

THE COMPRESSIVE STRENGTH OF FIBRE-REINFORCED PLASTICS

by

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June, 1975

A thesis submitted for the degree of Doctor of
Philosophy of the University of London and for
the Diploma of Imperial College.

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ABSTRACT

Unidirectional fibreglass and fibre carbon rods with various fibre contents were manufactured using the leaky mould technique. Some physical and mechanical properties of these rods were closely examined and measured in order to ascertain their effect on the axial compressive strength of the composites. The results are discussed theoretically in conjunction with established and proposed failure modes.

The development of the split Hopkinson bar used for compression testing of materials at high strain rates is discussed and in particular, the development of condenser gauges to measure the stress pulses in the bar. A split Hopkinson bar apparatus was used to measure the axial compressive strength of the composites at strain rates in the range $(10^3 \text{ to } 10^4) \text{ sec}^{-1}$ and the results are discussed with respect to possible dynamic failure modes.

ACKNOWLEDGEMENT

I would like to express my appreciation to all those people whose help has contributed to the research which is presented in this thesis.

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CHAPTER ONE

INTRODUCTION

Various modes of compressive failure are possible in fibre-reinforced plastics. The work presented in this thesis is essentially confined to the case of the dominant mode apparent in a unidirectional composite in which a load is applied in the fibre direction. This mode is generally accepted as a buckling phenomenon. However, theoretical analyses based on the buckling failure mode (see for instance ROSEN (1964)) predict strengths usually much higher than those experimentally realized. This research is undertaken to investigate, experimentally and theoretically, this lack of agreement.

The most probable explanation would be that one or more of the other failure modes, such as the shear failure of the fibre found in fibrecarbon, have lower strengths and cause failure. Each of the failure modes envisaged for fibreglass are eliminated, except possibly for one, consisting of a stable form of buckling which can account for the experimentally observed compressive strengths. Additional compression testing at high strain rates suggest that the same failure mode applies, independently of strain rate.

Fibreglass, as used in the text, refers solely to unidirectional glass-fibre-reinforced epoxy resin unless otherwise stated: fibrecarbon is similarly defined. Each component is specified in the following manner; the first letter refers to the type of resin matrix (either A or B), the second letter refers to the type of fibre (E for E-glass, R for R-glass, C for carbon) and the following number refers to the fractional fibre content by volume percent, (eg. BR37). All fractional contents which are not specified are by volume. Strength refers to the axial compressive strength unless otherwise stated. In general, S.I. units are used throughout. However, as all workshop machinery used in the research programme was calibrated in inches (units of 0.001 in.), lengths are sometimes specified in inches for convenience.

A general list of the symbols used in the text is given. Due to the diversive nature of the subject matter, a few symbols

are duplicated in different chapters and symbolize different parameters; however, the correct parameter is readily ascertained from the context.

LIST OF SYMBOLS

c_o	Wave velocity ($= \sqrt{E/\rho}$)
C	Capacitance
C.R.O.	Cathode ray oscilloscope
E	Young's modulus (axial) or electric field
G	Shear modulus (longitudinal-transverse)
I	Moment of inertia or electric current
L	Length
M	Mass
n	Positive integer
P	Load or pressure
q	Lateral reaction
Q	Lateral load or electric charge
U.C.S.	Ultimate compressive strength
t	Time or thickness
V	Fractional volume, velocity or voltage
W	Energy function
$\dot{X}$	dX/dt
$\ddot{X}$	d^2X/dt^2
(x,y,z)	Cartesian coordinates
(r,z, θ)	Cylindrical polar coordinates
α	Angle
β	Angle
γ	Shear strain
Δ	Incremental increase
ϵ	Strain
ρ	Density
θ	Angle
λ	Wavelength
ν	Poisson's ratio (longitudinal-transverse)
σ	Stress
σ_n	Normal component of stress
γ	Shear stress

Subscripts

c	Composite or critical
f	Fibre
I	Incident
m	Matrix
R	Reflected
s	Specimen
T	Transmitted
v	Voids

Dimensions

in.	Inch
revs.	Revolutions
X°	X degrees (angular)
°C	Degrees centigrade
%	Percent
min.	Minute
sec	Second
μsec	Microsecond
kg.	Kilogram
m	Metre
mm	Millimetre
μ	Micron (10^{-6} m)

PART ONE

STATIC STRENGTH

CHAPTER TWO

MANUFACTURE OF FIBRE-REINFORCED RODS

2.1. RAW MATERIALS

All the work was confined to composites made from two types of glass fibre and three types of carbon fibre. All fibres had a circular cross-section with a standard diameter of 10μ (ten microns) for glass fibres and 8μ for carbon fibres. The fibres were embedded in one of two types of epoxy resin matrix.

2.1.1. GLASS FIBRES

The glass fibres were in the form of a continuous roving wound onto a cylindrical base called a cheese. A roving consists of a large number of individual parallel fibres bound together with a size. The size consists of a variety of chemicals coated onto the fibres and accounts for only a few percent of the total weight of the roving. The size not only holds the roving together but also acts as a lubricant and protects the fibre surface from environmental and mechanical damage. In particular it prevents the fibres from abrading against each other. A small proportion of the size consists of a coupling agent which helps to produce a chemical linkage between the glass and the surrounding matrix within the composite. Such a linkage helps to increase the bond strength of the glass fibre to the matrix, otherwise the bond would be purely mechanical.

The E-glass fibres (XG1229 roving) were supplied by Turner Brothers Asbestos Ltd. and the R-glass fibres were supplied by St.-Gobain Industries. The compositions of both glasses are given in table 2.1. The E-glass roving was thicker, consisting of about 5,900 fibres (1200 tex) in contrast to the 3,700 fibres (800 tex) of the R-glass roving. An epoxy resin-compatible size was used on the E-glass while the R-glass size consisted mainly of epoxy resin and, at the recommendation of the manufacturer, the R-glass was stored at a temperature below 4°C .

All the glass was unwound from the same two cheeses, (one R-glass, the other E-glass). Checks were made on the glass content

TABLE 2.1

Chemical composition of glass fibres (% by weight)*

	<u>E glass</u>	<u>R glass</u>
Si O ₂	54	60
Al ₂ O ₃	15	25
Ca O	17	9
Mg O	5	6
B ₂ O ₃	8	-
Na O	1	-

Impurities amount to less than 1 %

TABLE 2.2

Composition of matrix resins (parts by weight)

		<u>resin A</u>	<u>resin B</u>
Araldite	MY750	100	100
Hardener	HT907	80	0
Hardener	HT972	0	27
Accelerator	DY063	1	0
Initial viscosity at 60°C		0.7 poise*	3 poise*

* Manufacturers' values

of unit length rovings before and after the manufacture of the fibreglass rods and were found to be consistent. The size content of the R-glass roving was 1.5 % by weight while it varied between one and two percent in the E-glass roving.

2.1.2. CARBON FIBRES

The carbon fibres were manufactured by Morganite Ltd. and were received in the form of straight tows. Each tow was one metre in length and consisted of about 10,000 individual parallel fibres. In the "as received" condition, each tow had three revolutions of twist which helped to keep the tow together and improved handling characteristics.

Three types of carbon fibres were used, labelled as type IS, type II and type IIIS. "S" designates that the fibres have been surface treated (i.e. chemically etched) to improve the bond strength between the fibre and the surrounding matrix in the composite.

The basic carbon fibre produced during the manufacturing process is type III. Type I (higher stiffness) and type II (higher strength) fibres are produced by further heat treatments of the type III fibre. The carbon fibre differs from the glass fibre in that it is anisotropic and much stiffer; (i.e. the longitudinal Young's modulus of the carbon fibre is much greater than both the transverse Young's modulus and the Young's modulus of the glass fibre). The carbon fibre is also opaque and black in appearance. The label "S" is omitted from future references to the fibres.

2.1.3. EPOXY RESINS

The fibres were embedded in one of two types of matrix. Both were hot setting resins and were labelled for convenience as resin A and resin B. They were supplied by Ciba Geigy Ltd. and their compositions are set out in table 2.2. The ingredients for each resin were accurately weighed and mixed according to the manufacturer's instructions. The usable lives of the two resins at room temperature were limited to two weeks (resin A) and ten hours (resin B) and the resins were therefore prepared

in small quantities at a time. The resins were stored in a dessicator because they were hygroscopic.

2.2. MANUFACTURE OF RODS

The steel mould used to produce all the composite material is shown in Fig. 2.1. The lower portion of the mould consisted of a base, two sides and two ends which were screwed together. To close the open mould, the close fitting upper portion was lowered into it. The method of composite production utilised the standard "leaky mould" technique after manual laying up of the fibres. This mould produced a cylindrical rod 8 in. long with a cross-sectional diameter of about 0.395 in.

2.2.1. INTRODUCTION OF FIBRES INTO THE MOULD

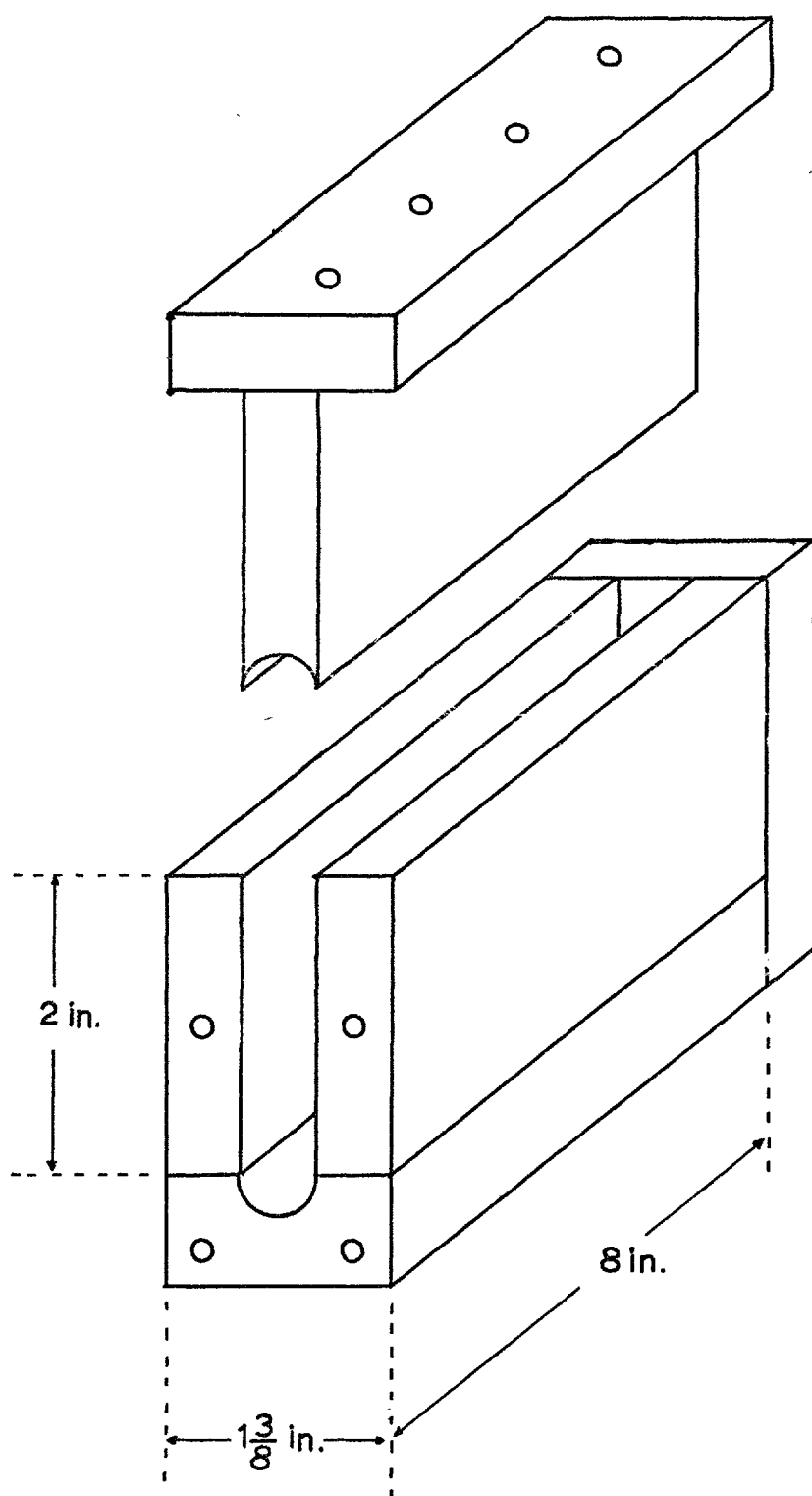
First, the ends of the clean open mould were removed. A length of roving was unwound from the cheese and laid in the mould with an overlap of at least several inches at each end. The roving was then cut and the process repeated. This was continued until the desired number of rovings were present in the mould. Care was taken to ensure that the rovings were well aligned, often by temporarily pulling the rovings in tension. This operation is shown in Fig. 2.2 together with the cheese and mould.

The upper portion of the mould was then placed in position to hold the rovings together while their ends protruding out of the mould were cut off using a pair of scissors. The lengths of the rovings were cut to about $\frac{1}{2}$ in. less than the length of the mould. The mould was then opened and the bunch of rovings carefully removed and put to one side. The mould was coated with a suspension of teflon in acetone to act as a release agent. The bunch of rovings was replaced in the mould and the ends of the mould replaced. All the screws were lubricated with a high temperature grease.

The same method was used in the production of fibre carbon rods. The only difference was that bunches of about ten tows were cut together into five equal lengths (slightly less than the length of the mould) outside the mould.

Fig. 2.1

Mould used for production of composite rods,
one end removed



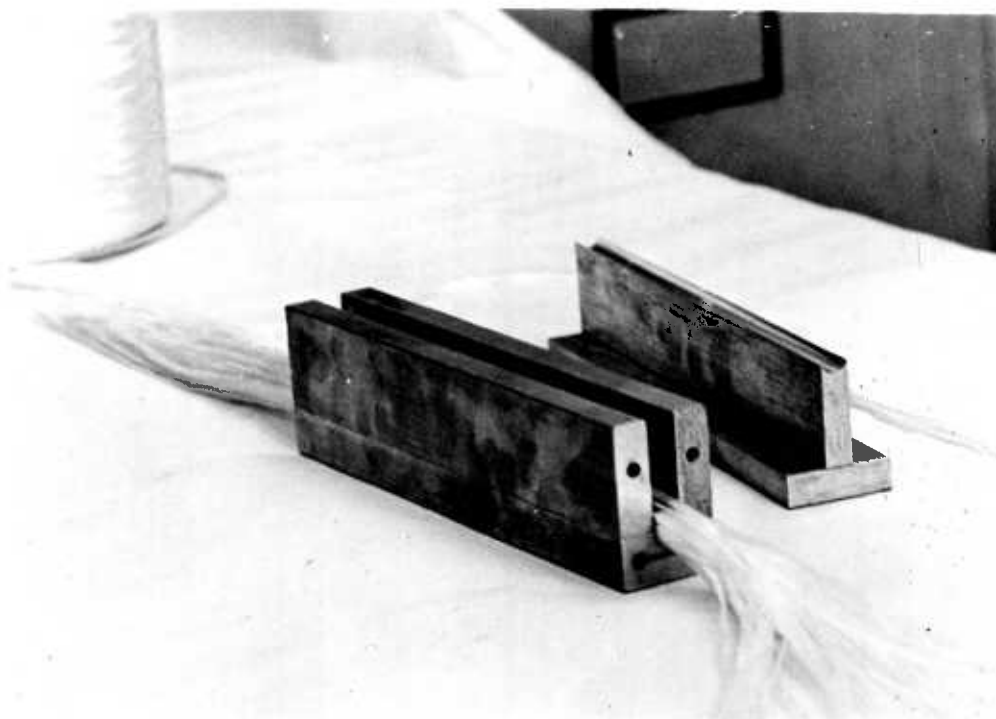


Fig. 2.2 Open mould with glass fibre strands

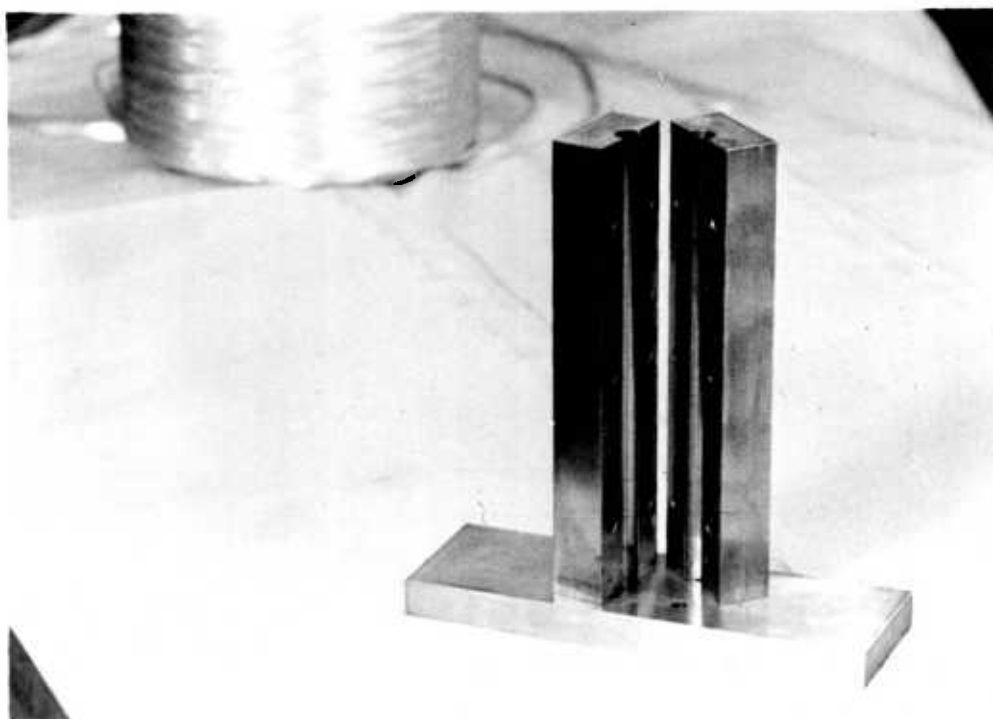


Fig. 2.3 Vertical mould

The cut tows were then ready to be impregnated with the resin, usually within a vacuum chamber.

2.2.2. USE OF VACUUM CHAMBER

The vacuum chamber built for the purpose of impregnating the fibres is shown in Fig. 2.4, complete with the mould. The vacuum chamber consisted basically of a glass dome resting on a flat metal base. The open mould was secured in position on an electric heater which raised the temperature of the mould to the desired level. The heater consisted of an electrical resistance wire running inside, and insulated from, a metal tube of rectangular cross-section. The two electrical terminals were connected to a variac via two insulated attachments in the base plate of the chamber. The variac was used to vary the voltage and hence the power output of the heater. The temperature was measured using a mercury thermometer attached to the exterior of the heater.

Above the open mould was secured the device for introducing the resin into the mould. The resin was poured into the hopper which consisted of an open rectangular box-shaped container with a circular hole (1/16 in. diameter) in the bottom. It rested on, and travelled along, the length of a flat plate from which the region directly above the mould had been removed so that when the hopper crossed this region, the resin was free to pour into the mould. The hopper could be moved in either direction by means of wires attached to the hopper and to a wheel. The wheel could be rotated manually from outside the chamber using a special attachment. When the chamber was in use, a metal screen was placed over the dome as a safety precaution in case the dome collapsed.

2.2.3. IMPREGNATION OF FIBRES

The impregnation procedure was as follows. First the mould was secured in place on the heater and the resin poured into the hopper. The dome was replaced and a vacuum of between 0.5 and 1.0 torr applied. This was measured using a vacuum gauge connected to the chamber. The mould was heated to and maintained at a temperature of 70°C. The pressure in the chamber was always

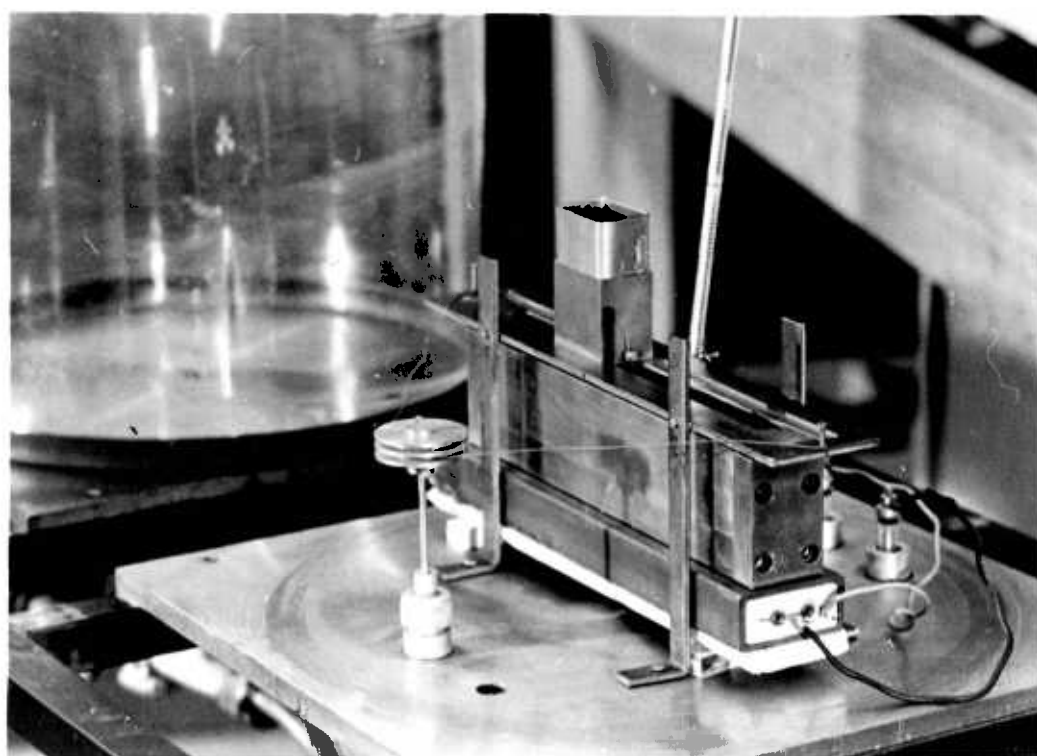
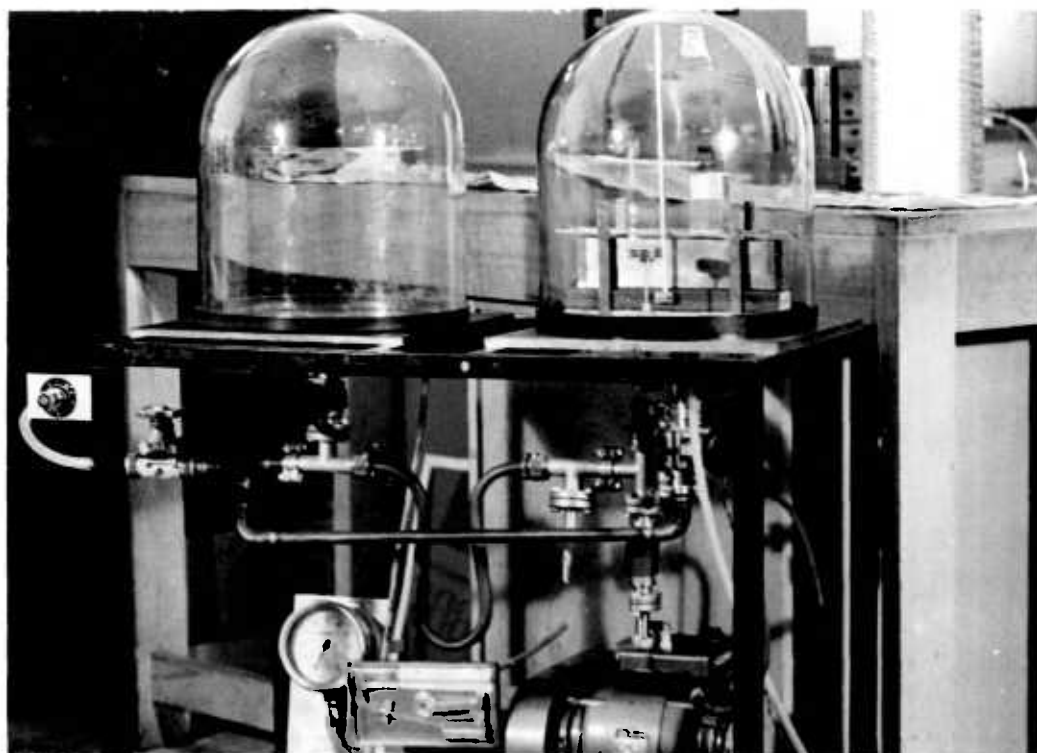


Fig. 2.4 Vacuum Chamber impregnating apparatus and mould

well above the saturated vapour pressure of the epoxy resin (10^{-2} torr) but any dissolved air and water in the resin boiled off. This was allowed to continue for about 15 minutes by the end of which time nearly all boiling had ceased. The resin was then poured evenly over the fibres using the method previously described. Another 30 minutes were allowed for the resin to fully impregnate all the fibres. Sufficient resin was used to ensure that all the fibres were fully immersed. The vacuum was then released, the mould removed and the upper portion of the mould inserted into position. The composite was then ready to be cured in the hot press.

The impregnating apparatus was cleaned with acetone after use. Thirty minutes impregnation time at a temperature of 70°C was standard although this could be varied because a longer impregnation time was required if more fibres were present. Increasing the temperature reduced the viscosity of the resin and hence improved the impregnation. However this also reduced the usable life of the resin and so a compromise was reached.

2.2.4. CURING OF COMPOSITE RODS

The hot press used for the curing process consisted of two electrically heated flat plates. The plates were parallel to each other and lay in a horizontal position. They could be pressed together using a hydraulic ram attached to the lower plate. The ram was controlled by a manually-operated hydraulic pump.

The complete mould was placed in the press which was maintained at a temperature of 125°C . The press was closed until the mould was in contact with both plates. A dwell time of 15 minutes was allowed to let the mould plus contents reach the temperature of the press and the lower plate of the press was then slowly raised by hydraulic pressure until the mould was fully closed.

During this process, excess resin was forced out of the mould through the gaps between the two portions. Most of this resin came from the loose-fitting ends rather than from the close-fitting sides of the mould. Only a relatively few fibres escaped into the side gaps of the mould. This was due to the sharp edges of the upper portion which forced fibres away from the gaps.

After one hour, by which time the resin had set hard, the

hydraulic pressure was released and the mould removed. A previous coating of grease on the plates helped in the removal of the mould from the press and also helped in the removal of the excess resin, which had escaped from the mould, from the plates. The mould was then dismantled and the composite rod removed. In order to complete the cure of the resin, the rod was placed in an oven maintained at a temperature of 150°C for at least three hours.

The dismantled mould was cleaned using first a razor blade and then (320 grade) wet silicon carbide paper. The curved parts of the mould were best cleaned by wrapping the paper around a cylinder with a diameter chosen so that the curvature of the exposed paper fitted in the mould exactly. The dimensions of each composite rod were checked and all were found to have a uniform diameter along their lengths to within ± 0.001 in.

2.2.5. PRODUCTION OF RESIN RODS

The previously described mould could be used to produce fibreglass rods with a fibre content as low as 35 %. Below this value the glass tended to sink to the bottom half of the rod, which resulted in a non-uniform distribution of fibres over the circular cross-section. Large air bubbles also tended to be trapped inside the mould. For this reason most of the pure resin rods were made in a different mould, shown in Fig. 2.3. The resin could be poured directly into the closed mould, which stood vertically so that no air bubbles could be trapped. For the sake of consistency the resin rods were taken through the same impregnating and cure cycles as the fibre-reinforced rods. The mould was dismantled into its three components in order to release the rod from the mould. The same operating techniques were used on both moulds.

CHAPTER THREE

PREPARATION OF TEST SPECIMENS

3.1. PRELIMINARY TREATMENT OF RODS

The diameter of each rod varied with the resin content. The initial cross-section was only approximately circular and included two lips running the length of the rod. The surface 0.015 in. contained a higher proportion of resin than the bulk of the rod. It was therefore desirable to remove this outer layer and produce exactly cylindrical rods, each with a diameter of 0.375 in.

Initial attempts to achieve this uniformity included "turning down" using a lathe and water-based coolant. This proved to be unsatisfactory because it was found to be almost impossible to obtain a constant diameter along the length of the rod and its cross-section tended to be oval rather than circular. Microscopic examination showed that cutting with a lathe's tool piece damaged the surface layer of the rod causing debonding cracks to penetrate deep into the interior.

For this reason all the rods were "centreless ground" which operation was undertaken, in two separate batches, by an outside firm. All the rods were ground down to a diameter of 0.375 in. with a specified tolerance of ± 0.0005 in. The centreless grinding process produced "perfectly cylindrical" rods while microscopic examination showed that there was no surface damage. The diameter of each rod, when measured, was found to be well within the specified tolerance with a maximum error estimated at 0.0002 in. The variations within each of the two batches were probably even less because all the rods in each batch were ground down with the grinder fixed at the same setting. Approximately $\frac{1}{2}$ in. of material from each end of every rod was always discarded.

3.2. COMPRESSION SPECIMENS (AXIAL)

In this type of specimen the load was applied to the composite material in a direction parallel to the axis of the rod, which was also parallel to the fibres. The standard specimen consisted of a disc-shaped piece of composite material which was obtained simply by cutting off a piece of the rod. The majority of specimens were cut off using a silicon carbide slitting wheel attached to a lathe.

3.2.1. CUTTING OF SPECIMENS

The lathe chuck was replaced by a collet which secured the attachment which in turn held the slitting wheel. The disc-shaped wheel had an outside diameter of 5 in. and a uniform thickness of 0.01 in. and was lubricated with a water-based coolant; it was rotated at the maximum speed available (2,200 revs./min.). The rod to be cut was held firmly in position by screws pressing the rod into a V block. The portion of the rod to be cut off protruded out of the V block while the V block was secured in the tool holder of the lathe. This operation is shown in Fig. 3.1.

The axis of the rod had to be aligned with the axis of the slitting wheel before the cutting could commence. This was done so that the two flat faces of the disc lay in planes normal to the fibre direction. To achieve the required alignment, the composite rod was first replaced by a straight steel cylindrical rod about one metre in length held firmly in the V block. The nut securing the V block was then slackened, so that the block could be moved. The steel rod was then aligned with the geometry of the lathe by applying a set square at each end of the rod in turn. The nut was tightened and the composite rod replaced in the block. The accuracy of this method improved as the length of the steel rod increased; using the one-metre steel rod, the rods were estimated to be less than $1/8^\circ$ off-axis with the slitting wheel. It was assumed that the geometry of the lathe was well aligned with the axis of the rotating collet as is always true for any well built lathe.

The composite rod could be moved in a horizontal plane using the two calibrated screw threads of the lathe. The use of one screw thread determined the length of the disc to be cut off

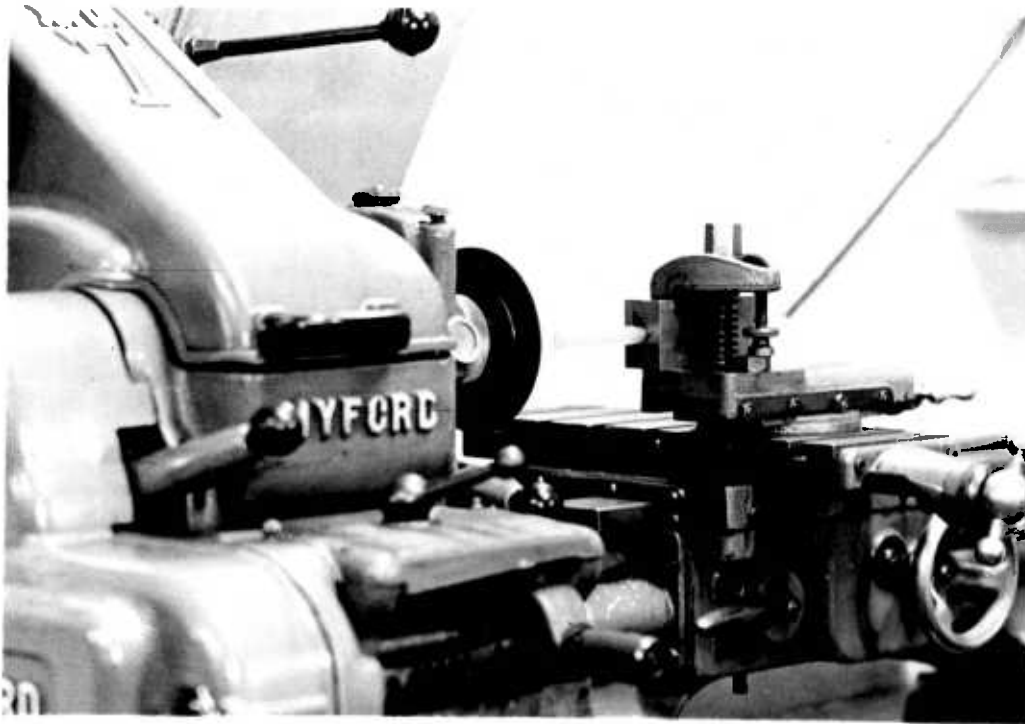


Fig. 3.1 Cutting off disc specimens from composite rod.

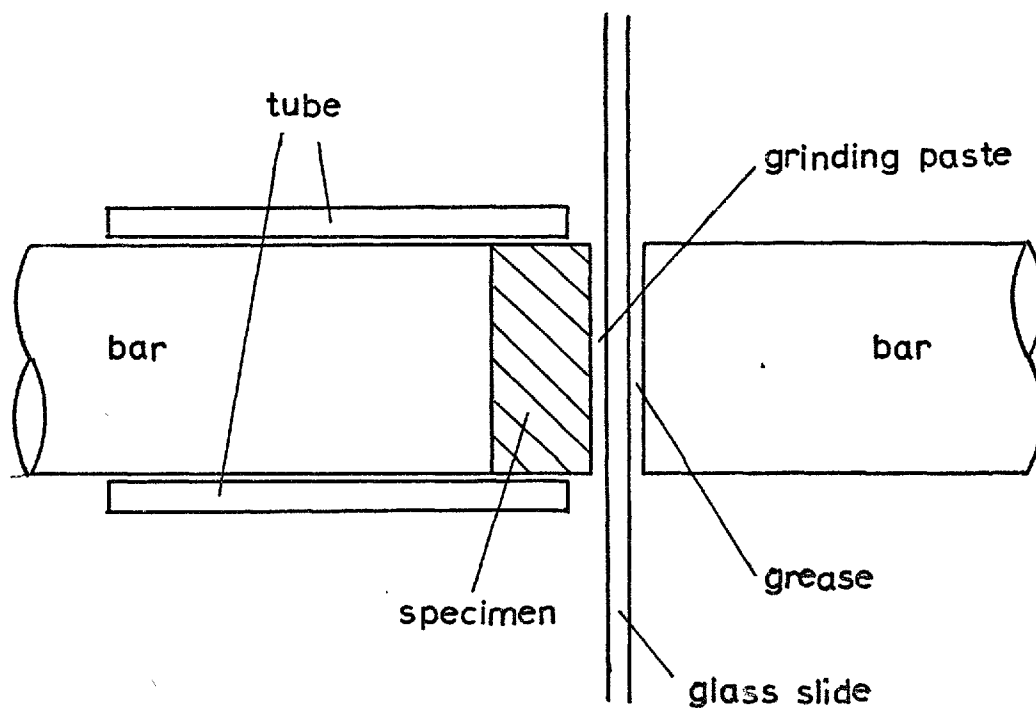
while use of the other moved the rod into the cutting edge of the slitting wheel. The second screw thread was turned manually at a uniform rate slow enough to prevent overheating at the cutting edge. Both faces of all specimens were cut on the same occasion to ensure that they were as parallel as was practically possible. If the wheel ceased to "run true" (due to warping) or if any other defect was found, it was replaced. Microscopic examination showed that the slitting wheel caused no surface damage (i.e. no debonding of fibres) even if the discs were wafer thin. The slitting wheel produced a good finish on the specimen faces with a high degree of flatness although a "faint grain" produced by the wheel could be visually detected.

3.2.2. POLISHING OF SPECIMENS

Most of the disc specimens were tested "as cut". In addition, the end faces of some specimens were polished flat and parallel using a manual technique which is now described.

A series of V blocks mounted in line to the same base were recut on a milling machine so that the V grooves had the best alignment possible. The piece of apparatus adapted for use was a split Hopkinson bar apparatus similar to that described in chapter 12. Two cylindrical rods each of diameter 0.375 in. and made of hardened steel, were mounted and secured in line in the V blocks. The ends of the bars had been previously ground flat and parallel, normal to the axis of the rods, using a surface grinder. The specimen to be polished was sandwiched between the two bars as shown in Fig. 3.2. A plate glass microscope slide was sandwiched between the specimen and one rod. The glass face touching the rod was lubricated with grease while the other face was coated with a paste of grinding powder and water. The slide was moved manually with a reciprocating motion in its one plane of freedom and the contact pressure was adjusted to suit by pressing one rod against the specimen before securing. The tube slid over one bar and the specimen and prevented the specimen from moving. Polishing was continued until visual inspection showed that the whole surface had the same texture, then the specimen was inverted and the other face was polished. The slide was replaced when it became worn. From time to time as the bar end faces became worn, they were

Fig. 3.2 Longitudinal section of polishing process



polished using grinding powder on both faces of the slide and no specimen. Both rods were often rotated into new positions in order to eliminate any preferential polishing due to non-uniform pressure acting at the interface. Long specimens could be polished by securing the specimen itself in a V block.

Initial rough polishing involved the use of 280 grade silicon carbide powder. Successively finer grades (400, 600, 800) were used depending on the required finish. A final polishing using 1200 grade (average grit size 3μ) boron carbide powder was usually only confined to specimens for dynamic testing and the use of finer abrasive powders in order to obtain flatter surfaces was considered to be unnecessary. This polishing technique was a somewhat laborious process but proved to be very successful for producing flat and parallel specimen faces.

3.2.3. TAPERED SPECIMENS

In addition to the standard disc-shaped compression specimens, a few specimens were made with tapered central regions. These were up to 50 mm longer than the standard (3 mm) length of disc specimens and required additional end support in the form of steel rings which were held in position with a cold-setting epoxy resin. Compressive failure occurred without the rings, with a brush-like appearance at one end as described by EWINS (1970, 1971) for similar types of compression specimens. A variety of different geometries were used, all having an axis of symmetry, some of which are shown in Fig. 8.15. The tapered regions were produced by machining in a lathe. Specimen type (b) of Fig. 8.15 was produced by using a tool piece especially adapted to move in a circular arc of radius 100 mm or 125 mm.

From a practical point of view, the disc specimen offered four distinct advantages over the tapered specimens :-

- 1) Ease of fabrication.
- 2) Economic usage of composite material.
- 3) Suitability for high strain rate testing.
- 4) Potential for direct microscopic observation of fibre ends in reflected and transmitted light. (Carbon fibres observed in reflected light only due to their being opaque).

3.3. THREE-POINT-BEND SPECIMENS

This type of specimen is widely used for the determination of interlaminar shear strength. It has a rectangular block shape with the fibres lying parallel to the longest faces. Such shaped specimens can be cut easily from the flat boards in which shape composites are usually made but in order to be consistent, they were all cut from the same cylindrical rods from which the compression specimens were obtained.

Cylindrical pieces of the composite rod ($5/8$ in. long) were mounted, using tam wax, in V grooves cut into a block of asbestos. The grooves were aligned with the faces of a slitting wheel which formed part of an industrial cutting machine. Three longitudinal cuts were made to produce two specimens of thickness $1/8$ in. each. The two remaining curved faces of each specimen were removed using a surface grinder to produce a specimen of $1/4$ in. width.

3.4. COMPRESSION SPECIMENS (TRANSVERSE)

These specimens were in the form of rectangular blocks and were cut from the cylindrical rods using similar techniques to those described for three-point-bend specimens. The load was applied to two faces lying parallel to the fibres and to each other. Care was taken to ensure that these two faces, in particular, were flat and parallel.

CHAPTER FOUR

MEASUREMENT OF THE PHYSICAL PROPERTIES OF COMPOSITES AND THEIR CONSTITUENTS

4.1. FIBRE CONTENT

4.1.1. GLASS FIBRE CONTENT

The glass fibre content of composites was measured using the "burn off" technique. A disc-shaped piece of composite rod was used, the thickness of which had been measured using a micrometer screw gauge in order to determine its volume. Its mass was measured using a sensitive beam balance. The disc was gently heated in a crucible over a bunsen burner flame so that the resin smouldered but remained below its flash point. When all smoking had ceased, the temperature was slowly increased until all traces of the resin were removed but never allowing the glass to melt. The disc, still retaining its integrity, was then composed of glass only and possessed an off-white colour; it was reweighed after cooling. The fractional mass content of glass per unit mass and volume of the composite were determined from these measurements. The glass content of a roving was determined in a similar manner except it was found to be more convenient to heat the roving in a furnace maintained in the temperature range 400°C to 500°C.

4.1.2. CARBON FIBRE CONTENT

The fibre content of fibre carbon was obtained in a similar manner to that of fibreglass except that the resin was removed by acid. The disc was heated in a mixture of concentrated sulphuric acid and hydrogen peroxide as described by STURGEON (1971).

4.2. DENSITY

4.2.1. FIBRE DENSITY

Fibre density was measured using a relative density bottle. The accuracy of the results was sensitive to temperature fluctuations during measurement and so all weighings were performed in an air-conditioned room maintained at a temperature of 20°C. Care was taken to remove all air bubbles when the fibres were immersed in the distilled water.

In the case of glass fibres, the size was removed from the roving using the method described in section 4.1.1. Some measurements were taken with the size intact in which case the rovings were immersed in paraffin because of the poor wetting characteristics of water with the size. Paraffin, however, has the disadvantage of having a higher coefficient of thermal expansion than water. The difference between the roving densities with and without size agreed with the calculated value using the known weight fraction and density of the size. This showed that that the heat cleaning process did not change the glass density. The calculations used to determine the fibre density are given by STURGEON (1971).

4.2.2. COMPOSITE DENSITY

Composite density was easily determined because the volumes of the geometrically regular composite rods and discs could be directly calculated. The density could be determined from the same disc which would be later used for fibre content measurement, as described in section 4.1.1. The density values of discs determined before and after polishing showed that the volume of a disc "as cut" is given by using the measured value of the disc thickness less 0.01 mm. The density of each rod was determined in a similar manner except that a pair of calipers were used to measure the thickness (length).

4.2.3. RESIN DENSITY

The density of the resin within the composite was found to vary with the fibre content; the reasons for this are discussed in chapter 5. Its value could be calculated once the fractional fibre content, fibre density and composite density were known.

The following notation is used :-

ρ density

M mass

V fractional volume

A fractional area of cross-section

Subscripts f, m and c refer to fibre, resin matrix and composite respectively. The fibre volume fraction was calculated for each rod from data obtained from a disc specimen.

$$V_f = A_f = M_f/\rho_f \quad (1)$$

Due to the nature of the manufacturing process, this value was expected to be uniform over the length of the rod. The matrix density is given by :-

$$\rho_m = (\rho_c - A_f \rho_f)/(1 - A_f) \quad (2)$$

The value of ρ_m could be calculated for a single disc or an average value over the whole rod could be calculated depending on whether the disc or rod density was used.

4.3. VOID CONTENT

A void inside a homogeneous piece of material is defined as a volume surrounded by an internal boundary and not occupied by the material. It is usually occupied by air. The fractional void content of the piece of material is then defined as the total volume of all voids divided by the volume of the piece. A 100 % void content therefore means that no material is present. For the fractional void content to have any real meaning, the piece of material must be large enough to include a large number of voids. It is then possible to express the void content as a macroscopic function of the space coordinates bounded by the

external faces of the piece. The void content gives no information about the shape or size distributions of the voids.

In a composite material, void content needs to be more rigorously defined because account must be taken of the "void content" in each constituent material. In fibre composites, the fibres are considered to be void-free and hence voids can only occur in the matrix. The fractional void content is then defined as the volume of all voids in the matrix divided by the total volume of the composite including voids, fibres and matrix. The maximum possible void content is therefore equal to $(1 - V_f)$ and corresponds to a bundle of fibres with no matrix present.

The void content is determined by subtracting the measured density of the voided composite from the value expected for a composite with the same fibre content and no voids. Using the same notation as in section 4.2.3. and including subscript v referring to voids, the fractional void content is given by :-

$$V_v = (V_f (\rho_f - \rho_m) + \rho_m - \rho_c) / \rho_m \quad (3)$$

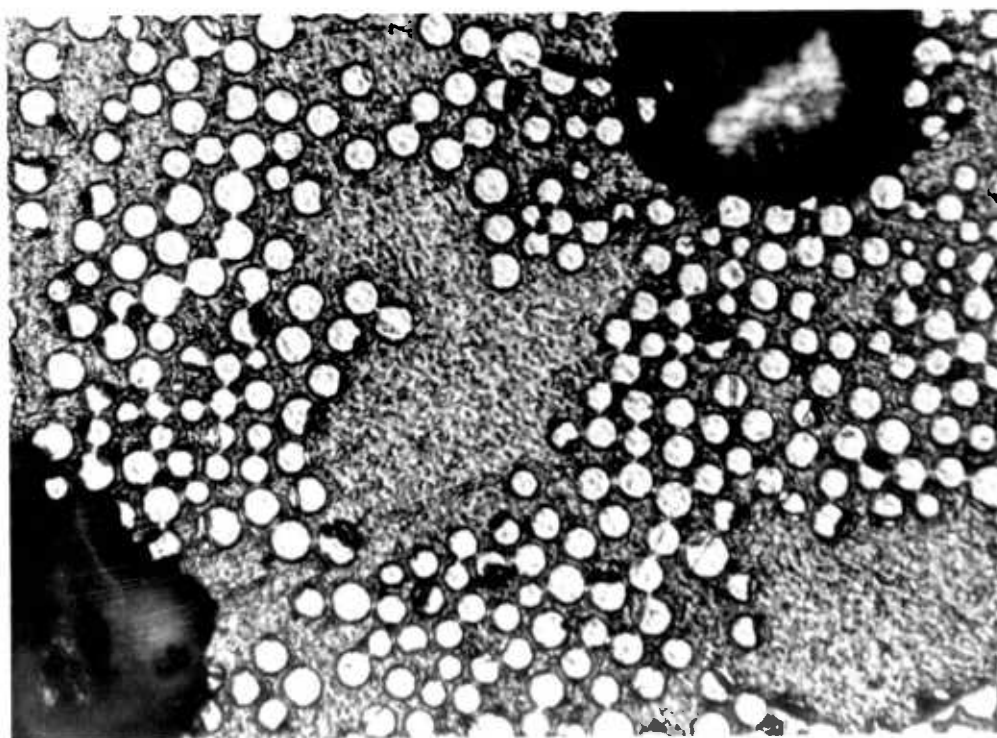
The value of ρ_m varies with the fibre content and is determined from equation (2) using a microscopically-verified void-free composite with the same fibre content. The value of V_f is determined in a similar manner as for void-free discs.

This method gives poor accuracy for void contents less than $\frac{1}{2}$ % due mainly to uncertainty in the value of ρ_m . The scatter in resin densities shown in Fig. 5.1 for a particular fibre content could account for a calculated void content of $\pm \frac{1}{2}$ %. Void-free composites can therefore only be verified by microscopic examination. The density of the rod ρ_c is used in equation (3) and hence the average void content of the whole rod is determined. The exposed voids at the ends of the composite disc specimens act as capillaries and absorb liquid debris. This makes the use of short specimens for density measurement undesirable.

Polishing techniques similar to those described by McNEIL and BENNETT (1964) were employed on disc specimens prior to examination using a light microscope. Three types of voids were observed and photographed as shown in Figs. 4.1, 4.2, 4.3 and 7.4. These were interstitial, general and planar voids as defined and described by PAUL and THOMSON (1965). The first two were

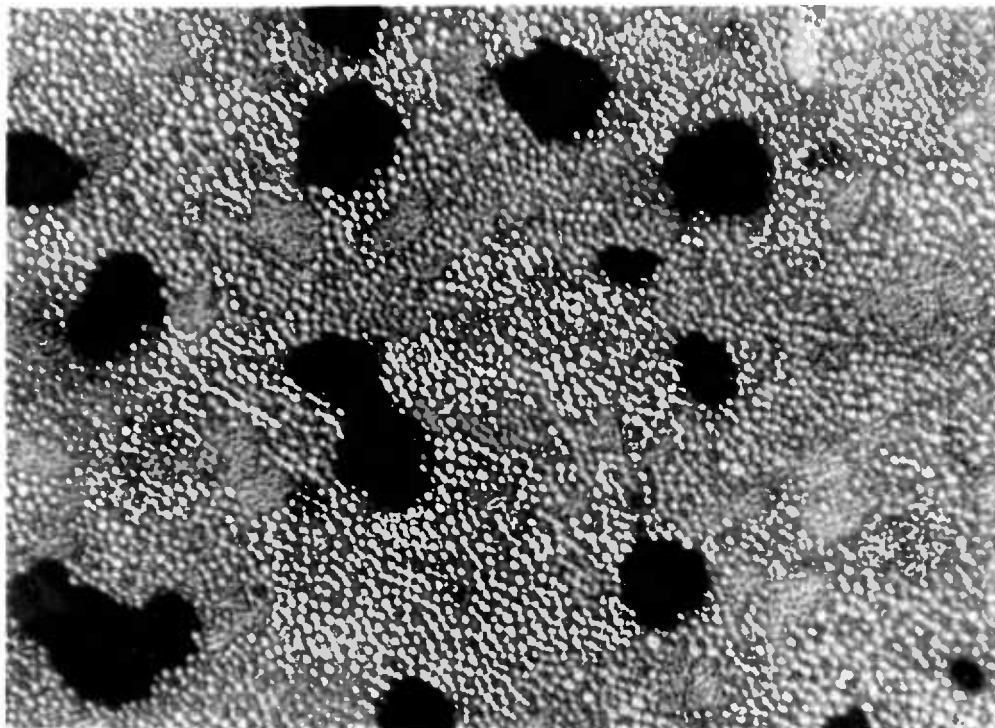


Transmitted light (x 140)



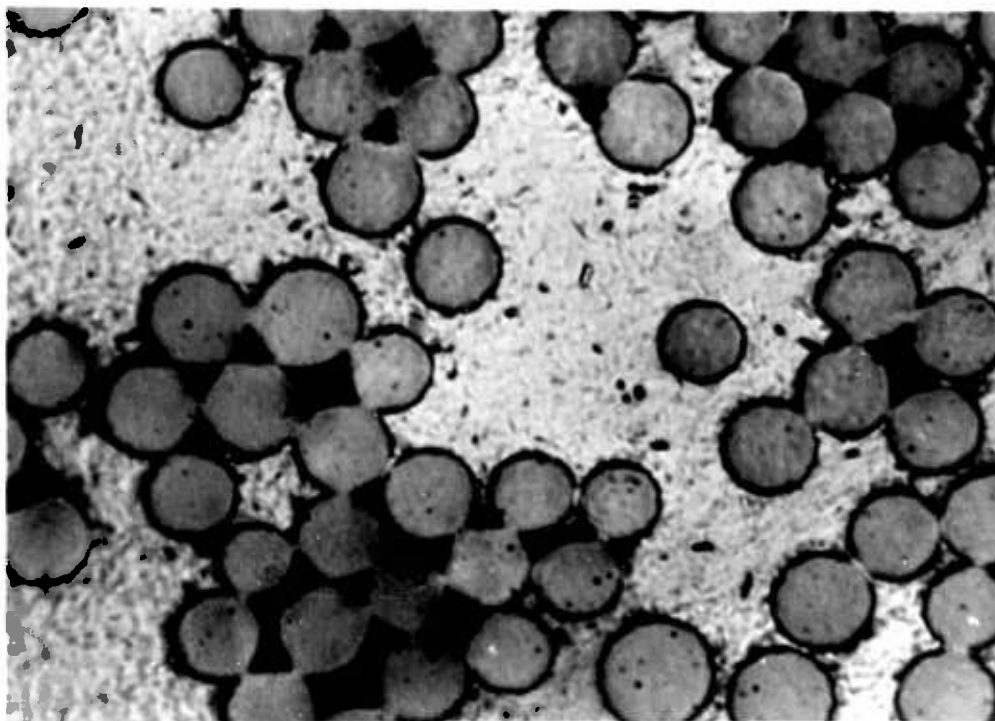
Reflected light (x 140)

Fig. 4.1 General voids in VAE 47 specimen



Reflected light (x 60)

Fig. 4.2 General voids in VAE 47 specimen



Transmitted light (x 560)

Fig. 4.3 Interstitial voids

essentially one-dimensional, elongated in the fibre direction. The interstitial void occupied the tricorn caused by three fibres in mutual contact although this could be extended to the case of several fibres in close proximity. It was usually caused by poor impregnation of the fibres. A general void occupied a much larger region involving many fibres. It was usually caused by the entrapment of air bubbles or by the evolution of a volatile by-product during the cure cycle. The planar void was essentially a two-dimensional void lying parallel to the fibres as shown in Fig. 7.4. This type of void was created by a volatile reaction between resin A and type II carbon fibres and was characteristic of the AC_{II} composites only. The gas had collected in the resin-rich region separating two adjacent tows similar to the region shown in Fig. 7.3.

The most direct and accurate method of measuring void content is by stereographic techniques, viewing polished cross-sections of the composite through a microscope as carried out by SANDS et al. (1967). In the general voids shown in Fig. 4.1, fibres are not visible. This could have been due to surface tension preventing wetted fibres from entering the interface boundaries of the voids during impregnation. Another possibility is that the ends of fibres initially present in the void lacked support and broke off during the cutting and polishing processes. In Fig. 4.1 can be seen two resin-rich regions devoid of fibres. These correspond to regions either just above or just below a general void. Therefore the first possibility is probably correct. This cannot be regarded as a general rule for as the void or the fibre content increases, the void is forced to engulf some fibres. In addition an observed general void can be a region with a large density of interstitial voids where the fibre ends did not receive enough support and broke off. Stereographic measurements of void content can hence lead to erroneous results.

It was noted that many voids traversed the thickness of the disc specimen and it was assumed that the fibres inside these voids, if any, did not "fall out". The validity of this assumption depends on the shape and size of the voids. A few fibreglass rods made by a process involving the evolution of gaseous acetone during the cure cycle, possessed very large voids and direct light could be seen by the naked eye through them in the disc

specimen. In this case the fibres were considered to have "fallen out" and the measured fibre mass could be corrected accordingly.

The vacuum impregnation system produced perfectly void-free composites except in some fibreglass specimens where a few interstitial voids were observed. These were confined to the closely packed central fibres of glass rovings where the size had failed to dissolve in the resin. Under these conditions, resin could only enter the voids from the exposed ends of the rovings. Although atmospheric pressure forced the resin to fill such voids after impregnation, the resin was restricted from flowing along these narrow voids due to its viscosity and by any debris, including size, blocking the voids. The use of Poiseuille's equation to approximate the flow conditions showed that the time required to fill such gaps could be larger than the cure schedule allowed. Such void contents were estimated to be always less than 0.1 %.

Some AE rods were impregnated at atmospheric pressure. Their void contents were almost completely in the form of general voids and varied in the range 7.4 % for a 36 % fibre content to 3.8 % for a 74 % fibre content. The void content was approximately proportional to $(1 - V_f)$, as expected, because more air bubbles were forced out of the composite with more resin during the cure cycle to give the higher fibre content composites. Four AE60 rods were impregnated under 1, $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ atmospheric pressures which gave void contents of 5.1 %, 1.2 %, 0.5 % and 0.4 % respectively. It is therefore apparent that only a moderate vacuum is required to give good quality composites. PAUL and THOMSON (1965) fabricated tension-wound fibreglass using a vacuum impregnating system. They found that a vacuum of 10 torr was sufficient to give completely void-free composites.

CHAPTER FIVE

PHYSICAL STATE OF THE INTERNALLY STRESSED MATRIX

Epoxy resin shrinks during the process of polymerisation (cure) to form a solid typically 5 % more dense than the liquid resin. However the resin is inhibited from contracting within a composite by the presence of the much stiffer fibres to which the resin adheres. The composite rods were cured at a temperature of 125°C and allowed to cool to room temperature. The cured resin was largely inhibited from contracting, when cooled, by the presence of the fibres because the coefficient of thermal expansion of the fibre was much less than that of the resin. Both of these reasons caused the resin density in the fibre composites to be less than the resin density when no fibres were present.

The cure shrinkage values of the resin rods shown in table 5.1 were obtained after subtracting the thermal contractions from the measured contractions (at room temperature). The original dimensions of the rods were those of the steel mould at the cure temperature.

TABLE 5.1

Cure shrinkage of resin rods (measured values)

	<u>longitudinal</u>	<u>transverse</u>	<u>volume</u>
resin A	0.8 %	2.0 %	4.7 %
resin B	0.0 %	0.5 %	1.0 %

Linear coefficients of expansion ($10^{-6}/^{\circ}\text{C}$) (manufacturers' values)

<u>resin A</u>	<u>resin B</u>	<u>glass fibres</u>	<u>carbon fibres</u>
60	60	10	-1 (longitudinal) 30 (transverse)

It was noted that all the resin and composite rods adhered to the mould until manually released. It therefore appeared that the resin solidified into a state complete with tensile strains which would cause the measured cure shrinkage once the rod was released from the mould. Inhibited thermal and cure contractions were hence grouped together and regarded as elastic strains. In order to calculate the stresses, strains and density of the resin in the composite, the resin was first hypothetically postulated to contract freely as in simple resin rods. Then it was strained according to Hookean mechanics to make contact with the rigid fibres as in its real condition. It is apparent from table 5.1 that the resin rods were anisotropic, with preferential cure shrinkage in planes normal to the rod's axis. These results were consistent for different rods of the same resin. The thermal contractions were assumed to be isotropic.

The following calculations refer specifically to resin A reinforced with glass fibres although the methods used apply equally well to resin B and carbon fibres. Three principle axes, 1, 2 and 3 are defined which are either cartesian or cylindrical with the 1 axis in the fibre direction. The Hookean equations relating stresses and strains in the resin are :-

$$E \epsilon_1 = \sigma_1 - \nu (\sigma_2 + \sigma_3) \quad (4)$$

$$E \epsilon_2 = \sigma_2 - \nu (\sigma_1 + \sigma_3) \quad (5)$$

$$E \epsilon_3 = \sigma_3 - \nu (\sigma_1 + \sigma_2) \quad (6)$$

where ν is Poisson's ratio, E is Young's modulus, ϵ is strain, σ is stress and subscripts 1, 2, 3 denote the three principle axes. Positive stresses and strains are tensile.

The strain field in the resin is essentially two-dimensional, occurring in the plane normal to the fibre axis. Strains acting in a direction parallel to the fibre axis are constant over the whole composite and are controlled by the axial fibre strain. Three regions of interest exist in the resin and are denoted as regions (a), (b) and (c). All three regions are in a state of axial tensile strain equal to 1.3 %, comprising 0.8 % cure and 0.5 % thermal strains.

Region (a) is a resin-rich region free from fibres. This region has a free two-dimensional contraction in the plane 2,3 due to the tensile strain ϵ_1 . Under these conditions the results were as follows :-

$$\epsilon_1 = 1.3 \% \quad \sigma_2 = \sigma_3 = 0$$

Using equations (4) to (6),

$$\begin{aligned} \epsilon_2 &= \epsilon_3 = -0.49 \% \\ \sigma_1 &= 21 \text{ MN/m}^2 \end{aligned}$$

Once the three principal strains were known, the change in volume and hence density could be calculated. The density decreased by 0.3 % in this region.

Region (b) is the thin cylindrical resin tube surrounding a fibre. In this case the resin has only a one-dimensional free contraction which is in the radial direction. The hoop strain is controlled by the fibre. Under these conditions the results were as follows :-

$$\begin{aligned} \epsilon_1 &= 1.3 \% \quad \sigma_2 = 0 \\ \epsilon_3 \text{ (hoop strain)} &= 2.0 \% \text{ (transverse cure strain)} + 0.5 \% \text{ (thermal strain)} \\ &= 2.5 \% \end{aligned}$$

Using equations (4) to (6),

$$\begin{aligned} \sigma_1 &= 43 \text{ MN/m}^2 \\ \sigma_3 &= 57 \text{ MN/m}^2 \\ \epsilon_2 &= -2.3 \% \end{aligned}$$

The density decreased by 1.4 %. The radial compressive stress acting at the glass-resin interface due to the tensile hoop stress is given by :-

$$\sigma = \sigma_3 \frac{t}{r} \quad (7)$$

where t is the thickness of the resin tube and r is the radius of the fibre.

Region (c) is the resin within the tricorner created by three fibres in mutual contact when there is no interstitial void. This region has no free-dimensional contraction and is limited to contracting in the same manner as the fibre. Under these

conditions the results were as follows :-

$$\epsilon_1 = 1.3 \% \quad \epsilon_2 = \epsilon_3 = 2.5 \%$$

Using equations (4) to (6), $\sigma_2 = \sigma_3 = 150 \text{ MN/m}^2$

The decrease in density was 6.4 %. It may be noted that the above-quoted values for tensile stresses acting in plane 2,3 of region (c) are larger than the value for the resin yield strength (see Fig. 12.9). It is doubtful whether stresses larger than the yield strength of the resin can exist in a composite.

A similar situation to that in region (c) of composites occurred in resin rods which remained adhered to the mould at room temperature. The elastic strains stored in the resin during cure were most probably not linear and acted with an effectively reduced Young's modulus. Although equations (4) to (6) assume linear response, the strains calculated are independent of the value of E provided E remains constant.

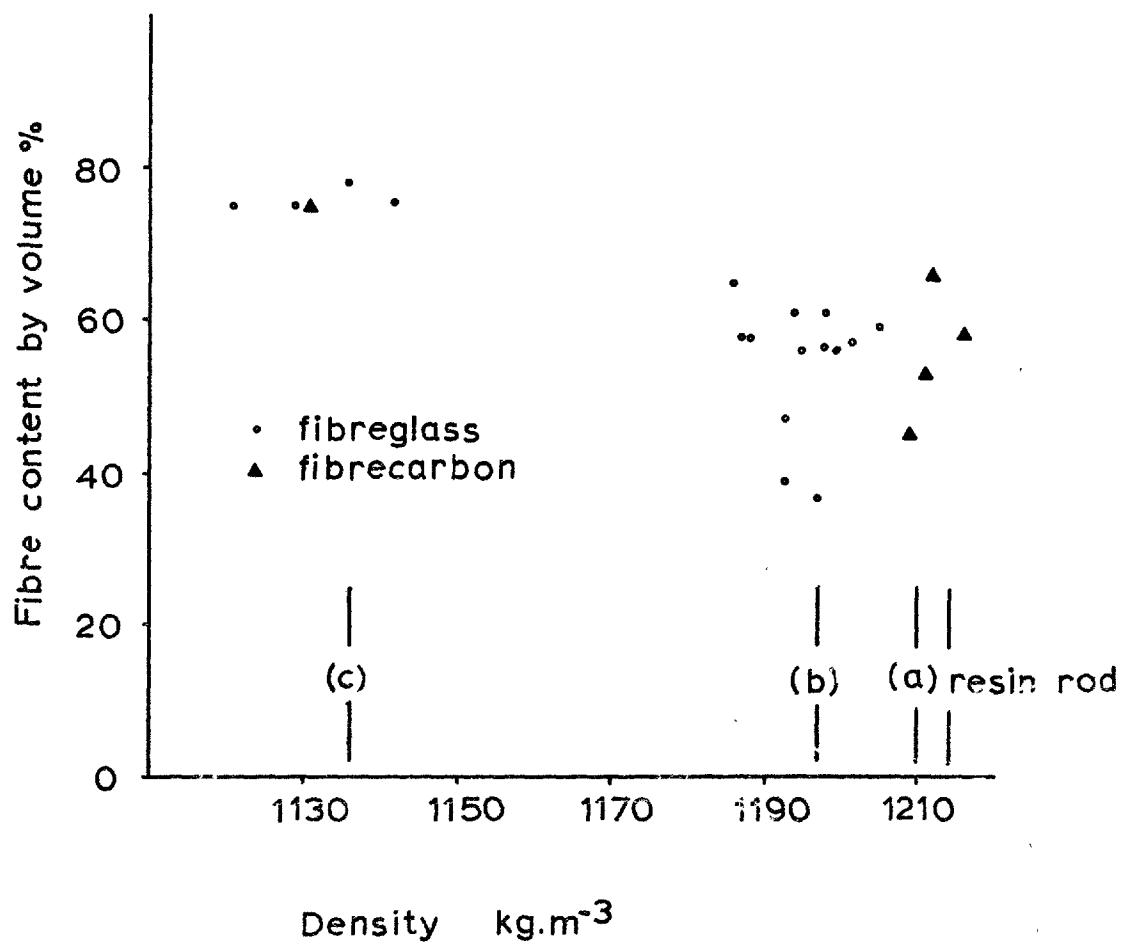
In region (c) there were tensile stresses acting on the fibre-resin bond which must have reduced the effective bond strength. Microscopic examination revealed that the resin in a few tricornes had managed to contract, breaking the resin-fibre bond. The breaking of this bond could have been facilitated by the degradational effect of water acting on the exposed surface of the specimen.

The compressive stresses acting at the resin-glass interface in region (b) enhanced the bond strength, in contrast to region (c). More rigorous analyses of the stress and strain fields surrounding such fibres show them to be non-uniform and that the fibre's influence can penetrate one fibre diameter or more into the matrix; see for instance CHAWLA and METZGER (1972). The fields around neighbouring fibres therefore interact in composites with a fibre content exceeding 10 % to give the matrix a blend of the properties of regions (a) and (b).

Fig. 5.1 shows the variation of resin density with the fibre content in fibreglass and fibre-carbon rods using the average resin density of each individual rod. The measured resin density within disc specimens from the same rod usually agreed to within $\pm 0.5 \%$. The calculated densities for regions (a), (b) and (c) and the

Fig. 5.1

Density of resin A matrix within
the fibre composites



measured density of pure resin rods are marked. The scatter present is more than that due to experimental error alone and hence represents the variation in resin density between different rods. It can be seen that the density is independent of fibre content except for the rods with maximum fibre content. This suggests that the total volume of type (c) regions (which must increase with increasing fibre content) is negligible. A sharp reduction in resin density is apparent in the rods with maximum fibre content. The mould was inhibited from closing fully when these rods were made, independently of the applied pressure, by the presence of the compressed fibres. Under these conditions all the resin was confined to region (c) whether or not it was in a tricorn.

Reasonable agreement was obtained between the experimental and calculated densities. The fibreglass rods (except those with maximum fibre content) had an approximately average resin density of region (b). The changes in calculated resin densities due to the different thermal expansion coefficients of carbon fibres were small. These changes increased the density of region (b) for instance by 1 kg./m^3 , but this was not enough to explain the measured higher density of resin within fibre-carbon rods. A possible source of systematic errors could be that of acid dissolving some of the carbon during the experimental measurement of fibre content. For this reason the fibre content of a carbon fibre tow was measured in a similar manner which showed that no loss in fibre mass was encountered. A calculated resin density of 1210 kg./m^3 is changed to 1205 kg./m^3 if the manufacturer's quoted density of 1980 kg./m^3 for type I fibres is used instead of the measured density of 1970 kg./m^3 . It is thus conceivable that systematic errors could account for the higher resin densities in fibre-carbon rods.

CHAPTER SIX

MEASUREMENT OF MECHANICAL PROPERTIES OF COMPOSITES AND THEIR CONSTITUENTS

6.1. ULTIMATE COMPRESSIVE STRENGTH (U.C.S.)

All compression testing was carried out in an Instron ten tonne testing machine fitted with a ten tonne tension-compression load cell, except for AC_{III}75 specimens which were tested using an Instron 25 tonne frame due to their high failure loads.

The compression specimens were sandwiched between two flat and parallel plates made of hard steel. The lower plate rested on the crosshead of the frame while the upper one was secured to the load cell. The crosshead could be moved mechanically towards or away from the upper plate at various speeds and the load, which could be calibrated electronically, was recorded by the pen of a strip chart recorder. The sensitivity of the recorded load could be varied between 20 kg. and 10 tonnes for full scale deflection while the speed of the chart could be varied to give different sensitivities of the recorded crosshead extensions. The faces of the plates were measured to be parallel to within 0.05° .

The ultimate compressive strength (U.C.S.) was obtained by dividing the maximum load recorded prior to specimen failure by its constant cross-sectional area in a plane parallel to the plate faces. In the case of tapered specimens, the minimum cross-sectional area was used. The U.C.S. measured in this way assumed the stress to be uniform over the cross-sectional area. Except in the case of tapered specimens, this assumption was only correct if the steel of the plates was much stiffer than the compression specimen tested. This was normally true for a transverse compression specimen where its stiffness was controlled by the resin modulus. For axial compression discs, this was only true for resin specimens. A uniform stress field was obtained in the disc specimen by sandwiching it between two hard steel discs with the same diameter as the specimen so that their three axes coincided. Both steel discs were 3 mm thick with their faces ground flat and parallel and their yield strength was greater

than that of any specimen tested.

A few disc specimens were compressed at a temperature of -196°C , the boiling point of liquid nitrogen. A special compression jig was used for these specimens, which could be attached to the underneath of the crosshead; compression was obtained by a tensile coupling from the jig through the crosshead to the load cell. Specimens were compressed between two flat and parallel steel faces as previously described. A large thermoflask containing liquid nitrogen was placed under the crosshead so that the specimen and jig were immersed in the liquid.

6.2. SHEAR STRENGTH

6.2.1. THREE-POINT-BEND METHOD

This method is the standard technique for the determination of the interlaminar shear strength of fibre-reinforced composites; see for instance A.S.T.M. (1963). A schematic diagram of the apparatus used is shown in Fig. 6.1. The specimens had a span:depth ratio of 5:1 with the fibres lying in the span direction. Each specimen had a span of $5/8$ in. and a breadth of $1/4$ in. The three $1/8$ in. diameter tungsten carbide rollers fitted loosely into cylindrical grooves. The upper roller was self-aligning in a plane normal to the fibre direction and was equidistant from the other two rollers. The apparatus was attached to the compression jig mentioned in section 6.1. which enabled testing to be performed at a temperature of -196°C in a similar manner. Compression was transmitted to the specimen via the rollers causing a maximum shear stress in the plane equidistant from the axes of the three rollers given by :-

$$\gamma = \frac{3}{4} \frac{P}{d b} \quad (8)$$

where P is load, d is depth, and b is the breadth of the specimen. The shear strength is the maximum value of the shear stress which the specimen can withstand.

Fig. 6.1 Three-point-bend apparatus

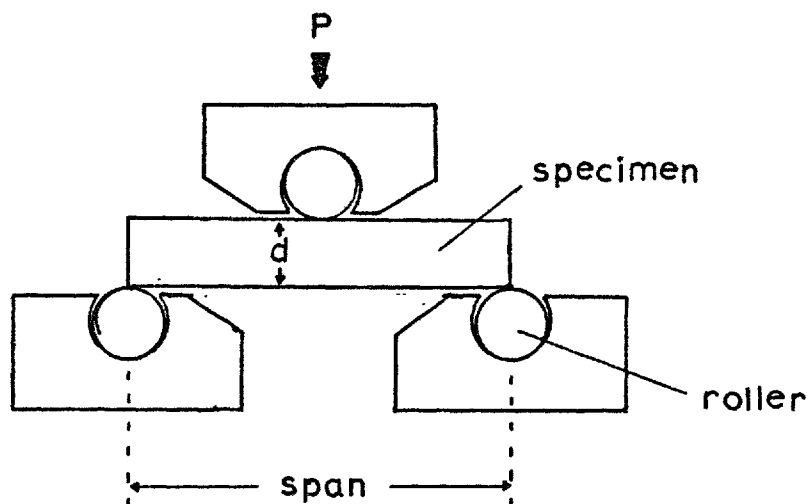


Fig. 6.2 Shearing rod apparatus

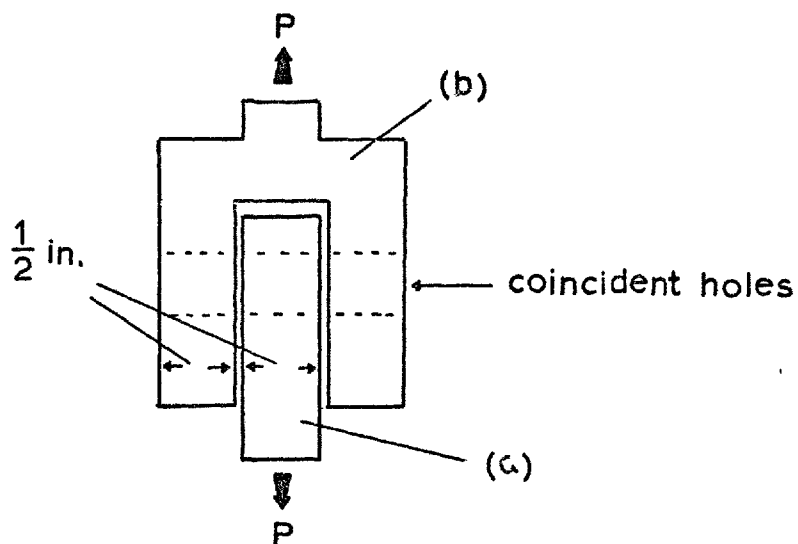
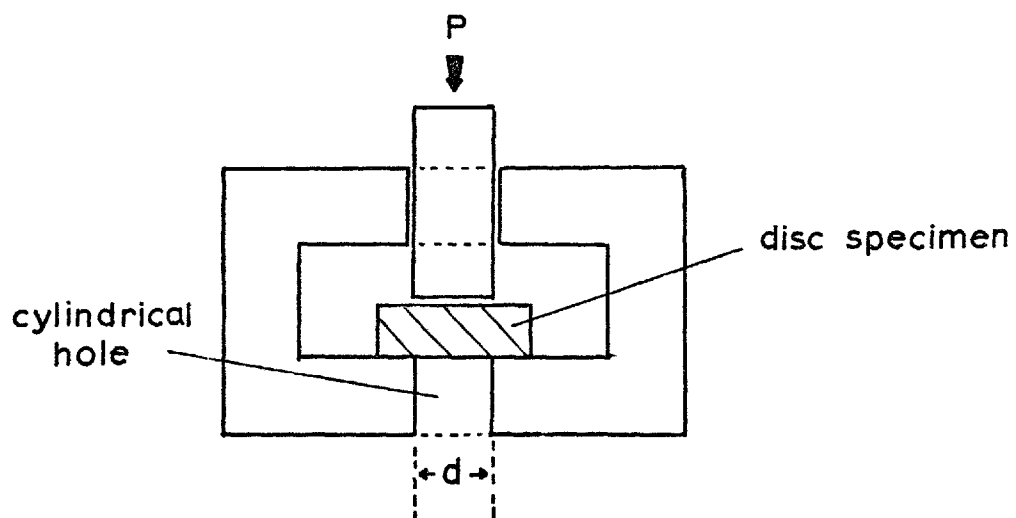


Fig. 6.3 Disc-punch apparatus



6.2.2. SHEARING ROD METHOD

This method was used solely for the determination of resin shear strength. A schematic diagram of the apparatus built for this method is shown in Fig. 6.2. Section (a) slid closely between the two arms of section (b) so that the three cylindrical holes, each of 3/8 in. diameter, were aligned. The close fitting resin rod was pushed into the combined hole and so united section (a) with the two arms of section (b). The rod had sufficient length to fully occupy all three holes. A tensile load P was applied so that the rod failed in shear. The shear strength is given by :-

$$\gamma = \frac{P}{2A} \quad (9)$$

where A is the cross-sectional area of the rod. The shear strength was also determined at -196°C using the technique described in section 6.1.

6.2.3. DISC-PUNCH METHOD

This method used a polished disc specimen. A schematic diagram of the apparatus built for this method is shown in Fig. 6.3. The hardened steel rod of 4.75 mm diameter could pass through the two aligned and close fitting cylindrical holes in the steel body. The specimen was placed as shown with its axis coincident with the axis of the rod. The ends of the rod were ground flat, normal to the rod's axis, so that a good contact was made between the specimen and the rod. A compressive load P was applied as shown so that a cylinder from within the specimen, with the same diameter as the rod, was pushed into the empty hole. The shear strength is given by :-

$$\gamma = \frac{P}{\pi d L} \quad (10)$$

where L is the length (thickness) of the specimen and d is the diameter of the rod.

6.2.4. TRANSVERSE COMPRESSION METHOD

In this method, a compressive stress is applied to the specimen in a direction normal to the fibre axis. The failure mode of fibreglass specimens due to transverse compression was by shear on a shear plane parallel to the fibres and inclined at an angle $(90 - \theta)^\circ$ to the applied stress σ as shown in Fig. 6.4. In fibre carbon specimens, shear failure occurred on multiple planes each inclined at approximately the same angle. The shear stress acting on the plane(s) is given by :-

$$\gamma = \frac{1}{2} \sigma \sin 2\theta \quad (11)$$

The normal compressive stress σ_n acting on this plane is given by :-

$$\sigma_n = \sigma \cos^2 \theta \quad (12)$$

Maximum shear stress occurs when $\theta = 45^\circ$. However, for the composites tested, θ was estimated to equal 70° which showed that shear strength was dependent on the value of σ_n . This dependency was described and explained by COLLINGS (1973).

6.2.5. COMPARISON OF SHEAR STRENGTH MEASURING TECHNIQUES

The shear strengths of resins A and B were measured to be 64 and 60 MN/m² respectively using the shearing rod method. The load-deflection curves obtained for resin A are shown in Fig. 6.5. Curve (a) shows that the maximum stress due to resin yield was obtained before catastrophic failure by crack propagation in the shear planes. A similar curve was obtained for resin B at room temperature. These room temperature shear strengths are approximately half their compressive strengths. Curve (b) was obtained by testing resin A at -196°C . An audible click could be heard corresponding to each peak; these were probably due to intermittent crack extensions. Catastrophic failure occurred, at a shear stress of 112 MN/m², before any sign of resin yield. However, when tested in compression at this temperature, resin yield did occur. (See Fig. 12.9). If the shear strength was half the compressive strength, the "true value"

Fig. 6.4

Failure mode of transverse compression specimen

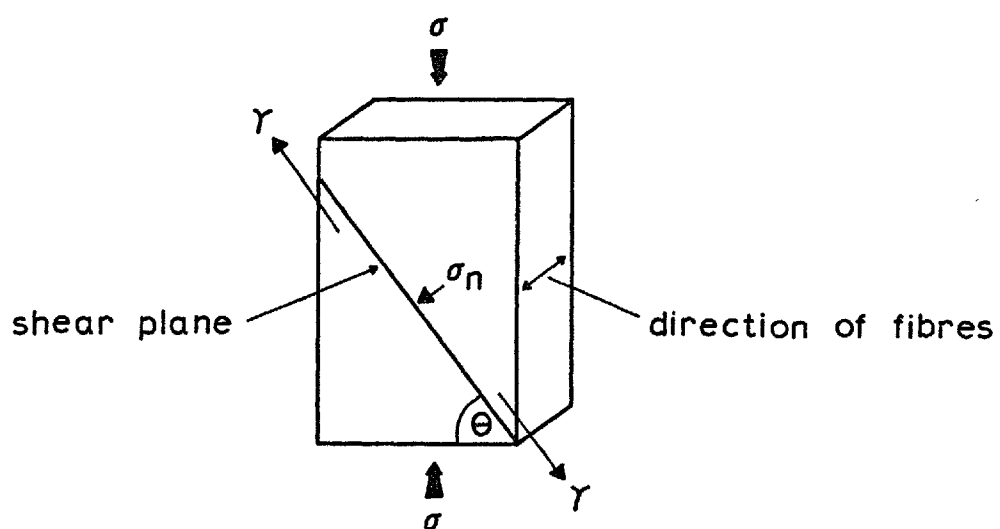
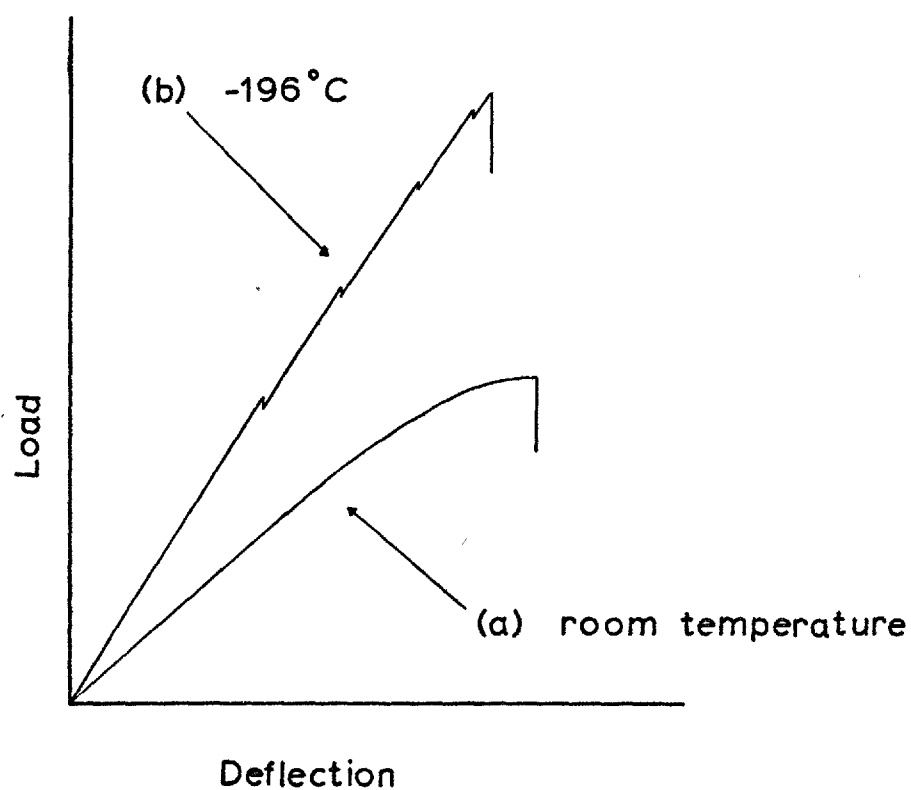


Fig. 6.5

Load-deflection curves of resin A rods in shearing rod apparatus



of compressive strength due to resin yield was about 240 MN/m^2 . This rule was also true at room temperature and is approximately true for numerous materials.

The load-deflection curves for three-point-bend testing followed one of two distinct patterns as shown in Fig. 6.6. All specimens exhibiting pattern (a) failed at a shear stress approximately equal to the shear strength of the resin. These were always fibreglass specimens. This showed that the glass-resin bond was stronger than the resin itself. This bond is described as glass-resin for simplicity but it can be subdivided into glass-size and size-resin bonds. Specimens exhibiting pattern (b) failed at stresses less than the resin shear strength suggesting that the fibre-resin bond failed first. Type (b) failures were confined to fibre-carbon specimens and exhibited more scatter for each composite (typically 20 % of the mean strength) than type (a) failures (typically 10 %). Most of the specimens tested had a fibre content of about 57 %. No distinct variation of strength with fibre content was expected or observed, see for instance BADER et al. (1973). AE75 specimens had a mean shear strength of 66 MN/m^2 compared with 57 MN/m^2 for AE57 specimens.

AE57 specimens tested at -196°C showed a mean shear strength of 132 MN/m^2 , exhibiting pattern (b). This was greater than the measured resin shear strength of 112 MN/m^2 at this temperature, but less than the projected value of 240 MN/m^2 for resin shear strength. These results suggest that the resin shear strength at this temperature was at least 132 MN/m^2 .

The three-point-bend method suffers from the disadvantage of giving shear strengths dependent on the specimen geometry as pointed out by DAVIS (1964). In addition, the linear elastic analysis which gives equation (8) does not take into account local damage occurring in the composite beneath the rollers prior to failure. Moreover, the failure plane did not always coincide with the predicted failure plane, with maximum shear stress, equidistant from the three rollers. A more elaborate analysis of the stress-strain fields in the specimen was given by BERG et al. (1971). A few composite rods were made with glass fibres which had had their size removed as described in section 4.1.1. The mechanical damage sustained by these

Fig. 6.6 Load-deflection curves of three-point-bend specimens

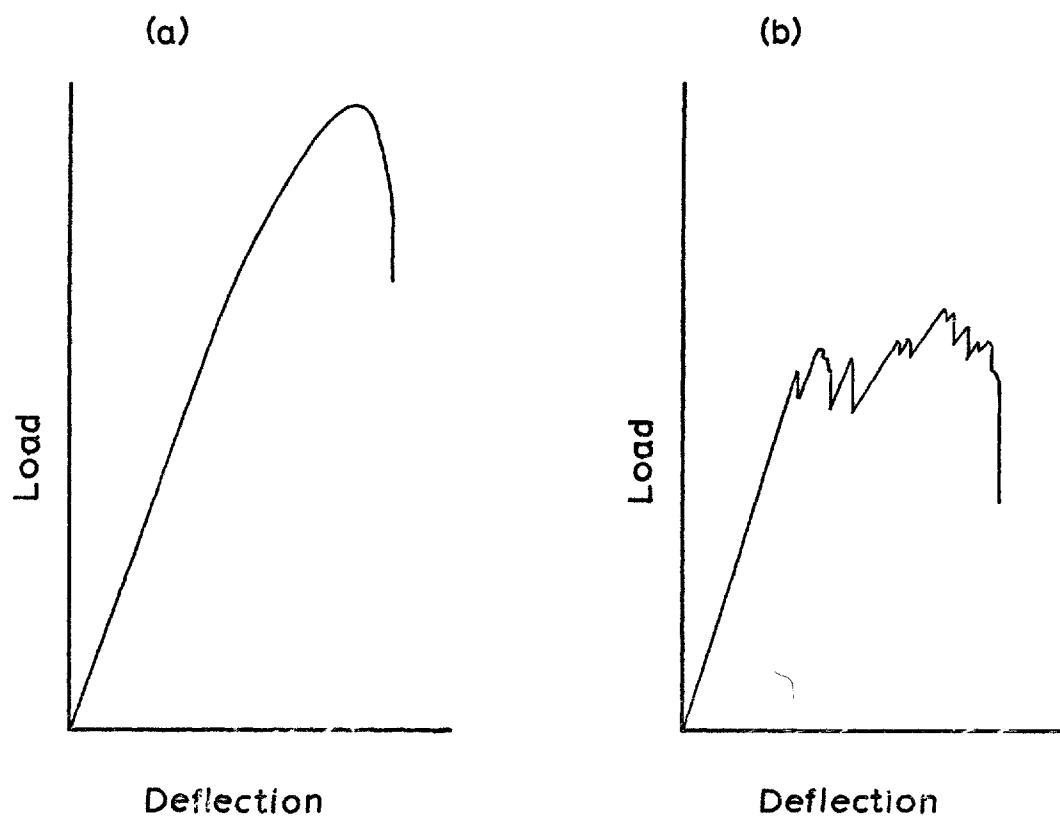
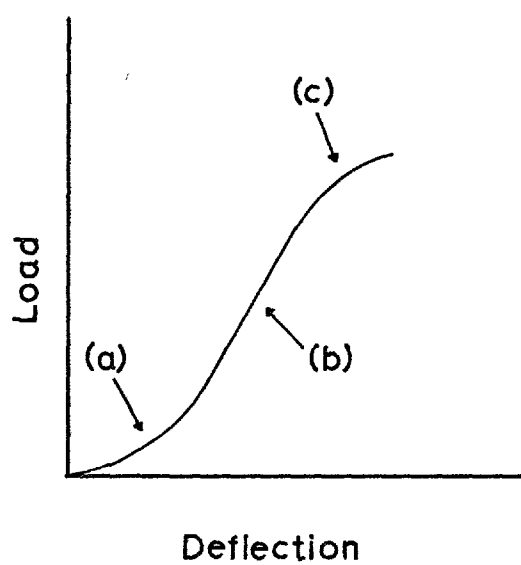


Fig. 6.7 Load-deflection curve of glass roving



unprotected fibres during handling seriously reduced their tensile strength. Three-point-bend specimens obtained from these rods failed in tension and therefore only a minimum value of shear strength could be recorded.

The disc-punch method did not suffer from the above-mentioned disadvantages and failure was confined to a well defined cylindrical surface. Moreover, specimens were much easier to obtain from cylindrical rods and used less material than three-point-bend specimens. The load-deflection curves followed the same patterns as described for the three-point-bend specimens and gave similar scatter for their strength values. A slight correlation of shear strength with specimen length was detected. The mean strengths of AE57 specimens of lengths 5, 3 and 2 mm were 64, 58 and 57 MN/m² respectively. This compared with the value 57 MN/m² obtained for AE57 three-point-bend specimens. Such a correlation would be expected if the fibres were imperfectly aligned with the axis of the steel rod. Therefore only specimens less than 3 mm long were used to determine shear strengths.

The original reason for removing the size from glass fibres was to reduce the shear strength of composites made from such fibres. In fact the reverse appeared to be true. For instance, an AE57 rod made from size-free fibres had a mean shear strength of 67 MN/m². It therefore appeared that a purely mechanical glass-resin bond could explain the measured shear strength of fibreglass.

A shear strength of 66 MN/m² was obtained for an AE57 specimen using the transverse compression method, inherently inaccurate due to inaccuracies in the measured values of θ . The load-deflection patterns were similar to those previously described. The shear stress acted in a direction normal to the fibres' axes within the shear plane whereas in the two previous methods described, the shear stress acted parallel to the fibres. These results show that the two different modes of failure gave similar shear strengths. A typical value for σ_n was 20 MN/m². In the three-point-bend method, the average values of σ_n over the shear planes were typically 20 MN/m² although the actual values at the points of shear failure were probably much less. In the disc-punch method, σ_n was approximately zero.

6.3. AXIAL YOUNG'S MODULUS OF FIBRES (TENSILE)

It has been assumed that the stress-strain curves of all fibres were linear and elastic up to failure.

6.3.1. GLASS FIBRES

The Young's modulus of the fibres, E_f , was obtained in tension. A single roving, about 80 cm in length, was used which was the maximum length which could be accommodated in the testing machine. Each end was coated with a cold-setting epoxy resin and sandwiched between two aluminium tabs of typical dimensions 5 cm by 2 cm. The longest dimensions were aligned parallel to the strand. Once the resin had set hard, the roving was clamped, via the two tabbed ends, between the two tension grips which were attached one to the crosshead, and the other to the load cell of the frame. It was necessary to clamp the roving via the tabs otherwise premature failure of the glass fibres occurred at the grips. The roving was pulled in tension, by a typical load of 50 kg., until failure occurred. A load-deflection curve was obtained from the strip chart recorder. E_f was given by the slope of this curve where :-

$$E_f = \frac{P L}{\delta A} \quad (13)$$

P = load

δ = deflection

L = length of roving measured between the tabs

A = cross-sectional area of the glass in the roving

A was calculated from the glass mass per unit length of the roving and the glass density. The accuracy of this method increases with increasing L .

The load-deflection curve of a glass roving shown in Fig. 6.7 is distorted in order to emphasise three distinct regions. Region (a) is where the slack in individual fibres was being taken up. Region (b) is a straight portion where all the fibres were taut and the slope of this region gives the correct value

of E_f . Region (c) is where individual fibres were breaking prior to complete failure.

6.3.2. CARBON FIBRES

The Young's moduli of carbon fibre tows were obtained in a similar manner to those of glass fibre rovings. Carbon fibres are axially more than five times as stiff as glass fibres with comparable strengths. The failure strain of carbon fibres is hence much less and was unable to take up all of the slack in all of the fibres before fibres began to break. This resulted in region (a) merging directly into region (c) of Fig. 6.7 and thus only a minimum value of E_f could be obtained. The load-deflection curve of the frame itself was estimated to be negligible in the case of glass rovings and produced an error, which could be corrected for, in the E_f value of carbon fibre tows of about 0.3 %.

6.4. AXIAL YOUNG'S MODULUS OF COMPOSITES (COMPRESSIVE)

6.4.1. FIBREGLASS

In order to determine the axial Young's modulus of fibreglass E_c , a polished disc specimen was compressed as described in section 6.1. between two steel discs. The deflection recorded on the strip chart was the total deflection in the specimen plus the frame, the two steel discs being regarded as part of the frame. In order to determine the load-deflection curve of the frame alone, the specimen was replaced by a specimen made, with the same dimensions, from steel of known modulus. The load-deflection curve (straight line) of the steel specimen could be calculated and was subtracted from the load-deflection curve of the frame plus steel specimen. In other words, the deflections were subtracted at common loads. The load-deflection curve of the frame alone was therefore recorded. This curve in turn was then subtracted from the load-deflection curve of the frame plus composite specimen thus giving the load-deflection curve of the specimen alone. The stress-strain curve of the specimen could be obtained from this curve and hence E_c could be measured. The same crosshead and chart speeds were used throughout as well as

the same recorded load sensitivity to eliminate errors due to discrepancies in their specified values. The deflection in the specimen must be at least a third of the deflection in the frame in order to obtain accurate results. A minimum specimen length of 6 mm was used for this reason. It was found that the stress-strain curves of all composites were linear up to failure.

6.4.2. FIBRECARBON

The basic method described in section 6.4.1. was used with fibre carbon discs. The specimens had to be much longer if the specimens' deflections were to be at least a third of that of the frame because fibre carbon is axially much stiffer than fibreglass. Whole rods were used as specimens but the stress in each rod did not exceed 25 % of its U.C.S. in order to conserve material. The rods were aligned between the plates with the aid of a V block. All the recorded stress-strain curves were observed to be linear in the stress ranges used.

TABLE 6.1

General data

<u>Young's modulus (GN/m²)</u>	<u>Measured value</u>	<u>Manufacturer's value</u>
E-glass	78	65 to 80
R-glass	86	86
carbon type IS (batch G44)	≥387	414 ± 34
carbon type II (batch H720)	≥264	272
carbon type IIIS (batch H516)	-	231
resin A (static, r.t.)	1.64	2.5 to 3.5
resin B (static, r.t.)	2.77	3.5 to 5.0

Density (kg./m³)

E-glass	2580	2550
R-glass	2550	2550
carbon type IS	1970	1980
carbon type II	1800	1780
carbon type IIIS	1780	1780

Poisson's ratio

E-glass	-	0.22
R-glass	-	0.25
resins A and B	-	0.38

CHAPTER SEVEN

DEGREE OF ALIGNMENT IN UNIDIRECTIONAL COMPOSITES

All the previously mentioned composites have been described as unidirectional with the axial load applied in this direction. This statement must be regarded as an approximation. The degree of alignment between the fibres and the applied load can be separated into three components :-

- a) The alignment between the applied load and the axis of the composite rod (or disc).
- b) The alignment between the mean fibre direction and the rod's axis.
- c) The scatter of all fibres about this mean direction.

Component a) has already been dealt with and could be made as accurate as required.

Component b) is expected to be very good. This is a virtue of the fact that the rod was identical to the moulded product except for the removal of a thin external layer during the centreless grinding process. The mean fibre direction averaged over the length of the rod must therefore have coincided with the rod's axis because no fibres entered or left the mould except at its ends.

Component c) is difficult to assess but an indirect method is to measure the axial Young's modulus of the composite. Extensive mathematical treatments have been carried out by several workers to determine the axial Young's modulus of a unidirectional composite as a function of the moduli of its constituents and of the fibre content. See for instance, HASHIN and ROSEN (1964), HALPIN (1969) and GOGGIN (1973). For all practical purposes where the fibre moduli are much greater than the matrix moduli, a simple mixture rule may be applied :-

$$E_c = V_f E_f + (1 - V_f) E_m \quad (14)$$

Subscripts c, f and m denote composite, fibre and resin matrix respectively. The maximum error in this equation applied to the composites used in practice, was always less than 1 %. This equation assumes that both constituents are linearly elastic

which was always true for the fibres used, but at large strains the tangent modulus of the resin matrix differed from the value of E_m and had to be used instead to give the tangent modulus of the composite. However, this correction to the value of E_c was always negligible as the resin only contributes a few percent at most to it.

COOK (1968) calculated the axial Young's modulus of a unidirectional composite as a function of the root mean square of the fibre scatter about its mean direction. As a result of this scatter, values of E_c calculated by Cook are less than the values predicted by equation (14). Thus the measured reduction in modulus from the mixture rule can be correlated with the scatter in fibre directions.

The axial Young's moduli of composites in the AE range are shown in Fig. 7.1 together with the mixture rule prediction. The average reduction in modulus corresponds to a root mean square value of about 6° . Other composites gave similar values; except in the case of fibre carbon where the errors present in the used values of fibre moduli prevented any reasonable estimation from being made. The reduction varied considerably for different specimens from different rods and this variation could also be observed in the moduli of whole rods measured very accurately using ultrasonic methods. Therefore the values plotted in Fig. 7.1 represent individual rods rather than individual specimens. The scatter seen in Fig. 7.1 is typical of composites made by the hand-lay-up leaky mould technique as shown by COLLINGS and FINBOW (1970) who measured the axial Young's moduli of unidirectional fibre carbon specimens with various fibre contents and compared them with the values obtained using the mixture rule. The root mean square values obtained from their graphs were only a few degrees; also some moduli values plotted were even larger than the mixture rule predictions. However they used the manufacturer's quoted mean values of fibre moduli to predict mixture rule values rather than measuring the moduli values independently. Their mixture rule values must therefore be regarded as being suspect because scatter of moduli values for carbon fibres is known to exist even within the same batch.

GOGGIN (1973) did similar work with fibre carbon made by the hand-lay-up and tension-wound techniques. Some of his results

Fig. 7.1 Axial Young's modulus of composites in the AE range

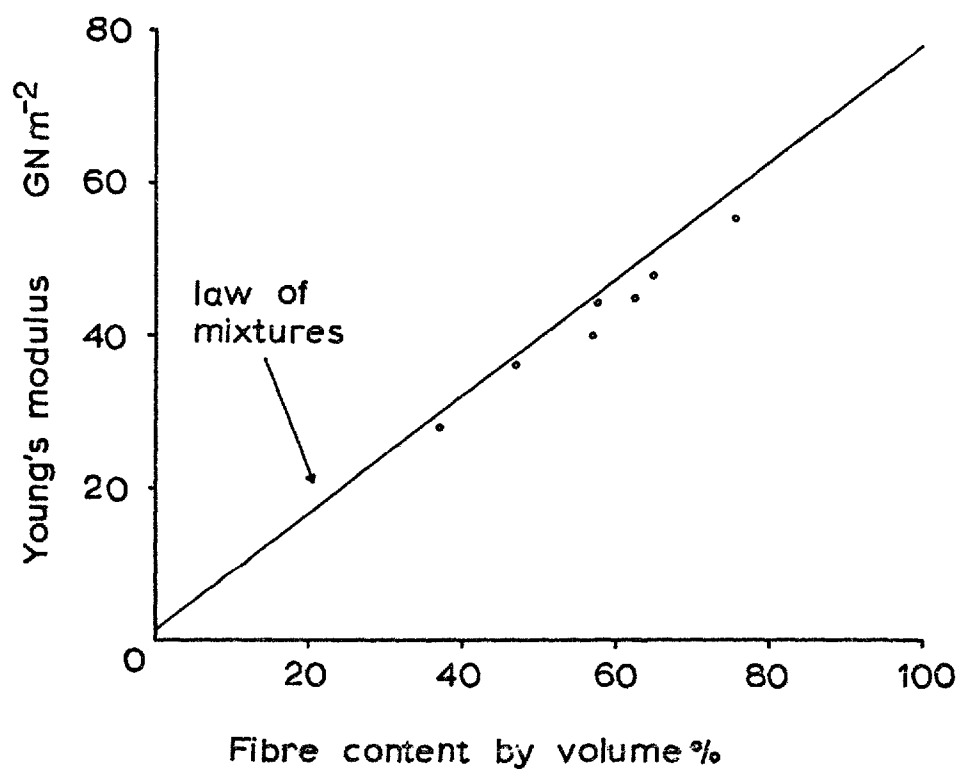
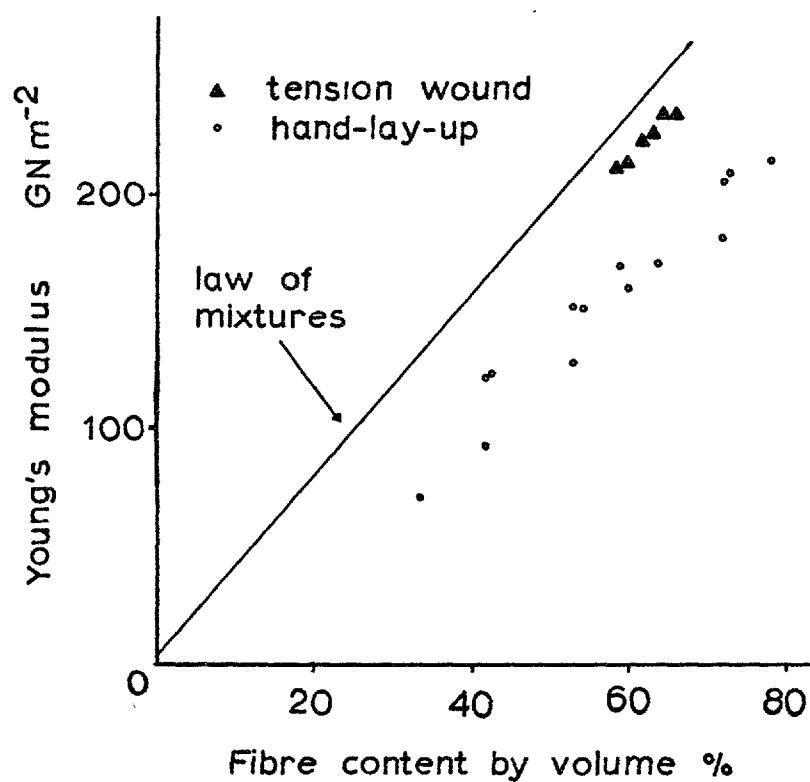


Fig. 7.2 Axial Young's modulus of Goggin's fibre carbon composites



for type I carbon fibre composites are reproduced in Fig. 7.2. Goggin made his own measurements of fibre moduli and hence his mixture rule values do not suffer from the previously mentioned source of error. From Fig. 7.2 the root mean square values of fibre scatter for the tension-wound and hand-lay-up fibre carbon can be measured to be 7° and 10° respectively. FRIED and KAMINETSKY (1964) performed similar experiments on tension-wound fibreglass. A value of about 7° was obtained from their results. This value agrees well with the results from the tension-wound fibre carbon of Goggin.

It must be remembered that the measurement of moduli is an indirect method of measuring fibre scatter and relies completely on the validity of Cook's work. For small modulus reductions, this method is very inaccurate as a minimum error of 1 % must be placed on all moduli measurements. The scatter in published moduli values of tension-wound composites is less than in hand-lay-up composites. (See for instance Fig. 7.2). This is to be expected as the degree of alignment in tension-wound composites is independent of any manual skill and is hence more reproducible. From the above-mentioned results, it appears that the alignment in the fibreglass rods was good and comparable with the alignment in the tension-wound composites of other workers.

An attempt was made to measure the degree of alignment directly using one of the faces of a three-point-bend specimen parallel to the fibres. This face was observed through a microscope fitted with a stage that could be rotated through any specified angle and also could be moved in a plane normal to the field of view. The stage was rotated so that one of the cross-wires in the eyepiece was aligned parallel to the fibres which it traversed and the angle noted. Fifty separate readings were taken, uniformly spread over the maximum square area (40 mm^2) over which the specimen could be viewed. The root mean square angle was calculated and corrected to take into account that the angles measured were the projections of a three-dimensional array of fibres with radial symmetry onto a two-dimensional plane. (For small angles, the obtained value could be corrected by multiplying it by $\sqrt{2}$). The results are given in table 7.1.

TABLE 7.1

Fibre scatter

<u>Specimen</u>	<u>Root mean square angle (degrees)</u>
AE57	2.9
AE57 (same rod, different specimen)	2.7
AC _I 60	2.1
AC _{III} 75	2.1

The values of table 7.1 are about half the predicted values of the previous method. However the eye tended to align the length of wire in the field of view (1 mm diameter) parallel to the average direction of the fibres in its immediate vicinity. Also the values given represented the scatter of fifty readings relative to their mean direction which itself might have differed from the axis of the rod. Due to these factors, the measurements represented minimum rather than mean values. In the case of fibre carbon, a few individual fibres had a large misorientation (in excess of 30°) and none of these fibres were included in the measured orientations; the measured scatter would have been significantly increased if only a few had been included.

The fibres within a single roving or tow were observed visually to be, and, by their very nature, must have been, well aligned to each other. Nearly all the fibre scatter therefore was between individual rovings or tows within the composite. Microscopic examination showed that individual rovings or tows could be detected within the composite, each surrounded by a resin rich layer as shown in Fig. 7.3. This figure also shows an individual fibre in the resin rich layer with a large misorientation which is demonstrated by its elliptical cross-section. The composite rod could hence have been regarded as a bonded collection (typically 100) of long thin unidirectional composites with a typical root mean square scatter between them of 6° .

Another physical parameter dependent upon the alignment is the maximum fibre content of composites achieved when the mould failed to fully close in the hot press independent of the applied pressure. This condition is obtained by introducing an excess

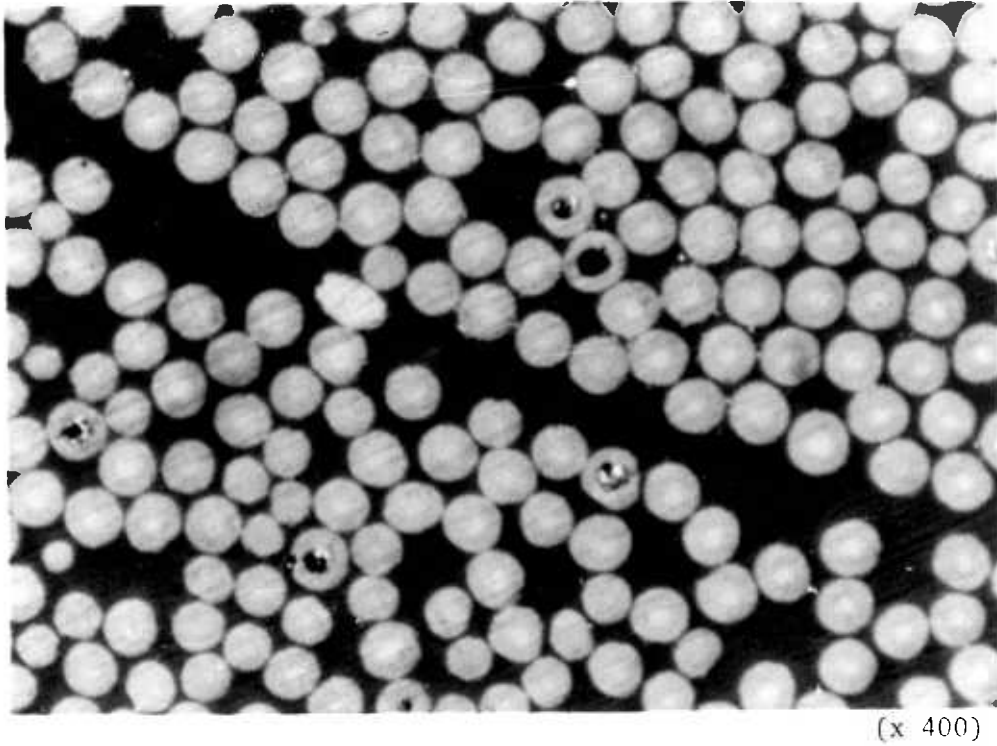


Fig. 7.3 Resin rich region between adjacent tows in AC_I58 specimen

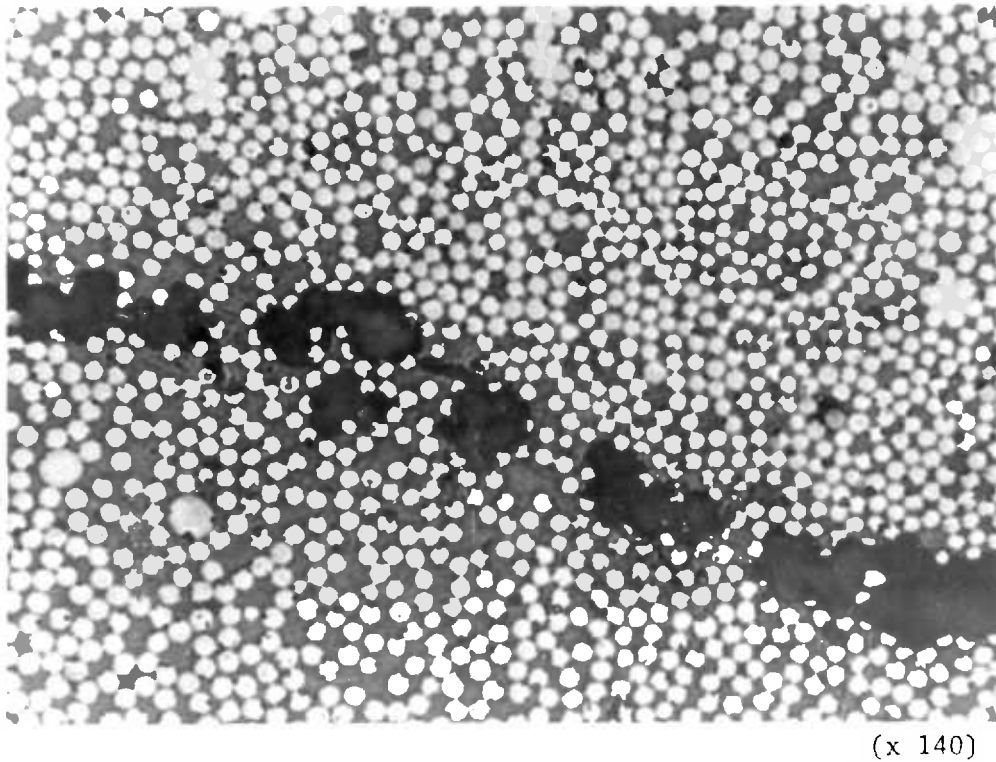


Fig. 7.4 Planar voids in AC_{II} 58 specimen

of fibres into the mould. Perfectly aligned fibres arranged in a hexagonal array have a maximum fibre content of 91 %, the theoretical limit. Composites with misorientated fibres must have a fibre content less than 91 % which decreases as the degree of misorientation increases. However, a practical composite, even if perfectly aligned, cannot be expected to achieve such a high degree of order due to the low entropy of such a well ordered hexagonal array and hence a 91 % fibre content is unobtainable. A distribution of fibre diameters also causes a lower fibre content.

The maximum fibre content of E-glass, R-glass and C_{III} composites was 75 %, 78 % and 75 % respectively and was also reproducible for all rods produced when the mould failed to fully close. It was noticed that while all R-glass fibres appeared to have the same diameter, the E-glass fibres had a detectable range of diameters and it was believed that this caused the larger maximum fibre content of the R-glass composites. The maximum fibre content can therefore be regarded as a comparative measure of quality control for unidirectional composites.

CHAPTER EIGHT

AXIAL COMPRESSIVE STRENGTH (EXPERIMENTAL)

8.1. GENERAL CONSIDERATIONS

The maximum strain rate at which the testing machine could reach the required loads for fibreglass specimen failure was limited to $4 \times 10^{-4} \text{ sec}^{-1}$. No strength dependence on strain rate could be detected in the range $(1 \text{ to } 4) \times 10^{-4} \text{ sec}^{-1}$ and testing was done typically at a strain rate of $1.5 \times 10^{-4} \text{ sec}^{-1}$. Fibrecarbon specimens were tested at the same crosshead speeds and hence at a slower strain rate. Failure always occurred within a few minutes for all specimens tested. The strain rate was almost independent of the specimen's length because most of the deflection occurred in the frame and not the specimen. Pure resin specimens possessed much lower strengths than composite specimens and were tested at strain rates of about $(1.5 \text{ and } 300) \times 10^{-4} \text{ sec}^{-1}$ at which a significant strength and modulus dependence on strain rate were detected.

The composite rods, having been centreless ground, were stored in a ventilated laboratory at room temperature. The original testing programme was completed within two months although some additional testing was carried out up to eighteen months later. In one rod set aside to test time-strength dependence, no strength degradation could be detected for specimens cut, then tested up to eighteen months later. Specimens were cut from the same rod, if possible, in one batch on the same day in order to measure the effect of one particular parameter on strength properties. They were also tested in one batch on the same day. This procedure minimized the comparative effect of other parameters, known or unknown, on strength properties. The standard length adopted for disc specimens was about 3 mm unless the effect of length variation was being investigated.

8.2. FAILURE MODE

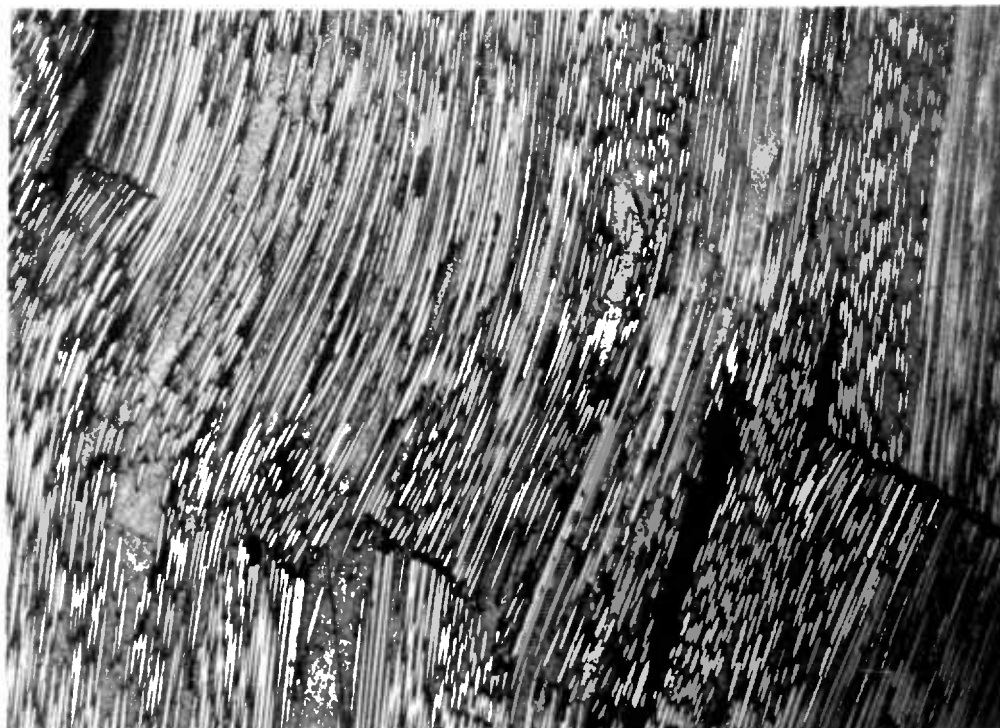
8.2.1. FIBREGLASS

The mode of failure of fibreglass disc specimens was quite distinct and is shown in Fig. 8.1 and, schematically, in Fig. 8.4a. The failed region had the appearance of a two-dimensional shear failure on a shear plane. This region always touched two opposite edges of the specimen independently of its length and hence the inclination of the shear plane to the end faces varied between a few degrees for short specimens up to a maximum 45° for long specimens.

The failed region had the appearance shown schematically in Fig. 8.4b for specimens longer than 10 mm with longitudinal splitting connecting the two failed regions. A similar appearance is shown by FRIED and WINAS (1963) in unidirectional fibreglass specimens which had failed in axial compression. The rectangular block shaped specimens (6 mm by 19 mm by 25 mm) were compressed between flat and parallel plates with the fibre axis parallel to the longest dimension. The failure pattern in the lamina-reinforced composite shown by GRESZCZUK (1973), (Fig. 15; $V_f = 49\%$), is also similar in appearance to Fig. 8.4b.

The thickness of the failed region, within the disc specimens, was consistent and affected a planar region (band) about 1 to 2 mm thick. This description of the failure mode is a generalization and many variations were observed. The failure mode was distinct only for composites with a low glass fibre content. Complete disintegration occurred for specimens with a high glass content and the same failure pattern was more difficult to establish from the debris.

The failure mode of tapered specimens, which was distinct for all glass contents, showed a similar pattern except that the failed region did not touch the specimen's ends. A typical failure plane is shown in Fig. 8.3 for an AR39 tapered specimen on which the loose debris had "fallen off". The photographs were taken with the aid of a scanning electron microscope after the specimen had been gold plated using vacuum deposition techniques. The fracture surfaces of individual fibres are distinctly visible showing the initial tensile and compressive regions. These



Reflected light (x 30)

Fig. 8.1 Shear failure band in AE 57 specimen



Mixed light (x 30)

Fig. 8.2 Shear failure band in AE 57 specimen with fibres orientated at 10° to applied load

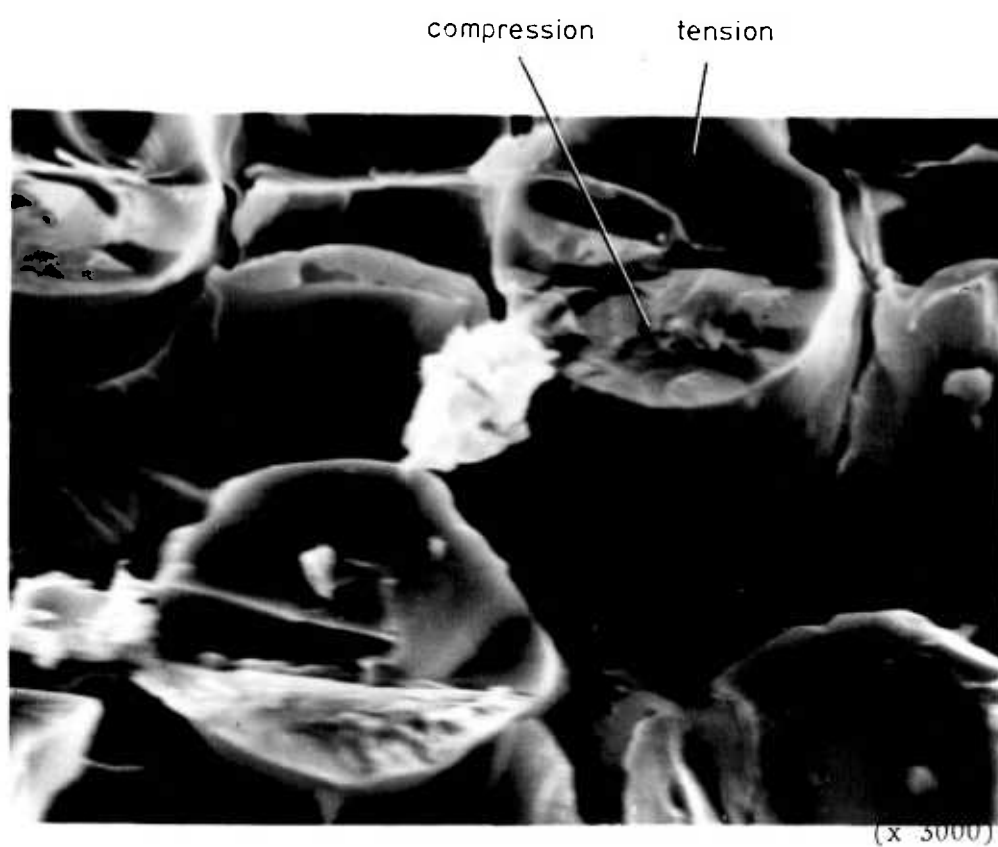
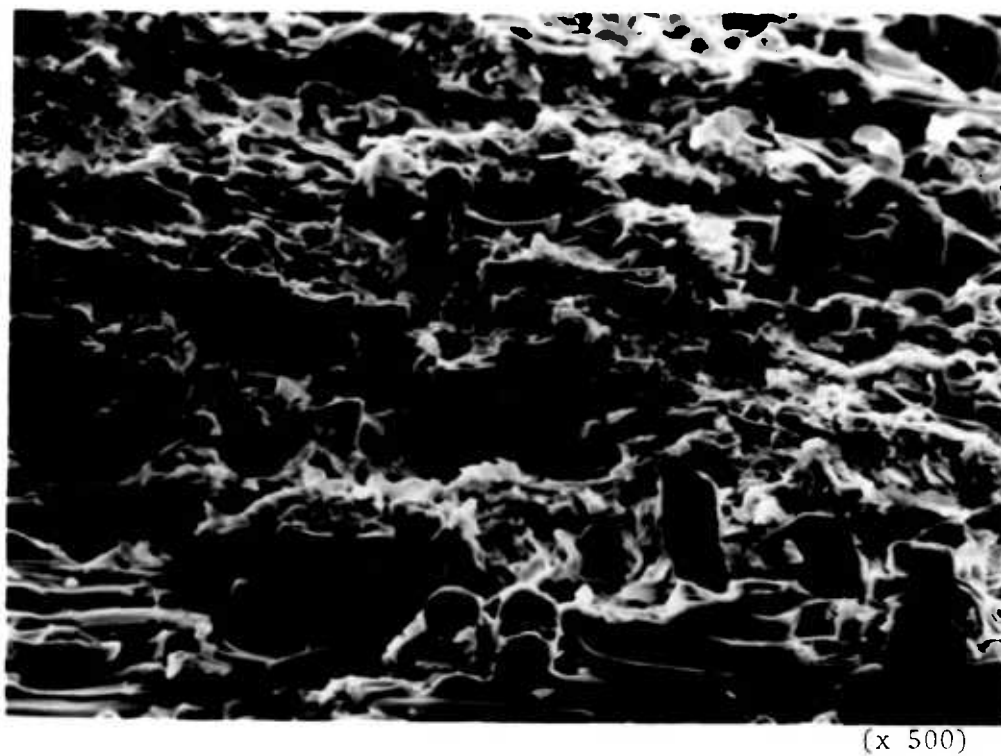
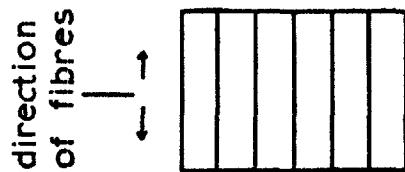


Fig. 8.3 Compression fracture surface of AR 39 specimen

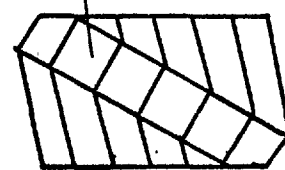
Fig. 8.4 Failure pattern of fibreglass disc

(a) Short specimen



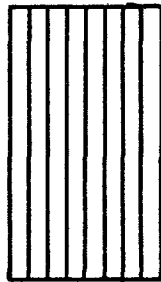
before failure

failed region (or band)



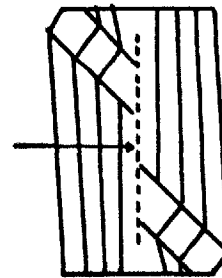
after failure

(b) Long specimen



before failure

longitudinal splitting



after failure

Fig. 8.5

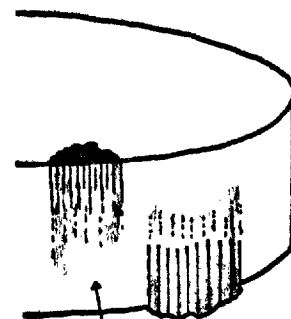
**Failed regions in
fibreglass disc**



failed regions

Fig. 8.6

**Failed regions in fibreglass
disc tested at -196°C**



failed regions

fracture surfaces are similar in appearance to those shown by EWINS and HAM (1973) for carbon fibres which had failed in the same mode..

Once the stress in the specimen had reached the U.C.S., failure was instantaneous. However, in a few disc specimens with low glass fibre contents (AE37), failure commenced up to 2 seconds before it became catastrophic. In this time it was possible to observe that damage was occurring in the two regions shown in Fig. 8.5 which correspond in relative position to the failure pattern of Fig. 8.4a. One AE37 disc specimen was loaded up to 95 % of its U.C.S. and then unloaded. Microscopic examination showed that the damaged region (only one in this case) had not penetrated as far as the specimen's surface.

A few polished AE37 disc specimens were tested at a temperature of -196°C . An increase of 100 % in the U.C.S. was obtained over room temperature values. Small instantaneous decreases in load were observed on the load-deflection curve which corresponded in time to audible clicks and occurred at stresses above 80 % of the U.C.S. One specimen was unloaded, before failure, after the first click was heard. The damaged region is shown schematically in Fig. 8.6 and differs from the room temperature mode of failure. Failed specimens had completely disintegrated and no deductions about the failure mode could be made.

8.2.2. FIBRECARBON

Disc specimens disintegrated almost completely when tested and the failure pattern was difficult to establish. In tapered specimens, the failed region consisted of one or two inclined and intersecting shear planes similar to the failure pattern of fibre carbon as described by EWINS and HAM (1973).

8.3. END CONSTRAINTS OF DISC SPECIMENS

The strength of a disc specimen depends upon the loading conditions at its ends. Under perfect loading conditions, the stress-strain fields in the specimen are identical to those present if the specimen was continuous through and beyond the loading interfaces. Specimens cut from an AE57 rod were compressed under three different loading conditions :-

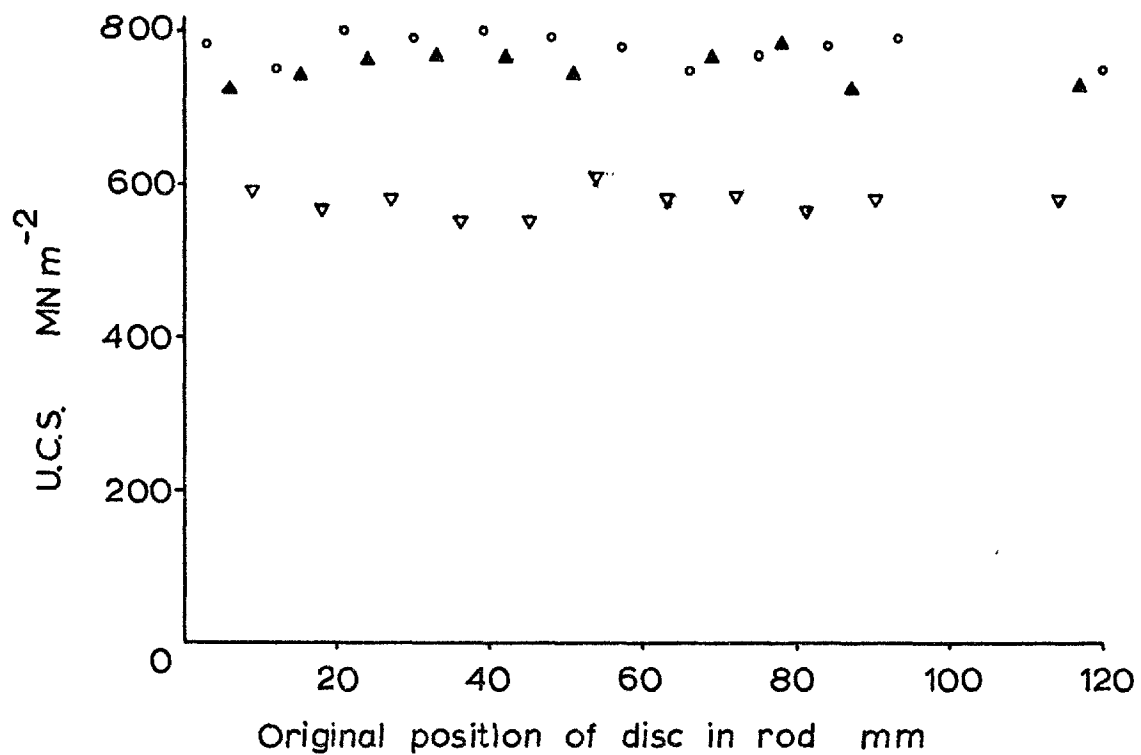
- a) Tested, "as cut", between the plates.
- b) Tested, "as cut", sandwiched between 3 mm thick steel discs.
- c) Polished with 800 grade silicon carbide powder and then tested as in b).

The results are shown in Fig. 8.7. If metal discs were not used, the specimen produced an elastic indentation in each plate which caused a non-uniform strain field in the specimen. For this reason, all subsequent testing was done with metal discs except for the pure resin specimens. The polished specimens were only 4 % stronger than the "as cut" specimens, also the scatter was the same for both types. These results showed that the U.C.S. was insensitive to the condition of the surface. No strength variation was observed when the 3 mm steel discs were replaced by 6 mm ones. This showed that the 3mm discs were thick enough to produce a uniform stress field at the specimen interface. AE75 specimens tested under type b) and c) conditions gave similar trends. Fig. 8.7 also shows the strength of disc specimens to be independent of their original position in the composite rod.

Measurements of the coefficient of static friction at the specimen-steel interface had a minimum value of 0.25 when normally stressed at a value of 4 MN/m^2 . This value of 0.25 was expected to increase with increasing normal stress. As the transverse Young's modulus of unidirectional fibre-reinforced plastics is low, it is inconceivable that the large transverse forces required to allow relative movement to take place at the interface were present. The transverse expansion of the specimen near the interface was hence identical to the free transverse expansion of the metal discs. A possible cause of strength degradation is therefore the mismatch of free transverse

Fig. 8.7

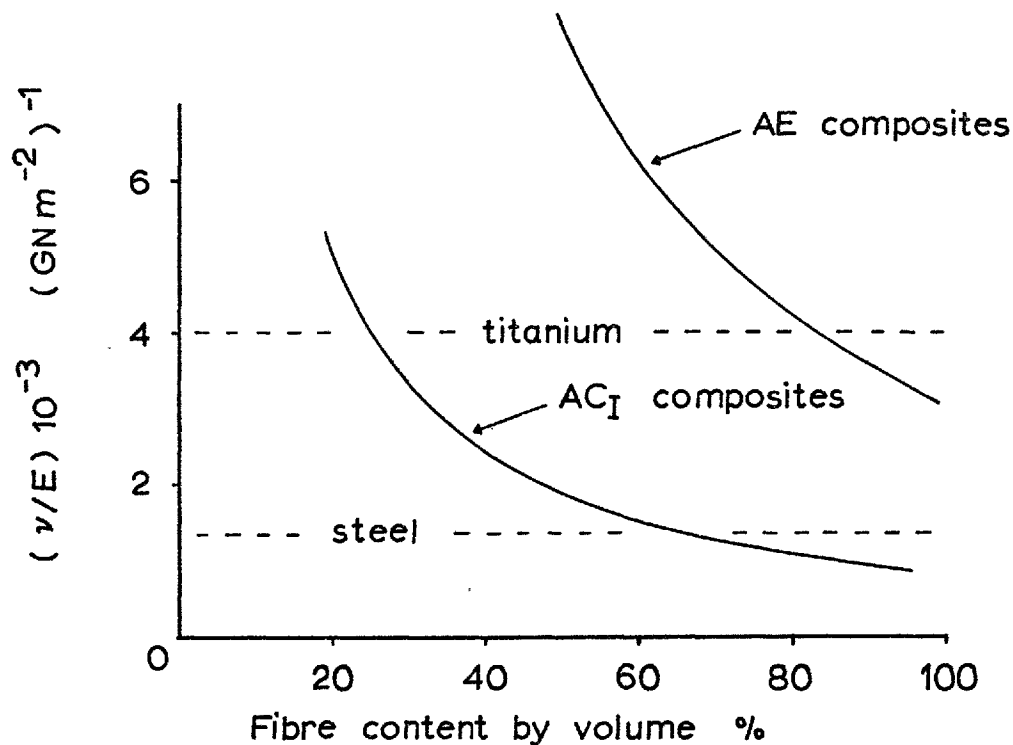
U.C.S. of discs cut from an AE56 rod



specimen type	U.C.S. (mean)	standard deviation
◦ (c)	777	18
▲ (b)	748	19
▼ (a)	576	15

Fig. 8.8

Transverse expansion of discs



expansion between the specimen and the metal discs. The free transverse strain in the composite is equal to $\sigma (\nu/E_c)$ where ν is the longitudinal-transverse Poisson's ratio and σ is the longitudinal (axial) stress. A measure of the mismatch was given by the size of the discontinuity in (ν/E) at the interface because the normal stress σ was continuous at the interface. This mismatch is shown in Fig. 8.8. The values of ν for the composites were calculated by using the manufacturers' values of ν for the resin and fibre and assuming a mixture rule. The transverse stresses in the composites were always compressive provided the value of (ν/E) was greater than that of the metal discs. AE disc specimens with various fibre contents were tested between 6 mm thick high-strength titanium alloy discs (IMI 700) with the same diameter as the specimens and compared with similar tests using steel discs. The end faces of the titanium discs were polished flat and parallel using 800 grade silicon carbide powder. No significant difference was noticed with AE37 specimens but an average strength increase of 13 % in the range AE57 to AE75 was measured using titanium alloy discs. If this effect was critical then the strength of AE75 specimens, where there is practically no mismatch, would be expected to increase significantly. This was not the case, because AE57 specimens gave similar strength increases as AE75 specimens.

Steel discs had the advantage of being much harder and less prone to surface damage. Also the strongest specimens tested could exceed the yield strength of the titanium alloy. The use of titanium discs with fibre-carbon specimens would have caused transverse tensile stresses in the specimens which would have been undesirable because of the low transverse tensile strength of these composites. Therefore, for convenience, all testing was carried out using steel discs.

8.4. DISC SPECIMEN GEOMETRY

An AE57 rod was cut in half in order to test the effect of disc diameter on strength properties. One half was centreless ground down to half its original cross-sectional area and specimens cut from this half were tested between steel discs with the same diameter as the specimens. The measured mean

strengths and standard deviations of five specimens each, cut from the thin and thick halves of the rod were $(796 \pm 45) \text{ MN/m}^2$ and $(796 \pm 34) \text{ MN/m}^2$ respectively. These results showed that the effect of exposed specimen surface, if any, was small and could be represented by a thin strengthless surface layer with a thickness less than 50μ .

AE57 specimens were cut off with various lengths in the range 0.8 mm to 15 mm and tested. The strength was independent of the length in the range 1.5 mm to 15 mm but a distinct dependence was observed for lengths less than 1.5 mm as shown in Fig. 8.9. All specimens were polished because it was expected that the effect of surface condition would be more pronounced for thin specimens. Extra care had to be taken not to damage these wafer-thin specimens during handling. One specimen, 1 mm in length, tested in the "as cut" condition had a strength 20 % less than the polished 1 mm specimens. This compared with a 4 % reduction for 3 mm specimens. A significant increase in strength for thin specimens was observed, as shown in Fig. 8.9, by using a different pair of steel discs which had been used much less and hence most probably had flatter surfaces. This increase was not apparent when used with 3 mm specimens.

8.5. OFF-AXIS LOADING

Specimens were cut from an AE57 rod at various angles in the range $(10^\circ \geq \theta \geq 0^\circ)$ where θ is the angle between the axis of the rod and the normal of the two flat faces of the specimen. The results of strength testing are shown in Fig. 8.10. No significant loss in strength occurred provided the specimen was less than 0.5° off-axis. Alignment was therefore not considered to be a critical parameter because it was possible to cut specimens well within a tolerance of 0.5° . As a final check, a few specimens were produced by first polishing the end of the rod using the techniques described in section 3.2.2., then cutting off the specimen and finally polishing the cut face. This method produced specimens with "exact" alignment. No significant change in strength was obtained for these specimens.

The appearance of the failure mode was similar to that of the aligned specimens, as shown in Fig. 8.2 for a failed specimen

Fig. 8.9 Strength-length correlation of AE57 discs

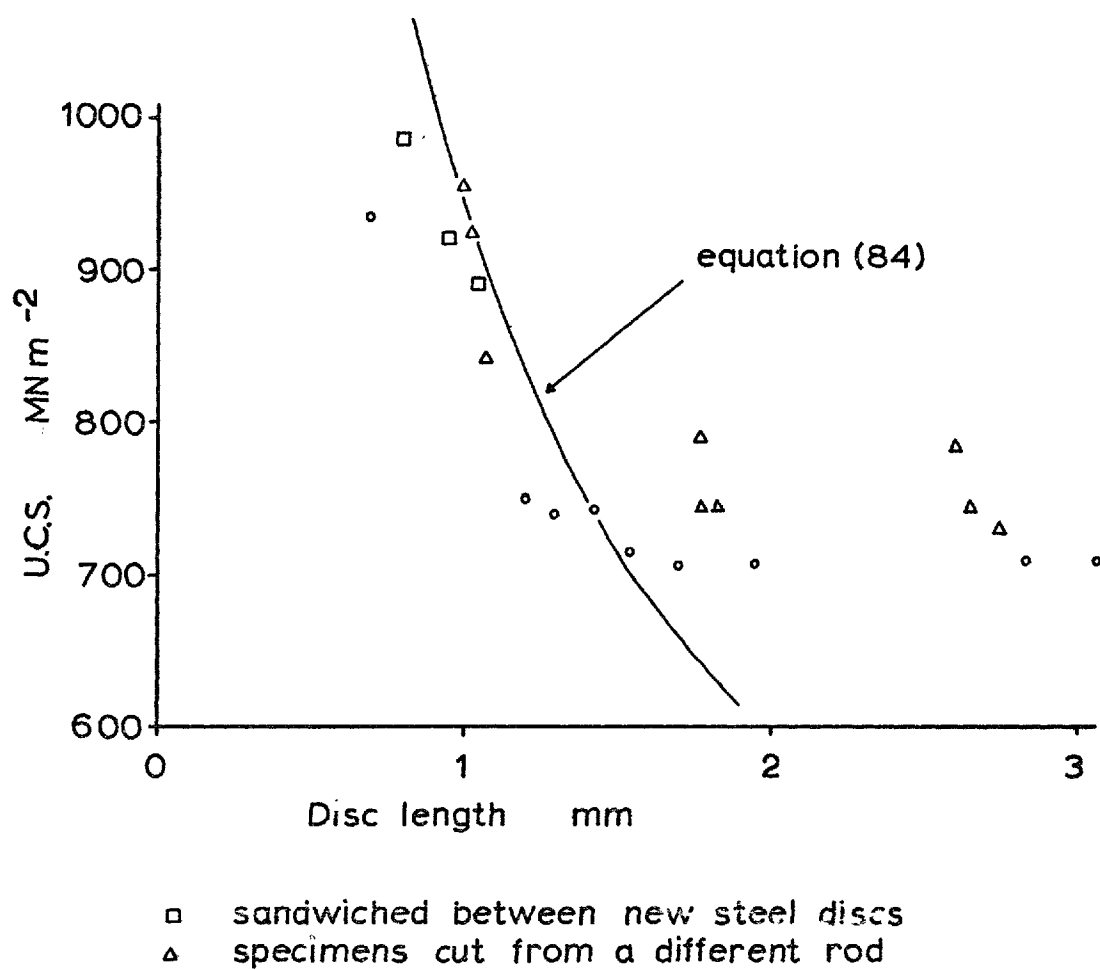
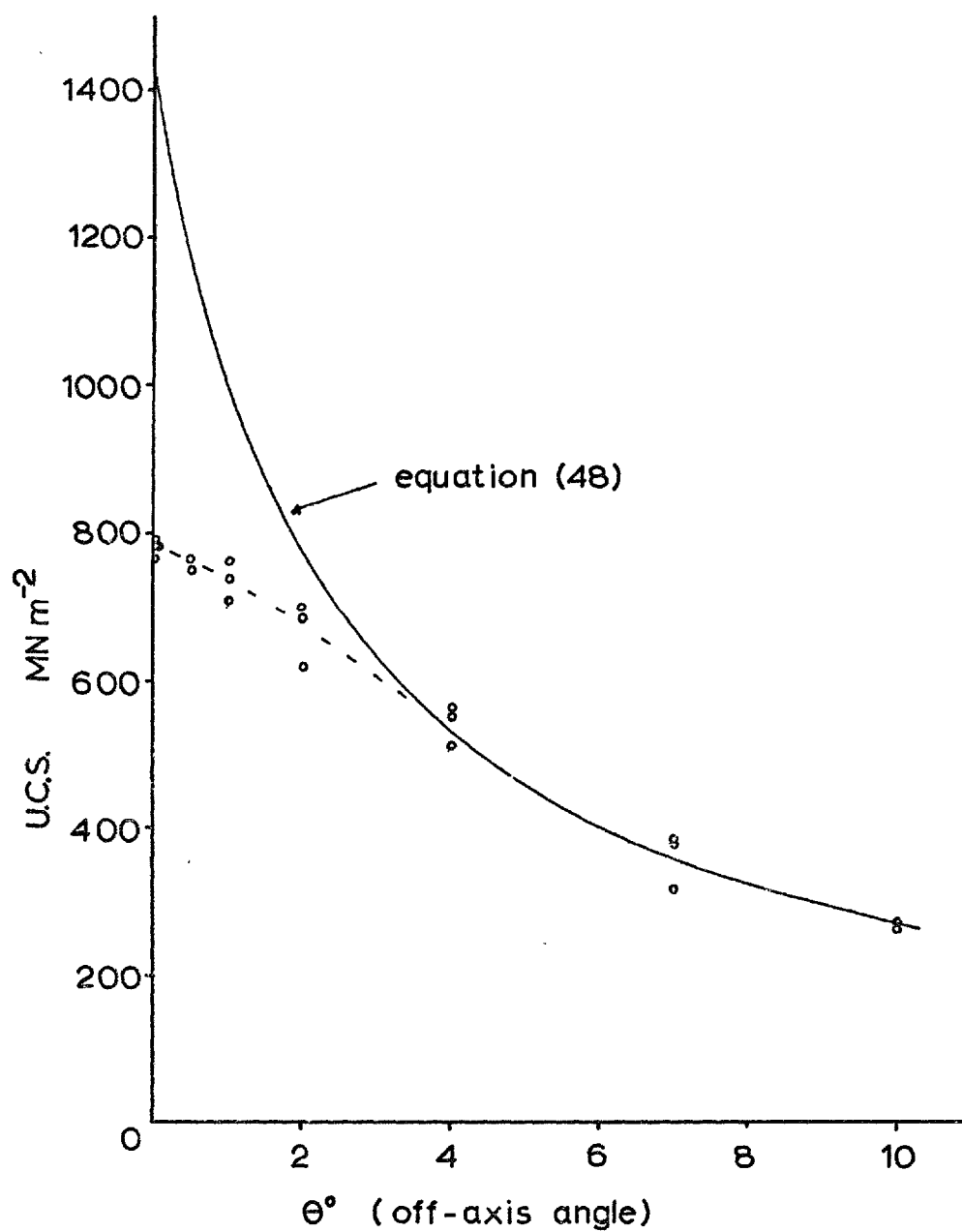


Fig. 8.10 Strength-orientation correlation of AE57 discs



orientated at 10° off-axis. The dark region corresponds to the shear failure band.

8.6. FIBRE CONTENT (DISC SPECIMENS)

The strength variation with fibre content for the AE range is shown in Fig. 8.11. The mean and spread of strength values are plotted. The strength values in the AR range are plotted in Fig. 8.12 which shows that AR composites were slightly superior to AE composites. Composites in the BE and BR ranges produced identical results to the AE and AR ranges respectively and are not shown.

The strengths of fibre-carbon discs are indicated in Fig. 8.13 which shows that BC_I composites possessed similar strengths to AC_I composites. BC_{II} composites were significantly stronger than AC_{II} composites which was attributed to the 3 % void content of the AC_{II} composites. The cause of this void content was unknown but was produced during the cure schedule by a reaction between resin A and the fibres. This reaction did not occur when resin B was used.

8.7. TAPERED SPECIMENS

A limited amount of testing was performed using tapered fibreglass specimens. The aim of these tests was to confine the failed region to a volume not in contact with the specimen ends and to compare these results with those from the disc specimens. It was necessary to give each end support in the form of a steel ring to prevent premature failure occurring at the ends. Several types of geometry are shown in Fig. 8.14 and their comparative strengths recorded for AE57 specimens. Maximum strengths were recorded using specimen geometry type (a) and all subsequent testing was done with this geometry. The cross-sectional area of the central region always had to be reduced by $2/3$ to prevent one of the steel rings from breaking and it had to be reduced by $3/4$ for specimens with a 75 % fibre content. Under this condition, these maximum fibre content specimens failed at one end where the failed region was contained by the ring as shown in Fig. 8.15b; hence only a minimum strength could be recorded.

Fig. 8.11

Strength of AE discs

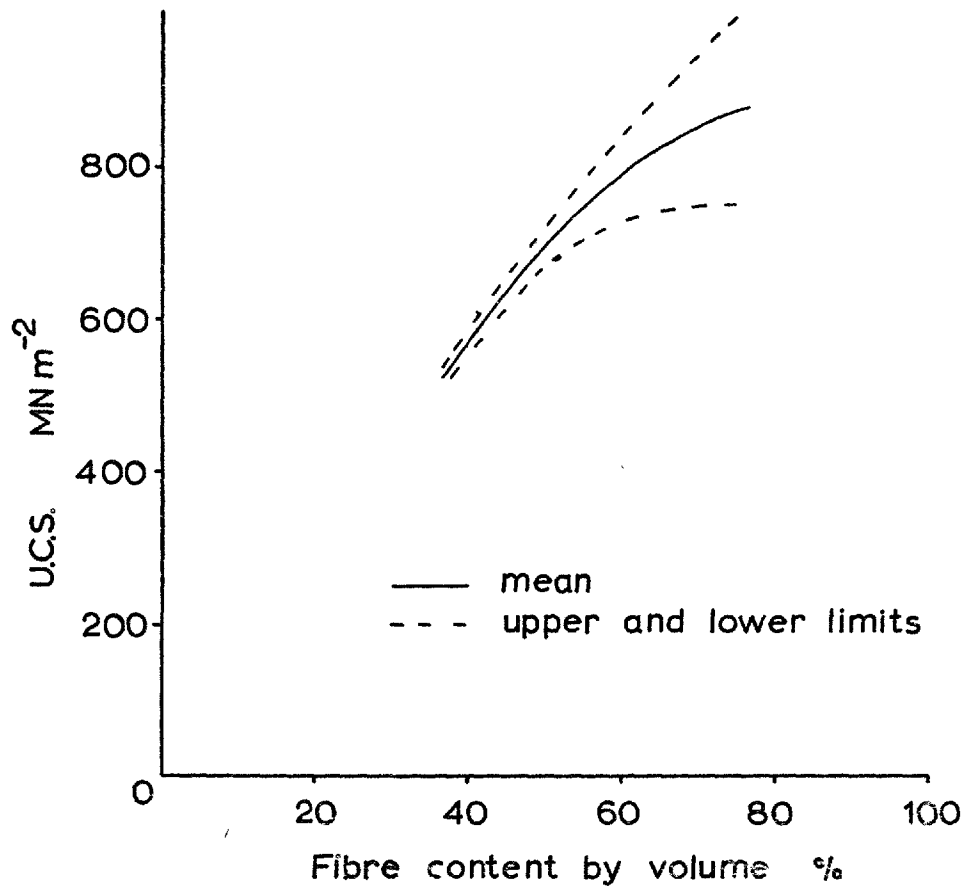


Fig. 8.12

Strength of AR discs

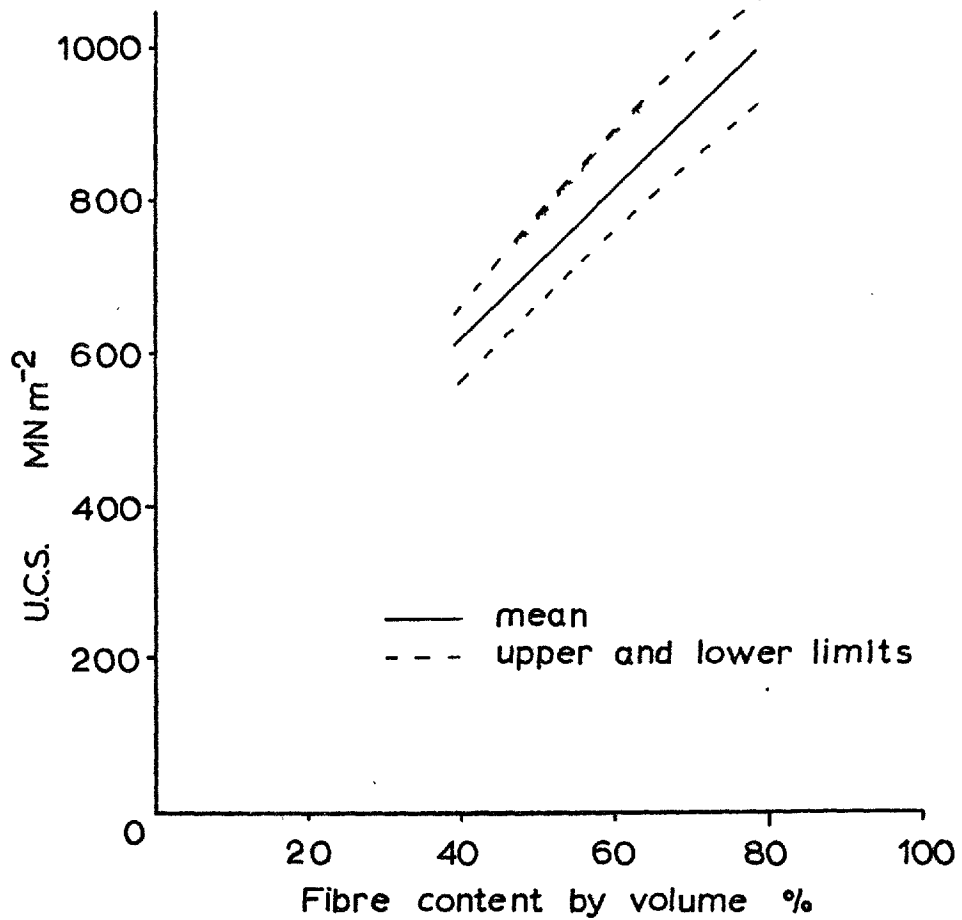
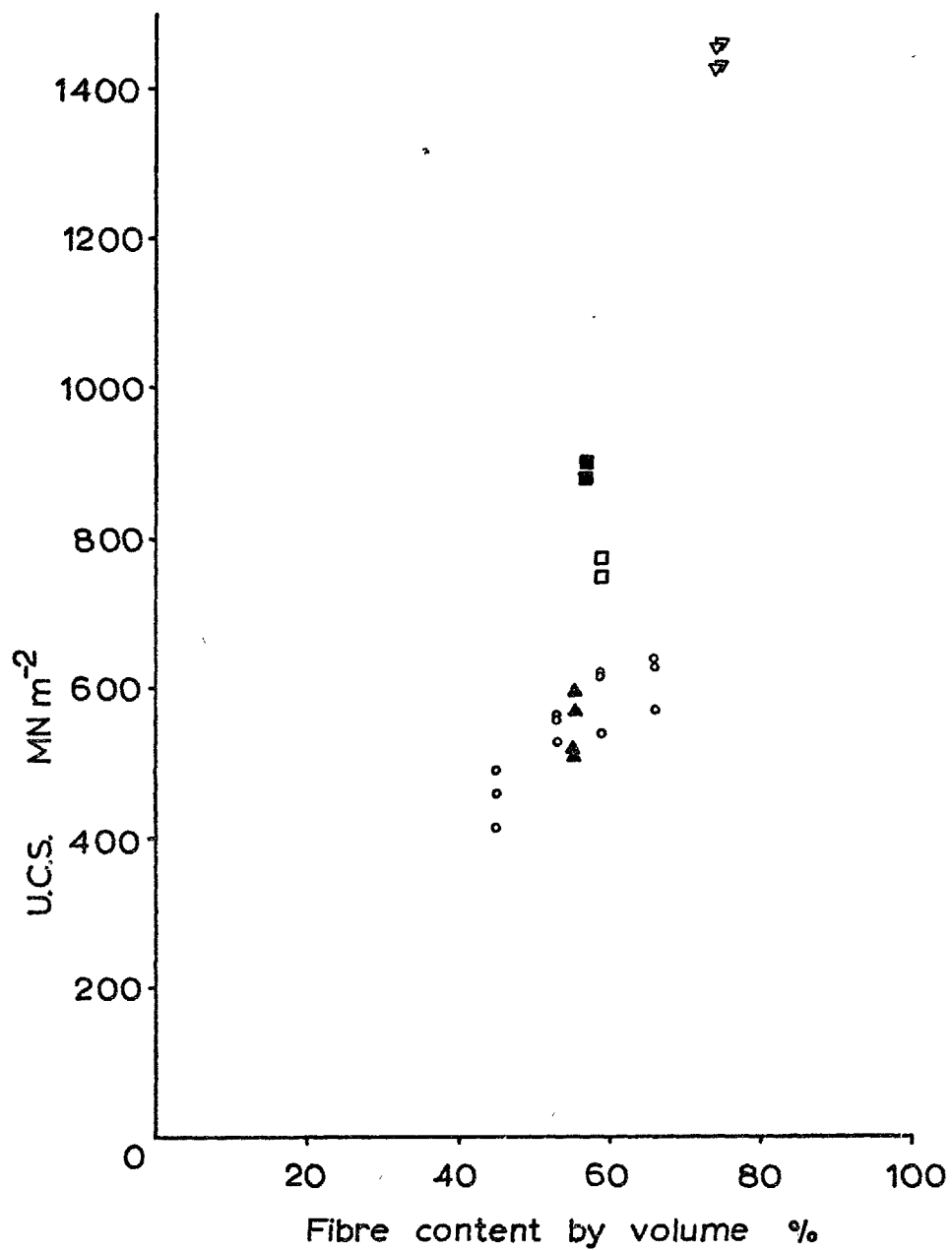


Fig. 8.13

Strength of fibre carbon discs



- AC_I composites
- ▲ BC_I composites
- AC_{II} composites
- BC_{II} composites
- ▼ AC_{III} composites

Fig. 8.14 Comparative strengths and geometries of tapered AE specimens (scale 1:1)

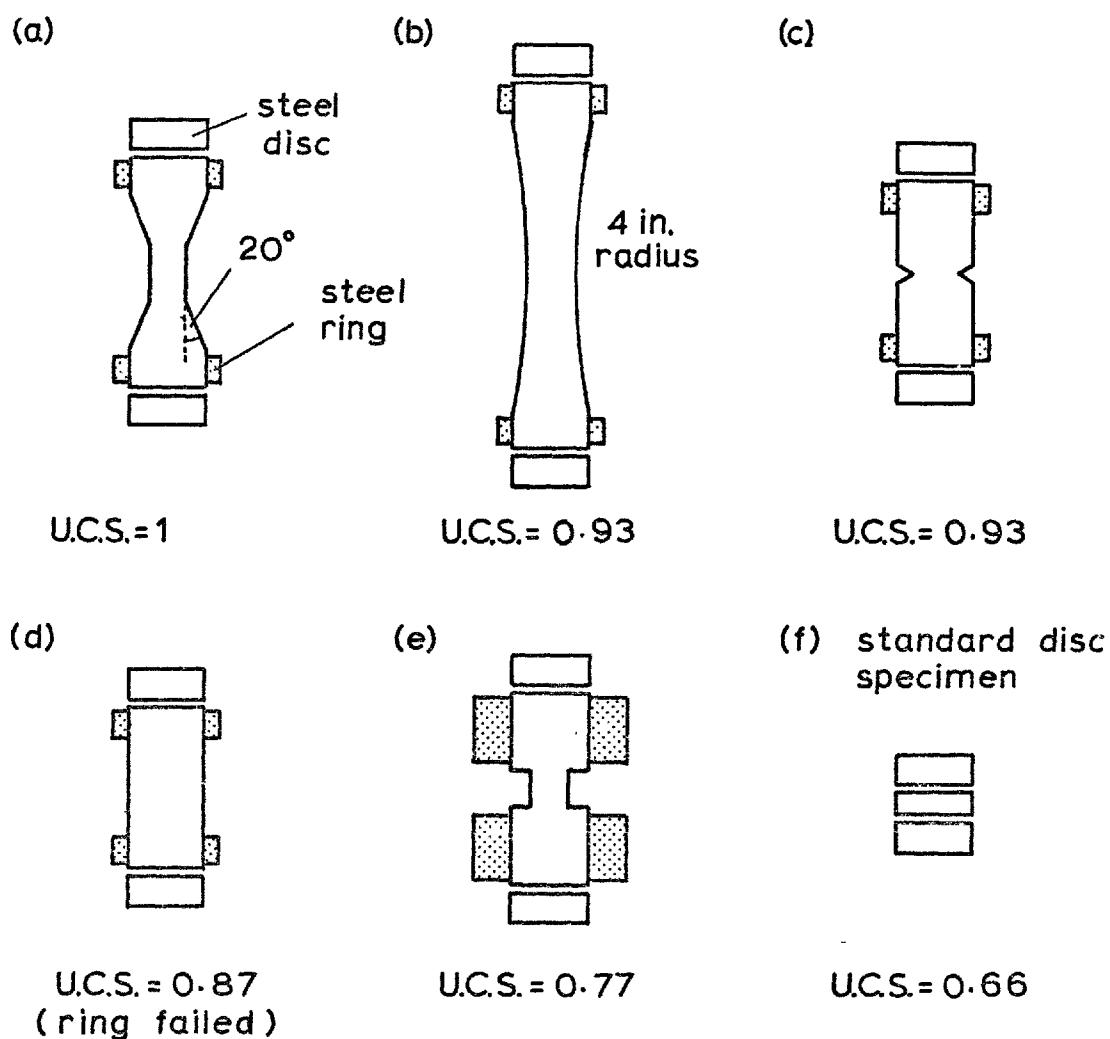
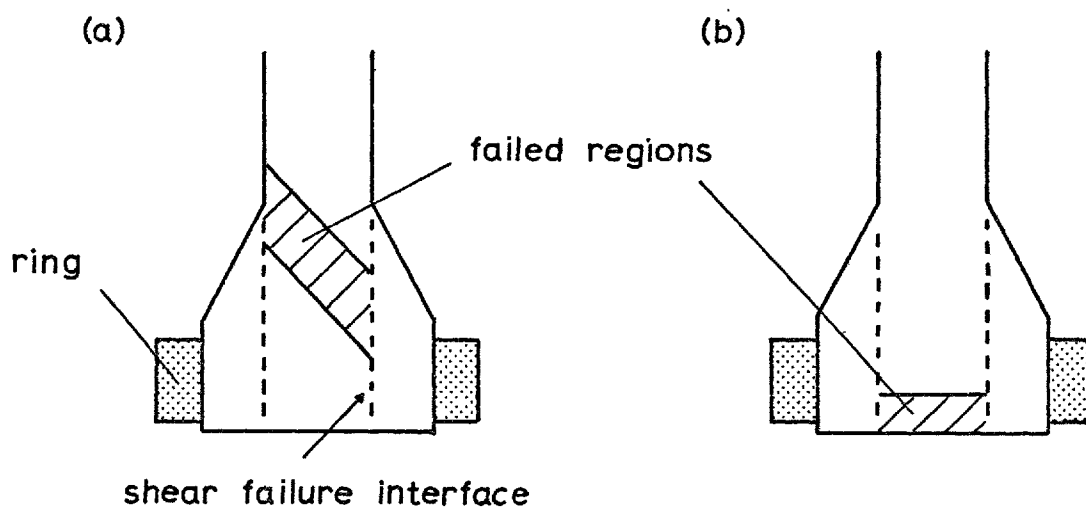


Fig. 8.15 Failure patterns in tapered AE specimens



Failure always occurred within the tapered region as shown in Fig. 8.15a (except, as stated, for maximum fibre content specimens). Shear failure started at 20 % of the U.C.S. as the central column grew through the tapered regions to the ends. The failed region was similar in appearance to that in disc specimens. For the other geometries shown in Fig. 8.14, failure was only confined to the central region in specimen (e) which had the lowest strength of the tapered specimens. Failure in specimens (b) and (c) occurred in regions between one end and the minimum cross-sectional area in a similar manner to that described for specimen (a).

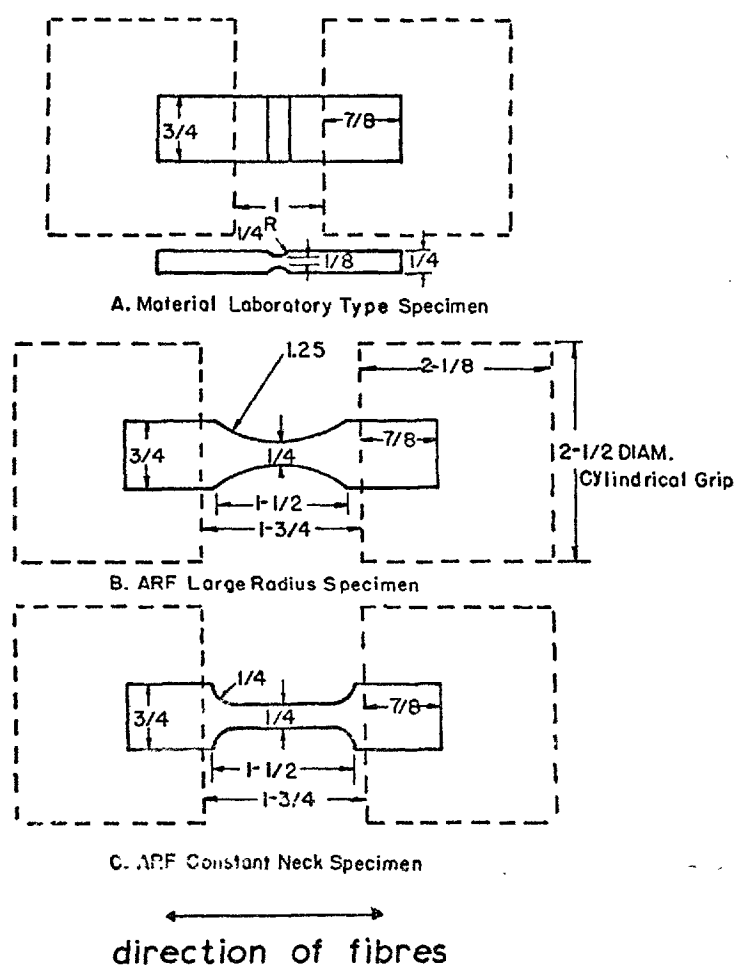
BROUTMAN (1964) tested three specimen geometries, shown in Fig. 8.16, cut from tension-wound unidirectional composites consisting of E-glass fibres and epoxy resin. All specimens had a 70 % fibre content and were compressed axially. Load was applied to the ends through wedge grips. The mean strengths for specimen types A, B and C were 982, 864 and 825 MN/m² respectively.

LEVENETZ (1964, 1965) tested similar composites in the form of tension-wound NOL rings. The NOL ring consists of a ring of 146 mm inside diameter, 6 mm in width and 3 mm in thickness with the fibres aligned in the hoop direction. He used the split-disc-ring method shown schematically in Fig. 8.17 which gave a mean strength of 930 MN/m² for a 65 % fibre content. It does have the disadvantage of compressing two initially curved regions. This method as well as various others are described by A.S.T.M. (1963). He also used a special compressive fixture designed to mechanically apply uniform radial loads to the NOL rings while the ring is kept circular at all times. The radial load is balanced by the compressive hoop load in the ring. This fixture gave increases in strength of 70 % over the split-disc-ring method for standard fibreglass.

The strengths of AE and AR composites using tapered type (a) specimens are shown in Fig. 8.18. Specimens tested in the BR and BE ranges had similar strengths. The only specimen tested in the AC range (AC_I45) had a strength slightly less than that of AC_I45 disc specimens.

Fig. 8.16

Specimen geometries used by Broutman
(all measurements in inches)



specimens (B) and (C) are two-dimensional
and 1/4 in. thick

Fig. 8.17

Split-disc-ring apparatus for NOL rings

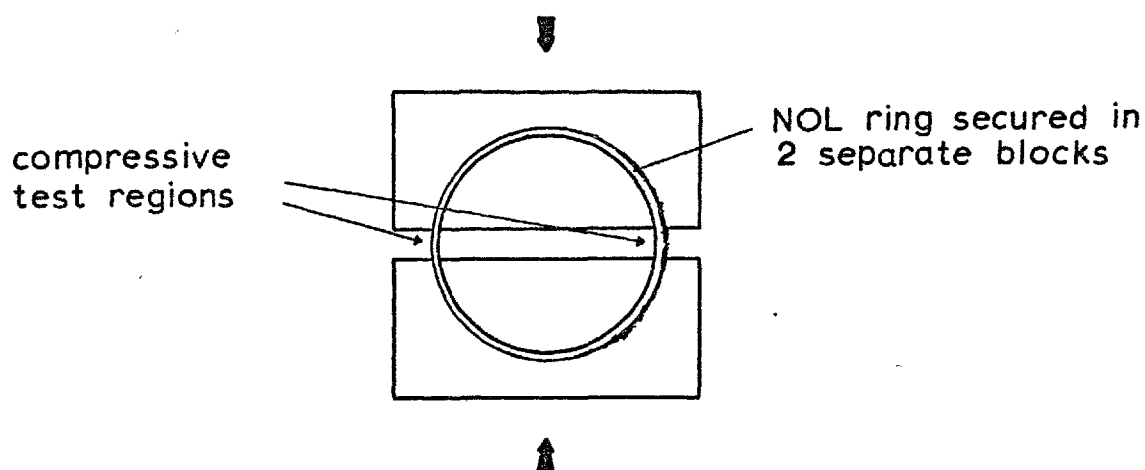
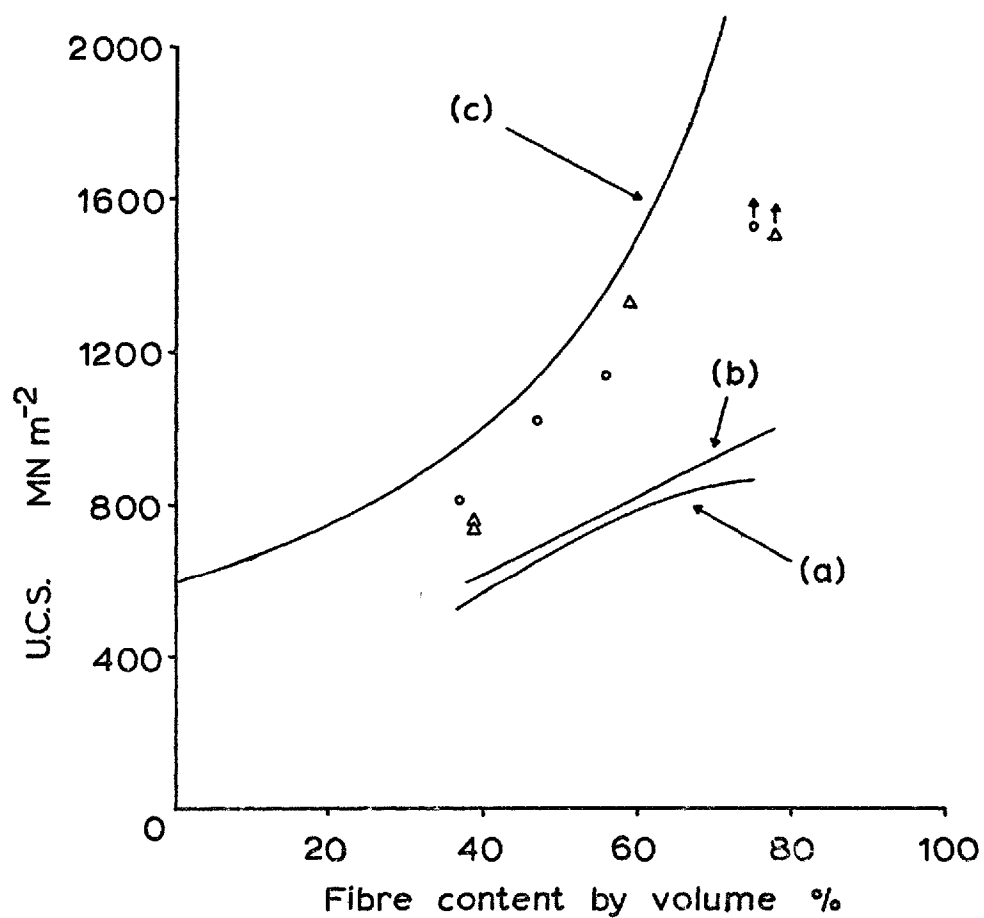


Fig. 8.18

Strength of tapered AE and AR specimens



(a) AE discs

(b) AR discs

(c) equation (33) using resin A

○ AE tapered specimens

△ AR tapered specimens

8.8. VOID CONTENT

The effect of void content on the U.C.S. of disc specimens in the AE range was small. The fractional loss in strength was approximately equal to the void content independent of the fibre content. However, with tapered specimens, the effect was more pronounced. An AE75 specimen with a void content of 4 % failed with a 17 % loss in strength as compared to a 30 % loss in strength for an AE37 specimen with a 7 % void content.

Several workers, PAUL and THOMSON (1965), SANDS et al. (1967), PETKER (1965) and FRIED (1965) reported that large decreases in interlaminar shear strength occur with only small void contents. However, reported data concerning the effect of void content on compressive strength is confined to bidirectional composites (FRIED (1965)) and is not directly comparable with data obtained from unidirectional composites.

CHAPTER NINE

AXIAL COMPRESSIVE STRENGTH (THEORETICAL)

9.1. GENERAL CONSIDERATIONS

The experimental work was confined to fibre-reinforced composites in which the fibre modulus is greater than the matrix modulus by a factor of at least 20. Hence theoretical considerations are confined to failure modes subject to this condition and cannot necessarily be expected to apply to other types of composites. In addition it is assumed that the failure strain of the composite is within the elastic limits of both constituents. If this is not true, then the compressive strength is governed by the composite modulus and the failure strain of the constituent which fails first.

The generally accepted theory regards the failure mode as a buckling phenomenon. The buckling strength of a column subjected to an axial load is well documented (TIMOSHENKO (1961)). Two possible methods can be used to determine the buckling load in both of which the length of the column is regarded as remaining constant during bending.

The first method is to calculate the lateral deflection of the column due to an applied lateral load while it is subjected to an axial load. The buckling load can then be defined as the value of the axial load when very small lateral loads produce infinitely large deflections.

The second method utilises energy considerations. With reference to Fig. 9.1, the column is in equilibrium with a central deflection, a , while subjected to the combined axial and lateral loads of P and Q respectively. The additional work done by the loads P and Q in producing a small additional deflection δa must equal the additional increase in elastic strain energy stored in the column. As P is increased, Q must be decreased to maintain the same value of " a " until P reaches the buckling load and Q is zero. If P is increased above this value then Q must reverse direction in order to maintain equilibrium and the column is then supported laterally to prevent it from buckling. The buckling load P_c is therefore reached when the column changes

Fig. 9.1

Column subjected to
biaxial loading

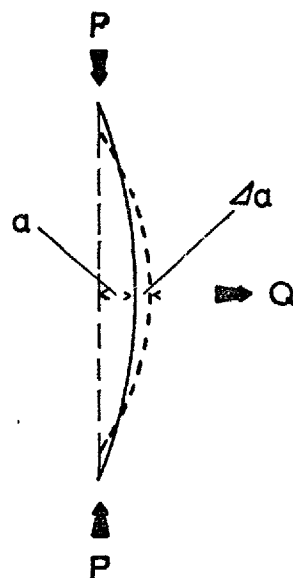


Fig. 9.2

Shear mode

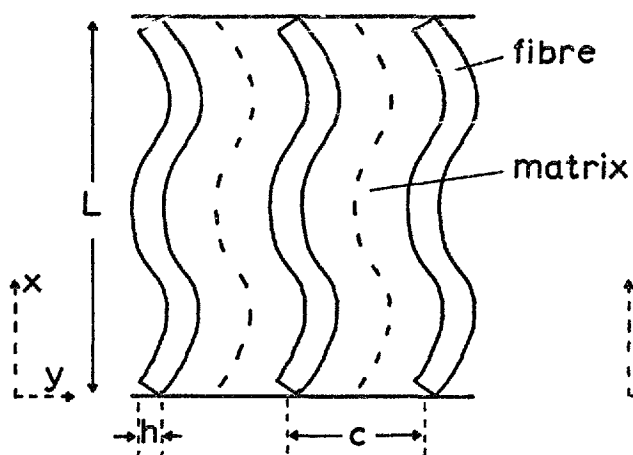


Fig. 9.3

Geometry of fibres
in shear mode

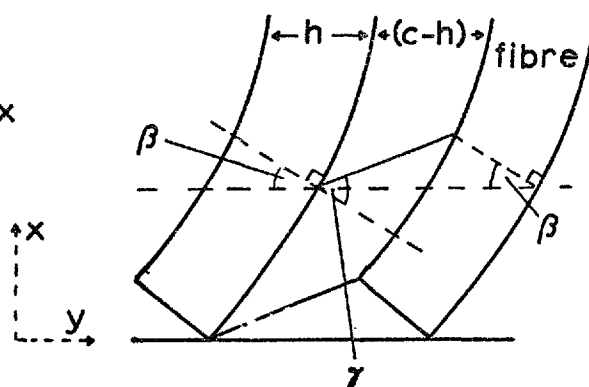
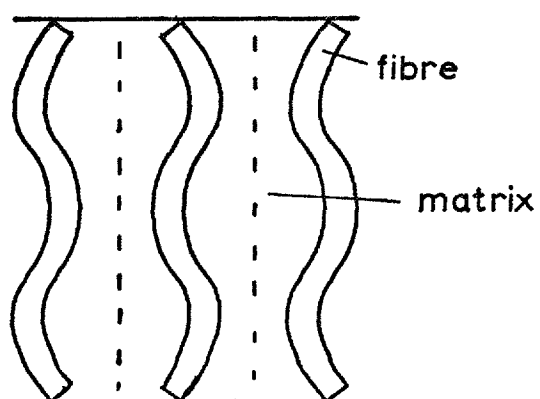


Fig. 9.4

Extension-compression
mode



from a state of stable to unstable equilibrium. P_c is determined by equating the additional work done by P ($P = P_c$) to the additional increase in strain energy due to the deflection Δa .

Using these two methods, it is possible to envisage various buckling configurations subjected to various constraints and calculate the buckling loads. The composite is expected to buckle in the mode that has the minimum buckling load.

9.2. MICRO-ANALYSIS

The envisaged two-dimensional shear mode of failure where all the fibres buckle into the same shape is shown in Fig. 9.2. The unit of volume of the composite considered is one fibre of thickness h and one adjoining matrix volume of thickness $(c - h)$, both with length L . This unit is repeated throughout a composite of any thickness so that the fibre volume fraction V_f is given by :-

$$V_f = h/c \quad (15)$$

The two-dimensional analysis of TIMOSHENKO (1961) of the buckling of a column on an elastic foundation is used. The lateral displacement, y , of the fibre is given by the series :-

$$y = \sum_n a_n \sin(n\pi x/L) \quad (16)$$

for any curve where n has only positive integer values.

TIMOSHENKO (1961) proved that the buckling load is always a minimum for one value of n only provided the lateral reaction on the fibre due to the foundation is proportional to y . Making this condition an initial assumption gives :-

$$y = a \sin(n\pi x/L) \quad (17)$$

The shear strains in the matrix are assumed to have no effect on the fibre strains due to the relatively low modulus of the matrix. With reference to Fig. 9.3, it can be shown from the geometry that the shear strain γ in the matrix is given by :-

$$\gamma = \beta \frac{c}{c-h} = \frac{dy}{dx} \frac{c}{c-h} \quad (18)$$

The shear energy W_m of deformation of the matrix is given by :-

$$W_m = \frac{1}{2}(c-h) \int_0^L G_m \gamma^2 dx \quad (19)$$

where G_m is the shear matrix modulus. Substituting equations (15), (17) and (18) into equation (19) and integrating gives :-

$$W_m = \frac{G_m}{1-V_f} \frac{\pi^2 n^2 a^2 c}{4L} \quad (20)$$

The extensional energy of deformation of the matrix is equal to :-

$$\frac{E_f}{6} \frac{1-V_f}{V_f} \frac{h^3}{L^3} n^2 a^2$$

where E_f is the Young's modulus of the fibre. The value of this energy is much less than W_m and is hence ignored. The strain energy W_f in the elastic column (fibre) due to bending is given by TIMOSHENKO (1961) as :-

$$W_f = \frac{\pi^4}{4} E_f I n^4 \frac{a^2}{L^3} \quad (21)$$

where I is the moment of inertia of the cross-sectional area about its neutral axis, and which in this two-dimensional case is given by :-

$$I = h^3/12 \quad (22)$$

Substituting equations (15) and (22) into equation (21) gives :-

$$W_f = \frac{\pi^4}{48} \frac{h^2 c}{L^3} E_f V_f n^4 a^2 \quad (23)$$

The total strain energy W is given by :-

$$\begin{aligned} W &= W_f + W_m \\ &= \frac{\pi^2 n^2}{4L} a^2 c \left[\frac{G_m}{1 - v_f} + \frac{\pi^2}{12} v_f E_f (h/L)^2 n^2 \right] \end{aligned} \quad (24)$$

The axial displacement of the composite due to its lateral displacement is given by :-

$$\frac{1}{2} \int_0^L (dy/dx)^2 dx = \frac{\pi^2}{4L} n^2 a^2 \quad (\text{provided } 0 < \frac{a n}{L} \ll 1)$$

The work done H by the average axial stress σ acting over the cross-sectional area of the composite unit is therefore given by :-

$$H = \sigma \frac{\pi^2}{4L} n^2 a^2 c \quad (25)$$

Equilibrium was maintained, for the finite displacement a , by the transverse stress Q acting on the composite unit where :-

$$Q = Q_0 \sin(n\pi x/L) \quad (26)$$

The work done T by Q during the deformation is given by :-

$$T = \int_0^L Q y dx = Q_0 a \int_0^L \sin^2(n\pi x/L) dx = \frac{1}{2} Q_0 a L \quad (27)$$

If a is increased by the increment Δa , then the corresponding energy changes are given by :-

$$\Delta T = (dT/da) \Delta a \quad \text{etc.} \quad (28)$$

and to maintain equilibrium :-

$$\Delta T + \Delta H = \Delta W \quad (29)$$

Substituting equations (24), (25), (27) and (28) into equation (29) yields :-

$$Q_0 = \pi^2 n^2 c (a/L^2) \left[\frac{G_m}{1 - V_f} + \frac{\pi^2}{12} V_f E_f (h/L)^2 n^2 - \sigma \right] \quad (30)$$

Therefore Q_0 is proportional to "a" and hence Q is proportional to y . If $\sigma = 0$, then Q represents the reaction. The foundation is therefore elastic agreeing with the initial assumption. The buckling stress σ_c is then given by equating Q to zero for finite displacements :-

$$\sigma_c = \frac{G_m}{1 - V_f} + \frac{\pi^2}{12} V_f E_f (h/L)^2 n^2 \quad (31)$$

Equation (31) was derived by ROSEN (1964) using a similar analysis. GRESZCZUK (1972) used a model of cylindrical fibres in a square array and assumed that all of the matrix deformed in a similar pattern to the fibres. The same technique was used throughout with the appropriate substitutions for equations (15) and (22) which finally yielded :-

$$\sigma_c = \frac{G_m}{1 - V_f} + \frac{\pi^2}{16} V_f E_f (d/L)^2 n^2 \quad (32)$$

which can be regarded as a refinement of equation (31) where d is the fibre diameter. Provided the buckling wavelength is at least an order of magnitude greater than the fibre diameter, the second of the two terms on the right of equations (31) and (32) are negligible. Therefore, in this model the composite buckles into comparatively large wavelengths with a strength given by :-

$$\sigma_c = \frac{G_m}{1 - V_f} \quad (33)$$

ROSEN (1964) also envisaged an "extension-compression" failure mode shown in Fig. 9.4. The predicted strengths for this mode exceed the shear mode strengths for all values of V_f greater than 0.1 and therefore the shear mode is dominant for all practical composites.

9.3. OFF-AXIS LOADING (MACRO-ANALYSIS)

The macro-analysis regards the composite as a macroscopic material with no reference to its micro-structure; hence the name. The two-dimensional model used in this analysis is shown in Fig. 9.5 and corresponds to the actual loading conditions experienced in disc specimens described in section 8.5. The applied constant stress, σ , is inclined at an initial angle, θ , to the fibre axis. The model deforms in simple shear due to this stress, with a shear strain γ , and with the fibres remaining rigid. As the two loading faces are not coincident, a clockwise torque is produced which must be balanced by additional stresses acting on the loading faces. These stresses were comparatively small for the specimen geometries used in section 8.5. and a uniform stress, σ , was considered to act on the specimen ends. With reference to Fig. 9.6, the distance moved by the applied load is equal to $(x + y)$. From geometrical considerations :-

$$x = L (1 - \cos\gamma) \quad (34)$$

$$y = L \tan\theta \sin\gamma \quad (35)$$

Therefore the work done, H , by the applied load is given by :-

$$H = \sigma L c (1 - \cos\gamma + \sin\gamma \tan\theta) \quad (36)$$

which reduces to :-

$$H = \sigma L c (\gamma^2/2 + \tan\theta \gamma) \quad (37)$$

for small shear strains. The increase in strain energy W is given by :-

$$W = \frac{1}{2} L c G_c \gamma^2 \quad (38)$$

where G_c is the shear modulus of the composite. It is convenient to define the energy density function, U , where :-

$$U = \frac{W - H}{L c} \quad (39)$$

Fig. 9.5

Off-axis loading

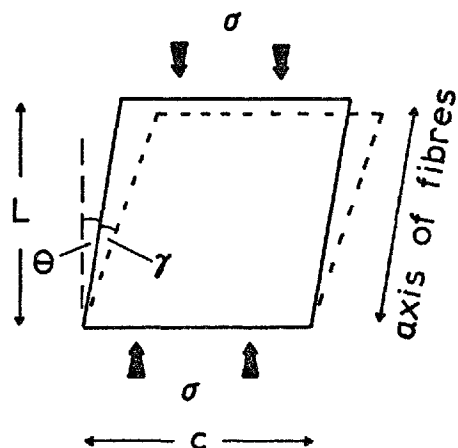


Fig. 9.6

Geometry of (Fig. 9.5)

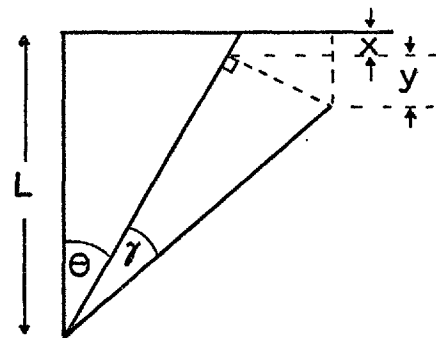


Fig. 9.7

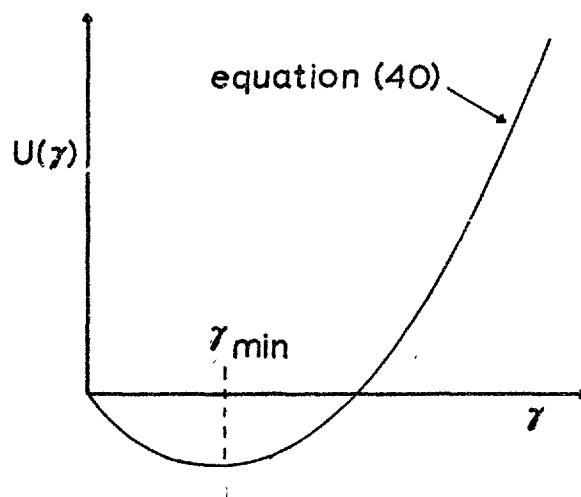
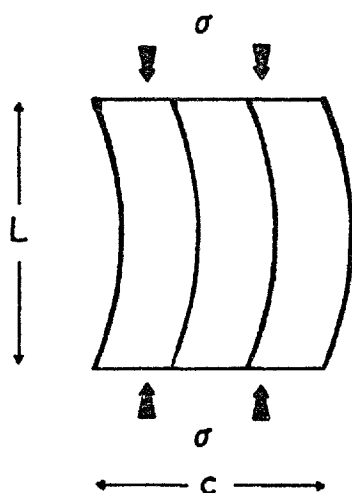
Variation of energy U with shear strain γ 

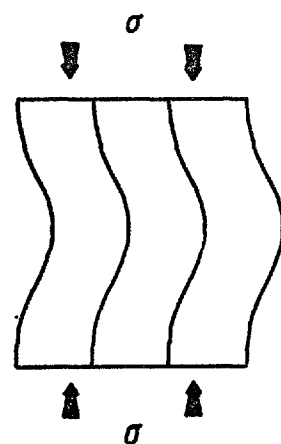
Fig. 9.8

Shear modes

(a)



(b)



$$\text{or } U = \frac{1}{2}G_c \gamma^2 - \sigma \left(\frac{1}{2}\gamma^2 + \gamma \tan\theta \right) \quad (40)$$

A typical curve of the function $U(\gamma)$ is shown in Fig. 9.7 for finite values of σ and θ . For equilibrium, the unbalanced torque acting on the composite, which is proportional to $dU/d\gamma$, must equal zero or :-

$$dU/d\gamma = G_c \gamma - \sigma (\gamma + \tan\theta) = 0 \quad \text{when } \gamma = \gamma_{\min} \quad (41)$$

$$\text{Therefore } \gamma_{\min} = \frac{\sigma \tan\theta}{G_c - \sigma} \quad (42)$$

$U(\gamma_{\min})$ is a minimum value, therefore $\gamma_{\min}$ is the stable equilibrium shear strain of the composite when subjected to a stress σ . If σ approaches the value of G_c from zero, $\gamma_{\min}$ approaches infinity and hence G_c is a limiting value of σ . Before σ can reach this value, $\gamma_{\min}$ reaches the critical value γ_c given by :-

$$\gamma_c = \gamma_c / G_c \quad (43)$$

where γ_c is the shear strength of the composite. Therefore shear failure occurs when $\gamma_{\min}$ reaches the value γ_c and the composite strength is given by :-

$$\sigma_c = \frac{\gamma_c}{\gamma_c / G_c + \tan\theta} \quad (44)$$

Equation (44) reduces to the equation :-

$$\sigma_c = G_c \quad (45)$$

for the special case when θ equals zero (FOYE (1966)). A simple two-dimensional series model is used to determine G_c in terms of the constituent properties where the stress is assumed to be uniform throughout the composite. This analysis gives :-

$$G_c = \frac{G_m}{(1 - V_f) + V_f (G_m / G_f)} \quad (46)$$

which reduces to :-

$$G_c = \frac{G_m}{1 - v_f} \quad \text{when } G_f \gg G_m \quad (47)$$

Therefore equation (44) becomes :-

$$\sigma_c = \frac{\gamma_c}{(1 - v_f) \gamma_c / G_m + \tan \theta} \quad (48)$$

Equation (48) is plotted in Fig. 8.10 as a function of θ using the measured values in table 9.1 for the AE57 composite whose strength is also shown in Fig. 8.10.

TABLE 9.1

Properties of AE57 composites

$$\begin{aligned} \gamma_c &= 57 \text{ MN/m}^2 \\ G_m &= 0.60 \text{ GN/m}^2 \\ v_f &= 0.57 \end{aligned}$$

G_m was measured indirectly using the relationship :-

$$G_m = \frac{E_m}{2(1 + \nu)} \quad (49)$$

for isotropic materials. The value of Poisson's ratio, ν , was quoted as 0.38 by the resin manufacturer. Excellent agreement is obtained with the experimental results for values of θ greater than 4° but the experimental results fall far short of the theoretical prediction for values of θ less than 4° . It therefore appears that the theory represents an upper limit, and for small values of θ the strength is controlled by another mechanism. However, the mode of failure had the same basic appearance described in section 8.2.1., and shown in Fig. 8.2 independent of θ . Of particular interest is the critical dependence of theoretical strength on θ for small values of θ . Thus even if the theoretical curve did represent experimental results for all values of θ , the practical limitations of alignment alone would prevent the peak value from being reached.

The validity of equation (48) was also checked by testing various composites for one value of θ (7°). The results are shown in table 9.2 compared with the theoretical results.

TABLE 9.2

Strength of (7° off-axis) discs

<u>Composite</u>	<u>Strength (MN/m²)</u>	
	<u>Experimental</u> (mean of 3 tests)	<u>Theoretical</u>
AR59	382	388
BE56	327	359
AC _I 45	265	300
AC _{III} 75	400	383

It has been assumed that free lateral movements of the specimen ends can take place in the theoretical analysis; however, the frictional forces between the specimen ends and the steel discs were too large to allow this to happen and therefore the small lateral deflections required for shear failure must have taken place in the testing machine itself. If no lateral movement occurs, then shear forces at the specimen ends combine with the applied stress to give a resultant compressive stress in the direction of the fibres. Under this condition, the compressive strength would be proportional to $\cos^2\theta$, which was experimentally observed to be untrue. This restriction can be removed theoretically by letting the fibres deform into the sinusoidal shape given in Fig. 9.8a where :-

$$y = a \sin(n\pi x/L) \quad (50)$$

instead of letting the fibres remain rigid as in Fig. 9.5. The shear energy, W , is given by :-

$$W = \frac{1}{2}c G_c \int_0^L \gamma^2 dx \quad (51)$$

where the shear strain, γ , is equal to dy/dx . Equation (51)

reduces to :-

$$W = G_c \frac{\pi^2 a^2 n^2 c}{4L} \quad (52)$$

Substituting this value of W into equation (24) and continuing as before in the micro-analysis gives :-

$$Q_o = \pi^2 c (a/L^2) (G_c - \sigma) \quad (53)$$

$$\text{and therefore :-} \quad \sigma_c = G_c \quad (54)$$

Equation (54) is identical to equation (45) for the case when θ is equal to zero but equation (54) was derived assuming no lateral movement of the specimen ends. The same equation can be derived assuming a sinuoidal buckling pattern such as that shown in Fig. 9.8b.

9.4. COMPARISON OF MACRO- AND MICRO-ANALYSES

It can be seen when comparing the strength due to the macro-analysis (equations (45) and (54)) with the strength due to the micro-analysis (equation (33)) that they are identical provided that :-

$$G_c = \frac{G_m}{1 - V_f} \quad (55)$$

This relationship was derived previously (equation (47)) for the case of a simple two-dimensional model. The two analyses are in fact the same because the same fibre-matrix boundary conditions are used in both. The micro-analysis does have the disadvantage that it requires a theoretical model of a real composite; also it must involve deformation in only two dimensions. The macro-analysis still uses a two-dimensional failure mode but can use a real composite itself as the model and hence use experimental values of G_c obtained from real composites. The equality of the strengths given by the two analyses can therefore be verified by measuring the experimental validity of equation (55). The comparison of equation (55) with the experimental results of GOGGIN (1973) for unidirectional

carbon fibre epoxy resin composites shows that it is reasonably accurate, predicting values of G_c about 10 % less than those measured experimentally. Helical buckling is a possible three-dimensional failure pattern but the failure pattern established from the debris of broken specimens always appeared to be two-dimensional.

9.5. EFFECT OF COMPOSITE SHEAR STRENGTH

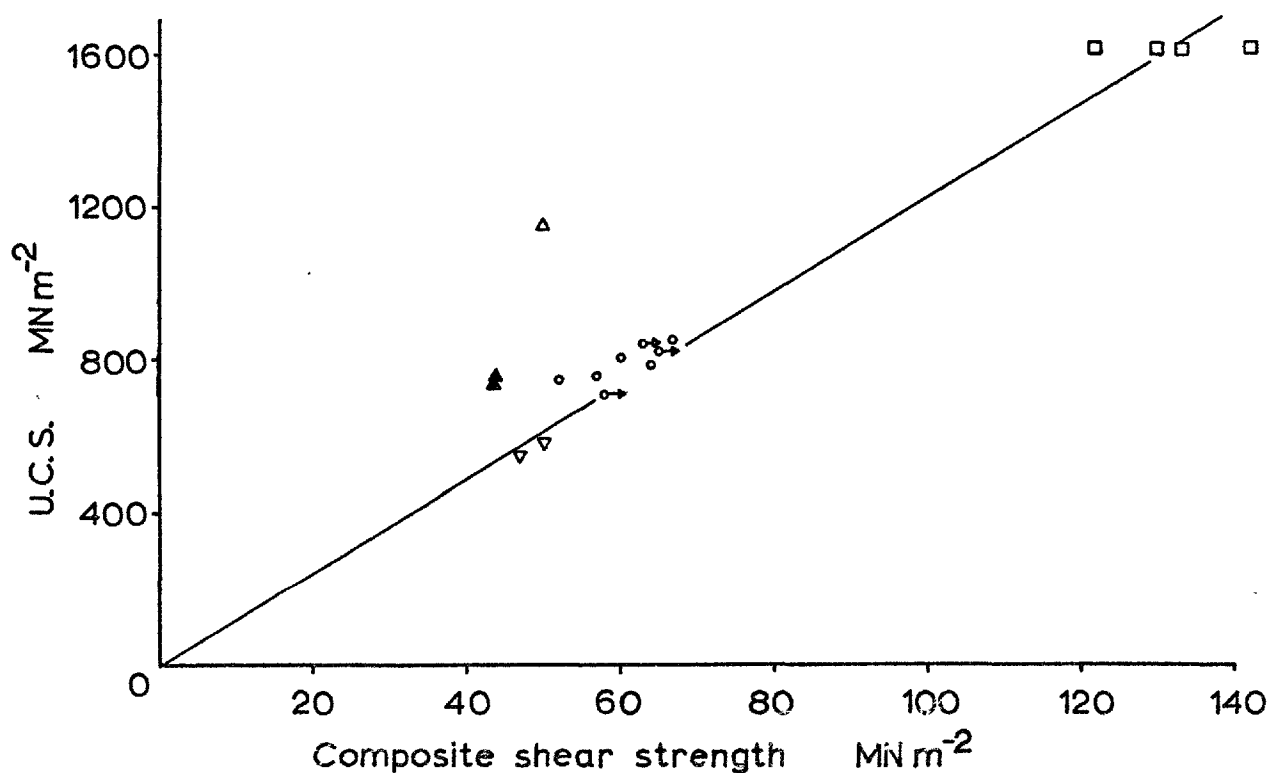
The theoretical analysis deduces that σ_c is proportional to G_m . However, resin B matrix had a 70 % increase in shear modulus over resin A, although identical strengths were recorded for all composites in the A and B ranges. These two resins had the advantage, from a comparative viewpoint, of giving similar composite shear strengths and hence the results were not affected by any possible strength dependence on shear strength. This lack of correlation with theory agrees with the conclusions of section 9.3. where, for small values of θ , the strength appears to be controlled by another failure mechanism.

The shape of equation (48) as plotted in Fig. 8.10 has a strong dependence on the composite shear strength. As γ_c approaches zero, σ_c , via equation (48), approaches a delta function with a zero value of σ_c for all values of θ except ($\theta = 0$). Therefore for any practical composite, the increasing sensitivity of compressive strength with alignment due to decreasing shear strength would indirectly cause a shear-compressive strength dependence. Experimental results are shown in Fig. 9.9. Each point represents a mean position for a particular composite except for the low-temperature results where the shear strength of each three-point-bend specimen is marked. At this temperature (-196°C), the matrix shear modulus of resin A increased by a factor of four over its room temperature value while the composite shear strength doubled.

Excluding the fibrecarbon results which are mentioned in section 9.7., an approximately linear compressive strength dependence on shear strength is apparent from Fig. 9.9. A similar dependence has been observed by FRIED (1965) for bidirectional fibreglass. Therefore, the composite must possess a minimum shear strength in order that the theoretical strength

Fig. 9.9

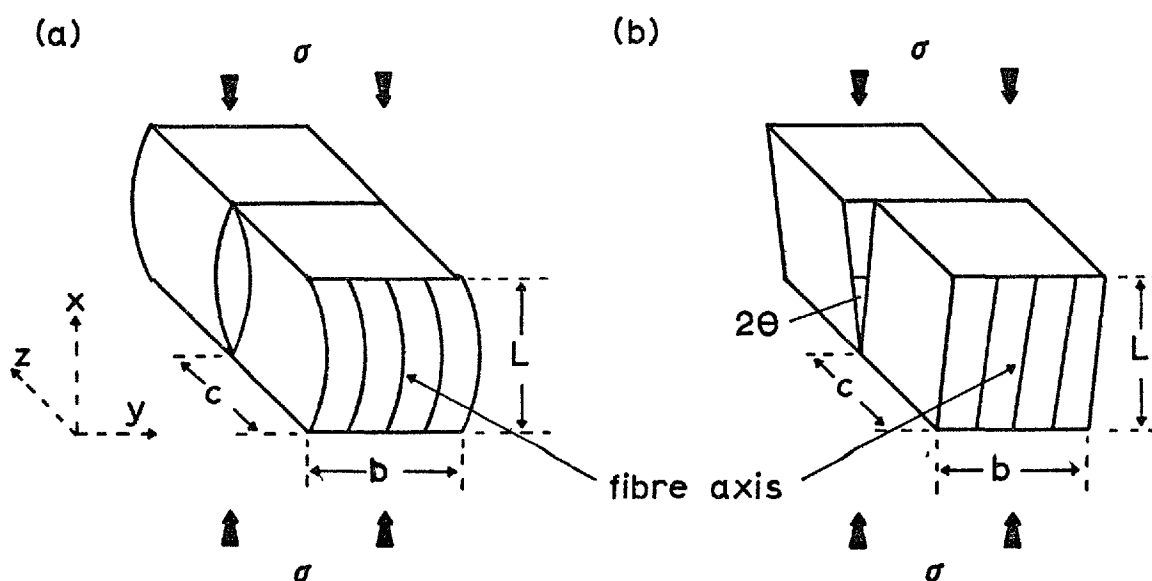
Mean (U.C.S.-shear strength) data of individual rods adjusted (where necessary) for a fibre volume fraction of 57 %



- ∇ C_I fibre carbon
- $\blacktriangle$ C_{II} fibre carbon
- $\triangle$ C_{III} fibre carbon
- $\circ$ fibreglass
- $\square$ AE fibreglass tested at -196°C and represent individual shear specimens

Fig. 9.10

Representation of misorientated rovings within the composite



may be reached.

A possible cause of failure is the progressive breaking up of a composite due to internal transverse stresses; such stresses are on a micro- and macro-scale. The micro-scale consists of the compressive radial stresses which bond a fibre to its surrounding matrix. The radial stress at the interface becomes tensile for a large enough longitudinal compressive strain because the Poisson's ratio of the matrix (0.38) exceeds that of glass fibre (0.22). This allows for the possibility of debonding between matrix and fibres and hence a degradation in strength as described by LANIR and FUNG (1972). However, the thermal and cure contraction strains of the resin matrix are large enough (see chapter 5) to prevent this from happening at the longitudinal compressive strains encountered at failure.

The composites with maximum fibre contents differ from the rest in that there are tensile radial stresses acting at the interface before any load is applied. However, these tensile stresses decrease as the compressive load is increased. The U.C.S. of these composites did not differ from extrapolated values determined from the strengths of other composites with lower fibre contents. In addition, microscopic examination showed that no debonding occurred in any of the examined disc specimens prior to failure.

The macro-scale consists of transverse stresses due to the misorientation between fibres or in particular between rovings or tows. The internal geometry of real composites is described in chapter 7. The misorientation causes transverse stresses to occur at the interfaces of these rovings or tows. Simple models are shown in Fig. 9.10 which provide relationships between the misorientation and the subsequent shear stresses acting between adjacent rovings due to an applied mean axial stress. In Fig. 9.10a, two composites are shown from a continuous series of composites in the z direction, each with a cross-sectional area (c b) normal to the applied stress, σ , and with a fibre misorientation given by the curve :-

$$y = \pm a_0 \sin(\pi x/L) \quad (56)$$

Adjacent composites in the series are misorientated in opposite directions. Assuming initially that restricted lateral displacements can take place between adjacent composites, the composite deforms into the shape given by :-

$$y = (a_0 + a) \sin(\pi x/L) \quad (57)$$

due to the applied stress σ . The composite shear strain, γ , is given by :-

$$\gamma = dy/dx - dy/dx_{(a=0)} = \frac{a\pi}{L} \cos(\pi x/L) \quad (58)$$

The increase in strain energy W for one composite is given by :-

$$W = \frac{1}{2} G_c \int_0^L \gamma^2 dx L b c = G_c L b c \frac{\pi^2}{4L} a^2 \quad (59)$$

The axial displacement (parallel to σ) is given by :-

$$\frac{1}{2} \int_0^L (dy/dx)^2 dx - \frac{1}{2} \int_0^L (dy/dx)_{(a=0)}^2 dx = \frac{\pi^2}{4L} (a^2 + 2a a_0)$$

Therefore the work done, T , by σ for one composite is given by :-

$$T = \frac{\pi^2}{4L} (a^2 + 2a a_0) \sigma b c \quad (60)$$

The energy function U is defined as :-

$$U = \frac{W - T}{L b c} \quad (61)$$

Substituting equations (59) and (60) into equation (61) gives :-

$$U = \frac{\pi^2}{4L} b c (G_c a^2 - (a^2 + 2a a_0) \sigma) \quad (62)$$

The shear force F acting at the interface to prevent lateral displacement is given by :-

$$F = dU/da_{(a=0)} = - \frac{\pi^2}{2L} b c \sigma a_0 \quad (63)$$

The shear stress, γ , acting at the interface takes the form :-

$$\gamma = \gamma_o \sin(\pi x/L) \quad (64)$$

Therefore the shear force is also given by :-

$$F = \int_0^L \gamma b dx = \frac{2}{\pi} \gamma_o b L \quad (65)$$

Substituting for F in equations (63) and (65) gives :-

$$\gamma_o = \pi^3 c (a_o/L^2) \sigma \quad (66)$$

The standard angular deviation, θ , is given by :-

$$\theta = \left(\frac{1}{L} \int_0^L (dy/dx)^2 dx \right)^{\frac{1}{2}} = \frac{\pi a_o}{L \sqrt{2}} \quad (67)$$

Therefore the compressive strength, σ_c , is given by :-

$$\sigma_c = \frac{1}{14} (L/c) (\gamma_c/\theta) \quad (68)$$

where γ_c is the composite shear strength and is hence the maximum allowable value for γ_o . A similar equation :-

$$\sigma_c = \frac{1}{14} (L/b) (\gamma_c/\theta) \quad (69)$$

can be derived where the composites of Fig. 9.10a are arranged, without rotation, in a continuous series in the y direction. In this case γ_c represents the composite transverse tensile strength. A value of c or b is approximated to the square root of the cross-sectional area of the roving or tow. Using typical values for an AE50 composite :-

$$\gamma_c = 60 \text{ MN/m}^2, \quad \theta = 0.05 \text{ radians}, \quad c = 0.5 \text{ mm}$$

$$\text{gives :-} \quad \gamma_c \approx (100 L \text{ mm}) \text{ MN/m}^2 \quad (70)$$

L represents the characteristic half wavelength of the misaligned roving. From the appearance of rovings lying in the open mould,

a rough estimate of the minimum value of L is 20 mm, which corresponds to a minimum value of σ_c equal to 2,000 MN/m².

If the length of the specimen is less than 20 mm as in the case of all disc specimens, a more representative model is shown in Fig. 9.10b where L is the length of the specimen and each composite consists of straight fibres orientated at an angle of $\pm \theta$ to the applied stress. A similar analysis gives the equation :-

$$\sigma_c = \frac{1}{6} (L/c) (\gamma_c/\theta) \quad (71)$$

which is in the same form as equations (68) and (69). In this model the maximum shear stresses occur at the specimen ends where the frictional forces between the composite and the metal discs prevent such stresses from occurring. This effect is more dominant for shorter specimens and hence nullifies the strength dependence on length.

This mode of failure is not expected to be dominant because of the large estimated values of σ_c using equation (70). If it is, then cracking in the specimen is expected to occur at stresses significantly less than the U.C.S. From the results of section 8.2.1., no damage occurred to disc specimens before the U.C.S. was reached. BROUTMAN (1964) used specimens identical to type (c) of Fig. 8.16 to test unidirectional and bidirectional fibreglass epoxy resin composites. Portions were cut from these specimens, which had previously been loaded up to various fractions of their U.C.S. and were examined using light and scanning electron microscopes. BROUTMAN (1964) stated that "crack initiation was not observed in any of the materials tested below 80 % of the compressive strength." He concluded that "crack initiation at the resin-glass interface occurs in a simple compression test at stresses representing less than 80 % of the ultimate stress." In comparison, shear failure at the ends of the tapered specimens of Fig. 8.14 started at a stress of about 20 % of the U.C.S. and was dependent on the specimen geometry rather than the type of composite.

A marked dependence of strength on fibre misorientation should be apparent if the proposed failure mode is dominant. Therefore a large scatter in strength values, particularly for

short specimens, should occur but which in practice did not occur. The two different models of Fig. 9.10 give similar equations (68) and (71). It is reasonable to believe therefore that an equation of the form :-

$$\sigma_c = K \frac{1}{c} (\gamma_c / \theta) \quad (72)$$

where K is a constant, can be used to approximate the relationship between the relevant parameters in a real composite in which rovings are orientated randomly to each other in three dimensions. From equation (72) it can be deduced that this failure mode does not occur provided a critical value of (γ_c / θ) is exceeded. In addition if this failure mode does apply then the strength can be increased by using a roving consisting of fewer fibres, by improving the alignment and by increasing the composite shear strength.

9.6. EFFECT OF SPECIMEN GEOMETRY

Of particular interest is the strength difference between disc and tapered specimens shown in Fig. 8.18. This difference at first sight might be attributed to the "poor" transfer of load between the disc specimen and the metal discs. However the results of section 8.3. demonstrate that no matter how "perfect" this transfer of load is, the strength of a disc specimen can never reach the strength of a tapered specimen. Any viable strength theory must therefore be able to predict this effect of a free interface. Moreover, the theory must be able to predict the distinct length-strength dependence shown in Fig. 8.9. The micro-analysis predicts strength increases for short specimens, (see equation (32)) but the dependence is small, being only 0.5 % and 0.1 % for lengths of 1 mm and 2 mm respectively.

A model is presented which gives the required length-strength dependence. The theory is by no means rigorous and relies on unfounded assumptions but does provide an insight into the failure mechanism. This two-dimensional analysis refers to the buckling of a section of the composite with thickness t , shown in Fig. 9.11. This column is loaded to values higher than its free buckling load but is prevented from buckling by the reaction

Fig. 9.11 Model of proposed failure mode for short disc specimens

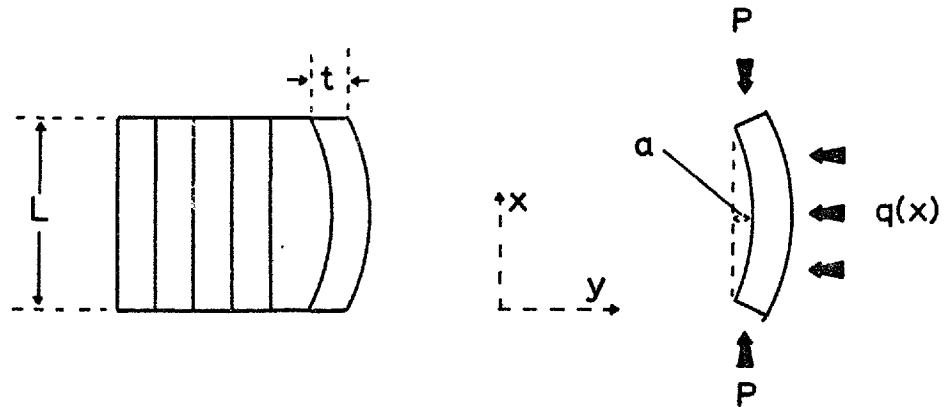


Fig. 9.12 Envisaged buckling mode of a disc specimen

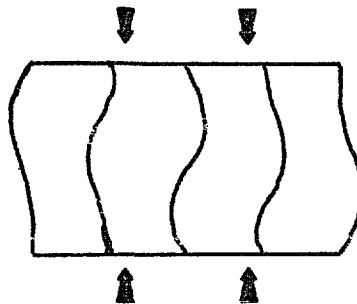
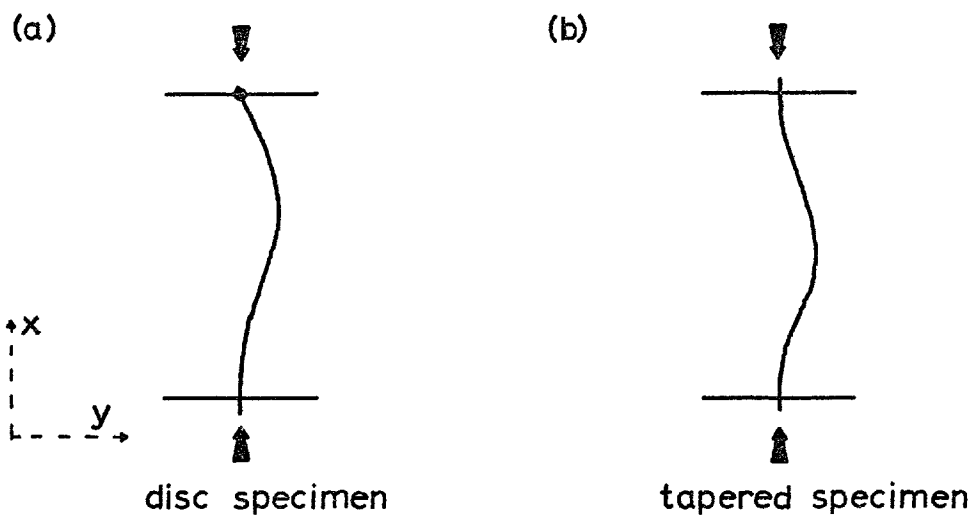


Fig. 9.13 (Euler column)-specimen buckling analogy



on the column due to the rest of the composite. This reaction remains positive so long as the applied stress is less than G_c so that the analysis applies to the failure mode only when the predicted strength is less than G_c . The distinction between the column and the remaining composite is quite arbitrary but it must be small enough to allow this external layer to deform by simple bending and not by shear. Using the analysis of TIMOSHENKO (1961), the equation of static equilibrium for this column is given by :-

$$E_c I \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} + q(x) = 0 \quad (73)$$

P is the axial load, $E_c I$ is the bending moment and q is the lateral reaction acting on the column. Because the foundation is assumed to be elastic (i.e. $q(x)$ is assumed to be proportional to the displacement y), the exact solution is given by :-

$$y = a \sin(\pi x/L) \quad (74)$$

$$\text{and } q(x) = q_0 \sin(\pi x/L) \quad (75)$$

Substituting equations (74) and (75) into equation (73) gives the solution :-

$$P = E_c I \frac{\pi^2}{L^2} + \frac{L^2}{\pi^2} \frac{q_0}{a} \quad (76)$$

for finite values of y . Reference is made to equation (53) where Q_0 (which is equivalent to the reaction in this case) is calculated for a composite deforming in shear when the deformation is given by equation (74) and the reaction is given by equation (75).

$$Q_0 = \pi^2 c (a/L^2) (G_c - \sigma) \quad (53)$$

It is therefore assumed that q_0 in this present case can be represented by the equation :-

$$q_0 = K (a/L^2) \quad (77)$$

where K is a constant over a small range of σ . Equation (76) therefore reduces to :-

$$P = E_c I \frac{\pi^2}{L^2} + \frac{K}{\pi^2} \quad (78)$$

P is independent of y and is hence the buckling load. The critical buckling stress σ_c is given by :-

$$\sigma_c = P/t \quad (79)$$

and I is given by :-

$$I = t^3/12 \quad (80)$$

Therefore equation (78) reduces to :-

$$\sigma_c = \frac{\pi^2}{12} E_c \frac{t^2}{L^2} + \frac{K}{\pi^2} \frac{1}{t} \quad (81)$$

The strength depends upon the arbitrary thickness of the column and hence the minimum strength, which is dominant, is given by the condition :-

$$d\sigma_c/dt = 0 \quad (82)$$

$$\text{or} \quad t^3 = \frac{6}{\pi^4} \frac{K}{E_c} L^2 \quad (83)$$

Substituting equation (83) into equation (81) gives :-

$$\sigma_c = (0.385 K^{\frac{2}{3}} E_c^{\frac{1}{3}}) L^{-\frac{2}{3}} \quad (84)$$

The best fit of equation (84) to the data in Fig. 8.9 is obtained using a value ($K = 570 \text{ (MN/m}^2\text{) mm}$). A typical value of t is 0.1 mm which agrees with the initial condition that t/L must be small. A more elaborate analysis would show this model to be inadequate. Particularly the use of equation (77) to approximate the value of the reaction. A more accurate value could be obtained by letting q_0 equal Q_0 in equation (53) where ($\sigma = \sigma_c$). In this case, c represents the variable thickness of a transition

region between the buckling and rigid composite sections of Fig. 9.11. However, by carrying out these corrections, the original simplicity of the mathematical treatment is lost.

This model fails to predict the critical length 1.5 mm above which the strength is independent of length. It is suggested therefore that a disc specimen buckles with a characteristic half buckling wavelength of 1.5 mm dependent on the composite properties. If the length of the specimen is reduced to less than 1.5 mm, the composite can only buckle with a maximum half wavelength equal to the length of the specimen. This suggestion agrees well with the visual observations of section 8.2.1. concerning the thickness of the failed region. The envisaged failure mode of a short disc specimen whose length exceeds 1.5 mm is shown in Fig. 9.12 which can be superimposed onto Figs. 8.4a and 8.5. The damaged regions of Fig. 8.5 correspond to buckled columns separated from the remaining composite. In this proposed failure mode, no strength dependence on shear strength has been mentioned. However, it is suggested that a more elaborate analysis can give values of the reaction which in turn can give a stable form of buckling where stable displacements occur before catastrophic failure. (This is true for the case of the buckling of a bent column). This mode allows the possibility of the critical shear strain being reached before a stability failure is reached similar to the model of Fig. 9.5. Under these conditions, the U.C.S. is controlled by the composite shear strength. The initial damage prior to failure, where observed, (see Fig. 8.5), always occurred near opposite corners of the specimen. This suggests that a buckling pattern existed in the specimen before failure otherwise there was no means by which the relative positions of the damaged regions could be interrelated. ORRINGER (1971) was able to detect such a pattern occurring prior to failure in a model composite consisting of parallel piano wires embedded in a plastic matrix (stycast).

Using this model of a buckling column within the composite, the possibility of a strength difference between disc and tapered specimens immediately becomes apparent. The Euler buckling load of the column in Fig. 9.13b, which represents a tapered specimen, is twice that of the identical column in Fig. 9.13a which represents a disc specimen. The only difference

between the two configurations is that one of the fixed ends in Fig. 9.13b has been pivoted to obtain the configuration in Fig. 9.13a and hence represents a free interface. The effect of a free interface can then be stated to be the allowing of a finite value of dy/dx at this interface while in a tapered specimen dy/dx must always equal zero at its ends.

9.7. FIBRECARBON

The strength data obtained from fibre carbon cannot be correlated with the strength of fibreglass. Although type I and type II carbon fibres gave composites a shear-compression strength correlation similar to fibreglass (see Fig. 9.9), type III carbon fibre composites are twice as strong as this correlation predicts.

EWINS and HAM (1973) measured the axial compressive strength of unidirectional fibre carbon manufactured by the hand-lay-up technique using an epoxy resin matrix. Their composites had various fibre contents consisting of either high modulus (HM) or high strength (HS) fibres with Young's moduli of 350 GN/m^2 and 280 GN/m^2 respectively and they used tapered specimens similar in shape to Fig. 8.14b. The measured composite shear strengths of the HM and HS composites were 72 MN/m^2 and 86 MN/m^2 respectively. Their results showed that the strengths of HM and HS composites were similar to the strengths of the type I and type III carbon fibre composites respectively, measured with disc specimens and shown in Fig. 8.13. No improvement in strength over these disc specimen strengths occurred despite their use of tapered specimens with much higher shear strengths.

The only tapered specimen, (AC_I45), which was tested in this study had a strength slightly less than the AC_I45 disc specimens. It is therefore concluded that there is no strength dependence on shear strength and there is no difference between the strengths of disc and tapered specimens.

EWINS and HAM (1973) produced convincing evidence to suggest that the compressive failure mode of fibre carbon is controlled by a shear failure in the fibre itself. If this is true, then it explains the observed results, because the composite will fail at a particular strain for a particular type of fibre,

independently of fibre content, shear strength or specimen shape. The results obtained from fibrecarbon cannot therefore be correlated with the buckling modes envisaged for fibreglass. EWINS and HAM (1973) did obtain buckling failures in fibrecarbon by increasing the temperature to above 100°C and hence reducing the shear strength considerably. Because of the low shear strength, the buckling strength was less than the fibre strength and hence was dominant. The change in failure mode was marked by the discontinuity in the slope of the strength-temperature correlation. The measured strength at room temperature therefore represented the minimum buckling strength; that is, if the fibres did not fail first, the composite would fail by buckling at a stress in excess of the measured U.C.S. Using this reasoning and a comparison of the relative strengths of fibreglass disc and tapered specimens, the buckling strength of a tapered fibrecarbon specimen can be estimated to have a minimum value approaching twice the measured disc U.C.S. Comparison of the buckling strengths of AC_{III} fibrecarbon and fibreglass shows that the former is at least twice as strong despite its lower shear strength. The strength of the AE and AR tapered specimens is given approximately by :-

$$\sigma_c = 0.9 \frac{G_m}{1 - V_f} \quad (85)$$

(see Fig. 8.18). Hence the buckling strength of AC_{III} tapered specimens is given by :-

$$\sigma_c = 1.8 \frac{G_m}{1 - V_f} \quad (86)$$

which exceeds the theoretical strength. Therefore the buckling strength is controlled by the matrix shear modulus via equation (33).

9.8. FIBRE MODULUS

The Young's modulus of the fibres is the only mechanical parameter which is significantly different between fibreglass and fibrecarbon. Therefore it is suggested that this parameter caused the buckling strength difference between the two composites. A three-fold increase in E_f causes at least a two-fold increase in buckling strength provided the theoretical strength is not exceeded and other parameters such as alignment and shear strength exceed certain minimum values. Superimposing the expected buckling strength data of fibrecarbon onto Fig. 8.9 suggests that either the value of K in equation (84) must increase or the critical length must decrease or that both must occur. This discussion predicts that the minimum shear strength required to allow the compressive strength to reach the theoretical limit decreases as the fibre modulus increases.

LAGER and JUNE (1969) tested unidirectional high modulus boron fibre ($E_f = 450 \text{ GN/m}^2$) composites with an epoxy resin matrix in axial compression. The fibres within their composites appeared to have diameters of 50 microns, to have a high degree of alignment, and to be uniformly spaced. Composites with fibre contents in the range 15 % to 45 % were tested using a special test configuration in which a thin rectangular block shaped specimen, with the fibres lying in the lengthwise direction, was loaded at opposite ends and supported laterally over all of the specimen except for a short central region where failure occurred. This region was not necked down. They used two epoxy resins with shear moduli (G_m) of 1.32 GN/m^2 and 0.66 GN/m^2 respectively. The measured strengths of all composites made with either of these two resins were found to agree well with the formula :-

$$\sigma_c = 0.63 \frac{G_m}{1 - V_f} \quad (87)$$

Unfortunately, no mention of composite shear strength was made, but it appeared to be strong enough to allow the compressive strength to be controlled by the matrix shear modulus. This occurred despite the high value of the shear modulus for one of the resins (30 % higher than resin B).

The correlation factor, 0.63, was low, particularly when

compared with the value 0.9 obtained for AE and AR composites, (see equation (85)). This value was in itself a minimum value due to a different failure mechanism controlled by the shear strength. LAGER and JUNE (1969) attributed this low value to anisotropic properties of the matrix while it also could be attributed either to off-axis loading (which is known to be critical for shear modulus controlled U.C.S.) or to the characteristics of their specimen shape and testing conditions.

LENOE et al. (1969) tested identical composites with various geometries and loading configurations in axial compression. They concluded that specimens with rectangular cross-sections gave considerably lower strength values with more scatter than specimens with circular cross-sections.

9.9. FIBRE DIAMETER

LEVENETZ (1964, 1965) tested tension-wound fibreglass NOL rings with an epoxy resin matrix in compression. He fabricated composites consisting of fibres with various diameters in the range 10 to 250 microns. The fibres within each composite had the same diameter. He used the split-disc-ring and the NOL fixture mentioned in section 8.7. to determine compressive strengths. No variation in strength was noted in the split-disc-ring test results for composites with different fibre diameters. Using the NOL fixture, a maximum strength of 2000 MN/m^2 was recorded for composites with a fibre diameter of 120 microns. The recorded strength of standard composites with a 10 micron fibre diameter using the NOL fixture was 1040 MN/m^2 . Although his results indicated a strength dependence on fibre diameter, any conclusions were clouded by the various other parameters, known and unknown, which were altered during the testing programme. The fibre contents varied between 65 % for 10 micron fibre composites and 77 % for 120 micron fibre composites while no information was given concerning the matrix moduli and composite shear strengths. In addition the Young's modulus of the 120 micron fibres exceeded that of the standard 10 micron fibres by 40 %.

The loading geometry of the NOL fixture must induce transverse compressive stresses in the composite. This was

confirmed by the experimentally-measured longitudinal Young's moduli of the composites being always greater than the predicted values using the mixture rule. This increase is expected if the composite is inhibited from free lateral expansion. The NOL fixture measures the longitudinal compressive strength under the influence of a biaxial stress field and is therefore expected to give higher strength values over composites tested in uniaxial compression. The indicated trend of increasing strength with increasing fibre diameter gives another reason for the high strength of large diameter boron fibre composites.

9.10. VOID CONTENT

The exact failure mechanism of void-free composites must be known before any theoretical approach to the strength degradation due to voids can be attempted. Also, different types of voids must control the strength degradation in different ways. For instance, the planar voids mentioned in section 4.3. must strongly enhance the possibility of the type of failure envisaged in section 9.5. occurring. It is therefore unlikely that the loss of strength can be specified simply as a function of the total void content.

FOYE (1966) calculated the loss in shear modulus for unidirectional composites due to general voids. He assumed that :-

$$\sigma_c = G_c(\text{void content}) \quad (88)$$

This formula can only apply to those composites whose strength is controlled by the composite shear modulus; such composites are the exception rather than the rule. FOYE (1966) also assumed that the fibre density within the voids was the same as outside them and that the fibres in these voids did not strengthen the composite. This did not apply to the voided AE composites where the voids tended to push the fibres aside, except those with the maximum fibre content, (see section 4.3.). With these above-mentioned assumptions, σ_c , from equation (88) must pass through a peak value when plotted as a function of the fibre content for a finite and constant void content.

FOYE (1966) suggested that this was the reason for the peak value obtained experimentally by FRIED and KAMINETSKY (1964) for the strengths of unidirectional fibreglass with various fibre contents and unspecified void contents. This is unlikely because the void content is expected to decrease as the fibre content increases, as mentioned in section 4.3.

GRESZCZUK (1967) obtained a good correlation for bidirectional composites. He calculated the loss in composite shear strength due to the reduction in matrix area of an average shear plane due to voids. He assumed that :-

$$\sigma_c = K \gamma_c(\text{void content}) \quad (89)$$

where K is a constant. An equation similar to (89) might well apply for unidirectional composites because a correlation between σ_c and γ_c is known to exist. However, equation (89) could not explain the observed different sensitivities of strength degradation due to voids for disc and tapered specimens, (see section 8.8.).

PART TWO

DYNAMIC STRENGTH

CHAPTER TEN

HOPKINSON BAR - DEVELOPMENT AND ANALYSIS

10.1. ORIGINAL APPARATUS

The Hopkinson bar, as the name implies, was first used by HOPKINSON (1914) to measure stress pulses produced by impact loading. The impact load to be measured is imparted normally to one end of a long cylindrical steel bar. At the other end, and in line, is placed a shorter cylindrical steel bar of equal diameter, which is known as the time piece. A good contact is maintained between the two bars, the ends of which are ground flat and parallel and smeared with grease. This enables a compression pulse, once initiated at the impact end, to traverse into the time piece without reflection or distortion. The two bars remain in contact as long as the join remains in compression. The compressive stress at the join falls to zero due to the addition of the reflected pulse, now tensile, returning from the free end of the time piece. The time piece then flies off, because the join cannot sustain a tensile force, and is caught in a ballistic pendulum which measures the trapped momentum left in it. Deductions concerning the maximum stress and the duration of the pulse can be made by varying the length of the time piece using similar impact loads. The exact stress-time relationship of the pulse cannot be measured and the stress amplitude must remain within the elastic range of the bars.

10.2. IMPROVEMENTS DUE TO DAVIES

With the advent of the electronic age, DAVIES (1947) was able to replace the time piece with a cathode ray oscilloscope (C.R.O.) and a condenser gauge mounted at the free end of the bar. Schematic diagrams of two types of gauge used are shown in Fig. 10.1; both of them are of loose fit and slide onto the end of the bar. An electric potential is maintained between the brass gauge and the earthed bar. The end of the bar moves a short distance to the right as the compression pulse is reflected. The rate at which this displacement takes place is proportional

Fig. 10.1a End ring condenser gauge

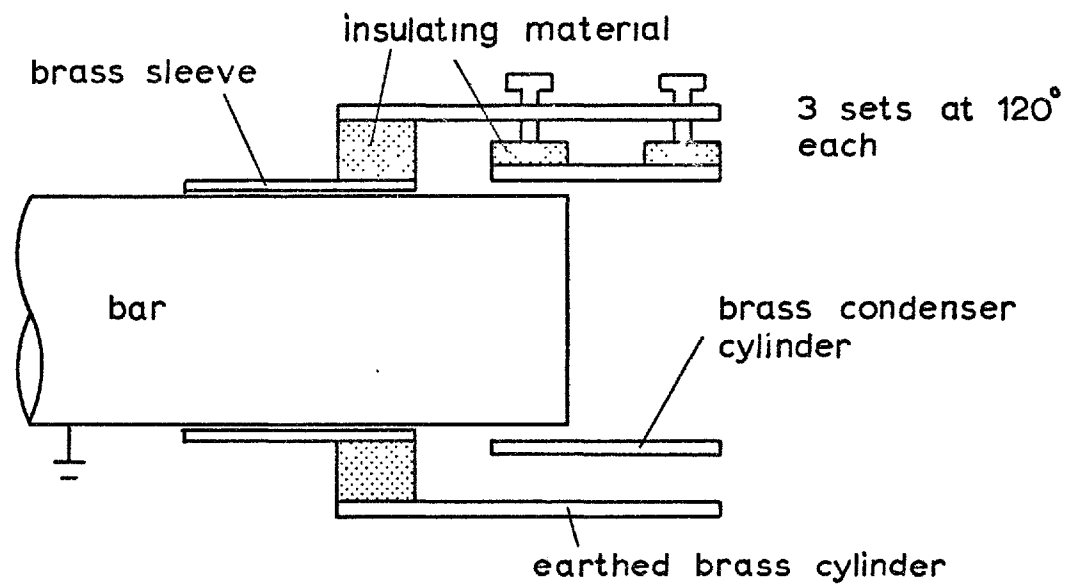
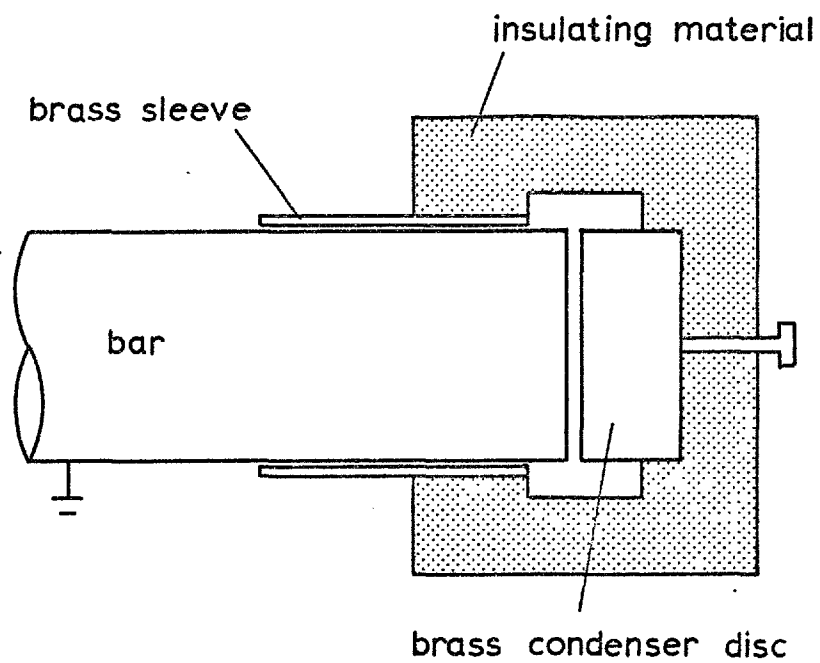


Fig. 10.1b Parallel-plate condenser gauge



to the magnitude of the stress pulse reflecting at the free end of the bar at that instant in time.

This displacement also causes a change in capacitance between the brass gauge and the bar which can be measured using a C.R.O. The single sweep can be photographed and from it the stress-time profile of the pulse can be measured. A condenser ring gauge consisting of a concentric ring placed around the central region of the bar was also described by DAVIES (1947). This type measures the radial expansion of the bar and is shown schematically in Fig. 11.1.

10.3. LIMITATIONS OF ONE-DIMENSIONAL WAVE THEORY

DAVIES (1947) made a critical survey on the limitations of simple stress wave theory as applied to cylindrical bars. Use was made of the theories developed by POCHHAMMER (1876) and CHREE (1889) on the dispersive nature of such waves. A more general account can be found in KOLSKY (1963).

Four types of wave, longitudinal, flexural, torsional and Rayleigh surface waves can propagate in elastic bars. The present discussion is confined to longitudinal waves. The wave equation of motion :-

$$\frac{d^2 u}{dt^2} = c_o^2 \frac{\partial^2 u}{\partial z^2} = \frac{E}{\rho} \frac{\partial^2 u}{\partial z^2} \quad (90)$$

can be derived assuming :-

- a) Plane cross-sections remain plane.
- or a) The stress and strain fields are uniform over the cross-section.
- b) The stress and strain fields are one-dimensional.
- c) No damping exists.

u is the particle displacement in the axial (z) direction. The wave velocity c_o is independent of the wavelength and hence no dispersion occurs. Condition c) can usually be met to within reasonable tolerances. Condition b) can only be met if the Poisson's ratio, ν , of the bar is zero, a state which exists in no known material. Conditions a) and b) are found to be interrelated.

If the lateral strains are taken into account, the wave equation becomes :-

$$\frac{d^2 u}{dt^2} - \nu^2 K^2 \frac{\partial^4 u}{\partial z^2 \partial t^2} = c_o^2 \frac{\partial^2 u}{\partial z^2} \quad (91)$$

where K is the radius of gyration of the cross-section of the bar about its axis and the second term on the left represents the effects due to lateral motion. From this expression the approximate formula :-

$$c_p/c_o = 1 - \nu^2 \pi^2 (a/\lambda)^2 \quad (92)$$

where a is the bar's radius, can be derived. It can be seen that the phase velocity, c_p , is dependent on the wavelength, λ , giving rise to dispersion. The phase velocity lies very close to c_o if the wavelength is large compared with the radius of the bar and no significant dispersion occurs.

DAVIES (1947) discussed the effect wavelength has on the variation of the stress field over the cross-section of the bar. The variation increases as the wavelength decreases. For values of $(a/\lambda < 0.1)$, this variation is less than five percent.

10.4. KOLSKY'S SPLIT HOPKINSON BAR

The split Hopkinson bar was first used by KOLSKY (1949) to measure the dynamic stress-strain behaviour of various materials. The apparatus consisted of two well-aligned cylindrical steel bars with a specimen of the material to be tested sandwiched between them. The ends of the two bars, together with the two opposite faces of the disc shaped specimen, were ground flat and parallel. This enabled a good contact to be maintained between the specimen and the bars. The two faces of the specimen were smeared with grease in order to minimize friction effects due to the different lateral expansions of the bars and specimen. KOLSKY (1949) obtained the required impact load by exploding a detonator at the free end of the first bar. Other workers imparted high velocity projectiles such as bullets onto the free end in order to obtain the required impact load.

A ring condenser gauge was placed on the first bar sufficiently distant from the specimen to enable the whole incident pulse to be recorded before the pulse reflected from the specimen could interfere. A parallel plate condenser gauge, which was placed at the free end of the second bar, recorded the stress pulse transmitted through the specimen. The second bar had to be sufficiently long for this gauge to record the whole pulse before secondary reflections could occur. The stress-strain curve of the specimen could be calculated once the incident, reflected and transmitted pulses were measured. The radius of the specimen was usually a little less than the identical radii of the two bars to allow for the lateral expansion of the specimen when it was deforming plastically.

10.5. FURTHER DEVELOPMENTS

The basic arrangement of Kolsky's split Hopkinson bar has been used up to the present for high strain rate testing, (i.e. for strain rates in the range 10^3 sec^{-1} to 10^4 sec^{-1}) with various modifications. Strain measurements on both bars can be made with condenser gauges, electrical resistance gauges or diffraction grating gauges, all of which are described in chapter 11.

Higher strain rates have been achieved by using the first bar itself as a projectile, impacting it onto the specimen. In this case direct displacement measurements of the impact face of the first bar must be made. This was carried out by SAMANTA (1971) who used a photomultiplier to monitor a diminishing beam of light which was being cut off by the advancing end of the projectile. WULF (1974) used a stationary end ring condenser gauge through which the projectile passed while SIERAKOWSKI (1971) monitored the impact end using high speed photographic techniques. A photoelectric technique similar to that employed by SAMANTA (1971) was used by GRIFFITHS and MARTIN (1974) who monitored both end faces of the specimen in a stationary split Hopkinson bar apparatus.

The split Hopkinson bar has been adapted by various workers to test specimens in tension, shear, and torsion as has been reviewed by CAMPBELL (1973). A general account of the split Hopkinson bar can be found in LINDHOLM (1971).

10.6. STRESS-STRAIN ANALYSIS OF SPECIMEN

With reference to Fig. 10.2, consider a compressive strain pulse (negative strain) propagating along the first bar, through the specimen with time dependent dimensions, length h , radius a , and into the second bar. ϵ_I , ϵ_R , ϵ_T denote the incident, reflected and transmitted pulses respectively, measured at their respective specimen faces. Subscripts 1, 2, denote the incident and transmitted faces of the specimen respectively while a cylindrical polar frame of reference (r, z, θ) is used with its origin at the centre of face 2. The velocities of faces 1 and 2 are $-(U + V)$ and $-V$ respectively while V_r and V_z are the radial and longitudinal velocities within the specimen relative to the origin. It is assumed that there is uniform deformation over the whole specimen. Therefore :-

$$V_z = -U \frac{z}{h} \quad (93) \quad \text{and} \quad V_r = \nu U \frac{r}{h} \quad (94)$$

where ν is the effective Poisson's ratio of the specimen. Differentiating with respect to time (t) gives :-

$$\frac{dV_z}{dt} = -\frac{z}{h} \quad (95)$$

$$\text{and} \quad \frac{dV_r}{dt} = \frac{\nu^2 r}{h^2} U^2 + \frac{\nu r}{h} \frac{dU}{dt} + \frac{\nu r}{h^2} U^2 \quad (96)$$

$$\text{Also} \quad U = -h \dot{\epsilon} \quad (97) \quad \text{and} \quad \frac{dU}{dt} = -h \ddot{\epsilon} - h \dot{\epsilon}^2 \quad (98)$$

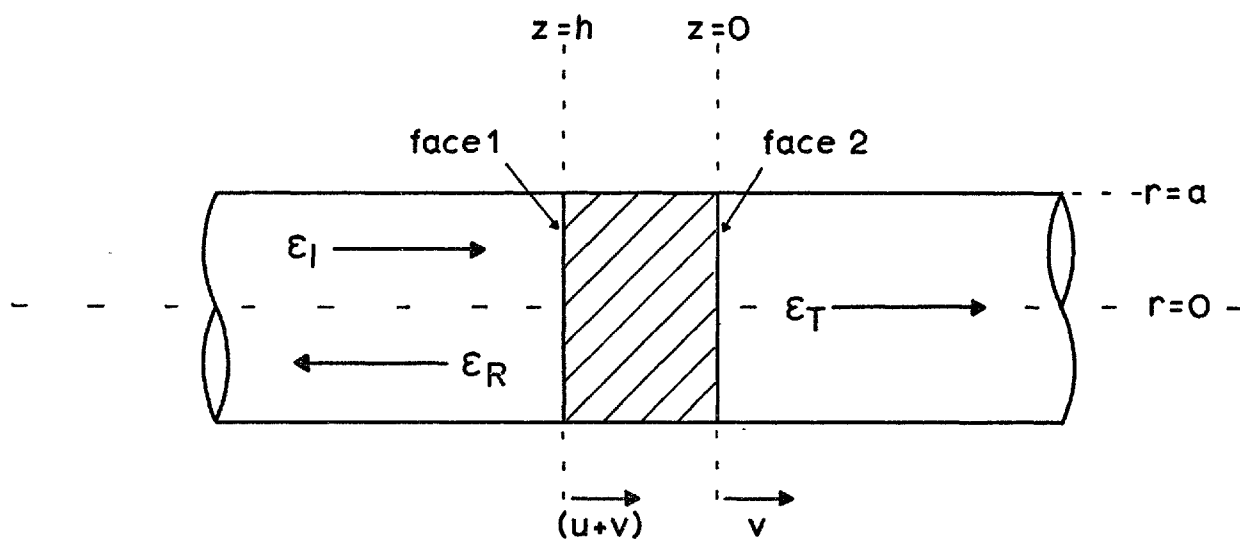
where ϵ is the uniform longitudinal strain in the specimen. The total kinetic energy, K , of the specimen with volume Ω is given by :-

$$K = \frac{1}{2} \int_{\Omega} (V_r^2 + (V_z - V)^2) \rho_s d\Omega \quad (99)$$

where ρ_s is the specimen density.

Fig. 10.2

Strain wave propagation through specimen



Therefore :-

$$\begin{aligned}\dot{K} &= \frac{dK}{dt} = \frac{1}{2} \int_{\Omega} \frac{d}{dt} (v_r^2 + v_z^2 + v^2 - 2v v_z) \rho_s d\Omega \\ &= \int_{\Omega} \left(\frac{v^2 r^2}{h^2} U \frac{dU}{dt} + (v^2 + v^3) \frac{r^2}{h^3} U^3 + \frac{z^2}{h^2} U \frac{dU}{dt} \right. \\ &\quad \left. + v \frac{dV}{dt} + \frac{z}{h} U \frac{dV}{dt} + \frac{z}{h} v \frac{dU}{dt} \right) \rho_s d\Omega\end{aligned}\quad (100)$$

Carrying out the integration :-

$$\int_{\Omega} d\Omega = \int_0^a r dr \int_0^h dz \int_0^{2\pi} d\theta \quad (101)$$

$$\begin{aligned}\text{gives :- } \dot{K} &= \pi a^2 \rho_s U \left(\frac{v^2 a^2}{2h} \frac{dU}{dt} + \frac{1}{2} (v^2 + v^3) \frac{a^2}{h^2} U^2 \right. \\ &\quad \left. + h \frac{V}{U} \frac{dV}{dt} + \frac{1}{2} h \frac{dV}{dt} + \frac{1}{2} h \frac{V}{U} \frac{dU}{dt} \right)\end{aligned}\quad (102)$$

The rate of work done $\dot{T}$ on the specimen by the external forces acting at the two ends is given by :-

$$\dot{T} = \pi a^2 ((U + V) P_1 - V P_2) \quad (103)$$

where P is the pressure. The axial equation of motion gives :-

$$\frac{\partial \sigma_z}{\partial z} = \rho_s \frac{d}{dt} (v_z - v) = -\rho_s \left(\frac{z}{h} \frac{dU}{dt} + \frac{dV}{dt} \right) \quad (104)$$

Integrating with respect to z over the whole specimen gives :-

$$P_1 - P_2 = \rho_s h \left(\frac{1}{2} \frac{dU}{dt} + \frac{dV}{dt} \right) \quad (105)$$

Substituting for P_1 in equation (103) gives :-

$$\begin{aligned}\dot{T} &= \pi a^2 U \left(P_2 + \frac{1}{2} \rho_s h \frac{dU}{dt} + \frac{1}{2} \rho_s h \frac{V}{U} \frac{dU}{dt} \right. \\ &\quad \left. + \rho_s h \frac{dV}{dt} + \rho_s h \frac{V}{U} \frac{dV}{dt} \right)\end{aligned}\quad (106)$$

The rate of energy dissipation, $\dot{W}$, or elastic energy storage in the specimen is given by :-

$$\dot{W} = -\pi a^2 U \sigma \quad (107)$$

where σ is the spatial mean value of stress due to material properties alone. Conservation of energy requires that :-

$$\dot{K} + \dot{W} = \dot{T} \quad (108)$$

Substituting equations (102), (106) and (107) into equation (108) finally gives :-

$$\begin{aligned} \sigma = & -P_2 + \rho_s \left(\frac{1}{2} \nu^2 \frac{a^2}{h} - \frac{h}{6} \right) \frac{dU}{dt} + \frac{1}{2} \rho_s (\nu^2 + \nu^3) \frac{a^2}{h^2} U^2 \\ & - \frac{1}{2} \rho_s h \frac{dV}{dt} \end{aligned} \quad (109)$$

Substituting for U and dU/dt from equations (97) and (98) gives :-

$$\begin{aligned} \sigma = & -P_2 - \frac{1}{2} \rho_s a^2 \left(\nu^2 - \frac{1}{3} \frac{h^2}{a^2} \right) \ddot{\epsilon} + \frac{1}{2} \rho_s a^2 \left(\nu^3 + \frac{1}{3} \frac{h^2}{a^2} \right) \dot{\epsilon}^2 \\ & - \frac{1}{2} \rho_s h \frac{dV}{dt} \end{aligned} \quad (110)$$

This can be simplified by noting that :-

$$\begin{aligned} P_2 + \frac{1}{2} \rho_s h \frac{dV}{dt} &= P_2 + \frac{1}{2} \rho_s h \frac{c_0}{E} \frac{dP_2}{dt} \\ &= P_2(t) + k \frac{dP_2(t)}{dt} = P_2(t + k) \end{aligned} \quad (111)$$

provided that k , a time constant $(= (\rho_s h)/(2 \rho c_0))$ is small compared to the duration of the applied pulse. E is the Young's modulus, c_0 is the wave velocity and ρ is the density, of the pressure bars. Therefore :-

$$\begin{aligned} \sigma(t) = & -P_2(t + k) - \frac{1}{2} \rho_s a^2 \left(\nu^2 - \frac{1}{3} \frac{h^2}{a^2} \right) \ddot{\epsilon} \\ & + \frac{1}{2} \rho_s a^2 \left(\nu^3 + \frac{1}{3} \frac{h^2}{a^2} \right) \dot{\epsilon}^2 \end{aligned} \quad (112)$$

This equation gives the stress-time profile in the specimen where P_2 equals $(-E \epsilon_T)$ and the other four terms are inertial corrections. The first of these correction terms was derived by KOLSKY (1949) and the second by DAVIES and HUNTER (1963) in similar analyses. SAMANTA (1971) derived a term proportional to $\dot{\epsilon}^2$ for the case when the specimen is deforming plastically ($\nu = \frac{1}{2}$). He makes the erroneous conclusion however that this term has a minimum value in equation (112) when h equals a $\sqrt{\frac{3}{4}}$.

It was found experimentally for all of the specimens tested, that the inertial correction terms represented at most a few percent of $\sigma(t)$ and could be ignored. The pulse duration was about 10 microseconds (10 μ secs) and k was about 0.05 μ secs. The stress-time behaviour of the specimen is therefore given by :-

$$\sigma(t) = -E \epsilon_T(t + k) \quad (113)$$

where k represents a small movement of the pulse along the time axis.

The displacements of the first and second faces are given by :-

$$c_o \int_0^t (\epsilon_I - \epsilon_R) dt \quad \text{and} \quad c_o \int_0^t \epsilon_T dt$$

respectively. Integration is started at any instant in time before the arrival of the pulse. The mean spatial strain in the specimen ϵ is therefore given by :-

$$\epsilon = \frac{c_o}{h} \int_0^t (\epsilon_I - \epsilon_R - \epsilon_T) dt \quad (114)$$

It is not convenient to record ϵ_R as well as the other two pulses; its value must therefore be determined in terms of the other parameters. Recalling equation (105) :-

$$P_1 - P_2 = \rho_s h \left(\frac{1}{2} \frac{dU}{dt} + \frac{dV}{dt} \right) \quad (105)$$

$$\text{or} \quad E(-\epsilon_I - \epsilon_R + \epsilon_T) = \frac{1}{2} \rho_s h \frac{dU}{dt} - \rho_s h c_o \dot{\epsilon}_T$$

$$\text{Therefore} \quad -\epsilon_R = \epsilon_I - \epsilon_T - 2k \dot{\epsilon}_T + \frac{k}{c_o} \frac{dU}{dt} \quad (115)$$

Substituting for ϵ_R in equation (114) gives :-

$$\epsilon = 2 \frac{c_0}{h} \int_0^t (\epsilon_I - \epsilon_T - k \dot{\epsilon}_T) dt - k \dot{\epsilon} \quad (116)$$

The value of $\dot{\epsilon}$ to a first approximation is obtained from equation (116) by differentiating with respect to t and ignoring second order differentials.

$$\dot{\epsilon} = 2 \frac{c_0}{h} (\epsilon_I - \epsilon_T - k \dot{\epsilon}_T) \quad (117)$$

Substituting for $\dot{\epsilon}$ in equation (116) and differentiating with respect to t gives :-

$$\begin{aligned} \dot{\epsilon} &= 2 \frac{c_0}{h} (\epsilon_I - k \dot{\epsilon}_I - \epsilon_T + k \ddot{\epsilon}_T) \\ &= 2 \frac{c_0}{h} (\epsilon_I(t - k) - \epsilon_T) \end{aligned} \quad (118)$$

by ignoring second order derivatives. Therefore :-

$$\epsilon(t) = 2 \frac{c_0}{h} \int_0^t (\epsilon_I(t - k) - \epsilon_T(t)) dt \quad (119)$$

Equation (119) was also derived by DAVIES and HUNTER (1963).

10.7. LIMITATIONS ON SPECIMEN GEOMETRY

If the relative value of k is small enough to be ignored, then the relationship :-

$$\epsilon_I + \epsilon_R = \epsilon_T \quad (120)$$

can be used. This can be verified experimentally for any particular case by recording all three pulses as reported by BILLINGTON and BRISSENDEN (1971) and LINDHOLM and YEAKLEY (1968). The latter two authors also checked their results by using, in addition, a ring condenser gauge placed over the specimen. The radial strain of the specimen, measured by using this gauge, behaved as expected.

BELL (1966) made similar checks by using a diffraction

grating strain gauge scratched onto a small central area of an aluminium specimen. This type of gauge which measures longitudinal strain is described in chapter 11. BELL (1966) found that the measured surface strain in the centre of the specimen coincided with the average strain calculated from equation (119) when the specimen was deforming elastically. The average strain exceeded the measured surface strain at the specimen's centre for large plastic deformations. However, as reported by LINDHOLM and YEAKLEY (1968), this effect can be explained in terms of complicated static stress-strain fields caused by poor lubrication at the specimen's faces.

Equation (120) implies that there is a maximum length of specimen for which it can be considered valid. This condition was given by DAVIES and HUNTER (1963) as :-

$$T > \pi t_s \quad (121)$$

where T is the duration of the loading pulse in the specimen and t_s is the transit time. To obtain higher strain rates, T must be reduced and hence the specimen's length must also be reduced.

Frictional effects between the specimen and pressure bars present an error of unknown size which becomes relatively more important as the length of the specimen is reduced; this produces an effective lower limit to the length of specimens used. DAVIES and HUNTER (1963) estimated this lower limit as :-

$$a/h \sim 1 \quad (122)$$

This condition agrees well with the results of MAIDEN and GREEN (1966) who tested lubricated aluminium specimens of various lengths which were deformed plastically during the tests. Their results varied drastically from test to test for short specimens. Reduction of the specimen's length also reduces the accuracy of measuring the specimen's strain.

The predominant use of the split Hopkinson bar is to produce large plastic deformation in metal specimens and relative motion must therefore take place at the specimen-bar interfaces. However, in all of the composite specimens tested, deformation was elastic up to the failure point which always occurred at a

strain less than 4 %. Lack of relative motion at the interfaces, as in the static case, did not therefore present a problem. The lubrication at dynamic strain rates is probably superior than at static strain rates because there is not enough time for the lubricant at the interface to be forced out.

Many workers used specimen diameters down to half that of the pressure bars. This is usually a necessity in order to accomodate large radial strains in the specimen. The results of LINDHOLM (1964) showed that aluminium specimens with various diameters gave little variation in their stress-strain behaviour. This occurred despite the stress wave perturbations occurring in the specimen and at the strain gauges which must have been caused by the mismatch of area at the specimen-bar interfaces. The effect is expected to be more pronounced for small deformations as in composite specimens. In addition, the static stress-strain fields due to area mismatch are known to cause large strength losses in fibreglass specimens, (see section 8.3.). For these reasons, all of the specimens tested using the split Hopkinson bar had the same diameter as the pressure bars. Reduction in specimen diameter does have the advantage of increasing the applied stress in the specimen for any particular incident stress pulse.

CHAPTER ELEVEN

COMPARISON OF DYNAMIC STRAIN MEASURING TECHNIQUES

11.1. DIFFRACTION GRATING STRAIN GAUGE

It is possible to rule a diffraction grating onto the curved surface of the bar with lines perpendicular to the axis of the bar and their lengths short compared with the radius of curvature so that the grating occupies an effectively flat area. If a monochromatic beam of light with wavelength λ is directed onto the grating in a plane perpendicular to the ruled lines, the angle of maximum intensity of reflection of the beams is governed by the equation :-

$$p (\sin\alpha - \sin\theta_n) = n\lambda \quad (123)$$

where n is zero or a positive integer, p is the separation of adjacent lines, α is the angle of incidence and θ_n is the angle of reflection of order n .

When a stress pulse is transmitted along the bar, the strain produced at the surface causes the spacing p to change to $p + dp$. This causes an angular shift in the reflected beams from θ_n to $\theta_n + d\theta_n$. The longitudinal strain in the bar to a first approximation is given by :-

$$\frac{dp}{p} = \epsilon = \frac{\cos\theta_n}{\sin\alpha + \sin\theta_n} d\theta_n \quad (124)$$

The variation, $d\theta_n$, can be monitored by using a photomultiplier to measure the power of a reflected beam which has first passed through a V shaped notch. As the beam moves across the notch due to the variation in θ_n , the fraction of light cut out by the notch also varies; this causes a change in light power received by the photomultiplier. This technique has been perfected by BELL (1966) but considerable expertise is required for its successful operation. The resolution of the stress pulse is equal to the total separation of grating lines which affect the reflecting beam. This length can be as small as 0.005 in.

11.2. ELECTRICAL RESISTANCE STRAIN GAUGE

This gauge consists of a long continuous piece of thin metal strip arranged into a two-dimensional rectangular shape such that nearly all of the strip lies parallel to one side of the rectangle. Straining the strip affects its electrical resistance, R , which can be measured between the two ends. This relationship can be expressed as :-

$$\epsilon = \frac{\Delta R}{R} k \quad (125)$$

where k is the gauge factor and, for most commercial gauges, has a strain-independent value of about 2. $(\Delta R/R)$ is the fractional change in resistance of the gauge.

The gauge is glued to the bar's surface with the aligned portion of the strip laid parallel to the bar's axis so that the gauge strains in an identical manner to the bar's surface: the gauge is only sensitive to longitudinal strains in the bar because of its geometry. Hoop strains can be measured by rotating the gauge through 90° before glueing. The resistance of the gauge is proportional to the average surface strain over the area of the gauge. Therefore the resolution of longitudinal pulses is equal to the length of the gauge measured parallel to the bar's axis. Any response to flexural waves can be eliminated by placing the gauge in series with a second identical gauge glued onto the bar in a diametrically opposite position to the first.

The reliability of the gauge depends on good adhesion between the gauge and bar. Catastrophic failure of the bond is obviously detectable but partial failure is not and can lead to erroneous results. ELST (1965) and KRAFFT (1962) noted the tendency of this type of gauge to become detached when subjected to high strain rates. The dynamic response of the gauge also depends on the ability of the straining gauge to follow the straining bar closely. The inertia of the gauge causes shear loads on the glue and a tendency for the strain of the gauge to lag behind the strain of the bar's surface.

Type 3/120/E/C foil gauges manufactured by Tinsley Telcon Ltd. were used throughout. This type has a gauge length of

2.9 mm (resolution length) and a width of 1.7 mm. The specified gauge factor for each gauge was assumed to be correct.

11.3. RING CONDENSER GAUGE

Three types of condenser gauge, all of which are described in chapter 10, have been used in the past by various workers. They all have radial symmetry and possess the advantage of averaging the recorded pulse over the cross-section of the bar; optical and resistance strain gauges record only that part of the pulse traversing a small surface element. Another advantage is the insensitivity of condenser gauges to flexural and torsional waves. The ring condenser gauge has the additional advantage that it is not restricted to a fixed position on the bar and can readily be moved to new positions along the length of the bar as required. A precise account of the different sensitivities of the three types of condenser gauges to longitudinal stress waves with short wavelengths is given by DAVIES (1947).

The present discussion is confined to the description of the ring condenser gauge which measures the radial expansion of the bar. This gauge is shown schematically in Fig. 11.1. The electric field E can be calculated using Gauss's law by making use of the radial symmetry and assuming that E has no axial component.

$$E = \frac{Q}{2\pi \epsilon_0 b} \frac{1}{r} \quad (126)$$

where r is the radial coordinate and lies between the limits r_1 and r_2 . Q is the charge and ϵ_0 is the permittivity of air. The electric potential V between the ring and the bar is given by :-

$$V = \frac{Q}{2\pi \epsilon_0 b} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{Q}{2\pi \epsilon_0 b} \ln(r_2/r_1) \quad (127)$$

and hence the capacitance C is given by :-

$$C = \frac{Q}{V} = \frac{2\pi \epsilon_0 b}{\ln(r_2/r_1)} \quad (128)$$

Fig. 11.1 Schematic diagram of ring condenser gauge

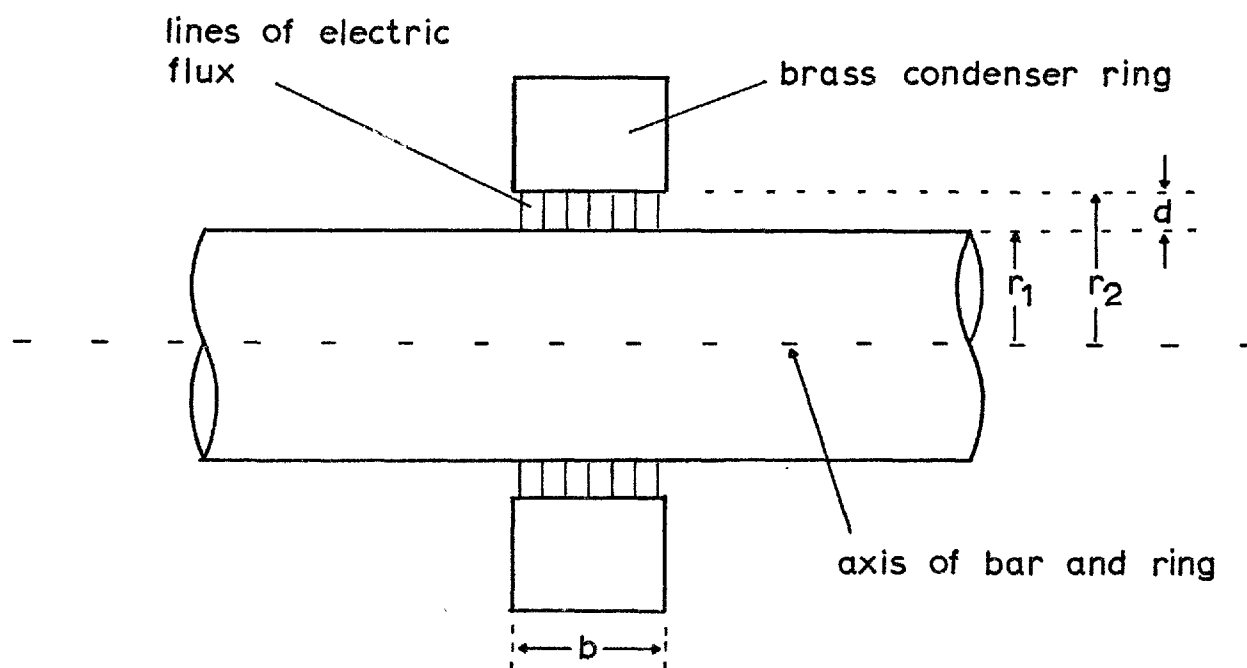
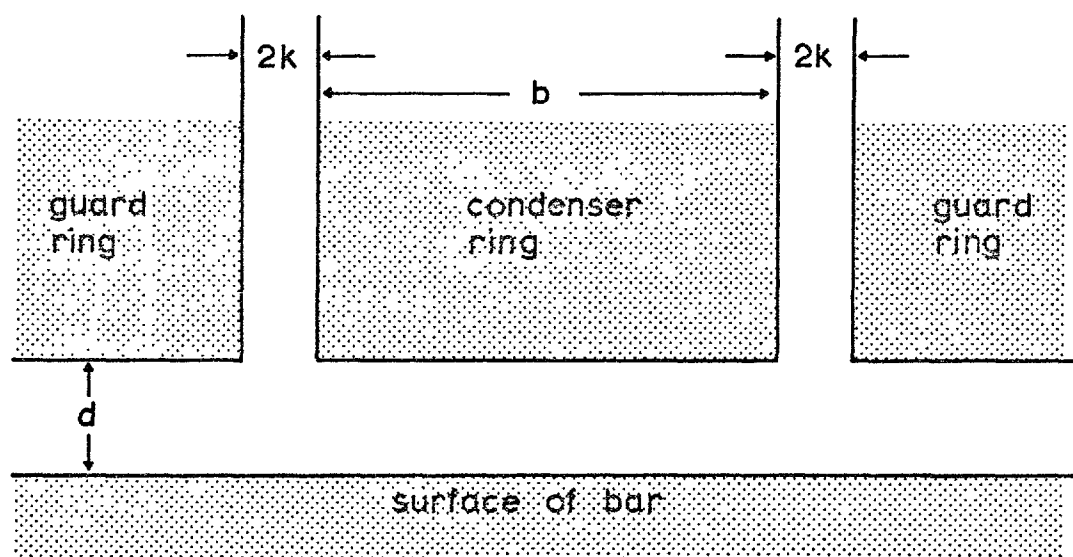


Fig. 11.3 Region in between guard rings



Letting ($a = r_2$) and ($d = r_2 - r_1$), enables equation (128) to take the form :-

$$C = \frac{2\pi \epsilon_0 b}{\frac{d}{a} + \frac{1}{2}\left(\frac{d}{a}\right)^2} \quad (129)$$

by expanding the logarithmic term and knowing that $d \ll a$. If a change in capacitance ΔC due to a small change Δr in the bar's radius is considered, then :-

$$C + \Delta C = \frac{2\pi \epsilon_0 b}{\frac{d - \Delta r}{a} + \frac{1}{2}\left(\frac{d - \Delta r}{a}\right)^2} \quad (130)$$

Subtracting equation (129) from equation (130) and simplifying using the binomial expansion gives :-

$$\Delta C = \frac{2\pi \epsilon_0 a b \Delta r}{d (d - \Delta r)} \quad (131)$$

The displacement Δr can then be written as :-

$$\Delta r = \frac{d \Delta C}{C} \left(1 - \frac{\Delta r}{d}\right) \quad (132)$$

Therefore, a linear relationship between Δr and ΔC can be assumed only if $\Delta r \ll d$.

11.4. IMPORTANCE OF STRAY FIELDS IN CONDENSER RING GAUGE

The electric field can be separated into two parts :-

- 1) The required uniform field which exists directly between the bar and the ring.
- 2) The stray field which originates on the sides and the external face of the ring.

The capacitance of the gauge using equation (129) can be expressed approximately as :-

$$C = 2\pi \epsilon_0 b \frac{a}{d} (1 + f) \quad (133)$$

where $f \geq 0$ and is a measure of the error if a simple field solution is used in which there is no stray field, (i.e. if

$f = 0$, there is no error). No solution of equation (133) as a function of ring geometry could be found in the literature but KIRCHOFF, quoted by DAVIES (1947), calculated an exact solution for the case of two identical discs which are circular and parallel to each other. In the particular case when the common radius of the discs equals the separation between them equals their common thickness, $f = 1.37$ which corresponds to an error of 137 % if a simple field solution is used. f rapidly approaches zero as the size of the disc separation is reduced.

It should be noted that there are several opposing criteria to be imposed on the geometry of the ring gauge. To a first approximation, the change in capacitance is proportional to the average radial strain of the bar in the region directly adjacent to the ring. The resolution of the stress pulse is therefore equal to the length b in Fig. 11.1. Even this statement is suspect because the stray field and hence the capacitance is affected by the straining bar some distance from the ring. To increase the resolution, b must be decreased which automatically increases f . d must be increased to produce a more linear response of the gauge (see equation (132)) which again increases f . In addition the sensitivity of the gauge which is proportional to (b/d^2) , is decreased by increasing d and decreasing b . (See equation (144)).

The stray field becomes more important if the ring is mounted, for practical reasons, in an insulating material most examples of which have a dielectric constant greater than 3. This has the effect of increasing the intensity of the electric field inside the insulator by a factor equal to the dielectric constant.

Another practical problem which might occur is deviation from concentricity of the bar and ring. Errors in the calculated capacitance due to non-concentric cylinders can be expressed as :-

$$f = \frac{1}{2}(s/d)^2 \quad (134)$$

(See for instance DAVIES (1947)). d is the average separation of the two cylinders and s is the displacement between the axes of the two cylinders which remain parallel. The sensitivity of the ring gauge to flexural waves, which corresponds to a

fluctuating value of s via equation (134), is therefore reduced by increasing d . This is another reason for increasing d .

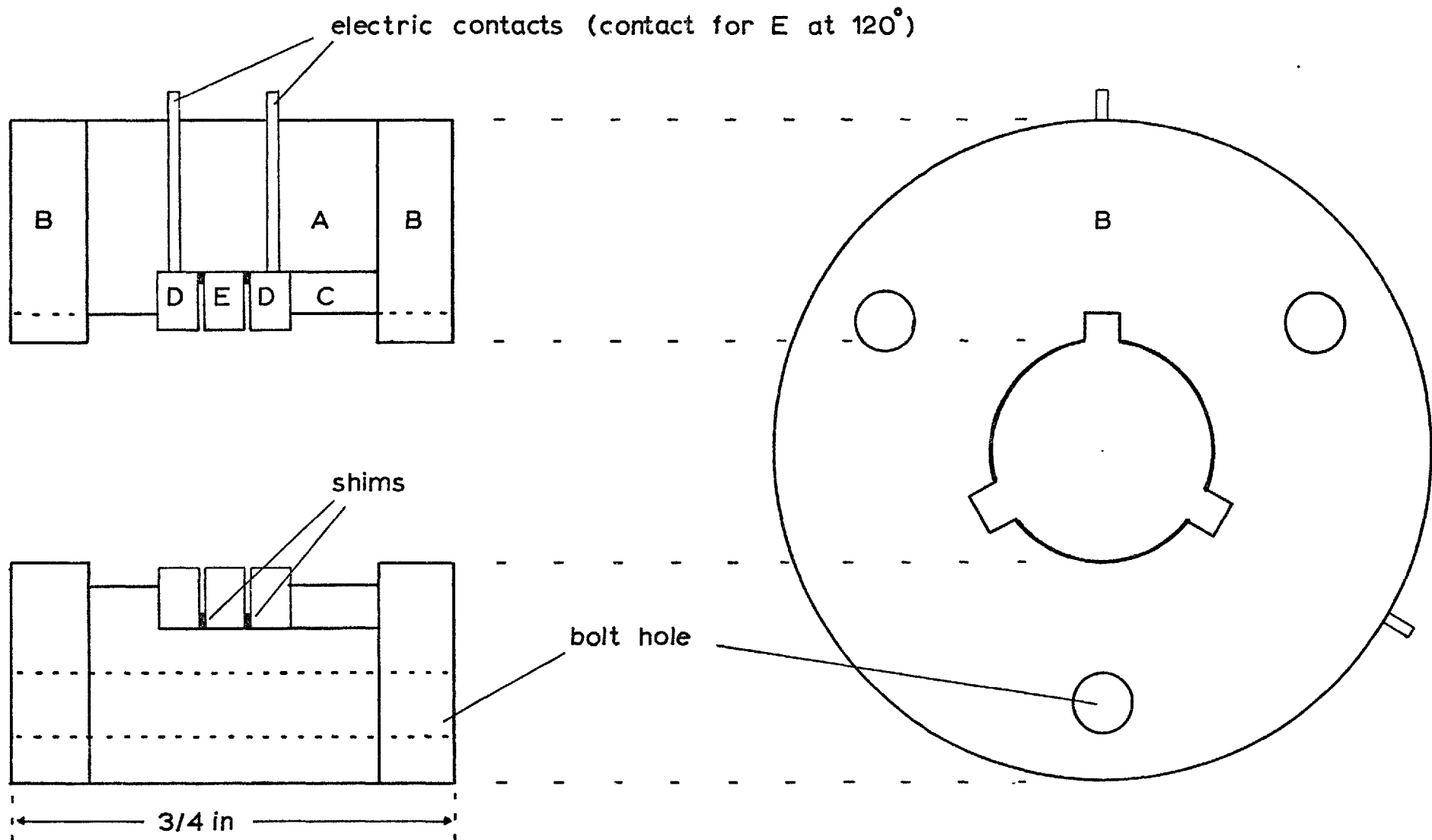
11.5. DESCRIPTION OF PRACTICAL RING CONDENSER GAUGE

A cut away view of the gauge constructed for use with the split Hopkinson bar of chapter 12 is shown in Fig. 11.2. All components except the three brass rings were made from perspex. Any problems caused by stray electric fields were eliminated by using two guard rings D, one each side of the gauge ring E. All three brass rings were concentric and had the same diameter. Rings D, E, D, C, in that order, were pushed into the close fitting body, A. The brass rings were separated from each other by thin circular shims punched out from a sheet of polythene. The ends, B, were secured to the body, A, by means of three bolts. The fit between the bar and the ends was as tight as is possible while allowing manual movement of the gauge along the bar.

The body had to be centred with the ends before the bolts were tightened. To do this the gauge was first placed on the bar and three long shims of suitable dimensions were then manoeuvred into position along the aligned slots in the gauge so that they were wedged between the rings and the bar. The bolts were then tightened and the shims removed. The gap between the rings and the bar could be checked by viewing the rings through the three slots at each end B in turn using a microscope. Separate electric contacts with the three brass rings were made by screwing thin brass bolts (No. 10 B.A.) through the threaded perspex body A to touch them.

The radius of curvature of the bar's surface was much greater than the small gap between the bar and the rings. The electrostatic problem of calculating the electric field in the region of interest could therefore be regarded as two-dimensional. The solution was calculated by THOMSON (1893) who used the method of conjugate functions with the Schwarian transformation. It is assumed that the three rings are maintained at the same potential; this is true in practice to within 0.03 %.

Fig. 11.2 Construction of ring condenser gauge (scale 4:1)



With reference to Fig. 11.3, the capacitance C of the gauge ring with respect to the bar is given by :-

$$C = 2\pi \epsilon_0 \frac{a}{d} \left(b - \frac{2}{\pi} k \sin^{-1} \frac{k}{(d^2 + k^2)^{\frac{1}{2}}} - \frac{d}{\pi} \ln \frac{d^2}{d^2 + k^2} \right) \quad (135)$$

where the first term on the right gives the capacitance of the gauge ring if $f = 0$. Substituting for the used values :-

$$d = 0.01 \text{ in.}, \quad b = 0.025 \text{ in. or } 0.055 \text{ in.}, \quad k = 0.0025 \text{ in.}$$

gives a maximum value of ($f = 0.026$). Equation (135) can be rewritten, neglecting $(k/d)^2$ and higher terms, as :-

$$C = 2\pi \epsilon_0 \frac{a}{d} (b + 2k) \quad (136)$$

This is equivalent to saying that the capacitance is the same as that of a "perfect" condenser with a uniform field where the length b is increased by half the breadth of the slit at each end. The maximum resolution of the stress pulse is therefore given by :-

$$b + 2k = 0.03 \text{ in.}$$

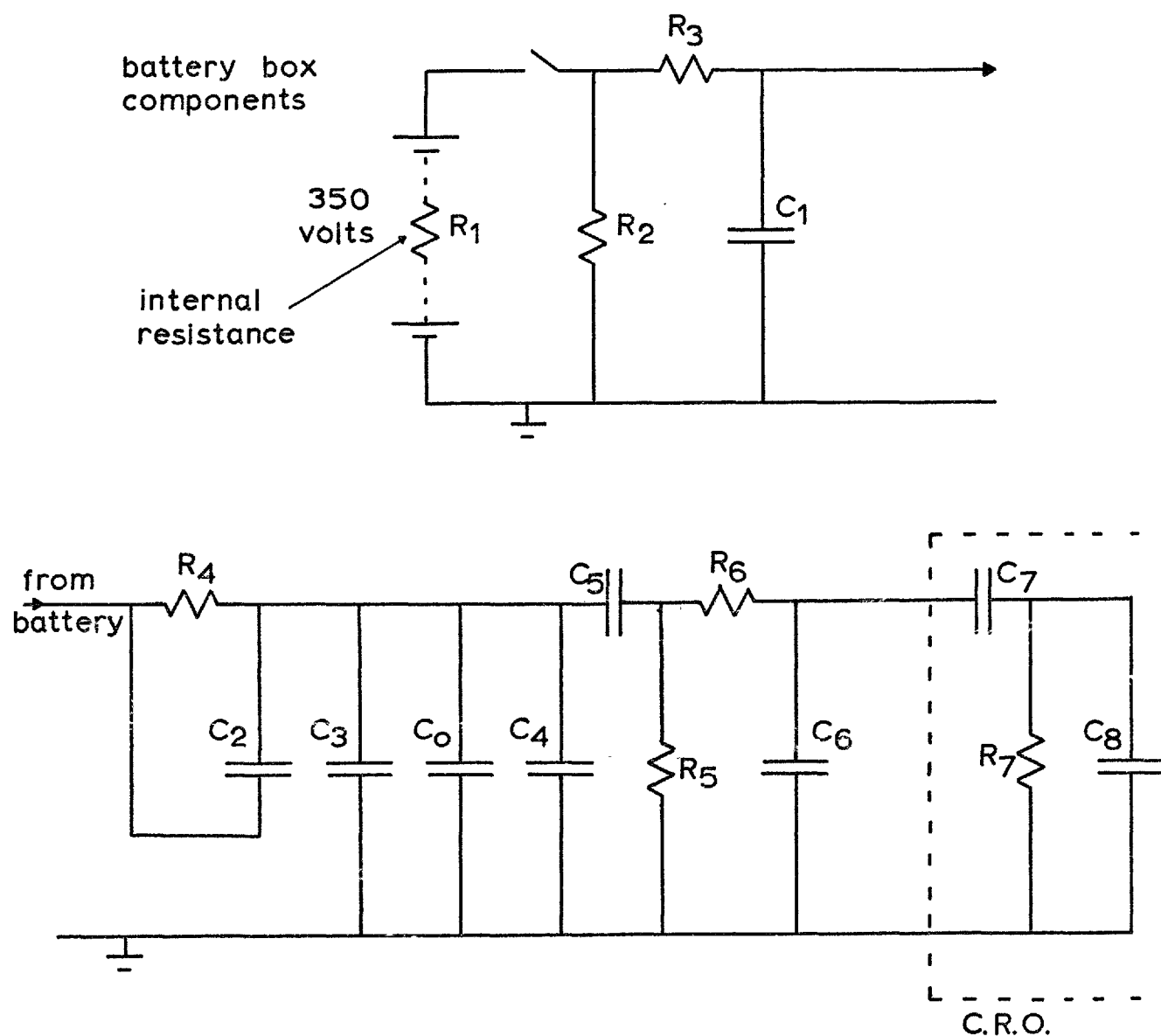
It should be noted that the resolution is reduced if the axes of the ring and bar are not parallel. This reduction becomes relatively more significant as the length, b , of the ring is reduced.

11.6. ELECTRONIC CIRCUITS USED WITH CONDENSER GAUGE

The gauge was placed in an electronic circuit in order to measure the fluctuating capacitance. This circuit produced a corresponding fluctuating voltage which was monitored using a Tektronix 551 dual beam cathode ray oscilloscope (C.R.O.) fitted with type L and W plug-in amplifiers. The circuit used is shown in Fig. 11.4.

All components, except the gauge itself, were housed in earthed metal boxes. Connections outside the boxes were made with antimicrophonic coaxial cable and all cables were independently earthed at both ends. The battery box components

Fig. 11.4 Electronic circuit used with condenser gauge



$R_1 = 2.5 \text{ k}\Omega$
 $R_2 = 1 \text{ M}\Omega$
 $R_3 = 2.4 \text{ M}\Omega$
 $R_4 = 20 \text{ M}\Omega$
 $R_5 = 100 \text{ M}\Omega$
 $R_6 = 1 \text{ k}\Omega$
 $R_7 = 1 \text{ M}\Omega$

$C_1 = 1 \text{ }\mu\text{F}$
 $C_2 = 30 \text{ pF}$ (guard rings)
 $C_3 = 300 \text{ pF}$
 $C_0 = (3 \text{ or } 6) \text{ pF}$ (gauge)
 $C_4 = 160 \text{ pF}$ (cable)
 $C_5 = 1000 \text{ pF}$
 $C_6 = 70 \text{ pF}$ (cable)
 $C_7 = 0.1 \text{ }\mu\text{F}$
 $C_8 = 20 \text{ pF}$

acted as a low-pass filter to give a noise-free 350 volts supply to the gauge. C_2 is the capacitance between the active gauge ring and the guard rings all of which were initially maintained at the same potential. The analysis is complicated because the guard rings themselves had a capacitance with respect to the earthed bar and were hence affected by the straining bar. However it can be shown that provided $C_1 \gg C_2$ this effect can be ignored. The total capacitance, C , in parallel with the gauge, C_0 , including the cables was about 600 picofarads, (600 pF). The charge on this capacitance was isolated from earth by the resistances $(R_4 + R_3 + R_1)$, R_5 and R_7 so that the charge remained constant during a short transient pulse of 10 μ secs. The leakage of charge was controlled through the smallest of these resistances by the time constant, $(C R_7)$, which caused a maximum distortion of the peak value of the recorded pulse of $\frac{1}{3}$ %. As the charge Q remained constant :-

$$C = \frac{Q}{V} \quad (137) \quad \text{and} \quad C + \Delta C = \frac{Q}{V + \Delta V} \quad (138)$$

$$\text{Therefore :-} \quad \frac{\Delta C}{C} = - \frac{\Delta V}{V} \left(1 + \frac{\Delta V}{V} \right) \quad (139)$$

where ΔC is connected to Δr by equation (132).

$$\Delta r = d \frac{\Delta C}{C_0} \left(1 - \frac{\Delta r}{d} \right) = - \frac{C d}{C_0 V} \left(1 - \frac{\Delta r}{d} + \frac{\Delta V}{V} \right) \Delta V \quad (140)$$

$$\text{Now,} \quad \Delta r = -\nu a \epsilon \quad (141)$$

where ϵ is the longitudinal strain in the bar and a is its radius.

$$\text{Therefore :-} \quad \epsilon = \frac{C d}{\nu a V C_0} \Delta V \left(1 - \frac{\Delta r}{d} + \frac{\Delta V}{V} \right) \quad (142)$$

The term $(\Delta V/V)$ could be ignored because ΔV did not exceed 0.1 volts. The maximum strains recorded in the bars were about $\frac{1}{2}$ % which corresponded to a value of the term $(\Delta r/d)$ of 3 %. In order to simplify the analysis, this term is ignored so that a linear relationship :-

$$\epsilon = \frac{C d}{\nu a V C_0} \Delta V \quad (143)$$

was used throughout. The sensitivity of the circuit is given by :-

$$\begin{aligned}\Delta V &= -\frac{V}{C} \Delta C = -2\pi \epsilon_0 a b \frac{V}{C d^2} \Delta r \\ &= 2\pi \nu \epsilon_0 b \left(\frac{a}{d}\right)^2 \frac{V}{C} \epsilon\end{aligned}\quad (144)$$

A large sensitivity can be obtained by reducing the value of C . However, C , together with the value of R_7 (input impedance of the C.R.O.), controls the rate of charge leakage via the time constant $C R_7$; the distortion is therefore increased by reducing C . A large sensitivity can also be obtained by reducing the value of d causing more distortion if the linear relationship of equation (143) is used. A compromise must therefore be reached. Ten percent of the sensitivity was lost across C_5 due to its comparatively low value but this allowed C_5 together with R_7 to become a high-pass filter and prevented low frequency noise from reaching the C.R.O.

A gauge resolution length of 0.03 in. was initially used, but such sharp resolution was not required due to the highly dispersive nature of the bars to high frequency waves. Subsequently a length of 0.06 in. was therefore used. In addition, the rise time of the C.R.O. (0.015 μ secs) was unnecessarily fast for resolving the stress waves and therefore the low-pass filter (R_6 together with $C_6 + C_8$) was incorporated in the circuit. This effectively reduced the rise time by a factor of five and prevented high frequency noise from reaching the C.R.O.

11.7. ELECTRONIC CIRCUITS USED WITH RESISTANCE GAUGE

The circuit diagram is shown in Fig. 11.5. All components excluding the gauge were housed in an earthed metal box and external connections were made with coaxial cable as mentioned in the previous section. The initial current, I , passing through the gauge was equal to 25 milliamps. and was determined by measuring the voltage, V , across the gauge. The change in resistance, ΔR , of the gauge due to the stress pulse caused corresponding changes in voltage, ΔV , and current, ΔI , where :-

$$V = I R_2 \quad (145)$$

$$V + \Delta V = (I + \Delta I) (R_2 + \Delta R) \quad (146)$$

$$I (R_1 + R_2) = (I + \Delta I) (R_1 + R_2 + \Delta R) \quad (147)$$

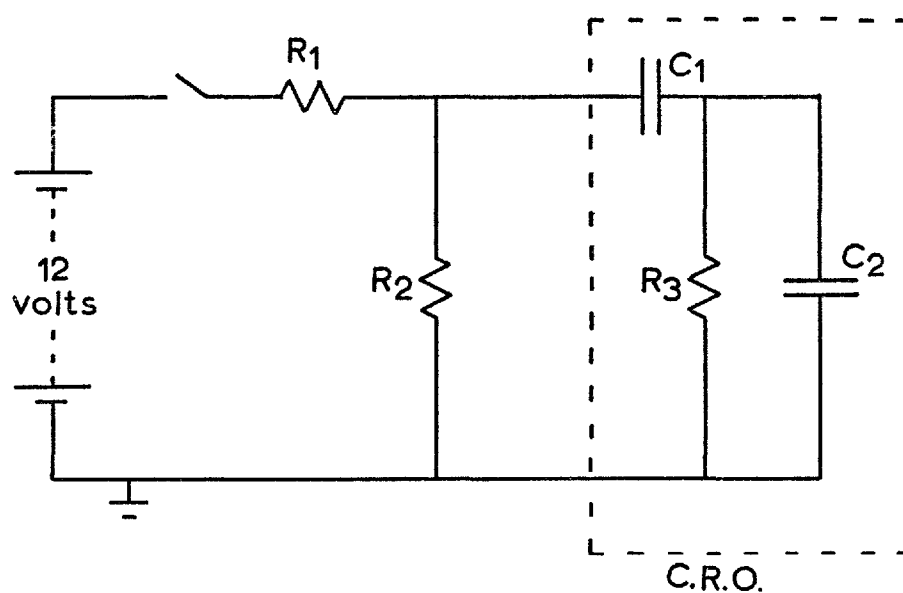
From these equations can be derived :-

$$\Delta V = I \Delta R \left(1 - \frac{R_2}{R_1 + R_2} \right) - I \Delta R^2 \frac{R_1}{R_1 + R_2} \quad (148)$$

which gives a linear relationship between ΔV and ΔR provided that the non-linear term is small enough to be ignored. The error introduced by ignoring this term never exceeded $\frac{1}{3}$ %. ΔR is related to the strain by the linear equation (125).

Fig. 11.5

Electronic circuit used with resistance gauge



$$\begin{aligned} R_1 &= 360 \quad \Omega \\ R_2 &= 120 \quad \Omega \text{ (gauge)} \\ R_3 &= 1 \quad \text{M}\Omega \end{aligned}$$

$$\begin{aligned} C_1 &= 0.1 \quad \mu\text{F} \\ C_2 &= 20 \quad \text{pF} \end{aligned}$$

CHAPTER TWELVE

EXPERIMENTAL TECHNIQUES AND RESULTS

12.1. DESCRIPTION OF APPARATUS

The split Hopkinson bar used to test the composite specimens is shown in Fig. 12.1 and is the same as that used by FITZPATRICK (1968). It consists essentially of two 12 in. long cylindrical bars made of hardened tool steel and centreless-ground to a common diameter of 0.375 in. The ends of the bars were polished flat, normal to the axis of the bars, using the process described in section 3.2.2. The whole assembly was secured on a milling machine and the aligned V grooves were recut in order to obtain the best possible alignment.

The required stress pulse was obtained by exploding an electric impulse detonator inside a disposable cylindrical anvil made of hardened silver steel. The 6 in. long by 0.375 in. diameter anvil had its confined end ground flat normal to its axis. All the bars were secured in the aligned V blocks by finger tight bolts and their common interfaces were lubricated with light engine oil. A schematic diagram of the apparatus complete with the electronic components is shown in Fig. 12.2.

The detonator was exploded by connecting its two leads across a six volt battery housed in D. The stress pulse caused by the explosion passed through the first of the three condenser gauges which was used as a triggering device to activate a single sweep of the two electron beams across the C.R.O. screen. This gauge, which had no guard rings, had a simpler design and a much higher sensitivity than the one described in section 11.5. In order to improve the triggering reliability of the C.R.O., the voltage pulse from C1 was fed into the monostable multivibrator circuit A complete with power supply. This circuit incorporated a Signetics type N74121 integrated circuit which in turn incorporated a Schmitt-trigger. It was designed to produce a square voltage pulse of 5 volts amplitude when the input voltage exceeded 0.1 volts for a duration of 4 μ secs. The duration of the pulse from C1 could be varied between 4 and 10 μ secs. and allowed the possibility of this circuit being used to delay the

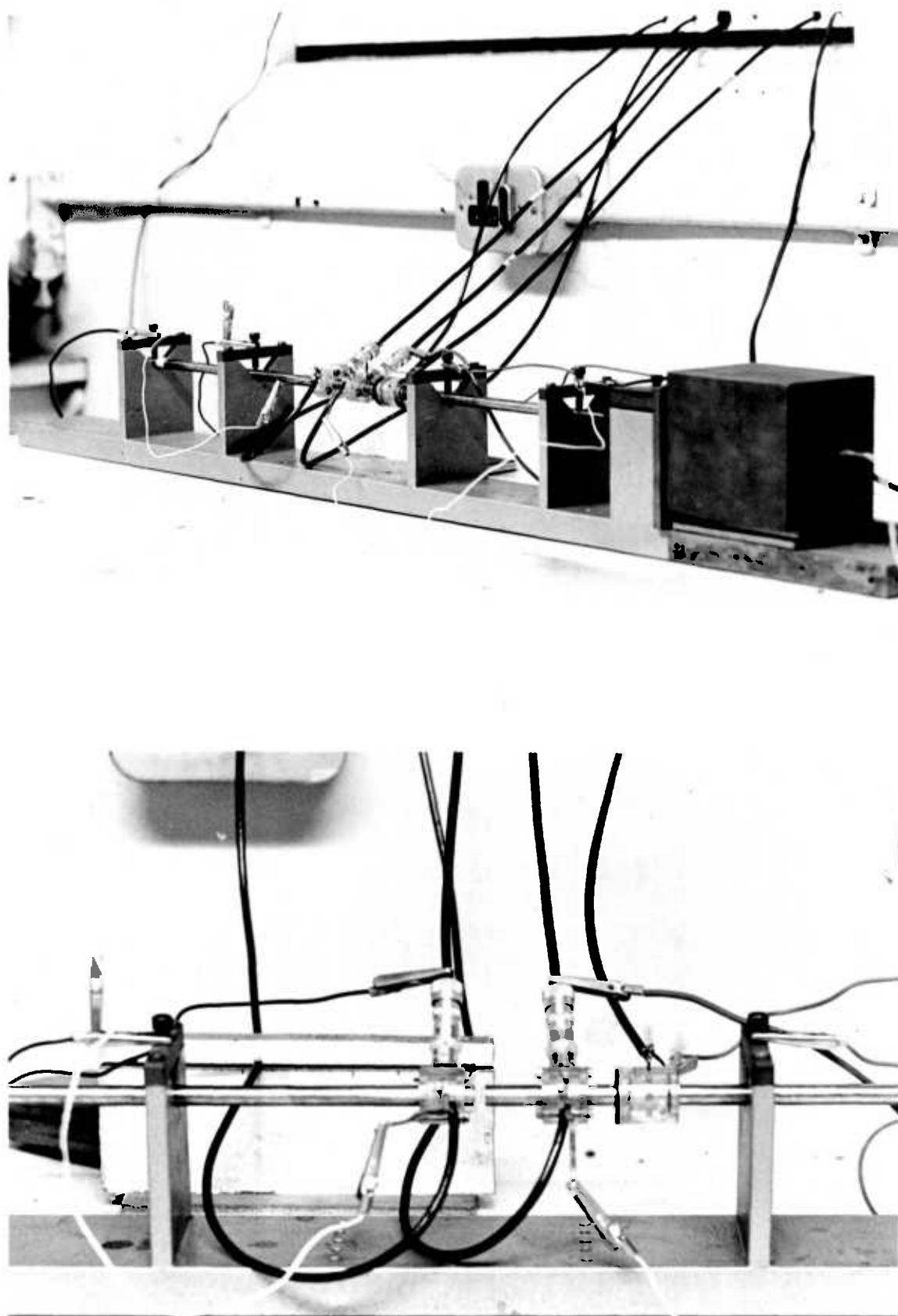
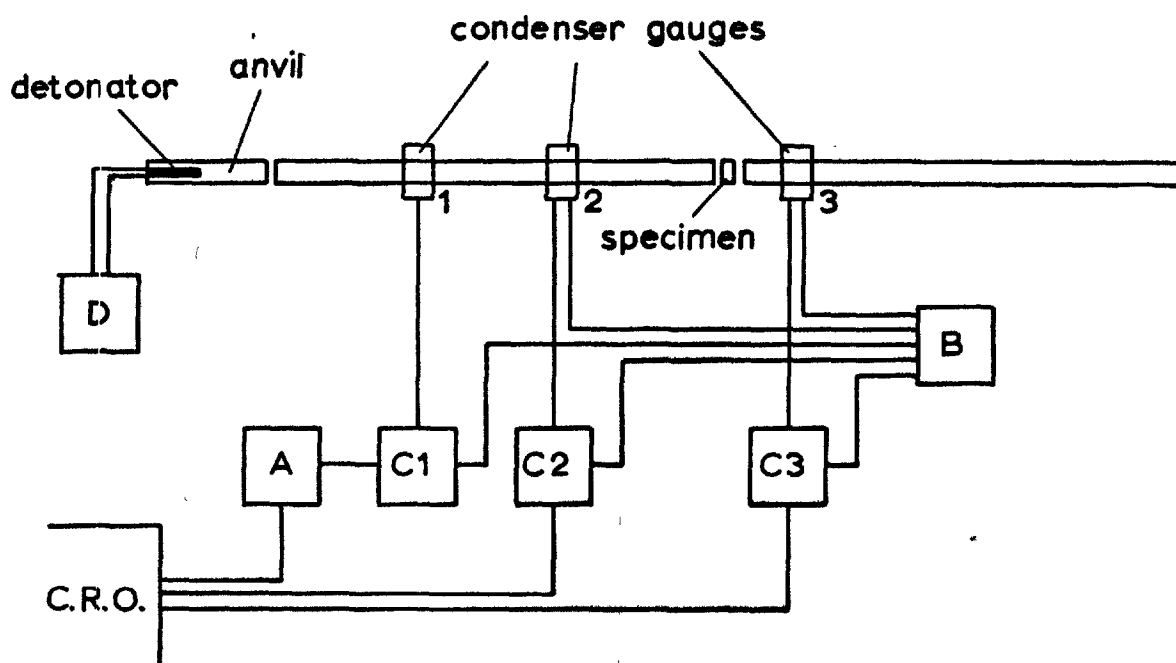


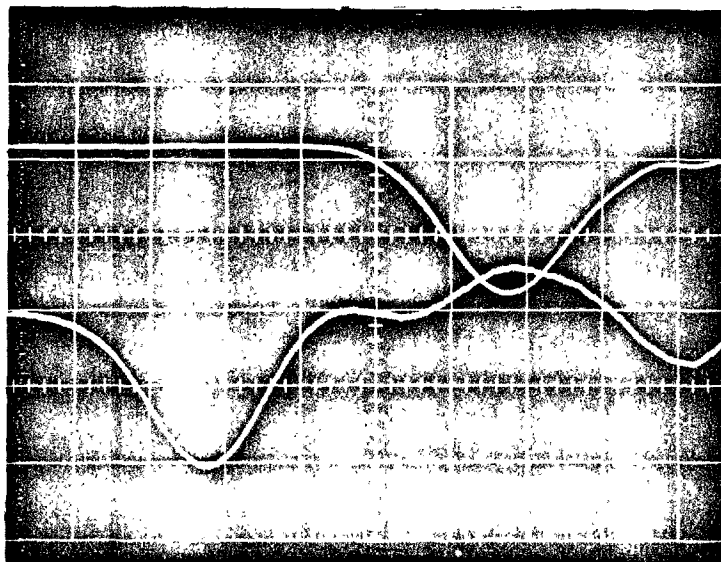
Fig. 12.1 Split Hopkinson bar apparatus

Fig. 12. 2 Schematic lay out of apparatus



- A monostable multivibrator
- B battery box
- C feed circuits
- D firing box for detonator

Fig. 12. 5 Photographic recording of stress pulses



triggering of the C.R.O.

The single sweep was photographed with a camera using Polaroid type 107 Land pack film. The grid on the screen was photographed separately on the same film using a different focus setting.

12.2. EXPERIMENTAL TECHNIQUES

A time scale of $4 \text{ mm}/\mu\text{sec}$. (velocity of electron beam across screen) was used on the C.R.O. for most of the work. The accuracy of the calculated stress-strain curves was very dependent on the accurate supposition of the incident and transmitted pulses before subtracting one from the other to obtain the reflected pulse. This in turn required the accurate positioning of gauges 2 and 3 on the bars and their positions were thus checked after each explosion. The required time shift between the two recorded pulses on the photograph could be determined by superimposing the identical pulses recorded when no specimen was present. This also allowed a periodic relative calibration between the two gauges to be made, which was always consistent to within a few percent. The quality of the contact between the two bars was demonstrated by the lack of distortion of the stress pulse which crossed the common interface and by the inability to detect a reflected pulse from this interface. The brass rings of the gauges together with the portions of the bars which they surrounded were maintained in a very clean condition.

The absolute calibration of gauge 3 was obtained by gluing close to it, in the axial direction, a resistance strain gauge on the same bar. The amplitudes of the two recorded pulses were then compared. The wave velocity in the pressure bars was determined by recording the time required for the transmitted pulse to traverse twice the length of the bar. The time scale of the C.R.O. was calibrated using an accurate oscillator. The attenuation of the pulse amplitude due to dispersion was also measured from the photographic recording to equal $0.06 \text{ \%/cm}$. The Young's modulus of the pressure bars was calculated to equal 200 GN/m^2 using equation (90).

12.3. DETONATOR PULSE

Five different sizes of electric detonators were used as mentioned in table 12.1.

TABLE 12.1.

I.C.I. electronic detonators

<u>Type</u>	<u>Symbol in text</u>	<u>Mass of explosives (gms.)</u>
No. 1	a	.05
No. 2	b	.1
No. 6 Star	c	.4
No. 8 Charge	d	.6
No. 8 Star	e	.8

Each detonator produced an approximately triangular pulse of 8 μ secs. duration (4 cm length in the pressure bars) with blunted edges, although the smaller detonators produced pulses which were a little shorter than this. Each pulse was followed by a tail of high frequency waves which varied distinctly from pulse to pulse. Detonators (a) and (b) were prone to produce pulses with a comparatively large tensile tail. The high frequency waves were very dispersive (as mentioned in section 10.3.) and could not be correlated between the same pulse as recorded by gauges 2 and 3 and even less so between gauge 3 and the resistance strain gauge positioned axially.

Detonators (c), (d) and (e) produced pulses of about the same amplitude (0.6 % strain = 1200 MN/m² stress) while detonators (a) and (b) produced pulses of about $\frac{1}{4}$ and $\frac{1}{2}$ this value respectively. The duration of pulse (e) could be increased by surrounding the anvil with a close fitting steel cylinder, although its amplitude remained the same. Although pulses produced by the same type of detonator were similar, they were never identical.

12.4. RESULTS

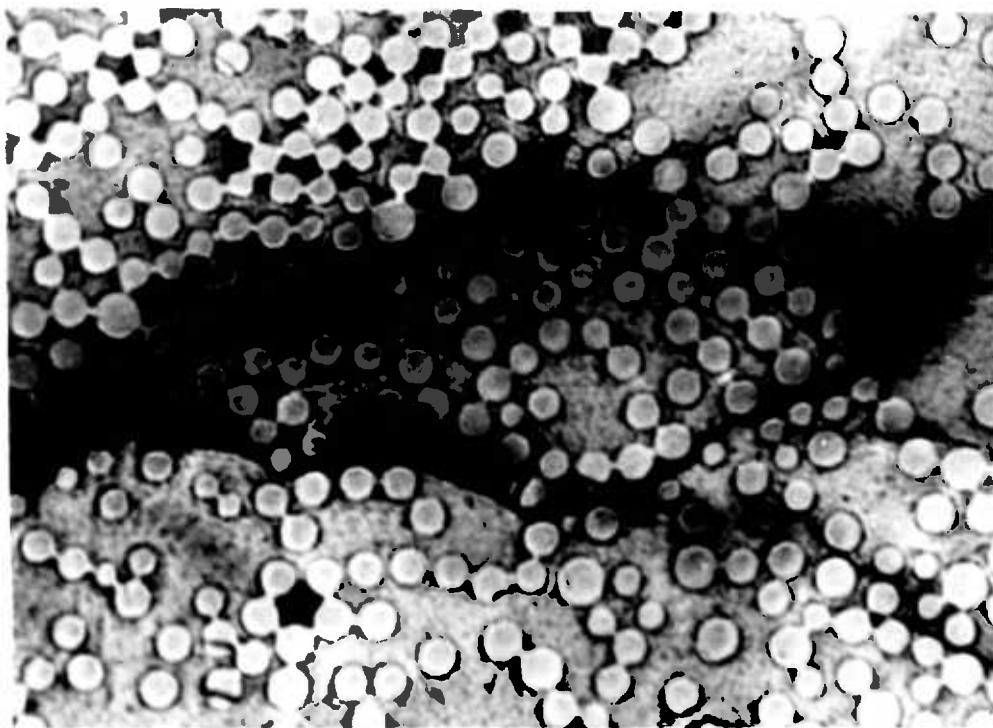
The end faces of all specimens were polished flat and parallel using the techniques described in section 3.2.2. Most of the specimens tested had a length of about 2.8 mm. A few fibreglass and fibre carbon specimens with lengths of 5 mm and 10 mm respectively were tested and gave similar stress-strain curves to those determined from 2.8 mm specimens. Relatively long fibre carbon specimens can conform to the inequality of equation (121) due to the faster axial wave velocity in these specimens :- 12 mm/ μ sec. as opposed to 5 mm/ μ sec. in steel and fibreglass. Strain rates varied in the range $(6 \text{ to } 10) \times 10^3 \text{ sec}^{-1}$ for fibreglass specimens and $(1 \text{ to } 3) \times 10^3 \text{ sec}^{-1}$ for fibre carbon specimens.

12.4.1. FIBREGLASS SPECIMENS

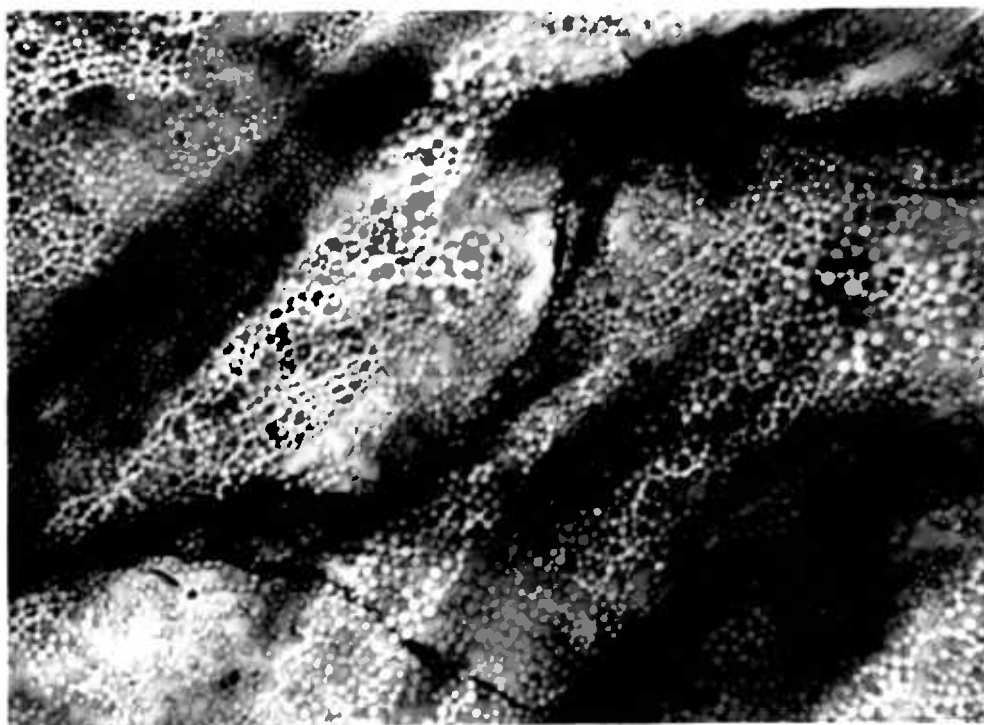
The 1000 MN/m² stress pulse to which the fibreglass specimens were subjected was not large enough to damage any specimen with a glass content exceeding 57 %. Below this value, longitudinal cracking in the matrix occurred which became more extensive for lower glass contents. These cracks usually traversed the whole length of the specimen but did not necessarily cut the curved surface and did not cause any loss of strength when subsequently tested at a static strain rate. Typical cracks are shown in Fig. 12.3.

In only two specimens (AE37) did complete failure occur, in which case the specimens disintegrated into powder. The stress-strain curve of one of these specimens is shown in Fig. 12.6 although this curve is taken from a preliminary series of tests in which strain measurements were regarded as being suspect. In one AE37 specimen, failure had only just occurred and the failure mode could be deduced from this failed specimen which still retained its integrity. The failure pattern is similar to the static pattern, consisting of a single inclined shear band traversing the whole specimen as shown in Fig. 12.4. The dynamic shear band is much thinner than the static band :- 50 μ as opposed to 2 mm.

A typical photograph of the C.R.O. screen is shown in

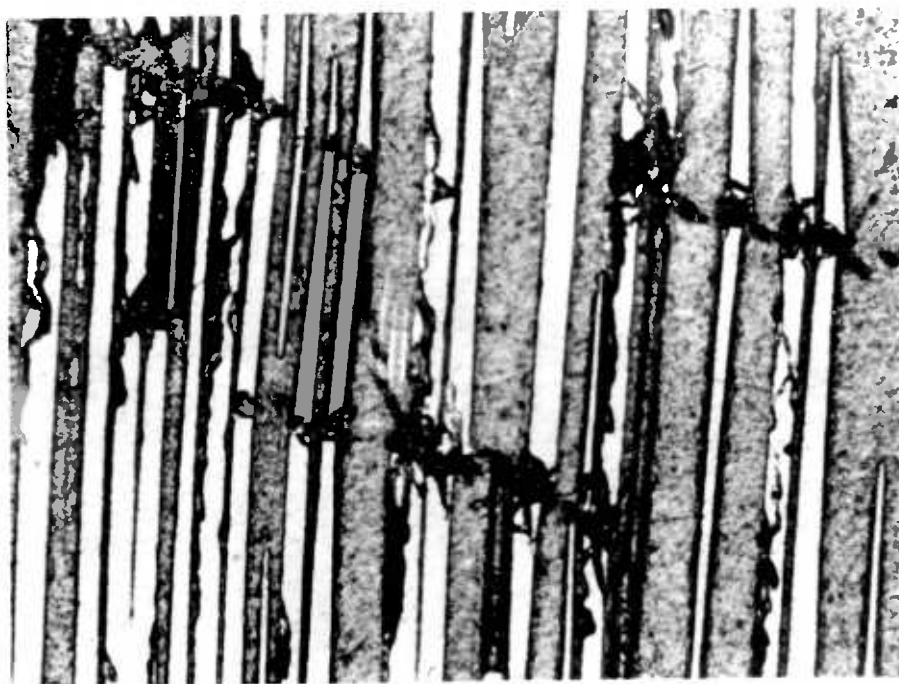


Transmitted light (x 140)

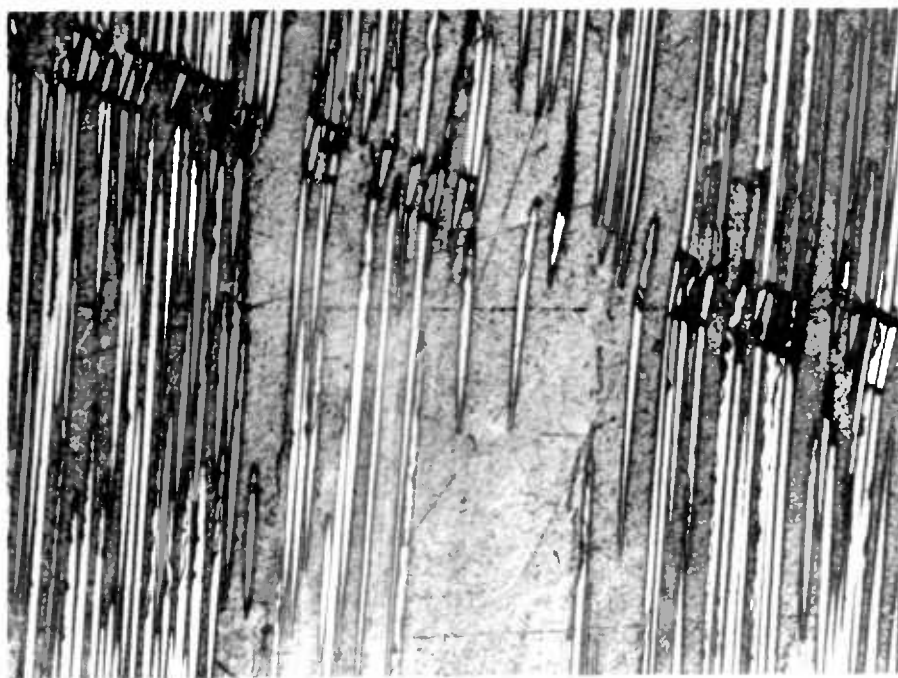


(x 60)

Fig. 12.3 Cracks in AE 57 specimen due to high strain loading



(x 140)

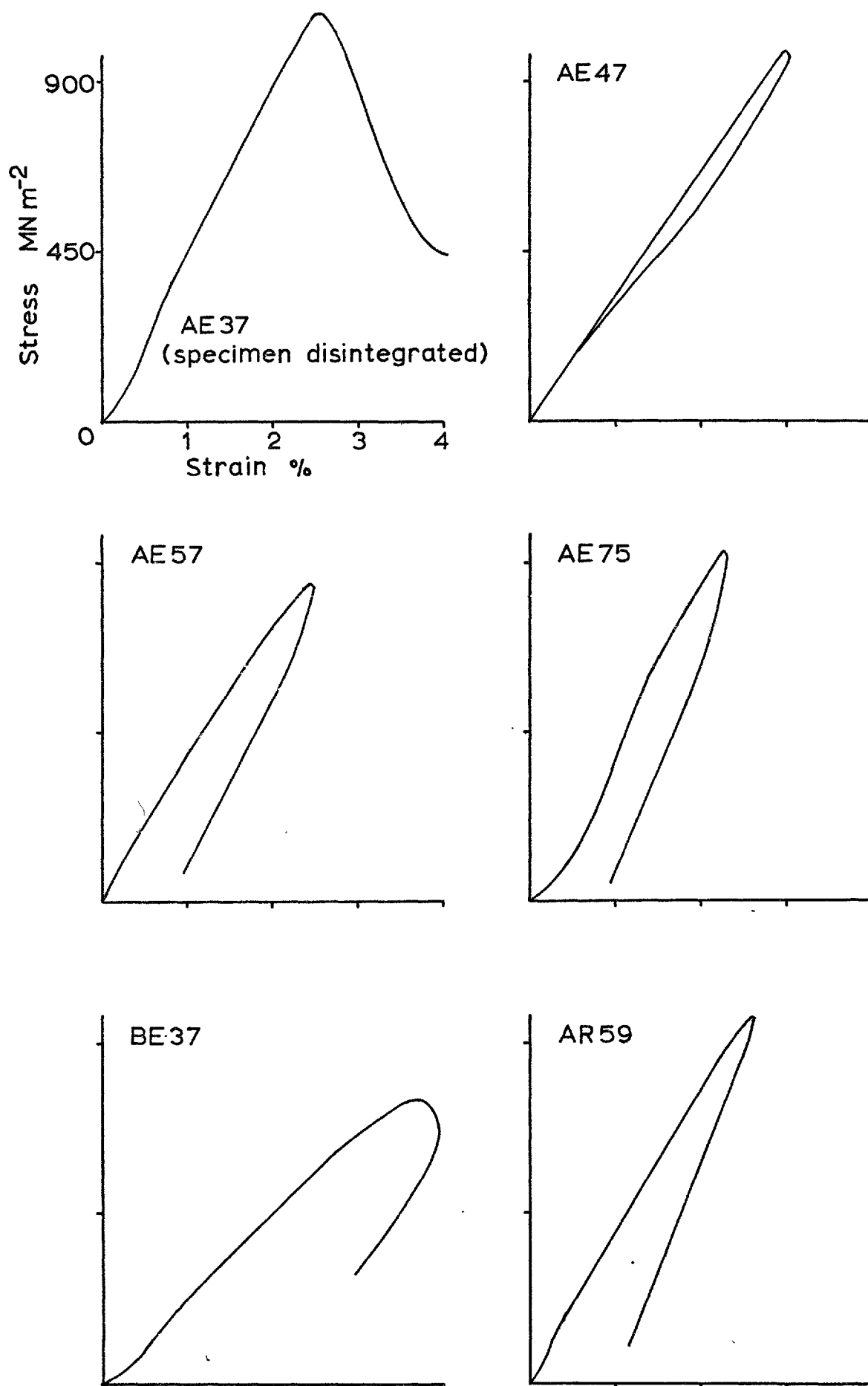


(x 60)

Fig. 12.4 Dynamic failure mode in AE 37 specimen

Fig. 12.6

Dynamic stress-strain curves of fibreglass



Where is it?

Fig. 12.5 which recorded the transmitted pulse on the upper beam and the incident and reflected pulses on the lower beam. The required curves were transposed onto graph paper by moving a calibrated microscope stage, on which the photograph rested, so that the cross-wires of the eyepiece coincided with the curve. This could be done with an error of less than ± 0.1 mm.

The stress-strain curves were calculated using equations (113) and (119). The value of k for most specimens was about $0.05 \mu\text{secs.}$ and represented a small shift of the recorded pulses on the time scale. The inertial correction terms in equation (112) were always small enough for them to be ignored. Typical stress-strain curves are shown in Fig. 12.6. The area within each hysteresis loop is proportional to the energy absorbed by the specimen. However, the size of the loop varied from test to test even for similar specimens and no correlation between the cracks' area, or any other known parameter, and energy absorption could be found. The dynamic Young's moduli of all specimens, defined as the maximum modulus over the loading curve, were the same as their static values within the accuracy of the tests. All the curves in Fig. 12.6 were obtained using (c), (d) or (e) detonators. Similar shapes were obtained with (a) and (b) detonators except that the loading curve terminated at a lower stress.

12.4.2. FIBRECARBON SPECIMENS

The incident stress wave was large enough to cause failure in all the fibre carbon specimens except type AC_{III}75. The maximum recorded stresses exceeded their static strengths by up to 20 % although the failure was most probably initiated at a lower stress when a rapid loss of modulus commenced on the stress-strain curve. In this case the dynamic strengths of all the fibre carbon specimens tested were approximately the same as their static strengths. The dynamic Young's modulus, defined as the maximum modulus on the stress-strain curve, was the same as its static value in every case to within the accuracy of the tests. The accuracy of the strain measurements, however, was much less than that for fibreglass specimens due to the much reduced deformation required for failure. In this case it would

be more convenient and accurate to record the reflected and transmitted pulses. This was performed by placing the two gauges equidistant from the specimen and further apart from each other as carried out by BILLINGTON and BRISSENDEN (1971), but the experiment proved to be unsuccessful due to the mechanical noise, left in the first bar by the incident pulse, which interfered with the recorded reflected pulse.

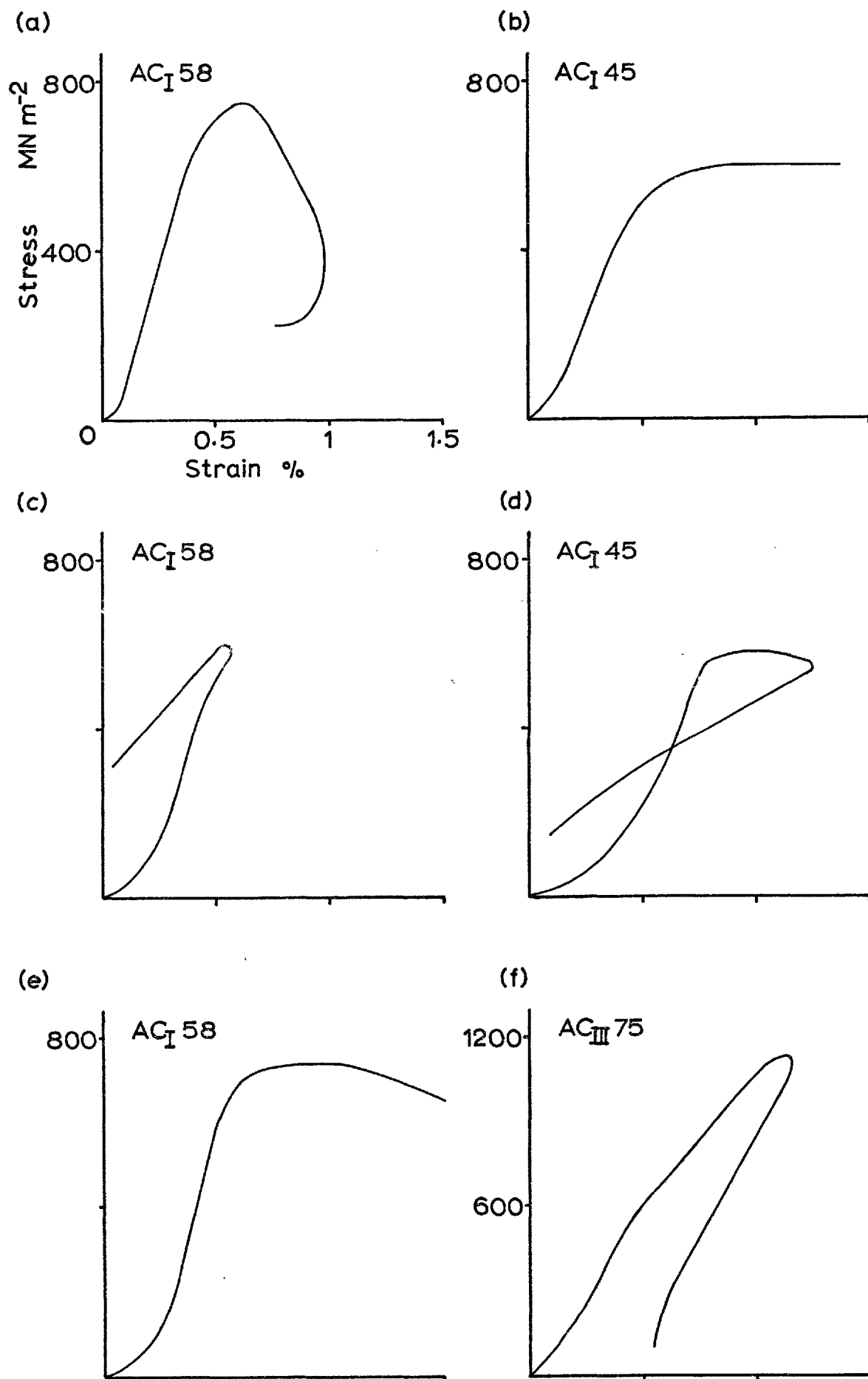
GRIFFITH and MARTIN (1974) used a split Hopkinson bar to axially compress fibre-carbon specimens with various fibre contents to within their elastic limits. They found that two linear regions with initial and secondary moduli respectively could be distinguished from the loading portion of each of the stress-strain curves. The initial modulus increased linearly with increasing fibre content, as in the static case, but with values typically 50 % higher. The secondary modulus was always much less than the initial modulus and independent of the fibre content.

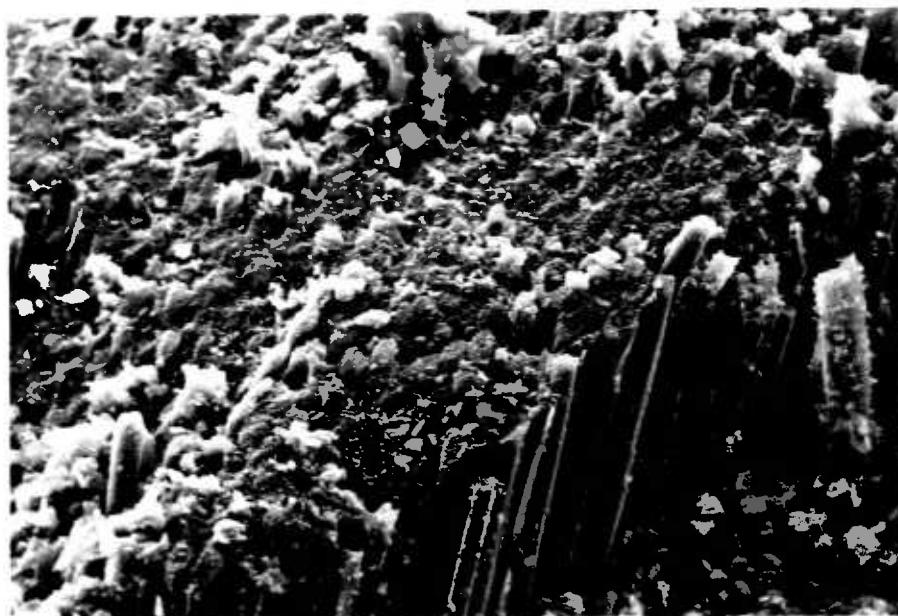
Typical stress-strain curves are shown in Fig. 12.7. Curves (c) and (d) were obtained by using type (b) pulses. The other curves were obtained with one of the larger pulses. Curves (c) and (d) have pronounced negative hysteresis loops which cannot be regarded as a physical reality otherwise the specimen must have emitted mechanical energy during its deformation. The cause of this phenomenon is not known but it was reproducible with type (b) detonator pulses and not observed with type (c), (d) or (e) pulses.

The specimens which gave curves (b) and (e) disintegrated into powder but the specimen (10 mm in length) which gave curve (a) broke up into several large fragments which showed the same distinctive failure pattern observed with static testing. The only difference was that several shear planes existed independently of each other and were interconnected by longitudinal splitting. The fracture surface of this specimen viewed with a scanning electron microscope is shown in Fig. 12.8. This surface is similar in appearance to those shown by EWINS and HAM (1973) for fibre-carbon which failed in static testing.

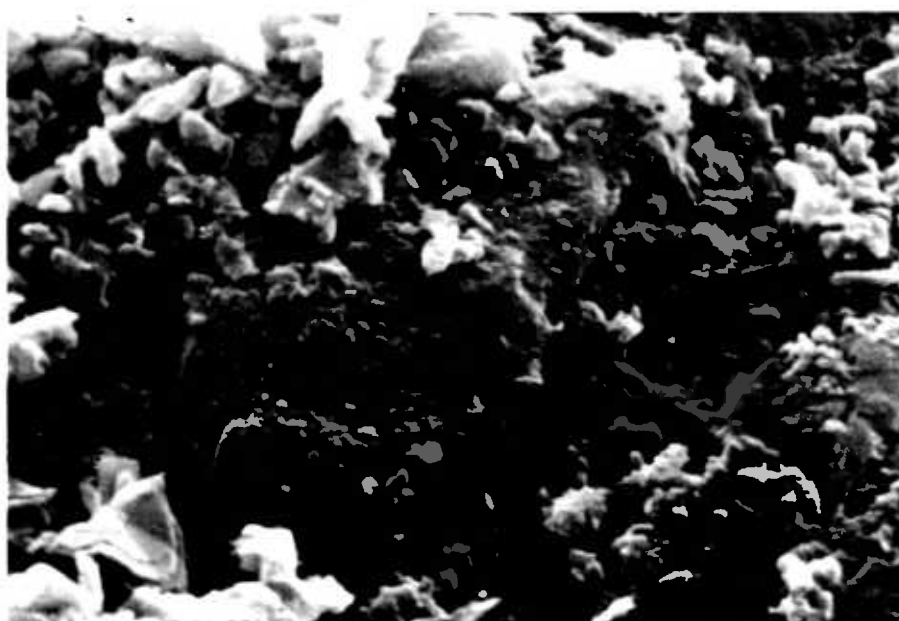
The specimen which gave curve (c) (Fig. 12.7) retained all of its static strength when subsequently tested while the specimen which gave curve (d) lost 20 % of its static strength.

Fig. 12.7 Dynamic stress-strain curves of fibre carbon





(x 400)



(x 3000)

Fig. 12.8 Dynamic fracture surface of AC_I60 specimen

It appears therefore that the shear failure was progressive and was taking place on the horizontal portion of curve (d), during which time at least 20 % of the fibres had sheared. The only visible signs of damage were a few faint cracks. Although the maximum stress on curve (f) never reached the static strength of the specimen, a few small pieces of the specimen were chipped off indicating the damaging nature of high strain rate loading.

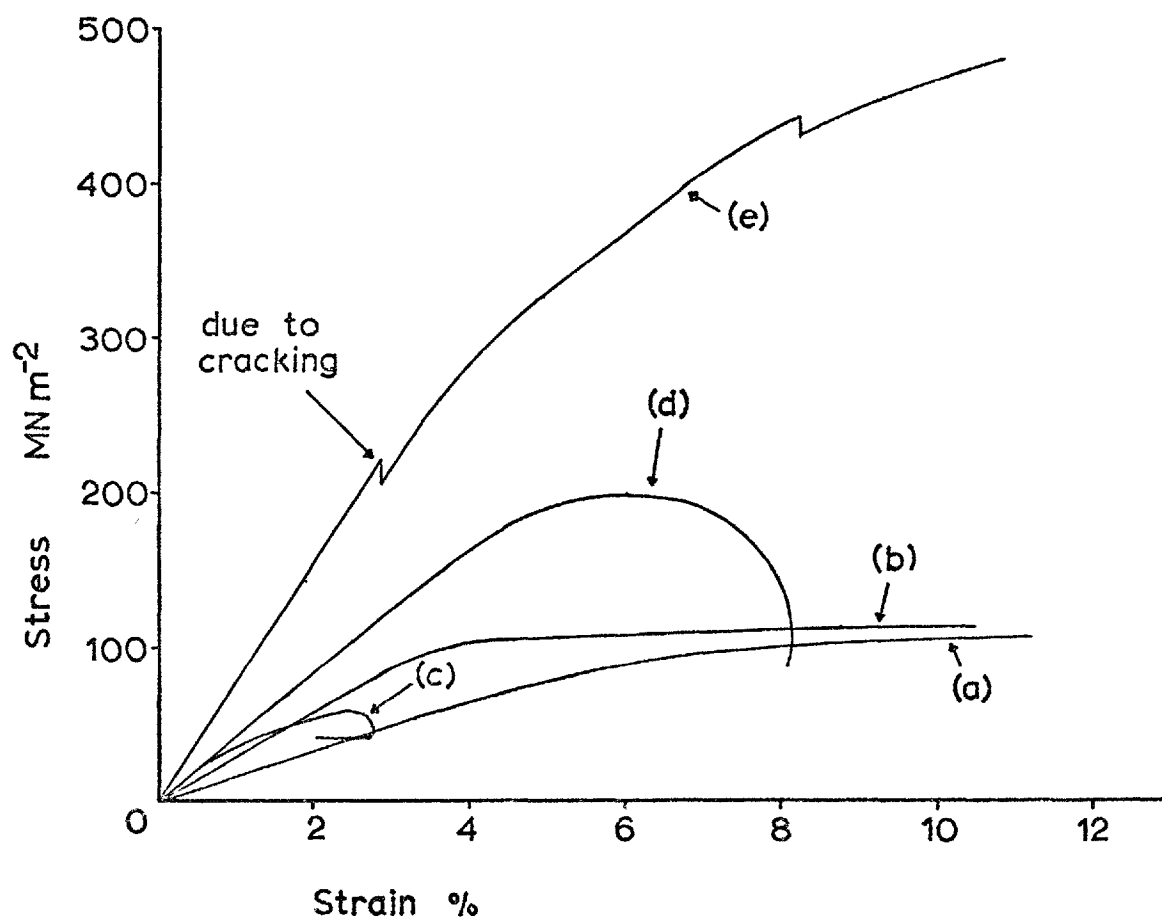
12.4.3. RESIN SPECIMENS

The dynamic stress-strain curves of resin A obtained with type (a) and (c) pulses, together with other data, are shown in Fig. 12.9. The shape of the dynamic curves are typical of a viscoelastic material as demonstrated by KOLSKY (1949). The specimens subjected to type (c) pulses shattered completely into fragments, while the specimens subjected to type (a) pulses remained intact.

In an ideal viscoelastic material, identical properties can be obtained by decreasing the temperature and the strain rate. For this reason, specimens were tested statically at a temperature of -196°C (curve (e) of Fig. 12.9) to see whether the resin's high strain rate properties could be reproduced. Extensive cracking occurred in the specimen prior to resin yield indicating that cracking is due to the brittle nature of the resin at high strain rates or low temperatures rather than the testing techniques. The initial dynamic Young's modulus (4.2 GN/m^2) represents an increase of 150 % over the static value.

Resin B gave similar dynamic stress-strain curves with about the same modulus. However, type (c) pulses caused the resin to break into only two or three pieces with the pattern of the cracks similar to those found in fibreglass specimens. No permanent deformation occurred in the specimen. It therefore appears that the mechanical properties of resin B are less sensitive to strain rate. This is supported by the fact that no change in modulus occurred when resin B was tested at the two different static strain rates which were used to obtain curves (a) and (b) in Fig. 12.9.

Fig. 12.9 Stress-strain curves of resin A



<u>Curve</u>	<u>Strain rate</u> <u>(sec^{-1})</u>	<u>Temperature</u>	<u>Initial Young's</u> <u>modulus (GN/m^2)</u>	<u>Pulse</u>
(a)	0.14×10^{-3}	r.t.	1.64	-
(b)	30×10^{-3}	r.t.	2.92	-
(c)	5×10^3	r.t.	4.2	(a)
(d)	5×10^3	r.t.	4.2	(c)
(e)	static	-196°C	7.8	-

12.4.4. WAVE PROPAGATION

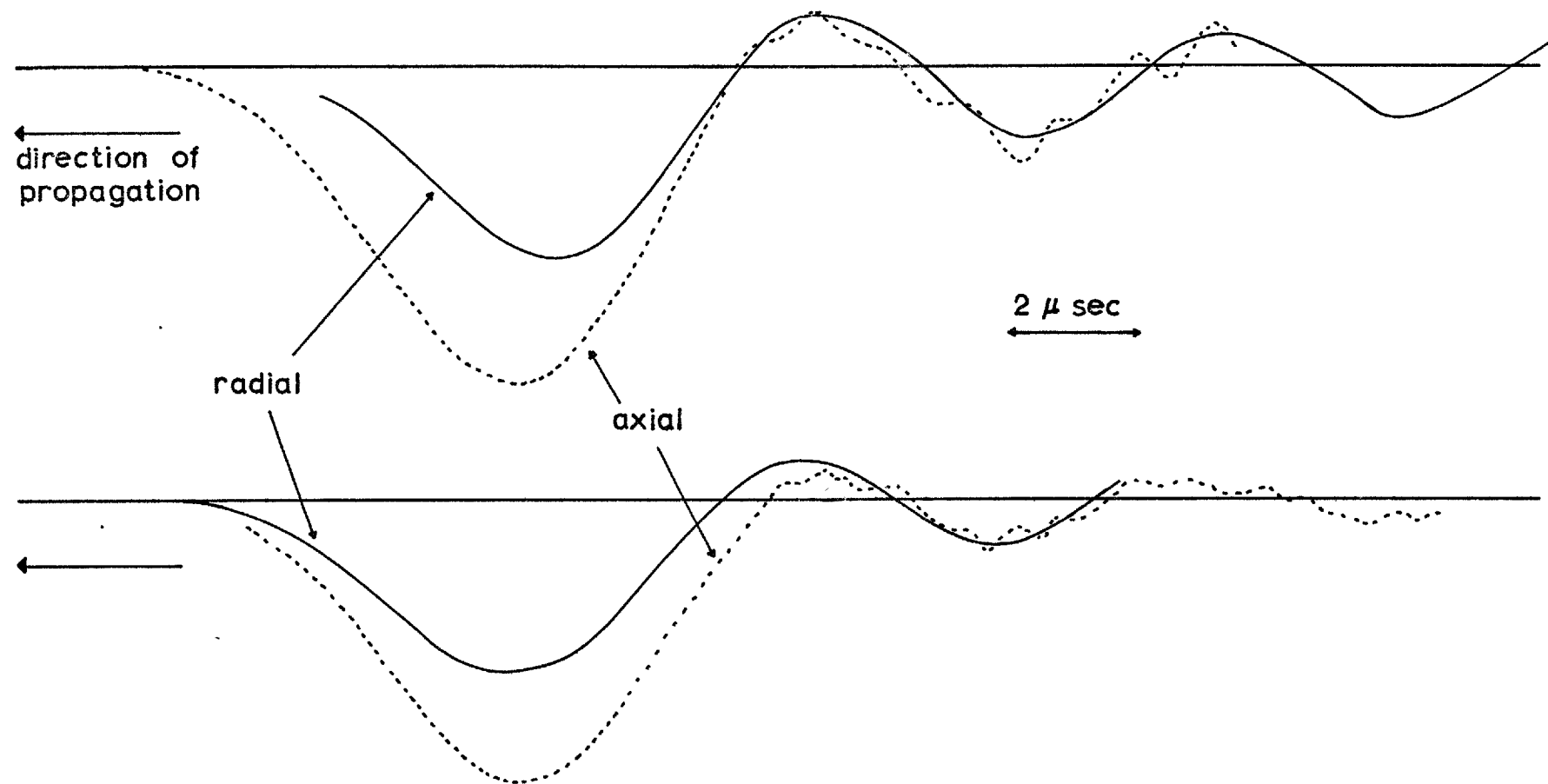
Type (a) pulses were propagated along an AE57 rod, 18 cm long, and were monitored with two resistance strain gauges glued in the axial and hoop directions at the centre of the rod. The fibreglass rod was secured in two of the V blocks of the split Hopkinson bar with one end exposed and the other end touching the anvil. Both ends were polished flat, normal to the axis of the rod using the techniques of section 3.2.2. Type (a) detonators produced pulses in the rod with an axial strain amplitude of $\frac{1}{2}$ %. The wave velocity and pulse amplitude attenuation were determined by monitoring multiple internal reflections from the ends of the rod using the C.R.O. which was triggered internally in the auto trigger mode. The attenuation was determined as 0.23 %/cm which is four times the attenuation in the steel pressure bars. The wave velocity was determined as 4.56 mm/ μ sec. This corresponds to a Young's modulus of 41 GN/m², by using equation (90), which is the same as its expected static value within the limits of experimental error. The same conclusion was drawn by ISRAELI and LIFSHITZ (1974) who measured the axial wave velocity of stress pulses in unidirectional fibreglass rods with various fibre contents.

Fig. 12.10 shows the hoop or radial, and axial strains, superimposed together, of two compression pulses which traversed the rod in opposite directions. The larger pulse of each pair represents the axial strain although the two strains of each pulse are not to the same scale. Poisson's ratio was determined to equal 0.28 from the ratios of the amplitudes of both pulses. This value is the same as the expected static value determined by using the manufacturers' values of ν for the resin and glass and assuming a mixture rule.

The radial and axial strains in the tail of the pulse appear to differ considerably, which is most probably a consequence of the three-dimensional nature of high frequency waves. (See for instance DAVIES (1947) and KOLSKY (1963)). The same behaviour was observed in the steel pressure bars where the radial and axial strains were monitored with condenser and resistance gauges respectively. In one of the pulses depicted in Fig. 12.10, the radial strain appears to lag behind the axial strain. The cause of this phenomenon is not known.

Fig. 12.10

Strain-time profiles of type (a) pulses in fibreglass rod



12.5. DISCUSSION OF RESULTS

12.5.1. STRENGTH OF FIBREGLASS

In order to predict the dynamic strength, the exact cause of static failure must be known and hence adapted to take into account dynamic effects. This cannot be done because a complete static strength theory was not obtainable. However, in order to give some insight into the mechanisms of dynamic failure, it is possible to adapt the static model of a buckling fibre supported on an elastic foundation. The static equation of equilibrium was given by TIMOSHENKO (1961) as :-

$$E_f I \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} + q(x) = 0 \quad (149)$$

where q is the reaction on the fibre due to the supporting matrix, I is the moment of inertia of the cross-section of the fibre about its neutral axis, P is its axial load and x and y are the axial and transverse coordinates respectively. The solution of equation (149) is given by :-

$$y = a \sin(n\pi x/L) \quad (150)$$

for the case of an elastic foundation where L is the length of the fibre, n is a positive integer and a is the amplitude of the buckling fibre. The reaction can be expressed in the form :-

$$q = q_0 \sin(n\pi x/L) \quad (151)$$

For the case of cylindrical fibres in a square array,

$$I = \frac{\pi}{64} d^4 \quad (152) \quad \text{and} \quad P = \frac{\pi}{4} \frac{d^2}{V_f} \sigma \quad (153)$$

where d is the fibre diameter and σ is the axially applied composite stress. q_0 is given by :-

$$q_0 = \frac{\pi^3}{4} \frac{d^2}{L^2} \frac{1}{V_f} a n^2 G_c \quad (154)$$

and can be derived using the energy methods described in chapter 9. Substituting equations (150) to (154) into equation (149) finally gives the micro-analysis strength of the composite given by equation (32). In the case of time dependent (dynamic) applications, equation (149) becomes :-

$$E_f I \frac{\partial^4 y}{\partial x^4} + P \frac{\partial^2 y}{\partial x^2} + q(x) + \rho \frac{d^2 y}{dx^2} = 0 \quad (155)$$

where the additional term represents the inertial forces due to the laterally accelerating fibre. ρ is the mass per unit length of the fibre and includes the surrounding matrix. The solution of equation (155) is given by :-

$$y = a \sin(n\pi x/L) T(t) \quad (156)$$

where $T(t)$ is a time dependent function. Substituting equations (151) to (154) and (156) into equation (155) gives :-

$$\frac{d^2 T}{dt^2} - \Omega^2 T(t) = 0 \quad (157)$$

$$\text{where } \Omega^2 = \frac{\pi^3 d^2}{4L^2} \left(\frac{n^2}{V_f} (\sigma - G_c) - E_f \frac{\pi^2 d^2 n^4}{16L^2} \right) \quad (158)$$

If σ is less than the theoretical static strength, Ω^2 is negative and the solution of equation (157) allows oscillatory motion of the fibre about its mean position. The composite therefore remains intact as predicted in the static case. If σ exceeds the theoretical static strength, Ω^2 is positive and the general solution of equation (157) is given by :-

$$T(t) = A e^{\Omega t} + B e^{-\Omega t} \quad (159)$$

where A and B are constants dependent upon the boundary conditions. If both the boundary conditions ($y = 0$, $t = 0$) and ($dy/dt = 0$, $t = 0$) are applied, only a trivial solution $T = 0$ exists. This is a consequence of the fact that the straight fibre is always in equilibrium, whether stable or unstable, and hence that there are no transverse forces acting on the fibre to initiate buckling. If the boundary condition that the fibre is

stationary when $t = 0$ is applied, the normalized solution is given by :-

$$T = \frac{1}{2}(e^{-\Omega t} + e^{\Omega t}) \quad (160)$$

where an initial transverse displacement exists at the time $t = 0$. The first term rapidly diminishes and becomes insignificant; hence the buckling amplitude increases exponentially with respect to time where :-

$$T = \frac{1}{2}e^{\Omega t} \quad (161)$$

can be maximized with respect to n in equation (158) to give the expected buckling wavelength because the fibre is expected to buckle at the fastest possible rate. This analysis gives :-

$$(L/n)^2 = \frac{\pi^2}{8} E_f V_f \frac{d^2}{(\sigma - G_c)} \quad (162)$$

where (L/n) is half of the buckling wavelength and G_c is the dynamic shear modulus of the composite. As expected, this equation only has real values when σ exceeds G_c . However, fibreglass is known to fail statically at stresses much less than G_c and therefore G_c is substituted by the static disc specimen strength.

The predicted buckling wavelength becomes smaller as the stress exceeds the static strength. This was confirmed experimentally by noting the thinner shear band obtained with high strain rates. (Figs. 12.4 and 8.1). If the thickness of the shear band is equivalent to half the buckling wavelength, then (L/n) measured experimentally lies in the range :-

$$10 d > L/n > 6 d$$

which is equivalent to an applied stress exceeding the static strength by σ^* where :-

$$360 \text{ MN/m}^2 < \sigma^* < 1000 \text{ MN/m}^2$$

In the failed AE37 specimens the dynamic strength was about

twice the static strength and :-

$$\sigma^* \sim 500 \text{ MN/m}^2$$

Substituting equation (162) into equation (158) gives :-

$$\Omega^2 = \frac{\pi}{\rho} \frac{(\sigma - G_c)^2}{E_f v_f^2} \quad (163)$$

Therefore ($\Omega = 2 \times 10^7 \text{ sec}^{-1}$) in this particular case and by using equation (161), it can be seen that the amplitude of the buckling fibre increases by a factor of (5×10^8) in one microsecond. The results show that the AE37 specimens appear to sustain a σ^* value of about 500 MN/m^2 for one microsecond. Therefore, by estimating a failure amplitude of two microns, the initial amplitude at time ($t = 0$) can be calculated at $4 \times 10^{-15} \text{ m}$.

The condition that an initial displacement, however small, must exist in order to produce failure might seem to invalidate the analysis. However, reducing the initial displacement to less than $4 \times 10^{-15} \text{ m}$ causes an increase in the uncertainty of the fibre's transverse velocity through the Heisenberg uncertainty principle. Such an initial transverse velocity can cause failure in one microsecond even if there is no initial displacement.

Of particular interest is the existence of only one shear band in the failed specimen and also the fact that it touches both ends of the specimen. This suggests, in an even more pronounced manner than in the static case, that the buckling pattern established itself in the specimen prior to failure initiation. The failure mode appears to be independent of the strain rate.

12.5.2. STRENGTH OF FIBRECARBON

The evidence given in section 12.4.2. shows that the dynamic failure mode is the same as the static mode and gives rise to similar strengths. It can therefore be concluded that the shear strength of carbon fibres is independent of the strain rate.

CHAPTER THIRTEEN

SUMMARY

The essential theme of this research programme was to ascertain the parameters which affect the axial compressive strength of unidirectional fibre-reinforced plastics and how they do so. In addition, attempts were made to discover why these parameters affect the U.C.S. in the way they do. A better understanding of failure mechanisms is necessary for improving the strength and hence the acceptance of fibre-reinforced plastics as a structural material.

Although the name "fibre-reinforced plastics" has been used in the title of this work, fibreglass is the composite most extensively used in this investigation. The generality of the name "fibre-reinforced plastics" allows other composites with some similarities to fibreglass to be included because their strength is controlled by the same failure mechanism. In addition the use of other composites allows a wider range of parameter variations to be used; for instance, the use of carbon fibres instead of glass gives a five fold increase in fibre modulus while the variation of fibre modulus in different types of glass fibres is much more limited. Unfortunately the strength of carbon fibre composites is controlled by a shear failure in the fibre itself which is not characteristic of the fibreglass failure mechanism but this still allowed the carbon fibre results to be used comparatively except that the strengths recorded represented "minimum" strengths. If the carbon fibre had not failed first, the composite would have failed at a higher strength than the one recorded, in a manner similar to that observed in fibreglass.

The initial work consisted of manufacturing suitable composites under controlled conditions so that a high degree of uniformity, quality and reproducibility could be achieved. Moreover the required parameters could be varied between different composites in a controlled manner.

The relevant parameters were well defined and measured as accurately as possible. These fell into two categories; physical properties such as fibre content, void content, densities and alignment; and mechanical properties such as shear strength, compressive strength and moduli of the composite and constituents.

An elementary account is given in chapter five of the relationship between the measured resin densities of the composites and the stress-strain fields of the resin matrix within the composites.

The interlaminar shear strength of the composite was measured using both the three-point-bend and disc-punch methods. Both of these methods gave the same strengths for specimens cut from the same composite. The three-point-bend method suffered from the disadvantage that the ratio of axial composite tensile strength to shear strength must exceed a minimum value otherwise only a minimum value of shear strength can be recorded; hence the disc-punch method was preferred. In addition the disc-punch specimens were more easily cut from cylindrical rods than three-point-bend specimens although in the case of flat plate laminates, the reverse is true.

The generally-accepted compressive strength theory regards the failure mode as a buckling phenomenon. This theory gives the strength as :-

$$\sigma_c = \frac{G_m}{1 - V_f} \quad (33) \text{ and } (47)$$

However, all measured strengths fell far short of this prediction and the use of a different resin matrix with a 75 % increase in shear modulus produced no change in strength. It therefore appears that this equation represents an upper limit and the strength is controlled by another failure mechanism. This conclusion can be drawn from section 9.3. where the basic theory adapted for off-axis loading gives excellent agreement with experimental results for composites loaded at 4° or more to the fibre axis.

The failure pattern did agree with the theory and showed a distinct buckling mode with a buckling half wavelength of about 2 mm and was characteristic of both disc and tapered specimens. By reducing the length of disc specimens to less than 1.5 mm the composite could only buckle with a maximum half wavelength equal to its length and this resulted in a strength increase with decreasing length as expected for an Euler column.

The strengths of disc specimens were only about a half that of tapered specimens cut from the same rod. Experiments showed that this loss of strength was not due to "poor" transfer of load

at the disc ends but showed similar characteristics to the Euler column which is pivoted instead of laterally secured at its ends.

A linear correlation between shear strength and compressive strength is apparent from Fig. 9.9 and it appears that a minimum shear strength is required to enable the compressive strength to reach the theoretical strength given by equation (33). The shear strength is a measure of the ability of the composite to resist damage due to internal stresses. A possible cause of failure and therefore a subsequent reason for this dependence is the progressive breaking up of a composite due to internal transverse stresses induced by the axial compressive stress; such stresses are on a micro- and macro-scale. The micro-scale consists of the radial stresses which bond a fibre to its surrounding matrix. It is possible, due to the mismatch of Poisson's ratios of the fibre and matrix, for tensile stresses to occur at this interface which reach the bond strength. The macro-scale consists of transverse stresses due to the misorientation between fibres or in particular between rovings. However, the experimental evidence discussed in conjunction with theoretical models suggests that this proposed failure mode does not apply to the composites used in this investigation.

A possible reason for the compressive-shear strength dependence is discussed in section 9.6. It is suggested that a stable form of buckling exists where stable displacements occur before catastrophic failure. This mode allows the possibility of the critical shear strain being reached before a stability failure is reached in a similar manner to the model of Fig. 9.5.

The effect of fibre modulus and diameter on the compressive strength is discussed in sections 9.8. and 9.9. respectively. The experimental evidence suggests that increasing either of these parameters increases the compressive strength. More specifically, that minimum shear strength is reduced which is required for the compressive strength to reach the theoretical limit of equation (33).

Disc specimens were also axially compressed at strain rates approaching 10^4 sec^{-1} using a split Hopkinson bar apparatus. The strength of fibreglass increased by a factor of two under dynamic loading while the strength of fibre carbon remained the same. This showed that the shear strength of the carbon fibre (which controls the compressive strength of fibre carbon) is

independent of strain rate. In the case of fibreglass, strain rate effects can be taken into account by adding the inertial force term into the equation of static equilibrium, (149), of a buckling column. Solving the resulting equation predicts that the composite will buckle at a higher load with a decreasing wavelength as the strain rate increases. This was confirmed experimentally by noting that the thickness of the shear band due to dynamic failure was approximately 80μ (see Fig. 12.4) as compared to 2 mm due to static failure (see Fig. 8.1).

In addition, internal damage in the form of cracks confined to the matrix phase, was observed in disc specimens dynamically loaded but which had not failed. No loss in strength was observed when subsequently tested statically. This cracking was attributed to the brittle nature of the resin matrix at high strain rates and although it did not cause a loss in strength, this must be regarded as undesirable as such cracks leave the composite more susceptible to environmental attack.

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