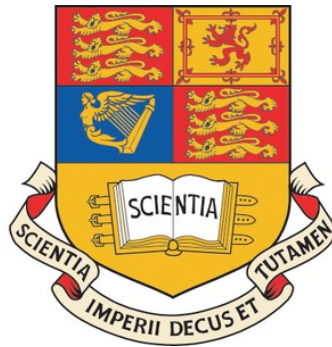


Unsteady and Conjugate Heat Transfer in Convective-Conductive Systems



Richard Mathie
Department of Chemical Engineering
Imperial College London

Supervised by Dr. Christos N. Markides
Collaborating academic: Prof. Serafim Kalliadasis

A thesis submitted for the degree of
Doctor of Philosophy
August 2012

Publications

To date these are the publications that have been presented to the public domain in relation to the works herein.

- R. Mathie and C.N. Markides, Augmentation in Unsteady Conjugate Heat Transfer Part I: Semi-Analytical 1-D Framework, *International Journal of Heat and Mass Transfer*, In press (2012).
- R. Mathie, H. Nakamura and C.N. Markides, Heat Transfer Augmentation in Unsteady Conjugate Thermal Systems Part II: Applications, *International Journal of Heat and Mass Transfer*, In press (2012).
- R. Solanki, R. Mathie, A. Galindo and C.N. Markides, Modelling of a Two-Phase Thermofluidic Oscillator: Effects of Irreversible Thermal Loss, *Applied Thermal Engineering*, Submitted (2012).
- R. Mathie and C.N. Markides, Electronics Cooling Review, Technical Report, Meeting with Chris Hole (The Technology Partnership), London (2008).
- R. Mathie and C.N. Markides, Unsteady 1D Thermal Conduction in Finite Solids, and Composite Bodies, *11th UK Heat Transfer Conference*, London (2009).
- R. Mathie, T. Smith and C.N. Markides, Heat Transfer-Thermodynamic Cycle Interactions in the Non-Inertive Feedback Thermofluidic Engine, Poster Presentation, *21st International Thermodynamics Conference*, London (2009).
- R. Mathie, C.N. Markides and A.J. White, A Framework for the Analysis of Thermal Losses in Reciprocating Compressors and Expanders, *9th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics*, HEFAT2012, Malta (2012). Award of “Best Paper in Thermodynamics”.

Abstract

Unsteady (time-varying) heat transfer is an important transport phenomenon that is found in many engineering and industrial applications. In such systems, generic spatiotemporal variations in the flow give rise to variations in the heat flux for a given fluid-solid temperature difference, which can be interpreted as spatiotemporal fluctuations of the instantaneous heat transfer coefficient. These variations can lead to unsteady and conjugate heat transfer, in which the exchanged heat flux arises from an interaction between the bulk fluid temperature and the temperature in the solid. Further, the non-linear coupling between the fluctuating temperature differences and the heat transfer coefficients can lead to an effect we refer to as augmentation, which quantitatively describes the ability of a particular arrangement to have a different time-mean heat flux from the product between the mean heat transfer coefficient and the mean temperature difference across the fluid. In this thesis we investigate unsteady conductive-convective heat transfer, and the resulting augmentative and conjugate effects. The overriding purpose is to propose a simple framework for the description of the effect of unsteadiness on the overall heat exchange performance, leading to the improved understanding and prediction of related processes.

An analytical model is developed that describes the thermal interaction between the solid and the fluid domains with the use of a time-varying heat transfer coefficient, and assuming 1-D conductive heat transfer in the solid. It is found that the degree of augmentation can be defined in terms of key independent problem variables, including: a time-averaged Biot number, a dimensionless solid thickness (normalised by an unsteady thermal diffusion length), a heat transfer coefficient fluctuation intensity (amplitude normalised by the mean), and a heat capacity ratio between the fluid and solid domains. The model is used to produce regime maps that describe the range of conditions in which augmentation effects are exhibited. Such maps can be used in the design of improved heat exchangers or thermal insulation, for example through the novel selection of materials that can exploit these augmentation effects. Cases are considered for which the bulk fluid temperature is fixed, and for which the bulk fluid temperature is allowed to respond to the solid, both in thermally developing and fully developed flows. Generally the augmentation effect is found to be negative, reflecting a reduction in the heat ex-

change capability. However, regions of positive augmentation are uncovered in thermally developing convective flows, which has important implications for heat exchanger design.

The approach is used to model two different thermodynamic cycles; gas springs and two-phase thermodynamic oscillator engines. Firstly, for the gas spring it is found that at low Péclet numbers the addition of an insulating layer exacerbates the thermal losses in the spring as it shifts the system away from the isothermal ideal operation. Conversely at high Péclet numbers thicker insulating layers reduces the loss as it shifts the system towards the adiabatic ideal. It is also found that there is an intermediate thickness of material thickness which maximises the loss in the gas spring, by up to 20 % of the nominal maximum loss for an isothermal cylinder lining. Secondly the heat transfer and resulting shuttle loss in the vapour space of a two-phase thermofluidic oscillator was studied. This model was compared to experimental data from a working test bed and resulted in a substantial improvement in the calculation of the cycle efficiency of the engine.

Detailed flow measurements were also conducted on a fluid film flowing down a heated incline, to investigate the effects of unsteady heat transfer in these flows. These wavy interfacial flows exhibit large and periodic fluctuations in heat transfer and the frequency and amplitude of the waves was controlled by a specially constructed flow preparation arrangement. To enable the temperature and heat flux measurements the heated incline consisted of a thin titanium foil. A novel measurement technique was developed (here, for the first time) to measure the film interface height (film thickness), film temperature and instantaneous heat flux through the heated surface. This was achieved with a combination of spatiotemporally resolved Laser Induced Florescence (LIF) measurements and Infra-red (IR) thermography. In the case of steady flow conditions (without forced waves) the formation of Marangoni driven rivulet structures are observed on the film surface. In the case of unsteady flow the formation of waves on the film surface result in visible mixing of the rivulet structures. The mixing and the unsteady motion of the waves give rise to a periodic fluctuation in the heat transfer coefficient, with fluctuation intensities of up to 35 % being recorded. The model predictions of the augmentation ratio for these problems are also compared to direct measurements from each case. Good agreement is observed with the experimental results for the global heat transfer trends. In both cases the augmentation ratio is negative, reflecting a reduction in time-averaged heat transfer. Finally, a backwards-facing step flow is also studied, for which a low magnitude of augmentation was observed ($< 1\%$), considerably lower than the augmentation measured in the thin film flows which were up to 10 %.

I would like to dedicate this thesis to my loving parents for all their help and support

Acknowledgements

And I would like to acknowledge ...

- Wendy, for standing by me through the tough times, cooking me food, and giving me so much love and support
- Dr. Christos Markides, who has given so much help
- All my friends in the windowless box that is ACXE405, good times
- The members of ICCAC and TriIC; it was good to see some familiar faces out in the mud and later in the pub, when I could make it
- All my ex-Selwynite friends; Its really great that we still meet up and have not gone our separate ways. I cant believe its been 4 years since we graduated. I feel older, I am not sure if I am wiser yet
- The DTA studentship (Doctoral Training Award) awarded to me by the Department of Chemical Engineering, Imperial College London, and EPSRC without which I would not have been able to conduct this research
- I would also like to acknowledge the use made of the FLIR SC3000 IR camera, and Cedip Titanium IR camera, which were borrowed from the EPSRC (Engineering and Physical Sciences Research Council) Instrument Pool (EIP)

*“Heat is work and work’s a curse,
And all the heat in the universe,
Is gonna coooooool down
'Cos it can’t increase,
Then there’ll be no more work
And there’ll be perfect peace”*

– Author: Flanders & Swann From: “At the Drop of Another Hat”

Contents

Contents	vii
List of Figures	xi
List of Tables	xviii
Nomenclature	xix
I Introduction	1
I.1 Background and Motivation	2
I.1.1 Unsteady heat transfer	3
I.1.2 Non-Inertive-Feedback Thermofluidic Engine (NIFTE) . . .	4
I.1.3 Film flows	7
I.2 Thesis Overview	11
II Foundations of Unsteady Conjugate Heat Transfer	15
II.1 Unsteady Thermal Losses in Thermodynamic Cycles	17
II.1.1 Rankine cycle expanders	17
II.1.2 Piston compressors and expanders	18
II.1.3 Irreversible heat transfer losses	19
II.1.4 Heat transfer in reciprocating systems	20
II.2 Newton’s Law of Cooling in Unsteady Flows	21
II.3 Unsteady Conjugate Heat Transfer	24
II.4 Temperature Measurements in Film Flows	25
II.4.1 Nusselt film flow	27
II.4.2 Wave instabilities on the film flow surface	28
III An Analytical Framework for Unsteady Conduction in Solid Domains	34
III.1 Time-varying Heat Transfer Coefficient	34
III.1.1 Implications of unsteadiness and conjugation	36
III.1.2 Heat transfer augmentation	38
III.2 1-D Solid Thermal Response	39
III.2.1 1-D heat equation in the solid	40

CONTENTS

	III.2.2 Conjugation and the Fourier and Biot numbers	41
	III.2.3 Fourier method	44
	III.2.4 Boundary conditions	46
	III.2.5 Thermal response	47
	III.2.6 Thermal impedance	48
III.3	Laplace Method for the Solution of the IBVP	52
	III.3.1 Uniqueness of the solution	53
	III.3.2 General solution	55
	III.3.3 Boundary conditions	56
	III.3.4 Transfer functions	58
	III.3.5 Inverting the Laplace transform	61
III.4	Composite Solids	66
III.5	Convective Boundary Conditions	69
IV	Experimental Methods	72
IV.1	Experiment Overview	72
IV.2	Apparatus	76
	IV.2.1 Flow loop	76
	IV.2.2 Foil holder	82
	IV.2.3 Liquid flow preparation	84
	IV.2.4 Power supply	91
	IV.2.5 LIF system	99
	IV.2.6 IR thermography	102
	IV.2.7 Data acquisition	105
IV.3	Foil Material Selection	106
IV.4	Fluid Selection	109
IV.5	Measurement Technique	112
	IV.5.1 LIF measurements	113
	IV.5.2 IR measurements	117
IV.6	LIF Calibration	117
IV.7	Assumptions	121
	IV.7.1 Assurances over convective losses	122
	IV.7.2 Dye concentration gradients	124
	IV.7.3 Surface tension effects	126
IV.8	Error Analysis	129
	IV.8.1 IR measurements	129

CONTENTS

V	Low-Dimensional Convective-Conductive Systems	132
V.1	Problem Formulation	132
V.2	Solution Methodology	133
V.2.1	Domain matching	135
V.3	Bulk Fluid Temperature Fluctuations	137
V.3.1	Case II	139
V.3.2	Case III	140
V.3.3	Case IV	140
V.3.4	Case V	141
V.3.5	Case VI	142
V.4	Results and Discussion	144
V.4.1	Case I	145
V.4.2	Case IV	149
V.4.3	Case V	153
V.4.4	Case VI	155
V.5	Summary	156
VI	Compression/Expansion-Work in Convective-Conductive Systems: Gas Springs and the NIFTE	159
VI.1	Gas Spring: Problem Formulation	159
VI.2	Results and Discussion	165
VI.2.1	Time-varying heat transfer coefficient	166
VI.2.2	Case 1: Unity material property ratios	168
VI.2.3	Case 2: Conventional material properties	170
VI.2.4	Case 3: Materials with unfavourable properties	173
VI.3	NIFTE	174
VI.3.1	Modelling the cycle	175
VI.3.2	Comparison to experimental results	176
VI.4	Summary	180
VII	Broadband Convective-Conductive Systems: Backwards Facing Step	181
VII.1	Nakamura experiment	181
VII.2	Modelled broadband augmentation factors	183
VII.3	Characterising the HTC	185
VII.4	Augmentation Predictions and comparison to experiment	187
VII.5	Location of the flow stagnation point	188
VII.5.1	Predicted reattachment points	191

CONTENTS

VII.5.2	Mean stream-wise thermal velocity profiles	195
VII.5.3	Velocity profiles	198
VII.5.4	Velocity from spatio-temporal distributions	200
VII.6	Summary	202
VIII	Experimental Results in Film Flows	204
VIII.1	Temperature and Flow Field on the Fluid Surface	204
VIII.1.1	Comparison between LIF and IR measurements	205
VIII.1.2	Rivulet and trough formation	207
VIII.1.3	Film temperature against thickness	208
VIII.1.4	Adjacent foil temperature	209
VIII.1.5	Conclusions on the rivulet flow structure	209
VIII.2	Effect of Flow rate and Heat Flux	211
VIII.2.1	Rivulet oscillation	214
VIII.3	Films With Flow Pulsation	215
VIII.4	Film Surface Temperature, Thickness and the Temperature of the Back Surface of the Foil	217
VIII.5	Heat Transfer Augmentation in the Film Flow	225
VIII.5.1	Analysis and modelling	228
VIII.6	Summary	234
IX	Conclusions	237
	References	244

List of Figures

I.1	Schematic diagram of the NIFTE device and the T – S diagram of the cycle.	5
I.2	Model of the evolution of a film flow surface.	9
I.3	Marangoni film flow instability on a locally heated plate.	10
I.4	Marangoni rivulets, and temperature distribution on the film surface at $Re = 1$	10
I.5	Thesis overview; Four model flows are studied which exhibit different physical characteristics. The cases considered either exhibit a high or low Biot number, or the situations when the fluid side of the heat transfer process undergo compressible work.	14
II.1	Capillary driven instability resulting in hydrodynamic waves on the surface of a fluid film flowing across a plate.	29
II.2	Neutral stability curves for wave flow on a plate at different inclination angles.	30
II.3	Wave velocities c_r (a) and growth rate $c_i \alpha_K$ (b) down a thin film on a flexible surface for wave number α_K	31
II.4	Wave evolution on a falling film obtained by the numerical solution of the boundary layer equations with pressure due to capillarity forces excited by noise	32
III.1	Thermal diffusion through a solid. Schematic of the thermal wave envelop.	35
III.2	Thermal input (at inner boundary) impedance with different output (outer) boundary conditions.	47
III.3	Thermal impedance across the material.	48
III.4	Infinitesimal resistor capacitor, impedance network.	49

LIST OF FIGURES

III.5	Equivalent contraction of network in Fig. III.4 to representation by 3 impedances.	50
III.6	Graph showing the internal thermal impedance Z_1 and Z_2 as function of $\frac{a}{\delta}$	51
III.7	Diagram of the Boundary Conditions.	57
III.8	Thermal temperature response on the outer wall, $T(y = 0)$, of a 1-D solid to a step and impulse in temperature on the inner wall, $T(y = a)$, for isoflux boundary condition, $q(y = 0) = 0$	63
III.9	Thermal response of a 1-D solid to a step and impulse in heat flux for isothermal and isoflux boundary conditions.	66
III.10	Two solids in thermal contact with each other produce a composite thermal solid, essentially altering the input and output impedance to thermal oscillation.	67
III.11	A selection of material properties, with $\epsilon = \sqrt{\kappa\rho c_v}$ on the x-axis and diffusion length (m) δ over $\sqrt{\omega}$, $\left(\sqrt{\frac{2\kappa}{\rho c_v}}\right)$ on the y-axis.	68
III.12	Magnitude of thermal impedance for an aluminium-steel composite with a thermal oscillation at 1 rad s^{-1}	70
III.13	Magnitude of thermal impedance for composite solid consisting of a layer of stagnant water on concrete with a thermal oscillation at 1 rad s^{-1}	71
IV.1	Diagram of the film flow experiment.	73
IV.2	Schematic diagram of the fluid flow loop for the experiment test section.	77
IV.3	Pump schematic and operating curves, courtesy of Grundfos.	78
IV.4	Flow oscillator valve.	80
IV.5	Stepper motor controller.	81
IV.6	Picture of the pump, heat exchanger and filter physical layout on the experimental rig.	82
IV.7	Foil holder.	83
IV.8	Flow distribution box. smooths and distributes the flow into an even film over the foil.	85
IV.9	Approximate decay of turbulence in the distribution box based on a $k-\epsilon$ turbulence model.	87
IV.10	Various conditions experienced by the flow at the knife-edge at the distribution box outlet.	91
IV.11	Power 10's P60-10100 2U rack mounted 10 kW high current DC power supply. Capable of outputting 1000 A at 10 V.	92

LIST OF FIGURES

IV.12	The copper bus bars which collect the current from the six output terminals on the DC power supply and connects them to the cables routing power to the foil electrodes.	94
IV.13	The voltage drop across the supply cables and foil at different supply currents.	95
IV.14	The temporal deviation of foil power at different applied powers. . . .	96
IV.15	3-phase 415 VAC to 210 VAC. transformer. Rated at 18 kV A.	97
IV.16	Calibration graticule for correcting and calibrating against the distortion of the LIF images captured by the CMOS sensors.	100
IV.17	The IR cameras used for the measurement of the temperature of the back side of the foil.	102
IV.18	Effect of background radiation inclination angle and emissivity on the IR camera temperature measurement.	103
IV.19	Calibration plate for correcting the perspective distortion of the IR images.	105
IV.20	Material selection charts. Foil dimensionless thickness $\frac{a}{\delta}$ at a nominal wave frequency of 10 Hz shown as a function of foil resistance, R , for a foil width $W = 300$ mm and length, $L = 450$ mm and a range of thicknesses, $a = 25$ μ m to 1 mm.	109
IV.21	Film LIF measurements. Successive snapshots of a fluid wave propagating across the film as it flows down the heated foil.	114
IV.22	Film profile.	114
IV.23	Typical IR measurements using the FLIR SC3000 at $Re = 360$, a heat flux $\bar{q} = 2.1$ W cm ⁻²	118
IV.24	Calibration method of the laser intensity for the temperature measurement of the fluid film.	120
IV.25	Temperature of the foil with no fluid flow.	123
IV.26	Contact angles measurement using the pendant drop method on a goniometer.	127
IV.27	Contact angle over time.	128
IV.28	Signal to noise ratios for various thicknesses of titanium foil at different frequencies and foil thicknesses normalised by the IR measurement noise η_{IR} and the “signal source”.	130
V.1	Schematic of heat transfer for and incompressible fluid flow over a solid of finite thickness	133

LIST OF FIGURES

V.2	Diagram showing the lines of characteristics in the fluid domain. . . .	142
V.3	Time evolution of unsteady heat transfer behaviour: Case I.	146
V.4	Variation of the augmentation ratio with Biot number: Case I.	147
V.5	Variation of the augmentation ratio with material thickness: Case I. .	147
V.6	Augmentation map, Biot number against wall thickness: Case I. . . .	148
V.7	Spatial temperature profile: Case IV	150
V.8	Spatial profiles of various heat transfer characteristics: Case IV. . . .	151
V.9	Augmentation map, Biot number against wall thickness: Case IV. . .	151
V.10	Time evolution of unsteady heat transfer behaviour: Case V.	153
V.11	Time evolution of unsteady heat transfer behaviour: comparison Case I to Case V.	154
V.12	Variation of the augmentation ratio with Biot number and wall thick- ness: Case V.	155
V.13	Augmentation map, Biot number against wall thickness: Case V. . . .	156
V.14	Case VI augmentation	157
VI.1	Schematic of the gas spring problem.	160
VI.2	Complex Nusselt number	162
VI.3	Cyclic temperature difference and wall heat flux	165
VI.4	Cauchy Nusselt number distrobutions	167
VI.5	Modelled gas spring losses and pressure ratios	169
VI.6	Modelled gas spring losses: Conservative case.	171
VI.7	Modelled gas spring losses: Conservative case. Effect of wall thickness	172
VI.8	Modelled gas spring losses: Extreme case.	173
VI.9	Gas spring pressure ratio: Extreme case.	174
VI.10	Electrical circuit representations of the NIFTE (see Fig. I.1).	177
VI.11	The effect of the timescale parameter $R_t C_d$ at constant displacer hydro- static capacitance C_d on the predictions of the NIFTE: (a) frequency f_0 ; and (b) exergetic efficiency η_{ex} . Showing results from the LTP model (solid line), nominal DHX model (dashed line), and DHX model with $2\tilde{C}_{hx}$ (dotted line).	178

LIST OF FIGURES

VI.12	The effect of the timescale parameter $R_f C_d$ at constant displacer hydrostatic capacitance C_d on the predictions of the NIFTE: (a) frequency f_0 ; and (b) exergetic efficiency η_{ex} . Showing results for the revised models featuring the unsteady thermal loss impedance Z_{TL} with an <i>adiabatic</i> boundary condition, namely LTP_{ADTL} (solid line), DHX_{ADTL} (dashed line), and DHX_{ADTL} with $2\tilde{C}_{hx}$ (dotted line).	180
VII.1	Schematic diagram of the backwards facing step.	181
VII.2	Experimentally measured variations in the HTC.	184
VII.3	Statistical distribution of the measured HTC.	186
VII.4	Matching the frequency content of the measured and synthesised HTC.	187
VII.5	Backward step augmentation loss, modelled and experimental	188
VII.6	Two consecutive snapshots of the surface HTC	189
VII.7	HTC comparing the difference between frames to calculate a surface thermal velocity field.	190
VII.8	Average of the velocity field over the backwards facing step.	191
VII.9	Backward facing step reattachment contours.	192
VII.10	Reattachment positions estimated from velocity fields of the various scalars. One line corresponds to the intersection of the span-wise mean of the velocity field with zero (Method-1), and the other corresponds to the mean position of the contours of zero velocity (Method-2).	194
VII.11	Time-averaged stream-wise heat transfer profiles for the backward facing step.	196
VII.12	Time-averaged stream-wise heat transfer profiles for the forward facing step.	197
VII.13	Mean (time- and span-wise averaged) profile of h velocity for the backward facing step.	199
VII.14	Mean (time- and span-wise averaged) profile of h velocity for the forward facing step.	199
VII.15	Spatio-temporal distribution of the heat transfer coefficient, h , at $z = -30$ mm, behind the reattachment region for a $H = 10$ mm backward facing step with a free-stream velocity of $U_\infty = 6$ m/s.	200
VII.16	The velocity of the thermal structures was estimated as the average gradient from best line fits through a number of heat transfer coefficient contours. Here one contour is highlighted in red, and the best line for that contour is shown.	201

LIST OF FIGURES

VII.17	The observed structure velocities for the backward facing step, using several methods of analysis.	202
VIII.1	IR image, LIF measurement location	205
VIII.2	Film temperature measurements from the LIF and IR techniques and the film thickness, d	206
VIII.3	Temperature profiles across the surface of the fluid film, as measured by the IR camera, at different locations downstream of the knife edge. Reynolds number of 179 and a heat flux of 3.5 W cm^{-2}	207
VIII.4	Relationship between film thickness and temperature.	208
VIII.5	Temperature profile of the fluid surface as measured by the LIF technique, and the heated foil as measured by the IR camera, for a flow Reynolds number $Re = 251$ and a heat flux of $\bar{q} = 3.58 \text{ W cm}^{-2}$	209
VIII.6	Schematic of the span-wise circulation in the film flow as a result of the Marangoni effect.	210
VIII.7	Qualitative presentation of IR measurements of the back of the foil with steady film flow	212
VIII.8	Qualitative presentation of IR measurements of the fluid film surface with steady flow	213
VIII.10	Expanded image of the flow structure at $t = 0.2 \text{ s}$ from Fig. VIII.9. . .	215
VIII.11	Temperature distribution in the foil with pulsation in the film flow. . .	216
VIII.12	Temperature distribution of the film surface with pulsation.	217
VIII.13	Temperature IR and LIF and thickness D for $Re = 179$, $\bar{q} = 2.38$, $x = 285 \text{ mm}$	218
VIII.14	Temperature IR and LIF and thickness D for $Re = 179$, $\bar{q} = 2.38$, $x = 185 \text{ mm}$	219
VIII.15	Temperature IR and LIF and thickness D for $Re = 179$, $\bar{q} = 3.58$, $x = 185 \text{ mm}$	220
VIII.16	Heat flux, HTC and Nusselt number for $Re = 179$, $\bar{q} = 3.58$, $x = 185 \text{ mm}$	221
VIII.17	Temperature IR and LIF and thickness D for $Re = 251$, $\bar{q} = 2.38$, $x = 285 \text{ mm}$	222
VIII.18	Temperature IR and LIF and thickness D for $Re = 251$, $\bar{q} = 2.38$, $x = 185 \text{ mm}$	223
VIII.19	Temperature IR and LIF and thickness D for $Re = 251$, $\bar{q} = 3.58$, $x = 185 \text{ mm}$	224

LIST OF FIGURES

VIII.20	Profile of a wave and the resulting heat transfer as the wave flows down plate the heated plate, for the flow conditions of $Re = 300$, $Pr = 5.0$ and $a_1^* = a/\delta_{s,1} = 0.056$	226
VIII.21	Instantaneous HTC characteristics for a set of 12 runs with the same flow condition $Re = 300$, $Pr = 5.0$ and $a_1^* = 0.056$, but with different (uniform, steady) heat inputs $\bar{q}_w$ to the foil substrate over the range 200 W to 5 kW. Showing joint probability density functions (PDFs) of the dependence of instantaneous local h on the instantaneous local film thickness d	227
VIII.22	A view of the augmentation domain with experimental points.	231
VIII.23	Augmentation contours as a function of $Bi = \bar{h}a/k_s$ and $h'' = h_\sigma/\bar{h}$, with the positions of experimental data shown.	232
VIII.24	Measured augmentation effect against h'' including the corresponding modelled points.	233

List of Tables

II.1	Positive-displacement compressor and expander indicated and isentropic efficiencies for a leakage gap size of 10-15 μm	19
III.1	Comparison between thermal and electrical conduction.	49
III.2	Input and total impedance of impedance network in Fig. III.5.	52
III.3	Table of transfer functions for 1-D thermal conduction.	59
IV.1	Thermal properties of materials and fluid in the film flow experiments. .	75
IV.2	Electrical requirements of the DC Power supply at full power.	93
IV.3	Electrical requirements step-down transformer.	98
IV.4	IR camera specifications.	104
IV.5	Properties of materials considered for the foil [1].	107
IV.6	Possible fluid properties selection table.	111
V.1	The six different condition cases considered.	145
VII.1	Overview of experimental conditions.	183
VII.2	Thermal properties of materials in the backwards step experiments. . .	184

Nomenclature

Material Properties

α	Thermal diffusivity, $[\text{m}^2 \text{s}^{-1}]$
c	Specific heat capacity, $[\text{J kg}^{-1} \text{K}^{-1}]$
$\mathcal{D}$	Mass diffusivity, $[\text{m}^2 \text{s}^{-1}]$
ϵ	Thermal effusivity, $[\text{J/m}^2/\text{K/s}^{1/2}]$
κ	Thermal conductivity, $[\text{W m}^{-1} \text{K}^{-1}]$
ν_f	Fluid kinematic viscosity, $[\text{m}^2/\text{s}]$
ρ	Density, $[\text{kg m}^{-3}]$
σ_f	Surface tension of the fluid, $[\text{N m}^{-1}]$

Dimensionless Variables

Bi	Biot number, $\frac{ha}{k_s}$
Ka	Kapitza number, $\frac{\sigma_f}{\rho_f \nu_f^{4/3} g^{1/3}}$
Mg	Marangoni number, $-\frac{d\sigma_f}{dT} \frac{1}{\mu_f \alpha_f} D \Delta T$
Sc	Schmidt number, $\frac{\nu_f}{\mathcal{D}}$
Sr	Soret number, $\frac{\mathcal{D}_T k_T \Delta T}{\nu_f \bar{T} \Delta C}$
St	Strouhal number, $\frac{ha}{k_s}$
We	Weber number, $\frac{\rho_f U^3 l}{\sigma_f}$

Bu	Unsteady Biot number, $\frac{\text{Bi}}{a^*}$
Fo	Fourier number, $\frac{\alpha_s}{\omega L_s^2}$
Nu	Nusselt number, $\frac{h L_f}{k_f}$
Pe	Péclet number, $\frac{U_b D_h}{\alpha_f} [= \text{Re Pr}]$
Pe_ω	Péclet number for oscillating flows, $\omega D_h^2 / \alpha_f [= \text{Wo}^2 \text{Pr}]$
Pr	Prandtl number, $\frac{\nu_f}{\alpha_f}$
Re	Reynolds number, $\frac{U_b D_h}{\nu_f}$
C_H	Stanton number, $\frac{h}{\rho_f c_f U_b} \left[= \frac{\text{Nu}}{\text{Pe}} = \frac{\text{Nu}}{\text{Re Pr}} \right]$
Wo	Womersley number, $\sqrt{\frac{\omega}{\nu_f}} D_h \left[= \sqrt{\frac{\text{Pe}_\omega}{\text{Pr}}} \right]$
A_{loss}	Augmentation loss, $1 - A$
A	Augmentation ratio, $\frac{\overline{h \Delta T_f}}{\bar{h} \overline{\Delta T_f}}$
a^*	Normalised solid thickness, $\frac{a}{\delta_s}$
C^*	Volumetric (or volume-specific) heat capacity ratio, $\frac{\rho_f c_f}{\rho_s c_s}$
d^*	Normalised fluid thickness, $\frac{d}{a}$
C_R	Heat capacity ratio, $C^* d^* = \frac{\rho_f c_f}{\rho_s c_s} \frac{d}{a}$
θ^+	Normalised temperature difference across the fluid, $\Delta T_{f,b} / \overline{\Delta T_f}$
U^*	Normalised fluid velocity, $\frac{\nu_f}{\omega d^2} \text{Re}$
t^+	Normalised time, $\frac{\omega t}{2\pi}$

x^+	Normalised position in the stream-wise direction, x/d
Z^*	Normalised thermal impedance of the solid, $\bar{h}Z$

Variables

$\tilde{y}$	Coordinate normal to the wall and starting from the inner (wetted) solid surface, $y - a$, [m]
Z	Thermal impedance in the frequency domain, [$\text{m}^2 \text{K W}^{-1}$]
z	Thermal impedance in the time domain, [$\text{m}^2 \text{K W}^{-1}$]
a	Solid thickness, [m]
d	Fluid domain thickness, [m]
D_h	Hydraulic diameter, [m]
δ	Thermal diffusion length, $\sqrt{\frac{2\alpha}{\omega}}$, [m]
g	Gravitational acceleration, [m s^{-2}]
Γ	Fluid flow-rate per unit channel width, Q/w , [$\text{m}^3 \text{s}^{-1} \text{m}^{-1}$]
h	Heat transfer coefficient, [$\text{W m}^{-2} \text{K}^{-1}$]
L_{th}	Thermal development length, [m]
ω	Oscillation frequency, [rad s^{-1}]
$\dot{q}$	Heat flux, [W m^{-2}]
Q	Volumetric flow-rate, [$\text{m}^3 \text{s}^{-1}$]
$\dot{Q}$	Heating rate, [W]
$\dot{Q}_{gen}$	Heat generation inside the solid per unit volume, [W m^{-3}]
S	Surface area, [m^2]

T	Temperature, [K]
ΔT_f	Fluid temperature difference, $T_{f,b} - T_w$, [K]
t	Time, [s]
u	Fluid velocity field, [m s^{-1}]
U_b	Bulk fluid velocity field, [m s^{-1}]
w	Width of flow perpendicular to the flow direction, [m]
x	Coordinate in the streamwise direction, [m]
y	Coordinate normal to the wall and starting from the outer (unwetted) solid surface, $\tilde{y} + a$, [m]
Z_{in}	Input impedance
σ	Standard deviation of sub scripted variable

Subscripts

0	Outer wall position (at the solid surface away from the fluid), $y = 0$
w	Inner wall position (in contact with the fluid), $y = a$
∞	At fluid free-stream conditions, $y \rightarrow \infty$
s	In the solid domain
f	In the fluid domain
b	Bulk (flow-averaged) fluid parameter
A	Isoflux outer wall boundary condition
I	Isothermal outer wall boundary condition
in	Inlet condition, $x = 0$

bw	Coefficients for a backwards travelling wave
fw	Coefficients for a forwards travelling wave
n	n^{th} component in the frequency domain

Operators

$\overline{(\cdot)}$	Time-mean component of $(\cdot)$
$(\cdot)'$	Fluctuating component, $(\cdot) - \overline{(\cdot)}$
$(\cdot)''$	Normalised root mean square (rms) fluctuation intensity, $\sigma_{(\cdot)}/\overline{(\cdot)}$
$\widehat{(\cdot)}$	Amplitude of fluctuation of periodic $(\cdot)$
$(\cdot)^+$	Normalised value with respect to the time-mean, $(\cdot)/\overline{(\cdot)}$
$(\cdot)^*$	Normalised amplitude of fluctuation, $\widehat{(\cdot)}/\overline{(\cdot)}$
$\Re\{\cdot\}$	Real part of complex $(\cdot)$
$\frac{D(\cdot)}{Dt}$	Convective or material derivative, $\frac{\partial(\cdot)}{\partial t} + \underline{u} \cdot \nabla(\cdot)$
$(\cdot) \star (\cdot)$	Circular convolution

Chapter I

Introduction

Unsteady (time-varying) heat transfer is an important transport phenomenon that is found in many engineering and industrial applications. Examples range from micro-scale cooling schemes for microprocessor chips [2–4]; to conventional internal combustion engines, heat exchangers [5, 6], compressors and expanders [7–10]; to novel processes of heat integration for improved efficiency [11–13].

The understanding and consideration of unsteady heat transfer effects is also an important factor in development of a number of clean or renewable energy technologies. These include numerous external heat engines [9], thermoacoustic devices [6, 14–17] and other machines that are capable of operating with external heat sources such as solar, geothermal and waste heat [13], whose common characteristic is that they operate with internally oscillating thermal–fluid flows. It is expected that the relevance of these devices both in industrial and domestic settings will increase as the cost of conventional energy sources continue to rise due to the ever increasing price of oil [18, 19] and as other non-renewable as they become ever scarce [20]. Additionally the energy security [21] of an economy is improved by diversifying energy usage into renewable technology, and waste energy recovery.

This thesis is aimed at developing a simple framework to understand and predict unsteady conjugate heat transfer. The framework is applied to several convecting flows, and thermodynamic cycles. We build on the work of Zudin [22] who developed a method for modelling conjugate heat transfer in convecting flows for a fixed fluid temperature, and we extend this to include situations where the fluid temperature can change, either due to a thermodynamic process, or because of the heat transfer itself. To describe the unsteady heat transfer first an analytical solution to the unsteady heat transfer in the

solid is found. There have been many numerical [23, 24] and analytical [25] studies into the heat transfer through a solid. However the approach taken here is different in that a transfer function is derived to represent the solid domain, and this is used as the boundary condition to the simplified fluid domain description.

Heat transfer in the fluid domain was modelled using either a fluctuating heat transfer coefficient [26, 27] (HTC) or a complex heat transfer coefficient [28], and the evolution of temperature in the fluid is described using the energy conservation equation. The developed framework is then applied to convecting flows with oscillating HTC's including; wavy fluid film flows on a heated incline; the heat transfer behind a backwards facing step [29]; a gas spring [28]; and a two-phase thermofluidic oscillator [30] (see Section I.1.2).

Additionally in order to help validate the resulting predictions, and to further our understanding of unsteady heat transfer, it is also the aim to measure the unsteady heat transfer process in a fluid film flowing down a heated incline, and compare the model predictions to the experimental results. Film flows exhibit unsteadiness due to the surface instabilities that develop [31, 32], which result in wave formation on the film surface and hence fluctuations in the HTC [33]. The large degree of control over the frequency and amplitude of these fluctuations makes film flows a good candidate for studying unsteady conjugate heat transfer. A novel experiment with advanced measurement techniques was devised to measure the instantaneous temperature and heat flux at the fluid–solid interface and the temperature of the fluid using infra red cameras and laser induced fluorescence of Rhodamine B dye in the fluid. These flows also exhibit a number of other interesting phenomena due to Marangoni surface instabilities, which will also be discussed in this thesis.

I.1 Background and Motivation

The initial motivation for the work presented, stemmed from trying to understand the performance and behaviour of the Non-Inertive-Feedback Thermofluidic Engine (NIFTE) [14, 15, 34, 35]. The NIFTE, is an example of a two-phase thermofluidic oscillator heat engine. This cycle (described in Section I.1.2) undergoes unsteady heat transfer, and so is subject to a number of unsteady heat transfer effects. These effects tend to be detrimental to the performance of the device, and so there is a desire to alleviate them.

A model is needed to understand these unsteady effects, and provide greater insight into the physical reasons for the losses. Primarily an analytical model for the thermal conduction problem was pursued, as numerical methods make parametric investigation

Background and Motivation

difficult due to the long execution time. Ultimately the developed tools and understanding are not limited to the NIFTE, which is only one example of an unsteady heat transfer problem. The models and techniques can be extended to other unsteady heat transfer applications; specifically in this thesis they are applied to the unsteady thermal loss in a gas spring, and thermally convecting interfacial flows with heat transfer.

In addition, it was decided that it was important to complement the model predictions with experimental measurements of a heat transfer process which exhibited significant unsteady heat transfer. This would allow the unsteady heat transfer effects to be measured directly for comparison with the models. Fluid film flows on inclined heated surfaces were chosen as they exhibit large fluctuations in heat transfer due to film instabilities and waves [36, 37].

I.1.1 Unsteady heat transfer

Unsteadiness in the heat transfer process introduces several fundamental effects into the heat transfer problem. The first effect is “augmentation”, an improvement or deterioration in heat transfer performance relative to an equivalent steady problem. The second effect is “conjugation” and occurs when the temperature fluctuations in both the solid domain as well as the fluid domain become important in the solution of the heat transfer problem. These two effects can also combine to link boundary conditions together which is what makes unsteady heat transfer more of a challenge when compared to its steady state counterpart.

Augmentation

To explain briefly the effect of augmentation, consider an unsteady convective heat transfer problem. The heat flux from the solid surface to the fluid can be written in terms of Newton’s law of cooling as $\dot{q} = h\theta_f$, where $\dot{q}$ is the heat flux, h is the convective heat transfer coefficient and $\theta_f = T_f - T_w$ is the temperature difference between the wall and the fluid. The issues arise when the values of heat transfer and temperature are time-varying [22, 27, 38].

By decomposing the HTC, heat flux and temperature difference into their mean and unsteady components the following expression is obtained,

$$\dot{q} = (\bar{q} + q') = (\bar{h} + h')(\bar{\theta}_f + \theta'_f), \quad (\text{I.1})$$

$$= \bar{h} \bar{\theta}_f + \bar{\theta}_f h' + \bar{h} \theta'_f + h' \theta'_f. \quad (\text{I.2})$$

Here $\bar{q}$ is the mean heat flux and q' is the fluctuating component. The mean heat flux can be found by the rules of averages,

$$\bar{q} = \bar{h} \bar{\theta}_f + \overline{h' \theta'_f}. \quad (\text{I.3})$$

It can be seen that when h' is non zero, the mean heat flux no-longer equates to $\bar{h} \bar{\theta}_f$ and now depends upon the correlation between h' and θ'_f . This occurs even when the temperature of the solid is constant as long as the fluid temperature is able to fluctuate.

Conjugation

Conjugate heat transfer occurs when the solution of the conductive heat transfer in the solid domain becomes as significant as the solution of the convective heat transfer in the fluid domain, to the solution of the overall heat transfer problem. In steady heat transfer this is exemplified by cases when it is no-longer adequate to impose an isothermal or adiabatic boundary condition at the interface (boundary) between the solid and the fluid [39]. Conversely the case when the heat transfer solution in the fluid domain becomes unimportant is considered here as non-conjugate (i.e. when it becomes possible to impose an isothermal or adiabatic boundary condition instead of solving the temperature and heat flux within the fluid and matching these at the common boundary). In unsteady heat transfer this effect is exacerbated by the ability of the temperature of solid-fluid boundary to fluctuate.

An example where the unsteady thermal solution in the solid is as important as that in the fluid domain in predicting the heat transfer is given by He and Oldfield [23]. There, the temperature of gas turbine blades are estimated, subject to “hot streaks” of gas exiting the nozzle guide vanes [40], and the fluctuating heat transfer coefficient associated with the periodic passing of the turbulent wakes. He and Oldfield [23] are primarily interested in predicting the blade metal temperature. This is because the key failure mechanisms for turbine blades, that in creep, corrosion and loss of strength of the blade materials at high temperatures, are highly temperature sensitive. Nevertheless He and Oldfield [23] do model the effect of the fluctuating heat transfer on the solid, and find the interaction of the steady and unsteady solution to be significant.

I.1.2 Non-Inertive-Feedback Thermofluidic Engine (NIFTE)

A schematic diagram of the NIFTE is shown in Fig. I.1(a) that depicts the device which consists of two vertical fluid filled columns. The two main vertical columns in the NIFTE

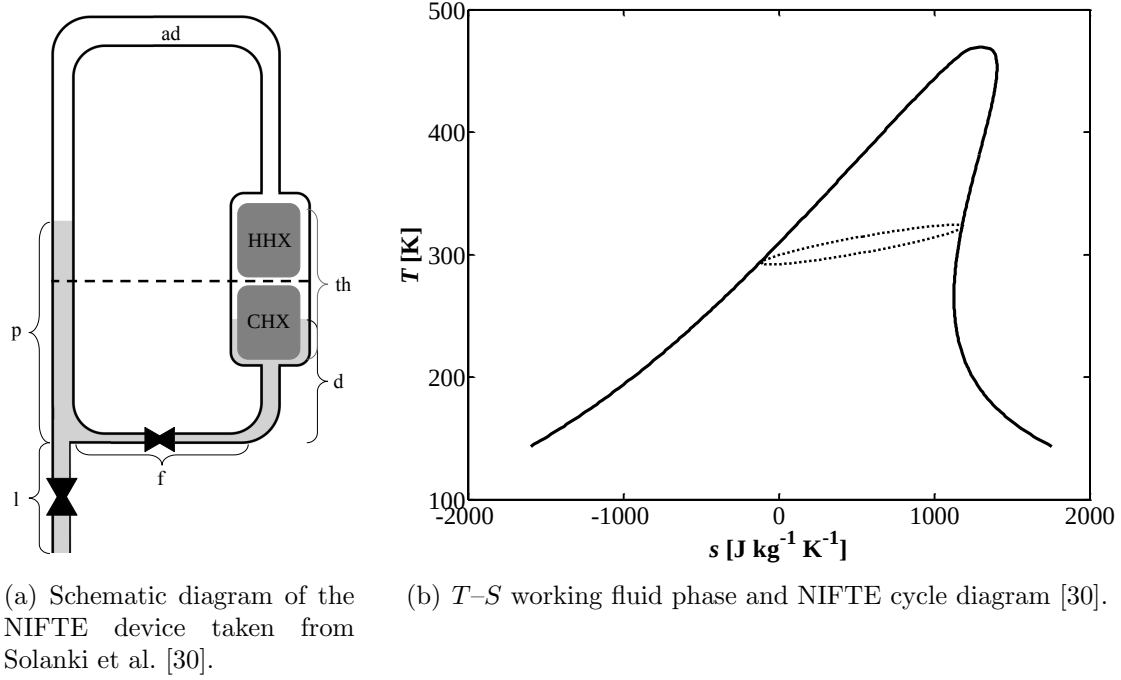


Figure I.1: Schematic diagram of the NIFTE device and the T - S diagram of the cycle. In (a) the hot heat exchanger (HHX) and cold heat exchanger (CHX) blocks are within the displacer cylinder ‘d’. The power cylinder is denoted by ‘p’, the feedback connection and valve by ‘f’, and the load line by ‘l’. The “adiabatic” vapour volume, denoted by ‘ad’. The horizontal dashed line indicates the equilibrium (time-mean) liquid-vapour interface level. In (b) the working fluid was n -pentane (solid line) and T - S cycle diagram of a typical linearised thermodynamic cycle (dotted line).

are known as the power cylinder (p), which is directly connected to the load line (l) where the useful work of the engine is done, and the displacer cylinder (d), in which the hot heat exchanger (HHX) and cold heat exchanger (CHX) are located. These two vertical cylinders are connected by two horizontal tubes. Upon heating of the hot heat exchanger (HHX) to a temperature above the boiling point of the working fluid, the resulting vapour phase collects in the upper portion of the device. This vapour filled region has been termed the ‘adiabatic vapour volume’ (ad) [15, 41]. This notation follows from the developed of the NIFTE. However, in reality the vapour volume is only *nominally* adiabatic, and in fact experiences parasitic cyclic heat transfer [30].

Due to gravity the liquid phase of the fluid is located in the lower parts of the device. The white area in Fig. I.1(a) represents the vapour phase and the grey area represents the liquid phase of the working fluid. The connecting tube that contains the working fluid in

the liquid phase is referred to as the feedback connection (f), while the connection that contains the working fluid in the vapour phase is referred to as the vapour connection. The feedback tube includes a valve that can be used to adjust the operating condition of the engine during operation.

Operation

For the NIFTE to operate the HHX temperature must be above the boiling point of the working fluid at the outlet pressure, and the CHX temperature must be below the boiling point at the inlet pressure. During a cycle, when the fluid-vapour interface in the displacer cylinder is in contact with the HHX, vapour is generated. This results in an increase in pressure in the adiabatic vapour volume which then displaces the liquid level in power cylinder and fluid is displaced out through the load line (l). The lowering of the fluid-vapour interface in the power cylinder leads to a hydrostatic pressure difference between the liquid levels in the power and displacer cylinders. This causes the level in the displacer cylinder to drop and overshoot its equilibrium position (the position between the two heat exchangers). The height of the liquid-vapour interface in the displacer cylinder now exposes the vapour phase to the CHX resulting in the condensation of the vapour phase. As the vapour mass in the adiabatic vapour volume (ad) is reduced the pressure drops. Fluid flows back in through the load line (l) and the liquid level in the power cylinder (p) rises. Once more, the hydrostatic pressure difference between the liquid levels in the power and displacer cylinders drives the liquid level to return back to the equilibrium position (horizontal dashed line in Fig. I.1(a) and the fluids inertia results in the interface in the displacer cylinder overshooting this position. The cycle then repeats giving a sustained oscillation of fluid in the load line. The load line can be connected to a pair of non-return valves that create a unidirectional pumping action from the pure oscillating motion.

Performance

Unsteady vapour cycle engines, such as the NIFTE, are advantageous for low grade heat utilisation, because of the near isothermal heat addition and rejection at the heat exchangers. This allows such vapour cycles to achieve theoretical cycle efficiencies close to the Carnot [17, 35, 41]. Additionally the high heat transfer coefficients for evaporation and condensation allow the use of compact and economical heat exchangers with greater power density. The NIFTE has few mechanical moving parts and which make it simple to manufacture and easy to maintain. Currently developed NIFTEs can pump flow rates

Background and Motivation

of 500 to 1000 kWh^{-1} into heads of about 1 to 5 m [30, 35, 42]. However, the current generation of NIFTE devices can only achieve maximum efficiencies of 30 % of the Carnot efficiency [35], though most report considerably lower values [30].

There are two main loss mechanisms which are associated with the NIFTE as a consequence of unsteady heat transfer. Firstly in the heat exchanging components [30, 42] due to the oscillating wetting of the fluid on the HHX and CHX there is a loss in available work associated not only from the temperature drop across the heat exchangers due to steady state heat fluxes, but also from the lowering of temperature during the evaporation/condensation stage of the cycle. The difference between the temperature of the heat source and the mean temperature during the boiling at the hot heat exchanger results in this loss of exergy. The thermal charge-discharge characteristics of the heat exchangers are important in minimising this loss, where both the thermal capacity and the thermal conductivity of the heat exchanger are of importance. This is also true for the condenser (cold side) heat exchanger.

Secondly the “shuttle loss” arising from the alternating parasitic condensation and subsequent re-evaporation of the working fluid vapour either onto the inner walls and surfaces of the “adiabatic” vapour region or even via free droplet nucleation (cloud formation) in the vapour void [30]. The timing of this condensation and evaporation counteracts the operation of the device and is out of phase with the ongoing thermodynamic cycle. That is vapour condenses on the walls as the HHX is generating vapour (and the pressure is increased in the vapour region) and the droplets evaporate as the CHX condenses vapour (and the pressure drops in the vapour region). Thus there are significant thermal losses associated with the adiabatic volume despite the zero mean heat flux through the walls over the cycle. For this reason this component has been previously referred to as “nominally” adiabatic.

By understanding the unsteady conjugate heat transfer problem it is hoped that these parasitic loss mechanisms in the NIFTE and other novel thermodynamic cycles can be minimised. This will lead to devices and systems with greater efficiencies and improved overall performance.

I.1.3 Film flows

The high surface-to-volume ratios of thin liquid film flows make them useful in processes where high heat and mass transport rates are required [43]. Application include, but are not limited to; feed stock vapour generators in chemical process [44], industrial evaporators for the food industry [45], desalination plant [46], cooling/refrigeration [43]. The

heating effect on the film may arise from a number of sources such as; chemical reactions occurring in the fluid films or from heating of the substrate/wall.

The first major progress in the understanding of heat and mass transfer in liquid film flows was made by Nusselt [47]. Here film formation results by the condensation of vapour onto a vertical or inclined wall. These films, under the action of gravity, flow down the wall, and as such have become known as falling film flows [48]. At low Reynolds numbers ($30 \leq Re_d$ [49, p. 647]), when the film is thin and steady, the flow is laminar and wave free. In this condition heat transfer across the film is dominated by conduction. The velocity profile across the film and consequently the heat transfer coefficient of the flow can be derived by considering the momentum force balance across the fluid. This gives the familiar Nusselt film flow solution (see Section II.4.1), where the Nusselt number of 2.5 is deduced for a constant heat flux boundary condition.

Wave instabilities

Above a Reynolds numbers of $Re_d \sim 30$ and up to 1800 [50] the film flow exhibits a laminar wave regime [49]. Beyond a Reynolds number of 1800 the film flow enters the turbulent regime [51]. The wave instabilities at the interface occur as the inertial forces overcome the restoring viscous and surface tension forces in the fluid [52]. This leads to an increase in the heat and mass transfer rates [53].

An example model, from Chang et al. [54], of the evolution of a wavy film surface is shown in Fig. I.2. Here the typical features of the wave flow are apparent. There is a main wave front, preceded by a number of smaller capillary waves, and succeeded by a smooth trailing region after the main peak. This is similar to that observed in the current film flow experiment, as seen in Chapter VIII.

Marangoni effect

The Marangoni effect is a surface tension gradient driven flow which appears on the fluid free surface. Variation in surface tension can arise from thermal gradients across the film [55], as surface tension is temperature dependant. Alternatively surface tension gradients can arise when the fluid consists of mixture (binary mixture or more) of components. This is because the surface tension of a fluid mixture is dependent on the relative concentration of the fluids components. Such differences in the fluid concentration can be driven by the Soret effect [56, 57], which itself is driven by thermal gradients in the fluid.

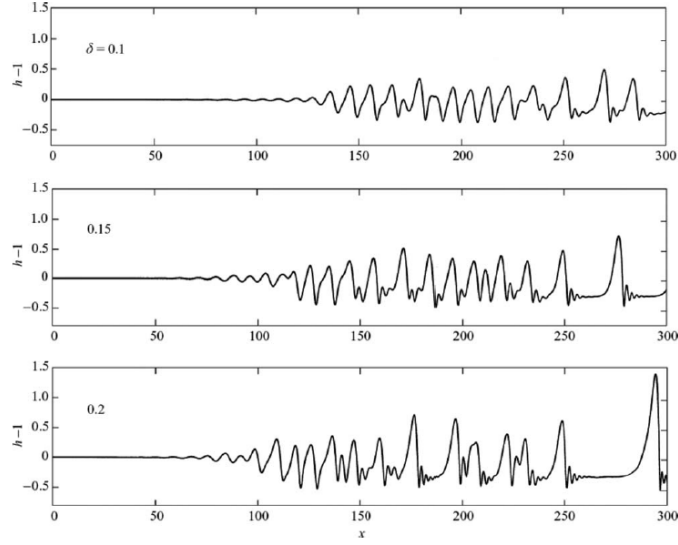


Figure I.2: Model of the evolution of a film flow surface from Chang et al. [54].

Skotheim et al. [55] shows an example which illustrates the Marangoni effect in a locally heated film flow (see Fig. I.3 and Fig. I.4). Here a 25 % ethyl-alcohol water solution is flowing down a vertical plate and over a 4 mm long (in the streamwise direction) heating element. The flow is then subject to local heating and a bump is formed in the film's surface as shown in Fig. I.3. This bump is generated by the Marangoni effect. As the fluid flows over the heating element its temperature increases. Consequently the surface tension of the film is reduced, this produces a force on the fluids surface, acting in the upstream direction, against gravity. The flow at the hottest part of the film is pulled up the plate leading to a thickening of the film. Conversely the film thickness immediately ahead of the bump is slightly reduced, as here the force is in the opposite direction pulling down the film. The Marangoni effect has been shown to steepen the wave fronts of waves travelling down a heated incline [58] in a similar manner.

It was shown in Kabov et al. [33] that for higher heat fluxes the Marangoni effect manifests itself as a 3D distortion of the flow with the formation of longitudinal rivulets and troughs (see Fig. I.4). This flow conditions is sustainable as the troughs contain fluid at a higher temperature than the thick rivulets. The troughs being thinner have a higher heat transfer coefficient and less mass flow to heat than the fluid in the rivulets. The surface tension of the heated fluid is lower than the surface tension on the rivulets, so as a consequence fluid is pulled towards the rivulets, which exacerbates the effect.

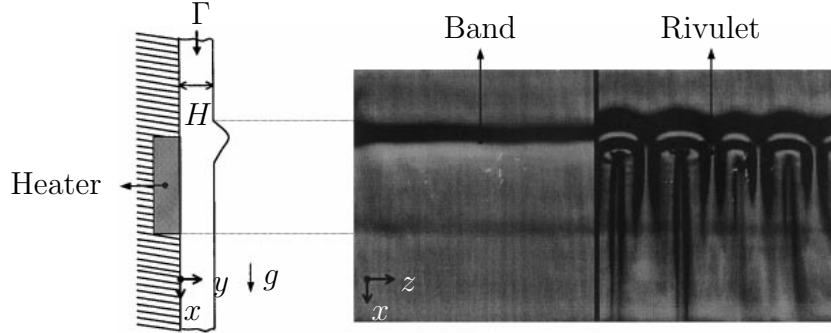
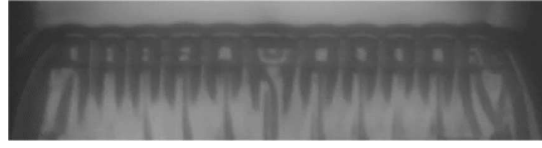


Figure I.3: Marangoni film flow instability on a locally heated plate from Skotheim et al. [55].



Rivulet structures (25 % ethyl-alcohol solution).

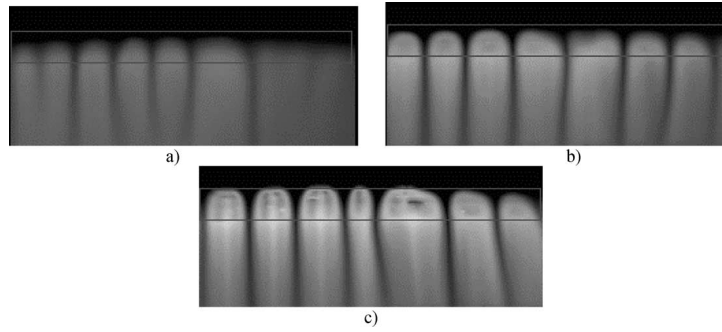


Figure I.4: Marangoni rivulets, and temperature distribution on the film surface at $Re = 1$ from Kabov et al. [33]. Heat fluxes (a) $\bar{q} = 4.7 \text{ W cm}^{-2}$ onset of the instability, (b) $\bar{q} = 5.5 \text{ W cm}^{-2}$, (c) $\bar{q} = 5.8 \text{ W cm}^{-2}$.

Film rupture

When the fluid exhibits an inability to compliantly wet the solid surface, the film flow can break down due to the aforementioned thermocapillary effects. This rupture of the film surface leads to the formation of dry spots on the heated surface, leading to a deterioration in the overall heat and mass transfer. In some cases the temperature of the solid material will increase dramatically, as heat is not removed, putting significant thermal stress on this component.

Fujita and Ueda [59] showed the effect of the applied heat flux on the breakdown of annular film flow. It was shown that for a given flow rate there was a critical heat flux at which breakdown occurred, and that this breakdown occurred because of rivulet formation on the annulus. For a comparable flow Reynolds number of the planar film flow experiment to be presented later the critical heat flux was between 1 W cm^{-2} and 3 W cm^{-2} .

The stability of the dry patch can be determined by the force balance at the stagnation point as shown in Hartley and Murgatroyd [60]. However, Fujita and Ueda [59] concluded that the passing of fluid waves over the dry spot would delay the onset of breakdown as the fluctuating mass flow could temporarily overcome the impingement of the interface on the solid surface.

I.2 Thesis Overview

In this thesis we aim to describe the thermal performance of devices when subject to unsteady heat transfer. The initial motivation for this work was to understand the unsteady thermal losses in oscillation thermodynamic cycles, such as the NIFTE (as outlined in Chapter I). However we extend this to also consider the general heat exchange performance of unsteady convecting heat transfer problems.

The thesis is structured as follows. In Chapter II and Chapter III a theoretical framework for unsteady heat transfer, which is to provide the basis for the rest of the work, is presented. That is, in Chapter II a detailed overview of the issues involved in unsteady heat transfer is given. Here the concepts of conjugation and augmentation is discussed, and the instances in which these effects become important is deduced by looking at the Biot number Bi and Fourier number Fo .

Next, in Chapter III, we consider the 1-D thermal conduction through a solid and derive the thermal impedance and transfer functions in the solid domain. These transfer functions are then used to deduce the impulse and step responses of the solid for a number

of boundary conditions, and will be used subsequently in the measurements for the film flow experiment, and modelling of a number of unsteady heat transfer processes.

A film flow experiment was performed to demonstrate the effects of unsteadiness on the heat transfer problem. The experimental method for the film flow measurements was discussed in Chapter IV. Here a film flow experiment was devised as wavy film flows exhibit well defined periodic heat transfer fluctuations. The experiment consisted of a 300 mm wide 450 mm long test section spanned by a 50 μm thick titanium foil, over which the film flowed over. The foil was heated electrically by passing current a large (~ 500 A) current through it, the resulting resistive (I^2R) losses providing the heating. The flow preparation technique and choices behind material and fluid selection were also discussed.

A novel non-invasive measurement technique was developed which involved Laser Induced Fluorescence (LIF) measurements of Rhodamine B dye in the fluid film to obtain both film temperature and thickness. Because the foil was thin, Infrared (IR) thermography measurements were used on the underside of the foil to obtain the temperature and heat flux at the fluid solid boundary (using the responses derived in Chapter III). This was then used to evaluate the film's heat transfer coefficient, and observe the interesting behaviour of the film flow results of which were presented in Chapter VIII.

By using the 1-D solid response derived in Chapter III the behaviour of convecting flows with fluctuating heat transfer coefficient, (such as film flows) was modelled in Chapter V. The solution methodology involved balancing a circuitry convolution of the solids transfer response (from Chapter III) against Newton's law of cooling, $\dot{q}_f = h\theta_f$, and solving for the wall temperature T_w . The evolution of the fluid side temperature was also considered by using energy conservation. In total six models were derived and explored, each with slight increases in complexity. In each case examples were given as to what real world conditions the model resembles. Of the models only three were truly conjugate, and four displayed augmentation effects, and in only one case was it found that augmentation could become positive.

The same modelling technique was then employed, in Chapter VI, to investigate the effect of the solid impedance on the thermal losses in a gas spring. A gas spring consists of a piston and cylinder enclosing a fixed volume of gas. When the piston is displaced an opposing force is produced, much like a conventional spring, however here the force displacement relationship is governed by the ideal gas law. And the resulting change in pressure from the displacement of the piston leads to a change in the fluid temperature giving rise to heat transfer. To account for the pressure work done in the

gas, a complex Nusselt [28] representation of the heat transfer coefficient was used, as described in Section II.2. This complex Nusselt number was constant throughout the cycle, as opposed to the real but unsteady representation from Chapter V. The complex Nusselt representation however incorporated a capacitive term, to account for the thermal storage of the thin layer of gas at the wall, resulting in a difference in phase between the fluid wall temperature difference θ_f and the heat flux at the wall. The resulting heat transfer problem is conjugate but not augmentative. The thermal loss was then calculated from the work input into the spring and the total amount of the work out over a cycle.

The impedance model was also used to predict the efficiency and oscillation frequency of the original NIFTE thermodynamic cycle [30] introduced at the start of the chapter. This was applied to try to qualify the thermal shuttle losses between the vapour and the solid walls. These predictions were compared to experimental results [30], and found to be an improvement on previous models which had not included the thermal impedance of the solid components [61].

In Chapter VII the convective augmentation models from Chapter V were applied to the results obtained from Nakamura [29, 62] for heat transfer behind a backward facing step. Case V from Chapter V was extended to handle a broad band aperiodic heat transfer coefficient signal. The model of the heat transfer coefficient was parametrised from simple statistical measures from the backwards step experiments. Comparisons were then made between the predictions and actual degree of augmentation. As an aside, various methods were investigated for locating the flow stagnation point down stream of the step by observing features in the temperature and heat flux profiles. It was noticed that features in the heat flux profiles could readily be observed migrating across the heated foil, and it was deduced that these must be associated with the movement of fluid in the recirculation region. Methods for tracking the features were explored, and the stagnation point was located and compared to measurements made using a hot wire anemometer.

Finally in Chapter VIII the results from the film flow experiment are explored. Several interesting flow phenomena were observed, and the effects on heat transfer was discussed. The measured augmentation factors were then compared to the modelled predictions from Case V. Our concluding remarks are then given in Chapter IX.

The four different unsteady heat transfer problems considered in this thesis are used to evaluate the theoretical framework presented in Chapters III and V. Each of these heat transfer problems exhibit slightly differing characteristics. For instance in Chapter V it

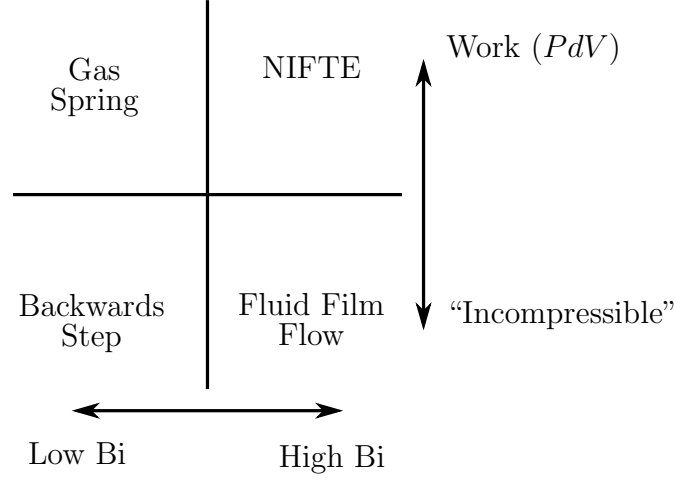


Figure I.5: Thesis overview; Four model flows are studied which exhibit different physical characteristics. The cases considered either exhibit a high or low Biot number, or the situations when the fluid side of the heat transfer process undergo compressible work.

is determined that a large Biot number tends to exacerbate the unsteady heat transfer effects. Conversely in Chapter VI we investigate the effect of work done in the fluid domain on the unsteady heat transfer problem. In the case of the NIFTE and fluid film flows the heat transfer coefficients are relatively high, leading to high Biot numbers. Similarly only the NIFTE and the gas springs experience periodic work done in the fluid domain. The diagram in Fig. I.5 shows how the each of the chosen heat transfer problems overlap these four different intersecting problem regimes.

Chapter II

Foundations of Unsteady Conjugate Heat Transfer

With the exception of a small number of cases involving purely laminar steady flows, the vast majority of flows in engineering applications are unsteady. Unsteady flows can be: (i) periodic, which can be ‘pulsating’ (non-zero mean flow) or ‘oscillating’ (zero mean flow); (ii) quasiperiodic; or (iii) aperiodic (broadband, stochastic, turbulent). In this Chapter we examine *unsteady, forced* convective heat transfer in the *absence of compressibility*. Earlier studies have considered heat transfer in various such arrangements involving unsteady flows and have provided a wealth of information concerning their heat exchange characteristics. Unfortunately however, they have not yet led to a consensus as to the most useful way to describe and present the heat exchange enhancement (or abatement where this is desired) capabilities of these arrangements, nor have they provided a clear insight into the crucial effects of unsteadiness and conjugation on these capabilities. Thus, it is important to examine the difficulties that arise in unsteady problems in detail. Furthermore, on a practical level, there is a lack of reference literature and data to which design engineers can turn to in order to predict the temperature or heat flux in standard geometries, as they would for steady heat transfer [63]. In the first instance one would be interested in the time-averaged heat transfer quantities, which are the focus of the current work.

Much of our understanding of heat transfer in these flows has been motivated by the specific desire to optimise the performance of energy-conversion machines that involve an interchange between heat and work, resulting in simultaneous pressure, flow and heat transfer fluctuations. For instance, in reciprocating thermoacoustic machines (engines,

CHAPTER II. FOUNDATIONS OF UNSTEADY CONJUGATE HEAT TRANSFER

coolers) and devices such as expanders, compressors and combustion engines, or even gas turbine blades [40], the goal is to improve the heat and work transfer characteristics, while avoiding thermal failure. A typical example of this is encountered in Stirling engine regenerators (which consist of stacks, foams and porous media), where there exist conflicting requirements of maximising the heat transfer between the gas and the solid regenerator material, while minimising the pressure drop in the flow through the device [5, 26, 64]. Recently, White [10] also considered the implications of unsteady heat transfer in the wider context of heat pump performance.

Nevertheless, energy-conversion devices involve for the most part unsteady heat transfer in the presence of *compressible gaseous* flows. In the treatment of most gaseous flows the assumption of a constant wall temperature is adequate [65], and as such the heat transfer process does not exhibit conjugation (as defined in Section II.3). The effect of conduction in the solid (and hence of any solid parameters, i.e. thermophysical properties, or geometric variables) drops out of the problem, which is simplified to obtaining information on the heat transfer coefficient (HTC) and using this to quantify the convective heat transfer in the fluid. On the other hand, compressibility introduces an additional complexity into the problem that arises from the contribution of work towards the total energy budget. Perhaps for this reason, together with the general challenges that exist in establishing a formal link between the heat flux and the temperature so that a reliable HTC definition can emerge, due to the natural non-linearity that exists in the underlying flows, a consensus as to the effect of unsteadiness (e.g. pulsation/oscillation frequency) on the HTC continues to escape the community due to contradicting findings [38].

Even in simple *incompressible* flows where pressure work is absent, the question of whether the heat exchange is higher or lower than that of the equivalent steady flow with otherwise similar conditions has not been answered definitively. And this, despite numerous targeted (and carefully performed) experimental investigations, analytical studies and numerical simulations. For example, Sorin et al. [27] measured a HTC for intermittent flow in an exhaust pipe that was consistently higher than the steady flow equivalent, increasing with increasing frequency, reaching a maximum and decreasing as the frequency was increased further. Conversely, the experimental and numerical study performed in an incompressible oscillatory pipe flow with uniform heat flux by Zhao and Cheng [38] demonstrated a monotonic increase in the HTC with increasing dimensionless frequency, in agreement with the experiments of Leong and Jin [66] performed in a foam-filled channel. Yu et al. [67] solved analytically the heat transfer problem in an incompressible pulsating pipe flow with uniform heat flux. Interestingly, the authors reported no effect

on the time-averaged HTC due to pulsation. By contrast, Habib et al. [68] reported a complex non-monotonic relationship between the HTC and the frequency in pulsating pipe flow experiments with a constant heat flux boundary condition (B/C). Generally, it can be said that previous studies can be classified into four different groups, according to which flow pulsations can either: (i) enhance [69, 70]; (ii) deteriorate [71]; (iii) have no effect on [72, 73]; or (iv) have a complex effect (i.e. either enhance or deteriorate) on heat transfer depending on the character of the flow [74, 75].

II.1 Unsteady Thermal Losses in Thermodynamic Cycles

Numerous engines, thermal-fluid devices and other systems are capable of utilising clean and renewable energy sources such as solar, geothermal and waste heat, which is expected to render them increasingly relevant in domestic, as well as industrial settings. A common feature of these systems concerns their interaction (externally) with inherently time-varying fluid and heat streams (e.g., flow rates, pressures, temperatures), and/or the occurrence (internally) of naturally time-varying thermodynamic processes during operation, even when the external conditions are steady. These interactions can readily give rise to unsteady and conjugate heat exchange, in which heat transfer occurs simultaneously by convection between the fluid streams and the solid walls of the relevant components and by conduction through the solid walls. In many cases the unsteady (i.e., time-varying) heat transfer can take place simultaneously with pressure and density variations in the fluid, for example in thermoacoustic engines [76–78], dry free-liquid-piston (Fluidyne) engines [41, 79, 80] or two-phase thermofluidic oscillator equivalents [35, 42, 61], free-piston Stirling engines [16, 81, 82] and pulse-tubes [83–85], as well as in stand-alone compressors and expanders or when these units appear as components within larger systems. In some cases the conditions can give rise to conjugate heat transfer in the presence of the time-varying variations of pressure, density, and temperature. This is the subject of the present chapter.

II.1.1 Rankine cycle expanders

Amongst other, compressors and expanders can be employed in Rankine Cycle engines, including organic (ORC) equivalents [86], for power generation. In fact, an efficient and low-cost expander is a crucial component of any economically efficient ORC system.

CHAPTER II. FOUNDATIONS OF UNSTEADY CONJUGATE HEAT TRANSFER

Typical breakdowns of the thermodynamic exergy destruction in the case of a basic and a regenerative ORC system reveal that, after the evaporator(s), the expander is the component associated with the largest performance penalty, with an exergetic loss of about 20 % [87]. A variety of expanders are available and these are usually categorised as belonging to two main classes of machines:

- Positive-displacement machines (also known as ‘volume’ type, such as pneumatic converters, or screw, scroll or reciprocating piston expanders); and,
- Turbomachines (also ‘velocity’ or ‘dynamic’ type, such as axial or axial-flow turbine expanders).

Positive-displacement expanders are further classified according to the mechanism used for fluid flow (motion), into: (i) rotary expanders (including multi-vane or rotating piston expanders [88], screw or helical expanders [89] and scroll expanders [89–92]); and, (ii) reciprocating (or reciprocal) piston expanders [93]. Turbomachines include axial- and radial-flow (or centrifugal) turbine expanders (or turbo-expanders) [17]. Operational and specification data from expansion devices employed in over 2,000 Rankine engines were collected by Curran [94]. These data, that span power outputs from 0.1 to 1,120 kWe, reveal that low-speed expansion devices (<5,000 rpm) tend to be predominantly positive-displacement (e.g., reciprocating) machines producing power outputs of up to 10 kWe, while turbomachines on the other hand are adopted at higher speeds (>5,000 rpm) for meeting greater demands, with output shaft powers in excess of 10 kWe. A second compilation of specification and performance information was employed by O.E. [95] in order to identify the optimal geometries and maximum obtainable design efficiencies for different types of expander. The resulting compilation of performance characteristics indicates that at low speeds positive-displacement expanders offer superior performance (i.e., exhibit greater efficiencies) when compared to single-stage turbomachines. Rotary positive-displacement expanders achieve similar isentropic efficiencies η_{is} to single-stage turbines at specific diameters (and consequently sizes) that are only 1/4 to 1/3 of those required for single-stage turbo-expanders, while reciprocating positive-displacement expanders can achieve similar efficiencies at even lower speeds [95].

II.1.2 Piston compressors and expanders

As was discussed in the previous section, positive-displacement machines offer the potential of higher efficiencies compared to axial and centrifugal turbomachines, especially in

Unsteady Thermal Losses in Thermodynamic Cycles

Table II.1: Positive-displacement compressor and expander indicated and isentropic efficiencies for a leakage gap size of 10-15 μm , taken from Ref. [97]

Machine	Type	η_{ind}	η_{is}
Compressor	Reciprocating piston	0.85-0.91	0.72-0.76
	Rotary piston	0.85-0.90	0.72-0.75
	Scroll	0.39-0.64	0.36-0.56
Expander	Reciprocating piston	0.81-0.87	0.66-0.72
	Rotary piston	0.78-0.84	0.63-0.69
	Scroll	0.47-0.75	0.32-0.69

smaller-scale applications. For example, Moss et al. [96], employed isentropic efficiency values η_{is} of 97.5 % for compression and 99.3 % for expansion in the context of a reciprocating Joule/Brayton combined heat and power (CHP) plant, though these figures are only estimates and not direct measurements [10]. Similarly, White [10] proceeds to use theoretical values of 98 % and 99 % of the polytropic efficiency of reciprocating compressors and expanders as part of a reverse Joule/Brayton cycle heat pump for domestic applications. Indeed, for small scale expanders manufactured with high tolerance/low precision (and hence also, low cost), and thus medium to large leakage gaps, Table 1 (compiled from data found in Ref. [97]) suggests that, beyond turbo-machines, reciprocating and rotary piston compressors and expanders also significantly outperform their scroll counterparts. Specifically, piston machines have indicated η_{ind} efficiencies in the range 85–91 % for compression and 78–87 % for expansion, and isentropic η_{is} in the range 72–76 % for compression and 63–72 % for expansion. In Ref. [97] it is also suggested that for even smaller leakage gap sizes of 5 μm , η_{ind} and η_{is} reach ~ 95 % and ~ 90 % for both compression and expansion, respectively.

Clearly, reciprocating and rotary piston compressors and expanders are of interest in the above mentioned applications. The present study focuses on reciprocating piston machines, however the results are transferable to rotary piston equivalents, as well as related engines and devices (e.g., Refs. [16, 35, 41, 42, 61, 76–85]).

II.1.3 Irreversible heat transfer losses

Performance losses in reciprocating engines, compressors and expanders arise from mechanical irreversibilities (i.e., mechanical friction, pressure drops through the intake and exhaust valves), but also from thermal irreversibilities (i.e., heat transfer between the fluid and walls of the device). In order to improve their efficiency it becomes essential to

minimise all loss mechanisms. Valve losses may be minimised by a combination of careful valve timing, rapid opening and large open areas. Once mechanical losses have been minimised, thermal losses become the dominant loss mechanism. The present investigation focuses on this aspect of performance.

A performance loss in reciprocating systems arising as a consequence of unsteady heat transfer occurs even if the overall process is globally adiabatic (i.e., for zero time-mean heat transfer), since any heat exchange between the gas and cylinder walls is an inherently irreversible process. The exegergetic loss due to (zero-mean) heat transfer fluctuations arises as a consequence of the fact that heat from the fluid (gas, liquid or vapour) at a high temperature is temporarily stored in the solid walls at a lower temperature, and then returned (in its entirety) to the fluid at an even lower temperature. Thus, although the *net fluctuating* heat loss is zero, an exergetic penalty is paid.

For this reason, beyond the steady (time-averaged) heat losses, it is important to be able to account for, characterise and predict accurately the time-varying heat exchange fluctuations between the fluid and solid. A further characteristic of interest concerns the timing (phase difference) of the heat transfer relative to the temperature fluctuations; in some systems the heat is lost and gained by the fluid at the wrong time in the cycle, thus adversely affecting performance and efficiency.

II.1.4 Heat transfer in reciprocating systems

Unsteady heat transfer in reciprocating systems (engines, compressors, etc.) has been the subject of various investigations that have shown that it is not possible to describe the convective heat transfer process with the employment of a constant heat transfer coefficient (HTC) [7–9, 28, 98–100]. In fact, the employment of such a steady flow HTC in the quantitative description of the unsteady thermal interaction between a solid and fluid, although a simple and powerful approach can be seriously inadequate in many cases. Yet, many theoretical and modelling approaches assume a constant value for the HTC taken from steady flows in order to quantify the unsteady convective heat exchange. In other studies tempts are made to modify the Nusselt number correlations generated in steady flows, through the definition of suitable Reynolds numbers, and have shown that this goes some way towards offering a correction, but that important deviations remain [26]. Those that do account for the unsteady heat transfer process, often continue to neglect the full, conjugate problem by assuming either a constant temperature or a constant heat flux boundary condition at the common solid-fluid interface. Consequently, computational results are often in considerable disagreement with experiments. The

Newton's Law of Cooling in Unsteady Flows

assumption of a constant wall temperature is often adequate when dealing with gases, but this is not always the case and becomes strenuous at best in the case of phase-change (or equivalently, of high HTC's) and also under certain geometrical conditions [101]. The former is a process that can readily take place, for example, in a host of Rankine Cycle engines and other machines in which water, an organic fluid or a refrigerant is expanded to produce work. In this chapter a simple, one-dimensional framework is presented that has been developed for the quantitative description of unsteady and conjugate heat transfer in the presence of displacement work. This framework applies the approach presented in Chapter III to a generalised model of a reciprocating compression and/or expansion process. The specific process that will be examined is that of a gas spring, in which a fixed mass undergoes sequential compression and expansion within the same close space (volume) [7, 8, 28, 100].

II.2 Newton's Law of Cooling in Unsteady Flows

To gain insight into the difficulties associated with unsteady heat transfer, attention must be drawn to its characterisation. The classical description of convective heat transfer rests on Newton's Law of Cooling, according to which the heat flow rate *into* a wall $\dot{Q}_w$ is proportional to the wall surface area available for heat transfer S_w and the temperature difference across the fluid $\Delta T_f = T_{f,b} - T_w$, where $T_{f,b}$ is the bulk (flow-averaged) fluid temperature and T_w is the wall temperature, i.e.,

$$\dot{Q}_w = hS_w (T_{f,b} - T_w) = hS_w \Delta T_f. \quad (\text{II.1})$$

Here, the constant of proportionality h is the convective HTC. At the same time, Fourier's Law of Conduction states that the heat flow rate is also proportional to the spatial temperature gradient at the wall, either in the fluid $\partial T_f / \partial y|_w$, or in the solid domain $\partial T_s / \partial y|_w$,

$$\dot{Q}_w = -k_f S_w \frac{\partial T_f}{\partial y} \Big|_w = -k_s S_w \frac{\partial T_s}{\partial y} \Big|_w, \quad (\text{II.2})$$

such that, from Eqs. (II.1) and (II.2),

$$h = \frac{-k_f \frac{\partial T_f}{\partial y} \Big|_w}{\Delta T_f} = \frac{-k_s \frac{\partial T_s}{\partial y} \Big|_w}{\Delta T_f}, \quad (\text{II.3})$$

CHAPTER II. FOUNDATIONS OF UNSTEADY CONJUGATE HEAT TRANSFER

and, given characteristic thermal lengthscales L_f in the fluid and L_s in the solid,

$$Nu = \frac{hL_f}{k_f} = \frac{-\frac{\partial T_f}{\partial y}|_w L_f}{\Delta T_f}; \quad Bi = \frac{hL_s}{k_s} = \frac{-\frac{\partial T_s}{\partial y}|_w L_s}{\Delta T_f}, \quad (\text{II.4})$$

for the Nusselt Nu and Biot numbers Bi , respectively, both of which are dimensionless temperature differences (or equivalently, gradients) as indicated by these expressions.

Now, for a given (constant) HTC h , Eq. (II.1) requires that the heat flux $\dot{q}_w = \dot{Q}_w/S_w$ and the bulk fluid–wall temperature difference ΔT_f are always in phase. In many common steady flow situations the temperature gradient in the fluid at the wall $\partial T_f/\partial y|_w$ and hence also the heat flux at the wall $\dot{q}_w$ (from Fourier’s Law; Eq. (II.2)) are indeed proportional to ΔT_f . Therefore, h and also its dimensionless description Nu are real constants [102]. Unfortunately, many studies in oscillating arrangements have shown that $\dot{q}_w$ can be out of phase with ΔT_f , in direct violation of Newton’s Law with a constant h .

This possibility was demonstrated first by Pfriem [103] and later by Lee [7] in the context of purely oscillating compressible flows. The authors predicted theoretically that heat transfer in a reciprocating piston-in-cylinder arrangement with oscillating pressure would be out of phase with both the cylinder centreline temperature $T_{f,\infty}$ [103] and the mean gas temperature $T_{f,b}$ [7]. The prediction has since been confirmed experimentally by a number of investigators [28, 104–106]. For example, in Refs. [28, 106] it was noted that in an oscillating in-cylinder flow, heat flowed from the wall into the chamber at moments when the gas temperature (both $T_{f,\infty}$ [107] and $T_{f,b}$ [106]) was higher than that of the wall. Specifically it was observed that $\dot{q}_w$ exhibited a phase *lead* relative to ΔT_f . Of direct relevance to the present study was the extension of the theoretical findings of Pfriem [103] and Lee [7] to heat transfer in oscillating *incompressible* flows by Kurzweg [108] and Gedeon [109].

In order to describe the aforementioned phase shift between $\dot{q}_w$ and ΔT_f it is possible to allow h to time-vary, yet in problems where there is a reversal in the sign of $\dot{q}_w$ or ΔT_f , the expression $h(t) = \dot{q}_w/\Delta T_f$ results momentarily in negative or infinite h , respectively. This approach has been avoided in the past, perhaps due to the difficulty of interpreting the physical meaning of a negative or infinite h , and by extension of a negative or infinite Nu . We will return to this point in Section III.1.

Instead, to circumvent this problem and to capture the necessary heat transfer information, many authors [7, 28, 103, 107] have employed a complex formulation for Nu as a function of an oscillating flow Péclet number $Pe = \omega D_h^2/\alpha_f$ (a dimensionless angular

Newton's Law of Cooling in Unsteady Flows

frequency ω , where D_h is the hydraulic diameter and α_f the thermal diffusivity of the fluid). In this approach h and $T_{f,b}$ are written in complex form, $h = h_{\text{Re}} + i \cdot h_{\text{Im}}$ and $T_{f,b} = \bar{T}_{f,b} + \hat{T}_{f,b}(\cos \omega t + i \cdot \sin \omega t)$, with h_{Re} and h_{Im} being real constants, while the wall temperature T_w is considered constant. Since $\dot{q}_w = \bar{\dot{q}}_w + \dot{q}'_w = \Re\{h\Delta T_f\}$, this leads to,

$$\bar{\dot{q}}_w = h_{\text{Re}} (\bar{T}_{f,b} - T_w) = h_{\text{Re}} \overline{\Delta T_f}; \quad (\text{II.5})$$

$$\dot{q}'_w = h_{\text{Re}} \hat{T}_{f,b} \cos \omega t - h_{\text{Im}} \hat{T}_{f,b} \sin \omega t. \quad (\text{II.6})$$

In fact, recent linear thermoacoustic treatments are resorting increasingly to the presentation of unsteady heat transfer in complex form [64, 110, 111], though it must be noted that it is entirely equivalent to presenting magnitude and phase directly, as for example Nield and Kuznetsov [112] did in their treatment of laminar incompressible pulsating flow.

Nevertheless, although Eq. (II.6) allows one to account for a phase lag between the fluctuating $\dot{q}'_w$ and $\Delta T'_f$ (or equivalently, between the time-varying $\dot{q}_w$ and ΔT_f), it still restricts $\dot{q}_w$ to having the same spectral (or frequency) content as ΔT_f , and in the case of constant T_w it restricts $\dot{q}_w$ to having the same content as $T_{f,b}$. This is a noteworthy limitation of this approach, as it cannot account for any ability of the unsteady fluid flow field to affect the character of $\dot{q}_w$ independently of $T_{f,b}$.

In summary, heat transfer studies in unsteady flows have been the subject of several investigations that yielded conflicting and inconsistent conclusions. Many past observations imply, crucially, that classical correlations involving constant dimensionless parameters of the form $Nu = f(Re, Pr)$, where Re is the Reynolds number and Pr is the Prandtl number, cannot be used to predict unsteady problems, and suggest the need for a revised definition of a HTC that can be used reliably for engineering purposes. Driven primarily by specific interests, many of these studies have focused on compressible gaseous flows: (i) that do not exhibit conjugation effects, therefore not allowing a complete understanding of the underlying phenomena; and also (ii) in which pressure work is a dominant energy term that influences the energy exchange, limiting the extension of this understanding to incompressible situations. Furthermore, due to their inherent difficulty (including the need for careful spatiotemporally resolved information from either measurements and/or simulations, and a much extended list of dependent parameters), these investigations have covered only a narrow and disparate parameter range and have not considered closely the contribution of the full array of fluid and flow parameters on the problem, while the effect of the solid was often neglected. In liquid and even more so in two-phase flows, the

heat transfer processes become considerably more complex due to the strong presence of unique features of unsteady heat transfer such as conjugation, whereby the solid begins to have an effect. This effect must be understood if we are to clarify the inconsistencies in our current knowledge, and consequently if we are to overcome key design limitations exhibited by many related energy technologies that are currently under development.

II.3 Unsteady Conjugate Heat Transfer

‘Conjugate’ heat transfer involves heat exchange that occurs simultaneously by convection between a fluid and an adjacent wall, and by conduction through the solid. In the current work we focus specifically on unsteady (time-varying) heat transfer [23]. In this context, an unsteady conjugate heat transfer (UCHT) problem, or simply ‘conjugate’ problem, is defined herein as one which exhibits an unspecified and significant:

1. Time-varying temperature; *and*,
2. Time-varying heat flow rate (or heat flux),

at the common fluid–solid interface. Given these conditions, the interface temperature and heat flux arising from heat transfer processes in the fluid and solid domains can only be found by solving simultaneously for both of these processes given suitable boundary conditions external to the two domains.

In UCHT, the coupling between the fluid flow, the convective heat transfer process in the fluid and the conduction in the solid, can give rise to phenomena that are of great interest in the context of improved heat exchange or insulation. However, the phenomena are complex and non-linear (see Chapter V and VI) even in the absence of phase change, compressibility and turbulence. As a consequence, their fundamental understanding is far from complete, and so our ability to harness their potential is lacking.

Beyond its fundamental importance UCHT is ubiquitous and highly significant in key applications that are expected to become increasingly relevant both in industrial and domestic settings. UCHT is a controlling process in numerous external combustion engines, thermoacoustic devices and other machines capable of utilising clean and renewable energy forms such as solar, waste and geothermal heat. Indeed, the common and central feature of these devices is their inherent reliance on internally oscillating thermal-fluid flows which result in UCHT. The emergence of these clean energy technologies is being held up by a gap between their current performance and their predicted

Temperature Measurements in Film Flows

capabilities, which arises as a consequence of an inability to predict UCHT in the first instance, and by extension its effect on their overall performance and efficiency. The unsteady transport phenomena inherent to these technologies also exist in a diverse range of other engineering applications, but also environmental processes, physiological systems and biomedical applications. Examples range from microscale reactors and microprocessor cooling schemes; to multi-scale respiratory/circulatory flows; to engine combustors and especially microcombustors and new low-emission concepts, e.g. Homogeneous Charge Compression Ignition (HCCI) engines, gas and steam turbines, heat exchangers and compressors/expanders, film flow evaporators and coolers; to thermal management in buildings; to large scale flows in low-grade heat recovery systems or high-efficiency power plants and novel heat integration processes in industrial plants for improved efficiency.

Despite its extended practical relevance and fundamental importance, UCHT continues to receive far less attention than its steady counterpart. Even in problems that are clearly unsteady, the vast majority of non-isothermal fluid flow models assume an isothermal or adiabatic B/C at the fluid–solid boundary, without considering or establishing the validity of these assumptions. This allows one to bypass the more involved coupled fluid–solid problem, but effectively ignores altogether the true conjugate nature of the problem. Furthermore, the reliance of nearly all treatments of UCHT on a conventional HTC definition that arises conceptually from steady flow situations, although a simple and powerful approach in many cases, can be inadequate, misleading, or even completely inaccurate in others, as outlined in Section II.2.

Improvements to our understanding of UCHT and subsequently to the performance of many of the aforesaid systems must come from studying actual cases of UCHT within a framework that can capture the time-varying interplay between fluid flow and heat transfer, and account for the presence of conjugate fluid–solid interactions. The overriding aim of this work is to present a framework for such an approach.

II.4 Temperature Measurements in Film Flows

Film flows with periodic (externally excited) or stochastic interfacial wave structures have been studied extensively. Lel et al. [113] performed point measurements with high temporal resolution and accuracy of film thickness and interfacial wave velocity using a confocal lens technique and compared it to a fluorescence intensity method, and reported satisfactory agreement. Further, Moran et al. [114] made local measurements of the velocity profile within the liquid film as hydrodynamic waves travel down the film surface,

CHAPTER II. FOUNDATIONS OF UNSTEADY CONJUGATE HEAT TRANSFER

and found that for large waves the velocity profiles were significantly different to Nusselt theory [47]. These flows are of interest to the present work, since: (i) the interfacial wave dynamics arising from hydrodynamic instabilities, and the associated varying liquid film thickness, will give rise naturally to spatiotemporally varying HTC's; and further (ii) temperature gradients induced within the film, as this undergoes heating or cooling, can result in significant surface tension gradients and give rise to thermocapillary 'Marangoni' instabilities [33, 115, 116], which will also affect the interfacial, flow and heat transfer processes. Generally thinner films exhibit the largest heat and mass transfer rates, however in very thin films these instabilities can lead to dewetting or dry spot formation, which will degrade the overall heat and mass transfer characteristics.

The temperature of a fluid film can be measured with laser fluorescence techniques, which have allowed non-intrusive temperature measurements with accuracies better than 0.5 K to be achieved. Such a method is employed herein, by using the temperature-sensitive dye Rhodamine B. Fluorescence methods have proven popular in evaluating the heat exchange of microchannel devices [117, 118], and has also been used to measure fluid film temperatures elsewhere [36, 119]. In particular, Schagen et al. [36] and, Schagen and Modigell [119] employed a combined ultraviolet (UV) fluorescence-phosphorescence technique with a Diacetyl marker at a single point in the flow. The authors used these measurements to recover simultaneous information on the liquid film thickness and temperature profile within the film, and hence the HTC, at the position of the point measurement. However, these measurements were recovered by assuming a Nusselt velocity profile in the film, which may not account for deviations from this theoretical flow (e.g. as reported by Moran et al. [114]), due to two or three-dimensional flow effects or unsteadiness (including turbulence) in the film.

The free-surface temperature of liquid films has been measured directly using infrared (IR) thermography, sometimes with an IR filter placed in front of the camera lens to block the wavelengths of radiation to which the working fluid is transparent. The key aspect of this measurement is the absorptivity of the fluid, which is required to be high at the wavelengths used for the thermography. This ensures the observation of a thin layer of fluid near the interface (i.e. of the free-surface), without contamination from the bulk of the fluid deeper within the film, or from the solid substrate. For example, Lel et al. [113, 120] used an IR camera to measure the interfacial temperature of silicone oil films. In those studies silicon fluids were selected as they allowed a wide range of viscosities to be spanned, with a corresponding range of Reynolds and Kapitza numbers. In this case the absorption coefficient allowed 99 % of the radiation to be absorbed within a thin

Temperature Measurements in Film Flows

(10 – 15 μm) layer at the surface of the films [120]. IR thermography has also been used extensively in water film flows (e.g. Zhang et al. [121, 122], and many others), and is also employed in the present measurements.

II.4.1 Nusselt film flow

The flow of a steady, fully developed, laminar liquid film on an inclined plate is described by the Nusselt film flow solution [47]. Nusselt flow forms a natural basis for comparison of the heat transfer measured in the wavy film flows investigated in the present experiments, although differences should be expected since our flows are not steady, or laminar at higher Reynolds numbers, and also may not be fully developed by the measurement location. The HTC in a steady and fully developed Nusselt flow can be obtained by considering that the body forces acting on the fluid due to gravity must be in balance with the viscous forces due to shear across the film, such that the fluid velocity profile satisfies the analytical Nusselt solution [47],

$$U_x = \frac{g \sin \beta}{\nu_f} \tilde{y} \left(d - \frac{\tilde{y}}{2} \right), \quad (\text{II.7})$$

which can be re-written in terms of Reynolds number Re , or the bulk film velocity U_b as,

$$U_x = \frac{3\nu_f \text{Re}}{d} \left(\frac{\tilde{y}}{d} \right) \left[1 - \frac{1}{2} \left(\frac{\tilde{y}}{d} \right) \right]; \quad \frac{U_x}{U_b} = 3 \left(\frac{\tilde{y}}{d} \right) \left[1 - \frac{1}{2} \left(\frac{\tilde{y}}{d} \right) \right], \quad (\text{II.8})$$

where g is gravitational acceleration and ν_f the kinematic viscosity of the fluid. Here, the coordinate $\tilde{y} = y - a$ is perpendicular to and defined as zero at the inner wall. The Reynolds number has been obtained from $\text{Re} = \int_0^d U_x d\tilde{y} / \nu_f = U_b d / \nu_f = \Gamma / \nu_f$ where Γ is the fluid flow-rate per unit channel width Q/w , such that $\text{Re} = g \sin \beta d^3 / 3\nu_f^2$, and the bulk flow velocity of the film is given $U_b = g \sin \beta d^2 / 3\nu_f$.

Now, writing down the energy equation in the fluid,

$$\frac{DT_f}{Dt} = \alpha_f \nabla^2 T_f, \quad (\text{II.9})$$

and noting that the flow is thin, fully developed, steady-state and laminar allows some of the terms in Eq. II.9 to be neglected. Furthermore, for a constant heat flux B/C the flow will be self-similar in the streamwise direction, which means that the streamwise temperature gradient $\partial T_f / \partial x$ will be constant. On applying these simplifications, the

CHAPTER II. FOUNDATIONS OF UNSTEADY CONJUGATE HEAT TRANSFER

governing equation of the temperature profile in the film becomes,

$$U_x \frac{\partial T_f}{\partial x} = \alpha_f \frac{\partial^2 T_f}{\partial \tilde{y}^2}. \quad (\text{II.10})$$

Substituting Eq. II.8 into Eq. II.10 and integrating subject to the B/Cs $\partial T_f / \partial \tilde{y}(\tilde{y} = 0) = -\dot{q}_w / k_f$ on the solid wall and $\partial T_f / \partial \tilde{y}(\tilde{y} = d) = 0$ at the free surface, while noting that $\dot{q}_w = -U_b \rho_f c_f d (\partial T_f / \partial x)$ in the fluid with $\dot{q}_w$ defined as +ve from the fluid to the wall, we obtain the following temperature profile in the film,

$$T_f = T_w + \frac{\dot{q}_w}{k_f} d \left[-\frac{1}{8} \left(\frac{\tilde{y}}{d} \right)^4 + \frac{1}{2} \left(\frac{\tilde{y}}{d} \right)^3 - \left(\frac{\tilde{y}}{d} \right) \right]. \quad (\text{II.11})$$

from which the bulk temperature is thus,

$$T_b = \frac{1}{d} \int_0^d T_f d\tilde{y} = T_w + \frac{2}{5} \frac{\dot{q}_w}{k_f} d, \quad (\text{II.12})$$

which gives a film Nusselt number Nu of,

$$Nu = \frac{hd}{k_f} = 2.5. \quad (\text{II.13})$$

II.4.2 Wave instabilities on the film flow surface

As the heat transfer coefficient of the fill flow is strongly linked to the local film depth (Section II.4.1). The formation of periodic waves in the film flow will generate large periodic fluctuations in the film heat transfer coefficient. The formation of these waves can occur naturally, or through some forcing effect.

When a fluid film flows down an inclined plate, the surface of the film can experience an instability in response to a perturbation of the film surface. The perturbation in the surface height will grow, or decay depending on the flow conditions, as it travels down the film surface. Perturbations which grow will then form waves which will continue to travel down the plate. Different wave regimes exist, as first demonstrated by Kapitza and Kapitza [123], having characteristic wavelengths, amplitudes, velocities and surface shapes along the film for given liquid flow rate and physical properties [124]. These experiments also showed that forcing the flow rate in the inlet region allowed the selection of various wave regimes depending on the forcing frequency and amplitude. However the

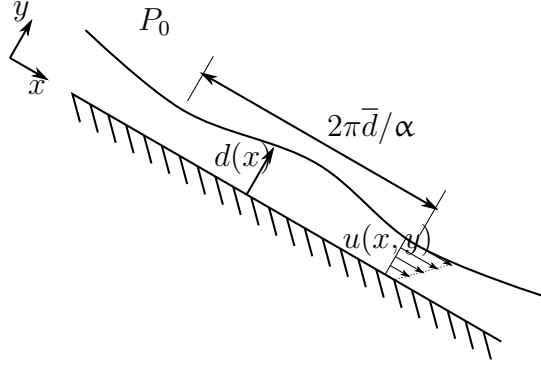


Figure II.1: Capillary driven instability resulting in hydrodynamic waves on the surface of a fluid film flowing across a plate.

forcing frequency had to be smaller than the neutral wave number for wave growth to occur [52].

The characteristics of these wave regimes, and the waves growth rate can be predicted by considering the linear stability analysis of the governing flow equations [54, 125, 126]. Typical solution methodologies start by taking the boundary layer equations for thin fluid films, as derived by Chang et al. [32],

$$\frac{D\vec{u}}{Dt} = \frac{1}{5\delta_{Re}} \left(\frac{\partial^3 d}{\partial x^3} + \frac{1}{3} \frac{\partial^2 u}{\partial y^2} + 1 \right) - 3\chi \frac{\partial d}{\partial x}, \quad (\text{II.14})$$

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}, \quad (\text{II.15})$$

$$y = d(x, t) : \quad \frac{\partial d}{\partial t} = v - u \frac{\partial d}{\partial x}, \quad \frac{\partial u}{\partial y} = 0, \quad (\text{II.16})$$

$$y = 0 : \quad u = v = 0. \quad (\text{II.17})$$

One of the spatial dimensions, normally the depth direction y , is then eliminated by imposing an approximation, such as imposing a self similar velocity profile in the film. This is achieved by applying a Galerkin projection,

$$u(x, y, t) = \sum_{n=1}^N u_n(x, t) \psi(\eta) \quad (\text{II.18})$$

as used in Chang et al. [54]. These models are then either solved numerically [54], or by performing linear stability analysis on the nonlinear PDE's [52, 124–126].

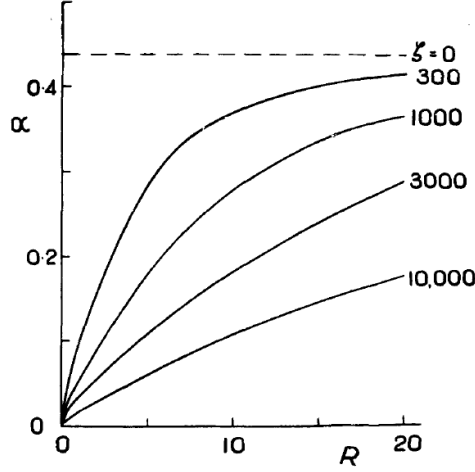


Figure II.2: Neutral stability curves for wave flow on a plate at different inclination angles from Benjamin [52]. Here R is the Reynolds number, α is the dimensionless wave number, and ζ is the Kapitza number.

In Benjamin [52] the growth of a disturbance in the film thickness $d(x, t)$, as illustrated in Fig. II.1, with dimensionless wave number α was studied. A map of the unstable wave numbers for a given flow Reynolds number Re was presented in Benjamin [52] and is shown in Fig. II.2. The map shows that the region of stability for the film flow depends on the Kapitza number, denoted here as $\zeta = \sigma_f \rho_f^{-1} \nu_f^{-4/3} g^{-1/3} \sin \theta^{-1/3}$, of the fluid flowing down the incline. It is also apparent that for any finite Reynolds number there exists a finite range of wave numbers α at which the film is unstable, and that at lower Reynolds numbers this range is smaller. Remember that the wave number is the inverse of the wave length $\alpha = 2\pi\bar{d}/\lambda$ and this upper bound on α means there is a minimum wavelength of wave which will grow on the film. Typically for the film flow experiment described in this chapter the nominal film thickness was $\sim 2\text{si mm}$ and the Reynolds numbers were greater than 10 which equates to a minimum wave length of 31 mm.

In the unstable region the growth rate of the waves can be calculated by further investigating the solution to the boundary layer equations, as done for failing films on flexible inclines by Matar et al. [128] and Sisoiev et al. [127]. The experiment considered in this chapter of a fluid flowing down a inclined foil, which is held in tension and heated electrically, is similar in some respect to the investigation by Matar et al. [128], in that the foil could be considered a flexible incline. The foil however has a tension far in excess of the surface tension of the fluid surface, $\Sigma \gg 1$, and should be assumed rigid. Nevertheless the results from Sisoiev et al. [127] for large Σ will tend to that for a ridged incline and

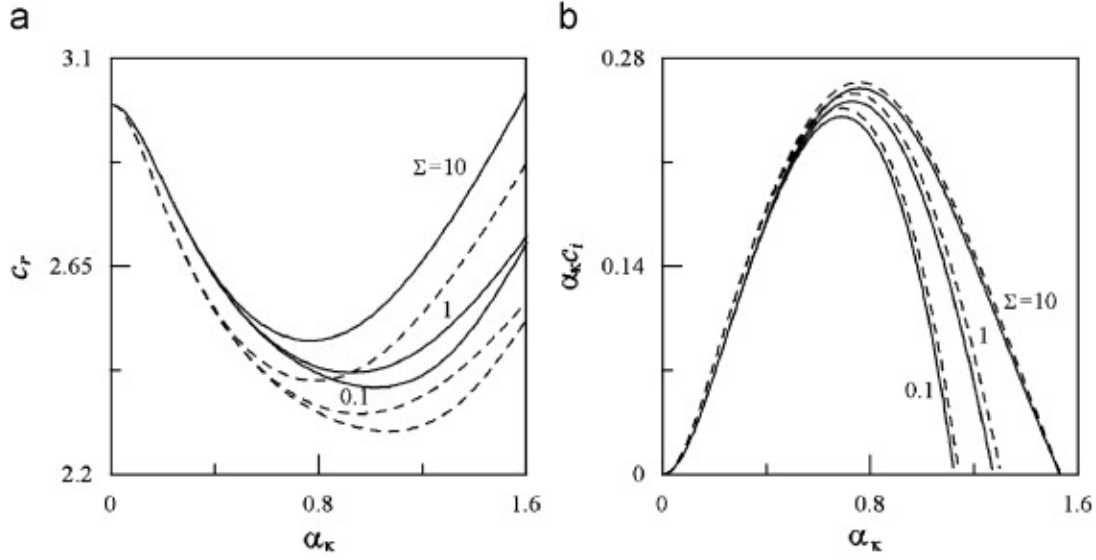


Figure II.3: Wave velocities c_r (a) and growth rate $c_i \alpha_K$ (b) down a thin film on a flexible surface for wave number α_K taken from Sisoiev et al. [127]. Here $\Sigma = \sigma_s/\sigma_f$ is the ratio of tension of the solid surface, σ_s , to the surface tension of the fluid film σ_f . Solid lines are from linear stability analysis of the governing flow equations [128] and the dashed are from on Orr-Sommerfeld solution.

should be comparable to the experimental arrangement presented here. To illustrate the expected wave behaviour for the film flow experiment, the growth rate and wave speed calculated by Sisoiev et al. [127] are shown in Fig. II.3.

It is shown in Fig. II.3 that the peak growth rate for wave instabilities is roughly at the midpoint of the unstable region, between the neutral stability wave number (from Fig. II.2) and a wave number α of 0. Also the growth rate of the waves drops off to zero at the extremes of this unstable region. As the governing boundary layer equations are nonlinear, once the waves have grown large, the assumptions behind the nonlinear stability analysis becomes invalid. Left unforced the waves most likely to form are at the wave number with the highest growth rate, however by forcing an oscillation of the flow rate at the inlet region, waves with wave numbers up to the neutral stability wave number can be preferentially selected, allowing a large range of wave frequencies to be investigated. As an additional benefit, by imposing a large forcing amplitude the development length of the waves is reduced, reducing the length of the experiment needed.

The non-linear boundary layer equations for film flows were solved numerically in Chang et al. [54], and the evolution of one of these solutions is shown in Fig. II.4. In this investigation a very small random perturbation to the flow interface was introduced at the inlet to the computational region and the development of the fluid interface was observed.

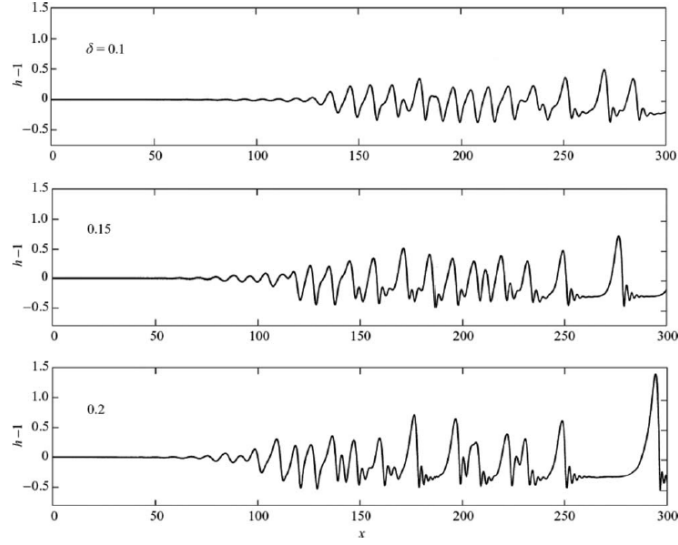


Figure II.4: Wave evolution on a falling film obtained by the numerical solution of the boundary layer equations with pressure due to capillarity forces excited by noise, for different values of the flow parameter δ , from Chang et al. [54]. Here $\delta = \delta_{\text{Re}} = \frac{(3\text{Re}^2\text{We}^{-1})^{1/3}}{15} = (\rho d_c^3 g^4 / \sigma)^{1/3} / 45\nu^2$ is a Reynolds number normalised by the Weber number, and represents the ratio of inertial forces to the damping of the wave motion. Increasing δ can be seen as a reduction in the damping of the wave motion (due to surface tension and viscous forces), or an increase in flow velocity.

Here we can see the effect of the rescaled Reynolds number $\delta_{\text{Re}} = (3\text{Re}^2\text{We}^{-1})^{1/3}/15$ on the development of the waves. Higher Reynolds numbers result in the waves developing earlier on in terms of the scaled streamwise length scale x . Also the wave amplitude is larger at higher Reynolds number and exhibits greater nonlinearities, in the form of the capillary waves proceeding the main wave front. Generally the waves seen in the model consist of a main wave front, preceded by a number of smaller capillary waves, and succeeded by a smooth trailing region after the main peak. This is similar to that observed in the current fill flow experiment, as seen in Chapter VIII.

When heat transfer is imposed on the film flow the interface stability problem becomes non-isothermal. Temperature gradients, or temperature driven concentration gradients (Soret effect) result in surface tension variation on the fluid interface. The resulting surface tension driven Marangoni flows effects add significant complexity to heated film flows. Skotheim et al. [55] showed how a locally heated film exhibited a standing wave due to this Marangoni effect, even resulting in the formation of rivulets. The resulting thermocapillary force also alters the wave stability as shown by Goussis and Kelly [53].

Temperature Measurements in Film Flows

For reference a more general review of the stability of film interfaces is presented by Craster and Matar [31], including a range of other topics relating to film instabilities.

The effect of the thermocapillary action on fluid waves travelling down heated or cooled inclines was investigated by Miladinova et al. [58]. It was found that for waves travelling down a heated incline, in which the mean temperature increases linearly in the stream wise direction, the fluid in the trough proceeding the wave front will be hotter than that at the wave crest. Generally, as the surface tension of a fluid is lower at higher temperatures, the surface tension gradient produces a force on the surface towards the wave peak. This thermocapillary force acts to pull the fluid in the trough towards the crest, opposite to gravity. This results in a steepening of the wave front and an amplification of the wave front. It should be expected to see this effect in the current experiments results when comparing wave fronts in the heated and non heated cases.

Chapter III

An Analytical Framework for Unsteady Conduction in Solid Domains

In this chapter we investigate the thermal response of a solid to thermal fluctuations and develop an impedance model representation of this response. The impulse and step responses in various conditions are derived. and the effect of composite solids is investigated.

III.1 Time-varying Heat Transfer Coefficient

Many systems exhibit unsteady convective heat transfer behaviour due to fluctuations in the fluid flow [65]. This can occur, for example, in slug flows [120, 129], internally oscillating/pulsating flows [5, 27, 38] or wavy film flows [36]. Similarly unsteady heat transfer may be caused by fluctuations in a thermodynamic process, such as a pressure change [107]. In such systems, spatiotemporal variations in the fluid mixing, vapour pressure or fluid thickness result in fluctuations in the heat transfer rate for a given fluid-solid temperature difference, which can be interpreted as fluctuations of the instantaneous heat transfer coefficient [26].

Consider the heat transfer between a fluid domain and a solid domain of finite thickness a , as show in Fig. III.1. Here the fluid is in contact with the “inner” wall or boundary condition (B/C) along the length of the solid domain. The heat flux at the inner wall

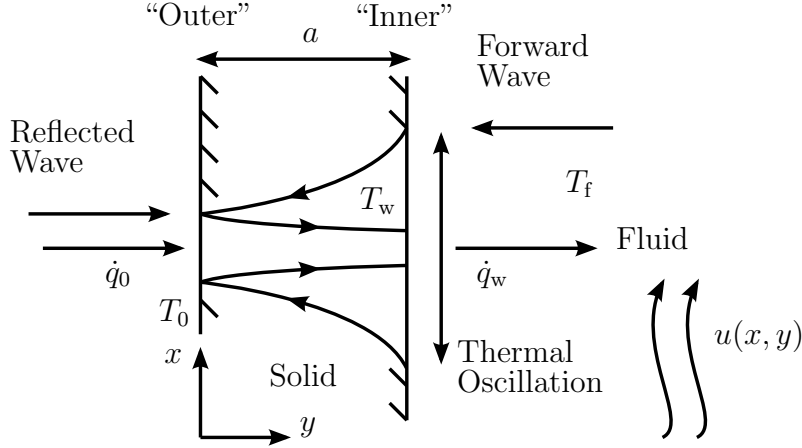


Figure III.1: Thermal diffusion through a solid undergoing some heat transfer process. The diagram shows the envelopes of the thermal waves and the inner and outer boundary conditions.

where $y = a$, $\dot{q}_w$, is given by both Fourier's Law (see Eq. II.2) and Newton's Law (see Eq. II.1),

$$\dot{q}_w(x) = -k \frac{\partial T}{\partial y} \Big|_w = h [T_{f,b}(x) - T_w(x)] , \quad (\text{III.1})$$

as mentioned previously, the wall term $-k(\partial T/\partial y)_w$ can be evaluated either on the fluid or on the solid side of the inner wall, as long as the corresponding material conductivity is used. In Eq. III.1, h is the HTC, $T_{f,b}(x)$ is the bulk temperature of the fluid defined here as a normalised average enthalpy flow rate,

$$T_{f,b}(y) = \frac{\int_{y=a}^{y=a+d} \rho_f(x, y) u(x, y) T_f(x, y) dy}{\int_{y=a}^{y=a+d} \rho_f(x, y) u(x, y) dy} , \quad (\text{III.2})$$

and $T_w(x) = T(x, y = a)$ is the temperature of the inner wall.

Now, at any location on the wall x , Eq. III.1 can be interpreted as the definition (see Eq. II.3) of a *time-varying* heat transfer coefficient,

$$h(t) \stackrel{\text{def}}{=} \frac{\dot{q}_w(t)}{\Delta T_f(t)} = \frac{-k_f \frac{\partial T_f}{\partial y} \Big|_w(t)}{\Delta T_f(t)} . \quad (\text{III.3})$$

This h is not itself a property of the flow, and in general, h will not be constant, but a function of the flow and solid geometries, the fluid and solid properties and even temperatures. The consequences of this fact become crucial in unsteady problems, where unlike in steady problems where it is standard practice to employ correlations of the

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

type $Nu = f(Re, Pr)$, many complications emerge [23]. Nonetheless, the viewpoint is taken in the current approach that a time-varying HTC can be an adequate means of quantifying $\dot{q}_w$ in our selected problems of interest, i.e. in forced convective heat transfer, in the absence of compressibility and with constant properties. This can be justified, as long as $(\partial T_f / \partial y)_w \propto (T_{f,b} - T_w)/y$, which would arise as asymptotically correct to first order if $T_{f,b}$ were expanded as a power series in y and higher order terms were omitted in accordance with their small magnitude. The case when this assumption on h breaks down, due to the application of thermodynamic work in the fluid domain, is studied later in Chapter VI.

This definition can result in $h(t)$ becoming negative or infinite momentarily. Although it is possible to interpret the physical meaning of a momentarily negative or infinite h or Nu (for example, h may become negative as a consequence of the signs of $\dot{q}_w$ and ΔT_f being opposite, leading to brief instances in which heat transfer occurs in the opposite direction to the *bulk* temperature difference, as was shown to be the case in Ref. [107]), we restrict the present analysis to the case where h is positive and finite at all times.

III.1.1 Implications of unsteadiness and conjugation

As stated previously, in most gas-solid systems the temperature in the solid T_s cannot be affected significantly by fluctuations in the fluid temperature T'_f or in the HTC h' , due to the relatively low h (e.g. Nakamura [29]), equivalent to a constant temperature B/C at the interface with the fluid T_w . This is a typical paradigm of a non-conjugate heat transfer problem. However, fluctuations of temperature and heat flux can penetrate from the fluid into the solid, depending on the flow conditions (e.g. at higher h , such as in evaporating/condensing flows), the thermophysical properties of the fluid and solid, and the geometry of the solid. The presence of the solid will then have an effect on the temperature (and heat flow) solution, giving rise to a coupling of the convection and conduction processes. In this *conjugate* problem the energy equation must be solved both in the fluid and in the solid, and the two solutions matched at the common boundary (i.e. the inner wall). Here, we consider the UCHT problem of convection in a fluid and conduction through a solid wall, described by time-varying temperature $T(t)$ and heat flux $\dot{q}(t)$ fields, and time-varying heat transfer coefficient $h(t)$. In this problem we observe non-zero T'_w and $\dot{q}'_w$ and it is not possible to apply an isothermal or isoflux B/C at the inner wall.

Essentially, this problem has a number of potential differences from the steady case:

Time-varying Heat Transfer Coefficient

1. The instantaneous local $\dot{q}(y, t)$ throughout the solid is not the same everywhere, due to the thermal storage capacity of the solid, which is characterised by the Fourier number ($Fo = \alpha_s / \omega a^2$, with ω a characteristic frequency). Consequently, the heat flux at the inner wall $\dot{q}_w(t)$ is not instantaneously equal to the heat flux at the outer wall $\dot{q}_0(t)$. Also, in the steady problem, the only relevant solid property is the thermal conductivity k_s , however in the unsteady case, the density ρ_s and heat capacity c_s are equally important (see Section III.2.1).
2. The solid geometry (i.e. the solid thickness a , but also the character of the surface) has a greatly more complicated effect on $\dot{q}$ (both $\bar{\dot{q}}$ and $\dot{q}'$) than in the steady problem where an increase in a always results in a decrease in $\dot{q}$. This effect also depends crucially on the B/C on the outer wall ($\dot{q}_0$ and/or T_0), away from the fluid-solid interface.
3. In the fluid, $h = \bar{h} + h'$ can involve a fluctuating component to the HTC. Extreme examples include two-phase and evaporating/condensating phenomena such as slug flow, dry spot formation and de-wetting of film flows. Due to coupling effects this leads to the possibility that not only the fluctuating $\dot{q}'_w$, but also the mean $\bar{\dot{q}}_w$ exchanged with the solid depends on the fluctuating part of the process as described by h' and T' :

$$\begin{aligned}
 \dot{q}_w &= h \Delta T_f \\
 \Rightarrow \bar{\dot{q}}_w + \dot{q}'_w &= (\bar{h} + h') (\bar{\Delta T}_f + \Delta T'_f) \\
 \Rightarrow \bar{\dot{q}}_w &= \bar{h} \bar{\Delta T}_f + \overline{h' \Delta T'_f}.
 \end{aligned} \tag{III.4}$$

The appearance of the covariance term between the HTC and temperature fluctuations $\overline{h' \Delta T'_f}$ in Eq. III.4 is of critical significance (see also Section III.1.2 below). It implies that the presence of unsteadiness can result in a modification of $\bar{\dot{q}}_w$ in the unsteady fluid/heat flow case and a deviation from expectations based on the known steady heat transfer performance. For instance, a negative $\overline{h' \Delta T'_f}$ would result in a reduction in $\bar{\dot{q}}_w$ relative to the steady equivalent $\bar{h} \bar{\Delta T}_f$. This is important whether one is interested in enhancing heat transfer or reinforcing an adiabatic condition. Further, in spite of the linearity of the heat equation (assuming constant solid properties), it is of note that a non-zero h' in the fluid can give rise to a non-linear coupling between the mean and fluctuating temperature solutions in the solid, leading to interesting phenomena and effects. Clearly, the term

$\overline{h'\Delta T'_f}$ is a crucial parameter and deserves particular attention.

III.1.2 Heat transfer augmentation

Heat transfer enhancement, or ‘augmentation’, is a term that has been used widely in the literature to refer to the process of improving the performance of a heat transfer system or device by increasing its ability for heat exchange. In itself, the concept of augmentation is a relative one and requires the selection of a quantity for the characterisation of heat transfer, as well as a benchmark for comparison with respect to which an increase can be achieved. In steady heat transfer problems, augmentation is often quantified in terms of the heat flow rate, or the heat flux (heat flow rate per unit area), or the HTC (or equivalently the Nusselt number), while the reference benchmark is often a particular geometric configuration and flow condition. Passive augmentation (i.e. not requiring an energy input) can be achieved in the fluid domain by an improvement of the HTC, for example by the addition of surface structures (dimples, fins, pins, etc.) [130, 131] or the insertion of coils, helical or other structures [132, 133] into the flow channel.

On the other hand, in unsteady heat transfer where the number of problem variables is greater, there is also a wider range of options for the definition of the benchmark and for improvements to be made. In the present work we use the term augmentation to refer to an increase in time-averaged heat flux into the wall $\bar{q}_w$ relative to the heat flux experienced in an equivalent *steady* problem $\dot{q}_{st}$ exhibiting a constant HTC h_{st} equal to the time-averaged HTC in the unsteady problem $\bar{h}$, and, resulting from a constant temperature difference across the fluid ΔT_{st} equal to the time-averaged temperature difference in the unsteady problem $\overline{\Delta T_f}$. Mathematically, we define the ‘augmentation ratio’ A as,

$$A = \frac{\bar{q}_w}{\dot{q}_{st}} = \frac{\bar{h}\overline{\Delta T_f}}{h_{st}\Delta T_{st}} = 1 + \frac{\overline{h'\Delta T'_f}}{\bar{h}\overline{\Delta T_f}}, \quad (\text{III.5})$$

where the subscript ‘st’ denotes conditions in an equivalent steady problem. As can be seen from Eq. III.4, the augmentation ratio is linked directly to non-linearity in the conduction problem in the solid. It is important to note further, that in this definition:

1. A simple increase in the heat exchange area does not in itself result in augmentation;
2. Augmentation cannot exist unless there are finite fluctuations in h' and $\Delta T'_f$.

In this sense, it has been proven theoretically [22] that in cases where the flow is fully developed there can only be *negative* augmentation ($0 \leq A \leq 1$), reflecting a *reduction* in

1-D Solid Thermal Response

heat transfer due to unsteadiness. Still, this may not be the case in developing regimes, which are dealt with in the present study (see Case IV in Section V.4.2).

In summary, it is evident that the employment of a time-averaged $\bar{h}$, although still of primary relevance to unsteady engineering calculations, is plagued by a loss of information regarding the effects of the fluctuations of the various quantities on the $\bar{q}$ in the system. Unless these effects are taken into account, this approach will remain ineffective. Here, we will quantify the ‘degree of augmentation’ by evaluation of the term $A = \overline{h' \Delta T_f'} / \bar{h} \Delta T_f$.

III.2 1-D Solid Thermal Response

To understand conjugation and the thermal response of the “solid” domain must be studied. Considerable effort has gone into the problem of transient heat flow through solid bodies, both numerical and analytical. However much of this work has involved relatively simple boundary conditions with constant heat transfer coefficients (HTCs) [24, 134]. A better analytical approach is needed to tackle more complex boundary conditions, such as fluctuating heat transfer coefficients or when work is being done on the fluid side. This analytical solution will give greater insight into the heat transfer problems that a numerical method. Later on in Chapter IV and subsequently in Chapter VIII this model will be used to calculate the heat flux and temperature at the fluid–solid boundary in a fluid flow, using measurements of the temperature on the opposite surface of the solid.

In the simplest heat transfer scenario, a solid materials is in thermal contact with some heat transfer process. This process applies a thermal oscillation to one (the “inner”) solid boundary, and the other (“outer”) boundary is subject to either an isothermal or isoflux boundary condition (see Fig. III.1). That is either the temperature is fixed or the heat flux is constant, respectively. This could represent a large number of physical engineering applications, for example, a heat exchanger in an oscillating thermal cycle [5]. Such heat exchangers need careful design to avoid shuttle (unsteady) losses from the oscillating heat transfer.

In this section, the solution to 1-D thermal diffusion in a single continuous solid of finite thickness is obtained and used to define the thermal transfer function of the solid. Unsteady thermal diffusion is described by a thermal diffusion wave equation which is derived in Section III.2.3. This equation (Eq. III.30) gives the evolution of the temperature inside the solid. However, generally the relationship between temperature and heat flux at the solids boundaries is of most interest. This relationship is calculated in Section III.2.6 as a thermal impedance (thermal transfer function). The thermal

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

impedance is only dependant upon the “outer” boundary condition material properties and frequency and is independent of the thermal fluctuation applied to the “inner” solid boundary. These analytic models are used to simulate the thermal response of a 1-D solid as the frequency, material and thickness are changed. The addition of different “inner” boundary conditions, such as convection are investigated later in Chapter V and Chapter VI.

III.2.1 1-D heat equation in the solid

The governing equation for heat transfer in the solid domain, $0 \leq y \leq a$, is the 1-D unsteady heat diffusion (conduction) equation,

$$\frac{\partial T_s}{\partial t} = \alpha_s \frac{\partial^2 T_s}{\partial y^2} + \frac{\dot{Q}_{\text{gen}}}{\rho_s c_s}. \quad (\text{III.6})$$

Here, $\alpha_s = k_s / \rho_s c_s$ is the thermal diffusivity of the solid material, with k_s , ρ_s and c_s the thermal conductivity, density and specific heat capacity of the solid, respectively, all of which are considered constant. Also, $\dot{Q}_{\text{gen}}$ represents heat generation inside the solid, which could come from a number of sources such as chemical or nuclear reactions, radiation or electrical heating.

The present analysis is confined to situations in which $\dot{Q}_{\text{gen}}$ is spatially uniform and constant with time. It is worth noting that Eq. III.6 is linear and can be solved subject to two boundary conditions. In the presented work one of these boundary conditions arises from the thermal interaction with the fluid flow (inner B/C).

The temperature of the solid T_s and resulting heat flux $\dot{q}$ are time-varying. These unsteady quantities can be decomposed into mean and fluctuating components, e.g. for the instantaneous local heat flux in 1-D, $\dot{q}(y, t) = \bar{\dot{q}}(y) + \dot{q}'(y, t)$. As the PDE is linear, when substituting this decomposition into Eq. III.6, the governing equation in the solid can be split into steady state and fluctuating components,

$$\frac{\partial^2 \bar{T}_s}{\partial y^2} = -\frac{\dot{Q}_{\text{gen}}}{k_s}; \quad (\text{III.7})$$

$$\frac{\partial T'_s}{\partial t} = \alpha_s \frac{\partial^2 T'_s}{\partial y^2}. \quad (\text{III.8})$$

Solving the steady component (Eq. III.7) is trivial and results in the well-known linear

1-D Solid Thermal Response

temperature profile through the solid when $\dot{Q}_{\text{gen}} = 0$,

$$\bar{T}_s(y) = \frac{\bar{T}_w - \bar{T}_0}{a} y + \bar{T}_0; \quad (\text{III.9})$$

$$\bar{q}_w(y) = -\frac{k_s}{a} (\bar{T}_w - \bar{T}_0). \quad (\text{III.10})$$

Additionally a parabolic temperature profile and a linear heat flux profile are obtained in the more general case when $\dot{Q}_{\text{gen}}$ is a positive constant. From Eq. III.9 and Eq. III.10 it is apparent that k_s is the only important material property in steady problems and that a is the only important physical lengthscale. As the unsteady part of the problem is independent of the steady state component and because $\dot{Q}_{\text{gen}}$ is constant in terms of the fluctuating solution, the case where the heat is generated internally through $\dot{Q}_{\text{gen}}$ or the case where heat is applied heat flux through the outer wall boundary condition $\bar{q}_0$ are equivalent. This is useful as the cases with an adiabatic outer boundary condition $\bar{q}'_0 = 0$ and constant internal heat generation in the solid, such as in the foil experiment (Chapter IV) can be treated the same as an isoflux $\bar{q}'_0 = 0$ boundary condition with no internal heat generation.

The solution to the steady problem can become coupled to the fluctuating component due to the application of a non-linear fluid-side B/C on the solid (from Eq. III.4). In such a case the steady and fluctuating components of heat transfer in the solid can no longer be solved independently and is discussed in Chapter V.2.1.

III.2.2 Conjugation and the Fourier and Biot numbers

The diffusion equation (Eq. III.6) can be normalised by taking a characteristic frequency for the heat transfer problem ω_1 which in a periodic system would be the first harmonic, and a length scale a the material thickness. That is making the substitutions;

$$t^* = t\omega_1, \quad (\text{III.11})$$

$$y^* = \frac{y}{a}, \quad (\text{III.12})$$

gives,

$$\frac{\partial T}{\partial t^*} = \text{Fo} \frac{\partial T}{\partial y^{*2}}. \quad (\text{III.13})$$

Here Fo is the Fourier number,

$$\text{Fo} = \frac{\alpha}{\omega_1 a^2}. \quad (\text{III.14})$$

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

The Fourier number is important as it indicates the relative importance of solving the diffusion equation in the solid [135].

Large Fourier number approximations

If the Fourier number is large, that is $Fo \gg 1$, then, for a finite fluctuation in the solid's temperature, the second derivative of the temperature profile in the solid must be small,

$$\frac{\partial^2 T}{\partial y^{*2}} = 0. \quad (\text{III.15})$$

This can be seen as the thermal conduction being more important than the thermal capacity of the solid. The temperature profiles in the solid are therefore straight. This means that for the case of an isothermal boundary condition, $T'_0 = 0$, on the “outer” wall the heat flux into the solid can be approximated as,

$$\dot{q}_w = -\kappa \frac{T_w - T_0}{a}. \quad (\text{III.16})$$

Here $\dot{q}_w$ and T_w are the heat flux and temperature on the inner wall respectively, and T_0 is the temperature on the isothermal “outer” boundary. Similarly for an isoflux boundary condition, $\dot{q}'_0 = 0$, the temperature should be uniform across the solid domain should be the same and so the heat flux into the solid can be approximated by,

$$\dot{q}_w = \rho c_v a \frac{dT_w}{dt}. \quad (\text{III.17})$$

In both of these cases the full solution in the solid is not required, and the approximation can be used as the boundary condition to the heat transfer problem.

Small Fourier number approximations

Conversely, when the Fourier number is small, that is $Fo \ll 1$, then the second derivative of the temperature profile in the solid can become large. This means that the decay of the thermal fluctuation into the solid is quick, and the envelope of the temperature fluctuation decays rapidly into the solid. Effectively the problem becomes semi-infinite in for both isoflux and isothermal boundary conditions. That is the outer boundary condition is decoupled from the solution and so both boundary conditions are equivalent. If δ is the thermal penetration length scale of this decaying envelope then it

1-D Solid Thermal Response

can be seen that the heat flux scales with,

$$\dot{q}_w \sim \kappa \frac{T_0 - T_w}{\delta} \sim \rho c_v \frac{\delta}{2} \frac{dT_w}{dt}. \quad (\text{III.18})$$

That is the heat flux at the wall $\dot{q}_w$ should be proportional to the temperature gradient which scales with $(T_0 - T_w)/\delta$. Also the heat flux at the wall $\dot{q}_w$ will scale with the amount of heat energy which passes across the inner boundary it to change the temperature of the region (δ) containing the fluctuation. It should be expected however for there to be a significant phase shift between the heat flux and temperature difference (See Section III.2.5).

Conjugation

When considering both the fluid and solid domains for the case of large Fourier number with an isothermal boundary condition the following balance is obtained,

$$h(T_f - T_w) = \frac{\kappa}{a}(T_w - T_0). \quad (\text{III.19})$$

By rearranging the relative temperature difference across the two domains is found,

$$\frac{T_w - T_0}{T_f - T_w} = \frac{ha}{\kappa} = \text{Bi}, \quad (\text{III.20})$$

which is the Biot number. At very high Biot number the temperature difference across the fluid is negligible, and so the resulting heat flux can be calculated nearly by considering the solid domain. Similarly at very low Biot numbers, the solid domain temperature is effectively the same as the external temperature T_0 and the heat flux can be calculated by considering the fluid domain only. It can be seen that for the cases of very high, or very low Biot numbers one of the domains can be neglected from the solution, that is the problem becomes non-conjugate [135].

Now consider the case of small Fourier numbers with both isothermal and isoflux outer boundary conditions (remember that small Fourier number decouples the outer boundary condition). Using Eq. III.18 the balance becomes,

$$\frac{T_w - T_0}{T_f - T_w} \sim \frac{h\delta}{\kappa} = \text{Bu}, \quad (\text{III.21})$$

where Bu is the unsteady Biot number. As before the solution becomes non-conjugate for small and large Bu.

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

Finally, the case where there is a large Fourier number and an isoflux outer boundary condition. Using Eq. III.17 to obtain,

$$h(T_f - T_0) \sim \rho c_v a \omega_1 \frac{dT_w}{dt^*}, \quad (\text{III.22})$$

where t^* is the normalised time as before. Rearranging gives,

$$\frac{dT_w}{dt^*} \frac{1}{T_f - T_0} \sim \frac{ha}{\kappa} \frac{\alpha}{a^2 \omega} = \text{Bi Fo}. \quad (\text{III.23})$$

This shows that when the product Bi Fo is large the temperature difference across the fluid domain is insignificant, and the fluctuation in temperature at the wall is most important. Conversely when Bi Fo is small the relative fluctuation at the wall is insignificant, and the heat transfer in the fluid is most important.

In all these cases we see that for there to be conjugate effects the Biot number and Fourier number should be neither large nor small. That is for conjugation to occur;

$$Bi \sim O(1), \quad (\text{III.24})$$

$$Fo \sim O(1). \quad (\text{III.25})$$

III.2.3 Fourier method

For a 1-D solid of finite thickness the solution to Eq. III.6 subject to the unsteady boundary conditions at T_0 and T_w can be split into the steady and unsteady state solution components. The steady state solution has all ready been stated. Ignoring the initial transient, the general solution of Eq. III.8 in terms of the unsteady components T' can be found by separation of variables. First suppose that the unsteady temperature can be written as the product of two independent functions,

$$T' = \Theta(t)Y(y). \quad (\text{III.26})$$

Substituting this into Eq. III.8 and rearranging gives,

$$\frac{\dot{\Theta}(t)}{\Theta(t)} = \alpha \frac{\ddot{Y}(y)}{Y(y)}. \quad (\text{III.27})$$

1-D Solid Thermal Response

Since the right hand side is independent of the left hand side both sides must be equal to a constant λ . The solution for Θ and Y is thus ;

$$\Theta(t) = Ae^{\lambda t}, \quad (\text{III.28})$$

$$Y(y) = Be^{\sqrt{\frac{\lambda}{\alpha}}y} + Ce^{-\sqrt{\frac{\lambda}{\alpha}}y}. \quad (\text{III.29})$$

Therefor for a given fluctuation in T' the solution to the diffusion equation is given as a Fourier series,

$$T'_s(y, t) = \Re \sum_{n=-\infty}^{\infty} \left[e^{i\omega_n t} \left\{ T_{\text{fw},n} e^{-(1+i)\frac{y}{\delta_{s,n}}} + T_{\text{bw},n} e^{(1+i)\frac{y}{\delta_{s,n}}} \right\} \right]; \quad n \neq 0, \quad (\text{III.30})$$

where $\delta_{s,n} = \sqrt{2\alpha_s/\omega_n}$ is the thermal diffusion lengthscale. Equation (III.30) describes two exponentially decaying or growing waves, $T_{\text{fw},n}$ travelling in the “forward” and $T_{\text{bw},n}$ “backward” direction through the material with temporal frequency ω_n .

The heat flux can be found from Fourier’s law $\dot{q} = -\kappa \frac{\partial T}{\partial y}$. Substituting into Eq. III.30 results in an expression for the unsteady heat flux at any point in the material,

$$\dot{q}'(t, y) = \Re \sum_{n=0}^{\infty} \left[(1+i) \frac{\kappa}{\delta_n} e^{i\omega_n t} \left\{ -T_{\text{fw},n} e^{(1+i)\frac{y}{\delta_{s,n}}} + T_{\text{bw},n} e^{-(1+i)\frac{y}{\delta_{s,n}}} \right\} \right], \quad (\text{III.31})$$

and depends on the imposed B/Cs which fix the values of the coefficients $T_{\text{fw},n}$ and $T_{\text{bw},n}$. Also here, $\delta_{s,n} = \sqrt{2\alpha_s/\omega_n}$ is a thermal diffusion lengthscale and $\omega_n = 2\pi f_n$ is the n^{th} angular frequency component of the fluctuating response.

Figure III.1 shows schematically how the thermal oscillation propagates through the finite solid. There are two exponentially decaying envelopes of the thermal “wave”. Often the conduction problem in a semi infinite solid is considered and so the backward travelling wave is neglected, as its exponent is seen to be growing in the positive y direction. This is valid for cases when the material is “thick”, however the backward travelling “wave” is the result of reflection from the second boundary and must be included in the solution for conduction with finite thickness. This is used to derive a thermal impedance model for unsteady heat conduction in a solid.

Since all of the terms are summed linearly over the Fourier domain it is therefore possible to drop the summation for clarity, and only consider the solution at particular frequencies. Also, to make notation easy to read the $e^{i\omega t}$ term can also be left out, and

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

only the coefficients of the Fourier series considered. It should be remembered that these temperature and heat flux values are the coefficients to a Fourier series and encode the magnitude and phase of the time domain oscillation.

III.2.4 Boundary conditions

Equation III.30 and Eq. III.31 give the general form of the solution to the diffusion equation. However, the two unknown coefficients of the thermal waves must be determined by imposing boundary conditions on the solid domain. This can be done by solving for the set of simultaneous equations for various boundary conditions.

For example, if the temperature fluctuations on either surface are known the wave coefficients $T_c = [T_f T_b]^T$ can be found from,

$$\underline{T} = \begin{bmatrix} T_w \\ T_0 \end{bmatrix} = M_T \begin{bmatrix} T_f \\ T_b \end{bmatrix} \quad (\text{III.32})$$

and, similarly, if the heat fluxes are known,

$$\underline{\bar{q}} = \begin{bmatrix} \dot{q}_w \\ \dot{q}_0 \end{bmatrix} = M_q \begin{bmatrix} T_f \\ T_b \end{bmatrix}, \quad (\text{III.33})$$

where the equations for temperature or heat flux at each surface of the solid have been described by the matrices in Eq. III.34 and Eq. III.35 ;

$$M_T = \begin{bmatrix} M_{T(0)} \\ M_{T(a)} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ e^{(1+i)\frac{a}{\delta}} & e^{(1+i)\frac{a}{\delta}} \end{bmatrix}, \quad (\text{III.34})$$

$$M_q = \begin{bmatrix} M_{q(0)} \\ M_{q(a)} \end{bmatrix} = \frac{\kappa(1+i)}{\delta} \begin{bmatrix} -1 & 1 \\ -e^{(1+i)\frac{a}{\delta}} & e^{-(1+i)\frac{a}{\delta}} \end{bmatrix}. \quad (\text{III.35})$$

The solutions to these equations are dependent on material and geometric properties. By examining the governing equations certain dimensionless parameters of importance are revealed. Firstly, there is the ratio of thermal diffusion length to material thickness $\frac{a}{\delta}$. Secondly, there is the ratio of $\frac{\kappa}{\delta}$ which relates the heat flux to the thermal oscillation. This ratio can usefully be split up into separate material and geometric/problem properties. $\frac{\kappa}{\delta} = \epsilon\sqrt{\omega}$ where $\epsilon = \sqrt{\kappa\rho c_v}$ is the thermal effusivity.

Notice how the dimensionless lengthscale $\frac{a}{\delta}$ and the Fourier number from Section III.2.2

1-D Solid Thermal Response

are related by,

$$\text{Fo} = \frac{\alpha}{\omega_1 a^2} = 2 \left(\frac{a}{\delta} \right)^{-2}. \quad (\text{III.36})$$

Thus it should be expected that the response between $\dot{q}_w$ and T_w will tend to those derived in Section III.2.2 for small and large $\frac{a}{\delta}$

III.2.5 Thermal response

Using dimensional analysis it can be seen that of the seven parameters $\{T_w, T_0, \dot{q}_w, \dot{q}_0, \epsilon, \frac{a}{\delta}, \omega\}$, which describe the conduction problem, there are four dimensions: temperature, mass, length and time $[\theta, M, L, T]$. Therefore, three dimensionless parameters can completely describe the thermal conduction through the solid, which could be $\epsilon\sqrt{\omega}\frac{T_w}{q_w}$, $\epsilon\sqrt{\omega}\frac{T_0}{q_0}$, $\frac{a}{\delta}$. The range of solutions is unbounded as there are many possible permutations for applying flux or thermal boundary conditions, or combinations of the two. However a selection of solutions are investigated for a few interesting cases. In particular, the cases for measured, or induced thermal oscillation at T_w , and either an adiabatic $q_0 = 0$ or isothermal $T_0 = 0$ condition on the other side of the solid.

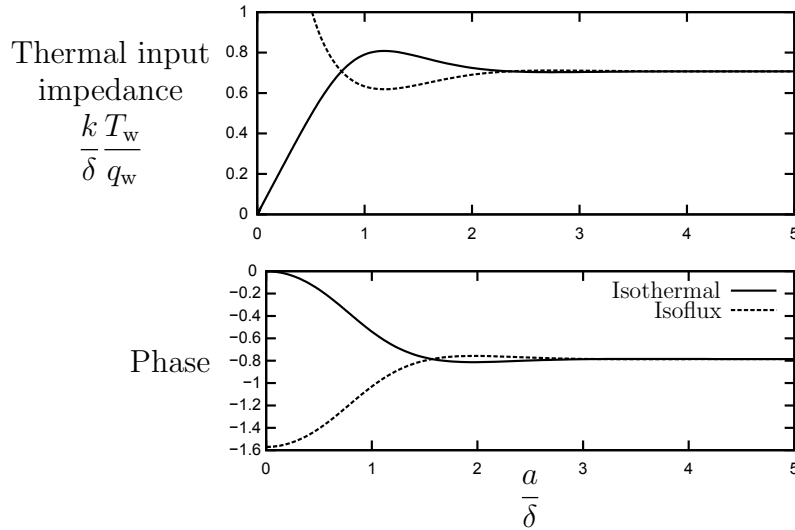


Figure III.2: Thermal input (at inner boundary) impedance with different output (outer) boundary conditions.

The graphs in Fig. III.2 and Fig. III.3 show the magnitude and phase of the thermal impedance across the material defined as $Z = \frac{T_w - T_0}{q_w}$ and the thermal input impedance $Z_{in} = \frac{T_w}{q_w}$. It is interesting to note that Fig. III.3 the thermal impedance in the adiabatic

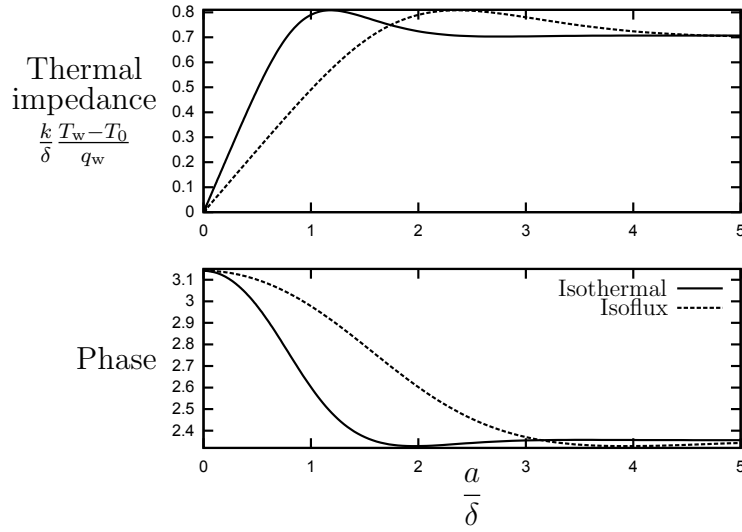


Figure III.3: Thermal impedance across the material.

case is the same as for the isothermal case, but stretched in $\frac{a}{\delta}$ by a factor of 2.

The figures show dimensionless impedance, normalised by the solids thickness and thermal conductivity, for constant material properties ϵ , and frequency and show how the impedance would vary with changing thickness of the solid. This representation was chosen because material properties and oscillation frequency tend to be set by the circumstance of the heat transfer, and so the effect of thickness needs to be known. It is interesting to note that the material has maximum thermal impedance when the thickness $\frac{a}{\delta}$ is between 1 and 2, depending on the boundary condition imposed on the outer face of the solid. The heat flux is always lagging the thermal oscillation, but the phase and impedance tends to a constant when the thickness becomes very large, the solid effectively becomes semi-infinite, and the outer face boundary condition is decoupled from the problem.

It should also be noted that when ω tends to 0 and $\delta \rightarrow \infty$, Fig. III.3 could be misinterpreted as the impedance tending to 0. The trend shown is essentially for $\frac{\kappa}{\delta} \frac{T_w - T_0}{q_w}$ to tend towards $\frac{0}{0}$, and so the impedance really tends to $\frac{\kappa}{a}$ per unit area, as expected.

III.2.6 Thermal impedance

For a continuous solid the thermal conduction in the solid is analogous to the electrical conduction through a resistive wire with electrical capacity distributed along its length and no inductance. Table III.1 shows a comparison between the governing variables and equations. Using this model the material can be approximated by a network of

1-D Solid Thermal Response

infinitesimally small impedance, see Fig. III.4, resistors along the top linking capacitors linked to “ground”. The network of impedance can be converted to the equivalent circuit shown in Fig. III.5 consisting of 3 thermal impedance. By inspection it can be seen that $Z_1 = Z_3$ due to the symmetry of the material.

Table III.1: Comparison between thermal and electrical conduction.

	Thermal conduction	Electrical conduction
Potential	T	V
Current	q	I
Impedance	$\Delta T = Z_{th}q$	$\Delta V = Z_e I$
Conductivity per unit length	κ	$\frac{1}{\rho_r}$
Capacitance per unit length	ρc_v	C
Resistance equation	$q = -\kappa \frac{dT}{dy}$	$I = -\frac{1}{\rho_r} \frac{dV}{dy}$
Capacitance equation	$\frac{dq}{dy} = \rho c_v \frac{dT}{dt}$	$\frac{dI}{dy} = C \frac{dV}{dt}$
Differential equation	$\rho c_v \frac{dT}{dt} = \kappa \frac{d^2 T}{dy^2}$	$C \frac{dV}{dt} = \frac{1}{\rho_r} \frac{d^2 V}{dy^2}$

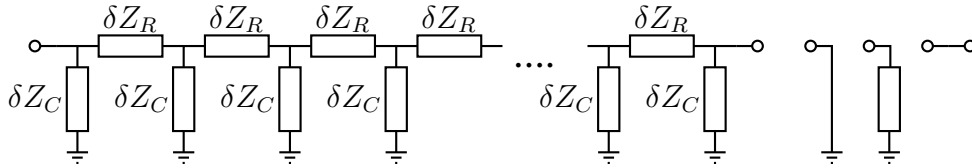


Figure III.4: Infinitesimal resistor capacitor, impedance network.

The impedances Z_1 and Z_2 in Fig. III.5 can be found by comparison to the solution to the simultaneous equations III.32 and III.33. By substituting $\begin{bmatrix} T_f & T_b \end{bmatrix}^T = M_T^{-1} \underline{T}$ into Eq. III.33 we get,

$$\underline{q} = M_q \begin{bmatrix} T_f \\ T_b \end{bmatrix} = M_q M_T^{-1} \underline{T}. \quad (\text{III.37})$$

This represents the thermal impedance of the solid as a matrix $Z = M_T M_q^{-1}$.

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

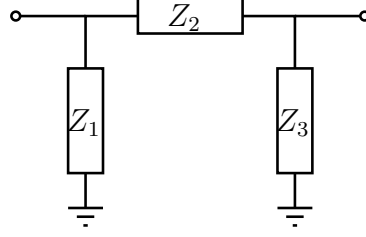


Figure III.5: Equivalent contraction of network in Fig. III.4 to representation by 3 impedances.

by looking at the circuit in Fig. III.5 it can be deduced that ;

$$q_w = \frac{T_w}{Z_1} + \frac{1}{Z_2}(T_w - T_0), \quad (\text{III.38})$$

$$q_0 = -\frac{T_0}{Z_3} + \frac{1}{Z_2}(T_w - T_0). \quad (\text{III.39})$$

This can be written as the matrix equation,

$$\mathbf{q} = \begin{bmatrix} \frac{1}{Z_1} + \frac{1}{Z_2} & -\frac{1}{Z_2} \\ \frac{1}{Z_2} & -\frac{1}{Z_2} - \frac{1}{Z_3} \end{bmatrix} \underline{\mathbf{T}}, \quad (\text{III.40})$$

which is equivalent to Eq. III.37. The impedance's Z_1 and Z_2 can now be found by comparison;

$$Z_1 = Z_3 = \frac{\delta}{\kappa(1+i)} \frac{e^{(1+i)\frac{a}{\delta}} - e^{-(1+i)\frac{a}{\delta}}}{e^{(1+i)\frac{a}{\delta}} + e^{-(1+i)\frac{a}{\delta}} - 2}, \quad (\text{III.41})$$

$$Z_2 = \frac{\delta}{\kappa(1+i)} \frac{1}{2} \{e^{(1+i)\frac{a}{\delta}} - e^{-(1+i)\frac{a}{\delta}}\}. \quad (\text{III.42})$$

To check the results are realistic, we consider the limits as $\frac{a}{\delta}$ tends to 0 and ∞ ;

$$\lim_{\frac{a}{\delta} \rightarrow 0} Z_1 \rightarrow \infty, \quad (\text{III.43})$$

$$\lim_{\frac{a}{\delta} \rightarrow \infty} Z_1 \rightarrow \frac{\delta}{\kappa(1+i)}, \quad (\text{III.44})$$

$$\lim_{\frac{a}{\delta} \rightarrow 0} Z_2 \rightarrow 0, \quad (\text{III.45})$$

$$\lim_{\frac{a}{\delta} \rightarrow \infty} Z_2 \rightarrow \infty. \quad (\text{III.46})$$

1-D Solid Thermal Response

This is as expected as when $\frac{a}{\delta} \rightarrow \infty$ the solid can be thought of as very thick, and so Z_2 must get very large with both sides of the solid becoming effectively decoupled. Also, Z_1 will tend to a constant lagging impedance due to the thermal capacitance and resistance of the material. Conversely when $\frac{a}{\delta} \rightarrow 0$, Z_2 must tend to zero as both sides of the solid are effectively one, and $Z_1 \rightarrow \infty$ as there is no thermal mass associated with the infinitely thin solid. The phase of Z_2 will increase with $\frac{a}{\delta}$ as more wavelengths can fit into the width of the solid.

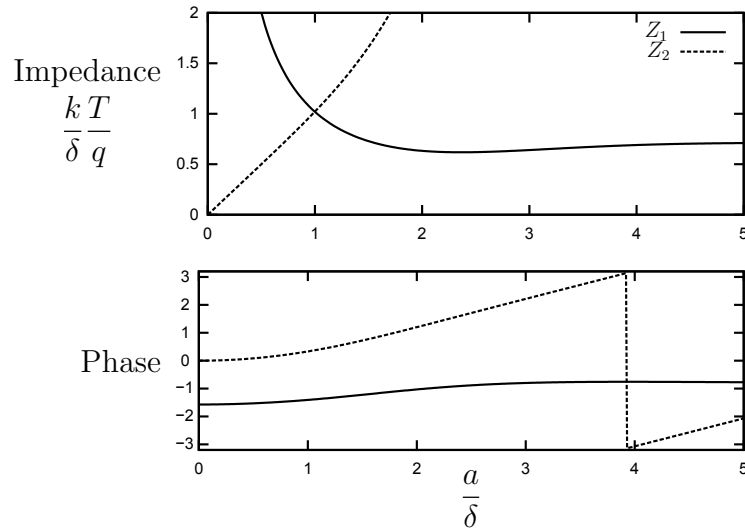


Figure III.6: Graph showing the internal thermal impedance Z_1 and Z_2 as function of $\frac{a}{\delta}$.

The bode plot in Fig. III.6 shows how the solid impedance varies with $\frac{a}{\delta}$. Impedance Z_2 increases progressive with thickness, growing exponentially. Z_1 is vertically asymptotic to $\frac{a}{\delta} = 0$ and the value $Z = \frac{\delta}{\kappa(1+i)}$ but has a minimum magnitude between $\frac{a}{\delta}$ equals 2 and 3.

Using this model any oscillating thermal boundary condition can readily be applied to the 1-D solid material, as it is a matter of applying the thermal circuit analogy. For example, the thermal input impedance for both the adiabatic and isothermal boundary condition cases can be found in Table III.2.

Table III.2 also shows how the impedance would change when other thermal impedance are attached to the outer face of the solid. The isothermal and adiabatic cases are just extremes of placing zero or infinite impedance at that boundary condition. Any impedance Z_{out} placed on the output effectively modulates Z_3 by $Z'_3 = \frac{Z_3 Z_{out}}{Z_3 + Z_{out}}$.

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

Table III.2: Input and total impedance of impedance network in Fig. III.5.

	Isothermal, $T_0 = 0$	Adiabatic, $q_0 = 0$
$Z_{in} = \frac{T_w}{q_w}$	$\frac{Z_1 Z_2}{Z_1 + Z_2}$	$\frac{Z_1(Z_2 + Z_3)}{Z_1 + Z_2 + Z_3}$
$Z = \frac{T_w - T_0}{q_w}$	$\frac{Z_1 Z_2}{Z_1 + Z_2}$	$\frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$

III.3 Laplace Method for the Solution of the IBVP

The Fourier method provides an approach for solving periodic disturbances in the heat transfer through a solid, and gives a good insight into how the system behaves. However this periodic approach is inadequate in dealing with transient heat transfer, and involves a cumbersome matrix notation. Greater insight into the nature of the unsteady heat transfer in a solid can be obtained by considering the problem in the Laplace domain. By using the Laplace method the transient thermal response of the solid can be investigated, as apposed to the purely periodic response from the Fourier method.

Thermal conduction in a homogeneous solid across a finite 1-D domain is a classic example of an Initial bound value problem (IBVP). To solve this IBVP the thermal conduction in a isotropic solid material must be considered. The heat flux conducted through a solid material is described by Fourier's law,

$$\dot{q}(y, t) = -\kappa_s \frac{\partial T(y, t)}{\partial y}. \quad (\text{III.47})$$

In the problem considered here the thermal conductivity, κ_s , which is a property of the solid material, is isotropic across the spatial domain and is independent of time and temperature. Fourier's law can be combined with the energy equation in the solid to produce a partial differential equation (PDE) describing the evolution of temperature across the solid domain,

$$\frac{\partial T(y, t)}{\partial t} = \alpha \frac{\partial^2 T(y, t)}{\partial y^2}. \quad (\text{III.48})$$

Taking the Laplace transform $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$ of the PDE in Eq. III.48 gives a

Laplace Method for the Solution of the IBVP

representation of the IBVP in the Laplace domain,

$$s\hat{T}(y, s) - T(y, 0) = \alpha \frac{\partial^2 \hat{T}(y, s)}{\partial y^2}. \quad (\text{III.49})$$

III.3.1 Uniqueness of the solution

Before proceeding further in the solution of the heat equation Eq. III.49 using the Laplace transform method, information about the initial boundary condition $T(y, 0)$ must be deduced. First, in order for this to be achieved, the uniqueness of the solution to the IBVP Eq. III.48 must be investigated. This is important for two reasons. Firstly uniqueness is a reassurance that a only one solution exists for a particular set of boundary conditions. Secondly, if the solution to the IBVP, Eq. III.48, is unique and because the IBVP is linear the solution can be found as the summation of solutions to different boundary conditions. For instance it is convenient to consider the initial value problem;

$$T_s(y, 0) = g(y), \quad (\text{III.50})$$

$$T_s(0, t) = 0, \quad (\text{III.51})$$

$$T_s(a, t) = 0, \quad (\text{III.52})$$

and the bound value problem;

$$T_s(y, 0) = 0, \quad (\text{III.53})$$

$$T_s(0, t) = \alpha(t), \quad (\text{III.54})$$

$$T_s(a, t) = \beta(t), \quad (\text{III.55})$$

separately. By separating the initial value from bound value problem an initial temperature boundary condition of $T(y, 0) = 0$ can be assumed when solving the bound value problem using the Laplace method, which greatly simplifies results.

Following a method outlined in Elżanowski [136, Section 2.3], the uniqueness of the solution to the IBVP can be deduced with a proof by contradiction. First it is assumed that there are multiple valid solutions to the IBVP and the imposed boundary conditions, and then through a number of manipulations it will be shown that this assumption can not be true.

Consider the heat equation III.48 which is valid in the solid domain $0 < y < a$ and

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

for time $0 < t < \infty$ and is subject to the following boundary conditions:

$$T(y, 0) = g(y), \quad (\text{III.56})$$

$$T(0, t) = \alpha(t), \quad (\text{III.57})$$

$$T(a, t) = \beta(t). \quad (\text{III.58})$$

It can be assumed that two solutions $T = \theta_1(y, t)$ and $T = \theta_2(y, t)$ exist. As Eq. III.48 is linear we can combine the proposed solutions to get a third solution to the heat equation,

$$w(y, t) = \theta_1 - \theta_2. \quad (\text{III.59})$$

Since the solutions θ_1 and θ_2 were subject to the same boundary conditions the boundary conditions for the new solution w are;

$$w(y, 0) = 0, \quad (\text{III.60})$$

$$w(0, t) = 0, \quad (\text{III.61})$$

$$w(a, t) = 0.. \quad (\text{III.62})$$

Multiplying the heat equation for w , Eq. III.48, by w and integrating with respect to y over the domain $0 < y < a$ results in,

$$\int_0^a w \frac{\partial w}{\partial y} dy = \int_0^a w \frac{\partial^2 w}{\partial y^2} dy. \quad (\text{III.63})$$

Then with integration by parts, and noticing that $\frac{\partial w^2}{\partial t} = 2w \frac{\partial w}{\partial t}$, Eq. III.63 becomes,

$$\frac{1}{2} \int_0^a \frac{\partial w^2}{\partial t} dy = \left[w \frac{\partial w}{\partial y} \right]_0^a - \int_0^a \left(\frac{\partial w}{\partial y} \right)^2 dy. \quad (\text{III.64})$$

Note that, as a result of the boundary conditions imposed on w , Eq. III.61 and III.61, the first term on the right hand side evaluates to $\left[w \frac{\partial w}{\partial y} \right]_0^a = 0$, and because the square of any real valued number is positive Eq. III.64 results in the inequality,

$$\frac{1}{2} \int_0^a \frac{\partial w^2}{\partial t} dy = - \int_0^a \left(\frac{\partial w}{\partial y} \right)^2 dy \leq 0. \quad (\text{III.65})$$

However by integrating again but this time with respect to t over the interval $[0, t]$ gives

Laplace Method for the Solution of the IBVP

rise to the following contradiction,

$$\left[\frac{1}{2} \int_0^a w^2 dy \right]_0^t = - \int_0^t \int_0^a \left(\frac{\partial w}{\partial y} \right)^2 dy dt \leq 0. \quad (\text{III.66})$$

This is a contradiction as right hand side of the expression evaluates to $\frac{1}{2} \int_0^a w(y, t)^2 dy$ due to the boundary condition Eq. III.60 and so is always greater than or equal to zero,

$$\left[\frac{1}{2} \int_0^a w^2 dy \right]_0^t \geq 0. \quad (\text{III.67})$$

Therefor only one solution exists:

$$w = 0, \quad (\text{III.68})$$

$$\theta_1 = \theta_2. \quad (\text{III.69})$$

Now that it is confirmed that solutions to the IBVP are unique and considering that the PDE is linear we can proceed to solve for the B/Cs

III.3.2 General solution

For the case when the initial temperature across the solid domain is uniform,

$$T(y, 0) = 0, \quad (\text{III.70})$$

the Laplace representation of the IBVP problem in Eq. III.49 becomes a linear homogeneous ordinary differential equation (ODE) of the Laplace representation of the temperature $\hat{T}$ across the spatial domain y . Substituting a generic exponential function,

$$\hat{T}(y, s) = Ae^{\lambda y}, \quad (\text{III.71})$$

into Eq. III.49 results in a relation for the coefficients of the exponential,

$$\lambda = \pm \sqrt{\frac{s}{\alpha_f}}. \quad (\text{III.72})$$

Here α_f is the thermal diffusivity of the solid and s is the Laplace parameter in the complex s domain.

As there are two coefficients for the exponential the complimentary function for the

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

temperature profile in the solid is,

$$\hat{T}(y, s) = Ae^{py} + Be^{-py}. \quad (\text{III.73})$$

Then, substituting Eq. III.73 into Fourier's law, which is shown in Eq. III.47, the distribution of heat flux through the solid is described as,

$$\hat{q}(y, s) = -\kappa p (Ae^{py} - Be^{-py}), \quad (\text{III.74})$$

where $p = \sqrt{\frac{s}{\alpha}}$ is the Laplace parameter in the p domain.

III.3.3 Boundary conditions

As before in the Fourier method (see section III.2.3) the response will be considered for two archetypal Boundary Conditions (B/C) on the “back”, $y = 0$, surface of the solid domain. These B/Cs being considered are the isothermal, constant temperature, and isoflux, constant heat flux, boundary conditions. Again it is worth noting that, as the heat transfer problem in the solid is linear, the constant heat flux B/C is equivalent to an adiabatic B/C with internal heat generation in the solid domain, with the addition of a correction to account for the different steady state solutions.

Figure III.7 shows a schematic representation of the location of these B/Cs on the solid domain. In Fig. III.7 $T(t, 0)$ and $\dot{q}(t, 0)$ are the temperature and heat flux on the “back” surface of the solid domain respectively. As the “back” surface of the solid domain is governed by one of these applied boundary conditions, it will respond in a passive manor to the fluctuations in temperature $T(t, a)$ or heat flux $\dot{q}(t, a)$ on the “inner” surface of the solid, $y = a$. These fluctuations in temperature $T(t, a)$ or heat flux $\dot{q}(t, a)$ could be due to some thermal or thermodynamic proses occurring in the fluid in contact with the “inner” surface, such as fluctuating heat transfer coefficient, $h(t)$, or fluctuating fluid temperature $T_f(t)$.

To find the two coefficients for the general solution in Eq. III.74 a total of 2 B/C equations must be known. These boundary conditions can be any two of the following

Laplace Method for the Solution of the IBVP

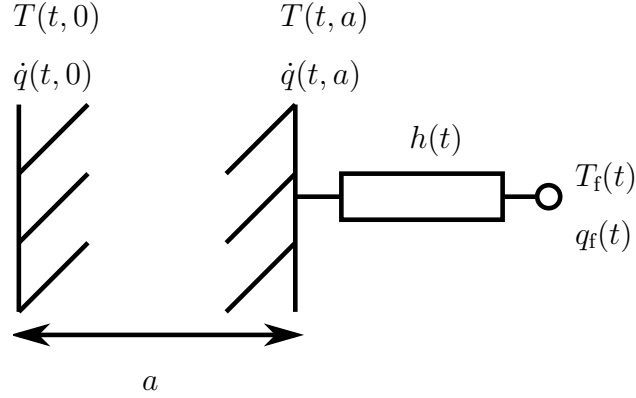


Figure III.7: Diagram of the Boundary Conditions.

four equations in the Laplace domain;

$$\hat{q}(0, s) = -\kappa p(A - B), \quad (\text{III.75})$$

$$\hat{q}(a, s) = -\kappa p(Ae^{pa} - Be^{-pa}), \quad (\text{III.76})$$

$$\hat{T}(0, s) = A + B, \quad (\text{III.77})$$

$$\hat{T}(a, s) = Ae^{pa} + Be^{-pa}. \quad (\text{III.78})$$

Alternatively a boundary condition equation involving the heat transfer coefficient and fluid temperature can be used,

$$\hat{q}_w(s) = \hat{q}(a, s) = h(\hat{T}_f(s) - \hat{T}(a, s)). \quad (\text{III.79})$$

In the case where an isothermal boundary condition is imposed on the “back” solid surface, opposite from the fluid,

$$\hat{T}(0, s) = 0, \quad (\text{III.80})$$

then the coefficients of the general solution must satisfy,

$$A = -B. \quad (\text{III.81})$$

Conversely if an iso-fluxual condition is imposed on the outer solid surface,

$$\hat{q}(0, s) = 0, \quad (\text{III.82})$$

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

then the coefficients must take the form,

$$A = B. \quad (\text{III.83})$$

The application of the isothermal or isoflux boundary conditions on the “back” surface allows the relationship between the coefficients of the general solution, shown in Eq. III.81 and Eq. III.83, to be deduced. To go further and find the value of the coefficients, and hence obtain the solution for the temperature profile across the solid, one of the other B/Cs must be applied on the “inner” surface. However, as shall be illustrated shortly in Section III.3.4, it is not necessary to solve in this manner every time a different variation in the B/C is considered. Instead a transfer function relating the solution output to an applied input can be found. Further more this allows impulse responses to be deduced, which allows the output to be calculated from a simple convolution with the input.

III.3.4 Transfer functions

As noted before when considering the Fourier method for the solution to the heat equation, the temperature profile in the solid is actually of little interest in the heat transfer problem (see section III.2.3). What is of much greater importance to the heat transfer problem is the temperature and heat flux at the boundaries of the solid domain. If transfer functions relating these quantities at the solids interfaces can be found the heat equation inside the solid no longer needs to be solved and the solids reaction to the imposed thermal fluctuation can be reduced to an impulse or step response.

The impedance relating the heat flux seen at the interface with the fluid side of the system, to the temperature of the same interface can be defined as

$$\hat{Z}(s) = \frac{\hat{T}(a, s)}{\hat{q}(a, s)}. \quad (\text{III.84})$$

By substituting Eq. III.73 and Eq. III.74 for the isothermal case, $A = -B$, into Eq. III.84 gives

$$\hat{Z}(s) = \frac{1}{p\kappa} \tanh(pa). \quad (\text{III.85})$$

Similarly for the iso-fluxual case, $A = B$ we get

$$\hat{Z}(s) = \frac{1}{p\kappa} \coth(pa). \quad (\text{III.86})$$

Laplace Method for the Solution of the IBVP

Table III.3: Table of transfer functions for 1-D thermal conduction. Here $p = \sqrt{s/\alpha}$ and two imposed boundary conditions are considered; isoflux $\hat{q}(s, 0) = 0$ and isothermal $\hat{T}(s, 0) = 0$.

		Outer Boundary Condition	
		Isoflux $\hat{q}(s, 0) = 0$	Isothermal $\hat{T}(s, 0) = 0$
Transfer functions		$\frac{\hat{q}(s, a)}{\hat{T}(s, a)} = p\kappa \tanh\{pa\}$	$\frac{\hat{q}(s, a)}{\hat{T}(s, a)} = p\kappa \coth\{pa\}$
		$\frac{\hat{q}(s, a)}{\hat{T}(s, 0)} = p\kappa \sinh\{pa\}$	$\frac{\hat{q}(s, a)}{\hat{q}(s, 0)} = \cosh\{pa\}$
		$\frac{\hat{T}(s, a)}{\hat{T}(s, 0)} = \cosh\{pa\}$	$\frac{\hat{T}(s, a)}{\hat{q}(s, 0)} = \frac{1}{p\kappa} \sinh\{pa\}$

Similar transfer functions can be derived to relate each permutation of temperature and heat flux at either side of the solid for the isoflux and isothermal wall boundary conditions and are listed in table III.3. As there are two boundaries, and a total of two properties of interest, heat flux $\dot{q}$ and temperature T , and two applied boundary conditions, there are a total of 8 possible transfer functions. However the trivial cases, where in the imposed boundary condition is included in the transfer relation, can be omitted resulting in a total of only 6 transfer functions.

Comparison to the Periodic Response

As before

$$p = \sqrt{\frac{s}{\alpha}}. \quad (\text{III.87})$$

Note that when looking the response to a periodic heat transfer, as before, then $s = i\omega$ which gives

$$\sqrt{\frac{s}{\alpha}} = p = (1 + i) \frac{1}{\delta} \quad (\text{III.88})$$

$$\frac{T_{a,s,n}}{T_{0,s,n}} = \cosh \left\{ (1 + i) \frac{a}{\delta_n} \right\}, \quad (\text{III.89})$$

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

and the heat flux response at the fluid solid boundary is

$$\frac{\dot{q}_{a,s,n}}{T_{0,s,n}} = (1 + i) \frac{\kappa}{\delta_n} \sinh \left\{ (1 + i) \frac{a}{\delta_n} \right\}. \quad (\text{III.90})$$

Finlay by combining Eq. III.89 and Eq. III.90 the thermal impedance seen by the fluid is found,

$$\frac{\dot{q}_{a,s,n}}{T_{a,s,n}} = \frac{1}{Z_n} = (1 + i) \frac{\kappa}{\delta_n} \tanh \left\{ (1 + i) \frac{a}{\delta_n} \right\}. \quad (\text{III.91})$$

Using the Impedance

By calculating the inverse Laplace transform of the impedance the impulse response of the system can be found. The system's response to any perturbation, q , can then be found by the convolution of the impulse response with the perturbation.

$$T(a, t) = z(t) * q(a, t) \quad (\text{III.92})$$

Initial and final value theorem

Before going further we can deduce the initial and final value of the impedance response, without having to invert the Laplace transforms, and determine if they are realistic.

The initial value theorem,

$$\lim_{t \rightarrow \infty} z(t) = \lim_{s \rightarrow 0} sZ(s). \quad (\text{III.93})$$

The final value theorem,

$$\lim_{t \rightarrow 0} z(t) = \lim_{s \rightarrow \infty} sZ(s). \quad (\text{III.94})$$

Using the initial and final value theorems the impulse and step response for the Isothermal case can be calculated,

The impulse response;

$$\lim_{t \rightarrow \infty} z_I * \delta() = 0, \quad (\text{III.95})$$

$$\lim_{t \rightarrow 0} z_I * \delta() = \infty. \quad (\text{III.96})$$

Laplace Method for the Solution of the IBVP

The step response;

$$\lim_{t \rightarrow \infty} z_I * H() = \frac{a}{\kappa}, \quad (\text{III.97})$$

$$\lim_{t \rightarrow 0} z_I * H() = 0. \quad (\text{III.98})$$

Here $\delta()$ is the Dirac delta function and $H()$ is the Heaviside step function. This makes sense as it would be expected, in the isothermal case, that the temperature of the wall would decay back to the initial temperature after being subject to an impulse, as the heat flows through the isothermal boundary condition. Similarly for the step response the temperature tends to the steady state thermal resistance of the solid a/κ . Also, a singularity is seen at time of the impulse, this should be expected as instantaneously the temperature of the wall will match the singularity of the impulse.

Finally for the Isoflux case,

The impulse response;

$$\lim_{t \rightarrow \infty} z_A * \delta() = \frac{1}{a\rho c_p}, \quad (\text{III.99})$$

$$\lim_{t \rightarrow 0} z_A * \delta() = \infty. \quad (\text{III.100})$$

The step response;

$$\lim_{t \rightarrow \infty} z_A * H() = \infty, \lim_{t \rightarrow 0} z_A * H() = 0. \quad (\text{III.101})$$

In the case of the isoflux impulse response the change in temperature of the wall tends to a value which would satisfy conservation of energy into the the solid, rising the internal energy of the solid $\rho c_p a T$ by 1, with a uniform temperature across the solid. Furthermore the step response shows that for an isoflux response the temperature of the wall will heat up indefinitely.

III.3.5 Inverting the Laplace transform

To get the solution to a Laplace transform of the form $\hat{Z}_t(p)$ we use the identity which transposes between different Laplace domain (s, p) and time domain (t, τ) variables as described by Novikov [137],

$$z_s(t) = \int_0^\infty z_p(\tau) a(t, \tau) d\tau, \quad (\text{III.102})$$

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

where,

$$a(t, \tau) = \mathcal{L}_{s(t)}^{-1} e^{-p\tau}. \quad (\text{III.103})$$

This is because,

$$\mathcal{L}_{s(t)} \left\{ \int_0^\infty z_p(\tau) a(t, \tau) d\tau \right\} = \int_0^\infty z_p(\tau) d\tau \int_0^\infty e^{-st} a(t, \tau) dt, \quad (\text{III.104})$$

$$= \int_0^\infty e^{-p\tau} z_p(\tau) d\tau, \quad (\text{III.105})$$

$$= \mathcal{L}_{p(\tau)} \{z_p\} = \hat{Z}(p). \quad (\text{III.106})$$

This identity is useful as the inverse Laplace transforms for the impedance can be found in terms of p and then transformed by the identity to the time domain. In the case of the thermal impedance, $p = \sqrt{s/\alpha}$ so we can calculate $a(t, \tau)$ as,

$$a(t, \tau) = \mathcal{L}_{s(t)}^{-1} \left\{ e^{-\tau \sqrt{\frac{s}{\alpha}}} \right\}, \quad (\text{III.107})$$

$$= \frac{\tau}{2\sqrt{\pi}\sqrt{\alpha}t^{3/2}} e^{-\frac{\tau^2}{4\alpha t}}. \quad (\text{III.108})$$

The temperature response with an isoflux boundary condition

As we want to measure thermal fluctuations on the back side of the foil from the front side of the foil is it important to know the impulse response of a temperature change on one side of the foil to that on the opposite side.

From table III.3 the transfer function for an isoflux outer wall condition relating the temperature of the outer wall to the inner wall is,

$$\frac{\hat{T}(s, 0)}{\hat{T}(s, a)} = \frac{1}{\cosh(pa)} = 2(e^{pa} + e^{-pa})^{-1}. \quad (\text{III.109})$$

Using binomial expansion this can be rewritten as a series summation of exponential functions,

$$\frac{\hat{T}(s, 0)}{\hat{T}(s, a)} = 2e^{pa} + \sum_{k=0}^{\infty} (-1)^k e^{-(2k+1)pa}. \quad (\text{III.110})$$

From the p Laplace domain this can be transformed into a summation of delta functions

Laplace Method for the Solution of the IBVP

with in time τ and delay $(2k+1)a$,

$$\mathcal{L}_{p(\tau)}^{-1} \frac{\hat{T}(s, 0)}{\hat{T}(s, a)} = 2 \sum_{k=0}^{\infty} (-1)^k \delta(\tau - (2k+1)a). \quad (\text{III.111})$$

From Eq. III.108 and Eq. III.102 the impulse and step response can be derived. Impulse response,

$$\delta() * \mathcal{L}_{s(t)}^{-1} \frac{\hat{T}(s, 0)}{\hat{T}(s, a)} = 2 \sum_{k=0}^{\infty} (-1)^k (2k+1) \frac{ae^{-\frac{(2k+1)^2 a^2}{4\alpha_s t}}}{2\sqrt{\pi\alpha_s t^{\frac{3}{2}}}}, \quad (\text{III.112})$$

where $\delta()$ is the Dirac delta function, and $*$ represents the convolution between two functions. Step response,

$$H(t) * \mathcal{L}_{s(t)}^{-1} \frac{\hat{T}(s, 0)}{\hat{T}(s, a)} = -2 \sum_{k=0}^{\infty} (-1)^k \operatorname{erf}\left(\frac{(2k+1)a}{2\sqrt{\alpha_s t}}\right) + C, \quad (\text{III.113})$$

where erf represents the error function and C is a constant.

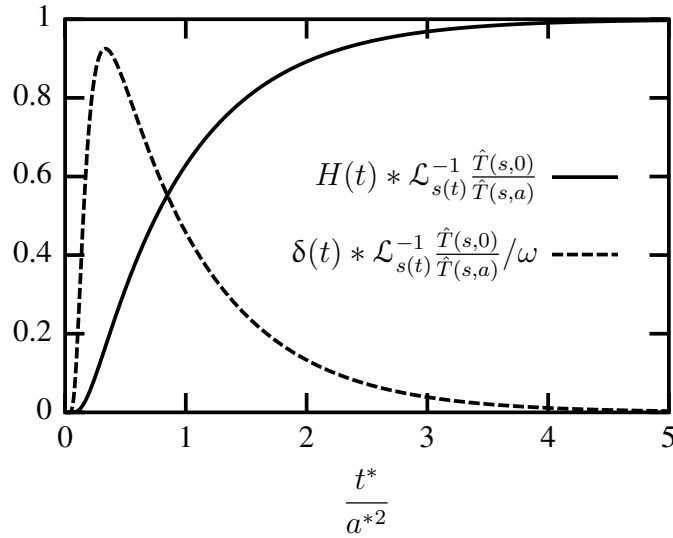


Figure III.8: Thermal temperature response on the outer wall, $T(y=0)$, of a 1-D solid to a step and impulse in temperature on the inner wall, $T(y=a)$, for isoflux boundary condition, $q(y=0)=0$.

Both Eq. III.112 and Eq. III.113 can be normalised if a fundamental time constant ω is introduced, such that: $t^* = t\omega$ and $a^* = a/\delta_s$ where, as before, δ_s is the thermal

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

penetration depth $\sqrt{2\alpha_s/\omega}$.

Figure III.8 shows the resulting impulse and step response. It appears as there is an initial delay for the temperature on the outer B/C to respond to fluctuations occurring on the inner B/C. In the case of the impulse response the temperature quickly rises, and then slowly decays back to 0. In the step response, which is the integral of the impulse response, the outer wall temperature after an initial delay rises quickly, and then gradually reaches equilibrium with the new inner wall temperature.

The inverse Laplace transform of $\frac{1}{p} \tanh(pa)$

By first observing that the inverse Laplace transform of $\tanh(s)$ is a summation of Dirac delta functions,

$$\mathcal{L}_p^{-1}[\tanh(p)](\tau) = \delta(\tau) + 2 \sum_{k=1}^{\infty} (-1)^k \delta(\tau - 2k). \quad (\text{III.114})$$

We can calculate the thermal response of the solid in the p domain with an Isothermal boundary condition to be,

$$\mathcal{L}_p^{-1}\left[\frac{1}{p} \tanh(pa)\right](\tau) = z_I(\tau) = H(\tau/a) + 2 \sum_{k=1}^{\infty} (-1)^k H(\tau/a - 2k). \quad (\text{III.115})$$

The inverse Laplace transform of $\frac{1}{p} \coth(pa)$

Similarly for the isoflux case we find that in the p domain the impedance looks like,

$$\mathcal{L}_p^{-1}\left[\frac{1}{p} \coth(pa)\right](\tau) = z_A(\tau) = H(\tau/a) + 2 \sum_{k=1}^{\infty} H(\tau/a - 2k). \quad (\text{III.116})$$

The integration of functions of the form $a(t, \tau)H(\tau - 2ka)$

From the above we can see we are looking for solutions of the form,

$$\int_0^{\infty} a(t, \tau) H(\tau - 2ka) d\tau = \int_{2ka}^{\infty} a(t, \tau) d\tau. \quad (\text{III.117})$$

Which can be expanded to,

$$\int_{2ka}^{\infty} \frac{\tau}{2\sqrt{\pi}\sqrt{\alpha}t^{3/2}} e^{-\frac{\tau^2}{4\alpha t}} = \sqrt{\frac{\alpha}{\pi t}} e^{-\frac{a^2 k^2}{\alpha t}}. \quad (\text{III.118})$$

Isothermal impedance response

The impulse response in the isothermal case is given by,

$$z_I(t) * \delta(t) = \frac{1}{\kappa} \sqrt{\frac{\alpha}{\pi t}} \left(1 + 2 \sum_{k=1}^{\infty} (-1)^k e^{-\frac{a^2 k^2}{\alpha t}} \right), \quad (\text{III.119})$$

and the equivalent step response is,

$$z_I(t) * H(t) = \frac{2}{\kappa} \sqrt{\frac{\alpha t}{\pi}} \left(1 + 2 \sum_{k=1}^{\infty} (-1)^k \left[ak \sqrt{\frac{\pi}{\alpha t}} \operatorname{erf} \left(\frac{ak}{\sqrt{\alpha t}} \right) + e^{-\frac{a^2 k^2}{\alpha t}} \right] \right). \quad (\text{III.120})$$

As before it can be seen that the impulse response tends to 0 as $t \rightarrow \infty$ and ∞ as $t \rightarrow 0$. Whereas the step response tends to

Isoflux impedance response

Finally the isoflux impulse response is given by,

$$z_A(t) * \delta(t) = \frac{1}{\kappa} \sqrt{\frac{\alpha}{\pi t}} \left(1 + 2 \sum_{k=1}^{\infty} e^{-\frac{a^2 k^2}{\alpha t}} \right), \quad (\text{III.121})$$

while the step response is,

$$z_A(t) * H(t) = \frac{2}{\kappa} \sqrt{\frac{\alpha t}{\pi}} \left(1 + 2 \sum_{k=1}^{\infty} \left[ak \sqrt{\frac{\pi}{\alpha t}} \operatorname{erf} \left(\frac{ak}{\sqrt{\alpha t}} \right) + e^{-\frac{a^2 k^2}{\alpha t}} \right] \right). \quad (\text{III.122})$$

The impedance responses

Both the isoflux and isothermal impulse and step responses are plotted in Fig. III.9. These responses show the effect of applying either, a step change in the heat flux on the inner wall, or by applying an impulse of heat. The initial thermal response for the isoflux and isothermal boundary conditions are similar when subject to the step or impulse excitations. In the case of the impulse excitation the response at $t = 0$ is asymptotic, reflecting the singularity of the delta function. This makes numerical convolution with the impulse response difficult. However by integrating with the step response this difficulty can be overcome [137].

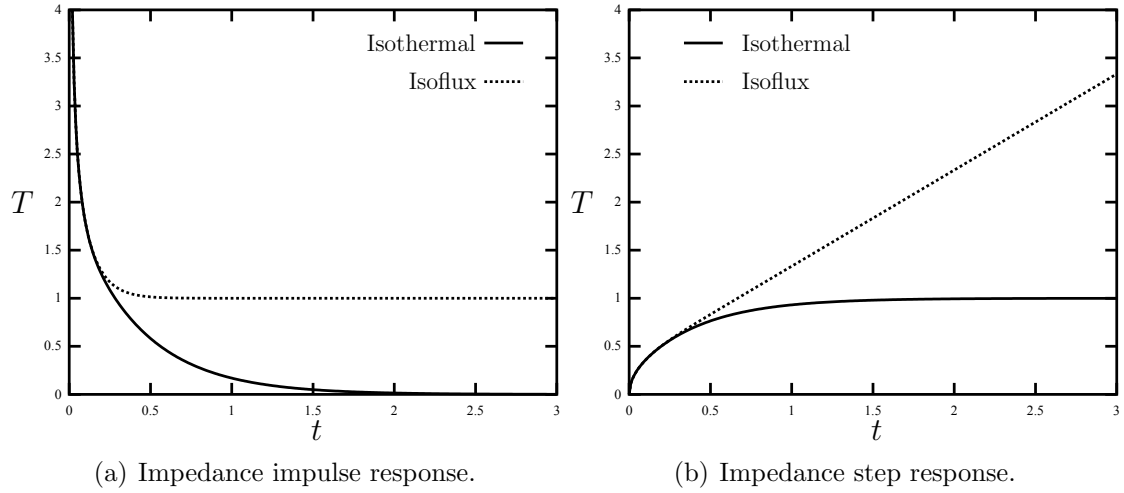


Figure III.9: Thermal response of a 1-D solid to a step and impulse in heat flux for isothermal and isoflux boundary conditions.

In the case of the impulse response for the isothermal outer boundary condition the temperature eventually returns to the external temperature T_0 . On the other hand the response for the isoflux B/C dose not return to its initially temperature and settles down to a new temperature as the impulse of heat spreads through the solid domain. For the step response it is the isothermal B/C which settles on a new temperature, due to the heat flux through the solid. In the isoflux case the temperature continues to increase as the domain is heated up.

III.4 Composite Solids

Previous studies such as De Monte [25] and others [24, 134] have investigated the transient thermal conduction through a “composite” slab consisting of layers of different materials. Such analysis tends to focus on the transition of the internal temperature, this is important mainly from a materials perspective, for such processes as tempering in the case of metallurgy and curing in the case of plastics and composites. However for the purposes of characterising the thermal response of the composite slab only the response at the slabs boundaries of important. Additionally by recognising that each layer of the slab has an equivalent thermal circuit, the resulting impedance network of the composite slab can be transformed to a system of three components, as before.

When two layers of different materials are sandwiched together, creating a composite

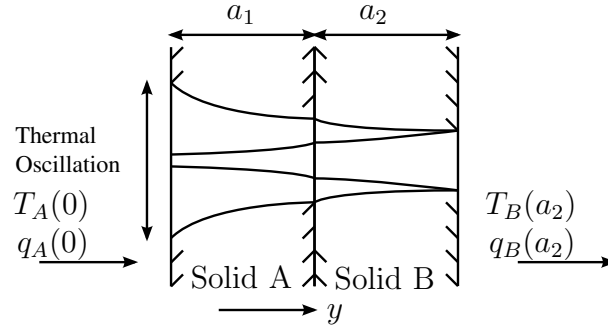


Figure III.10: Two solids in thermal contact with each other produce a composite thermal solid, essentially altering the input and output impedance to thermal oscillation.

thermal solid, the thermal impedances of the new solid can be quite different to the thermal impedances of the original materials. The thermal impedance of the combined materials depend on the range of thickness of the layers in the slab and the frequency of the thermal oscillation. Also, the equivalent impedance network of the combined material slab is asymmetric, the impedance Z_1 and Z_3 of the equivalent thermal circuit will no longer be equal. This is of interest as even a thin layer of material applied to a surface can change the thermal properties, allowing better thermal matching of materials where unsteady heat transfer is important.

To solve the thermal diffusion equation in this new case the problem can be split into the separate domains of the two materials, and the equations for the single material case applied to each of these domains. This results in the need to solve four simultaneous equations, for the four wave coefficients. These are the two boundary conditions which need to be imposed on either surface of the 1-D solid, and the thermal contact condition where the two materials meet. This condition is for the temperature and heat flux through both materials to be equal at the contact point;

$$T_A(y = a_1) = T_B(y = 0), \quad (\text{III.123})$$

$$q_A(y = a_1) = q_B(y = 0). \quad (\text{III.124})$$

Using the matrix notation from before the contact condition can be written;

$$\begin{bmatrix} M_{T_A(a_1)} \\ M_{q_A(a_1)} \end{bmatrix} \begin{bmatrix} T_f \\ T_b \end{bmatrix}_A = \begin{bmatrix} M_{T_B(0)} \\ M_{q_B(0)} \end{bmatrix} \begin{bmatrix} T_f \\ T_b \end{bmatrix}_B. \quad (\text{III.125})$$

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

The thermal wave coefficients in each material are therefore linked by,

$$T_{cA} = M_{cA,B} T_{cB}, \quad (\text{III.126})$$

where,

$$M_{cA,B} = \begin{bmatrix} M_{T_A(a_1)} \\ M_{q_A(a_1)} \end{bmatrix}^{-1} \begin{bmatrix} M_{T_B(0)} \\ M_{q_B(0)} \end{bmatrix}. \quad (\text{III.127})$$

If the thermal oscillations were known on either side of the composite solid the coefficients of the thermal waves in material B can be found from,

$$\begin{bmatrix} T_A(0) \\ T_B(a_2) \end{bmatrix} = \begin{bmatrix} M_{T_A(0)} M_{cA,B} \\ M_{T_B(a_2)} \end{bmatrix} T_{cB}. \quad (\text{III.128})$$

The coefficients for material A can then be found from Eq. III.126, and the heat fluxes can be found from these coefficients.

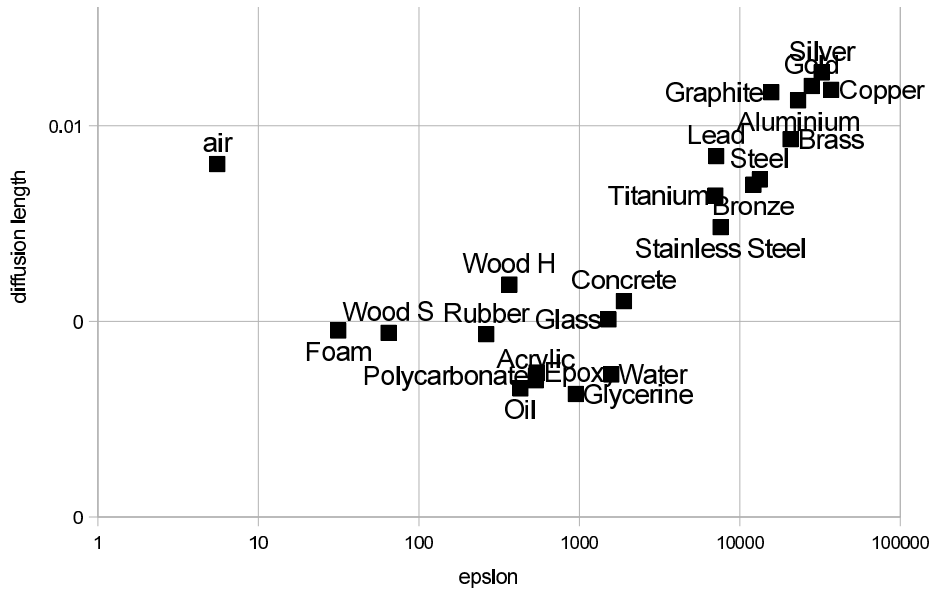


Figure III.11: A selection of material properties, with $\epsilon = \sqrt{\kappa\rho c_v}$ on the x-axis and diffusion length (m) δ over $\sqrt{\omega}$, $\left(\sqrt{\frac{2\kappa}{\rho c_v}}\right)$ on the y-axis.

It can be seen from inspecting Eq. III.30 and Eq. III.31 for both materials that the solution is dependent on the ratio of thicknesses $\frac{a_1}{\delta_A} / \frac{a_2}{\delta_B}$, and the ratio of thermal effusivities ϵ_A/ϵ_B , remembering that the frequency of the oscillation is the same in both materials and that $\epsilon\sqrt{\omega} = \frac{\kappa}{\delta}$. The ratio of effusivities describes how conductive to thermal oscilla-

Convective Boundary Conditions

tions the materials are to each other. Figure III.11, computed from material properties listed in Ref. [138], shows how the effusivity and diffusion length compare for a selection of engineering materials.

It is interesting that air has a very large diffusion length compared to its low effusivity. This is due to air having a low volumetric heat capacity. It can be seen that the effusivity and thermal length scales for solids are mostly dependent on thermal conductivity, hence most solid materials lie on a line on the log-log plot. However, addition of air into the materials, such as in foams, reduces the effusivity.

Since there are two new parameters which can be varied, namely the ratio of the effusivity and the ratio of non-dimensional thickness between the two materials, it is difficult to show all the general trends. Still, a few examples of materials which may commonly be sandwiched together can be chosen for investigation. The effect of the ratio of thickness, total thickness can then be investigated on the magnitude of the input impedance.

Firstly, a composite solid consisting of steel and aluminium is considered. Steel has a lower effusivity and diffusion length compared to aluminium. However, the difference is small as their quantities are of the same order of magnitude. It can be seen in Fig. III.12(b) that steel has a larger impedance when compared to aluminium and the peak impedance is reached at a smaller thickness. In the contour plot it can be observed that the variation of thermal impedance with the thickness ratios of the two materials is not linear. Also, the contour shows that only a thin layer, about 20% of total thickness of aluminium, can drop the impedance to thermal oscillation quite significantly.

A second example is chosen, this time with similar effusivity, but different diffusion lengths. The composite “solid” consists of concrete with a thin layer of water, which may be experienced in many structural applications. It is reasonable to assume the water as being solid if the layer is very thin, so that there are minimal convection currents in the fluid and most of the heat transfer is conductive. This time the input impedance is shown from both surfaces of the composite solid. Figure III.13(a) and Fig. III.13(b) show how the effect of the thickness ratio on input impedance is different from both sides.

III.5 Convective Boundary Conditions

A convective boundary condition can be applied when there is a fluid passing over one of the solid surfaces. The heat flux is defined as $q_w = h(T_\infty - T_w)$, where T_∞ is the

CHAPTER III. AN ANALYTICAL FRAMEWORK FOR UNSTEADY CONDUCTION IN SOLID DOMAINS

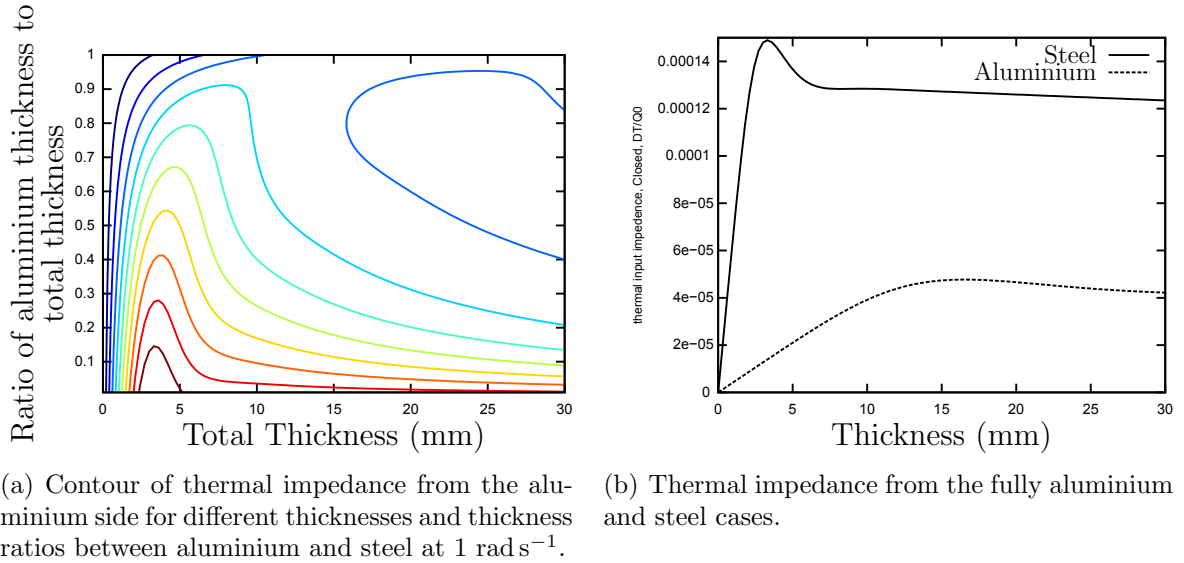


Figure III.12: Magnitude of thermal impedance for an aluminium-steel composite with a thermal oscillation at 1 rad s^{-1} .

temperature of the fluid in the free stream and h is the HTC. The heat transfer coefficient is typically around 10 to $100 \text{ W K}^{-1} \text{ m}^{-2}$ for air and 500 to $10\,000$ for water. This can be simply incorporated into the equivalent thermal impedance circuit of the model, in the form of an impedance $Z_h = 1/h$ connected between the surface of the solid and the free stream temperature T_∞ .

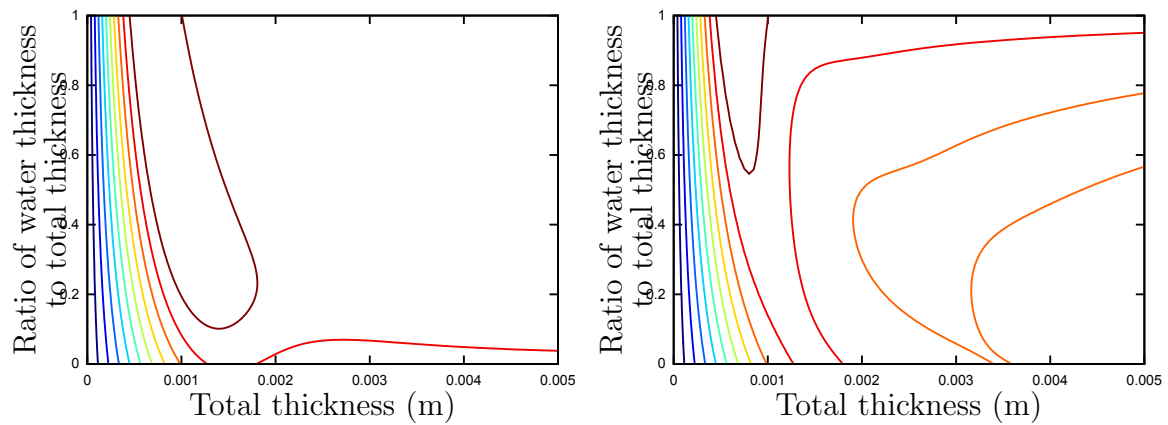
Now the input and total impedance for the air and solid is the superposition of the two,

$$Z = Z_{solid} + Z_h, \quad (\text{III.129})$$

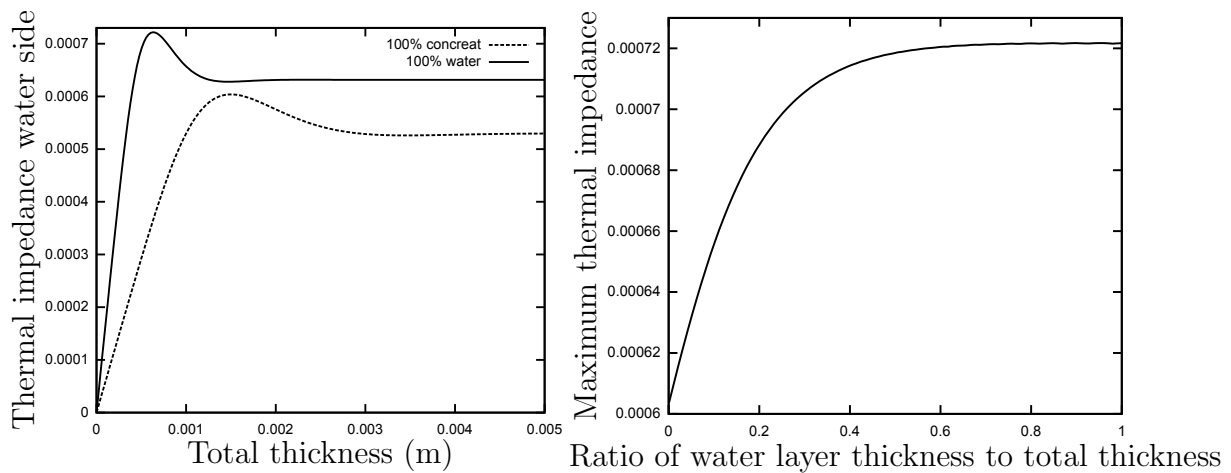
$$Z_{in} = Z_{in,solid} + Z_h. \quad (\text{III.130})$$

Also if the convective condition is on the outer surface, connecting T_0 to T_∞ , then Z_3 is modified by Z_h as in Section III.2.6.

Convective Boundary Conditions



(a) Contour of thermal impedance from the water side for different thickness and thickness ratios between a composite solid comprising of water and concrete at 1 rad s^{-1} .
 (b) Contour of thermal impedance from the concrete side.



(c) Thermal impedance from the fully water and concrete cases.

(d) Maximum thermal impedance.

Figure III.13: Magnitude of thermal impedance for composite solid consisting of a layer of stagnant water on concrete with a thermal oscillation at 1 rad s^{-1} .

Chapter IV

Experimental Methods

An experiment was performed to demonstrate the effects of unsteadiness on the heat transfer problem. Interfacial heated film flows were selected for these measurements as these are canonical flows that are ideal for the study of unsteady convective heat transfer. These flows provide a well defined environment for the establishment of a range of perturbations; from low dimensional instabilities (at lower Reynolds numbers), to full spatiotemporal broadband turbulence (at higher flow rates). Film flows exhibit unsteady heat transfer due to the periodic waves which can form on the film (see Section II.4.2). In this chapter we describe the film flow experiment and measurement technique developed to measure the heat transfer.

IV.1 Experiment Overview

An experiment was conducted to measure the instantaneous temperature and heat flux of a liquid (water) film as it flows down a flat, inclined heated foil. The foil was made from 99.6+ % pure titanium and had a thickness a of 50 μm . Heating was achieved by holding the foil taut between two 10 mm thick copper electrodes, and applying a current from a DC power supply. The apparatus was capable of a maximum heat flux of 4.2 W cm^{-2} over a 1200 cm^2 area. At a distance of 300 mm upstream of the imaged area the water was distributed over the foil through a $w = 300 \text{ mm}$ wide knife-edge slot to form a thin fluid film with a uniform thickness across the heated area. The water flow that fed the distributor was a combination of two separate supplies. One was a steady supply whose flow-rate was measured by rotameters. The second, through which a prescribed

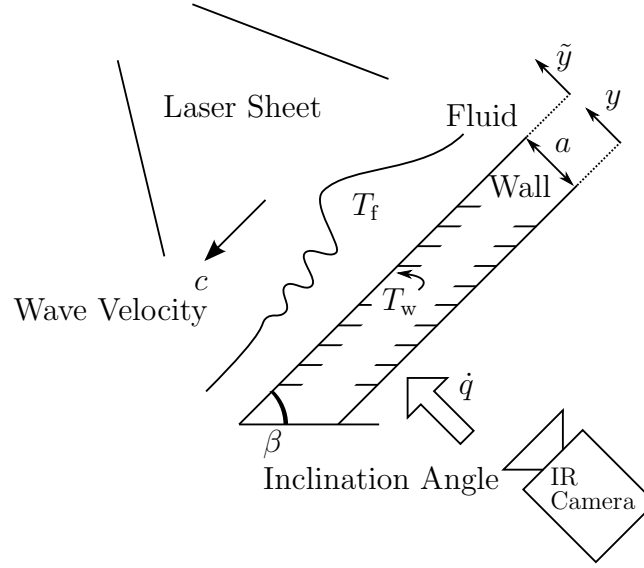


Figure IV.1: Diagram of the film flow experiment.

proportion of total flow-rate was by-passed, was a pulsating supply, modulated by a rotating throttle valve to excite coherent large-scale waves that flowed down the foil at a controlled frequency. A simplified schematic of the flow and measurement arrangement is shown in Fig. IV.1.

Having established a flow condition, a laser-induced fluorescence (LIF) technique was then used to measure the film thickness and free-surface temperature simultaneously. The water phase was marked by pre-dissolving a temperature-dependent fluorescent dye, Rhodamine B, to form a perfectly mixed aqueous-dye solution of low concentration (~ 1 g/L). The aqueous liquid was excited by a light sheet of thickness $100\text{ }\mu\text{m}$ that was generated by a dedicated sheet optics lens combination. The sheet optics combination was connected, via a guiding arm, to a frequency-doubled 532 nm Nd:YAG laser that produced pulses of light at 100 Hz. The resulting fluorescent light was captured by a pair of CMOS cameras which were synchronised to capture images upon each laser pulse. The captured images were then stored as an image sequence for later processing. An optical long pass dichroic filter was positioned in front of the camera lenses to block the incident laser light but pass the fluorescent light from the Rhodamine B dye. Based on the temperature dependence of the quantum efficiency of Rhodamine B fluorescence [117, 118], imaging of the emitted light allowed not only the identification of the interface height of the waves travelling down the film (i.e. film depth), but also the direct measurement of the temperature of the free-surface of the film. Since the intensity of the fluorescence emission from the dye is temperature dependant the film interface temperature was inferred from the intensity

CHAPTER IV. EXPERIMENTAL METHODS

of the imaged film at the known interfacial location. Simultaneous to the capture of LIF images, temperature measurements were made on the underside of the foil using an IR camera (see Fig. IV.1) and in the fluid inlet with thermocouples. In addition, in a separate set of experiments, both the IR camera and a series of fine (75 μm) thermocouples were used to make independent measurements of the film and foil temperatures for the purposes of calibration and verification.

The use of electrodes that were much thicker, and therefore with a considerably lower resistance than the foil, ensured that the power supplied was distributed evenly over the foil, so that the heating effect was uniform. This was checked by measuring the voltage distribution over the foil using point measurements with a multimeter. No noticeable voltage distortion was observed, with a worst case deviation in power of 0.1 % over the 300×400 mm foil area. Moreover, the uniform heating was confirmed by the observation of a uniform temperature distribution across the foil in dry test cases, wherein the foil is heated in the absence of any fluid flow and the foil temperature was measured with the IR camera. In these tests it was also found that convection of heat to the air was minimal, being less than 1 % of the heat flux into the liquid that was experienced in the falling film flows, in worst case conditions. Specifically it was found that, in the absence of a liquid film flow, 50 W of heat were required to maintain a steady temperature of 60 $^{\circ}\text{C}$ over the surface of the foil. However after establishing a typical water film flow over the foil, up to 5 kW were required to obtain the same foil temperatures (see Section IV.7.1). Hence, it is important to note that, since the heat flux into the film at the inner wall (foil surface in contact with the fluid) is much much larger than the losses to the surroundings from the outer wall (un-wetted surface on the opposite, underside of the foil), the *outer* side of the foil can be considered as an *adiabatic* boundary condition (B/C).

The liquid temperature was kept low, below 40 $^{\circ}\text{C}$, throughout the experimental runs in order to minimise evaporation and to focus the investigation on the forced convective heat transfer processes in liquid films. For example, at the upper range of achievable foil heat fluxes (which is expected to correspond with the highest heat losses), 3.2 W cm^{-2} of heating power was applied with the film flow rate at 8 l min^{-1} , across the 300 mm wide test section. Assuming complete transfer of heat from the foil to the fluid with no other heat losses to the surroundings, a flow-wise temperature gradient of 17.4 K m^{-1} should be expected. However, a temperature gradient of 16.7 K m^{-1} was measured with the IR camera in the thermal profile of the fully developed portion of the flow. This equates to a heat loss of 0.13 W cm^{-2} to the surroundings, which amounts to less than 5 % of the total heat input. Even if all of this heat went into evaporation of the fluid at the

Experiment Overview

Table IV.1: Thermal properties of materials and fluid in the film flow experiments.

	Density, ρ (kg m^{-3})	Heat capacity, c ($\text{J kg}^{-1} \text{K}^{-1}$)	Thermal conductivity, k ($\text{W m}^{-1} \text{K}^{-1}$)
Water	1 000	4 200	0.6
Titanium	4 510	540	21.9

film surface, which is unlikely as other convective heat transfer processes occur at the film interface [36], then the evaporating flow rate would be $0.5 \text{ ml s}^{-1} \text{ m}^{-2}$. This equates to a maximum evaporative flow rate of 4 ml min^{-1} which is 3 to 4 orders of magnitude smaller than the total flow rate. Furthermore, note that the heat transfer coefficient in this case, between the foil and the fluid film, is about $2400 \text{ W K}^{-1} \text{ m}^{-2}$ whereas the heat transfer coefficient between the air and the fluid film is an order of magnitude lower at about $200 \text{ W K}^{-1} \text{ m}^{-2}$ from direct measurement. This is in general agreement with Nusselt theory for the film flow, and expected values of these heat transfer coefficients from Ref. [102].

As the foil used was thin, that is, the foil was over an order of magnitude thinner than any thermal feature observed on its surface with the IR camera. Consequently it is assumed that heat transfer in the foil is dominated by 1-D thermal conduction through its thickness (refer to Nakamura [139] for a full explanation of the spatial resolution of IR thermography measurements). With this assumption the model for 1-D thermal conduction in the solid (as performed in Chapter III) can be used to describe the foil's thermal response to temperature fluctuations, as seen in Section III.2. This response will depend upon the imposed outer wall boundary condition and, as it was stated previously, the thermal boundary condition on the back/underside surface of the foil (or, outer wall '0') can effectively be assumed to be adiabatic. Note that as, in the current problem, the solid diffusivity is constant and so the thermal conduction in the solid is linear: the problem of finite 1-D thermal conduction with an adiabatic boundary condition imposed at one surface and constant heat generation in the solid (through resistive heating) is equivalent to the system considered in Chapter III where an isoflux wall boundary condition is imposed at one end of the domain and with no heat generation in the solid. The resulting thermal response that relates the common fluid-solid interface temperature (inner wall 'w') to the measured temperature fluctuations on the outer wall for a given frequency, as

shown in Section III.3.4, is,

$$\frac{T_{w,n}}{T_{0,n}} = \cosh \{(1 + i)a_n^*\} , \quad (\text{IV.1})$$

while the corresponding heat flux response is given by,

$$\frac{\dot{q}_{w,n}}{T_{0,n}} = \frac{(1 + i)}{\sqrt{2}} \epsilon_s \sqrt{\omega_n} \sinh \{(1 + i)a_n^*\} , \quad (\text{IV.2})$$

where $a_n^* = a/\delta_{s,n}$ is a normalised solid thickness with a the foil thickness and $\delta_{s,n} = \sqrt{2k_s/\rho_s c_s \omega_n} = \sqrt{2\alpha_s/\omega_n}$ the thermal diffusion length in the solid associated with the n^{th} harmonic of the thermal fluctuations, α_s is the thermal diffusivity and $\epsilon_s = \sqrt{k_s \rho_s c_s}$ the thermal effusivity of the solid material.

By combining Eq. IV.1 and Eq. IV.2 the thermal impedance of the solid as seen by the fluid is,

$$\frac{\dot{q}_{w,n}}{T_{w,n}} = \frac{1}{Z_A(\omega_n)} = \frac{(1 + i)}{\sqrt{2}} \epsilon_s \sqrt{\omega_n} \tanh \{(1 + i)a_n^*\} . \quad (\text{IV.3})$$

Thus, the temperature at the inner wall T_w can be found by implementing an inverse filter with the response in Eq. IV.1 on the temperature of the foil measured on the outer wall by the IR camera, T_0 . Similarly the instantaneous heat flux at the inner wall $\dot{q}_w$ can be evaluated from the IR temperature measurement, the response in Eq. IV.2 and the electrical power supplied to the foil. Eq. IV.1 indicates (after using the material properties listed in Table IV.1) that the attenuation in the amplitude of the temperature fluctuations through the foil thickness at 50 Hz, which is the Nyquist frequency for the image capture rate, is 0.1 %.

IV.2 Apparatus

IV.2.1 Flow loop

Referring to Fig. IV.2 a closed loop cycle was used to deliver the fluid to the experiment test section. The advantage of the closed loop system as opposed to an open loop system being that the fluid is recycled and so the concentration of the florescent dye remains constant. An 18 litre holding tank underneath the test section acted as a reservoir for the fluid being used in the test. This reservoir was the storage location of the fluid when the experiment was not active and compensated for the change of volume when the rest

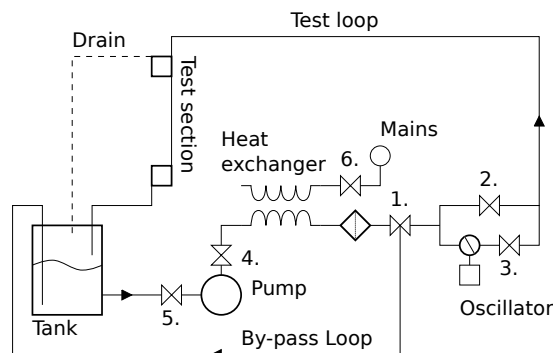


Figure IV.2: Schematic diagram of the fluid flow loop for the experiment test section. The diagram shows the locations of the flow loop's pump, control valves, and holding tank.

of the flow loop and test section were filled during operation. When operating the fluid was pumped through a heat exchanger before entering the rest of the flow loop and the test section. The heat exchanger was used to remove the waste heat from the experiment and the heat generated from pumping work imparted on the fluid to force it around the loop. Water from the mains was used as the cooling stream, passing through the heat exchanger, and down the drain. By controlling the flow of cooling water through the heat exchanger using valve (6) the temperature of the flow at the inlet to the test section was kept constant throughout the experimental run.

To minimise the build up of particulates in the flow loop a 10 μm filter was placed between the pump and the heat exchanger. Particulates and other contaminants in the fluid such as dust, ionic compounds and surfactants can change the fluid's physical properties, for instance the surface tension of the liquid (see Section IV.7.3). Such a change in fluid properties would be counteractive to achieving consistent experimental results. These contaminants could arise from many sources, for example dust in the air or corrosion of metal parts in the flow loop. In addition to the filter the use of deionised water in the flow loop helped reduced the rate of corrosion and so also reduced the amount of contamination present.

After passing through the filter the flow then passed through the bypass valve (1) and the two main control valves (2) and (3), as shown in Fig. IV.2. By using a bypass to divert a proportion of the surplus flow back to the storage tank, a higher flow-rate can be maintained through the pump than that required by the experiment. By maintaining a high flow-rate in the pump the likelihood of cavitation on the pump impeller or of the pump stalling, which would result in excessive wear on the pump, is reduced. Additionally a higher flow-rate and large recirculation around the loop results in more effective

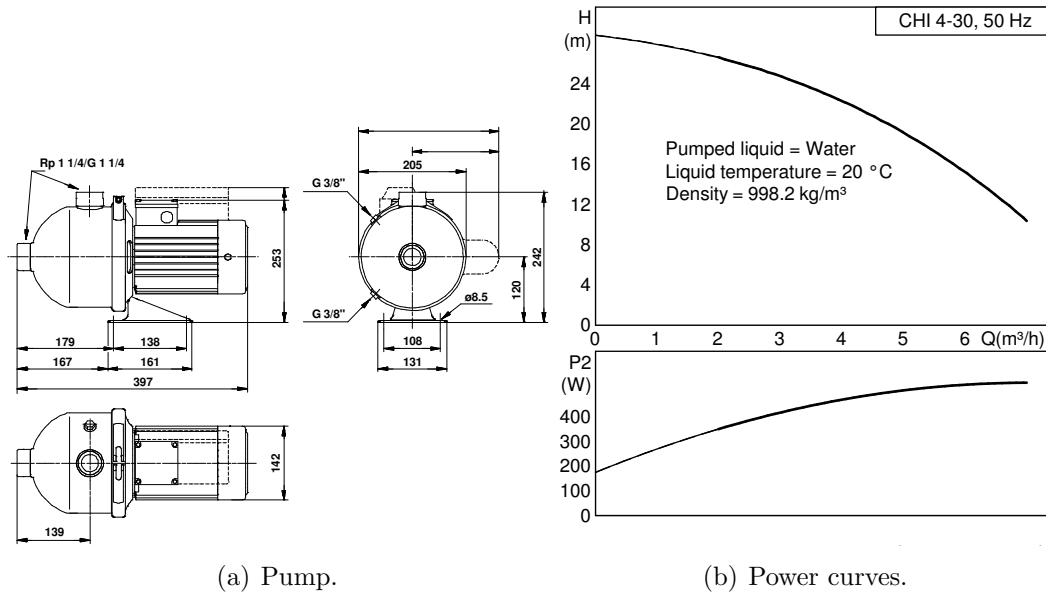


Figure IV.3: Pump schematic and operating curves, courtesy of Grundfos.

operation of the heat exchanger, and so a low temperature in the feed tank is easier to maintain.

Pump

The pump used was a Grundfos CHI 4-30. It has a stainless steel impeller and pump housing so it is resistant to corrosion which, again helps reduce the probability of contamination of the fluid. The pump is rated at a flow rate of up to $4 \text{ m}^3 \text{ h}^{-1} = 66 \text{ L min}^{-1}$ and a pressure of up to 2.1 bar (see Fig. IV.3(b)). The primary seals of the pump are silicon carbide/silicon carbide mechanical shaft seals which are highly corrosion and wear resistant. These seal consists of two silicon carbide (SiC) disk shaped faces which are pushed together by a spring and the action of pressure inside the pump. One of the faces is fixed to the pump body and the other is connected to the rotating shaft. A secondary seal is used to prevent leakage through the connection between the shaft and its SiC face. These secondary rubber seals are made from FKM fluorocarbon monomer, which is resistant to most oils and chemicals. During operation the primary SiC sealing faces are lubricated by the formation of a thin fluid layer between the faces, and as such running the pump dry should be avoided.

Flow oscillator valve

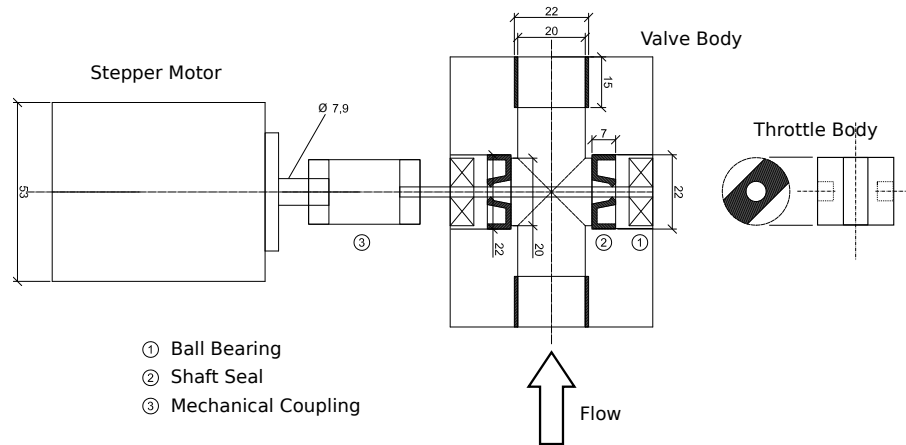
The free-surface of the film flow will naturally form waves above the critical flow condition. However, low inclination angles and low Reynolds numbers will tend to dampen any hydrodynamic wave formation. The waves occurring as a result of natural flow instabilities in the flow will have a characteristic frequency and may even be broadband in nature depending on the conditions. For more control over the wave amplitude and frequency, and to obtain more regular fluid waves an oscillator valve is used to impart pulsation on the flow and so force the interface waves at a particular frequency. As the formation of waves is nonlinear left unforced the most energetically favourable waves will grow largest quickest. By forcing the system at a particular frequency with a large enough amplitude waves of that frequency will form selectively. In addition, by using a pulsed inlet condition, fully developed wave flow can be triggered to occur much further upstream, reducing the length of the test section necessary to observe heat transfer in wavy flows.

The oscillator valve shown in Fig. IV.4, constructed by the author, consists of a metal spindle, or throttle body, with a hole drilled transversely through its axis and mounted in the valve body. Depending on the orientation of the throttle body the port through the valve body is either fully open or fully closed. The throttle body can then be rotated at a given frequency to produce a pulsating flow through the valve. The frequency of pulsation of the fluid flow will be twice the rotation rate of the valve as the valve is “open” twice per rotation. Contact seals, rated to 10 bar, and ball bearings were used to hold the throttle body in place allowing the valve body to rotate smoothly and preventing liquid from escaping from the valve body.

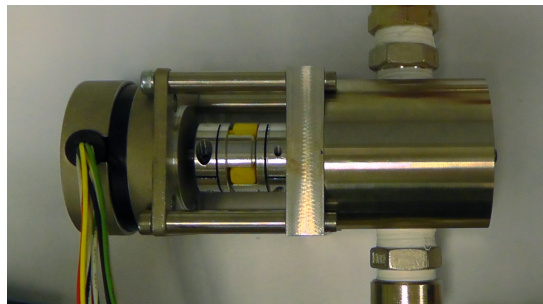
Valve motor and controller

The valve axle was driven by a 5 V, 1 A stepper motor and this motor was controlled via the stepper motor controller shown in Fig. IV.5. The controller was assembled by the author from an RS electronics unipolar stepper motor drive board, two discrete high power resistor and a number of toggle switches to select the operation configuration.

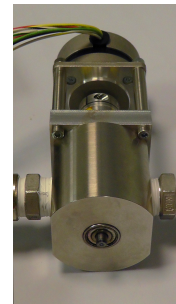
To improve performance the stepper motor was driven from a 24 V, 60 W power supply using two 22 Ω current limiting resistance for each coil pair on the motor. The higher supply voltage allows the driver circuit to overcome the inductance of the motor windings more quickly when switching the coils, allowing the motor to run at higher speeds. Due to the 5 Ω resistance of the motor coils it can be calculated that the stall



(a) Flow oscillator schematic.



(b) Image of the complete flow oscillator valve assembly, with stepper motor, motor coupling and plumbing fittings protruding from the valve inlet and outlet.



(c) Second image, showing detail of the valve spindle and the ball bearings which have been push fit into the valve body.

Figure IV.4: Flow oscillator valve.

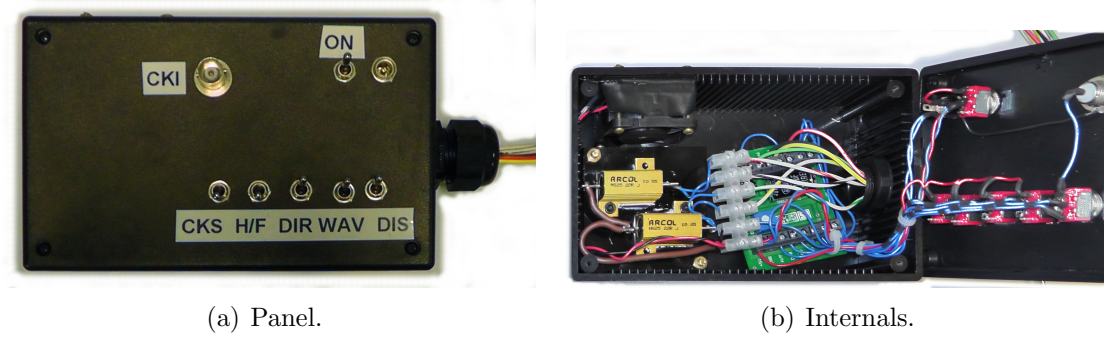


Figure IV.5: Stepper motor controller.

current for the motor with a $22\ \Omega$ resistor should be $0.89\ \text{A}$, which equates to a power dissipation of $19.6\ \text{W}$ per resistor. The current limiting resistors, which were rated to $25\ \text{W}$ each, were mounted on an extruded aluminium heatsink, for heat dissipation. The resistor and heatsink assembly was then mounted inside an enclosure and a dc fan was mounted to blow air across the finned heatsink and out through two holes punched into the enclosure for cooling.

The circuit was able to offer a wide selection of operating arrangements which can be configured by using the 5 toggle switches on the front panel of the enclosure. The picture in Fig. IV.5(a) shows the front panel of the stepper motor control circuit, the “CLK” switch allows selecting the internal clocking frequency or for an external clocking signal “CKI” provided by a function generator, this allowed the motor speed to be controlled with a high degree of accuracy and stability. “DIR” sets the direction of rotation of the motor, “WAV” sets wave motor drive mode and “H/F” sets half stepping drive mode, as apposed to full stepping drive. “DIS” disables the motor so that it stops turning. A rotation rate of up to $350\ \text{rpm}$ was achievable which results in a maximum wave forcing frequency of $11\ \text{Hz}$. The degree of pulsation of the flow can be adjusted using valves (2) and (3), by mixing the mean and pulsating components of the flow.

Physical layout

Following this, the fluid travels $1.5\ \text{m}$ upwards through a $\frac{3}{4}$ inch diameter pipe, through a digital ultrasonic rotameters before being delivered to the distribution box, discussed in Section IV.2.3, which distributes the fluid across the width of the metal foil. The fluid then flows down the test section and falls into a collector and back into the tank.

The pump was mounted on a block of vibration absorbing material to diminish any

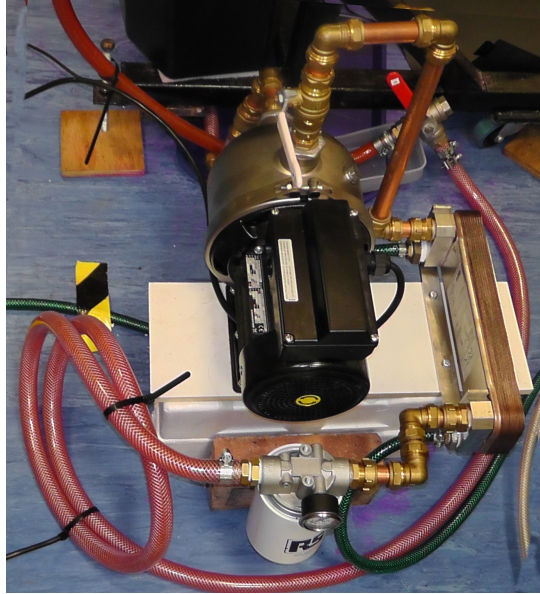


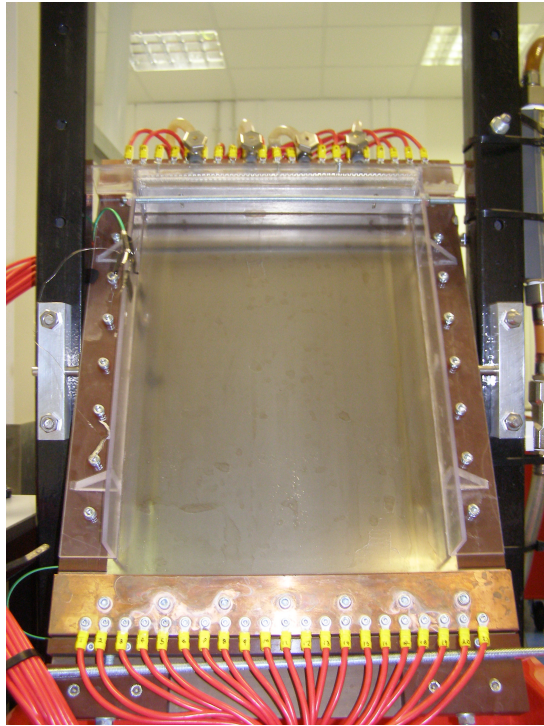
Figure IV.6: Picture of the pump, heat exchanger and filter physical layout on the experimental rig.

noise transmission to the supporting frame on which the experiment test section was mounted. As the pump was a large source of vibration this could promote the growth of waves at only the specific vibrational frequency emitted by the pump which is undesirable. The pump was also separated from the main test section by a 2 m long flexible pipe to help minimise transmission of vibrations.

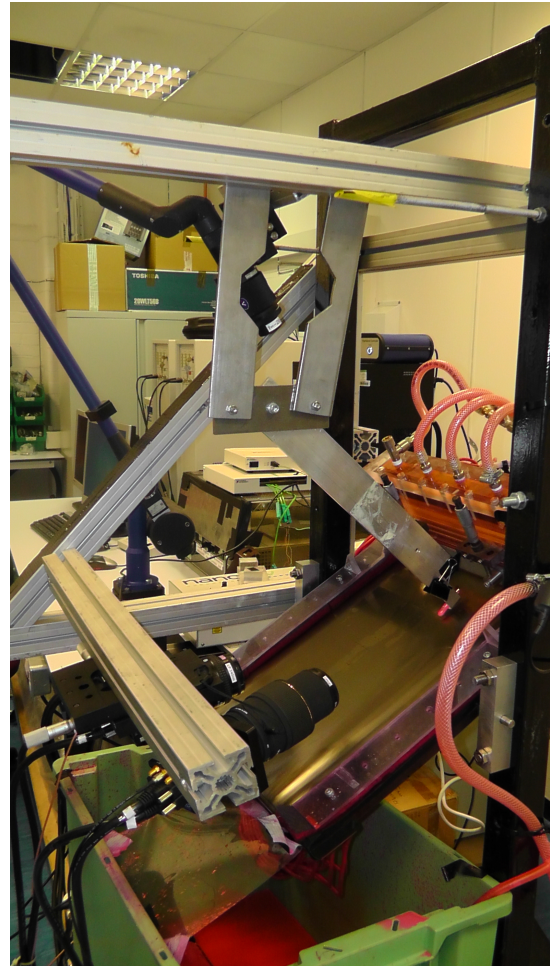
A gravity-fed system could have been used and would have provide lower vibrational noise to the distribution box input, however there would not have been enough back pressure to force the fluid through flow loop. The pump was connected to a variac to allow us to adjust its rotation speed. By minimising the rotational speed of the pump the vibrations would also be minimised. Two valves (4 and 5) were used to facilitate in priming the pump and to isolate the pump when not in use to ensure that air does not get back into the pump housing which would require the pump to be re-primed.

IV.2.2 Foil holder

The metal foil was held in place by clamping two ends of the foil between two large copper electrodes and a frame, referred to as the foil holder. In order to maintain a good clamping action on the foil, the copper electrodes were shaped so that the only points of contact occur along the long edges of the electrode as can be seen in Fig. IV.8. This



(a) Initial arrangement with both copper electrodes on the top surface of the foil and with the foil lying flat on the holder.



(b) Complete foil assembly, cameras and laser head.

Figure IV.7: Foil holder.

ensures that the foil will not slip out from the electrode, as would occur if the electrodes were completely flat. The foil holder was machined to the authors design out of a 20 mm thick Tufnol sheet. This material is a laminate of linen fabric held together by phenolic resin, and as such it is reasonably strong, corrosion resistant and electrically insulating. One of the electrodes was mounted on a sliding section on the foil holder which enabled the foil tension to be adjusted by tightening the two retaining screws of the slider.

The original intention was to keep the foil straight on the holder, to avoid warping, clamping both electrodes on to the same side of the foil as the film flow, as shown in Fig. IV.7(a). This however proved problematic as the hydrodynamic jump in the flow,

caused by the lower electrode, would backed up on the plate when operating at shallow angles. In the revised arrangement the foil holder was rotated by 180 degrees so that the slider was located at the top, instead of at the bottom of the test section. The foil was then wrapped around the bottom edge of the foil holder with the lower electrode clamping the foil to the under surface of the foil holder. The bottom edge of the foil holder was routed to a rounded profile so as to limit the radius of curvature that the foil experienced when being bent around the bottom edge, as this was found to minimise the formation of wrinkles in the foil. Thus, this rearrangement provided two benefits; as the electrode was no longer in the flow path it no longer disturbed the flow, and curving the foil round the bottom edge was found to help keep the foil straight.

IV.2.3 Liquid flow preparation

In order to provide well characterised, and reliable data it is necessary for the velocity distribution of the fluid flow to be uniform over the width of the foil. By ensuring a spatially uniform flow it can be guaranteed that the flow condition does not vary across the width of the test section. Uniform inflow with minimal velocity fluctuation or turbulence intensity in the flow will also be necessary if spatially coherent wave structure on the film are to be observed. Such waves will occur either as a result of the forced pulsation of the flow or from naturally occurring flow waves due to the interface instability over some critical flow condition.

To achieve a uniform flow distribution the cross-section of the flow must be expanded from that of the delivery pipe from the pump (~ 10 mm diameter, ~ 80 mm² area) to the relatively large and wide cross-section of the film (300 mm wide and 1–3 mm high, ~ 300 to 900 mm²) as equally as possible. In order to achieve this a large distribution box, constructed by the author out of 20 mm thick acrylic sheets, was positioned at the inlet of the test section. In the case of the experiment fluid is delivered from the pump, flow regulating valve gear and rotameters through a $\frac{3}{4}$ inch flexible pipe for 1.5 m up to the distribution box. Near the distribution box, to help improve the spreading of the flow across the width of the test section, the flow from the single pipe is then split by a manifold of T-junctions into 4 separate $\frac{1}{2}$ inch pipes which then inject the fluid at the top of the distribution box manifold, at regular 5 cm intervals along the width of the box (see Fig. IV.8).

Effectively the distribution box is acting like the settling chamber before the test section of a very low-speed, low Reynolds number, wind tunnel, and as such the design

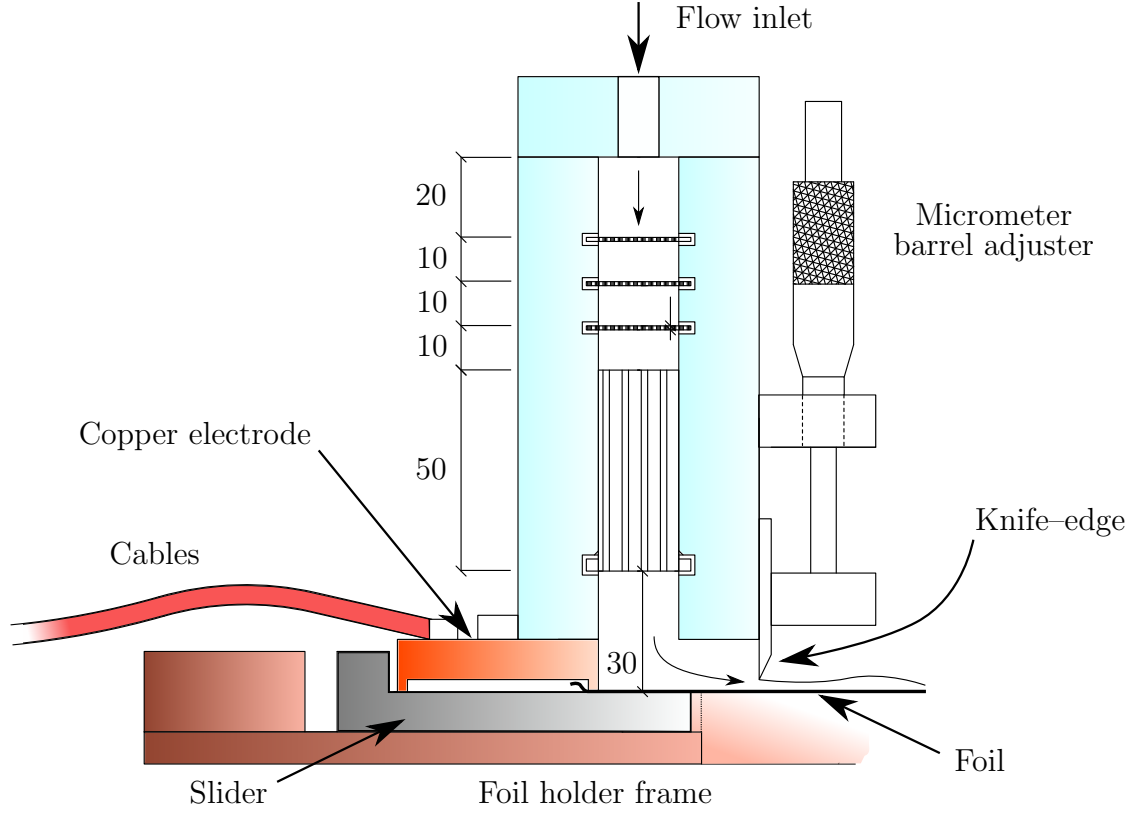


Figure IV.8: Flow distribution box. smooths and distributes the flow into an even film over the foil.

rules such as those presented by Mehta and Bradshaw [140] can be employed here.

To achieve a uniform flow profile large scale eddies and turbulence in the flow should be broken up and dissipated. In low-speed wind tunnels the large-scale swirling motions in the flow are disrupted with the placement of honeycomb sections in the settling chamber. Meshes are then employed to flatten the velocity profile, by imposing a pressure drop which is proportional to the flow speed squared [140, 141]. The resulting flow is a more uniform velocity profile over the cross-section of the settling chamber and has thinner boundary layers build up on the chamber and passageway walls. Additionally the meshes act to break up turbulent structures in the flow to smaller scales, which are then dissipate more quickly, reducing the turbulence intensity of the flow [141].

The cavity inside the distribution box was 300 mm wide, 20 mm deep and 130 mm long. Ideally the flow box would gradually expand out to the width of the meshes and honeycomb, however for such a large expansion (from 10 to 300 mm) the flow box would have to be inconveniently long. For this reason the flow was distributed by using a manifold and porous foam arrangement. At the top section of the box, the foam and first

CHAPTER IV. EXPERIMENTAL METHODS

mesh act as a manifold taking in the 4 fluid inlets and spreading their flows out across the width of the flow box. This arrangement was much shorter, but resulted in a larger pressure drop than a gradual expansion.

Three additional meshes and a honeycomb section were then employed to smooth the flow and flatten the velocity profile across the box. According to the empiric design rules of Mehta and Bradshaw [140], “meshes” or “screens” should be spaced by at least 0.2 chamber diameters apart to allow for the flow disturbances to decay. This equates to a spacing of about 4 mm or greater between successive meshes in the distribution box. Here a spacing of 10 mm between meshes was chosen. From this it can be seen that the residence time of the fluid in this space between meshes will be between 0.15 to 1.5 s depending on the flow-rate and between 4.5 to 45 s for the whole distribution box. These meshes should have an open area ratio greater than 0.57 to prevent a flow instability occurring due to the coalescence of jets emerging from the holes in the screen [140, 141]. The open area ratio of the screens used in the the distribution box were 0.58 so this type of flow instability should be avoided.

Additionally the range of the Reynolds number experienced in the distribution box,

$$\text{Re}_D = \frac{DU}{\nu_f}, \quad (\text{IV.4})$$

based on channel depth $D = 20$ mm and fluid kinematic viscosity ν_f and the average flow velocity in the box U , was between 30 and 550 for the water and water-ethanol mixtures and 1 to 60 for the flows involving glycerol (see Section IV.4). Interestingly the Reynolds number in the distribution box, is the same as the film flow Reynolds number based on film thickness,

$$\text{Re} = \frac{DU_{\text{box}}}{\nu_f} = \frac{\Gamma}{\nu_f} = \frac{dU_{\text{film}}}{\nu_f} \quad (\text{IV.5})$$

where Γ is the flow rate per unit width of the channel, d is the film height and, U_{film} and U_{box} are the average flow velocities in the film flow and distribution box respectively. This occurs between the two flows locations despite the change in cross-section as the width of the flow remains unchanged.

Turbulence in the distribution box

The turbulent fluctuations of the flow in the distribution box at such low Reynolds numbers should be very small. However it is important to understand how large these fluctuations may be, and whether the distribution box is achieving its purpose in re-

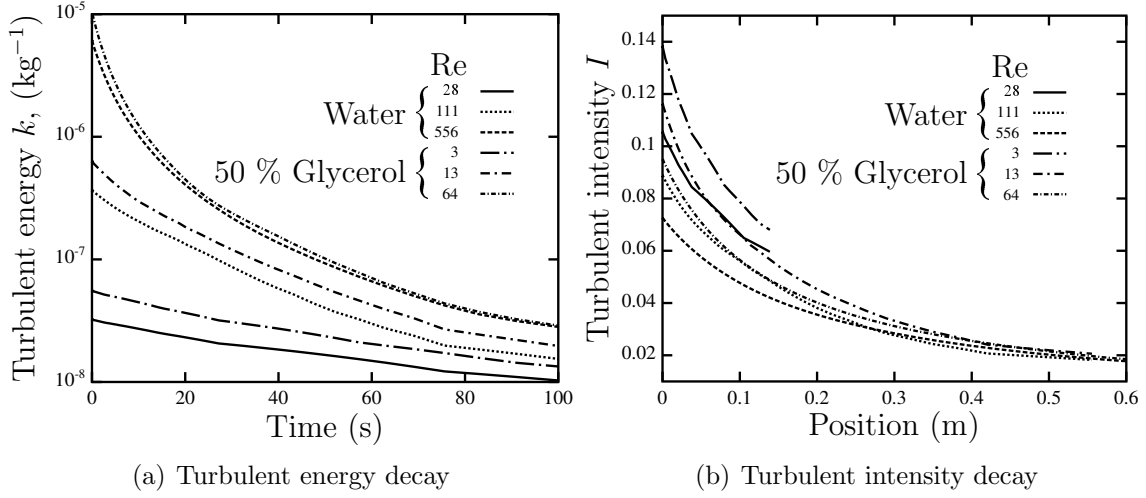


Figure IV.9: Approximate decay of turbulence in the distribution box based on a k - ϵ turbulence model.

ducing the overall turbulence intensity in the flow. In Groth and Johansson [141] an experiment was conducted using a wind tunnel to measure the turbulence intensity, and decay behaviour behind a mesh-like grid structure. It was shown that the square of the turbulence intensity behind the grid decayed with the inverse of the distance from the mesh, and an empirical relation for the decay of the turbulence was presented,

$$I^2 = \left(\frac{u}{U}\right)^2 = \frac{1}{25.2} \left(\frac{x}{M}\right)^{-1}. \quad (\text{IV.6})$$

Here $u = \langle u'^2 \rangle^{1/2}$ is the rms of the flow velocity fluctuation, U is the mean flow velocity, I is the turbulent intensity, x is the distance downstream of the mesh and M is the mesh width which is the distance between holes in the mesh. From Eq. IV.6 it can be seen that 10 mesh widths downstream of the screen the turbulence intensity is predicted to be 6 % whereas $x/M = 100$ down stream the turbulence will have diminished to 1.99 %. It is expected here however that the turbulence intensity in the flow box should be much lower as the measurements in Groth and Johansson [141] are for much higher Reynolds numbers than experienced in the distribution box.

An alternate method for estimating the turbulence intensity and the residence time in the flow box can be obtained by investigating the analytical relations for turbulence. In order to calculate the residence time of the turbulence it is important to calculate the rate of dissipation of the turbulent kinetic energy. According to Davidson [142] and

CHAPTER IV. EXPERIMENTAL METHODS

Lesieur [143] the turbulent kinetic energy in an isotropic turbulence can be estimated by,

$$k = \frac{3}{2}(u)^2, \quad (\text{IV.7})$$

where k is the kinetic energy in the fluid per unit mass and u is the rms of the flow velocity fluctuation as before. Dissipation occurs in a turbulent flow by the cascade of kinetic energy from the larger scale turbulent eddies to smaller eddies. This continues until a length scale is reached (the Kolmogorov micro-scale η) whereby the energy is dissipated by viscous forces [143] into heat. Hence the rate at which kinetic energy is dissipated is equal to the rate at which kinetic energy cascades down the length scales. At the largest eddy length scale, L_{turb} , the rate of dissipation is proportional to the total kinetic energy in the eddy, u^2 , over the eddy turnover time, L_{turb}/u . This gives a useful estimate of the turbulent dissipation rate,

$$\epsilon = C \frac{k^{\frac{3}{2}}}{L_{\text{turb}}} \quad (\text{IV.8})$$

where C is an unknown constant.

Unfortunately the constant in Eq. IV.8 would have to be found through experimentation, and so provides us with only a basic idea of the dissipation term. In Lesieur [143, Section 6.4.3] and others [144–146] the turbulent energy dissipation rate, ϵ , is estimated by considering the turbulent dissipation length scale l ,

$$\epsilon = C_{\mu}^{\frac{3}{4}} \frac{k^{\frac{3}{2}}}{l}. \quad (\text{IV.9})$$

where $C_{\mu} = 0.09$. Here the turbulence integral length scale, l , is approximated by using an empiric relation [144, 147] such that l is 0.07 the channel depth, or 1.5 mm in the case of the distribution box.

The evolution of turbulence in the fluid domain is governed by the following transport equation,

$$\frac{Dk}{Dt} + \nabla \cdot T' = P - \epsilon, \quad (\text{IV.10})$$

where $\nabla \cdot T'$ is the turbulent transport rate and P is the turbulence production rate. In the case where the turbulence is low and there is no production of turbulence we can neglect the P and $\nabla \cdot T'$ terms. By substituting Eq. IV.9 for the dissipation rate, a non linear ODE in k is obtained which can be integrated numerically.

Firstly, in order to integrate Eq. IV.10 the initial turbulence intensity must be es-

Apparatus

timated. According to Sandborn and Center [148] and others [144, 147] the initially turbulence intensity in to the flow box can be estimated from the empiric relation for turbulence intensity in a pipe flow,

$$I = \frac{u}{\bar{U}} = 0.16 \text{Re}_D^{-\frac{1}{8}}. \quad (\text{IV.11})$$

This gives an initially turbulence intensity of between 7 to 16 % for the Reynolds numbers 550 to 1 respectively. These Initial turbulent intensities appear to be slightly larger than those seen from Groth and Johansson [141], however this should be expected as the flow Reynolds numbers are significantly lower here. That is at lower flow rates the ratio of flow fluctuation and flow velocity will become greater.

By integrating Eq. IV.10 the decay of the turbulent kinetic energy k in the flow was estimated, and is shown in Fig. IV.9. It can be seen that at the lower Reynolds numbers the turbulent kinetic energy is very low and dissipates relatively slowly whereas at the higher Reynolds numbers the dissipation rate is higher. This generates in the interesting result that the turbulence intensity will have decayed to about half the initial intensity at the start of the flow box independent of flow speed, Fig. IV.9(b), (Though we are not Modelling turbulence generation here).

Another useful way to asses whether any turbulence will occur inside the flow box is to compare the flow box dimensions, $D \sim L_{\text{turb}}$, with the smallest turbulent eddy scale, the Kolmogorov micro scale,

$$\eta = \left(\frac{\nu}{\epsilon}\right)^{1/4} = L_{\text{turb}} \text{Re}_{\text{turb}}^{-3/4}. \quad (\text{IV.12})$$

Here Re_{turb} is the turbulent Reynolds number defined as uL_{turb}/ν . With the addition of Eq. IV.11 it can be seen that at a low flow Reynolds numbers, $\text{Re}_D < 8.12$, the Kolmogorov scale becomes larger than the length scale of the channel $\eta > L_{\text{turb}} \sim D$, its therefor highly likely that the flow condition will be laminar below this Reynolds number, even with the meshes, as any disturbance will decay rapidly.

Mesh open area ratio

According to Mehta and Bradshaw [140] the optimal open area ratio, β , for the screens or meshes is 0.57. If the area ratio is too low there will be a tendency for the flow profile to overshoot at the boundary layer and the pressure drop will be very high. An additional problem experienced with small open area ratios is the risk of exciting a flow

CHAPTER IV. EXPERIMENTAL METHODS

instability due to the coalescence of the fluid jets exiting the holes in the mesh [140, 141]. Conversely if the area ratio is too high more screens will be necessary to flatten the flow profile which can be counterproductive. The screen used here consisted of a thin metal sheet with 2 mm diameter holes at regular 3.5 mm intervals in a hexagonal pattern; this corresponds to an open area ratio, β , of 0.59, or a solidity of 41 %. Using the relation from Wieghardt [149] the ratio of pressure drop across the mesh to the dynamic pressure can be calculated

$$K_P = 6.5 \frac{1 - \beta}{\beta^2} \text{Re}_\beta^{-1/3}, \quad (\text{IV.13})$$

where $\text{Re}_\beta = \frac{Ud}{\beta\nu}$ is the Reynolds number based on the screen wire thickness d and the open area ratio β . The Reynolds number, Re_β , for the distribution box mesh are between 1 and 70 which is in the range of valid Reynolds numbers for Eq. IV.13 and gives a pressure loss ratio of between 8 and 2.

Honeycomb

In addition to the meshes, a honeycomb section was used to disrupt swirl and straighten the fluid flow. Mehta and Bradshaw [140] state that ideally the honeycomb should be 6-8 times the cell diameter to be effective. Shorter sections fail to straighten the flow adequately and longer sections of honeycomb will suffer from excessive boundary layer growth on the cell walls. To dissipate all lateral variations in the flow the cell diameter should be smaller than the lateral wavelength of any velocity variation approximately 1/150 the diameter of the settling chamber. Unfortunately the requirement for a cell diameter of > 2 mm is a little unattainable with the scale of the distribution box. As a compromise a honeycomb with a cell diameter of 6 mm and a cell length of 36 mm was used.

Flow contraction

After the settling chamber the flow experiences a reduction in cross-section area formed by the foil and the knife-edge at the mouth of the distribution box. The contraction ratio seen by the flow is at least a factor of 10 as the flow leaves the distribution box cavity with a depth of 20 mm and forms a film with a thickness of about 2 mm. The contraction increases the mean flow velocity and as a result the relative velocity fluctuations and lateral variations in flow velocity are reduced. The knife-edge height can be adjusted by two micrometer barrel adjusters at both ends of the distribution box to match the film thickness and to correct for any first order distortion in the flow across the width of the

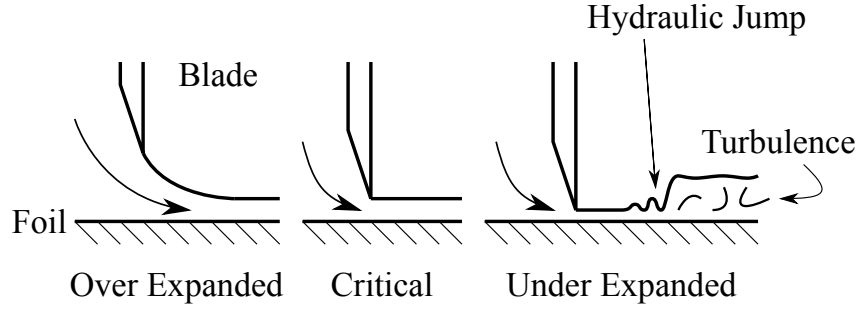


Figure IV.10: Various conditions experienced by the flow at the knife-edge at the distribution box outlet.

foil. It is important to be able to adjust the knife-edge height to match the film height as if the gap is too small a hydraulic jump will form, resulting in the generation of turbulence and unsteadiness in the flow, as illustrated in Fig. IV.10. Equally if the gap is too large the surface tension of the film will be insufficient to prevent air from flowing back into the distribution box. Such an event requires the box to be re-primed as air bubbles in the distribution box can disrupt the fluid flow. The correct positioning of the knife edge height for a certain flow condition was adjusted by eye. This approach made it possible to ascertain whether the flow was underexpanded due to the jump, which appeared in the form of flow disturbances on the surface of the film. Initially the knife edge was placed in a low position in order to allow the distribution box to fill with liquid and to prevent air leaking back into the distribution box. The knife edge was then raised until the flow was free from disturbances. The knife edge height was then read off the micrometer so that the same height could be reproduced when repeating runs of that flow condition.

Finally due to geometric constraints, because of the position of the clamping electrode and foil, a 90° turn in the flow direction close to the outlet was necessary to clear the electrode. This would be detrimental to the regularity of the flow, however as the channel width was much wider than the film height it was assumed that such a turn would not introduce a large irregularity to the flow, and as long as the flow-rate across the width of the channel could be made reasonable uniform up to this point, this was a reasonable compromise.

IV.2.4 Power supply

A high current switch mode DC power supply was used to provide electrical (resistive, or Joule) heating to the foil. The advantage of such a DC power supply over using

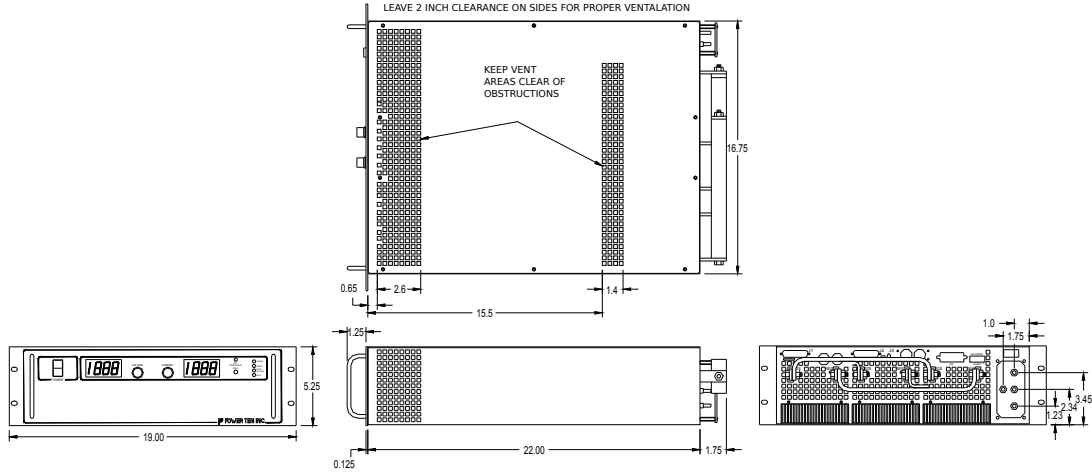


Figure IV.11: Power 10's P60-10100 2U rack mounted 10 kW high current DC power supply. Capable of outputting 1000 A at 10 V.

a variac or transformer is that the DC power supplied does not oscillate at the mains supply frequency. This is important as we are trying to measure fluctuations in the heat flux between the foil and the fluid film which would otherwise be obscured by the 50 Hz components and its harmonics and noise of an AC supply. In addition to this a switch mode power supply provides a high degree of control and stability over the applied power.

The Power 10, 2U rack mounted 10 kW, 1000 A DC power supply (see Fig. IV.11) manufactured by the Elgar Electronics Corporation [150] was sourced from a reclaim merchant in Singapore. Unfortunately this meant that the supply was configured to take a US mains style 3-phase input at 230 VAC line to line instead of the UK voltage rating of 415 VAC. For this reason a 3-phase step down transformer was required (see IV.2.4).

According to the manufacturers manual [150] the efficiency of the supply at full load is 80 % and the power factor is 0.72 minimum. From this the input power requirements can be calculated, the results of which are displayed in Table IV.2. It can be seen that an electrical rating of about 50 A per phase is needed for wiring and equipment connected to the input of the DC supply at peak power.

The output from the supply could be controlled in either current limiting or voltage limiting mode, by adjusting the current and voltage limit potentiometers on the front panel of the supply. The supply automatically switches between the modes so that the current and voltage never exceed their set limits. The output of the supply's voltage and current is displayed on the front panel of the unit. The current readout is accurate to

Apparatus

Table IV.2: Electrical requirements of the DC Power supply at full power.

Parameter	Value	Unit
DC Power Output	10	kW
Supply efficiency	0.8	-
Power factor	0.7	-
Real Power at input to supply	12.5	kW
Apparent Power	17.9	kV A
Reactive Power	12.8	kW
Phase Voltage	208	V
Phase Current	49.6	A

1 A and the voltage readout is accurate to 0.1 V.

To save on cost, the DC power supply was obtained from a second hand supplier and as such it was missing the bus bar connecting the output terminals and was in need of repair. The author personally performed the refurbishment of the supply, replacing a number of broken internal components and ensuring the electrical output of the supply was within specification. Additionally a set of new bus bars, shown in Fig. IV.12, to connect to the power supply outputs was machined along with the two copper clamping electrodes.

High current electrical connections

To heat the foil the current from the terminals on the back of the DC power supply (see Fig. IV.11) was collected by a bus bar and connected to the foil electrodes on the film flow rig. The power supply has three terminals for positive which need to be connected together and three terminals for the negative side of the supply. According to Ref. [151], a bus bar with a cross-section of 2 inches by $\frac{1}{2}$ inch should have sufficient ampacities to carry 1000 A, with only a 30° C rise in temperature above ambient. Rectangular copper bars with this cross-section were used to construct the two bus bars for connecting to the power supply and the copper electrodes for clamping the foil to the foil holder. In order to provide a flexible connection between the the bus bar and the foil electrode, so that the frame could easily be rotated, a bundle of 22 individual wires was used to connect each electrode to one of the corresponding bus bars (see Fig. IV.12(b)). Each wire was rated to carry a maximum of 30 A, which gives a total current carrying capacity of 660 A. Ultimately the maximum current passed through the foil was about 500 A so the current carrying capacity of the cables was sufficient.

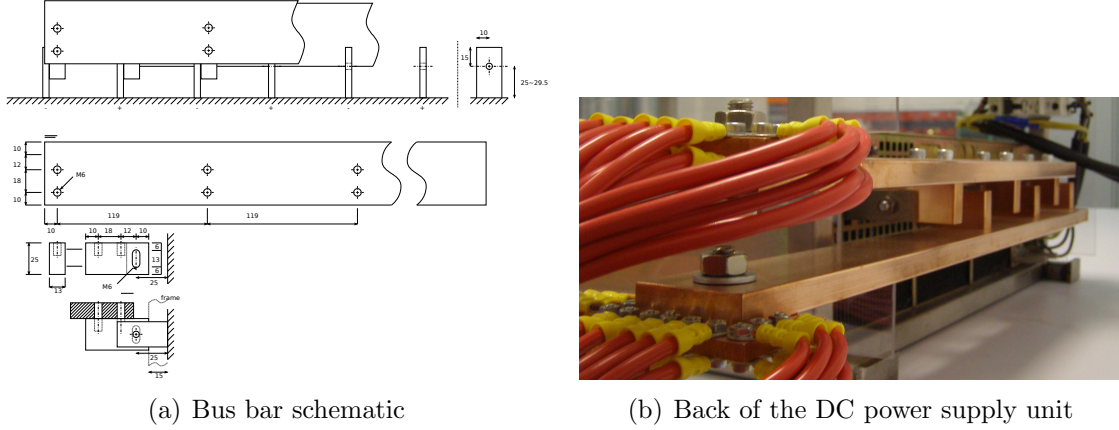


Figure IV.12: The copper bus bars which collect the current from the six output terminals on the DC power supply and connects them to the cables routing power to the foil electrodes.

The power supply provides a front panel voltage readout of the potential between its output terminals, however this will not take into account the losses in the cables. During the experiment the voltage was measured between the copper electrodes connected to the foil by the data acquisition unit (see Fig. IV.13). This is important as the power input to the foil needs to be accurately measured. From the experiments, Fig. IV.13, the nominal resistance of the connecting cables R_c and the total resistance of the foil R_f were found to be $540 \mu\Omega$ and $18.3 \text{ m}\Omega$ respectively.

It should be expected that when in operation the foil and cables will heat up; This will result in a slight change of resistance of the foil and cables. This effect is most apparent in the resistance of the cables and can be seen in the variation in cable resistance seen in Fig. IV.13(a) and is most likely due to the cables heating up between runs. The variation in foil resistance is less noticeable due to the fact the foil is being cooled by the film flow, and so is at a more consistent temperature. The power losses in the cables is about 3 % of the power delivered to the foil. When reporting the values of the heating power (or heat flux) it is important to correct for this power loss in the cables, reporting on the supplied to the foil not the total power from the DC power supply.

This change in resistance of the electrical cables and foil will change the power delivered to the foil during an experimental run. It is important to minimise this fluctuation in power delivery and ensure that the measurement of the power input to the foil is

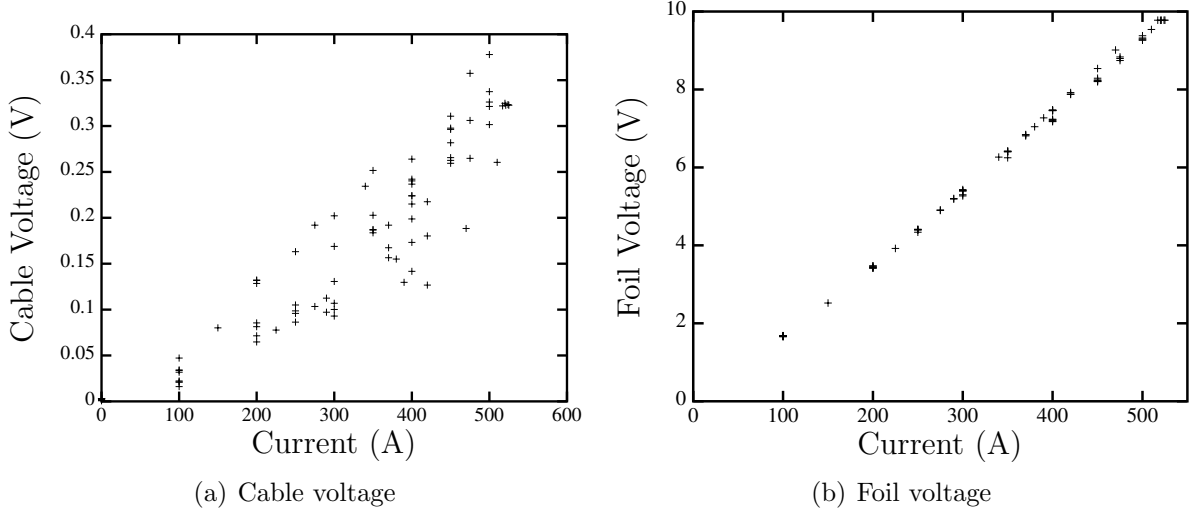


Figure IV.13: The voltage drop across the supply cables and foil at different supply currents.

accurate. In constant current mode the power delivered to the foil will be,

$$P_f = I^2(R_f + \epsilon_{R,f}), \quad (\text{IV.14})$$

where $\epsilon_{R,f}$ is a perturbation in the foil resistance due to say temperature change in the experiment. The sensitivity of the heating power to the foil as a result of $\epsilon_{R,f}$ will be,

$$\frac{dP_f}{d\epsilon_{R,f}} = I^2. \quad (\text{IV.15})$$

In contrast to constant current mode, in constant voltage mode the power delivered to the foil will be,

$$P = \frac{V^2}{R_f + \epsilon_{R,f}}, \quad (\text{IV.16})$$

and the sensitivity to the foil power to a perturbation in the foil resistance $\epsilon_{R,f}$ will be,

$$\frac{dP_f}{d\epsilon_{R,f}} = \frac{-V^2}{(R_f + \epsilon_{R,f})^2}. \quad (\text{IV.17})$$

By using the nominal power for each case we can compare the two modes. In constant voltage mode the nominal power is $P = V^2/R_{foil}$ and so the sensitivity will be,

$$\frac{dP_f}{d\epsilon_{R,f}} = -\frac{P_f R_f}{(R_f + \epsilon_{R,f})^2}. \quad (\text{IV.18})$$

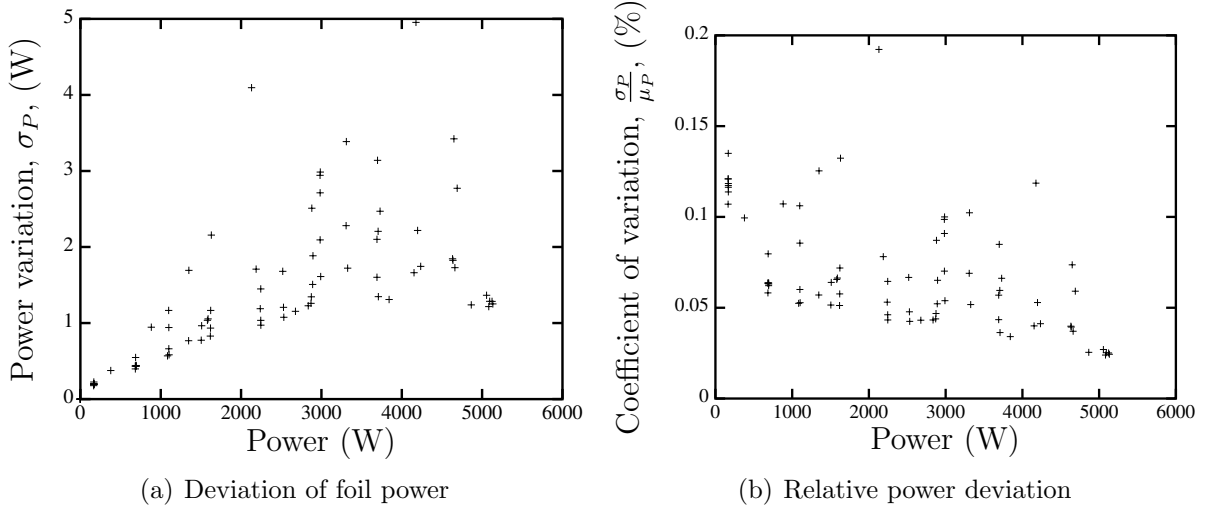


Figure IV.14: The temporal deviation of foil power at different applied powers.

In constant current mode the nominal power is $P = I^2 R_f$ giving a sensitivity of,

$$\frac{dP_f}{d\epsilon_{R,f}} = \frac{P_f}{R_f}. \quad (\text{IV.19})$$

This is interesting as it shows that the sensitivity of the power delivery in constant current mode is independent of the change in foil resistance. In contrast, when using constant voltage mode the change in power worsens as the resistance gets smaller. In addition, having the supply in current mode means the power delivered to the foil will be independent of changes to the cable resistance, due to heating, as the foil voltage is measured directly such that changes in the cable resistance or foil resistance will not effect the measured power input to the foil. To minimise measurement errors due to changes in the foil resistance the foil should be allowed to heat up to a quasi-steady state before each experimental run.

Figure IV.14(a) shows the deviation in the power supplied to the foil at different power inputs. The deviation in power is quite small relative to the total power input and the average coefficient of variation in the power supplied to the foil was about 0.07 %. This low value of variation means that, for example, the power supplied to the foil at 5 kW is stable to within 3 W.

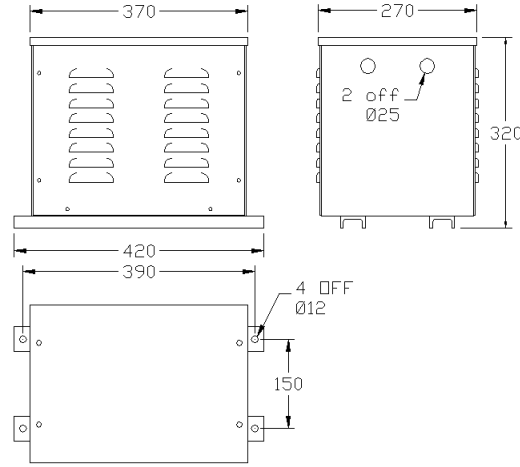


Figure IV.15: 3-phase 415 VAC to 210 VAC. transformer. Rated at 18 kV A.

Step down transformer

As mentioned previously the DC power supply used required a step down transformer to convert UK mains 415 VAC line voltage to 208 VAC US-style line voltage. Transformers are rated by the amount of apparent power (which is the product, $S = |V||I|$, of the maximum current and coil voltage) that the transformer is able to convert from one voltage to another. This is more useful than an absolute current rating for a transformer as it is independent of whether the limit applies to the the primary or secondary coil. It should be noted, however, that the rating is determined by the heating effect from the I^2 power losses in the coils, and losses in the transformer core due to magnetic effects and eddy losses, which should also scale with I^2 .

A transformer rating of 18 kV A is required to supply the DC power supply when operating at full output. To reduce cost and for a smaller lighter easier to move transformer a rating smaller than 18 kV A can be used as long as the transformer is operated intermittently with a duty cycle to allow for sufficient cooling. The transformer used here is only able to sustain an output of 18 kV A for 30 min before overheating and requires a duty cycle of 25 % between successive runs, i.e. 30 mins on 90 mins off, to maintain a safe operational temperature. This should be adequate for the purposes of the experiment as it takes less than a minute to reach the equilibrium set point, quasi-steady condition on the film flow, necessary before data acquisition.

CHAPTER IV. EXPERIMENTAL METHODS

Table IV.3: Electrical requirements step-down transformer.

Parameter	Value	Unit
Input	415	V
Output	208	V line - line
Rating continuous	9	kV A
Rating @25 % duty cycle	18	kV A
Max operation time @18 kV A	30	min
Input Current	25	A

As the transformer is limited by the heating from the the I^2 power losses the relationship between the continuous rating of the transformer and its rating at a duty cycle is,

$$S_{\text{cont.}} = S_{\text{cycle}} * \sqrt{\text{cycle}}. \quad (\text{IV.20})$$

From this and knowing that the duty cycle at 18 kV A is 0.25, it can be seen that the contiguous rating for this transformer is 9 kV A. The total wall plug rating required for the 3-phase supply to the transformer when operating at the maximum output, 18 kV A, is 25 A.

Electrical safety

High power electrical systems can expose the user to the risk of electric shock, and in the case of equipment failure presents a fire risk. The power supply is connected to a high current high voltage mains electricity supply and should be treated with caution.

The nature of the experiment, whereby the fluid is in direct contact with the electrically energised foil can prove hazardous. The low impedance, high current output of the supply can be particularly dangerous if a circuit is accidental made across the body of the operator, or bystander, as only a small current can prove fatal, even at low voltages. It also is worth noting that although the maximum output voltage is only 10 V, this voltage may not be fully isolated from the mains, and the negative supply voltage could be up to 300 V from earth.

For these reasons, to protect the user and the experiment, adequate circuit protection and safety precautions are required. In this work, a 63 A circuit breaker and residual current device was used at the 230 VAC input to the supply to act as short circuit protection in case of equipment failure and to protect the operator from any electrical leakage through them to earth. It was also important that there was a preferential low

Apparatus

impedance current path (provided by the foil) connected between the output terminals, and that the negative terminal had a path to ground. All other conductive parts, such as the frame and pipework were earthed. The operator also avoided contact with the fluid when the power supply was in use. In addition an emergency isolation switch was located in an accessible location near to the working area, in order to isolate the supply if needed.

IV.2.5 LIF system

A LIF/PIV system manufactured by LaVision was used for the measurement of the film thickness and temperature by imaging the emission from a fluorescence dye in the fluid. Here LIF stands for Laser Induced Fluorescence, and PIV stands for Particle Image Velocity. This system, and its software, was designed to measure the velocity field of a fluid flow by seeding the fluid with small ($\sim 50\text{ }\mu\text{m}$ diameter) reflective glass beads or fluorescent particles. By illuminating the flow with a sheet of light from a laser, and capturing and comparing images of the illuminated particles, the fluid velocity in the plane of the light sheet can be measured.

In the presented work the camera and laser system was customised to measure the temperature and interface height of a fluid film flowing down a heated plate. Instead of using particles to seed the flow, a temperature dependant fluorescent dye, Rhodamine B, was used to stain the fluid. By illuminating the flow with the laser light sheet a cross section of the film could be imaged from the fluorescent emission of the dye. The intensity of the fluorescence allowed the temperature of the film to be deduced, and the location of the film interface was measured by locating the film interface in the captured images. A custom algorithm was developed to identify the location of the interface, and the intensity of the light which was then used to evaluate the fluid temperature.

Laser optics

The light source for the system consisted of a dual cavity Nd:Yag frequency doubled 532 nm laser. The pulse width of the laser output pulse was between 5–8 ns with each pulse contained up to 50 mJ of energy and the maximum attainable pulse rate was 100 Hz per laser cavity.

To transform the circular beam of light emitted by the laser into a thin planar sheet of light a sheet optic laser head provided by LaVision was used. This laser head consists of a number of lenses which act to shape and focus the light. The minimum achievable

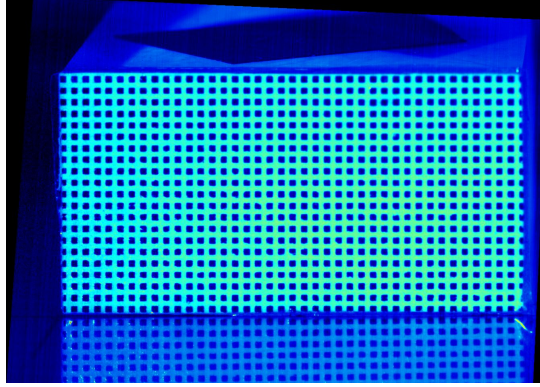


Figure IV.16: Calibration graticule for correcting and calibrating against the distortion of the LIF images captured by the CMOS sensors.

thickness of the light sheet when focused on the target area was 0.3 mm and the divergence of the sheet's width is about 20° from the laser head. A beam guide or "guiding arm", the blue tubular structure seen in the background of Fig. IV.7(b), was used to connect the output of the laser to the sheet optics. This guiding arm consisted of a series of tubes hinges and mirrors which allows the laser beam to be pointed in almost any direction. This has the advantage that the laser sheet can be easily positioned and adjusted, and the laser beam is safely concealed within the guiding arm before being expanded by the sheet optics. The laser head was then mounted on a "linear transition stage" so that the laser sheet position could be adjusted, with a high degree of accuracy, in the lateral direction. This stage was then held in the required location relative to the frame and foil holder using extruded aluminium section, as can be seen in Fig. IV.7(b).

Image capture

Images of the emission from a fluorescent dye dissolved in the fluid and illuminated by the laser light sheet were captured using two 1280 x 1024 pixel CCD 8-bit high-speed cameras. The exposure time used when capturing images of the film was set to 1000 μ s, which was long enough to have a large time window for the firing of the laser pulse, yet small enough that the image was insensitive to external light sources, that is anything but the flash of light from the laser pulse. The timing of the image frames was determined by the laser pulse. Two cameras were used to expand the field of view of the films interface being captured and increase the effective spatial resolution.

Only the light from the fluorescent emission is needed in the captured images as light from the laser contains no information about the temperature of the fluid. The laser light

Apparatus

scattered directly off the film surface, and fluid-foil interface contained many reflections, was quite noisy and could be intermittent due to deformation in the film surface. Also any light at the wave length of the laser which did reach the camera was at a much higher intensity than the fluorescence emission from the dye which would overwhelm the desired signal from the fluorescence. Therefor a long pass dichroic interference filter was fitted to the cameras. This filter only passed light with wavelengths above 540 nm, which includes the light from the fluorescent emission, but blocked the light at the the wavelength of the laser, 532 nm.

Due to the maximum pulse rate of the laser the LIF measurements can only be made at up to 100 Hz which corresponds to the measurements of signals with frequencies of up to a 50 Hz, the Nyquist frequency. It is important to note that no phase information can be deduced at the Nyquist frequencies, that is the phase of the signal will be measured as either 0° in phase or 180° out of phase with the measurement. It is therefor unsuitable to measure signals at 50 Hz, to be able to resolve the shape and phase of any fluctuation adequately it is best to be able to measure at least 2-3 harmonics of the signal.

On each laser pulse the LIF system provides the synchronisation signal for the IR camera and the DAQ system so that the IR image capture and data acquisition occurred at the same time as the laser pulse. When using the Cedip camera, which had a maximum frame rate of 50 Hz a frequency divider circuit was used to half the triggering frequency for that camera.

LIF image camera correction

When taking a picture of the planar laser sheet, the projected image on the CCD sensor of the camera will have undergone significant distortion. To correct for this the images were transformed back onto the coordinates of the laser plane using the pinhole camera projection model. In order to undertake this inverse transform and correct for the image distortion the camera model was calibrated using an image calibration graticule. The graticule, seen in Fig. IV.16, consisted of a precision machined block of acrylic with graticule markings in a regular cubic pattern. Each graticule mark was 0.5 mm thick and the spacing between markings was 1.0 mm. The accuracy of the calibration was able to correct for the distortion within a pixel or around 50 μm .



(a) Cedip Titanium



(b) FLIR SC3000

Figure IV.17: The IR cameras used for the measurement of the temperature of the back side of the foil.

IV.2.6 IR thermography

To facilitate with the measurements of the temperature on the back of the foil an IR camera was loaned from the EPSRC Engineering Equipment Pool (EPI). The equipment pool has two camera models available for loan which meet the measurement requirements. These were a Cedip Titanium and the FLIR SC3000 (see Fig. IV.17). Over the course of the project, depending on availability, one of either camera was used to make the temperature measurements on the back surface of the foil.

An IR thermal imaging works by measuring the incident infrared thermal radiation emitted from a surface which can be used to infer the temperature of that surface. The radiative energy emitted by a surface can be calculated using Stefan–Boltzmann law for thermal radiation on a grey body [152],

$$I = \varsigma \varepsilon T^4. \quad (\text{IV.21})$$

where I is the incident thermal radiation, $\varsigma = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$ is the Stefan–Boltzman constant, and ε is the emissivity of the surface of the material. The incident radiation seen by the camera will be a combination of the radiation emitted from the surface and any reflected radiation from the background and surroundings (see Fig. IV.18).

$$I_{\text{IR}} = \varsigma \varepsilon T_{\text{w}}^4 + \varsigma (1 - \varepsilon) I_{\text{bg}} \quad (\text{IV.22})$$

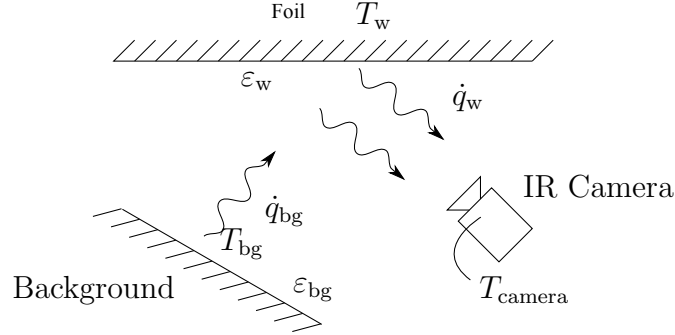


Figure IV.18: Effect of background radiation inclination angle and emissivity on the IR camera temperature measurement.

Bare polished titanium metal has a relatively low emissivity of about 0.1 [152]. Nevertheless due to a thin oxide layer that forms on the metal the emissivity of the unpolished foil is slightly higher at about 0.3. Still the IR camera must be offset at an angle to the foil to prevent the camera capturing an image of itself due to reflections from the foil surface. To prevent imaging of further reflections of the environment the area that would be in the line of sight of the camera due to the reflection from the foil was obscured by black card backed with aluminium. The aluminium blocked any other sources of IR radiation and contributed to keep the temperature of the card even by spreading any heat. As the card is carbon black it has an emissivity of approximately 1 so any radiation from its surface is only a result of its temperature, T_{bg} , thus suppressing signals from further reflections. The IR radiation measured by the camera is therefore a combination of this background radiation and the emission from the foil due to its temperature. The actual surface temperature as a function of the apparent temperature from the IR camera can be calculated from,

$$T_{\text{apparent}}^4 = \epsilon T_w^4 + (1 - \epsilon) T_{bg}^4. \quad (\text{IV.23})$$

The relationship between the apparent temperature and the real temperature of the foil was checked and calibrated by using thermocouples on the foil. As an additional reference point, thermal targets consisting of squares of electrical tape, which is known to have a high emissivity, were positioned on the foil. By measuring the temperature of the tape with the IR camera, which would be very close to its true temperature, this could be used to find the correct foil mean temperature, and so the correct the local emissivity of the foil.

CHAPTER IV. EXPERIMENTAL METHODS

Table IV.4: IR camera specifications.

Imager	Cedip Titanium	FLIR SC3000	
resolution	640×512	320×240	pixel
full frame rate	100	50	Hz
max frame rate	4980 (@16×4)	750	Hz
accuracy	±1 % or 1 °C		
thermal sensitivity	4.20	20	mK
wavelength range	3.6–5.1	8.0–9.0	μm

IR camera specifications

The specification of each camera used is listed in Table IV.4. The thermal sensitivity is a measure of the noise in the output from each pixel of the sensor which is an indication of how repeatable the temperature measurements are and what level of temperature fluctuation can be measured reliably, whereas the accuracy shows how close the measurement is to the actual temperature of the surface. This means that whereas the camera can only measure the temperature to an accuracy of within 1 K, changes in the surface temperature from 4 mK or 20 mK can be measured.

IR image camera correction

As with the LIF images an image correction had to be employed to correct for the perspective distortion of the foil to the image plane [153]. Here the perspective transformation from the plane of the foil to the image plane of the camera is modelled as the homogeneous matrix operation:

$$\mathbf{X} \sim \begin{bmatrix} \frac{1}{f}\mathbf{R} & \mathbf{t} \\ \mathbf{p}^T & 1 \end{bmatrix} \tilde{\mathbf{X}}. \quad (\text{IV.24})$$

Where the points on the foil are represented by the vector $\tilde{\mathbf{X}} = (X, Y, W)^T$ and the points on the camera image are $\tilde{\mathbf{X}} = (x, y, 1)^T$. The matrix in Eq. IV.24 represents the affine transform for projecting the foil plane onto an image plane and has been broken down into several constituent parts. $\mathbf{R}$, represents the rotation and scaling between the two planes, f is the focal length, $\mathbf{t}$ is the vector relating the translation between the two planes and $\mathbf{p}$ is related to the focal point of the image. As the projection is an affine transform the scale of the matrix does not effect the evaluation, and so the matrix only has eight degrees of freedom. Therefore the matrix can be fully determined by knowing

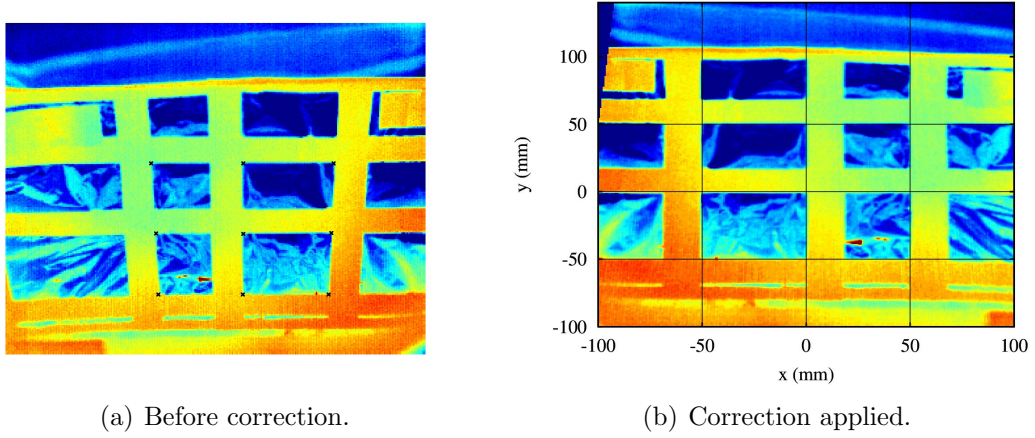


Figure IV.19: Calibration plate for correcting the perspective distortion of the IR images.

only 4 coordinates (a coordinate consists of two values) on both the foil and image plane.

To calibrate the camera model a graticule, shown in Fig. IV.19, was consisted from a sheet of thick foil covered card. Electrical tape was then used to make the graticule lines, which gave a high contrast ratio when observed by the IR camera. The graticule was held onto the foil using magnets and was then carefully aligned to markings on the foil holder which in turn were used to align the LIF cameras and laser sheet. The lines of tape were aligned onto the card using a ruler and constructed so that the tape intersected at 9 control points, in a grid with a spacing of 50 mm. These control points were then used to calculate the transform matrix in Eq. IV.24 [153]. By using 9 control points instead of the minimum of 4 the error in the prediction of the transform matrix is reduced, however the definition of the matrix is over constrained and was solved by using a least squares approach. The accuracy of spatial alignment between the LIF image and the IR image is within 2 mm.

IV.2.7 Data acquisition

A number of K-type thermocouples were used to measure the fluid temperature in the distribution box and the temperature of the foil at a number of locations. These were used for calibration of the IR and LIR temperature measurements, and to monitor and correct the accuracy of these measurements through the experimental runs. In addition to this a digital ultrasonic flow meter was used to measure the flow-rate and pulsation amplitude of fluid going into the distribution box.

CHAPTER IV. EXPERIMENTAL METHODS

The K-thermocouples work by the Seebeck effect, by which the junction between two dissimilar metals will produce a small voltage when heated. In order to measure this voltage another pair of junctions are made at the cold end of the thermocouple, and the temperature of this junction T_{cj} was measured with a high accuracy thermistor. The voltage produced by the thermocouple is proportional to the difference between the cold and the hot junctions, and effectively obeys a linear relationship in the temperature range of interest,

$$V = (T - T_{cj}) * 41 \text{ } \mu\text{V K}^{-1}. \quad (\text{IV.25})$$

An accuracy of 0.1 K can be expected from the thermocouple measurements.

The ultrasonic flow meter works by measuring the Doppler shift of sound due to the motion of the fluid. By measuring the transit time of an ultrasonic sound pulse in both directions across an in-line cell, which the fluid is flowing through the device is able to compensate for changes in the fluid composition and temperature. This is because the down flow velocity of the sound pulse is $c + U_f$ and the up-flow velocity of the is $c - U_f$, the difference between these two velocities is U_f which is independent of the speed of sound in the fluid c . The meter outputs a pulse for every 1 mL and has an accuracy of 0.1 L min^{-1} .

To collect and record the information from these sensors a National Instruments USB data acquisition card and associated support board was used to measure the thermocouple voltages, foil voltage, and record the pulse train from the digital ultrasonic flow meter. The board was configured to start recording data on a trigger signal from the LIF system, and data points were recorded upon each sync pulse.

The data acquisition (DAQ) card had 32 analog inputs which were sampled at 16-bit resolution. The voltage range of the input was selectable from $[-100, 100] \text{ mV}$ to $[-10, 10] \text{ V}$. The corresponding accuracy of the voltage measurement was $52 \text{ } \mu\text{V}$ and 1.92 mV respectively. As the thermocouple voltage would be of the order of the lowest range accuracy, a National Instruments thermocouple preamplifier, with cold junction compensation, was used to boost the thermocouple signal into the measurement range of the DAQ card. The preamplifier had a gain of 100 accurate to 0.08 % with a non-linearity of 0.004 %, and the accuracy of the cold-junction thermistor was $0.4 \text{ } ^\circ\text{C}$.

IV.3 Foil Material Selection

The foil selected for the experiments needs to comply with a number of requirements. Firstly, the resistance of the foil needed to match the power supply output so that the

Foil Material Selection

Table IV.5: Properties of materials considered for the foil [1].

Material	Electrical resistivity ρ [Ω m]	Heat capacity c_P [J kg ⁻¹ K ⁻¹]	Thermal conductivity κ [W m ⁻¹ K ⁻¹]	Density ρ [kg m ⁻³]	Thermal diffusivity α @ 25°C [m ² s ⁻¹]
Stainless Steel	1.18×10^{-7}	450	16.3	7870	4.6×10^{-6}
Titanium	4.20×10^{-7}	503	21.9	4510	9.7×10^{-6}
Bronze	1.68×10^{-8}	385	42	8920	1.2×10^{-5}
Steel	9.61×10^{-8}	450	50	7870	1.4×10^{-5}
Gold	2.44×10^{-8}	840	318	19 300	2.0×10^{-5}
Iron	9.61×10^{-8}	450	80	7870	2.3×10^{-5}
Nickel	6.99×10^{-8}	444	90.9	8910	2.3×10^{-5}
Lead	2.20×10^{-7}	127	35.3	11 300	2.5×10^{-5}
Brass					
Cu 70 %	3.50×10^{-8}	385	110	8920	3.2×10^{-5}
Brass					
Cu 68 %	3.50×10^{-8}	385	125	8920	3.6×10^{-5}
Tin	1.09×10^{-7}	228	66.8	7290	4.0×10^{-5}
Zinc	5.90×10^{-8}	388	116	7140	4.2×10^{-5}
Tungsten	5.60×10^{-8}	134	173	19 300	6.7×10^{-5}
Magnesium	4.39×10^{-8}	1020	156	1740	8.8×10^{-5}
Aluminium	2.65×10^{-8}	897	220	2700	9.1×10^{-5}
Copper	1.68×10^{-8}	385	370	8920	1.1×10^{-4}
Graphite	1.38×10^{-6}	710	165	2100	1.1×10^{-4}
Silver	1.59×10^{-8}	233	418	10 500	1.7×10^{-4}
Calcium	3.30×10^{-8}	647	201	1540	2.0×10^{-4}

maximum power could be attained. The foil also needed to be thin enough to allow the temperature fluctuations of the liquid film to be measured from its under surface, but thick enough to allow finite heat flux fluctuations and the conjugate effects to be measured. A list of materials considered for the experiment, which were readily available in foil form and with large enough dimensions, is presented in Table IV.5.

It was desirable to keep the foil voltage low in order to prevent excessive electrolysis or breakdown occurring in the fluid, and also from the perspective of safety. However, if the foil resistance was too low it would become difficult to provide sufficient current through the foil. This is because power has to be stepped down from the mains supply and if the output voltage is very low, the cost of the variac or DC power supply required would be high. Additionally the currents involved would require an inconveniently large

set of bus bars and cables to avoid heating losses there.

The DC power supply used, as seen in Section IV.2.4, has the lowest voltage rating for its power output in its product range. This assisted with the material selection process, as metals tend to have low resistivities. To match the foil perfectly to the power supply, and thus to maximise the achievable power input to the foil, the total foil resistance should be about $10 \text{ V}/1000 \text{ A} = 10 \text{ m}\Omega$. The foil resistance can be found from the foil dimensions and material resistivity ρ as,

$$R = \frac{\rho L}{W a}, \quad (\text{IV.26})$$

where L is the foil length, W the foil width and a its thickness.

The second requirement on the foil was that it should be thin enough to allow the temperature and heat flux fluctuations on its fluid side to be evaluated from the IR measurements on its underside (see Section IV.8.1) yet thick enough to permit some conjugate effects to arise. A dimensional wall thickness, a^* , of about 0.1 was estimated as being a sufficient compromise.

The resulting foil resistance, R , is plotted against the normalised foil thermal thickness, $\frac{a}{\delta}$, at a nominal wave frequency of 10 Hz in Fig. IV.20. Most foils which can be commercially sourced come in certain standard thicknesses. The range of sizes depends on the material, but large sheets of materials with thicknesses as thin as 25 μm can be obtained relatively easily and at a reasonable cost. The plot in Fig. IV.20(a) shows the resistance and dimensionless thermal thickness of a selection of foils, which could be used for the heating element on the experiment. In this plot a range of foil thicknesses between 25 and 1000 μm was assumed. It becomes immediately apparent that the majority of the materials considered have a resistivity that is far too low, even for the obtained power supply. For instance an aluminium foil with the same thermal thickness $\frac{a}{\delta}$ as a titanium foil requires several orders of magnitude more current, (and a much lower supply voltage). This presents a significant problem in terms of supplying the foil with such high currents for heating since any bus bars and cables would need to be very large to prevent excessive power losses.

Titanium and graphite provide the best match to the power supply and geometric configuration. Interestingly a graphite “foil” would be significantly thicker than the equivalent titanium foil, however the structural strength of such a “foil” was uncertain. In addition it was not known to what extent the thermal and electrical properties of graphite foil would be anisotropic due to the layered molecular structure of graphite. For

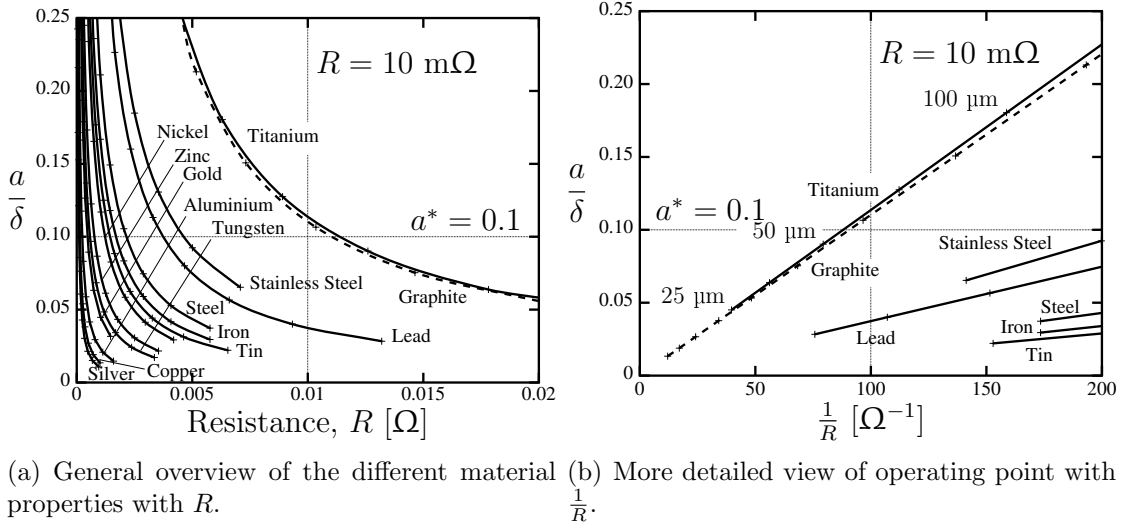


Figure IV.20: Material selection charts. Foil dimensionless thickness $\frac{a}{\delta}$ at a nominal wave frequency of 10 Hz shown as a function of foil resistance, R , for a foil width $W = 300$ mm and length, $L = 450$ mm and a range of thicknesses, $a = 25$ μm to 1 mm.

this reason a titanium foil with a 50 μm thickness (marked on Fig. IV.20(b)) was chosen. An additional benefit of this selection is that titanium is corrosion resistant and has a high work function which promotes wettability.

IV.4 Fluid Selection

Previous investigations have used silicon oils [113, 120], as they have a greater propensity to wet the surface of the solid substrates. Additionally, silicon oils can be manufactured with a large range of viscosities so enable a large range of Reynolds numbers to be covered and effectively undergo no evaporation when heated. However it proved to be hard to find a suitable temperature sensitive fluorescent dye for 532 nm light source, as required by the desired optical temperature measurement technique. These oils also proved to be considerably more expensive than other options. More common oils such as paraffin could be used, however they present a fire and vapour risk, due to the heating arrangement.

A selection of fluids, and their physical characteristics, considered for use in the experiment is shown in Table IV.6. For comparison the Reynolds number, Marangoni number and Kapitza number was calculated for each fluid for nominal experimental conditions. The Marangoni number,

$$\text{Mg} = -\frac{d\sigma_f}{dT} \frac{1}{\mu_f \alpha_f} D \Delta T, \quad (\text{IV.27})$$

is a ratio of convective flow driven by temperature induced surface tension gradients on the surface of the fluid, and the inertial forces in the fluid, as a result of the Marangoni effect. The Kapitza number,

$$\text{Ka} = \frac{\sigma_f}{\rho_f \nu_f^{\frac{4}{3}} g^{\frac{1}{3}}}, \quad (\text{IV.28})$$

is the ratio of surface tension forces to inertial forces which is an indicator of the hydrodynamic waves regime. It is interesting that water–ethanol mixtures and low viscosity silicon oils all exhibit high Marangoni numbers.

In the case of using paraffin oil the entire test section should be sealed. This would be required to prevent the contamination of the lab due to the oil evaporating from the foil and then condensing else where, or even from splashing of the fluid, and creep flow over surfaces due to the low viscosities and surface tension. For comparison even with the water and water–ethanol water–glycerol solutions which were eventually used with an open test section, there was noticeable contamination in the form of staining from the dye on the lab floor passed the drip tray. This would have proved to be unsafe in the case of oils, as the floors would become slippery, and there possibility of spreading the contamination further due to the operator walking would be high.

Having a completely closed test section would provide some difficulties in the chosen observation technique, due to reflections. Additionally the lack of access to the foil with an enclosed test section means that very little can be done to re-wet the foil when the film surface becomes broken.

Water was chosen for the current study because of its non-toxic and non-hazardous nature. In fact, deionised water was preferred as it is free from particulates, ions and surfactants, which may affect its surface tension properties. In addition, since deionised water has no ions it reduces the rate of corrosion in the various parts of the flow loop, and prevents electrolysis occurring on the electrically energised foil, which would lead to changes in the properties of the fluid and the foil surface. Also if electrolysis were to occur, a proportion of the electrical power supplied would be dissipated in the fluid instead of the foil due to the resistive losses involved in the movement of ions through the fluid. This would change the overall arrangement of the heat transfer problem.

Water has a very high surface tension which means it is difficult to keep the fluid film to completely wet the foil surface under heating. This presents a problem as the experiment exhibits a failure mode of catastrophic runaway if the film becomes broken as the un-wet section of foil heats up rapidly further inhibiting wetting. The un-wet foil temperature can heat up rapidly above even the fluids boiling temperature preventing

Fluid Selection

Table IV.6: Possible fluid properties selection table. Where ρ_f is the fluid density, μ_f is the dynamic viscosity, ν_f is the kinematic viscosity, σ_f is the surface tension of the fluid at 20 °C, $d\sigma/dT$ is the rate of change of surface tension with temperature at 20 °C, c_f is the heat capacity of the fluid, κ_f is the thermal conductivity of the fluid, and n is the refractive index of the fluid. Some comparative Reynolds number, Re (at the lowest flow-rate 1 L min⁻¹), Kapitza number Ka, and Marangoni numbers Mg for the flows are also shown.

	ρ_f $\frac{\text{kg}}{\text{m}^3}$	μ_f Pa s	ν_f cSt	σ_f $\frac{\text{mN}}{\text{m}}$	$\frac{d\sigma_f}{dT}$ $\frac{\text{mN}}{\text{m K}}$	c_f $\frac{\text{JK}}{\text{kg}}$	κ_f $\frac{\text{W}}{\text{m}^2 \text{K}}$	n	Re	Ka	Mg
water	1000	0.000 890	0.890	72.8	−0.17	4180	0.56	1.33	62.4	3970	1690
Glycerol	1260	1.41	1120	63.4	−0.08	1620	0.29	1.47	0.05	0.20	1.1
50/50	1150	0.0084	7.32	68.1	−0.13	2900	0.29	1.40	7.6	195	268
Ethanol	789	0.001 20	1.52	12	−0.08	2420	0.17	1.36	36.5	406	2050
10 %	987	0.001 00	1.01	51	−0.14	4010	0.52	1.34	54.8	2370	320 000
15 %	982	0.001 25	1.27	46.5	−0.12	3920	0.5	1.34	43.7	1600	915 000
20 %	978	0.001 50	1.53	42	−0.09	3830	0.48	1.34	36.2	1130	639 000
Silicone oils											
1.5c	853	0.0012	1.50	18.0	−0.03	1550	0.10	2.18	37.0	574	1420
5c	918	0.0045	5.00	19.7	−0.03	1550	0.12	2.49	11.1	117	331
20c	950	0.019	20.0	20.7	−0.03	1550	0.14	2.68	2.8	18.8	68.6
100c	967	0.096	100	20.9	−0.03	1550	0.15	1.4	0.56	2.18	12.6
Paraffin	810	0.0016	2.0	27.0	−0.12	1860	0.15	1.45	27.4	607	2350

re-wetting and growing the de-wet rejoin and will eventual result in damaging the foil as it overheats, warps and oxidises. Additionally as the foil heats up it expands and this will course the foil to warp, and tends to bulge the foil out of the film. Care must be taken to prevent this condition, and power reduced with the onset of dewetting immediately.

To counter this effect solutions of water with 5, 10 , and 20 % ethanol were also used, which increased the fluids wettability and so increased the power input achievable before dewetting occurred. A 50 % glycerol solution was also tried, this reduced the film flow Reynolds number significantly and so correspondingly increased the film thickness due to the larger viscosity which also helped preventing the dewetting condition. It should be noted that ethanol–water mixtures have higher viscosity than there pure constituents components, as shown by Satomi et al. [154]. The peak viscosity being attained for a concentration of about 35 %.

Silicon oils have a transmission bandwidth from 200 to 2600 nm which means that for the IR cameras silicone oil would effectively be opaque. Similarly for water the absorption of IR radiation is approximately 100 cm^{-1} between the wavelength of 3.6 to 5.1 μm , the IR pass band for the Cedip camera (see table IV.17). The absorption at a wavelength of 8 to 9 μm (the FLIR camera) is about 500 cm^{-1} . This corresponds to an attenuation of 10 to 50 for any IR radiation from the foil surface passing through a fluid film of water, depending on which camera was used. So again water is effectively opaque to transmitted IR radiation from the foil and should have a high radiative emissivity, i.e. temperature measurements using the IR camera should correlate well with the temperature of the fluids surface.

IV.5 Measurement Technique

As stated previously in Section IV.1, the aim of the experiment was to measure the heat transfer process in a falling film flow. The temperature of the fluid film was to be determined using a LIF technique, and the temperature and heat flux at the fluid–solid interface was to be determined by making IR temperature measurements on the back surface of the foil.

To make a measurement the desired fluid flow rate through the test section was set, and any air was purged from the distribution box. If any pulsation was required at the inlet, the oscillator valve, Section IV.2.1, was used and the pulsation frequency was controlled by a function generator. Power was then provided to the foil using the DC supply (see Section IV.2.4).

Measurement Technique

Once the desired operating point was reached, and any initial transient, which was observed using the IR camera, in the foil temperature had decayed, the LIF and IR measurements were started. On each LIF image capture the CCD camera was triggered by the laser system. Within the exposure period of the CCD, a laser pulse was triggered, providing the illumination of the fluid film. As the intensity of the resulting fluorescence was far greater than any background light, and the fluorescence duration was far shorter than the camera exposure time, effectively the LIF image constituted only of light originating from the laser flash. Up to 1000 images at a time could be captured with a frame rate of up to 100 Hz to form an image sequence. This equates to a total time of 10 s. Lower frame rates could be used to capture longer sequences.

The trigger for the IR camera to capture an image, and for the the DAQ card to sample the thermocouple voltages was provided by the LIF system. The sync signal was transmitted upon each laser pulse through a 50 Ω coax cable to both pieces of equipment. This ensured that the IR camera images and thermocouple measurements were synchronised with the captured LIF images.

The LIF images were then used to determine the film interface height, as shown in Section IV.5.1, and the intensity of the fluorescent light emitted by the Rhodamine B dye. The intensity of the fluorescence is proportional to I_{laser} the intensity of the laser light used to excite the dye, the dye concentration C_{dye} and the quantum efficiency ϕ of the dye,

$$I_{\text{LIF}} = C_{\text{dye}} I_{\text{laser}}(x, t) \phi(T). \quad (\text{IV.29})$$

As the quantum efficiency, ϕ , of Rhodamine B fluorescent dye is temperature dependant it can be used to measure the fluid temperature. That is if a value of C_{dye} and I_{laser} , and the relationship for the dye efficient $\phi()$ is known, the temperature of the fluid T can be determined from the intensity of the fluorescence I_{LIF} .

Finally the IR images were used to determine the temperature and heat flux at the fluid–foil interface.

IV.5.1 LIF measurements

The film interface height and fluid temperature were measured by LIF technique. An example of successive images of the film interface from the LIF system is shown in Fig. IV.21. Here a cross section of a wave front can clearly be seen travelling across the fluid domain, from right to left. It is also apparent that the intensity of the LIF image is higher at the thicker part of the film than the thinner. This is expected, as the emission intensity of

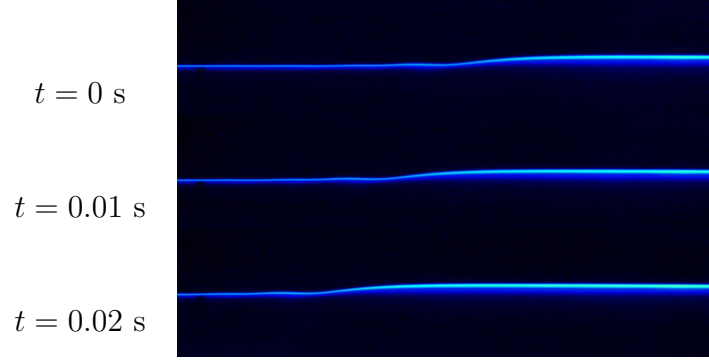


Figure IV.21: Film LIF measurements. Successive snapshots of a fluid wave propagating across the film as it flows down the heated foil.

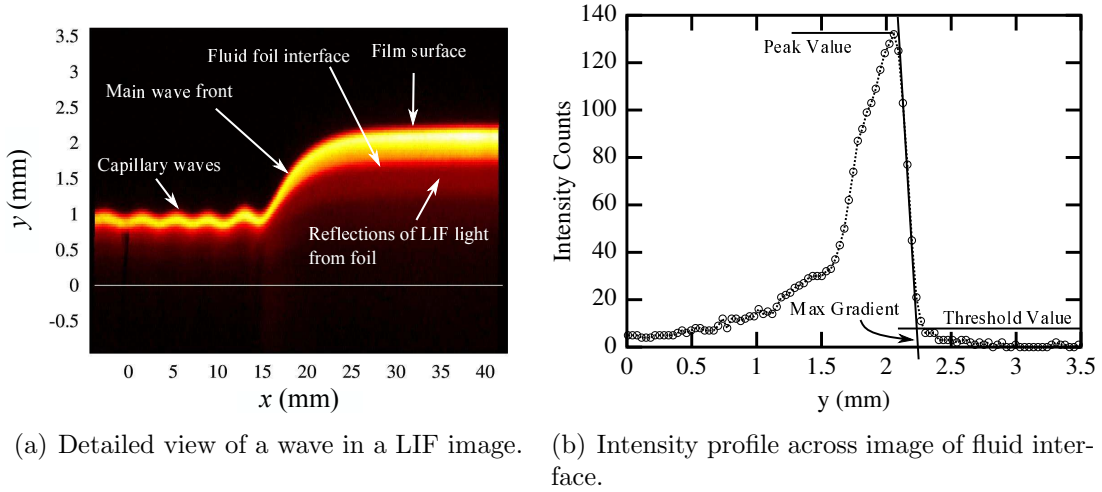


Figure IV.22: Film profile.

the dye decreases with temperature, and the film is hottest when it is thin.

A view of the imaged film profile with greater detail can be seen in Fig. IV.22(a). The image has been rescaled in the y direction making features on the film more apparent, for instance the capillary waves proceeding the wave front can now be seen. In the LIF image the location of the film surface is easily identified as the top edge of the bright emission from the dye being illuminated by the laser sheet.

The bottom interface between the fluid and foil can also be seen, though this is distorted as the light has to travel through the film and so is bent upwards by refraction. As the film thickness varies so to is the amount the light is bent, resulting in the image of the film–foil interface appearing deformed in the same shape as the film surface and

Measurement Technique

raised up from the line $y = 0$. Secondary reflections of the LIF light can also be seen in the image and this occurs as light from the LIF emission reflects off the foil and back up towards the camera. The reflection appears as a slightly dull patch following below the original profile.

Again it can be seen here that the intensity of the emission is brightest where the fluid is thickest, and not so bright where the film is thin. As before this is because the emission from the dye is lower at higher temperatures, in this image the foil was being heated. The variation of the emission intensity inside the LIF profile is also apparent. This could be used to evaluate the temperature variation through the thickness of the film. However for flows with more random wave motion the noisy interface will distort the image of anything inside the film by random amounts making temperature measurement difficult, as this lensing changes the intensity and location of the measured dye emission. To avoid this intensity measurements were only sampled very close to the upper surface of the film.

Another noticeable feature which can be seen in Fig. IV.22(a) is that the image of the interface is not very sharp, with the transition from the dark pixels above the interface to the bright pixels below taking place over the distance of about 5 pixels. This could be due to a number of reasons, such as the blurring of the image by the camera, the fact that the emission is volumetric, and has a finite thickness. A vertical slice through the image of the film is presented in Fig. IV.22(b). Here the intensity is plotted against the vertical position for each pixel. This shows more clearly the increase in intensity at the interface.

Thickness algorithm

A number of methods were devised for systematically detecting the interface location across the image:

1. A threshold intensity was set based on the noise signal in the dark region of the image, typically a value of 12 counts was found. Then, by interpolation, the location was found where the intensity profile intercepted this threshold,

$$y_{\text{threshold}} = \arg_y (0 = I_{\text{LIF}}(y) - I_{\text{threshold}}). \quad (\text{IV.30})$$

2. The location of the peak intensity on the profile,

$$y_{\text{max},I} = \arg \max_y I_{\text{LIF}}(y). \quad (\text{IV.31})$$

CHAPTER IV. EXPERIMENTAL METHODS

3. The location of maximum gradient of the intensity profile in Fig. IV.22(a) was found,

$$y_{\max,G} = \arg \max_y - \frac{\partial I_{\text{LIF}}}{\partial y}(y). \quad (\text{IV.32})$$

The location of the interface was then taken as intercept of this line of maximum gradient with $I = 0$,

$$d_{\text{film}} = y_{\max,G} \left(1 + \frac{I_{\text{LIF}}(y_{\max,G})}{\left(\frac{\partial I_{\text{LIF}}}{\partial y}(y_{\max,G}) \right)} \right). \quad (\text{IV.33})$$

The first “threshold” and third “gradient” method both produced estimates of the thickness which were closest to the position of the initial increase in the intensity profile which was assumed as the best estimate of the film interface location. These two methods also produced the smoothest measures of the thickness along the film, with the “gradient” method providing the smoothest results. It was therefor to use the “gradient” method to systematically find the interface location in the captured images.

Intensity algorithm

The images were also used to determine the intensity of the fluorescent emission from the dye in the fluid film, and therefor measure the fluid temperature. There is a difficulty, however, as there is an uncertainty in the intensity and origin of light which originates any distance below the interface due to distortion of the light as it passes through the uneven film surface. Areas where there is significant curvature to the film can appear brightened, which would result in incorrect temperature measurement. On the other hand the emission in the immediate vicinity of the interface is less sensitive to these distortions but the intensity is low, reducing the signal to noise ratio of the measurement.

As a compromise an average intensity sample was made from the interface, d_{film} to some location further into the film,

$$I_{\text{Film}} = \frac{1}{b(d_{\text{film}} - y_{\max,I})} \int_{d_{\text{film}} - b(d_{\text{film}} - y_{\max,I})}^{d_{\text{film}}} I_{\text{LIF}}(y) dy. \quad (\text{IV.34})$$

Here b is the proportion of the depth between the “interface location” and the position of peak intensity. Values for b of 0.25, 0.5 and 1.0 were tried, and it was found that $b = 0.5$ gave the best compromise between measurement noise and sensitivity to the film curvature.

IV.5.2 IR measurements

A typical example of an IR measurement of the back surface of the foil is shown in Fig. IV.23. Here the flow rate was set to 10 l/min which corresponded to a flow Reynolds number of $Re = 360$ for the 20 % ethanol–water mixture. A heat flux of 2.1 W cm^{-2} was applied to the foil and the flow pulsation was 1.76 Hz.

The movement of waves travelling down the film surface can clearly be seen as signal lines in the bottom right panel. This panel shows vertical pixel position on the vertical axis and frame number on the horizontal axis. The waves in the temperature profile move from the top left to the bottom right as the fluid waves travel down the foil.

Similarly the lower left panel in Fig. IV.23 shows the temperature profile across the width the film. However here the axes are arranged so that the vertical axis shows the frame number and the horizontal pixel position is on the horizontal axis. High temperatures are shown as red and cold as blue in this colour mapping.

IV.6 LIF Calibration

In order to measure the fluid temperature by the LIF technique, the parameters describing the measured intensity of the Rhodamine B dye fluorescence must be characterised. Looking at Eq. IV.29 the factors which effect the output of the fluorescence are the dye concentration C_{dye} , the laser intensity I_{laser} and the quantum efficiency $\phi(T)$. In order to be able to calculate the temperature of the illuminated dye using Eq. IV.29 the relative values of C_{dye} and I_{laser} and the relationship $\phi(T)$ must be known.

If the dye concentration C_{dye} is constant through the fluid it becomes easier to recover the fluid temperature. As there is a large flow recirculation in the flow loop it is assumed that the fluid is well mixed and so the dye concentration should be constant through the fluid. However there are a number of effects which could alter the dye concentration, such as; evaporation of the fluid film, thermally driven diffusion known as the Soret effect, or uneven mixing of the dye through the flow loop. In isothermal cases (without heating) the observed variation in fluorescence intensity with time was minimal therefor the assumption about the dye being well mixed with the fluid must be valid. The effect of evaporation and Soret were investigated in Section IV.7.2 and it was found that the dye concentration was low enough that there was little effect on dye concentration. Therefor assumption that C_{dye} was constant appears to be valid.

The intensity of a real laser output is not perfect and exhibits several distortions

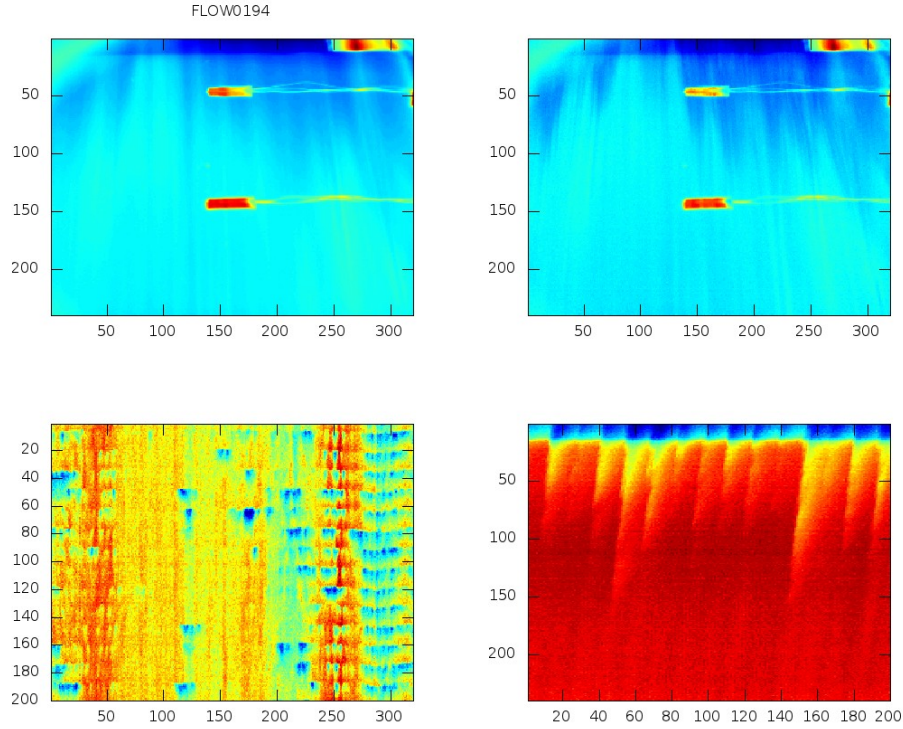


Figure IV.23: Typical IR measurements on the back surface of the foil using the FLIR SC3000 at a flow Reynolds number of $Re = 360$, a heat flux $\bar{q} = 2.1 \text{ W cm}^{-2}$, and a pulsation of 1.76 Hz, for a 20 % by volume water-ethanol mixture. The top two images show the mean temperature field, and a snapshot at a particular instance in time respectively. The bottom two images show the temperature over 200 frames across a horizontal and vertical section respectively.

which need to be characterised and corrected for, in order to make reliable and accurate temperature measurements. The first of these distortions is that the laser cavity exhibits flicker whereby the laser intensity fluctuates from one pulse to the next. The variation in intensity between laser pulses can be as much as 5 % due to flicker, which equates to a worst case uncertainty in temperature measurement of up to 3 K. In order to correct for this a fluorescent test sample, which was stained with a temperature insensitive dye, was placed in the field of view of one of the cameras and outside of the fluid. This was used to meter the laser intensity which could then be used to normalise the the measured fluorescent intensity of the emission from the film. The use of the cameras to meter the laser level had the additional benefit that any uncertainty in the cameras sensitivity would also be compensated for.

Additional unavoidable spatial distortion of the laser beam occurs as it passes through the laser optics to form the laser sheet. This results in a laser sheet that has a significant intensity variation along its width with a spatial distortion that is roughly Gaussian. That is a peak intensity directly in line with the beam optics, and with the intensity gradually drops off towards the edges of the sheet.

During the LIF calibration procedure the film flow rig was set at the desired flow-rate with no pulsation. The temperature in the loop was controlled by either removing heat using the heat exchanger or heating the fluid with the power supply. Once the desired temperature was reached for a calibration run, the power supply was shut down and the heat exchanger closed. Some time was allowed for the foil to reach thermal equilibrium. Images were then taken with the LIF camera. Simultaneously temperature measurements of the back of the foil were made with the IR camera to ensure the temperature uniformity of the measurement area. The temperatures of the foil and fluid were also recorded using fine (0.2 mm) K-type thermocouples. The process was repeated at different temperatures until a sufficient data set had been gathered.

Raw intensities from a set of calibration data are shown in Fig. IV.24(a). Each line shows data from a single run for the measured intensity at locations between 0 and 40 mm. The colour of the lines corresponds to the temperature of the fluid as measured by the thermocouples; red being hot and blue being cold. Each line corresponds to a single run with a fluid temperature which ranged from 13 to 39 °C.

It is clear that the laser intensity from the sheet optics is brighter on the right side of the LIF image resulting in larger fluorescent intensities there. The relationship between the emission intensity and the fluid temperature can also be seen; the hotter the fluid the

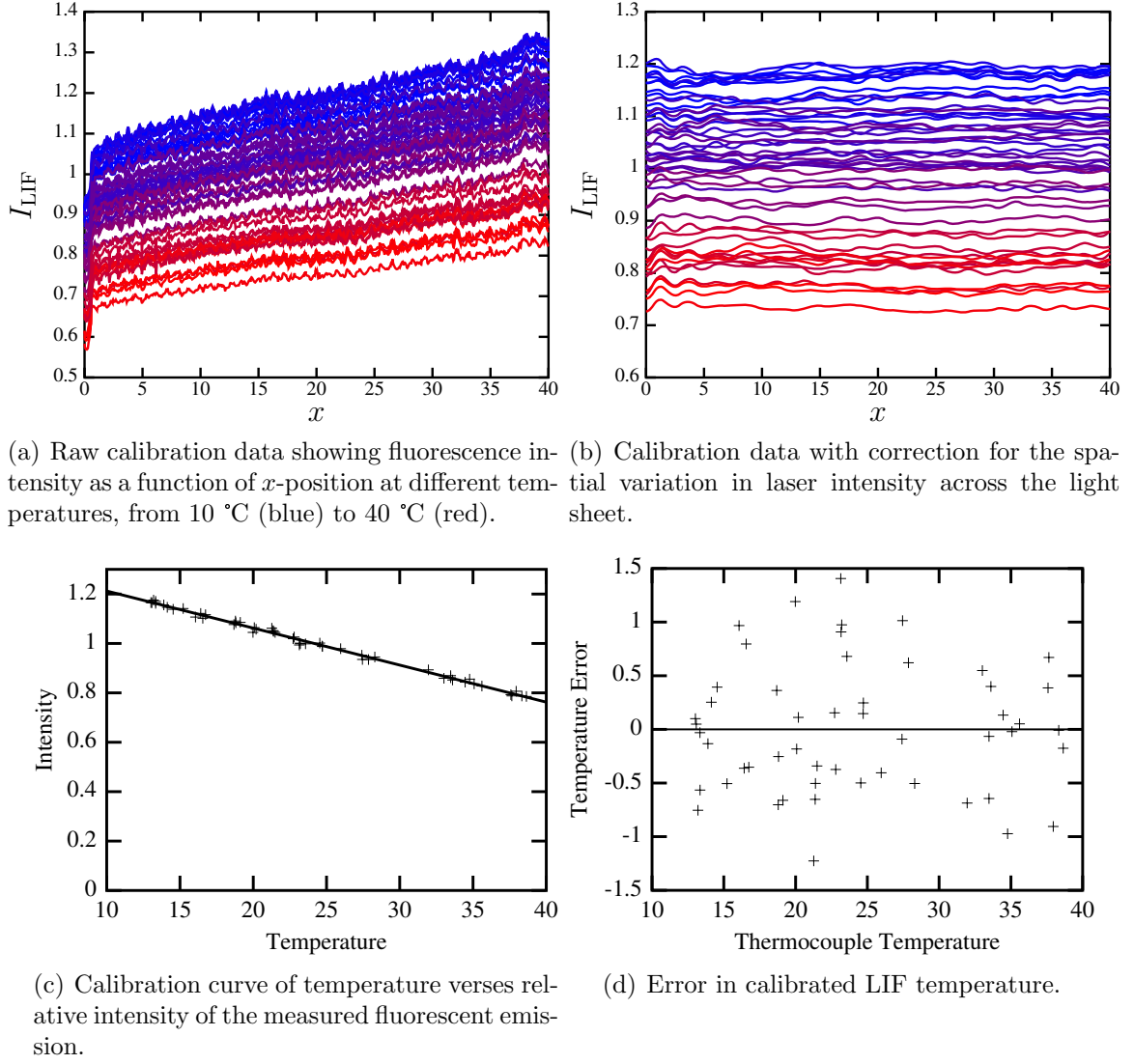


Figure IV.24: Calibration method of the laser intensity for the temperature measurement of the fluid film.

Assumptions

lower the quantum efficiency of the dye. As a final point of interest the high frequency signal fluctuations seen on the intensity profiles in Fig. IV.24(a) are a result of aliasing in the LIF image. This occurs because the image line of the fluid interface is in very close alignment with the row boundaries of the image resulting in a blip or dip in intensity each time the line crosses a row boundary in the image. This is unavoidable as the fluid surface is very close to horizontal. It is possible to take the measurement image at 45° to the interface. However, the image would still have to be rotated back to the horizontal position for post processing, resulting aliasing as before. Ultimately the answer is to filter the image signal which results in some loss of spatial resolution.

By averaging over the calibration runs a corrective profile for the laser intensity distribution can be formed,

$$I_{\text{correct}}(x) = \frac{1}{N} \sum_{n=1}^N I_{\text{LIF}, n}(x), \quad (\text{IV.35})$$

where n is the run number and N is the number of runs. This is used to normalise the raw data images to give,

$$I_{\text{LIF, norm}}(x, t) = \frac{I_{\text{LIF}}(x, t)}{I_{\text{correct}}(x)}. \quad (\text{IV.36})$$

The resulting normalised intensity profiles for the calibration data is shown in Fig. IV.24(b).

As the spatial variation in the intensity profiles for the calibration has now been normalised out, the intensity can be averaged in the spatial, x direction to determine the effect of the fluid temperature on the fluorescence intensity Fig. IV.24(c). The relationship appears to be linear with a gradient of -0.015 K^{-1} and a error in the temperature measurement between runs of 0.6 K (see Fig. IV.24(d)). It should be expected that the film temperature in any particular run would be uniform, however the variation in temperature in each individual LIF image was 0.7 K.

IV.7 Assumptions

To conduct the experiment and make the measurements several key assumptions were made:

1. The convective losses on the back side of the foil were minimal allowing the boundary condition on the back of the plate to be assumed adiabatic.
2. The spatial thermal gradients in the plane of the foil were small allowing a 1-D thermal solution in the solid.

3. Uniform heating over the foil.
4. Span-wise uniformity of the fluid flow. Otherwise there would be uncertainty in the flow condition measured.
5. The concentration of the dye is homogeneous for the temperature measurements using LIF.
6. The fluid properties are constant through the experiment.

Generally it was found that these assumptions were correct, and each issue is explored in the following sections.

IV.7.1 Assurances over convective losses

In order to simplify the calculation of the fluid side heat flux and temperature from the IR temperature measurements at the back side of the foil it has been asserted that the convective losses on the back side of the foil were minimal, and therefore an adiabatic boundary condition could be assumed there. In order to test this assumption the convective heat transfer on the back side of the foil in the absence of fluid flow was measured. The foil assembly was fixed at a 45° angle to the horizontal and a 50 mm thick section of insulating foam was fixed to the foils upper surface. The foam covered the entire area of the foil, effectively preventing heat transfer from that side. The foil was then heated electrically until thermal equilibrium was reached, after which the temperature was measured using the IR camera and 3 thermocouples mounted on the foil centre line at 50 mm intervals.

The resulting temperature difference between the ambient air T_∞ and the foil T_w at the three locations is shown in Fig. IV.25. The increase in the air-foil temperature differences is approximately linear with the applied heat flux to the foil. Also, the temperature measurements at all three thermocouple locations were approximately the same at every heat flux. Only at the highest applied heat flux is it clear that the uppermost thermocouple location on the foil (3) is hottest, and the lowest location the coolest. This should be expected as any heat transfer occurring on the back of the foil will be the result of natural convection driven by buoyancy forces in the air, and so the air will convect up the foil increasing in temperature. The Grashof number for such a convective flow,

$$\text{Gr} = \frac{g\beta(T_w - T_\infty)L^3}{\nu_{\text{air}}^2} \quad (\text{IV.37})$$

Assumptions

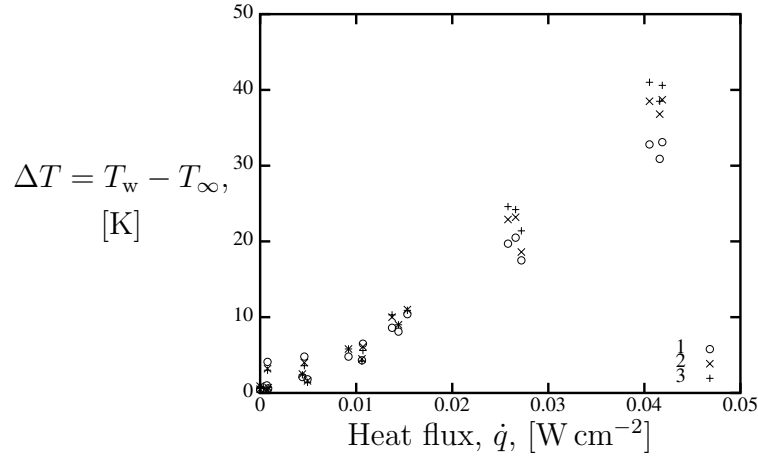


Figure IV.25: Temperature of the foil with no fluid flow.

will be between 4.0×10^7 to 4.0×10^8 . For naturally convecting flows the transition to turbulence should occur between $10^8 < Gr < 10^9$ [155] so the flow probably is not turbulent here, however it is a good idea to keep the foil temperature as close to the ambient air temperature as possible, not just to minimise the deviation from the assumed boundary condition but also to avoid the increased noise and unsteadiness any turbulence may cause.

In relative terms the heat loss seen through the the foil in Fig. IV.25 is small. In our experiments heat fluxes of up to 4 W cm^{-2} were applied which is 80 times the maximum heat flux loss seen here. The heat transfer coefficient between the air and the foil was about $13 \text{ W m}^{-2} \text{ K}^{-1}$, whereas the heat transfer coefficient between the foil and the fluid film was between 1000 and $4000 \text{ W m}^{-2} \text{ K}^{-1}$. Additionally the foil temperatures in the film flow experiments were not allowed to reach very high temperatures above the ambient, for the reasons mentioned above. Finally it should be expected that the fluctuating component of any heat loss on the back side of the foil will be much less than the absolute values of heat loss shown here. So in conclusion, the approximation,

$$|\dot{q}(s, a)| \gg |\dot{q}(s, 0)| \quad (\text{IV.38})$$

seems valid.

IV.7.2 Dye concentration gradients

In order to achieve accurate and repeatable temperature measurements with the LIF technique it is important to ensure that the dye concentration is constant throughout the fluid domain. The main effects which could lead to variation in the dye concentration are evaporation at the fluids surface, and the Soret effect.

It has been stated previously that the surface temperature of the film was kept as low as possible to minimise evaporation, as evaporation would adversely change the heat transfer problem. Consider the worst case scenario which, for evaporation, occurs at the maximum possible heat flux of 4 W cm^{-2} . Assuming that 1 % of the heat flux from the foil went into evaporating the fluid, then as the enthalpy of vaporisation of water at 320 K is 44 kJ mol^{-1} the molar evaporate flux would be $0.009 \text{ mol s}^{-1} \text{ m}^{-2}$. Typically at these heat fluxes the minimum flow rate was 4 l/min across the 300 mm width of the test section, this equates to a molar flow rate per unit width of $12 \text{ mol s}^{-1} \text{ m}^{-1}$. Therefore the percentage change in concentration at the location furthest down stream where LIF measurements were made is 0.02 %, which is insignificant.

The Soret effect, also known as thermal diffusion, is a mass transport phenomenon which occurs in mixtures of solids, liquids, gases or other particle systems. First observed over 150 years ago by Ludwig et al. [156] and later refined by Soret [157], after which the effect is named. The effect occurs upon the application of a temperature gradient on a mixture. Due to the difference in masses of the constituent molecules in the mixture, one of the chemical species will experience a force in the direction of the temperature gradient due to the spatial distribution of molecular collision speed. That species will respond by diffusing up or down the thermal gradient changing the concentration of the mixture [158, Section 21.5]. Thermal diffusion is a complex subject due to the collision mechanics of the molecules involved, however the diffusion equation for the species can be expressed as,

$$\frac{DC}{Dt} = \mathcal{D}_m \nabla^2 C + \mathcal{D}_m S_T C(1 - C) \nabla^2 T, \quad (\text{IV.39})$$

where C is the molar concentration of the dissolved species (in this case Rhodamine B dye), $\mathcal{D}_m$ is the mass diffusivity of the dye in the solute in the solvent and S_T is the Soret coefficient.

A dimensionless form of Eq. IV.39 can be constructed by introducing the following

Assumptions

dimensionless variables:

$$\tau = \frac{\nu t}{a}, \quad (\text{IV.40})$$

$$\phi = \frac{C - C_\infty}{\Delta C}, \quad (\text{IV.41})$$

$$\theta = \frac{T - T_\infty}{\Delta T}, \quad (\text{IV.42})$$

where ν is the kinematic viscosity and a is some length scale. By substituting Eq. IV.40 to IV.42 into Eq. IV.39 we get,

$$\frac{D\phi}{D\tau} = \text{ScPr} \nabla^2 \phi + \text{Sr} \nabla^2 \theta. \quad (\text{IV.43})$$

Here $\text{Sc} = \mathcal{D}_m/\alpha$ is the Schmidt number, $\text{Pr} = \alpha/\nu$ is the Prandtl number, and

$$\text{Sr} = \frac{\mathcal{D}_m S_T \bar{C} (1 - \bar{C}) \Delta T}{\Delta C}, \quad (\text{IV.44})$$

is the Soret number.

The Soret coefficient, S_T , and mass diffusivity D_m of a solute in a solvent is very hard to measure or predict, requiring complicated molecular dynamics models [159, 160]. There are many theoretical models for calculating the thermal diffusion ratio [161] and is dependant on many parameters such as the concentration temperature relative molecular weights, molecular radii thermal expansion coefficients and more. The Soret coefficient can be positive or negative depending on the direction the species migrates (with or against the temperature gradient). Typically for liquids the Soret coefficient is $1 \times 10^{-5} < |S_T| < 1 \times 10^{-3} \text{ K}^{-1}$ [162]. According to Rani et al. [163] and Jacobson et al. [164] the mass diffusivity for Rhodamine B dye in water at 21 °C is $0.36 \times 10^{-9} \text{ m}^2/\text{s}$.

To find how large the concentration difference across the fluid for a given temperature difference would be, consider Eq. IV.43 when it has reached equilibrium,

$$0 = \text{ScPr} \Delta \phi + \text{Sr} \Delta \theta. \quad (\text{IV.45})$$

By a scaling argument $|\Delta \phi| \sim 1$ and $|\Delta \theta| \sim 1$. Therefor equilibrium will occur when,

$$\text{ScPr} \sim \frac{D_m S_T \bar{C} (1 - \bar{C}) \Delta T}{\nu \Delta C}. \quad (\text{IV.46})$$

CHAPTER IV. EXPERIMENTAL METHODS

By rearranging it is found that the relative variation in the molar concentration of C is,

$$\frac{\Delta C}{\overline{C}} \sim S_T(1 - \overline{C})\Delta T \quad (\text{IV.47})$$

The Rhodamine B dye concentration in the fluid was $\sim 1 \text{ g l}^{-1}$ which equates to 2 mmol l^{-1} or a molar fraction of 4×10^{-5} , this number is quite small due to the dye molecules relatively large atomic weight of 479 g mol^{-1} . Typically temperature differences of the order of 10°C were imposed across the fluid domain due to the foil heating. Using the upper bound for the value of the Soret coefficient S_T of 1×10^{-3} [162] it is found that the variation in the dye concentration could be as high as 1 %, which equates to a Soret number of 3.6×10^{-04} . It should be noted, however that this is the very worst case, as the time dependant terms in Eq. IV.43 will act to reduce the dye concentration difference, and the Soret coefficient may be much lower. Additionally a 1 % change in the dye concentration would result in a 1 % change in the intensity of the fluorescence emission which equates to a maximum error in the temperature measurement due to this effect of only 0.2 K. Therefor the effect of a <1 % change in dye concentration will not have a significant impact on the accuracy of the of the LIF technique.

The Soret effect can lead to surface tension driven Marangoni instabilities as demonstrated in Borcia et al. [56] and Bergeon et al. [57]. As noted in Section IV.7.3 however, the dye concentration used in the film flow was so low that it had little effect on the surface tension of the liquid in the film flow, so Marangoni effects are unlikely to occur because of the dye. On the other hand the ethanol–water mixtures and water–glycerol mixtures, introduced to improve the wetting of the film to the foil, may exhibit such behaviour. Borcia et al. [56] modelled the Soret effect on surface tension and observed the formation of stripes holes and droplets on the surface. Bergeon et al. [57] also modelled the Marangoni effect in a binary mixture and calculate bifurcation diagrams of the different flow behaviour.

IV.7.3 Surface tension effects

The choice of water introduces a possibility that contamination from dust in the air and surfactants can build up over time changing the surface tension properties of the fluid [165]. This is particularly problematic for water as it readily dissolves ionic compounds in the dust. In addition, due to its high surface tension dust particles tend to congregate at the free–surface (fluid–air interface), reducing the surface tension of the fluid, whereas this is not the case for other fluids as the surface tension are lower, and

Assumptions

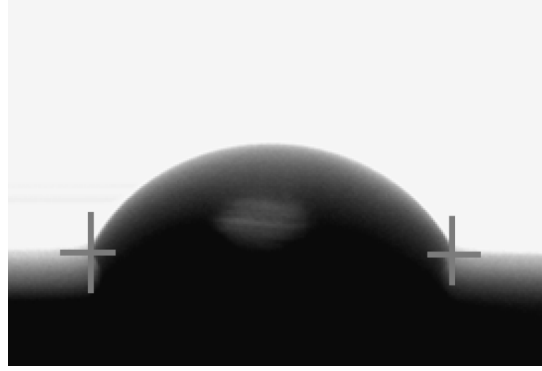


Figure IV.26: Contact angles measurement using the pendant drop method on a goniometer.

so the particulates become more evenly dispersed, and ionic compounds do not dissolve. Scheid [165] has suggested that for pure water the surface tension will degrade rapidly with time, on the order of seconds, even in relatively clean laboratory environments due to microscopic dust in the air. This will make experimentation difficult, since the interfacial flows of interest would be taking place while the key fluid property for these flows, that is surface tension, is changing.

To determine this effect, due to the possible contamination, an experiment was conducted with fresh deionised water, with the normal dye concentration, running continuously through the test section. Samples were taken from the flow loop at regular time intervals as shown in Fig. IV.27. The contact angle of the sampled fluid was then measured on a clean titanium substrate using the pendant drop method with a goniometer, as shown in Fig. IV.26.

In the pendant drop method a fixed volume of the fluid being tested is dropped from a fixed height onto the substrate which is held at a constant temperature of 20 °C. An image of the resulting droplet is then used to determine the contact angle between the fluid and substrate. From this contact angle the surface tension of the fluid can be ascertained by using the Young equation,

$$0 = \sigma_{SG} - \sigma_{SL} - \sigma_{LG} \cos(\theta_C). \quad (\text{IV.48})$$

Here σ_{LG} is the surface tension, or free-surface energy, of the air-liquid interface, σ_{SG} is the surface tension of the air-solid interface, and σ_{SL} is the surface tension between the liquid and the solid, and θ_C is the contact angle of the droplet. Though in our case we do not know the air-solid surface tension terms, variation in the contact angle will indicate

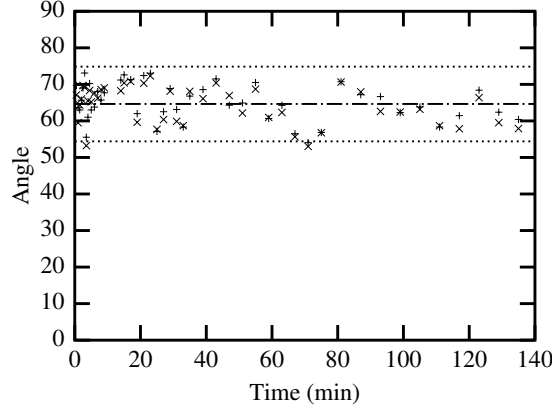


Figure IV.27: Contact angle over time of sampled fluid from the flow loop as the fluid was passed through the test section to check for contamination effects. The lines showing the mean and $\pm 2\sigma$ from the measurements are also drawn.

changes of the surface tension of the fluid.

By looking at Fig. IV.27 it is apparent that there is little variation of the surface tension of the fluid with time. The contact angle for pure water on a pure titanium substrate at 20 °C was found to be 60°, whereas the average contact angle seen here for the water dye combination was approximately $64.6 \pm 5.7^\circ$.

The effect of the dye on the interfacial relationships is a complex function of the dye concentration, the solid phase composition, and the dynamic rate of new interface formation [166]. According to Tuck [166] the interfacial tension decreases as dye concentration increases and the contact angles through the aqueous phase increase with increasing dye concentration. This study was conducted with dye concentrations in the range of 1.1 to 1.7 g L⁻¹. Other work such as Thijssen et al. [167] found that Rhodamine B dye solutions with a concentration of 0.24 g L⁻¹ changed the contact angle from an un-dyed dodecane–water solution by 5.4 degrees. This is very close to the change in contact angles that observed in the current study.

The concentration of Rhodamine B dye used in our experiment was ~ 1 g l⁻¹ for water. The lack of change of the surface tension with time suggests that fears over this problem were unfounded and that the combination of the 10 μ m filter in the flow loop, the short residence time of the fluid in the test section and the dissolved dye in the fluid were adequate in stabilising the variation of fluid surface tension as a result of particulate contamination from the air.

IV.8 Error Analysis

IV.8.1 IR measurements

The technique used to measure the heat flux and temperature at the fluid–solid foil interface relies on the accuracy and sensitivity of the IR camera. The error in the deduction of the fluid side foil temperature as a result of noise in the temperature measured by the IR camera, $\eta_{T,IR}$, and the implementation of an inverse filter, can be calculated by considering the solid temperature response across the foil for an isoflux back boundary condition Eq. III.89,

$$\eta_{T,w} = \cosh \left[(1 + i) \frac{a}{\delta} \right] \eta_{T,IR}. \quad (IV.49)$$

Similarly the error in the deduced wall heat flux as a result of the noise in the IR temperature measurement of the wall can be found by considering the transfer response of the back wall temperature, $T(s, 0)$, to the front wall heat flux, $\dot{q}(s, 0)$, for an isoflux back wall boundary condition given in Eq. III.90,

$$\eta_{\dot{q},w} = (1 + i) \frac{\kappa}{\delta} \sinh \left[(1 + i) \frac{a}{\delta} \right] \eta_{T,IR}. \quad (IV.50)$$

The signal–to–noise ratio, of the temperature or heat flux measurement, depends on the size of the signal being measured as well as the measurement noise. Ideally the signal to noise ratio should be much greater than one. We could call the value of the signal for which the signal equals the measurement noise the sensitivity, as its not possible to sense anything below this. It is interesting to note that because of the coupling between the wall heat flux and the wall temperature in Eq. III.91, the signal to noise ratio of the measurement depends on the source of the heat transfer fluctuations. If we assume that the heat transfer fluctuations appear as a result of a constant magnitude oscillation in the wall temperature $|T(s, a)|$, then the signal to noise ratio of the measurement of that wall temperature is found as,

$$\text{SNR}_{T(s,a)} = \frac{|T(s, a)|}{\eta_{T,w}} = \frac{1}{\cosh(pa)} \frac{|T(s, a)|}{\eta_{T,IR}}. \quad (IV.51)$$

However the signal to noise ratio of the measured wall heat flux is slightly more complicated since Eq. III.91 is needed to obtain $|\dot{q}(s, a)|$ from $|T(s, a)|$. Observing that for a constant magnitude fluctuation in the wall temperature $|T(s, a)|$, the magnitude of the wall heat flux “signal”, $|\dot{q}(s, a)|$ will have to become larger at higher frequencies. The

CHAPTER IV. EXPERIMENTAL METHODS

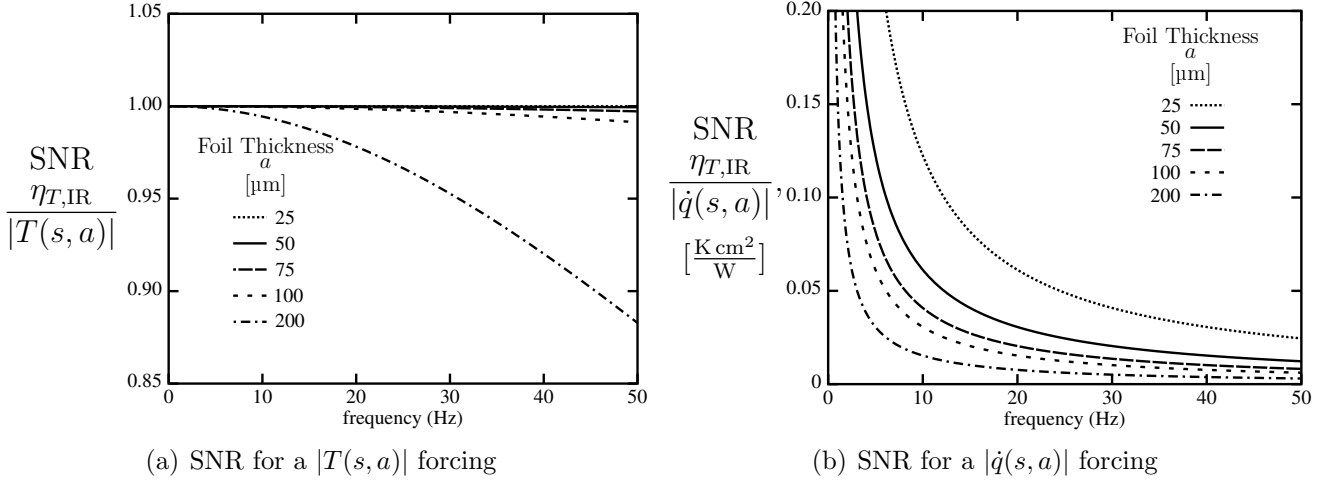


Figure IV.28: Signal to noise ratios for various thicknesses of titanium foil at different frequencies and foil thicknesses normalised by the IR measurement noise η_{IR} and the “signal source”.

resulting signal to noise ratio will be increased,

$$\text{SNR}_{\dot{q}(s,a)} = \frac{|\dot{q}(s, a)|}{\eta_{\dot{q},w}} = \frac{1}{\kappa p \sinh(pa)} \frac{|\dot{q}(s, a)|}{\eta_{T,\text{IR}}} = \frac{1}{\cosh(pa)} \frac{|T(s, a)|}{\eta_{T,\text{IR}}}. \quad (\text{IV.52})$$

Conversely if the heat transfer fluctuations appear as a result of a constant magnitude oscillation in the wall heat flux $|\dot{q}(s, a)|$, then the signal to noise ratio of the measured heat flux is,

$$\text{SNR}_{\dot{q}(s,a)} = \frac{|\dot{q}(s, a)|}{\eta_{\dot{q},w}} = \frac{1}{\kappa p \sinh(pa)} \frac{|\dot{q}(s, a)|}{\eta_{T,\text{IR}}} \quad (\text{IV.53})$$

while the signal to noise ratio of the measured wall temperature becomes,

$$\text{SNR}_{T(s,a)} = \frac{|T(s, a)|}{\eta_{T,w}} = \frac{1}{\cosh(pa)} \frac{|T(s, a)|}{\eta_{T,\text{IR}}} = \frac{1}{\kappa p \sinh(pa)} \frac{|\dot{q}(s, a)|}{\eta_{T,\text{IR}}}. \quad (\text{IV.54})$$

Observing Eq. IV.54 and IV.52 it is clear that the signal to noise ratio is the same for the both the temperature and heat flux measurements. However the response of the signal to noise ratio depends on the type of forcing ($|T(s, a)|$ or $|\dot{q}(s, a)|$) that produces the heat transfer fluctuations.

Both signal to noise ratio models for the measurements are shown in Fig. IV.28. It can be seen that if the temperature fluctuations has a uniform frequency spectrum then the signal to noise ratio is uniform with frequency, only dropping off noticeably at higher

Error Analysis

frequencies if the foil used is of considerable thickness. As the SNR (normalised here by the signal and IR measurement noise η_{IR}) is of the order one, the temperature measurement sensitivity will be approximately equal to the same as the IR camera measurement noise.

On the contrary if the fluctuations are a result of a heat flux fluctuation $|\dot{q}(s, a)|$ that has a uniform frequency spectrum, then the signal to noise ratio drops of rapidly with frequency as shown in Fig. IV.28(b). This implies that the sensitivity to heat flux fluctuations decreases quickly with frequency. The sensitivity at 1 Hz on a 50 μm titanium foil ($\frac{a}{\delta} = 0.02$) with 0.02 mK of camera noise will be about 0.03 W cm^{-2} , whereas at 10 Hz ($\frac{a}{\delta} = 0.09$) the sensitivity will only be 0.33 W cm^{-2} .

The maximum mean heat flux which could be applied to the foil was about 4 W cm^{-2} and it should be expected that any fluctuation in the foil heat flux will be up to 10 % this value (see Section VIII.5). So it can be seen that above 10 Hz the measured heat flux will have a very poor signal to noise ratio in this case.

It is likely that the actual measurement noise exhibited in the experiment here will be a somewhere between the uniform $|T(s, a)|$ spectrum and the uniform $|\dot{q}(s, a)|$ cases. This introduces the following dilemma, in order to be able to resolve the heat flux fluctuations occurring at a given frequency it is best if the foil is thin $\frac{a}{\delta} > 0.1$, however, the effects of the wall that are of interest occur when the foil is $\frac{a}{\delta} \sim 1$. The sensitivity can be improved by spatially averaging and temporal filtering, however the resolution of the results will be reduced.

Chapter V

Results 1:

Low-Dimensional Convective-Conductive Systems

In this chapter we will look at a number of simplified models of the unsteady conjugate heat transfer process in convecting flows (such as film flows). In each case the same general method is followed. The heat transfer response of the solid walls was modelled using 1-D thermal conduction, as investigated in Chapter III. This boundary condition was then matched against convective heat transfer. Finally the energy equation was used to deduce the evolution of the fluid temperature. A number of special cases are solved, and the resulting behaviour discussed.

V.1 Problem Formulation

Consider the unsteady forced convection heat transfer problem illustrated in Fig. V.1 between a fluid stream of extent d and an adjacent planar solid of finite thickness a . The flow can be external or internal, but in either case incompressible. In an external flow d would represent a boundary layer thickness or the depth from the free-surface, whereas in an internal flow d would represent the half-distance to a symmetric wall, or half the hydraulic diameter D_h . Figure V.1 shows an example of the former.

Generally, the flow velocity u (with non-zero mean $\bar{u}$), and the temperatures of the fluid T_f and solid T_s are time-varying. The unsteady quantities can be decomposed into

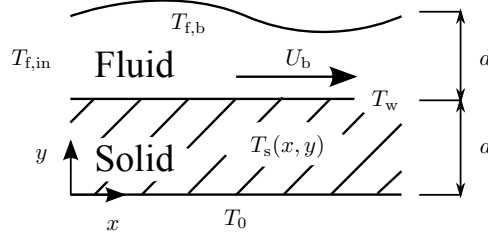


Figure V.1: Incompressible fluid flow over a solid of finite thickness; conjugate problem definition. Here, d is the fluid thickness, a is the solid material thickness and U is the bulk fluid flow velocity. Further, $T_{f,b}$ is the bulk fluid temperature, T_w and T_0 are the wall temperatures at the fluid-solid interface (inner wall) and the far side of the solid (outer wall), respectively, and $T_{f,in}$ is the inlet (i.e. at $x = 0$) bulk fluid temperature.

mean and fluctuating components, e.g. for the instantaneous local heat flux in 1-D, $\dot{q}(y, t) = \bar{\dot{q}}(y) + \dot{q}'(y, t)$. A time-varying heat flux $\dot{q}_w(t) = \dot{q}(y = a, t)$ at the interface between the fluid and the solid (inner wall, $y = a$) can be caused by temperature fluctuations in the fluid $T'_f(t)$, temperature fluctuations in the solid $T'_s(t)$, or fluctuations in the HTC h' (e.g. from fluctuations in u') in the fluid. This is explained in Section III.1.

An assumption is made in the current work that the convection problem is long and that the solid wall and fluid depth are relatively thin so that $(\partial T / \partial y) / (\partial T / \partial x) \sim L_x / L_y \gg 1$ [102]. This isolates the heat transfer problem from conduction occurring along the length of the domain, and establishes the solid thickness as the only dimension of importance. In this case, the conduction problem in the solid can be considered one-dimensional (1-D) in the normal direction to the wall and axial conduction in the fluid can be ignored. Hence in the context of Fig. V.1, we focus on a 1-D problem in the y -direction (as seen in Chapter III). We further assume that diffusion of heat is described adequately by Fourier's Law of Heat Conduction (Eq. II.2), and finally, that all thermal properties of the solid are constant.

V.2 Solution Methodology

The governing equation for heat transfer in the solid domain, $0 \leq y \leq a$, is the 1-D unsteady heat diffusion (conduction) equation Eq. III.6 as investigated previously in Chapter III. It is worth noting that Eq. III.6 is linear and can be solved subject to two boundary conditions, one of which arises in our problem from the thermal interaction with the fluid flow. The linearity implies that the solution in the solid can be decomposed into

CHAPTER V. LOW-DIMENSIONAL CONVECTIVE-CONDUCTIVE SYSTEMS

separate steady and fluctuating components,

$$\frac{\partial^2 \bar{T}_s}{\partial y^2} = -\frac{\dot{Q}_{\text{gen}}}{k_s}; \quad (\text{V.1})$$

$$\frac{\partial T'_s}{\partial t} = \alpha_s \frac{\partial^2 T'_s}{\partial y^2}. \quad (\text{V.2})$$

In this chapter we consider the application of two different “outer” boundary conditions;

- when the non-fluid side of the solid (“outer” B/C) is held at a constant temperature, $T'_0 = 0$ (that is, an isothermal B/C);
- or when the “outer” B/C is held at the constant heat flux, $\dot{q}'_0 = 0$ (that is, an isoflux B/C).

The inner wall heat flux on the fluid side $\dot{q}'_w$ can then be calculated from the imposed convective heat transfer condition in the fluid and the thermal response (shown in Eq. V.3) of the solid.

The fluctuating component described by Eq. V.2, is identical to Eq. III.6, but with T'_s replacing T_s , as expected from the linearity of the 1-D unsteady heat diffusion equation. Nevertheless, even though the governing PDE for the thermal conduction in the solid is linear, which allows the problem to be decomposed into steady ($\bar{T}_s$) and fluctuating (T'_s) components, the solution to the steady component becomes coupled to the fluctuating component due to the non-linear fluid-side convective B/C imposed on the solid (from Eq. III.4). In such a case the steady and fluctuating components of heat transfer in the solid can no longer be solved independently. Section V.2.1 is dedicated to the approach that was used in this work to deal with the non-linear convective B/C.

In Section III.2.3 we concluded that the general solution of Eq. III.6 to a periodic thermal perturbation is a summation of responses at different fluctuation frequencies. That is each frequency component of the heat flux can be calculated from the wall temperature by,

$$\dot{q}_{w,n} = \frac{T_{w,n}}{Z_{\text{in}}(\omega_n)}, \quad (\text{V.3})$$

where $T_{w,n}$ and $\dot{q}_{w,n}$ are the n^{th} frequency components of the temperature at the inner wall and the heat flux through it, respectively. Here $Z_{\text{in}}(\omega_n)$ is the solid thermal input impedance. This impedance depends on the outer B/C on the non-fluid side of the solid, and is given in Table III.2 in Section III.2.6. This impedance can be written as; *Isothermal*

Solution Methodology

B/C on the outer wall:

$$Z_I = \left[(1+i) \frac{\kappa}{\delta_n} \coth \left\{ (1+i) \frac{\kappa}{\delta_n} \right\} \right]^{-1}; \quad (\text{V.4})$$

Isoflux B/C on the outer wall:

$$Z_A = \left[(1+i) \frac{\kappa}{\delta_n} \tanh \left\{ (1+i) \frac{\kappa}{\delta_n} \right\} \right]^{-1}. \quad (\text{V.5})$$

Here $\delta_n = \sqrt{2\alpha/\omega_n}$ is the thermal diffusion length at frequency n .

The impedance is a function of material thermal properties (i.e. k_s , ρ_s and c_s), geometry (i.e. thickness a) and the frequency of the imposed oscillations ω_n . This enables the magnitude and phase of the heat flux oscillation into the solid domain to be calculated, at a given frequency, from a given temperature fluctuation on the two surfaces of the solid, i.e. the inner and outer wall in Fig. V.1. The full solution is found by summing all the solutions over all the oscillation frequencies, and including the mean component,

$$\dot{q}_w(t) = \bar{q} + \Re \sum_{n=1}^{\infty} \frac{T_{w,n}}{Z(\omega_n)} e^{i\omega_1 n} \quad (\text{V.6})$$

Since it is of interest to consider problems with a non-zero mean heat transfer $\bar{q}_w$ and applying an adiabatic boundary condition on the solid would contradict this, the isoflux condition is assumed instead so that $\bar{q}_0 \neq 0$ and $\bar{q}_w \neq 0$ while $\dot{q}'_0 = 0$ and $\dot{q}'_w = 0$. It can be shown that the fluctuating solution of our selected problem, where the isoflux condition is imposed externally on a solid with zero internal heat generation/removal $\dot{Q}_{\text{gen}} = 0$, is equivalent to the case of constant $\dot{Q}_{\text{gen}}$ with an adiabatic outer boundary condition ($\bar{q}_0 = 0$ and $\dot{q}'_0 = 0$), though the two time-mean solutions are not equivalent.

V.2.1 Domain matching

The overall UCHT problem requires that the solid and fluid domains be matched by the condition at the fluid–solid interface (inner wall). The convection of heat on the fluid side of the problem is modelled here by using a simple time-varying heat transfer coefficient $h(t)$ (Eq. III.3), such that,

$$h = \bar{h} + \hat{h} \sin \omega t; \quad 0 \leq h^* = \frac{\hat{h}}{\bar{h}} \leq 1. \quad (\text{V.7})$$

CHAPTER V. LOW-DIMENSIONAL CONVECTIVE-CONDUCTIVE SYSTEMS

In practical situations such fluctuations in the HTC would arise from purely fluid-mechanical processes and independently of the temperature field, such as from fluctuations in the fluid flow velocity, bursts of turbulence, or wave propagation over a fluid film. The dimensionless ratio h^* is effectively a normalised HTC fluctuation intensity. Naturally, the magnitude of the fluctuation intensity h^* would depend on the flow and on the particular flow conditions, and may have to be found in real applications by direct measurements [6, 168]. Alternatively, if a quick analysis is required to check whether the full UCHT problem needs to be considered, a rough estimate of the magnitude of h^* can be obtained from knowledge of an appropriate Nusselt relation for the flow, $Nu = f(Re)$, and the fluctuations in the flow velocity $\hat{u}/\bar{u}$.

As mentioned previously (at the end of Section III.1.1 and in Section V.2) the non-linearity of the convective B/C introduces a complication in the solution of the unsteady problem. Specifically, the multiplication between $h(t)$ and $\Delta T_f(t)$ in Eq. III.3 and Eq. III.4 gives rise to multiple frequencies (or, ‘frequency spreading’) in the solution of $\dot{q}_w(t)$ (and therefore $T_w(t)$, since this is allowed to respond to $\dot{q}_w(t)$), even though there is only one frequency in the imposed HTC fluctuation.

Now, the solution of the solid problem (Eq. V.3, with either Eq. V.4 or Eq. V.5 as the outer wall B/C) is posed in the frequency domain, while the convective heat transfer process that sets the inner wall B/C (Eq. III.3, with Eq. V.7 as the imposed HTC) is posed in the time domain. The solid domain solution that involves a multiplication in the frequency domain is equivalent to convolution in the time domain. Since the imposed HTC (Eq. V.7) is a periodic function, a circular convolution can be used as a straightforward and efficient way to obtain the response of the wall heat flux $\dot{q}_w$ from the fluctuations in the temperature of the solid’s surface T_w due to conduction in the solid.

In more detail, because an analytical description of the solid’s transfer function $Z(\omega_n)$ already exists in the frequency domain (Eq. V.4 and Eq. V.5), and because it is more numerically efficient, the circular convolution operation is not implemented directly in the time domain. Instead, the convolution theorem is invoked to perform the equivalent operation in the frequency domain. In this approach, the circular convolution solution in the time domain is obtained as the inverse transform of the product of the wall temperature in the frequency domain $T_w(\omega_n)$ and the impulse response of the solid $Z(\omega_n)$ (see Eq. V.3).

The challenge in calculating the unknown $T_w(t)$ through the period of the HTC fluctuation arises due to the fact that this variable appears both inside and outside the kernel of circular convolution (see Eq. V.8). In the numerical solution method employed here,

Bulk Fluid Temperature Fluctuations

an initial guess for $T_w(t)$ is firstly discretised in the time domain. A Fourier transform is used to evaluate T_w in the frequency domain. The solid transfer function $Z(\omega_n)$ is then applied by multiplication with $T_w(\omega_n)$. Following this, the time domain wall heat flux $\dot{q}_w(t)$ is calculated from the inverse Fourier transform of this product. Additionally, the wall heat flux in the time domain $\dot{q}_w(t)$ is also calculated by using Newton's law with the imposed time-varying $h(t)$ (Eq. III.3). The difference between these two evaluations of $\dot{q}_w(t)$ represents the error in the initial estimate of $T_w(t)$ through the oscillation period. This error can be used with a non-linear equation solver, such as the Newton–Raphson method or a simplex method, to calculate T_w such that,

$$h(t) (T_{f,b} - T_w) - z^{-1}(t) \star (T_w - T_0) = 0, \quad (\text{V.8})$$

which is a Fredholm equation of the second kind. In Eq. V.8 the circular convolution of the solid's frequency response with the periodic temperature difference across the solid is represented by,

$$z^{-1}(t) \star (T_w - T_0) = \sum_{n=-\infty}^{\infty} \frac{T_w(\omega_n) - T_0}{Z(\omega_n)} e^{i\omega_n t}, \quad (\text{V.9})$$

where $Z(\omega_n)$ is the solid's frequency response for either the isothermal or isoflux B/C on the outer wall. Eq. V.8) can be solved numerically for T_w once $T_{f,b}$ is known. The heat transfer augmentation A can then be calculated for different $\text{Bi} = \bar{h}a/k_s$, $a^* = a/\delta_{s,1}$ and $h^* = \hat{h}/\bar{h}$, where $\delta_{s,1}$ is based on the fundamental fluctuation frequency of the imposed HTC (e.g. simply ω in the particular case represented by Eq. V.7).

V.3 Bulk Fluid Temperature Fluctuations

In the cases where the fluid domain is large enough so the bulk fluid temperature $T_{f,b}$ (defined in Eq. III.2), can be assumed constant, Eq. V.8 can be solved directly without requiring any further information from the fluid domain. This is a similar condition to the work presented by Zudin [22]. By contrast, this may not be the case in some flows, e.g. thin film flows or pipe flows in which the channel is small, so a model needs to be developed to account for the response of the fluid temperature $T_{f,b}$ to the unsteady heat transfer with the solid wall. Zudin [169] has more recently considered situations where the fluid temperature is fluctuating, however, the author concluded that augmentation is always negative. We propose here that this is not necessarily the case (see Section V.4.2).

Consider the situation where fluid passes over a heated or cooled surface, such as

CHAPTER V. LOW-DIMENSIONAL CONVECTIVE-CONDUCTIVE SYSTEMS

that illustrated in Fig. V.1. This arrangement could describe an internal (e.g. pipe), or external (e.g. film or boundary layer) flow. At each streamwise position x along the plate an enthalpy flow rate is advected with the fluid immediately in contact with the inner wall at $y = a$. The fluid's temperature response to the unsteady heat flux $\dot{q}_w(x, t)$ at the wall can be evaluated by considering the energy balance on a suitable control volume in the fluid domain. Assuming that heat exchange only occurs between the fluid and solid, and that there is no heat loss from the fluid stream at the extreme vertical extent of the fluid domain $y \rightarrow d$, we can derive the following heat advection equation,

$$\frac{DT_{f,b}}{Dt} = \frac{\partial T_{f,b}}{\partial t} + U_b \frac{\partial T_{f,b}}{\partial x} = -\frac{\dot{q}_w(x, t)}{\rho_f c_f d}, \quad (\text{V.10})$$

where ρ_f is the fluid density, c_f is the specific heat capacity of the fluid, d is the characteristic fluid depth and U_b is the bulk fluid velocity, as defined in Fig. V.1. Also, recall from Eq. III.1 that the term $\dot{q}_w$ on the right-hand side of Eq. V.10 is defined positive into the solid.

The heat advection equation (i.e. Eq. V.10) gives rise to a developing region, wherein the solution is dependent on the inlet conditions of the fluid domain, and a fully developed region far away from the inlet. The fully developed case is of most interest in the current study, as the solution here becomes decoupled from the inlet conditions and independent of the streamwise direction, turning the thermal conduction in the solid into a quasi-1-D problem. However, the developing case will also be derived and investigated as this gives rise to some interesting results.

Eq. V.10 can be solved by the method of characteristics along the line $t = x/U_b + t_0$, for simplicity it will be assumed that U_b is constant and that the flow is incompressible. Together with knowledge of the inlet condition $T_{f,b}(x = 0) = T_{f,in}$ (see Fig. V.1), the general solution for the bulk fluid temperature is,

$$T_{f,b}(s) = \frac{\int \left\{ \frac{C_H(x, t_0)}{d} T_w(x) \exp \left\{ \int \frac{C_H(x, t_0)}{d} dx \right\} dx \right\}}{\exp \left\{ \int \frac{C_H(x, t_0)}{d} dx \right\}}. \quad (\text{V.11})$$

where the instantaneous local Stanton number C_H is defined as,

$$C_H(t) = \frac{Nu(t)}{Re Pr} = \frac{h(t)}{\rho_f c_f U_b}. \quad (\text{V.12})$$

The expression (V.11) with a time varying C_H can be calculated analytically in some

Bulk Fluid Temperature Fluctuations

cases:

Case I: The trivial solution in which $T_{f,b}$ is unaffected by $\dot{q}_w$, for example due to a large C_R (see Sections V.3.2 or V.3.4) or small Bi ; or,

Case II: A known arbitrary fluctuation in $\dot{q}_w$ is prescribed; or,

Case III: A known arbitrary fluctuation in T_w is prescribed and h is constant; or,

Case IV: The non-conjugate problem where h is allowed to fluctuate, but T_w is held constant;

Case V: The flow field is fully developed, such that the convection problem is independent of the x -direction and the problem can be treated as 1-D in the y -direction. In this case, $T_{f,b}$, T_w and h can all fluctuate, but h is instantaneously uniform across the entire fluid domain; or finally,

Case VI: The same as Case V except the fluctuation in the heat transfer coefficient convects with the fluid flow.

Finally, it is worth stating that the presented results are for $T_{f,in} > 0$ in the isothermal outer wall B/C case, when $T'_0 = 0$, and for $\bar{\dot{q}}_0 > 0$ (i.e. into the solid at the outer, non-fluid boundary) in the isoflux one when $\dot{q}'_0 = 0$.

V.3.1 Case II

In Case II we impose an arbitrary fluctuation in the wall heat flux of the form $\dot{q}_w = \bar{\dot{q}}_w + \Re\{\sum_{n=-\infty}^{\infty} \dot{q}_{w,n} e^{i\omega_n t}\}$ which has no variation in the (streamwise) x -direction. Taking Eq. V.10 this results directly in the following response of the bulk fluid temperature,

$$T_{f,b} = T_{f,in} + \frac{\bar{\dot{q}}_w}{\rho_f c_f d} x + \frac{1}{\rho_f c_f d} \Re \left\{ \frac{1}{i} \sum_{n=-\infty}^{\infty} \frac{\dot{q}_{w,n}}{\omega_n} e^{i\omega_n t} (1 - e^{-i\omega_n x/U_b}) \right\}. \quad (V.13)$$

In this case it is less meaningful to consider augmentation since the heat flux has been arbitrarily imposed, as has the phase between the heat flux and the HTC. However, it is useful to note the thermal response, as it can be used in the later cases. The response has a spatial variation due to the periodic $e^{-i\omega_n x/U_b}$ term, which results from the inlet condition. This behaviour does persist away from the inlet, even at long x . In reality it may be expected that such a spatial profile would decay in the streamwise direction due to effects not modelled here, such as heat loss from the fluid at $y \rightarrow d$.

V.3.2 Case III

In Case III we employ Newton's Law $\dot{q}_w = h(T_{f,b} - T_w)$ with constant h , and impose a uniform but time-varying wall temperature of the form $T_w = \bar{T}_w + \Re\{\sum_{n=-\infty}^{\infty} T_{w,n} e^{i\omega_n t}\}$, as before with no variation in the x -direction. The response of the fluid is given by,

$$T_{f,b} = T_{f,in} e^{-C_H x/d} + \Re\left\{ \sum_{n=-\infty}^{\infty} \frac{T_{w,n}}{1 + 2i a_n^{*2} C_R / \text{Bi}} e^{i\omega_n t} [1 - e^{-(i/U^* + C_H)x/d}] \right\}, \quad (\text{V.14})$$

where $C_R = (\rho_f c_f / \rho_s c_s)(d/a)$ is the heat capacity ratio between the fluid and solid domains, $U^* = U_b / \omega_n d$ is a dimensionless fluid velocity, C_H is the (constant) Stanton number from Eq. V.12 and $\text{Bi} = ha/k_s$ is the Biot number from Eq. II.4, for this problem. The heat capacity ratio can also be written in the form $C_R = C^* d^*$ so as to separate the effects of the material properties and the problem geometry, with $C^* = \rho_f c_f / \rho_s c_s$ the volumetric (or volume-specific) heat capacity ratio and $d^* = d/a$ the fluid depth non-dimensionalised by the solid plate thickness.

In this case the spatial variation in $T_{f,b}$ decays along the flow along with the local time-mean $\bar{q}_w(x)$, such that when the thermal problem becomes fully developed ($x \rightarrow \infty$) the mean heat transfer becomes zero. In addition, given that the HTC has been set to be constant, it is not possible to experience any heat transfer augmentation (as defined in Section III.1.2), and the augmentation ratio is zero $A = 0$.

V.3.3 Case IV

In this case the *non-conjugate* condition where T_w is constant and h is allowed to fluctuate according to Eq. V.7 is considered, in order to demonstrate that augmentation effects can indeed result even in non-conjugate situations. The Stanton number (Eq. V.12) can then be written as $C_H(t) = \overline{C_H} + \widehat{C_H} \sin \omega t$. This isothermal B/C at the *inner wall* gives the following response for the bulk fluid temperature,

$$T_{f,b} = T_w + (T_{f,in} - T_w) e^{-\overline{C_H} x/d - \widehat{C_H} U^* \cos \omega t + \widehat{C_H} U^* \cos \omega(t-x/U_b)}. \quad (\text{V.15})$$

As before for the fully developed case when $x \rightarrow \infty$, the local time-mean heat flux $\bar{q}_w(x)$ tends to zero, meaning a lack of augmentation. However, in the thermally developing region $\bar{q}_w(x)$ is still finite and there is a non-zero phase difference between the heat transfer coefficient h and the fluid temperature fluctuation $T_{f,b}$. So, even though in this case there is no fluctuation in the wall temperature T_w , there is still a potential for heat

Bulk Fluid Temperature Fluctuations

transfer augmentation in the developing region due to the fluid's fluctuating temperature, $\Delta T'_f = T'_f$. This gives rise to exponentially decaying sinusoidal bands of positive and negative augmentation, as presented in detail in Section V.4.2.

V.3.4 Case V

Consider Case V in which one must solve the problem described by Eq. V.8 and (V.11) simultaneously. Since the fully developed response is of most interest, using Eq. V.13 and assuming that the spatial term will disappear with large x , it can be deduced that the fluid temperature's response to a fluctuating heat flux will be,

$$T_{f,b} = \bar{T}_{f,b} + \frac{1}{\rho_f C_f d} \Re \left\{ \frac{1}{i} \sum_{n=-\infty}^{\infty} \frac{\dot{q}_{w,n}}{\omega_n} e^{i\omega_n t} \right\}, \quad (\text{V.16})$$

that can be evaluated by substituting $\dot{q}_{w,n}$ for the Fourier components of $\dot{q}_w$ from Eq. V.3,

$$T_{f,b} = \bar{T}_{f,b} + \frac{\text{Bi}}{2C_R} \Re \left\{ \frac{1}{i} \sum_{n=-\infty}^{\infty} \frac{T_{w,n}}{Z_n^* a_n^{*2}} e^{i\omega_n t} \right\}. \quad (\text{V.17})$$

Here, $Z_n^* = \bar{h}Z(\omega_n)$ is the non-dimensional solid response described in Eq. V.4 and Eq. V.5, depending on the B/C, while as before: $a_n^* = a/\delta_{s,n}$ is a dimensionless solid plate thickness, $\text{Bi} = \bar{h}a/k_s$ is the Biot number based on the time-averaged h , and $C_R = C^*d^*$ is the fluid to solid heat capacity ratio. Hence, T_w can be solved numerically to match the heat flux $\dot{q}_w$ at the common fluid–solid boundary (Eq. V.8),

$$\dot{q}_w = z^{-1}(t) \star (T_w - T_0) = h(t) (T_{f,b} - T_w). \quad (\text{V.18})$$

This solution assumes that the flow and the thermal profile within it are fully developed, simplifying the problem into one dimension, in the y -direction. The heat transfer can still be unsteady in this flow, however, the assumptions require that there is no spatial variation of the wall heat flux $\dot{q}_w$ in the flow direction, so that the thermal profile remains self-similar.

In the isoflux outer wall condition this is relatively straightforward to imagine as the mean temperature profile would increase linearly along the flow direction and the temperature fluctuation T' would be independent of position. This could occur, for example, if the entire solid was electrically heated. In the isothermal outer wall condition it is required that the fluctuation of the wall temperature must be zero $T'_w = 0$, while

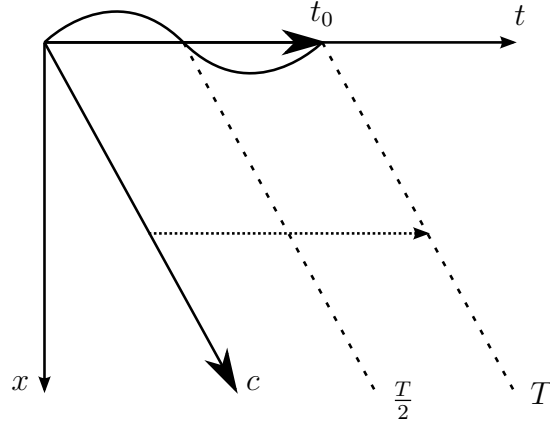


Figure V.2: Diagram showing the lines of characteristics in the fluid domain.

the mean wall temperature $\bar{T}_w$ must scale with the bulk fluid temperature $\bar{T}_f$ in the flow direction. For example, such a situation could be found in a cross-flow heat exchanger.

V.3.5 Case VI

Finally, consider Case VI where the heat transfer coefficient convects with the fluid flow. In the previous cases the heat transfer coefficient was assumed to be constant across the entire domain, even when undergoing a fluctuation. This may be an unrealistic assumption as generally the heat transfer coefficient is associated with some convective process. A possible example where instantaneously and simultaneously fluctuate of the heat transfer coefficient over the entire fluid domain could occur is in the case of evaporating/condensing heat transfer subject to a pressure fluctuation in the fluid. If the pressure was close to the vapour pressure of the fluid a drop in the pressure would promote evaporation increasing the heat transfer coefficient over the entire fluid domain.

When the heat transfer coefficient is associated with a convective effect however, such as with waves on a heated plate or with slug flow in a pipe, the fluctuation in heat transfer coefficient will convect with the flow. In some cases it is a reasonable approximation to assume that the heat transfer coefficient convects with the fluid at a constant bulk fluid velocity U_b . As waves, turbulence or fluid slugs would convect with the fluid bulk velocity. The equation relating the evolution of bulk fluid temperature can be found from the energy equation for an incompressible fluid,

$$\rho_f c_{p,f} \frac{DT_f}{Dt} = \kappa_f \nabla^2 T_f. \quad (\text{V.19})$$

Bulk Fluid Temperature Fluctuations

Integrating in the direction of the thickness of the fluid domain results in,

$$\rho_f c_{p,f} \frac{D \int_0^d T_f dy}{Dt} = \kappa \left[\frac{\partial T_f}{\partial y} \right]_0^d + \int_0^d \frac{\partial^2 T_f}{\partial x^2} dy. \quad (\text{V.20})$$

It is expected that the spatial temperature gradient in the stream-wise direction will be small therefor that term in Eq. V.20 can be neglected. Also assuming that no heat transfer occurs at the fluid air interface the convection equation becomes,

$$\frac{DT_{f,b}}{Dt} = \frac{\partial T_{f,b}}{\partial t} + U_b \frac{\partial T_{f,b}}{\partial x} = -\frac{\dot{q}_w}{\rho_f c_f d}. \quad (\text{V.21})$$

As U_b is a constant Eq. V.21 can be expressed using the method of characteristics (see Fig. V.2). With the substitution $t = c + t_0$ and $x = cU_b$ Eq. V.21 becomes,

$$\frac{\partial T_{f,b}}{\partial c} = -\frac{\dot{q}_w}{\rho_f c_f d} = -\frac{h(t_0)}{\rho_f c_f d} (T_{f,b} - T_w). \quad (\text{V.22})$$

Here the heat transfer coefficient is constant along the characteristic lines, shown in Fig. V.2, as it assumed to be convecting with the flow. The heat transfer coefficient is therefor only dependant on the initial time t_0 . By substituting $c^* = c\omega_1$ Eq. V.22 can be transformed to the dimensionless form,

$$\frac{\partial T_{f,b}}{\partial c^*} = -\frac{\text{Nu}}{\text{Pe}} h^*(t_0) (T_{f,b} - T_w), \quad (\text{V.23})$$

where $\text{Pe} = \omega_1 d^2 / \alpha_f$ is the Péclet number, $\text{Nu} = \bar{h}d / \kappa_f$ is the Nusselt number, and $h^* = h / \bar{h}$ is the HTC characteristics shape. This equation is harder to solve than the previous cases, as the solution has to be found across both c^* and t_0^* simultaneously.

To reform Eq. V.23 in terms of just the fluid temperature $T_{f,b}$, first the wall temperature T_w is expressed in terms of the wall heat flux $\dot{q}_w$,

$$T_w = Z * \dot{q}_w. \quad (\text{V.24})$$

Substituting Eq. V.22 into Eq. V.24 and rearranging gives an expression for T_w as fluctuations in the bulk fluid temperature $T_{f,b}$,

$$T_w = Z * \rho_f c_f d \frac{\partial T_f}{\partial c}, \quad (\text{V.25})$$

or in dimensionless form,

$$T_w = Z^* * \frac{\text{Pe}}{\text{Nu}} \frac{\partial T_f}{\partial c^*}. \quad (\text{V.26})$$

Substituting this back into Eq. V.22 gives a PDE in terms of the fluid temperature,

$$\frac{\partial T_{f,b}}{\partial c^*} = -\frac{\text{Nu}}{\text{Pe}} h^*(t_0) T_{f,b} + h^*(t_0) Z^* * \frac{\partial T_{f,b}}{\partial c^*}. \quad (\text{V.27})$$

This PDE can be integrated numerically to find the evolution of $T_{f,b}$ with c^* . That is,

$$T_{f,b,n+1} = T_{f,b,n} + \left(\frac{\partial T_{f,b}}{\partial c^*} \right)_n \delta c^*, \quad (\text{V.28})$$

where n is the n^{th} time step, and δc^* is the step size. Here the derivative $\partial T_{f,b}/\partial c^*$ was calculated implicitly by solving,

$$0 = \left(\frac{\partial T_{f,b}}{\partial c^*} \right)_n + \frac{\text{Nu}}{\text{Pe}} h^*(t_0) T_{f,b,n-1} - h^*(t_0) Z^* * \left(\frac{\partial T_{f,b}}{\partial c^*} \right)_n. \quad (\text{V.29})$$

Once the fluid temperature is known the other variables such as the heat flux and wall temperature can easily be deduced. This case is the closest to the film flow experiment described in Chapter IV and whose results were presented in Chapter VIII.

V.4 Results and Discussion

In the analysis described above five cases were considered for the response of the bulk temperature of the fluid flow $T_{f,b}$ to the heat exchanged at the inner wall (i.e. fluid–solid boundary) $\dot{q}_w$. Case I assumes a large fluid depth d , so that the heat transfer process cannot affect $T_{f,b}$. In Cases II and III the wall temperature T_w and heat flux $\dot{q}_w$ are set explicitly, and the effect of $\dot{q}_w$ on $T_{f,b}$ is examined. In Case IV T_w is held constant. With this isothermal B/C at the *inner* fluid–solid interface, it is observed that augmentation can occur in the thermally developing region of the flow. In the final case, Case V, the limit of Case II is taken at long x , so as to obtain the fully developed fluid’s response to unsteady heat transfer (still, with no spatial variation in $\dot{q}_w$). Cases I and V are conjugate in that a solution needs to be found in both the fluid and solid domains, with matching required at the inner wall, $y = a$. Case III cannot give rise to any augmentation since h is constant, while in Case II the augmentation is arbitrary and not of interest as h and $\dot{q}_w$ have both been chosen arbitrarily. Table V.1 summarises the cases.

Results and Discussion

Table V.1: The six different condition cases considered.

Case	$ h' $	$ T'_w $	$ T'_{f,b} $	Conjugation	Augmentation	Notes
I	+	+	0	Yes	Yes	‘Large’ fluid domain
II	N/A	N/A	+	N/A	N/A	Imposed $\dot{q}_w$
III	0	+	+	N/A	No	Imposed T_w
IV	+	0	+	No	Yes	Developing flow
V	+	+	+	Yes	Yes	Fully developed only
VI	+	+	+	Yes	Yes	Convecting h

The following sections will discuss results for Cases I, IV, V and VI as these are capable of exhibiting non-zero augmentation, as stated in Table V.1.

V.4.1 Case I

The condition of constant bulk fluid temperature $T_{f,b}$ corresponds to situations involving a large fluid domain compared to the solid thickness ($d \gg a$), or heat transfer close to the inlet where the bulk fluid temperature has not had time to respond to $\dot{q}'_w$. Figure V.3 shows normalised temporal traces of ΔT_f , $\dot{q}_w$ and h generated for this case.

Note here that,

$$\dot{q}_w^+(t) = \frac{\dot{q}_w(t)}{\bar{\dot{q}}_w} = \frac{(h/\bar{h})(\Delta T_f/\overline{\Delta T_f})}{\bar{\dot{q}}_w/\bar{h} \overline{\Delta T_f}} = \frac{h^+\theta^+}{A}. \quad (\text{V.30})$$

It can be seen in Fig. V.3 that $\dot{q}_w(t)$ generally follows $h(t)$. For example, $\dot{q}_w$ is minimum when h is at its minimum, even though at this particular instant ΔT_f is rising and would act to counteract the effect of the decrease in h . A discrepancy emerges when $\dot{q}_w$ and h are at their maxima, with the maximum $\dot{q}_w$ encountered just before the maximum h , but also just after the maximum ΔT_f . This is because the internal energy of the solid domain has increased (or, the solid has ‘charged up’) at this time and so there is a larger ΔT_f to drive heat transfer into the fluid. It can also be seen that the isothermal outer wall B/C with $T'_0 = 0$ has a shallower and broader peak in $\dot{q}_w$ and a shallower trough in ΔT_f compared to the isoflux condition $\dot{q}'_0 = 0$, but otherwise the general behaviour for these two types of B/C is similar.

The effects of h^* , Bi and a^* on the augmentation ratio $A = \bar{\dot{q}}_w/\bar{h} \overline{\Delta T_f}$ (from Eq. III.5) can also be investigated. Recall that when A is unity, i.e. when there is no augmentation, the time-averaged heat flux at the wall $\bar{\dot{q}}_w$ is expected to be equal to the mean HTC $\bar{h}$

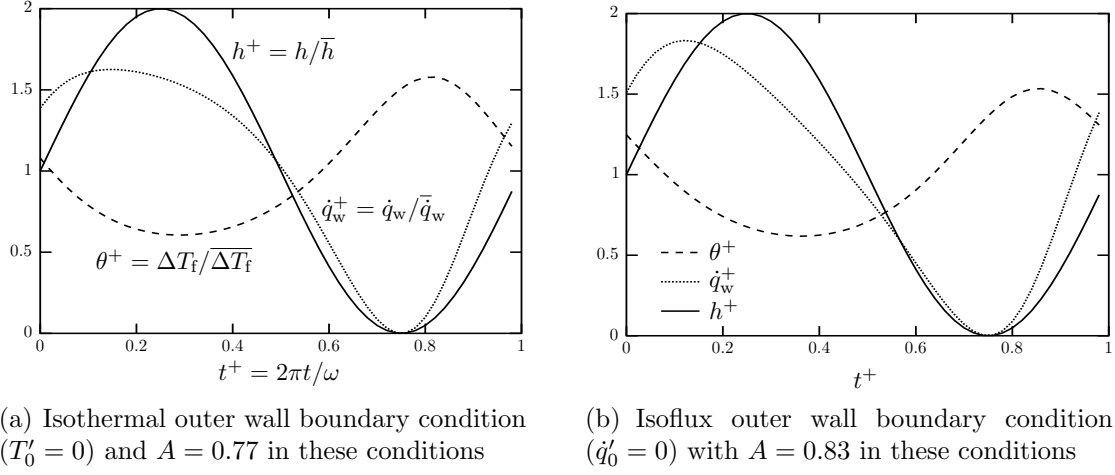


Figure V.3: Time evolution of unsteady heat transfer behaviour with constant $T_{f,b}$ (Case I). The conditions are: $h^* = 1$, $Bi = 1$ and $a^* = 1$. The temperature difference across the fluid domain is ΔT_f and the heat flux at the interface is $\dot{q}_w$. Note that, from Eq. V.30, $\dot{q}_w^+ = h^+ \theta^+ / A$.

multiplied by the mean temperature difference across the fluid $\overline{\Delta T_f}$. The augmentation ratio A is plotted as a function of the dependent variables in Fig. V.4 for the two B/Cs. The two types of B/C do not show significant differences in terms of the effect of Bi on A when the normalised material thickness a^* is set equal to unity (see also Fig. V.4). The augmentation ratio A is affected more strongly at larger values of Bi and larger HTC fluctuation intensities h^* , varying between unity and some minimum value, which for each value of h^* is reached at the highest Bi . At low Bi the heat transfer problem is dominated by the solid domain, and so the fluctuations in h have little effect on the wall temperature T_w . Hence, the augmentation tends to zero and $A \rightarrow 1$. As Bi is increased the wall temperature T_w is increasingly affected by the fluid domain fluctuations. This leads to an increase in the augmentation effect until a point is reached when the temperature fluctuations and heat flux fluctuations are completely in phase with each other and the maximum augmentation is achieved. This condition corresponds to a minimum $A = 0.15$.

Consider now the variation of the augmentation ratio with the dimensionless solid plate thickness a^* at a constant Bi in Fig. V.5. A much stronger augmentation effect is observed for the isoflux condition compared to the isothermal equivalent. The greatest augmentation (i.e. lowest augmentation ratio A) is observed at small solid thicknesses a^* and disappears ($A \rightarrow 1$) when $a^* \gg 1$.

This can be understood as follows: at low frequencies the wall temperature has enough

Results and Discussion

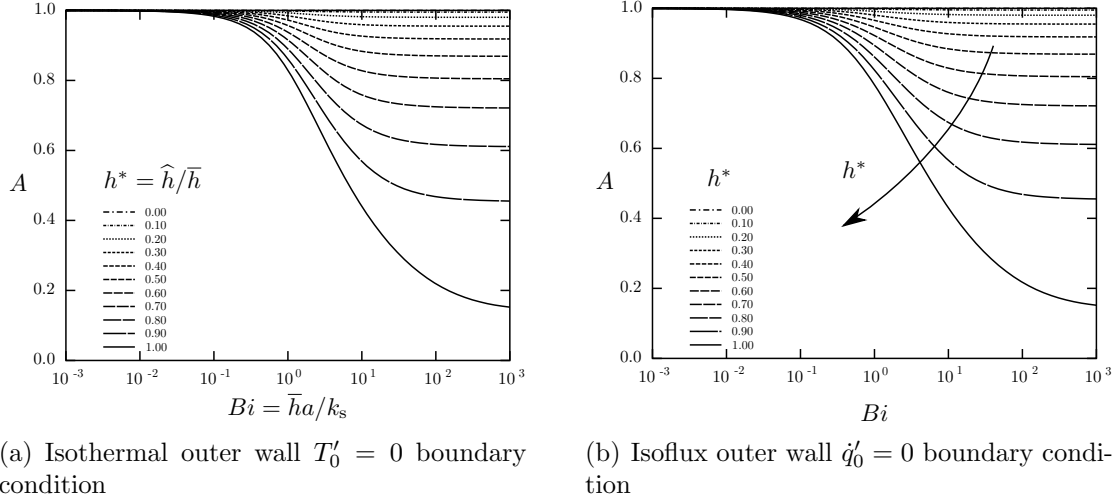


Figure V.4: Variation of the augmentation ratio A for constant fluid temperature $T_{f,b}$ (Case I) and $a^* = 1$, as a function of Bi for various h^* . (a) Isothermal outer B/C. (b) Isoflux outer B/C.

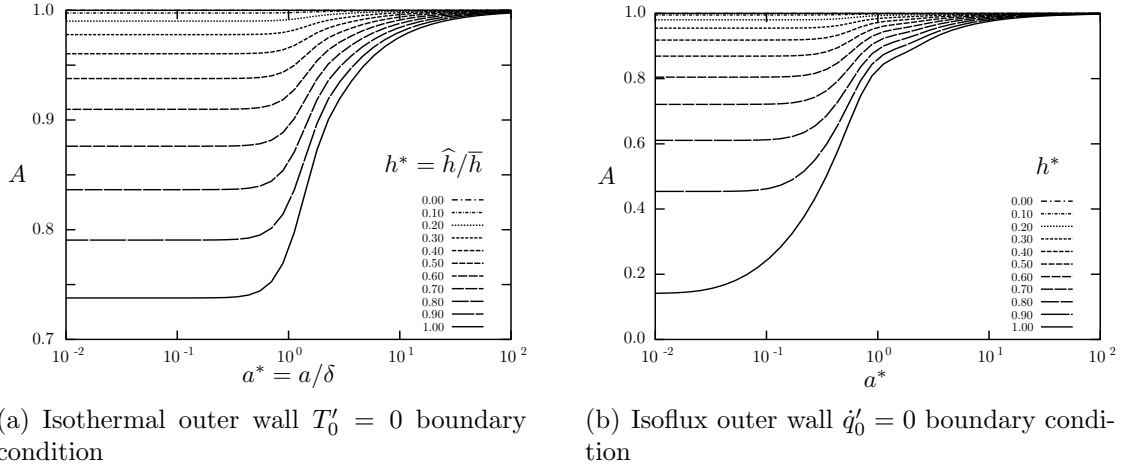


Figure V.5: Variation of the augmentation ratio A for constant fluid temperature $T_{f,b}$ (Case I) and $Bi = 1$, as a function of a^* for various h^* . (a) Isothermal outer B/C. (b) Isoflux outer B/C.

time to react to the fluctuations in the HTC, and so the time profiles of h and T_w will be more out of phase, resulting in greater augmentation (and smaller values of the augmentation ratio A). It is also interesting to note that the contours generated with the isoflux outer wall B/C are not as smooth around the point where the dimensionless thickness is unity as those generated with the isothermal condition.

Further insight into the results presented in Figs. V.4 and V.5 can be gained by plotting a map of augmentation as a function of Bi and dimensionless solid thickness a^* ,

which is done in Fig. V.6. Inspection of this contour plot reveals that the augmentation is similar for the isothermal and isoflux outer wall boundary conditions when a^* is sufficiently greater than one (i.e. when the solid is sufficiently thicker than δ). This is due to the fact that for thick solids the inner wall is sufficiently far away from the outer wall B/C, so that fluid-solid interface (inner wall) becomes decoupled from and independent of the outer wall B/C. On the other hand, when $a^* \ll 1$ the solutions diverge, with the isoflux outer B/C having a much larger range of Bi susceptible to significant augmentation (minimum achieved A of 0.14, as opposed to 0.74 for the isothermal condition at the extreme $h^* = 1$). This information helps to explain the similar profiles in Fig. V.4 that were generated for $a^* = 1$ and the dissimilar profiles in Fig. V.5 that were generated for $Bi = 1$. The former profiles (Fig. V.4) were generated by cutting vertically down the contours in Fig. V.6, whereas the latter (Fig. V.5) are formed by cutting horizontally across the same contours.

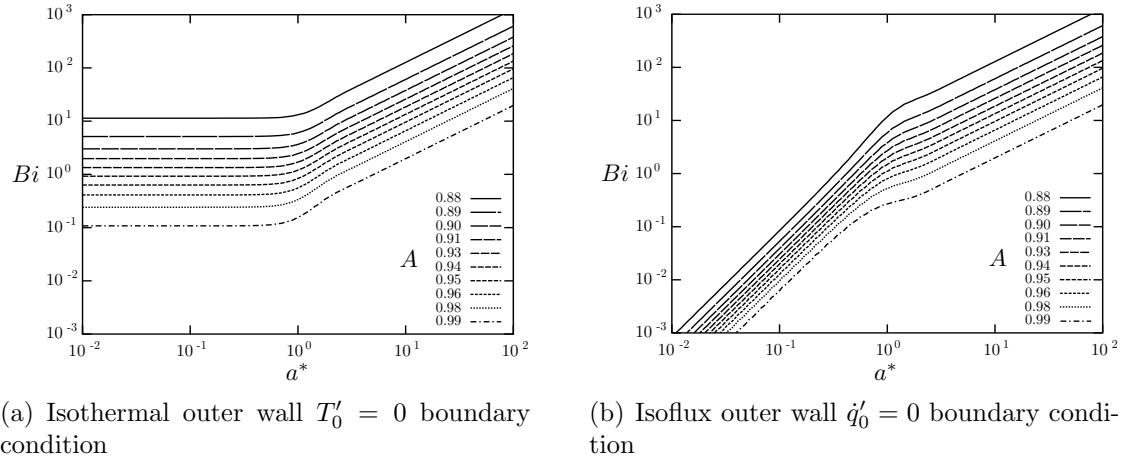


Figure V.6: Map of the magnitude of augmentation A in Biot number Bi and normalised material thickness a^* for Case I, at a fluctuation intensity in the heat transfer coefficient of $h^* = 0.5$.

Rule of thumb limits that are valuable for engineering purposes can be deduced from the above plots concerning conditions in which augmentation effects become negligible. For an isothermal outer wall B/C augmentation is insignificant (that is when $A \geq 0.99$ approximately) when,

$$Bi < 0.1 \text{ or } Bi < 0.3 a^*, \quad (\text{V.31})$$

Results and Discussion

and for the isoflux outer wall B/C approximately when,

$$\text{Bi} < a^{*2} \text{ and } \text{Bi} < 0.3 a^*. \quad (\text{V.32})$$

These limits have direct practical usefulness. For example, consider the design of a heat exchanger that will experience a periodic fluctuating heat transfer component. In this situation the material properties of the heat exchanger can be chosen in the design stage so as to minimise the augmentation effect and maximise the total time-averaged heat transfer. Conversely, where heat exchange is not wanted, such as for example pipe-work carrying a vapour with an oscillating pressure, augmentation should be promoted as this will reduce the overall heat losses.

By looking at the augmentation maps in Fig. V.6 for the two B/Cs, it can be deduced that there are two factors which ought to be considered in terms of the thermal properties of the solid material and their effect on unsteady conjugate heat transfer: (i) changes in the heat capacity $\rho_s c_s$, which corresponds to horizontal changes on these plots; and (ii) changes in the thermal conductivity, which corresponds to diagonal movements, with a gradient of two decades per decade on this logarithmic plot.

In the instance of an isothermal B/C imposed on the outer wall, augmentation becomes insensitive to changes in the material heat capacity $\rho_s c_s$ when the solid is ‘thin’, $a^* < 1$. Conversely, when an isoflux outer B/C is imposed, augmentation is insensitive to changes in solid’s thermal conductivity when the solid is ‘thin’. This derives from the stronger coupling of the fluid–solid interface (inner wall) temperature T_w to the outer wall B/C T_0 when the solid is thin. In this thin condition, since the solid’s ‘thermal mass’ (the total heat capacity of the solid per unit area in the wall normal y -direction, $\rho_s c_s a$) is insignificant, thermal conduction dominates thermal storage in the isothermal outer wall condition, whereas in the isoflux condition only the thermal capacity plays a role, as there is little conduction through the wall.

V.4.2 Case IV

When the solid domain is such that the inner wall temperature is effectively constant, $T'_w = 0$, only the fluid’s response to the fluctuating HTC needs to be considered. In this case, given that the heat transfer solution only depends on the fluid domain, the problem becomes non-conjugate. However, the fluid temperature $T_{f,b}$ (here, $\Delta T'_f = T'_{f,b}$) and heat transfer coefficient h can still fluctuate with respect to each other, and hence the problem can still exhibit augmentative effects.

Taking the fluid response described in Section V.3, where the fluid is advected over an active heat transfer area, a constant wall temperature B/C and a fluctuating HTC are imposed. The resulting equation for the fluid temperature is given by Eq. V.15 in Section V.3.3. Since the fluid temperature and the wall temperatures are known, the wall heat flux can be calculated, and the augmentation investigated.

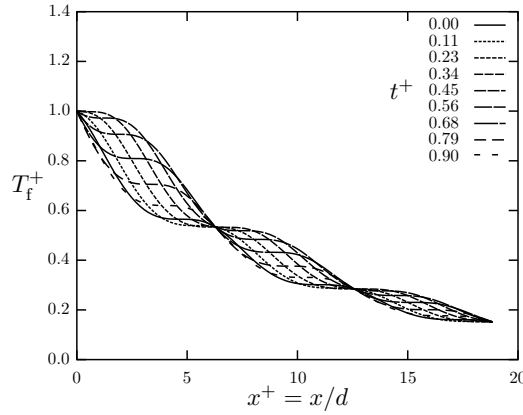


Figure V.7: Spatial profile of the normalised temperature $T_f^+ = \Delta T_{f,b}/(T_{f,in} - T_w)$ at different time instances $t^+ = 2\pi t/\omega$ for the constant wall temperature case (Case IV), with conditions $C_H = 0.1$, $h^* = 1$ and $U^* = 1$.

Figures V.7 and V.8 show examples of spatial temperature profiles for the constant inner wall temperature condition. The fluctuation in $T_{f,b}$ rises from zero at the inlet to a maximum and then back to a minimum node with a period of $2\pi U^* = x/d$ in the x -direction. The augmentation profile varies sinusoidally around unity, also with a spatial period of $2\pi U^* = x/d$. At the inlet $A = 1$, which corresponds to no augmentation. The augmentation is then alternately positive and negative. Maximum augmentation is achieved at positions $(\pi/2 + 2\pi n) U^* = x/d$ along x , and minimum augmentation is found every $(3\pi/2 + 2\pi n) U^* = x/d$, though the total heat transfer rate diminishes with x .

Figure V.9 shows that the *amplitude* of the augmentation ratio in the spatial dimension $\hat{A}$ increases with C_H . At low values of C_H the augmentation amplitude is negligible. Then, at some critical C_H the augmentation amplitude begins to rise until it reaches a maximum value, at which it plateaus. The point of maximum augmentation amplitude $\hat{A}$ corresponds to the condition in which the phase difference between h and the fluid temperature difference ΔT_f approaches zero. Figure V.9 reveals further that $\hat{A}$ is determined by the HTC fluctuation intensity h^* , while the critical C_H is dependent on U^* .

Results and Discussion

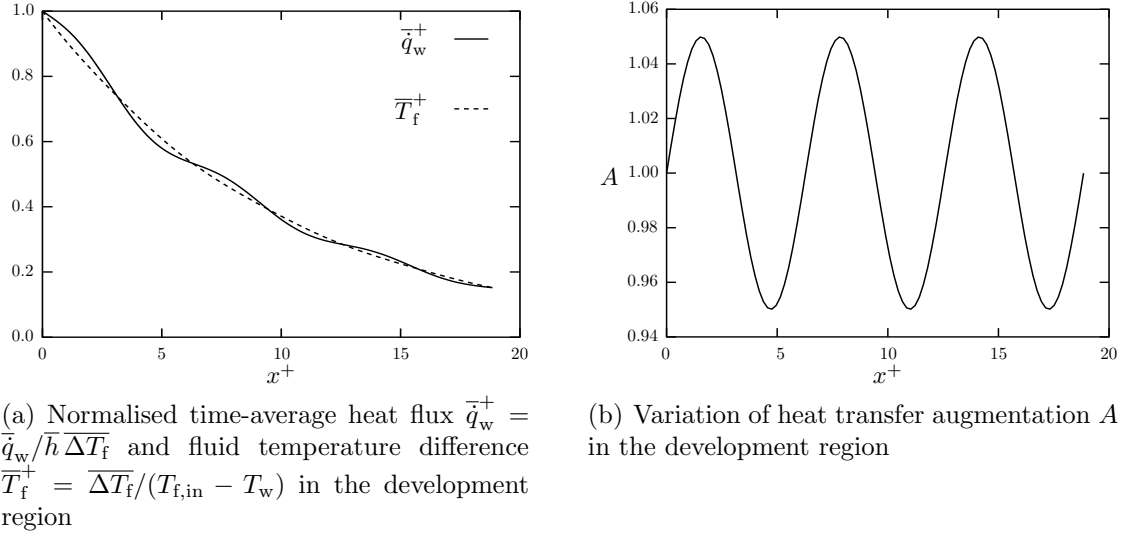


Figure V.8: Example spatial profiles of characteristic heat transfer quantities for Case IV with a constant inner wall temperature $T'_w = 0$. Showing heat transfer behaviour for conditions $C_H = 0.1$, $h^* = 1$ and $U^* = 1$.

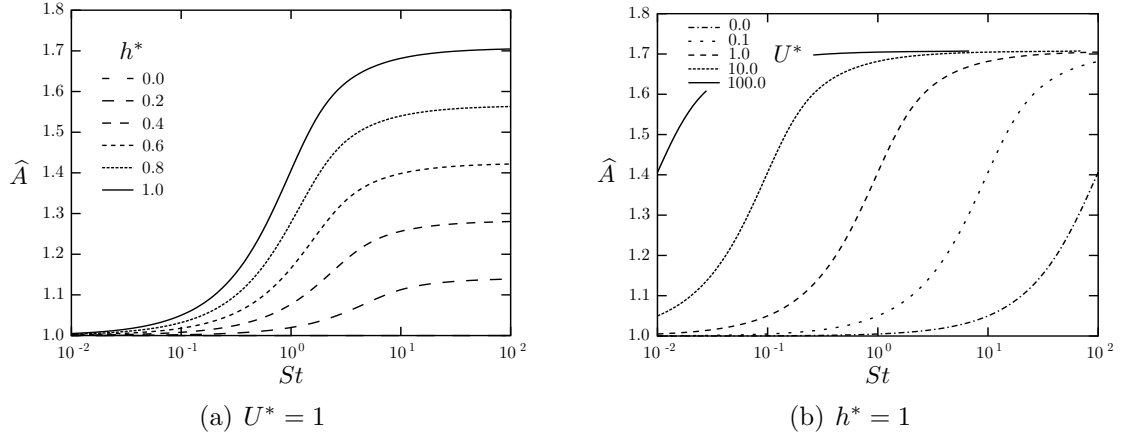


Figure V.9: Map of augmentation spatial amplitude $\hat{A}$ for Case IV against Stanton number C_H for different: (a) h^* , and (b) U^* , at the position of maximum A .

This observation concerning the spatial variation in A is significant and has important implications. In his pioneering work, Zudin [169] concluded that augmentation could not be positive for 1-D unsteady heat transfer, such that the presence of unsteadiness always leads to a deterioration in heat transfer performance. Nevertheless, here it is seen that heat transfer augmentation can be positive ($A > 1$) if fluid advection is considered in the thermally developing heat transfer problem, i.e. if the problem is in fact 2-D. This is the only scenario considered that exhibited positive augmentation. By investigating

the mechanism that gives rise to this behaviour we find that positive augmentation can only be experienced if the fluid domain is undergoing thermal development. Normally the temperature fluctuations in the fluid and solid (and hence also, the fluctuations in the temperature difference across the fluid) lag behind the fluctuations in the heat transfer coefficient, resulting in negative augmentation (or an augmentation loss), as in Fig. V.3. However, the convection of fluid along a developing temperature profile allows, in some regions, the fluctuations in fluid temperature to be advanced in phase with respect to the fluctuations in the HTC, resulting in augmentation ratios greater than unity. Physically this reflects a process in which, when the HTC is low instantaneously at some spatial location, warmer fluid is able to convect downstream into a cooler region, thus increasing the available temperature difference and driving a higher heat flux into the fluid when the heat transfer coefficient becomes high again.

Importantly, the positive augmentation effect can be utilised to improve heat exchange in engineering processes. For example and with reference to Fig. V.8(b), a heat exchanger experiencing an oscillating heat transfer coefficient (e.g. due to inlet flow variations) and made of a certain length (i.e. $x = \pi U^* d = \pi U / \omega$) would experience an overall positive augmentation effect. This would allow either more heat to be exchanged or a smaller heat exchanger to be used, which has obvious benefits. Moreover, such a heat exchanger made of length $x = 2\pi U / \omega$ would have a constant outflow temperature, despite experiencing unsteadiness in the heat transfer, which may have an application in situations where a steady temperature is desirable, such as in a process control application.

So far the HTC was assumed to have no spatial variation and the velocity of convection was assumed constant. Generally, the fluctuation of h is linked to fluctuations in the bulk fluid velocity U , which would imply non-constant advection. In other cases the cause of the fluctuation in h , such as bursts of turbulence, slug flow, or waves in a thin interfacial flow (film), would advect with the background flow which implies a spatial variation in h . Nevertheless, and although there is an added complexity in deriving the solution depending on the relationship chosen between U and h , the general conclusion drawn here should still apply to the scenario in which the bulk fluid velocity is time-varying. This is due to the fact that the same underlying mechanism of fluid advection advancing the phase of temperature fluctuations relative to those in the HTC is still expected to be present. The results presented here are for the special case when the fluctuations in U tend to zero, i.e. U is constant, but h still fluctuates. On the other hand, spatial variations in h may generate different results and this should be studied further.

V.4.3 Case V

In the limit of large x , far from the inlet, the fluid flow becomes fully developed, the heat transfer problem becomes self-similar and there is no longer a variation in the x -direction. Case V combines this type of non-isothermal flow, with fluctuating wall and fluid temperatures, as well as an unsteady HTC. The fully developed condition requires a heat flux at the inner wall $\dot{q}_w$ that is instantaneously uniform along the flow. This corresponds to a uniform time-averaged wall heat flux $\bar{\dot{q}}_w$ and linearly increasing time-averaged bulk fluid temperature $\bar{T}_{f,b}$ in the x -direction.

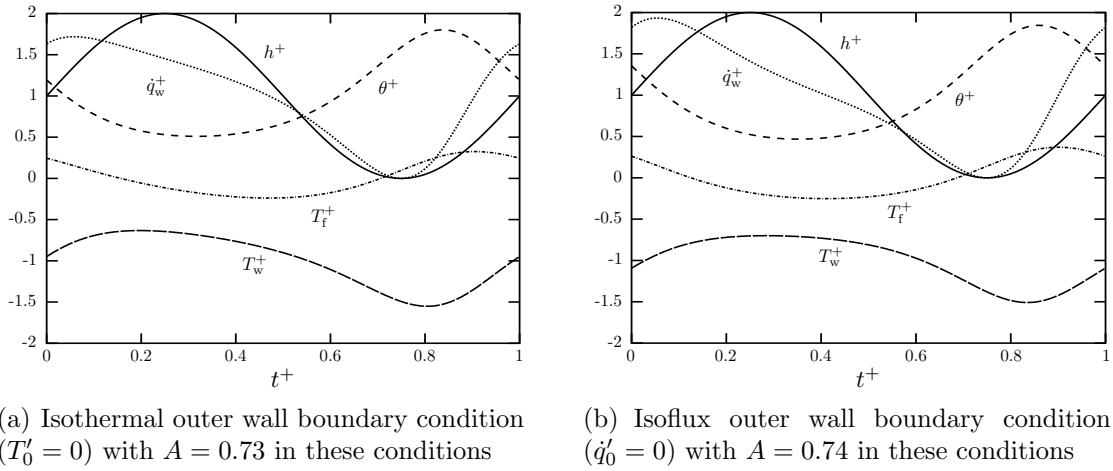


Figure V.10: Time evolution of the heat transfer behaviour in Case V with a time-varying $T_{f,b}$ for conditions: $h^* = 1$, $Bi = 1$, $a^* = 1$, $C_R = 1$. The temperature difference across the fluid domain is $\Delta T_f = T_{f,b} - T_w$ and the heat flux at the interface is $\dot{q}_w$. Note that here, $T_f^+ = (T_{f,b} - \bar{T}_{f,b})/\Delta \bar{T}_f$, $T_w^+ = (T_w - \bar{T}_{f,b})/\Delta \bar{T}_f$, $\theta^+ = \Delta T_{f,b}/\Delta \bar{T}_f = T_f^+ - T_w^+$, $h^+ = h/\bar{h}$ and $\dot{q}_w^+ = \dot{q}_w/\bar{\dot{q}}_w$.

Figure V.10 shows example time plots of the quantities of interest. As before, the inner wall heat flux $\dot{q}_w$ is observed leading the HTC and lagging behind ΔT_f , going through a maximum before h peaks and after ΔT_f passes its maximum value. This can be understood as follows: at the instant when h is minimum (that occurs at $t^+ \approx 0.75$ in Fig. V.10 when $h = 0$), there is no heat flux removed from the fluid and deposited into the wall, i.e. $\dot{q}_w = 0$. The bulk fluid temperature $T_{f,b}$ is high and the temperature of the wall T_w is low, even though the driving temperature difference ΔT_f for convective heat transfer is high. As the HTC gradually increases it allows an increasing $\dot{q}_w$ into the solid. The solid T_w responds after a small time by increasing gradually, while the fluid $T_{f,b}$ also responds after some time by slowly decreasing. As a consequence of these two processes,

ΔT_f across the fluid begins to decrease. There are now two competing effects on $\dot{q}_w$, from: (i) the increasing h ; and (ii) the decreasing ΔT_f . Initially, the former is stronger because ΔT_f is still relatively high, which leads to $\dot{q}_w$ increasing along with h , although lagging behind due to the aforementioned heat transfer dynamics. The increasing $\dot{q}_w$ leads to an increasing T_w and a decreasing $T_{f,b}$, and thus an attenuation in the driving potential for heat transfer ΔT_f . Eventually, at some time (this occurs at $t^+ \approx 0.06$ in Fig. V.10) the decrease in ΔT_f more than overcomes the continuing rise in h and results in a maximum $\dot{q}_w$. After this point the strongly decreasing ΔT_f causes $\dot{q}_w$ to decay ahead of h , even though h continues to increase.

Figure V.10(a) shows the temporal evolution in Case V for the isothermal outer wall B/C $T'_0 = 0$. Such a condition may be hard to conceive, given that the mean heat flux must be constant to satisfy the assumptions that Case V is based upon. The requirement for the mean heat flux condition would require that T_0 matched $T_{f,b}$, increasing linearly so that $\bar{T}_f - T_0$ was held constant. Although this is possible, an isoflux outer wall B/C will more readily satisfy the assumptions made in Case V.

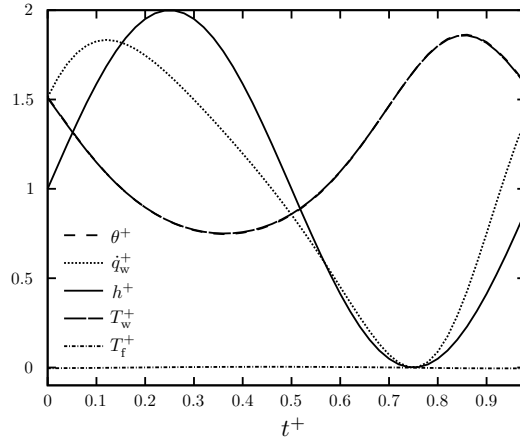


Figure V.11: Time evolution of the heat transfer behaviour in Case V (time-varying $T_{f,b}$), with an isoflux outer wall boundary condition $\dot{q}_0$ and: $h^* = 1$, $Bi = 1$, $a^* = 1$, $C_R = 100$.

The magnitude of the heat transfer augmentation A in Case V is dependent on the Biot number Bi , the normalised amplitude of the fluctuations in the HTC h^* , and the normalised material thickness a^* , as before in Case I. Case V also introduces the effect of the heat capacity ratio between the fluid and solid domains $C_R = C^* d^*$. For large C_R (e.g. because the fluid domain is large $d/a \gg 1$) the bulk fluid temperature $T_{f,b}$ will be constant and so Case V should tend to Case I. This can be seen in Fig. V.11, which demonstrates the similarity between Case V and Case I for large C_R (compare to the time

plots shown in Fig. V.3 generated at the same conditions). In addition, the similarity between these cases can be seen in Fig. V.12(a) where the results for A as a function of Bi tend to those obtained in Fig. V.4(b) for Case I, and Fig. V.12(a) where the results for A as a function of the solid domain thickness a^* tend to those in Fig. V.5(b).

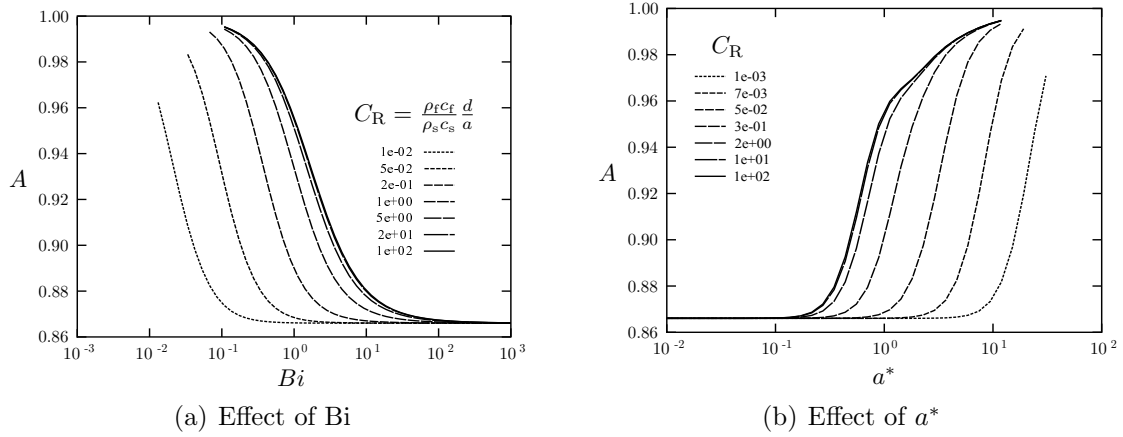


Figure V.12: Variation of augmentation A as a function of a^* , Bi and C_R for Case V with an isoflux outer wall boundary condition $\dot{q}'_0 = 0$ and $h^* = 0.5$.

Finally, consider Fig. V.13 that shows the full augmentation map for Case V. A decreasing heat capacity ratio C_R causes the region in this map that is affected by augmentation to spread to lower Bi and higher a^* . This would imply that in situations where C_R is low, more care should be taken to account for augmentation effects and greater scope exists for these effects to modify the resulting heat transfer behaviour. In Chapter VII and VIII we will compare and discuss the accuracy of this model to actual experimental results in flows featuring both periodic and broadband fluctuating HTC's.

V.4.4 Case VI

With Case VI the fluctuations in the HTC convect with the bulk fluid flow, and so h is not uniform over the fluid domain. The governing equation V.27 was integrated along the c^* for the case of an isoflux boundary condition, $\dot{q}'_0 = 0$.

Figure V.14 shows the evolution of the calculated heat transfer augmentation along the length of the fluid domain c^* for a range of different Biot numbers and dimensionless solid thickness a^* . We can see that for the case of $c^* = 0$ the augmentation behaves in the same manner as Case I. Compare the case of $a^* = 1$ and $c^* = 0$ in Fig. V.14 to

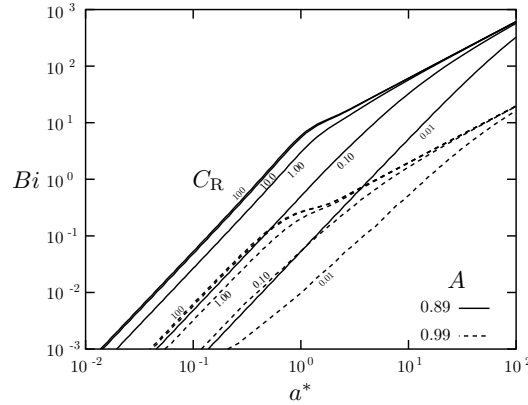


Figure V.13: Case V with an isoflux outer wall B/C. Movement of augmentation region with variations in C_R . An augmentation ratio of $A = 0.99$ corresponds to a 1 % negative augmentation, or augmentation loss, and a ratio of 0.89 corresponds to an 11 % negative augmentation ($A = 0.86$ is the minimum obtainable with $h^* = 0.5$).

Fig. V.5; at low Bi the augmentation effect is small and A tends to 1; at high Bi the augmentation factor is high and A drops. A similar relationship to Case I for a^* is also seen by comparing the case of $Bi = 1$ and $c^* = 0$ in Fig. V.14 to Fig. V.5. This is because the temperature at the inlet to the fluid domain has not had time to respond, and so $T_{f,b}$ is effectively constant.

When the material thickness a^* is large the augmentation effect seems to start of small with $A \sim 1$ and then increases (A decreasing) as the flow progresses in c^* . The evolution in A along c^* , at large a^* , looks like an exponential decay with the augmentation factor tending to a fix value of A . However at lower material thickness a^* the augmentation factor seems to overshoot this value and exhibit a decaying oscillation along c^* . The change with a^* between the smooth profile in A and the “under damped” profile appears to depends on the Biot number, with the transition appearing later in lower Biot numbers.

V.5 Summary

A simple semi-analytical model has been composed to describe one-dimensional conjugate heat transfer with a time-varying heat transfer coefficient, taking into account fluctuations in the solid and fluid temperatures. The model was used to investigate the effects of material properties, geometry and frequency on the heat transfer process and specifically on the heat transfer augmentation, through a set of reduced dimensionless problem variables. Maps of augmentation were produced for cases where the bulk fluid temperature was either fixed or free to respond to the heat exchange process with the

Summary

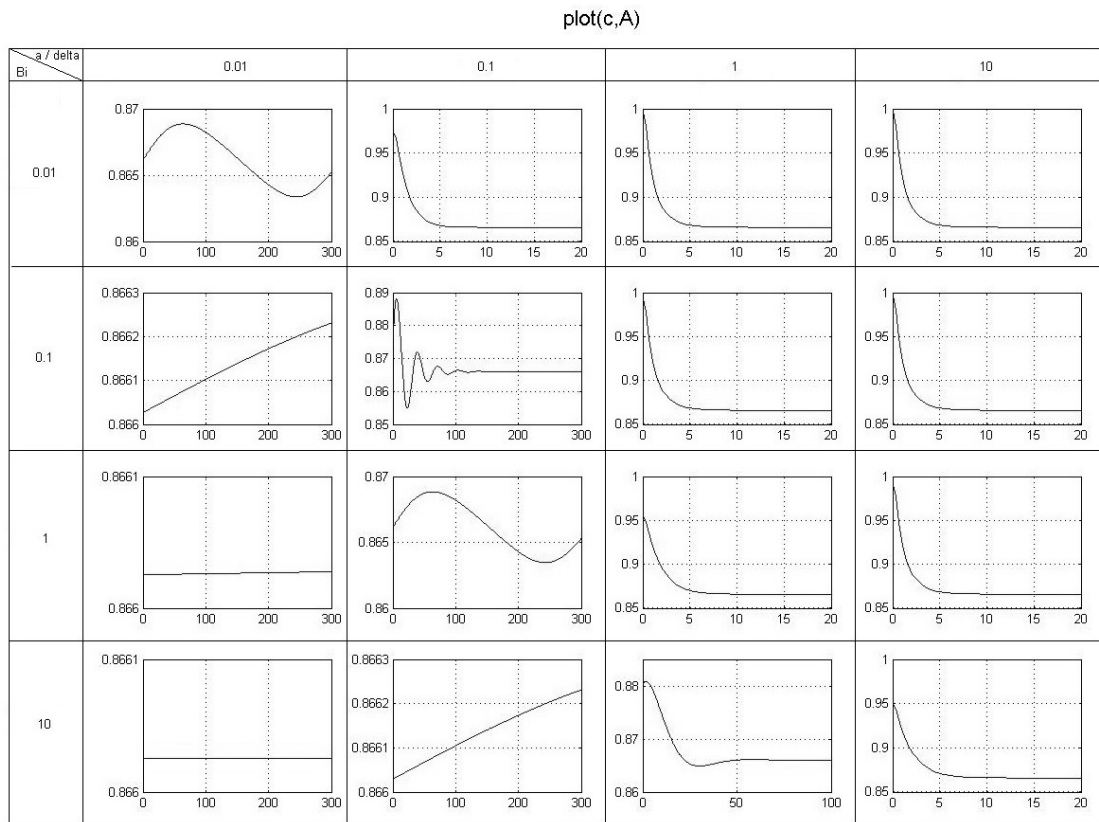


Figure V.14: Case VI augmentation $A = \bar{q}/h\delta\theta_f$ along c^* for different Biot numbers, Bi , and dimensionless solid thickness, a^* .

CHAPTER V. LOW-DIMENSIONAL CONVECTIVE-CONDUCTIVE SYSTEMS

solid, with isothermal and isoflux boundary conditions on the non-fluid side of the solid. Both developing and fully developed solutions were considered. It was noted that the externally imposed boundary condition on the non-fluid side of the solid only affected the results when the walls of the solid were thin, due to the increased coupling of the common fluid-solid boundary on the boundary condition.

In general it was found that the region of greatest (negative) augmentation occurred at high Biot numbers, when the solid walls were thin relative to the unsteady thermal diffusion length scale (e.g. this would occur at low frequencies), when the fluctuations in the heat transfer coefficient were large compared to its mean and when the heat capacity of the fluid domain was smaller than that of the solid (e.g. for thin films). The augmentation effect was negative in these cases, meaning a loss in ability to transfer heat in the mean, i.e. a reduction in the time-averaged heat flux relative to that expected from the multiple of the time-averaged heat transfer coefficient and the time-averaged temperature difference in the fluid. It is proposed that these maps can be used to choose material properties, or the solid geometry, to avoid or promote augmentation (i.e. heat transfer) as necessary.

As part of the investigation the thermal response of the fluid to unsteady heat exchange with the solid was derived for developing flows. It was found in this case that augmentation can be positive in some spatial regions, which is a new finding not reported in previous related studies. The underlying mechanism that gives rise to this effect was investigated. The enhanced heat transfer was found to arise by the convection of fluid along a developing thermal field which allowed, in some regions, the fluctuations in fluid temperature to be advanced in phase with respect to the fluctuations in the heat transfer coefficient (e.g. due to the flow). This finding is of great interest and can be utilised in heat exchanger designs, by tuning their length in order to promote greater net heat transfer or a smaller heat exchanger.

Chapter VI

Results 2:

Compression/Expansion—Work in Convective—Conductive Systems: Gas Springs and the NIFTE

A rigorous understanding and reliable methodology for the prediction of the behaviour and performance of thermodynamic systems demands the development of a framework that can describe formally the necessary unsteady and conjugate heat transfer processes. The purpose of this chapter is to present such a framework and to show results of its application for the characterisation of a reciprocating compression and expansion process.

VI.1 Gas Spring: Problem Formulation

The full problem treated here deals with the thermodynamic process undergone by a fixed mass of gas trapped in a reciprocating piston-in-cylinder configuration in the presence of finite heat transfer from/to the gas, with a fixed temperature imposed on the *external* (rather than the internal) wall of the cylinder. The latter has been done in order to estimate the effects of conjugate heat transfer. This is also an approximation, but one that allows assessment of the thermal properties of the wall. The model that is developed is valid for operation at low volumetric compression ratios ($1 < C_R < 6$), due to the description used for the convective heat transfer coefficient in the fluid, however, this can

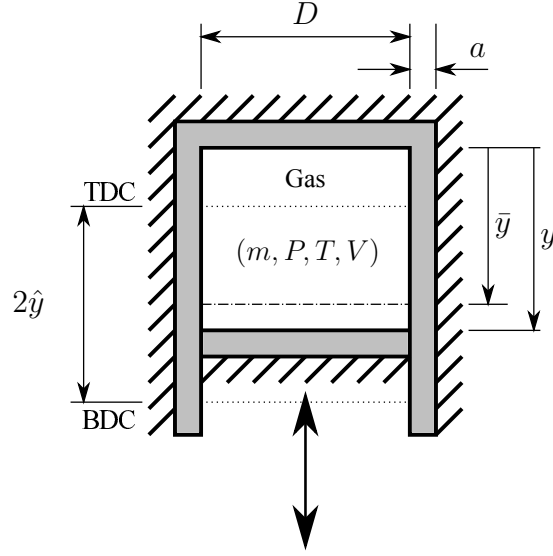


Figure VI.1: Schematic of the gas spring problem.

be easily extended to higher compression ratios. Radiative heat transfer, mechanical and fluid viscous losses are not considered.

Consider the sinusoidal compression and expansion with frequency ω_1 of a mass of gas m enclosed by the cylinder and the reciprocating piston shown in Fig. VI.1.

The geometry features a piston of diameter D following an *imposed* reciprocating motion (e.g., by connection via a mechanism to a motor rotating at constant angular speed ω_1), with amplitude $\hat{y}$ about a mean position $\bar{y}$ described by,

$$y(t) = \bar{y} + \hat{y} \cos(\omega_1 t). \quad (\text{VI.1})$$

During a cycle the piston makes one full movement from bottom dead centre (BDC) to top dead centre (TDC) and back again. As the piston progresses from BDC to TDC the volume enclosed by the piston $V(t) = Ay(t)$ decreases, which results in a corresponding increase in the pressure $P(t)$ and temperature $T_f(t)$ of the enclosed gas. Here, $A = \pi D^2/4$ is the cross-sectional area of the cylinder. Conversely, as the piston travels back to BDC the pressure P and temperature of the gas T_f will decrease. It is assumed that the mass of gas enclosed by the spring m is fixed, i.e. there is no leakage between the piston and the cylinder, and that the gas is ideal.

The fluctuations in the fluid temperature $T_f(t)$ during the cycle give rise to time-varying heat transfer between the fluid and the walls of the cylinder $\dot{Q}_f(t)$, and consequently also, through the cylinder walls to the environment, which is assumed to be

Gas Spring: Problem Formulation

isothermal at *constant* (external) temperature T_0 . In general, the solid wall temperature $T_w(t)$ will be fluctuating in response to the time-varying heat transfer rate $\dot{Q}_f(t)$.

Following Lee [7], Kornhauser and Smith Jr [8], Kornhauser and Smith [28] and others, cyclic convective heat transfer in a gas spring may be described by Newtons law of cooling while employing complex temperatures, heat fluxes and Nusselt numbers Nu . This results in a complex formulation of Newtons law,

$$\dot{q}_f = \frac{\kappa_f}{D} \left[Nu_{\Re} (T_f - T_w) + Nu_{\Im} \frac{1}{\omega_1} \frac{d(T_f - T_w)}{dt} \right]. \quad (VI.2)$$

Note that in Eq. VI.2, $\dot{q}_f$ is a flux, i.e. the heat transfer rate $\dot{Q}_f(t)$ per unit *time-varying* surface area available to heat transfer $S(t)$, which includes the sides of the cylinder, as well as its base and the face of piston, such that $S(t) = \pi D y(t) + \pi D^2/2$. In addition, and contrary to convention, note that $\dot{q}_f$ is defined as positive when leaving the gas (see also Eq. VI.9 and VI.21)). The imaginary part of the complex Nusselt number formulation in Eq. VI.2 arises physically from a turning point in the temperature profile through the gas [9], caused by the work done by/on the gas, which acts to introduce a phase difference between the heat flux at the wall $\dot{q}_w$ and the bulk temperature difference across the gas $T_f - T_w$. For $Pe_\omega < 100$, the Nusselt number Nu was derived by solving the energy equation in the gas after simplification [7],

$$Nu = Nu_{\Re} + i Nu_{\Im} = \sqrt{2Pe_\omega} \frac{(1 + i) \tanh(p)}{1 - \frac{\tanh(p)}{p}}, \quad (VI.3)$$

which is a function of the Péclet number Pe_ω ,

$$Pe_\omega = \frac{\omega_1 D^2}{4\alpha_{f,0}}, \quad (VI.4)$$

and where $p = (1 + i) \sqrt{Pe_\omega/8}$. For $Pe_\omega > 100$, the Nusselt number Nu was taken from a correlation derived empirically by Kornhauser and Smith [28],

$$Nu_{\Re} = Nu_{\Im} = 0.56 Pe_\omega^{0.69}. \quad (VI.5)$$

The complex Nusselt number Nu relationships are shown together in Fig. VI.2 as a function of the Péclet number Pe_ω . The heat flux through the wall $\dot{q}_w$ can be found as the cyclic convolution (operation denoted by ‘*’) of the solid’s thermal response to

CHAPTER VI. COMPRESSION/EXPANSION-WORK IN CONVECTIVE-CONDUCTIVE SYSTEMS

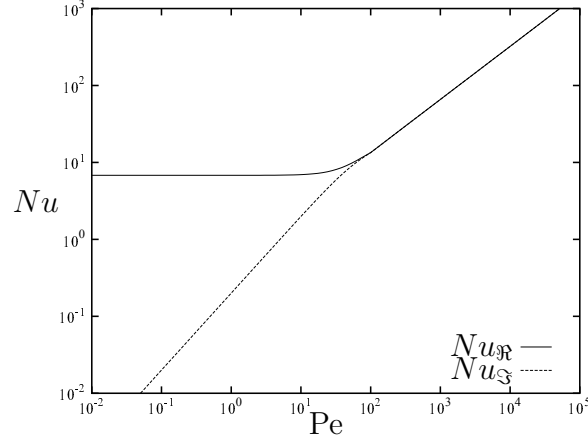


Figure VI.2: Complex Nusselt number Nu as a function of Péclet number

fluctuations in the wall temperature as shown in Chapter III,

$$\dot{q}_w = (T_f - T_w) * \frac{1}{z}, \quad (\text{VI.6})$$

where the thermal response, or “impedance” of the solid for an isothermal outer boundary condition $Z_n = Z(\omega_n)$ at a given frequency ω_n can be found by solving the one-dimensional unsteady heat diffusion equation in the solid,

$$Z_n = \frac{(1 - i)}{2} \frac{\delta_{s,n}}{\kappa_s} \tanh \left\{ (1 + i) \frac{a}{\delta_{s,n}} \right\}. \quad (\text{VI.7})$$

Here the thermal diffusion lengthscale in the solid, δ_s , sometimes referred to as the “thermal penetration depth”, at a given wave number n is,

$$\delta_{s,n} = \sqrt{\frac{2\kappa_s}{\rho_s c_s \omega_n}}. \quad (\text{VI.8})$$

The thickness of the cylinder wall is represented by a , and, ρ_s and c_s are the solid density and specific heat capacity respectively.

Equations VI.2 and VI.6 can be solved for T_w since $\dot{q}_f = \dot{q}_w$, while integrating the energy equation (i.e., Eq. VI.9) to solve for the evolution of the bulk fluid temperature T_f over the cycle,

$$mc_v dT_f + S\dot{q}_f dt + \rho_f RT_f dV = 0. \quad (\text{VI.9})$$

Performing dimensional analysis on the above equations it can be seen that the system

Gas Spring: Problem Formulation

can be described by 9 independent variables. The set of chosen variables is shown below,

$$f \left\{ \begin{array}{l} R_{\text{SB}} = \frac{\bar{y}}{D}, y_a = \frac{\hat{y}}{\bar{y}}, a^* = \frac{a}{\delta_{\text{s},1}}, \\ \kappa_r = \kappa_{\text{s}}/\kappa_{\text{f}}, \rho_{r,0} = \frac{\rho_{\text{s}}}{\rho_{\text{f},0}}, c_r = \frac{c_{\text{s}}}{c_v}, \gamma, \\ Pe_{\omega} = \frac{\omega_1 D^2}{4\alpha_{\text{f},0}}, T_0^* = \frac{c_v T_0}{\frac{1}{2}(\omega_1 D)^2} \end{array} \right\}. \quad (\text{VI.10})$$

where $\rho_{\text{f},0}$ and $\alpha_{\text{f},0}$ are the density and thermal diffusivity of the gas phase when the piston is in the mean (central) position, $y(t) = \bar{y}$. The variables include: (i) ratios of geometric parameters, and of fluid and solid thermophysical properties; (ii) the Péclet number Pe_{ω} ; and (iii) a dimensionless external (environment) temperature T_0^* , with T_0 constant.

The dimensionless geometric parameters that describe the geometry of the gas spring are found by normalising against each variables mean value, giving the displacement (y^*), area (S^*), volume (V^*) and density (ρ^*) variables given below:

$$y^* = \frac{y}{\bar{y}} = 1 + y_a \sin(\omega^* t^*), \quad (\text{VI.11})$$

$$S^* = \frac{2y^* R_{\text{SB}} + 1}{2R_{\text{SB}} + 1}, \quad (\text{VI.12})$$

$$V^* = \frac{V}{\bar{V}} = \frac{1}{y^*}, \quad (\text{VI.13})$$

$$\rho^* = \frac{\rho_{\text{f}}}{\rho_{\text{f},0}} = \frac{1}{V^*}, \quad (\text{VI.14})$$

where $t^* = \omega_1 t$ is the dimensionless time and $\omega^* = 1$ the dimensionless harmonic frequency. Normalising all temperatures by $(\omega_1 D)^2/2c_v$, which is a measure of the amount of energy given to the system scaled as a temperature, results in the dimensionless temperature,

$$T^* = \frac{c_v T}{\frac{1}{2}(\omega_1 D)^2}, \quad (\text{VI.15})$$

and the dimensionless pressure,

$$P^* = P \frac{c_v}{\frac{1}{2}(\omega_1 D)^2 \rho_{\text{f},0} R} = \rho^* R^* T_{\text{f}}^*. \quad (\text{VI.16})$$

Accordingly, the *real part* of the heat flux $\dot{q} = \dot{q}_{\text{f}} = \dot{q}_{\text{w}}$ in Eq. VI.2) becomes,

$$\dot{q}^* = \dot{q} \frac{c_v}{\frac{1}{2}(\omega_1 D)^2 \kappa_{\text{f}}} \frac{D}{\kappa_{\text{f}}} = \text{Nu}_{\text{R}} (T_{\text{f}}^* - T_{\text{w}}^*) + \text{Nu}_{\text{S}} \frac{d(T_{\text{f}}^* - T_{\text{w}}^*)}{dt^*}, \quad (\text{VI.17})$$

CHAPTER VI. COMPRESSION/EXPANSION–WORK IN CONVECTIVE–CONDUCTIVE SYSTEMS

and Eq. VI.6 is re-written as,

$$\dot{q}^* = (T_w^* - T_0^*) * \frac{1}{Z^*}, \quad (\text{VI.18})$$

where Z^* is the dimensionless thermal impedance of the solid at a given dimensionless frequency $n = \omega_n/\omega_1$, such that,

$$Z^* = Z \frac{\kappa_f}{D} = \frac{(1 - i)}{2\kappa_r} \sqrt{\frac{\alpha_{r,0}}{2\text{Pe}_\omega n}} \tanh \{(1 + i)a^*\}. \quad (\text{VI.19})$$

Furthermore,

$$\alpha_{r,0} = \frac{\kappa_r \gamma}{\rho_{r,0} c_r} = \left(\frac{\delta_{s,1}}{\delta_{f,0}} \right)^2, \quad (\text{VI.20})$$

in Eq. VI.19 is the solid to fluid thermal diffusivity ratio, with the former evaluated at the harmonic frequency ω_1 and the latter evaluated when the piston is in the central position $y(t) = \bar{y}$. Finally, Eq. VI.9 that describes the energy balance in the gas domain becomes,

$$\frac{1}{1 - \gamma} dT_f^* + \frac{\gamma}{\gamma - 1} \frac{2 + R_{\text{SB}}^{-1}}{2\text{Pe}_\omega} S^* \dot{q}^* dt^* + \rho^* R^* T_f^* dV^* = 0. \quad (\text{VI.21})$$

On examination of Eq. VI.21 it is observed that for low Péclet numbers Pe_ω the majority of the work done by the gas will be exchanged in the form of heat with the solid walls, and the system will effectively undergo isothermal compression and expansion. On the other hand, at high Péclet numbers Pe_ω the heat exchanged will be minimal, and the system will experience adiabatic compression and expansion.

Consequently, we would expect that at intermediate Péclet numbers Pe_ω the work done on the gas will result from an interplay between both the internal energy of the gas and the heat transfer to the solid and, by extension, the environment. A phase lag between the temperature of the fluid T_f^* and the heat transfer $\dot{q}^*$ will result in a net loss of work around the cycle. A useful metric to gauge the relative importance of the fluid and solid domains in the heat transfer solution is the Biot number, which can be interpreted as a ratio of temperature changes in the fluid to temperature changes across the solid,

$$\text{Bi} = \text{Nu}_{\mathcal{R}} a^* \frac{1}{\sqrt{\epsilon_r \text{Pe}_\omega}}. \quad (\text{VI.22})$$

By definition the solution to the gas spring problem will deviate from the isothermal boundary condition imposed in all previous studies on the internal walls of the cylinder when the Biot number Bi is large. In this condition the temperature of the wall will begin

Results and Discussion

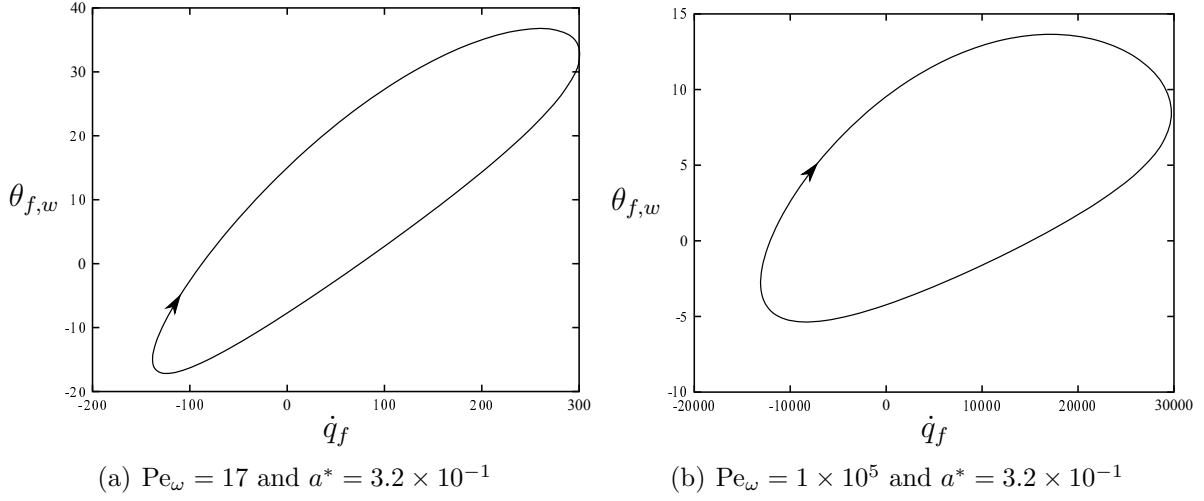


Figure VI.3: Modelled heat flux $\dot{q}_f$ and temperature difference θ_f around the cycle for a gas spring with $\kappa_r = 1$, $\rho_r = 1$ and c_r

to fluctuate. Due to the relationship of $Nu_{\mathfrak{R}}$ to the Péclet number Pe_ω , the Biot number Bi will become large when the Péclet number is either much less than one $Pe_\omega \ll 1$ or much greater than one $Pe_\omega \gg 1$. Two additional parameters of importance in determining the Biot number Bi are the effusivity ratio $\epsilon_r = \kappa_r \rho_{r,0} c_r \gamma$ between the solid and the fluid, and the ratio a^* of the wall thickness to the thermal diffusion length in the solid. Both small effusivity ratios ϵ_r and/or large lengthscale ratios a^* lead to large Biot numbers Bi .

VI.2 Results and Discussion

Figure VI.3 shows typical temperature difference-heat flux ($\dot{q}-\theta$) plots for two gas spring cases: (a) $Pe_\omega = 17$ and $a^* = 3.2 \times 10^{-1}$; and (b) $Pe_\omega = 1 \times 10^5$ and $a^* = 3.2 \times 10^{-1}$. The temperature difference refers to the difference between the fluid and the internal surface of the solid wall,

$$\theta_{f,w} = T_f - T_w. \quad (\text{VI.23})$$

In a (quasi-) steady heat transfer scenario one would expect to see straight lines passing through the origin, with a gradient that is not dependent on the direction of the process (i.e., heating *vs.* cooling). The closed area inscribed by the resulting curves is an indicator of the (imposed, through Eq. VI.2 to VI.5) phase difference between θ and $\dot{q}$.

Moreover, a *positive asymmetry* of this plot with respect to the vertical axis at $\dot{q} = 0$, would suggest a net heat flow from the gas to the solid, and hence also from the solid to

the surroundings, which can only come from a net work *input* to the gas spring cycle. This work input is a pure loss, with the ideal gas spring requiring exactly the same amount of work to compress the gas as is gained by its subsequent expansion.

VI.2.1 Time-varying heat transfer coefficient

It is useful to be able to evaluate the expected value of the heat transfer coefficient in thermodynamic processes such as the one presented in this chapter so as to evaluate the heat transfer performance. However the heat transfer coefficient, $h = \dot{q}/\theta$, is a ratio of two variables which fluctuate about zero, namely the heat flux $\dot{q}$ and the temperature difference across the fluid θ . This will result in a heat transfer coefficient h which has a probability distribution that can be approximated by a Cauchy distribution function,

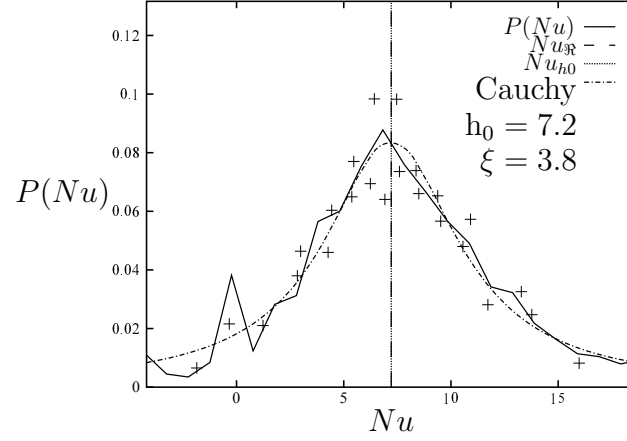
$$P_c(h) = \frac{1}{\pi\xi \left[1 + \left(\frac{h-h_0}{\xi} \right)^2 \right]}, \quad (\text{VI.24})$$

where h_0 is the location parameter, or expected value of h , describing the location of the peak probability density and ξ is the scale, or width, of the distribution. Such a distribution will arise if the ratio of two independent normally distributed variables is taken (heat flux $\dot{q}$ and temperature difference θ). It can be seen from Fig. VI.4 that the Cauchy distribution fits the modelled Nusselt number Nu remarkably well, providing a level of confidence in this choice of distribution function.

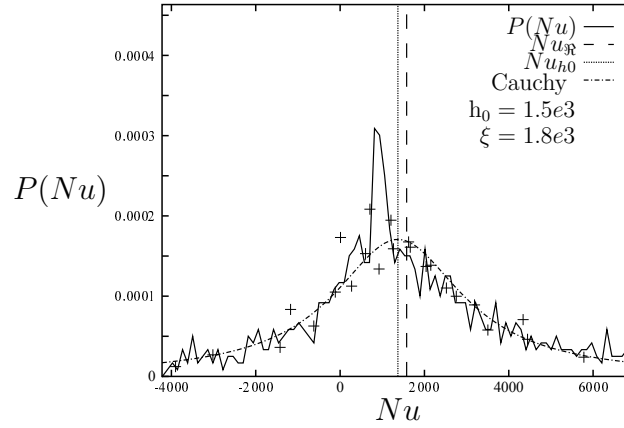
A Cauchy distribution poses a problem though, as it has no mean value, or higher moments. Additionally when using numerical methods or experimental data the mean is evaluated using a sample mean estimator which, for a Cauchy distribution, is itself a Cauchy distribution. Furthermore even if expressions for $\dot{q}$ and θ are known analytically the mean value cannot be calculated as the integral to find the expected value of h will not converge. Therefore the mean value of the Nusselt number $\overline{Nu}$ (and consequently also, of $\bar{h}$) cannot be used to estimate the expected value, and a sample mean will give unpredictable results.

It is important to note that despite this limitation, the parameters h_0 and ξ for a Cauchy distribution can still be estimated by maximising the log likelihood estimator,

$$l(h_0, \xi | h_1, h_2, h_3, \dots, h_n) = n [\log(\xi) - \log(\pi)] - \sum_{i=1}^n \log(\xi + (h_i - h_0)^2). \quad (\text{VI.25})$$



(a) Distribution of Nu for $Pe_\omega = 17$ and $a^* = 3.2 \times 10^{-1}$, showing a Cauchy approximation with expected value $Nu_{h,0}$ and $\xi = 3.8$



(b) Distribution of Nu for $Pe_\omega = 1 \times 10^5$ and $a^* = 3.2 \times 10^{-1}$, showing a Cauchy approximation with expected value $Nu_{h,0} = 1.5 \times 10^3$ and $\xi = 1.8 \times 10^3$

Figure VI.4: Distribution of the instantaneous Nusselt number Nu and Cauchy approximations $P_C(Nu)$

This estimation is performed herein in order to provide a value for the expected values of the heat transfer coefficient h_0 and a corresponding value for the Nusselt number $Nu_{h,0}$.

VI.2.2 Case 1: Unity material property ratios

We now proceed to investigate the gas spring case with the three key material property ratios set to unity, that is, with $\kappa_r = 1$, $\rho_r = 1$ and $c_r = 1$. The main results are shown in Fig. VI.5.

The thermal loss ψ in Fig. VI.5(a) is defined as,

$$\psi = \frac{\oint W dt}{\oint |W| dt}, \quad (\text{VI.26})$$

in accordance with the earlier discussion concerning this parasitic net work input to the thermodynamic cycle. Also, recall that in the present model no account is made for losses due to turbulence or viscous dissipation in the fluid.

Considering the case when the wall thickness is small, i.e., $a^* = 0.01$ in Fig. VI.5(a), the thinness of the wall gives rise to a strong coupling of the internal surface of the solid wall to the external (isothermal) boundary condition T_0 , which in turn results in the internal wall-fluid boundary condition being effectively isothermal. This is the condition that has been solved by Lee [7], and almost all other similar studies.

At low Péclet numbers Pe_ω the frequency is low enough to allow heat transfer to take place between the fluid contained within the spring and the solid walls of the cylinder and piston, thus allowing the fluid to remain at (or near) thermal equilibrium with the solid. As such, the gas effectively experiences isothermal compression and expansion. In this condition all of the work done on the gas is transferred as heat to the walls but across a very small temperature difference, resulting in a low cycle loss. Similarly, at high Péclet numbers Pe_ω the process is fast enough not to allow sufficient time for heat exchange. In these conditions the system is, in effect, undergoing adiabatic compression and expansion with the majority of the work being exchanged with the internal energy of the gas. The thermal cycle loss is again low.

When the Péclet number Pe_ω is at an intermediate value, however, heat is exchanged out of phase with the (finite) temperature difference θ_f across the gas resulting in an exegetic cycle loss. From Fig. VI.5(a), the peak thermal loss with an isothermal external wall boundary condition, $a^* \ll 1$, is about 10 % and occurs at a Péclet number Pe_ω of around 10. This compares well to the result from Lee [7], though the loss here appears

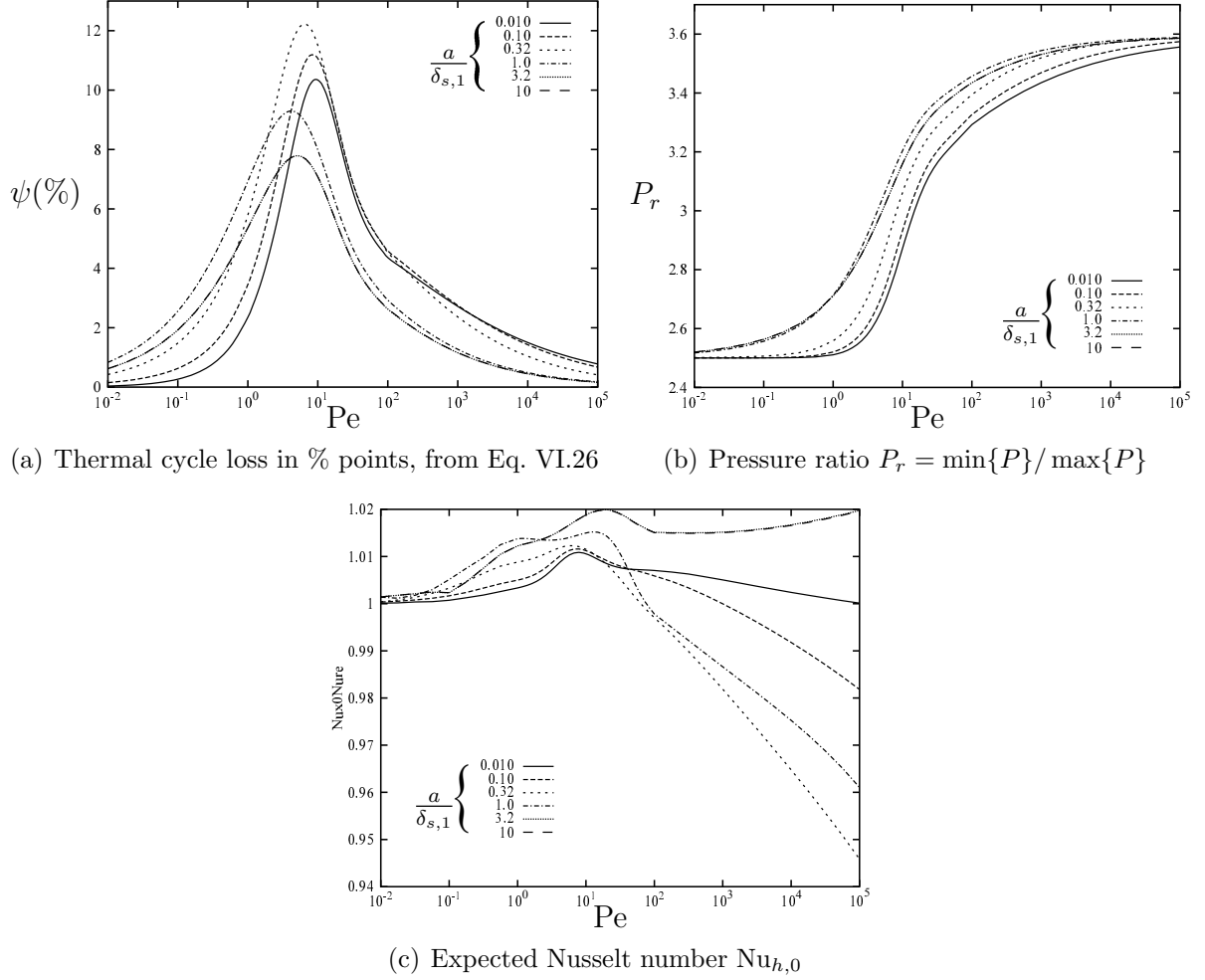


Figure VI.5: Response of a gas spring with the material ratios $\kappa_r = 1$, $\rho_{r,0} = 1$ and $c_r = 1$ as a function of Péclet number Pe_ω and for different insulation layer thickness a^* from 0.010 to 10

shifted to a slightly higher Pe_ω due to the fact that in the present work a volumetric variation is imposed on the gas (Eq. VI.1), whereas in Lee [7]] (and elsewhere) a pressure variation is imposed with a given pressure ratio. This is associated with higher pressure ratios at higher Pe_ω , as demonstrated in Fig. VI.5(b), where, as the system moves from isothermal (low Pe_ω) to adiabatic (high Pe_ω) operation the pressure ratio for the same volumetric compression ratio increases.

In the case of having a thick wall with $a^* = 10$ it is observed that the internal wall condition is only weakly coupled to (insulated from) the external boundary condition, and so the temperature at the interface between the wall T_w and fluid temperatures T_f are free to fluctuate. At high Péclet numbers Pe_ω the effect of the thicker cylinder wall is to make the system more adiabatic and to reduce the thermal losses. Nevertheless, at lower Pe_ω the effect of the finite fluctuations in wall temperature T_w act so as to move the cycle away from the ideal isothermal process, leading again to higher thermal losses. This is somewhat counter-intuitive: one would expect the addition of material to have an insulating effect and to reduce the thermal loss, through an increase in the thermal resistance due to conduction. In fact, the peak thermal loss has reduced, and the peak has migrated to a lower value of Pe_ω .

When the wall thickness is chosen to be of order unity, or that $a^* \sim \mathcal{O}(1)$, the magnitude of the thermal loss is greater than that observed at both low and high a^* . This arises from a thermal interaction in the solid (in the unsteady conduction) between the internal and external walls and leads to a modification of the phase of the heat flux out of the fluid $\dot{q}_f$. Thus, even though the magnitude of the heat flux $\dot{q}_f$ is reduced, the phase at which it occurs relative to the temperature across the fluid θ_f gives rise to a greater cycle loss.

VI.2.3 Case 2: Conventional material properties

The ratios of material properties in conventional systems do not typically equal unity, with the solid walls of the piston and cylinder likely to be considerably denser than the gas. The thermal conductivity of the solid is also likely to be higher, though the specific heat capacities of many gases tend to be higher than that of common solids. We consider a set of more typical values for our selected solid and gas properties by imagining that the walls of the cylinder are coated in a plastic such as PTFE or acrylic, and that the spring is filled (when extended to BDC) with hydrogen at 50 bar at 300 K. The material ratios become $\kappa_r = 1.06$, $\rho_{r,0} = 145.1$ and $c_r = 0.15$.

Results and Discussion

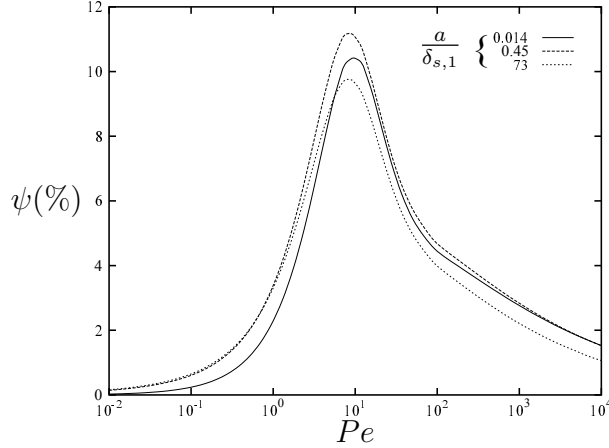


Figure VI.6: Thermal cycle loss in a gas spring (in % points) as a function of Péclet number Pe_ω , with the material property ratios set to $\kappa_r = 1.06$, $\rho_r = 145.1$ and $c_r = 0.15$, and over a range of wall thicknesses a^*

In this case the effect of the wall (see Fig. VI.6) is very much reduced compared to our earlier results in Fig. VI.5. Nevertheless, the exergetic loss exhibits similar characteristics to those observed previously, according to which $a^* \sim \mathcal{O}(1)$ produces the maximum loss and thicker walls makes the fluid boundary condition less isothermal, increasing the cycle losses at low Péclet numbers Pe_ω and reducing them at high ones.

Figure VI.7 shows more clearly the dependence of the irreversible thermal loss on the dimensionless wall thickness a^* . A wall thickness exists for which the maximum loss arises for a given Péclet number Pe_ω . The shape of these plots leads to a non-trivial conclusion with significant implications. Starting from a thin wall, it is interesting to observe that increasing the thickness of the wall, or insulation, is expected to lead to *higher* cycle losses, after which the losses decrease again. At low Pe_ω the loss reduction at the highest a^* does not return the overall loss to its minimum value observed at the lowest a^* , as opposed to the higher values of $Pe_\omega > 5$.

By comparing the wall heat flux $\dot{q}_f$ from Eq. VI.17 and VI.18 it can be seen that,

$$\dot{q}^* \sim \frac{\theta_{w,0}}{Z^*} \sim \text{Nu} \theta_{f,w}, \quad (\text{VI.27})$$

where $\theta_{j,k}^* = T_j^* - T_k^*$ is the temperature difference across the solid or fluid domain. Due to the way in which Z^* scales with a^* in VI.19, and specifically given that $\tanh(x) \sim x$ for $x \ll 1$, Eq. VI.27) leads to a scaling for the ratio of the temperature difference across

CHAPTER VI. COMPRESSION/EXPANSION-WORK IN CONVECTIVE-CONDUCTIVE SYSTEMS

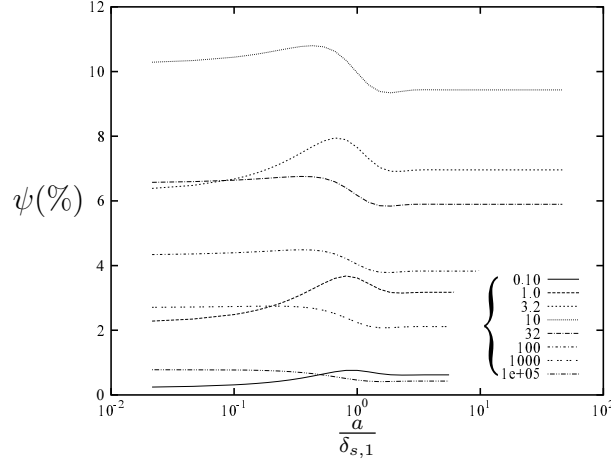


Figure VI.7: Thermal cycle loss in a gas spring (in % points) as a function of dimensionless wall thickness a^* , with the material property ratios set to $\kappa_r = 1.06$, $\rho_r = 145.1$ and $c_r = 0.15$, and over a range of Péclet numbers Pe_ω

the fluid to the temperature difference across the solid domain, $\theta_{w,0}/\theta_{f,w}$, of the form,

$$\frac{\theta_{w,0}}{\theta_{f,w}} \sim \frac{Nu}{\sqrt{2\epsilon_r Pe_\omega}} a^* = Bi, \quad (VI.28)$$

when a^* is less than one. Eq. VI.28 also applies to the steady state (mean temperature difference) part of the solution, since the steady state problem is associated with an a^* of zero. When a^* is greater than one the ratio of the fluctuations in the temperature differences across the domains will still scale with Bi , but with a modification due to $\tanh(x) \sim 1$ for $x \gg 1$,

$$\frac{\theta_{w,0}}{\theta_{f,w}} \sim \frac{Nu}{\sqrt{2\epsilon_r Pe_\omega}} = Bu = Bi/a^*, \quad (VI.29)$$

where Bu is an “unsteady” Biot number.

Expressions VI.28 and VI.29 show the relative importance of the solid to the fluid domain on the solution, and are essentially a function of the Péclet number Pe_ω and the material thermal effusivity ratio ϵ_r (since the Nusselt number Nu is also a function of the Péclet number Pe_ω). When the Biot or unsteady Biot Bi numbers are less than one the fluctuations of the wall temperature T_w^* are minimal and the problem is equivalent to having an isothermal wall temperature. Conversely when the Bi or Bu numbers are greater than one the wall temperature T_w^* will fluctuate significantly, thus affecting the solution. It can be seen therefore that the solid walls of the gas spring cylinder do affect the cycle undertaken by the gas when the effusivity ratio ϵ_r between the solid and the gas is low. In Fig. VI.6 the effusivity ratio was $\epsilon_r = 32.3$, whereas in Fig. VI.5 this was

Results and Discussion

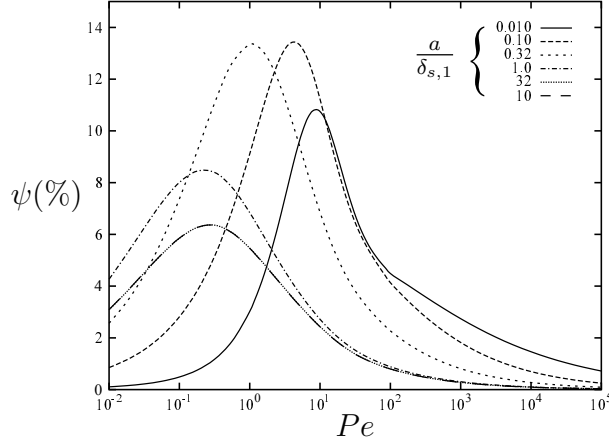


Figure VI.8: Thermal cycle loss in a gas spring (in % points) as a function of Péclet number Pe_ω , with the material property ratios set to $\kappa_r = 0.033$, $\rho_r = 10$ and $c_r = 0.07$, and over a range of wall thicknesses a^*

$\epsilon_r = 1.40$. As expected, at lower ratios of ϵ_r the walls of the cylinder have a greater effect on the cycle losses.

VI.2.4 Case 3: Materials with unfavourable properties

The gas spring can exhibit more extreme effects due to the solid wall when the effusivity ratio ϵ_r is lowered below unity. To obtain a low effusivity the heat capacity, thermal conductivity and density of the gas must be high relative to the solid. Hydrogen has the highest specific heat capacity c_v and thermal conductivity κ_f of any gas. Gases with higher atomic weights have higher densities ρ_f for the same pressure, however, this is outweighed by their lower heat capacities c_v and lower thermal conductivities κ_f , thus leading to lower thermal effusivities ϵ . In addition, if turbulence is introduced into the gas spring chamber, such as would be expected when operating with a gas intake and exhaust through suitable valves, the thermal conductivity can be higher still. Cantelmi et al. [98] showed that this the effect of turbulence in the gas can be modelled by a modified “turbulent” thermal conductivity for the gas which can be up to 30 times larger than the non-turbulent (molecular) case. Finally, if the cylinder walls are made of a low density material, such as a foam, the thermal conductivity κ_s , heat capacity c_s and density ρ_s of the solid side are also significantly reduced.

Using hydrogen gas at high pressure, specifically ~ 100 bar, an equivalent turbulent thermal conductivity multiplier guided by Cantelmi et al. [98] and with a foam layer insulating the inside of the cylinder and a cylinder and piston faces with internal surfaces

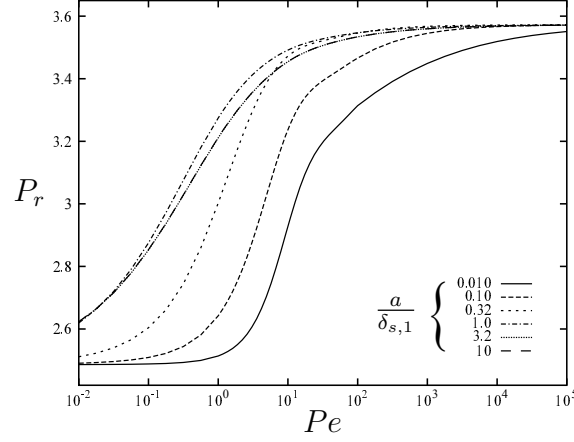


Figure VI.9: Pressure ratio P_r in a gas spring as a function of Péclet number Pe_ω , with the material property ratios set to $\kappa_r = 0.033$, $\rho_r = 10$ and $c_r = 0.07$, and over a range of wall thicknesses a^*

insulated by a foam (this can be fitted within a metal outer section for structural strength) it is feasible to obtain material ratios of $\kappa_r = 0.033$, $\rho_{r,0} = 10$ and $c_r = 0.07$, which correspond to a thermal effusivity ratio of $\epsilon_r = 0.032$. The results obtained from the gas spring described in the above paragraphs above are shown in Fig. VI.8 and VI.9.

Clearly the losses are more pronounced, reaching a maximum value of more than 14 % at intermediate dimensionless wall thickness a^* and Péclet numbers Pe_ω values.

An interesting feature can also be seen in Fig. VI.9, pressure ratio P_r is higher at a intermediate wall thicknesses $a^* \sim \mathcal{O}(1)$ than when the wall is much thicker than the diffusion length $\delta_{s,1}$. It is noted from that the loss gives rise to a requirement for a larger pressure ratio P_r to drive the same volumetric compression of the gas.

VI.3 NIFTE

The 1-D thermal impedance model was also applied to predict the behaviour of a vapour cycle engine in the form of the Non-Inertive-Feedback Thermofluidic Engine (NIFTE). The NIFTE, as presented by Smith [14, 15, 34], is an example of a two-phase thermofluidic oscillator heat engine. A schematic diagram of the NIFTE was shown in Fig. I.1 and a full description of the cycle was given in Section I.1.2.

A model to describe the behaviour of the NIFTE was outlined in Solanki et al. [42, 61] using a linear circuit analogy to predict the systems oscillation frequency, power output and cycle efficiency. We worked in conjunction with Solanki et al. [30] to construct a

model which incorporated the 1-D thermal response of the solid materials in the walls of the “adiabatic” vapour space and the heat exchanger components. This model was then compared to results from an experimental test rig.

VI.3.1 Modelling the cycle

In Solanki et al. [42, 61] a linear component analogy was proposed for modelling the NIFTE cycle dynamics. This is advantageous as linear systems theory can be applied to determine the behaviour of the system. By defining the open loop and closed loop gains of the system the poles and zeros can be used to determine the point when the marginal stability of the system.

In this analogy parts of the system which act to store energy, such as the column height in the power and displacement cylinders, or the compressibility of the vapour volume are equated to capacitive components. Similarly inductors describe the inertia experienced by the oscillating liquid flow and resistive components describe heat transfer and viscous effects (both associated with pure exergetic dissipation and loss). With this analogy volumetric flow rate ($\dot{V}_i$) are equated to electrical currents (I_i) and pressure differences (P_i) across a physical NIFTE device component in the fluid domain are equated to voltages (V_i). In Solanki et al. [42, 61] temperature differences and heat fluxes are also related to mass fluxes and pressure differences.

The heat transfer processes in the NIFTE are one of the most important aspects of the device, and are not well understood due to the complex nature of two-phase heat transfer. The action of the fluid interface wetting and then receding over the hot and cold heat exchangers in the displacement cylinder is inherently nonlinear. That is the temperature seen by the system is a step function against position. As a first order approximation a linear temperature profile (LTP) is assumed between the two heat exchanger blocks, as this allows a linear response of temperature and interface position. However, this description allows an indefinite temperature increase in the HHX and decrease in the CHX, and hence indefinite temperatures in the heat source and sink, which can lead to erroneous predictions.

In a recent study [42] two alternative heat exchanger models were considered to address the issues with the LTP model. The first model, known as the constant temperature difference (CTD) model, assumed a constant temperature difference between the heat exchangers and the working fluid. The second model, known as the dynamic heat exchanger (DHX) model, allowed the solid heat exchanger blocks to store and release heat dynamically (in the form of internal energy) as they interacted thermally with the working

fluid.

Here we will consider the dynamic (DHX) model and linear (LTP) models only. Figure VI.10 shows the equivalent linear circuit representations of these two models. The voltage source, P_{th} , represents the temperature of vapour production/condensation at the heat exchangers which is proportional to the the interface height in the displacement cylinder, P_d . The heights of the liquid levels in the power and displacement cylinders are represented by P_p and P_d respectively, and P_{ad} represents the pressure in the vapour volume. The location of the thermal loss as a result of the vapour condensation and evaporation, as mentioned in Section I.1.2, due to interaction with the solid walls of the “adiabatic” region is donated by the thermal impedance Z_{TL} . As before the thermal impedance for an adiabatic outer boundary condition is,

$$Z_{TL}(s) = \frac{1}{nk_s} \coth \left(nl_s \sqrt{\frac{s}{\alpha_s}} \right) . \quad (VI.30)$$

By using Thévenin’s equivalent theorem it is possible to simplify the circuits in Fig. VI.10 to one component with an open loop transfer function $G(s)$ which relates P_{th} (the pressure input representing the volumetric flow-rate in U_{th}) to output P_d . This was then used to calculate the closed loop transfer function, using gain factor k ,

$$C(s) = \frac{G(s)}{1 - kG(s)} . \quad (VI.31)$$

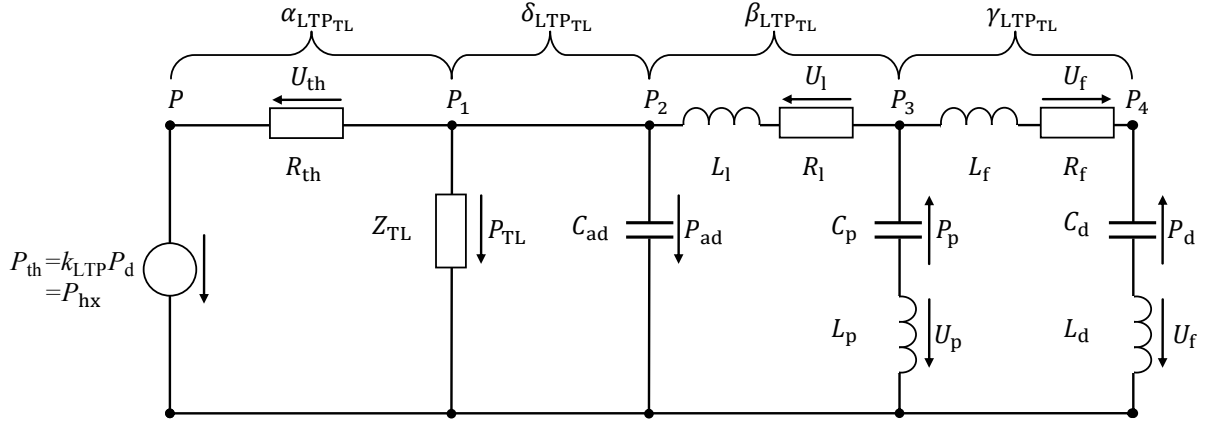
With the closed loop transfer function the point of marginal stability of the system could be found by solving,

$$\Re \{C(s)\} = 0 , \quad (VI.32)$$

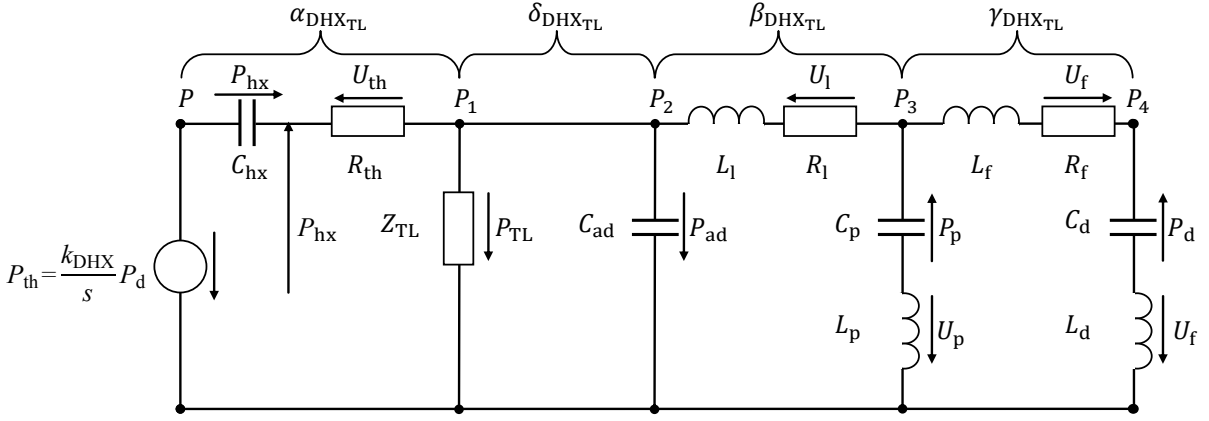
and finding the root with the lowest k . Once the oscillation frequency of the system was known calculating the efficiency and losses in the system can be calculated from the resulting pressures and flow rates around the equivalent circuit [30, 42, 61].

VI.3.2 Comparison to experimental results

Here we shall compare the predictions from the LTP and DHX models including and excluding the thermal loss impedance Z_{TL} (as presented by Markides and Smith [35] and Solanki et al. [42, 61, 61]) to experimental data for a NIFTE pump prototype from Ref. [15]. The effects of the feedback valve resistance R_f (and consequently of the time constant $R_f C_d$) on the oscillation frequency f_0 and the exergetic efficiency η_{ex} predicted by



(a) Linear heat exchanger temperature profile: LTP_{TL}



(b) Dynamic heat exchanger: DHX_{TL}

Figure VI.10: Electrical circuit representations of the NIFTE (see Fig. I.1). Here k_j corresponds to the feedback gain for model ‘j’, R_i to a resistance, C_i to a capacitance, L_i to an inductance, P_i to a pressure and U_i to a volumetric flow-rate. Subscript ‘hx’ refers to the heat exchangers, ‘th’ to the thermal domain, ‘TL’ to the unsteady thermal loss, ‘ad’ to the adiabatic vapour volume, ‘l’ to the load, ‘p’ to the power cylinder, ‘d’ to the displacer cylinder, and ‘f’ to the feedback line and valve. These networks were developed by using the methodology described in Refs. [15, 35, 42, 61].

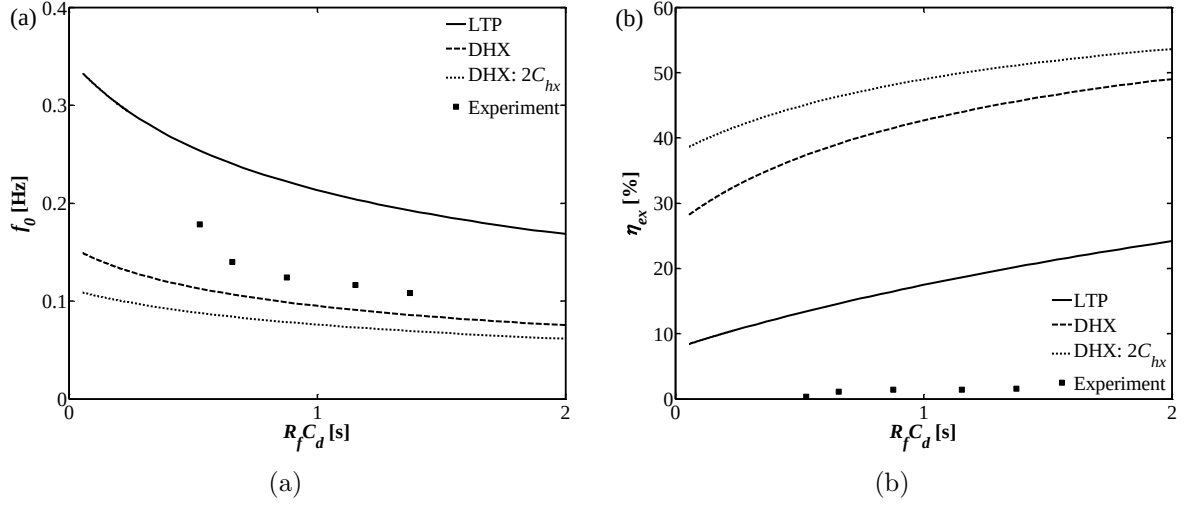


Figure VI.11: The effect of the timescale parameter $R_f C_d$ at constant displacer hydrostatic capacitance C_d on the predictions of the NIFTE: (a) frequency f_0 ; and (b) exergetic efficiency η_{ex} . Showing results from the LTP model (solid line), nominal DHX model (dashed line), and DHX model with $2\tilde{C}_{hx}$ (dotted line).

the LTP and DHX models are shown in Fig. VI.11(a) and (b), respectively. All parameters are set to their nominal values, with the exception of the feedback valve resistance R_f which is varied. The results are presented as a function of the varying time constant $R_f C_d$ in Fig. VI.11, though it is important to note that the results were generated by varying the feedback resistance parameter R_f with a *constant* displacer cylinder “capacitance” C_d .

Without the thermal loss impedance Z_{th}

Figure VI.11(a) shows that both the LTP and DHX models predict the correct qualitative trend for the frequency f_0 , whereby an increase in the feedback resistance R_f always leads to a decrease in frequency f_0 . The DHX model at the nominal value of heat exchanger capacitance $\tilde{C}_{hx}$ is associated with a slightly better prediction of the magnitude of the frequency f_0 compared to those of the DHX model with twice the nominal heat exchanger capacitance $2\tilde{C}_{hx}$ and those of the LTP model. Specifically, the nominal DHX model with $C_{hx} = \tilde{C}_{hx}$ under-predicts the experimentally obtained frequencies f_0 by $\sim 20 - 35$ % over the range of investigated $R_f C_d$, while the DHX model with $C_{hx} = 2\tilde{C}_{hx}$ under predicts the experimental f_0 by $35 - 45$ % and the LTP model over-predicts the same data by $45 - 80$ %.

The corresponding predictions of the exergetic efficiency η_{ex} by the LTP and DHX NIFTE models, over the same range of $R_f C_d$, show weak agreement with the experimental data, as shown in Fig. VI.11(b). Both the LTP and DHX models indicate that an increase in feedback resistance R_f leads to an increase in the NIFTE efficiency η_{ex} . However, although both models capture this trend, they both significantly overestimate the exergetic efficiency of the device η_{ex} . In particular, the LTP model predicts an efficiency 11 times the experimentally obtained value, whilst the DHX model predicts an efficiency 30 times that obtained experimentally.

With the thermal loss impedance Z_{th}

The effects of varying the feedback resistance R_f on the model frequency and efficiency predictions when the thermal loss impedances Z_{TL} are inserted into the NIFTE models are shown in Fig. VI.12 for the adiabatic outer boundary condition. Figure VI.12(a) shows the oscillation frequency f_0 predicted by the model as the feedback resistance R_f is varied. On comparison to the predictions from the same model but without the thermal loss, shown in Fig. VI.11(a), the inclusion of the thermal loss impedance Z_{TL} has little effect on oscillation frequency predicted by the LTP model. On the other hand, though the inclusion of this impedance in the DHX model does result in a slightly better prediction of the oscillation frequency f_0 , with the experimental data points now lying between the model results from the nominal $\tilde{C}_{\text{hx}}$ (dashed line) and $2\tilde{C}_{\text{hx}}$ (dotted line).

The corresponding exergetic efficiency η_{ex} results from the LTP_{TL} and DHX_{TL} models are shown in Fig. VI.12(b). The thermal loss impedance Z_{TL} has a significant effect on the evaluated efficiencies, compared to those predicted by the same models while neglecting thermal losses, shown in Fig. VI.11(b). From Fig. VI.12 it can be seen that both the LTP_{TL} and DHX_{TL} models predict efficiencies with the correct trend, whereby a higher feedback resistance leads to a higher efficiency as demonstrated experimentally. In addition to this, both models predict efficiency values in the same order of magnitude as the experimental results. The LTP_{TL} model underestimates the efficiencies, while the DHX_{TL} model predicts the experimental data better, not only in terms of efficiency but also while simultaneously matching the measured frequency operation without adjustments to any of the parameters. In conclusion the addition of the thermal loss perimeter Z_{th} into the LTP and DHX models results in significant improvement in the models predations.

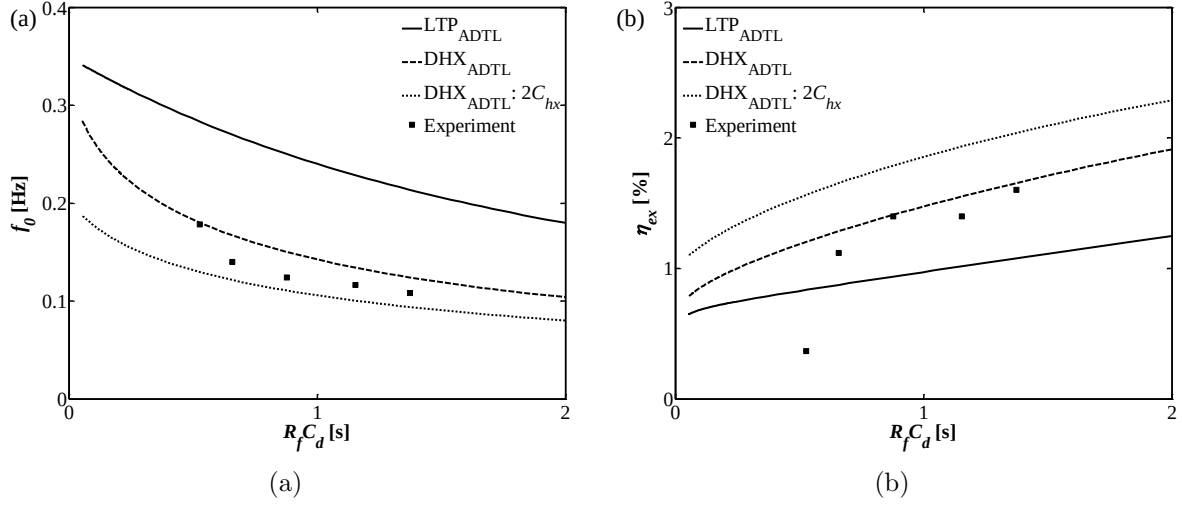


Figure VI.12: The effect of the timescale parameter $R_f C_d$ at constant displacer hydrostatic capacitance C_d on the predictions of the NIFTE: (a) frequency f_0 ; and (b) exergetic efficiency η_{ex} . Showing results for the revised models featuring the unsteady thermal loss impedance Z_{TL} with an *adiabatic* boundary condition, namely LTP_{ADTL} (solid line), DHX_{ADTL} (dashed line), and DHX_{ADTL} with $2\tilde{C}_{hx}$ (dotted line).

VI.4 Summary

A conjugate model that solves the one-dimensional unsteady heat conduction equation in the solid simultaneously with the first law in the gas phase has been applied to the study of gas springs. An imposed heat transfer coefficient has been used, which was taken from relevant experimental studies in the literature. Beyond the explicit inclusion of conjugate heat transfer, the model goes beyond previous investigations by considering the case of imposed volumetric compression and by allowing the required gas pressure to vary accordingly. Notable effects of the solid walls of the gas spring are uncovered, with cycle losses of up to 15 % for cases in which unfavourable materials (solid/gas) are selected, and losses closer to 10-12 % for more common material choices. The contribution of the solid itself through the cylinder wall thickness was about 10 % and 2-4 %, respectively. These results suggest that, in designing high-efficiency machines, the full conjugate and unsteady problem must be considered.

Chapter VII

Results 3:

Broadband Convective–Conductive Systems: Backwards Facing Step

VII.1 Nakamura experiment

Nakamura [29, 62] has recently conducted detailed spatio-temporally resolved measurements of heat transfer behind a backwards facing step. In this configuration a fluid, in this case air in a wind tunnel, flows over a long plate with a trip-wire at the leading edge to form a developing turbulent boundary layer. The flow then passes over a backward (downward) step on the plate's surface, positioned perpendicular to the flow direction. The developing boundary layer separates as air passes over the step and results in a turbulent recirculating flow region behind the step, as illustrated in Fig. VII.1. The resulting

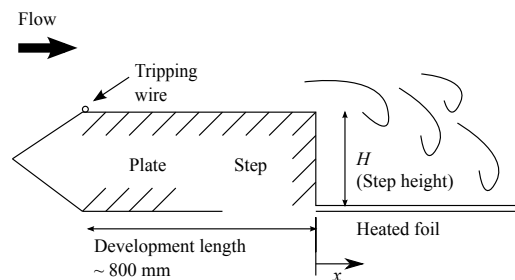


Figure VII.1: Schematic diagram of the backwards facing step.

CHAPTER VII. BROADBAND CONVECTIVE-CONDUCTIVE SYSTEMS: BACKWARDS FACING STEP

flow has been studied extensively [170, 171] and is a model problem for many simulations [39, 172] and experiments [173]. It is conventional in these problems to characterise the size of the turbulent recirculation region behind the step by the step height H , and to characterise the flows by a Reynolds number based on the free-stream velocity U_∞ and by the step height H , $Re_H = U_\infty H / \nu$, where ν is the kinematic viscosity of the fluid.

The heat transfer taking place behind such a step may be enhanced due to the increased mixing of the flow, which has obvious applications to heat transfer. Furthermore, the unsteadiness of the heat transfer process can also result in temperature fluctuations on the surface of the wall. The amplitude of these fluctuations could be important, for example, when avoiding thermal damage in a combustion chamber. In addition, the complex velocity field will be unsteady and will result in a time-varying h . This in turn could lead to augmentation, and is the subject of the present study.

In the experiments performed by Nakamura [29, 62, 139] the area behind the step was electrically heated by a thin (2 μm thick) titanium foil. A 4 mm thick copper plate was placed under the heated surface with a 1 mm air gap to impose an isothermal boundary condition. The foil temperature was measured with an infra-red camera at high frequency and the instantaneous heat flux through the foil was calculated by evaluating an inverse heat transfer problem. It is then possible to calculate the instantaneous HTC at any position on the foil surface, as in Fig. VII.2(a). Using data generated by these experiments we will demonstrate the ability of our technique, from Chapter V, to model the behaviour of this unsteady heat transfer problem, and consequently to predict the effect of changing the solid material or thickness on the heat transfer process.

Table VII.1 lists the conditions of the experimental data provided to us by Nakamura [29]. The measurements were conducted in two separate experimental campaigns separated by a considerable period of time, with the second campaign also considering the case of a forward facing step. In order to check the experimental repeatability, the experimental conditions for the backwards facing step were repeated in a second experimental campaign.

Direct numerical simulations (DNS) have been performed on these flows [39, 172] and have provided great insight into the turbulent flow and its effect on the heat transfer behaviour. Nevertheless, these computations require great resources and computation time to complete, with each single simulation corresponding to only one operation point. Following this a considerable effort needs to be made on spanning a range of variables and problem parameters, that can then allow a designer to extract and understand any under-

Modelled broadband augmentation factors

Table VII.1: Overview of experimental conditions.

Name	U_∞ (m/s)	H (mm)	Backward / Forward Facing Step
BS $U = 2$	2.0	10	Backward
BS $U = 3$	3.0	10	Backward
BS $U = 4$	4.0	10	Backward
BS $U = 6$	6.0	10	Backward
BS $U = 2$ (rep)	2.0	10	Backward (repeat)
BS $U = 3$ (rep)	3.0	10	Backward (repeat)
BS $U = 4$ (rep)	4.0	10	Backward (repeat)
BS $U = 6$ (rep)	6.0	10	Backward (repeat)
FS $U = 2$	2.0	7.5	Forward
FS $U = 3$	3.0	7.5	Forward
FS $U = 4$	4.0	7.5	Forward
FS $U = 6$	6.0	7.5	Forward

lying heat transfer trends. Using a simple lumped model of the heat transfer behaviour, such as that proposed herein, can lead to a faster extraction of the trends, though of course there is the inevitable loss of some information relative to the DNS.

VII.2 Modelled broadband augmentation factors

The modelling approach will aim to predict the augmentation effect from statistical information about h using Case V from Chapter V. Our model requires the heat capacity ratio $C_R = (\rho_f c_f / \rho_s c_s)(d/a)$ between the titanium foil and the air flow to be calculated, as well as the characteristic length scale of the fluctuations in the solid, $a^* = a/\delta_{s,n}$. The fluid domain extent is chosen to be the thickness of the thermal boundary layer. The height of the turbulent recirculation region is known to be 3 mm greater than the step height ($H = 10$ mm), from Ref. [62]. Scaling this by the Prandtl number for air, for which a value of $Pr = 0.71$ is used, gives an approximation for the thickness of the thermal boundary layer behind the step of 9.3 mm. The data in Table VII.2 are used to evaluate C_R that for this configuration is found to be equal to about 2.3, and the Biot number $Bi = \bar{h}a/k_s$, which is found to be between 1.9×10^{-6} and 4.2×10^{-6} depending on $\bar{h}$ that is a function of position, but also of the flow Re_H . The Biot number is small because of the small thickness of the heated foil a .

Now, returning to the frequency content of h in Fig. VII.2(b), it can be seen that the

CHAPTER VII. BROADBAND CONVECTIVE-CONDUCTIVE SYSTEMS: BACKWARDS FACING STEP

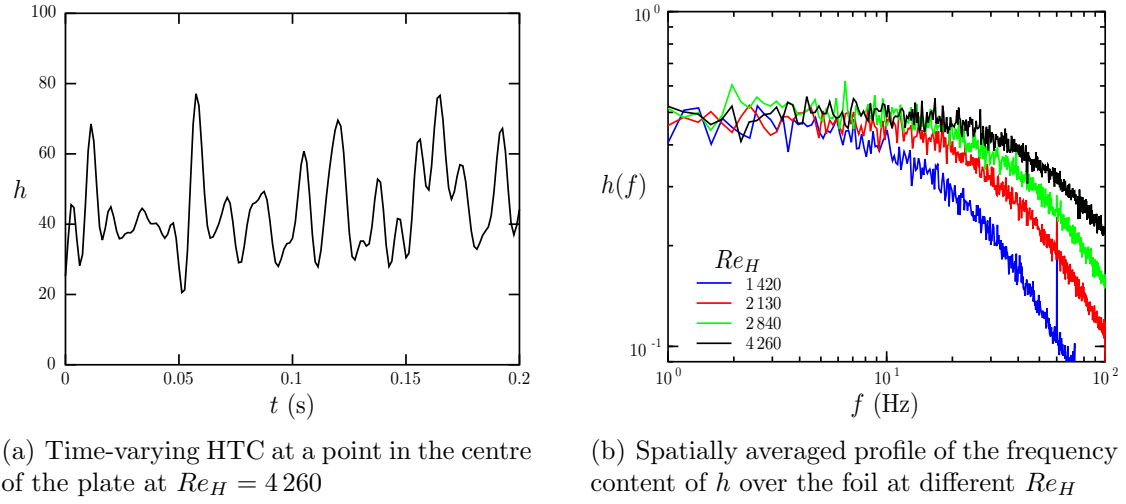


Figure VII.2: Experimentally measured variations in the HTC.

Table VII.2: Thermal properties of materials in the experiments by Nakamura [29, 62]. The thermal properties of air are at the film temperature, i.e. the mean temperature between the wall and the freestream.

	Heat capacity, c (J kg ⁻¹ K ⁻¹)	Thermal conductivity, k (W m ⁻¹ K ⁻¹)	Density, ρ (kg m ⁻³)
Titanium	540	21.9	4510
Air	1005	0.026	1.18

signal is largely broadband in nature. There is no dominant peak in the spectrum, only a slow first-order roll-off above a certain frequency that changes with the flow condition, i.e. Re_H . The 3-dB bandwidth frequency f_c related to this first order roll-off is quantified in Fig. VII.4(a). Our model was initially developed for sinusoidal time variations at a single frequency, but it is relatively straightforward to modify the algorithm so as to accommodate arbitrary profiles of h through a Fourier decomposition.

The broadband nature of the signals also lead to a range of thermal diffusion length scales over which unsteady heat transfer occurs since $\delta_{s,n} = \sqrt{2\alpha_s/\omega_n}$. By substituting the physical values from Table VII.2, an expression for the dimensionless foil thickness can be found as a function of frequency, $a^*(\omega) = a/\delta_s(\omega) = 4.7 \times 10^{-4}\sqrt{\omega}$. By taking at the largest 3-dB roll off frequency f_c of the broadband thermal response from the experimental data ~ 40 HZ, see Fig. VII.2(b), the size of the largest dimensionless solid thickness a^* observed in all of the experiments was about 7.4×10^{-3} . For comparison the rate at which the experimental data was sampled results in a Nyquist frequency of 200 HZ which corresponds to a dimensionless solid thickness a^* of 1.7×10^{-2} . This highlights that the material thickness was significantly shorter than the observed thermal diffusion length in the solid. The small value of a^* would suggest that the solid surface in contact with the thermal boundary layer will be strongly coupled to the constant heat flux boundary condition imposed on the outer (or back) side of the foil that is facing away from the flow, and that it is unlikely to see a high value of *conjugation* in this experiment, although this is not likely to be the case for thicker foils.

In the case of a thicker foil the temperature fluctuation on the foil surface should decrease, approaching a steady wall temperature condition, however, that the same time it will result in an increase in the heat flux fluctuation as the solid becomes more decoupled from the boundary condition on the outer side of the foil. Recall from Chapter III.2.2 that our definition of a “conjugate” problem involves both an unspecified time-varying temperature and heat flux at the common fluid-solid interface. That is $Fo \sim O(1)$ and $Bi \sim O(1)$.

VII.3 Characterising the HTC

The next step requires the generation of a synthesised h , which has a similar frequency content and a similar probability distribution to that measured in the experiments. Although the measured HTC signals can be used directly, an important aim here is not to

CHAPTER VII. BROADBAND CONVECTIVE-CONDUCTIVE SYSTEMS: BACKWARDS FACING STEP

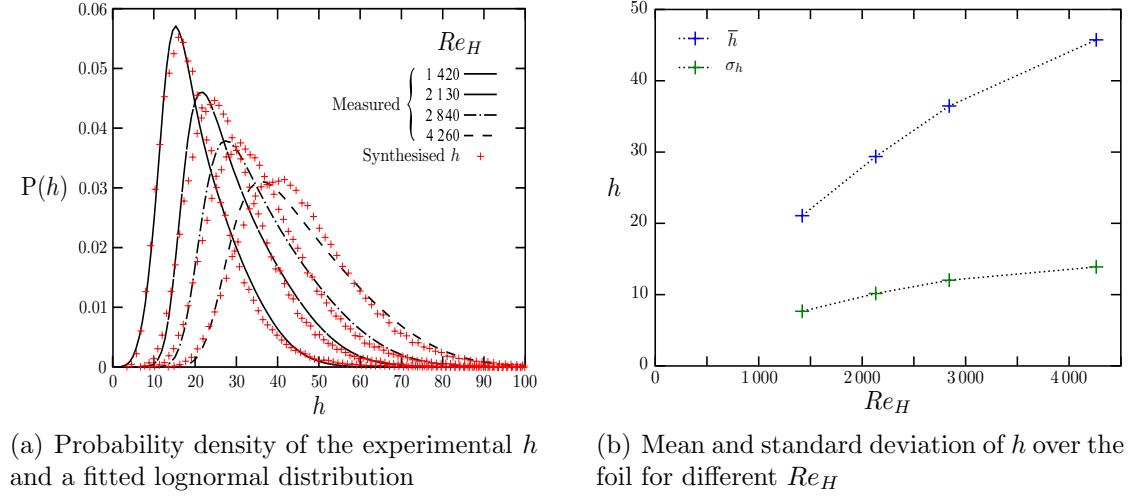


Figure VII.3: Statistical distribution of the measured HTC.

devise a modelling strategy that depends on the provision of this highly detailed information. Probability densities of the measured h are shown in Fig. VII.3(a) for 4 flows with different Re_H , together with log-normal distributions based on the corresponding means and variances provided by the experimental data. The log-normal approximations resemble closely the directly measured probability densities of h . At this point it is noted that the frequency content of a normal distribution is flat, so this must be low-pass filtered in order to exhibit a roll-off similar to that in the measured HTC's in Fig. VII.2(b). An example of the spectral density of a synthesised h after this filtering procedure has been performed is shown in Fig. VII.4(b). The cut-off frequencies were taken from the measured h spectra (i.e. from Fig. VII.2(b)) and are plotted in Fig. VII.4(a) for the 4 different flow conditions.

The method employed for synthesising the HTC is computationally simple. Firstly, a sufficiently large number of randomly distributed samples are generated, using a log-normal distribution with the same statistical parameters seen in the experimental data (Fig. VII.3(b)). These samples are then low-pass filtered with the corresponding roll-off frequency seen in Fig. VII.4(a). Taking this synthesised h it is possible to calculate the resulting temperature and heat flux response of the conjugate fluid-solid system for a given mean heat flux, as before in Case V. The statistics of the heat transfer behaviour can then be generated from this modelled response, such as the variation in temperature or the augmentation ratio A . Finally, since the HTC was generated randomly, a number of repeated trials are run in order to smooth out any random bias that may have been

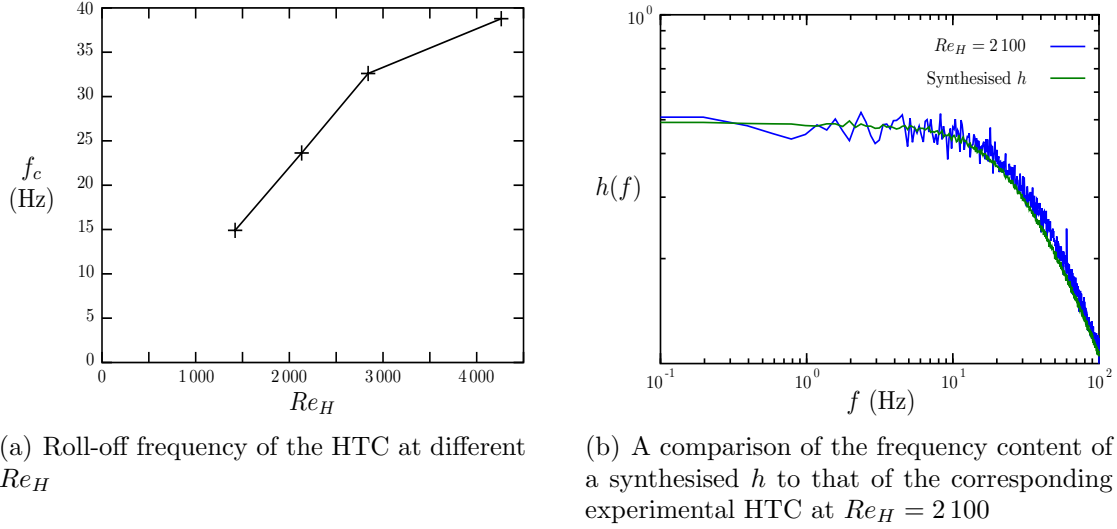


Figure VII.4: Matching the frequency content of the measured and synthesised HTC. The frequency content of h at $Re_H = 2100$ is spatially averaged over the foil.

generated by the procedure for the synthesis of h .

VII.4 Augmentation Predictions and comparison to experiment

Comparing the spatially averaged measurement of augmentation loss, $A_{\text{loss}} = 1 - A$, directly from the experimental data (Fig. VII.5(a)), with the stochastic prediction from our 1-D lumped conjugate model employing some basic statistical information on the expected HTC as described in Section VII.2 (Fig. VII.5(b)), it can be seen that the model has a reasonably close fit to the observed data. The trend as well as the magnitude of the augmentation are captured well. And this, even though the model uses a relatively simple description for convective heat transfer based on Newton's Law, and as such contains only superficial information on the underlying process in the fluid. It can be concluded that if the distribution of the frequency content of h is known to some approximation, for which the mean and variance of h for a certain flow condition are required, the effect of the solid wall on the conjugate heat transfer can be modelled quite accurately.

There are many fluid applications where measurements of the turbulence or unsteady flow behaviour have been made. Using the technique presented here it may be possible to predict the effect of unsteadiness on the conjugate heat transfer performance, and to cal-

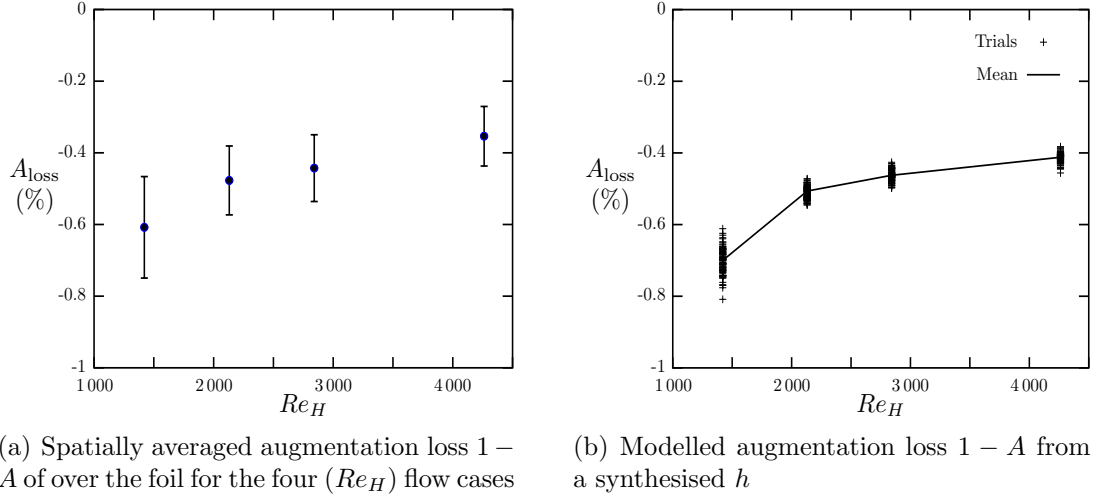


Figure VII.5: Augmentation loss $A_{\text{loss}} = 1 - A = -\overline{h'\Delta T_f'} / \overline{h} \overline{\Delta T_f}$ comparison: (a) from experimental data, and (b) modelled based on a synthesised h . The error bars represent, in: (a) the standard deviation of the variation in the augmentation loss over the foil, and in (b) results from different trial runs from which the average result is obtained.

culate quantities unique to these problems such as the augmentation ratio. Beyond this, the effects of simple geometrical modifications (i.e. thickness) and of the material properties of the solid and fluid can be better understood. In this particular demonstration the magnitude of augmentation is relatively small. Yet, the model reveals that in applications with a smaller dimensionless foil thickness a^* (e.g. in unsteady flows with slower frequencies), a larger Biot number Bi (e.g. a larger mean h), a smaller heat capacity ratio C_R , and a larger fluctuation in h , this effect could become much more significant.

VII.5 Location of the flow stagnation point

Qualitative observations of the measured surface heat transfer coefficient (HTC) h (e.g., Fig. VII.6); heat flux $\dot{q}$; and surface to free-stream temperature difference $\Delta T = T_\infty - T_w$; show large scale structures propagating across the surface. These structures can be tracked and used to calculate a surface thermal velocity field. Such a velocity field should be linked to the velocity of the shear layer in contact with the foil, and any temperature fluctuations in the fluid. It is not proposed here how these velocity fields are linked, but it is supposed that positions of zero mean velocity in a quantity should correspond to zero shear, or the reattachment point of the flow field.

There are many available algorithms for tracking objects in video streams. Examples

Location of the flow stagnation point

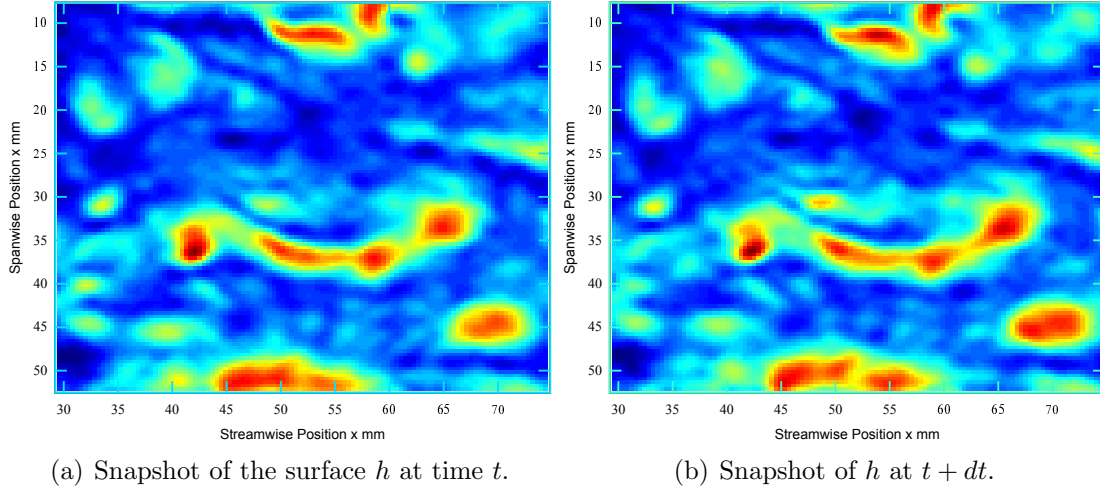


Figure VII.6: Two consecutive snapshots of the surface HTC h . The smooth large scale structures appear to move across the surface.

are the optical flow algorithms implemented in the OpenCV library such as the methods by Lucas and Kanade [174] or Horn and Schunck [175]. Initially an attempt was made to utilise these library in order to calculate the surface thermal velocity field. However as these algorithms rely on tracking features in the image with spatial frequency content, such as corners or texture, it was found that they were poorly suited to the present data-sets. In the measured experiments the scalar fields over the heated surface appear smooth, lack strong gradients and contrast, and have a poor repetition of texture from one frame to the next.

Instead, another method was proposed which did not rely on features in the images. Suppose the scalar quantity ϕ is undergoing some advection process $\underline{u}$ in the 2D image plane,

$$\frac{D\phi}{Dt} = 0. \quad (\text{VII.1})$$

This expands to give,

$$\frac{\partial \phi}{\partial t} + \underline{u} \cdot \nabla \phi = 0. \quad (\text{VII.2})$$

Here $\underline{u}$ is the velocity field seen by the scalar field. To invert this we can write,

$$|\underline{u}| \cos(\theta) = -\frac{1}{|\nabla \phi|} \frac{\partial \phi}{\partial t}, \quad (\text{VII.3})$$

where θ is the angle between the velocity vector $\underline{u}$ and the scalar field gradient vector $\nabla \phi$.

CHAPTER VII. BROADBAND CONVECTIVE-CONDUCTIVE SYSTEMS: BACKWARDS FACING STEP

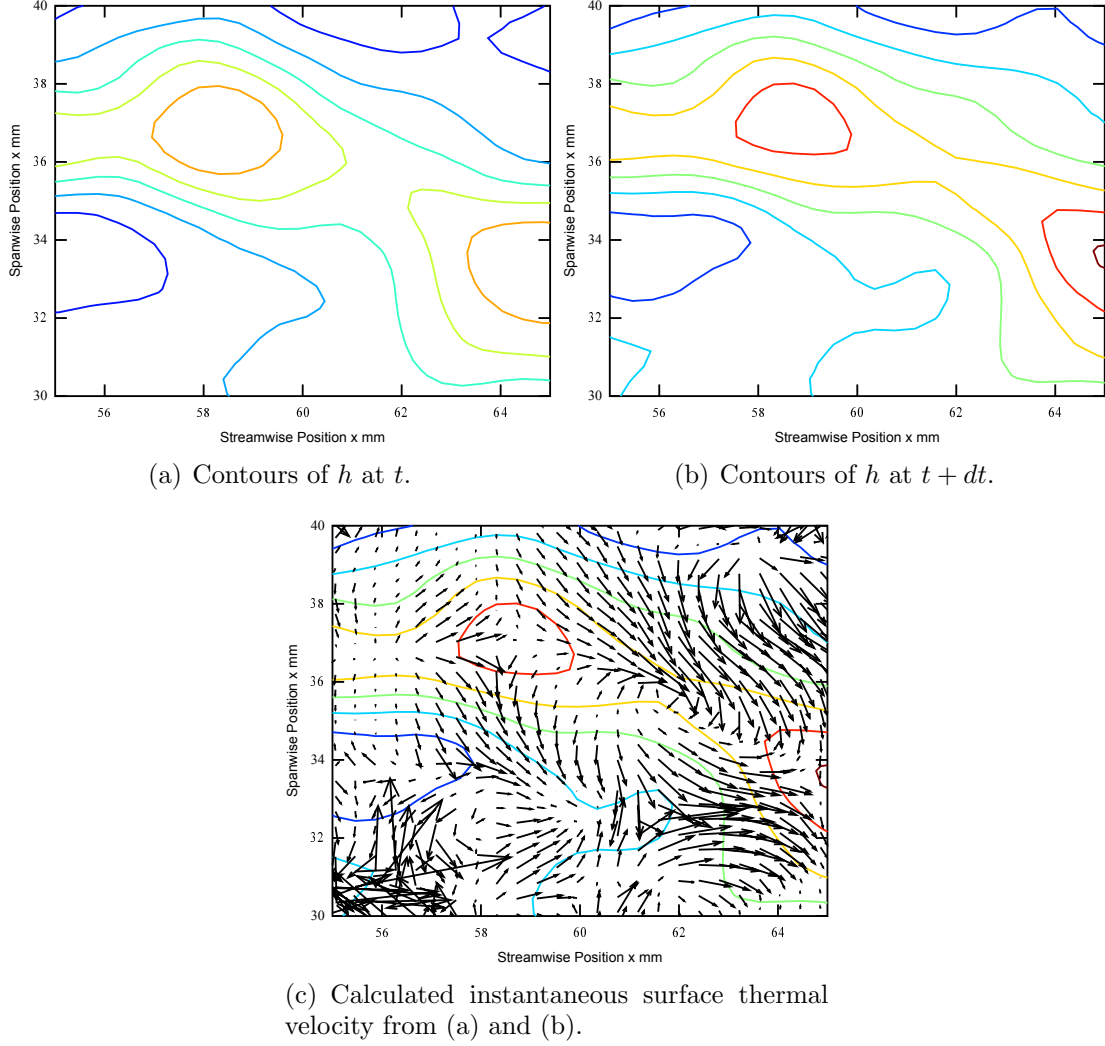


Figure VII.7: By comparing the difference between frames, in this case of h , it is possible to calculate a surface thermal velocity field.

Unfortunately information on θ is lost by the inner (dot) vector multiplication. Essentially this means that information on the component of velocity tangential to the scalar's gradient is irrecoverable. Neglecting the tangential component of velocity we can now write,

$$\underline{u} = -\frac{1}{|\nabla\phi|} \frac{\partial\phi}{\partial t} \underline{n}, \quad (\text{VII.4})$$

where $\underline{n}$ is the unit vector adjacent to the scalar gradient.

An example of an instantaneous velocity field calculated via this operation from two consecutive instantaneous h fields is shown in Fig. VII.7. By averaging over time to find

Location of the flow stagnation point

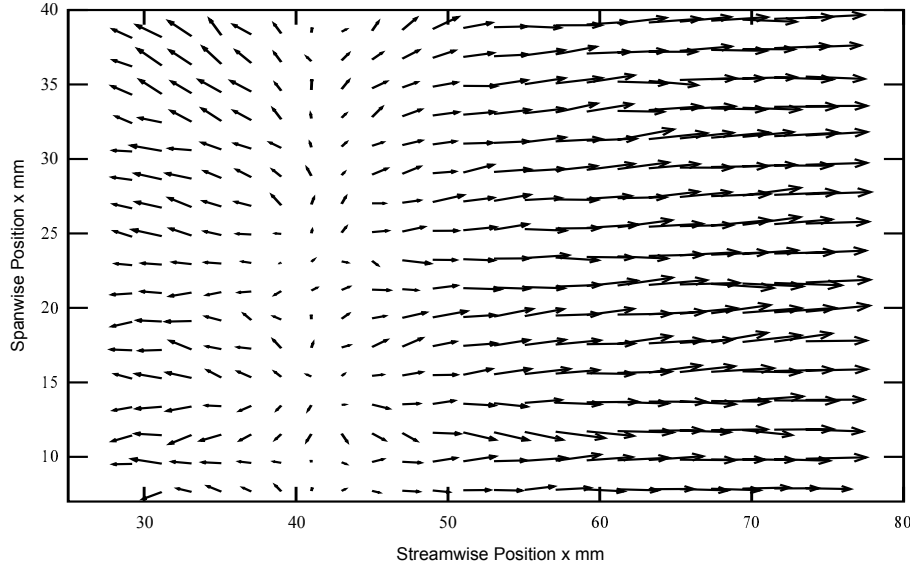


Figure VII.8: Average of the velocity field over all time steps for h at $U_\infty = 3$ m/s for the backwards facing step.

the mean 2D velocity field (e.g., Fig. VII.8) the fluctuations in the span-wise directions are removed. The mean position of flow reversal can be seen clearly and most of the vectors indicate the flow direction.

It is possible that a representation of the tangential velocity can be recovered, potentially by implementing a multi-grid method to find the velocities of patches at different length scales. However, for the purposes of this study the above analysis should be sufficient, as the velocity at each instant can be averaged to find the mean flow field.

VII.5.1 Predicted reattachment points

The 2D velocity fields were calculated from the surface temperature, heat flux and HTC by using the above method. It was found that each time-averaged 2D velocity field was approximately uniform in the span-wise direction. This allowed a meaningful span-wise averaging of each time-averaged velocity field that resulted in a mean stream-wise velocity profile, which indicated a location where the velocity reversed. For comparison purposes, the velocity fields generated from the temperature difference ΔT , heat flux $\dot{q}$ and HTC h are all considered. Furthermore, velocity fields were also calculated for the fluctuating components of surface temperature $\Delta T'$, heat flux $\dot{q}'$ and HTC h' .

In the time-averaged 2D velocity fields a position of zero stream-wise velocity is observed, with the flow reversing behind this point. It was deduced that this corresponds to

CHAPTER VII. BROADBAND CONVECTIVE-CONDUCTIVE SYSTEMS: BACKWARDS FACING STEP

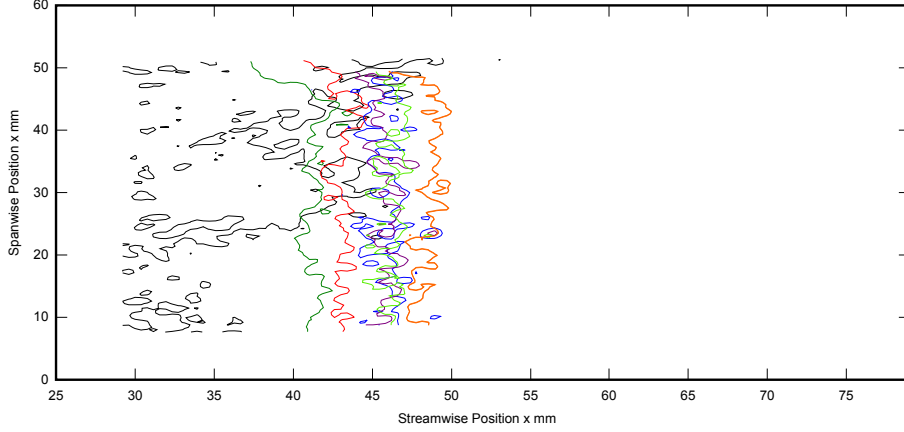


Figure VII.9: Reattachment contours for the mean stream-wise velocity profile of h for the backward facing step. The full experimental viewable range is 40×80 mm, though part of this range is omitted in this figure. Each different colour line corresponds to a different run, with the black line corresponding to the backward facing step with $U_\infty = 2$ m/s repeat run.

a mean flow reattachment position with respect to the quantity being investigated. Two methods were employed for recovering this mean reattachment position:

Method-1 The contour of zero velocity was found in the time-averaged 2D velocity field and the x positions of this contour were averaged to find the mean stream-wise reattachment position (see Fig. VII.9). In this case, we also calculated the rms (root mean square) of variation (or standard deviation) in the reattachment position.

Method-2 The time-averaged 2D velocity field was averaged in the span-wise direction (resulting in profiles that can be seen in Section VII.5.3) from which the stream-wise x position the intersection of the mean profile with $v = 0$ was found.

Method-1 provides information on the variation of the reattachment position and this was used to present error bars of 2 standard deviations (i.e., at 95 % confidence interval) in Fig. VII.10.

Figure VII.10(a) summarises the results for the mean reattachment positions based on h . It can be seen that the forward and backward facing steps behave quite differently. For the backward facing step the location of reattachment is more or less constant within the region of uncertainty, at $4 - 5H$ downstream of the step position. This is in reasonable agreement with the literature (e.g., Le et al. [176], Tihon et al. [177]). In Ref. [177] the

Location of the flow stagnation point

authors show much of the recent literature concerning experimental data on the location of the reattachment region ([177]; Fig. 6), as well as its relation to the position of peak wall shear stress ([177]; Fig. 5). They show that the length of the reattachment region behind a backward facing step x_r grows with Reynolds number Re from zero to a length of $17 - 18H$ at $Re = 500 - 600$, and that the flow is predominantly laminar. Here, Re is based on the free-stream velocity U_∞ and step height H . The length of the reattachment region then shortens to a plateau of about $7H$ from $Re = 1000 - 1500$, where the flow was found to be turbulent. Interestingly for the Reynolds number range of the present data, the position of peak wall shear stress fluctuations is $0.75x_r$, which thus corresponds to about $5H$.

This is possibly due to the self-similarity of the shear layer from the separation point. To draw an analogue, the divergent angle of a turbulent jet is independent of Re , even though the range of turbulent lengthscales increases with increasing Re . If the separated flow from the step is interpreted as a jet length, the recirculation region would stay constant as U_∞ changed. Of course, at low U_∞ this argument would eventually break down, and the attachment position would migrate upstream as the flow becomes laminar.

On the other hand, the forward facing step shows a significant downstream migration of the reattachment with increasing Re . This might be explained in terms of the higher shear rate at the higher Re , which would act to deform the recirculation region and so pull the reattachment downstream.

The data point at $U_\infty = 2$ m/s for the repeated backwards facing step experiment shows significant uncertainty (see also the variation in the corresponding time-averaged reattachment contour in Fig. VII.9) when compared to the other datasets. It is not known if this is due to some external disturbance to the flow, or a transition in flow regime, as there seems to be a significant span-wise distortion on the flow. For the forward facing step at $U_\infty = 2$ m/s the mean reattachment position was outside of the visible region. An approximate reattachment position was estimated by extrapolating the mean velocity profile to $v = 0$. This is marked by a square box and a discontinuous line.

Mean reattachment positions were also estimated from $\dot{q}$ (Fig. VII.10(b)), $\dot{q}'$, $\Delta T'$ (Fig. VII.10(c)) and h' . These showed broadly the same trends as those revealed by h , though the uncertainty was lowest for $\dot{q}$. The mean velocity profiles calculated from ΔT resulted in unsatisfactory results due to the large spatial gradient in the mean profile of ΔT . Consequently, the analysis involving $\Delta T'$ did render satisfactory results, although with a significantly larger uncertainty.

CHAPTER VII. BROADBAND CONVECTIVE-CONDUCTIVE SYSTEMS: BACKWARDS FACING STEP

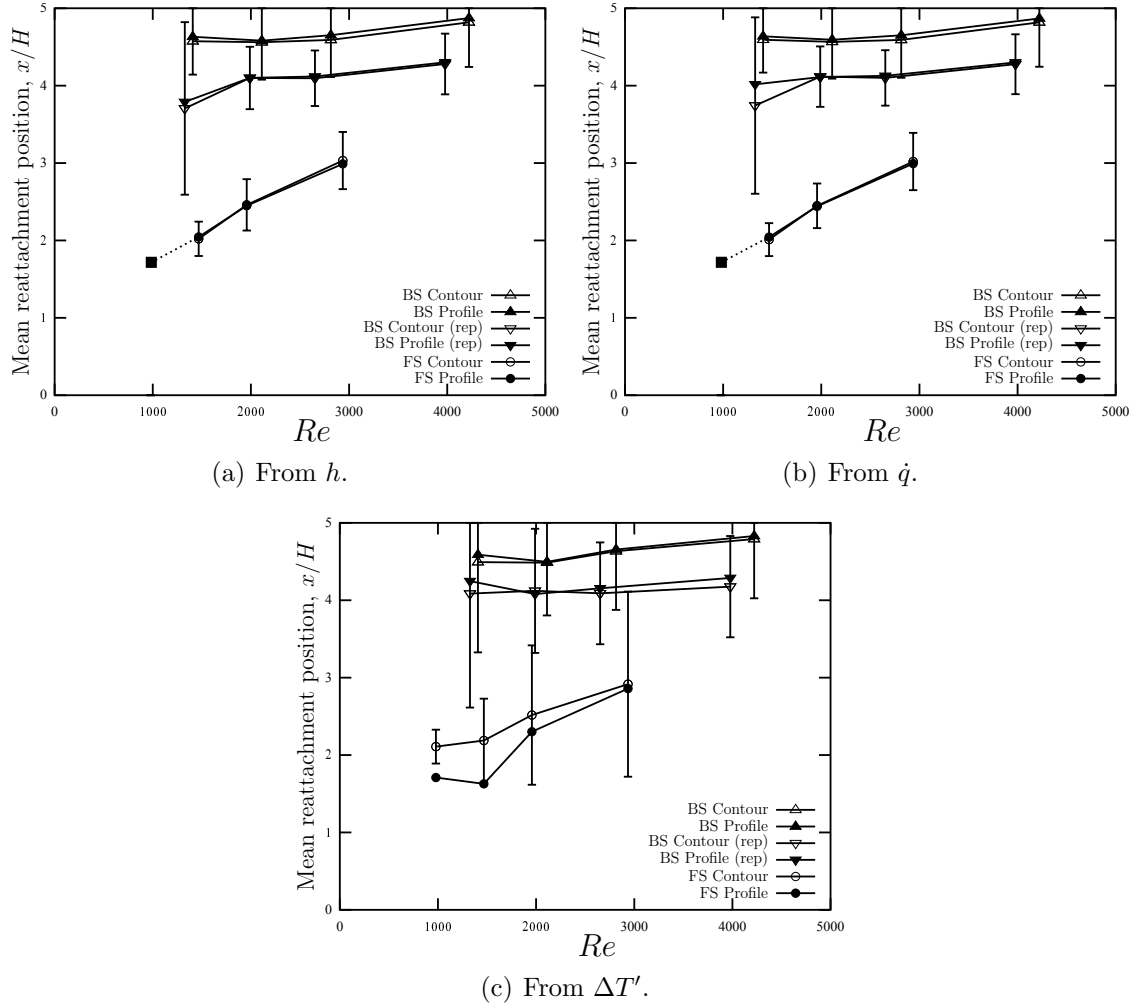


Figure VII.10: Reattachment positions estimated from velocity fields of the various scalars. One line corresponds to the intersection of the span-wise mean of the velocity field with zero (Method-1), and the other corresponds to the mean position of the contours of zero velocity (Method-2).

VII.5.2 Mean stream-wise thermal velocity profiles

The profiles of $\dot{q}$, ΔT and h can be compared to identify discrepancies between the two backward facing step experiments, and the one forward facing step data-set.

Backward facing step

The time-averaged stream-wise profiles of $\dot{q}$ for the backward facing steps are shown in Fig. VII.11(a). The red lines show the span-wise mean and the blue lines show the profile at the semi-span position. This was done to investigate any discrepancies between these profiles, which would indicate a large degree of variability in the span-wise direction. As can be seen this is not the case.

The $\dot{q}$ profiles are flat in the stream-wise direction, with a slightly reduced value immediately after the step. This is due to the temperature of the foil being higher there, and so a greater proportion of the resistive heating is conducted across the air gap to the copper plate, resulting in a lower $\dot{q}$ into the flow. As the flow speed is increased a larger $\dot{q}$ is seen. This is because a higher $\dot{q}$ must be applied to maintain a similar ΔT between the foil and the free-stream in each experiment. Other than the profiles at $U_\infty = 2$ m/s, there is not much variation between the two separate sets of data, though there is a small but consistent increase in $\dot{q}$ in the repeat experiments.

In Fig. VII.11(b) the stream-wise profiles of ΔT for each run are shown. The highest ΔT is observed immediately behind the step. This corresponds to the recirculating region where a reduced heat transfer would be expected. In all cases but that for $U_\infty = 2$ m/s, ΔT then reaches a minimum at around $5 - 6H$ from which it increases further downstream. This agrees with the estimation of the reattachment position from the velocity profiles (see Fig. VII.9), from which we would expect: (i) the highest HTC and hence lowest ΔT to be near the reattachment position; (ii) the ΔT to then increase again as the thermal boundary layer grows in the direction of the flow. As before, we note that the two profiles at $U_\infty = 2$ m/s are significantly different.

Finally, Fig. VII.11(c) shows the stream-wise HTC profile. As expected the HTC increases with increasing velocity. It is also lowest immediately behind the step where the recirculating region should be, highest at the position of reattachment at $5 - 7H$, and decreases downstream again as the thermal boundary layer grows. The profiles from the two separate sets of experiments show good agreement, except for the case of $U_\infty = 2$ m/s.

CHAPTER VII. BROADBAND CONVECTIVE-CONDUCTIVE SYSTEMS: BACKWARDS FACING STEP

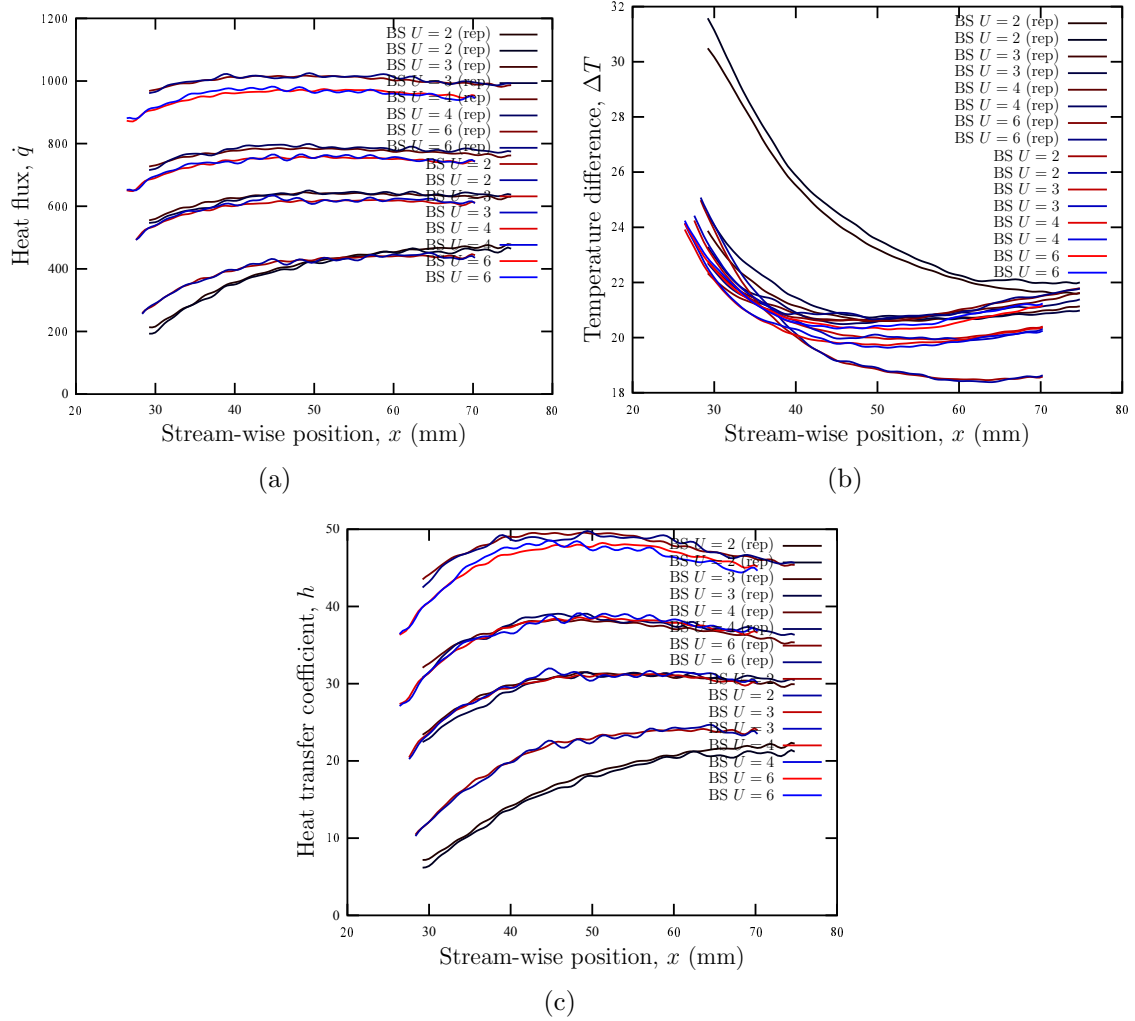


Figure VII.11: Time-averaged stream-wise heat transfer profiles for the backward facing step.

Location of the flow stagnation point

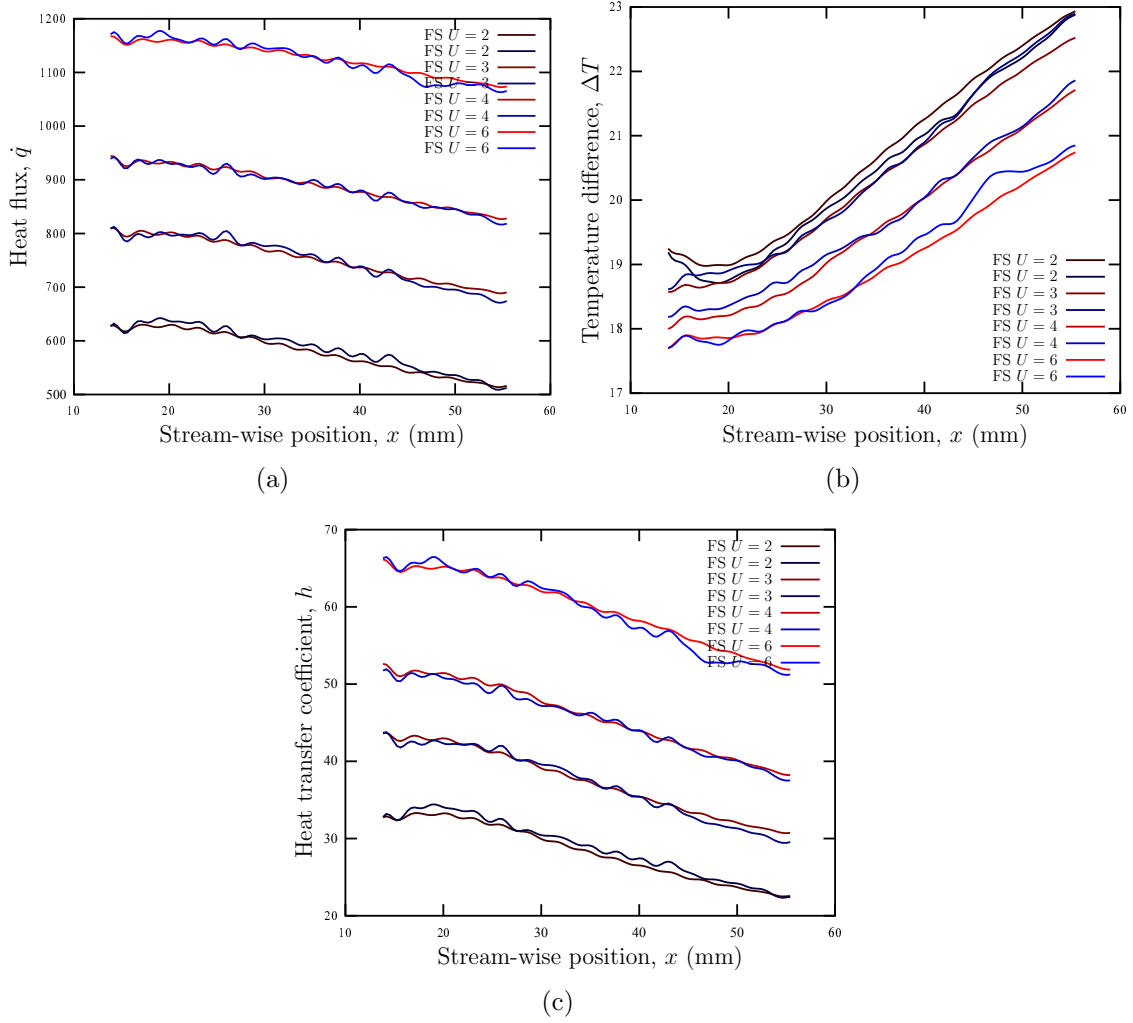


Figure VII.12: Time-averaged stream-wise heat transfer profiles for the forward facing step.

Forward facing step

In the forward facing step significantly different $\dot{q}$, ΔT and h profiles are observed. As before, the profile of $\dot{q}$ (Fig. VII.12(a)) is largely flat and is lowest where the foil temperature is highest, due to heat losses from the foil to the copper plate.

The ΔT profiles (Fig. VII.12(b)), however, do not follow the same trends as those in the backward facing step. The minimum ΔT position, roughly corresponding to peak h , is largely fixed at about $2H$. Interestingly, this does not reflect the migration in velocity field reattachment position seen in Fig. VII.9, which moves between $1.5H$ and $3H$.

Referring to Fig. VII.12(c), the peak value of h is near the lip of the forward facing

step and then decreases as the flow propagates down the plate and the thermal boundary layer thickens. It can be seen that the h profile is flatter towards the front of the plate (upstream, at shorter x), which may be due to the proximity to the recirculating flow ahead of the step, but the effect is much less pronounced than in the case of the backward facing step. Furthermore, contrary to the finding concerning a moving reattachment position, there is no significant horizontal displacement of the profiles of either h or $\dot{q}$ as Re is changed. We hypothesise that it is possible that in the forward facing step the turbulent fluctuations in the flow are enough to reduce the residence time of fluid in the recirculation bubble, thus reducing its effect on the heat transfer. On the other hand, the recirculation bubble in the backward facing step may be more stable, with much longer residence times. This would cause the air in the recirculating bubble to heat up, reducing the rate of heat transfer. Measurements of the air temperature and velocity in this region should be conducted in order to confirm this hypothesis.

VII.5.3 Velocity profiles

In Fig. VII.13 the mean (time- and span-wise averaged) stream-wise profiles of the propagation velocity of h across the heated surface of the backward facing step can be seen. The velocity values shown are a fraction (maximum of 5 %) of the free-stream velocity U_∞ . The peak surface velocity (v) is located towards the rear of the viewable area of the heated surface. The location of the peak surface v does not seem to change much with U_∞ although the peak is higher for the higher U_∞ runs, as expected. The profiles clearly show that on average there is a reversal of flow in the area where the recirculating region should be and that the position of zero surface v is around 40 to 50 mm (or 4 – 5 H) downstream of the step, as described in Section VII.5.1.

The positions of zero and maximum v in the forward facing step (Fig. VII.14) are seen to move downstream (to higher x) with increasing U_∞ . As before, the peak surface v is higher for higher U_∞ . The area of recirculation is once again indicated by the reversal in the direction of v .

Interestingly v reaches a plateau at long x and even decreases in the case of $U_\infty = 2$ m/s. This is indicative of the growth of the boundary layer, which acts to reducing the shear on the surface with increasing x . This, together with the low magnitude of v when compared to the free-stream U_∞ , suggests that the calculated surface v is linked to transport processes within the shear layer and not the free-stream.

Location of the flow stagnation point

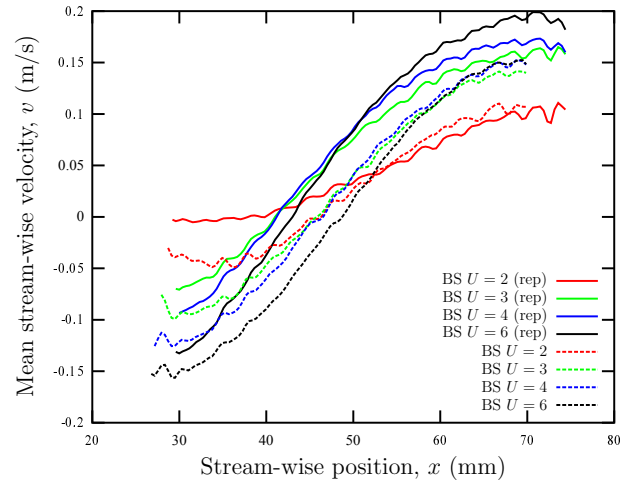


Figure VII.13: Mean (time- and span-wise averaged) profile of h velocity for the backward facing step.

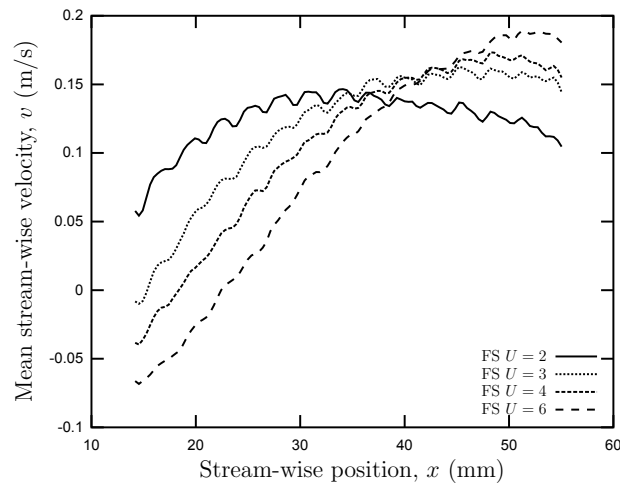


Figure VII.14: Mean (time- and span-wise averaged) profile of h velocity for the forward facing step.

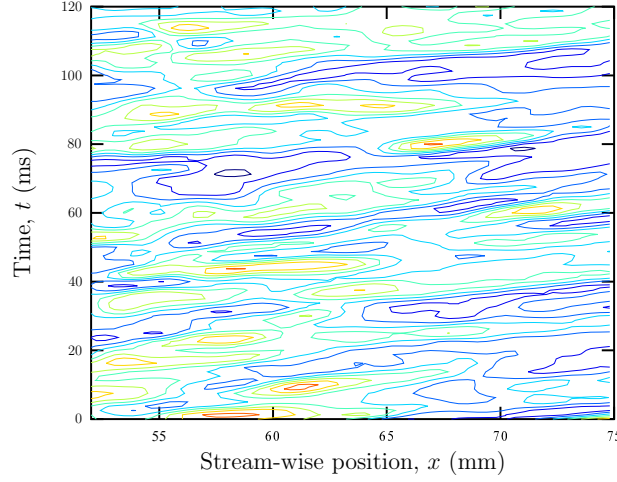


Figure VII.15: Spatio-temporal distribution of the heat transfer coefficient, h , at $z = -30$ mm, behind the reattachment region for a $H = 10$ mm backward facing step with a free-stream velocity of $U_\infty = 6$ m/s.

VII.5.4 Velocity from spatio-temporal distributions

Further to our previous analysis, we have tried to replicate the observations made by Nakamura [62] concerning the velocity of thermal structures seen in the spatio-temporal distribution of the heat transfer coefficient (h) (see Fig. VII.15 taken from Ref. [62]). The author states [178] that the streaky patterns seen here indicate the velocities of thermal structures moving across the surface of the heated plate and that these structures propagate downstream at about half the free-stream velocity.

In our previous analysis we showed how the change in measured heat transfer coefficient (h), or heat flux, from one frame to the next can be used to estimate such a velocity field. By averaging this field with respect to time, and span-wise direction, a velocity profile along the plate is obtained, which gives an indication of several key features such as the reattachment position, and the general trend for the near-wall velocity behind the step. However, this approach results in peak velocities in the averaged profiles of about 5-10 % of the free-stream velocity, which is considerably lower than that mentioned in Nakamura [62].

With reference to the contours of constant h in Fig. VII.16, a line of best fit through each individual contour can be used to estimate the velocity of the associated structure. As the contours tend to pick out structures in the distribution of h , the velocity of all the contours in a given region can be averaged to find a measure of the mean structural

Location of the flow stagnation point

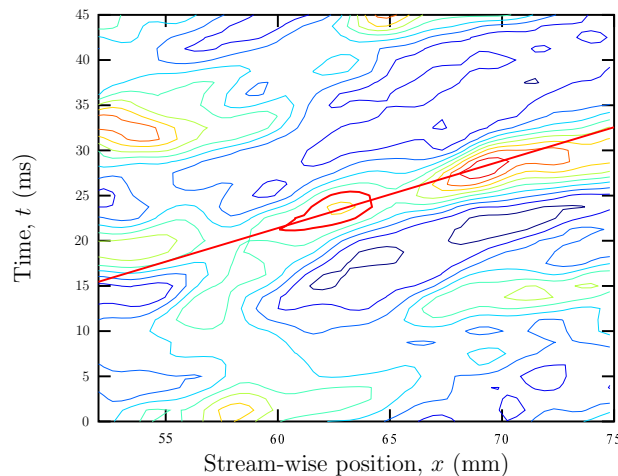


Figure VII.16: The velocity of the thermal structures was estimated as the average gradient from best line fits through a number of heat transfer coefficient contours. Here one contour is highlighted in red, and the best line for that contour is shown.

velocity. The line labelled “Slice” in Fig. VII.17 shows the mean structural velocity, calculated by this contour method, in the region 55-75 mm from the backward facing step which is behind the reattachment position. It can be seen that this measure of velocity is 20-30 % of the free-stream velocity, which is concordant with the estimation of 50 % of the free-stream velocity.

Also shown in Fig. VII.17 are velocities in the same region calculated from our previous method. The line denoted by “Max. avg.” shows the maximum of the time-averaged velocity within the region 55-75 mm behind the backward facing step, whereas the line “Avg.” shows the spatial-average (both in the stream- and span-wide direction) over the same region of the time-averaged velocity. The difference between the two indicates that there is a significant spatial variation in the calculated velocities. The reason for the “Slice” velocities being much greater than the “Avg.” velocities is that by taking measurements from the contours in the spatio-temporal distribution of h the observation of that velocity is strongly correlated to the structures which are sweeping across the region. On the other hand, the previous method sees low or zero velocities when the structures are not present, at any given point, and this significantly reduces the observed average velocity.

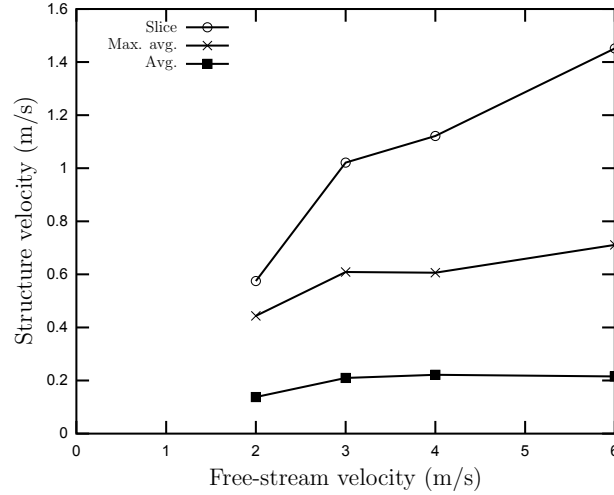


Figure VII.17: The observed structure velocities for the backward facing step, using several methods of analysis.

VII.6 Summary

High-quality spatiotemporally resolved data has been provided by Nakamura [29] of the heat flux and temperature on a surface behind a backward and forward facing step. The simple heat transfer model for Case V from Chapter V was extended to investigate broadband heat transfer coefficients and results were compared to high-resolution experimental data provided for a backwards-facing step.

Using simple statistical information on the heat transfer coefficient from the measurements in this flow (the mean, variance and spectral frequency cut-off), the model correctly predicted the observed trends and the approximate magnitude of augmentation exhibited in this experimental application over a range of Reynolds numbers.

Generally, agreement was found between the experimental and predicted augmentation ratios. The augmentation effect was negative in the considered flow cases, with the region of greatest (negative) augmentation occurring at high values of the Biot number and high heat transfer coefficient fluctuation intensities which occurred at low Reynolds numbers in the backwards-facing step flows.

Additionally in these measurements large scale features were observed convecting across the surface and a scheme was devised to convert these observations into a velocity fields. A location of zero surface velocity was observed in these fields and it was proposed that these were related to the reattachment region of the flow.

The reattachment point calculated from our scheme correlate well between the different variables; we employed the HTC, heat flux and surface temperature fluctuations.

Summary

This suggests that the recovered velocity is independent of these properties, though better signal to noise ratios were attained with the first two of these variables. Secondly, the presented reattachment lengths agreed well with values of velocity reattachment length reported in the literature.

For the backward facing step, at the presented Reynolds numbers (based on the free-stream velocity and step height), the reattachment point is largely fixed. For the forward facing step the reattachment position migrates backwards with increased Reynolds number. This would be expected as the higher Reynolds number will act to increase the shear experienced by the recirculating region behind the step. However, the location of peak heat transfer is not seen to migrate with the reattachment position.

Chapter VIII

Results 4:

Experimental Results in Film Flows

In this chapter we explore the results from the film flow experiments, as described in Chapter IV. First we consider the main features present in the flows including the Marangoni effect and the appearance of the surface waves. We then proceed to the resulting heat transfer coefficients, and finally compare the measured heat transfer augmentation in the experiments to those from the predictions made with our analytical models, as presented earlier in Section V.3.4.

VIII.1 Temperature and Flow Field on the Fluid Surface

The IR camera and LIF system were both arranged to make observations on the fluid surface that is on the top side of the foil. This enabled the temperature and thickness of the film to be measured and a comparison to be made between the temperature measurements from the LIF and IR techniques. Some rudimentary observations of the film flow characteristics can also be made. The LIF measurements were made in this case along a line 45 mm wide located 285 mm downstream of the knife edge outlet from the distribution box, as indicated by the black line in Fig. VIII.1.

In the case of Fig. VIII.1 the foil was positioned at an inclination angle of 40° from horizontal, and a 20 %, by volume, water ethanol mixture was allowed to flow down the

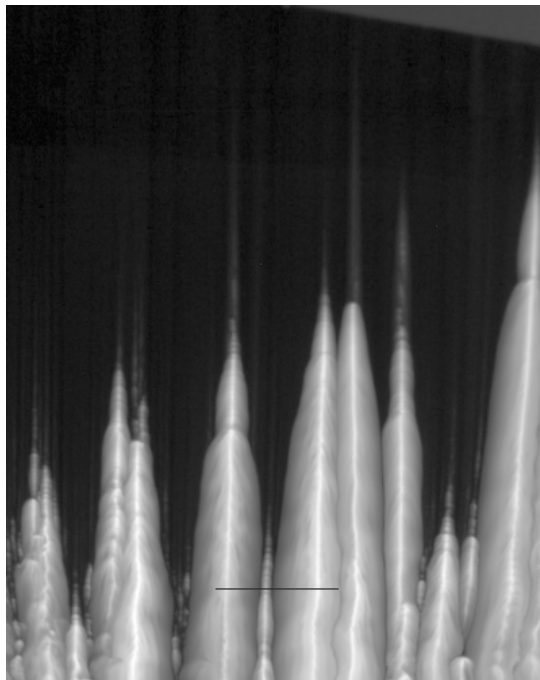


Figure VIII.1: IR image of the film surface, white corresponds to high temperature and black to cooler regions. The LIF measurement location is indicated by the black line is 285 mm downstream from the distribution box and is 50 mm wide. This image was taken at a Reynolds number of 179 and a heat flux of 3.5 W cm^{-2} with no flow forcing.

plate, in this case at 6 l min^{-1} which corresponds to a Reynolds number of $\text{Re} = 179$ over the 300 mm wide test section. An ethanol water mixture was used as it reduced the susceptibility of the fluid film to de-wet from the foil. It was important to avoid dewetting in this experiment as the formation of dry spots on the foil would result in a loss of heat transfer from the foil, and the eventual destruction of the foil due to excessive heating. It is interesting to note that the viscosity of a binary mixture of water and ethanol is higher than that of its constituent components [154, 179], with a peak viscosity occurring at approximately 50 % by volume or a molar concentration of 0.25. In using a water ethanol mixture not only is the Reynolds number reduced for the same flow rate by the higher viscosity but the Marangoni number is also increased. This helps to keep the fluid in contact with the foil up to relatively high heat fluxes.

VIII.1.1 Comparison between LIF and IR measurements

In Fig. VIII.2 the film temperature and thickness as measured by the LIF technique are compared against the observed temperature profile from the IR measurements at the

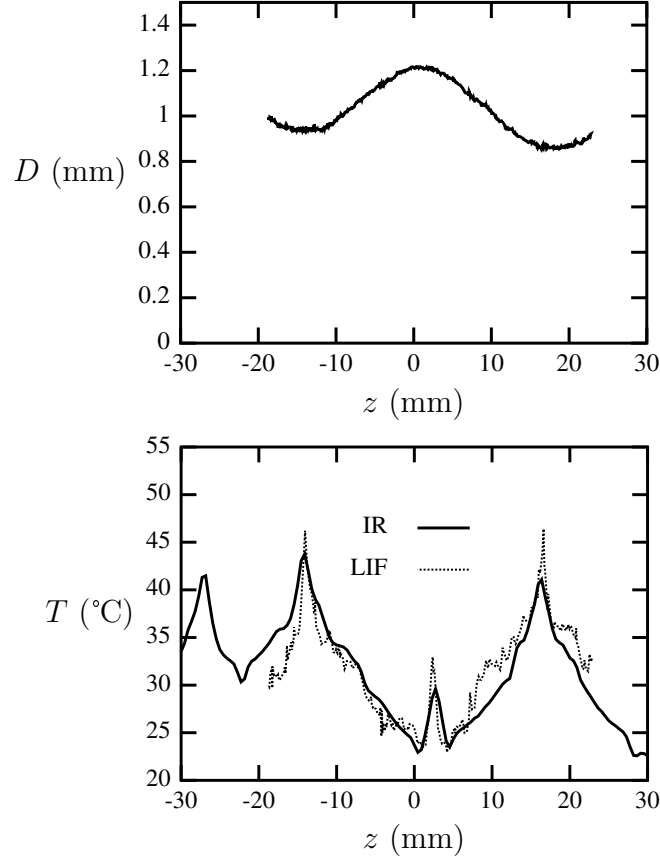


Figure VIII.2: Film temperature measurements from the LIF and IR techniques and the film thickness, d , from the LIF on a horizontal slice along the width of the film, 300 mm downstream from the inlet. Reynolds number of 179 and a heat flux of 3.5 W cm^{-2} .

location marked by the black line in Fig. VIII.1. Both temperature measurements are in good agreement, however the LIF technique shows slightly more measurement noise, and obviously does not provide the same field of view of the IR measurements. Nevertheless one advantage of the LIF technique is that it allows us to measure the film thickness and temperature simultaneously. Here we can see that the hot region is associated with a thinning of the fluid film of up to 20 % compared to the thickness at the cooler part of the flow. It is interesting to note that the third smaller peak in surface temperature at the centre of Fig. VIII.2 which is the beginning of a “hot region”, has very little to no thinning associated with it on the film profile. This indicates that the deformations or “rivulets” in the film are the result of the mass transport of fluid driven by this Marangoni/heat transfer effect which results in fluid accumulating at the cooler regions, and not the other way around. That is, deformation in the film profile is not necessary for there to be a Marangoni driven surface flow.

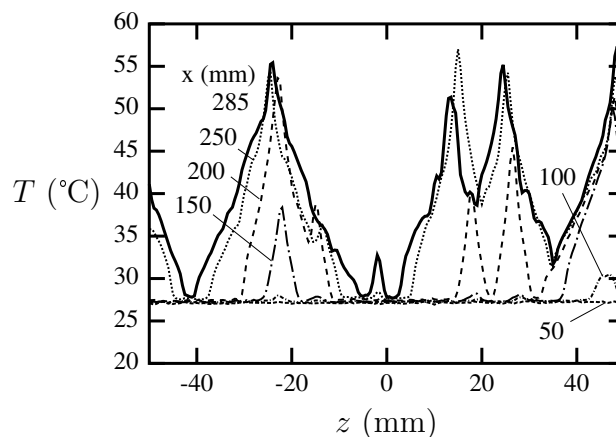


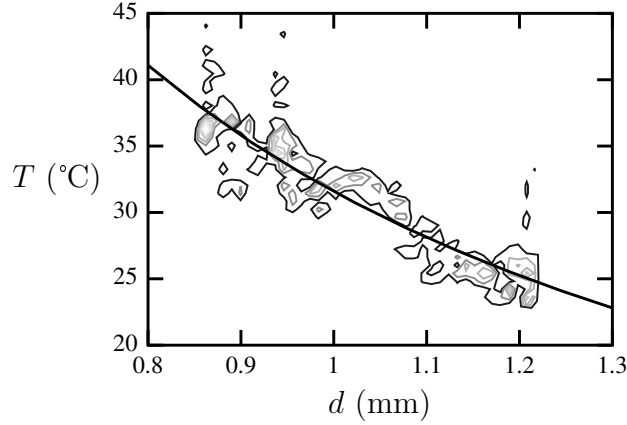
Figure VIII.3: Temperature profiles across the surface of the fluid film, as measured by the IR camera, at different locations downstream of the knife edge. Reynolds number of 179 and a heat flux of 3.5 W cm^{-2} .

VIII.1.2 Rivulet and trough formation

There are several distinctive features apparent in the temperature, measured by the IR camera, of the fluid surfaces as seen in Fig. VIII.1. Here the flow is moving from the top to the bottom of the image. At the top of the image the temperature profile of the surface is mostly flat and cooler than further downstream. There are a number of slight peaks or fingers in the temperature, aligned with the flow, but the amplitude is very small and they are very faint in the image.

As we look downstream the fingers get hotter than the surrounding fluid and start to appear brighter in the image, but still very narrow. Then at some point the finger suddenly widens and appears much brighter and therefore hotter than before. There is also a noticeable discontinuity between the temperature of this hot fluid and the surrounding surface with the temperature rising almost linearly in the span-wise, z , direction from the boundary to the peak in the middle of the finger, as shown in Fig. VIII.3 and VIII.2. The boundary of this region grows outward with a Gothic arch like profile, and there are faint diagonal streak marks in the temperature profile pointing outwards from the centre-line.

As heat transfer in the fluid is dominated by fluid convection streak lines in the temperature profile tend to suggest a direction of fluid flow. This would imply that there is significant span-wise component of flow at the surface, travelling away from the peak temperature at the centre of this hot rejoin. It is likely that this span-wise flow is driven by the Marangoni effect. Where in the surface tension of the film will be much lower in the hot region at the centre of the finger than the surrounding fluid, and this will pull



(a) Film thickness verses film temperature of the profile.

Figure VIII.4: Relationship between film thickness and temperature.

the fluid at the surface in the span-wise direction.

Furthermore the streaks in the flow of the cool fluid, travelling vertically down the film, and the streaks in the hot region, travelling diagonally across the flow, meet at an acute angle, and there seems to be little deviation from the paths that the streaks take. That is the streaks are very straight which would suggest that the flow lines are straight. This implies that the hot region is travelling outwards in a layer on top of the cooler fluid which passes underneath.

A final observation from the image in Fig. VIII.1, is that the “hot region” only initiate or form within the cooler surrounding film surface. That is no fingers or “hot region” appear or poke through other “hot region” on the surface. It is likely that due to the span-wise flow and disturbance in surface tension in these regions that it is unstable for extra regions to form within them.

VIII.1.3 Film temperature against thickness

The temperature of the film surface was plotted against the film thickness measured using the LIF technique and is shown in Fig. VIII.4. Here we can see the general trend where the thickest part of the film has the coolest surface temperature and the area with the highest surface temperature is thinnest. In a Nusselt flow it would be expected that the heat transfer coefficient would be inversely proportional to the film thickness, and so for a constant heat flux into the fluid the temperature difference across the fluid should be proportional to the fluid thickness. In the film flow studied here there are

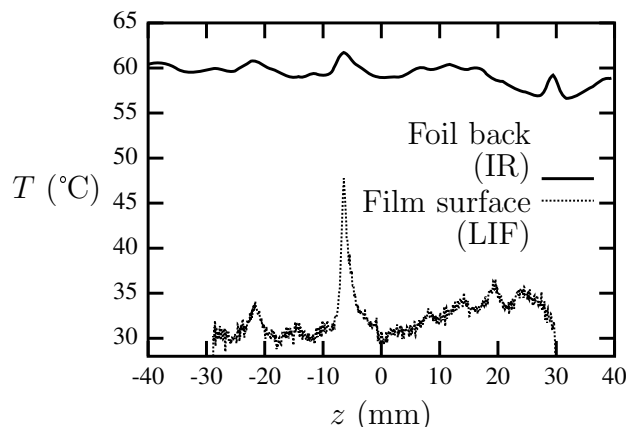


Figure VIII.5: Temperature profile of the fluid surface as measured by the LIF technique, and the heated foil as measured by the IR camera, for a flow Reynolds number $Re = 251$ and a heat flux of $\bar{q} = 3.58 \text{ W cm}^{-2}$.

additional complications arising from 2D and 3D flow effects, however in general the surface temperature of the film was inversely proportional to the film thickness.

VIII.1.4 Adjacent foil temperature

By arranging the IR camera so that it is positioned to observe the temperature of the underside of the foil, instead of the surface of the fluid film, the temperature difference across the fluid film can be determined. In Fig. VIII.5 a temperature profile in the spanwise direction, z , of the fluid surface, as measured by the LIF technique, and the heated foil, as measured by the IR camera, are shown. Here it can be seen that the sharp peak in fluid surface temperature coincides with a slight peak in the foil temperature. The temperature difference between the foil and fluid at this point is reduced, which means that there must be a significantly higher heat transfer coefficient at that point than in surrounding fluid.

VIII.1.5 Conclusions on the rivulet flow structure

Finally the fact that both the fluid surface and foil temperature are higher in this region, and so the bulk film temperature in this region is higher than on the surrounding film, indicates one of two possible flow conditions. As the foil is transferring a constant heat flux into the film over its area, a raised bulk fluid temperature could indicate that the film thickness is lower along that hotter region. A second possibility is that due to the

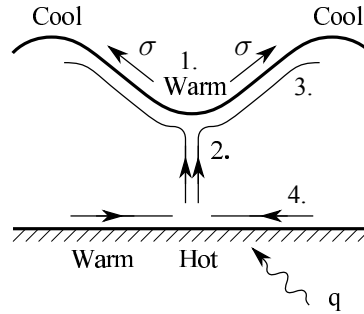


Figure VIII.6: Schematic of the span-wise circulation in the film flow as a result of the Marangoni effect.

Marangoni flow at the surface, the lower layers of fluid in the film will experience a span-wise pressure gradient in the opposite direction pushing the fluid towards the hotter region, resulting in a build up of heat in the hotter region. In the experiments it was observed that the “fingers” of hotter fluid were normally associated with a thinning of the film at that location, however this was not always the case and some films had quite flat thickness profiles, and yet still had these areas of higher film temperature. Perhaps the film temperature is raised due to a combination of both mechanisms.

To recap the finger effects seen in the film surface temperature are probably the result of a Marangoni driven flow as a result of the temperature gradients, and so surface tension gradients on the film surface. For the Marangoni flow to occur some initial difference in surface temperature, or perturbation in flow results in a slight variation in the surface tension of the fluid (1. see Fig. VIII.6). This surface tension gradient drives a span-wise component of flow at the fluid interface, which reinforces the temperature profile at the surface by the advection of hotter fluid from within the fluid film, closer to the heated foil, to the surface (2.). This flow starts off small, early on in the film, with only a very slight variation in temperature. At some critical point further downstream however, the region is seen to expand rapidly. The fluid of the heated region is seen to be flowing over the cooler surrounding fluid, the resulting accumulation of fluid in the cooler area will cause the formation of finger like rivulets some distance downstream of the initially thermal structure (3.). In the case when there is a significant restoring force, such as by gravity, the rivulet amplitude will be diminished, or even completely absent, and there should be significant span-wise return flow in the lower layers of the film (4.).

VIII.2 Effect of Flow rate and Heat Flux

To gain insight on the effect of the fluid flow rate and the applied heating flux on the structure of the film flow pattern, heat transfer some images of the temperature distribution over the fluid surface and the back of the foil are displayed in Fig. VIII.7 and VIII.8. The operation point for each image varies over a range of flow rates from a Reynolds number of 100 to 250 and heat fluxes from 0.36 to 3.6 W cm^{-2} . By changing the heat flux the Marangoni number of each flow will be different due to the change in the flow-wise and depth-wise temperature gradient in the fluid. It is important to note here that due to the small depth of these films the Rayleigh number, the ratio of buoyancy forces to viscose dissipation, is very low ~ 0.01 and so buoyancy driven convection is not expected.

The stream-wise variation in the foil temperature measured by the IR camera, Fig. VIII.7, appears minimal, and the peaks in the foil temperature, which we associate with the peaks in fluid temperature (see the span-wise profiles in Fig. VIII.5), appear regularly spaced and straight, pointing in the direction of the flow. It is apparent that increasing the flow rate and heat flux increases the amplitude and spatial frequency at which these “fingers” appear in the span-wise direction. From the intermediate Reynolds number an unsteady, oscillatory behaviour was observed in the finger pattern with a darker patch appearing either side of the hotter “finger” and proceed to race up and down the finger with an amplitude of 5 to 10 cm and a frequency of between 0.5 to 2 Hz. These oscillations were not apparent at the lower Reynolds numbers or low heat fluxes. Their position on the foil was also determined by the flow rate and heat flux, with higher Reynolds number pushing the oscillating region further down stream and higher heat fluxes moving them further upstream.

A better illustration of what is occurring in the fluid flow is obtained by positioning the IR camera to view the top surface of the film, Fig. VIII.8. There are missing panels for the higher heat fluxes at the lower flow rate in this figure because it became impossible to run the experiment at those flow conditions. When running the higher heat fluxes at the low flow rate the film began dewetting which risked damaging the experiment. In the previous runs from the investigation of the back surface Fig. VIII.7 these flow conditions could be met, however here the dewetting behaviour was observed. This was probably because of the accumulation of imperfections in the lower portion of the foil, from repeated heating and cooling, which meant the conditions could no longer be sustained.

Here in Fig. VIII.8 the triangular “fingers” of hot fluid, associated with the thinning of

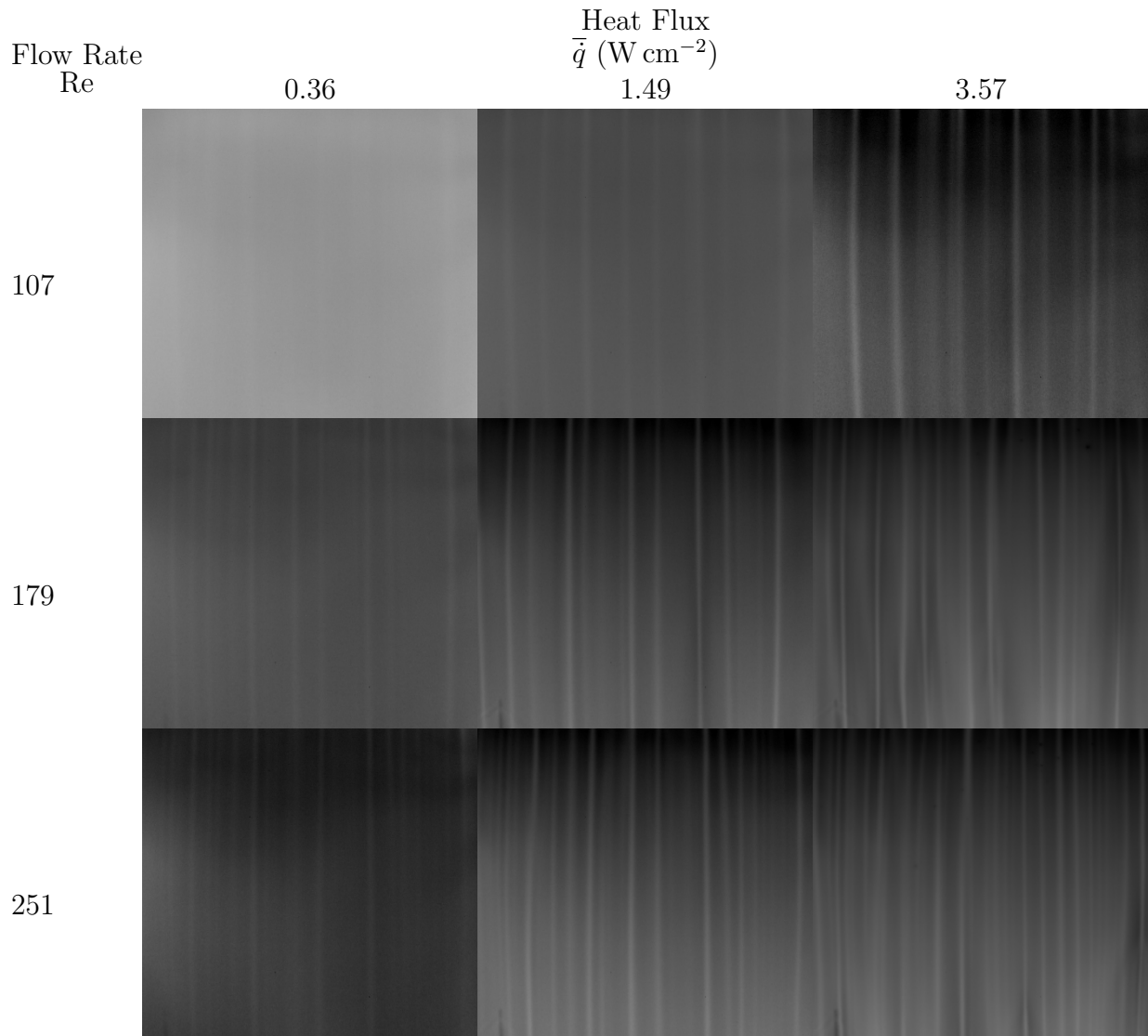


Figure VIII.7: Qualitative presentation of the temperature distribution measured using the IR camera on the back surface of the heated foil at different heat fluxes and film flow rates, without pulsation. The fluid here is 20 % ethanol and the inclination angle of the foil was 40°.

Effect of Flow rate and Heat Flux

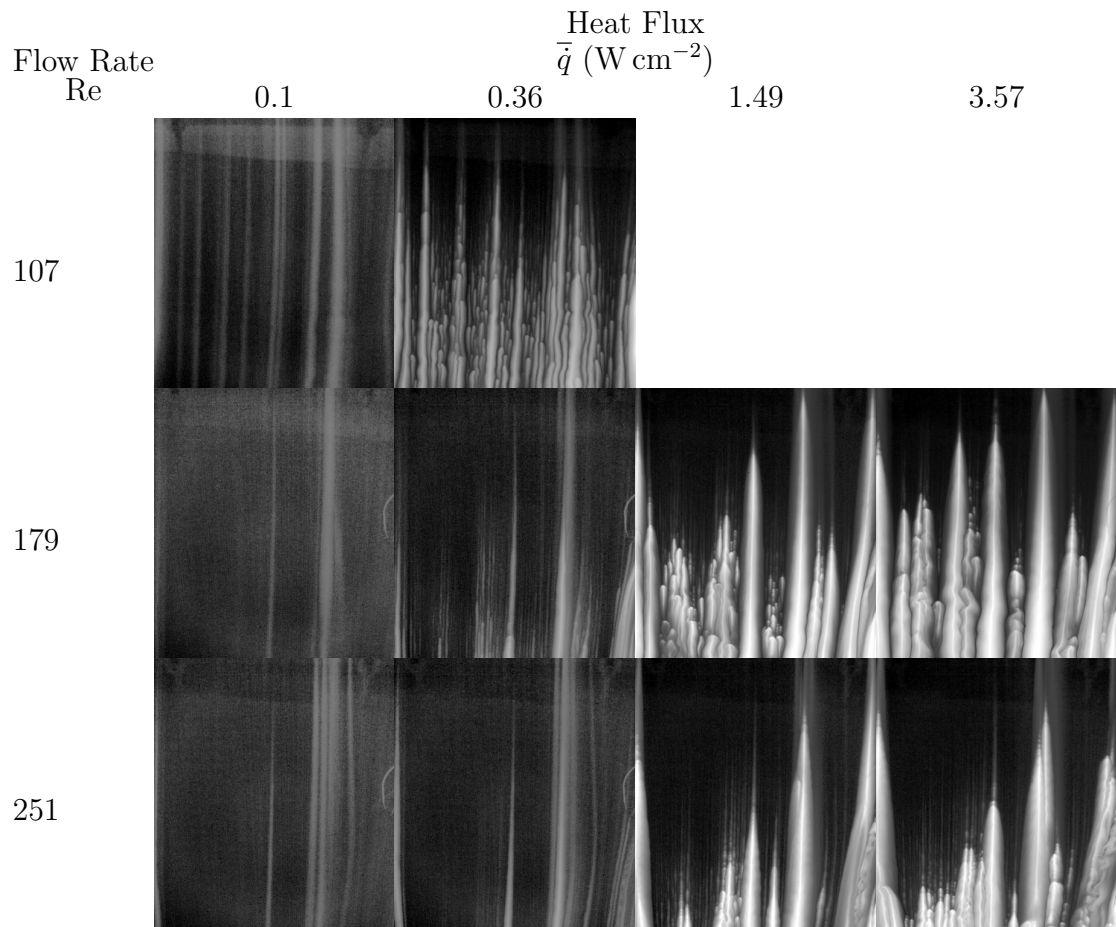


Figure VIII.8: Qualitative presentation of the temperature distribution, measured using the IR camera, on the surface of the fluid film at different heat fluxes and film flow rates, without pulsation. The fluid here is 20 % ethanol and the inclination angle of the foil was 40° .

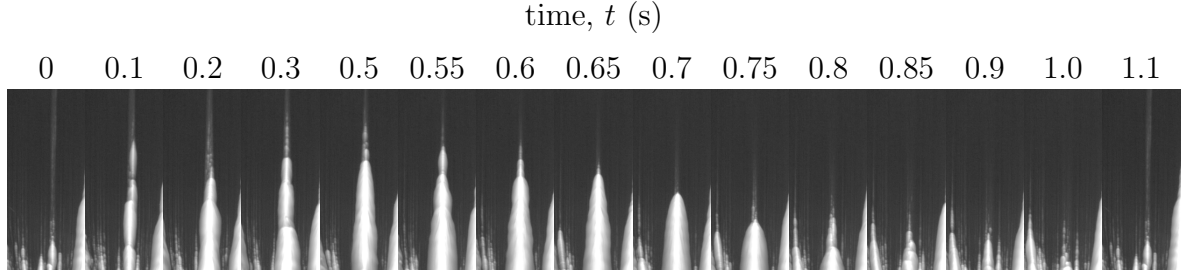


Figure VIII.9: Oscillation of a rivulet at a Reynolds number Re of 251 and a heat flux of 2.4 W cm^{-2} .

the films, are clearly visible. As before we can see that increasing the heat flux increases the number of fingers and moves the structures further upstream. whereas increasing the flow rate moves the structures further down stream. The triangular or “Gothic arch”–like structures of hot surface temperature seemed to oscillate up and down the film, in a similar manor to the “dark patches” observed previously It is assumed that they must be related.

The effect of flow rate and heat flux on the structures on the films shows that low flow rates and high heat fluxes increase the rivulet effect. This would suggest that the it is the thermal gradient in the stream wise direction which is important in driving the flow condition. That is lowering the flow rate and increasing the heat flux both result in a steepening of the thermal gradient in the streamwise direction.

VIII.2.1 Rivulet oscillation

An example of an oscillation in a rivulet on the film surface has be presented in Fig. VIII.9. Here we can see the Gothic-arch shaped region of high temperature surface fluid moving up and down stream of the film over a period of about a second. The region off hot fluid starts of very faint and thin but slowly grows up the flow and gets wider, as the rejoin grows it appears to have a sharp leading edge. Upon reaching its maximum extent at 0.5 to 0.6 s the region seems to get washed back down the stream and has a blunt leading edge. The faint line of hot fluid on the surface now diminishes until it is reestablished at 1.1 s, the cycle then repeats as before.

As the hot region “grows” up stream several interesting features are apparent. Firstly small wave like structures appear to flow down the “finger” a good example of this is seen in Fig. VIII.9 between the images 0.2 to 0.3 s. These are probably due to waves of fluid refolding the area where the film starts to thin. Possibly what is occurring is that

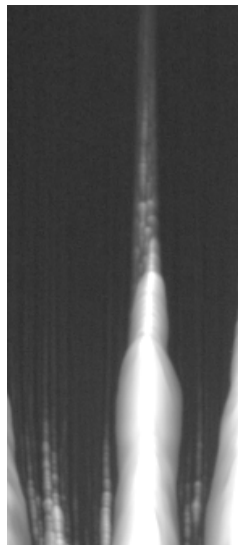


Figure VIII.10: Expanded image of the flow structure at $t = 0.2$ s from Fig. VIII.9.

the Marangoni cell is reducing the fluid thickness locally, and enhancing the heat transfer there, this pulls the foils temperature down. Without a large temperature gradient to sustain the Marangoni flow the surrounding fluid collapses in to fill the depression in the surface generating a hydrodynamic wave which sweeps down the film.

A second interesting feature is that the region just in front of the main growing Marangoni cell seems to contain many short-lived cells which are much smaller in size than the main cell. These cells either dissipate, or are merged into the main structure. This is particularly apparent in Fig. VIII.10.

VIII.3 Films With Flow Pulsation

A pulsation was introduced to the flow which generated hydrodynamic waves which travelled down the film. This was done to obtain a more coherent fluctuation in the heat transfer coefficient with a greater amplitude. Instigating a pulsation also enabled more control over the frequency of the HTC fluctuation. In Fig. VIII.11 and VIII.12 we explore the effect of various parameters on the distribution of temperature on the film surface and back of the foil as before for the steady state case. The effect of Pulsation amplitude and heat flux is shown for a flow of a 20 %b.v. ethanol water mixture down a heated plate with an inclination angle of 40° and at a mean flow Reynolds number of 179 and a frequency of 1.8 Hz. The forcing pulsation amplitude is displayed as a proportion of the mean flow rate.

CHAPTER VIII. EXPERIMENTAL RESULTS IN FILM FLOWS

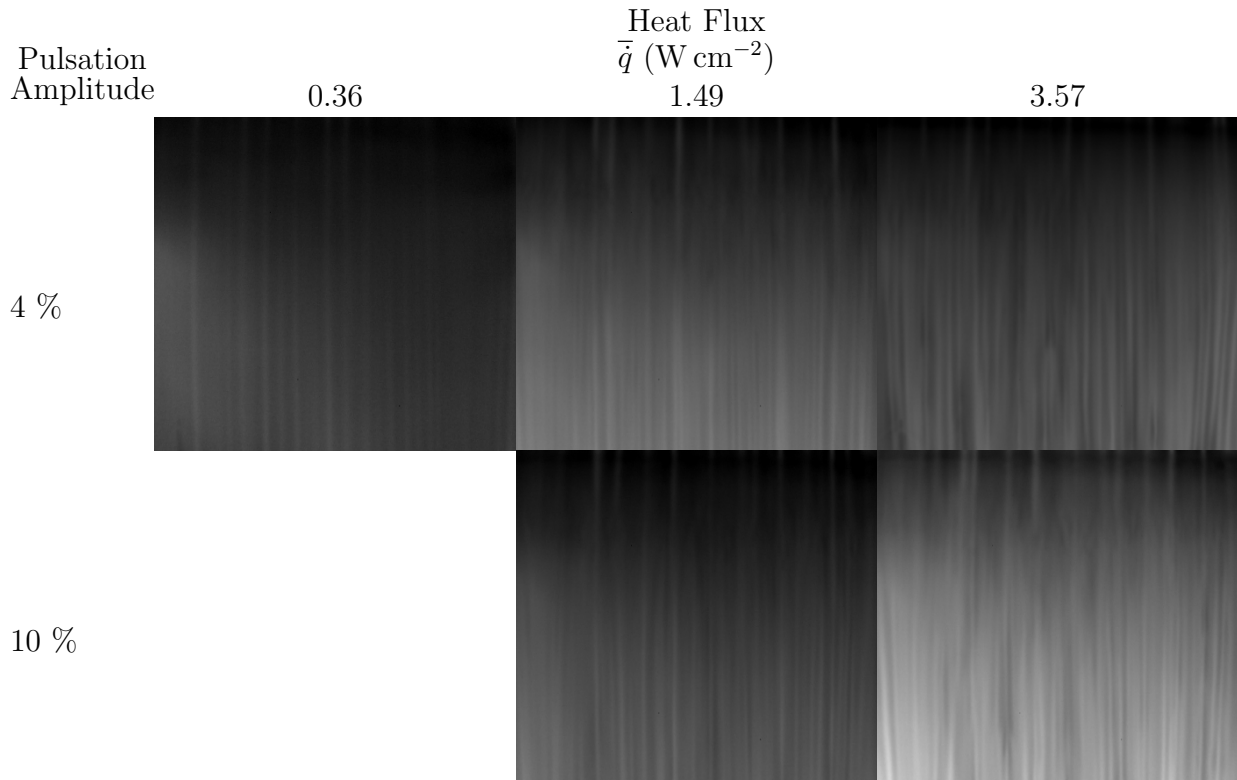


Figure VIII.11: Temperature distribution of the back surface of the foil, measured by the IR camera, with pulsation in the film flow. The average flow rate in this case was $Re = 179$.

Looking at the back surface of the foil, Fig. VIII.11, it is immediately apparent that the flow has become much more unsteady with the introduction of the waves. Only at the lowest pulsation amplitude and heat flux did the image appear smooth with little variation in the flow wise direction, and regular “fingers” of high temperatures along the direction of flow. At the higher heat fluxes patches of rougher texture in the temperature distribution were seen to pulse down the foil, in time with the waves.

On the films surface we can see directly the effect that the waves are having on the temperature distribution in the fluid, Fig. VIII.12. At the lower heat flux the temperature profiles appear uniform, the wave spacing and smaller capillary waves can just be made out in these images. The distortion in the film interface is probably visible here as a result of reflections off of the film from the background, as the film here was at a low temperature. As the heat flux is increased the regions of high surface temperature form as before, however the passing of each wave front can be observed mixing the hot area into the surrounding fluid. The size and randomness of the hot zones seems to increase

Film Surface Temperature, Thickness and the Temperature of the Back Surface of the Foil

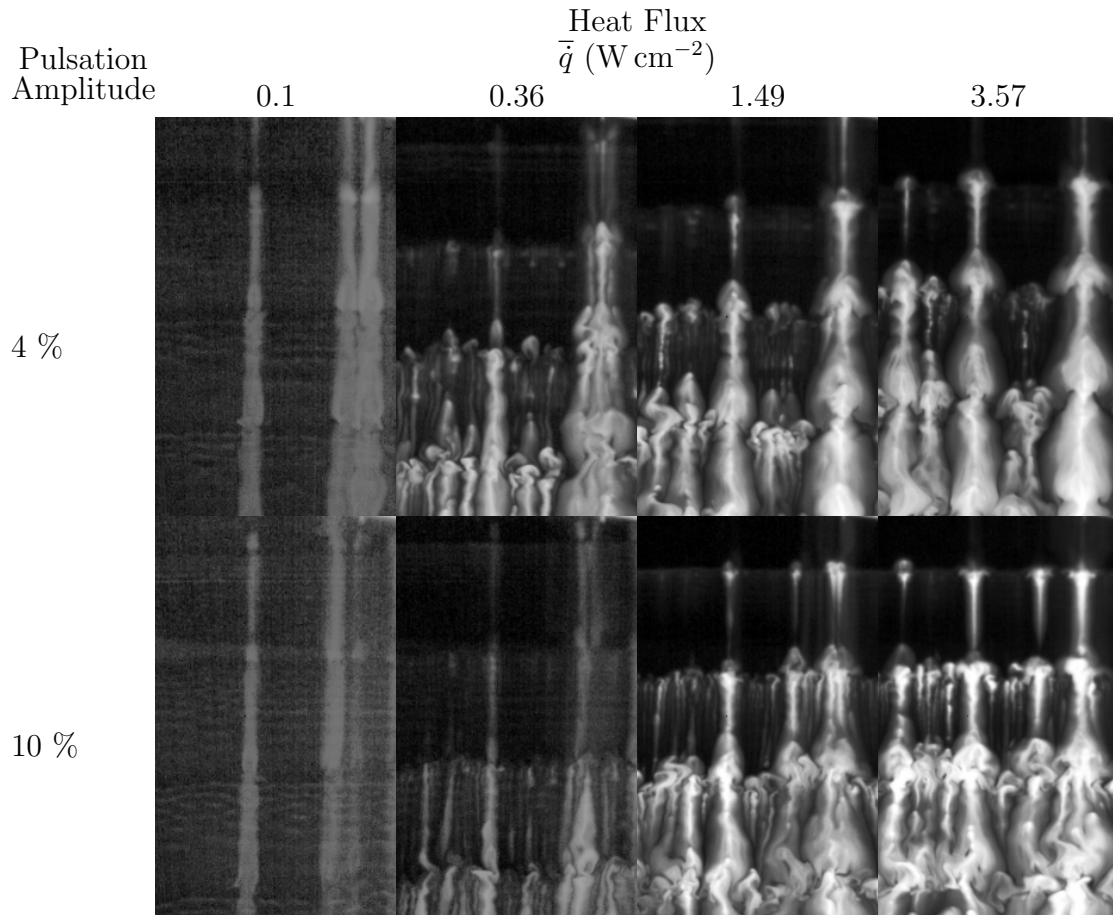


Figure VIII.12: Temperature distribution of the fluid film surface, measured by the IR camera, with pulsation in the flow. The average flow rate in this case was $Re = 179$.

as they flow down stream. At the higher heat fluxes and pulsation amplitudes the mixing appears stronger with a larger number of eddies appearing.

VIII.4 Film Surface Temperature, Thickness and the Temperature of the Back Surface of the Foil

In Fig. VIII.13 the time evolution of the depth, D , and temperature, T_{LIF} , of the film surface as measured by the LIF technique are plotted next to the temperature of the foil. Two locations which were used for the position of LIF measurements, in this case the measurement was located at the furthest position downstream at 285 mm. The images show the temperature or depth, with bright areas representing high temperature or film thickness. The LIF profile measurements were made in the span-wise, z , direction, cutting

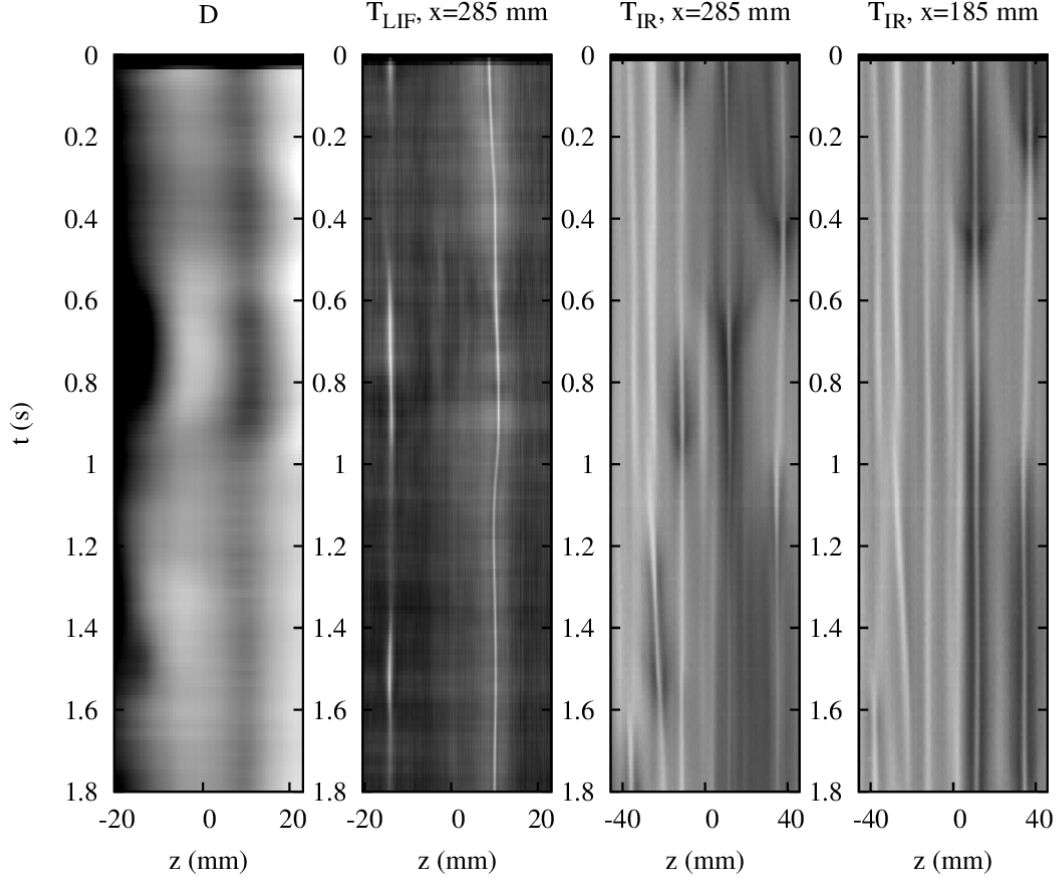


Figure VIII.13: Comparison of the LIF measurements of the fluid film temperature, T_{LIF} , and depth, D and IR temperature measurements, T_{IR} , on the back of the foil at a Reynolds number, $Re = 179$, a heat flux of $\bar{q} = 2.38 \text{ W cm}^{-2}$, for a 20 % water ethanol mixture on a plate at an inclination angle of 40° , 285 mm downstream from the distributor knife edge (lower test location).

across the flow.

The image in Fig. VIII.13 clearly shows the thinning of the film at the location of peak surface temperature T_{LIF} , at about $z = +10$ mm. The depth of this trough appears to increase at 0.6 to 0.9 s which coincides with the dark patches in the back foil temperature T_{IR} , $x = 285$ mm.

By looking at the same condition further upstream the oscillation in the trough position can be observed. As the trough migrates up and down stream it cuts through the profile at $x = 185$ mm creating the dots we see in Fig. VIII.14.

At a yet higher heat flux of 3.58 W cm^{-2} at the upper location $x = 185$ mm, the

Film Surface Temperature, Thickness and the Temperature of the Back Surface of the Foil

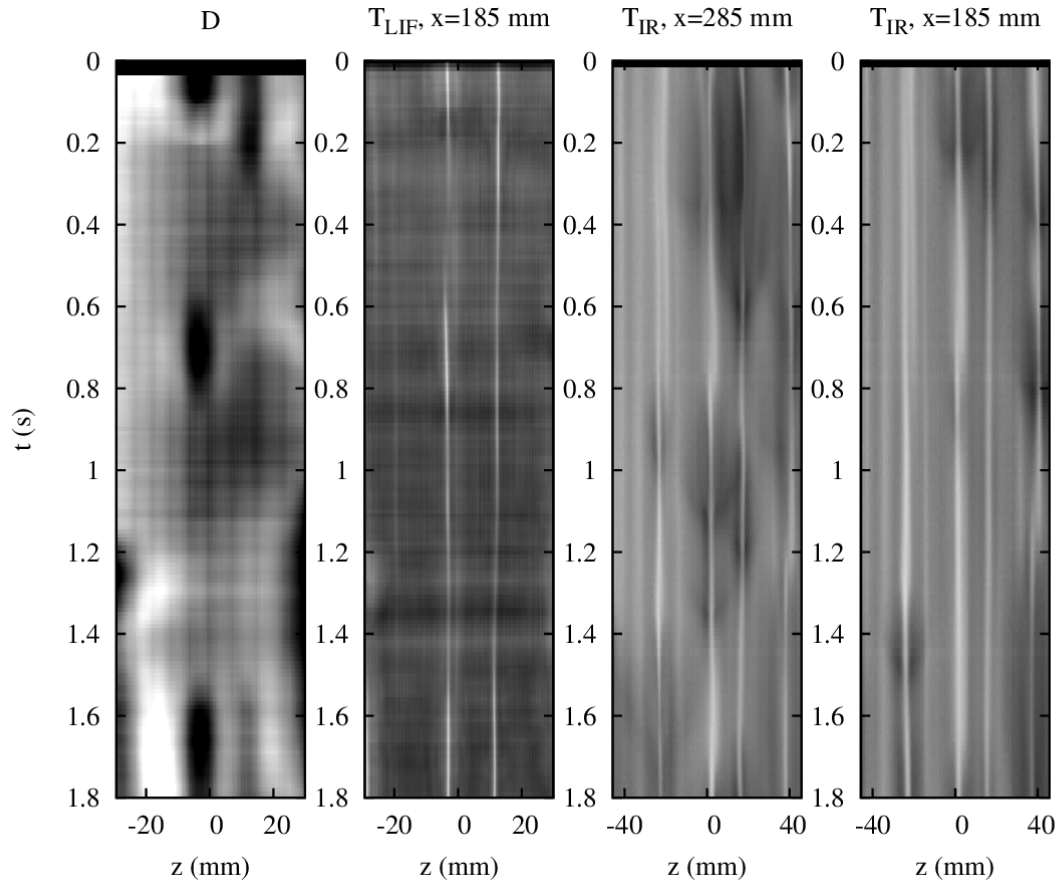


Figure VIII.14: Comparison of the LIF measurements of the fluid film temperature, T_{LIF} , and depth, D and IR temperature measurements, T_{IR} , on the back of the foil at a Reynolds number, $\text{Re} = 179$, a heat flux of $\bar{q} = 2.38 \text{ W cm}^{-2}$, for a 20 % water ethanol mixture on a plate at an inclination angle of 40° , 185 mm downstream from the distributor knife edge (upper test location).

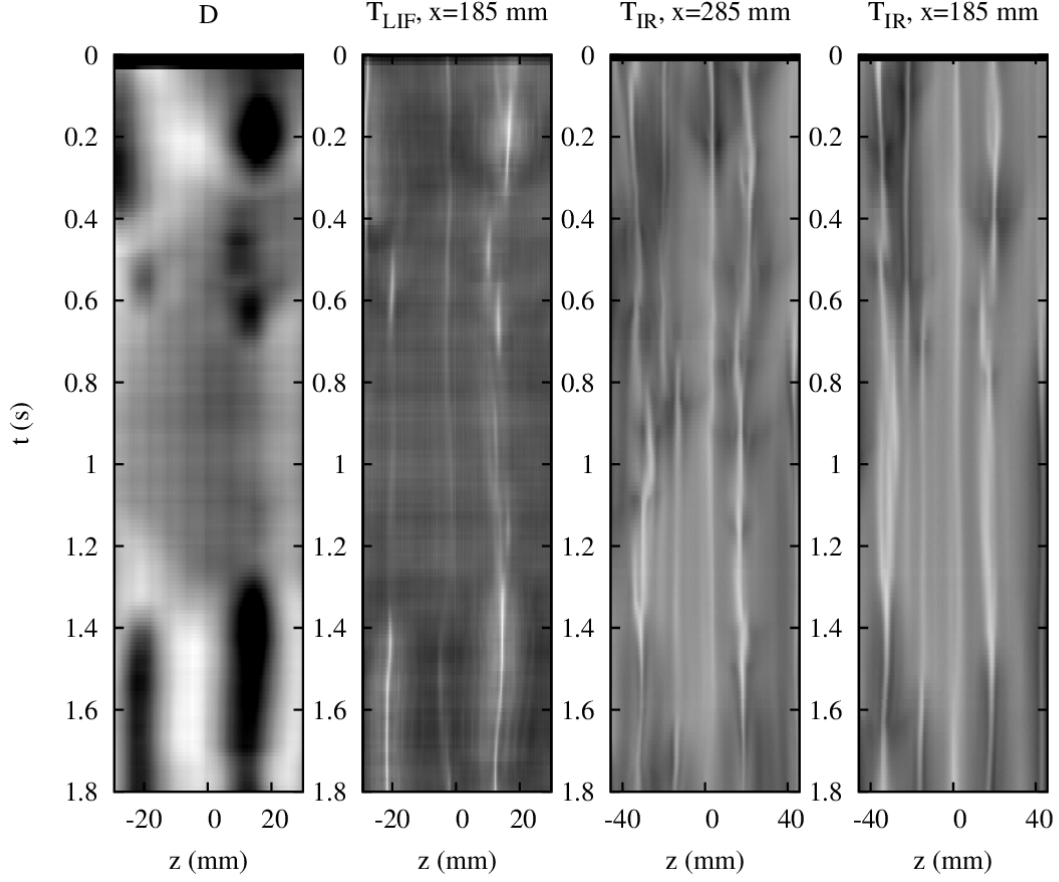


Figure VIII.15: Comparison of the LIF measurements of the fluid film temperature, T_{LIF} , and depth, D and IR temperature measurements, T_{IR} , on the back of the foil at a Reynolds number, $\text{Re} = 179$, a heat flux of $\bar{q} = 3.58 \text{ W cm}^{-2}$, for a 20 % water ethanol mixture on a plate at an inclination angle of 40° , 185 mm downstream from the distributor knife edge (upper test location).

troughs become more pronounced, Fig. VIII.15 and oscillate as before. Small bow waves are visible, on the depth image, either side of the trough as it moves up and down film.

In Fig. VIII.16 the time evolution of the instantaneous heat flux, heat transfer coefficient and Nusselt number over the LIF profile are shown for the results from Fig. VIII.15.

Film Surface Temperature, Thickness and the Temperature of the Back Surface of the Foil

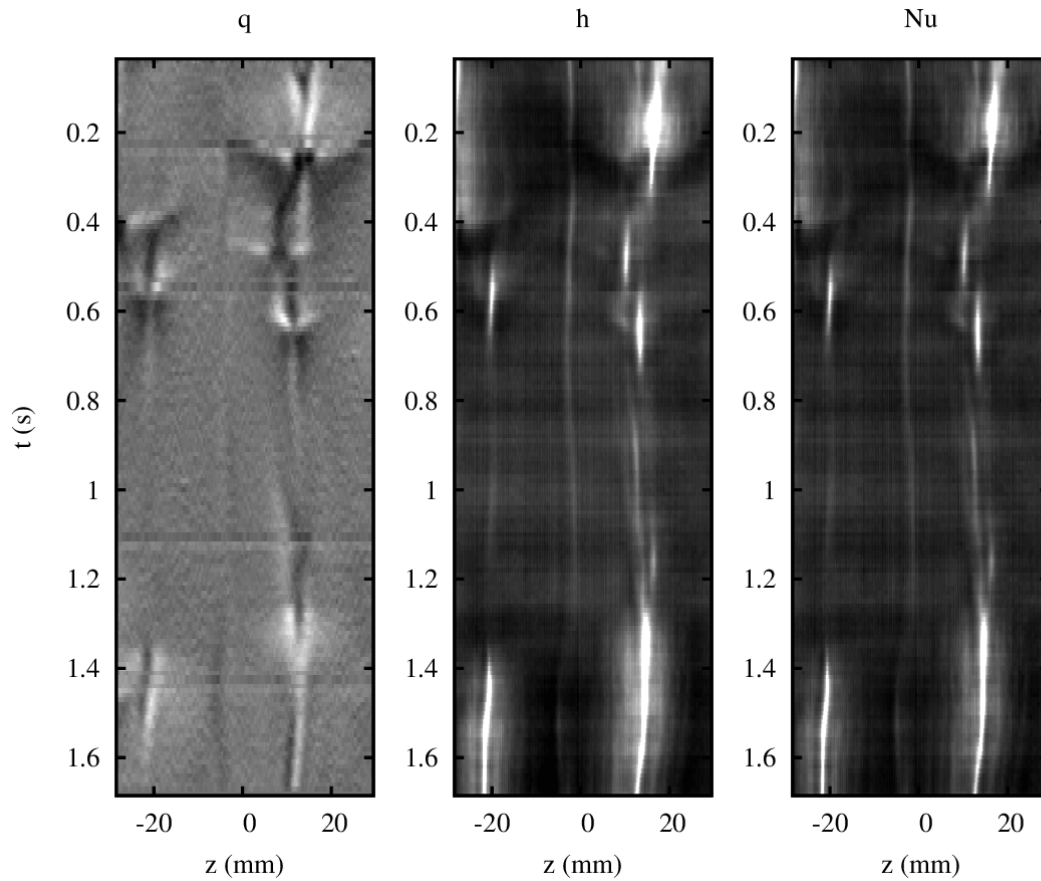


Figure VIII.16: Heat flux, heat transfer coefficient and Nusselt number at a Reynolds number, $Re = 179$, a heat flux of $\bar{q} = 3.58 \text{ W cm}^{-2}$, for a 20 % water ethanol mixture on a plate at an inclination angle of 40° , 185 mm downstream from the distributor knife edge (upper test location). The heat flux variation here was about 4 %, the Nusselt number was about 2.8, and the HTC was in the range of 1000 to $3000 \text{ W K}^{-1} \text{ m}^{-2}$.

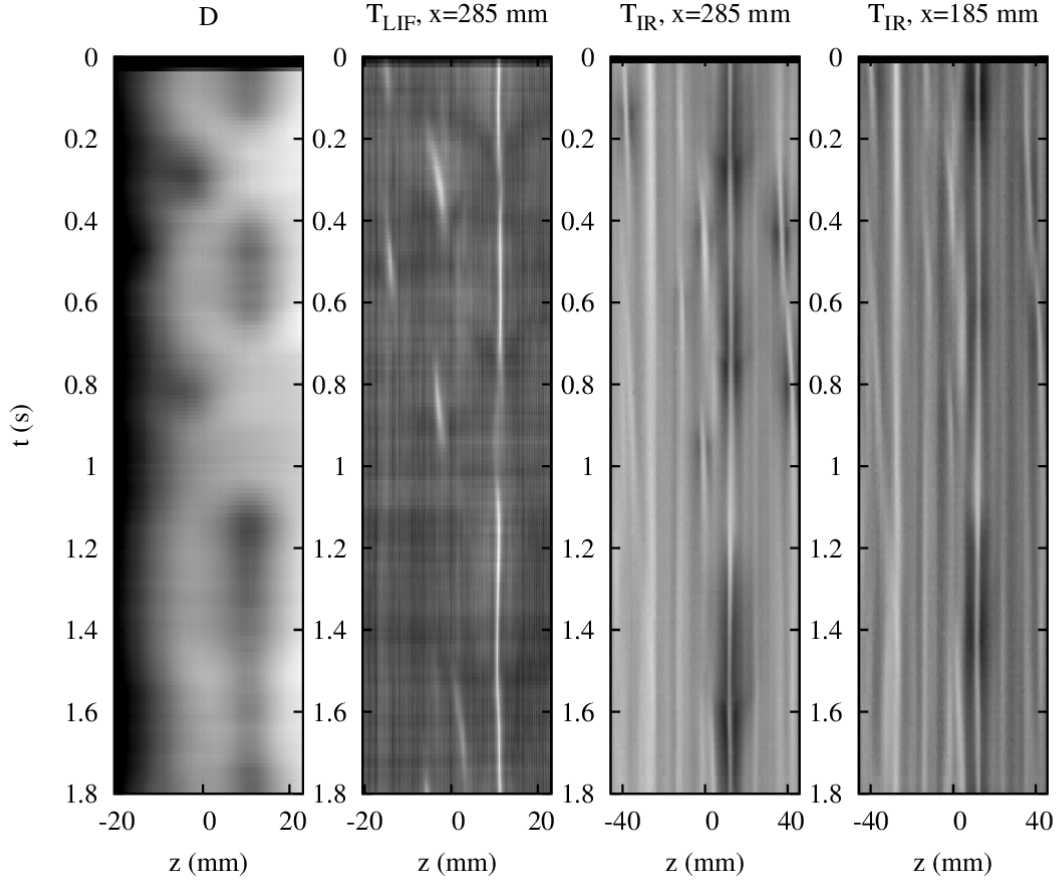


Figure VIII.17: Comparison of the LIF measurements of the fluid film temperature, T_{LIF} , and depth, D and IR temperature measurements, T_{IR} , on the back of the foil at a Reynolds number, $\text{Re} = 251$, a heat flux of $\bar{q} = 2.38 \text{ W cm}^{-2}$, for a 20 % water ethanol mixture on a plate at an inclination angle of 40° , 285 mm downstream from the distributor knife edge (lower test location).

Film Surface Temperature, Thickness and the Temperature of the Back Surface of the Foil

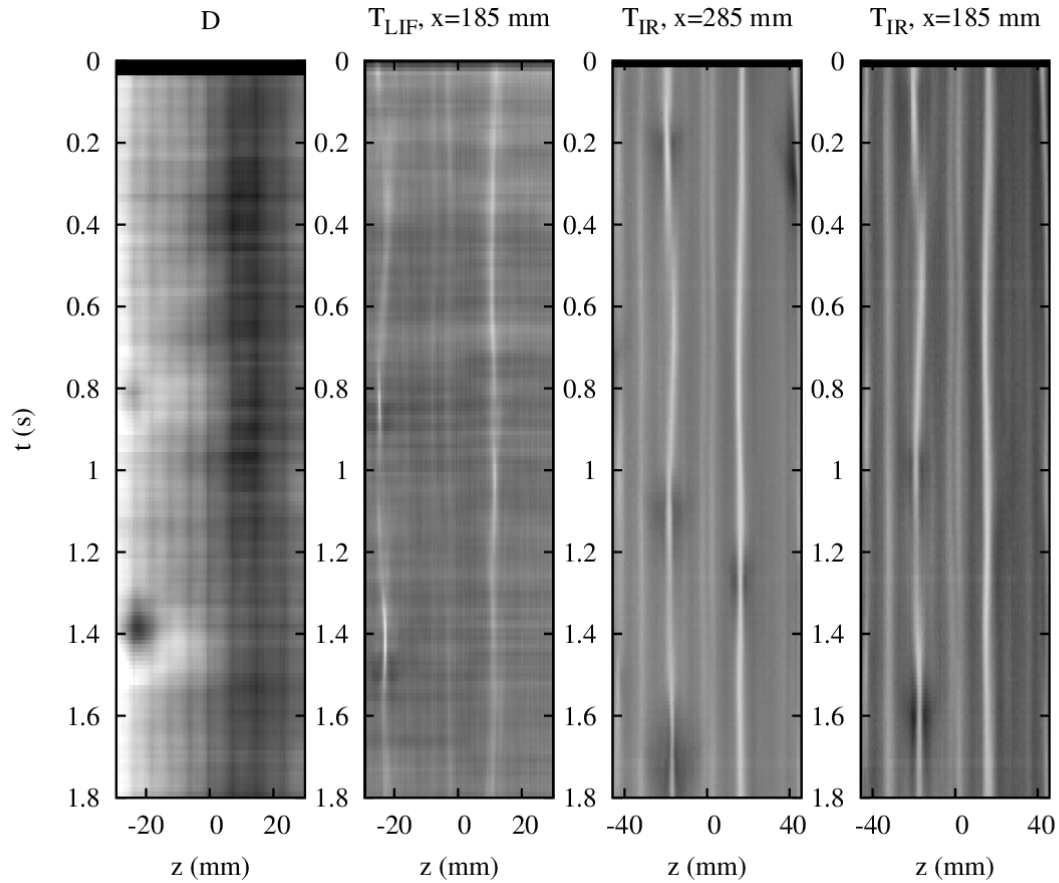


Figure VIII.18: Comparison of the LIF measurements of the fluid film temperature, T_{LIF} , and depth, D and IR temperature measurements, T_{IR} , on the back of the foil at a Reynolds number, $\text{Re} = 251$, a heat flux of $\bar{q} = 2.38 \text{ W cm}^{-2}$, for a 20 % water ethanol mixture on a plate at an inclination angle of 40° , 185 mm downstream from the distributor knife edge (upper test location).

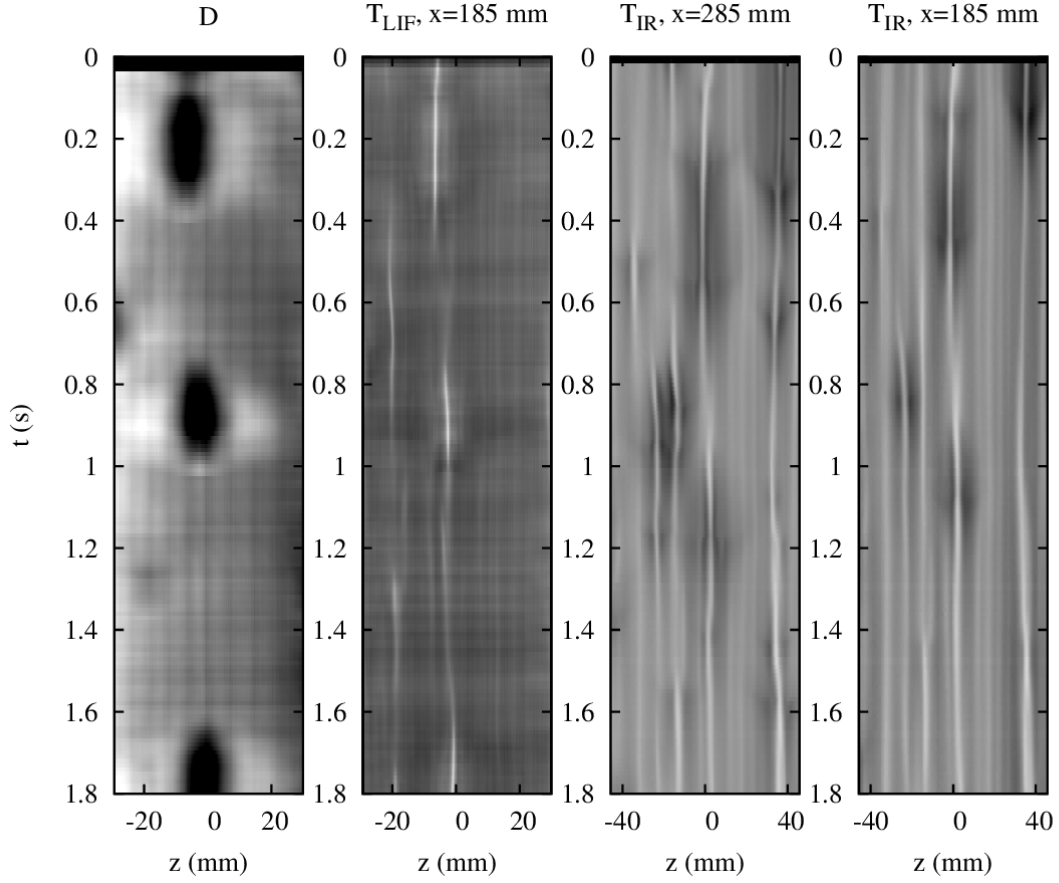


Figure VIII.19: Comparison of the LIF measurements of the fluid film temperature, T_{LIF} , T_{LIF} , and depth, D and IR temperature measurements, T_{IR} , on the back of the foil at a Reynolds number, $\text{Re} = 251$, a heat flux of $\bar{q} = 3.58 \text{ W cm}^{-2}$, for a 20 % water ethanol mixture on a plate at an inclination angle of 40° , 185 mm downstream from the distributor knife edge (upper test location).

VIII.5 Heat Transfer Augmentation in the Film Flow

The IR camera and LIF system were arranged so that the IR camera could image the underside of the foil, and the LIF measurements made a vertical slice along the stream-wise direction of film travel. Pulses of hydrodynamic waves were made to flow down the films at different flow rates and pulsation frequencies, the aim being to see what the effect of pulsation frequency and amplitude was on augmentation and whether these results fitted the models from Chapter V.

Typical results can be seen in Fig. VIII.20. The film interface height d , temperature T_f , and heat flux at the common fluid-solid surface (inner wall) $\dot{q}_w$, all vary with time and spatial position within the imaged area. Note that the local, time-varying heat flux $\dot{q}_w$ is defined herein as +ve from the fluid into the solid wall. The instantaneous HTC h , is also evaluated from these data and shown. In Fig. VIII.20(a) several spatial snapshots at different time instances are shown as a wave progresses through the 40 mm long interrogation (viewing) area. The main wave front can be clearly seen, and is preceded by several, smaller capillary waves. Where the film is thickest, the temperature difference between the water and the foil is largest, and where the film is thin the temperature difference is small. Conversely the HTC is lowest where and when the film is thick and much higher when it is thin, as we would expect (see Eq. II.13). Figure VIII.20(b) shows how the heat flux $\dot{q}_w$ rises and falls locally with each passing wave. In this case the fluctuation in $\dot{q}_w$ has an amplitude of 0.6 % of the mean heat flux. In correspondence to the wall heat flux, the wall temperature T_w is also approximately constant. The majority of the fluctuations in the bulk fluid temperature difference $\Delta T_f = T_f - T_w$ in this case emerges from fluctuations in the bulk fluid temperature T_f . This result arises in measurements taken far downstream of the inlet; closer to the fluid inlet considerably larger fluctuations in T_w and $\dot{q}_w$ were found.

In section II.4.1 the Nusselt number for a thin film was derived, and was found to be 2.5 for a lamina steady film flow over a uniformly heated plate. By comparing this value of Nusselt number Nu to that corresponding to the measured HTCs from our film flows, it is possible to investigate whether the waves in the film flows result in enhanced or reduced heat transfer compared to Nusselt flow. It is expected that the waves will increase mixing, by inducing unsteadiness in the flow and turbulence at higher Re , enhancing heat transport and resulting in a higher Nu than that in Eq. II.13. Other effects may also occur, for example the unsteadiness of the waves may result in velocity profiles different

CHAPTER VIII. EXPERIMENTAL RESULTS IN FILM FLOWS

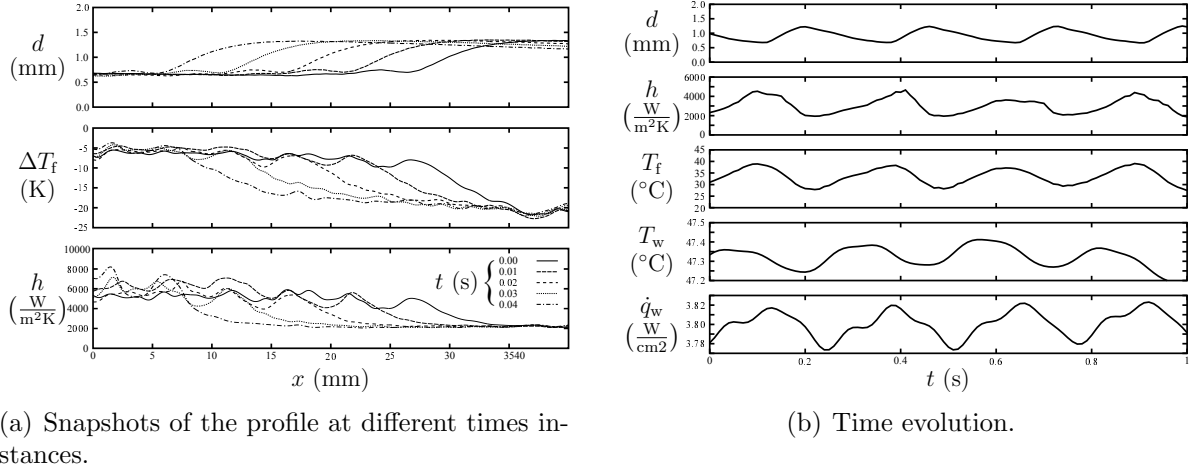


Figure VIII.20: Profile of a wave and the resulting heat transfer as the wave flows down plate the heated plate, for the flow conditions of $Re = 300$, $Pr = 5.0$ and $a_1^* = a/\delta_{s,1} = 0.056$.

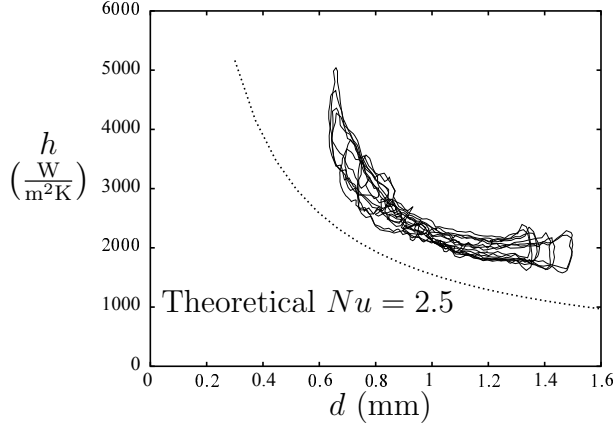
from those described by Eq. II.8, which in turn would result in a different temperature profile in the film. In any case, this comparison can give an indication of how well the investigated film flows can be described by the idealised steady-state Nusselt flow.

Fig. VIII.21 shows joint probability density function (PDF) contours of the instantaneous, local HTC h and corresponding instantaneous, local film thickness d at the measurement location for a particular flow condition. Contours of higher probability indicate that the HTC and film thickness were more likely to be in the region enclosed by that curve. A definite inverse relationship between the instantaneous HTC and the film thickness emerges. Further, the curves from each different foil heat input setting lie on top of each other, meaning that the measured HTC is independent of the heat flux applied¹. The independence of the HTC from the applied heat flux for a given flow condition is important as it validates the usefulness of the definition of a local and instantaneous HTC, and enables its meaningful application to the description (and the subsequent comparison) of heat transfer in different flow conditions².

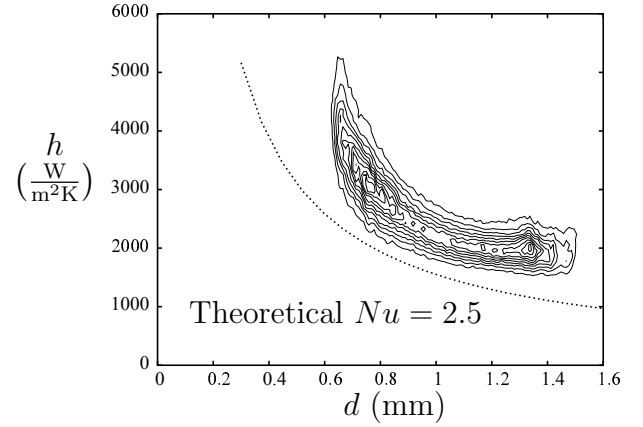
Also shown in Fig. VIII.21(a) and in the combined results in Fig. VIII.21(b) is the theoretical HTC obtained for Nusselt film flow, corresponding to $Nu = 2.5$ from Eq. II.13.

¹However, even though h has been shown to be independent of $\dot{q}_w$, the converse (not shown here) is not true; at a given flow condition ΔT_f and $\dot{q}_w$ are both dependent on h .

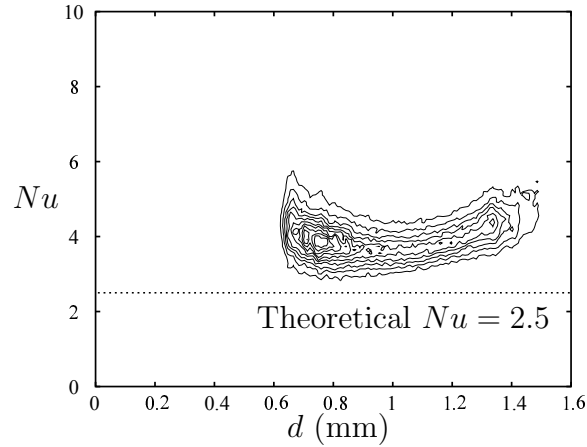
²There are instances when this independence can be lost, and the heat flux can affect the HTC. For example, this can occur if the heat flux is high enough to instigate boiling, or to give rise to significant gradients in the fluid properties (e.g. viscosity, or density such as in naturally convecting flows).



(a) Overlay of 50 % probability joint-PDF contours of HTC h and film thickness d , one contour extracted per run of $\bar{q}_w$ setting, with each contour signifying the region within which h and d lay for $\geq 50\%$ of the time in that particular run.



(b) Joint-PDF contours of h and d compiled over the whole set of 12 runs with different (uniform, steady) heat inputs to the foil substrate $\bar{q}_w$ from 200 W to 5 kW, but the same flow conditions. The data from all runs are combined.



(c) Joint-PDF of Nu and d corresponding to (b).

Figure VIII.21: Instantaneous HTC characteristics for a set of 12 runs with the same flow condition $Re = 300$, $Pr = 5.0$ and $a_1^* = 0.056$, but with different (uniform, steady) heat inputs $\bar{q}_w$ to the foil substrate over the range 200 W to 5 kW. Showing joint probability density functions (PDFs) of the dependence of instantaneous local h on the instantaneous local film thickness d .

CHAPTER VIII. EXPERIMENTAL RESULTS IN FILM FLOWS

The experimental data and the theoretical Nusselt result are in good agreement. The measured HTC follows closely an inversely proportional relationship with the film thickness, showing slight deviations when the films are at their smallest and largest thickness. Generally, the magnitude of the measured HTC is only slightly larger than that predicted by Nusselt theory, though the experiments do show consistently elevated values compared to the theoretical result. This is probably a direct consequence of increased mixing and transport within the films, from the passing of the wave fronts and the unsteadiness of the velocity profile in the fluid, leading to enhanced heat transfer relative to the purely steady and laminar Nusselt flow. A higher HTC would also be expected if the flow is still developing. The measurements were made a distance $\sim 300 d_0$ initial film thicknesses downstream of the inlet, whereas a standard relation for the thermal development length in the idealised Nusselt flow, $L_{th}/d = 0.03 \text{ Re Pr}$, gives an estimate of the development length of approximately $60 d$ fluid thicknesses.

Fig. VIII.21(c) shows the same data as Fig. VIII.21(b), but this time with the Nusselt number Nu instead of the HTC h plotted against d . Generally Nu is slightly greater than the theoretical Nusselt value of 2.5, with the difference between the two increasing when the film height is at its thickest, and at its thinnest. A possible explanation for the increased Nu when the film is thick may concern the increased flow (velocity component) in the normal direction to the wall and the increased space for mixing under the waves, and perhaps, an increase in the turbulence behind the wave front leading to a heat transfer enhancement. Conversely, the enhancement when the film is thin may be due to the modification of the temperature and velocity profiles in the film due to the unsteadiness in the flow [114], resulting in increased thermal gradients at the wall.

VIII.5.1 Analysis and modelling

Firstly, the analytical solution to 1-D conductive heat transfer in a finite solid domain is used to find a thermal impedance of the solid. The heat transfer in various 1-D convecting flows is then modelled by using this transfer function as a boundary condition to the fluid domain. Once a time-varying periodic HTC is imposed, the wall heat flux and temperatures are found numerically by simultaneously solving Newton's law of cooling, that describes the forced convective heat transfer in the fluid, and the aforementioned transfer function governing the heat conduction in the solid. Energy conservation in the fluid is used to calculate the evolving temperature of the fluid within the domain from an initial condition, and thus the transfer function relating the temperature of the

Heat Transfer Augmentation in the Film Flow

fluid and the heat flux at the wall. This model is then used to examine the effects of the dimensionless wall thickness $a^* = a/\delta_s$, the Biot number $\text{Bi} = \bar{h}a/k_s$, the HTC fluctuation intensity $h'' = h_\sigma/\bar{h}$, and the heat capacity ratio $C_R = (\rho_f c_f/\rho_s c_s)(d/a)$ on the heat transfer augmentation ratio,

$$A = \frac{\bar{q}_w}{\bar{h} \Delta T_f} = 1 + \frac{\overline{h' \Delta T_f'}}{\bar{h} \Delta T_f}. \quad (\text{VIII.1})$$

In this section we will investigate the predictive ability of the modelling framework developed in Chapter V with respect to the direct measurements of augmentation in the heated falling film flow experiments that were described above. To this end, the measured HTC in each experimental run, defined by a prescribed set of flow and heating conditions, is approximated by a pure sinusoid, $h = \bar{h} + \hat{h} \sin(\omega t)$, such that $\hat{h} = \sqrt{2}\sigma_h$, and the known parameters a^* , Bi , $h'' = h^*/\sqrt{2}$ and C_R from each experimental run are supplied the above model and the predicted and measured augmentation ratios A are compared.

In the film flow experiments it is expected that the temperature of the fluid will increase in the downstream direction as it contacts the hot substrate for longer, and also that perturbations in the fluid temperature will convect downstream while growing or decaying as seen in Case V in Chapter V. However, unlike Case V where the HTC was modelled as changing instantaneously but *uniformly* over the entire domain, here the HTC will convect, in phase, with the fluid waves as these travel down the foil. Due to this convection of the fluctuations in the HTC with the fluid, the coupling between the fluid temperature and the HTC will become stronger with increasing distance from the initial condition, i.e. the fluid inlet.

By taking the energy equation in the fluid (Eq. II.9) and integrating in the direction normal to the heated surface, the response of the bulk film temperature $T_{f,b}$ to a wall heat flux $\dot{q}_w$ can be approximated by,

$$\rho_f c_f d \frac{DT_{f,b}}{Dt} = -\dot{q}_w. \quad (\text{VIII.2})$$

If a constant flow velocity is assumed Eq. II.9 can be solved by the method of characteristics. The governing equation of the fluid film temperature is then found to be,

$$\frac{\partial T_{f,b}}{\partial c} = -\frac{h(t_0)}{\rho_f c_f d} \left(T_{f,b} - \rho_f c_f d \left[z(t_0) * \frac{\partial T_{f,b}}{\partial c} \right] \right), \quad (\text{VIII.3})$$

CHAPTER VIII. EXPERIMENTAL RESULTS IN FILM FLOWS

where c is a characteristic line of a fluid element and t_0 is the initial time when the fluid element was at the inlet. Since the HTC is modelled as convecting with the fluid element it is only a function of t_0 .

Eq. VIII.3 is nonlinear due to the convolution with the solid response $z(t_0)$, which is the time-domain representation of the thermal impedance of the solid $Z(\omega_n)$. The equation can be solved numerically, though this is increasingly time consuming for regions a long way from the fluid inlet. Still, numerical experimentation reveals that the thermal solution of the fluid will start as being a response representative of an isothermal condition on the *inner* wall, in which $T'_w = 0$ at the inlet (a stiff response), and tend to a response representative of an isoflux (but not adiabatic, $\bar{q}_w \neq 0$) condition on the *inner* wall, in which $\dot{q}'_w = 0$ when the flow is fully thermally developed (a soft response). It is also found that a reasonable approximation can be made for the transfer function of the fluid by taking the transfer function from Case V (see Chapter V) and adding a scale factor to the C_R to match this change in response. The modified fluid temperature response becomes,

$$T_{f,b} = \bar{T}_{f,b} + \frac{\text{Bi } a_1^*}{2C_R U^*} \frac{x}{a} \Re \left\{ \frac{1}{i} \sum_n \frac{1}{Z_n^* a_n^*} T_{w,n} e^{i\omega_n t} \right\}, \quad (\text{VIII.4})$$

where the Biot number is based on the time-averaged HTC such that $\text{Bi} = \bar{h}a/k_s$, the dimensionless fluid velocity is $U^* = (\nu_f/\omega d^2)\text{Re}$ and $\text{Re} = U_b d/\nu_f$, the dimensionless foil thickness for a given harmonic is $a_n^* = a/\delta_{s,n}$, and the heat capacity ratio between the fluid film and the foil is $C_R = (\rho_f c_f/\rho_s c_s)(d/a)$. Also note that the thermal impedance of the solid has been normalised by the mean HTC to give $Z_n^* = \bar{h}Z(\omega_n)$.

The heat transfer problem can now be solved to calculate the time-varying inner wall temperature T_w and heat flux $\dot{q}_w$, given an prescribed fluctuation in the HTC. As the set of equations is nonlinear, a numerical method is employed to find the solution, in which the variables are discretised at regular steps in the time domain. First an arbitrary wall temperature, T_w , is picked across the time domain. The fluid temperature T_f is then found from the fluid domain response in Eq. VIII.4 using this wall temperature. Furthermore the wall heat flux, $\dot{q}_w$, can be calculated from Newton's law of cooling, using the known imposed HTC and the temperature difference across the fluid film, $\Delta T_f = T_f - T_w$. Similarly the wall heat flux, $\dot{q}_w$, can also be calculated from the solid response in Eq. IV.3, using a fast Fourier transform to switch between the frequency and time domains. A nonlinear multidimensional equation solver can be used to solve for T_w so that the heat fluxes calculated by both methods are equivalent. Any nonlinear equation solver can be used and many computational libraries are capable of implementing fast

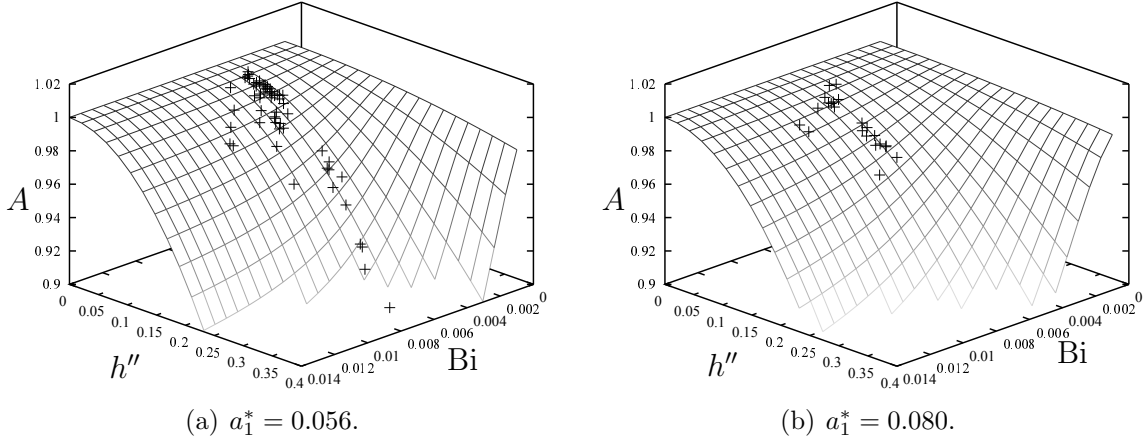


Figure VIII.22: A view of the augmentation domain with experimental points.

algorithms for such solvers. An Euler stepping method can be slow and possibly unstable, so a Newton–Raphson like method was used, as implemented by Matlab’s inbuilt `fsolve` function which chooses from a selection of algorithms (Fortran MINPACK library). Once a solution for T_w has been found $\dot{q}_w$ and T_f can be calculated as before.

Figure VIII.22 shows individual data points (one per experimental run) from two sets of experiments with different dimensionless foil thickness, or different fundamental (forcing) wave frequencies $a_1^* = a/\delta_{s,1}$. The measured nonlinear augmentation ratio A is displayed against the HTC fluctuation intensity $h'' = h_\sigma/\bar{h}$, and the Biot number Bi based on the mean HTC magnitude $\bar{h}$. A surface of the predicted augmentation from our theoretical model is also shown. The points generally follow the surface trend described by the model. We observe that, in the film flow experiments, an augmentation effect up to a 10 % is observed when the Biot number Bi and h'' are at their largest respective values. The values of h'' and Bi were varied by the liquid film mean flow-rate and pulsation amplitude setting in each run. On comparing the two plots, one must consider that the amplitude of the waves was attenuated at higher frequencies (i.e. higher a_1^*), leading to depreciated fluctuation intensities at the higher wave frequencies.

A better picture of the augmentation behaviour predicted by the model, and the region of the parameter space that has been spanned by the experimental data is shown in Fig. VIII.23. Interestingly, the model predicts that at high Bi the degree of augmentation is dependent only on the HTC fluctuation intensity h'' , and becomes independent of the mean Bi . In this region of the map the wall heat flux and also the wall temperature are constant, such that the heat transfer problem is predominantly *non-conjugate*. The

CHAPTER VIII. EXPERIMENTAL RESULTS IN FILM FLOWS

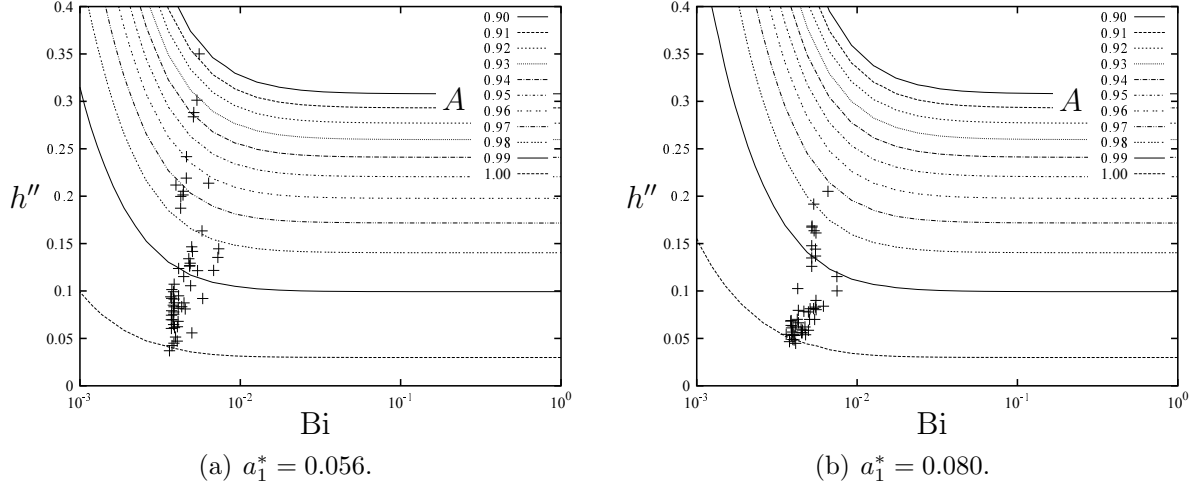


Figure VIII.23: Augmentation contours as a function of $Bi = \bar{h}a/k_s$ and $h'' = h_\sigma/\bar{h}$, with the positions of experimental data shown.

response of the augmentation ratio tends to that described by Eq. VIII.7, which was derived for a constant inner wall heat flux (isoflux condition). This arises in these thermally developing flows due to the high values of the Bi , which result in a strong coupling between the fluid temperature and the HTC at the measurement point, far downstream of the inlet, that act to constrain the fluctuations in the foil heat flux. At lower Bi the heat transfer problem becomes increasingly conjugate, with the wall temperature and heat flux responding increasingly to the fluctuations in the fluid. As the constraint on the foil heat flux is gradually removed, the increased conjugation gives rise to greater fluctuating foil temperatures and heat fluxes. This manifests in Fig. VIII.23 as a rise in the contours of constant A , which implies that larger HTC fluctuation intensities h'' are required for the same level of A . The value of Bi at which this transition to conjugation occurs depends on the foil thickness and wave frequency. In the lower wave frequency data in Fig. VIII.23(a) the transition occurs at lower Bi than in the higher wave frequency data in Fig. VIII.23(b). In both cases the data from the experiments are straddling this transitional region, with $4 \times 10^{-3} < Bi < 9 \times 10^{-3}$.

The augmentation ratio A is examined in terms of Bi in Fig. VIII.24. Experimental data points (denoted by '+') are shown next to the modelled points (denoted by 'o'). The latter required the input of the values of h'' and Bi , which were taken directly from the experiments. Generally a very good agreement is found between the experimental and predicted augmentation ratios. The value of A decreases gradually with increasing h'' , falling to about 90 % in the experiments with the smaller dimensionless foil thickness

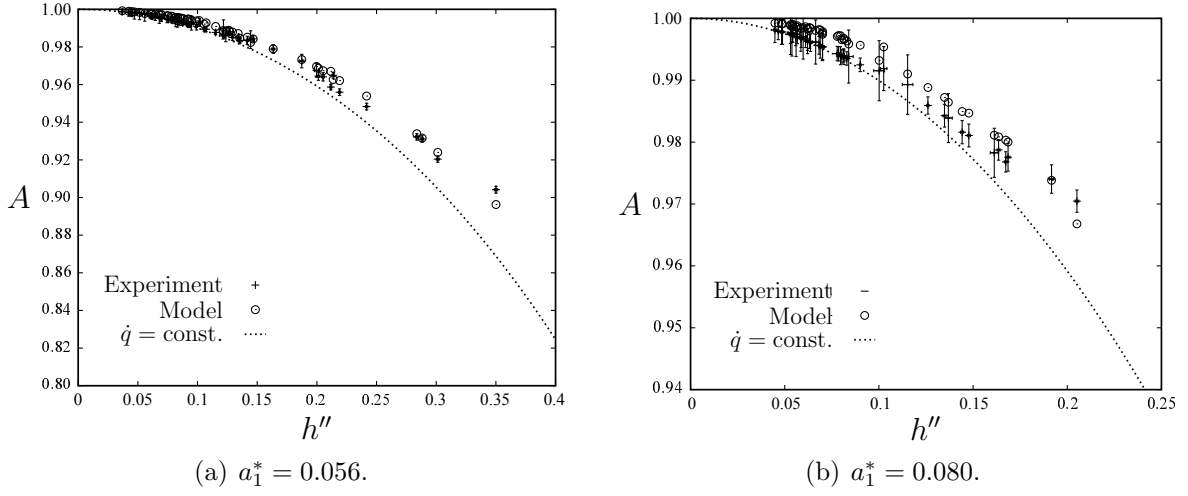


Figure VIII.24: Measured augmentation effect against h'' including the corresponding modelled points.

$a_1^* = 0.056$, and 97 % in the experiments with the larger dimensionless foil thickness $a_1^* = 0.080$. The theoretical prediction indicates that the deviation from unity is accelerated at increasingly higher h'' , suggesting that the accurate and reliable prediction of the time-averaged heat transfer in strongly unsteady flows with large variations in the HTC must consider the contribution of augmentation.

Finally, as noted in Section VIII.5, the heat flux fluctuations were relatively small, although still finite, at the measurement position, which in these experiments was chosen to be $\sim 300 d_0$ downstream of the inlet. In the long length limit, when the film flow becomes fully thermally developed, the inner wall heat flux fluctuations will tend to zero, and the inner wall heat flux a steady value. Consider this case, when the wall heat flux has become constant: $\dot{q}'_w = 0$, $\dot{q}_w = \bar{\dot{q}}_w$. In this case, the fluctuations in the wall-fluid temperature difference ΔT_f are completely dependent on the HTC. The augmentation ratio A then becomes,

$$A = \frac{\bar{\dot{q}}_w}{\bar{h} \Delta T_f} = \frac{\bar{\dot{q}}_w}{\bar{h} (\dot{q}_w/h)} = \frac{1}{\bar{h} (1/h)} = \frac{1}{\int h P(h) dh \int P(h)/h dh}. \quad (\text{VIII.5})$$

Assuming a sinusoidal HTC, $h = \bar{h} + \hat{h} \sin(\omega t)$, and using the binomial expansion of Eq. VIII.5 gives,

$$\frac{1}{A} = \frac{1}{1 - h^* \sin(\omega t) + h^{*2} \sin^2(\omega t) - \dots}, \quad (\text{VIII.6})$$

CHAPTER VIII. EXPERIMENTAL RESULTS IN FILM FLOWS

where $h^* = \hat{h}/\bar{h}$ is the normalised HTC amplitude. By evaluating the mean over one cycle Eq. VIII.6 becomes,

$$\frac{1}{A} = 1 + \frac{h^{*2}}{2} + \sum_n \frac{1}{2^{2n}} \binom{2n}{n} h^{*2n}; \quad n = 2 \dots \infty \quad (\text{VIII.7})$$

The series in Eq. VIII.7 will converge for $h^* < 1$, and is shown as a function of $h'' = h^*/\sqrt{2}$ in Fig. VIII.24. This relation will be considered when the modelled and measured augmentation ratios are compared.

Retuning to Fig. VIII.24, we can now compare the actual experimental results, and hence also the similar theoretical results from the full conjugate model, to the simplified model above which assumes $\dot{q}'_w = 0$ thus leading to Eq. VIII.7. A small difference can be identified between the measured values of A , and the corresponding values obtained for the limiting case described by Eq. VIII.7. Specifically, this perfectly isoflux inner wall approximation always leads to a notable overprediction of the augmentation effect, or an underprediction of the value of A , even though the amplitude of the heat flux fluctuations $\dot{q}'_w$ on the inner wall was confirmed by the measurements to amount to only 0.6 % of $\bar{\dot{q}}_w$, as stated in Section VIII.5. Recall that deviations of A from unity signify a greater augmentation effect. Hence, one may conclude that, depending on the accuracy required, it may be necessary to solve the full conjugate problem for the prediction of the heat exchange characteristics, even in problems with limited conjugation.

VIII.6 Summary

The problem considered was that of a fluid film flowing over a heated flat incline. In the case of steady flow conditions (without forced waves) the formation of rivulet structures were observed on the film surface. As the film flowed down the plate these structures first became apparent in the temperature field of the films surface but later in the flow, manifested a deformation of the film interface. As the thermal structure was observed in the absence of the interface deformation, at location close to the flow inlet, it was deduced that the thermal non-uniformity was driving the flow and interface deformation was a secondary artifact.

These thermal structures were initially long thin and faint lines of warm surface fluid which then opened out into a hotter region of fluid which had a Gothic arch like shape. The span-wise temperature profile in the hotter region was triangular in shape with a peak value of temperature at the centre of the region. A depression in the film

Summary

interface thickness was associated with the higher surface temperatures. However at locations closer to the film inlet little or no deformation was observed despite observing the triangular temperature profile.

It was deduced that aforementioned structures were the result of Marangoni driven flow. This effect drives flow at the surface of the film away from the region of higher surface temperature as the surface tension of the interface is reduced there. This Marangoni flow has the effect of pulling fluid up from the foil wall (Increasing heat transfer), and forcing a smaller cross flow close to the wall towards the location of peak temperature on the surface. Such flows result in an elevation of the foil temperature, directly under the location of peak temperature on the films surface, as fluid has travelled a greater distance along the heated wall. Additionally the effect reinforces itself as the upwelling increases the local surface temperature by convecting fluid from the hotter wall. It is the migration of the fluid on the films surface away from the peak temperature which eventually leads to the depression or trough in the interface height. leaving rivulets on either side. It is thought that the action of gravity delays the formation of the trough. An additional effect observed was that at the larger heat fluxes, but prior to film breakdown, the positions of the rivulets troughs oscillated up and down the film.

Waves would form on the film interface when forced at the inlet with an oscillation in flow rate (forced), or under certain flow rates and inclination angles (natural). These waves resulting in visible mixing of the rivulet structures mentioned previously. The mixing and the unsteady motion of the waves gave rise to fluctuations in the heat transfer coefficient, with fluctuation intensities h'' of up to 35 %.

The heat transfer coefficient was found to have an inverse relationship with film thickness as expected from Nusselt theory. However, the measured Nusselt number was higher than that predicted by the idealised Nusselt flow (i.e. fully developed, completely steady and laminar flow). The higher Nusselt number was expected and observed by others [113, 114] and is due to the increased mixing and unsteady, non parabolic flow profile in the wavy film. Nusselt numbers of 2.8 were experienced in the case of the steady film flows and 4 in the case of the forced wavy flows.

It was found that the heat flux fluctuation at the measurement location furthest from the inlet ($y = 285$ mm) was low, but finite. This resulted due to the coupling of the fluid temperature to the fluctuations in the heat transfer coefficient as the fluid convected with the fluctuations in the heat transfer caused by the waves.

The simple semi-analytical 1-D model from Case V of conjugate heat transfer with a time-varying heat transfer coefficient was applied to the film flow problem. In generally,

CHAPTER VIII. EXPERIMENTAL RESULTS IN FILM FLOWS

agreement was found between the measured and predicted augmentation ratio. The augmentation ratio was always negative, resulting in a reduction in heat transfer with respect to the steady case of up to 10 %. It was observed that the augmentation ratio depended on both the amplitude of the fluctuation of the heat transfer coefficient and, to a lesser extent, the Biot number. The region of greatest (negative) augmentation occurring at high values of the Biot number and high heat transfer coefficient fluctuation intensities. An estimate of augmentation ratio for a given heat transfer coefficient fluctuation was also calculated under the assumption of a perfectly isoflux inner wall condition. However, this assumption consistently over predicted the augmentation effect, despite the fact that inner wall heat flux fluctuations were confirmed by the measurements as being small. It can be inferred that the convective–conductive model correctly accounts for fluctuations in fluid or solid temperatures, and the heat transfer coefficient.

Chapter IX

Conclusions

In this thesis we developed a framework for the modelling of unsteady conductive–convective heat transfer, and for the evaluation of the resulting augmentative and conjugate heat effects. This was incorporated into models of various convective flow problems and thermodynamic cycles to predict their behaviour and performance. Finally based on this work a novel experiment was conducted to measure the instantaneous heat transfer coefficient on a fluid film flowing down a heated inclined surface. These experimental results were then compared to the augmentation predictions from the models.

We derived the 1-D transfer functions and subsequent impulse response of thermal conduction through a solid in Chapter III. This 1-D model is adequate for cases when the solid domain can be considered thin, that is when,

$$\frac{\partial T_s}{\partial x} \ll \frac{\partial T_s}{\partial y}. \quad (\text{IX.1})$$

The heat transfer in the fluids was then modelled using either an unsteady heat transfer coefficient, in the case of the unsteady convective flows, or a complex Nusselt number in the case of the gas spring and NIFTE. Matching and solving for a convective boundary condition, based on Newton’s law of cooling was conducted for a number of convective flow cases in Chapter V.

Maps of augmentation were produced for changing material properties, thicknesses and Biot numbers. In general it was found that the region of greatest (negative) augmentation occurred at high Biot numbers, when the solid walls were thin relative to the unsteady thermal diffusion length scale (e.g. this would occur at low frequencies), when the fluctuations in the heat transfer coefficient were large compared to its mean and when

CHAPTER IX. CONCLUSIONS

the heat capacity of the fluid domain was smaller to that of the solid (e.g. for thin films). The augmentation effect was negative in these cases, meaning a loss in ability to transfer heat in the mean, i.e. a reduction in the time-averaged heat flux relative to that expected from the multiple of the time-averaged heat transfer coefficient and the time-averaged temperature difference in the fluid. It is proposed that these maps can be used to chose material properties, or the solid geometry, to avoid or promote augmentation (i.e. heat transfer) as necessary.

As part of the investigation the thermal response of the fluid to unsteady heat exchange with the solid was derived for developing flows. It was found in this case that augmentation can be positive in some spatial regions, which is a new finding not reported in previous related studies. The underlying mechanism that gives rise to this effect was investigated. The enhanced heat transfer was found to arise by the convection of fluid along a developing thermal field which allowed, in some regions, the fluctuations in fluid temperature to be advanced in phase with respect to the fluctuations in the heat transfer coefficient (e.g. due to the flow). This finding is of great interest and can be utilised in heat exchanger designs, by tuning their length in order to promote greater net heat transfer or a smaller heat exchanger

Rule of thumb limits which are valuable for engineering purposes were deduced, from the model in Case V, concerning conditions in which augmentation effects become negligible. For an isothermal outer wall B/C augmentation is insignificant when (approximately),

$$\text{Bi} < 0.1 \text{ or } \text{Bi} < 0.3 a^* , \quad (\text{IX.2})$$

and for the isoflux outer wall B/C approximately when,

$$\text{Bi} < a^{*2} \text{ and } \text{Bi} < 0.3 a^* . \quad (\text{IX.3})$$

For a fully developed convective flow with isoflux boundary condition, $\bar{q}_f$, and sinusoidal h the augmentation factor will tend to,

$$A = \left(1 + \frac{h'^2}{2} \right)^{-1} . \quad (\text{IX.4})$$

The same approach from Chapter III and Chapter V was used to model two different thermodynamic cycles. First a gas spring was modelled. The cylinder walls were modelled as isothermal, as has been done historically [8] however using our 1-D transfer response of a solid, a thin layer of “insulating” material was placed on the inside of the cylinder.

It was found that at low Péclet numbers the addition of an insulating layer exacerbated the thermal loss of the spring as it moved the system away from the isothermal ideal. Conversely at high Péclet numbers thicker insulating layers reduced the loss as they moved the system closer to the adiabatic ideal. It was also found that there was an intermediate thickness of material thickness which maximised the loss in the gas spring, by up to 20 % of the nominal maximum loss for an isothermal cylinder lining. In the case of the NIFTE the addition of the parasitic cyclic heat transfer between the vapour volume and the solid walls in the device model resulted in a significant improvement in the prediction of the device efficiency over previous models, with close agreement to experimental data.

Flow measurements were conducted on a fluid film flowing down an inclined electrically heated incline, to investigate the effects of unsteady heat transfer. These wavy film flows exhibit large and periodic fluctuations in heat transfer and the frequency and amplitude of the waves was controlled by a specially constructed flow preparation arrangement. Additionally it was expected that interesting Marangoni effects would be observed. To enable the temperature and heat flux measurements the heated incline consisted of a 50 μm thick titanium foil.

A novel technique was presented in Chapter IV to measure the film interface height, film temperature and instantaneous heat flux through the foil. This was achieved with a combination of Laser Induced Florescence (LIF) measurements, using a Rhodamine B dye in the fluid film, and Infrared (IR) measurements of the underside of the foil. Temperature measurements were possible with the LIF technique due to temperature dependence of the florescence emission of Rhodamine B. Both measurements were undertaken simultaneously and were geometrically aligned to each other. The LIF Rhodamine B measurements were used to determine film height and temperature whereas the IR measurements were used to measurement the temperature and instantaneous heat flux at the interface between the fluid film and the heated foil. The temperature sensitivity of the LIF technique was found to be 0.6 K whereas the sensitivity of the IR temperature and heat flux measurements were dependant on the forcing frequency and amplitude (see Section IV.6 and Section IV.8.1). In Section VIII.1.1 the LIF temperature measurements were shown to be comparable to IR measurements taken on the films surface.

The results from these film flow measurements were explored in Chapter VIII. In the case of steady flow conditions (without forced waves) the formation of rivulet structures were observed on the film surface. As the film flowed down the plate these structures first became apparent in the temperature field of the films surface but later in the flow, manifested a deformation of the film interface. These thermal structures were initially

long thin and faint lines of warm surface fluid which then opened out into a hotter region of fluid which had a Gothic arch like shape. The span-wise temperature profile in the hotter region was triangular in shape with a peak value of temperature at the centre of the region. A depression in the film interface thickness was associated with the higher surface temperatures. However at locations closer to the film inlet little or no deformation was observed despite observing the triangular temperature profile.

It was deduced that aforementioned structures were the result of Marangoni driven flow. This effect drives flow at the surface of the film away from the region of higher surface temperature as the surface tension of the interface is reduced there. This Marangoni flow has the effect of pulling fluid up from the foil wall, and forcing a smaller cross flow close to the wall towards the location of peak temperature. Such flows result in an elevation of the foil temperature as fluid has travelled a greater distance along its surface. Additionally the effect reinforces itself as the upwelling increases the local surface temperature. It is the migration of the fluid on the films surface away from the peak temperature which eventually leads to the depression or trough in the interface height. leaving rivulets on either side. It is thought that the action of gravity delays the formation of the trough. An additional effect observed was that at the larger heat fluxes, but prior to film breakdown, the positions of the rivulets troughs oscillated up and down the film.

Waves would form on the film interface when forced at the inlet with an oscillation in flow rate (forced), or under certain flow rates and inclination angles (natural). These waves resulting in visible mixing of the rivulet structures mentioned previously. The mixing and the unsteady motion of the waves gave rise to fluctuations in the heat transfer coefficient, with fluctuation intensities h'' of up to 35 %.

The heat transfer coefficient was found to have an inverse relationship with film thickness as expected from Nusselt theory. However, the measured Nusselt number was higher than that predicted by the idealised Nusselt flow (i.e. fully developed, completely steady and laminar flow). The higher Nusselt number was expected and observed by others [113, 114] and is due to the increased Nusselt number and unsteady, non parabolic flow profile in the wavy film. Nusselt numbers of 2.8 were experienced in the case of the steady film flows and 4 in the case of the forced wavy flows.

Finally the simple semi-analytical 1-D model from Case V of conjugate heat transfer with a time-varying heat transfer coefficient was applied to two different non-isothermal or diabatic problems; The aforementioned fluid film falling down a heated planar incline (Section VIII.5), and the separated and re-attached flow of a turbulent boundary layer behind a backwards-facing step (Chapter VII). The model predictions of the augmenta-

tion effects in these problems were then compared to actual experimental results from each case. Such simple and efficient models allow for an understanding of the dominant underlying mechanisms of the heat transfer problem, and allow the effects of a wide range of materials selection and geometry parameters to be evaluated quickly and reliably for engineering design.

In the case of the film flow it was found that the heat flux fluctuation at the measurement locations were low, but finite. This resulted due to the coupling of the fluid temperature to the fluctuations in the heat transfer coefficient as the fluid convected with (and at the speed as) the fluctuations in the heat transfer caused by the waves. A good agreement was found between the measured and predicted augmentation ratio. The augmentation ratio was always negative, resulting in a reduction in heat transfer with respect to the steady case of up to 10 %. At the range of values of Biot number and dimensionless solid thickness that the experiment was performed, it was observed that the augmentation ratio depended on both the amplitude of the heat transfer coefficient and, to a lesser extent, the Biot number. Under the assumption of a perfectly isoflux inner wall condition, an augmentation ratio for a given heat transfer coefficient was also calculated. However, this assumption consistently over predicted the augmentation effect, despite the fact that inner wall heat flux fluctuations were confirmed by the measurements as being small. It can be inferred that the model correctly accounts for fluctuations in fluid or solid temperatures, and the heat transfer coefficient.

We have produced and demonstrated a simple and viable framework for modelling a range of unsteady heat transfer problems including; simple convecting flows, gas springs, vapour cycles, and wavy film flows. In addition a novel measurement technique was devised to measure spatiotemporal resolved heat transfer measurements in unsteady flows (in this case a periodic wavy film flow). Results from this experiment and others were used to check the predictions of the framework model. Generally there was very good agreement between the experimental and predicted augmentation ratios or cycle efficiencies.

Over the course of this work, a large set of recorded data has been generated (1 TB of image files over 1300 experimental runs) which are yet to be fully analysed. It is hoped that with some further work, using the data gathered here, the effects observed in the film flows can be fully characterised. Of interest at this stage would be the characteristics of the novel measurement technique developed in this thesis, as well as a deeper consideration of the augmentative effects in film flows. The simplistic models from Chapter V predicted that augmentative effects would be largest at the inlet to the fluid domain. It

is therefore of great importance to investigate the unsteady heat transfer process closer to the distribution box.

Fluid film flows are an exciting topic of research and there are currently many developments in the measurement of the highly interesting interfacial phenomenon associated with them. In this thesis we did not investigate in great detail these surface tension driven flows as we were primarily concerned with fluctuating and periodic heat transfer effects. However a future extension to the film flow experiments should investigate these effects and flow regimes further. In the current work a number of different fluids were used, primarily pure water, water ethanol and water glycerol mixtures. These working fluids exhibit temperature dependent surface tension variations either directly or due to differential evaporation of the binary mixture. These fluids also exhibited a tendency to de-wet the substrate, which made working at larger heat fluxes difficult. Work should be conducted to find a temperature sensitive fluorescent dye which is compatible with non-water-based fluids such as paraffin and silicone oils. These fluids will exhibit different surface tension characteristics, substrate wettability and fluid viscosities which will change the fluid flow regime.

At a more involved level, the test section should be fully enclosed, and sealed, to allow for flows with a greater degree of evaporation, or even condensation at the film surface to be studied. Filling such an enclosure with vapour from the working fluid would also effectively allow an isothermal temperature condition to be imposed on the film interface. The range of geometries studied should also be expanded, including inverting the flow and annular geometries. The inverted geometry was tested briefly with the current flow section, and gave rise to some interesting flow patterns. However, due to time constraints, it was not possible to make the flow arrangement workable (Significant leakage and dewetting problems manifested) and so no measurements were made in this configuration. Additionally it would be interesting to conduct PIV measurements so that the velocity distribution in the wavy flow can be ascertained simultaneously with the thermal profile through the film.

Finally we will extend the work on the thermal losses in gas springs to losses in energy storage systems with reciprocating components. One such machine is currently under development by Isentropic Ltd [181]. The round trip performance of these pumped thermal energy storage systems is highly dependent on the efficiency of the compressors and expander components [10, 180, 181]. We intend to measure the gas temperature, pressure and wall heat flux throughout the cycle in an experimental test section and to compare these results to predictions from 2D and 3D CFD models. In the Film flows experiments

the heat transfer coefficient was measured directly, whereas when we considered the gas spring problem the heat transfer coefficients was modelled from correlations in the literature. An interesting approach will be to instead evaluate these heat transfer coefficients from the direct measurements mentioned above.

References

- [1] Goodfellow. Goodfellow material properties, 2012. URL www.goodfellow.com/PDF/TAB001E.pdf.
- [2] R. Wälchli, T. Brunschwiler, B. Michel, and D. Poulikakos. Self-contained, oscillating flow liquid cooling system for thin form factor high performance electronics. *Journal of Heat Transfer*, 132:051401, 2010.
- [3] K. Gi, S. Maezawa, Y. Kojima, and N. Yamazaki. Cpu cooling of notebook pc by oscillating heat pipe. In *Proceedings of the 11th International Heat Pipe Conference, Tokyo, Japan*, pages 166–169, 1999.
- [4] V.G. Pastukhov, Y.F. Maidanik, C.V. Vershinin, and M.A. Korukov. Miniature loop heat pipes for electronics cooling. *Applied Thermal Engineering*, 23(9):1125–1135, 2003.
- [5] Y. Chen, E. Luo, and W. Dai. Heat transfer characteristics of oscillating flow regenerator filled with circular tubes or parallel plates. *Cryogenics*, 47(1):40–48, 2007.
- [6] A.J. Jaworski and A. Piccolo. Heat transfer processes in parallel-plate heat exchangers of thermoacoustic devices—numerical and experimental approaches. *Applied Thermal Engineering*, 2012.
- [7] K.P. Lee. A simplistic model of cyclic heat transfer phenomena in closed spaces. In *IECEC’83; Proceedings of the Eighteenth Intersociety Energy Conversion Engineering Conference*, volume 1, pages 720–723, Orlando, FL, USA, August 1983. 21-26.
- [8] A.A. Kornhauser and J.L. Smith Jr. The effects of heat transfer on gas spring performance. *Journal of Energy Resources Technology*, 115:70–75, 1993.

REFERENCES

- [9] B. Lawton. Effect of compression and expansion on instantaneous heat transfer in reciprocating internal combustion engines. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 201(3):175–186, 1987.
- [10] A.J. White. Thermodynamic analysis of the reverse joule-brayton cycle heat pump for domestic heating. *Applied Energy*, 86(11):2443–2450, 2009.
- [11] C.N. Markides. The role of pumped and waste heat technologies in a high-efficiency sustainable energy future for the uk. *Applied Thermal Engineering*, 2012.
- [12] M.A. Duran and I.E. Grossmann. Simultaneous optimization and heat integration of chemical processes. *AIChE Journal*, 32(1):123–138, 1986.
- [13] T.C. Hung, T.Y. Shai, and S.K. Wang. A review of organic rankine cycles (orcs) for the recovery of low-grade waste heat. *Energy*, 22(7):661–667, 1997.
- [14] T.C.B. Smith. Power dense thermofluidic oscillators for high load applications. In *Proceedings of the 2nd International Energy Conversion Engineering Conference (IECEC)*. Providence, RI, USA, 2004.
- [15] T.C.B. Smith. *Thermally driven oscillations in dynamic applications*. PhD thesis, PhD thesis. Cambridge: University of Cambridge, UK, 2006.
- [16] J.G. Wood and N. Lane. Advanced 35 w free-piston stirling engine for space power applications. In *AIP Conference Proceedings*, volume 654, pages 662–667, Albuquerque, NM, USA, February 2003.
- [17] T. Yamamoto, T. Furuhashi, N. Arai, and K. Mori. Design and testing of the organic rankine cycle. *Energy*, 26(3):239–251, 2001.
- [18] M.R. Darby. The price of oil and world inflation and recession. *The American Economic Review*, 72(4):738–751, 1982.
- [19] M.A. Adelman. User cost in oil production. *Resources and Energy*, 13(3):217–240, 1991.
- [20] R. Hirsch, R. Bezdek, and R. Wendling. Peaking of world oil production. In *Proceedings of the IV International Workshop on Oil and Gas Depletion*, pages 19–20, 2005.
- [21] D. Yergin. Ensuring energy security. *Foreign Affairs*, pages 69–82, 2006.

REFERENCES

- [22] Y.B. Zudin. *Theory of periodic conjugate heat transfer*. Springer, 2011.
- [23] L. He and M.L.G. Oldfield. Unsteady conjugate heat transfer modeling. *Journal of Turbomachinery*, 133:031022, 2011.
- [24] F. De Monte. Transient heat conduction in one-dimensional composite slab. a ‘natural’ analytic approach. *International Journal of Heat and Mass Transfer*, 43(19):3607–3619, 2000.
- [25] F. De Monte. An analytic approach to the unsteady heat conduction processes in one-dimensional composite media. *International Journal of Heat and Mass Transfer*, 45(6):1333–1343, 2002.
- [26] A. Piccolo and G. Pistone. Estimation of heat transfer coefficients in oscillating flows: The thermoacoustic case. *International Journal of Heat and Mass Transfer*, 49(9):1631–1642, 2006.
- [27] A. Sorin, F. Bouloc, B. Bourouga, and P. Anthoine. Experimental study of periodic heat transfer coefficient in the entrance zone of an exhaust pipe. *International Journal of Thermal Sciences*, 47(12):1665–1675, 2008.
- [28] A.A. Kornhauser and J.L. Smith. Application of a complex nusselt number to heat transfer during compression and expansion. *Journal of Heat Transfer*, 116(3): 536–542, 1994.
- [29] H. Nakamura. Spatio-temporal measurements of convective heat transfer for a backward-facing step flow. *ICHMT Digital Library Online*, 2009.
- [30] R. Solanki, R. Mathie, A. Galindo, and C.N. Markides. Modelling of a two-phase thermofluidic oscillator: effects of irreversible thermal loss. *Applied Energy*, 2012.
- [31] R.V. Craster and O.K. Matar. Dynamics and stability of thin liquid films. *Reviews of Modern Physics*, 81(3):1131, 2009.
- [32] H.C. Chang, E.A. Demekhin, and D.I. Kopelevich. Nonlinear evolution of waves on a vertically falling film. *Journal of Fluid Mechanics*, 250:433–433, 1993.
- [33] O.A. Kabov, B. Scheid, I.A. Sharina, and J.C. Legros. Heat transfer and rivulet structures formation in a falling thin liquid film locally heated. *International Journal of Thermal Sciences*, 41(7):664–672, 2002.

REFERENCES

- [34] T.C.B. Smith. Asymmetric heat transfer in vapour cycle liquid-piston engines. stirling engine. In *ISEC 2005: Proceedings of the 12th International Stirling engine Conference and technology Exhibition; 2005 Sept 7*, volume 9, pages 302–314, 2005.
- [35] C.N. Markides and T.C.B. Smith. A dynamic model for the efficiency optimization of an oscillatory low grade heat engine. *Energy*, 36:6967–6980, 2011.
- [36] A. Schagen, M. Modigell, G. Dietze, and R. Kneer. Simultaneous measurement of local film thickness and temperature distribution in wavy liquid films using a luminescence technique. *International Journal of Heat and Mass Transfer*, 49(25): 5049–5061, 2006.
- [37] O.V. Kabov, I.V. Marchuk, and V.M. Chupin. Thermal imaging study of the liquid film flowing on vertical surface with local heat source. *Russian Journal of Engineering Thermophysics*, 6:105–138, 1996.
- [38] T.S. Zhao and P. Cheng. Oscillatory heat transfer in a pipe subjected to a laminar reciprocating flow. *Transactions-American Society of Mechanical Engineers Journal of Heat Transfer*, 118:592–595, 1996.
- [39] P. Rajesh Kanna and M.K. Das. Conjugate heat transfer study of backward-facing step flow—a benchmark problem. *International Journal of Heat and Mass Transfer*, 49(21):3929–3941, 2006.
- [40] S.J. Thorpe, S. Yoshino, R.W. Ainsworth, and N.W. Harvey. An investigation of the heat transfer and static pressure on the over-tip casing wall of an axial turbine operating at engine representative flow conditions.(ii). time-resolved results. *International Journal of Heat and Fluid Flow*, 25(6):945–960, 2004.
- [41] C.W. Stammers. The operation of the fluidyne heat engine at low differential temperatures. *Journal of Sound and Vibration*, 63(4):507–516, 1979.
- [42] R. Solanki, A. Galindo, and C.N. Markides. The role of heat exchange on the behavior of an oscillatory two-phase low-grade heat engine. *Applied Thermal Engineering*, 2012.
- [43] H.R. Engelhorn and A.M. Reinhart. Investigations on heat transfer in a plate evaporator. *Chemical Engineering and Processing: Process Intensification*, 28(2): 143–146, 1990.

REFERENCES

- [44] J. Lutisan, J. Cvengros, and M. Micov. Heat and mass transfer in the evaporating film of a molecular evaporator. *Chemical Engineering Journal*, 85(2-3):225–234, 2002.
- [45] P. Hoffman. Plate evaporators in food industry—theory and practice. *Journal of Food Engineering*, 61(4):515–520, 2004.
- [46] F. Kafi, V. Renaudin, D. Alonso, and JM Hornut. New med plate desalination process: thermal performances. *Desalination*, 166:53–62, 2004.
- [47] W. Nusselt. Die oberflächenkondensation des wasserdampfes the surface condensation of water. *Zetrschr. Ver. Deutch. Ing.*, 60:541–546, 1916.
- [48] P.L. Spedding. Falling film flow. *Thermapedia*, 2011. URL http://dx.doi.org/10.1615/AtoZ.f.falling_film_flow.
- [49] F.P. Incropera, T.L. Bergman, A.S. Lavine, and D.P. DeWitt. *Fundamentals of heat and mass transfer*. Wiley, 2011.
- [50] S.S. Kutateladze. *Fundamentals of heat transfer*. Academic Press, New York, 1964.
- [51] D.A. Labuntsov. Heat transfer in film condensation of pure steam on vertical surfaces and horizontal tubes. *Teploenergetika*, 4(7):72–79, 1957.
- [52] T.B. Benjamin. Wave formation in laminar flow down an inclined plane. *J. Fluid Mech*, 2(2):554–574, 1957.
- [53] D.A. Goussis and R.E. Kelly. Surface wave and thermocapillary instabilities in a liquid film flow. *Journal of Fluid Mechanics*, 223:25–45, 1991.
- [54] H.C. Chang, E.A. Demekhin, and S.S. Saprikin. Noise-driven wave transitions on a vertically falling film. *Journal of Fluid Mechanics*, 462:255–283, 2002.
- [55] J.M. Skotheim, U. Thiele, and B. Scheid. On the instability of a falling film due to localized heating. *Journal of Fluid Mechanics*, 475(1):19, 2003.
- [56] I.D. Borcia, R. Borcia, and M. Bestehorn. Soret effect in thin liquid films: Localized running drops, holes and stripes. *EPL (Europhysics Letters)*, 75:112, 2006.
- [57] A. Bergeon, D. Henry, H. BenHadid, L.S. Tuckerman, et al. Marangoni convection in binary mixtures with soret effect. *Journal of Fluid Mechanics*, 375:143–177, 1998.

REFERENCES

- [58] S. Miladinova, S. Slavtchev, G. Lebon, and J.C. Legros. Long-wave instabilities of non-uniformly heated falling films. *Journal of Fluid Mechanics*, 453:153–175, 2002.
- [59] T. Fujita and T. Ueda. Heat transfer to falling liquid films and film breakdown–i:: Subcooled liquid films. *International Journal of Heat and Mass Transfer*, 21(2): 97–108, 1978.
- [60] D.E. Hartley and W. Murgatroyd. Criteria for the break-up of thin liquid layers flowing isothermally over solid surfaces. *International Journal of Heat and Mass Transfer*, 7(9):1003–1015, 1964.
- [61] R. Solanki, A. Galindo, and C.N. Markides. Dynamic modelling of a two-phase thermofluidic oscillator for efficient low grade heat utilization: Effect of fluid inertia. *Applied Energy*, 89:156–163, 2012.
- [62] H. Nakamura. Spatio-temporal measurement of convective heat transfer for the separated and reattaching flow. In *14th International Heat Transfer Conference*, Washington, WA, 2010. ASME. Paper IHTC14-22753.
- [63] W.M. Kays and A.L. London. *Compact heat exchangers*. McGraw-Hill, New York, NY, 1984.
- [64] K. Nam and S. Jeong. Measurement of cryogenic regenerator characteristics under oscillating flow and pulsating pressure. *Cryogenics*, 43(10):575–581, 2003.
- [65] P. Bouvier, P. Stouffs, and J.P. Bardon. Experimental study of heat transfer in oscillating flow. *International Journal of Heat and Mass Transfer*, 48(12):2473–2482, 2005.
- [66] K.C. Leong and L.W. Jin. An experimental study of heat transfer in oscillating flow through a channel filled with an aluminum foam. *International Journal of Heat and Mass Transfer*, 48(2):243–253, 2005.
- [67] J.C. Yu, Z.X. Li, and TS Zhao. An analytical study of pulsating laminar heat convection in a circular tube with constant heat flux. *International Journal of Heat and Mass Transfer*, 47(24):5297–5301, 2004.
- [68] M.A. Habib, A.M. Attia, A.I. Eid, and A.Z. Aly. Convective heat transfer characteristics of laminar pulsating pipe air flow. *Heat and Mass Transfer*, 38(3):221–232, 2002.

REFERENCES

-
- [69] M. Faghri, K. Javdani, and A. Faghri. Heat transfer with laminar pulsating flow in a pipe. *Letters in Heat and Mass Transfer*, 6(4):259–270, 1979.
 - [70] M.R. Mackley and P. Stonestreet. Heat transfer and associated energy dissipation for oscillatory flow in baffled tubes. *Chemical Engineering Science*, 50(14):2211–2224, 1995.
 - [71] H.N. Hemida, M.N. Sabry, A. Abdel-Rahim, and H. Mansour. Theoretical analysis of heat transfer in laminar pulsating flow. *International Journal of Heat and Mass Transfer*, 45(8):1767–1780, 2002.
 - [72] R. Siegel and M. Perlmutter. Heat transfer for pulsating laminar duct flow. *Journal of Heat Transfer*, 84:111, 1962.
 - [73] H. Chattopadhyay, F. Durst, and S. Ray. Analysis of heat transfer in simultaneously developing pulsating laminar flow in a pipe with constant wall temperature. *International Communications in Heat and Mass Transfer*, 33(4):475–481, 2006.
 - [74] H.W. Cho and J.M. Hyun. Numerical solutions of pulsating flow and heat transfer characteristics in a pipe. *International Journal of Heat and Fluid Flow*, 11(4):321–330, 1990.
 - [75] S.Y. Kim, B.H. Kang, and J.M. Hyun. Heat transfer in the thermally developing region of a pulsating channel flow. *International Journal of Heat and Mass Transfer*, 36(17):4257–4266, 1993.
 - [76] S. Backhaus and G.W. Swift. A thermoacoustic-stirling heat engine: Detailed study. *Journal of the Acoustical Society of America*, 107:3148–3166, 2000.
 - [77] J. Wheatley, T. Hoffer, G.W. Swift, and Migliori A. An intrinsically irreversible thermoacoustic heat engine. *Journal of the Acoustical Society of America*, 74:153–170, 1983.
 - [78] P.H. Ceperley. A pistonless stirling engine the traveling wave heat engine. *The Journal of the Acoustical Society of America*, 66(5):1508–1513, 1979.
 - [79] C.D. West and R.B. Pandey. Laboratory prototype fluidyne water pump. In *Proceedings of the 16th Intersociety Energy Conversion Engineering Conference*, volume 2, pages 1916–1918, Atlanta, GA, USA, August 1981. AERE, Harwell, Oxf, Engl.

REFERENCES

- [80] C.D. West. Dynamic analysis of the fluidyne. In *Proceedings of the 18th Intersociety Energy Conversion Engineering Conference*, volume 2, pages 779–784, Orlando, FL, USA, 1983. Oak Ridge National Laboratory, Oak Ridge, Tennessee.
- [81] R.W. Redlich and D.M. Berchowitz. Linear dynamics of free-piston stirling engines. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 199(3):203–213, 1985.
- [82] C. Walker. *Free piston Stirling engines*. Springer-Verlag New York, Inc., New York, NY, 1985.
- [83] B.J. Huang and M.D. Chuang. System design of orifice pulse-tube refrigerator using linear flow network analysis. *Cryogenics*, 36(11):889–902, 1996.
- [84] J.A.C. Kentfield. Fundamentals of idealized airbreathing pulse-detonation engines. *Journal of Propulsion and Power*, 18(1):77–83, 2002.
- [85] Allan J. Organ. *Stirling and pulse-tube cryo-coolers*. John Wiley and Sons, Bury St. Edmunds, November 2004.
- [86] A. Mobarak, N. Rafat, and M. Saad. Turbine selection for a small-capacity, solar-powered generator. In *Solar Energy International Progress Proceedings of the International Symposium Workshop on Solar Energy*, volume 3, pages 1351–1367, Cairo, Egypt, 1978. 16–22.
- [87] P.J. Mago, K.K. Srinivasan, L.M. Chamra, and C. Somayaji. An examination of exergy destruction in organic rankine cycles. *International Journal of Energy Research*, 32(10):926–938, 2008.
- [88] O. Badr, P.W. O’Callaghan, M. Hussein, and S.D. Probert. Multi-vane expanders as prime movers for low-grade energy organic rankine-cycle engines. *Applied Energy*, 16(2):129–146, 1984.
- [89] H. Leibowitz, I.K. Smith, and N. Stosic. Cost effective small scale orc systems for power recovery from low grade heat sources. In *Proceedings of IMECE 2006 ASME International Mechanical Engineering Congress and Exposition*, Chicago, IL, USA, November 2006.
- [90] H. Wang, R.B. Peterson, and T. Herron. Experimental performance of a compliant scroll expander for an organic rankine cycle. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 223(7):863–872, 2009.

REFERENCES

- [91] G. Xiaojun, L. Liansheng, Z. Yuanyang, S. Pengcheng, and J. Shen. Research on a scroll expander used for recovering work in a fuel cell. *International Journal of Thermodynamics*, 7(1):1–8, 2010.
- [92] R.B. Peterson, H. Wang, and T. Herron. Performance of a small-scale regenerative rankine power cycle employing a scroll expander. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 222(3):271–282, 2008.
- [93] M. Badami and M. Mura. Preliminary design and controlling strategies of a small-scale wood waste rankine cycle (rc) with a reciprocating steam engine (se). *Energy*, 34(9):1315–1324, 2009.
- [94] H.M. Curran. Use of organic working fluids in rankine engines. *Journal of Energy*, 5:218–223, 1981.
- [95] Balje O.E. A study on design criteria and matching of turbomachines, Part A: Similarity relations and design criteria of turbines. *Journal of Engineering for Power*, pages 83–102, 1962.
- [96] R.W. Moss, A.P. Roskilly, and S.K. Nanda. Reciprocating joule-cycle engine for domestic chp systems. *Applied Energy*, 80(2):169–185, 2005.
- [97] H.J. Huff and R. Radermacher. Co2 compressor-expander analysis. *Air-Conditioning and Refrigeration Technology Institute*, 2003.
- [98] F.J. Cantelmi, D. Gedeon, and A.A. Kornhauser. An analytical model for turbulent compression-driven heat transfer. *Journal of Heat Transfer*, 120:617–623, 1998.
- [99] J.M. Vignon and D. Mazon. Modèle de transfert de chaleur périodique en compression-détente pure. *International Journal of Thermal Sciences*, 38(1):89–97, 1999.
- [100] A.G. Catto and A.T. Prata. A numerical study of instantaneous heat transfer during compression and expansion in piston-cylinder geometry. *Numerical Heat Transfer, Part A: Applications*, 38(3):281–303, 2000.
- [101] R. Mathie and C.N. Markides. Heat transfer augmentation in unsteady conjugate thermal systems, Part I: Semi-analytical 1-d framework. *International Journal of Heat and Mass Transfer*, 2012. in print.

REFERENCES

- [102] L.M. Jiji. *Heat convection*. Springer Verlag, 2010.
- [103] H. Pfriem. Periodic heat transfer at small pressure fluctuations. *NACA Technical Memorandum*, 11(2):67–75, 1943. NACA-TM-1048.
- [104] V.D. Overbye, J.E. Bennethum, O.A. Uyehara, and P.S. Myers. Unsteady heat transfer in engines. *SAE Trans*, 69(461), 1961.
- [105] W.J.D. Annand, D. Pinfold, and Society of Automotive Engineers. Heat transfer in the cylinder of a motored reciprocating engine. Technical report, Society of Automotive Engineers, 400 Commonwealth Dr, Warrendale, PA, 15096, USA,, 1980.
- [106] D.J.O. Nijeweme, J.B.W. Kok, C.R. Stone, and L. Wyszynski. Unsteady in-cylinder heat transfer in a spark ignition engine: experiments and modelling. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 215(6):747–760, 2001.
- [107] A.A. Kornhauser and J.L. Smith Jr. Heat transfer with oscillating pressure and oscillating flow. In *Energy Conversion Engineering Conference, 1989. IECEC-89., Proceedings of the 24th Intersociety*, pages 2347–2353. IEEE, 1989.
- [108] U.H. Kurzweg. Enhanced heat conduction in oscillating viscous flows within parallel-plate channels. *Journal of Fluid Mechanics*, 156(1):291–300, 1985.
- [109] D. Gedeon. Mean-parameter modeling of oscillating flow. *Journal of Heat Transfer*, 108:513, 1986.
- [110] P. Nika, Y. Bailly, and F. Guermeur. Thermoacoustics and related oscillatory heat and fluid flows in micro heat exchangers. *International Journal of Heat and Mass Transfer*, 48(18):3773–3792, 2005.
- [111] J. Liu and S.L. Garrett. Relationship between nusselt number and the thermoviscous (roth) functions. *The Journal of the Acoustical Society of America*, 119:1457, 2006.
- [112] D.A. Nield and A.V. Kuznetsov. Forced convection with laminar pulsating flow in a channel or tube. *International Journal of Thermal Sciences*, 46(6):551–560, 2007.

REFERENCES

- [113] V.V. Lel, F. Al-Sibai, A. Leefken, and U. Renz. Local thickness and wave velocity measurement of wavy films with a chromatic confocal imaging method and a fluorescence intensity technique. *Experiments in Fluids*, 39(5):856–864, 2005.
- [114] K. Moran, J. Inumaru, and M. Kawaji. Instantaneous hydrodynamics of a laminar wavy liquid film. *International Journal of Multiphase Flow*, 28(5):731–755, 2002.
- [115] O.A. Kabov, E.A. Chinnov, and J.C. Legros. Three-dimensional deformations in non-uniformly heated falling liquid film at small and moderate reynolds numbers. In *2nd International Berlin Workshop on Transport Phenomena with Moving Boundaries (IBW2)*, Berlin, Germany, 2004.
- [116] B. Scheid, S. Kalliadasis, C. Ruyer-Quil, and P. Colinet. Interaction of three-dimensional hydrodynamic and thermocapillary instabilities in film flows. *Physical Review E*, 78(6):066311, 2008.
- [117] P. ChamCarthy, S.V. Garimella, and S.T. Wereley. Measurement of the temperature non-uniformity in a microchannel heat sink using microscale laser-induced fluorescence. *International Journal of Heat and Mass Transfer*, 53(15):3275–3283, 2010.
- [118] D. Ross, M. Gaitan, and L.E. Locascio. Temperature measurement in microfluidic systems using a temperature-dependent fluorescent dye. *Analytical Chemistry*, 73(17):4117–4123, 2001.
- [119] A. Schagen and M. Modigell. Local film thickness and temperature distribution measurement in wavy liquid films with a laser-induced luminescence technique. *Experiments in Fluids*, 43(2):209–221, 2007.
- [120] V.V. Lel, A. Kellermann, G. Dietze, R. Kneer, and A.N. Pavlenko. Investigations of the marangoni effect on the regular structures in heated wavy liquid films. *Experiments in Fluids*, 44(2):341–354, 2008.
- [121] F. Zhang, X. Zhao, J. Geng, Y.T. Wu, and Z. Zhang. A new insight into marangoni effect in non-isothermal falling liquid films. *Experimental Thermal and Fluid Science*, 31(4):361–365, 2007.
- [122] F. Zhang, Y.T. Wu, J. Geng, and Z.B. Zhang. An investigation of falling liquid films on a vertical heated/cooled plate. *International Journal of Multiphase Flow*, 34(1):13–28, 2008.

REFERENCES

- [123] P.L. Kapitza and S.P. Kapitza. Wave flow of thin viscous liquid films. *Zhurnal Eksperimentalnoi I Teoreticheskoi Fiziki*, 18:3–28, 1949.
- [124] H. Chang. Wave evolution on a falling film. *Annual Review of Fluid Mechanics*, 26(1):103–136, 1994.
- [125] C. Yih. Stability of liquid flow down an inclined plane. *The Physics of Fluids*, 1963.
- [126] V.Y. Shkadov. Wave flow regimes of a thin layer of viscous fluid subject to gravity. *Fluid Dynamics*, 2(1):29–34, 1967.
- [127] G.M. Sisoiev, O.K. Matar, R.V. Craster, and S. Kumar. Coherent wave structures on falling fluid films flowing down a flexible wall. *Chemical Engineering Science*, 65(2):950–961, 2010.
- [128] O.K. Matar, R.V. Craster, and S. Kumar. Falling films on flexible inclines. *Physical Review E*, 76(5):056301, 2007.
- [129] D. Brutin, F. Topin, and L. Tadrist. Experimental study of unsteady convective boiling in heated minichannels. *International Journal of Heat and Mass Transfer*, 46(16):2957–2965, 2003.
- [130] S.D. Hwang, H.G. Kwon, and H.H. Cho. Heat transfer with dimple/protrusion arrays in a rectangular duct with a low reynolds number range. *International Journal of Heat and Fluid Flow*, 29(4):916–926, 2008.
- [131] N. Sahiti, F. Durst, and A. Dewan. Heat transfer enhancement by pin elements. *International Journal of Heat and Mass Transfer*, 48(23):4738–4747, 2005.
- [132] S. Pethkool, S. Eiamsa-Ard, S. Kwankaomeng, and P. Promvonge. Turbulent heat transfer enhancement in a heat exchanger using helically corrugated tube. *International Communications in Heat and Mass Transfer*, 38(3):340–347, 2011.
- [133] P. Naphon and T. Suchana. Heat transfer enhancement and pressure drop of the horizontal concentric tube with twisted wires brush inserts. *International Communications in Heat and Mass Transfer*, 38(2):236–241, 2011.
- [134] X. Lu and P. Tervola. Transient heat conduction in the composite slab-analytical method. *Journal of Physics A: Mathematical and General*, 38:81, 2005.

REFERENCES

- [135] J.L.M. Hensen and A.E. Nakhi. Fourier and biot numbers and the accuracy of conduction modelling. *Proceedings of Building Environmental Performance: Facing the Future, University of York: BEPAC*, pages 247–256, 1994.
- [136] M.Z. Elżanowski. *An introduction to applied partial differential equations*. Portland State University, 2003.
- [137] I.A. Novikov. Several properties of the laplace transform and structure of solutions of the bound value problems of thermal conductivity with memory. *Journal of Engineering Physics and Thermophysics*, 42(3):345–350, 1982.
- [138] G.W.C. Kaye and T.H. Laby. *Tables of physical and chemical constants: and some mathematical functions*. Longmans, Green and co., 1921.
- [139] H. Nakamura. Frequency response and spatial resolution of a thin foil for heat transfer measurements using infrared thermography. *International Journal of Heat and Mass Transfer*, 52(21):5040–5045, 2009.
- [140] R.D. Mehta and P. Bradshaw. Design rules for small low speed wind tunnels. *Aeronautical Journal*, 827:443–449, 1979.
- [141] J. Groth and A. Johansson. Turbulence reduction by screens. *Journal of Fluid Mechanics*, 197:139–155, 1988.
- [142] P.A. Davidson. *Turbulence: an introduction for scientists and engineers*. Oxford University Press, USA, 2004.
- [143] M. Lesieur. *Turbulence in fluids*, volume 84. Springer Verlag, 2008.
- [144] J. Wojtkowiak and C. Oleskowicz-Popiel. Investigations of butterfly control valve flow characteristics. *Foundation of Civil and Environmental Engineering*, 7:328–395, 2006.
- [145] M. Nallasamy. Turbulence models and their applications to the prediction of internal flows: a review. *Computers & Fluids*, 15(2):151–194, 1987.
- [146] D. Kuzmin, O. Mierka, and S. Turek. On the implementation of the κ - ε turbulence model in incompressible flow solvers based on a finite element discretisation. *International Journal of Computing Science and Mathematics*, 1(2):193–206, 2007.

REFERENCES

- [147] J. Akroyd, A.J. Smith, R. Shirley, and L.R. McGlashan and Markus. A coupled cfd-population balance approach for nanoparticle synthesis in turbulent reacting flows. *Chemical Engineering Science*, 2011.
- [148] V.A. Sandborn and NACA. Glenn Research Center. *Experimental evaluation of momentum terms in turbulent pipe flow*. National Advisory Committee for Aeronautics, 1955.
- [149] K.E.G. Wieghardt. On the resistance of screens. *The Aeronautical Quarterly*, 4: 186–192, 1953.
- [150] Elgar Electronics Corporation. *P60, P80, D3 series DC power supplies: operation manual*. Elgar Electronics Corporation, 9250 Brown Deer Road, San Diego, CA 92121-2294, November 2007.
- [151] D.M. Boothman, B.J. Popham, and V. Callcut. Copper for busbars, 1984. URL <http://www.copperinfo.co.uk/busbars/pub22-copper-for-busbars/homepage.shtml>. URL http://www.copper.org/applications/busbar/ampacity/busbar_ampacities.html.
- [152] O. Manual. *ThermaCAM Reporter*. Flir Systems AB, 2002.
- [153] A. Criminisi, I. Reid, and A. Zisserman. A plane measuring device. *Image and Vision Computing*, 17(8):625–634, 1999.
- [154] Y.T.T.Y.Y. Satomi, H. Kcbota, and T. Makita. Specific volume and viscosity of ethanol-water mixtures under high pressure. *Review of Physical Chemistry of Japan*, 47(1), 1977.
- [155] K. Hiroharu, N. Nüichi, and H. Masaru. On the turbulent heat transfer by free convection from a vertical plate. *International Journal of Heat and Mass Transfer*, 11(7):1117–1125, 1968.
- [156] C. Ludwig, K.K.H. Staatsdruckerei, Kaiserl. Akademie der Wissenschaften in Wien, and W. Braumüller. *Diffusion zwischen ungleich erwärmten Orten gleich zusammengesetzter Lösung*. Aus der KK Hof-und Staatsdruckerei, in Commission bei W. Braumüller, Buchhändler des KK Hofes und der K. Akademie der Wissenschaften, 1856.

REFERENCES

- [157] C. Soret. Influence de la temperature sur la distribution des sels dans leurs solutions. *CR Acad Sci Paris*, 91:289–291, 1880.
- [158] E.L. Cussler. *Diffusion: Mass transfer in fluid systems*. Cambridge Univ Pr, 1997.
- [159] P. Polyakov. *Study of the thermal diffusion behavior of simple binary mixtures*. University of Twente, 2008.
- [160] K.I. Morozov. Soret effect in molecular mixtures. *Physical Review E*, 79(3):031204, 2009.
- [161] T.G. Cowling. Approximate theories of thermal diffusion. *Journal of Physics A: General Physics*, 3:774, 1970.
- [162] S. Wiegand. Thermal diffusion in liquid mixtures and polymer solutions. *Journal of Physics: Condensed Matter*, 16:R357, 2004.
- [163] S.A. Rani, B. Pitts, and P.S. Stewart. Rapid diffusion of fluorescent tracers into staphylococcus epidermidis biofilms visualized by time lapse microscopy. *Antimicrobial Agents and Chemotherapy*, 49(2):728–732, 2005.
- [164] S.C. Jacobson, R. Hergenroder, L.B. Koutny, and J.M. Ramsey. High-speed separations on a microchip. *Analytical Chemistry*, 66(7):1114–1118, 1994.
- [165] B. Scheid. personal communication, 2010.
- [166] D.M. Tuck. Interfacial properties of a hydrophobic dye in the tetrachloroethylene-water-glass systems. Technical report, Savannah River Site (US), 1999.
- [167] J.H.J. Thijssen, A.B. Schofield, and P.S. Clegg. How do (fluorescent) surfactants affect particle-stabilized emulsions? *Soft Matter*, 7(18):7965–7968, 2011.
- [168] L. Shi, X. Mao, and A.J. Jaworski. Application of planar laser-induced fluorescence measurement techniques to study the heat transfer characteristics of parallel-plate heat exchangers in thermoacoustic devices. *Measurement Science and Technology*, 21:115405, 2010.
- [169] Y.B. Zudin. Analysis of the processes of heat transfer with periodic intensity with allowance for temperature fluctuations in the heat carrier. *Journal of Engineering Physics and Thermophysics*, 73(2):243–247, 2000.

REFERENCES

- [170] D.W. Etheridge and P.H. Kemp. Measurements of turbulent flow downstream of a rearward-facing step. *Journal of Fluid Mechanics*, 86(03):545–566, 1978.
- [171] E.V. Shisnov, P.S. Roganov, S.I. Grabarnik, and V.P. Zabolotsky. Heat transfer in the recirculating region formed by a backward-facing step. *International Journal of Heat and Mass Transfer*, 31(8):1557–1562, 1988.
- [172] R.V.R. Avancha and R.H. Pletcher. Large eddy simulation of the turbulent flow past a backward-facing step with heat transfer and property variations. *International Journal of Heat and Fluid Flow*, 23(5):601–614, 2002.
- [173] S. Terhaar, A. Velazquez, J.R. Arias, and M. Sanchez-Sanz. Experimental study on the unsteady laminar heat transfer downstream of a backwards facing step. *International Communications in Heat and Mass Transfer*, 37(5):457–462, 2010.
- [174] B.D. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the 7th International Joint Conference on Artificial Intelligence*, 1981.
- [175] B.K.P. Horn and B.G. Schunck. Determining optical flow. *Artificial Intelligence*, 17(1-3):185–203, 1981.
- [176] H. Le, P. Moin, and J. Kim. Direct numerical simulation of turbulent flow over a backward-facing step. *Journal of Fluid Mechanics*, 330(1):349–374, 1997.
- [177] J. Tihon, V. Penkavova, and M. Pantzali. The effect of inlet pulsations on the backward-facing step flow. *European Journal of Mechanics-B/Fluids*, 29(3):224–235, 2010.
- [178] H. Nakamura. personal communication, 2010.
- [179] I.S. Khattab, F. Bandarkar, M.A.A. Fakhree, and A. Jouyban. Density, viscosity, and surface tension of water, ethanol mixtures from 293 to 323k. *Korean Journal of Chemical Engineering*, pages 1–6, 2012.
- [180] J.S. Howes, J. Macnaghten, and R.G. Hunt. Improved heat storage apparatus, September 28 2012. WO Patent 2,012,127,178.
- [181] J.S. Howes and J. Macnaghten. Energy storage, October 3 2008. US Patent App. 12/681,586.

REFERENCES

- [182] R. Bennitt. Vip compressor. In *Dresser-Rand Technology Conference, presented October*, pages 22–23, 1990.
- [183] W.C. Wirz. The vip compressor-a solution to reliable high speed operation.