

THE INFLUENCE OF SLOT GEOMETRY
ON THE TWO DIMENSIONAL TURBULENT WALL-JET

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ABSTRACT

An experimental investigation of the influence of slot geometry parameters on the effectiveness of film cooling downstream of a two-dimensional film cooling slot is presented for uniform density flows. Existing correlations were found to be incapable of satisfactorily predicting the influence of slot geometry parameters on effectiveness.

Measurements of mean and fluctuating hydrodynamic properties of the flow are presented for four values of the velocity ratio, $\bar{U}_C/U_G$, viz. 0.75, 0.91, 1.33 and 2.33; for values of the ratio of lip thickness to slot height, t/y_C , of 0.126 and 1.14; and for values of non-dimensional distance x/y_C , between zero and 150. The mean properties include static pressure, velocity and wall-shear-stress; the fluctuating properties include the three fluctuating components of velocity, the local turbulent-shear-stress, $\overline{uv}$, and u -spectra. Prandtl mixing-length and the Prandtl-Kolmogorov turbulent length scale are deduced from measurements and shown to be very non-similar for all values of x/y_C .

A procedure to predict the influence of slot geometry parameters on the effectiveness of film cooling and on other important properties of the flow in the initial region of the flow is described in detail. The procedure is based upon the finite-difference solution of time-averaged two-dimensional Navier-Stokes equations. The Prandtl-Kolmogorov model of turbulence is employed to calculate the effective viscosity. The solution procedure has been found to yield predictions which are in good agreement with experimental results.

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CHAPTER 1: INTRODUCTION

1.1 The problem considered

The work described in this thesis is concerned with the prediction of the flow properties downstream of a two-dimensional film cooling slot. It represents an early phase in a long-term objective to provide a procedure for predicting flow properties downstream of practical three-dimensional film cooling slots. Such a procedure should be capable of predicting the influence of initial and boundary conditions on the wall temperature, skin friction and profiles of mean velocity, turbulent kinetic energy and temperature downstream of the slot exit. The process of film cooling is employed in many practical situations, for example, blade cooling, rocket nozzles and combustion chambers; the process of film heating is also found in practice, for example, the de-icing of aircraft wings and windscreens. The hydrodynamic aspects of film cooling are of considerable practical significance in boundary layer control. The present work is particularly related to film cooling in gas turbine combustion chambers.

The film cooling slots employed in combustion chambers are, for structural reasons, three-dimensional. The practical configuration is further complicated since the walls of the combustion chamber are non-adiabatic; the slots are subjected to severe temperature gradients; and large density gradients prevail. In addition, the absence of separate control on the film cooling mass flow implies that the slot to free-stream velocity ratio frequently varies between 0.1 and 3.0 under operating conditions. The complexity of the practical problem has led to the adoption of design procedures which are based almost entirely on empirical correlations and past experience. Thus, there is a need for

the development of a design procedure based on fundamental equations and embodying a minimum of empiricism.

In order to provide a well-tested basis for a general prediction procedure, simplifying assumptions have been made in the present study. Thus, it has been assumed that the flow is two-dimensional, incompressible and that the wall is adiabatic. It must be emphasised, however, that the present procedure is based on differential equations and physical flux laws and is, therefore, on a sound foundation. Extension to account for the influence of density gradients and heat transfer at the wall is relatively simple; extension to three-dimensional flow is rather more complicated and requires more storage facilities than are available in the present generation of computers.

Since a prediction procedure must be tested carefully against experimental information, a substantial part of the present thesis is concerned with describing an experimental programme designed to provide the required test data. This experimental programme was also of a two-dimensional, incompressible nature with zero flux at the wall. In practice, the boundary condition of zero wall mass flux was preferred to that of zero wall heat flux. Thus the adiabatic wall was replaced by an impervious wall and, in the prediction procedure, the species equation was solved rather than the thermal energy equation. This mass transfer analogy was invoked because it permitted greater control over the initial and boundary conditions.

The specific objectives of the present work; previous work; the chosen plan of attack; and a brief outline of the thesis are described in the succeeding subsections of this introduction.

1.2 Specific objectives and restrictions of the study

The specific objective of the present study is to provide a prediction procedure for the flow downstream of constant property, two-dimensional wall jets. In particular to provide a method to predict the impervious-wall effectiveness (or adiabatic wall effectiveness) as a function of slot geometry parameters. It is necessary to test the procedure by comparing predicted properties with experiments.

The physical problem considered here is shown in figure 1.1. A secondary stream of air, at the same temperature and density as the main stream, is injected, from a slot of height y_c , tangentially into a turbulent boundary layer. The secondary stream contains a homogeneously mixed small amount of a chemically inert tracer gas (the mass fraction of the tracer is small enough to neglect density variations). The main stream and the secondary stream are separated by impervious material of thickness t . The study is restricted to the following conditions:

1. Flow is plane, steady, two-dimensional and turbulent.
2. No density, temperature and laminar viscosity gradients are present in the flow.
3. The wall on which the boundary layer develops is smooth and impervious.
4. All body forces are neglected.

1.3 Previous relevant work

For the sake of convenience, this section has been divided into two subsections: the first subsection deals with the review of the experimental investigations and the second gives a brief account of existing prediction methods.

1.3.1 Experimental investigation of hydrodynamics and ϕ^* -properties of the flow

1.3.1a Hydrodynamic properties of the wall-jet

The hydrodynamic properties which are of particular interest here are skin friction and the mean and fluctuating components of velocity. Most previous detailed investigations have been carried out in the range of velocity ratios from 2 to ∞ in contrast to the range of 0.1 to 2 which is relevant to gas turbines. Prominent among these previous investigations are those of Sigalla (95)[±], Bradshaw and Gee (10), Myers, Schauer and Eustis (61), Eskinazi and Kruka (26) and Tailand and Mathieu (106). The reason for the concentration of effort on the higher velocity ratio is understandable; in this region the velocity profile retains a maximum and the flow can be analysed approximately by similarity methods.

1.3.1b ϕ -properties

The effectiveness of film cooling is one of the most important ϕ -properties of the flow and this section is primarily concerned with it.

Investigations of film cooling can be broadly divided into two groups: the first pertaining to the practical slots which are three dimensional in form and the second to idealised slots which are two dimensional. In the former case, the work of Halls (34), Nobbs and Rice (66), Barnwell(2) and Holve (43) is of direct practical relevance to gas turbine practice. At present, there are some practical limitations

* ϕ stands for the conserved properties such as total enthalpy or mass concentration of a chemically inert gas. Total enthalpy can, of course, be replaced by temperature for a low speed flows with constant specific heat.

± Numbers in the bracket denote the references.

in the path of obtaining a successful prediction method, based upon the solution of Navier-Stokes equations, for the flow downstream of a three-dimensional cooling slot. Some of the important limitations are:

(1) the storage facilities available on the present day computers are not sufficient, (2) the existing experiments are not methodical and do not, therefore, allow the various dependent variables to be separated and their individual contribution assessed, (3) the experiments are not of the form which allow flux laws to be deduced. In contrast to the three-dimensional slots, the two-dimensional slots are within the reach of our present capacity to predict the flow and also to provide a solid foundation on which to start building a prediction method capable of future extension to three-dimensional situations. Therefore, from here on, the thesis is concerned with two-dimensional slots only.

There exists a wide variety of two-dimensional slots which have been investigated over the last 25 years. Whitelaw (118) has given a summary of various investigations up to 1965. Table 1 presents a brief and up-dated account of the slot geometry parameters investigated, their range and relative importance. It must be emphasised that the table is, by no means, complete and exhaustive. The investigations included in the table have been chosen with two main points in mind: (1) they should be of direct relevance to the present problem (2) it should be possible to deduce from the experiments the influence of slot geometry parameters on effectiveness or on hydrodynamic properties.

The effect of the initial upper lip boundary layer thickness on film cooling effectiveness was investigated by Chin et. al. (16) and Seban (92) for velocity ratio less than unity and nominally in zero density gradients. Burns and Stollery (12) conducted experiments for the influence of upper lip boundary layer thickness on effectiveness in

presence of density gradients. Seban (93) and Samuel and Joubert (89) carried out experiments with three slot heights at various velocity ratios. No attempt was made to separate out the influence of slot height on effectiveness. In a recent investigation, Carlson and Talmor (14) have found that the free stream turbulence and hot gas acceleration have very serious detrimental effect on the adiabatic-wall effectiveness. Their results show that the adiabatic-wall effectiveness at a free stream turbulence level of 22% was up to 20% (of unity) lower than in the case of a free stream turbulence level of 2%.

In addition to the present study, two main parallel investigations, one by Pai and the other by Burns, are in progress at Imperial College. Different aspects of film cooling are under investigation in these studies and the overlap of the work in many instances provides a welcome check on the results. There has been co-operation between these investigations and the findings of one investigation have influenced the direction and development of other investigations. The present findings for the influence of lip thickness and the upper lip boundary layer thickness, in absence of density gradients, have subsequently been confirmed by Burns and Stollery (12), who extended this investigation to include the influence of density gradients. They concluded that for large x/y_c , the effectiveness varies as slot Reynolds number raised to 0.2 power but for small x/y_c , the slot Reynolds number effect was small. They also found that for low values of p_c/p_g , there was a more pronounced decrease in the effectiveness due to the thicker lip and, for lower values of p_c/p_g , the influence of the upper lip boundary layer thickness can become important. Sivasegaram's experiments (97) on the influence of lip thickness were carried out to provide an independent check on the findings of reference 49 and also to compare adiabatic-wall effectiveness with impervious-wall effectiveness measurements of reference 49.

From the preceding paragraph, it is evident that apart from the influence of δ on effectiveness for the $\bar{U}_C/U_G$ less than unity, other slot geometry parameters such as t , y_C , slot velocity profile and turbulence level of slot and free stream fluid are yet to be methodically investigated. The present work includes such an investigation.

1.3.2 Theoretical investigations

The differential equations appropriate to the present problem are the time-dependent Navier-Stokes equations. During the last decade it has become possible to obtain complete solutions of these equations for a few selected flow solutions. This has been made possible through the use of advances in the computer technology. Most of the early theoretical work was based upon some simple correlations, derived from dimensional arguments. Experimental results were used to obtain the relevant constants in these correlations. For example, the form of the skin friction law for the wall jets, given by Sigalla (95), is based upon the Blasius skin friction law and some dimensional arguments. Similarly, most of the correlations for the prediction of the effectiveness of film cooling have been devised in order to obtain correct functional relationship for relevant dimensionless groups such as velocity ratio, slot Reynolds number and dimensionless distance. In between the two extremes of simple correlation and sophisticated solution of the Navier-Stokes equations, there is an intermediate stage in which predictions are based upon the solution of integral boundary layer equations. In these prediction methods, it is required to solve total differential equations rather than partial differential equations.

In the previous paragraph, prediction methods have been broadly classified into three groups. The prediction methods, in their respective groups, are discussed briefly in the subsequent paragraphs.

1. Simple correlations

The correlations for the prediction of effectiveness of film cooling are of two types:

1. Purely empirical,
2. Semi-empirical.

Chin et.al. (16) developed the following purely empirical correlation to predict the effectiveness of film cooling:

$$\eta = f\left[\left(\frac{\rho_C}{\rho_G}\right)^{-1.5} \left(\frac{\bar{U}_C}{U_G}\right)^{-1.0} Re_C^{-0.3} \left(\frac{U_{GL}}{v_G}\right)^{0.19} \left(\frac{x}{y_C}\right)\right] \text{ if } \frac{\bar{U}_C}{U_G} < 1$$

$$\eta = f\left[\left(\frac{\rho_C}{\rho_G}\right)^{-1.5} Re_C^{-0.3} \left(\frac{U_{GL}}{v_G}\right)^{0.19} \left(\frac{x}{y_C}\right)\right] \text{ if } 1 \leq \frac{\bar{U}_C}{U_G} \leq 2 \quad 1.3-1$$

Many correlations (14, 40, 93, 99, 104) of the semi-empirical type exist in the literature. They have been developed using two main analyses. In the first analysis, it is assumed that the slot flow behaves like a boundary layer and is partially or fully mixed with free stream fluid. Correlations based upon this approach are partially successful where $\bar{U}_C/U_G$ is less than approximately 1.5. For high slot velocity ratios, the second analysis is more successful; in this, it is assumed that the slot flow exists as a discrete layer (no mixing with freestream fluid). Spalding (99) has combined the two approaches to obtain a correlation which has been claimed to be valid in the range $0 \leq \bar{U}_C/U_G \leq 14$.

The correlations discussed above are simple and easy to use. They have the advantage of providing quick approximate answers, but it should be noted that these correlations are limited in their range of

application. There is large scatter in experimental results, due mainly to differences in slot geometry and it is not surprising that the correlations of the type discussed above do not have much chance of general success in predicting effectiveness.

2. Integral methods

Integral methods for the solution of boundary layer equations are of two general types: (1) parametric integral procedures and (2) explicit integral procedures. In both these methods total differential equations are obtained from the partial differential equations but they differ significantly in the physical inputs.

In parametric integral methods, velocity profiles are expressed in terms of algebraic functions with free parameters. In order to get the total differential equations from the partial differential equations, three steps are necessary:

1. the partial differential equations are multiplied by weighting functions,
2. profile assumptions in terms of free parameters are inserted,
3. the resulting matrix is inverted.

In addition, an expression for effective viscosity is needed to relate shear stress to the velocity gradients. Myers, Schauer and Eustis (61) predicted the maximum velocity decay, growth rate and shear stress distribution for $\bar{U}_C/U_G$ equal to ∞ , using a parametric integral method. A similar approach was adopted by Harris (38) in developing a method which is applicable to flows with $\bar{U}_C/U_G$ greater than 1 and arbitrary pressure gradients. Both these methods use flow similarity assumptions and are not applicable to cases when the velocity profile loses its maximum. Moses (60) used a parametric integral method for predictions where $U_C/U_G < 1$. Patankar and Spalding (73), on the other

hand, developed a parametric integral method applicable both to boundary layers and wall-jets. One of the main draw-backs of the parametric integral method is that the matrix to be inverted often becomes singular and remedies for this are difficult to find.

Explicit integral methods are free from profile assumptions, matrix inversion and direct use of an effective viscosity hypothesis. On the other hand, this method depends on empirical relations between drag coefficient, shape factors, entrainment rate, etc. Rotta (85), Truckenbrodt (113), Head (42), Escudier and Spalding (22), Walz (115) and Escudier and Nicoll (23) have used explicit integral method for boundary layer predictions.

Nicoll and Whitelaw (64) extended the method given by Escudier, Nicoll et.al.(24) for the prediction of effectiveness and hydrodynamic properties of wall-jets. Total differential equations for Reynolds number based upon mean momentum and kinetic energy thickness were solved using the Runge-Kutta method together with explicit auxiliary relations for skin friction and shape factor in terms of momentum and energy thickness Reynolds number. In order to solve the thermal boundary layer equations, it was assumed that the ratio of thermal to hydrodynamic boundary layer thickness was 1.25 and was kept constant in the flow field. Later on Kacker, Nicoll and Whitelaw (45) presented an improved version of the method where an additional equation was solved and the need for the constant ratio of thermal to hydrodynamic boundary layer thickness removed. In the course of these developments, it became obvious that this method suffers from the following basic disadvantages which are difficult to remedy:

1. The equations become singular when the momentum thickness passes through zero.

2. The extension of the method, to take into account the slot geometry parameters, would require re-development of auxiliary equations.
3. The equations are only valid in the region where the boundary layer approximations hold good and hence additional correlations are required to link slot conditions with those at some downstream station where the boundary layer assumptions are valid.

Due to the above limitations, further work on this method was abandoned.

3. Methods based upon solution of differential equations

Using finite-difference techniques, Thoman and Szewczyk (110) solved the two-dimensional, time dependent Navier-Stokes equation for the prediction of Karman Vortex Streets in laminar flow behind a circular cylinder. Solutions of this type, for turbulent flows present formidable numerical problems and the less complicated time averaged equations are considered likely to be the basis of prediction procedures for many years to come. Runchal, Spalding and Wolfshtein (88) have developed a method which is capable of solving these time averaged equations. It should be noted that these equations are elliptic in form and apply to recirculating flows. The application of boundary layer assumptions reduces these elliptic Navier-Stokes equations to parabolic equations which are comparatively easy to solve. Spalding (75), Bradshaw, Ferris and Atwell (9), Glushko (30) and Beckwith and Bushnell (4) have developed computer programs which are capable of solving the parabolic boundary layer equations using finite-difference techniques.

From the preceding paragraph, it is evident that the mathematical tools exist to solve both the time averaged Navier-Stokes equations and the boundary layer equations. There is no doubt that the prediction

methods based upon solution of differential equations are superior to the correlations and integral methods. As will be shown later in the thesis, the time averaged two-dimensional Navier-Stokes equations have been chosen to be the basis of the prediction method of the present study.

1.4 Chosen plan of attack

A close examination of the past experimental work in film cooling revealed discrepancies in results, sometimes of order of 100%. This could not be entirely due to the experimental errors. Various investigators had different slot geometries and it appeared likely that the influence of slot geometry on effectiveness was substantial. No systematic attempt has been made to examine the influence of slot geometry. Hence the first step was to systematically investigate the influence of slot geometry parameters on effectiveness and to find out what important implications, if any, slot geometry has for prediction procedures. As a result of this investigation, it was established that the flow in the immediate vicinity of the slot is elliptical in nature, and, after realising that the integral methods have no chance of general success in such a situation, integral methods (45) were abandoned in favour of finite difference solution of time averaged Navier-Stokes equations.

In the course of the development of the prediction procedure, experiments were conducted to yield the mean and turbulent properties of the wall jet ($\bar{U}_C/U_G > 1$) and wall wake ($\frac{\bar{U}_C}{U_G} < 1$) flows. These experimental results formed the basis of testing of prediction procedure. In the early stage of development of prediction method, Prandtl's mixing length model was adopted. But later on it was rejected in favour of the Prandtl-Kolmogorov model of turbulence, because the later model is capable of taking in account the upstream history of the flow as well as the effect of turbulence level on the mixing of the two streams. This approach was found to produce satisfactory predictions. As will be seen, this model

is not perfect. It does, however, permit satisfactory prediction to be obtained in many cases. General guide lines are provided for the development of a new model.

1.5 Outline of thesis

The main body of the present thesis is divided into 6 chapters; this introduction being the first chapter.

In chapter 2, the equations of motion are first presented and the necessity of a physical model for the solution of these equations is established. This is followed by a short description of physical models for turbulent flows. Then, stream function and vorticity are introduced as new variables and the equations are re-written in terms of ψ and ω . The physical model adopted for the present prediction method is also introduced in the equations. Finally, available methods for the solution of elliptic equations are reviewed.

Chapter 3 is concerned with the experimental investigation of effectiveness of film cooling. First, important slot geometry parameters are identified. The apparatus and the experimental techniques employed in the present investigations are described. This is followed by the presentation of experimental results for the influence of slot geometry parameters on the effectiveness. The chapter ends with a discussion of the present results and the conclusions drawn from them.

The experimental results of chapter 3 are concerned only with the wall concentration and they are not sufficient to satisfactorily test the prediction method; consequently, detailed measurements of various mean and fluctuating flow properties were carried out and these are presented in chapter 4. The experiments described in this chapter were carried out for four velocity ratios and two slot geometries: one with a lip of $1/32$ "

and the second with a lip of $9/32$ ". The flow properties measured are static pressure variation, mean velocity, skin friction and three components of fluctuating velocities and shear stress distribution. These measurements yield the data against which the prediction method described in chapter 5 is tested. The chapter ends with conclusions drawn from the measurements.

In chapter 5, the equations presented in chapter 2 are further simplified. All the equations are reduced to a similar form. This is followed by a description of a finite-difference scheme for the solution of these equations. Computing details such as the control volume, boundary conditions, number of iterations, convergence criterion, etc. are also described. The predictions obtained with the present solution procedure are presented and compared with the experimental results of chapter 4. In cases where experimental results are not available only the predicted trends are presented. For laminar flows, two cases of exact solutions are compared with finite-difference solutions and the agreement is found to be satisfactory.

Each of the chapters 3, 4 and 5 ends with a section on conclusions drawn from that chapter. The inter-relationships of these conclusions are discussed in the last chapter. This last chapter also re-appraises the work described in the previous chapters and makes suggestions for the future development of the experimental and theoretical work.

CHAPTER 2: MATHEMATICAL PROBLEM

2.1 Introduction

In the present chapter, the basic equations, on which the prediction procedure is based, are presented and discussed.

First, the time averaged Navier-Stokes equations for two-dimensional constant density flows are presented. These partial differential equations are non-linear and elliptic in nature. In a few laminar flow cases, they can be reduced to total differential equations for which exact solution may be possible. However, for turbulent flows it is not possible to obtain their solution without a postulated relationship for $\overline{uv}$ in terms of dependent and independent variables or their gradients. In the literature there are many suggested relationships of this type. These are discussed in the section dealing with physical models.

Having obtained a set of differential equations in which the total number of unknowns is equal to the number of equations, a mathematical tool is needed to solve them. Available methods of solutions are described in the last part of this chapter.

The method of solution which has been adopted is presented in chapter 5.

2.2 Relevant equations

In order to predict the time averaged temperature (or mass concentration ϕ of a chemical species), it is necessary to solve the equations of conservation of mass, momentum and the quantity ϕ . The time averaged conservation equations in cartesian coordinates may be written as:

Continuity Equation

$$\frac{\partial u_i}{\partial x_i} = 0$$

2.2-1

Momentum Equation

$$\rho U_j \frac{\partial U_i}{\partial x_j} = - \frac{\partial p_s}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \quad 2.2-2$$

Conservation of ϕ

$$\rho U_j \frac{\partial \phi}{\partial x_j} = - \frac{\partial J_j}{\partial x_j} + S_\phi \quad 2.2-3$$

where i and j can take values of 1, 2 and 3 (summation convention is to be used where repetition of subscripts take place),

p_s is pressure,

τ_{ij} is shear stress

J_j is mass-flux of ϕ

S_ϕ is source term for ϕ

2.3 Physical model

In order to solve the momentum equation, it is necessary to make an assumption for the functional relationship between τ_{ij} and other dependent and independent variables. This assumption should be based upon knowledge of the physical mechanisms of the turbulent motion and should describe these mechanisms sufficiently well to permit accurate predictions to be made.

Physical models, which have been described in the literature for the solution of momentum equation, may be broadly divided into two groups. In the first group, shear stress is related to mean velocity gradient according to the Boussinesq (8) equation:

$$\tau = \mu_{\text{eff}} \frac{\partial U}{\partial y}$$

2.3-1

In the turbulence models of the second group, the shear stress is calculated from one or more differential equations and the effective viscosity concept is no longer necessary.

Some theories of the turbulence are presented in the table below. All the turbulence models which are above the dashed line in the table belong to the first group and the ones below the dashed line belong to the second group.

No.	Turbulence Model	Equation for μ_{eff}	Proportionality Constant	Velocity Scale	Length Scale	Equations to be solved	Comments
1	Uniform effective viscosity theory (a) Prandtl's formula for jet (1942), reference <u>81</u> (b) Clauser's formula for boundary layer (1956), reference <u>17</u>	$.01\rho \left U_{\text{max}} - U_{\text{min}} \right x$ (width of jet) $.016\rho U_G x$ (displacement thickness)	 .01 .016	$U_{\text{max}} - U_{\text{min}}$ U_G	width of jet displacement thickness	(1)x-momentum	1. According to the experimental evidence, μ_{eff} is not constant across the layer. Also the constants in μ_{eff} vary from one geometry to another. 2. No convection and diffusion of turbulent kinetic energy is allowed. 3. Displacement thickness becomes negative in wall-jets.
2	Prandtl's mixing length theory (1925), reference 80	$\rho \left l \frac{\partial U}{\partial y} \right l$	1	$l \frac{\partial U}{\partial y}$	l	(1)x-momentum	1. Experiments show that length scale l varies from one geometry to another. 2. According to this theory μ_{eff} cannot be finite when velocity gradient is zero. 3. No convection and diffusion of turbulent kinetic energy is allowed.

No.	Turbulence Model	Equation for μ_{eff}	Proportionality Constant	Velocity Scale	Length Scale	Equations to be solved	Comments
3	Kolmogorov's energy-frequency theory (1942), reference 56	$\rho A \frac{\sqrt{q^2/2} \sqrt{q^2/2}}{f^*}$	A	$\sqrt{q^2/2}$	$\frac{\sqrt{\frac{1}{2}q^2}}{f^*}$	(1)x-momentum (2)frequency f^* (3)turbulent-kinetic energy	1. f^* is some mean frequency. 2. Convection and diffusion of turbulent kinetic energy is allowed. 3. Shear stress is zero when $\partial U / \partial y = 0$
4.	Prandtl's energy-length theory (1945), reference 82	$\rho C_\mu \sqrt{q^2/2} \ell$	C_μ	$\sqrt{\frac{1}{2}q^2}$	ℓ	(1)x-momentum (2)turbulent-kinetic energy	1. ℓ is taken as function of y/y_G and is not universal. 2. Convection and diffusion of turbulent kinetic energy is allowed. 3. According to this theory shear stress is zero when $\partial U / \partial y = 0$
5	Rotta's energy-length and shear stress theory (1951), reference 86	<u>Comments</u> 1. Equations for x-momentum, $q^2/2$, ℓ and τ are solved. 2. It is possible for τ to be finite where $\partial U / \partial y = 0$. 3. For the prediction of fully developed pipe flow, Rotta's method needs 6 constants. 4. Application of Rotta's method to a 2-dimensional recirculating flow is, at present, some way off.					

No.	Turbulence Model	Comments
6	Bradshaw's energy-stress theory (1967), reference 9	1. x-momentum and turbulent-kinetic energy equations are solved. 2. It is assumed that $\tau / (\rho \sqrt{q^2/2})$ is a Universal constant. 3. Diffusion and convection of turbulence is allowed. 4. It is not possible for τ to change sign, as it does in reality

In addition to the effective viscosity concept, there is another common thread through the turbulence models of the first group: all of them are based upon the assumption that the state of the turbulence at a point can be defined by two local parameters - (1) velocity scale (2) length scale. For example, in Prandtl's (1945) model these scales are respectively $\sqrt{q/2}$ and l . In the table, for each appropriate turbulence model, the equivalent quantity for the proportionality constant, velocity scale and length scale are listed. Spalding's suggestions (101) for the names of turbulence models have been followed as their names illustrate the nature of the theory.

The model due to Prandtl (1945) and Kolmogorov (1942) are similar to each other and in the present thesis, they shall be jointly referred to as Prandtl-Kolmogorov model of turbulence. According to this joint model:

$$\mu_{\text{eff}} = \rho C_{\mu} l \sqrt{q/2} \quad 2.3-2$$

where C_{μ} = function of $(\rho \sqrt{\frac{1}{2}q}^2 l/\mu)$;

and is constant at high values of $(\rho \sqrt{\frac{1}{2}q}^2 l/\mu)$.

All the turbulence models which use the Boussinesq (8) equation for the calculation of shear stress inherently assume that the shear stress is zero when the velocity gradient is zero. However, in wall-jets and a few other types of flow there is evidence to the contrary. Of all the models listed in the above table, only Rotta's model allows the shear stress to be finite when the velocity gradient is zero. Rotta obtained a differential equation for the double velocity correlation $\overline{u_i u_j}$,

from which differential equations for $\overline{uv}$, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$ and $\overline{q^2}/2$ may be obtained. These equations do not form a system where the number of equations are equal to the number of unknowns. They contain triple velocity correlations and pressure velocity correlations for which assumptions have to be made before these equations can be solved. Only in a few simple cases, such as fully developed pipe flow, Rotta has been able to obtain their solution. Rotta has pointed out in his paper (86) that the validity of the assumptions leading to the solution of the shear stress equation are yet to be checked experimentally. Since this has not been done for a simple pipe flow, the application of Rotta's method to the complex problem of recirculating flows can hardly be justified at the present stage.

Beckwith and Bushnell (4) and Glushko (30) applied the Prandtl-Kolmogorov model to the prediction of boundary layer flows. They specified the length scale distribution as a function of (y/y_G) . Ng and Spalding (63) and Rodi and Spalding (84) replaced the empirical input for ℓ by a differential equation which is a simplified form of Rotta's differential equation for the length scale. This equation may be written as:

$$\begin{aligned} \frac{U}{2} \frac{\partial}{\partial x} (\overline{q^2} \ell) + \frac{V}{2} \frac{\partial}{\partial y} (\overline{q^2} \ell) = \frac{\partial}{\partial y} \left(\frac{C_\mu (\frac{1}{2} \overline{q^2})^{3/2}}{\sigma_1} \ell \right) \frac{\partial \ell}{\partial y} + \frac{C_\mu \sqrt{\frac{1}{2} \overline{q^2}} \ell^2}{\sigma_2} \\ \frac{\partial}{\partial y} (\frac{1}{2} \overline{q^2}) + C_S \sqrt{\frac{1}{2} \overline{q^2}} C_\mu \ell^2 \left(\frac{\partial U}{\partial y} \right)^2 - \frac{C_B}{C_\mu} (\frac{1}{2} \overline{q^2})^{3/2} \\ - \frac{C_W}{C_\mu} \left(\frac{\ell}{y} C_\mu \right) \overline{q^2} (\frac{\overline{q^2}}{2})^{3/2} \end{aligned}$$

The solution of the above equation, together with momentum and turbulent kinetic energy equation, requires the constants σ_1 , C_B , C_W , σ_2 , q' and C_S to be specified. In the predictions of free-jets, Rodi and Spalding (84) have taken $C_W = 0$.

Recently, Spalding (103) has presented a new method of calculation where, instead of above length scale equation, the equation for time-mean value of the square of the vorticity fluctuations (w') is solved. The Prandtl-Kolmogorov model is retained for the calculation of effective viscosity. In addition to the x-momentum equation, the two equations which he solves are:

$$\frac{U}{2} \frac{\partial \overline{q^2}}{\partial x} + \frac{V}{2} \frac{\partial \overline{q^2}}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(\frac{\rho \overline{q^2}}{2} \frac{\partial \overline{q^2/2}}{\partial y} \right) +$$

$$\frac{1}{2} \frac{\overline{q^2}}{\sqrt{w'}} \left(\frac{\partial U}{\partial y} \right)^2 - C_D' \left(\frac{\overline{q^2}}{2} \right) \sqrt{w'} \quad 2.3-4$$

$$\text{and} \quad U \frac{\partial w'}{\partial x} + V \frac{\partial w'}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(\frac{\rho \overline{q^2}}{2} \frac{\partial w'}{\partial y} \right) +$$

$$C_1 \frac{\overline{q^2/2}}{\sqrt{w'}} \left[\left(\frac{\partial^2 U}{\partial y^2} \right)^2 + C_4 \left(\frac{1}{r} \frac{\partial U}{\partial y} \right)^2 \right]$$

$$- C_2 w'^{3/2} + C_3 \sqrt{w'} \left(\frac{\partial U}{\partial y} \right)^2 \quad 2.3-5$$

where C_D' , C_1 , C_2 , C_3 and C_4 are supposedly universal constants or functions; and σ_ω is the effective Prandtl-Schmidt numbers for the diffusion of entity ω'

If the length scale l is defined by the equation 2.3-2, then it may show that the quantity ω' is equivalent to $\frac{1}{2} \overline{u^2} / l^2$.

Harlow and Nakayama (36) proposed a method of prediction for boundary layer flows which is also based upon Prandtl-Kolmogorov model of turbulence. In addition to x-momentum equation, they solved the differential equations for turbulent kinetic energy and dissipation length scale. The last named equation was obtained from the Navier-Stokes equation. In order to solve this equation, they had to make a series of simplifying assumptions some of which are difficult to justify on physical grounds.

Nee and Kovasznay (62) developed a method of solution where the effective viscosity is calculated directly from a differential equation. This equation is based upon a "simple educated guess" (quote from their paper) and its application and chances of success in a general flow situation are doubtful.

The approach in the present prediction program is to employ a simple physical hypothesis, i.e. the one due to Prandtl-Kolmogorov. An empirical input is supplied for l in the form of an additional algebraic equation. This empirical input is in accord with the experimental information and embodies the recommendations of Glushko (30) and Wolfshtein (120).

For the future development of the present method, the next step should be to build in gradual refinements in the method. These should be incorporated in order of increasing complexity. First, the empirical

input for l should be replaced by a differential equation for l and then, at some later date, the Prandtl-Kolmogorov model can be abandoned in favour of the solution of the shear stress equation on the lines suggested by Rotta (86).

2.4 Simplification of the relevant equations

Equations 2.2-2 and 2.2-3 may be simplified by introducing the following flux relations:

$$\tau_{ij} = \mu_{\text{eff}} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad 2.4-1$$

$$J_j = \frac{\mu_{\text{eff}}}{\sigma_{\phi, \text{eff}}} \left(\frac{\partial \phi}{\partial x_j} \right) \quad 2.4-2$$

where μ_{eff} is the total effective viscosity (including the contributions from laminar and turbulent motions) and $\sigma_{\phi, \text{eff}}$ is effective Schmidt or Prandtl number for diffusion of ϕ .

For two dimensional flows, by introduction of stream function ψ and vorticity ω defined as:

$$U = \frac{\partial \psi}{\partial y} ; \quad V = -\frac{\partial \psi}{\partial x} \quad 2.4-3$$

$$\text{and } \omega = - \left(\frac{\partial U}{\partial y} - \frac{\partial V}{\partial x} \right), \quad 2.4-4$$

it is possible to eliminate the pressure between the x and y momentum equation 2.2-2. The definition of the stream function automatically satisfies the continuity equation. Utilising the definitions of ψ and ω , equations 2.2-2 ^{and} ~~to~~ 2.2-3 may be written as:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = - \rho \omega \quad 2.4-5$$

$$\rho \left(U \frac{\partial \omega}{\partial x} + V \frac{\partial \omega}{\partial y} \right) = \frac{\partial^2}{\partial x^2} (\mu_{\text{eff}} \omega) + \frac{\partial^2}{\partial y^2} (\mu_{\text{eff}} \omega) + S_\omega \quad 2.4-6$$

$$\rho \left(U \frac{\partial \phi}{\partial x} + V \frac{\partial \phi}{\partial y} \right) = \frac{\partial^2}{\partial x^2} \left(\frac{\mu_{\text{eff}}}{\sigma_{\phi, \text{eff}}} \phi \right) + \frac{\partial^2}{\partial y^2} \left(\frac{\mu_{\text{eff}}}{\sigma_{\phi, \text{eff}}} \phi \right) + S_\phi \quad 2.4-7$$

$$\text{where } S_\omega = \frac{2\partial^2}{\partial x \partial y} [\mu_{\text{eff}} (\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x})] + \frac{2\partial^2}{\partial x^2} (\mu_{\text{eff}} \frac{\partial u}{\partial y}) - \frac{2\partial^2}{\partial y^2} (\mu_{\text{eff}} \frac{\partial v}{\partial x}) \quad 2.4-8$$

and S_ϕ is source term for production or depletion of ϕ . It should be noted that S_ω is zero only for uniform viscosity flows and boundary layers.

The equation for the turbulent kinetic energy is usually written in the form:

$$\begin{aligned} & \left[\frac{U}{2} \frac{\partial \overline{q^2}}{\partial x} + \frac{V}{2} \frac{\partial \overline{q^2}}{\partial y} \right] + \left[\overline{uv} \frac{\partial v}{\partial x} + \overline{uv} \frac{\partial u}{\partial y} + (\overline{u^2} - \overline{v^2}) \frac{\partial u}{\partial x} \right] \\ & + \left[\frac{\partial}{\partial x} \left\{ u \left(\frac{p}{\rho} + \frac{q^2}{2} \right) \right\} + \frac{\partial}{\partial y} \left\{ v \left(\frac{p}{\rho} + \frac{q^2}{2} \right) \right\} - \right. \end{aligned}$$

$$\left. \frac{v}{2} \left\{ \frac{\partial^2 \overline{q^2}}{\partial x^2} + \frac{\partial^2 \overline{q^2}}{\partial y^2} \right\} \right] + \epsilon = 0 \quad 2.4-9$$

$$\text{where } \epsilon = \nu \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right.$$

$$\left. + \left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] \quad 2.4-10$$

Partial differential equations 2.4-5, 2.4-6 and 2.4-7 are non-linear and elliptic. As will be shown in chapter 5, equation 2.4-9 can be simplified to a form which is similar to that of equations 2.4-5 to 2.4-7. In the present prediction method, these four equations together with Prandtl-Kolmogorov model are solved simultaneously by employing a finite-difference technique for the solution of elliptic equations.

2.5 Available methods of solution for elliptic equations

Flow past a circular cylinder has been one of the most popular subjects of theoretical and experimental study in the field of fluid mechanics. Thom (109) made an attempt to solve the Navier-Stokes equations for laminar, steady flow past a circular cylinder as early as 1933. It is commendable to note that he attempted to solve the Navier-Stokes equations without the help of high-speed computers, which did not exist at that time. Interest in the solution of two-dimensional steady and unsteady Navier-Stokes equations was revived in the fifties with the advent of high-speed computers. Rather than relating a detailed history of the various attempts to solve the Navier-Stokes equations, the highlights and differences in various approaches are outlined.

There has been a difference of opinion regarding the dependent variables which are most suitable for numerical methods. Some workers (31, 105, 109, 120) prefer to use vorticity and stream function as the dependent variables whereas others (37) think that there are advantages in the use of pressure and velocities as the dependent variables.

Most of the early attempts to obtain solutions of the Navier-Stokes equations were hampered by instabilities of the numerical methods at high Reynolds number. Central difference schemes were used by Thom and Apelt (108), Burggraf (11), Kawaguti (53) and Simuni (96). Burggraf tried to get the solution for a Reynolds number of 400 by use of heavy under-

relaxation and small cell size, but the resulting computation time was too great.

Success in obtaining solutions for high Reynolds number was achieved by Barakat and Clark (1) and Blair (5); both adopted flow oriented finite-difference schemes. Such schemes were later termed "upwind differences". Spalding (102) has shown that the "upwind differences" are unconditionally stable at all Reynolds numbers and his suggestion was incorporated by Runchal and Wolfshtein (87) in their numerical procedure. This approach has made possible stable solutions at high Reynolds number with reasonable computation time. It has been shown by Pun and Spalding (79) that the method of solution developed by Runchal and Wolfshtein (87) can also be used in presence of strong density gradients (for example in combustion problems). Later on, the combined efforts of the Imperial College team resulted in a general prediction program for recirculating flows (31).

The numerical procedure developed by Wolfshtein and Runchal (87) has been adopted in the present prediction program. The two main reasons for adopting their method were the stability of the method for high Reynolds number flows and its applicability to both laminar and turbulent flows.

CHAPTER 3: EXPERIMENTAL INVESTIGATION OF FILM COOLING EFFECTIVENESS

3.1 Introduction

A close examination of previous experimental work (16, 39, 92, 93, 94, 116) on film cooling effectiveness reveals that no comprehensive investigation of the influence of slot geometry parameters on film cooling effectiveness has been carried out previous to the present work. As will be shown in this chapter, the large discrepancies which exist between measurements made by different authors under apparently similar conditions may be attributed to the influence of slot geometry parameters. The experimental work described in this chapter is of direct practical relevance and can be helpful in the design of practical cooling slots. No attempt has been made to develop correlations for the experimental data presented here, as in the author's view the effort may be better spent in developing a general prediction procedure for the film cooling effectiveness.

Aims of chapter 3

1. To identify the slot-geometry parameters which significantly influence the impervious-wall effectiveness.
2. To present experimental results which demonstrate quantitatively the influence of slot geometry parameters on the impervious-wall effectiveness.

Slot geometry parameters

Figure 1.1 shows two parallel streams of air, separated by a distance t , mixing downstream of an ideal two-dimensional film cooling slot. From dimensional arguments, the following parametric relation can be justified:-

$$\eta_{ad} = \frac{T_w - T_G}{T_C - T_G} = f \left(\frac{x}{y_C}, \frac{\bar{U}_C}{U_G}, R_{ec}, R_x, \frac{v_C}{\alpha_C}, \frac{v_G}{\alpha_G}, \frac{\rho_C}{\rho_G}, \right.$$

$$\left. \frac{dv}{dT} \left(\frac{T_G - T_C}{v} \right), \frac{d\rho}{dT} \left(\frac{T_G - T_C}{\rho_G} \right), \frac{T_G}{T_G - T_C}, G \right) \quad 3.1-1$$

where G is a parameter to express the slot geometry effects; a similar expression has been mentioned in reference 89. For the present work, both the streams are air and if the temperature difference between the two streams is small, the above expression can be simplified by neglecting terms containing density, temperature and other property variations.

Although the above parametric expression has been written for adiabatic-wall effectiveness, a similar relation is also valid for impervious-wall effectiveness provided the thermal diffusivities α_C and α_G are replaced by the corresponding mass-transfer diffusivities.

The slot geometry parameter G is related to the profiles of the various conserved properties in the plane of the slot exit. In the case of hydrodynamic properties, this may be sub-divided into:

1. the slot height, y_C ,
2. the upper-lip boundary layer thickness, δ ,
3. the shape of the upper-lip velocity profile,
4. the shape of the slot velocity profile,
5. the thickness and shape of the top edge of the slot and the slot contraction profile,
6. the turbulence level of the free stream and injected fluid,
7. the turbulent length scale of the slot and free stream fluid.

3.2 Present experimental program, apparatus and experimental technique

The experimental program has been carried out in order to investigate systematically the influence of slot geometry parameters on film-cooling effectiveness.

The measurements reported in the present study were obtained from the apparatus described in reference 117. It consists of a two-dimensional, moderate turbulence (0.5%), open circuit wind tunnel with provision for secondary air to be blown parallel to the base plate of the tunnel. A general arrangement of the injection region and the tunnel base plate is shown in figure 3.1; the slot height was adjustable, by moving the lip of the slot up and down, as required, and arranging the lip to ensure that a 1" parallel section existed before the slot exit.

The impervious-wall effectiveness was obtained by injecting a tracer of helium gas into the secondary flow and measuring the wall concentration of helium at various downstream locations. This was accomplished by sucking samples of the mixture of air and helium through a suitably spaced number of 0.02" diameter static-pressure holes, located in the base plate of the wind tunnel, and by measuring the concentration of helium in the samples with the aid of a gas chromatograph.

To ensure that the air-helium mixture was homogeneous at the slot exit, the tracer of the helium was injected into the secondary stream on the suction side of the secondary blower. Concentration profiles at the slot exit both in ^{Ch}y and z direction were measured from time to time and indicated that the mixture was homogeneous within the precision of the measurements ($\pm 1.5\%$). Care was also taken to ensure that the rate of sampling through the 0.02" diameter holes was sufficiently low. Preliminary tests (117) had indicated that sampling rates below a certain value resulted in constant helium concentration and that sampling rates above this critical value resulted in lower helium

concentration. The sampling rates employed to obtain the measurements were at all times below the critical value. For all measurements of concentration, the gas chromatograph employed a two meter long column, packed with Linde-type molecular sieve, to separate the permanent gases and a thermal conductivity cell to measure their concentration. The carrier gas was nitrogen; consequently, helium and oxygen were the only two significant peaks to be traced by the recorder. The column temperature was approximately 60°C. This combination of column and temperature permitted the helium and oxygen to pass through the column in less than three minutes. The thermal conductivity cell was calibrated at regular intervals for helium detection over the working range of concentrations. The calibration curve was found to be linear for helium concentration above approximately 0.1% by volume; all measurements were made in this linear range.

The impervious-wall effectiveness was measured at various locations down-stream of the slot and for constant values of the velocity ratio $\bar{U}_C/U_G$. The mean velocity profile at the slot exit from the wall to the free stream, was measured for each run and this yielded the value of $\bar{U}_C$. For all the present measurements, except those described in section 3.3.3.2-b, the free stream velocity was nominally 63 ft/s and the free stream turbulence level approximately 0.5%.

DISA constant temperature hot wire equipment was used for all measurements of turbulence properties. Two anemometers (55A01), a random signal correlator (55A06), cross-wire probe (55A32) and single inclined probe (55A29) were used to measure $\sqrt{v^2}$, $\sqrt{w^2}$, and $\overline{uv}$; $\sqrt{u^2}$ was measured with a single wire probe (55A25). Mean velocity measurements were also obtained from a single wire probe and agreed with the values obtained from the pressure measurements (using a flattened tip probe of internal dimension 0.004" x 0.04") to within $\pm 2\%$.

The hot wires were calibrated in accordance with the law of Collis and Williams (19); this may be written in the form:-

$$E^2 = A + B U_{\text{eff}}^{0.45} \quad 3.2-1$$

The calibration was carried out in a free stream of relatively low turbulence intensity (0.5%) and the constants A and B were calculated from six measurements of the voltage drop E and the effective velocity U_{eff} . The calibration was carried out before and after each run.

For a sloping wire of angle θ , the above equation may be modified to:-

$$E^2 = A + B [U (\cos^2 \theta + C^2 \sin^2 \theta)^{0.5}]^{0.45} \quad 3.2-2$$

and, in accordance with the suggestion of Champagne and Sleicher (15), C has been taken as 0.2.

Recent papers by Davis and Bruun (20) and Kjellström (54) indicate that the above equation may be incorrect, they suggest that U and $(\cos^2 \theta + C^2 \sin^2 \theta)^{0.5}$ should be raised to powers which are different from each other but which both depend upon the mean velocity. Further confirmation of this suggestion is necessary but it is useful to note that in the present case, where the calibration is over a limited range of velocity, the resulting differences are unlikely to be large.

3.3 Influence of slot geometry parameters

This section presents the experimental results of the influence of slot geometry parameters on film cooling effectiveness. For convenience previous work, the present contribution and discussion of each of the slot geometry parameters have been described in separate sub-sections with similar format.

3.3.1 Influence of t/y_C

3.3.1.1 Previous work

No previous systematic investigation of the influence of the parameter t/y_C on effectiveness has been carried out. Seban (93) and Samuel et. al. (89) made experiments with three slot heights while keeping the lip thickness constant, but unfortunately they did not try to separate out the influence of t/y_C on effectiveness, as both were primarily concerned with the influence of the velocity ratio. Seban (93) did not keep the free stream velocity constant while doing the runs for different velocity ratios; consequently, in his data the influence of Reynolds number is mixed up with the influence of other slot geometry parameters. Sivasegaram's experiments (97, 98) were carried out to confirm the findings of reference 49 concerning t/y_C , whereas Burns and Stollery (13) extended the findings of reference 49 to include the influence of density gradients.

3.3.1.2 Present contribution

The ratio t/y_C can be varied either by varying t or y_C . In the first set of experiments the lip thickness t was kept constant at $1/32$ " and the slot height, y_C varied from 0.074 " to 0.5 ". In the second set of measurements y_C was kept constant nominally at 0.25 " and the lip thickness varied from $1/32$ " to $15/32$ ".

3.3.1.2-a Effect of variation of the slot height

Four values of slot height were selected, viz. 0.074 ", 0.132 ", 0.25 " and 0.5 ". The range of velocity ratio covered by the measurements was:-

$$0.288 \leq \bar{U}_C/U_G \leq 2.66 ;$$

the corresponding range of slot Reynolds number was:-

$$745 \leq R_{ec} \leq 44000 .$$

The above ranges for velocity ratio and slot height are of practical interest in gas turbine practice.

Measured values of effectiveness are usually presented in figures which show logarithmic plots of effectiveness against downstream distance for constant values of velocity ratio. The data were first plotted in this way, but this form of presentation tends to compress the data. Consequently, the data have been cross plotted and are presented in figures 3.2 to 3.5 in the form of effectiveness against velocity ratio for constant downstream distance; the various figures indicate the data obtained using each of the four slot heights. The experimental points marked on the figure are interpolated values of effectiveness at the measured value of the velocity ratio.

3.3.1.2-b Effect of variations in lip thickness

The measurements described in this section were carried out with a slot height of 0.247" which was constant to within $\pm .003$ " over the central 12" of the wind tunnel. The thickness of the slot lip was increased by locating additional steel plates on the top of 1/32" thick lip (which was integral part of the slot). These plates had thickness which permitted values of lip thickness of 3/32", 5/32", 7/32", 9/32" and 15/32" to be attained. They were located with fitted screws.

For experiments with a thin lip ($t = 1/32$ "), the upper lip boundary layer had a virtual origin approximately 7" upstream of the slot exit: this was obtained by splitting the flow as shown in figure 3.1. However, for experiments with thick lips, the 'splitting' facility was removed with the aid of sheet metal and tape. The resulting upper lip boundary layer was fully turbulent at the slot exit and had a virtual origin 14" from the slot exit. The influence of upper lip

boundary layer thickness on effectiveness is discussed in section 3.3.2 and, in the present case, is not greater than 5% of unity.

The effectiveness is known to depend on the velocity ratio $\bar{U}_C/U_G$, and special care was therefore taken to ensure that measurements of the impervious-wall effectiveness were obtained for the same values of $\bar{U}_C/U_G$ with each value of t . This could not readily be achieved by integrating the mean velocity profile measured in the plane of the slot exit, since in thick lip situations substantial normal pressure gradients were present. Consequently, the following sequence was adopted: with the thinnest lip and therefore, with smallest normal pressure gradients*, a velocity profile was measured in the plane of the slot exit and integrated to yield a value of $\bar{U}_C$; measurements of impervious-wall effectiveness were obtained for this value of t and $\bar{U}_C$; without changing the position of the throttle valve on the secondary fan, the slot lip was thickened by the addition of a plate and measurements of the impervious-wall effectiveness were obtained for the new value of t and the same value of $\bar{U}_C$. This procedure was repeated for all values of t and thus, since the main stream velocity was maintained constant at 63 ft/s, values of impervious-wall effectiveness were obtained for sensibly constant values of $\bar{U}_C/U_G$ with each lip thickness. In fact, due to variations in the static pressure at the slot exit, slight variations in secondary flow was noted with the aid of an orifice plate located upstream of the secondary fan; these variations did not exceed $\pm 1\%$ and were considered negligible.

* Typical static pressure profiles for a thin lip are presented in reference 45; they indicate that the values of $\bar{U}_C$ should not be significantly affected by normal pressure gradient.

In addition to their utility in the calculation of $\bar{U}_C$, the slot velocity profiles are important from the prediction procedure point of view as they provide the boundary conditions at the slot exit. Typical velocity profiles obtained with the thinnest slot lip are shown in figure 3.6.

Measured values of the impervious-wall effectiveness are shown in figure 3.7 for a value of $\bar{U}_C/U_G$ of 1.07 and for six values of t/y_C : it can be readily seen that the influence of t/y_C is significant. Figure 3.7 also shows the maximum deviation of any measured point from the corresponding smooth curve is 0.06 and the mean deviation is considerably smaller. The coordinates of figure 3.7 neither permit all the present measurements to be conveniently shown, nor do they permit the influence of t/y_C to be clearly shown for values of velocity ratio not close to unity. Consequently, figure 3.8 has been prepared; in this figure the points represent values of impervious-wall effectiveness, interpolated from figures similar to figure 3.7, and corresponding to measured values of $\bar{U}_C/U_G$ and t/y_C .

3.3.1.3 Discussion

Figures 3.2 to 3.5 indicate that the value of effectiveness depends on the slot height in the region of velocity ratio close to unity. Figure 3.9 demonstrates the effect more clearly by bringing together in one diagram the results obtained with the four slot heights for one value of the downstream distance. It may be seen from figure 3.9 that the effectiveness against velocity ratio curve has a maximum at a velocity ratio close to unity for the measurements made with the largest slot height; this maximum diminishes with the slot height and is not present for the measurements obtained with the smallest slot height. It is estimated that the value of the velocity ratio $\bar{U}_C/U_G$ was measured with a precision which, for values of velocity ratio greater than 0.5, was not worse than 3%. The concentration of helium in

any sample was measured to a precision of $\pm 1.5\%$ of corresponding value at the slot.

The foregoing estimates are substantiated by the consistency of the results shown in figures 3.2 to 3.5. The reproducibility of the measurements was verified by making check measurements subsequent to the measurements which led to the results presented in figure 3.2 to 3.5. For these check measurements, the slot height was reset both to 0.25" and to approximately 0.074". Values of the impervious-wall effectiveness were obtained for a number of values of the velocity ratio including (for each of the two heights) one which was nominally unity; the maximum deviations of values shown in figures 3.2 and 3.4 were 4% of unity, i.e. of the effectiveness at the slot exit.

The results described in sub-section 3.3.1.2-b are in good agreement with those of sub-section 3.3.1.2-a and reference 98; this is demonstrated in figure 3.10. Some differences may be seen at the lower values of t/y_C but the maximum deviation of any measured point from a smooth curve is 0.05. The deviation at low values of t/y_C is not surprising since, in this region, the influence of a small variation in the upstream geometry becomes important. The upstream geometry in the experiments of sub-section 3.3.1.2-b was essentially the same as that of sub-section 3.3.1.2-a, except that the slot had been dismantled and reassembled.

The implication of the results of section 3.3.1.2-b are obvious: the larger the value of the lip thickness, the greater the region of separated flow behind the slot lip, the greater the mixing between the free stream and the secondary stream and the lower the impervious-wall effectiveness. The effect of lip thickness is greatest in the region where the velocity ratio is unity.

The substantial agreement between the results of 3.3.1.2-a with constant t and those of 3.3.1.2-b with constant y_C confirms that the variations in effectiveness were caused by the variation in t/y_C .

3.3.2 Influence of upper lip boundary layer thickness

3.3.2.1 Previous work

Two experimental investigations of the present type have previously been carried out: the first by Chin et. al. (16) and the second by Seban (92). These investigations were carried out by measuring the adiabatic-wall effectiveness: in each case the velocity ratio was restricted to values less than unity. Although both investigations indicated that the effect of the thickness of the upper lip boundary layer was small (i.e. a maximum of 8% of unity) the trends observed by these authors were opposite. Chin et. al. observed that the difference between measurements made with thin upper lip boundary layer (equal to eight slot heights) and those made with a thicker one (equal to twenty slot heights) was smaller for low values of $\bar{U}_C/U_G$ and tended to a maximum as this ratio tended to unity. This tendency suggested that the thickness of the upper lip boundary layer might be significant in deciding if the effectiveness against the velocity ratio curve should have a maximum. Seban, on the other hand, observed that the difference was a maximum for low values of the velocity ratio at a distance of 100 slot heights downstream: the difference appeared more constant at a distance of two hundred slot heights. Seban employed upper lip boundary layers with thickness equal to one slot height and equal to ten slot heights.

3.3.2.2 Present contribution

The two slot heights used for the present measurements were 0.074" and 0.25" (nominally). For the case of the smaller one, the upper lip boundary layer thickness were 0.18", 0.36" and 0.75". The 0.18" thick boundary layer was obtained as a result of the design of the slot

and its upstream geometry. The other two boundary layer thicknesses were obtained by extending the upper lip of the slot into the contraction of the wind tunnel. With the 0.25" slot, a boundary layer thickness of 0.91" was obtained in two ways:

1. by extending the lip of the slot into the contraction,
2. by extending the lip of the slot into the contraction and placing a 0.25" diameter rod upstream of the slot exit.

The resulting profiles had the same thickness but different shape and consequently different values of momentum thickness and other integral quantities. Figures 3.11 and 3.12 show the upper lip velocity profiles with a slot height of 0.074" and 0.25" respectively. The free stream velocities for the 0.074" and 0.25" slots were approximately 68 ft/s and 60 ft/s respectively. It is believed that the change in Reynolds number due to a 13% difference in free stream velocity should have very little influence on the present results.

Figure 3.13 shows the measurements of effectiveness obtained with the 0.074" slot for x/y_C of 50, 100 and 200. Similar results for the 0.25" slot are presented in figure 3.14.

3.3.2.3. Discussion

The results presented in figure 3.13, indicate that for 0.074" slot, the effect of the upper lip boundary layer on the impervious-wall effectiveness is of second order for the range of velocities tested. This effect is not greater than 5% of unity in the range of velocity ratios, $0.3 \leq \bar{U}_C/U_G \leq 2.05$. The tendency is for the effectiveness to decrease as the upper lip boundary layer thickness is increased. This is in accord with the general conclusion of Chin et. al.(16) and of Seban (92) for the values of velocity ratio less than unity and extends this result to

higher velocity ratio. The absolute values of the effectiveness agree closely with those of Seban as was to be expected since the 0.063" slot used by Seban clearly resembled the 0.074" slot used here.

In contrast to the results discussed in the previous paragraph, measurements with the 0.25" slot indicate a much larger influence of the upper lip boundary layer thickness on effectiveness. Boundary layer BL 3 and BL 4 (see figure 3.12) have nearly the same thickness (≈ 0.91 ") but they differ in shape. It is interesting to note that for unity velocity ratio, BL 4 profile yields an effectiveness which is around 15% (of unity) lower than the effectiveness given by BL 3. This suggests that the larger defect in the upstream boundary layer causes the faster decrease in the effectiveness.

It is possible to explain qualitatively why the upper lip effect is more important for the larger slot than for the smaller slot. For the smaller slot ($t/y_c \approx 0.41$), the turbulent mixing region caused by the lip acts as a buffer layer which does not allow the momentum defect of the upper lip velocity profile to influence the slot fluid. On the other hand, for the 0.25" slot, t/y_c is only 0.129 and the influence of the upper lip velocity profile enhances the mixing between the two streams and consequently causes a faster decrease in the effectiveness.

From the practical point of view, the maximum effectiveness will be obtained if the upper lip boundary layer is as thin as possible.

3.3.3 Influence of turbulent intensity

3.3.3.1 Previous work

It is likely that the turbulent intensity and length scale of the injected fluid and free stream play an important part in the mixing of the two streams, especially when the velocity ratio is around unity.

Carlson and Tamor (14) have recently suggested that the adiabatic-wall effectiveness is strongly influenced by free turbulence intensity. No previous work on the influence of slot turbulence intensity exists in the literature. There is some evidence (58) to suggest that if the scale of the turbulence is small as compared to the dimensions of the slot, it is likely to be of minor significance.

3.3.3.2 Present contribution

3.3.3.2-a Effect of slot turbulence intensity

The profile of the turbulence intensity at the slot exit was varied either by locating a fine grid upstream of the slot or by injecting air upstream of the slot and normal to the main direction of the flow. The measurements of effectiveness for two different slot turbulence intensity profiles are reported in this section. The slot height used for these measurements was 0.25". One of the two turbulence intensity profiles was obtained from the unaltered configuration of the slot and the second by introducing a 20 mesh 28 SWG copper screen upstream of the slot exit; the unaltered slot configuration yielded the lower values of the turbulence intensity.

Measured values of the impervious-wall effectiveness are shown in figure 3.15 for values of x/Y_C of 50 and 100; the measurements at each velocity ratio were obtained with the two slot turbulence intensity profiles. The two mean velocity and turbulence intensity profiles, measured in the plane of the slot exit, and corresponding to the velocity ratio of 1.33 are shown in figure 3.16; the differences between these profiles are typical of those for other velocity ratios. It may be seen from figure 3.16 that the mean velocity profiles were not identical although the maximum deviation is only 5%. This reflects the difficulty which was experienced in attempting to change the turbulence intensity while maintaining the mean velocity profile the same. Values of minimum turbulent intensities (nearly at the centre of the slot) are quoted in figure 3.15.

Although four different slot turbulence intensity profiles were employed at each velocity ratio, with only small variation in the mean velocity profile, the resulting values of effectiveness did not deviate significantly from the curves shown in figure 3.15. Only when the mean velocity profile was noticeably asymmetric did the effectiveness for a given velocity ratio and downstream distance change; the asymmetric profile in question indicated a larger boundary layer thickness on the base plate and resulted in a reduction in effectiveness. This asymmetric profile occurred in cases when the turbulent intensity was increased by normal injection upstream in the slot. In these cases, it is difficult to say whether the reduction in effectiveness was caused by asymmetric velocity profile or by non-tangential injection. Due to this uncertainty the measurements in which turbulent intensity was enhanced by normal injection have been omitted from figure 3.15.

3.3.3.2-b Effect of free stream turbulence

The natural free stream turbulence level ($\sqrt{u^2/U_G}$) in the test section was approximately 0.005. The turbulence intensity of the free stream was artificially increased by introducing turbulence promoting grids at $x = -7$ "; these were located between the contraction section and the test section. Two grids were constructed using 1/4" and 1/8" round steel rods and having mesh size (centre to centre distance between the rods) of 1" x 1" and 3/4" x 3/4" respectively. These grids are referred to as grid No. 2 and grid No. 3 in the text. Grid No. 2 and 3 produced free-stream turbulence levels of 0.07 and 0.035 respectively at the slot exit. It was noticed that the mean velocity profile of the upper lip at $x = 0$ was affected due to the presence of the grids. In order to make sure that the grids did not disturb the two-dimensionality of the flow, mean velocity profiles and skin friction measurements were carried out on either side of

the centre line at an x-value of $-0.25''$. It was observed that the skin-friction (measured by a Preston tube) was uniform to within $\pm 1.7\%$, in the z-direction whereas a maximum deviation of $\pm 1.5\%$ were found in the mean velocity profile over the central $7''$ of the tunnel.

It was found necessary to adjust the throttle on the main flow in order to keep the same free stream velocity when different grids were introduced. For the tests described in this sub-section, the free stream velocity was 60 ft/s and a slot height of $0.25''$ together with a lip thickness of $1/32''$ was employed.

Experimental results for the effectiveness using three different free stream turbulence intensities are presented in figure 3.17 for x/y_c of 25, 50 and 100. The corresponding upper lip velocity profiles are shown in the same figure. The trends of the effectiveness curves for x/y_c more than 100 are similar to those presented in figure 3.17.

3.3.3.3 Discussion

From the experimental results presented in figure 3.15, it is obvious that within the range of slot turbulence intensities investigated, the effectiveness is not significantly influenced by the intensity profile. The range of the slot turbulence intensities is small but, for developed flows, it is as high as can be expected in practice. For the free-stream, both the intensity and scale are much lower than found in combustion chambers. However, for blade cooling applications, the present range of free-stream turbulence intensities and length scales are appropriate.

In the experiments, described in section 3.3.3.2-b, where higher free-stream turbulence levels were obtained by introducing

turbulence promoting grids upstream of the slot, it was observed that in addition to the increased level of the turbulence, a thicker upper lip boundary layer was also obtained. From the measurements of section 3.3.2, it is obvious that for the 0.25" slot, a large upper lip boundary layer causes a faster drop in the effectiveness. These results are now used to eliminate the effect of changes in upper lip boundary layer thickness in order to determine only the influence of freestream turbulence level.

Figure 3.18 shows the effectiveness plotted against the displacement thickness of upper lip boundary layer for values of $\bar{U}_C/U_G$ of 0.6, 1.0 and 2.0 and for values of x/y_C of 50 and 100. Results from section 3.3.2 and 3.3.3.2-b are included in figure 3.18. In section 3.3.2, the upper lip boundary layer thickness is the parameter of interest, whereas in section 3.3.3.2-b a combined influence of upper lip boundary layer thickness and free stream turbulence level on effectiveness has been investigated. If the principle of superposition is used, it can be inferred from figure 3.18 that free stream turbulence has no significant influence on effectiveness. It is possible that the large influence of the free stream turbulence level noted by Carlson and Talmar (14) is due to the changes in the upper lip boundary layer thickness produced by turbulence promoting grids but, since $\sqrt{u^2}$ was not measured and the tunnel aspect ratio was small, their results are in considerable doubt.

3.3.4 Influence of other parameters

With the present set-up, it was not possible to examine the effect of slot Reynolds number and slot velocity profiles on impervious wall effectiveness over a wide range. However, these parameters were examined theoretically and the results are discussed in chapter 5. The effect of slot Reynolds number has been reported by Burns and Stollery (12) for the case of Arcton injection. They concluded that for large x/y_C , the slot Reynolds number influence is in agreement with prediction

of the boundary layer model i.e. effectiveness is proportional to $R_{ec}^{0.2}$ but for small x/y_C , R_{ec} has very little influence on effectiveness. Similar trends have been observed by Pai and Whitelaw (69) in absence of density gradients.

3.4 General discussion

In section 3.3, the influence of various slot geometry parameters were examined and discussed separately in different sub-sections. The purpose of this section is to discuss the inter-relationship between the different slot geometry parameters.

Of the slot geometry parameters investigated, the ratio of lip thickness to the slot height was found to be most important.

When studying the influences of a particular slot geometry parameter, only that parameter should be varied while keeping all other parameters constant. However, this is not always practical. For example, when the slot height is changed, the velocity profile of the slot does not remain the same. For this reason, it is probable that the results for the effect of t/y_C , to some extent, are influenced by changes in slot velocity profile. Substantial agreement between the results of section 3.3.1.2-a, 3.3.1.2-b and reference 98 together with the theoretical calculations of chapter 5 suggest, however, that the effect of slot velocity profile is of minor importance.

In section 3.3.2.2, an inter-action between the influence of upper lip boundary layer thickness and the slot geometry parameter t/y_C has been observed. For the 0.074" slot height ($t/y_C \approx 0.41$), the upper lip boundary layer is of minor importance, whereas for the 0.25" slot ($t/y_C \approx 0.129$) a considerable influence of the upper lip boundary layer was noted. From the above results, it may be expected that with a 0.25" slot and a lip of 0.125" or more (so that $t/y_C > 0.41$), the upper lip boundary

layer should not significantly influence the effectiveness.

The present results for the influence of free stream and slot turbulence level should be considered in conjunction with the existing slot conditions such as the slot velocity profile, shape and thickness of lip, etc. If an almost square velocity profile existed in the slot and the free stream together with a knife edged lip, the turbulent intensities would have significantly greater influence on the effectiveness than was observed in the present experiments. In the present investigation the boundary layer on the top and bottom surfaces of the slot filled approximately 85% of the 0.25" slot and it is likely that downstream of the slot, mixing is mainly caused by the mean velocity gradients and the turbulent intensities produced by the velocity gradients. Probably, for this reason, the free stream and slot turbulent intensities were found to have no important influence on the effectiveness.

The experimental results presented in this chapter show that the slot geometry parameters such as slot height, lip thickness and upper lip boundary layer thickness have important influence on the effectiveness and can account for discrepancies which exist between the effectiveness measurements of various authors and between their resulting correlations. Certainly, it is clear that the much used correlations of the form:

$$\eta \propto \left(\frac{U_G x}{\bar{U}_C y_C} \right)^a \left(\frac{\rho_C y_C U_G}{\mu_C} \right)^b \quad 3.4-1$$

are unsuitable for accurate prediction of effectiveness over a wide range of $\bar{U}_C/U_G$; they must include, at least, t/y_C and δ^*/y_C .

3.5 Conclusions

1. The effect of decreasing the slot height, while retaining the same lip thickness and ratio of slot to free stream velocity, is to decrease the effectiveness, provided the velocity ratio is close to unity. On the other hand, the effect of increasing the lip thickness for a given value of slot height is to decrease the effectiveness for all velocity ratios. Some differences in effectiveness value at low values of t/y_C , obtained with t constant and with y_C constant, are probably due to the small variations in upstream geometry which occurred in the two sets of measurements.
2. For a prescribed mass flow rate through the slot, the maximum slot height which is compatible with a value of velocity ratio close to unity will generally result in the optimum effectiveness.
3. Increasing the slot-turbulence intensity from 3% to 9% at a velocity ratio of 1.33 does not significantly affect the downstream effectiveness.
4. Within the range of present investigations (0.5% to 7.0%), the free stream turbulence level has no significant influence on effectiveness.
5. For a slot of height 0.074" and t/y_C of 0.43, the effect of the upper lip boundary layer thickness on the impervious-wall effectiveness is not greater than 5% of unity. A much larger influence of upper lip boundary layer exists for the 0.25" slot and t/y_C of 0.126.

CHAPTER 4: EXPERIMENTAL INVESTIGATION OF TURBULENT PROPERTIES OF WALL-JETS AND WALL-WAKE FLOWS

4.1 Introduction

In gas turbine combustion chambers, there are rows of cooling slots separated by distances of the order of fifty slot heights. The designer of the combustion chamber is, therefore, interested in this upstream region ($0 \leq x/y_c \leq 50$) of the wall-jet. This is also the region of the flow which is most strongly influenced by slot geometry effects. The present experimental and theoretical investigation concentrates on the upstream region of the wall jet because of its practical importance. However, there are other practical situations where the farther downstream (up to around x/y_c of 200) flow properties of the wall-jets are of interest. In view of this requirement, it was considered useful to include the downstream region in the experimental investigation.

The experiments described in this chapter have been carried out to provide test data for comparison with prediction and also to furnish the information regarding various forms of viscosity laws which could be employed in the prediction procedure. For this reason, profiles of static pressure, total pressure, the three components of the turbulent velocity and turbulent shear stress were measured. These measurements permit the direct testing of various physical hypothesis; that is, the measurements permit the direct evaluation of Prandtl's mixing length and the Prandtl-Kolmogorov length scale as a function of $\bar{U}_c/U_G$, x/y_c and of the ratio t/y_c . In addition, the detailed measurements of mean-velocity and fluctuating-velocity profiles together with shear stress profiles permit the indirect testing of physical hypothesis. That is, using a particular hypothesis, the prediction procedure may be used to predict

profiles of U , $\overline{q^2}$ and $\overline{uv}$ and these may be compared with the present results.

Some of the results for the influence of slot geometry parameters on the adiabatic-wall effectiveness, presented in chapter 3, are more adequately explained and further confirmed by the detailed measurements of the present chapter. For example, in chapter 3, it was reported that the effect of increasing the lip thickness is to cause a faster drop in effectiveness: this trend is further substantiated by the measurements of skin friction and decay of velocity maxima, presented in this chapter.

4.2 Aims of chapter 4

In the light of the preceding paragraphs, the aims of the present chapter may be stated as follows:

1. to present measured values of mean-velocity, shear stress and turbulent kinetic energy profiles for different two-dimensional slot geometries and initial conditions,
2. to present distributions of mixing length, ℓ_1 , and Prandtl-Kolmogorov length scale, ℓ_2 , deduced from the experimental data.

4.3 Parameters investigated and their range

Mean and turbulent properties of the flow were investigated for a nominal slot height, 0.25" and two lip thicknesses, 1/32" and 9/32", giving t/y_c ratios of 0.126 and 1.14. The free stream velocity was nominally 63 ft/s. Experiments were conducted for four blowing ~~rates~~ ^{ratios} 0.75, 0.91, 1.33 and 2.3 for each of the lip thicknesses. In the case of the thin lip, various profiles were measured at six non-dimensional

distances (x/y_C) from the slot exit viz. 0, 10, 20, 50, 80 and 150 whereas for the thick lip case, measurements were carried out at x/y_C of 0, 4, 10, 20, 50 and 150. For t of $1/32$ " some additional measurements of mean flow and correlations were carried out at $\bar{U}_C/U_G$ of 2.74.

Since these measurements have resulted in a considerable quantity of data, it is not possible to present all the data here. The data have been selected to present a comprehensive picture of the flow situation. All the data is available in tabulated form from the author.

4.3.1 Apparatus

The experiments were carried out in the wind tunnel described in chapter 3. Preliminary tests showed that the temperature of the air leaving the injection slot was higher by approximately 4°C than that of the main stream and, since the response of a hot wire is temperature sensitive, a small automobile radiator was used to cool the secondary air. The flow of the water to the radiator was controlled and this control ensured that the temperature profile in the plane of the slot exit was uniform to within $\pm 0.5^\circ\text{C}$. Measurements of u - spectra were obtained with the aid of a Bruel and Kjar frequency analyser (type 2105). The band-width of this analyser varies with the frequency and is a fixed percentage of an octave. In all cases, a Wayne-Kerr analyser (type A321) was used to check the spectrum measurements for frequencies above 150Hz: this instrument had a constant band width of 10 Hz. The result of the two analysers were in close agreement.

4.3.2 Experiments

4.3.2.1 Mean properties of the flow

In this section, measurements of the static pressure, mean velocity and wall shear stress are presented and discussed: representative integral properties are also presented.

Static pressure distribution

It is convenient to examine first the longitudinal static pressure distribution in the working section. Figure 4.1 shows the static pressure distribution on the base plate of the working section for two values of $\bar{U}_C/U_G$ and two values of lip thicknesses: the free stream static pressure distributions are shown on the same figure. It may be seen that the free stream static pressure and the base plate static pressure are not of the same magnitude. The difference has a maximum close to the slot exit and can still be observed 100 slot heights downstream: the static pressure distribution normal to the base plate and implied by figure 4.1 has been confirmed using a static pressure probe and is presented in detail in reference 45. These measurements include static pressure profiles in y-direction at various x-locations for slot heights of 0.074" and 0.25", and for a wider range of velocity ratio. The rise in the static pressure, shown in figure 4.1, is significantly greater for larger values of t/y_C and is more prominent immediately downstream of the slot lip.

These results show that the boundary layer assumption of zero normal pressure gradient is invalid for values of x/y_C less than around 100 although little error is likely to be incurred by invoking the assumption for x/y_C greater than 20. In the region upstream of the 20 slot heights, however, the normal pressure gradient implies that the flow must be described by the Navier-Stokes equations without simplifying boundary layer assumptions. As we shall see in the coming sections, for large values of t/y_C this requirement is even more necessary due to the enlarged region of separated flow immediately downstream of the slot lip.

Mean velocity profiles

Figure 4.2 presents typical mean velocity profiles at three values of x/y_C for the velocity ratios 0.75 and 2.3 and for both values

of lip thicknesses. For $x/y_C > 10$, the values of mean velocity were calculated from the measurements of the total pressure and from the mean of the wall and free stream static pressure appropriate to that x-station. The maximum error in velocity incurred by using this static pressure was 1%. In the vicinity of the slot and in the case of the thick lip, hot wire velocity measurements were considered more reliable as they do not depend upon the static pressure variation.

The shape of the mean velocity profile in the plane of the slot exit is of special interest since it represents one boundary condition for any prediction procedure designed to predict the downstream flow. The initial profiles shown in figure 4.2 indicate that the secondary flow profiles are not flat. It can be seen that the four profiles at x/y_C of 10 show no velocity maxima for the lowest velocity ratio and the thick lip although a velocity maximum does occur for the lower velocity ratio and the thin lip: for the highest velocity ratio the effect of the thick lip is to cause the velocity minimum to be located at a larger value of y/y_G . These effects are consistent with the greater mixing downstream of the thick lip.

Figure 4.3, which shows the decay of the velocity maximum for $\bar{U}_C/U_G$ of 2.3, indicates that the thick lip results in a more rapid initial decay of the velocity maximum than the thin lip. In other words, the larger region of separated flow and the larger momentum defect behind the slot lip cause a substantial increase in mixing rate. It is clear, however, that at x/y_C of 150 the velocity profiles retain a maximum for both values of slot lip thickness.

The profiles of mean velocity were plotted on the coordinates recommended by Clauser (17) and revealed that, for the values of x/y_C greater than approximately 20, a logarithmic region existed for all ratios

and both lip thicknesses. The logarithmic region proved to be well represented by the law of the wall with Patel's (77) constants.

Figure 4.3 shows the integral quantities H_{12} and R_2 obtained from the mean velocity profiles, for two values of velocity ratios. The influence of the slot lip is again obvious, for instance, the values of momentum thickness Reynolds number are significantly larger for the thicker lip cases. The shape factor exhibits a singularity as the momentum thickness passes through zero: i.e. as the mean velocity profile loses its maxima. This property of wall jets is a source of difficulty in prediction procedures based on integral equations. It may also be observed from the figure that, for lower velocity ratios, the shape factor tends towards a value of approximately 1.35, i.e. the usual value for a flat plate boundary layer.

Wall-shear stress

The measurements of the wall shear stress are shown on figure 4.4 for thick and thin lip and two values of velocity ratios. These have been obtained from a Preston tube (dia. 0.049") with Head and Rechenberg's calibration (41). It has previously been shown in reference 70 that, in wall jet and wall wake flows, values of shear stress obtained from Preston tubes agree well with those from calibrated Stanton tubes. However, in order to provide a direct check on the present measurements, the razor blade technique described in reference 70 was applied in the case of the lowest and highest velocity ratios. The razor blade values agreed with the Preston tube values to within 3%. The substantial decrease in the value of $C_F/2$ for $\bar{U}_C/U_G$ of 0.75 and t/y_C of 1.14 is consistent with the suggestion that for low values of $\bar{U}_C/U_G$ and higher values of t/y_C , a separated region may form on the wall, close to the slot.

Figure 4.5 shows a comparison of the present skin friction measurements with the results of Myers et.al.(61), Patel (76), Guitton (33), Parthasarathy (72) and Tailland and Mathieu(106). The results are plotted in terms of $\tau_w / \frac{1}{2} \rho U_{\max}^2$ versus $(U_{\max} y_1) / \nu$. These are the same non dimensional parameters used by Sigalla (95) who proposed a Blasius type of skin friction law for wall jet flows. Only the results of $\bar{U}_C / U_G$ of 2.3 are included here. The results are shown on two graphs - one for $\frac{x}{y_C} \leq 10$ and the other for $\frac{x}{y_C} > 10$. It may be noted that for $\frac{x}{y_C} > 10$, the present measurements lie in between the results of Parthasarathy (72) and Tailland and Mathieu(106). On the other hand, very near to the slot ($x/y_C < 10$), there is a tendency for the present measurements to lie closer to the curve representing the results of Tailland and Mathieu(106). The results of Myers et. al. appear to be slightly on the high side. It is encouraging to note that there is very good agreement with the hot film and Preston tube results of Guitton (33).

Momentum balance

The measurements recorded in the previous three sections, together with the integral momentum equation, provide a useful check on the two dimensionality of the flow. The integral momentum equation may be written in the form:

$$\frac{dR_2}{dR_x} + R_2 (1 + H_{12}) \frac{d \ln U_G}{dR_x} = C_F / 2 \quad 4.3-1$$

With known experimental values of H_{12} , U_G and $C_f/2$, it is possible to integrate the above equation and compare the calculated values of R_2 with experimentally measured R_2 . The results of this procedure indicated a small non two-dimensionality of the flow. The maximum discrepancy proved to be 16% of R_2 at x/y_C of 150. This discrepancy occurred for $\bar{U}_C/U_G$ of 0.91 and t/y_C of 0.126: for the other blowing rates the discrepancy was smaller and for the smaller values of x/y_C was much smaller.

The alternative procedure of using equation 4.3-1 to evaluate $C_f/2$ proved to be very unreliable due to the strong dependance of the result on the procedure used to fit R_2 , H_{12} and U_G as functions of R_x .

Measurements of $C_f/2$, U/U_G , $\sqrt{\frac{u^2}{U_G^2}}$ and $\overline{uv}/U_G^2$ at $\frac{x}{y_C}$ of 150 and

cross stream stations 3 inches on either side of the centre line revealed maximum discrepancies of 0.0001, 0.06, 0.007 and 0.0002 respectively.

These tests were carried out for a velocity ratio of 0.91 and t/y_C of 1.14; similar tests for other velocity ratios revealed smaller discrepancies and, for value of x/y_C of 50 and less, much smaller discrepancies.

4.3.2.2 Turbulent properties of the flow

The distribution of the fluctuating components of velocity are shown in figures 4.6-a and 4.6-b for values of the velocity ratio of 0.75 and 2.3 and t/y_C of 1.14 and 0.126. The turbulent intensities at x/y_C of 4 for t/y_C of 1.14 are shown for comparative purposes and, in view of the large magnitude of these intensities, must be regarded with

caution. Indeed all values of turbulence intensity in excess of approximately 20% (normalised with local mean velocity) should be regarded as qualitative.

Figure 4.6 (a, b) shows that the turbulence intensities are considerably greater at low values of x/y_C than at high values of x/y_C . It can also be seen that the thick lip causes the three fluctuating velocity components to be of similar order of magnitude for low values of x/y_C ; with increasing x/y_C this tendency towards isotropy disappears. At a downstream distance of 150 slot heights, the distribution resemble those measured by Klebanoff(55) in a zero pressure gradient turbulent-boundary layer. There are, however, significant differences, particularly in the case of the jet like flows where the distribution of turbulent kinetic energy in the inner region, approach the developed condition from smaller values of the turbulence intensity, and also in the relative magnitudes of $\sqrt{\overline{u^2}}$, $\sqrt{\overline{v^2}}$ and $\sqrt{\overline{w^2}}$. This trend is observed for both the lip thicknesses investigated. The results of figure 4.6a and figure 4.6b indicate that, far downstream, the distributions of the fluctuating components of velocity are less dependant on the lip thickness than on the velocity ratio.

Distributions of turbulent-kinetic energy are shown on figures 4.7a and 4.7b for four velocity ratios and for values of x/y_C of 10 and 150; figure 4.7a shows the measurements corresponding to t/y_C of 1.14 and figure 4.7b to t/y_C of 0.126. Again, it is clear that, at x/y_C of 10, the thicker lip results in higher values of turbulent kinetic energy and for x/y_C of 150, the magnitudes are less affected by the lip thickness than by the velocity ratio. However, it can be observed that for $\overline{U}_C/U_G$ of 2.3, the thicker lip causes the greater kinetic energy at x/y_C of 10 but the smaller kinetic energy at x/y_C of 150.

Turbulent shear stress

The turbulent shear stress distributions were obtained by direct measurements of $\overline{uv}$ for each of the four values of velocity ratio; typical distributions for the thick and thin lip cases are shown on figures 4.8a and 4.8b respectively. The extrapolation of these curves to the wall yielded values of wall shear stress which are in good agreement with those presented in figure 4.4. It is clear that, at x/y_C of 10 all but one shear stress distribution passes through zero; in each case this occurred closer to the wall than the point of zero mean velocity gradient. At x/y_C of 150, points of zero shear stress are retained for $\overline{U}_C/U_G$ of 2.3 and both lip thicknesses and for $\overline{U}_C/U_G$ of 1.33 and t/y_C of 0.126. The remaining combinations of velocity ratio and lip thickness result in turbulent shear stress profiles at x/y_C of 150 which through of similar magnitude revealed significant differences from the developed, zero pressure gradient, turbulent boundary layer distribution of Klebanoff.

The measured values of shear stress and mean velocity gradient, permit the evaluation of the production of turbulent kinetic energy $(\rho \overline{uv} \frac{\partial U}{\partial y})$ and of the shear work integral $\int_0^{y_G} \tau \frac{dU}{dy} dy$; typical values are shown on figures 4.9a and 4.9b for t/y_C of 1.14 and 0.126 respectively. The figures indicate the wide variation in shear work which may be found in turbulent wall flows. The distribution of the production of turbulent kinetic energy will be discussed in the section titled 'energy balance'.

Position of zero shear stress and velocity maximum

It has been experimentally observed in plane channels with walls of different roughness, curved ducts and wall-jets with zero free stream that the position of zero shear stress does not coincide with the

velocity maxima. This results in a region where the conventional definition of effective viscosity breaks down. Also this is the cause of negative production term in turbulent kinetic energy balance (as we shall see later in this chapter). Generally the distance between the location of zero shear stress and maximum velocity is only a small fraction of the total boundary layer thickness and hence no serious difficulty has been encountered in prediction of velocity profiles (68). On the other hand, this region acts as an artificial barrier to the fluxes of heat and mass and causes unsatisfactory prediction of temperature and concentration profiles. This difficulty has been overcome in reference (68) by artificially bridging the viscosity profiles. Another possibility suggested by Launder (57) is to include second derivative of velocity in the definition of effective viscosity.

Figure 4.10 shows the location of zero shear stress, velocity maxima and minimum turbulent kinetic energy for $\bar{U}_C/U_G = 2.3$ and both lip thicknesses. It is obvious from the diagram that the location of zero shear stress is always nearer to the wall than the location of maximum velocity. The position of minimum turbulent kinetic energy is closer to the point of zero shear stress for the thin lip case whereas for the thicker lip, it is closer to the position of maximum velocity. It is likely that the increased turbulence production, caused by the thicker lip, may play an important part in shifting the position of minimum in turbulent kinetic energy profile.

The position of the maximum velocity has been found by noting the maximum in pitot tube reading and its determination is subject to an error of approximately $\pm .005$ " in those cases where the velocity maxima is not very sharp.

Turbulent length scale

It has been shown in the literature (52, 58, 111) that the scale and intensity of the turbulence may affect the mean properties of the flow. However, Lavender and Pei (58) have shown that although the free stream turbulence intensity influences the heat transfer from a sphere, the scale of turbulence, in their experiments, was of minor significance. In the present experiments, the length scale measurements were carried out in order to investigate the size of eddies and any probable influence of their size on other mean properties.

Values of the correlation coefficient $R_{(0, r, 0)}$, were obtained at only two values of velocity ratio, 0.75 and 2.74; typical results are shown in figure 4.11. The correlation coefficient shown in figure 4.11 is defined as:

$$R_{(0, r, 0)} = \frac{\overline{u_1 u_2}}{\overline{u_1^2}} \quad 4.3-2$$

and was obtained using two single wire probes, one of which was maintained in a fixed position while the second was traversed in the y-direction and in the same x-plane. In above equation, the subscript 1 refers to the fixed probe and subscript 2 to the moving probe. The mean turbulent length scale, defined by:

$$L_y = \int_0^\infty R_{(0, r, 0)} dy \quad 4.3-3$$

has been evaluated and presented in figure 4.12. With the exception of the reference measurements in the plane of the slot exit the results have been plotted on both dimensional and non-dimensional coordinates.

Figure 4.12 shows that, in the plane of the slot exit, the scale of the turbulence in the free stream is greater than that in the slot. However, the scale decays with the distance from the slot and, for the wake like case, has attained a constant value of approximately $0.15 y_G$ at 80 slot heights; this is approximately the same value which Grant (32) measured in a zero pressure gradient boundary layer. The jet like flow, on the other hand, has not attained the characteristic length scale of the zero pressure gradient boundary layer within 80 slot heights.

Tailand, Sunyach and Mathieu (107) measured the correlation coefficient in a jet with zero free stream velocity. In this case the coefficient was normalised with $\sqrt{u_1^2} \sqrt{u_2^2}$ rather than $\overline{u_1^2}$ and this resulted in negative values of the correlation coefficient in the outer region of the flow. Tailand et. al. (107), in evaluating the length scale, integrated only between the zero limit and $y_{1/2}$. In spite of this difference, their results, when plotted non-dimensionally against $y_{1/2}$, are in close agreement with the present measurements: the range of the measurements is, however, not large ($0 < y/y_{1/2} < 1$). When the present measurements were used to evaluate the correlation coefficient normalised with $\sqrt{u_1^2} \sqrt{u_2^2}$, it was found that the resulting length scale was negative for small values of y and the velocity ratio of 2.74.

u - energy spectra

The present measurements of energy spectra were carried out mainly to determine whether or not the finite thickness of the slot lip gave rise to flow periodicity and, if so, to determine the extent to which this periodicity prevailed with distance from the slot. As will be seen, however, the measurement of energy spectra has additional, important benefits. It permits the dissipation of turbulent energy to be estimated, provides a check on the validity of the $-5/3$ law for wall-jet and wall-

wake flows and may be transformed to yield space-correlation coefficients.

Vortex shedding behind a rectangular cross-sectioned, bluff body has previously been examined by Bearman (3) who also determined the influence of various 'splitter plates' on the amplitude and frequency of the shedding. Bearman showed that, with no splitter plate, the vortices were shed at a frequency corresponding to a Strouhal number of 0.23 and that the Strouhal number increased with length of splitter plate until the ratio of splitter-plate-length to the half-height of the trailing edge reached 1.5: at this value the Strouhal number decreased and at a ratio value of 2.5 no vortex shedding could be detected. These results indicate the influence of a nearby wall on the periodicity of the flow.

In the present case, measurements of spectra were obtained for each of the four velocity ratios at values of x/y_c of 1 and 150 and for t/y_c of 1.14. Since shedding is most likely to take place when the velocity ratio is close to unity, the measurements with a velocity ratio of 0.91 will be discussed first. Figure 4.13a shows u-spectra for five y - values. The lines shown on this figure are smooth lines drawn through the experimental points. The figure shows that maximum-energy-peaks exist for y -values of 0.246 in. and 0.526 in., i.e. approximately the same y -values as the lower and upper corners of the slot lip. Away from these y -values, the energy peaks diminish. The peaks exist at a frequency of approximately 600 Hz compared with a shedding frequency, for a blunt body of this shape, of approximately 100 Hz. It is important to note that the energy contained within the peaks represents, at most, 25% of the energy bounded by a smooth curve excluding the peaks. Thus, although a significant frequency can be observed, it is likely to be of little significance to prediction procedures. It must be emphasised that particular care was exercised in the determination of the energy maxima

and it is highly improbable that higher energy levels have been missed.

Recent measurements of Peterka and Richardson (78) behind a cylinder have confirmed the findings of Bloor (7) that the separated shear layers have transition wave frequencies which can (depending on the Reynolds number) be 4 to 20 times larger than the characteristic shedding frequency. According to the work of Bloor (7) and Gerrard (28), these frequencies are characteristics of the instabilities of the shear layer and probably can also be identified with Tollmien-Schlichting waves observed by Sato and Kuriki (90). Bloor (7) has shown, from dimensional arguments, that the transition wave frequencies observed in the flow around a cylinder should be proportional to $U_\infty^{3/2}$ and consequently the ratio of the transition wave frequency to the Strouhal frequency should be proportional to Reynolds number (based upon U_∞ and the diameter of cylinder) raised to the power 1/2. Reasonably good agreement has been observed between the results of Bloor (7) and Peterka and Richardson (78) in this respect; this is shown in figure 4.14-b.

An attempt has been made to correlate the preferred frequencies observed in present experiments with the transition wave frequencies noted by Bloor (7) and Gerrard (28). For the present experiments, the reference velocity for the calculation of Reynolds number has been taken as an average of slot and free stream velocity and the characteristic length has been taken as the slot lip thickness. Figure 4.14a confirms the $u^{3/2}$ dependance of the preferred frequencies (f_t).

Figure 4.14b reveals that the ratio of f_t/f_s lies slightly below the Bloor's correlation. It is likely that the definition of Reynolds number is responsible for this discrepancy. However, it is interesting to note that there is general agreement in the order of magnitude of these frequencies. It has also been noted in the present study that the preferred frequency (f_t) peaks are not very sharp and have slight band spread. This behaviour is to be

expected as the transition layer is not excited with oscillations at a fixed frequency to which an instability frequency may lock. This is in agreement with findings of Peterka and Richardson (78).

Measurements of spectra, corresponding to x/y_C of 150 and the same flow conditions as figure 4.13a are shown on figure 4.13b. It can readily be seen that no peaks exist and that the shape of the spectra closely resemble that of a zero-pressure-gradient, turbulent boundary-layer. It may be seen that, close to the wall, more energy is contained at the higher frequencies; as the y -value increases, the energy content of the higher frequencies is decreased. This result is in accord with the findings of Klebanoff for the developed, zero-pressure-gradient, turbulent boundary-layer. The measurements also verify that, away from the wall, the $-5/3$ law is valid over a greater region of the spectra than close to the wall. At higher frequencies there is a tendency for the curves to have a -7 slope. However although eddies exist at frequencies up to and above 14 kHz, finite wire dimensions and inadequate compensation limit the accuracy of hot-wire measurements above 14 kHz.

The u -spectra obtained in jet- and wake-like flows are shown in figures 4.15a and 4.15b respectively. Far downstream, the spectra for the two situations are similar but at x/y_C of 1 the wake-like flow reveals the more pronounced energy-peaks. They are, however, still small when compared with the peaks encountered downstream of a blunt body and in the absence of a wall.

As mentioned earlier, the turbulence scale may be estimated from the spatial correlation functions and these may be estimated from the u -spectra measurement. The Fourier transform of the u -spectrum is

$$R_{(r,0,0)} = \int_0^{\infty} \frac{E_1(k_1)}{2} \cos(k_1 x) dk_1 \quad 4.3-4$$

u

and figure 4.16 shows values of $R_{(r,o,o)}$ obtained from this equation and u-spectra measurements at x/y_C of 150 and $\bar{U}_C/U_C$ of 2.3. The figure clearly shows the increase in scale with distance from the wall. No absolute value can, however, be attributed to the magnitude of the scale at any y -location: the lack of $E_1(k_1)$ measurements at low values of k_1 prevent this. If local isotropy is assumed, $R_{(o,r,o)}$ and $R_{(o,o,r)}$ may be obtained from the values of $R_{(r,o,o)}$.

Turbulent-energy balance

The equation of turbulent-kinetic energy appropriate to the upstream region of a wall-jet or wall-wake flow has been presented in chapter 2 (equation 2.4-9). The terms in square brackets are generally referred to, respectively, as advection, production and diffusion: ϵ is the dissipation. Only the advection and production terms of equation 2.4-9 can be completely determined from hot-wire measurements. The advection term, for velocity ratios of 0.75 and 2.3 and for x/y_C of 10, is shown on figure 4.17. To obtain the advection term, $\overline{\frac{\partial q^2}{\partial x}}$ and $\overline{\frac{\partial q^2}{\partial y}}$ were determined by fitting a parabola through three measured turbulent-kinetic-energy values and evaluating the gradient at the central point: values of V were obtained from the U -profile and the continuity equation. The distributions shown on figure 4.17 were obtained with the aid of measured values of $\overline{q^2}$ at x/y_C of 4, 10 and 20. The first component of the turbulent production term was presented on figure 4.9; the second component has been neglected since the measurements indicate that, for values of x/y_C greater than and equal to 10, it is never greater than 5% of the first component; and the third component has been neglected since it is only

greater than 5% of the first component in regions where the first term itself is very small. The inclusion of the second and third components would in no way alter the implications of figure 4.9, for example, the region of negative production would remain.

The experimental determination of the diffusion term is difficult and has not been attempted here. In addition, the experimental determination of the nine terms of dissipation has not been attempted. Consequently, dissipation has been evaluated with the aid of the assumption of local isotropy and the diffusion calculated as the outstanding term in equation 2.4-9. Since the integral of diffusion across the layer should equal zero, the assumption used in the evaluation of dissipation is subject to an indirect check.

The local-isotropy assumption implies that:

$$\overline{\left(\frac{\partial u}{\partial x}\right)^2} = \frac{1}{2} \overline{\left(\frac{\partial v}{\partial x}\right)^2} = \frac{1}{2} \overline{\left(\frac{\partial w}{\partial x}\right)^2} \quad 4.3-5$$

and leads to the expression

$$\epsilon = 15 \nu \overline{\left(\frac{\partial u}{\partial x}\right)^2} \quad 4.3-6$$

and it is further assumed that

$$\overline{\left(\frac{\partial u}{\partial x}\right)^2} = \frac{1}{U^2} \overline{\left(\frac{\partial u}{\partial t}\right)^2} \quad 4.3-7$$

then

$$\epsilon = 15 \frac{\nu}{U^2} \overline{\left(\frac{\partial u}{\partial t}\right)^2} \quad 4.3-8$$

and this value of ϵ can be obtained from measurements. Klebanoff examined

the local-isotropy assumption and, following his arguments, in the vicinity of the wall the resulting value of ϵ may be low by a factor of 4: the discrepancy should diminish with distance from the wall.

The energy balances for velocity ratios of 0.75 and 2.3, t/y_c of 1.14 and x/y_c of 10 are shown on figure 4.17*. For the lower velocity ratio, the results are reassuring. The production is high near the wall and in the wake of the slot lip: in between, and in the outer region, the production is low. As suggested by the shear-stress measurements, the production is always positive. The advection is negative over most of the layer and, in the wake of the lip, is of the same order of magnitude as the production: in the outer region it is, however, positive. The dissipation does not show a minimum corresponding to the production term. The ratio of the integral of production to that of diffusion was - 1.3; that is, the integral of

*The four terms shown on this figure are as follows:

$$\text{Advection} = \left(\frac{U}{2} \frac{\partial}{\partial x} \overline{q^2} + \frac{V}{2} \frac{\partial}{\partial y} \overline{q^2} \right) \frac{y_G}{U_G^3}$$

$$\text{Diffusion} = \left[\frac{\partial}{\partial y} v \left(\frac{P}{\rho} + \frac{\overline{q^2}}{2} \right) \right] \frac{y_G}{U_G^3}$$

$$\text{Dissipation} = \epsilon \frac{y_G}{U_G^3}$$

$$\text{Production} = \left(\overline{uv} \frac{\partial U}{\partial y} \right) \frac{y_G}{U_G^3}$$

diffusion is not zero.

Townsend (112) and Klebanoff (55) obtained corresponding values of - 2.2 and - 2.8 for wake and flat-plate-boundary-layer flows respectively. For the higher blowing rate, however, the results are less reassuring. The production, as expected from the shear stress measurements, is very large in the wake of the lip and a small region of negative production can be observed. Advection and dissipation are relatively small, although of the same order of magnitude as for the lower velocity case. Thus the production is virtually balanced by diffusion. In this case, the ratio of the integral of production to that of diffusion was found to be - 4.0.

In order to provide a check of this unusual result, the dissipation for the higher velocity ratio was also determined from the measured u-spectra. Again, local isotropy is assumed and, on this assumption the dissipation may be written in the form:

$$\epsilon = 15 \nu \int_0^{\infty} k_1^2 E_1(k_1) dk_1 \quad 4.3-9$$

Using measured u-spectra at y-values of 0.078, 0.153, 0.553 and 0.953 inch, the resulting values of dissipation agreed with those previously obtained to within 0.0005 (i.e. approximately 15% at the higher values).

Poor frequency response of the hot wire is sometimes responsible for the under-estimation of dissipation measurements. The frequency response of the hot wire was flat up to 14 KHz and the spectrum analysis was made up to 20 KHz. Almost at the completion of the present work, a new DISA hot wire set (55D01) became available and this, with a

bridge ratio of 1:1, is capable of much better frequency response than the older hot wire set (55A01) used in the present experiments. With careful adjustment of the bridge balance and gain of the amplifier, a flat hot wire frequency response was obtained up to 105 KHz at an operating velocity of 200 ft/s whereas at zero velocity the frequency response was flat only up to 40 KHz. A Hewlett-Packard wave analyser (model 310A) was used to analyse the signal between 20 KHz and 100 KHz. The band width of the filter was 200 Hz at all tuning frequencies. Below 20 KHz the Wayne-Kerr frequency analyser was used. Thus, the spectrum function was obtained for frequencies up to 100 KHz. Integration of the spectrum function yielded the dissipation function. Both the dissipation function obtained with the hot wire of frequency response 14 KHz and that obtained with hot wire having a frequency response of 105 KHz, are shown in figure 4.17. It is obvious that, very near to the wall, the previous results were underestimated.

However, even after applying the correction for poor frequency response, it is clear that the local isotropy assumptions lead to values of dissipation which are of the order of 20 times less than production. It is certain that the local isotropy results are incorrect (see figure 4.6) but a factor of 20 is unlikely and it must be concluded that diffusion is of considerable significance in wall-jet flows. This implies, for example, that prediction procedures which use effective viscosity laws based on the Prandtl mixing length concept will fail since this concept assumes that production and dissipation are equal.

The low magnitude of the dissipation term can, perhaps, be explained by the undeveloped nature of the turbulence at x/y_c of 10. In this region, the eddies are likely to be comparatively large and certainly a very large proportion of the turbulence energy exists at wave numbers

less than those at which the $-5/3$ law is approached (see figure 4.15a); both these conditions imply that the energy available for dissipation is comparatively small.

Any lack of two-dimensionality would, of course, influence the turbulent-energy-balance since z-components of the energy transfer have been neglected. However, in view of the magnitude of the x-gradient of the x-components of mean and fluctuating velocities, the resulting error should be small.

Distribution of mixing length (ℓ_1) and Prandtl-Kolmogorov
length ℓ_2

Mixing length (ℓ_1)

Mixing length theory assumes a local equilibrium of the flow i.e. the turbulent production and dissipation balance one another. At this stage it can be anticipated that local equilibrium is unlikely to exist in wall jet flows and consequently, that complete similarity of mixing length profile is unlikely to exist.

It is also obvious that the mixing length profile cannot be the same for jet like ($\bar{U}_C/U_G > 1$) and wake like ($\bar{U}_C/U_G < 1$) flows since in the former case the mixing length has a local infinite value where the velocity gradient is zero and, in the latter, no such singularity exists.

A previous survey of mixing length profile was carried out by Escudier (25) but was restricted mainly to flows without secondary injection. For such flows the results showed some scatter but Escudier concluded that a satisfactory fit to the data was achieved by the equation:

$$l_1 = ky \quad y < (\lambda G1/k) y_G$$

$$l_1 = (\lambda G1) (y_G) \quad y > (\lambda G1/k) y_G \quad 4.3-10$$

where k and $\lambda G1$ equal to 0.41 and 0.075. Patankar and Spalding (75) have recommended a value of 0.09 for $\lambda G1$.

Escudier also considered four profiles taken from flows with secondary injection. These profiles were restricted to the outer region of the flow (values of y greater than that at which $\frac{\partial u}{\partial y}$ is zero)

and appeared to support a constant $\lambda G1$ of 0.1; this value was greater than the 0.075 value recommended by Escudier for boundary layer flows.

It should be noted, however, that three of these profiles were determined using values of shear stress obtained from mean velocity profiles and the integral momentum equation (an unreliable procedure) and that all four were taken from measurements made in a flow with a high value of the velocity ratio $\bar{U}_C/U_G$.

The present measurements allowed the calculation of Prandtl's mixing length, l_1 , and these are presented in figure 4.18 for four values of blowing rates and two lip thicknesses. It is evident from these figures that the length scale l_1 tends towards a constant value in the outer region of the flow only for the lowest and the highest blowing rates; even the value of the constant changes at various x locations. For the flows with velocity maxima, the different y -location of the zero shear stress and the zero mean velocity gradient result in singularities. The length scale distributions are very non-similar for flows with a $\bar{U}_C/U_G$ of 0.91

and 1.33. Figure 4.19 shows the distribution of λG_1 for thick and thin lip cases and $\bar{U}_C/U_G = 2.3$. It is interesting to note that the larger value of λG_1 for the thick lip is consistent with the increased mixing produced by the thick lip.

Prandtl-Kolmogorov length scale (ℓ_2)

The Prandtl-Kolmogorov theory of turbulence has been introduced in chapter 2. Values of the turbulent length scale* (ℓ_2) can be calculated from experimental data and are shown in figure 4.20. They exhibit general trends which are similar to those shown on figure 4.18 (for ℓ_1). In the calculation of these distributions, C' has been taken as 0.2. The accuracy of the ℓ_2 distribution will, in general, be poorer than that of ℓ_1 , as it requires the additional measurement of $\overline{q^2}$ which itself is the sum of three experimentally measured fluctuating velocities.

If the plateau in the curve of ℓ_2/y_G versus y/y_G is denoted as λG_2 , it may be observed that λG_2 exhibits trends similar to λG_1 . For the velocity ratio of 2.3, the variation of λG_2 in the x direction for thick and thin lip is shown in figure 4.19.

In the determination of ℓ_1 and ℓ_2 the velocity gradient has been obtained by fitting a parabola to three points and finding the slope in the centre. No smoothing of the velocity profile has been done. Some idea of the scatter in the distribution of ℓ_1 and ℓ_2 can be obtained from

* ℓ_2 is defined as follows:

$$\ell_2 = \frac{\mu_{\text{eff}}}{C' \rho \sqrt{\frac{1}{2} \overline{q^2}}}$$

the results presented in reference 46. In figures 4.18 and 4.20, experimental points have been omitted for the sake of clarity. Their omission in no way alters the conclusions drawn from these figures.

The ratio of turbulent shear stress to turbulent kinetic energy ($-\overline{uv} / q^2$)

Figure 4.21 presents distributions of the function $a_1 (\equiv -\overline{uv} / q^2)$. In reference 9, it was shown that, for wall boundary-layers in the absence of velocity maxima, a_1 can be regarded as a constant ($a_1 = 0.15$). It is clear from figure 4.21, however, that a_1 is not generally constant for wall-jet and wall-wake flows but that far downstream, for wall-wake flows, a_1 tends to a constant of around 0.15.

No data points have been shown on figure 4.21 and it should be stated that, particularly for values of y/y_G greater than 0.7, the scatter is large. Nevertheless, the trends shown by the figures are representative of the data.

4.4 Summary and conclusions

The experimental investigations described in this chapter permit the following conclusions to be drawn:

1. The static pressure is not constant in the flow, mainly due to the effect of finite lip thickness. The variation is a maximum in the plane of the slot exit but can be detected as far as 50 slot height downstream.
2. The existence of the "law of the wall" is supported by the present measurements in the region downstream of 20 slot heights.
3. For flows with a velocity maximum, the point of zero shear stress is found to be closer to the wall than that to zero velocity gradient. The point of minimum turbulent kinetic energy lies between the

point of zero shear stress and the velocity maximum.

4. The measured distributions of ℓ_1 and ℓ_2 do not exhibit general similarity although the tendency towards similarity is greatest for values of $\bar{U}_C/U_G$ far from unity.

5. The preferred frequencies observed for low values of x/y_C imply that the time-averaged Navier-Stokes equations are not entirely appropriate to this region.

6. For wall jet flows, the diffusion term in the turbulence energy balance cannot be neglected.

CHAPTER 5: SOLUTION PROCEDURE

5.1 Introduction

The basic equations for the flow down-stream of a two-dimensional film cooling slot have been presented in chapter 2. These equations need further simplification before they can be solved. The simplification used in this chapter are; the neglect of the vorticity source term and normal stresses; assumed viscosity hypothesis; and the form of diffusion and dissipation terms. The simplified equations have a common form and are solved using a finite-difference scheme. The finite-difference scheme, together with the control volume, boundary conditions, convergence criterion and other computing details are described in detail in the present chapter.

The experimental results presented in chapters 3 and 4 provide the necessary data against which the predictions are compared. The complex behaviour of the experimental length scale distributions demonstrated in chapter 4 are difficult to represent by algebraic equations. Hence, a simplified form of the length scale distribution is presented in this chapter.

The calculations, which are presented here, can be broadly divided into two groups - turbulent flow calculations and laminar flow calculations: the latter make it possible to compare the results with exact solutions and thus to prove the validity of the finite-difference scheme. The turbulent flow calculations can be further classified into two sections: in the first section, a direct comparison with the present experimental results is possible and, hence, these calculations require experimental profiles for velocity, mass concentration and turbulent kinetic energy to be supplied as boundary conditions at the slot exit whereas in the second section only the trends for the flow variables are predicted without direct comparison with experiments as in this case no experimental data are

available.

The assumed length scale distribution together with the simplifications discussed in the first paragraph of this introduction have yielded predictions which are in good agreement with the experimental results over a wide range of variables.

5.2 Aims of chapter 5

The aims of the present chapter may be stated as follows:

1. To present details of the computational procedure,
2. To test the prediction method against experimental results and available exact laminar flow solutions.

5.3 Requirements and limitations of the prediction method

The solution procedure should be capable of predicting the influence of slot-geometry parameters (as listed in section 3.1) on the following flow properties:

1. skin-friction and wall temperature (or concentration ϕ) downstream of a two-dimensional film cooling slot,
2. velocity and temperature (or ϕ) field.

The following are the acceptable (pro term) limitations of the present method:

1. no density gradients in the flow,
2. no heat or mass flux at the wall,
3. plane two-dimensional flow.

A further requirement is that it must be possible to extend the method to take account of density gradients and heat or mass transfer through the wall.

5.4 Identification of two regions of the flow

The distributions of static pressure presented in section 4.3.2.1 clearly demonstrate that, very near to the slot, the flow is governed by the time averaged Navier-Stokes equations whereas in the further downstream

region, boundary layer equations may be employed. For the sake of convenience and future reference, these two regions will be called the 'upstream region' and 'downstream region' respectively.

In the 'upstream region', the differences in static pressure arise from the lip which causes the stream-lines to curve. Equations 2.4-5, 2.4-6 and 2.4-7 are valid for this region, and are elliptic in nature. In general, these equations may be employed in separated flows which have recirculation and in flows, such as a jet impinging on a flat plate, where there is no recirculation but the pressure field contravenes the boundary layer assumptions.

The parabolic-boundary layer equations are valid in the 'downstream region'; these are a special case of the elliptic equations and, by solving the elliptic equations for the whole field, one can ignore different flow regions. However, the solutions of elliptic equations is less economical of computer time and hence it is desirable to divide the flow into the above mentioned two regions. The boundary between the two regions is not distinct. The distributions of static pressure suggest that, if this boundary is located between x/y_c of 25 and 50, the error due to the boundary layer assumptions should not be significant in the 'downstream region'.

A computer program for the 'downstream region' has been developed by Pai and Whitelaw (67). This program depends upon the starting conditions at x/y_c of 20, which have to be supplied from experiments. Later on, Pai (68) presented an improved version of the program in which the integration of boundary layer equations is started from the slot: the use of boundary layer equations in the 'upstream region' is, however, not strictly correct. It is possible to employ the present elliptic program to provide the starting velocity profile for the parabolic program (67) in order to obtain

a complete program which can predict starting from the slot conditions. This is of prime importance, as in almost all applications only the slot conditions are known. As already mentioned, the designers of gas turbine combustion chambers are only interested in the region of the flow which extends up to approximately x/y_C of 50 and consequently the control volume of the elliptic program is arranged to cover the region up to x/y_C of 50.

5.5 Physical assumptions

Before equations 2.4-5, 2.4-6, 2.4-7 and 2.4-9 can be solved, it is necessary to make the following assumptions:

1. The source term, S_ω , in the vorticity equation (equation 2.4-6) is taken as zero for turbulent and laminar flows. This assumption introduces only minor error for turbulent flows and no error for laminar flows since, for laminar flows S_ω can be shown to be identically zero and for turbulent flows the components of the S_ω term are of second order.

2. The form of the diffusion term in the turbulent kinetic energy (equation 2.4-9) is unsuitable for direct simultaneous solution with equations 2.4-5, 2.4-6 and 2.4-7. Partly for this reason, but mainly because the complete equation introduces more unknowns than can be solved for the diffusion term is simplified to:

$$\left[\frac{\partial}{\partial x} \left(\frac{\mu_{eff}}{\sigma_{q,eff}} \frac{\partial \overline{q^2}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\mu_{eff}}{\sigma_{q,eff}} \frac{\partial \overline{q^2}}{\partial y} \right) \right]$$

where $\frac{\mu_{eff}}{\sigma_{q,eff}}$ is the effective transport coefficient for the diffusion of

turbulent kinetic energy and $\sigma_{q,eff}$ is the equivalent of the effective Prandtl or Schmidt number.

3. The reasons for simplifying the dissipation term ϵ are similar to those for simplifying the diffusion term. The dissipation is written as:

$$\left[\frac{C_D}{l_D} \rho \left(\frac{\overline{\sigma^2}}{2} \right)^{3/2} \right]$$

where l_D is dissipation length scale and C_D is supposedly a universal constant at high Reynolds number (R).

This assumption for dissipation is valid provided the high frequency eddies (which dissipate energy) are isotropic; it can not be tested directly against experiments but only indirectly by comparing the results of a prediction procedure with experiments.

4. Experiments (see chapter 4, section 4.3.2.2) have shown that the normal stress terms are unimportant in the present flow situation and consequently they have been omitted from the solutions presented here.

With assumptions 2, 3 and 4, the equation of turbulent kinetic energy may be rewritten as

$$\begin{aligned} & \left[\frac{U}{2} \frac{\partial \overline{\sigma^2}}{\partial x} + \frac{V}{2} \frac{\partial \overline{\sigma^2}}{\partial y} \right] + \left[\overline{uv} \frac{\partial U}{\partial y} + \overline{uv} \frac{\partial V}{\partial x} \right] + \\ & \left[\frac{\partial}{\partial x} \left(\frac{\mu_{eff}}{\sigma_{q,eff}} \frac{\partial \overline{\sigma^2}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\mu_{eff}}{\sigma_{q,eff}} \frac{\partial \overline{\sigma^2}}{\partial y} \right) \right] + \\ & \left[\frac{C_D}{l_D} \rho \left(\frac{\overline{\sigma^2}}{2} \right)^{3/2} \right] = 0 \end{aligned}$$

5.5-1

This form of the turbulent kinetic energy equation is similar to that of equations 2.4-6 and 2.4-7 provided the production and dissipation terms are treated as source terms: this is done in the solution procedure.

The effective viscosity is calculated according to the Prandtl-Kolmogorov model. This model has already been discussed in chapter 2, but for convenience and continuity, it is repeated here.

$$\mu_{\text{eff}} = \rho C_{\mu} l_{\mu} \sqrt{q^2/2} \quad 5.5-2$$

It should be noted that the symbols l_2 and C' used in section 4.3.2.2 are identical to l_{μ} and C_{μ} used here. Different symbols are used in order to distinguish the experimental results (l_2 and C') from the physical input to the prediction program (C_{μ} and l_{μ}).

It is necessary to provide a suitable distribution of l_{μ} , l_D and also to give suitable values to the constants C_{μ} , C_D , $\sigma_{q,\text{eff}}$, $\sigma_{\phi,\text{eff}}$. The choice of these constants and their effect on the solutions obtained are discussed later in this chapter.

In the light of experimental information (section 4.3.2.2), it can be anticipated that the assumptions for diffusion, dissipation and turbulent viscosity will not be entirely satisfactory in all the regions of flow. The form of the diffusion term is justified only by the assumption that the turbulent-kinetic energy diffuses down its own gradient; there is no direct experimental evidence for this assumption. The form of the dissipation term assumes that small eddies are present to dissipate energy; as a result, the dissipation term is likely to be over-estimated in the region behind the slot lip where large eddies prevail. The effective viscosity hypothesis cannot exactly represent the flows where zero shear stress and zero mean velocity gradient occur at different locations; this happens in wall jet flows and also in flow in channels with walls of unequal roughness.

Although defects can be anticipated, it is hoped that these will not greatly influence the prediction of the practically important dependent variables such as effectiveness and skin-friction.

5.6 Finite-difference scheme

Only a brief outline of the finite-difference scheme will be presented here: it has previously been described in detail in references 31, 87, 88 and 120.

5.6.1 Control volume

In common with most procedures to solve elliptical equations, a control volume is defined; a rectangular mesh of grid points is described. The grids are so arranged that outermost rows or columns coincide with the boundaries of flow field. It is not necessary for the grids to be equally spaced. However, it has been found that non-monotonic variation of grid lines may cause instabilities in the solution. The number of the grid lines is dictated by the problem concerned, the accuracy desired and the computer time available to the user.

5.6.2 Algebraic equations

A set of algebraic equations are derived from the differential equations 2.4-5, 2.4-6, 2.4-7 and 5.5-1, by integrating them by a method called 'tank and tube' in reference 31, 87 and 88. A small cell is constructed around a node P in such a way that cell boundaries lie half-way between point P and its four neighbours (see figure 5.1). In order to integrate the equation an assumption is necessary regarding the distribution of the variable ϕ . In the present method it is assumed that inside a cell, ϕ is uniform and at a cell boundary (see figure 5.1):

$$\left. \frac{\partial \phi}{\partial x} \right|_w = \frac{\phi_w - \phi_P}{x_w - x_P}$$

5.6-1

* In equation 5.6-1 w-subscript refers to the point half-way between P and W (see figure 5.1).

whereas the calculation of first derivative at P is obtained from the equation:

$$\left(\frac{\partial \phi}{\partial Y}\right)_P = \frac{(\phi_N - \phi_P) \frac{Y_P - Y_S}{Y_N - Y_P} + (\phi_P - \phi_S) \frac{Y_N - Y_P}{Y_P - Y_S}}{Y_N - Y_S} \quad 5.6-2$$

It is to be noted that the above expression is exact only when ϕ varies linearly between points N, P and S. The integral containing the convective term is evaluated using an 'upwind difference' whereas, for the diffusive term, 'central difference' is used. These formulations ensure that the conservation laws are preserved over arbitrarily small or large volumes and the criteria for convergence is also satisfied. For further details of integration, the reader should read reference 31.

The algebraic equations obtained by the integration of differential equations relate the flow variables at one grid node to the neighbouring nodes and are of the following form:

$$\phi_P = C_E \phi_E + C_W \phi_W + C_N \phi_N + C_S \phi_S + D \quad 5.6-3$$

where ϕ denotes the flow variables like vorticity, stream function, turbulent kinetic energy, etc. C's represent the combined contribution of the convection and diffusion terms whereas D represents the source term. If the grid is of dimension N x M and one is solving for K variables then the total number of algebraic equations would be N x M x K.

5.6.3 Solution of the algebraic equations

In general, there are two iterative methods for the solution of a set of simultaneous algebraic equations: one involves matrix inversion and the other uses successive substitution; the later is employed in the present

computer program. There are various ways in which successive substitution can be used. Gauss-Seidel procedure is used in the present program. It must be mentioned that it is possible to make the program more efficient with respect to the rate of convergence, if efforts are directed to the optimization of the solution of the algebraic equations, perhaps by utilization of the block relaxation technique.

The order in which the flow variables are solved is as follows: ω , ψ , q^2 and ϕ . The whole control volume is scanned point by point in order to calculate new values of the variables by successive substitution. These values are compared with values from previous iteration and when it is observed that the changes from one iteration to the next iteration are so small that there is practically no change in values of variables, i.e. the convergence criterion is satisfied, the computations are terminated. If the boundary conditions are properly specified and grids have been chosen to be sufficiently fine, the solution obtained by finite difference method should be negligibly different from the solution of the differential equations.

5.7 Control volume employed

Unlike the parabolic equations, elliptic equations require a closed domain where the equations are to be solved with proper boundary conditions. A rectangular control volume has been chosen in the present case. The size of the control volume is dictated by the slot height, lip thickness and the downstream region of interest. A typical control volume is shown in figure 5.2 for the case of $t/y_C = 1.14$. The control volume is further divided into rectangular cells. A 21 x 21 mesh has been used for the calculations of section 5.12.1-a; a 21 x 26 mesh was used for the results of section 5.12.1-b and 5.12.2. The grid spacing in both cases was non-uniform

but increased monotonically in the positive x and y directions. The maximum increase in the grid spacing was 1.3 times the previous spacing.

The number of grid lines in the y direction was determined by the 1.3 factor mentioned in the previous paragraph and the thickness of the slot lip: thus the thin lip calculations led to a choice of 21 grid lines. The number of grid lines in the x-direction is more arbitrary since x-direction gradients are less important than y-direction gradients.

5.8 Boundary conditions

The elliptic form of equations 2.4-5, 2.4-6, 2.4-7 and 5.5-1 implies that boundary conditions are required in the plane of the slot exit, on the wall, in the free stream and in contrast to parabolic equations, at some downstream location in the flow; i.e. on the four sides of the rectangle used to bound the region of interest as shown in figure 5.2. These boundary conditions may be in the form of prescribed values of the dependent variables or their gradients.

In the present study an impervious-wall and zero longitudinal pressure gradient are assumed. Thus, the wall and free-stream boundary conditions may be written as:

$$\begin{array}{llll}
 \frac{\partial \psi}{\partial y} = U_G &) & & \\
 &) & & \\
 &) & & \\
 &) & & \\
 \omega = m_j = 0 &) & \text{free-stream} & \\
 &) & \text{boundary condition} & 5.8-1 \\
 &) & & \\
 \overline{q^2} = \text{a prescribed value} &) & &
 \end{array}$$

$$\begin{array}{lcl}
 \psi = \frac{\partial \psi}{\partial y} = 0 &) & \\
 &) & \\
 &) & \text{wall-boundary} \\
 &) & \text{condition} \\
 \overline{q^2} = \frac{\partial m_j}{\partial y} = 0 &) &
 \end{array}
 \quad 5.8-2$$

The down-stream boundary condition is assumed to be:-

$$\frac{\partial \psi}{\partial x} = \frac{\partial \omega}{\partial x} = \frac{\partial m_j}{\partial x} = \frac{\partial \overline{q^2}}{\partial x} = 0
 \quad 5.8-3$$

which assumes that the flow becomes fully developed in the length of the control volume. This assumption is not borne out by experiments; on the other hand, the x-station of the downstream boundary condition may be chosen further downstream than the region of interest and does not strongly influence the properties of the flow in the region of interest.

The downstream boundary condition for the calculations of section 5.12.1-a was assigned at x/y_C of 24; for the calculations of section 5.12.1-b and 5.12.2, a value of 53 was used. Test calculations for the case of turbulent flows, showed that the assignment of the downstream boundary condition at x/y_C of 10 and 20 led to sensibly identical values of ψ and ω at x/y_C of 5. Further calculations indicated that at x/y_C of 20 and 30 the stream lines were almost parallel; thus for subsequent calculations, it was considered that only a small number of grid lines was necessary between the further downstream station at which results are required and that at which the downstream boundary condition is assigned. Consequently, one grid line was employed between x/y_C of 20 and 24 and 3 between 30 and 53.

It remains to specify the boundary condition in the plane of the slot exit. Specific values of ψ , ω , m_j and $\overline{q^2}$ can readily be specified everywhere except at the corners of the slot lip where the boundary conditions for ω are double valued.

If it is assumed that all the vorticity contained in the boundary layers on both sides of the lip (see figure 5.2) feed respectively to control volumes P_1 and P_2 and conduction of vorticity from W_1 to P_1 and W_2 to P_2 is negligible then it can be shown (see reference 119) that:-

$$\omega_{W1} \approx \frac{U C_1}{2h_{p1}} ; \quad \omega_{W2} \approx \frac{-U G_1}{2h_{p2}} \quad 5.8-4$$

The above equations are suitable for both thick and thin boundary layers.

It is also possible to calculate the vorticity on lip corners by assuming that the stream lines leave the lip corners parallel to the base plate (i.e. $\psi_{p1} = \psi_{W1}$ and $\psi_{p2} = \psi_{W2}$) and making use of $\nabla^2 \psi = -\rho \omega$ relation. This approach has not been used here as it is felt that the assumptions of $\psi_{p1} = \psi_{W1}$ and $\psi_{p2} = \psi_{W2}$ are unrealistic for the cases where $\overline{U}_C/U_G$ is not near unity.

In the laminar flow calculations for the effect of upper lip boundary layer thickness on skin friction, it was observed that the lip corner vorticity can appreciably affect the skin friction values near the slot. In such cases, the above method of calculation of ω_{W2} was replaced by the known vorticity profile (using the Blasius solution (6)) for the upper lip boundary layer. It is worthwhile to note that only the convex corners in the control volume present the problem of double valued ω whereas

the concave corners ($I = 1$ and $J = 1$ is a concave corner of the control volume) do not present such a problem.

5.9 Wall functions

In the case of a laminar flow, the property gradients in the vicinity of the wall are steep but, with small sacrifice in economy, the finite-difference equations can be solved with only a minor change in the general procedure. A minor change is required in order to determine the value of vorticity at the wall and involves the neglect of x -convection and diffusion in the interval adjacent to the wall. In addition to this, if it is assumed that the vorticity variation is linear near the wall, then it can be shown that:-

$$\omega_1 = \frac{3(\psi_2 - \psi_1)}{(y_2 - y_1)^2} - \frac{\omega_2}{2} \quad 5.9-1$$

where subscripts 1 and 2 refer to J values of the grid, i.e. 1 denotes the wall and 2 denotes the second grid line. The same expression can be used for the calculation of vorticity on the lip (except for the lip corners) if y 's and J -subscripts are replaced by x 's and I -subscripts respectively.

In the case of turbulent flows, however, the property gradients in the vicinity of the wall can be so steep as to require a very large number of grid lines. This is usually the case with velocities and hence a special procedure is used to link the wall boundary conditions for ω and $\overline{q^2}$ with the grid line adjacent to the wall ($J = 2$). The procedure is such that the grid line adjacent to the wall is sufficiently far away from the wall to permit the remainder of the control volume to contain an economical number of grid lines. Thus, as will be indicated, algebraic expressions link the wall boundary condition ($J = 1$) with the vorticity and the kinetic-energy at a point away from the wall ($J = 2$).

Wall functions are, in effect, an application of a 'law of the wall' to finite difference procedure and their purpose, in this context, is to maintain economy. If the Pradtl mixing length was used as the basis for the effective viscosity hypothesis, the turbulent kinetic energy equation would not be solved, and the wall functions developed by Patankar and Spalding (75) would be appropriate. Here, the Prandtl-Kolmogorov hypothesis is the basis for the effective viscosity hypothesis, the equation of turbulent-kinetic energy is solved, and the wall functions must be based on $\overline{q^2}$ and U rather than on U alone. The wall functions used in the present calculations were developed by Wolfshtein and are described in detail in reference 120. They may be written in the following form:

$$S' = 1 \quad ; R < 19.8$$

$$S' = 0.0565(R+5)^{0.895} \quad ; a = 0, R > 19.8$$

$$S' = 0.097 a^{-0.34} (R+5)^{0.58} \quad ; 0.252 > a > 0; R > 19.8$$

$$S' = 0.155 (R+5)^{0.58} \quad ; a > 0.252 ; R > 19.8$$

where $S' \approx (\tau_w y) / (\mu U)$

$$R = \sqrt{\overline{q^2}/2} \rho y / \mu$$

$$K_1 = \frac{\overline{q^2}}{2} \rho / \tau_w$$

$$a = K_1 / R^{0.82}$$

$$\text{and } \omega_1 = \omega_w = S' U / y = \tau_w / \mu$$

The value of turbulent shear stress at the second grid line ($J = 2$)

is evaluated as the arithmetic mean of τ_w and turbulent shear stress at the third grid line ($J = 3$). This value together with the relation

$\omega = \frac{-\tau}{\mu_{\text{eff}}}$, yields ω_2 , and the values of ω_3 , ω_4 , etc. are calculated according

to the finite-difference procedure. These wall functions are appropriate in the flow regime where Couette flow is appropriate. They are valid up to the y^+ values where the 'law of the wall' holds good. Thus the grid line adjacent to the wall must be located within this region. The wall-functions developed by Wolfshtein are only valid for flows with no mass transfer through the wall or strong longitudinal free stream pressure gradients. Further development is necessary to confirm the applicability of the wall-functions to flows with mass transfer or a strong free-stream pressure gradient.

The above wall functions permit the determination of ω_w and τ_w without sacrificing economy. They allow the grid lines adjacent to the wall to be some distance from the wall and hence avoid the need for a large number of grid lines in the region of steep gradients. Similar functions can be derived for other dependent variables but this has not been done. The stream function normally varies slowly close to a wall and, in this case, no special procedure is necessary; in the present impervious-wall configuration, the same is true for mass concentration. Thus, it remains only to derive a procedure for the turbulent-kinetic energy close to a wall: this is outlined below but is explained in greater detail in reference 100.

The turbulent-kinetic energy at the wall is zero and thus the use of a linear distribution of $\overline{q^2}$ in the interval close to the wall would lead to error. Consequently, in the finite-difference scheme, a false value of $\overline{q^2}$ is used at the wall instead of zero and is derived in a manner which implies that a linear distribution of $\overline{q^2}$ will not lead to errors in the wall region.

Spalding has shown (100) that in the fully turbulent region ($\mu_{\text{eff}} \gg \mu_{\text{laminar}}$) of couette flow, if $\overline{q^2}$ varies as $y^{p'}$, then by solution of one dimensional turbulent kinetic energy equation the following expression for p' may be written:

$$p' = \left[\frac{2}{3} \sigma_{q, \text{eff}} \frac{C_D}{C_\mu} \left(1 - \frac{\tau_*}{C_\mu C_D} \right) \right]^{1/2} \quad 5.9-3$$

$$\text{where } \tau_* = \left| \frac{\tau}{\rho \overline{q^2/2}} \right| \quad J = 2$$

It is not possible to satisfy the power law assumption over the whole of the region between the wall and second grid line; consequently equation 5.9-3 is obtained with the assumption that the power law is valid on the second grid line ($J = 2$) only.

Knowing the exponent p' , it is easy to show that by matching the ordinate and slope of the $\overline{q^2}$ profile at the second grid line, the following expression for $\overline{q_{w, \text{false}}^2}$ may be written:

$$\overline{q_{w, \text{false}}^2} = (\overline{q^2}_{\text{second grid line}})^{1-p'} \quad 5.9-4$$

A more detailed discussion of the validity of this procedure may be found in reference 100.

5.10 Effective transport coefficient

In laminar flow, the solution of equations 2.4-5, 2.4-6 and 2.4-7 requires a knowledge of the laminar transport properties such as viscosity, thermal conductivity and mass diffusion: these are readily available in the literature. In turbulent flow, the same set of

equations may be solved but they require the specification of effective transport properties such as viscosity, thermal conductivity and mass diffusion. In the present case, equation 5.5-1 was solved in addition to equations 2.4-5, 2.4-6 and 2.4-7, and the Prandtl-Kolmogorov hypothesis was used to link shear stress, mean-velocity gradients and turbulent-kinetic energy. As may be seen from the earlier considerations, the following quantities must be specified before the equations are complete:

$$\ell_{\mu}, \ell_D, C_{\mu}, C_D, \sigma_{q,eff}, \sigma_{m,eff}.$$

These will be considered in the following sub-sections.

As will be noted, the values assigned to these quantities are, in many cases, the same as in reference 120. Absolute universality of these constants cannot be expected. It is likely that they are slowly varying functions of local Reynolds number ($\frac{1}{2} \overline{q^2} y/\nu$), proximity to the wall and perhaps some other parameters. Lack of experimental results does not allow these functional relationships to be incorporated in the present solution procedure. This may be postponed for future refinement. It is desirable to determine to what extent these quantities have universal values and it is to be hoped that one set of values can be used over a wide range of flow configurations. In the present work, the approach was, therefore, to first assume values used successfully in other flow configurations and subsequently to determine if changes would lead to significant improvement in predictions. In this section the values used initially will be introduced and in the later part of the section the effect of changes will be discussed.

5.10-1 Constants for the present computations

5.10.1-a Prandtl-Kolmogorov turbulent viscosity law

The Prandtl-Kolmogorov form of turbulent viscosity requires

the specification of the constant C_μ and the length scale l_μ . In the work described in reference 120, C_μ was taken as 0.22 and l_μ given by the equation:

$$l_\mu = y [1 - \exp. (- A_\mu \sqrt{q^2/2} y \rho/\mu)] \quad 5.10-1$$

where A_μ was 0.016. This length scale equation allowed l_μ to vary linearly with y at large values of y but introduced a damping term to reduce the influence of the turbulent-kinetic energy close to the wall: it is similar in form to Van Driest's (114) mixing length hypothesis and clearly resembles that used by Patanker and Spalding in reference 75.

The values of C_μ and A_μ used in reference 120 have been retained in the present case but the distribution of l_μ has been modified in accord with the experimental evidence. The distributions of l_μ , obtained from measurements and presented in chapter 4, show that l_μ suffers singularities when local turbulent shear stress and mean velocity gradients become zero at different y -locations. However, this section seeks to demonstrate the precision of predictions obtained with an empirical specification of l_μ and the form noted below is in reasonable accord with the experimental evidence (although, of course, not accounting for the presence of singularities):

$$l_\mu = y [1 - \exp. \{-A_\mu (\sqrt{q^2/2} y \rho/\mu)\}], \quad 0 < y < \lambda_{2t} y_G \quad 5.10-2$$

$$l_\mu = \lambda_{2t} y_G \quad \lambda_{2t} y_G < y < y_G \quad 5.10-3$$

constant, C_D , and the dissipation length scale, ℓ_D : both must be obtained from experimental data. In accord with the suggestions of reference 120, the constant C_D is taken as 0.416 and the dissipation length scale is given by the equation:

$$\ell_D = y [1 - \exp. \{-A_D (\sqrt{q^2/2} \ y^p/\mu)\}], \quad 0 < y < \lambda_{2t} \ y_G \quad 5.10-4$$

$$\ell_D = \ell_\mu = \lambda_{2t} \ y_G, \quad \lambda_{2t} \ y_G < y < y_G \quad 5.10-5$$

where the damping constant, A_D , is assigned the value 0.263. The form of equation 5.10-4 is similar to that of equation 5.10-2.

At first glance, it appears that the constants C_D and C_μ are independent. This is not the case, however, since it can be shown that:

$$U^+ = \frac{C_D^{\frac{1}{4}}}{C_\mu^{\frac{3}{4}}} \ln (E \ y^+) \quad 5.10-6$$

i.e. the ratio $C_D^{\frac{1}{4}}/C_\mu^{\frac{3}{4}}$ must equal the Von Karman constant in the law of the wall. The present choice of C_D and C_μ is consistent with a value of the Von-Karman constant of 0.40.

5.10.1-c Diffusivities for kinetic energy and species

It remains to specify $\sigma_{q,eff}$ and $\sigma_{m,eff}$: they are 1.53 and 0.4 respectively. The former is in accord with reference 120 and the latter is in reasonable agreement with the admittedly sparse experimental information of reference 46.

5.10.2 Influence of changes in constants

In the present procedure, the quantities l_μ , C_μ , l_D , C_D , $\sigma_{q,eff}$ and $\sigma_{m,eff}$ have been specified: all except l_μ and $\sigma_{m,eff}$ in accord with those found satisfactory for the impinging jet. The question arises as to the influence of the particular choice for each of these quantities on the predictions and it is the purpose of this section to provide an answer. The succeeding paragraphs, therefore, describe the influence of each of these constants as determined by performing systematic calculations with the prediction procedure.

The calculations presented here to show the influences of C_μ , C_D , $\sigma_{q,eff}$ and $\sigma_{m,eff}$ were carried out for a velocity ratio of 0.75 and a ratio, t/y_C , of 0.126: l_μ and l_D were not changed in these calculations. The particular choice of the velocity ratio and lip thickness is arbitrary and any other combination of the velocity ratio and lip thickness could have been chosen for these calculations. The results of these calculations are shown in figure 5.4.

5.10.2-a Influence of C_μ

The constant value of C_μ was varied from 0.22 to 0.18 while C_D , $\sigma_{q,eff}$ and $\sigma_{m,eff}$ were held constant at 0.313, 1.5 and 0.4 respectively. No significant change in the velocity or turbulent kinetic-energy profile was observed in the outer region of the flow, but as might be expected, the higher value implies a higher effective viscosity in the inner region with a consequent increase in the rate of drop of effectiveness (approximately 2% of unity) and in the values of skin friction (approximately 10%).

The values of C_μ tested were, in absolute terms, very similar in

magnitude: they correspond to the values recommended by Wolfshtein (120) and Spalding (100) respectively. It should be realised, however, that values outside this range will lead to incompatible values of the Von Karman constant in the law of the wall or in the constants in Jayatilaka's p-function (44).

5.10.2-b Influence of C_D

The possible variation in C_D is rather greater than in C_μ : values varying from 0.224 to 0.416 have been proposed. The value of C_D is, however, dependent on the value of C_μ since the equation:

$$\frac{C_\mu^{\frac{3}{4}}}{C_D^{\frac{1}{4}}} \frac{U}{\sqrt{\tau_w/\rho}} = \ln \left[E \frac{y \sqrt{\tau_w/\rho}}{\mu} \right] = \frac{1}{K} \frac{U}{\sqrt{\tau_w/\rho}} \quad 5.10-7$$

must hold (120) for the constant shear, non-diffusional boundary layer. The choice of a value of C_μ of 0.22 thus implies a value of C_D of 0.416. The effect of a decrease in C_D ; with C_μ held constant is to increase the turbulent kinetic energy in the inner region of the flow: a change from 0.416 to 0.313 (with $C_\mu = 0.22$) results in values of turbulent kinetic energy which are higher by approximately 30%

The choice of C_μ and C_D equal to 0.22 and 0.416 respectively implies that for constant shear-stress non-diffusive layer the constant a_1 (refer to section 4.3.2.2) is equal to 0.15 and which is in agreement with many experimental results and also with the recommendations of Bradshaw et. al. (9).

5.10.2-c Influence of $\sigma_{q,eff}$

Calculations showed that the variation of $\sigma_{q,eff}$ from 1.0 to 2.0 (with $C_D = 0.416$ and $C_\mu = 0.22$) does not result in a significant change in either the mean velocity profile or the profile of turbulent kinetic energy. Thus, it appears that the diffusion term is not of great importance in the test calculations: this result is not in agreement with the present experimental findings (chapter 4) and it must be the consequence of the assumptions implied in the calculations of the diffusion term.

5.10.2-d Influence of $\sigma_{m,eff}$

The influence of the effective Schmidt number on concentration profiles is known to be substantial. The lower the value, the more rapid the growth of concentration profiles will be. Some measurements of $\sigma_{m,eff}$ presented in reference 46, indicated a value of 0.5 ± 0.2 . The present predictions show that constant value of 0.4 leads to satisfactory agreement with measured profiles of mass concentration and effectiveness.

5.10.2-e Influence of λ_μ and λ_D

In the calculations described in the preceding subsections, the distributions of λ_μ and λ_D have been considered to be equal except in the vicinity of the wall where the different values of A_μ and A_D result in differences of λ_μ and λ_D . In the computer program, the distribution of λ_μ is supplied and that of λ_D derived from it. If all other constants are fixed and λ_μ is increased, the effect is to increase production and decrease dissipation; the result is an increase in the overall level of turbulent kinetic energy.

5.11 Computing details

A listing of the computer program, together with explanation of the Fortran symbols and the function of various subroutines, is given in Appendix 1. This program has been used for the calculations of results presented in section 5.12.1-b and with small modifications, it has also been used for the results of section 5.12.1-a and 5.12.2. The program has been run on IBM 7094, IBM 360 and CDC 6600 machines. The computer time taken by the program varies from machine to machine and also depends upon the flow conditions, convergence criterion, etc. Typically, on IBM 7094 machine, the program takes 7 minutes for 250 iterations when 4 equations (ψ , ω , q^2 and m) are solved. The total number of iterations required for the convergence can vary by a factor of 2 depending upon Reynolds number. A figure of 350 for the total number of iterations to satisfy the convergence criterion, may be taken as an average of many runs.

If $\Delta\phi$ is the change in the value of a dependent variable ϕ from one iteration to the next iteration and $\phi_{\max}$ is the maximum value of ϕ over the whole of the field then computations are stopped provided the following convergence criterion is satisfied for all dependent variables:

$$\frac{\Delta\phi}{\phi_{\max}} < C_c$$

A value of C_c has to be chosen in order to obtain results of finite-difference procedure which are close to the converged solution. Choosing a too big value for C_c results in the finite difference solution which are far away from the converged solution, and on the other hand,

choosing a too small value requires large computing time with only a marginal improvement in answers. Past experience allowed the value of C_c to be selected as .0001 and .00005 for turbulent and laminar flows respectively.

Some difficulty was encountered in the initial stages of program development due to divergence: this was caused by too rapid changes in the effective viscosity from one iteration to the next. Under-relaxation in effective viscosity was found necessary to achieve convergence. In some instances (for example in laminar flow calculations with a thick lip), oscillations were noticed in the residuals and it was not possible to obtain convergence. It is believed that these oscillations were due to local vorticity changes near the thick lip and did not affect the whole field. These calculations have not been included here.

5.12 Predictions

As already mentioned in section 5.1, the computations, which are presented here, are of two types: one pertaining to laminar flows and the other to turbulent flows. The turbulent flow calculations can further be subdivided into two sections: in the first section comparison with experimental results are demonstrated, whereas in the second section only the trends are predicted without direct comparison with experiments.

5.12.1 Turbulent flow predictions

5.12.1-a Comparison of predictions and experimental measurements

Table 2 shows the comparisons which have been made with the data of references (43) and (50): they are presented in figures 5.5, 5.6, 5.7 and 5.8. It should be remembered that the constants λ_w and $\sigma_{m,eff}$ have been obtained with the aid of these data so it is to be expected

that the predictions should not differ from the measurements by order of magnitude except, perhaps, in the region where the local shear stress and mean velocity gradient becomes zero. On the other hand, plausible assumptions have been made regarding the functional form of l_μ and $\sigma_{m,eff}$ and the remaining constants have been taken from the impinging jet studies of Wolfshtein (120). Thus the comparisons represent a necessary test of the validity of the procedure.

Figure 5.5 presents mean velocity, concentration and turbulent-kinetic energy measurements at two values of x/y_C and the corresponding predictions based on measured initial and downstream conditions, as described in section 5.8, located at x/y_C of 24: these measurements refer to a value of t/y_C of 0.126. The results shown in figure 5.6 correspond to those shown on figure 5.5 and to a value of t/y_C of 1.14. Particularly in the case of turbulent kinetic energy, discrepancies exist between the predictions and measurements shown on figure 5.5 and 5.6: they are, however, comparatively small and are certainly small enough to be neglected for most practical purposes. Indeed, in view of the simple form of l_μ , the predictions are very satisfactory.

Corresponding to the profile shown on figure 5.5 and 5.6, figure 5.7 and 5.8 present predictions of the wall-shear-stress coefficient and impervious-wall effectiveness, respectively. The predictions of wall-shear-stress are in good agreement with experiments: the maximum discrepancy is less than 10%. The minimum in the shear-stress-coefficient close to the slot exit and for the measurement with the thicker lip should be noted: it will be discussed again in the section 5.12.1-b. The predictions of effectiveness show clearly that the effect of an increase in the slot-lip-thickness is to reduce the effectiveness. The

agreement between experiment and prediction is very satisfactory except for the thick slot lip and wake-like flows.

In general the comparison presented in this subsection may be considered to be very satisfactory. Some observed discrepancies may be attributed to the simple form of ℓ_μ distribution which cannot be entirely representative of the complex flow pattern under investigation.

5.12.1-b Predicted trends

The previous section was intended to establish the validity of the present numerical procedure and the physical hypothesis. In the present section, the influence of various independent variables on dependent variables, as predicted by the present procedure, are demonstrated. Where possible, the predicted trends are compared with those observed by experimenters and provide further evidence of the validity of the procedure. It should be noted that it is the absence of detailed experimental initial conditions which prevents the direct comparison of predicted and measured data in this section. Those direct comparisons which are possible were described in section 5.12.1-a.

Influence of velocity ratio

The influence of velocity ratio can be deduced from the measurements and predictions presented in section 5.12.1-a. It is desirable, however, to determine the influence of velocity ratio on practically important dependent variables over a wider range of velocity ratios than those presented in section 5.12.1-a and based upon reference initial and boundary conditions. Figure 5.9 A and B show this influence in the range of velocity ratios from 0 to 5 at downstream distances of 8, 20 and 35. The initial conditions conformed to square secondary and freestream velocity, concentration and turbulent-kinetic-energy profiles. The last named was assigned a value of 0.1% from the wall to the free

stream and the same value at the external boundary. The velocity profile, which has been termed 'square' here, is shown in figure 5.10. The thicknesses of the boundary layers on the wall, and on both sides of the lip are indicated in the same figure. The downstream boundary conditions were assigned at x/y_c of 53 and conformed to those described in section 5.8.

The predictions shown on figure 5.9 A and B are in accord with the measurements of references 49, 89 and 93. In addition, the general trends are in agreement with expectations and with the measurements of previous authors.

Influence of Reynolds number

The influence of velocity ratio, demonstrated in the previous section, corresponds to a free-stream velocity of 62 ft/s. If, for a constant velocity ratio, the free-stream velocity is changed, the slot velocity will also change and with it, for instance, the value of the wall-shear stress. This may be regarded as the influence of slot Reynolds number and its magnitude is shown in figure 5.11. The calculations shown on figure 5.11 were performed with similar initial and boundary conditions to those of previous section. Property values were appropriate to air at atmospheric pressure and 20°C. The range of slot Reynolds number is 4000 to 40,000 corresponding to secondary velocities of 31 ft/s and 310 ft/s—a practically useful range.

Figure 5.11 B and C show that the influence of slot-Reynolds number on the effectiveness is small. Stollery and El Ehwany (104) have shown that as the flow asymptotes to a zero-pressure gradient boundary layer flow, the slot-Reynolds number dependence should asymptote to:

$$\eta \propto R_{ec}^{0.2} ;$$

The present results show that this situation has not been reached at a value of x/y_C of 47.

Influence of upper-lip boundary layer thickness

The measurements reported in references 16 and 92 demonstrated that the influence of the upper lip boundary layer thickness was small. All of these measurements were, however, carried out with comparatively large values of the ratio t/y_C . Thus any influence of a momentum defect due to boundary layer thickness was hidden by the large influence of the slot lip itself. Recent measurements (described in section 3.3.2) with a low value of t/y_C , show that the influence of the upper lip boundary layer thickness is appreciable when the ratio, t/y_C , is small. This trend is confirmed by the predictions shown in figure 5.12. It is demonstrated in figure 5.12 B that, at t/y_C of 1.14 and $\bar{U}_C/U_G$ of 1.0, the influence of increasing the upper lip boundary layer thickness from $0.12 y_C$ to $4.0 y_C$ is to reduce the effectiveness by approximately 8% of unity. On the other hand, figure 5.12 D shows that for t/y_C of 0.126 and $\bar{U}_C/U_G$ of 1.0, the effectiveness is reduced by up to 25% of unity for the same change in thickness of the upper lip boundary layer. The same figure also demonstrates that for the case of thinner lip, the influence of upper lip boundary layer at $\bar{U}_C/U_G$ of 0.5 is very similar to that of $\bar{U}_C/U_G$ of 1.0. The results of figure 5.12 A and C also show that an increase of δ/y_C , from 0.12 to 4.0 can have a significant effect on the wall shear stress.

Influence of the shape of the slot-velocity profile

Figures 5.13 A and B show the influence of change in slot velocity profile on skin friction and effectiveness for $\bar{U}_C/U_G$ of 0.5 and 1.0. The slot velocity profiles are shown in figure 5.10. It is evident from these figures that the influence of the profile shape on effectiveness is relatively small. However, there is appreciable effect on skin friction for $\bar{U}_C/U_G$ of 1.0.

Influence of slot and free-stream turbulence intensity

The influence of the various turbulence intensities is shown in figure 5.14 for $\bar{U}_C/U_G = 0.5$ and 1.0. According to the predictions, a increase in the value of slot or free stream turbulence intensity up to 20% lowers the effectiveness by not more than 5% of unity. The skin friction is only affected appreciably for $x/y_C < 5$. The present results for effectiveness are in agreement with the measurements described in chapter 3.

Influence of slot lip thickness

It has been demonstrated in chapter 3 that the lip thickness is an important slot geometry parameter and it significantly influences the film cooling effectiveness and skin friction. This is further confirmed by the predictions presented in figure 5.15 (A to F) for three values of $\bar{U}_C/U_G$; viz 0.5, 1.0 and 5.0; and three value of t/y_C , viz 0.126, 0.63 and 1.14. The length scale distribution for t/y_C of 0.63 was obtained by linear interpolation between the length scales for t/y_C of 0.126 and 1.14. The effectiveness results taken from figure 5.15 B, D and F have been cross plotted in figure 5.16 in terms of η versus $\bar{U}_C/U_G$; this figure clearly shows the predicted influence of t/y_C and $\bar{U}_C/U_G$ on

effectiveness. The trends of figure 5.16 are in close agreement with the experimental results presented in chapter 3 (figure 3.9). The dip in skin friction curve noted previously for $\bar{U}_C/U_G \approx 0.75$, is much more evident in figure 5.15 A for a lower velocity ratio of 0.5 and it suggests that separation may take place on the base plate for the case of a thick lip if $\bar{U}_C/U_G$ is small enough. Following this trend, computations were done for $\bar{U}_C/U_G$ of 0.25 and $t/y_C = 1.14$ and are shown in figure 5.17. The existence of separation is, indeed, confirmed by these predictions. Burns and Stollerly (13) have also demonstrated the presence of separation on base plate in the case of low helium injection rates using a Schlieren system. No experimental data is available to confirm the trends of skin friction curves of figure 5.15 C and E for $\bar{U}_C/U_G$ of 1 and 5.

5.12 Laminar flow predictions

From the practical point of view, laminar flow predictions are of minor importance. However, they are very useful for checking the accuracy and validity of the finite-difference method since, as in some laminar flow situations, it is possible to compare the finite difference results with the exact solutions of differential equations. Two well known exact solutions, with which the present predictions have been compared, are Blasius solution (6) for boundary layers and Glauert's solution (29) for wall jet flows with zero free stream.

Figure 5.18 shows the velocity and stream function distribution for $\bar{U}_C/U_G$ of 0.333 and 0.5 at x/y_C of 26. From the velocity profile, it is obvious that for these velocity ratios and far downstream the flow reverts to a boundary layer type flow and the predictions can be compared with the Blasius exact solution; this is done in the same figure. In the present solution, it is difficult to find the origin of boundary layer

(as it is needed in the calculation of $\bar{\eta}$ and $f(\bar{\eta})$). To overcome this difficulty, the computed velocity profile and the exact velocity profile from the Blasius solution were matched at one value of $\bar{\eta}$; the value of effective distance from the origin was then obtained. The agreement between the finite-difference solution and the exact solution is very satisfactory.

Figure 5.19 shows the comparison between Glauert's exact solution ($\bar{U}_C/U_G = \infty$) and the finite-difference solution with values of $\bar{U}_C/U_G$ of 5 and 100 at x/y_C of 26. The agreement between the exact solution and the finite difference calculations for $\bar{U}_C/U_G$ of 100 demonstrates that, for all practical purposes, the velocity ratio of 100 is high enough to be compared with the Glauert solution for $\bar{U}_C/U_G$ of ∞ . Figure 5.19 also indicates the free stream value of $U/U_{\max}$ appropriate to x/y_C of 26.

The overall agreement between the finite-difference solutions and exact laminar flow solutions are satisfactory and confirm the validity and accuracy of the present method.

Figure 5.20 shows the results obtained with the finite-difference procedure for the influence of upper-lip boundary layer thickness on skin friction in laminar flows. For the $\bar{U}_C/U_G$ less than unity, the skin friction decreases if the upper lip boundary layer is thickened; this trend is in agreement with the results of Schetz and Jannone (91). However, for $\bar{U}_C/U_G$ of 1.0 and 5.0, the trend is reversed, i.e. the thicker upper lip boundary layer gives higher skin friction than the thinner one. There are no experimental or theoretical results for $\bar{U}_C/U_G > 1$ to compare with.

5.13 Conclusions

1. A computer program has been presented for the solution of elliptic equations in the upstream region of wall jet flows.

2. The accuracy and validity of the computer program has been checked by comparing the finite-difference results with exact laminar flow solutions: it has been found to be satisfactory.

3. The Prandtl-Kolmogorov turbulence model, together with a simple specification of the length scale of turbulence, has been used to predict wall-jet and wall-wake flows and have been found to give predictions which are in good agreement with experimental results.

4. There is scope for further improvement of the physical input to the program and of the economy of the computational scheme.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The aims of the present study, as listed in chapter 1, have largely been achieved. Conclusions specific to the work described in each of chapters 3, 4 and 5 are listed in the last section of the relevant chapter. The conclusions presented in this section are more general in that they explicitly bring together the conclusions of individual chapters and indicate the inter-relationships.

1. The effectiveness of film cooling is significantly influenced by slot geometry parameters. The present measurements imply that the values of lip thickness, slot height and upper-lip boundary layer thickness must always be included in any prediction procedure which claims to be general. The magnitude of the influence of slot geometry parameters and the complexity of their influence make it necessary to base a prediction procedure on the relevant differential equations.

2. A prediction procedure based on the elliptic partial differential equations of conservation of mass, momentum and species and incorporating the Prandtl-Kolmogorov model of turbulence, together with a numerical procedure proposed by Runchal, Wolfshtein and Spalding is shown to yield satisfactory prediction of effectiveness, wall-shear stress, mean velocity and turbulent kinetic energy in the region between the slot exit and 50 slot heights downstream. It permits, for example, the influence of slot geometry parameters on effectiveness to be satisfactorily predicted. The turbulent length scale, which must be specified in the Prandtl-Kolmogorov model, is empirical in the nature and based on the present measurements of length scale.

3. The preferred frequencies observed behind the lip imply that the solution of the time averaged Navier-Stokes equations is a simplification of the complex flow existing downstream of the slot. Inadequate understanding of the physics of the flow and the limitations on the computer size do not justify, at present, the solution of the time dependant equations.

4. The differences observed in the experimentally measured values of the dissipation and diffusion of turbulent kinetic energy and the values calculated and used in the prediction method are to some extent responsible for the discrepancies between the experimental results and the predictions. The particular formulation of the dissipation and diffusion terms used in the predictions needs experimental verification in different flow situations.

6.2 Recommendations for future work

Although the aims of the present work have been achieved, the long term aim to provide a prediction procedure for flow properties downstream of practical three-dimensional film cooling slots is some way from completion. The main purpose of this section is to provide suggestions for the future research required to aid the achievement of this long term aim. At the same time, the opportunity is taken to suggest further explorations of less important questions which have arisen as a result of the present work.

1. The present solution procedure is applicable to constant density flows in the absence of pressure gradients. In order to predict practically important flow situations, the solution procedure should be extended to take into account the effect of pressure and density gradients on the flow. The wall functions of the solution procedure should be modified accordingly and the predictions checked against the experimental data. To the author's

knowledge, no systematic experimental investigation exists for the influence of slot-geometry parameters in presence of density and pressure gradients in the 'upstream region' of the flow. Consequently, a new experimental investigation should be undertaken to fulfil the particular needs of the extended version of the solution procedure.

2. The walls of combustion chambers are non-adiabatic and are exposed to hot gases. In order to predict the combustion chamber wall temperatures accurately, it is necessary to include in the solution procedure the effect of radiation from hot gases and finite heat-flux at the wall.

3. The algebraic input for the length scale should be replaced by a length scale equation on the lines suggested by Rotta (86). Some limitations of the Prandtl-Kolmogorov model can only be overcome by solving a differential equation for the shear stress. In the author's view, the shear stress equation must first be solved for simple boundary layer type flows and only subsequently applied to the more complicated problems of elliptic flows. This development should go hand in hand with a carefully designed experimental program.

4. A more detailed investigation of the 'preferred frequencies' should be undertaken to precisely determine the limitations which their presence imposes on the solution of time averaged equations. An experimental investigation for different lip thicknesses and velocity ratios should be carried out to further confirm the correlations between f_t and f_s proposed in section 4.3.2.2 and also to determine the value of x/y_c downstream of which these frequencies may be ignored. These experiments should be carried out for two or three Reynolds numbers, since the presence of transition waves is known to depend upon the Reynolds number of the flow.

5. In the present experimental investigation, it was not possible to obtain a total isolation of all the slot geometry parameters. For example, the introduction of turbulence promoters affected the upper lip velocity profile. In order to deduce the influence of one parameter from the combined influence of two or more parameters, generally some additional experiments have to be undertaken. This can be avoided by more effective control of the slot geometry parameters. An independent control on the upper lip velocity profile, by means of suction or blowing on the top plate, will be useful in this respect. A similar control on the slot velocity profile will enable the investigation of the influence of the slot velocity profile on effectiveness of film cooling.

6. To date only ad hoc investigations exist for three-dimensional slots; consequently, there is need for a systematic experimental investigation of the film-cooling effectiveness of three-dimensional slots. In 10 - 20 years time, it is likely that a suitable method for the prediction of three-dimensional flows will be available. Thus, any investigation of three-dimensional film cooling should be designed to provide information suitable for the testing of three-dimensional prediction methods while, at the same time, providing information of direct value to designers.

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NOMENCLATURE

<u>Symbol</u>	<u>Meaning</u>	<u>Equation of first appearance</u>
a	constant	3.4-1
a_1	Bradshaw's parameter ($\equiv -uv/\overline{q^2}$)	-
A	constant	3.2-1
A_μ	constant	5.10-1
A_D	constant	5.10-4
b	constant	3.4-1
B	constant	3.2-1
C	constant	3.2-2
C'	constant	4.3-11
C_B	constant	2.3-3
C_C	convergence criterion	-
C_D	constant	5.5-1
C_D'	constant	2.3-4
$C_f/2$	drag coefficient ($= \tau_w/\rho U_G^2$)	4.3-1
C_S	constant	2.3-3
C_W	constant	2.3-3
C_1, C_2, C_3 and C_4	constants	2.3-5
C_μ	constant	2.3-2
D	source term	5.6-3
E	d.c. voltage drop across hot wire	3.2-1
E	also constant in law of the wall	5.10-6
$E_1(k_1)$	u-spectrum function ($\equiv \overline{u^2} = \int_0^\infty E_1(k_1) dk_1$)	4.3-4
f	frequency	-
f^*	reference frequency in Kolmogorov-Prandtl model	-

<u>Symbol</u>	<u>Meaning</u>	<u>Equation of first appearance</u>
f_s	Strouhal frequency	-
f_t	Preferred frequency	-
$f(\dots)$	means function of	1.3-1
$f(\eta)$	non-dimensional function used by Blasius (6) ($\equiv \psi \sqrt{v_x U_\infty}$)	-
G	Parameter for slot geometry effects	3.1-1
h_{p1}, h_{p2}	width of the control volume in figure 5.2	5.8-4
H_{12}	shape factor ($\equiv R_1/R_2$)	4.3-1
I	any integer number (Fortran notation)	-
J	any integer number (Fortran notation)	-
$JSLOT, JFREE$	refer to Fortran symbols in Appendix 1	-
J_j	mass-flux in j-direction	2.4-2
k_1	wave number ($\equiv 2\pi f/U$)	4.3-4
k	Von-Karman constant	4.3-10
K	any integer number (Fortran notation)	-
K_1	a dimensionless parameter ($\equiv \frac{1}{2} \overline{q^2} \rho / \tau_w$)	-
l	a general term for any length scale	2.3-2
l_1	Prandtl's mixing length	4.3-10
l_2	turbulent length scale due to Prandtl-Kolmogorov	4.3-11
l_D	dissipation length scale	5.5-1
l_μ	Prandtl-Kolmogorov length scale	5.5-2
L	starting length of the upper lip	1.3-1

<u>Symbol</u>	<u>Meaning</u>	<u>Equation of first appearance</u>
L_y	Integral length scale ($\equiv \int_0^\infty R(0,r,0) dy$)	4.3-3
m	mass concentration of a chemically inert substance	5.8-1
M	any interger number (Fortran notation)	-
N	any interger number (Fortran notation)	-
p'	constant	5.9-3
p	fluctuating pressure	2.4-9
p_s	static pressure	2.2-2
p_o	total pressure of free stream	-
p_{atm}	atmospheric pressure	-
P_1, P_2	refers to control volume P_1 and P_2 in figure 5.2	-
$\overline{q^2}/2$	turbulent kinetic energy ($\equiv \frac{\overline{u^2 + v^2 + w^2}}{2}$)	2.3-2
q'	constant	2.3-3
r	distance between the two hot wire in correlation measurements.	4.3-1
r	also radius of curvature	2.3-5
R	Reynolds number based upon turbulent kinetic energy ($\equiv \sqrt{\frac{1}{2} \overline{q^2}} \rho y / \mu$)	5.9-2
R_1	Reynolds number based upon displacement thickness ($\equiv \frac{U_G}{\nu} \int_0^{\delta_G} (1 - u/U_G) dy$)	-
R_2	Reynolds number based upon momentum thickness ($\equiv \frac{U_G}{\nu} \int_0^{\delta_G} \frac{u}{U_G} (1 - u/U_G) dy$)	4.3-1
R_x	Reynolds number based upon x	3.1-1
R_{ec}	Slot Reynolds number ($\equiv U_C Y_C / \nu$)	1.3-1

<u>Symbol</u>	<u>Meaning</u>	<u>Equation of first appearance</u>
$R_{(0,r,)}$	correlation coefficient $(\overline{u_1 u_2} / \overline{u_1^2})$	4.3-2
S	shear work integral $(\equiv (1/\rho U_G^3) \int_0^{y_G} \rho u v (\partial U / \partial y) dy)$	-
S'	dimensionless shear stress	5.9-2
S_ω	source term for vorticity	2.4-6
S_ϕ	source term for production or depletion of ϕ	2.2-3
t	thickness of the lip	-
t	also time	4.3-8
T	temperature	3.1-1
T_{UC}	slot turbulence level $(\equiv \sqrt{u^2} / U_{C,max})$	- -
T_{UG}	free stream turbulence level $(\equiv \sqrt{u^2} / U_G)$	-
$u)$ $)$ $v)$ $)$ $w)$	fluctuating components of velocity in x, y and z directions	2.4-9
u_1	u-fluctuating velocity at fixed probe in $R_{(0,r,0)}$ measurements	4.3-2
u_2	u-fluctuating velocity for moving probe in $R_{(0,r,0)}$	4.3-2
U	time-mean velocity in x-direction	2.3-1
$\overline{U}_C$	average velocity at the slot exit $(\equiv (1/y_C) \int_0^{y_C} U dy)$	1.3-1

<u>Symbol</u>	<u>Meaning</u>	<u>Equation of first appearance</u>
U_{Cl}	velocity at $J=JSLOT-1$ and $I=1$	5.8-4
$U_{C,max}$	slot maximum velocity	-
U_{eff}	effective cooling velocity for the hot-wire	3.2-1
U_{Gl}	velocity at $J=JFREE+1$ and $I=1$	5.8-4
U_G	free stream velocity	1.3-1
U_i	U_1 , U_2 , and U_3 stand for U , V , and W when $i = 1, 2$ and 3	2.2-1
U_{max}	maximum velocity at any x-station	-
U_{min}	minimum velocity at any x-station	-
U_{ref}	$\equiv (\bar{U}_C + U_G)/2$	-
u^+	non-dimensional velocity $(\equiv U/\sqrt{\tau_w/\rho})$	5.10-6
U_∞	free stream velocity	-
V	time mean velocity in y-direction	2.3-3
W	time mean velocity in z-direction	2.3-3
x	distance measured along the wall from the slot exit and in the direction of main stream.	1.3-1
x_i	x_1 , x_2 and x_3 stand for x , y and z when $i = 1, 2$ and 3	2.2-1
y	distance normal to the wall	2.3-1
y_A	distance of the fixed probe from the wall in correlation measurements	-
y_C	slot height	1.3-1
y_G	boundary layer thickness (defined as the distance from the wall unto a point where velocity is 99% of U_G)	4.3-10

<u>Symbol</u>	<u>Meaning</u>	<u>Equation of first appearance</u>
y_{q2min}	distance from the wall to a point of minimum turbulent kinetic energy	-
$y_{uv=0}$	distance from the wall to the point of zero shear stress	-
y^+	non-dimensional distance ($\equiv y (\tau_w/\rho)^{1/2}/\nu$)	5.10-6
y_1	distance of velocity maxima from the wall	-
$y_{\frac{1}{2}}$	distance from the wall to the point where $U = (U_{max} - U_G)/2$	-
z	cross stream direction	2.4-10
<u>Greek Symbols</u>		
α	thermal diffusivity	3.1-1
δ	upper lip boundary layer thickness	-
δ^*	displacement thickness of upper lip boundary layer	-
θ	angle of the hot wire with respect to U	3.2-2
ϵ	dissipation	2.4-9
η	effectiveness ($\equiv (m_I - m_C)/(m_C - m_G)$)	1.3-1
η	also non-dimensional function used by Blasius (6) ($\equiv y \sqrt{U_\infty/\nu_x}$)	-
λ_{G2}	a constant in Prandtl-Kolmogorov model (defined on page 78 and similar to λ_{2t})	-
λ_{G1}	a constant in mixing length model	4.3-10
λ_{2t}	a constant in Prandtl-Kolmogorov model	5.10-3
μ_{eff}	effective viscosity	2.3-1

<u>Symbol</u>	<u>Meaning</u>	<u>Equation of first appearance</u>
μ	laminar viscosity	3.4-1
ν	kinematic viscosity	1.3-1
ρ	density	1.3-1
σ_1, σ_2	constants	2.3-3
$\sigma_{m,eff}$	effective Schmidt number for diffusion of ϕ	-
σ_q	constant	2.3-4
$\sigma_{\phi,eff}$	effective Prandtl of Schmidt number for diffusion of ϕ	2.4-2
$\sigma_{\omega'}$	constant	2.3-5
τ	shear stress	2.2-2
τ^*	non-dimensional shear stress	5.9-3
ϕ	temperature of mass concentration of a chemically inert substance	2.2-3
ϕ	also stands for flow variables such as ψ , ω , $\frac{1}{2}q^2$ or m	5.6-1
ψ	stream function	2.4-3
ω	vorticity	2.4-4
ω'	time-mean value of the square of the vorticity fluctuations	2.3-4

<u>Subscripts</u>	<u>Refers to</u>
ad.	adiabatic conditions
C	slot conditions
G	free stream
I	impervious wall condition
max	maximum of a variable
P	node P of a tank or cell

SubscriptsRefers to

S, E, N and W

south, east, north, and west corners around the node P

 w_1, w_2

lip corners in figure 5.2

w

wall condition

Overbars, etc.- (e.g. $\overline{u^2}$)temporal mean of a time dependant quantity
also stands for average ($\overline{U_C}$) of a quantity
which is not a function of time.

TABLE 1: SUMMARY OF PREVIOUS EXPERIMENTAL INVESTIGATIONS

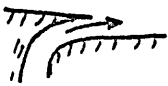
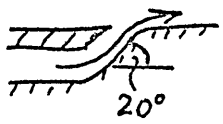
No.	Author	Reference	Range of Variables										Slot Geometry	Comments
			$\bar{U}_C/U_G$	δ	t	y_C	R_{ec}	T_{UC}	T_{UG}	dp/dx	x/y_C	$T_C - T_G$		
1	J. H. Chin et.al.	16	0.22 to 3.3	0.8" and 2.0"	knife edge ≈ 0	0.106"	1400 to 8200	-	-	-	up to 233	-89°F to 101°F		η for hot and cold injection can be correlated by same equations.
2	R. A. Seban	92	0.27 to 0.76 (mass velocity)	.06" and 0.6"	0.02"	.063"	930 to 2830	-	-	-	up to 300	70°F		Small effect of δ on η has been reported.
3	R. A. Seban	93	0.17 to 20.70	.06"	.02"	.063", .125" and 0.25"	620 to 6960	-	-	-	up to 300	70°F	Same as in reference 92	No attempt was made to separate out the influence of y_C on η
4	J. P. Hartnett et.al.	39	0.28 to 1.1	0.20"	knife edge	0.123"	2400 to 8500	-	-	three pressure gradients tried	up to 270	100°F		(1) Slot geometry similar to Wieghardt (116). (2) Small influence of dp/dx on η noted
5	A. E. Samuel and P. N. Joubert	89	0.25 to 3.18	$\approx 0.8"$	knife edge	0.115", 0.25" and 0.312"	Not given	-	-	-	up to 275	Not given		Same comments as for reference 93.

TABLE 1 (Continued)

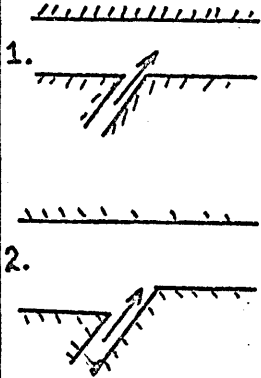
No.	Author	Reference	Range of Variables										Slot Geometry	Comments
			$\bar{U}_C/U_G$	δ	t	Y_C	R_{ec}	T_{UC}	T_{UG}	dp/dx	x/Y_C	$T_C - T_G$		
6	L. W. Carlson and E. Talmor	14	0.17 to 0.67	Not given	knife edge	0.062"	1000 to 6000 (estimated)	-	2% to 22%	Favourable to pressure gradient	128	930°F	 1. 2.	(1) Angle of injection 20°. (2) Two dimensional channel only $\frac{1}{2}$ " wide (3) Appreciable density gradients present.
7	B. R. Pai and J. H. Whitelaw	68 69	0.3 to 3.1	0.25"	knife edge	0.1"	70 to 14000	-	-	Favourable and adverse pressure gradients	212	-		)))) Experiments are mostly concerned with density gradients))
8	W. K. Burns and J. L. Stollery	12	0.53 to 4.0	0.06" and 0.16"	0 and 0.125"	0.062"	230 to 17450	-	-	-	500	-	Similar to reference 68 and 69	))))))
9	S. Sivasegaram and J. H. Whitelaw	98	0.5 to 5.0	Not given	0.16" and 0.34"	0.2"	5000 to 15000	-	0.7%	0	100	Not given	Similar to reference 92	(1) Non-tangential injection gives lower effectiveness. (2) Thicker lip causes faster drop of η .

TABLE 1 (Continued)

No.	Author	Reference	Range of Variables										Slot Geometry	Comments
			$\bar{U}_C/U_G$	δ	t	y_C	R_{ec}	T_{UC}	T_{UG}	dp/dx	x/y_C	$T_C - T_G$		
10	Present study	-	0.4 to 3.0	0.18" to 0.91"	0.032" to 0.48"	0.063", 0.125", 0.25" and 0.50"	980 to 23000	2.3% to 9.5%	0.5% to 7.5%	0	200	-	Similar to reference 92	-

TABLE 2: SUMMARY OF FLOW CONDITIONS FOR WHICH PREDICTIONS ARE COMPARED WITH EXPERIMENTAL RESULTS

$\bar{u}_C/u_G$	0.75		0.91		1.33		2.3	
t/y_C	0.13	1.14	0.13	1.14	0.13	1.14	0.13	1.14
U at x/y_C of	10,20	10,20	10,20	10,20	10,20	10,20	10,20	10,20
$\overline{q^2}$ at x/y_C of	10,20	10,20	10,20	10,20	10,20	10,20	10,20	10,20
m_j at x/y_C of	-	10,20	-	10,20	-	10,20	10,20	10,20
cf/2 at s/y_C of	10,20	10,20	10,20	10,20	10,20	10,20	10,20	10,20
η at x/y_C of	4,12,20	4,12,20	4,12,20	4,12,20	4,12,20	4,12,20	4,12,20	4,12,20

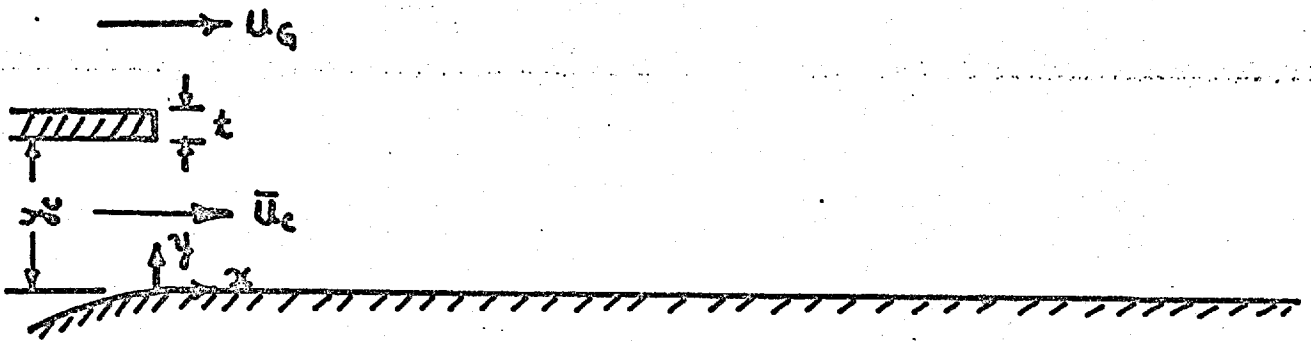


FIG. 1.1 A SCHEMATIC REPRESENTATION OF THE PROBLEM

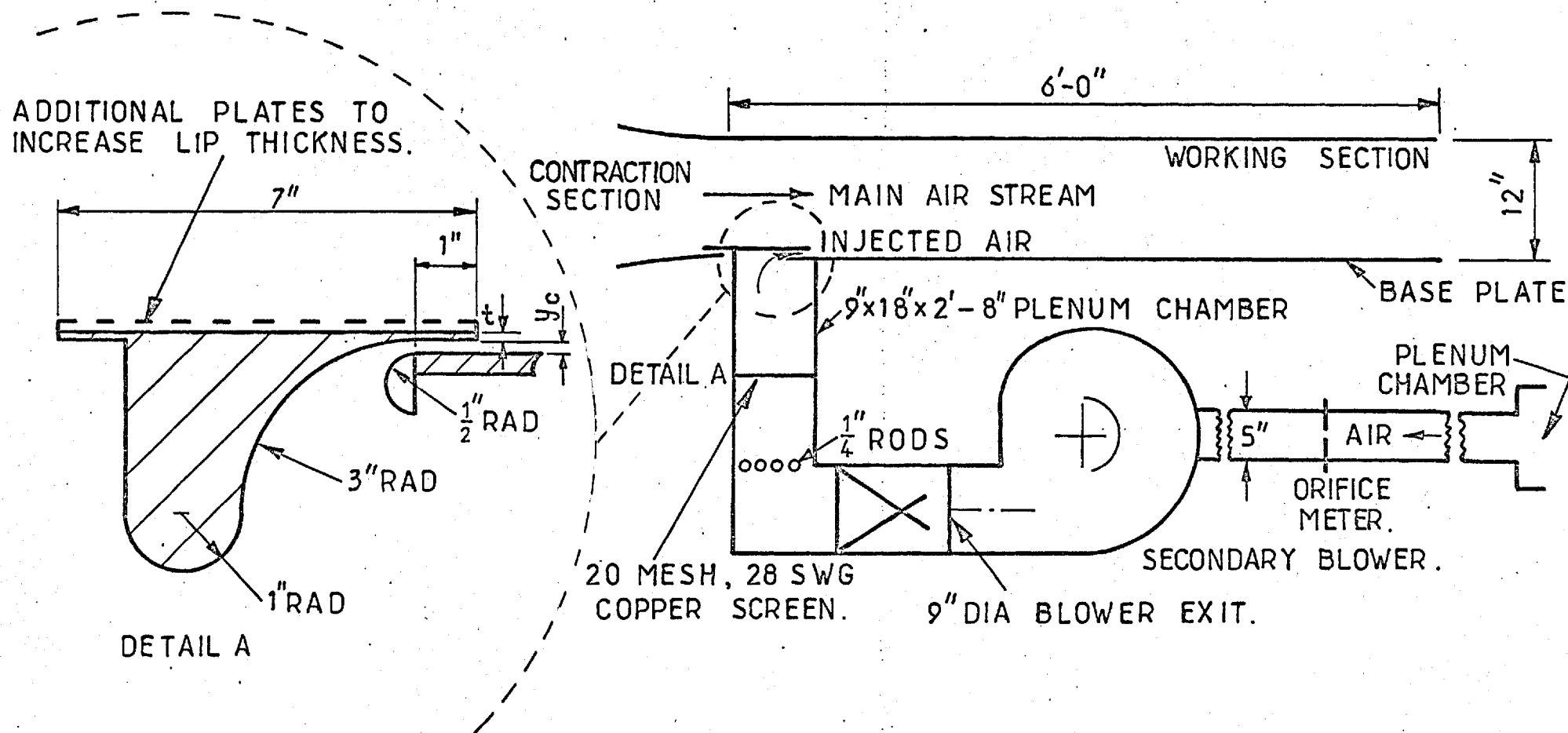


FIG. 3.1 GENERAL ARRANGEMENT OF WORKING SECTION AND SLOT CONFIGURATION.

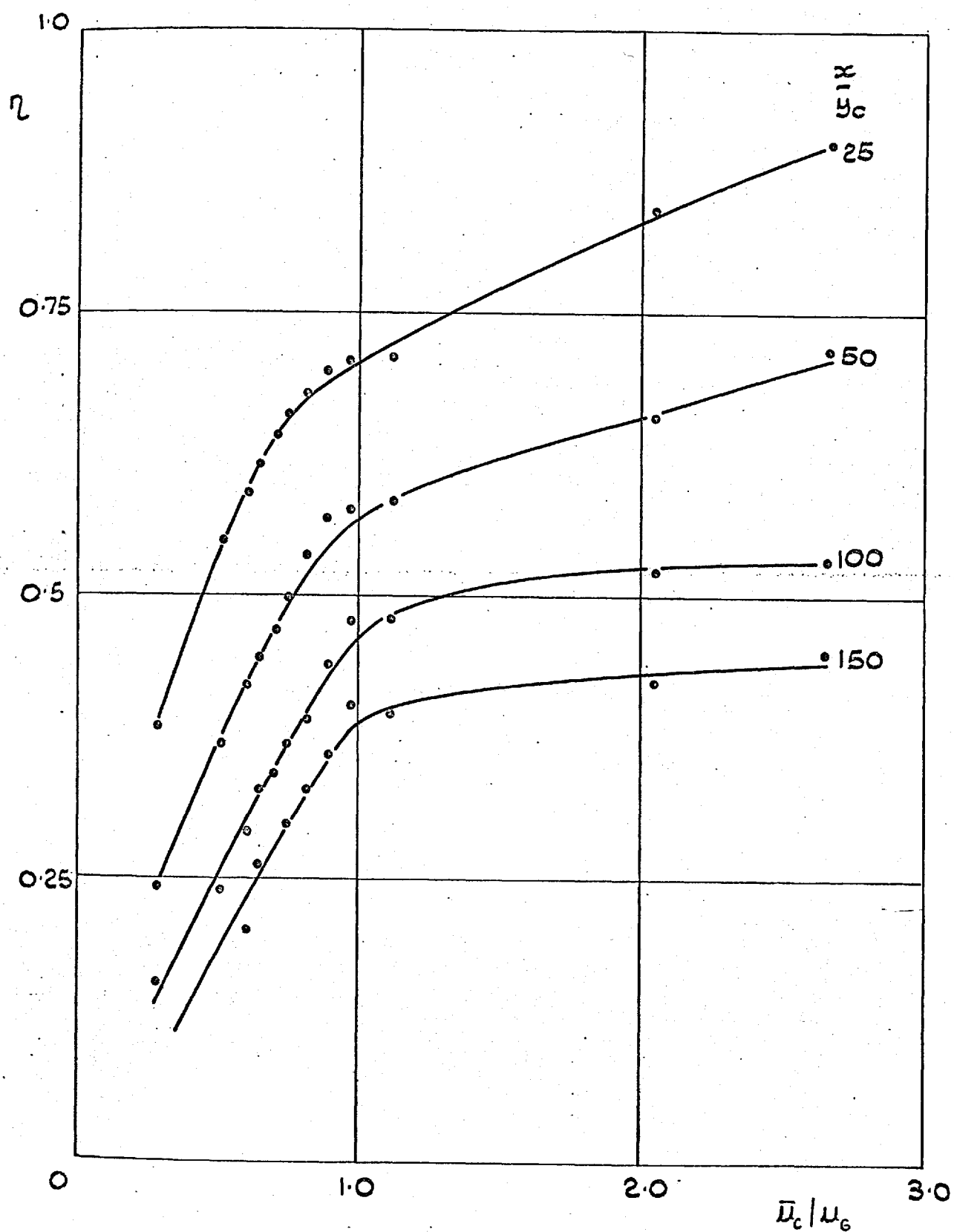


FIG.3.2 IMPERVIOUS WALL EFFECTIVENESS — 0.074 INCH SLOT

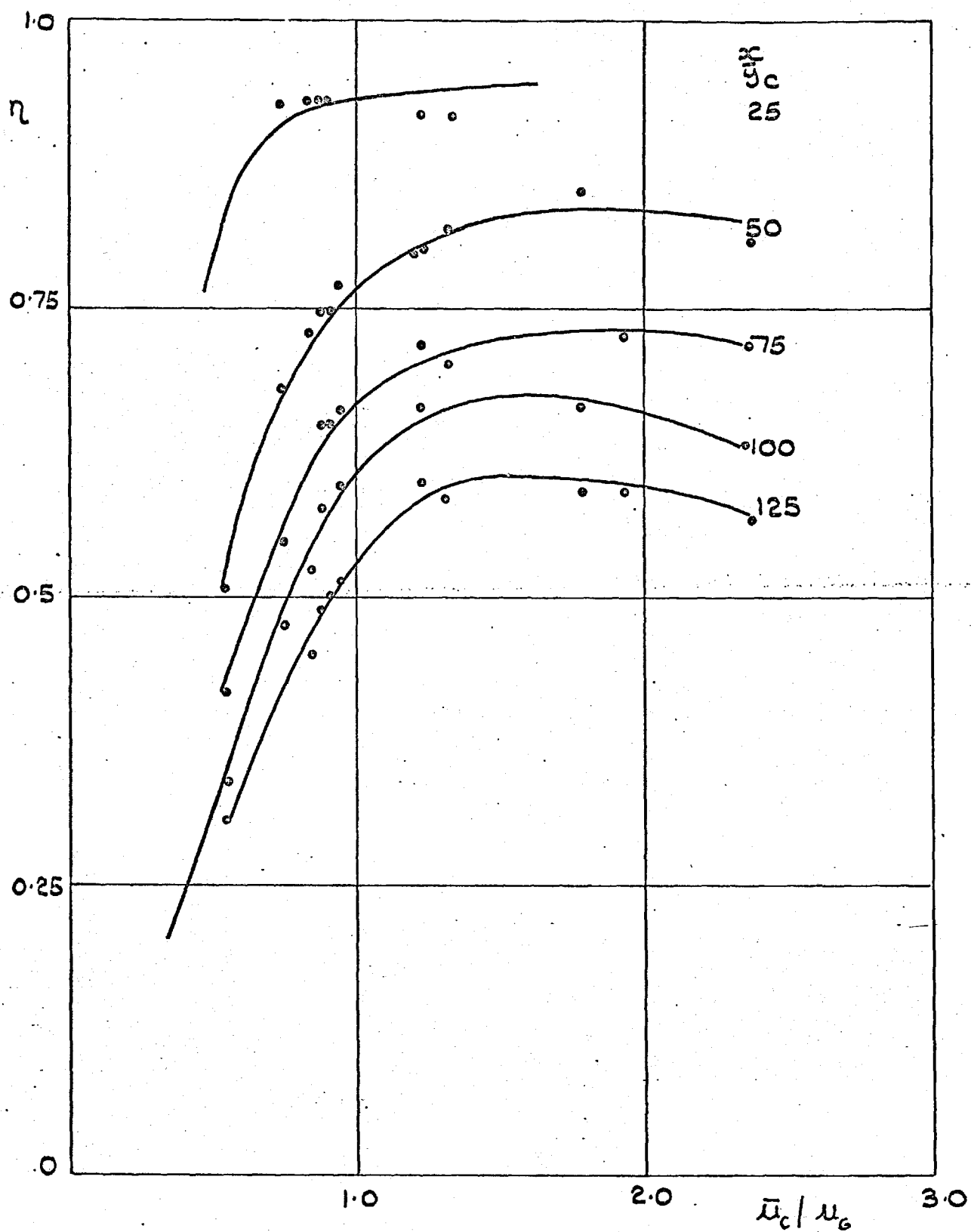


FIG. 3.3 IMPERVIOUS WALL EFFECTIVENESS — 0.132 INCH SLOT

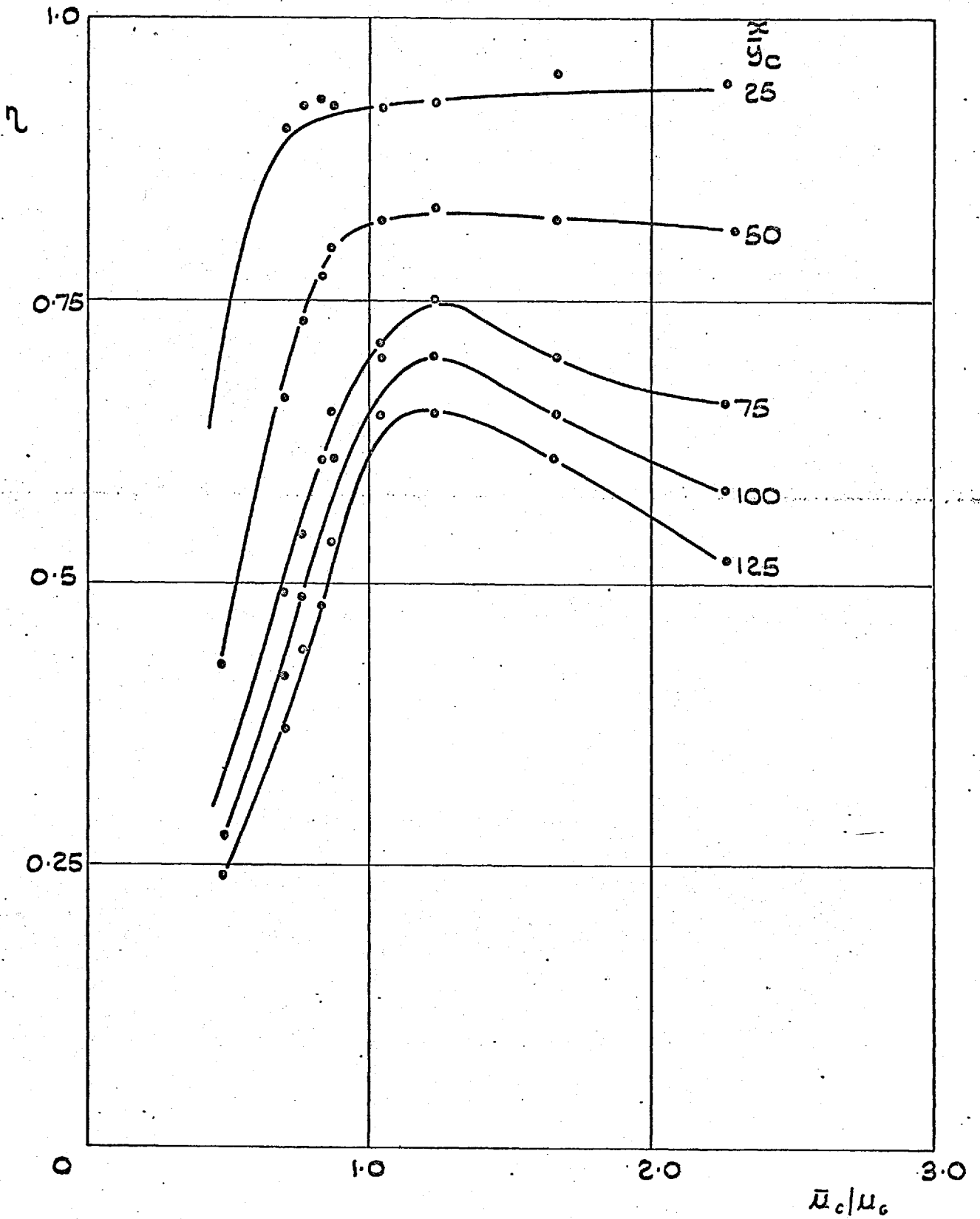


FIG. 3.4 IMPERVIOUS WALL EFFECTIVENESS — 0.25 INCH SLOT.

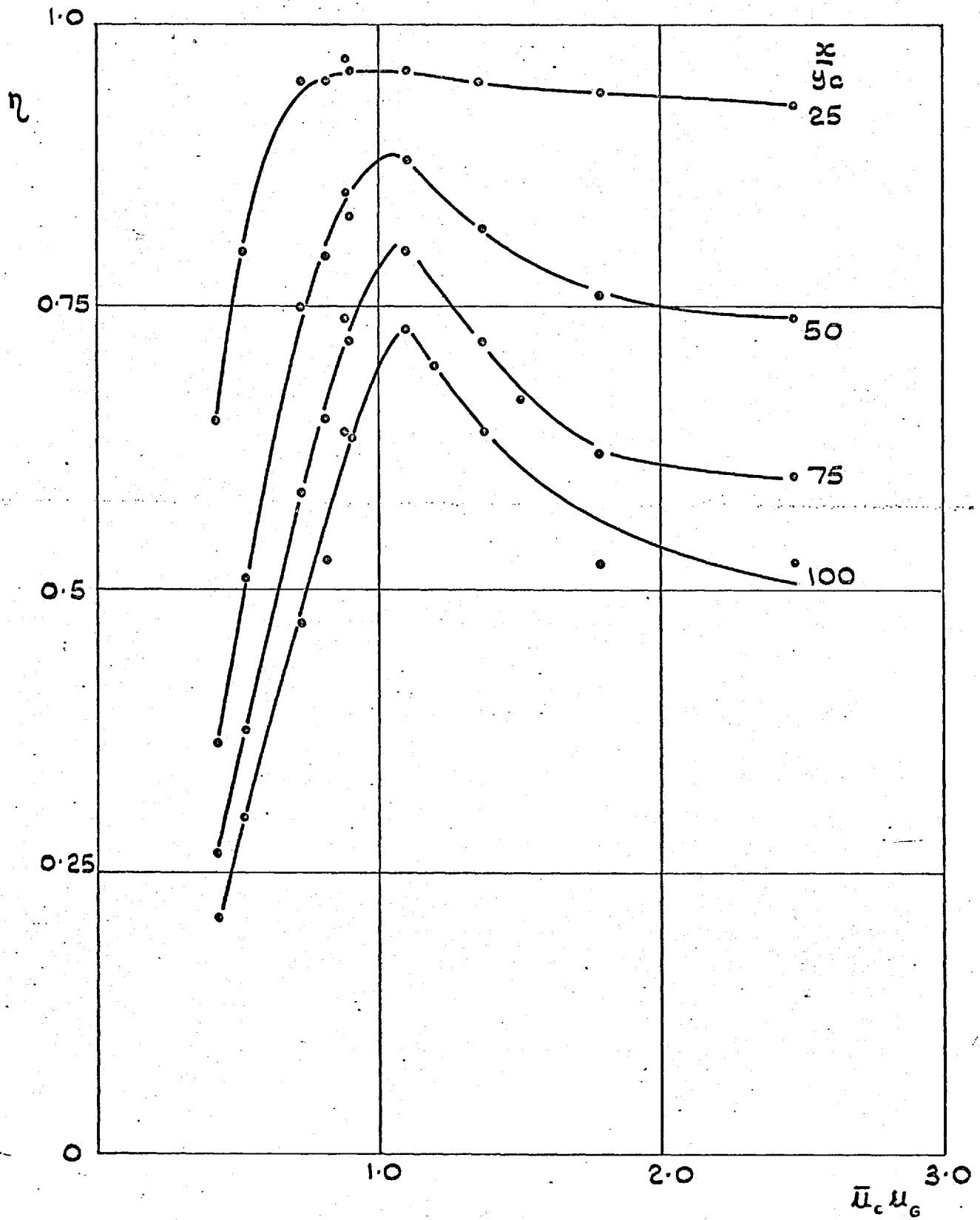


FIG.3.5 IMPERVIOUS WALL EFFECTIVENESS — 0.50 INCH SLOT

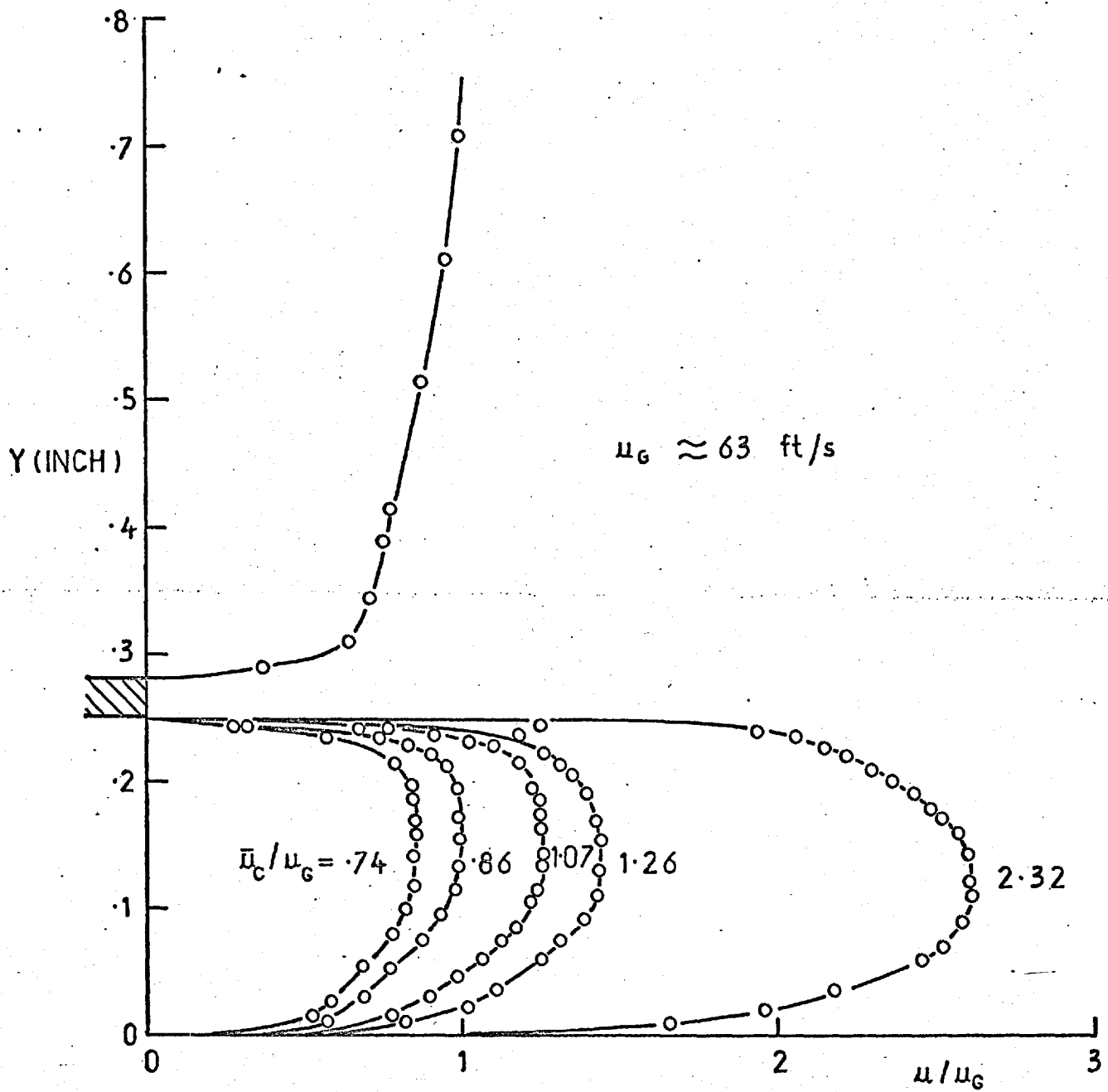


FIG. 3.6 MEASURED VELOCITY PROFILES AT THE SLOT EXIT.
 $y_c = 0.25''$; $t = 1/32''$

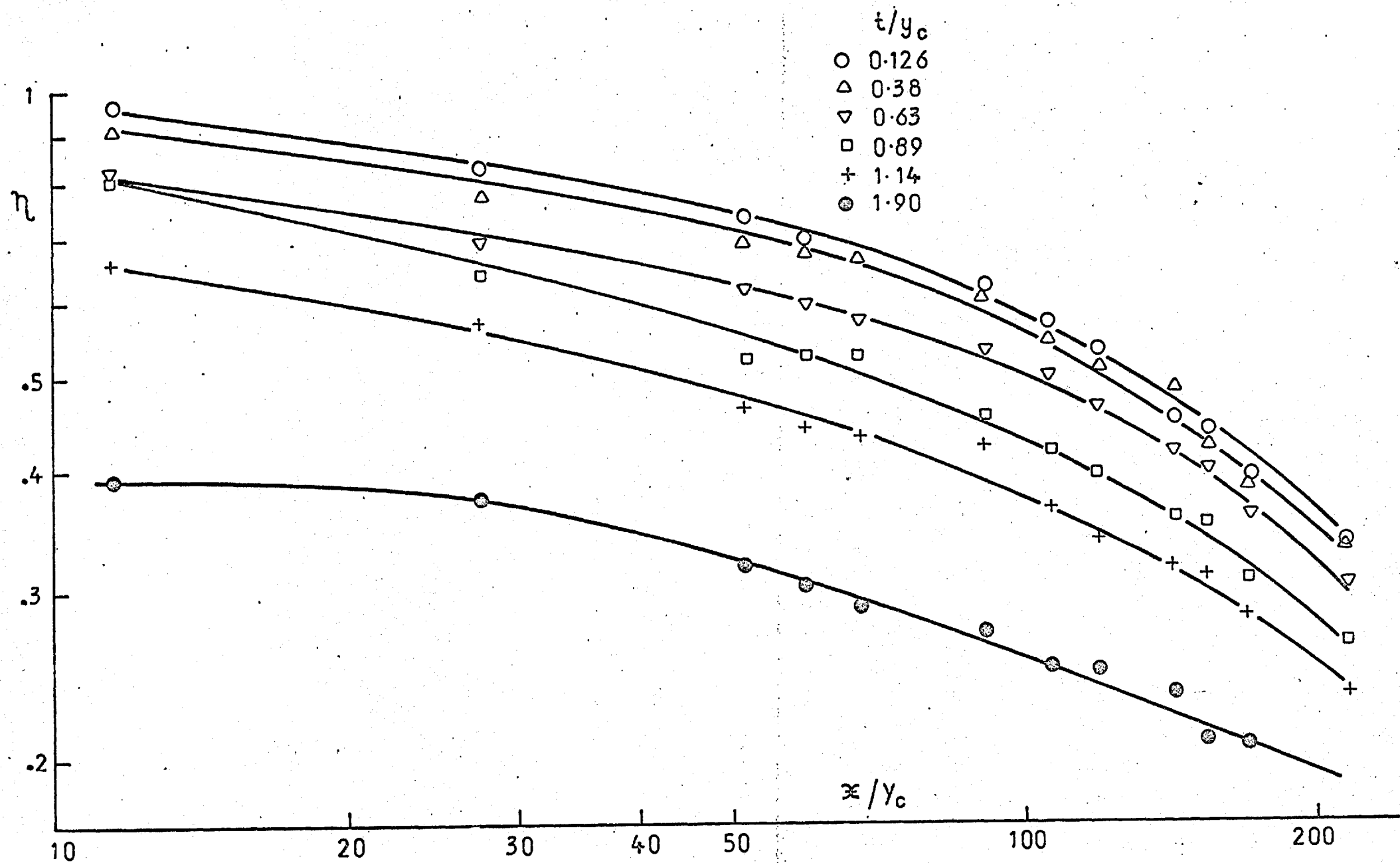
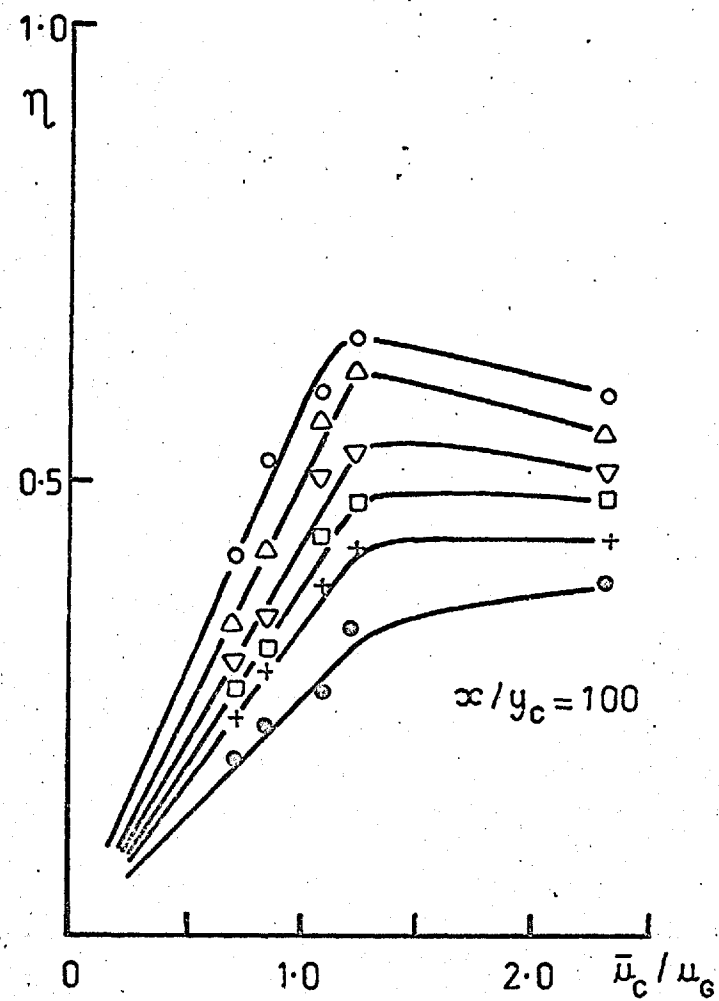
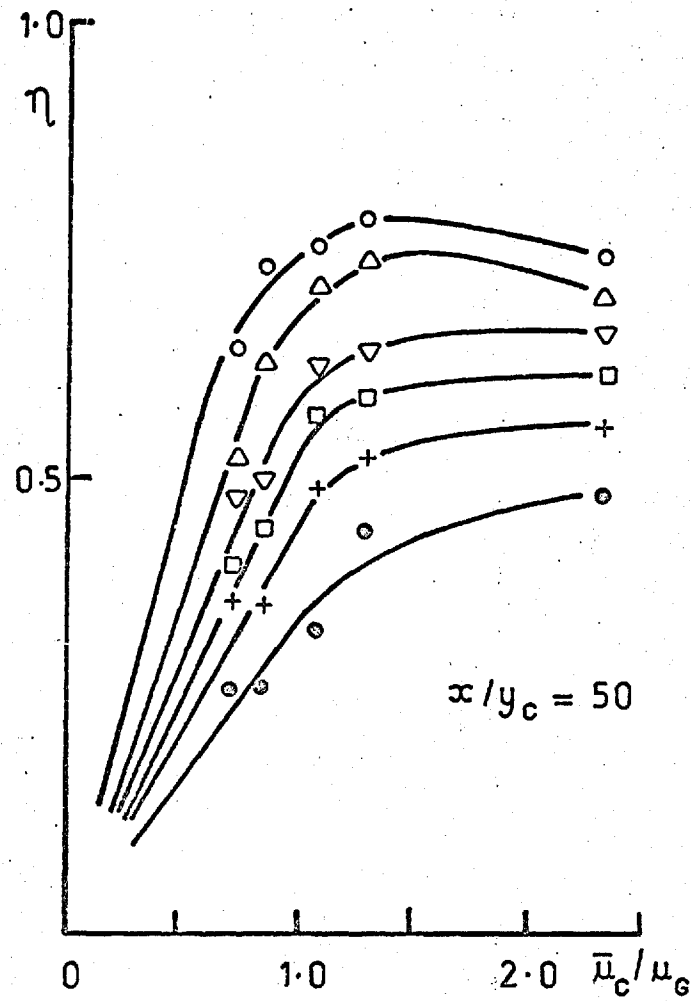
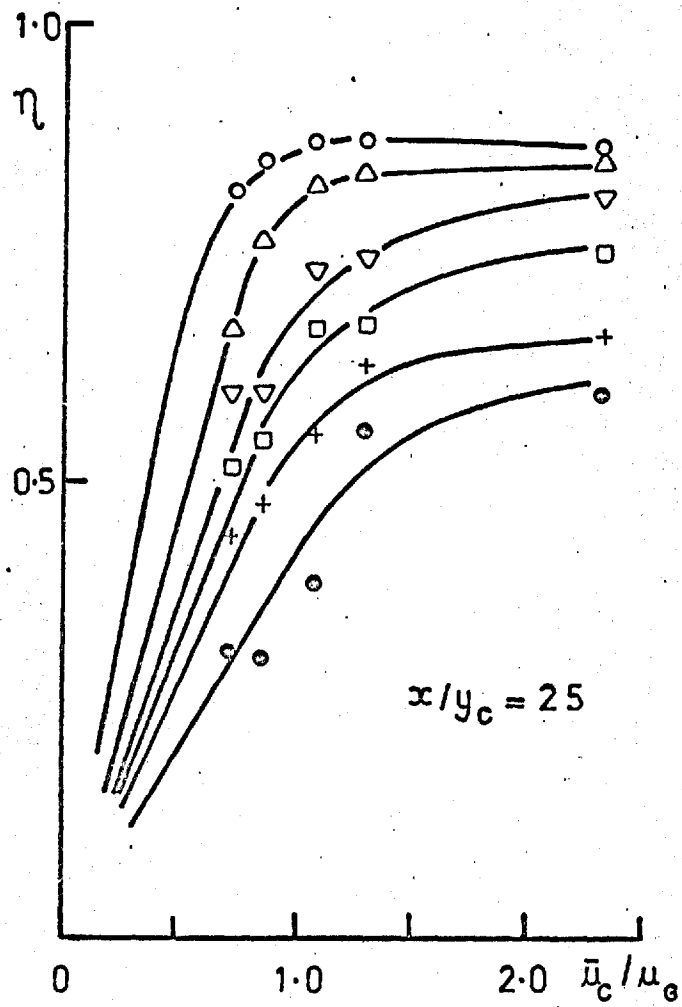


FIG. 3.7 IMPERVIOUS - WALL EFFECTIVENESS ; $\bar{\mu}_c/\mu_c = 1.07$



t/y_c	t/y_c
○ 0.126	□ 0.89
△ 0.38	+ 1.14
▽ 0.63	● 1.90

FIG. 3.8 IMPERVIOUS WALL EFFECTIVENESS AT $x/y_c = 25, 50$ AND 100 ; $y_c = 0.247''$

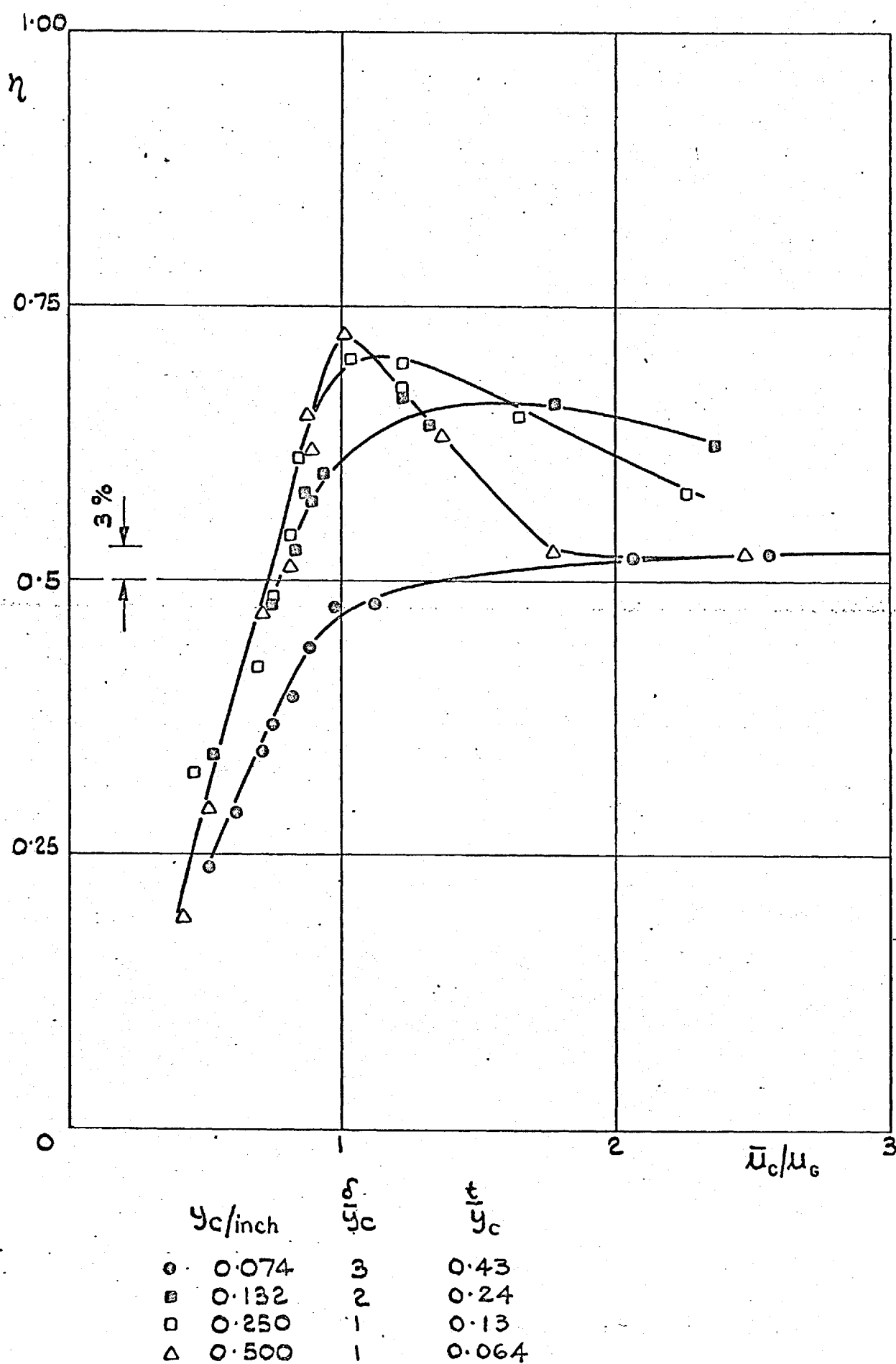


FIG. 3.9 IMPERVIOUS WALL EFFECTIVENESS AT A DISTANCE OF 100 SLOT HEIGHTS DOWNSTREAM.

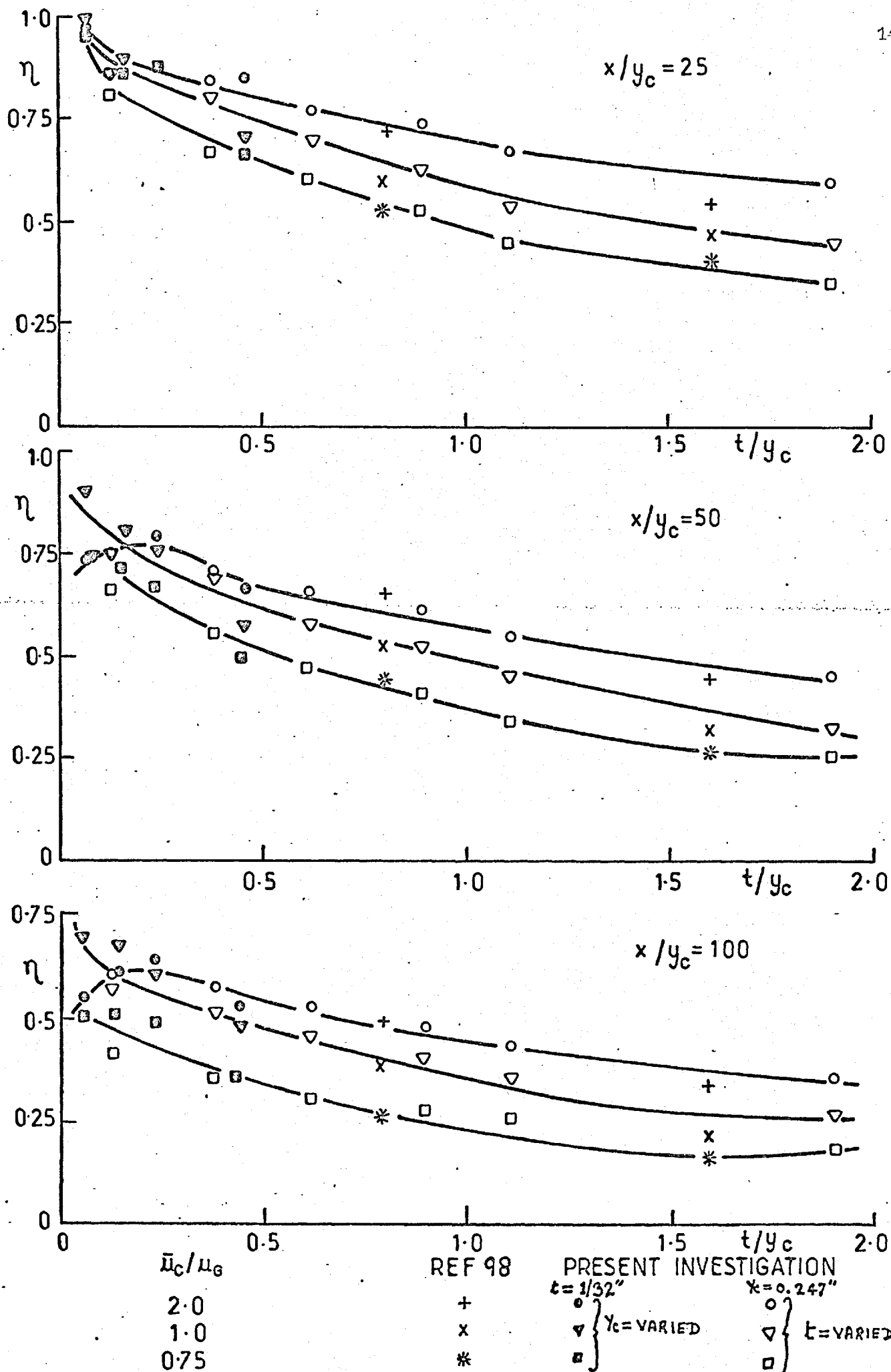
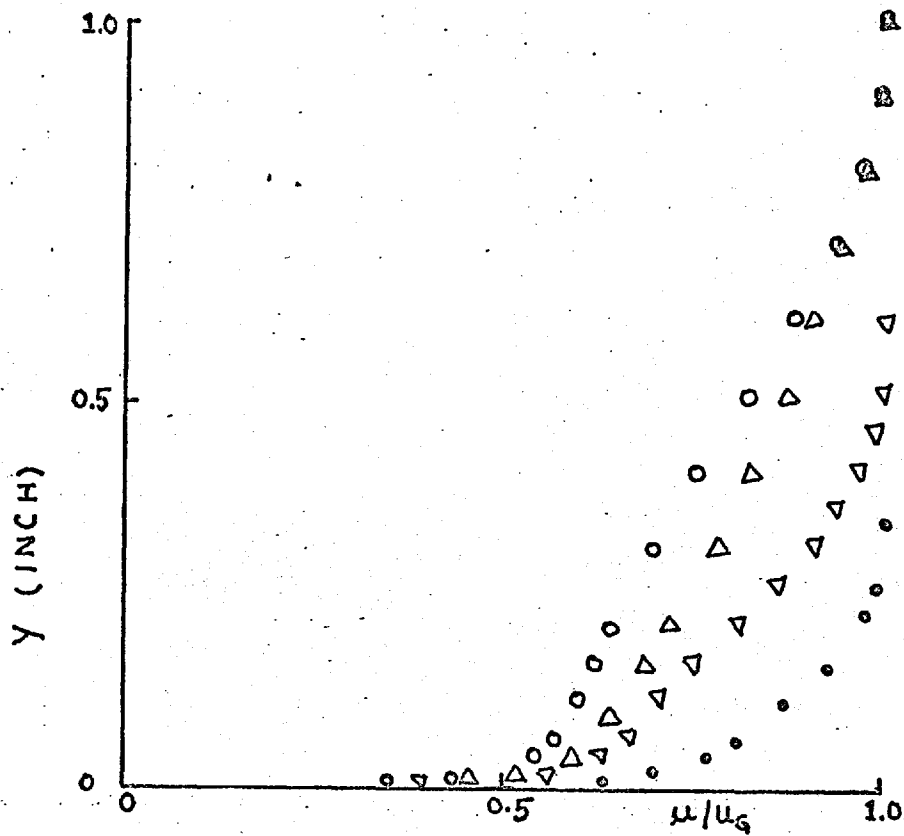


FIG.3.10 COMPARISON OF RESULTS OF REFERENCE 98 AND OF THE PRESENT INVESTIGATION.

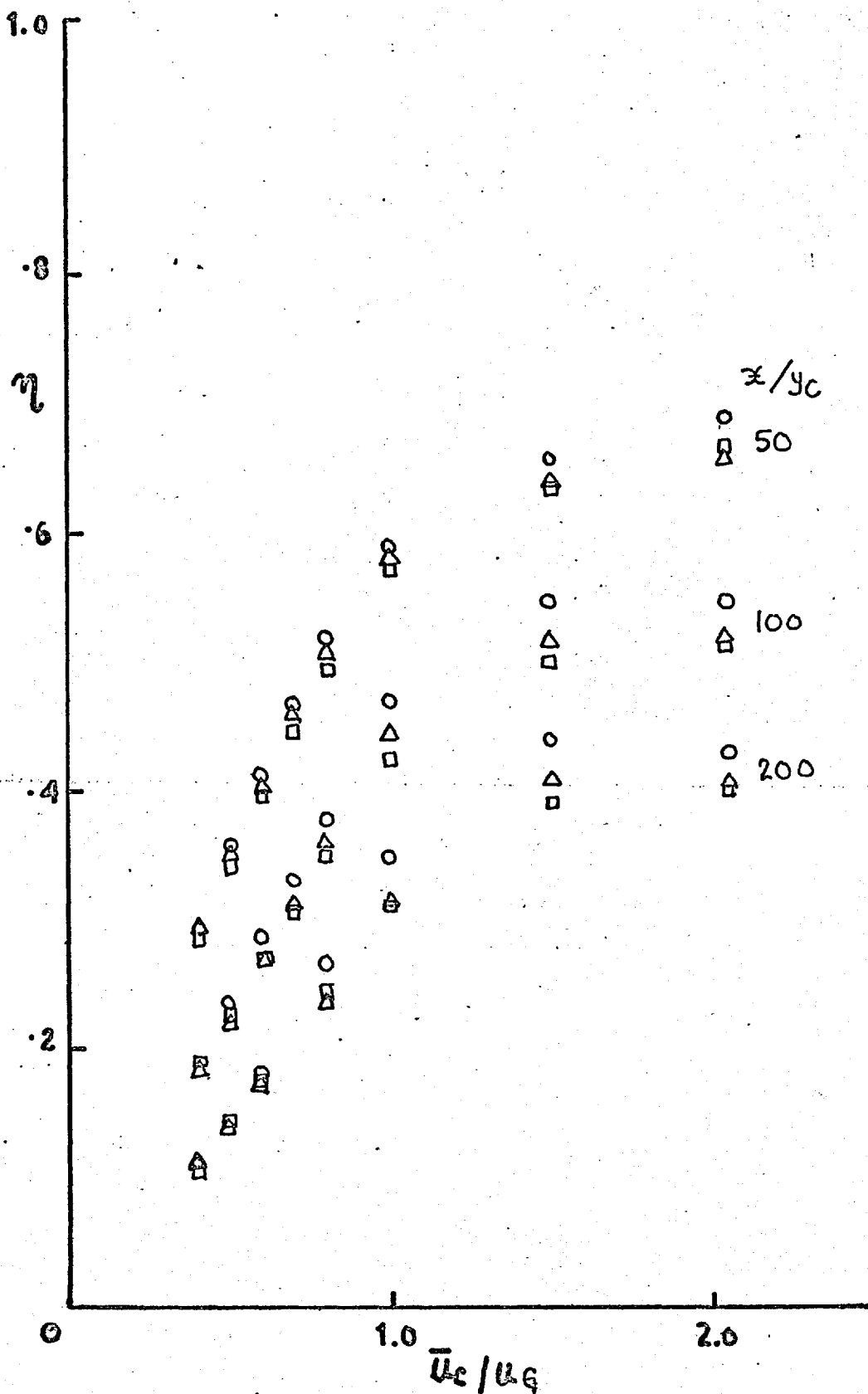
FIG. 3.11 TYPICAL UPPER LIP BOUNDARY LAYER PROFILES; $y_c = 0.074''$;
 $t = 1/32''$



SYMBOL	δ''	δ'''	CODE NO.	METHOD OF OBTAINING
○	0.26	.034	BL 1	NATURAL
▽	0.5	.087	BL 2	SAND PAPER UPSTREAM OF THE SLOT
△	0.91	.164	BL 3	SAND PAPER WITH 1/4" DIA. ROD AT $x = -12''$
○	0.91	.202	BL 4	SAND PAPER WITH 1/4" DIA. ROD AT $x = -7''$

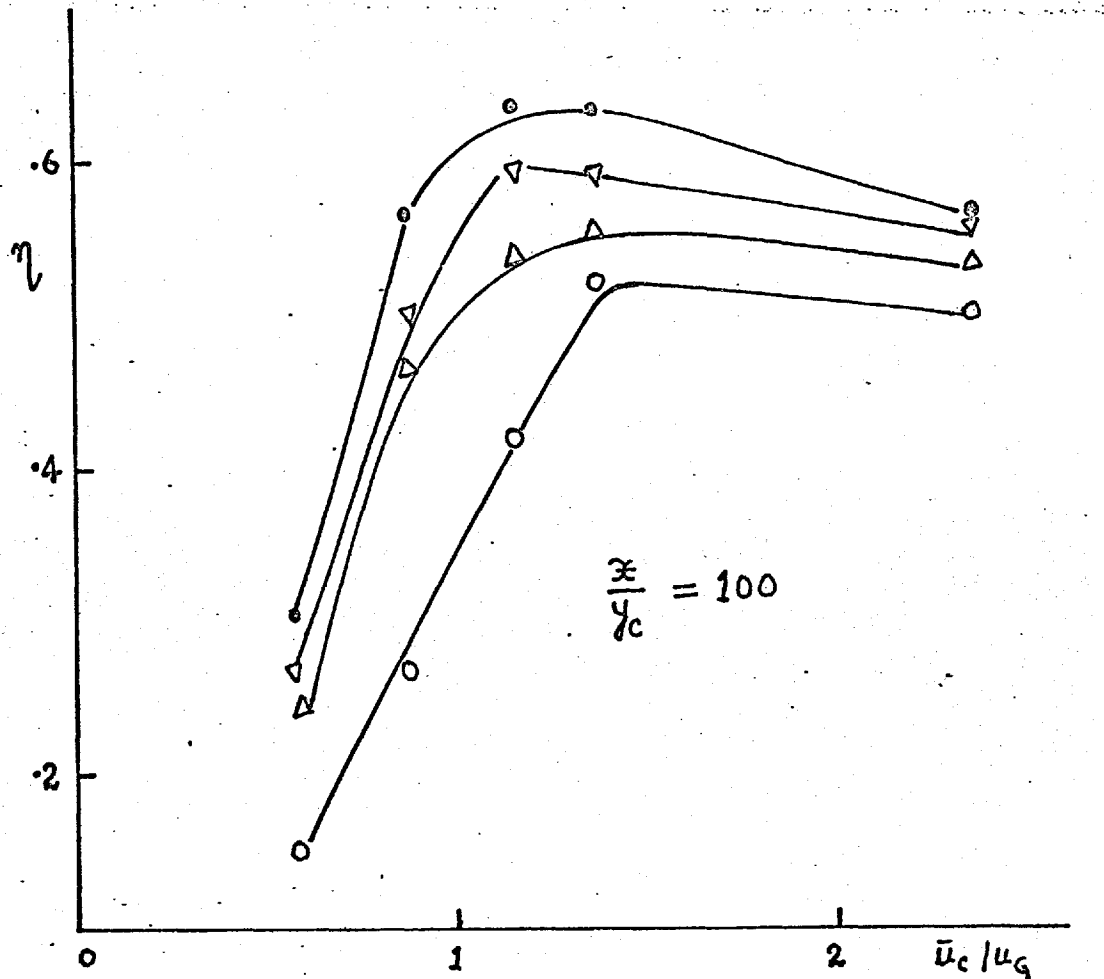
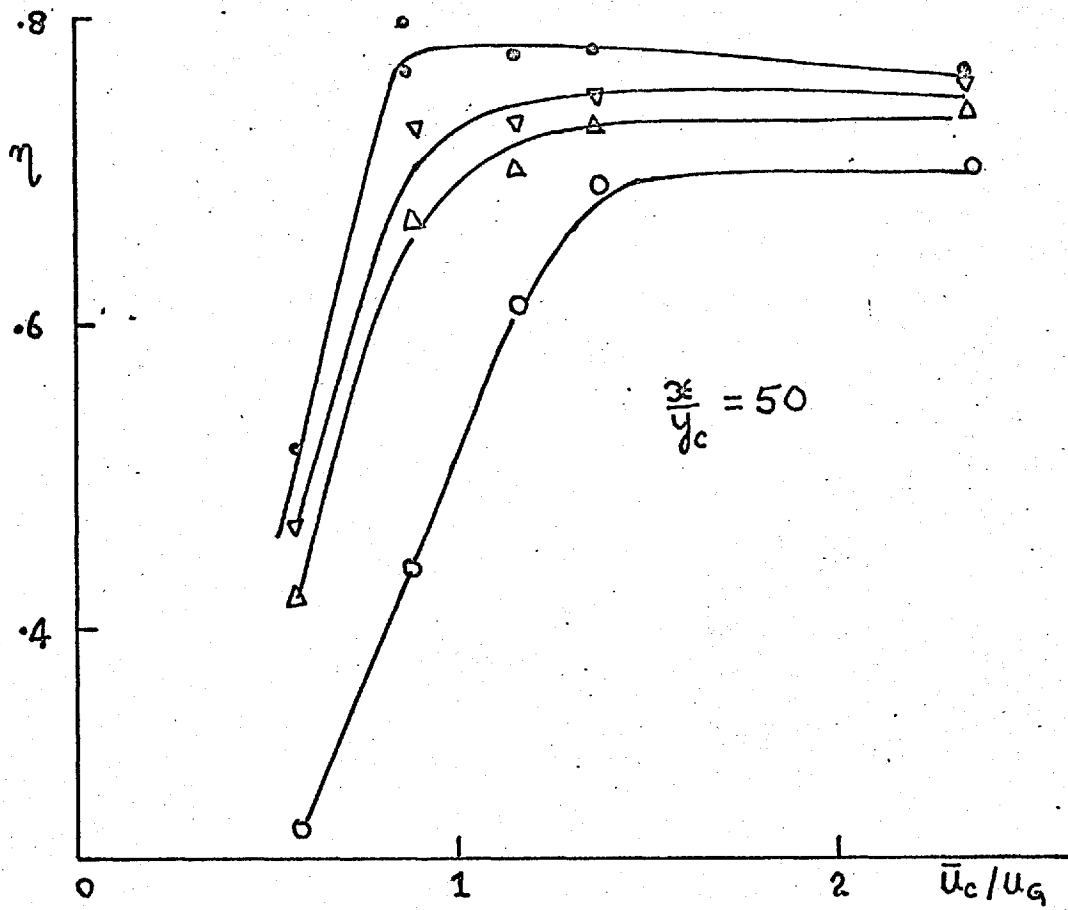
FIG.3.12 UPPER-LIP BOUNDARY LAYER PROFILES; $\gamma_c = 0.25''$,

$$t = 1/32''$$



$\circ$	δ	δ / y_c
$\square$	0.18"	2.4
$\triangle$	0.36"	4.8
	0.75"	10.2

FIG. 3.13 MEASURED VALUES OF η ; $y_c = 0.074$ "; $t = 1/32$ "



UPPER LIP BOUNDARY LAYER

- BL1
- ▽ BL2
- △ BL3
- BL4

FIG.3.14 EFFECTIVENESS AT $x/y_c = 50$ AND 100 ; $y_c = 0.25''$; $t = 1/32''$

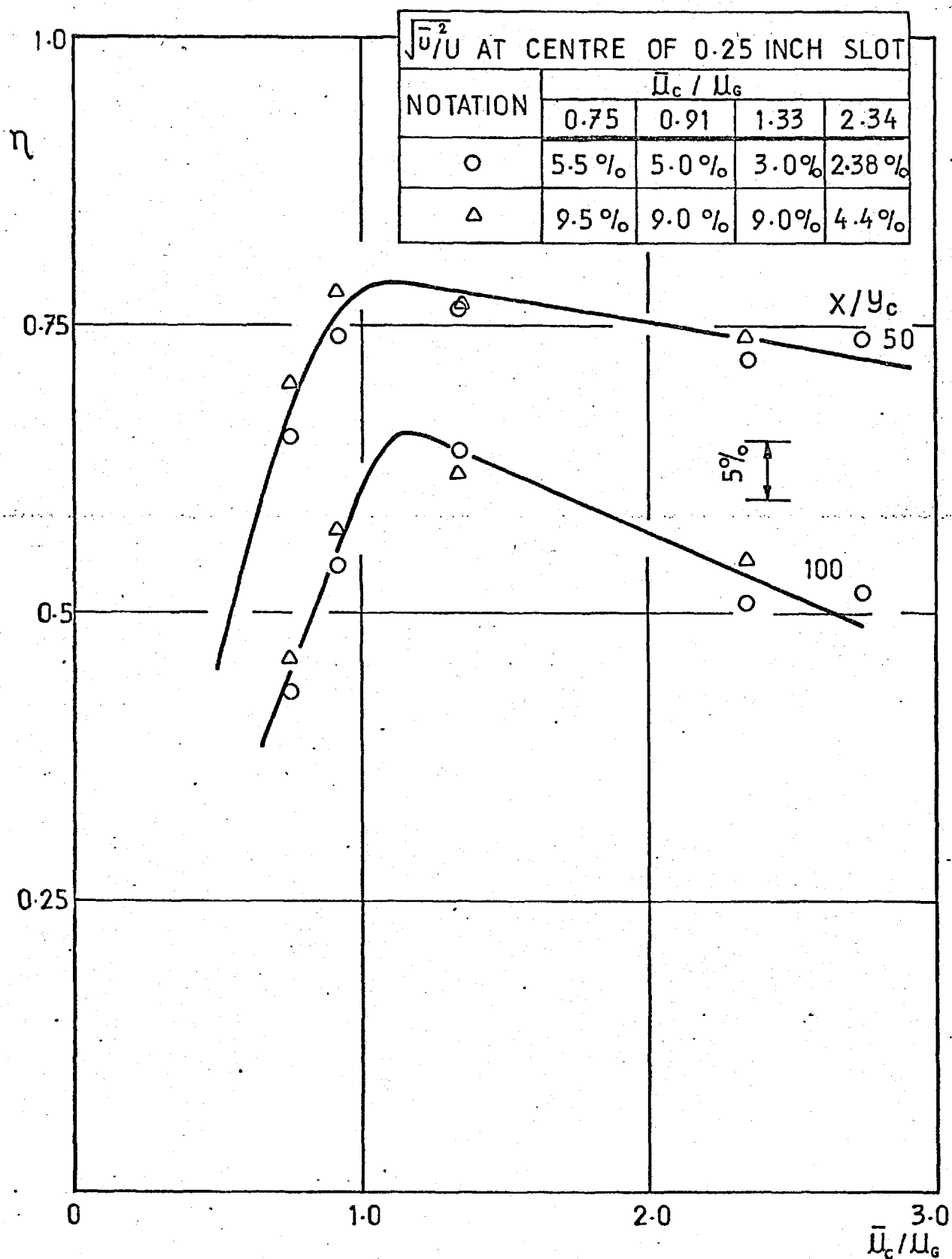


FIG.3.15 THE INFLUENCE OF SLOT-TURBULENCE-INTENSITY ON THE IMPERVIOUS-WALL EFFECTIVENESS; $t=1/32''$

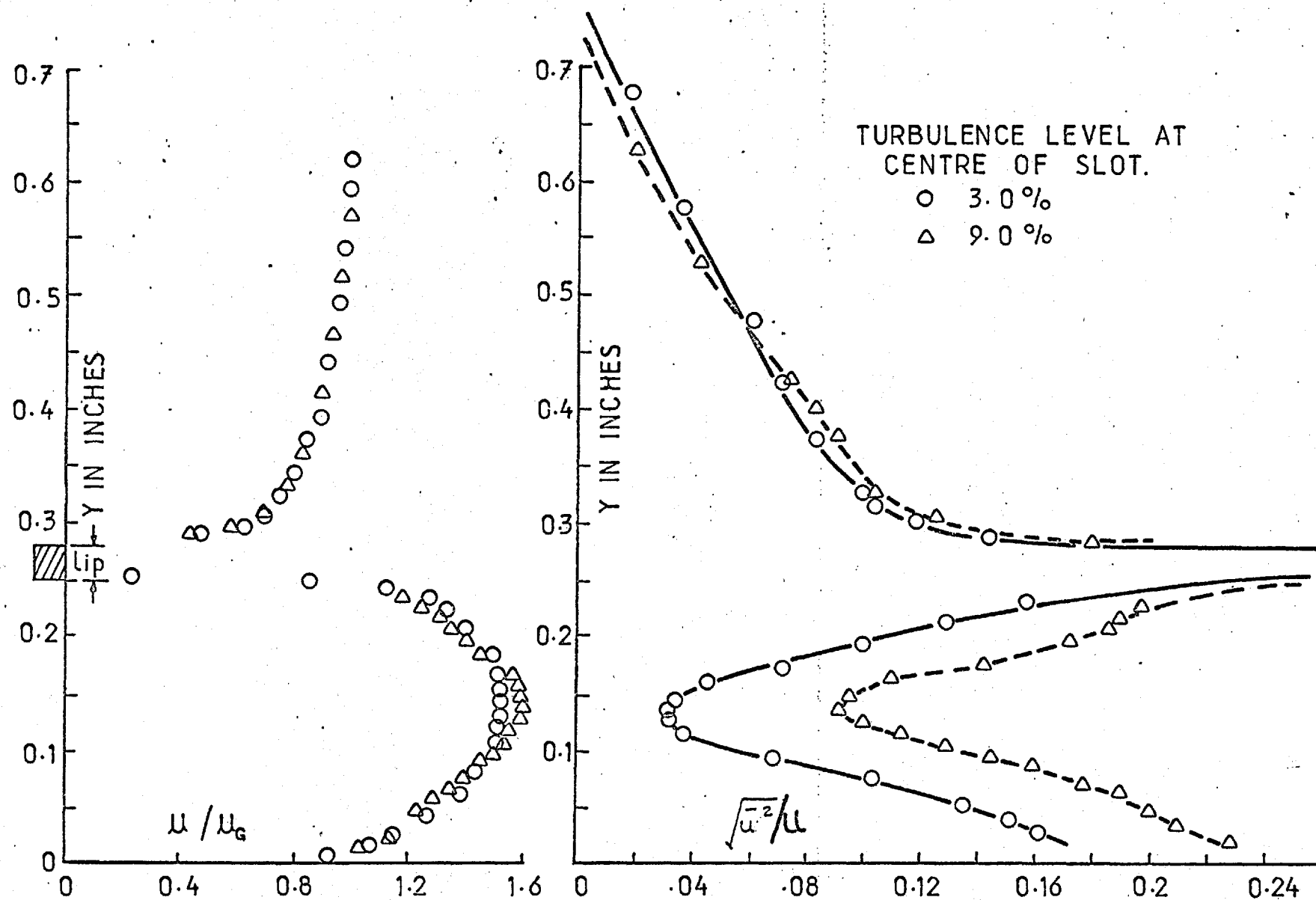
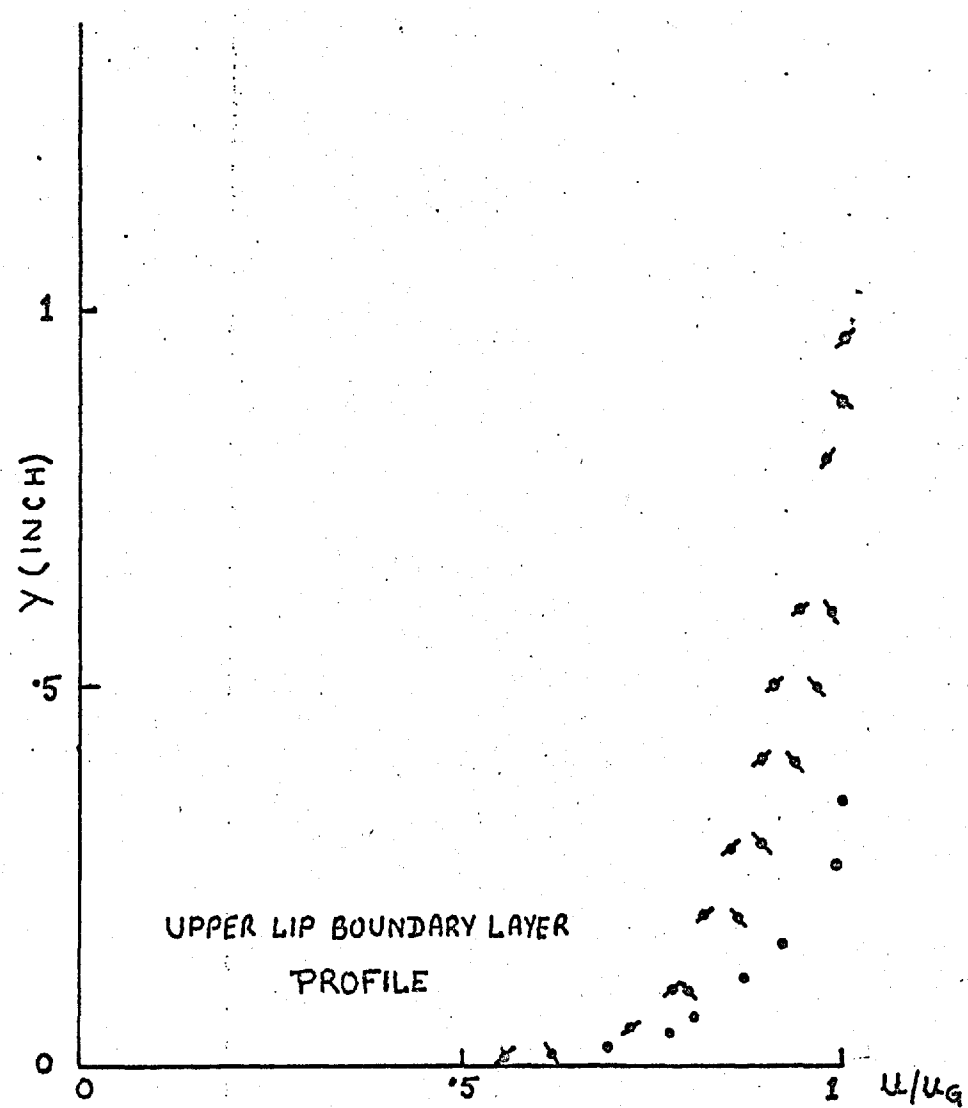
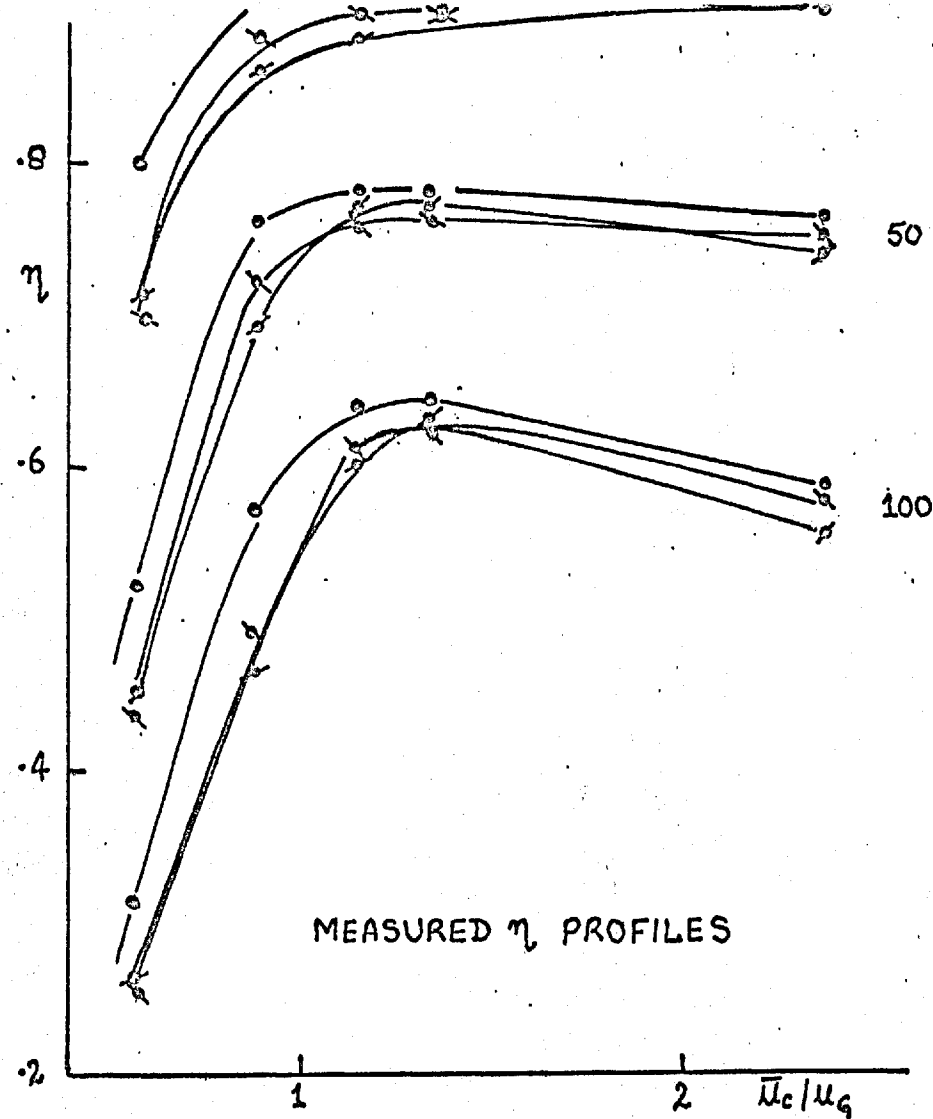


FIG.3.16 MEAN VELOCITY AND TURBULENCE-INTENSITY PROFILES AT $x/y_c = 0$; $\bar{u}_c / u_c = 1.33$;
 $t = 1/32''$; $y_c = 0.25''$



SYMBOL	δ''	$\delta^{*''}$	CODE NO.	METHOD OF OBTAINING	FREE STREAM TURBULENCE LEVEL
—○—	.26	.034	BL1	NATURAL	.005
—▽—	.65	.068	BL5	GRID NO.3 AT $x=-7''$	.035
—□—	.85	.105	BL6	GRID NO.2 AT $x=-7''$	.07

FIG.3.17 INFLUENCE OF TURBULENCE PROMOTING GRIDS ON η ; $y_c=0.25''$; $t=1/32''$

FREE STREAM TURBULENCE LEVEL	SYMBOL
0.005	○
0.035	⊗
0.07	⊠

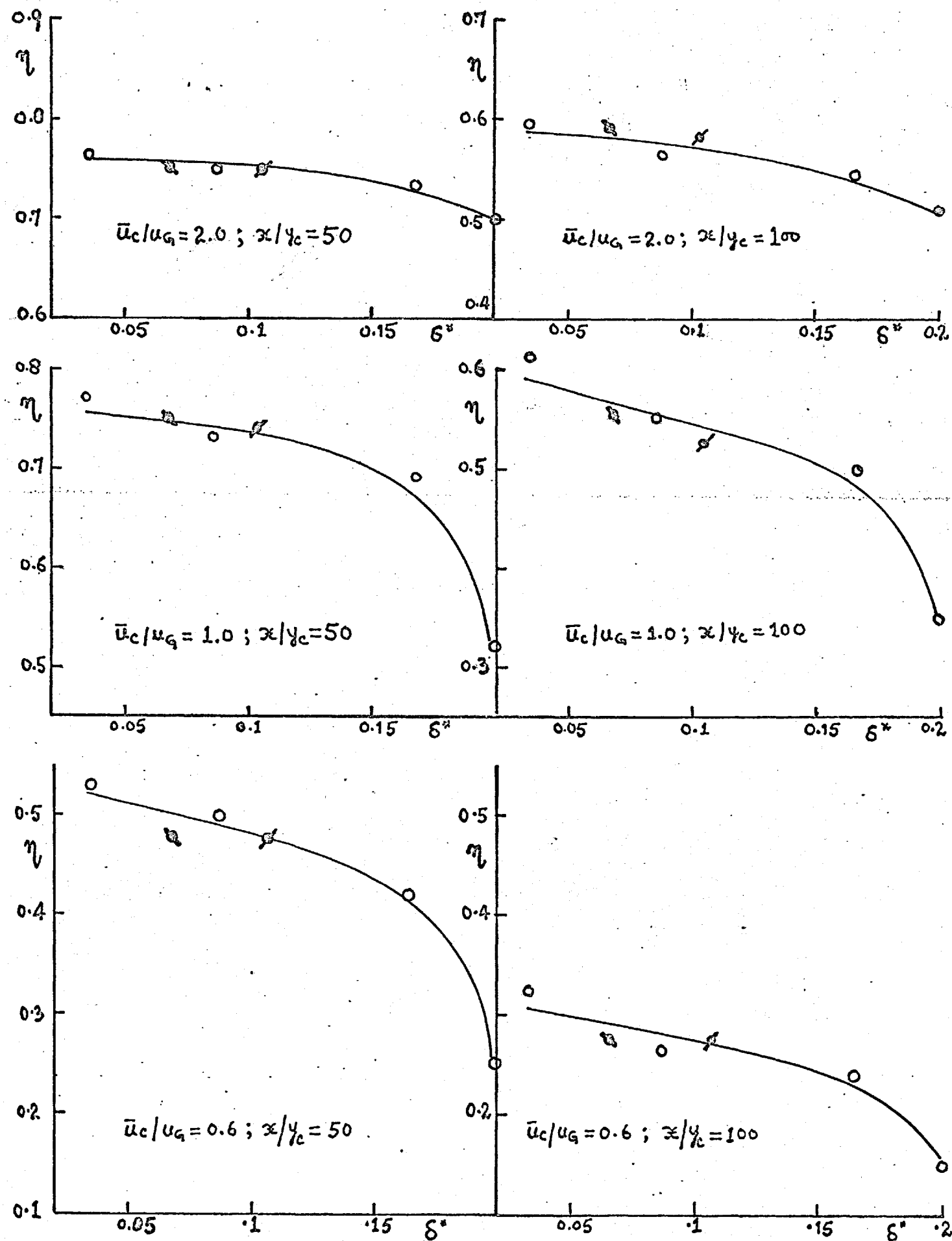


FIG. 3.18 EFFECTIVENESS VS. DISPLACEMENT THICKNESS; $\gamma_c = 0.25$
 $t = 1/32''$

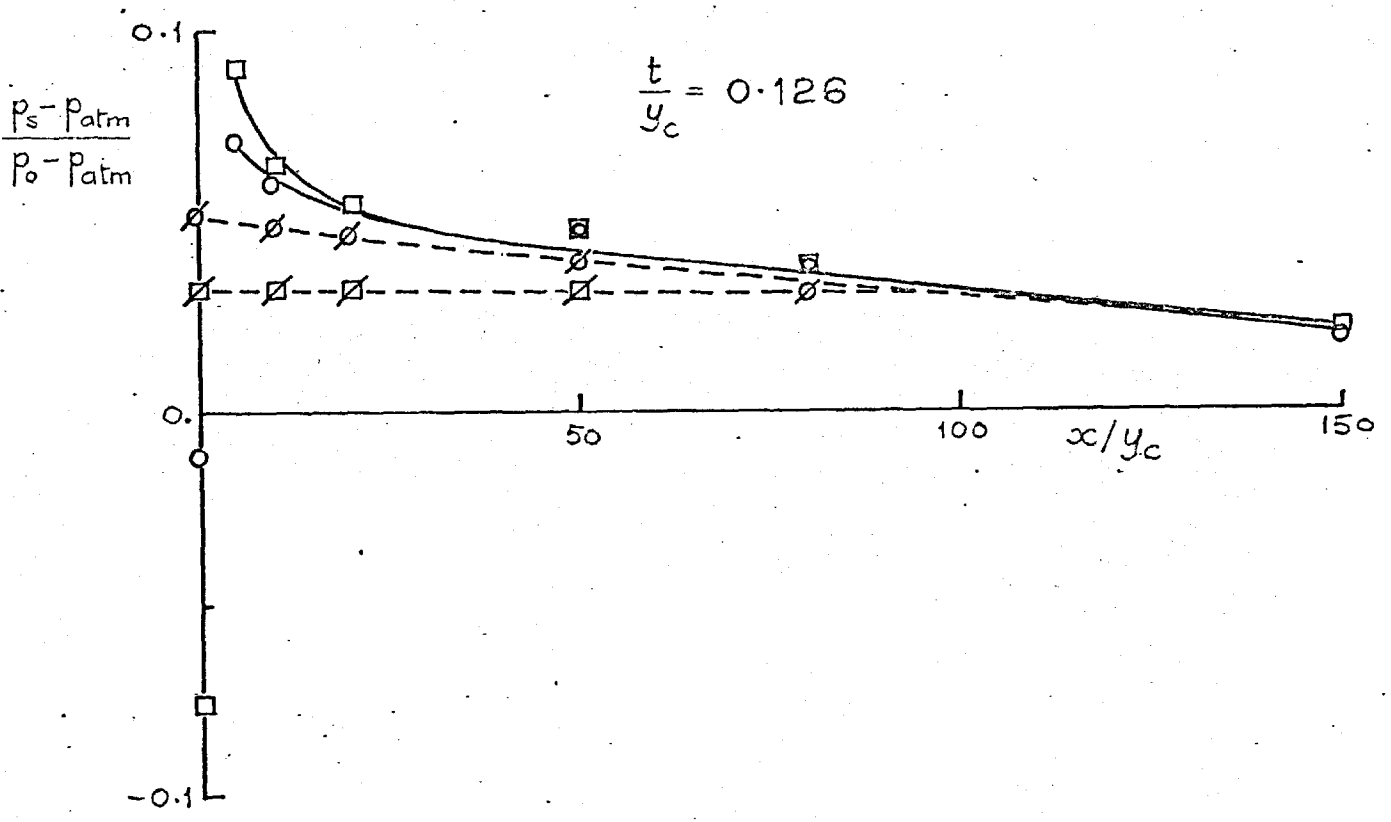
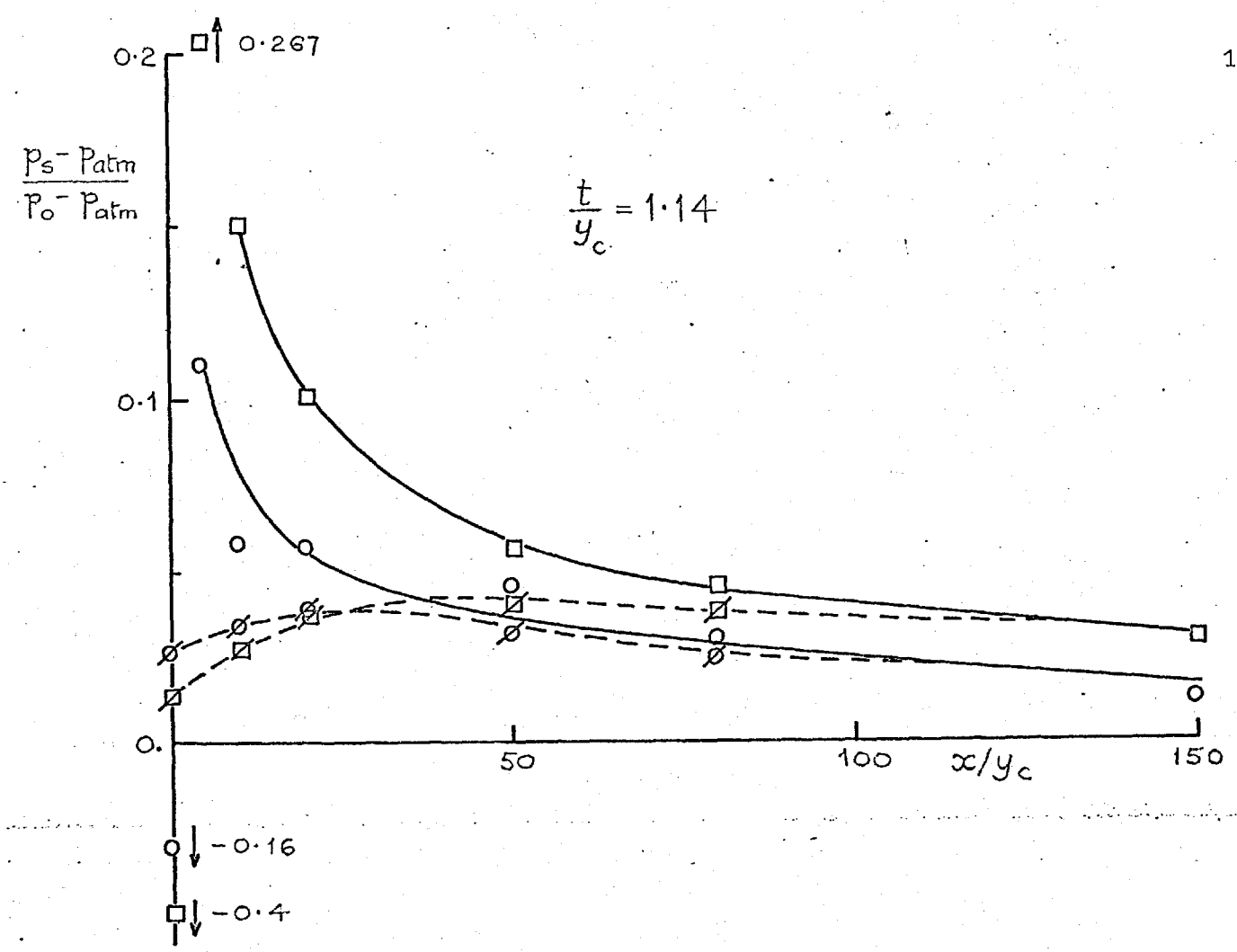
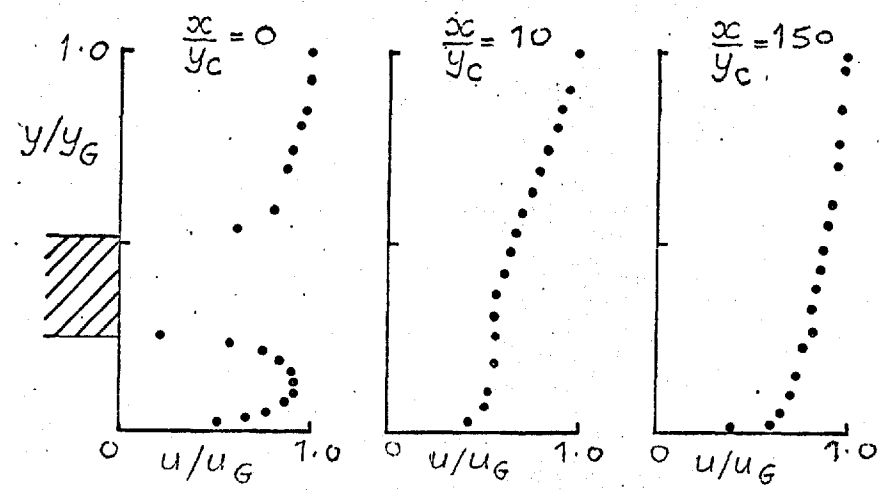
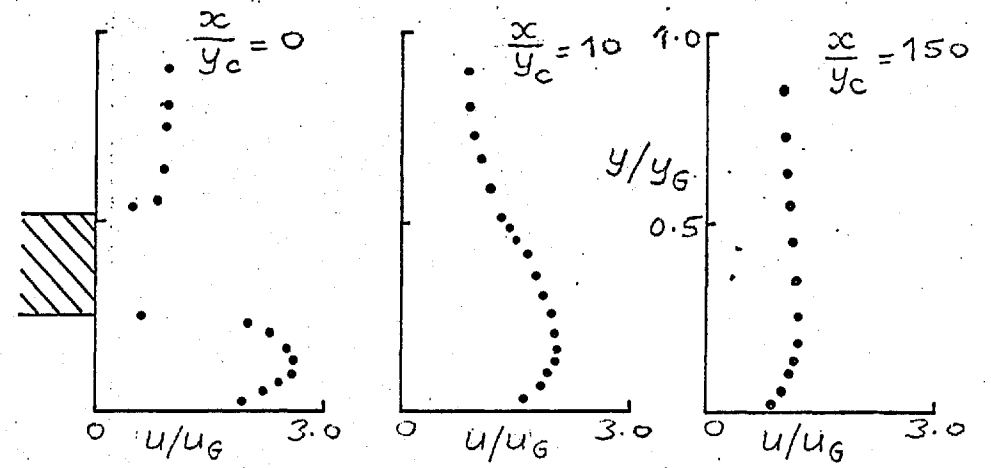


FIGURE 4.1: STATIC-PRESSURE DISTRIBUTIONS

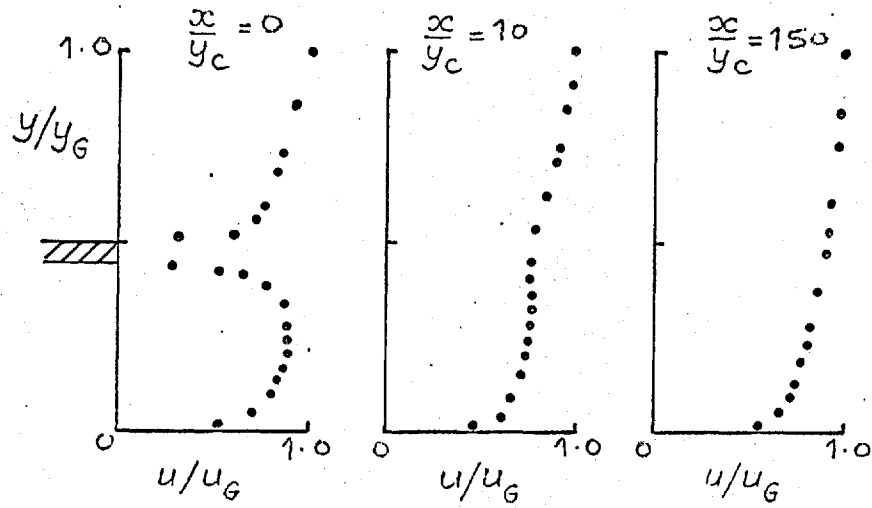
$\bar{u}_c/\bar{u}_o$	BASE PLATE	FREE STREAM
0.75	○	○
2.30	□	□



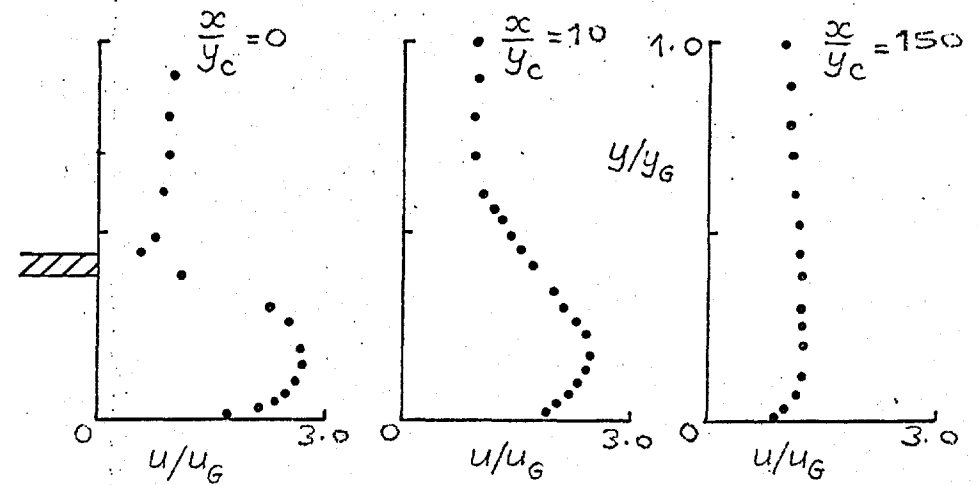
$$\frac{\bar{u}_c}{u_G} = 0.75$$



$$\frac{\bar{u}_c}{u_G} = 2.3$$



$$\frac{\bar{u}_c}{u_G} = 0.75$$



$$\frac{\bar{u}_c}{u_G} = 2.3$$

FIGURE 4.2: PROFILES OF MEAN VELOCITY

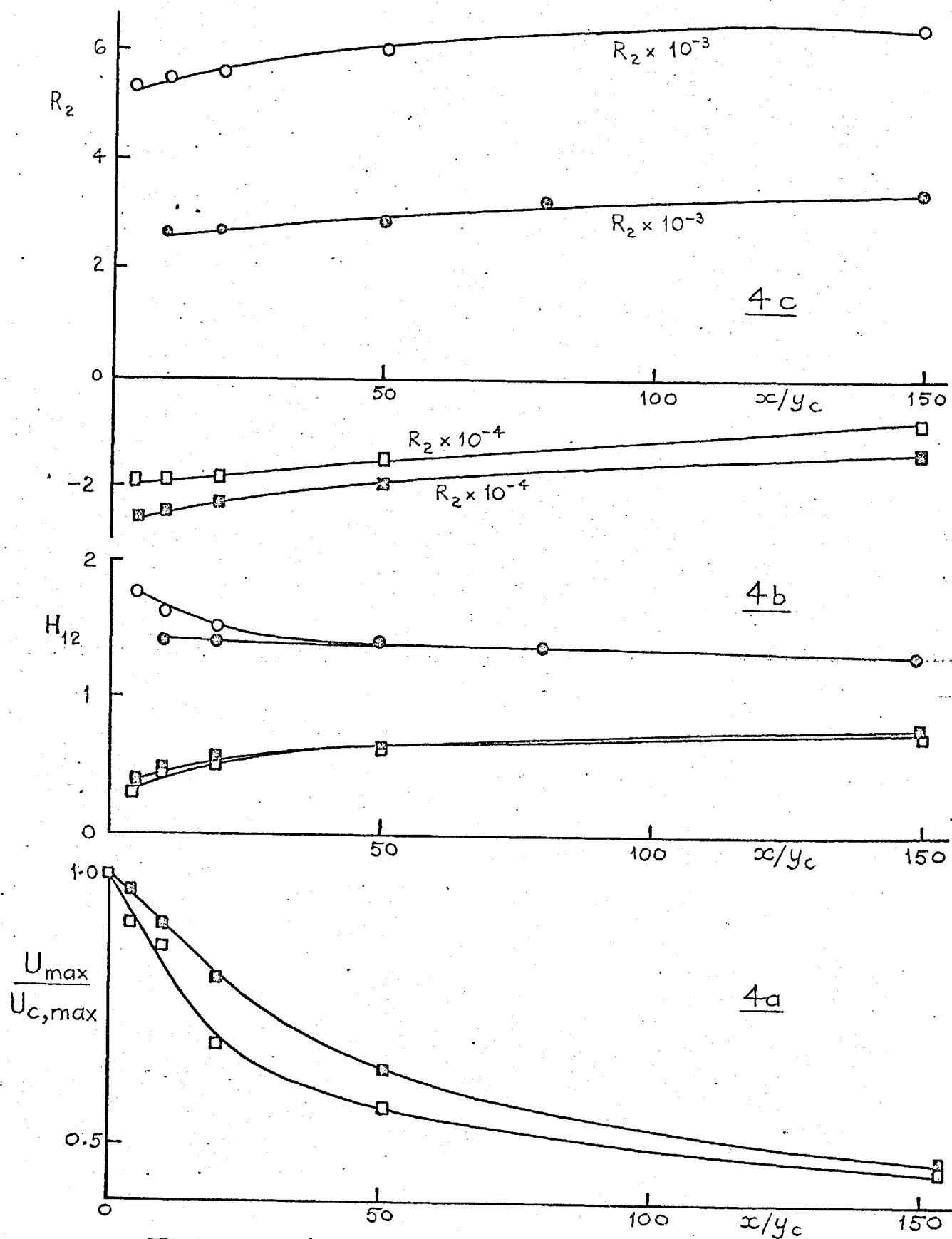


FIGURE 4.3: R_2 , H_{12} and $U_{\max}/U_{c,\max}$ as functions of x/y_c .

$\bar{u}_c$ u_g	t/y_c	
	1.14	0.126
0.75	○	●
2.3	□	■

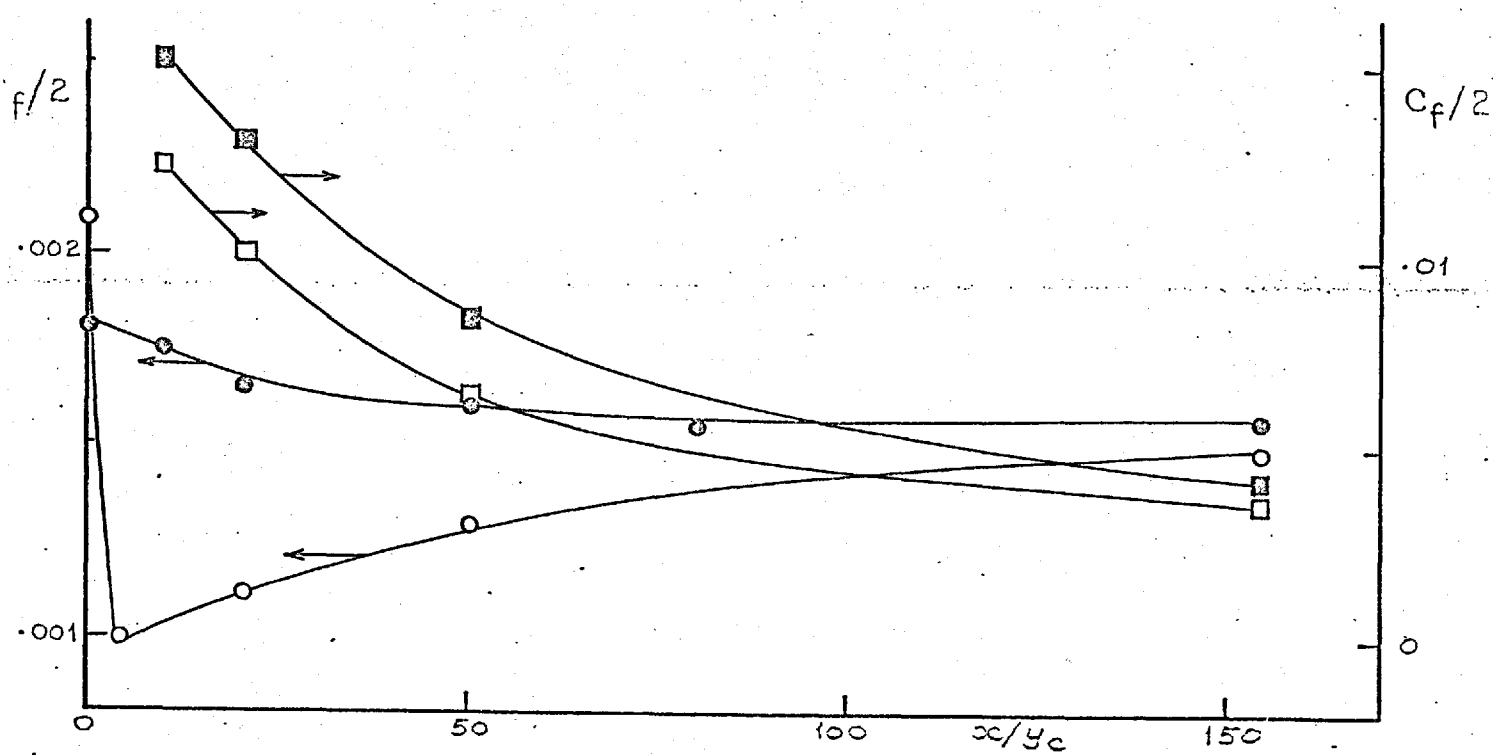


FIGURE 4.4: $C_f/2$ VARIATION AS FUNCTION OF x/y_c

$\frac{\bar{u}_c}{u_6}$	t/y_c	
	1.14	0.126
0.75	○	●
2.3	□	■

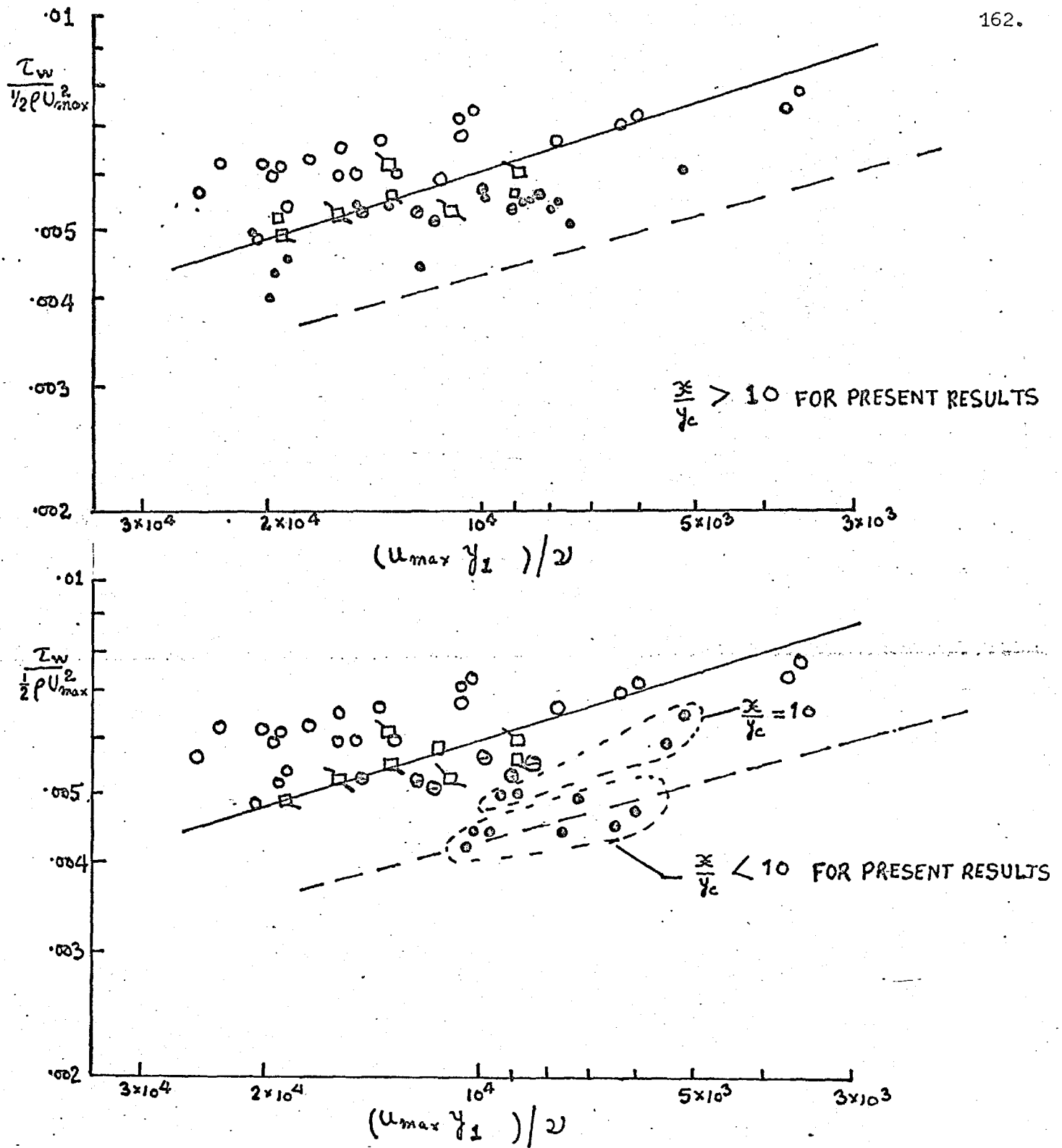


FIGURE 4.5: COMPARISON OF SKIN FRICTION RESULTS

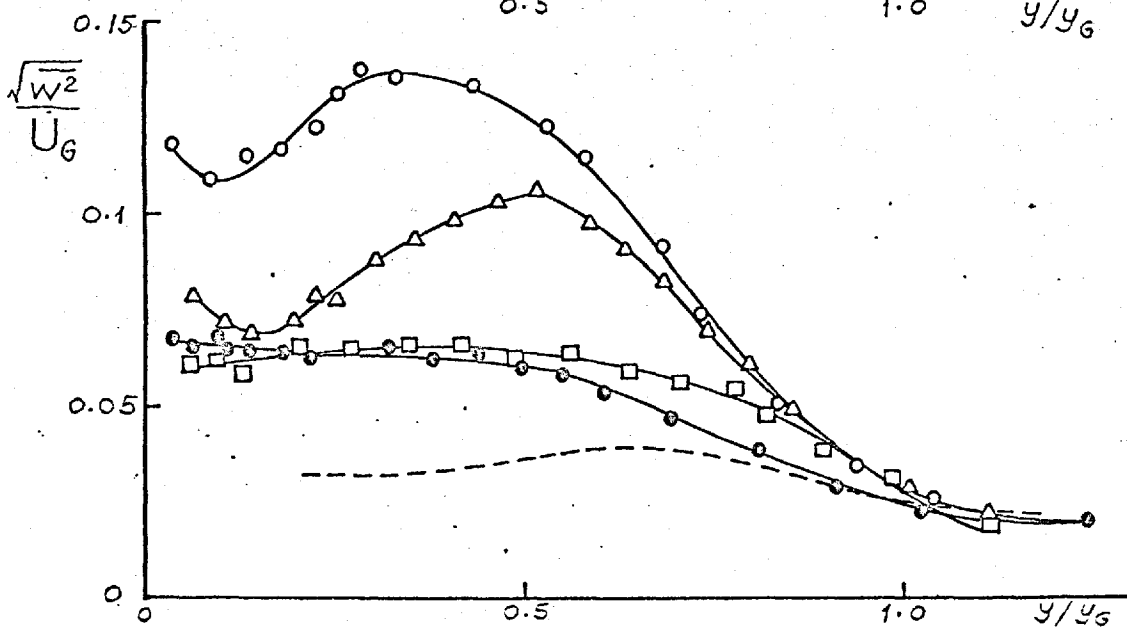
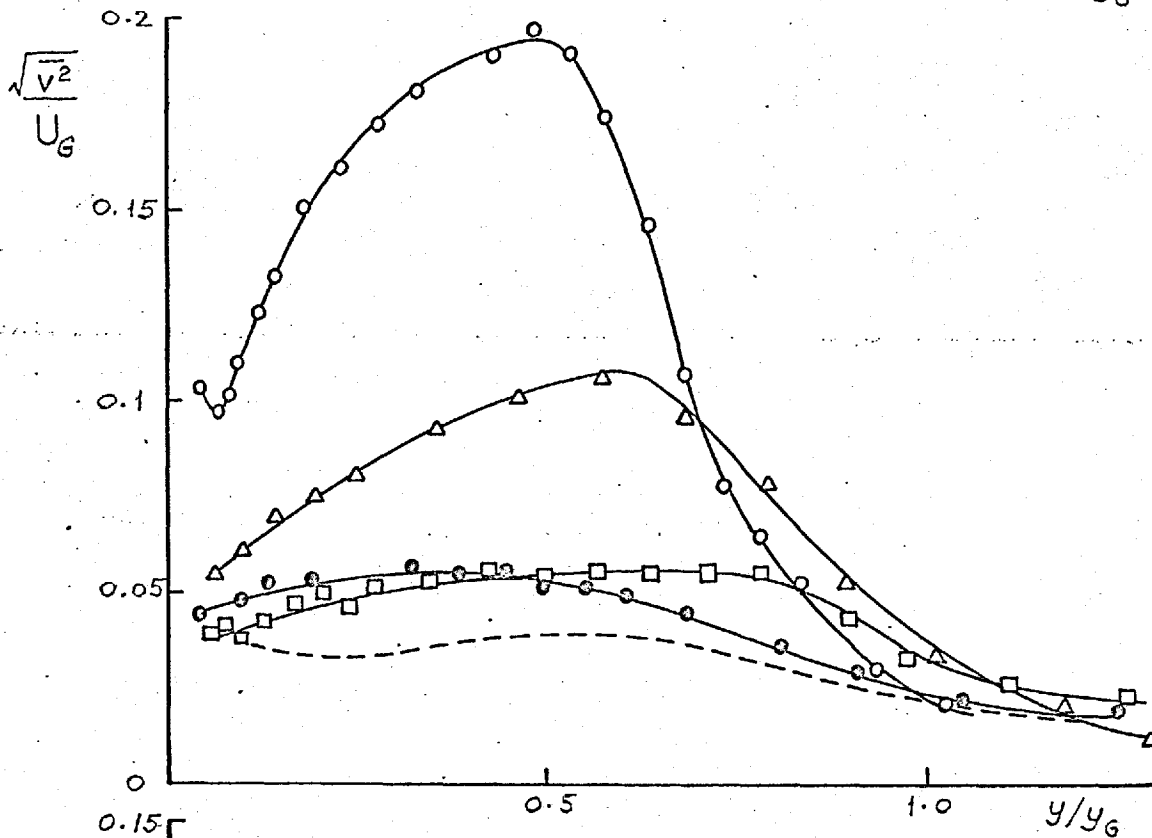
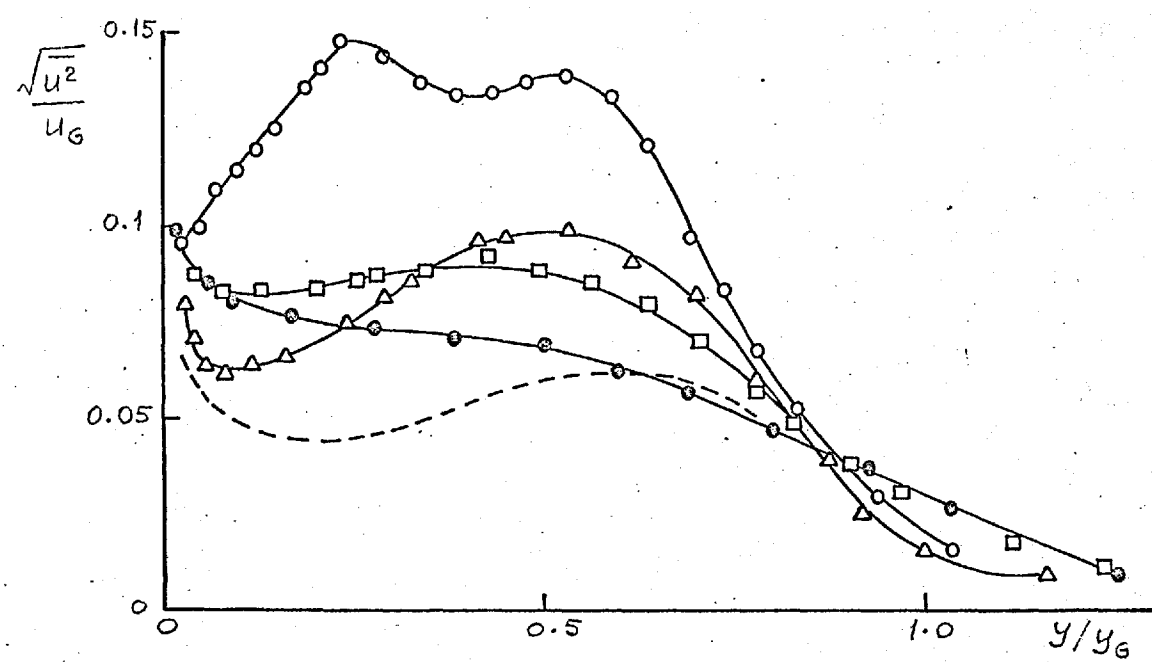


FIGURE 4.6a: PROFILES OF THE FLUCTUATING COMPONENTS OF VELOCITY; $\bar{u}_c/u_G = 0.75$

$t/y_c = 1.14$: $x/y_c = 4$, $\circ$; 10 , Δ ; 50 , $\square$; 150 , $\diamond$ | $t/y_c = 0.126$: $x/y_c = 10$, $\cdots$.

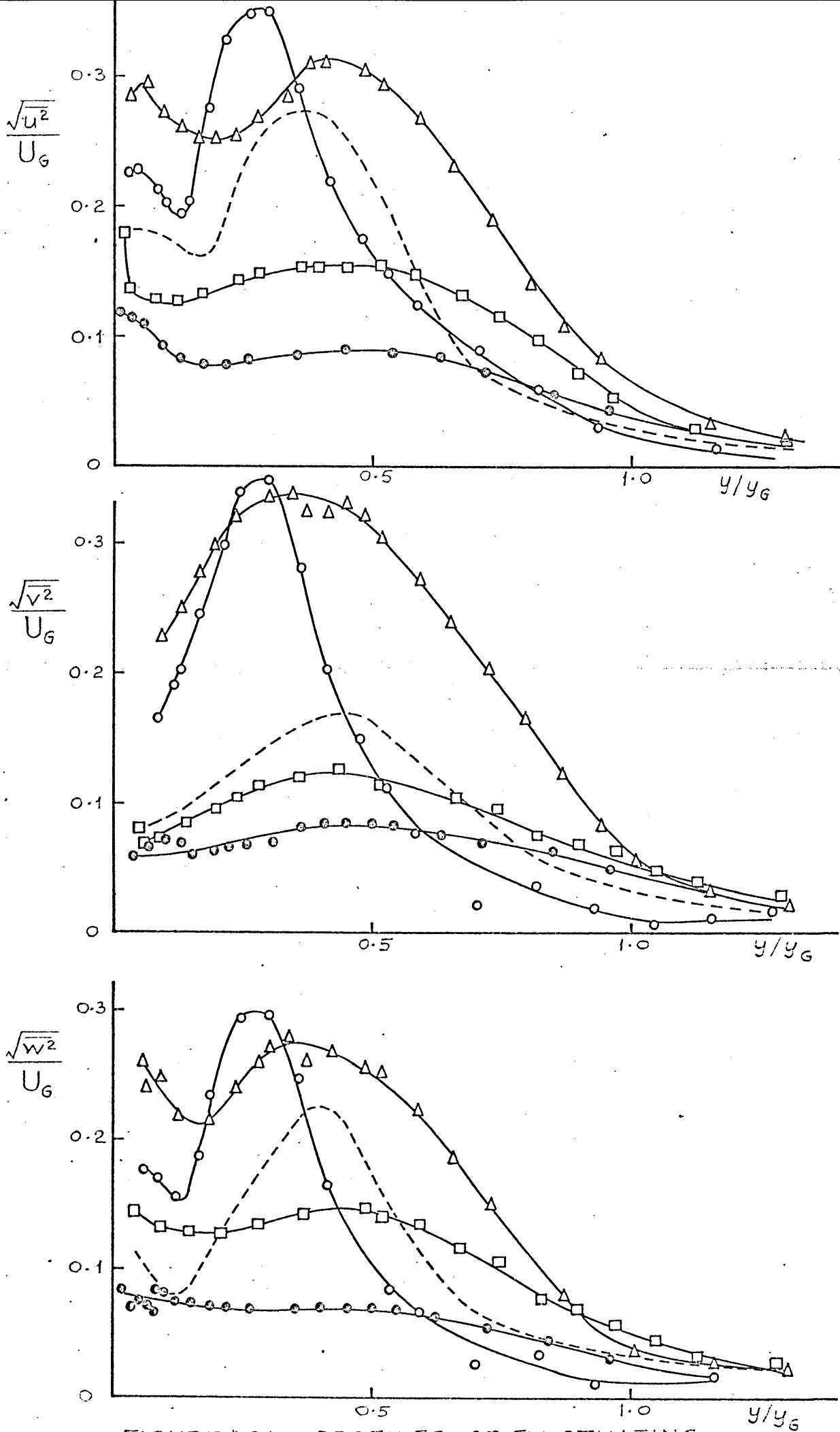


FIGURE 4.6b: PROFILES OF FLUCTUATING COMPONENTS OF VELOCITY ; $\bar{u}_c/u_s = 2.3$

$t/y_c = 1.14 : x/y_c = 4, \circ ; 10, \Delta ; 50, \square ; 150, \bullet$ $t/y_c = 0.126 : x/y_c = 10, \circ$

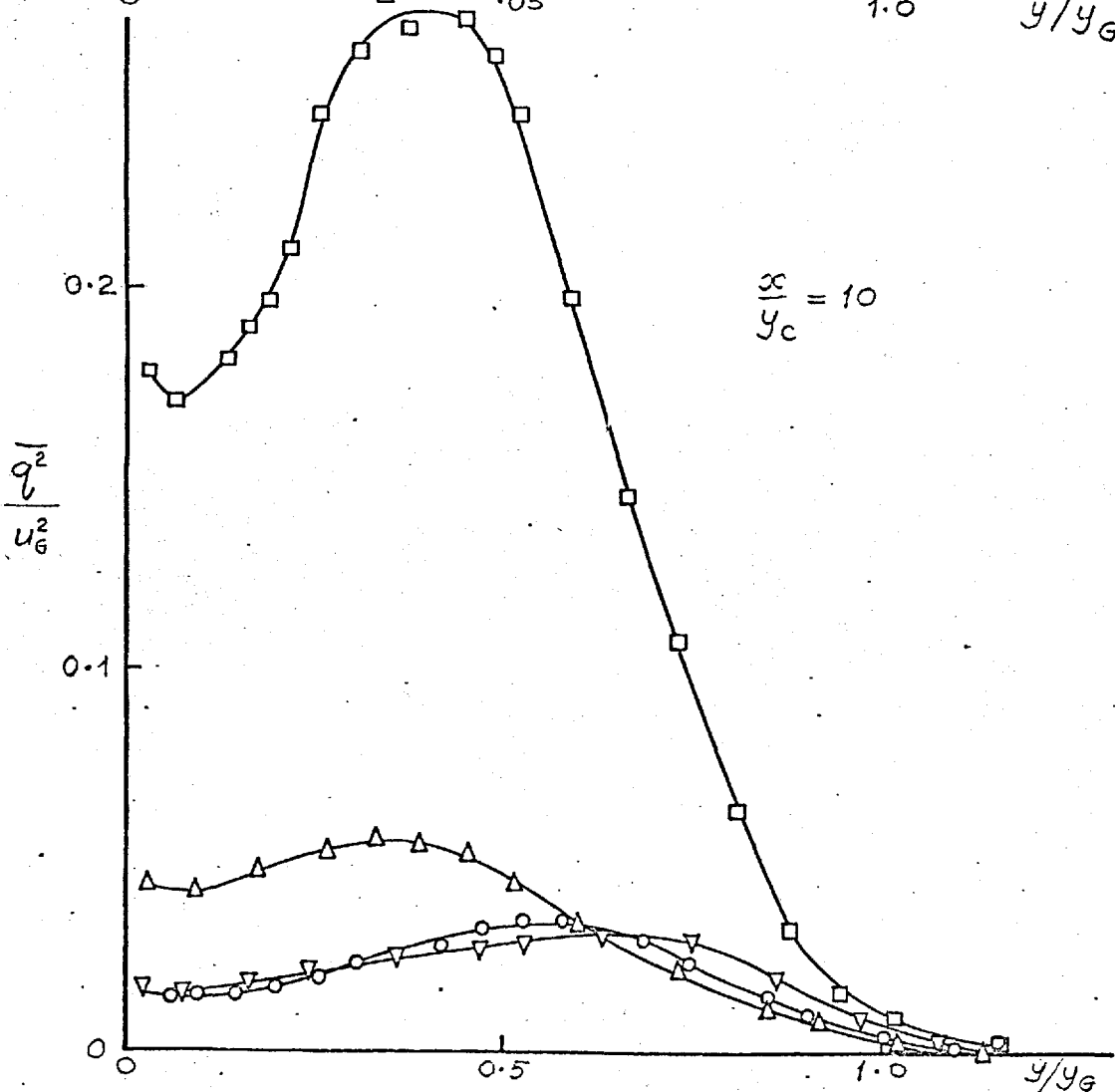
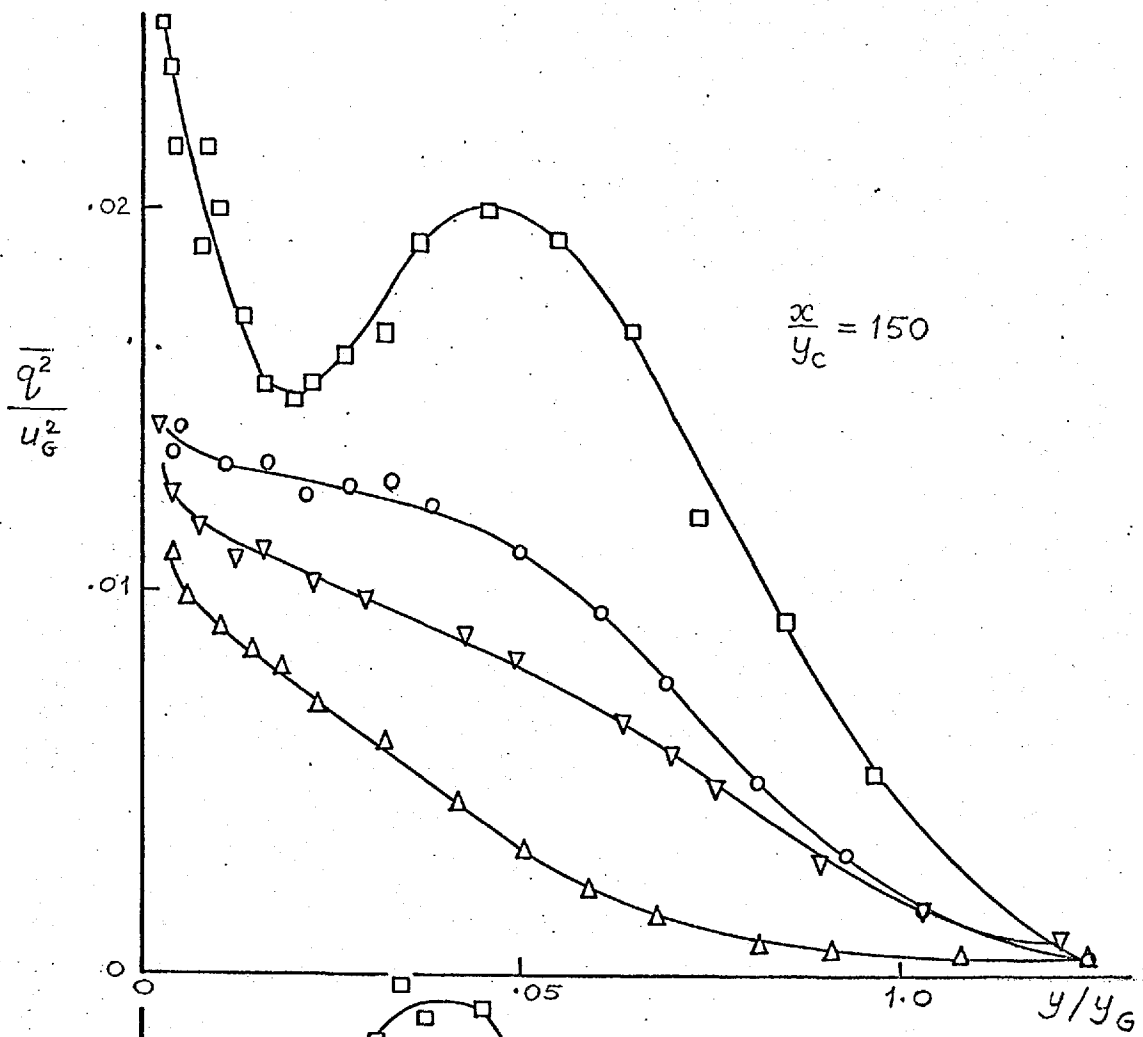


FIGURE 4.7a: PROFILES OF TURBULENT KINETIC-ENERGY;

$t/y_c = 1.14$

$\bar{u}_c/u_G = 0.75, \circ ; 0.91, \nabla ; 1.33, \Delta ; 2.30, \square$

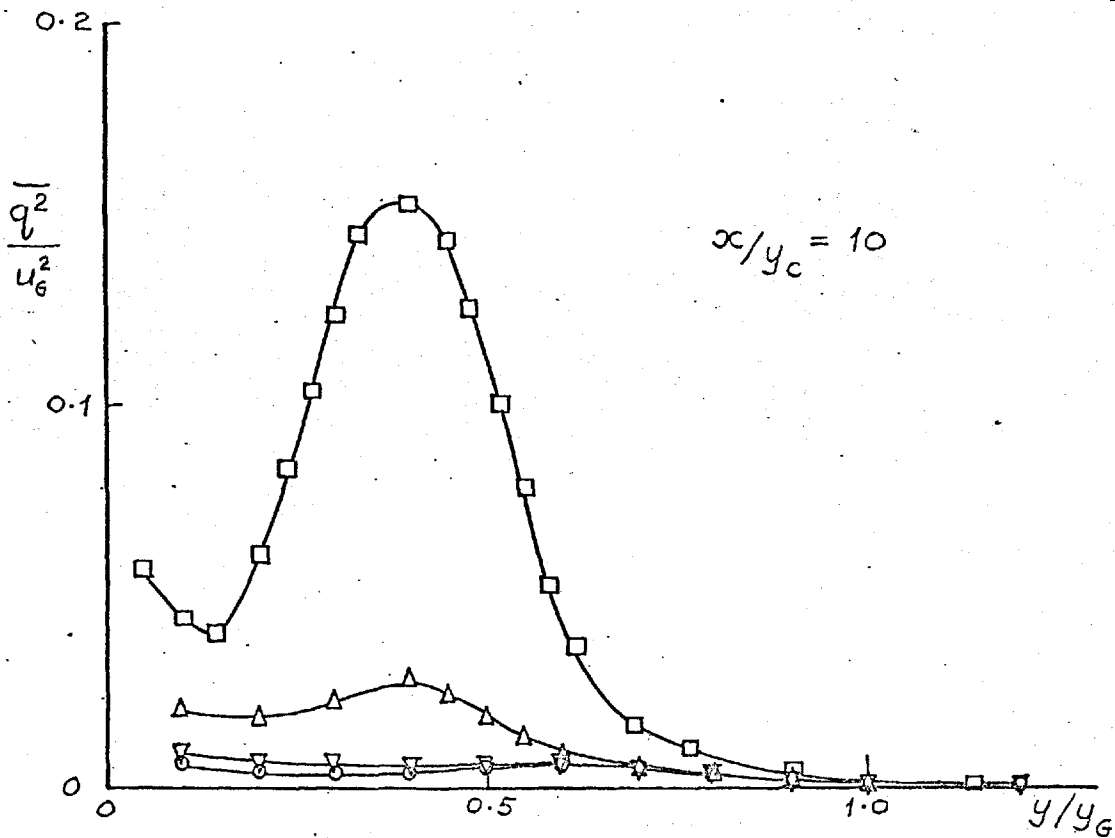
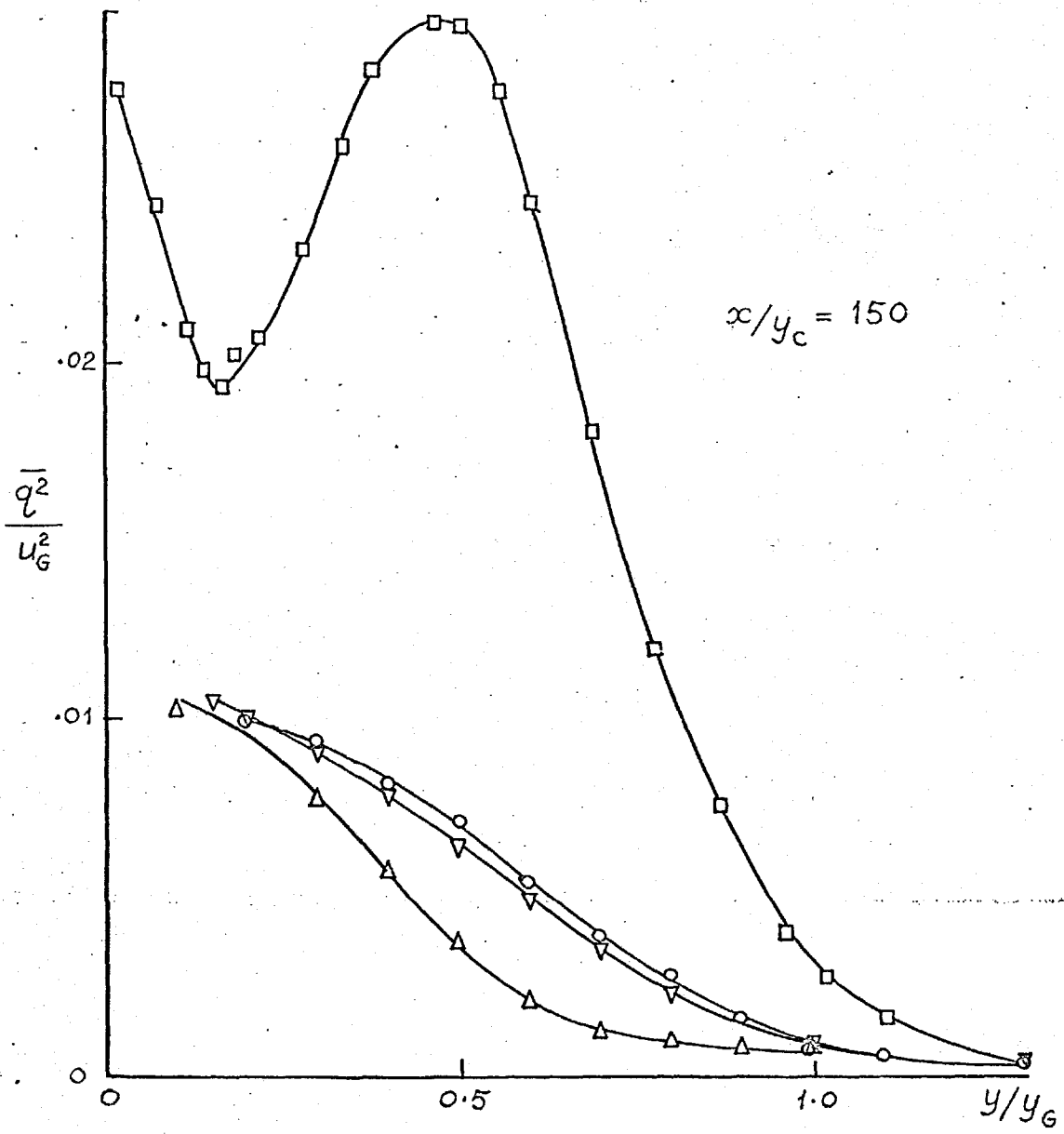


FIGURE 4.7b: PROFILES OF TURBULENT KINETIC-ENERGY; $t/y_c = 0.126$

$\bar{u}_c/u_6 = 0.75, o ; 0.91, \nabla ; 1.33, \Delta ; 2.30, \square$

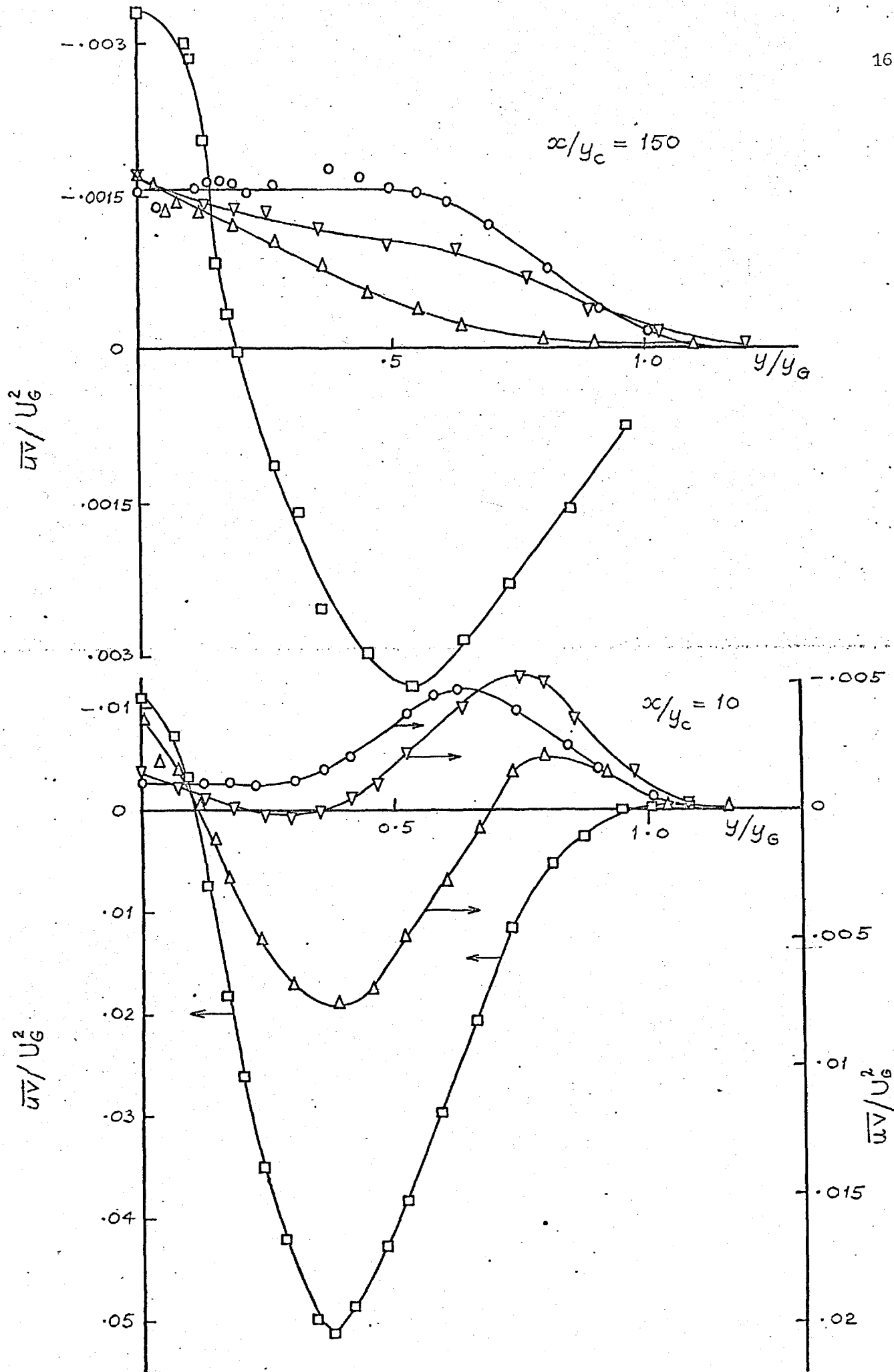


FIGURE 4.8a: PROFILES OF TURBULENT SHEAR STRESS; $\bar{t}/y_c = 1.1 \pm$

$\bar{u}_c/u_6 = 0.75, \circ$; $0.91, \nabla$; $1.33, \Delta$; $2.30, \square$

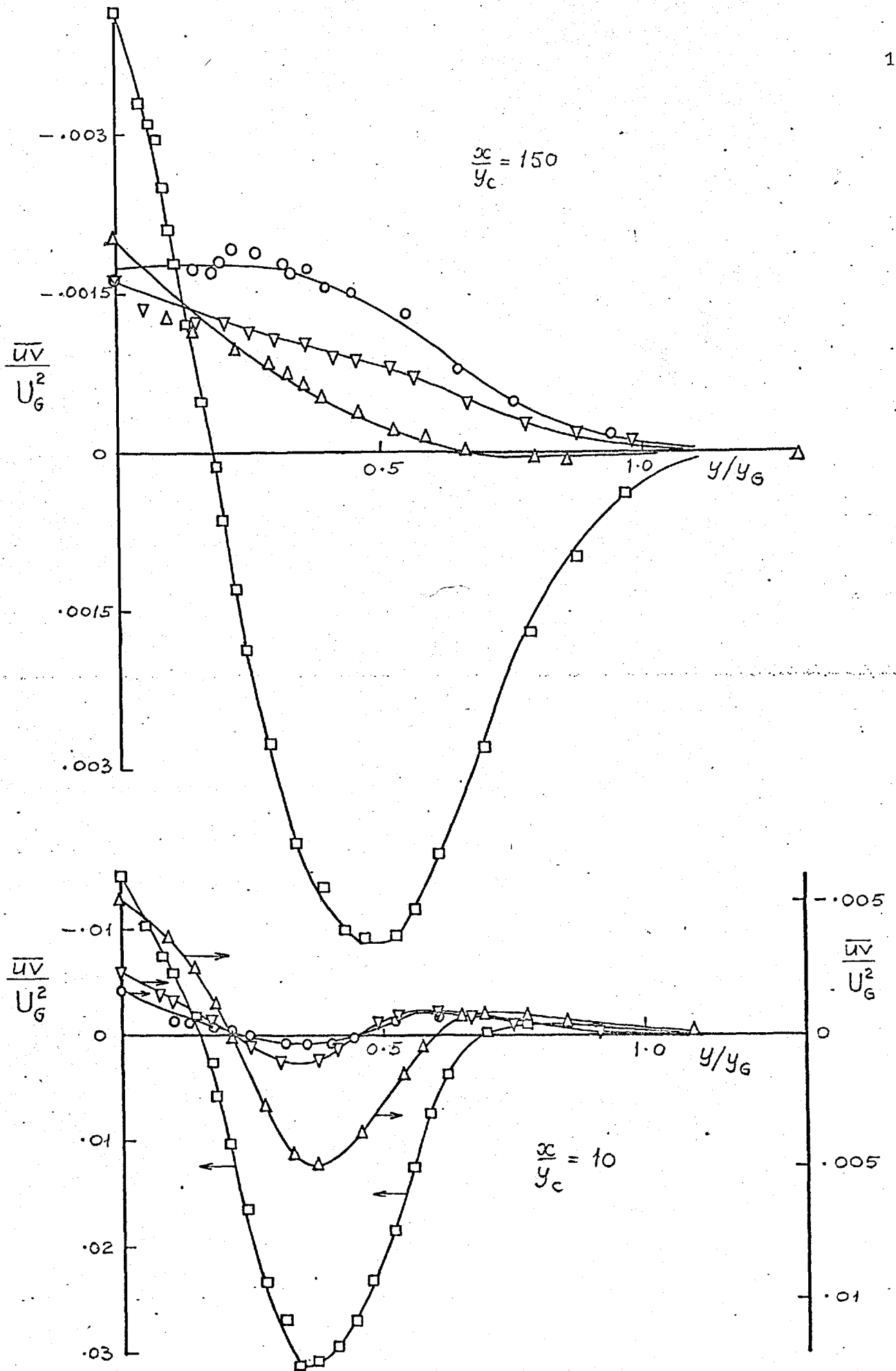


FIGURE 4.8b: PROFILES OF TURBULENT SHEAR STRESS

$$t/y_c = 0.126$$

$$\bar{u}_c/u_G = 0.75, \circ; 0.91, \nabla; 1.33, \Delta; 2.30, \square$$

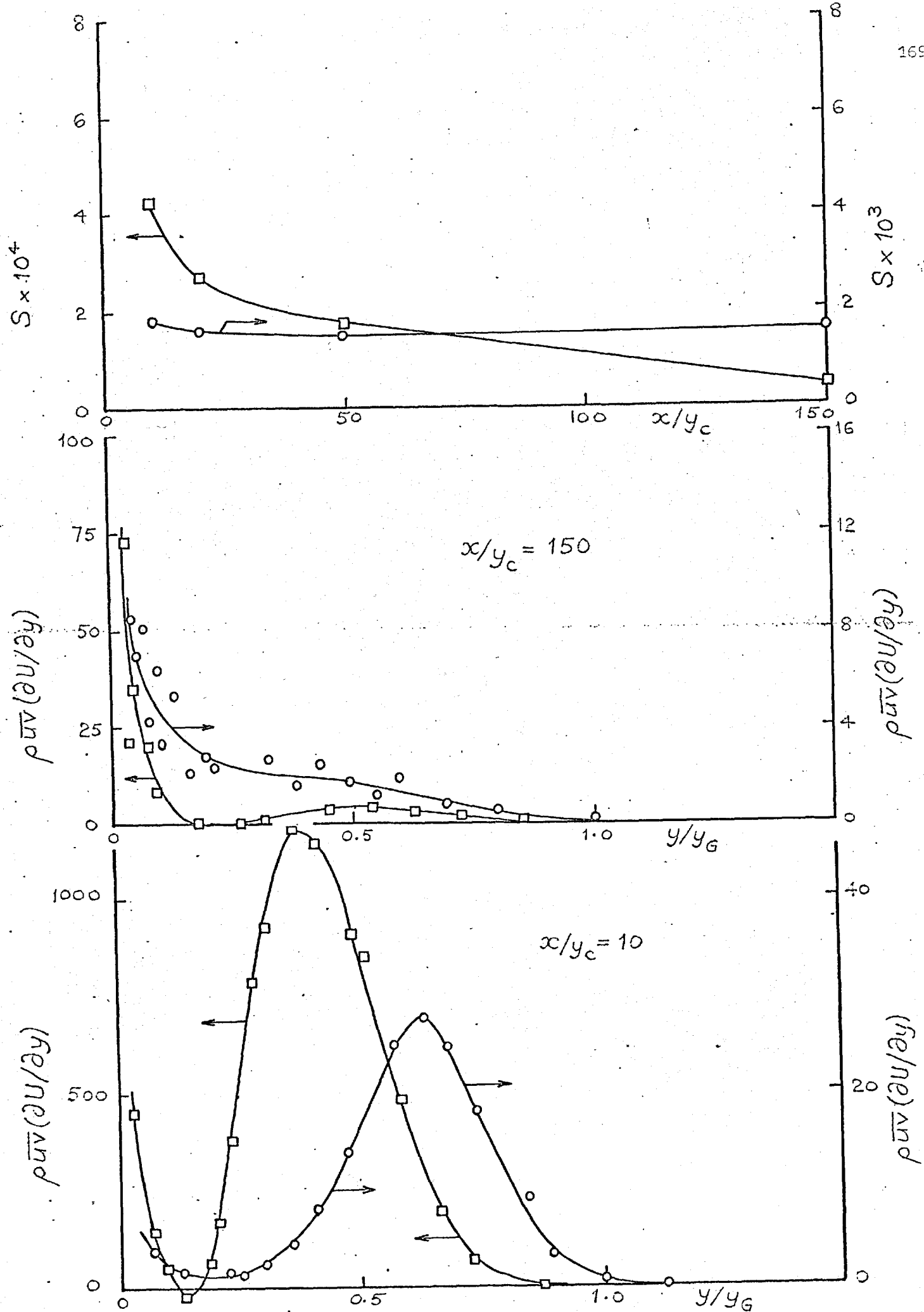


FIGURE 4.9a: PRODUCTION OF TURBULENT KINETIC ENERGY AND SHEAR WORK INTEGRAL

$$t/y_c = 1.14$$

$$\bar{u}_c/u_G = 0.75, \circ ; 2.30, \square$$

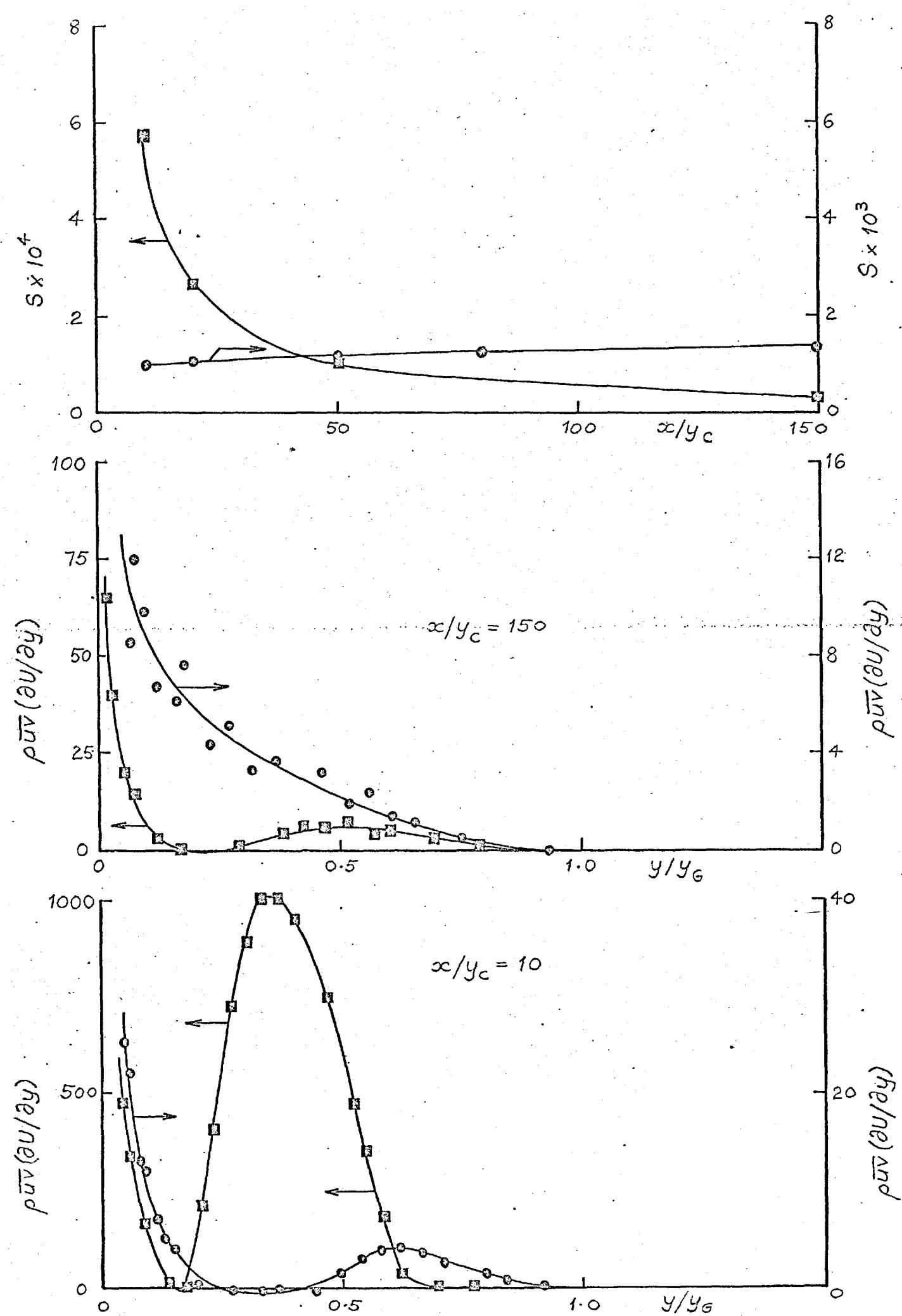


FIGURE 4.9 b: PRODUCTION OF TURBULENT KINETIC ENERGY AND SHEAR WORK INTEGRAL,

$$t/y_c = 0.126$$

$$\bar{u}_c/u_c = 0.75, \circ ; 2.30, \blacksquare$$

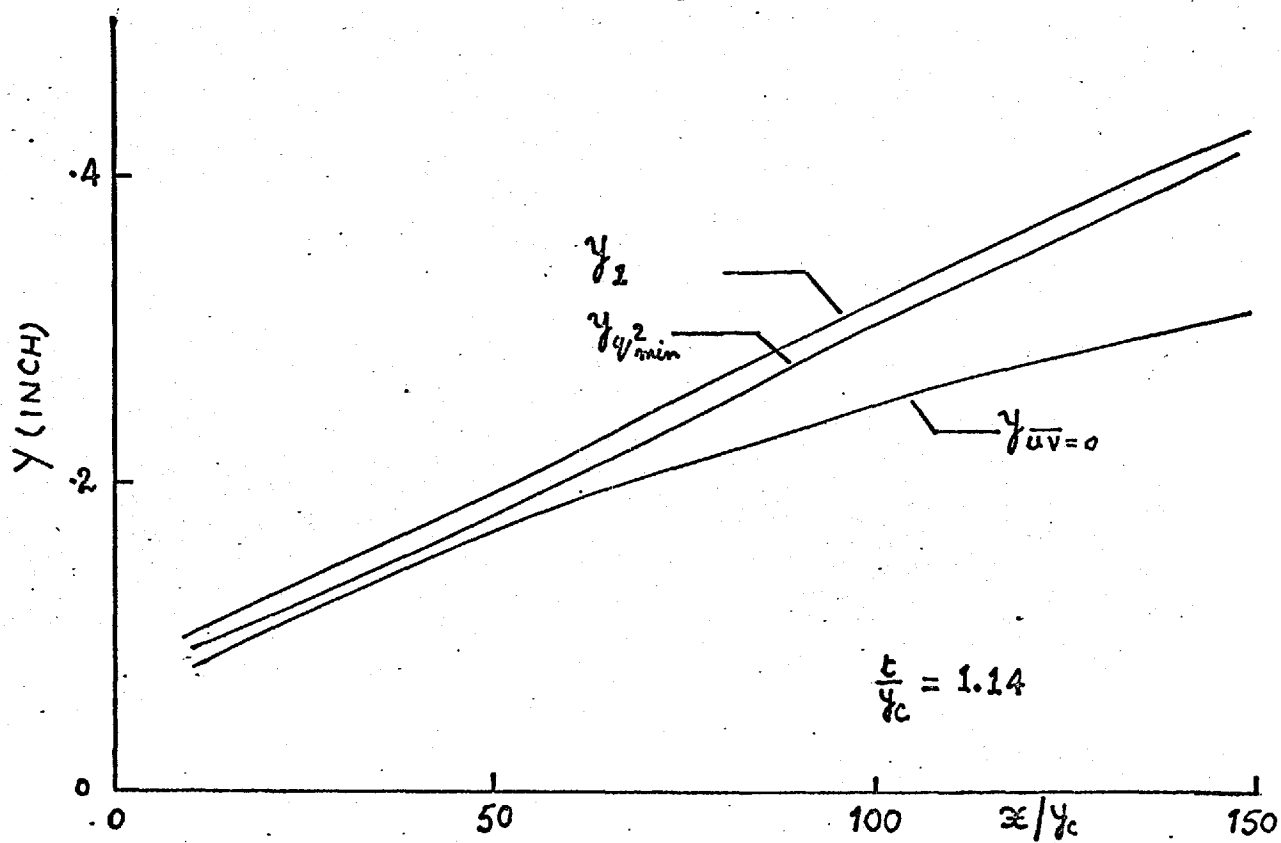
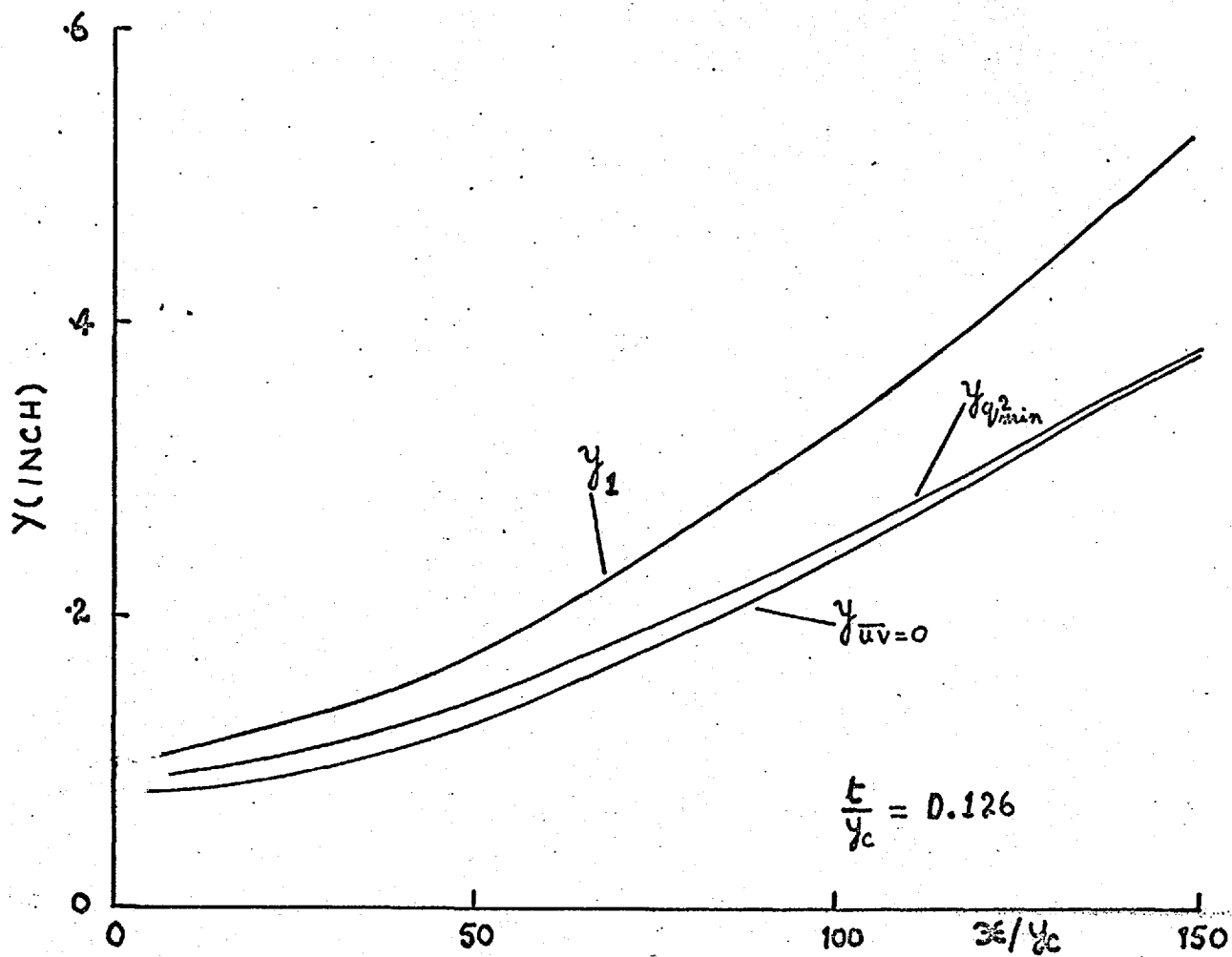


FIG.4.10 LOCATION OF VELOCITY MAXIMA, ZERO SHEAR STRESS AND MINIMUM KINETIC ENERGY PLANES ; $\bar{u}_c/u_a = 2.3$

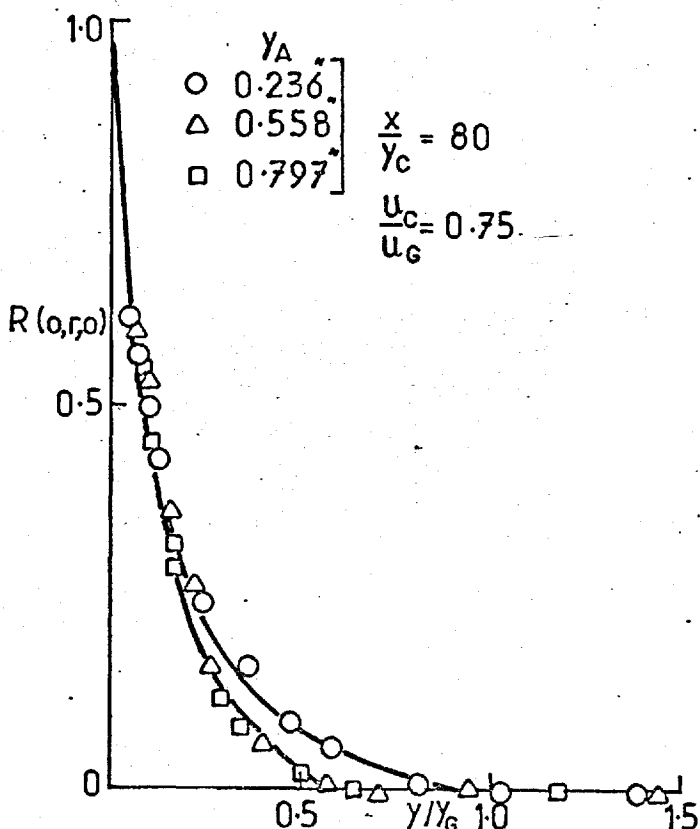
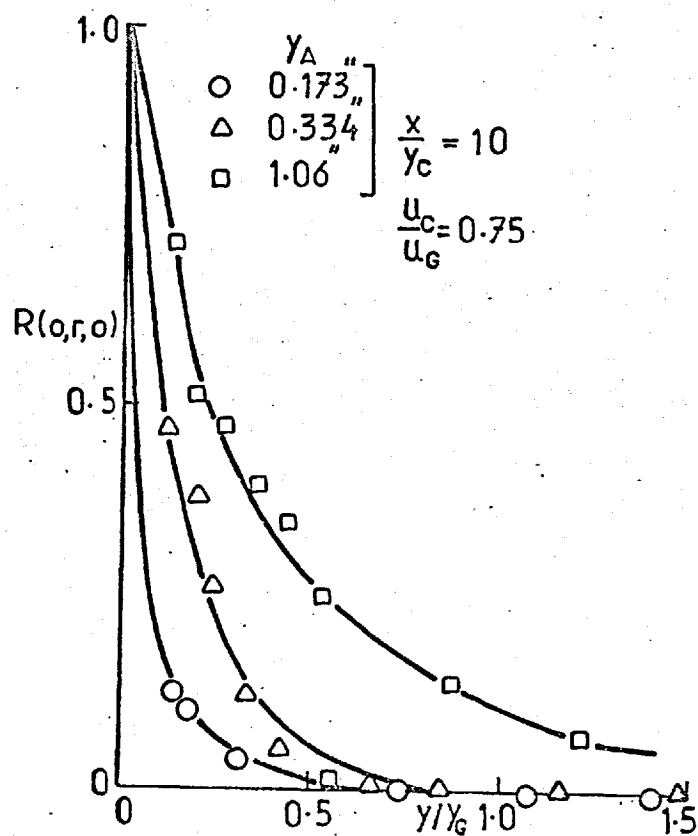
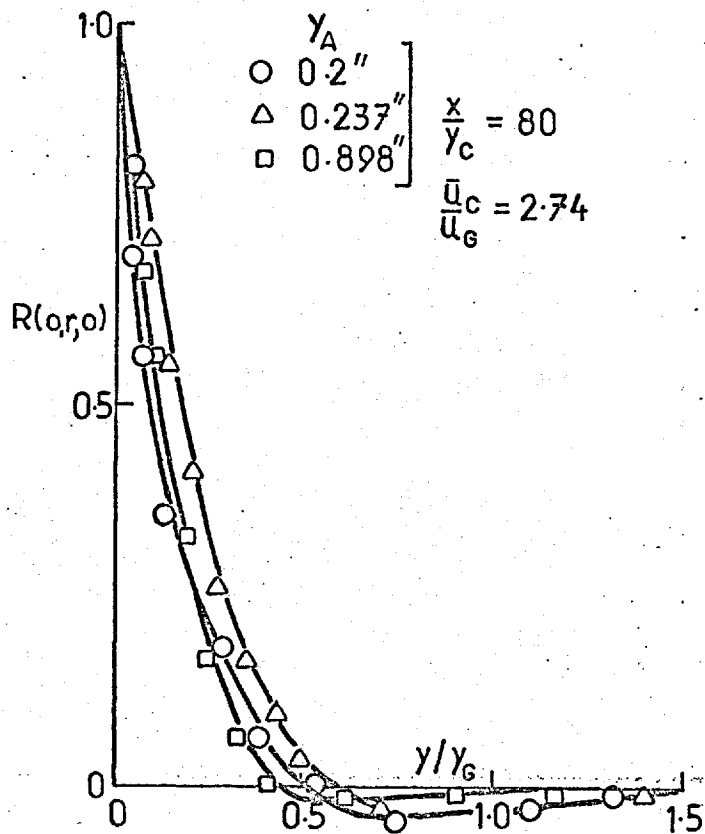
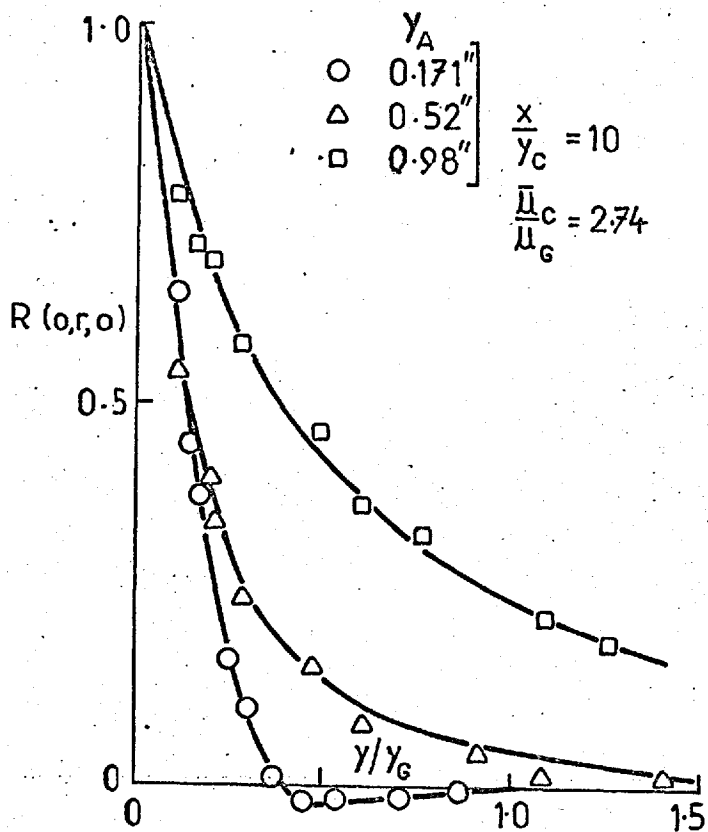


FIG. 4.11 PROFILES OF THE CORRELATION COEFFICIENT, $R(0, r, 0)$;
 $k = 1/32''$, $y_c = 0.250''$

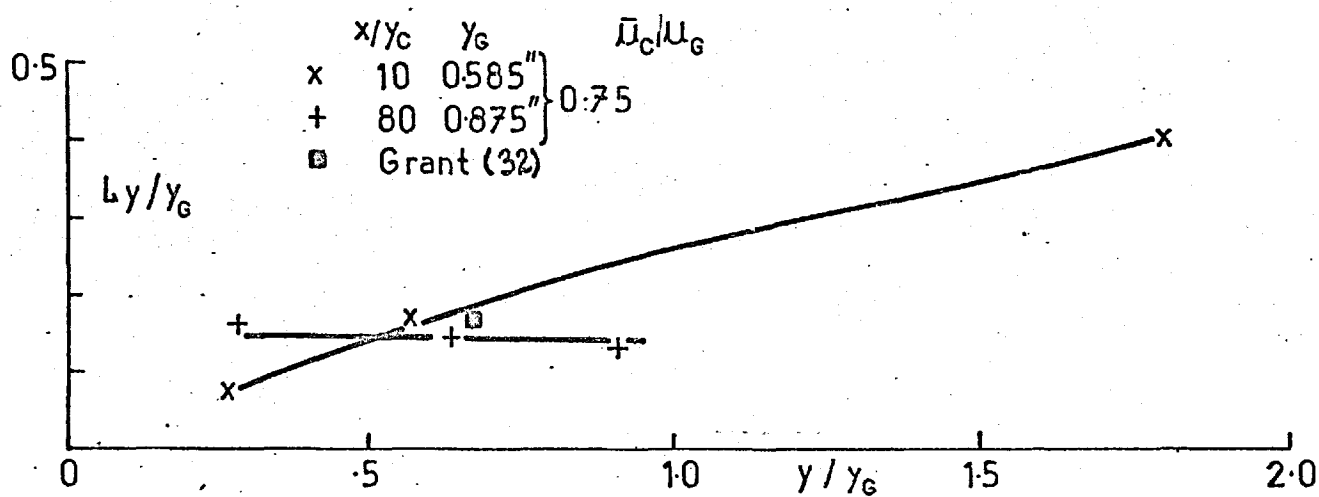
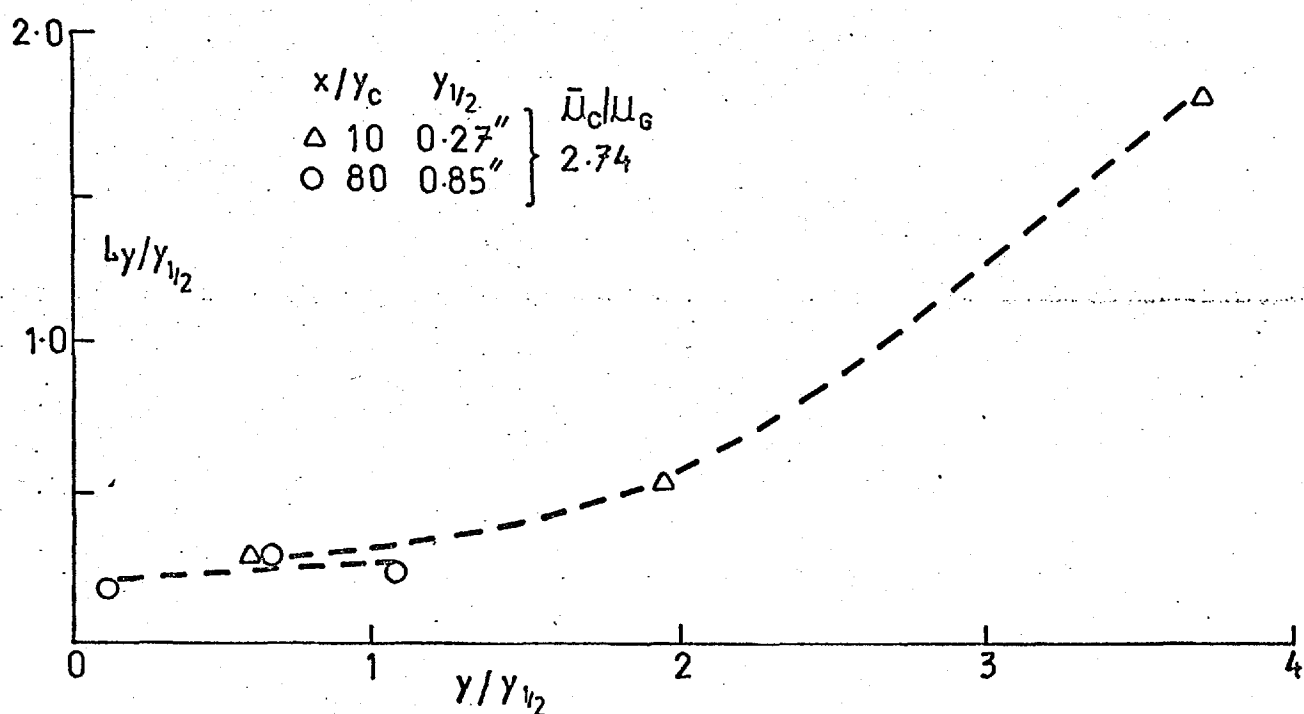
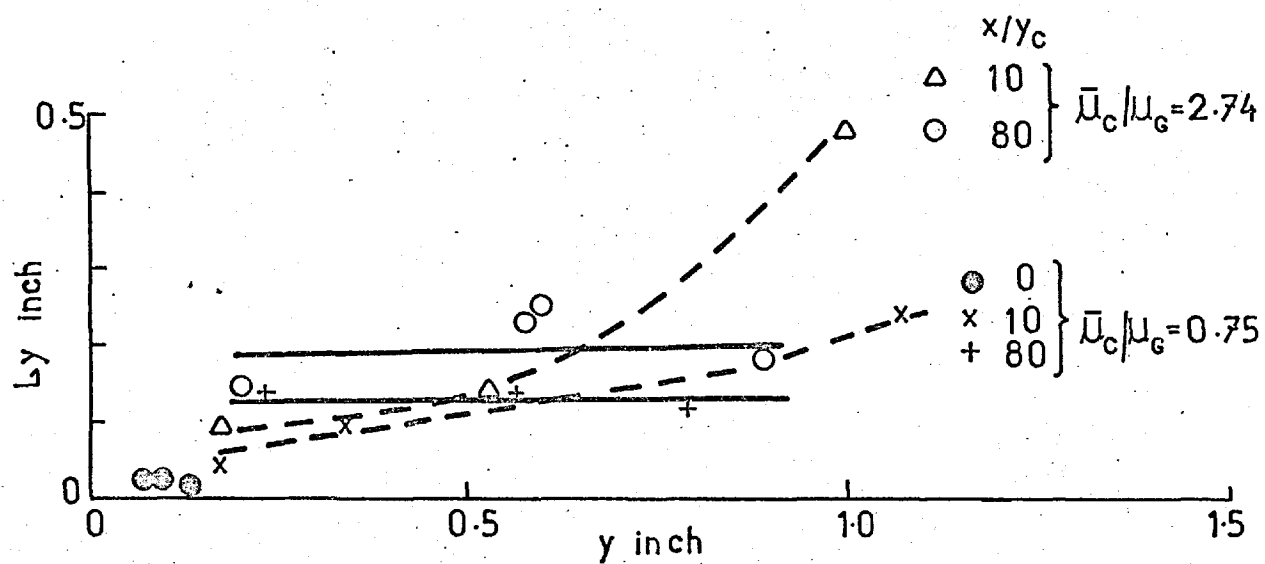


FIG.4.12 DISTRIBUTIONS OF THE TURBULENT LENGTH SCALE, $\gamma_c = 0.25''$, $t = 1/32''$

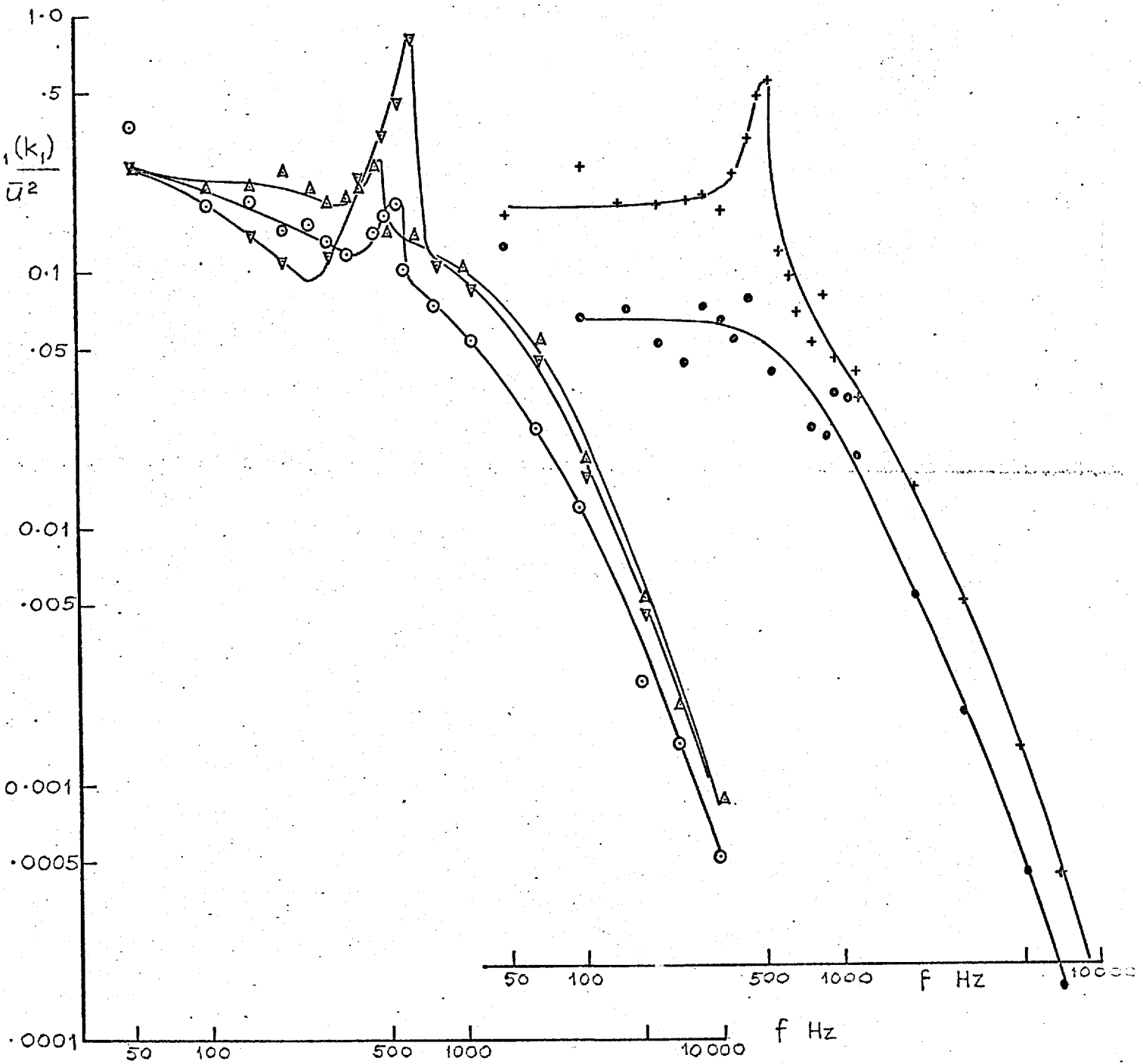


FIGURE 4.13a: u -SPECTRA MEASUREMENTS;

$\bar{u}_c/\bar{u}_g = 0.91$, $t/y_c = 1.14$, $\bar{x}/y_c = 1$.

y (in)	
0.023	○
0.096	△
0.196	▽
0.296	●
0.526	+

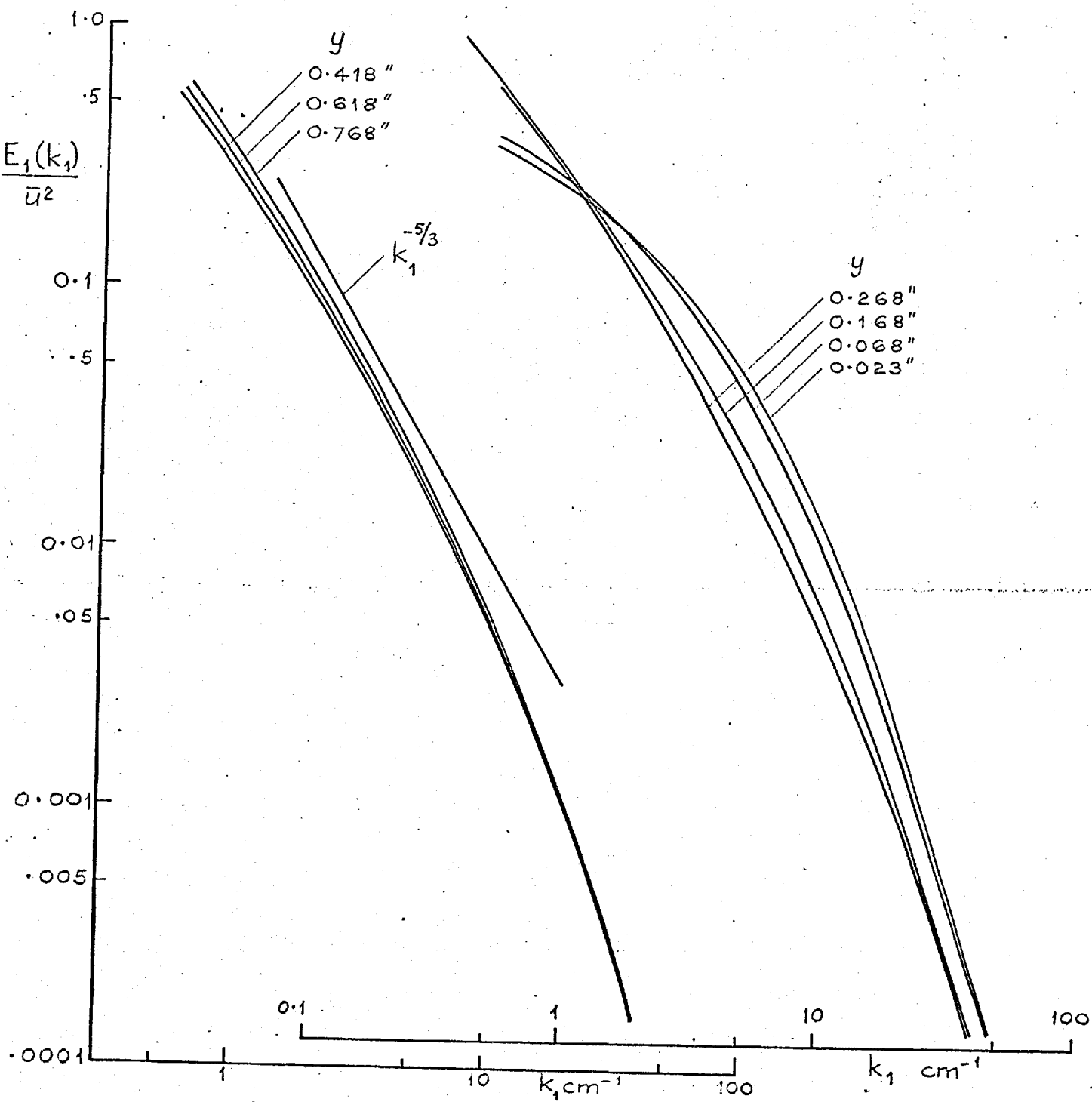


FIGURE 4.13b: u -SPECTRA MEASUREMENTS;
 $\bar{u}_c/\bar{u}_e = 0.91$, $t/y_c = 1.14$, $x/y_c = 150$

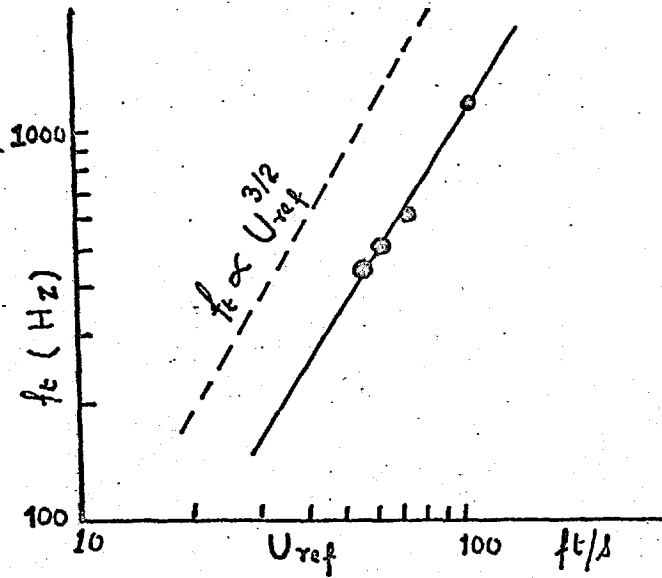


FIG. 4.14a RELATION BETWEEN PREFERRED FREQUENCY f_t AND U_{ref}

$$U_{ref} = (\bar{U}_c + U_G)/2$$

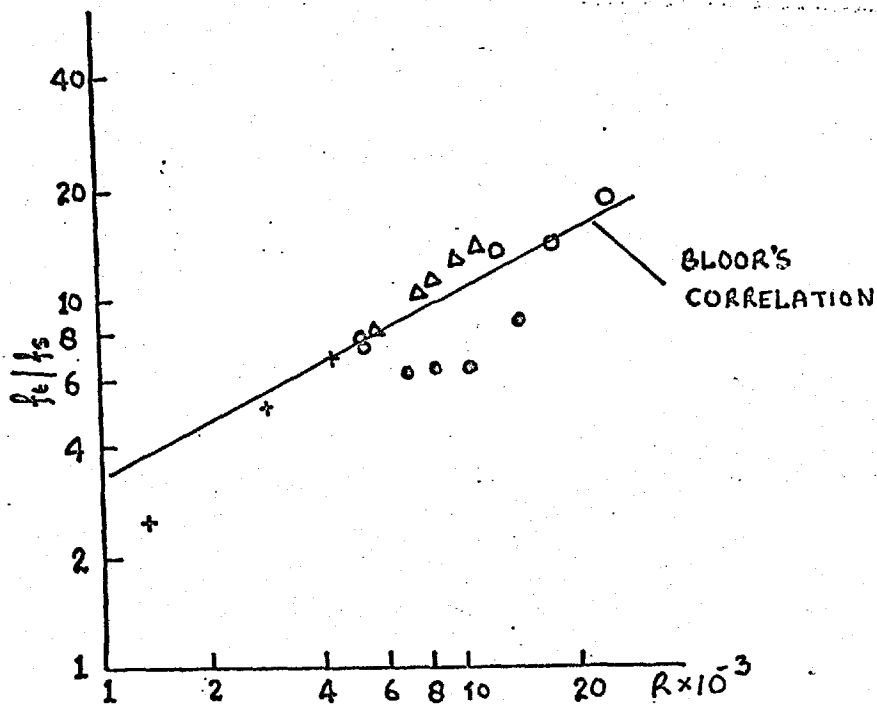


FIG. 4.14b REYNOLDS NUMBER VERSUS f_t/f_s

$$R = U_{ref} t / \nu$$

	AUTHOR	REF.
+ ○ }	BLOOR'S DATA	7
△	PETERKA AND RICHARDSON'S DATA	78
●	PRESENT DATA	

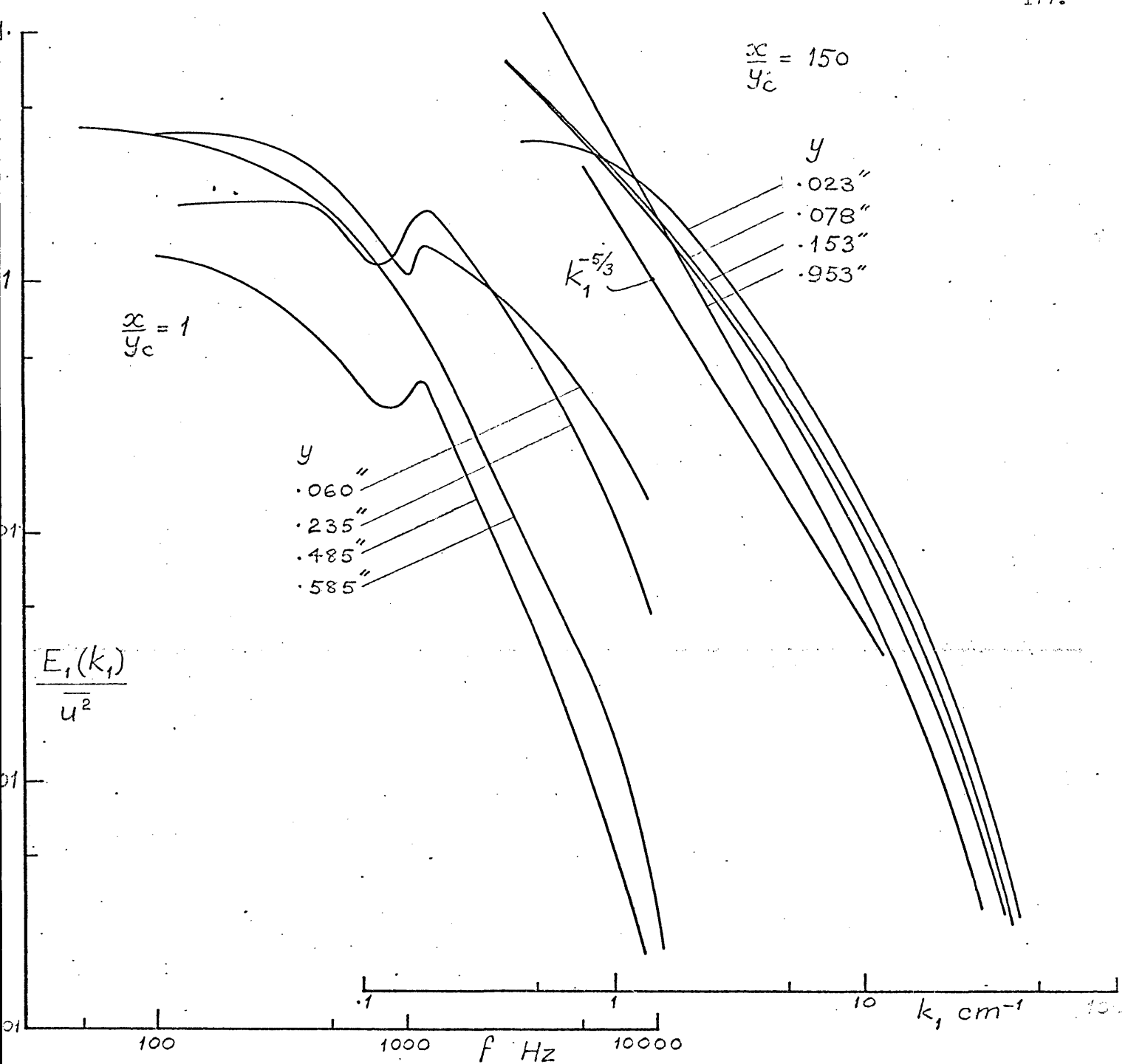


FIGURE 4.15a: u -SPECTRA MEASUREMENTS;
 $\bar{u}_c/\bar{u}_g = 2.3$; $t/y_c = 1.14$

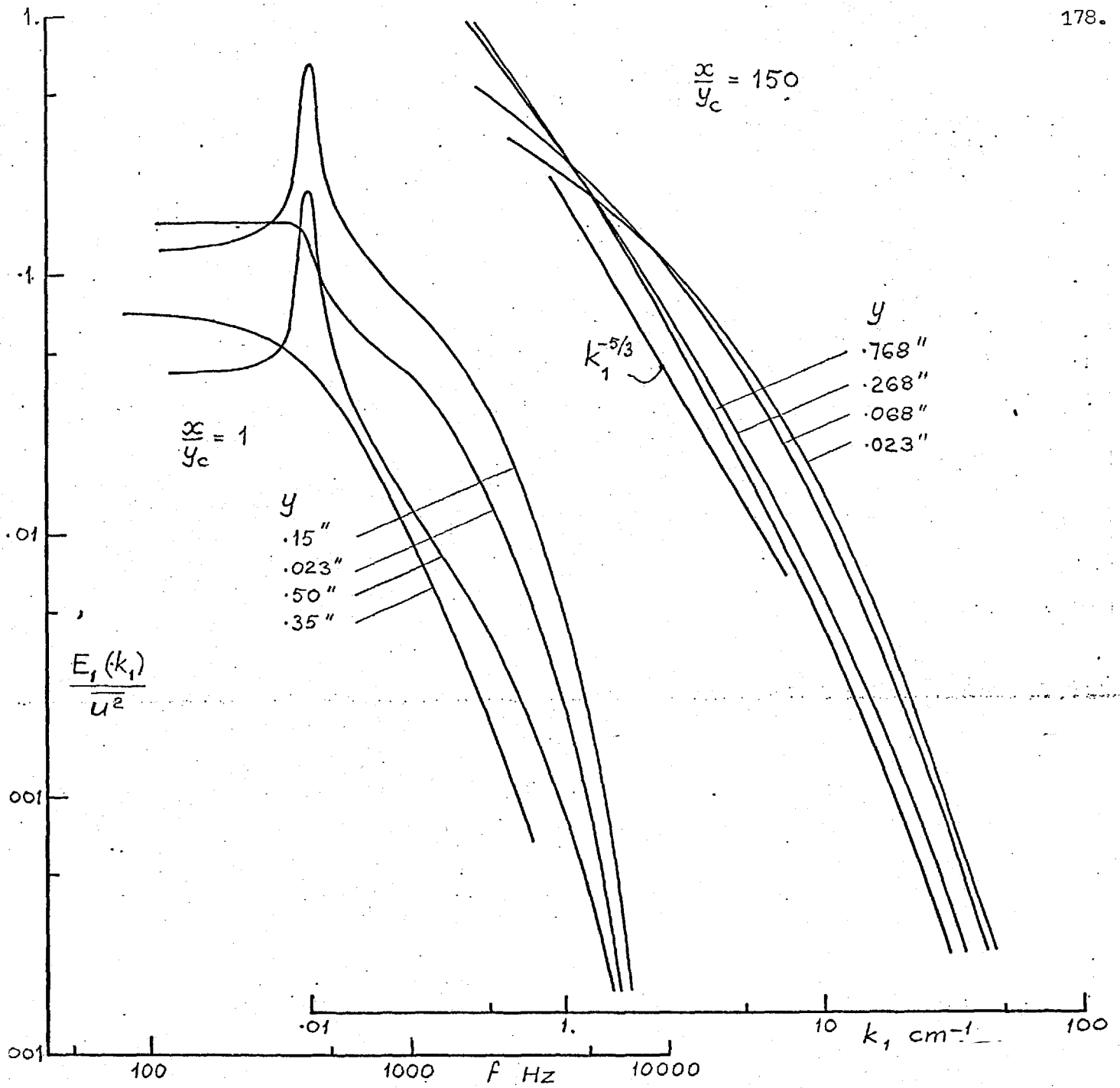


FIGURE 4.15b: u -SPECTRA MEASUREMENTS;

$\bar{u}_c / u_G = 0.75$; $t / y_c = 1.14$

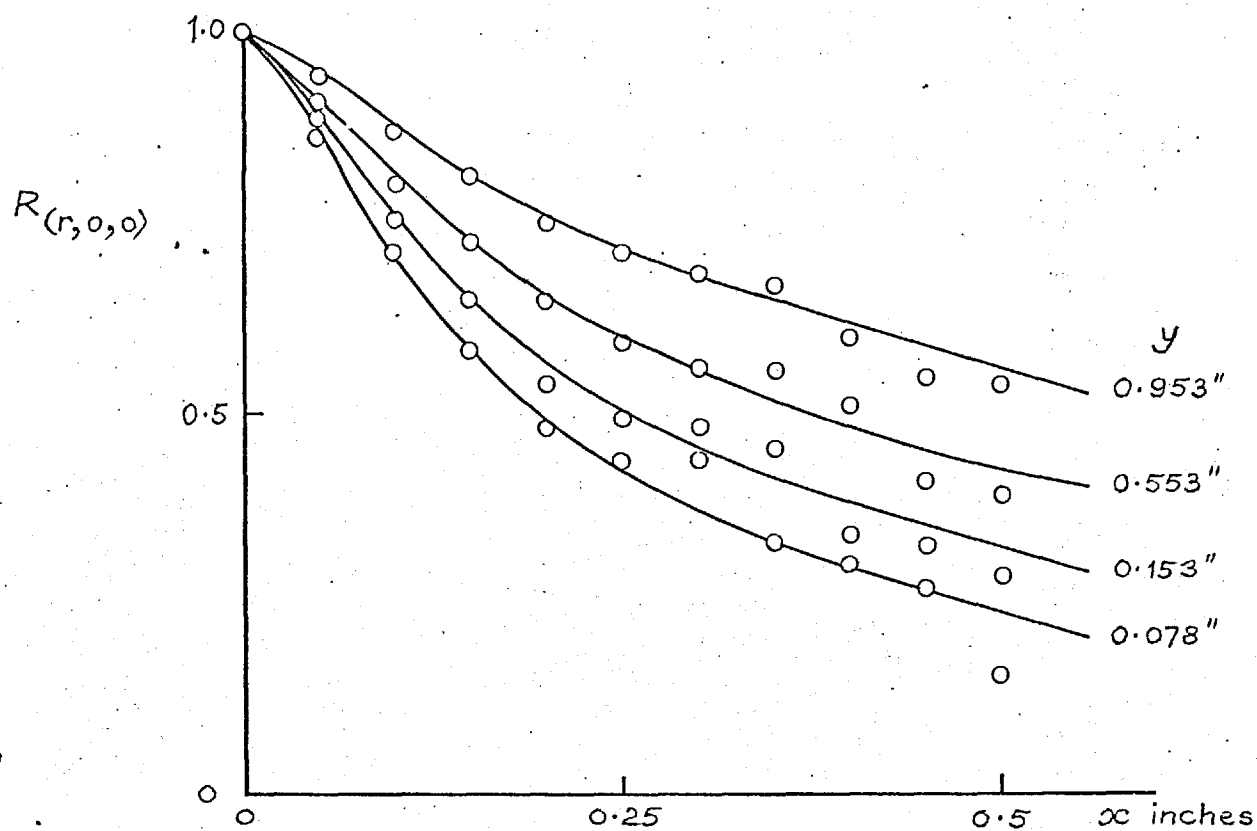


FIGURE 4.16: THE SPATIAL CORRELATION $R(r, 0, 0)$
OBTAINED FROM u -SPECTRA MEASUREMENTS AT
 x/y_c of 150 ; $\bar{u}_c/u_g = 2.3$, $t/y_c = 1.14$

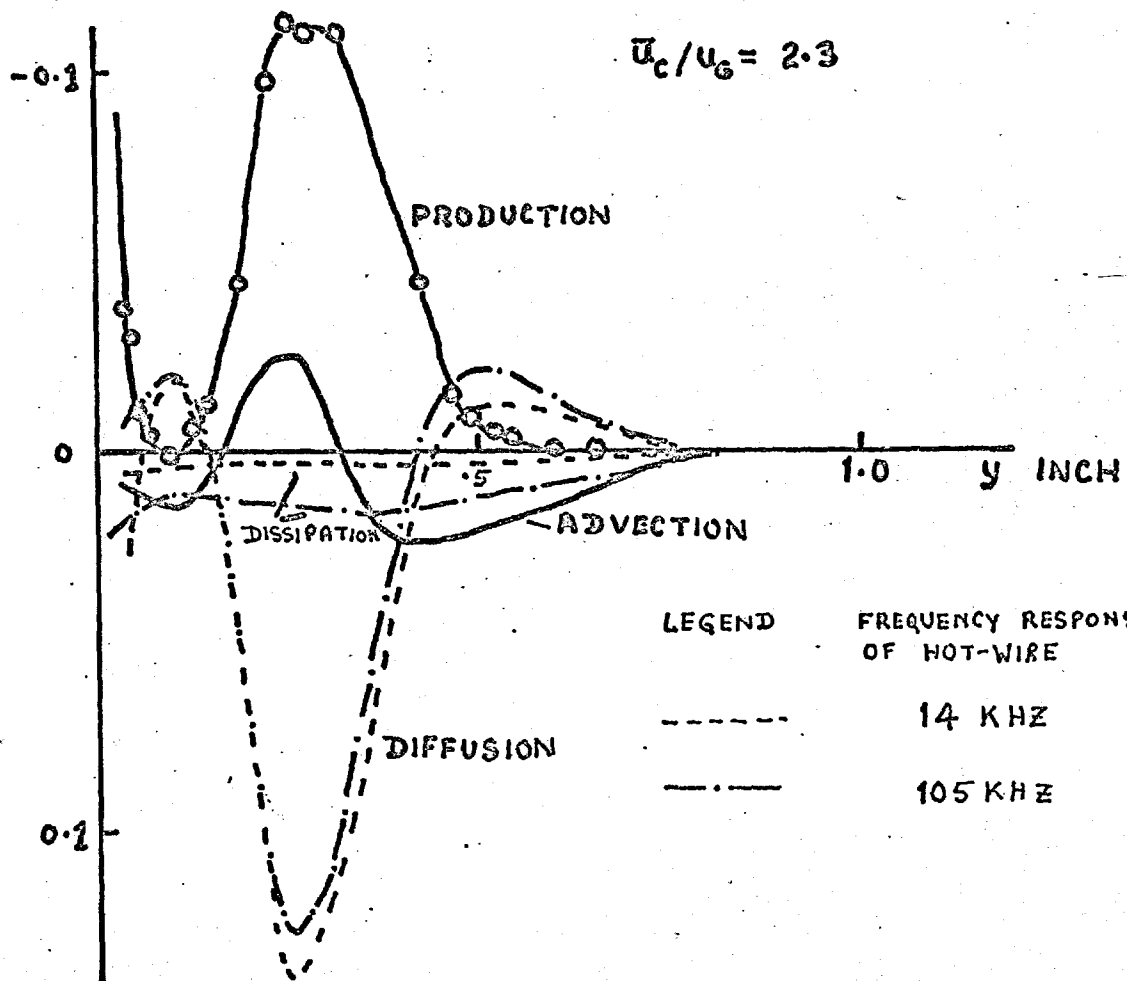
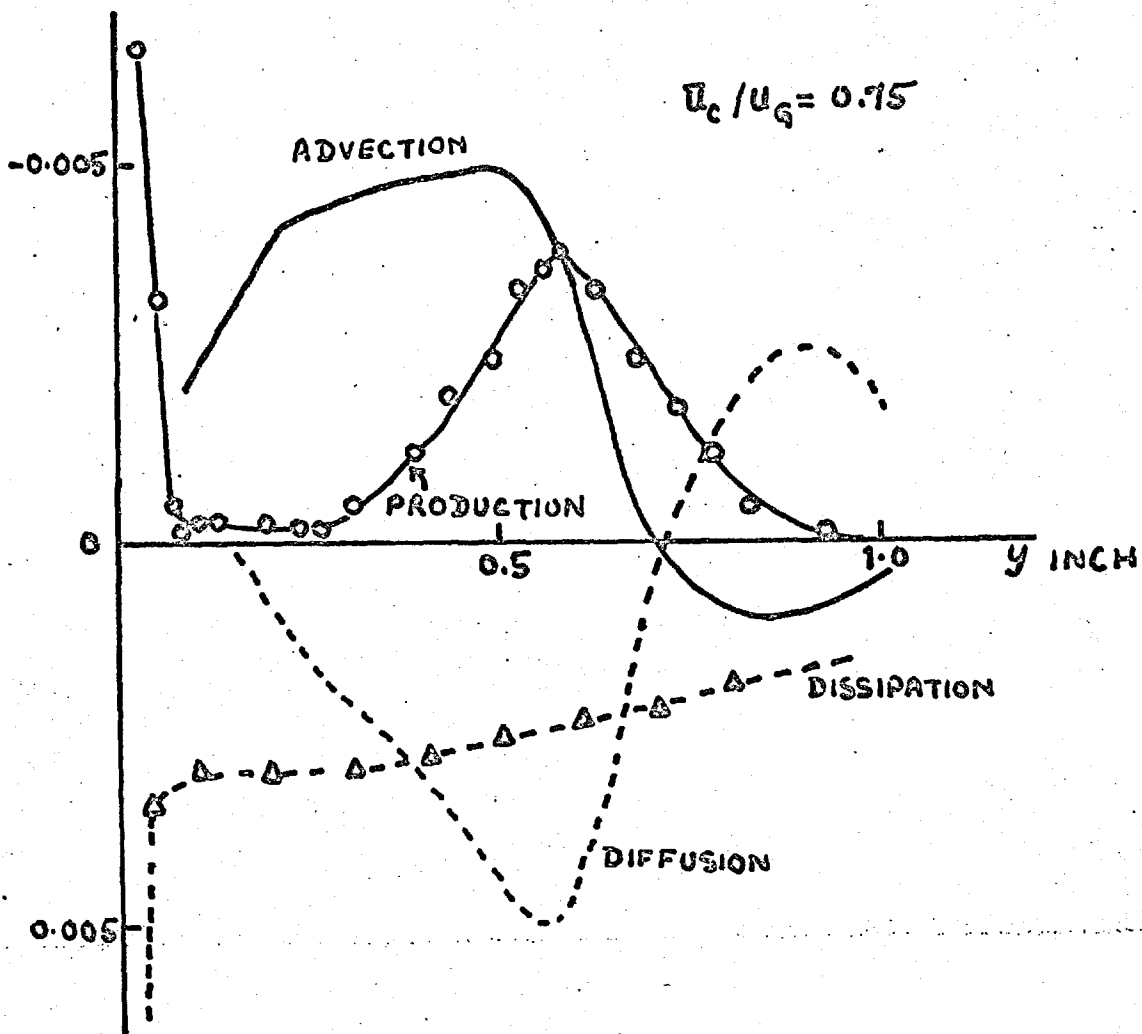


FIGURE 4.17 ENERGY BALANCE: $x/y_c = 10$, $t/y_c = 1.14$

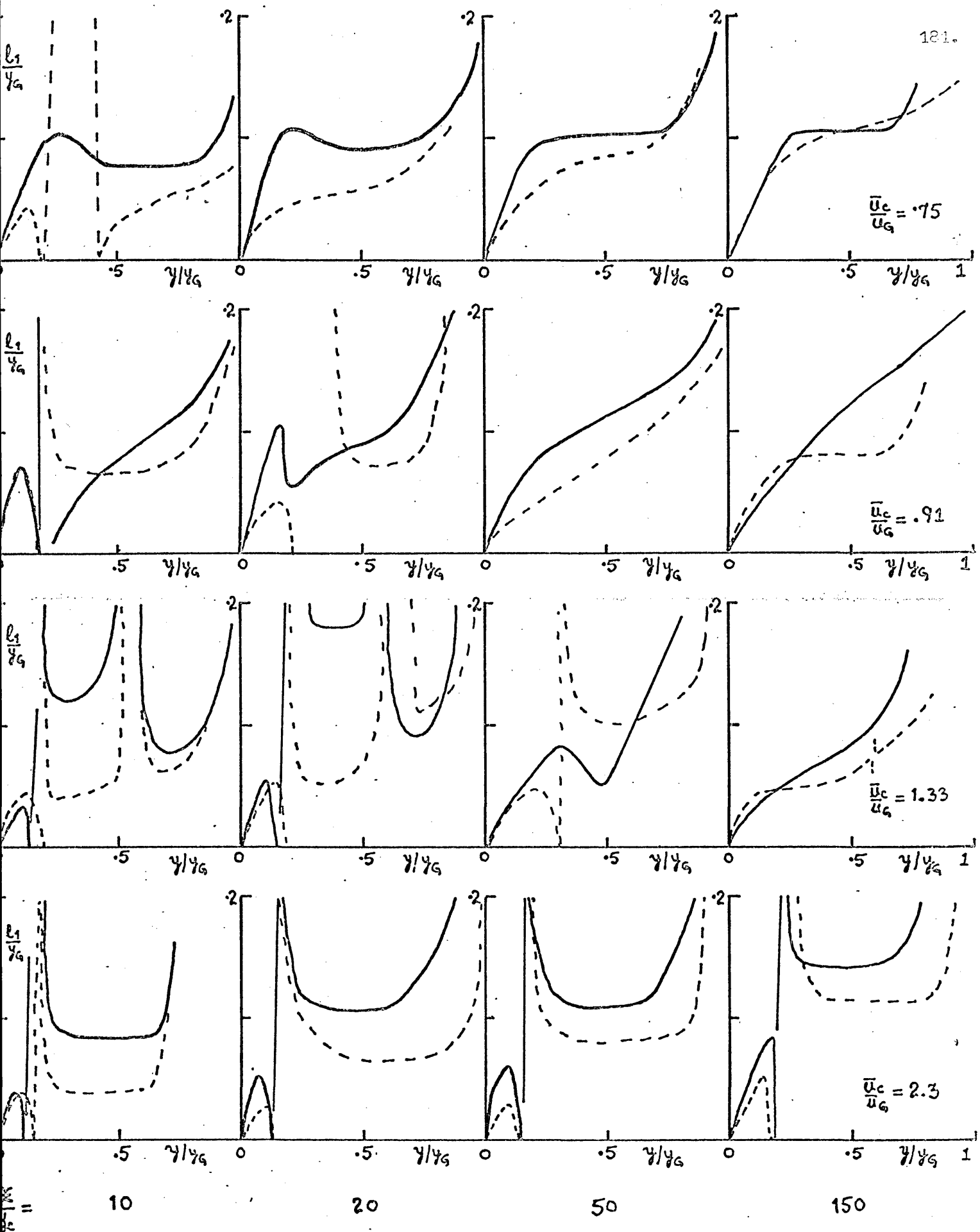


FIG.4.18 PROFILES OF PRANDTL MIXING LENGTH

— $k/\gamma_c = 1.14$
 - - - $t/\gamma_c = 0.126$

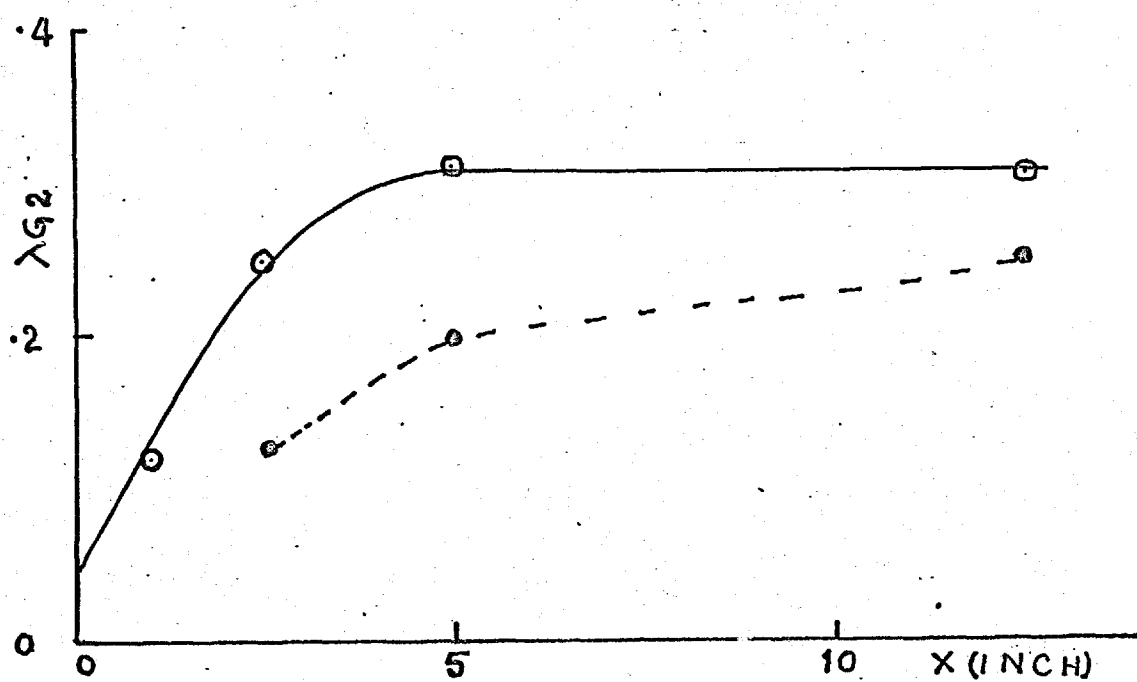
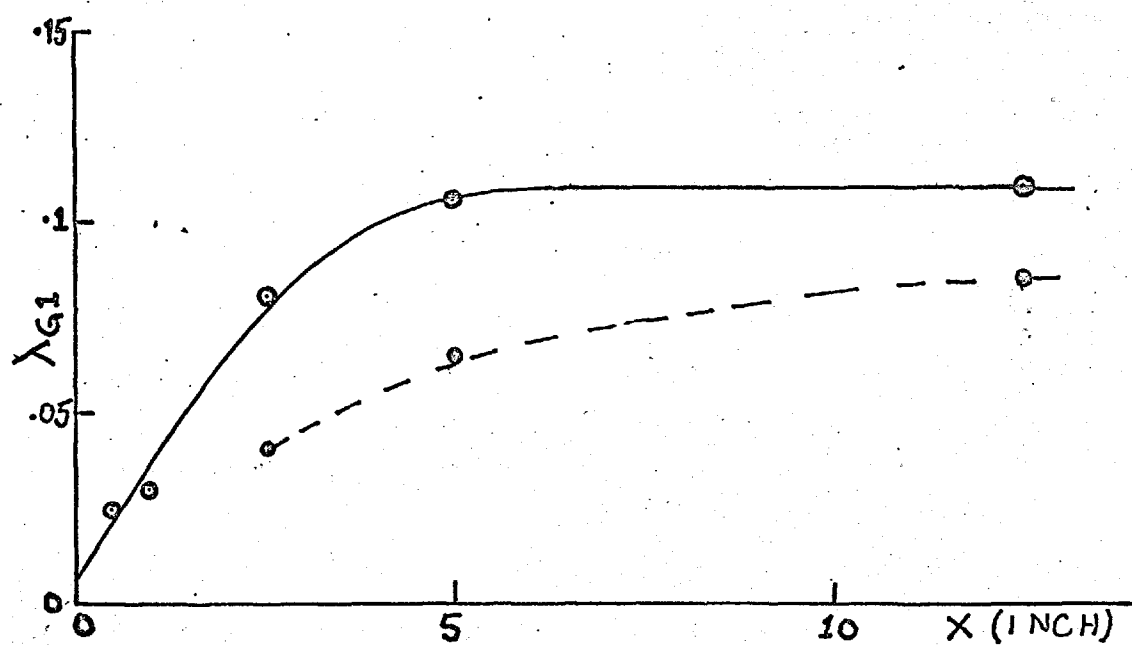


FIG.4.19 VARIATION OF CONSTANTS λ_{G1} AND λ_{G2} ; $\bar{u}_c/u_G=2.3$

— $t/\gamma_c = 1.14$ } SMOOTH CURVE THROUGH
 --- $t/\gamma_c = 0.126$ } EXPERIMENTAL POINTS
 ○ } EXPERIMENTAL POINTS

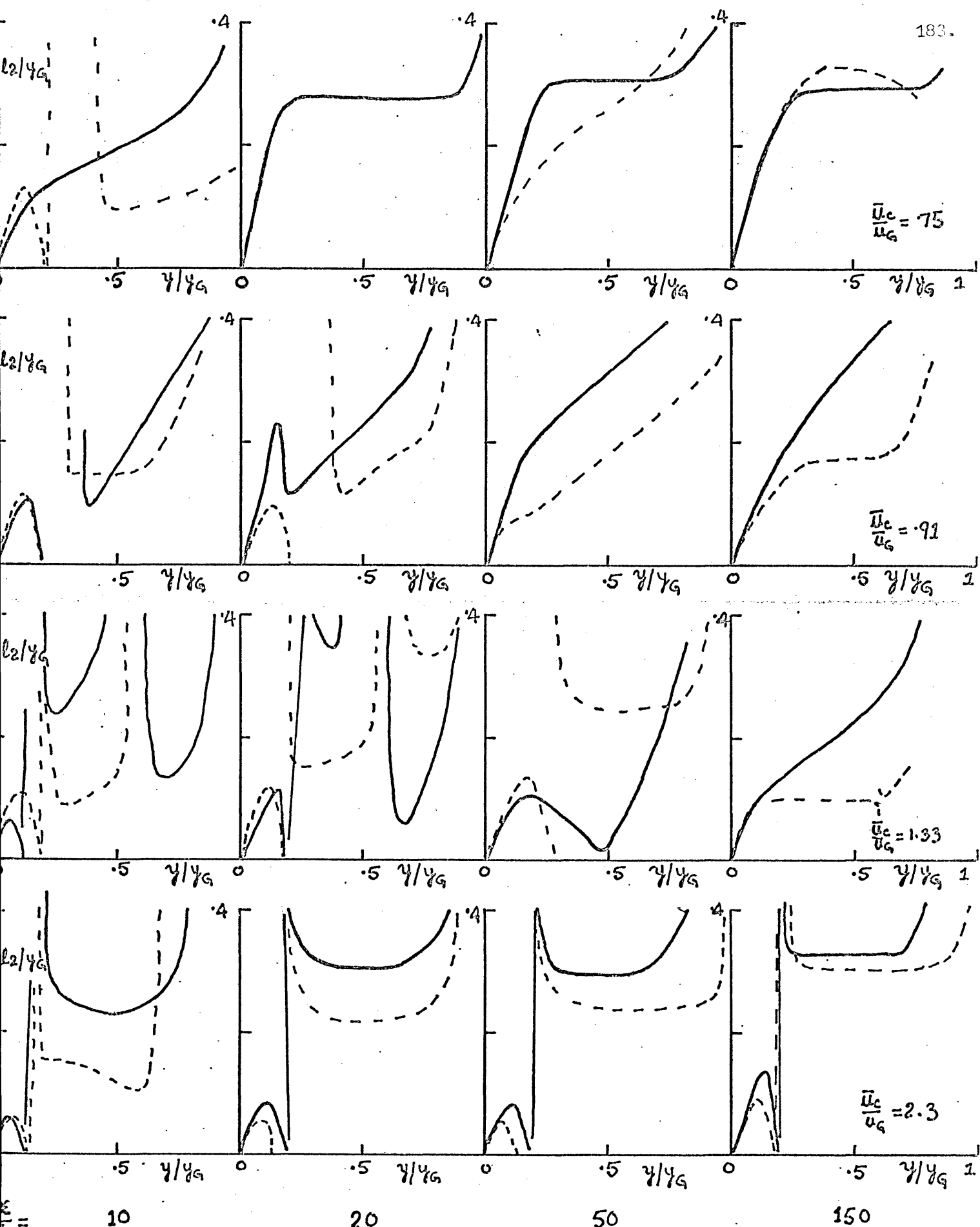


FIG. 4.20 PROFILES OF PRANDTL-KOLMOGOROV LENGTH SCALE

— $\frac{t}{y_c} = 1.14$
 - - - $\frac{t}{y_c} = 0.126$

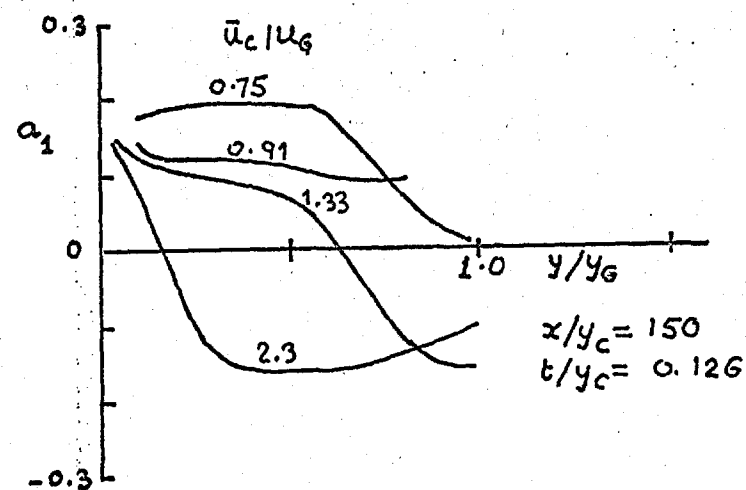
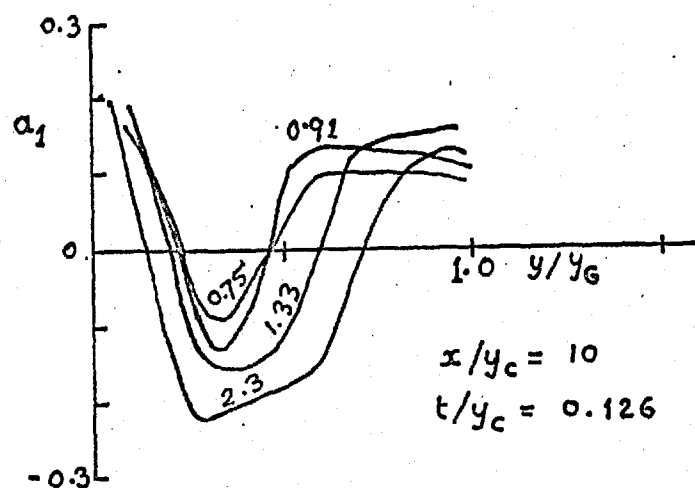
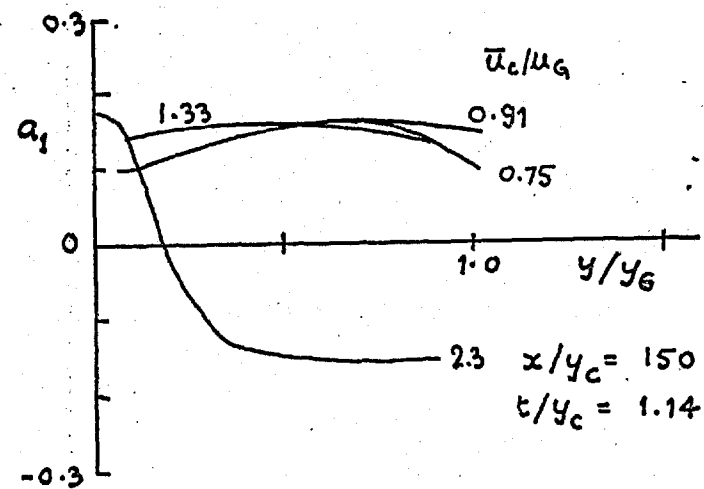
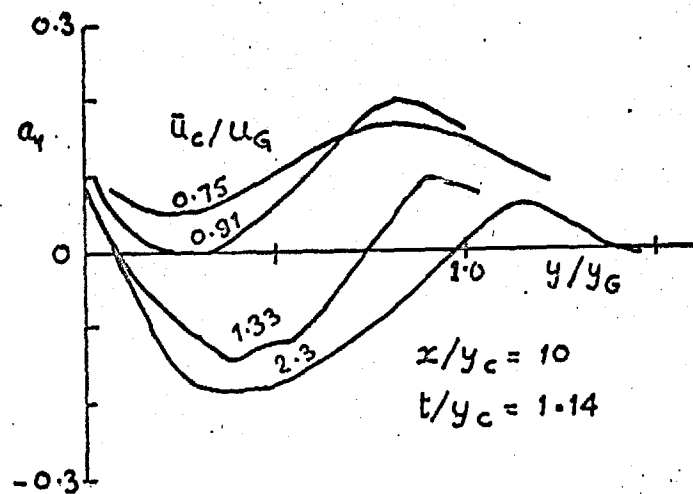


FIGURE 4.21 PROFILES OF BRADSHAW'S TURBULENT INTENSITY PARAMETER

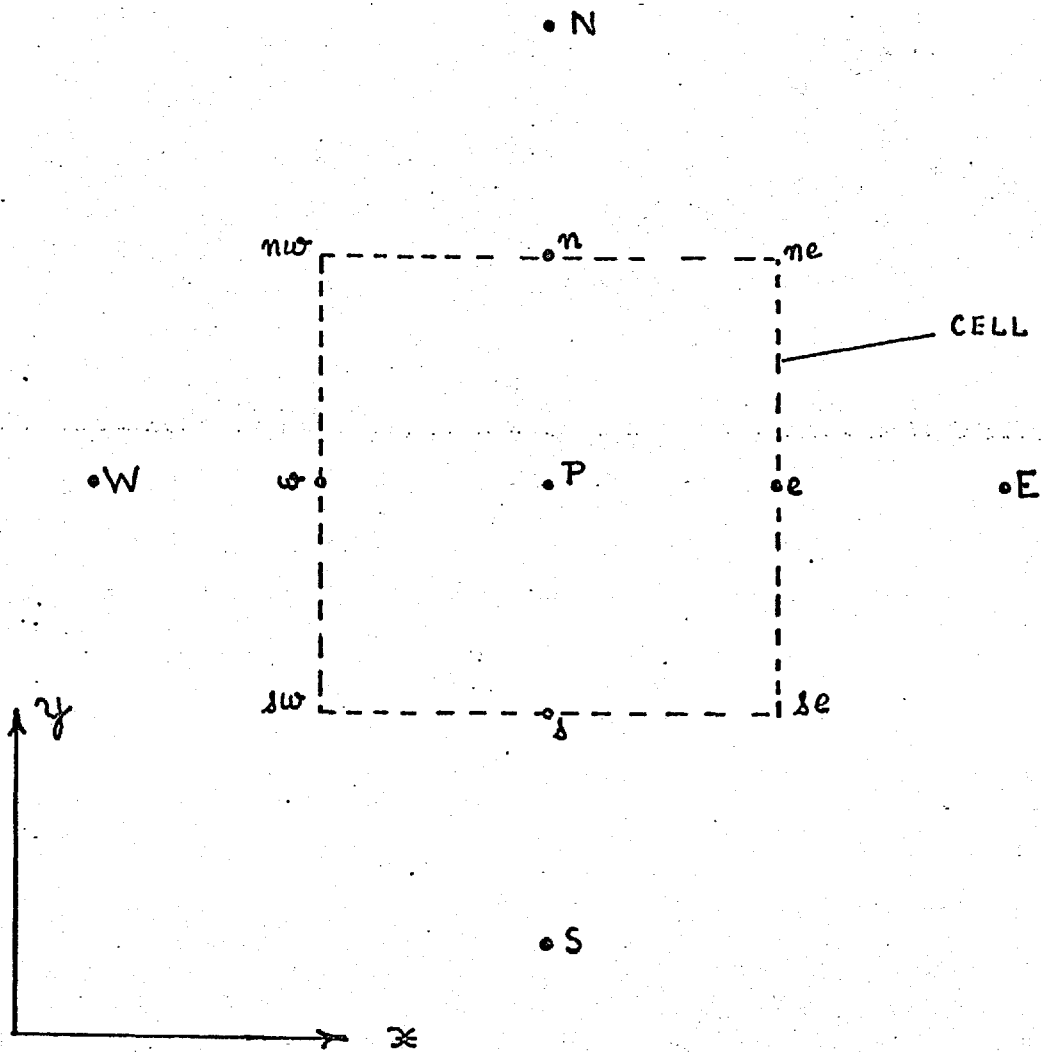


FIG. 5.1: A CONTROL CELL OR TANK AT POINT 'P'

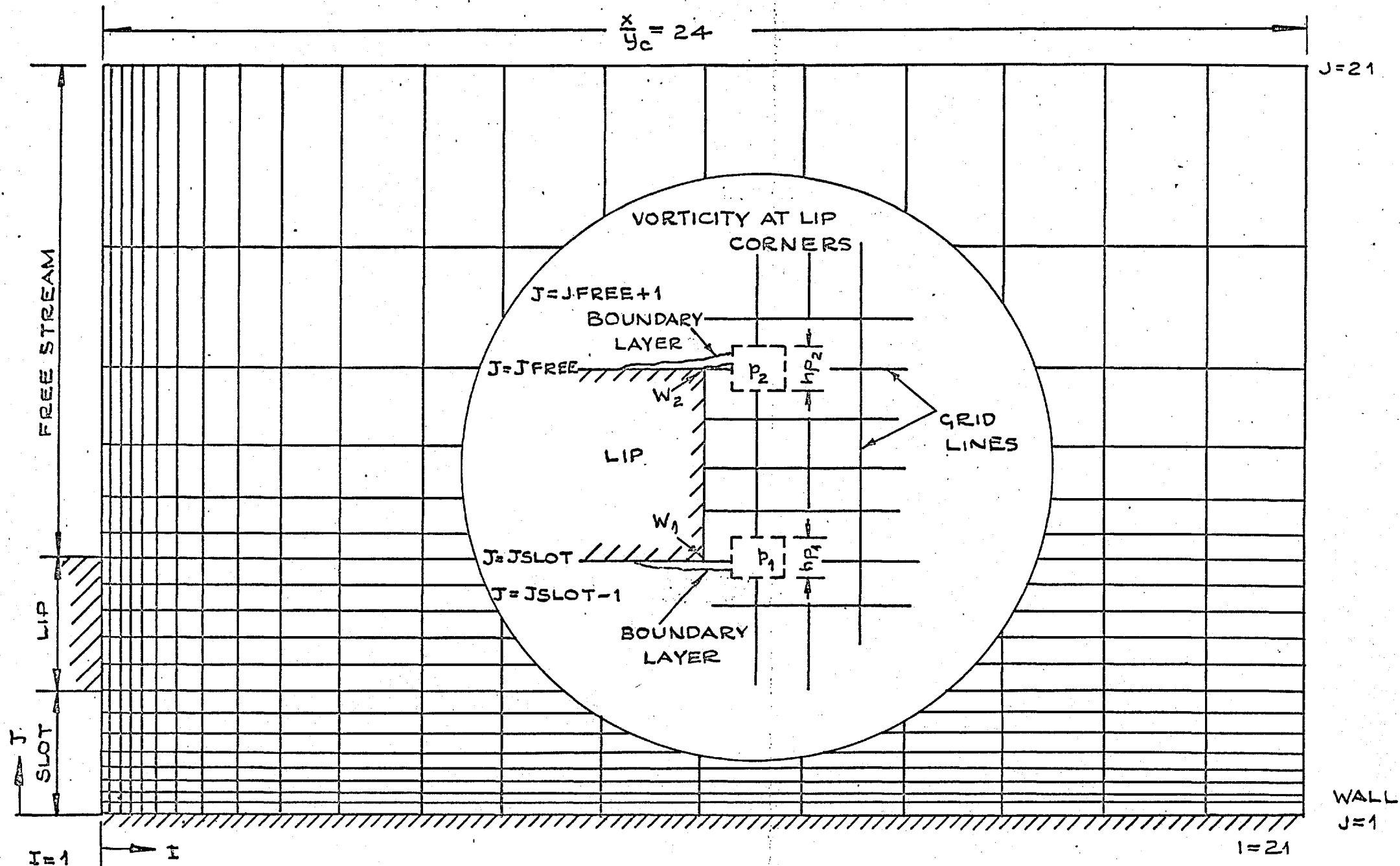


FIG. 5.2 A TYPICAL GRID SYSTEM FOR FINITE-DIFFERENCE SOLUTION

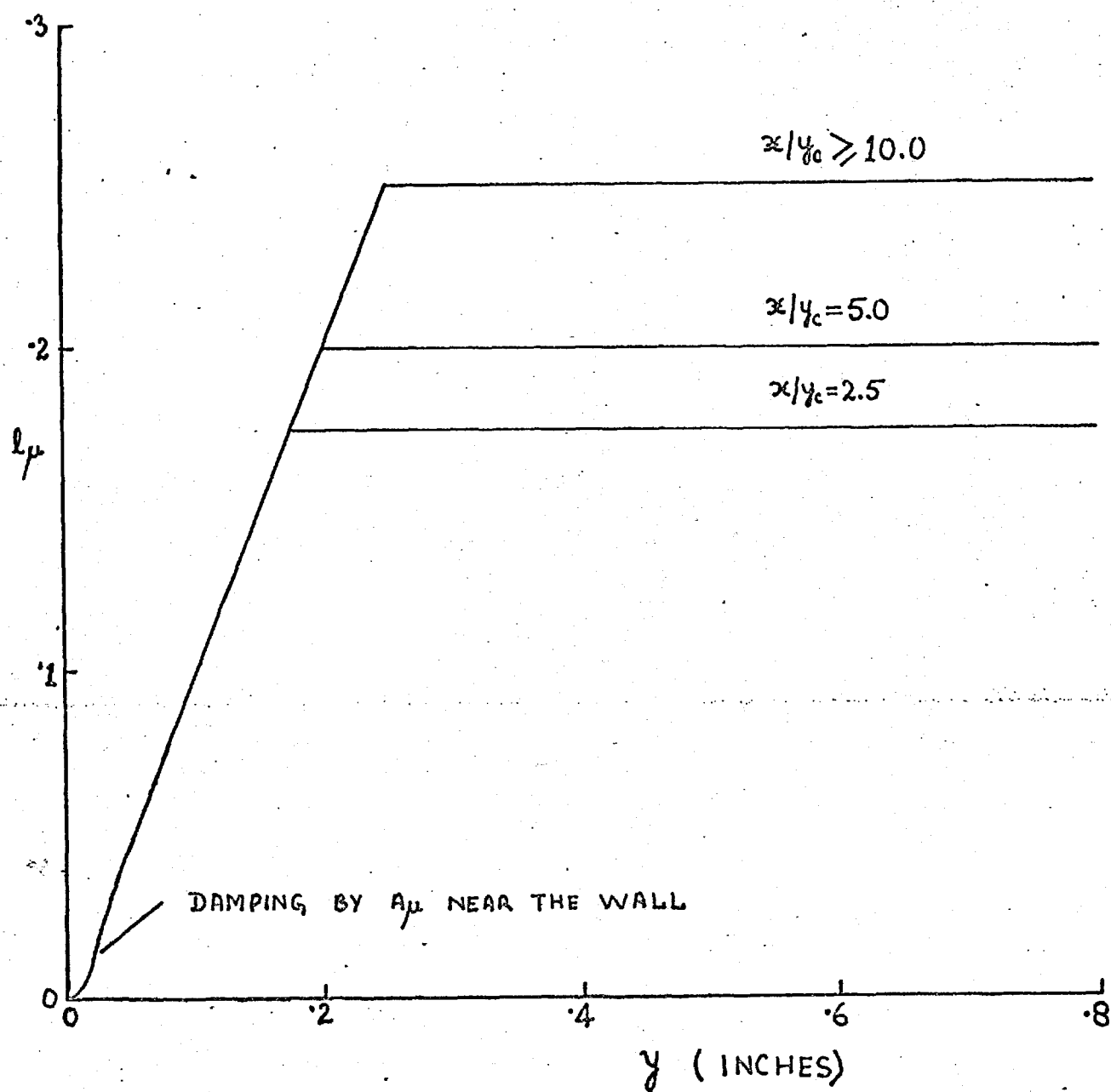


FIG. 5.3 DISTRIBUTION OF l_μ IMPLIED BY EQUATIONS 5.10-2 AND 5.10-3 FOR $t/y_c = 1.14$; $y_G = 1.0''$

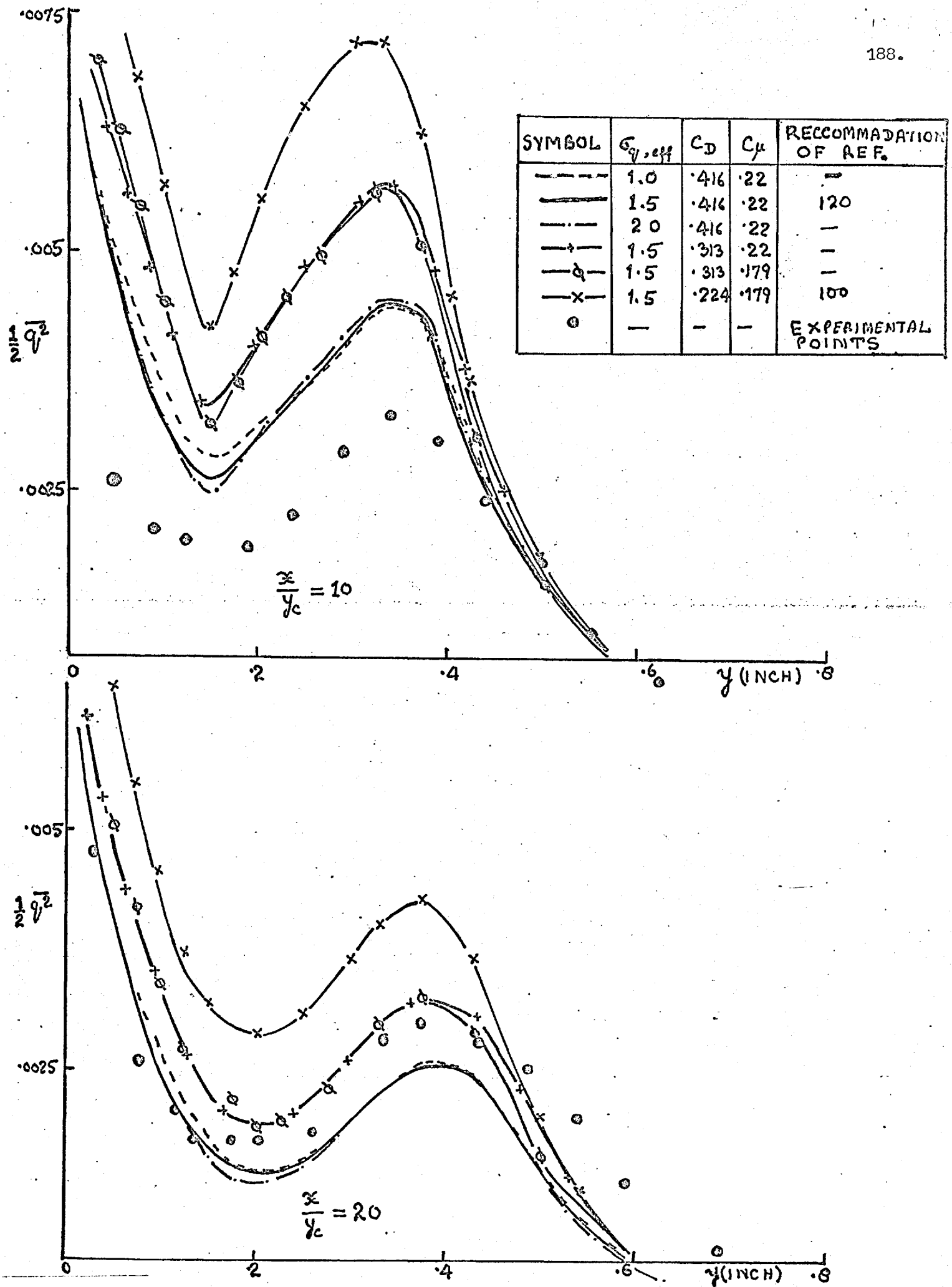


FIG. 5.4-a: INFLUENCE OF CHANGES IN CONSTANTS ON KINETIC ENERGY ($\bar{q}^2$) PROFILE ; $\bar{u}_c/u_c = 0.75$; $t/y_c = 0.126$

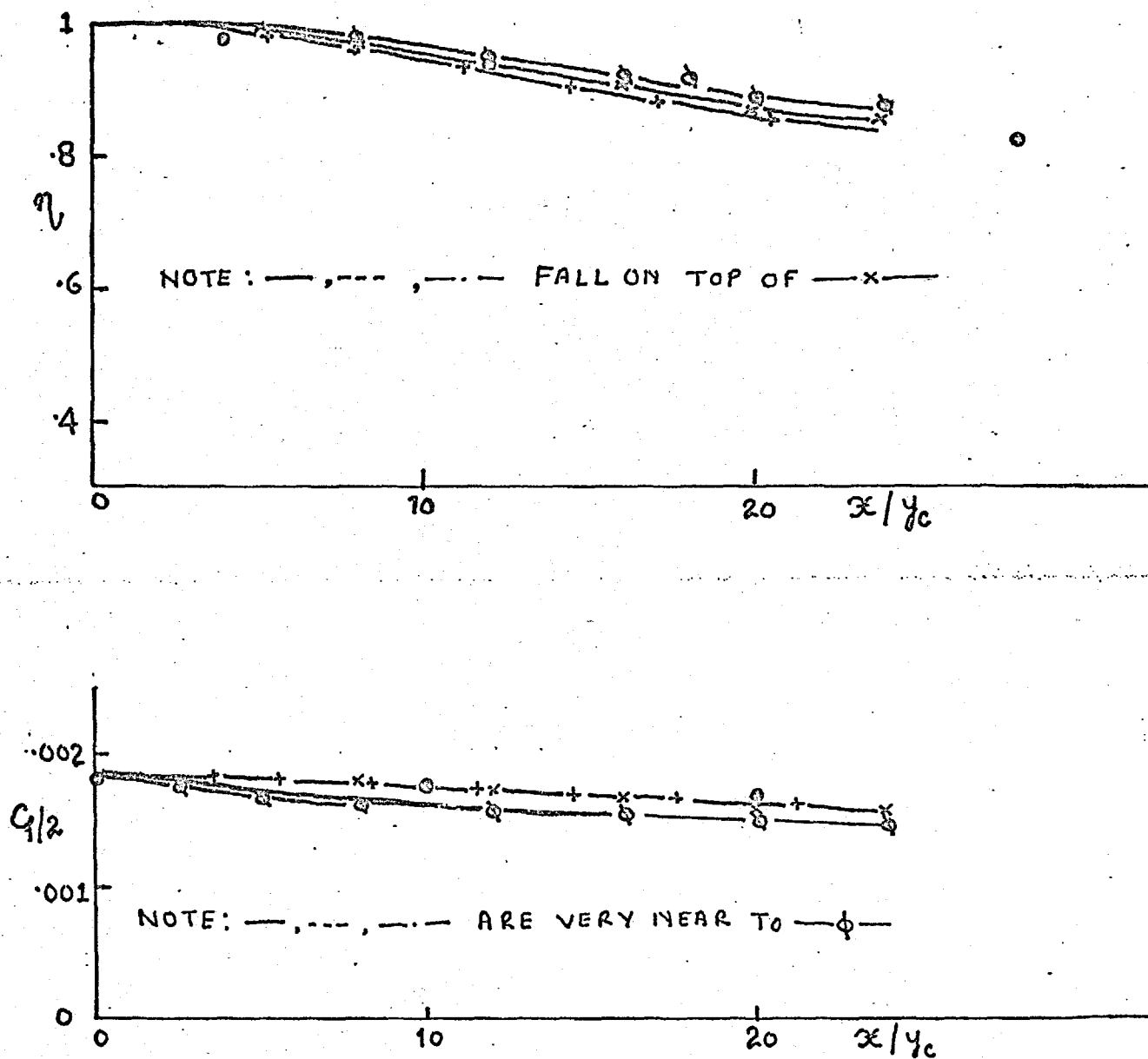


FIG. 5.4b INFLUENCE OF CHANGES IN CONSTANTS ON $C_{1/2}$ AND η ; $\bar{u}_c/u_0 = 0.75$, $t/y_c = 0.126$
 SYMBOLS SAME AS ON FIG. 5.4-a

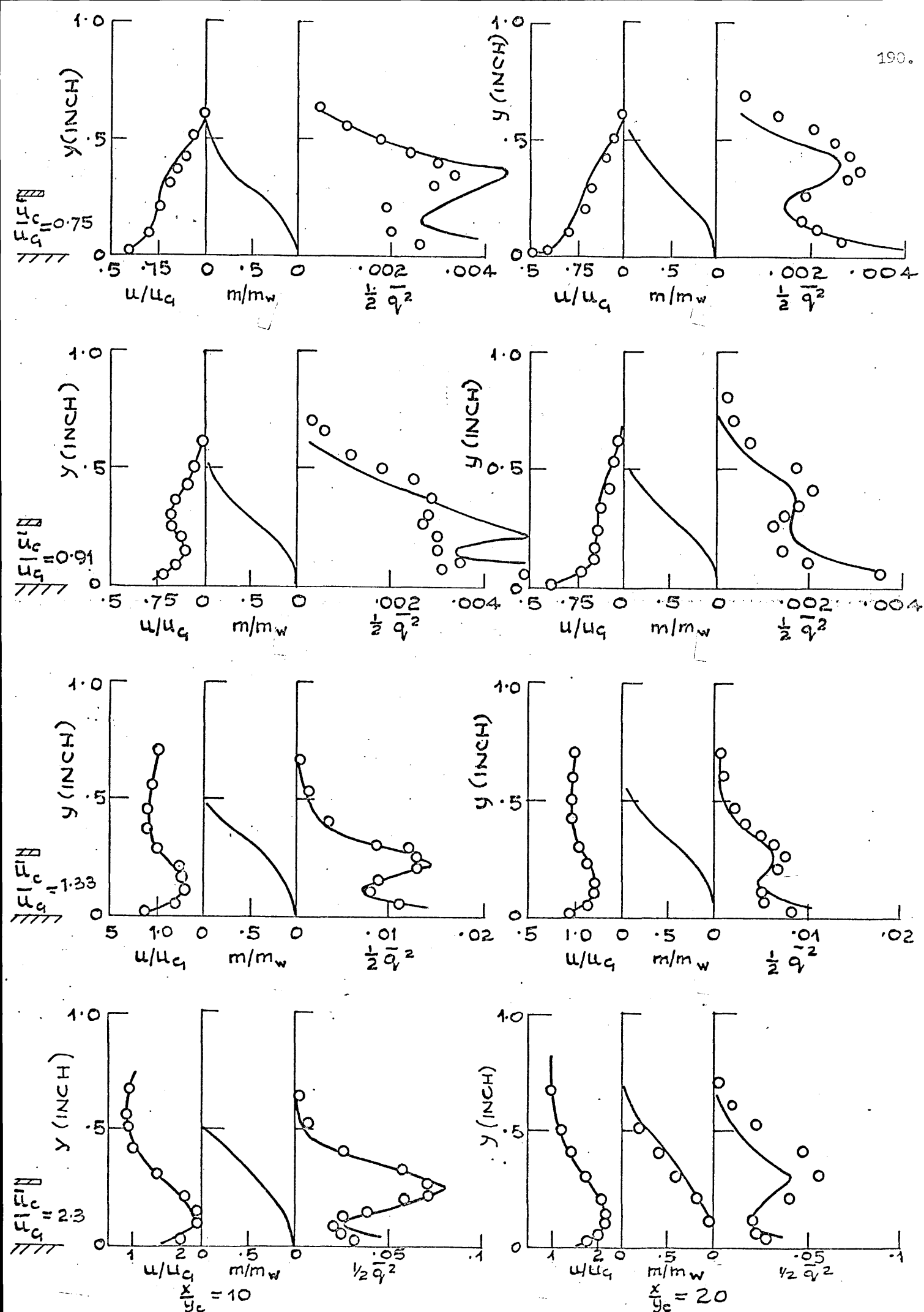


FIG. 5.5 PREDICTED AND MEASURED u/u_c , m/m_w AND q^2 PROFILES, $t/y_c = 0.126$, $\delta = .95$, $y_c = 0.25$ inches; O MEASUREMENTS OF REF. (50), — PREDICTIONS.

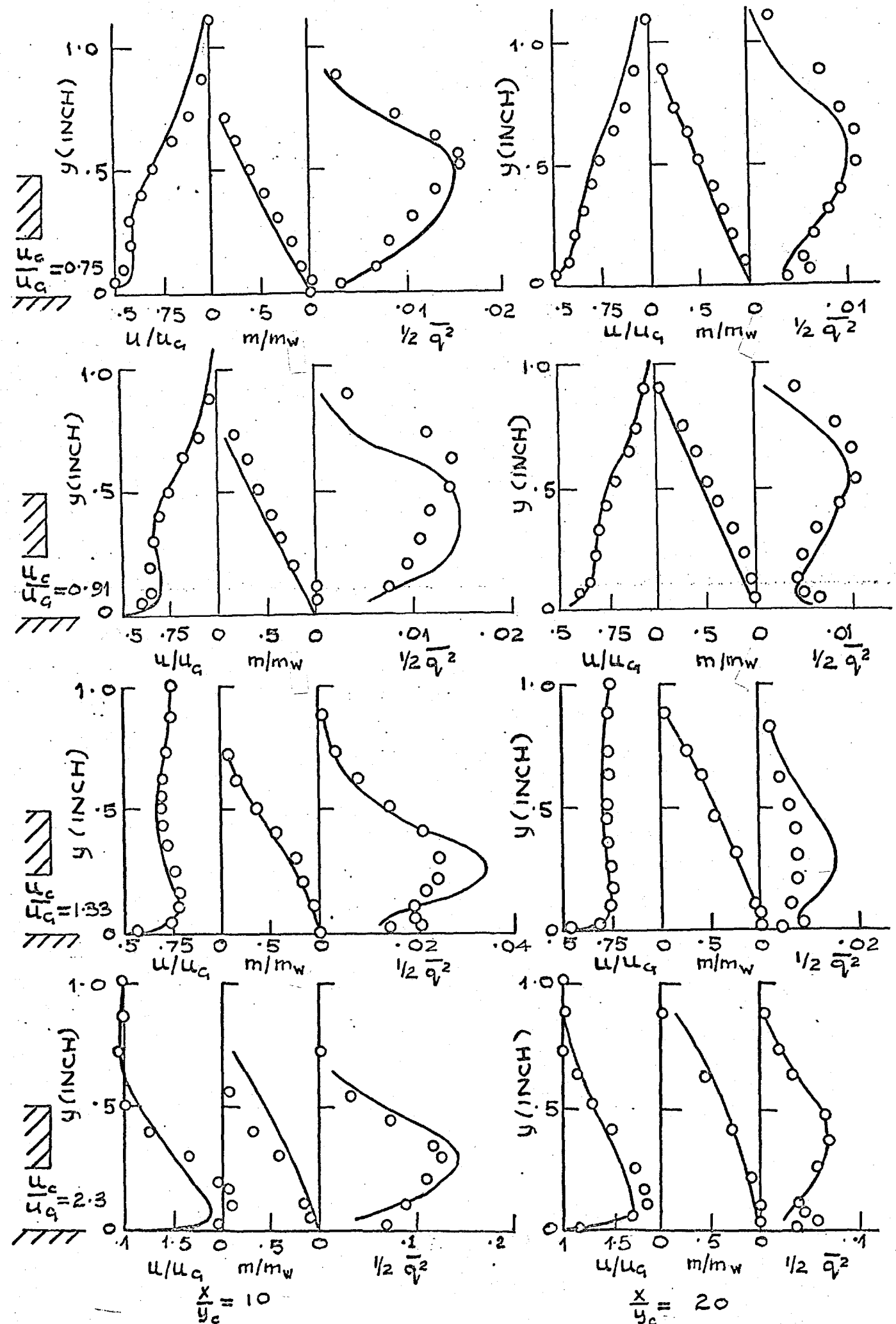


FIG. 5.6 PREDICTED AND MEASURED u/u_c , m/m_w AND $\bar{q}^2$ PROFILES, $\delta/y_c = 1.52$; $t/y_c = 1.14$, $y_c = 0.25$; O MEASUREMENTS OF REF. (50) — PREDICTIONS.

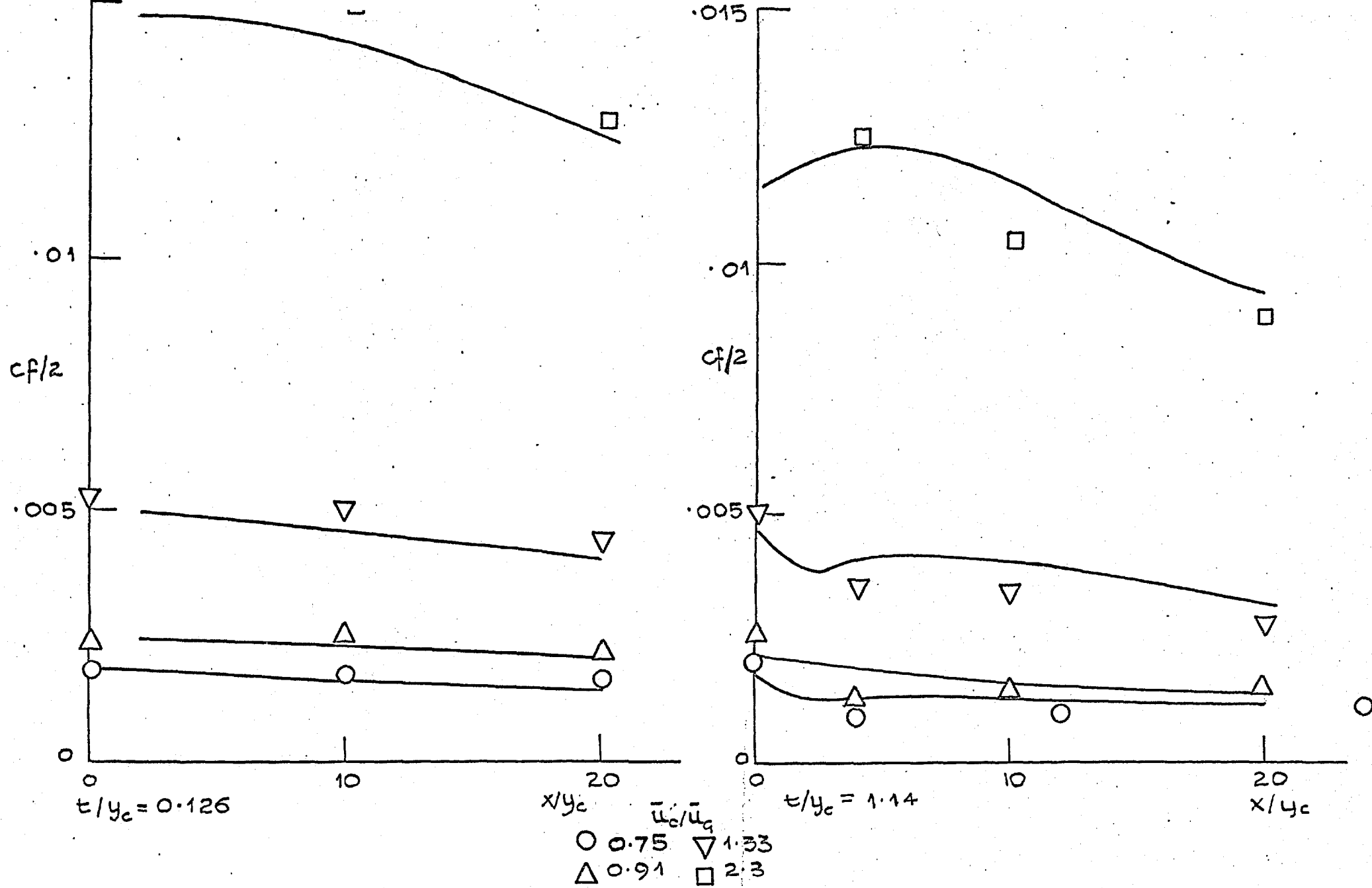


FIG. 5.7 SKIN FRICTION DISTRIBUTION. — PREDICTIONS, ($\bigcirc, \triangle, \nabla, \square$) MEASUREMENTS OF REF. 48 AND 50.

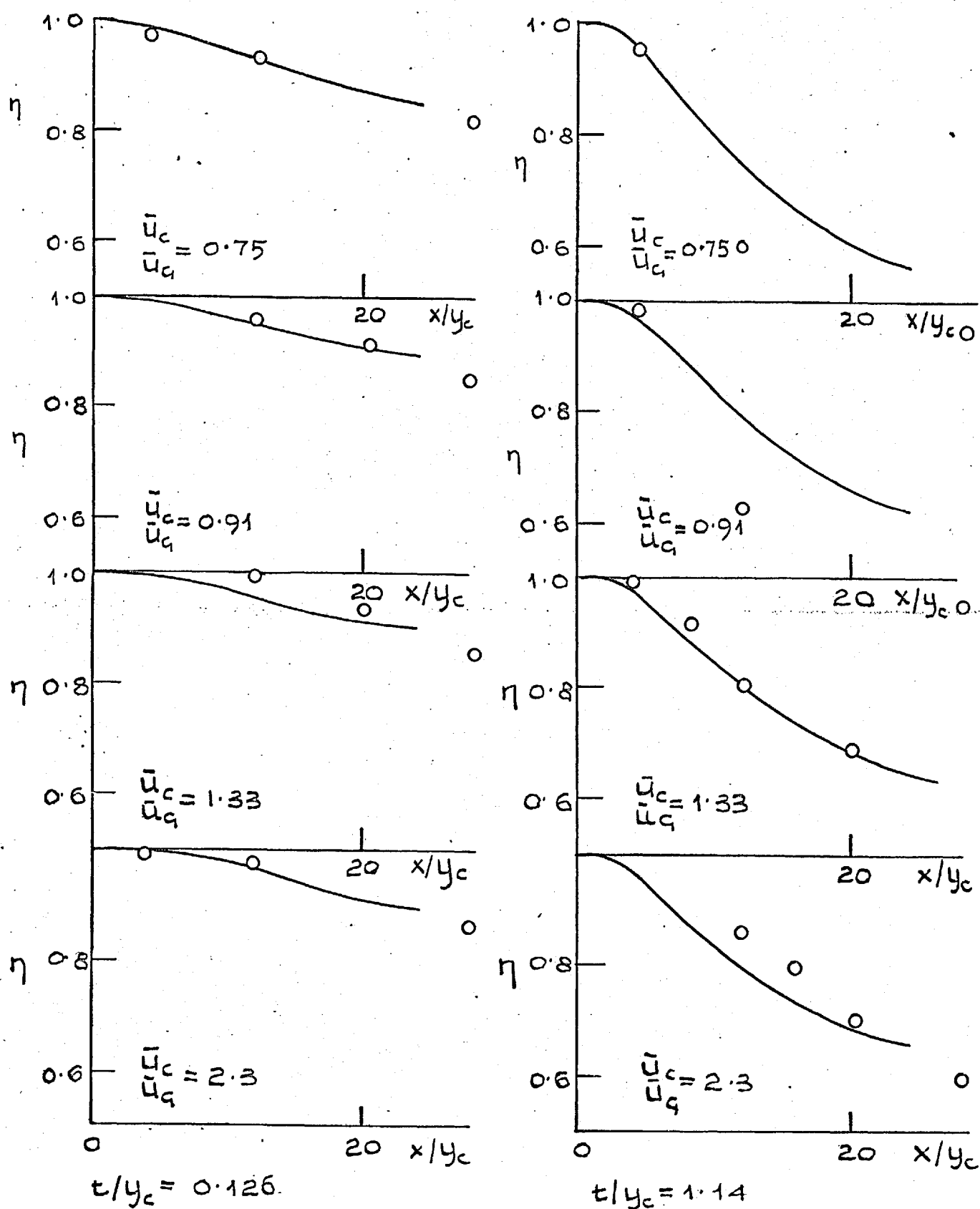


FIG. 5.8 EFFECTIVENESS DISTRIBUTION
 — PREDICTIONS
 ○ MEASUREMENTS OF REF. 47

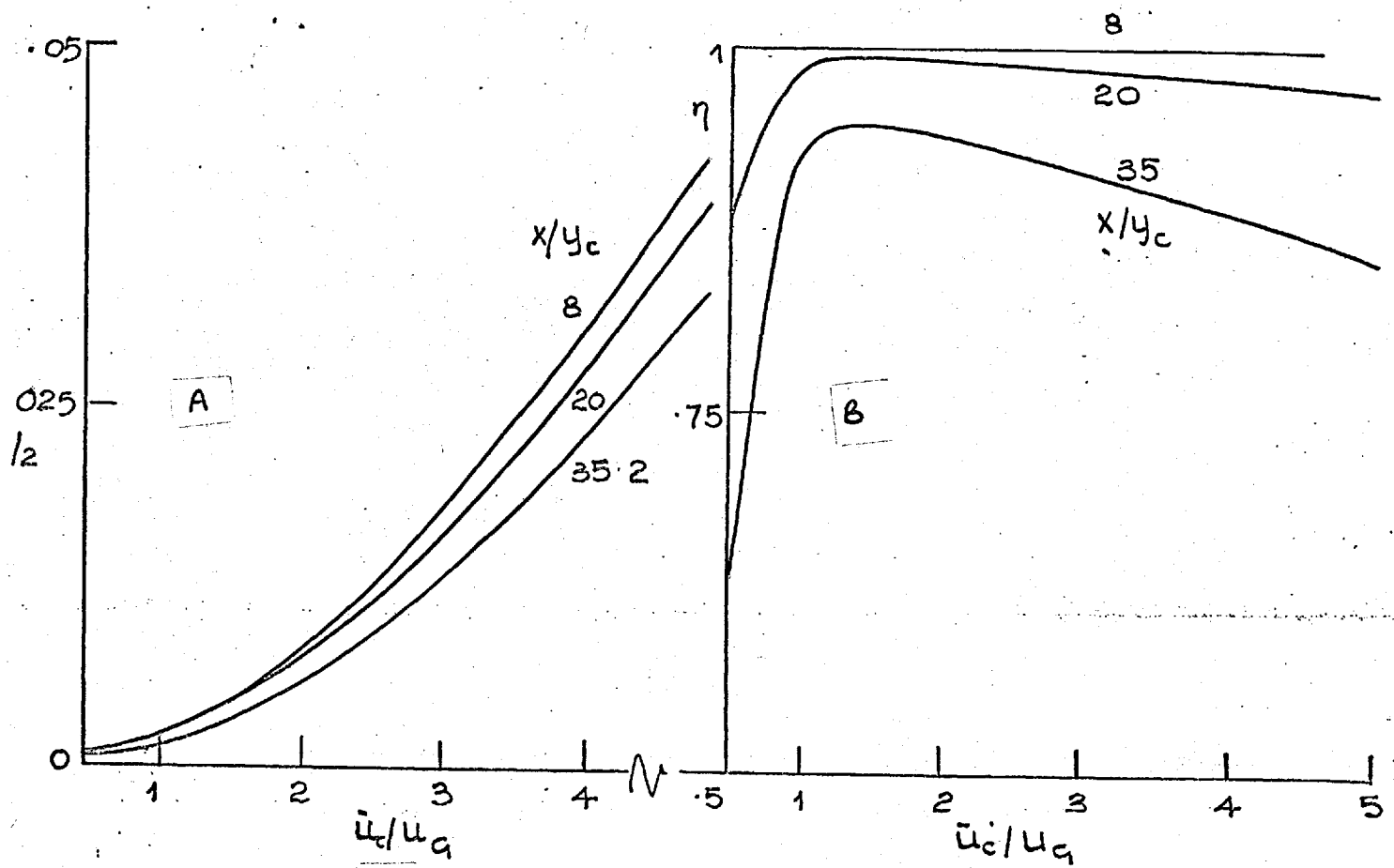
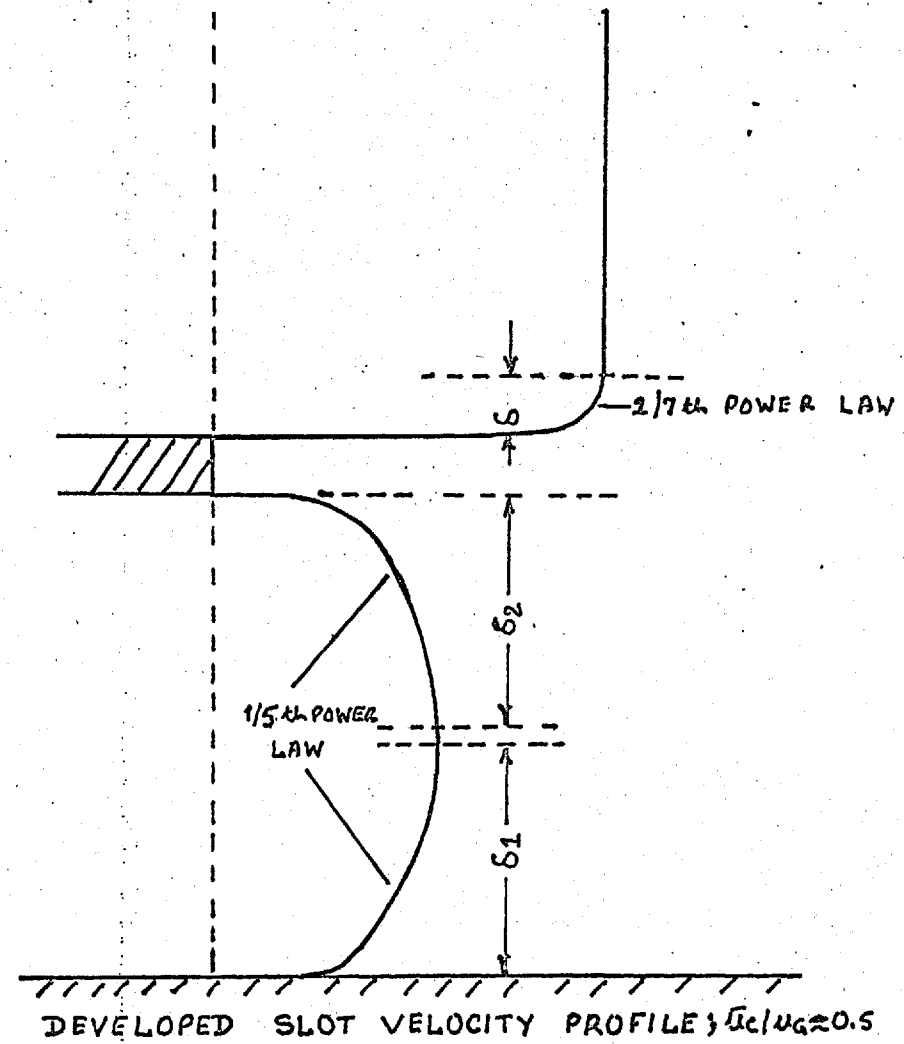
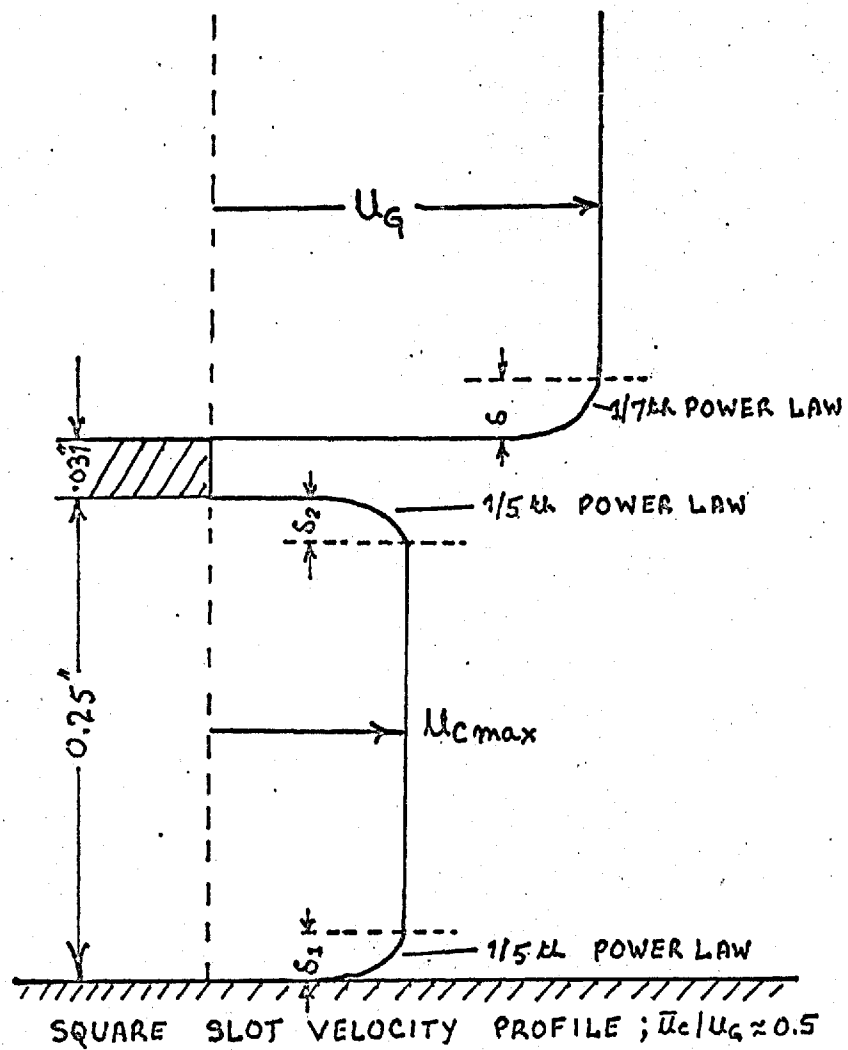


FIG. 5.9 EFFECT OF SLOT VELOCITY RATIO
ON $C_f/2$ AND η ; $t/y_c = 0.126$; $\delta/y_c = 0.12$;
 $y_c = 0.25''$



δ_1 , δ_2 AND δ REPRESENT BOUNDARY LAYER THICKNESSES

FIG.5.10 SLOT VELOCITY PROFILES

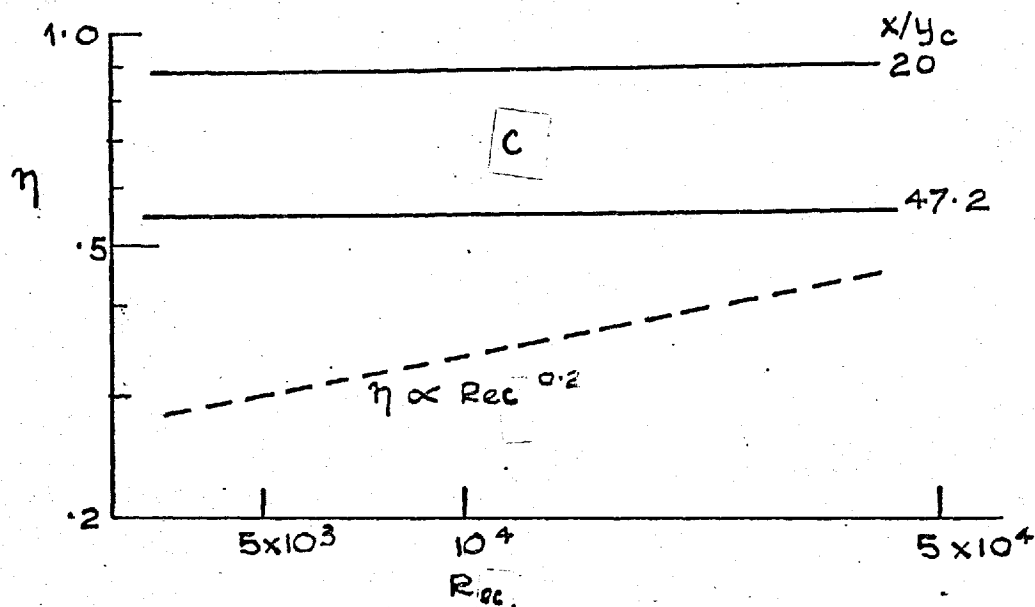
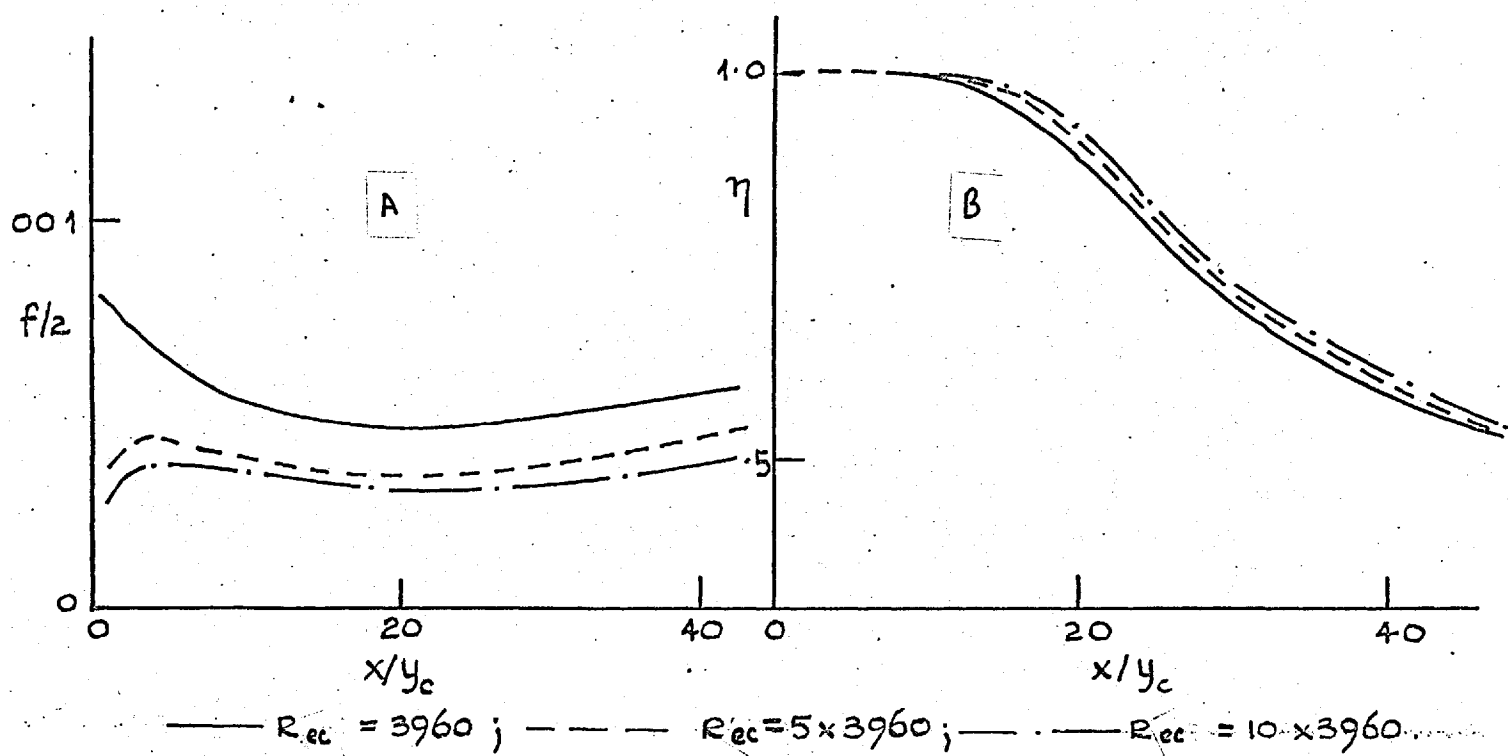
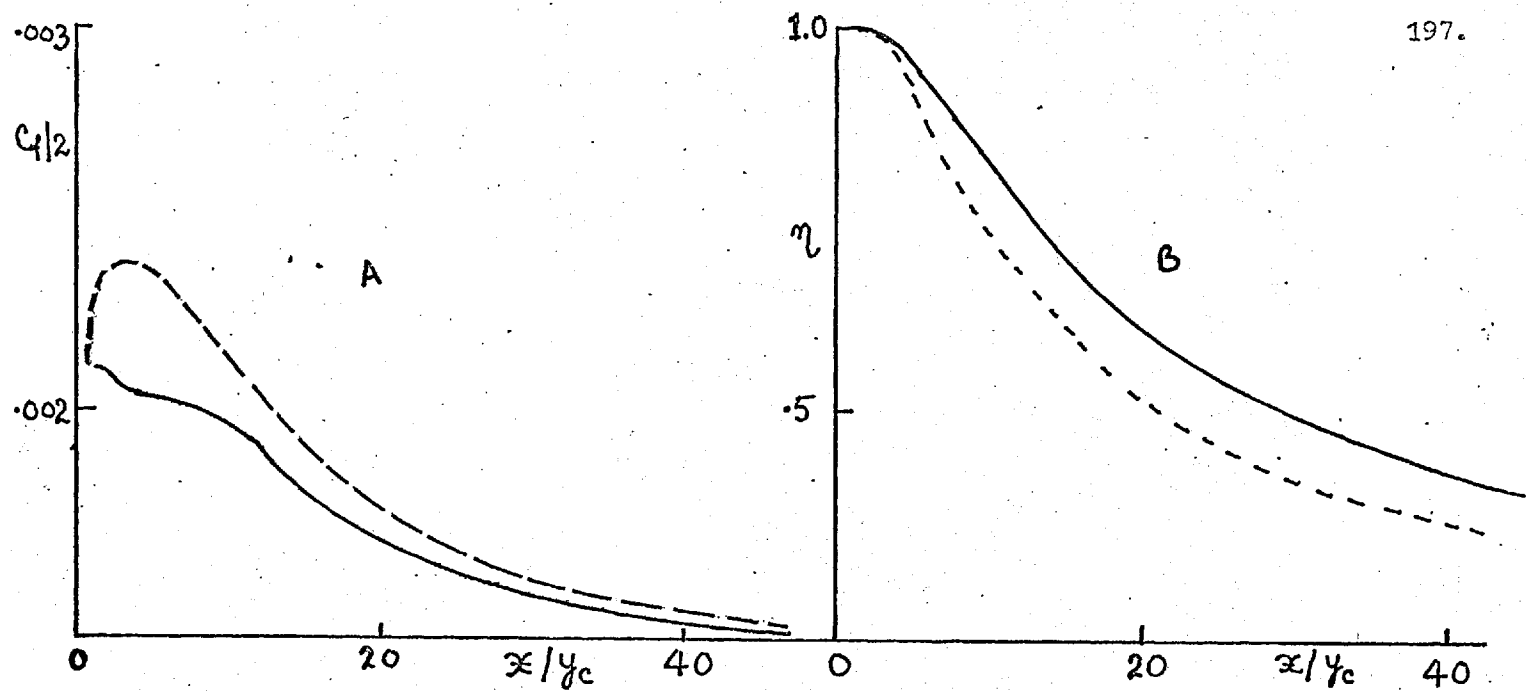
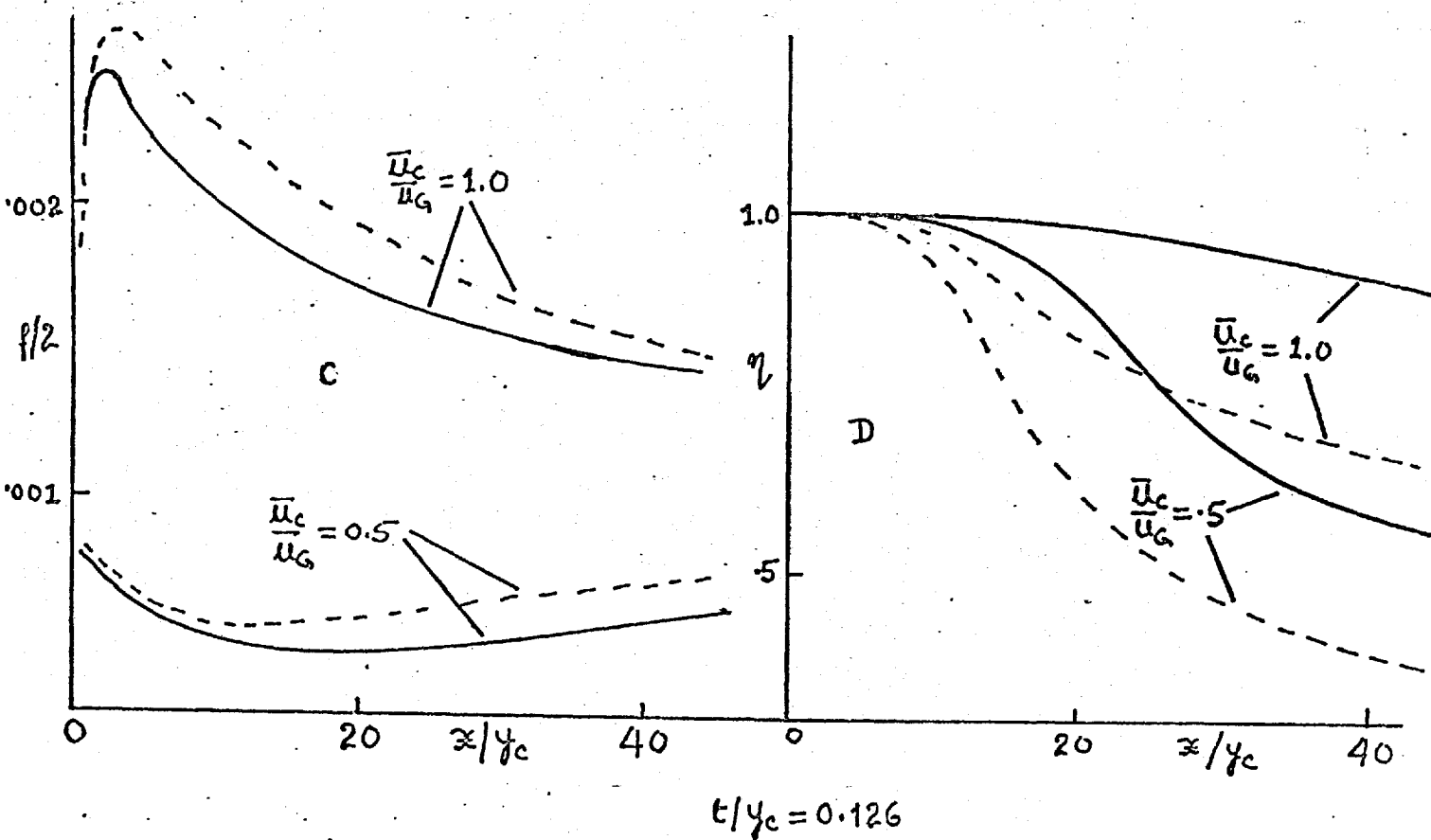


FIG. 5. EFFECT OF SLOT REYNOLDS NUMBER ON $Cf/2$ AND η ; $\bar{u}_c/u_c = 0.5$; $t/y_c = 0.126$; $\delta/y_c = 0.12$; $y_c = 0.25''$



$$\bar{u}_c / u_G = 1.0 ; t/y_c = 1.14$$



$$t/y_c = 0.126$$

FIG. 5.12 EFFECT OF UPPER LIP BOUNDARY LAYER THICKNESS
(— $\delta/y_c = 0.12$; ---- $\delta/y_c = 4$) ; $y_c = 0.25''$

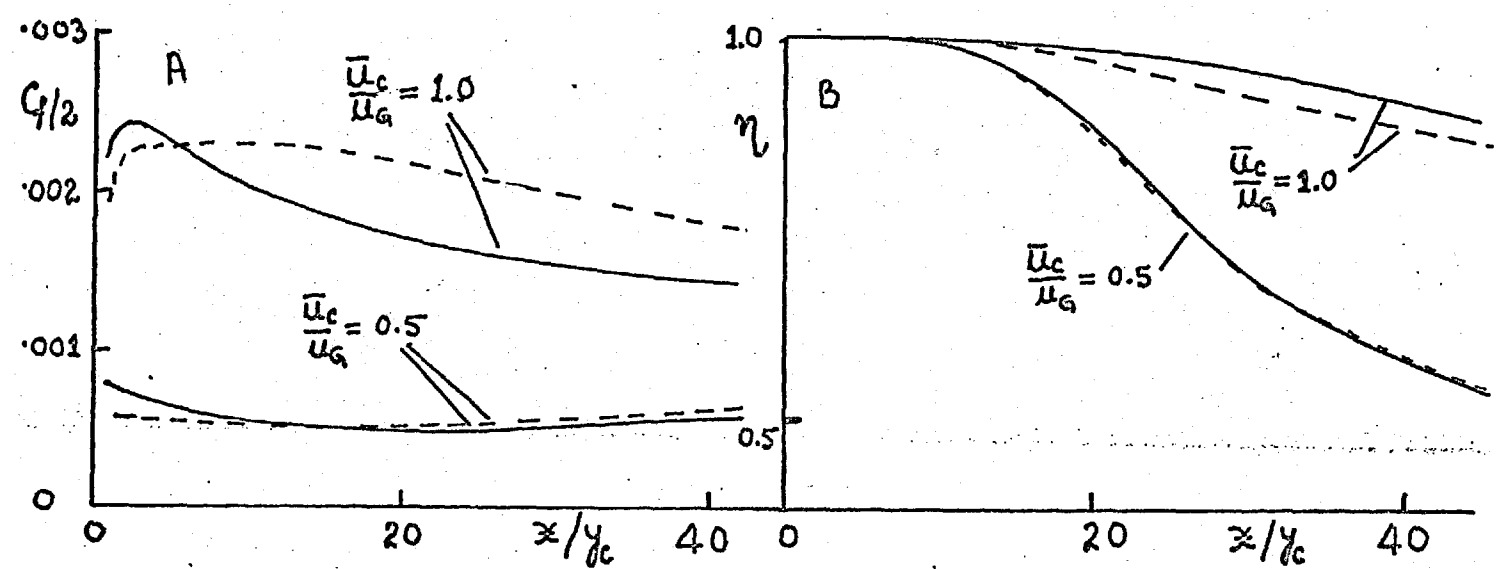


FIG. 5.13 EFFECT OF SLOT VELOCITY PROFILE; (—SQUARE VELOCITY PROFILE, ----DEVELOPED VELOCITY PROFILE); $t/y_c = 0.126$
 $\delta/y_c = 0.12$

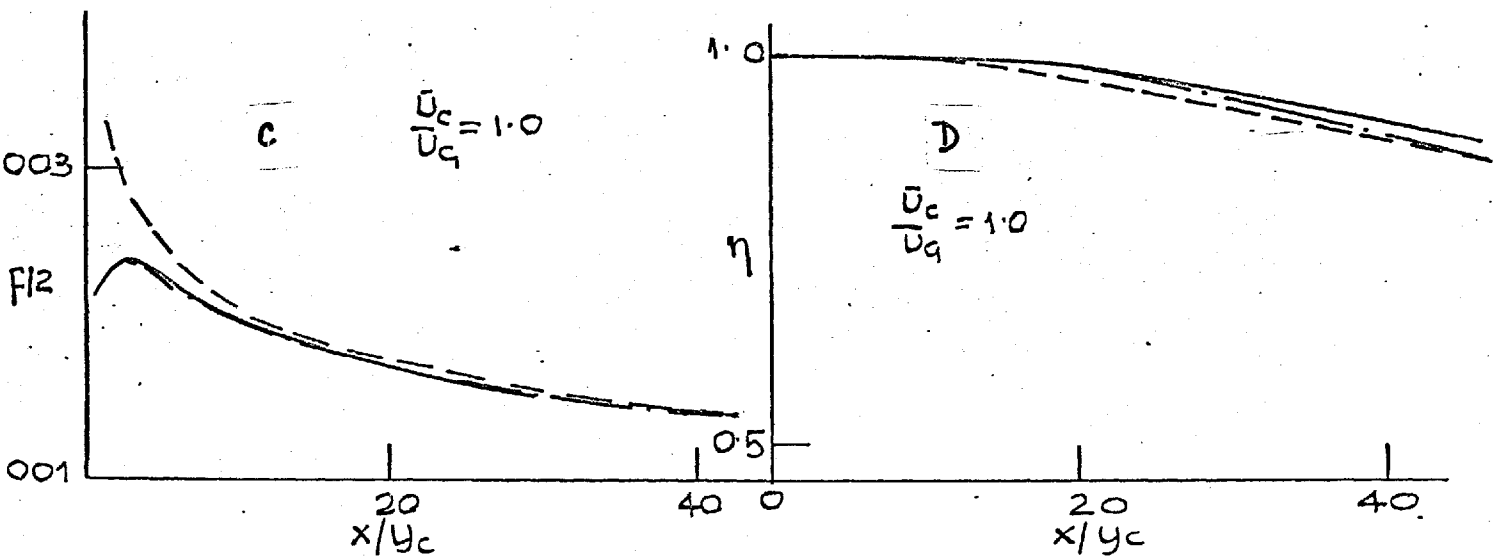
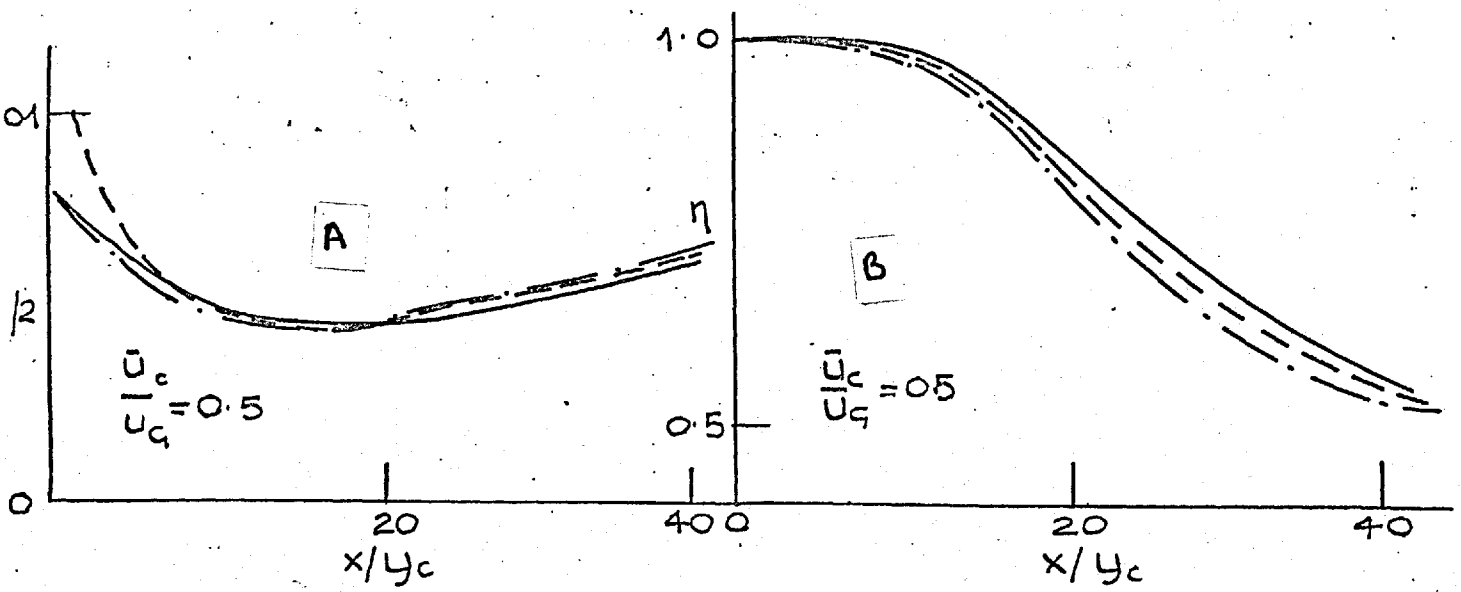


FIG. 5.4 EFFECT OF SLOT AND FREE STREAM TURBULENCE LEVEL; $t/y_c = 0.126$; $\delta/y_c = 0.12$; $y_c = 0.25$ ";
 (— $Tu_q = Tu_c = 0.01$); (--- $Tu_q = 0.01, Tu_c = 0.2$);
 (— · — $Tu_q = 0.2, Tu_c = 0.01$)

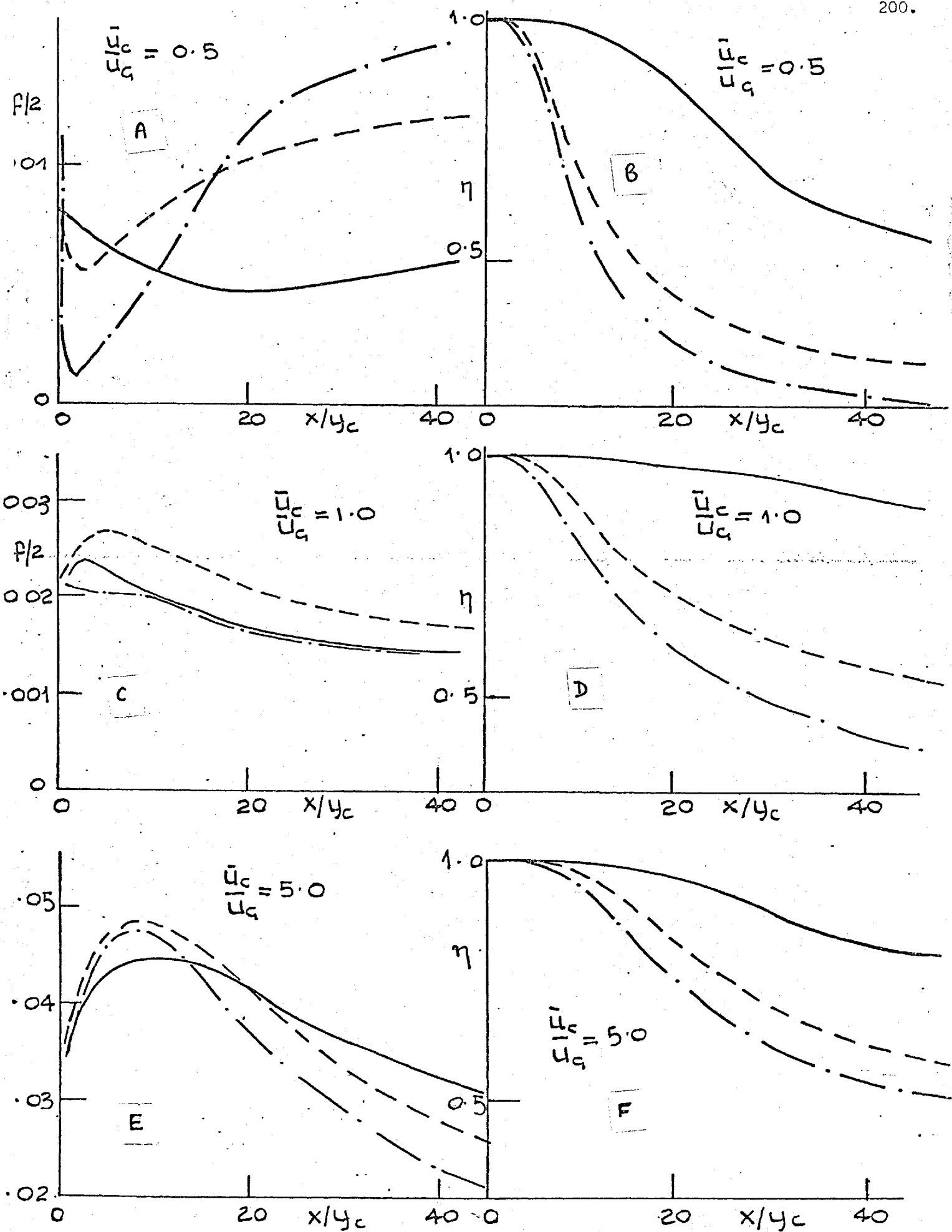


FIG. 5.15 EFFECT OF LIP THICKNESS ON $C_f/2$ AND η ;
 (—) $t/y_c = 0.126$; (---) $t/y_c = 0.63$; (-·-) $t/y_c = 1.14$; $\delta/y_c = 0.12$
 " = 0.25"

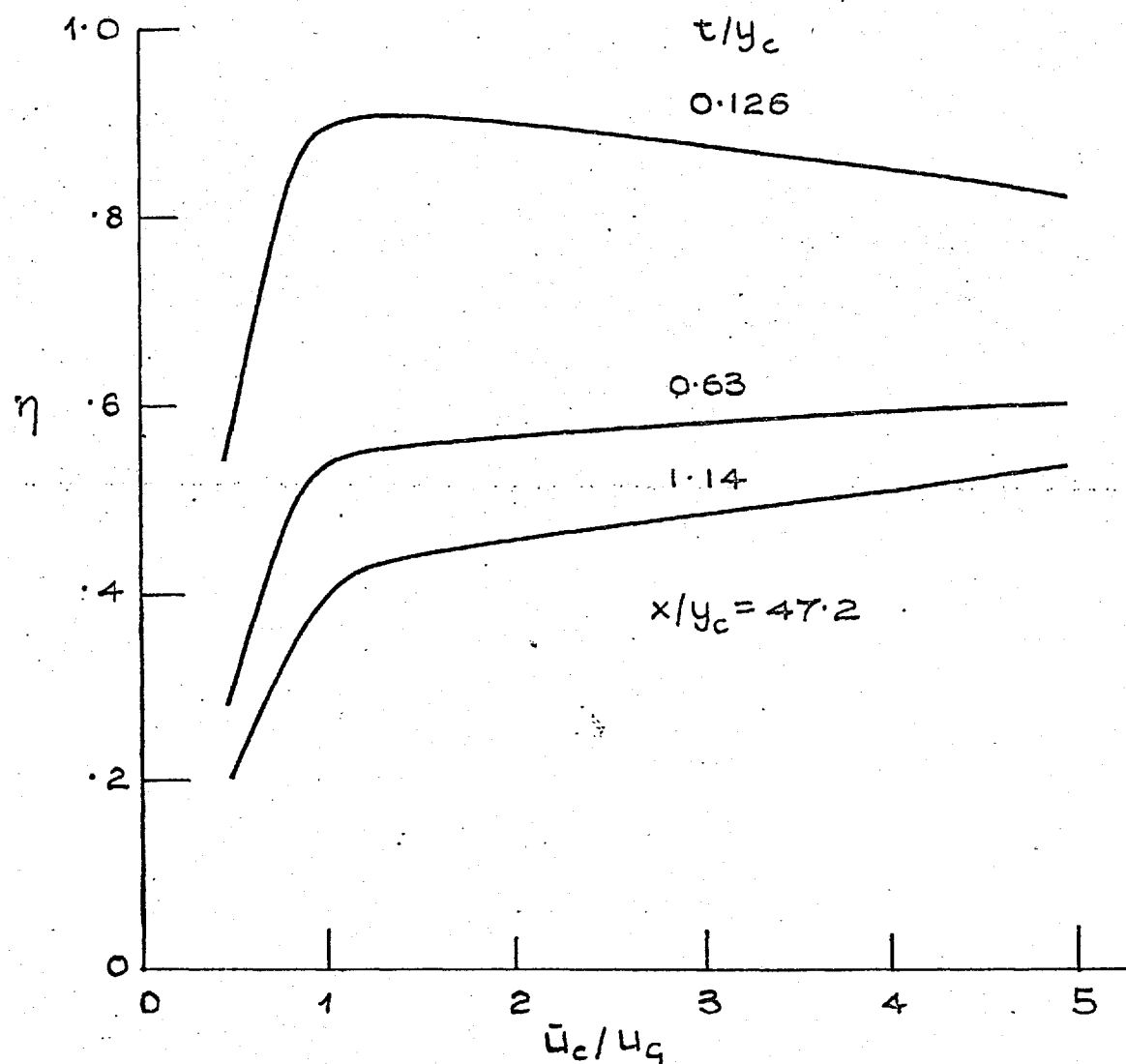


FIG. 5.16 PREDICTED η VERSUS $\bar{u}_c/u_c$ CURVES; $y_c = 0.25$; $\delta/y_c = 0.12$

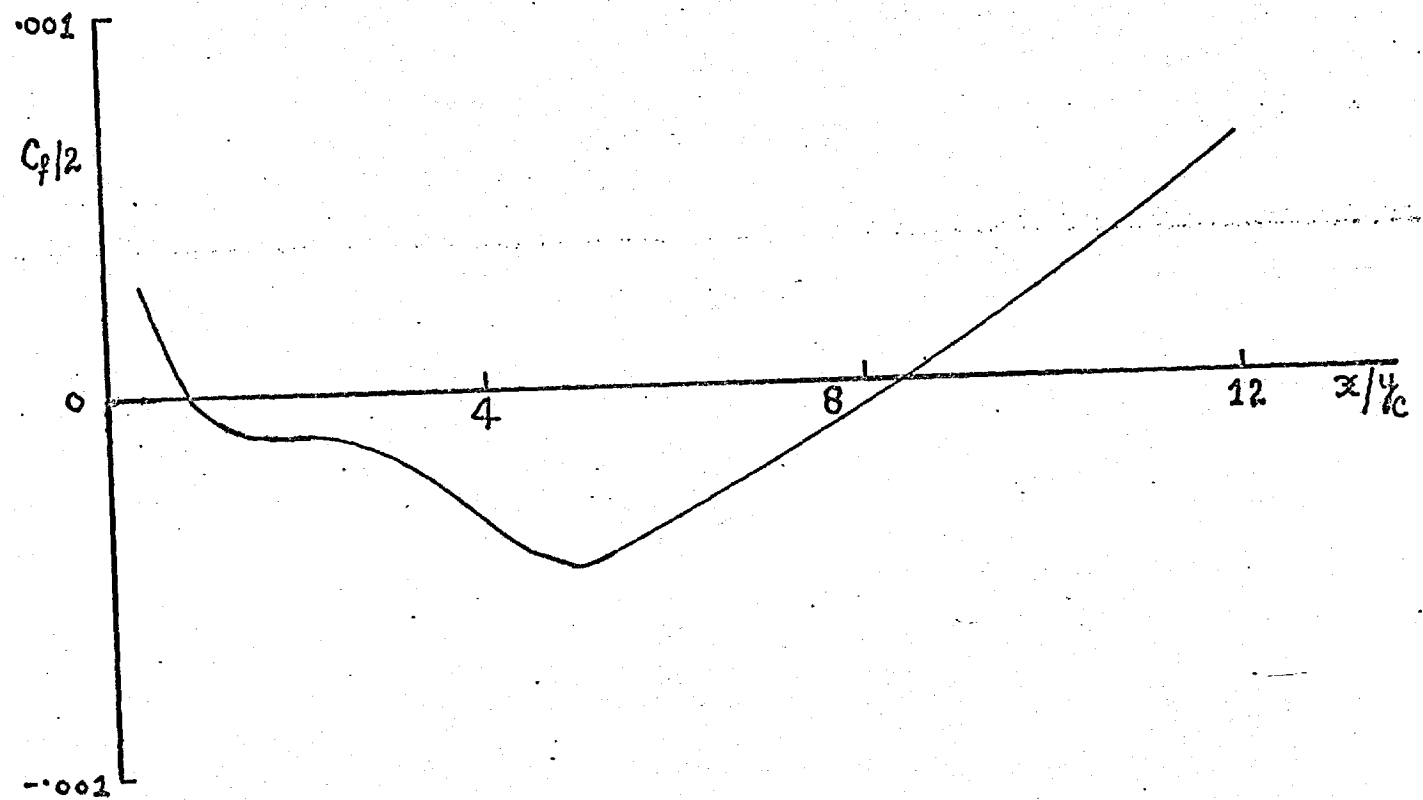
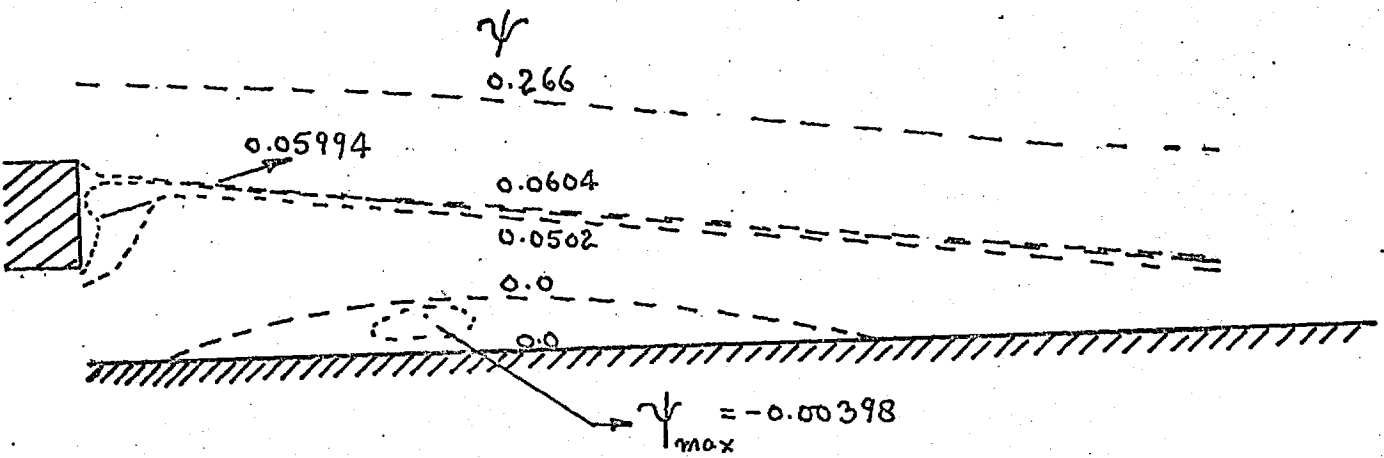


FIG. 5.17 SEPARATION ON BASE PLATE : $\bar{u}_c/u_G = 0.25$; $y_c = 0.25''$; $t/y_c = 1.14$

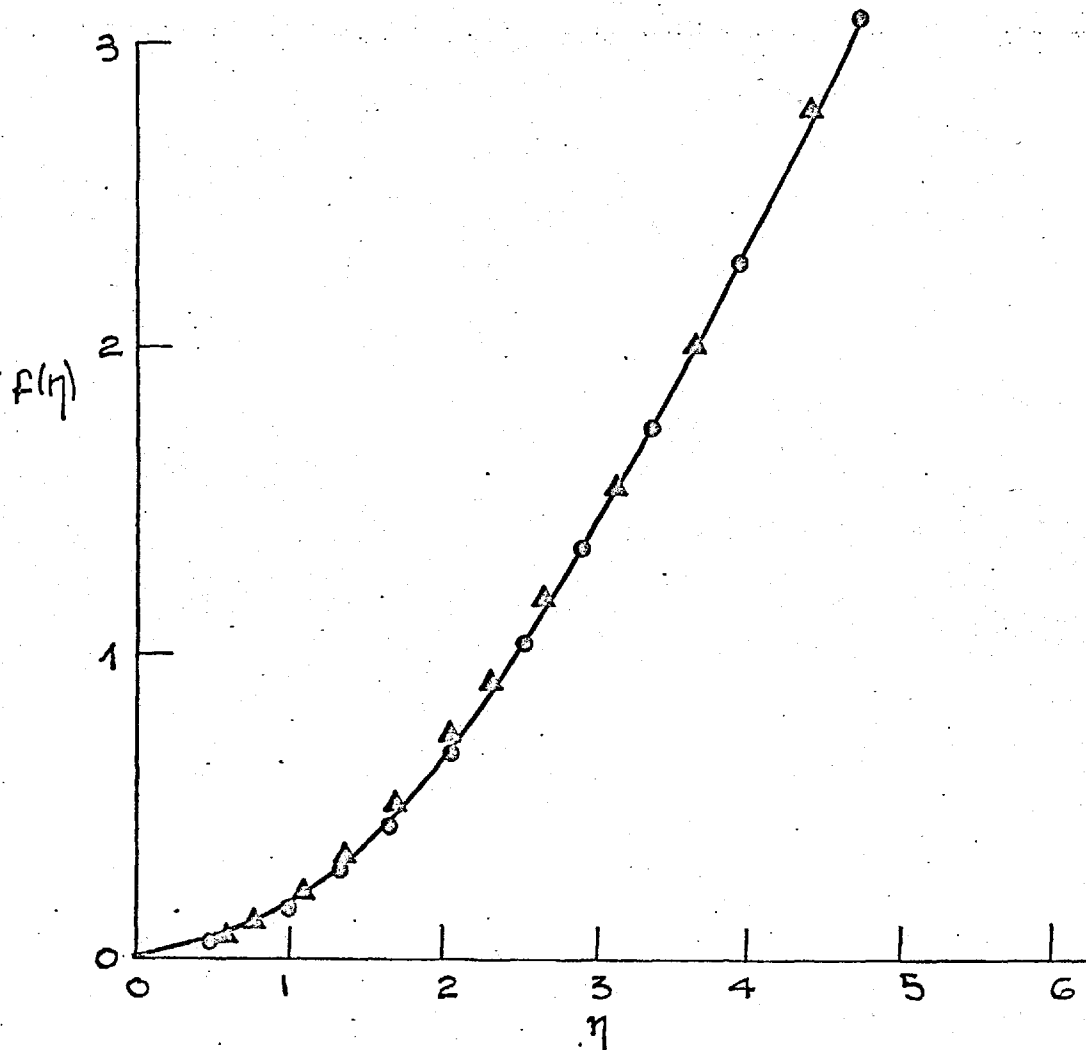
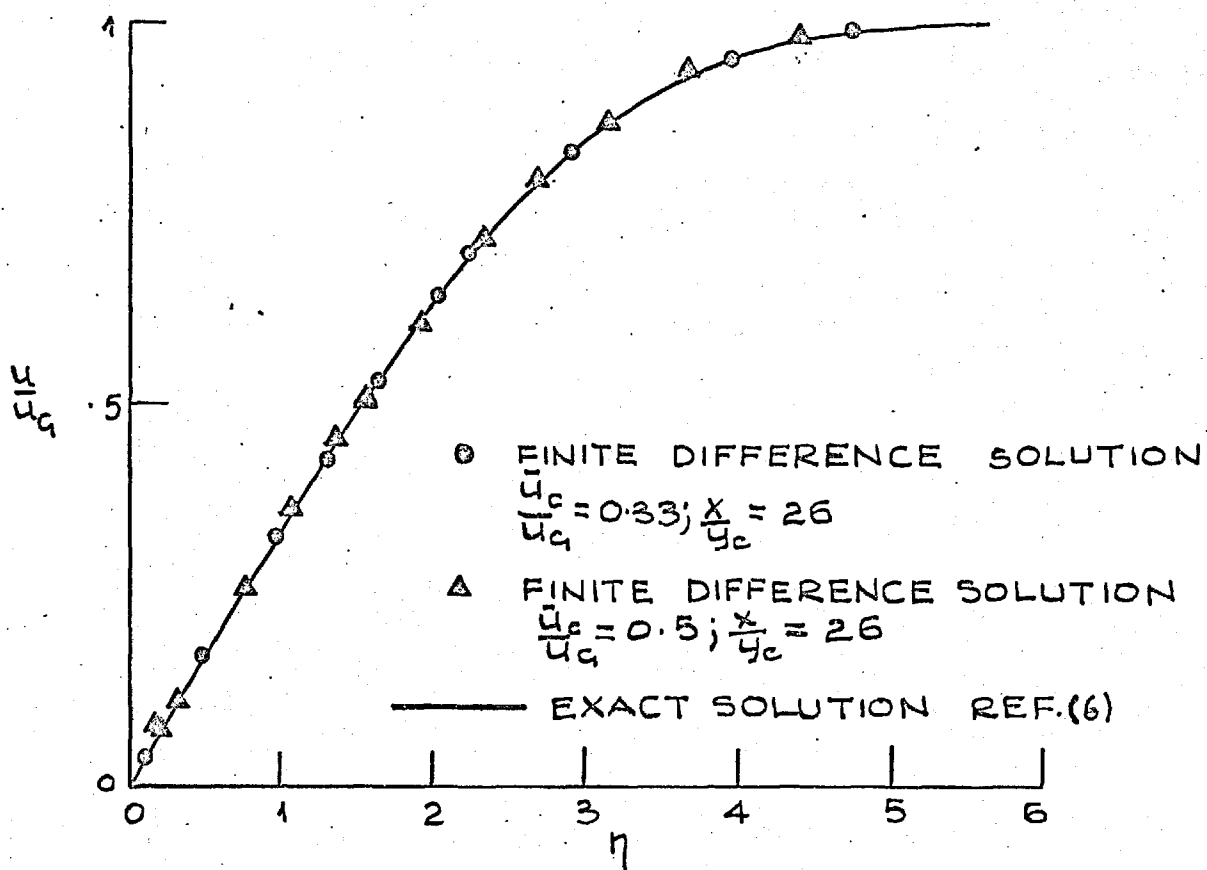


FIG. 5.18 COMPARISON OF EXACT LAMINAR BOUNDARY LAYER SOLUTION (6) WITH FINITE-DIFFERENCE SOLUTION

$$y_c = 0.25'', t/y_c = 0.126; \mu = 0.0025; \rho = 1; u_c = 1; \delta/y_c = 0.0$$

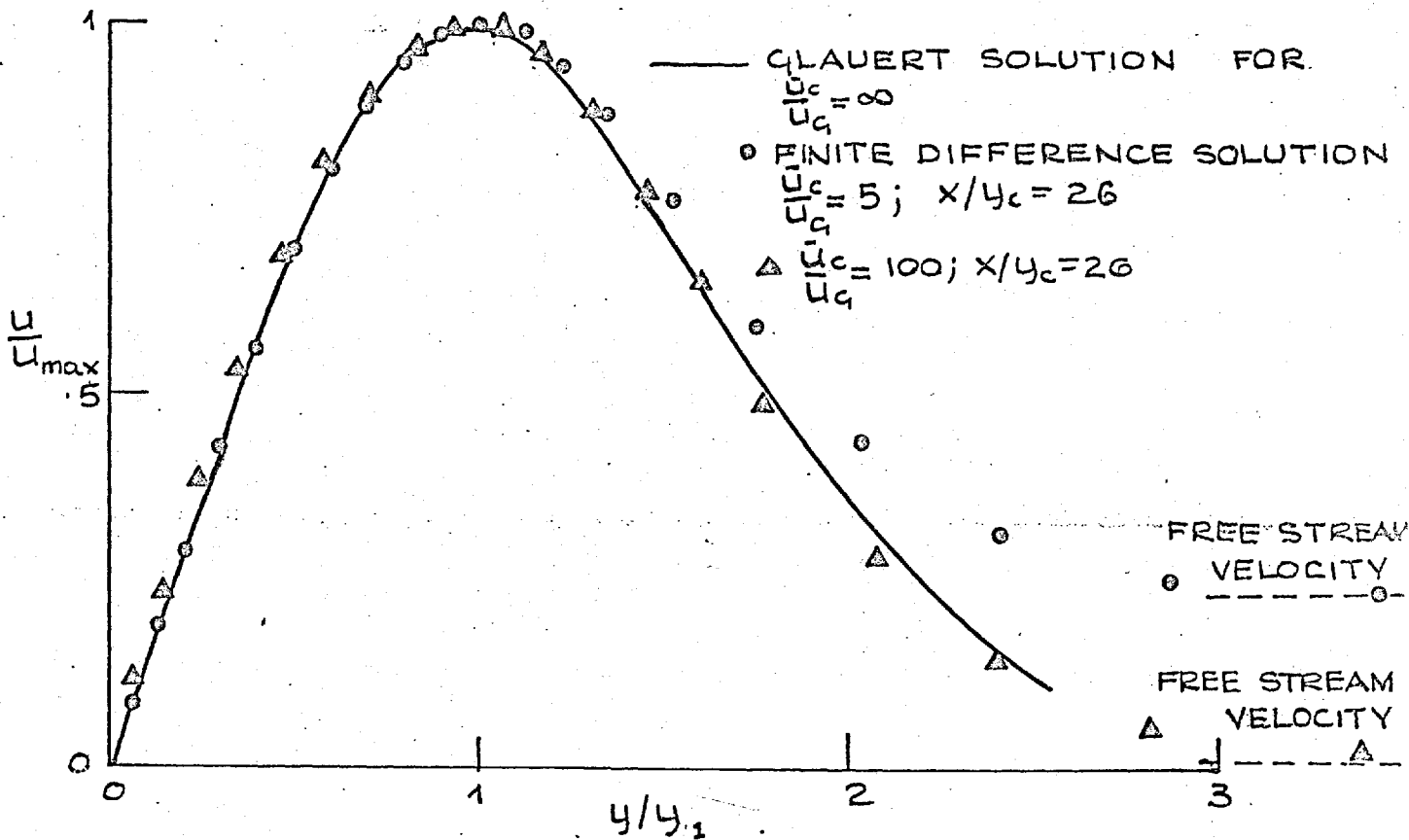


FIG. 5.19 COMPARISON OF CLAUERT (29) WALL-JET SOLUTION WITH FINITE DIFFERENCE SOLUTION

NOTE: y_1 REPRESENTS LOCATION OF y WHERE $U = U_{\max}$;

$y_c = 0.25''$, $t/y_c = 0.126$, $\mu = 0.0025$, $\rho = 1$, $U_0 = 1$, $\delta/y_c = 0.0$

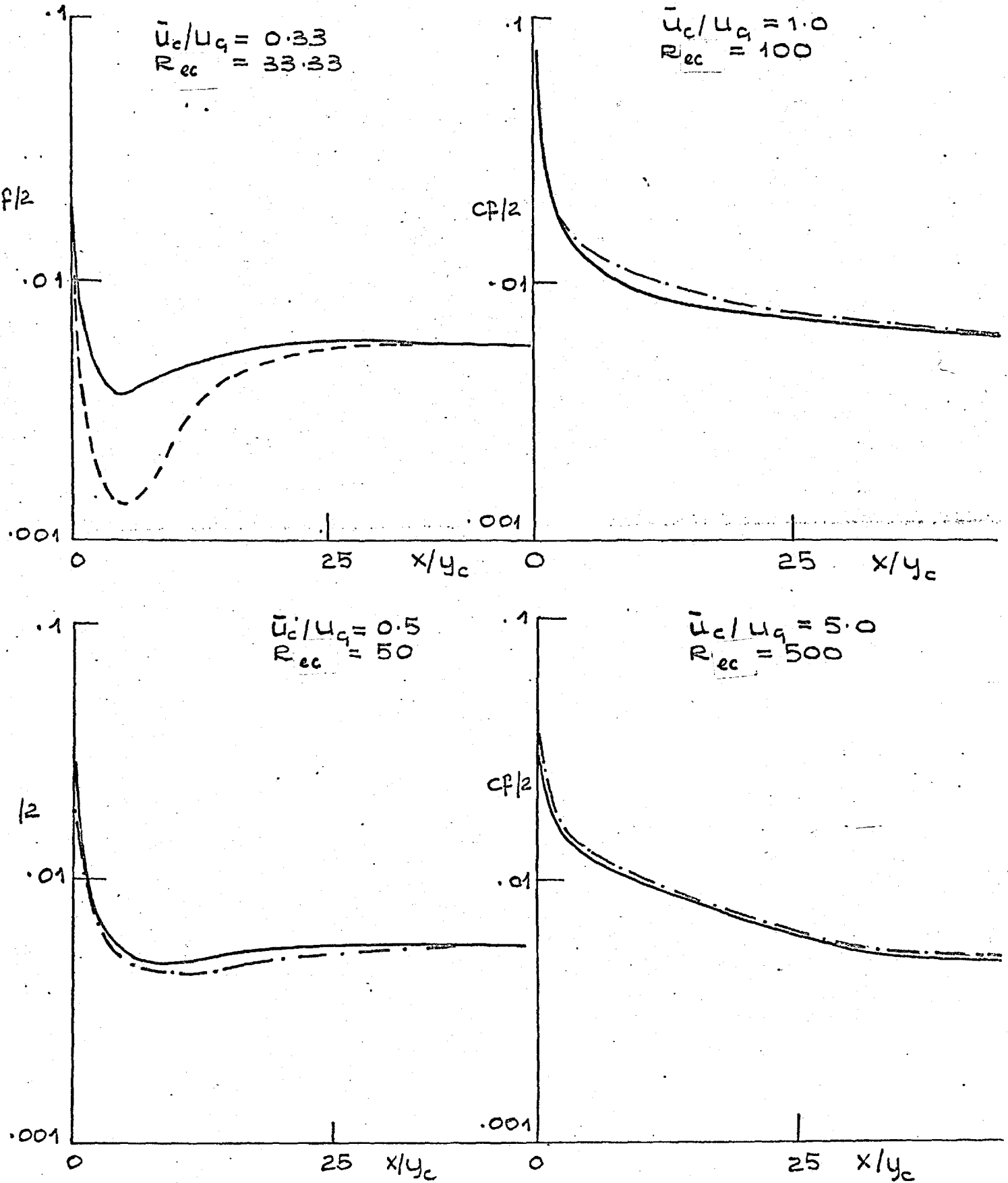


FIG. 5.20 PREDICTED EFFECT OF UPPER-LIP BOUNDARY LAYER THICKNESS ON SKIN-FRICTION FOR LAMINAR FLOW; $\epsilon/y_c = 0.126$; $y_c = 0.25''$ (— $\delta/y_c = 0$, --- $\delta/y_c = 0.5$, — — — $\delta/y_c = 1$)

APPENDIX 1: The Computer Program for the Initial Region of Wall-Jet

The purpose of this appendix is to present information relevant to the computer program used to obtain the results described in chapter 5. The function of the various sub-routines is described; the important Fortran symbols explained; some comments made on the means of starting the computations; and the listing of the computer program presented.

Function of various sub-routines

HEAD	Starts the computations, calculates the length scales and velocities and controls the iteration cycle.
EQN	Performs the finite-difference computations within the control volume and calculates the boundary conditions.
OUT	Gives the output listings and calculates pressure distribution.
DPX	Calculates $\partial p / \partial x$
DPY	Calculates $\partial p / \partial y$
INPDAT	Supplies all the input and constants to the program.
VISCO	Calculates viscosity
VELINP	Calculates and supplies to sub-routines HEAD the velocity, ψ and ω profiles at the slot exit.

Meaning of Fortran Symbols

<u>Symbol</u>	<u>Meaning</u>
A(I,J,K)	Array containing all the dependant variables; I and J denote the location of nodes in x and y direction and K stands for the dependant variable.
ALG	Equivalent to λG_2 for l_2 length scale.
AK	Von Karman constant
AM(K)	Array containing the maximum value of A(I,J,K) array.

<u>Symbol</u>	<u>Meaning</u>
ANAME(6,K)	Array containing a thirty six letter description of each of the variables denoted by K.
ASYMBL(K)	Array containing a four letter name for each of the variables denoted by K.
ATITLE(19)	Array containing a 114 letters heading for the problem.
CC	Convergence criterion
CF(I)	Skin-friction coefficient $C_f/2$
CK	Under-relaxation parameter for effective viscosity.
C2	Constant A_D in equation 5.10-4
C3	Constant A_μ in equation 5.10-2
DELTA 1	Boundary layer thickness on the wall at slot exit
DELTA 2	$(X2(JSLOT) - DELTA2) =$ boundary layer thickness on the under side of the lip at slot exit.
DELTA 3	$(DELTA5 - X2(JFREE)) =$ upper-lip boundary layer thickness at slot exit.
DV11	$\partial U / \partial x$
DV12	$\partial U / \partial y$
DV21	$\partial V / \partial x$
DV22	$\partial V / \partial y$
FRTURB(4)	Free stream turbulence level $\sqrt{u^2}/U_G$
FRV	Value of velocity at outer edge where the boundary layer ends.
HE	$x_{i+1} - x_i$
HN	$y_{j+1} - y_j$
HS	$y_j - y_{j-1}$
HW	$x_i - x_{i-1}$
H1	$x_{i+1} - x_{i-1}$
H2	$y_{j+1} - y_{j-1}$

<u>Symbol</u>	<u>Meaning</u>
ICNTR1*	Should be equal to 1 if input for A(I,J,IE) is supplied.
ICNTR2*	Should be equal to 1 if punched output for A(I,J,IE) is desired.
ICNTR3*	Should be equal to 1 if starting boundary conditions are to be employed.
IE	Number of conserved property equations to be solved.
ILINE	Running number of lines already written since the last change in the index J.
IN	Number of mesh point in x-direction.
INM	IN - 1
IPAGE	Running number of lines already written from the beginning of the page.
IPT	Control index for sub-routine specifying the form of output listing.
ITBCNT	Control for the number of slot-exit turbulence profiles for which computations are required.
IX	Control for the output in x-direction; IX=1 prints output for all values of I (from I = 1 to IN); IX = 2 prints output for alternate values of I (i.e. I = 1,3,5,7 . . . up to INM or IN)
IV	Number of variables in the array A (I,J,K).
JN	Number of grid points in y-direction.
JNM	JN-1
JSLOT	J value of the grid point where slot finishes.
JFREE	J value of the grid point where the lip finishes.
JX	Control for the output in y-direction; JX = 1 prints output for all values of J (from J = 1 to JN); JX = 2 prints output for alternate values of J (i.e. J = 1,3,5,7 . . . up to JNM or JN).

* Refer to "A note on starting the computation" on page 210.

<u>Symbol</u>	<u>Meaning</u>
NI(N)	Array containing output control information; when NITER = NI(N) output is produced.
NITER	Running number of iterations.
NMAX	Maximum number of iterations to be done.
NPRIN	Number of variables for which output is produced.
NDP,NF,NG1,NG2, NK,NMH,NML, NMU,NP,NRO, NT,NV1,NV1, NW	These numbers stand for the following variables: dynamic head, stream function, mass velocity in x-direction, mass velocity in y-direction, turbulent-kinetic energy, turbulent length scale, mass fraction of ϕ , turbulent-viscosity, static pressure, density, shear stress, velocity in x-direction, velocity in y-direction, vorticity.
POWN	Power for the slot velocity profile (taken as 1/5th in the present program.)
PR(K)	Array containing the turbulent-Prandtl numbers.
RES	Maximum value of the residual defined as $(A(I,J,K)_{NITER} - A(I,J,K)_{NITER-1}) / AM(K)$.
ROH	Density of the free stream fluid.
ROL	Density of the slot fluid.
SLTURB(4)	Slot turbulence level $\sqrt{u^2}/U_C$
SOURCE	Source term. S_ϕ and S_w in equation 2.4-7 and 2.4-6
SS(I)	Patankar-Spalding (75) wall function (equation 4.3-6 of reference 75) used to calculate the power β in expression $U \propto y^\beta$
STRFC1(26)	Array containing values of stream functions at slot exit.
ST(I)	Effectiveness.
T(I)	Wall shear stress.
TAUSTR	τ^* given by equation 5.9-3
Q	Power p' in the expression $\overline{q^2} \propto y^{p'}$ (equation 5.9-3)
UC	Slot maximum velocity
UG	Free stream velocity
VELIN1(26)	Array containing the velocity profile at the slot exit.

<u>Symbol</u>	<u>Meaning</u>
WORCT1(26)	Array containing vorticity profile at the slot exit.
X1(I)	Array containing x-coordinates of the mesh points.
X2(J)	Array containing y-coordinates of the mesh point.
ZA	Constant C_D in dissipation term
ZC	Constant C_μ of equation 5.5-2
ZL	Turbulent length scale for viscosity near the wall.
ZLP	Turbulent length scale away from the wall.
ZL3	Turbulent length scale for dissipation near the wall.
ZMU	Laminar viscosity.

A note on starting the computations

The computer program is usually run in two stages. In the first stage, ICNTR3 is put equal to 1; this control ensures that;

- (2) The wall vorticity is calculated according to equation 5.9-1
- (2) $A(I, 1, NK) = 0.0$
- (3) In the length scale calculations, the boundary layer thickness at any x-station is taken equal to the boundary layer thickness at $x = 0$.

In order to maintain the stability of the calculations, it is necessary to start the program with ICNTR3 = 1 and obtain an approximate solution in punched cards (i.e. ICNTR2 = 1). In the second stage the approximate solution is fed as input (i.e. ICNTR1 = 1) and the control ICNTR3 is set equal to any integer other than 1. If ICNTR3 $\neq$ 1, the following calculations are done:

- (1) Vorticity and turbulent kinetic energy are calculated according to the wall functions described in chapter 5.
- (2) The boundary layer thickness at any x-location is calculated from the velocity profile at that x-location and this local value of y_G is used in the calculation of length scale.

In the laminar flow computations, ICNTR3 is always put equal to 1 as no calculation of turbulent viscosity is needed.

Apart from the stability, running the computer program in two steps has another advantage: it allows the examination of computations and at this stage the decision to continue the computations or not may be taken.

If it is desired to obtain the solution in one step, a condition may be introduced in the program by which after a particular number of iterations, ICNTR3 is automatically changed from the starting value of 1 to 2 (or any other number except 1).

\$IBFTC HEAD

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PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,PUNCH)
COMMON/CVRBLE/A(26,26,14),X1(26),X2(26),IMIN(26),IMAX(26),RP(4),
1 PR(4),IN,JN,IE,IV,IR
COMMON /CNUMBR/NW,NF,NK,NHS,NVT,NML,NMH,NP,NG1,NG2,NMU,NRO
COMMON /CN2/ NV1,NV2,NDP,NT
COMMON/C/ANAME(6,14),ASYMBL(14),NI(50),NPRIN,INDG,CC,NMAX,INM,JNM
COMMON/CTURB/ZA,ZB,ZC,C1,C2,C3,ZLP
COMMON/SLT/SLTURB(4),FRTURB(4) ,DELTA1,DELTA2,DELTA5
COMMON /CBOU/ JSLOT,JFREE,UG,UC,ROL,ROH,ZMU
COMMON/SFV/SV,SHS,CF(41),ST(41)
COMMON/CB1/R(26),C(26),S(26),ZK(26) ,T(26),ZJ(26)
COMMON/CONTL/ICNTR1,ICNTR2,ICNTR3 ,ITBCNT ,NITER
DIMENSION RSDU(10),AM(10),ATITLE(19) ,FHS(81)
DIMENSION WORCT1(26),WORCT2(26),VELIN1(26),VELIN2(26),STRFC1(26),
1STRFC2(26)

```

C***

C READING, PREPARATION AND WRITING OF CONSTANTS

C***

```

INM=IN-1
JNM=JN-1
READ (5,100) ATITLE
READ(5,70) (ASYMBL(I),(ANAME(K,I),K=1,6),I=1,14)
70 FORMAT(A4,6A6)
100 FORMAT (12A6)
WRITE (6,101) ATITLE,((ANAME(L,K),L=1,6),K=1,1E)
101 FORMAT (12H1SOLUTION OF/1H0.19A6.3HFOR/(1H0.10X,6A6/))
WRITE(6,102)NMAX,CC,IN,JN
102 FORMAT (6H0NMAX=,14,3X,3HCC=,1PE11.4,3X,3HIN=,13,3HJN=,13)
N=1
NITER=1
WRITE (6,15) X1
WRITE (6,16) X2
15 FORMAT (4H0X1=,15F8.3)
16 FORMAT (4H0X2=,15F8.3)
DO 200 K=1,IV
DO 200 I=1,IN
DO 200 J=1,JN
200 A(1,J,K)=0.

700 FORMAT(8F9.3)
CALL VELINP(X2,JN,VELIN1,VELIN2,WORCT1,WORCT2,STRFC1,STRFC2)
DO 171 J=1,JN
A(1,J,NF)=STRFC1(J)
A(1N,J,NF)=STRFC2(J)
A(1,J,NW)=WORCT1(J)
171 A(1N,J,NW)=WORCT2(J)
DO 202 J=2,JSLOT
A(1,J,NML)=1.0
A(1,J,NRO)=ROL
202 A(1,J,NG1)=ROL*VELIN1(J)
A(1,1,NML)=1.0
JFP=JFREE+1
DO 203 J=JFP,JN
A(1,J,NML)=0.0
A(1,J,NRO)=ROH
203 A(1,J,NG1)=ROH*VELIN1(J)
A(1,JSLOT,NK)=0.
A(1,JFREE,NK)=0.

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A(1,JFREE,NG1)=0.
A(1,JSLOT,NG1)=0.0
A(1,1,NG1)=0.0
DO 205 I=2,IN
205 A(1,JN,NK)=A(1,JN,NK)
DO 204 I=2,IN
DO 204 K=1,IV
DO 204 J=1,JN
A(I,J,K)=A(1,J,K)
204 CONTINUE
IF(ICNTR1.NE.1) GO TO 1000
READ(5,703)((A(1,J,NW),I=1,IN),J=1,JN)
READ(5,703)((A(1,J,NF),I=1,IN),J=1,JN)
READ(5,703)((A(1,J,NK),I=1,IN),J=1,JN)
READ(5,703)((A(1,J,NML),I=1,IN),J=1,JN)
1000 CONTINUE
703 FORMAT(6E12.5)
DO 75 ITURB=1,ITBCNT
DO 98 J=1,JN
IF(J.LE.JFREE) A(1,J,NK)=1.5*(SLTURB(1)*UC)**2.0
98 IF(J.GE.JFREE) A(1,J,NK)=1.5*(FRTURB(1)*UG)**2.
IF(ICNTR1.EQ.1) GO TO 99
DO 97 I=1,IN
DO 97 J=1,JN
97 A(1,J,NK)=A(1,J,NK)
99 CONTINUE
CALL OUT (A,IMIN,IMAX,IN,JN,IE,IV,1,X1,X2)
WRITE(6,103)
103 FORMAT(1H1,3X,1HN,9X,5HRSTRM,7X,5HRVORT,7X,5HRESSV,8X,2HSV,8X,4HRK
1EN,14X,4HRENT,7X,6HRESSHS,7X,3HSHS)
C***
C THIS IS CALCULATION OF TURBULENT LENGTH SCALE
C***
IF(UC.LE.1.0) FRV=0.99
IF(UC.GT.1.0) FRV=1.01
Y2=DELTA5
CMU=ZC
1 CONTINUE
DO 50 I=2,IN
IF(ICNTR3.EQ.1) GO TO 2000
DO 198 J=12,JN
IF(UC.LE.1.0.AND.A(1,J,NV1).LT.FRV) GO TO 198
IF(UC.GT.1.0.AND.A(1,J,NV1).GT.FRV) GO TO 198
GO TO 197
198 CONTINUE
197 IF(NITER.GT.5)
1 Y2G=(X2(J)-X2(J-1))/(A(1,J-1,NV1)-A(1,J,NV1))
IF(NITER.GT.5)
1 Y2G=Y2G*(A(1,J-1,NV1)-FRV)+X2(J-1)
IF(Y2G.LT.Y2) Y2G=Y2
2000 CONTINUE
ALG=.10+0.15*X1(1)/2.5
IF(X1(1).GT.2.5) ALG=0.25
DO 50 J=1,JN
Y=X2(J)-X2(1)
IF(A(1,J,NK).LT.0.0) A(1,J,NK)=0.0
REK=SQRT(A(1,J,NK))*Y*ROL/ZMU
AL=(1.-EXP(-C3*REK))*Y
AL=CMU*AL

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ALP=ALG*Y2*CMU
IF(ICNTR3.EQ.1) GO TO 2001
IF(NITER.LE.5) Y2G=Y2
ALP=ALG*Y2G*CMU
2001 CONTINUE
IF(AL.GT.ALP)AL=ALP
50 A(I,J,NMH)=AL
IF(NITER.NE.1) GO TO 79
DO 78 J=JSLOT,JFREE
78 A(I,J,NK)=0.0
79 CONTINUE
C***
C COMPUTATION OF THE VELOCITIES
C***
DO 10 J=2,JNM
HN=X2(J+1)-X2(J)
HS=X2(J)-X2(J-1)
A(IN,J,NG1)=((A(IN,J+1,NF)-A(IN,J,NF))*HS/HN
1 +(A(IN,J,NF)-A(IN,J-1,NF))*HN/HS)/(HS+HN)
DO 10 I=2,INM
HE=X1(I+1)-X1(I)
HW=X1(I)-X1(I-1)
A(I,J,NG1)=((A(I,J+1,NF)-A(I,J,NF))*HS/HN
1 +(A(I,J,NF)-A(I,J-1,NF))*HN/HS)/(HN+HS)
A(I,J,NG2)=((A(I-1,J,NF)-A(I,J,NF))*HE/HW
1 +(A(I,J,NF)-A(I+1,J,NF))*HW/HE)/(HE+HW)
10 CONTINUE
DO 11 I=2,INM
HE=X1(I+1)-X1(I)
HW=X1(I)-X1(I-1)
A(I,JN,NG2)=((A(I-1,JN,NF)-A(I,JN,NF))*HE/HW
1 +(A(I,JN,NF)-A(I+1,JN,NF))*HW/HE)/(HE+HW)
11 CONTINUE
DO 12 I=1,IN
DO 12 J=1,JN
A(I,J,NRO)=1.
A(I,J,NV1)=A(I,J,NG1)/A(I,J,NRO)
A(I,J,NV2)=A(I,J,NG2)/A(I,J,NRO)
12 A(I,J,NDP)=(A(I,J,NG1)*A(I,J,NG1)+A(I,J,NG2)*A(I,J,NG2))
1 /A(I,J,NRO) *0.5
C***
C COMPUTATION OF THE DENSITY AND THE VISCOSITY
C***
21 DO 32 J=1,JN
DO 32 I=1,IN
32 A(I,J,NRO)=A(I,J,NML)*ROL+(1.-A(I,J,NML))*ROH
CALL VISCO
C***
C THE ITERATION CYCLE
C***
RES=0.
SHSO=SHS
SV0=SV
DO 2 K=1,IE
IF(NITER.EQ.1)AM(K)=1.
ES=0.
RELAX=RP(K)
CALL EQN (K, ES,RELAX,A,X1,X2,IMIN,IMAX,IN,JN,IE,PR,IV,AM)
RSDU(K)= ES

```

```

2 IF (ABS(RSDU(K)).GT.ABS(RES)) RES=RSDU(K)
  RESSV=1.-SVO/SV
  RESSH=1.-SHSO/SHS
  IF (NITER.NE.NI(N)) GO TO 301
  CALL OUT (A,IMIN,IMAX,IN,JN,IE,IV,1,X1,X2)
  WRITE (6,103)
  N=N+1
301 WRITE (6,104) NITER,RSDU(NF),RSDU(NW),RESSV,SV,RSDU(NK),RSDU(NML),
1 RESSH,SHS
104 FORMAT (1H,15,5X,1P5E12.3,5X,3E12.3)
  IF (NITER.GT.NMAX) GO TO 5
  NITER=NITER+1
  IF (ABS(RES).GT.CC.OR.NITER.LE.5) GO TO 1
  CALL OUT (A,IMIN,IMAX,IN,JN,IE,IV,1,X1,X2)
  IF (ICNTR2.NE.1) GO TO 1001
  PUNCH 702 ((A(I,J,NW),I=1,IN),J=1,JN)
  PUNCH 702 ((A(I,J,NF),I=1,IN),J=1,JN)
  PUNCH 702 ((A(I,J,NK),I=1,IN),J=1,JN)
  PUNCH 702 ((A(I,J,NML),I=1,IN),J=1,JN)
1001 CONTINUE
  N=1
  NITER=1
  GO TO 75
5 WRITE(6,106) NITER
106 FORMAT (32H0THE PROCESS DID NOT CONVERGE IN,15,13H ITERATIONS)
  CALL OUT (A,IMIN,IMAX,IN,JN,IE,IV,1,X1,X2)
  IF (ICNTR2.NE.1) GO TO 1002
  PUNCH 702 ((A(I,J,NW),I=1,IN),J=1,JN)
  PUNCH 702 ((A(I,J,NF),I=1,IN),J=1,JN)
  PUNCH 702 ((A(I,J,NK),I=1,IN),J=1,JN)
  PUNCH 702 ((A(I,J,NML),I=1,IN),J=1,JN)
1002 CONTINUE
702 FORMAT(6E12.5)
  N=1
  NITER=1
75 CONTINUE
  STOP
  END
$IBFTC SMEQN7
SUBROUTINE EQN (K,RES,RELAX,A,X1,X2,IMIN,IMAX,IN,JN,IE,PR,IV,AM)
COMMON/C/ANAME(6,14),ASYMBL(14),NI(50),NPRIN,INDG,CC,NMAX,INM,JNM
COMMON /CNUMBR/NW,NF,NK,NHS,NVT,NML,NMH,NP,NG1,NG2,NMU,NRO
COMMON /CN2/ NV1,NV2,NDP,NT
COMMON /CB0U/ JSLOT,JFREE,UG,UC,ROL,ROH,ZMU
COMMON/CTURB/ZA,ZB,ZC,C1,C2,C3,ZLP
COMMON/SFV/SV,SHS,CF(41),ST(41)
COMMON/CB1/R(26),C(26),S(26),ZK(26),T(26),ZJ(26)
COMMON/CONTL/ICNTR1,ICNTR2,ICNTR3,ITBCNT,NITER
DIMENSION A(IN,JN,IV),X1(IN),X2(JN),IMIN(JN),IMAX(JN),PR(IE)
DIMENSION AM(IE)
DIMENSION RT(40),SS(40),CA(26)
AK=0.42
AKS=AK*AK
DUDX=0.0
CMIN=0.155
AMAX=AM(K)
AM(K)=0.
39 DO 40 J=2,JNM
DO 40 I=2,INM

```

```

      IF (J.EQ.2 .AND.K.EQ.NW) GO TO 40
C***
C   SOURCE TERMS
C***
      SOURCE=0.
      ZQ=0.
      H1=      X1(I+1)-X1(I-1)
      H2=      X2(J+1)-X2(J-1)
      HE=X1(I+1)-X1(I)
      HW=X1(I)-X1(I-1)
      HN=X2(J+1)-X2(J)
      HS=X2(J)-X2(J-1)
      IF (K.NE.NK) GO TO 789
      DV21=((A(I+1,J,NV2)-A(I,J,NV2))*HW/HE
1    +(A(I,J,NV2)-A(I-1,J,NV2))*HE/HW)/(HE+HW)
      DV12=((A(I,J+1,NV1)-A(I,J,NV1))*HS/HN
1    +(A(I,J,NV1)-A(I,J-1,NV1))*HN/HS)/(HN+HS)
      DV11=((A(I+1,J,NV1)-A(I,J,NV1))*HW/HE
1    +(A(I,J,NV1)-A(I-1,J,NV1))*HE/HW)/(HE+HW)
      DV22=((A(I,J+1,NV2)-A(I,J,NV2))*HS/HN
1    +(A(I,J,NV2)-A(I,J-1,NV2))*HN/HS)/(HN+HS)
      DV12=-A(I,J,NW)+DV21
      A(I,J,NT)=A(I,J,NMU)*(DV12+DV21)
      GEN=(2.*(DV11*DV11+DV22*DV22)+(DV12+DV21)*(DV12+DV21))*A(I,J,NMU)
      IF (A(I,J,NK).LT.0.) A(I,J,NK)=0.
      Y=X2(J)-X2(1)
      ZLP=A(I,JN,NMH)/ZC
      IF (Y-ZLP) 101,102,102
102  ZL3=ZLP
      GO TO 113
101  REK=A(I,J,NRO)*SQRT(A(I,J,NK))*Y/ZMU
      ZL3=Y*(1.-EXP(-C2*REK))
113  DIF=ZA*A(I,J,NRO)          *SQRT(A(I,J,NK))/ZL3
      SOURCE=GEN
      ZQ=DIF
789  IF (K.EQ.NF) SOURCE=A(I,J,NRO)*A(I,J,NW)
C***
C***
C   CONDUCTIVE TERMS
C***
      IF (K-NF) 65,62,65
65  IF (K-NW) 63,62,63
62  DP=1.
      DN=1.
      DS=1.
      DE=DP
      DW=DP
      GO TO 35
63  DP =A(I ,J ,NMU)/PR(K)
      DE =A(I+1,J ,NMU)/PR(K)
      DN =A(I ,J+1,NMU)/PR(K)
      DW =A(I-1,J ,NMU)/PR(K)
      DS =A(I ,J-1,NMU)/PR(K)
35  CE=H2*(DE+DP)/(4.*HE)
      CW=H2*(DW+DP)/(4.*HW)
      CN=H1*(DN+DP)/(4.*HN)
      CS=H1*(DS+DP)/(4.*HS)
      ADNM=CE+CW+CN+CS +ZQ*H1*H2/4.0
25  IF (K-NW) 42,94,42

```



```

94 CE=CE*A(I+1,J,NMU)
   CW=CW*A(I-1,J,NMU)
   CN=CN*A(I,J+1,NMU)
   CS=CS*A(I,J-1,NMU)
   ADNM=ADNM*A(I,J,NMU)
42 ANUM=CE*A(I+1,J,K) +CW*A(I-1,J,K) +CN*A(I,J+1,K) +CS*A(I,J-1,K)
   2      +SOURCE *H1*H2 /4.
   IF (K-NF) 41,44,41
C***
C   CONVECTIVE TERMS
C***
41 ZCU=0.
   AU=0.
   DFW=(A(I,J+1,NF)+A(I-1,J+1,NF)-A(I,J-1,NF)-A(I-1,J-1,NF)) /4.
   DFN=-(A(I-1,J,NF)+A(I-1,J+1,NF)-A(I+1,J,NF)-A(I+1,J+1,NF)) /4.
   DFS=-(A(I+1,J,NF)+A(I+1,J-1,NF)-A(I-1,J,NF)-A(I-1,J-1,NF)) /4.
   DFE=(A(I,J-1,NF)+A(I+1,J-1,NF)-A(I,J+1,NF)-A(I+1,J+1,NF)) /4.
   IF (DFW) 911,912,913
911 ZCU=-DFW+ZCU
   GO TO 912
913 AU=AU+DFW*A(I-1,J,K)
912 IF (DFE) 921,922,923
921 ZCU=ZCU-DFE
   GO TO 922
923 AU=AU+DFE*A(I+1,J,K)
922 IF (DFS) 931,932,933
931 ZCU=ZCU-DFS
   GO TO 932
933 AU=AU+DFS*A(I,J-1,K)
932 IF (DFN) 941,942,943
941 ZCU=ZCU-DFN
   GO TO 942
943 AU=AU+DFN*A(I,J+1,K)
942 CONTINUE
C***
C   COMPUTATION OF THE NEW VALUES
C***
   ADNM=ADNM+ZCU
   ANUM=ANUM+AU
44 Z=A(I,J,K)
   A(I,J,K)=ANUM/ADNM
   IF (ABS(A(I,J,K)).GT.ABS(AM(K))) AM(K)=A(I,J,K)
   RS=(A(I,J,K)-Z)/AMAX
   IF (ABS(RS).GT.ABS(RES)) RES=RS
40 CONTINUE
C***
C   THE VARYING BOUNDARY CONDITIONS
C***
C***
C   LIP
C***
   IF (K.EQ.NK) GO TO 300
   IF (K.NE.NW.AND.K.NE.NML) GO TO 211
   JL=JSLOT+1
   JH=JFREE-1
   DF1=A(1,JSLOT,NF)-A(1,JSLOT-1,NF)
   DF2=A(2,JSLOT,NF)-A(1,JSLOT,NF)
   DX1S=X1(2)-X1(1)
   DX1S=DX1S*DX1S

```

```

DX2S=(X2(JSLOT)-X2(JSLOT-1))*(X2(JSLOT+1)-X2(JSLOT-1))
A(1,JSLOT,NW)=2.*DF1/DX2S-DF2/DX1S
DF1=A(1,JFREE+1,NF)-A(1,JFREE,NF)
DF2=A(2,JFREE,NF)-A(1,JFREE,NF)
DX2S=(X2(JFREE+1)-X2(JFREE))*(X2(JFREE+1)-X2(JFREE-1))
A(1,JFREE,NW)=-2.*DF1/DX2S-DF2/DX1S
IF (JH,LT,JL) GO TO 300
DO 235 J=JL,JH
H1=X1(2)-X1(1)
A(1,J,NW)=-0.5*A(2,J,NW)-3.*(A(2,J,NF)-A(1,J,NF))/H1/H1/A(1,J,NRO)
Y1=X1(2)-X1(1)
Y2=X1(3)-X1(1)
F1=A(2,J,NML)
F2=A(3,J,NML)
Y1S=Y1*Y1
Y2S=Y2*Y2
A(1,J,NML)=(F1*Y2S-F2*Y1S)/(Y2S-Y1S)
235 CONTINUE
300 CONTINUE
C***
C WALL
C***
BETA=1./7.
DO 301 I=2,IN
H2=X2(2)-X2(1)
A(1,1,NW)=-0.5*A(1,2,NW)-3.*(A(1,2,NF)-A(1,1,NF))/H2/H2/A(1,1,NRO)
IF(ICNTR3.EQ. 1) A(1,1,NT)=-A(1,1,NW)*ZMU
IF(ICNTR3.EQ. 1) GO TO 1000
IF(NITER.LT.5) GO TO 1000
USLIP=(1.0-BETA)*A(1,2,NF)/H2
U1P5=(A(1,2,NF)*(BETA+1.0)/H2+USLIP)/2.0
RT(1)=U1P5*H2*ROH*AKS/(2.0*ZMU)
SS(1)=1.0/RT(1)-0.1561*RT(1)**(-0.45)+0.08723*RT(1)**(-0.3)
1 +0.03713*RT(1)**(-0.18)
BETA=(SS(1)*AKS)**0.5/AK
T(1)=A(1,1,NT)
IF(A(1,2,NK).LT.0.0)A(1,2,NK)=0.0
R(1)=SQRT(A(1,2,NK))* H2*A(1,2,NRO)/ZMU
CMAX=0.0565*(R(1)+5.0)**0.895
IF(R(1)-19.8)2250,2250,2253
2253 IF(T(1))2254,2255,2254
2255 ZK(1)=10000.
GO TO 2256
2254 ZK(1)=A(1,2,NK)*A(1,2,NRO)/ABS(T(1))
2256 CONTINUE
CA(1)=ZK(1)/R(1)**0.827
CAA=0.097/CA(1)**0.34
IF(CAA-CMIN)2260,2260,2259
2259 C(1)=CAA*(R(1)+5.0)**0.58
IF(C(1).GT.CMAX)GO TO 2263
GO TO 2264
2260 C(1)=CMIN*(R(1)+5.0)**0.58
GO TO 2264
2250 C(1)=1.
A(1,1,NK)=-A(1,1,NK)
GO TO 2267
2263 C(1)=CMAX
2262 A(1,1,NK)=A(1,2,NK)
GO TO 2261

```

```

2264 A(I,1,NK)=-A(I,2,NK)*0.39
2261 CONTINUE
2267 CONTINUE
      UART=A(I,2,NF)*(BETA+1.)/H2
      A(I,1,NT)=C(I)*ZMU*UART/H2
1000  CONTINUE
      A(IN,3,NT)=-A(IN,3,NW)*A(IN,3,NMU)
      PP=(A(I,1,NT)-A(I,3,NT))/(X2(3)-X2(1))
      A(I,2,NT)=A(I,1,NT)-PP*H2
      A(I,2,NW)=-A(I,2,NT)/A(I,2,NMU)
      A(I,1,NW)=-A(I,1,NT)/ZMU
      CF(I)=A(I,1,NT)
      IF(ICNTR3.EQ. 1) A(I,1,NK)=0.0
      IF(ICNTR3.EQ. 1) GO TO 1001
      IF(NITER.LT.5) GO TO 1001
      TAUSTR=A(I,2,NT)/A(I,2,NK)/A(I,2,NRO)
      Q=2.0*PR(3)*ZA*(1.0-TAUSTR*TAUSTR/(ZA*ZC))/(3.0*ZC)
      IF(Q.LT.0.0) Q=0.0
      Q=SQRT(Q)
      A(I,1,NK)=A(I,2,NK)*(1.0-Q)
1001  CONTINUE
      Y1=X2(2)-X2(1)
      Y2=X2(3)-X2(1)
      F1=A(I,2,NML)
      F2=A(I,3,NML)
      Y1S=Y1*Y1
      Y2S=Y2*Y2
      A(I,1,NML)=(F1*Y2S-F2*Y1S)/(Y2S-Y1S)
301  ST(1)=(A(I,1,NML)-A(1,JN,NML))/(A(1,1,NML)-A(1,JN,NML))
C***
C    MAIN STREAM
C***
211  CONTINUE
      IF (K.NE.NF) GO TO 212
      DO 51 I=2,IN
51    A(I,JN,NF)=ROH*A(I,JN,NV1)*(X2(JN)-X2(JNM))*(1.0+DUDX*(X1(I)-X1(1)
      1))+A(1,JNM,NF)
C***
C    WALL JET
C***
212  CONTINUE
      DO 5 J=1,JN
5    A(IN,J,K)=A(IN-1,J,K)
C***
C    MEAN WALL FLUXES
C***
      ST(1)=1.0
      SHS=0.
      SV=0.
      INMM=IN-2
      DO 391 I=1,INMM,2
      SV=SV+CF(I)+4.*CF(I+1)+CF(I+2)
391  SHS=SHS+ST(I)+4.*ST(I+1)+ST(I+2)
      RETURN
      END
$IBFTC SMOUT1
      SUBROUTINE OUT (A,IMIN,IMAX,IN,JN,IE,IV,IPT,X1,X2)
      COMMON/C/ANAME(6,14),ASYMBL(14),NI(50),NPRIN,INDG,CC,NMAX,INM,JNM
      COMMON /CN2/ NV1,NV2,NDP,NT

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```

COMMON /CNUMBER/NW,NF,NK,NHS,NVT,NML,NMH,NP,NG1,NG2,NMU,NRO
COMMON/CONTL/ICNTR1,ICNTR2,ICNTR3,ITBCNT,NITER
DIMENSION A(IN,JN,IV),IMIN(JN),IMAX(JN),X1(IN),X2(JN)
DIMENSION PP(41)
COMMON/SFV/SV,SHS,CF(41),ST(41)
EXTERNAL DPX,DPY
DATA PO/0./
IX=2
JX=1
IF(NITER.EQ. 1) GO TO 723

```

```

C***
C   COMPUTATION OF THE PRESSURE FOR WALL JET WHEN THE ORIGIN IS ON THE
C   JUNCTION OF THE SLOT AND THE WALL.
C***

```

```

DO 31 J=1,JN
  A(IN,J,NT)=-A(IN,J,NW)*A(IN,J,NMU)
31 A(1,J,NT)=-A(1,J,NW)*A(1,J,NMU)
  DO 32 I=2,INM
    A(I,1,NT)=-A(I,1,NW)*A(I,1,NMU)
32 A(I,JN,NT)=-A(I,JN,NW)*A(I,JN,NMU)
    A(2,2,NP)=PO
    DO 21 I=2,INM
      21 CALL DPX(I,2,PP(I),A,X1,X2,IN,JN,IV)
      DO 22 I=3,INM
        22 A(I,2,NP)=(PP(I-1)+PP(I))*0.5*(X1(I)-X1(I-1))+A(I-1,2,NP)
          DO 23 I=2,INM
            HE=X1(I+1)-X1(I)
            HW=X1(I)-X1(I-1)
            DV11=((A(I+1,2,NV1)-A(I,2,NV1))*HW/HE
            1  +(A(I,2,NV1)-A(I,1,NV1))*HE/HW)/(HE+HW)
          23 A(I,2,NP)=A(I,2,NP)-A(I,2,NDP)+2.*A(I,2,NMU)*DV11
            DO 27 I=2,INM
              HN=X2(3)-X2(2)
              HS=X2(2)-X2(1)
              DV22=((A(I,3,NV2)-A(I,2,NV2))*HS/HN
              1  +(A(I,2,NV2)-A(I,1,NV2))*HN/HS)/(HN+HS)
            A(I,2,NP)=A(I,2,NP)+A(I,2,NDP)-2.*A(I,2,NMU)*DV22
            DO 25 J=2,JNM
              25 CALL DPY(I,J,PP(J),A,X1,X2,IN,JN,IV)
              DO 26 J=3,JNM
                26 A(I,J,NP)=A(I,J-1,NP)+(PP(J)+PP(J-1))*0.5*(X2(J)-X2(J-1))
                  DO 27 J=2,JNM
                    HS=X2(J)-X2(J-1)
                    HN=X2(J+1)-X2(J)
                    DV22=((A(I,J+1,NV2)-A(I,J,NV2))*HS/HN
                    1  +(A(I,J,NV2)-A(I,J-1,NV2))*HN/HS)/(HN+HS)
                  27 A(I,J,NP)=A(I,J,NP)-A(I,J,NDP)+2.*A(I,J,NMU)*DV22
                    DO 28 I=2,INM
                      DO 28 J=2,JNM
                        28 A(I,J,NP)=A(I,J,NP)-A(INM,JNM,NP)

```

```

723   CONTINUE

```

```

C***
C   GEOMETRICAL DISPLAY OF THE RESULTS
C***
  IF (IPT.EQ.2) GO TO 201
  DO 10 K=1,NPRIN
    IF(K.EQ.6)GO TO 10
    IF(K.EQ.9)GO TO 10
    IF(K.EQ.11)GO TO 10

```

```

WRITE(6,100) (ANAME(L,K),L=1,6)
100  FORMAT(1H130X,21HTHE DISTRIBUTION OF 6A6//4H J//)
      DO 2 J=1,JN,JX
        2  WRITE(6,111) J,(A(I,J,K),I=1,IN,IX)
111  FORMAT(1H0,I2,1X,1P13E9.2)
      WRITE(6,112) (I,I=1,IN,IX)
112  FORMAT(1H0/4H I=. 12(I2,7X),I2)
      IF (K.EQ.NW) WRITE (6,233) (CF(I),I=1,IN,IX)
      IF (K.EQ.NML) WRITE (6,233) (ST(I),I=1,IN,IX)
233  FORMAT(1H0,3X,1P13E9.2)
      10  CONTINUE
201  IF (IPT.EQ.1) RETURN
C***
C    TABULAR DISPLAY OF THE RESULTS
C***
      IPAGE=0
      DO 12 J=1,JN
        ILINE=0
        IL=IMIN(J)
        IH=IMAX(J)
        DO 1 I=IL,IH
          IF (IPAGE.EQ.0) WRITE (6,101) (ASYMBL(K),K=1,NPRIN)
101  FORMAT(10H1 I      J,7(9X,A6))
          IF (ILINE.EQ.0) WRITE (6,103)
103  FORMAT(1H )
          WRITE(6,102) I,J,(A(I,J,K),K=1,NPRIN)
102  FORMAT(1H ,I2,5X,I2,7(1PE15.5))
          ILINE=ILINE+1
          IF (IPAGE.GE.40) IPAGE=0
          IPAGE=IPAGE+1
1    IF (ILINE/5*5.EQ.ILINE) WRITE(6,103)
      12  WRITE (6,103)
          WRITE (6,233) CF
          WRITE (6,233) ST
          RETURN
      END
$IBFTC SDPX
      SUBROUTINE DPX (I,J,DPDX,A,X1,X2,IN,JN,IV)
      DIMENSION A(IN,JN,IV),X1(IN),X2(JN)
      COMMON /CN2/ NV1,NV2,NDP,NT
      COMMON /CNUMBR/NW,NF,NK,NHS,NVT,NML,NMH,NP,NG1,NG2,NMU,NRO
      HE=X1(I+1)-X1(I)
      HN=X2(J+1)-X2(J)
      HS=X2(J)-X2(J-1)
      HW=X1(I)-X1(I-1)
      DI=A(I,J,NDP)*((A(I+1,J,NRO)-A(I,J,NRO))*HW/HE
1    +(A(I,J,NRO)-A(I-1,J,NRO))*HE/HW)/(HE+HW)/A(I,J,NRO)
      DII=A(I,J,NG2)*A(I,J,NW)
      DIII=((A(I,J+1,NT)-A(I,J,NT))*HS/HN
1    +(A(I,J,NT)-A(I,J-1,NT))*HN/HS)/(HN+HS)
      DPDX=DI+DII+DIII
      RETURN
      END
$IBFTC SDPY
      SUBROUTINE DPY (I,J,DPDY,A,X1,X2,IN,JN,IV)
      DIMENSION A(IN,JN,IV),X1(IN),X2(JN)
      COMMON /CN2/ NV1,NV2,NDP,NT
      COMMON /CNUMBR/NW,NF,NK,NHS,NVT,NML,NMH,NP,NG1,NG2,NMU,NRO
      HE=X1(I+1)-X1(I)

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```

HW=X1(I)-X1(I-1)
HS=X2(J)-X2(J-1)
HN=X2(J+1)-X2(J)
DI=A(I,J,NDP)*((A(I,J+1,NRO)-A(I,J,NRO))*HS/HN
1  +(A(I,J,NRO)-A(I,J-1,NRO))*HN/HS)/(HN+HS)/A(I,J,NRO)
DII=-A(I,J,NG1)*A(I,J,NW)
DIII=((A(I+1,J,NT)-A(I,J,NT))*HW/HE
1  +(A(I,J,NT)-A(I-1,J,NT))*HE/HW)/(HE+HW)
DPDY=DI+DII+DIII
RETURN
END

```

\$IBFTC INPDAT

BLOCK DATA

COMMON/CVRBLE/A(26,26,14),X1(26),X2(26),IMIN(26),IMAX(26),RP(4),

1 PR(4),IN,JN,IE,IV,IR

COMMON/C/ANAME(6,14),ASYMBL(14),NI(50),NPRIN,INDG,CC,NMAX,INM,JNM

COMMON /CNUMBR/NW,NF,NK,NHS,NVT,NML,NMH,NP,NG1,NG2,NMU,NRO

COMMON /CN2/ NV1,NV2,NDP,NT

COMMON /CBOU/ JSLOT,JFREE,UG,UC,ROL,ROH,ZMU

COMMON/SFV/SV,SHS,CF(41),ST(41)

COMMON/CB1/R(26),C(26),S(26),ZK(26) ,T(26),ZJ(26)

COMMON/CTURB/ZA,ZB,ZC,C1,C2,C3,ZLP

COMMON/CONTL/ICNTR1,ICNTR2,ICNTR3 ,ITBCNT ,NITER

COMMON/SLT/SLTURB(4),FRTURB(4) ,DELTA1,DELTA2,DELTA5

C ICNTR1 SHOULD BE 1 IF INPUT DATA FOR WORTICITY ,STREAM FUNCTION

C KINETIC ENERGY AND MASS CONCENTRATION IS TO BE SUPPLIED

C ICNTR2 IS TO BE 1 IF PUNCHED OUT PUT IS DESIRED

C ICNTR3 IS TO BE 1 IF STARTING BOUNDARY CONDITIONS ARE DESIRED

C ITBCNT IS COUNTER FOR THE TURBLANCE PROFILES AT THE SLOT

DATA NI/80,49*500/,NMAX,CC/75,0.0001/

DATA NI/10,49*500/,NMAX,CC/55,0.0001/

DATA NI/50*500/,NMAX,CC/200,0.0001/

DATA X2/0.0,0.015,0.032,0.052,0.075,0.1,0.130,0.165,0.205,0.25,
1 0.3,0.35,0.4,0.46,0.531,0.62,0.74,0.89,1.08,1.3 ,1.63,2.03,2.43,
2 2.8,3.2,3.6/

DATA X1/0.0,0.05,0.1,0.15,0.21,0.285,0.385,0.515,0.685,

1 0.905,1.2,1.6,2.0,2.5,3.0,3.5,4.0,4.5,5.0,5.65,6.5,7.5,8.8,10.3,
2 11.8,13.3/

DATA IN,JN,IMIN,IMAX/26,26,26*1,26*26/

DATA JSLOT,JFREE,UC,UG/10,15,1.0,1.0/,ROL,ROH/1.0,1.0/

DATA ZA,ZC,C2,C3/0.416,0.22,0.263,0.016/

DATA NW,NF,NK,NML,NP/1,2,3,4,7/,IV/14/,NV1,NV2,NDP,NT/10,12,8,13/
1 ,NG1,NG2,NMU,NRO/9,11,5,6/ ,IE/3/ , NPRIN/14/

2 ,PR(3),PR(4)/1.5,0.4/,INDG/1/

DATA ZMU/0.0000316/

DATA T/26*1./

DATA NMH/14/

DATA IE/4/

DATA ICNTR1,ICNTR2,ICNTR3/1,1,1/

DATA ICNTR1,ICNTR2,ICNTR3/1,2,2/

DATA ICNTR1,ICNTR2,ICNTR3/2,2,1/

DATA ICNTR1,ICNTR2,ICNTR3/2,1,1/

DATA ICNTR1,ICNTR2,ICNTR3/1,1,2/

DATA SLTURB(1),SLTURB(2),SLTURB(3),SLTURB(4)/0.01,0.1,0.2,0.3/

DATA FRTURB(1),FRTURB(2),FRTURB(3),FRTURB(4)/0.01,0.1,0.2,0.3/

DATA DELTA1,DELTA2,DELTA5/0.025,0.225,0.1,53/

DATA ITBCNT/ 1/

END

\$IBFTC SRVIC

```

SUBROUTINE VISCO
COMMON/CVRBLE/A(26,26,14),X1(26),X2(26),IMIN(26),IMAX(26),RP(4),
1 PR(4),IN,JN,IE,IV,IR
COMMON /CNUMBER/NW,NF,NK,NHS,NVT,NML,NMH,NP,NG1,NG2,NMU,NRO
COMMON /CN2/ NV1,NV2,NDP,NT
COMMON/C/ANAME(6,14),ASYMBL(14),NI(50),NPRIN,INDG,CC,NMAX,INM,JNM
COMMON /CBOU/ JSLOT,JFREE,UG,UC,ROL,ROH,ZMU
COMMON/CTURB/ZA,ZB,ZC,C1,C2,C3,ZLP
COMMON/CB1/R(26),C(26),S(26),ZK(26) ,T(26),ZJ(26)
COMMON/SFV/SV,SHS,CF(41),ST(41)
COMMON/CONTL/ICNTR1,ICNTR2,ICNTR3 ,ITBCNT ,NITER
CK=0.7
DO 40 J=2,JN
DO 40 I=2,INM
ZMEFOL=A(I,J,NMU)
A(I,J,NMU)=A(I,J,NRO)*SQRT(A(I,J,NK))*A(I,J,NMH)
A(I,J,NMU)=A(I,J,NMU)+ZMU
A(I,J,NMU)=CK*ZMEFOL+(1.0-CK)*A(I,J,NMU)
IF(ICNTR3 .EQ. 1.AND.NITER.LT. 35) A(I,J,NMU)=20. *ZMU
40 CONTINUE
DO 16 J=1,JN
A(1,J,NMU)=A(2,J,NMU)
16 A(IN,J,NMU)=A(INM,J,NMU)
DO 15 I=1,IN
A(I,JN,NMU)=ZMU
15 A(I,1,NMU)=ZMU
RETURN
END
$IBFTC SRVEL
SUBROUTINE VELINP(X2,N0,VELIN1,VELIN2,WORCT1,WORCT2,STRFC1,STRFC2
1)
COMMON /CBOU/ JSLOT,JFREE,UG,UC,ROL,ROH,ZMU
COMMON/SLT/SLTURB(4),FRTURB(4) ,DELTA1,DELTA2,DELTA5
DIMENSION X2(N0)
DIMENSION Y1(50),Y2(50),STRFC1(50),STRFC2(50),WORCT1(50),WORCT2(50
1),VEL1(50),VEL2(50)
DIMENSION VELIN1(50),VELIN2(50)
POWN=0.2
PWM=1.0-POWN
POWS=1.0+POWN
DO 1 I=1,N0
Y1(I)=X2(I)
1 Y2(I)=X2(I)
UCM=UC
DELTA3=X2(JSLOT)-X2(1)
DELTA4=X2(JFREE)-X2(1)
DO 10 I=1,N0
YS=Y1(I)-Y1(1)
IF (YS.LE.DELTA1) GO TO 3
IF (YS.LE.DELTA2) GO TO 4
IF (YS.LE.DELTA3) GO TO 5
IF (YS.LE.DELTA4) GO TO 6
IF (YS.LE.DELTA5) GO TO 7
GO TO 8
3 YG=DELTA1
Y=YS
IF(Y.LT. 0.0005) VEL1(I)=0.0
IF(Y.GT.0.0005) VEL1(I)=((Y/YG)**POWN)*UCM
STRFC1(I)=((1.0/YG)**POWN)*Y**POWS/POWS

```

```

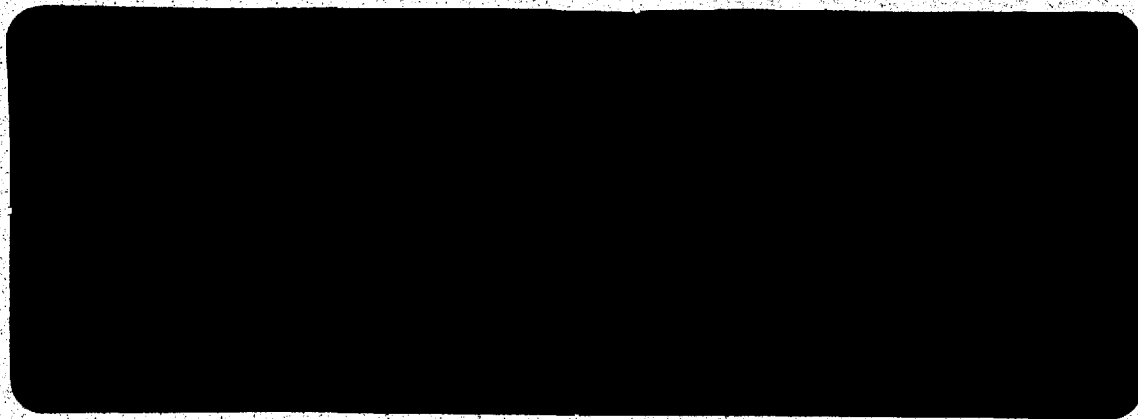
C1=YG/1.2
C1=C1*UCM
IST3=I
STRFC1(I)=STRFC1(I)*UCM
IF(I.EQ. 1) WORCT1(I)=0.0
IF(I.NE. 1) WORCT1(I)=-UCM*POWN*(YG/Y)**PWM/YG
GO TO 10
4  IST4=I
   Y=YS-DELTA1
   VEL1(I)=UCM
   STRFC1(I)=C1+UCM*Y
   WORCT1(I)=0.0
   C2=C1+(DELTA2-DELTA1)*UCM
   GO TO 10
5  IST5=I
   YG=Y1(JSLOT)-DELTA2
   Y=Y1(JSLOT)-Y1(I)
   STRFC1(JSLOT)=C2+UCM*YG/POWS
   IF(Y.LT.0.0005) VEL1(I)=0.0
   IF(Y.GT.0.0005) VEL1(I)=((Y/YG)**POWN)*UCM
   STRFC1(I)=
     -UCM*((1.0/YG)**POWN)*Y**POWS/POWS
1  +STRFC1(JSLOT)
   IF(Y.LT.0.0005) WORCT1(I)=0.0
   IF(Y.GT.0.0005) WORCT1(I)=UCM*(1.0/YG)**POWN*POWN/Y**PWM
   GO TO 10
6  IST6=I
   VEL1(I)=0.0
   STRFC1(I)=STRFC1(JSLOT)
   WORCT1(I)=0.0
   GO TO 10
7  IST7=I
   YG=DELTA5-Y1(JFREE)
   Y=Y1(I)-Y1(JFREE)
   IF(Y.LT.0.0005) VEL1(I)=0.0
   IF(Y.GT.0.0005) VEL1(I)=(Y/YG)**0.143*UG
   STRFC1(I)=STRFC1(JFREE)+((1.0/YG)**0.143)*(Y**1.143/1.143)*UG
   IF(Y.LT.0.0005) WORCT1(I)=0.0
   IF(Y.GT.0.0005) WORCT1(I)=-0.143*(YG/Y)**0.857/YG
   WORCT1(I)=WORCT1(I)*UG
   C7=STRFC1(JFREE)+(YG/1.143)*UG
   GO TO 10
8  VEL1(I)=UG
   Y=Y1(I)-DELTA5
   STRFC1(I)=C7+UG*Y
   WORCT1(I)=0.0
10  CONTINUE
   DO 15 I=1,N0
   VELIN1(I)=VEL1(I)
15  CONTINUE
   WRITE (6,20) (Y1(I),VELIN1(I),STRFC1(I),WORCT1(I),I=1,N0)
20  FORMAT(4E20.4)
   RETURN
   END

```

***** FOLLOWING ARE THE DATA CARDS
 THE INITIAL REGION OF A TWO DIMENSIONAL WALL JET, TURBULENT VISCOSITY
 PRANDTL-KOLMOGOROV 1945 TURBULENCE THEORY
 VORTHE VORTICITY
 STRM THE STREAM FUNCTION
 KINETHE TURBULENT KENETIC ENERGY

MASS THE MASS CONCENTRATION
TURV THE TURBULENT VISCOSITY
DENSTHE MEAN DENSITY
PRESTHE STATIC PRESSURE
DYNATHE DYNAMIC PRESSURE
G1 THE MASS VELOCITY IN DIRECTION 1
U THE VELOCITY COMPONENT IN DIR 1
G2 THE MASS VELOCITY IN DIRECTION 2
V THE VELOCITY COMPONENT IN DIR 2
STR THE SHEAR STRESS
MIXL THE MIXING LENGTH SCALE
***** END OF DATA CARDS

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THE DEPENDENCE OF THE IMPERVIOUS WALL EFFECTIVENESS OF A TWO-DIMENSIONAL WALL-JET ON THE THICKNESS OF THE UPPER LIP BOUNDARY LAYER

S. C. KACKER and J. H. WHITELAW

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(Received 15 February 1967 and in revised form 24 May 1967)

NOMENCLATURE

- C , mass concentration;
 $\bar{u}_c$, mean velocity of the flow at exit from the slot;
 u_G , free-stream velocity;
 x , distance measured along the wall in the free-stream direction;
 y_c , slot height;
 δ , thickness of the upper lip boundary layer;
 η_i , impervious wall effectiveness defined by $\eta_i = \frac{C_s - C_G}{C_c - C_G}$

Subscripts

- C , pertaining to the flow through the slot;
 G , pertaining to the free-stream;
 S , pertaining to the wall.

INTRODUCTION

IN AN earlier paper [1], one of the present authors presented measurements of the impervious wall effectiveness of a two-dimensional wall-jet, made with a slot height of 0.25 in. These measurements were reported in less detail in a subsequent paper by Nicoll and Whitelaw [2] which also described an explicit method for predicting the effectiveness of the uniform property, two-dimensional wall-jet. The measurements described in the above two papers indicate that the effectiveness at any downstream station has a maximum at a value of the ratio of the mean velocity in the slot to the free-stream velocity, $\bar{u}_c/u_G$, close to unity. The magnitude of this maximum has been found in [3] to decrease substantially as the slot height is decreased. These observations are in keeping with the trends indicated by the measurements of Seban [4].

The cause of the decrease in the maximum value of the effectiveness can be ascribed to several sources all of which are associated with the geometry of the injection region or of the region upstream of this point; the present authors are presently conducting an experimental programme to

investigate these effects. As part of this investigation, the effect of the thickness of the boundary layer on the upper lip of the slot has been investigated and it is the purpose of this note to present the findings. The situation considered is shown schematically on Fig. 1.

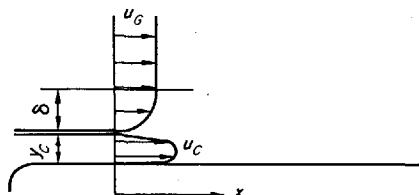


FIG. 1. The injection region of a two-dimensional wall-jet.

PREVIOUS MEASUREMENTS

Two experimental investigations of the present type have previously been carried out: the first by Chin *et al.* [5] and the second by Seban [6]. These investigations were carried out by measuring the adiabatic wall effectiveness: in each case the velocity ratio $\bar{u}_c/u_G$ was restricted to values less than unity. Although both investigations indicated that the effect of the thickness of the upper lip boundary layer was small (i.e. a maximum of 8 per cent of unity) the trends observed by these authors were opposite. Chin *et al.* observed that the difference between measurements made with a thin upper lip boundary layer (equal to eight slot heights) and those made with a thicker one (equal to twenty slot heights) was smaller for low values of the ratio, $\bar{u}_c/u_G$, and tended to a maximum as this ratio tended to unity. This tendency suggested that the thickness of the upper lip boundary layer might be significant in deciding if the effectiveness against velocity ratio curve should have a maximum. Seban, on the other hand, observed that the difference was a maximum for low values of the velocity ratio at a distance of 100 slot heights downstream: the difference appeared more constant

at a distance of two hundred slot heights. Seban employed upper lip boundary layers with thicknesses equal to one slot height and equal to ten slot heights.

It should be stressed that within the range of measurement, the differences reported in [5] and [6] were not much greater than the experimental precision. It should also be stated that the absolute values of effectiveness obtained by these two experimenters were significantly different: the measurements of Chin *et al.* being generally higher than those of Seban. This is in keeping with the trends found by Whitelaw [3] since Chin *et al.* used a 0.108-in slot whereas Seban used a 0.063 in slot.

PRESENT MEASUREMENTS

The present measurements were made using the apparatus described in [1] and [2]. The free stream velocity in all cases was nominally 68 ft/s and, as in the previous measurements, the impervious wall effectiveness was determined by injecting a tracer of helium into the secondary stream, sampling the fluid at the wall at downstream locations and measuring the mass fraction of helium in each sample with the aid of a gas chromatograph. Considerable experience with this technique suggests that the impervious wall effectiveness can be determined to a precision of ± 1.5 per cent. The value of the velocity ratio is obtained by measuring the velocity profile across the slot and integrating the result: the precision of this procedure diminishes as the value of the velocity ratio diminishes but should not be worse than ± 5 per cent. The measurement of downstream distance is accurate to at worst 0.005 in, i.e. less than 0.1 per cent.

The slot height used for the present investigation was 0.074 in and the upper lip boundary-layer thicknesses used were 0.18 in, 0.36 in and 0.75 in. The 0.18-in thick boundary layer was obtained as a result of the design of the slot and its upstream geometry. The other boundary-layer thicknesses were obtained by extending the upper lip of the slot into the contraction of the wind-tunnel. In all cases, the mean velocity profile of the upper lip boundary-layer proved to have the normal turbulent boundary-layer shape.

Figure 2 shows the measurements of effectiveness obtained from the present investigation and indicates that the effect of the upper lip boundary layer on the impervious wall effectiveness is of a second order for the range of values tested.

CONCLUSIONS

The results of the present investigation show that the effect of the thickness of the upper lip boundary layer on the impervious wall effectiveness is not greater than 5 per cent of unity in the range of velocity ratios $0.3 \leq \bar{u}_c/u_g \leq 2.05$. The tendency is for the effectiveness to decrease as the upper lip boundary-layer thickness is increased. This confirms the general conclusions of Chin *et al.* and of Seban for values of the velocity ratio less than unity and extends this conclusion

to higher values of the velocity ratio. The absolute values of the effectiveness agree closely with those of Seban which was to be expected since the present investigation used a similar slot height to that of Seban.

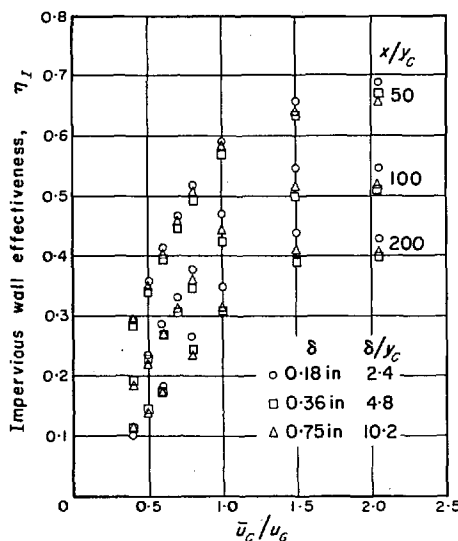


FIG. 2. Measured values of the impervious wall effectiveness.

ACKNOWLEDGEMENT

This work was supported by the Ministry of Technology (Agreement PD/37/043) to whom grateful acknowledgement is made.

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2. W. B. NICOLL and J. H. WHITELAW, The effectiveness of the uniform density, two-dimensional wall jet, *Int. J. Heat Mass Transfer* **10**, 623 (1967).
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6. R. A. SEBAN, Effects of initial boundary-layer thickness on a tangential injection system, *J. Heat Transfer* **82**, 392 (1960).

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Lecturer.

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Some Properties of the Two-Dimensional, Turbulent Wall Jet in a Moving Stream

Measurements of the mean and fluctuating properties of a two-dimensional, constant property, plane turbulent wall jet in a moving stream are presented and their implications discussed.

Introduction

THE BEHAVIOR of wall jets is of practical interest mainly because of their application to boundary-layer control and to film cooling. In spite of this, most detailed experimental investigations have been carried out in the range of velocity ratios from 2 to infinity; the range of practical interest is approximately from 0.1 to 2. Prominent among these previous investigations are those of Bradshaw and Gee [1],¹ Myers, Schauer, and Eustis [2], Eskinazi and Kruka [3], and Tailland and Mathieu [4]. The reason for the concentration of effort on the higher velocity ratios is understandable; in this region the velocity profiles retain a maximum and the flow can be analyzed, albeit approximately, by methods of similarity.

The present paper describes detailed measurements of the mean and fluctuating properties of a two-dimensional, constant property, plane turbulent wall jet in a moving stream. The measurements are confined to the range of velocity ratios from 0.75 to 2.74 and to the region from the slot exit to a downstream distance of 150 slot heights. The measurements form part of an investigation of the effects of slot geometry on the effectiveness of film cooling and were undertaken with a view to providing in-

formation regarding the various forms of viscosity laws which could be employed in a prediction procedure suitable for turbulent wall flows with secondary injection. For this reason, profiles of static pressure, total pressure, the three components of the turbulent velocity, and turbulent shear stress were measured; these yielded profiles of effective viscosity, Prandtl's mixing length, and the Prandtl-Kolmogorov turbulent length scale.

The measurements, and their implications, are presented and discussed in the following sections.

Apparatus and Experimental Techniques

The open-circuit wind tunnel used for the present investigation had an 18-in. $\times$ 12-in. working section which was located downstream of an 8:1 area contraction ratio. Upstream of the contraction were three 20 gauge by 28 S.W.G. copper screens and a gyro-flow blower which was used to deliver air to the working section at a velocity of approximately 63 fps. The air entering the suction side of the blower was passed through a felt filter.

Secondary air was injected tangentially to the 18-in-wide base plate of the working section. A small blower was used to create this secondary mass flow and a plenum chamber with two 20 gauge $\times$ 28 S.W.G. copper screens was located between the blower and the injection slot; an orifice plate was located on the suction side of the secondary blower in order to measure the mean flow. Preliminary tests showed that the temperature of the air leaving the injection slot was higher than that of the mainstream and, since the response of a hot wire is temperature sensitive, a small automobile radiator was used to cool the secondary air. The flow rate of water to the radiator was controlled and this control insured that the temperature profile in the plane of the slot exit was uniform to within ± 0.5 deg C.

¹ Numbers in brackets designate References at end of paper.

Contributed by the Applied Mechanics Division for presentation at the Winter Annual Meeting, New York, N. Y., December 1-5, 1968, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

Discussion of this paper should be addressed to the Editorial Department, ASME, United Engineering Center, 345 East 47th Street, New York, N. Y. 10017, and will be accepted until one month after final publication of the paper itself in the JOURNAL OF APPLIED MECHANICS. Manuscript received by the ASME Applied Mechanics Division, April 26, 1968. Paper No. 68—WA/APM-13.

Nomenclature

a_1 = constant, equation (9)
 a_s = constant, equation (19)
 a_D = constant, equation (19)
 A_d = damping constant in van Driest law for mixing length
 A = constant, equation (1)
 B = constant, equation (1)
 C = constant, equation (2)
 C_e = constant, equation (18)
 $c_f/2$ = drag coefficient, $\tau_s/\rho u_G^2$
 E = d-c voltage drop across hot wire
 G = equation (11)
 H_{12} = shape factor, R_1/R_2
 H_{32} = shape factor, R_3/R_2
 K = von Karman constant
 l = Prandtl's mixing length, equation (12)
 l_t = turbulent length scale, equation (18)

L_y = lateral integral length scale, equation (6)
 L = equation (10)
 p_s = static pressure
 p_0 = total pressure of free stream
 p_∞ = static pressure of free stream
 q^2 = turbulent kinetic energy, $u'^2 + v'^2 + w'^2$
 $R(0, r, 0)$ = correlation coefficient, equation (5)
 r = distance between two hot wires in correlation measurements
 R_1 = Reynolds number based upon displacement thickness, $\delta_1 u_G/\nu$
 R_2 = Reynolds number based upon momentum thickness, $\delta_2 u_G/\nu$

R_2 = Reynolds number based upon energy thickness, $\delta_3 u_G/\nu$
 R_x = Reynolds number based upon distance x , $u_G x/\nu$
 $\bar{s}$ = shear work integral,

$$\frac{1}{\rho u_G^3} \int_0^{y_G} \left(\tau \frac{\partial u}{\partial y} \right) dy$$

 l = lip thickness
 u = velocity in main stream direction
 u_G = free-stream velocity
 $\bar{u}_G$ = average slot velocity,

$$\frac{\int_0^{y_G} u dy}{y_G}$$

(Continued on next page)

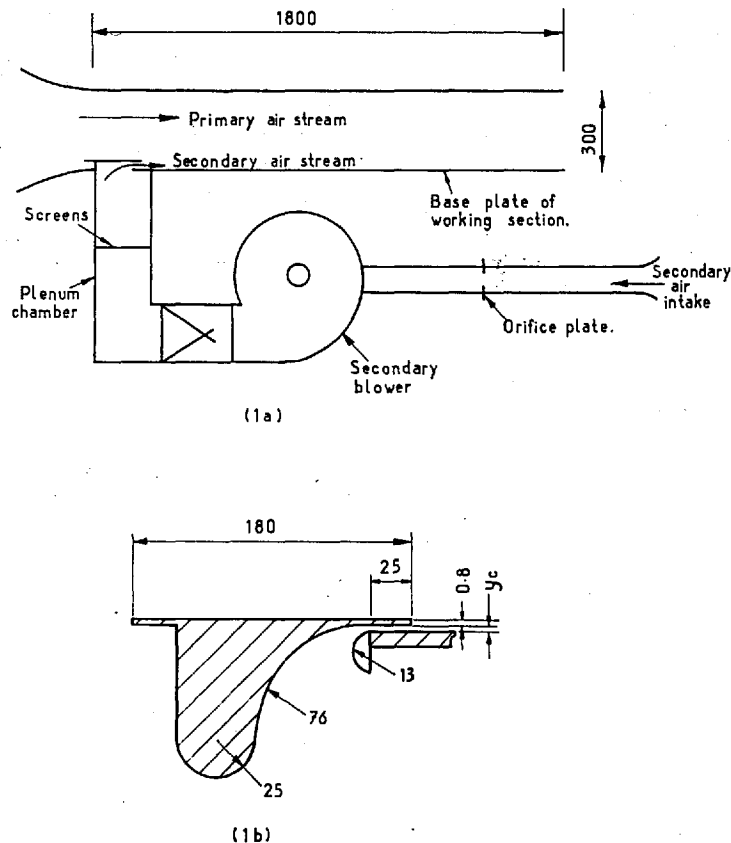


Fig. 1 General arrangement of apparatus (dimensions in mm): $y_c = 6.5$

The apparatus is shown schematically in Fig. 1(a), and Fig. 1(b) shows the injection slot in detail.

The total pressure, total temperature, and hot-wire probes were carried on traversing equipment which moves longitudinally along a 4-in-wide gap in the roof of the wind tunnel. The traversing head could be clamped in any longitudinal location and the resulting 4-in-wide gap closed with a close-fitting length of Dural plate. A slide plate permitted the probe holder to be moved laterally by up to 3 in. on either side of the center line of the working section. The slide plate could be clamped in any position within this range. Vertical movement was controlled by a micrometer head which could be read to the nearest 0.0002 in.

All mean velocities were calculated from measurements of static pressure and of total pressure; the latter were obtained with a total pressure probe of internal dimensions 0.004 in. $\times$ 0.040 in. Static pressure holes designed in accordance with the recommendations of Shaw [5] were located in the base plate of the working section both on and off the center line; additional static pressure holes were located on one side wall of the working section. The MacMillan [6] correction was applied to all values of mean velocity.

DISA constant temperature hot-wire equipment was used for all measurements of all turbulence properties. Two anemometers (55A01), a random signal correlator (55A06), and a cross-wire probe (55A32) were used to measure v' , w' and $u'v'$; u' was measured with a single-wire probe (55A25). Mean velocity measurements were also obtained from a single-wire probe and agreed with the values obtained from pressure measurements to within ± 2 percent.

The hot wires were calibrated in accordance with the law of Collis and Williams [7]; this may be written in the form

$$E^2 = A + Bu_n^{0.45} \quad (1)$$

The calibration was carried out in a free stream of relatively low turbulence intensity (0.5 percent) and the constants A and B were calculated from six measurements of the voltage drop, E , and the normal velocity, u_n . The calibration was carried out before and after each profile.

In all the measurements reported in the present paper, the free-stream velocity was nominally the same and the secondary flow rate was varied to give velocity ratios, $\bar{u}_c/\bar{u}_0$ in the range 0.75 to 2.74. The pressure gradient in the working section (obtained

Nomenclature

u', v', w' = fluctuating components of velocity in x , y , and z -directions
 x = distance measured along wall in main stream direction from slot exit
 y = distance normal to wall
 y_A = distance of fixed hot wire from wall in correlation measurements
 y_c = slot height

y_1 = distance from wall to a point where velocity $u = \frac{u_{max} - u_0}{2}$
 y_0 = constant, equation (8)
 ρ = density
 μ = molecular viscosity
 μ_{eff} = effective viscosity
 $\nu = \mu/\rho$
 γ = intermittency
 ϵ = dissipation
 δ_1 = displacement thickness,

δ_2 = momentum thickness, $\int_0^{y_0} (1 - u/u_0) dy$
 δ_3 = energy thickness, $\int_0^{y_0} \frac{u}{u_0} \left(1 - \frac{u}{u_0}\right) dy$
 τ = shear stress
 τ_s = wall shear stress

from the static-pressure holes in the side wall of the working section) was so small as to be negligible for all computational purposes.

A more detailed description of the present apparatus and of preliminary measurements may be found in reference [8].

Mean Properties of Flow

In this section, measurements of the static pressure, mean velocity and wall shear stress are presented and discussed; representative integral properties are also presented. The measurements were carried out at four values of the velocity ratio $\bar{u}_c/u_0$, viz., 0.75, 0.915, 1.33, and 2.74; and at six values of the nondimensional distance from the slot exit, viz., 0, 10, 20, 50, 80, and 150. Since these measurements have resulted in a considerable quantity of data, it cannot all be presented in a published paper. Where omissions have been made, the data which are presented, are intended to be representative of the whole body of data.

Static Pressure Distributions. It is convenient to examine, first, the longitudinal static-pressure distribution in the working section. Fig. 2 shows the static pressure distribution on the base plate of the working section for the four values of $\bar{u}_c/u_0$; the free-stream static-pressure distributions, measured on the side wall of the working section, are also shown in the figure. It may be seen that the free-stream static pressure and the base-plate static pressure are not of the same magnitude. The difference is a maximum close to the slot exit and can still be observed 100 slot-heights downstream; the static-pressure distribution normal to the base plate and implied by Fig. 2 has been confirmed using a static-pressure probe.²

These results show that the boundary-layer assumption of zero normal pressure gradient is invalid for values of x/y_c less than

² Measurements of this type, and for a wider range of velocity ratio, have been reported in reference [9].

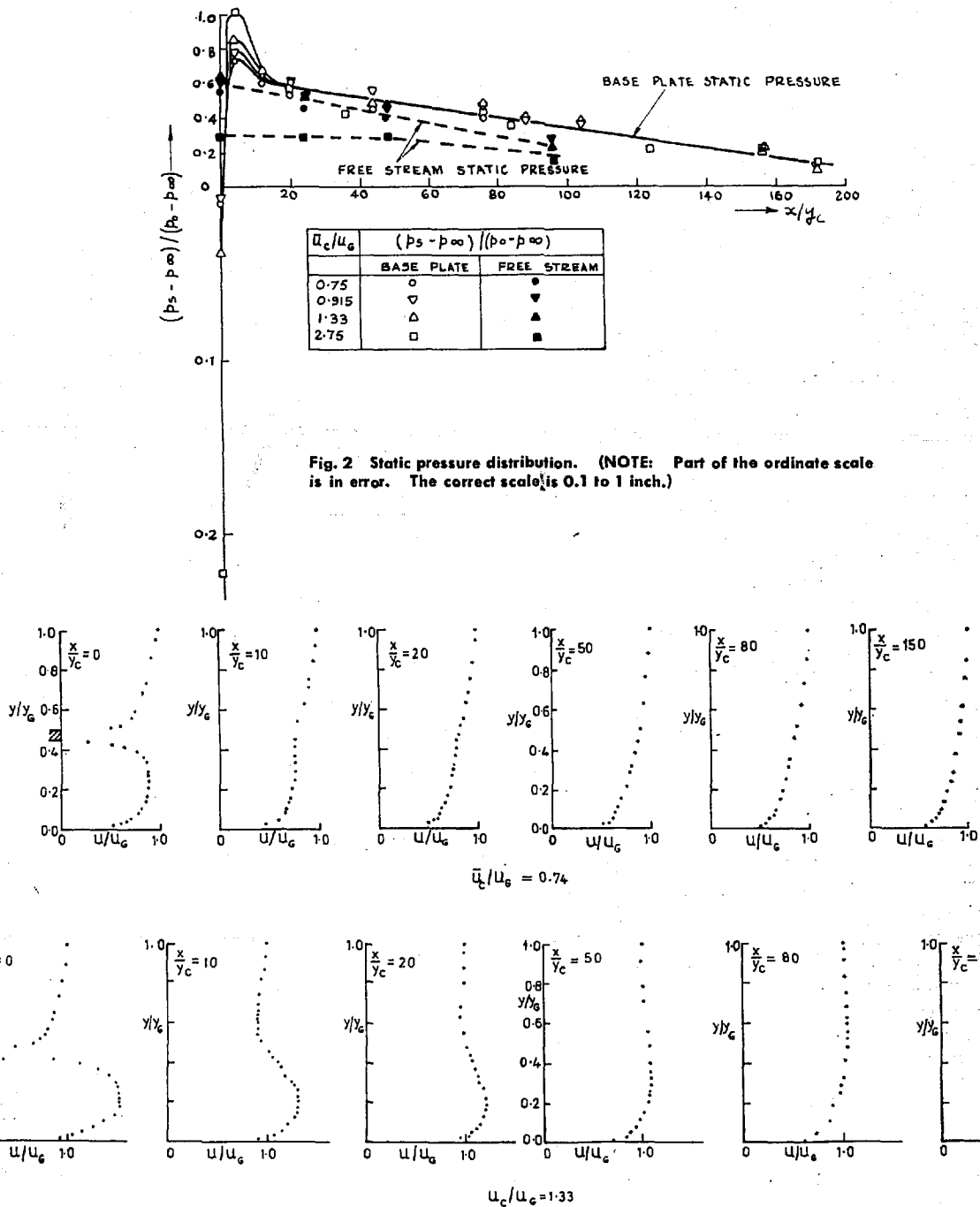


Fig. 2 Static pressure distribution. (NOTE: Part of the ordinate scale is in error. The correct scale is 0.1 to 1 inch.)

Fig. 3 Profiles of mean velocity

around 100 although little error is likely to be incurred by invoking the assumption for x/y_c greater than 20. In the region upstream of 20 slot heights, however, the flow must be described by the Navier-Stokes' equations without the simplifying boundary-layer assumptions. For larger values of t/y_c , this requirement is even more necessary due to the enlarged region of separated flow immediately downstream of the slot lip.

Mean-Velocity Profiles. Fig. 3 presents typical mean-velocity profiles for the velocity ratios 0.75 and 1.33. The values of mean velocity were calculated from measurements of total pressure and from the mean of the wall and free-stream static pressures appropriate to that x -station. The maximum error in velocity incurred (for $x/y_c > 10$) by using this static pressure was ± 1 percent.

The shape of the mean-velocity profile in the plane of the slot exit is of special interest since it represents one boundary condition for any procedure designed to predict the downstream flow properties. The initial profiles shown in Fig. 3 indicate similar boundary layers on the upper lip of the slot and symmetrical, developed secondary-flow profiles. It can be seen that the momentum defect caused by the finite lip thickness is recognizable for a considerable distance downstream; in the case of the wake flow, this defect leads to a region of very small velocity gradient. It is useful to note that the profiles at 150 slot-heights downstream are significantly different in shape for the jet and wakelike flows.

The precision of the measurements can be estimated from Fig. 3 but can more readily be assessed from Fig. 4 which presents the mean-velocity profiles plotted on the coordinates proposed by Clauser [10]. The lines of constant shear stress have been calculated from the law of the wall

$$u^+ = \frac{1}{K} \ln y^+ + C \quad (2)$$

using the constants recommended by Patel [11]; viz., $K = 0.4187$ and $C = 5.45$. Fig. 4 shows that the measured values of mean velocity exhibit a scatter of not more than 1 percent.

Wall Shear Stress. Previous measurements of wall-shear stress in wall-jet flows have been carried out using conservation of momentum [12], the Preston tube [1, 3, 13], the wall balance [4], and the hot film [2]. These measurements indicate consistent trends although the magnitude of the wall-shear stress appears to vary with the experimental technique. The profiles shown in Fig. 4 indicate that a logarithmic region does exist for the wall-jet flow, especially for values of x/y_c in excess of 50. Clauser charts for other velocity ratios exhibit similar trends although, as might be expected, the wake flows tend to exhibit a logarithmic region closer to the slot exit. Since the value of K in the law of the wall is qualitatively supported by the present measurements, it is reasonable to assume that the values of shear stress suggested by the Clauser chart will not be greatly in error. Consequently, values of the drag coefficient obtained from Clauser charts are presented in Fig. 5(a) and it can be seen that these results are consistent with one another. As might be expected, Preston tube measurements, obtained with a round total pressure probe and the calibration constants of Patel, were identical with those obtained from the Clauser chart.

The present measurements are outside the range of previous investigations but it is interesting to note that Bradshaw and Gee calibrated their Preston tube in the wall jet with zero free-stream velocity and found that the calibration was within 6 percent of that obtained in pipe flow. This conclusion is consistent with the present measurements. The present results are well predicted by the drag laws of Escudier, Nicoll, and Spalding [14] but this is hardly surprising since this law is based upon the measurements of references [1, 2, 3, and 13]. The maximum deviation of the correlation formula,

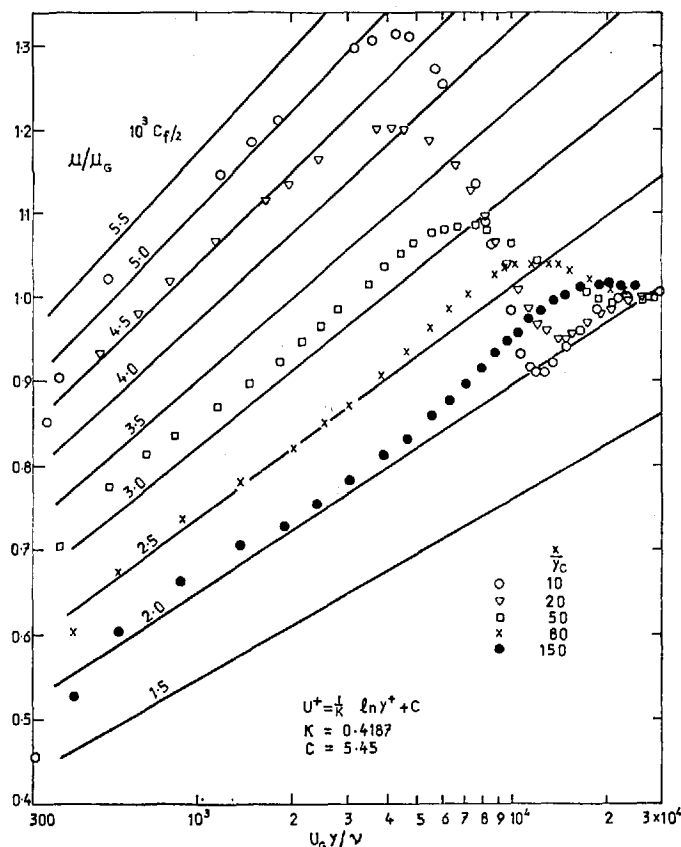


Fig. 4 Clauser chart: $\bar{u}_c/\bar{u}_0 = 1.33$

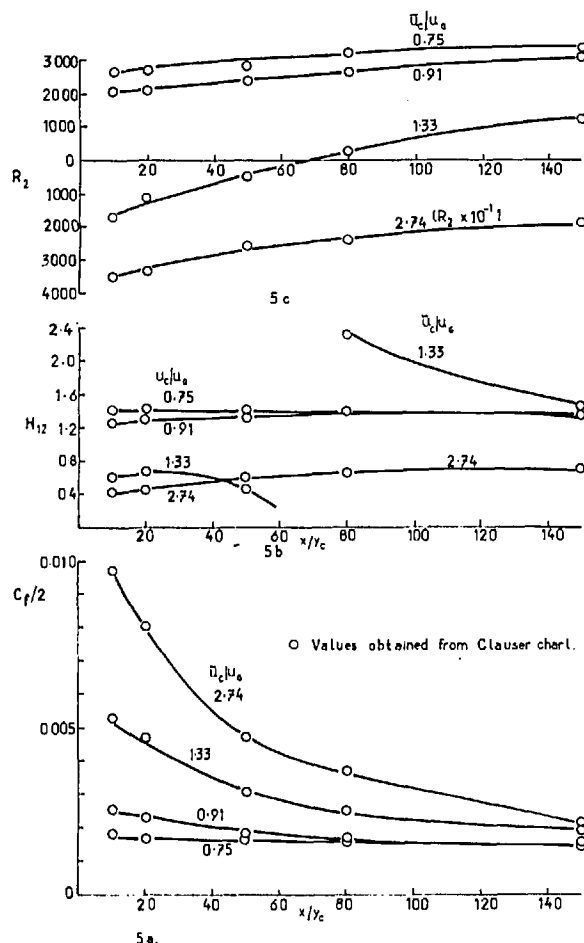


Fig. 5 R_2 , H_{12} , and $c_f/2$ as functions of x/y_c

$$c_f/2 = \frac{0.243\zeta^2 + 0.0376\zeta - 0.00106 + \frac{0.0914\zeta^2}{1 + 65/\zeta}}{\left[\ln \left\{ \frac{3.389R_2\zeta}{(1 - \zeta)(1 + 2\zeta)} \right\} \right]^2} \quad (3)$$

where

$$\zeta = \frac{2}{3}H_{32} - 1 + \left[\frac{2}{3}H_{32} \left(\frac{2}{3}H_{32} - 1 \right) \right]^2$$

from the present experimental results is ± 10 percent for all velocity ratios except 1.33; in this last case, the momentum thickness Reynolds number R_2 passes through zero and equation (3) is inapplicable.

Integral Properties. Figs. 5(b, c) show the variation of the shape factor, H_{12} , and of the momentum-thickness Reynolds number, R_2 , with nondimensional distance from the slot. The shape factor exhibits a singularity as the momentum-thickness Reynolds number passes through zero; i.e., as the mean-velocity profile loses its velocity maximum. This property of wall jets is a source of difficulty in prediction procedures based on integral equations: One method of overcoming the difficulty is described in reference [15]. It may also be observed from Fig. 5(b) that, for the lower velocity ratios, the shape factor tends to a value of approximately 1.35; i.e., the usual value for a flat-plate boundary layer.

As remarked in the previous subsection, the integral momentum equation

$$\frac{dR_2}{dR_x} + (1 + H_{12}) \frac{d \ln u_G}{dR_x} = c_f/2 \quad (4)$$

can be used in conjunction with mean-velocity profiles to determine the shear stress coefficient. This was attempted using the

measurements shown in Figs. 5(b, c) together with the known distribution of free-stream velocity. The values of momentum thickness were subjected to least-squares fitting procedures using both polynomial and logarithmic equations but the resulting gradients together with the pressure gradient term did not lead to values of $c_f/2$ which closely resembled the values obtained from the Clauser chart: Discrepancies of up to 40 percent were observed. Close examination of the results suggested that a satisfactory momentum balance would require velocity profiles at much smaller x -intervals.

Turbulence Properties of Wall Jet

Turbulence Intensities. The distributions of the fluctuating velocity components are shown in Figs. 6(a, b) for values of $\bar{u}_C/u_G$ of 0.75 and 1.33. The corresponding distributions of turbulent kinetic energy are shown in Fig. 7 and it can be seen that for the wakelike flow the kinetic energy initially decreases with x and, far downstream, begins to increase: The far-downstream distribution is similar in shape to that of a flat-plate boundary layer. The jetlike flow exhibits the same rapid decrease in kinetic energy but, in the region between 50 and 150 slot-heights downstream, the total kinetic energy is almost constant although the distribution across the boundary layer has changed noticeably. At 150 slot heights the kinetic-energy distribution of the jetlike flow is tending toward a shape similar to that for the wakelike flow but has not attained it. The positions of the zero mean-velocity gradient and minimum kinetic energy are indistinguishable.

Turbulent Length Scale. Values of the correlation coefficient, $R_{(0, \tau, 0)}$, were obtained at only two values of velocity ratio, 0.75 and 2.74; typical results are shown in Fig. 8. The correlation coefficient shown in Fig. 8 is defined as

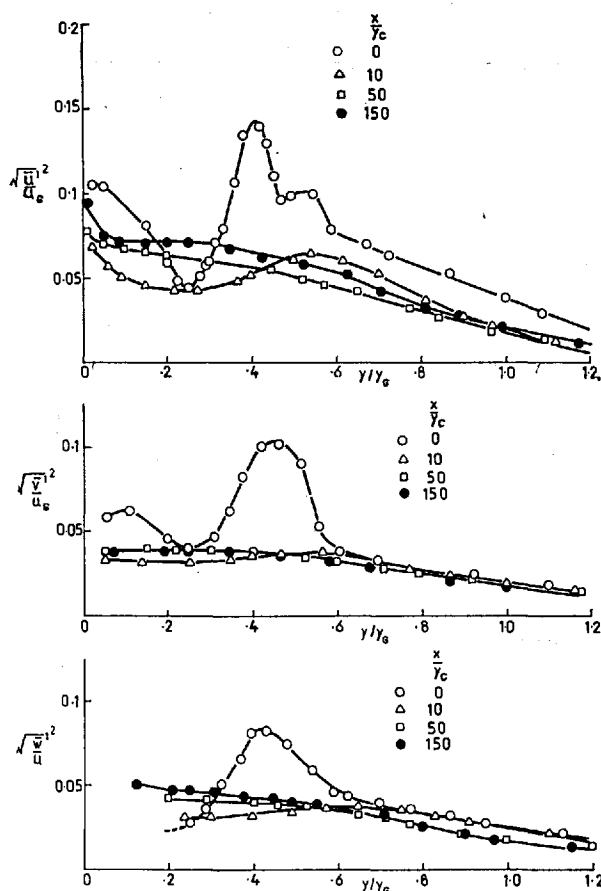


Fig. 6(a) $\bar{u}_C/u_G = 0.75$

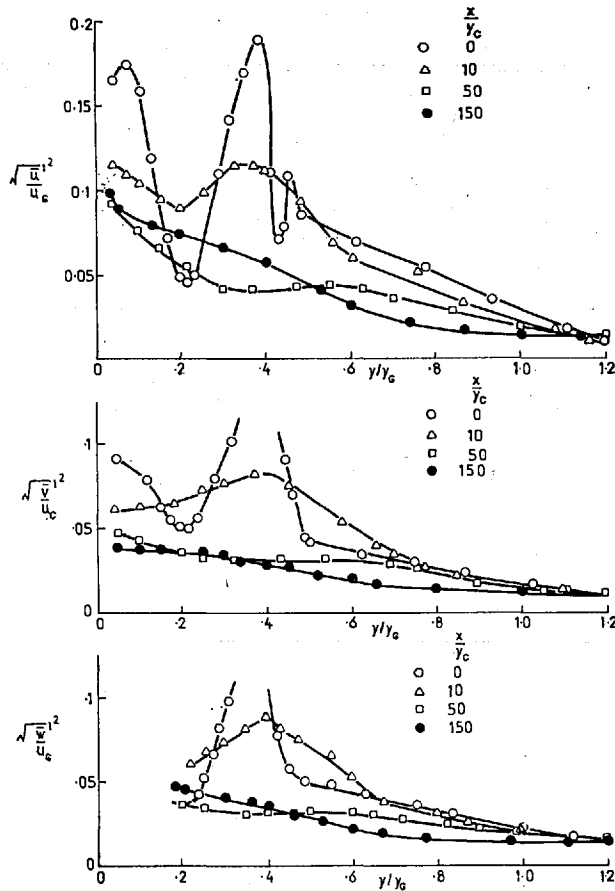


Fig. 6(b) $\bar{u}_C/u_G = 1.33$

Fig. 6 Profiles of fluctuating components of velocity

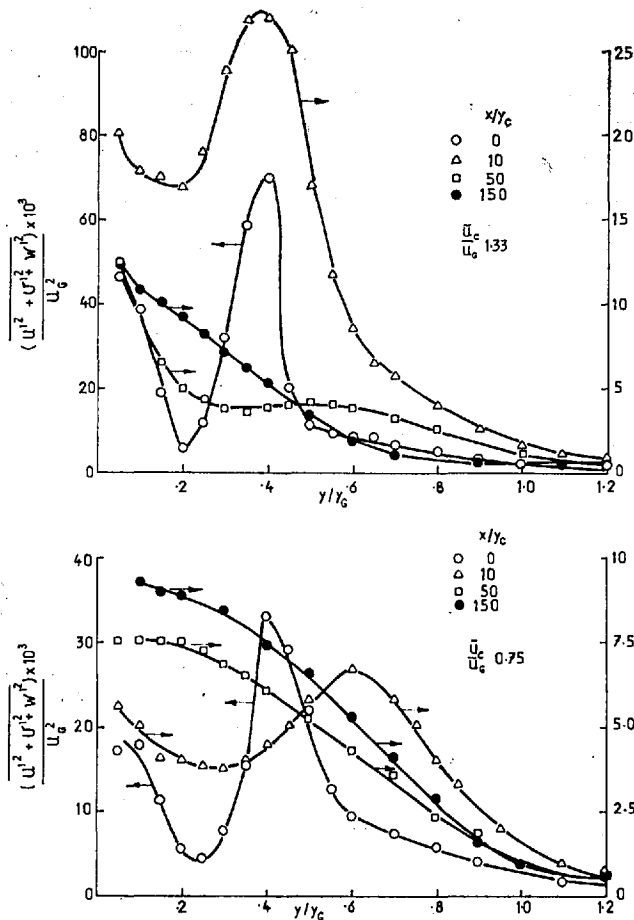


Fig. 7 Profiles of turbulent kinetic energy

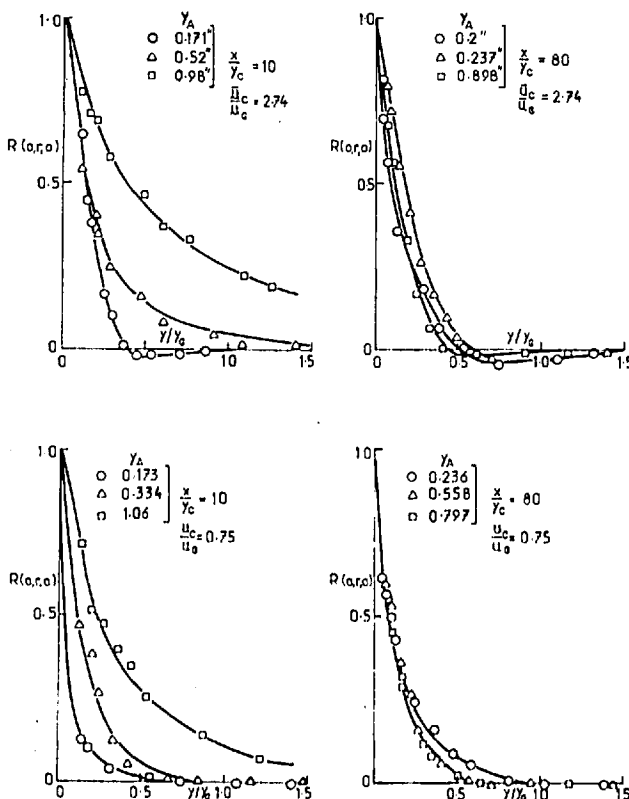


Fig. 8 Profiles of correlation coefficient, $R(0, y, 0)$

$$R_{(0, y, 0)} = \frac{u_1' u_2'}{u_1'^2} \quad (5)$$

and was obtained using two single-wire probes, one of which was maintained in a fixed position while the second was traversed in the y -direction and in the same x -plane. In equation (5), the subscript 1 refers to the fixed probe and subscript 2 to the moving probe. The mean turbulence length scale, defined by

$$L_y = \int_0^\infty R_{(0, y, 0)} dy \quad (6)$$

has been evaluated and presented in Fig. 9: with the exception of the reference measurements in the plane of the slot exit, the results have been plotted on both dimensional and nondimensional coordinates.

Fig. 9 shows that, in the plane of the slot exit, the scale of the turbulence in the free stream is greater than that in the slot. However, the scale decays with distance from the slot and, for the wakelike case, has attained a constant value of approximately $0.15 y_0$ at 80 slot heights; this is approximately the same value which Grant [16] measured in a zero-pressure-gradient boundary layer. The jetlike flow, on the other hand, has not attained the characteristic length scale of the zero-pressure-gradient boundary layer within 80 slot heights.

Tailland, Sunyach, and Mathieu [17] measured the correlation coefficient in a jet with zero free-stream velocity. In this case, the coefficient was normalized with $\sqrt{u_1'^2 u_2'^2}$ rather than $u_1'^2$ and this resulted in negative values of the length scale in the outer region of the flow. Mathieu, in evaluating the length scale, integrated only between the zero limit and y_1 . In spite of this difference, Mathieu's results, when plotted nondimensionally against y_1 , are in close agreement with the present measurements; the range of measurements is, however, not large ($0 < y/y_1 < 1$). When the present measurements were used to evaluate the correlation coefficient normalized with $\sqrt{u_1'^2 u_2'^2}$, it was found that the resulting length scale was negative for small values of y and the velocity ratio of 2.74.

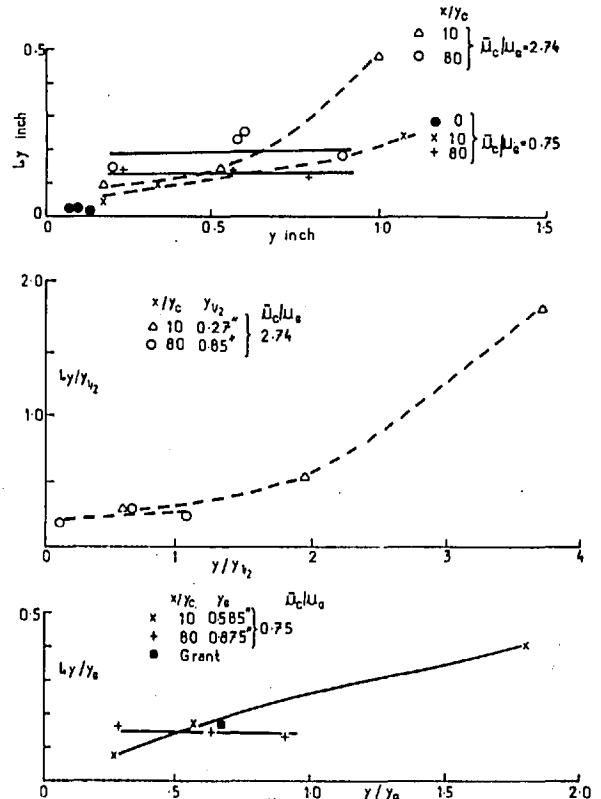


Fig. 9 Distributions of turbulent length scale

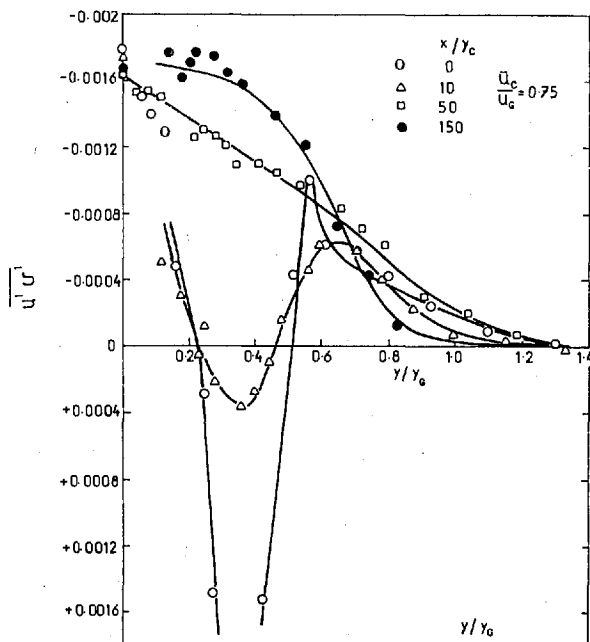


Fig. 10(a)

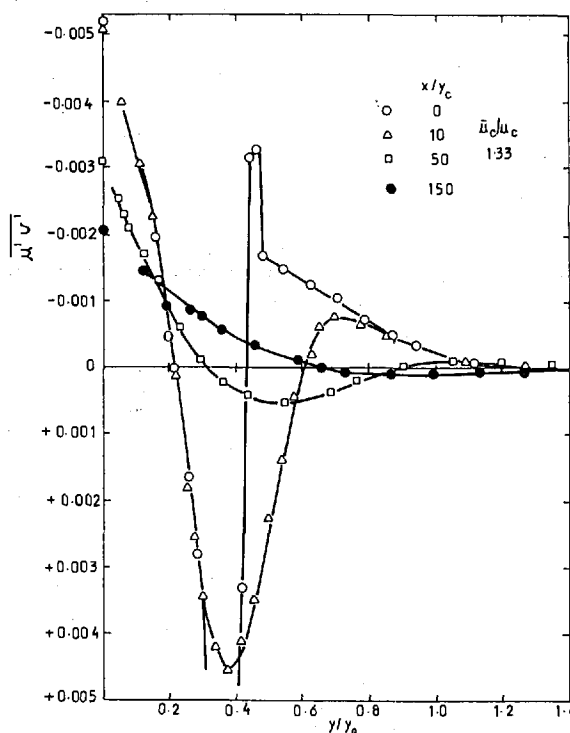


Fig. 10(b)

Fig. 10 Profiles of turbulent shear stress. (NOTE: Ordinate label should read $\overline{u'v'}/u_c$.)

Turbulent Shear Stress. The turbulent shear stress distributions were obtained by direct measurement of $\overline{u'v'}$ for the four values of velocity ratio: Typical values are shown in Figs. 10(a, b). The shear stress distributions may be extrapolated to the wall to yield values of the wall shear stress. Values of wall shear stress, obtained from Clauser charts, are shown in Fig. 10 and it may be seen that the extrapolation of the distributions of $\overline{u'v'}$ will lead to reasonable agreement. The method appears to be reasonably reliable in the downstream region where the gradients are less steep. For values of x/y_c close to the slot exit it is less reliable, especially in the jetlike flows.

It is clear from Fig. 10, that the shear stress profile passes

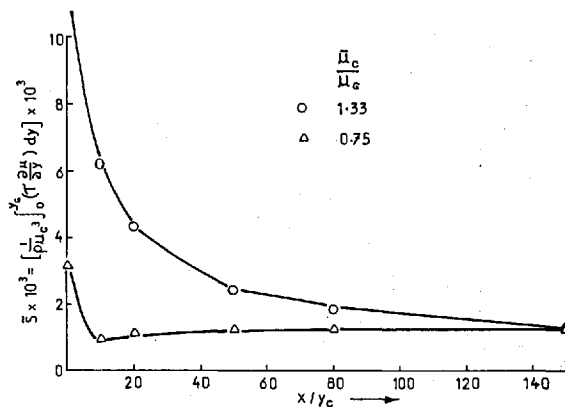


Fig. 11 Distribution of shear work integral

through zero for all values of x/y_c in the case of the jetlike flow and for low values of x/y_c in the case of the wakelike flow. The inner point of zero shear is at all times closer to the wall than the point of zero velocity gradient. The shear stress profile for the wakelike flow is similar in form and in magnitude to that for a zero-pressure-gradient boundary layer far downstream; the profile for the jetlike flow is still not of boundary-layer shape at 150 slot heights from the slot. In both cases, the shear stress is finite for values of y/y_0 greater than unity, but it should be remembered that no correction has been applied for the intermittency.

The measurements of shear stress and of the mean properties of the flow permit the evaluation of the shear work integral; the variation of this quantity with x -distance is shown in Fig. 11. No attempt has been made to measure the terms required to form an energy balance but the advection term can be estimated from the data shown in Figs. 7 and 3.

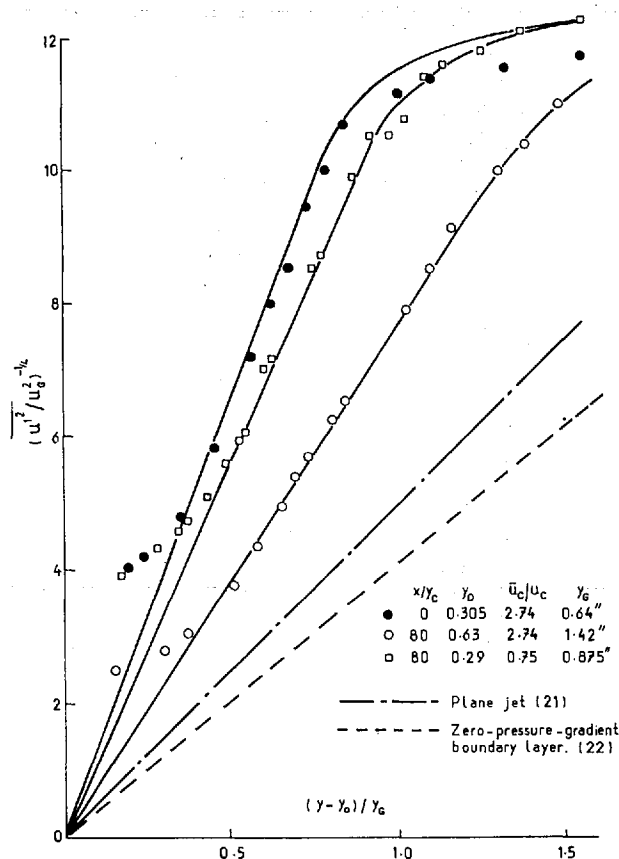


Fig. 12 Profiles of longitudinal, fluctuating velocity component in free stream

The Irrotational Free Stream. The theories of Phillips [18] and of Stewart [19] suggest that the edge of a turbulent flow induces irrotational velocity fluctuations in the free stream such that

$$\overline{v'^2} = \overline{u'^2} + \overline{w'^2} \quad (7)$$

and that

$$\overline{u'^2} \propto (y - y_0)^{-4} \quad (8)$$

y_0 is the apparent origin of the irrotational fluctuations. Theoretically the foregoing suggestions are only asymptotically true, but the measurements of Townsend [20] in a wake, Bradbury [21] in a free jet, and Bradshaw [22] in a boundary layer have tended to confirm them.

Detailed measurements of v' and w' were not made in the free stream, but measurements of u' were made for values of y/y_0 up to 2.0. A sample of these measurements is shown in Fig. 12 together with indications of the results obtained by Bradbury and Bradshaw. It may be seen that the present measurements also indicate a fourth-power law and that the constant of proportionality varies with the particular flow configuration. The value of y_0 has been chosen to make all the curves pass through zero and, consequently, no physical significance can be claimed.

Prediction Procedures

In the past few years numerical prediction procedures for turbulent flows, and based on the differential forms of the conservation equations, have been reported by Bradshaw, Ferris, and Atwell [23], Mellor [24], Patankar and Spalding [25], and Smith and Cebeci [26]. These methods represent a significant advance over previous methods based on the integral form of the conservation equations and have in common that empirical inputs must be provided in some form.

In reference [23], the equations of mass continuity, mean-momentum, and turbulent kinetic energy are solved and the empirical input is in the form of three functions:

$$a_1 \equiv \overline{u'v'}/\rho q^2 = 0.15 \quad (9)$$

$$\frac{L}{y_0} \equiv (\overline{u'v'})^{1/2}/\epsilon y_0 = f_1(y/y_0) \quad (10)$$

$$\frac{G}{y_0} \equiv \left(\frac{\overline{pv}}{\rho} + \frac{1}{2} \overline{q^2 v} \right) / \left[\left(\frac{\overline{u'v'}}{\rho} \right)_{\max} \frac{\overline{u'v'}}{\rho} y_0 \right] = f_2(y/y_0) \quad (11)$$

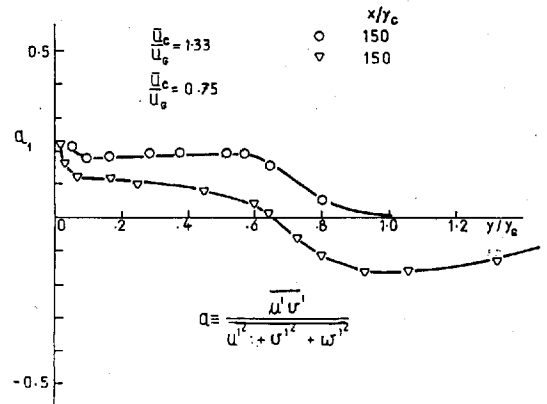


Fig. 13 Profiles of Bradshaw's turbulent intensity parameter

which represent the turbulent intensity, dissipation, and diffusion, respectively. In its present form this procedure is not valid for flows with velocity maxima and, as a result, the present experiments were not devised to investigate the foregoing functions. However, the function a_1 can be obtained from the present measurements and typical profiles are shown in Fig. 13. Profiles of a_1 at smaller values of x/y_0 , exhibit considerable deviations from constancy, particularly when a velocity maximum is present. In such cases a_1 is negative over most of the profile.

In references [24] and [16], the equations of mass continuity and mean momentum are solved; the empirical input is in the form of an "effective viscosity" law. Both use the distribution of effective viscosity devised by Mellor and Gibson [27] on the basis of a study of equilibrium boundary layers. This distribution uses the Prandtl mixing length [28] form of effective viscosity,

$$\mu_{eff} = \rho l^2 \left| \frac{\partial u}{\partial y} \right| \quad (12)$$

for the inner region, with the mixing length expressed in the form recommended by van Driest [29], i.e.,

$$l = Ky[1 - \exp(-y/\Delta_d)], \quad (13)$$

and the form recommended by Clauser [30] for the outer region, i.e.,

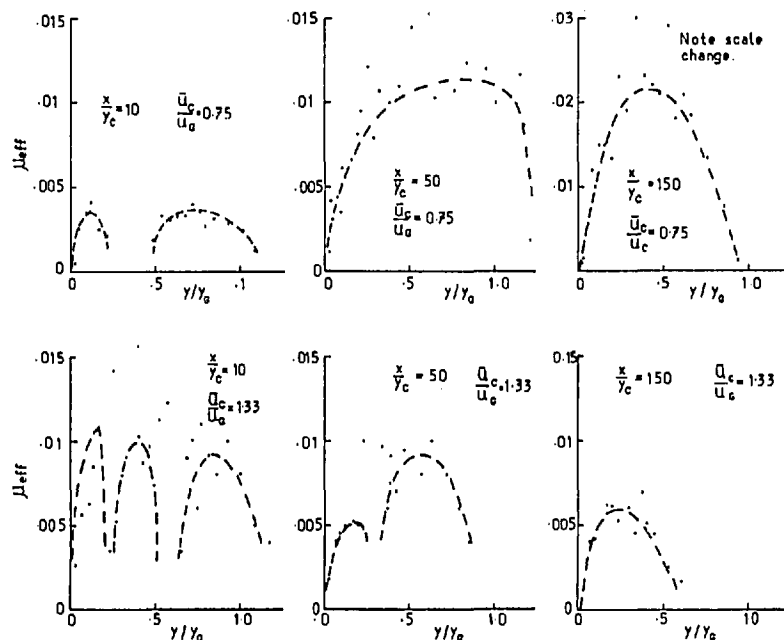


Fig. 14 Profiles of effective viscosity

$$\mu_{eff} = K\rho u_G \delta_1 \quad (14)$$

In the case of reference [26], the equation for the outer region was further modified by the inclusion of the intermittency factor as recommended by Klebanoff [31]; i.e.,

$$\mu_{eff} = K\rho u_G \delta_1 \gamma \quad (15)$$

$$\text{where } \gamma = 0.5[1 - \text{erf } 5(y/y_G - 0.78)]. \quad (16)$$

The inclusion of the Clauser form of effective viscosity implies

that the distribution of effective viscosity is not suitable for flows with velocity maxima, since the displacement thickness is negative in such cases. Indeed, as may be seen from the measurements shown in Fig. 14, the effective viscosity in the outer region is by no means a function of y/y_G alone, even for wakelike flows. The measurements display a considerable scatter, due largely to the three-point fitting procedure used in the evaluation of the gradient of mean velocity, but it is clear that a simple effective viscosity hypothesis has little hope of succeeding within the present range of $\bar{u}_G/u_G$ and x/y_G .

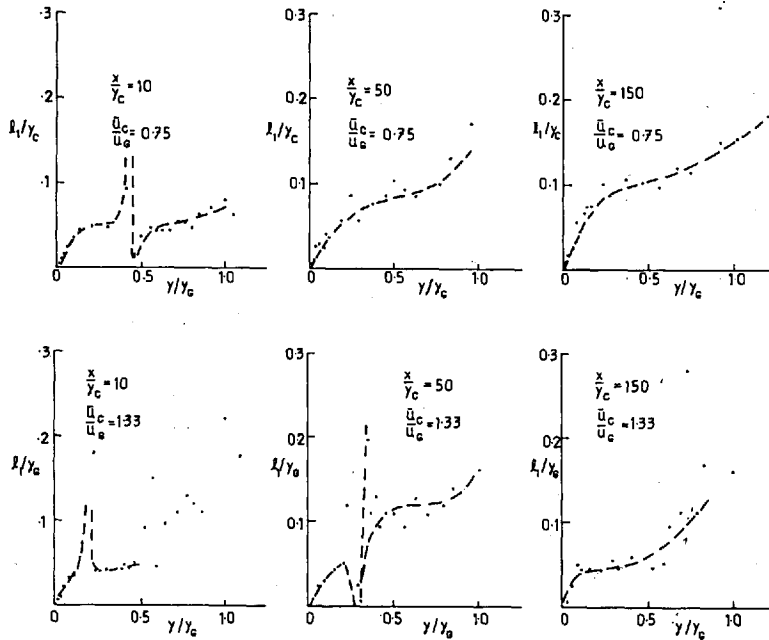


Fig. 15 Profiles of Prandtl mixing length

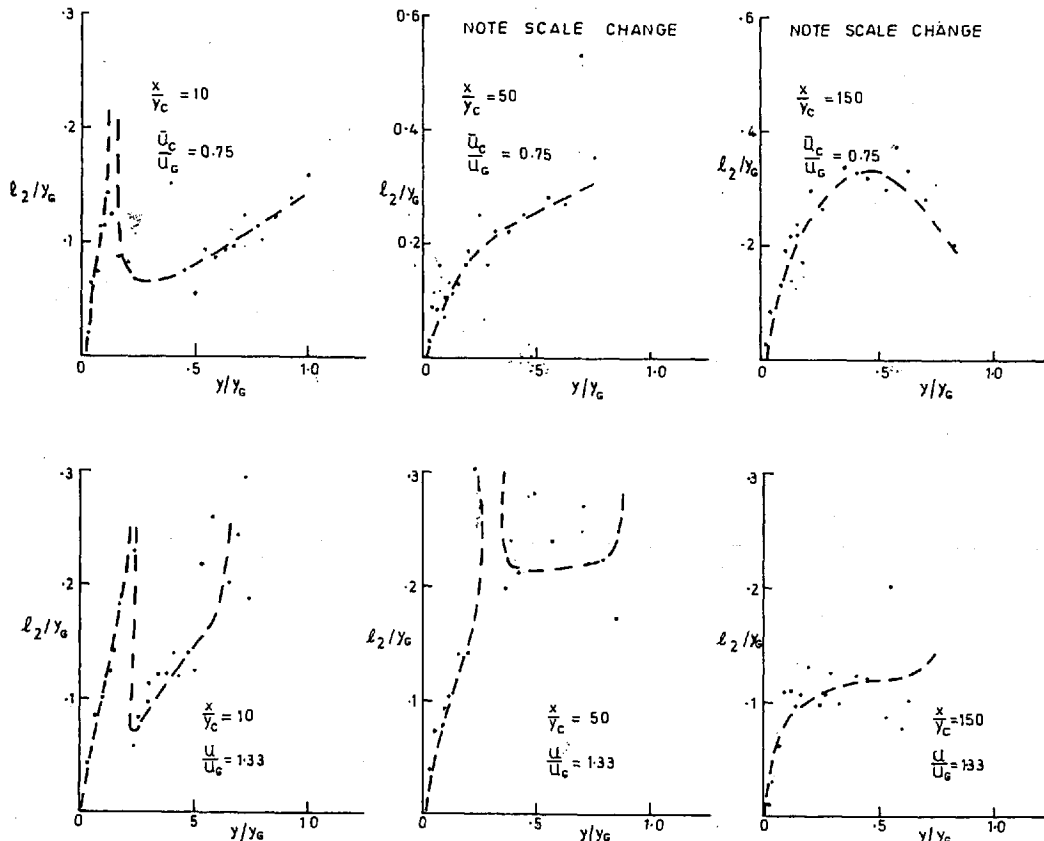


Fig. 16 Profiles of Prandtl-Kolmogorov length scale

The method of Patankar and Spalding is capable of solving either the equations of continuity and mean momentum or these equations plus the turbulent-kinetic-energy equation. In the former case, the empirical input is in the form of the Prandtl mixing length, i.e.,

$$\mu_{\text{eff}} = \rho l^2 \left| \frac{\partial u}{\partial y} \right| \quad (17)$$

where

$$l = Ky, \quad y \leq (\lambda/K)y_G$$

and

$$l = \lambda y_G, \quad y \geq (\lambda/K)y_G,$$

which is used everywhere but for the finite increment close to the wall where an explicit form of the van Driest hypothesis is used: In the latter case, the empirical input is in the form of the Prandtl [32]-Kolmogorov [33] turbulent length scale defined by the equation

$$l_2 = \overline{u'v'}/[\partial u/\partial y C_\rho \sqrt{q^2}] \quad (18)$$

and, at the present time, has not been used in the vicinity of walls. In free flows, Spalding [34] has used a differential equation

$$\bar{u} \frac{dl_2}{dx} = a_D \bar{q}^{1/2} - a_S (\bar{\tau}/p)^{2/3} \bar{q}^{-2/3} \quad (19)$$

to express a spatial average value of l_2 as a function of x and spatial average values of u , q , and $\overline{u'v'}$. This approach is based on the work of Rotta [35] and led Spalding to the conclusion that, although the use of an average value of l_2 explained many features of free flows, the constants a_D and a_S were likely to be functions of the Reynolds number of turbulent ($\equiv \bar{q} l_2/\nu$) for wall flows.

Most important from the viewpoint of the present flow configuration is that, in principle, the empirical inputs recommended by Patankar and Spalding and by Spalding are suitable for the prediction of flows with velocity maxima.

The values of mixing length and turbulent length scale shown in Figs. 15 and 16 were obtained from the present measurements and clearly do not exhibit any clear tendency toward similarity. In the case of the mixing length, the jetlike flows, as could have been anticipated, indicate points of zero and infinite mixing length but, what is more disturbing, the downstream values of λ are different for the jet and wakelike flows. Similar trends are exhibited by the distributions of turbulent length scale although, in this case, the scatter is worse due to the inclusion of three independently measured quantities in the definition of l_2 ; it should be noted that the constant, C_ρ , has been taken as 0.2 in Fig. 16. The distributions of l_2 suggest that spatial average values of l_2 are unlikely to lead to good predictions of shear stress in the present flow configuration.

In spite of the discouraging lack of similarity between the mixing length profiles, considerable success has already been achieved [34] in predicting the shape of velocity profiles in flows similar to those reported here. In reference [36] it is shown that, starting with a measured velocity profile at a value of x/y_G of 20, the Patankar-Spalding procedure predicts velocity profiles which are in excellent agreement with the measurements for all but one value of velocity ratio; viz., 0.58. For higher values of velocity ratio, viz., 0.70, 1.23, and 2.10, and for values of k and λ of 0.435 and 0.09, respectively, the agreement is within the precision of the measurement.

The apparently contradictory evidence presented in the previous two paragraphs can be reconciled if it is recognized that the dependent variable, u , is not strongly dependent upon the mixing length and is unlikely to be strongly dependent upon the turbulent length scale. Present evidence indicates that the mixing length approach, together with values of k and λ of around 0.435 and 0.09, will lead to satisfactory predictions of wall shear stress,

growth rate, and mean-velocity profiles provided the boundary-layer assumptions are valid and provided the velocity ratio is not too high: For a jet in zero free stream, these values lead to an enhanced growth rate. The simple mixing length assumption will not lead to good predictions of shear stress distribution.

The paradox introduced in the previous paragraph is partly resolved when it is realized that the region close to the wall is described by a "wall law" which determines the velocity profile for values of velocity up to 0.5 to 0.6 of the maximum velocity. In the case of reference [25], this law is an explicit form of the Van Driest hypothesis but, for the present flow configuration, it might equally well be the more usual "logarithmic law of the wall." Thus, provided the velocity defect in the outer region is not too great, flow properties such as the wall shear stress and the integral thicknesses are likely to be well predicted.

Conclusions

The investigation permits the following detailed experimental conclusions to be drawn:

1 The static pressure is not constant in the flow, due mainly to the effect of a finite lip thickness. The variation is a maximum in the plane of the slot exit but can be detected as far downstream as 50 slot heights.

2 The existence of a "law of the wall" is supported by the present measurements in the region downstream of 20 slot heights.

3 For flows with a velocity maximum, the point of zero shear is found to be closer to the wall than that of zero velocity gradient; the point of minimum turbulent kinetic energy is identical, within the experimental precision, with the point of zero shear stress.

4 The shear work integral decreases rapidly with downstream distance. The decay is exponential for flows with velocity maxima but for wakelike flows, it decreases rapidly to a minimum and then increases slowly toward the downstream value for the jetlike flow.

5 The characteristic size of eddies, defined as $\int R_{(0,r,0)} dy$, attains a value similar to that for a zero-pressure-gradient boundary layer in 80 slot heights. For jetlike flows, the turbulent shear stress and kinetic energy profiles do not resemble those for a zero-pressure-gradient boundary layer in 150 slot heights.

6 Phillip's fourth-power law for the turbulent velocities in the irrotational flow is confirmed for all velocity ratios.

Two more general and more practically important conclusions may also be drawn:

1 At a distance of 150 slot heights from the slot exit, the flow characteristics of a wakelike flow ($\bar{u}_C/\bar{u}_G < 1$) are similar to those of a boundary layer in the absence of secondary injection. At the same downstream location, however, the flow characteristics of a jetlike flow ($\bar{u}_C/\bar{u}_G > 1$) do not resemble those of a boundary layer in the absence of secondary injection.

2 The profiles of effective viscosity, Prandtl's mixing length and turbulence length scale do not exhibit an obvious similarity and, consequently, prediction procedures which depend on these concepts are unlikely to be wholly satisfactory for wall flows with secondary injection.

Acknowledgments

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The Effect of Slot Height and Slot-Turbulence Intensity on the Effectiveness of the Uniform Density, Two-Dimensional Wall Jet

This paper presents measurements of the impervious-wall effectiveness of a two-dimensional wall jet obtained using a constant value of lip thickness, 0.032 in. and four slot heights, viz., 0.5, 0.25, 0.132, and 0.074 in. The measurements are in the range of slot-to-free-stream velocity ratio

$$0.288 \leq \bar{u}_c/u_G \leq 2.66$$

and clearly demonstrate that, in the region where the velocity ratio is close to unity, the effectiveness is greatly dependent upon the slot height. The reason for this dependence is investigated and additional measurements are presented to show that the influence of the slot turbulence intensity on the effectiveness is small. It is concluded that the ratio of slot lip thickness to slot height is the most significant parameter and that the effect of an increase in this ratio is to decrease the effectiveness.

Introduction

THE cooling of a surface which is exposed to a stream of hot gas is frequently accomplished by injecting a stream of cooler gas along the surface in question. This process is generally referred to as film cooling and, as a result of many experimental investigations [1-8],¹ is qualitatively understood. The measurements reported in the foregoing papers were of the adiabatic-wall effectiveness, η_{aw} , defined by

$$\eta_{aw} = \frac{T'_{aw} - T_G}{T_C - T_G}$$

and, in spite of the fact that the measurements were obtained in wind tunnels and employed simple two-dimensional injection slots, the results show considerable scatter—hundreds of percent

in some cases. The present investigation was undertaken in an effort to determine the cause of these discrepancies. Preliminary results were reported in reference [10]: These measurements were of the impervious-wall effectiveness, η_I , defined by

$$\eta_I = \frac{m_I - m_G}{m_C - m_G}$$

and were obtained in a low-turbulence wind tunnel and with a two-dimensional slot of height 0.25 in. The impervious-wall effectiveness is analogous to the adiabatic-wall effectiveness and the two are identical if the total Lewis number is unity. The measurements of Zakkay, Krause, and Wood [9] suggest that, for mass concentrations of the order used in the present measurement, the total Lewis number is unity. It should be stressed, however, that the present investigation is concerned with a comparison of the effectiveness measurements obtained by the same technique, so that the absolute value of effectiveness is not of direct importance.

The measurements reported in [10] were presented and discussed in a subsequent paper by Nicoll and Whitelaw [11], which also described an explicit method for predicting the effectiveness of the uniform-property, two-dimensional wall jet.

Nomenclature

a = constant
 A = constant
 b = constant
 B = constant
 c = constant
 d = constant
 D = constant
 E = potential drop across a hot wire
 m = mass concentration
 R_C = slot Reynolds number, defined by $\rho C \bar{u}_c \bar{u}_c / \mu_C$

t = thickness of the slot lip
 T' = temperature
 u = mean component of velocity along the plate
 u' = fluctuating component of velocity along the plate
 $\bar{u}_c$ = average value of velocity in the slot, defined by $\frac{1}{\bar{u}_c} \int_0^{\bar{u}_c} u \, dy$
 x = distance along the plate, measured from the slot exit
 y = distance normal to the plate surface

z = distance across the plate, normal to x and y
 δ = thickness of boundary layer on the upper lip of the slot
 ρ = density
 μ = absolute viscosity
 η = effectiveness

Subscripts

aw = adiabatic wall
 C = slot
 G = free stream
 I = impervious wall

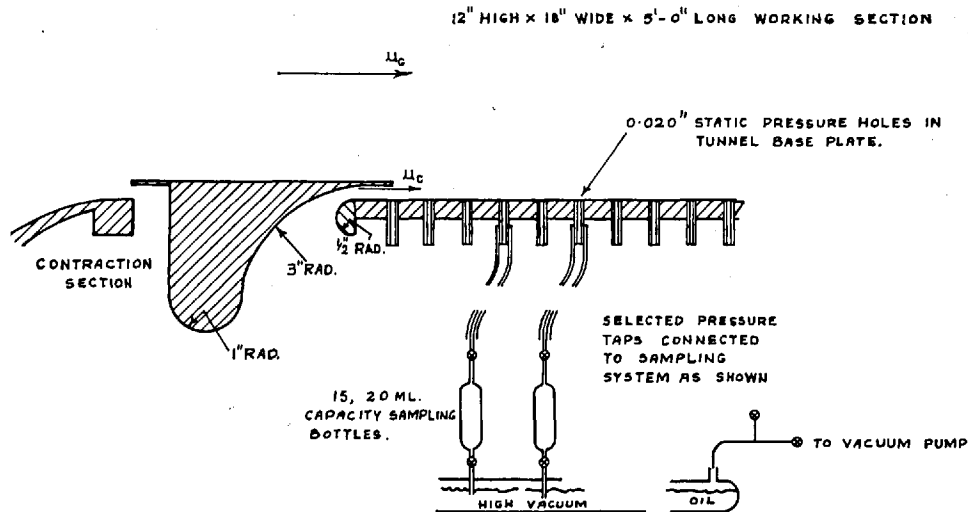


Fig. 1 General arrangement of injection region and tunnel base plate

That paper compared the impervious-wall effectiveness measurements with the predictions of the theory and also made comparison with the adiabatic-wall effectiveness measurements of Seban and Back [6] and Chin, et al. [2]. The comparisons were generally satisfactory but discrepancies appeared to exist in the region close to values of the velocity ratio, $\bar{u}_c/u_\infty$, of unity.

The measurements described in [10] and [11] exhibit a maximum value of effectiveness when this quantity is plotted against the velocity ratio, $\bar{u}_c/u_\infty$, for constant values of the dimensionless distance, x/y_c , greater than 25. The maximum occurs in the vicinity of a velocity ratio of unity. The measurements of Chin, et al. [2], do not exhibit a maximum of this type but the measurements of Seban [4] do, in certain cases. Seban used three different slot heights and his results show a maximum for slot heights of 0.250 in. and 0.125 in.; the measurements obtained with a 0.063-in. slot height do not exhibit a maximum. The slot height employed by Chin, et al., was 0.106 in.

The results discussed in the previous paragraph suggest that the slot height, in a suitable nondimensional form, may play an important part, directly or indirectly, in determining the effectiveness downstream of the slot when the value of $\bar{u}_c/u_\infty$ is close to unity. The purpose of the present paper is to present measurements which confirm this suggestion and identify the ratio of slot lip thickness to slot height, l/y_c as the significant parameter. The experimental investigation was carried out in two parts. In the first, the impervious-wall effectiveness was measured for a single value of the slot lip thickness (0.032 in.) and for three values of slot height (0.5, 0.132, and 0.074 in.). The range of velocity ratios covered by these measurements was

$$0.288 \leq \bar{u}_c/u_\infty \leq 2.66;$$

the corresponding range of slot Reynolds number was

$$745 \leq Re \leq 44,000.$$

The earlier measurements, made with the 0.25-in. slot height, are repeated in the present paper for completeness. In the second part of the experimental investigation, measurements were made to determine the influence of the slot turbulence intensity on the effectiveness; measurements were carried out within the range of turbulence intensity, at the center of the slot, from 1 to 10 percent.

Experimental Apparatus and Procedure

The measurements reported in the present paper were obtained from the apparatus described in [10] and [11]. It consisted of a two-dimensional, low-turbulence, open-circuit wind tunnel with

provision for secondary air to be blown parallel to the base plate of the tunnel. A general arrangement of the injection region and tunnel base plate is shown in Fig. 1; the slot height was varied by moving the lip of the slot up or down, as required, and adjusting the lip to insure that a 1-in. parallel section existed before the slot exit. For all the present measurements, the free-stream velocity was nominally 70 fps and the free-stream turbulence level approximately 0.5 percent.

The impervious-wall effectiveness was obtained by injecting a tracer of helium gas into the secondary flow and measuring the wall concentration of helium at various locations downstream of the slot. This was accomplished by sucking samples of the mixture of air and helium through a suitably spaced number of the 0.020-in.-dia static-pressure holes, which were located in the base plate of the wind tunnel, and by measuring the concentration of helium in the samples with the aid of a gas chromatograph.

To insure that the air-helium mixture was homogeneous at the slot exit, the tracer of helium was injected into the secondary air on the suction side of the secondary blower. Concentration profiles at the slot exit, both in the y and z -directions, were measured from time to time and indicated that the mixture was homogeneous within the precision of measurement. Care was also taken to insure that the rate of sampling through the 0.020-in. holes was sufficiently low. Preliminary tests indicated that sampling rates below a certain value resulted in a constant helium concentration and that sampling rates above this critical value resulted in lower helium concentrations. The sampling rates employed to obtain the measurements reported in this paper were at all times below the critical value. For all measurements of concentration, the gas chromatograph employed a two-meter-long column, packed with Linde-type molecular sieve to separate the permanent gases, and a thermal conductivity cell to measure their concentration. The carrier gas was nitrogen; consequently, helium and oxygen were the only two significant peaks to be traced by the recorder. The column temperature was approximately 60 deg C. This combination of column and temperature permitted the helium and oxygen to pass through the column in less than 3 min. The thermal-conductivity cell was calibrated at regular intervals for helium detection over the working range concentrations. The calibration curve was found to be linear for helium concentrations above approximately 0.1 percent by volume; all measurements were made in this linear range.

The impervious-wall effectiveness was measured at various locations downstream of the slot and for constant values of the velocity ratio, $\bar{u}_c/u_\infty$. The mean-velocity profile at the slot exit, from the wall to the free stream, and at a distance of 10 slot heights downstream, were measured for each run; the former

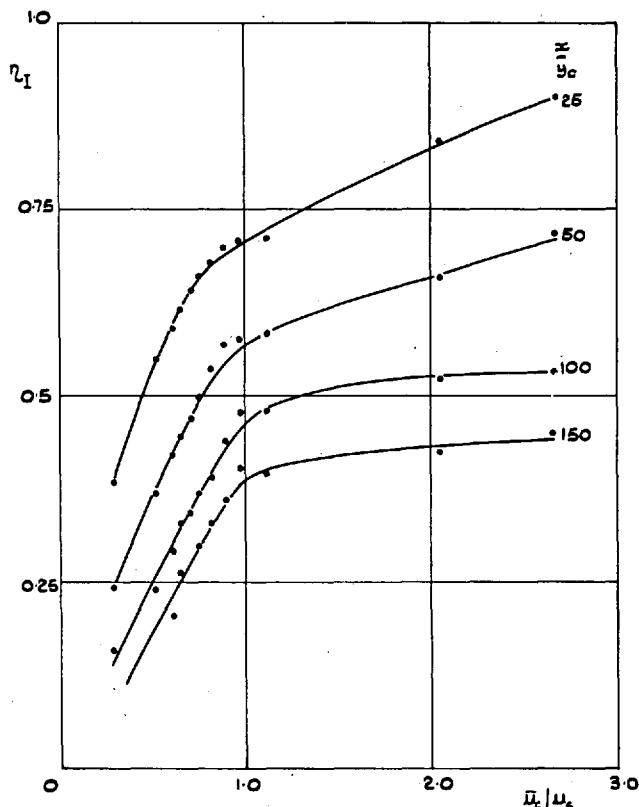


Fig. 2 Impervious-wall effectiveness—0.074-in. slot

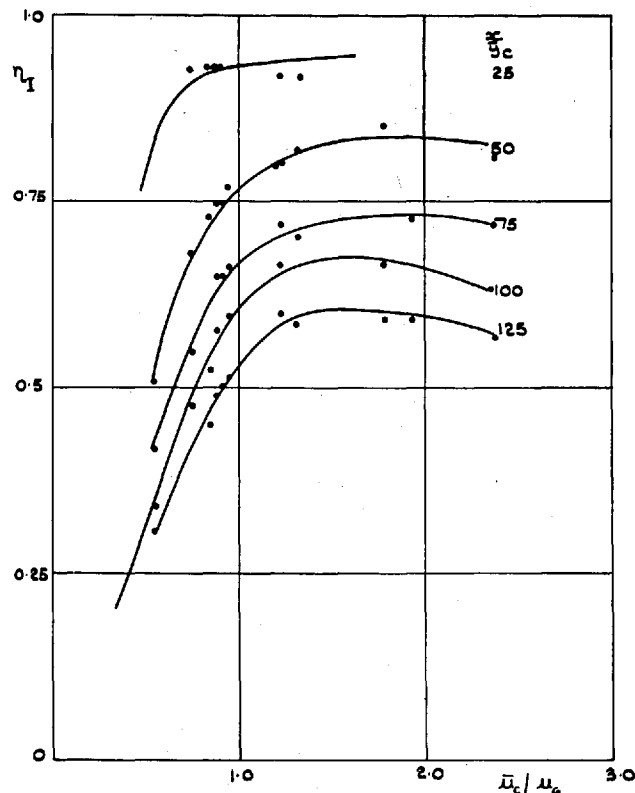


Fig. 3 Impervious-wall effectiveness—0.132-in. slot

yielded the value of $\bar{u}_c$. Occasional measurements with a static probe indicated that there were no significant pressure gradients normal to the tunnel base plate and, consequently, wall taps were used to measure the static pressure.

In addition to the mean-velocity measurements, the turbulent component of velocity, $\sqrt{u'^2}$, was measured in the plane of the slot exit and at a value of x/y_c of 10; these detailed measurements were made only in the case of the 0.25-in. slot height. A DISA constant temperature hot-wire anemometer was used for the turbulence intensity measurements. The wires were calibrated using the equation proposed by Collis and Williams [12], i.e.,

$$E^2 = C + Du^{0.45}$$

where C and D are determined from the calibration and u is the velocity normal to the wire. The hot-wire was calibrated before and after each profile; the calibration was carried out in the low turbulence free stream.

The profile of turbulence intensity at the slot exit was varied either by locating a fine grid upstream of the slot or by injecting air upstream of the slot and normal to the main direction of the flow. The measurements of effectiveness for two different slot-turbulence intensity profiles are reported in the section, "Effect of Variations in the Slot-Turbulence Intensity Profile"; these results typify the greater volume of results which were obtained. One of the two-turbulence intensity profiles was obtained from the unaltered configuration of the slot and the second by introducing a 20 mesh 28 SWG copper screen upstream of the slot exit; the unaltered slot configuration yielded the lower values of turbulence intensity.

Results

The results will be presented in two sections. The first section deals with the effect of variations in the slot height and the second with the effect of variations in the slot-turbulence intensity profile.

Effect of Variations in the Slot Height. Measured values of effectiveness are usually presented in figures which show logarithmic plots of effectiveness against downstream distance for constant values of the velocity ratio. The present data were first plotted in this way, but this form of diagram tends to compress the data. Consequently, the data have been cross-plotted and are presented in Figs. 2, 3, 4, and 5 in the form of effectiveness against velocity ratio for constant downstream distance; the various figures indicate the data obtained using each of the four slot heights.² The "experimental" points marked on the figures are interpolated values of effectiveness at the measured value of the velocity ratio.

Figs. 2 to 5 indicate that the value of effectiveness does depend on the slot height in the region of velocity ratio close to unity. Fig. 6 demonstrates the effect more clearly by bringing together in one diagram the results obtained with the four slot heights for one value of the downstream distance. It may be seen from Fig. 6 that the effectiveness-against-velocity ratio curve has a maximum at a velocity ratio of unity for the measurements made with the largest slot heights; this maximum diminishes with slot height and is not present for the measurements obtained with the smallest slot height. It is estimated that the value of the velocity ratio $\bar{u}_c/u_0$ was measured with a precision which, for values of the velocity ratio greater than 0.5, was not worse than 3 percent. The concentration of helium in any sample was measured to a precision of ± 1.5 percent of the corresponding value at the slot.

The foregoing estimates are substantiated by the consistency of the results shown in Figs. 2, 3, 4, and 5. The reproducibility of the measurements was verified by making check measurements subsequent to the measurements which led to the results presented in Figs. 2 to 5. For these check measurements, the slot height was reset both to 0.25 in. and to approximately 0.074 in. Values of the impervious-wall effectiveness were obtained for a number of values of the velocity ratio including (for each of the two slot heights) one which was nominally unity; the maximum

² The data are available in tabulated form in reference [13].

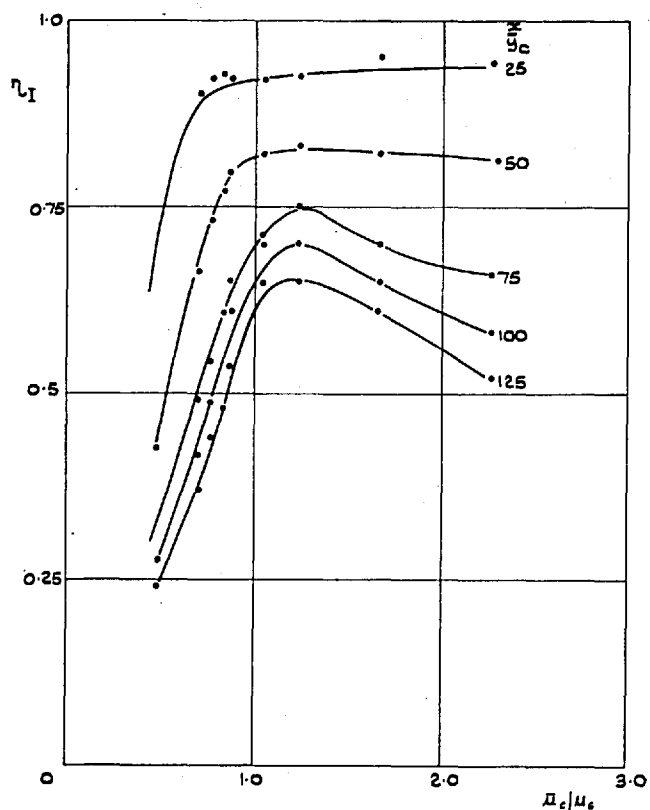


Fig. 4 Impervious-wall effectiveness—0.25-in. slot

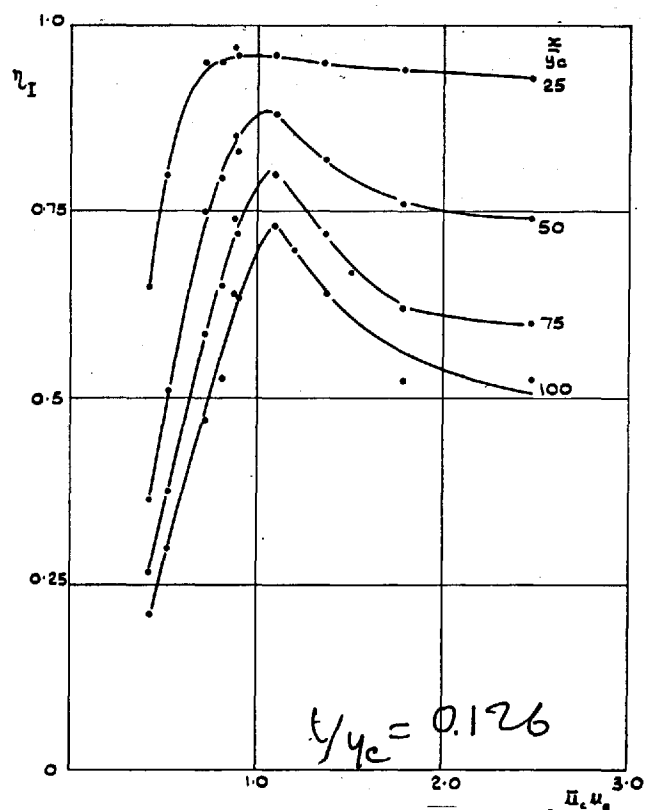


Fig. 6 Impervious-wall effectiveness at a distance of 100-slot heights downstream

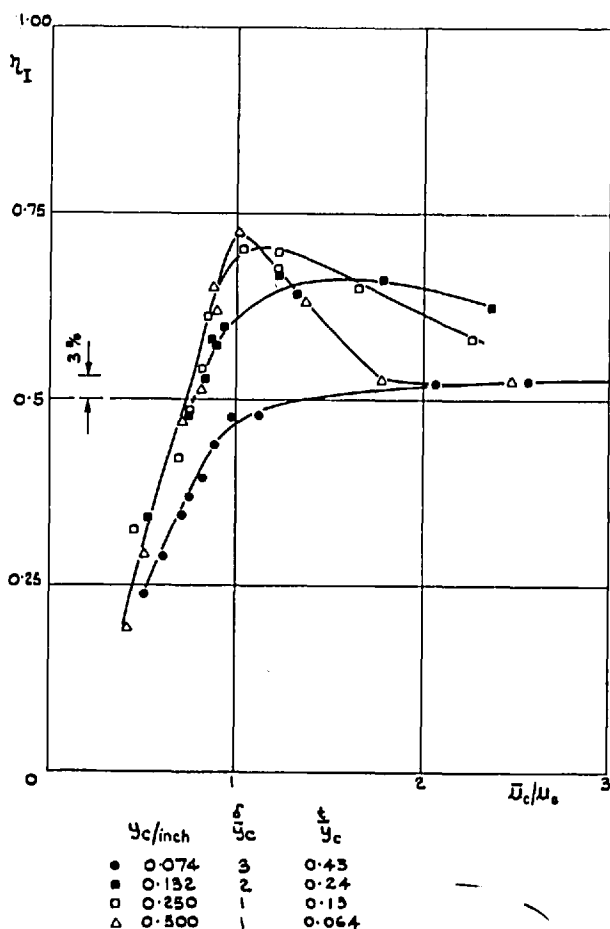


Fig. 5 Impervious-wall effectiveness—0.5-in. slot

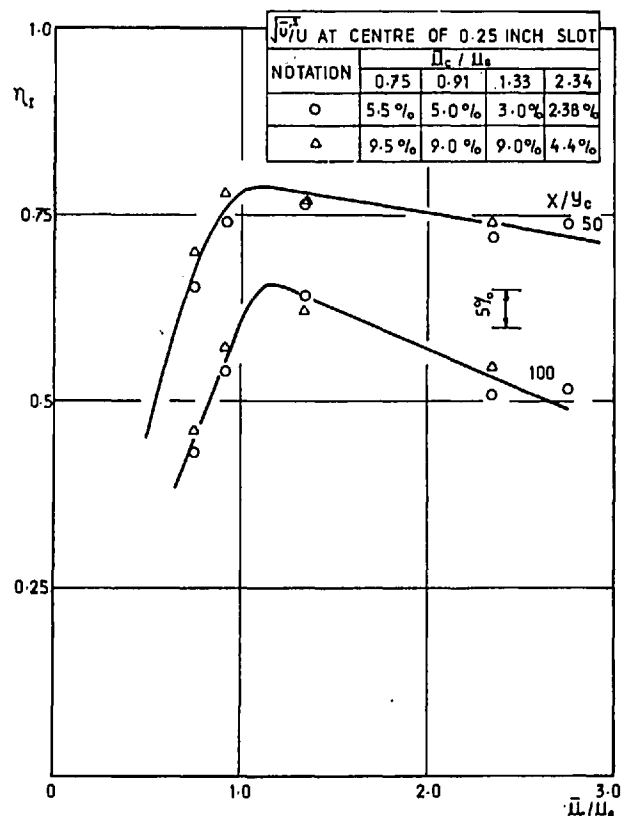


Fig. 7 The influence of slot-turbulence intensity on the impervious-wall effectiveness

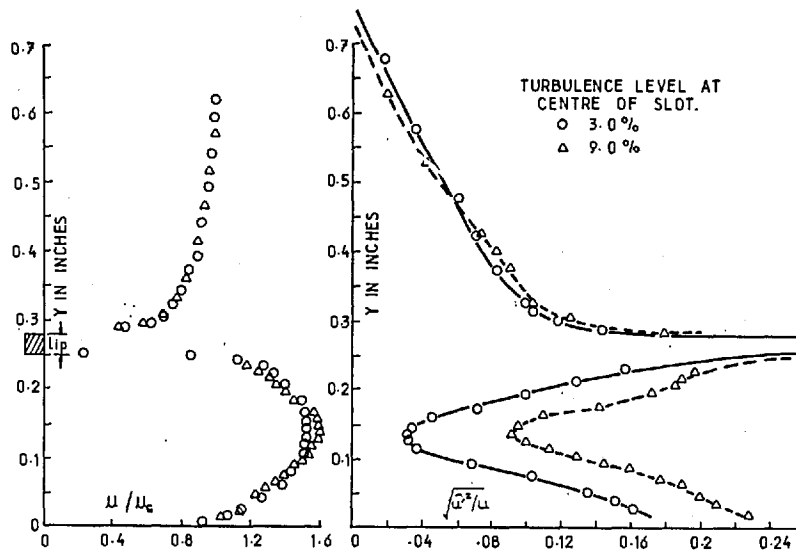


Fig. 8 Mean velocity and turbulence-intensity profiles at $x/y_c = 0$; $u_c/u_g = 1.33$

deviation of these check measurements from the corresponding values shown in Figs. 2 and 4 was 4 percent of unity, i.e., of the effectiveness at the slot exit.

Effect of Variations in the Slot-Turbulence Intensity Profile. Measured values of the impervious-wall effectiveness are shown in Fig. 7 for values of x/y_c of 50 and 100; the measurements at each velocity ratio were obtained with the two slot-turbulence intensity profiles mentioned in the section, Experimental Apparatus Procedure. The two mean-velocity and turbulence-intensity profiles, measured in the plane of the slot exit, and corresponding to the velocity ratio of 1.33, are shown in Fig. 8; the differences between these profiles are typical of those for the other velocity ratios. It may be seen from Fig. 8 that the mean-velocity profiles are not identical although the maximum deviation is only 5 percent. This reflects the difficulty which was experienced in attempting to change the turbulence intensity while maintaining the mean-velocity profile the same. However, although four different slot-turbulence-intensity profiles were employed at each velocity ratio, with only small variations in mean-velocity profiles, the resulting values of effectiveness did not deviate significantly from the curves shown in Fig. 7. Only when the mean-velocity profile was noticeably asymmetric did the effectiveness, for a given velocity ratio and downstream distance, change; the asymmetric profile in question indicated a larger boundary-layer thickness on the base plate and resulted in a reduction in effectiveness.

The measurements shown in Fig. 7 are representative of measurements at different values of x/y_c and for four different slot-turbulence intensity profiles within the range of 3 to 10 percent. They lead to the conclusion that, in this range, the effectiveness is not significantly influenced by the intensity profile.

Discussion of Results

The decrease in effectiveness with decrease in slot height must be directly related to the velocity and turbulence-intensity profiles at the slot exit. These profiles consist of three distinct parts, the upper lip boundary layer, the thickness of the slot lip, and the flow within the slot; each of these can be varied independently of the others. The three parts will now be considered in turn with a view to deciding their possible contribution to the effect characterized by Fig. 6.

The effect of the upper lip boundary layer has been investigated by Seban [14], Chin, et al. [2], and Kacker and Whitelaw [15]. The measurements contained in references [14] and [2] are for

values of the velocity ratio less than unity; those in reference [15] cover the range

$$0.3 \leq \bar{u}_c/\bar{u}_g \leq 2.05.$$

All of the measurements indicate that the effect of thickening the upper lip boundary layer is to decrease the effectiveness. The magnitude of the decrease appears to be not more than 5 percent even when the ratio of the upper lip boundary-layer thickness to slot height, δ/y_c , is increased from unity to more than 10.

The thickness of the slot lip, t , was approximately 0.032 in. in the present measurements; this is a significant proportion of the 0.074-in. slot height and its possible effect cannot be discounted. The immediate effect of the slot lip is to create a region of separated flow downstream of the slot. It is likely that an increase in t/y_c will enhance the mixing between the two streams and consequently decrease the effectiveness. The recent measurements reported in reference [16] confirm this trend but for values of t/y_c greater than those used in the present measurements.

The velocity profile within the slot affects the downstream properties in several ways. First, the magnitude of the slot Reynolds number is likely to characterize the departures from similarity caused by the presence of the wall; the theoretical investigations of Cole, Spalding, and Stollery [17] suggest that this effect is small. Second, the shape of the velocity profile may have a direct effect: Since the influence of the thickness of the upper lip boundary layer is small, it is likely that the influence of the shape of the velocity profile within the slot will be small provided it remains symmetrical. Third, the turbulence-intensity profile within the slot will affect the rate of mixing of the two streams, particularly when the mean velocities of the two streams are of the same order; this turbulence-intensity profile is, to some extent, dependent upon the mean-velocity profile.

Of the possible variables discussed in the previous paragraphs, the ones which appear to be the most likely causes of the effect shown in Fig. 6 are t/y_c and the turbulence-intensity profile at the slot exit. In these measurements, and for $\bar{u}_c$ equal to $\bar{u}_g$, the minimum value of $\sqrt{(u')^2}/\bar{u}_c$, located close to the midpoint of the slot, increased from 0.04 with the 0.5-in. slot height to 0.085 with the 0.074-in. slot height. The measurements, shown in Fig. 7, however, show that variations in the turbulence intensity of this order do not give rise to substantial changes in the downstream effectiveness. Thus it can be concluded that the effect shown in Fig. 6 is due to the variation of t/y_c ; an increase in the value of

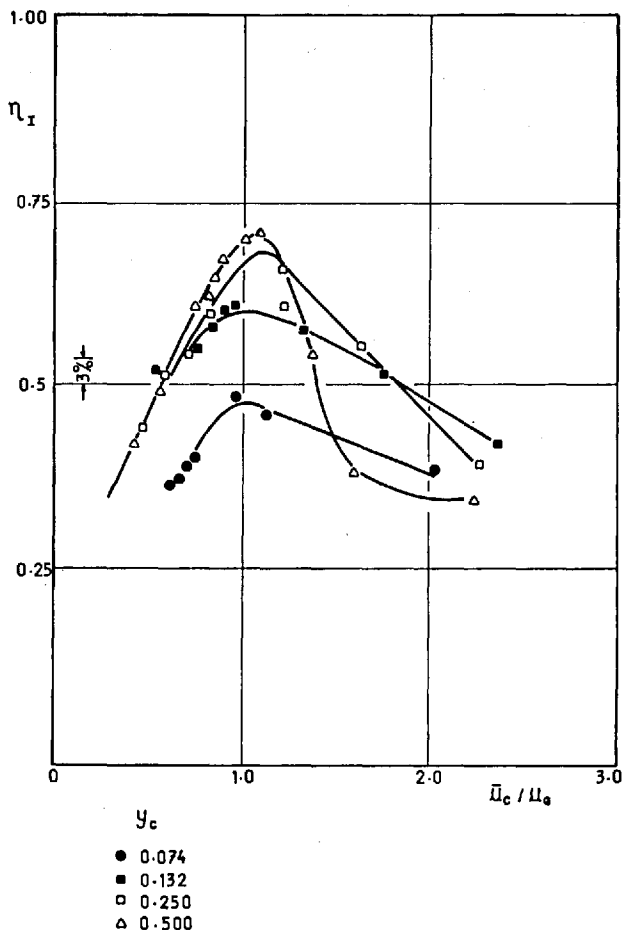


Fig. 9 Impervious-wall effectiveness at a value of $xu_0/y_c\bar{u}_c$ of 100

t/y_c leads to a decrease in effectiveness in the region where u_c is of the same order as u_0 .

Implications of Results

From the practical standpoint, there are two important deductions to be made from the present measurements:

- 1 In the region of a velocity ratio of unity and for a prescribed lip thickness, the effectiveness depends upon slot height to a considerable extent.
- 2 For a specified mass flow, the optimum downstream effectiveness is obtained with the maximum possible slot height compatible with a value of $\bar{u}_c/u_0$ close to unity; this may be confirmed from Fig. 9 which presents the data given in Fig. 6 for a value of $xu_0/y_c\bar{u}_c$ of 100.

It is believed that the foregoing deductions could lead to the design of more effective film cooling slots. Certainly, velocity ratios close to unity are frequently found in practice, e.g., in the combustion chambers of gas-turbine engines, and the present desire to improve the combustion process in these chambers is likely to require more sophisticated film cooling slot design and possibly film cooling which is controlled independently of the mass flow from the compressor.

The foregoing deductions also imply that the empirical correlations which are generally postulated to predict the effectiveness of film cooling, i.e., equations of the form

$$\eta = A(u_0x/\bar{u}_cy_c)^n$$

or

$$\eta = B(u_0x/\bar{u}_cy_c)^n R_c^d$$

are unsuitable for the accurate prediction of effectiveness except in the region of low values of the velocity ratio, $\bar{u}_c/u_0$. Similarly, two-parameter, integral prediction methods cannot predict the effect of slot height shown in Fig. 6. The present authors believe that a solution of the elliptic vorticity and energy equations is required in the region downstream of the slot although far downstream ($x/y_c > 20$) a solution of the parabolic, x -momentum, and energy equations will suffice. The framework for these solutions has been provided in references [18] and [19] but the solution procedures described in the papers require an empirical input for the effective viscosity; the present authors are now directing their experimental efforts in this direction.

Conclusions

- 1 The effect of decreasing the slot height, while retaining the same lip thickness and ratio of slot to free-stream velocity, is to decrease the effectiveness provided the velocity ratio is close to unity. The maximum decrease, at a distance of 100 slot heights from the slot, is of the order of 25 percent of unity; i.e., a reduction in effectiveness from 0.75 to 0.5.
- 2 For a prescribed mass flow through the slot, the maximum slot height which is compatible with a value of velocity ratio close to unity will generally result in the optimum effectiveness.
- 3 The effect of changing the intensity of the turbulence of the slot flow by approximately 100 percent does not significantly affect the downstream effectiveness.
- 4 The parameter which characterizes the effect described in number 1 is the ratio of lip thickness to slot height.

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