

# **The dynamic response of blast-loaded monolithic and composite plated structures**

A thesis submitted for the degree of Doctor of Philosophy (Ph. D.)

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## Declarations

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Karl Micallef  
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## Abstract

The scope of the thesis is to investigate the dynamic behaviour of plated structures when subjected to blast loading, focusing in particular on localised blast loading. Two main categories of plated structures are investigated, viz. monolithic plates and composite plates.

The thesis includes a literature survey of existing works on the subject, which concludes that there is no available method of describing a localised blast load arising from a given charge size, geometry and stand-off distance from target. The review also investigates analytical methods for predicting displacement of plates subjected to blast, assuming rigid-plastic behaviour and the concept of using dimensionless parameters to predict such displacements. The survey also reviews material models for composites and damage mechanisms for these materials.

On the basis of these findings, the thesis proposes a systematic method of mathematically describing the spatial and temporal variations of a localised blast load from a known set of threat parameters (explosive type, quantity, size and stand-off distance). The method is validated by comparison of numerical results using the proposed loading function implemented in a finite element analysis software with experimental results of blast loads on steel plates.

This leads to the first study, which focuses on the performance of monolithic plates subjected to a blast load of the form described above. Existing formulations for uniform loading found in the literature are extended to consider this new form of loading. Various plate thicknesses are investigated (thick, moderately thick and thin) and it is found that good correlation is achieved with numerical results, even when the blast load is simplified into an impulsive one.

The performance of composite plates under blast loading is also investigated, focusing primarily on a new high-performance composite material (namely, Dyneema HB26). Material characterisation and blast loading tests were carried out and these were used to develop a material model for Dyneema, which is validated using finite element simulations. Its performance is numerically compared with mild and armour (Armox 370T Class 1) steel plates of equal areal density and it is found that Dyneema offers an improvement over mild steel, but armour steel plates lead to the least permanent midpoint deflection. Using dimensionless parameters, a simple design guideline is provided to estimate the deflection for a given plate geometry made of a monolithic or composite material subjected to a specific blast load. The use of this guideline was also illustrated by considering various threats and using the proposed method to recommend various plate thicknesses required for different material systems to meet a specified damage limitation.

Furthermore, a numerical-analytical method is proposed to predict the occurrence of Mode I delamination (or separation between the plies) in the early-time response of laminated composite materials, by means of stress propagation analysis.

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Lilkom it-tlieta, grazzi hafna mill-qalb; inħobbkom hafna, issa u għal dejjem.

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# Chapter 1. Introduction

## 1.1 Preamble

The main objective of this chapter is to provide a setting for the research presented in this thesis. It will establish the scope and limits of this research and outline the main aspects which will be investigated in the subsequent chapters.

## 1.2 Background

In today's world, scarred by many undertakings widely acknowledged to be epitomised (but definitely not limited to) by the events of 11<sup>th</sup> September 2001, the threat of terrorism is more relevant to the engineer than ever. The structural engineer, in particular, often needs to expand traditional limit state design to encompass the effects of such terrorist acts, which very often include designing structures to resist blast and/or impact loadings, or at least limit the damage to the structure such that the loss of property and life is minimised; this thesis will only focus on the study of blast loading and its effects.

### 1.2.1 Localised air blast loading

The blast load itself can arise from a number of sources. For typical engineering structures, these can be conventional bombs and truck bombs or local explosive sources such as IEDs (improvised explosive devices). The latter give rise to localised blast, affecting a limited area of the structure, in contradistinction to global blast, where the effects of the blast tend to be more uniformly distributed over the target area. It has been shown (e.g. in [1]) that the response of structures to such loading forms leads to additional failure modes not encountered in globalised loading scenarios and thus the effects of localised blasts on structures need to be separately investigated.

In military applications, not only structures but also vehicles are potentially subjected to localised blast sources, particularly from buried landmines. For example, 4191 casualties, including no less than 1155 fatalities, have been attributed to IEDs, landmines and other war equipment during 2010, 70% of which were civilians and over 25% of which occurred solely during the war in Afghanistan in 2010 [2]. However, the effects of buried landmines are not studied in this thesis, since this involves significant soil-structure interaction, soil fragmentation and concurrent high velocity multiple fragment impact and blast overpressure effects, which would need to be studied separately and fall outside the scope of the current work. Thus, this thesis will focus on air blast explosions and, in particular, localised blasts and their effects on structures.

### 1.2.2 Materials used for armoured structures

Protection against blast loads can be achieved either by removing the actual threat (e.g. demining efforts in war-afflicted areas) when possible or else by developing and engineering structures which can actually resist the effects of such threats (e.g. armour design).

The first method is typically dangerous, time-consuming and also expensive. For example, a mine-clearing operation along a 27 km road in Angola took 6 months to complete and cost \$475000, or almost £295000 [3]. Furthermore, there may be involuntary disruption imposed due to armed conflict in the zone.

However, in many cases, it is impossible to know a priori the location of a blast source, especially when these are placed at ad hoc locations and at an unspecified time (e.g. IEDs). Thus, the alternative (and more technical) solution is to design structures which can mitigate the blast effects, examples of which are MRAP (mine resistant ambush protected) vehicles for the case of buried landmines and/or IEDs.

Such vehicles and, indeed, most typical structural engineering systems could be idealised into a series of plated elements enclosing a supporting skeletal frame. Protective design against blast loading has traditionally been associated with the use of monolithic plates, typically metals such as steel. This leads to a major difficulty with such structures: weight. In fact, it is well-recognised that while a typical MRAP vehicle provides very good protection to its crew, it is too heavy for off-road travel, in particular the high bearing pressures that its weight would impose on poor ground, which is often unavoidable in extreme environmental locations, such as sand in deserts or mud in jungles [4]. Heavy MRAP vehicles also pose difficulties in navigating, due to their high centre of gravity and rigid suspensions, affecting vehicle stability even at slow speeds and slight slopes, limiting their turn rate and turn speed and practically eliminating the possibility of travelling on side slopes [4]. In fact, between November 2007 and September 2008, some 122 incidents related to MRAP vehicles have been reported, including 7 fatalities from vehicle roll-overs [5].

In addition, such vehicles, are typically very expensive (up to \$500000 [6] or almost £310000 each) and proper engineering design is required to design the components of such armoured vehicles to be as efficient as required in terms of material performance.

Notwithstanding these disadvantages, steel is still the most commonly used material in protective structures [7]. Thus, the performance of monolithic plates subjected to blast loading will be another aspect studied in this thesis.

Reduction in weight is no new matter in structural engineering and the strength-to-weight ratio of a material is very often thought of as a measure of a material's efficiency. Nevertheless, in blast resistant design of armoured vehicles, there are stringent requirements associated with strength-to-weight and stiffness-to-weight ratios dictating a restricted possible class of materials. The argument is thus extended to the design of such blast-resistant entities, mainly by the use of ceramics and polymeric composite materials [8]. Indeed, the latter have been used for military purposes in a number of CAV (composite armoured vehicles), with the CAV-ATD (advanced technology demonstration), produced by United Defence of the USA and the corresponding British counterpart ACAVP (advanced composite armoured vehicle platform), produced by Qinetiq (formerly DERA) [9]. Indeed, the latter has a full glass fibre composite hull, leading to significant improvements in weight.

Newer types of polymeric composites, such as polyethylene and polypropylene based materials, which are even lighter than conventional glass or carbon composites, have been used mostly for personal protection in the form of bullet-proof vests and combat helmets, but also as liners in armoured vehicles.

For example, in 2009 the Dutch Ministry of Defence army commissioned Royal TenCate to develop a ceramic strike face backed with a Dyneema liner for the Patria XA-188 APC (armoured personnel carrier), for operation in Afghanistan [10].

However, due to commercial and military sensitivity, very limited (if any) data on the response of such high performance composites when subjected to blast loading. Thus, another element studied in this thesis will be the performance of such composites under blast loading.

### **1.3 Aims and scope**

In the light of the discussion in Section 1.2, it is evident that there is the need to study how plated structures respond when subjected to blast loads, particularly if these are of a localised nature. It is also apparent that various material systems can be used and the response and even failure will vary depending on the properties of these materials.

The dynamic nature of blast load also implies that the structure is loaded at high strain-rates, which is known to affect both strain and stress levels in a structure and also material properties. In addition, the high loadings associated with blast would suggest that the structure will be subjected to stresses which exceed its elastic capacity. Designing a structure not to fail and to remain elastic under such high loads will make the structures thicker and thus heavier and more expensive, not to mention the higher level of loads transferred to the supports in that case will raise further issues. Thus, it would be favourable to assess the reserve energy absorption capacity which a material has beyond its initial yield in order to realistically estimate its capacity up to failure.

Thus, the scope of this thesis is to investigate the dynamic behaviour of plated structures which are subjected to blast loading, focusing in particular on localised blast loading, and to provide the design engineer with reliable analysis tools and simple design methods to predict and assess potential damage to monolithic and composite plated structures.

Fitting within this scope, the aims of the thesis are:

- To carry out a survey of existing literature related to the subject and identify key areas which need further investigation to meet the scope described above.
- To understand the mechanism behind blast loading and develop a mathematical description of such an event, i.e., spatial and temporal distributions of incident overpressure induced by global and localised blasts.
- To assess the performance of both monolithic and composite plates when subjected to localised blast loads, with such an assessment being done experimentally, numerically and, where possible, analytically.
- To compare the performance of various plated systems of different materials but with the same areal density.
- To provide a simple design process to select an adequate thickness of a plated structure made of a specified material such that it does not exceed an acceptable level of damage when subjected to a given blast threat.
- To assess failure mechanisms in composites and provide the framework for a constitutive model for composite materials which encompasses the effects of high loading rate and plastic damage.

## **1.4 Organisation of the thesis**

This thesis is organised into 8 chapters, as follows.

Following this brief introduction, Chapter 2 presents an overview of the existing literature related to the subject. From this survey, certain aspects which are missing in the literature are identified and thus the literature review sets the context for contributions made in subsequent chapters.

Chapter 3 proposes a generalised method of describing the spatial and temporal components of a blast load, which can be either global or localised, depending on the charge type, size, mass and its stand-off from the target. The proposed method is verified by comparison of experimental data with numerical results from finite element analysis software.

A direct application of the results from Chapter 3 is presented in Chapter 4, where an analytical analysis of the performance of monolithic rigid-plastic plates subjected to the blast load described by the form discussed above is presented. Using a rigid-plastic approximation to the actual elastic-plastic behaviour, existing formulations in literature for uniform loading are extended and various plate thicknesses (viz. thick, moderately thick and thin, compared to the plate's dimensions) are considered.

Chapter 5 investigates the performance of composite plates under blast loading, focusing primarily on new grades of high performance composites, namely Dyneema HB26. Its performance is numerically compared with monolithic plates of equal areal density and a simple guideline is provided to estimate the deflection for a given plate geometry made of a monolithic or composite material subjected to a specific blast load.

An analytical-numerical method to predict the occurrence of delamination in laminated composite materials is presented in Chapter 6, where through-thickness stress levels are assessed by means of wave propagation analysis.

Finally, Chapter 7 provides a summary of the main conclusions, alongside recommendations for future work related to the main topics addressed in this thesis.

## **1.5 Collaboration and dissemination**

### **1.5.1 Funding and collaboration**

The research presented in this thesis is part of a project jointly funded by EPSRC (Engineering and Physical Sciences Research Council) and DSTL (Defence Science and Technology Laboratory), under grant number EP/G042861/1, started in October 2009 in collaboration with the Department of Aeronautics of Imperial College London and BISRU (Blast and impact survivability research unit) within the Department of Mechanical Engineering of the University of Cape Town (UCT) in South Africa.

Whilst the research at the Department of Civil and Environmental Engineering focused on blast loading, the Department of Aeronautics focused on the response of structures under impact loading.

### 1.5.2 Dissemination

The results from this research, including significant portions presented in this thesis, were presented in international conferences and published in international peer-reviewed journals and reference to published works is made in appropriate parts of subsequent chapters.

The list of publications is presented hereunder.

- Conferences

15<sup>th</sup> International Conference on Computational Methods and Experimental Measurements [11]

3<sup>rd</sup> International Workshop on Performance, Protection and Strengthening of Structures under Extreme Loading [12, 13]

16<sup>th</sup> International Conference on Composite Structures [14-16]

15<sup>th</sup> European Conference on Composite Materials [17, 18]

- Journals

Composite Structures [19, 20]

International Journal of Mechanical Sciences [21]

Journal of Engineering Mechanics (ASCE) [22]

## Chapter 2. Literature review

### 2.1 Nomenclature

In this chapter, the following notation will be used:

#### *Latin lower case*

|            |                                                       |
|------------|-------------------------------------------------------|
| $p_e$      | Youngdahl's effective pressure, [ $M L^{-1} T^{-2}$ ] |
| $p(t)$     | instantaneous blast pressure, [ $M L^{-1} T^{-2}$ ]   |
| $t$        | time, [T]                                             |
| $t_{mean}$ | Youngdahl's mean time, [T]                            |
| $t_d$      | blast duration, [T]                                   |
| $t_y$      | time of initial yield, [T]                            |

#### *Latin upper case*

|           |                                                             |
|-----------|-------------------------------------------------------------|
| $B$       | plate width, [L]                                            |
| $D_n$     | Johnson's damage number, [1]                                |
| $E_i$     | elastic modulus, [ $M L^{-1} T^{-2}$ ]                      |
| $G_{ij}$  | shear modulus, [ $M L^{-1} T^{-2}$ ]                        |
| $G$       | energy release rate, [ $M L^2 T^{-3}$ ]                     |
| $G_c$     | fracture toughness, [ $M T^{-2}$ ]                          |
| $G_t$     | total energy release rate, [ $M L^2 T^{-3}$ ]               |
| $H$       | plate thickness, [L]                                        |
| $I$       | impulse, [ $M L T^{-1}$ ]                                   |
| $I_e$     | Youngdahl's effective impulse, [ $M L T^{-1}$ ]             |
| $L$       | plate length, [L]                                           |
| $M_0$     | plate static plastic collapse moment, [ $M L T^{-2}$ ]      |
| $P$       | pressure, [ $M L^{-1} T^{-2}$ ]                             |
| $P_0$     | ambient pressure, [ $M L^{-1} T^{-2}$ ]                     |
| $P_i$     | incident pressure, [ $M L^{-1} T^{-2}$ ]                    |
| $P_m$     | peak overpressure, [ $M L^{-1} T^{-2}$ ]                    |
| $P_r$     | reflected pressure, [ $M L^{-1} T^{-2}$ ]                   |
| $P(r, t)$ | loading function, [ $M L^{-1} T^{-2}$ ]                     |
| $P_r(r)$  | spatial part of pressure pulse load, [ $M L^{-1} T^{-2}$ ]  |
| $P_t(t)$  | temporal part of pressure pulse load, [ $M L^{-1} T^{-2}$ ] |
| $R$       | plate radius, [L]                                           |
| $R_e$     | radius of explosive charge, [L]                             |
| $R_n$     | Zhao's response number, [1]                                 |
| $S$       | stand-off distance, [L]                                     |
| $S'$      | scaled distance, [ $M^{-1/3} L$ ]                           |
| $T$       | structural response time, [T]                               |
| $V_0$     | impulsive velocity, [ $L T^{-1}$ ]                          |
| $W$       | plate width, [L]                                            |
| $W_e$     | mass of explosive charge, [M]                               |

### *Greek lower case*

|                |                                                               |
|----------------|---------------------------------------------------------------|
| $\alpha$       | waveform parameter, [1]                                       |
| $\delta$       | gap opening, [L]                                              |
| $\delta_0$     | onset displacement, [L]                                       |
| $\delta_{max}$ | failure displacement, [L]                                     |
| $\lambda$      | Jones' dimensionless impulse, [1]                             |
| $\xi_{c,l}$    | Jacob et al.'s loading parameter for circular plates, [1]     |
| $\xi_{q,l}$    | Jacob et al.'s loading parameter for quadrangular plates, [1] |
| $\xi_s$        | Jacob et al.'s stand-off parameter, [1]                       |
| $\theta$       | angle of incidence of blast wave, [1]                         |
| $\rho$         | density of plate material, [M L <sup>-3</sup> ]               |
| $\phi_c$       | Jacob et al.'s damage parameter for circular plates, [1]      |
| $\phi_q$       | Jacob et al.'s damage parameter for quadrangular plates, [1]  |
| $\sigma_{ij}$  | stress component, [M L <sup>-1</sup> T <sup>-2</sup> ]        |
| $\sigma_0$     | yield stress, [M L <sup>-1</sup> T <sup>-2</sup> ]            |
| $\tau$         | traction across surface, [M L T <sup>-2</sup> ]               |
| $\tau_c$       | failure traction, [M L T <sup>-2</sup> ]                      |

## **2.2 Introduction**

In this chapter, a brief literature survey of previous works done by others which are of relevance to the work in this thesis is presented. The aim of this review is to provide an overview of the main aspects related to the problems undertaken in subsequent chapters and to provide a context for these same problems.

It is beyond the scope of this chapter to provide a full and exhaustive review of all the engineering aspects related to blast effects on structures but only the relevant issues pertaining to this work.

It should also be noted that each chapter includes a brief review of relevant literature which would be of immediate use in that chapter and these are generally not included in this chapter to avoid repetition.

Thus, the aim of this chapter is to provide a general discussion on:

- Blast loading and effects of extreme dynamic loading on material performance and properties
- Traditional armour design using monolithic materials, such as metals
- Dimensionless comparison of structural response to impulsive loading
- Laminated polymeric composite materials, their constituents and material models
- Failure mechanisms models for composite materials
- Use of composite materials (unaccompanied or in hybrid systems) in armour systems

## 2.3 Blast loading

Although blast loads have a low probability of occurrence in most engineering applications, except in military infrastructure, nevertheless blast loads are among the most significant types of dynamic loads commonly encountered in many wide-ranging engineering applications, including aircraft, ships, topside structures and civil structures.

Thus, a study and understanding of the mechanism of how a blast load is generated and how the shock wave propagates is necessary.

### 2.3.1 The physics of blast loading

A blast load on a structure is invariably the result of an explosion and explosions can be broadly classified into 3 categories, viz., physical, nuclear and chemical explosions [23]:

- Physical explosions

Physical explosions can occur, for example, when a hot liquid flows through a frozen pipe or the failure of a cylinder full of compressed gas or even the eruption of a volcano. In all of these examples, there is no chemical reaction but the explosion simply results from a physical conversion of state or some other such phenomenon.

- Nuclear explosions

Nuclear explosions, on the other hand, represent a blast resulting from a significant release of energy, famously described by Einstein's  $E = mc^2$ , arising from subatomic activity. As history has demonstrated, such explosions exert extremely large levels of energy when compared to conventional (or chemical) explosions<sup>1</sup>, but the study of such explosions is beyond the scope of this work.

- Chemical explosions

Finally, a chemical explosion arises from the rapid oxidation of fuel elements (viz. carbon and hydrogen atoms) which are present in an explosive compound. The latter also contains any oxygen required for the reaction. When this reaction is triggered, the explosive will decompose violently, with a significant evolution of heat (e.g. 4389 kJ/kg of TNT [24]) and the production of gas. The rapid expansion of this gas interacts with its surrounding medium, generating either a shock pressure if in contact with a solid or a shock wave in the case of contact with a fluid medium, i.e., air, for air blasts, and water, for underwater blasts. This discussion will be limited to the former case.

It should be noted that the chemical explosion can either be a deflagration or a detonation. If the explosive material decomposes (or burns) at a rate which is much lower than the speed of sound in the material, then the combustion process is called deflagration and this is propagated by the heat released during the reaction [23]. Detonation, on the other hand, is the explosive reaction which produces a high intensity shock wave. This leads to a classification of the chemical explosives themselves.

---

<sup>1</sup> For example, the bombs dropped on Hiroshima and Nagasaki in 1945 had the equivalent of 12.5 and 22 kilotonnes of TNT, respectively.

Chemical explosives are classified as being either low or high explosives [25]. The former include explosive powders, such as gunpowder. The action of such materials is by burning or deflagration. Low explosives have been used in history for a variety of applications, ranging from mining to pyrotechnics and even military applications, particularly as propellants for cannonballs and, of course, the infamous attempt to blow up the House of Lords on 5<sup>th</sup> November 1605. However, most conventional modern explosives are derived from nitrate-based compounds, such as TNT and PETN, and these are termed high explosives, since they detonate, rather than deflagrate. Within the context of this thesis, the term explosion will thus be restricted to chemical detonation processes.

The detonation process leads to the development of the detonation wave and this is described by a number of parameters, namely, the Champan-Jouget detonation pressure [26], the detonation temperature [27] and the detonation velocity [28]. For example, for TNT with a density of 1650 kg/m<sup>3</sup>, the detonation pressure is 21000 MPa, the detonation temperature is 3720 K and the detonation velocity is 6950 m/s.

Once detonation has occurred, the interaction process starts. The product gases expand and a pressure wave is generated. To describe this process, an equation of state is used to relate energy, pressure and volume. One of the widely used equations of state is the JWL (Jones-Wilkins-Lee) equation due to its simplicity and since it describes well the behaviour of many explosives [29]. The full derivation is shown in [30] and further discussion is made in Chapter 3.

The propagation process itself varies significantly depending on whether it occurs in air or water. For the purpose of this study, only propagation in air is considered.

A schematic representation of a blast wave is shown in Figure 2-1. Immediately after detonation, at a specific point in the vicinity of the charge, the wave has an instantaneous rise and then decays exponentially until the pressure reaches ambient (i.e. atmospheric) pressure again. A partial vacuum is formed behind the blast front and this creates a negative pressure (with respect to the ambient pressure). The duration of the negative phase is generally longer than that of the positive phase, but the amplitude is often much smaller and thus it is ignored.

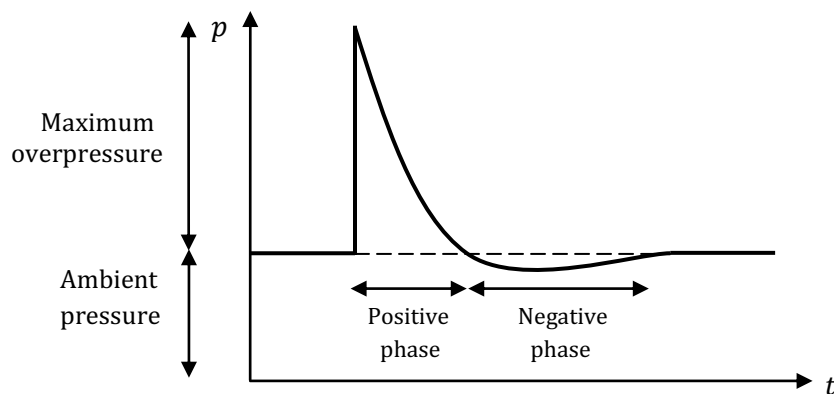


Figure 2-1: Schematic representation of a blast wave (after [29])

The instantaneous pressure,  $p(t)$ , of the positive phase of an ideal blast wave is described by the Friedlander equation given by [31]:

$$p(t) = P_0 + P_m \left[ 1 - \left( \frac{t}{t_d} \right) e^{-\frac{\alpha t}{t_d}} \right] \quad (2-1)$$

where  $P_0$  is the ambient (often atmospheric) pressure,  $P_m$  is the peak overpressure,  $t$  is the instantaneous time,  $t_d$  is the duration of the positive phase and  $\alpha$  is referred to as the waveform parameter. The waveform parameter,  $\alpha$ , depends on the peak overpressure and is chosen such that the overpressure-time relationships will yield a suitable value of impulse.

For a given stand-off distance,  $S$ , in m, from the explosion, the peak overpressure is [31]:

$$\frac{P_m}{P_0} = \frac{808 \left[ 1 + \left( \frac{S'}{4.5} \right)^2 \right]}{\sqrt{\left[ 1 + \left( \frac{S'}{0.048} \right)^2 \right]} \sqrt{\left[ 1 + \left( \frac{S'}{0.32} \right)^2 \right]} \sqrt{\left[ 1 + \left( \frac{S'}{1.35} \right)^2 \right]}} \quad (2-2)$$

where  $S'$  (often also denoted by  $Z$ ) is the scaled distance given by:

$$S' = \frac{S}{W_e^{\frac{1}{3}}} \quad (2-3)$$

where  $W_e$  is the TNT equivalent of the explosive charge in kg. TNT equivalent weights for different charges are obtained, for example, from [32]. This is further discussed in Section 2.3.2.

The duration of the shock pulse,  $t_d$ , in ms, is given by [31]:

$$\frac{t_d}{W_e^{\frac{1}{3}}} = \frac{980 \left[ 1 + \left( \frac{S'}{0.54} \right)^{10} \right]}{\left[ 1 + \left( \frac{S'}{0.02} \right)^3 \right] \left[ 1 + \left( \frac{S'}{0.74} \right)^6 \right] \sqrt{\left[ 1 + \left( \frac{S'}{6.9} \right)^2 \right]}} \quad (2-4)$$

Further blast parameters, such as the time of arrival of shock wave, the shock wave velocity, the peak wind velocity behind the shock front, the density of the air behind the shock front and the kinetic energy per unit air volume behind the shock front are given in [33].

The peak overpressure,  $P_m$ , in MPa, may also be given as [34]:

$$P_m = \begin{cases} 1.13 S'^{-2.1}, & 1 \leq S' \leq 10 \\ 0.183 S'^{-1.16}, & 10 \leq S' \leq 200 \end{cases} \quad (2-5)$$

Finally, the specific impulse,  $I$ , in Ns/m<sup>2</sup>, of the shock wave is given by [34]:

$$I = \begin{cases} 203 S'^{-0.91}, & 1 \leq S' \leq 10 \\ 335 S'^{-1.06}, & 10 \leq S' \leq 200 \end{cases} \quad (2-6)$$

It should be noted that a representation of a blast wave such as that shown in Figure 2-1 is only true for an uncased charge in free air, i.e., away from any nearby reflecting surfaces [29].

As the blast wave reaches a surface and is reflected, the behaviour changes and will lead to significant enhanced pressures, with the reflected pressure being even up to 20 times higher than the incident pressure [35].

Similarly, the charge casing has an effect on the blast characteristics, but such effects are not well understood and few works exist on how to describe them. One such method is given in [34], which gives an equivalent uncased charge based on the casing weight to the charge weight ratio. However, further investigation of such loadings is beyond the scope of this thesis.

### 2.3.2 Blast scaling laws

Scaling or model laws are used to predict the properties of blast waves from large-scale explosions based on tests at a much smaller scale. The scaled distance expression in (2-3) was developed by B. Hopkinson [36] and Cranz [37] during World War I and it states that when two explosives of similar geometry and of the same material but of different weights are detonated under the same conditions, they will produce identical waves at the same scaled distance.

Another scaling law commonly used is that of Sachs [38], who developed his model during World War II to allow prediction of blast effects at different ambient conditions.

On the basis of scaled distance, blast curves were developed by Kingery and Bulmash [39] to provide the blast parameters related to the detonation of TNT charges. These curves, first provided in TM 5-1300 [40], which was superseded by UFC 3-340-02 [41] in 2008, give various blast parameters, as shown in Figure 2-2.

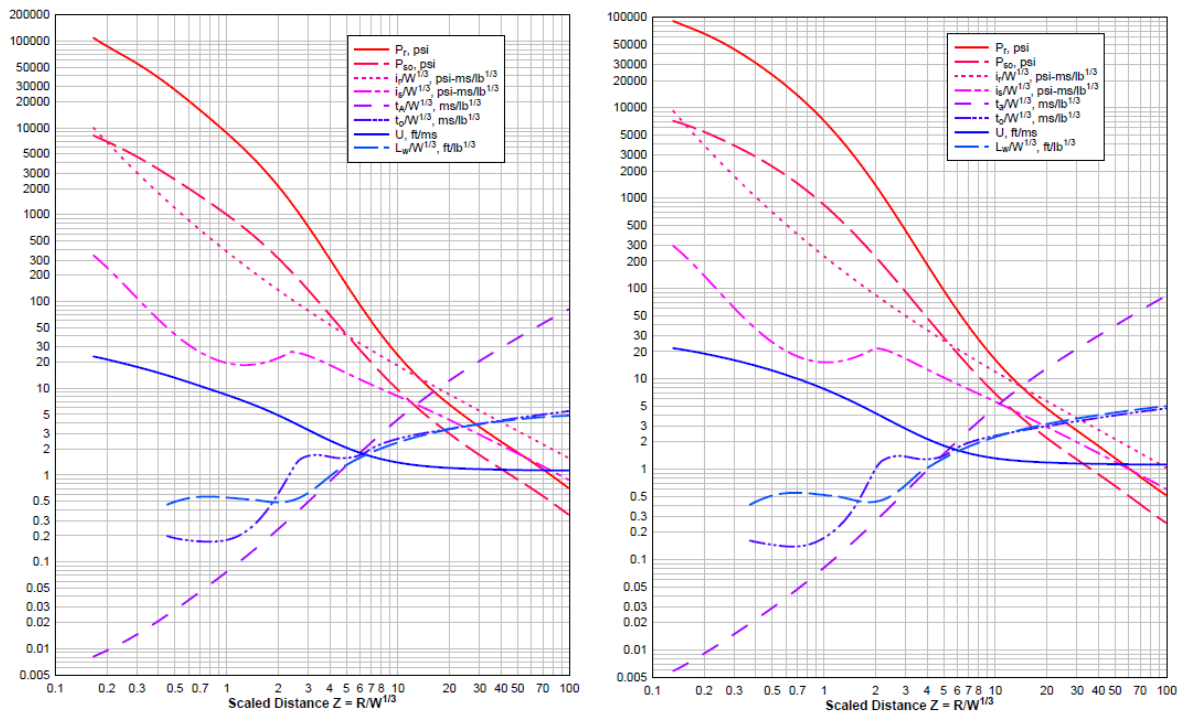


Figure 2-2: Positive phase shockwave parameter charts (after [41]) for TNT explosions in free air at sea level (left; hemispherical charge; right: spherical charge)

### 2.3.3 Numerical representation of a blast load

The complexity of blast wave reflections and interactions is generally resolved by means of load-blast functions implemented in commercial software packages.

The most popular of these are the code developed by the US Army, CONWEP (CONventional Weapons Effects Program) [42], which is based on the empirical blast loading functions of Kingery and Bulmash [39] and the use of LS-DYNA of Livermore Software Technology Corporation, which has an implementation the same load-blast function, done by Randers-Pehrson and Bannister [43]. The load-blast function is given as:

$$P(t) = P_r \cos^2 \theta + P_i (1 + \cos^2 \theta - 2 \cos \theta) \quad (2-7)$$

where  $\theta$  is the angle of incidence (defined by the tangent to the wave front and tangent to the target surface),  $P_r$  is the reflected pressure and  $P_i$  is the incident pressure.

CONWEP allows the computation of the impulse from a blast, on the basis of a charge weight and distance to target. For example, for a charge of 15 kg of TNT at a distance of 1 m from target, the peak incident overpressure would be 5017 kPa, the normally reflected pressure 41430 kPa, the incident impulse is 316 kPa ms and the reflected impulse 4147 kPa ms [44].

Another package which allows numerical simulation of blast events is Ansys AUTODYN, a package which has been available since the 1980s specifically designed for non-linear dynamic analysis incorporating various solvers, viz. Eulerian, Lagrangian and ALE (Arbitrary Lagrangian Eulerian), and thus capable of handling FSI (fluid structure interaction). AUTODYN has been widely used to simulate blast loading on a number of scales (e.g. [2, 45, 46]) and its use will be demonstrated in Chapter 3.

### 2.3.4 Localised blast loading

As briefly discussed in Section 1.2.1, the study of localised blast loads and their effects on plated structures is one of the main themes of this thesis and thus will be discussed hereunder.

A localised blast may be thought of as the effect on a structure resulting from a close-in explosion which affects a localised region of the structure, as opposed to uniform loading which occurs over the whole area [29].

Gharababaei et al. [47] define a localised blast load as one where the stand-off distance is less than the radius of a circular plate whereas Jacob et al. [48] attribute the localised nature to a blast load when the stand-off distance is less than the radius of the explosive charge.

Several researchers have investigated this aspect, particularly the effects on metal plates.

A leading researcher in this area is Nurick, who, with many collaborators, has studied the effects of localised loading on a variety of structural forms (e.g. [1, 46, 48-56]). Other research related to localised blast has been carried out by Lee and Wierzbicki [57, 58] and Neuberger et al. [59, 60]. More recently, analytical studies which have been corroborated through experiments have been carried out by Gharababaei et al. [47, 61-63], where the proposed expression for the central displacement is obtained for fully clamped plates subjected to localised blast loads arising from charges at a stand-off of up to a plate radius away from the plate.

Karagiozova et al. [52] and Langdon et al. [64] have investigated the effects of localised blasts on FMLs (fibre metal laminates) and sandwich panels and have also proposed a loading function to describe the localised blast load by extracting from AUTODYN the temporal and spatial variations for a specific charge size and stand-off.

In general, an infinite series with terms which are products of functions of space and time can express any loading function. In these works, the series has been truncated after the first term, and the localised blast loading function is assumed to be composed of 2 independent parts (as proposed in e.g. [1, 46, 52, 64]) as follows:

$$P(r, t) = P_r(r)P_t(t) \quad (2-8)$$

where  $P_r(r)$  reflects the effects of the spatial variation and  $P_t(t)$  is the temporal function. In the case of a localised blast, each of these functions is postulated to be an exponentially decaying function.

However, in previous studies, no systematic way on how to obtain a specific spatial and temporal variation which can model blast loads representing any charge at any stand-off distance has been established and this will be investigated in this thesis.

## **2.4 Blast loading on monolithic plates**

As mentioned briefly in Chapter 1, the traditional armour design has mainly focused on monolithic plated structures, notably using steel. Thus, a review of the response of steel plates to blast loading is imperative.

### **2.4.1 Plastic behaviour of monolithic plates**

The first study of the (static) behaviour of steel plates was done by Hopkins and Prager [65] in 1953, who found the static collapse pressures for perfectly plastic (or rigid plastic) circular plates of various support conditions and subjected to various cases of symmetrical loading (a central concentrated load, annular loading, circular loading and uniform loading).

However, it is also necessary to investigate the dynamic response of monolithic plates, given that blast loading is of a dynamic or impulsive nature. Considerable studies have been carried out on the plastic behaviour of such plates of various geometries and support conditions when subjected to dynamic loading, as shall be outlined in the following section.

An early study on the dynamic response of plates was done by Wang and Hopkins [66] in 1954, who studied clamped circular plates subjected to a uniformly distributed ideal impulse. Many similar studies were subsequently conducted by other researchers who studied the performance of plates under dynamic loading for different boundary and loading conditions.

Hopkins and Prager [67] carried out a similar study to that of Wang and Hopkins but for a simply supported plate.

Wierzbicki et al. [68, 69] investigated clamped steel plates and considered strain-rate effects.

Florence [70] studied a similar problem but with the outer edge subjected to a transverse impulse rather than a constant velocity. Florence [71] later generalised the work of Wang and Hopkins for any arbitrary uniform loading but only for the case of a rectangular pulse and later [72, 73] studied the response of a clamped plate subjected to a centrally located uniform load. Conroy [74] and Liu [75] examined the same problem but for simply supported conditions.

Batra and Dubney [76] examined plates with deflections in the range where both bending moments and membrane effects are important but ignoring transverse shear and rotatory inertia effects and compared the analytical results with experimental data.

A number of investigators, such as Onat and Haythornthwaite [77], Jones [78-80] and Symonds and collaborators [81-84], have considered the inclusion of finite displacements in their analytical formulations while Symonds and Wierzbicki [83] compared some of the theoretical results with experimental data on steel and titanium plates.

Jones and Gomes de Oliveira [85] and later Li and co-workers [86-88] extended the studies outlined in the on-going discussion by considering transverse shear effects in plates and shells. Other works which have included transverse shear and/or rotatory inertia effects include those by Quanlin [89], who studied the effect of a heavy impactor on an infinite plate, and Sathyamoorthy [90].

Cox and Morland [91] are the only authors who obtained solutions for dynamic plastic behaviour of non-axi-symmetrically loaded plates.

Jones working with other researchers [78-80, 85, 86, 92-96] carried out extensive work in the field on both beams and also circular and rectangular plates and compared some of the theoretical results with experimental data.

Nurick and many collaborators [1, 48, 49, 51, 97-107] also performed several investigations related to rectangular and circular steel plates, including a comprehensive and comparative review (up to 1989) of both theoretical and experimental work on the subject [98, 99].

These include a number of dimensionless expressions which are used to compare experimental data and used as a guide to assess damage resulting from dynamic loading, as discussed in the next section.

#### **2.4.2 Dimensionless damage numbers**

Non-dimensionalised representation of data is a useful engineering tool to investigate the scaling effects of various parameters, such as load and also the response of a structure to that load. This is also important when converting results from the study of a model to those of a prototype. It is also particularly practical and relevant when comparison of various scenarios needs to be done but without unnecessary repetition of scenarios which would otherwise give the same results in the space of non-dimensionalised variables.

It was Maxwell [108] who, in 1871, first devised a system to define symbols (such as [M] and [L] to represent mass and length respectively and so on) and subsequently products of powers of such symbols, which he called dimensions.

Shortly after him, in 1899, Rayleigh [109] presented the first use of dimensionless variables to assess the information embedded in an equation's dimensions. However, it was Buckingham [110] who, in 1914, then devised a method – Buckingham's  $\Pi$  Theorem – to extract a complete set of dimensionless quantities which can uniquely define a physical problem.

Jones defined a dimensionless impulse,  $\lambda$ , given in terms of the plate's static plastic collapse moment capacity,  $M_0$ , and then used this to predict displacement of metal plates of density,  $\rho$ , and yield stress,  $\sigma_0$ , subjected to an impulsive velocity,  $V_0$ . Jones proposed expressions both for quadrangular plates [92] with dimensions  $L$  and  $B$  as well as for circular plates [78] of radius  $R$ , given by (2-9) and (2-10) respectively:

$$\lambda = \frac{\rho V_0^2 L B}{M_0} \quad (2-9)$$

$$\lambda = \frac{\rho V_0^2 R^2}{M_0} \quad (2-10)$$

Further discussion and illustration of the use of these dimensionless impulse parameters is presented in subsequent chapters.

With respect to the study of the response of structures to dynamic loading, the classic dimensionless quantity is Johnson's damage number [111], which describes the damage imparted to a metal structure, given by:

$$D_n = \frac{\rho V_0^2}{\sigma_0} \quad (2-11)$$

This has been the basis of many later works utilising such a concept, the most relevant of which are described hereunder.

The basic Johnson damage number was extended by Nurick and Martin [98] in 1989 to include the effects of target plate geometry (differentiation between quadrangular and circular plates), the plate's aspect ratio and its thickness,  $H$ , and also the effects of the load source (a parameter relating the explosive's radius,  $R_e$ , to the plate dimensions).

More recently, Jacob et al. [48, 51] extended the dimensionless parameter proposed in [98] by taking into account the effect of stand-off distance.

For quadrangular plates, the damage number is given by [51]:

$$\phi_q = \frac{I}{2H^2 \sqrt{LB} \rho \sigma_0} \left( \frac{\xi_{q,l}}{\xi_s} \right) \quad (2-12)$$

where  $\xi_{q,l}$  is a loading parameter given by:

$$\xi_{q,l} = 1 + \ln \left( \frac{LB}{\pi R_e^2} \right) \quad (2-13)$$

and  $\xi_s$  is a stand-off distance parameter given by:

$$\xi_s = 1 + \ln\left(\frac{S}{R_e}\right) \quad (2-14)$$

Similarly, for circular plates of radius  $R$ , the damage number is given by [48]:

$$\phi_c = \frac{I}{\pi R H^2 \sqrt{\rho \sigma_0}} \left( \frac{\xi_{c,l}}{\xi_s} \right) \quad (2-15)$$

where  $\xi_{c,l}$  is a loading parameter given by:

$$\xi_{c,l} = 1 + \ln\left(\frac{R}{R_e}\right) \quad (2-16)$$

It should also be noted that (2-12) and (2-15) can also be written in terms of (2-9) and (2-10) respectively, as demonstrated in [48, 51, 98].

In light of the research in this thesis (where the performance of both monolithic and composite plates under blast loading is investigated) it should also be noted that the above dimensionless damage numbers have also been used [55, 112-115] to compare the performance of both material systems and also hybrids, as shall be discussed in Sections 2.10.1 and 2.10.2.

Further discussion on and application of these dimensionless parameters is presented in Chapter 3 and also Chapter 5.

Another dimensionless number to describe the dynamic response of beams and plates was proposed by Zhao [116], termed response number:

$$R_n = \frac{\rho V_0^2}{\sigma_0} \left( \frac{L}{H} \right)^2 \quad (2-17)$$

Zhao expressed many of the existing solutions for plates and beams subjected to various forms of dynamic loading (including many of those outlined in Section 2.4.1) in terms of the response number,  $R_n$ . However, Li and Jones [117] question the validity of this damage number for cases where transverse shear, strain-rate, hardening and temperature effects are important. These phenomena, applied to various structural systems subjected to impulsive loading, are studied using dimensional analysis by the same authors.

In addition, the effects of pulse shape are also discussed in [117]. This leads to another important aspect of dynamic loading: for cases when the loading time is comparable to the structure's response time, then the effect of the shape of the pulse load becomes relevant.

### 2.4.3 Effects of pulse shape

In blast and impact loading scenarios, more often than not, the engineer has to deal with large plastic deformations. It is well-known from structural dynamics that, for the case of dynamic loading, the shape of the pulse load can significantly affect the dynamic plastic response of a structure.

Although the temporal variation of a blast loads is typically simplified and idealised into linear or exponentially decaying curves, as discussed in Section 2.3, it is often practically difficult to actually measure the time history of a dynamic load, not to mention the fact that repeatability of time-histories is questionable.

Perzyna [118] first explored the effect of pulse shapes on the dynamic response of circular plates but it was Youngdahl [119] who first proposed a number of correlation parameters to relate results for different pulse shapes. Thus, Youngdahl defined an effective impulse,  $I_e$ , a mean time,  $t_{mean}$  and an effective pressure,  $p_e$ , as defined in [119]:

$$I_e = \int_{t_y}^T p(t) dt \quad (2-18)$$

$$t_{mean} = \frac{1}{I_e} \int_{t_y}^T t p(t) dt \quad (2-19)$$

$$p_e = \frac{I_e}{2t_{mean}} \quad (2-20)$$

where  $t_y$  is the time of first yield and  $T$  is the structure's response time.

Krajcinovic [120, 121] later confirmed Youngdahl's work, which was originally empirically derived. It was Li and Jones [122] who provided a theoretical foundation for Youngdahl's correlation parameter method using bound theorems of plasticity for skeletal and plated elements.

## 2.5 Introducing composite materials

In the world of civil engineering, the most widely-used materials are perhaps reinforced concrete and steel. The former is a combination of concrete and steel reinforcement acting together taking compression and tension respectively, while the latter is itself an alloy of a number of elements, mainly iron and carbon. Thus, the idea of utilising a "composite" material in civil engineering is not an entirely novel idea.

In more recent years, the term "composite" material has become more associated with fibre reinforced polymers, where a polymeric matrix (e.g. epoxy, vinyl-ester or polyurethane) is reinforced with strands of fibre or a combination of fibres (such as carbon, glass, aramid or polyethylene). Typically, the fibres in the matrix are arranged in a lamina, which in turn are bonded to create a stack of laminae, called a laminate [123].

Thus, this creates a new material, the composite material. The uses of such materials range from personal protective equipment to aerospace fuselages, from sport equipment to ships and racing cars to aircraft. The range of composite materials being considered in this study will be fibre-reinforced composites consisting of fibre reinforcement embedded in a matrix.

The aim of this section is to review the various systems which exist for both constituents and to evaluate the significance of changing parameters within the different architectures.

### 2.5.1 Matrix systems

The matrix is typically a resin and it transfers the externally applied load to the fibres which are embedded in it. Matrix systems are generally classified as either thermoplastic or thermosetting.

Thermoplastic resins have the advantage of being capable to take reprocessing upon the application of successive heating and cooling. They are generally low-cost materials but have low mechanical properties. Typical materials include polyvinylchloride, polyethylene, polypropylene, polystyrene, polyamide and polycarbonate.

Thermosetting resins can only be processed once and have higher mechanical properties than thermoplastic ones. Typical thermosets are condensed polyesters, vinylesters, allylic derivatives, phenolics, aminoplasts, furanes and epoxides.

### 2.5.2 Fibre reinforcement

Fibres are typically of a very small scale with a diameter of about 10  $\mu\text{m}$  and thus single fibres (called elementary filaments or mono-filaments) are typically bundled together to form a strand (also referred to as a yarn).

Yarns are described by their linear density, which is a measure of the fineness of the yarns. This measurement is given by the tex value, which is the mass per kilometre. Thus, 1 tex = 1 g/km =  $10\text{E-}6$  kg/m.

The strands are woven to form a fabric, which consists of fibres in the lengthwise direction (referred to as the warp) and fibres in the crosswise direction (referred to as the weft or fill).

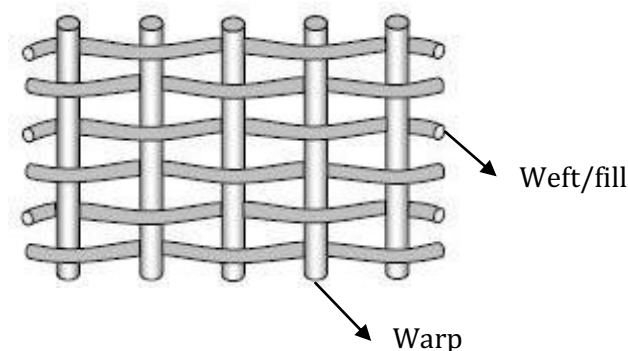


Figure 2-3: A woven fabric (after [124])

### 2.5.3 Weave patterns

Various different weave patterns exist, but the main ones will be examined in this section.

- Plain weave

In a plain weave (also known as taffeta), each warp and weft thread passes over a thread and under the next, leading to a fabric which is not very deformable. Such a fabric has properties which are virtually identical in the two directions.

A variation of a plain weave is the basket weave, in which two or more warp and weft threads are woven side by side.

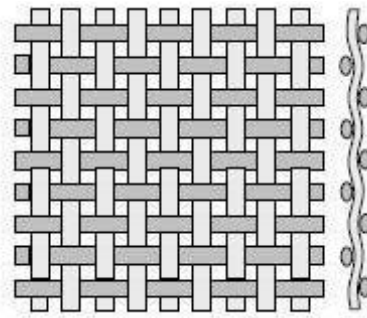


Figure 2-4: Plain weave (after [124])

- Twill weave

In a twill weave (also known as serge), the number of warp threads and weft threads interlacing each other are varied. For example, in a 2x1 twill weave, weft threads are woven over one and under two warp threads; in a 2x2 twill weave, weft threads pass over two and under two warp threads. This type of fabric permits slippage between the warp and weft.

A variation of the twill weave is the satin weave, where the number of warp threads and weft threads that pass over each other before interlacing is much greater. The satin weave is described by a number which indicates how much weft threads the warp thread passes over, viz., an 8 satin weave is a fabric where the warp threads pass over 8 weft threads.

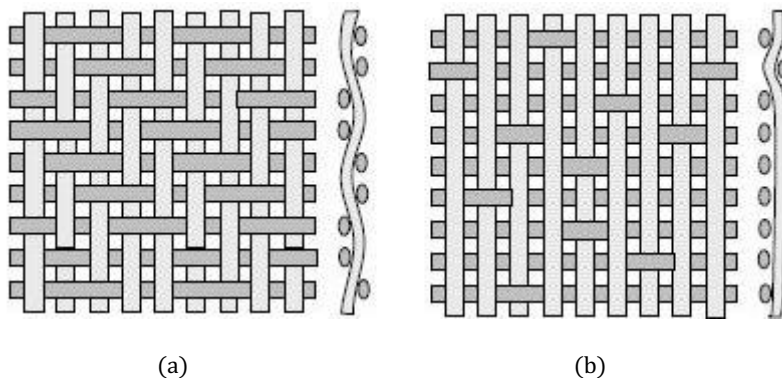


Figure 2-5: 2x2 twill weave (left); 8 satin weave (right) (after [124])

- Cross-ply weave

In a cross-ply weave, two layers of threads are placed on top of each other without any form of interlacing, except for very fine warp and weft threads. The lack of interlacing restrains shear effects.

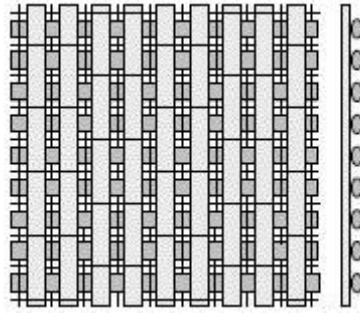


Figure 2-6: Cross-ply weave (after [124])

- Uni-directional

Although a UD (uni-directional) fabric does not really consist of a woven architecture, it is included here for the purpose of completeness.

In a UD fabric, the threads are only present in the warp direction, with very fine weft threads present to hold the main fibres together. Thus, the fabric has high performance in the warp direction with very little (if any) in the weft direction.

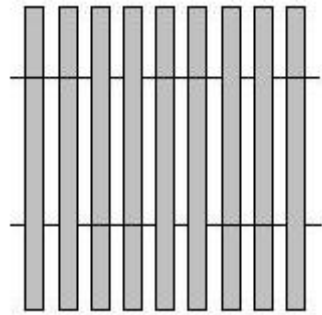


Figure 2-7: UD fabric (after [124])

## 2.6 Dynamic effect on composite material response

The mechanical response of materials under dynamic loading conditions has been a subject of interest for a significant time. In fact, the propagation of stress waves in a cylindrical bar subjected to tension impact was investigated as early as in 1807 by Thomas Young [125]. Young established that the velocity of impact is directly proportional to the elastic strain and the proportionality constant gives the velocity of propagation of the elastic deformation.

The earliest works related to the effect of strain-rate is attributed to John Hopkinson in 1872 and of his son Bertram in 1905 [126] and by Landon and Quinney in 1923 [127]. Progress in the field was initially slow but considerable development was made on the subject during World War II [128]. Initially, such work was done on metallic materials but studies on composites embarked in the 1970s; the first attempts to study the response of composites to dynamic load were by Rotem and Lifshitz [129] and Sierakowski et al. [130].

Considerable research has been carried out to investigate the effect of strain-rate on material properties and a selection of these works is subsequently highlighted in the next sections.

### 2.6.1 Effect of strain-rate on material properties (normal stress)

In the earliest work related to strain-rate effect on composites, Rotem and Lifshitz [129] studied the tensile behaviour of UD glass/epoxy composites under various strain-rates, ranging from  $10\text{E-}6$  to  $30$  /s and found that the dynamic modulus is 50% higher than the static value in the latter case.

Later, Peterson et al. [131] studied the performance of chopped glass fibre-reinforced styrene/maleic anhydride (S/MA) materials in the range of  $10^{-1}$  to  $10$  /s and observed a 50 to 70% increase in the elastic modulus with increase in strain rate.

Todo et al. [132] carried out tensile testing of plain weave carbon/polyamide-6 (CF/PA6), carbon/modified polyamide-6 (CF/mPA6), glass/polyamide-6 (GF/PA6) and glass/modified polyamide-6 (GF/mPA6) composites at strain-rates ranging from  $10\text{E-}2$  to  $40$  /s. It was found that, except for the GF/mPA6, the tensile strength and failure strain increased with an increase in strain-rate.

More recently, Shokrieh et al. [133] have shown through experimental results that there is a noticeable increase in the tensile modulus of UD glass/epoxy composites with increasing strain-rates ranging from  $0.001$  to  $100$  /s.

Schoßig et al. [134] tested glass fibre reinforced polypropylene and polybutene-1 at rates ranging from  $0.007$  to  $174$  /s and an increase of tensile strength was found for both materials with increasing strain-rate.

Very recently, following tests on plain weave E-glass/epoxy composites in the strain range of  $140$  to  $400$  /s, Naik et al. [135] observed an increase of 75 to 93% in through-thickness tensile strength compared to static values. Also, the tensile strength increased up to 11% along the thickness direction with increase in strain-rate.

Daniel et al. [136] investigated the response of UD carbon/epoxy composites at strain-rates up to  $500$  /s. The modulus increased moderately with strain-rate in the longitudinal direction and more considerably in the transverse direction. The effect of strain-rate on modulus in the transverse direction was more pronounced due to the fact that the mechanical behaviour in this direction is mainly matrix-dominated.

Weeks and Sun [137] investigated an AS4/PEEK (APC-2) composite at strain-rates ranging from  $100$  to  $1000$  /s. The material was found to exhibit significant strain-rate sensitivity.

Brown et al. [138] studied E-glass/polypropylene composites over a strain range of  $10\text{E-}3$  to  $10\text{E}2$  /s and found that the tensile and compressive modulus and strength increased with increasing strain-rate.

Tensile tests were also performed by Vashchenko et al. [139] on glass/polyamide composites within the strain-rate range of  $10\text{E-}3$  to  $10\text{E}5$  /s and found a linear relationship between the tensile strength and the log of strain-rate.

McKown and Cantwell [140] studied the effect of strain rate ( $1\text{E-}4$  to  $1\text{E}10$  /s) on a self-reinforced polypropylene composite. The results indicate an increase in the initial elastic stiffness, yield strength and maximum strength with increasing strain-rate.

### **2.6.2 Effect of strain-rate on material properties (shear stress)**

The effect of strain-rate on shear response was also investigated by a number of researchers.

Papadakis et al. [141] studied the effect of strain-rate on a continuous glass fibre/polypropylene composite at various testing speeds (5, 50 and 500 mm/min). They found that the modulus increased with increasing loading rate while the failure strain decreased.

Tsai and Sun [142] tested UD S2/8852 glass/epoxy composites at strain-rates up to 600 /s and found that in-plane shear strength increases when strain-rate increased but shear failure strain decreased as shear strain-rate increases.

Shokrieh et al. [143] carried out tests on UD glass/epoxy composites at strain-rates ranging from 0.0216 to 1270 mm/s and found that the dynamic shear strength increased by 37% compared to the static value. However, the shear modulus decreased with increase in strain-rate and the shear strain to failure also decreased with strain-rate.

## **2.7 Constitutive models for composite materials**

The constitutive model of a material can be defined as a relation used to characterise its physical behaviour and such a model will be used to describe the material response when subjected to loading.

Composites are, in general, anisotropic materials and also tend to exhibit non-linear behaviour under various kinds of loading. Also, the severe loading nature of dynamic loads implies that strain-rate effects have a repercussion on the stress-strain response of the material, as discussed in Section 2.6. In addition, it is common for a material to be loaded beyond its elastic limit and deform plastically under large magnitude loading, giving rise to plastic strains which lead to progressive damage of the material.

### **2.7.1 Classification of constitutive models**

Constitutive models are classified as either explicit or implicit [144, 145] and the classification depends on the form of the model and the way the model relates to physically-derived models of behaviour [144].

Explicit constitutive models can be considered to be the classical method of defining constitutive relationships, linking the material properties to its behaviour using physically-based models. The material's mechanical behaviour is expressed as a stress-strain relationship, with stress being, in general, a function of strain, strain-rate, strain history and plasticity, temperature and material properties. Such explicit models describe material behaviour on either a macroscopic or a microscopic level.

A macroscopic model involves analysis at a fairly large scale (e.g. the laminate level) and uses averaged properties for a composite ply which are then applied to the composite laminate. The material stress and strain fields are derived from an equation derived from the work potential of the system and are achieved by solving this equation with the strain-displacement, compatibility and equilibrium equations, typically using finite element analysis [146-148].

On a microscopic level (or at the level of, say, the fibre scale), a constitutive relationship is developed based on the properties of a unit cell composite ply and how these depend on the properties of the fibre and the matrix. The model can also include the presence of voids, fibre initial imperfections and the interface properties between the fibre and matrix [146-148].

Implicit constitutive models describe the material behaviour in a purely mathematical way by considering only the inputs and outputs of a system without direct representation of the physical phenomena which occur in the process. Such models typically employ neural networks to represent approximation functions to produce outputs from a set of given inputs [149-151]. Implicit models are usually suited for very complex material behaviour, where explicit models based on simple phenomenological findings might not capture all the relevant behaviour [144].

There exist also a class of models which are a hybrid, combining explicit and implicit relations, an example of which is the dissipated energy density approach [152-154]. In such a model, an arbitrary polynomial function (implicit) is used to define mathematically the dissipated energy density, which is a material property (explicit) describing energy lost due to non-linear behaviour.

### **2.7.2 Homogenisation**

As described in Section 2.7.1, composite materials can be studied on either a macroscopic or a microscopic level. On a macroscopic scale, the material is assumed to be homogenous, with the properties of the composite constituents being averaged (using the rule of mixtures) to give an apparent property. In micromechanics, the behaviour of the composite is studied at the level of interaction between its constituents, thus the composite is considered to be a heterogeneous material.

All materials are, at a sufficiently small scale, heterogeneous, but should such a consideration be taken as far down as the atomic level, this would make material behaviour descriptions difficult to achieve. Thus, the notions of continuity and homogenisation are introduced.

In a composite material, the main constituents, i.e., the matrix and the fibre are assumed to be isotropic and homogenous. Heterogeneity exists at the discontinuity at the fibre-matrix interface, due to the change of mechanical properties. However, if one considers the properties on a sample of a small size, it is found that these can be fairly averaged with reasonable accuracy. Thus, properties measured on a sample of this small size can be considered to be independent of the location of the sample.

In such a case, the material is considered to be effectively homogenous and a larger structure can be solved by considering the averaged properties measured at the small scale. If such a scale exists such that it is intermediate between the scale of the constituents and the structure, the material is said to be homogenised. Thus, the material is now converted from a microscopically heterogeneous state to a macroscopically homogenous one.

Although it might appear desirable to predict stress-strain response of composites on a micromechanical scale, i.e., based on the properties of the fibres and the matrix, many workers have shown that it is difficult to achieve this, even for UD composites [155]. Thus, in this thesis, only macroscale constitutive modelling of composite materials will be investigated.

## 2.8 Failure mechanisms and criteria for composite materials

The earliest failure theories have been developed in 1773 by Coulomb [156], who proposed the famous Mohr-Coulomb failure theory for isotropic materials. This was only valid for isotropic materials whose strength increases linearly with increasing hydrostatic stress, making it ideal for soils.

Further yield criteria were proposed, notably those by Tresca [157], Von Mises [158] and Torre [159], in the late 19<sup>th</sup> and early and mid 20<sup>th</sup> centuries respectively, but these were limited to isotropic materials. Subsequently, a number of failure criteria for anisotropic materials were developed, with the first set up specifically for composites being that of Tsai [160] in 1965.

Due to the nature of composite materials, there is a variety of ways in which failure can occur and it is probably best to classify failure mechanisms in terms of the different constituents, namely, fibre and matrix and the failure associated with de-bonding between plies in a laminate.

The simplest failure criteria are the non-interactive maximum stress or maximum strain envelopes, using a simple material limiting value taken from experimental testing under tension, compression or shear for the various constituents. The maximum stress criterion assumes that the composite fails when the stress level exceeds a particular allowable value, typically  $\sigma_{11}$  (fibre failure),  $\sigma_{22}$  (matrix failure) or  $\sigma_{12}$  (transverse shear failure). Similarly, the maximum strain criterion assumes that the composite fails when the strain exceeds the respective allowable strain value for a particular direction. However, most criteria in the literature involve more sophisticated relationships, reviewed hereunder.

### 2.8.1 Ply failure

Several criteria were proposed which describe the failure of an entire ply, rather than the separate ply failure modes. Selections of such criteria [161-164] do not differentiate between fibre or matrix failure while the more popular (interactive) ones [160, 165-170] produce failure surfaces and are widely popular, simple to use and implement in finite element codes and thus commonly used in design applications, as seen from the survey carried out by the AIAA (American Institute of Aeronautics and Astronautics) in Figure 2-8:

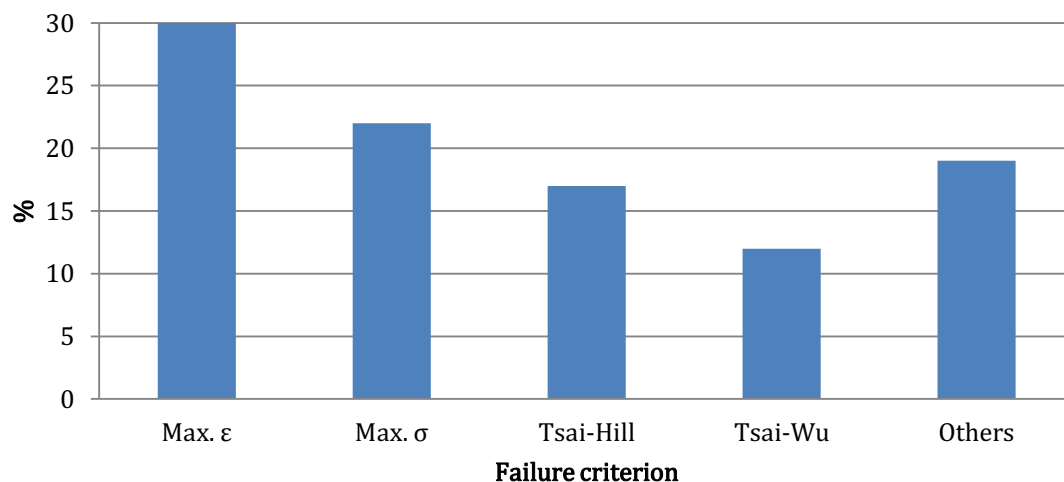


Figure 2-8: Results of AIAA failure criteria survey (after [171])

However, a number of failure criteria describe failure occurring in a particular constituent of the ply, namely, the fibre, the matrix or shear failure between the two. An extensive review of composite failure theories is provided in [144] and briefly summarised in the subsequent sections.

### **2.8.2 Fibre failure**

Fibre failure can occur either in tension or in compression.

Fibre failure in tension is classically described by, but not limited to, attaining a maximum stress or maximum strain, although other (interactive) criteria are also available.

Fibre failure in compression is initiated by fibre micro-buckling and kink band formation, with the latter being the most commonly accepted failure feature, and besides the usual maximum stress/strain failure criteria, there are various other approaches developed by various authors.

### **2.8.3 Matrix failure**

Matrix failure is an intricate phenomenon and its complexity meant that many criteria have been developed throughout the years, including the use of fracture mechanics to describe crack initiation and growth.

Tension failure of the matrix is generally described by an interaction between the tensile normal and the in-plane shear stresses, in addition to simple maximum stress/strain criteria.

Similarly, failure criteria for matrix compression exist as well as combined tension-compression criteria.

### **2.8.4 Shear failure**

A number of criteria were developed to describe in-plane shear failure between the fibre and the matrix. In addition to the maximum shear stress and strain criteria, the various theories have much inconsistency as to how shear failure is analysed, using tensile or compressive strength.

### **2.8.5 Delamination**

Delamination is a predominant and unique form of failure in laminated composites due to the lack of reinforcement in the thickness direction.

Its importance implied the development of a number of criteria predicting the initiation of delamination as well as growth of existing delaminations.

The latter are based on fracture mechanics and are discussed further in Sections 2.9.2 and 2.9.3.

## 2.9 Damage and fracture in composite materials

The failure criteria described in Section 2.8 deal with failure in a lamina. However, failure of a lamina does not lead to total loss of performance but an accumulation of damage will lead to final failure. Thus, failure is a progressive phenomenon and one must take into account the loss in stiffness of the lamina as damage is sustained.

At this point, it is useful to note that the progressive damage is studied using either fracture mechanics (i.e. discrete crack propagation) or by using damage mechanics (i.e. homogenised volumetric crack accumulation using, for example, the smeared crack model); the latter approach is more readily available in most finite element analysis packages.

### 2.9.1 Progressive ply damage

Progressive ply damage is an implementation of damage accumulation by means of reduction of the material elastic properties at the ply level. Typically,  $E_1$  is reduced for fibre failures and  $E_2$  and  $G_{12}$  for matrix transverse shear cracking [172]. The material is loaded until the limit of a selected failure criterion is reached, after which a corresponding material property (or properties) is/are reduced and the analysis proceeds using the reduced value/s (and thus reduced stiffness) by means of damage variables (e.g. [173-176]).

The gradual reduction of properties is considered to have good correlation with physical phenomena, especially in the case of transverse matrix cracking, since it represents the gradual accumulation of cracks until a state of crack density saturation is achieved [177]. The process continues until final failure is attained, when another limit is reached, usually fibre fracture.

However, such a limit may be inadequate in laminates which are matrix-dominated, such as  $(\pm 45^\circ)_s$  or in cases where stress concentrations exist, such as in a bolted joint [178]. Indeed, in such situations, local fibre fractures would effectively relieve stress concentrations and laminate failure would not occur.

These damage mechanisms are considered to be intra-ply, or belonging to the ply's constituents (e.g. matrix cracking or fibre fracture). However, there can also be inter-ply failure mechanisms, such as delamination, or separation between the plies forming the laminate.

The initiation of delamination is often considered to be a failure criterion, as outlined in Section 2.8.5, and is usually based on stresses and a quadratic interaction of interlaminar stresses and a characteristic distance [179]. Such distance is a function of both geometry and material properties and is determined through testing.

On the other hand, delamination propagation is modelled using fracture mechanics and the strain energy released in crack propagation is differentiated into three mechanisms, or "modes", of crack growth, namely, peeling, shearing and tearing, as shown in Figure 2-9.

Whilst Modes I and II are simple to measure using a double cantilever beam and an end notched beam respectively, the Mode III energy is less straightforward to obtain and it is sometimes ignored by some researchers or combined with Mode II [180].

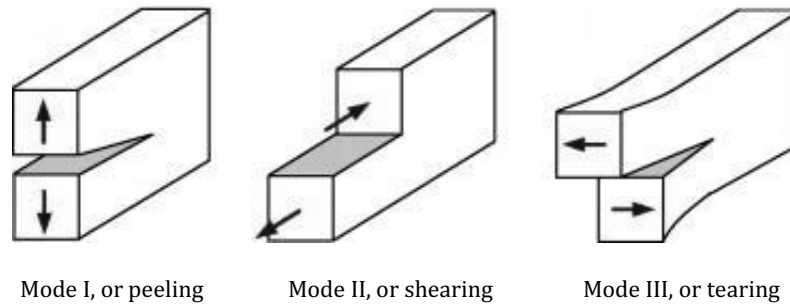


Figure 2-9: Crack growth modes (after [144])

Delamination affects the through-thickness integrity of the laminate and hinders the later integral plate response which would otherwise develop when a composite plate is subjected to out of plane loading. The applied pulse load, such as blast, will cause compressive waves to propagate through the thickness of such a plate during its early-time response and the reflected tensile stresses can potentially exceed the cohesive strength between composite plies, causing debonding and Mode I fracture thus a loss in stiffness which adversely affects plate action under blast loads. Thus, it is required to study the wave propagation in a composite and predict the compressive and (more importantly) the tensile stresses in it and this will be investigated in this thesis.

On the other hand, delamination growth is usually performed with one of two approaches, namely the VCCT (virtual crack closure technique) and the CZM (cohesive zone model), described hereunder.

### 2.9.2 The virtual crack closure technique

The VCCT was developed by Rybicki and Kanninen [181] and allows calculation of energy released at the crack tip with a single calculation, based on the assumption that when a crack extends, the energy absorbed in the process is equal to the work required to close the crack to its original length. The strain energy release rate,  $G$ , from the various crack components, Mode I, Mode II and Mode III, are combined to give the total strain energy release rate,  $G_t$ , which is compared to the critical value or fracture toughness,  $G_c$ , and the crack is propagated further.

VCCT is based on Irwin's crack tip energy analysis [182] and, unlike the J-integral method of Rice [183], which is based on linear-elastic fracture mechanics [184-186], the VCCT is applicable to 3D structures and does not have stringent mesh size requirements [187].

The difficulty with implementing the VCCT in a finite element model is that each time after the crack advances, a new mesh is required and this is computationally expensive. However, the VCCT performs well with relatively large elements when compared to the CZM.

### 2.9.3 The cohesive zone model

The CZM was developed by Dugdale and Barenblatt [188, 189] and conceptually uses cohesion tractions along the deformation zone ahead of a crack tip.

It was implemented using the finite element method by Hillerborg [190] and improved by Schellekens et al. [191], who proposed using a separate cohesion element.

Cohesive elements define the behaviour of the interface between plies. This behaviour is usually described by a material model for the cohesive zone, hence the term CZM.

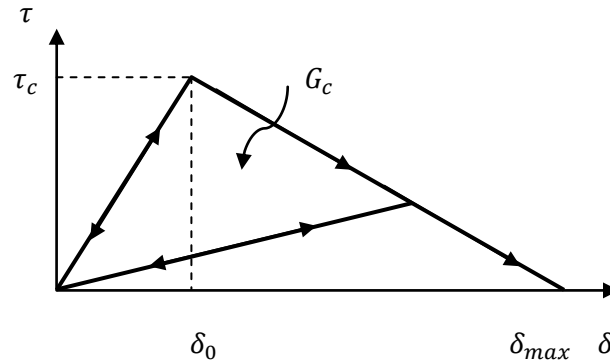


Figure 2-10: Cohesive zone (bi-linear) model (after [144])

A relationship between the gap opening,  $\delta$ , and the traction across the surface,  $\tau$ , is given. After the element surpasses the strength limit,  $\tau_c$ , and the onset displacement,  $\delta_0$ , is attained, the stiffness is reduced gradually, according to the chosen relationship. This decrease continues until the interface reaches  $\delta_{max}$ , after which the element has no more stiffness and the plies are completely separated. The work done in reducing the element stiffness to zero is equal to  $G_c$ , which is equal to the area under the traction-separation curve, shown in Figure 2-10.

Figure 2-10 shows a simple bi-linear model, but many other relationships exist, such as linear-exponential and exponential, and it has been shown [192] that many solutions give very similar results for energy release rate and thus there is no sensitivity to the chosen traction law.

The CZM is considered to be superior to the VCCT [193], primarily because it can handle both crack initiation and propagation and does not require re-meshing. The major drawback of CZM is the size of the cohesive element, which typically needs to be 0.5 mm or less in order to achieve convergence [194].

However, Turon et al. [195] proposed a method which increases the mesh size up to 1 mm and was further improved by Guiamatsia et al. [196], whose formulation has provided excellent agreement between predicted and experimental results for a typical Mode I failure using elements of 5 mm in size through the use of enhanced element basis functions.

## 2.10 Blast loading on composite plates

It has been mentioned in Chapter 1 that composite materials have been utilised for construction of armour vehicles and other military equipment including ships and vehicles and, in particular, for ballistic protection in the form of personal protective equipment such as bulletproof vests and helmets made from a range of materials including Kevlar, Spectra, Twaron, Zylon and Dyneema [197]. This section will focus on existing research related mainly to the study of composite materials used in the context of blast resistant design.

As discussed in Section 2.4, extensive work has been carried out on blast loading of monolithic plates and, compared with the extent of these studies, the blast performance of composite materials is relatively little and recent.

Early studies on the response of composite materials to blast include that of Rajamani [198] in the early 1980s, followed by many similar works up to recent times (e.g. [199-204]) mostly related to glass and carbon based composites.

With the increase in the use of composites for the construction of hulls and other parts of boats [205, 206] in the 1990s, a significant impetus towards research on blast performance of composites was the increase in design and fabrication of hulls for military marine structures (e.g. [207-211]). More recently, new high performance materials, such as Dyneema and Tegril, have been introduced in the market and their use for blast resistance is being explored; this is further discussed in Chapter 5.

However, due to the limited ductility of plain brittle composite structures (such as glass and carbon) and also the increased cost of high performance materials (such as aramids or polyethylene-based composites) when compared to monolithic or inferior composite panels, more often than not plated structures incorporating composites are used in conjunction with other materials, to form hybrid systems, as discussed in the following sections.

### **2.10.1 Sandwich panels**

In the context of composite materials, a sandwich panel typically refers to a panel with 2 relatively stiff (composite) facing sheets separated by a core, typically made of some compressible material such as foam or balsa, for obvious weight savings. The obvious advantage of such a system is the reduction in weight, which makes sandwich systems particularly appealing to the aviation industry. Whilst the latter has its own threats arising from terrorist action or mechanical malfunctions which could potentially induce blasts, sandwich panels are also becoming relevant in military applications due to their attractive properties. Indeed, it has been shown [212] that, in some cases, sandwich panels are superior than monolithic plates, compared on an equal weight basis.

Xue and co-workers [213-216] have worked extensively in the field on both sandwich beams and plates, including the development of a constitutive model for sandwich panels. Fleck and Deshpande and their co-workers [217-223] have also worked extensively in the field on sandwich structures, including the development of a constitutive model for the foam material [224].

Langdon et al. [64, 115, 225] also investigated extensively such sandwich panels, particularly experimentally and also used [115] the dimensionless damage numbers described in Section 2.4.2 to predict the permanent displacement of monolithic plates to compare the performance of both materials. The sandwich panels were found to achieve smaller displacements, although the extent of damage was found to be greater.

More recently, other researchers (e.g. [226-231]) have also done further work related to sandwich panels.

### **2.10.2 Fibre metal laminates**

FMLs are a form of sandwich system, with the outer skins in this case being metallic materials, such as steel or aluminium, and the core being a composite.

Typical FML systems whose response under blast loading have been studied include ARALL (aramid reinforced aluminium laminates) [232, 233] and GLARE (glass laminate aluminium reinforced epoxy), a commercially-available FML having a glass-fibre reinforced epoxy core with aluminium sheets which is widely used in the aviation and aerospace industries [113, 234].

The leading researcher in this field is Langdon, who has extensively worked with Lemanski, Schleyer and others [52-56, 112, 113, 235] on the subject, including studies of localised blasts on FMLs and use of dimensionless damage numbers [55, 112, 113] described in Section 2.4.2. In this case, smeared values for density and yield stress were obtained using a rule of mixtures. Recently, Vo et al. [114] have also investigated blast response of FMLs and also made use of the dimensionless damage numbers to predict back and front face displacement of FMLs.

### **2.10.3 Hybrid composite structures**

Hybrid composite structures involve the use of monolithic and composite materials in a single structure, similarly to FMLs but not necessarily following a stacking sequence or even none at all. Many structures, including ships and cars, would typically involve a metallic skeletal frame with composite infill panels supported by the frame and this raises the issue of methods of joining the two components.

In fact, a number of researchers have studied this problem (e.g. [236-239]), including also frequency filtering at the joint to control wave propagation through the joint [240]. Other researchers [241, 242] have also studied optimisation of the hybrid system itself, including the use of genetic algorithms, although such optimisation techniques are still not well developed for engineering applications.

## **2.11 Concluding remarks**

This chapter has reviewed previous research related to the investigation of blast response of plated structures, broadly related to the actual blast loading, the response of monolithic plates to such loading, composite materials and response of composite and hybrid plates to such loading. From this review, these remarks are deduced and have served as the motivation for the work in the subsequent chapters:

- Although the behaviour of explosives, their detonation process and globalised blast loading are relatively well understood, the corresponding work related to localised blast loading is still lacking. Thus, there is a need to study localised blast loads; this will be tackled in Chapter 3.
- There is extensive literature available on analysis of monolithic plates subjected to dynamic and impulsive loads but there is a need to extend existing literature to assess the performance of such plates when subjected to localised blast loading; these issues are addressed in Chapter 4.

- Although various composite systems subjected to blast loading have been studied, such an assessment of new high performance materials is not available in the open literature. In addition, the performance of such materials compared with that of equal weight monolithic plates has not been carried out; this is investigated in Chapter 5.
- The issue of delamination in laminated composite systems was identified but from literature it was found that its onset and progression relies on use of cohesive elements in finite element analysis packages; a method to assess its initiation which does rely on the use of cohesive elements will be explored in Chapter 6.

## Chapter 3. Dimensionless loading parameters for close-in blasts

### 3.1 Nomenclature

In this chapter, the following notation will be used:

#### *Latin lower case*

|       |                                                   |
|-------|---------------------------------------------------|
| $b$   | loading parameter, $[L^{-1}]$                     |
| $d$   | stand-off distance, $[L]$                         |
| $f_i$ | functions of $\left(\frac{d}{D_e}\right)$ , $[1]$ |
| $i$   | non-dimensional impulse, $[1]$                    |
| $m_e$ | mass of explosive, $[M]$                          |
| $r$   | radial axis direction, $[L]$                      |
| $t$   | time, $[T]$                                       |
| $t_d$ | blast duration, $[T]$                             |
| $w$   | loading parameter, $[T^{-1}]$                     |
| $w_f$ | permanent transverse displacement, $[L]$          |
| $z$   | scaled distance, $[M^{-1/3} L]$                   |

#### *Latin upper case*

|           |                                                                  |
|-----------|------------------------------------------------------------------|
| $A$       | explosive material constant, $[M L^{-1} T^{-2}]$                 |
| $B$       | explosive material constant, $[M L^{-1} T^{-2}]$                 |
| $C_p$     | gas specific heat at constant pressure, $[L T^{-2} \theta^{-1}]$ |
| $C_v$     | gas specific heat at constant volume, $[L T^{-2} \theta^{-1}]$   |
| $D_e$     | diameter of explosive, $[L]$                                     |
| $E$       | energy, $[M L^2 T^{-2}]$                                         |
| $E_0^e$   | specific energy of explosive, $[L^2 T^{-2}]$                     |
| $H$       | plate thickness, $[L]$                                           |
| $I$       | impulse, $[M L T^{-1}]$                                          |
| $L$       | plate length, $[L]$                                              |
| $P$       | pressure, $[M L^{-1} T^{-2}]$                                    |
| $P_0$     | maximum overpressure, $[M L^{-1} T^{-2}]$                        |
| $P_a$     | overpressure, $[M L^{-1} T^{-2}]$                                |
| $P(r, t)$ | loading function, $[M L^{-1} T^{-2}]$                            |
| $P_s(r)$  | spatial part of pressure pulse load, $[M L^{-1} T^{-2}]$         |
| $P_t(t)$  | temporal part of pressure pulse load, $[M L^{-1} T^{-2}]$        |
| $Q_e$     | explosive heat of combustion, $[M T^{-2}]$                       |
| $R$       | radius of target, $[L]$                                          |
| $R_0$     | radius of central uniformly-loaded region, $[L]$                 |
| $R_1$     | explosive material constant, $[1]$                               |
| $R_2$     | explosive material constant, $[1]$                               |
| $R_e$     | radius of explosive, $[L]$                                       |
| $T$       | gas temperature, $[\theta]$                                      |
| $W$       | plate width, $[L]$                                               |

### *Greek lower case*

|               |                                                                                    |
|---------------|------------------------------------------------------------------------------------|
| $\alpha_i$    | dimensional analysis variables, [1]                                                |
| $\beta$       | charge-shape constant, [ $M^{1/3} L$ ]                                             |
| $\varepsilon$ | strain, [1]                                                                        |
| $\lambda$     | loaded radius ratio, [1]                                                           |
| $\lambda_0$   | uniformly-loaded radius ratio, [1]                                                 |
| $\xi_{q,d}$   | Jacob et al.'s stand-off parameter, [1]                                            |
| $\xi_{q,l}$   | Jacob et al.'s loading parameter, [1]                                              |
| $\rho$        | density of plate material, [ $M L^{-3}$ ]                                          |
| $\rho_a$      | density of air, [ $M L^{-3}$ ]                                                     |
| $\rho_e$      | density of explosive, [ $M L^{-3}$ ]                                               |
| $\rho_p$      | density of explosive product, [ $M L^{-3}$ ]                                       |
| $\phi_{q,S}$  | Jacob et al.'s damage parameter for quadrangular plates with stand-off effect, [1] |
| $\sigma$      | stress, [ $M L^{-1} T^{-2}$ ]                                                      |
| $\sigma_0$    | yield stress, [ $M L^{-1} T^{-2}$ ]                                                |
| $\omega$      | explosive material constant, [1]                                                   |

### *Greek upper case*

|         |                                        |
|---------|----------------------------------------|
| $\Pi_i$ | Buckingham's dimensionless groups, [1] |
|---------|----------------------------------------|

## **3.2 Background**

As discussed in Chapters 1 and 2, proximal blasts cause particular threats to both human life and civil and military infrastructure. The source of such blasts can be, for example, IEDs (improvised explosive devices) and buried land mines exploding onto the underside of military vehicles. When blasts occur in close proximity to engineering structures, the localised effect of the load causes particular forms of damage to the structure and thus require particular consideration which is different from the case of global (or uniform) blast loading [1].

### **3.2.1 Localised blast loading**

Various researchers have investigated the effect of localised blast on a variety of structural forms and material systems.

Qin [2] studied the response of sandwich and monolithic beams under localised impulsive loading. Florence [3, 4] and Conroy [5] provide analytical expressions describing the response of circular steel plates subjected to uniform localised loadings. Lee and Wierzbicki [6, 7] studied particular forms of failure in steel plates under localised blast loading.

Many other researchers (e.g. [8-12]) carried out considerable work related to localised blasts on steel plates, including the influence of stand-off on the response [10] and the effect of boundary conditions on plates subjected to localised blast loading [12].

Langdon et al. [13-20] have extensively studied localised blast loading on sandwich panels and FMLs (fibre-metal-laminates), including studies on mathematical descriptions of a localised blast load [15, 20], described by (2-8) in Chapter 2.

### 3.2.2 Non-dimensionalisation

Dimensionless parameters are useful tools for scaling the effects of load and a structure's response to it and also to avoid unnecessary repetition of, for example, experiments and various researchers have used non-dimensional parameters in their works.

However, dimensional analyses found in literature (the most relevant of which have been outlined in Chapter 2) typically deal with providing expressions for the response of structures subjected to loading, rather than expressions to describe the actual blast loading, given a set of threat parameters.

### 3.3 Aim of current work

Thus, the objective of the work in this chapter is to develop dimensionless expressions for a set of unique parameters which will define the variation of a localised blast, both spatially and temporally.

These parameters are functions of a set of known loading inputs (viz. explosive type, charge mass and diameter and stand-off distance). The dimensionless parameters are then used to define a loading distribution on a structure such that it can be easily input into commercial finite element packages for analysis.

The results from the exercise are validated by comparison with experimental results from blast load tests on steel panels recently tested at UCT (University of Cape Town) in South Africa.

### 3.4 Non-dimensionalised loading parameters

#### 3.4.1 Loading description

Throughout this chapter and forthcoming chapters, it is assumed that a localised blast will be generated by means of a certain mass,  $m_e$ , of explosive material having a heat of explosion,  $Q_e$ , having a diameter,  $D_e$ , and which is acting at a stand-off distance,  $d$ . Thus, the parameters which uniquely describe the load source are  $d$ ,  $m_e$ ,  $Q_e$  and  $D_e$ .

It is also assumed that the localised blast can be described by an analytical function combining the effects of the temporal,  $P_t(t)$ , and spatial,  $P_s(r)$ , variations to define a loading distribution function  $P(r, t)$  of the form:

$$P(r, t) = P_s(r)P_t(t) \quad (3-1)$$

where

$$P_s(r) = \begin{cases} P_0, & 0 \leq r \leq R_0 \\ P_a e^{-br}, & r > R_0 \end{cases} \quad (3-2)$$

and

$$P_t(t) = \begin{cases} e^{-wt}, & 0 \leq t \leq t_d \\ 0, & t > t_d \end{cases} \quad (3-3)$$

For the purpose of this work, a zero rise time is assumed and  $t_d$  is the duration of the positive phase of the blast,  $P_0$  is a value of constant pressure acting over a central circular region of radius  $R_0$  and  $b$  and  $w$  are exponential decay factors describing the variation of the spatial and temporal functions respectively. Thus, the parameters which describe the loading profile completely are  $R_0$ ,  $P_0$ ,  $t_d$ ,  $b$  and  $w$ .

It should be noted that the parameter  $P_a$  in (3-2) is not a further independent loading parameter, since  $P_a$  attains the value of  $P_0$  when the post-multiplier  $e^{-br}$  is evaluated at  $r = R_0$ . Thus,  $P_a = P_0 e^{bR_0}$ .

The loading distribution is shown in Figure 3-1:

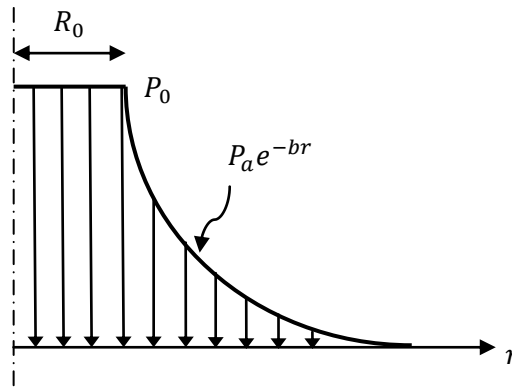


Figure 3-1: Load spatial distribution,  $P_s(r)$

Such a loading description defined by (3-1) has been used by various other researchers in the past (e.g. [8, 12, 15, 20]), as described in Chapter 2. Whilst this has been shown to be a reasonable and realistic function for localised blast loads, there are various justifications for such an assumption.

Whilst an intrinsic assumption is that the loading description is valid only for air blasts, experimental and numerical work (e.g. [20]) has shown that the overpressure is observed to decrease rapidly with increasing distance from underneath the charge. Thus, the ratio of the maximum pressure amplitude to the minimum one is very high. In addition, if simultaneous maximum loading is assumed, then this leads to a conservative loading scenario, making the assumption that the pressure acting on the target can be well-described by the product of two exponentially decaying functions,  $P_s(r)$  and  $P_t(t)$ .

This would, however, be only realistic and accurate for blast loading acting onto a reflecting surface in an unconfined space, i.e., where no blast wave diffraction occurs, there is no occurrence of drag loading and there is no pressure re-reflection.

### 3.4.2 Dimensional analysis

A non-dimensionalisation exercise was carried out employing Buckingham's  $\Pi$  theorem [21] and the non-dimensional parameters were extracted, using the procedure below.

|       | <b>M</b> | <b>L</b> | <b>T</b> | <b><math>\alpha_i</math></b> |
|-------|----------|----------|----------|------------------------------|
| $P_0$ | 1        | -1       | -2       | $\alpha_1$                   |
| $t_d$ | 0        | 0        | 1        | $\alpha_2$                   |
| $m_e$ | 1        | 0        | 0        | $\alpha_3$                   |
| $D_e$ | 0        | 1        | 0        | $\alpha_4$                   |
| $b$   | 0        | -1       | 0        | $\alpha_5$                   |
| $R_0$ | 0        | 1        | 0        | $\alpha_6$                   |
| $d$   | 0        | 1        | 0        | $\alpha_7$                   |
| $Q_e$ | 0        | 2        | -2       | $\alpha_8$                   |
| $w$   | 0        | 0        | -1       | $\alpha_9$                   |

Table 3-1: Dimensional analysis of loading parameters

From Table 3-1, it can be seen that the rank of the dimensional matrix of equations is 3 and thus, from the 9  $\alpha_i$  variables defining the problem, it can be concluded that there are  $(9 - 3 =) 6$  dimensionless variables which describe this particular problem.

Writing the co-efficients for each dimensional quantity, the following equations are extracted:

$$\begin{cases} \alpha_1 + \alpha_3 = 0 \\ -\alpha_1 + \alpha_4 - \alpha_5 + \alpha_6 + 2\alpha_1 + \alpha_8 = 0 \\ -2\alpha_1 + \alpha_2 - 2\alpha_8 - \alpha_9 = 0 \end{cases} \quad (3-4)$$

By letting  $\alpha_1 = -\alpha_3$ ,  $\alpha_2 = 2\alpha_1 + 2\alpha_8 + \alpha_9$  and  $\alpha_4 = \alpha_1 + \alpha_5 - \alpha_6 - \alpha_7 - 2\alpha_8$ , then the following 6 dimensionless parameters are extracted by combining co-efficients for each  $\alpha_i$  as follows:

$$\Pi_1 = \frac{P_0 t_d^2 D_e}{m_e} \quad (3-5)$$

$$\Pi_2 = \frac{Q_e t_d^2}{D_e^2} \quad (3-6)$$

$$\Pi_3 = \frac{R_0}{D_e} \quad (3-7)$$

$$\Pi_4 = D_e b \quad (3-8)$$

$$\Pi_5 = w t_d \quad (3-9)$$

$$\Pi_6 = \frac{d}{D_e} \quad (3-10)$$

Through some algebraic manipulation, it was found that the 5 unknown parameters  $R_0, P_0, t_d, b$  and  $w$  are all functions of  $\left(\frac{d}{D_e}\right)$  as follows:

$$P_0 = \frac{Q_e m_e}{D_e^3} \left\{ f_1 \left( \frac{d}{D_e} \right) \right\} \quad (3-11)$$

$$t_d = \frac{D_e}{\sqrt{Q_e}} \left\{ f_2 \left( \frac{d}{D_e} \right) \right\} \quad (3-12)$$

$$w = \frac{\sqrt{Q_e}}{D_e} \left\{ f_3 \left( \frac{d}{D_e} \right) \right\} \quad (3-13)$$

$$R_0 = D_e \left\{ f_4 \left( \frac{d}{D_e} \right) \right\} \quad (3-14)$$

$$b = \frac{1}{D_e} \left\{ f_5 \left( \frac{d}{D_e} \right) \right\} \quad (3-15)$$

Equations (3-11) to (3-15) suggest that the loading parameters depend on the ratio  $\left( \frac{d}{D_e} \right)$ , which can very easily be shown to be a dimensionless version of the scaling parameter (or scaled distance,  $z$ ) used to obtain such loading parameters.

Recalling that  $z = \frac{d}{\sqrt[3]{m_e}}$  as described in Chapter 2, then:

$$z = \beta \left( \frac{d}{D_e} \right) \quad (3-16)$$

where  $\beta$  is a charge-shape constant given by:

$$\beta = \left( \frac{4}{\pi \rho_e} \right)^{\frac{1}{3}} \left( \frac{D_e}{H_e} \right)^{\frac{1}{3}} \quad (3-17)$$

for a cylindrical charge and

$$\beta = \left( \frac{6}{\pi \rho_e} \right)^{\frac{1}{3}} \quad (3-18)$$

for a spherical charge.

Thus, it is now necessary to establish the form of the functions  $f_i \left( \frac{d}{D_e} \right)$  shown in equations (3-11) to (3-15).

### 3.5 Parametric studies using AUTODYN

For the purpose of this study, the range of  $\left( \frac{d}{D_e} \right)$  being considered is within the range  $0.5 \leq \left( \frac{d}{D_e} \right) \leq 6$ , which is deemed to be a representative range of practical loading scenarios which can be described as being “localised”.

A set of 12 models was compiled using AUTODYN v.13.0 [22], a hydrocode specifically designed for non-linear dynamic analysis and routinely used to simulate blast loadings. A 2D axially-symmetric model was set up representing a cylindrical space of 300 mm diameter and 500 mm height and meshed uniformly with an Eulerian mesh of uniform 1 mm size, as shown in Figure 3-2.

Flow-out boundary conditions were assigned to the curved face and 1 flat face while the other flat face was modelled as a rigid boundary.

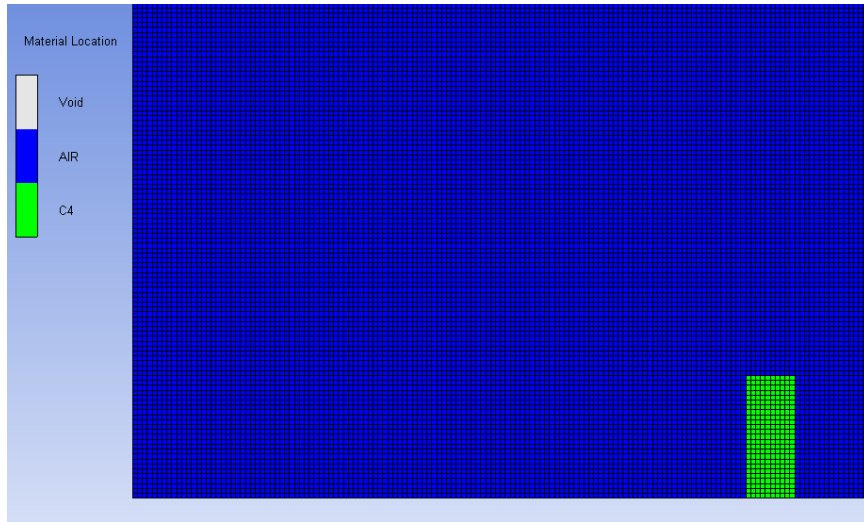


Figure 3-2: Partial view of model showing finite element mesh

### 3.5.1 Air modelling

The space was filled with air using material data from the AUTODYN material library using an “ideal gas” equation of state, described by:

$$P = \rho_a \left( \frac{C_p}{C_v} - 1 \right) C_v T \quad (3-19)$$

where  $P$  is the gas pressure and the material properties for air were taken as the default values assigned by AUTODYN, viz.,  $\rho_a = 1.225 \text{ kg/m}^3$  is the density of air,  $C_p = 1.005 \text{ kJ/kgK}$  is the specific heat at constant pressure,  $C_v = 0.7176 \text{ kJ/kgK}$  is the specific heat at constant volume and  $T = 288.2 \text{ K}$  is the gas temperature.

The internal energy of air was assigned as  $2.068\text{E}5 \text{ kJ/kg}$ , which sets the ambient pressure to  $101 \text{ kPa}$  (or atmospheric pressure at room temperature and at sea level).

### 3.5.2 Explosive modelling

The explosive charge was modelled as a shallow cylindrical block of C4 (PE4) explosive of constant diameter,  $D_e$ , of  $50 \text{ mm}$  and constant mass,  $m_e$ , of  $31.4 \text{ g}$ . The JWL (Jones-Wilkins-Lee) equation of state was used to describe the explosive’s detonation behaviour:

$$P = A \left( 1 - \frac{\omega \rho_p}{R_1 \rho_e} \right) e^{-\frac{R_1 \rho_e}{\rho_p}} + B \left( 1 - \frac{\omega \rho_p}{R_2 \rho_e} \right) e^{-\frac{R_2 \rho_e}{\rho_p}} + \omega \rho_e E_0^e \quad (3-20)$$

where  $P$  is the pressure,  $\rho_e = 1601 \text{ kg/m}^3$  is the density of the explosive,  $\rho_p$  is the density of the explosive product,  $E_0^e = 5.621488\text{E}6 \text{ kJ/kg}$  is the explosive's specific internal energy and  $A = 609.77 \text{ GPa}$ ,  $B = 12.95 \text{ GPa}$ ,  $R_1 = 4.5$ ,  $R_2 = 1.4$  and  $\omega = 0.25$  are empirically-derived material constants. The detonation point was assumed to be at the centre of mass of the explosive.

### 3.5.3 Modelling results

In each of the models, the stand-off distance was varied, from 25 mm to 300 mm in 25 mm increments. In each case, the pressure value on the rigid face was monitored at 60 gauge points placed at 5 mm intervals, as shown in Figure 3-3:

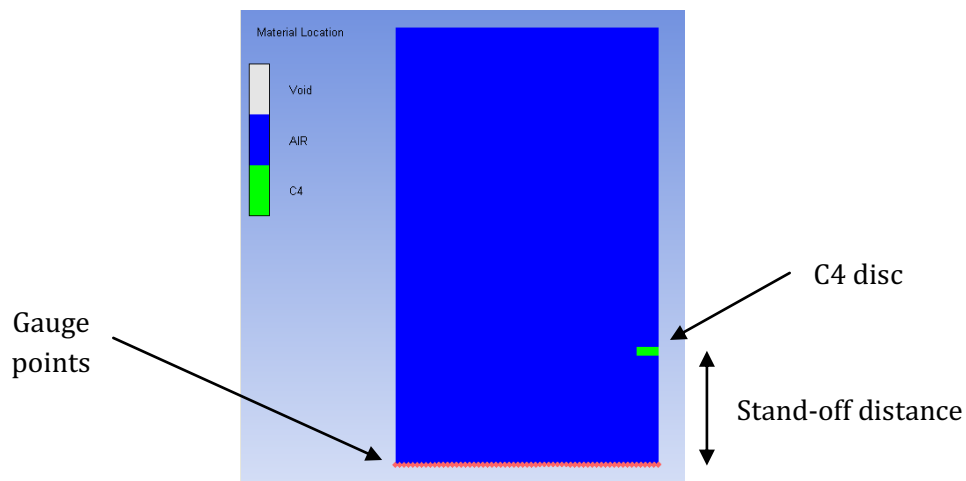


Figure 3-3: AUTODYN model setup (half model shown)

In each case, the detonation was numerically simulated and the event was modelled up to a duration of 0.2 ms, monitoring the pressure in the domain (e.g. Figure 3-4 to Figure 3-6) at the gauge points.

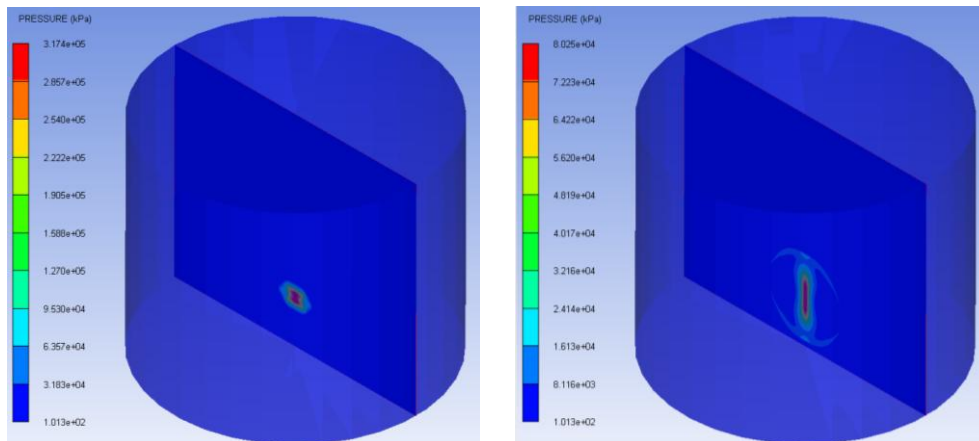


Figure 3-4: Pressure distribution for 100 mm stand-off (left: 0.01 ms, right: 0.02 ms)

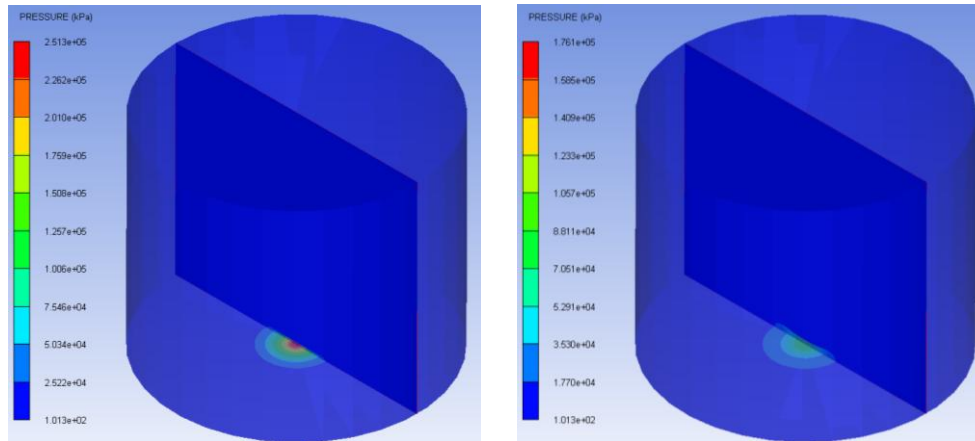


Figure 3-5: Pressure distribution for 100 mm stand-off (left: 0.03 ms, right: 0.04 ms)

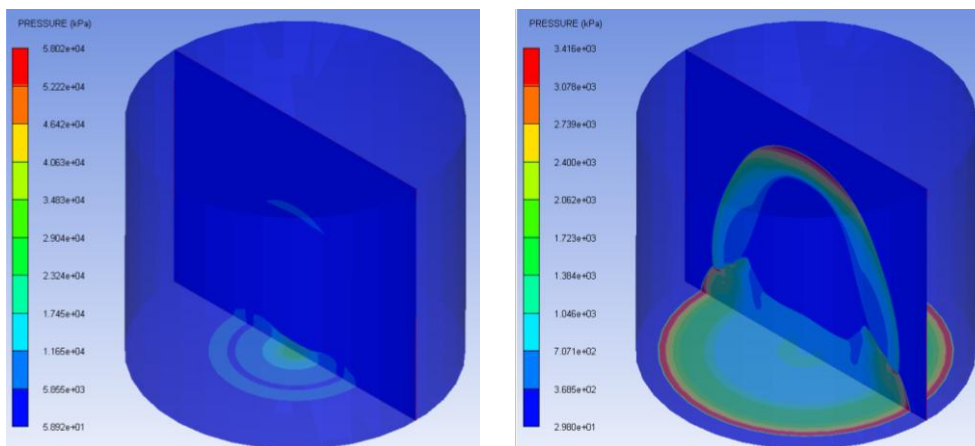


Figure 3-6: Pressure distribution for 100 mm stand-off (left: 0.05 ms, right: 0.1 ms)

From each of the gauge points, the pressure-time histories were obtained. A selection of these results from one of the models is shown in Figure 3-7.

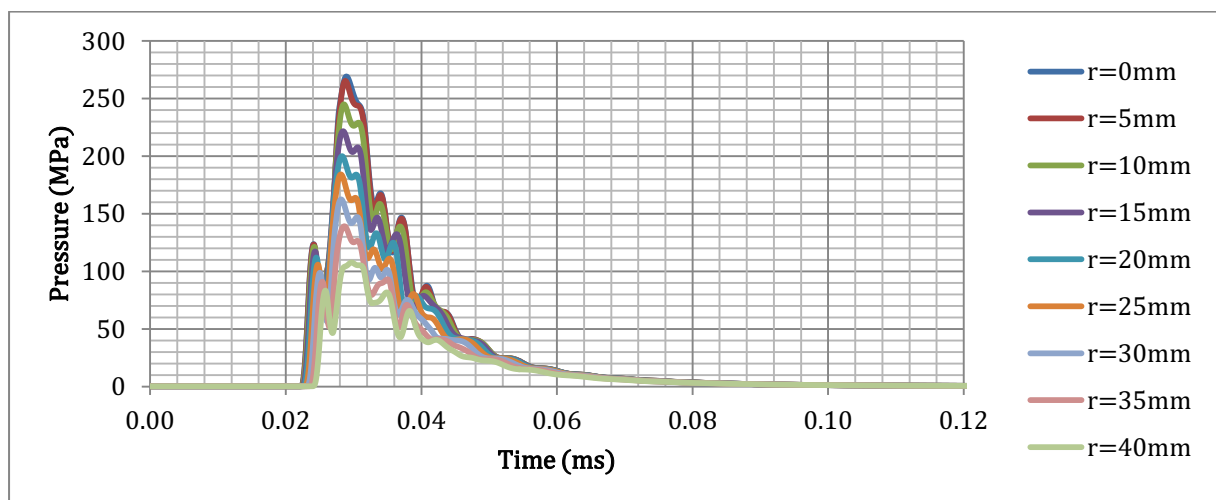


Figure 3-7: Selection of pressure-time histories for 100 mm stand-off

For each model, the variation of pressure with space and time could be established, assuming the relationships in (3-2) and (3-3). The curve fitting was done using built-in curve fitting tool in MATLAB R2010b [23] and the parameters  $R_0$ ,  $P_0$ ,  $t_d$ ,  $b$  and  $w$  were extracted.

An example of the fits is shown in Figure 3-8 and Figure 3-9:

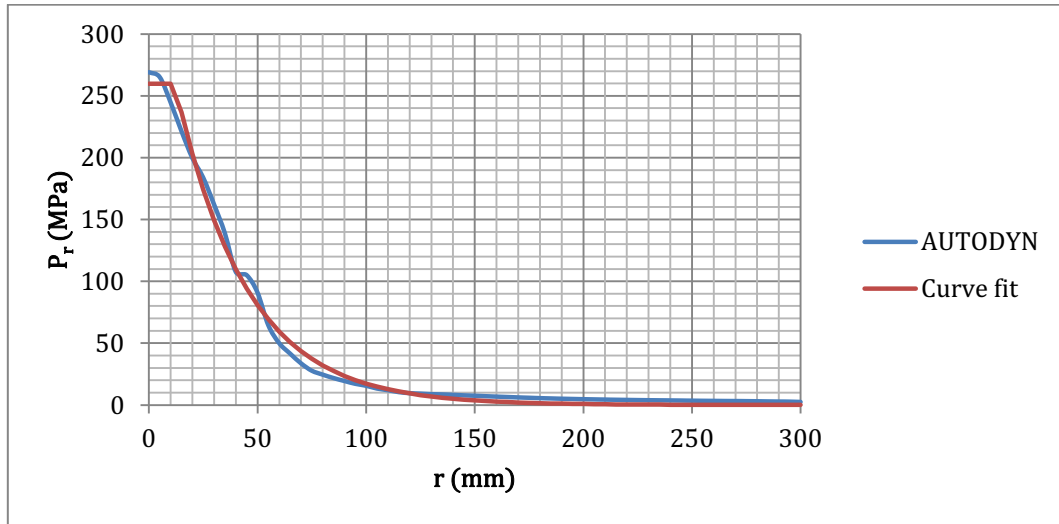


Figure 3-8: Spatial load variation for 100 mm stand-off

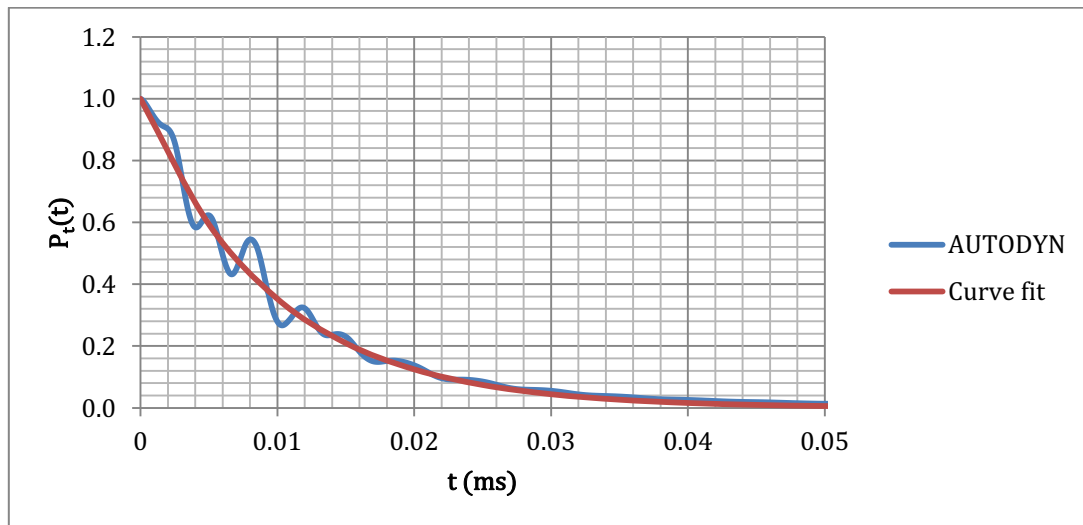


Figure 3-9: Temporal load variation for 100 mm stand-off

### 3.6 Loading profile equations and charts

Using the results obtained during the process outline in Section 3.5, a series of least-square fitting exercises were carried out using the built-in curve fitting tool in MATLAB [23].

Thus, the functions  $f_i$  in equations (3-11) to (3-15) were established as follows:

$$f_1 = (0.4994) \left( \frac{d}{D_e} \right)^{-1.391} \quad (3-21)$$

$$f_2 = (0.9114)e^{0.4418(\frac{d}{D_e})} \quad (3-22)$$

$$f_3 = \begin{cases} 6.055 e^{-2.17(\frac{d}{D_e})} + 2.389 e^{-0.03591(\frac{d}{D_e})}, & 0 \leq \left(\frac{d}{D_e}\right) \leq 3.5 \\ -1.76\left(\frac{d}{D_e}\right) + 8.25, & 3.5 \leq \left(\frac{d}{D_e}\right) \leq 4.0 \\ -0.226\left(\frac{d}{D_e}\right) + 2.136, & 4.0 \leq \left(\frac{d}{D_e}\right) \leq 6.0 \end{cases} \quad (3-23)$$

$$f_4 = \begin{cases} -0.07\left(\frac{d}{D_e}\right) + 0.375, & 0 \leq \left(\frac{d}{D_e}\right) \leq 2.0 \\ 0.2547\left(\frac{d}{D_e}\right) - 0.2542, & 2.0 \leq \left(\frac{d}{D_e}\right) \leq 6.0 \end{cases} \quad (3-24)$$

$$f_5 = (2.528)\left(\frac{d}{D_e}\right)^{-0.7119} \quad (3-25)$$

These are shown graphically in Figure 3-10 to Figure 3-14.

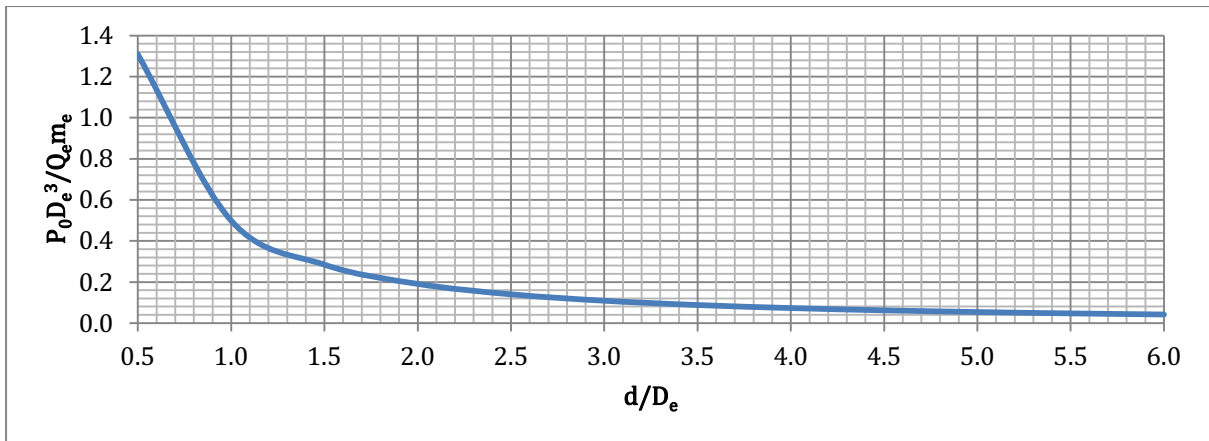


Figure 3-10: Non-dimensionalised constant pressure

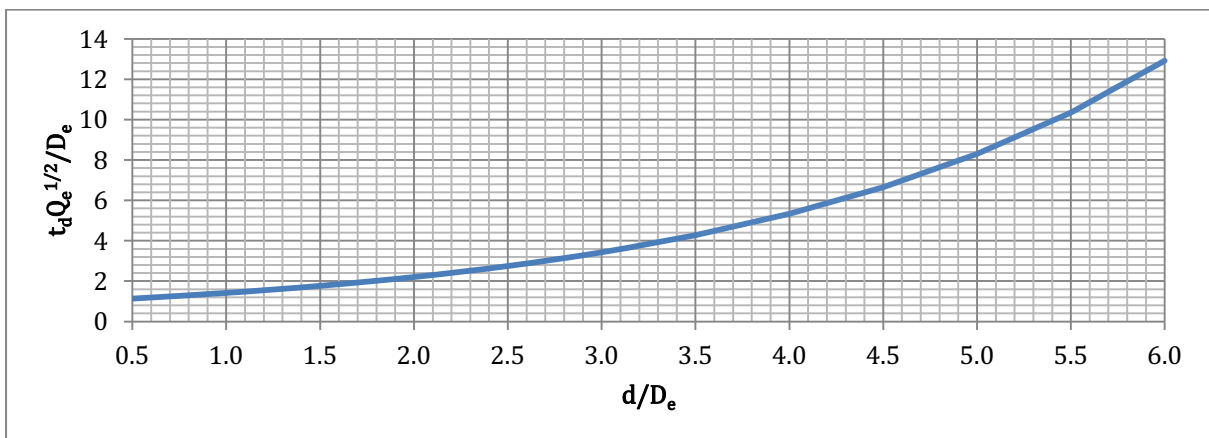


Figure 3-11: Non-dimensionalised duration

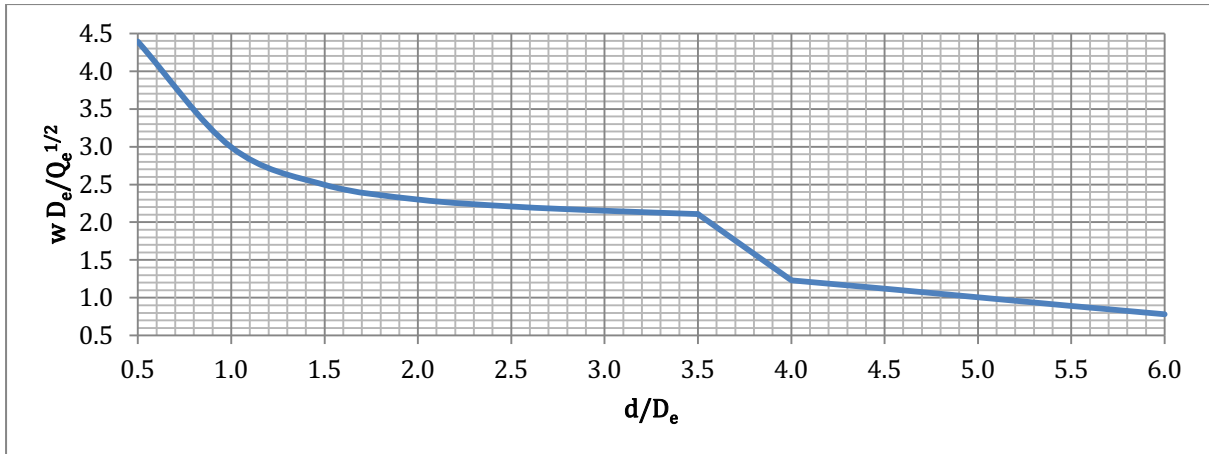


Figure 3-12: Non-dimensionalised temporal decay factor

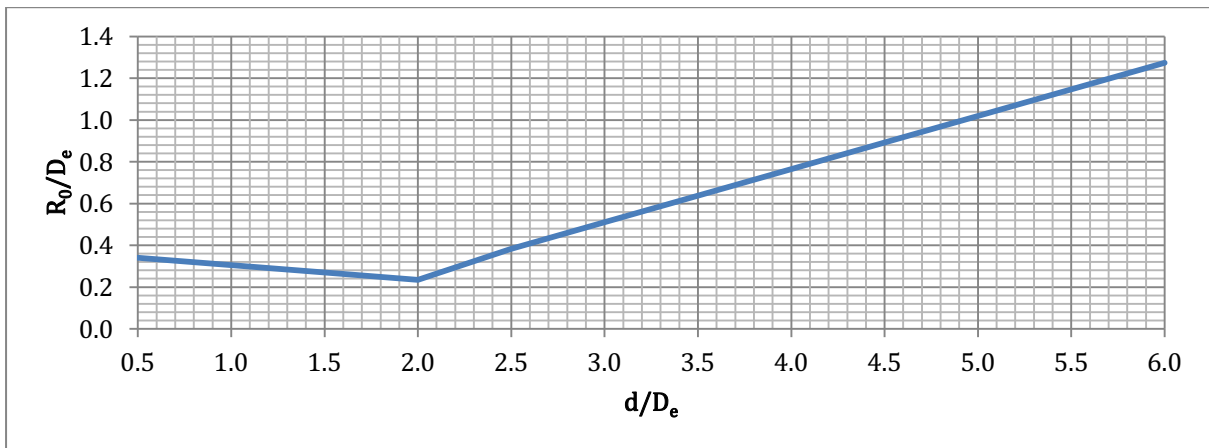


Figure 3-13: Non-dimensionalised constant pressure radius

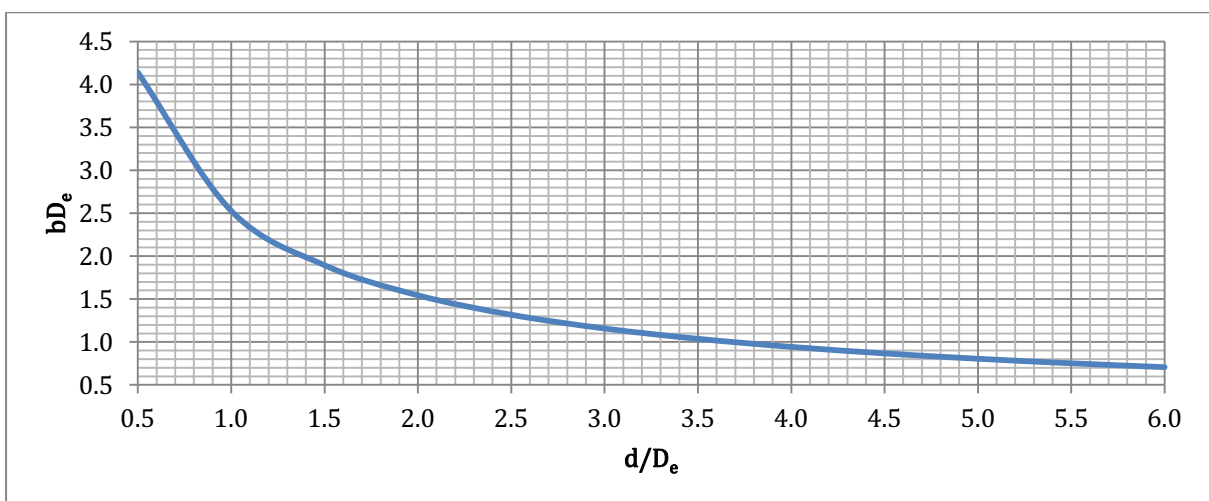


Figure 3-14: Non-dimensionalised spatial decay factor

Although these relationships have been calibrated using a series of models with an explosion scenario with  $D_e = 50$  mm and where  $d$  is varied, the results are still valid and applicable for any other charge diameter/stand-off distance combination, provided that the  $d/D_e$  ratio falls within the applicable range discussed in Section 3.5.

To illustrate this, an explosion scenario with a C4 cylindrical charge having  $D_e = 40$  mm,  $d = 38$  mm and  $m_e = 40$  g was modelled in AUTODYN and the pressure and temporal distributions obtained are compared with the corresponding ones obtained by using the derived relationships. In this case, one gets  $R_0 = 10.75$  mm,  $P_0 = 1317$  MPa,  $t_d = 0.0269$  ms,  $b = -54.72$  /m and  $w = 159162$  /s. These are shown in Figure 3-15 and Figure 3-16.

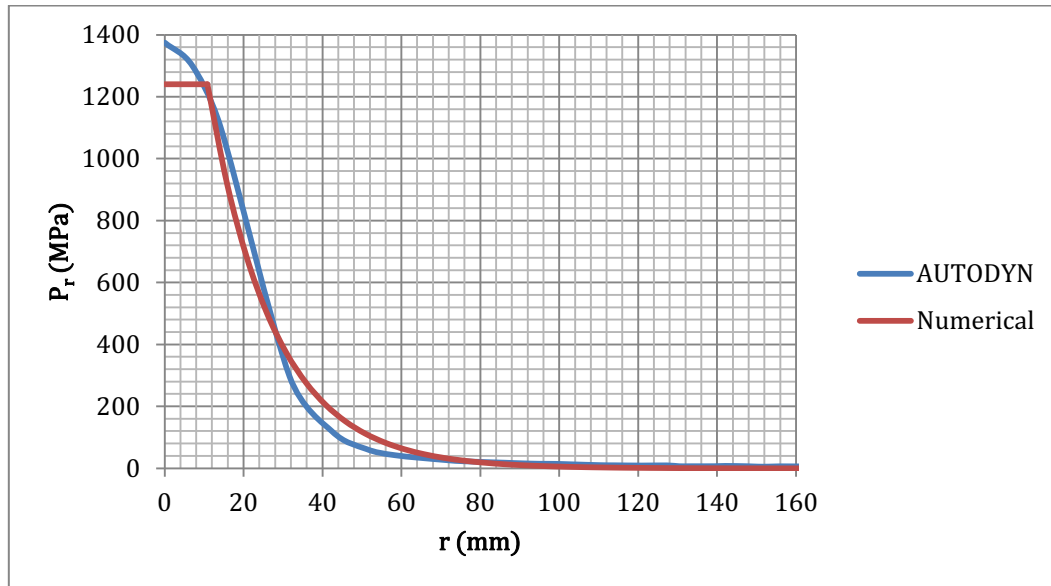


Figure 3-15: Spatial load variation for case with  $D_e = 40$  mm

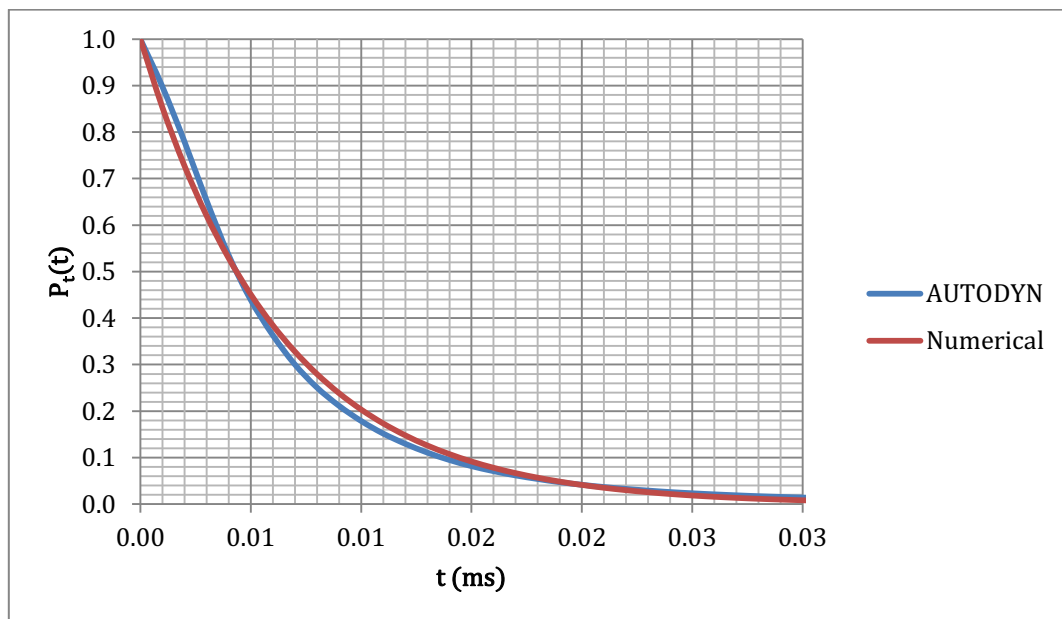


Figure 3-16: Temporal load variation for case with  $D_e = 40$  mm

It can be seen that there is good agreement in both spatial and temporal distributions, demonstrating that the proposed relationships are applicable for any  $d/D_e$  combination within the limiting range  $0.5 \leq \left(\frac{d}{D_e}\right) \leq 6$ .

### 3.7 Variation of impulse with loading

Having established the form of the loading (spatial and temporal) distributions, a discussion on the variation of impulse with the various loading parameters is presented hereunder.

From (3-1) and using (3-2) and (3-3), an expression for the impulse in terms of the loading radius  $r$  could be written as:

$$I(r) = 2\pi \int_0^{t_d} \int_0^r P(r, t) r dr dt \quad (3-26)$$

Considering a target of radius  $R$  and defining the ratio  $\lambda = \frac{r}{R}$  and  $\lambda_0 = \frac{R_0}{R}$  and substituting in (3-26), then  $I(\lambda)$  could be written as:

$$I(\lambda) = \begin{cases} \frac{\pi P_0 (1 - e^{-wt_d}) (\lambda R)^2}{w}, & \lambda < \lambda_0 \\ \frac{2\pi P_0 (1 - e^{-wt_d}) e^{-\lambda_0 R b}}{w b^2} \{e^{-\lambda_0 R b} [1 + \lambda_0 R b] - e^{-b\lambda R} [1 + \lambda R b]\}, & \lambda \geq \lambda_0 \end{cases} \quad (3-27)$$

The total impulse which a given charge could potentially deliver to the target would be given by:

$$I(\infty) = \lim_{\lambda \rightarrow \infty} \{I(\lambda)\} = \frac{(1 - e^{-wt_d})}{w} \left\{ \pi P_0 (\lambda_0 R)^2 + \frac{2\pi P_0 (1 + \lambda_0 R b)}{b^2} \right\} \quad (3-28)$$

Thus, a non-dimensional impulse,  $i$ , could be defined as the ratio of the impulse imparted to a target to the total impulse which the threat generates:

$$i = \frac{I(\lambda)}{I(\infty)} = \begin{cases} \frac{(\lambda_0 R b)^2}{2 + 2\lambda_0 R b + (\lambda_0 R b)^2}, & \lambda < \lambda_0 \\ \frac{2 + 2\lambda_0 R b + (\lambda_0 R b)^2 - 2e^{-Rb(\lambda - \lambda_0)} [1 + \lambda R b]}{2 + 2\lambda_0 R b + (\lambda_0 R b)^2}, & \lambda \geq \lambda_0 \end{cases} \quad (3-29)$$

The variation of  $i$  with  $\lambda$  for  $\lambda \geq \lambda_0$  (which is the case for practical applications) for various values of  $\left(\frac{D_e}{R}\right)$  at different values of  $\left(\frac{d}{D_e}\right)$  is shown graphically in Figure 3-17 to Figure 3-20.

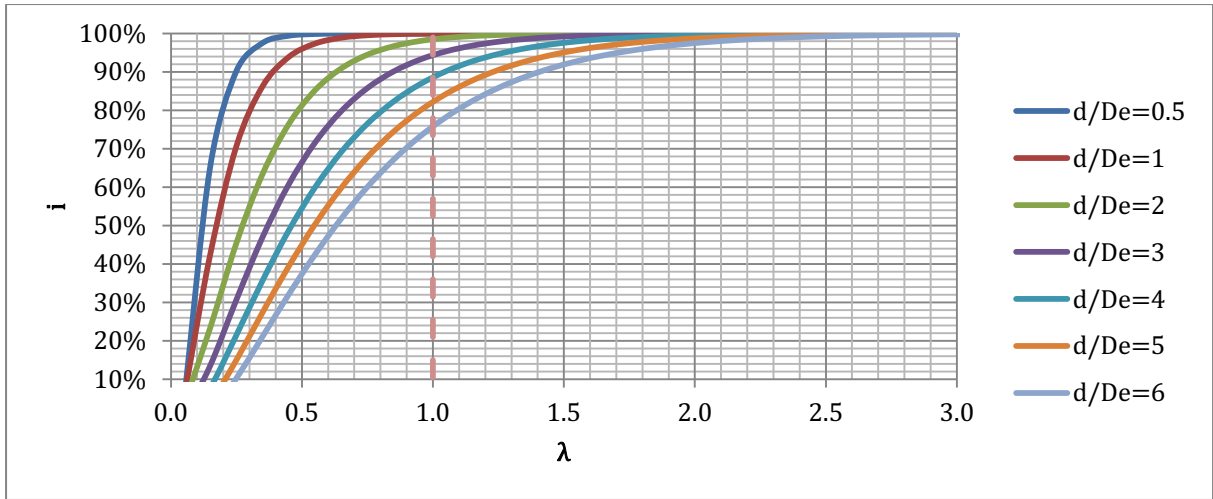


Figure 3-17: Variation of  $i$  with  $\lambda$  for  $De/R = 0.25$

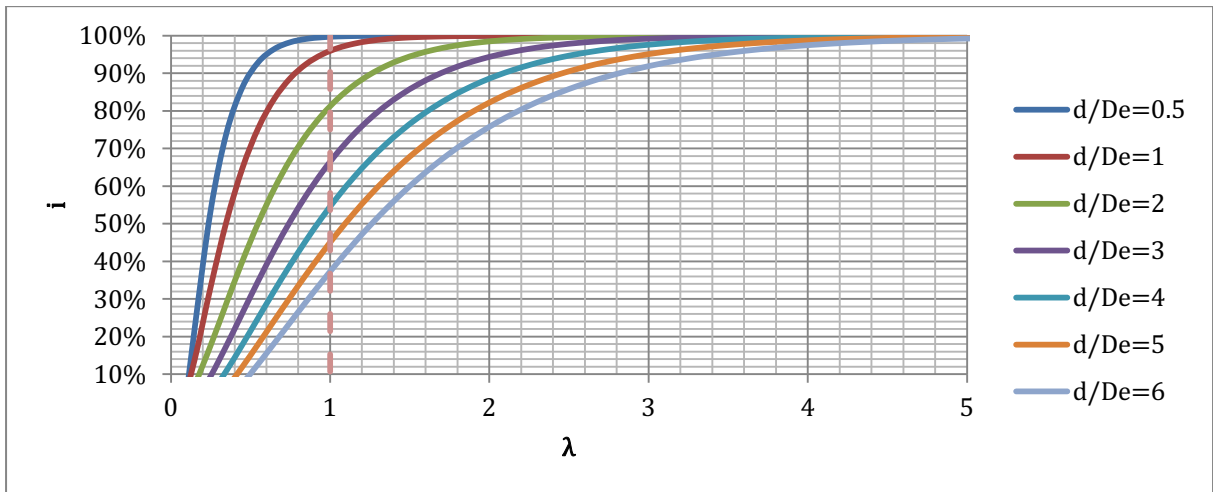


Figure 3-18: Variation of  $i$  with  $\lambda$  for  $De/R = 0.5$

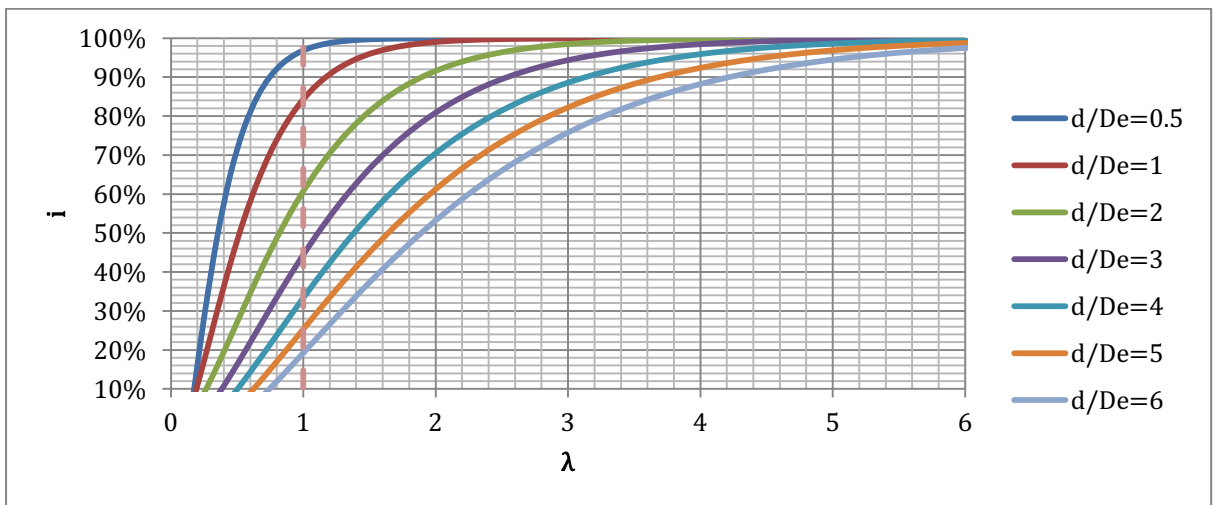


Figure 3-19: Variation of  $i$  with  $\lambda$  for  $De/R = 0.75$

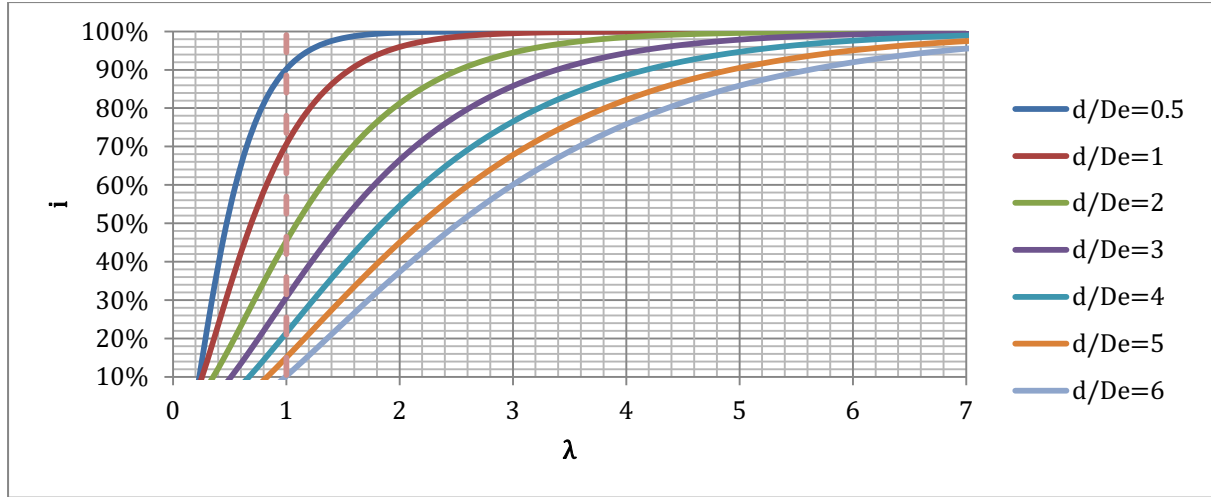


Figure 3-20: Variation of  $i$  with  $\lambda$  for  $D_e/R = 1$

It can be observed that, as  $\left(\frac{D_e}{R}\right)$  increases, then for a given stand-off to charge diameter ratio, i.e.,  $\left(\frac{d}{D_e}\right)$ , the ratio of the total impulse imparted to the target decreases. Thus, it can be concluded that the more significant threat scenario would be a small stand-off to charge diameter ratio, rather than a large charge diameter to target plate diameter ratio.

It is also worth noting that, for low values of  $\left(\frac{D_e}{R}\right)$ , there is little reduction in the relative impulse with increasing charge stand-off and the impulse ratio is still reasonably high even for distant charges. For example, for  $\left(\frac{D_e}{R}\right) = 0.25$ ,  $i = 75\%$  for  $\left(\frac{d}{D_e}\right) = 6$ .

### 3.8 Correlation of numerical results with experimental data

The results derived in Section 3.6 were verified by comparing experimental test data with numerical results which utilise the proposed model parameters.

The validation was done using metallic plates, whose material performance is well-known and understood in order to gain confidence in the numerical results.

#### 3.8.1 Test setup

Laboratory testing<sup>2</sup> of localised blast loading on steel plates was carried out at the BISRU (Blast and Impact Survivability Research Unit) at the Department of Mechanical Engineering within UCT using the ballistic pendulum setup therein.

This setup has long-been used for laboratory-scale blast testing of plated structures of various materials, including metallic [8-10, 24-28] and also composite-based materials, such as FMLs [13, 15-19] and sandwich panels [29, 30].

<sup>2</sup> The work of Professor Genevieve S. Langdon of BISRU is hereby acknowledged.

Typically, the test panel is clamped and mounted at one end of the ballistic pendulum via spacer rods which allow the test panel to deform without touching the pendulum itself. At the other end of the pendulum, a recording pen is fixed, which traces the swing of pendulum as the blast occurs, thus recording the oscillation. From the amplitude of this oscillation and knowing the mass of the pendulum, the impulse can be determined, using standard single-degree-of-freedom structural dynamics theory.

The setup is shown schematically in Figure 3-21.

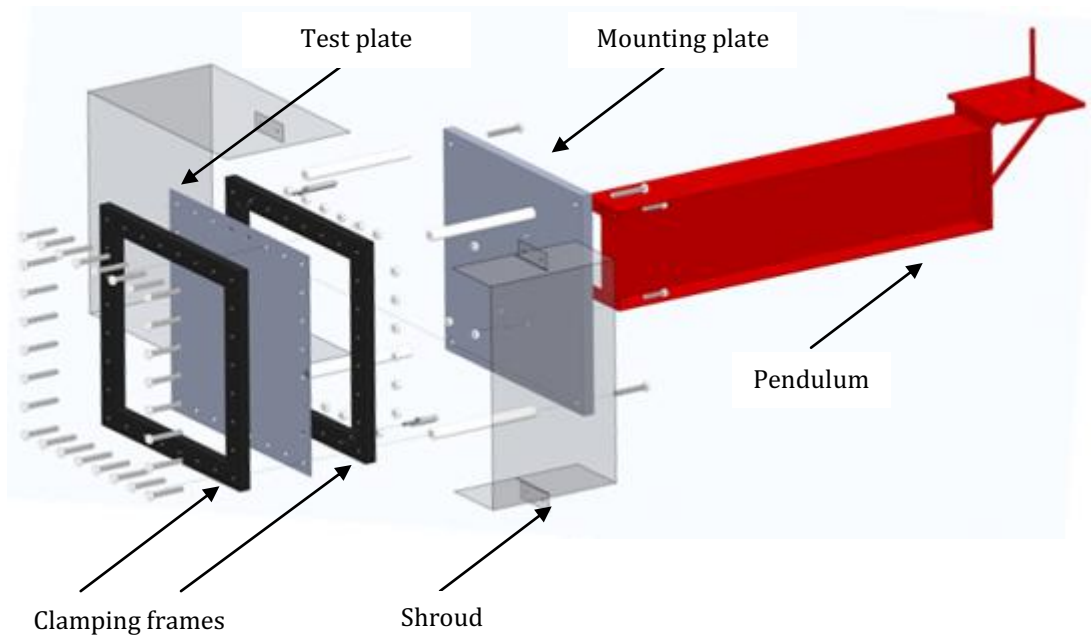


Figure 3-21: Schematic test arrangement

The actual blast is delivered via a centrally-located disc of PE4 (or C4) plastic explosive of known mass. The charge is mounted on a polystyrene bridge to achieve the required stand-off distance, as shown in Figure 3-22.

Detonation is remotely initiated at the centre of the charge by means of an electrical detonator, shown in Figure 3-23.

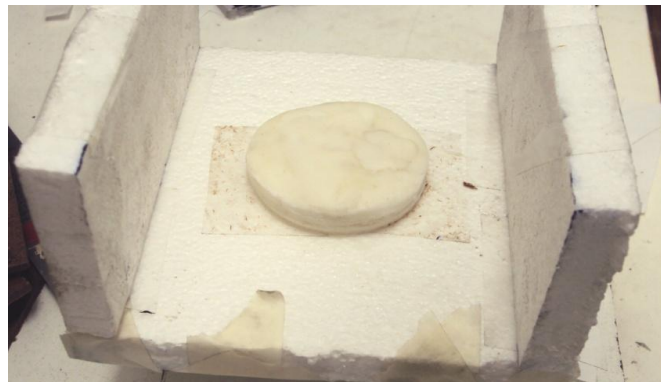


Figure 3-22: PE4 charge mounted on polystyrene bridge

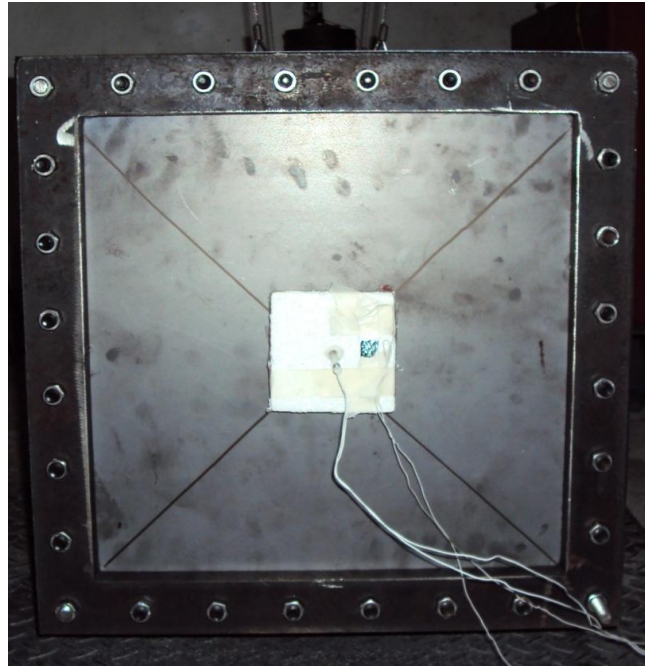


Figure 3-23: Polystyrene bridge and detonator in position

### 3.8.2 Test materials

In this set of experiments, 400x400x3.8 mm fully-clamped panels with an exposed area of 300x300 mm were subjected to a centrally-located cylindrical disc of PE4 explosive.

Two types of plates were tested, viz. mild steel sourced locally in South Africa (with a known static yield stress of 300 MPa) and a variety of armour steel, namely, Armox 370T Class 1, sourced from SSAB AB of Sweden.

SSAB, a firm based south of Stockholm on the Baltic coast, develops and manufactures high-performance steel plates, including the Armox range, known for its high toughness in relation to hardness [31].

Various grades of Armox steel exist, including the 370T Class 1, which was used in this study. The chemical composition of the material is shown in Table 3-2 while the mechanical properties quoted by the manufacturer are shown in Table 3-3.

| B     | C    | Cr | Si  | Mn  | Mo  | Ni  | P     | S    |
|-------|------|----|-----|-----|-----|-----|-------|------|
| 0.005 | 0.32 | 1  | 0.4 | 1.2 | 0.7 | 1.8 | 0.015 | 0.01 |

Table 3-2: Maximum percentage chemical composition for Armox 370T Class 1 [32]

| Hardness<br>(HB) | 0.2% yield strength<br>(MPa) | Tensile strength<br>(MPa) | Elongation<br>(%) |
|------------------|------------------------------|---------------------------|-------------------|
| 380-430          | 1000                         | 1150-1350                 | 8                 |

Table 3-3: Mechanical properties for Armox 370T Class 1 (3 to 3.9 mm thickness) [32]

More complete material data was obtained from tensile testing<sup>3</sup> of ArmoX 370T Class 1 specimens carried out in the Structures Laboratory within the Department of Civil and Environmental Engineering of Imperial College London.

The tests were carried out using an Instron 8802 servo-hydraulic loading rig fitted with a 250 kN load cell. Strain measurements were made using an Instron AVE (Advanced Video Extensometer), shown in Figure 3-24, capable of measuring strains in 2 directions. Computer-controlled methods were used to set the loading rate and to manage data acquisition.



Figure 3-24: Test setup and AVE apparatus

For some tests, a second set of strain measurements was also made using traditional foil-type strain gauges on the opposite face of the specimen, as shown in Figure 3-25.

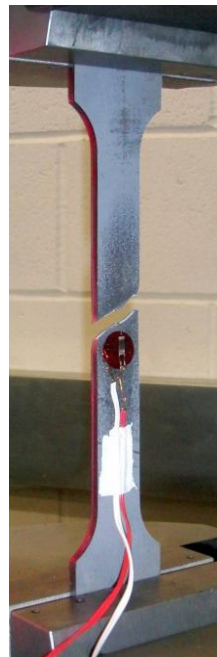


Figure 3-25: Test specimen with affixed strain gauges

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<sup>3</sup> The work of Mr. Andrew D. Pullen of the Structures Laboratory at the Department of Civil and Environmental Engineering of Imperial College London is hereby acknowledged.

A typical stress-strain curve is shown in Figure 3-26, where the initial elastic modulus was found to be 202.5 GPa and Poisson's ratio of 0.29. The yield and strength values were found to be in excellent agreement with the quoted manufacturer's value.

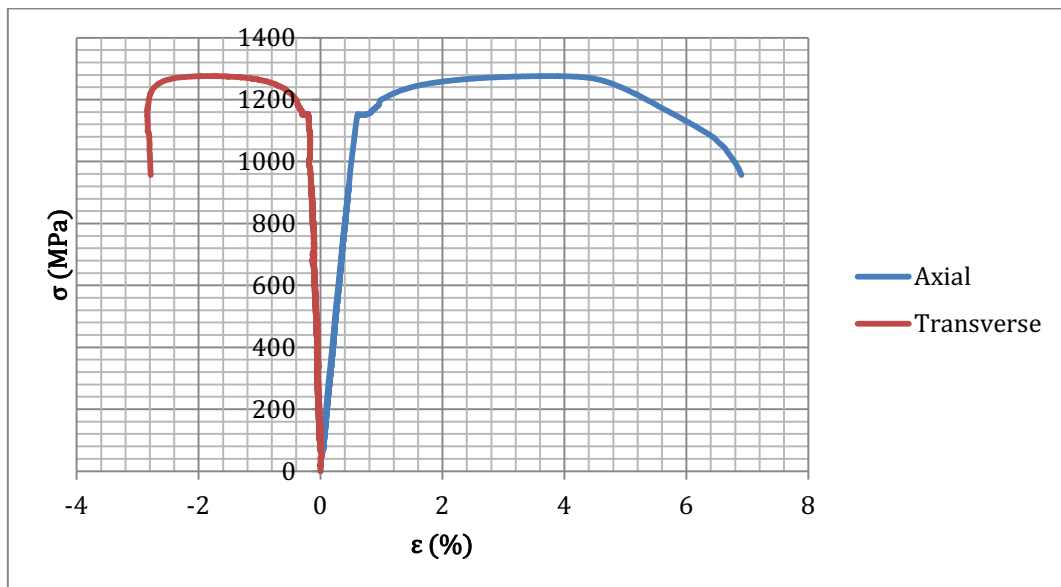


Figure 3-26: Stress-strain curve for Armox 370T Class 1 armour steel

These tests were carried out at a strain-rate of 0.006 /s and no further tests at higher rates were performed, due to lack of availability of further test coupons.

However, it has been found by other researchers [33, 34] that high-performance steels, such as Armox, do not exhibit strain-rate sensitivity.

Other researchers [35, 36] have found Johnson-Cook parameters for other varieties of Armox steel, viz. 440T and 500T, and found that the constant representing strain-rate effects, i.e.,  $C$ , attains a value of 0.018 and 0.00549 respectively, which are not very significant (cf.  $C = 0.076$  for mild steel [34]).

Figure 3-27 compares the effect of strain-rate on the yield stress for these materials using the cited  $C$  parameters and it can be seen that whilst the yield stress of mild steel increases by up to 23% for high strain-rates, the Armox steel yield stress does not increase by more than 6%, even at a strain-rate of 1000 /s, confirming the above assertion that Armox steels do not exhibit strong strain-rate sensitivity.

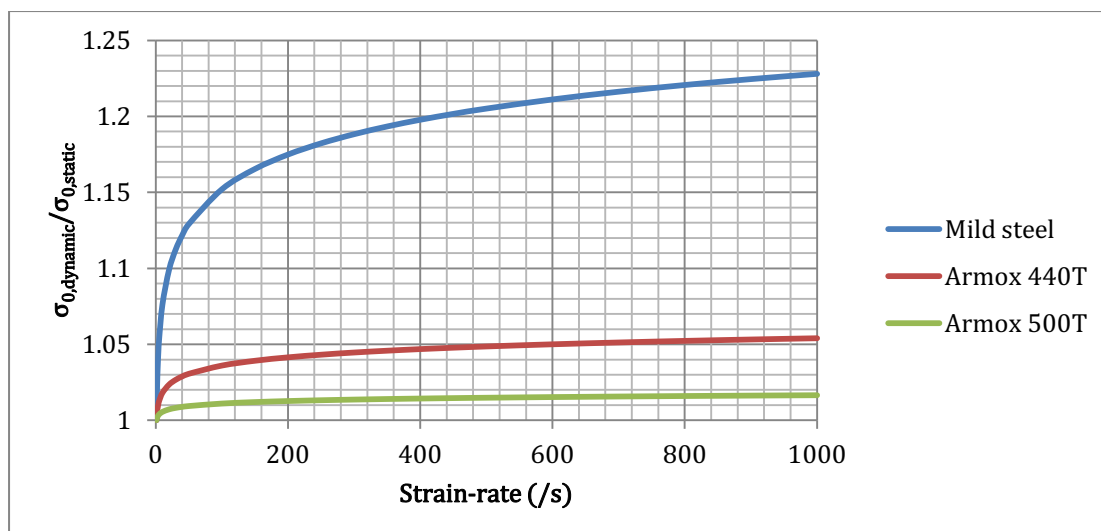


Figure 3-27: Comparison of strain-rate effects on various steels

Thus, the measured static data for Armox 370T Class 1 will be used in this study.

### 3.8.3 Test matrix and results

Various stand-off distance/charge mass/charge diameter combinations were tested and the test matrix and results are summarised in Table 3-4 and Table 3-5.

| Test | $d$ (mm) | $D_e$ (mm) | $m_e$ (g) | $I$ (Ns) | $w_f$ (mm)        |
|------|----------|------------|-----------|----------|-------------------|
| AS1  | 25       | 50         | 40        | 80.0     | 29.5 <sup>4</sup> |
| AS2  | 25       | 50         | 33        | 66.1     | 21.9              |
| AS3  | 50       | 50         | 40        | 82.2     | 14.4              |
| AS4  | 50       | 50         | 70        | 143.2    | 25.6              |
| AS5  | 38       | 50         | 40        | 80.5     | 21.3              |
| AS6  | 38       | 50         | 50        | 100.1    | 27.1              |
| AS7  | 38       | 40         | 40        | 82.2     | 17.6              |
| AS8  | 38       | 40         | 50        | 91.0     | 18.6              |

Table 3-4: Test results for Armox 370T Class 1

| Test | $d$ (mm) | $D_e$ (mm) | $m_e$ (g) | $I$ (Ns)         | $w_f$ (mm)        |
|------|----------|------------|-----------|------------------|-------------------|
| MS1  | 25       | 75         | 25        | 56.4             | 30.2              |
| MS2  | 25       | 75         | 40        | 88.6             | 48.9              |
| MS3  | 25       | 75         | 50        | 116.8            | 59.8              |
| MS4  | 25       | 50         | 40        | 82.9             | 53.7 <sup>5</sup> |
| MS5  | 25       | 50         | 25        | N/A <sup>6</sup> | 33.3              |
| MS6  | 25       | 50         | 33        | 68.2             | 42.0              |

Table 3-5: Test results for mild steel

<sup>4</sup> Panel ruptured.

<sup>5</sup> Panel ruptured.

<sup>6</sup> Did not record.

It was observed, from the tables above and Figure 3-28 that there is a linear increase of impulse with increasing charge mass, with the relationship being almost 2:1 and which is almost independent of the stand-off/charge diameter ratio.

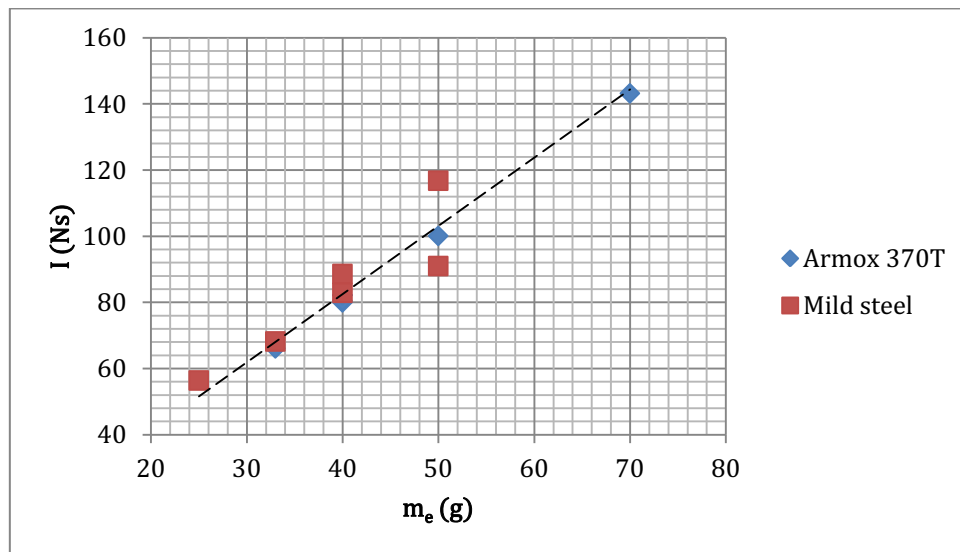


Figure 3-28: Variation of impulse with charge mass

It was also observed that both steels rupture at the same charge size/stand-off/mass combination (AS1 and MS4), as shown in Figure 3-29.



Figure 3-29: Panel rupture (left: armour steel; right: mild steel)

The similar tearing threshold for both materials suggests that the higher yield stress of armour steel is offset by the larger ductile range of the mild steel.

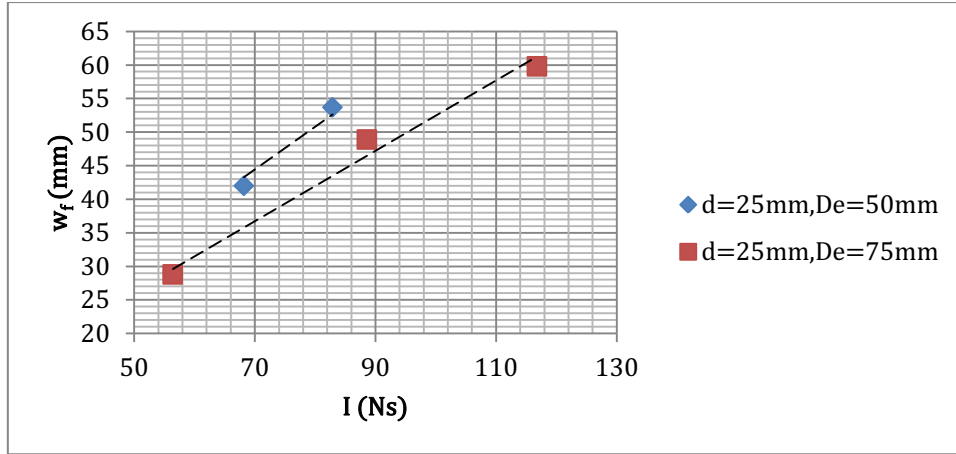


Figure 3-30: Variation of permanent displacement with impulse for mild steel

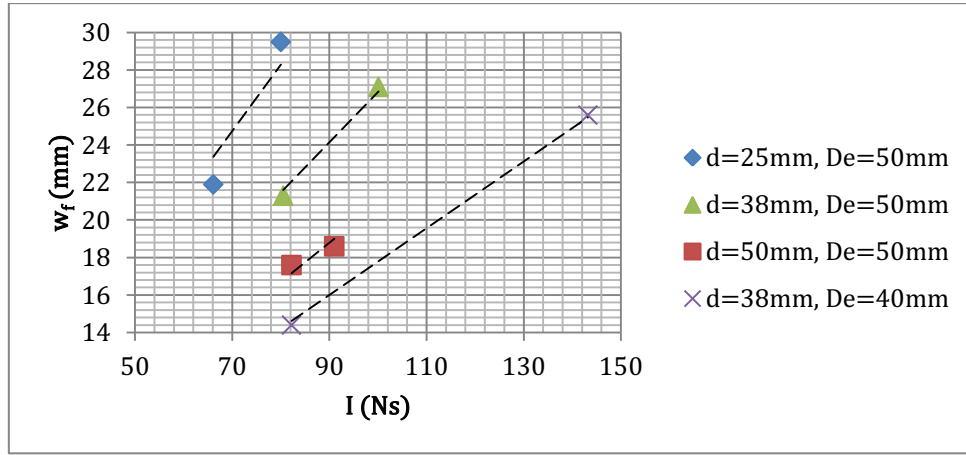


Figure 3-31: Variation of permanent displacement with impulse for Armox 370T Class 1

In terms of the variation of permanent displacement with impulse, it was observed there is a strong dependence on  $\left(\frac{d}{D_e}\right)$ , as evident from Figure 3-30 and Figure 3-31.

Using the dimensionless damage number (or dimensionless impulse),  $\phi_{q,s}$ , proposed by Jacob et al. [10], this dependency is accounted for by means of a stand-off distance parameter for quadrangular plates of dimensions  $L$  and  $W$ , thickness  $H$ , static yield stress  $\sigma_0$  and density  $\rho$ , subjected to an impulse  $I$  resulting from a charge of radius  $R_e$ , given by [9]:

$$\phi_{q,s} = \frac{I}{2H^2\sqrt{LW\rho\sigma_0}} \left( \frac{\xi_l}{\xi_d} \right) \quad (3-30)$$

where  $\xi_l$  is a loading parameter given by:

$$\xi_l = 1 + \ln \left( \frac{LW}{\pi R_e^2} \right) \quad (3-31)$$

and  $\xi_d$  is a stand-off distance parameter given by:

$$\xi_d = 1 + \ln\left(\frac{d}{R_e}\right) \quad (3-32)$$

The experimental results are now shown in terms of dimensionless quantities in Figure 3-32.

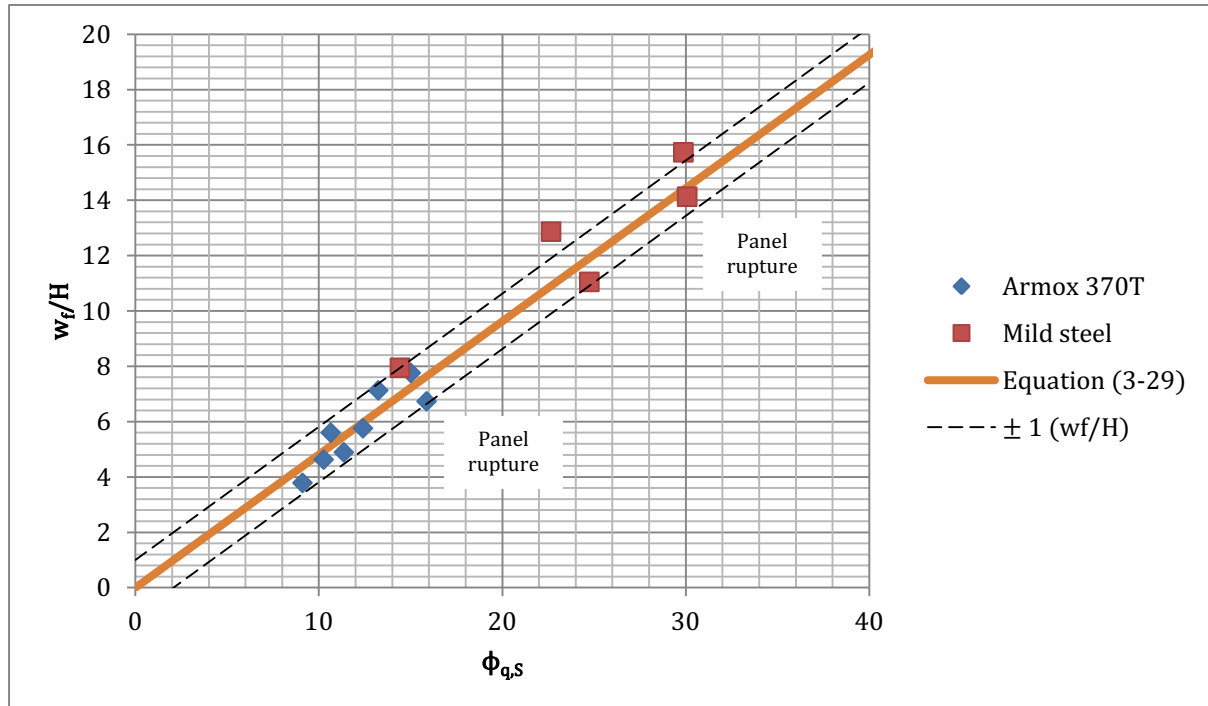


Figure 3-32: Variation of dimensionless displacement with dimensionless impulse

It can be observed that all results, independent of material, stand-off distance and charge diameter, can be described by a single linear function which can be obtained by a least-squares fit analysis, falling within a displacement/thickness ratio of  $\pm 1$  and with a confidence level of  $R^2 = 0.943$ , given by:

$$\left(\frac{w_f}{H}\right) = 0.4812 \phi_{q,s} \quad (3-33)$$

It can be observed that (3-33) is consistent with a similar expression proposed by Nurick and Martin [28, 37] for impulsively loaded quadrangular plates.

By utilising (3-26), the impulse in each test was evaluated and compared with the experimentally measured value obtained by means of the ballistic pendulum. The results are summarised in Figure 3-33.

It can be observed that there is a good correlation between the experimental and numerical values, with a maximum difference of 13.5% and a mean difference of 9.2%.

A further verification was done by comparison of the plate response, as discussed in the following section.

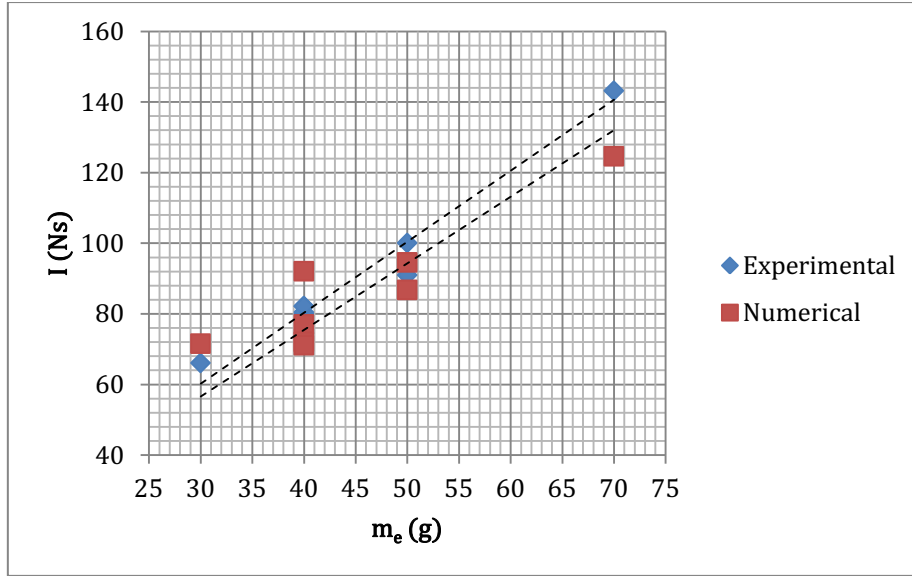


Figure 3-33: Comparison between experimental and numerical results

### 3.8.4 Numerical modelling

Having obtained a set of experimental data, the test scenarios were modelled in a commercial finite element analysis package, defining the blast load by means of the function proposed in (3-1) to (3-3).

Given the lack of complete material data available for the mild steel used in the tests, the numerical modelling was limited to the armour steel for the purpose of illustration.

For the various  $d$ ,  $D_e$  and  $m_e$  values described in Table 3-4, the loading parameter values were obtained from Figure 3-10 to Figure 3-14 or from equations (3-21) to (3-25). These loading distributions were implemented in a user-defined loading subroutine (VDLOAD) in the commercial finite element analysis package ABAQUS/Explicit v.6.9-1 [38] and applied onto the plate surface. A typical VDLOAD subroutine is shown in Appendix A.

The plate itself was modelled using 8-noded linear brick elements with reduced integration and hourglass control (C3D8R). Due to symmetry, a quarter plate was modelled for computational efficiency and a uniform mesh size of 3 mm was used.

The mesh size was established by assessing and ensuring numerical convergence, as shown in Figure 3-34.

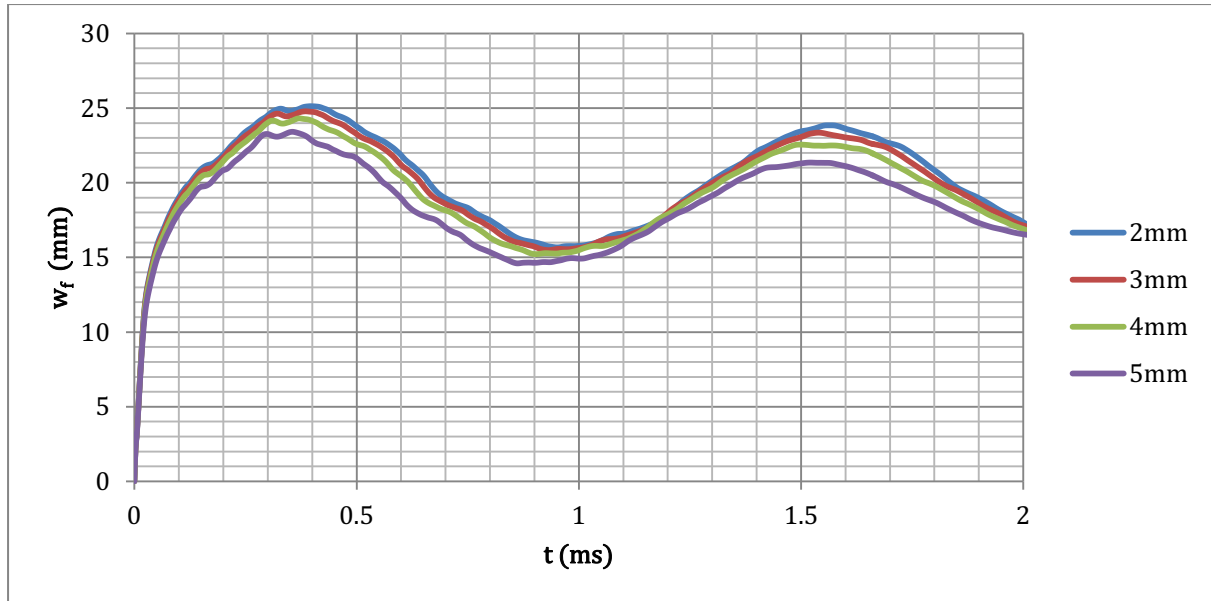


Figure 3-34: Mesh sensitivity analysis (AS2)

The 3 mm mesh was chosen due to its numerical convergence and its significantly decreased computational cost when compared with the 2 mm mesh, as seen in Table 3-6.

| Mesh size (mm) | Number of elements for quarter model | Run-time <sup>7</sup> for 2 ms simulation time (min) | File size (GB) |
|----------------|--------------------------------------|------------------------------------------------------|----------------|
| 2              | 11250                                | 11                                                   | 2.5            |
| 3              | 5000                                 | 5                                                    | ≈ 1            |
| 4              | 2888                                 | 3                                                    | ≈ 0.7          |
| 5              | 1800                                 | 2                                                    | ≈ 0.5          |

Table 3-6: Summary of mesh sensitivity analysis

In terms of the material model, an elasto-plastic model was used for the armour steel, using the elastic values given in Section 3.8.2, and for the plastic behaviour, the true stress-true strain curve was used as an input in ABAQUS/Explicit, assuming isotropic hardening.

A selection of the outputs from ABAQUS is shown in Figure 3-35 to Figure 3-39 and the results are summarised in Figure 3-36.

<sup>7</sup> Desktop machine with 4 CPUs at 2.67 GHz and 4 GB of RAM.

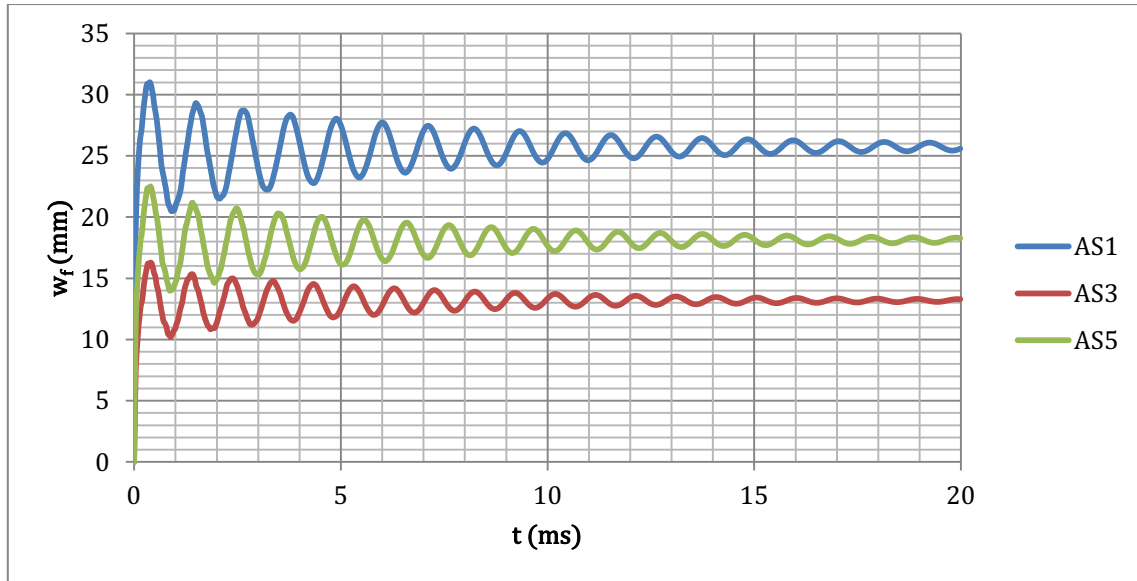


Figure 3-35: Selection of displacement-time histories from ABAQUS

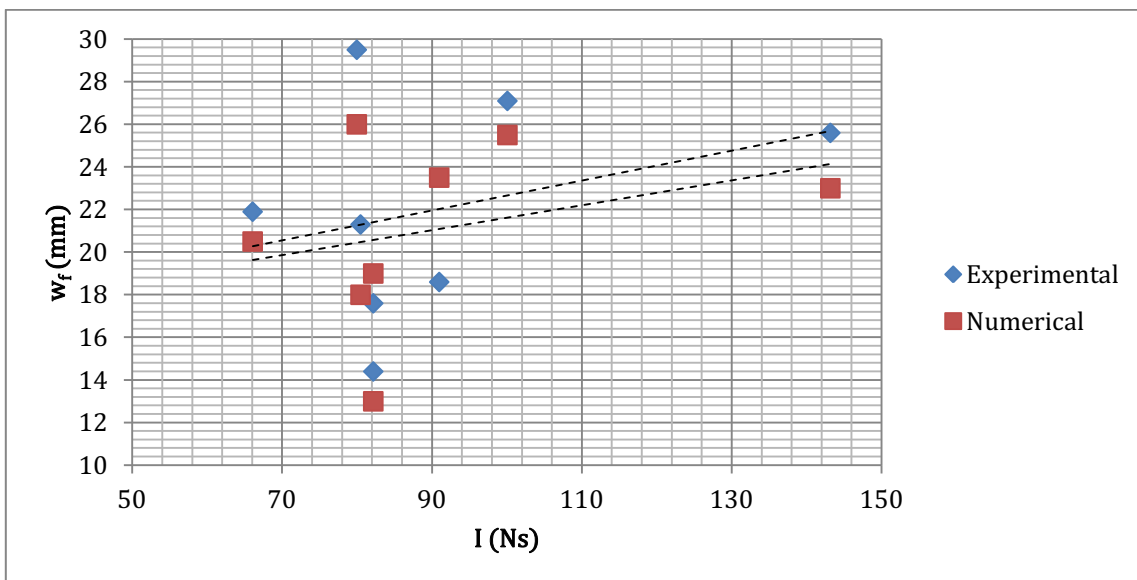


Figure 3-36: Comparison between experimental and numerical results

It can be seen that there is good correlation between the experimental and numerical results, with a maximum difference of 11.9% and a mean difference of 8%.

In addition to loading uncertainties discussed in Section 3.8.3, the discrepancy can be attributed to the simplifications associated with the material model (i.e. ignoring rate effects) and also due to difference in support conditions, i.e., between the (fully-clamped) model and the (bolted) test plate. Also, the polystyrene bridge in the test setup could enhance the blast effect due to further reflected pressures not represented by the proposed loading function.

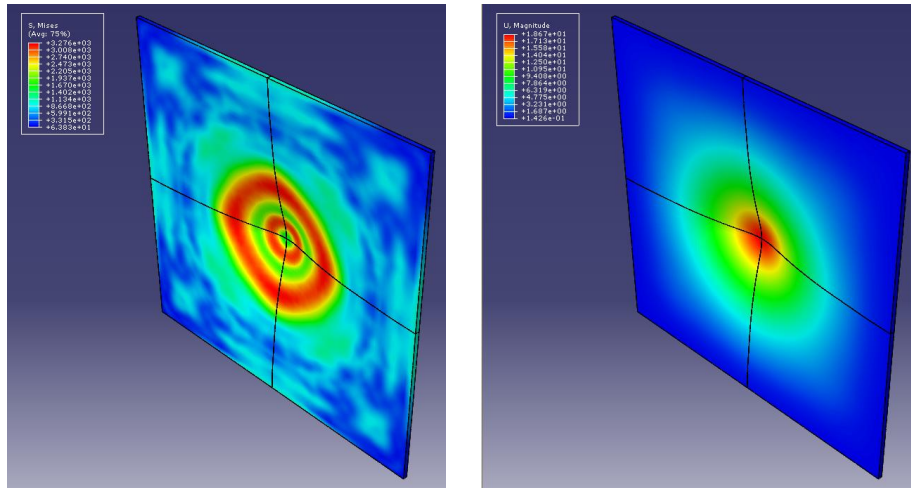


Figure 3-37: Mises stress (left) and displacement (right) contour plots for AS2

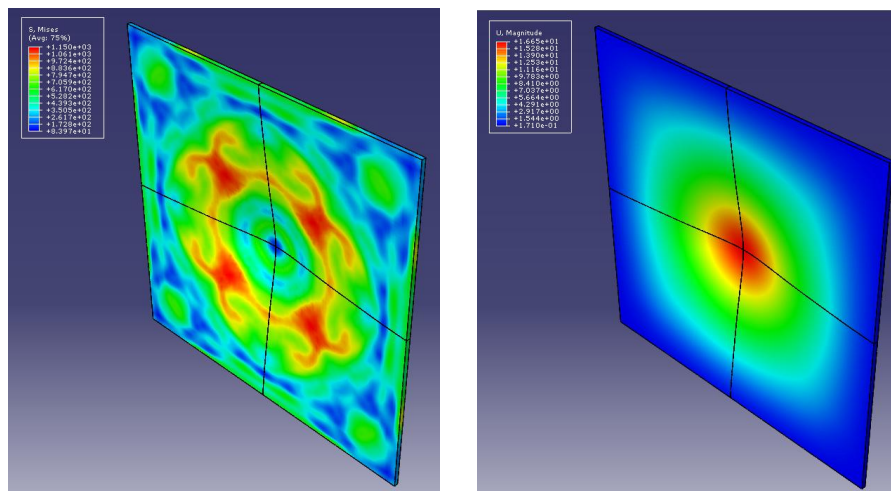


Figure 3-38: Mises stress (left) and displacement (right) contour plots for AS3

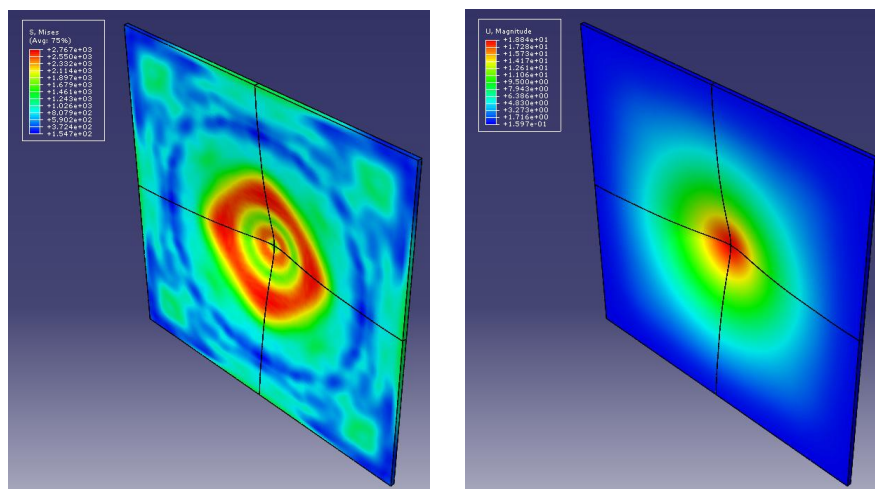


Figure 3-39: Mises stress (left) and displacement (right) contour plots for AS7

### 3.9 Concluding remarks

In this chapter, a generalised form of spatial and temporal description of a localised blast load is presented. It is postulated that the two parts (spatial and temporal) are independent, an assumption supported by AUTODYN simulations and literature.

A non-dimensionalisation exercise is carried out, through which 6 dimensionless parameters which uniquely describe the load are extracted. These are all found to be a function of the ratio of the charge stand-off distance to the charge diameter.

A number of numerical simulations using AUTODYN are carried out, in which the pressure variation on a rigid barrier for various charge stand-off/diameter combinations and the temporal and spatial variations are obtained for each case.

It should be noted that the relationships are obtained based on the assumption that the load is applied on a rigid barrier and ignoring the flexibility of the loaded structure. In effect, the structure would deform as it is loaded, making the results obtained through this method an upper bound to the exact solution. This is further discussed in Chapter 7. The accuracy of the method, nonetheless, was tested via corroboration of test results with subsequent simulations.

Least-square regression is performed to obtain the relationship between stand-off/charge diameter ratio and the various loading parameters and dimensionless charts for each of these parameters are produced.

The proposed method is verified by comparing experimental data with numerical models of a blast load on steel plates using ABAQUS/Explicit.

In addition to comparison of measured and predicted impulse, further verification is done by modelling the load via a user-defined loading subroutine (VDLOAD) and utilising the loading parameters obtained from the equations or charts.

From the results obtained in Section 3.8, it is clear that the proposed method accurately describes localised blast loads and can be used to simulate the response of structures in finite element packages, such as ABAQUS/Explicit, and also to obtain analytical results using exact solutions where these exist. An application of this will follow in Chapter 4.

## Chapter 4. Blast loading on monolithic plates

### 4.1 Nomenclature

In this chapter, the following notation will be used:

#### *Latin lower case*

|              |                                                                 |
|--------------|-----------------------------------------------------------------|
| $a$          | loading parameter, [1]                                          |
| $b$          | loading parameter, [ $L^{-1}$ ]                                 |
| $p_0$        | maximum overpressure, [ $M L^{-1} T^{-2}$ ]                     |
| $p_1(r)$     | spatial part of pressure pulse load, [ $M L^{-1} T^{-2}$ ]      |
| $p_2(t)$     | temporal part of pressure pulse load, [ $M L^{-1} T^{-2}$ ]     |
| $p_c$        | static collapse pressure, [ $M L^{-1} T^{-2}$ ]                 |
| $p_e$        | Youngdahl's effective pressure, [ $M L^{-1} T^{-2}$ ]           |
| $p_l$        | lower bound static collapse pressure, [ $M L^{-1} T^{-2}$ ]     |
| $p_u$        | upper bound static collapse pressure, [ $M L^{-1} T^{-2}$ ]     |
| $r$          | radial axis direction, [L]                                      |
| $t$          | time, [T]                                                       |
| $t_{mean}$   | Youngdahl's mean time, [T]                                      |
| $w$          | plate transverse displacement, [L]                              |
| $w_f$        | final plate transverse displacement, [L]                        |
| $w_i$        | plate transverse displacement at the $i^{th}$ phase, [L]        |
| $\dot{w}_i$  | plate transverse velocity at the $i^{th}$ phase, [ $L T^{-1}$ ] |
| $\ddot{w}_i$ | plate acceleration at the $i^{th}$ phase, [ $L T^{-2}$ ]        |

#### *Latin upper case*

|             |                                                                 |
|-------------|-----------------------------------------------------------------|
| $A$         | plate constant, [ $T^{-1}$ ]                                    |
| $B$         | plate constant, [ $L T^{-2}$ ]                                  |
| $C$         | plate constant, [ $L T^{-2}$ ]                                  |
| $A_i - K_i$ | integration constants, [various]                                |
| $\dot{D}$   | internal energy dissipation rate, [ $M L^2 T^{-3}$ ]            |
| $\dot{E}$   | external work rate, [ $M L^2 T^{-3}$ ]                          |
| $I$         | impulse per unit surface area, [ $M L^{-1} T^{-1}$ ]            |
| $I_e$       | Youngdahl's effective impulse, [ $M L T^{-1}$ ]                 |
| $M_0$       | plastic collapse moment per unit width, [ $M L T^{-2}$ ]        |
| $M_r$       | radial bending moment per unit width, [ $M L T^{-2}$ ]          |
| $M_\theta$  | circumferential bending moment per unit width, [ $M L T^{-2}$ ] |
| $N_0$       | plastic membrane force, [ $M L T^{-2}$ ]                        |
| $N_r$       | radial membrane force, [ $M L T^{-2}$ ]                         |
| $N_\theta$  | circumferential membrane force, [ $M L T^{-2}$ ]                |
| $Q_0$       | plastic transverse shear force per unit width, [ $M T^{-2}$ ]   |
| $Q_r$       | lateral shear force per unit width, [ $M T^{-2}$ ]              |
| $R_0$       | radius of central uniformly-loaded region, [ $M L T^{-2}$ ]     |
| $R$         | plate radius, [L]                                               |
| $T$         | plate natural period, [T]                                       |

|              |                                                                                      |
|--------------|--------------------------------------------------------------------------------------|
| $V_0$        | initial impulsive velocity, [L T <sup>-1</sup> ]                                     |
| $W_f$        | final maximum plate transverse displacement, [L]                                     |
| $W_i$        | maximum plate transverse displacement at the $i^{\text{th}}$ phase, [L]              |
| $W_s$        | final maximum plate transverse displacement at support, [L]                          |
| $\dot{W}_i$  | maximum plate transverse velocity at the $i^{\text{th}}$ phase, [L T <sup>-1</sup> ] |
| $\ddot{W}_i$ | maximum plate acceleration at the $i^{\text{th}}$ phase, [L T <sup>-2</sup> ]        |
| $X$          | load parameter, [1]                                                                  |
| $Y$          | load parameter, [1]                                                                  |

#### *Greek lower case*

|                       |                                                                                 |
|-----------------------|---------------------------------------------------------------------------------|
| $\alpha$              | plate parameter, [L <sup>3</sup> ]                                              |
| $\beta$               | plate parameter, [L <sup>3</sup> ]                                              |
| $\gamma$              | transverse shear strain, [1]                                                    |
| $\delta$              | plate parameter, [L <sup>2</sup> ]                                              |
| $\varepsilon$         | plate parameter, [L <sup>3</sup> ]                                              |
| $\zeta$               | plate parameter, [1]                                                            |
| $\eta$                | dynamic load factor, [1]                                                        |
| $\eta_{crit}$         | critical dynamic load factor, [1]                                               |
| $\eta_e$              | effective dynamic load factor, [1]                                              |
| $\kappa_r$            | radial curvature, [1]                                                           |
| $\kappa_\theta$       | circumferential curvature, [L <sup>-1</sup> ]                                   |
| $\dot{\kappa}_r$      | rate of change of radial curvature, [T <sup>-1</sup> ]                          |
| $\dot{\kappa}_\theta$ | rate of change of circumferential curvature, [L <sup>-1</sup> T <sup>-1</sup> ] |
| $\lambda$             | dimensionless kinetic energy, [1]                                               |
| $\mu$                 | plate mass per unit area, [M L <sup>-2</sup> ]                                  |
| $\nu$                 | ratio of plastic shear capacity to plastic bending capacity, [1]                |
| $\xi$                 | plastic hinge position, [L]                                                     |
| $\dot{\xi}$           | travelling plastic hinge velocity, [L T <sup>-1</sup> ]                         |
| $\xi_0$               | initial plastic hinge position, [L]                                             |
| $\sigma_0$            | yield stress, [M L <sup>-1</sup> T <sup>-2</sup> ]                              |
| $\tau$                | pulse load duration, [T]                                                        |
| $\varphi$             | load parameter, [1]                                                             |
| $\psi$                | rotation of plate centre line, [1]                                              |
| $\omega$              | loaded area ratio, [1]                                                          |

## 4.2 Background

Most ductile engineering materials, such as steel, usually have considerable reserve strength beyond their yield point and such strength may be used in practical applications to realistically estimate the capacity up to failure when such a material is subjected to dynamic load. A blast event, although typically an unlikely loading source, is nonetheless one of high intensity and plastic deformation is a reality which the engineer must encompass in analysis and design in order to render these effects incorporable with economy.

In such an analysis, it is desirable to idealise the material behaviour as being, for example, elastic-perfectly plastic or rigid-perfectly plastic. In the latter case, the material is rigid whenever the stress is less than the uni-axial yield stress value,  $\sigma_0$ .

Such a simplification permits the analytical study of the main characteristics of structures with relative simplicity and without great loss of accuracy.

In most engineering applications, structural elements can be geometrically classified as being either beams or plates. Beams are members whose lengths are large when compared to their cross-sectional sizes and whilst idealisation of frame structures into beam elements is justified, many engineering structures can be more readily idealised as being composed of plates.

#### **4.2.1 Blast loading on plated structures**

In fact, the performance of plated steel has been the subject of a number of studies which have been carried out in the past and many have focused on the dynamic performance of such structures, as reviewed extensively in Chapter 2. Various researchers have also investigated the effect of localised blast on plated structures.

In fact, localised blast loading is a particular case of blasts generated from close-range charges, such as those from IEDs (improvised explosive devices) or buried land mines [253]. In addition to the work by Nurick and collaborators [1, 48-51, 104], other research related to localised blast has been carried out by Lee and Wierzbicki [57, 58], Neuberger et al. [59, 60] and Gharababaei et al. [47, 61-63].

However, in the studies encountered in literature, all dynamic loads considered are particularly useful for the case of either globalised blast loading or localised blast loading, where the pressure and/or impulse is of a uniform or restricted nature with no study including a specific spatial variation which can model blast loads representing any general stand-off.

In Chapter 3, an expression which can describe a blast load of any generic form has been presented. By the correct and judicious use of the loading parameters in this function, both localised (e.g. IEDs) and globalised (e.g. far-field explosion) sources of blast loads can be described.

#### **4.3 Aim of current work**

Thus, the objective of this chapter is to extend the works available in the literature and develop analytical expressions for the permanent transverse displacement of a circular plate which is subjected to a blast load of the form described in Chapter 3.

Differentiation will be made between moderately thick, thick and thin plates, whose response would be dominated by bending, transverse shear and rotatory inertia and membrane effects, respectively. In addition, for each case, the applied load is simplified into an impulsive velocity and the response is compared to that obtained with the full loading spatial and temporal distributions. The effects of pulse shape will also be investigated.

In the absence of suitable experimental data, the analytical results are compared with numerical results for validation.

#### 4.4 Problem definition

In this section, the study under consideration will be outlined: all assumptions listed and clarified and the fundamental equations which will be used in this chapter are presented.

##### 4.4.1 Loading distribution

In this chapter, it is assumed that an axi-symmetrical lateral load of the form  $p(r, t) = p_1(r)p_2(t)$ , which was described in Section 3.4.1, is applied on a circular plate of thickness  $H$ , diameter  $2R$ , and mass per unit area  $\mu$ , which has a continuous pinned support along its perimeter. The circular geometry has been chosen for ease of mathematical computation, given the radial nature of the loading.

It is also assumed that the spatial distribution of the loading function is of the form described in Chapter 3, namely:

$$p_1(r) = \begin{cases} p_0, & 0 \leq r \leq R_0 \\ p_0 a e^{br}, & R_0 \leq r \leq R \end{cases} \quad (4-1)$$

It should be noted that the parameter  $a$  is a multiple of the constant pressure  $p_0$  and is given by  $a = e^{-bR_0}$  since  $a = 1$  at  $r = R_0$ . The temporal variation of the pressure pulse is described by the function  $p_2(t)$ , whose form is described in Section 4.6.

##### 4.4.2 Plate thickness to radius ratio

A plate's geometry is defined by thickness to radius ratio. In this chapter, 3 cases will be considered, namely:

Case A: Moderately thick plates

Case B: Thick plates

Case C: Thin plates

Whilst lateral shear forces are retained in the governing equilibrium reactions for all cases, in Case A the plates examined are assumed to be sufficiently thin such that the influence of transverse shear strains and rotatory inertia may be neglected entirely but not so thin that in-plane or membrane action can provide a noticeable effect on the overall behaviour of the plate.

In Case B, the plates studied are of a thickness such that the influence of transverse shear strains and rotatory inertia play an important role in the overall behaviour of the plate.

Finally, in Case C, the plates are considerably thin such that their behaviour is dominated by in-plane or membrane action.

These statements are investigated further in Section 4.8.4, where a range of applicability for each case is identified.

#### 4.4.3 Material properties

It is assumed that the plate is made from a rigid-plastic material. Thus, the material is rigid whenever the stress is less than the uni-axial yield stress value,  $\sigma_0$ . Such a simplification permits the study of the main characteristics of structures with relative simplicity.

It is also assumed that the plastic flow is controlled by Tresca's yield criterion, shown in Figure 4-1, which for Case A reduces to [254]:

$$\text{Max} \{|M_r|, |M_\theta|, |M_r - M_\theta|\} \leq M_0 \quad (4-2)$$

where  $M_0$  is the plastic collapse moment per unit width of the plate, given by:

$$M_0 = \frac{\sigma_0 H^2}{4} \quad (4-3)$$

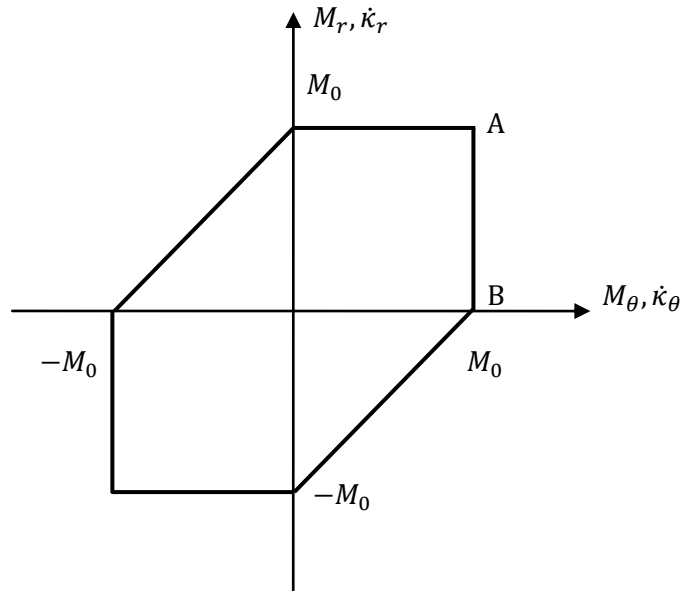


Figure 4-1: Tresca yield condition (after [254])

For Case B, it is assumed that the plastic flow is controlled by the interactive yield criterion shown in Figure 4-2, which was used by Sawczuk and Duszek [255] when examining the static plastic behaviour of circular plates.

In this yield criterion,  $M_0$  is given by (4-3) and  $Q_0$  is the plastic transverse shear force of the plate, given by:

$$Q_0 = \frac{\sigma_0 H}{2} \quad (4-4)$$

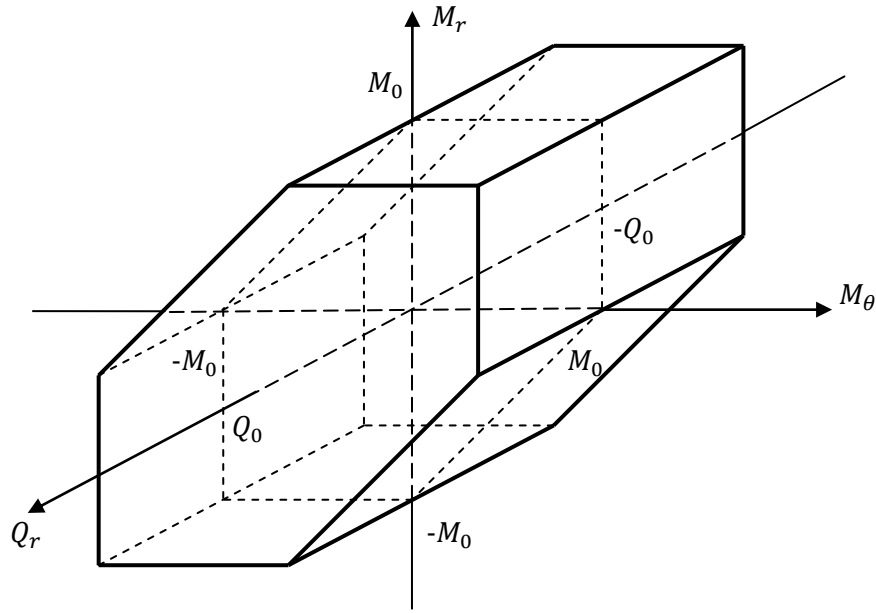


Figure 4-2: Interactive yield condition (after [255])

For Case C, the plastic flow is assumed to be controlled by Tresca's yield criterion for a combination of bending moment,  $M$ , and axial membrane force,  $N$ , as shown in Figure 4-3, which is given by [254]:

$$\frac{M}{M_0} + \left(\frac{N}{N_0}\right)^2 = 1 \quad (4-5)$$

where  $M_0$  is given by (4-3) and  $N_0$  is the plastic axial membrane force of the plate, given by:

$$N_0 = \sigma_0 H^2 \quad (4-6)$$

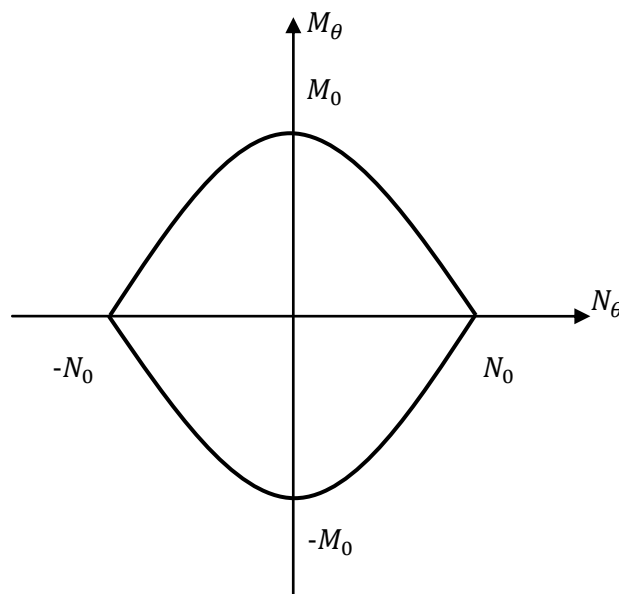


Figure 4-3: Interactive Tresca yield condition (after [254])

In addition, the effects of material strain-rate sensitivity are not considered in this chapter. This simplification is justified for high-performance steels such as armour steel (as discussed in Section 3.8.2), which are likely to be used in protective structures subjected to blast loading.

However, exclusion of strain-rate sensitivity would be unsuitable for other metallic plate structures made of, for instance, mild steel and permanent deflections derived on the basis of the static yield value will typically be over-conservative. This limitation is discussed and addressed in Section 4.9.

## 4.5 Static rigid-plastic behaviour of monolithic plates

In this section, the static plastic collapse pressure will be predicted using the 2 limit theorems of plasticity and a value for the exact plastic collapse pressure will then be obtained using these theorems.

### 4.5.1 Governing equilibrium equations

For infinitesimal displacements, the total internal energy dissipation rate,  $\dot{D}$ , is given by [254]:

$$\dot{D} = \int 2\pi r (M_r \dot{\kappa}_r + M_\theta \dot{\kappa}_\theta) dr \quad (4-7)$$

where  $\dot{\kappa}_r$  is the radial curvature of the plate's middle surfaces given by [254]:

$$\dot{\kappa}_r = -\frac{d^2 w}{dr^2} \quad (4-8)$$

and  $\dot{\kappa}_\theta$  is the circumferential curvature given by [254]:

$$\dot{\kappa}_\theta = -\frac{1}{r} \left( \frac{dw}{dr} \right) \quad (4-9)$$

Similarly, for an infinitesimal lateral or transverse displacement,  $w$ , the total external energy dissipation rate,  $\dot{E}$ , is given by [254]:

$$\dot{E} = \int 2\pi r p \dot{w} dr + 2\pi r (\bar{Q} \dot{w} - \bar{M} \dot{w}') \quad (4-10)$$

where  $\bar{M}$  and  $\bar{Q}$  are the moment and shear at the boundaries, respectively.

By the principle of virtual velocities,  $\dot{D} = \dot{E}$  and thus from (4-7) and (4-10), the equilibrium equations are obtained, viz., the moment and lateral equilibrium equations below [254]:

$$\frac{d(rM_r)}{dr} - M_\theta - rQ_r = 0 \quad (4-11)$$

$$\frac{d(rQ_r)}{dr} + r p(r, t) = 0 \quad (4-12)$$

#### 4.5.2 Lower bound calculation

For the problem defined in Section 4.4, the lower bound,  $p_l$ , to the exact collapse pressure,  $p_c$ , must produce a bending moment distribution which satisfies the equilibrium equations in Section 4.5.1 with  $p(r, t) = p_1(r)$  for the static case and nowhere violates the Tresca yield criterion. The distribution should also satisfy the (natural) boundary condition at  $r = R$ , namely,  $M_r = 0$  for a simply supported condition.

By using side AB of Tresca yield criterion in Figure 4-1, it is seen that at A,  $M_\theta = M_r$ , which is necessary for symmetry and at B,  $M_r = 0$ , corresponding to the requirement for  $r = R$ . Thus, it is postulated that:

$$M_\theta = M_0 \quad (4-13a)$$

$$0 \leq M_r \leq M_0 \quad (4-13b)$$

Substituting (4-1) in (4-12) and then (4-13a) into (4-11) and (4-12), gives:

$$\frac{d^2(rM_r)}{dr^2} = \begin{cases} -p_l r, & 0 \leq r \leq R_0 \\ -p_l a r e^{br}, & R_0 \leq r \leq R \end{cases} \quad (4-14)$$

Thus:

$$M_r = \begin{cases} -\frac{p_l r^2}{6} + A_1 + \frac{B_1}{r}, & 0 \leq r \leq R_0 \\ \frac{p_l a e^{br}}{b^2} (2 - br) + C_1 + \frac{D_1}{r}, & R_0 \leq r \leq R \end{cases} \quad (4-15)$$

where the integration constants:

$$A_1 = M_0 \quad (4-16a)$$

$$B_1 = 0 \quad (4-16b)$$

$$C_1 = \frac{p_l a e^{bR_0}}{b^2} (bR_0 - 1) - \frac{p_l R_0^2}{2} + M_0 \quad (4-16c)$$

$$D_1 = \frac{p_l R_0^3}{3} - \frac{p_l a e^{bR_0}}{b^3} (b^2 R_0^2 - 2bR_0 + 2) \quad (4-16d)$$

are obtained from the conditions  $Q_r = 0$  and  $M_r = M_0$  at  $r = 0$  and from continuity of  $Q_r$  and  $M_r$  at  $r = R_0$ .

Satisfying the requirement  $M_r = 0$  at  $r = R$  and re-arranging, an expression for  $p_l$  is found:

$$p_l = \frac{6M_0 R}{\alpha} \quad (4-17)$$

where the parameter  $\alpha$  is given by:

$$\alpha = R_0^2 (3R - 2R_0) - \frac{6a}{b^3} [e^{bR} (bR - 2) - e^{bR_0} (b^2 R_0 \{R - R_0\} - b \{R - 2R_0\} - 2)] \quad (4-18)$$

In the special case when  $a = 1$ ,  $R_0 = R$  and as  $b \rightarrow 0$ , then it can be seen from (4-18) that  $\alpha = R^3$  and, from (4-17),  $p_l = \frac{6M_0}{R^2}$ , which corresponds to the case of a uniformly distributed pressure given in literature, e.g. [65, 67, 254].

#### 4.5.3 Upper bound calculation

The upper bound,  $p_u$ , to the exact collapse pressure is found by equating the rate of internal energy dissipation to the external rate of work carried out during any kinematically admissible velocity profile which the plate undergoes as the load is applied.

It is assumed that the velocity profile is of the form:

$$\dot{w} = \dot{W} \left(1 - \frac{r}{R}\right) \quad (4-19)$$

where  $\dot{W}$  is the maximum transverse velocity at the plate centre, as per Figure 4-4:

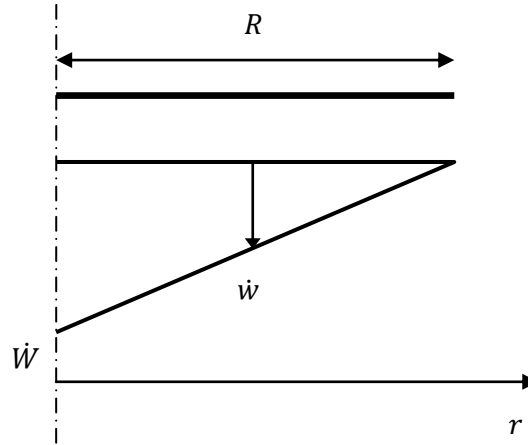


Figure 4-4: Velocity profile

Although it is not physically unreasonable to assume that the velocity profile is of such a conical shape for the form of loading being considered, it is also mathematically evident that for both the case when  $b \rightarrow \infty$  (i.e., the case of a point load) and also the case when  $b \rightarrow 0$  (i.e., the case of a uniformly distributed load), the velocity profile is of the form described in (4-19) and thus it is plausible to use this same profile for the interval  $0 < b < \infty$ , namely, the case under consideration.

From (4-8) and (4-9),  $\dot{\kappa}_r = 0$  and  $\dot{\kappa}_\theta = \frac{\dot{W}}{rR}$  and, equating the external work and internal energy dissipation rates:

$$\int_0^{R_0} 2\pi r \dot{W} p_u \left(1 - \frac{r}{R}\right) dr + \int_{R_0}^R 2\pi r \dot{W} p_u a^{ebr} \left(1 - \frac{r}{R}\right) dr = \int_0^R 2\pi r \frac{\dot{W}}{rR} M_0 dr \quad (4-20)$$

Thus:

$$p_u = \frac{6M_0 R}{\beta} \quad (4-21)$$

where the parameter  $\beta$  is given by:

$$\beta = R_0^2(3R - 2R_0) - \frac{6a}{b^3} [e^{bR}(bR - 2) - e^{bR_0}(b^2 R_0 \{R - R_0\} - b\{R - 2R_0\} - 2)] \quad (4-22)$$

#### 4.5.4 Exact static plastic collapse pressure

It can be seen from (4-18) and (4-22) that  $\alpha = \beta$  and thus, since the upper and lower bounds are identical, then the exact static plastic collapse pressure can be found from (4-17) or (4-21):

$$p_c = p_l = p_u = \frac{6M_0 R}{\beta} \quad (4-23)$$

Defining the loaded area ratio,  $\omega$ , as  $\omega = \frac{R_0}{R}$ , the variation is shown in Figure 4-5 in non-dimensional form for various values of  $b$ .

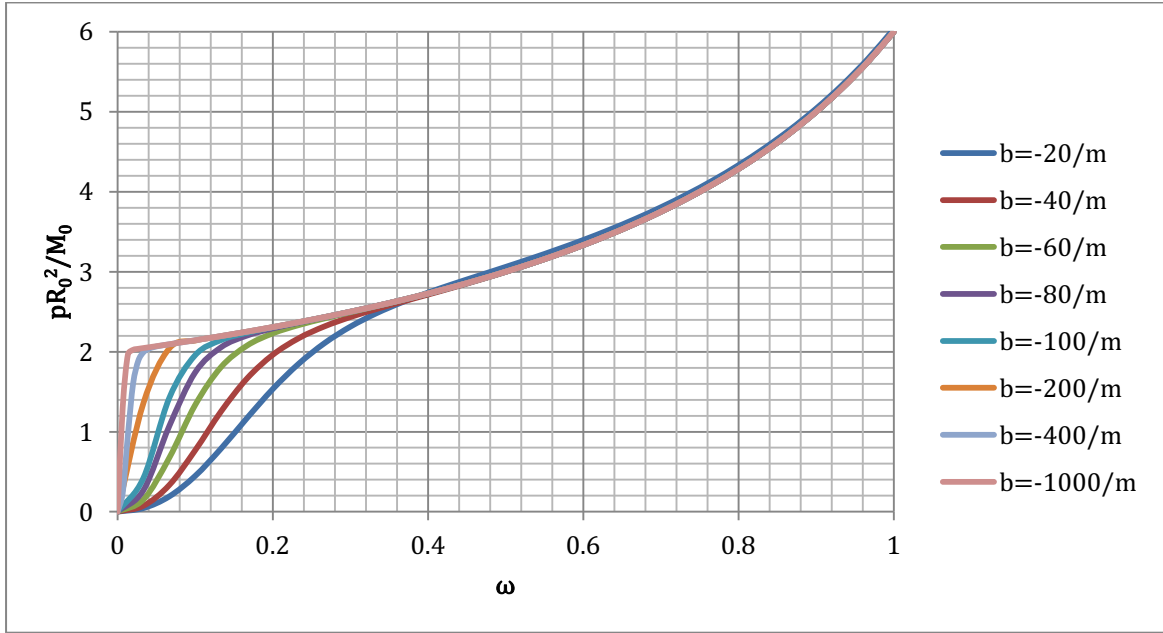


Figure 4-5: Static plastic collapse pressure

It can be seen that, while the variation is highly dependent on the value of  $b$  (especially for low values of this load parameter) for  $\left(\frac{R_0}{R}\right) \lesssim 0.35$ , it collapses into a single curve for all  $b$  values as the plate becomes more uniformly loaded. It can also be seen that, for the case of  $R_0 = R$  (i.e.,  $\omega = 1$ ), which corresponds to a uniformly fully loaded plate, the collapse pressure is equal to  $\left(\frac{6M_0}{R^2}\right)$  for all values of  $b$ , which is the collapse pressure for a uniformly distributed load on a circular plate, mentioned earlier. In addition, it can be seen that, for the case of  $\omega \rightarrow 0$  and for  $b \rightarrow \infty$ , then the collapse pressure tends to  $\left(\frac{2M_0}{R_0^2}\right)$ , or a collapse load of  $2\pi M_0$ , which corresponds to the result for the collapse (point) load given in literature [65].

## 4.6 Case A – Dynamic rigid-plastic behaviour of moderately thick plates

In this section, the foregoing static analysis is extended to take into account the transverse inertia term in the lateral equilibrium equation, assuming that the dominant mode is bending and thus assuming that only infinitesimal displacements occur.

### 4.6.1 Governing equilibrium equations

For the dynamic case, the equilibrium equations presented in Section 4.5.1 are still valid, with the inclusion of the inertia term in (4-10) and (4-12), which are replaced respectively by [254]:

$$\dot{E} = \int 2\pi r (p - \mu \ddot{w}) \dot{w} dr + 2\pi r (\bar{Q} \dot{w} - \bar{M} \dot{w}') \quad (4-24)$$

$$\frac{d(rQ_r)}{dr} + r p(r, t) - \mu r \ddot{w} = 0 \quad (4-25)$$

In the dynamic case, the applied pressure  $p(r, t)$  has to include the temporal variation  $p_2(t)$ . The first case being considered is a rectangular pulse shape, as shown in Figure 4-6 and described by (4-26); the analysis is extended in Section 4.6.5 for alternative pulse shapes.

$$p_2(t) = \begin{cases} 1, & 0 \leq t \leq \tau \\ 0, & t \geq \tau \end{cases} \quad (4-26)$$

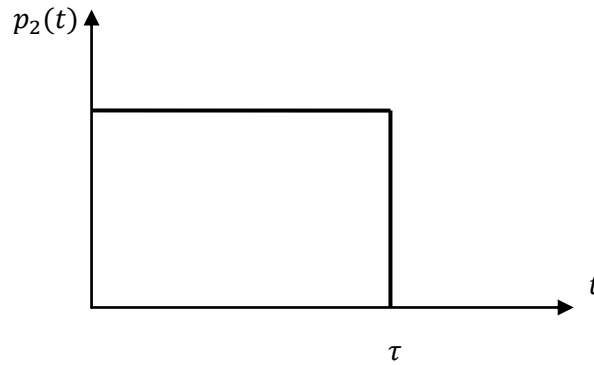


Figure 4-6: Rectangular pulse shape

The dynamic response is considered for 2 different scenarios which depend on the ratio of the applied load to the static collapse load, namely:

Case 1:  $1 \leq \eta \leq \eta_{crit}$

Case 2:  $\eta > \eta_{crit}$

where  $\eta$  is a dynamic load factor defined as:

$$\eta = \frac{p_0}{p_c} \quad (4-27)$$

and  $\eta_{crit}$  will be defined in (4-41).

In each case, the response consists of distinct phases of motion, as described hereunder.

#### 4.6.2 Case 1: $1 \leq \eta \leq \eta_{\text{crit}}$

- First phase of motion:  $0 \leq t \leq \tau$

The same velocity profile shown in Figure 4-4 is assumed to be valid for the dynamic problem; in this case, the velocity is taken to be  $\dot{W}_1$ .

Since from (4-8) and (4-9),  $\dot{\kappa}_r = 0$  and  $\dot{\kappa}_\theta = \frac{\dot{W}}{rR} > 0$ , then the generalised stress field lies on the side AB of the Tresca yield criterion in Figure 4-1 and thus:

$$M_\theta = M_0 \quad (4-28a)$$

$$0 \leq M_r \leq M_0 \quad (4-28b)$$

Substituting (4-1) in (4-25) and then (4-28a) into (4-11) and (4-25), gives:

$$\frac{d^2(rM_r)}{dr^2} = \begin{cases} \mu r \dot{W}_1 \left(1 - \frac{r}{R}\right) - p_0 r, & 0 \leq r \leq R_0 \\ \mu r \dot{W}_1 \left(1 - \frac{r}{R}\right) - p_0 a r e^{br}, & R_0 \leq r \leq R \end{cases} \quad (4-29)$$

Thus:

$$M_r = \begin{cases} \mu r^2 \ddot{W}_1 \left(\frac{1}{6} - \frac{r}{12R}\right) - \frac{p_0 r^2}{6} + A_2 + \frac{B_2}{r}, & 0 \leq r \leq R_0 \\ \mu r^2 \ddot{W}_1 \left(\frac{1}{6} - \frac{r}{12R}\right) + \frac{p_0 a e^{br}}{b^3} \left(\frac{2}{r} - b\right) + C_2 + \frac{D_2}{r} - M_0, & R_0 \leq r \leq R \end{cases} \quad (4-30)$$

where the integration constants:

$$A_2 = M_0 \quad (4-31a)$$

$$B_2 = 0 \quad (4-31b)$$

$$C_2 = \frac{p_0 a e^{bR_0}}{b^2} (bR_0 - 1) - \frac{p_0 R_0^2}{2} + M_0 \quad (4-31c)$$

$$D_2 = \frac{p_0 R_0^3}{3} + M_0 R_0 - \frac{p_0 a e^{bR_0}}{b^3} (2 + bR_0) \quad (4-31d)$$

are obtained from the conditions  $Q_r = 0$  and  $M_r = M_0$  at  $r = 0$  and from continuity of  $Q_r$  and  $M_r$  at  $r = R_0$ .

Satisfying the requirement  $M_r = 0$  at  $r = R$  and re-arranging, an expression for  $\mu \ddot{W}_1$  is found:

$$\mu \ddot{W}_1 = \frac{12M_0}{R^2} \left( \eta - \frac{R_0}{R} \right) \quad (4-32)$$

Integrating (4-32) twice gives:

$$\mu W_1 = \frac{6M_0}{R^2} \left( \eta - \frac{R_0}{R} \right) t^2 + E_2 t + F_2 \quad (4-33)$$

where the integration constants  $E_2 = F_2 = 0$  are obtained from the initial conditions  $W_1 = \dot{W}_1 = 0$  at  $t = 0$ .

- Second phase of motion:  $\tau \leq t \leq T$

The same analysis as in the preceding section is valid, except that  $W_1$  is now replaced by  $W_2$  and since the direct loading phase is now complete,  $p_0 = 0$ . Thus, (4-32) is re-written as:

$$\mu \ddot{W}_2 = -\frac{12M_0R_0}{R^3} \quad (4-34)$$

Integrating (4-34) twice, noting that the integration constants are obtained from the continuity of velocity and displacement at  $t = \tau$  and re-arranging gives:

$$W_2 = \frac{6M_0R_0}{\mu R^2} \left( \left\{ 2\tau\eta - \frac{R_0}{R}t \right\} t - \tau^2\eta \right) \quad (4-35)$$

The motion ceases when  $\dot{W}_2 = 0$ , which occurs when  $t = T$ , given by:

$$T = \frac{R}{R_0} \eta \tau \quad (4-36)$$

Thus, the maximum permanent deflection at any point  $0 \leq r \leq R$  is found by substituting (4-36) in (4-35) to give:

$$w_2 = \frac{6M_0\eta\tau^2}{\mu R_0 R} \left( \eta - \frac{R_0}{R} \right) \left( 1 - \frac{r}{R} \right) \quad (4-37)$$

For the special case when  $R_0 = R$ , it is readily seen that (4-36) reduces to  $T = \eta\tau$  and (4-37) reduces to  $w_2 = \frac{6M_0\eta\tau^2}{\mu R^2} (\eta - 1) \left( 1 - \frac{r}{R} \right) = \frac{p_c\eta\tau^2}{\mu} (\eta - 1) \left( 1 - \frac{r}{R} \right)$ , in accordance with the available results for the case of a uniformly distributed pressure.

- Static admissibility

Although it is evident that inequality (4-28a) is satisfied throughout the response due to normality requirements, it is necessary to verify whether the expression for  $M_r$  obtained in (4-30) satisfies the requirement in (4-28b).

Thus, it is necessary to show that:

$$\frac{dM_r}{dr} = 0 \quad (4-38a)$$

$$\frac{d^2M_r}{dr^2} \leq 0 \quad (4-38b)$$

For the first phase of motion, the bending moment distribution is obtained by re-arranging (4-30):

$$\frac{M_r}{M_0} = \begin{cases} 1 + 2\left(\eta - \frac{R_0}{R}\right)\left(\frac{r}{R}\right)^2 - \left(\eta - \frac{R_0}{R}\right)\left(\frac{r}{R}\right)^3 - \frac{\eta r^2 R}{\beta}, & 0 \leq r \leq R_0 \\ \frac{R_0}{r} + 2\left(\eta - \frac{R_0}{R}\right)\left(\frac{r}{R}\right)^2 - \left(\eta - \frac{R_0}{R}\right)\left(\frac{r}{R}\right)^3 - \frac{3\eta R R_0^2}{\beta} + \zeta\eta, & R_0 \leq r \leq R \end{cases} \quad (4-39)$$

where  $\zeta = \frac{6aR}{\beta b^3} \left( e^{br} \left\{ \frac{2}{r} - b \right\} + e^{bR_0} \left\{ b^2 R_0 \left[ 1 - \frac{R_0}{r} \right] + \frac{2}{r} [bR_0 - 1] - b \right\} \right)$ .

This is shown graphically in Figure 4-7 and Figure 4-8 for values of  $\omega = 0.1$  and  $0.2$  respectively. It is seen that the influence of  $\eta$  is less pronounced on the smaller value of  $\omega$ .

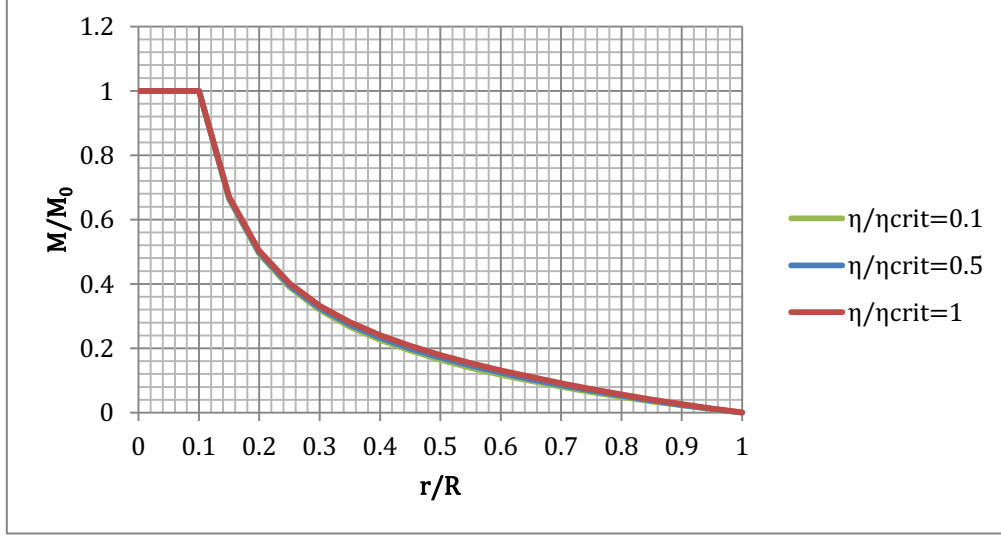


Figure 4-7: Radial bending moment distribution with  $\omega = 0.1$  for  $0 \leq t \leq \tau$

From (4-39), it is seen that:

$$\left( \frac{dM_r}{dr} \right)_{r=0} = 0 \quad (4-40a)$$

$$\left( \frac{d^2 M_r}{dr^2} \right)_{r=0} = M_0 \left( \frac{4\{\eta R - R_0\}}{R^3} - \frac{2\eta R}{\beta} \right) \geq 0 \quad (4-40b)$$

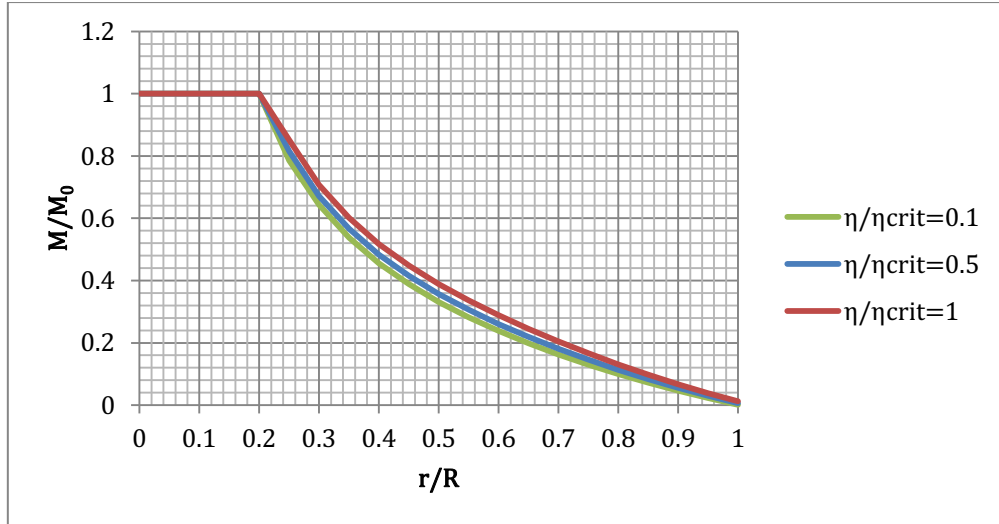


Figure 4-8: Radial bending moment distribution with  $\omega = 0.2$  for  $0 \leq t \leq \tau$

Thus,  $\left(\frac{4\{\eta R - R_0\}}{R^3} - \frac{2\eta R}{\beta}\right) \leq 0$  in order to avoid yield violation at  $r = 0$ . This implies that:

$$\eta \leq \frac{2\beta R_0}{R(2\beta - R^3)} = \eta_{crit} \quad (4-41)$$

In the special case when  $R_0 = R$  and  $\beta = R^3$ , it is readily seen that (4-41) reduces to  $\eta_{crit} = 2$ , which, from literature (e.g. [254]), is the limit for the case of a uniformly distributed pressure.

For the second phase of motion, (4-30) with  $p_0 = 0$  gives a bending moment distribution which is independent of  $\eta$ :

$$\frac{M_r}{M_0} = \begin{cases} 1 + \frac{R_0}{R} \left(\frac{r}{R}\right)^2 - \frac{2R_0}{R} \left(\frac{r}{R}\right)^3, & 0 \leq r \leq R_0 \\ \frac{R_0}{r} + \frac{R_0}{R} \left(\frac{r}{R}\right)^2 - \frac{2R_0}{R} \left(\frac{r}{R}\right)^3, & R_0 \leq r \leq R \end{cases} \quad (4-42)$$

This is shown graphically in Figure 4-9. It can easily be seen that for this phase, the bending moment distribution is statically admissible since:

$$\left(\frac{dM_r}{dr}\right)_{r=0} = 0 \quad (4-43a)$$

$$\left(\frac{d^2 M_r}{dr^2}\right)_{r=0} = -\frac{2M_0 R_0}{R^3} < 0 \quad (4-43b)$$

Thus, the analysis in this section is admissible and exact throughout the entire plate, provided that  $1 \leq \eta \leq \frac{2\beta R_0}{R(2\beta - R^3)}$ .

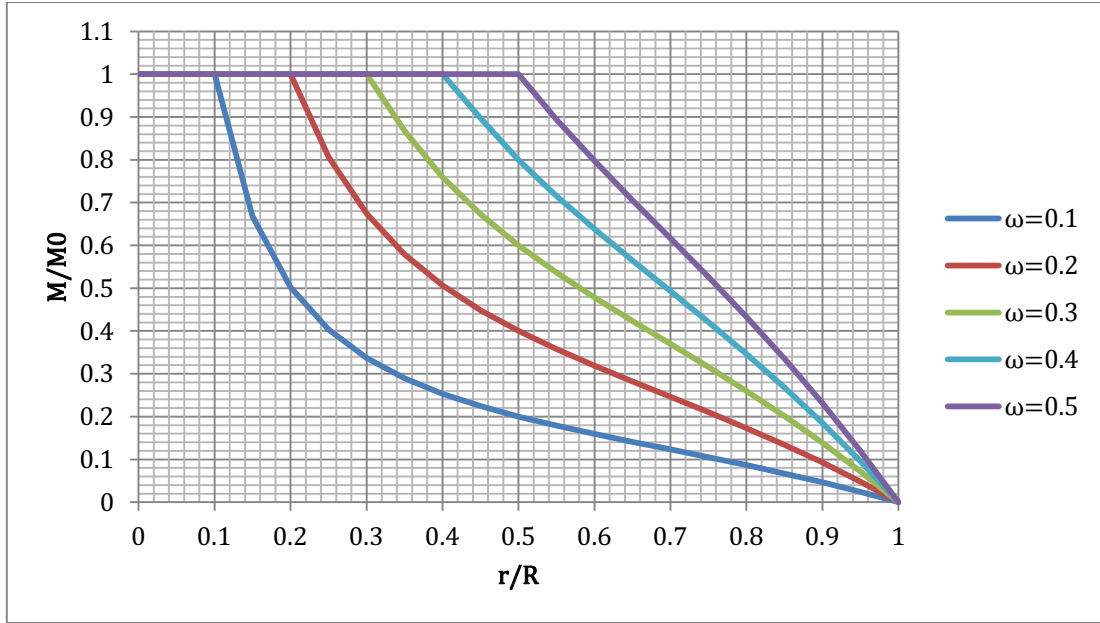


Figure 4-9: Radial bending moment distribution for  $0 \leq t \leq \tau$  for various values of  $\omega$

#### 4.6.3 Case 2: $\eta > \eta_{crit}$

For large pressure pulses with  $\eta > \eta_{crit}$  given in (4-41), it is assumed that a central portion of the plate within a region  $0 \leq r \leq \xi_0$  is controlled by the corner A of the yield criterion in Figure 4-1 while the remaining outer region  $\xi_0 \leq r \leq R$  is controlled by the side AB of the same Figure 4-1. It is also assumed that  $\xi_0 \geq R_0$ .

In this case, 3 stages of motion are considered, viz., the direct loading phase, during which a stationary plastic hinge forms at  $r = \xi_0$  until  $t = \tau$ , at which point the second phase starts and the plastic hinge starts to move towards the plate centre until  $t = T_1$ , at which point the third and final phase starts when the plastic hinge is at the middle and motion continues until it ceases at  $t = T$ .

- First phase of motion:  $0 \leq t \leq \tau$

It is assumed that the velocity profile is of the form:

$$\dot{w} = \begin{cases} \dot{W}_1, & 0 \leq r \leq \xi_0 \\ \dot{W}_1 \frac{(R-r)}{(R-\xi_0)}, & \xi_0 \leq r \leq R \end{cases} \quad (4-44)$$

where  $\dot{W}_1$  is the maximum transverse velocity at the plate centre, as per Figure 4-10:

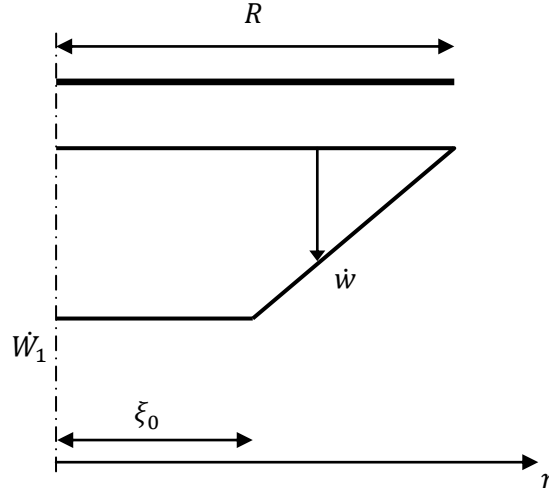


Figure 4-10: Velocity profile for first phase

From (4-8) and (4-9),  $\dot{\kappa}_r = 0$  and  $\dot{\kappa}_\theta = \frac{\dot{w}_1}{r(R-\xi_0)} > 0$ , thus (4-28a) and (4-28b) are again satisfied. Substituting (4-1) in (4-25) and then (4-28a) into (4-11) and (4-25), gives:

$$\frac{d^2(rM_r)}{dr^2} = \begin{cases} \mu r \dot{w}_1 - p_0 r, & 0 \leq r \leq R_0 \\ \mu r \dot{w}_1 - p_0 a r e^{br}, & R_0 \leq r \leq \xi_0 \\ \mu r \dot{w}_1 \frac{(R-r)}{(R-\xi_0)} - p_0 a r e^{br}, & \xi_0 \leq r \leq R \end{cases} \quad (4-45)$$

Thus:

$$M_r = \begin{cases} \frac{\mu r^2 \dot{w}_1}{6} - \frac{p_0 r^2}{6} + A_4 + \frac{B_4}{r}, & 0 \leq r \leq R_0 \\ \frac{\mu r^2 \dot{w}_1}{6} + \frac{p_0 a e^{br}}{b^3} \left( \frac{2}{r} - b \right) + C_4 + \frac{D_4}{r}, & R_0 \leq r \leq \xi_0 \\ \frac{\mu r^2 \dot{w}_1 (2R-r)}{12(R-\xi_0)} + \frac{p_0 a e^{br}}{b^3} \left( \frac{2}{r} - b \right) + E_4 + \frac{F_4}{r}, & \xi_0 \leq r \leq R \end{cases} \quad (4-46)$$

where the integration constants:

$$A_4 = M_0 \quad (4-47a)$$

$$B_4 = 0 \quad (4-47b)$$

$$C_4 = \frac{p_0 a e^{b\xi_0}}{b^2} (b\xi_0 - 1) - \frac{p_0 \xi_0^2}{2} + M_0 \quad (4-47c)$$

$$D_4 = M_0(1 - \xi_0) - \frac{\mu \xi_0^3 \dot{w}_1}{6} + \frac{p_0 \xi_0^3}{2} - \frac{p_0 a e^{b\xi_0}}{b^3} (2 - b\xi_0 + b^2 \xi_0^2) \quad (4-47d)$$

$$E_4 = \frac{\frac{p_0 a}{b^3} \{e^{bR}(bR - 2) - e^{b\xi_0}(b\xi_0 - 2)\} - \frac{p_0}{12} \{R^3 + \xi_0 R^2 - \xi_0^3 + \xi_0^2 R\} - M_0 \xi_0}{R - \xi_0} \quad (4-47e)$$

$$F_4 = \frac{\frac{p_0 a}{b^3} \{R e^{b\xi_0} (b\xi_0 - 2) - \xi_0 e^{bR} (bR - 2)\} + \frac{\xi_0 R}{12} \{12M_0 + p_0 R^2 + p_0 R \xi_0 - p_0 \xi_0^2\}}{R - \xi_0} \quad (4-47f)$$

are obtained from the conditions  $Q_r = 0$  and  $M_r = M_0$  at  $r = 0$ ,  $Q_r = 0$  and  $M_r = M_0$  at  $r = \xi_0$  and from the requirement  $M_r = 0$  at  $r = R$ .

From (4-46) for  $0 \leq r \leq R_0$  and using (4-47a) and (4-47b),  $M_r = M_0$  only when  $(\mu \dot{W}_1 - p_0) = 0$  or:

$$\dot{W}_1 = \frac{p_0}{\mu} \quad (4-48)$$

Integrating (4-48) twice gives the displacement:

$$W_1 = \frac{p_0 t^2}{2\mu} + G_4 t + H_4 \quad (4-49)$$

with both integration constants  $G_4$  and  $H_4$  equal to zero since  $W_1 = \dot{W}_1 = 0$  at  $t = 0$ .

Since  $Q_r = 0$  in the interval  $0 \leq r \leq \xi_0$ , then for  $Q_r$  in the interval  $\xi_0 \leq r \leq R$  and ensuring continuity of  $Q_r$ , it transpires that at  $r = \xi_0$ ,  $Q_r = 0$ , which is satisfied when using (4-11) and (4-25) with (4-46) and with the use of (4-48):

$$\frac{p_0}{p_c} = \eta = \frac{2\beta}{\delta} \quad (4-50)$$

where the parameter  $\delta$  is given by:

$$\delta = \frac{12a}{b^3} \{e^{b\xi_0} (b^2 \xi_0 [R - \xi_0] - b[R - 2\xi_0] - 2) - e^{bR} (bR - 2)\} - \frac{(R^4 - 5\xi_0^4 - 6\xi_0^2 R^2 + 8\xi_0^3 R)}{(R - \xi_0)} \quad (4-51)$$

It can be seen from (4-51) that when  $\xi_0 = 0$ ,  $a = 1$ ,  $b \rightarrow 0$ , then  $\delta = R^3$  and, with  $R_0 = R$  and with the same conditions for  $a$  and  $b$ , then, from (4-22),  $\beta = R^3$ . In this case, from (4-50) it is readily seen that  $\eta = 2$ , which is the limit  $\eta_{crit}$  for the case of a uniformly distributed pressure.

It can also be seen that for very high loadings, the plastic hinge forms at the supports since when  $p_0 \gg p_c$  (or  $\eta \rightarrow \infty$ ),  $\xi_0 \rightarrow R$ . Thus, (4-50) and (4-51) give the position of the stationary plastic hinge  $\xi_0$  in terms of the dynamic load factor  $\eta$ . It must be noted that the analytical solution for  $\xi_0$  is highly non-linear and must be obtained by means of numerical methods.

- Second phase of motion:  $\tau \leq t \leq T_1$

Since  $p_0 = 0$ , the plastic hinge now starts to move inwards from  $\xi_0$  at a velocity  $\dot{\xi}$  according to this velocity profile:

$$\dot{w} = \begin{cases} \dot{W}_2, & 0 \leq r \leq \xi \\ \dot{W}_2 \frac{(R-r)}{(R-\xi)}, & \xi \leq r \leq R \end{cases} \quad (4-52)$$

where  $\dot{W}_2$  is the maximum transverse velocity at the plate centre and  $\xi$  is the time-dependent position of the moving plastic hinge, as per Figure 4-11.

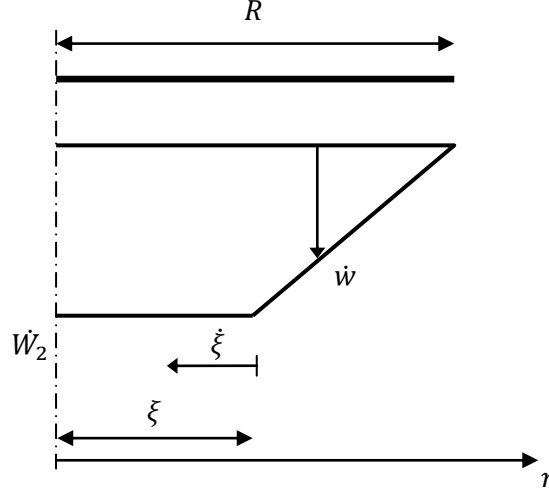


Figure 4-11: Velocity profile for second phase

From (4-8) and (4-9),  $\dot{\kappa}_r = 0$  and  $\dot{\kappa}_\theta = \frac{\dot{W}_1}{r(R-\xi)} > 0$ , thus (4-28a) and (4-28b) are again satisfied. Substituting (4-1) in (4-25) and (4-28a) into (4-11) and (4-25), gives the governing differential equation:

$$\frac{d^2(rM_r)}{dr^2} = \begin{cases} \mu r \ddot{W}_2, & 0 \leq r \leq \xi \\ \mu r \dot{W}_2 \frac{(R-r)}{(R-\xi_0)} + \mu r \dot{W}_2 \dot{\xi} \frac{(R-r)}{(R-\xi_0)^2}, & \xi \leq r \leq R \end{cases} \quad (4-53)$$

Considering (4-53) initially in the interval  $0 \leq r \leq \xi$  gives:

$$M_r = \frac{\mu r^2 \ddot{W}_2}{6} + A_5 + \frac{B_5}{r}, \quad 0 \leq r \leq \xi \quad (4-54)$$

where the integration constants  $A_5 = M_0$  and  $B_5 = 0$  are obtained from the conditions  $Q_r = 0$  and  $M_r = M_0$  at  $r = 0$ .

From (4-54) and using (4-47a) and (4-47b),  $M_r = M_0$  if  $\left(\frac{\mu r^2 \ddot{W}_2}{6}\right) = 0$  or:

$$\mu \ddot{W}_2 = 0 \quad (4-55)$$

Integrating (4-55) twice gives the displacement:

$$W_2 = \frac{E_5 t}{\mu} + F_5 \quad (4-56)$$

with both integration constants  $E_5$  and  $F_5$  obtained by ensuring velocity and displacement continuity with the previous phase, giving  $E_5 = p_0\tau$  and  $F_5 = -\frac{p_0\tau^2}{2\mu}$ .

Considering now (4-53) in the interval  $\xi \leq r \leq R$  and substituting (4-55) and its integral gives:

$$M_r = \frac{p_0\tau\dot{\xi}r^3}{12(R-\xi_0)^2}(2R-r) + G_5 + \frac{H_5}{r}, \quad \xi \leq r \leq R \quad (4-57)$$

where the integration constants:

$$G_5 = -\frac{M_0\xi}{R-\xi^2} - \frac{p_0\tau\dot{\xi}}{12(R-\xi)^2}(R^3 + \xi R^2 + \xi^2 R - \xi^3) \quad (4-58a)$$

$$H_5 = \frac{M_0\xi R}{R-\xi} + \frac{p_0\tau\dot{\xi}\xi R}{12(R-\xi)^2}(R^2 + \xi R - \xi^2) \quad (4-58b)$$

are obtained using  $M_r = M_0$  at  $r = \xi$  and  $M_r = 0$  at  $r = R$ .

Since  $Q_r = 0$  in the interval  $0 \leq r \leq \xi$ , then for  $Q_r$  in the interval  $\xi \leq r \leq R$  and ensuring continuity of  $Q_r$ , it transpires that at  $r = \xi$ ,  $Q_r = 0$ , which is satisfied when using (4-11) and (4-25) with (4-57), (4-58a) and (4-58b):

$$\dot{\xi} = -\frac{2p_c\beta}{p_0\tau(R+3\xi)(R-\xi)} \quad (4-59)$$

Thus, (4-59) gives the velocity of the travelling plastic hinge, which upon integration gives:

$$\left(\frac{\xi}{R}\right)^3 - \left(\frac{\xi}{R}\right)^2 - \left(\frac{\xi}{R}\right) = \frac{2\beta t}{\eta\tau R^3} - \frac{\delta}{R^3} + \frac{(R+\xi_0)(R-\xi_0)^2}{R^3} - 1 \quad (4-60)$$

where the integration constant is found using the initial condition  $\xi = \xi_0$  at  $t = \tau$ .

It can be seen from (4-51) that when  $\xi_0 = 0$ ,  $a = 1$ ,  $b \rightarrow 0$ , then  $\delta = R^3$  and, with  $R_0 = R$  and with the same conditions for  $a$  and  $b$ , then, from (4-22),  $\beta = R^3$ , in which case the right hand side of (4-60) reduces to  $\left(\frac{2t}{\eta\tau} - 1\right)$ , which is the case for a uniformly distributed pressure.

The motion ceases when the plastic hinge is at the centre of the plate, i.e.,  $\xi = 0$ , which occurs when  $t = T_1$ , given by:

$$T_1 = \frac{\eta\tau R^3}{2\beta} \left( 1 + \frac{\delta}{R^3} - \frac{(R+\xi_0)(R-\xi_0)^2}{R^3} \right) \quad (4-61)$$

With the same special conditions described above, (4-61) reduces to  $T_1 = \frac{\eta\tau}{2}$ , which again is the result for the case of a uniformly distributed pressure.

- Third phase of motion:  $T_1 \leq t \leq T$

At the start of the final phase of motion, the plastic hinge is at the centre of the plate, thus the velocity profile is of the form:

$$\dot{w} = \dot{W}_3 \left(1 - \frac{r}{R}\right) \quad (4-62)$$

where  $\dot{W}$  is the maximum transverse velocity at the plate centre, as per Figure 4-12.

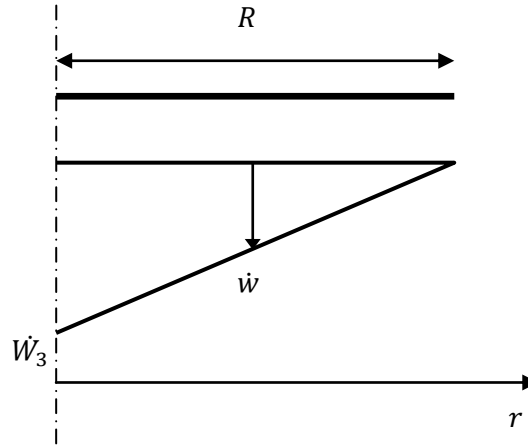


Figure 4-12: Velocity profile for final phase

The analysis is similar to that presented earlier, where the governing differential equation is given by:

$$\mu \ddot{W}_3 = -\frac{12M_0 R_0}{R^3} \quad (4-63)$$

Thus, the maximum permanent deflection is given by:

$$W_f = \frac{p_0 \eta \tau^2}{4\mu R_0 \beta} \left( R^4 - 2R_0 \left\{ \frac{\beta}{\eta} + [\xi_0^3 - R\xi_0^2 - R^2\xi_0 - \delta] \right\} \right) \quad (4-64)$$

occurring at:

$$T = \frac{p_0 \tau R^3}{12M_0 R_0} + T_1 \quad (4-65)$$

For the case of  $R_0 = R$ ,  $\beta = \delta = R^3$  and  $\xi_0 = 0$ , (4-64) reduces to  $W_f = \frac{p_0 \tau^2}{2\mu} \left( \frac{3}{2\eta} - 1 \right)$ , which is the case for a uniformly distributed pressure.

- Static admissibility

Whilst inequality (4-28a) is satisfied throughout the response due to normality requirements, it is necessary to show that the expressions for  $M_r$  obtained in (4-46), (4-54) and (4-57) satisfy the requirement in (4-28b). For the case under consideration, i.e.  $\eta > \eta_{crit}$ , it is not possible to proof static admissibility analytically due to the highly non-linear governing equations and thus admissibility will be demonstrated by considering various travelling hinge scenarios and showing admissibility in each of these cases.

For the first phase of motion, the bending moment distribution is shown graphically in Figure 4-13 for various values of  $\xi_0$ . It is seen that in the entire interval  $0 \leq r \leq R$ ,  $M_r \leq M_0$ .

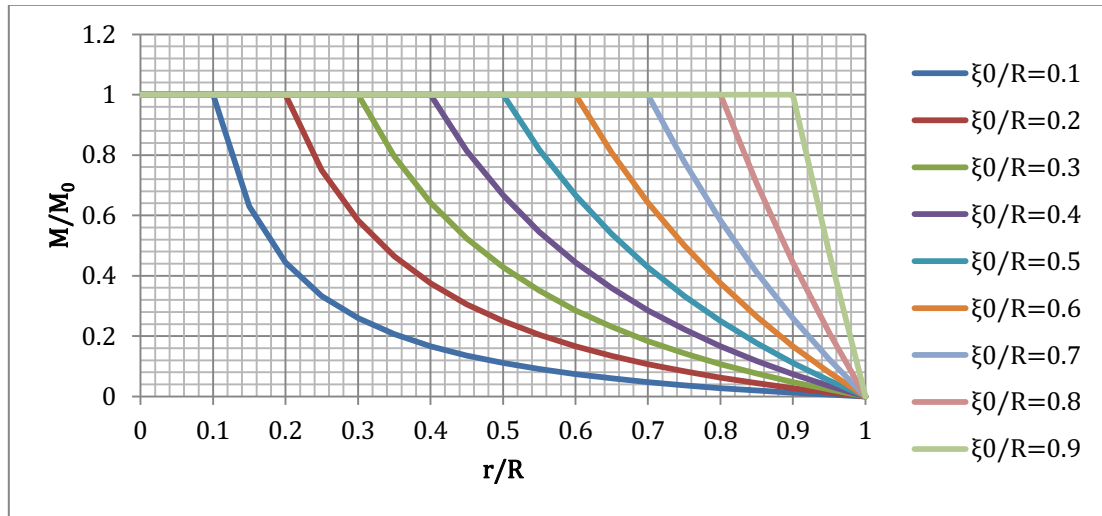


Figure 4-13: Radial bending moment distribution for  $0 \leq t \leq \tau$  for various values of  $\xi_0$

For the second phase of motion, the bending moment distribution is shown graphically in Figure 4-14 for various values of  $\xi$ . It is seen that in the entire interval  $0 \leq r \leq R$ ,  $M_r \leq M_0$ .

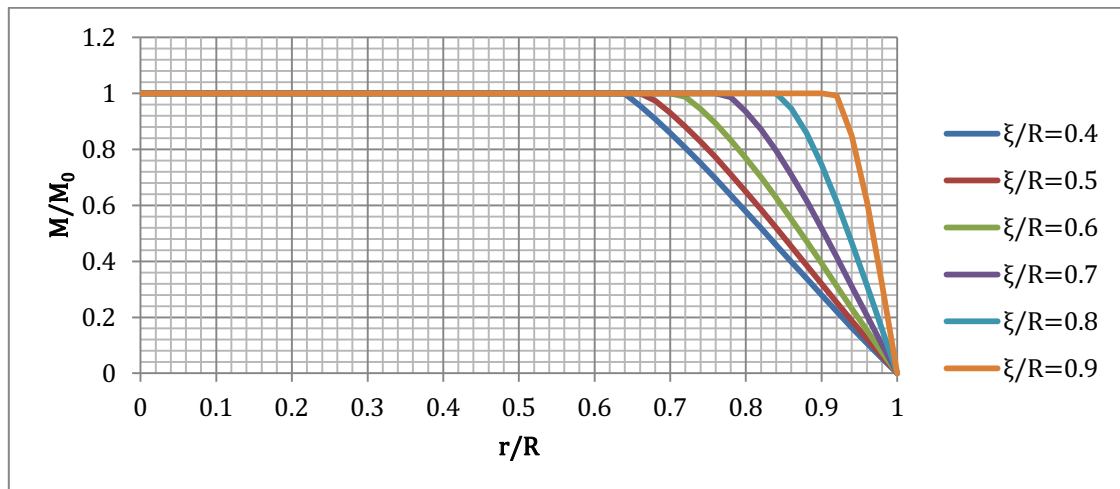


Figure 4-14: Radial bending moment distribution for  $\tau \leq t \leq T_1$  for various values of  $\xi$

For the third and final phase of motion, the bending moment distribution is shown graphically in Figure 4-15, where the result shown is independent of any other parameters. It is seen that in the entire interval  $0 \leq r \leq R$ ,  $M_r \leq M_0$ .

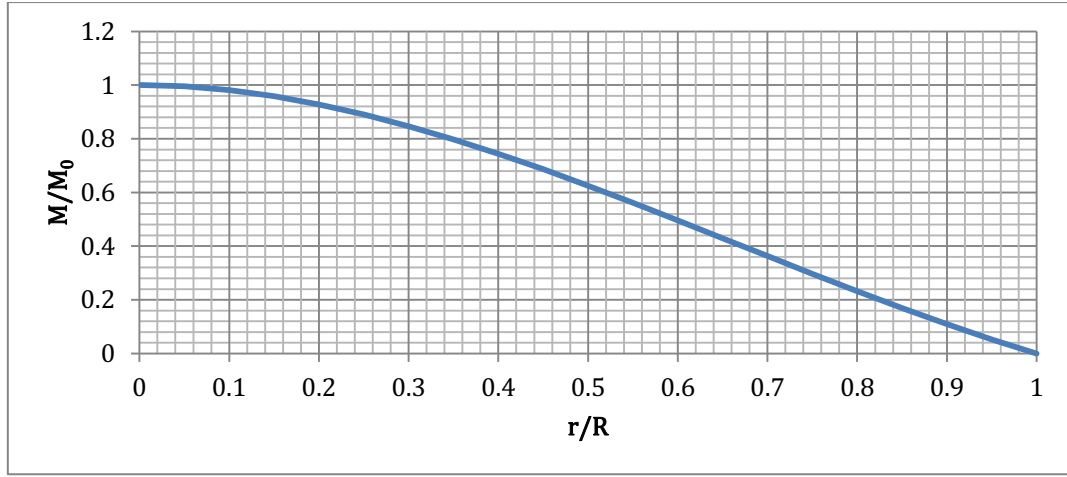


Figure 4-15: Radial bending moment distribution for  $T_1 \leq t \leq T$

Thus, it has been shown that the solution developed in Section 4.6.3 is statically admissible throughout the entire plate during the whole response  $0 \leq t \leq T$ .

#### 4.6.4 Idealisation into an impulsive load

A pulse load of a very short duration (i.e.,  $\tau \rightarrow 0$ ) and a very high amplitude (i.e.,  $\left(\frac{p_0}{p_c}\right) \gg 1$  or  $\eta \rightarrow \infty$ ) can be described as an impulsive load [254]. In this case, the pulse load is replaced by an impulsive velocity loading, in which the total impulse is equal to the change in linear momentum at the start of motion. Thus, conservation of linear momentum at  $t = 0$  implies that, for a rectangular pulse:

$$\int_0^{R_0} 2\pi r p_0 \tau \, dr + \int_{R_0}^R 2\pi r p_0 a e^{br} \tau \, dr = \int_0^R 2\pi r \mu V_0 \, dr \quad (4-66)$$

where  $V_0$  is an initial impulsive velocity distributed over the plate, which, from (4-66), is given by:

$$V_0 = \left(\frac{\varepsilon}{R^2}\right) \left(\frac{\eta p_c \tau}{\mu}\right) = \left(\frac{\varepsilon}{R^2}\right) \left(\frac{p_0 \tau}{\mu}\right) \quad (4-67)$$

where the parameter  $\varepsilon$  is given by:

$$\varepsilon = R_0^2 + \frac{2a}{b^2} (e^{bR_0} \{1 - bR_0\} - e^{bR} \{1 - bR\}) \quad (4-68)$$

If  $\left(\frac{p_0}{p_c}\right) \gg 1$  or  $\eta \rightarrow \infty$ , then (4-64) reduces to:

$$W_f = \frac{p_0 \eta \tau^2}{4\mu R_0 \beta} (R^4 - 2R_0 \{\xi_0^3 - R \xi_0^2 - R^2 \xi_0 - \delta\}) \quad (4-69)$$

Thus, using (4-67):

$$\frac{W_f}{H} = \frac{\lambda}{24} (R^4 - 2R_0\{\xi_0^3 - R\xi_0^2 - R^2\xi_0 - \delta\}) \left(\frac{R}{\beta}\right)^2 \frac{R}{R_0} \quad (4-70)$$

where  $\lambda$  is a dimensionless initial kinetic energy given by:

$$\lambda = \left(\frac{\beta}{\varepsilon}\right)^2 \left(\frac{\mu V_0^2}{M_0 H}\right) \quad (4-71)$$

For the usual case of  $R_0 = R$ ,  $\beta = \delta = R^3$  and  $\xi_0 = 0$ , (4-70) reduces to  $\frac{W}{H} = \frac{\lambda}{8}$ , which is the case for a uniformly distributed pressure given by Jones [254].

Defining  $I = p_0 \tau$  as the impulse per unit surface area of the plate, then (4-70) may be written as:

$$\frac{W_f}{H} = \frac{\lambda}{24} \left(\frac{I}{\mu V_0}\right)^2 (R^4 - 2R_0\{\xi_0^3 - R\xi_0^2 - R^2\xi_0 - \delta\}) \left(\frac{\varepsilon}{\beta}\right)^2 \frac{1}{R_0 R} \quad (4-72)$$

The maximum displacement is plotted against a dimensionless initial kinetic energy in Figure 4-16 for the case of  $\omega = 0.5$ .

Similar graphs can be plotted for various values of  $\omega$ , but in each case it is seen that there exist a value of  $\eta$  (in this case,  $\eta = 30$ ) which, when using (4-64), the results are only marginally smaller than the impulsive load simplification with  $\eta = \infty$  using (4-72).

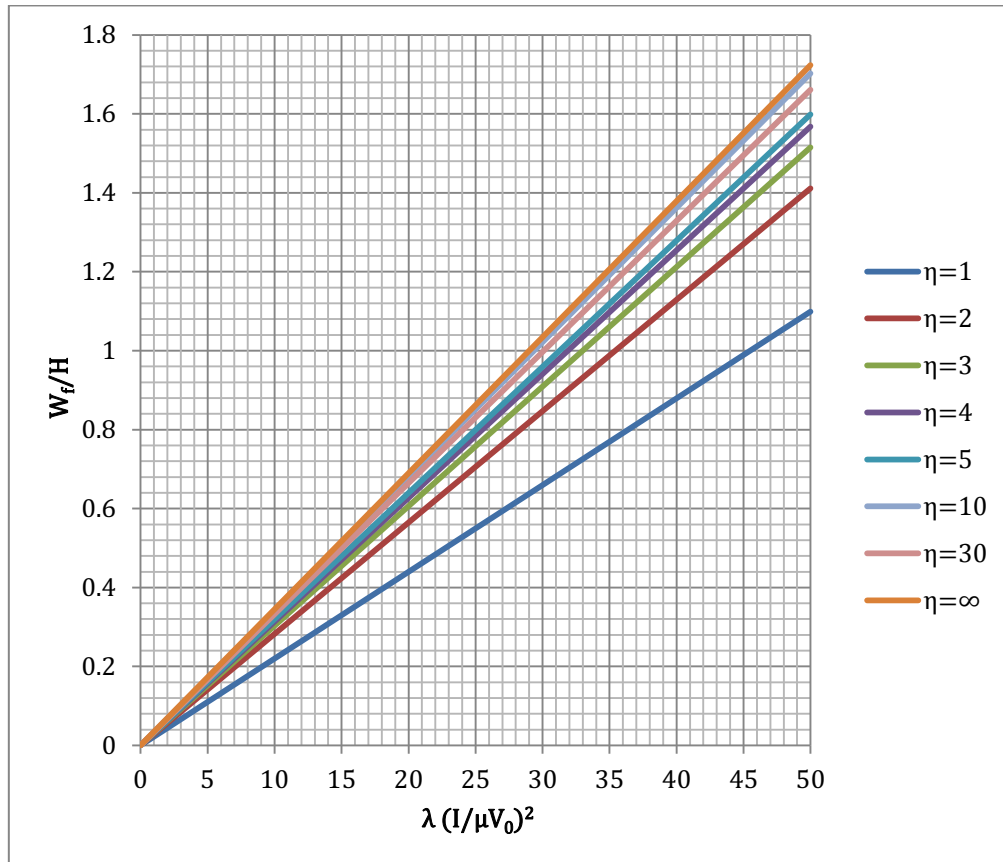


Figure 4-16: Dimensionless  $W_f$  for impulsive loading case (for  $\omega = 0.5$ )

#### 4.6.5 Pulse shape independent interaction diagram

The dynamic analysis presented in Sections 4.6.2 to 4.6.4 has been limited to a rectangular pulse shape. A similar analysis can be performed for alternative temporal variation functions which will provide a better description to a real blast load. Two further functions were considered, discussed hereunder.

A simple linearly decaying function, shown in Figure 4-17 and described by (4-73), is the first alternative pulse shape.

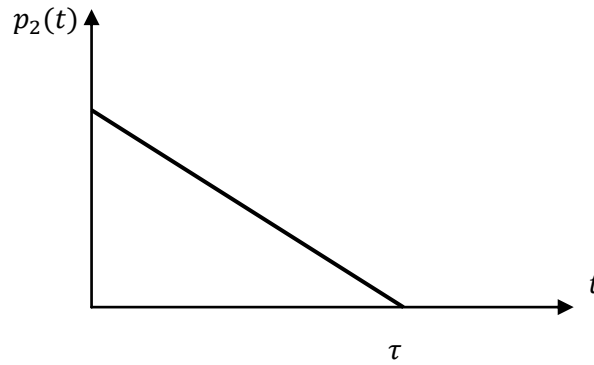


Figure 4-17: Linearly decaying pulse shape

$$p_2(t) = \begin{cases} 1 - \frac{t}{\tau}, & 0 \leq t \leq \tau \\ 0, & t \geq \tau \end{cases} \quad (4-73)$$

A more generalised exponentially decaying function, suggested by Li and Meng [256, 257] and which presented is in Figure 4-18 and (4-74), is also considered.

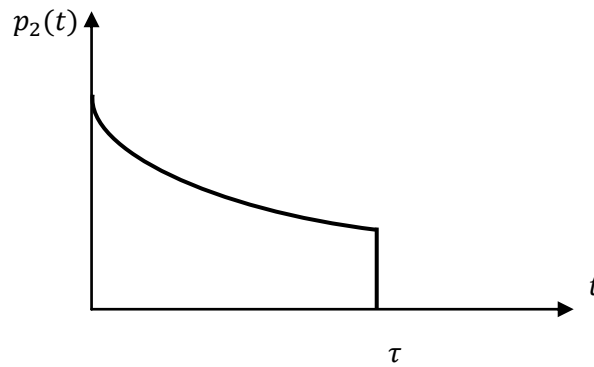


Figure 4-18: Generalised exponentially decaying pulse shape

$$p_2(t) = \begin{cases} \left(1 - X \frac{t}{\tau}\right) e^{-\frac{Yt}{\tau}}, & 0 \leq t \leq \tau \\ 0, & t \geq \tau \end{cases} \quad (4-74)$$

By correct choices of the parameters  $X$  and  $Y$ , it can be shown that (4-74) reduces to the previous 2 cases, e.g., with  $X = 1$  and  $Y = 0$ , (4-74) reduces to (4-73) and with  $X = Y = 0$ , (4-74) reduces to (4-26).

Indeed, (4-74) can also be reduced to the temporal function proposed in (3-3), by taking  $X = 0$  and  $Y = w\tau$  (or  $wt_d$  using the notation of Chapter 3).

Expressions for the final permanent displacements in each case are found using a similar procedure to that in Section 4.6.3, giving the following results:

For the linearly decaying pulse load,

$$W_f = \frac{p_0 \tau^2}{96 \mu R_0 \beta} \left( 6\eta R^4 + 12R_0 \left\{ \eta [\delta + R^2 \xi_0 + R \xi_0^2 - \xi_0^3] - \frac{4\beta}{3} \right\} \right) \quad (4-75)$$

occurring at:

$$T = \frac{p_0 \tau R^3}{24 M_0 R_0} + \frac{\eta \tau R^3}{4\beta} \left( 1 + \frac{\delta}{R^3} - \frac{(R + \xi_0)(R - \xi_0)^2}{R^3} \right) \quad (4-76)$$

For the exponentially decaying pulse load,

$$W_f = \frac{p_0 \tau^2}{24 \mu Y^3} \left( \frac{p_0 \varphi^2 Y^3 R^3}{M_0 R_0} + \frac{12}{\beta} \{ \eta \varphi^2 Y^3 [\delta + R^2 \xi_0 + R \xi_0^2 - \xi_0^3] + 2\beta [2X - Y] \} \right. \\ \left. - 24e^{-Y} \{ 2X + Y[2X - 1] + Y^2[X - 1] \} \right) \quad (4-77)$$

occurring at:

$$T = \frac{p_0 \varphi \tau R^3}{12 M_0 R_0} + \frac{\varphi \eta \tau R^3}{2\beta} \left( 1 + \frac{\delta}{R^3} - \frac{(R + \xi_0)(R - \xi_0)^2}{R^3} \right) \quad (4-78)$$

where  $\varphi$  is a parameter given by  $\varphi = \frac{e^{-Y}(X[Y+1]-Y)-X+Y}{Y^2}$ .

The results from (4-64), (4-75) and (4-77) are plotted in non-dimensional form in Figure 4-19.

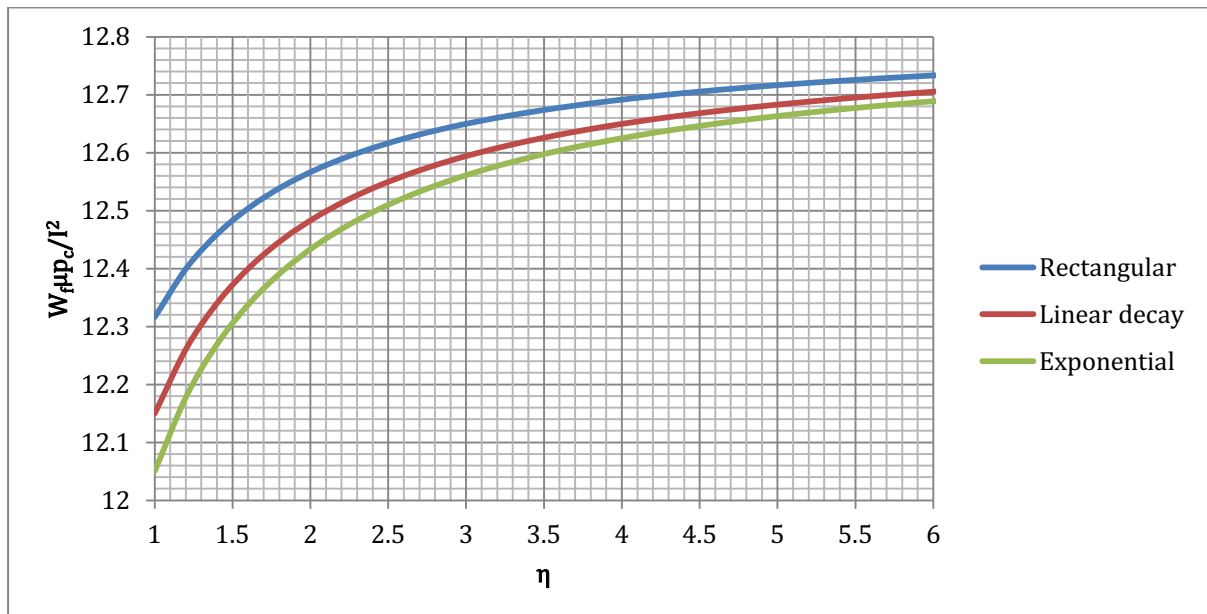


Figure 4-19: Dimensionless  $W_f$  for different pulse shapes ( $\omega = 0.1$ )

These results are shown as a function of the maximum applied pressure  $p_0$  and it can be readily seen that there is a strong dependence on the pulse shape, especially in the region where  $\eta \approx 1$ .

This poses a difficulty from a practical point of view since it is often complex to record accurately a pressure-time history of a blast load and to model the actual blast load in a laboratory test or in the field.

The pulse-shape effect is eliminated by expressing the results of Figure 4-19 in terms of the Youngdahl parameters [119] rather than the maximum pressure and impulse. Thus, for each of the pulse loads, the effective impulse,  $I_e$ , the mean time,  $t_{mean}$  and the effective pressure,  $p_e$  (which were discussed in Chapter 2), are found using (2-18) to (2-20), as defined in [119]:

$$I_e = \int_0^T p(t) dt \quad (4-79)^8$$

$$t_{mean} = \frac{1}{I_e} \int_0^T t p(t) dt \quad (4-80)^8$$

$$p_e = \frac{I_e}{2t_{mean}} \quad (4-81)$$

Computing the above for each pulse shape and plotting the new results in terms of the effective pressure and impulse, a single curve is obtained which is independent of pulse shape, as per Figure 4-20, and which is given in terms of the effective dynamic load factor,  $\eta_e = \frac{p_e}{p_c}$ .

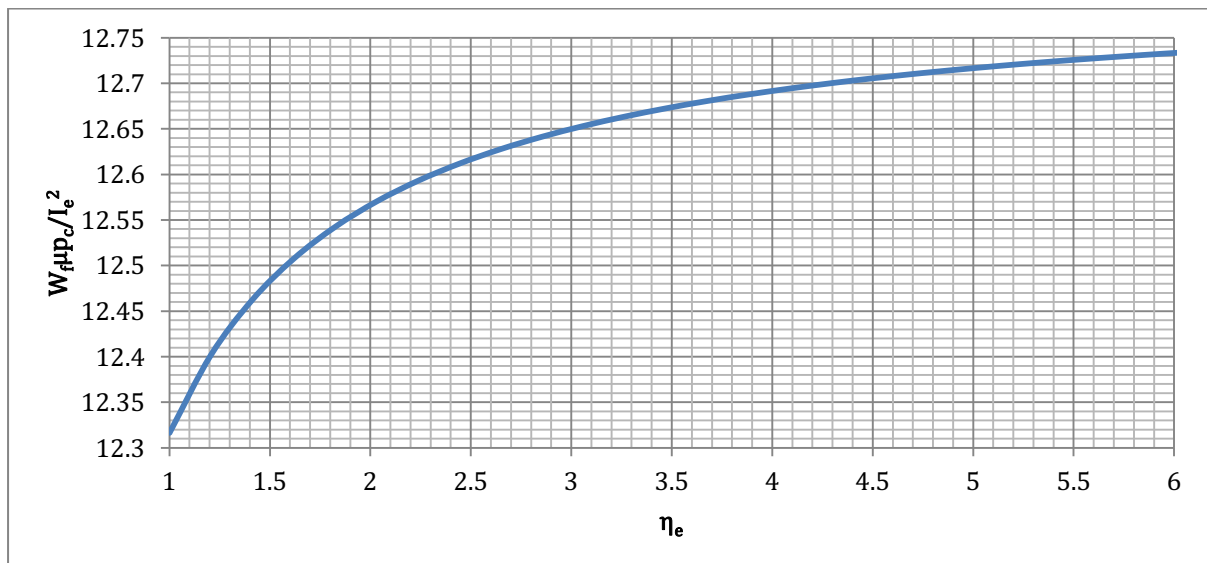


Figure 4-20: Dimensionless pulse-shape independent  $W_f$  ( $\omega = 0.1$ )

In Figure 4-21, the single curve for various values of  $\omega$  is given while in Figure 4-22 the interaction surface for all 3 parameters is shown.

<sup>8</sup> For the case of rigid plasticity, the lower integration limit ( $t_y$  in Youngdahl's paper) is taken to be zero since plasticity commences at the onset of loading.

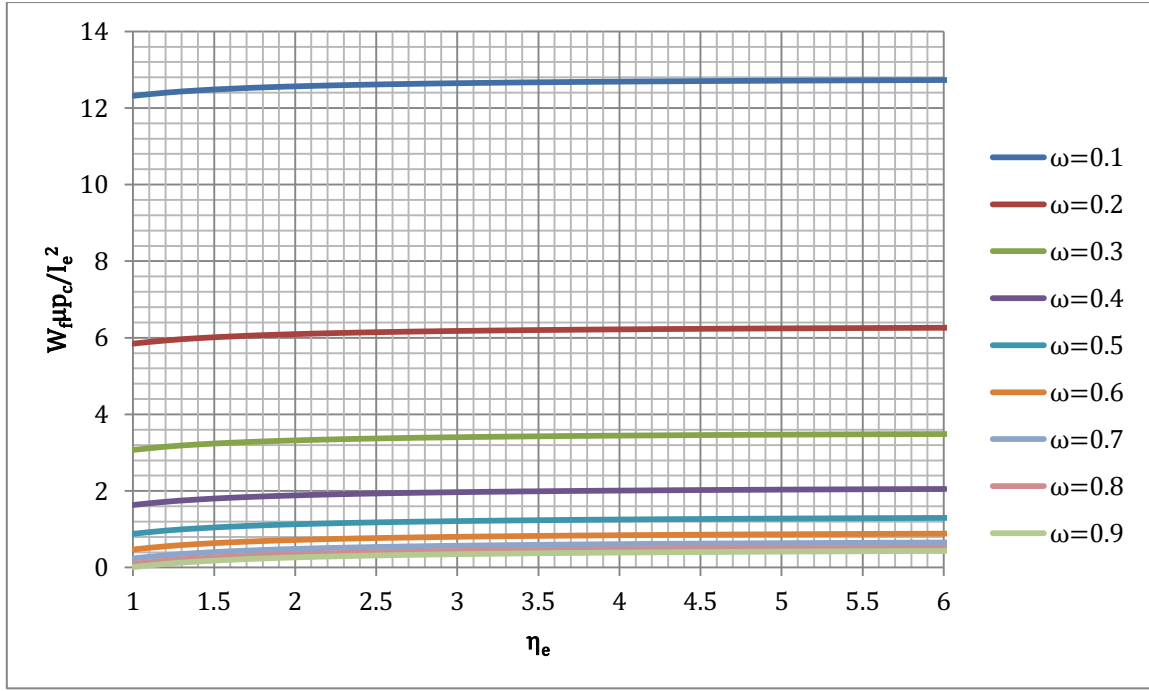


Figure 4-21: Pulse-shape independent dimensionless  $W_f$  for various  $\omega$  values

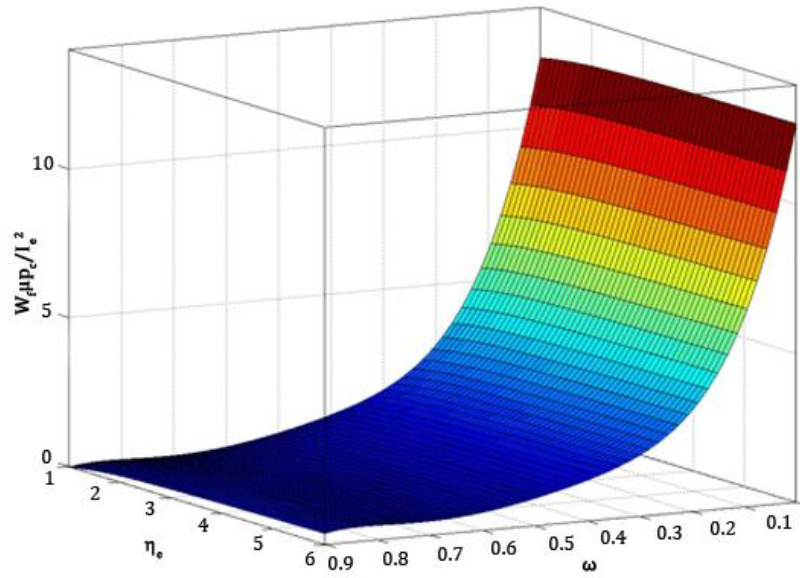


Figure 4-22: Pulse-shape independent dimensionless  $W_f$  interaction surface

#### 4.6.6 Concluding remarks

In this section, an analytical model is presented which predicts the transverse displacement of a simply supported circular steel plate exhibiting rigid plastic behaviour when subjected to a blast load, which is described by means of a function combining spatial and temporal variations, presented in Chapter 3.

The formulation is based on the assumption that the axi-symmetrical lateral load causes infinitesimal deflections and that the plate thickness  $H$  is such that the contributions of membrane, transverse shear and rotatory inertia effects are negligible.

On the basis of these assumptions, the static plastic collapse pressure found earlier using the upper and lower bounds of plasticity theory was extended to consider dynamic loading and the study focuses on finding the transverse displacement of the plate at various stages of its motion. A critical value of applied pressure to the plate's static collapse pressure is found, which differentiates between "low" loads and "high" loads in the terminology adapted by Hopkins and Prager [67].

It was seen that for high applied loads and/or short duration loads, the pulse can be replaced by an equivalent impulsive velocity and it was found that the permanent transverse displacement can be approximated by means of such an approach for high values of applied loading by means of dimensionless plots of applied initial kinetic energy against displacement.

Three temporal variation forms of the load are assumed, viz., the general decreasing pulse load proposed by Li and Meng [257] and two special cases of this load form, i.e., rectangular and linearly decaying pulses. It was found that the predicted final transverse displacements for the above pulses loads are highly dependent on the pulse shape.

Thus, pulse-shape effects are eliminated by replacing the actual applied pressure with the "effective pressure" as defined by Youngdahl [119] to give a single pulse shape-independent curve for the final plate deflection.

However, the effects of pulse shape are only relevant for the case of either low loading levels (i.e. low values of  $\eta$ , as evident in Figure 4-19) or long duration loads (i.e. large values of  $\tau$ ). For the case of impulsive loadings, such as blast loading being considered, it is possible to idealise the pulse loading into an impulse and thus the response is independent of pulse shape. Since the concept of defining effective loading parameters is aimed at eliminating the effects of pulse shape (i.e. the temporal description of the load), any further consideration of defining effective loading parameters will not be considered in this study.

Nevertheless, this formulation is only limited to bending action and thus valid only for small plate deflections. This implies that the current equations would highly overestimate the response of plates subjected to blast loading, where the maximum transverse displacement would be favourably affected by the effects of transverse shear, rotatory inertia and in-plane membrane forces. These are studied in turn in the following sections.

#### **4.7 Case B – Dynamic rigid-plastic behaviour of thick plates**

The analysis of Section 4.6 is only valid for plates whose behaviour is dominated by bending effects. Although it has been shown [255] that transverse shear effects are not significant in the case of static plastic collapse behaviour of circular plates (in the case of plates whose radius is at least 1.5 times the plate thickness), the influence of transverse shear can be significant for dynamic plastic response, particularly for stocky plates.

This is evident from Figure 4-23 (after [254]), where the transverse shear strain,  $\gamma$ , is the slope of the centre line of the plate.

It is evident that, as the plate thickness increases, the relevance of  $\gamma$  also increases.

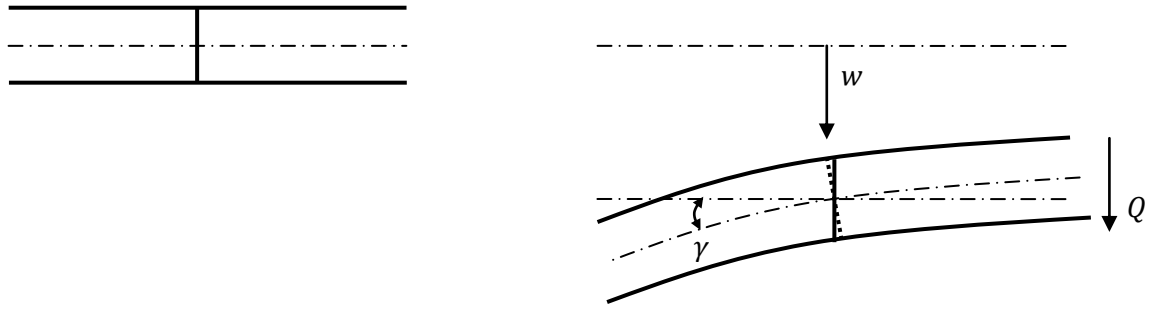


Figure 4-23: Pure plate shearing (left: undeformed state; right: deformed state)

The inclusion of rotatory inertia will also be discussed in the context of the problem under current consideration. Rotatory inertia will contribute towards the moment equilibrium equation, as shown in Figure 4-24 (after [254]) and as stated in (4-84a).

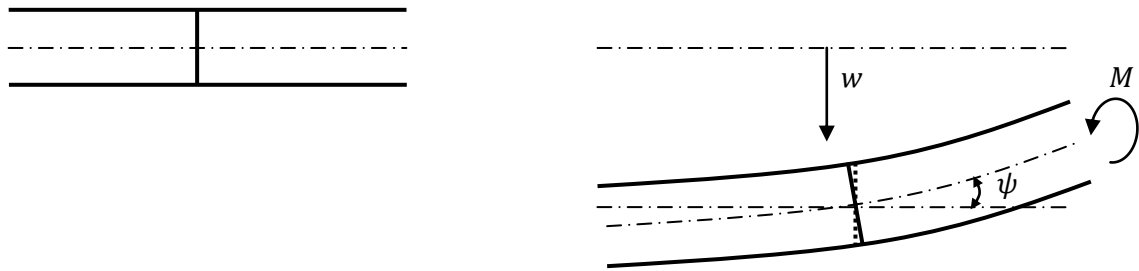


Figure 4-24: Pure plate bending (left: undeformed state; right: deformed state)

In this figure,  $\psi$  is the rotation of the plate's centre line and, thus, the radial curvature of the plate is given by [85]:

$$\kappa_r = \frac{\partial \psi}{\partial r} \quad (4-82)$$

Thus, the total slope of the plate is given by [85]:

$$\frac{\partial w}{\partial r} = \gamma - \psi \quad (4-83)$$

Thus, in this section, the analyses presented in Sections 4.5 and 4.6 are extended to take into account the transverse inertia term in the lateral equilibrium equation.

#### 4.7.1 Governing equilibrium equations

The governing equilibrium equations, in polar co-ordinates, which need to be satisfied are given by [85]:

$$\frac{\partial M_r}{\partial r} + \frac{(M_r - M_\theta)}{r} + Q_r = I_r \ddot{\psi} \quad (4-84a)$$

$$\frac{\partial Q_r}{\partial r} + \frac{Q_r}{r} = \mu \ddot{w} - p(r, t) \quad (4-84b)$$

In this equation,  $M_r$  and  $M_\theta$  are the radial and circumferential bending moment quantities respectively,  $Q_r$  is the radial shear force,  $I_r$  is the second moment of area of the plate cross section (per unit width) and  $\psi$  is the rotation of the plate centre line due to bending, where the following relations hold:

$$\dot{\kappa}_r = \frac{\partial \dot{\psi}}{\partial r} \quad (4-85a)$$

$$\dot{\kappa}_\theta = \frac{\dot{\psi}}{r} \quad (4-85b)$$

where  $\dot{\kappa}_r$  and  $\dot{\kappa}_\theta$  are the rates of radial and circumferential changes in curvature respectively.

While vertical equilibrium, i.e. (4-84b), is independent of transverse shear and rotatory inertia, it is clear that rotatory inertia effects contribute to the bending equilibrium equation, i.e. (4-84a). In the absence of rotatory inertia effects, then the right hand side of (4-84a) reduces to zero, which is the first scenario which will be studied in this section.

#### 4.7.2 Range of plate geometries for influential transverse shear effects

Considering a simply supported circular plate subjected to an applied load  $p(r, t)$  as described in Section 4.4.1, and using (4-39) and (4-84a) with  $I_r = 0$ , then for  $\eta \leq \eta_{crit}$  and in the interval  $0 \leq r \leq R_0$ , it can be shown that:

$$\frac{Q_r}{Q_0} = \frac{1}{2\nu} \left( \frac{r}{R} \right) \left( 6 \left\{ \eta - \frac{R_0}{R} \right\} - \frac{4r}{R} \left\{ \eta - \frac{R_0}{R} \right\} - \frac{3\eta R^3}{\beta} \right) \quad (4-86)$$

where the parameter  $\nu$  is a ratio of the plastic shear strength to the plastic bending strength of the plate and is defined as:

$$\nu = \frac{Q_0 R}{2M_0} \quad (4-87)$$

It should be noted that, for the case of a prismatic section,  $\nu$  reduces to  $\nu = \frac{R}{H}$ , which is a direct measure of the plate geometry.

It can be seen from (4-86) that  $Q_r$  attains its maximum value at  $r = R_0$  and that, for  $\eta = \eta_{crit}$ , then:

$$\frac{Q_r}{Q_0} = - \frac{2R_0^3}{\nu(2\beta - R^3)} \quad (4-88)$$

Thus, it is seen that  $\left| \frac{Q_r}{Q_0} \right| \geq 1$  when  $\nu \leq \frac{2R_0^3}{(2\beta - R^3)}$  which implies that transverse shear effects will influence the behaviour of the plate for a plate geometry  $\frac{R}{H} \leq \frac{2R_0^3}{(2\beta - R^3)}$ . For the case of  $R_0 = R$  and  $\beta = R^3$  (i.e.  $a = 1$  and  $b \rightarrow 0$ ), then this limit reduces to the expression for a uniformly distributed load found in literature [254], i.e.,  $\nu \leq 2$ .

Similarly, for the case of  $R_0 \leq r \leq R$ , it can be shown that at  $r = R$  and for  $\eta = \eta_{crit}$ :

$$\frac{Q_r}{Q_0} = \frac{1}{2\nu} \left( \frac{b^2 \{6R_0^3 - 2R_0R^2 + 2\alpha - R^3\} + 12aR_0 \{e^{bR}[bR - 1] - e^{bR_0}[bR_0 - 1]\}}{b^2 \{2\beta - R^3\}} \right) \quad (4-89)$$

Thus,  $\left| \frac{Q_r}{Q_0} \right| \geq 1$  when  $\nu \leq \frac{b^2 \{6R_0^3 - 2R_0R^2 + 2\alpha - R^3\} + 12aR_0 \{e^{bR}[bR - 1] - e^{bR_0}[bR_0 - 1]\}}{2b^2 \{2\beta - R^3\}}$  or, for the case of  $R_0 = R$  and  $\beta = R^3$ , when  $\nu \leq 5$ .

For higher loading values, i.e.,  $\eta > \eta_{crit}$ , then it can be shown that:

$$\begin{aligned} \frac{Q_r}{Q_0} = \frac{1}{2\nu} & \left( \frac{2R^2r}{\delta \{R - \xi_0\}} + \frac{12R^2ae^{br}}{rb^2\delta} \left\{ \frac{1}{br} + 1 - br \right\} \right. \\ & + \frac{R}{(R - \xi_0)r} \left\{ \frac{12Ra}{b^3\delta} [e^{bR}(bR - 2) - e^{b\xi_0}(b\xi_0 - 2)] \right. \\ & \left. \left. - \frac{R}{\delta} (R^3 + \xi_0R^2 - \xi_0^3 + \xi_0^2R) - \xi_0 \right\} - \frac{R}{r} \right) \end{aligned} \quad (4-90)$$

where  $\xi_0$  is the radius of the stationary plastic hinge, defined by the relation in (4-50) and  $\delta$  is defined in (4-51).

Evaluating (4-90) at  $r = R$  implies that:

$$\begin{aligned} \frac{Q_r}{Q_0} = \frac{1}{2\nu} & \left( \frac{2R^4}{\delta \{R - \xi_0\}} + \frac{12Rae^{bR}}{b^2\delta} \left\{ \frac{1}{bR} + 1 - bR \right\} \right. \\ & + \frac{1}{R - \xi_0} \left\{ \frac{12Ra}{b^3\delta} [e^{bR}(bR - 2) - e^{b\xi_0}(b\xi_0 - 2)] \right. \\ & \left. \left. - \frac{R}{\delta} (R^3 + \xi_0R^2 - \xi_0^3 + \xi_0^2R) - \xi_0 \right\} - 1 \right) \end{aligned} \quad (4-91)$$

For very high loadings, i.e., as  $\eta \rightarrow \infty$ , then  $\xi_0 \rightarrow R$ , in which case, as it can be seen from (4-91),  $\left| \frac{Q_r}{Q_0} \right| \rightarrow \infty \forall \nu \in \mathbb{R}$ . This means that, for a plate subjected to a high impulsive load, the transverse shear is infinitely large at the start of the event, independently of the magnitude of its shear strength or, conversely, of its geometry.

#### 4.7.3 Impulsive loading for very stocky plates ( $\nu \leq 1.5$ )

As it has also been shown in Section 4.7.2, for low loads (i.e.,  $\eta < \eta_{crit}$ ), then transverse shear effects are only relevant for very low  $\nu$  values, which are not often encountered in the case of plated structures resisting blast.

Commonly, such plated structures would have slenderness values (i.e. corresponding  $\nu$  values) in the order of 100 and thus transverse shear effects are only significant for cases of very high loads.

In this case, it is possible to idealise the blast load as a uniform impulsive velocity. This is a valid assumption, provided that the load is of a very short duration (i.e.,  $\tau \rightarrow 0$ ) and has a very high amplitude (i.e.,  $\frac{p_0}{p_c} \gg 1$  or  $\eta \rightarrow \infty$ ). Using the results derived in Section 4.6.4, then the initial impulsive velocity,  $V_0$ , is given by (4-67).

The variation of the parameter  $\varepsilon$ , given by (4-68), with various load parameters is shown in Figure 4-25:

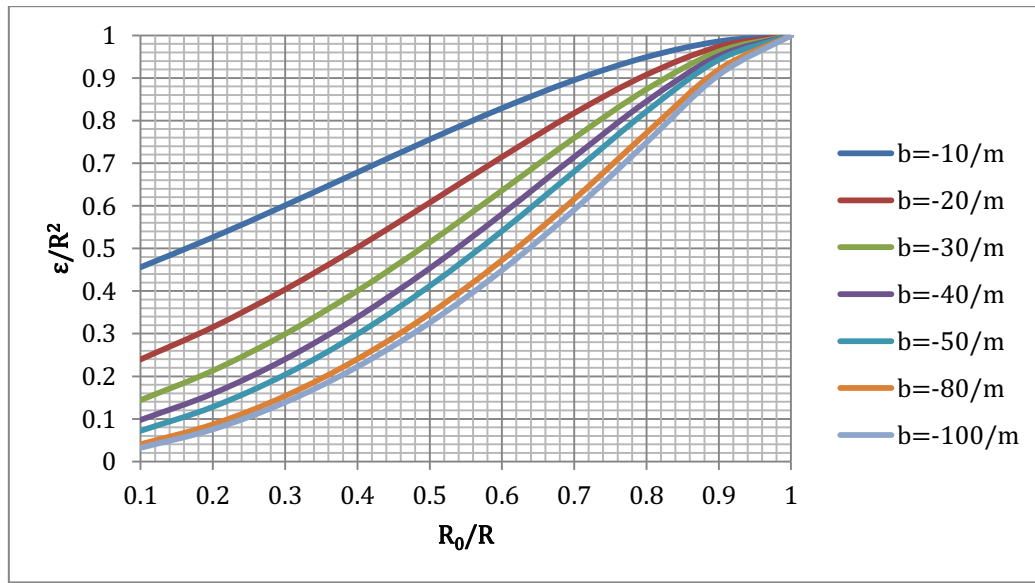


Figure 4-25: Variation of  $\varepsilon$  with  $R_0/R$

It can be seen that in the limit as  $R_0 \rightarrow R$ , then  $\varepsilon \rightarrow R^2$  irrespective of the value of the parameter  $b$ , which then leads to  $V_0 = \frac{p_0 \tau}{\mu}$ , which is the value found in literature for uniform loads, and as  $b \rightarrow 0$ , then  $\varepsilon \rightarrow R^2$  irrespective of  $\left(\frac{R_0}{R}\right)$ .

For typical close-in blasts (stand-off to charge diameter ratio between 0.5 and 5), then it can be seen from the results in Chapter 3 that  $\left(\frac{R_0}{R}\right)$  assumes small values (i.e.  $\left(\frac{R_0}{R}\right) \lesssim 0.2$ ) and  $b$  takes values in the range  $-30/m \leq b \leq -80/m$ , which leads to the parameter  $\varepsilon$  assuming values which are typically less than  $0.2R^2$ . Thus, this implies that, for typical close-in blasts, the impulsive velocity is directly proportional to the peak applied pressure and the load duration, irrespective of the loading distribution.

If transverse shear effects are to dominate bending effects, then a suitable velocity profile describing the motion of the plate will be the one shown in Figure 4-26 [85, 254], where the whole plate remains rigid, except at the boundary:

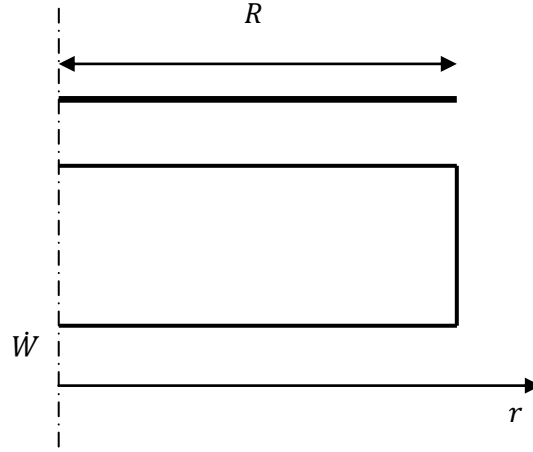


Figure 4-26: Velocity profile for very stocky plates

From the moment equilibrium equation and without considering rotatory inertia:

$$\frac{d^2(rM_r)}{dr^2} = \mu r \ddot{W} \quad (4-92)$$

Thus:

$$M_r = \frac{\mu r^2 \ddot{W} (r^2 - R^2)}{6} \quad (4-93)$$

since both integration constants are found to be equal to zero to satisfy  $M_r = 0$  at  $r = R$  and for  $M_r$  to remain finite at  $r = 0$ .

Thus, using (4-84a), the transverse shear force is given by:

$$Q_r = \frac{\mu \ddot{W}}{6} \left( 3r - \frac{R^2}{r} \right) - \frac{M_\theta}{r} \quad (4-94)$$

Satisfying symmetry at  $r = 0$ , then  $Q_r = 0$ , which gives:

$$M_\theta = -\frac{\mu R^2 \ddot{W}}{6} \quad (4-95)$$

Also, for  $Q_r = -Q_0$  at  $r = R$ , then it implies that:

$$\ddot{W} = -\frac{2Q_0}{\mu R} \quad (4-96)$$

Integrating (4-96) twice gives:

$$W = -\frac{Q_0}{\mu R} t^2 + V_0 t \quad (4-97)$$

where the integration constants are obtained from the initial conditions, namely,  $\dot{W} = V_0$  and  $W = 0$  at  $t = 0$ .

The duration of the response is found by setting  $\dot{W} = 0$ , to give the response time as:

$$T = \frac{\mu V_0 R}{2Q_0} = \frac{3}{2} \left( \frac{\varepsilon R}{\beta} \right) \frac{\eta \tau}{\nu} \quad (4-98)$$

Thus, the permanent displacement is found by evaluating (4-97) at  $t = T$  to give:

$$W_f = \frac{\mu V_0^2 R}{4Q_0} = \frac{1}{4} \left( \frac{\varepsilon^2}{R^3} \right) \frac{(p_c \tau)^2}{\mu Q_0} \quad (4-99)$$

For the case of  $\beta = R^3$  and  $\varepsilon = R^2$  (i.e.  $R_0 = R$ ,  $a = 1$  and  $b \rightarrow 0$ ), then (4-98) and (4-99) reduce to the corresponding expressions for a uniformly distributed load found in literature [85].

From (4-99) it can be seen that the displacement is constant throughout the plate within the range  $0 \leq r \leq R$ .

- Static admissibility

Using (4-93) to (4-97), then it can easily be shown that:

$$\frac{M_r}{M_0} = \frac{2\nu}{3} \left( 1 - \left\{ \frac{r}{R} \right\}^2 \right) \quad (4-100a)$$

$$\frac{M_\theta}{M_0} = \frac{2\nu}{3} \quad (4-100b)$$

$$\frac{Q_r}{Q_0} = -\frac{r}{R} \quad (4-100c)$$

Thus, it can be seen from (4-100) that  $Q_r \leq Q_0$  in the whole interval  $0 \leq r \leq R$  but, to ensure that  $M_r$  and  $M_\theta$  do not exceed  $M_0$ , then  $\nu \leq \frac{3}{2}$ . Thus, the result is statically admissible for plates having  $\nu \leq \frac{3}{2}$ , termed "Class I" plates in [85].

#### 4.7.4 Impulsive loading for stocky plates ( $1.5 \leq \nu \leq 2$ )

A similar procedure to that described in Section 4.7.3 will yield results for plates with different  $\nu$  values.

However, an alternative velocity profile to that shown in Figure 4-26 is required in order to avoid yield violation. This implies that the response of the plate is spread over two phases of motion: one with bending across the plate but with transverse shear slides at the supports and a final one with plate bending.

These are shown in Figure 4-27 and Figure 4-28 respectively.

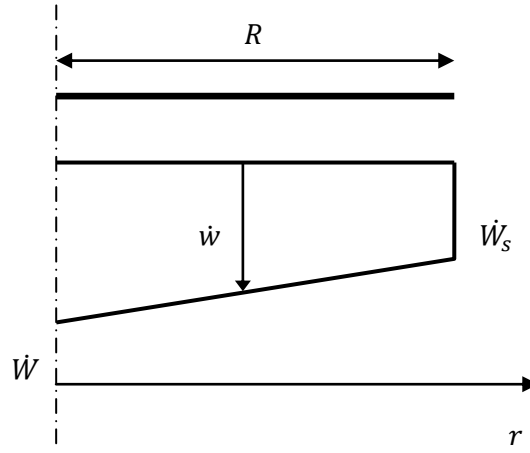


Figure 4-27: Velocity profile for first phase of motion of stocky plates

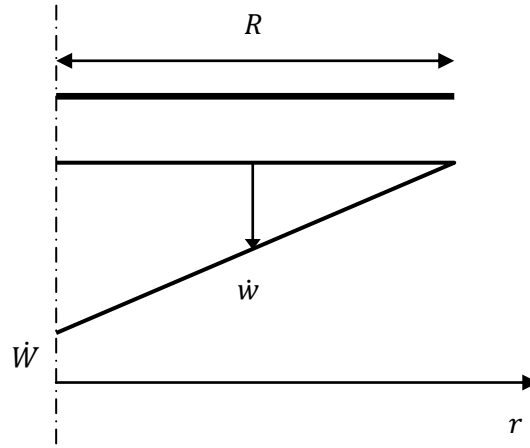


Figure 4-28: Velocity profile for second phase of motion of stocky plates

This has been studied in [85] and the results are modified here for the problem under study. The response ends when the velocity is equal to zero at a time given by:

$$T = \frac{\mu V_0 R}{6M_0} = \left(\frac{\varepsilon R}{\beta}\right) \eta \tau \quad (4-101)$$

It should be noted that the response time here is independent of the value of  $\nu$ .

The permanent displacement at any point is given by:

$$w_f = \frac{\mu V_0^2 R^2}{12M_0(\nu - 1)} \left( \{3 - 2\nu\} \frac{r}{R} + 2\nu - \frac{5}{2} \right) \quad (4-102)$$

Thus, at the supports, or at  $r = R$ :

$$W_s = \frac{\mu V_0^2 R^2}{24M_0(\nu - 1)} \quad (4-103)$$

At midpoint, or at  $r = 0$ :

$$W_f = \frac{\mu V_0^2 R^2 (4\nu - 5)}{24 M_0 (\nu - 1)} \quad (4-104)$$

- Static admissibility

It has been shown [85] that, in order to satisfy static admissibility, the above results are valid for the range of plates within the interval  $\frac{3}{2} \leq \nu \leq 2$ , termed “Class II” plates in [85].

#### 4.7.5 Impulsive loading for less stocky plates ( $\nu \geq 2$ )

A similar procedure to that described in Sections 4.7.3 and 4.7.4 gives results for plates with higher slenderness values, by considering three phases of motion, viz., a first phase during which there is shear sliding at the supports and during which the plate has a rigid central portion of radius  $r_1$ , followed by a second phase where the position of the plastic hinge moves inwards towards the centre, leading to a final phase, where the velocity profile is that of a single central hinge. These are shown in Figure 4-29 to Figure 4-31.

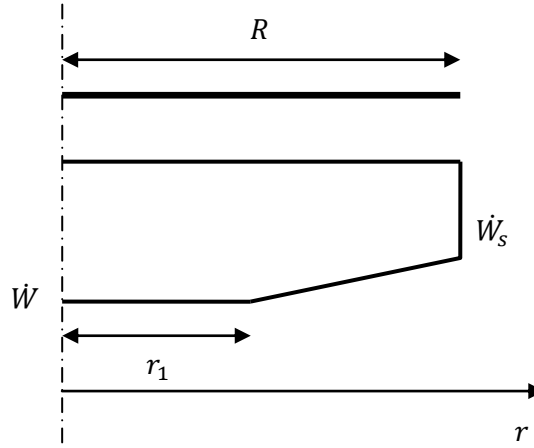


Figure 4-29: Velocity profile for first phase of motion of slender plates

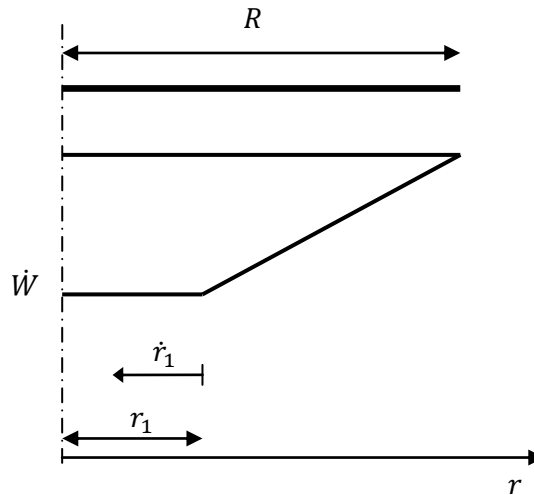


Figure 4-30: Velocity profile for second phase of motion of slender plates

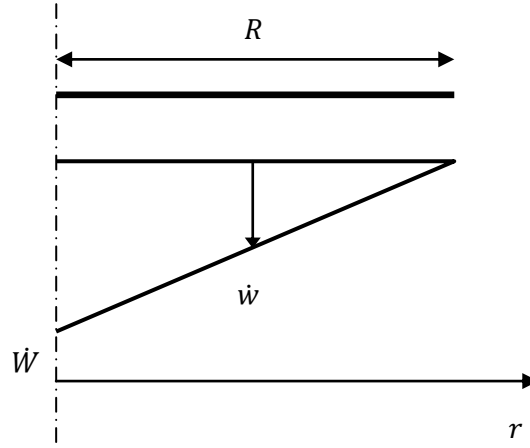


Figure 4-31: Velocity profile for third phase of motion of slender plates

The inner radius  $r_1$  is given by [85]:

$$r_1 = R \left( \frac{\sqrt{4v^2 - 8v + 1} - 1}{2v} \right) \quad (4-105)$$

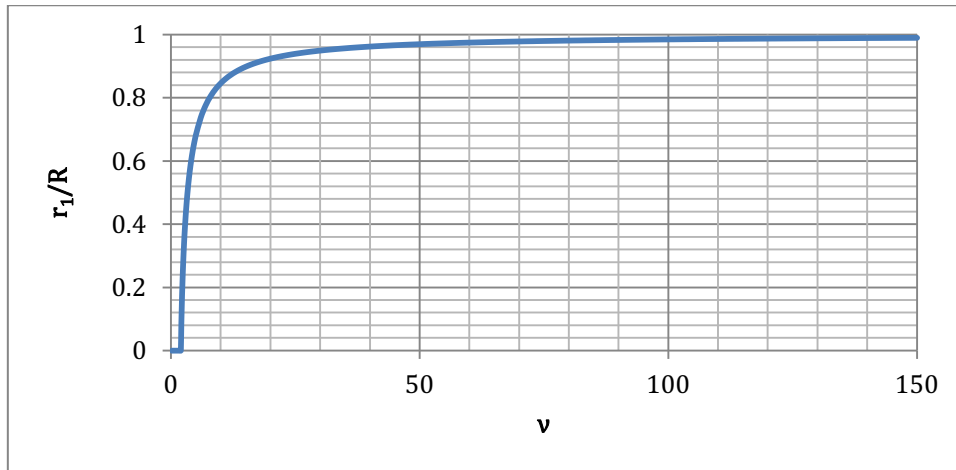


Figure 4-32: Variation of  $r_1$  with  $v$

It can be seen from (4-105) and Figure 4-32 that, as  $v \rightarrow \infty$ , then  $r_1 \rightarrow R$ , which leads to a “bending” problem and thus independent of transverse shear effects.

The response ends when the velocity is equal to zero at a time given by [85]:

$$T = \frac{\mu V_0 R}{6M_0} = \left( \frac{\varepsilon R}{\beta} \right) \eta \tau \quad (4-106)$$

It should be noted that the response time here is the same as (4-101), i.e., that for the range  $\frac{3}{2} \leq v \leq 2$ , and is independent of the value of  $v$ .

The permanent displacement at any point is given by:

$$w_f = \begin{cases} \frac{\mu V_0^2 R^2}{24 M_0} \left( 3 + \frac{2r}{R} + \left\{ \frac{r}{R} \right\}^2 \right) \left( 1 - \frac{r}{R} \right), & 0 \leq r \leq r_1 \\ \frac{\mu V_0^2 R^2}{24 M_0} \left( \left[ 1 + \frac{2r_1}{R} + 3 \left\{ \frac{r_1}{R} \right\}^2 \right] \left[ 1 - \frac{r}{R} \right] + \left[ 2 - \frac{r_1}{R} - \frac{r}{R} \right] \left[ 1 - \left\{ \frac{r_1}{R} \right\}^2 \right] \right), & r_1 \leq r \leq R \end{cases} \quad (4-107)$$

Thus, at the supports, or at  $r = R$ :

$$W_s = \frac{\mu V_0^2 R^2}{24 M_0} \left( \left\{ \frac{r_1}{R} \right\}^3 - \left\{ \frac{r_1}{R} \right\}^2 - \left\{ \frac{r_1}{R} \right\} + 1 \right) \quad (4-108)$$

At midpoint, or at  $r = 0$ :

$$W_f = \frac{\mu V_0^2 R^2}{8 M_0} \forall v \in \mathbb{R} \quad (4-109)$$

The results are summarised in Figure 4-33:

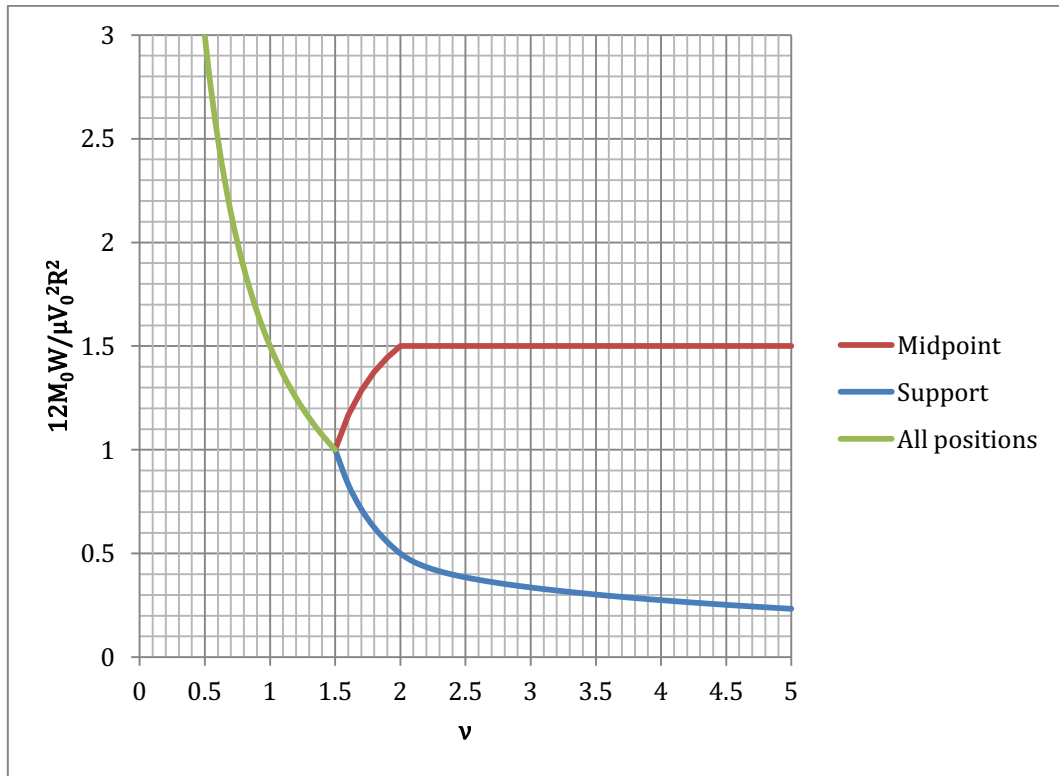


Figure 4-33: Dimensionless permanent transverse displacements (after [85])

From Figure 4-33, it can be seen that the displacement at the supports becomes less significant for values of  $\nu \gtrsim 5$  to being almost non-existent for  $\nu \gtrsim 10$ . This is also evident in Figure 4-34. It can also be seen that, for  $\nu \geq 2$ , the midpoint displacement does not vary with  $\nu$ . Jones and Oliveira [85] have shown that the result converges to that for a “bending only” formulation, which confirms that transverse shear effects are only relevant for plates of  $\nu$  not exceeding 5.

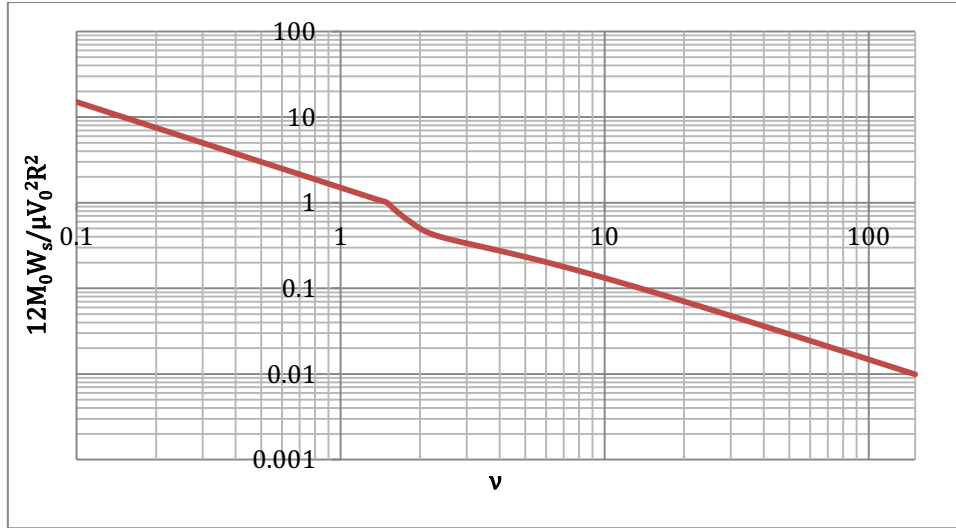


Figure 4-34: Variation of dimensionless  $W_s$  with  $v$

From Figure 4-33 and Figure 4-34, it can be observed that the displacements are a function of the applied impulsive velocity  $V_0$ , as expected. The latter is a function of the applied localised blast load and its spatial distribution, as evident from (4-67) and (4-68) and as illustrated in Figure 4-25.

Thus, an alternative representation for the dimensionless displacements is shown in Figure 4-35 and Figure 4-36 for the midpoint and support displacements respectively, where a series of response plots for various  $\left(\frac{\varepsilon}{R^2}\right)$  values are shown.

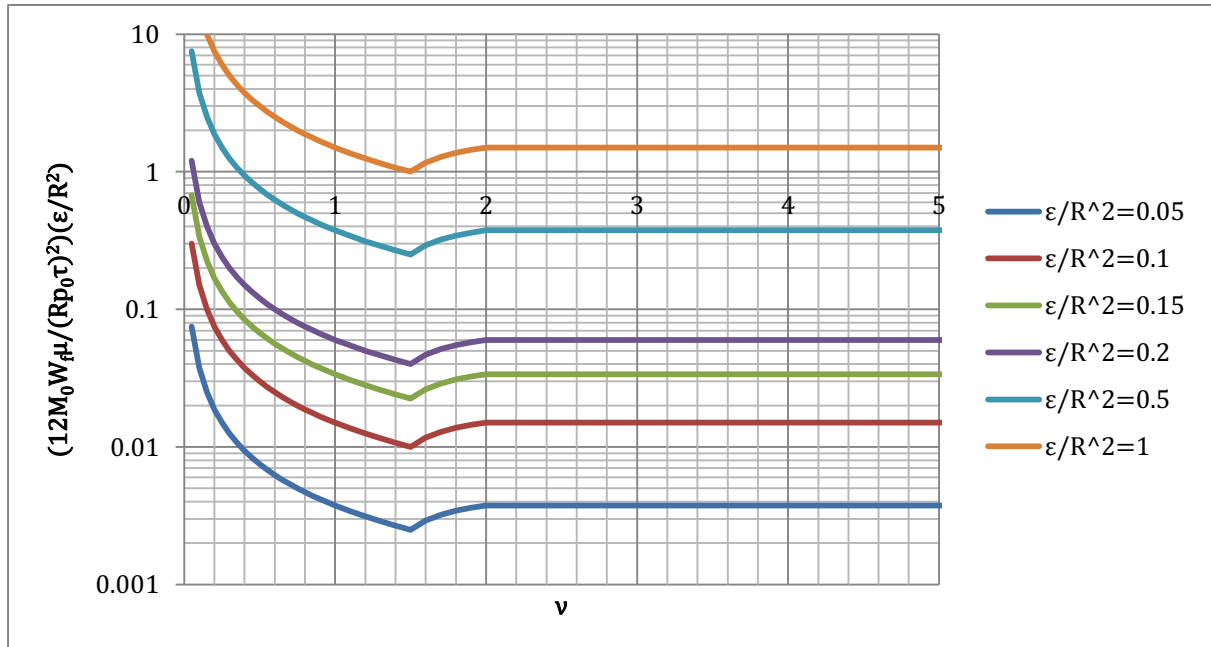


Figure 4-35: Dimensionless  $W_f$  for various  $\varepsilon/R^2$  values

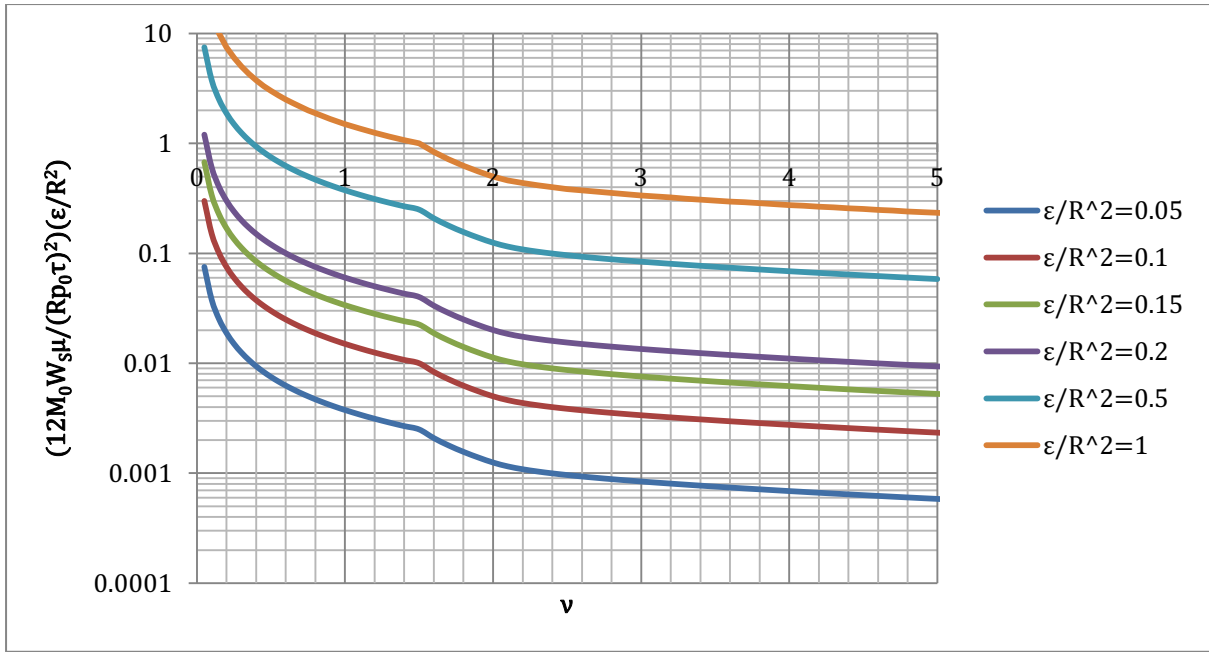


Figure 4-36: Dimensionless  $W_s$  for various  $\epsilon/R^2$  values

The full interaction between all parameters is shown in Figure 4-37 and Figure 4-38.

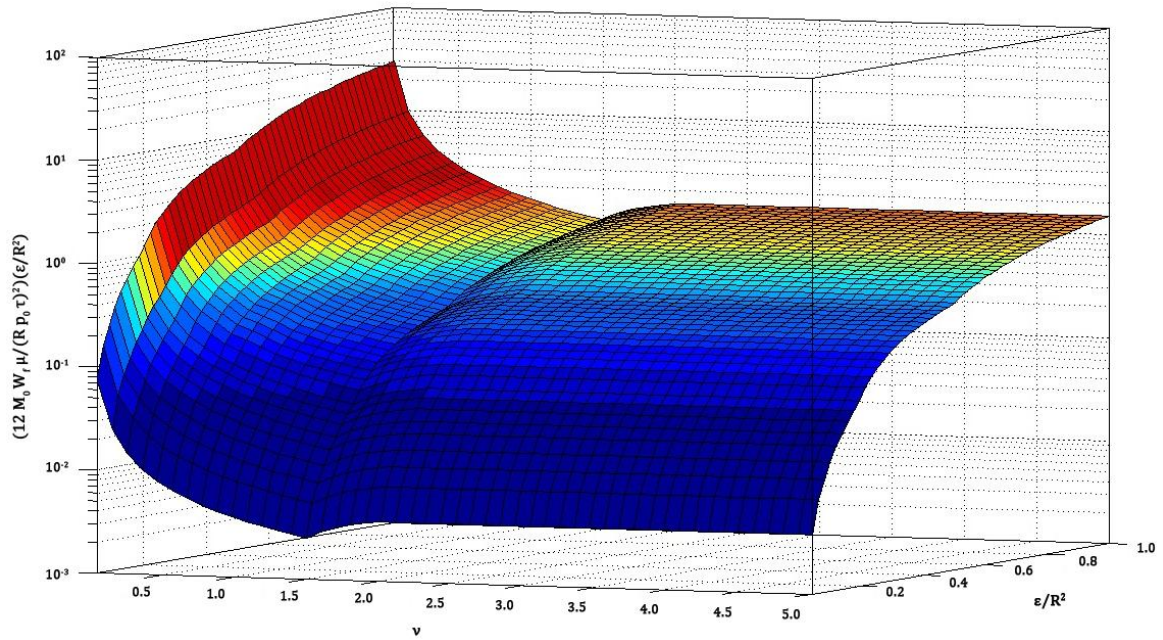


Figure 4-37: Interactive surface for dimensionless  $W_s$

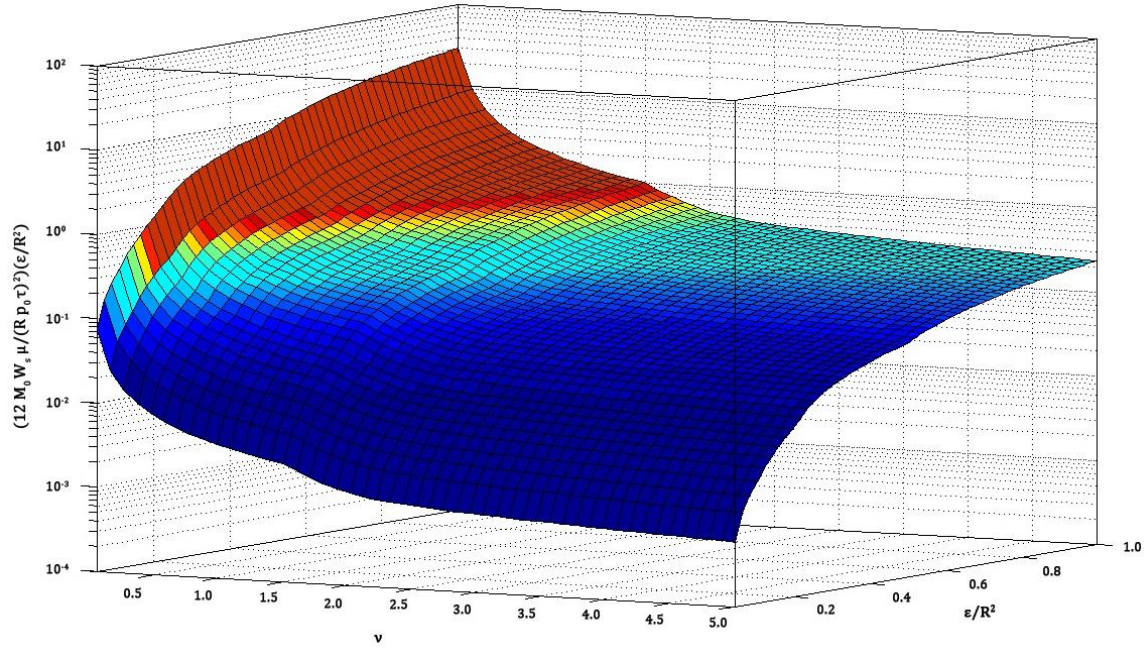


Figure 4-38: Interactive surface for dimensionless  $W_f$

The range of depicted  $\left(\frac{\epsilon}{R^2}\right)$  values are typical for close-in blasts (refer to Figure 4-25) and  $\left(\frac{\epsilon}{R^2}\right) = 1$  represents the case of a uniformly distributed load in curves in Figure 4-35 and Figure 4-36. It can be observed that for such typical loads the influence of transverse shear is even less significant than for the case of a uniform load, even for very small  $\nu$  values.

#### 4.7.6 Dynamic analysis considering rotatory inertia effects

Rotatory inertia effects can be maintained in the equilibrium equations by allowing  $I_r \neq 0$  in (4-84a).

This has been studied fully in [85] and it can be seen that two phases of motion are necessary: the first which has a stationary central plastic zone and shear sliding at the supports and ending when the shear sliding ceases, followed by the movement of the plastic zone until it reaches the centre, yielding velocity profiles similar to those in Figure 4-27 and Figure 4-28.

Inclusion of rotatory inertia will lead to an overall increase in the support displacement and a decrease in the midpoint displacement [254], making it a conservative assumption to ignore rotatory inertia effects for typical plate geometries, where displacement at the supports is almost negligible and where bending and/or membrane action is governing.

#### 4.7.7 Concluding remarks

In this section, the analytical model presented in Sections 4.5 and 4.6 is extended to consider the influence of transverse shear and rotatory inertia.

The analytical formulation is based on existing theoretical expressions found in literature, which have been extended for the case of localised blast loads, or the equivalent impulsive velocity which such a load could be idealised into.

It was found that the influence of transverse shear is only relevant for small values of  $\nu$ , which do not correspond to typical plate geometries found in plated structures subjected to blast loading. With respect to midpoint deflection, the present theoretical method predicts that the solution converges to a 'bending only' solution beyond  $\nu = 2$  and is only larger than such a solution for  $\nu \leq 1$ , which represents a very impractical plate geometry and therefore not applicable in practice. In terms of transverse shear slide at the supports, this is found to be only noticeably significant for  $\nu \leq 5$ .

This effect is even less noteworthy for close-in blasts, where the effect of transverse shear is significantly reduced with decreasing  $\varepsilon$  values, where  $\varepsilon$  can be considered to be a measure of the localised nature of the load.

Although the analysis in this section has considered the effects of transverse shear, this has been limited to the case where shear sliding may happen at the supports. The possibility of shear slides occurring underneath the explosive has not been studied in this work and this would lead to considering an alternative velocity profile during an initial phase of the motion, where the plastic hinges would then move from the centre of the plate to the supports. However, previous works considering such a mechanism in beams (e.g. [258]) suggest that such a failure mechanism would be more relevant to an impact problem, where a falling mass impacts the beam or plate with an incident striking velocity.

Inclusion of rotatory inertia was not studied in this work but reference was made to existing literature on the subject, whereby it is found that consideration of rotatory inertia would decrease midpoint displacement and marginally increase support slide for low  $\nu$  values.

Thus, it is now necessary to complete the discussion by considering the effects of finite deflections, or in-plane forces.

#### **4.8 Case C – Dynamic rigid-plastic behaviour of thin plates**

In Sections 4.6 and 4.7, the effects of finite deflections have been ignored. In this section the influence of such displacements is included in the formulation. This essentially implies that the governing equations are based on the displaced (deformed) structure, rather than on the original (undeformed) one, as shown in Figure 4-39.

The main difference in this case is that, for equilibrium to be maintained, an in-plane (membrane) force,  $N$ , needs to be developed. This force can lead to significant reductions in the overall final permanent displacement obtained by considering bending and/or shear effects only since, as the displacement  $w$  increases, the contribution of  $M$  and  $Q$  becomes less dominant than that of  $N$  due to a phenomenon known as string response.

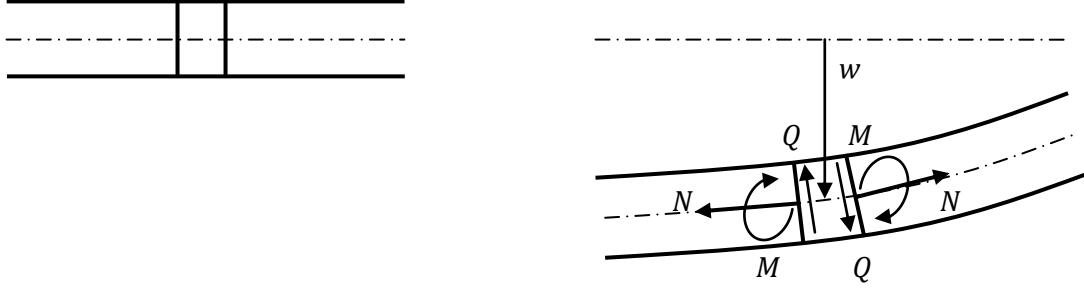


Figure 4-39: Large deflection phenomenon (left: undeformed state; right: deformed state)

Thus, in this section, the static analysis presented in Section 4.5 is extended to take into account both the transverse inertia term and the in-plane forces in the lateral equilibrium equation.

It should be noted that this approach is an approximate one, since the adopted transverse displacement field is the same used for infinitesimal displacements and once the rate of internal energy dissipation is worked out using the normality rule, it is equated to the external work rate without fully satisfying equilibrium. However, this simplification has been shown [77] to be adequate.

#### 4.8.1 Governing equilibrium equations

The equilibrium equation needs to take into account any possible plastic deformation between the plastic hinge lines and thus a generalised equilibrium equation describing axi-symmetric behaviour needs to be satisfied, which, using polar co-ordinates, is given by [80, 254]:

$$\begin{aligned}
 & \int_A [\{p(r, t) - \mu \ddot{w}\} \dot{w}] dA \\
 &= \sum_{m=1}^r \left\{ \int_{C_m} [(M_r + N_r w) \frac{\partial \dot{w}}{\partial r}] dC_m \right\} \\
 &+ \int_A [(M_r + N_r w) \dot{\kappa}_r + (M_\theta + N_\theta w) \dot{\kappa}_\theta] dA
 \end{aligned} \tag{4-110}$$

In this equation,  $\frac{\partial \dot{w}}{\partial r}$  is the angular velocity across an axi-symmetric hinge,  $N_r$  and  $N_\theta$  are the radial and circumferential membrane forces respectively,  $M_r$  and  $M_\theta$  are the corresponding bending moment quantities and  $\dot{\kappa}_r$  and  $\dot{\kappa}_\theta$  are the corresponding changes in curvature.

The complete derivation of (4-110) is given in [80] and essentially arises from the assumption that if the total area  $A$  of a plate with a boundary length  $C$  is assumed to be consisting of  $r$  smaller areas each surrounded by a boundary of length  $C_m$  then, by Green's Theorem, the internal work terms of the equilibrium equation can be written as shown in (4-110), ignoring any in-plane deformations since these are insignificant when compared to the out-of-plane displacements,  $w$ .

It can be seen that the work done due to the external (dynamic) load and the inertia force (left-hand side of (4-110) above) are equated to the energy dissipated by the plate, given in 2 parts, viz., the first term of the right-hand side describing the energy dissipated at each of the hinges (from  $m = 1$  to  $r$ ) and the last term is the energy dissipated in the continuous deformation fields.

Even for simple problems, solving (4-110) in closed form can be complicated using the full yield condition given in Figure 4-3 and a further simplification is carried out, namely, by assuming that the plastic flow is controlled by either of two simple square yield conditions, viz., an inscribing one and a circumscribing one, as shown in Figure 4-40:

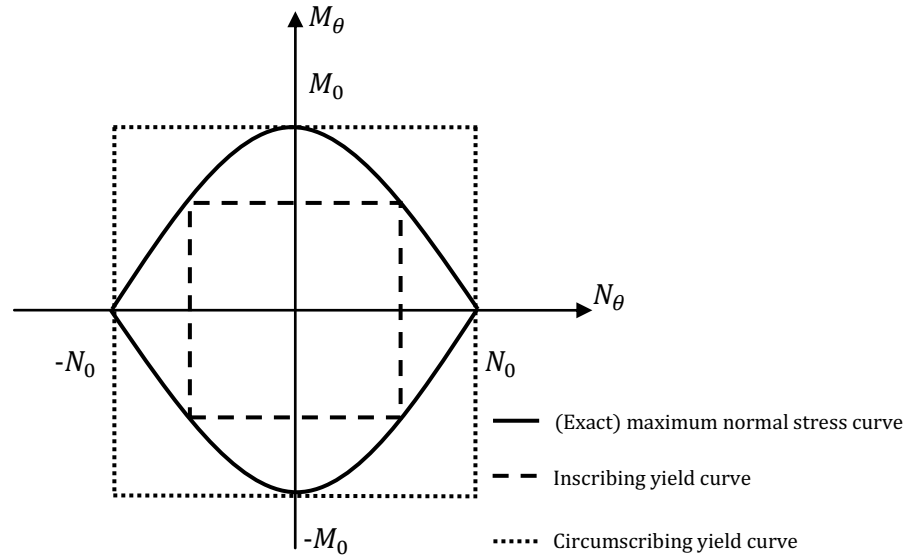


Figure 4-40: Simplified square yield condition (after [254])

The dynamic case is first solved for the case of a rectangular pulse shape, shown in Figure 4-6; the analysis is extended in Section 4.8.5 for alternative pulse shapes.

#### 4.8.2 Combined bending and membrane effects

Assuming that the velocity profile shown in Figure 4-4 is assumed to be valid for the dynamic problem, then  $\dot{\kappa}_r = 0$  and  $\dot{\kappa}_\theta = \frac{\dot{W}}{rR}$  while  $M_\theta = M_0$  and  $N_\theta = N_0$  using the square yield condition of Figure 4-40. Thus, (4-110) can be re-written for the current problem as:

$$\begin{aligned}
 & \int_0^{R_0} \left[ \left\{ p_0 - \mu \ddot{W} \left( 1 - \frac{r}{R} \right) \right\} \dot{W} \left( 1 - \frac{r}{R} \right) r \right] dr \\
 & + \int_{R_0}^R \left[ \left\{ p_0 a e^{br} - \mu \ddot{W} \left( 1 - \frac{r}{R} \right) \right\} \dot{W} \left( 1 - \frac{r}{R} \right) r \right] dr \\
 & = \int_0^R \left[ \left\{ M_0 + N_0 W \left( 1 - \frac{r}{R} \right) \right\} \left( \frac{\dot{W}}{R} \right) r \right] dr
 \end{aligned} \tag{4-111}$$

The equation is solved for 2 instances in time, namely, during the direct loading phase ( $0 \leq t \leq \tau$ ) and the free vibration phase until motion ceases at time  $t = T$ .

- First phase of motion:  $0 \leq t \leq \tau$

If the integrals in (4-111) are evaluated and the resulting expressions are simplified, then it can be shown that the equation reduces to:

$$\ddot{W} + A^2 W = B \quad (4-112)$$

where:

$$A^2 = \frac{24M_0}{\mu R^2 H} \quad (4-113)$$

$$B = \frac{12ap_0}{\mu R^3 b^3} [e^{bR}(bR - 2) - e^{bR_0}(b^2 R_0 \{R - R_0\} + b\{2R_0 - R\} - 2)] \\ + \frac{6}{\mu R^3} \left[ p_0 R_0^2 \left( R - \frac{2R_0}{3} \right) - 2RM_0 \right] \quad (4-114)$$

Through some algebraic manipulation, it can be shown that  $B$  can be written as:

$$B = \frac{12(\eta - 1)M_0}{\mu R^2} \quad (4-115)$$

where  $\eta$  is the load factor, defined by (4-27).

Clearly, (4-112) is a non-homogenous second order ODE (ordinary differential equation) with constant co-efficients and thus a solution to this ODE is given by:

$$W = A_1 \cos At + B_1 \sin At + \frac{B}{A^2} \quad (4-116)$$

where the constant co-efficients are given by  $A_1 = -\frac{B}{A^2}$  and  $B_1 = 0$  when satisfying the initial conditions that  $W = \dot{W} = 0$  at  $t = 0$ .

- Second phase of motion:  $\tau \leq t \leq T$

The same analysis as in the preceding section is valid, except that the direct loading phase is now complete, i.e.,  $p_0 = 0$ . Thus, (4-111) reduces to:

$$\ddot{W} + A^2 W = C \quad (4-117)$$

where:

$$C = -\frac{12M_0}{\mu R^2} \quad (4-118)$$

The solution to (4-112) is given by:

$$W = C_1 \cos At + D_1 \sin At + \frac{C}{A^2} \quad (4-119)$$

The constants  $C_1$  and  $D_1$  are obtained by ensuring continuity of  $W$  and  $\dot{W}$  at  $t = \tau$ , i.e.,  $C_1 = \frac{\eta \sin A\tau}{2}$  and  $D_1 = \frac{(B-C) \cos A\tau - B}{A^2}$ .

Substituting these constants into (4-119) and re-arranging, then the displacement is given by:

$$\frac{W}{H} = \frac{1}{2} \{ (1 - \eta + \eta \cos A\tau) \cos At + (\eta \sin A\tau) \sin At - 1 \} \quad (4-120)$$

The motion ceases when  $\dot{W} = 0$  at  $t = T$ , given by:

$$T = \frac{1}{A} \left\{ \tan^{-1} \left( \frac{\eta \sin A\tau}{1 - \eta + \eta \cos A\tau} \right) \right\} \quad (4-121)$$

Thus, the final permanent displacement is given by substituting (4-121) into (4-120) to give:

$$\frac{W_f}{H} = \frac{1}{2} \left\{ \sqrt{1 + 2\eta(\eta - 1)(1 - \cos A\tau)} - 1 \right\} \quad (4-122)$$

From (4-113), it can be easily shown that  $A$  can be re-written in the form  $A = \frac{1}{R} \sqrt{\frac{6\sigma_0}{\rho}}$  which implies that practical limits on values of the parameter  $A$  can be established for typical values of  $\rho$ ,  $\sigma_0$  and  $R$ .

For most monolithic materials used in blast-resistant structures,  $\rho$  would typically be around 7850 kg/m<sup>3</sup>, while  $\sigma_0$  varies from around 300 MPa for mild steel to around 1200 MPa for armour steel systems.

The plate's radius  $R$  would depend on the structure's geometric configuration, but typically larger than (say) 0.2 m for realistic structural sizes and not exceeding around 2 m.

Thus,  $A$  would have typical values in the range between 450 /s (for large diameter armour steel plates) to 7600 /s (for small diameter mild steel ones).

On the other hand, value for the duration,  $\tau$ , typically vary from 0.05 for very close-in blasts to 0.2 ms for more globalised blast load scenarios.

These practical considerations set limits to expected values of  $A\tau$  to range from 0.035 to 1.5.

The response is shown in non-dimensional graphical form in Figure 4-41 for various values of  $\eta$ .

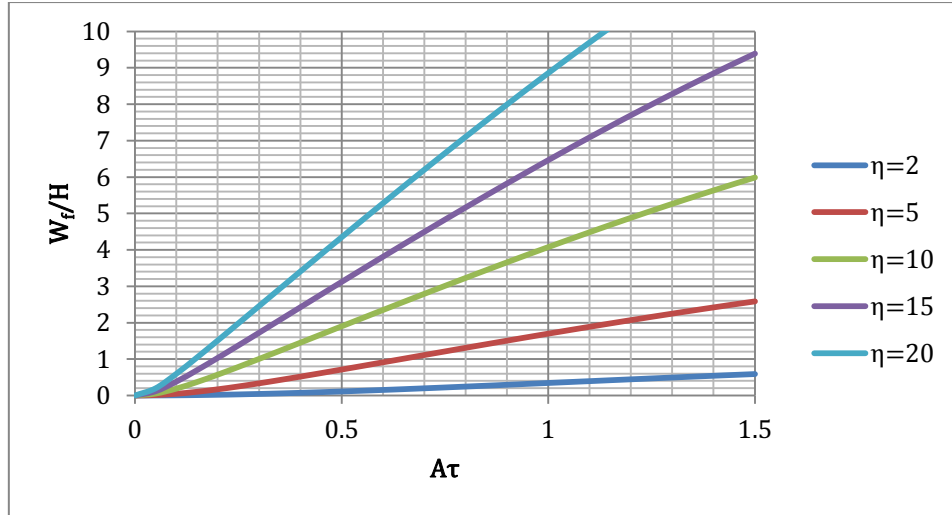


Figure 4-41: Dimensionless  $W_f$  for rectangular pulse load

Idealising the rectangular pulse load as an impulsive load, assuming it has a very short duration (i.e.,  $\tau \rightarrow 0$ ) and a very high amplitude (i.e.,  $\frac{p_0}{p_c} \gg 1$  or  $\eta \rightarrow \infty$ ), then as  $\tau \rightarrow 0$ ,  $\cos A\tau \approx 1 - \frac{(A\tau)^2}{2}$  and (4-122) reduces to:

$$\frac{W_f}{H} = \frac{1}{2} \left\{ \sqrt{1 + (\eta A\tau)^2} - 1 \right\} \quad (4-123)$$

Using the definition of  $\lambda$  given by (4-71), then, using the approach developed in Section 4.6.4, it can easily be shown that (4-123) can be written in the form:

$$\frac{W_f}{H} = \frac{1}{2} \left\{ \sqrt{1 + \frac{2\lambda}{3}} - 1 \right\} \quad (4-124)$$

If the simplified circumscribing yield criterion shown in Figure 4-40 is replaced by the inscribed one in the same figure, then the solution changes to:

$$\frac{W_f}{H} = \frac{1}{2} \left\{ \sqrt{1 + \frac{2\lambda}{3} \left( \frac{1}{0.618} \right)} - 1 \right\} \quad (4-125)$$

This is obviously a more conservative estimate of the permanent transverse displacement.

#### 4.8.3 Simplification into a circular membrane

It is evident that in the case of extreme loads, the influence of finite displacements has a significant role due to the large permanent displacement associated with such loads. Thus, the contribution of membrane action exceeds that of bending for the case of large transverse displacements. This leads to a further simplification of the response by neglecting completely the influence of moment and assuming that the response is controlled solely by membrane action, in which case (4-110) reduces to:

$$\int_A [\{p(r, t) - \mu \ddot{w}\} \dot{w}] dA = \sum_{m=1}^r \left\{ \int_{C_m} [(N_r w) \frac{\partial \dot{w}}{\partial r}] dC_m \right\} + \int_A [N_r w \dot{\kappa}_r + N_\theta w \dot{\kappa}_\theta] dA \quad (4-126)$$

For the current problem of a rectangular pulse load, (4-111) reduces to:

$$\begin{aligned} & \int_0^{R_0} \left[ \left\{ p_0 - \mu \ddot{W} \left( 1 - \frac{r}{R} \right) \right\} \dot{W} \left( 1 - \frac{r}{R} \right) r \right] dr \\ & + \int_{R_0}^R \left[ \left\{ p_0 a e^{br} - \mu \ddot{W} \left( 1 - \frac{r}{R} \right) \right\} \dot{W} \left( 1 - \frac{r}{R} \right) r \right] dr \\ & = - \int_0^R \left[ \left\{ N_0 W \left( 1 - \frac{r}{R} \right) \right\} \left( \frac{\partial^2 \dot{W}}{\partial r^2} + \frac{1}{r} \frac{\partial \dot{W}}{\partial r} \right) r \right] dr \end{aligned} \quad (4-127)$$

This may be written as:

$$\ddot{W} + A^2 W = D \quad (4-128)$$

where:

$$A^2 = \frac{24M_0}{\mu R^2 H} \quad (4-129)$$

$$D = \frac{12\eta M_0}{\mu R^2} \quad (4-130)$$

Following a procedure similar to that outlined in Section 4.8.2 gives:

$$\frac{W}{H} = \frac{\eta}{2} (1 - \cos A\tau) \quad (4-131)$$

for the first phase of motion  $0 \leq t \leq \tau$  and:

$$\frac{W}{H} = \frac{\eta}{2} \{ (\cos A\tau - 1) \cos At + (\sin A\tau) \sin At \} \quad (4-132)$$

for the second phase of motion  $\tau \leq t \leq T$  with the total response time  $T$  given by:

$$T = \frac{1}{A} \left\{ \tan^{-1} \left( \frac{\sin A\tau}{\cos A\tau - 1} \right) \right\} \quad (4-133)$$

Thus, the maximum permanent displacement is given by evaluating (4-132) at  $t = T$ , i.e.:

$$\frac{W_f}{H} = \frac{\eta}{2} \{ \sqrt{1 - \cos A\tau} \} \quad (4-134)$$

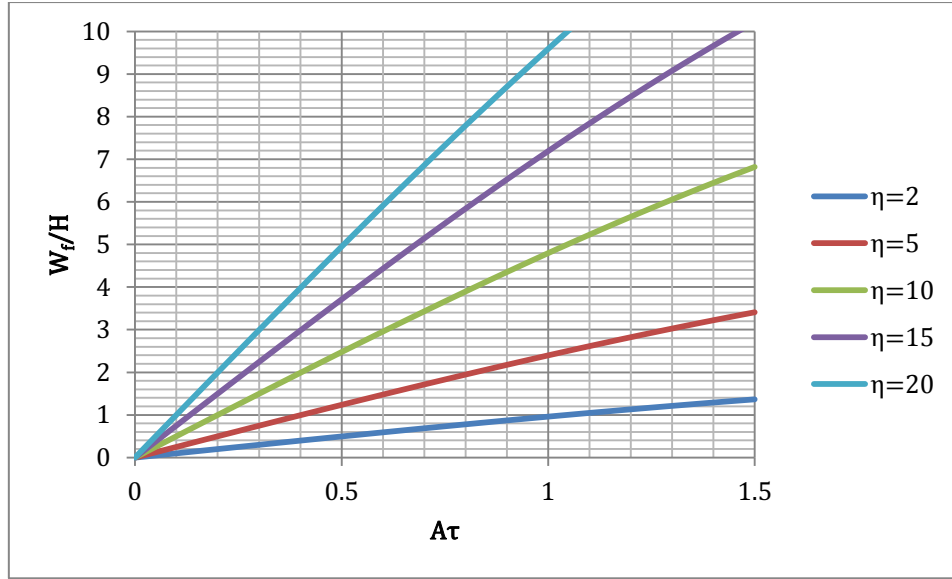


Figure 4-42: Dimensionless  $W_f$  for a membrane under rectangular pulse load

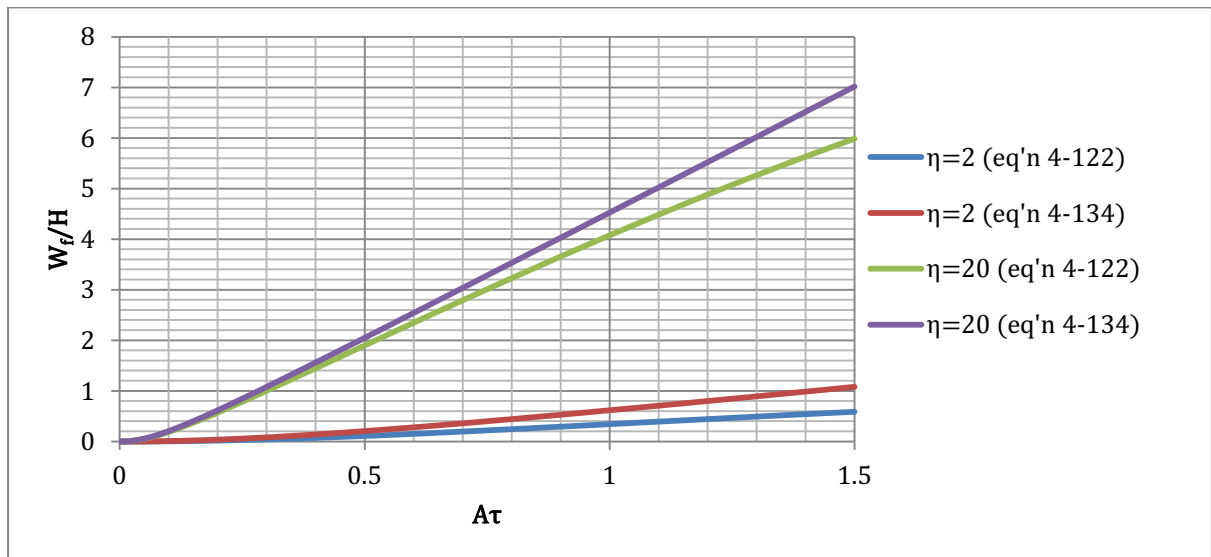


Figure 4-43: Dimensionless  $W_f$  for various  $\eta$  values with/out bending effects (rectangular pulse)

Comparing Figure 4-41 with Figure 4-42, or the plots in Figure 4-43, it can be observed that the difference between the full (bending and membrane) solution and the simplified (membrane only) solution is small for the smaller values of  $A\tau$  but is significantly higher in the case of small values of  $\eta$  (or small loadings), in which case bending effects will still play a major role in the plate's response.

Idealising the load as an impulsive one, then (4-134) is simplified into:

$$\frac{W_f}{H} = \sqrt{\frac{\lambda}{6}} \quad (4-135)$$

For the alternative yield criterion shown in Figure 4-40, then (4-135) would be written as:

$$\frac{W_f}{H} = \sqrt{\frac{\lambda}{6} \left( \frac{1}{0.618} \right)} \quad (4-136)$$

The results using the full (membrane and bending) and simplified (membrane only) approaches are plotted together in non-dimensional form in Figure 4-44.

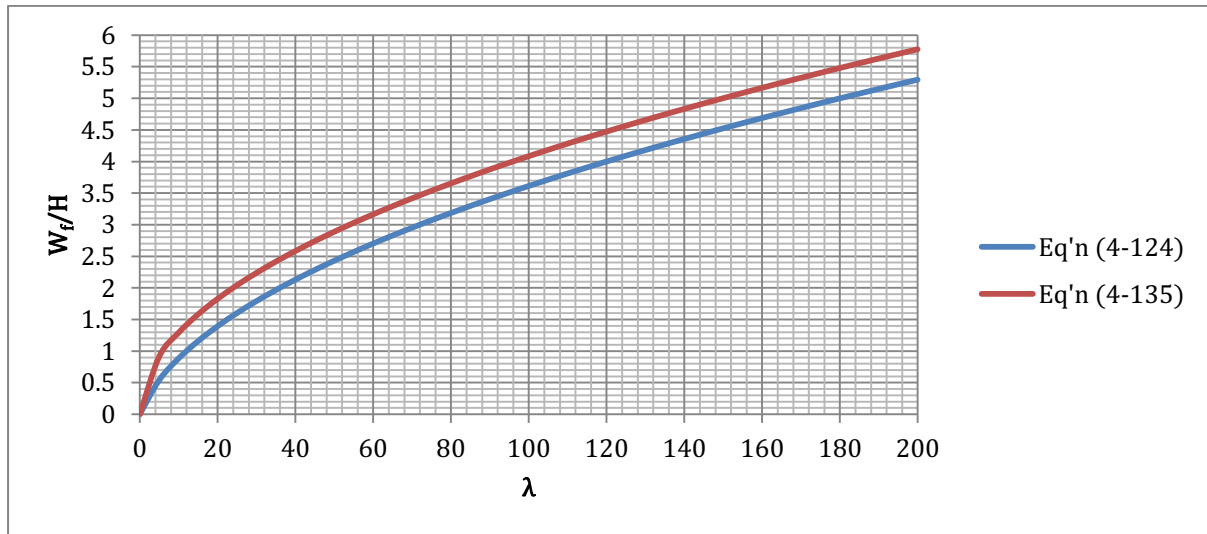


Figure 4-44: Dimensionless  $W_f$  for various impulse values with/out bending effects (rectangular pulse)

As seen in Figure 4-44, the difference between the 2 approaches is more significant in the case of low impulses (or low permanent displacements), where bending effects are not negligible.

However, for high impulse values, the difference between the 2 approaches is less significant; the difference decreases exponentially with impulse, as shown in Figure 4-45, with the maximum difference being roughly 5% for very large impulses.

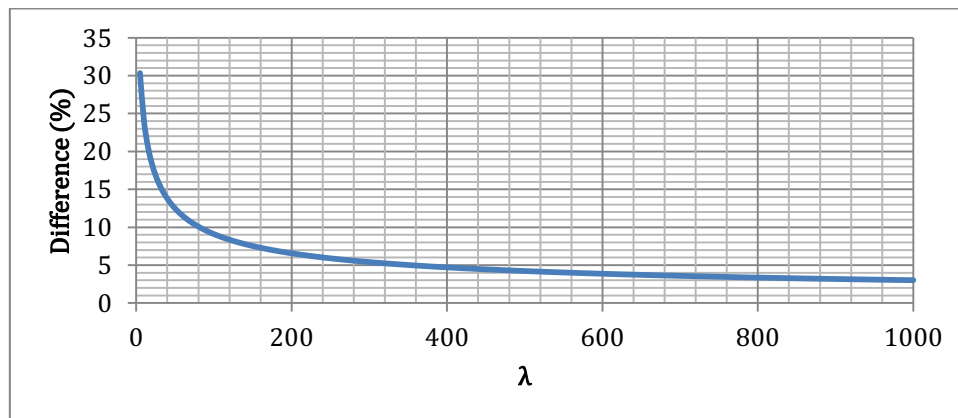


Figure 4-45: Difference in  $W_f$  with/out bending effects (rectangular pulse)

#### 4.8.4 Numerical simulation with ABAQUS

In this section, numerical validation of the analytical method derived in Sections 4.8.2 and 4.8.3 will be described. In the current model, transverse shear and rotatory inertia are ignored and thus a range of plate thicknesses for which the presented analysis is suitable is identified.

The calibration was carried out using the analytical formula [65, 67] for the response to a uniform pressure on a simply supported circular plate, where only bending effects are considered, and by comparison with numerical results from ABAQUS/Explicit v.6.9-1 [259].

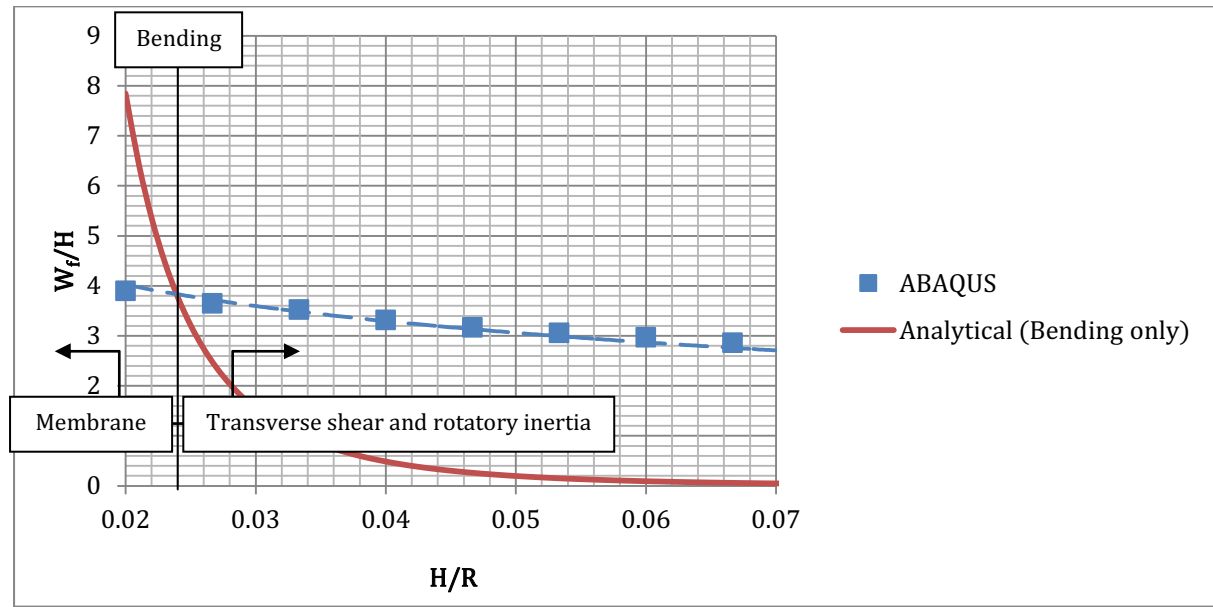


Figure 4-46: Comparison of ABAQUS and analytical results (uniform pressure,  $\eta = 1.5$ )

From Figure 4-46, it is seen that the numerical and analytical curves intersect at around  $\left(\frac{H}{R}\right) = 0.025$ , while the analytical (bending only) result grossly overestimates the permanent displacement for smaller values (due to the favourable effect of membrane action) and underestimates the same displacements for larger values (due to the additive effects of transverse shear).

For practical applications, such as the blast-loaded plates described in Chapter 3, plate thickness/radius ratios (or  $\left(\frac{H}{R}\right)$  values) of around 0.01 are typical, which suggests that, membrane effects are dominant in such cases. This is further corroborated by the large displacements observed in the tested plates, indicating that membrane effects are not insignificant and that bending consideration alone leads to over-conservative results.

In the light of the above discussion, for the purpose of this numerical study, the following plate geometric parameters are used:  $R = 0.15$  m,  $H = 0.0015$  m, to give  $\left(\frac{H}{R}\right) = 0.01$ .

The plate is assumed to be made of a high-performance steel, such as the armour steel described in Chapter 3, and the material properties are assumed to be similar to those stated in Section 3.8.2, viz.,  $\sigma_0 = 1000$  MPa and  $\mu = 15.7$  kg/m<sup>2</sup>.

The loading parameters were chosen such that for fixed values of  $a = 10$  and  $b = -50$  /m and various values of  $p_0$  (5, 10, 15, 20, 25 and 30 MPa) and for a constant value of  $R_0 = 50$  mm. The load spatial distributions chosen are intended to represent different blast loads at various stand-off distances and thus of various impulse values.

A load duration of  $\tau = 5 \times 10^{-5}$  s was chosen, such that the ratio  $\left(\frac{\tau}{T_n}\right)$  was kept low in order to ensure that the load is of an impulsive nature. Non-dimensionalised impulse values for each of these loads are found to be in the set  $\lambda = \{5, 20, 45, 75, 120, 170\}$ .

A 3D finite element model with a uniform mesh size of 5 mm was set up in ABAQUS/Explicit using 8-noded linear brick elements with reduced integration and hourglass control (C3D8R), as shown in Figure 4-47. The loading distribution was implemented by means of the VDLOAD user-defined subroutine in ABAQUS, as described in Section 3.8.4, for the various load cases.

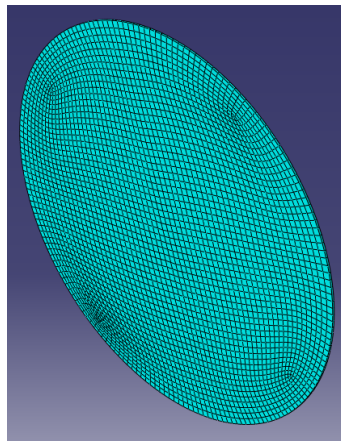


Figure 4-47: Finite element model in ABAQUS

Typical outputs from ABAQUS are shown in Figure 4-48 to Figure 4-50. The results obtained from the analytical and numerical models are compared and summarised in Table 4-1.

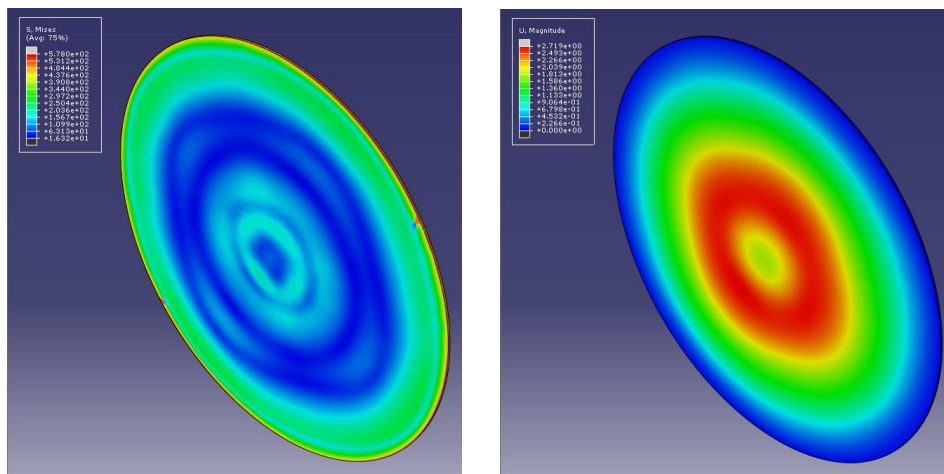


Figure 4-48: Typical output for  $p_0 = 10$  MPa (left: Mises stress, right: displacement)

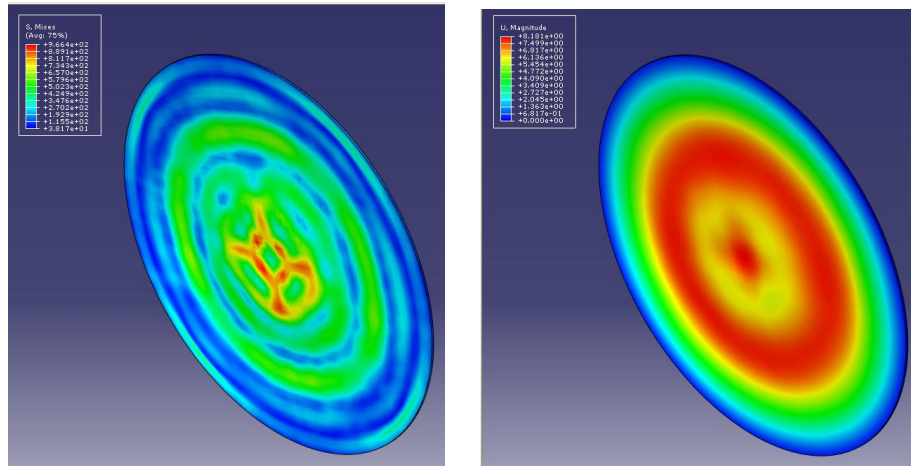


Figure 4-49: Typical output for  $p_0 = 30$  MPa (left: Mises stress, right: displacement)

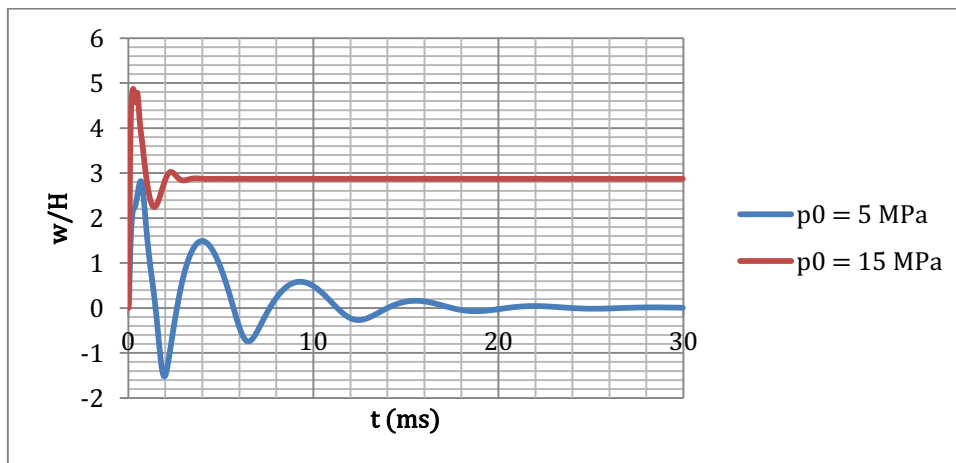


Figure 4-50: Typical displacement-time histories from ABAQUS

| $\lambda$ | $\frac{W_f}{H}$              |                                   |                                  |                            |                                   |                                  |        |
|-----------|------------------------------|-----------------------------------|----------------------------------|----------------------------|-----------------------------------|----------------------------------|--------|
|           | Membrane and bending effects |                                   |                                  | Membrane effects only      |                                   |                                  | ABAQUS |
|           | Full equation eq'n (4-122)   | Impulsive load                    |                                  | Full equation eq'n (4-134) | Impulsive load                    |                                  |        |
|           |                              | Using circum'd yield eq'n (4-124) | Using inscr'd yield eq'n (4-125) |                            | Using circum'd yield eq'n (4-135) | Using inscr'd yield eq'n (4-136) |        |
| 5         | 0.45                         | 0.52                              | 0.74                             | 0.89                       | 0.89                              | 1.13                             | 0.01   |
| 20        | 1.28                         | 1.35                              | 1.82                             | 1.78                       | 1.78                              | 2.27                             | 1.33   |
| 45        | 2.14                         | 2.22                              | 2.94                             | 2.67                       | 2.68                              | 3.4                              | 2.87   |
| 75        | 3.02                         | 3.1                               | 4.07                             | 3.56                       | 3.57                              | 4.54                             | 4.40   |
| 120       | 3.9                          | 3.99                              | 5.2                              | 4.45                       | 4.46                              | 5.67                             | 5.47   |
| 170       | 4.78                         | 4.88                              | 6.33                             | 5.33                       | 5.35                              | 6.81                             | 6.60   |

Table 4-1: Summary of ABAQUS and analytical results

A plot of the maximum permanent displacements against non-dimensional impulse, with the analytic and ABAQUS results is shown in Figure 4-51:

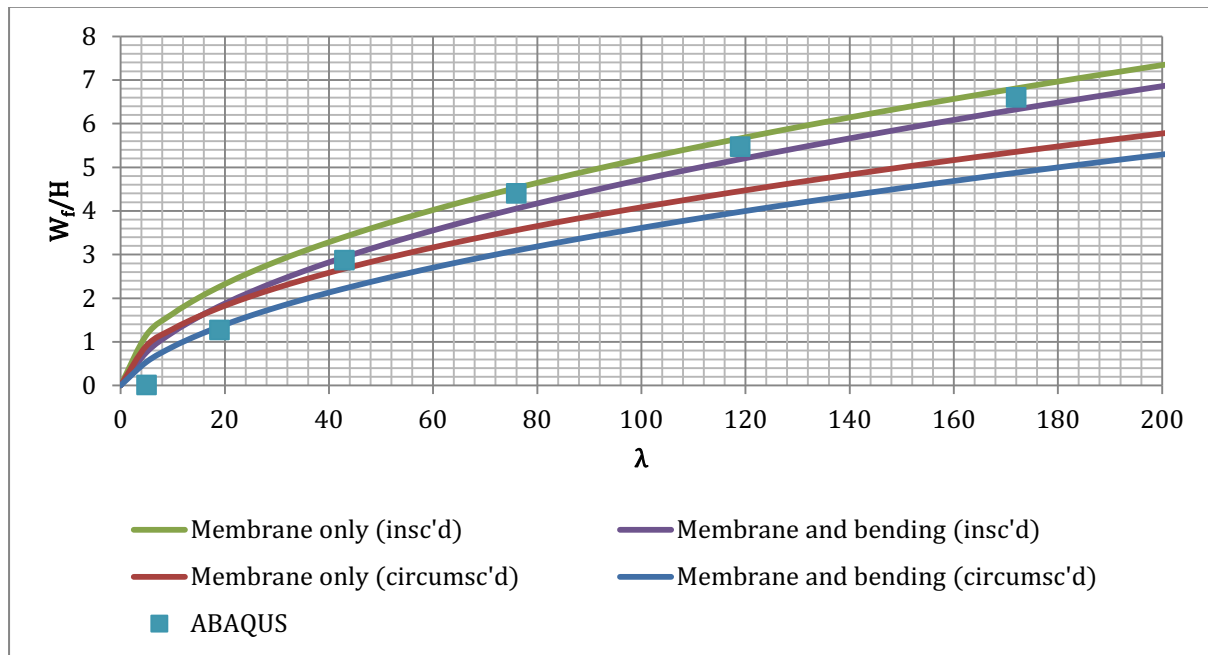


Figure 4-51: Comparison of ABAQUS and analytical results

From the results shown in Table 4-1 and Figure 4-51, a number of conclusions can be drawn.

Considering the differences between the results using the full expressions and the impulsive simplification for each case, it can be observed that, for the case of considering combined bending and membrane effects, the difference between the circumscribed and the inscribed solutions is fairly constant (with an average of 25%) but the average difference between the 'full result' and the impulsive simplification (using the circumscribed yield criterion) is only 4.9%. Thus, the impulsive simplification is justified for the case of blast loading being considered.

It can also be seen that for both yield criteria, the results obtained by considering both membrane and bending effects and membrane effects alone are not extremely dissimilar, with an average difference of 22.5% when using the full equation and, for the impulsive simplification, an average difference of 19.2% for the circumscribed case and 15.6% using the inscribed case. However, the differences decrease considerably with increasing impulse, with a difference of less than 10% in all cases for  $\lambda = 170$ . Thus, the 'membrane only' result would suffice for high impulse values, which are typical of blast loads.

Comparing now the analytical results with the numerical results from ABAQUS, it can be observed that, for low impulse values ( $\lambda \lesssim 50$ ), considering bending and membrane effects while using the circumscribed yield criterion provides a more realistic value of the permanent displacement. This is due to the fact that at low impulse values, bending would still be dominant and thus its contributions cannot be ignored. In addition, yield is more governed by the  $M_\theta$  component rather than the  $N_\theta$  part in Figure 4-3 and thus would be tending to the yield criterion shown in Figure 4-1.

Another explanation for the discrepancy between the analytical and numerical results is that, for the low impulse values ( $\lambda = 20$  and, in particular,  $\lambda = 5$ ), elastic response was observed and thus the rigid-plastic model would overestimate the predicted displacements. These elastic vibrations were found to cease in the higher impulse ranges, as observed in Figure 4-50.

For higher impulse values than  $\lambda = 50$ , it can be observed that the permanent displacement obtained using the inscribed yield criterion is an excellent estimate of the plate's response, with the average difference being less than 5% when considering both membrane and bending effects and just over 6% when using the impulsive simplification.

Thus, it can be concluded that, for high impulse values, the analytical results provide excellent approximations of the permanent displacement by using the impulsive load simplification and considering only membrane effects, to give an upper bound to the displacement value.

#### 4.8.5 Response for alternative pulse shapes

The analysis presented in Sections 4.8.2 and 4.8.3 has been limited to a rectangular pulse shape. A similar analysis can be performed for the 2 temporal functions described in Section 4.6.5, viz., the simple linearly decaying function and the exponentially decaying function, suggested by Li and Meng [256, 257].

Expressions for the final permanent displacements in each case are found using a similar procedure to that described earlier, giving the following results.

- Linearly decaying load

For the sake of brevity, only the results are presented in this section. Considering both bending and membrane effects, the governing ODE is given by:

$$\ddot{W} + A^2 W = \frac{B \left( \eta \left\{ 1 - \frac{t}{\tau} \right\} - 1 \right)}{\eta - 1} \quad (4-137)$$

where  $A$  and  $B$  are defined in (4-113) and (4-115) respectively.

Thus, the maximum permanent displacement is given by:

$$\frac{W_f}{H} = C_2 \cos At + D_2 \sin At + \frac{C}{A^2} \quad (4-138)$$

where the constants  $C_2$  and  $D_2$  are given by:

$$C_2 = E_2 \cos A\tau - F_2 \sin A\tau \quad (4-139)$$

$$D_2 = E_2 \sin A\tau + F_2 \cos A\tau \quad (4-140)$$

where:

$$E_2 = -\frac{B}{A^2(\eta-1)} - \frac{C}{A^2} - \frac{B \cos A\tau}{A^2} + \frac{B\eta \sin A\tau}{A^3\tau(\eta-1)} \quad (4-141)$$

$$F_2 = -\frac{B\eta}{A^3\tau(\eta-1)} + \frac{B \sin A\tau}{A^2} + \frac{B\eta \cos A\tau}{A^3\tau(\eta-1)} \quad (4-142)$$

and  $C$  is defined in (4-118).

The response time is given by:

$$T = \frac{1}{A} \left\{ \tan^{-1} \left( \frac{D_2}{C_2} \right) \right\} \quad (4-143)$$

In the case of an idealisation as an impulsive load, by using the approximations  $\cos A\tau \approx 1 - \frac{(A\tau)^2}{2}$  and  $\sin A\tau \approx A\tau$  for  $\tau \rightarrow 0$ , then (4-138) would reduce to:

$$\frac{W_f}{H} = \sqrt{1 + \frac{\lambda}{6}} - 1 \quad (4-144)$$

It can be shown that the difference between the results obtained using (4-124), i.e., the corresponding equation for the rectangular pulse, and (4-144) above are very similar, which is expected since, for impulsive loads, the final midpoint displacement should be independent of pulse-shape.

The difference between results decreases as the impulse increases, as shown in Figure 4-52.

If the bending effects are neglected and only membrane effects are considered, then the governing ODE is given by:

$$\ddot{W} + A^2 W = D \left( 1 - \frac{t}{\tau} \right) \quad (4-145)$$

where  $A$  and  $D$  are defined in (4-113) and (4-130) respectively.

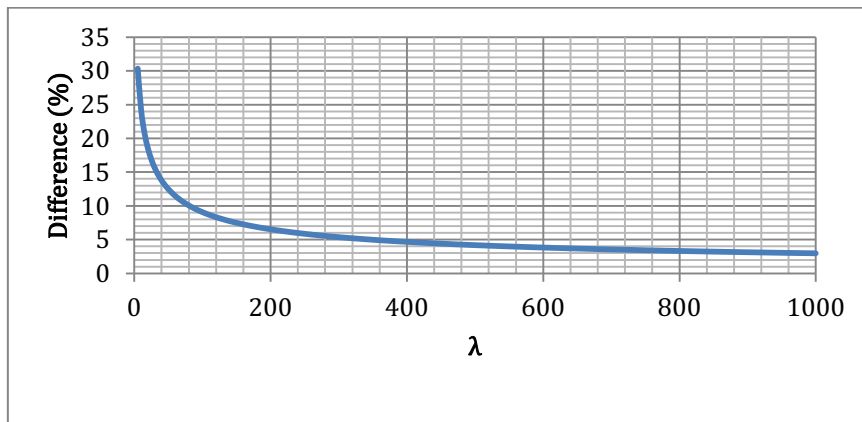


Figure 4-52: Difference in  $W_f$  using rectangular and triangular pulses

Thus, the maximum permanent displacement is given by:

$$\frac{W_f}{H} = C_3 \cos At + D_3 \sin At \quad (4-146)$$

where the constants  $C_3$  and  $D_3$  are given by:

$$C_3 = E_3 \cos A\tau - F_3 \sin A\tau \quad (4-147)$$

$$D_3 = E_3 \sin A\tau + F_3 \cos A\tau \quad (4-148)$$

where:

$$E_3 = \frac{D}{A^3\tau} (\sin A\tau - A\tau \cos A\tau) \quad (4-149)$$

$$F_3 = \frac{D}{A^3\tau} (\cos A\tau + A\tau \sin A\tau - 1) \quad (4-150)$$

The response time is given by:

$$T = \frac{1}{A} \left\{ \tan^{-1} \left( \frac{D_3}{C_3} \right) \right\} \quad (4-151)$$

In the case of an idealisation as an impulsive load, by using the approximations  $\cos A\tau \approx 1 - \frac{(A\tau)^2}{2}$  and  $\sin A\tau \approx A\tau$  for  $\tau \rightarrow 0$ , then (4-138) would reduce to:

$$\frac{W_f}{H} = \frac{\sqrt{15\lambda}}{10} \quad (4-152)$$

It can be shown that the difference between the results obtained using (4-135), i.e., the equation for the rectangular pulse, and (4-152) above is only 5.1%, which is expected since, for impulsive loads, the final midpoint displacement should be independent of pulse shape.

A non-dimensional plot of maximum permanent displacement against impulse, for the rectangular and linear decay pulse shapes (using the circumscribed yield value) is shown in Figure 4-53.

It should be noted that, in each case, the value of  $\lambda$  in (4-144) and (4-152) can be substituted by  $\left( \frac{\lambda}{0.618} \right)$  in order to obtain a more accurate result for the case of high values of  $\lambda$  using the inscribed yield value, in accordance with the observations made in Section 4.8.4. These results are shown in Figure 4-54.

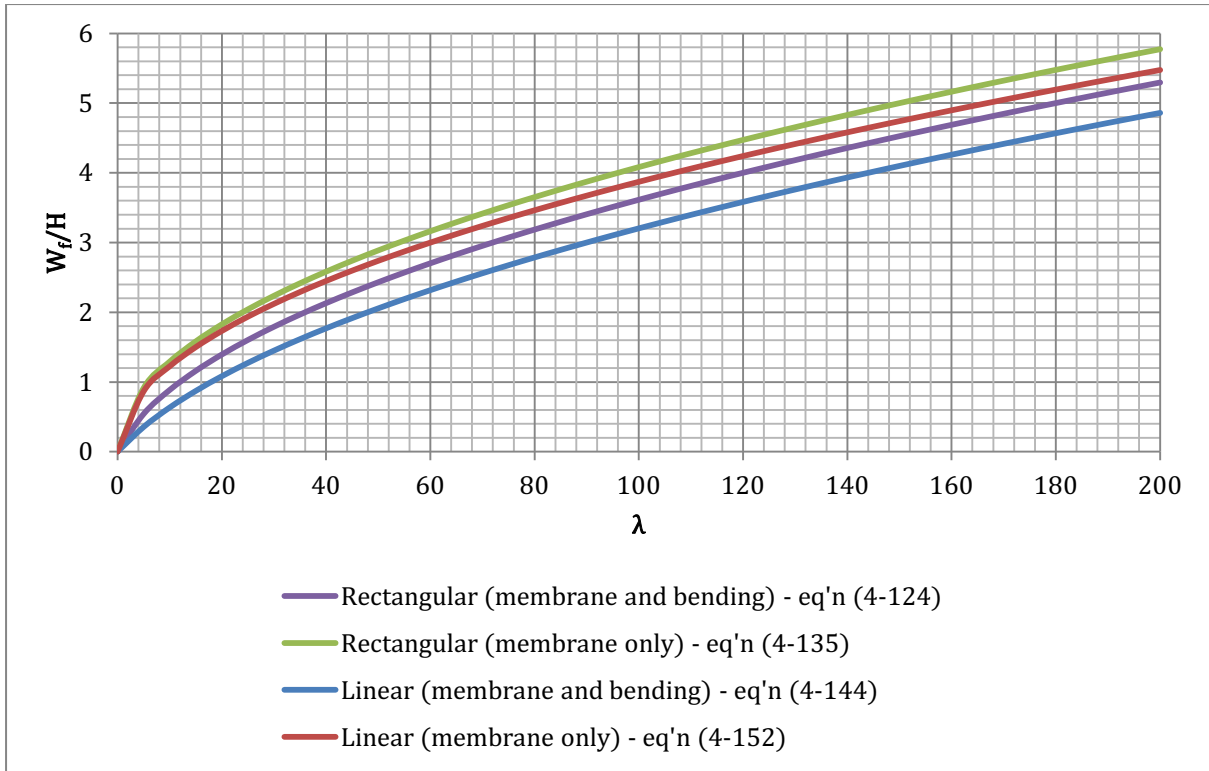


Figure 4-53: Dimensionless  $W_f$  for various impulse values with/out bending effects for 2 pulse shapes using circumscribed yield criterion

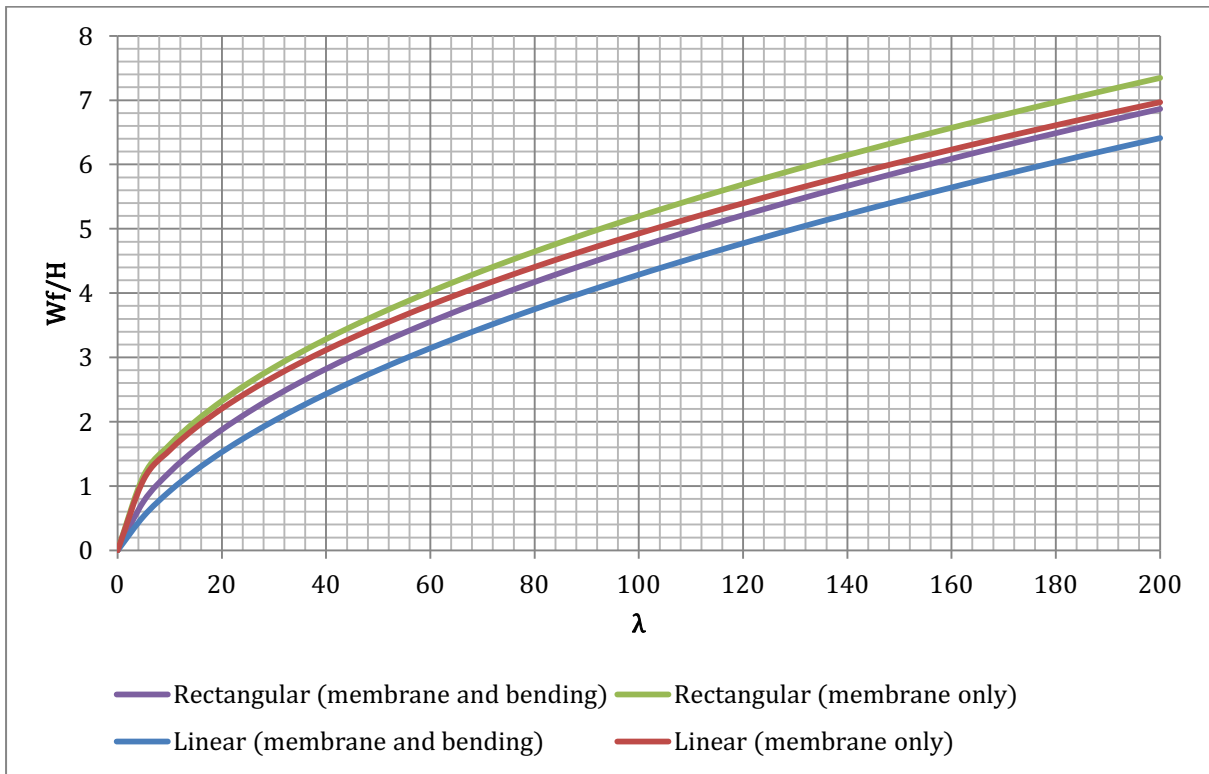


Figure 4-54: Dimensionless  $W_f$  for various impulse values with/out bending effects for 2 pulse shapes using inscribed yield criterion

Thus, it can be concluded that, using the impulsive loading approximation, the final displacement is virtually independent of the pulse shape and the impulsive simplification is not necessary for the exponentially decaying load.

- Exponentially decaying load

A similar procedure for the more generalised exponentially decaying load is summarised below.

Considering bending and membrane effects, for the exponentially decaying pulse load, the governing ODE is given by:

$$\ddot{W} + A^2 W = \frac{B}{\eta - 1} \left( \eta \left\{ 1 - X \frac{t}{\tau} \right\} e^{-\frac{Yt}{\tau}} - 1 \right) \quad (4-153)$$

where  $A$  and  $B$  are defined in (4-113) and (4-115) respectively.

Thus, the maximum permanent displacement is given by:

$$\frac{W_f}{H} = C_4 \cos A\tau + D_4 \sin A\tau + \frac{C}{A^2} \quad (4-154)$$

where the constants  $C_4$  and  $D_4$  are given by:

$$C_4 = E_4 \cos A\tau - F_4 \sin A\tau \quad (4-155)$$

$$D_4 = E_4 \sin A\tau + F_4 \cos A\tau \quad (4-156)$$

where:

$$E_4 = K_1 \cos A\tau - K_2 \sin A\tau - \frac{e^{-Y}}{A\tau} (K_3\tau + Y\{K_4 - K_3\tau\}) \quad (4-157)$$

$$F_4 = -\frac{B}{A^2(\eta - 1)} - \frac{C}{A^2} + \left( \frac{K_3}{A} + \frac{K_4 Y}{A\tau} \right) \sin A\tau + \left( \frac{B}{A^2(\eta - 1)} - K_4 \right) \cos A\tau e^{-Y} (K_4 - K_3) \quad (4-158)$$

and:

$$K_1 = \frac{1}{A\tau} (K_3\tau + K_4 Y) \quad (4-159)$$

$$K_2 = \frac{B}{A^2(\eta - 1)} - K_4 \quad (4-160)$$

$$K_3 = \frac{B\eta X\tau}{(Y^2 + \{A\tau\}^2)(\eta - 1)} \quad (4-161)$$

$$K_4 = \frac{B\eta\tau^2(Y^2 - 2XY + \{A\tau\}^2)}{(Y^2 + \{A\tau\}^2)^2(\eta - 1)} \quad (4-162)$$

and  $C$  is defined in (4-118).

The response time is given by:

$$T = \frac{1}{A} \left\{ \tan^{-1} \left( \frac{D_4}{C_4} \right) \right\} \quad (4-163)$$

If the bending effects are neglected and only membrane effects are considered, then for the exponentially decaying pulse load, the governing ODE is given by:

$$\ddot{W} + A^2 W = D \left\{ 1 - X \frac{t}{\tau} \right\} e^{-\frac{Yt}{\tau}} \quad (4-164)$$

where  $A$  and  $D$  are defined in (4-113) and (4-130) respectively.

Thus, the maximum permanent displacement is given by:

$$\frac{W_f}{H} = C_5 \cos At + D_5 \sin At \quad (4-165)$$

where the constants  $C_5$  and  $D_5$  are given by:

$$C_5 = E_5 \cos A\tau - F_5 \sin A\tau \quad (4-166)$$

$$D_5 = E_5 \sin A\tau + F_5 \cos A\tau \quad (4-167)$$

where:

$$E_5 = K_5 \cos A\tau - K_6 \sin A\tau - \frac{D\tau^2 e^{-Y} (Y^2 - 2XY + \{A\tau\}^2 - X[Y^2 + \{A\tau\}^2])}{(Y^2 + \{A\tau\}^2)^2} \quad (4-168)$$

$$F_5 = K_5 \cos A\tau - K_6 \sin A\tau + \frac{De^{-Y} (Y^3 \tau \{X - 1\} + XY^2 \tau + A^2 Y \tau^3 \{X - 1\} - A^2 X \tau^3)}{A(Y^2 + \{A\tau\}^2)^2} \quad (4-169)$$

and:

$$K_5 = \frac{D\tau(\{A\tau\}^2 \{X + Y\} + Y^2 \{Y - X\})}{(Y^2 + \{A\tau\}^2)^2} \quad (4-170)$$

$$K_6 = -\frac{D\tau^2 (Y^2 - 2XY - \{A\tau\}^2)}{A(Y^2 + \{A\tau\}^2)^2} \quad (4-171)$$

The response time is given by:

$$T = \frac{1}{A} \left\{ \tan^{-1} \left( \frac{D_5}{C_5} \right) \right\} \quad (4-172)$$

#### 4.8.6 Concluding remarks

In this section, the analytical model for the problem under study is completed by considering the effects of finite deflections or in-plane forces. As discussed earlier, for high applied loads and/or short duration loads, an applied pulse load can be replaced by an equivalent impulsive velocity and it was found that the permanent transverse displacement can be accurately approximated by means of such an approach for high values of applied loading by means of dimensionless plots of applied impulse against displacement.

For a plate geometry which falls within specified geometric limits where transverse shear and/or rotatory inertia effects are negligible, a series of numerical simulations were carried out using ABAQUS/Explicit and excellent correlation with the derived analytical results (with an error of less than 5% for values of  $\lambda$  in excess of 50) are obtained.

The effects of pulse shape were again investigated and analytical expressions for the final permanent displacement were presented for 3 pulse shapes.

It was found that, for the case of the linearly decaying pulse, the impulsive simplification produces results which are similar to those for the rectangular pulse load (just over 5% difference) and thus it can be concluded that the final transverse displacements can be obtained using the impulsive simplification for a rectangular pulse shape, irrespective of pulse shape, for high values of impulse (i.e. as the duration of the load decreases and/or the pressure increases).

It was also found that membrane action is the dominant mode of energy dissipation and for the case of blast loading, and localised blasts in particular, where the transverse displacements are expected to be quite large compared to the plate thickness, it would suffice to consider only membrane action and ignore the contribution of bending altogether, with little loss of accuracy or, conversely, with little overestimation of the response.

#### **4.9 Final remarks**

In this chapter, an analytical model is presented which predicts the transverse displacement  $w$  of a simply supported circular steel plate of diameter  $2R$  exhibiting rigid plastic behaviour when subjected to a blast load. The form of this blast load is described by means of a piecewise continuous function with a central circular zone of constant pressure over a diameter  $2R_0$  and an exponentially decaying spatial variation on the remainder of the plate, which variation is a function of the stand-off distance and size of the charge initiating the blast, as described in Chapter 3.

Such a blast load function is a generalised form of loading which can model very close-in (localised) blasts for small values of  $R_0$  and also quasi-uniform loads as  $R_0$  approaches  $R$  for distant (global) blast loads, making the presented model a very adaptable solution which is adequate for a variety of loading scenarios not previously investigated in the literature.

On the basis of these assumptions, the (unique) static plastic collapse pressure is presented found by using the upper and lower bounds of plasticity theory and the static analysis is extended to consider dynamic loading and the study focuses on finding the transverse displacement of the plate at various stages of its motion for the case of various pulse loads, for 3 cases, viz., assuming infinitesimal deflections, considering transverse shear and rotatory inertia effects and lastly assuming that the lateral load causes finite deflections.

For high applied loads and/or short duration loads, which are typical of blast loads, the pulse can be replaced by an equivalent impulsive velocity and it was found that the permanent transverse displacement can be accurately approximated by means of such an approach.

Thus, it can be concluded that the results in this chapter can provide very useful information to the designer regarding the response of blast-loaded monolithic plates.

Whilst it was shown that considering infinitesimal deflections alone would overestimate the permanent displacements and that the effects of transverse shear effects are negligible for usual plate geometries, it was found that the effects of membrane action are dominant in the response of the plate and this was verified by means of a series of numerical simulations using ABAQUS/Explicit. Excellent correlation of the numerical results with the derived analytical results was achieved.

Whilst these results are suitable for plates made from materials which are not very sensitive to the rate of loading, it should be noted that these results could, however, prove to be too conservative when applied to materials which exhibit rate-sensitivity (or visco-plasticity), such as ductile metals.

Thus, the current formulation would need to be modified such that the analytical expression are not based on a static yield stress but on a dynamic yield stress using, for example, the Cowper-Symonds [260] or Johnson-Cook [261] constitutive equations or simplified approaches, such as the Perrone-Badra approximation [262].

## Chapter 5. Blast loading on composite plates

### 5.1 Nomenclature

In this chapter, the following notation will be used:

#### *Latin lower case*

|       |                                            |
|-------|--------------------------------------------|
| $b$   | loading parameter, $[L^{-1}]$              |
| $m$   | plate slenderness exponent, $[1]$          |
| $n$   | normal direction, $[1]$                    |
| $r$   | radial axis direction, $[L]$               |
| $s_i$ | shear direction, $[1]$                     |
| $t_d$ | load positive phase duration, $[T]$        |
| $w$   | loading parameter, $[T^{-1}]$              |
| $w_f$ | final plate transverse displacement, $[L]$ |

#### *Latin upper case*

|           |                                                                                      |
|-----------|--------------------------------------------------------------------------------------|
| $A - C$   | low-rate Dyneema HB26 shear stress-strain fitting parameters, $[M L^{-1} T^{-2}]$    |
| $A' - C'$ | medium-rate Dyneema HB26 shear stress-strain fitting parameters, $[M L^{-1} T^{-2}]$ |
| $B$       | plate width, $[L]$                                                                   |
| $D$       | low-rate Tegriss shear stress-strain fitting parameter $[M L^{-1} T^{-2}]$           |
| $D_n$     | Johnson's damage number, $[1]$                                                       |
| $E$       | low-rate Tegriss shear stress-strain fitting parameter $[1]$                         |
| $F_{ij}$  | strain-rate constant, $[1]$                                                          |
| $G$       | work done, $[M L T^{-2}]$                                                            |
| $G_c$     | fracture energy, $[M L^{-1} T^{-2}]$                                                 |
| $H$       | plate thickness, $[L]$                                                               |
| $I$       | impulse, $[M L T^{-1}]$                                                              |
| $L$       | plate length, $[L]$                                                                  |
| $M_0$     | plastic collapse moment per unit width, $[M L T^{-2}]$                               |
| $P_0$     | maximum overpressure, $[M L^{-1} T^{-2}]$                                            |
| $P_a$     | decaying overpressure multiplier, $[M L^{-1} T^{-2}]$                                |
| $P_s(r)$  | spatial part of pressure pulse load, $[M L^{-1} T^{-2}]$                             |
| $P_t(t)$  | temporal part of pressure pulse load, $[M L^{-1} T^{-2}]$                            |
| $R_0$     | radius of central uniformly-loaded region, $[L]$                                     |
| $R_e$     | radius of explosive charge, $[L]$                                                    |
| $R$       | plate radius, $[L]$                                                                  |
| $S$       | stand-off distance, $[L]$                                                            |
| $T$       | traction, $[M L T^{-2}]$                                                             |
| $T_0$     | damage initiation traction, $[M L T^{-2}]$                                           |
| $V_0$     | initial impulsive velocity, $[L T^{-1}]$                                             |
| $W$       | mass of explosive charge, $[M]$                                                      |
| $Z$       | scaled distance, $[M^{-1/3} L]$                                                      |

### *Greek lower case*

|                            |                                                                                        |
|----------------------------|----------------------------------------------------------------------------------------|
| $\varepsilon$              | normal strain, [1]                                                                     |
| $\varepsilon_{ij}$         | $ij^{\text{th}}$ strain component, [1]                                                 |
| $\dot{\varepsilon}_{0,ij}$ | reference static strain-rate for $ij^{\text{th}}$ strain component, [T <sup>-1</sup> ] |
| $\dot{\varepsilon}_{eff}$  | effective strain-rate, [T <sup>-1</sup> ]                                              |
| $\gamma$                   | shear strain, [1]                                                                      |
| $\delta$                   | ply separation, [L]                                                                    |
| $\delta_0$                 | damage initiation separation, [L]                                                      |
| $\delta_{cr}$              | failure separation, [L]                                                                |
| $\lambda$                  | Jones' dimensionless impulse, [1]                                                      |
| $\lambda_{sl}$             | modified Jones' dimensionless impulse, [1]                                             |
| $\mu$                      | plate mass per unit area, [M L <sup>-2</sup> ]                                         |
| $\xi_{q.d}$                | Jacob et al.'s stand-off parameter, [1]                                                |
| $\xi_{q.l}$                | Jacob et al.'s loading parameter, [1]                                                  |
| $\xi_{q.sl}$               | slenderness parameter, [1]                                                             |
| $\rho$                     | density, [M L <sup>-3</sup> ]                                                          |
| $\sigma$                   | normal stress, [M L <sup>-1</sup> T <sup>-2</sup> ]                                    |
| $\sigma_0$                 | yield stress, [M L <sup>-1</sup> T <sup>-2</sup> ]                                     |
| $\tau$                     | shear stress, [M L <sup>-1</sup> T <sup>-2</sup> ]                                     |
| $\phi_{q.l}$               | Jacob et al.'s damage parameter for quadrangular plates with load effect, [1]          |
| $\phi_{q.s}$               | Jacob et al.'s damage parameter for quadrangular plates with stand-off effect, [1]     |
| $\phi_{q.sl}$              | damage parameter for quadrangular plates with slenderness effect, [1]                  |
| $\omega$                   | loaded radius ratio, [1]                                                               |

## 5.2 Background

Engineering materials invariably undergo damage when subjected to extreme loading, such as blast. In the case of laminated polymer-based composite materials, the initial occurrence of damage does not necessarily indicate ultimate failure. The progressive build-up in loss of material strength and stiffness occurs as a result of various damage mechanisms which are unique to such materials, such as inter-ply delamination, fibre debonding or matrix failure, as discussed in Chapter 2.

The use of such composite materials or metal-composite hybrids such as FMLs (fibre metal laminates) for protection against severe loads is becoming more relevant, as researchers and designers and users seek lighter protection systems than conventional (heavier) ones, such as steel. The range of composite materials used for this purpose has recently broadened from the traditional carbon and glass fibre systems to new high performance polymeric materials, such as aramids, polypropylene and polyethylene fibres.

### 5.2.1 High performance polyethylene composites

Dyneema is the trade name of a variety of UHMwPE (ultra high molecular weight polyethylene) fibres spun using a process called gel spinning patented by the Dutch company DSM in 1979 and long marketed as “the world’s strongest fibre”.

It is claimed that Dyneema is 10 times stronger than steel and 50% stronger than aramid fibre, on an equal-weight basis [263]. It is extremely light, with its specific gravity being 0.97. Being a rather new material, relatively little work related to Dyneema has been carried out.

Rodriguez et al. [264] carried out static and dynamic tests on SK66 and UD66 Dyneema composites.

Jacobs and Van Dingenen [197] developed a simple empirically-based model for 2 grades of Dyneema subjected to various ballistic threats, assuming that the absorbed energy is proportional to the mass below the strike face.

Chocoron et al. [265] carried out a study on the mechanical properties of Dyneema and found that the fibres behave essentially as a linear elastic material until failure, exhibiting brittle behaviour, while non-woven Dyneema shows significant plasticity and softening behaviour but little variation with strain-rate in the range of  $10^{-3}$  /s to 1 /s.

Iannucci et al. [266] carried out tests on 250x25x5 mm Dyneema uni-directional composite samples to determine its tensile, compressive and shear properties under monotonically increasing loading and cyclic loading. These are discussed in further detail in Section 5.5.1. The complex combination of tensile, compressive and shear response has been successfully implemented in a finite element commercial software (LS-DYNA) by Iannucci and Pope [267] who have made use of the work of Cunniff [268] based on fibre properties linking the mesoscale behaviour of the fibres to the macroscale behaviour of the laminate.

Jena et al. [269] conducted tests on layered systems with Dyneema and steel while more recently Ong et al. [270, 271] have conducted testing and numerical simulations of hybrid systems of steel and Dyneema HB25 under ballistic loading.

More limited work has been carried out on the performance of Dyneema under blast loading, with the exception of Kirkpatrick et al. [272], who have used Dyneema cylinders to mitigate the effects of a pipe explosive and Kandal et al. [273], who have studied Dyneema and carbon systems subjected to impacting metal foam projectiles to simulate shock loading.

### **5.2.2 High performance polypropylene composites**

Another high performance composite is Tegrise, which is the trade name of a polypropylene thermoplastic composite manufactured by the American company Milliken and launched in 2004 [274]. Tegrise is a self-reinforced product, made from a co-extruded polypropylene tape yarn having core (which is highly drawn to achieve its high strength) which is placed within a lower melt polymer matrix for composite processing. The yarn is woven into a fabric and multiple sheets are then stacked and thermally consolidated under pressure to form rigid sheets. The sheets are then used to manufacture panels or moulded using pressure thermoforming techniques, thus the former name with which Tegrise was known, viz., MFT (Mouldable Fabric Technology). Tegrise is also extremely light, quoted as having a specific gravity of 0.78 [275].

Although Tegrise was initially developed for use in the transportation, motorsport and other recreational industries [276, 277], its use for defence applications has also been of significance. However, to date, very little open literature is available on the subject.

Grosch et al. [278] recently tested 4 hybrid (Tegris and aluminium/steel) panels under blast loadings arising from a C4 charge of almost 1 kg applied at various stand-off distances. The performance was compared to that of a 10 mm thick mild steel plate and it was found that the composite-metallic assemblies resulted in less maximum deflections.

Very recently, Valluri et al. [279] carried out static tensile testing of Tegris samples and found that the strengths and stiffnesses measured compare very well with those specified by the manufacturer, viz., a stiffness of over 6 GPa, tensile strength of just over 200 MPa and a strain at failure of almost 6%. Fatigue testing was also performed and it was found that at stress ratios over 70%, the material fails at less than 2000 cycles, but for a stress ratio of less than 60%, over 1000000 cycles were carried out and failure did not occur.

### **5.3 Aim of current work**

The objective of the present study is to investigate the performance of a new grade of Dyneema (HB26) under blast loading and compare its performance with that of monolithic plates.

Results of blast loaded Dyneema panels tested at UCT (University of Cape Town) in South Africa are presented and material characterisation tests are also performed on Dyneema samples. From these tests, a simple constitutive model for Dyneema is developed and implemented through a user-defined material subroutine in the commercial finite element package ABAQUS/Explicit. Material characterisation tests on Tegris samples are also carried out in order to establish material properties for a further constitutive model.

In the case of Dyneema HB26, the results from the model are validated by comparison with the experimental results. The same blast loads are applied on metallic (mild and armour steel) panels of the same areal density and non-dimensional analysis is used to compare the performance of the materials.

### **5.4 Blast testing of Dyneema HB26 panels**

Laboratory testing<sup>9</sup> was carried out at the BISRU (Blast and Impact Survivability Research Unit) within UCT using the ballistic pendulum setup therein, which was described in Section 3.8.1.

#### **5.4.1 Experimental setup**

In typical blast tests, the test panel is clamped but, in the case of Dyneema panels, the panels were simply held in position and not fully clamped.

In this set of tests, the PE4 (or C4) charge diameter was fixed at 50 mm the stand-off distance was also maintained as 50 mm (refer to Section 5.4.3 for details) using a polystyrene bridge. Detonation is remotely initiated centrally by means of an electrical detonator. The arrangement is shown in Figure 5-1.

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<sup>9</sup> The work of Professor Genevieve S. Langdon and Mr. Wei-Chi Lee of BISRU is hereby acknowledged.

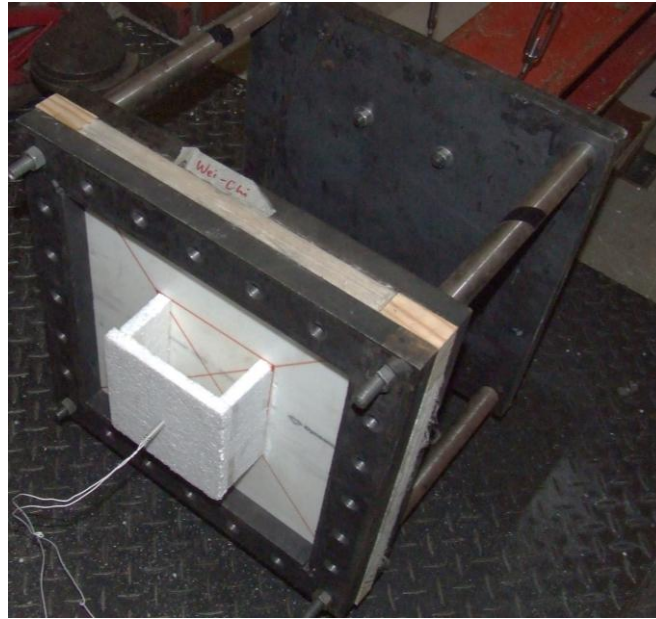


Figure 5-1: Dyneema HB26 panel with polystyrene bridge in position

#### 5.4.2 Test specimens

The choice of specimen thickness was primarily dictated by areal density considerations, with the intention of comparing the performance of Dyneema with mild and/or armour steel having the smallest feasible thickness, viz.  $\approx 3$  mm, or an areal density of  $\approx 23$  kg/m<sup>2</sup>.

Based on the above, the chosen thickness of Dyneema panels was 24 mm.

The test panels had an exposed area of 300x300 mm, determined by the dimensions of the clamping frame of the testing rig.

#### 5.4.3 Test matrix

Having fixed the panel size, the stand-off distance was chosen, on the basis of geometric scaling, such that the laboratory tests would represent full scale tests on typical armour components. Assuming that the full scale test corresponds to a typical MRAP (Mine Resistant Ambush Protected) vehicle of dimension 2500 mm, then a stand-off distance,  $S$ , of 400 mm (representing an anti-vehicle buried mine threat) would be scaled to 50 mm.

Similarly, the mass,  $W$ , of the PE4 charges were chosen to represent various threat scenarios and determined using the Hopkinson-Cranz scaling law [280]:

$$Z = \frac{S}{\sqrt[3]{W}} \quad (5-1)$$

The range of threats considered ranged from 8 kg to 15 kg of TNT, which was scaled down to equivalent PE4 charges assuming a TNT equivalence of 1.3 using (5-1), as follows:

| Threat scenario |                |                            | Test     |               |
|-----------------|----------------|----------------------------|----------|---------------|
| $S$ (m)         | $W_{TNT}$ (kg) | $Z$ (m/kg <sup>1/3</sup> ) | $S$ (mm) | $W_{PE4}$ (g) |
| 0.4             | 8              | 0.20                       | 50       | 12            |
| 0.4             | 10             | 0.19                       | 50       | 15            |
| 0.4             | 15             | 0.16                       | 50       | 23            |
| 0.4             | 30             | 0.13                       | 50       | 45            |

Table 5-1: Test matrix for scaled mine threats

Another set of tests were devised by considering scaling down of a threat scenario caused by IEDs (improvised explosive devices) at a stand-off distance of 300 mm and also ranging from 8 kg to 15 kg of TNT. These are summarised in Table 5-2 as follows:

| Threat scenario |                |                            | Test     |               |
|-----------------|----------------|----------------------------|----------|---------------|
| $S$ (m)         | $W_{TNT}$ (kg) | $Z$ (m/kg <sup>1/3</sup> ) | $S$ (mm) | $W_{PE4}$ (g) |
| 0.3             | 8              | 0.15                       | 50       | 28            |
| 0.3             | 10             | 0.14                       | 50       | 36            |
| 0.3             | 15             | 0.12                       | 50       | 53            |

Table 5-2: Test matrix for scaled IED threats

Thus, the final test matrix is as follows:

| Test | $S$ (mm) | $W_{PE4}$ (g) |
|------|----------|---------------|
| 1    | 50       | 12            |
| 2    | 50       | 15            |
| 3    | 50       | 23            |
| 4    | 50       | 28            |
| 5    | 50       | 36            |
| 6    | 50       | 45            |
| 7    | 50       | 53            |

Table 5-3: Summary of tests

#### 5.4.4 Test results

In each test, the impulse,  $I$ , and the final midpoint displacement,  $w_f$ , were measured. The test results are summarised in Table 5-4:

| Test | $I$ (Ns) | $w_f$ (mm) |
|------|----------|------------|
| 1    | 17.7     | 5.96       |
| 2    | 28.48    | 10.52      |
| 3    | 43.94    | 16.34      |
| 4    | 53.74    | 19.80      |
| 5    | 68.86    | 28.08      |
| 6    | 85.19    | 33.40      |
| 7    | 98.54    | 39.31      |

Table 5-4: Summary of test results

It can be seen from Table 5-4 and Figure 5-2 that there is a linear increase of impulse with increasing charge mass, with the relationship being almost 2:1.

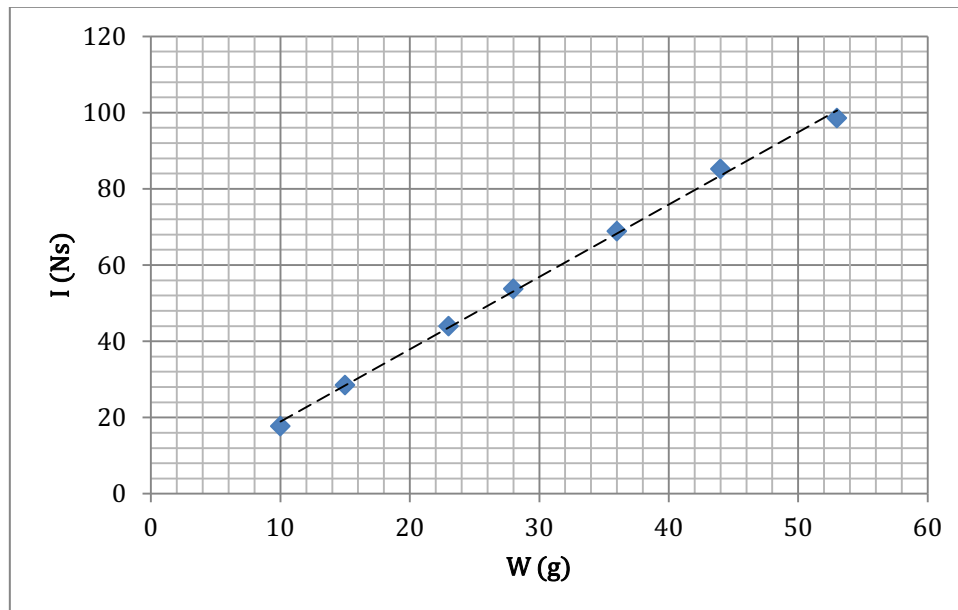


Figure 5-2: Variation of impulse with charge mass

It can be also seen from Table 5-4 and Figure 5-3 that there is also a linear increase of permanent midpoint displacement with increasing impulse.

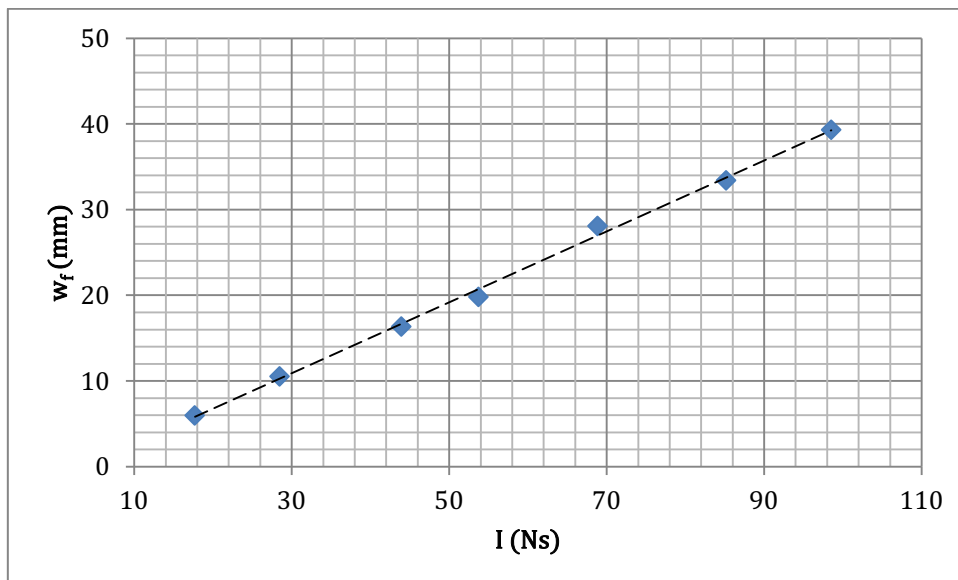


Figure 5-3: Variation of permanent displacement with impulse for Dyneema HB26

In all cases, no tearing or penetration was observed, but front face damage of varying degree was observed on all test panels, from superficial scarring for the lower charges to more significant damage for the larger charges. Examples are shown in Figure 5-4 to Figure 5-6.

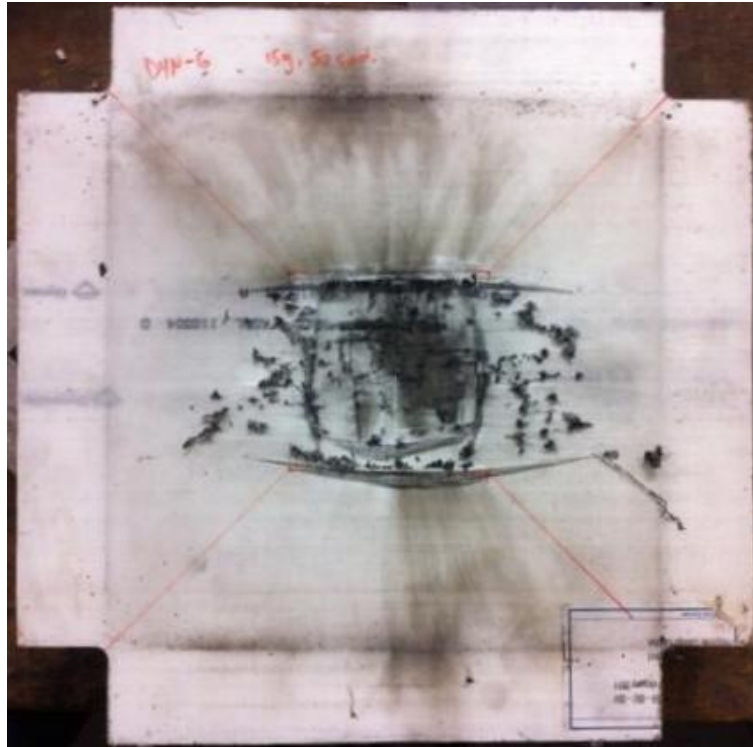


Figure 5-4: Dyneema HB26 panel (test 1) front face

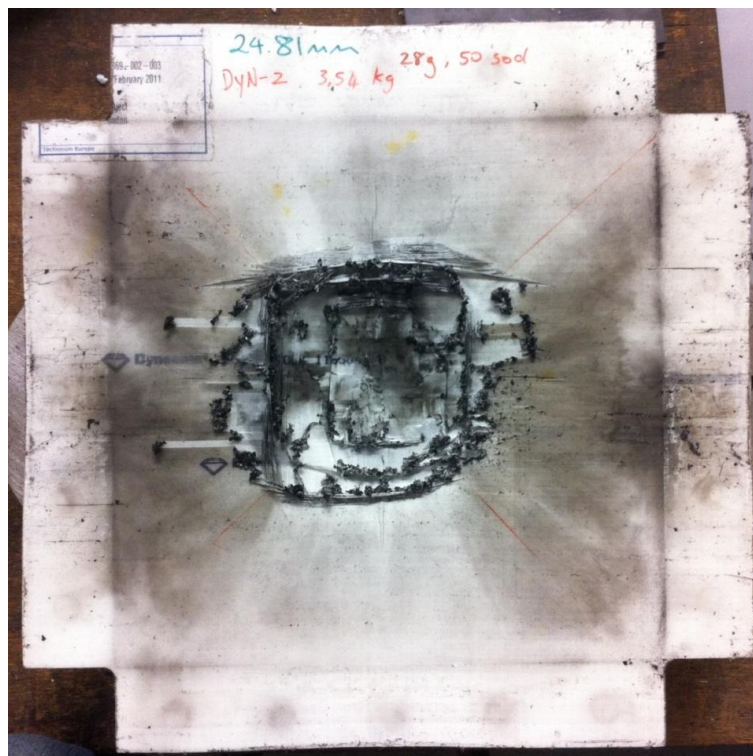


Figure 5-5: Dyneema HB26 panel (test 4) front face

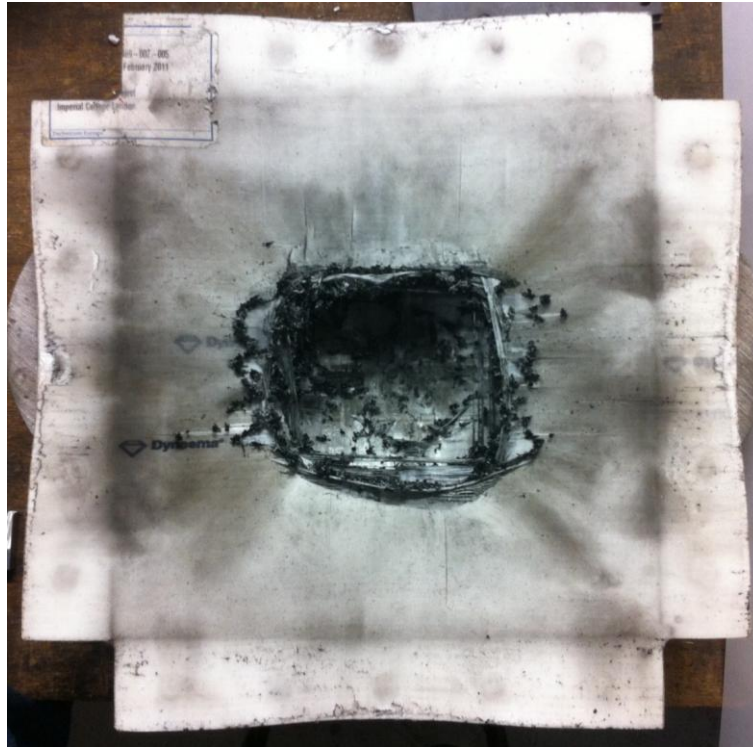


Figure 5-6: Dyneema HB26 panel (test 7) front face

Back face permanent deformation was observed in all specimens, with minimal deformation for the smaller charges to a permanent transverse displacement of over 1.5 plate thickness for the largest charge. The latter is shown in Figure 5-7 and Figure 5-43, where the test result is compared with the corresponding numerical one.

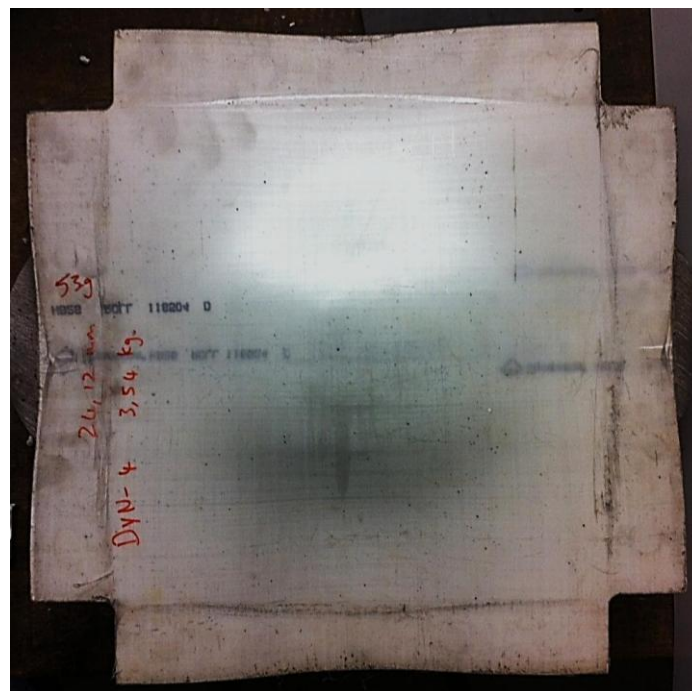


Figure 5-7: Dyneema HB26 panel (test 7) back face

Another significant observation is the pulling-in at the edges, which is particularly evident in the panel subjected to the largest charge (refer to Figure 5-6 and Figure 5-7). The pulling-in effect is characteristic of Dyneema, although in this case it is observed to be not as significant as in the case of Dyneema subjected to impact by fast-moving projectiles (e.g. [266, 271]).

## 5.5 Material characterisation of Dyneema

### 5.5.1 Existing material data

As discussed earlier, material characterisation tests [266] have been carried out on Dyneema uni-directional composite samples to determine its tensile, compressive and shear properties under monotonically increasing loading and cyclic loading. Monotonic loading results are summarised in Figure 5-8 to Figure 5-10.

The tensile behaviour, depicted in Figure 5-8, is essentially linearly elastic with an initial modulus of almost 13 GPa until a yield strain of approximately 3.75%, after which softening occurs until a failure strain of approximately 8.5% [266].

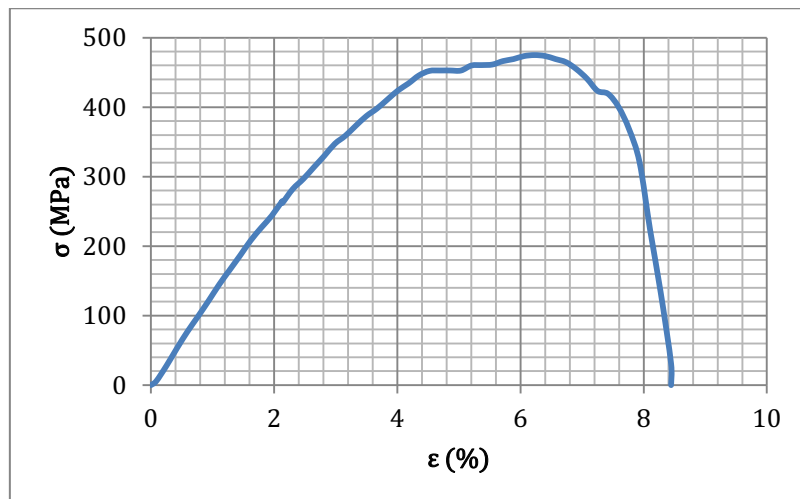


Figure 5-8: Tensile behaviour of Dyneema sample (after [266])

The compressive behaviour, shown in Figure 5-9, has an initially linear elastic stress-strain response with a modulus of 1.25 GPa until a stress of 8 MPa, followed by a long plastic plateau [266]. The peak before the plateau is probably due to the fibre micro-kinking and can be ignored in constitutive equations [266, 267].

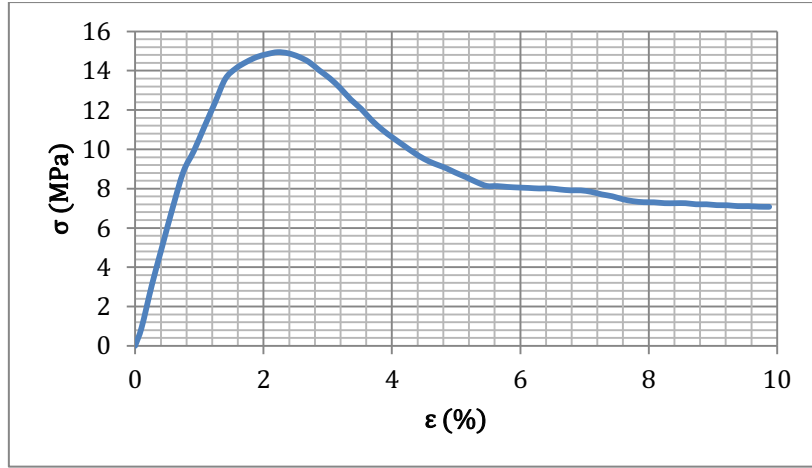


Figure 5-9: Compressive behaviour of Dyneema sample (after [266])

The shear response of Dyneema is highly non-linear as seen in Figure 5-10 and a cubic function is fitted to the test data of the form [266]:

$$\tau = A\gamma^3 + B\gamma^2 + C\gamma \quad (5-2)$$

where  $A = 1120.77 \text{ MPa}$ ,  $B = -580.42 \text{ MPa}$  and  $C = 109.61 \text{ MPa}$  [266].

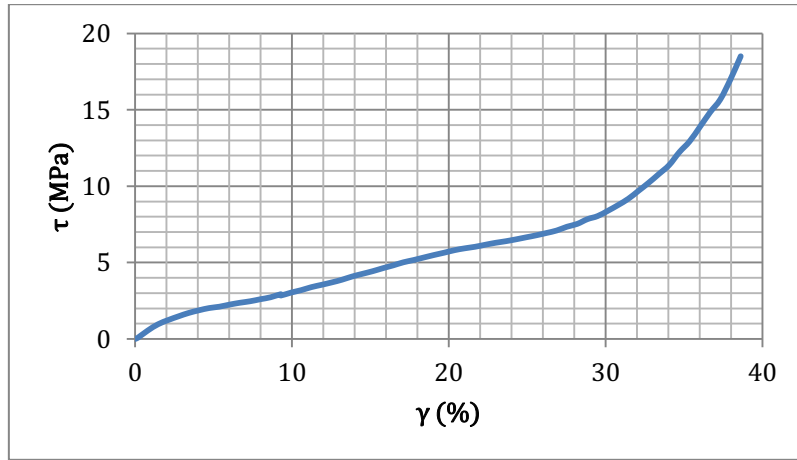


Figure 5-10: Shear behaviour of Dyneema sample (after [266])

### 5.5.2 Material data for Dyneema HB26

Another set of tests<sup>10</sup> have been recently conducted at the Department of Civil and Environmental Engineering at Imperial College London on Dyneema HB26 samples.

The tests were carried out using an Instron 8802 load rig and a 250 kN load cell and all measurements made using the Instron AVE (advanced video extensometer), capable of measuring strains in 2 directions. Computer-controlled methods were used to set the loading rate and to manage data acquisitioning.

<sup>10</sup> The work of Mr. Andrew D. Pullen of the Structures Laboratory is hereby gratefully acknowledged.

The various tests are described hereunder.

- Tensile testing

Tensile testing of Dyneema posed several logistical difficulties, particularly due to the extremely slippery nature of the material, making it difficult to adequately grip the test specimens and ensure that no slippage occurred prior to tensile failure of the material.

Tensile tests were carried out to BS EN ISO 527-5:2009 [281] initially on 250x25x2.2 mm specimens loaded at 1 mm/min. Successful tests were carried out on such samples but with a reduced width of 5 mm, in order to attain failure within the gauge length, as shown in Figure 5-11.



Figure 5-11: Reduced-width tensile specimen of Dyneema HB26

The results showed good repeatability and had similar qualitative trends to the results reported in [266], but with an improvement in stiffness, with a modulus of 25.8 GPa, a yield stress of 700 MPa and a failure strain of 3.5%, with a very short plastic region. A typical stress-strain plot is shown in Figure 5-12.

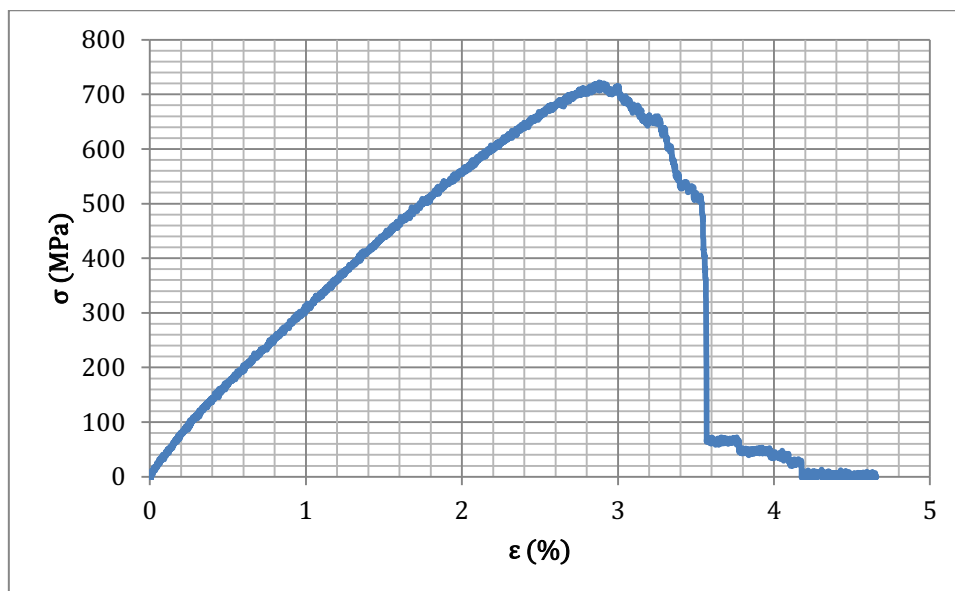


Figure 5-12: Tensile behaviour of Dyneema HB26 sample loaded at 1 mm/min

- Compressive testing

In-plane compression testing to BS EN ISO 14126:1999 [282] was carried out on similar specimens with an ungripped length of 25 mm, at 1 mm/min and 100 mm/min, as shown in Figure 5-13.

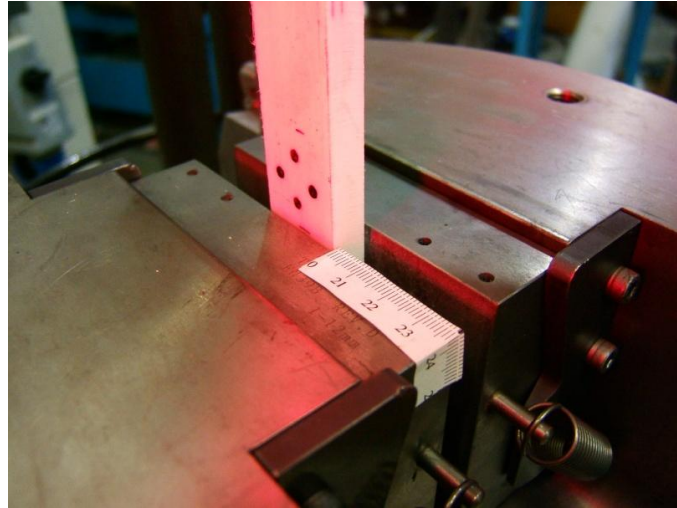


Figure 5-13: Compression testing of Dyneema HB26 sample

Failure, in the form of buckling as seen in Figure 5-14, was observed, exhibiting behaviour which is very similar to that reported in [266] and shown in Figure 5-9, but with the plastic plateau increasing to approximately 12 MPa, irrespective of loading rate. Typical stress-strain plots are shown in Figure 5-15 and Figure 5-16.

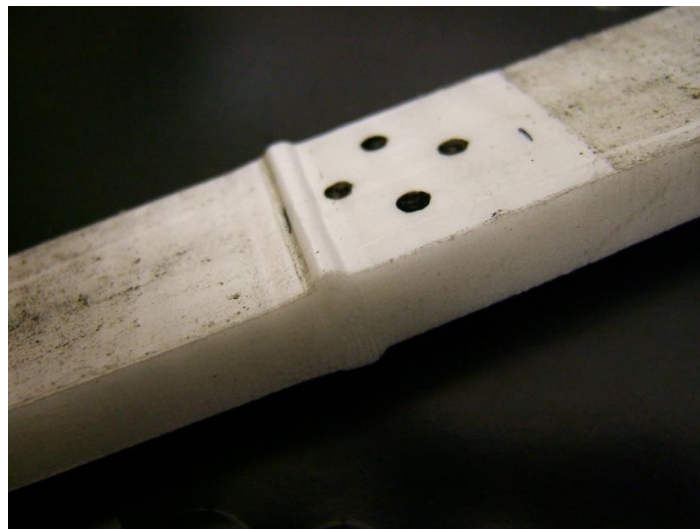


Figure 5-14: Compression failure of Dyneema HB26 sample

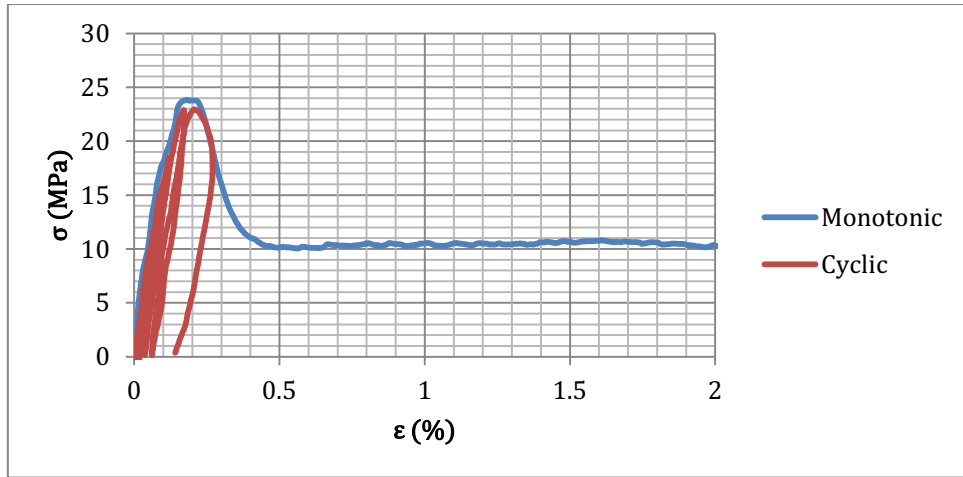


Figure 5-15: Compressive behaviour of Dyneema HB26 sample under monotonic and cyclic loading

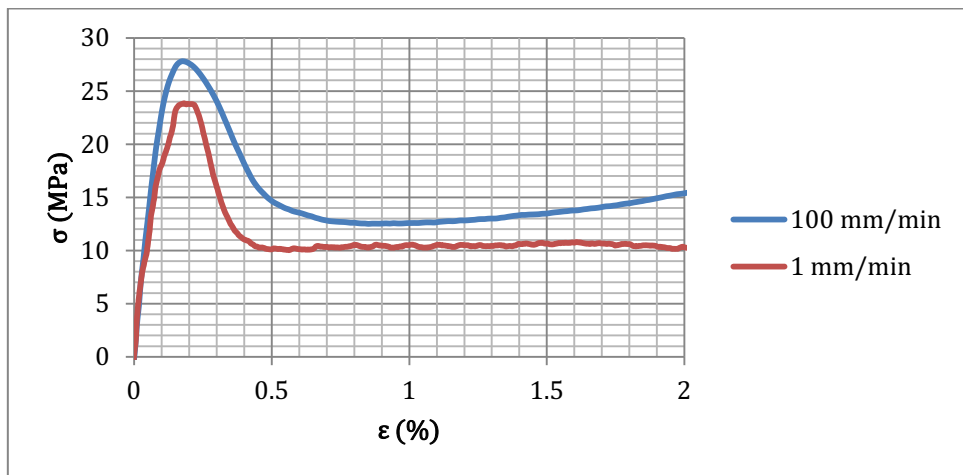


Figure 5-16: Compressive behaviour of Dyneema HB26 sample at various rates

- Through-thickness compressive testing

Although no published standard exists for through-thickness compressive testing of fibre-reinforced plastics, such tests were carried out on 30 mm diameter samples of 24 mm thickness, as shown in Figure 5-17.

Tests were carried out at 2 loading rates, viz.,  $10^{-2}$  /s and  $10^{-3}$  /s. The test specimens are shown in Figure 5-18 and the results are summarised in Figure 5-19. It can be seen that there is little change in the response with loading rate.



Figure 5-17: Through-thickness compression testing of thick Dyneema HB26 sample



Figure 5-18: Pristine (left) and tested (middle, right) Dyneema HB26 samples after through-thickness compression tests

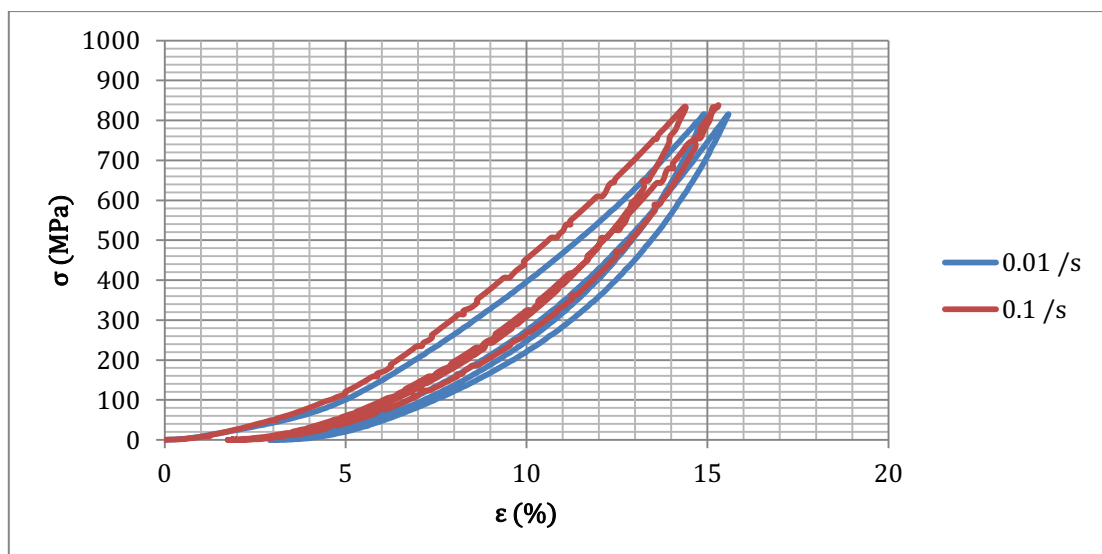


Figure 5-19: Through-thickness compressive behaviour of Dyneema HB26 samples

- Shear testing

In terms of shear behaviour, monotonic and cyclic loading of Dyneema HB26 specimens was carried out to BS EN ISO 14129:1998 [283], using the  $\pm 45^\circ$  tensile test method. 250x25x2.2 mm specimens were tested at 2 mm/min. Typical (failed) specimens are shown in Figure 5-20.

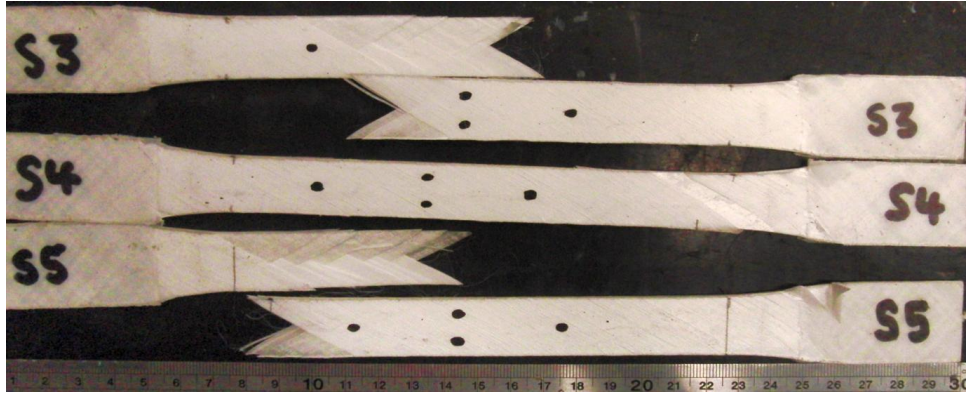


Figure 5-20: Shear failure of Dyneema HB26 samples

A typical stress-strain curve is shown in Figure 5-21, with the cyclic loading curve indicating that the unloading curve is essentially linear and the modulus is equal to the initial tangent modulus (approximately 110 MPa).

Using the same cubic fit as proposed in (5-2), the material constants were found to be  $A = 576$  MPa,  $B = -308.33$  MPa and  $C = 71.94$  MPa.

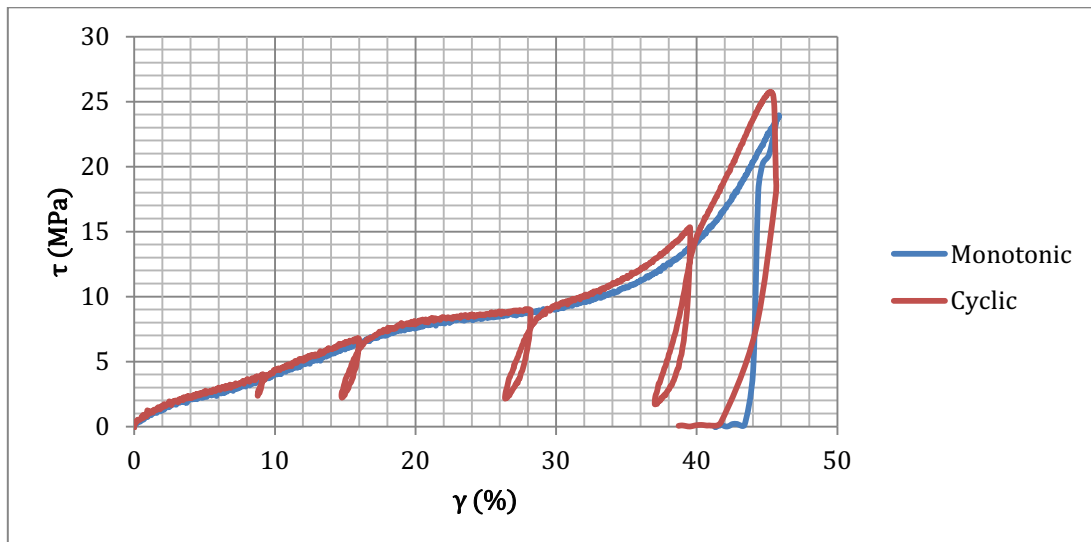


Figure 5-21: Shear behaviour of Dyneema HB26 sample under monotonic and cyclic loading

Further tests were carried out at 200 mm/min, with the result being shown in Figure 5-22. This shows a shift in the baseline curve (changing the constants  $A$ ,  $B$  and  $C$  to  $A' = 802.77$  MPa,  $B' = -492.48$  MPa and  $C' = 117.1$  MPa respectively) and also an increase in the ultimate shear stress (approximately 20%) but with no change in the ultimate shear strain.

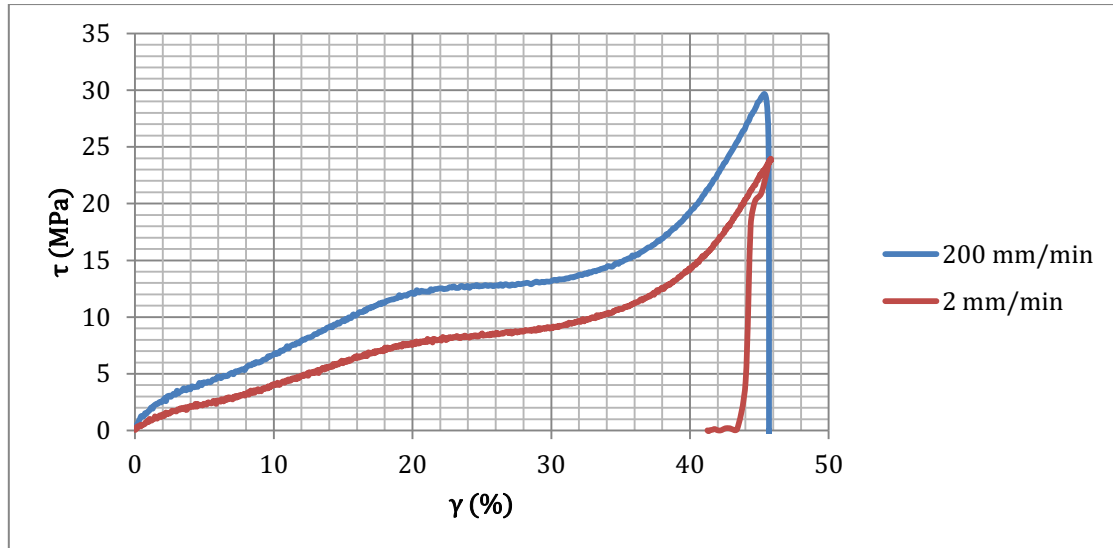


Figure 5-22: Shear behaviour of Dyneema HB26 sample at various rates

The material characterisation data extracted from these tests was used to formulate a constitutive model for Dyneema HB26, as described in the next section.

### 5.5.3 Material model for Dyneema HB26

From the material data obtained from the tests described above, the tensile and compressive behaviours were idealised as elastic-perfectly plastic rate-independent relationships, as shown in Figure 5-23, with their respective yield stress values and a tensile failure strain of 3.5%. It was postulated no failure occurs in compression, as assumed in [267].

The cubic shear stress-strain relation of (5-2) was retained without simplification, as shown in Figure 5-24.

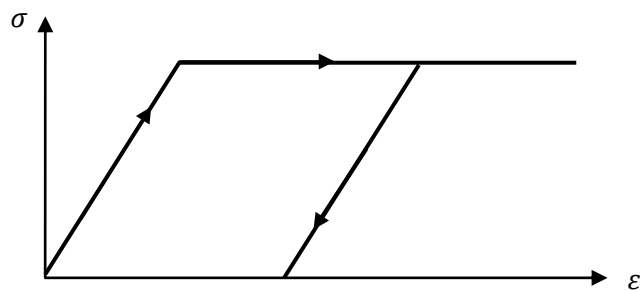


Figure 5-23: Assumed  $\sigma$ - $\epsilon$  relationships for tension and compression (Dyneema HB26)

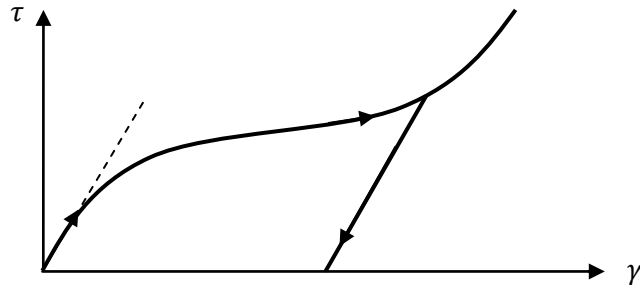


Figure 5-24: Assumed  $\tau$ - $\gamma$  relationship for shear (Dyneema HB26)

## 5.6 Material characterisation of Tegriss

### 5.6.1 Existing material data

As noted in Section 5.2.2, material tensile characterisation tests [279] have been carried out on Tegriss samples. Monotonic loading results are shown in Figure 5-25.

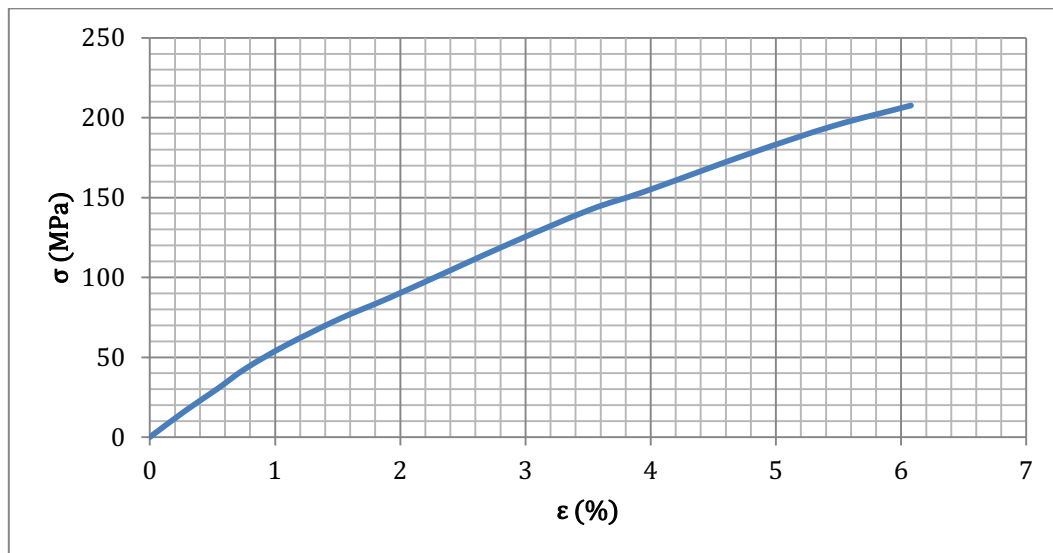


Figure 5-25: Tensile behaviour of Tegriss sample (after [279])

However, such tests are not enough to provide sufficient information on the behaviour of Tegriss to enable a constitutive model to be formulated and thus further tests were carried out, as described in Section 5.6.2.

### 5.6.2 Material data for Tegriss

A fresh set of tests<sup>11</sup> have been recently conducted at the Department of Civil Engineering at Imperial College London on Tegriss samples. The tests were carried out using the same testing setup described in Section 5.5.2 and the results are discussed hereunder.

<sup>11</sup> The work of Mr. Andrew D. Pullen is again hereby acknowledged.

- Density

The density of Tegriss was experimentally found to be slightly higher than the value quoted by the manufacturer ( $830 \text{ kg/m}^3$  c.f.  $780 \text{ kg/m}^3$ ).

- Tensile testing

Tensile tests were carried out to BS EN ISO 527-5:2009 [281] initially on  $250 \times 50 \times 6.1 \text{ mm}$  specimens loaded at  $1 \text{ mm/min}$  and also at  $100 \text{ mm/min}$ . A typical test specimen being tested is shown in Figure 5-26. Other tests were carried out on specimens of an alternative geometry, as shown in Figure 5-27, but these were found to fail prematurely close to the fillet edges, probably due to stress concentrations at the reduction in cross-sectional area.

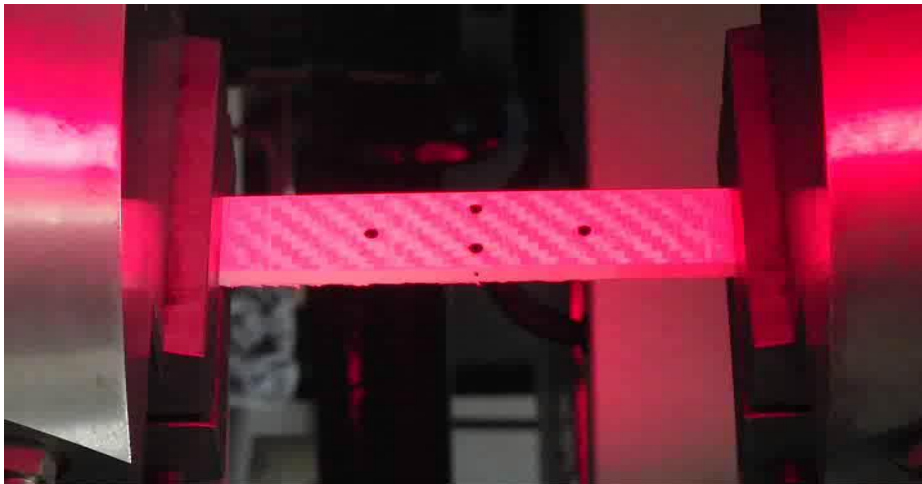


Figure 5-26: Tensile testing of Tegriss sample

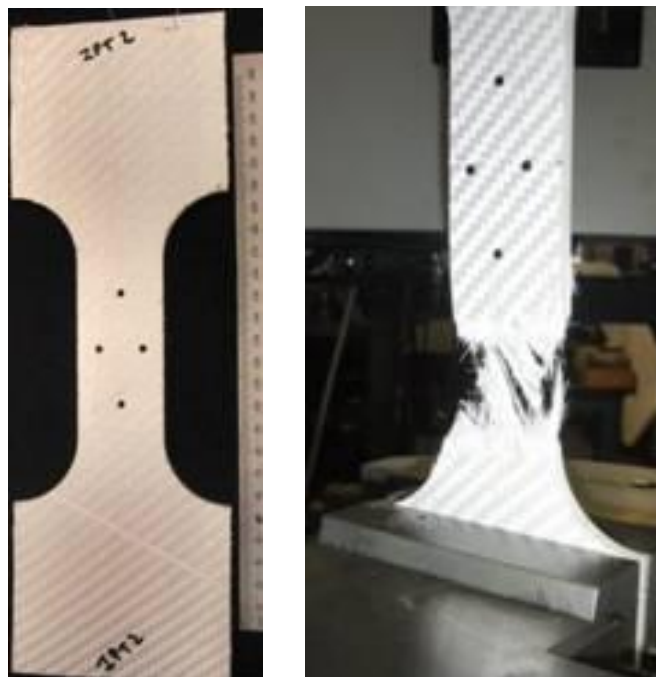


Figure 5-27: Alternative Tegriss specimen geometry (left: intact, right: failed)

The results showed good repeatability and had similar qualitative trends to the results reported in [279], with an initial linear elastic zone with a rate-independent modulus of 6 GPa, followed by a long plastic (hardening) zone which is rate-dependent but with failure being fairly rate-independent at around 200 MPa. A typical stress-strain plot is shown in Figure 5-28.

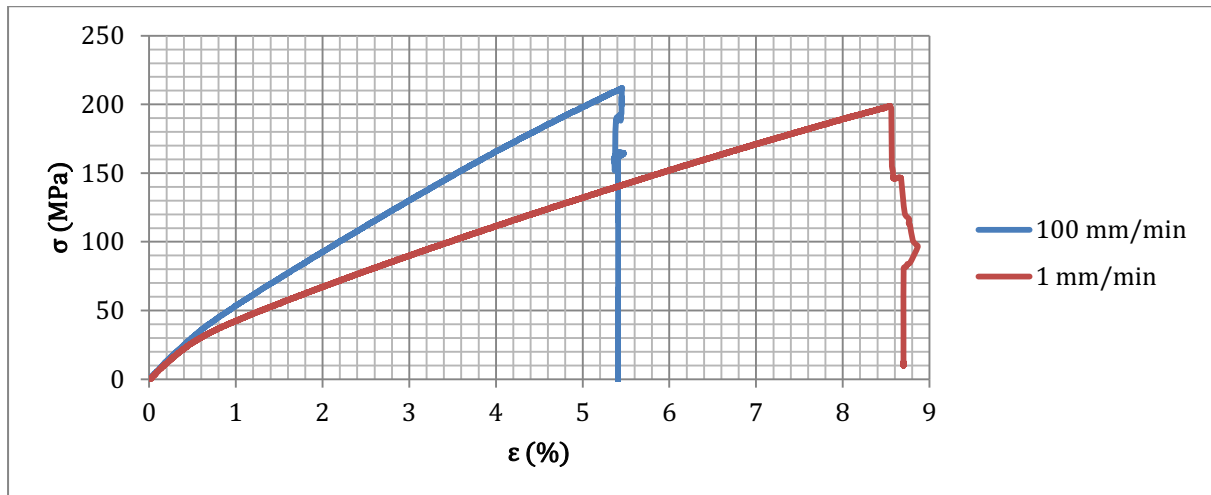


Figure 5-28: Tensile behaviour of Tegriss sample at various rates

- Compressive testing

In-plane compression testing to BS EN ISO 14126:1999 [282] was carried out on similar specimens with an ungripped length of 25 mm. Failure, in the form of buckling as seen in Figure 5-29, was observed to occur at low stress levels, with no significant rate effects in both modulus and failure. A typical stress-strain plot is shown in Figure 5-30.

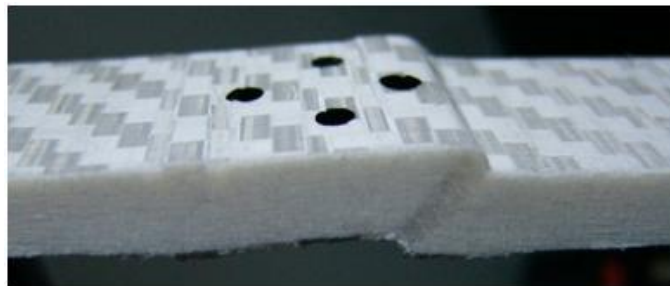


Figure 5-29: Tegriss specimen showing compressive failure

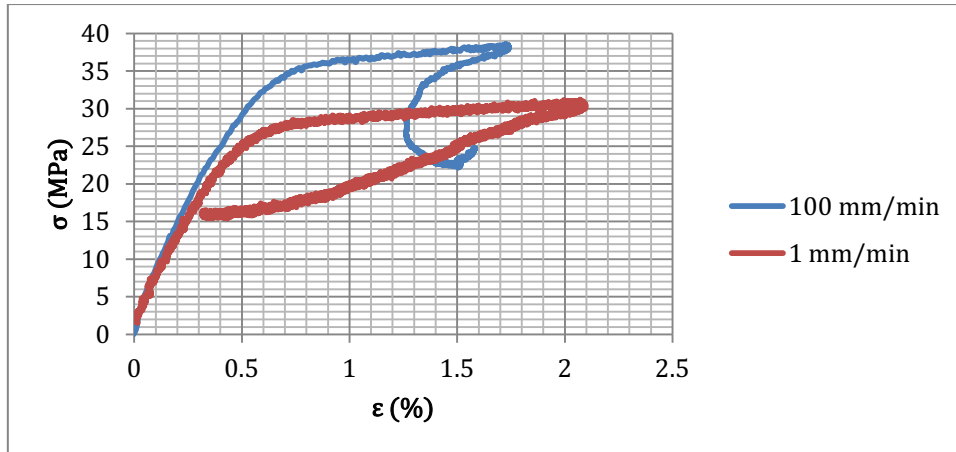


Figure 5-30: Compressive behaviour of Tegriss sample at various rates

- Through-thickness compressive testing

Through-thickness tests were carried out on cylindrical samples of Tegriss laminate, similarly to the assembly described in Section 5.5.2, as shown in Figure 5-31. The results are summarised in Figure 5-32.

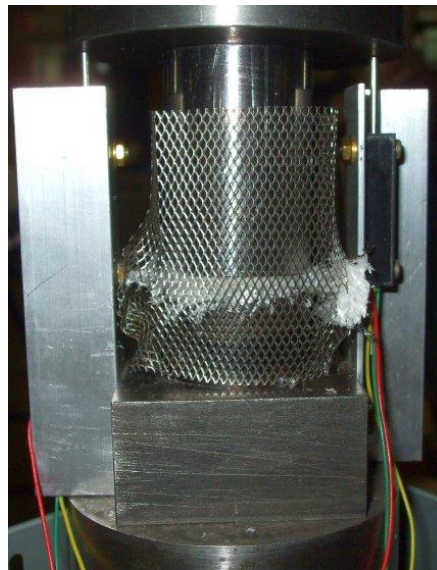


Figure 5-31: Through-thickness compression testing of Tegriss specimen

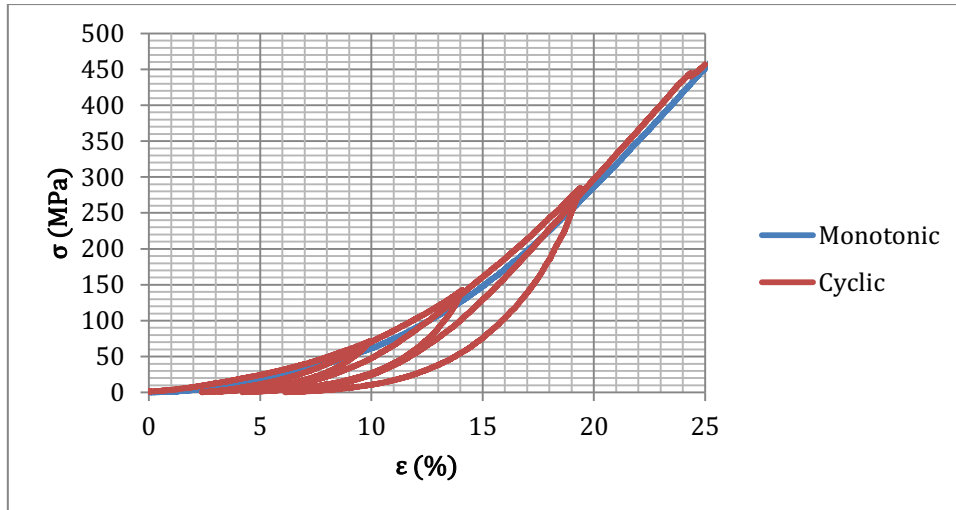


Figure 5-32: Through-thickness compression behaviour of Tegriss sample under monotonic and cyclic loading

- Shear testing

Finally, tension-shear tests to BS EN ISO 14129:1998 [283] were carried out at 2 mm/min and 200 mm/min, resulting in non-linear shear stress-strain relationships shown in Figure 5-33, with failure occurring in both cases at a strain of around 70%.

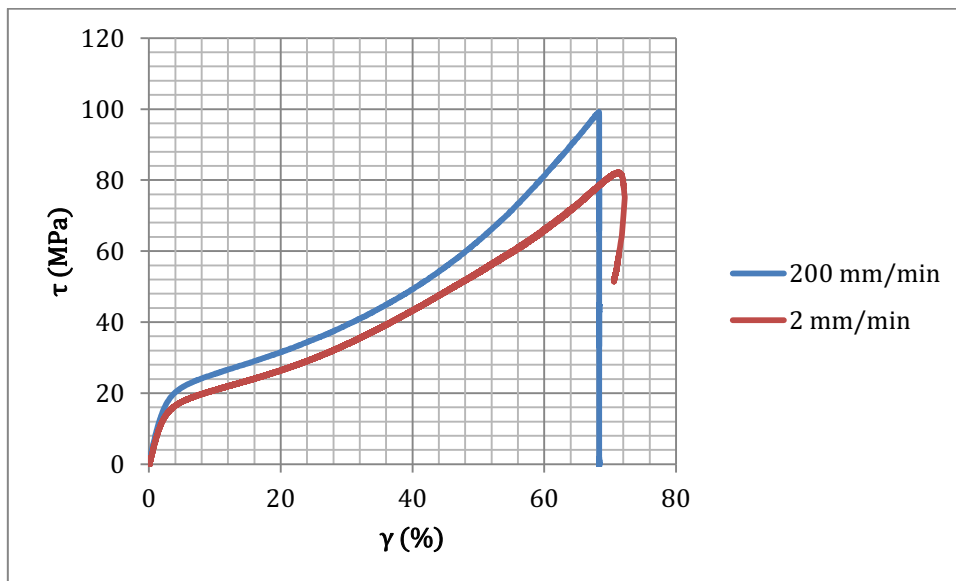


Figure 5-33: Shear behaviour of Tegriss sample at various rates

The material characterisation data extracted from these tests was used to formulate a constitutive model for Tegriss, as described in Section 5.6.3.

### 5.6.3 Material model for Tegrís

Using the material data described in Section 5.6.2, the following material relationships for Tegrís are proposed.

The (static) tensile behaviour was idealised as a bi-linear relationship to capture the plastic hardening zone, as shown in Figure 5-34, with an initial modulus of 6 GPa, a yield strain value of 0.5% and a tensile failure stress of 200 MPa.

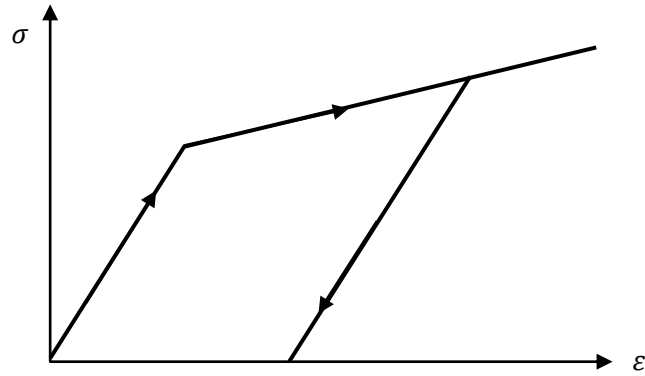


Figure 5-34: Assumed  $\sigma$ - $\epsilon$  relationship for tension (Tegrís)

The compression behaviour was idealised as elastic-perfectly plastic, as shown in Figure 5-35, with an initial modulus of 6 GPa and a yield strain of 0.5%.

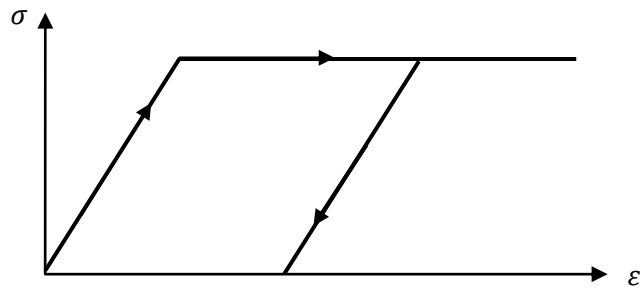


Figure 5-35: Assumed  $\sigma$ - $\epsilon$  relationship for compression (Tegrís)

The shear behaviour was idealised as initially linear elastic, with a modulus of 450 MPa, followed by a non-linear plastic zone, as shown in Figure 5-36. The non-linear relationship is described by an exponential function given in (5-3):

$$\tau = De^{E\gamma} \quad (5-3)$$

where  $D = 16.7$  MPa and  $E = 2.3$ .

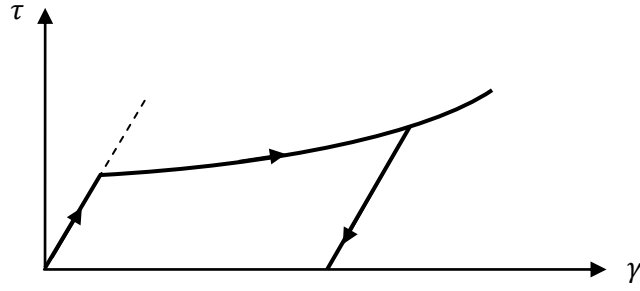


Figure 5-36: Assumed  $\tau$ - $\gamma$  relationship for shear (Tegris)

Strain-rate effects were also included by using a formulation proposed by Yen [284] and Daniel [285], where the associated static values are multiplied by a simple strain-rate dependent factor of the form:

$$1 + F_{ij} \ln \left( \frac{\dot{\epsilon}_{eff}}{\dot{\epsilon}_{0,ij}} \right) \quad (5-4)$$

where  $F_{ij}$  is a strain-rate constant (0.0608 for tension, 0.0478 for in-plane compression and 0.0456 for shear),  $\dot{\epsilon}_{0,ij}$  is the reference static strain-rate ( $\frac{1}{3000}$  /s for tension and  $\frac{1}{1500}$  /s for in-plane compression and shear) and  $\dot{\epsilon}_{eff}$  is defined as an effective strain-rate given by:

$$\dot{\epsilon}_{eff} = \sqrt{\dot{\epsilon}_{ij} \dot{\epsilon}_{ij}} \quad (5-5)$$

It should be noted that the above values were obtained from regression but by considering only the static ( $\approx 10^{-4}$  /s) and medium ( $\approx 10^{-2}$  /s) rate data only. Further testing at higher loading rates would confirm (or otherwise) the proposed relationships.

## 5.7 Numerical modelling and results

Numerical analysis was carried out using the commercial finite element analysis package ABAQUS/Explicit v.6.9-1 [252].

### 5.7.1 Loading profile

Using the loading function,  $P(r, t)$ , described in Section 3.4.1, to model the blast load, the loading parameters  $R_0$ ,  $P_0$ ,  $t_d$ ,  $b$  and  $w$  were obtained by considering charge details, i.e., the C4 charge mass at the stand-off distance of 50 mm. The loading distribution was implemented in a FORTRAN-coded user-defined loading subroutine (VDLOAD) in ABAQUS/Explicit and applied onto the test panel. The loading distributions are shown in Figure 5-37:

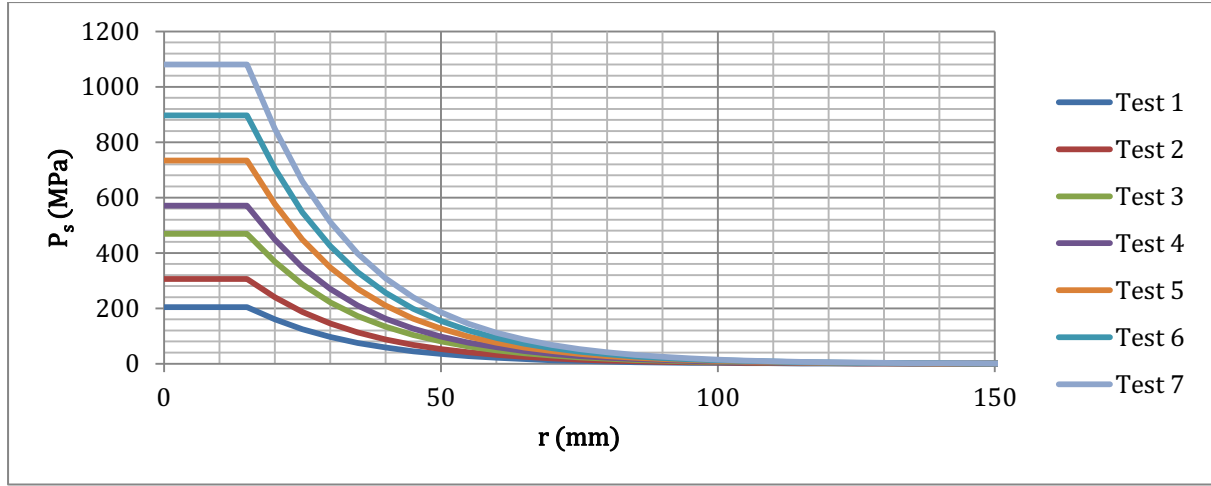


Figure 5-37: Loading spatial distributions for test scenarios

For the test configurations in the current study, the non-dimensional impulse,  $i$ , given in (3-25), is shown in graphical form in Figure 5-38. It can be seen that virtually all of the charge's impulse potential is imparted onto the test panel, since at  $\omega = 1$ ,  $i = 99.5\%$ . This implies that the current studies represent significantly aggressive blast threats which could demonstrate the suitability (or otherwise) of the proposed materials under study to resist blast loading.

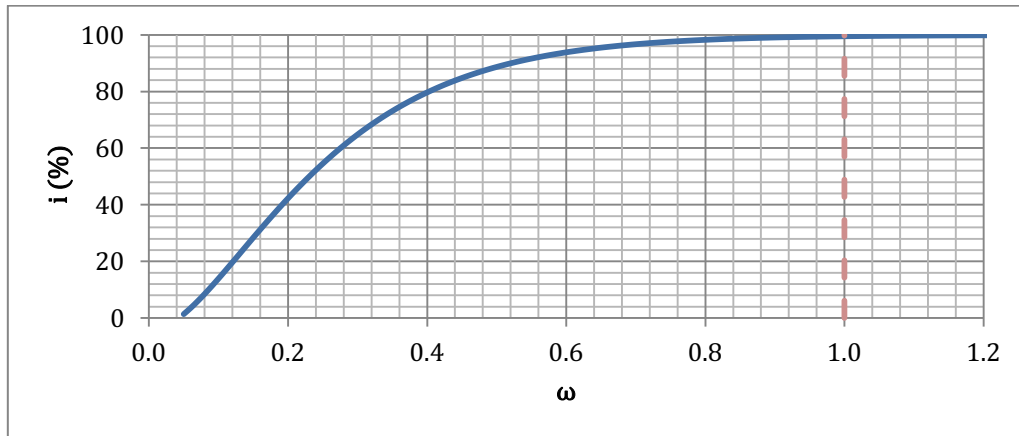


Figure 5-38: Dimensionless impulse variation with radial distance

### 5.7.2 Numerical modelling of Dyneema HB26 laminate

The material stress-strain relationships described in Section 5.5.3 were implemented through a FORTRAN-coded user-defined material subroutine (VUMAT) in ABAQUS/Explicit, which is included in Appendix B. In this work, the constitutive model used is a simplified physically-based model in which the stresses are computed using a series of independent stress-strain relationships for the normal and shear stress components, taking into account loading, unloading and reloading phenomena. This is an approximation to the actual (coupled) behaviour and is intended to provide a preliminary assessment of the response of Dyneema panels subjected to blast loading. The various responses and un/loading behaviours were verified by means of single element tests.

The uni-directional cross-ply laminate was modelled as a series of 1 mm thick plies using 8-noded linear brick elements with reduced integration and hourglass control (C3D8R) and using 0.05 mm cohesive layers in between the plies.

The cohesive layers were modelled using the built-in cohesive elements in ABAQUS/Explicit used to simulate the inter-ply failure. The traction separation approach was used to model the constitutive relationship of the cohesive material since the thickness of such layers is very small. The basis of the model relates the traction,  $T$ , with the displacement discontinuity (or separation),  $\delta$ , as per Figure 5-39 and the relationship is split into 2 parts. Initially, the relationship is linear-elastic until damage first occurs, controlled by a quadratic stress-based initiation criterion, given by:

$$\left(\frac{T}{T_0}\right)_n^2 + \left(\frac{T}{T_0}\right)_{s1}^2 + \left(\frac{T}{T_0}\right)_{s2}^2 = 1 \quad (5-6)$$

where  $n$  is the normal direction traction and  $s_1$  and  $s_2$  are the shear direction tractions.

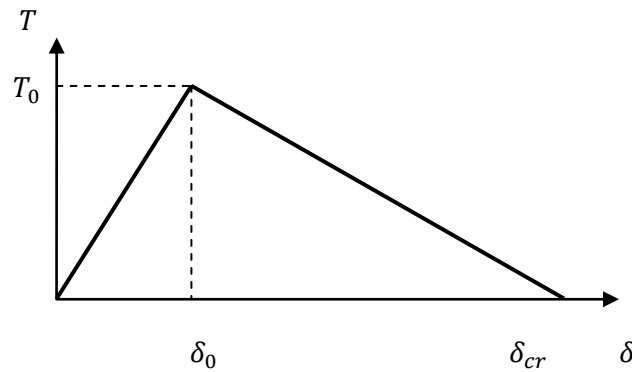


Figure 5-39: Bi-linear traction-separation relationship

Once the damage is initiated, the material is degraded according to a mixed-mode power law using an energy-based evolution approach, according to:

$$\left(\frac{G}{G_c}\right)_n^2 + \left(\frac{G}{G_c}\right)_{s1}^2 + \left(\frac{G}{G_c}\right)_{s2}^2 \quad (5-7)$$

where  $G$  is the work done by the tractions in the various directions and  $G_c$  is the critical fracture energy required to cause failure in each of the directions, which, with reference to Figure 5-39, is given by the total area under the curve, or:

$$G_c = \int_0^{\delta_{cr}} T d\delta \quad (5-8)$$

In this case, the cohesive layers were modelled using 8-noded three-dimensional cohesive elements (COH3D8) and the nominal failure stress values used for the normal and shear directions were 2 and 1.5 MPa respectively while the modal energy values used were 5.45 and 14.6 kJ/m<sup>2</sup> for mode I and mode II fractures respectively, in accordance with literature [270, 271].

### 5.7.3 Numerical results for Dyneema HB26

The models in ABAQUS/Explicit were meshed uniformly with an element size of 5 mm.

Typical outputs from ABAQUS/Explicit are shown in Figure 5-40 to Figure 5-42 and summarised in Figure 5-44.

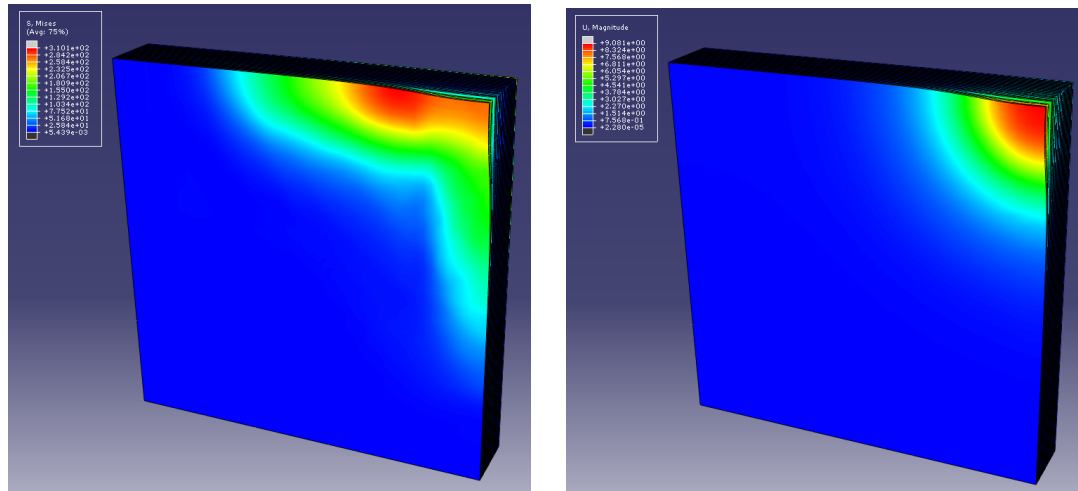


Figure 5-40: Dyneema quarter model (test 2) showing Mises stress (left) and displacement (right)

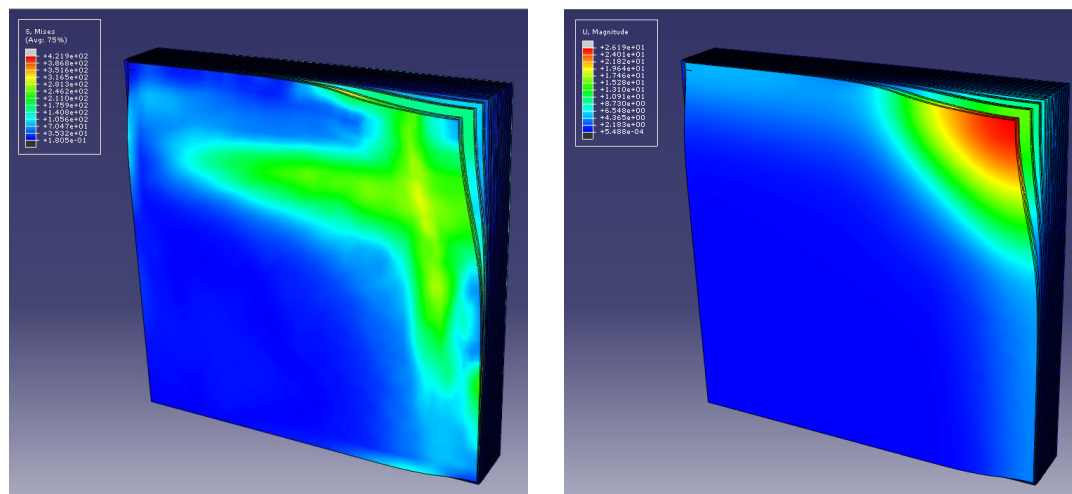


Figure 5-41: Dyneema quarter model (test 4) showing Mises stress (left) and displacement (right)

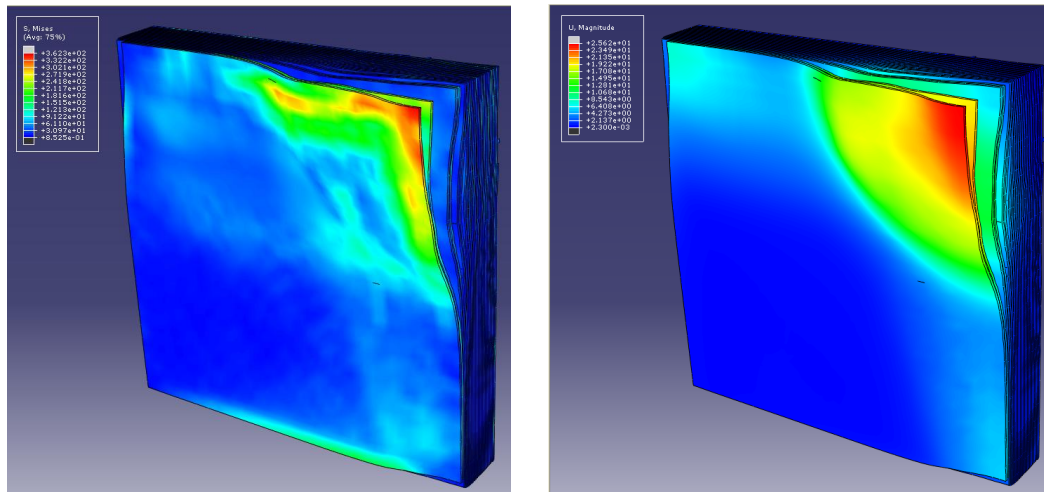


Figure 5-42: Dyneema quarter model (test 7) showing Mises stress (left) and displacement (right)

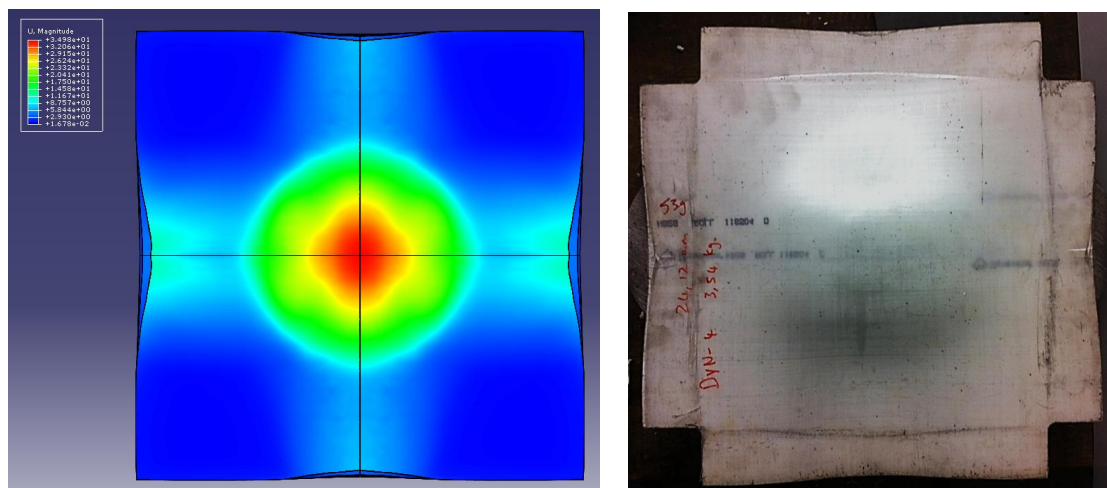


Figure 5-43: Dyneema panel (test 7) showing displacement numerical result (left) and test specimen (right)

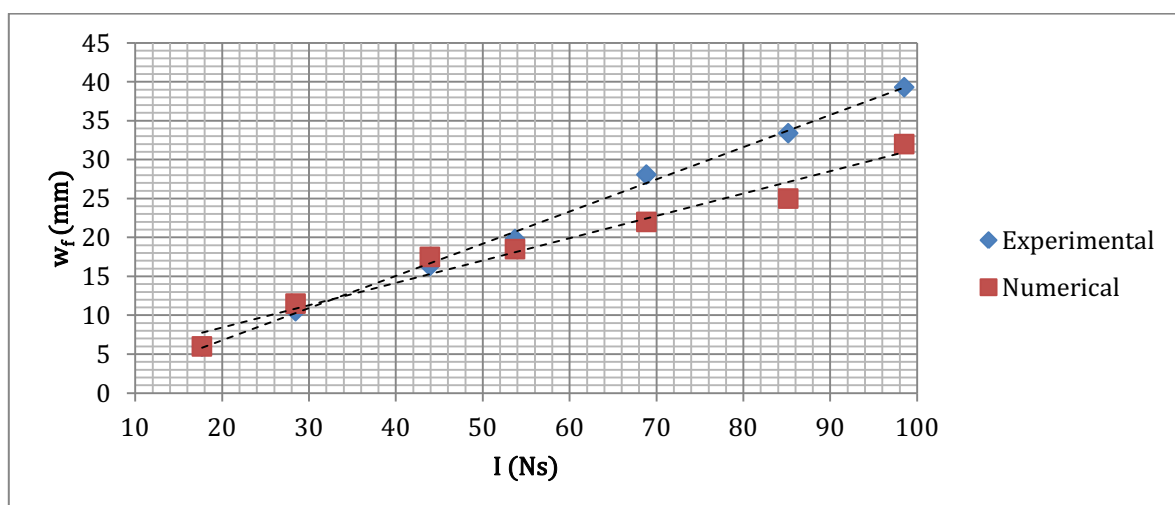


Figure 5-44: Summary of test and numerical results

The permanent transverse displacements from the experiments were compared with the numerical results and it was found that good correlation is achieved, with a mean difference of 12.5%. Deformation patterns from the numerical models were also compared with the tested panels and good corroboration was observed in terms of the final displaced shape and deformation mechanisms (e.g. pulling-in), as seen in Figure 5-43.

It was also observed that the larger discrepancies between the experimental and numerical results occurred for the higher charges (particularly tests 5 to 7). The discrepancy is attributed to a number of factors, as discussed hereunder.

As noted in Section 5.4.4, for the higher impulse ranges, drawing-in was observed and this implies that large shear deformation took place. Such deformation leads to fibre rotation and this is known to have an effect on the shear response [286]. Thus, a fibre angle re-orientation (also called scissoring) algorithm would need to be implemented to take this into account.

In addition, after the blast tests, it was observed that the target face suffered significant scarring and possible burning, as evident from Figure 5-4 to Figure 5-6. In fact, a pungent smell was reported<sup>12</sup> in the blast chamber subsequent to the blast tests. This suggests that thermoplastic damage was also incurred and the constitutive law should also possibly incorporate these effects.

Another source for the discrepancy could be in the interlaminar properties used in the numerical model. These were selected from literature and it is unknown whether these are adequate for this grade of Dyneema. Thus, interlaminar testing should be carried out to accurately assess the properties for the cohesive layers.

Finally, there is also the possibility of numerical error in the form of (finite element) mesh sensitivity. A mesh sensitivity analysis was carried out and the numerical results were found to converge reasonably well with the experimental results with a mesh size of 5 mm (as per Figure 5-45).

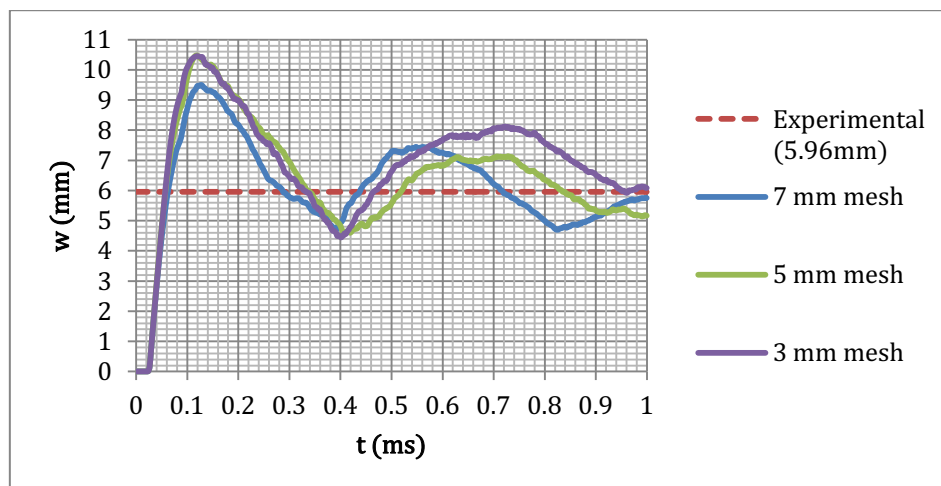


Figure 5-45: Displacement-time history (test 1) for various mesh sizes

<sup>12</sup> Personal communication by Dr. Luke. A. Louca of Imperial College London, August 2012.

However, with reduced integration elements (such as the ones used in the numerical analysis), the convergence from below typically associated with finite elements is not guaranteed. Thus, a stiffer response is not guaranteed and further mesh refinement might be necessary. However, a balance between accuracy and computational cost also needs to be met, as evident from Table 5-8, illustrating the mesh sensitivity analysis for test 1.

| Mesh size (mm) | Number of elements for quarter model | Maximum deflection (mm) | Permanent deflection (mm) | Run-time <sup>13</sup> for 1 ms simulation time (min) | File size (GB) |
|----------------|--------------------------------------|-------------------------|---------------------------|-------------------------------------------------------|----------------|
| 3              | 117500                               | 10.5                    | 7                         | 178                                                   | > 42           |
| 5              | 42300                                | 10.5                    | 6                         | 46                                                    | ≈ 11           |
| 7              | 20727                                | 9.5                     | 5.5                       | 22                                                    | ≈ 5            |

Table 5-5: Summary of mesh sensitivity analysis

## 5.8 Comparison of composites with mild steel and armour steel

For the purpose of comparison, the response of mild and (Armox 370T Class 1) armour steel plates subjected to the same loading scenarios described in Table 5-3 were studied. The studied metallic panels were of equal areal density as the 24 mm thick Dyneema HB26, i.e., 3.1 mm thick.

The response of the plates was done by means of two approaches, viz. by using a finite element analysis package and by utilising the analytical expressions developed in Chapter 4. Although the latter were developed for blast-loaded circular plates, their validity (or otherwise) for application in the case of square plates will be assessed. This will be verified by comparison with the numerical results.

### 5.8.1 Numerical analysis

The plates were modelled in ABAQUS/Explicit using 8-noded linear brick elements with reduced integration and hourglass control (C3D8R).

In the case of mild steel, an elasto-plastic model was used, with the initial elastic modulus being 200 GPa and a Poisson's ratio of 0.33, with the hardening behaviour defined using the Johnson-Cook hardening model, using data from literature [249] with the following material parameters:  $A = 217$  MPa,  $B = 234$  MPa,  $C = 0.0756$  and  $n = 0.6428$ .

For the armour steel, the same material data described in Section 3.8.2 was used.

The results are summarised in Figure 5-46 and typical outputs from ABAQUS/Explicit are shown in Figure 5-47 to Figure 5-49.

<sup>13</sup> Desktop machine with 4 CPUs at 2.67 GHz and 4 GB of RAM.

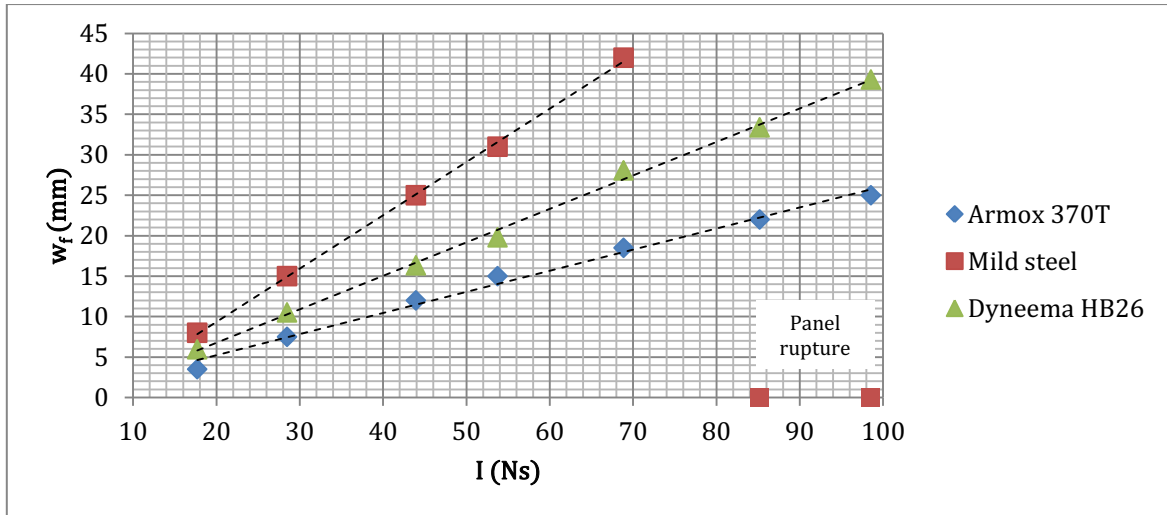


Figure 5-46: Variation of permanent displacement with impulse for all materials

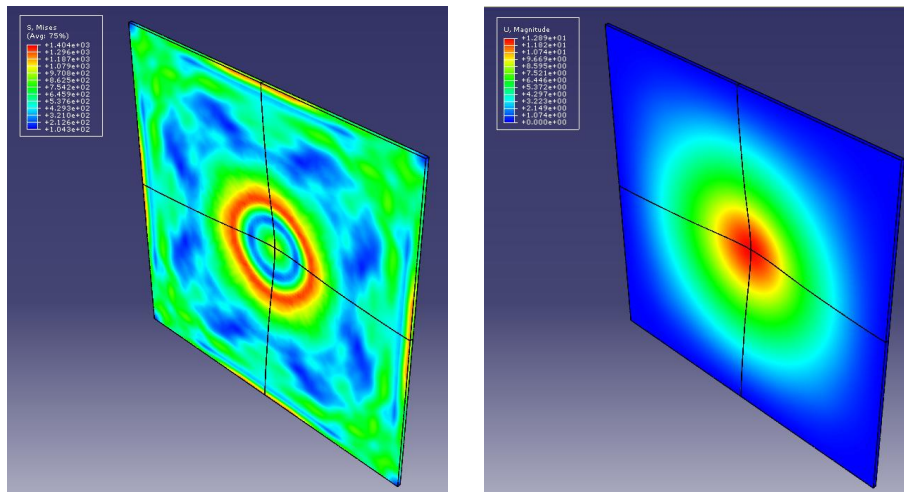


Figure 5-47: Armour steel model (test 4) showing Mises stress (left) and displacement (right)

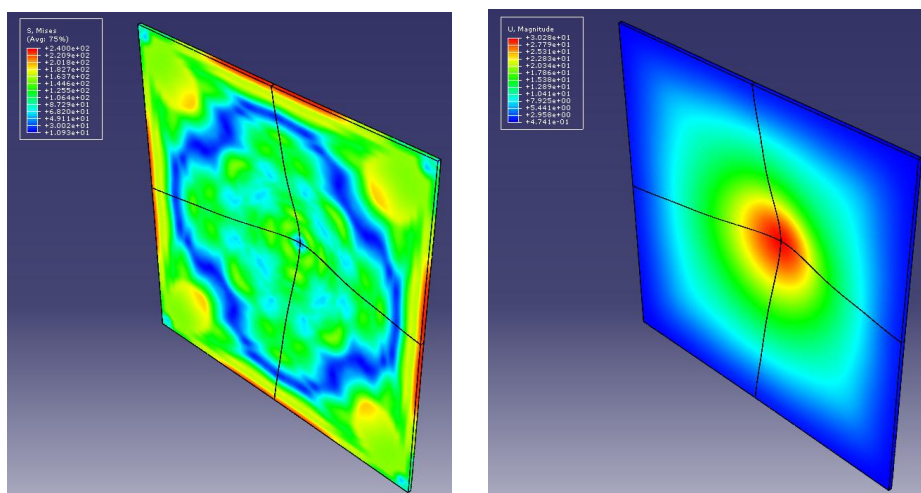


Figure 5-48: Mild steel model (test 4) showing Mises stress (left) and displacement (right)

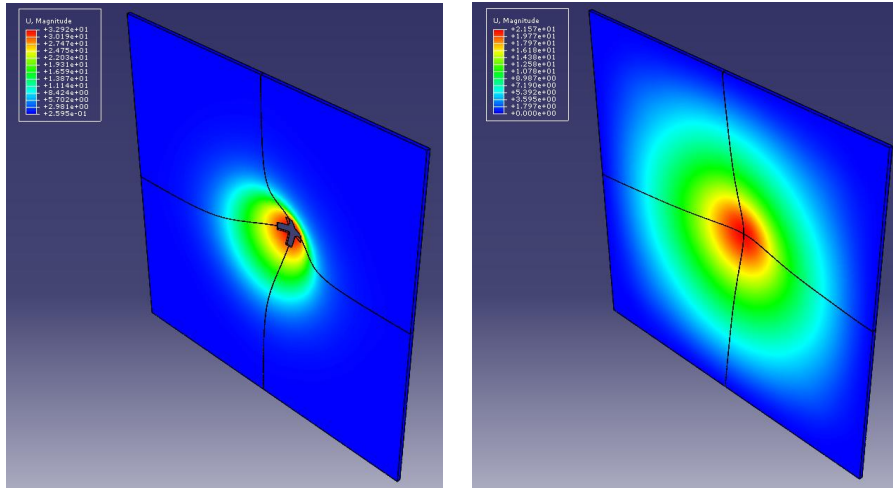


Figure 5-49: Models (test 7) showing ruptured mild steel panel (left) and intact armour steel panel (right)

From Figure 5-46, it can be seen that all materials exhibit a linear increase in permanent transverse displacement with increasing impulse, with mild steel predicting to have the largest plastic deformation, armour steel the least and Dyneema fairly in between. It is also predicted that rupture of the mild steel would occur when subjected to the largest 2 charges for the given stand-off distance.

### 5.8.2 Analytical analysis

The responses of the armour and mild steel plates were assessed by means of the analytical expressions developed in Chapter 4. It is postulated that the edge conditions (i.e., the geometry) do not have a significant effect on the permanent midpoint displacement and this conjecture is verified in this section.

In the first instance, the ‘membrane only’ expression (i.e. equation 4-136) is used to predict the permanent midpoint displacement for the armour steel plates and the results are summarised in Table 5-6:

| Test | $\lambda$ | $w_f$ (mm) |            |
|------|-----------|------------|------------|
|      |           | Numerical  | Analytical |
| 1    | 10        | 3.5        | 5.0        |
| 2    | 25        | 7.5        | 8.1        |
| 3    | 61        | 12         | 12.5       |
| 4    | 91        | 15         | 15.3       |
| 5    | 149       | 18.5       | 19.6       |
| 6    | 228       | 22         | 24.3       |
| 7    | 304       | 25         | 28.1       |

Table 5-6: Comparison of numerical and analytical results for armour steel plates

It can be observed that the analytical results provide a very good estimate of the displacement, with a mean difference of only 12.5% when compared with the numerical results.

However, from observations made in Section 4.8.4 and Figure 4-51, for the case of  $\lambda = 10$ , bending effects are expected to have a significant role in the plate's response and thus the displacement in this case is evaluated by means of the 'combined bending and membrane' expression (i.e. equation 4-124), giving  $w_f = 3.7\text{mm}$ , leading to an average difference of 7.2%.

This procedure was also used to estimate the permanent displacement for the case of the mild steel plates, with the results summarised in Table 5-7:

| Test | $\lambda$ | $w_f$ (mm) |            |
|------|-----------|------------|------------|
|      |           | Numerical  | Analytical |
| 1    | 46        | 8          | 10.0       |
| 2    | 120       | 15         | 17.6       |
| 3    | 285       | 25         | 27.2       |
| 4    | 426       | 31         | 33.2       |
| 5    | 699       | 42         | 42.6       |

Table 5-7: Comparison of numerical and analytical results for mild steel plates

It can be observed that, even in this case, the analytical results provide a good estimate of the displacement, with a mean difference of 14.2% when compared with the numerical results.

The greater discrepancy in this case is probably due to the fact that whilst the numerical results are derived from a finite element model where strain-rate effects are considered, the analytical results are based on the static yield stress, leading to slightly more conservative results.

Thus, it has been demonstrated that the equations developed in Chapter 4 can be used to provide a relatively accurate (upper bound) estimate of the permanent displacement of a square plate subjected to blast loading.

## 5.9 Discussion of results

The performance of mild steel, armour steel and Dyneema HB26 will be compared in this section.

A plot of the reduction in permanent transverse displacement compared to that achieved with mild steel is shown in Figure 5-50.

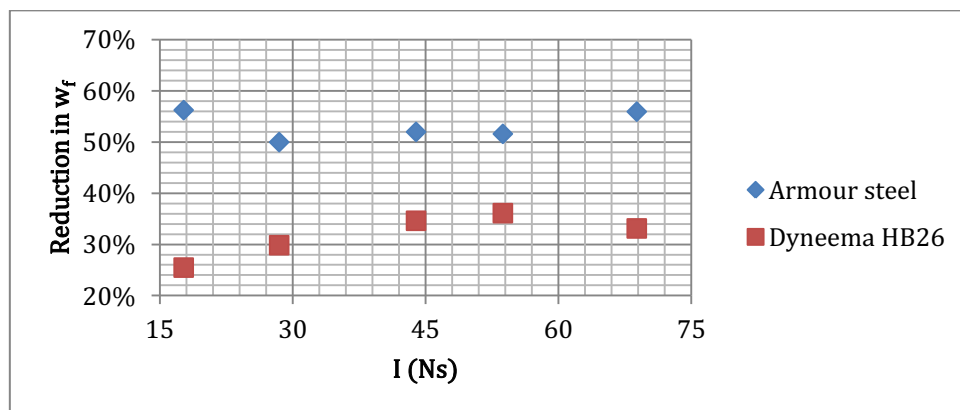


Figure 5-50: Reduction in permanent displacement

It can be observed that Dyneema HB26 shows an improvement in performance, with a mean improvement of 32% when compared to mild steel. For the same areal density, armour steel is the most efficient in reducing the permanent displacement for all cases, with a mean improvement of 53% and with the reduction increasing with applied impulse.

As evident from Figure 5-46, the results show no particular trend, except that each material follows an individual linear relationship between impulse and permanent displacement. An attempt will be made to unify the results for better performance comparison using dimensional analysis.

### 5.9.1 Empirical analysis

As discussed in Chapter 2, many researchers have proposed dimensionless numbers to describe the behaviour of metallic plates subjected to dynamic loads (e.g. [111, 116, 117, 254, 287]) but Nurick and co-workers extended these formulations to take into account both plate geometry, i.e., for both quadrangular [51] and circular plates [48] and loading conditions, particularly for localised blasts.

This has been used in Chapter 3 and given hereunder:

$$\phi_{q,d} = \frac{I}{2H^2 \sqrt{LB\rho\sigma_0}} \left( \frac{\xi_l}{\xi_d} \right) \quad (5-9)$$

where  $\xi_l$  and  $\xi_d$  are loading and stand-off distance parameters respectively, given by (3-27) and (3-28) respectively.

This damage parameter has been shown to be not only applicable to metals but also successfully used to compare metallic plates with FMLs [112, 113].

In the present study, the damage parameter in (5-9) was modified in order to compare the different materials under consideration, as discussed below.

Defining  $\mu$  as the areal density, then it can easily be shown that (5-9) can be rewritten as:

$$\phi_{q,sl} = \phi_{q,d} \xi_{sl} = \frac{I}{2H^{1.5} \sqrt{LB\mu\sigma_0}} \left( \frac{\xi_l}{\xi_d} \right) \xi_{sl} \quad (5-10)$$

where  $\xi_{sl}$  is a slenderness parameter given by:

$$\xi_{sl} = \left( \frac{H}{B} \right)^m \quad (5-11)$$

For the current problem, it was found that all results from the 3 materials under study collapse to a single linear relationship for non-dimensionalised displacement when  $m = 1.6$ , which was established by means of linear regression techniques.

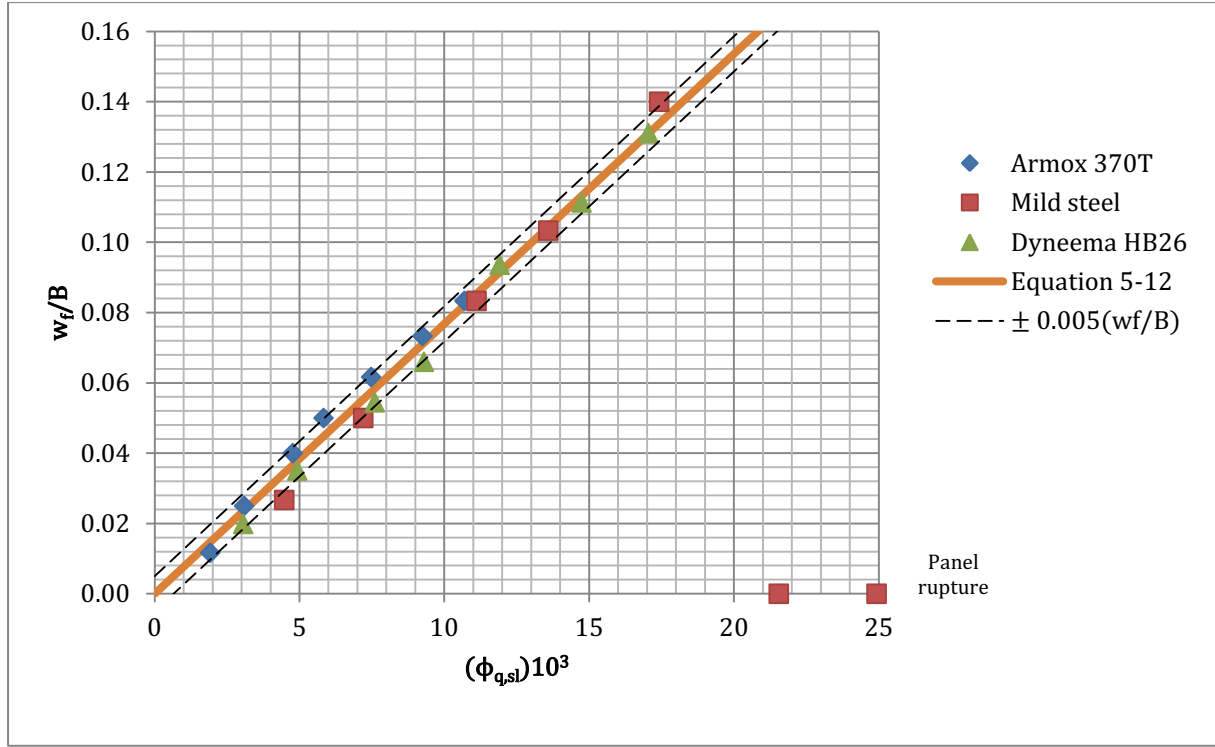


Figure 5-51: Variation of  $w_f$  with  $\phi_{q,sl}$

Figure 5-51 shows the results from the tests (for Dyneema HB26) and numerical models (for the 2 steels).

It can be observed that all results, independently of material, can be described by a single linear function, falling within a displacement/width ratio of 0.5% with a confidence level of  $R^2 = 0.989$ . Such a line was obtained by a least-squares fit analysis and is described by:

$$\left(\frac{w_f}{B}\right) = 7.68 \phi_{q,sl} \quad (5-12)$$

Such an equation provides a simple and useful guideline for predicting permanent transverse displacement at midpoint and also gives an indication of the impulse threshold at which tearing could occur for each material. This is further discussed in Section 5.9.3.

### 5.9.2 Theoretical analysis

It can easily be shown that the dimensionless damage parameter in (5-12) could be written in terms of the theoretical dimensionless impulse,  $\lambda$ , devised by Jones [92] as follows:

$$\left(\frac{w_f}{B}\right) = 3.84\sqrt{\lambda} \left(\left\{\frac{\xi_l}{\xi_s}\right\} \xi_{sl}\right) = 3.84\sqrt{\lambda_{sl}} \quad (5-13)$$

where  $\lambda$  is given in terms of an impulsive velocity  $V_0$  and the plate's static plastic collapse moment capacity  $M_0$  given by:

$$\lambda = \frac{\rho V_0^2 L B}{M_0} = \frac{4I^2}{\mu \sigma_0 L B H^3} \quad (5-14)$$

and  $\lambda_{sl}$  is a modified Jones' dimensionless impulse taking into account the effects of loading distribution, stand-off distance and plate slenderness, defined as:

$$\lambda_{sl} = \lambda \left( \left\{ \frac{\xi_l}{\xi_s} \right\} \xi_{sl} \right)^2 \quad (5-15)$$

The prediction of (5-13) is shown in graphical form in Figure 5-52 and compared with the test data from this study. It can be seen that there is excellent agreement even in this case.

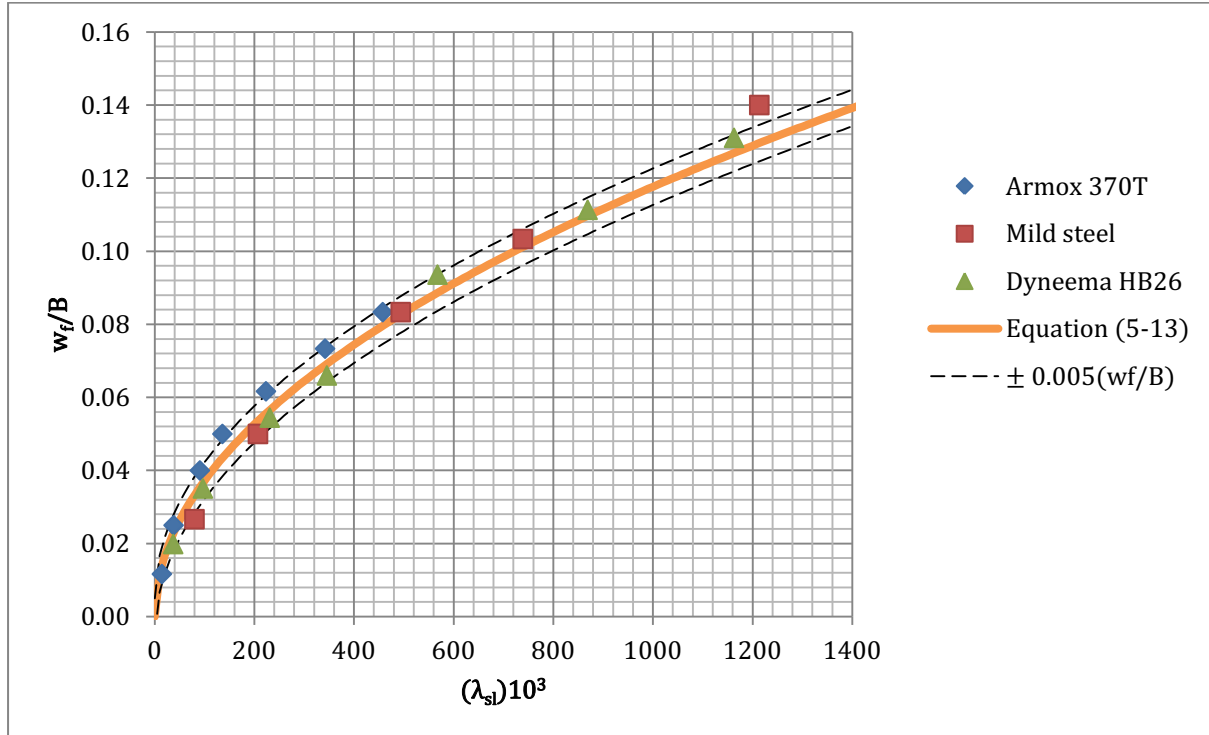


Figure 5-52: Variation of  $w_f$  with  $\lambda_{sl}$

Strain-rate effects can easily be introduced into (5-13) by replacing the static yield stress  $\sigma_0$  in (5-10) by a rate-sensitive equivalent, such as by using the Perrone-Bhadra simplification [262], but no tests at higher loading rates have been performed to date and thus cannot presently be included.

However, as discussed in Section 3.8.2, armour steel does not exhibit significant strain-rate effects and a preliminary estimate for the permanent displacement using the proposed approach would not be overly conservative.

On the other hand, it can also be observed that the results for Dyneema HB26 are in excellent agreement with the predictions of both (5-12) and (5-13), suggesting that there is little strain-rate influence in the material's dynamic performance under such loadings.

### 5.9.3 Example of a preliminary design application

The simple equation given in (5-12), used in conjunction with (3-27), (3-28), (5-10) and (5-11), can be used to give a preliminary thickness size for plated structures subjected to localised blast loads. An illustration of this is presented hereunder.

Consider a simply-supported square plate of dimensions 2x2 m which is subjected to a blast originating from a charge of diameter 0.5 m at a stand-off distance of 0.5 m and generating an impulse of 12300 Ns (approximately equivalent to 8 kg of TNT or 6.2 kg of C4 explosive using the relationship established from Figure 5-2). This would correspond to a Level 3B blast threat, in accordance with STANAG 4569 [288], which standardises the protection levels required for the floors of armoured vehicles.

Using the design equation derived in Section 5.9.1, it is possible to provide a preliminary plate thickness which would be required to restrict the permanent deformation to a specified limit. Assuming that it is required that the deflection does not exceed 100 mm, or  $\left(\frac{w_f}{B}\right) = 0.05$ , then from (5-12), this would correspond to a dimensionless damage value of  $\phi_{q,sl} = 0.0065$ .

Various materials could provide such a resistance and using the appropriate density and yield values, it can easily be shown that the structure can be one of the following options:

| Material   | Thickness (mm) | Areal mass (kg/m <sup>2</sup> ) |
|------------|----------------|---------------------------------|
| Armox 370T | 5              | 41                              |
| Mild steel | 29             | 230                             |
| Dyneema    | 124            | 121                             |
| Tegris     | 723            | 600                             |

Table 5-8: Summary of plate options

This clearly suggests that the most efficient solution in terms of weight would be the armour steel structure, with Tegris being the worst-performing material in terms of weight penalty.

Knowing the dimensionless impulse value corresponding to failure of a specific material (e.g.  $\phi_{q,sl} \approx 0.02$  for mild steel) would also exclude certain materials from being considered for use in a blast-resistant structure.

Consider now a 1x1 m plate whose permanent deflection tolerance is 150 mm with the blast threat kept as above. In this case,  $\phi_{q,sl}$  increases to 0.0202 and clearly this exceeds the rupture threshold for mild steel as established in Figure 5-51. Thus, the structure could only be made of armour steel, Dyneema or Tegris panels.

From the experimental data available, the rupture limit for Dyneema is not known, although it can readily be seen that this definitely exceeds 0.017 from the available data. Similarly, to date, no blast tests on Tegris panels have been carried out and thus the impulse limits for these composite systems would need to be established.

It should be noted that whilst this method is applicable for plates with all boundary conditions, the numerical values are only valid for simply supported plates.

From the blast tests described in Section 3.8.3, it can be established that, for the case of fully clamped plates, the tearing thresholds correspond to  $\phi_{q,sl} \approx 0.015$  for Armox 370T and  $\phi_{q,sl} \approx 0.028$  for mild steel, with (5-12) being replaced by:

$$\left(\frac{w_f}{B}\right) = 6.88 \phi_{q,sl} \quad (5-16)$$

### 5.10 Concluding remarks

In this chapter, the dynamic performance of high performance composite systems was investigated. Low and medium rate axial and shear testing of high performance composite material specimens was carried out and the material properties were extracted. From these, a constitutive model for Dyneema HB26 was proposed and implemented in ABAQUS/Explicit.

A separate set of experimental tests, namely blast tests, were carried out on Dyneema HB26 panels to cover a range of charge masses at a fixed stand-off distance to achieve scaled distances simulating various threat scenarios. Significant delamination was observed in most cases and damage to the back face was also noted but no rupture occurred, even for the larger charges.

The experimental results were compared to numerical results obtained from ABAQUS, using the above-mentioned constitutive model and a loading profile defined using the method proposed in Chapter 3. Good correlation was observed, although the discrepancy between the test and numerical results for the larger impulses is somewhat large and possible improvements to the constitutive model have been suggested.

The same blast loadings were numerically simulated but being applied on mild and armour (Armox 370T Class 1) steel panels of equal dimensions but increased and reduced thicknesses respectively, such that a constant areal density is maintained. The numerical results suggest that mild steel would rupture at the higher impulse regimes. Both armour steel and Dyneema HB26 offer an improved resistance to localised blast loading when compared to mild steel, with the former reducing the permanent displacement by over 50% and the latter by over 30%.

A new non-dimensionalised impulse parameter is proposed, incorporating earlier similar parameters found in literature (e.g. [48, 98]) but adding a term to account for plate slenderness. It was found that there is a good correlation between the responses of all 3 materials considered in this study and simple dimensionless expressions are proposed to predict permanent midpoint transverse displacement.

The same parameter is written also in terms of Jones' dimensionless impulse [92], which is modified accordingly to take into account the plate slenderness. The analytical solution also corresponds satisfactorily with the results.

A demonstration of the applicability of the dimensionless impulse equation proposed in this chapter is included, which shows how a preliminary sizing for the thickness of a plated structure subjected to blast could be approximated, given the threat from the blast, the plate geometry and the tolerable damage limits.

## Chapter 6. Early-time response of dynamically-loaded composites

### 6.1 Nomenclature

In this chapter, the following notation will be used:

#### *Latin lower case*

|           |                                                                   |
|-----------|-------------------------------------------------------------------|
| $c$       | elastic longitudinal wave velocity, [L T <sup>-1</sup> ]          |
| $f$       | frequency, [T <sup>-1</sup> ]                                     |
| $k$       | material constant related to damping, [T <sup>-1</sup> ]          |
| $m$       | mass, [M]                                                         |
| $t$       | time, [T]                                                         |
| $t_d$     | pulse load duration, [T]                                          |
| $u$       | displacement in the longitudinal axis direction, [L]              |
| $\dot{u}$ | velocity in the longitudinal axis direction, [L T <sup>-1</sup> ] |
| $u'$      | strain in the longitudinal axis direction, [1]                    |
| $x$       | longitudinal axis direction, [1]                                  |

#### *Latin upper case*

|            |                                                                                         |
|------------|-----------------------------------------------------------------------------------------|
| $A$        | cross-sectional area of bar, [L <sup>2</sup> ]                                          |
| $B$        | material constant relating modulus to strain-rate, [M L <sup>-1</sup> T <sup>-1</sup> ] |
| $D$        | damping co-efficient, [M T <sup>-1</sup> ]                                              |
| $E$        | Young's elastic modulus, [M L <sup>-1</sup> T <sup>-2</sup> ]                           |
| $E_0$      | static Young's elastic modulus, [M L <sup>-1</sup> T <sup>-2</sup> ]                    |
| $F(x)$     | pulse load spatial part, [M L <sup>-1</sup> T <sup>-2</sup> ]                           |
| $G(t)$     | pulse load temporal part, [M L <sup>-1</sup> T <sup>-2</sup> ]                          |
| $L$        | length of bar, [L]                                                                      |
| $O$        | differential operator, [1]                                                              |
| $P_0$      | initial load, [M L T <sup>-2</sup> ]                                                    |
| $T$        | dimensionless time, [1]                                                                 |
| $T_{crit}$ | critical time step, [T]                                                                 |
| $T_{min}$  | minimum time step for stability, [T]                                                    |

#### *Greek lower case*

|                     |                                                               |
|---------------------|---------------------------------------------------------------|
| $\alpha^2(\nu)$     | function of Poisson's ratio, [1]                              |
| $\beta$             | material density to shear modulus ratio, [L <sup>-1</sup> T]  |
| $\delta(x)$         | Dirac delta function, [1]                                     |
| $\varepsilon$       | strain, [1]                                                   |
| $\dot{\varepsilon}$ | strain-rate, [T <sup>-1</sup> ]                               |
| $\mu$               | Lamé's first parameter, [M L <sup>-1</sup> T <sup>-2</sup> ]  |
| $\zeta$             | damping ratio, [1]                                            |
| $\lambda$           | Lamé's second parameter, [M L <sup>-1</sup> T <sup>-2</sup> ] |
| $\nu$               | Poisson's ratio, [1]                                          |
| $\rho$              | material density, [M L <sup>-3</sup> ]                        |

|             |                                              |
|-------------|----------------------------------------------|
| $\sigma$    | stress, [M L <sup>-1</sup> T <sup>-2</sup> ] |
| $\varphi$   | displacement shape function, [1]             |
| $\varphi'$  | strain shape function, [1]                   |
| $\varphi''$ | curvature shape function, [1]                |

*Greek upper case*

|          |                           |
|----------|---------------------------|
| E        | dimensionless strain, [1] |
| $\Sigma$ | dimensionless stress, [1] |

## 6.2 Background

As discussed in previous chapters, dynamic loads on a structure, such as blast, are typically of a very short (or zero) rise time and of short duration, implying that the load attains a high amplitude at  $t = 0$ . This leads to the importance of the early-time response of the structure when subjected to such dynamic loads. In this chapter, particular focus is put on such response in laminated composite materials.

### 6.2.1 Delamination in composites

The experimental and numerical results described in Chapter 5 suggest that, as anticipated, delamination is an important damage mode in laminated composites. From the numerical simulations, it was also observed that such failure typically occurs very early during the response to the applied blast loads, particularly due to the fact that the applied loads are of a very short duration and thus the load reaches its maximum almost immediately, as well as the low interlaminar properties of UHMwPE (ultra high molecular weight polyethylene) composites, such as Dyneema.

The response of composites to dynamic loadings has been widely studied (e.g. [173, 198, 202, 203, 207, 209, 210, 289-292]) and delamination is typically numerically modelled either by using the VCCT (virtual crack closure technique) or more readily by means of cohesive elements, as discussed in Chapter 2. Indeed, the latter approach has been used in the finite element models described in Chapter 5. However, the use of cohesive elements presents numerical difficulties related to the definition of the properties of the cohesive layer itself and also the mesh sensitivity associated with such elements [195]. Measuring the interlaminar properties by means of the DCB (double cantilever beam) test for Mode I [293] and using a short-beam test for Mode II [294] can often be practically difficult due to slippage at the gripping jaws, particularly for materials such as Dyneema.

Govender et al. [292] have effectively characterised the interlaminar properties of fibre reinforced composites by means of a SHPB (split Hopkinson pressure bar) and assessed the response by means of a hyperbolic partial differential equation with one space variable and a constant co-efficient whose solution gives the undamped propagation of a wave through a linear elastic medium of density  $\rho$  and elastic modulus  $E$ , given by:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (6-1)$$

where the constant,  $c$ , is defined as the elastic longitudinal wave velocity and is given by:

$$c = \sqrt{\frac{E}{\rho}} \quad (6-2)$$

Thus, it is one of the main scopes of this chapter to be able to assess the incidence of delamination by comparison with through-thickness properties of a laminated composite by such an analytical method, rather than by using cohesive elements and interlaminar properties.

### 6.2.2 Effect of damping on composite performance

When a dynamic load is applied on a composite, energy is dissipated through various processes, such as visco-elastic behaviour of the matrix, damping at the fibre-matrix interface and damping due to damage [295]. Thus, damping has a significant role in the dissipation of energy.

The first works on the effect of damping on composites were reviewed by Gibson and Plunkett [296]. A number of researchers have carried out work on the effect of damping [297-308] but in most of the works, the investigation of damping is limited to inclusion of visco-elastic layers within laminates to enhance damping characteristics.

However, the effect of material damping has not been investigated, neither numerically nor experimentally. Indeed, in their review of damping studies related to composites, Chandra et al. [309] have noted that there is a need to study damping in terms of the effect of stress or amplitude of vibration.

Thus, it is also desirable to include the effects of damping in the analytic description of the wave equation given by (6-1).

### 6.3 Aim of current work

Thus, the objective of this chapter is to model the propagation of one-dimensional compression waves caused by short-duration pulse loads  $P(x, t)$  of various temporal shapes through elastic and visco-elastic media. The response of the material will be investigated through analysis of the early-time response in terms of displacement, strain and stress. Thus, a failure mechanism initiated through spalling will be described in terms of a maximum stress.

The response will be described mathematically by means of partial differential equations which describe the longitudinal body waves through a bar of material under consideration. In this study, the effect of surface waves will be ignored, assuming that the waves travel through a medium whose length is at least an order of magnitude greater than the cross-sectional dimension. Two different cases will be considered, namely:

Case 1 – Undamped vibrations

Case 2 – Vibrations including the effect of visco-elasticity (i.e. damping)

#### 6.4 Analytical model of spalling failure in the 3-D case without strain-rate effects

The phenomenon of propagation of elastic waves in 3-D linear-elastic isotropic strain-rate-independent media can be studied by considering the relevant governing differential equations.

Consider Navier's equation of elasticity:

$$\mu \nabla^2 \vec{u} + (\mu + \lambda) \bar{\nabla}(\bar{\nabla} \cdot \vec{u}) = \rho \vec{\ddot{u}} \quad (6-3)$$

where  $\mu$  and  $\lambda$  are Lamé's constants defined as:

$$\mu = \frac{E}{2(1 + \nu)} \quad (6-4a)$$

$$\lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \quad (6-4b)$$

Dividing the two constants, implies that the ratio is independent of modulus but only a function of Poisson's ratio, given by:

$$\alpha^2(\nu) = \frac{\lambda}{\mu} = \frac{\nu}{2(1 - 2\nu)} \quad (6-5)$$

Defining a parameter  $\beta^2$  as the ratio of density to shear modulus, as follows:

$$\beta^2 = \frac{\rho}{\mu} \quad (6-6)$$

Considering the identity  $\nabla^2 \vec{A} = \bar{\nabla}(\bar{\nabla} \cdot \vec{A}) - \bar{\nabla} \times (\bar{\nabla} \times \vec{A})$ , then Navier's equation can be written in a new form as:

$$\bar{\nabla}(\bar{\nabla} \cdot \vec{u}) - \bar{\nabla} \times (\bar{\nabla} \times \vec{u}) + \{1 + \alpha^2(\nu)\} \bar{\nabla}(\bar{\nabla} \cdot \vec{u}) - \beta^2 \vec{\ddot{u}} = \vec{0} \quad (6-7)$$

After expansion and simplification:

$$\{2 + \alpha^2(\nu)\} \bar{\nabla}(\bar{\nabla} \cdot \vec{u}) - \bar{\nabla} \times (\bar{\nabla} \times \vec{u}) - \beta^2 \vec{\ddot{u}} = \vec{0} \quad (6-8)$$

By applying Helmholtz Decomposition on the displacement field,  $\vec{u}$ , an irrotational vector field,  $\varphi$ , and a solenoidal vector field,  $\bar{\nabla} \times \vec{\psi}$ , are ensued. Thus,  $\vec{u}$  is considered to be linked to a scalar potential and a vector potential such that:

$$\vec{u} = \bar{\nabla}\varphi + \bar{\nabla} \times \vec{\psi} \quad (6-9)$$

This implies that the divergence of  $\vec{u}$  (i.e., dilatation), can be expressed as  $\bar{\nabla} \cdot \vec{u} = \nabla^2 \varphi$ . Similarly, the curl of  $\vec{u}$  can be expressed as  $\bar{\nabla} \times \vec{u} = \bar{\nabla} \times (\bar{\nabla} \times \vec{\psi}) = \bar{\nabla}(\bar{\nabla} \cdot \vec{\psi}) - \nabla^2 \vec{\psi}$ .

Substituting  $\vec{u}$ ,  $(\bar{\nabla} \cdot \vec{u})$  and  $(\bar{\nabla} \times \vec{u})$  into the new form of Navier's equation:

$$\{2 + \alpha^2(\nu)\} \bar{\nabla}(\nabla^2 \varphi) - \bar{\nabla} \times [\bar{\nabla}(\bar{\nabla} \cdot \vec{\psi}) - \nabla^2 \vec{\psi}] - \beta^2(\bar{\nabla}\ddot{\varphi} + \bar{\nabla} \times \ddot{\vec{\psi}}) = \vec{0} \quad (6-10)$$

It can also be shown that  $\bar{\nabla} \times [\bar{\nabla}(\bar{\nabla} \cdot \vec{\psi}) - \nabla^2 \vec{\psi}] = -\bar{\nabla}(\nabla^2 \vec{\psi})$  and thus the equation becomes:

$$\{2 + \alpha^2(\nu)\} \bar{\nabla}(\nabla^2 \varphi) + \bar{\nabla} \times (\nabla^2 \vec{\psi}) - \beta^2(\bar{\nabla} \ddot{\varphi} + \bar{\nabla} \times \ddot{\vec{\psi}}) = \vec{0} \quad (6-11)$$

Factorising  $\bar{\nabla}$  gives:

$$\bar{\nabla}[\{2 + \alpha^2(\nu)\}(\nabla^2 \varphi) + (\nabla^2 \vec{\psi})] - \beta^2(\bar{\nabla} \ddot{\varphi} + \bar{\nabla} \times \ddot{\vec{\psi}}) = \vec{0} \quad (6-12)$$

Collecting  $\varphi$  and  $\vec{\psi}$  terms separately yields two equations, namely:

$$\begin{cases} \bar{\nabla}[\{2 + \alpha^2(\nu)\} \nabla^2 \varphi - \beta^2 \ddot{\varphi}] = \vec{f}(x_i, t) \\ \bar{\nabla}(\nabla^2 \vec{\psi}) - \beta^2 (\bar{\nabla} \times \ddot{\vec{\psi}}) = -\vec{f}(x_i, t) \end{cases} \quad (6-13)$$

The solution of  $\varphi$  and  $\vec{\psi}$  from the two equations above will give the solution to  $\vec{u}$  and thus the solution to Navier's equation. Unfortunately, more often than not, solving this equation analytically is impossible unless simple geometry, boundary conditions and loading are concerned.

## 6.5 Analytical model of spalling failure in the 2-D case

In this chapter, the solution of the wave equation in two dimensions (one spatial variable and time) will be considered. The two cases described in Section 6.3 have closed form analytical solutions and these will be studied in this section.

### 6.5.1 Case 1 – Undamped vibrations

The first case is essentially described by the “classic” wave equation, shown in (6-1). In the problem being addressed, the equation is subject to two initial conditions (initial displacement and velocity) and two boundary conditions (strain values at bar ends), namely:

$$u_{t=0} = f_0(x) \quad (6-14a)$$

$$\dot{u}_{t=0} = f_1(x) \quad (6-14b)$$

$$u'_{x=0} = g_1(t) \quad (6-14c)$$

$$u'_{x=L} = g_2(t) \quad (6-14d)$$

The solution to this equation was obtained from Polyanin [310], where the solution is presented in terms of the Green's function,  $G(x, \xi, \tau)$ , as:

$$u(x, t) = \frac{\partial}{\partial t} \int_0^L f_0(\xi) G(x, \xi, t) d\xi + \int_0^L f_1(\xi) G(x, \xi, t) d\xi + c^2 \int_0^t g_1(\tau) \left[ \frac{\partial}{\partial \xi} \{G(x, \xi, t - \tau)\} \right]_{\xi=0} d\tau + c^2 \int_0^t g_2(\tau) G(x, L, t - \tau) d\tau \quad (6-15)$$

where

$$G(x, \xi, \tau) = \frac{2}{cL} \sum_{n=1}^{\infty} \frac{1}{\lambda_n} \sin(\lambda_n x) \sin(\lambda_n \xi) \sin(\lambda_n c \tau) \quad (6-16a)$$

$$\lambda_n = \frac{\pi(2n+1)}{2L} \quad (6-16b)$$

The results for displacement from this mixed boundary value problem was decomposed into two piecewise functions, namely  $0 \leq t \leq t_d$  and  $t > t_d$ .

For the duration of the loading, the initial and boundary conditions are:

$$f_0(x) = f_1(x) = g_2(t) = 0 \quad (6-17a)$$

$$g_1(t) = \frac{P(x, t)}{EA} \quad (6-17b)$$

From the solution for displacement, it was easy to obtain similar expressions for strain and velocity by differentiating with respect to  $x$  and  $t$  respectively. These were used to provide new initial and boundary conditions for the remainder of the time history:

$$g_1(t) = g_2(t) = 0 \quad (6-18a)$$

$$f_0(x) = u(x, t_d) \quad f_0(x) = u'(x, t_d) \quad f_1(x) = \dot{u}(x, t_d) \quad (6-18b)$$

### 6.5.2 Case 2 – Damped vibrations

The second case is a modified version of the first one, with the inclusion of a damping term described by a velocity-proportional resistance co-efficient given by:

$$\frac{\partial^2 u}{\partial t^2} + k \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (6-19)$$

where the constant,  $k$ , is the damping term related to the damping co-efficient,  $D$ , by:

$$k = \frac{D}{EA} \quad (6-20)$$

The solution to this equation is also available analytically and is obtained from Polyanin [310]:

$$u(x, t) = \frac{\partial}{\partial t} \int_0^L f_0(\xi) G(x, \xi, t) d\xi + \int_0^L [f_1(\xi) + k f_0(\xi)] G(x, \xi, t) d\xi - c^2 \int_0^t g_1(\tau) G(x, 0, t - \tau) d\tau + c^2 \int_0^t g_2(\tau) G(x, L, t - \tau) d\tau \quad (6-21)$$

where

$$G(x, \xi, \tau) = e^{-\frac{k\tau}{2}} \left[ \frac{\sinh(t\sqrt{p})}{L\sqrt{p}} + \frac{2}{L} \sum_{n=1}^{\infty} \frac{\cos(\mu_n x) \cos(\mu_n \xi) \sin(t\sqrt{c^2 \mu_n^2 - p})}{\sqrt{c^2 \mu_n^2 - p}} \right] \quad (6-22a)$$

$$\mu_n = \frac{\pi n}{L} \quad (6-22b)$$

$$p = \frac{k^2}{4} \quad (6-22c)$$

The above relation holds for  $p > 0$  and  $c^2\mu_n^2 - p < 0$ . Thus, it was deduced that, in this formulation,  $c^2\mu_n^2 - p = 0$  represents the critical damping. From this equation, the value of critical damping was obtained as:

$$p = \frac{k^2}{4} = c^2\mu_n^2 \quad (6-23)$$

Thus:

$$k = 2c\mu_n = \frac{2c\pi n}{L} \quad (6-24)$$

For  $n = 1$ , the critical damping is given by:

$$k_{crit} = \frac{2c\pi}{L} \quad (6-25)$$

For the purpose of this study, the response was investigated at damping levels corresponding to  $\zeta = 1\%$ ,  $2\%$  and  $5\%$ , where  $\zeta$  is the damping ratio of the level of damping provided in a system to the value of critical damping for the system.

A similar method as described in Section 6.5.1 was adopted, whereby the solution is obtained for the different boundary and initial conditions as a function of time.

### 6.5.3 Truncation and accuracy

In each case, it was found that the infinite series which describes the Green's function converges if it is truncated at a value of  $n = 15$  and a value of  $n = 20$  was chosen for improved accuracy.

The displacement  $u(x, t)$  was converted into a non-dimensional form by application of Buckingham's Pi Theorem [110]. The non-dimensional space and time quantities are as follows:

$$\bar{x} = \frac{x}{L} \quad (6-26)$$

and

$$\bar{t} = \frac{ct}{L} \quad (6-27)$$

Thus,  $u(x, t)$  is easily converted to  $\bar{u}(\bar{x}, \bar{t})$ . From this, all relevant quantities are expressed in non-dimensional form, e.g., strain as  $\bar{u}' = \frac{\partial u}{\partial x} = L \frac{\partial \bar{u}}{\partial \bar{x}}$ .

## 6.6 Finite element model of spalling failure

An alternative method to predicting the strain (and stress) response can be done by means of a finite element formulation to define the problem, which will be described in this section.

This includes the procedure followed for determination of sufficient degrees of freedom, the assembly of relevant matrices, the integration scheme adopted and stability thereof. An isoparametric consistent Lagrangian element based finite element is used to study the wave propagation through the material.

### 6.6.1 Lagrangian mesh description

In the model devised in this chapter, the bar is represented by a continuous system with  $N$  degrees of freedom, as shown in Figure 6-1:

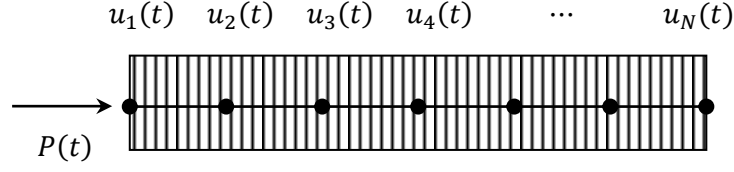


Figure 6-1: Idealisation of element into  $N$  degrees of freedom

The displacement of this system is obtained by employing the global assumed-modes method, whereby a shape function  $\varphi_i(x)$  will be used to represent a displacement shape for the whole system and a time-dependent function  $u_i(t)$  obtained from the numerical solution of the dynamic equation would give an approximation for the displacement as:

$$u(x, t) \approx \sum_{i=1}^N \varphi_i(x) u_i(t) \quad (6-28)$$

The functions  $\varphi_i(x)$  require being admissible functions, such that they form a linearly independent set and that each possesses continuity in its derivative. A set of polynomial functions was chosen having the form:

$$\varphi_i(x) = \frac{(x - x_1)(x - x_2) \cdots (x - x_{i-1})(x - x_{i+1})(x - x_N)}{(x_i - x_1)(x_i - x_2) \cdots (x_i - x_{i-1})(x_i - x_{i+1})(x_i - x_N)} \quad (6-29)$$

From these shape functions, the generalised mass, damping and stiffness co-efficients are derived using the following generalised equations from standard structural dynamics [311]:

$$m_{ij} = \rho A \int_0^L \varphi_i \varphi_j dx \quad (6-30a)$$

$$c_{ij} = D \int_0^L \varphi_i \varphi_j dx \quad (6-30b)$$

$$k_{ij} = EA \int_0^L \varphi'_i \varphi'_j dx \quad (6-30c)$$

For a chosen number of freedom degrees,  $N$ , the corresponding  $[\mathbf{M}]$ ,  $[\mathbf{C}]$  and  $[\mathbf{K}]$  matrices are assembled using the above formulae utilising the software Maple v.12 [312].

The value of  $N$  was found such that the natural frequencies of the system obtained from the eigenvalues resulting from the determinantal equation  $|[\mathbf{K}] - [\mathbf{M}]\omega^2| = 0$  are within the range of frequencies over which the load amplitude exceeds 95% of the maximum. It was assumed that by employing such a number of nodes, it would be sufficient to capture an accurate response.

### 6.6.2 Pulse loads in frequency domain

As described in Section 6.6.1, it was necessary to obtain the frequency content of the excitation in order to choose an adequate number of vibration modes.

For the purpose of this study, four pulse load shapes are being considered, of this form:

$$P(x, t) = F(x)G(t) \quad (6-31)$$

where  $G(t)$  is the temporal part and  $F(x)$  is the spatial part, given by the Dirac delta function  $\delta(x)$ .

The temporal distributions for the different pulse shapes are shown in Figure 6-2 and given by:

a) Rectangular pulse

$$G(t) = \begin{cases} P_0, & 0 \leq t \leq t_d \\ 0, & t > t_d \end{cases} \quad (6-32)$$

b) Half sine pulse

$$G(t) = \begin{cases} P_0 \sin\left(\frac{\pi t}{t_d}\right), & 0 \leq t \leq t_d \\ 0, & t > t_d \end{cases} \quad (6-33)$$

c) Linearly decaying pulse

$$G(t) = \begin{cases} P_0 \left(1 - \frac{t}{t_d}\right), & 0 \leq t \leq t_d \\ 0, & t > t_d \end{cases} \quad (6-34)$$

d) Quarter cosine pulse

$$G(t) = \begin{cases} P_0 \cos\left(\frac{\pi t}{2t_d}\right), & 0 \leq t \leq t_d \\ 0, & t > t_d \end{cases} \quad (6-35)$$

The loads were transformed into the frequency domain, by means of a Fourier Transform [313]:

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^{t_d} G(t) e^{-i\omega t} dt \quad (6-36)$$

For example, the rectangular pulse is expressed as:

$$G(\omega) = \frac{P_0 \sqrt{\sin^2(\omega t_d) + (\cos(\omega t_d) - 1)^2}}{\omega \sqrt{2\pi}} \quad (6-37)$$

This is shown in Figure 6-3. This enabled the determination of the relevant frequency content to be considered and thus the number of degrees of freedom which the model must include for accurate representation.

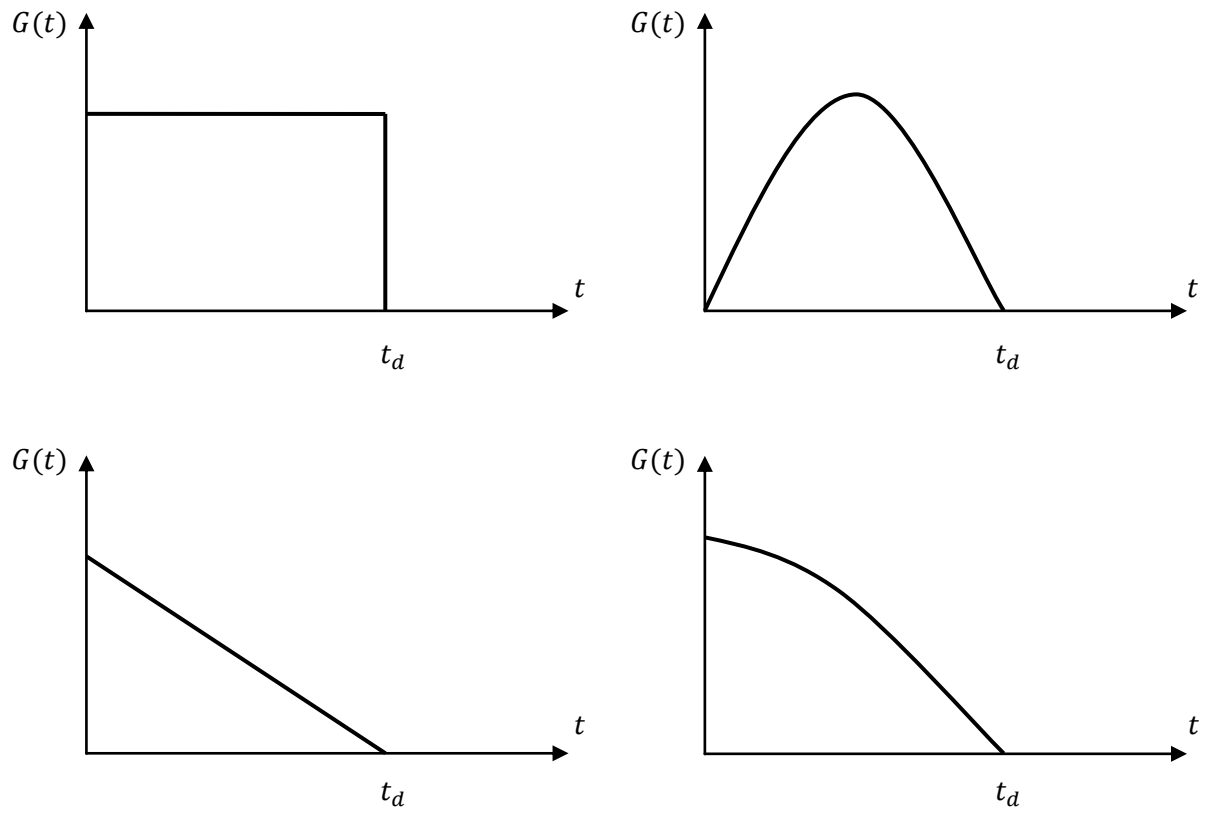


Figure 6-2: Pulse shapes considered

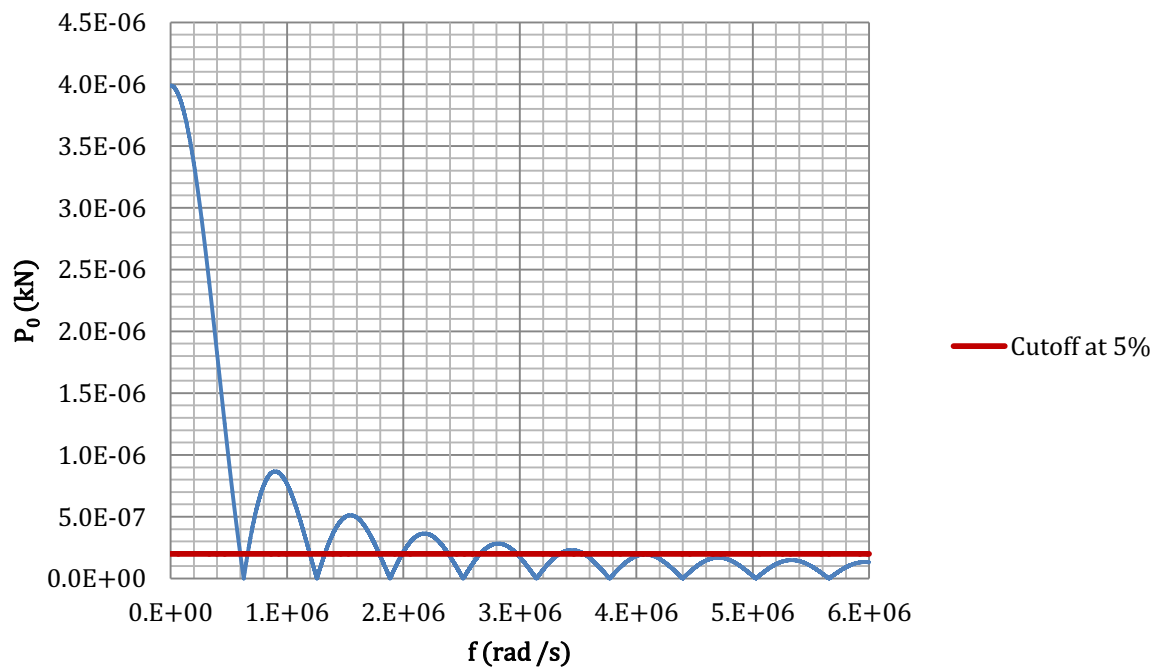


Figure 6-3: Fourier Transform for the rectangular pulse load

It was found that with  $N = 11$ , the model would capture the relevant range of frequencies for all four loads. From this solution, system natural modes were found and from the highest frequency, the lowest period was also computed using  $T_{min} = \frac{2\pi}{\omega_{max}}$  and this result was used to establish an adequate time step in the numerical time-integration scheme adopted in the solution of the dynamic equation describing the wave propagation.

### 6.6.3 Case 1 – Undamped vibrations

The equation of motion describing a pulse load  $P(x, t)$  acting on an infinitesimal element of size  $dx$ , shown in Figure 6-4, is considered.

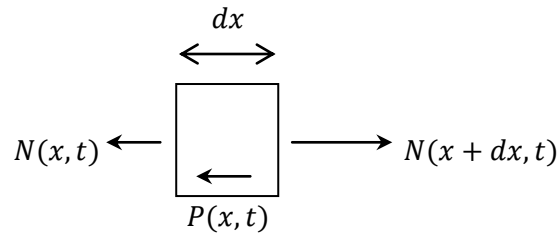


Figure 6-4: Free-body diagram of forces acting on element

For equilibrium, by Newton's second law of motion,

$$\sum_{\vec{F}} F_x = ma_x \quad (6-38)$$

$$N(x + dx, t) - N(x, t) + P(x, t)dx = (\rho A dx) \frac{\partial^2 u}{\partial t^2}$$

Considering that force is related to strain by  $N(x, t) = EA \frac{\partial u}{\partial x}$  and thus  $\frac{dN}{dx} = EA \frac{\partial^2 u}{\partial x^2}$  then:

$$EA \frac{\partial^2 u}{\partial x^2} dx + P(x, t)dx = \rho A \frac{\partial^2 u}{\partial t^2} dx \quad (6-39)$$

Substituting (6-2) and rearranging,

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = \frac{P(x, t)}{\rho A} \quad (6-40)$$

This corresponds to the wave equation described in Section 6.5.1, which can be expressed as a dynamic equation in matrix form as:

$$[\ddot{U}] + [M]^{-1}[K][U] = [M]^{-1}[P(t)][\delta(x)] \quad (6-41)$$

The solution for the displacement  $[U]$  was obtained using an explicit time-integration scheme, namely, the central difference method [314], implemented in a MATLAB [245] script. An extract of the script is given hereunder:

```

% velocity
qdot(:,i) = qdot(:,end) + (qdoubledot(:,end))*timestep;

% displacement
q(:,i) = q(:,end) + (qdot(:,end))*timestep + 0.5*qdoubledot(:,end))*((timestep)^2);

% acceleration
qdoubledot(:,i) = inv(M)*(P(:,i) - K*q(:,i) - C*qdot(:,i));

```

The relevant initial conditions at  $t = 0$  were inputted:

$$[U] = [\dot{U}] = [0] \quad (6-42a)$$

$$[\ddot{U}] = [U]^{-1}[P(t)][\delta(x)] \quad (6-42b)$$

Achievement of stability of the integration scheme was guaranteed by selecting a time step which is not larger than the critical time step,  $T_{crit}$  [314]:

$$T_{crit} = \frac{T_{min}}{\pi} \quad (6-43)$$

A time step was chosen such that it is less than 10% of the value of  $T_{min}$  in order to guarantee stability. Convergence of the solution was verified by comparing results from using different time steps.

From the solution for  $[U]$ , it was possible to compute the numerical values of displacement and strain using the relations:

$$u(x, t) \approx \sum_{i=1}^{11} \varphi_i(x) u_i(t) \quad (6-44)$$

$$u'(x, t) = \frac{\partial u}{\partial x} = \varepsilon(x, t) \approx \sum_{i=1}^{11} \varphi'_i(x) u_i(t) \quad (6-45)$$

In this case, the stress is readily obtained by the simple relation:

$$\sigma(x, t) = E \varepsilon(x, t) \quad (6-46)$$

#### 6.6.4 Case 2 – Damped vibrations

The second case considered corresponds to the equation presented in Section 6.6.3, with the wave equation expressed as:

$$[\ddot{U}] + [M]^{-1}[C][\dot{U}] + [M]^{-1}[K][U] = [M]^{-1}[P(t)][\delta(x)] \quad (6-47)$$

This was similarly solved using the same explicit time-integration scheme. The displacements, strains and stresses are obtained in the same way as described above.

## 6.7 Case study

In this section, the formulation is illustrated with a case study of a typical glass/vinylester composite. The results from both the analytical solution, where applicable, and the finite element model will be correlated and compared. The results are also compared with those obtained using a commercial finite element package.

### 6.7.1 Material and geometric properties and loading parameters

The material being considered is a uni-directional E-glass in a vinylester resin 1110 (Fibre Glast) cured with 1% by weight MEKP [315]. Typical densities for these materials are 2550 kg/m<sup>3</sup> for the E-glass and 1030 kg/m<sup>3</sup> for the matrix. The density of the composite material works out to be  $\rho = 1940$  kg/m<sup>3</sup> assuming that the composite has a 60% volume fraction.

Recently, Daniel et al. [315] carried characterisation of the material described above and a modulus of  $E = 11$  GPa was measured. Thus, the material has an elastic longitudinal wave velocity of  $c = 23.8$  mm/s.

A specimen length such that the wave generated by the load would have travelled along the entire length of the specimen and be reflected back during the duration of the load was chosen. Thus, a length of  $L = 40$  mm was chosen.

The area was chosen such that the length to diameter ratio is not less than 10 and assumed to be circular in cross-section, thus a diameter of 4 mm and an area of  $A = 12.6$  mm<sup>2</sup>.

The four pulse loads described in Section 6.6.2 were considered and, in all cases, values of  $P_0 = 1$  kN and  $t_d = 10$   $\mu$ s were adopted.

### 6.7.2 Case 1 results

Non-dimensionalised results from the analytical and numerical solutions of the undamped variations are presented hereunder. In this case, the stress-time variation follows the same profile as the strain-time variation since the elastic modulus is constant and thus the stress-time plots are not all presented. In this case, only the results for a node close to the middle of the element will be provided (node 4).

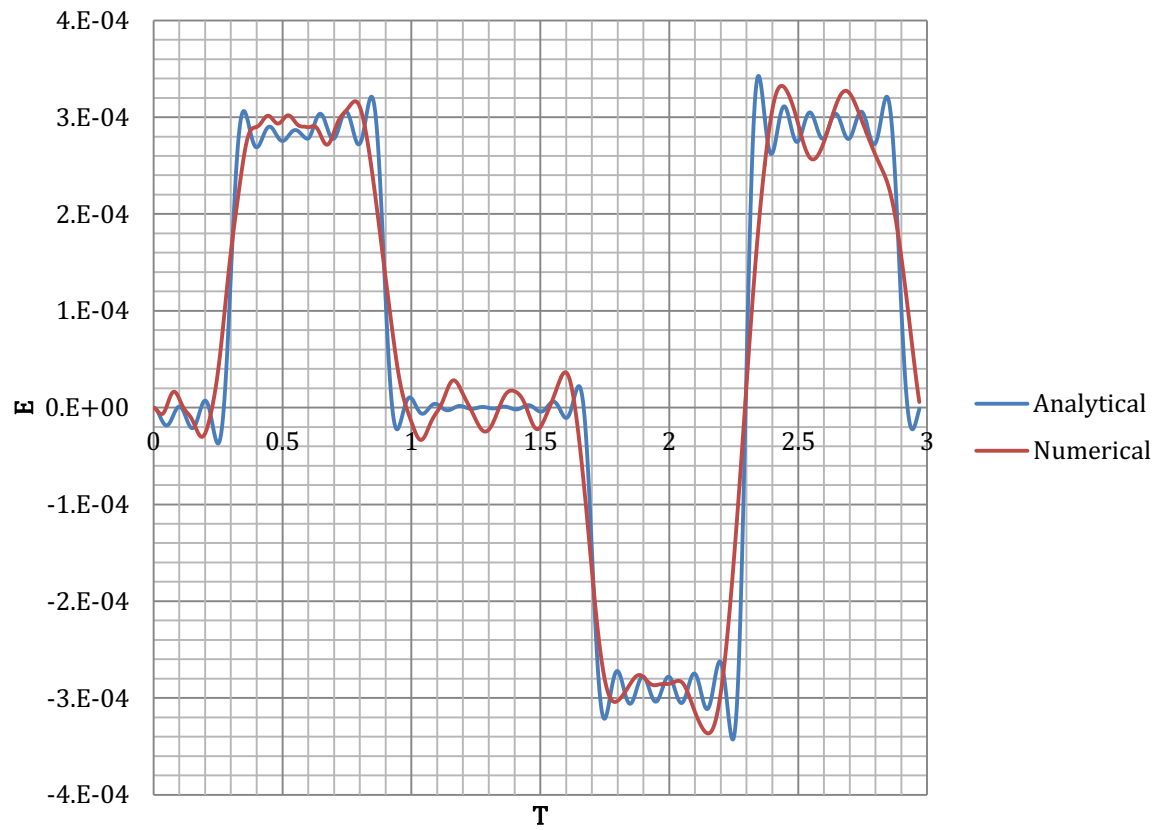


Figure 6-5: Dimensionless strain history for Case 1 (node 4, rectangular pulse load)

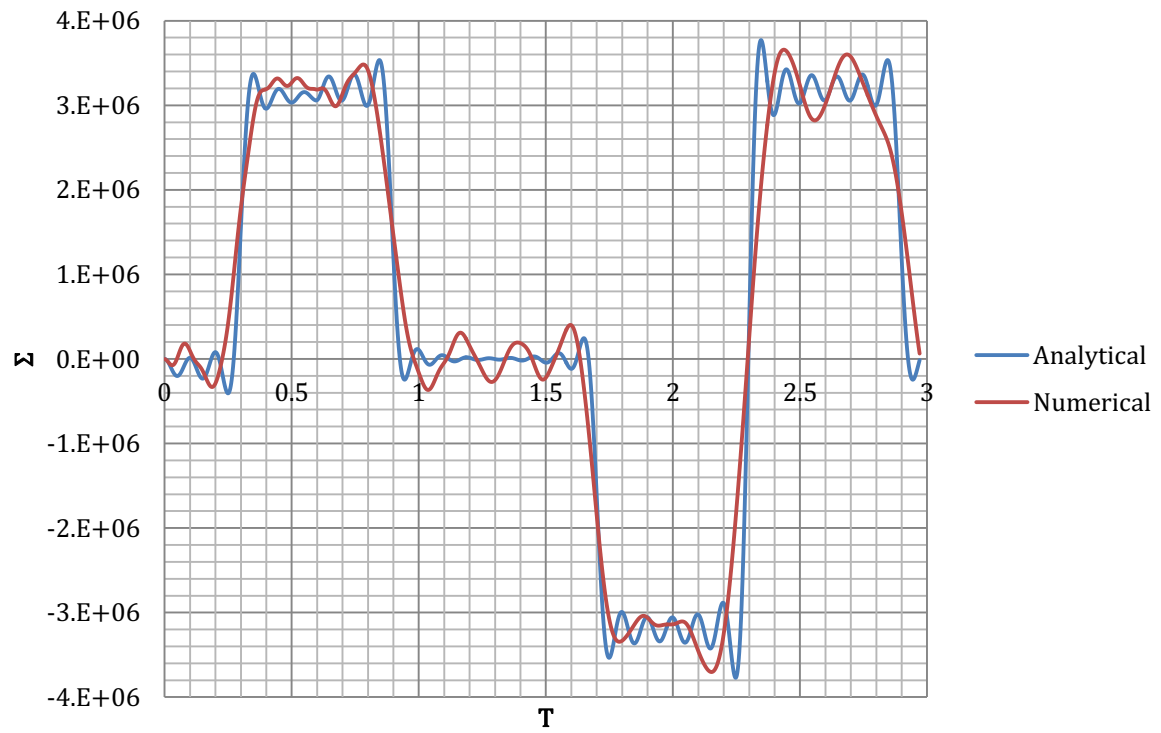


Figure 6-6: Dimensionless stress history for Case 1 (node 4, rectangular pulse load)

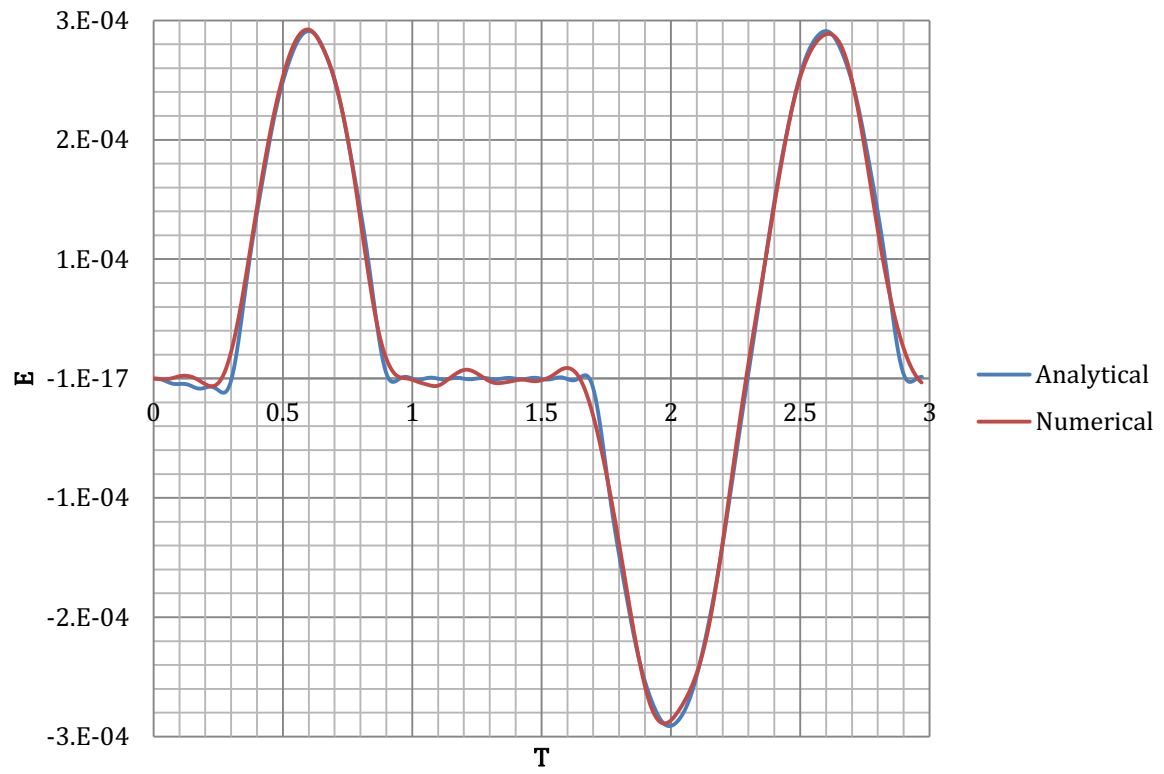


Figure 6-7: Dimensionless strain history for Case 1 (node 4, half sine pulse load)

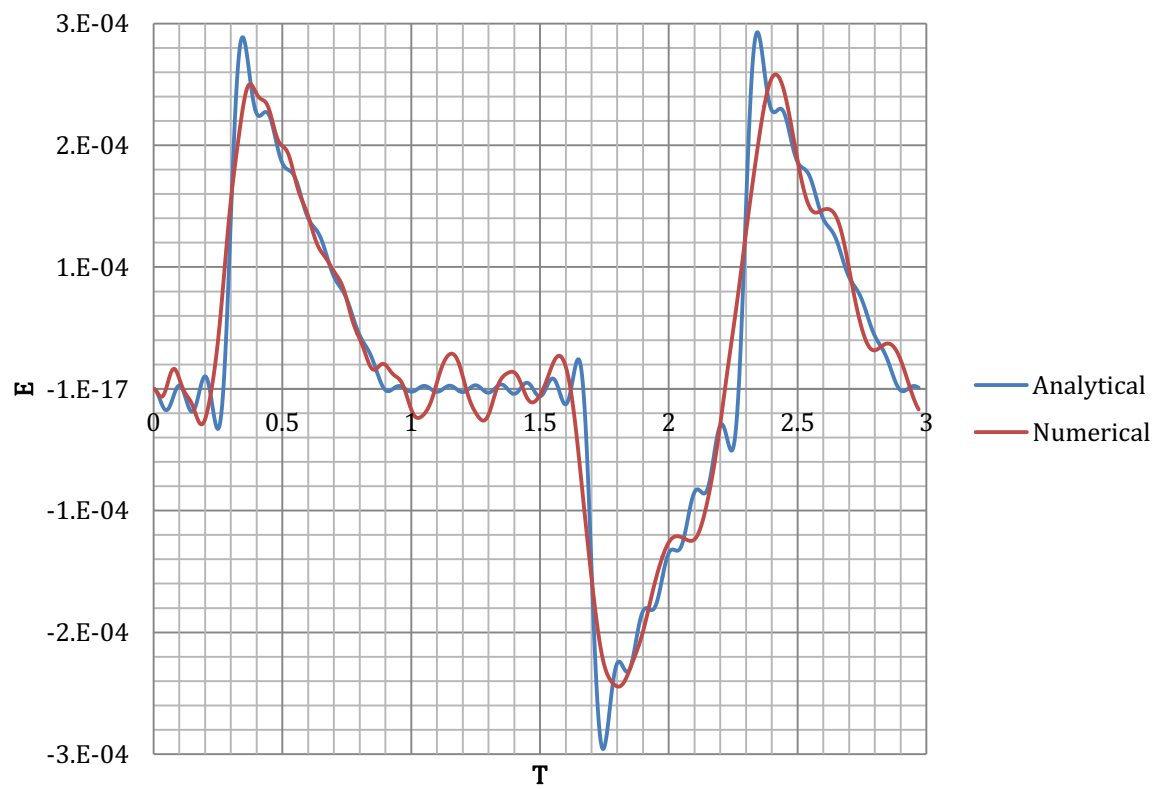


Figure 6-8: Dimensionless strain history for Case 1 (node 4, linearly decaying pulse load)

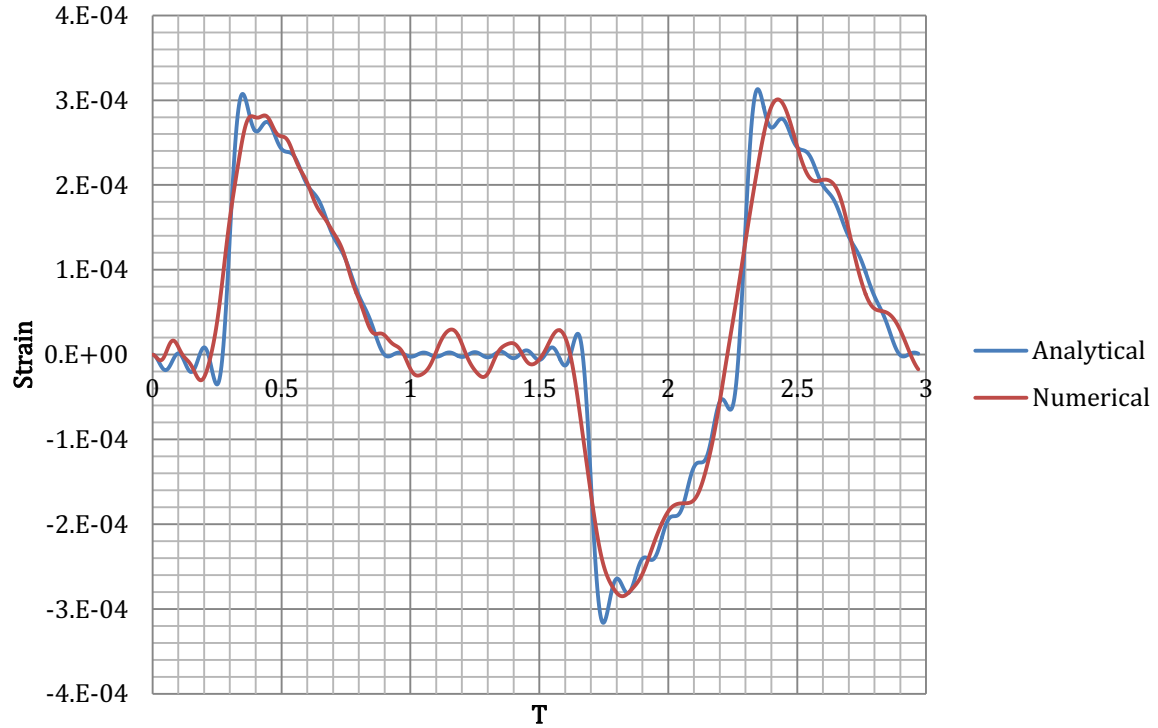


Figure 6-9: Dimensionless strain history for Case 1 (node 4, quarter cosine pulse load)

### 6.7.3 Case 2 results

The results for Case 2 are presented hereunder. As for Case 1, the results for stress are identical to those for strain but multiplied by the constant value of the elastic modulus and thus for brevity, only the strain-time plots are given. The stress profile is presented only once for illustration.

For the purpose of brevity, throughout the remainder of the results, only plots for node 4 and under the rectangular pulse shape will be presented. In addition, only the effect of  $\zeta = 5\%$  will be shown for all pulse load shapes since the effects of  $\zeta = 2\%$  and  $\zeta = 1\%$  was found to be less pronounced.

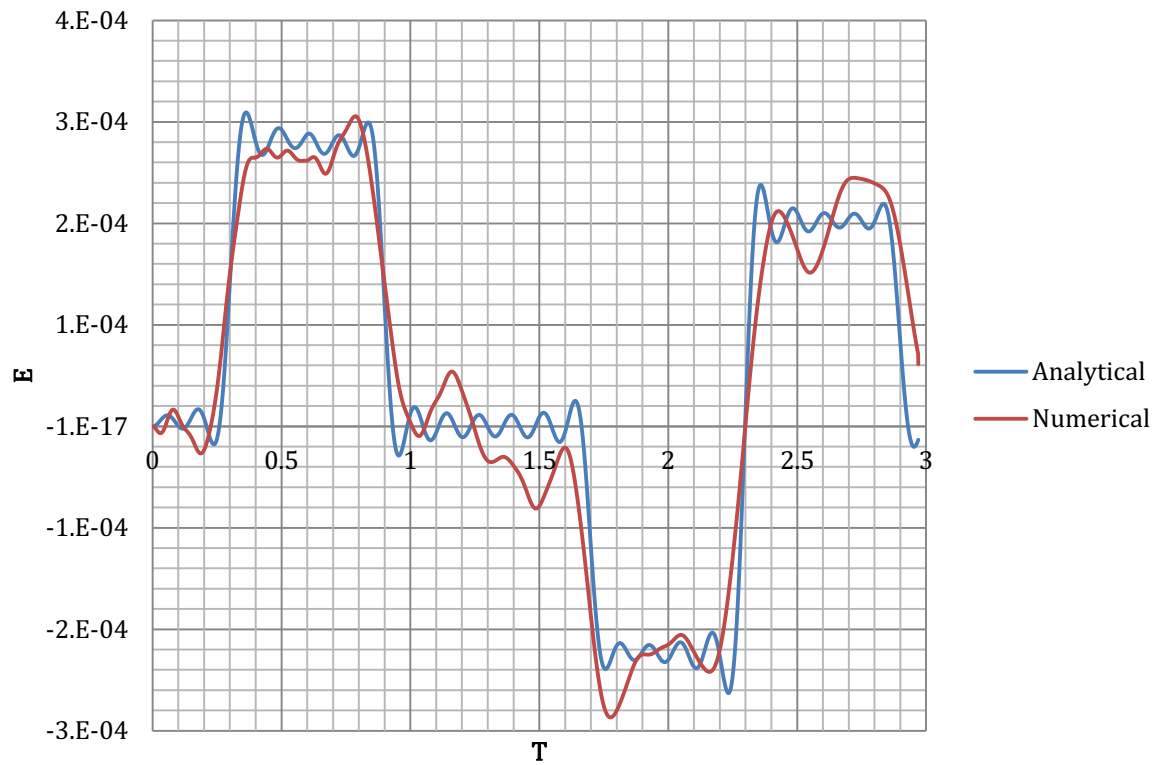


Figure 6-10: Dimensionless strain history for Case 2 with  $\zeta = 5\%$  (node 4, rectangular pulse)

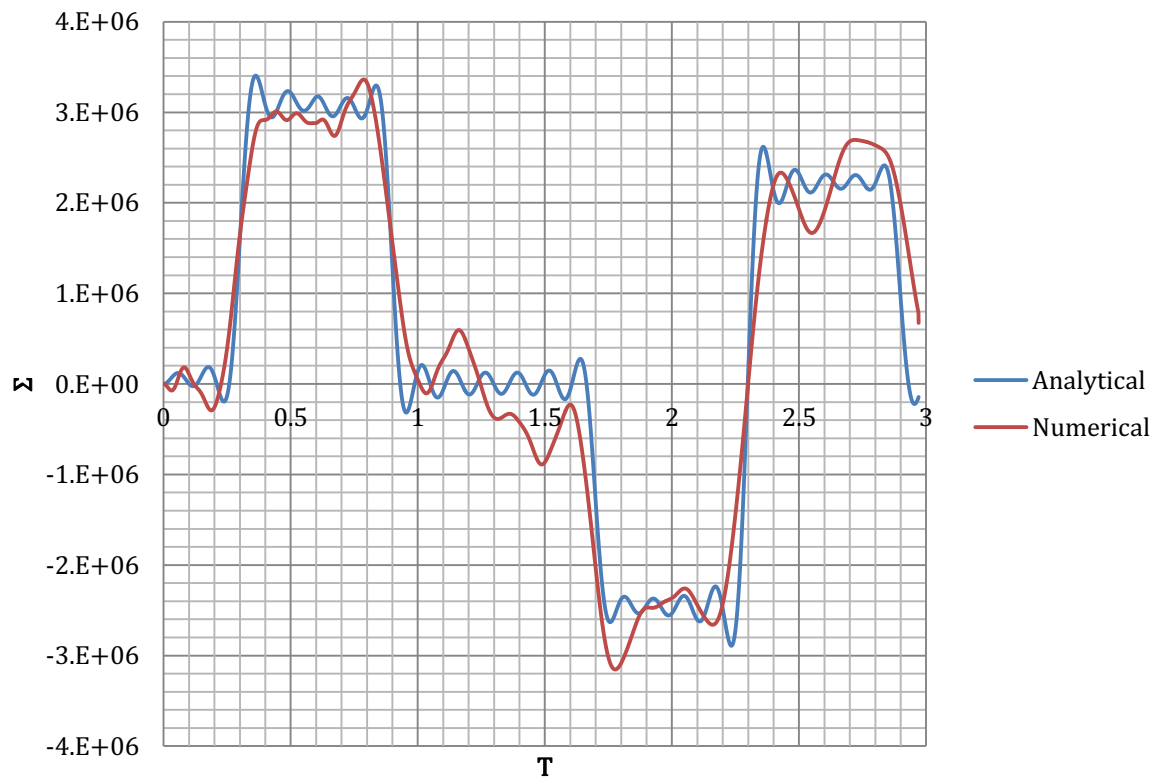


Figure 6-11: Dimensionless stress history for Case 2 with  $\zeta = 5\%$  (node 4, rectangular pulse)

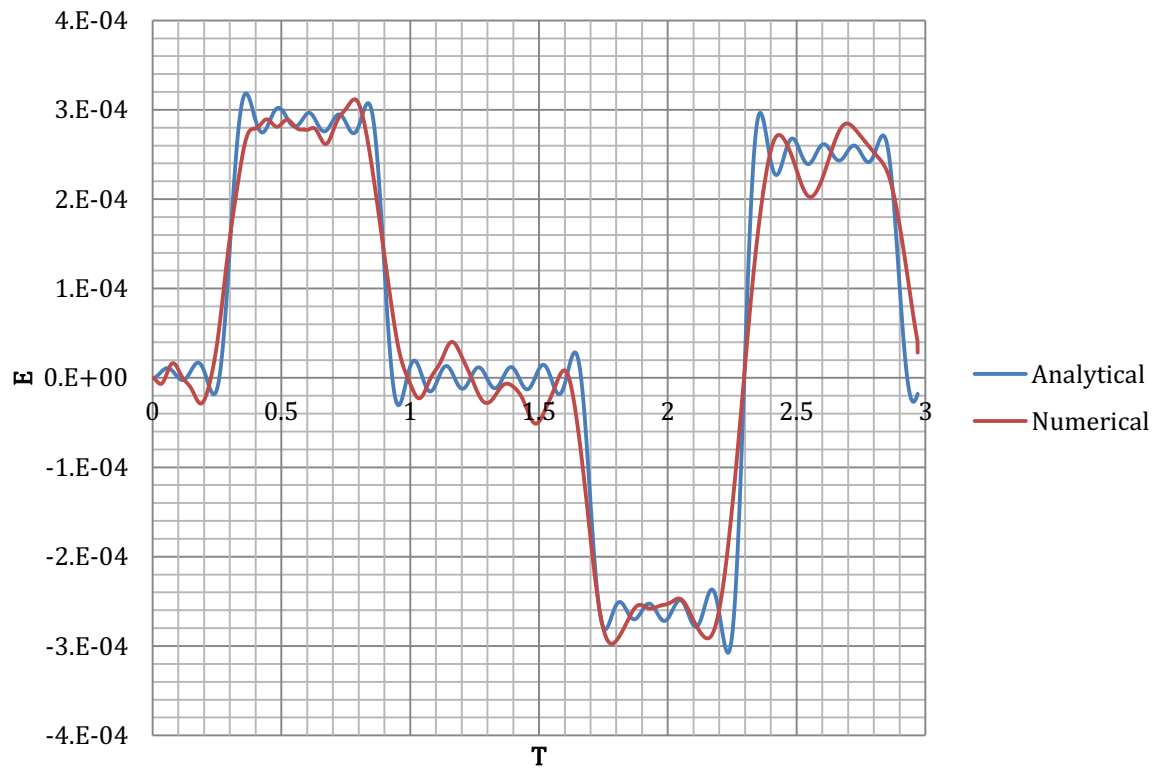


Figure 6-12: Dimensionless strain history for Case 2 with  $\zeta = 2\%$  (node 4, rectangular pulse)

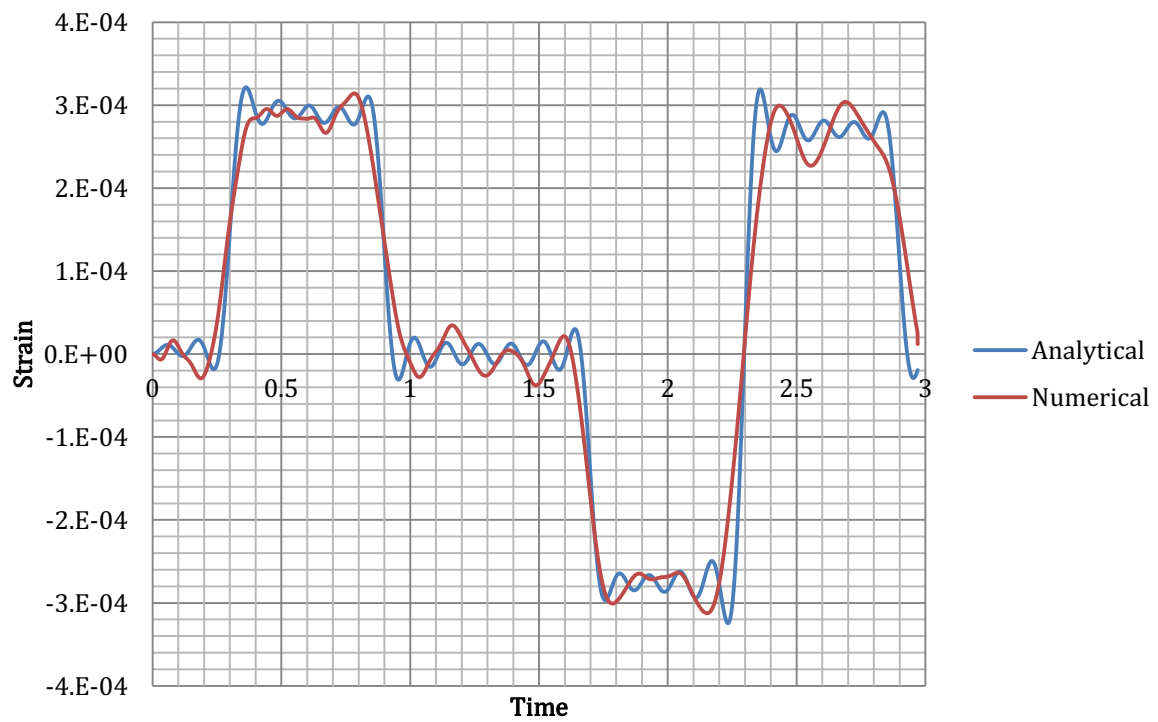


Figure 6-13: Dimensionless strain history for Case 2 with  $\zeta = 1\%$  (node 4, rectangular pulse)

#### 6.7.4 Results from ABAQUS

From the earlier results, it is evident that the presence of damping is not significant and thus only Case 1 was investigated using the commercial finite element package ABAQUS/Explicit v.6.9-1 [252]. The specimen was modelled using 8-noded linear brick (solid) elements with reduced integration (C3D8R). A uniform mesh size of 0.4 mm was used and the material properties described in Section 6.7.1 were inputted.

Typical outputs from ABAQUS are shown in Figure 6-14 and Figure 6-15.

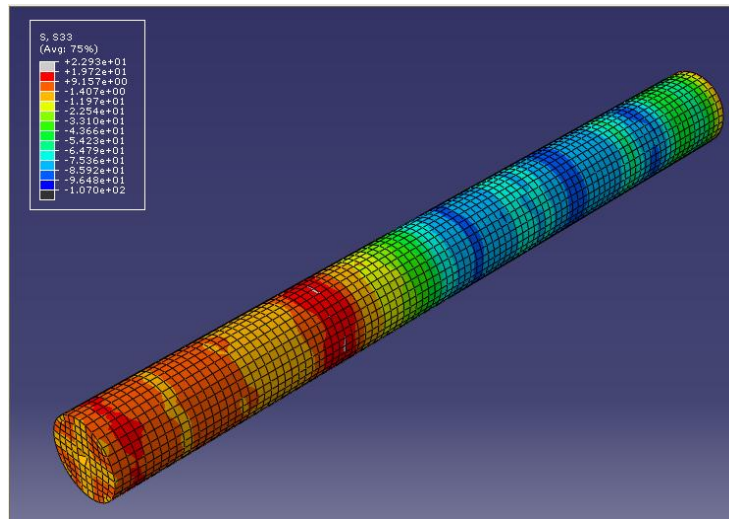


Figure 6-14: Typical stress distribution from ABAQUS (Case 1, rectangular pulse)

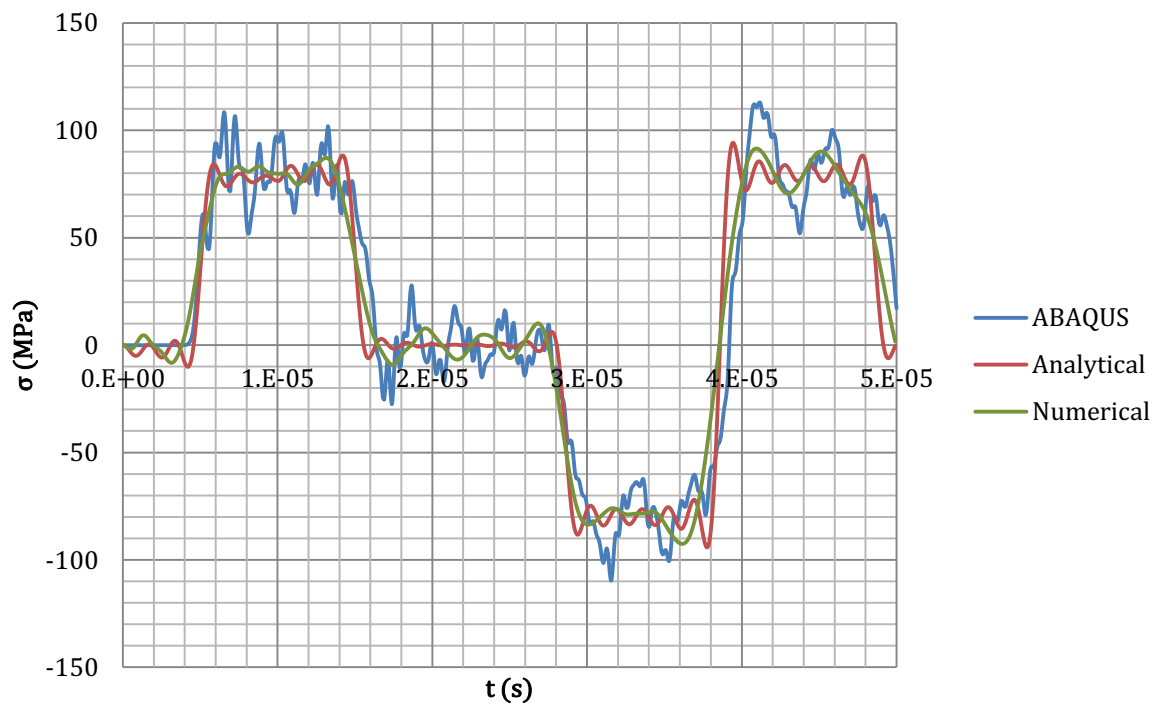


Figure 6-15:  $\sigma$ - $t$  history for Case 1 showing all results (node 4, rectangular pulse)

## **6.8 Discussion of results**

In this section, the results from Section 6.7 are compared and discussed. The salient arguments considered are the correlation between analytical and numerical results, the influence of the pulse load shape and the effect of visco-elasticity.

### **6.8.1 Comparison of analytical and numerical results**

For Cases 1 and 2, the analytical solutions were available and these are compared to the numerical results from the method proposed.

It is seen that there is excellent correlation between the two sets of results. Agreement is excellent in both temporal and spatial terms. The difference between the two with respect to maximum strains and stresses is almost negligible in the half sine pulse, less than 5% in the rectangular pulse, less than 10% in the quarter cosine pulse and just over 10% in the case of the linearly decaying pulse.

Thus, it can be concluded that the number of degrees of freedom chosen is adequate for satisfactory results.

The good correlation with the output from ABAQUS also suggests that the proposed numerical method gives significantly accurate results which are closer to the analytical output than the results from ABAQUS, due to the higher order of element used.

### **6.8.2 Influence of pulse load shape**

It is seen that, for a given node, the maximum strains and stresses attained do not vary with the pulse load shape. For example, for Case 1, a maximum normalised strain of approximately  $3\text{E-}4$  was achieved in all cases, by comparing Figure 6-5 with Figure 6-7 to Figure 6-9.

### **6.8.3 Comparison of Case 1 with Case 2**

When comparing Case 1 with Case 2, it is seen that with damping consideration, a reduction in the peak strain (or stress) is attained when comparing the same nodal histories. This reduction is up to 25% with  $\zeta = 5\%$  and approximately 10% with  $\zeta = 2\%$  and 1%. This can be observed in Figure 6-16.

Further reduction occurs with progressive wave cycles. As expected, the rate of decay is more pronounced with the higher damping values. It is also observed that the reduction in peak values occurs even during the early-time response and thus it is not insignificant to consider the effect of damping.

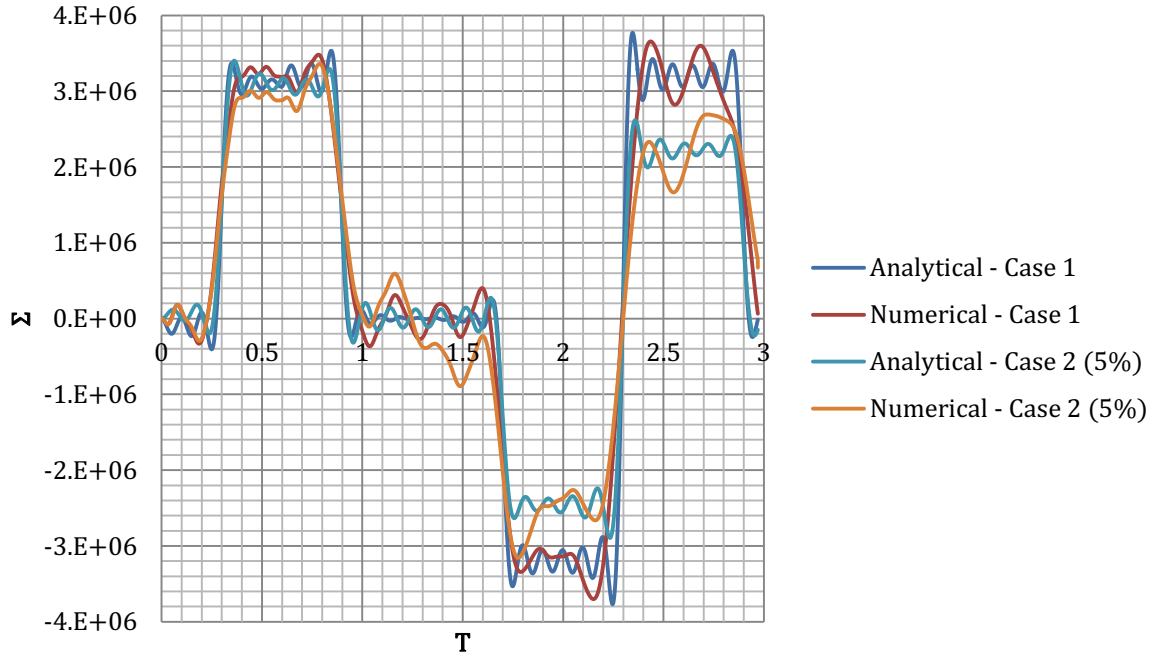


Figure 6-16: Dimensionless stress history for Cases 1 and 2

## 6.9 Assessment of delamination in blast-loaded composite panels

The aim of this chapter was to develop a method which could predict the onset of delamination in a laminated composite material which is subjected to a pulse load of short duration, such as blast.

The previous sections described a method by which the through-thickness stresses induced by an applied pulse load can be estimated.

In this section, the developed tool is applied to the blast-loaded Dyneema HB26 panels described in Chapter 5 and the predicted results are compared with the observed behaviour of these panels.

### 6.9.1 Material and geometric properties

As described in Section 5.2.1, Dyneema has a density of  $970 \text{ kg/m}^3$ . In addition, a through-thickness modulus,  $E$ , of approximately  $8 \text{ GPa}$  was measured, as described in Section 5.5.2. Thus, the wave velocity is  $2871 \text{ m/s}$ .

In terms of geometry, the plates had a thickness,  $L$ , of  $24 \text{ mm}$ , as per Section 5.4.2 and a loaded area of  $300 \times 300 \text{ mm}$ , thus an area,  $A$ , of  $0.09 \text{ m}^2$ .

In terms of the cohesive strength of the material, as described in Section 5.7.2, a maximum stress of  $2 \text{ MPa}$  in the normal direction is expected in Dyneema [271]. Although this has not been verified experimentally for Dyneema HB26, this magnitude of stress is compared with the maximum stress predicted and used as a measure of the likelihood of the onset of delamination.

### 6.9.2 Loading parameters

Although various blast tests were carried out on several panels, in this study 2 tests in particular will be studied, viz., test 2 and test 7.

With reference to Table 5-4, test 2 represents the smallest impulse (28.54 Ns) and which was observed to cause very small permanent displacements and thus very little delamination. Test 7 represents the most aggressive blast test, with an impulse of 98.54 Ns and having attained a significant permanent displacement, associated with widespread delamination.

The results in previous sections suggest that the maximum strain (and, thus, stress) levels are independent of pulse shape, as demonstrated in Section 6.8.2. Thus, although the temporal distribution of the blast load is known to be of an exponentially decaying form, in each case, the load was idealised as a linearly decaying pulse with a duration,  $t_d$ , of 0.0314 ms and thus corresponding to a load,  $P$ , of 1814 kN and 6276 kN for tests 2 and 7 respectively.

### 6.9.3 Test 2 results

The analytical equations described in Section 6.5 were used to compute the strains and stresses in the specimen subjected to the load described above.

Although the stress-time history can be obtained at various positions through the thickness, for the sake of brevity, only histories for positions close to the back face are shown, since these locations are where delamination is mostly expected.

A selection of these histories is shown in Figure 6-17 to Figure 6-19.

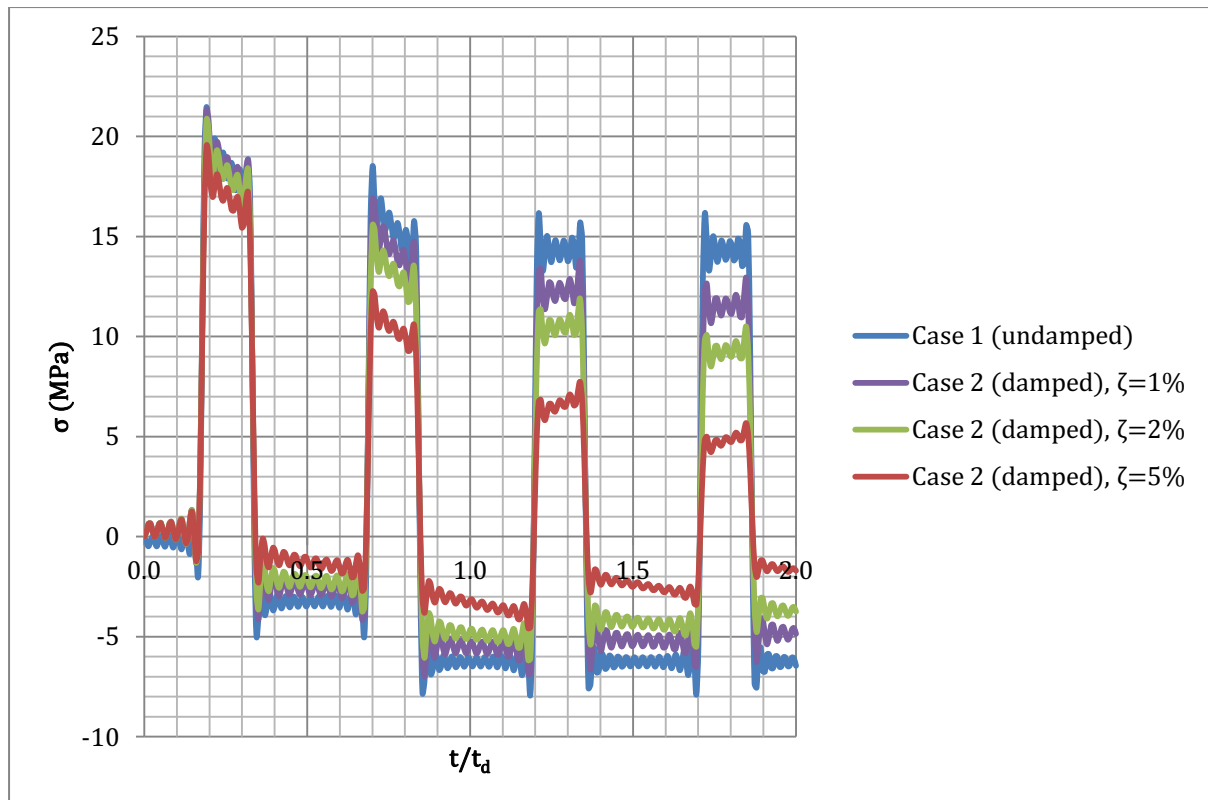


Figure 6-17:  $\sigma$ - $t$  history for test 2 (16 mm position)

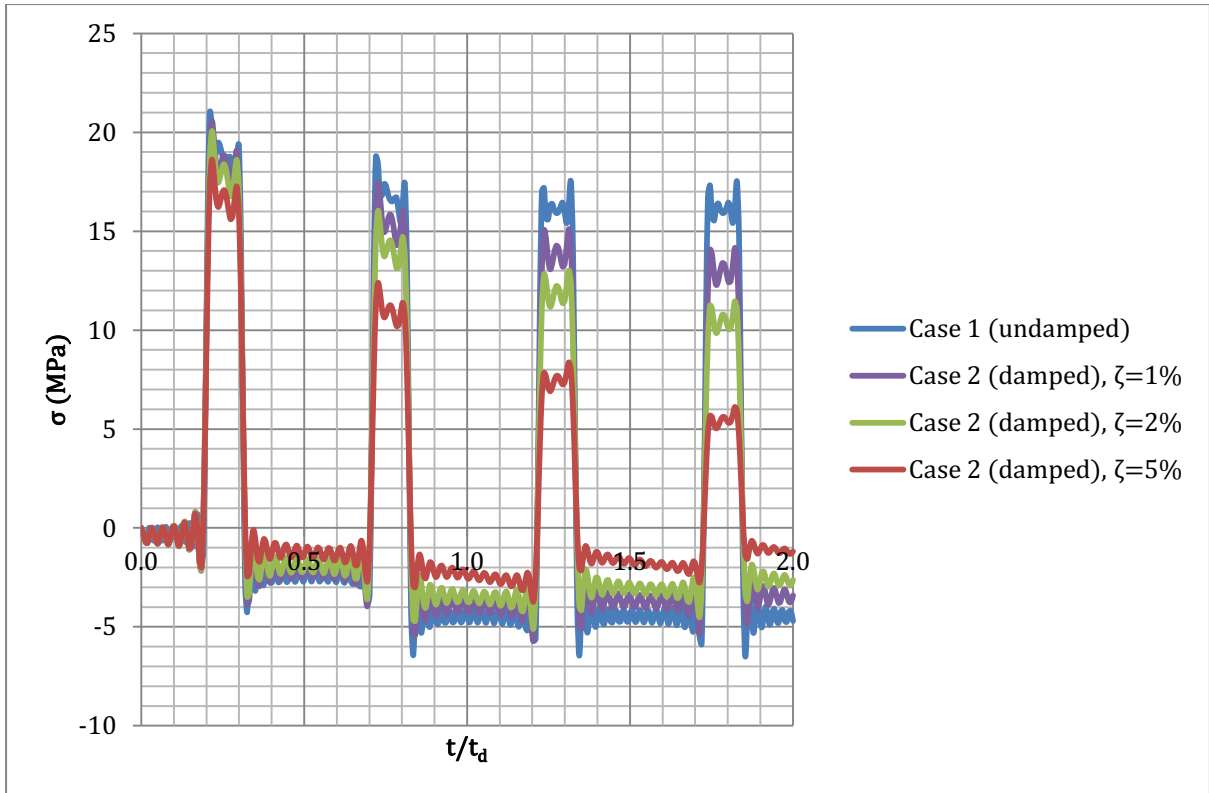


Figure 6-18:  $\sigma$ - $t$  history for test 2 (18 mm position)

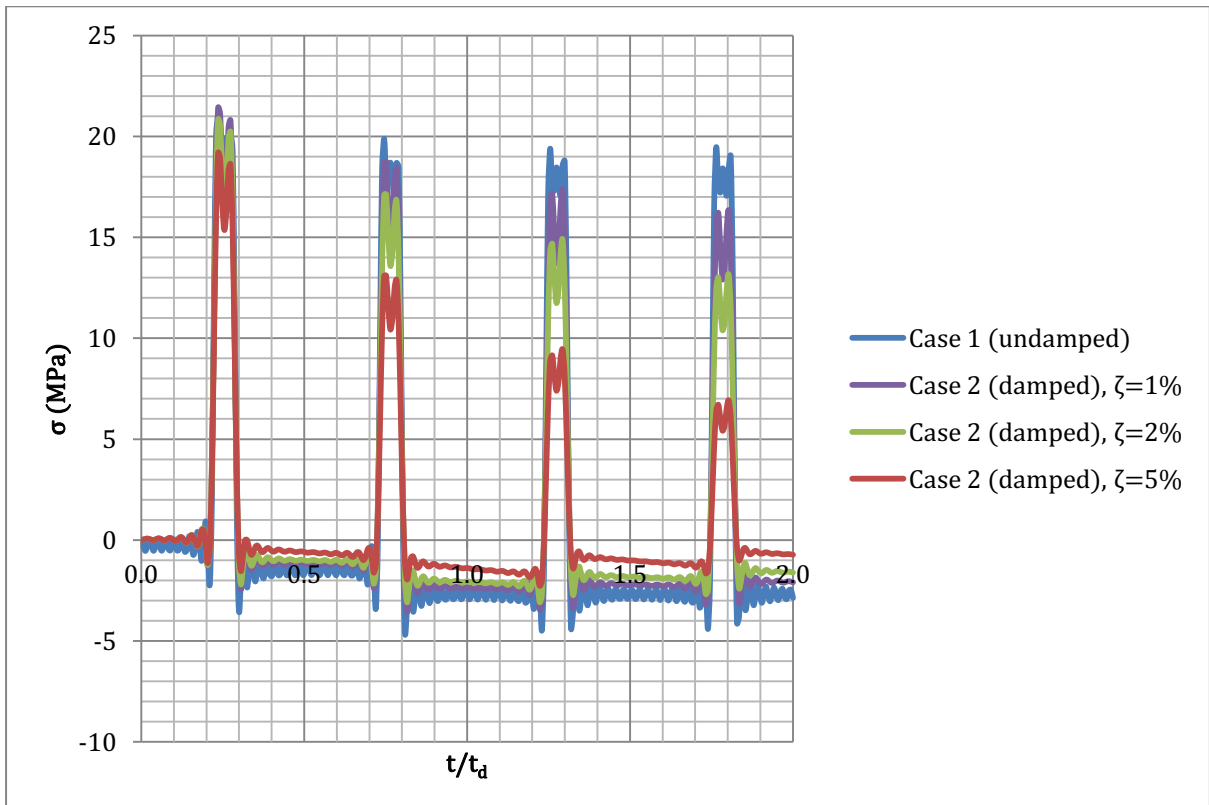


Figure 6-19:  $\sigma$ - $t$  history for test 2 (20 mm position)

From these plots, it can be observed that during the plate's early-time response, i.e.,  $\left(\frac{t}{t_d} \approx 1\right)$ , although the inclusion of damping effects decreases the maximum tensile stress by up to 50% for  $\zeta = 5\%$  and even up to 12% for  $\zeta = 1\%$ , the analytical model predicts maximum stress of over 2 MPa in each of these locations.

Whilst attaining 2 MPa does not necessarily imply that delamination has occurred (since this only occurs when the cohesive layer exceeds its maximum separation, as discussed in Sections 2.9 and 5.72), it does suggest that damage to the cohesive layers has initiated and that delamination is very likely to occur.

It must also be noted that in the actual plate, mixed-mode behaviour will occur, implying that inter-ply failure could well be achieved, even with normal stresses less than 2 MPa due to the unfavourable effects of shear stresses. In fact, the numerical results from ABAQUS suggest that delamination does indeed occur at these locations, as shown in Figure 6-20.

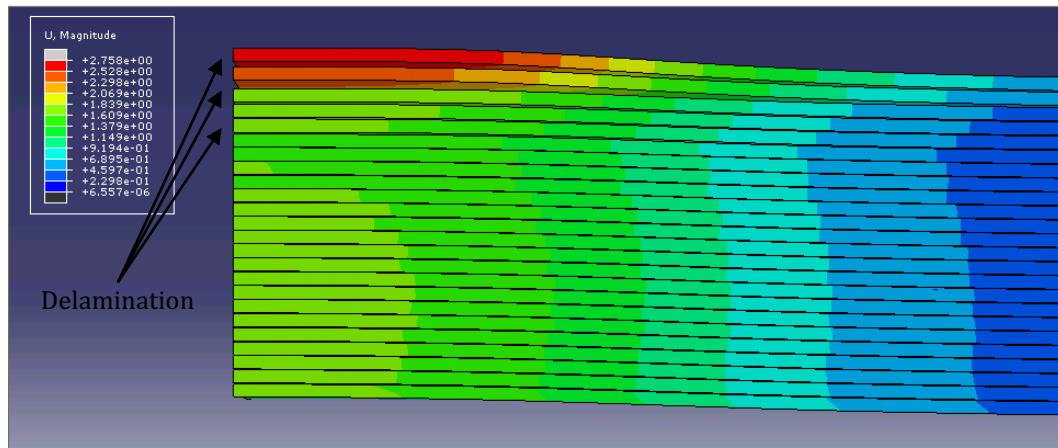


Figure 6-20: Plate cross-section from ABAQUS showing initial stages of delamination for test 2 (at  $t/t_d \approx 1.25$ )

#### 6.9.4 Test 7 results

A similar assessment was done on the test 7 plate and a sample of the stress-time histories are shown in Figure 6-22 and Figure 6-23.

Similar observations can be made in this case, where it can be seen that the tensile stresses are well in excess of 2 MPa, even at  $\left(\frac{t}{t_d} < 1\right)$ , suggesting that delamination occurs even during the direct loading phase. This is also demonstrated by the results from ABAQUS shown in Figure 6-24, where delamination can be observed at  $\frac{t}{t_d} \approx 0.8$ .

The maximum tensile stresses are predicted to occur at  $\frac{t}{t_d} \approx 1.2$ , when more widespread delamination is expected, even up to halfway through the thickness, which is also confirmed by the results from ABAQUS, shown in Figure 6-25.

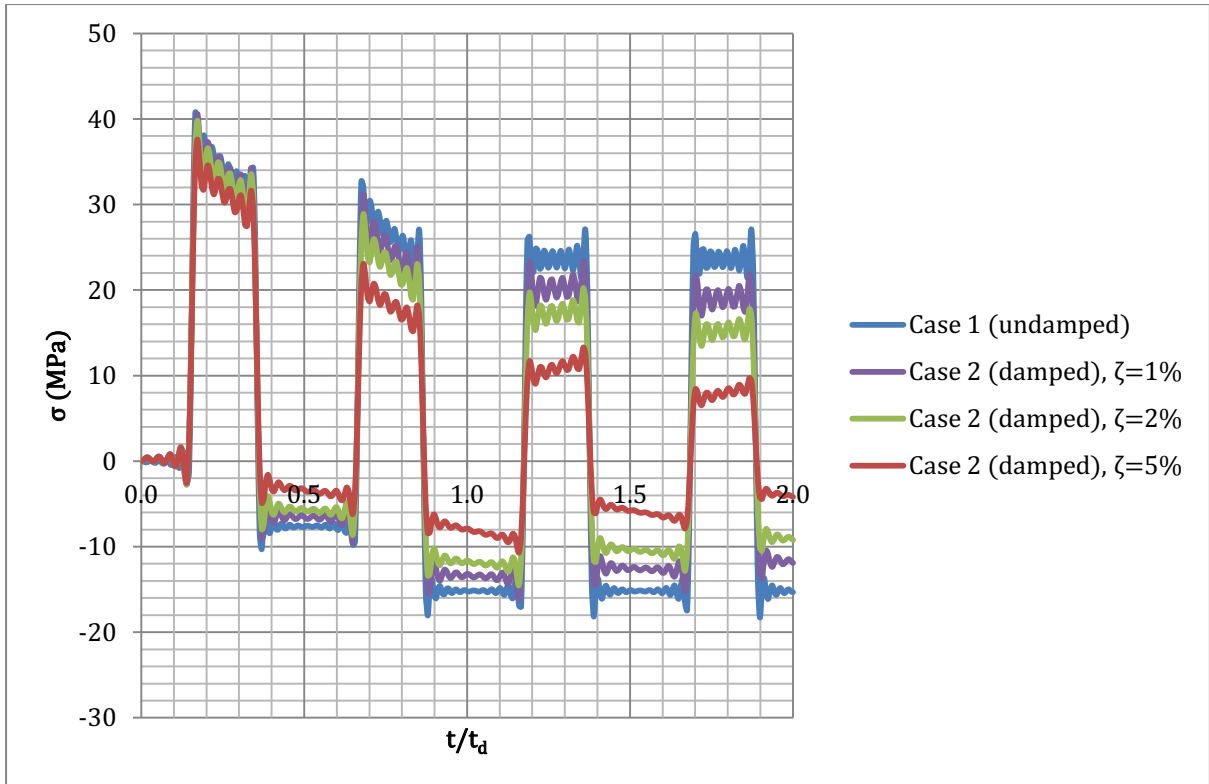


Figure 6-21:  $\sigma$ - $t$  history for test 7 (14 mm position)

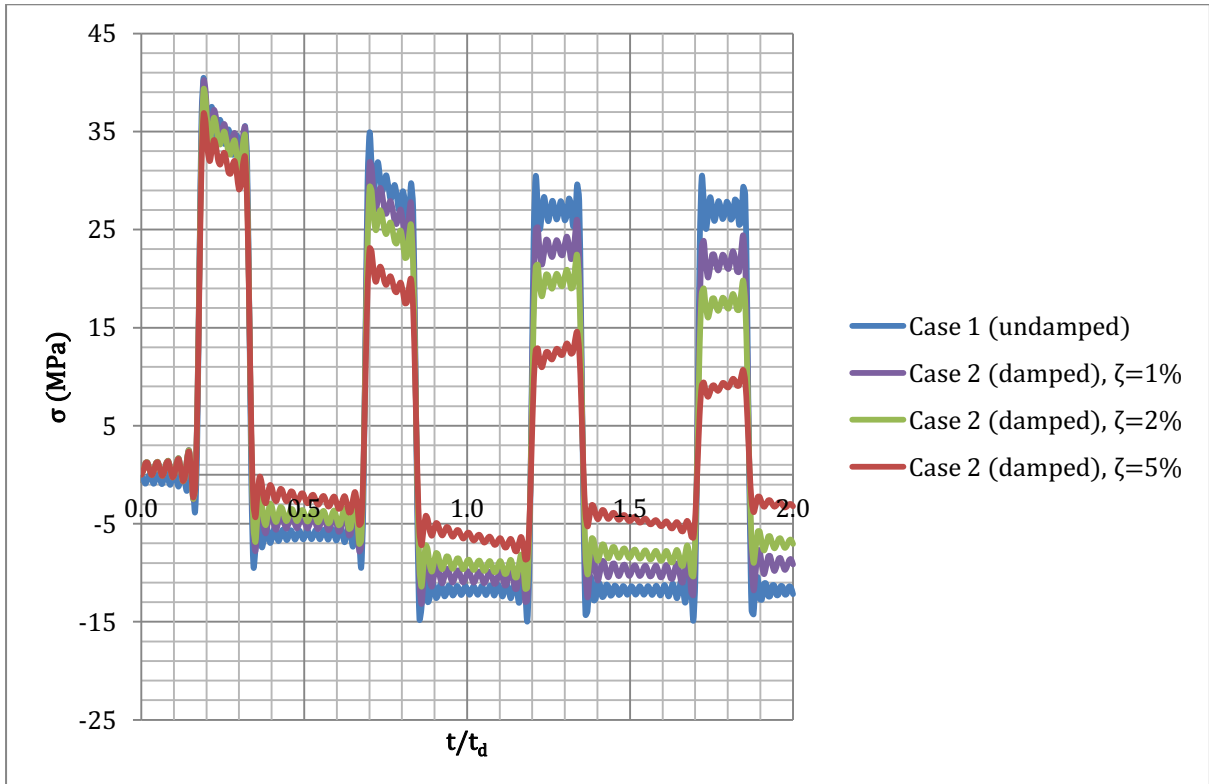


Figure 6-22:  $\sigma$ - $t$  history for test 7 (16 mm position)

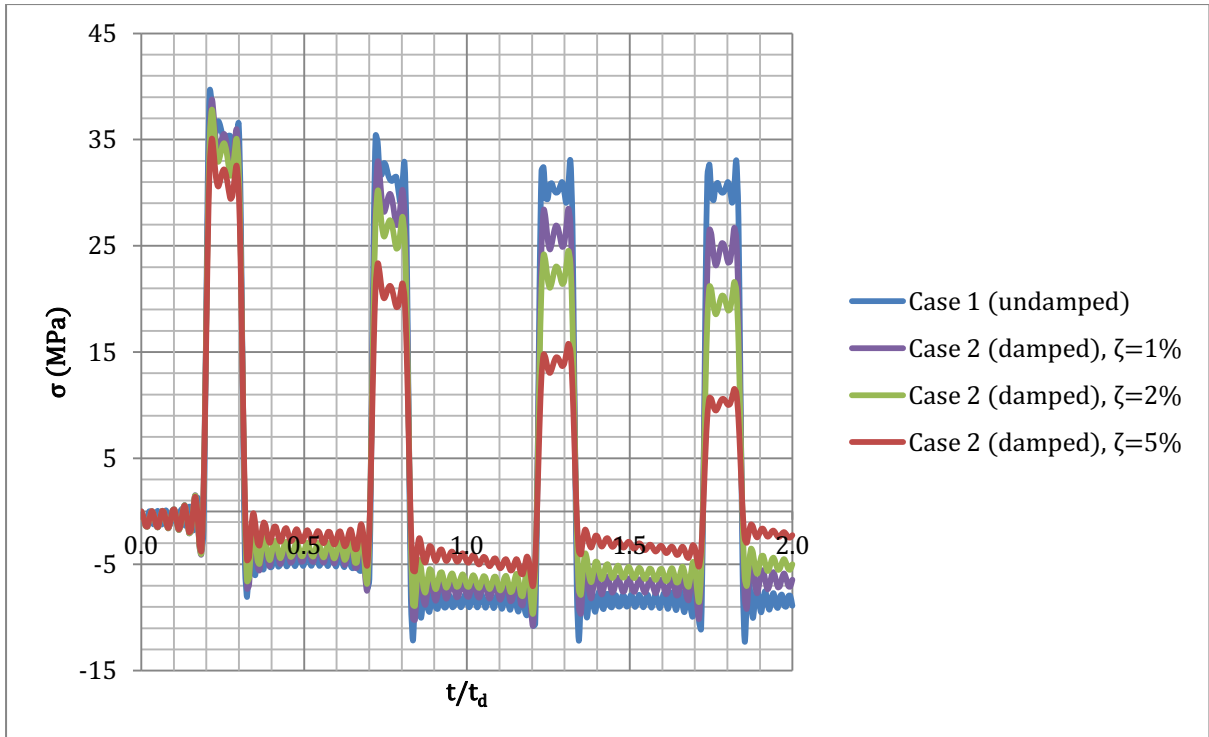


Figure 6-23:  $\sigma$ - $t$  history for test 7 (18 mm position)

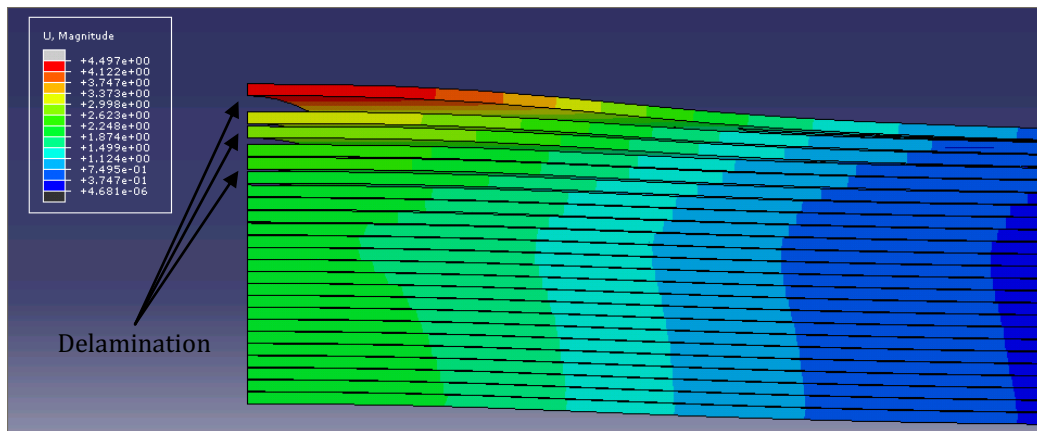


Figure 6-24: Plate cross-section from ABAQUS showing initial stages of delamination for test 7 (at  $t/t_d \approx 0.8$ )

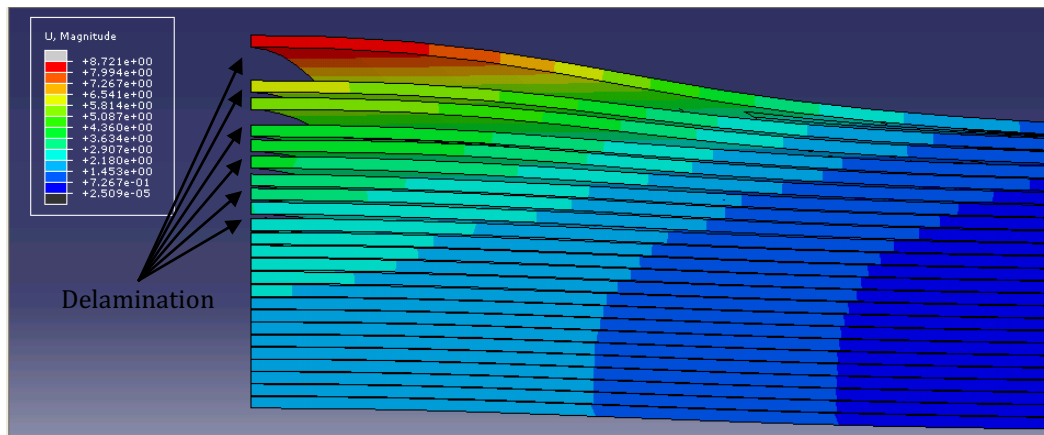


Figure 6-25: Plate cross-section from ABAQUS showing initial stages of delamination for test 7 (at  $t/t_d \approx 1.2$ )

## 6.10 Concluding remarks

In this chapter, wave propagation analysis is used to investigate the stress and strain variation through the thickness of an elastic composite medium. This is done by both analytical and numerical methods. The response for different pulse load shapes of the same amplitude and duration was found to give similar maximum strains and stresses.

It was found that the numerical results obtained (using the proposed method) for the damped and undamped elastic cases are in excellent agreement with the analytical results for displacement, strain and stress. Similarly good correlation for Case 1 has been achieved with output from ABAQUS.

With respect to Case 2 results, it was found that the reduction in peak strain and stress is only significant with  $\zeta = 5\%$ , albeit the reduction is not too marked with low damping levels and can be (conservatively) neglected.

By means of the proposed model, the magnitude of the (tensile) stresses which a propagating wave generated by a pulse load, such as blast, will induce in a laminated composite will be predicted in a simple way. Such stresses are then compared with the cohesive strength of the laminate, which governs whether delamination between the individual plies occurs or not.

In the event of the tensile stresses exceeding such strength, then delamination will occur during the early-time response of the composite, severely affecting the through-thickness integrity of the laminate and thus any energy dissipation by means of plate action cannot be achieved

This has been demonstrated by assessing a selection of the Dyneema HB26 composite plates subjected to blast loading described in Chapter 5. It has been shown that in the case of the plate where little delamination was observed in the tests, the maximum tensile stresses were only just in excess of the maximum tensile capacity of the cohesive layer while in the plate with the most severe loading, the maximum predicted stresses are well in excess of the maximum tensile capacity, suggesting severe delamination.

Thus, by means of the simple 1-D model utilising wave propagation analysis described in this chapter, one can identify the onset (or otherwise) of delamination in laminated composite plates subjected to blast loading.

Although comparing the magnitude of the maximum tensile stress predicted by the model with the cohesive strength of the material can provide an estimate of whether delamination is likely or not, the model does not account for the traction-separation evolution of the cohesive layer and thus this will give a lower bound to the load necessary for delamination to occur.

However, for composite materials such as Dyneema, it is postulated that there is little softening behaviour after the initiation of failure and thus an assessment using the proposed method does not lead to overly conservative results.

## Chapter 7. Conclusions and recommendations for future work

### 7.1 Recapitulation

As outlined in Section 1.3, the scope of this thesis has encompassed an investigation of the dynamic behaviour of monolithic and composite plated structures subjected to blast loading, with particular focus on localised blast loading. This has been achieved by a combination of developments in advanced analyses techniques using finite element procedures and simple design methods to predict and assess potential damage to such plates. To this end, analytical, numerical and experimental methods have been used.

Firstly, a literature survey of existing works on the subject was carried out, which concluded that there is no available method of describing a localised blast load arising from a given charge size, geometry and stand-off distance from target. The review also investigated analytical methods for predicting displacement of plates subjected to blast, assuming rigid-plastic behaviour and the concept of using dimensionless parameters to predict such displacements. The survey also reviews the constituents of polymeric composite materials, material models and damage mechanisms for such materials and the use of these materials in applications involving blast loading.

On the basis of these findings, the thesis proposed a systematic method of providing the (mathematical) spatial and temporal descriptions of a localised blast load from a known set of threat parameters (explosive quantity and stand-off distance) and by the application of dimensional analysis. The results produced using the proposed loading function implemented in a finite element analysis package were validated by comparison with experimental data of blast loads on steel plates.

This led to the first study, which focused on the performance of monolithic plates subjected to a blast load of the form described above. Existing formulations for uniform loading found in the literature were extended to consider this new form of loading. Various plate thicknesses were investigated (thick, moderately thick and thin) and it was found that good correlation is achieved with numerical results when the blast load is simplified into an impulsive one.

The performance of composite plates under blast loading was also investigated. Material characterisation and blast loading tests were carried out on Dyneema HB26 specimens and plates respectively and these were used to develop a material model for Dyneema, which was validated using finite element simulations.

Its performance was numerically compared with mild and armour (Armox 370T Class 1) steel plates of equal areal density and it was found that Dyneema HB26 offers an improvement over mild steel, but armour steel plates lead to the least permanent midpoint deflection. It was also seen that mild steel ruptures when subjected to an impulse beyond a specific impulse threshold, at which impulse both Dyneema HB26 and armour steel did not tear.

Using dimensionless parameters, a simple guideline was provided to estimate the permanent midpoint deflection for a given plate geometry made of a monolithic or composite material subjected to a specific blast load. The use of this guideline was also illustrated by considering various threats and using the proposed method to recommend various plate thicknesses required for different material systems to meet a specified damage limitation.

Furthermore, a numerical-analytical method was proposed to predict the occurrence of Mode I delamination (or separation between the plies) in a laminated composite material during its early-time response, by means of stress propagation analysis. The effects of damping were considered and it was found that these are relatively insignificant.

The main conclusions drawn from this thesis are summarised in the follow section.

## **7.2 Summary of principal findings**

In this section, the main results from this thesis are outlined.

### **7.2.1 Spatial and temporal description of a blast load**

The main conclusions in this respect are:

- A mathematical description of a blast load having 2 parts (independent temporal and spatial exponential distributions) was proposed; the form of these functions is supported by AUTODYN simulations and literature.
- Expressions and corresponding charts have been derived from a series of dimensional analyses and presented in (3-17) to (3-21) to obtain the blast parameters to describe the temporal and spatial distributions for a blast load for a given explosive charge of specified material, geometry and stand-off distance.
- By considering the dimensionless impulse quantity expressed in (3-25) in terms of the impulse imparted to the exposed area of the target plate as a fraction of the total impulse generated by a given threat, it was possible to study the extremity of the blast load and which are the most critical parameters.
- 

### **7.2.2 Material characterisation tests**

From these tests, the following was achieved:

- Tensile stress-strain curves for Armox 370T Class 1 armour steel were obtained, which can be readily used in numerical and/or analytical modelling. A modulus of just over 200 GPa was measured and a yield stress of 1100 MPa and rupture at 7% was observed.
- Tensile, in-plane compressive, through-thickness compressive and shear low-rate tests on Dyneema HB26 samples were also carried out and material properties extracted for use to develop a constitutive model for the material. An elastic tensile modulus of around 23 GPa was measured, with a yield stress of around 700 MPa and limited plastic deformation. Shear behaviour was found to be highly non-linear and with plastic strain deformation in excess of 40%.
- Similar tests (at low and medium rates of loading) were carried out on Tegriss samples, with a much lower tensile yield stress of 200 MPa and non-linear shear deformation up to a failure strain of around 70%.

### **7.2.3 Blast testing of mild and armour steel plates**

The main findings are:

- Both metallic plates tested (viz., mild steel and Armox 370T Class 1 armour steel) ruptured at the same stand-off and charge size combination, suggesting that the high yield stress of the armour steel is offset by the increased ductility of the mild steel. Nevertheless, armour steel plates offer reduced permanent transverse displacement when compared to mild steel for the same threat level.
- The permanent displacement can be estimated using (3-29) in terms of the non-dimensionalised damage parameter,  $\phi_{q,s}$ , proposed by Jacob et al. [48, 51].
- Fully clamped mild steel plates subjected to a localised blast load rupture at a  $\phi_{q,s}$  value of around 30. Fully clamped Armox 370T Class 1 (armour steel) plates subjected to a localised blast load rupture at  $\phi_{q,s} \approx 16$ .

#### 7.2.4 Blast testing of Dyneema HB26 plates

It can be concluded that:

- Under the range of blast loadings considered, Dyneema HB26 did not rupture; thus, it can be concluded that its rupture damage number exceeds a  $\phi_{q,s}$  value of 1.2.
- For the blast loads tested, no back face damage was incurred, with the superficial damage being limited to the front face, primarily due to the incident heat wave from the explosive materials. However, significant delamination and a fair extent of pulling-in could be observed.

#### 7.2.5 Analytical rigid-plastic analysis of monolithic plates

In this regard, the conclusions are as follows:

- An expression for the static plastic collapse load for a monolithic simply supported circular plate subjected to a localised blast load is presented in (4-23).
- An expression for the permanent plastic deformation of such a plate under a rectangular pulse load considering only bending action is given by (4-37), provided that the load ratio  $\eta$  is less than the limit given in (4-41). For cases exceeding the critical load ratio, the permanent displacement is given by the expression in (4-64).
- Results for alternative pulse shapes are given in (4-75) for linearly decaying loads and (4-77) for exponentially decaying loads.
- Expressions for the permanent plastic deformation of thick plates (i.e. with consideration of transverse shear and rotatory inertia effects) are also given by (4-99), (4-104) or (4-107), depending on the plate's ratio of plastic shear capacity to plastic bending capacity (or its slenderness).
- However, it was shown that consideration of such effects is not particularly relevant for plates of typical slenderness values which are used in applications related to blast resistance.
- Consideration of large deflections, or membrane effects, was shown to be the most relevant for blast loaded plates, where deformations are expected to be significantly large compared to the plate's thickness. It was shown that ignoring bending effects and considering only membrane effects would suffice and an expression for permanent displacement is provided in (4-134), (4-146) and (4-165) for different pulse shapes.

- It was also found that by idealising the blast load as being impulsive, one could provide estimates for displacement using (4-135) or (4-136) for a more conservative result, without loss of accuracy.
- The derived analytical results showed excellent agreement with corresponding numerical results for various values of dimensionless impulse,  $\lambda$ , proposed by Jones [254], in the range up to  $\lambda \approx 200$ . It can be concluded that for high impulse values, the analytical results provide excellent approximations of the permanent displacement by using the impulsive load simplification and considering only membrane effects, with an accuracy of up to 6%. This eliminates the need for more sophisticated analytical methods or finite element analysis simulations.
- It can also be concluded that the rigid-plastic approximation is not suitable for low values of impulse  $\lambda \lesssim 5$ , where the elastic range would still be relevant.

### 7.2.6 Comparison between monolithic and composite plates

It can be concluded that:

- For the same blast threat and equal areal density, Dyneema HB26 offers better performance when compared to mild steel, with a reduction in permanent midpoint displacement of 30% and preventing rupture.
- However, for the same blast threat and equal areal density, Armox 370T Class 1 armour steel offers the best reduction in permanent midpoint displacement, with 50% reduction when compared to mild steel and also preventing rupture.
- A new dimensionless damage number,  $\phi_{q,sl}$ , was proposed in (5-10) in order to compare the behaviour of monolithic and composite plates subjected to blast loading, where the performance criterion being permanent displacement. This parameter accounts for plate slenderness, which becomes a significant parameter when comparing materials with significantly different densities.
- In this case, the dimensionless displacement was given in terms of the plate's dimensions, rather than its thickness, since monolithic and composite plates of equal areal density will, more often than not, have slenderness values varying considerably (in this case, by a factor of almost 10).
- The proposed damage number can be used by designers to obtain a preliminary plate thickness for a given blast threat and a specified limit for permanent displacement for various material systems, as illustrated in Section 5.9.
- The identification of limits on  $\phi_{q,sl}$  corresponding to rupture for the various materials would provide the designer with further knowledge of which material system/s is suitable for a particular blast application (e.g. rupture limits for Armox 370 Class 1 armour steel and mild steel are  $\phi_{q,sl} = 0.015$  and  $\phi_{q,sl} = 0.028$  respectively, for fully clamped plated structures).

### 7.2.7 Assessing onset of delamination failure in composites

In this respect, it can be concluded that:

- The onset of spalling in a laminated composite can be predicted by wave propagation analysis and assessing the through-thickness stresses.
- In addition to analytical equations presented in Section 6.5, an 11-noded Lagrangian element developed in Section 6.6, could also be used to evaluate stress values resulting from wave propagation through a visco-elastic medium subjected to a pulse load.
- It was found that the stresses predicted by this method qualitatively suggest the occurrence of delamination (or otherwise) in blast-loaded Dyneema HB26 panels.

### **7.3 Limitations and recommendations for future work**

The findings and contributions arising from this thesis have also highlighted the need for further research related to the subject, particularly to address some of the limitations of this work.

Thus, recommendations for future investigations are provided in this section.

#### **7.3.1 Dimensionless loading parameters**

Whilst comparison of experimental and numerical results using the blast loading function proposed in Chapter 3 has been shown to be a plausible, the loading parameters are only valid for explosive charges which have a shallow cylindrical form. Thus, it is recommended that further charge shapes, such as spherical ones, are investigated.

Also, for relatively thicker charges, it is postulated that the location of the initial detonation point might also affect the generated blast wave and this is deemed to be worthy of investigation.

In addition, the pressures derived from the method in Chapter 3 are based on reflections off a rigid surface which is normal to the shock wave. This is known to produce conservative loads [316], particularly if the target is very flexible or compressible.

Thus, it is suggested that the extent of reduction in the pressure value is studied, by means of consideration of FSI (fluid-structure interaction) in a package such as AUTODYN or by simplified methods such as those proposed by Kambouchev et al. [317-319].

#### **7.3.2 Rigid-plastic analysis for monolithic plates**

The results in Chapter 4 related to the rigid-plastic analyses of monolithic plates subjected to blast loading are valid for simply supported circular plates. This imposes a restriction on the applicability of the results for, say, quadrangular plates or for plates with fully clamped boundary conditions. Thus, it is suggested that the analysis is extended for alternative geometries and support boundary conditions.

Also, the current analysis does not include strain-rate effects. Although this has been shown to be a reasonable simplification for less ductile materials such as armour steel, it is not a valid assumption for materials such as mild steel.

This can be done by introducing a strain-sensitive constitutive model in the formulation, such as the Cowper-Symonds model, and by using appropriate simplifications, such as the Perrone-Bhadra solution [262], which has been shown to be adequate for visco-plastic response of impulsively loaded membranes [320] and various other structures [254].

### **7.3.3 Blast testing of new material systems**

The experimental testing in this research has been limited to one grade of armour steel (Armox 370T Class 1) and one high performance composite (Dyneema HB26). It would be desirable to extend the experimental work to cover other materials, including (but not limited to) Armox 440T and Tegril. Further Dyneema HB26 panels subjected to alternative charge size/stand-off combinations could also be tested to establish a rupture threshold for the material.

In addition, it would also be advantageous to study the behaviour of hybrid systems, including fibre metal laminates, as these are known to be particularly relevant in the field of blast resistant design, as shown in Chapter 2.

### **7.3.4 Optimisation of material systems**

Extending the argument in Section 7.3.3, it would also be worth investigating the possibility of optimising the layup of a hybrid system.

Although the use of Dyneema HB26 has been shown in Chapter 5 to be promising, the material is, to date, extremely expensive, costing €300/kg<sup>14</sup> (or approximately £245/kg), especially when compared to monolithic systems (c.f. 8 No. 400x400x3.8 mm Armox 370T Class 1 panels purchased from SSAB of Sweden for £500 (or approximately £13/kg), which represents only 5% of the cost).

It has been argued that the apparent high purchase cost of composites when compared to other (monolithic) materials by means of a direct comparison on a unit price basis may not be appropriate, since when installation, transportation and life cycle costs are included in the cost comparison, composites can compete with conventional materials [316]. However, this is unlikely to be the case with the high performance composites being developed and it is imperative that optimisation is carried out.

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<sup>14</sup> Personal communication by Professor Lorenzo Iannucci of Imperial College London, January 2013. Further cost verifications (e.g. via online sources) were not possible, due to commercial sensitivity.

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## Appendix A: VDLOAD subroutine

```

subroutine vdload(
C
C read only (unmodifiable) variables:
    1 nblock, ndim, stepTime, totalTime,
    2 amplitude, curCoords, velocity, dirCos, jltyp, sname,
C
C write only (modifiable) variable:
    1 value)
C
    include 'vaba_param.inc'
C
    dimension curCoords(nblock, ndim), velocity(nblock, ndim),
    1 dirCos(nblock, ndim, ndim), value(nblock)
C
    character*80 sname
C
C
C *****
C input parameters
C *****
C Ro: constant part, mm
C blastTime: duration of blast event, s
C Po: constant pressure under charge diameter, MPa
C W: parameter for temporal distribution, /s
C A: parameter for spatial distribution, MPa
C B: parameter for spatial distribution, /mm
C
    parameter (Ro = 15.3, blastTime = 0.0000314)
    parameter (Po = 306)
    parameter (W = -135322)
    parameter (A = 661, B = -0.0506)
C
C
C *****
C temporal distribution
C *****
C assumes an exponential decay over time of the form  $P(t) = \text{EXP}(W*t)$ 
C
    if (stepTime .LE. blastTime) then
        ampTime = EXP(W*stepTime)
    else
        ampTime = 0
    endif
C
C
C *****
C spatial distribution
C *****
C assumes an exponential decay over space of the form  $P(r) = A*\text{EXP}(B*r)$ 
C
    do 100 k = 1, nblock

```

```

C
C works out radial distance from centre
C
    r = SQRT(((curCoords(k,1))**2) + ((curCoords(k,2))**2))
C
    if (r .LE. Ro) then
        value(k) = Po*ampTime
    else
        value(k) = (A*EXP(B*r))*ampTime
    endif
C
C
100 continue
C
    return
end
C
C*****

```

## Appendix B: VUMAT subroutine for Dyneema HB26

```
subroutine vumat(
C
C read only (unmodifiable) variables:
  1 nblock, ndir, nshr, nstatev, nfieldv, nprops, lanneal,
  2 stepTime, totalTime, dt, cmname, coordMp, charLength,
  3 props, density, strainInc, relSpinInc,
  4 tempOld, stretchOld, defgradOld, fieldOld,
  5 stressOld, stateOld, enerInternOld, enerInelasOld,
  6 tempNew, stretchNEW, defgradNew, fieldNew,
C
C write only (modifiable) variables:
  7 stressNew, stateNew, enerInternNew, enerInelasNew)
C
  include 'vaba_param.inc'
C
C
  dimension props(nprops), density(nblock), coordMp(nblock),
  1 charLength(nblock), strainInc(nblock, ndir + nshr),
  2 relSpinInc(nblock, nshr), tempOld(nblock),
  3 stretchOld(nblock, ndir + nshr), defgradOld(nblock, ndir + nshr + nshr),
  4 fieldOld(nblock, nfieldv), stressOld(nblock, ndir + nshr),
  5 stateOld(nblock, nstatev), enerInternOld(nblock),
  6 enerInelasOld(nblock), tempNew(nblock),
  7 stretchNew(nblock, ndir + nshr), defgradNew(nblock, ndir + nshr + nshr),
  8 fieldNew(nblock, nfieldv), stressNew(nblock, ndir + nshr),
  9 stateNew(nblock, nstatev), enerInternNew(nblock),
  1 enerInelasNew(nblock)
C
  character*80 cmname
C
C the state variables are stored as:
C   state(*,1) = strain 11
C   state(*,2) = strain 22
C   state(*,3) = strain 33
C   state(*,4) = strain 12
C   state(*,5) = strain 23
C   state(*,6) = strain 13
C
C   state(*,7) = element deletion status (1=active, 0=deleted)
C
C
C the user needs to input:
C   props(1-3):   tensile properties (eps0t, epsmaxt, sig0t)
C   props(4-6):   compressive properties (eps0c, epsmaxc, sig0c)
C   props(7-13):  shear properties (G12, eps0s, epsmaxs, sigmaxs, A, B, C)
C   props(14-20): other properties (E1, E3, G23, G13, nu12, nu23, nu31)
C
  double precision k, nu12, nu23, nu31, E, E1, E2, E3, G12, G13, G23,
  1 eps0t, epsmaxt, sig0t,
  2 eps0c, epsmaxc, sig0c,
  3 eps0s, epsmaxs, sig0s, sigmaxs, eps1lim, eps2lim,
```

```

4   phi, cst, cubic, linear,
5   nu21, nu32, nu13, C11, C12, C13, C33, C44, C55, C66, A, B, C,
6   alpha, beta, gamma, delta, epsilon, zeta, eta, theta, iota, kappa,
7   lambda, mu, opt1, opt2, term1, term2, term3, term4, term5, term6
C
    parameter (zero = 0.d0, one = 1.d0, half = 0.5d0)
C
C
C*****
C input parameters
C*****
C
C tensile properties
C
    eps0t = props(1)
    epsmaxt = props(2)
    sig0t = props(3)
C
C
C compressive properties
C
    eps0c = props(4)
    epsmaxc = props(5)
    sig0c = props(6)
C
C
C shear properties
C
    G12 = props(7)
    eps0s = props(8)
    epsmaxs = props(9)
    sigmaxs = props(10)
    A = props(11)
    B = props(12)
    C = props(13)
C
C
C other material parameters
C
    E = props(14)
C
C unequal moduli in 1 and 2 directions can be overridden below
C otherwise assumed E1=E2 for 0/90 X-ply laminate
C
    E1 = E
    E2 = E
C
    E3 = props(15)
C
    G23 = props(16)
    G13 = props(17)
C
    nu12 = props(18)
    nu23 = props(19)

```

```

        nu13  = props(20)
C
        nu21  = E2*nu12/E1
        nu32  = E3*nu23/E2
        nu31  = E3*nu13/E1
C
C
C define stiffness terms
C
        cst = 1 - nu12*nu21 - nu23*nu32 - nu31*nu13 - 2*nu21*nu32*nu13
        phi = 1/cst
C
C
C check correctness of input data
C
        if (phi .LT. zero) then
            write (6,*) '**** ERROR with C matrix, check the input elasticity'
            call XPLB_EXIT
        endif
C
C
        C11 = E1*(1 - nu23*nu32)*phi
        C12 = E1*(nu21 + nu31*nu23)*phi
        C13 = E1*(nu31 + nu21*nu32)*phi
        C21 = C12
        C22 = E2*(1 - nu13*nu31)*phi
        C23 = E2*(nu32 + nu12*nu32)*phi
        C31 = C13
        C32 = C23
        C33 = E3*(1 - nu12*nu21)*phi
        C44 = G12
C
        if (nshr .GT. one) then
            C55 = G23
            C66 = G13
        endif
C
C
C*****
C 1st increment required to be elastic by ABAQUS
C*****
C
        if (StepTime .EQ. zero) then
C
            do k = 1, nblock
C
                alpha = C11*strainInc(k,1)
                beta = C12*strainInc(k,2)
                gamma = C13*strainInc(k,3)
                delta = C21*strainInc(k,1)
                epsilon = C22*strainInc(k,2)
                zeta = C23*strainInc(k,3)
                eta = C31*strainInc(k,1)
                theta = C32*strainInc(k,2)

```

```

        iota = C33*strainInc(k,3)
        kappa = C44*2*strainInc(k,4)
        lambda = C55*2*strainInc(k,5)
        mu = C66*2*strainInc(k,6)
C
C
C update the elastic stress
C
        stressNew(k,1) = stressOld(k,1) + alpha + beta + gamma
        stressNew(k,2) = stressOld(k,2) + delta + epsilon + zeta
        stressNew(k,3) = stressOld(k,3) + eta + theta + iota
        stressNew(k,4) = stressOld(k,4) + kappa
        stressNew(k,5) = stressOld(k,5) + lambda
        stressNew(k,6) = stressOld(k,6) + mu
C
C
C update state variables
C
        stateNew(k,1) = stateOld(k,1) + strainInc(k,1)
        stateNew(k,2) = stateOld(k,2) + strainInc(k,2)
        stateNew(k,3) = stateOld(k,3) + strainInc(k,3)
        stateNew(k,4) = stateOld(k,4) + 2*strainInc(k,4)
        stateNew(k,5) = stateOld(k,5) + 2*strainInc(k,5)
        stateNew(k,6) = stateOld(k,6) + 2*strainInc(k,6)
C
C
        if (stressNew(k,1) .LT. zero) then
            eps1lim = epsmaxc
        else
            eps1lim = epsmaxt
        endif
C
C
        if (stressNew(k,2) .LT. zero) then
            eps2lim = epsmaxc
        else
            eps2lim = epsmaxt
        endif
C
C
        if (abs(stateNew(k,1)) .GE. eps1lim .OR.
1      abs(stateNew(k,2)) .GE. eps2lim .OR.
2      abs(stressNew(k,4)) .GE. sigmaxs) then
C
C failure occurs
C
            stateNew(k,7) = zero
C
        else
C
C no failure occurs
C
            stateNew(k,7) = one
C

```

```

endif
C
C
end do
C
C
C
else
C
C
C
C*****
C subsequent to 1st increment
C*****
C
do k = 1,nblock
C
C calculate stresses
C
alpha = C11*strainInc(k,1)
beta = C12*strainInc(k,2)
gamma = C13*strainInc(k,3)
delta = C21*strainInc(k,1)
epsilon = C22*strainInc(k,2)
zeta = C23*strainInc(k,3)
eta = C31*strainInc(k,1)
theta = C32*strainInc(k,2)
iota = C33*strainInc(k,3)
kappa = C44*2*strainInc(k,4)
lambda = C55*2*strainInc(k,5)
mu = C66*2*strainInc(k,6)
C
C
C *****
C direct stresses
C *****
C
C sigma11
C *****
C
C check if tension/compression
C
if (stressOld(k,1) .LT. zero) then
C
C compression zone
C *****
C
if (abs(stressOld(k,1)) .LT. sig0c .OR.
1      abs(stressOld(k,1)) .GE. sig0c .AND.
2      stressOld(k,1)*strainInc(k,1) .LT. zero) then
C
C elastic loading or unloading
C
stressNew(k,1) = stressOld(k,1)

```

```

1                                + alpha + beta + gamma
C
1                                else if (abs(stressOld(k,1)) .GE. sig0c .AND.
1                                stressOld(k,1)*strainInc(k,1) .GE. zero) then
C
C loading in plastic plateau
C
1                                stressNew(k,1) = stressOld(k,1)
C
C                                endif
C
C                                else
C
C                                tension zone
C *****
C
1                                if (abs(stressOld(k,1)) .LT. sig0t .OR.
1                                abs(stressOld(k,1)) .GE. sig0t .AND.
2                                stressOld(k,1)*strainInc(k,1) .LT. zero) then
C
C elastic loading or unloading
C
1                                stressNew(k,1) = stressOld(k,1)
1                                + alpha + beta + gamma
C
1                                else if (abs(stressOld(k,1)) .GE. sig0t .AND.
1                                stressOld(k,1)*strainInc(k,1) .GE. zero) then
C
C loading in plastic plateau
C
1                                stressNew(k,1) = stressOld(k,1)
C
C                                endif
C
C                                endif
C
C sigma22
C *****
C
C check if tension/compression
C
1                                if (stressOld(k,2) .LT. zero) then
C
C compression zone
C *****
C
1                                if (abs(stressOld(k,2)) .LT. sig0c .OR.
1                                abs(stressOld(k,2)) .GE. sig0c .AND.
2                                stressOld(k,2)*strainInc(k,2) .LT. zero) then
C

```

```

C elastic loading or unloading
C
      stressNew(k,2) = stressOld(k,2)
      1      + delta + epsilon + zeta
C
      else if (abs(stressOld(k,2)) .GE. sig0c .AND.
      1      stressOld(k,2)*strainInc(k,2) .GE. zero) then
C
C loading in plastic plateau
C
      stressNew(k,2) = stressOld(k,2)
C
      endif
C
C
C
      else
C
C
C tension zone
C *****
C
      if (abs(stressOld(k,2)) .LT. sig0t .OR.
      1      abs(stressOld(k,2)) .GE. sig0t .AND.
      2      stressOld(k,2)*strainInc(k,2) .LT. zero) then
C
C elastic loading or unloading
C
      stressNew(k,2) = stressOld(k,2)
      1      + delta + epsilon + zeta
C
      else if (abs(stressOld(k,2)) .GE. sig0t .AND.
      1      stressOld(k,2)*strainInc(k,2) .GE. zero) then
C
C loading in plastic plateau
C
      stressNew(k,2) = stressOld(k,2)
C
      endif
C
      endif
C
C
C sigma33
C *****
C
C simple elastic updating; not crucial (delamination occurs before sigma33 failure)
C
      stressNew(k,3) = stressOld(k,3) + eta + theta + iota
C
C
C *****
C shear stresses

```

```

C *****
C
C shear (tau12)
C *****
C
    sig12_1 = 3*A*(stateOld(k,4))**2
    sig12_2 = 2*B*stateOld(k,4)
    sig12_3 = C
    cubic = (sig12_1 + sig12_2 + sig12_3)*2*strainInc(k,4)
C
C
    if (stressOld(k,4)*strainInc(k,4) .GT. zero) then
C
C loading
        stressNew(k,4) = stressOld(k,4) + cubic
C
    else
C
C unloading
C
        stressNew(k,4) = stressOld(k,4) + C44*2*strainInc(k,4)
C
    endif
C
C
C shear (tau23, tau13)
C *****
C
C simple elastic update; not crucial
C
C
    stressNew(k,5) = stressOld(k,5) + lambda
    stressNew(k,6) = stressOld(k,6) + mu
C
C
C
C update state variables
C *****
C
    stateNew(k,1) = stateOld(k,1) + strainInc(k,1)
    stateNew(k,2) = stateOld(k,2) + strainInc(k,2)
    stateNew(k,3) = stateOld(k,3) + strainInc(k,3)
    stateNew(k,4) = stateOld(k,4) + 2*strainInc(k,4)
    stateNew(k,5) = stateOld(k,5) + 2*strainInc(k,5)
    stateNew(k,6) = stateOld(k,6) + 2*strainInc(k,6)
C
C
    if (stressNew(k,1) .LT. zero) then
        eps1lim = epsmaxc
    else
        eps1lim = epsmaxt
    endif
C
C

```

```

        if (stressNew(k,2) .LT. zero) then
            eps2lim = epsmaxc
        else
            eps2lim = epsmaxt
        endif
C
C
        if (abs(stateNew(k,1)) .GE. epsmaxt .OR.
1      abs(stateNew(k,2)) .GE. epsmaxt .OR.
2      abs(stressNew(k,4)) .GE. sigmaxs) then
C
C failure occurs
C
            stateNew(k,7) = zero
C
        else
C
C no failure occurs
C
            stateNew(k,7) = one
C
        endif
C
        end do
C
    endif
C
C
    return
C
end
C
C*****

```

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