

SOME ASPECTS OF NEUTRINO INTERACTIONS

BY

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ABSTRACT

The work contained in this thesis can be divided into two essentially distinct parts which deal with spins and polarizations for particles of arbitrary spin and with the neutrino induced reactions.

Following an introduction giving a brief outline of the current situation and aims, chapter two contains a discussion of a covariant formulation for the description of polarization and the construction of specific spin and polarization states for arbitrary spin particles through a projection operator technique which results in a covariant spin density matrix. This is required for spin three-half particles in subsequent chapters.

The general structure of the weak interaction vertex is developed on a current - current theory foundation in chapter three for the elastic and inelastic neutrino induced reactions, with emphasis on the best choice of form factors required to simplify the somewhat lengthy expressions of the following two chapters. These deal with the most general forms of the cross-sections and polarizations, which are examined with a view to suggesting possible tests of time reversal invariance. For the inelastic process the test is discussed from the knowledge of the form of the most general spin density matrix as developed in chapter two.

Some unification of the elastic and inelastic processes through the application of $\tilde{U}(12)$ symmetry is used in chapter six to obtain numerical estimates, and results for the differential and total cross-

-sections are given with reference to the nucleon processes. Finally the subsequent decay of the final hadrons is considered briefly to see what tests of time reversal invariance are indicated by the decay distributions.

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PREFACE

The work described in this thesis was developed in the Department of Physics, Imperial College of Science and Technology, University of London between October 1962 and September 1965 under the supervision of Professor P.T. Matthews.

Except where otherwise indicated the material contained in this thesis is original and has not been previously presented for a degree in this or any other university.

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1. INTRODUCTION

In recent years high energy neutrinos have played a great role in the development of the theory of weak interactions, and it is now becoming more apparent that they can also serve as a useful probe of the strong interactions. Much information about the weak interactions has been obtained from experiments at CERN⁽¹⁻³⁾ and Brookhaven⁽⁴⁻⁶⁾ since Schwartz⁽⁷⁾ and Pontecorvo⁽⁸⁾ independently proposed using neutrinos from the decay of pions as a probe. It is now known that there are two different neutrinos (ν_μ and ν_e)^(2,4) associated with the two leptons μ and e , and that there are two separate conservation laws for these two types of lepton. No neutral lepton currents are yet apparent, suggesting their complete absence, and there does not appear to be an intermediate vector boson of mass less than 2 Gev.⁽⁹⁾ The present CERN anti-neutrino experiment⁽¹⁰⁾ should provide additional information about the weak interactions.

The main concern of this report is the study of the most general amplitudes (in a current-current theory) for reactions of the type

$$\nu_e(\bar{\nu}_e) + B_i \Rightarrow B_f + l \quad 1.1$$

and

$$\nu_e(\bar{\nu}_e) + B_i \Rightarrow B_f^* + l, \quad 1.2$$

(that is where an incident neutrino (ν_e) or anti-neutrino ($\bar{\nu}_e$) associated with a particular lepton (l) interacts with a target baryon (B_i) to produce a final baryon (B_f) or baryon resonance (B_f^*) and the lepton (l) corresponding to the neutrino or anti-neutrino), and in particular

those reactions where the target is a nucleon, and also the part that neutrinos can (and may be able to) play in the study of the strong and weak interaction processes. It is reasonably well established that there is a point interaction between leptons⁽¹²⁾ and thus a current-current type theory⁽¹¹⁾ (one which essentially separates the weak part (leptonic) of the interaction from the hadronic (strong) part) gains ground. Once this hadron part is isolated from the lepton part predictions and relations from the various strong interaction symmetry schemes can be incorporated in the hadron vertex, making it possible to connect and correlate the form factors occurring in the various allowed weak interaction processes.

On the basis of the $SU(3)^{(13,14)}$ symmetry scheme it is possible to inter-relate the coupling constants in the allowed processes of the type 1.1 (where B can be any one of the eight spin one-half baryons) and those in type 1.2 (with B^* one of the ten spin three-half resonances, but not to connect those of 1.1 and 1.2 . However under the larger group involved in the $SU(6)^{(15-17)}$ scheme or its relativistic extensions (for example $\tilde{U}(12)^{(18)}$) it is possible to relate the currents of 1.1 and 1.2 in addition to those within 1.1 and 1.2 . The two most favourable reactions for comparison from an experimental viewpoint are

$$\begin{array}{ll} \nu_e + n \Rightarrow p + l & 1.3 \\ \text{and} & \\ \nu_e + N \Rightarrow N^* + l, & 1.4 \end{array}$$

and it is these which provide the only relevant data to date⁽¹⁾. It is in this context that high-energy neutrinos can be used as a probe of the

strong interactions and make tests of the the recently proposed higher symmetry schemes (provided there is a current-current type interaction where the leptons and hadrons are at separate vertices). In this respect the neutrinos play much the same role as the electron in high-energy electron scattering experiments, but have the added advantage that they can give information on both vector and axial-vector type couplings since parity is not conserved^(19,20) in weak interactions.

Until the experiments of Christenson et al.⁽²¹⁾ it seemed that eventhough weak interactions were not invariant under space inversion, the operation of time reversal did hold. Many explanations and theories have been put forward⁽²²⁾ to explain this apparent violation of time reversal invariance (T) in the decay of the kaon. If T is violated in this weak decay then tests must be made to see if it is violated elsewhere in the realm of the weak interactions, and it is shown that neutrinos may be used to test the validity of this invariance principle through measurements on the polarization of the final particles in the reactions 1.1 and 1.2 . However it seems unlikely that such experiments would be able to indicate the manner in which any violation occurred.

In order to discuss these polarization properties of the final particles it is found convenient to introduce covariant spin and polarization operators following Fronsdal⁽²³⁾ and Michel⁽²⁴⁾ . It is of course only necessary to use those relating to spin one-half and spin three-half particles to discuss reactions 1.1 and 1.2 , but it seems worthwhile to discuss their construction for general spin and to indicate

the connection between a covariant density matrix formulation and the helicity description by using spin raising and lowering operators. This technique is developed in chapter two, and whilst it is not necessary now to use explicit forms for the wave functions in momentum space (operators being used throughout), it is a simple matter to construct specific wave functions to represent a particle with given spin component in any required direction.

The processes considered here can be conveniently divided into two groups, the "elastic" processes (two initial and two final particles) and the "inelastic" (to include those reactions where there is a baryon resonance present in the final state—here particles or resonances are used to describe those "particles" which are stable or unstable with respect to the strong interactions). Chapter three is devoted to the study of the structure of the lepton vertex and the elastic and inelastic hadron vertices, with discussions on the implications of a conserved vector current⁽²⁵⁾ (C. V. C.), a conserved iso-vector current (C. I. V. C.) hypothesis⁽²⁶⁾ and a partially conserved axial-vector current (P. C. A. C.) — a Goldberger-Treiman relation^(27,28). The form of the lepton vertex (point interaction) is well known, but for the hadron vertex several choices of the six independent elastic currents and the eight inelastic currents are available (in each case this is the total number of vector and axial-vector form factors required to describe the process, there being an equal number of each). For the elastic process 1.1, that form of the current which gives the minimum number of terms in the

product of form factors occurring in the expression for the transition probability for unpolarized particles is chosen⁽²⁹⁾ — in the manner familiar in the electro-magnetic interactions of nucleons^(30,31). This is aimed at simplifying experimental analysis. A similar aim is involved in the choice of the form factors for the inelastic processes.

Using these form factors, the most general results for the cross-sections and polarizations of the final particles are given for the elastic reactions^(29,32) (chapter four) and the inelastic ones⁽³³⁾ (chapter five). Possible tests of time reversal invariance are suggested— involving the transverse polarization of the lepton^(29,34) and baryon⁽²⁹⁾ (or baryon resonance⁽³⁵⁾). Due to the number of form factors, only qualitative results are indicated in these sections. All expressions for the elastic case are derived and given retaining all the six form factors, but because of the very lengthy expressions obtained for the baryon resonance polarization with the eight inelastic form factors, the test for CP invariance is indicated using the most general form for the density matrix and polarization vector for a spin three-half particle as provided by chapter two.

With the present interest in symmetry schemes for strong interactions, chapter six contains results derived on the basis of applying $\tilde{U}(12)$ to the hadron vertex⁽³⁶⁾. $\tilde{U}(12)$ groups the baryon octet and the baryon resonance decouplet in one multiplet, thus relating their strong interaction form factors, hence the hadron currents for 1.2 are expressible in terms of the form factors of 1.1. With the help of

the C. V. C. and P. C. A. C. the currents are expressed in terms of one vector and one axial-vector form factor plus one seemingly arbitrary parameter. This vector form factor is related to the electro-magnetic form factor of the nucleon by the C. I. V. C. hypothesis, and results are given with reference to 1.3 and 1.4 for various choices of the two form factors and the parameter still present.

Finally in chapter seven the subsequent weak decay (non-leptonic) of the final baryon— for example when it is a lambda particle— and the strong decay of the final baryon resonance are studied, again with the hope of providing a means of testing time reversal invariance in the neutrino interactions.

2. SPIN AND POLARIZATION

(a) The spin density matrix

This has been considered by many authors^(24,37-46), so only a brief introduction is given here.

The wave functions Φ associated with a particle will always describe a particle completely polarized in some direction. If this pure state is normalised to unity then the projection operator P_Φ onto this state has the properties:

$$\text{Tr}(P_\Phi) = 1,$$

$$(P_\Phi)^2 = P_\Phi$$

and

$$(P_\Phi)^\dagger = P_\Phi,$$

with the expectation value of any observable A given by:

$$\langle A \rangle = \langle \Phi | A | \Phi \rangle = \text{Tr}(A P_\Phi).$$

A beam of identical particles of the same momenta can in general be represented as a statistical mixture of pure states $|\Phi^\alpha\rangle$ (it is clear that a partially polarized beam cannot be represented by a single pure state). If each state is expanded in terms of normalized eigenstates $|\Phi_i\rangle$ of the spin operator in a specified direction (the polarization operator), then

$$|\Phi^\alpha\rangle = \sum_i c_i |\Phi_i\rangle,$$

and

$$\langle A \rangle = \text{Tr}(\rho A)$$

where

$$\rho = \sum_i \sum_j c_i^* c_j |\Phi_i\rangle \langle \Phi_j|. \quad 2.1$$

ρ is defined to be the spin density matrix of the beam, and by

observation has the properties⁽⁴⁰⁾:

(a) For a pure state ρ has the same form as the spin polarization operator (hence $\rho^n = \rho$ for each integer $n > 0$)

(b) ρ is hermitian

(c) $\langle \bar{q}_m | \rho | \bar{q}_n \rangle = q_n^* q_m$, so that if $|\bar{q}_m\rangle$ represents the state with spin $+m$ in some specified direction this leads to the usual representation of ρ in spin-space by the elements ρ_{nm} .

In the rest system of a particle the general structure of ρ is well known⁽²⁴⁾. For a particle of spin j there are $(2j + 1)$ independent states, hence under a rotation

$$\rho \rightarrow \rho' = D(J) \rho D(J)^*,$$

where $D(J)$ is the proper rotation matrix, which means that the elements of ρ transform like the components of tensors under $D(J) \otimes \bar{D}(J)$. Since $D(J) \sim \bar{D}(J)$, it is clear that ρ can be written as a sum of irreducible tensors of ranks $0, 1, \dots, 2j-1, 2j$ ($D(J) \otimes \bar{D}(J) \sim D(2j) \oplus D(2j-1) \oplus \dots \oplus D(1) \oplus D(0)$). Space inversion commutes with all rotations, so all the irreducible tensors are even under space reflection and

$$\rho = \text{Tr}(\rho) \mathbb{I} + R_i S_i + T_{ij} (S_i S_j + S_j S_i) + \dots, \quad 2.2$$

where S_i are the components of the spin vector, R_i a pseudo-vector and T_{ij} a rank two tensor.

The problem now is to find a covariant description of this density matrix for particles of any spin. This necessitates the construction of a spin four-vector and corresponding spin and polarization operators for a particle of spin j . The approach used here follows that of

Fronsdal⁽²³⁾ in that the starting point is the existence of a rank j (n) tensor to describe a particle of integer spin j (half-integer spin $n+\frac{1}{2}$), rather than that of Weaver, Hammer and Good⁽⁴⁷⁾ who use a Hamiltonian formulation, or that of Weinberg⁽⁴⁸⁾ who exhibits a manifestly covariant wave equation.

(b) Wave equations

The prescription adopted by Fronsdal⁽²³⁾ for the description of higher spin fields is used here. The general spin field is defined by the conditions

$$(p^2 - m^2)\phi_{(\mu)j} = 0$$

$$\phi_{\mu_1\mu_2(\mu)j-2} - \phi_{\mu_2\mu_1(\mu)j-2} = 0$$

$$p_{\mu_1}\phi_{\mu_1(\mu)j-1} = 0$$

$$\xi_{\mu_1\mu_2}\phi_{\mu_1\mu_2(\mu)j-2} = 0 \quad 2.3$$

for integer spin j (in the usual tensor representation⁽⁴⁹⁾), and by

$$(\not{p} - m)\psi_{(\mu)n} = 0$$

$$\psi_{\mu_1\mu_2(\mu)n-2} - \psi_{\mu_2\mu_1(\mu)n-2} = 0$$

$$p_{\mu_1}\psi_{\mu_1(\mu)n-1} = 0$$

$$\gamma_{\mu_1}\psi_{\mu_1(\mu)n-1} = 0 \quad 2.4$$

for half-integer spin j ($=n+\frac{1}{2}$) in the Rarita-Schwinger⁽⁵⁰⁾ formalism.

Here $(\mu)_j$ is used to signify $\mu_1\mu_2\mu_3\cdots\mu_{j-1}\mu_j$.

The fields $\phi_{(\mu)_J} (\psi_{(\mu)_n})$ do not only contain spin j ($n+\frac{1}{2}$) particles, hence the need for the subsidiary equations, but there is a convenient unique projection operator $\Phi_{(\mu)_J(\nu)_J}(p, j)$ ($\Psi_{(\mu)_n(\nu)_n}(p, j)$) which projects out the relevant part of the tensor $\phi_{(\mu)_J} (\psi_{(\mu)_n})$. Suitable forms⁽⁵¹⁾ for the spin projection operator are

$$\Phi_{(\mu)_J(\nu)_J}(p, j) = (j!)^{-2} \sum_{\mu} \sum_{\nu} \left(\sum_{r=0}^j \left\{ a_r \prod_{k=1}^r \Phi_{\mu_{2k-1}\mu_{2k}}(p) \prod_{\ell=1}^r \Phi_{\nu_{2\ell-1}\nu_{2\ell}}(p) \right. \right. \\ \left. \left. \times \prod_{m=r+1}^j \Phi_{\mu_m\nu_m}(p) \right\} \right) \quad 2.5$$

where

$$a_r = (-1)^r \frac{j!}{(2j)!} \frac{j!}{r!(j-r)!} \frac{(2j-2r)!}{(j-2r)! j!},$$

$$j' = \begin{cases} \frac{j}{2} & \text{if } j \text{ is even} \\ \frac{(j-1)}{2} & \text{if } j \text{ is odd,} \end{cases}$$

$$\Phi_{\mu\nu}(p) = \xi_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

and the first summations are carried out over all permutations of the sets of indices $(\mu)_J$ and $(\nu)_J$,

and

$$\Psi_{(\mu)_n(\nu)_n}(p, j) = \frac{(2j+1)}{4(j+1)} \gamma_{\mu_1} \Phi_{\mu_1}(\mu)_{n-1}(\nu)_{n-1} \gamma_{\nu_n} \quad 2.6$$

Now $\Phi_{(\mu)_J(\nu)_J}(p, j) \phi_{(\nu)_J}(p)$ and $\Psi_{(\mu)_n(\nu)_n}(p, j=n+\frac{1}{2}) \psi_{(\nu)_n}(p)$ can be

taken as the appropriate wave functions for integer and half-integer spin particles respectively. For half-integer spin it is still

necessary to use the positive energy projection operator $(\not{p} + m)$ in addition to Ψ . (Note that $(\not{p} + m)$ commutes with Ψ .)

(c) Polarization operators

In addition to spin projections it is necessary to be able to project out states which have specific "magnetic" quantum numbers. Whilst it is usual to choose the axis of quantization to be in the z-direction it seems advisable to retain an arbitrary direction for this axis in developing a covariant prescription. Such an operator will be termed a polarization (projection) operator.

The study of the Inhomogeneous Lorentz Group leads to the study of the Lie Algebra defined by

$$[P_\mu, P_\nu] = 0$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i[\xi_{\nu\rho} M_{\mu\sigma} - \xi_{\mu\rho} M_{\nu\sigma} - \xi_{\nu\sigma} M_{\mu\rho} + \xi_{\mu\sigma} M_{\nu\rho}]$$

$$[P_\lambda, M_{\mu\nu}] = i[\xi_{\mu\lambda} P_\nu - \xi_{\nu\lambda} P_\mu] ,$$

where P_μ and $M_{\mu\nu}$ are hermitian operators. From this set of operators it is required to find a set of commuting observables. It is well known that $P^2 (=P_\mu P_\mu)$ and $W^2 (=W_\mu W_\mu)$ where $W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} P_\nu M_{\rho\sigma}$ is the Pauli-Lubanski spin tensor) belong to this set, and that

$$[P_\mu, W_\nu] = 0$$

$$[W_\mu, W_\nu] = -i\epsilon_{\mu\nu\rho\sigma} P_\rho W_\sigma .$$

From this last expression it follows that there are three four-vectors

$n_{\mu}^{(1)}$, $n_{\mu}^{(2)}$, $n_{\mu}^{(3)}$ satisfying

$$n^{(1)}_{\mu} n^{(j)}_{\nu} = -\delta_{\mu\nu}$$

and $n^{(1)}_{\mu} p = 0$,

such that

$$[W_{\mu} n^{(1)}, W_{\nu} n^{(j)}] = i\epsilon_{\mu\nu k} W_k n^{(k)}, \quad 2.7$$

thus the algebra defined by the three generators $W_{\mu} n^{(1)}$ is isomorphic to the rotation group in three dimensions, and

$$\begin{aligned} W^2 &= -W_{\mu} n^{(1)} W_{\mu} n^{(1)} \\ &= -s(s+1)\hbar^2. \end{aligned} \quad 2.8$$

Furthermore, for any $n_{\mu} p = 0$, $W_{\mu} n$ has eigen-values m with $-m < j < m$, so that W_{μ} can be interpreted as the covariant spin tensor.

Letting $\phi(j)$ denote the wave function for a particle of spin j and $\phi(j, \xi)$ a spin j particle with component ξ in the direction s_{μ} , it follows that

$$\begin{aligned} W^2 \phi(j) &= -j(j+1)\hbar^2 \phi(j) \\ S^{(j)} \phi(j, \xi) &= \xi \phi(j, \xi), \end{aligned} \quad 2.9$$

where

$$mS^{(j)} = W^{(j)} \cdot s, \quad \text{with } s^2 = -1 \text{ and } s \cdot p = 0.$$

(d) Construction of spin vectors

Explicit forms for W_{α} can be found for any spin j , starting from those for $j = \frac{1}{2}$ and $j = 1$.

$$iW^{(\frac{1}{2})}_{\alpha} = \epsilon_{\alpha\beta\mu\nu} p_{\beta} \sigma_{\mu\nu} \quad 2.10$$

$$(W^{(1)})_{\alpha\mu\nu} = -i\epsilon_{\alpha\beta\mu\nu}p_\beta \quad 2.11$$

satisfy

$$W^{(\frac{1}{2})} \cdot W^{(\frac{1}{2})} = -\frac{3p^2}{4}$$

$$(W^{(1)} \cdot W^{(1)})_{\mu\nu} = -2p^2 \Phi_{\mu\nu}(p)$$

(the term $\Phi_{\mu\nu}(p)$ is just the spin projection operator for a spin one particle, so gives no difficulty), the correct forms for $j = \frac{1}{2}$ and $j = 1$.

For integer spin j , a spin tensor having the correct properties is

$$(W^{(j)})_{\alpha(\mu)j(\rho)j} = i \sum_{\ell=1}^j \left\{ (W^{(1)})_{\alpha\mu_1\nu_1} \prod_{\ell=1}^j \Phi_{\mu_\ell\nu_\ell}(p) \right\} \Phi_{(\nu)j(\rho)j}(p,j), \quad 2.12$$

and for half-integer spin

$$(W^{(j)})_{\alpha(\mu)n(\rho)n} = \left\{ W^{(\frac{1}{2})}_{\alpha\mu_1\nu_1} \prod_{\ell=1}^n \Phi_{\mu_\ell\nu_\ell}(p) + \sum_{\ell=1}^n \left\{ (W^{(1)})_{\alpha\mu_1\nu_1} \right. \right. \\ \left. \left. \times \prod_{\ell=1}^n \Phi_{\mu_\ell\nu_\ell}(p) \right\} \right\} \Psi_{(\nu)n(\rho)n}(p,j), \quad 2.13$$

has the required properties.

Written in this form 2.12 and 2.13 are cumbersome, but are not really difficult to work with since

$$[W^{(j)}_{\alpha}, \Phi(p,j)] = 0 \quad 2.14$$

$$\text{and} \quad [W^{(j=n+\frac{1}{2})}_{\alpha}, \Psi(p,j=n+\frac{1}{2})] = 0,$$

thus a projection operator could be written explicitly on the left of $W^{(j)}_{\alpha}$ to give a symmetric form, and one which clearly satisfies the subsidiary conditions for spin j . Such definitions (2.12 and 2.13) agree with the "conventional" non-relativistic forms (all terms with μ or ρ equal to zero go to zero in the rest frame of the particle).

(e) Construction of polarization projection operators

A projection operator can now be constructed⁽²³⁾ to project out states with a specific value ξ of the spin in a particular direction s_α .

Consider a particle of spin j . Construct $S^{(j)}$ (see 2.9). Since $\overline{\Gamma}_\xi(S^{(j)} - \xi) = 0$ with $-j < \xi < j$, it is clear that

$\overline{\Gamma}_{\xi_1}(S^{(j)} - \xi_1)$ with $\xi_1 \neq \xi$ will project out that part of the wave function with spin ξ in the direction s_α , and the normalised ($P^2 = P$) projection operator is

$$P(j, \xi) = \overline{\Gamma}_{\xi_1}(S^{(j)} - \xi_1) \left(\overline{\Gamma}_{\xi_1}(\xi_1 - \xi) \right)^{-1} \overline{\Phi}(j, j). \quad 2.15$$

Thus starting from a general tensor of rank j (n) it is possible to project out that part which describes a particle with spin j and s_α component ξ :-

$$\begin{aligned} \phi_{(\mu)j}(p, j, \xi) &= P(j, \xi)_{(\mu)j}(\nu)_j \phi_{(\nu)j} \\ \text{and} \quad \psi_{(\mu)n}(p, j, \xi) &= P(j, \xi)_{(\mu)n}(\nu)_n \psi_{(\nu)n} \end{aligned} \quad 2.16$$

In particular it is worth noting that

$$P(j, 0)_{(\mu)j}(\rho)_j = \frac{(2j)!}{2^j j! j!} \left(\overline{\Phi}_{(\mu)j}(\nu)_j(p, j) \prod_{l=1}^j s_{\nu_l} \right) \left(\prod_{l=1}^j s_{\rho_l} \right) \times \overline{\Phi}_{(\sigma)j}(\rho)_j(p, j)$$

$$\text{and} \quad 2.17$$

$$P(j, j)_{(\mu)j}(\rho)_j = (-1)^j \prod_{l=1}^j t_{\mu_l} \prod_{l=1}^j t_{\rho_l}^*$$

for integer spin j (t and t^* are orthogonal to s , see next section), and for half-integer spin j

$$P(j, \pm j)_{(\mu)n(\rho)n} = \frac{\pm 1}{2^n} \left(S^{(\pm)} \pm \frac{1}{2} \right) \prod_{i=1}^n \chi_{\mu_i \rho_i} \quad 2.18$$

where

$$\chi_{\mu\rho} = \gamma_\alpha (\bar{\Phi}_{\mu\beta}(P) + s_\mu s_\beta) (\bar{\Phi}_{\rho\alpha}(P) + s_\rho s_\alpha) \gamma_\beta \cdot$$

This last expression (2.18) contains products of γ at this stage, but it is found to be particularly easy to handle in this form (see next section) rather than reducing it to a form containing $\gamma\gamma$ and γ_5 terms only. The forms of $P(j, \xi)$ for $2j = 1, 2, 3$ and 4 are given in table 1.

(f) Spin raising and lowering operators

Whilst

$$\phi_{(\mu)j}(P, j, \xi) \phi_{(\rho)j}^\dagger(P, j, \xi) = P(j, \xi)_{(\mu)j(\rho)j}$$

and

$$\psi_{(\mu)n}(P, j, \xi) \bar{\psi}_{(\rho)n}(P, j, \xi) = P(j, \xi)_{(\mu)n(\rho)n} (j + m)$$

it is not at all clear what

$$\phi_{(\mu)j}(P, j, \xi) \phi_{(\rho)j}^\dagger(P, j, \eta) \text{ and } \psi_{(\mu)n}(P, j, \xi) \bar{\psi}_{(\rho)n}(P, j, \eta)$$

look like for ξ not equal to η (these terms occur in the spin density matrix). However there exist operators which will raise or lower the spin component in the direction s_α by one unit, a fact which can be seen quite easily by considering instead of the $W_n^{(1)}$ the combinations

$$W_n(n^{(1)} + i n^{(2)}) = i/2 S_+$$

$$W_0(n^{(1)} - in^{(2)}) = \sqrt{2}S_-$$

$$W_0 n^{(3)} = S_- \quad 2.19$$

These have the same commutation relations as the usual angular momentum operators⁽⁵²⁾:-

$$[S, W^2] = 0$$

$$[S, S_{\pm}] = \pm S_{\pm}$$

$$[S_+, S_-] = S_- \quad 2.20$$

Also by analogy with the angular momentum operators, if $|j, \xi\rangle$ are normalised states, then

$$\sqrt{2} \langle j, \xi | S_{\pm} | j, \xi \pm 1 \rangle = \sqrt{((j \pm \xi)(j \mp \xi + 1))},$$

giving

$$R = \frac{\sqrt{2}}{\sqrt{((j + \xi)(j - \xi + 1))}} S_+$$

and

$$L = \frac{\sqrt{2}}{\sqrt{((j - \xi)(j + \xi + 1))}} S_- \quad 2.21$$

as the raising and lowering operators which act on normalised states to give normalised states.

Setting $n^{(3)}_{\alpha} = s_{\alpha}$ and $n^{(1)}_{\alpha} + in^{(2)}_{\alpha} = \sqrt{2} t_{\alpha}$ (now $s^2 = -1$, $t^2 = 0$, $t \cdot t^* = -1$, $t \cdot s = 0$ and $p \cdot t = p \cdot s = 0$), it follows that

$$\phi_{(\mu)j}(p, j, \xi) = \bar{\phi}_{(\mu)j}(p, j, \xi) \prod_{l=1}^j t_{\nu_l}^* \phi_{(\nu)j} \quad 2.22$$

with

$$\begin{aligned} \bar{\phi}_{(\mu)j}(p, j, \xi) = & \sqrt{\left(\frac{(2j)! (j-\xi)!}{2^{j-\xi} j! j! (j+\xi)!} \right)} \frac{1}{j!} \sum_{(\mu)} \left\{ \sum_{r=0}^{(j-\xi)'} (-1)^{(j-\xi)} \frac{(j-2r)!}{(j-\xi-2r)!} \right. \\ & \times (-1)^r \sum_{k=1}^r \bar{\phi}_{\mu_{2k-1} \mu_{2k}}(p) \prod_{l=2}^{j-\xi} s_{\mu_l} \left. \sum_{m=j-\xi+1}^j t_{\mu_m} \right\} \end{aligned}$$

for $\xi \geq 0$, and

$$\bar{\psi}(p, j, -\xi) = (-1)^\xi \bar{\psi}^*(p, j, \xi) \quad .$$

The corresponding result for half-integer spin is (from 2.18 and 2.21)

$$\psi_{(\mu)_n}(p, j, \xi) = \psi_{(\mu)_n}(p, j, \xi) \prod_{k=1}^n t_k^* \psi_{\nu_k}(\nu)_n \quad 2.23$$

with

$$\begin{aligned} \sqrt{(2j)} \psi_{(\mu)_n}(p, j, \xi) = & \left\{ \frac{1}{2} i \gamma_5 \sqrt{\xi} / (2(j - \xi)) \bar{\psi}_{(\mu)_n}(p, j - \frac{1}{2}, \xi + \frac{1}{2}) \right. \\ & \left. + \frac{1}{2} \sqrt{\xi} / (j + \xi) \bar{\psi}_{(\mu)_n}(p, j - \frac{1}{2}, \xi - \frac{1}{2}) \right\} . \end{aligned}$$

Particular results of interest are

$$\phi_{(\mu)_J}(p, j, j) = \prod_{l=1}^J t_{\mu_l} \left\{ \prod_{l=1}^J t_{\nu_l}^* \phi_{(\nu)_J} \right\} \quad 2.24$$

for the highest integer spin state, and

$$\psi_{(\mu)_n}(p, j, j) = \frac{1}{2} \sqrt{\xi} \prod_{l=1}^n t_{\mu_l} \prod_{l=1}^n t_{\nu_l}^* \psi_{(\nu)_n} \quad 2.24$$

for the highest half-integer spin state.

It is clear that 2.18 would give

$$\psi_{(\mu)_n}(p, j, -j) = \frac{1}{2} \sqrt{\xi} \prod_{l=1}^n t_{\mu_l}^* \prod_{l=1}^n t_{\nu_l} \psi_{(\nu)_n} ,$$

an undesirable form since the appearance of a common factor

$$\left(\prod_{l=1}^n t_{\nu_l}^* \psi_{(\nu)_n} \right) \text{ is essential in evaluating } \psi_{(\mu)_n}(p, j, \xi) \bar{\psi}_{(\rho)_n}(p, j, \eta) .$$

Successive application of L (2.21), or 2.23, give the preferable form

$$\phi_{(\mu)_n}(p, j, -j) = \frac{1}{2} i \gamma_5 \sqrt{\xi} \prod_{l=1}^n t_{\mu_l}^* \prod_{l=1}^n t_{\nu_l} \psi_{(\nu)_n} .$$

Explicit wave functions constructed from 2.22 and 2.23 are in

complete agreement with those displayed in a recent paper by Frishman and Gotsman⁽⁵³⁾. Those for $2j = 1, 2, 3$ and 4 are listed in table 2.

$2j$	1	2	3	4
4				$t_\mu t_\nu$
3			$\frac{\sqrt{2}t_\mu}{2}$	
2		t_μ		$\frac{\sqrt{2}(t_\mu s_\nu + s_\mu t_\nu)}{2}$
1	$\sqrt{\frac{1}{2}}t$		$-\frac{\sqrt{6}}{3}\left(\frac{1}{2}i\gamma_5 t_\mu^* - \frac{1}{2}t_\mu^* s_\mu\right)$	
0		s_μ		$\frac{\sqrt{6}}{6}(2s_\mu s_\nu - t_\mu^* t_\nu - t_\mu t_\nu^*)$
-1	$\sqrt{\frac{1}{2}}t^*$		$-\frac{\sqrt{6}}{3}\left(\frac{1}{2}i\gamma_5 t_\mu^* - \frac{1}{2}t_\mu^* s_\mu\right)$	
-2		t_μ^*		$\frac{\sqrt{2}}{2}(t_\mu^* s_\nu + s_\mu t_\nu^*)$
-3			$\frac{\sqrt{2}t_\mu^*}{2}$	
-4				$t_\mu^* t_\nu^*$

TABLE 1

The polarization operators $P(j, \xi)$ (2.15) are given by

$$P(j, \xi)_{(\mu)_J (\rho)_J} = (-1)^J Q(j, \xi)_{(\mu)_J} Q^*(j, \xi)_{(\rho)_J}$$

and

$$P(j, \xi)_{(\mu)_n (\rho)_n} = (-1)^{n+1} Q(j, \xi)_{(\mu)_n} \bar{Q}(j, \xi)_{(\rho)_n},$$

where the $Q(j, \xi)_{(\mu)}$ are listed in Table 1 for the lower spins.

2ξ	$2j$	1	2	3	4
4					$t_\mu t_\nu$
3				$\frac{1}{2} t_\mu^* t_\mu$	
2			$-t_\mu$		$\sqrt{\frac{2}{2}} (s_\mu t_\nu + t_\mu s_\nu)$
1	$-\frac{1}{2} t_\mu^*$			$\sqrt{\frac{6}{6}} (iy s_\mu^* t_\mu - t_\mu^* s_\mu)$	
0			s_μ		$\sqrt{\frac{6}{6}} (2s_\mu s_\nu - t_\mu t_\nu^* - t_\mu^* t_\nu)$
-1	$\sqrt{\frac{1}{2}} iy s_\mu^*$			$\sqrt{\frac{6}{6}} (2iy s_\mu^* s_\mu + t_\mu^* t_\mu)$	
-2			t_μ^*		$\sqrt{\frac{2}{2}} (s_\mu t_\nu^* + t_\mu^* s_\nu)$
-3				$\sqrt{\frac{1}{2}} iy s_\mu^* t_\mu^*$	
-4					$t_\mu^* t_\nu^*$
	$\times \phi$	$\times t_\nu^* \phi_\nu$	$\times t_\nu^* \phi_\nu$	$\times t_\rho^* t_\sigma^* \phi_{\rho\sigma}$	

TABLE 2

The explicit forms of the wave functions $\phi_{(\mu)j}(p, j, \xi)$ for $2j = 2$ and 4 , and $\phi_{(\mu)n}(p, j, \xi)$ for $2j = 1$ and 3 .

(g) A covariant spin density matrix

It is now clear that the generalisation⁽⁵⁴⁾ of the form for ρ given in 2.2 is

$$\rho_{(\mu)(\nu)} = \text{Tr}(\rho) \left\{ \Phi(\text{or } \Psi)_{(\mu)(\nu)}(p, j) + \left(P_\alpha \bar{W}_\alpha \right)_{(\mu)(\nu)} + \left(T_{\alpha\beta} (\bar{W}_\alpha \bar{W}_\beta + \bar{W}_\beta \bar{W}_\alpha) \right)_{(\mu)(\nu)} + \left(T_{(\alpha)} (\bar{W}_\alpha \bar{T}_\alpha \bar{W}_\alpha) \right)_{(\mu)(\nu)} + \dots \right\}, \quad 2.26$$

with P_α , $T_{\alpha\beta}$, $T_{(\alpha)}$ satisfying

$$P \cdot p = 0$$

$$T_{\alpha\beta(\alpha)} = T_{\beta\alpha(\alpha)}$$

$$\xi_{\alpha\beta} T_{\alpha\beta(\alpha)} = 0$$

$$p_\alpha T_{\alpha(\alpha)} = 0, \quad 2.27$$

($\bar{W}_\alpha$) is to be taken as the sum over all permutations of the indices α),

to give the $(2j+1)^2 - 1$ parameters. In the rest frame this does indeed reduce to the form given earlier (since $W_0 = 0$ in the rest frame).

The polarization tensors P_α (the dipole⁽⁵⁵⁾ polarization), $T_{\alpha\beta}$ (the quadrupole polarization), , $T_{(\alpha)2\ell}$ (the " 2^ℓ -pole" polarization) can be obtained, if ρ is known, from

$$P_\alpha = \langle W_\alpha \rangle = \text{Tr}(W_\alpha \rho) / \text{Tr}(\rho)$$

$$T_{(\alpha)} = \langle \bar{W}_\alpha \bar{T}_\alpha \bar{W}_\alpha \rangle. \quad 2.28$$

It is of course clear that if the density matrix is written in the form

$$\rho_{(\mu)(\nu)} = \sum_{\xi\eta} \rho_{\xi\eta} |p, j, \xi\rangle \langle p, j, \eta|$$

then

$$\rho_{\xi\eta} = \Phi_{(\mu)j}^* (p, j, \xi) \rho_{(\mu)j} (p, j, \eta) \quad \text{for integer spin,}$$

and

$$\rho_{\xi\eta} = \bar{\Psi}_{(\mu)n} (p, j, \xi) \rho_{(\mu)n} (p, j, \eta) \quad \text{for half-integer spin.} \quad 2.29$$

With these definitions and formalisms there is complete agreement with the spin one-half results of Bouchiat and Michel⁽⁵⁶⁾:

$$\begin{aligned} \rho^{(\pm)} &= \left((\rho_{11} + \rho_{-1-1}) + i\gamma_5 \{ (\rho_{11} - \rho_{-1-1}) \not{\beta} - \rho_{1-1} \not{\not{x}} - \rho_{-11} \not{\not{x}}^* \} \right) \\ &\quad \times (\not{p} + m) \\ &= (\rho_{11} + \rho_{-1-1}) (1 - W \cdot \langle W \rangle) (\not{p} + m) . \end{aligned}$$

An initial particle- one used as a target or the incoming beam- will have well defined and known polarization properties, that is have a well determined density matrix. The density matrix for a final particle- one produced in the interaction- will give all the information regarding the particle. How is such a density matrix defined and determined? Suppose a spin j particle is produced in some interaction which has matrix element

$$M = \phi_{(\mu)j}^* (p, j) A_{(\mu)j} \quad (\text{or } \bar{\psi}_{(\mu)n} (p, j) B_{(\mu)n}) .$$

Then

$$|M|^2 = (\phi^* A) (A^\dagger \phi) \quad (\text{or } (\bar{\psi} B) (B^\dagger \psi)) ,$$

with the transition probability proportional to

$$\begin{aligned} &\text{Tr} \left(A_{(\mu)j} A_{(\nu)j}^\dagger \Phi_{(\nu)j} (\nu, j) \rho_{(\mu)j} (p, j) \right) \\ &(\text{or } \text{Tr} \left(B_{(\mu)n} B_{(\nu)n}^\dagger \Psi_{(\nu)n} (\nu, j) \rho_{(\mu)n} (p, j) (\not{p} + m) \right)) . \end{aligned}$$

By considering the subsequent decay of this spin j particle (this will be a real particle, it is produced and then decays), the decay rate will be proportional to

$$\text{Tr} \left(C_{(\mu)}^{(\rho(j))} C_{(\mu)(\nu)}^{\dagger} \right),$$

where

$$\rho_{(\mu)(\nu)}^{(j)} = \bar{\Phi}_{(\mu)(\rho)}(p, j) A_{(\rho)}^{\dagger} A_{(\sigma)}^{\dagger} \bar{\Phi}_{(\sigma)(\nu)}(p, j) \quad 2.30$$

$$(\text{or } (2m)^{-1} (\not{p} + m) \Psi_{(\mu)(\rho)} B_{(\rho)}^{\dagger} B_{(\sigma)}^{\dagger} \Psi_{(\sigma)(\nu)} (\not{p} + m)),$$

as can be seen using the usual Feynman Rules techniques.

This is now readily adaptable to chains of decays, for example $A + B \Rightarrow C + D$, $D \Rightarrow E + F$, $F \Rightarrow G + H$,, since at each stage it will be easy to go to the relevant frame of reference (usually the rest frame of the decaying particle) with the covariantly defined density matrix.

In the context of this thesis these definitions are used for the reactions

$$\nu(\bar{\nu}) + B \Rightarrow B + l, B \Rightarrow B + \pi \quad 2.31a$$

and

$$\nu(\bar{\nu}) + B \Rightarrow B^* + l, B^* \Rightarrow B + \pi, \quad 2.31b$$

where in 2.31a $B \Rightarrow B + \pi$ is a weak decay of a spin one-half particle, and in 2.31b $B^* \Rightarrow B + \pi$ is a strong decay of a spin three-half resonance. For these two cases 2.30 gives for the density matrices of the final B and B^*

$$(2m)^{-1} (\not{p} + m) \Gamma_{\alpha}^{\rho} (R) \bar{\Gamma}_{\beta} (\not{p} + m) \cdot \text{Tr} \left\{ (\not{\chi} + \mu) \Gamma'_{\alpha} \rho(\nu) \bar{\Gamma}'_{\beta} \right\} \quad 2.32a$$

and

$$(2m)^{-1} (\not{p} + m) \Psi_{\mu\alpha} \left(\frac{3}{2} \right) \Gamma_{\alpha\beta}^{\rho} (R) \bar{\Gamma}_{\gamma\delta} \Psi_{\delta\nu} \left(\frac{3}{2} \right) (\not{p} + m) \cdot \\ \times \text{Tr} \left\{ (\not{\chi} + \mu) \Gamma'_{\beta} \rho(\nu) \bar{\Gamma}'_{\gamma} \right\}, \quad 2.32b$$

assuming a current-current interaction, where $\rho(B)$ and $\rho(\nu)$ are the density matrices of the initial baryon (B) and the neutrino (ν).

$\rho(1)$ is of course defined in a similar manner:

$$(2\mu)^{-1} (\chi + \mu) \Gamma'_\alpha \rho(\nu) \bar{\Gamma}'_\beta (\chi + \mu) \cdot \text{Tr} \{ \not{p} + m \} \Gamma_\alpha \rho(B) \bar{\Gamma}_\beta \} \quad 2.33a$$

and

$$(2\mu)^{-1} (\chi + \mu) \Gamma'_\beta \rho(\nu) \bar{\Gamma}'_\gamma (\chi + \mu) \cdot \text{Tr} \{ \not{p} + m \} \psi_{\delta\alpha} \left(\frac{3}{2} \right) \Gamma_{\alpha\beta} \rho(B) \bar{\Gamma}_{\gamma\delta} \} \cdot \quad 2.33b$$

At this stage it seems necessary to note that all definitions apply to particles with non-zero mass, so care must be taken if the lepton mass is to be neglected. One way out of this apparent difficulty is to use the fact that in the rest frame of the particle the polarization three-vector has the form⁽⁵⁸⁾

$$\underline{P}_R = \underline{P} - p(p_0 + m)^{-1} p_0 ,$$

and this will be used as the definition of the polarization three-vector of a particle. Here P_μ is the polarization four-vector of the particle of mass m and momentum p_μ . This $\underline{P}_R$ is defined even for a zero mass particle.

3. THE GENERAL STRUCTURE OF THE WEAK INTERACTION LAGRANGIAN

(a) Effective Lagrangian

All the well-known weak interaction processes can be classified phenomenologically into three classes - purely leptonic, partly leptonic or non-leptonic, depending on the proportion of leptons present in the reaction. So far only reactions over a relatively small range of momentum transfer ($\leq 1-2$ Gev.) have been extensively studied, and over this range all the observed weak interactions can be adequately described by an effective Lagrangian density which may be written as⁽²⁶⁾

$$L_{\text{eff.}} = \frac{G}{\sqrt{2}} J_{\alpha} J_{\alpha}^{\dagger} , \quad 3.1$$

where J_{α} is the appropriate current and G the Universal Fermi coupling constant. The weak interaction is assumed to be due to a very short-range force, so that the two currents interact only at the same space-time point (a current-current interaction).

J_{α} is a combination of currents due to

(a) Leptons : the muon current $j^{(\mu)}$ and the electron current $j^{(e)}$, and

(b) Hadrons : a strangeness conserving current $J(\Delta S=0)$ and a strangeness changing one $J(\Delta S \neq 0)$,

that is

$$J_{\alpha} = j^{(e)}_{\alpha} + j^{(\mu)}_{\alpha} + aJ(\Delta S=0)_{\alpha} + bJ(\Delta S \neq 0)_{\alpha} . \quad 3.2$$

All the currents in 3.2 are essential and the form 3.1 is

required, since all the following are known reactions⁽¹¹⁾

(a) neutron beta-decay ($n \rightarrow p + e^- + \bar{\nu}_e$) , requiring $j^{(e)}$ and $J(\Delta S=0)$ (and likewise $\pi \rightarrow e + \nu_e$)

(b) muon decay ($\mu \rightarrow \nu_\mu + e + \bar{\nu}_e$) , requiring $j^{(e)}$ and $j^{(\mu)}$

(c) $\pi \rightarrow \mu + \nu_\mu$, requiring $j^{(\mu)}$ and $J(\Delta S=0)$

(d) hyperon beta-decay ($\Lambda \rightarrow p + e^- + \bar{\nu}_e$) , requiring $J(\Delta S \neq 0)$ in addition to $j^{(e)}$ (and likewise $K \rightarrow \mu + \nu_\mu$) .

From 3.1 and 3.2 it is clear that the weak interaction between leptons and hadrons (partly leptonic) is described by the effective Lagrangian

$$L \sim j^{(e)} J^\dagger(\Delta S=0) + \text{h.c.} \quad 3.3a$$

for strangeness non-changing reactions (e.g. neutron beta-decay), and by

$$L \sim j^{(e)} J^\dagger(\Delta S \neq 0) + \text{h.c.} \quad 3.3b$$

for strangeness changing reactions (e.g. hyperon beta-decay).

These types are the ones of main concern in this report.

Some reactions predicted by 3.1 and 3.2 are extremely difficult to observe, for example $e \rightarrow e + \nu_e + \bar{\nu}_e$ has a very small cross-section. Some experiments in nuclear physics can be designed to detect parity violating contributions to nuclear forces⁽⁵⁹⁾ (the $J(\Delta S=0) J^\dagger(\Delta S=0)$ terms), but it is impossible to ascertain whether this is in agreement with a current-current theory - the currents contain parity violating terms in addition to parity conserving ones since space-inversion is not an invariant operation in weak interactions^(19,20).

It should be pointed out that no neutral lepton currents have been found, so are assumed to be non-existent.

All the currents in 3.2 are taken at the same space-time point. The lowest order matrix elements of the interaction Lagrangian give directly the observed reaction amplitudes, the higher order elements being in general divergent. It is in this sense that 3.1 is phenomenological. Even as an effective Lagrangian this cannot be adequate at very high energies, as may be demonstrated by observing that the reaction $\nu_{\mu} + e^{-} \Rightarrow \nu_e + \mu^{-}$ can occur, and consists of only S-wave scattering. From 3.6 it can be shown that the cross-section increases as the square of the energy of the μ -neutrino, whilst by unitarity it is bounded⁽¹¹⁾. Thus L_{eff} has to be modified at high momentum transfers by generating an effective non-locality, leading to the convenient postulate of the existence of an Intermediate Vector Boson⁽⁶⁰⁾ W having charge ± 1 unit. W is in effect the current "carrier", and can be thought of as quite consistent with a current-current theory.

So far as is known there is complete symmetry over all leptonic reactions with respect to the interchange of electron and muon (apart from their masses), referred to as μ -e universality. This is the reason for $j^{(e)}$ and $j^{(\mu)}$ in 3.2 occurring with the same weight. However the $\Delta S \neq 0$ part of the hadron contribution to the weak current is weighted relative to the lepton currents and the $\Delta S = 0$ part of the hadron current - for example $K \Rightarrow \mu + \nu$ is weighted relative to $\pi \Rightarrow \mu + \nu$. This weighting is remarkably well predicted and taken into account by the Cabbibo

hypothesis⁽⁶¹⁾ which states that the weak current of the strongly interacting particles is composed of currents, which are members of SU(3) octets, in the form

$$(\cos \theta)J(\Delta S=0) + (\sin \theta)J(|\Delta S|=1), \quad 3.4$$

θ being determined experimentally. No weak interactions in which the strangeness is changed by more than one unit have been found^(62,63), and are thought not to exist. Thus $\Delta S \neq 0$ becoming $|\Delta S| = 1$ does not pose any difficulties experimentally. There is a further experimentally tested rule - the $\Delta S = +\Delta Q$ rule^(63,64) - which restricts the number of possible strangeness changing reactions, so it seems to be permissible to write the hadron current as

$$(\cos \theta)J(\Delta S=0) + (\sin \theta)J(\Delta S=\Delta Q) .$$

(b) Lepton current

The most general Lagrangian to describe the interaction of four leptons (the 4-Fermi⁽⁶⁵⁾ interaction) has the form

$$L = \sum_{i=1}^5 G_i (\bar{\nu} O_i \mu) (\bar{e} O_i \nu) + \text{h.c.} ,$$

where O_i stands for the five possible types of interaction (S, P, V, A and T). Using the Fierz⁽⁶⁶⁾ transformation whereby

$$L = \sum_i G'_i (\bar{\nu} O_i \nu) (\bar{e} O_i \mu) + \text{h.c.} ,$$

and the fact that the neutrino is massless and has negative helicity⁽⁶⁷⁾ (an anti-neutrino has positive helicity), only the V and A terms remain. Experimental evidence indicates that for μ -decay $G_V = G_A = G_\mu$ ⁽⁶⁸⁾, resulting in

$$\sqrt{2}L = G_\mu (\bar{\nu} \gamma_\alpha (1 - i\gamma_5) \mu) (\bar{e} \gamma_\alpha (1 - i\gamma_5) \nu) . \quad 3.5$$

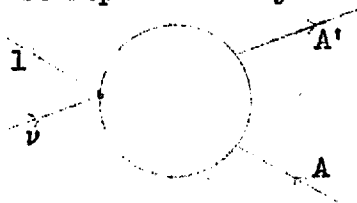
It is now generally accepted that the lepton current takes the form

$$j^{(e)} + j^{(\mu)}$$

with

$$j^{(\ell)} \sim \bar{l}\gamma_{\alpha}(1 - i\gamma_5)\nu_{(\ell)} \quad 3.6$$

in momentum space for $\ell = e$ or μ , and where $\nu_{(\ell)}$ refers to the relevant neutrino. If an anti-neutrino is involved the $(1 - i\gamma_5)$ factor must be replaced by $(1 + i\gamma_5)$. This lepton current is seen to have a point structure, and diagrammatically any reaction involving leptons and other particles will be represented by



(c) Elastic baryon current

Parity is not conserved in weak interaction processes, so it is clear that $J(\Delta S=0)$ and $J(\Delta S \neq 0)$ have both axial-vector and vector terms. Thus the form of J_{α} due to two baryons B_1 and B_2 of momenta p_1 and p_2 is

$$J_{\alpha} \sim \bar{u}(p_2)(V_{\alpha} + A_{\alpha}i\gamma_5)u(p_1) \quad 3.7$$

in momentum space. This will be referred to as the elastic baryon current as opposed to the inelastic where there is one baryon and one baryon resonance in the final state, for example $\nu + B \Rightarrow B + l$ as opposed to $\nu + B \Rightarrow B^* + l$. From Lorentz invariance alone it is seen that the vector and axial-vector currents, V_{α} and A_{α} , can be

expressed as a linear combination of γ_α , p_α and q_α . Thus the usual forms taken are

$$V_\alpha = g_V \gamma_\alpha + f_V p_\alpha + h_V q_\alpha$$

and

$$A_\alpha = g_A \gamma_\alpha + f_A p_\alpha + h_A q_\alpha, \quad 3.8$$

or sometimes

$$V_\alpha = g'_V \gamma_\alpha + f'_V q_\beta i\sigma_{\alpha\beta} + h'_V q_\alpha \quad 3.9$$

to exhibit an analogy with electro-magnetic interactions. Here

$$p = p_1 + p_2$$

and

$$q = p_1 - p_2.$$

However these do not seem to be the most useful combinations⁽²⁹⁾ to take in view of work on the electro-magnetic theory of nucleons. The (vector) current associated with this electro-magnetic interaction between two nucleons of mass M and momenta p_1 and p_2 is best⁽⁶⁹⁻⁷²⁾ written in the form

$$J^{(e)}_\alpha = \bar{u}(p_2) \left(2M p_\alpha F_C + r_\alpha F_M \right) u(p_1) (P^2)^{-1} \quad 3.10$$

with

$$r_\alpha = \epsilon_{\alpha\nu\rho\sigma} q_\nu p_\rho \gamma_5 \gamma_\sigma.$$

This form eliminates the terms in the product of the "conventional" Dirac and Pauli form factors (F_1 and F_2) in cross-section formulae for unpolarised nucleons^(30,31). A choice of V_α and A_α analogous to 3.10 is

$$V_\alpha = \left(2P_\alpha A(q^2) + R_\alpha B(q^2) + 2Q_\alpha E(q^2) \right) (P^2)^{-1}$$

and

$$A_\alpha = \left(2Q_\alpha C(q^2) + R_\alpha D(q^2) + 2P_\alpha F(q^2) \right) (Q^2)^{-1}, \quad 3.11$$

where

$$P = \frac{p_1}{m_1} + \frac{p_2}{m_2} ,$$

$$Q = \frac{p_1}{m_1} - \frac{p_2}{m_2}$$

and

$$R = \frac{x}{m_1 m_2} .$$

It is felt that the combinations P_α and Q_α are better than p_α and q_α when the masses differ - they are proportional when $m_1 = m_2$, and $P \cdot Q \equiv 0$ whilst $p \cdot q = 0$ only when the masses are equal⁽⁷³⁾. The form factors in 3.8 and 3.11 are related by the following:

$$g_V = B$$

$$m_1 m_2 P^2 f_V = (m_1 + m_2)(A - B) + (m_2 - m_1)E$$

$$m_1 m_2 P^2 h_V = (m_2 - m_1)(A - B) + (m_1 + m_2)E$$

and

$$g_A = D$$

$$m_1 m_2 Q^2 f_A = (m_2 - m_1)(C - D) + (m_1 + m_2)F$$

$$m_1 m_2 Q^2 h_A = (m_1 + m_2)(C - D) + (m_2 - m_1)F .$$

The form factors entering in 3.11 lead to simplified formulae for cross-sections and polarizations (see chapter 4), and have the following properties and/or advantages over those in 3.8 and 3.9:

(a) The hadron part of the weak current in the processes

$$\nu + n \rightarrow p + l^-$$

and

$$\bar{\nu} + p \rightarrow n + l^+$$

3.12

is thought to obey the isovector current (I. V. C.) hypothesis⁽²⁶⁾.

This states that V_α and $V_\alpha^\dagger$ together with the iso-spin part of the electro-magnetic current $J^{(e)}_\alpha$ form the three components of a single

iso-spin one triplet. This I. V. C. hypothesis implies a conserved vector current (C. V. C.), but the C. V. C. does not imply an I. V. C. .

The I. V. C. hypothesis requires

$$A \sim F_C$$

$$B \sim F_M$$

and

$$E \sim 0$$

for the processes 3.12 .

(b) If there is a C. V. C. for $\Delta S = 0$ reactions

$$E(q^2) = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \frac{F^2}{Q^2} A(q^2) ,$$

so apart from mass differences $E(q^2)$ is zero.

(c) The divergence of the axial-vector current $(\partial_\alpha A_\alpha)$ is

$$\bar{u}(p_2) \gamma_5 \left\{ (m_1 + m_2) C(q^2) + (m_2 - m_1) \frac{F^2}{Q^2} F(q^2) \right\} u(p_1) ,$$

so in the case of equal masses ($m_1 = m_2 = M$)

$$\partial_\alpha A_\alpha = \bar{u}(p_2) \gamma_5 u(p_1) 2MC(q^2) .$$

This is true for 3.12, and the form factors associated with $\langle p | \partial_\alpha A_\alpha | n \rangle$ are identical to those associated with $\langle \text{vac.} | \partial_\alpha A_\alpha | \bar{p}n \rangle$, except that $q_\alpha = -\bar{p}_\alpha - n_\alpha$ in the latter case. Thus the assumptions that the amplitude $\langle \text{vac.} | \partial_\alpha A_\alpha | \bar{p}n \rangle$ is dominated by the exchange of a single pion (for small q^2) and that the axial-vector is asymptotically conserved, lead to a Goldberger-Treiman relation⁽²⁷⁾ in one form factor only, of the form

$$C(q^2) \sim \frac{1}{2M} (m_\pi^2) \xi_{\pi np} / \left(\frac{q^2}{m_\pi^2} - 1 \right) ,$$

where $\xi_{\pi np}$ is the pion-neutron-proton strong coupling constant, and

the form factor a_π gives the pion decay amplitude, determined from $\pi \Rightarrow \mu + \nu$ ⁽⁷⁴⁾.

If A_α for 3.12 is written as

$$\gamma_\alpha F_A(q^2) + \frac{a_\pi}{\mu} F_P(q^2),$$

comparison with 3.11 leads to the identification

$$\begin{aligned} D(q^2) &= F_A(q^2) \\ \text{and} \quad C(q^2) - D(q^2) &= \frac{q^2}{2M\mu} F_P(q^2), \end{aligned} \quad 3.13$$

and hence to $C(0) = D(0)$.

(d) If time reversal invariance (T) is valid, all six functions A, B, C, D, E, F are real. The appearance of imaginary parts of the products of the form factors in any expression would thus suggest a possible test of T.

(e) The terms $E(q^2)$ and $F(q^2)$ in V_α and A_α are the second class currents of Weinberg ⁽⁷⁵⁾. If V_α and A_α have the same G-parity as their leading terms (those which have no derivative ⁽⁷⁶⁾) and might be expected to occur in the weak Lagrangian ⁽⁷⁷⁾, then $E = F = 0$ for 3.12.

(f) If J_α and $J_\alpha^\dagger$ transform like an isovector, then E and F are imaginary whilst A, B, C and D are real for $\Delta S = 0$ processes. Cabbibo has suggested ⁽⁷⁸⁾ that this behaviour be true for the $\Delta S \neq 0$ processes too, and that this be the manner in which T is violated.

It is clear from (a) and (c) that for the reactions 3.12 something is known about the form factors. For other possible reactions (see 4.1, 4.2, 5.1 and 5.2) it is necessary to appeal to symmetry schemes

($SU(3)^{(79)}$ or $\tilde{U}(12)^{(36)}$ for instance) and to relate the form factors to those in the known processes 3.12 in order to make any predictions (see chapter 6).

(d) The inelastic baryon current

The most general form for the current J_α for reactions of the type $\nu + B \Rightarrow B^* + l$, referred to here as inelastic (though quasi-elastic is sometimes used) is

$$J_\alpha = \bar{u}(p_2)_\beta (V_{\beta\alpha} i\gamma_5 + A_{\beta\alpha}) u(p_1) \quad 3.14$$

where $u(p)_\beta$ is the spinor describing the spin three-half baryon resonance B^* , and satisfying

$$(\not{p} - m)u(p)_\beta = 0 ,$$

$$\text{and} \quad \gamma_\beta u(p)_\beta = 0$$

$$p_\beta u(p)_\beta = 0 .$$

The terms $V_{\beta\alpha}$ and $A_{\beta\alpha}$ give rise to vector and axial-vector terms in J_α , the $i\gamma_5$ entering in V because the B^* has positive parity.

As in the elastic case it is possible to choose the possible independent terms entering V and A in many different ways. It is easily seen that there is a maximum of four different independent couplings for each. The usual ones taken⁽⁸⁰⁾ are

$$V_{\beta\alpha} = a_1 g_{\beta\alpha} + q_\beta (a_2 \gamma_\alpha + a_3 p_\alpha + a_4 q_\alpha)$$

and

$$A_{\beta\alpha} = a'_1 g_{\beta\alpha} + q_\beta (a'_2 \gamma_\alpha + a'_3 p_\alpha + a'_4 q_\alpha) \quad 3.15$$

for computational purposes, with

$$V_{\beta\alpha} = a_1 \varepsilon_{\beta\alpha} + q_\beta (a'_1 \gamma_\alpha + a'_2 i\sigma_{\alpha\delta} q_\delta + a_4 q_\alpha)$$

being used in definitions.

If $J_\alpha J_\beta^\dagger$ is formed and the spin sums over all baryon and baryon resonance states taken, this leads to 24 terms $\text{Re}(a_i a_j^*)$. There is a choice analogous to 3.11, namely

$$V_{\beta\alpha} = b_1 \varepsilon_{\beta\alpha} + q_\beta (b_2 P_\alpha + b_3 Q_\alpha + b_4 R_\alpha),$$

with $A_{\beta\alpha}$ defined similarly, which reduces these 24 possible combinations to 20, as does

$$V_{\beta\alpha} = c_1 \varepsilon_{\beta\alpha} + q_\beta (c_2 P_\alpha + c_3 Q_\alpha) + c_4 \gamma_5 \epsilon_{\mu\nu\beta\alpha} p_\mu q_\nu$$

with $A_{\beta\alpha}$ as expected. This latter form simplifies the trace evaluations. However there is a form which gives still less terms $\text{Re}(a_i a_j^*) - 16$ - and looks like

$$V_{\beta\alpha} = c_\beta (d_1 P_\alpha + d_2 q_\alpha + d_3 r_\alpha) + \gamma_5 d_4 \epsilon_{\mu\nu\beta\alpha} p_\mu q_\nu$$

and

$$A_{\beta\alpha} = q_\beta (d'_1 P_\alpha + d'_2 q_\alpha + d'_3 r_\alpha) + \gamma_5 d'_4 \epsilon_{\mu\nu\beta\alpha} p_\mu q_\nu. \quad 3.16$$

It should be noted that whilst q_β terms are written in all forms for V and A above it is only the $(p_1)_\beta$ which contribute.

The imposition of the C. V. C. hypothesis to this transition gives the linear relations

$$a_1 - (m_1 + m_2) a_2 + (m_1^2 - m_2^2) a_3 + q^2 a_4 = 0 \quad 3.17$$

and

$$(m_1^2 - m_2^2) d_1 + q^2 d_2 = 0 \quad 3.18$$

when applied to the vector currents in 3.15 and 3.16, the d_i giving the simpler form. The two sets of form factors a_i and d_i are

connected in the following manner:

$$\begin{aligned}
 a_1 &= -P^2 m_1 m_2 d_4 \\
 a_2 &= (Q^2 m_1 m_2 d_3 - 2m_2 d_4) \\
 a_3 &= d_1 + d_4 + (m_2 - m_1) d_3 \\
 a_4 &= d_2 - d_4 + (m_1 + m_2) d_3
 \end{aligned}
 \tag{3.18a}$$

and

$$\begin{aligned}
 a'_1 &= -Q^2 m_1 m_2 d'_4 \\
 a'_2 &= (2m_2 d'_4 - P^2 m_1 m_2 d'_3) \\
 a'_3 &= d'_1 - d'_4 + (m_1 + m_2) d'_3 \\
 a'_4 &= d'_2 + d'_4 + (m_2 - m_1) d'_3
 \end{aligned}
 \tag{3.18b}$$

All form factors in 3.15 and 3.16 are functions of the momentum transfer q^2 , and again all are relatively real if time reversal invariance holds. Very little else can be said about their form from a general viewpoint. For the various reactions in this inelastic grouping $SU(3)$ gives some correlations⁽⁸¹⁾, but the larger symmetry scheme $\tilde{U}(12)$ ⁽¹⁸⁾ allows these form factors to be compared with and related to the elastic ones, as well as reducing the number. The only way, apart from these symmetry schemes, of obtaining information about them seems to be from photo-production⁽⁸²⁾ and electro-production⁽⁸³⁾.

4. CROSS-SECTIONS AND POLARIZATIONS FOR $\nu + B \Rightarrow B + l$

This section contains a study of the elastic reaction $\nu(\bar{\nu}) + B \Rightarrow B + l$ in general terms, with emphasis on those processes where both the initial and final baryons are nucleons. The processes of greatest interest are those in which the target baryon is a nucleon:

$$\nu + n \Rightarrow p + l^- \quad 4.1a$$

$$\bar{\nu} + p \Rightarrow n + l^+ \quad 4.1b$$

$$\bar{\nu} + p \Rightarrow \Sigma^0 + l^+ \quad 4.1c$$

$$\bar{\nu} + p \Rightarrow \Lambda + l^+ \quad 4.1d$$

$$\bar{\nu} + n \Rightarrow \Sigma^- + l^+ . \quad 4.1e$$

Only those reactions allowed by the $\Delta S = \Delta Q$ rule for strange particle production and by conservation of lepton number have been listed in 4.1.

There are of course the possible reactions

$$\nu + Y \Rightarrow Y + l^- \quad 4.2a$$

$$\bar{\nu} + Y \Rightarrow Y + l^+, \quad 4.2b$$

where Y is a suitable hyperon (of non-zero strangeness) - Σ , Λ or Ξ , which would also be described by the general formulae obtained, though these are not of current experimental interest.

With very little modification the results are also applicable to the decays

$$n \Rightarrow p + e^- + \bar{\nu} \quad 4.3a$$

$$\Xi \Rightarrow \Sigma + e^- + \bar{\nu} \quad 4.3b$$

$$\Lambda \Rightarrow p + e^- + \bar{\nu} \quad 4.3c$$

among others.

(a) Transition probability

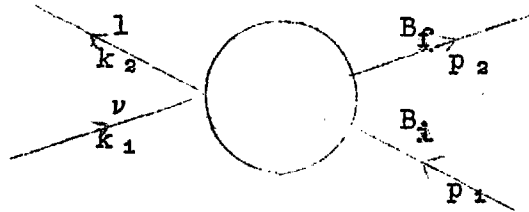
From 3.3, the invariant matrix element is given by

$$M = \sqrt{\frac{1}{2}} G j_{\alpha}^{(\ell)} J_{\alpha}^{\dagger(B)}$$

i.e.

$$M = \sqrt{\frac{1}{2}} G \bar{u}(k_2) \gamma_{\alpha} (1 \pm i \gamma_5) u(k_1) \bar{u}(p_2) (V_{\alpha} + A_{\alpha} i \gamma_5) u(p_1) \quad 4.4$$

in momentum space, using 3.6 and 3.7, where the connection between particles and momenta is represented diagrammatically below



Averaging over the initial spin states and summing over final spin states (denoted by $\Sigma \Sigma$),

$$\frac{\Sigma \Sigma}{I_F} |M|^2 = \frac{1}{2} G^2 \left(\frac{\Sigma \Sigma}{\nu \ell} j_{\mu}^{(\ell)} j_{\nu}^{\dagger(\ell)} \right) \left(\frac{\Sigma \Sigma}{B B} J_{\mu}^{\dagger(B)} J_{\nu}^{(B)} \right), \quad 4.5$$

where

$$\frac{\Sigma \Sigma}{\nu \ell} j_{\mu}^{(\ell)} j_{\nu}^{\dagger(\ell)} = 8 (k_2 \cdot L)_{\mu \nu} \quad 4.6$$

with

$$(L_{\alpha})_{\mu \nu} = (k_{1\mu} g_{\nu\alpha} - g_{\mu\nu} k_{1\alpha} + k_{1\nu} g_{\mu\alpha} \pm i \epsilon_{\mu\nu\rho\alpha} k_{1\rho}), \quad 4.7$$

the '+' ('-') sign referring to an initial ν ($\bar{\nu}$).

If only a certain selection of leptons is taken, for example those with a specific polarization vector S_{α} , then $k_{2\alpha}$ in 4.6 is to be replaced by $(k_{2\alpha} - \mu S_{\alpha})$ where μ is the lepton mass. The expression for the baryon spin sum corresponding to 4.6 is not written down as it

is of no use by itself and very lengthy by virtue of 3.11.

At this stage the initial baryon (B_i) is considered to be polarized (polarization vector T_α) and all spin states of the final baryon (B_f) summed over, with the result that

$$\frac{\sum \epsilon^2 |M|^2}{\sum F} = 8G^2 K_\alpha (k_2)_\alpha - \mu S_\alpha) m_1 m_2, \quad 4.8$$

where

$$\begin{aligned} K_\alpha = & \left\{ U^{(1)}_\alpha \pm U^{(2)}_\alpha \langle QPk_1(\alpha) \rangle \right\} + \left\{ V^{(1)}_\alpha \pm V^{(2)}_\alpha \langle QPk_1(\alpha) \rangle \right\} P \cdot T \\ & + \left\{ W^{(1)}_\alpha \pm W^{(2)}_\alpha \right\} k_1 \cdot T + \left\{ W^{(1)} \cdot k_1 \mp W^{(2)} \cdot k_1 \right\} T_\alpha \\ & + Y_\alpha \langle k_1 PQT \rangle + Y \cdot k_1 \langle (\alpha) PQT \rangle \pm Z^{(1)} \langle Qk_1 T(\alpha) \rangle \\ & \pm Z^{(2)} \langle Pk_1 T(\alpha) \rangle, \end{aligned}$$

with $\langle (\mu)(\nu)(\rho)x \rangle$ denoting $\epsilon_{\mu\nu\rho\sigma} x_\sigma$, and the coefficients given in 4.R1.

It is at once clear that the terms in $U^{(2)}$ and $V^{(2)}$ are missing when all the spin states of the lepton are taken ($S_\alpha = 0$). These terms are combinations of the imaginary parts of the products of the elastic baryon form factors - indeed all terms containing an $\langle ABC(\rho) \rangle$ have as coefficient such an imaginary part, and they vanish if time reversal invariance (or equivalently CP invariance) holds.

Such a transition probability can now be used to calculate the cross-sections for 4.1 and 4.2 and the decay rates for 4.3 .

(b) Cross-section

The differential cross-section for 4.1 and 4.2 when the initial baryon has polarization 4-vector T_α and both the final particles (the

lepton and baryon) are unpolarized is⁽³¹⁾

$$\frac{d\sigma}{dt} = \frac{G^2 K_0 k_2 m_1 m_2}{2\pi (s - m_1^2)^2} \quad 4.9$$

using 4.8, where the numerator is to be expressed as a function of the Mandelstam variables s , t and u .

If this initial baryon is polarized in some general direction 4.9 contains all possible combinations of the form factors, so it is very unlikely that any useful information could be obtained from such a situation. For this reason the differential cross-section is considered for zero polarization and also specific directions of polarizations.

In covariant notation the differential cross-section for unpolarized initial baryons is

$$\left(\frac{d\sigma}{dt} \right)_{T=0} = \frac{G^2}{2\pi} \frac{U^{(1)} k_2 m_1 m_2}{(s - m_1^2)^2}, \quad 4.10$$

where $U^{(1)} k_2$ is given by 4.R2 in terms of s , t and u . This expression contains all six form factors, but it is to be noticed that they enter as $|F_1|^2$ apart from the two terms $\text{Re}(AE^*)$ and $\text{Re}(CF^*)$ - which are absent if there are no second class currents - and the characteristic parity violating term $\pm \text{Re}(BD^*)$.

At very high energies

$$\left(\frac{d\sigma}{dt}(s \rightarrow \infty) \right)_{T=0} = \frac{G^2}{2\pi} \alpha \text{ for all } t, \quad 4.11$$

where

$$\alpha = |(m_1 + m_2)A + (m_1 - m_2)E|^2 x + |(m_1 - m_2)C + (m_1 + m_2)F|^2 y - t|B|^2 x - t|D|^2 y.$$

(For definitions of x and y see 4.R1). Thus at very high energies

and low momentum transfers

$$\left(\frac{d\sigma}{dt}(s \rightarrow \infty, t \rightarrow 0) \right)_{T=0} = \frac{G^2}{2\pi} \left\{ \left| A + \frac{(m_1 - m_2)}{(m_1 + m_2)} E \right|^2 + \left| C + \frac{(m_1 + m_2)}{(m_1 - m_2)} F \right|^2 \right\} \quad 4.12$$

for $m_1 \neq m_2$, and

$$= \frac{G^2}{2\pi} \{ |A|^2 + |D|^2 \} \quad 4.13$$

for $m_1 = m_2$, where it is assumed that, for $m_1 = m_2$, $F(t) \sim t$ for small values of $|t|$ in order that the current be defined there (see 3.11).

It is clear also in the equal mass case that $G^2(0) - D^2(0)$ must vanish, agreeing with the implications of a Goldberger-Treiman relation⁽²⁷⁾ (3.13).

At very high energies there is no difference between the neutrino induced and anti-neutrino induced reactions (the term in $\text{Re}(BD^*)$ does not occur in 4.12 or 4.13), a fact which seems to be well supported by experiment. So far expression 4.13 only deals with the case $t=0$. The modified expression for low values of the momentum transfer in the limit of zero lepton mass and equal baryon masses ($m_1 = m_2$) is

$$\begin{aligned} \left(\frac{d\sigma}{dt} \right)_{T=0} \approx \frac{G^2}{2\pi} & \left\{ (|A|^2 + |D|^2) + \frac{t}{m^2} \left(\frac{1}{4} (|A|^2 - |B|^2) - 4 \left| \frac{Fm^2}{t} \right|^2 \right. \right. \\ & \left. \left. + \frac{sm^2}{(s-m^2)^2} |A|^2 + \frac{(s-2m^2)m^2}{(s-m^2)^2} |D|^2 + \frac{2m^2}{(s-m^2)} \text{Re}(BD^*) \right) \right\} \quad 4.14 \end{aligned}$$

for all energies, containing only the four form factors A, B, D and F.

Neutrino and anti-neutrino reactions at the same energies could thus provide information on $|A|^2$, $|B|^2$, $|D|^2$ and $\left| \frac{Fm^2}{t} \right|^2$ (by looking at

$d\sigma(\nu) + d\sigma(\bar{\nu})$ and $\text{Re}(BD^*)$ (by finding $d\sigma(\nu) - d\sigma(\bar{\nu})$).

From 4.14 also, at $t=0$ and all energies

$$\left(\frac{d\sigma}{dt} \right)_{T=0} \approx \frac{G^2}{2\pi} (1 + \lambda^2)$$

if $A(0) = 1$ and $D(0) = \lambda$ - which appears to be reasonable if λ is taken to be 1.15. With this value of λ $\frac{d\sigma}{dt} \approx 1.91 \times 10^{-38} \text{ cm}^2 (\text{Gev/c})^{-2}$, a result which gives a simple method of measuring the neutrino flux⁽⁸⁴⁾, since this can be compared with the observed number of low (zero) q^2 events as a function of the measured neutrino energy.

Assuming some suitable parameterization for the form factors 4.14 will provide a very rough order of magnitude estimation for the total cross-section for unpolarized nucleons at high energies (in the reaction $\nu(\bar{\nu}) + N \Rightarrow N + \mu$). If

$$A \sim \left(1 - \frac{t}{M_V^2} \right),$$

$$B \sim \kappa A,$$

$$D \sim 1.15 \left(1 - \frac{t}{M_A^2} \right)$$

and

$$F = 0,$$

integrating 4.14 directly (which is rigourously correct if $E_\nu \gg m$ - that is for high energy neutrinos)

$$\sigma(E_\nu \rightarrow \infty) = \frac{G^2}{2\pi} \left\{ \frac{M_V^2}{3} + 1.15^2 \frac{M_A^2}{3} + \frac{1}{24} \left(-M_V^2 \left(\frac{M_V}{M} \right)^2 (1 - \kappa^2) \right) \right\}. \quad 4.15$$

With $M_V = M_A = 0.84$ and $\kappa = 4.7$ (M_V and κ , and the form of A and B, are "obtained" assuming an I. V. C. hypothesis and M_A is arbitrary),

$$\sigma(E_\nu \rightarrow \infty) \approx 0.85 \times 10^{-38} \text{ cm}^2,$$

and if a larger value of M_A , say 1.0, is taken this would increase to approximately $0.98 \times 10^{-38} \text{ cm}^2$. Of course the actual form of the form factors would alter this, but it appears that a cross-section of the order of 10^{-38} cm^2 is to be expected. An indication of the high energy

behaviour of the cross-section due to various choices of form factors is given in table 3 for several forms of the q^2 dependence. It is seen that the form

$$F_1^2(q^2) \sim \frac{F_1^2(0)}{(1 - q^2/q_a^2)^4}$$

gives a constant high energy behaviour, the other choices indicated giving at least a logarithmically increasing behaviour in some contributions.

The effect of introducing an Intermediate Vector Boson (W) into the calculations would result in corrections to the given formulae of the order of $\mu^2/(M_W)^2$, and for this reason the results with a W present are not displayed here.

When the initial baryon has arbitrary polarization (T_α) $\frac{d\sigma}{dt}$ is given by 4.9, and if all the form factors are relatively real there can be no contribution from that polarization perpendicular to the production plane, for in the laboratory frame

$$\left(\frac{d\sigma}{dt}\right)_{\substack{p_1=0 \\ I_{pol.}}} = \left(\frac{d\sigma}{dt}\right)_{\substack{p_1=0 \\ unpol.}} - \text{Im}(I) \frac{G^2}{2\pi} 8(\underline{p}_2 \wedge \underline{k}_1) \cdot \underline{T}(s - m_1^2)^{-2}, \quad 4.16$$

where I is given by 4.R3 and $U^{(1)} \cdot k_2(p_1=0)$ by 4.R4. From 4.16

$$\left(\frac{d\sigma^1 - d\sigma_1}{dq^2}\right)_z \approx \frac{G^2}{2\pi} \sin \theta \ 8m|\underline{p}_2| \ \text{Im}\left(\frac{DF^*}{q^2} + \frac{AB^*}{(4m^2 - q^2)}\right) \quad 4.17$$

for large E_ν , with $|\underline{p}_2|^2 = q^2(q^2 - 4m^2)(4m^2)^{-1}$ and θ the angle between the incoming neutrino 3-momentum ($\underline{k}_1$) and the outgoing nucleon direction ($\underline{p}_2$). Here σ^1 corresponds to a nucleon polarized in the $(\underline{p}_2 \wedge \underline{k}_1)$ direction (the z-direction) and σ_1 in the $(-z)$ -direction in the

reactions $\nu(\bar{\nu}) + N \Rightarrow N + l$.

The ratio

$$\left(\frac{d\sigma^1 - d\sigma_1}{dq^2} \right)_Z \left\{ \left(\frac{d\sigma^1 + d\sigma_1}{dq^2} \right)_Z \right\}^{-1}$$

can become quite large if F is purely imaginary and contributes to the same extent as A , B , C and D . This of course leads to a possible test T (the appearance of $\text{Im}(I)$ in 4.16 and 4.17) and in particular to a direct test of the Cabbibo hypothesis (Cabbibo, at Dubna⁽⁷⁸⁾, introduced the specific hypothesis that the second class currents were relatively imaginary with respect to the first class, giving, in a sense, a maximal violation of T). Similar tests are available in the final particle polarizations (N and l) which are given in following sections.

Finally the cross-section for initial baryons polarized in the direction transverse to the final baryon's direction and in the production plane (i.e. in the direction of $(\underline{p}_2 \wedge \underline{k}_1) \wedge \underline{p}_2$) is looked at, with the result that

$$\left(\frac{d\sigma^1 - d\sigma_1}{dq^2} \right)_X = \text{Re}(J) \frac{G^2}{2\pi} \frac{4E_\nu \sin \theta}{(2m_1 E_\nu)^2} \quad 4.18$$

in the lab. frame, where J is given by 4.R5. In the equal mass case and at high energies of the neutrino (E_ν) this becomes

$$\left(\frac{d\sigma^1 - d\sigma_1}{dq^2} \right)_X \cong \frac{G^2}{2\pi} 2\sin \theta \text{Re}(BF^* + AD^*). \quad 4.19$$

From 4.14 and 4.19 with $F=0$, $D=1.15A$ and $B=4.7A$ the ratio R (below) depends strongly on q^2 , being approximately 1 at $q^2 = 0$. If $\text{Re}(BF^*)$ is zero, 4.19 measures directly $\text{Re}(AD^*)$.

$$R = \left(\frac{d\sigma^1 - d\sigma_1}{dq^2} \right)_X \left\{ \left(\frac{d\sigma^1 + d\sigma_1}{dq^2} \right)_X \right\}^{-1}$$

	$n = 1$	$n = 2$	$n = 2$	$n = 1$
(A,A)	$4a$	$\frac{4}{3}a$	$\frac{2a\alpha}{3m^2}$	$4 \frac{a^2}{m^2} \log\left(\frac{s}{a}\right)$
(B,B)	$a^2 \log\left(\frac{s}{a}\right)$	$\frac{1}{6}a\alpha$	$\frac{a^2\beta}{6m^2}$	$\frac{2a^2 s}{3m^2}$
(C,C)	$\frac{\mu^2 a^2}{s^2} \log\left(\frac{s}{a}\right)$	$\frac{\mu^2 a\alpha}{6s^2}$	$\frac{\mu^2 a \gamma}{3s^2 m^2}$	$\frac{\mu^2 a^2}{sm^2}$
(D,D)	$a^2 \log\left(\frac{s}{a}\right)$	$\frac{1}{6}a\alpha$	$\frac{a \gamma}{3m^2}$	$\frac{2a^2 s}{3m^2}$
(B,D)	$\frac{3}{2} a^2$	$\frac{a^2\beta}{3s}$	$2 \frac{a^4}{sm^2} \log\left(\frac{s}{a}\right)$	$\frac{5a^2 s}{6m^2}$
(E,E)	$\frac{\mu^2 a^2}{s^2} \log\left(\frac{s}{a}\right)$	$\frac{\mu^2 a^2}{6s^2}$	$\frac{\mu^2 a\beta}{6s^2 m^2}$	$\frac{\mu^2 a^2}{sm^2}$
$\left(\frac{F}{t}, \frac{F}{t}\right)$	$2a^2 s$	$\frac{2}{3}a^2\beta$	$4 \frac{a^4}{m^2} \log\left(\frac{s}{a}\right)$	$\frac{2a^2 s^2}{3m^2}$
(A,E)	$4 \mu^2 \frac{a}{s}$	$\frac{4 \mu^2 a}{3s}$	$\frac{2 \mu^2 a\alpha}{3sm^2}$	$4 \frac{\mu^2 a^2}{sm^2} \log\left(\frac{s}{a}\right)$
$\left(C, \frac{F}{t}\right)$	$4 \frac{\mu^2 a^2}{s} \log\left(\frac{s}{a}\right)$	$\frac{2 \mu^2 a\alpha}{3s}$	$\frac{4 \mu^2 a\gamma}{3sm^2}$	$3 \frac{\mu^2 a^2}{m^2}$
	$F_1 \sim \sqrt{\frac{(P^2) F_1(0)}{(1-ta^{-1})^n}}$		$F_1 \sim \frac{P^2 F_1(0)}{(1-ta^{-1})^n}$	

TABLE 3

The high energy contribution to the total cross-section due to the indicated choice of form factor with given behaviour. Here F_i is any one of the form factors listed in the first column, where by (F_i, F_j) is implied $\text{Re}(F_i F_j^*)$. Terms of the order of μ^4 have been omitted in all these leading terms, and the expressions are for $m_1 = m_2$. $\alpha = (a + 2m^2)$, $\beta = (2a + m^2)$ and $\gamma = (a^2 + m^2 a + m^4)$.

(c) Lepton polarization

If the final baryon is not observed (that is it is assumed to be unpolarized and all the spin states summed over), the lepton polarization 4-vector (S_α) is given by

$$S_\alpha = \pm \frac{k_2}{\mu} \alpha + \mu \frac{K}{K_0 k_2} \alpha, \quad 4.20$$

using 2.28 and 2.33. For the rest frame polarization vector this gives

$$\underline{S}_R = \pm \frac{1}{\mu + k_{20}} \underline{k}_2 + \left(\underline{K} - \frac{K_0}{\mu + k_{20}} \underline{k}_2 \right) \frac{1}{K_0 k_2}, \quad 4.21$$

so in the limit of zero lepton mass ($\mu = 0$) $\underline{S}_R = \pm \hat{\underline{k}}_2$, in agreement with the expected values for a massless spin- $\frac{1}{2}$ particle (from 4.20 it is seen that

$$S^2 = -1 + \mu^2 K^2 (K_0 k_2)^{-2},$$

giving $S^2 = -1$ for $\mu = 0$ - i.e. completely polarized in some direction).

For an unpolarized initial baryon ($T_\alpha = 0$) 4.20 becomes

$$\mu S_\alpha = \pm k_{2\alpha} + \mu^2 \left(U^{(1)}_\alpha \pm U^{(2)} \langle QP k_1(\alpha) \rangle \right) \left(U^{(1)} \cdot k_2 \right)^{-1}, \quad 4.22$$

thus a non-zero value for the transverse (i.e. perpendicular to the plane containing the neutrino and lepton) lepton polarization indicates a violation of time reversal invariance. The expression for this transverse polarization is

$$\underline{S}^T = \frac{(\underline{k}_1 \wedge \underline{k}_2)}{U^{(1)} \cdot k_2} 8 \text{Im} \{ A E^* \pi - C F^* y \}, \quad 4.23$$

and at very high energies with the muon coming out parallel to the neutrino this can become quite appreciable, with $(\underline{S}^T)^2 \sim \frac{1}{2}$ if it is assumed that F is relatively imaginary with respect to the other form

factors, $E = 0$ and $|Ft^{-1}| \sim A \sim \lambda D$. Due to the many assumptions about the behaviour of all these form factors that are required to obtain numerical results, it seems pointless to try and give a quantitative study of $\underline{S}^T$ - see Berman and Veltman⁽³⁴⁾ however for some estimates.

If a polarization study of the lepton is carried out with an unpolarized target much information could be gathered on the form factors from the three independent measurements $\underline{S}_R \cdot \underline{k}_2$, $\underline{S}_R \cdot (\underline{k}_1 \wedge \underline{k}_2)$ and $\underline{S}_R \cdot [\underline{k}_2 \wedge (\underline{k}_1 \wedge \underline{k}_2)]$ when the final baryon is not considered.

(d) Baryon polarization

When the final baryon is a hyperon its polarization (Σ_α) is readily accessible from the decay products. For this reason Σ_α is of greater interest than S_α . This problem has been considered by Adler⁽³¹⁾ using a different polarization formalism and with a different choice of currents, but the results are given and studied in the framework developed here.

For an unpolarized lepton and an unpolarized target the final baryon polarization is found to be given by

$$(\underline{U}^{(1)} \cdot \underline{k}_2) \Sigma_\alpha = \text{Re}(\Sigma_\alpha^{(1)} + \Sigma_\alpha^{(2)}) + \text{Im}(\Sigma^{(3)} + \Sigma^{(4)}) \langle p_1 k_1 k_2 (\alpha) \rangle_{4.24},$$

where $\Sigma_\alpha^{(1)}$ and $\Sigma^{(3)}$ contain products of class one currents and products of class two currents and $\Sigma_\alpha^{(2)}$ and $\Sigma^{(4)}$ are mixtures of products of class one with class two currents. The relevant expressions for the $\Sigma^{(1)}$ are given in 4.R6. If there are no second class currents $\Sigma^{(2)}$ and $\Sigma^{(4)}$ are absent in 4.24, whilst $\Sigma^{(3)}$ and $\Sigma^{(4)}$ vanish if T is valid.

Thus if there is any baryon polarization normal to the production plane time reversal invariance is violated. It does not seem possible however to say how this violation occurs - in the Cabbibo manner or otherwise - for if $\underline{S}^T \neq 0$ then $\underline{\Sigma}^T \neq 0$ of necessity (assuming B and D are non-zero).

In the laboratory frame the three independent polarization correlations are $\underline{\Sigma}_R \cdot \underline{p}_2$, $\underline{\Sigma}_R \cdot [\underline{p}_2 \wedge (\underline{k}_1 \wedge \underline{p}_2)]$ and $\underline{\Sigma}_R \cdot \hat{\underline{n}}$ (mentioned above). The first of these contains far too many combinations of the form factors to be of any great interest. From the second

$$\left(U^{(1)} \cdot k_2 \right) \underline{\Sigma}_R \cdot \hat{\underline{t}} = E_\nu \sin \theta \operatorname{Re}(L), \quad 4.25$$

with L listed in 4.R7, giving at high energies

$$\alpha \underline{\Sigma}_R \cdot \hat{\underline{t}} = 2 \sin \theta \operatorname{Re} \left\{ \left(1 + \frac{m_2^2}{m_1^2} \right) (AD^* + FB^*) + \left(1 - \frac{m_2^2}{m_1^2} \right) (BC^* - ED^*) \right\}.$$

It is interesting to note the similarity between this form for $m_1 = m_2$ and 4.19 .

Finally the transverse polarization of the final baryon is

$$\left(U^{(1)} \cdot k_2 \right) \underline{\Sigma}_R \cdot \hat{\underline{n}} = E_\nu \frac{|\underline{p}_2|}{m_b} \sin \theta \operatorname{Im}(M), \quad 4.26$$

where M is given in 4.R8 . This reduces at high energies to

$$\alpha m_2 \underline{\Sigma}_R \cdot \hat{\underline{n}} = |\underline{p}_2| \sin \theta \operatorname{Im} \{ (A-E)B^*x + (C-F)D^*y \}.$$

(e) Decay rates

The total decay rate for processes of the type $B \Rightarrow B' + e^- + \bar{\nu}$ (any one of the decays 4.3) expressed in the usual notation is

$$\Gamma = \frac{1}{2m_1} \frac{1}{(2\pi)^5} \int_{\mathbb{R}^F} \{ \Sigma \Sigma' |M|^2 d^4 p_2 \delta(p_2^2 - m_2^2) d^4 k_1 \delta(k_1^2) d^4 k_2 \delta(k_2^2 - \mu^2) \\ \times \delta^4(p_1 - p_2 - k_1 - k_2) \} , \quad 4.27$$

where $\Sigma \Sigma' |M|^2$ is obtained from 4.8 by changing the sign of k_1 , the lower sign being relevant for the final anti-neutrino throughout.

There are several ways of proceeding with this integral, but it is at once clear that if all the lepton configurations and all directions and magnitudes of p_2 are integrated over the result 4.27 will be independent of T_α and reduce to

$$\Gamma = \frac{4G^2 m_2}{(2\pi)^4 2\pi} \int (U^{(1)} \cdot k_2) \frac{d^3 p_2}{2p_{20}} \frac{d^3 k_1}{2k_{10}} \frac{d^3 k_2}{2k_{20}} \delta^4(p_1 - p_2 - k_1 - k_2). \quad 4.28$$

This can now be readily reduced in the rest frame of the decaying particle ($p_1 = 0$), for $U^{(1)} \cdot k_2$ can be expressed in terms of q^2 and $p_1 \cdot k_1$ which are independent of angles ($q^2 = (p_1 - p_2)^2$ and $p_1 \cdot k_1 = m_1 E_\nu$), with the result that

$$\Gamma = \frac{G^2 m_2}{64\pi^3 m_1} \int U^{(1)} \cdot k_2 dq^2 dE_\nu \quad 4.29$$

with $U^{(1)} \cdot k_2$ essentially given by 4.84. The ranges of integration are

$$(q_0 - |q|)(q^2 - \mu^2) < 2q^2 E_\nu < (q_0 + |q|)(q^2 - \mu^2)$$

and

$$\mu^2 < q^2 < (m_1 - m_2)^2,$$

giving a relatively simple $\int dE_\nu$. However $\int dq^2$ is, even in the simple case where all the form factors are approximated by constants, complicated.

If $U^{(1)} \cdot k_2$ had not already been written down explicitly in terms of q^2 and E_ν , 4.28 could have been used directly by making use of the following easily derivable results⁽⁸⁵⁾:-

$$(a) \quad 24(q^2)^3 \int k_{1\mu} k_{1\nu} \rho(k; q) = \pi \sqrt{\{q^2 - \mu^2\}}^5 (4q_\mu q_\nu - q^2 \delta_{\mu\nu})$$

$$(b) \quad 4(q^2)^2 \int k_{1\mu} \rho(k; q) = \pi \sqrt{\{q^2 - \mu^2\}}^3 q_\mu$$

$$(c) \quad 2q^2 \int \rho(k; q) = \pi \sqrt{\{q^2 - \mu^2\}}$$

where

$$\rho(k; q) = d^4 k_1 \delta(k_1^2) d^4 k_2 \delta(k_2^2 - \mu^2) \delta^4(q - k_1 - k_2),$$

to give

$$\Gamma = \frac{G^2}{64\pi^3} \frac{m^2}{m_1} \int_{\mu^2}^{(m_1 - m_2)^2} \left\{ \sqrt{\{q^2 - \mu^2\}}^3 \sqrt{\{(m_1 + m_2)^2 - q^2\}} / \sqrt{\{(m_1 - m_2)^2 - q^2\}} \right. \\ \left. \times \left(6(q^2)^3\right)^{-1} F(q^2) dq^2 \right\} \quad 4.30$$

where $F(q^2)$ is given by 4.29. This integral can be evaluated numerically if a suitable behaviour for the form factors is inserted.

To obtain the result in a form where some approximations can be made it seems best to proceed⁽⁸⁶⁾ via 4.29 with the order of the integrals reversed:

$$\Gamma = \frac{G^2}{64\pi^3} \frac{m^2}{m_1} \int U^{(1)} \cdot k_2 dE_\nu dq^2 \quad 4.31$$

with

$$X - Y < q^2 < X + Y$$

$$0 < 2m_1 E_\nu < m_1^2 - m_2^2 + \mu^2$$

where

$$(m_1^2 - 2m_1 E_\nu)X = 2m_1^2 E_\nu^2 - E_\nu m_1 (m_1^2 - m_2^2 - \mu^2) - \mu^2 m_1^2$$

$$(m_1^2 - 2m_1 E_\nu)Y = m_1 E_\nu \sqrt{\{(m_1^2 - m_2^2 - \mu^2 - 2m_1 E_\nu)^2 - 4m_1^2 \mu^2\}}.$$

Since the range of q^2 is small, it seems permissible to approximate the form factors by constants in the following manner:

$$Aa, Bb, |C|^2 y |c|^2, |D|^2 y |d|^2, Ee, |F|^2 y |f|^2.$$

Whilst the inclusion of $y \left(= m_1 m_2 \{ (m_1 - m_2)^2 - q^2 \}^{-1} \right)$ is arbitrary, it seems best to remove the denominator in $U^{(1)} \cdot k_2$. This results in

$$U^{(1)} \cdot k_2 m_1 m_2 \sim (2m_1 E_\nu)^2 \alpha' + 2m_1 E_\nu \{ (q^2 + m_1^2 - m_2^2) \alpha' - 2\beta' \mu^2 \} \\ + (q^2 - \mu^2) \{ (m_1^2 - m_2^2) \beta' + m_1 m_2 \gamma' + q^2 \delta' \}, \quad 4.32$$

where α' , β' , γ' and δ' are independent of E_ν and q^2 and are listed in 4.R10. Now q^2 and q^4 can be replaced by X and $X^2 + \frac{Y^2}{3}$ respectively if $\int dq^2$ is to be performed first. This gives a lengthy result for $U^{(1)} \cdot k_2 m_1 m_2$ (see 4.R11), but $m_1 - m_2$ ($= \delta$) is small compared to m_1 for all the interesting processes 4.3, and furthermore E_ν is of order δ , as is μ . Thus to lowest order in δ (i.e. δ^2) only the term

$$(2m_1 E_\nu)^2 \alpha' + 2m_1 E_\nu (m_1^2 - m_2^2) \alpha' + \frac{m_1^2}{m_1} \left(2m_1^2 E_\nu^2 - m_1 E_\nu (m_1^2 - m_2^2) \right) \gamma' \quad 4.33$$

survives from 4.R11, and in particular only

$$|a|^2 - |b|^2 + \frac{(m_1 + m_2)^2}{m_1 m_2} |f|^2 \text{ from } \alpha'$$

$$\text{and } \frac{m_1 m_2}{(m_1 + m_2)^2} |a|^2 + \frac{(m_1 + m_2)^2}{m_1 m_2} |f|^2 \text{ from } \gamma'.$$

This is good enough for neutron beta-decay where $\frac{\delta}{m_1} \sim 10^{-3}$, but the annoying presence of the term $4m_1^2 \mu^2$ in Y makes the final E_ν integration awkward in this case (since μ^2 may not be neglected). For the Λ , Σ and Ξ electronic decays it is possible to take $\mu^2 = 0$, with the result that

and

$$\begin{aligned} \left(1 - 2\frac{E_\nu}{m_1}\right)X &= 2E_\nu(E_\nu - \delta\Delta) \\ \left(1 - 2\frac{E_\nu}{m_1}\right)Y &= 2E_\nu(\delta\Delta - E_\nu), \end{aligned}$$

where $2m_1\Delta = m_1 + m_2$. The integral over q^2 may now be evaluated to give

$$\Gamma \sim \frac{G^2}{16\pi^3} \frac{m_2}{m_1} \int_0^{\delta\Delta} U^{(1)} \cdot k_2 E_\nu (\delta\Delta - E_\nu) \left(1 - 2\frac{E_\nu}{m_1}\right)^{-1} dE_\nu. \quad 4.34$$

Since $\frac{\delta}{m_1} \sim 0.2$, $U^{(1)} \cdot k_2$ must be evaluated more accurately than in 4.33, the next term being required, to give

$$\Gamma \sim \frac{G^2}{\pi^3} \left\{ \left(1 + \frac{1}{2}\Delta\frac{\delta}{m_1}\right)\frac{\alpha^2}{30} - \frac{1}{2}\Delta\frac{\delta}{m_1}\frac{\beta^2}{24} - \left(1 + 2\Delta\frac{\delta}{m_1}\right)\frac{\gamma^2}{240} \right\} (\delta\Delta)^5,$$

which reduces to

$$\begin{aligned} \Gamma \sim \frac{G^2}{30\pi^3} \left\{ \left(1 - \frac{1}{32}\frac{\delta}{m_1}\right)|a|^2 + \left(1 + \frac{1}{32}\frac{\delta}{m_1}\right)|b|^2 + \left(1 - \frac{9}{32}\frac{\delta}{m_1}\right)|f|^2 \right. \\ \left. + \frac{11}{16}\frac{\delta}{m_1} \operatorname{Re}(ae^* + 4cf^*) \right\} \delta^5. \end{aligned}$$

(f) Result appendix (4.R)

4.R1

The transition probability is proportional to $K(k_2 - \mu S)$ where K is a combination of the following terms as in 4.8 :-

$$\begin{aligned} U_{\alpha}^{(1)} = & \left\{ (4|A|^2x + |B|^2xy^{-1} + 4|F|^2y + |D|^2)P \cdot k_1 P_{\alpha} \right. \\ & + (4|C|^2y - |D|^2yx^{-1} + 4|E|^2x - |B|^2)Q \cdot k_1 Q_{\alpha} \\ & - 2(|A|^2 - |C|^2 - |E|^2xy^{-1} + |F|^2yx^{-1})k_{1\alpha} \\ & + 4\operatorname{Re}(CF^*y + AE^*x)(P \cdot k_1 Q_{\alpha} + Q \cdot k_1 P_{\alpha}) \\ & \left. + 2\operatorname{Re}(BD^*)(P \cdot k_1 Q_{\alpha} - Q \cdot k_1 P_{\alpha}) \right\} \end{aligned}$$

$$U^{(2)} = 4\text{Im}(AE^*x - CF^*y)$$

$$\begin{aligned} V_{\alpha}^{(1)} = & \left\{ 4\text{Re}\left((B-A)D^* - (B-4Ay)F^*\right)xP \cdot k_1 P_{\alpha} + 8\text{Re}(AF^*y - CE^*x)k_1 \alpha \right. \\ & - 4\text{Re}\left((D-C)B^* - (D-4Cx)E^*\right)yQ \cdot k_1 Q_{\alpha} \\ & + 2\text{Re}\left((Ay - Ex)D^* - (Cx - Fy)B^* - 4(AC^* + EF^*)xy\right)(P \cdot k_1 Q_{\alpha} + Q \cdot k_1 P_{\alpha}) \\ & \left. + 2\text{Re}\left((A-B+E)B^*x + (C-D+F)D^*y\right)(P \cdot k_1 Q_{\alpha} - Q \cdot k_1 P_{\alpha}) \right\} \end{aligned}$$

$$V^{(2)} = 2\text{Im}\left((Ay + Ex)D^* + (Cx + Fy)B^* + 4(AC^* - EF^*)xy\right)$$

$$W_{\alpha}^{(1)} = 2\text{Re}\left((DE^* + BC^*)Q_{\alpha} + (BF^* + AD^*)P_{\alpha}\right)$$

$$W_{\alpha}^{(2)} = 2\text{Re}\left((AB^* + FD^*yx^{-1})Q_{\alpha} + (CD^* + EB^*xy^{-1})P_{\alpha}\right)$$

$$Y_{\alpha} = -2\text{Im}\left((EB^*x + CD^*y)Q_{\alpha} + (AB^*x + FD^*y)P_{\alpha}\right)$$

$$Z^{(1)} = 2\text{Im}(CB^* + ED^*)$$

$$Z^{(2)} = 2\text{Im}(AD^* + FB^*),$$

$$\text{with } m_1 m_2 x^{-1} = (m_1 + m_2)^2 - q^2 \quad \text{and} \quad m_1 m_2 y^{-1} = (m_1 - m_2)^2 - q^2.$$

4. R2

In terms of the Mandelstam variables s , t and u ($s = (p_1 + k_1)^2$, $t = (p_1 - p_2)^2$ and $u = (p_1 - k_2)^2$ with $p_1 + k_1 = p_2 + k_2$ and $s + t + u = m_1^2 + m_2^2 + \mu^2$)

$$\begin{aligned} m_1 m_2 U^{(1)} \cdot k_2 = & (s - m_2^2 - \mu^2) \left(\frac{m_1}{m_2} (m_2^2 - u) \{ (|A + E|^2 - |B|^2)_x \right. \\ & \left. + (|C + F|^2 - |D|^2)_y \} + (s - m_1^2) \{ (|A|^2 - |E|^2 - |B|^2)_x \right. \end{aligned}$$

$$\begin{aligned}
& - (|C|^2 - |F|^2 - |D|^2)y\} + (m_1^2 + \mu^2 - u) \left(\frac{m_2^2}{m_1} (s - m_1^2) \{ (|A - E|^2 \right. \\
& - |B|^2)x + (|C - F|^2 - |D|^2)y\} + (m_2^2 - u) \{ (|A|^2 - |E|^2 - |B|^2)x \\
& - (|C|^2 - |F|^2 - |D|^2)y\} \Big) - (\mu^2 - t)m_1 m_2 \left((|A|^2 - |C|^2) \right. \\
& - |E|^2 xy^{-1} + |F|^2 yx^{-1} \Big) + \frac{1}{2}|B \mp D|^2 (s - m_1^2)(s - m_2^2 - \mu^2) \\
& + \frac{1}{2}|B \mp D|^2 (m_2^2 - u)(m_1^2 + \mu^2 - u)
\end{aligned}$$

with $m_1 m_2 x^{-1} = (m_1 + m_2)^2 - t$ and $m_1 m_2 y^{-1} = (m_1 - m_2)^2 - t$.

4.R3

The expression required for the polarization $\underline{T} \cdot \underline{\hat{n}}$ (4.16) is

$$\begin{aligned}
2m_2 I &= (m_1^2 - m_2^2 + 4m_1 E_\nu) \left\{ \{ (m_1 + m_2)A - (m_1 - m_2)E \} B^* x \right. \\
&- \{ (m_1 - m_2)C - (m_1 + m_2)F \} D^* y \Big\} + m_2 q^2 \left\{ (A + E)B^* x \right. \\
&+ (C + F)D^* y \Big\} - \mu^2 \frac{m_2^2}{m_1} \left\{ (A - E)B^* x + (F - C)D^* y \right\} \\
&\mp 2m_1 m_2 \left\{ (m_1 - m_2)C + (m_1 + m_2)F \right\} B^* \\
&\mp 2m_1 m_2 \left\{ (m_1 + m_2)A + (m_1 - m_2)E \right\} D^* .
\end{aligned}$$

4.R4

The unpolarized cross-section is given by $\frac{G^2}{2\pi} \frac{U^{(1)} \cdot k_2 m_1 m_2}{(2m_1 E_\nu)^2}$

in the lab. frame, where

$$\begin{aligned}
& m_1 m_2 U^{(1)} \cdot k_2 (p_1 = 0) \\
&= (2m_1 E_\nu)^2 \alpha + 2m_1 E_\nu \left\{ (m_1^2 - m_2^2) \alpha - 2\mu^2 \beta + q^2 (\alpha \mp 2\text{Re}(BD^*)) \right\} \\
&+ (q^2 - \mu^2) \left\{ (m_1^2 - m_2^2) (\beta + \frac{1}{2}|B \mp D|^2) + m_1 m_2 (|A|^2 - |C|^2) \right. \\
&+ (m_1 - m_2)^2 |E|^2 x + (m_1 + m_2)^2 |F|^2 y - \mu^2 \frac{m_1}{m_2} \left\{ (|A + E|^2 - |B|^2)x \right. \\
&+ (|C + F|^2 - |D|^2)y \Big\} + q^2 \left\{ (|A|^2 - |B|^2)x - (|C|^2 - |D|^2)y \right. \\
&+ \frac{1}{2}|B \mp D|^2 \Big\} \Big\} ,
\end{aligned}$$

with

$$m_1 m_2 \alpha = |(m_1 + m_2)A + (m_1 - m_2)E|^2 x + |(m_1 - m_2)C + (m_1 + m_2)F|^2 y \\ - q^2 |B|^2 x - q^2 |D|^2 y$$

and

$$m_2 \beta = (m_1 + m_2) \left((|A|^2 - |B|^2)x + |F|^2 y \right) + (m_1 - m_2) \left((|C|^2 - |D|^2)y \right. \\ \left. + |E|^2 x \right) + 2m_1 \operatorname{Re}(AE^* x + CF^* y) .$$

4.5R5

The expression required for the polarization $\underline{T}_0 \hat{t}$ (4.18) is

$$J = (m_1^2 - m_2^2 + 4m_1 E_\nu) \left\{ \left((m_1 + m_2)A - (m_1 - m_2)E \right) D^* \right. \\ \left. + \left((m_1 + m_2)F - (m_1 - m_2)C \right) B^* \right\} + q^2 \left\{ \left((m_1 + m_2)A - (m_1 - m_2)E \right) D^* \right. \\ \left. + \left((m_1 + m_2)F - (m_1 - m_2)C \right) B^* + (m_1 + m_2)(AB^* + FD^* y x^{-1}) \right. \\ \left. + (m_1 - m_2)(CD^* + EB^* x y^{-1}) \right\} + 2m_1 \mu^2 \left((A + E)D^* + (C + F)B^* \right) \\ + (m_1^2 - m_2^2) \left((m_1 + m_2)(CD^* + EB^* x y^{-1}) - (m_1 - m_2)(AB^* + FD^* y x^{-1}) \right) .$$

4.6R6

The baryon polarization 4-vector is the sum of the following terms as indicated in 4.24

$$\Sigma^{(1)}_\alpha = (P \cdot k_1 Q \cdot k_2 + P \cdot k_2 Q \cdot k_1) \left(BC^* x P_\alpha + AD^* y Q_\alpha + (BD^* - AC^* - EF^*)(x P_\alpha - y Q_\alpha) \right) \\ - BC^* (2Q \cdot k_1 Q \cdot k_2 y Q_\alpha + Q \cdot k_1 k_2_\alpha + Q \cdot k_2 k_1_\alpha) \\ - AD^* (2P \cdot k_1 P \cdot k_2 x P_\alpha + P \cdot k_1 k_2_\alpha + P \cdot k_2 k_1_\alpha) \\ + \frac{1}{2} BD^* (-P \cdot k_1 P \cdot k_2 Q^2 + Q \cdot k_1 Q \cdot k_2 P^2)(x P_\alpha - y Q_\alpha) \\ \pm \frac{1}{4} (|B|^2 + |D|^2) (P \cdot k_1 Q \cdot k_2 - Q \cdot k_1 P \cdot k_2) (P_\alpha - Q_\alpha) \\ \pm |B|^2 (P \cdot k_1 k_2_\alpha - P \cdot k_2 k_1_\alpha) \pm |D|^2 (Q \cdot k_1 k_2_\alpha - Q \cdot k_2 k_1_\alpha)$$

$$\begin{aligned}\Sigma^{(2)}_{\alpha} = & (A - B)F^{*}(P.k_1 P.k_2 - P^2 k_1.k_2)(xP_{\alpha} - yQ_{\alpha}) \\ & + (C - D)E^{*}(Q.k_1 Q.k_2 - Q^2 k_1.k_2)(xP_{\alpha} - yQ_{\alpha}) \\ & + ED^{*}\left\{(2Q.k_1 Q.k_2 - P.k_1 Q.k_2 - P.k_2 Q.k_1)xP_{\alpha} - k_1.k_2 Q^2(xP_{\alpha} \right. \\ & \left. - yQ_{\alpha}) + (Q.k_1 k_{2\alpha} + Q.k_2 k_{1\alpha})\right\} - FB^{*}\left\{(2P.k_1 P.k_2 - P.k_1 Q.k_2 \right. \\ & \left. - P.k_2 Q.k_1)yQ_{\alpha} + k_1.k_2 P^2(xP_{\alpha} - yQ_{\alpha}) - (P.k_1 k_{2\alpha} + P.k_2 k_{1\alpha})\right\}\end{aligned}$$

$$\begin{aligned}\Sigma^{(3)} = & \pm \left\{(m_1 - m_2)CB^{*} - (m_1 + m_2)AD^{*}\right\} + 2AB^{*}xP.(k_1 + k_2) \\ & + 2CD^{*}yQ.(k_1 + k_2)\end{aligned}$$

$$\begin{aligned}\Sigma^{(4)} = & \pm \left\{(m_1 + m_2)FB^{*} - (m_1 - m_2)ED^{*}\right\} - 2FD^{*}yP.(k_1 + k_2) \\ & - 2EB^{*}xQ.(k_1 + k_2) \quad .\end{aligned}$$

4.R7

The expression for $\hat{\Sigma}_R \cdot \hat{t}$ (4.25) is given in terms of L :-

$$\begin{aligned}2m_1 m_2 L = & (m_1^2 - m_2^2 + 4m_1 E_{\nu}) \left\{ \left\{ (m_1 + m_2)A - (m_1 - m_2)E \right\} D^{*} \right. \\ & + \left\{ (m_1 + m_2)F + (m_1 - m_2)C \right\} B^{*} \left. \right\} \mp (m_1^2 - m_2^2) \left\{ (m_1 + m_2)|B|^2 \right. \\ & + (m_1 - m_2)|D|^2 \left. \right\} - m_1 \mu^2 \left\{ (A - E)D^{*} + (F + C)B^{*} \pm |B|^2 \mp |D|^2 \right\} \\ & + Q^2 \left\{ \left\{ (m_1 + m_2)A - (m_1 - m_2)E \right\} D^{*} + \left\{ (m_1 + m_2)F + (m_1 \right. \right. \\ & \left. \left. - m_2)C \right\} B^{*} \mp (m_1 - m_2)|B|^2 \mp (m_1 + m_2)|D|^2 \right\} .\end{aligned}$$

4.R8

The expression for $\hat{\Sigma}_R \cdot \hat{n}$ (4.26) is given in terms of M :-

$$\begin{aligned}m_1 m_2 M = & (m_1^2 - m_2^2 + 4m_1 E_{\nu}) \left\{ \left\{ (m_1 + m_2)A + (m_1 - m_2)E \right\} B^{*} x \right. \\ & - \left\{ (m_1 + m_2)F + (m_1 - m_2)C \right\} D^{*} y \left. \right\} \mp m_1 m_2 \left\{ \left\{ (m_1 + m_2)A \right. \right. \\ & \left. \left. + (m_1 - m_2)E \right\} D^{*} - \left\{ (m_1 + m_2)F + (m_1 - m_2)C \right\} B^{*} \right\}\end{aligned}$$

$$\begin{aligned}
& -\mu^2 m_1 \left\{ \left((m_1 + m_2)A - (m_1 - m_2)E \right) B^* x - \left((m_1 + m_2)F \right. \right. \\
& \quad \left. \left. - (m_1 - m_2)C \right) D^* y \right\} + q^2 \left\{ \left((m_1 + m_2)A + (m_1 - m_2)E \right) B^* x \right. \\
& \quad \left. - \left((m_1 + m_2)F + (m_1 - m_2)C \right) D^* y \right\} .
\end{aligned}$$

4cR9

For the integral 4c30

$$\begin{aligned}
F(q^2) = & \left\{ \left| (m_1 - m_2)A - (m_1 + m_2)E x y^{-1} \right|^2 - (m_1 - m_2)^2 |B|^2 \right\} x^{-1} (q^2 + 2\mu^2) \\
& + \left\{ \left| (m_1 + m_2)C - (m_1 - m_2)F y x^{-1} \right|^2 - (m_1 + m_2)^2 |D|^2 \right\} y^{-1} (q^2 + 2\mu^2) \\
& + \left(|C|^2 - |A|^2 + 2|E|^2 x y^{-1} + 2|F|^2 y x^{-1} \right) q^2 (2q^2 + \mu^2) \\
& + 2(|D|^2 - |B|^2) q^2 (q^2 - \mu^2) .
\end{aligned}$$

4cR10

The coefficients α' , β' , γ' and δ' for 4c32 are

$$\begin{aligned}
\alpha' = & \left| a + \frac{(m_1 - m_2)}{(m_1 + m_2)} e \right|^2 + \frac{(m_1 + m_2)^2}{m_1 m_2} \left| \frac{(m_1 - m_2)}{(m_1 + m_2)} c + f \right|^2 - |b|^2 \\
& - \frac{(m_1 - m_2)^2}{m_1 m_2} |d|^2
\end{aligned}$$

$$\begin{aligned}
\beta' = & \left(1 + \frac{m_1}{m_2} \right) \left(\frac{m_1 m_2}{(m_1 + m_2)^2} (|a|^2 - |b|^2) + |f|^2 \right) - \left(1 - \frac{m_1}{m_2} \right) (|c|^2 - |d|^2) \\
& + \frac{m_1 m_2}{(m_1 + m_2)^2} |e|^2 + 2 \frac{m_1}{m_2} \operatorname{Re} \left(\frac{m_1 m_2}{(m_1 + m_2)^2} a e^* + c f^* \right)
\end{aligned}$$

$$\begin{aligned}
\gamma' = & \frac{m_1 m_2}{(m_1 + m_2)^2} |a|^2 + \frac{(m_1 - m_2)^2}{m_1 m_2} (|e|^2 - |c|^2) + \frac{(m_1 + m_2)^2}{m_1 m_2} |f|^2 \\
& - \frac{\mu^2}{m_2^2} \left(\frac{m_1 m_2}{(m_1 + m_2)^2} (|a + e|^2 - |b|^2) + (|c + f|^2 - |d|^2) \right)
\end{aligned}$$

$$\delta' = \frac{m_1 m_2}{(m_1 + m_2)^2} (|a|^2 - |b|^2) + |d|^2 .$$

4.R11On performing $\int dq^2$ in 4.31

$$\begin{aligned}
U^{(1)} \cdot k_2 m_1 m_2 \rightarrow & (2m_1 E_\nu)^2 \alpha^i + 2m_1 E_\nu \left\{ 2m_1^2 E_\nu^2 - m_1 E_\nu \left(3(m_1^2 - m_2^2) - \mu^2 \right) \right. \\
& + m_1^2 (m_1^2 - m_2^2 - \mu^2) \left. \right\} (m_1^2 - 2m_1 E_\nu)^{-1} - 4m_1 E_\nu \mu^2 \beta^i \\
& + \left\{ (m_1^2 - m_2^2) \delta^i + m_1 m_2 \gamma^i \right\} \left\{ 2m_1^2 E_\nu^2 - E_\nu m_1 \left((m_1^2 - m_2^2) - 3\mu^2 \right) - 2m_1^2 \mu^2 \right\} (m_1^2 - 2m_1 E_\nu)^{-1} \\
& - \mu^2 \left(2m_1^2 E_\nu^2 - E_\nu m_1 \left\{ (m_1^2 - m_2^2) - \mu^2 \right\} - m_1^2 \mu^2 \right) \delta^i \\
& \quad \times (m_1^2 - 2m_1 E_\nu)^{-1} \\
& + \frac{1}{3} \left\{ 16m_1^4 E_\nu^4 - 16m_1^3 E_\nu^3 (m_1^2 - m_2^2 - \mu^2) + 4m_1^2 E_\nu^2 \left(\right. \right. \\
& \quad (m_1^2 - m_2^2 - \mu^2)^2 - 4m_1^2 \mu^2 \left. \right) + 6E_\nu m_1^3 \mu^2 (m_1^2 - m_2^2 - \mu^2) \\
& \quad \left. \left. + m_1^4 \mu^2 \right\} \delta^i (m_1^2 - 2m_1 E_\nu)^{-2} .
\end{aligned}$$

5. THE INELASTIC PROCESS $\nu + B \Rightarrow B^* + l$

The processes $\nu(\bar{\nu}) + B \Rightarrow B^* + l$ are studied in a similar manner to the elastic processes of the previous chapter, and it is shown that there exist analogous tests of time reversal invariance. Again the emphasis is on a detailed phenomenological study of the reactions

$$\nu + n \Rightarrow N^{*+} + l^- \quad 5.1a$$

$$\nu + p \Rightarrow N^{*++} + l^- \quad 5.1b$$

$$\bar{\nu} + p \Rightarrow N^{*0} + l^+ \quad 5.1c$$

$$\bar{\nu} + n \Rightarrow N^{*-} + l^+ \quad 5.1d$$

and the SU(3) symmetric counterparts

$$\nu + N \Rightarrow Y^* + l^- \quad 5.2a$$

$$\bar{\nu} + N \Rightarrow Y^* + l^+ \quad 5.2b$$

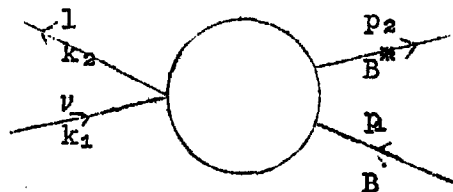
The formalism as developed also applies to the $\nu + Y \Rightarrow Y^* + l$ processes and the leptonic decays $B^* \Rightarrow B + \nu + l$, which includes, among others, the Ω^- leptonic decay.

(a) The transition probability

The invariant matrix element for these reactions is of the form

$$M = \lambda \sqrt{\frac{1}{2}} G \bar{u}(k_2) \gamma_\alpha (1 \mp i\gamma_5) u(k_1) \bar{u}_\beta(p_2) (V_{\beta\alpha} i\gamma_5 + A_{\beta\alpha}) u(p_1) \quad 5.3$$

in momentum space, using 3.6 and 3.14, where the connection between particle and momenta is given by



Averaging over initial spin states and summing over final spin states ($\Sigma\Sigma$) leads to

$$\frac{\Sigma\Sigma}{\text{IF}} |M|^2 = \frac{1}{2} \lambda^2 G^2 8(k_2 \cdot L)_{\mu\nu} H_{\mu\nu} \quad 5.4$$

where

$$H = \frac{1}{2} \text{Tr} \left\{ (V_{\beta\mu} i\gamma_5 + A_{\beta\mu}) (\not{p}_1 + m_1) (\overline{V}_{\alpha\nu} i\gamma_5 + \overline{A}_{\alpha\nu}) \Psi_{\alpha\beta}(p_2, 2j=3) \right\} \quad 5.5$$

with

$$\overline{S} = \gamma_0 S^\dagger \gamma_0$$

and

$$\Psi_{\alpha\beta} = (\Phi_{\alpha\beta} - \frac{1}{3} (\Phi \cdot \gamma)_\alpha (\Phi \cdot \gamma)_\beta) (\not{p}_2 + m_2), \quad \text{the spin-} \frac{3}{2} \text{ projection}$$

operator obtained from 2.6, with $\Phi_{\alpha\beta} = \xi_{\alpha\beta} - p_{2\alpha} p_{2\beta} (m_2)^{-2}$.

This form of $\Psi_{\alpha\beta}$ is much easier to handle in trace evaluations than the "standard" form

$$\Psi_{\alpha\beta} = \left(\xi_{\alpha\beta} - \frac{1}{3} \gamma_\alpha \gamma_\beta + \frac{1}{3m_2^2} (p_{2\alpha} \not{p}_2 \gamma_\beta + p_{2\beta} \gamma_\alpha \not{p}_2) - \frac{4}{3} p_{2\alpha} p_{2\beta} \right) (\not{p}_2 + m_2).$$

Here the initial baryon is assumed to be unpolarized so that manageable formulae may be obtained, though mention is made of polarizations (briefly) in section 5 of this chapter. The λ factor is included in the above so that the various reactions 5.1 can be classified together with only this coupling constant differing. On performing the trace calculation 5.4 reduces to

$$\begin{aligned} \frac{\Sigma\Sigma}{\text{IF}} |M|^2 &= 4G^2 \lambda^2 k_2 \cdot X_\alpha \\ &= 4G^2 \lambda^2 \frac{4}{3} \left\{ p_1 \cdot k_1 p_1 \cdot k_2 X^{(1)}(t) + p_2 \cdot k_2 p_2 \cdot k_1 X^{(2)}(t) \right. \\ &\quad + (p_2 \cdot k_1 p_1 \cdot k_2 + p_1 \cdot k_1 p_2 \cdot k_2) X^{(3)}(t) + k_1 \cdot k_2 X^{(4)}(t) \\ &\quad \left. + (p_2 \cdot k_1 p_1 \cdot k_2 - p_1 \cdot k_1 p_2 \cdot k_2) X^{(5)}(t) \right\}, \quad 5.6 \end{aligned}$$

where the $X^{(l)}(t)$ are listed in 5.R1 and 5.R2 for the two choices of

V and A given in 3.15 and 3.16. $X^{(5)}(t)$ contains only V-A interference terms.

(b) Differential cross-section

The invariant differential cross-section for B^* production is

$$\frac{d\sigma}{dt} = \lambda^2 \frac{G^2}{2\pi} \frac{k_2 \cdot X}{(s - m_1^2)^2}, \quad 5.7$$

assuming all particles are unpolarized (excepting the neutrino, of course, which is completely polarized) and where the coefficients of the $X^{(i)}(t)$ are to be expressed in terms of s and t (see 5.R3).

With the choice

$$\begin{aligned} V_{\beta\alpha} &= a_1 g_{\beta\alpha} + q_\beta (a_2 \gamma_\alpha + a_3 P'_\alpha + a_4 q_\alpha) \\ A &= a'_1 g_{\beta\alpha} + q_\beta (a'_2 \gamma_\alpha + a'_3 P'_\alpha + a'_4 q_\alpha), \end{aligned} \quad 5.8$$

(where now $P' = p_1 + p_2$), $\frac{d\sigma}{dt}$ is seen to be a combination of 24 terms $\text{Re}(a_i a_j^*)$ of the eight form factors . Little can be said about the behaviour of the differential cross-section until some assumptions are made for the t-dependence of the functions a_i . However for small momentum transfers $\frac{d\sigma}{dt}$ depends only on the normalisation of the a_i at $t = 0$. In the limit $t \rightarrow 0$ it takes the form

$$\begin{aligned} \frac{d\sigma}{dt}(t=0) &= \frac{\lambda^2 G^2}{12\pi} \frac{(s - m_2^2)^2}{(s - m_1^2)^2} \left\{ \left(1 - \frac{m_1}{m_2}\right)^2 |a_1 - (m_1 + m_2)(a_2 + (m_2 - m_1)a_3)|^2 + \left(1 + \frac{m_1}{m_2}\right)^2 |a'_1 - (m_2 - m_1)(a'_2 + (m_1 + m_2)a'_3)|^2 \right\} \\ & \quad 5.9 \end{aligned}$$

when terms in μ^2 ($\mu =$ the lepton mass) are neglected. This is the

reason for the non-occurrence of the doubly-induced form factors a_1 and a'_4 (it is worth noting that the 'same' form factors - those multiplying the q_α currents - in the elastic processes also disappeared in this approximation). It is interesting to observe that if there is a C. V. C. then the first term in 5.9 vanishes, for $q_\alpha V_{\beta\alpha}$ implies that

$$a_1 - (m_1 + m_2)a_2 - (m_2^2 - m_1^2)a_3 + q^2 a_4 = 0 \text{ for all } q^2,$$

and at $t = 0$ only terms in the axial-vector coupling constants are present - just the divergence of the axial-vector current in fact.

For the other choice of form factors suggested in chapter 3, viz.

$$\begin{aligned} V_{\beta\alpha} &= q_\beta (d_1 P'_\alpha + d_2 q_\alpha + d_3 \epsilon_{\alpha\mu\nu\rho} P'_\mu q_\nu \gamma_5 \gamma_\rho) + d_4 \epsilon_{\beta\alpha\mu\nu} P'_\mu q_\nu \gamma_5 \\ A_{\beta\alpha} &= q_\beta (d'_1 P'_\alpha + d'_2 q_\alpha + d'_3 \epsilon_{\alpha\mu\nu\rho} P'_\mu q_\nu \gamma_5 \gamma_\rho) + d'_4 \epsilon_{\beta\alpha\mu\nu} P'_\mu q_\nu \gamma_5, \end{aligned}$$

5.10

$\frac{d\sigma}{dt}$ is a combination of 16 terms $\text{Re}(d_i d_j^*)$ of the eight form factors.

In view of the connection between the a_i and the d_i , the invariant differential cross-section at low (zero) q^2 now takes the very concise form

$$\frac{d\sigma}{dt}(0) = \frac{\lambda^2 G^2}{12\pi} \frac{(s - m_2^2)}{(s - m_1^2)} \frac{(m_2^2 - m_1^2)^2}{m_2^2} \left\{ (m_2 - m_1)^2 |d_1|^2 + (m_1 + m_2)^2 |d'_1|^2 \right\}$$

5.11

as may be seen from 5.R2 or by using 3.18 and 5.9. Whilst it would be difficult to assess the value of any one of the a_i from 5.9, 5.11 seems to give a means of finding $|d'_1|$ if $|d_1|$ is comparable to $|d'_1|$, which does not appear to be an unreasonable assumption. Indeed $\frac{d\sigma}{dt}$ for $t \neq 0$ takes on a very much shorter form written in terms of the

d_1 form factors. At very high energies and all momentum transfers

$$\frac{d\sigma}{dt}(s \rightarrow \infty, t) \sim \frac{\lambda^2 G^2}{12\pi} \left\{ \frac{1}{m_2^2} \left((m_1 + m_2)^2 - t \right) \left((m_1 - m_2)^2 - t \right) \left(\{ (m_1 - m_2)^2 - t \} |d_1|^2 + \{ (m_1 + m_2)^2 - t \} |d'_1|^2 \right) + t E m_1 m_2 \right\}, \quad 5.12$$

thus it is not difficult to find the high energy behaviour of the total cross-section for specific choices of d_1 and d'_1 . For example, with the most "popular" type of behaviour $d_1(t) \sim d_1(0)(1 - tb^{-1})^{-n}$, the total cross-section increases quartically, quadratically or logarithmically with s according as $n = 0, 1$ or 2 . Experimental data does not give a very good guide at the moment, but if b is suitably chosen then the logarithmic behaviour could not be ruled out (σ would behave as $b^4 \log(\frac{s}{b})$, so this contribution would be heavily damped). Indeed with the factors $\left((m_1 \pm m_2)^2 - t \right)$ occurring one could conjecture $|d_1| \sim [(m_1 - m_2)^2 - t]^{-1} - [(m_1 + m_2)^2 - t]^{-1}$ and still obtain a reasonable fit - this is just to demonstrate how unsatisfactory the present situation is. Indications of the high energy behaviour of the total cross-section for various choices of n are given by Albright and Liu⁽⁸⁰⁾ for the currents defined by 5.8. Here for completeness this behaviour is given for the currents 5.10 (see table 4).

As in the elastic case, experiments with a polarized target would lead to a test of time reversal invariance in these inelastic reactions. The differential cross-section for the processes with the initial baryon polarized transversely (i.e. perpendicular to the production plane) is dependent solely on the imaginary parts of the

	n = 1	n = 2	n = 2	n = 1
(1,1)	$\frac{1}{6}a^2 s^2$	$a^4 \log\left(\frac{s}{a}\right)$		
(1,2)	$\frac{1}{3}a^2 s \mu^2$	$a^4 \frac{\mu^2}{s} \log\left(\frac{s}{a}\right)$		
(2,2)	$\frac{1}{12}a^2 s \mu^2$	$\frac{1}{4}a^4 \frac{\mu^2}{s}$		
(3,3)	$\frac{11}{60}a^2 s^3$	$\frac{2}{3}a^4 s$	$\frac{1}{6}a^2 m_1^2 m_2^2 \alpha$	$\frac{2}{3}a^2 m_1^2 m_2^2 s$
(3,4)	$\frac{7}{12}a^2 s^2 m_2$	$2a^4 m_2 \log\left(\frac{s}{a}\right)$	$\frac{1}{3}a^2 m_1 m_2^2 \alpha$	$\frac{4}{3}a^2 m_1 m_2^2 s$
(4,4)	$\frac{8}{3}a^2 s m_2^2$	$\frac{2}{3}a^2 m_2^2 \alpha$		
(5,5)	= (1,1)	= (1,1)		
(5,6)	= (1,2)	= (1,2)		
(6,6)	= (2,2)	= (2,2)		
(7,7)	= (3,3)	= (3,3)	$\frac{1}{6}a^2 m_1^2 m_2^2 \beta$	= (3,3)
(7,8)	= (3,4)	= (3,4)	$\frac{1}{3}a^2 m_1 m_2^2 \beta$	= (3,4)
(8,8)	= (4,4)	$\frac{2}{3}a^2 m_2^2 \beta$		
(3,7)	$\frac{3a^2 s^3}{10m_1 m_2}$	$\frac{2a^4 s}{3m_1 m_2}$	$2m_1 m_2 \frac{a^4}{s} \log\left(\frac{s}{a}\right)$	$\frac{2}{3}a^2 m_1 m_2 s$
(3,8)	$\frac{5a^2 s^2}{12 m_1}$	$\frac{3a^4}{2m_1}$	$2m_2 \frac{a^4}{s} \log\left(\frac{s}{a}\right)$	$\frac{2}{3}a^2 m_2 s$
(4,7)	= (3,8)	= (3,8)	= (3,8)	= (3,8)
(4,8)	$\frac{4m_2 a^2 s}{3m_1}$	$\frac{4a^4 m_2}{s m_1} \log\left(\frac{s}{a}\right)$		

with $d_i(t) = d_i(0)(1-t/a)^{-n}$, all i . $\left\{ \frac{d^2 d_s}{P^2 d'_s} \right\} = \frac{d_s}{d'_s} (1-t/a)^{-n}$, others as before.

TABLE 4

$\sigma(s) \sim \frac{G^2}{12\pi} d_i(0) d_j(0) \times \{(i,j) \text{ entry above}\}$ at high energies,

where (1,5), for example, refers to d_1 and d'_1 , and $\frac{\alpha}{\beta} = 2a + (m_1 \pm m_2)^2$.

products of the form factors $a_i a_j^*$ or $d_i d_j^*$. For both choices of form factors this cross-section is a function of 22 of the possible 28 combinations, and it is because of the large number of terms that the expression is not given here, it being sufficient to indicate the possibility of testing time reversal invariance.

(c) Lepton polarization

Since the matrix element of the reaction splits into a lepton current and a hadron current ($M \propto L_\beta H_\beta$ from 5.3), the density matrix of the final lepton is of the form (see 2.33b)

$$\rho(1) \propto (k_2 + \mu) \gamma_\alpha (1 \mp i\gamma_5) k_1 \gamma_\beta (1 \mp i\gamma_5) (k_2 + \mu) H_{\alpha\beta}, \quad 5.13$$

where $H_{\alpha\beta}$ is given explicitly in equation 5.4. It is then a simple matter to find the polarization 4-vector for this final lepton using 2.28, 2.10 and 5.12 :

$$P_\mu(1) = \text{Tr} \left(\frac{1}{2} i\gamma_5 \sigma_{\mu\nu} k_{2\nu} \rho(1) \right),$$

leading to

$$P_\mu(1) k_{2\cdot} X = \pm \frac{k_{2\cdot} X}{\mu} k_{2\mu} \mp \mu X_\mu - \langle (\mu) p_1 k_1 p_2 \rangle Y, \quad 5.14$$

where $k_{2\cdot} X$ and X_μ are listed in 5.6, 5.R1 and 5.R2 and Y is the imaginary part of the combinations of form factors as given in 5.R4 and 5.R5 - most simply given in terms of the d_i .

The presence of the term $\langle (\mu) p_1 k_1 p_2 \rangle$ in 5.14 leads to a test of T , as in the elastic case, owing to the entry of imaginary parts (only) in Y . Y is seen to depend only on those terms entering the

divergences of the vector and axial-vector currents, and if the vector current is conserved

$$Y \propto \text{Im}(d'_1 d'^2) .$$

If there is a C. V. C. and a P. C. A. C. then a measurement of this transverse polarization will give some idea of the size of $\text{Im}(d'_1)$ and $\text{Im}(d'^2)$ (if time reversal invariance is violated).

(d) Baryon resonance polarization

Making use of the results of chapter 2, and in particular 2.32b and 2.13, the polarization 4-vector of the baryon resonance can be found after a somewhat lengthy and tedious calculation. The dependence on the eight form factors present in the currents gives the polarization vector in a complicated form, from which only very general observations can be made. Since these observations can be inferred from a study of the most general expression for the B^* spin density matrix - and its resulting polarizations -, they will be derived in this fashion here.

The four possible magnetic substates of a spin three-half particle corresponding to the values $2m = \pm 3, \pm 1$ of the spin component in the direction s_μ are

$$\psi_\mu(3,3) = -\frac{1}{2} t_\mu^* (t_\nu^* \psi_\nu)$$

$$\psi_\mu(3,1) = \frac{\sqrt{6}}{6} (i\gamma_5 t_\mu^* - t_\mu^* s_\mu) (t_\nu^* \psi_\nu)$$

$$\psi_\mu(3,-1) = \frac{\sqrt{3}}{6} (2i\gamma_5 s_\mu + t_\mu^* t_\mu^*) (t_\nu^* \psi_\nu)$$

$$\psi_\mu(3,-3) = \frac{1}{2} i\gamma_5 t_\mu^* (t_\nu^* \psi_\nu)$$

as found from chapter 2, where t and t^* are orthogonal to s and to the momentum p of the particle ($p \cdot s = 0$) and the wave functions $\psi(2j, 2m)$ refer to the particle of spin j and s component m .

The most general form of the density matrix of the spin $3/2$ particle can then be found using

$$\rho_{\mu\nu} = \sum_{mn} \rho_{2m, 2n} \psi_{\mu}(p, m) \bar{\psi}_{\nu}(p, n), \quad 5.16$$

where $2m$ and $2n$ take the values $+3, +1, -1, -3$ and $\rho_{\mu\nu}$ is hermitian ($\rho_{\mu\nu}^\dagger = \gamma_0 \rho_{\nu\mu} \gamma_0$) so that $\rho_{2m, 2n}^* = \rho_{2n, 2m}$, with the result that

$$\rho_{\mu\nu} = \frac{1}{2} \rho_{\mu\nu}' (\not{p} + m),$$

where

$$\begin{aligned} \rho_{\mu\nu}' = & \left(\rho_{3,3} + \frac{1}{3} \rho_{1,1} \right) t_{\mu} t_{\nu}^* + \left(\rho_{-3,-3} + \frac{1}{3} \rho_{-1,-1} \right) t_{\mu}^* t_{\nu} + \frac{2}{3} \left(\rho_{1,1} + \rho_{-1,-1} \right) s_{\mu} s_{\nu} \\ & + \frac{1}{\sqrt{3}} \left\{ \left(\rho_{-1,3} - \rho_{1,-3}^* \right) t_{\mu}^* t_{\nu}^* + \text{h.c.} \right\} + \frac{\sqrt{2}}{\sqrt{3}} \left\{ \left(\rho_{1,3} s_{\mu} t_{\nu}^* + \rho_{-1,-3} s_{\mu} t_{\nu} \right) \right. \\ & \left. + \text{h.c.} \right\} + \frac{\sqrt{2}}{3} \left\{ \left(\rho_{-1,1} (s_{\mu} t_{\nu} - t_{\mu} s_{\nu}) \right) + \text{h.c.} \right\} \\ & + i \gamma_5 \left\{ \left(\rho_{3,3} - \frac{1}{3} \rho_{1,1} \right) t_{\mu} t_{\nu}^* + \left(-\rho_{-3,-3} + \frac{1}{3} \rho_{-1,-1} \right) t_{\mu}^* t_{\nu} \right. \\ & \left. + \frac{2}{3} \left(\rho_{1,1} - \rho_{-1,-1} \right) s_{\mu} s_{\nu} + \frac{1}{\sqrt{3}} \left(\left(\rho_{-1,3} + \rho_{1,-3}^* \right) t_{\mu}^* t_{\nu}^* + \text{h.c.} \right) \right. \\ & \left. + \frac{\sqrt{2}}{\sqrt{3}} \left(\left(\rho_{1,3} s_{\mu} t_{\nu}^* - \rho_{-1,-3} s_{\mu} t_{\nu} \right) + \text{h.c.} \right) + \frac{\sqrt{2}}{3} \left(\rho_{-1,1} (s_{\mu} t_{\nu} + t_{\mu} s_{\nu}) + \text{h.c.} \right) \right\} \\ & + i \gamma_5 \gamma_{\alpha} \left\{ t_{\mu}^* \left(-\sqrt{2} \rho_{-3,3} t_{\nu}^* + \frac{\sqrt{2}}{3} \rho_{-1,1} (t_{\mu} t_{\nu} - 2 s_{\mu} s_{\nu}) + \frac{\sqrt{2}}{\sqrt{3}} (\rho_{1,3} t_{\mu} t_{\nu}^* \right. \right. \\ & \left. \left. - \rho_{-1,-3} t_{\mu}^* t_{\nu}) - \frac{2}{\sqrt{3}} (\rho_{-1,3} s_{\mu} t_{\nu}^* + \rho_{1,-3}^* t_{\mu}^* s_{\nu}) + \frac{2}{3} (\rho_{1,1} t_{\mu} s_{\nu} \right. \right. \\ & \left. \left. - \rho_{-1,-1} s_{\mu} t_{\nu}) \right) + \text{h.c.} \right\}, \quad 5.17 \end{aligned}$$

where complex conjugation and interchange of μ and ν is represented by h.c.,

for example

$$\begin{aligned} t_{\mu} t_{\nu}^{*} &\xrightarrow{h.c.} t_{\mu} t_{\nu}^{*} \\ t_{\alpha}^{*} t_{\mu} t_{\nu} &\xrightarrow{h.c.} t_{\alpha}^{*} t_{\mu}^{*} t_{\nu}^{*} \end{aligned}$$

This form is seen to automatically satisfy

$$p_{\mu} \rho_{\mu\nu} = \rho_{\mu\mu} p_{\mu} = 0$$

and

$$\gamma_{\mu} \rho_{\mu\nu} = \rho_{\mu\mu} \gamma_{\mu} = 0$$

by virtue of the specific wave functions 5.15, and contains in general 16 independent elements $\rho_{2m, 2n}$. It reduces in the rest frame of the spin 3/2 particle (87) to

$$\begin{aligned} \rho_{IJ}/m &= A_1 z_{IzJ} + A_2 x_{IxJ} + A_3 y_{IyJ} + A_4 (x_{IyJ} + y_{IxJ}) \\ &+ iB_1 (x_{IyJ} - y_{IxJ}) + B_2 (x_{IzJ} + z_{IxJ}) + iA_5 (x_{IzJ} - z_{IxJ}) \\ &+ B_3 (y_{IzJ} + z_{IyJ}) + iA_6 (y_{IzJ} - z_{IyJ}) \\ &+ \sigma_z \{ B_4 z_{IzJ} + B_5 x_{IxJ} + B_6 y_{IyJ} + B_7 (x_{IyJ} + y_{IxJ}) \\ &+ iA_7 (x_{IyJ} - y_{IxJ}) + A_8 (x_{IzJ} + z_{IxJ}) + iB_8 (x_{IzJ} - z_{IxJ}) \\ &+ A_9 (y_{IzJ} + z_{IyJ}) + iB_9 (y_{IzJ} - z_{IyJ}) \} \\ &+ \sigma_x \{ A_{10} z_{IzJ} + A_{11} x_{IxJ} + A_{12} y_{IyJ} + A_{13} (x_{IyJ} + y_{IxJ}) \\ &+ iB_{10} (x_{IyJ} - y_{IxJ}) + B_{11} (x_{IzJ} + z_{IxJ}) + iA_{14} (x_{IzJ} \\ &- z_{IxJ}) + B_{12} (y_{IzJ} + z_{IyJ}) + iA_{15} (y_{IzJ} - z_{IyJ}) \} \\ &+ \sigma_y \{ A_{16} z_{IzJ} + A_{17} x_{IxJ} + A_{18} y_{IyJ} + A_{19} (x_{IyJ} + y_{IxJ}) \end{aligned}$$

$$\begin{aligned}
& + iB_{13}(x_1 y_J - y_1 x_J) + B_{14}(x_1 z_J + z_1 x_J) + iA_{20}(x_1 z_J \\
& - z_1 x_J) + B_{15}(y_1 z_J + z_1 y_J) + iA_{21}(y_1 z_J - z_1 y_J) \} \quad 5.18
\end{aligned}$$

upon the substitution $s \rightarrow (0, z)$, $\sqrt{2} t \rightarrow (0, x + iy)$, $\frac{E}{2} + m \rightarrow 2m$ and $iy \frac{E}{2} (\frac{E}{2} + m) \rightarrow 2m \underline{\sigma} \cdot \underline{a}$. The coefficients A_k and B_k (listed in 5.R5) are not all independent, the 21 A_k and 15 B_k reduce to 10 independent A_k and 6 independent B_k (see 5.R6) to give the 16 elements $\rho_{2m, 2n}$. All the coefficients here listed are real-as required by hermiticity-, and under time reversal

$$A_k^f \downarrow J \rightarrow A_k^f \downarrow J$$

while

$$B_k^f \downarrow J \rightarrow -B_k^f \downarrow J ,$$

Thus if time reversal invariance is valid in the production process the B_k will vanish. Under T , $z \rightarrow z$, $x \rightarrow -x$, $y \rightarrow -ya$ and $\underline{\sigma} \rightarrow -\underline{\sigma}$ with complex conjugation in addition, giving the six constraints (if T is not violated)

$$\rho_{3, 3} = \rho_{-3, -3}$$

$$\rho_{1, 1} = \rho_{-1, -1}$$

$$\rho_{1, 3} = -\rho_{-1, -3}^*$$

$$\rho_{-1, 3} = -\rho_{1, -3}^* , \quad 5.19$$

as may be verified directly from 5.17 or 5.18 .

The expectation value of the spin vector W_α for the spin 3/2 particle is given by

$$\langle W_\alpha \rangle = \text{Tr} \{ (W_\alpha)_{\mu\nu} \rho_{\nu\mu} \} / \text{Tr} \{ \rho_{\mu\mu} \} \quad 5.20$$

where

$$m(W_\alpha)_{\mu\nu} = -i\epsilon_{\mu\nu\alpha\beta} p_\beta - \frac{1}{4} i\epsilon_{\rho\alpha\beta} p_\beta \gamma^\sigma 5^\sigma \epsilon_{\rho\sigma} \epsilon_{\mu\nu} ,$$

resulting in

$$\begin{aligned} \langle W_\alpha \rangle = & s_\alpha (\rho_{3,3} - \rho_{-3,-3} + \frac{1}{3} \rho_{1,1} - \frac{1}{3} \rho_{-1,-1}) + t_\alpha \{ 2/2 \rho_{-1,1} + \sqrt{6} (\rho_{-1,-3} \\ & - \rho_{1,3}^*) \} - t_\alpha^* \{ 2/2 \rho_{-1,1}^* + \sqrt{6} (\rho_{-1,-3}^* - \rho_{1,3}) \} \end{aligned} \quad 5.21$$

when the normalisation is taken as

$$\rho_{3,3} + \rho_{-3,-3} + \rho_{1,1} + \rho_{-1,-1} = 4 .$$

Equation 5.21 in conjunction with 5.19 implies that the produced spin 3/2 particle is unpolarized in the direction of spin quantization s_α if time reversal invariance holds. There may of course still be some spin alignment in this direction since $\rho_{3,3} \neq \rho_{1,1}$ of necessity. Thus if the axis of quantization is taken as the normal to the production plane, any polarization of the produced B^* (in the reactions $\nu + B \Rightarrow B^* + l$) in this direction will give a test of T . Unfortunately a null result for this measurement will not imply the validity of T , since it does not test the final two conditions of 5.19. The angular distribution of the decay products in $B^* \Rightarrow B + \pi$ may be more helpful in this respect (see chapter 7) .

(e) Result appendix (5.R)

5.R1

With the vector and axial-vector currents

$$\begin{aligned} V_{\alpha\beta} &= a_1 \epsilon_{\alpha\beta} + q_\beta (a_2 \gamma_\alpha + a_3 P'_\alpha + a_4 q_\alpha) \\ A_{\alpha\beta} &= a'_1 \epsilon_{\alpha\beta} + q_\beta (a'_2 \gamma_\alpha + a'_3 P'_\alpha + a'_4 q_\alpha) \end{aligned}$$

the $X^{(1)}(t)$ for 5.6 are :-

$$X^{(1)}(t) = -m_2 \operatorname{Re}(a_1 a_2^* + a_1' a_2'^*) + \frac{1}{8} m_1^3 m_2 xy (y|b_1|^2 + x|b_2|^2) \\ + \frac{1}{2} m_1^2 m_2 xy \operatorname{Re}(b_1 a_2^* + b_2 a_1'^*) + m_1 m_2 \operatorname{Re}(y b_1 a_1^* + x b_2 a_1'^*)$$

$$X^{(2)}(t) = \frac{1}{2} \frac{m_1}{m_2} (y|a_1|^2 + x|a_1'|^2) + \frac{1}{8} m_1^3 m_2 xy (y|c_1|^2 + x|c_2|^2) \\ - \frac{1}{4} \frac{m_1^2}{m_2} \operatorname{Re}\{(x+3y)a_1 a_2^* - (3x+y)a_1' a_2'^*\} + \frac{1}{4} m_1^2 (x+y) \operatorname{Re} \\ (y c_1 a_1^* + x c_2 a_1'^*) - \frac{1}{2} x y m_1^3 \operatorname{Re}(c_1 a_2^* - c_2 a_1'^*)$$

$$X^{(3)}(t) = \frac{1}{4} m_1^2 xy (|a_2|^2 + |a_2'|^2) + m_1 \operatorname{Re}(a_1 a_2^* - a_1' a_2'^*) \\ + \frac{1}{8} m_1^3 m_2 xy \{y(|a_3|^2 - |a_4|^2) + x(|a_3'|^2 - |a_4'|^2)\} \\ + \frac{1}{4} m_1^2 m_2 xy \operatorname{Re}(c_1 a_2^* + c_2 a_1'^*) - \frac{1}{2} m_1 m_2 \operatorname{Re}(y c_1 a_1^* + x c_2 a_1'^*) \\ + \frac{1}{8} (x+y) m_1^2 \operatorname{Re}(y b_1 a_1^* + x b_2 a_1'^*) - \frac{1}{4} m_1^3 xy \operatorname{Re}(b_1 a_2^* - b_2 a_1'^*)$$

$$X^{(4)}(t) = \frac{1}{4} m_1 m_2 (y|a_1|^2 + x|a_1'|^2) + \frac{1}{4} m_1^3 m_2 xy (|a_2|^2 - |a_2'|^2) \\ - \frac{1}{4} m_1^2 m_2 xy \operatorname{Re}(a_1 a_2^* + a_1' a_2'^*) - \frac{1}{4} m_1^3 m_2 xy \operatorname{Re}(y b_1 a_1^* \\ + x b_2 a_1'^*) - \frac{1}{4} m_1^3 m_2^2 xy \operatorname{Re}(y c_1 a_2^* + x c_2 a_1'^*) \\ + \frac{1}{4} m_1^4 m_2 xy \operatorname{Re}(y b_1 a_2^* - x b_2 a_1'^*) - \frac{1}{4} m_1^2 m_2^2 (x+y) \{y(|a_3|^2 \\ - |a_4|^2) + x(|a_3'|^2 - |a_4'|^2)\} + \frac{1}{8} m_1^3 m_2 xy \{y(m_1^2 |b_1|^2 \\ + m_2^2 |c_1|^2) + x(m_1^2 |b_2|^2 + m_2^2 |c_2|^2)\}$$

$$X^{(5)}(t) = \operatorname{Re}(2a_1 a_1'^* - m_1 x a_2 a_1'^* - m_1 y a_1 a_2'^* - m_1^2 x y a_2 a_1'^*),$$

where here

$$m_1 m_2 x = (m_1 + m_2)^2 - t,$$

$$m_1 m_2 y = (m_1 - m_2)^2 - t$$

and

$$b_1 = a_3 + a_4, \quad b_2 = a_3' + a_4', \quad c_1 = a_3 - a_4, \quad c_2 = a_3' - a_4'.$$

5.R2

With the vector and axial-vector currents:-

$$V_{\beta\alpha} = q_{\beta}(d_1 P'_{\alpha} + d_2 q_{\alpha} + d_3 \epsilon_{\alpha\mu\nu\rho} P'_{\mu} q_{\nu} \gamma_5 \gamma_{\rho}) + d_4 \epsilon_{\mu\nu\beta\alpha} P'_{\mu} q_{\nu} \gamma_5$$

$$A_{\beta\alpha} = q_{\beta}(d'_1 P'_{\alpha} + d'_2 q_{\alpha} + d'_3 \epsilon_{\alpha\mu\nu\rho} P'_{\mu} q_{\nu} \gamma_5 \gamma_{\rho}) + d'_4 \epsilon_{\mu\nu\beta\alpha} P'_{\mu} q_{\nu} \gamma_5$$

$$X^{(1)}(t) = \frac{1}{8} m_1^3 m_2 xy(y|b_1|^2 + x|b_2|^2) - \frac{1}{2} m_1 m_2^3 E$$

$$X^{(2)}(t) = \frac{1}{8} m_1^3 m_2 xy(y|c_1|^2 + x|c_2|^2) - \frac{1}{2} m_1^3 m_2 E$$

$$X^{(3)}(t) = \frac{1}{8} m_1^3 m_2 xy\{y(|d_1|^2 - |d_2|^2) + x(|d'_1|^2 - |d'_2|^2)\} \\ + \frac{1}{8} m_1^2 m_2^2 (x + y)E$$

$$X^{(4)}(t) = \frac{1}{8} m_1^3 m_2 xy\{y(m_1^2 |b_1|^2 + m_2^2 |c_1|^2) + x(m_1^2 |b_2|^2 + m_2^2 |c_2|^2)\} \\ - \frac{1}{4} (x + y) m_1^2 m_2^2 \{y(|d_1|^2 - |d_2|^2) + x(|d'_1|^2 - |d'_2|^2)\}$$

$$X^{(5)}(t) = m_1 m_2 xy \text{Re}(4d_4 d_1'^* - m_1 x d_4 d_1'^* - m_1 y d_3 d_1'^* - m_1^2 xy d_3 d_1'^*) ,$$

where x and y are as for 5.R1,

$$E = 4(x|d_4|^2 + y|d'_4|^2) - 2m_1 xy \text{Re}(d_3 d_4'^* + d_3' d_4'^*) \\ + m_1^2 xy(y|d_3|^2 + x|d'_3|^2)$$

and

$$b_1 = d_1 + d_2, \quad b_2 = d'_1 + d'_2, \quad c_1 = d_1 - d_2, \quad c_2 = d'_1 - d'_2$$

5.R3

In terms of the Mandelstam variables s , t and u

$$4p_1 \cdot k_1 p_1 \cdot k_2 = (s - m_1^2)(s - m_2^2 + t)$$

$$4p_2 \cdot k_1 p_2 \cdot k_2 = (s - m_2^2 - \mu^2)(s - m_1^2 - \mu^2 + t)$$

$$4(p_2 \cdot k_1 p_1 \cdot k_2 \pm p_2 \cdot k_2 p_1 \cdot k_1) = (s - m_1^2 - \mu^2 + t)(s - m_2^2 + t) \\ \pm (s - m_2^2 - \mu^2)(s - m_1^2)$$

$$2k_1 \cdot k_2 = \mu^2 - t$$

5.R4

The relevant expression for the lepton polarization (5.14) contains

$$Y = m_1 x \left\{ a'_2 + m_2 c_2 + \frac{1}{4} m_1 (x + y) b_2 \right\} a'^*_1 + m_1 y \left\{ a_2 + m_2 c_1 + \frac{1}{4} m_1 (x + y) b_1 \right\} a^*_1 + \frac{1}{2} m_1^2 xy \left\{ m_1 (a_2 b'_1 - a'_2 b_2) + m_2 (a_2 c'_1 - a'_2 c_2) - m_1 m_2 (y a_3 a'_4 + x a'_3 a_4) \right\}$$

in terms of the currents 5.8, with the defining relations in 5.R1.

In terms of the currents 5.10

$$Y = -\frac{1}{2} m_1^3 m_2 xy (y d_1 d'_2 + x d'_1 d_2) .$$

5.R5

The coefficients A_k and B_k for the density matrix of the spin 3/2 particle (in its rest frame) are given by

$$\begin{aligned} A_1 &= \frac{2}{3} (\rho_{1,1} + \rho_{-1,-1}) , \quad A_2 = \frac{1}{6} \left((\rho_{1,1} + \rho_{-1,-1} + 3 \rho_{3,3} + 3 \rho_{-3,-3}) + 2/3 \operatorname{Re}(\rho_{-1,3} - \rho_{1,-3}) \right) , \\ A_3 &= \frac{1}{6} \left((\rho_{1,1} + \rho_{-1,-1} + 3 \rho_{3,3} + 3 \rho_{-3,-3}) - 2/3 \operatorname{Re}(\rho_{-1,3} - \rho_{1,-3}) \right) , \\ A_4 &= \frac{1}{\sqrt{3}} \operatorname{Im}(\rho_{-1,3} + \rho_{1,-3}) , \quad A_5 = -\frac{1}{3} \operatorname{Im}(\sqrt{3} (\rho_{1,3} + \rho_{-1,-3}) + 2 \rho_{-1,1}) , \\ A_6 &= -\frac{1}{3} \operatorname{Re}(\sqrt{3} (\rho_{1,3} - \rho_{-1,-3}) + 2 \rho_{-1,1}) , \quad A_7 = \frac{1}{6} (\rho_{1,1} + \rho_{-1,-1} - 3 \rho_{3,3} - 3 \rho_{-3,-3}) , \\ A_8 &= -A_6 , \quad A_9 = A_5 , \quad A_{10} = -\frac{4}{3} \operatorname{Re} \rho_{-1,1} , \quad A_{11} = \frac{1}{3} \operatorname{Re} \left((\rho_{-1,1} - 3 \rho_{-3,3}) - \sqrt{3} (\rho_{-1,-3} - \rho_{1,3}) \right) , \\ A_{12} &= -\frac{1}{3} \operatorname{Re} \left((\rho_{-1,1} - 3 \rho_{-3,3}) + \sqrt{3} (\rho_{-1,-3} - \rho_{1,3}) \right) , \\ A_{13} &= -\frac{1}{3} \operatorname{Im}(3 \rho_{-3,3} + \rho_{-1,1}) , \quad A_{14} = A_4 , \quad A_{15} = \frac{1}{3} \left(\sqrt{3} \operatorname{Re}(\rho_{1,-3} - \rho_{-1,3}) + (\rho_{1,1} + \rho_{-1,-1}) \right) , \end{aligned}$$

$$A_{16} = -\frac{4}{3} \operatorname{Im} \rho_{-1,1} , \quad A_{17} = \frac{4}{3} \operatorname{Im} \left((\rho_{-1,1} - 3 \rho_{-3,3}) - \sqrt{3} (\rho_{1,3} + \rho_{-1,-3}) \right) ,$$

$$A_{18} = \frac{4}{3} \operatorname{Im} \left((\rho_{-1,1} - 3 \rho_{-3,3}) + \sqrt{3} (\rho_{1,3} + \rho_{-1,-3}) \right) , \quad A_{19} = -A_{13} ,$$

$$A_{20} = -A_{15} , \quad A_{21} = -A_4 ,$$

and

$$B_1 = \frac{1}{6} (-\rho_{1,1} + \rho_{-1,-1} - 3 \rho_{3,3} + 3 \rho_{-3,-3}) , \quad B_2 = \frac{1}{\sqrt{3}} \operatorname{Re}(\rho_{1,3} + \rho_{-1,-3}) ,$$

$$B_3 = \frac{1}{\sqrt{3}} \operatorname{Im}(\rho_{-1,-3} - \rho_{1,3}) , \quad B_4 = \frac{2}{3} (\rho_{1,1} - \rho_{-1,-1}) ,$$

$$B_5 = \frac{1}{6} \left((3 \rho_{3,3} - 3 \rho_{-3,-3} - \rho_{1,1} + \rho_{-1,-1}) + 2\sqrt{3} \operatorname{Re}(\rho_{-1,3} + \rho_{1,-3}) \right) ,$$

$$B_6 = \frac{1}{6} \left((3 \rho_{3,3} - 3 \rho_{-3,-3} - \rho_{1,1} + \rho_{-1,-1}) - 2\sqrt{3} \operatorname{Re}(\rho_{-1,3} + \rho_{1,-3}) \right) ,$$

$$B_7 = \frac{1}{\sqrt{3}} \operatorname{Im}(\rho_{-1,3} - \rho_{1,-3}) , \quad B_8 = B_3 , \quad B_9 = -B_2 , \quad B_{10} = -B_2 ,$$

$$B_{11} = \frac{1}{6} \left((\rho_{1,1} - \rho_{-1,-1}) - \sqrt{3} \operatorname{Re}(\rho_{-1,3} + \rho_{1,-3}) \right) , \quad B_{12} = -B_7 , \quad B_{13} = B_3 ,$$

$$B_{14} = B_7 , \quad B_{15} = \frac{1}{3} \left((\rho_{1,1} - \rho_{-1,-1}) + \sqrt{3} \operatorname{Re}(\rho_{-1,3} + \rho_{1,-3}) \right) .$$

5.R6

The 21 A_k are reduced to 10 independent ones by the 6 relations indicated in 5.R5 and the additional constraints

$$A_2 - A_3 = A_1 - 2 A_{15}$$

$$A_2 + A_3 = A_1 - 2 A_7$$

$$A_{18} + A_{17} = 2 A_{13} - A_{16}$$

$$A_{18} - A_{17} = A_{16} - 2 A_5$$

$$A_{11} + A_{12} = A_{10} - 2 A_6 ,$$

and the 15 B_k are reduced to 6 by the additional 3 relations

$$B_5 + B_6 = -B_4 - 2B_1$$

$$B_5 - B_6 = B_{15} - B_{11}$$

$$B_{11} + B_{15} = B_4 \quad .$$

6. $\tilde{U}(12)$ AND NEUTRINO INDUCED REACTIONS

In the earlier chapters it has been found that the number of form factors entering the hadron current of the weak interaction vertex is too great for meaningful complete calculations to be attempted. On making arbitrary ad hoc assumptions and introducing a large number of parameters (which could be varied), results could be obtained, but a reasonable fit to experiment in this manner would not be very conclusive or indeed instructive. On the basis of the $SU(3)$ symmetry scheme the octet - octet transitions can be related (considering only the hadrons) and the octet - decuplet, but the two transitions cannot be correlated. However if the $\tilde{U}(12)$ scheme of S.D.S.⁽¹⁸⁾ is adopted the baryon octet and decuplet can be grouped into the same supermultiplet - the $\underline{36}_1$ - and (in particular) the two reactions $\nu + N \rightarrow N + l$ and $\nu + N \rightarrow N^* + l$ become intimately connected (in a current - current theory). It is on this basis that calculations are performed here, and since, with the assumption of an I.V.C. hypothesis, the elastic vector form factors can be related to the iso-vector electro-magnetic form factors of the nucleon an acceptable form for the form factors is available.

(a) The currents

The effective strangeness conserving and strangeness changing hadron currents are assumed to transform like the $\Delta S = 0, \Delta I = 1$ (i.e. ' ρ '- like) and the $\Delta S = 1, \Delta I = 1$ (' K^* '- like) component of the $\underline{14}_3$ regular representation of $\tilde{U}(12)$ respectively, with relative weights

given by the Cabibbo⁽⁶¹⁾ angle. The vector and axial-vector currents are then of the form

$$V_\mu \sim F(q^2) \left(J_\mu + J_{\mu\nu} \frac{q_\nu}{i\langle\mu\rangle} + J \frac{q_\mu}{\langle\lambda\rangle} \right) \quad 6.1$$

and

$$A_\mu \sim G(q^2) \left(J_{5\mu} + J_{5i} \frac{q_i}{\langle\pi\rangle} + \frac{1}{2} J_{\pi\rho} \epsilon_{\pi\rho\mu\nu} \frac{q_\nu}{\langle\sigma\rangle} \right), \quad 6.2$$

where the currents $J_\mu, \dots, J_5$ are defined in appendix B and the parameters $\langle\mu\rangle, \langle\lambda\rangle, \langle\pi\rangle$ and $\langle\sigma\rangle$ have the dimension of mass. V_μ and A_μ are assumed to transform like members of two different $\underline{143}$'s of $\tilde{U}(12)$. Because $\underline{364} \otimes \underline{364}$ (B and B^* belong to the $\underline{364}$ regular representation of $\tilde{U}(12)$) contains $\underline{143}$ once only, the form of the hadron current is unique, with fixed D to F ratios. Physical masses for particles have been inserted in the currents, and the various couplings can be written down (Kawarabayashi and White⁽⁸⁸⁾ have considered this for the leptonic decays of the baryons and baryon resonances).

The current V_μ (6.1) is the most general allowed with the inclusion of derivative couplings. Using the C. V. C. hypothesis ($q_\mu V_\mu = 0$) it is clear that $\langle\lambda\rangle$ can be fixed :

$$\langle\lambda\rangle^{-1} = - (m_1 - m_2) (q^2)^{-1}, \quad 6.3$$

so that whilst V_μ does not appear to give a C. I. V. C. theory in the form 6.1 (the C. I. V. C. is experimentally acceptable), it does taken as

$$F(q^2) \left(J_\mu + J_{\mu\nu} \frac{q_\nu}{i\langle\mu\rangle} + \frac{(m_2 - m_1)}{q^2} J q_\mu \right), \quad 6.4$$

since in the equal mass case (i.e. for N-N transitions) it gives the form of the nucleon electro-magnetic transition. Now by the C. I. V. C.

hypothesis $F(q^2)$ can be interpreted as the electro-magnetic iso-vector form factor of the nucleon, and using a normalisation such that $\mu_p = 2.8$ bohr magnetons $\langle \mu \rangle$ can be fixed to be approximately 1.04 Gev.. For the form of $F(q^2)$

$$\frac{F(q^2)}{\sqrt{(P^2)F(q^2)}} \Big\} = \left(1 - \frac{q^2}{a}\right)^{-2} \quad 6.5 \left\{ \begin{array}{l} a \\ b \end{array} \right.$$

are taken. These imply $F_M(q^2) \sim F(q^2)$ whilst $F_C(q^2) \sim \left(1 + \frac{q^2}{2m\langle \mu \rangle}\right) F_M(q^2)$, forms not in total disagreement with experimental evidence for high values of $|q^2|$ ⁽⁸⁹⁾. Apart from the parameter 'a' in 6.5 (~ 0.74 using nucleon electro-magnetic data) the vector current for both elastic and inelastic processes is now fixed.

By considering the divergence of A_μ and assuming that there is a partially conserved axial-vector current (a Goldberger-Treiman relation), it is possible to deduce that $\langle \pi \rangle \propto m_\pi$. In detail

$$2m_N G(q^2) \frac{5}{3} \left(1 + \frac{q^2}{2\langle \pi \rangle m_N}\right) \approx m_\pi a_\pi g_{\pi np} \left(1 + \frac{q^2}{(m_\pi)^2}\right) \quad 6.6$$

for small q^2 (for neutron β -decay the whole range is small enough), implying that $\langle \pi \rangle m_N \sim m_\pi^2$.

The relation 6.6 is in remarkably good agreement with the value of $G(0)$ from neutron β -decay experiments ($G(0) = \frac{3}{5} 1.15$). It is here assumed that $G(q^2)$ has a similar behaviour to $F(q^2)$, viz.

$$\frac{G(q^2)}{\sqrt{(P^2)G(q^2)}} \Big\} = G(0) \left(1 - \frac{q^2}{b}\right)^{-2} \quad 6.7 \left\{ \begin{array}{l} a \\ b \end{array} \right.$$

It is not too clear what q^2 dependence G should have since there is an overall P^2 ($= (m_1 + m_2)^2 - q^2$) factor in A_μ , but 6.7a leads to a logarithmic increase in the total cross-section as against a constant high energy behaviour with 6.7b (as a function of the energy).

The current $J_{\pi\rho} \epsilon_{\mu\nu\rho} q_\nu$ is not a 'basic' current of the S. D. S. theory of $\tilde{U}(12)$, nor is it a current of the first kind⁽⁴⁸⁾. These may be good reasons for neglecting it ($\langle\sigma\rangle^{-1} = 0$), but it is here retained for completeness and the remaining parameter $\langle\sigma\rangle$ varied to show its effect on the total cross-section. The relatively large value of $\langle\pi\rangle^{-1}$ from 6.6 might be thought to cause large corrections to the cross-section, but fortunately it always occurs with a μ^2 factor (μ is the lepton mass) so that its effect is still small.

With these choices of parameters the currents reduce to

$$V_\mu = F(q^2) \left\{ \left(\frac{1}{2} P_\mu \left\{ 1 + \frac{(m_1 + m_2)}{\langle\mu\rangle} \frac{1}{4} q^2 - \frac{(m_1 - m_2)^2}{q^2} \frac{1}{4} P^2 \right\} - \frac{1}{2} Q_\mu \left\{ \frac{(m_1 - m_2)}{\langle\mu\rangle} + \frac{(m_1^2 - m_2^2)}{q^2} \frac{1}{4} P^2 \right\} \right) (\bar{N}(p_2) N(p_1))_F + \frac{1}{4} \left(1 + \frac{(m_1 + m_2)}{\langle\mu\rangle} \right) (\bar{N}(p_2) \gamma_\mu N(p_1))_D + 2/3 F \right\} \quad 6.8a$$

and

$$A_\mu = G(q^2) \frac{P^2}{4} \left\{ \bar{N}(p_2) \left(\gamma_\mu \left\{ 1 - \frac{(m_1 - m_2)}{\langle\sigma\rangle} \right\} - \left\{ \frac{(m_1 + m_2)}{\langle\sigma\rangle} + \frac{(m_1 - m_2)}{\langle\pi\rangle} \right\} \frac{1}{2} P_\mu - \left\{ \frac{(m_1 + m_2)}{\langle\pi\rangle} + \frac{(m_1 - m_2)}{\langle\sigma\rangle} \right\} \frac{1}{2} Q_\mu \right) i \gamma_5 N(p_1) \right\}_D + 2/3 F \quad 6.8b$$

for the octet - octet transitions, and to

$$V_\mu = i F(q^2) \left(1 + \frac{(m_1 + m_2)}{\langle\mu\rangle} \right) \frac{1}{2} \epsilon_{\mu\kappa\lambda} P_\kappa Q_\lambda \bar{N}(p_2) N(p_1) \quad 6.9a$$

and

$$A_\mu = -iG(q^2) \left\{ \frac{1}{2}P^2 \left(1 - \frac{(m_1 - m_2)}{\langle \sigma \rangle} \right) \bar{D}_\mu(p_2) N(p_1) + \left(\frac{1}{2}Q_\mu \left\{ 1 - \frac{(m_1 + m_2)}{\langle \pi \rangle} \right\} \right. \right. \\ \left. \left. + \frac{1}{2}P_\mu \left\{ 1 + \frac{(m_1 - m_2)}{\langle \pi \rangle} - 2 \frac{m_1}{\langle \sigma \rangle} \right\} \right) Q_K \bar{D}_K(p_2) N(p_1) \right\} \quad 6.9b$$

for the octet - decuplet ones.

Apart from the kinematic factors included in the above, the relevant couplings for the various processes are just the SU(3) ones given in table 5. Now all currents are known in terms of $F(q^2)$ and $G(q^2)$, so results can be indicated for different choices of 'a', 'b' and $\langle \sigma \rangle$.

The identification of the form factors A, B, C, D, E and F of 3.11 and those implicit in 6.8 is as follows :

$$A = \frac{1}{4}P^2 \left(1 + \frac{1}{4} \frac{(m_1 + m_2)}{\langle \mu \rangle} Q^2 - \frac{1}{4} \frac{(m_1 - m_2)^2}{Q^2} P^2 \right) \lambda F(q^2) \quad 6.10a$$

$$B = \frac{1}{4}P^2 \left(1 + \frac{(m_1 + m_2)}{\langle \mu \rangle} \right) \rho F(q^2) \quad 6.10b$$

$$E = \frac{1}{4}P^2 \left(\frac{(m_2 - m_1)}{\langle \mu \rangle} + \frac{1}{4} \frac{(m_2^2 - m_1^2)}{Q^2} P^2 \right) \lambda F(q^2) \quad 6.10c$$

$$C = \frac{1}{4}P^2 \left(1 - \frac{1}{4} \frac{(m_1 + m_2)}{\langle \pi \rangle} Q^2 - \frac{1}{4} \frac{(m_1 - m_2)}{\langle \sigma \rangle} P^2 \right) \rho G(q^2) \quad 6.10d$$

$$D = \frac{1}{4}P^2 \left(1 - \frac{(m_1 - m_2)}{\langle \sigma \rangle} \right) \rho G(q^2) \quad 6.10e$$

and

$$F = -\frac{1}{4}P^2 \frac{1}{4}Q^2 \left(\frac{(m_1 + m_2)}{\langle \sigma \rangle} + \frac{(m_1 - m_2)}{\langle \pi \rangle} \right) \rho G(q^2) \quad , \quad 6.10f$$

where λ is the F type coupling factor and ρ the D + 2/3F factor relevant for the particular process.

For the inelastic processes the relations take the form

$$d_1 = d_2 = d_3 = 0 \quad 6.11a$$

$$2m_1 m_2 d_4 = \left(1 + \frac{(m_1 + m_2)}{\langle \pi \rangle}\right) \lambda F(q^2) \quad 6.11b$$

$$m_1^2 m_2 Q^2 d'_1 = \left((m_1 - m_2) - \frac{q^2}{\langle \sigma \rangle}\right) \lambda G(q^2) \quad 6.11c$$

$$m_1^2 m_2 Q^2 d'_2 = \frac{m_1 m_2}{\langle \pi \rangle} Q^2 - (m_1 + m_2) + \frac{(m_1^2 - m_2^2)}{\langle \sigma \rangle} \lambda G(q^2) \quad 6.11d$$

$$m_1^2 m_2 Q^2 d'_3 = \left(1 - \frac{(m_1 - m_2)}{\langle \sigma \rangle}\right) \lambda G(q^2) \quad 6.11e$$

$$2 d'_4 = m_1 P^2 d'_3 \quad 6.11f$$

in terms of the currents defined in 3.16, with λ the relevant SU(3) factor.

Equations 6.10 give, for $m_1 = m_2$, all the conditions on A, B, C, D, E and F which were required in chapter 3 ($D(0) = C(0)$ and $F \propto q^2$), with the built-in conditions that A and B are related to the nucleon electro-magnetic form factors and $E = 0$. It is worth noting also that, again for $m_1 = m_2$, $\langle \sigma \rangle$ occurs in F only and $\langle \pi \rangle$ in C only, with

$$\frac{C}{D} = \left(1 + \frac{q^2}{2m_N \langle \pi \rangle}\right) \\ \sim \left(1 + \frac{q^2}{m_\pi^2}\right)$$

if there is a P. C. A. C. (see 6.6). Since the C. I. V. C. relates the vector form factors for the weak $n \rightarrow N^{*0}$ process with those in the N^{*0} photo-production from nucleons reaction, it is of interest to see how $\tilde{U}(12)$ compares with the Gourdin and Salin⁽⁸²⁾ estimate of their

form factors C_1 and C_2 . They obtain $C_1(0) = 5.6$ and $C_2(0) = 0.37$ as against the $\tilde{U}(12)$ values of 3.5 and 0.25 on the assumption that the electro-magnetic current transforms like $\lambda^{(3)} + 1/\sqrt{3} \lambda^{(8)}$.

From equations 6.11, the dominant high energy behaviour of the cross-section is provided by the d'_1 term (see 5.12) and is dependent on $\langle\sigma\rangle$. At $q^2 = 0$

$$\frac{d'_3}{d'_1} = \left(\frac{1}{(m_1 - m_2)} - \frac{1}{\langle\sigma\rangle} \right),$$

and a value of $\langle\sigma\rangle \sim m_B - m_{B^*}$ ($\langle\sigma\rangle$ is a constant for all B-B and B-B* processes) would give very simple approximate forms for 6.11 (only d_4 , d'_1 , and d'_2 non-zero, with the q^2 dependence of $F(q^2)$ and $G(q^2)$), whilst even such a small value would affect only the F of 6.10 and make $F \sim \frac{3}{2} q^2 D$ for the $m_1 = m_2$ processes.

(b) Results

(1) Elastic

The resulting expression for the differential cross-section for the elastic processes is given by (substituting 6.10 in 4.9)

$$\begin{aligned} \frac{d\sigma}{dt} = & \frac{G^2}{2\pi} \frac{F^2}{16m_2^2} \left\{ \left(\{ M^2 (1 + \frac{1}{4} M Q^2 \langle\mu'\rangle)^2 + 2 M m^2 \langle\mu'\rangle (1 + \frac{1}{4} M Q^2 \langle\mu'\rangle) \right. \right. \\ & \left. \left. + m^4 \langle\mu'\rangle^2 \right\} \lambda^2 - \frac{t}{m_1^2} (1 + M \langle\mu'\rangle)^2 \rho^2 \right\} F^2(t) \\ & + F^2 \left(\frac{m^2}{m_1^2} - \frac{1}{16} (M^2 + m^2)^2 \langle\sigma'\rangle^2 + m \langle\sigma'\rangle \{ M^2 + m (M^2 - \frac{1}{4} [M^2 - m^2]) \} \right) \rho^2 G^2(t) \\ & \times \left\{ (s - m_1^2)(s - m_2^2) + ts \right\} (s - m_1^2)^{-2} \end{aligned}$$

	λ	ρ
$n \Rightarrow p$	1	$\frac{5}{3}$
$\bar{n} \Rightarrow \bar{p}$	-1	$\frac{1}{3}$
$\Sigma^- \Rightarrow \Sigma^0$	$\sqrt{2}$	$\frac{2}{3}$
$\Lambda \Rightarrow \Sigma^+$	0	$\frac{\sqrt{2}}{3}$
$\Sigma^- \Rightarrow \Lambda$	0	$\frac{\sqrt{2}}{3}$
$\Sigma^0 \Rightarrow \Sigma^+$	$-\sqrt{2}$	$-\frac{2}{3}$
$(\Delta S = 0) \times \cos \theta$		

	λ	ρ
$\bar{n} \Rightarrow \Sigma^0$	$\sqrt{\frac{1}{2}}$	$\frac{1}{3\sqrt{2}}$
$\bar{n} \Rightarrow \Lambda$	$\frac{\sqrt{3}}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$
$\bar{p} \Rightarrow \Sigma^+$	1	$\frac{1}{3}$
$\Sigma^0 \Rightarrow p$	$-\sqrt{\frac{1}{2}}$	$\frac{1}{3\sqrt{2}}$
$\Lambda \Rightarrow p$	$\frac{\sqrt{3}}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$n \Rightarrow \Sigma^+$	-1	$\frac{1}{3}$
$(\Delta S = 1) \times \sin \theta$		

$$\left(\bar{B} \bar{B} \right)_F = \lambda \text{ and } \left(\bar{B} \bar{B} \right)_{D + 2/3F} = \rho \text{ for 6.10 .}$$

$(\bar{B} B)$ for the $|\lambda|$ of 6.11 :-

$n \Rightarrow N^{*+}$	$\frac{1}{\sqrt{3}}$
$p \Rightarrow N^{*++}$	1
$\bar{n} \Rightarrow \bar{N}^{*0}$	$\frac{1}{\sqrt{3}}$
$\Sigma^- \Rightarrow Y_1^{*0}$	$\frac{1}{\sqrt{6}}$
$\Sigma^0 \Rightarrow Y_1^{*+}$	$\frac{1}{\sqrt{6}}$
$\Lambda \Rightarrow Y_1^{*+}$	$\frac{1}{\sqrt{2}}$
$(\Delta S = 0) \times \cos \theta$	

$Y_1^{*0} \Rightarrow p$	$\frac{1}{\sqrt{6}}$
$Y_1^{*-} \Rightarrow n$	$\frac{1}{\sqrt{3}}$
$\bar{N}^{*0} \Rightarrow \Lambda$	$\frac{1}{\sqrt{2}}$
$\bar{N}^{*-} \Rightarrow \Sigma^0$	$\frac{1}{\sqrt{6}}$
$\bar{N}^{*0} \Rightarrow \Sigma^+$	$\frac{1}{\sqrt{3}}$
$\Omega^- \Rightarrow \bar{p}$	1
$(\Delta S = 1) \times \sin \theta$	

TABLE 5

The SU(3) coefficients for the elastic and inelastic hadron vertices.

$$\begin{aligned}
& + \frac{G^2}{2\pi} \frac{t(P^2)^2}{32(s - m_1^2)^2} \left\{ Q^2 (1 + M\langle\mu'\rangle)^2 \rho^2 F^2(t) - P^2 (1 + m\langle\sigma'\rangle)^2 \rho^2 G^2(t) \right\} \\
& \pm \frac{G^2}{2\pi} \frac{t(P^2)^2}{m_1 m_2} (1 + M\langle\mu'\rangle)(1 + m\langle\sigma'\rangle) \rho^2 F(t)G(t) \left\{ \frac{1}{(s - m_1^2)} \right. \\
& \quad \left. + \frac{(s - m_2^2 + t)}{(s - m_1^2)^2} \right\}, \quad 6.12
\end{aligned}$$

where $M = \left(1 + \frac{m_2^2}{m_1^2}\right)$, $m = \left(\frac{m_2^2}{m_1^2} - 1\right)$, $\langle\mu'\rangle = \frac{m_1}{\langle\mu\rangle}$ and $\langle\sigma'\rangle = \frac{m_1}{\langle\sigma\rangle}$.

This differential cross-section for the nucleon processes $\nu(\bar{\nu}) + N \Rightarrow N + 1$ is plotted in figure 2 with the indicated choices of $F(t)$ and $G(t)$. For small values of t $\frac{d\sigma}{dt}$ does not distinguish between $F^2 P^2 \sim (1 - t/a)^{-4}$ or $F^2 \sim (1 - t/a)^{-4}$ (the curves for the former choice are not given as their shape is much the same as the latter apart from a slightly greater slope). This choice could be made from the total cross-section (figures 5 and 8) which increases logarithmically at high energies for $F^2 \sim (1 - t/a)^{-4}$ (as given by the usual computation with similar form factors⁽⁹⁰⁻⁹²⁾ but behaves as a constant as $s \rightarrow \infty$ for $F^2 P^2 \sim (1 - t/a)^{-4}$ (see table 3 for the high energy behaviour of the individual form factors). No structure ($F \sim \text{constant}$) results in highly divergent cross-sections and a simple pole approximation also gives a behaviour worse than logarithmic divergence. If one of these two forms can be established from the total cross-section then the parameters a and b in F and G might be found from $\frac{d\sigma}{dt}$, but the experimental uncertainties at this stage are such that no really sensible guess

can be made. The parameter $\langle\sigma\rangle$ again adds an element of uncertainty into these comparisons, any finite value reducing the size of the b parameter in G. For the elastic processes $\nu(\bar{\nu}) + N \Rightarrow N + 1$, one way of eliminating the dependence on $\langle\sigma\rangle$ would be to consider the difference of the neutrino and anti-neutrino cross-sections which is independent of this $\langle\sigma\rangle$, but again experimental errors would be of the same order as the variations induced by changes in a and b. The overall evidence seems to point to a and b being of the same order of magnitude.

With the $\tilde{U}(12)$ model the results for the differential cross-section ratios for the processes where a Λ or Σ are produced - $R(\Sigma) = \sigma(\bar{\nu} + p \Rightarrow n + 1) : \sigma(\bar{\nu} + p \Rightarrow \Sigma^0 + 1)$ and $R(\Lambda) = \sigma(\bar{\nu} + p \Rightarrow n + 1) : \sigma(\bar{\nu} + p \Rightarrow \Lambda + 1)$ - as given below, differ little from the SU(3) results of Sirlin⁽⁹³⁾ since the only basic difference is one of inserting physical mass factors which give small corrections.

s	R(Σ)		R(Λ)	
	t = 0	t = -0.2M ²	t = 0	t = -0.2M ²
4M ²	40	48	18	17
6M ²	37	48.5	17.5	18
∞	32	50	17	18.5
(M is the nucleon mass)				

(2) Inelastic

The results for the inelastic process $\nu(\bar{\nu}) + N \Rightarrow N^* + 1$ are very similar to those of Albright and Liu⁽³²⁾ (for $F^2 \sim (1 - t/a)^{-4}$) who use SU(6) to find the relative magnitudes of the form factors. The values for $\frac{d\sigma}{dt}$ given here agree with theirs at $q^2 = 0$, as of course they should. The results presented here are completely consistent with their later paper⁽⁸⁰⁾ which includes a relativistic extension of SU(6).

The differential cross-section for the inelastic processes is now given by

$$\begin{aligned} \frac{d\sigma}{dt} = & \lambda^2 \frac{G^2}{12\pi} \left\{ P^2 \left(-\frac{t}{m_1 m_2} (1 + M\langle\mu'\rangle)^2 F^2 + P^2 (1 - t\langle\sigma'\rangle^{-2}) G^2 \right) \right. \\ & \times \left((s - m_1^2)(s - m_2^2) + \frac{1}{2}t(2s - m_1^2 - m_2^2) \right) \\ & \left. \pm (P^2)^2 \left(t(1 + M\langle\mu'\rangle)(1 + m\langle\sigma'\rangle)FG \right) (2s - m_1^2 - m_2^2 + t) \right\}, \end{aligned}$$

6.13

with the neglect of terms in μ^2 (see 6.12 for the terms M , m , $\langle\mu'\rangle$ and $\langle\sigma'\rangle$).

The results here presented are for the reactions

$$\nu_\mu + n \Rightarrow N^{*+} + \mu^- \quad 6.14a$$

$$\bar{\nu}_\mu + p \Rightarrow N^{*0} + \mu^+, \quad 6.14b$$

the corresponding result for $\nu_\mu + p \Rightarrow N^{*++} + \mu^-$ being three times that of 6.14a . The CERN data^(1,10) being the result per nucleon must be divided by two for comparison of the experimental results and the theory. Again the total cross-section resulting from the

choice 6.5b and 6.7b is constant at high energies, that for 6.5a and 6.7a being logarithmically increasing. Since the V-A interference term contains some $\langle\sigma\rangle$ dependence (admittedly probably small) no means of deciding the sizes of a and b seem available in this inelastic process, though verification of the choices indicated by the elastic reaction would be possible. $\langle\sigma\rangle$ occurs linearly in the difference of the cross-sections for the neutrino and anti-neutrino induced reactions, and this might afford a means of fixing its sign if a sufficiently accurate assessment of a and b could be made from the elastic reaction and experimental errors drastically reduced.

To sum up, it does not seem possible to decide the best value of $\langle\sigma\rangle$ though an infinite value (its complete absence) would not be in disagreement with experiment. Likewise it is not improbable that a and b are comparable with F and G behaving as $\left(\sqrt{(P^2)(1 - t/a)^2} \right)^{-1}$ being most acceptable if the cross-section is found not to increase with the neutrino energy at very high energies.

(c) Figure captionsFigure 1

The invariant differential cross-section for the inelastic processes

$$\begin{aligned} \nu_{\mu} + n &\Rightarrow N^{*+} + \mu^{-} \quad (\text{continuous line}) \\ \text{and } \bar{\nu}_{\mu} + p &\Rightarrow N^{*0} + \mu^{+} \quad (\text{broken line}) \end{aligned}$$

with

$$F(t) = (1 - t/a)^{-2} \quad \text{and} \quad G(t) = (1 - t/b)^{-2} \times (0.69).$$

The curves (a), (b), (c), (d) and (e) are for the (a,b) of (F,G) equal to (0.74,0.74), (0.74,0.86), (0.74,1.00), (0.86,0.86) and (1.00,1.00) respectively.

Figure 2

The invariant differential cross-section for the elastic processes

$$\begin{aligned} \nu_{\mu} + n &\Rightarrow p + \mu^{-} \quad (\text{continuous line}) \\ \text{and } \bar{\nu}_{\mu} + p &\Rightarrow n + \mu^{+} \quad (\text{broken line}) \end{aligned}$$

with F(t) and G(t) and the curves (a), (b), (c), (d) and (e) as for figure 1.

Figure 3

The total cross-section corresponding to figure 1.

Figure 4

An indication of the variation of the total cross-section for $\nu_{\mu} + n \Rightarrow N^{*+} + \mu^{-}$ with $a = b = 0.74$ and $\langle \sigma \rangle^{-1} = -2, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$ and 2 for the curves (a), (b), (c), (d), (e), (f) and (g).

Figure 5

The total cross-section corresponding to figure 2.

Figure 6

An indication of the variation of the total cross-section f for $\nu_{\mu} + n \Rightarrow p + \mu^{-}$ with $a = b = 0.74$ and $|\langle \sigma \rangle^{-1}| = 2, 1, \frac{1}{2}$ and 0 for the curves (a), (b), (c) and (d). The separation for $\langle \sigma \rangle$ positive or negative is dependent on terms in μ^2 (μ is the lepton mass), and is too small to demonstrate.

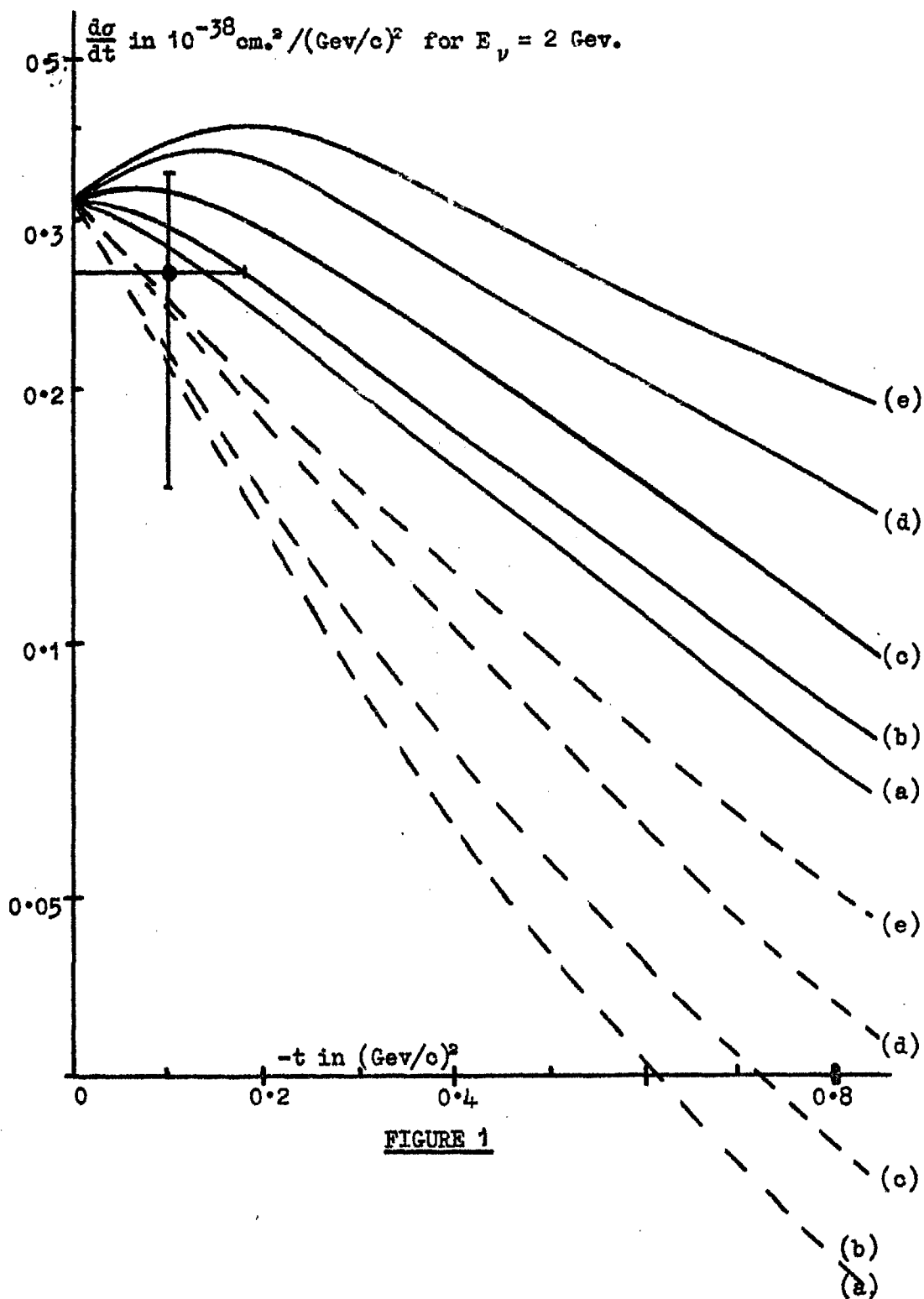
Figure 7

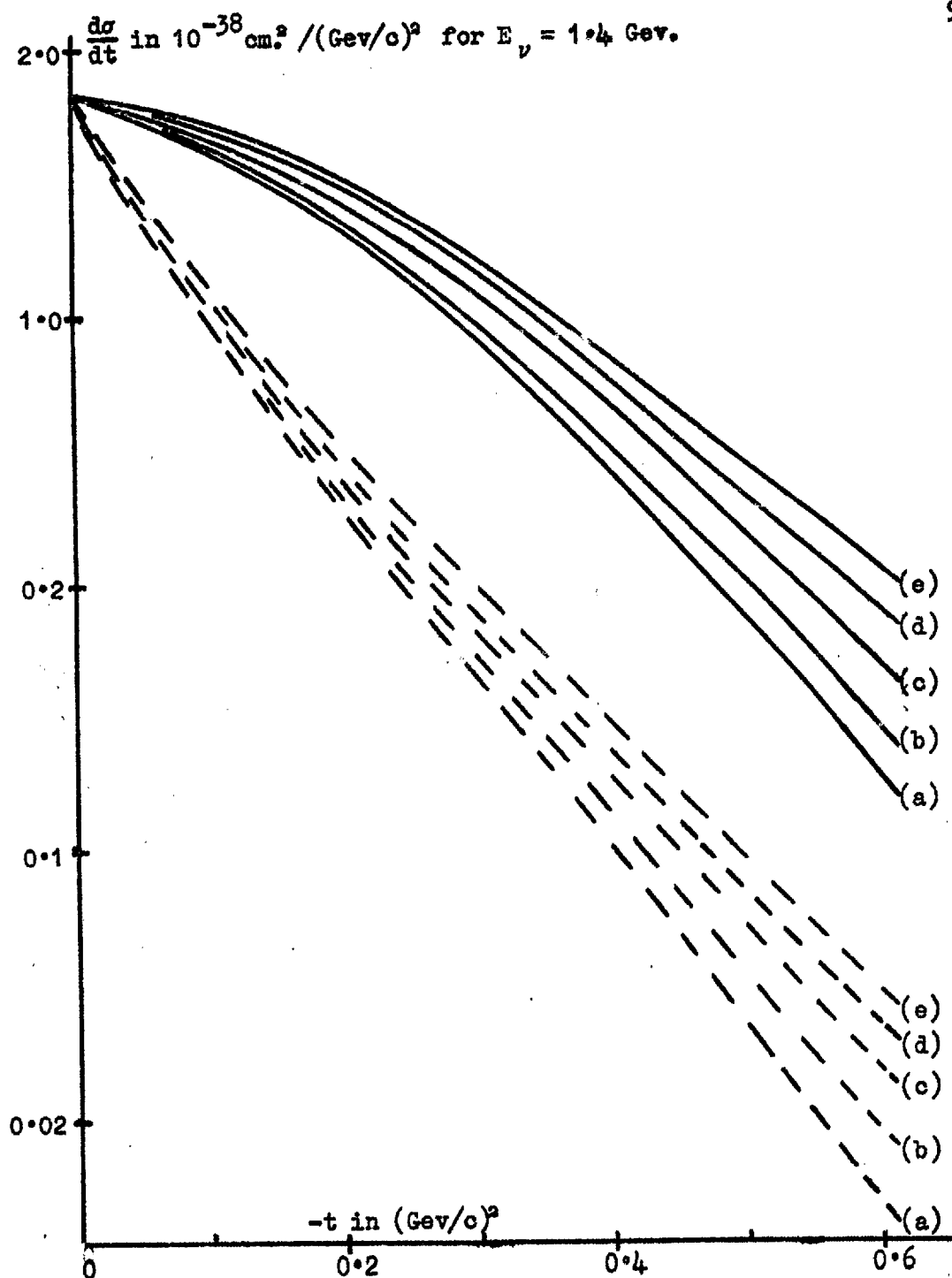
The inelastic cross-sections with $F(t) = (1 - t/a)^{-2} \sqrt{(1 - t/4m^2)}$ and $G(t) = 0.69(1 - t/b)^{-2} \sqrt{(1 - t/4m^2)}$. The curves (b) and (c) are for $a = b = 0.74$ and $a = b = 0.86$ respectively. (a) is the curve (a) of figure 3, given for comparison.

Figure 8

The elastic cross-sections with $F(t)$ and $G(t)$ and the curves (b) and (c) as for figure 7. (a) is the curve (a) of figure 5.

The experimental values indicated are those from reference 10. Neutrino experiments are indicated by ■, anti-neutrino by □.



FIGURE 2

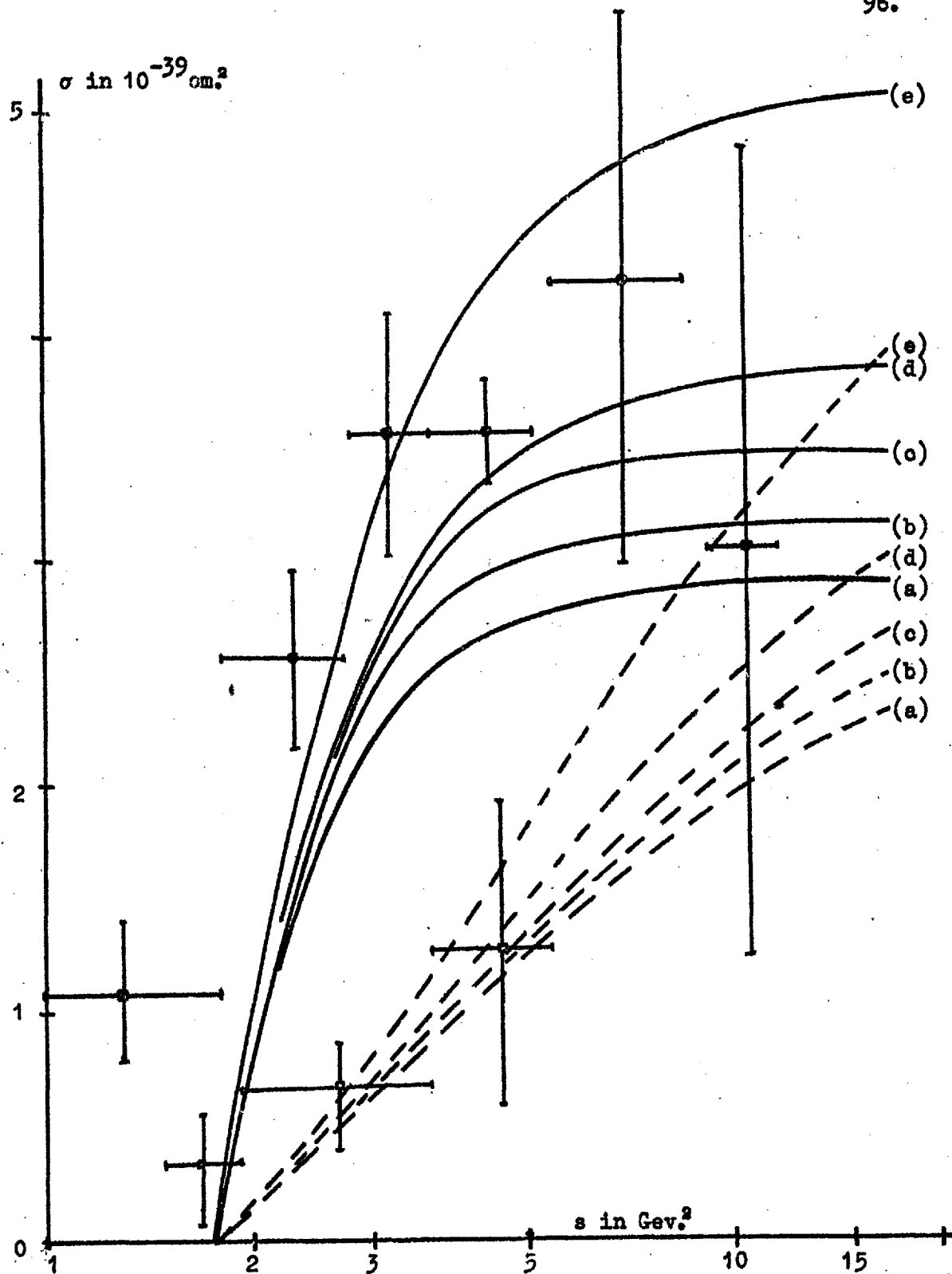


FIGURE 3

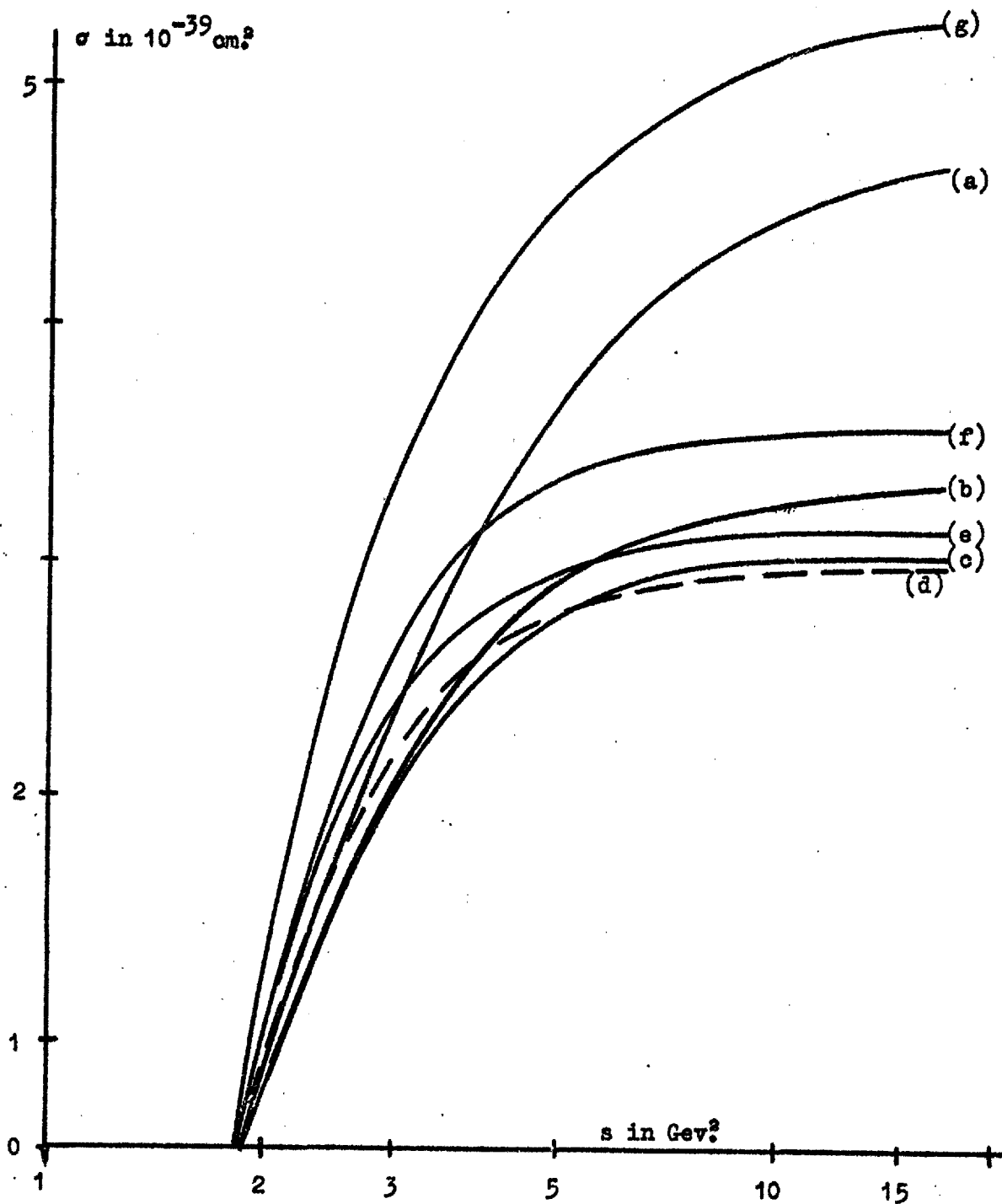


FIGURE 4.

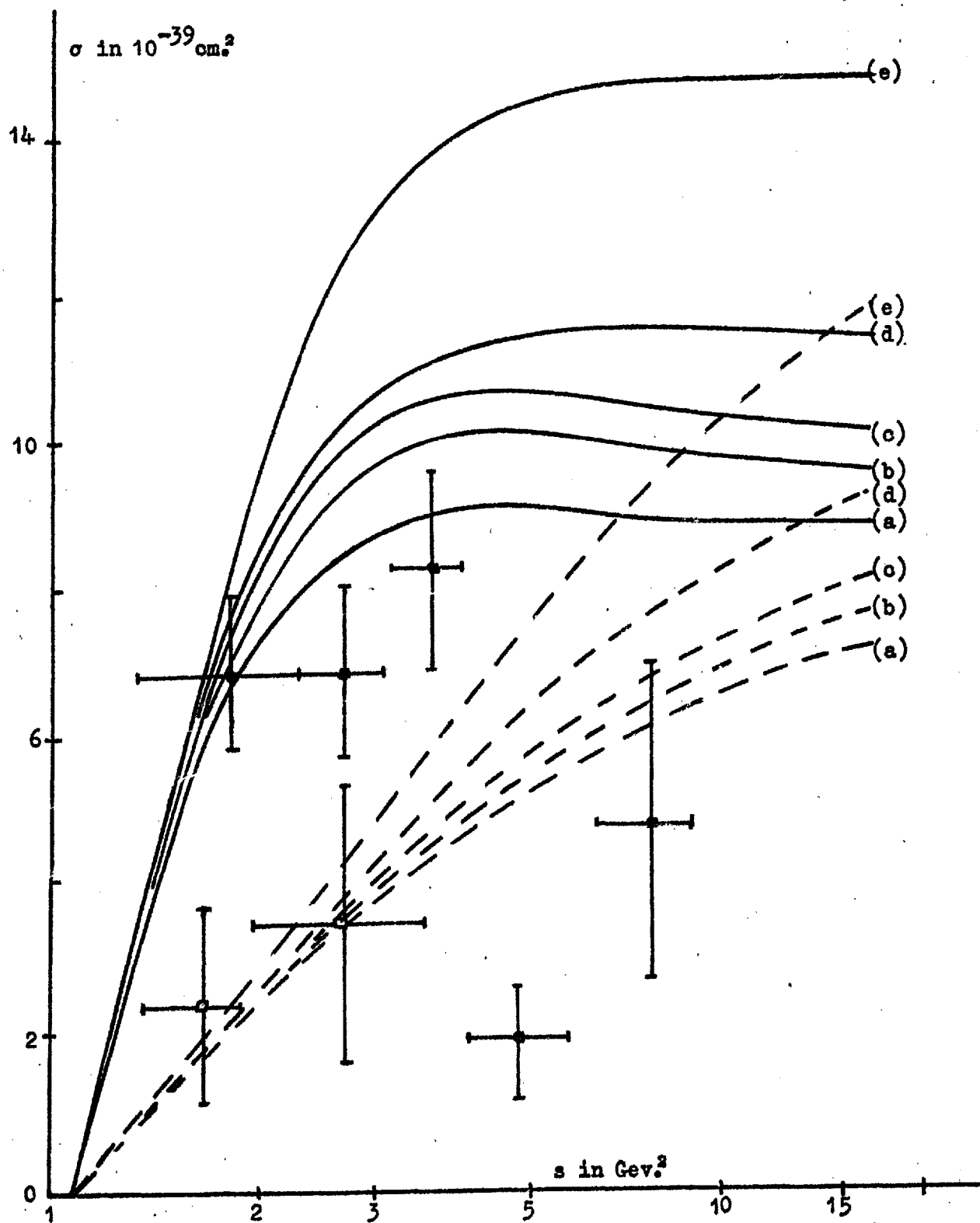
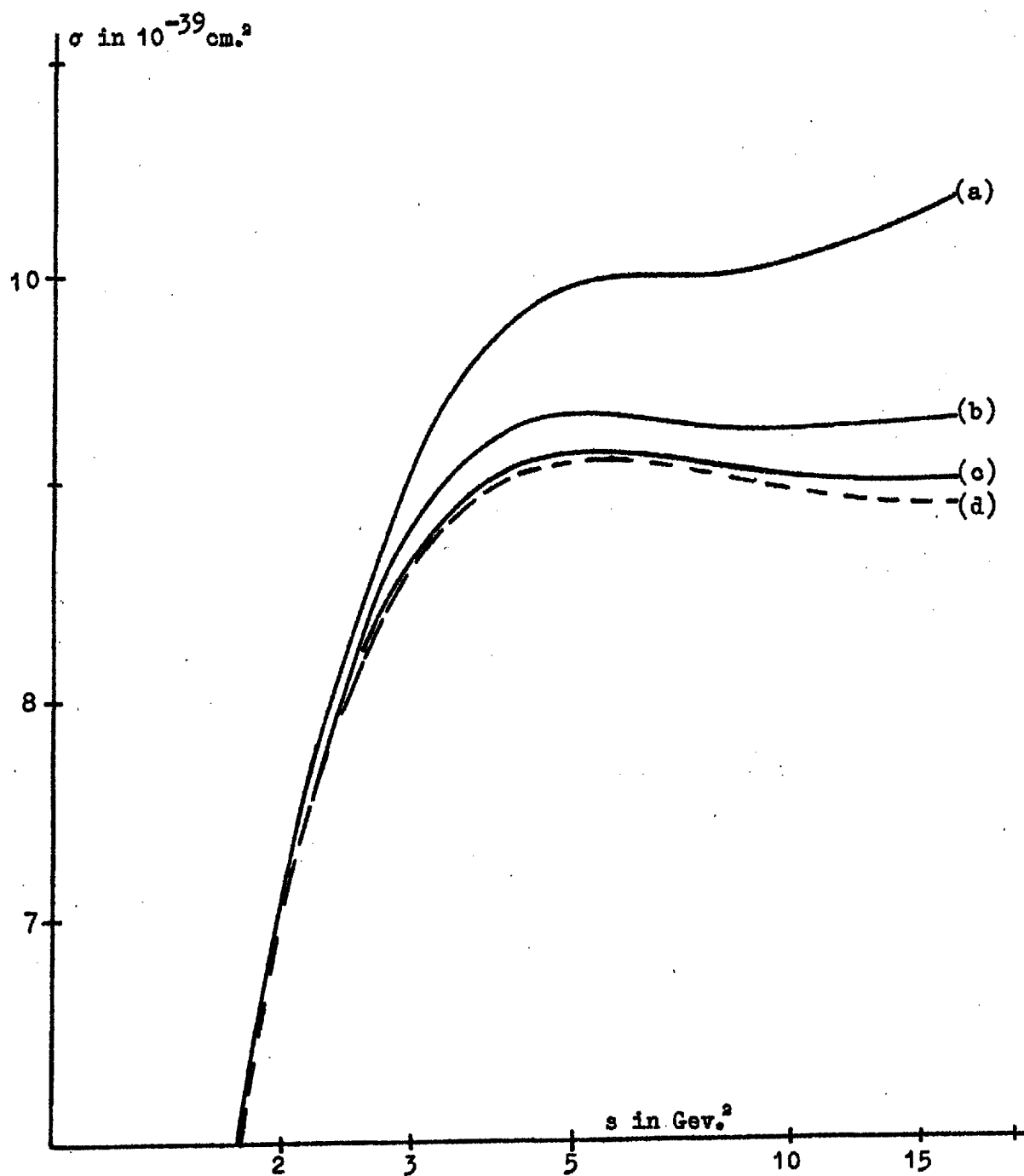
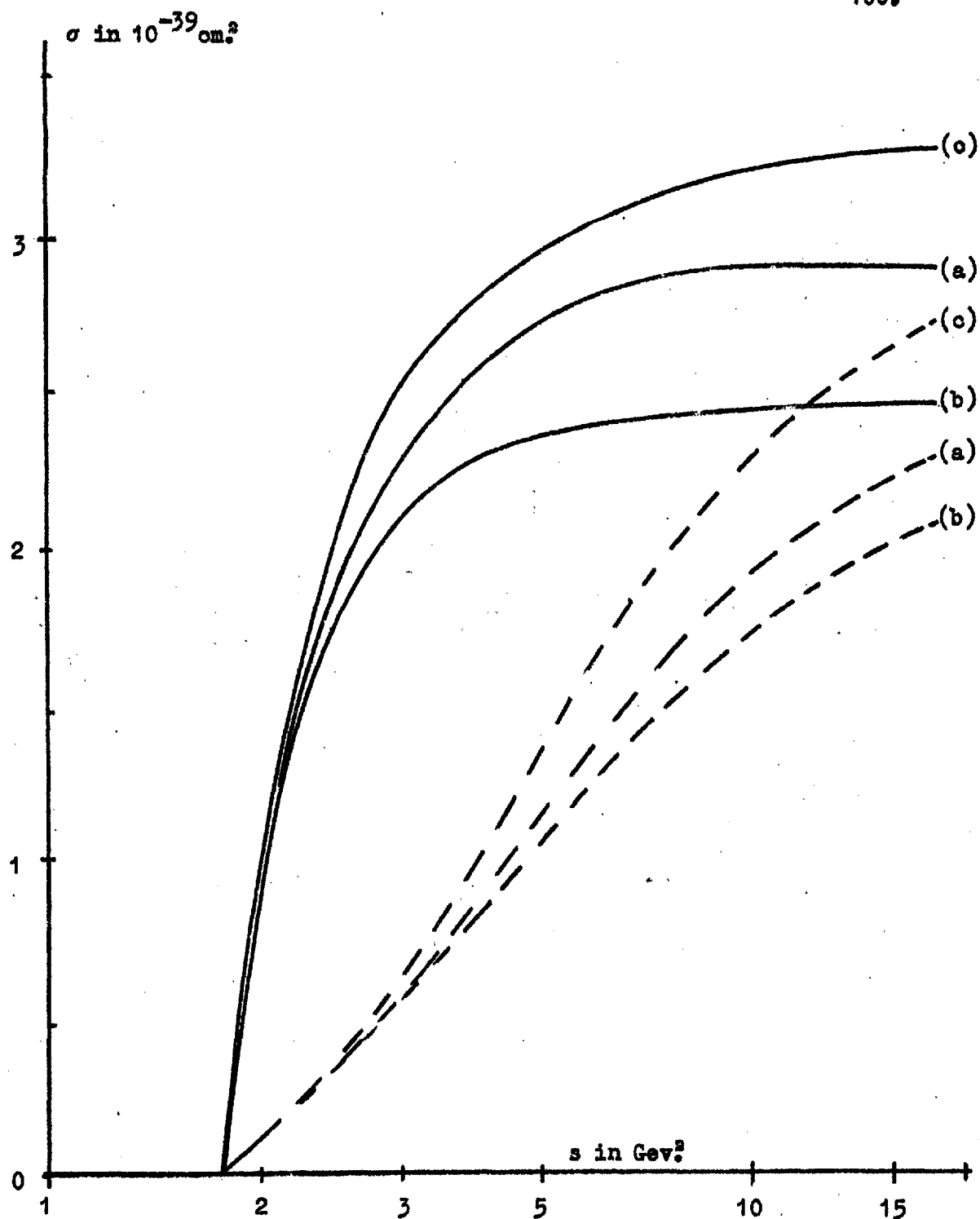
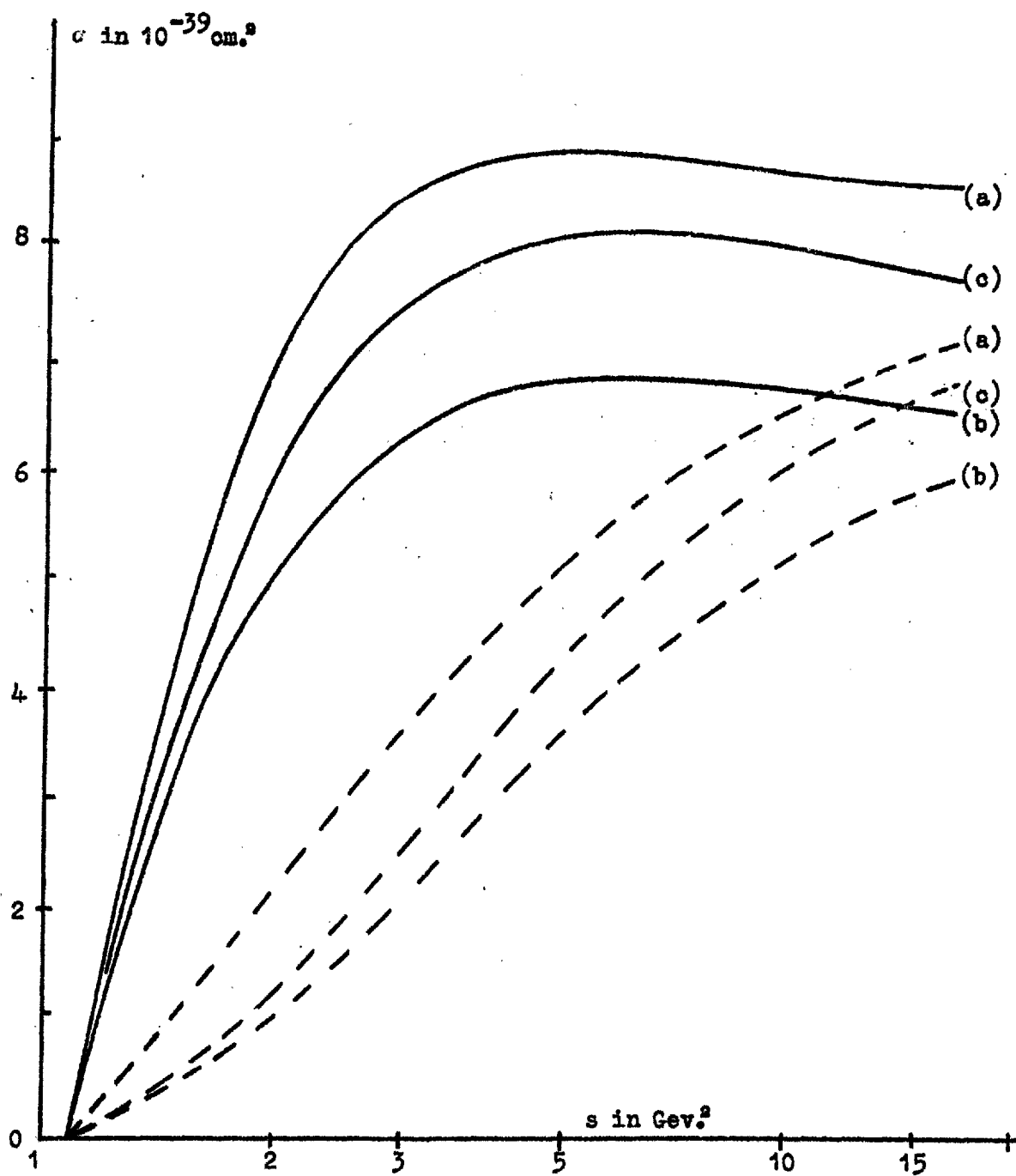


FIGURE 5

FIGURE 6

FIGURE 7

FIGURE 8

7. SUBSEQUENT DECAYS

(a) The weak decay $B \Rightarrow B^* + \pi$

Much has been written on the non-leptonic hyperon decays⁽⁹⁵⁾ using various models and symmetry schemes. Such discussions would be out of place here, but in view of the expression for the polarization 4-vector of the final hadron obtained in chapter 4 - with the indicated tests of time reversal invariance - it seems sensible to ask if these tests manifest themselves in the decay of this hadron. Furthermore, violation of CP invariance might show itself in this weak decay.

The most general form for the decay matrix element is

$$M = \bar{u}(f)(A_D + i\gamma_5 B_D)u(p), \quad 7.1$$

where A_D and B_D are scalar and pseudo-scalar form factors (functions of the covariant momentum transfer $(p - f)^2$), which are here allowed to be complex, and p and f are the momenta of the initial and final hadrons. The initial hadron is now assumed to be produced in a neutrino reaction, $\nu + N \Rightarrow B + l$, and hence has a well defined density matrix

$$\rho(B_i) = \frac{1}{2}(1 - i\gamma_5 \Sigma)(\not{p} + m), \quad 7.2$$

where Σ is the polarization vector given by 4.24 and 4.R6 .

Thus the differential decay rate ($d\Gamma$) is proportional to

$$|A_D|^2(p \cdot f + mm') + |B_D|^2(p \cdot f - mm') - 2\text{Re}(A_D B_D^*) \Sigma \cdot f m \quad 7.3$$

and the polarization vector of the final hadron (W_α) proportional to

$$\begin{aligned}
& |A_D|^2 \left\{ \Sigma.f \left(\frac{m}{m'}, f_\alpha + p_\alpha \right) - (p.f + mm') E_\alpha \right\} + |B_D|^2 \left\{ \Sigma.f \left(\frac{m}{m'}, f_\alpha - p_\alpha \right) \right. \\
& \quad \left. + (p.f - mm') E_\alpha \right\} + 2\text{Re}(A_D B_D^*) \left\{ \frac{p.f}{m'} f_\alpha - m' p_\alpha \right\} \\
& \quad - 2\text{Im}(A_D B_D^*) \langle (\alpha) \Sigma p f \rangle .
\end{aligned}
\tag{7.4}$$

The terms which contain possible T violating effects are now

$\Sigma.f$ from $d\Gamma$

and $\Sigma.f$ and $\text{Im}(A_D B_D^*) \langle (\alpha) \Sigma p f \rangle$ from W_α .

From 7.3, the angular distribution of the final baryon (or pion) in the rest frame of the decaying baryon is of the form

$$1 + \alpha \cos \theta + \beta \sin \theta \cos \phi + \gamma \sin \theta \sin \phi , \tag{7.5}$$

where θ is the angle between the direction of one of the decay products and the normal to the production plane ($\hat{N}$), the azimuthal angle ϕ is measured with respect to the plane of $\hat{p}_1$ and $\hat{N}$ ($\hat{p}_1$ is the direction of the target nucleon in the rest frame of the decaying baryon) and the coefficients α , β and γ are scalar - pseudo-scalar interference terms. If time reversal invariance is not violated in the production process then the term in $\cos \theta$ is absent in 7.5 - that is, if the decay distribution for a baryon polarized perpendicular to the production plane is not isotropic then time reversal invariance is not valid.

$\underline{W}^R \cdot \hat{N}$ contains time reversal invariant correlations

$$\cos \theta, \cos \theta \sin \theta \cos \phi, \cos \theta \sin \theta \sin \phi$$

and time reversal violating correlations

$1, \cos^2 \theta$ from the production process

and $\sin \theta \cos \phi, \sin \theta \sin \phi$ from the decay.

The other two polarization correlations $\underline{W}^R \cdot \hat{p}_1$ and $\underline{W}^R \cdot (\hat{p}_1 \wedge \hat{N})$ give rise to T invariant terms

$1, \sin \theta \sin \phi, \sin \theta \cos \phi, \sin^2 \theta \cos^2 \phi, \sin^2 \theta \sin^2 \phi, \sin^2 \theta \sin \phi \cos \phi$

and T violating terms

$\cos \theta \sin \theta \sin \phi, \cos \theta \sin \theta \cos \phi$ from the production

and $\cos \theta$ from the decay.

Of course not all these polarization correlations are independent - the density matrix of the decaying baryon has only four elements :

$$\rho(\frac{1}{2}) \sim (1 + i\gamma_5 \not{P} + i\gamma_5 \not{Q} + i\gamma_5 \not{Q}^* Q^*)(\not{p} + m),$$

and T invariance implies that $P \left(= \frac{\rho_{1,1} - \rho_{-1,-1}}{\rho_{1,1} + \rho_{-1,-1}} \right)$ is zero (see 2.28).

There are thus only 4 independent correlations, but all possible terms are given here to indicate the variety of possible tests of time reversal invariance.

(b) The strong decay $B^* \Rightarrow B + \pi$

Again the main concern is how the time reversal violating terms occur in the angular distribution and polarization of the final baryon. These decays are strong, so parity violating effects do not arise from the decay process but from the production. The

decay matrix element is of the form

$$\bar{u}(f)A_D u_\mu(p)k_\mu, \quad 7.6$$

where p , f and k are the momenta of the decaying baryon resonance, the decay product baryon and the pion.

The transition probability for this decay is then proportional to

$$\begin{aligned} |A_D|^2 \text{Tr}(f_\alpha \rho_{\alpha\beta} f_\beta (f + m')) \\ = |A_D|^2 2(p \cdot f + mm') \text{Tr}(f_\alpha \rho_{\alpha\beta} f_\beta), \end{aligned} \quad 7.7$$

where $\rho_{\alpha\beta}$ is the density matrix of the B^* relevant to the production process. The most general forms of ρ were given in 5.15 and 5.16. From 7.6 and 5.16, the angular distribution of the decay baryon contains at most four arbitrary constants, and is of the form⁽³⁵⁾

$$D(\theta, \phi) = \alpha_1 \cos^2 \theta + \alpha_2 \sin^2 \theta \cos^2 \phi + \alpha_3 \sin^2 \theta \sin^2 \phi + \alpha_4 \sin^2 \theta \sin \phi \cos \phi \quad 7.8$$

(with θ and ϕ defined as in 7.5) in the rest frame of the B^* .

The dominant contribution to α_4 is the vector - axial-vector interference term, but there is a small non-interference contribution⁽⁹⁶⁾ proportional to $(E_1^2 - E_\nu^2)$, where E_1 and E_ν are the energies of the lepton and neutrino in the production process. The form 7.8 is the contribution to $D(\theta, \phi)$ excluding the T violating terms which are of the form

$$\beta_1 \cos \theta \sin \theta \sin \phi + \beta_2 \cos \theta \sin \theta \cos \phi.$$

The expectation value of the spin of the final baryon (its polarization) is given by

$$\langle \underline{W}_\mu \rangle \propto \text{Tr} \left\{ i\gamma_5 \gamma_\mu (\not{x} + m') f_\alpha \rho_{\alpha\beta} f_\beta (\not{x} + m') \right\} ,$$

so clearly only those terms containing an $i\gamma_5 \not{a}$ (where 'a' = s, t or t^*) in 5.15 contribute , and these are of the form

$$a_\mu (p \cdot f + m m') = a \cdot f (p_\mu + f_\mu \frac{m}{m'}) .$$

Now from 5.15,

$$\underline{W}^R \cdot \hat{\underline{N}} \propto \{ A_8 \cos \theta \sin \theta \sin \phi + A_9 \cos \theta \sin \theta \cos \phi + B_4 \sin^2 \theta \cos^2 \phi \\ + B_5 \sin^2 \theta \sin^2 \phi + B_6 \cos^2 \theta + B_7 \sin^2 \theta \sin \phi \cos \phi \} ,$$

$$\underline{W}^R \cdot \hat{\underline{p}}_1 \propto \{ A_{10} \sin^2 \theta \cos^2 \phi + A_{11} \sin^2 \theta \sin^2 \phi + A_{12} \cos^2 \theta \\ + A_{13} \sin^2 \theta \sin \phi \cos \phi + B_{11} \sin \theta \cos \theta \cos \phi \\ + B_{12} \sin \theta \cos \theta \sin \phi \} ,$$

and

$$\underline{W}^R \cdot (\hat{\underline{p}}_1 \wedge \hat{\underline{N}}) \propto \{ A_{16} \sin^2 \theta \cos^2 \phi + A_{17} \sin^2 \theta \sin^2 \phi + A_{18} \cos^2 \theta \\ + A_{19} \sin^2 \theta \cos \phi \sin \phi + B_{14} \sin \theta \cos \theta \cos \phi \\ + B_{15} \sin \theta \cos \theta \sin \phi \} ,$$

where the A_k and B_k are time reversal invariant and violating coefficients listed in 5.R5 .

A set of sixteen independent correlations (found by picking a set of independent A_k and B_k from 5.R5) could be chosen as the six of $D(\theta, \phi)$ (4 T-conserving and 2 T-violating), the six of $\underline{W}^R \cdot \hat{\underline{N}}$ (2 T-conserving and 4 T-violating) and the A_{10} , A_{11} and A_{13} terms

of $\underline{W}^R \cdot \hat{p}_1$ and the A_{18} term of $\underline{W}^R \cdot (\hat{p}_1 \wedge \hat{N})$. This gives 10 independent T-conserving and 6 independent T-violating correlations as required.

In terms of the production parameters (the eight form factors) the forms of $D(\theta, \phi)$ and $\langle \underline{W}_\mu \rangle$ are very lengthy. They shed no light on the structure of the T-violating terms, and even in the case where most of the form factors are neglected, no further inferences can be made to aid the interpretation. For these reasons these lengthy expressions are not tabulated, the general properties indicated above being considered sufficient.

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APPENDIX A

Notation and normalisations

The natural system of units in which $\hbar = c = 1$ is used throughout. With this convention, to obtain cross-sections for the processes listed here in (cm.)² a conversion factor is needed. The relevant value of $\frac{G^2}{2\pi}$ used in formulae is thus 0.83×10^{-38} - the usual result for G (the 'Universal four-Fermi coupling constant') is $GM^2 \approx 1.0 \times 10^{-5}$ where M is the proton mass.

Four - vectors are denoted by p , p being used for a 3-vector ($p = (p_0, \mathbf{p})$). The metric tensor $\xi_{\mu\nu}$ is such that

$$a_\mu b_\nu \xi_{\mu\nu} = a_0 b_0 - \underline{a} \cdot \underline{b}$$

(lower indices are used in all sections with the exception of appendix B, where both upper and lower are used for clarity) and the totally anti-symmetric symbol $\epsilon_{\mu\nu\lambda\rho}$ is defined by $\epsilon_{0123} = +1$.

The specific representation for the Dirac γ -matrices used in chapter 5 is that of Schweber⁽⁹⁷⁾, with

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}$$

$$\text{and } i\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\text{where } \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

elsewhere the defining relation $\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu = 2\xi_{\mu\nu}$ is all that is

required. With the usual convention $\not{p} = \gamma_\mu p_\mu$. Four-spinors $u(p)$ are normalised to $2m$, so that the spin sum $\sum_s u_s \bar{u}_s$ is $(\not{p} + m)$. The adjoint spinor $\bar{u}(p)$ is conveniently defined to be $u^\dagger(p)A$, where ' $\dagger$ ' represents hermitian conjugation and A is such that $\gamma_\mu^\dagger A = A \gamma_\mu$. In the above specific representation $A = \gamma_0$.

Single particle states have been normalised covariantly so that the result of integrating any $f(p_1, p_2, \dots, p_n)$ over n -particle phase space is to use the form

$$\frac{1}{\text{Flux}} \int f(p_1, p_2, \dots, p_n) \left(\prod_{i=1}^n d^4 p_i \delta(p_i^2 - m^2) \right) \delta^4(P - \sum_i p_i)$$

where $2\pi d \equiv d$ and $2\pi \delta = \delta$.

APPENDIX B

$\tilde{U}(12)$ wave functions and currents

In chapter 6 it is assumed that the effective currents are given in a form which assumes the results of S. D. S.⁽¹⁸⁾ modified, to include symmetry breaking, by the insertion of physical masses in place of a common mass for each member of the multiplet. Here the relevant details and results are given.

The fully symmetric wave function $\Psi_{\{ABC\}}$ for the 364 representation of $\tilde{U}(12)$ decomposes in terms of $\tilde{U}(4) \otimes SU(3)$ as

$$\begin{aligned} \Psi_{\{ABC\}} = & \frac{\sqrt{3}}{2/2} D_{\{\alpha\beta\gamma\}\{pqr\}} + \epsilon_{pqr} V_{[\alpha\beta\gamma]} + \frac{1}{2/6} \left(\epsilon_{pqs} N_{[\alpha\beta]\gamma, r}^s \right. \\ & \left. + \epsilon_{qrs} N_{[\beta\gamma]\alpha, p}^s + \epsilon_{rps} N_{[\gamma\alpha]\beta, q}^s \right), \end{aligned} \quad B1$$

where A, B and C are split into 'Dirac' and 'SU(3)' indices α, p, β, q and γ, r (α, β and $\gamma = 0, 1, 2$ or 3 , p, q and $r = 1, 2$ or 3). Here $\{\dots\}$ denotes symmetry in the appropriate indices and $[\dots]$ anti - symmetry.

Suppressing the SU(3) indices, the wave functions are defined by

$$D_{\{\alpha\beta\gamma\}} = \left(\gamma_\mu C \right)_{\alpha\beta} D_{\mu\gamma} + \frac{1}{2} \left(\sigma_{\mu\nu} C \right)_{\alpha\beta} D_{\mu\nu\gamma} \quad \} \quad B2a$$

$$\text{with } \gamma_\mu D_{\mu\gamma} = 0 \text{ and } D_{\mu\gamma} - i\gamma_\nu D_{\mu\nu\gamma} = 0,$$

$$N_{[\alpha\beta]\gamma} = C_{\alpha\beta} K_\gamma + \left(\gamma_5 C \right)_{\alpha\beta} N_\gamma + i \left(\gamma_\mu \gamma_5 C \right)_{\alpha\beta} N_{\mu\gamma} \quad \} \quad B2b$$

$$\text{with } K_\gamma - \gamma_5 N_\gamma + i\gamma_\mu \gamma_5 N_{\mu\gamma} = 0$$

and

$$\left. \begin{aligned} V_{[\alpha\beta\gamma]} &= C_{\alpha\beta} U_\gamma + (\gamma_5 C)_{\alpha\beta} V_\gamma + i(\gamma_\mu \gamma_5 C)_{\alpha\beta} V_{\mu\gamma} \\ \text{with } U_\gamma + \gamma_5 V_\gamma &= 0 \text{ and } U_\gamma + \frac{1}{4} i \gamma_\mu \gamma_5 V_{\mu\gamma} = 0 \end{aligned} \right\} \text{B2c}$$

where C is the anti-symmetric matrix with the property that $C^{-1}\psi^T$ (ψ^T is the transpose of ψ) transforms like $\bar{\psi}$.

The Bargmann - Wigner⁽⁹⁸⁾ equations of motion are such that $D_{\{\alpha\beta\gamma\}}$ describes a particle of spin $3/2$, $N_{[\alpha\beta]\gamma}$ of spin $\frac{1}{2}$ whilst $V_{[\alpha\beta\gamma]}$ vanishes identically. For particles of mass m and momentum p these wave functions reduce to

$$m D_{\{\alpha\beta\gamma\}}(p) = ((\not{p} + m)\gamma_\mu C)_{\alpha\beta} D_{\mu\gamma}(p) , \quad \text{B3a}$$

$$m N_{[\alpha\beta]\gamma}(p) = ((\not{p} + m)\gamma_5 C)_{\alpha\beta} N_\gamma(p) \quad \text{B3b}$$

and

$$V_{[\alpha\beta\gamma]}(p) = 0 . \quad \text{B3c}$$

Now the baryon octet and decuplet can be assigned to the 364 representation⁽⁹⁹⁾ of $\tilde{U}(12)$. To obtain the form for the 'baryon current' the expectation of the 144-vector $(\gamma^R \lambda^i)_A^B$ between 364 states is found (364 $\otimes$ 364 contains 143 once only). Here γ^R represents the 16 γ -matrices and λ^i the unitary spin matrices of Gell-Mann⁽¹³⁾. If unequal masses are used for the baryons the results obtained for the currents are

$$J^i = \frac{1}{4} P^2 (\bar{N}N)_F + \frac{3}{4} P^2 \bar{D}_\lambda D_\lambda + \frac{3}{2} Q_\lambda \bar{D}_\lambda Q_\lambda D_\lambda , \quad \text{B4a}$$

$$\begin{aligned} J_\mu^i &= \frac{1}{2} P_\mu (\bar{N}N)_F + \frac{1}{4} (\bar{N} r_\mu N)_{D+2/3F} + \frac{1}{2} (\epsilon_{\mu\kappa\lambda} P_\kappa Q_\lambda \bar{D}_\nu N + \text{h.c.}) \\ &\quad + \frac{3}{4} P^2 \bar{D}_\lambda \gamma_\mu D_\lambda + \frac{3}{2} Q_\lambda \bar{D}_\lambda \gamma_\mu Q_\lambda D_\lambda , \end{aligned} \quad \text{B4b}$$

$$J_{\mu\nu}^I = \frac{1}{4}P^2 \left\{ \left(\bar{N} \sigma_{\mu\nu} N \right)_{D+2/3F} + 3\bar{D}_\lambda \sigma_{\mu\nu} D_\lambda \right\} + \frac{3}{2}Q_\lambda \bar{D}_\lambda \sigma_{\mu\nu} Q D_k + \frac{1}{4}i(P_\mu Q_\nu - Q_\mu P_\nu) \left(\bar{N} N \right)_{1/3F-D} + i\epsilon_{\mu\kappa\lambda} (P_\lambda \bar{D}_\kappa N + \text{h.c.}) , \quad B4c$$

$$J_{\mu 5}^I = \frac{1}{4}P^2 \left(i\bar{N} \gamma_\mu \gamma_5 N \right)_{D+2/3F} + \frac{3}{4}i\bar{D}_\lambda \gamma_\mu \gamma_5 D_\lambda + \frac{3}{2}iQ_\lambda \bar{D}_\lambda \gamma_\mu \gamma_5 Q D_k - \frac{1}{4}iP^2 \left(\bar{D}_\mu N + \text{h.c.} \right) + i \left(\frac{P'_\mu Q}{3} \bar{D}_\lambda N + \text{h.c.} \right) \quad B4d$$

and

$$J_5^I = \frac{1}{4}P^2 \left(\bar{N} \gamma_5 N \right)_{D+2/3F} + \left(Q_\lambda \bar{D}_\lambda N + \text{h.c.} \right) + \frac{3}{4}P^2 \bar{D}_\lambda \gamma_5 D_\lambda + \frac{3}{2}Q_\lambda \bar{D}_\lambda \gamma_5 Q D_k , \quad B4e$$

where the initial particle has mass m and momentum p , the final particle has mass m' and momentum p' and

$$P = \frac{p}{m} + \frac{p'}{m'} , \quad Q = \frac{p}{m} - \frac{p'}{m'} \quad \text{and} \quad r_\mu = \epsilon_{\mu\kappa\lambda} p_\nu q_\kappa \gamma_\lambda \gamma_5 .$$

These reduce to equations 5.10 - 5.16 of S. D. S. when equal masses are inserted - i.e. when the symmetry is not intrinsically broken by the Bargmann - Wigner equations.

In these currents B4a - B4e the $SU(3)$ content has been suppressed, the relevant expressions to be used in J_R^I being

$$\left(\bar{N} N \right)_F = \bar{N}_r^p \Gamma(\lambda^I)_{p q}^q N_q^r - \bar{N}_r^p \Gamma N_p^q (\lambda^I)_q^r ,$$

$$\left(\bar{N} N \right)_D = \bar{N}_r^p \Gamma(\lambda^I)_{p q}^q N_q^r + \bar{N}_r^p \Gamma N_p^q (\lambda^I)_q^r ,$$

$$\left(\bar{D} \Gamma N \right) = \bar{D}^{pqr} \Gamma(\lambda^I)_p^t \epsilon_{tqs} N_r^s$$

and

$$\left(\bar{D} \Gamma D \right) = \bar{D}^{pqr} \Gamma(\lambda^I)_p^s D_{sqr} .$$

For completeness the matrices λ^I , N and D are listed :

$$\lambda^1 = \begin{pmatrix} \cdot & 1 & \cdot \\ 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}, \quad i\lambda^2 = \begin{pmatrix} \cdot & 1 & \cdot \\ -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot \end{pmatrix},$$

$$i\lambda^5 = \begin{pmatrix} \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot \\ -1 & \cdot & \cdot \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 \\ \cdot & 1 & \cdot \end{pmatrix}, \quad i\lambda^7 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 \\ \cdot & -1 & \cdot \end{pmatrix} \quad \text{and} \quad \sqrt{3}\lambda^8 = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & -2 \end{pmatrix},$$

$$N = \begin{bmatrix} \frac{\Sigma^0 + \Lambda^0}{\sqrt{2} + \sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0 + \Lambda^0}{\sqrt{2} + \sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\sqrt{\frac{\Lambda^0}{6}} \end{bmatrix}$$

and

$$\begin{array}{ccccccc} D_{333} & & & & \Omega^- & & \\ D_{233} & D_{133} & & & = & \frac{1}{\sqrt{3}} \Xi^{*-} & \frac{1}{\sqrt{3}} \Xi^{*0} \\ D_{223} & D_{123} & D_{113} & & & \frac{1}{\sqrt{3}} Y_1^{*-} & \frac{1}{\sqrt{3}} Y_1^{*0} & \frac{1}{\sqrt{3}} Y_1^{*+} \\ D_{222} & D_{122} & D_{112} & D_{111} & N^{*-} & \frac{1}{\sqrt{3}} N^{*0} & \frac{1}{\sqrt{3}} N^{*+} & N^{*++} \end{array}.$$

It is now clear how the relevant coupling strengths given in table 5 are obtained. The assumption about the transformation properties of the weak interaction currents requires the combination $\lambda^1 + i\lambda^2$ for the $\Delta S = 0$, $\Delta I = 1$ current (' ρ' - like) and $\lambda^4 + i\lambda^5$ for the $\Delta S = 1$, $\Delta I = 1$ current (' K^{*} '- like).