Imperial College London
Imperial College Business School

Product Release Strategies in the Digital Economy

Esma Koca

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Declaration

I, Esma Koca, declare that the research presented in this thesis is my own work, except where acknowledged. No part of this thesis has been submitted before for any degree or examination at this or any other university.

Abstract

Digital technologies have been transforming the economy and are challenging many traditional frameworks. In this thesis, we contribute to the research stream that revisits conventional models in the light of the digital economy. Our focus is on the releases of new products, a widespread business practice disrupted by digitally-driven consumer preferences and by an enhanced toolbox for firms. First, we model analytically the releases of different versions of a durable product in a monopoly. A firm selects the price, time to release a new version, and whether to continue selling the old product at a discount or not. Consumers, on the other hand, have distinct foresight and perception of product obsolescence. We show that the firm's release strategy crucially depends on obsolescence perception of consumers and that the firm can influence obsolescence by a prudent timing of the new version. Secondly, we provide an analytical framework for the release of a new product category when two asymmetric firms engage in price and quality competition. One of the firms, the ecosystem firm, can leverage its previous market coverage. The other firm, the category firm, is new to the market. The ecosystem firm can add on to the stand-alone quality of the new product by linking the two classes of products via a software while the category firm relies only on the intrinsic connection between the products. We characterize the conditions when the ecosystem leverage keeps the competitor at bay and deters innovation. Finally, we discuss release strategies of online content providers, such as streaming services and e-learning platforms. We provide an optimization model that covers three aspects of the release strategy: release of an individual series, order of multiple series within each subscription period, and personalization of the content. Further, we test our model using data from online education.

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'When wireless is perfectly applied the whole earth will be converted into a huge brain, which in fact it is, all things being particles of a real and rhythmic whole. We shall be able to communicate with one another instantly, irrespective of distance. Not only this, but through television and telephony we shall see and hear one another as perfectly as though we were face to face, despite intervening distances of thousands of miles; and the instruments through which we shall be able to do his will be amazingly simple compared with our present telephone. A man will be able to carry one in his vest pocket.'

Nikola Tesla (Kennedy, 1926)

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Chapter 1

Introduction

In this thesis, we focus on product release strategies, a theme that has received the attention of researchers from various fields including operations management (OM) and industrial organization (IO). We emphasize the impact of digital technologies on product releases. To this end, we examine conventional product release frameworks of the existing literature and provide extensions to these models that help explaining recent product release practices further. In addition, we highlight the need to develop new models to explain new business models of the digital economy.

1.1 The Digital Economy

Digital technologies have been rapidly diffusing into our lives and changing effectively how we interact with the economy as its stakeholders. While the digital transformation is taking place at different paces in different markets, the common argument is that digitization impacts all type of industries, including traditional capital-intensive sectors such as construction (Agarwal et al., 2016) or steel (Koch, 2018). Besides, the impact is on various economic agents: consumers, firms, policymakers and intermediaries (Choi and Whinston, 2000; Illing et al., 2006). As Neelie

Kroes[1] puts it: "There is no separate digital economy, we have an economy that is *digital*".

The natural question that follows immediately is "What is the difference between the new digital economy and the old economy?". Starting with what is *not* different, the fundamental forces in the new economy are still demand and supply, and there exist cases when the digital economy fails to function efficiently. As a result, policymakers continue to intervene, as they had been doing before digitization, to correct for digital market failures. For instance, the European Commission has fined Google with €2, 42 billion in 2017 and €4, 34 billion in 2018 (Richter, 2018). Instead, the difference between the new and the old economy stems from the blurring boundaries within which the market forces shape the economy (Goldfarb et al., 2015; Dawson et al., 2016).

To illustrate the long-run impact of digitization on the definition of industries and on the capabilities of economic agents when contributing to economic output, we take the case of smartphones. Let us imagine that an economist, who has just watched Steve Job's Keynote presentation about the first generation iPhone in January 2007, attends the Apple's iPhone launch event in September 2018.[2] What would be the significant differences she would observe immediately when comparing the two iPhone markets except a new Apple CEO and that the iPhone has transformed technologically into a better product? Perhaps, one of the critical differences she would mention is the other product categories introduced in reference to the smartphone during the same event (smartwatch, smart TV, iPad, HomePod, MacBook, etc.). Commonly referred to as Apple's product ecosystem (Markman, 2017), this family of goods seamlessly connect with Apple's operating systems, iOS or watchOS. This observation extends to various industry examples where products and services communicate via an underlying software, and it enhances consumers' access to the same experience from one good to another. Amazon's Alexa or Google Home are other digital infrastructures that enable connection among various types of goods, intrinsically related or not, such as household appliances,

[1] Neelie Kroes is the former Vice President of the European Commission and Commissioner for Digital Agenda.

[2] We refer the reader to Haskel and Westlake (2017) for an evaluation of the intangible economy, as the umbrella term that includes the digital economy, with a comparison of two gyms from 1977 and 2017.

internet search, online grocery or online streaming. Thus, we highlight *connected goods* as one of the significant differences between the new and the old economy.

Revisiting our imaginary economist who has time-traveled from 2007 to 2018, she would be surprised also by the new ways consumers are responding to and, in fact, shape Apple's iPhone strategy. While most of the consumers watching the 2007 event were not informed about what Steve Jobs was going to launch that day, our economist would find that the new consumers, present at the 2018 launch event, are well-informed and predictive about future products, their prices, and the discounts thanks to online word-of-mouth platforms such as Mac Rumors. Formally, the literature refers to the consumers in the digital economy as strategic or forward-looking (Tapscott, 1996; Belleflamme and Peitz, 2015). Thirdly, our economist would also point to the demand-side network effects that are more visible in 2018. Put it differently, she would identify an Apple community that keeps on buying from Apple as more people buy iPhones or other products from Apple. Demand-side network effects are even more present in sharing economy platforms such as Facebook, Airbnb, Uber or in the video games sector where consumers actively contribute to the creation of the good itself. However, it is important to note that the demand-side network effect becomes more influential as goods embody communication platforms, such as FaceTime, iMessage, iCloud or Family Sharing in the iPhone example. Fourthly, our economist would also notice that some consumers consider old versions of the iPhone to be obsolete even though they work very well physically and those consumers would use this perception of obsolescence to justify their eagerness to upgrade to the new version (Bellezza et al., 2017). The literature refers to the consumer-driven obsolescence as "perceived" obsolescence and empirical research confirms that in many industry settings of the digital economy, perceived obsolescence is the primary determinant of the consumers' upgrade decisions (Cooper, 2004; Slade, 2009; Wilhelm et al., 2011; Bellezza et al., 2017). Finally, our economist would notice the impact of the product ecosystem on other consumption habits such as watching movies or making digital payments. As one example, she would note that it is no luxury now for the consumers to determine when, what and how to consume in the online content settings such as streaming TV shows (Statista, 2018). We have discussed so far the observations about consumers of the digital economy that would suffice for the outline of this

thesis. However, it is important to note that if we had taken another example, we would have been specifying other types of digitally-driven consumption phenomena such as showrooming (Gensler et al., 2017), online reviews (Zhu and Zhang, 2010) and collaborative consumption (Belk, 2014). To summarize, the consumers of the digital economy are *strategic, inter-dependent and well informed* about firms and products, and in many cases, they are actively involved in designing the consumption experience themselves.

Up to this point, we have covered the nature of the goods and the consumers in the digital economy. The next group of economic agents we will include are from the supply-side of the economy. Regarding the smartphones, the economist in Apple's launch event would readily mention competitors such as Samsung and LG in the smartphone category that produce almost identical products, which is despite Steve Jobs' emphasis at the launch of the first generation iPhones that the product is patented. However, she would also note that each firm seems to have its own community of consumers that are sufficiently loyal and locked-in to the firm's product ecosystem. The literature identifies the type of consumers that allow for the firm to act as if there existed no competition as "locked-in consumers" (Varian et al., 1999; Dubé et al., 2009).

Secondly, our economist would also notice the emphasis of Apple on its operating systems and complementary services, and hence huge upfront investments in the product development phase. On the other hand, she would also observe that the marginal production costs of the products decrease as they embody digital features. This observation is, in fact, a key feature of any intangible good, and Haskel and Westlake (2017) refer to it as "sunkenness" and "scalability". Related to this, our economist would also highlight the network of firms that work in collaboration with Apple to produce Apple's goods but also Apple's contracts with competitors such as Wacom that can interact with Apple's ecosystem. That is an example illustrating that the digital technologies build complementarity between the products. Haskel and Westlake (2017) call these supply-side complementarities "synergies" between the intangible goods and the ecosystem of firms. Another important observation would be the positive externalities between the various product categories in the production side or the "spillover" effect of the intangibles (Haskel and Westlake, 2017). Therefore, the industry borders are not

that rigid anymore, and firms can enter into new categories by connecting them to their current umbrella of goods with the underlying digital infrastructure (Bouwman et al., 2018; Coyle, 2018). Thereby, they would benefit from the consumers' previous appreciation for other goods in the new category. To summarize, our economist would probably contrast Apple 2007 with Apple 2018 along the 4S's of the digital economy (sunkenness, scalability, synergy, spillover) (Haskel and Westlake, 2017), and conclude that digital technologies transformed Apple into a firm with a family of product categories that is more *connected* with its consumers, suppliers, and competitors.

We have discussed the new economy within a specific perspective and a time frame (2007-2018). We aim to introduce the reader to the concepts that are relevant in the context of *product releases in the digital economy*, which is the main theme of this thesis. As we mentioned before, this is *not* the complete story of the digital economy. While we aim to focus solely on product releases, we briefly introduce some frequently discussed research topics of the digital economy. First, we observe that the conversation intensifies around new forms of competition. Among these settings, "multi-sided platforms" are one type of market setting that is wide-spread across various sectors such as application development (e.g., Google's Android operation system), marketplace matching (e.g., Uber or Airbnb), social media platforms (e.g., Facebook) or blended platforms (e.g., WeChat) that combine various activities (Hagiu, 2014). The literature handles platforms from various aspects such as pricing in the presence of network externalities between the platform users (e.g., Rochet and Tirole, 2003; Armstrong, 2006; Chakravorti and Roson, 2006), impact of intermediation on platform competition (e.g., Belleflamme and Peitz, 2010; Edelman and Wright, 2015), comparability and net neutrality (e.g., Njoroge et al., 2013; Bourreau et al., 2015; Greenstein et al., 2016) or digital piracy (e.g., Peitz and Waelbroeck, 2006; Rasch and Wenzel, 2013; Belleflamme, 2016). We refer the reader to Peitz and Waldfogel (2012) for a collection of research topics in the digital economy with an industrial organizations perspective, and recommend Statista (2018) for recent practices.

1.2 Product Release Strategies

It is crucial for the survival of any businesses to introduce new products, and it is *rare* to come up with a counterexample of an industry where there exists no new product launches. That is why product releases have taken the attention of researchers from various fields including operations management, industrial organization, and marketing.

Operations management handles the product releases typically with an emphasis on supply chain management, the durability of the goods, and product development decisions. One of the common themes is the launch of successive generations in a monopoly setting, commonly referred to as "management of product rollovers" (Billington et al., 1998). Since the emphasis is more on the supply chain and inventory aspects, the models tend to give more weight to cost related elements instead of consumer preferences. However, the existing models successfully address crucial questions of the traditional economy such as whether to keep an old version when releasing a new version for which the marginal costs of production are significant (Lim and Tang, 2006) or in the presence of inventory risks and costs (Koca et al., 2010; Ferguson and Koenigsberg, 2007). Durability and product obsolescence are other aspects of the product releases covered by the OM literature (e.g., Levinthal and Purohit, 1989; Desai and Purohit, 1998; Ferguson and Koenigsberg, 2007). In particular, product obsolescence is typically exogenous to the firm in these models while in few of the papers the firm can choose when to observe the exogenous product obsolescence (e.g., Arslan et al., 2009). However, in the digital economy, the consumers' perception of obsolescence (Bellezza et al., 2017), as well as the firm's release timing, plays a crucial role in determining the novelty of a product (Cooper, 2004). Finally, the literature often assumes that consumers are myopic while in the digital economy consumers are becoming more strategic with quick access to consumer forums, company announcements, and the information about release practices of the firms. However, the OM literature has recently begun to be concerned with the predictive outlook of consumers (e.g., Liang et al., 2014; Lobel et al., 2015) in the rollover literature. To summarize, the product release models of the OM field focus on supply chain and manufacturing aspects instead of the consumers' role, assume that obsolescence is exogenous to the firms as well as to the consumers, and the time-to-market

of the product is informed by the industry clockspeed. In the digital economy, however, the impact of production costs subside while consumers are more strategic and active in shaping the firm's release strategy.

The industrial organization field, on the other hand, has been including various elements of the digital economy in the product release models earlier than the OM field. For instance, strategic consumer behavior and its implications on pricing (e.g., Moorthy and Png, 1992; Bhargava and Choudhary, 2008) and timing (e.g., Calzada and Valletti, 2012) of the new products received attention, especially by the literature that considers the management of different versions of a product line, typically in a monopoly setting. The underlying assumption of the version management is that once a new version is released, the older generation is removed immediately (solo rollover). This is in contrast to the OM field, which regards the decision of whether or not to keep the old version as a crucial element of the problem. In the second chapter, we justify as to why a solo rollover would be suitable for information goods settings. Secondly, as in the OM field, the version management literature assumes that the quality of the products is exogenous to the consumers. In the digital economy, on the other hand, both firms and consumers influence the novelty of goods. We address this matter with the perception of obsolescence and show how it impacts the firm's pricing and timing decisions in the second chapter.

Quality is another topic that has received the attention of economists (e.g., Motta, 1993; Johnson and Myatt, 2018). The literature typically focuses on a single type of product category over which the firms compete in a two-stage game that involves price and quality competition. However, the current setting may not be sufficient to explain some of the new practices in the digital economy. With the improvements in the digital technology, firms such as Apple or Samsung become more interested in investing on operating systems or bundling software to connect new products to their family of goods, and hence transforming the conventional business models of quality competition. In the third chapter, we extend the classical two-stage game of the literature to address this new form of competition.

To summarize, the existing models address various elements present in the traditional economy. However, digital transformation is evolving rapidly and resulting in new types of firm and

consumer behavior over time, which requires the need to revisit current models.

1.3 Product Release Strategies in the Digital Economy

In this thesis, we revisit the product release models and integrate them with new behavioral phenomena encountered in the digital economy. In particular, we contribute to the literature by reexamining the product rollovers (Chapter 2) and price-quality competition models (Chapter 3) in the light of the digital economy. Further, we provide a welfare analysis that may extend to fruitful normative policy discussions as future studies. We also develop a new framework for the release of a series of online content (Chapter 4), which has been a crucial business problem for various sectors such as education and media. Figure 1.1 summarizes the setting of each chapter.

The first part focuses on the releases of different versions of the same product and contribute to the versioning and product rollover research streams. When releasing a new version of a durable product, a firm considers whether to convince its consumer base to upgrade while attracting new consumers. To this end, the firm designs a rollover strategy, which comprises identifying the price of the new product and deciding on whether to continue to sell the old product at a discount (dual rollover) or not (solo rollover). We argue that the effectiveness of a rollover strategy crucially depends on the obsolescence perception of consumers and that the company can influence obsolescence by a prudent timing of the new version. We explore the impact of the timing decision in solo and dual rollovers in markets composed of myopic (inattentive) and strategic (attentive) consumers. We find that the release time provides the firm with the leverage to induce upgrading behavior and reinforces the solo rollover as the optimal policy unless the marginal cost of producing the late version is high compared to the first version. We also characterize the release strategy (pricing and release timing) of the firm in a market that consists of consumers with different foresight (myopic and strategic). We extend the current models by de-emphasizing the role of production costs and highlighting the active role of consumers in shaping a firm's release strategy and by enhancing the firm's toolbox with

release timing.

In the second part, we switch our focus from a single product in a monopoly to multiple products in the presence of competition. In particular, we provide a framework for the release of a new product category when two asymmetric firms engage in price and quality competition. One of the firms, the ecosystem firm, has the leverage of previous market coverage. The other firm, the category firm, is new to the market. This particular structure of the market is the reason for the asymmetry. The ecosystem firm can add on to the stand-alone quality of the new product by linking the two products via a software, while the category firm relies only on the intrinsic connection between the products. We allow for the possibility that the two products can be intrinsically substitutes, complements or independent. We investigate if the ecosystem leverage enables the ecosystem firm to keep its rival at bay, as some industry experts have argued. We conclude that the effect of the ecosystem leverage on the category firm is not necessarily adverse, but under some conditions, even facilitates the welfare of the rival. Secondly, we examine the influence of the ecosystem leverage on the stand-alone innovation in the new category. We find that the ecosystem leverage, in particular, the built-in connection, improves the category innovation unless the two goods are strong substitutes. The same applies when the ecosystem firm is the quality leader as long as the goods are weak substitutes. Our results suggest that, when the ecosystem firm is the quality follower, ironically, the ecosystem leverage threatens both the category firm and the stand-alone innovation the most. However, we also confirm that the two firms are willing to share the leverage, and hence "coopete" in this situation, unless they are stuck in the asymmetric setting for another reason such as contractual frictions.

In the third part, we delve more into consumer-driven product releases. In particular, we discuss release strategies of online content providers, such as streaming services and e-learning platforms. We cover three aspects of the release strategy: (1) release of an individual series, (2) order of multiple series within each subscription period, and (3) personalization of the content. Online content providers need to overcome two challenging tasks: attracting new viewers and retaining existing users of the platforms. Practitioners point out to "viewer engagement" as a metric of success that indicates to what degree the online content provider utilizes the two levers.

We argue that release strategies are crucial in maximizing "viewer engagement" by inducing two fundamental viewing phenomena: binge-watching and water-cooler effect. Typically, online content providers either release episodes of content periodically (episodic release) or make all the content available upfront (entire season at once). The traditional episodic release strategy creates viewer generated chatter around the story (water-cooler talk), induces engagement with the content and improves customer retention. On the other hand, an entire season released at once enables viewers to watch multiple episodes of the material in one sitting (binge-watching), which in turn increases the value they attach to the content, and hence attracts more subscribers as the value of the content improves. We argue that a hybrid strategy, deployed by content providers such as Amazon Prime and Hulu, may activate both levers and maximize the viewers' engagement depending on the content and the viewers' personalities. We also discuss personalization of the content, another aspect commonly addressed by practitioners, through digital nudges. We provide a network model for designing hybrid, personalized content release strategies, and we test our model on data from online education. To our knowledge, this is the first framework that considers the content release problem in the online setting, and we believe that our methodology could inspire other models as well as provide useful insights to practitioners.

Chapters	Problem Setting	Consumer Characteristics	Firm Toolbox
Chapter 2: Designing Digital Rollovers	Versions of a product category —*Substitutes* Monopoly	Perceived obsolescence Myopic versus strategic consumers Heterogeneous quality appreciation	(1) Solo versus dual rollover? (2) When to release? (3) At which prices to release?
Chapter 3 Double Utilization of Consumer Appreciation	A family of products —*Complements* —*Substitutes* Duopoly	Double appreciation of connected features of the products Heterogeneous quality appreciation	(1) At which qualities to release ? (2) How much to connect the goods? (3) At which prices to release?
Chapter 4: Optimal Content Release Strategies	Multiple types of interdependent content —*Complements* —*Substitutes* Monopoly	Water cooler talk versus binge-watching Heterogeneous viewing behavior	(1) How to release a series? (2) How to release multiple series? (3) How much to personalize?

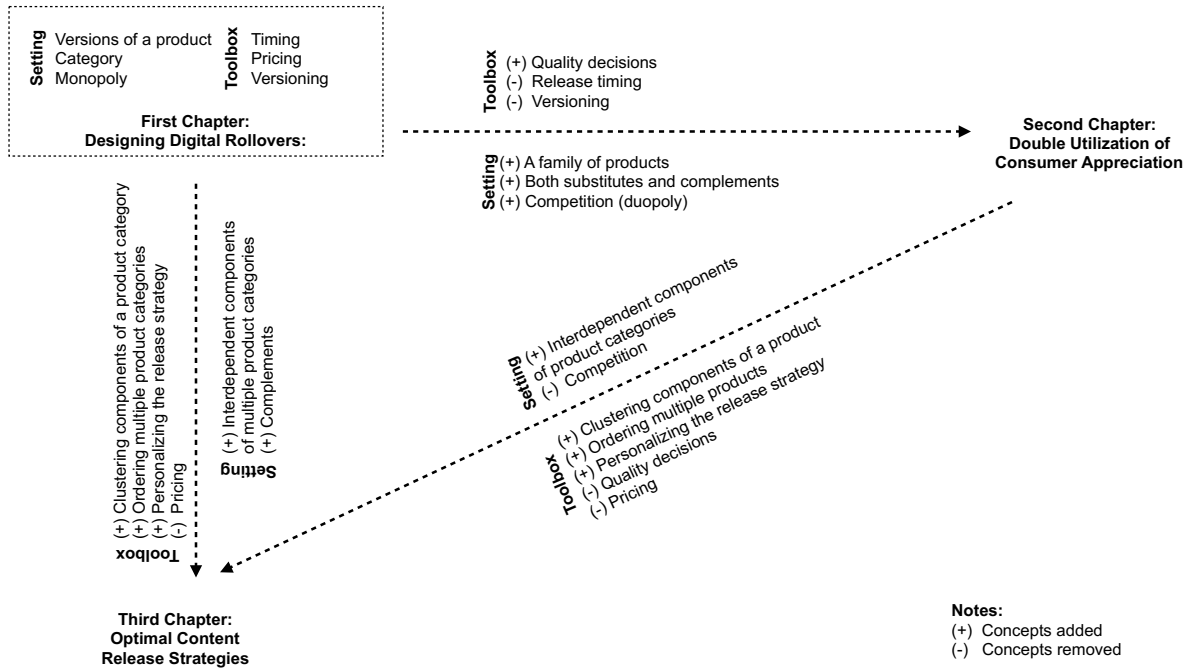


Figure 1.1: The Three Chapters on Product Release Strategies in the Digital Economy

Chapter 2

Designing Digital Rollovers: Managing Obsolescence with Release Time

2.1 Introduction

This chapter examines the release strategies of firms selling different versions of a digital good. Ideally, firms aim for new releases "that will drive people to upgrade", as Apple's CEO puts it (Gibbs, 2016), while attracting new customers from the remainder of the market. Versioning, in the form of second price discrimination, suggests benefiting from the heterogeneous nature of consumers' willingness to pay. Typically, firms create an upgrader base from the high-value types with an early version, and subsequently persuade these consumers to obtain the new versions with comparatively lower future release prices (Dhebar, 1994; Kornish, 2001). As an example, the recent pricing regime Apple implements when releasing versions of the iPhone (such as iPhone 7 Plus and 8 Plus) is to charge a low premium for the advanced version relative

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to its bill of materials cost (Rassweiler and Schneemann, 2017). Enhancing the upgrader base gradually over time, at the expense of shrinking profit margins, may work well if the market "perceives" the new versions sufficiently novel relative to the previous versions. Otherwise, the firm may find itself trapped in a situation where the owners of the previous version do not upgrade.[1]

An essential aspect of releasing the new versions is whether to keep the old versions in the market (dual rollover) at a discounted price, or not (solo rollover) (Billington et al., 1998; Erhun et al., 2007). Dual rollovers (e.g., smartphones, smart TVs) can be a viable strategy to generate additional revenue stream from the low-value consumers to support the sales of the new version. This is the case especially when the market does not see the new version as sufficiently novel (Billington et al., 1998), where the high value consumers are hesitant to upgrade. On the other hand, dual rollovers risk the sales of the new version due to cannibalization threat of the old version. Thus, the firms may prefer a solo rollover instead (e.g., professional tablets, educational software).

We argue that the firm is capable of "driving people to upgrade" without incurring the risks associated with a dual rollover if it responds to the market with the proper release time. The existing literature, which we discuss in the reminder of this section, assumes that the timing is dictated by technological progress of the firm as observed in the releases of the new generation of smart TVs, where the new version of the Apple TV/Amazon Fire TV are released after the new resolution technology "4K" is developed.

Counterexamples are also plentiful, such as iPhones/iPads/Apple Watches or FIFA games. Typically, the release cycles are driven by the "perception of obsolescence" of consumers.[2] In particular, the market perceives the previous version as "old" at the launch of the new version even if it is not obsolete in objective terms (Cooper, 2004; Nahm, 2004; Koenigsberg et al., 2011). In fact, Cooper (2004) and Slade (2009) suggest that perceived obsolescence is the

[1] According to a survey, only 16% of the previous iPhone owners state that they would upgrade to the new version (iPhone X) and the most cited reason is "my iPhone works just fine" (Hartmans, 2018).

[2] LG Electronics CEO Cho puts it as "we will unveil new smartphones when it is needed." (Cheng, 2018).

primary factor determining the novelty of a product while absolute obsolescence is only the secondary aspect. As an example, even when there exists no apparent battery slowdown, hence no "objective" quality deterioration, "perceived obsolescence" is documented to drive users to experience a phantom battery slowdown, namely the "Slow iPhone" phenomenon precisely at the release of the new version (Richter, 2016b). It was argued that obsolescence as perceived by consumers increases with longer release cycles (Bulow, 1982; Dhebar, 1994) because the consumers' attachment to the early version along with the perception of irreplaceability/indispensability decrease over time (Schifferstein and Zwartkruis-Pelgrim, 2008). In addition, the perception of obsolescence may become psychologically "justifiable" in the eyes of consumers (Bellezza et al., 2017). The link between the release time and the novelty gap of the versions, enables the firm to control the "perception of obsolescence". Along with price discrimination, the firm can use it as leverage to create a high volume of upgraders. Thus, a firm that is capable of setting the release intervals and convincing the market to perceive that the new version is novel may not even need the dual rollover strategy, as we discuss in this paper.

Another real-life complication relevant to durable goods, both digital and non-digital, is consumers' foresight about the firm's future actions. For the inattentive (myopic) consumers that focus on the present time, the firm needs to convince them once at a time and align the price only with what they see. Attentive (strategic) consumers, however, can predict the future and consider prospective actions of the firm while they decide. As a result, the firm's pricing power would be limited. Nevertheless, the release time lever may support the firm to create long gaps between versions so that a high volume of strategic consumers may upgrade even when the market is hesitant to see an obsolescence.

In this paper, we propose a model to comprehend the optimal release strategy of a firm that is equipped with price discrimination, release timing, and the design of the rollovers (the solo or dual rollovers). To understand the effect of "perceived obsolescence" in isolation, we start with a simple model with multiple assumptions. In particular, we assume that the firm is a monopoly and sells "pure digital goods" without costs of reproduction (the revenues are equal to the profits) or capacity constraints, which are typically the case for the digital goods (Shapiro et al., 1998; Varian, 2001b). In Section 5, we relax the assumption of pure digital goods and

discuss the implications for semi-digital goods. We assume that there exist two versions and a continuous timeline along which the two versions can be released, as well as that the consumers can buy both versions. We also assume that the consumers derive no salvage value from the old version when switching to the new version. While salvage value may be relevant in traditional goods (such as books or machines), the resale of the old version in a secondary market may be infeasible for the digital goods for the reasons including but not limited to personal licensing, data security concerns, or consumers' attitude to keep the old product as a back up, which we discuss further in Section 2.

We examine the release strategy of the firm under various prospects of the consumers. We also consider the consumers' heterogeneous appreciation of quality, hence different willingness to pay. These two dynamics reveal which consumer type does what and how the market responds to the rollover strategy. We first analyze the release strategy under two distinct consumer foresights (myopic and strategic). Second, we consider both groups simultaneously and examine which groups to focus: the myopic consumers that are easily hooked into the product category with high prices or the strategic consumers that are willing to wait unless the prices are low. We also investigate the robustness of our analysis by relaxing the assumption of the pure "digital goods" as well as considering the implications when the release time is fixed.

We hope that our work provides insights to the industry managers in better understanding the levers they have when releasing new digital goods over time. A critical insight from our model is that firms should not keep the old version when the new version is released. In a digital product category, focusing on the release timing and management of the obsolescence instead of targeting low-value consumers with the old version provides more fruitful outcomes. The reason behind the result is that the firm is capable of setting the release windows, to manage the relative novelty of the versions and consumers' upgrading. Even when the market is reluctant to see a novelty difference between the versions of a product category, the firm can overcome this challenge by delaying the release of the new version. Consequently, the market would perceive high quality gap between the versions and will become more eager to upgrade. Thus, the firm can execute a solo rollover instead of navigating more complicated setting of a dual rollover. This finding aligns with the solo rollover practices commonly observed in the releases

of purely digital goods (e.g., educational software such as Wolfram Research's Mathematica or StataCorp's Stata) or semi-digital products (e.g., tablets such as Apples iPad Pro or Wacom's Intuos) for which the hardware costs are increasing only gradually over time (Torres, 2017).

We also confirm that the release time is a crucial lever regardless of the composition of the market between myopic and strategic consumers. The firm builds release intervals with lengths depending on consumers' perception of obsolescence and foresight. The only exception is when the market is short-sighted and extremely confident in the durability of goods. In this case, the firm releases the new version immediately and targets low value types with a high price discount on the new version. Otherwise, the managers should build release intervals to create consumers that buy multiple versions. If the market is strategic, the firm should establish longer release intervals and decrease the first-period price as a response to consumers' strategic waiting. Doing so, the high value consumers perceive that the first version has a reasonable life cycle prior to the obsolescence and is offered at an affordable price while the novelty gap is sufficiently large justifying upgrading behavior. In this way, the firm can create a consumer base willing to upgrade when the new release is out.

If the market is composed of both myopic and strategic consumers, the firm's release strategy depends on the consumers' intrinsic perception of the obsolescence and the portion of the strategic consumers in the market. In particular, the pricing and the release timing span each group. We note that the "primary pricing regime" (the relative prices), as a form of firm's effort to induce upgrading, is driven by the dominant group. In a market with a high degree of obsolescence perception, the firm's primary pricing regime depends on the dominant consumer group. If the market is predominantly myopic, then the firm should primarily focus on myopic consumers. If, instead, the strategic consumers dominate, then the firm replicates the pricing regime of the polar case associated with the strategic consumers. The threshold determining which consumer group (myopic or strategic) is dominant depends on the obsolescence inclination of the market; the more the market is willing to perceive an obsolescence, the less focus the firm puts on the strategic consumers since the less influential the waiting behavior for the new version. An important observation here is that the polar cases, with neat evaluations, provide a high-level perspective to the pricing strategy of a more complicated setting. Even in a situation

where the firm is not certain about the exact portion of the strategic consumers, evaluating the dominance of one group over another would suffice to determine the relative prices. If, instead, the consumers are *ex-ante* confident in the durability of the first version, the relative prices should be set to target all possible segments (single and multiple buyers) from both groups. We also cover welfare, and show that the firm and the strategic consumers are obsolescence-seekers but the myopic consumers are hurt from the obsolescence. Thus, a policy maker should ensure that consumers are educated about the firm's future strategies.

The contributions of this paper are summarized as follows:

1. We propose a model for product rollovers that incorporates solo/dual settings, myopic/strategic consumers, and the release time. Our model also captures heterogeneous quality perception that enables us to reveal market segmentation.

2. We derive the optimal release strategy and segmentation for a market composed of consumers with distinct foresights and appreciation of quality.

3. We extend the discussion by relaxing the "pure digital goods" assumption.

4. We provide managerial and policy implications of different rollover strategies.

The firm's problem to release different versions of a product category has received the attention of two main research streams: the version management and the rollover literature.

The literature on versioning has revealed that the firm should release the versions sequentially (solo setting), instead of simultaneously, if the consumers are strategic and more impatient than the firm (Moorthy and Png, 1992). Extending this setting to digital goods, Kornish (2001) considers an exogenous technological improvement in between the two time periods in which different versions are offered. Calzada and Valletti (2012) focus on the upgrading behavior arising from the consumers' willingness to obtain both versions. They extend the findings of previous literature by showing that the sequential release is optimal for the digital goods as well, if the consumers are less patient than the firm. Lobel et al. (2015) complement the discussion of the sequential releases by studying consumers that are either buying now

or waiting for the future (no upgrading) and confirms that the length of the release intervals depends on the patience of consumers. To this end, the literature shows that the release cycles improve the firm's profit when responding to the strategic waiting behavior of the consumers.

All the works mentioned above implicitly make three main assumption: the firm does not keep offering the old version in the second period (solo rollover), the market consists solely of strategic consumers, and the qualities of the products are exogenous to both the firm and the consumers. In contrast, we consider the choice of rollover types (solo versus dual) when designing the release strategy of the firm. We confirm that solo rollover is the optimal setting for digital products, and provide a posteriori justification for the focus on solo rollovers in the version management literature. Secondly, we consider myopic consumers in addition to forward-looking consumers and extend the firm's pricing and release timing to a more comprehensive context, where the market has different degree of consumer foresight. Finally, we extend the pricing and release timing discussion of the literature with the obsolescence dimension. We endogenize relative qualities of the versions for both the firm and consumers.

Similarly, the product rollover literature discusses the firm's release strategy of different versions made progressively available over time. In contrast to the versioning literature, however, they make the typical assumptions of fixed release intervals and existence of myopic consumers. Furthermore, consumer choices are often not explicitly linked with the firm's pricing regime and the resulting market segmentation was omitted (e.g. Ferguson and Koenigsberg, 2007; Koca et al., 2010). Lim and Tang (2006) and Arslan et al. (2009) include the release time in their analysis while the impact of the release time is on the volume of an exogenous group of consumers rather than on the individual consumer behavior per se. The studies of the rollover stream, however, contribute to the releasing problem and to the version management research with an important dimension, the choice of the rollover types (solo versus dual). The primary attention of the literature when comparing the rollover types relates to the supply-side, such as the costs of production and the capacity constraints (e.g. Ferguson and Koenigsberg, 2007; Lim and Tang, 2006; Koca et al., 2010).

In addition, there exists a small body of literature within the rollover stream capturing the

demand-side dynamics, such as the consumer driven obsolescence or the consumers' strategic behavior under fixed release intervals. Levinthal and Purohit (1989) show that the solo rollover strategy is the optimal strategy when strategic consumers are buying either the old or the new version, assuming homogeneous quality appreciation of consumers. Liang et al. (2014) extend this setting with two types of consumer groups that are homogeneous within, high and low value types. They suggest that the dual rollover is optimal if the market is dominated by myopic consumers and the exogenous innovation is high. They also note that supply side aspects, such as capacity constraints and firm's disposal value of the old version, play a major role in the choice of the rollovers. Zhou et al. (2015) extend this setting further with upgrading behavior and remark that the dual rollover is optimal if the market is willing to see a quality gap between the versions, while the implicit assumptions are fixed short release intervals and homogeneous valuation of the quality among the consumers. We reconcile these studies by showing that the upgrading behavior reinforces the solo rollover while the fixed short release intervals posit the dual rollover.

In contrast to the rollover literature, which considers exogenous deterioration (e.g. Levinthal and Purohit, 1989; Ferguson and Koenigsberg, 2007; Zhou et al., 2015) or technological improvement (Liang et al., 2014), we link the perception of the obsolescence to the firm's release timing. We show that the release timing is a crucial lever in a market with both myopic and strategic consumers, and the firm can use it to manage the consumer perception. Secondly, we relax the assumptions of consumer homogeneity, and include real-life complications, namely a market with distinct consumers' foresight and heterogeneous appreciation of quality. By doing so, we observe market segmentation and delve into the reasons behind the upgrading behavior. As a result, we provide insights for the firm's segmentation strategy: which consumer groups (myopic, strategic or both) and willingness to pay types (high or low value) that the primary pricing regime should focus on.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 and 4 cover the analysis on myopic and the strategic consumers, respectively. Section 5 considers a market composed of both the myopic and strategic consumers. Section 6 provides extensions and presents welfare analysis. Section 7 concludes.

2.2 Model

We consider a monopolist producer that, at time $t_1 = 0$, introduces a product at a price p_1 and is planning to introduce a new version of the same product at time t_2 at a price p_2. Both prices and $t_2 \in (0, \infty)$ are decision variables for the firm. If the firm prefers to execute a *solo rollover policy*, it combines the launch of the new version with the phase-out of the previous version. Instead of phasing out the first product at time t_2, the company has the option to sell the first product at a discounted price βp_1, where $\beta \in [0, 1]$. In this case, the firm employs a *dual rollover policy*. Under both of the rollover types, the firm discounts the future cash flows with a discount factor δ_s. Decisions of the firm to maximize its expected profits include: (1) the introduction time of the second product t_2, (2) pricing of the first and the second product, p_1 and p_2, and (3) the choice of solo or dual rollover and the discounted price βp_1 in case it prefers to execute a dual rollover.

First, we consider digital goods whose marginal costs are typically assumed to be zero by the literature (Shapiro and Varian, 1998). Thus, the firm's profit and revenue maximization problems are equivalent. We further discuss semi-digital goods with positive marginal costs of production in Section 6. We assume that the decision to develop the second product was taken as we focus on product launch. Fixed costs such as product development costs are then sunk costs and do not affect the rollover strategy. Similar to the version management literature (e.g. Mussa and Rosen, 1978; Bhargava and Choudhary, 2001; Johnson and Myatt, 2003; Haruvy et al., 2013), we assume that there is no capacity constraint and, thereby, we disregard the inventory and production quantity aspects of the problem (Yayla-Küllü, 2013). The absence of the capacity constraint seems justified for the digital goods as they can easily be reproduced and made on order (e.g., a software or a digital content) (Shapiro et al., 1998; Pei et al., 2012).

Next, we explore the demand side of the problem, and start with high-level market behavior. At its initial launch, each version of the product is perceived as the most recent technology, with a utility rate of v per unit of time. If the product is accompanied by a more recent version, the utility rate of the product is assumed to decay with a discount factor α. We will refer to this parameter as "perceived obsolescence" in our further discussion. This is the main

innovation in our model: an existing product can become obsolete *only* by reference to a new product. This can be due to several factors. Most naturally, when a new version is available, the consumers "perceive" the previous version less novel (Cooper, 2004; Bellezza et al., 2017; Hill, 2017) because it is not stylish (Waldman, 1996; Slade, 2009; Hellmann and Luedicke, 2018) or the most recent technology (Nair and Hopp, 1992; Nair et al., 2004; Slade, 2009) anymore. Also, the new version could be optimized for new standards, for instance higher capacity/memory/bandwidth requirements, making the old version perform relatively worse (Cooper, 2004). To concentrate on this novel aspect of our model, we assume that the new product does not add *any* utility compared to the old version when it was first introduced. We could of course also introduce a standalone additional value for the new version, but we keep this modeling assumption to focus the attention on our contribution.

Therefore, for a product that was first launched at time t_1, the utility rate is reduced to $\alpha^{t_2-t_1}v$ at time t_2 when an upgrade is introduced. The earlier papers have considered a fixed obsolescence rate (e.g. Desai and Purohit, 1998; Ferguson and Koenigsberg, 2007; Agrawal et al., 2015). Our setting is different since the perceived obsolescence of our model is dependent on the time interval between the launch of the two versions. As the interval between the launches widens, the perceived obsolescence increases and vice versa (Bulow, 1982; Dhebar, 1994; Cooper, 2004). Elapsed time would justify consumers to perceive the previous version as less stylish since their attachment, and perception of irreplaceability/indispensability decreases over time (Schifferstein and Zwartkruis-Pelgrim, 2008). In addition, the new standards arising from the improvements in complimentary infrastructures would pull down the consumers' durability perception of the old version. We take into account both an exogenous component of obsolescence (α) that is dictated by the market, and an endogenous introduction time t_2 of the second product. The exogenous component of obsolescence (α) stems from market's evaluation of the product line concerning durability regardless of the release time. As an example, soccer video games (such as FIFA) may be perceived less durable due to the consumers' desire of alignment with the real-life situation of the football leagues. On the other hand, car racing games (such as Need for Speed) may be regarded more long-lasting as the perception of the best "race" sites might be less volatile. The introduction time t_2 allows the company to manage the overall

obsolescence of a previous version and influence buying behavior for the second version.

Next, we delve into the demand with the individual consumer behavior to shed light on consumer behavior of different rollover types. Following the seminal work of Mussa and Rosen (1978), we assume that the consumers agree in their preference ordering, but they are heterogeneous when it comes to the appreciation of quality. This heterogeneous behavior is reflected by θ that defines the consumer type in terms of her appreciation of a certain quality. In our discussion, a consumer type θ is assumed to be uniformly distributed over $[0, 1]$, which is common in the version management literature. For a product that was launched at $t_1 = 0$ to be used until the end of the product's life cycle and in the absence of any other future upgrades, the present total utility discounted with a rate δ_c for a customer type of θ corresponds to $\theta \int_0^\infty v\delta_c^t \, \mathrm{d}t = \theta u(\delta_c)$, where $\delta_c \in (0, 1)$ is the consumer discount factor as used in the versioning literature (e.g. Calzada and Valletti, 2012). Therefore, the net consumer surplus of this product corresponds to $\theta u(\delta_c) - p$, where $u(\delta_c) := \frac{v}{-\log(\delta_c)}$ is the present total utility and p is the price.

We assume that the consumers use at most one product at a time, and they derive no salvage value from the old version when they upgrade to another version. Put differently, the versions are close enough substitutes while reselling an old version is not feasible. Among the reasons why a second-hand market might not exist are personal licenses (software), the impracticality of the ownership transfer (apps), data & privacy concerns, and the motivation to keep the old version as a back up (apps and semi-digital goods such as smartphones, GSMA (2012)). When a consumer at time t_2 is using the primary product launched at $t_1 = 0$ and is contemplating whether to upgrade to the new version, she compares the surplus $\theta u(\delta_c) - p_2$ obtained from upgrading to the new product with the surplus $\theta \alpha^{t_2} u(\delta_c)$ that comes from keeping the first product and not upgrading to the second product. A consumer upgrades to the new product if and only if the difference between the two surpluses $\theta u(\delta_c)(1 - \alpha^{t_2}) - p_2$ is positive.

We distinguish between myopic and strategic consumer foresight. A *technologically myopic consumer* does not have an outlook to future releases or future price discounts since either she is unaware of, or not interested in, obtaining this information. This consumer only considers the options immediately available to her and the anticipatory behavior does not occur. In

contrast, a *strategic consumer* can predict the company's future actions, for instance through online channels: rumor websites or consumer forums (e.g., *Macrumors.com* or *Techradar.com*) and company pre-announcements. This forward-looking consumer would anticipate that, at a known point in time, a new version will be available, and by that time the current product will lose its originality. Thus, the consumers of this type may wait for the new release or a future price discount on the primary version.

In particular, a myopic consumer does not perceive any consumer surplus from waiting for the new release. Neither does she take into account that the first version would be obsolete at time $t = t_2$ nor does she perceive a price reduction for the early version (in a dual rollover). Her net surplus from obtaining the early version would be $\theta u(\delta_c) - p_1$ (early buying option) while she perceives no value from other options (late buying and discount buying options) at $t = 0$. On the contrary, a strategic consumer θ, predicts that there will be a new release at time t_2 and she perceives a surplus of $\left(\theta \int_{t_2}^{\infty} v\delta_c^t \, \mathrm{d}t - \delta_c^{t_2} p_2\right) = \delta_c^{t_2} \left(\theta u(\delta_c) - p_2\right)$ at $t = 0$ if she evaluates this option solely. Likewise, the strategic consumer can foresee that the first version will be technologically obsolete at $t = t_2$, and this would pull her total surplus by $\delta_c^{t_2} \theta u(\delta_c)(1 - \alpha^{t_2})$ or that there will be discount on the price of the old version in a dual rollover and this would provide a net surplus of $\delta_c^{t_2} \left(\theta u(\delta_c)\alpha^{t_2} - \beta p_1\right)$.

We characterize a consumer's foresight with the random variable I with parameter $\gamma \in [0, 1]$, that is $I \sim Bernoulli(\gamma)$, such that I and θ are independent. A fraction $\gamma \in (0, 1)$ of the market consists of strategic consumers, namely $I = 1$, and these consumers are randomly dispersed over the vertical differentiation line θ. Figure 2.1 summarizes all possible consumer surpluses both for the myopic and strategic consumers, in a market consisting of the strategic consumers with a share of γ and the myopic consumers with a share of $1 - \gamma$.

We start our analysis[3] with a market that only consists of the *myopic consumers* or *strategic consumers*. After providing the optimal rollover strategy for these extreme cases, we extend our analysis to a market consisting of both the myopic and strategic consumers.

[3]The reader is referred to the Appendix for proofs.

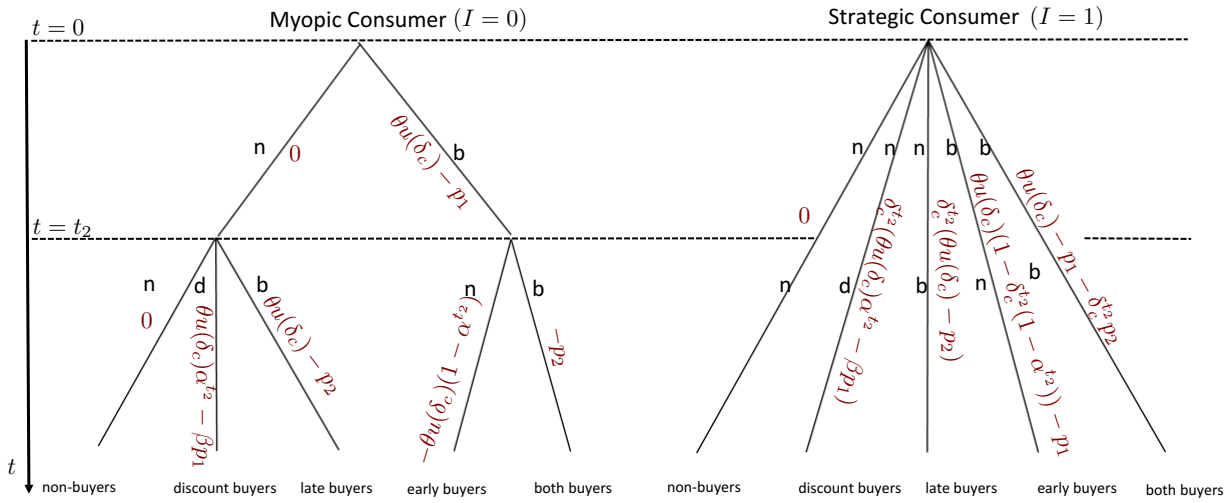


Figure 2.1: Consumer Behavior and Consumer Surpluses

2.3 Myopic (inattentive) Consumers, $\gamma = 0$

Myopic behavior is the rational consumption behavior when consumers do not account for the company's future actions such as the launch of a new version or mark-down on the previous version. They base their decisions solely on what is available in the market at any particular point in time.

At time $t_1 = 0$, consumers decide whether or not to buy the first product. If a consumer's surplus from the first product is positive, i.e., $\theta u(\delta_c) - p_1 > 0$, then that consumer buys the first product. At time t_2, the consumer has the option to buy the new product or the old version at a discount (dual rollover). Hence, the possible purchasing decisions are: buying only the first product at time $t_1 = 0$, buying only the second product at time t_2, buying the first product at time $t_1 = 0$ and upgrading to the second product at time t_2, buying the first product at t_2 at a discount, and buying nothing.

Based on these buying behaviors, we define $\theta_{ij} \in [0,1]$ as the threshold consumer that is indifferent between decision i and decision j. To simplify the notation, we drop i when it refers to the non-buying option. Otherwise, the notation is $i, j \in \{E$ (for early buying decision), L (for late buying decision), B (for both product decision), D (for discount buying decision)$\}$. When $\theta \geq \theta_{ij}$, decision j is preferred over decision i. To ensure that the threshold values θ_E and θ_L are in $(0,1)$, we make the usual assumptions $p_1 < u$ and $p_2 < u$ without loss of generality. The definition of each threshold is as follows.

In the first period:

- A consumer prefers to buy the first product over buying nothing if and only if

$$\theta u - p_1 \geq 0 \iff \theta \geq \frac{p_1}{u} =: \theta_E$$

In the second period:

- A consumer that did not purchase the first product prefers buying the second product over buying nothing if and only if

$$\theta u - p_2 \geq 0 \iff \theta \geq \frac{p_2}{u} =: \theta_L$$

- A consumer that purchased the first product prefers buying the second product over buying nothing if and only if

$$\theta u(1-\alpha^{t_2}) - p_2 \geq 0 \iff \theta \geq \min\left\{\frac{p_2}{u(1-\alpha^{t_2})}, 1\right\} =: \theta_B$$

- A consumer that did not purchase the first product prefers buying the first product at the discounted price βp_1 over buying nothing if and only if

$$\theta u\alpha^{t_2} - \beta p_1 \geq 0 \iff \theta \geq \min\left\{\frac{\beta p_1}{u\alpha^{t_2}}, 1\right\} =: \theta_D$$

- A consumer that did not purchase the first product prefers buying the second product over buying the first product at the discounted price βp_1 if and only if

$$\theta u(1-\alpha^{t_2}) - p_2 + \beta p_1 \geq 0 \iff \theta \geq \min\left\{\frac{p_2 - \beta p_1}{u(1-\alpha^{t_2})}, 1\right\} =: \theta_{DL}$$

At $t_1 = 0$, a consumer can either buy the first product or buy nothing. Hence, the critical consumer threshold for the first period is θ_E. At time t_2, as a consequence of the four different consumer options, there are four more threshold values, i.e., θ_B, θ_L, θ_D and θ_{DL}, that characterize the consumer segments. We immediately see that $\theta_B \geq \theta_L$ and $\theta_B \geq \theta_{DL}$ whereas the ordering between the other thresholds values depends on the decision variables p_1, p_2, β, t_2 and the utility decay parameter α. We discuss the ordering of the threshold consumer types and the possible market segmentation scenarios for solo and dual rollover in the next subsections, respectively. For now, we introduce a general profit function for a myopic consumption setting as follows:

$$\begin{aligned}\Pi_M(p_1, p_2, t_2, \beta) &= p_1(\theta_B - \theta_E)^+ + \delta_s^{t_2} p_2(\theta_E - \max\{\theta_L, \theta_{DL}\})^+ + (p_1 + \delta_s^{t_2} p_2)(1 - \max\{\theta_E, \theta_B\}) \\ &+ \delta_s^{t_2}\beta p_1(\min\{\theta_E, \theta_{DL}\} - \theta_D)^+ \end{aligned} \tag{2.1}$$

The profit function is composed of four revenue sources: early buyers, late buyers, both product buyers, and discount buyers. Since the marginal costs of the products are set to zero, we use the term revenues and profits interchangeably.

The first term corresponds to the profits generated from the early buyers, that is, the consumers that prefer to buy the first product at time $t_1 = 0$ over buying nothing, i.e., $\theta > \theta_E$, but that prefer not to upgrade to the second product at t_2, i.e., $\theta \leq \theta_B$. Therefore, the revenue generated from the early buyers segment is $p_1(\theta_B - \theta_E)^+$.

The second term of the profit function represents the revenues generated from the late buyers, that is, the consumers that do not buy the first product at time $t_1 = 0$, i.e., $\theta \leq \theta_E$, and that prefer to buy the second product at t_2, i.e., $\theta > \theta_L$, rather than the discounted product, i.e., $\theta \geq \theta_{DL}$. Therefore, the revenues generated from the late buyers segment are $\delta_s^{t_2} p_2(\theta_E -$

$\max\{\theta_L, \theta_{DL}\})^+$.

The third term of the profit function refers to the both product buyers, that is, consumers that buy the first product at time $t_1 = 0$, i.e., $\theta > \theta_E$, and that upgrade to the second product at time t_2, i.e., $\theta > \theta_B$. Thus, the revenues generated from this segment are $(p_1 + \delta_s^{t_2} p_2)(1 - \max\{\theta_E, \theta_B\})$.

The last term of the profit function is associated with the discount buyers, that is, consumers that do not buy the first product at time $t_1 = 0$, i.e., $\theta \leq \theta_E$, and that prefer to buy the first product at a discount at t_2, i.e., $\theta > \theta_D$, rather than the second product, i.e., $\theta < \theta_{DL}$. The revenue corresponding to this segment is $\delta_s^{t_2} \beta p_1 (\min\{\theta_E, \theta_{DL}\} - \theta_D)^+$.

Lemma 2.1 *If $\beta \in [\bar{\beta}, 1]$ where $\bar{\beta} = \min\left\{\frac{\alpha^{t_2} p_2}{p_1}, 1\right\}$, then the discount buyers segment disappears.*

An implication of Lemma 2.1 is immediate: If the company does not offer enough discount in the second period, then the second product cannibalizes the demand for the first product. This leads to the absence of dual rollover. Also, the more expensive the second product relative to the first product (p_2/p_1), the less discount the company has to give (i.e., the larger $\bar{\beta}$). Similarly, the larger the perceived quality depreciation (i.e., the smaller α and/or the larger t_2), the larger the discount has to be (i.e., the smaller $\bar{\beta}$).

2.3.1 Solo Rollover

In a solo rollover, the first product is available at time $t \in [0, t_2)$ at the price p_1 and the second product is available at time $t \in [t_2, \infty)$ at the price p_2. We observe four potential consumer segments: both product buyers, early buyers, late buyers and non-buyers. First, we characterize demand segmentation depending on the decision variables p_1, p_2 and t_2, and the utility decay parameter α.

Proposition 2.2 *For $\alpha \in (0, 1)$ and $t_2 \in (0, \infty)$, we have:*

1. *If* $p_2 \geq u(1-\alpha^{t_2})$, *none of the consumers buy both versions and*

   (a) *if* $p_2 \geq p_1$, *then the market consists solely of the early buyers (region E);*

   (b) *if* $p_2 < p_1$, *then the market consists of the early buyers and late buyers (region EL).*

2. *If* $p_2 < u(1-\alpha^{t_2})$, *there are consumers that buy both versions and*

   (a) *if* $p_2 \geq p_1$, *then the market consists of the early buyers and consumers that purchase both versions (region EB);*

   (b) *if* $p_1(1-\alpha^{t_2}) < p_2 < p_1$, *then the market consists of the early buyers, late buyers, and consumers that purchase both versions (region ELB);*

   (c) *if* $p_2 \leq p_1(1-\alpha^{t_2})$, *then the market consists of the late buyers and consumers that purchase both versions (region LB).*

Corollary 2.3 *For fixed prices* $p_1, p_2 \in (0, u)$, *we have:*

1. *If* $\alpha \to 0$ *and/or* $t_2 \to \infty$, *and*

   (a) *if* $p_2 > p_1$, *then the market consists of the early buyers and consumers that purchase both versions (region EB);*

   (b) *if* $p_2 = p_1$, *then the market consists solely of the consumers that purchase both versions (region B);*

   (c) *if* $p_2 < p_1$, *then the market consists of the late buyers and consumers that purchase both versions (region LB).*

2. *If* $\alpha \to 1$ *and/or* $t_2 \to 0$, *and*

   (a) *if* $p_2 \geq p_1$, *then the market consists solely of the early buyers (region E);*

   (b) *if* $p_2 < p_1$, *then the market consists of the early buyers and late buyers (region EL).*

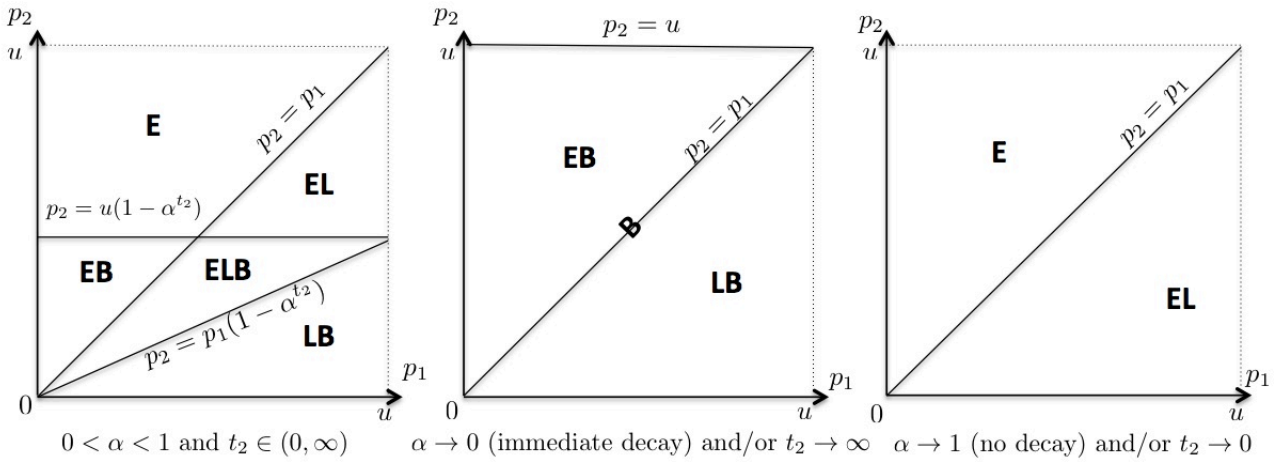


Figure 2.2: Segmentation Strategies in a Solo Rollover for Myopic Consumers

To summarize Proposition 2.2, Figure 2.2 visualizes demand segmentation as a function of the price levels p_1 and p_2. When the second product is more expensive than the first product, the late buyers segment disappears, since consumers that are not convinced to buy a new product with utility u and price p_1 in the first period would not want to wait to obtain the same utility for a price p_2 higher than the price p_1 they rejected earlier.

From the opposite angle, if the relative price of the second product $\frac{p_2}{p_1}$ drops below $1-\alpha^{t_2}$, the early buyers segment vanishes. We note that $1-\alpha^{t_2}$ is associated with the relative novelty of the new version over the previous version that becomes technologically obsolete in the second period. If the price of the second product is significantly low to justify the upgrading, every consumer that purchased in the first period will transform into both product buyers.

When the price of the second product p_2 is less than the extra utility it brings over the previous version $u(1-\alpha^{t_2})$, then there are consumers upgrading and, thereby, buying both versions. Otherwise, when $p_2 \geq u(1-\alpha^{t_2})$, this segment disappears.

As a special case, if there is immediate decay ($\alpha \to 0$) or infinitely delayed introduction ($t_2 \to \infty$), depending on the relative price of p_2, the market is segmented either as the early buyers and both product buyers or the late buyers and both product buyers. There are always consumers buying both products since either the rapid decay or the delayed introduction gives consumers an incentive to upgrade to the second product.

Another special case is obtained when there is no utility decay ($\alpha \to 1$) or immediate introduc-

tion of the second product ($t_2 \to 0$). In this case, the both product buyers segment disappears since there is no added value brought by the new version over the previous version. Therefore, in this special case there is no motivation for the early buyers to upgrade to the second product because either there is no decay in the utility of the first product or the introduction time is so immediate that the first product is still perceived to be too novel to justify upgrading. If the price of the new version is higher than the price of the previous version, there will be only the early buyers in the market. Otherwise it is segmented into the early buyers and late buyers.

We now discuss how the firm designs its solo rollover. The expected profit function of the firm, as implied by Proposition 2.2, is as follows:

$$\Pi_M(p_1, p_1, t_2) = p_1(\theta_B - \theta_E)^+ + \delta_s^{t_2} p_2(\theta_E - \theta_L)^+ + (p_1 + \delta_s^{t_2} p_2)(1 - \max\{\theta_E, \theta_B\}) \tag{2.2}$$

The first term of the profit function corresponds to the profits generated from the early buyers. The second term of the profit function represents the revenues generated from the late buyers. The third term of the profit function refers to the both product buyers.

Lemma 2.4 *The optimal prices satisfy* $p_1^* \geq p_2^* \geq p_1^*(1 - \alpha^{t_2})$.

As an implication of Lemma 2.4, for given α, δ_s and t_2, the optimal prices lead to market segmentation as (1) the early buyers and late buyers with optimal prices $u(1 - \alpha^{t_2}) < p_2^* < p_1^*$ or (2) the early buyers, late buyers, and both product buyers with optimal prices $p_1^*(1 - \alpha^{t_2}) < p_2^* < \min\{p_1^*, u(1 - \alpha^{t_2})\}$; or (3) the early buyers and both product buyers with optimal prices $p_1^* = p_2^*$; or (4) the late buyers and both product buyers with optimal prices $p_1^*(1 - \alpha^{t_2}) = p_2^*$. Next, we discuss how this market segmentation is further refined when we endogenize the introduction time t_2.

Proposition 2.5 *For* $\delta_s \in (0, 1)$, *there exist* α', $\alpha'' \in (0, 1)$, $\alpha' < \alpha''$, *such that:*

1. *If* $\alpha \in (0, \alpha']$, *it is optimal for the firm to serve the late buyers and consumers that purchase both versions (Region LB) and*

(a) for $\alpha \to 0$, it introduces the second product almost immediately ($t_2^* \to 0$);

(b) for $\alpha > 0$, it delays the introduction of the second product ($t_2^* > 0$).

2. *If $\alpha \in (\alpha', \alpha'')$, the firm serves the early buyers, late buyers, and consumers that purchase both versions (Region ELB) with the sequential introduction of the second product ($t_2^* > 0$).*

3. *If $\alpha \in [\alpha'', 1]$, the firm serves the early buyers and late buyers (Region EL) with immediate release of the second product ($t_2^* \to 0$).*

α' and α'' are depicted in Figure 2.3 (See proof in the appendix for expressions).

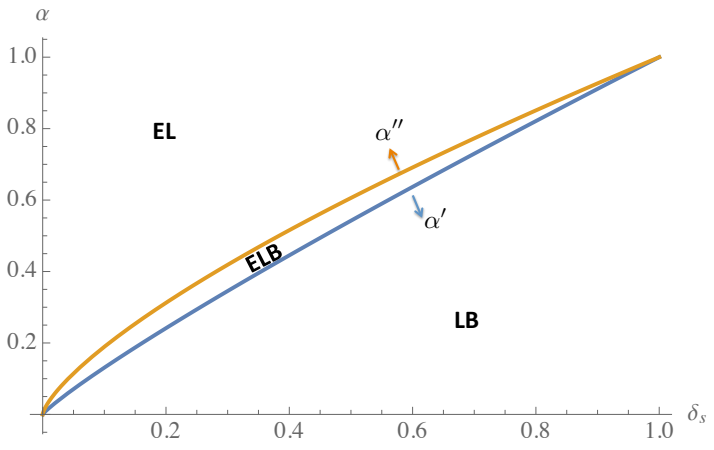


Figure 2.3: Optimal Segmentation Strategies

Proposition 2.5 shows the optimal segmentation strategy for various obsolescence rates $\alpha \in (0, 1)$. As a limiting case, imagine that the first product becomes perfectly obsolete with $\alpha \to 0$. Then, the firm introduces the second product almost immediately, $t_2^* \to 0$, and market segmentation takes place as every first period buyer upgrades to the second product. This is a case where every consumer "buys the same product twice", as it will become apparent below. As another limiting case, imagine that the first product never becomes obsolete $\alpha \to 1$. Then again the firm introduces the second product as soon as possible $t_2^* \to 0$, but for a different

reason: consumers buy the product only once, but the firm can practice a "limit intertemporal price discrimination" partitioning the early buyers from the late buyers. In these two limiting cases, the firm does not use introduction time as a lever to manage its rollover. However, when the obsolescence effect is significantly high with $\alpha \in (0, \alpha'')$, the firm utilizes introduction time $t_2^* \in (0, \infty)$ to manage its rollover policy. When the obsolescence effect is high and thereby, the obsolescence parameter is low with $\alpha \in (0, \alpha')$, the firm prefers to serve the late buyers and both product buyers with pricing policy of $p_2^* = p_1^*(1 - \alpha^{t_2^*})$ and introduction time $t_2^* \in (0, \infty)$ (the reader is referred to the proof in the appendix for the closed form expressions of p_1^*, p_2^* and the discussion of t_2^*). When the obsolescence effect is moderate with a parameter $\alpha \in (\alpha', \alpha'')$, the firm prefers to serve the early buyers, late buyers and the discount buyers with the optimal introduction time $t_2^* \in (0, \infty)$ (the reader is referred to the proof in the appendix for the closed form expressions of p_1^*, p_2^* and the discussion of t_2^*). With an exogenous market tendency to perceive obsolescence between the two versions, it is not optimal for the firm to induce every first period buyer to upgrade. Still, even with a moderate obsolescence the firm is able to motivate some of its first period consumers to upgrade. Finally, when the obsolescence effect is low with $\alpha \in (\alpha', \alpha'')$, it is optimal for the company to implement intertemporal price discrimination with an immediate release of the new version rather than using the introduction time to induce the first period buyers to upgrade.

Corollary 2.6 *It is always optimal to serve the late buyers, i.e., E and EB are never optimal strategies.*

As an immediate consequence of Proposition 2.5, Corollary 2.6 implies that it is always optimal for the firm to serve the late buyers segment with optimal prices satisfying $p_1^* > p_2^*$. Put it differently, the launch price of an early version should always be higher than the launch price of a successor. The intuition behind this result is the competition between versions when the market consists of only myopic consumers. Consumers then compare the late version with the early version only at the launch of the late version. To be able to motivate the current consumers to upgrade, the firm needs to set the price of the late version lower than the launch price of the early version. Having said that, as technological obsolescence causes the utility

from the early version to depreciate at the launch of the successor, the optimal relative price of the second product is always greater than or equal to the additional novelty $(1 - \alpha^{t_2^*})$ it brings over the predecessor. Next, Corollary 2.7 shows how the additional novelty $(1 - \alpha^{t_2^*})$ is influenced by the firm through the launch time t_2^* and derives the optimal prices, optimal profit level as well as the overall demand segmentation.

Corollary 2.7 *For $\delta_s \in (0, 1)$:*

1. *The optimal introduction time $t_2^*(\alpha) \to 0$, when $\alpha \to 0$, continuously increases with α for $\alpha \in (0, \alpha'')$, and $t_2^*(\alpha) \to 0$ for $\alpha \in [\alpha'', 1)$.*

2. *The optimal expected profit function $\Pi^*(\alpha) \to \frac{u}{2}$, when $\alpha \to 0$, continuously decreases with α for $\alpha \in (0, \alpha'')$, and remains constant at $\Pi^*(\alpha) = \frac{u}{3}^-$ for $\alpha \in [\alpha'', 1)$.*

3. *The optimal price for the first product $p_1^*(\alpha) \to \frac{u}{2}$, when $\alpha \to 0$, continuously increases with α for $\alpha \in (0, \bar{\alpha})$, continuously decreases with α for $\alpha \in (\bar{\alpha}, \alpha'')$, and remains constant at $p_1^*(\alpha) = \frac{2u}{3}^-$ for $\alpha \in [\alpha'', 1)$.*

4. *The optimal price for the second product $p_2^*(\alpha) \to \frac{u}{2}$, when $\alpha \to 0$, continuously decreases with α for $\alpha \in (0, \alpha'')$ and remains constant at $p_2^*(\alpha) = \frac{u}{3}^-$ for $\alpha \in [\alpha'', 1)$.*

(See proof in the appendix for expressions).

Figure 2.4 visualizes Corollary 2.7, where $u = 1$ and $\delta_s = 0.79$. The upper left panel shows the optimal profit with respect to α. At $\alpha = 0$, the optimal introduction time $t_2^* \to 0$ and the optimal profit level converges to $\frac{u}{2}$.[4] When $0 \leq \alpha \leq \alpha' \simeq 0.812$, it is optimal to set the prices

[4]This is the limiting case that we labelled earlier as each consumer "buys the same product twice". In fact, with complete decay, the firm optimally sets the price of its first version at its monopoly level, $p = u/2$, selling to consumers between 1/2 and 1. The firm thus makes total profit of 1/4 from the first version. The immediate introduction of the second version makes the old version totally obsolete, and the firm posts the same price for the new version which is sold to the very same set of customers, generating an additional profit of 1/4. There is no reason to delay the introduction of the second version, as the firm discounts its profits. This second version is then introduced as soon as possible (with an arbitrarily small lag). Note that, ex ante, some consumers make negative net consumer surplus in expectation, which is of course due to their myopic behavior.

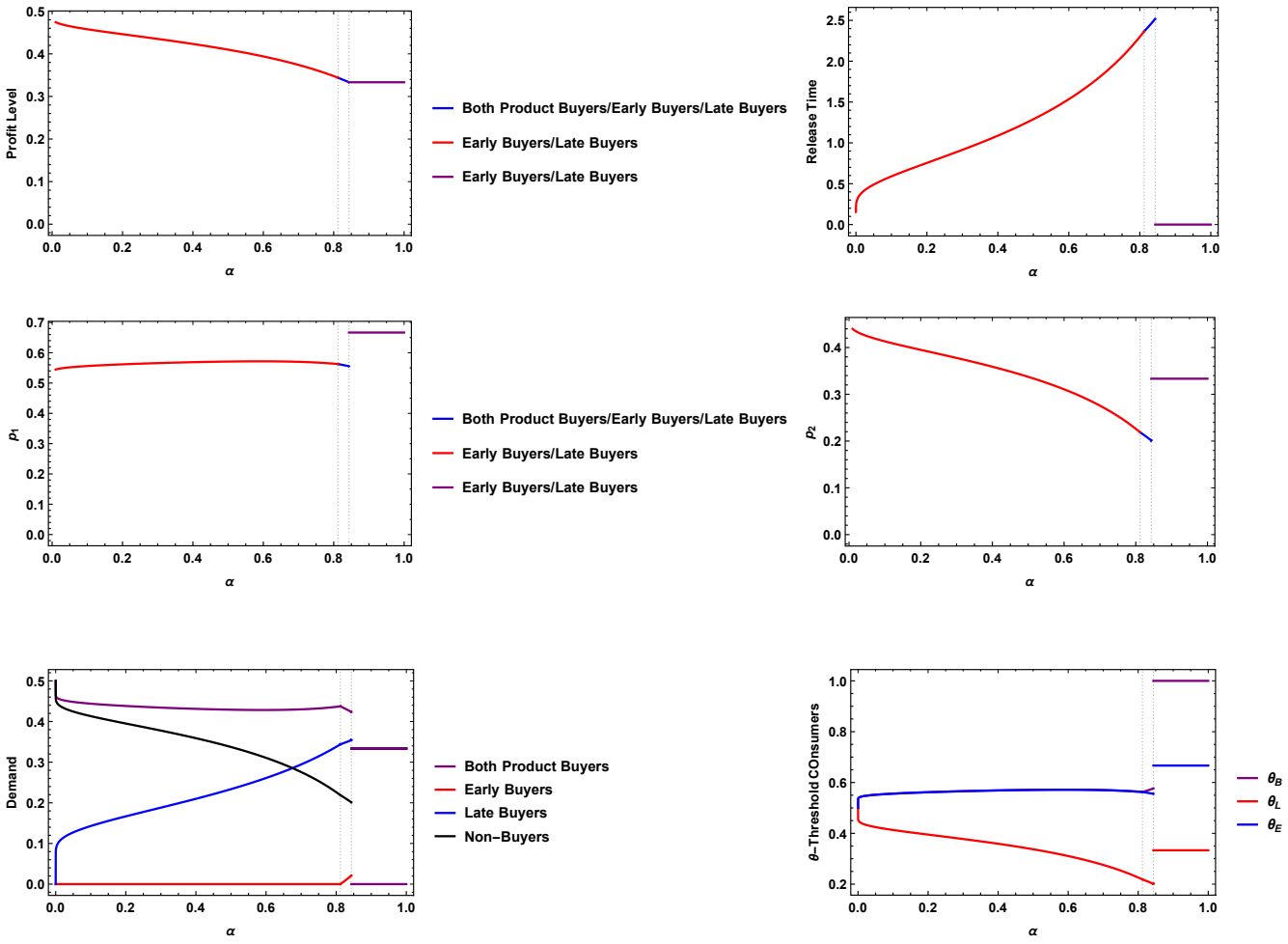


Figure 2.4: Optimal Rollover Strategy (Solo) with respect to α, where $u = 1$ ($v = 1, \delta_c = 0.37$) and $\delta_s = 0.79$

and introduction time such that the market segments as both product buyers and late buyers. The firm induces every buyer of the first period to upgrade. As the parameter α increases, the tendency of consumers to see a difference between the two products diminishes. Hence, the firm tries to influence the technological obsolescence through the introduction time t_2. Naturally, as the total obsolescence effect diminishes, the profits decrease. When $\alpha' < \alpha \leq \alpha'' \simeq 0.843$, the optimal segmentation strategy is serving all the segments: both product buyers, early buyers, and late buyers. Again, as α increases, the total obsolescence effect diminishes and this causes profits to fall. We note that up to α'', the company is able to smooth out the decrease in the perceived novelty of the second product over the previous version by delaying the introduction time of the second product. However, for very large α values, i.e., $\alpha \geq \alpha''$, the first product is perceived as almost new, even at high values of t_2. Therefore, it now becomes optimal to induce a different demand segmentation: the market is split between the early buyers and late buyers, and the second product is launched as soon as possible, with an expected profit converging to $\frac{u}{3}$.[5]

The upper right panel of Figure 2.4 summarizes the non-monotonic optimal introduction time for the second product with respect to α. There is an increase in the introduction time as α increases due to the decrease in the perceived quality difference between the two versions. To control for the technological obsolescence, the company delays the introduction of the second product. However, as mentioned earlier, for very large α values, it is best for the firm to introduce the second product almost immediately ($t_2^* \to 0$) and alter its pricing policy dramatically.

The middle panels of Figure 2.4 depict how optimal prices change with respect to α. From the panels, it can be observed that the optimal prices are $p_1^* = p_2^* = \frac{u}{2}$ by inducing the same group of consumers to buy twice when $\alpha = 0$. For the region $\alpha < \bar{\alpha} \simeq 0.591$, the optimal price for the first product increases with α due to the incentive to increase the profit margin. For $\bar{\alpha} < \alpha < \alpha'' \simeq 0.843$, p_1^* decreases with α due to the incentive to increase the consumer base.

[5] In this other limiting case without decay, versioning cannot be used to generate perceived quality differences, but it is still introduced as the firm can appraise the second version by targeting the residual demand of those that did not buy the first version. As the versions are again introduced with an arbitrarily small lag, we call this case "limit intertemporal price discrimination".

The optimal price for the second product decreases as α increases to attract consumers that do not perceive significant difference between the two versions. After α'', it is optimal for the firm to waive controlling the technological obsolescence but fix prices as $p_1^* \to \frac{2u}{3}$ and $p_2^* \to \frac{u}{3}$.

The lower panels depict the behavior of the demand (on the left) that is derived from the threshold consumer types (on the right).

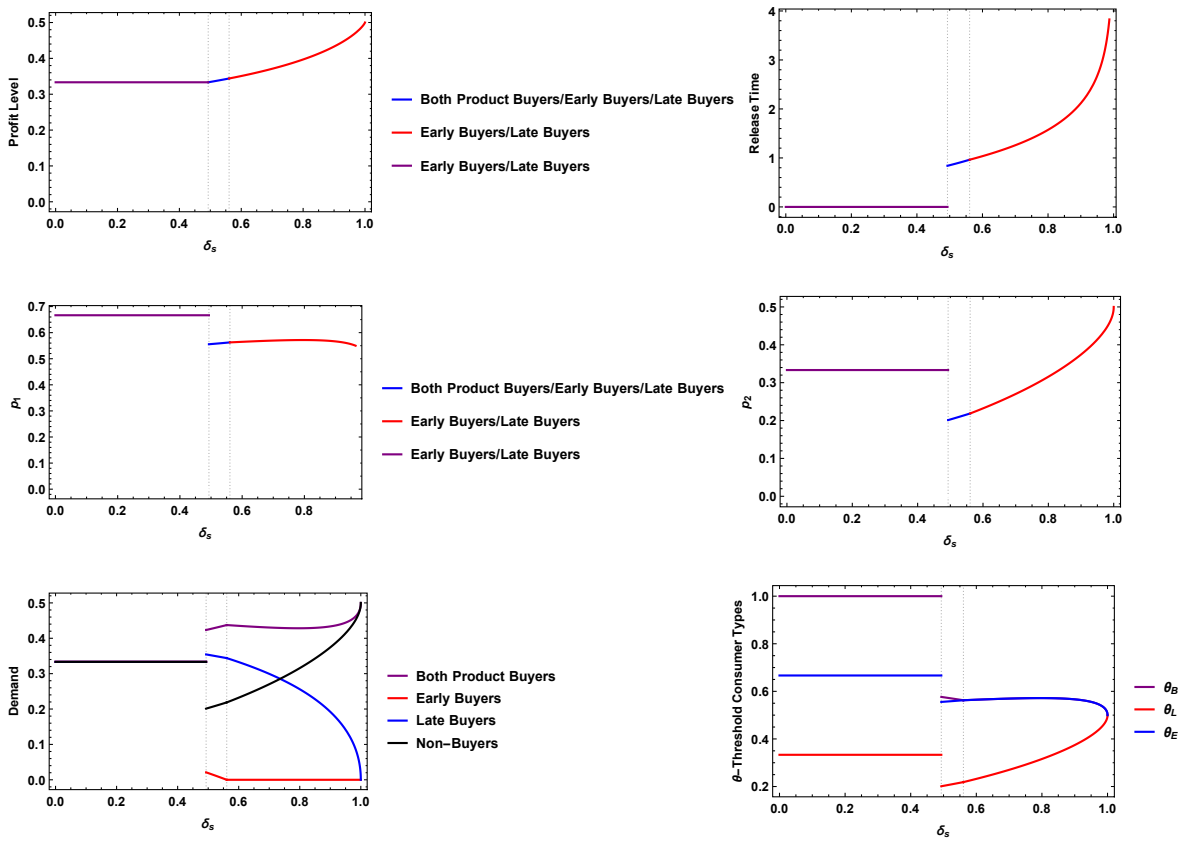


Figure 2.5: Optimal Solo Strategy with respect to δ_s, where $u = 1$ and $\alpha = 0.6$

Figure 2.5 visualizes the optimal solo strategy with respect to the firm's discount rate δ_s, where $u = 1$ and $\alpha = 0.6$. In the region $0 < \delta_s \leq \delta_s' \simeq 0.4933$ where the firm is very impatient, it is optimal to set the prices and the introduction time such that it mutes the technological obsolescence and the market segments into the early buyers and late buyers. As the firm becomes more patient, then designing the strategy through technological obsolescence, i.e., setting $t_2 \neq 0$, becomes more effective. For the region $\delta' < \delta_s < \delta_s'' \simeq 0.5607$, the optimal segmentation strategy is targeting the both product buyers, early product buyers, and late product buyers, whereas when the firm is extremely patient, i.e., $\delta_s \geq \delta_s''$, it is optimal to induce all the first version buyers to upgrade and attract some consumers as late product buyers. The

discussion of the various panels follows closely that of Figure 2.4 and is not repeated for the sake of space: essentially, a high value of δ_s corresponds to a low value of α, and vice versa.

2.3.2 Dual Rollover

In a dual rollover strategy, the first product is available at the price level p_1 during the time period $t \in (0, t_2)$ and at a discount βp_1 in the period $t \in [t_2, \infty)$, where $\beta \in [0, 1]$. The second product is available at the price level p_2 at time $t \in [t_2, \infty)$. In addition to the previous consumer segments, there is also a discount buyers segment that the firm may consider to serve. In this section, we assess the optimal segmentation, pricing, and timing strategy for a dual rollover. The ultimate aim is to investigate which strategy, solo or dual rollover, is more profitable with the myopic consumers.

The profit function for the dual rollover is shown in Equation (2.1). We reiterate that $\beta < \bar{\beta}$ is a necessary condition to avoid the cannibalization of the discount sales by the late buyers (see Lemma 2.1). Having said that, the firm does not guarantee the discount sales as there is also the possibility of cannibalization by the early product buyers when the late buyers segment does not appear in the market. Next, we discuss how the market segments when the condition $\beta < \bar{\beta}$ is satisfied.

Proposition 2.8 *For $\alpha \in (0, 1)$, $t_2 \in (0, \infty)$ and $\beta \in [0, \bar{\beta})$,*

1. *If $p_2 \geq u(1 - \alpha^{t_2})$, none of the consumers buy both versions, and*
   (a) *if $p_2 \geq p_1(1 - \alpha^{t_2} + \beta)$, then the market consists of the early buyers and if in addition $\beta < \alpha^{t_2}$, the discount buyers (region ED);*
   (b) *if $p_2 < p_1(1 - \alpha^{t_2} + \beta)$, then the market consists of the early buyers, late buyers, and discount buyers (region ELD).*
2. *If $p_2 < u(1 - \alpha^{t_2})$, there exist consumers buying both versions, and*

(a) *if* $p_2 \geq p_1(1-\alpha^{t_2}+\beta)$, *then the market consists of the early buyers, consumers that purchase both versions and if in addition* $\beta < \alpha^{t_2}$, *the discount buyers (region EBD);*

(b) *if* $p_1(1-\alpha^{t_2}) < p_2 < p_1(1-\alpha^{t_2}+\beta)$, *then the market consists of the early buyers, late buyers, consumers that purchase both versions, and discount buyers (region ELBD);*

(c) *if* $p_2 \leq p_1(1-\alpha^{t_2})$, *then the market consists of late buyers, consumers that purchase both versions and discount buyers (region LBD).*

Corollary 2.9 *For fixed prices* $p_1, p_2 \in (0, u)$,

1. *If* $\alpha \to 0$ *and/or* $t_2 \to \infty$ *(*$\bar{\beta} \to 0$*), there are no discount buyers and the market segments as in the case of the solo rollover.*

2. *If* $\alpha \to 1$ *and/or* $t_2 \to 0$ *(*$\bar{\beta} \to \min\{p_2/p_1, 1\}$*) and*

   (a) *if* $p_2 \geq p_1$, *then it follows that* $\bar{\beta} = 1$ *and*

      i. *if* $\beta < 1$, *then the market consists of the early buyers and discount buyers;*

      ii. *if* $\beta = 1$, *then the market consists of only the early buyers;*

   (b) *if* $p_2 < p_1$, *then it follows that* $\bar{\beta} = \frac{p_2}{p_1}$ *and*

      i. *if* $\beta < \bar{\beta}$, *then the market consists of the early buyers, late buyers, and discount buyers;*

      ii. *if* $\beta \geq \bar{\beta}$, *then the market consists of only the early buyers and late buyers.*

Figure 2.6 visualizes the market segmentation in a dual rollover as explained in Lemma 2.1, Proposition 2.8 and Corollary 2.9. The upper half of the figure shows the market segmentation when $\beta \geq \bar{\beta}$, where the demand of the discount buyers segment is cannibalized by the demand of the late buyers. Therefore, in this case, market segmentation occurs as in the case of solo rollover as implied by Lemma 2.1.

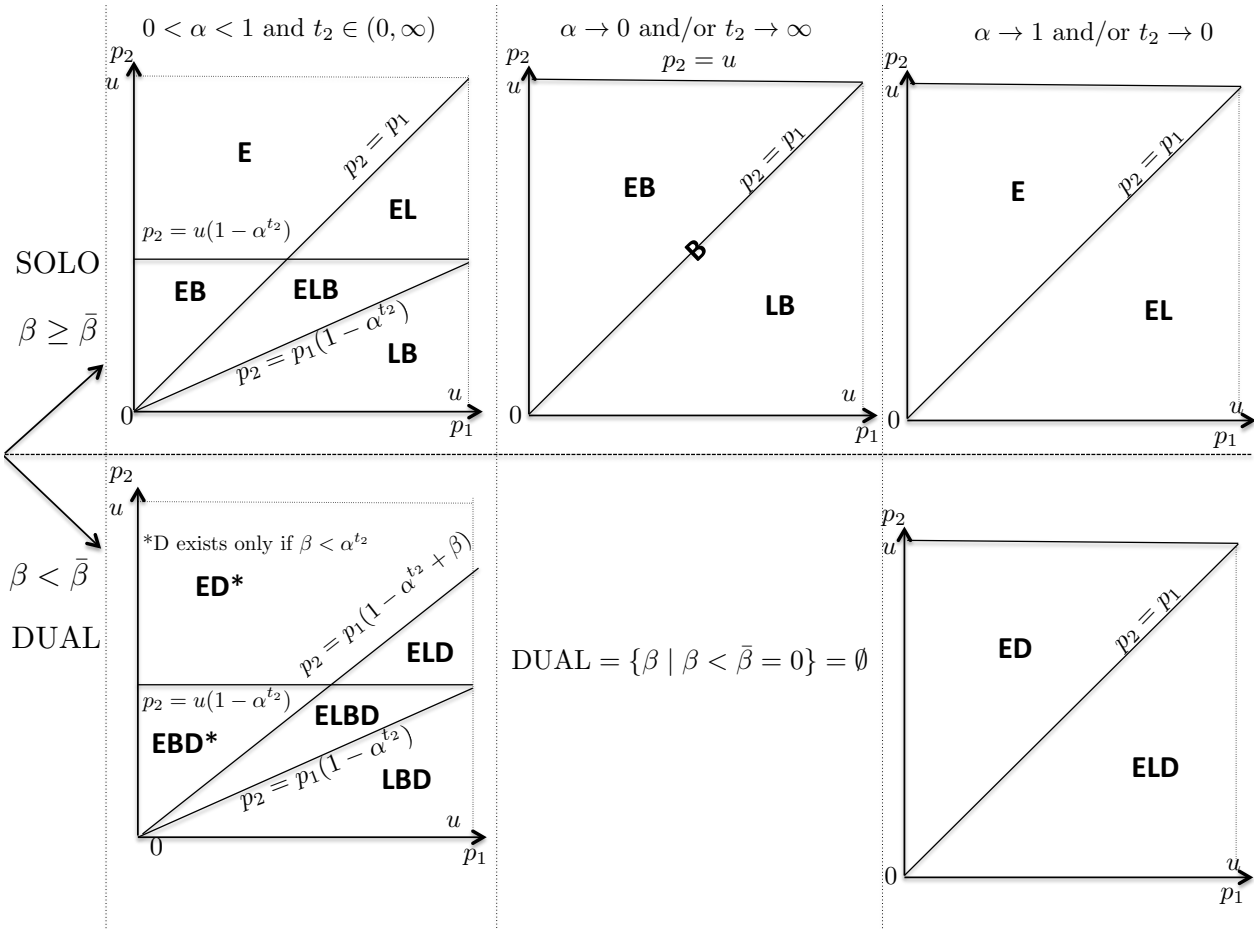


Figure 2.6: Segmentation Strategies

For $\beta < \bar{\beta}$, on the other hand, the firm ensures that the discount buyers segment is not cannibalized by the late buyers segment (see bottom half of the figure). Proposition 2.8 shows that the late buyers segment exists if and only if the relative price of the second product p_2/p_1 is less than $(1 - \alpha_2^t + \beta)$. First, we note that since $\beta < \bar{\beta}$, it follows that the launch price of the second product is higher than the discounted price of the first period product, i.e., $p_2 - \beta p_1 > 0$. For the late buyers to exist, the relative price of the second product $\frac{p_2 - \beta p_1}{p_1}$ should be less than the extra novelty it brings $(1 - \alpha^{t_2})$ to convince some non-buyers of the first period to buy the second product. Otherwise, the late buyers segment vanishes as a consequence of cannibalization by the demand of the early buyers. In this case, the discount buyers segment is in danger of being cannibalized by the demand of early buyers and it follows that if and only if the discount rate β is less than the utility depreciation α^{t_2}, the discount buyers segment appears. Figure 2.6 illustrates the case where $\beta < \alpha^{t_2}$. If this condition is not satisfied, the discount buyers segment is swallowed by the demand of the early buyers and we observe market segmentation either as the early buyers or the early buyers and both product buyers. If the relative price p_2/p_1 is less than $(1 - \alpha_2^t)$, then the early buyers segment vanishes since the relative price is too low compared to the novelty the second product brings $(1 - \alpha^{t_2})$. This is enough for the demand of early buyers to be cannibalized by the late buyers. Finally, if the price of the second product p_2 is higher than the additional utility it brings $u(1 - \alpha^{t_2})$, the both product buyers segment vanishes as in the case of the solo rollover.

When the obsolescence effect is extremely high ($\alpha \to 0$) and/or the introduction time is extremely delayed ($t \to \infty$), the threshold discount rate $\bar{\beta} \to 0$ implies that there is no discount rate securing the discount buyers segment to appear. In this case, the firm faces a market segmentation like the case of solo rollover as exhibited by the middle panel of Figure 2.2. When the obsolescence effect is extremely low ($\alpha \to 1$) and/or the introduction time is immediate ($t \to 0$), the threshold discount rate is $\bar{\beta} \to \min\left\{\frac{p_2}{p_1}, 1\right\}$ since the additional novelty brought by the second version is extremely low ($1 - \alpha^{t_2} \to 0$). When the launch price of the second product is higher than the launch price of the first product ($p_2 \geq p_1$), we observe the threshold discount rate $\bar{\beta} \to 1$, implying that if $\beta = 1$ we have only the early buyers whereas when $\beta < 1$ we have the early buyers and discount buyers. Put it differently, except the extreme case of

preserving the launch price of the first product in the second period, the firm guarantees that the discount buyers segment appears. On the other hand when $p_2 < p_1$, we observe $\bar{\beta} = \frac{p_2}{p_1}$, which in turn implies that if $\beta < \bar{\beta}$ we have the early buyers, late buyers, and discount buyers in the market, whereas we have the early buyers and late buyers if $\beta \geq \bar{\beta}$. The right panel of Figure 2.2 summarizes the discussion with the limiting case of extremely low obsolescence effect with $\alpha \to 1$ and/or $t \to 0$.

Next, we discuss the optimal segmentation strategy for a dual rollover strategy. First, we determine the optimal prices for a given launch time $t_2 \in (0, \infty)$.

Lemma 2.10 *Given fixed $\alpha, \delta_s \in (0,1)$ and $t_2 \in (0, \infty)$, the optimal prices in a dual rollover satisfy $p_1^*(1 - \alpha^{t_2} + \beta^*) \geq p_2^* \geq p_1^*(1 - \alpha^{t_2})$.*

Lemma 2.10 enables us to focus on the region $p_1^*(1-\alpha^{t_2}+\beta^*) \geq p_2^* \geq p_1^*(1-\alpha^{t_2})$ in determining the optimal launch time and optimal prices for a dual rollover strategy. In addition, we also discuss if the discount buyers segment exists given the optimal decision variables or if this segment is cannibalized by the demand from late buyers, leading the rollover policy to eventuate in a solo rollover.

Proposition 2.11 *For a market consisting of myopic consumers, the optimal rollover strategy is the solo rollover strategy.*

We conclude that the optimal dual rollover strategy collapses to a solo rollover with the optimal discount rate that equals to the threshold discount rate ($\beta^* = \bar{\beta}$). This implies that the solo rollover strategy is always preferred to a dual rollover. In a market with myopic consumers that have no motivation to forward-look and postpone their purchases expecting a better and/or a cheaper product, our findings suggest that the optimal policy should be solo rollover with launch time being a significant decision variable to manage both the upgrading and first time buying behavior of consumers.

Our findings for myopic consumers appear in contrast to those of Liang et al. (2014). They propose that, in the presence of only myopic consumers, the firm should employ a dual rollover.

We remark that their setting considers a capacity constraint that leads the pricing strategy to be affected largely by the market disposal rate and production quantities as opposed to our setting. Another difference is the modelling of the qualities of the two versions. In the mentioned paper, there is an innovation rate taken as an exogenous parameter. We, on the other hand, provide the decision maker the flexibility to influence the perceived quality differences between the versions with the introduction time t_2.

To conclude, the findings of this section are in line with the version management literature that typically assumes solo rollover, whereas we have shown it to be optimal. Our result though holds for the case of the myopic consumers. Next, we discuss the case where the market consists of the strategic consumers.

2.4 Strategic (attentive) Consumers, $\gamma = 1$

As opposed to the myopic consumers, the strategic consumers are conscious of the choices that will be available in the future such as the release of a new version or a discount on an early version. They are able to predict surpluses associated with each future choice. In addition, forward looking consumers are aware that the future actions of the company have an effect on their current choice. A consumer thinking of buying the early version apprehends that it will become technologically obsolete whenever the new version is available.

Figure 2.7 depicts a summary of aggregate surpluses associated with each choice. As a result of the comparison of the available choices at $t_1 = 0$, we derive the threshold values for consumer types that are indifferent between pairwise choices. For instance, a consumer that is indifferent between buying the early version alone with a surplus of $\theta u(1 - \delta_c^{t_2}(1 - \alpha^{t_2})) - p_1$ and buying the second version alone with a surplus of $\delta_c^{t_2}(\theta u - p_2)$ is denoted by θ_{LE} (in analogy to the discussion of the myopic consumers) and is located at $\theta_{LE} := \frac{p_1 - \delta_c^{t_2} p_2}{u(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$.

We note two possibilities in the ordering of the net present value of the utility attached to the second product $\delta_c^{t_2} u$ and the net utility attached to the first product $(1 - \delta_c^{t_2}(1 - \alpha^{t_2}))u$. When $\delta_c^{t_2} u \leq (1 - \delta_c^{t_2}(1 - \alpha^{t_2}))u$, the consumers that are located above the threshold consumer type

θ_{LE} prefer to buy the first product over the second version and vice versa. If the inequality is reversed, then the consumers located above the threshold type θ_{LE} would prefer to wait for the late product rather than buying the early version and vice versa. The comparison between the net utilities of the two products depends on the consumer patience level δ_c, the obsolescence parameter α and the decision when to introduce the second product t_2. The difference between the net utilities is partly endogenous through the introduction time t_2. In particular, if the firm sets an immediate introduction time, it is possible to observe a higher net utility with the second product relative to the first product. On the other hand, delaying the introduction time leads to a net utility of the first product exceeding the net utility of the second version.

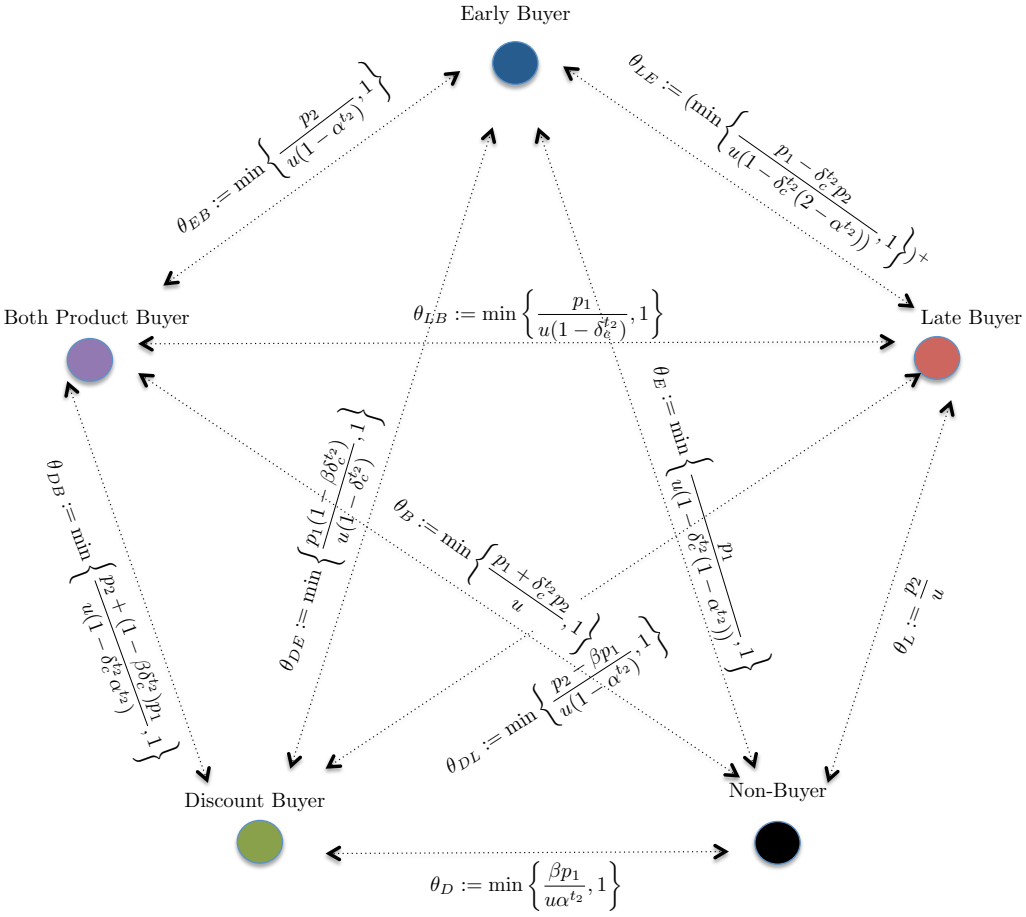


Figure 2.7: Threshold Consumer Types

2.4.1 Solo Rollover

At time $t_1 = 0$, the strategic consumer contemplates whether to buy the early version or to wait for the new version with an awareness that there will be a new version at t_2 with a price p_2. She may also decide not to buy. Next, we characterize market segmentation under a solo rollover strategy.

Proposition 2.12 *For $\alpha, \delta_c \in (0,1)$, there is $\tau(\alpha, \delta_c) \in [0, \infty)$ with the following properties:*

1. *If $t_2 > \tau(\alpha, \delta_c)$, then a consumer of type $\theta \in [\theta_{LE}, 1]$ prefers early buying to late buying (regular preference ordering).*
2. *If $t_2 < \tau(\alpha, \delta_c)$, then a consumer of type $\theta \in (0, \theta_{LE}]$ prefers early buying to late buying (irregular preference ordering).*

Moreover, we have $\tau(\alpha, \delta_c) = 0$ if and only if $\alpha \in [\delta_c, 1)$. (See proof in the appendix for expressions).

Proposition 2.12 discusses the effect of the introduction time on preference orderings of late buying ($\delta_c^{t_2} u$) and early buying ($u(1 - \delta_c^{t_2}(1 - \alpha^{t_2}))$). When the introduction time of the second product is relatively soon ($t_2 < \tau(\alpha, \delta_c)$), the reservation price of early buying is less than the reservation price of late buying for all consumer types $\theta \in [0, 1]$. The preference ranking occurs as both product buying (u) being superior to late buying ($\delta_c^{t_2} u$) and late buying preference dominating early buying ($u(1 - \delta_c^{t_2}(1 - \alpha^{t_2}))$), namely, an *irregular preference ordering*. In this case, a consumer with high marginal willingness to pay ($\theta > \theta_{LE}$) prefers late buying over early buying whereas a consumer with relatively low quality appreciation ($\theta \leq \theta_{LE}$) prefers early buying over late buying. To sum up, as the effect of obsolescence gets strong, the more a consumer appreciates quality, the more likely that she prefers late buying over early buying.

On the contrary, when the introduction time is late ($t_2 > \tau(\alpha, \delta_c)$), the exact opposite holds for preference orderings. The consumers with high willingness to pay for quality ($\theta \geq \theta_{LE}$) prefer early buying over late buying, whereas the consumers with low marginal appreciation ($\theta < \theta_{LE}$)

prefer late buying over early buying. As a result, both product buying dominates early buying and early buying is superior to late buying, a ranking that we call *regular preference ordering.*

We remark that when $\alpha \in [\delta_c, 1)$, it follows that $\tau(\alpha, \delta_c) = 0$ and, thereby, $t_2 > \tau(\alpha, \delta_c)$. Thus, one can only observe the regular preference ordering. On the other hand, when $\alpha \in (0, \delta_c)$, one can observe both types of preference orderings, depending on whether the release time is early or late. For $\alpha \in (0, \delta_c)$, we note that $\tau(\alpha, \delta_c)$ decreases with α and increases with δ_c.

Proposition 2.13 *For $\alpha, \delta_c \in (0, 1)$ and $t_2 \in \mathbb{R}^+$,*

1. *There are both product buyers if and only if $p_2 < u(1 - \alpha^{t_2})$ and $p_1 < u(1 - \delta_c^{t_2})$;*
2. *There are early buyers if and only if*
   (a) *$p_1 < p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})$ for $t_2 > \tau(\alpha, \delta_c)$ (regular preference ordering),*
   (b) *$p_1 < p_2(1 - \delta_c^{t_2}(1 - \alpha^{t_2}))$ for $t_2 < \tau(\alpha, \delta_c)$ (irregular preference ordering);*
3. *There are late buyers if and only if*
   (a) *$p_1 > p_2(1 - \delta_c^{t_2}(1 - \alpha^{t_2}))$ for $t_2 > \tau(\alpha, \delta_c)$ (regular preference ordering),*
   (b) *$p_1 > p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})$ for $t_2 < \tau(\alpha, \delta_c)$ (irregular preference ordering).*

Figure 2.8 visualizes market segmentation as stated in Proposition 2.13 and should be read in conjunction with Figure 2.2 to highlight differences compared to the myopic setting. The option of buying both products, the highest rank in the preference order, provides extra utility over other alternatives but with additional costs attached. Formally, both product buyers exist if and only if the extra novelty brought by the new version ($\delta_c^{t_2} u(1 - \alpha^{t_2})$) is high enough to cover the price of the second version ($\delta_c^{t_2} p_2$). Similarly, the benefit of not waiting in the first period ($u(1 - \delta_c^{t_2})$) is high enough to justify the price paid for the first product (p_1). This is different from the myopic consumer setting where myopic consumers are locked-in to the buying decision in the first period and their upgrading decision would be based only on the extra utility derived from the late version in relation to its price. As a result, in the presence of strategic consumers

that appreciate the option of waiting, the LB segment gradually shrinks as it is replaced by the L segment (see Figure 2.2 and 2.8 for a comparison).

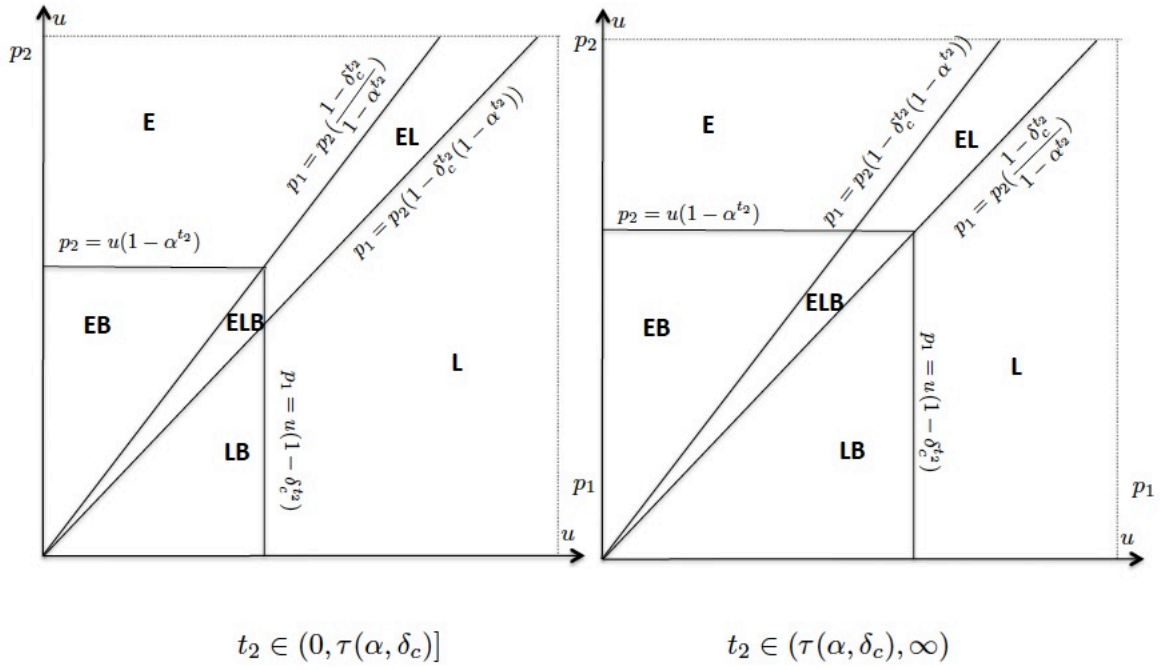


Figure 2.8: Segmentation Strategies in a Solo Rollover for Strategic Consumers

Corollary 2.14 *For fixed prices $p_1, p_2 \in (0, u)$,*

1. *If $t_2 \to 0$ for $\alpha, \delta_c \in (0, 1)$, and*

   (a) *if $p_2 \geq p_1$, then the market consists solely of the early buyers (region E);*

   (b) *if $p_2 < p_1$, then the market consists solely of the late buyers (region L).*

2. *If $t_2 \to \infty$ for $\alpha, \delta_c \in (0, 1)$, then the market segments as in the case of the myopic consumers (as summarized in Corollary 2.3).*

3. *If $\alpha \to 0$ for $\delta_c \in (0, 1), t_2 \in (0, \infty)$, and*

   (a) *if $p_1 \leq p_2(1 - \delta_c^{t_2})$, then the market consists of the early buyers and consumers that purchase both versions (region EB);*

(b) *if* $u(1-\delta_c^{t_2}) > p_1 > p_2(1-\delta_c^{t_2})$, *then the market consists of the late buyers and consumers that purchase both versions (region LB);*

(c) *if* $p_1 \geq u(1-\delta_c^{t_2})$, *then the market consists solely of the late buyers (region L).*

4. *If* $\alpha \to 1$ *for* $\delta_c \in (0,1), t_2 \in (0,\infty)$, *then the market segments as in the case of myopic consumers (as summarized in Corollary 2.3).*

5. *If* $\delta_c \to 0$ *for* $\alpha \in (0,1), t_2 \in (0,\infty)$, *then the market segments as in the case of myopic consumers (as summarized in Proposition 2.2)*

6. *If* $\delta_c \to 1$ *for* $\alpha \in (0,1), t_2 \in (0,\infty)$, *and*

(a) *if* $p_1 \leq \alpha^{t_2} p_2$, *then the market consists of the early buyers and late buyers (region EL);*

(b) *if* $p_1 > \alpha^{t_2} p_2$, *then the market consists solely of the late buyers (region L).*

Corollary 2.14 focuses on limiting cases. When the release of the second product is infinitely delayed ($t_2 \to \infty$) and there is no obsolescence ($\alpha \to 1$), or when the consumers are extremely impatient ($\delta_c \to 0$), attentive consumers behave in the same manner as myopic consumers.

In the case of full obsolescence ($\alpha \to 0$), the segmentation strategies transform. In the myopic case, full obsolescence forces the consumers with high willingness to pay to upgrade. the attentive consumers with high willingness to pay would position themselves as late buyers if the price would not cover the opportunity cost of not having a product in the first period. If instead, the introduction time of the second product is immediate ($t_2 \to 0$), both product buying would be perceived as an inferior strategy: depending on the prices of the products, the market consists only of a single segment. A similar situation arises when the consumers are fully patient and they buy one version at most.

We conclude our discussion with the optimal rollover strategy. Expected profit, as implied by Proposition 2.13, for given prices $p_1, p_2 \in [0,u]$ and $t_2 \in \mathbb{R}_+$, is:

$$\Pi_S(p_1,p_2,t_2)=\begin{cases}\Pi_S^1(p_1,p_2,t_2) & \text{if } t_2\in(0,\tau(\alpha,\delta_c)]\\ \Pi_S^2(p_1,p_2,t_2) & \text{if } t_2\in(\tau(\alpha,\delta_c),\infty)\end{cases}$$

where

$$\Pi_S^1(p_1,p_2,t_2)=p_1(\min\{\theta_{EB},\theta_{LE}\}-\theta_E)^+ +\delta_s^{t_2}p_2(\theta_{LB}-\max\{\theta_{LE},\theta_L\})^+ +(p_1+\delta_s^{t_2}p_2)(1-\max\{\theta_{EB},\theta_{LB}\}),$$

$$\Pi_S^2(p_1,p_2,t_2)=p_1(\theta_{EB}-\max\{\theta_{LE},\theta_E\})^+ +\delta_s^{t_2}p_2(\min\{\theta_{LB},\theta_{LE}\}-\theta_L)^+ +(p_1+\delta_s^{t_2}p_2)(1-\max\{\theta_{EB},\theta_{LB}\}).$$

The firm can move along the two branches of the profit function depending on whether or not it wants to emphasize the obsolescence effect with an early or late release time ($t_2 \lessgtr \tau(\alpha,\delta_c)$). The choice between the two options is not intuitive. Focusing on obsolescence with long release intervals would give the incentive to the first period consumers to upgrade, but also delays profit into the future. Conversely, an early release brings profits closer in time but can also induce early buyers to wait precisely because the new version will not be very obsolete.

We first discuss the optimal price policies when the release time is given.

Lemma 2.15 *The optimal prices satisfy*

- $p_2^*(1-\delta_c^{t_2}(1-\alpha^{t_2}))\le p_1^*\le p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})$ *for* $t_2>\tau(\alpha,\delta_c)$ *(regular preference ordering);*
- $p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})\le p_1^*\le p_2^*(1-\delta_c^{t_2}(1-\alpha^{t_2}))$ *for* $t_2<\tau(\alpha,\delta_c)$ *(irregular preference ordering).*

When the release interval is sufficiently long, the prices show similarities to the myopic setting. The lower bound for the first-period price would be limited by the relative utility of the early option over the late option, while the upper bound would reflect the policy for the upgraders. The difference comes from the price bracket as the optimal prices are squeezed in a smaller region when consumers have foresight, which limits the pricing power of the firm. A similar idea follows for the short release intervals. Next, we endogenize the introduction time t_2 with Proposition 2.16, and derive the optimal rollover strategy under various obsolescence and patience levels.

Proposition 2.16 *For* $\delta_s\in(0,1)$,

1. *If $\delta_c \geq \delta_s$, it is optimal for the firm to introduce only the first version with $t_2^* \to \infty$ (Region E).*

2. *If $\delta_c < \delta_s$, it is optimal for the firm to sequentially introduce the versions with a regular preference ordering ($t_2^* > \tau(\alpha, \delta_c)$). There exist $\alpha'(\delta_c) \in (0,1)$ and $\alpha''(\delta_c) \in (\alpha'(\delta_c), 1)$ such that:*

   (a) *If $\alpha \in (0, \alpha'(\delta_c)]$, it is optimal for the firm to serve the late buyers and consumers that purchase both products (Region LB),*

   (b) *If $\alpha \in (\alpha'(\delta_c), \alpha''(\delta_c))$, the company serves the early buyers, late buyers, and consumers that purchase both products (Region ELB),*

   (c) *If $\alpha \in [\alpha''(\delta_c), 1]$, the company serves the early buyers and late buyers (Region EL).*

$\alpha'(\delta_c)$ and $\alpha''(\delta_c)$ are illustrated in Figure 2.9 (See proof in the appendix for expressions).

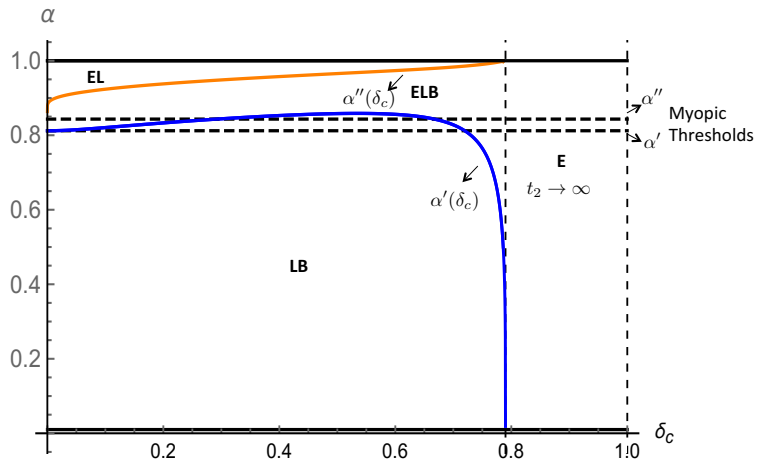


Figure 2.9: Optimal Segmentation Strategies for $\delta_s = 0.79$ and $v = 1$

As expected, strategic waiting behavior typically pulls down the firm's profits (see Figure 2.10 as an illustration, in comparison with Figure 2.4 in the case of myopic consumers). That said, the firm's optimal market segmentation is sensitive to the strategic consumer's patience level

(see Figure 2.9). When the consumers weigh future gains more than the firm does ($\delta_c > \delta_s$), the firm would delay the release of the version infinitely and the market would segment solely as early buyers (region E).[6] If instead, the consumers value future less than the firm does, releasing the late version makes sense but at a distant future ($t_2^* > \tau(\alpha, \delta_c)$). Note that the long release interval has mixed effects on the net present value of the obsolescence effect ($\delta_c^{t_2}(1-\alpha^{t_2})$). On the one hand, it widens the quality gap between the versions, and this is applicable to the myopic case as well. On the other hand, it softens the obsolescence effect by inducing attentive consumers to perceive it only in the distant future and thereby increasing the relative value of buying in the first period. Activating both forces, the firm can ensure that there exists substantial fractions of high-value upgraders as long as it keeps the prices reasonably low.

In particular, when the market is perceiving high obsolescence ($\alpha < \alpha'(\delta_c)$), the firm keeps the second period price ($\delta_c^{t_2} p_2$) sufficiently low to convince all the first period buyers to upgrade (region LB). The firm's motive here is similar to the myopic setting. The difference, however, is that the firm adjusts the release time due to the consumer attentiveness to the total price of upgrading ($p_1 + \delta_c^{t_2} p_2$). In addition, the more the consumers value the future (δ_c), the more the firm wants to induce every first period buyers to upgrade unless the value is not extremely large (see Figure 2.9).[7]

When the market perceives a moderate obsolescence ($\alpha''(\delta_c) > \alpha > \alpha'(\delta_c)$), the firm would need to lower both the first and second-period prices in order to convince the high-value consumers to upgrade. Even when the market perceives little quality deterioration ($\alpha \in [\alpha''(\delta_c), 1)$), the firm would use the release time as a lever since the "limit inter-temporal price discrimination" strategy would not make much sense in the presence of consumers' waiting behavior. That said, the firm would still be focusing solely on two segments: the early and late buyers (EL). Interestingly, the more the consumers are reluctant to perceive obsolescence, the higher the firm's profits in this region. This follows from the firm's high valuation of the future when

[6] Otherwise, the consumers would have a strong incentive to wait for the late version which would be sold at a reasonably lower price than the early version due to the firm's relative impatience. As a result, the firm prefers delaying the late version indefinitely.

[7] Even for $\alpha = 0$, the release time is finite ($t_2^* = \frac{\log\left(\frac{\log(\delta_s)}{\log(\delta_c)}\right)}{\log(\delta_c) - \log(\delta_s)}$), not arbitrarily small as in the myopic case.

compared to the consumers as well as from the market insensitivity to the release time and to the obsolescence. Therefore, the firm ensures that decrease in demand is mild while benefiting from increased profit margins to be collected sooner with shorter release interval.That said, the release time is still finite as opposed to the immediate release of the myopic case.

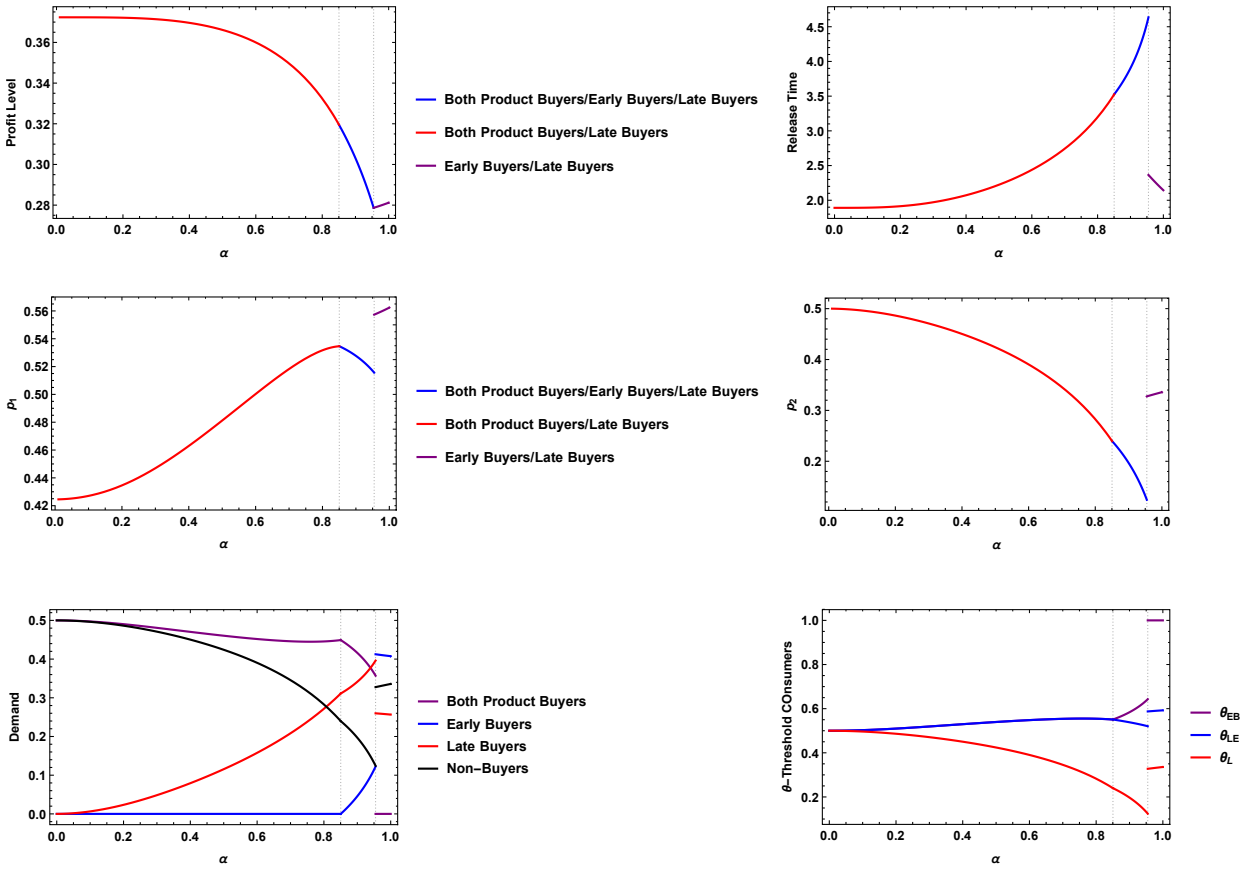


Figure 2.10: Optimal Rollover Strategy (Solo) with respect to α, where $u = 1$, $\delta_c = 0.37$ and $\delta_s = 0.79$

2.4.2 Dual Rollover

At time $t = 0$ in a dual rollover, a strategic consumer would be aware of a net surplus $\delta_c^{t_2}\theta(\alpha^{t_2}u - \beta p_1)$ that would be derived if she chooses to wait for the discount at time $t = t_2$. She would be comparing a deferred and degraded utility $\delta_c^{t_2}\alpha^{t_2}u$ with a discounted $\delta_c^{t_2}\beta p_1$. For this additional choice to be in the option set, the discounted price βp_1 should be reasonably low.

Lemma 2.17 *For $\alpha, \delta_c \in (0,1)$, $p_1, p_2 \in (0,u)$, and $t_2 \in \mathbb{R}^+$, there exists discount buyers if*

and only if $\beta \in [0, \bar{\beta})$ *where,*

$$\bar{\beta} = \begin{cases} \frac{\alpha^{t_2} p_2}{p_1} & \text{if } p_1 > p_2(1 - \delta_c^{t_2}(1 - \alpha^{t_2})) \\ \frac{\alpha^{t_2}}{1 - \delta_c^{t_2}(1 - \alpha^{t_2})} & \text{if } p_1 \leq p_2(1 - \delta_c^{t_2}(1 - \alpha^{t_2})). \end{cases}$$

If a potential consumer ($\theta : \delta_c^{t_2}(\theta \alpha^{t_2} u - \beta p_1) > 0$) is comparing the second period options (late buying versus discount buying), deciding at $t = 0$ is the same as at $t = t_2$. Hence, the discussion is equivalent to the myopic setting. If the comparison is inter-temporal, the discounted relative price ($\frac{\delta_c^{t_2} \beta p_1}{p_1}$) should justify waiting for the inferior relative utility ($\frac{\delta_c^{t_2} \alpha^{t_2}}{1 - \delta_c^{t_2}(1 - \alpha^{t_2})}$) and experiencing both the obsolescence and the delay. When the initial price is high ($p_1 > p_2(1 - \delta_c^{t_2}(1 - \alpha^{t_2}))$), the market segments the early buyers in an irregular order while ruling out the late buyers completely in the regular order. In both cases, the demand for the early option emerges as an immediate threat to the discounted option. Similar lines with a reverse ordering apply if instead, the second-period price is low. Next, we discuss the market segmentation as a result of the pricing policy:

Proposition 2.18 *For* $\alpha, \delta_c \in (0, 1)$, $t_2 \in \mathbb{R}^+$ *and* $\beta \in [0, \bar{\beta})$, *there exist discount buyers and;*

1. *There are both product buyers if and only if* $p_2 < u(1 - \alpha^{t_2})$ *and* $p_1 < u(1 - \delta_c^{t_2})$;
2. *There are early buyers if and only if*
   (a) $p_1 < p_2(\frac{1 - \delta_c^{t_2}}{1 - \alpha^{t_2}})$ *for* $t_2 > \tau(\alpha, \delta_c)$ *(regular preference ordering),*
   (b) $p_1 < \frac{p_2(1 - \delta_c^{t_2})}{1 - \alpha^{t_2} + \beta(1 - \delta_c^{t_2}(2 - \alpha^{t_2}))}$ *for* $t_2 < \tau(\alpha, \delta_c)$ *(irregular preference ordering);*
3. *There are late buyers if and only if*
   (a) $p_1 > \frac{p_2(1 - \delta_c^{t_2})}{1 - \alpha^{t_2} + \beta(1 - \delta_c^{t_2}(2 - \alpha^{t_2}))}$ *for* $t_2 > \tau(\alpha, \delta_c)$ *(regular preference ordering),*
   (b) $p_1 > p_2(\frac{1 - \delta_c^{t_2}}{1 - \alpha^{t_2}})$ *for* $t_2 < \tau(\alpha, \delta_c)$ *(irregular preference ordering).*

Different from the solo setting, the existence of the second most inferior option (late buying or early buying, depending on the length of the release intervals) is under threat also from

the discount buyers in addition to the de-facto competition from the next superior option. Therefore, the pricing power of the firm would be limited when compared to the solo case (see Lemma 2.19). In particular, the upper bound of p_1^* (p_2^*) is tighter if the release time is sufficiently short (long) and the pricing power would be more limited as the price discount (β) increases. To be more specific, when the release time is shorter, the inter-temporal competition between the early and the discounted versions brings the prices p_1^* and βp_1^* further down. On the contrary, if the release time is longer, the competition between the late and the discounted version forces the optimal prices βp_1^* and p_2^* to decline. Meanwhile, the reduced competition the early version facing from the discount buying provides the firm a partial pricing power that pushes the optimal price p_1^* up. These two forces, in turn, reflect as larger p_1^* and lower p_2^*.

Lemma 2.19 *For fixed $\alpha, \delta_c, \delta_s \in (0,1)$, $t_2 \in \mathbb{R}^+$, and $\beta \in [0, \bar{\beta})$, the optimal prices satisfy*

- $\frac{p_2^*(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))} \leq p_1^* \leq p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})$ *for $t_2 > \tau(\alpha, \delta_c)$ (regular preference ordering);*
- $p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}) \leq p_1^* \leq \frac{p_2^*(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$ *for $t_2 < \tau(\alpha, \delta_c)$ (irregular preference ordering).*

The optimal pricing policy always segments the market into late (early) buyers in a regular (irregular) ordering. This point is important to understand the choice of the firm between solo and dual rollovers. In particular, if the firm chooses a regular preference ordering (long release intervals), cannibalized revenue from the late buyers dominates the extra revenue generated from the discount buyers. Thus, the firm sets the limit discount rate to $\bar{\beta}$ ensuring that there exists no discount buyers, thereby no cannibalization effect. Practically, this is equivalent to a solo rollover. We conclude that a solo rollover is always better than a dual rollover if the long release interval regime is in effect. Lemma 2.20 outlines the discussion formally:

Lemma 2.20 *If the firm executes a regular preference ordering ($t_2 > \tau(\alpha, \delta_c)$) with the prices outlined in Lemma 2.19, then the optimal discount rate is $\beta^* = \min\left\{\frac{\alpha^{t_2} p_2}{p_1}, 1\right\}$, and thereby, the firm assures that the discount buyers vanish ($\Pi_D^R(\beta^* | p_1, p_2, t_2) = \Pi_S^R(p_1, p_2, t_2)$).*

If, instead, the short release regime is implemented, the cannibalization effect is on the early buying option and thereby, inter-temporal. As implied by Lemma 2.21, when a revenue loss

results from strategic waiting, the extra revenue generated due to the discounted version pulls up profits more than the reduction due to the cannibalization. Therefore, if a short release regime is in effect, dual rollover has the potential to outperform solo rollover.[8]

Lemma 2.21 *Provided that early buyers segment is the neighbouring segment of the discount buyers in a dual rollover, and with fixed prices (p_1, p_2), the optimal discount rate is $\beta^* = \left\{\frac{\alpha^{t_2}(\delta_s^{t_2}+\delta_c^{t_2})}{2(\delta_s^{t_2}-(1-\alpha^{t_2})\delta_s^{t_2}\delta_c^{t_2})}, 1\right\} < \bar{\beta}$. It follows that $\Pi_D^R(\beta^*|p_1, p_2, t_2) > \Pi_S^R(p_1, p_2, t_2)$.*

Finally, we discuss whether the firm prefers regular ordering, which practically implies the solo rollover, over irregular ordering when the release time is endogenous:

Proposition 2.22 *For $\delta_s, \delta_c \in (0,1)$, and $\beta \in [0, \bar{\beta}(t_2))$, there exist α'_D, $\alpha''_D \in (0,1)$, $\alpha'_D < \alpha''_D$, such that:*

1. *For $\delta_c \in [\delta_s, 1]$, it is optimal for the firm to introduce only the first version $t_2^* \to \infty$*
2. *For $\delta_c \in (0, \delta_s)$,*
   (a) *If $\alpha \in (0, \alpha'_D]$, it is optimal for the firm to serve the late buyers and consumers that purchase both versions (Region LB) with a regular order release policy, i.e., $t_2^* \in (\tau(\alpha, \delta_c), \infty)$;*
   (b) *If $\alpha \in (\alpha'_D, \alpha''_D)$, the firm serves the early buyers, late buyers, and consumers that purchase both versions (Region ELB) with a regular order release policy, i.e., $t_2^* \in (\tau(\alpha, \delta_c), \infty)$;*
   (c) *If $\alpha \in [\alpha''_D, 1]$, the firm serves the early buyers and late buyers (Region EL) with a regular order release policy, i.e., $t_2^* \in (\tau(\alpha, \delta_c), \infty)$.*

Here, $\alpha'_D > \alpha'$, $\alpha''_D > \alpha''$ for each value of $\delta_c < \delta_s$, where α', α'' are solo rollover thresholds outlined in Proposition 2.16.

[8] In fact, we show that if the release regime is constrained to short release intervals ($t_2 < \tau(\alpha, \delta_c)$), the dual rollover always outperforms a solo rollover. The reader is referred to Appendix A.2.1 for further details.

Proposition 2.22 immediately shows that the regular ordering would be preferred if the offer to the discount buyers is anticipated by the attentive consumers at $t = 0$. The firm is able to reduce the impact of second-period cannibalization between the old and the new versions by magnifying the obsolescence effect with a long release time, and increasing the value of the first version. Due to the extremely degraded quality, the effective discount should be very low ($\beta < \bar{\beta}(t_2)$) in order for the discount buyers to exist. Even then the discount buyers segment would be extremely small. If the firm has to execute a dual rollover, the release windows are further extended so that the discount segment is as small as possible.

We arrive at an important conclusion. Dual rollover can be optimal only if the firm is constrained in its ability to launch the second version. Once this choice is fully endogenized, solo rollover is the optimal strategy when releasing digital goods regardless of consumers' foresight.

2.5 Mixed Case, $\gamma \in (0,1)$

We have so far analyzed the case with the myopic and strategic consumers separately, to highlight the similarities and differences between these two extreme cases. As suggested by the literature[9], it more realistic to consider a market with a mix of both types of consumers which we analyze in this section.

Since the firm cannot identify the degree of consumer myopia, consumers must be offered the same price. Intuitively, the optimal strategy of the firm will be some weighted average of each optimal strategy in the polar cases. This is the case in our setting and actually true even for very small departures away from the polar cases, contrary to other settings where, in some ranges, the firm might still want to focus on only one type of consumers.[10] In our model, if

[9]See, Nair (2007); Li et al. (2014); Osadchiy and Bendoly (2015).

[10]Think, for instance, of a case where a pharmaceutical firm sets different prices in two countries, a rich and poor one, where the former is charged more than the latter. If price discrimination in not feasible (or it is undone by arbitrage, such as parallel trade), the firm will optimally set a weighted average of the two prices if it wants to supply both countries. But if instead the rich country is large enough compared to the poor country, the optimal response is to sell only in the rich country at the high original price. This latter type of situation cannot arise in our model.

the firm sets the prices at optimal level when there were only the myopic customers (which are optimal only for $\gamma = 0$), we can show that there will always be some strategic customers willing to purchase the first, second or both versions (if α is small) at those prices. Similarly, if the firm were to offer the optimal prices for the strategic customers (which are optimal only for $\gamma = 1$), then there would always be some myopic customers that would buy at least the first or the second version. The optimal response of the firm takes into account that there is always a mix of customers it is willing to cater for. Of course, the precise value of γ then determines if the optimal responses are more "tilted" towards myopic or strategic customers.[11]

From a higher point of view, however, the relative prices (the primary pricing regime) is set mainly based on the dominant group. The firm replicates the pricing regime of the polar case associated with the dominant group. In particular, when the myopic (strategic) consumers are dominant and the market is obsolescence-inclined, that is low α, then the relative prices are set such that all the myopic (strategic) first period buyers upgrade. If the market is obsolescence-hesitant, then the pricing regime ensures that some early buyers exist within the myopic (strategic) consumers.

We confirmed earlier that solo rollover is the best strategy regardless of the consumer type, and this carries over to the mixed case. Here, the pricing strategy and the release windows are sensitive to the degree of consumer foresight. Figure 2.11 describes how the solution adapts to the general case. As strategic waiting limits the firm's pricing power, the firm focuses primarily on myopic consumers unless the share of strategic consumers (γ) is high. In particular, when the consumers are more impatient than the firm ($\delta_c < \delta_s$) and the market is obsolescence inclined, the primary pricing regime ensures that the market segments myopic late and both product buyers (Region LB_M) of Figure 2.11), which is the pricing regime in the myopic-only consumers case. Note that, this pricing regime would cause the market for the strategic consumers to readily segment late and both product buyers because of the foresight they have. If, instead, the market is relatively hesitant to the obsolescence, then the pricing regime aims to serve all segments of the myopic consumers while focusing still only on the late and both

[11]The reader is referred to Proposition A.10 in the Appendix for a formal outline of the mixed case.

product buyers of the strategic consumers (Region ELB_M) of the Figure 2.11. Again, the strategic consumers would position themselves against the obsolescence effect and would prefer either being both product buyers (if they are high-willingness-to-pay types) or would wait for the late version (low-willingness-to-pay types).

In case the firm is more impatient than the consumers ($\delta_c \geq \delta_s$) and the strategic consumers dominate the market, then these consumers would be shaping the pricing regime, and there will be only early buyers in the market (as in the polar case associated with the strategic consumers). Otherwise, the pricing regime would be based on the myopic consumers, and the discussion mentioned above would apply.

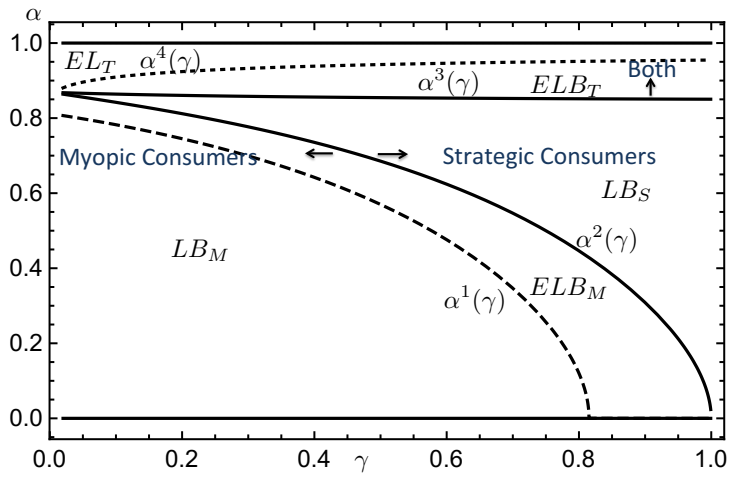


Figure 2.11: Optimal Segmentation, where $u = 1$, $\delta_c = 0.37$ and $\delta_s = 0.79$

Convincing the strategic consumers to upgrade, given the obsolescence-inclination is low, (α is high) is convenient for the firm as these consumers would be aware of the release windows and affordable prices in advance. In particular, they perceive high value from obtaining the early version that has a reasonable life cycle (length of the first period) as well as are willing to upgrade to the new version since the total price they pay for both versions are reasonable. That is why, when the market is reluctant to perceive obsolescence ($\alpha \geq \alpha^2(\gamma)$), the primary

pricing regime is based on the strategic consumers even if their portion in the market (γ) is low (Region LB_S) of the Figure 2.11).

If the market perceives almost no-obsolescence ($\alpha \geq \alpha^3(\gamma)$), the primary pricing regime would focus on both groups, the myopic and strategic consumers simultaneously. Note that, the prices per se would be tilted towards the group that is dominant. Despite the hesitant market, the firm still obtains both product buyers from both groups of customers (Region ELB_T) of the Figure 2.11) under this regime unless the market is extremely unwilling towards the obsolescence ($\alpha \geq \alpha^4(\gamma)$), where the market segments as early and late buyers (Region EL_T) of the Figure 2.11).

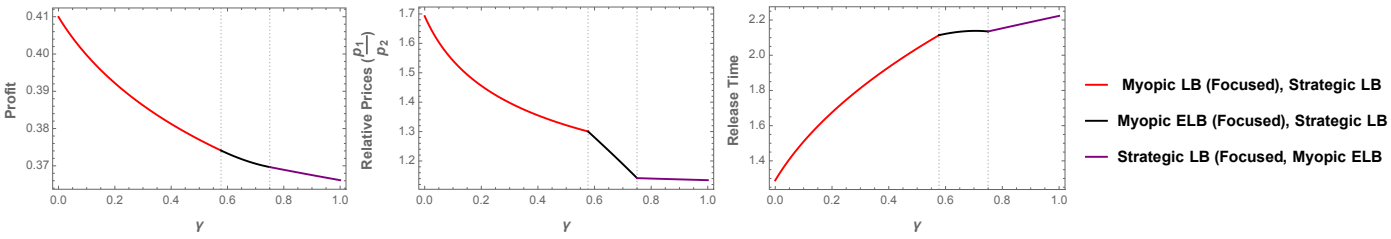


Figure 2.12: Optimal Release Policy, where $u = 1$, $\delta_c = 0.37$, $\alpha = 0.5$ and $\delta_s = 0.79$

As in the polar cases,the release time is still used strategically by the firm to affect the degree of perceived obsolescence, with delayed introduction arising, the greater the share of strategic consumers. An example of the optimal release policy is illustrated in Figure 2.12, for a case where the market is moderately keen on obsolescence ($\alpha = 0.5$). The main question is whether to emphasize relatively more myopic (with the LB_M or the ELB_M strategy) or strategic (with the LB_S strategy). Along the lines discussed above, when myopic consumers are dominant ($\gamma \leq 0.75$), the release regime targets more myopic consumers and the segmentation strategy is LB_M if myopic consumers are extremely dominant ($\gamma \leq 0.58$), and ELB_M otherwise. The presence of strategic consumers, though, progressively delays the release time of the second version compared to the pure myopic case. The underlying mechanism are shown with the relative price of the two versions (middle panel) and the release windows (right panel) of Figure 12. The relative price (p_1/p_2) incrementally decreases as the share of the strategic consumers (γ) increases, while the firm softens the negative pressure on profits with larger release windows.

2.6 Discussions

We have shown that the solo rollover is always optimal when the firm is selling digital goods (pure information goods) and has the release time leverage. Next, we discuss the choice of the rollover type when the goods are not digital, or the release time is exogenous to the firm. Following, we outline the welfare implications of the rollover strategy characterized in the earlier sections.

2.6.1 Extensions

Our results so far apply to the digital goods such as software, apps, video games, and e-books. That said, often these goods (such as a smartwatch or a tablet) are a bundle of digital and physical features, and the reproduction is not costless. This raises the question of whether the firm would still choose the solo over dual rollover or not if the goods are not purely digital. The answer depends on the relative marginal costs of production. As expected, if the late version is equally or less costly (such as professional tablets) compared to the early version, the firm would prefer the solo rollover (as in IPad Pro or Wacom Intuos). Thereby, the firm yields higher profit margins from the late version whose demand has not been cannibalized by the costlier early version.

If instead, the late version is more costly to reproduce than the early version, the dual rollover may be preferable. That said, we remark that the firm has full control on the release time, hence the obsolescence, and can induce the high-value consumers to buy both versions. The firm would choose to carry the early version to the second period if and only if the late version has very high costs of production. In that case, it makes sense for the firm to release the late version shortly after, and target the high willingness to pay types while giving up on some both product buyers.[12] Another real-life complication putting forward the dual rollover is the imposed

[12] An example would be smart TVs (with and without 4K resolution screening), where a late version is reasonably costlier to produce than an earlier version due to the incremental innovation arising from the complementary hardware suppliers. Also, the overall findings here may shed light on the practices where the firms roll-out intermediate generations as in Apple's smartwatches.

short release intervals that may emerge from the clockspeed of the complementary industries, commitment to previous release strategies, and the threat of prospective competitors.[13]

2.6.2 Welfare Implications

Recent industry chatter revolves around the obsolescence management and its effect on consumer welfare.[14] According to our findings, the effect depends on the consumers' foresight and the product obsolescence that is arising from the consumers (with α) and the firm (with t_2). The welfare of myopic consumers is hurt significantly from the obsolescence whether it is a perception of the market (perceived obsolescence) or built-in by the firm (planned obsolescence). In fact, planned or perceived high obsolescence results in negative consumer welfare due to consumer myopia *ex-ante* and regret *ex-post*. The firm, on the other hand, benefits most when obsolescence is full, hence the consumers are most likely to be hurt, if the planned obsolescence is in effect.

Strategic consumers, on the other hand, always benefit from partial obsolescence. Moreover, if they are sufficiently impatient, they would even seek "full obsolescence". We call this obsolescence seeking rationale as *the curse of being strategic*, and it stems from both product buyers that are enjoying continuous quality at low prices. From a policymaker point of view, this implies that the consumers should be educated about firm's obsolescence management policy so that they are attentive in advance and has price leverage against the firm.[15]

Firm's obsolescence seeking motive under the perceived obsolescence setting is also apparent when product obsolescence is exogenous to consumers and is set solely by the firm (planned obsolescence). This finding complies with the literature that low-durability versions would emerge when the firm can build the obsolescence (see references in Waldman (2003) and Agrawal et al. (2015)). However, consumer reaction to the forced quality deterioration may mitigate the

[13] A detailed discussion is in Appendix A.2.1.

[14] A recent example is a debate on Apple's forced battery slowdown with the release of the new versions (iPhone 8 and iPhone X). The concerns about the consumer welfare led to investigations in France (BBC *News*, 2017) and the US (Mickle and McKinnon, 2017).

[15] A detailed discussion is provided in Appendix A.2.2.

firm's motivation to build full obsolescence. In particular, consumers may react by being more attentive once the forced obsolescence is experienced (Cooper, 2004; Guiltinan, 2009). Under this setting, the firm would not be willing to build obsolescence and instead prefers partial obsolescence or even full durability depending on the consumers' sensitiveness (patience) to the durability.[16]

2.7 Conclusions

We have presented a model of product rollovers that provides insights on how a firm should release versions of a digital product category over time. Our model gives weight to consumers' foresight and the "perceived obsolescence", which are commonly observed behavioral phenomena in digital industries. We concentrated the analysis on digital goods, but our results are applicable to semi-digital products (e.g., smartphones, tablets, and computers) that are a bundle of digital and physical features.

Our main results characterize the pricing regime and the release windows that improve firm's profit when responding to consumers' foresight and the perception of obsolescence. Considering the release time as a lever, we discuss the firm's potential to manage the "perception of obsolescence", hence the willingness of the consumers to obtain multiple versions, under different attitudes of the market towards durability. If the consumers are myopic and hesitant to the durability, the firm should release the versions with short release windows while widening the intervals as the market becomes more content on the novelty of the early version at the launch of the new version. As for the strategic consumers, the firm's pricing power is limited but the release time (extended release interval) enables the firm to build high-value consumers that are willing to buy multiple versions. The polar cases may not be realistic per se, but provide the pricing regimes in general when the myopic (or strategic) groups dominate.

Given that the market is a mixture of the myopic and the strategic consumers, the release policy

[16] A detailed discussion is provided in Appendix A.2.3.

depends on both the fraction of the strategic consumers and the market's attitude towards the obsolescence. While the firm targets both groups, the higher level pricing regime primarily focuses on the dominant group. The pricing regimes (relative prices) discussed in the polar cases provide the firm the primary direction when setting the prices. If the consumers are reluctant to perceive obsolescence, the primary pricing regime is based on both the myopic and strategic consumers. When the market is inclined to see a novelty gap between the versions, the regime is based on the myopic consumers unless the fraction of the strategic consumers is above a threshold.[17]

Another main result of our analysis is that dual rollover is never optimal if the firm has full control on the release time. We verify the default solo rollover settings of the version management models (e.g. Calzada and Valletti, 2012; Lobel et al., 2015). The release intervals enable the firm to successfully allocate the consumers to the first and second period according to their willingness to pay. Hence, the relative value of the dual over the solo rollover, which is gathering extra revenue from the low-value consumers, decreases while the cannibalization effect on the late version persists. Therefore, only one version of a digital product category should exist at a given time. Our results extend to the goods that are bundled with physical features unless the costs attached to the late version are very high. If the release time is exogenously short (e.g., pre-committed release intervals, a threat of competition), then the firm should keep the old version in the second period, executing a dual rollover when consumers are strategic.

Several relevant aspects of the digital goods follow from our model. One fruitful extension would be to consider the demand-side network effects and demand uncertainty. Also, we assumed that the firm fully announces and commits to its rollover policy, which is realistic given the repeated nature of the digital releases and the firm's willingness to protect its reputation. That said, it would be interesting to extend the model with a more nuanced pre-announcement strategy, and its effect on the consumers' obsolescence perception.

In this paper, we focused on a monopolist that has already innovated and managed to protect its

[17] As an example, the threshold over which the primary pricing regime is based on the strategic consumers is 75% if the market has a moderate obsolescence perception ($\alpha = 0.5$).

intellectual property. This setting enables us to reveal fundamental trade-offs that would exist when the monopoly assumption is relaxed, such as the competition between the own versions of the firms and the effect of release time on the perception of the obsolescence. Nevertheless, inclusion of an outside competition would further cast light on the length of the release windows in competitive industries such as the smartphones market, where the consumers think that product releases are too frequent.[18]

[18] According to Richter (2016a), 63% of the smartphone users in the US think that too many new versions are released in a year.

Chapter 3

Double Utilization of Consumer Appreciation: Is the Seamless Connection the New Monopoly Leverage?

3.1 Introduction

Monopoly leverage, defined as power spillover from one market into a related market, has been at the core of antitrust debates. At times, it is found harmful to competition, an example being the EU fining of Google (European Commission, 2017). At other times, it is not perceived as an abuse of power because two related markets are complementary, and it does not make sense for the monopolist to manipulate the prices of one market since it would harm the related market (Edelman, 2014). The latter argument typically applies when the two markets are complements in nature, and the firms compete solely with prices. However, it needs to be revisited in the

Many thanks to conference participants at 4th "Industrial Organization in the Digital Economy"for their helpful comments, as well as Axel Gautier and Paul Belleflamme for their insights on an earlier version of this chapter.

context of the digital economy.

This paper examines quality and price competition of two distinct types of firms. One type is the "ecosystem firm" that served the market with a previous product category. The ecosystem firm has the advantage to relate the new category with its previous good (that we call the "base" category) and carry the previous consumer appreciation into the new category. We refer to this advantage as the "ecosystem leverage", which is typically established with a bundling software that connects the new category with the old product categories. An example to this setting is the smartwatch industry where ecosystem firms, such as Apple Inc., link their smartwatch to the base category, the smartphone, with their operating systems. On the other hand, the category firms, such as Tag Heuer, offer smartwatches that are independent of the base product category. A possible adverse impact of the ecosystem leverage is that overall competition may deteriorate as a result of the slow pace of the standalone category innovation. For instance, some industry experts state that smartwatches, as well as other smart wearables, are under the shadow of smartphones and that the category innovation lags due to Apple ecosystem (Beaver, 2017). Inspired by these industry examples, we extend the previous discussions on the leverage of a dominant firm with three tools typically used in the recent practices: price, stand-alone quality competition associated with the new category, and the endogenous connection designed by the ecosystem firm.

Further, we cover various types of intrinsic relation a new category product can have with the base category: partial substitutes, independent goods, and partial complements (Shocker et al., 2004). For instance, tablets and portable-laptops are categorized as partial substitutes (Eadicicco, 2017) while virtual reality headsets and smartphones are identified as partial complements (Grand View Research, 2017). The inherent link between the new and the old category can be crucial as it may inform to what extent the ecosystem firm exploits its leverage. For instance, if the new and the old categories are substitutes, the ecosystem firm may manage to deter the cannibalization effect by building complementarity while ensuring that its competitor is not posing a significant threat. Being constrained by the ecosystem firm, the profits of the category firm may diminish as a result of limited innovation it could offer to the market. To account for these dynamics, we extend the previous literature focusing on exogenous complementary

markets (Elhauge, 2009) by also including the substitute goods.

Finally, we consider a one-sided interaction between the two goods. The previous appreciation of the base category influences the new category instead of a mutual interaction forcing the ecosystem firm to align its policies with the benefits of the rival. If there were a mutual interaction between two product categories, the dominant firm would be cautious about its competition policy in the new category as how it competes in the new category would affect, in turn, on its welfare in the base category. The empirical evidence (e.g., Stremersch et al. (2007)) suggests otherwise for many high tech industries and claims that the power spillover is typically from the previously established good to the new product category but not the other way around. One explanation for this observation can be that the markets for the old categories are saturated (Thomas, 2018). Excluding the reciprocal effect enables us to unveil the primary impact of the ecosystem leverage that may not necessarily hurt the rival.

With the setting explained above, we examine to what extent an ecosystem firm uses the previous market appreciation as a strategic tool to keep its competitor at bay. Further, we investigate the influence of the ecosystem leverage on innovation in the new category (category innovation). The ecosystem leverage enables the previous category features to be accessible in the new category with a software (such as Watch OS, Wacom Bundled Software or Frog's Sensing Toys Software). Hence, the ecosystem firm offers an enhanced, rather than repeated, experience to the consumers that bought the previous product categories. The competitor cannot provide the same, but it can innovate on the standalone quality. By doing so, it would be competing against the overall bundled innovation of the ecosystem firm.

One outcome of this scenario could be that the new category slides under the shadow of the previous category instead of being a high-quality product category. At the extreme, the new category is just an accessory of the base category. The process behind this outcome may be an aggressive investment of the ecosystem firm on complementarity and overturning inherent substitution into complementarity. In this case, the competitor would be under pressure from both the previous category appreciation and the current category competition. As a result, the competitor may be disincentivized to innovate and diminish the standalone quality of its

product. It may even produce obsolete products (as in the case of Pepple (Staley, 2016)). The inevitable consequence of such a scenario would be that the competitor is forced to exit the market and the ecosystem firm seizes the dominance in both markets.

Another outcome may be that the competitor innovates aggressively. Subsequently, the ecosystem firm could focus on stand-alone quality. Recent developments in the smartwatch industry suggest a similar setting: SWatch, Tag Heuer, and FitBit as stand-alone new category producers are investing aggressively on stand-alone quality. As a response, Apple innovates and strengthens the smartwatch as a high-quality product category instead of designing it as an accessory of the smartphone (Atkins, 2017; Gurman et al., 2017). A possible consequence is that the two firms are willing to share the leverage, and hence "coopete", unless they are stuck in the asymmetric case, where only one firm can use the leverage, for external reasons such as contractual frictions.

We aim to understand the cases where the new category innovation speeds up. We also cover the conditions under which the ecosystem firm actually threatens the category firm. Our main question is how the market conditions, i.e., the relation between the two categories or the previous market coverage, inform the influence of the ecosystem leverage on innovation and on the competitor. We also investigate whether the firms' quality positions (being the leader or follower) have an influence on welfare and innovation, and to what extent the firms would leapfrog their rival. To our knowledge, this paper is the first to consider the double utilization of a previous consumer appreciation with an endogenously built complementarity under the setting explained above.

Contributing to the policy discussion on the monopoly leverage, we characterize the conditions when ecosystem leverage is harmful to the category firm. We find that it is beneficial to the category firm unless the base and the new category are strong substitutes or the category firm is the quality leader when the inherent substitution is weak. We confirm that monopoly leverage is not harmful to competition if there exists an intrinsic complementarity between the markets, even when the new category is a weak substitute of the old category. The underlying reason is the synergy between the stand-alone quality and the built-in complementarity instead of the

reciprocal effects of the pricing regimes in two product categories as focused by the literature. Besides, our results point to the quality positions of the firms as an indicator of whether or not the ecosystem leverage damages competition. Ironically, when the ecosystem firm is the stand-alone quality leader, the ecosystem leverage is not harmful but only beneficial to the category firm even if the related markets are substitutes. Thus, we reconcile two opposite views on whether or not monopoly leverage deters competition and harms the category firm. We extend this discussion further by showing that the adverse situations can vanish if the ecosystem firm shares its leverage. Interestingly, we find that it is most advantageous to both firms to share the seamless integration when the market experiences adverse situations. Therefore, the policymaker should persuade firms that collaboration is, in fact, beneficial so that firms engage in "coopetition"[1] instead of being stuck in an aggressive competition.

Secondly, we contribute to the industry discussion about the influence of the ecosystem leverage on the innovation of the new category. We conclude that the industry experts are partially correct in that the ecosystem firm slows the new product category innovation. In particular, the category innovation slows down as claimed by the industry experts only if the two product categories are inherently substitutes. If instead, the two product categories are complements or weak substitutes with the ecosystem firm being the quality leader, the ecosystem leverage, in essence, promotes quality competition, hence the category innovation. Furthermore, we also clarify that the firms never leapfrog their opponent's quality position. In other words, being the quality leader or follower are deep-rooted situations for each firm. Finally, we cover the consumer welfare implications of the ecosystem leverage and confirm that the consumer welfare is enhanced when the category innovation increases, and vice versa.

Through the analysis, we provide a detailed short-term outlook for the firms that focus on pricing regimes and characterize the conditions when to serve previous buyers of the base category for each firm. Interestingly, we find that the ecosystem firm should not serve its

[1]Coopetition is prevalent in the cases where the players collaborate on some competitive tools while at the same time compete on other elements (Nalebuff and Brandenburger, 1996; Belleflamme and Neysen, 2009). In our case, the firms share the seamless integration advantage whilst they engage in price and stand-alone quality competition.

previous consumers, the ecosystem market, if the new category is a strong substitute of the base category. We suggest that the category firm would focus on the ecosystem market unless it is negligibly small. On the contrary, when the ecosystem firm is the quality follower, it should always focus on the ecosystem market while its rival concentrate solely on the non-buyers of the base category if the the new and old category are strong substitutes.

The contributions of this paper are summarized as follows:

1. We propose a model that incorporates both quality and price as competition tools while capturing various market settings, i.e. substitute, independent and complementary product categories.

2. We derive the optimal quality and pricing regimes as well as the segmentation for each submarket, composed of the buyers and non-buyers of the base category with heterogeneous appreciation of quality.

3. We discuss the welfare implications of the ecosystem leverage and extend our analysis by relaxing the asymmetric ecosystem leverage with granting the category firm the option to connect the base category with additional investment.

3.1.1 Literature Review

Various research streams have discussed the release of different products. One of these research streams is the "new product introduction" literature from the management field that focuses on a monopoly setting and considers the release of new products over existing versions that are inherent substitutes (Papanastasiou and Savva, 2016). Typically, the pricing regime is the prominent competition tool in this literature. The studies also acknowledge indirect factors such as consumer's social learning (Bose et al., 2008; Jing, 2011; Yu et al., 2015; Papanastasiou and Savva, 2016) and word-of-mouth (WOM) (e.g. Lee et al., 2008; Cui et al., 2012; Ajorlou et al., 2016). In particular, the studies highlight the spillover effects of these externalities on the appreciation of consumers for new products. The main argument of these studies is that the

externalities occur due to a widespread consumer base of the early product, that is the more the consumers buy the early version, the more WOM or social learning. The trade-off is then between the inter-category substitution and spillover effect due to the widespread knowledge of the early product (Lee and O'Connor, 2003; Goldenberg et al., 2010; Liu and Schiraldi, 2012; Yu et al., 2015; Papanastasiou and Savva, 2016). While the substitution between the products drives the firm to canalize the consumers to the new product with a potentially higher profit margin, the spillover effect motivates the firm to increase the sales in the first period (Jing, 2011).

Another stream of research from economics that cover a similar setting (e.g. Katz and Shapiro, 1985, 1994; Srinivasan et al., 2004; Farrell and Klemperer, 2007) typically under competition is the "network effects" literature. Network effects refer to the increase in the consumer utility as the product becomes more widespread (Katz and Shapiro, 1985). Many studies have argued that the network effects may favor early products over superior versions and deter innovation (e.g. Katz and Shapiro, 1985, 1994; Economides, 1996; Hauser et al., 2006; Farrell and Klemperer, 2007; Molina-Castillo et al., 2011). On the other hand, there exist empirical studies (Liebowitz and Margolis, 1994; Tellis et al., 2009) claiming that the network effects enhance innovation instead of locking in the consumers to the early versions of the product. Our findings may reconcile these two conflicting arguments, that is if the two versions are weak substitutes the network effect may be innovation enforcing whereas if the versions are strong substitutes one may observe adverse effects on innovation.

While the settings (the new product introduction and the network effect models) described above explain many product release practices especially if the products are versions of the same product line (such as theatrical and non-theatrical versions of a movie), they do not fully explain the dynamics that exist in the product ecosystems. First, the connection between two product categories in a product ecosystem is not always in the form of substitution, but there exist many examples (such as headsets and smartphones or video games and consoles) that are complements in nature (Shocker et al., 2004). Secondly, the firm indirectly controls the consumer appreciation for the new product with the sales of the early version. In the product ecosystems, however, the emphasis is on the interconnection between the products and the

qualities, which inform the utilization of the appreciation for the early product (Jacobides and MacDuffie, 2013; Druce, 2014; Zhu and Furr, 2016; Babiolakis, 2016). Finally, the spillover effect emphasized by this literature is on the non-buyers of the early version while the product ecosystems focus on selling to their previous buyers second time using their previous appreciation (Jacobides and MacDuffie, 2013; Babiolakis, 2016). The literature on "indirect network effects" complements the new product introduction studies with the emphasis on the inherent complementarity between the product categories. Formally, the literature defines the "indirect network effect" as the increase in consumer appreciation for one group of the product category (typically hardware) as the complementary product categories (usually software) become widespread (Stremersch et al., 2007). The literature has documented indirect network effects in various industries (see Basu et al. (2003) for a review). One crucial aspect considered in this literature is the "chicken-egg problem", the reciprocal spillover effect between the primary and the secondary markets (Katz and Shapiro, 1994; Varian, 2001a). Empirical studies report conflicting evidence on whether or not these secondary effects exist. Some studies point out the second-order effects from secondary to the primary market and that the hardware (primary category) producers should invest in software to generate more sales in their primary market (such as Nair et al. (2004) for personal digital assistants). Nonetheless, a growing body of literature argues that the indirect effect is temporal and the direction of the spillover is from the primary to the secondary market (Stremersch et al., 2007; Tellis, 2008; Binken and Stremersch, 2009). We consider that the competition between the ecosystem and category firm is one-sided since the demand for the base categories is typically saturated.

In view of the analytical studies, the common setting is a duopoly offering products of a category compatible with different versions of another product category offered by complementors (Carrillo and Tan, 2008). This setting leads to analysis of many practically relevant research questions such as cooperation and integration with complementors (Farrell and Klemperer, 2007; Carrillo and Tan, 2008), compatibility and standardization (Katz and Shapiro, 1985; Markovich and Moenius, 2009) as well as of policy issues such as product tying (Choi, 2010), predatory pricing (Clements and Ohashi, 2005), tipping (Dubé et al., 2010), inertia (Markovich and Moenius, 2009), and net-neutrality (Economides and Hermalin, 2012; Bourreau et al.,

2015). These studies provide insights to many two-sided platform settings such as the video games and video consoles markets, where platforms (video consoles) compete on a product category while a second complementary category consists of products that are compatible with either one of the platforms (e.g. Clements and Ohashi, 2005; Derdenger, 2014; Kim et al., 2014; Kemerer et al., 2017). However, this research stream does not address the product ecosystem setting. Most importantly, in the product ecosystems, the connection between the two categories are endogenous to the ecosystem firm while the models mentioned above focus on exogenous inherent complementarity. Therefore, the focus of policy discussions is on the direct means such as product tying, that is when firms offer a good on the condition that the consumers buy the related good from the same firm. Yet, the industry experts highlight the strategical "indirect quality-tying" in the product ecosystems as a factor that may hurt category firms and deter innovation. Secondly, the inherent connection between the two categories in a product ecosystem setting can be in the form of substitution in addition to complementarity. As we mentioned before, among the factors that inform the competition between the firms of the product ecosystems is the type of intrinsic link between the new and the base category, whether they are substitutes or complements (Shocker et al., 2004; Jacobides and MacDuffie, 2013). Finally, the majority of the studies of this stream tend to focus on pricing regime instead of the quality competition while we cover both tools.

In particular, "price bundling" literature specializes in the pricing regimes of the firms with complementary goods. Specifically, the discussion revolves around in which combinations the related goods should be offered to the consumers: in a package with same prices (pure bundling), separately with different prices (pure components) or both (mixed bundling) (Prasad et al., 2010; Vamosiu, 2018). The literature provides a detailed characterization of the optimal price bundling strategies. A high-level summary would be as follows. Pure bundling is optimal when the demand for the related goods are negatively correlated, with the marginal costs of production are low (Bakos and Brynjolfsson, 1999; Vamosiu, 2018) while mixed bundling strategies should be preferred under a monopoly with complements (Anderson and Leruth, 1993), or under imperfect competition with independent goods (Carbajo et al., 1990). The literatures states that pure components pricing is optimal under a duopoly with complements

(Anderson and Leruth, 1993) (see Venkatesh and Kamakura (2003); Vamosiu (2018) for a literature review).

A growing body of literature examines the price bundling as a strategical tool when introducing new products over old product categories (e.g. Choi, 2003; Chen and Xie, 2007; Sheng and Pan, 2009; Prasad et al., 2010; Vamosiu, 2018). Choi (2003) considers bundling of an experience good with an established previous product quality in a monopoly setting. The main conclusion of this study is that the information leverage enhances welfare, especially when bundling motivates the firm to create the new market that it had deployed a pure components strategy. Sheng and Pan (2009) extends this setting to a two-firm case where each firm is serving a single market. In their work, one of the firms has a high reputation while the other is in a weak position. Investigating various cases from high-technology industries, they verify that bundling is beneficial unless consumers perceive that bundling informs a price discount on the weak product. Chen and Xie (2007), Vamosiu (2018), Prasad et al. (2010) study the interface between network effects and bundling. They confirm that mixed bundling is more profitable when the network effect is one-sided. This setting is closest to our setting among the research streams reviewed so far. The similarity arises from the effect of the size of the previous market (network effect) on the consumer appreciation for the new category and that the goods are related (complements, substitutes). While these studies help better understanding short-term pricing strategies of monopolies that offer new categories over previously released goods, their setting does not explain the business models of product ecosystems, nor they address the competition between an ecosystem and a category firm. We complement to this discussion with the "quality bundling" of the ecosystem firm while the pricing regime is pure components.

Price bundling has also been the topic of many policy discussions, especially in the context of tying. In particular, the studies show that price bundling can foreclose the competition in the new category, and deter the entry of category firms (e.g. Whinston, 1989; Carlton and Waldman, 1998; Bakos and Brynjolfsson, 2000; Choi and Stefanadis, 2001; Ghosh and Balachander, 2007; Choi, 2010; Edelman, 2015). We contribute to the policy discussions with the "ecosystem leverage" framework with the "indirect tying" of two product categories.

Up to this point, we covered the studies that focus solely on pricing regime. Another related stream of research, the "endogenous quality competition" literature, extends the discussions with the quality competition. However, the typical assumption in this stream is a single product category (e.g. Motta, 1993; Lehmann-Grube, 1997; Valletti, 2000). The concept of interdependency between the two markets does not exist in this literature. Therefore, we consider the settings described in this literature as a benchmark case to investigate the role of ecosystem leverage on firms' behavior and the category innovation.

To summarize, we integrate the relevant aspects mentioned in these various research streams that collectively explain the "ecosystem leverage" phenomenon existing in many industries with the increasing number of ecosystem firms such as Apple, Samsung, Amazon, Google, and Frog with the interconnected family of products, that is the product ecosystems (Jacobides and MacDuffie, 2013; Zhu and Furr, 2016).

The rest of the paper is as follows. Section 2 describes the framework, and Section 3 explains the various market segmentation cases under fixed prices and quality levels. Section 4 analyzes price competition and Section 5 examines quality decisions. Section 6 discusses some extensions, and Section 7 concludes.

3.2 Model Description

We consider the release of a "new good" over a previous product category, "base good". We assume that a monopolist, firm 1, has released the base good and a fraction ($\lambda \in [0,1]$) of the market has bought it. Market segments as users and non-users of the base category, and hence the fraction ($\lambda \in [0,1]$), is exogenous in our model. This setting may fit well into industries that release new categories (e.g., smart wearables) to the markets with a saturated consumer base of a previous product class (such as smartphones) (Thomas, 2018). Besides, empirical literature (e.g., Stremersch et al. (2007)) confirms that, for many high tech industries, previously established goods affect the sales of the new product category but not the other way around. Thus, the planning horizon of this model starts when the ecosystem firm provides the

new category along with a competitor, firm 2.

In our model, we take into account whether the new good is related to the base good or not. We denote the inherent relation between the new good and the base category with ($c \in (0, 2)$), and identify the goods as unrelated ($c = 1$), partial substitutes ($c < 1$), and complements ($c > 1$)[2]. Also, firm 1 has the option to alter the inherent category dependency by building seamless integration with the base category and set it to a level denoted by c_1.

The pair play a two-stage game: determining the *quality investments* (first stage) and the *prices* (second stage). In the first stage, the firms decide the stand-alone qualities of the products (u_i for firm i). Our departure from the existing literature on quality competition (e.g., Motta, 1993) is that we allow the ecosystem firm to build complementarity between the new and base goods, which are a priori related (c). Connecting the new and the previous goods seamlessly via software is typical among product ecosystems such as Apple and Samsung. An exemplary standard base category for these firms is the smartphone, where Apple connects the smartwatch with the iPhone through the iOS/watch OS while Samsung Flow (the bundling software of Samsung) links categories such as tablets or computers with the Samsung Galaxy smartphone. Doing so, the ecosystem firm (firm 1) obtains leverage since the buyers of the base category gain an additional utility from the built-in complementarity. The competitor, on the other hand, faces a "base category competition" as it is challenging to convert the buyers of the base category who would otherwise be enjoying seamless integration from firm 1. Thus, the ecosystem firm adopts the consumer appreciation of a previous category as a competitive advantage and keeps its competitor at bay in the "intra-category" competition.

Firm i (for $i = 1, 2$) incurs a fixed cost ($u_i^2/2$) to build a standalone quality (u_i). Note that the new good is naturally dependent on the base category (c). By investing an amount ($k(c_1-c)^2/2$), firm 1 has the option to connect the new good with its base category and alter the dependency between the goods ($c_1 > 0$)[3]. We assume that the firms have symmetric costs for the stand-

[2]We note that the natural lower limit for c is 0, and by setting the upper limit to 2 we ensure that symmetric attention is given to the complementarity and substitution cases.

[3]Firm 1 would always build $c_1 \geq c$ since otherwise it would incur a cost by degrading the total value of the quality bundle.

alone quality. Therefore, we avoid the cases where the firms would have an incentive to deviate from their quality position (being quality leader or follower) and leapfrog their rival solely due to the asymmetric costs of quality[4]. Instead, we investigate whether or not the ecosystem leverage would create an incentive for the firms to leapfrog their competitors.

Further, we posit that the quality of the seamless connection (c_1) is not dependent on the stand-alone qualities since we are allowing firm 1 to build value for its previous customers, even if the stand-alone value of the new category is low, which is often the case in high technology products (Deloitte, 2016b). At one end, the ecosystem firm (firm 1) can design a new category that has a low stand-alone value (low u_1) and high dependency on the base category. The consumers that have the base category would be benefiting much higher from the new good in comparison to the non-buyers of the base good, similar to the product "tying" concept (O'Donoghue and Padilla, 2006). The difference, however, is that the goods are tied "indirectly", which is common practice in wearable tech gadgets (Bilton, 2016). At the other end, the ecosystem firm (firm 1) can choose not to alter the natural connection between the goods ($c_1 \to c$), and the high-quality new category (high u_1) as in the case of tablets. Thus, our model comprises a range of real-life cases. The cost parameter $k > 0$ is a parameter showing if the complementarity is easy to achieve (low k) or very costly to build (high k). Following the quality decisions in the first stage, the firms play a Bertrand pricing game and set the prices (p_i).

Next, we discuss consumer preferences. Consumers have varied tastes for the stand-alone quality (u_i), and we describe their marginal willingness to pay for the quality with a random variable y that is uniformly distributed with unit density ($y \sim [0, 1]$). The consumers who do not have the base category gain a surplus ($yu_i - p_i$) from the new good of firm i, as in the standard single category settings (e.g., Mussa and Rosen, 1978; Motta, 1993).

In addition, the consumers possessing the base category consider the relationship between the product categories and perceive the quality of the new good in reference to the base category

[4]Quality leapfrogging occurs when the quality leader (or follower) re-position itself as the follower (or leader) as a reaction to the opponent's quality decision (Motta, 1993). The reader is referred to Lehmann-Grube (1997) for this type of leapfrogging.

(ycu_i). As a special case, when the goods are not related ($c = 1$), the consumers perceive the quality u_i and set a reservation price yu_i, regardless of having the base category or not. If the goods are partial substitutes ($c < 1$), the consumers would perceive some features of the new good as a "repetition" of the base category features (Shocker et al., 2004). For example, smartphone owners mention features such as gaming, viewing photos, and watching videos of tablets as functions they primarily use on their phones (Deloitte, 2016b). For a given level of inherent connecttion (c), the higher the quality of the new category (high u_i), the more likely it incorporates features of the base category.[5] Our model also allows for the new and base goods to be a priori partial complements ($c > 1$), which may reflect the case of virtual reality (VR) headsets and smartphones. Combining both the inherent dependency of the goods and built-in complementarity, we conclude that a consumer (y) with the base good generates a net surplus ($yc_1u_1 - p_1$) from the new good of firm 1 and ($ycu_2 - p_2$) of firm 2.

We define a characterization function (I) to represent if a consumer has bought the base category ($I = 1$) or not ($I = 0$). Thus, *net additional surplus* $V(y; i)$ derived by a consumer type (y) when buying the new category could be summarized as follows, where $i = \{1, 2, 0\}$ denotes the set of actions (buying from firm 1, 2 or not buying):

$$V(y;i) = \left\{ \begin{array}{ll} yu_1(1 - I(1 - c_1)) - p_1 & \text{if } i = 1 \\ yu_2(1 - I(1 - c)) - p_2 & \text{if } i = 2 \\ 0 & \text{if } i = 0 \end{array} \right\}$$

Therefore, firm 1 can overturn a partial substitution ($c < 1$) or enforce a complementarity ($c > 1$) from the base category (cu_1) with its ecosystem leverage c_1. In particular, high-value types (high y) are sensitive to the dependency between the categories, and firm 1 can transform this consumer reaction to an advantage with the complementarity investment. Also, firm 1 can do this without changing its stand-alone quality (u_1), hence controlling for the base

[5] As an example, more advanced versions of tablets and smartwatches include more features that overlap with the smartphone such as GPS or phone calls compared to the earlier versions.

category competition cu_1. For instance, Apple Watch Series 1 presents limited features (low u_1) if not synchronized with an iPhone while the seamless connection (c_1) provides access to the smartphone features such as messaging, e-mails or navigation. The competitor, however, directly faces "base category competition" proportional to its quality investment (u_2) while engaging in intra-category competition with the ecosystem firm. The greater the base category market coverage (high λ), the more firm 2 is squeezed in between the "base category" and the "intra-category" competition.

We start our analysis with consumers' reaction to firms' price and quality strategies. Following this, we solve the two-stage game with subgame perfection.

3.3 Market Segmentation

In this section, we discuss the demand of each firm given their release strategy. At the start of the planning horizon, there exists two groups of consumers: the buyers of the base category (of size λ) and the non-buyers (of size $1-\lambda$). The firms do not discriminate between the groups and charge a single price. Otherwise, they may experience adverse effects on the company image. On the other hand, the two groups perceive qualities differently because of the endogenous (built-in complementarity) and exogenous (inherent relatedness) connection the new good has with the base category. Motta (1993) already discusses the case when a duopoly serves the first time buyers, corresponding to the group of consumers that did not buy the base category of our setting. We refer to this group of consumers as "Motta's market" from this point onwards since we can immediately rely on his analysis for this limiting case. Further, we name the second group that has bought the base category as "ecosystem market".

As a reaction to price and quality decisions of the two firms, both groups decide whether to buy from firm 1, firm 2 or none of them. Thus, potentially we may observe six consumer segments with or without the base and the new good. The consumer types (with high or low willingness to pay) of each segment depend on the position of the firms in between the intra-category competition with the opponent and the spillover effect (c_1u_1 and cu_2) of the base category over

the new good.

Whether the low or the high-value consumers (with low or high y) obtain the new good from a particular firm depends on the stand-alone quality leader (either $u_1 > u_2$ or $u_2 > u_1$). We define $\bar{Y}_M$ (for Motta's market) and $\bar{Y}_E$ (for ecosystem market) as the consumers that are indifferent between the two firms. Further, we define $\underline{Y}_M$ and $\underline{Y}_E$ as the consumers that derive the same value from the new good of the follower or not buying at all. We also assume that the indifferent types ($\bar{Y}_M, \bar{Y}_E$ and $\underline{Y}_M, \underline{Y}_E$) buy the higher quality option. Thus, the high value types of both groups ($y \geq \bar{Y}_M$ and $y \geq \bar{Y}_E$) buy from the quality leader while the low value types ($\underline{Y}_M \leq y < \bar{Y}_M$ and $\underline{Y}_E \leq y < \bar{Y}_E$) select the follower.

When the goods are inherently unrelated ($c = 1$) and the ecosystem firm does not use its leverage ($c_1 = c$), the threshold of each group becomes equivalent. Otherwise, we would expect different values depending on which firm is the quality leader, the price and quality levels (p_i, u_i and c_1) as well as exogenous dependency parameter (c). When characterizing the threshold consumers, and hence the market's segmentation, we separately cover the three cases of the quality leadership ($u_1 > u_2$, $cu_2 > c_1u_1$ and $u_2 > u_1 > c/c_1u_2$) below.[6]

3.3.1 Ecosystem Firm is the Quality Leader, $u_1 > u_2$

If the ecosystem firm is the stand-alone quality leader ($u_1 > u_2$), it is intuitive that the high-value types in each sub-market appreciate the ecosystem firm (firm 1) more, while the low-value types prefer the category firm (firm 2).

Starting with *Motta's market*, a consumer type ($y = \frac{p_i}{u_i}$) would be indifferent between buying from a certain firm i since her net surplus would be ($V(y;i) = 0$). When comparing the two firms, if the markup of the quality leader ($p_1 - p_2$) is equivalent to the quality premium ($y(u_1 - u_2)$) the consumer (y) perceives, she would be indifferent between the two firms, hence it follows that $\bar{Y}_M = [\frac{p_1 - p_2}{u_1 - u_2}]_0^1$. Consumers with valuations ($y \geq \max\left\{\bar{Y}_M, \frac{p_1}{u_1}\right\}$) buy from the

[6]The reader is referred to the Appendix for proofs.

quality leader instead of the follower. Within similar lines, a consumer (y) in Motta's market obtains the new good from firm 2 if and only if the price paid (p_2) is less than or equal to her reservation price (yu_2), which implies that the consumers y with valuation ($\bar{Y}_M > y \geq \frac{p_2}{u_2} =: \underline{Y}_M$) buy the new good from firm 2.

For the *ecosystem market*, the inherent dependency the new good has with the base category (cu_i) and the built-in complementarity (c_1u_1) inform the decision of the consumers. When contemplating whether to obtain a good from firm 1 or not obtaining at all, she evaluates the price paid (p_1) in return of the effective stand-alone utility generated c_1u_1, and chooses buying from firm 2 over not buying if ($y \geq \frac{p_1}{c_1u_1}$). If the additional price a consumer (y) pay ($p_1 - p_2$) to firm 1 is equivalent to the extra stand-alone quality firm 1 provides ($c_1u_1 - cu_2$), she would be indifferent between the two firms, hence we have $\bar{Y}_E = [\frac{p_1-p_2}{c_1u_1-cu_2}]_0^1$. As a result, ($y \geq \max\left\{\bar{Y}_E, \frac{p_1}{c_1u_1}\right\}$) represents the consumers who buy from the ecosystem firm (firm 1). The low value types ($y < \bar{Y}_E$), on the other hand, weigh the effective stand-alone quality cu_2 against the price (p_2) and buy if $\bar{Y}_E \geq y \geq \frac{p_2}{cu_2} =: \underline{Y}_E$.

Next, we compare the threshold types of the two sub-markets (Motta's and the ecosystem market) and derive insights on the position of the high and low-value types of the two sub-markets. In particular, we are interested in whether or not the firms still can induce the same kind of consumers that they can persuade to buy in Motta's market. Further, we also determine for which values of the built-in complementarity (c_1) the ecosystem firm convinces, even the low-value types, and exploits the competitor's consumer base.

Lemma 3.1 *For given u_1, u_2, c_1 such that $u_1 > u_2$, $c \in (0, 2)$, and $c_1 > c$,*

1. *if $c > 1$, we have ($\underline{Y}_E < \underline{Y}_M$) and ($\bar{Y}_E < \bar{Y}_M$) (aggressive built-in complementarity);*
2. *else if $c \leq 1$, we have ($\underline{Y}_E \geq \underline{Y}_M$) and*
   - *($\bar{Y}_E \geq \bar{Y}_M$) if $(1 - c_1)u_1 \geq (1 - c)u_2$ (defensive built-in complementarity),*
   - *($\bar{Y}_E < \bar{Y}_M$) if $(1 - c_1)u_1 < (1 - c)u_2$ (aggressive built-in complementarity).*

If the new good is not intrinsically related to the base category ($c = 1$) and if the ecosystem firm builds no complementarity ($c = c_1$), then each firm can convince the same value types of both sub-markets. When the goods are natural complements of the base category ($c > 1$), then the position of the ecosystem firm improves in the ecosystem market, regardless of the built-in complementarity (c_1), as it can induce relatively lower value consumer types to buy. As a result, the category firm focuses instead on lower value consumer types.

If instead, the new good is a partial substitute of the base category, then the market power of each firm depends on the value of the built-in complementarity (c_1). Intuitively, one would expect the ecosystem firm to build complementarity to mitigate the base category substitution effect while the particular amount would depend on the cost parameter (k), which we discuss in Section 5. If the built-in complementarity is large $(1-c_1)u_1 < (1-c)u_2$, then the ecosystem firm can expand into relatively low value types when compared to Motta's market, which we call *aggressive built-in complementarity*. An aggressive complementarity drives the competitor to focus on the lower value types. Convincing low-value types is not easy as the substitution effect (cu_2) leads some of these consumers not to buy at all. Alternatively, when the ecosystem firm executes a *defensive built-in complementarity* $(1-c_1)u_1 \geq (1-c)u_2$, then its market power becomes weaker in the ecosystem market than Motta's market, which helps the competitor to settle down in the ecosystem market with a focus on relatively higher value types.

Demand ($D(i,j)$)[7] firm (i) obtains from the sub-market (j) is as follows:

$$D(i,j) = \left\{ \begin{array}{ll} (1-\lambda)\big(1 - \max\left\{ [\frac{p_1-p_2}{u_1-u_2}]_0^1, \frac{p_1}{u_1} \right\}\big) & \text{if } i=1, j=M \\ (1-\lambda)\big([\frac{p_1-p_2}{u_1-u_2}]_0^1 - \frac{p_2}{u_2}\big)^+ & \text{if } i=2, j=M \\ \lambda\big(1 - \max\left\{ [\frac{p_1-p_2}{c_1u_1-cu_2)}]_0^1, [\frac{p_1}{c_1u_1}]_0^1 \right\}\big)^+ & \text{if } i=1, j=E \\ \lambda\big([\frac{p_1-p_2}{c_1u_1-cu_2}]_0^1 - [\frac{p_2}{cu_2)}]_0^1\big)^+ & \text{if } i=2, j=E \end{array} \right\}$$

.

[7] We note that $[x]_0^1 := \min\{\max\{x, 0\}, 1\}$.

We make the standard assumption of the versioning literature (e.g., Mussa and Rossen, 1978): $p_i < u_i$. This relation ensures that there exists at least one firm having non-zero demand in Motta's market. For the ecosystem market, one needs to have either $(p_1 \leq c_1 u_1)$ or $(p_2 \leq c u_2)$ to ensure that there exists at least one firm. We note that when the inherent dependency is in the form of complementarity $(c > 1)$ or the goods are independent $(c = 1)$, then both of the conditions are readily satisfied. Otherwise, whether the two conditions are satisfied or not depends on the release strategy and the dependency parameters of c and c_1. To keep our discussion focused, we assume that $(p_1 \leq c_1 u_1)$ or $(p_2 \leq c u_2)$ in this section. Later, when we discuss the pricing policy, we revisit these assumptions and show whether and when these assumptions are subject to violation.

In the rest of the section, we focus on the potential cases of market segmentation depending on the release strategies (price and quality levels) and finally generate the profit function. Before this, we eliminate instances that would never arise:

Lemma 3.2 *For fixed quality levels u_1, u_2, c_1 such that $u_1 > u_2$ and $c_1 > c$,*

1. *pricing regime $(p_2 u_1 \geq p_1 u_2)$ is not a desirable pricing regime for the category firm, hence it leapfrogs $(p_2 u_1 < p_1 u_2)$;*

2. *if $c_1 > 1$, the pricing regime $(c_1 u_1 - c u_2 < p_1 - p_2)$ is not a desirable pricing regime for the ecosystem firm, hence it leapfrogs $(c_1 u_1 - c u_2 > p_1 - p_2)$;*

3. *if $c_1 < 1$ and $(1 - c_1) u_1 > (1 - c) u_2$ (defensive built-in complementarity), the pricing regime $(u_1 - u_2 < p_1 - p_2)$ is not a desirable pricing regime for the ecosystem firm, hence it leapfrogs $(u_1 - u_2 > p_1 - p_2)$.*

In particular, if $(p_2 u_1 \geq p_1 u_2)$, then the category firm (firm 2) does not exist in any of the sub-markets since it implies that $\bar{Y}_M \leq \underline{Y}_M$ and $\bar{Y}_E \leq \underline{Y}_E$. We remind the reader that before the pricing game, firms have already made their quality investments and the costs are sunk. Therefore, the category firm lowers its price $(p_2 u_1 < p_1 u_2)$ to exist at least in Motta's market. We note $(p_2 u_1 > p_1 u_2)$ as the primary pricing regime for the category firm. Along similar lines,

we observe that targeting the ecosystem market is a dominant strategy when the new good is a natural complement of the base category ($c_1 > 1$). If it follows that ($c_1u_1 - cu_2 > p_1 - p_2$), then we have $\bar{Y}_M, \bar{Y}_E > 1$. Therefore, $D_{1j} = 0$ for both $j = M$ and $j = E$. Firm 1 deviates from this situation by lowering its price ($c_1u_1 - cu_2 > p_1 - p_2$), hence serving the ecosystem market. On the contrary, if the new good is a natural substitute of the base good, ($u_1 - u_2 > p_1 - p_2$) becomes the dominant strategy with the condition that the ecosystem firm deploys a defensive built-in complementarity $(1 - c_1)u_1 > (1 - c)u_2$. It follows that firm 1 prefers serving Motta's market all the time. To conclude, if the new good is inherently a complement of the base good, the ecosystem firm always exists in the ecosystem market and the category firm always targets Motta's market. If instead, the goods are substitutes in nature, then Motta's market would always be the target for both firms unless the ecosystem firm is competing with an aggressive built-in complementarity. If instead the ecosystem firm is building aggressive complementarity, then one may observe only the category firm in Motta's market.

Next, we introduce potential market segmentation instances:

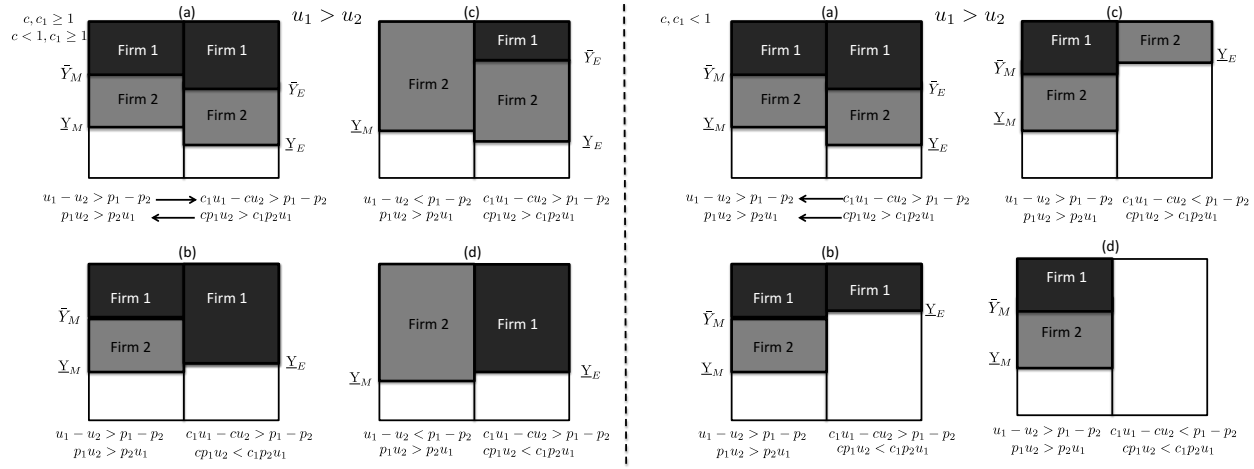
Lemma 3.3 *For fixed quality levels u_1, u_2, c_1 such that $u_1 > u_2$, $c \in (0, 2)$, and $c_1 > c$,*

1. *for all values of c_1, the category firm always exists in Motta's market, while serving the ecosystem market if and only if $p_1cu_2 > p_2c_1u_1$;*

2. *for $c_1 > 1$, the ecosystem firm always exists in the ecosystem market, while serving Motta's market if and only if $(p_1 - p_2 < u_1 - u_2)$;*

3. *for $c_1 < 1$, the ecosystem firm always exists in Motta's market if $(1-c_1)u_1 > (1-c)u_2$ or $(p_1-p_2 < u_1-u2)$ while serving the ecosystem market if and only if $(p_1-p_2 < c_1u_1-cu2)$.*

Thus, we have four potential cases of market segmentation for the inherent complementarity ($c > 1$) and the substitution ($c < 1$) with defensive built-in complementarity $(1 - c_1)u_1 > (1 - c)u_2$ scenarios depending on whether or not firm 1 exists in Motta's market and firm 2 in the ecosystem market(see Figure 3.1 for a summary of the discussion so far). If instead, the ecosystem firm is engaging in aggressive built-in complementarity to connect the two goods



(a) $(c_1 \geq 1)$ (b) $(c_1 < 1)$ and $(1 - c_1)u_1 > (1 - c)u_2$

Figure 3.1: Market Segmentation, $u_1 > u_2$

that are inherently substitutes, then one can also observe cases where the ecosystem firm is focusing only on the ecosystem market.

Thus, the total profit function Π_i of firm i can be written as:

$$\Pi_1 = p_1\left((1-\lambda)\left(1-[\frac{p_1-p_2}{u_1-u_2}]_0^1\right)+\lambda\left(1-\max\left\{[\frac{p_1-p_2}{c_1u_1-cu_2}]^1,\frac{p_1}{c_1u_1}\right\}\right)\right)-u_1^2/2-k(c_1-c)^2/2, \tag{3.1}$$

$$\Pi_2 = p_2\left((1-\lambda)\left([\frac{p_1-p_2}{u_1-u_2}]_0^1-\frac{p_2}{u_2}\right)+\lambda\left([\frac{p_1-p_2}{c_1u_1-cu_2)}]_0^1-\frac{p_2}{cu_2}\right)^+\right)-u_2^2/2. \tag{3.2}$$

3.3.2 Category Firm is the Quality Leader in Both Markets, ($u_2 > u_1$ and $cu_2 > c_1u_1$)

This section covers the case where the high-value types of both sub-markets are inclined to the category firm, the quality leader. In particular, Motta's consumers with willingness to pay ($y \geq \max\left\{\bar{Y}_M, \frac{p_2}{u_2}\right\}$) would buy from firm 2 while the low value consumers ($\bar{Y}_M \geq y \geq \frac{p_1}{u_1} =: \underline{Y}_M$) would obtain the new good from the ecosystem firm. In the ecosystem market, on the other hand, firm 1 has the ecosystem leverage (c_1) with which it may convince some of the high-value types to obtain the new good from itself instead of the category firm. In particular, consumers with valuation ($\bar{Y}_E \geq y \geq \frac{p_1}{cu_1} =: \underline{Y}_E$) would buy from the ecosystem firm while the high value types $y \geq \bar{Y}_E$ would buy from the category firm.

Lemma 3.4 *For given u_1, u_2, c_1 such that $u_2 > u_1$, $cu_2 > c_1u_1$, $c \in (0, 2)$, and $c_1 > c$,*

1. *if $(1 - c_1)u_1 \geq (1 - c)u_2$, we have ($\bar{Y}_E \leq \bar{Y}_M$) (defensive built-in complementarity);*
2. *if $(1 - c_1)u_1 < (1 - c)u_2$, we have ($\bar{Y}_E > \bar{Y}_M$) (aggressive built-in complementarity);*
3. *if $c_1 > 1$, we have ($\underline{Y}_E \leq \underline{Y}_M$) (expanding into the low value consumers), and*
4. *if $c_1 \leq 1$, we have ($\underline{Y}_E > \underline{Y}_M$) (shrinking to the moderate value consumers).*

If the ecosystem firm is the quality follower ($u_1 < u_2$), it can expand into the upstream high-value consumers of the ecosystem market if and only if its built-in complementarity is sufficiently high ($(1 - c_1)u_1 < (1 - c)u_2$). Otherwise, its position in terms of reaching into the high-value consumers would be worse than Motta's market, where nobody connects the new good with the base good. Looking at the lower end of the market, built-in complementarity ($c_1 > 1$) enables the ecosystem firm to reach into the low-value types that are potentially the non-buyers in Motta's market. In the existence of an inherent complementarity ($c > 1$), the ecosystem firm can always expand into the low-value consumers even without any extra effort (setting $c_1 = c$) while aggressive complementarity can be established only with additional cost. If, instead, the goods are natural substitutes, reaching into both the low and the high-value consumers is

costly while *aggressive built-in complementarity* is comparably easier to be deployed than the *expanding into the low-value consumers* policy. Intuitively we would expect the ecosystem firm to be more prone to build complementarity in the inherent substitution case ($c < 1$) than the case where the categories are intrinsically complementary ($c > 1$).

Comparing the cases of stand-alone quality leadership (Lemma 3.1 versus 3.4), the firm with the higher stand-alone quality is more advantageous to expand into the market if the new good is a natural complement of the base good due to the synergy between the natural complementarity and the stand-alone quality (cu_i). Therefore, expanding into the market with *aggressive built-in complementarity* comes with extra efforts when the ecosystem firm is the quality follower. Thus, one would expect the ecosystem leverage to be less influential on the "category firm" (firm 1) and its welfare. When the goods are natural substitutes, the relation flips: the more the stand-alone quality, the more the overlap between the two good categories. Deploying an *aggressive built-in complementarity* is possible even with low levels of endogenous integration (c_1) at the expense of low complementarity investment. One may expect the ecosystem leverage to keep the category firm at bay when the new good is a natural substitute of the base good.

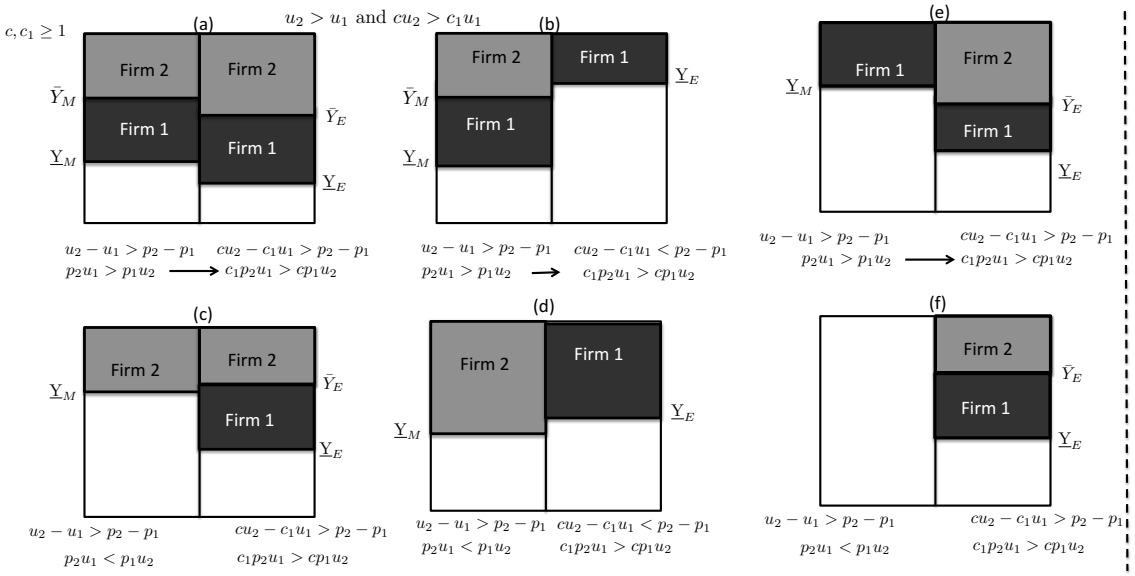


Figure 3.2: Market Segmentation under the Intrinsic Complementarity ($c > 1$) and for $cu_2 > c_1u_1$

Lemma 3.5 *For fixed quality levels u_1, u_2, c_1 such that $u_2 > u_1$, $c_1u_1 < cu_2$, and $c_1 > c$,*

1. *pricing regime ($c_1p_2u_1 < cp_1u_2$) is not a desirable pricing regime for the ecosystem firm, and it leapfrogs ($c_1p_2u_1 \geq cp_1u_2$);*

2. *if $c < 1$, the pricing regime ($u_2 - u_1 \leq p_2 - p_1$) is not a desirable pricing regime for the category firm, hence it leapfrogs ($u_2 - u_1 > p_1 - p_2$).*

From a higher point of view, the ecosystem firm would always aim to exist in the ecosystem market ($c_1p_2u_1 \geq cp_1u_2$), since otherwise, it would not be able to survive in any of the sub-markets. The category firm, on the other hand, would target at least Motta's market ($u_2 - u_1 > p_1 - p_2$), where the new good is an inherent substitute of the base category. If the category firm does not implement the primary pricing regime ($u_2 - u_1 > p_1 - p_2$), then it would vanish from the whole market since the inability to exist in Motta's market implies not being in the ecosystem market where each consumer carries an inherent substitute of the new good. For the complementarity case, however, the category firm may target either of the markets.

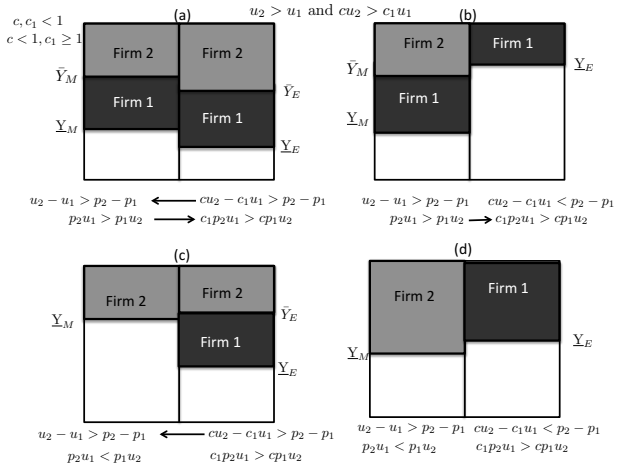


Figure 3.3: Market Segmentation under the Intrinsic Substitution ($c < 1$), and for $cu_2 > c_1u_1$

Thus, potential market segmentation instances follow as:

Lemma 3.6 *For fixed quality levels u_1, u_2, c_1 such that $u_2 > u_1$,$c_1u_1 < cu_2$, $c \in (0, 2)$, and $c_1 > c$,*

1. *for all values of c, c_1, the ecosystem firm always exists in the ecosystem market while serving Motta's market if and only if $p_1u_2 < p_2u_1$;*

2. *for $c < 1$, the category firm always exists in Motta's market while serving the ecosystem market if and only if $(p_2 - p_1 < cu_2 - c_1u_1)$;*

3. *for $c > 1$, the category firm exists in Motta's market if $p_2 - p_1 < u_2 - u_1$ while serving the ecosystem market if and only if $(p_2 - p_1 < cu_2 - c_1u_1)$.*

As a result, when the new good is a natural substitute of the old category ($c < 1$) there exist four possible segmentation of the market (Figure 3.3) depending on whether or not the ecosystem firm (firm 1) is in Motta's market and the category firm in ecosystem market. If instead, the new good is a complement of the base category then one can also observe cases where the category firm is focusing only on the ecosystem market (Figure 3.2). In other words, the natural complementarity between the goods has the potential to make the ecosystem market appealing for the category firm despite the ecosystem leverage its competitor possesses.

Thus, the total profit function Π_i of firm i can be written as:

$$\Pi_1 = p_1\left((1-\lambda)\left([\frac{p_2-p_1}{u_2-u_1}]_0^1-\frac{p_1}{u_1}\right)^+ +\lambda\left([\frac{p_2-p_1}{cu_2-c_1u_1)}]_0^1-\frac{p_1}{c_1u_1}\right)\right)-u_1^2/2-k(c_1-c)^2/2. \quad (3.3)$$

$$\Pi_2 = p_2\left((1-\lambda)\left(1-[\frac{p_2-p_1}{u_2-u_1}]_0^1\right)+\lambda\left(1-\max\left\{[\frac{p_2-p_1}{cu_2-c_1u_1)}]_0^1,\frac{p_2}{cu_2}\right\}\right)\right)-u_2^2/2. \quad (3.4)$$

3.3.3 Catching Up in the Ecosystem Market, ($u_2 > u_1$ and $c_1u_1 > cu_2$)

Finally, we cover the case where each firm is the quality leader in their natural markets ($u_2 > u_1$ and $c_1u_1 > cu_2$). We note that, by definition, this is the case where the ecosystem firm plays an aggressive built-in complementarity move ($\bar{Y}_E > \bar{Y}_M$) (see Lemma 3.4). Specifically, the quality leader of the ecosystem market is the ecosystem firm while the category firm is the quality leader in Motta's market. In our previous discussions, we implicitly omitted the cases ($p_2 > p_1$) for ($u_1 > u_2$), ($p_1 > p_2$) for ($u_2 > u_1$), and ($c_1u_1 > cu_2$) so that both firms exist in at least one submarket. In this case, however, it is not possible to eliminate these peculiar pricing regimes since firms still exist in at least one of the sub-markets.

Lemma 3.7 *For fixed quality levels u_1, u_2, c_1 such that $u_2 > u_1$, $c_1u_1 > cu_2$, and $c_1 > c$,*

1. *if $p_1 > p_2$, there exists only the category firm in Motta's market and the ecosystem firm targets the ecosystem market with the pricing regime ($c_1u_1 - cu_2 > p_1 - p_2$), while the category firm exists in the ecosystem market if and only if ($cp_1u_2 > c_1p_2u_1$);*

2. *if $p_2 > p_1$, there exists only the ecosystem firm in the ecosystem market and the category firm targets Motta's market with the pricing regime ($u_2-u_1 > p_2-p_1$), while the ecosystem firm exists in Motta's market if and only if ($p_1u_2 > p_2u_1$).*

As an implication of Lemma 3.7, we have the ecosystem firm in the ecosystem market and the category firm in Motta's market regardless of the prices. Both the prices and the qualities inform

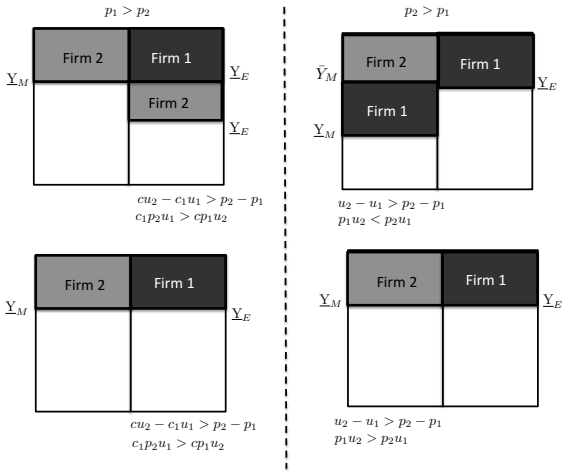


Figure 3.4: Catching Up the Competitor

the existence of each firm in the market where they have less leverage regarding quality. To be more specific, if the relative price of the category firm is sufficiently low such in comparison to the inferior quality it offers, then it also exists in the ecosystem market, serving the low-value consumers. Along the similar lines, the ecosystem firm can attract the low-value consumers by setting a low price to compensate for the inferior stand-alone quality. Thus, the total profit function Π_i of firm i can be written as:

$$\Pi_1 = p_1\left((1-\lambda)\left([\frac{p_2-p_1}{u_2-u_1}]_0^1-\frac{p_1}{u_1}\right)^+ +\lambda\left(1-\max\left\{[\frac{p_1-p_2}{c_1u_1-cu_2}]_0^1,\frac{p_1}{c_1u_1}\right\}\right)\right)-u_1^2/2-k(c_1-c)^2/2. \tag{3.5}$$

$$\Pi_2 = p_2\left((1-\lambda)\left(1-\max\left\{[\frac{p_2-p_1}{u_2-u_1}]_0^1,\frac{p_2}{u_2}\right\}\right)+\lambda\left([\frac{p_1-p_2}{c_1u_1-cu_2)}]_0^1-\frac{p_2}{cu_2}\right)^+\right)-u_2^2/2. \tag{3.6}$$

3.4 Price Competition

In this section, we focus on the second stage of the game where the firms set their prices given the quality investments (u_1, u_2, c_1) made in the first stage. We start with the case where the quality leader is the ecosystem firm $(u_1 > u_2)$:

Proposition 3.8 *For fixed quality levels u_1, u_2, c, c_1 such that $u_1 > u_2 > 0$ and $c_1 > c \in (0, 2)$, there exist $m = \frac{\frac{6u_1}{4u_1-u_2} - 1 - \frac{(1-c)u_2}{u_1}}{c}$ and $c' = \frac{2u_1-u_2}{4u_1-u_2}$ such that*

1. *if $c > c'$ or $c_1/c > m$, both firms target both sub-markets*
   (with relative prices $p_1^*/p_2^* = \frac{cc_1(1-\lambda)u_1+c_1\lambda(u_1-u_2)-2u_1\left(c^2(1-\lambda)u_2\right)}{c_1u_1(1-\lambda)+\lambda(u_1-u_2))-cu_2(c(1-\lambda)u_2}$) (region (a) in Figure 3.5);
2. *if $c_1/c < m$ and $c < c'$, there exist $\lambda_1, \lambda_2 \in (0, 1)$ such that*
   (a) *if $\lambda > \lambda_1$, both firms target both sub-markets*
   (with relative prices $p_1^*/p_2^* = \frac{cc_1(1-\lambda)u_1+c_1\lambda(u_1-u_2)-2u_1\left(c^2(1-\lambda)u_2\right)}{c_1u_1(1-\lambda)+\lambda(u_1-u_2))-cu_2(c(1-\lambda)u_2}$) (region (a) in Figure 3.5),
   (b) *if $\lambda_2 < \lambda < \lambda_1$, the quality follower targets both sub-markets while the quality leader focuses solely on Motta's market*
   (with relative prices $p_1^*/p_2^* = \frac{2c(1+\lambda)u_1+c\lambda u_2+2\lambda(u_1-u_2)}{c(1+\lambda)u_2}$) (region (c) in Figure 3.5), and
   (c) *if $\lambda < \min\{\lambda_1, \lambda_2\}$, both firms target only Motta's market*
   (with relative prices $p_1^*/p_2^* = \frac{2u_1}{u_2}$)(region (d) in Figure 3.5).

(The reader is referred to the proof for expressions.)

An immediate implication of Proposition 3.8 is that the two firms always exist in Motta's market[8]. It is intuitive to see both firms in Motta's market when its size is substantially high (λ is low). Having both firms in a small Motta's market (λ is high) is not trivial, and interestingly follows from the firms' focus on the ecosystem market if the two categories are substitutes. We mentioned earlier that when the two firms exist in the ecosystem market with goods that are substitute to the base category, they readily convince Motta's consumers since the prices designed to attract the ecosystem consumers are reasonably low as perceived by the neutral consumers.

[8]We remind the reader that we assume $p_i < u_i$ following the standard assumption of the literature. It is important to note that this result follows as long as the assumption $p_i < u_i$ holds. Otherwise, Motta's market may disappear when the two categories are extremely strong complements (c is extremely high), and the base category is sufficiently widespread (λ is large).

A second insight is that both firms, whether the stand-alone quality leader or the follower, exist in the ecosystem sub-market (and hence in both sub-markets) unless the new good is a strong substitute of the base category. The firms are attracted to the ecosystem market with different motivations: the ecosystem firm has the leverage to overcome the substitution effect while the category firm (the quality follower) is perceived less of a substitute and has the potential to attract the low-value consumers. When intrinsic substitution is high ($c < c'$), ironically, the ecosystem firm is the first firm to give up on the ecosystem market provided that its size is sufficiently small (or equivalently the built-in complementarity is not extremely large) ($\lambda < \lambda_1$). The underlying reason is that the ecosystem firm is at the same time the quality leader and possesses more features that overlap with the base category, and is exposed to substitution more than the competitor. For instance, it may be easier for a low-quality tablet producer to convince a graphical tablet owner to buy a similar product the second time than a professional tablet manufacturer. Finally, when the inherent substitution is large ($c < c'$) and the ecosystem market is reasonably small ($\lambda < \lambda_2$) both of the firms solely focus on Motta's market, and ignore the ecosystem market. Figure 3.5 illustrates the price equilibria regions of Proposition 3.8 for two instances: the built-in complementarity is weak ($c_1 = 1.2c$) and strong ($c_1 = 2c$). We note that the quality levels used in Figure 3.5 are the quality levels associated with the benchmark model (Motta, 1993) where there exist no ecosystem consumers.

Next, we discuss the pricing policies the firms adopt when the quality leader is the category firm in both sub-markets ($u_2 > u_1$ and $cu_2 > c_1u_1$):

Proposition 3.9 *For fixed quality levels u_1, u_2, c, c_1 such that $u_2 > c_1/cu_1 > 0$ and $c_1 > c \in (0, 2)$,*

1. *if $c_1/c < 2$, there exists $\lambda_1 \in (0, 1)$ such that*

   (a) *if $\lambda > \lambda_1$, both firms target both sub-markets*
   (with relative prices $p_1^*/p_2^* = \frac{c_1u_1(-\lambda((c-1)u_2+u_1)+cu_2+c_1(\lambda-1)u_1)}{2u_2\left(-cc_1(\lambda-1)u_2+c\lambda(u_2-u_1)+c_1^2(\lambda-1)u_1\right)}$) (region (a) in Figure 3.6),

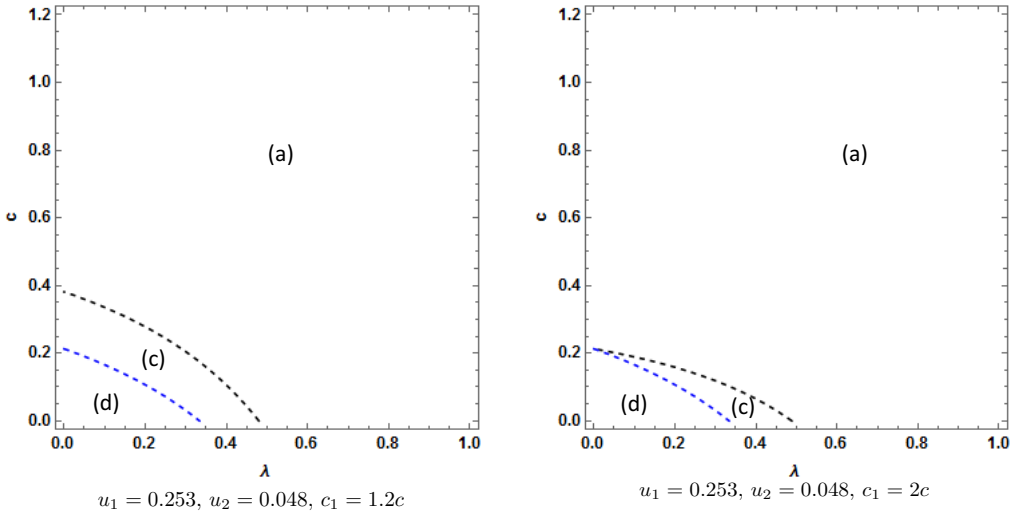


Figure 3.5: Optimal Pricing Regime when $u_1 > u_2$

(b) if $\lambda < \lambda_1$, the quality follower targets both sub-markets while the quality leader focuses solely on Motta's market
(with relative prices $p_1^*/p_2^* = \frac{c_1(\lambda+1)u_1}{c_1\lambda u_1 - 2c_1(\lambda-1)u_2 + 2\lambda(u_2-u_1)}$)(region (c) in Figure 3.6);

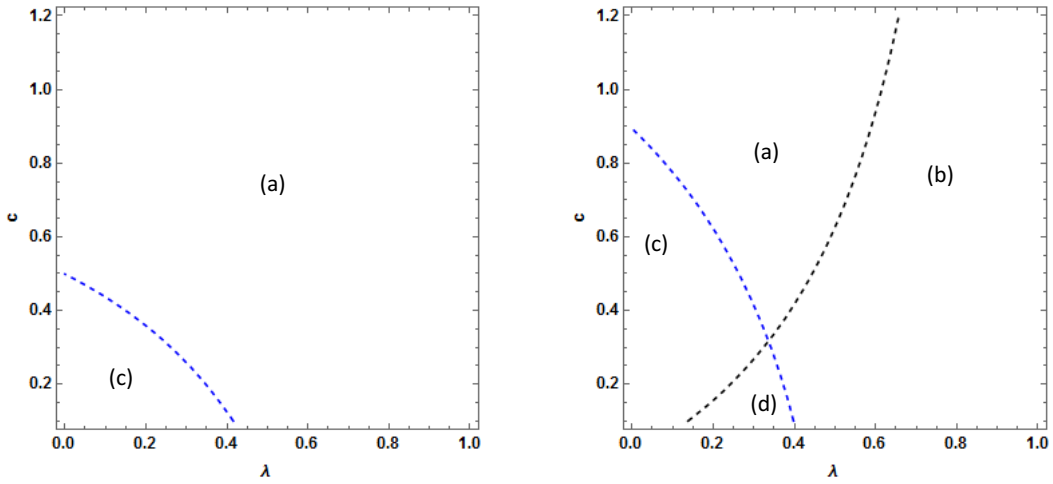
2. *if $c_1/c > 2$, there exist $\lambda_1, \lambda_2 \in (0,1)$ such that*

(a) if $\lambda_2 > \lambda > \lambda_1$, both firms target both sub-markets
(with relative prices $p_1^*/p_2^* = \frac{c_1 u_1(-\lambda((c-1)u_2+u_1)+cu_2+c_1(\lambda-1)u_1)}{2u_2\left(-cc_1(\lambda-1)u_2+c\lambda(u_2-u_1)+c_1^2(\lambda-1)u_1\right)}$) (region (a) in Figure 3.6),

(b) if $\lambda_2 < \lambda < \lambda_1$, the quality follower targets both sub-markets while the quality leader focuses solely on Motta's market
(with relative prices $p_1^*/p_2^* = \frac{c_1(\lambda+1)u_1}{c_1\lambda u_1 - 2c_1(\lambda-1)u_2 + 2\lambda(u_2-u_1)}$)(region (c) in Figure 3.6),

(c) if $\lambda_1 < \lambda < \lambda_2$, the quality leader targets both sub-markets while the quality follower focuses solely on th ecosystem market (with relative prices $p_1^*/p_2^* = \frac{c_1 u_1}{2cu_2}$)(region (b) in Figure 3.6),

(d) if $\lambda < \min\{\lambda_1, \lambda_2\}$, the quality leader targets only Motta's market while the quality follower focuses solely on the ecosystem market



$u_1 = 0.253,\ u_2 = 0.048,\ c_1 = 1.2c$ $u_1 = 0.253,\ u_2 = 0.048,\ c_1 = 3c$

Figure 3.6: Optimal Pricing Regime when $u_2 > c_1/cu_1$

(with relative prices $p_1^*/p_2^* = \frac{c_1 u_1}{u_2}$)(region (d) in Figure 3.6).

(The reader is referred to the proof for expressions.)

Figure 3.6 illustrates the price equilibria regions of Proposition 3.9 for two instances: the built-in complementarity is weak ($c_1 = 1.2c$) and strong ($c_1 = 3c$). An important observation of Proposition 3.9 is that we have the quality leader (firm 2) in Motta's market and the follower (firm 1) always in the ecosystem market. In other words, category firm charges sufficiently low prices ($p_2^* < u_2 - u_1 + p_1^*$) to generate demand from Motta's market all the time. Charging a low price ($p_2^* < u_2 - u_1 + p_1^*$) is still the optimal strategy for the category firm even when the ecosystem market is more advantageous than Motta's market due to the connection with the base category. The rationale behind the insistence of the category firm to stay in Motta's market is the competition it would face in the alternative market (the ecosystem market) from the ecosystem firm which would not be able to survive in the whole market otherwise. Thus, we have the ecosystem firm in ecosystem market and the category firm in Motta's market as a result of price competition.

Whether or not the category firm also targets ecosystem market depends on the intrinsic depen-

dency between the base and new good (c) as well as the size of the ecosystem market (λ). If the product categories are natural substitutes and the ecosystem market is small, then the category firm solely focuses on Motta's market. Similar reasoning applies to the ecosystem firm: the ecosystem leverage (c_1 and λ) informs its decision whether or not to include Motta's market in its demand base. When the ecosystem leverage is sufficiently high ($c_1/c > 2$ and $\lambda > \lambda_1$), then the ecosystem firm focuses on the ecosystem market and solely serves its previous consumers. If the built-in complementarity is limited, the ecosystem firm serves both sub-markets in hope to expand its demand base with neutral consumers.

When comparing the two cases where either firm is the quality leader in both sub-markets ($u_1 > u_2$ and $cu_2 > c_1u_1$), we observe that the quality leader leaves the ecosystem market only if the ecosystem market is exogenously less advantageous than Motta's market (c and λ are low). Besides, if the ecosystem firm is the quality leader, it may leave the ecosystem market since the consumers perceive the ecosystem product as "a strong substitute" of the base good. Aforesaid is never the case if the ecosystem firm is the quality follower as its product is inferior and hence perceived less substitutable with the base category.

Another important distinction is that when the ecosystem firm is the quality follower, it is prone to focus on the ecosystem market while leaving the category firm the Motta's market. We remind the reader that this is not the case when the category firm is the quality follower. The main reason behind this distinction is that when the ecosystem firm is the quality follower, it has both the ecosystem leverage and being perceived as a mild substitute in the ecosystem market, the tools to keep its competitor at bay in the ecosystem market. We observe cases where the ecosystem firm, as the quality follower, is in both sub-markets while the quality leader has no chance to survive in the ecosystem market. The ecosystem firm having the low stand-alone quality but a strong connection to the base good poses more risk to the category firm than an ecosystem firm with higher independent quality.

Up to this point, we considered the cases where each firm is the quality leader in both sub-markets. Finally, we turn our attention to the case where either firm is the quality leader of a single sub-market: the category firm is the quality leader in Motta's market ($u_2 > u_1$), and the

ecosystem firm is the quality leader in the ecosystem market ($c_1u_1 > cu_2$).

Proposition 3.10 *For fixed quality levels u_1, u_2, c, c_1 such that $u_2 > u_1 > 0$, $c_1u_1 > cu_2$, and $c_1 > c \in (0, 2)$, there exists λ_3, λ_4 such that*

1. *if $\lambda > \max\{\lambda_3, \lambda_4\}$, then both firms target the ecosystem market while Motta's market is served solely by the category firm (with relative prices $p_1/p_2 = \frac{-2c^2(\lambda-1)u_2+c(\lambda-1)(2c_1u_1+u_2)-2c_1\lambda u_1}{c(\lambda-2)u_2}$) (region (b) in Figure 3.7);*

2. *if $\lambda < \min\{\lambda_3, \lambda_4\}$, then both firms target Motta's market while the ecosystem market is served solely by the ecosystem (with relative prices $p_1/p_2 = \frac{c_1(\lambda+1)u_1}{c_1\lambda u_1-2c_1(\lambda-1)u_2+2\lambda(u_2-u_1)}$) (region (c) in Figure 3.7), and*

3. *for otherwise, category firm serves only Motta's market while the ecosystem firm targets only the ecosystem market (with relative prices $p_1/p_2 = \frac{c_1u_1}{u_2}$) (region (d) in Figure 3.7).*

Proposition 3.10 intuitively suggests that if a particular sub-market dominates the market, then both firms (low and high quality) serve that sub-market while only the high-quality firm exists in the small sub-market. When the sub-markets are of comparable sizes, then each firm focuses solely on the sub-market where they have a quality advantage. The numerical example suggests that the more aggressive the ecosystem firm (lower panel in Figure 3.7), the more probable that the firms focus only in the sub-markets where they have a quality advantage. Otherwise (upper panel in Figure 3.7), only if the ecosystem market is large we may observe cases where each firm is focusing on a single market. Besides, for a given ecosystem market only if the new good is not a close substitute of the base category, we would observe both firms targeting the ecosystem firm. The "catching up" case is a bridge between the two extreme cases of quality leadership, and in section 5 we show that firms never prefer to be in this hybrid quality position.

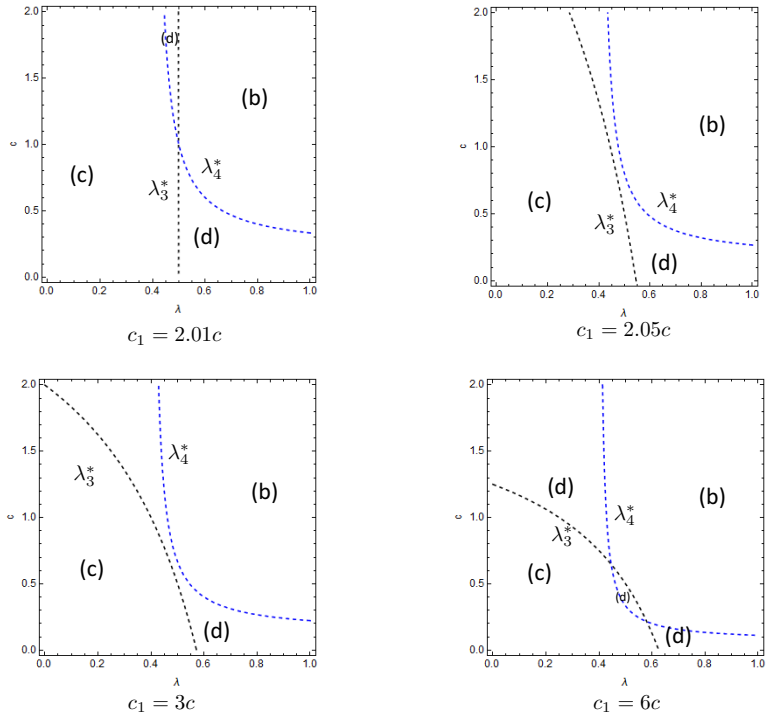


Figure 3.7: Optimal Pricing Regime when $u_2 > u_1$ and $c_1 u_1 > c u_2$ ($u_2 = 0.24$, $u_1 = 0.12$)

3.5 Quality Competition

In this section, we turn our attention back to the first period and analyze the quality investments of the firms. The ecosystem leverage (c_1 and λ) and the cost of quality building (or the return on the quality investments) shape how firms set the innovation levels in the new category.

3.5.1 Motta's Case, $\lambda = 0$

First, we cover the benchmark case where there exists no ecosystem leverage, that is no ecosystem market.

Lemma 3.11 *If the market consists solely of consumers not having the base category ($\lambda = 0$), the ecosystem firm does not build complementarity ($c_1 = c$) and both firms exist in the market: either with (1) prices* $p_1^* = \frac{u_2(u_1-u_2)}{4u_1-u_2}, p_2^* = \frac{2u_1(u_1-u_2)}{4u_1-u_2}$ *($p_1^*/p_2^* = \frac{2u_1}{u_2}$) and the quality levels* $u_1^* = 0.253$ *and* $u_2^* = 0.048$*, or with (2) the prices* $p_1^* = \frac{u_1(u_2-u_1)}{4u_2-u_1}, p_2^* = \frac{2u_2(u_2-u_1)}{4u_2-u_1}$ *($p_1^*/p_2^* = \frac{u_1}{2u_2}$) and the quality levels* $u_1^* = 0.048$ *and* $u_2^* = 0.253$ *(Motta, 1993).*

According to (Motta, 1993), there exist two Nash Equilibrium (NE) points where the firms may become either the quality leader, producing a premium good, or the follower with poor quality. In particular, without the ecosystem consumers, we observe heterogeneous levels of quality ($u_1^*/u_2^* = 5.27$ or $u_2^*/u_1^* = 5.27$) and price levels ($p_1^*/p_2^* = 10.54$ or $p_2^*/p_1^* = 10.54$) with which the quality leader focuses on the high value consumers (52.5 % of the market) while the quality follower serves the low value consumers (26.2% of the market). A priori access to the innovation may lead one firm to be the quality leader, but none of the factors (endogenous or exogenous) existing in the setting inform whether or not a particular firm is the quality leader. Another important insight is that being a quality leader or follower is sticky, that is, none of the firms has an incentive to change their position *ex post* their move as the quality leader or the follower (no leapfrogging) if the ecosystem consumers are absent in the market. In other words, none of the players regret their moves that lead to one of the two NE points (Motta, 1993). We refer to this discussion as "leapfrogging" in the rest of the paper. Aforementioned is our benchmark model when evaluating the effect of ecosystem leverage on the firms' price and quality game.

3.5.2 Ecosystem Market, $\lambda = 1$

As the other polar case, we consider a market that consists solely of the ecosystem buyers ($\lambda = 1$), and firm 1 with full ecosystem leverage.

Lemma 3.12 *If the market consists solely of the consumers having the base category ($\lambda = 1$), both firms exist in the market: either with (1) prices $p_1^* = \frac{2c_1u_1(cu_2-c_1u_1)}{cu_2-4c_1u_1}, p_2^* = \frac{cu_2(cu_2-c_1u_1)}{cu_2-4c_1u_1}$ ($p_1^*/p_2^* = \frac{2c_1u_1}{cu_2}$) and the quality levels $u_1^{l*}(c,k) > u_2^{f*}(c,k), c_1^*(k)$ (left panel of the Figure 3.8) or with (2) the prices $p_1^* = \frac{c_1u_1(c_1u_1-cu_2)}{c_1u_1-4cu_2}, p_2^* = \frac{2cu_2(cu_2-c_1u_1)}{4cu_2-c_1u_1}$ ($p_1^*/p_2^* = \frac{c_1u_1}{2cu_2}$) and the quality levels $u_2^{l*}(c,k) > u_1^{f*}(c,k), c_1^*(k)$ (right panel of the Figure 3.8).*

We note that the ecosystem market can simplify into Motta's case if the goods are independent ($c = 1$) and the ecosystem firm is not building endogenous complementarity ($c_1 = c$). Similar to Motta's case, there exist two NE points where either one of the firms is the quality leader.

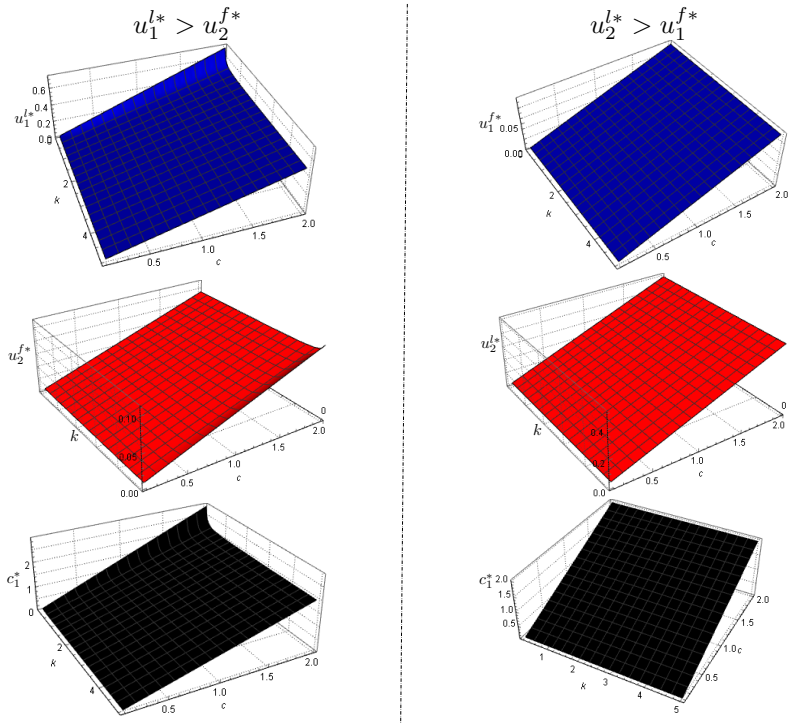


Figure 3.8: Optimal Quality Regime when $\lambda = 1$

Therefore, *ex post* the quality game, there exists no incentive for a firm to leapfrog the competitor's quality position regardless of its quality position given in either of the equilibria, for the reasons which we explain next.

When the ecosystem firm is the quality leader, it would have superior powers if the new good is a complement of the base category: the ecosystem leverage and the natural complementarity advantage. On the other hand, inherent substitution may limit its ecosystem power since the quality leader is affected more by the natural substitution than its competitor. Therefore, its profits are under risk to be suppressed by intrinsic substitution while it has the power to control the natural substitution through the built-in complementarity. Its competitor, however, wouldn't be able to control for the inherent substitution if it had leapfrogged and produced a higher stand-alone quality than the ecosystem firm. Thus, the category firm wouldn't have an incentive to leapfrog if it was a quality follower and the new good is a natural substitute of the base category. When the new good is a natural complement of the base good, we observe that the ecosystem firm (as the quality leader) invests substantially both in the stand-alone quality and the built-in connection due to the synergy between the two dimensions of the quality both

in terms of the value created and cost benefits.[9] For the category firm to leapfrog, it would need to invest a higher amount than its competitor while the return is not substantial to cover the high cost of being the quality leader.

If the category firm is the quality leader and the new good is a natural substitute of the base category, then the ecosystem firm would rather stay as the quality follower to mitigate the substitution effect with low stand-alone quality supported by the built-in complementarity. Given the competitor's cost-effective quality strategy, the category firm wouldn't have an incentive to leapfrog either. The ecosystem firm would still insist on being the quality follower for the opposite case, where the new good is a natural complement of the base category, with a similar motivation to establish a high level of total quality mostly with the built-in complementarity. Therefore, in either case, the firms would not have an incentive to deviate from their quality positions, meaning that being the quality follower or leader is sticky.

Figure 3.8 illustrates the overall trend where the stand-alone qualities as well as the built-in complementarity decrease with the inherent substitution (c) and the cost associated with the built-in complementarity (k). Put it differently, the new category innovation is under the shadow of the base category if two goods are inherent substitutes while the new category receives substantial innovation when it is a natural complement of a previously well-established product class. This partially clarifies the debate whether or not the ecosystem products shadow the category innovation. The answer is yes if the goods are inherent substitutes. The crucial question, however, is whether this is due to the intrinsic substitution that is exogenous to the firms or the complementarity built solely by the ecosystem firm. As illustrated in Figure 3.9, the role of built-in complementarity diminishes the adverse effect of the inherent market dynamics, that is pulling the stand-alone innovation upwards. Regardless of the natural connection type between the categories (substitutes or complements), built-in complementarity promotes category innovation. In general, the positive effect of the built-in complementarity is stronger for the ecosystem firm than the category firm since the former is utilizing the synergy between the

[9] We note that producing the same level total quality with two sources (stand-alone quality and the built-in complementarity) is cheaper than using one source since the costs functions associated with quality are convexly increasing.

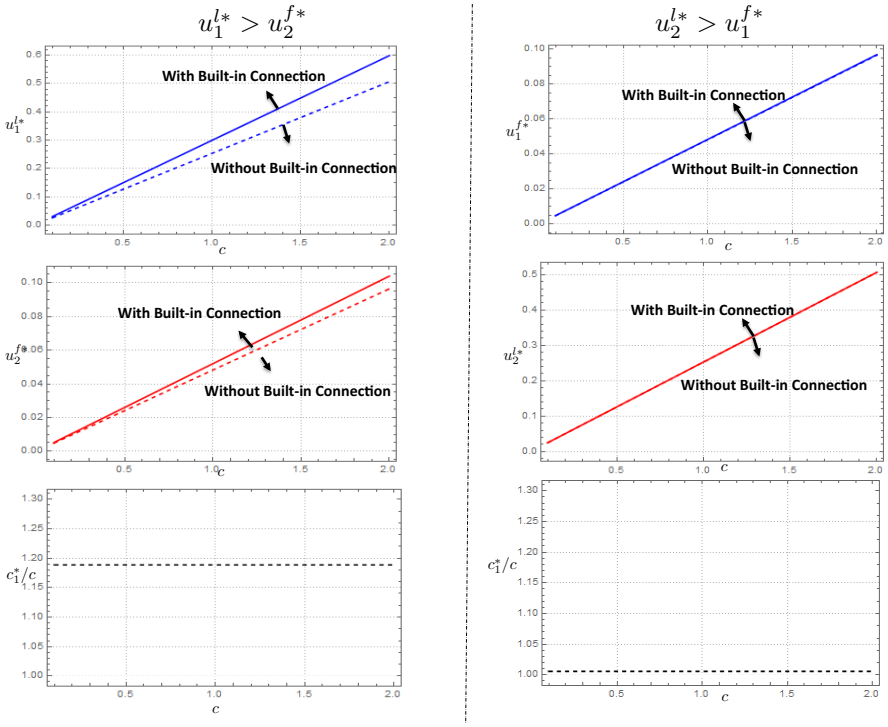


Figure 3.9: Influence of the Built-in Complementarity on the Stand-Alone Innovation when $\lambda = 1$ and $k = 0.2$

two dimensions of the quality bundle (built-in connection c_1 and the stand-alone quality u_1). When the ecosystem firm is the quality leader (left panel in Figure 3.9), the influence of the built-in complementarity on the stand-alone innovation is strong. The effect is insignificant for the opposite case, where the category firm is the quality leader (left panel in Figure 3.9) because the ecosystem firm builds very little connection if it's the quality follower at the same time. Finally, the higher the cost of endogenous connection (k), the less influential the ecosystem firm becomes and the more the stand-alone innovation is deterred when the goods are substitutes; and vice versa. We conclude that the built-in connection (c_1) promotes stand-alone innovation (stand-alone qualities of the new category, u_1 and u_2), contrary to the common perception of the industry, for the case where all the consumers in the market are buyers of the base category, that is when the ecosystem firm has the maximum leverage. The industry innovation improves with the built-in connection, and the innovation further inflates when the ecosystem firm is the quality leader.

Before turning our attention to the intermediate cases ($\lambda \in (0, 1)$), we compare the two polar cases by revisiting Lemma 3.1 and 3.4, the discussion on the aggressive/defensive complemen-

tarity building.

Corollary 3.13 *If the market consists solely of the users of the base category ($\lambda = 1$), there exists c', c'' such that $1 > c' > c''$ and*

1. *when the ecosystem firm is the quality leader ($u_1 > u_2$);*
   (a) *if ($c > c'$), the ecosystem firm deploys aggressive built-in strategy and otherwise ($c < c'$) it executes a defensive built-in strategy,*
   (b) *if ($c > c''$), the stand-alone quality levels built in the ecosystem case ($\lambda = 1$) are higher than Motta's case ($\lambda = 0$) and vice versa;*
2. *when the category firm is the quality leader ($u_2 > u_1$);*
   (a) *if ($c > 1$), the ecosystem firm deploys aggressive built-in strategy and the stand-alone quality levels built in the ecosystem case ($\lambda = 1$) are higher than Motta's case ($\lambda = 0$),*
   (b) *if ($c < 1$), the ecosystem firm deploys defensive built-in strategy and the stand-alone quality levels built in Motta's case ($\lambda = 0$) are higher than the ecosystem case ($\lambda = 1$).*

One can immediately observe a trend that the ecosystem case is associated with higher profits for both firms if the new good is a natural complement of the base category (middle and lower panels in Figure 3.10). On the contrary, if the new good is inherently a substitute of the base category, the opposite holds. The main dynamics behind improving/diminishing profits is the distinct quality investment strategies of the firms under the different type of relations the new good has with the base category as complement or substitute. In particular, a market solely consisting of previous customers (the ecosystem case) encourages the ecosystem firm to aggressively build complementarity (c_1) if the new good is already a natural complement of the previous category. The ecosystem firm targets lower value consumers that it would not target if it was competing in Motta's market as the quality leader. If instead, the ecosystem firm is the quality follower, then it targets high-value consumers that it would not serve if they

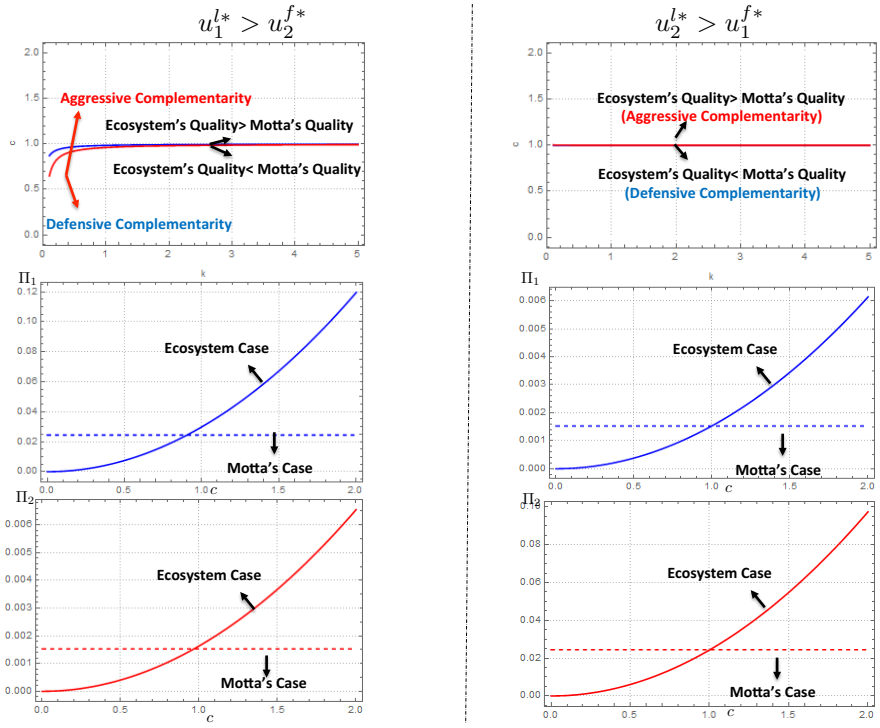


Figure 3.10: Ecosystem case versus Motta's Case, ($k = 0.2$ for the plots of Π_1, Π_2)

had not bought the base category. Therefore, the ecosystem firm is aggressive in building the complementarity only when the goods are natural complements. Likewise, the category firm is also benefiting from the previous appreciation of the consumers.

Aggressive complementarity inflates quality competition and induces both firms to build higher stand-alone quality. Increased innovation, in turn, improves the profits of both firms and the quality improvement is accompanied by higher profit margins. While the overall trend is similar between the two cases of the quality leadership, we note that if the ecosystem firm is the quality leader, aggressive complementarity is still the optimal strategy even if the new good is a natural "weak" substitute of the base category (see the upper panels of the Figure 3.10 for a comparison). This small difference stems from the quality synergy the ecosystem firm captures with the built-in connection as the quality leader.

To wrap up, the base category sales and the ecosystem firm's focus on the built-in complementarity improve overall innovation and the producer welfare when the new good is naturally a complement of the base good. In contrast, if the consumers perceive the two goods as partial substitutes, the ecosystem firm is in defense and focuses only on the high-value consumers

when it is the quality leader, or the low-value consumers when it is the quality follower. As a result, quality competition deteriorates and the profit margins, as well as the producer welfare, diminish. Whether or not the base category deters the new category innovation and the category firm's profits depends on the inherent relatedness between the categories (complements or substitutes) instead of the ecosystem firm's designated built-in complementarity.

3.5.3 Both Sub-Markets, $\lambda \in (0, 1)$

In this section, the market consists of two groups of consumers: the ecosystem market of size (λ) and the non-ecosystem consumers (Motta's market) of size $(1 - \lambda)$. The discussion of this section aims at understanding the effects of the ecosystem leverage (λ and c_1) on the stand-alone quality associated with the new category and the welfare of the category firm.

To this end, we first start with the setting where the firms are exogenously forced to be either the quality leader or the follower, that is, they are not able to leapfrog. This type of constraint on quality leapfrogging may be relevant in many digital and high tech industries due to the high fixed costs of creating incremental quality (convex costs functions) especially when the gap between being a leader and a follower is wide (Motta et al. (1997); Kim (1997)).

Proposition 3.14 *For $c \in (0, 2), k > 0$, there exists $\bar{c}_1, \bar{c}_2$ such that*

1. *when the ecosystem firm is the quality leader,*
   - (a) *if $c < \bar{c}_1$, there exists λ_1^*, λ_2^* such that*
     - i. *if $\lambda > \lambda_1^*$, both firms target both sub-markets (region (a) in Figure 3.11),*
     - ii. *if $\lambda_2^* < \lambda < \lambda_1^*$, the quality follower targets both sub-markets while the quality leader focuses solely on Motta's market (region (c) in Figure 3.11) and*
     - iii. *if $\lambda < \lambda_2^*$, both firms target only Motta's market (region (d) in Figure 3.11).*
   - (b) *if $c > \bar{c}_1$, then both firms target both sub-markets;*
2. *when the category firm is the quality leader, there exists λ_2^* such that*

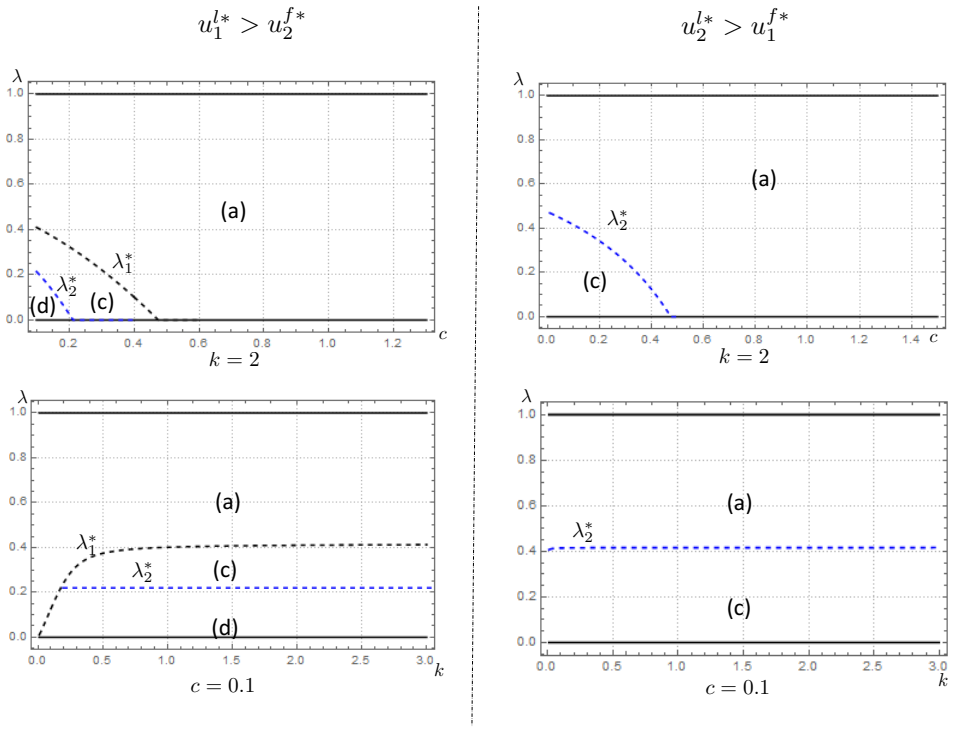


Figure 3.11: Quality Competition and Market Segmentation

(a) if $c < \bar{c}_2$, and

i. if $\lambda > \lambda_2^*$, both firms target both sub-markets,

ii. $\lambda < \lambda_2^*$, the quality follower targets both sub-markets while the quality leader focuses solely on Motta's market.

(b) if $c > \bar{c}_2$, then both firms target both sub-markets.

An interesting insight from Proposition 3.14 is that the quality leader, whether the ecosystem or category firm, is always the first to lose interest in the ecosystem market and focus on Motta's market. Especially when the ecosystem firm is the quality leader, one would expect the ecosystem leverage to induce the competitor to exit the ecosystem market. However, this is never the case since the firms exit the ecosystem market if and only if the new good is a close substitute of the base category and the ecosystem market is small. As the quality leader, the ecosystem firm always suffers more from substitution and prefer to focus on a bigger sized Motta's market where the ecosystem firm can position itself as the superior quality producer. Hence, the category firm is not forced to exit the ecosystem market, implying that leapfrogging is not attractive for the category firm nor the ecosystem firm while enjoying high-value demand

from a large sub-market (Motta's market).

Along the similar lines, when the category firm is the quality leader, it is the only one that would leave the ecosystem market provided that the new good is a close substitute of the base category and the ecosystem market is sufficiently small. In contrast to the case where the ecosystem firm is the quality leader, however, the quality follower always stays in the ecosystem market. The reason behind this is that the quality follower is the firm with the ecosystem leverage. The ecosystem leverage would not lead the category firm to exit the ecosystem market per se, but it would inflate the adverse effect of the partial substitution on the category firm, which suggests that none of the firms would have an incentive to leapfrog their competitor if they had given the option to deviate from their quality choice. We discussed the intuitive explanation on why firms would not quality leapfrog or catch-up the competitor in one of the sub-markets given they are the quality leader or follower, which we formally state next:

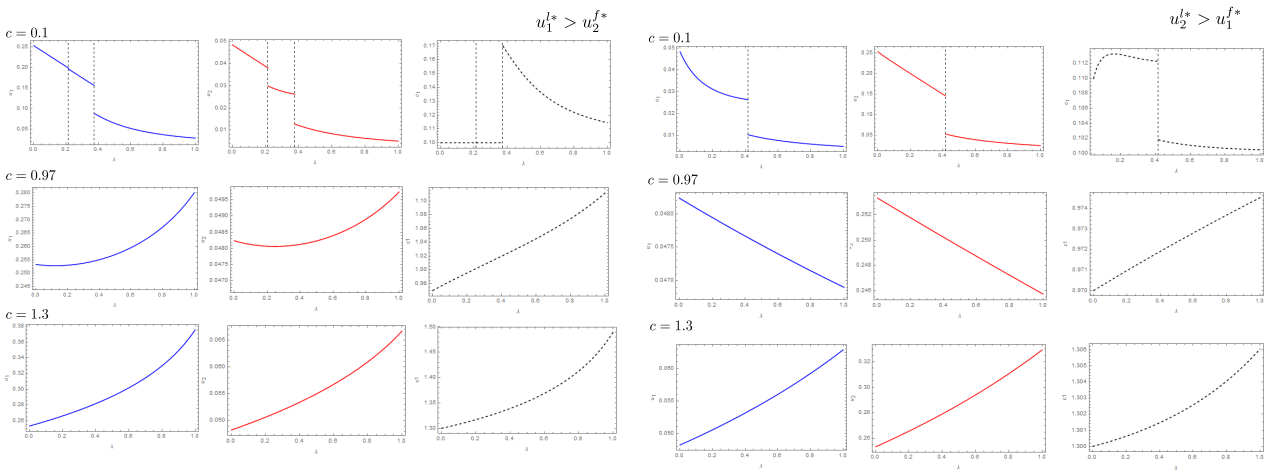
Lemma 3.15 *Given $\lambda \in (0,1)$, $k > 0$ and $c \in (0,2)$, there exist two NE points associated with quality competition where either the ecosystem or the category firm is the quality leader ($u_1^{l*}(c,\lambda,k) > u_2^{f*}(c,\lambda,k)$ or $u_2^{l*}(c,\lambda,k) > u_1^{f*}(c,\lambda,k)$).*

Next, we delve more into the effect of ecosystem leverage (c_1 and λ) on the stand-alone innovation in the new category. From a higher level perspective, distinct quality regimes (explained in Proposition 3.14 for $k = 2$) stem from limited or expanded market power of the firms under various market settings.

If the new good is intrinsically a strong substitute of the base category (upper panels in Figure 3.12) then category innovation decreases with the size of the ecosystem market (λ). Category innovation will be under the shadow of the base category if the new category has too many overlapping features with the base category (such as smartwatch and smartphones). Also, two types of quality regimes can be observed depending on the size of the ecosystem market: high category innovation when the ecosystem market is small and low category innovation when the ecosystem market is significant. When the ecosystem firm is the quality leader and the substitution among the goods is high (upper left panel in Figure 3.12), it does not activate

the ecosystem leverage ($c_1 = c$) and focus only on the stand-alone quality and target Motta's market. The main reason behind this is the strong substitution it faces in the ecosystem market. The ecosystem firm would benefit from the built-in connection between the categories if and only if the ecosystem market is sufficiently large. Interestingly, as the ecosystem market expands, the overall innovation, whether stand-alone or base category dependent, diminishes. As the firms move to the stand-alone regime with low quality, the natural substitution effect decreases (cu_i) as well as the ecosystem firm's incentive to control for it through the built-in connection (c_1). We observe a similar trend in the "category innovation" when the ecosystem firm is the quality follower. On the contrary, the base category driven built-in innovation has a reverse trend: it is highest when the ecosystem market is smallest and vice versa. When the ecosystem firm is alone in a small ecosystem market, it invests more on endogenous built-in connection mostly because of the various stand-alone quality levels present in the markets. When the ecosystem market is large, stand-alone quality gap decreases. The firms are exposed to a similar level of substitution threat from the base category and this, in turn, decreases the ecosystem firm's incentive to build a strong built-in connection.

If the goods are weak substitutes in nature (middle panel in Figure 3.12), both firms compete in both sub-markets. Ecosystem leverage influences the category innovation negatively unless the ecosystem firm is the quality leader and the base category is more widespread. To be more specific, the ecosystem firm faces a trade-off: keeping the competitor at bay with superior total quality ($c_1 u_1$) versus confronting the stronger substitution effect stemming from its quality position. As the ecosystem market expands above a sufficient level, the ecosystem firm becomes more quality-driven and invests in both dimensions of quality (the stand-alone quality and the built-in complementarity). Let us first assume that the category firm is the quality leader, and the new good is a weak substitute of the base good (middle left panel). In this instance, the ecosystem firm benefits from the ecosystem leverage supposing that the ecosystem market is dominant (left middle panel in Figure 3.12). The relation does not hold when the category firm is the quality leader since it has less of an incentive to innovate and engage in quality competition. As a result, the stand-alone quality levels gradually diminish whilst the ecosystem firm builds more complementarity as the ecosystem market expands.



(a) When the ecosystem firm is the quality leader (b) When the category firm is the quality leader

Figure 3.12: Innovation in the New Category, for $k = 0.5$

For the case when the new good is a natural complement of the base category, regardless of the quality positioning of the ecosystem firm, innovation in the new category and seamless connection with the base category increase substantially with the ecosystem market's size (lower panel in Figure 3.12). The ecosystem leverage promotes the new category instead of deterring the category innovation. Conversely, the ecosystem leverage restrains the stand-alone quality levels when the new category is a substitute of the base category. Despite the common industry belief that category innovation makes sense for a firm if the competitor is refrained from innovating (Aaker, 2010), we find that category innovation would occur as a result of fierce quality competition and is fueled by the ecosystem consumers with the previous category of the ecosystem firm, provided that the two categories are not strong substitutes. Our discussion to this end is formally stated in Lemma 3.16 and illustrated in Figure 3.12.

Lemma 3.16 *For $k > 0$, there exists $\bar{c}_1(k), \bar{c}_2(k) \in (0, 1)$ such that,*

1. *for $c < \bar{c}_1(k)$ and $c < \bar{c}_2(k)$,*

   (a) *the stand-alone qualities (u_1^{*l}, u_2^{*f} or u_1^{*lf}, u_2^{*l}) decrease with λ regardless of the quality position of the firms,*

   (b) *if the ecosystem firm is the quality leader, there exists no built-in connection ($c_1^* \to c$) if the ecosystem market is small ($\lambda < \lambda_1*$) and for otherwise ($\lambda > \lambda_1*$) the built-in connection decreases with the size of the ecosystem market (λ),*

(c) if the catgeory firm is the quality leader, the built-in connection decreases with the size of the ecosystem market (λ);

2. *for* $\max\{\bar{c}_1(k), \bar{c}_2(k) < c < 1\} < c < 1$,

   (a) if the ecosystem firm is the quality leader, the stand-alone quality levels (u_1^{*l},u_2^{*f}) diminish with the ecosystem market's size (λ) if the markets is small and increases with λ if the the ecosystem market is large,

   (b) if the category firm is the quality leader, the optimal stand-alone qualities (u_1^{*l}, u_2^{*f}) monotonically decreases with λ,

   (c) the optimal built-in connection (c_1^*) monotonically increases with λ, regardless of the quality positions of the firms;

3. *for $c > 1$, the optimal stand-alone qualities (u_1^{*l}, u_2^{*f} or u_1^{*lf}, u_2^{*l}) as well as the optimal built-in connection (c_1^*) monotonically increases with λ.*

To conclude, existence of the ecosystem consumers promote category innovation (u_1, u_2) and seamless connection (c_1) unless the new good is a strong substitute of the base good ($c < \bar{c}_1(k), \bar{c}_2(k)$) or when the goods are weak substitutes and the category firm is the quality leader.

Finally, we discuss producer welfare (profits) under the optimal quality regimes mentioned in Proposition 3.14. Considering the new good a complement of the base category, increasing stand-alone innovation and consequent pricing powers improve the profits of both firms (lower panel of the Figure 3.13). Our results confirm the argument that the ecosystem leverage is not a threat to the competitor provided that the new category is a complement of the related market, where the ecosystem firm has sufficient market power.

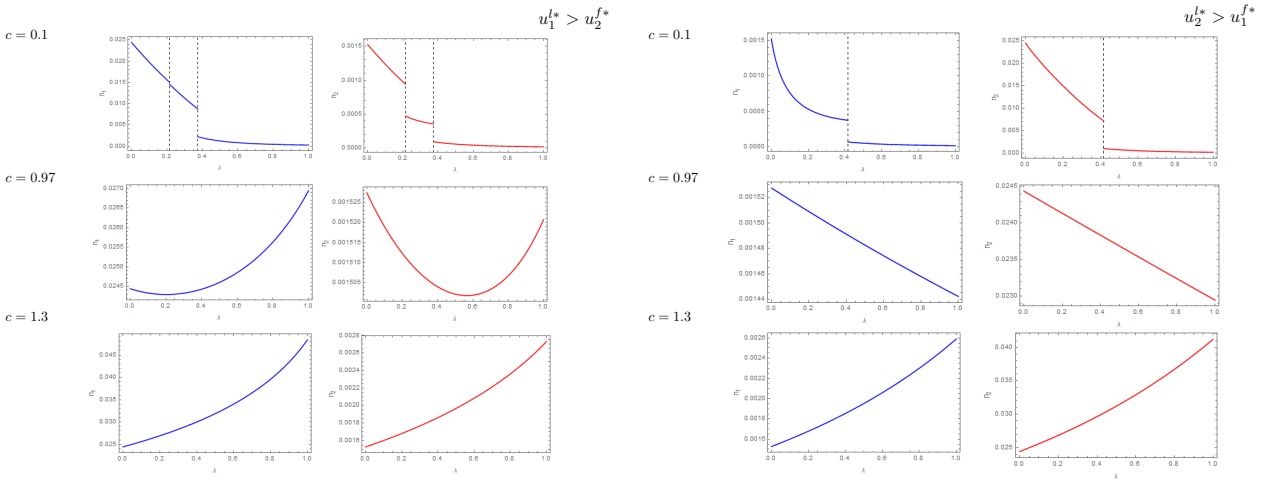
When the new good is a weak substitute of the base category, the category firm is negatively affected by the ecosystem consumers if the category firm is the quality leader (left-middle panel in Figure 3.13). In fact, aggressive quality competition and the inherent substitution factor causes the profits to shrink for both of the firms. When the category firm is the quality follower (right middle panel in Figure 3.13), and the ecosystem market is small, the category

firm is kept at bay as an inferior quality producer by the ecosystem firm. The ecosystem firm's strength stems from its superior total quality ($c_1 u_1$) that is the combination of both the stand-alone innovation and the built-in connection. Ironically, if the ecosystem market is large, the category firm benefits from the ecosystem leverage. The main reason behind this is that the ecosystem firm focuses on the upper end of the ecosystem market and this pulls up the prices and hence improves the category firm's profit margin. Whether or not the ecosystem leverage is harmful to the category firm depends on who is the quality leader, the size of the ecosystem market, and the underlying relatedness between the goods.

Regarding the case where the new good is a strong substitute of the base category, the profits of each firm diminish as the base category becomes more widespread, regardless of the quality position of the firms (upper panel in Figure 3.13). We note that the jumps in the profits indicate the shifts from one pricing regime to another (as specified with relative prices in Proposition 3.8). The decline in the profits becomes dramatic as the firms switch to different quality regimes (Figure 3.12) and primary pricing policies (the relative prices). The main reason for diminishing profits is the inherent substitution that becomes stronger with a broader base of the ecosystem consumers. However, as the quality leader enters into the ecosystem market, the profits decline further because of the built-in complementarity that forces down the stand-alone quality. We reconcile the debate about whether or not the ecosystem leverage is harmful to the competitor by showing that it is a threat to the category firms in many instances if the new good is a substitute of the base category while beneficial when the new category is a complement of the base category.

We infer that the ecosystem leverage (c_1 and λ) is neither necessarily an enemy for the competitor nor it always deters the category innovation. The stand-alone innovation may deter, or the category firm may be in a disadvantaged position because of the ecosystem leverage only if the new good is a substitute of the base category.

Next, we delve more into the ecosystem leverage and focus on the built-in complementarity (c_1) in isolation from the size of the ecosystem market (Figure 3.14). The built-in connection improves the profits of both firms if the ecosystem firm is the quality leader (Figure 3.14a). It is



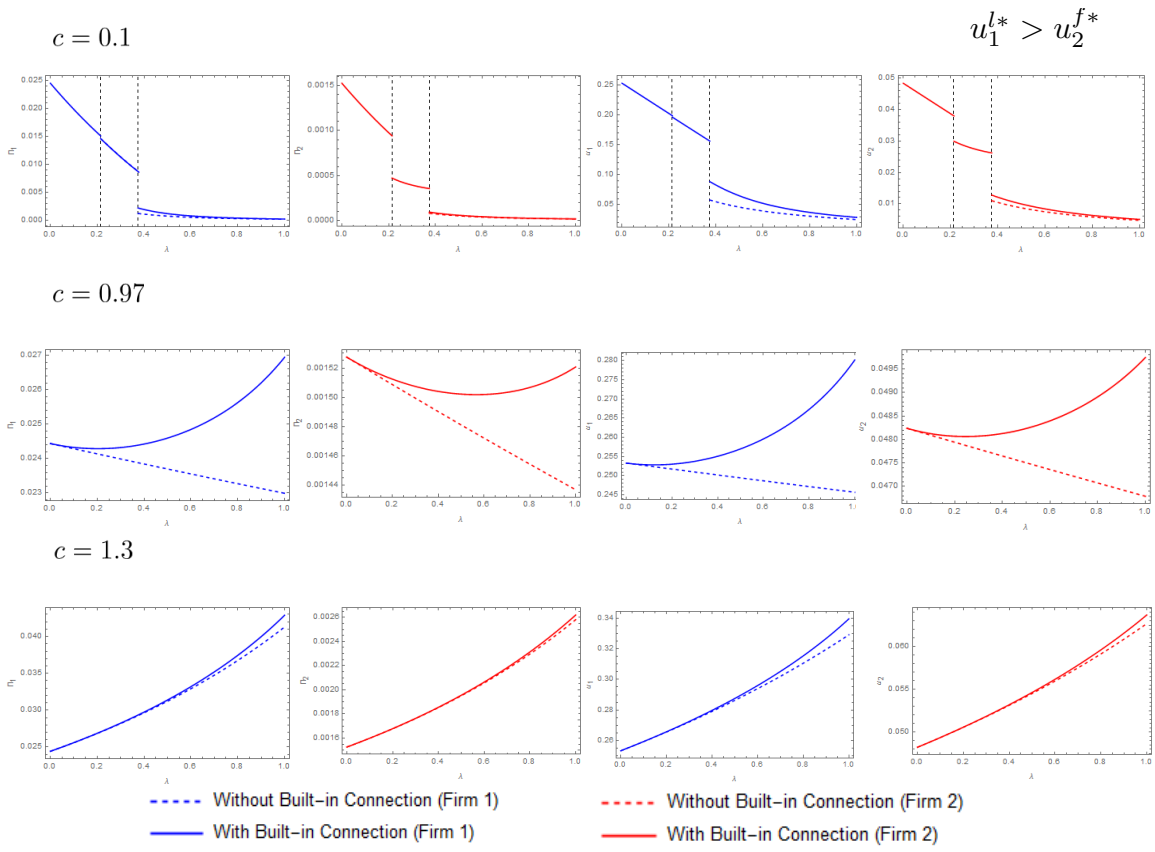
(a) When the ecosystem firm is the quality leader (b) When the category firm is the quality leader

Figure 3.13: Optimal Profits (Π_1^* and Π_2^*), for $k = 0.5$

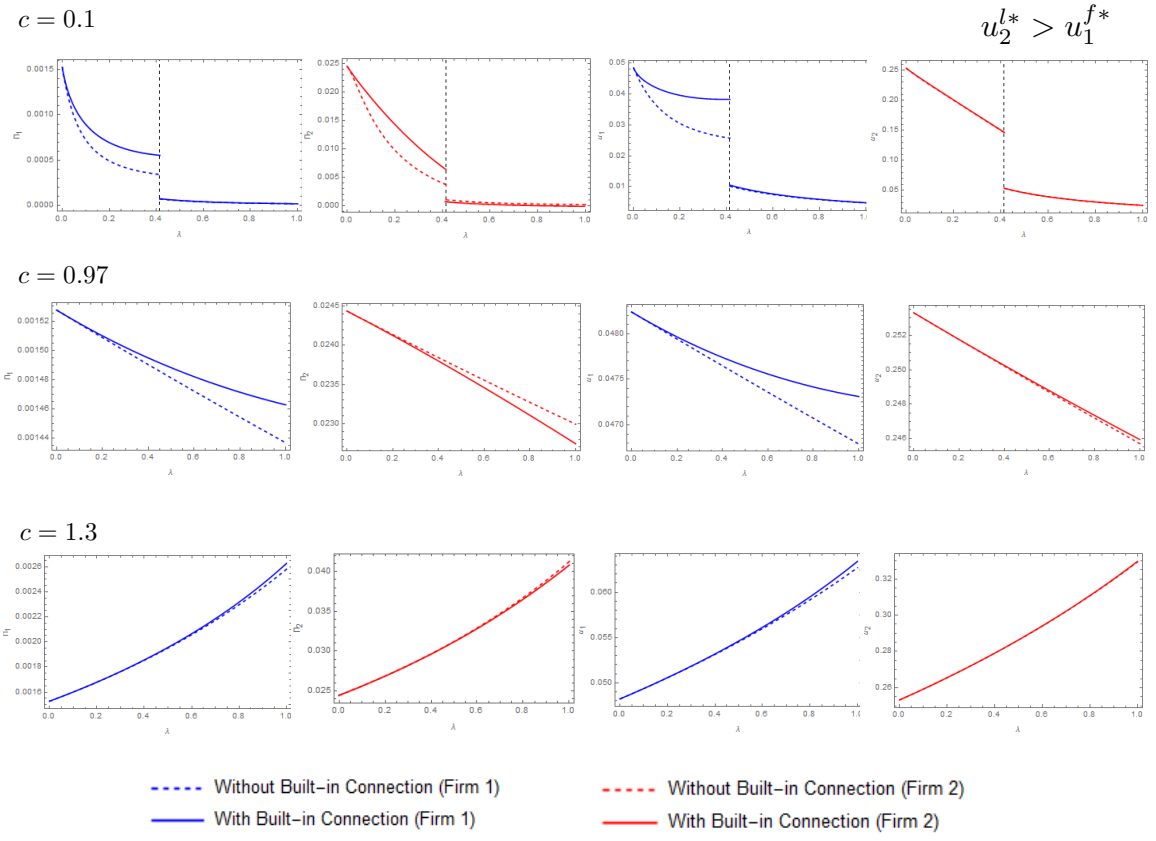
more intuitive to observe that the ecosystem firm's profits improve with the built-in connection by the synergy it creates with the stand-alone quality. The more interesting finding is that the category firm also benefits from the built-in connection. With increasing quality competition, the ecosystem firm charges higher prices, augmenting the profit margin of the category firm as well. Therefore, we conclude that the "built-in connection" promotes category innovation per se and improves the welfare of the firms when the ecosystem firm is the stand-alone quality leader.

If instead, the category firm is the quality leader (Figure 3.14b), then the extent of improvement in the welfare of the category firm depends on the size of the ecosystem market. For the category firm targeting the ecosystem consumers due to their significant share in the market, the built-in connection pulls down the profit, while the stand-alone innovation always serves its interests of the size of the ecosystem market. At the same time, the built-in connection improves the welfare of the ecosystem firm as the quality follower.

Thus, we conclude that the "built-in connection" per se is not necessarily a threat to the category firm nor the stand-alone innovation and only when the category firm has the advantageous position (the quality leader) then its profits diminish as a result of the built-in connection.



(a) When the ecosystem firm is the quality leader



(b) When the category firm is the quality leader

Figure 3.14: Influence of the Built-in Complementarity on the Innovation and the Producer Welfare, for $k = 0.5$

3.6 Discussions

We have concluded that producer welfare is not necessarily affected negatively by the ecosystem customers (λ) and the built-in connection (c_1). On the contrary, we find that the "ecosystem leverage" may improve the welfare of both firms as well as the category innovation under certain conditions depending on which firm is the quality leader and the connection the new good has with the base category. In this section, we elaborate more on the welfare implications with a discussion of the consumer surplus and when the category firm can also build a connection with the base category.

3.6.1 Consumer Welfare Analysis

Our results suggest that consumers may experience higher stand-alone qualities and well-connected ecosystem goods, unless the two classes of products are close substitutes or the ecosystem firm is the quality leader when the product classes are weakly related. However, one needs to consider the prices which dramatically change according to the market parameters (c and λ) (discussed in Section 3.4) to have a complete picture of consumer surplus. In this section, we characterize the conditions where the ecosystem leverage (λ and c_1) has adverse effects on the welfare of the consumers.[10] We provide a numerical discussion and acknowledge that an analytical setting can be a fruitful future extension of the product ecosystems framework presented in this paper.

We start with the case where the goods are intrinsically complementary goods. In the earlier sections, we confirmed that the category innovation and the built-in connection improves with the ecosystem leverage (λ and c_1). We first examine the case where the ecosystem firm is the quality leader ($u_1^{l*} \geq u_2^{f*}$), see the left panel of Figure 3.15. With the superior value of its quality bundle, the ecosystem firm focuses on the higher-end of the market while the category firm can only elevate to the higher value consumers with the improvement of its stand-alone quality.

[10]The reader is referred to the Appendix B.2 for the derivation of the total consumer welfare.

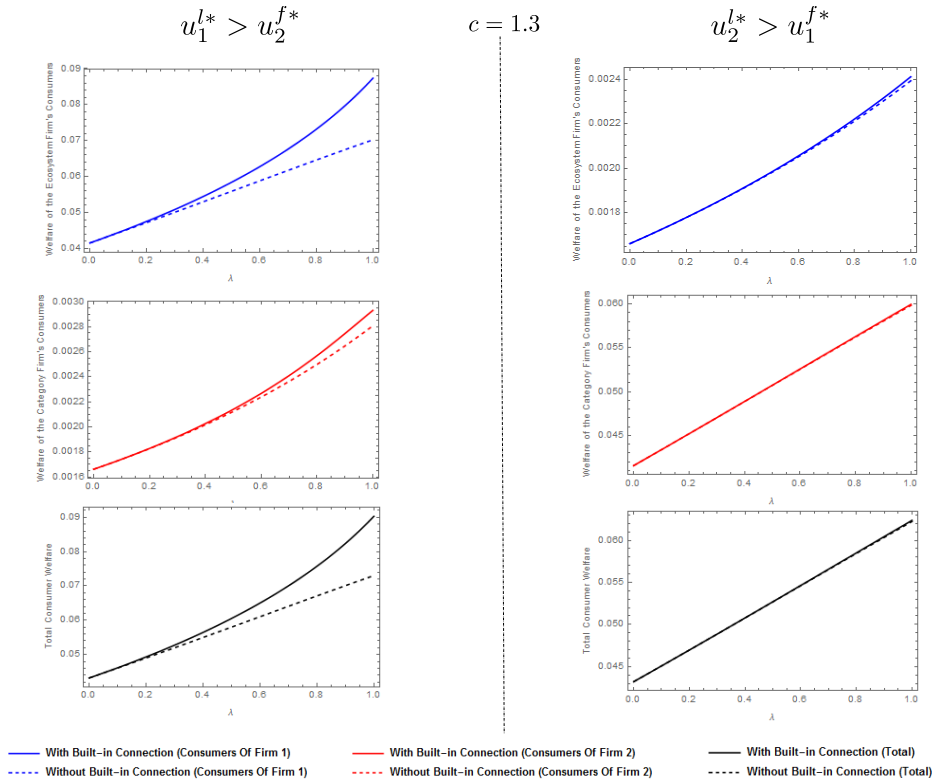


Figure 3.15: Consumer welfare when the product categories are complements for $k = 0.5$

Consumers also benefit from the ecosystem leverage since they receive superior innovation at reasonable prices. Therefore, the ecosystem leverage moderated with the price and quality competition improves the aggregate welfare. However, not every player in the market benefits from the ecosystem leverage. Specifically, the consumers of the Motta's market, regardless of from which firm they buy from, are hurt from the built-in connection (c_1) even if we eliminate the effect of (λ) on the scale of the consumer welfare when considering sub-markets with equal sizes ($\lambda = 1/2$) (see Figure 3.16). The reason is the increased prices as a result of the improved innovation that they can receive only partially since they do not own the base category. A similar situation occurs in the alternative case where the category firm is the quality leader ($u_2^{l*} \geq u_1^{f*}$) except that the effect of the ecosystem leverage on the consumer surplus is not as substantial as it was when the ecosystem firm is the quality leader (see the right panel of Figure 3.15). Therefore, we conclude that the ecosystem leverage promotes consumer welfare. Figure 3.15 summarizes the discussion on the complementary goods.

Next, we consider the case when the new product category is a substitute of the base category ($c < 1$). We remind the reader that depending on the level of substitution, the firms deploy

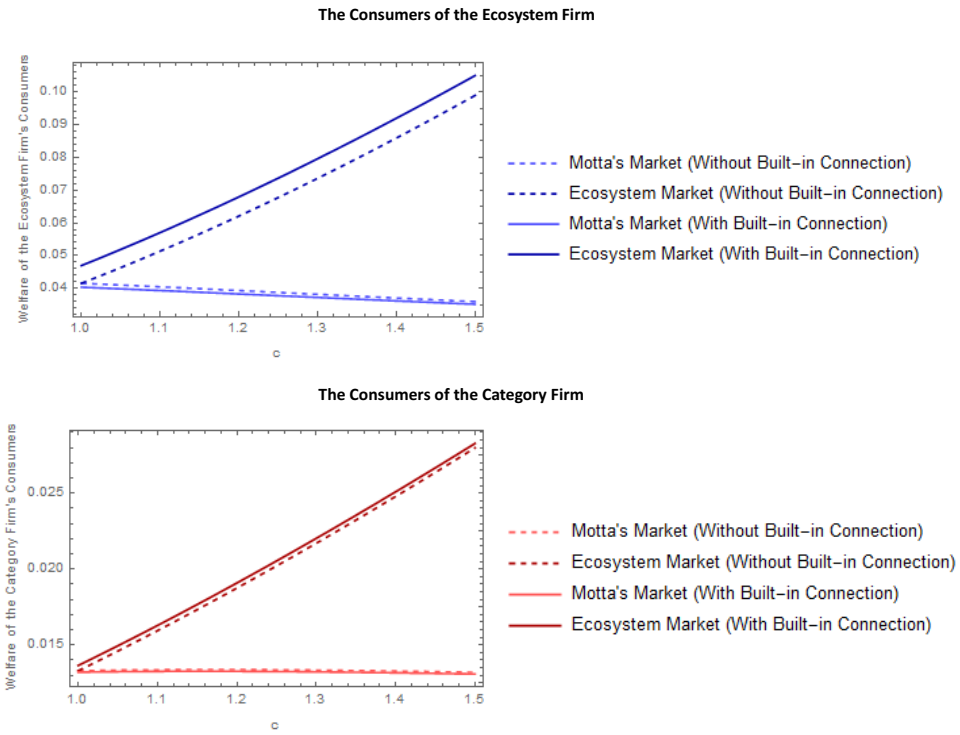


Figure 3.16: Consumer welfare of each firm when the sub-market sizes are equal ($\lambda = 0.5$ and $k = 0.5$)

different types of pricing regimes. In particular, when the intrinsic substitution is substantial, the quality leader, regardless of which firm it is, puts more emphasis on Motta's market while for the opposite case, both firms compete in both sub-markets.

We first focus on the weak substitution case that we covered in detail in Section 3.5, and investigate the influence of the ecosystem leverage on consumer welfare. In this case, the quality position of the firms plays a crucial role. If the quality leader is the category firm, then the category innovation degrades as the base category becomes widespread (increasing λ) while the built-in connection (c_1) enhances the innovation, and the overall impact of the ecosystem leverage is negative. That, in turn, influences consumers negatively and the total welfare diminishes as a result. However, if we evaluate the influence of built-in connection in isolation, we confirm that its effect is positive on both firms since the built-in connection increases the quality competition. Therefore, we conclude that the consumer welfare is negatively influenced from the ecosystem leverage while the deterioration is a result of the intrinsic connection between the categories and the quality position of the category firm that interfere with the positive effect of

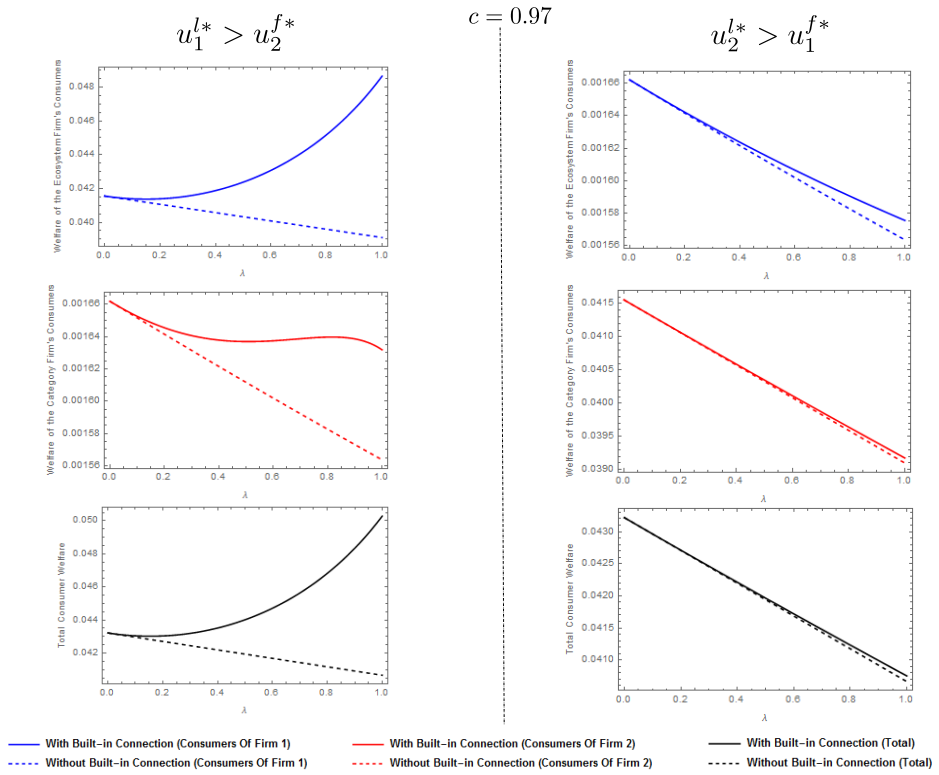


Figure 3.17: Consumer welfare when the product categories are weak substitutes for $k = 0.5$

the endogenous connection that could be built by the ecosystem firm. If instead, the ecosystem firm is the quality leader, the total welfare increases with the ecosystem leverage since the category innovation progresses with the ecosystem leverage. We note that the consumers of the category firm are affected negatively when the base category is more widespread mainly due to the prices. The category firm increases its prices in return for the increasing innovation, and its consumers from Motta's market are affected negatively.

Finally, we cover the case where the two product classes are strong substitutes. We remind the reader that this is the case when the category innovation degrades as the base category becomes more widespread (as λ increases) while the built-in connection of the ecosystem firm (c_1) decelerates this adverse effect on the innovation. Our results indicate that slow innovation reduces the welfare of the consumers despite the lower prices charged by both of the firms. However, the pricing regime the firms deploy has a significant impact on some consumer groups. When the category firm is the quality leader, it does not target the ecosystem market especially if the sub-market is small. In this case, the ecosystem firm is a monopoly in this sub-market and invests effectively on the quality bundle, that is both the stand-alone innovation as well as the

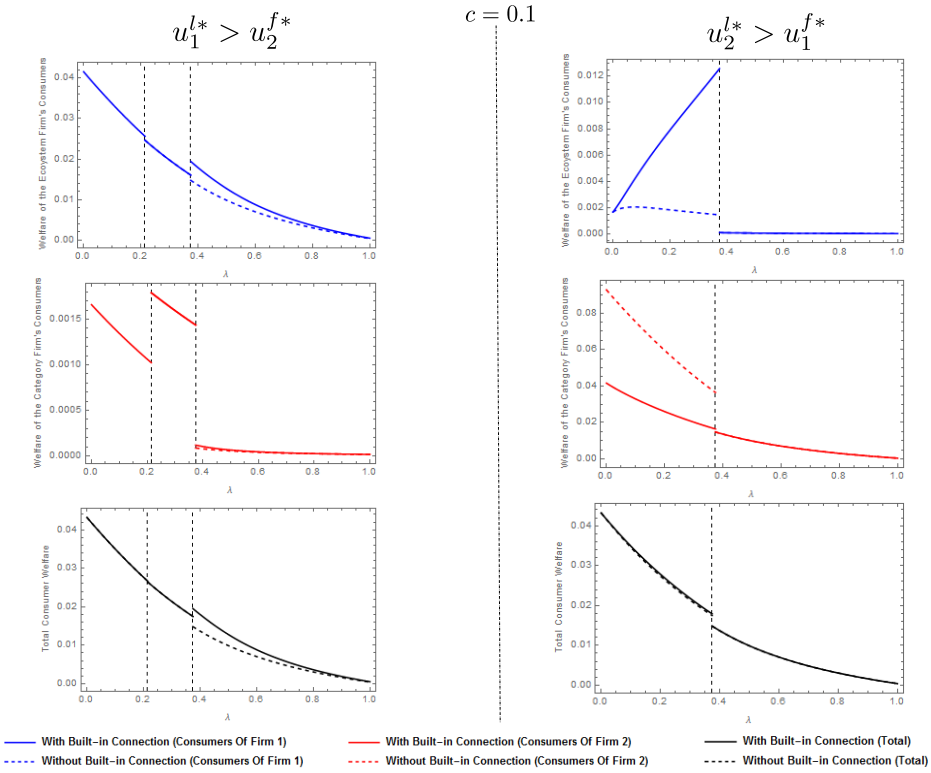


Figure 3.18: Consumer welfare when the product categories are strong substitutes for $k = 0.5$

built-in connection. Thus, the ecosystem consumers benefit from the improved quality bundle. The category firm, on the other hand, engages in quality competition with the ecosystem firm in Motta's market as a side effect of the superior quality bundle its rival is offering in the ecosystem market. That is not necessarily bad for the category firm since it induces higher profit margins. The consumers of the category firm, however, are affected negatively since the profit margin effect dominates. This case also clearly demonstrates how the built-in connection moderates the whole mechanism. In particular, we observe that the built-in connection inflates the quality competition, which in turn improves the ecosystem consumers' welfare while the situation of category firms' consumers worsens.

To summarize, our numerical results indicate that category innovation is the primary dynamics that determine the welfare of the consumers, that is when the category innovation degrades, so does the consumers' welfare. The pricing regime, on the other hand, plays a key role in determining the situations of various consumer segments. As a high-level observation, when the category firm is the quality leader its consumers, which consist only of the Motta's market, are affected negatively both from the built-in connection and the high prices they face in base

category.

3.6.2 Ecosystem Leverage for Sale

One central assumption of our model is that the category firm is not able to build connection endogenously between the new and the base good. While examples of such instances are plentiful (such as iPhone and smartwatches with an Android operating system), there exist industry examples, such as Wacom's graphical tablet pairing with Apple's IPad and competing with Apple's IPad Pro, that challenge our assumption.

In this section, we relax this assumption and consider that the category firm can connect the new good with the ecosystem firm's base category ($c_2 u_2$) by transferring an amount ($k_2(c_2 - c)^2$) to the ecosystem firm. We consider that both firms exist in both markets since this case provides us with the full set of competition dynamics (dynamics within a sub-market and between the two sub-markets) and focuses on the implications on the innovation. We start with the built-in connection:

Lemma 3.17 *For $c > \max\{\bar{c}_1, \bar{c}_2\}$ (see Proposition 3.14), $k_2, k > 0$, there exists λ_1^*, λ_2^* such that both firms exist in both markets if $\lambda > \lambda_1^*$ when the ecosystem firm is the quality leader and $\lambda > \lambda_2^*$ when the category firm is the quality leader, and*

1. *if the ecosystem firm is the quality leader and $\lambda > \lambda_1^*$, it follows that $c_1^* > c_2^*$;*
2. *if the category firm is the quality leader and $\lambda > \lambda_2^*$, it follows that $c_2^* > c_1^*$;*

and there exists $\hat{c} \in (0, 1)$ and $\hat{\lambda}$ such that

1. *if $c > \hat{c}$, then $\frac{\partial |c_1^* - c_2^*|}{\partial \lambda} > 0$, and*
2. *if $c < \hat{c}$, then $\frac{\partial |c_1^* - c_2^*|}{\partial \lambda} > 0$ for $\lambda < \hat{\lambda}$ and $\frac{\partial |c_1^* - c_2^*|}{\partial \lambda} < 0$ when $\lambda > \hat{\lambda}$.*

We start with the built-in connection and immediately observe the complementing nature of stand-alone innovation and the endogenous link between the categories (the quality bundle when combined). The quality leader is always the one building a stronger link with the base

category. Secondly, we observe that as the new good has less overlapping features or more complementing features with the base category (as c increases), the quality leader puts more emphasis on the built-in connection. The opposite holds when the goods are close substitutes (c is small) since the asymmetric substitution effect would limit the quality leader. That is why, when the new good is a close substitute of the base category, the quality gap between the inter-category connections (c_1^* and c_2^*) is smaller.

We also observe that the bigger the size of the ecosystem market (λ), the more the seamless connection the firms build (c_1^*, c_2^*) due to the complementing nature of the built-in connection (c_1^*, c_2^*) and the stand-alone quality levels (u_1, u_2), hence a high-value "quality bundle". We observe similar trends when the new good is a substitute of the base category ($c < 1$) as long as the ecosystem market is small (λ is low). We have an opposite situation when the size of the ecosystem market dominates (λ is large). The main reason behind this is the substitution effect arising from the inherent connection between the two categories, and the substitution effect is more influential in a large ecosystem market. We remind the reader that a similar mechanism is also observed in the the asymmetric case of leverage (the main discussion). Lemma 3.17 suggests that the category firm would build a stronger link with the base good if the category firm is the quality leader. However, that does not necessarily imply that the ecosystem firm would have an inferior seamless technology when compared with the asymmetric case. Would the ecosystem firm build more or less when compared to the asymmetric case? That is the next question we investigate with a numerical example.

Our results indicate that the ecosystem leverage granted to the category firm (symmetric case) inflates the seamless connection the ecosystem firm builds (see Figure 3.19) provided that the category firm is the quality leader. We remind the reader that when the category firm is the quality leader, it is more inclined to build seamless connection due to the synergistic nature of the built-in connection and the stand-alone innovation. We complete the analysis with the ecosystem firm's response, that is to build more integration from its end. Therefore, the new category emerges as a well integrated good to the base category. If instead the category firm is the quality follower, it has less motivation to build integration, and hence the ecosystem firm sustains its built-in policy of the asymmetric case.

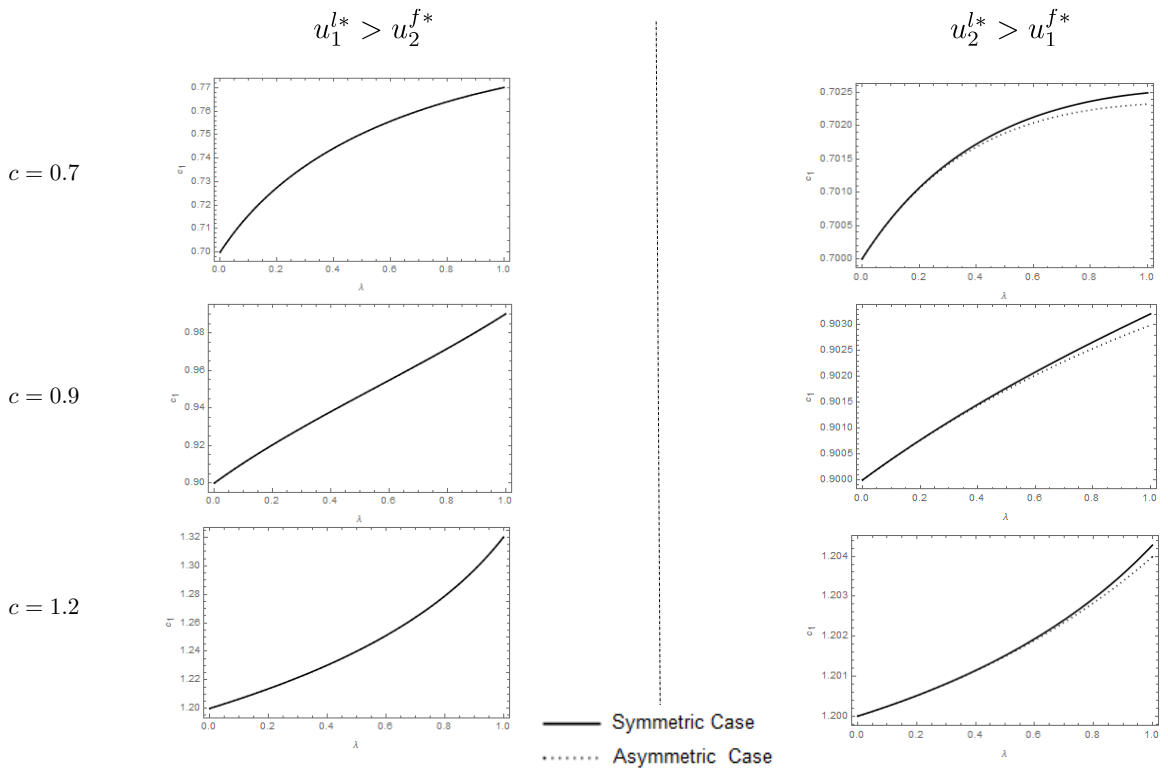


Figure 3.19: Built-in connection by the ecosystem firm for $k = 0.7$, $k_2 = 0.9$, and $\lambda = 0.9$

Next, we discuss the implications of the ecosystem technology (available to both firms) on the stand-alone innovation (u_1, u_2) (illustrated by Figure 3.20). The synergistic nature of the stand-alone and built-in connection plays a role for the ecosystem firm as well when responding to the symmetric case of leverage. Both firms are inclined to built superior quality bundles when compared to the asymmetric case, provided that the category firm is the stand-alone quality leader. If the ecosystem firm is the leader, however, the impact of the symmetric case is negligible on the stand-alone innovation since the category firm is not inclined to exploit the leverage due to its low quality position.

We assumed so far that the revenue from the seamless connection granted to the competitor is exogenously imposed (for fixed k_2) on the ecosystem firm. An important aspect to consider is whether or not the ecosystem firm would indeed provide the service to the category firm if it would have control over k_2:

Lemma 3.18 *For $c > \max\{\bar{c}_1, \bar{c}_2\}$ (see Proposition 3.14) and $\lambda \in [0, 1]$, if the ecosystem firm is the quality leader ($u_1^{l*} > u_2^{f*}$), the ecosystem firm does not share the leverage ($k_2^* \to \infty$) with*

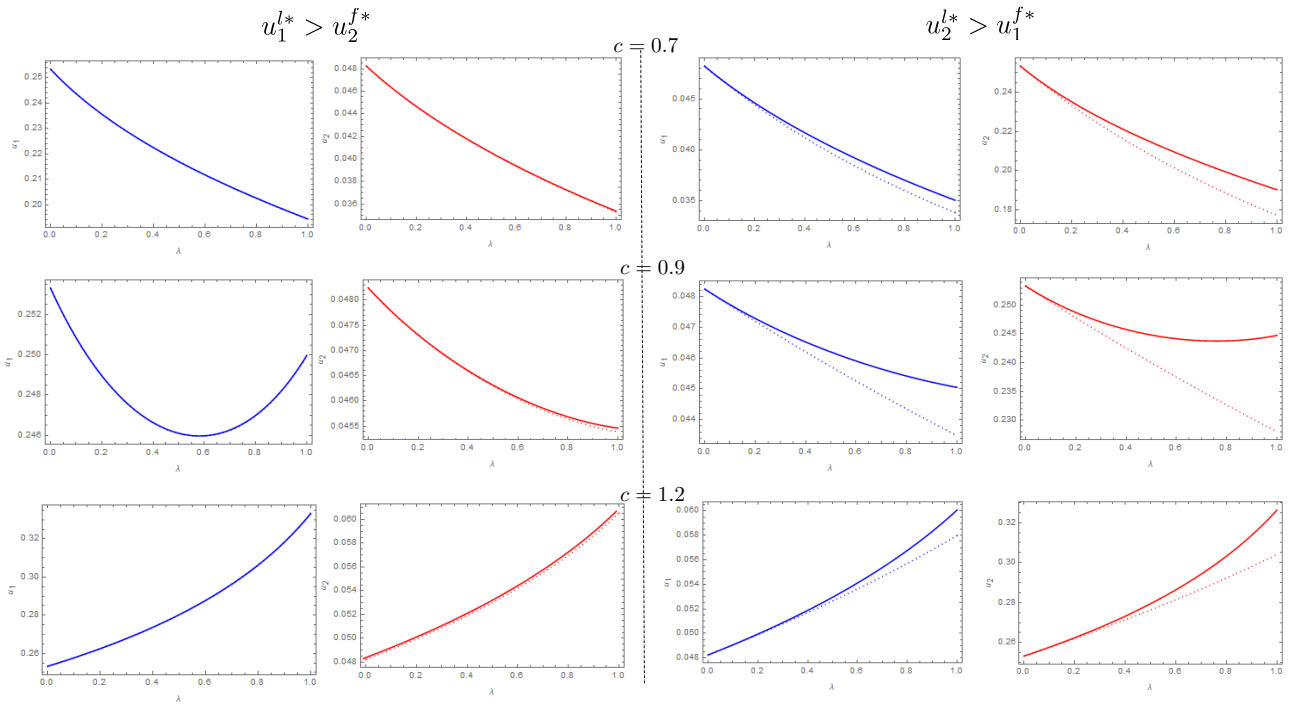


Figure 3.20: Category innovation $k = 0.7$, $k_2 = 0.9$, and $\lambda = 0.9$

the category firm, while if the category firm is the quality leader ($u_2^{l*} > u_1^{l*}$), the ecosystem firm shares the leverage for free ($k_2^* \to 0$).

The main reason for the ecosystem firm to share the ecosystem leverage with the category firm is the enhanced quality competition and higher profit margins it can derive from higher-valued consumers in return. That is a crucial insight, and it overturns the adverse effect of the ecosystem firm on the innovation and the welfare of the category firm that we have discussed in the asymmetric leverage case. In other words, the ecosystem firm positing the most risk to the welfare as the quality follower in an asymmetric leverage case is willing to share its leverage with the category firm for free. We remind the reader that this is also the case where the category firm prefers to build a strong connection between the two categories. Thus, the symmetric leverage is optimal for both firms, and they prefer "coopetition" over "aggressive competition".[11] In practice, the firms can get stuck at the asymmetric case due to the reasons

[11]Opel CE's CEO Karl-Thomas Neumann replies as "The rumours about the Apple car is a sign that the auto industry is in a process of upheaval. The car will become part of the internet of things and we want to be part of that. when he is asked what he thinks about Apple entering the car market, and other executives such as Carlos Ghosn from Renault-Nissan Alliance, Martin Winterkorn from Volkswagen and Oliver Zipse from BMW responds along the similar lines (Grothaus, 2015; Taylor and Julia, 2015).

such as contractual frictions or being unaware of the gains, and the policymaker may nudge the firms towards a shared innovation in seamless connection by eliminating these concealed costs.

Another interesting implication of Lemma 3.18 is that when the ecosystem firm is the quality leader, i.e. when it is not harmful to the welfare or the category innovation in an asymmetric case, it never prefers to collaborate with the category firm on the ecosystem leverage. Therefore, when it is not harmful in an asymmetric case, it favors the "soft competition" over coopetition.

The aggressive competition of the asymmetric case gives rise to the coopetition and the soft competition results as reluctance to collaboration. We conclude that the ecosystem leverage is not necessarily a threat to the society and emergence of new categories. If the firms are nudged to collaboration, the positive effects smoothly accelerates.

3.7 Conclusions

We have presented a model of price and quality competition of two asymmetric firms in a new product category. One of the firms, the ecosystem firm, has an advantage, the "ecosystem leverage". The leverage emerges from the consumers (the ecosystem market of size λ) possessing a previous product category that may or may not be intrinsically related to the new product category and the ecosystem firm's capability to link the two product classes (represented with a quality investment c_1). The ecosystem firm can utilize the appreciation of the buyers of the former product category and transform the inherent dependency to its advantage. In particular, the ecosystem firm can connect the new good with the previous product categories, hence offer a quality bundle, while the competitor has to serve its consumers with the stand-alone quality of the new product category. Our analysis focuses on the dynamics of standalone and built-in quality investments vis-a-vis prices. The result of this study can offer insights to many digital and innovation-driven settings, such as the smart-products[12], where new product categories can

[12]Smart-products comprise physical and digital features that are capable of adapting to consumers and other related products (Maass et al., 2007). These products exists in many industries including smartwearables (such as smarteyewear of *Snap* or smart jacket of *Levis, and Google's Project Jacquard*), health-diagnostics products

be seamlessly connected to existing products through a built-in software designed by ecosystem firms.

One of our main contributions is to evaluate whether the ecosystem leverage (c_1 and λ) posits a threat to the category firm. Intuitively, one would expect the ecosystem leverage to force the category firm to focus on the sub-market that didn't buy the previous categories (Motta's market) or the low-value consumers that are less sensitive to the ecosystem leverage. Our results, however, indicate that this is only partially correct: the inherent relatedness between the product categories (c), and the quality positions of the firms inform whether or not the ecosystem firm keeps its competitor at bay.

In particular, if the new and old goods are complements, the ecosystem leverage improves the welfare of both of the firms regardless of their quality position (stand-alone quality leader or follower). If instead, the new goods is a substitute to the old category ($c < 1$), then the ecosystem firm may keep the category firm at bay, especially when the goods are strong substitutes. Interestingly, the ecosystem leverage hurts the category firm most when it's the stand-alone quality leader. Even if the new good is a weak substitute of the old good the profits of the category firm diminishes both with λ and the built-in connection (c_1). Put it differently; the category firm may benefit from the ecosystem leverage especially when it is a "weak substitute" of the old category (when it's the stand-alone quality follower) while the ecosystem firm is the "strong substitute".

Secondly, we address the industry discussion on whether or not the ecosystem leverage deters the innovation in the new category. Some industry experts point at the ecosystem firms as the reason of slow innovation in connected product categories (Beaver, 2017). Our results indicate that the ecosystem leverage is not always harmful to the category innovation. This leverage can even promote innovation under some circumstances. In particular, if the new good is a complement of the old product class, then the ecosystem leverage increases the quality competition and improves the category innovation. Besides, the ecosystem leverage promotes

(such as diagnostics ecosystem of *Lumos Diagnostics*, smart-learning (such as smarttoys of *Frog*) or smartgadgets (such as 3D Printing Ecosystems of *Frog*).

the category innovation if the goods are weak substitutes and the ecosystem firm is the quality leader. The reason behind is the motivation of the ecosystem firm to increase the overall value of its quality bundle while the competitor, a "weak substitute" to the old category, easily increases its quality investments to cope with ecosystem leverage. However, if the category firm provides a good substitute of the old good (when it's the quality leader), then the quality competition deteriorates while the ecosystem firm focuses on the built-in connection. Further, when the goods are strong substitutes, the stand-alone quality competition decreases and the category innovation diminishes.

We conclude that the category innovation is under the shadow of the ecosystem connection, as industry experts claim, only if the new category is a substitute of the old product categories or when the ecosystem firm is the quality follower even if the new and old categories are weak substitutes. In other words, one can think the inherent connection the new product class has with the old product class as a parallel mechanism that intervenes the ecosystem leverage. When the moderation is in the form of complementarity, inherent connection transforms the ecosystem leverage into a quality enforcing mechanism leading to an increase in the profit margins of the firms improving the producer welfare. If instead, the intrinsic connection is in the form of substitution, the effect of moderation depends on whether or not the ecosystem firm is the quality leader as well as the strength of the substitution.

Furthermore, we also clarify that the firms, regardless of being the ecosystem or the category firm, never prefer "quality leapfrogging". In other words, being a quality leader or a follower is sticky as in the benchmark case where there is no ecosystem leverage. Therefore, we confirm that quality leapfrogging does not affect the dynamics of category innovation or the producer welfare. Our primary results shed light on many new category settings of the smart products industry. For instance, our framework helps in better understanding the recent upturn in category innovation in the smartwatch industry and insistence of the category firms such as Montblanc and Tag Heuer that the ecosystem leverage of Apple granted by its smartphone has not been a threat but a treat (Deloitte, 2017).

Another example is the headphones product category where industry experts discuss whether or

not Apple would position its new headset as a high-end or low-end product, and the implications of Apple's entrance on stand-alone producers such as Bose (Gurman and Wu, 2018). We foresee that in both cases (high-end or low-end); Apple, as the ecosystem firm, and the category firms would collectively benefit from Apple's entrance into the new category. The underlying reason is that the category innovation improves as the ecosystem firm with many previous product categories that are complementary to the headphones takes part in the quality competition, which, in turn, enhances the profit margins of both firms. Our results suggest that Apple benefits more from the ecosystem leverage if it is the stand-alone quality leader and we foresee that Apple would enter the market as quality leader.

Moreover, we provide a comprehensive analysis to understand the short term pricing regimes under preset quality levels. We identified that the ecosystem firm, as the standalone quality leader, should always serve the non-buyers of the base category (Motta's market). However, it should be the first one to leave the ecosystem market if the new product is the "strong substitute" of the base category so as to eliminate the pressure of the inherent substitution while the category firm keeps serving the ecosystem consumers. If instead, the ecosystem firm is the quality follower, it possesses the capability to engage in aggressive price competition and induce its competitor to leave the ecosystem market. These dynamics suggest that the ecosystem firm with low stand-alone quality, but a strong connectivity poses more risk to the category firm compared to an ecosystem firm offering high standalone quality with lower connectedness.

As an extension, we address how the consumer welfare is affected from the ecosystem leverage. Our results suggest that the consumer welfare improves with category innovation. Therefore, the policymakers should be attentive to whether or not the ecosystem firm is the quality leader as well as the type of natural connection the new category has with the previous product classes. It is important to note that when the ecosystem firm is the quality follower in the new category, that is when it is intrinsically a weak substitute of the base category, it possesses the most risk to the total welfare. We also note that if the new good is a complement of the base category or the ecosystem firm is the stand-alone quality leader, the ecosystem leverage is not an adversary of the welfare but instead an enhancer.

As the second extension, we relax our assumption of asymmetric leverage. We consider that the ecosystem firm grants the category firm the option to obtain the ecosystem leverage from its rival by paying a rent. Converting the setting to the symmetric leverage case has changed our results substantially. In particular, there is the potential to eliminate and even reverse the adverse effects of the ecosystem firm on the innovation as well as the welfare of its rival for the case where the category firm is the quality leader. In this case, the ecosystem firm, in fact, is ready to share its leverage for free with the category firm as it receives more in return through increased quality competition and high-profit margins, and hence serving higher willingness to pay consumers. At the same time, the category firm is also willing to receive this advantage. Thus, we find that the aggressive competition of the asymmetric case transform into coopetition if the ecosystem firm is aware of the positive effects of sharing the leverage with its rival. For other instances in which the ecosystem firm engages in a soft competition with its rival, neither the ecosystem firm nor its rival is willing to collaborate.

Thus, we conclude that asymmetric ecosystem leverage is not a threat to the market nor the innovation unless the category firm is the quality leader and the goods are weakly substitutes. Furthermore, the policymaker can eliminate this adverse situation by nudging the ecosystem firm to grant the leverage to its rival, and "coopete".

Aside from the contributions summarized above, some relevant aspects of the new product category competition have not been fully considered in this paper. Above all, we assumed that the ecosystem market is fixed (λ is fixed), that is there exists one-sided influence from the base category to the new category. This assumption reflects the empirical findings that the previously established goods affect the sales of the new product category but not the other way around (Stremersch et al., 2007). However, a fruitful extension would be to examine the two-sided interaction of the base and new category, and the effect of inter-category tension on the ecosystem leverage (c_1 and λ) as well as the category innovation. Besides, we have not included a normative/regulatory analysis where one analytically captures the effect of ecosystem leverage on the category firm that is as efficient as the ecosystem firm in producing the stand-alone quality. In addition, we have not included an analytical discussion that shows the impact of the ecosystem leverage on the consumers. Despite this, our model provides insights as the first

step for a normative/regulatory analysis that may follow as a future work from the framework presented here.

Chapter 4

Keep the Buzz and Binge On: Optimal Content Release Strategies

If binge is really what they want, then let them binge!

Netflix, 2015

4.1 Introduction

This chapter discusses how *release strategies* of an online content provider can boost *viewer engagement*, which informs the success of the platform. We cover three aspects of an online content release strategy: the release of an individual series, organizing the content library with the order of different series and personalization of the content according to specific viewing behavior. "Keeping consumers engaged and covering the content" is a critical metric for online content providers, such as online streaming companies (Lubin, 2016; Patel, 2016; Mosacho, 2017; Karr, 2018) or e-learning platforms. In particular, the ultimate goal of a streaming platform is to generate increasing revenue streams over time while optimizing the investment

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on the content with two levers: acquiring new subscribers and retaining the existing customers (Lombardo, 2018). The element that would activate these two levers is the engagement of the viewers, that is the extent to which users of the online content are interested and actively involved in the content (Limelight Networks, 2017). In fact, viewer engagement is also a key metric in online learning platforms where the primary emphasis is on learning rather than profit maximization. This paper considers both contexts and introduces a network-based model that suggests content release strategies maximizing the engagement of the viewers with the content.

Currently, two standard practices of content release are typical in both online streaming and e-learning platforms: *entire season at once* and *episodic release strategies*. Streaming services such as Netflix and FreeForm or online education programs including EdX and Khan Academy release all the episodes of a series or a course at once. On the other hand, major video-on-demand services such as HBO Now (de Souza, 2017) or e-platforms such as Udacity follow the traditional *episodic release strategy* even when there exists no legal constraints (e.g., contracts with third parties). To understand the motives of the content providers when deploying different release practices, one needs to examine two significant viewing phenomena of online content consumption: *binge-watching* and *water-cooler talk*.

Binge-watching refers to viewing three or more episodes of a series in one viewing session (Schweidel and Moe, 2016) and commonly exists among the users of the entire season at once platforms of online education (Lu et al., 2017) and online streaming (Matrix, 2014). According to a survey, 44% of the male and 56% of the female subscribers of online streaming services in the US are binge-watchers (Adweek, 2018) while 88% of Netflix users acknowledge that they are binge-watching the content (Nielsen, 2017b). The entire season at once strategy encourages viewers to binge-watch and improves their engagement with the story and commitment to watch the rest of the series (Matrix, 2014). This involvement increases the number of monthly subscribers as it improves the value of the whole content library. Therefore, the entire season at once policy has the potential to attract more users by providing them the flexibility to binge-watch which can scale up the subscription base. Indeed, with 51.92 million US subscribers

in 2017, Netflix seems to be performing slightly better than HBO and Amazon.[1] However, there exists a flip side of managing a subscription base with an entire season at once strategy, that is, consumers rapidly consuming the content and hence content providers' expenditures on refreshing their libraries increase.

Streaming services, typically provide users the flexibility to join and leave the platform whenever they wish. As a result of the flexibility, service cancellations tend to peak, with a rate of 50% of the user base for some platforms, when a favorite show is over (Friedman, 2017; Lombardo, 2018). Since the show would be consumed very quickly with binge-watching, the content provider becomes obliged to replenish its library continuously to compete for the attention of viewers by offering brand new content. Extensive investment on the content has the potential to trap the content provider with cash outflows and obligations rising more rapidly than the subscription revenue (Sweney, 2018). Since 2011, this has been the case for Netflix, with debts growing by 42% per year while the revenue growth lags behind by 24% (Trainer, 2017).

Water cooler talk, commonly referred as viewer chatter and excitement about the next chapters of a content (Nathan et al., 2008; Hanna et al., 2011), is observed commonly in episodic releases of education (Lu et al., 2017) and TV series (Matrix, 2014; Wilson, 2014). Twitter, a digital water-cooler, brings about 41% of its users to talk about the TV series and 40% of the tweets (around 110 million Tweets per year) are about a TV series in the UK at peak times (KantarMedia, 2014; Macmillan, 2015).[2] Water-cooler talk may increase the likelihood that viewers watch the series until the season finale, hence translate into increased "subscriber retention" according to the practitioners (Hill, 2012; Sciretta, 2013; Wilson, 2014) as well as the literature (Nathan et al., 2008). Besides, water-cooler talk induces new viewers to be attracted to the story and start watching the series (Nathan et al., 2008). Therefore, an episodic release strategy enables the content provider to benefit from the upfront expenditure over a longer

[1] According to Statista (2017), the number of US subscribers for HBO is 49 million in 2017. During the same year, Amazon Prime had 80 million US subscribers, which includes a bundle of services in addition to video content. Despite the unavailability of direct statistics for Amazon Prime TV, estimates suggest that 40% of the total subscribers use the video content at least once a week (Seitz, 2016).

[2] *Game of Thrones*, an episodically released series, has been the most tweeted about TV show in 2017 while *Stranger Things*, released with an entire season at once strategy, ranked second (Twitter, 2017).

duration due to the viewer-generated conversations around the story (water-cooler talk).

Dynamics described to this end work well under the assumption that viewers are interested in the series with only one episode, and that they would be willing to wait for the next episodes. According to Netflix statistics (2016), however, viewers become interested in a series not necessarily with the first episode, but with episodes 2-7 depending on the particular TV series. As an example, the majority of the viewers were attracted to S*tranger Things*, a popular Netflix show of 2016, as early as the third episode, while it took 7 episodes, on average, for viewers to become interested in *Gilmore Girls* (Netflix, 2016). Besides, according to a survey, only 22% of the viewers would wait for a new episode after watching just one episode whereas, on average, 74% of the binge viewers are committed to returning to the series to experience the rest of the season (Conviva, 2015).

We argue that a *hybrid strategy*, where clusters of episodes, but not necessarily the whole season, are released periodically, may activate both binge-watching and the water-cooler effect at a lower content expenditure. In fact, Amazon Prime and Hulu apply hybrid strategies occasionally, for example, Amazon Prime Video's *Alpha House* and Hulu's *The Handmaid's Tale*.[3] Hence, there seems to be a need for developing a deeper understanding of hybrid release strategies in online streaming context. Hybrid release strategies are also relevant in the online learning context (Garrison and Kanuka, 2004). An episodic release of an online module enforces users to cover the content with a joint pace (water-cooler effect) by design. Synchronous learning promotes both peer-to-peer and tutor-learner interaction, referred to as the most significant aspects of a learner's experience (Richardson and Swan, 2003; Swan, 2003). Besides, the cognitive and neuroscientific literature states that covering the content with breaks is more helpful in learning than massed (binge) learning (Gerbier and Toppino, 2015). On the other hand, an entire season at once strategy encourages learners to interact with inter-related content more, another significant aspect of online learning (Swan, 2003), and mass learn (binge-learning) effectively by linking various concepts, enabling a study flow and promoting a better learner

[3]First three episodes of *Alpha House* and *The Handmaid's Tale* were released together while the rest of the season was released as one episode per week.

experience (Lu et al., 2017). Moreover, an entire season at once strategy provides the viewers with the flexibility to cover the content whenever they want, which is one of the critical aspects of online learning that informs learning experience (Su et al., 2005).

In addition to *hybrid strategies*, personalization of the content among the viewers is another aspect we cover as part of the release strategy. According to the literature, viewers are heterogeneous in their propensity to view the content in bulk (Kubey and Csikszentmihalyi, 2002; Schweidel and Moe, 2016) and to binge learn (Lu et al., 2017). A purely personalized content has the potential to increase viewer engagement by addressing user-specific viewing preferences (Deloitte, 2016a; Friedlander, 2017) while at the same time conversations and excitement around the story (water-cooler effect) would probably diminish (Mosacho, 2017). We aim to capture the trade-off between personalization and group pace to improve viewer engagement with the optimal number of groups of viewers against which to personalize the content. It is important to note that we consider personalization of content through "digital nudges" (Fetherston et al., 2017; Weinmann et al., 2016) such as highlighting, recommending, or reminding a cluster of content (Bonk and Zhang, 2009), common tools used by streaming services as well as relevant tools in the online learning context, rather than refraining the viewers to access the content library.

In this paper, we focus on the challenge of content providers to maximize the engagement of the heterogeneous viewers with a fixed content library. We tackle this problem with a content release strategy having three dimensions: (1) release of an individual series over the planning horizon (multiple subscription periods), (2) order of multiple series within each subscription period, and (3) personalization of the content.

An important aspect and a prerequisite of building a content release strategy that maximizes viewer engagement is measuring the engagement itself. To measure engagement, we adopt the "network-driven contagiousness model" developed by Brockmann and Helbing (2013). Using this model, the literature discusses the spread and diffusion of various complex phenomena such as epidemics (Manitz et al., 2014; Iannelli et al., 2017), opinions (Liu et al., 2015), rumors (Jiang et al., 2016), and delays (Manitz et al., 2017). In their setting, typically the nodes represent

locations between which the underlying event spreads and the edges denote the diffusion of the event. In particular, diffusion (of a disease) is a combination of *the flux* (the population) and *the spread* (infection spread rates) in between two nodes (such as the airports). The "network-driven contagiousness model" produces *effective distance* that combines both the flux and the diffusion parameters to represent all aspects of the underlying network mobility (Iannelli et al., 2017; Manitz et al., 2017). Furthermore, the model unveils the locations where the phenomenon of interest originates and spreads to the rest of the network. To summarize, the "network-driven contagiousness model" converts the mobility in the underlying network into an "effective distance" between each nodes of the network and reveals the locations that have the shortest effective distance to the rest of the network.

According to the literature, viewing behavior is both user-specific and content-driven (Schweidel and Moe, 2016; Lu et al., 2017). Besides, the release of an inter-dependent content influences the viewing behavior (binge-watching versus water-cooler effect) dynamically. Therefore, consumer engagement can be considered as a complex system and hence represented as a complex network[4], where nodes are the episodes of the content, and the edges represent the diffusion of viewing in between the episodes. In our setting, the flux represents the number of viewers while the spread is inversely proportional to the break lengths viewers are giving in between the episodes. We locate the episodes (captivating episodes) where viewer engagement emerges and diffuses to the rest of the content. Similar to a Netflix study (Netflix, 2016), the captivating episodes point out the locations in the story where viewer commitment and willingness to wait for the next episode is maximized. Different than Netflix; however, our model allows for multiple captivating episodes instead of a single one. The captivating episodes, in turn, inform the optimal locations for a cluster finale of a series or for linking the clusters of a series that connect well. For example, if the captivating episodes of a series are 4 and 7, then the clusters of each subscription period would be episodes 1-4, 5-7, and 8-season finale.

A network-driven contagiousness model fits well with our setting since one of the key aspects

[4]We represent the viewer engagement with heterogeneous (various connectivity patterns) and multi-scale complex (many inter-connected nodes) networks.

we are interested in is to locate the captivating episodes through which viewer engagement propagates to the rest of the content. There exist other propagation source detection methods such as Gaussian source estimators, Bayesian beliefs (Pinto et al., 2012; Altarelli et al., 2014) or Jordan center models (Zhu et al., 2013). These methods may as well be used to reveal sources of propagations (see Jiang et al. (2017) for a methodological review), requiring information about the underlying propagation dynamics such as the infected nodes or the exact state of a node at any given time. These models are proven to be effective in epidemics spread especially when prior information on the state of specific nodes can be derived. Besides, the states are generally represented as binary, being infected or not. The network-driven contagiousness model, on the other hand, is based solely on the mobility (effective distance) instead of exogenously derived information on the state of the nodes. Put it differently, the model analyzes the mobility of the network and informs about the state of the nodes endogenously as well as the nature of spread. In the online viewing settings, analyzing engagement based on the mobility is crucial since the online content (episodes, series, courses) is typically an experience good, that is the attributes of the online content is invisible and can be assessed ex-post viewing and based on the viewing behavior (Ward and Ramachandran, 2010; Pine et al., 1998). Therefore, the states of the nodes are not available a priori while the viewing pace (mobility) enables us to infer the states of the nodes ex-post viewing. Thus, the network-driven contagiousness model that determines diffusion and state of the nodes based on the underlying mobility fits well with our setting and enables us to derive viewer engagement solely based on the network mobility (the pace of content coverage).

Once we have designed the contagiousness model, we sequence the episodes of an individual series so that the viewers are motivated to return back to the rest of the content even when they arrive at the cluster finale of their favorite show, and ensure viewer commitment until the next subscription period. We consider the content release design in two contexts, which are commonly cited forms of online content consumption (Google, 2015): online streaming and e-learning. But our model is also relevant in other settings such as digital music services. We apply our technique to the online learning context using the data from an online MBA programme for the period 2015-2017.

Despite the practical relevance, the literature on digital content release strategies is relatively scarce. Existing research addresses the sequential and simultaneous release of substitutable (Calzada and Valletti, 2012; Hennig-Thurau et al., 2007) and complementary versions (Paun, 1993; Choi, 2003) of a product line while the literature has not yet captured the episodic interdependence of online series in the context of release strategies. However, viewing, as a behavioral phenomenon, has received the attention of the literature. In particular, binge viewing and the water-cooler effect (social buzz) have been discussed both in the online streaming and e-learning contexts.

In the case of binge viewing, most studies focus on the existence of the behavior and the drivers behind it. Zhang et al. (2014) verify that the binge consumption behavior, which they refer to as clumpy consumption in the paper, is widespread in online settings. Lu et al. (2017) demonstrate that binge studying behavior is most prevalent when online course content is simultaneously released while the reverse holds for sequential release. Besides, both in the context of TV shows and online learning, binge behavior reflects the viewers' desire to continue watching the content as the story connects more with further viewing and to complete a story (Pena, 2015; Schweidel and Moe, 2016; Lu et al., 2017). Studies suggest that binge viewing is observed more frequently, both in terms of the number of binge-watchers and in terms of the number of episodes chain-watched, for "narrative-driven"(Trouleau et al., 2016; Schweidel and Moe, 2016) series. Considering binge-watching patterns both within and across series, Schweidel and Moe (2016) point out implications of this viewing behavior on advertising effectiveness.

In view of the water-cooler effect, research verifies that existing social interaction and chatter are influential in the context of TV series (Hill, 2012; Hennig-Thurau et al., 2015) as well as in online learning (Richardson and Swan, 2003; Rovai, 2004, 2007). In particular, the literature confirms that the water-cooler talk improves the viewer engagement since it enables viewers to express their thoughts about the content, serves as a "backchannel" complementing the story and creates a feeling of social presence (Nathan et al., 2008; Hill and Benton, 2012; Schreiber, 2012; Proulx and Shepatin, 2012; Nee and Dozier, 2017). In the online learning field, the terminology frequently used instead of water-cooler effect is "social interaction". Similar to the tv series context, the research suggest that interaction among the users of an online

learning platforms increases the engagement by creating a sense of social presence, facilitates deep learning and enables students to share their understanding and standpoints (McInnerney and Roberts, 2004; Anderson, 2004; Garrison and Cleveland-Innes, 2005). Therefore, the water-cooler effect, in both contexts, enhances the viewing experience beyond the content itself (Lovett and Staelin, 2016; Harrington et al., 2013; Richardson and Swan, 2003), and motivates the users to anticipate the future material (Xiong and Bharadwaj, 2014).

Movie recommendation is a popular research field, where the primary goal is to predict viewer-content matching based on the users' past behavior (information filtering techniques), the choices of similar users (collaborative filtering), or the keywords entered by the user (search engine method), see Yu et al. (2016) for a detailed review. In the online streaming services, typically hybrid methods that leverage from information and collaborative filtering techniques (Murthi and Sarkar, 2003; Adomavicius and Tuzhilin, 2005; Martínez et al., 2009; Barragáns-Martínez et al., 2010) are used to ensure that viewers find a content that matches best to their preferences (Krauss et al., 2013).

Our method is different than the existing movie recommendation systems. As a step further, we emphasize the viewer groups that cover the content with the same pace and deploy the same release schedule to each group such that the viewer engagement is maximized. We infer which material to recommend from the complex interaction of the varied content, heterogeneous user behavior (related to information filtering techniques), and the influence of the group of viewers (related to collaborative filtering) on the individual user. While the information of interest is typically self-reported data such as search queries or ratings in the recommendation systems, we solely focus on the time stamps, that is, when an episode of the library is started to be viewed and completed by each user. Using timestamps, we model viewers' engagement, both at the individual and the aggregate level, suggest viewer groups that have similar behavior, and recommend the content that would go well together for each group. Further, we take into account the influence of the recommendation or in general the release strategy on viewers' engagement with the content. Our method is not a substitute for the movie recommendation systems that predict the best content based on user demographics. Instead, we emphasize the link between the engagement and release schedules. Therefore, we complement movie

recommendation systems by picturing the discussion as a complex system while at the same time using limited data.

To our knowledge, we are the first to reflect upon empirically verified viewing behavior (binge-watching and water-cooler effect), and apply it to the release of interdependent series of content. We extend the product release literature with recent behavioral phenomena. We also delve into the release of interdependent episodes of a series linked with an underlying story and of multiple series connected with viewers' individual perceptions of complementarity. Characterizing the captivating episodes based on the past viewing data, we suggest optimal release strategies for online content providers based on recent viewing data. In particular, our findings point out strategy implications for online content designers, who are contemplating to refresh their libraries with new seasons, or considering to expand into new geographical regions. Further, our findings can suggest a release strategy for an entirely new series based on its similarity (in terms of its genre) with an already released content.

The rest of this chapter is as follows. In Section 2, we describe our methodology to measure viewer engagement and we illustrate the characterization of the captivate episodes. In Sections 3, we focus on the release strategy with the three dimensions mentioned above. In Section 4, we apply our method to an online learning setting, and we conclude in Section 5.

4.2 Measuring Viewer Engagement

The overarching idea of this chapter is to analyze consumer viewing behavior using network theory and design content release strategies that maximize the total engagement of the viewers. In this section, we focus on the viewing behavior and introduce the methodology to measure engagement and locate the captive episodes. To this end, we adopt the "network-driven contagiousness model" developed by Brockmann and Helbing (2013) that characterizes the spread of various phenomena (such as rumors, ideas, diseases) in complex networks. The novelty of the method comes from measuring the link between two nodes with the "effective distance" derived solely from the mobility of the underlying network. In our case, the underlying network has

two constituents: *break lengths* between any two episodes available in the content (which we will use to measure binge-watching) and the *flux of viewers* covering an episode after watching another one (which we will use to measure the water-cooler effect).

Running Example: We utilize a running example throughout Section 2 and 3 to illustrate our methodology. In this example, we consider a content library (synthetic data) that consists of a single soap opera, each of which consist of 10 episodes and 10 viewers who subscribed to the online streaming service. We provide the timestamps of content coverage associated with each user, and we refer the reader to Appendix C for this dataset. In this section, to simplify the discussion, we focus an a single soap opera denoted by "A" in the dataset and its episodes Ai, $i = 1, 2, \ldots, 10$.

4.2.1 *Engagement Network*

Definition: A ***network*** (or a *graph*) G is a tuple $G = (\mathcal{V}, \mathcal{E})$, consisting of the set of nodes $\mathcal{V}$ and the set of links $\mathcal{E}$ connecting them. The network can be described by its $V \times V$ adjacency matrix which indicates the existence and the weight of a connection (edge) between each pair of nodes. We consider an undirected weighted network, i.e. the adjacency matrix is symmetric and its elements reflect the strengths of the connections. The nodes are defined as episodes in a given series, which can consist of one or more seasons.[5] $V = |\mathcal{V}|$ is the number of nodes in the network (i.e., the number of episodes). The links between the episodes arise from two types of relations (leading to two different adjacency matrices):

- break distances (related to the binge viewing behavior, see Section 4.2.2);
- flux of viewers (related to the water-cooler effect, see Section 4.2.1.2),

that arise from the viewing behavior of the users ($n = 1, \ldots, N$) of the online library.

[5]In the online learning case, a series refers to a an online course, a season refers to a session in the course, and an episode refers to a lesson within a session.

Running Example: The engagement network associated with soap opera "A" consists of $V = 10$ nodes and the edge between any two nodes is a combination of break lengths (relating to the binge viewing behavior) and flux of viewers (relating to the water-cooler effect), which we derive in the subsequent sections. The links between the episodes of a soap opera is informed by two constituents, which we discuss next.

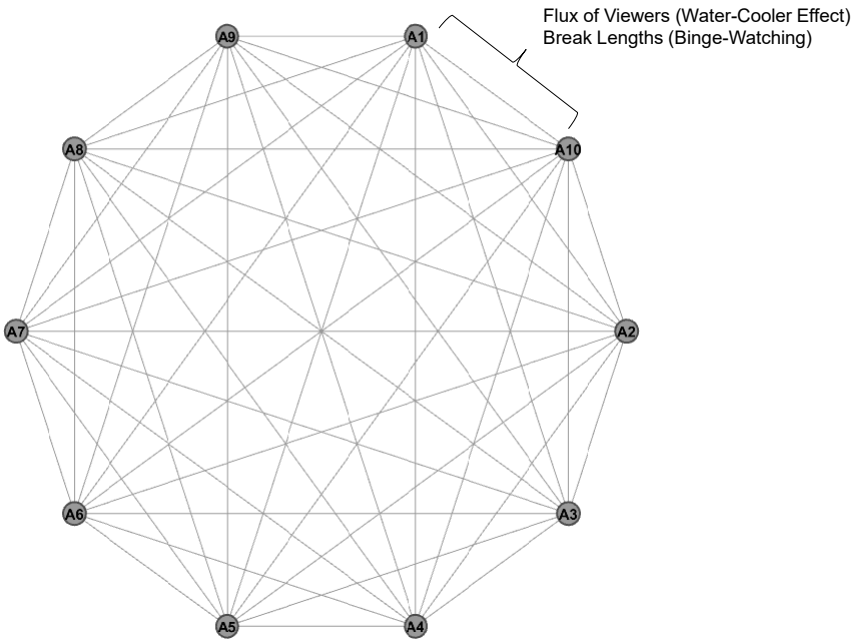


Figure 4.1: Engagement Network Associated with Soap Opera A

4.2.1.1 *Break Distance*

Intuitively, the distance between two episodes of the content library is determined by the breaks that viewers are taking between them. We represent the break distances with l, which is a $V \times V$ symmetric matrix whose elements $l_{i,j} = l_{j,i}$ describe how close episodes i and j are depending on whether (1) they were viewed in the same watching session with no break, and (2) the length of the break between them, if there was one. The matrix is constructed in two steps:

Step 1. Mapping the individual viewer network.

First, we derive the topological link associated with binge-watching behavior, namely the binge variable $b_{i,j}(n)$. We note that the topological link (or distance) is a terminology of network

theory referring to the conceptual way in which constituent nodes of the network are related, and in our case it is the viewing pace, and hence the binge watching behavior.

The first distinction we make when analyzing the binge-watching behavior of a viewer (n) when watching episodes (i, j) is whether or not these episodes are watched consecutively. To be more specific, if a viewer (n) has watched other episodes in between (i, j), then we evaluate these episodes non-binge watched. In that case, we assign $(b_{i,j}(n) = 0)$, the minimum value for the binge variable $(b_{i,j}(n) = 0)$.

If the viewer n has watched these episodes consecutively, then we make a further distinction based on whether or not the watching occurred in the same viewing session, that is in one sitting. The definition of the viewing session and its duration depend on the context, which we clarify further when we discuss the binge variable in our running example for soap operas and in Section 4 for the education context. If viewer (n) has covered episodes (i, j) in the same viewing session, then we consider them as binge-watched (first order binge watching). Formally, we assign $(b_{i,j}(n) = 1)$, and hence the binge variable attains its maximum. Apart from the first order binge watching, the viewers may still cover the episodes (i, j) in a short period of time by returning back to the platform with short breaks in between different viewing session (second order binge watching).

In this case, we assign the binge variable $b_{i,j} \in (0, 1)$ based on the break length $(break_{i,j}(n))$ between the two consecutive episodes (i, j) relative to the maximum and the minumum breaks $(break^{Max}(n))$, and $break^{Min}(n))$ the viewer n takes when consuming the overall content, that is $\frac{break_{i,j}(n)-break^{Min}(n)}{break^{Max}(n)-break^{Min}(n)}$. To map the relative break length as a binge variable, we utilize a uniformly distributed normalized link $(1 - \frac{break_{i,j}(n)-break^{Min}(n)}{break^{Max}(n)-break^{Min}(n)})$. We remind the reader that a viewing network is an undirected graph, and hence we consider $break_{i,j}(n) = break_{j,i}(n)$, and hence the adjacency matrix $b(n)$ that consists of entries $b_{i,j}(n)$ is symmetric. Formally, we

define $b_{i,j}(n)$ as in the following:

$$b_{i,j}(n) = \begin{cases} 1, & \text{if } i \text{ and } j \text{ were watched in same viewing session with no break} \\ 1 - \frac{break_{i,j}(n) - break^{Min}(n)}{break^{Max}(n) - break^{Min}(n)}, & \text{if } i \text{ and } j \text{ were not watched in same viewing session but are consecutive} \\ 0, & \text{if } i \text{ and } j \text{ were not watched in same viewing session and are not consecutive} \end{cases} \tag{4.1}$$

Running Example: In our example, we consider that a viewing session finishes if a viewer takes a break longer than one hour ($break_{i,j} > 1$ hour) between the two episodes (i, j) that were watched consecutively following the definition of Schweidel and Moe (2016). We consider episodes (i, j) to be binge-watched if and only if ($break_{i,j} \leq 1$ hour). As an example, we provide links $b_{i,j}(n)$ of the viewing network for soap opera A belonging to user $n = 1$ in Figure 4.2. We note that $break^{Max}(n) = 479.16$ and $break^{Min}(n) = 0.03$ hours for this user (see the table $break(n)$ in Figure 4.2). She watched episodes 1 and 2 within the same viewing session and hence $b_{1,2} = 1$. On the other hand, the user had a break of length 198.39 hours when switching from episode 5 to 6, and hence $b_{5,6} \neq 1$. In particular, the binge variable is calculated as $1 - \frac{break_{i,j}(n) - break^{Min}(n)}{break^{Max}(n) - break^{Min}(n)} = 1 - \frac{198.39 - 0.03}{479.16 - 0.03} = 0.5860$.

viewer (1)

Soap Opera	Episodes	Start Time	End Time
A	1	27/06/2018 20:55	27/06/2018 21:55
A	2	27/06/2018 22:05	28/06/2018 22:55
A	3	18/07/2018 22:05	18/07/2018 22:52
A	4	19/07/2018 00:51	19/07/2018 01:51
A	5	19/07/2018 01:53	19/07/2018 02:51
A	6	27/07/2018 09:14	27/07/2018 10:14
A	7	27/07/2018 11:04	27/07/2018 11:54
A	8	27/07/2018 14:31	27/07/2018 15:19
A	9	28/07/2018 06:13	28/07/2018 07:03
A	10	28/07/2018 10:45	28/07/2018 11:41

break length, $break(1)$

	1	2	3	4	5	6	7	8	9	10
1	0.00	0.17	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2	0.17	0.00	479.16	NaN	NaN	NaN	NaN	NaN	NaN	NaN
3	NaN	479.16	0.00	1.99	NaN	NaN	NaN	NaN	NaN	NaN
4	NaN	NaN	1.99	0.00	0.03	NaN	NaN	NaN	NaN	NaN
5	NaN	NaN	NaN	0.03	0.00	198.39	NaN	NaN	NaN	NaN
6	NaN	NaN	NaN	NaN	198.39	0.00	0.83	NaN	NaN	NaN
7	NaN	NaN	NaN	NaN	NaN	0.83	0.00	2.62	NaN	NaN
8	NaN	NaN	NaN	NaN	NaN	NaN	2.62	0.00	14.89	NaN
9	NaN	NaN	NaN	NaN	NaN	NaN	NaN	14.89	0.00	3.70
10	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	3.70	0.00

binge variable, $b(1)$

	1	2	3	4	5	6	7	8	9	10
1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	1.00	1.00	0.59	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.59	1.00	1.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.99	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.99	1.00	0.97	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.97	1.00	0.99
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99	1.00

Figure 4.2: Running Example: Mapping from break length to binge variable

Secondly, we derive the topological distances ($l_{i,j}(n) \in [0,1]$) from the binge variable ($b_{ij}(n) \in [0,1]$) for each viewer n. In our case, the nodes of the viewer engagement network are conceptually linked with the breaks the viewers are taking in between the episodes. When making the conversion $b_{i,j}(n) \rightarrow l_{ij}(n)$, we use a beta distribution $F(x, \alpha, \beta)$, frequently used when representing variability in fractions, to smoothen the corners. Put it differently, it decreases the weight given to the tails of the distribution, the extreme values. The parameters of the beta distribution denote the weight put into central values, and the bigger the parameters the less the tails are emphasized. In our case, this conversion reduces the importance of small breaks that a viewer takes, for instance, to drink some water or to bring more pop-corn, and it increases the weight of the longer breaks where a viewer is in fact disconnected from the content.

Therefore, we construct the $l(n)$ matrix for each viewer $n = \{1, 2, \ldots, sN\}$, where the break distance $l_{i,j}(n)$ is equal to $1 - F(b_{i,j}(n), \alpha, \beta)$, the mirror image of the beta cumulative distribution function (CDF) of the binge variable $b_{i,j}(n) \in [0,1]$. Figure 4.3 illustrates a comparison of the binge variable ($b_{i,j} \in [0,1]$) and the topological distance ($l_{i,j} \in [0,1]$).

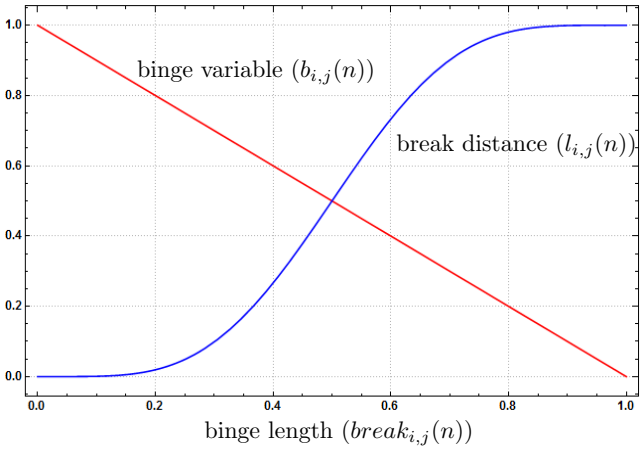


Figure 4.3: Binge variable ($b_{i,j}(n)$) versus Break Distance ($l_{i,j}(n)$), for $break^{Max} = 1$, $break^{Min} = 0$

The binge variable is strongest and the break distance $l_{i,j}(n)$ is smallest if episodes i and j were

watched in the same viewing session. The opposite holds when the episodes are not covered respectively.[6]

Running Example: We report the break lengths of our running example in the table titled $break(1)$ in Figure 4.2. We use the beta distribution $F(x, 5, 5)$, and our choice of parameters reflects the emphasis we would like to put on central values. As an illustration, we transform the binge variable associated with the first viewer ($n = 1$) in between the episodes $(5, 6)$ to the break length ($b_{5,6}(1) = 0.586 \rightarrow l_{i,j}(1)$) with the mapping $1 - F(0.59, 5, 5) = 0.2965$. We remind the reader that the choice of shape parameters of the beta distribution inform the emphasis we put on the central values and how much we neglect the tails. For instance, the break length ($b_{5,6}(1) = 0.0.586$) translates into a break distance $1 - F(0.586, 2, 2) = 0.3723$ when we increase the emphasis on tails by decreasing the value of the shape parameters.

binge variable, $b(1)$

	1	2	3	4	5	6	7	8	9	10
1	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	1.0000	0.9959	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.9959	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	1.0000	1.0000	0.5860	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.5860	1.0000	1.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.9946	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9946	1.0000	0.9690	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9690	1.0000	0.9923
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9923	1.0000

break distance, $l(1)$

	1	2	3	4	5	6	7	8	9	10
1	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	0.0000	0.0000	0.2965	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	0.2965	0.0000	0.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000

Figure 4.4: Running Example: Mapping from binge variable to break distance

Step 2. Aggregating over all viewers.

After constructing the individual $l(n)$ matrices, we obtain the aggregate l matrix by averaging

[6]We note that one can directly map $break_{i,j}(n) \rightarrow l_{i,j}$ instead of the mapping $break_{i,j}(n) \rightarrow binge_{i,j}(n) \rightarrow l_{i,j}$. We prefer to have the intermediary step where we show the link between the break lengths and the binge behavior $break_{i,j}(n) \rightarrow binge_{i,j}(n)$ since it differentiates between binge-viewing and taking small breaks without binge-watching. This distinction has also been emphasized by the empirical literature (Schweidel and Moe, 2016). In addition, we use the mapping $break_{i,j}(n) \rightarrow binge_{i,j}(n)$ when discussing the personalization of the content in Section 4.3.3.

across all N viewers.

$$l_{i,j} = \sum_{n=1}^{N} \frac{l_{i,j}(n)}{N} \tag{4.2}$$

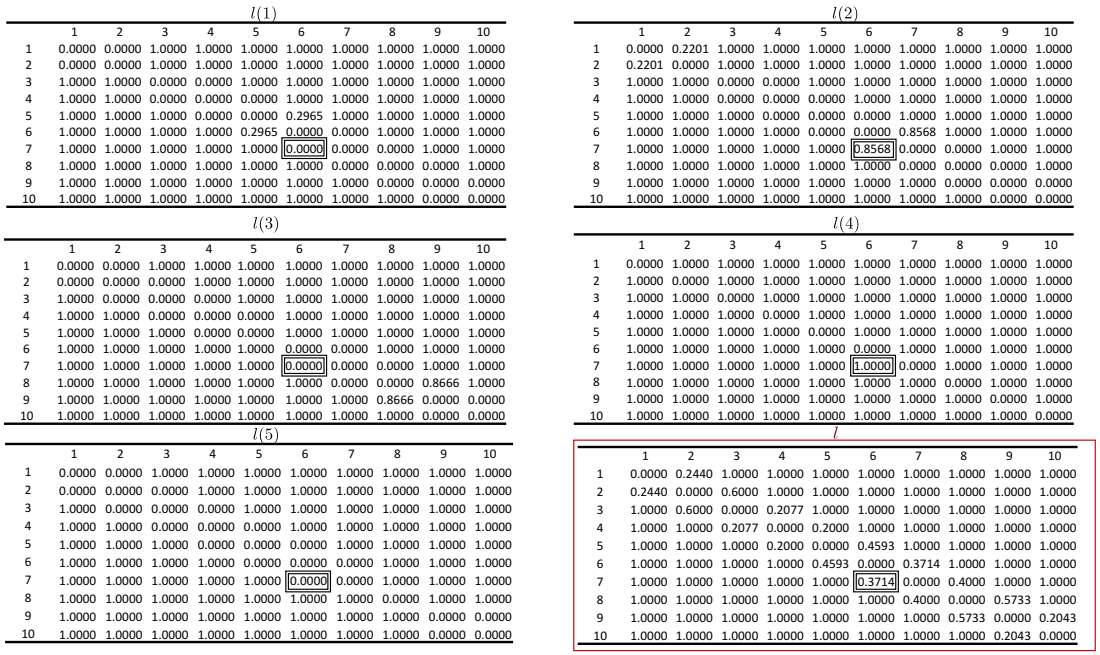
The elements $l_{i,j}$ of the resulting matrix can take values in $[0, 1]$, where a value close to zero (one) means that most or all of the viewers watched episodes i and j in the same viewing session (had a long break).

Running Example: We have five viewers ($n = 1, 2, , 5$) whose binge-watching behavior is summarized with the break distances $L(n)$. We aggregate over these viewers to compute the aggregate binge-watching behavior. As an example, the first, third and fifth viewer binge-watched episodes 6 and 7, that is $l_{6,7}(n) = 0$ for $n = 1, 3, 5$, while the second user had a long break with break distance $l_{6,7}(2) = 0.8568$ and the fourth viewer has not watched the episodes ($l_{6,7}(4) = 1$). The aggregate distance between episodes 6 and 7 is $l_{6,7} = 0.3714$, indicating a strong aggregate binge-watching behavior despite some viewers taking long breaks. We refer the reader to Figure 4.5 for a summary of the "break distance" matrix. Figure 4.6 illustrates the binge-watched episodes in the network, where the link weights $(1 - break_{i,j})$ represent how quickly two adjacent episodes are viewed in succession.[7] One can immediately observe from the graph that episodes $1, 2$ and $9, 10$ are most prone to be binge-watched while the viewers take long breaks between episodes 2 and 3, and hence it is a vulnerable point where engagement is low. However, as we argued before, binge-watching is only one aspect of engagement, and we next cover the water-cooler effect, or the flux of viewers.

4.2.1.2 *Flux of Viewers*

In this subsection, we cover the mobility of the underlying network, which arises due to the number of viewers moving from one episode (node) to another episode. In particular, we are interested in the transition probability associated between any two nodes i and j of the network. The transition probability discussion leads us to the link between any episodes of the content that arises due to the flux of viewers (water-cooler effect).

[7] We used Gephi Version 0.9.2 to visualize the network.



$l(1)$

	1	2	3	4	5	6	7	8	9	10
1	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	0.0000	0.0000	0.2965	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	0.2965	0.0000	0.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000

$l(2)$

	1	2	3	4	5	6	7	8	9	10
1	0.0000	0.2201	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.2201	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.8568	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.8568	0.0000	0.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000

$l(3)$

	1	2	3	4	5	6	7	8	9	10
1	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.8666	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8666	0.0000	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000

$l(4)$

	1	2	3	4	5	6	7	8	9	10
1	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000

$l(5)$

	1	2	3	4	5	6	7	8	9	10
1	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000

l

	1	2	3	4	5	6	7	8	9	10
1	0.0000	0.2440	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.2440	0.0000	0.6000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	0.6000	0.0000	0.2077	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	0.2077	0.0000	0.2000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	0.2000	0.0000	0.4593	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	0.4593	0.0000	0.3714	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.3714	0.0000	0.4000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.4000	0.0000	0.5733	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.5733	0.0000	0.2043
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.2043	0.0000

Figure 4.5: Running Example: Obtaining the aggregate break distances

Step 1. Flux of viewers between the episodes i, j.

Flux between any two nodes of a network can be passengers traveling between two terminals i and j (Brockmann and Helbing, 2013) or traffic load on a road connecting towns i and j (Manitz et al., 2017). In our case, $F_{i,j}$ is the number of viewers who have watched episodes j given that they have seen episode i, and hence the flux between i and j such that $i \neq j$. Formally, we have

$$F_{i,j} = \sum_{n=1}^{N} 1_{i,j \in e^V(n)} \tag{4.3}$$

where $e^V(n)$ is subset of nodes (episodes) that were watched by viewer n and $1_{i,j \in e^V(n)}$ is an indicator function which is equal to one if both episodes i and j were watched by viewer n and zero otherwise. Carrying out the discussion to the whole network, we build a symmetric matrix F whose elements $F_{i,j} = F_{j,i}$ reflect the flux of viewers between any two nodes i, j.

Running Example: In our dataset all of the episodes of the soap opera "A" have been covered by some of the viewers, and hence $e^V(1) = e^V(2) = e^V(3) = \mathcal{V}$, while viewer 4 and viewer 5 skipped some episodes and hence $e^V(4) \neq \mathcal{V}$, $e^V(5) \neq \mathcal{V}$. In particular, $e^V(4) = \{1, 2\}$ and $e^V(5) = \{1, 2, 3, 4, 5, 6, 7, 9, 10\}$. Therefore, the flux between episodes 1 and 2 is $F_{1,2} = 5$ viewers while the flux between the episodes 8 and 9 is $F_{8,9} = 4$ viewers. Figure 4.7 shows the matrix F

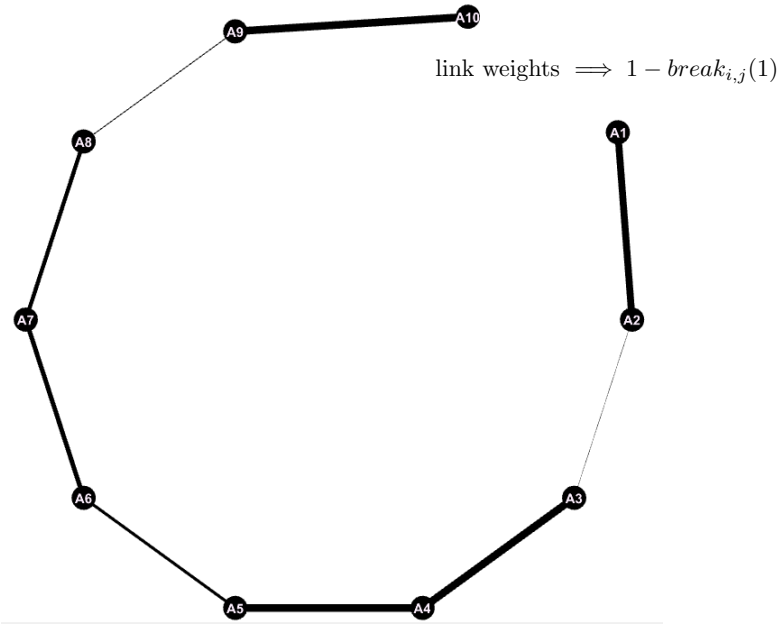


Figure 4.6: Running Example: Representing Binge Watching Behavior with "Break Distances"

associated with soap opera A.

Step 2. Generating the flux.

Next, we generate the transition probabilities from the flux matrix. The transition probability p_{ij} indicates the likelihood that the next episode, a viewer watches is j given that she has watched episode i. Formally, we have

$$p_{i,j} = \frac{F_{i,j}}{\sum_{j'=1}^{V \setminus i} F_{i,j'}}, \tag{4.4}$$

for $i \neq j$. The literature interprets the transition probability i, j as the shock that spreads to node j given that the shock is in the node i (Brockmann and Helbing, 2013; Manitz et al., 2017). The *shock* can be thought of as an infection in disease spread (Brockmann and Helbing, 2013) or as a travel delay (Manitz et al., 2017). In our case, the shock is the "viewer engagement": how likely is a viewer to watch episode j given that she has watched episode i.

Next, we construct a matrix P (the flux matrix) with entries p_{ij}. It is important to note that matrix P is not symmetric $p_{i,j} \neq p_{j,i}$: even though the flux between a given pair of nodes is the same, i.e., $F_{i,j} = F_{j,i}$, the total flux of node i might be different from that of node j. We note that in network terminology, the total flux of a node i is in fact the "node strength", that is

the sum of weights of links connected to the node i.

Running Example: Figure 4.7 shows the flux matrix P for the running example. To illustrate how we construct matrix P from F, the node strength of the first episode is $\sum_{j'}^{V} F_{1,j'} = 36$, and therefore the probability that a viewer watched, say episode 10 given that she has viewed episode 1 is $p_{1,10} = \frac{3}{36} = 0.083$. We illustrate the water-cooler effect through transition probabilities in Figure 4.7.

F

	1	2	3	4	5	6	7	8	9	10
1	NA	5	4	4	4	4	4	4	4	3
2	5	NA	4	4	4	4	4	4	4	3
3	4	4	NA	4	4	4	4	4	4	3
4	4	4	4	NA	4	4	4	4	4	3
5	4	4	4	4	NA	4	4	4	4	3
6	4	4	4	4	4	NA	4	4	4	3
7	4	4	4	4	4	4	NA	4	4	3
8	4	4	4	4	4	4	4	NA	4	3
9	4	4	4	4	4	4	4	4	NA	3
10	3	3	3	3	3	3	3	3	3	NA
Aggregated Weight	36	36	35	35	35	35	35	35	35	27

P

	1	2	3	4	5	6	7	8	9	10
1	NA	0.14	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
2	0.14	NA	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
3	0.11	0.11	NA	0.11	0.11	0.11	0.11	0.11	0.11	0.11
4	0.11	0.11	0.11	NA	0.11	0.11	0.11	0.11	0.11	0.11
5	0.11	0.11	0.11	0.11	NA	0.11	0.11	0.11	0.11	0.11
6	0.11	0.11	0.11	0.11	0.11	NA	0.11	0.11	0.11	0.11
7	0.11	0.11	0.11	0.11	0.11	0.11	NA	0.11	0.11	0.11
8	0.11	0.11	0.11	0.11	0.11	0.11	0.11	NA	0.11	0.11
9	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	NA	0.11
10	0.08	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09	NA

Figure 4.7: Running Example:Transition Probabilities (The Flux)

4.2.2 *Effective distance (Engagement)*

We have discussed two constituents of the underlying network, the break distance (the topological distance) and the flux of viewers (the mobility). We note that these two elements are not necessarily discount, that is the break distance (the topological distance) implies the flux of viewers (the mobility) while the opposite is not true. While we acknowledge this is as weakness of our model, it may at the same time be relevant in the online learning where forums discussions are used as the water-cooler retrospectively to cover the backchannel discussions.

Next, we integrate the topological and mobility aspects of the underlying phenomena, which

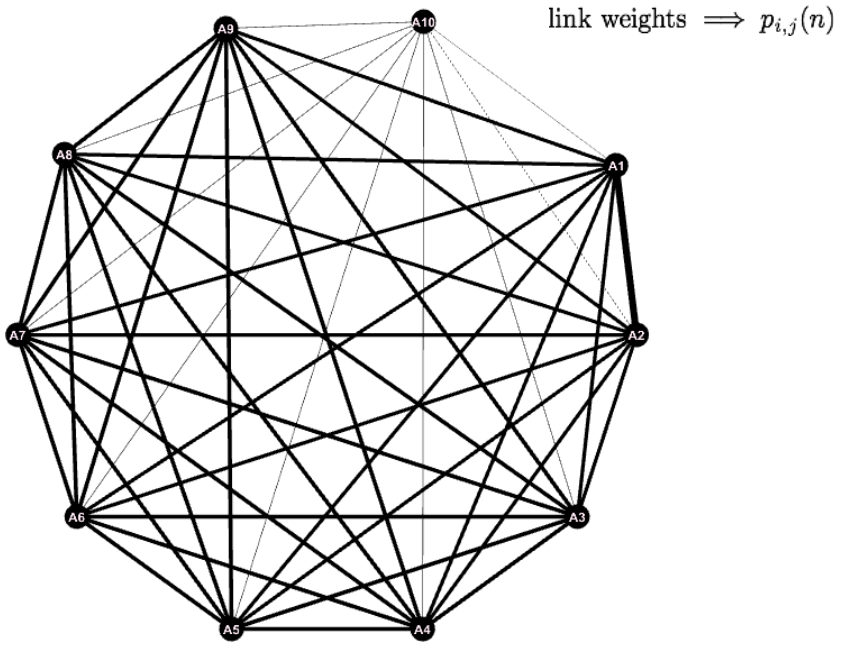


Figure 4.8: Running Example: Representing The Water-Cooler Effect with "flux"

the engagement of viewers in the content, and generate the "effective distance":

Definition: adopted from Brockmann and Helbing (2013); Manitz et al. (2017)

The **shortest effective distance** $d(i, j)$ is the distance of the shortest path from i to j among the collection of all possible paths $\Gamma_{i,j}$. In particular, the effective distance along a path γ from the collection of all possible paths $\Gamma_{i,j}$ is composed of two elements: the "topological distance" (break distances in our case) and the "network mobility" (flux transition probabilities in our setting).

We note that a path γ doesn't necessarily dictate a natural order between the episodes, that is one can have a path $1 \to 3 \to 2$ between the nodes 1 and 2. In the online streaming setting, this pace of content coverage may not sound realistic as one would except the most viewers to cover the content with respect to the natural order, that is $1 \to 2 \to 3$. This is not a problem in our setting since we have an optimization problem that minimizes the path lengths. Therefore, if the majority of the viewers covered the content within similar lines to the natural order of the episodes, the the optimization algorithm wouldn't pick this route. We note that the shortest effective distance will be also used when we order various series where there exists no natural order, and hence this type of modeling enables us to use the effective distance in

various settings.

The topological distance is denoted by $L(\gamma) := \left(\sum_{i',j' \in \gamma} l_{i',j'}\right)$ for a path γ, that is the sum of break distances between the nodes of the path. In view of the the network mobility, denoted by $P(\gamma)$ for a path γ, we utilize the transition probabilities $p_{i',j'}$ (see Equation 4.4) where $i', j' \in \gamma$. In particular, we compute the transition probability from a source node i to a target node j through the route say $\gamma := i \to \ldots \to j$ as $p_{i,j} = \prod_{i',j' \in \gamma} p_{i',j'}$ assuming that the transition probabilities $p_{i',j'}$ are independent from each other. We acknowledge that the Markovian assumption is a simplification in this setting as probability of switching from one episode to another may be dependent on the previous sidesteps watched.

We use logarithmic transformation to convert the multiplicative link weights $p_{i,j} = \prod_{i',j' \in \gamma} p_{i',j'}$ to additive distances $-p_{i,j} = -\sum_{i',j' \in \gamma} p_{i',j'}$, which, in turn, enables us to use shortest path algorithms that requires additive path distances. Therefore, the network mobility is denoted by $P(\gamma) := \left(\sum_{i',j' \in \gamma} \log p_{i',j'}\right)$.

The final step is to combine these two continents, the topological distance and the network mobility. We note that the scales of the two elements are not compatible that is the the topological distance $L(\gamma)$ is summation of terms $l_{i',j'}$ that are in the interval $[0, 1]$ while the network mobility $P(\gamma) \in (-\infty, 0]$ as we have $p_{i',j'} \in [0, 1]$. Thus, one needs to use a weighting factor w to account for the incompatibility. We note that the scale of w depends on the application and the range of the entries of $L(\gamma)$ and $P(\gamma)$ informs w. Thus, we combine the two constituents, the topological distance and the network mobility and derive the shortest effective distance as:

$$d(i, j) := \min_{\gamma \in \Gamma_{i,j}} [L(\gamma) - wP(\gamma)] \tag{4.5}$$

To simplify the discussion, we further define $e(\gamma) := \sum_{i',j' \in \gamma} (l_{i',j'} - w \log p_{i',j'})$, namely the effective distance of path (γ), and hence it follows that the shortest effective distance $d(i, j) := min_{\gamma \in \Gamma_{i,j}} e(\gamma)$.

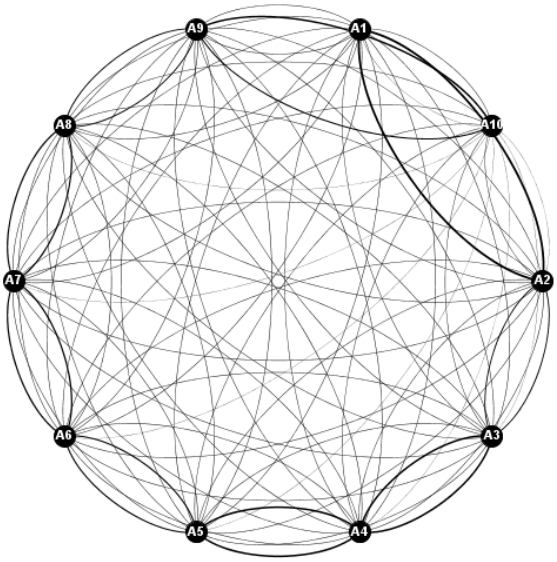
Intuitively, shorter (longer) effective distance between two nodes means that they are located "close" to each other ("distant" from one another) and likely (not likely) to diffuse the phe-

nomena of interest. In the online viewing context, a shorter effective distance between the two episodes implies that the viewer engagement is strong between them.

Running Example: Let us consider a path γ from episode 1 to 7, say $\gamma = 1 \rightarrow 2 \rightarrow 7$. First, we note that we use a weighting factor $w = 1$ in our example. The reason behind this is that our graph satisfies the triangular inequality and hence we have $d_{i,j} = l_{i,j} + p_{i,j}$, and the scales of $l_{i,j}$ and $p_{i,j}$ are relatively compatible. The sum of the break distances $L(\gamma)$ along this path is $l_{13} + l_{37} = 1 + 1 = 2$ and the sum of log transition probabilities equals to $\log p_{13} + \log p_{37} = 2 \times \log(0.1143) = -4.34$. Therefore, the effective distance associated with path γ equals to $e(\gamma) = L(\gamma) - \left(\sum_{(u_i u_{i-1}) \in \gamma} \log p_{u_i u_{i-1}}\right) = 6.34$. Alternatively, we can calculate the effective distance of the path $\gamma = 1 \rightarrow 2 \rightarrow 7$ as the sum of sub path distances, $e(\gamma) = e(1 \rightarrow 2) + e(2 \rightarrow 7)$. Figure 4.9 shows the effective distances for paths $\gamma = i \rightarrow j$.

$d_{i,j} = e(\gamma = i \rightarrow j)$

i \ j	1	2	3	4	5	6	7	8	9	10
1	NA	2.22	3.17	3.17	3.17	3.17	3.17	3.17	3.17	3.20
2	2.22	NA	2.77	3.17	3.17	3.17	3.17	3.17	3.17	3.20
3	3.20	2.80	NA	2.38	3.17	3.17	3.17	3.17	3.17	3.20
4	3.20	3.20	2.38	NA	2.37	3.17	3.17	3.17	3.17	3.20
5	3.20	3.20	3.17	2.37	NA	2.63	3.17	3.17	3.17	3.20
6	3.20	3.20	3.17	3.17	2.63	NA	2.54	3.17	3.17	3.20
7	3.20	3.20	3.17	3.17	3.17	2.54	NA	2.57	3.17	3.20
8	3.20	3.20	3.17	3.17	3.17	3.17	2.57	NA	2.74	3.20
9	3.20	3.20	3.17	3.17	3.17	3.17	3.17	2.74	NA	2.40
10	3.48	3.48	3.46	3.46	3.46	3.46	3.46	3.46	2.66	NA



link weights $\Longrightarrow 1/d_{i,j}$

Figure 4.9: Running Example:Shortest Effective Distances and Engagement Graph

Thus, we include both the binge-watching and the water-cooler effect when measuring engagement. If the probability of a path equals one (when the water-cooler effect is maximum), then the effective distance is the topological distance (informed solely by the binge-watching behavior). If the probability of a path is zero (no water-cooler effect), then the effective distance is maximum.

Running Example: In our example, the graph satisfies the triangular inequality, i.e., a path $i \to j$ is always shorter than a path $i \to k \to j$. Therefore, the shortest path to any target node j from and source i is the direct path $\gamma = i \to j$. Hence, the shortest effective distances $d_{i,j}$ are equal to $e(\gamma = i \to j)$, see Figure 4.9, left panel. We observe that the viewer engagement between the episodes 1 and 2 is the highest with shortest effective distance ($d_{i,j} = 2.22$) compared to the rest of the graph.

In real life cases, it is more likely to have disconnected content or uneven links even if all the nodes are connected. In these more general settings, one can employ Dijkstra's algorithm, which is a fast path search method, to find the shortest path $\gamma \in L(\gamma)$ from the source node u to the target node v (Manitz et al., 2017).

4.2.3 *Engagement Centrality and Captive Episodes*

Up to this point, we explained the shortest effective distance $d_{i,j}$ as a measure of viewer engagement between any two episodes i, j. In this section, we focus on individual nodes and compute a score, namely the "engagement centrality" score, that shows how captive the node is in terms of diffusing the viewer engagement to other nodes of the network. Formally, EC_i denotes the engagement centrality score for a node $i \in \mathcal{V}$.

When computing the engagement centrality scores and locating the captive nodes, we follow the methodology introduced by Brockmann and Helbing (2013). According to Brockmann and Helbing (2013), "concentric nodes" that have a combined low mean and standard deviation of shortest effective distances ($d_{i,j}$) are the points that are central in spreading the underlying phenomena. There exist two constituents implying that an underlying phenomenon is more likely to be spread from node i to the rest of the network: small average effective distance to other nodes (μ_i) and small variability (σ_i) of the distances across the nodes. Analogous to Brockmann and Helbing (2013), in our setting, the "captive episodes" are the episodes that are more likely to spread the viewing engagement to the rest of the content with a combination of the lowest distances and lowest variability of the distances. Put it differently, captive episodes

lock the viewers into the content with viewer generated conversations around the story and motivate the viewers to cover the rest of the library with the shortest breaks.

Different than Brockmann and Helbing (2013), however, we differentiate between different release strategies (such as episodic releases or entire season at once strategy) when computing the engagement centrality score of an episode. The main reason behind this is that different release strategies lead to different graph layouts.

On the one end, the content provider may deploy an entire season at once policy where all the content is available to the viewers simultaneously. This is equivalent to a layout where all nodes are in one single cluster from a graphical point of view, and hence the engagement centrality discussion is analogous to the setting of Brockmann and Helbing (2013). Therefore, we consider how effective the episode is in terms of diffusing the viewer engagement to all other episodes of the rest of the library in this release strategy. We refer to the engagement centrality measure of an episode released in an entire season at once policy as *unconditional engagement centrality score*, which we formally introduce next:

Definition: The ***unconditional engagement centrality*** score EC_i of episode i (or a global centrality) is computed based on the distances between i and all of the nodes in the network, i.e. to all other episodes in the graph:

$$EC_i = \frac{1}{\sqrt{\mu_i^2 + \sigma_i^2}} \tag{4.6}$$

where μ_i and σ_i are the mean and the standard deviation of the effective distances from node i to nodes $j \in \mathcal{V} \backslash \{i\}$. Hence:

$$\mu_i = \frac{\sum_{j=1, j\neq i}^{V} d(\gamma = i \to j)}{V-1}, \qquad \sigma_i = \sqrt{\frac{\sum_{j=1, j\neq i}^{V} d(\gamma = i \to j) - \mu_i)^2}{V-1}}. \tag{4.7}$$

We mentioned earlier that a combined low mean and a low standard deviation of the shortest effective distances lead to a higher engagement centrality score. From a mathematical point of view, one can interpret the two constituents (the mean and the standard deviation) as the

two coordinates that jointly inform the position of the engagement centrality score. Thus, the joint effect of the mean and the standard deviation can be represented with the inverse of the square root of the sum of the squares (SRSS) of the two constituents following the Pythagorean theorem.

Running Example: In this example, we assume that the viewers are aware that all episodes of series "A" is available, that is, the content provider deploys an entire season at once policy. Let us focus on episode 3 and derive EC_3 using Figure 4.9 (left panel). In this running example we have $d(i,j) = e(\gamma = i \rightarrow j)$. From the third row of the table, we compute the mean of the effective distances from episode 3 to the rest of the content $j = 1, 2, 4, 5, 6, 7, 8, 9, 10$, that is, $\mu_3 = \frac{\sum_{j=1, j\neq i}^{\mathcal{V}} e(\gamma=3\rightarrow j)}{9} = 3.045971$. The standard deviation of the effective distance equals to $\sigma_3 = 0.2648211$. Thus, the engagement centrality of the episode 3 equals to $EC_3 = \frac{1}{\sqrt{3.045971^2+0.2648211^2}} = 0.327069$. We refer the reader to Figure 4.10 for the unconditional engagement centrality scores of each episode, where the size of the nodes indicate how captive an episode i is.

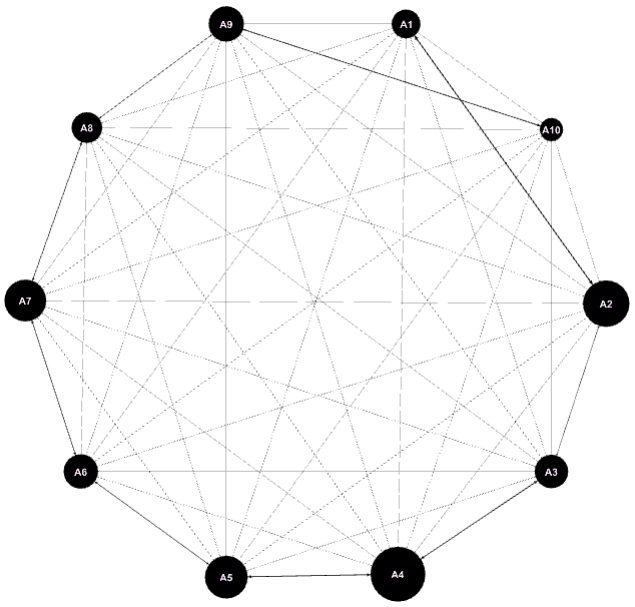


Figure 4.10: Running Example: Captive Episodes when All Episodes Released At Once

On the other hand, the content provider may deploy an episodic release strategy where the "clusters of episodes" are released, instead of entire library at once. In this case, the graph

consists of inter-dependent clusters of nodes and the viewer engagement is spread between these layers of episodes.[8] Therefore, the engagement score is conditional on the release strategy the content provider adopts and we capture this aspect by adapting the unconditional engagement centrality score to the multi-clustered layout of the episodic release strategy.

To be more specific, we represent a release strategy with two elements: the number of clusters, say k, in which we release the episodes simultaneously, and the episodes that are cluster finales, denoted by f_c for $c = 1, \ldots, k$. As an example, one can release a series that consists of 10 episodes in two clusters ($k = 2$) with cluster finales at episodes, $f_1 = 6$ for the first cluster and $f_2 = 10$ for the second cluster. Then the engagement centrality score of a cluster finale f_c, given an episodic release strategy with k clusters and cluster finales f_c for $c = 1, \ldots, k$, is computed based on the shortest distance to the subsequent cluster of episodes. In particular, we consider viewer engagement that flows from the finale of a cluster c, namely f_c, to the next cluster of episodes, $j' \in \{f_c + 1, \ldots, f_{c+1}\}$. To measure this, we focus on the effective distance from the cluster finale f_c to each node of the next cluster $(c + 1)$, i.e., $d(\gamma = f_c \to j')$ for $j' \in \{f_c + 1, \ldots, f_{c+1}\}$.

Definition: The ***conditional engagement centrality*** score of a cluster finale f_c, given an episodic release strategy with k clusters and cluster finales f_c for $c = 1, \ldots, k$ is

$$EC_{f_c|f_{c+1}} = \frac{1}{\sqrt{(\mu_{f_c|f_{c+1}})^2 + (\sigma_{f_c|f_{c+1}})^2}}. \tag{4.8}$$

where the mean and standard deviation of the effective distances $d_{f_c,j'}$ are computed as

$$\mu_{f_c|f_{c+1}} = \frac{\sum_{j'=f_c+1}^{f_{c+1}} d(\gamma = f_c \to j')}{|f_c, \ldots, f_c + 1|}, \qquad \sigma_{f_c|f_{c+1}} = \sqrt{\frac{\sum_{j'=f_c+1}^{f_{c+1}} (d(\gamma = f_c \to j') - \mu_{f_c|f_{c+1}})^2}{|f_c, \ldots, f_c + 1|}}. \tag{4.9}$$

Running Example: Let us consider a release schedule with two clusters where $c_1 = \{1, 2, 3, 4\}$

[8] This setting is similar to the "multi-layered network" discussion that has become popular in the propagation literature (e.g. Montagna and Kok, 2016; Bargigli et al., 2015; Poledna et al., 2015).

and $c_2 = \{5, 6, 7, 8, 9, 10\}$. We calculate the conditional engagement centrality scores using the shortest distance table of Figure 4.9. In particular, we have $j' = \{5, 6, 7, 8, 9, 10\}$ and hence $e(\gamma = 4 \rightarrow j') = \{2.37, 3.17, 3.17, 3.17, 3.17, 3.20\}$. These lead us to $\mu_{4|10} = 3.04$ and $\sigma_{c_1=4|10} = 0.30$. Thus, we have $EC_{4|10} = 0.3273$ (see Figure 4.11).

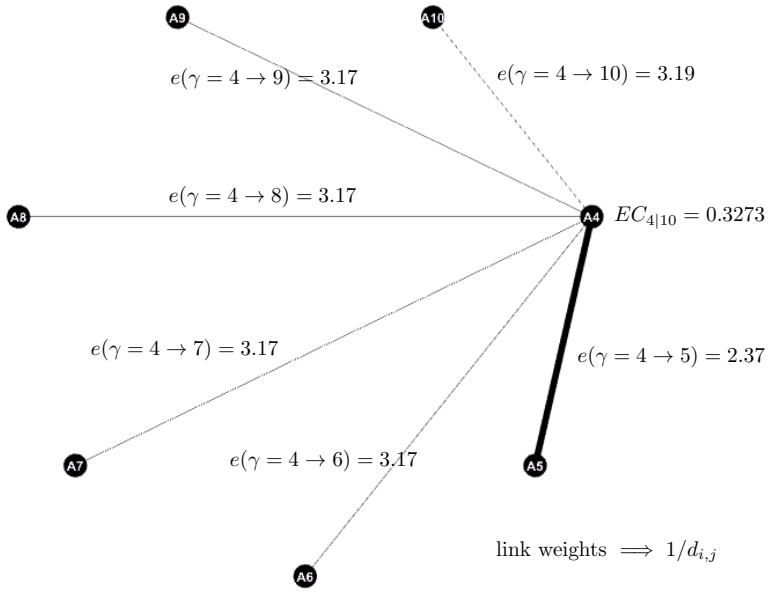


Figure 4.11: Running Example:Conditional Distance of Episode 4 to the Cluster $\{5, 6, 7, 8, 9, 10\}$

To summarize, we have discussed the viewer engagement measures under distinct release strategies (such as episodic release or entire season at once policy) as a preparation for the Section 3, where we discuss the release schedules that maximize the engagement.

4.3 *Content Release Strategy*

In this section, we focus on various content release strategies and their impact on viewer engagement. We aim to find release strategies that provide improved viewer engagement when compared to the benchmark strategies, which are the release of the *entire season at once* and the *episodic release* of the series.

In Section 2, we have shown the impact of the release policy on the engagement centrality

scores associated with episodes. In particular, we have discussed the engagement score of cluster finales and the effect of the set of episodes that subsequently follow on the engagement score of the cluster finales. In this section, we increase the span of our discussion and derive the total engagement score of a network given a release strategy. We start with the release of a series and then extend our analysis to multiple series and the personalization of the content across different viewer types.

One significant aspect of our discussion is that we evaluate the viewing behavior observed in an entire content released at once setting with no personalization of the material, and suggest how viewers' engagement would be affected by different release schemes. In an entire season at once setting, viewers are not nudged to watch the content in a specific pace nor in a particular order and hence demonstrate their viewing patterns without the influence of the nudges. From this unconditional behavior, we detect the episodes with high viewer engagement, as well as the vulnerable points where viewers take extended breaks or cover the content asynchronously to the rest of the users of the content library. These captive and uninvolving episodes inform the release policy that would improve viewer engagement. In particular, we base our analysis on making cluster finales at the most captive episodes such that these cluster finales produce the maximum aggregate engagement score together.

We focus on the impact of the release strategy on the viewer engagement assuming that the viewing pace of the users is fixed. We acknowledge that this is indeed a potential drawback of our setting since the viewer engagement may, in turn, affect the viewing pace. Ideally, one may tackle this second order effect by designing multiple release strategy settings, measuring viewing behavior under each release policy and comparing the viewer engagement scores using the unconditional engagement scores we have introduced in Section 2. In our online learning setting, designing experiments with various release setting has ethical implications as it is prone to creating educational advantages/disadvantages. Therefore, we focus on the viewing data generated under the entire season at once strategy and provide managerial implications solely based on viewer engagement while omitting the second order influence of the release strategy on the viewing pace through viewer engagement. As future research, we consider designing experiments of various release settings under the online streaming services context.

4.3.1 Release of a Series

We start with the release of a single series that consists of V episodes. In particular, we are interested in the optimal number of clusters (k) for the series and the episodes that should be assigned to each cluster, which, in turn, are released in consecutive subscription periods t. Since the series has a natural order for the episodes, one only needs to identify the episodes that are the cluster finales (f_c) of each cluster ($c \in \{1, \ldots, k\}$) to define a release strategy. Our aim is to find a release strategy (k, $\{f_c\}_{c=1}^{k}$) that maximizes the aggregate viewer engagement.

Section 2 discussed how to measure the engagement associated with a single episode for the entire season at once strategy with $k = 1$ ("unconditional engagement centrality") as well as engagement centrality "conditioned" on a release strategy where $k \geq 2$. Starting with the entire season at once strategy ($k = 1$), we derive the aggregate engagement score as the mean of the individual unconditional engagement scores (EC_i) associated with each episode i:

$$EC(b) = \frac{\sum_{i=1}^{V} EC_i}{V} \tag{4.10}$$

Next, we focus on the engagement scores of the release strategies with $k \geq 2$. We remind the reader that we denote an engagement score of a cluster finale f_c conditioned on the release strategy with ($k \geq 2$) clusters as $EC_{f_c|f_{c+1}}$ (see Equation 4.8). It is important to note that a cluster finale f_c is at the same time the last episode of the corresponding cluster. Based on the individual centrality scores ($EC_{f_c|f_{c+1}}$ for $c = 1, 2 \ldots, k-1$), we derive the engagement score of a release strategy (k, $\{f_c\}_{c=1}^{k}$) where $k \geq 2$ as in the following:

$$EC(k, \{f_c\}_{c=1}^{k}) = \frac{EC(b) + \sum_{c=1}^{k-1} EC_{f_c|f_{c+1}}}{k} \tag{4.11}$$

Running Example: Let us compute the engagement scores of various release schedules for soap opera A. If all episodes are released at once, the aggregate engagement score equals to $EC(b) = \frac{\sum_{i=1}^{10} EC_i}{10} = 0.3243$ (see Figure 4.10 for the individual engagement scores EC_i). Next, let us derive the engagement score of the release strategy (2, $f_1 = 4, f_2 = 10$), where

$EC(2, f_1 = 4, f_2 = 10) = EC_{4|10}$. Hence, it follows that $EC(2, f_1 = 4, f_2 = 10) = 0.3273$ (see Section 4.2.3 for the computation of $EC_{4|10}$). Therefore, the entire season at once strategy is inferior to the release strategy $(2,\ f_1 = 4, f_2 = 10)$ in terms of generating viewer engagement.

Next, we focus on the optimal release strategy $(k^*, \{f_c\}_{c=1}^{k^*})$ that produces maximum viewer engagement, which formally reads as:

$$(k^*, \{f_c\}_{c=1}^{k^*}) = \max_{k, f_c} EC(k, \{f_c\}_{c=1}^{k}) \tag{4.12}$$

We note that the size of the decision set for finale clusters $\{f_c\}_{c=1}^{k}$ for $k \in \{1, \ldots, V\}$ given a release setting with k clusters equals to $\binom{V-1}{k-1}$, and hence the cardinality of our decision space is $\sum_{k=1}^{V} \binom{V-1}{k-1} = 2^{V-1}$. In addition, given a release strategy one needs to calculate the engagement score $E(k^*, \{f_c\}_{c=1}^{k^*})$ that has a complexity of $\sum_{c=1}^{k} |f_{c+1} - f_c|$, since we need to calculate the mean and standard deviation of the shortest effective distance of the cluster finale f_c to the each episodes in the subsequent cluster of episodes $j' \in \{f_c + 1, \ldots, f_c\}$ (see Equation 4.9). Due to its conditional structure, any exact solution algorithm for this problem is likely to consume a runtime that is exponential to the size of the problem. For practical applications, we therefore suggest a polynomial time "series heuristics" that we discuss next.

Given the shortest effective distances $(d_{i,j})$ between any two nodes $i, j \in \mathcal{V}$, we first compute the engagement centrality score $EC(b)$ (see the Equation 4.10) for the entire season at once strategy $(k = 1)$. Secondly, we consider the release strategies with two clusters $(k = 2)$, which have the form $(2, f_1 = i, f_2 = V)$, that is, the location of the first cluster finale $(f_1 = i)$ fully specifies the release strategy. We derive the engagement scores $EC_{i|\mathcal{V}}$ (see the Equation 4.8) for all potential first cluster finales $f_1 = i \in \{1, \ldots, V\}$. Therefore, given a two-clustered release schedule $(k = 2)$, an optimal point to make a cluster finale is as follows: $(f_1^* = i' \in \arg\max_{i \in \{1, \ldots, V-1\}} EC_{i|\mathcal{V}})$, and hence the best engagement score we can attain in a release schedule with 2 clusters is $(EC_{f_1^*|\mathcal{V}})$.[9] We note that if we have $EC_{f_1 = i'|\mathcal{V}} \geq EC(b)$, then releasing series in two clusters is

[9]It is possible to have multiple solutions to $\arg\max_{i \in \{1, \ldots, V\}} EC_{i|\mathcal{V}}$. In that case, as a rule of thumb, we select the episode with the smallest index, that is $f_1^* = \min\{\arg\max_{i \in \{1, \ldots, V\}} EC_{i|\mathcal{V}}\}$.

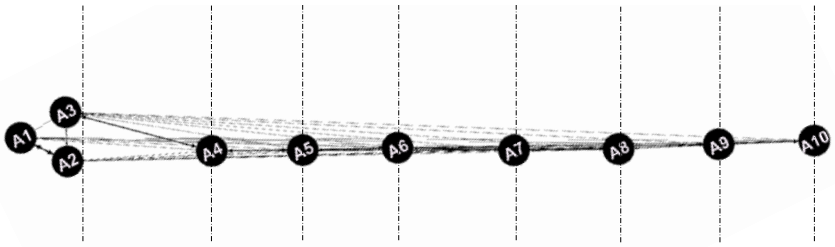
superior over the entire season at once policy.

As for the three clustered ($k = 3$) release strategy, we reuse the optimal $k = 2$ policy, i.e., $(2, f_1 = i', f_2 = V)$. In particular, we fix i' as a cluster finale, and hence it follows either that $f_1 = i'$ or $f_2 = i'$. In other words, we search for the best point to make a third cluster finale among the two groups of episodes $\{1, \ldots, i\}$ and $\{i + 1, \ldots, V\}$ that are associated with the $k = 2$ release strategy. As a result, we have either $(f_1 = j'_1 \in \arg\max_{j\in\{1,\ldots,i'-1\}} EC_{j|i}, f_2 = i', f_3 = V)$ or $(f_1 = i', f_2 = j'_2 \in \arg\max_{j\in\{i'+1,\ldots,V-1\}} EC_{j|i}, f_3 = V$. In either case, the aggregate engagement centrality score associated with the $k = 3$ release strategy is computed as $EC(3, \{f_c\}_{c=1}^3) = (EC_{f_1|f_2} + EC_{f_2|f_3})/2$ (see Equation 4.11). Consequently, we compare the two cases and assign $j' \in \arg\max_{j'_1,j'_2}\{EC(3, f_1 = j'_1, f_2 = i', f_3 = V), EC(3, f_1 = i', f_2 = j'_2, f_3 = V)\}$. Thus, we find the $k = 3$ policy, that is $(3, \{f_c^*\}_{c=1}^3)$.

We derive the policy $(k, \{f_c^*\}_{c=1}^k)$ for k clustering strategies, $k = 4, \ldots, V$, in analogous steps and obtain the optimal release strategy by choosing the clustering k that maximizes the engagement centrality score, that is $(k^*, \{f_c\}_{c=1}^{k^*}) \in \arg\max_{k\in\mathcal{V}} EC(k, \{f_c^*\}_{c=1}^k)$.

Running Example: Let us apply our release algorithm to soap opera A. We utilize the shortest effective distance matrix d_{ij} (see Figure 4.9). We start with an empty 10x10 table where the rows indicate potential cluster finales (i) and the columns show the number of clusters (k). For the first column we compute $EC_b = 0.324$. We note that all the entries in the first column are EC_b as $k = 1$ denotes the benchmark strategy, where we release everything upfront. Next, we fill in each row (i) of the second column ($k = 2$) with engagement scores $EC(2, f_1 = i, f_2 = k) = EC_{i|10}$ using the effective distance matrix d_{ij} along with Equations 4.8 and 4.11. We observe that $EC_{i|10} > EC_b$ for $i = \{1, 3, 4, 5, 7, 9\}$ while $EC_b > EC_{2|10}$. The maximum score for ($k = 2$) policy is obtained when ($i = 9$), that is, when cluster finale is the 9^{th} episode. We compute the optimal aggregate score associated with clustering policy ($k = 2$) as $\frac{EC_b+EC_{9|10}}{2} = \frac{0.324+0.416}{2} = 0.370$ by using Equation 4.11. Within similar lines, we locate the episodes of cluster finales using the series heuristics and subsequently compute the total score of the release policy using Equation 4.11 for $k = 3, 4, 5, 6, 7, 8, 9, 10$. Figure 4.12 shows the total score of each clustering policy. We note that ($k = 8$) is the best policy

with a total engagement centrality score 0.412. Therefore, the optimal release of series A is $(k^* = 8, f_1^* = 3, f_2^* = 4, f_3^* = 5, f_4^* = 6, f_5^* = 7, f_6^* = 8, f_7^* = 9, f_8^* = 10)$.



$(k^* = 8, f_1^* = 3, f_2^* = 4, f_3^* = 5, f_4^* = 6, f_5^* = 7, f_6^* = 8, f_7^* = 9, f_8^* = 10)$

Finales	1	2	3	4	5	6	7	8	9	10
1		0.325	0.326	0.328	0.330	0.333	0.338	0.346	0.366	
2		0.320	0.321	0.322	0.323	0.325	0.329	0.336	0.361	0.361
3		0.326	0.328	0.330	0.334	0.341	0.357	0.421		
4		0.327	0.331	0.335	0.342	0.357	0.422			
5		0.325	0.329	0.333	0.343	0.381				
6	0.324	0.330	0.336	0.348	0.394					
7		0.334	0.347	0.389						
8		0.336	0.365							
9		0.416								
10		NA								
Score	0.324	0.370	0.367	0.378	0.386	0.383	0.403	**0.412**	0.389	0.375

Figure 4.12: Running Example:Release Strategy for Soap Opera A

Our focus has been on a single series, and we derived a release schedule $(k^*, \{f_c\}_{c=1}^{k^*})$ based on the local search heuristic we introduced in this section. Section 4.3.2 extends the analysis to multiple series.

4.3.2 *Optimal Order of Multiple Series Within a Subscription Period*

Our focus in this section is on the release of multiple series. We argue that the decision maker can improve the viewer engagement further with the ordering of different types of content (such as soap operas in an online streaming platform or courses of an online education programme). With the combinations of the material that connect well, the decision maker can induce viewers to be actively involved with the content and discourage them from leaving the platform after watching the cluster finale of their favorite show.

We consider a content library with multiple series, say $s \in \mathcal{M} = \{1, \ldots, M\}$. We assume that the information about the release of each series s, that is, $(k_s^*, \{f_c^s\}_{c=1}^{k_s^*})$, is available (observe the additional subscript in the notation that we use to denote the series of interest with s). In other words, the content provider has already tackled the first challenge, the release of a series using the method in Section 4.3.1, and she knows the episodes of a series to release in each subscription period t.

Her challenge is now to determine the groups of series from the content library to be released simultaneously and the launch order of each group.[10] Formally, an ordering policy within a subscription period t is defined as the number of the groups g_t and the series to include in each group $S_{g_t'}$ such that $\cup_{g_t'=1}^{g_t} S_{g_t'} = \mathcal{M}$. To simplify the notation, we focus on a single subscription period t in the rest of the discussion, and hence $(g, \{S_{g'}\}_{g'=1}^{g})$ denotes the ordering of the series in the subscription period t of interest.[11] We note that the discussion on the ordering of multiple series is analogous to the clustering of an individual series, except now our unit of analysis is a series instead of the episodes. From now on, we drop the subscript t for convenience of notation.

Releasing All Series At Once ($g = 1$): First, we analyze the policy that releases all series of the library simultaneously.[12] To this end, we start with the engagement centrality score of each series, that represents the captiveness of a series in the whole library if all the content had been released upfront. That is indeed similar to the unconditional engagement centrality scores introduced in Section 4.3.1. Formally, the unconditional engagement centrality score of a series s for the rest of the library is:

$$EC_s^L = \frac{1}{\sqrt{\mu_s^2 + \sigma_s^2}} \tag{4.13}$$

where μ_s and σ_s are mean and standard deviation of effective distances from the finale f^s of

[10] Indeed, many digital platforms commonly announce the clusters of soap operas that will be available through monthly calendars such as "New This Month" by Hulu or "New Upcoming" by Netflix.

[11] One can repeat the same procedure for each subscription period to organize the library over time.

[12] It is important to note that releasing all series at once (second dimension of the release strategy) is not necessarily equivalent to entire season at once policy associated with individual series (the first dimension of a strategy).

series s to the pilots p^r of other series for $r \neq s; r, s \in \mathcal{M}$, that is

$$\mu_s = \frac{\sum_{r=1, r\neq s}^{M} d(f^s, p^r)}{M-1}, \qquad \sigma_s = \sqrt{\frac{\sum_{j=1, r\neq s}^{M} (d(f^s, p^r) - \mu_s)^2}{M-1}}. \tag{4.14}$$

Once we have generated the engagement centrality scores, we measure the aggregate viewer engagement associated with the benchmark policy, where all series of the library are released simultaneously, as

$$EC^L(b) = \frac{\sum_{s=1}^{M} EC_s}{M}. \tag{4.15}$$

Ordering Policies ($g \geq 2$): Next, we discuss the aggregate viewer engagement associated with an ordering policy $(g, S_{g'})$ where $g \geq 2$. First, we clarify the viewer engagement between two groups of series, say S_i and S_j provided that S_i is released prior to S_j as in the following.

The goal here is to measure the effective distance between the least captive series of the first group S_j and each series of the next group S_j. In other words, we consider the the least captive series of the first group S_j as the group finale as it is potentially the series that is watched the last. Therefore, our discussion follows in analogous steps to those in the previous section.

Let us assume that s is the least captive series of the first group, that is $s \in \arg\min_{s\in S_i} EC^L_{s'}$, with a cluster finale f^s, and p^r denotes the pilot of each series in the next group of series $r \in S_j$. Then, the engagement score associated with the case where the group S_j is available after the group of series S_i is

$$EC_{S_i, S_j} = \frac{\sum_{r\in S_j} EC_{f^s|p^r}}{|S_j|}. \tag{4.16}$$

Once generating the engagement score between each group, we compute the total engagement score associated with the ordering policy $(g, S_{g'})$ for $g' \in \{1, \ldots, g\}$ by aggregating the individual engagement scores as

$$EC(g, \{S_{g'}\}_{g'=1}^{g}) = \frac{EC^L(b) + \sum_{i\in\{1,\dots,g-1\}, j=i+1} EC_{S_i,S_j}}{g}. \tag{4.17}$$

Ideally, the decision maker needs to search the best combination of the series s from the library $s \in \{1, \dots, M\}$ for each potential number of groups $g \in \{1, \dots, M\}$, to maximize the aggregate engagement. Given a group number g, her choice set when selecting which series to allocate to which group is equivalent to $\binom{M}{g}$, and hence her total choice set is $\sum_{g=1}^{M} \binom{M}{g} = 2^M$. To solve this problem, we propose a heuristic, which we call as "order heuristic". This method is an adaptation of the series heuristic that we introduced in Section 4.3.1, and it consists of two steps: creating a captive series order and deploying the "series heuristic" with a higher dimension of analysis, that is clustering based on the series instead of the episodes.

Step 1: Creating the Captive Series Order ($\mathcal{M}'$)*:* We note that when clustering a series, hence grouping the episodes, we have a natural order of the components imposed by the story. When clustering the whole library, we don't have a natural order of series, and it is, in fact, one of the decisions we need to make. We tackle this complexity with an order based on the captiveness of the series. In particular, we rank the series according to their unconditional engagement centrality scores, that is the captiveness $EC_s(b)$ of a series if all of its episodes had been released upfront, as discussed in Section 4.3.1. Based on the engagement centrality scores $EC_s(b)$ of the individual series, we construct an ordered list $\mathcal{M}' = \{r_i : r_i < r_{i+1} \iff EC_{r_1}(b) > EC_{r_2}(b)\}$, that is the series are released in descending order of unconditional engagement centrality scores.

Running Example: We have five series in the content library $\mathcal{M} = \{A, B, C, D, E\}$ and the optimal release of each series is illustrated in Figure 4.13. First, we rank the series based on their captiveness and construct the ordered list $\mathcal{M}'$. To this end, we compute the unconditional engagement centrality score $EC_s(b)$ of each series $s \in \mathcal{M}$. We refer the reader to Section 4.3.1 for the derivation of $EC_A(b)$ as an example. Figure 4.13 shows unconditional scores of each series, where series D has the most captive content. Using the scores $EC_s(b)$ of Figure 4.13, we generate order of the series $\mathcal{M}' = \{D, E, A, C, B\}$.

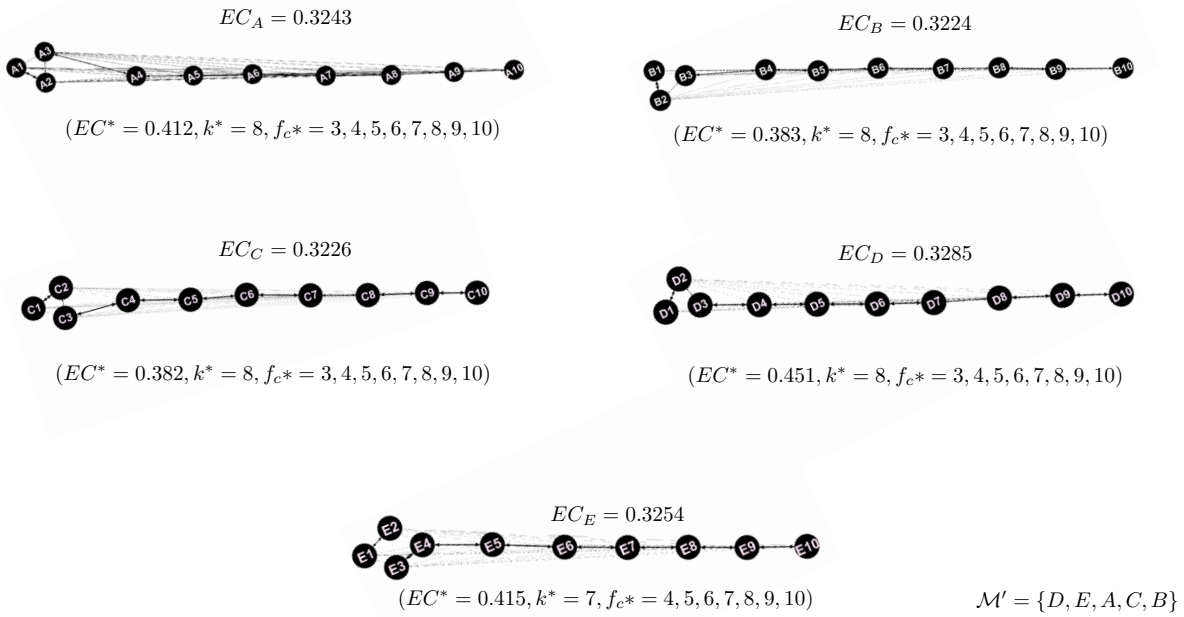


Figure 4.13: Running Example: Release strategy for each soap opera

Step 2: "Series Heuristics": After obtaining the order of the series in $\mathcal{M}'$, we apply the series heuristics that we have introduced in Section 4.3.1, except now we are focusing on the distances between the series instead of the episodes. We start with the case where the series that is when all series are available at once, i.e., $EC^L(b)$. Secondly, we consider an ordering with two groups $g = 2$ such that the total engagement $EC(2, S_1, S_2)$ is maximized. To this end, we compute the engagement scores for all potential $g = 2$ ordering policies, which are in the form $(2, S_1 = \{1, \ldots, s\}, S_2 = \{s + 1, \ldots, M\})$, where s is the least captive series of the first group (S_1). The engagement scores associated with $g = 2$ ordering policies are computed as $\big(EC^L(b) + EC_{S_1,S_2}\big)/2$ for $r \in S_2$ by Equation (4.17). Therefore, an ordering policy $(2, S_1 = \{1, \ldots, s\}, S_2 = \{s, \ldots, M\})$ is superior over another ordering policy $(2, S_1' = \{1, \ldots, s'\}, S_2' = \{s' + 1, \ldots, M\}$ if and only if $EC_{S_1,S_2} > EC_{S_1',S_2'}$. We select the best engagement score we can attain in a release schedule with $g = 2$ ordering strategies, that is, $s^* \in \arg\max_{s \in \mathcal{M}} EC_{S_1,S_2}$. Thus, we find the optimal $(g = 2)$ ordering policy denoted by $(2, \{S_{g'}^*\}_{g'=1}^2)$. We note that an $g = 2$ ordering is superior over the no-ordering policy $g = 1$ if and only if $EC_{S_1,S_2} > EC^L(b)$, that is, if the marginal contribution of the clustering into two is higher than the engagement score of the no-ordering policy.

In analogy to Section 4.3.1, we go over to $g = 3$ strategies by fixing the clustering point we have found in the $g = 2$ policy. In particular, we have two groups $S_1 = \{1, \ldots, s^*\}, S_2 = \{s^* + 1, \ldots, M\}$, and we consider to cluster one of these groups into two. Therefore, we have either $S_1' = \{1, \ldots, r'\}, S_2' = \{r' + 1, \ldots, s^*\}$, $S_3' = \{s^* + 1, \ldots, M\}$ or $S_1'' = \{1, \ldots, s^*\}, S_2'' = \{s^* + 1, \ldots, r''\}$, $S_3'' = \{r'' + 1, \ldots, M\}$. Equation (4.17) implies that an $g = 3$ ordering with a group of series (S_1, S_2, S_3) has an engagement score that can be computed as $\big(EC^L(b) + EC_{S_1,S_2} + EC_{S_2,S_3}\big)/3$. As a result, we have either $r' \in \arg\max EC_{S_1',S_2'}$ or $r'' \in \arg\max EC_{S_2'',S_3''}$. Therefore, we compare the two cases and assign

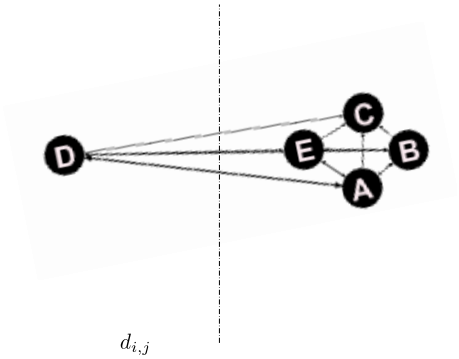
$$r^* \in \arg\max_{r',r''} \{EC(3, S_1', S_2', S_3'), EC(3, S_1'', S_2'', S_3'')\}$$

as the optimal point to cluster the collection of series into two further groups. This concludes our discussion for the $g = 3$ ordering policy denoted by $(3, \{S_{g'}^*\}_{g'=1}^3)$.

Along similar lines, we find near-optimal policies associated with each ordering scheme $g \in \mathcal{M}$. Finally, we compare the near-optimal policies of each ordering scheme and select the best policy, that is, $g^* \in \arg\max_{g \in \mathcal{M}} EC(g, \{S_{g'}\}_{g'=1}^g)$.

Running Example: In this part of the running example, we concentrate on the order of these series in the first subscription period $t = 1$, and we use the effective distances $d_{i,j}$ between the pilots and cluster finales of each series (see Figure 4.14). We note that to derive the effective distances $d_{i,j}$ between the finale of a series i and the pilot of a series j, we follow the steps in Section 4.2.1 and Section 4.2.2. Next, apply our order algorithm. We start with an empty 5x5 table (the bottom right panel of Figure 4.14) where the rows indicate series (i) that may potentially be the least captive series of a group and the columns the number of the groups (g). We start with the first column and compute $EC_b = 0.418$ with Equation 4.15. Next, we consider ordering policy $(g = 2)$ with engagement scores $EC(2, S_1, S_2)$ using the effective distance matrix d_{ij} along with Equations 4.16 and 4.17. We refer the reader to the second column of the table for the scores associated with each ordering policy. The maximum score for $(g = 2)$ ordering policy is obtained when $(i = D)$, that is, when $S_1 = \{D\}$ and $S_2 = \{E, A, C, B\}$. Put it differently, releasing the most captive soap opera D separately from the rest of library is the

best ordering policy since after watching series D viewers would be motivated to view the rest of the library that are relatively less captive. We compute the optimal aggregate score associated with clustering policy $(g = 2)$ as $\frac{EC_b^L + EC(2,S_1,S_2\})}{2} = \frac{0.418+0.435}{2} = 0.427$ by Equation 4.17. Figure 4.14 shows the total scores of remaining ordering polices, that are computed along the similar lines. We note that $(g = 2)$ is the best policy with a total engagement centrality score 0.427. Therefore, the optimal ordering for the fist subscription period is $(g^* = 2, S_1^* = \{D\}, S_2^* = \{E, A, C, B\})$.



$d_{i,j}$

	D	E	A	C	B
D	NA	2.322	2.163	2.386	2.322
E	2.386	NA	2.386	2.386	2.322
A	2.163	2.322	NA	2.386	2.322
C	2.674	2.609	2.674	NA	2.609
B	2.386	2.322	2.386	2.386	NA

$(g^* = 2, S_1^* = \{D\}, S_2^* = \{E, A, C, B\})$

Series in the First Subscription Period	1	2	3	4	5
D		0.435			
E		0.423	0.423	0.419	
A	0.418	0.425	0.425		
C		0.383	0.383	0.383	0.383
B					
Score	0.418	0.427	0.426	0.423	0.403

Figure 4.14: Running Example: Organizing the Library in the First Period

The content designer determines the ordering policy for all subscription periods in the planning horizon $t = \{1, \ldots, T\}$, and hence can repeat the steps we summarized for each period (see Figure 4.15 for example). We assumed that the library is fixed, that is, the number of series is given. Over the planning horizon, exogenous changes can occur, e.g., new series can be added or some of the current series can be cancelled. In that case, it is easy for the decision maker to reflect these changes by adding the optimal release schedule for the new series as input for the ordering heuristics or excluding a currently available series. However, adding the optimal release schedule for the new series can be challenging when the viewing data is limited. In that case, the decision maker can benefit from proxies such as using viewing data from a previous

release perhaps in a different market or a similar series of the same genre.

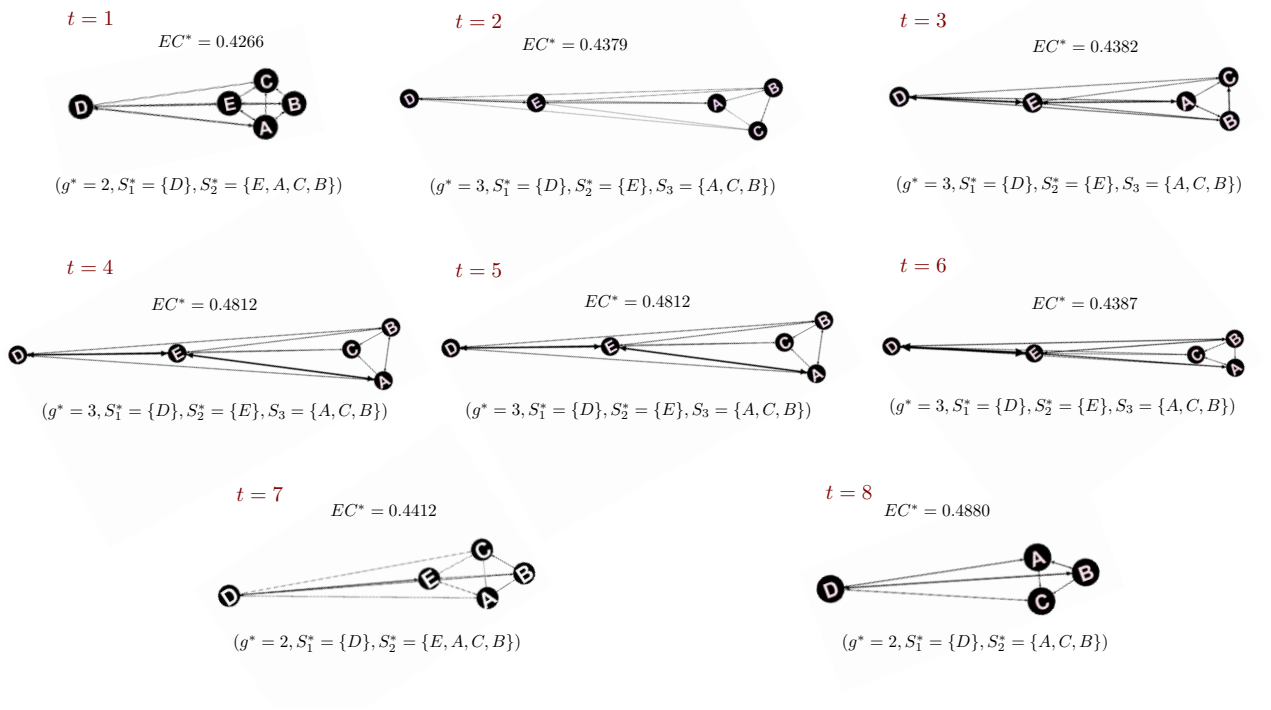


Figure 4.15: Running Example: Organizing the Library Across Subscription Periods

4.3.3 *Personalization of the Content*

Up to this section, we considered a "one size fits all" release strategy without personalization. The question in this section is whether or not to personalize the content and offer different release schedules for different types of viewers. To answer this question, we compare two extreme cases: the one size fits all release versus a purely heterogeneous content strategy. On the one hand, the one size fits all release enables viewers to view the same series at similar times and initiate the water-cooler talk. On the other hand, the purely heterogeneous content strategy allows the content provider to address the various binge-watching attitudes of each user.

An optimal amount of personalization on the release strategy should address the trade-off between binge-watching and water-cooler talk. In particular, the content provider determines two elements: the number of viewer groups w^* and the release strategy directed to the different viewer groups. We remind the reader that a release strategy for a given group of viewers,

say w', consists of the release of an individual series $(k_s^*(w'), \{f_c(w')\}_{c=1}^{k_s^*})$, and the ordering of multiple series in a subscription period t, that is , $(g_t(w'), \{S_{g'}(w')\}_{g'_t=1}^{g_t})$ for the content library for $w' \in \{1, \ldots, w\}$.

We start the discussion with the detection of viewer groups using a scale that we refer to as the "propensity for binge-watching". We compute the propensity for binge-watching for each viewer n from the binge matrix $b(n)$ that we discussed in Section 4.2.1.1. We remind the reader that the b matrix summarizes the inclination of a viewer to binge-watch and indicates the break distances if the viewer prefers not to binge-watch.We refer to the frequency of binge-watching as the first order binge behavior since it directly focuses on the binge-watching behavior itself. The duration of the breaks, on the other hand, is a second order binge behavior since it takes into account short breaks as a proxy for binge watching even though the viewer is not actually binge-watching the content. Therefore, we account for various viewer types such as the viewers who are not binge-watching but covering the content in small consistent breaks or viewers who binge-watch once they are on the platform but disappear for long intervals before they return to the platform.

To represent the first order binge behavior, we calculate the sparsity of the individual $b_s(n)$ matrices for each series s, that is $S[b_s(n)] = \frac{\sum_{i\neq j; i,j\in\{1,\ldots,V\}}[b_{i,j}(n)=0]}{V(V-1)}$ and use the mirror image of the sparsity $(1 - S[b_s(n)])$ to represent the first order binge behavior. On account of the second order binge behavior, we focus on the weight of individual $b_s(n)$ matrices for each series s, that is $W_s[b(n)] = \frac{\sum_{i\neq j; i,j\in\{1,\ldots,V\}} b_{i,j}}{V(V-1)}$. Combining the first and second order binge behavior, we obtain the propensity for binge-watching $binge_s(n)$ for a series s as

$$binge_s(n) = \alpha(1 - S[b_s(n)]) + (1 - \alpha)W[b_s(n)], \tag{4.18}$$

where $\alpha \in (0, 1)$ informs the weight the decision maker attaches to the first and the second order binge behavior. We note that $binge(n) \in [0, 1]$ since the sparsity $S[b(n)]$ and the weight $W[b(n)]$ of a matrix with entries $b_{i,j} \in [0, 1]$ are also in the interval $[0, 1]$. Following, we derive the aggregate scores showing a viewer's propensity to binge-watch as in the following,

$$binge(n) = \frac{\sum_{s \in \mathcal{M}} binge_s(n)}{M}. \tag{4.19}$$

Running Example: Let us compute the binge-watching propensity score of the first user for the soap opera A, namely $binge_A(1)$. To this end, we utilize the binge matrix $b_A(1)$ derived in Section 4.2.1.1 (Figure 4.4). Next, we compute the sparsity of $b_A(1)$, that is $S[b_A(n)] = 0.74$. Similarly, we find the weight of $b_A(1)$, $W[b_A(n)] = 0.15$. We use Equation 4.18 to compute first viewer's propensity for binge viewing the soap opera A, i.e., $binge_A(n) = \alpha \times 0.26 + (1-\alpha) \times 0.15$. We remind the reader that α indicates the weight given to the first order binge watching. In this example, we set $\alpha = 0.5$. Therefore, we derive $binge_A(n) = 0.205$. Along the similar lines, we generate propensity for binge-watching scores of each user for each series as well as the aggregate scores (see Figure 4.16) using Equation 4.19.

		$binge_s(n)$					
		A	B	C	D	E	$binge(n)$
Viewers	1	0.2054	0.2062	0.0500	0.2024	0.2024	0.1733
	2	0.1942	0.2024	0.0500	0.2062	0.2024	0.1710
	3	0.2033	0.2024	0.2062	0.2024	0.2023	0.2033
	4	0.0500	0.0500	0.2024	0.2024	0.0500	0.1110
	5	0.1891	0.0500	0.2056	0.2062	0.2023	0.1707
		0.1684	0.1422	0.1428	0.2039	0.1719	

Figure 4.16: Running Example: Propensity for Binge-Watching of the Viewers, for $\alpha = 0.5$

Next, we rank the viewers according to their propensity for binge-watching, $binge(n)$. At one extreme, we may have a viewer n_1 that is a pure binge-watcher, that is, she watches all episodes in the same watching session. The pure binge-watcher demonstrates perfect first order binge viewing with $1 - S[b(n_1)] = 1$ and perfect second order binge viewing $W[b(n_1)] = 1$. Her propensity for binge-watching would be $binge(n_1) = 1$. At the other extreme, we have a viewer n_2, a pure non-binge-watcher, that satisfies $binge(n_2) = 0$ with no episodes watched in the same watching session $1 - S[b(n_2)] = 0$ and she takes long break intervals or quits the platform

after watching some episodes $W[b(n_2)] = 0$. Depending on the data and context, we may not observe these extreme viewers, and for some series, we may have a homogeneous group of viewers (Schweidel and Moe, 2016). Once we have generated the individual viewer scores, we rank them in a list N^R, where indices i, j from N^R with $i < j$ implies the viewer i is more prone to binge-watching then viewer j.

Running Example: Using the propensity scores $binge(n)$ in Figure 4.16, we rank the viewers as in $N^R = \{3, 1, 2, 5, 4\}$, where the third viewer is most prone to binge-watch the content.

Next, the content provider who has profiled the viewers according to their binge-watching attitude decides on the level of personalization, that is, the number of viewer groups. Let us assume for now that she decided to have W groups with fixed group sizes. We assign the viewers from the set N^R to W sets denoted by $N^R(w')$ for $w' \in \{1, \ldots, W\}$. Next, we obtain aggregate break distance and flux transition probability matrices for each group of viewers w' in analogous steps to Section 4.2.1.1 and Section 4.2.1.2, and hence the effective distance matrix (see Section 4.2.2) for each series in the library.

Once we obtain the effective distance matrix, we apply the series heuristics for the release of individual series (see Section 4.3.1) to generate release strategies for the individual series s, that is $\big(k_s^*(w'), f^*_{c\in\{1,\ldots,k_s^*\}}(w')\big)$. Next, we apply the ordering heuristics to obtain the ordering policy $\big(g_t(w'), S_{g'_t\in\{1,\ldots,g_t\}}(w')\big)$ associated with each subscription period t. Thus, we generate the release strategy with a level w of personalization.

Running Example: Let us assume that the content provider chooses to have two viewer groups consisting of viewers $w_1 = \{3, 1, 2\}$ and $w_2 = \{5, 4\}$, where w_1 consists of viewers that are more prone to binge watching. Then, the content provider personalizes the content according to the viewing behavior of the two groups. Formally, she has the ordering policy $\big(2, w_1, w_2\big)$. As suggested above, we generate effective distance matrix for each group of viewers when deriving the release of individual series (series heuristics) and the order of multiple series for each subscription period (order heuristics). Figure 4.17 summarizes the impact of the personalization strategy chosen by the content provider (highlighted in red) in comparison to the benchmark case with no personalization (highlighted in black). It is important to note

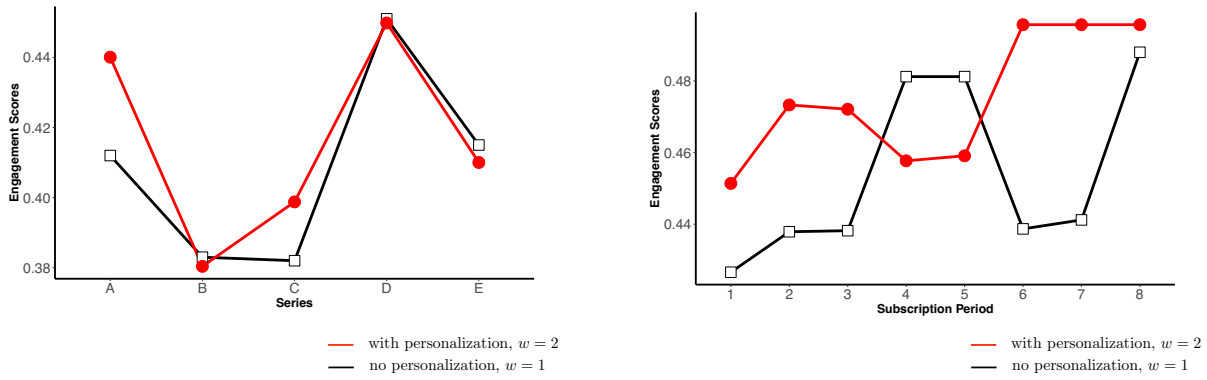


Figure 4.17: Running Example: Personalization of the Content

that, personalization of the content does not necessarily improve viewer engagement. As an example, viewer engagement with the soap opera E degrades with personalization (left panel of Figure 4.17). The intuition behind this is that viewer engagement is a combination of two elements: water cooler talk and binge-watching. The water cooler effect decreases and binge-watching driven engagement improves with personalization. Therefore, total viewer engagement with the content of interest may diminish with personalization if the impact of water cooler effect is higher as in the case of soap opera E. Similarly, viewer engagement may degrade in a subscription period with personalization as in the case of periods 4 and 5 (right panel of Figure 4.17). However, in this example, the overall viewer engagement score, that is the mean of the engagement scores of the subscription periods, associated with the personalization policy ($EC^* = 0.47$) is superior over the benchmark policy ($EC^* = 0.45$).

The question remains, "how far to personalize the content?". To provide an answer to this question, the content provider needs to generate the viewer engagement scores for each possible level of grouping, that is, $W \in 1, \ldots, N$ with potential group sizes, and hence she has a choice set $\sum_{W=1}^{N} \binom{N-1}{W-1} = 2^{N-1}$. To account for the complexity of the choice set, we apply a "personalization heuristic". The method is along the lines of the previous discussions, that is, we iterate by dividing groups into further subgroups. Formally, the viewer engagement

associated with a personalization level W for a given subscription period t is

$$EC_t^T(W) = \sum_{w'=1}^{W} \Big(EC(k^*(w'), f_c^*(w')) + EC(g_t(w'), S_{g_t'}(w')) \Big) / W, \tag{4.20}$$

where $EC(k^*(w'))$ and $EC(g_t(w'), S_{g_t'}(w'))$ are the viewer engagement scores associated with the release of the individual series (see equation (4.12)) and the ordering policy (see equation (4.17)), respectively. We start with the benchmark $W = 1$ policy and iterate to a two groups strategy by calculating the total viewer engagement score $EC_t^T(W = 2)$ by diving the viewers into two equal sizes and comparing it with $EC_t^T(W = 1)$. If we have $EC_t^T(W = 1) > EC_t^T(W = 2)$, the one size fits all policy is better than the $W = 2$ policy. We iterate further by focusing on each groups (W_1) and (W_2) of the $(W = 2)$ policy and divide each group into two further subgroups. If the total viewer engagement score $EC_t^{w_i}(W = 1)$ of the group (W_i) improves with grouping further into two segments, that is $EC_t^{w_i}(w = 2) > EC_t^{w_i}(w = 1)$, we increase the depth of personalization with the sub-groups W_{i1}, W_{i2}. The heuristics return the optimal solution by comparing different personalization levels $W \in \{1, \ldots, N\}$.

Thus, we conclude our model of content release that can be instrumental for platforms, such as over the top media services or education platforms, when designing their content libraries.

4.4 Online Learning

In this section, we apply our model of content release to online learning context. In particular, we use data from an online postgraduate programme that consists of 17 courses covered by 76 students in two years (Fall 2015-Summer 2017).[13] As in the tv series context, we have both binge-watching and water cooler talk in the online education setting. In particular, online

[13]The dataset is secondary and ensures full anonymity of the learners in both directions of communication: data provided by the education platform and results reported back to education platform. The methodology and anonymised data used in this study was reviewed and approved by the Education Ethics Committee (EEC) of Imperial College London with the reference number EERP17180-34, following submission of the General Data Protection Regulations compliance checklist provided by the education platform to the EEC. The primary data is stored at Imperial College Business School Hub Database by the Educational Technical Unit.

education platform enables viewers to watch the videos available in their own pace (and hence allowing for binge-watching) while at the same time there exist online forum discussions and exercises where students interact with their cohort and the lecturers (digital water cooler talk). It is also important to note that the data is obtained under an all the content released at once policy, and hence the content release model we have developed can be applied to this specific case. Figure 4.18 provides a descriptive summary of the data.

Data Content $\Longrightarrow$

				Student 1			Student 2			
Course Codes	Course Names	Activities	Activity Names	Average Activity Coverage Time	Start Time	End Time	Average Activity Coverage Time	Start Time	End Time	...

Start Time = End Time − Average Activity Coverage Time

Type	Course ID	Course Name	# of Students	# of Activities	Maximum Break Length (Hours)	Min Break Length (Hours)	% of Activites Completed	# of Students
Core Courses	1	Managerial Economics	76	121	8665.52	0.00	0.98	76
	2	Corporate Finance	76	94	8814.80	0.02	0.97	76
	3	Financial and Management Accounting	76	95	8431.67	0.02	0.99	76
	4	Global Marketing Management	76	122	8736.38	0.00	0.99	76
	5	Strategy	76	85	10227.58	0.02	0.98	76
Elective Courses	6	Digital Business	21	104	3144.50	0.00	0.78	21
	7	Advanced Strategy	27	86	2618.42	0.00	0.77	27
	8	Advanced Corporate Finance	24	102	1499.28	0.00	0.74	24
	9	Brand Management	17	90	3351.37	0.00	0.78	17
	10	Leadership	28	89	1787.72	0.00	0.87	28
	11	Venture Capital Finance	24	93	1439.40	0.00	0.82	24
	12	Entrepreneurship	30	75	4685.58	0.00	0.80	30
	13	Managing Negotiations	18	88	1278.12	0.00	0.85	18
	14	Private Equity	16	117	2370.08	0.00	0.68	16
	15	Mergers & Acquisitions	14	115	3291.58	0.00	0.57	14
	16	International Business	15	105	3293.55	0.00	0.81	15
	17	Operations Management	18	80	3021.85	0.00	0.68	18

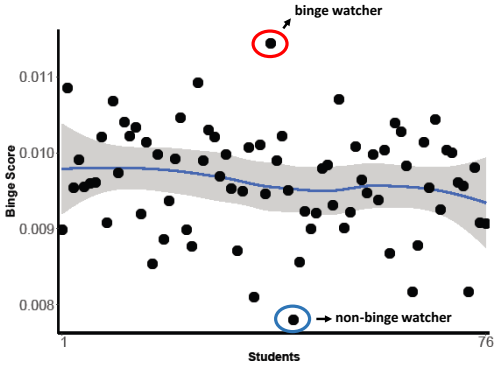
Figure 4.18: Summary of the Online Education Data

A significant component of the data is the time stamps at which learners complete activities of each course while it does not include the times at which students start covering an activity of interest. To account for the missing data on start times, we utilized "activity duration" estimates provided by the education platform based on surveys from past years, and thereby we generate estimated start times. Each course consists of 10 sessions (as in sessions of tv shows) each of which consists of activities (as in episodes of tv shows) where video content from lecturers is shown. As the summary table shows, there were students that took long breaks in some courses, which impacts the forum discussion participation negatively and hence diminish student engagement. Another reason for the education platform to reconsider its release schedule is the low coverage rates especially in the elective courses. In addition, student

feedback suggested that some materials should be reorganized as a session or some sessions should be clustered into smaller components as the material was relatively advanced. In this section, we share the summary of the results we have reported back to the education platform to this end, by randomising individual viewing behaviour to ensure data privacy. Further, we investigate the characteristics of the content (courses) and the viewers (students) using the propensity to binge watching scores we have introduced in Section 4.3.3. One important observation is that the students have heterogeneous viewing attitudes. In particular, some students cover several activities in few study sessions and take long breaks in between (such as the student highlighted in red, Figure 4.19a) while some other students cover the content regularly with small breaks (such as the student highlighted in red). Also, we observe that the students have different viewing attitudes across courses, that is some courses are relatively more binge-watched (such as the course highlighted in red, Figure 4.19b) while some others are covered in regular intervals by most of the viewers (such as the course highlighted in blue). Our results indicate that the core courses are more binge studied compared to the elective courses. As for the elective courses, qualitative and advanced courses, such as *Advanced Strategy*, are more periodically covered with medium breaks while quantitative courses such as *Venture Capital Finance* or *Operations Management* are more binge-studied compared to other courses.

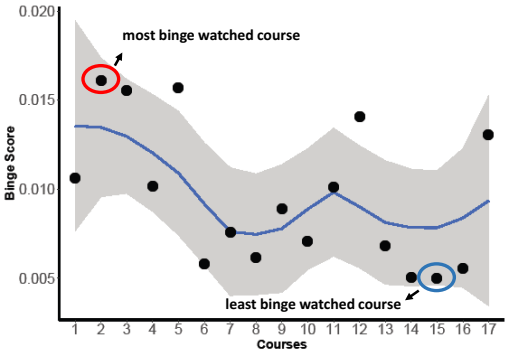
4.4.1 Release of Courses

In this section, we apply "series heuristics" to find the optimal release strategy for each course. Figure 4.20 (right panel) summarizes the optimal number of clusters (k_s^*) for courses $s = 1, \ldots, 17$. We have discussed that students have different viewing behaviors for different courses in the previous section (Figure 4.19b). As a result of the heterogeneous viewing patterns in each course, optimal release strategy varies depending on the course type (as in genre for tv series). In general, core courses should be released in several clusters while most of the electives should be in few clusters. However, qualitative courses with frequent case discussions such as *Entrepreneurship* or *Managing Negotiations* should be released in more clusters when compared



(a) Propensity of Binge Watching (with randomization)

Course ID	Courses	Binge Score
1	Managerial Economics	0.01065
2	Corporate Finance	0.01613
3	Financial and Management Accounting	0.01558
4	Global Marketing Management	0.01020
5	Strategy	0.01573
6	Digital Business	0.00581
7	Advanced Strategy	0.00760
8	Advanced Corporate Finance	0.00619
9	Brand Management	0.00890
10	Leadership	0.00711
11	Venture Capital Finance	0.01016
12	Entrepreneurship	0.01410
13	Managing Negotiations	0.00683
14	Private Equity	0.00507
15	Mergers & Acquisitions	0.00503
16	International Business	0.00557
17	Operations Management	0.01306



(b) Binge Scores of Courses (Real Data)

Figure 4.19: Summary on Binge Watching

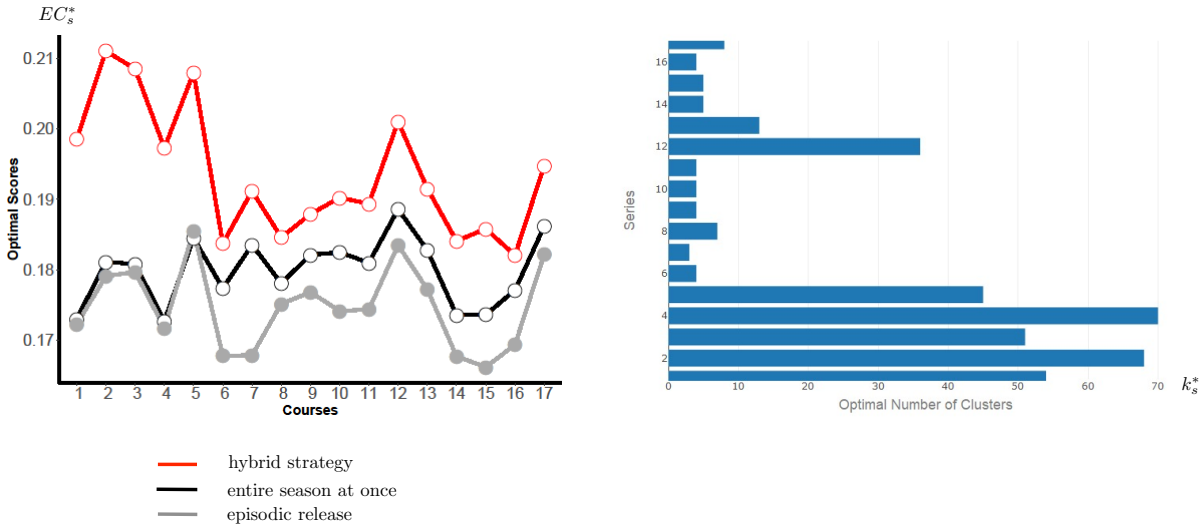
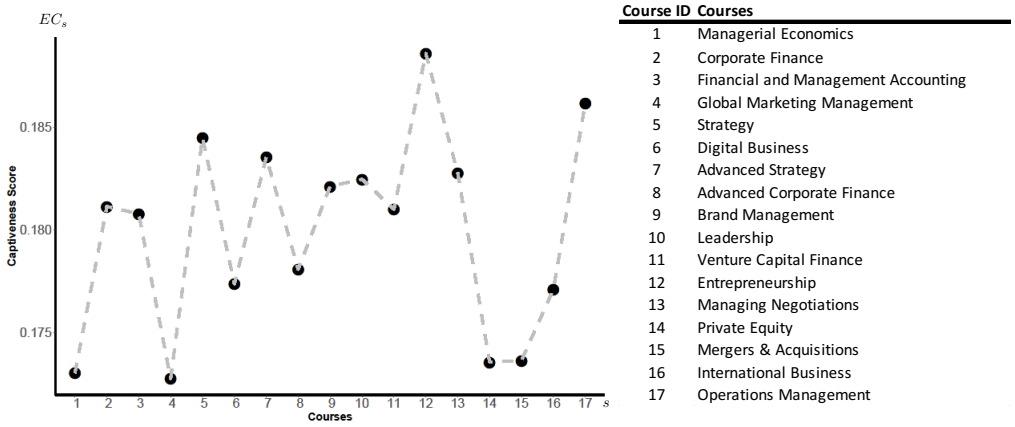


Figure 4.20: Releasing the Individual Courses

to the quantitative or theory based qualitative courses.

Next, we compare viewer engagement scores of the optimal strategies to the benchmark policies: releasing episodes of content periodically (episodic release) and making all the content available upfront (entire season at once). We observe that hybrid strategies dominate both the episodic release and entire season at once strategies. When compared to elective courses, hybrid strategies provide a substantial improvement to the viewer engagement in core courses. It is interesting to note that that entire season at once policy is always as good as the episodic release in this case, implying that binge watching effect dominates over water cooler effect. The intuition behind this is that students prefer online education because of the flexibility provided to cover the content in their own pace. However, we also observe the impact of the water cooler talk on the aggregate viewer engagement as the hybrid strategy always dominates the entire season at once policy.



Course ID	Courses
1	Managerial Economics
2	Corporate Finance
3	Financial and Management Accounting
4	Global Marketing Management
5	Strategy
6	Digital Business
7	Advanced Strategy
8	Advanced Corporate Finance
9	Brand Management
10	Leadership
11	Venture Capital Finance
12	Entrepreneurship
13	Managing Negotiations
14	Private Equity
15	Mergers & Acquisitions
16	International Business
17	Operations Management

Figure 4.21: Captiveness of the Courses

4.4.2 Designing the Order of Courses

We continue with "order heuristics" and derive the order according to which groups of courses should be available to students.[14] To this end, we start with the captiveness of courses in generating viewer engagement. Figure 4.21 shows how captivating a course is, that is how likely it is that students will continue to watch the rest of the content and initiate interaction with their cohort. In our case, elective courses such as *Entrepreneurship* or *Operations Management* are most captivating courses while core courses such as *Managerial Economics* or *Global Marketing Management* are among least captivating courses. This observation may be due to various reasons such as the content of the courses, teaching team or the previous background of the students about the course. In view of the content, whether or not a course is elective does not necessarily inform captiveness as we have both captive and non-captive courses in both groups.

Identifying the captiveness scores of the courses is the first step to organize library across subscription periods. Next, using the rank of captive courses, we form the semester design as summarized in Figure 4.22. The upper panel shows the number of course groups we suggest to the education platform in each time period. As an example, in the first month of the programme

[14]We used R Studio Version 1.1.419 to apply content release model in the online learning case.

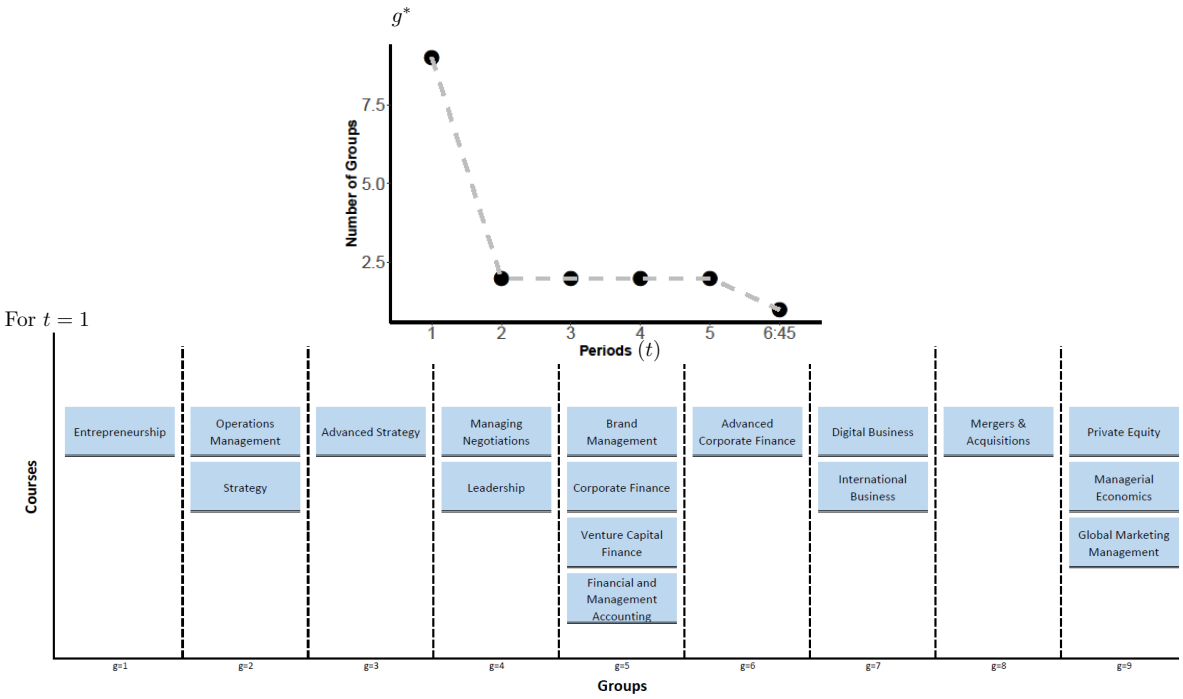


Figure 4.22: Optimal Ordering

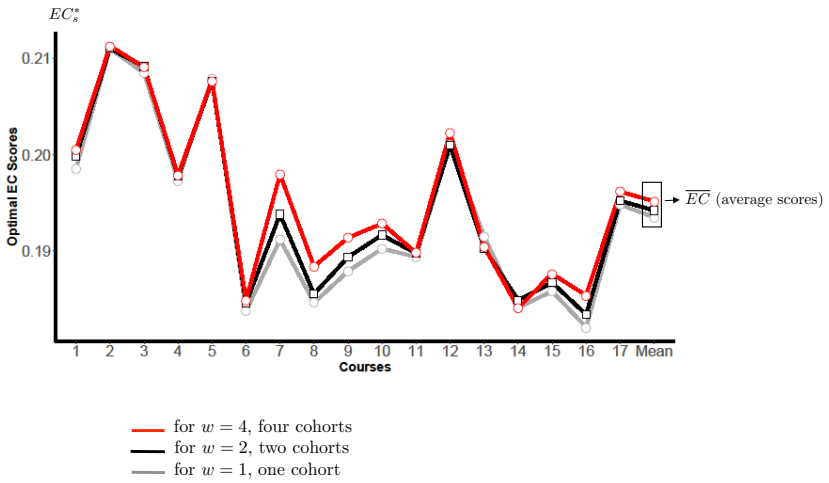
our model suggests that courses should be released in 7 groups. The lower panel of the figure elaborates further on content design of the first month and summarizes the courses to be released concurrently in order to ensure viewer engagement during that particular time period. The order of courses follows distinct patterns and that courses of same field should not necessarily be released simultaneously. For instance, our model suggests that clusters of activities from *Private Equity*, *Managerial Economics* and *Global Marketing Management* would be released together (see $g = 9$ in the Figure 4.22). On the other hand, we also observe that several courses from finance field (*Corporate Finance*, *Venture Capital Finance* and *Financial and Management Accounting*) should be released together with a marketing course. Potentially, one underlying reason would be that students seek variety when they cover core courses such as *Managerial Economics* and *Global Marketing Management* while they may be more content to have similar type of elective courses (such as finance electives) as they might have selected courses that they have background knowledge or interest.

4.4.3 Personalization of the Curriculum

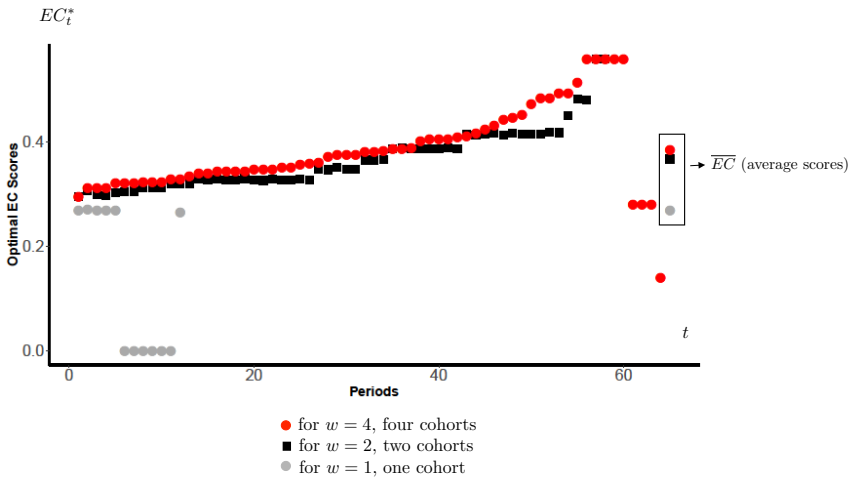
Finally, we investigate if personalization of the content would improve overall viewer engagement. In our case, the dataset consists of 76 students and we observe from Figure 4.18 that there are as few as 15 students enrolling in elective courses. As a result, the education platform would be interested in having at most four cohorts (personalization degree of four), that is to have maximum four groups of students for which to personalize the content. In this sample analysis, our heuristics (both series and ordering heuristics) indicate that personalizing the content to four cohorts of students produces the best aggregate viewer engagement score.

In particular, we find that the engagement associated with most of the courses is maximum with a personalization degree of four, that is there should be different hybrid release strategies for each group (Figure 4.23). The intuition behind this result is that in our case of online education binge studying aspect dominates water cooler talk in creating viewer engagement (Figure 4.23a). One exception to this general trend is the *Private Equity* course, which should be released with an entire content at once policy ($w = 1$). Therefore, the open forum discussions and interaction among the cohort plays key role in this course. Secondly, we find that releasing the *Managing Negotiations* and *Financial and Management Accounting* courses to two cohorts improves the viewer engagement most, that is both binge watching and water cooler talk elements play crucial role for these courses. Similarly, we find that personalizing the order of series to different viewer groups is crucial (Figure 4.23b). It is important to note that one element of personalization of content library is concurrently providing viewer specific content to each group of viewers. In the education context, personalizing the content in this degree may create concerns about educational advantage/disadvantage. To account for this crucial aspect, we suggest the educational platforms, and also the online streaming services, to use digital nudges to enhance viewing behaviour. As an example, in our case, educational platform may send notifications through the platform to remind that a cluster of content of a course is ready to be covered.

We elaborate further on the release of individual series for different viewer types (Figure 4.24). A general trend we observe is that content should be released more periodically for viewers that



(a) Optimal Release of Individual Courses



(b) Optimal Ordering in a Learning Period

Figure 4.23: Summary on Personalization

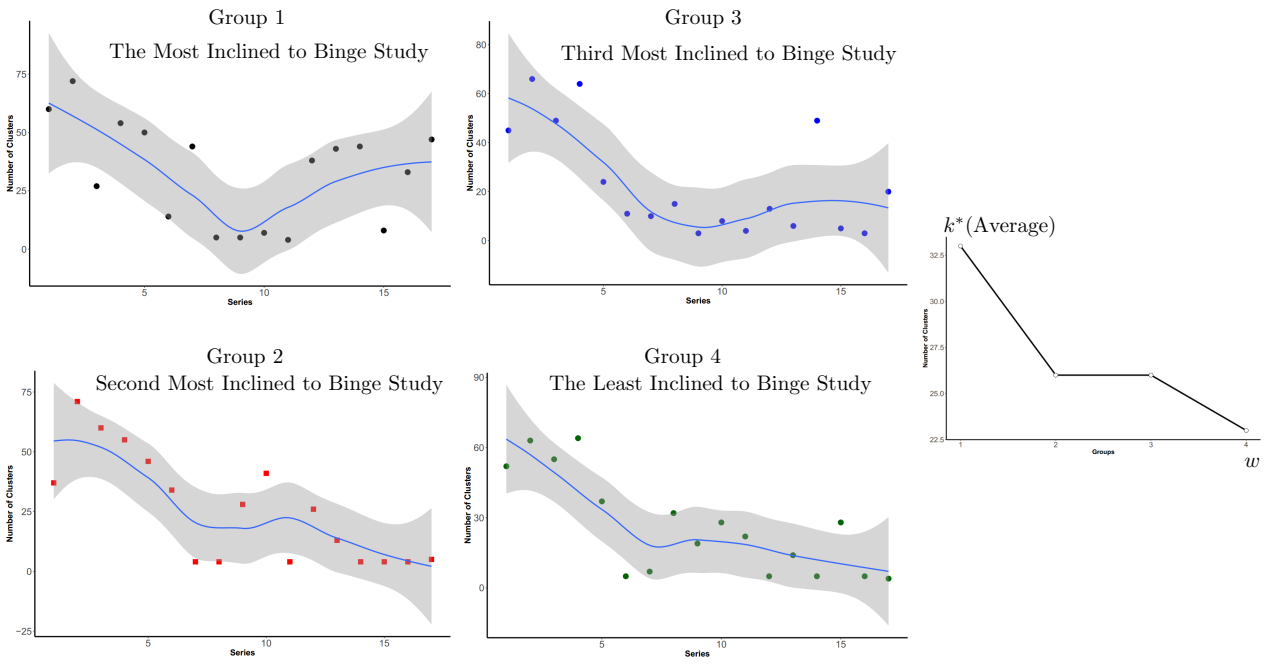


Figure 4.24: Optimal Release of Individual Courses for Each Cohort

are more inclined to binge study, while noting that there exists variations across courses. The intuition behind this trend is that binge studying students have a tendency to leave the platform for longer durations of time and releasing content periodically would ensure the students to cover the content more regularly and most importantly with similar paces with their cohort. That, in turn, improves the overall viewer engagement.

For some elective courses, however, our model suggests releasing the content in few clusters for the viewers that are inclined to binge watch when compared to other groups. For courses with certain characteristics, such as being advanced (e.g., *Advanced Strategy, Advanced Corporate Finance*) or framework-based (e.g., *Strategy*), binge watching element of viewer engagement dominates, and hence the content provider would ensure a content design that primarily induces viewers that are more inclined to cover the content in long viewing sessions while still encouraging students from the same cohort to align their viewing pace to some extent.

In view of the course orders, our model suggests that the education platform should ensure a variety of courses available at the same time for groups that are inclined to binge study (Figure 4.25). Therefore, we agree with Netflix that the content provider should let the students

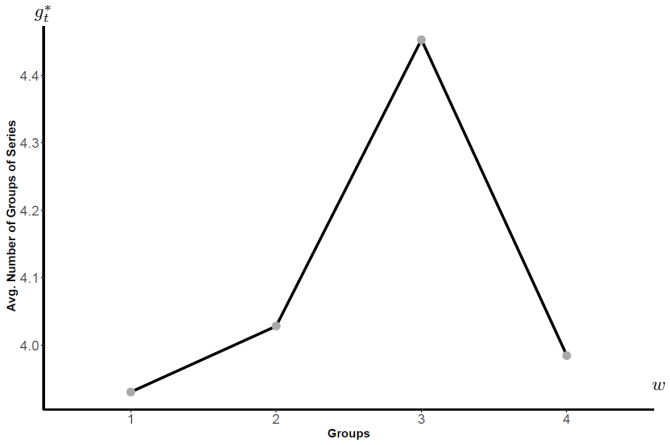


Figure 4.25: Optimal Ordering for Each Cohort

to binge study, or in their words: "If binge is really what they want, then let them binge!". However, our results indicate a key aspect, that is to let the students binge study as long as the content consists of a variety of courses instead of the same type of course. Therefore, binge studying motivation of students may be canalized to studying a range of courses with similar pace with the rest of the cohort. It is interesting to observe that the education platform should ensure a variety of courses to less binge studying group as well. The intuition behind this is to ensure a rich library consisting of various courses for the to non-binge studying group that is at the same time frequent visitors of the platform.

4.5 Conclusions

We have introduced a first model, to our knowledge, of online content release with practical implications for content providers such as the over the top media services or education platforms. Inspired by the comments of the industry practitioners, we specifically focused on three dimensions of the release strategy to improve total viewer engagement with the content: releasing individual series, ordering multiple series in each subscription period, and personalization of the content according to viewer types.[15] In addition, we have tested our model in a real setting, where the content is an online postgraduate course.

Our main contribution is to develop a model of online content release with the aim to maximize the viewer engagement with the content. To measure viewer engagement, i.e. the metric of success for online content providers, we have adopted the network-driven contagion model developed by Brockmann and Helbing (2013) explaining spreads of diseases. In particular, network-driven contagion model enables us to include empirically verified viewing behaviors in online content cases, binge-watching and water cooler talk. Further, we enhance the model to content release setting with the computation of viewer engagement scores conditioned on the network topology, which is the outcome of the content release strategy of the firm. Thus, we link binge-watching and water cooler talk effects from the consumer's end with the content release strategy of the firm. As a result, our model produces an optimal content release schedule that would maximize viewer engagement with the content library.

Typically, online content providers either release episodes of content periodically (episodic release) as in the case of HBO Now or Udacity, or make all the content available upfront (entire season at once) such as Netflix or EdX. An important implication of our model is that hybrid release strategies, that is, releasing clusters of episodes periodically, may dominate these practically observed release strategies. In addition, we also show that the optimal release of content is dependent on the type of the content (genre) and viewer characteristics. Indeed, we find

[15] In addition, in our meetings with three over the top media service providers and one education platform, managers have highlighted these three aspects as the dimensions of content release that they would consider as part of their release strategy.

that core courses should be released in several clusters in order to allow for students to discuss concepts with their cohort while elective courses with quantitative component should be released in few clusters to provide the opportunity to the students to connect various inter-linked concepts in long study sessions. Therefore, our model provides flexibility to content providers to design release schedules specific to content and viewers.

Another contribution we provide to the literature is personalization of the content. Our model is based on the trade-off between binge-watching and the water cooler talk. We argue that this tradeoff informs the degree of personalization that would improve viewer engagement. Therefore, personalization of content should be designed depending on how prone the viewers are to binge watch as well as the content of interest. In the online learning case, we find that dividing the students into four cohorts would improve the engagement of viewers. In particular, we suggest that the education platform should release courses periodically ensuring availability of a rich variety of courses for the students that are inclined to binge-study and leave the platform for long durations of time. By doing so, the content provider is able to induce binge studying students to cover a variety of courses in long viewing sessions instead of binge-study an individual course, which in turn addresses both the willingness of the students to binge study and participate in forum discussions and thereby improve the water cooler talk.

Despite our contributions to the literature and industry practices, several relevant aspects of online content releases have not been considered in full. Above all, our model of optimal content release strategy is based on viewers' past content coverage pattern while it does not include the impact of content release strategy on viewing times of the platform users. We complement studies such as Lu et al. (2017) (online learning) and Schweidel and Moe (2016) (online streaming) that focus on viewing behavior under either entire season released at once or episodic release settings by providing a design of content release with three dimensions. However, we acknowledge that one fruitful extension of our model would be to verify viewing response under various release settings (including hybrid releases, content ordering, and personalization). Secondly, we have used a network-driven contagion model to compute viewer engagement and the impact of a release strategy on the viewer engagement. Network-driven contagion model enabled us to include viewers' binge-watching and water cooler talk enabling us to study viewer engagement

under a mixed type of release settings. A fruitful extension to this framework would be to verify our results further with other modeling approaches such as Markov decisions processes that would include viewers' dynamic decision process. Finally, we acknowledge that our focus is primarily on developing an optimization model for content release while the empirical section of our study is for illustrating an application of our model. In addition, our data is based solely on viewing pace of the platform users and is relevant to various online content cases. Unique to the online learning setting, one may extend our model further with student performance on exams or quizzes as well as student feedback about each course. We also acknowledge that empirical studies with industry data in online streaming would enrich the discussion further.

Chapter 5

Conclusions

Digital technologies have been transforming the nature of industries and the role of economic agents as well as increasing the interdependence among various stakeholders. Observing these dynamics, researchers from multiple fields are revisiting the models of the traditional economy to include new characteristics of the firms and consumers, and new frameworks are introduced to describe the new setting of the digitally-driven economy. In this thesis, we aim to contribute to the literature by concentrating on product release strategies in the digital economy. To this end, we considered three frequently observed business practices of the digital economy: digital product rollovers, ecosystems of products, and release of online content.

5.1 Summary

In Chapter 2, we focused on product rollovers that have been previously studied with an emphasis on production-related constraints. Typically, the literature assumes that the durability of a product is exogenous to the firms and the consumers. Our model, instead, considers a setting with proactive and informed consumers and firms reacting to consumers' perception. In this setting, the consumers shape rollover policy of the firm with their outlook and their "perception of obsolescence", while the firm has the power to alter its rollover strategy even it

has already fixed the product development arrangements. We considered three dimensions of the rollover strategy: pricing of the versions, timing of the release of the new version, and the decision whether the old version sold alongside the latest release (dual rollover) or not (solo rollover).

We characterized the optimal rollover strategy under various cases of consumer foresight and perception of obsolescence. Most importantly, we observed that the current findings in the rollover literature are subject to change if the release time is regarded as a lever for a monopolist. Our results update the extant rollover literature concluding that dual rollover is an optimal strategy in the presence of myopic consumers. In particular, we found that release timing can be a tool for the monopolist to allocate the consumers to first and subsequent periods according to their perception of obsolescence. As a result, the monopolist can always deploy a solo rollover and avoid the risk of its demand for the new version to be cannibalized by the old version in a dual rollover. Further, we found that solo rollover is the optimal policy if the firm is able to intervene with the release time and when the cost of production of the new version is low. Thereby, we provided an explanation why the solo rollover settings of the version management literature may be relevant for the information goods settings (e.g., Calzada and Valletti, 2012; Lobel et al., 2015). We also presented a welfare analysis showing that the consumers inattentive to the firm's release policy are hurt from the perceived obsolescence while both the strategic consumers and the monopolist benefit from high obsolescence.

In Chapter 3, we covered the new competition setting involving an ecosystem firm offering a family of products (such as Apple with a smartwatch and a smartphone) and a category firm that has a single product (such as Tag Heuer with an independent smartwatch). Typically, the ecosystem firms possess an advantage over the category firms as they are able to connect the new goods to a family of existing products via digital technology. They can enhance the consumer experience of the new product with appreciation from previous category. We refer to the ecosystem firm's capability to connect the new goods with the old goods as "ecosystem leverage". The traditional setting of two-stage price-quality competition literature (e.g., Motta, 1993) studies one single product in isolation of previous products. In our setting, the goods can be substitutes, complements or independent products. Thus, we built a framework to contribute

to the industry discussion about the impact of ecosystem firms and single category firms on innovation levels in a new product. We showed that an ecosystem leverage is not necessarily a threat to the competitor. In some cases, it even improves the welfare of both firms. In particular, when the goods are not strong substitutes and the ecosystem firm is not the stand-alone quality leader, the ecosystem leverage increases the profits of both the ecosystem and category firms. Further, we found that there are cases when the ecosystem leverage does not deter innovation but enhances the quality levels of both goods. Another interesting finding is that when the ecosystem firm poses the most risk to its competitor, ironically, it is willing to share its leverage with the category firm in return for revenues.

In Chapter 4, we turned our attention to online content providers, i.e. over-the-top media services providers (e.g., Netflix, Amazon Prime, Hulu) and online education platforms (e.g., Coursera, EdX, Udacity). Industry experts point out that designing the release of online content library is a significant challenge for the content providers. The industry provides us with different release policies, such as the episodic release policy of HBO Now or the all at once strategy of Netflix, even if the content providers are of similar types and serve markets with overlapping characteristics. There exist conflicting views on which type of release policy is better. To initiate an understanding of optimal releasing strategy of online content providers, we introduced a framework of three dimensions: (1) release of an individual series, (2) order of multiple series within a subscription period, and (3) personalization of the content. The primary emphasis of our model is to maximize "viewer engagement", a frequently cited industry metric. To this end, we integrated two empirically verified viewing phenomena, binge-watching and water cooler talk, as indicators of viewer engagement. We tested our model on data from an online education programme and concluded that in most cases, hybrid release strategies, which release clusters of episodes periodically, outperform both the episodic and the entire season at once policies. Further, we showed that the viewing behavior is heterogeneous among users of the platforms, and it is highly dependent on the considered type of the series. Thus, we suggested content-based and viewer-specific release schedules that improve the engagement of users with the overall content library. We hope that our work contributes to a better understanding of the content release strategies for both industry practitioners and researchers. Multiple interviews

with managers from various online content providers revealed that our framework addresses critical challenges of the field and provide a methodology on designing online content release.

To conclude, we covered recent digital settings (digital product rollovers, product ecosystem firms, and online content providers) and proposed extensions over traditional frameworks from the literature (second and third chapters) and introduced a new framework to address some firm and consumer characteristics that emerge in the digital economy (fourth chapter). We refer the reader to Figure 5.1 for a summary of the chapters and conclusion sections of each article for a more detailed discussion of the results.

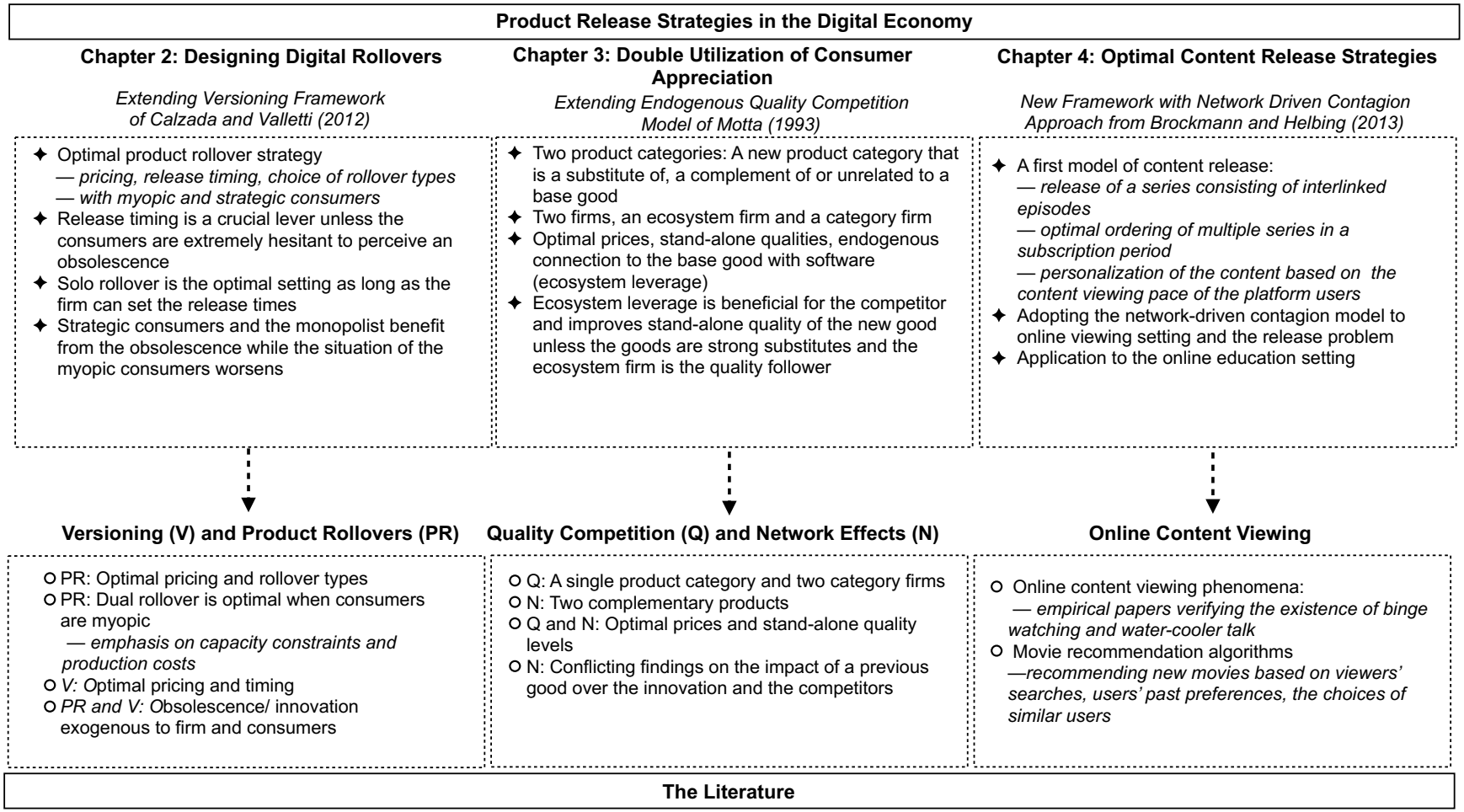


Figure 5.1: Summary of the Thesis and Its Contributions

5.2 Future Research Directions

Several relevant aspects have not been covered in our models. We discuss the limitations of each chapter and the relevant future research directions specific to each model in the conclusions sections of each chapter. In this section, we discuss general future research directions that may contribute to the discussion of product releases in the digital economy.

One future direction could be to investigate the complex ecosystems such as WeChat or Amazon, where various groups of product ecosystems are present under the same platform. One way of modeling these complex ecosystems could be the network-driven contagiousness models that illustrate the interdependence among multiple locations linked with human mobility (e.g., Brockmann and Helbing, 2013), which we also adopt in the fourth chapter of this thesis. In particular, one can locate the captive applications (or services) within the complex ecosystem or predict how users switch from one app to another. Perhaps more interestingly, one may model the interlinked groups of ecosystems as in WeChat using the multi-layered network theory where the spread of phenomena in-between interdependent networks is investigated (e.g., Huang et al., 2013). With a product tying perspective, this type of framework may provide practical implications for platform managers, advertisers as well as policymakers.

Another business model frequently observed in online platforms and experience goods is the "Freemium" strategy where a free trial of the service is provided to users with an option to upgrade to the full version. One crucial aspect of this model is to decide the quality and the duration of the free version. The tradeoff is that the free version may cannibalize the full version if the content is too advanced or the length of the free trial is too long, but, at the same time, it may signal a high-quality full version in the eyes of the consumers, which may increase their willingness to pay. This setting can be further extended by considering the effects of competition.

Further, there exist examples of hybrid products that include features of different types of standalone goods such as the laptop-tablet hybrids of Acer and Lenovo or Apple's iPhone XR, XS smartphone-tablet hybrids. It is intuitive to think that hybrid products may have

Topic	Problem Setting	Consumer Characteristics	Firm Toolbox
Closing the Movie Release Windows? The Role of Consumers' Word-of-Mouth with H. Ren and T. Huang	Versions of a product category —Substitutes Experience goods	Social learning of consumers Heterogeneous quality appreciation	(1) Which versions to release? (2) At which prices to release? (3) At which qualities to release?
Global Rollovers	Two interactive markets —Consumers' access to the goods of each market Monopoly	Different consumer outlooks: —Consumers attentive to firms' release strategies —Inattentive consumers	(1) In which markets to release? (2) At which prices to release? (3) When to release?
Game of Preannouncements	New product category Experience goods Duopoly	Pre-announcement triggered strategic consumer behavior	(1) What to preannounce? (2) When to preannounce?

Figure 5.2: Future Research Projects

advantage over independent goods as they serve to extended group of potential consumers. In addition, typical hybrid products provide broader set of features with additional utility of multiple different products in one single product. However, one challenge the firms offering hybrid products face is the competition from the firms producing the respective independent products. At the same time, the existence of hybrid products may divert the appreciation of consumers to perceive standalone goods as more related and hence make the independent good producers to become close competitors. Potentially, it could be interesting to investigate the implications of hybrid products on both consumers and producers of independent goods. We summarize some of our ongoing research projects on product releases in Figure 5.2.

Bibliography

Aaker, D. A. (2010). *Brand relevance: Making competitors irrelevant.* John Wiley & Sons.

Adalian, J. (2017). How the handmaids tale changed the game for hulu. `http://www.vulture.com/2017/06/hulu-the-handmaids-tale-how-it-changed-the-game.html`. Accessed on 23 August 2017.

Adomavicius, G. and A. Tuzhilin (2005). Toward the next generation of recommender systems: A survey of the state-of-the-art and possible extensions. *IEEE Transactions on Knowledge & Data Engineering* (6), 734–749.

Adweek (2018). Share of binge-watchers in the united states as of april 2018, by gender. `https://www.statista.com/topics/2508/binge-watching-in-the-us/`. Accessed on 26 July 2018.

Agarwal, R., S. Chandrasekaran, and M. Sridhar (2016). Imagining constructions digital future. *McKinsey Insights Report, McKinsey & Company and McKinsey Global Institute,Jun*.

Agrawal, V. V., S. Kavadias, and L. B. Toktay (2015). The limits of planned obsolescence for conspicuous durable goods. *Manufacturing & Service Operations Management 18*(2), 216–226.

Ajorlou, A., A. Jadbabaie, and A. Kakhbod (2016). Dynamic pricing in social networks: The word-of-mouth effect. *Management Science 64*(2), 971–979.

Altarelli, F., A. Braunstein, L. DallAsta, A. Lage-Castellanos, and R. Zecchina (2014). Bayesian inference of epidemics on networks via belief propagation. *Physical Review Letters. 112*(11), 118701.

Anderson, E. T. and J. D. Dana Jr (2009). When is price discrimination profitable? *Management Science 55*(6), 980–989.

Anderson, S. P. and L. Leruth (1993). Why firms may prefer not to price discriminate via mixed bundling. *International Journal of Industrial Organization 11*(1), 49–61.

Anderson, T. (2004). Towards a theory of online learning. *Theory and Practice of Online Learning 2*, 109–119.

Armstrong, M. (2006). Competition in two-sided markets. *The RAND Journal of Economics 37*(3), 668–691.

Armstrong, M. (2013). A more general theory of commodity bundling. *Journal of Economic Theory 148*(2), 448–472.

Arslan, H., S. Kachani, and K. Shmatov (2009). Optimal product introduction and life cycle pricing policies for multiple product generations under competition. *Journal of Revenue and Pricing Management 8*(5), 438–451.

Atkins, R. (2017). Swiss watchmakers play the long game as smartwatch sales slow. `https://www.ft.com/content/1563c94c-0fbf-11e7-b030-768954394623`. Accessed on 11 Jan 2018.

Babiolakis, M. (2016). Forget products. build ecosystems.how products are transforming to open interconnectable interfaces. `https://medium.com/@manolisbabiolakis/forget-products-build-ecosystems-792dea2cc4f2`. Accessed on 11 Jan 2018.

Bakos, Y. and E. Brynjolfsson (1999). Bundling information goods: Pricing, profits, and efficiency. *Management science 45*(12), 1613–1630.

Bakos, Y. and E. Brynjolfsson (2000). Bundling and competition on the internet. *Marketing science 19*(1), 63–82.

Bala, R. and S. Carr (2009). Pricing software upgrades: The role of product improvement and user costs. *Production and Operations Management 18*(5), 560–580.

Bargigli, L., G. Di Iasio, L. Infante, F. Lillo, and F. Pierobon (2015). The multiplex structure of interbank networks. *Quantitative Finance 15*(4), 673–691.

Barragáns-Martínez, A. B., E. Costa-Montenegro, J. C. Burguillo, M. Rey-López, F. A. Mikic-Fonte, and A. Peleteiro (2010). A hybrid content-based and item-based collaborative filtering approach to recommend tv programs enhanced with singular value decomposition. *Information Sciences 180*(22), 4290–4311.

Basu, A., T. Mazumdar, and S. Raj (2003). Indirect network externality effects on product attributes. *Marketing Science 22*(2), 209–221.

BBC *News* (2017). Apple investigated by france for "planned obsolescence". `http://www.bbc.co.uk/news/world-europe-42615378.` Accessed on 24 September 2018.

Beaver, L. (2017). Here's how apple could boost the smartwatch market. http://uk.businessinsider.com/apple-smartwatch-market-2017-8?r=US&IR=T. Accessed on 11 Jan 2018.

Belk, R. (2014). You are what you can access: Sharing and collaborative consumption online. *Journal of business research 67*(8), 1595–1600.

Belleflamme, P. (2016). The economics of digital goods: A progress report.

Belleflamme, P. and N. Neysen (2009). Coopetition in infomediation: General analysis and application to e-tourism. In *Advances in tourism economics*, pp. 217–234. Springer.

Belleflamme, P. and M. Peitz (2010). Platform competition and seller investment incentives. *European Economic Review 54*(8), 1059–1076.

Belleflamme, P. and M. Peitz (2015). *Industrial organization: markets and strategies*. Cambridge University Press.

Bellezza, S., J. M. Ackerman, and F. Gino (2017). be careless with that! availability of product upgrades increases cavalier behavior toward possessions. *Journal of Marketing Research 54*(5), 768–784.

Bhargava, H. K. and V. Choudhary (2001). Information goods and vertical differentiation. *Journal of Management Information Systems 18*(2), 89–106.

Bhargava, H. K. and V. Choudhary (2008). Research notewhen is versioning optimal for information goods? *Management Science 54*(5), 1029–1035.

Bhargava, H. K., B. C. Kim, and D. Sun (2013). Commercialization of platform technologies: launch timing and versioning strategy. *Production and Operations Management 22*(6), 1374–1388.

Bhattacharya, S., V. Krishnan, and V. Mahajan (2003). Operationalizing technology improvements in product development decision-making. *European Journal of Operational Research 149*(1), 102–130.

Bilginer, Ö. and F. Erhun (2015). Production and sales planning in capacitated new product introductions. *Production and Operations Management 24*(1), 42–53.

Billington, C., H. L. Lee, and C. S. Tang (1998). Successful strategies for product rollovers. *MIT Sloan Management Review 39*(3), 23.

Binken, J. L. and S. Stremersch (2009). The effect of superstar software on hardware sales in system markets. *Journal of Marketing 73*(2), 88–104.

Bonk, C. J. and K. Zhang (2009). *Empowering online learning: 100+ activities for reading, reflecting, displaying, and doing*. John Wiley & Sons.

Borland, J., P. Dawkins, D. Johnson, and R. Williams (2000). Returns to investment in higher education.

Bose, S., G. Orosel, M. Ottaviani, and L. Vesterlund (2008). Monopoly pricing in the binary herding model. *Economic Theory 37*(2), 203–241.

Boss, M., H. Elsinger, M. Summer, and S. Thurner 4 (2004). Network topology of the interbank market. *Quantitative finance 4*(6), 677–684.

Bourreau, M., F. Kourandi, and T. Valletti (2015). Net neutrality with competing internet platforms. *The Journal of Industrial Economics 63*(1), 30–73.

Bouwman, H., S. Nikou, F. J. Molina-Castillo, and M. de Reuver (2018). The impact of digitalization on business models. *Digital Policy, Regulation and Governance 20*(2), 105–124.

Brockmann, D. and D. Helbing (2013). The hidden geometry of complex, network-driven contagion phenomena. *science 342*(6164), 1337–1342.

Bruce, N. I., N. Z. Foutz, and C. Kolsarici (2012). Dynamic effectiveness of advertising and word of mouth in sequential distribution of new products. *Journal of Marketing Research 49*(4), 469–486.

Bulow, J. I. (1982). Durable-goods monopolists. *Journal of political Economy 90*(2), 314–332.

Calzada, J. and T. M. Valletti (2012). Intertemporal movie distribution: Versioning when customers can buy both versions. *Marketing Science 31*(4), 649–667.

Carbajo, J., D. De Meza, and D. J. Seidmann (1990). A strategic motivation for commodity bundling. *The Journal of Industrial Economics*, 283–298.

Carlton, D. W. and M. Waldman (1998). The strategic use of tying to preserve and create market power in evolving industries. Technical report, National Bureau of Economic Research.

Carrillo, J. and G. Tan (2008). Platform competition with complementary products. Technical report, Working paper.

Chakravorti, S. and R. Roson (2006). Platform competition in two-sided markets: The case of payment networks. *Review of Network Economics 5*(1).

Chen, Y. and J. Xie (2007). Cross-market network effect with asymmetric customer loyalty: Implications for competitive advantage. *Marketing Science 26*(1), 52–66.

Cheng, R. (2018). Lg shakes up its smartphone launch strategy. `http://uk.businessinsider.com/`

iphone-x-sales-piper-jaffray-survey-2018-3?r=US$\&$IR=T. Accessed on 24 September 2018.

Choi, J. P. (1994). Network externality, compatibility choice, and planned obsolescence. *The Journal of Industrial Economics*, 167–182.

Choi, J. P. (2003). Bundling new products with old to signal quality, with application to the sequencing of new products. *International Journal of Industrial Organization 21*(8), 1179–1200.

Choi, J. P. (2010). Tying in two-sided markets with multi-homing. *The Journal of Industrial Economics 58*(3), 607–626.

Choi, J. P. and C. Stefanadis (2001). Tying, investment, and the dynamic leverage theory. *RAND Journal of Economics*, 52–71.

Choi, S.-Y. and A. B. Whinston (2000). The future of the digital economy. In *Handbook on electronic commerce*, pp. 25–52. Springer.

Clements, M. T. and H. Ohashi (2005). Indirect network effects and the product cycle: video games in the us, 1994–2002. *The Journal of Industrial Economics 53*(4), 515–542.

Conviva (2015). Binge watching:the currency of video economics. http://www.conviva.com/conviva-customer-survey-reports/binge-watching-2015/. Accessed on 25 August 2017.

Cooper, T. (2004). Inadequate life? evidence of consumer attitudes to product obsolescence. *Journal of Consumer Policy 27*(4), 421–449.

Coyle, D. (2018). The digital economy is disrupting our old models. https://www.ft.com/content/3c94c25a-47a4-11e8-8c77-ff51caedcde6. Accessed on 09 Sep 2018.

Cui, G., H.-K. Lui, and X. Guo (2012). The effect of online consumer reviews on new product sales. *International Journal of Electronic Commerce 17*(1), 39–58.

Dawson, A., M. Hirt, and J. Scanlan (2016). The economic essentials of digital strategy. *McKinsey Quarterly 3*, 45.

de Souza, A. (2017). Who's afraid of netflix? not hbo, says its programming chief. http://www.straitstimes.com/lifestyle/entertainment/whos-afraid-of-netflix-not-hbo. Accessed on 23 August 2017.

Deloitte (2016a). Streaming tv, personalization, and the cio. https://deloitte.wsj.com/cio/2016/06/06/streaming-tv-personalization-and-the-cio/. Accessed on 22 July 2018.

Deloitte (2016b). There's no place like phone. consumer usage patterns in the era of peak smartphone. global mobile consumer survey 2016: Cut uk.

Deloitte (2017). The deloitte swiss watch industry study 2017, its all about digital. `https://www2.deloitte.com/ch/swisswatchstudy`. Accessed on 11 Jan 2018.

Derdenger, T. (2014). Technological tying and the intensity of price competition: An empirical analysis of the video game industry. *Quantitative Marketing and Economics 12*(2), 127–165.

Desai, P., O. Koenigsberg, and D. Purohit (2004). Strategic decentralization and channel coordination. *Quantitative Marketing and Economics 2*(1), 5–22.

Desai, P. and D. Purohit (1998). Leasing and selling: Optimal marketing strategies for a durable goods firm. *Management Science 44*(11-part-2), S19–S34.

Dhebar, A. (1994). Durable-goods monopolists, rational consumers, and improving products. *Marketing Science 13*(1), 100–120.

Druce, B. (2014). Are you designing a product ecosystem that engages the heart and mind? `https://planetinnovation.com.au/designing-product-ecosystem-engages-heart-mind/`. Accessed on 11 August 2018.

Dubé, J.-P., G. J. Hitsch, and P. E. Rossi (2009). Do switching costs make markets less competitive? *Journal of Marketing research 46*(4), 435–445.

Dubé, J.-P. H., G. J. Hitsch, and P. K. Chintagunta (2010). Tipping and concentration in markets with indirect network effects. *Marketing Science 29*(2), 216–249.

Eadicicco, L. (2017). Can apple make you want an ipad again? `http://time.com/4657459/apple-ipad-2017/`. Accessed on 11 Jan 2018.

Economides, N. (1996). The economics of networks. *International Journal of Industrial Organization 14*(6), 673–699.

Economides, N. and B. E. Hermalin (2012). The economics of network neutrality. *The RAND Journal of Economics 43*(4), 602–629.

Edelman, B. (2014). *Leveraging Market Power Through Tying and Bundling: Does Google Behave Anti-Competitively?* Harvard Business School.

Edelman, B. (2015). Does google leverage market power through tying and bundling? *Journal of Competition Law & Economics 11*(2), 365–400.

Edelman, B. and J. Wright (2015). Price coherence and excessive intermediation. *The Quarterly Journal of Economics 130*(3), 1283–1328.

Elhauge, E. (2009). Tying, bundled discounts, and the death of the single monopoly profit theory. *Harvard Law Review*, 397–481.

Erhun, F., P. Gonçalves, and J. Hopman (2007). The art of managing new product transitions. *MIT Sloan Management Review 48*(3), 73.

European Commission (2017). Antitrust: Commission fines google 2.42 billion for abusing dominance as search engine by giving illegal advantage to own comparison shopping service. `http://europa.eu/rapid/press-release_IP-17-1784_en.htm`. Accessed on 11 January 2018.

Farrell, J. and P. Klemperer (2007). Coordination and lock-in: Competition with switching costs and network effects. *Handbook of industrial organization 3*, 1967–2072.

Ferguson, M. E. and O. Koenigsberg (2007). How should a firm manage deteriorating inventory? *Production and Operations Management 16*(3), 306–321.

Fetherston, J., A. Bailey, S. Mingardon, and J. Tankersley (2017). The persuasive power of the digital nudge. `https://www.bcg.com/publications/2017/people-organization-operations-persuasive-power-digital-nudge.aspx`. Accessed on 22 July 2018.

Friedlander, J. (2017). Engaging each viewer with a personalised streaming service. `http://www.csimagazine.com/csi/Engaging-each-viewer-with-a-personalised-streaming-service.php`. Accessed on 22 July 2018.

Friedman, W. (2017). Ott churn rates pass 50%. `https://www.mediapost.com/publications/article/310974/ott-churn-rates-pass-50.html?utm_source=newsletter&utm_medium=email&utm_content=headline&utm_campaign=106386&hashid=0exonRml32f34jC-6vU2SR-IfcI`. Accessed on 22 July 2018.

Fudenberg, D. and J. Tirole (1998). Upgrades, tradeins, and buybacks. *The RAND Journal of Economics*, 235–258.

Garrison, D. R. and M. Cleveland-Innes (2005). Facilitating cognitive presence in online learning: Interaction is not enough. *The American Journal of Distance Education 19*(3), 133–148.

Garrison, D. R. and H. Kanuka (2004). Blended learning: Uncovering its transformative potential in higher education. *The Internet and Higher Education 7*(2), 95–105.

Gensler, S., S. A. Neslin, and P. C. Verhoef (2017). The showrooming phenomenon: it's more than just about price. *Journal of Interactive Marketing 38*, 29–43.

Gerbier, E. and T. C. Toppino (2015). The effect of distributed practice: Neuroscience, cognition, and education. *Trends in Neuroscience and Education 4*(3), 49–59.

Ghosh, B. and S. Balachander (2007). Research note?competitive bundling and counterbundling with generalist and specialist firms. *Management Science 53*(1), 159–168.

Gibbs, S. (2016). Tim cook:iphones that will drive people to upgrade are on their way. `https://www.theguardian.com/technology/2016/may/03/tim-cook-iphone-model-upgrade`. Accessed on 24 September 2018.

Gluckman, P. (2018). The digital economy and society: a preliminary commentary. *Policy Quarterly 14*(1).

Goldenberg, J., B. Libai, and E. Muller (2010). The chilling effects of network externalities. *International Journal of Research in Marketing 27*(1), 4–15.

Goldfarb, A., S. M. Greenstein, and C. E. Tucker (2015). *Economic analysis of the digital economy*. University of Chicago Press.

Google (2015). Most common motivations for watching online videos in the united states in 2015. `https://www.statista.com/statistics/354190/motivations-watching-online-videos-us/`. Accessed on 26 August 2017.

Grand View Research (2017). Virtual reality (vr) market analysis by device, by technology, by component, by application (aerospace & defense, commercial, consumer electronics, industrial, & medical), by region, and segment forecasts, 2018 - 2025. `https://www.grandviewresearch.com/industry-analysis/virtual-reality-vr-market`. Accessed on 11 August 2018.

Greenley, G. E. and B. L. Bayus (1994). Marketing planning processes in uk and us companies. *Journal of Strategic Marketing 2*(2), 140–154.

Greenstein, S., M. Peitz, and T. Valletti (2016). Net neutrality: A fast lane to understanding the trade-offs. *Journal of Economic Perspectives 30*(2), 127–50.

Grothaus, M. (2015). Heres what automakers think about apple entering the car market. `https://www.fastcompany.com/3043462/heres-what-automakers-think-about-apple-entering-the-car-market`. Accessed on 11th Jan 2018.

GSMA (2012). Mobile phone lifecycles use, take-back, reuse and recycle. https://www.gsma.com/iot/wp-content/uploads/2012/03/environmobilelifecycles.pdf. (Report).

Guiltinan, J. (2009). Creative destruction and destructive creations: environmental ethics and planned obsolescence. *Journal of business ethics 89*(1), 19–28.

Gurman, M., S. Moritz, and I. King (2017). Apple plans to release a cellular-capable watch to break iphone ties. https://www.bloomberg.com/news/articles/2017-08-04/apple-said-to-ready-cellular-capable-watch-to-break-iphone-ties. Accessed on 11 Jan 2018.

Gurman, M. and D. Wu (2018). Apple turns up audio ambition with high-end headphones. https://www.bloomberg.com/news/articles/2018-03-05/apple-is-said-to-turn-up-audio-ambition-with-high-end-headphones. Accessed on 11 June 2018.

Hagiu, A. (2014). *Strategic decisions for multisided platforms*. MIT.

Hanna, R., A. Rohm, and V. L. Crittenden (2011). Were all connected: The power of the social media ecosystem. *Business horizons 54*(3), 265–273.

Harrington, S., T. Highfield, and A. Bruns (2013). More than a backchannel: Twitter and television. *Participations 10*(1), 405–409.

Hartmans, A. (2018). People might not be buying the new $999 iphone because they're perfectly happy with the iphone they already have. http://uk.businessinsider.com/iphone-x-sales-piper-jaffray-survey-2018-3?r=US$\&$IR=T. Accessed on 24 September 2018.

Haruvy, E. E., D. Miao, and K. E. Stecke (2013). Various strategies to handle cannibalization in a competitive duopolistic market. *International Transactions in Operational Research 20*(2), 155–188.

Haskel, J. and S. Westlake (2017). *Capitalism without capital: the rise of the intangible economy*. Princeton University Press.

Hauser, J., G. J. Tellis, and A. Griffin (2006). Research on innovation: A review and agenda for marketing science. *Marketing science 25*(6), 687–717.

Hellmann, K.-U. and M. K. Luedicke (2018). The throwaway society: a look in the back mirror. *Journal of Consumer Policy 41*(1), 83–87.

Hennig-Thurau, T., V. Henning, H. Sattler, F. Eggers, and M. B. Houston (2007). The last picture show? timing and order of movie distribution channels. *Journal of Marketing 71*(4), 63–83.

Hennig-Thurau, T., C. Wiertz, and F. Feldhaus (2015). Does twitter matter? the impact of microblog-

ging word of mouth on consumers adoption of new movies. *Journal of the Academy of Marketing Science 43*(3), 375–394.

Hill, A. (2017). How looming product upgrades make us careless with what we have? `https://www.ft.com/content/8f77fd2a-e2b7-11e4-aa1d-00144feab7de`. Accessed on 24 September 2018.

Hill, S. (2012). Social tv: Linking content, buzz and sales. *Think Insights with Google. Last modified September*.

Hill, S. and A. Benton (2012). Analyzing the impact of social tv strategies on viewer engagement. In *KDD 2012 2nd SOMA Workshop on Social Media Analytics, 12th-16th August.*

Huang, X., I. Vodenska, S. Havlin, and H. E. Stanley (2013). Cascading failures in bi-partite graphs: model for systemic risk propagation. *Scientific reports 3*, 1219.

Iannelli, F., A. Koher, D. Brockmann, P. Hövel, and I. M. Sokolov (2017). Effective distances for epidemics spreading on complex networks. *Physical Review E 95*(1), 012313.

Illing, G., M. Peitz, et al. (2006). *Industrial organization and the digital economy.* Mit Press.

Jacobides, M. G. and J. P. MacDuffie (2013). How to drive value your way. *Harvard Business Review 91*(7), 92–100.

Jiang, J., W. Sheng, S. Yu, Y. Xiang, and W. Zhou (2016). Rumor source identification in social networks with time-varying topology. *IEEE Transactions on Dependable and Secure Computing*.

Jiang, J., S. Wen, S. Yu, Y. Xiang, and W. Zhou (2017). Identifying propagation sources in networks: State-of-the-art and comparative studies. *IEEE Communications Surveys & Tutorials 19*(1), 465–481.

Jing, B. (2011). Social learning and dynamic pricing of durable goods. *Marketing Science 30*(5), 851–865.

Johnson, J. P. and D. P. Myatt (2003). Multiproduct quality competition: Fighting brands and product line pruning. *American Economic Review 93*(3), 748–774.

Johnson, J. P. and D. P. Myatt (2018). The determinants of product lines. *The RAND Journal of Economics 49*(3), 541–573.

Jurgensen, J. (2013). Netflix says binge viewing is no house of cards,. *The Wall Street Journal*.

KantarMedia (2014). A year in the life of tv and twitter in the uk. `https://www.kantarmedia.`

com/global/thinking-and-resources/latest-thinking/a-year-in-the-life-of-tv-and-twitter-in-the-uk. Accessed on 20 July 2018.

Karr, D. (2018). How netflix increases customer engagement utilizing big data. https://martech.zone/netflix-big-data/. Accessed on 22 July 2018.

Katz, M. L. and C. Shapiro (1985). Network externalities, competition, and compatibility. *The American economic review 75*(3), 424–440.

Katz, M. L. and C. Shapiro (1994). Systems competition and network effects. *Journal of economic perspectives 8*(2), 93–115.

Kemerer, C. F., B. K. Dunn, and S. Janansefat (2017). Winners-take-some dynamics in digital platform markets: A reexamination of the video game console wars.

Kennedy, J. B. (1926). When woman is boss: an interview with nikola tesla. *Colliers, Seattle*.

Kim, J.-H., J. Prince, and C. Qiu (2014). Indirect network effects and the quality dimension: A look at the gaming industry. *International Journal of Industrial Organization 37*, 99–108.

Kim, L. (1997). The dynamics of samsung's technological learning in semiconductors. *California Management Review 39*(3), 86–100.

Koca, E., G. C. Souza, and C. T. Druehl (2010). Managing product rollovers. *Decision Sciences 41*(2), 403–423.

Koch, S. (2018). Metals industry braces for a technological disruption.

Koenigsberg, O., R. Kohli, and R. Montoya (2011). The design of durable goods. *Marketing Science 30*(1), 111–122.

Kornish, L. J. (2001). Pricing for a durable-goods monopolist under rapid sequential innovation. *Management Science 47*(11), 1552–1561.

Krauss, C., L. George, and S. Arbanowski (2013). Tv predictor: personalized program recommendations to be displayed on smarttvs. In *Proceedings of the 2nd international workshop on big data, streams and heterogeneous source mining: Algorithms, systems, programming models and applications*, pp. 63–70. ACM.

Kubey, R. and M. Csikszentmihalyi (2002). Television addiction is no mere metaphor. *Scientific American 286*(2), 74–80.

Lee, J., D.-H. Park, and I. Han (2008). The effect of negative online consumer reviews on product

attitude: An information processing view. *Electronic commerce research and applications 7*(3), 341–352.

Lee, Y. and G. C. O'Connor (2003). New product launch strategy for network effects products. *Journal of the Academy of Marketing Science 31*(3), 241–255.

Lehmann-Grube, U. (1997). Strategic choice of quality when quality is costly: the persistence of the high-quality advantage. *The RAND Journal of Economics*, 372–384.

Levinthal, D. A. and D. Purohit (1989). Durable goods and product obsolescence. *Marketing Science 8*(1), 35–56.

Li, J., N. Granados, and S. Netessine (2014). Are consumers strategic? structural estimation from the air-travel industry. *Management Science 60*(9), 2114–2137.

Liang, C., M. Çakanyıldırım, and S. P. Sethi (2014). Analysis of product rollover strategies in the presence of strategic customers. *Management Science 60*(4), 1033–1056.

Liebowitz, S. J. and S. E. Margolis (1994). Network externality: An uncommon tragedy. *Journal of economic perspectives 8*(2), 133–150.

Lim, W. S. and C. S. Tang (2006). Optimal product rollover strategies. *European Journal of Operational Research 174*(2), 905–922.

Limelight Networks (2017). Most common reasons for canceling online streaming service subscriptions in the u.s. 2017. `https://www.statista.com/statistics/785635/cancel-online-streaming-service-subscription-in-the-us/`. Accessed on 22 July 2018.

Liu, J., Y. Li, Z. Ruan, G. Fu, X. Chen, R. Sadiq, and Y. Deng (2015). A new method to construct co-author networks. *Physica A: Statistical Mechanics and its Applications 419*, 29–39.

Liu, T. and P. Schiraldi (2012). New product launch: herd seeking or herd preventing? *Economic Theory 51*(3), 627–648.

Lobel, I., J. Patel, G. Vulcano, and J. Zhang (2015). Optimizing product launches in the presence of strategic consumers. *Management Science 62*(6), 1778–1799.

Lombardo, C. (2018). Streaming tvs problem: Subscribers cancel service when their favorite shows are over. `https://www.wsj.com/articles/streaming-tvs-drop-in-drop-out-problem-1516284000`. Accessed on 22 July 2018.

Lovett, M. J. and R. Staelin (2016). The role of paid, earned, and owned media in building enter-

tainment brands: Reminding, informing, and enhancing enjoyment. *Marketing Science 35*(1), 142–157.

Lu, J., E. T. Bradlow, and J. W. Hutchinson (2017). Binge consumption of online content. Preprint on webpage at `https://faculty.wharton.upenn.edu/wp-content/uploads/2017/07/JoyLu_JobMarketPaper_2017.pdf`.

Lubin, G. (2016). How netflix will someday know exactly what you want to watch as soon as you turn your tv on. `http://uk.businessinsider.com/how-netflix-recommendations-work-2016-9?r=US&IR=T`. Accessed on 22 July 2018.

Maass, W., A. Filler, and S. Janzen (2007). Reasoning on smart products in consumer good domains. In *European Conference on Ambient Intelligence*, pp. 165–173. Springer.

Macmillan, G. (2015). Why twitter owns tv conversations. `https://marketing.twitter.com/emea/en_gb/insights/8-reasons-why-twitter-is-the-home-of-tv-conversation.html`. Accessed on 20 July 2017.

Mahajan, V., E. Muller, and R. A. Kerin (1984). Introduction strategy for new products with positive and negative word-of-mouth. *Management Science 30*(12), 1389–1404.

Manitz, J., J. Harbering, M. Schmidt, T. Kneib, and A. Schöbel (2017). Source estimation for propagation processes on complex networks with an application to delays in public transportation systems. *Journal of the Royal Statistical Society: Series C (Applied Statistics) 66*(3), 521–536.

Manitz, J., T. Kneib, M. Schlather, D. Helbing, and D. Brockmann (2014). Origin detection during food-borne disease outbreaks-a case study of the 2011 ehec/hus outbreak in germany. *PLoS currents 6*.

Markman, J. (2017). Apple grows its ecosystem, and its advantage.

Markovich, S. and J. Moenius (2009). Winning while losing: Competition dynamics in the presence of indirect network effects. *International Journal of Industrial Organization 27*(3), 346–357.

Martínez, A. B. B., J. J. P. Arias, A. F. Vilas, J. G. Duque, and M. L. Nores (2009). What's on tv tonight? an efficient and effective personalized recommender system of tv programs. *IEEE Transactions on Consumer Electronics 55*(1).

Matrix, S. (2014). The netflix effect: Teens, binge watching, and on-demand digital media trends. *Jeunesse: Young People, Texts, Cultures 6*(1), 119–138.

McInnerney, J. M. and T. S. Roberts (2004). Online learning: Social interaction and the creation of a sense of community. *Educational Technology & Society 7*(3), 73–81.

Mickle, T. and J. McKinnon (2017). U.s., french officials question apple over iphone battery slowdowns. `https://www.wsj.com/articles/u-s-french-officials-question-apple-over-iphone-battery-slowdowns-1515545073`. Accessed on 24 September 2018.

Molina-Castillo, F.-J., J.-L. Munuera-Alemán, and R. J. Calantone (2011). Product quality and new product performance: The role of network externalities and switching costs. *Journal of Product Innovation Management 28*(6), 915–929.

Montagna, M. and C. Kok (2016). Multi-layered interbank model for assessing systemic risk.

Moore, S. (2016). Gartner survey shows wearable devices need to be more useful. `https://www.gartner.com/newsroom/id/3537117`. Accessed on 11th Jan 2018.

Moorthy, K. S. (1984). Market segmentation, self-selection, and product line design. *Marketing Science 3*(4), 288–307.

Moorthy, K. S. and I. P. Png (1992). Market segmentation, cannibalization, and the timing of product introductions. *Management Science 38*(3), 345–359.

Mosacho, N. (2017). How binge-watching has changed the way we watch (and produce) television. `https://www.viaccess-orca.com/blog/binge-watching-changed-way-watch-produce-television`. Accessed on 22 July 2018.

Motta, M. (1993). Endogenous quality choice: price vs. quantity competition. *The Journal of Industrial Economics*, 113–131.

Motta, M., J.-F. Thisse, and A. Cabrales (1997). On the persistence of leadership or leapfrogging in international trade. *International Economic Review*, 809–824.

Murthi, B. and S. Sarkar (2003). The role of the management sciences in research on personalization. *Management Science 49*(10), 1344–1362.

Mussa, M. and S. Rosen (1978). Monopoly and product quality. *Journal of Economic theory 18*(2), 301–317.

Nahm, J. (2004). Durable-goods monopoly with endogenous innovation. *Journal of Economics & Management Strategy 13*(2), 303–319.

Nair, H. (2007). Intertemporal price discrimination with forward-looking consumers: Application to the us market for console video-games. *Quantitative Marketing and Economics 5*(3), 239–292.

Nair, H., P. Chintagunta, and J.-P. Dubé (2004). Empirical analysis of indirect network effects in the market for personal digital assistants. *Quantitative Marketing and Economics 2*(1), 23–58.

Nair, S. K. and W. J. Hopp (1992). A model for equipment replacement due to technological obsolescence. *European Journal of Operational Research 63*(2), 207–221.

Nalebuff, B. and A. M. Brandenburger (1996). *Coopetition-kooperativ konkurrieren: Mit der Spieltheorie zum Unternehmenserfolg*. Campus-Verlag.

Nathan, M., C. Harrison, S. Yarosh, L. Terveen, L. Stead, and B. Amento (2008). Collaboratv: making television viewing social again. In *Proceedings of the 1st international conference on Designing interactive user experiences for TV and video*, pp. 85–94. ACM.

Nee, R. C. and D. M. Dozier (2017). Second screen effects: Linking multiscreen media use to television engagement and incidental learning. *Convergence 23*(2), 214–226.

Netflix (2016). Youre still hooked and netflix knows why. `https://media.netflix.com/en/press-releases/youre-still-hooked-and-netflix-knows-why7`. Accessed on 26 August 2017.

Nie, C.-X. and F.-T. Song (2018). Constructing financial network based on pmfg and threshold method. *Physica A: Statistical Mechanics and its Applications 495*, 104–113.

Nielsen (2017a). Most popular television series in 2016/2017, sorted by average number of twitter interactions (in 1,000s). `https://www.statista.com/statistics/304363/us-twitter-tv-series-audience-buzz/`. Accessed on 24 August 2017.

Nielsen (2017b). Share of binge viewers among netflix and hulu plus users in the united states as of july 2013. `https://www.statista.com/statistics/289617/binge-viewing-users-netflix-hulu/`. Accessed on 23 August 2017.

Njoroge, P., A. Ozdaglar, N. E. Stier-Moses, and G. Y. Weintraub (2013). Investment in two-sided markets and the net neutrality debate. *Review of Network Economics 12*(4), 355–402.

O'Donoghue, R. and A. J. Padilla (2006). *The law and economics of Article 82 EC*. Hart Publishing.

OECD (2014). *Addressing the Tax Challenges of the Digital Economy*.

Osadchiy, N. and E. Bendoly (2015). Are consumers really strategic? implications from an experimental study. `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2593184`. (Working Paper).

Papanastasiou, Y. and N. Savva (2016). Dynamic pricing in the presence of social learning and strategic consumers. *Management Science 63*(4), 919–939.

Patel, N. (2016). How netflix maintains a low churn rate by keeping customers engaged & watching. https://neilpatel.com/blog/how-netflix-maintains-low-churn. Accessed on 26 August 2017.

Paun, D. (1993). When to bundle or unbundle products. *Industrial Marketing Management 22*(1), 29–34.

Pedram, M. and S. Balachander (2014). Increasing quality sequence: When is it an optimal product introduction strategy? *Management Science 61*(10), 2487–2494.

Pei, J., D. Klabjan, and F. Karaesmen (2012). Pricing of digital goods vs. physical goods. http://home.ku.edu.tr/fkaraesmen/public_html/pdfpapers/Pei_Klabjan_Karaesmen_pricing.pdf. Working Paper.

Peitz, M. and P. Waelbroeck (2006). Piracy of digital products: A critical review of the theoretical literature. *Information Economics and Policy 18*(4), 449–476.

Peitz, M. and J. Waldfogel (2012). *The Oxford handbook of the digital economy*. Oxford University Press.

Pena, L. L. (2015). *Breaking binge: Exploring the effects of binge watching on television viewer reception*. Ph. D. thesis, Syracuse University.

Pine, B. J., J. H. Gilmore, et al. (1998). Welcome to the experience economy. *Harvard Business Review 76*, 97–105.

Pinto, P. C., P. Thiran, and M. Vetterli (2012). Locating the source of diffusion in large-scale networks. *Physical Review Letters 109*(6), 068702.

Plambeck, E. L. and T. A. Taylor (2018). Testing by competitors in enforcement of product standards. *Management Science*.

Poledna, S., J. L. Molina-Borboa, S. Martínez-Jaramillo, M. Van Der Leij, and S. Thurner (2015). The multi-layer network nature of systemic risk and its implications for the costs of financial crises. *Journal of Financial Stability 20*, 70–81.

Prasad, A., R. Venkatesh, and V. Mahajan (2010). Optimal bundling of technological products with network externality. *Management Science 56*(12), 2224–2236.

Proulx, M. and S. Shepatin (2012). *Social TV: how marketers can reach and engage audiences by connecting television to the web, social media, and mobile*. John Wiley & Sons.

Ramachandran, K. and V. Krishnan (2008). Design architecture and introduction timing for rapidly

improving industrial products. *Manufacturing & Service Operations Management 10*(1), 149–171.

Rasch, A. and T. Wenzel (2013). Piracy in a two-sided software market. *Journal of Economic Behavior & Organization 88*, 78–89.

Rassweiler, A., L. W. and G. Schneemann (2017). Ihs markit teardown reveals what higher apple iphone 8 plus cost actually buys. `https://technology.ihs.com/595738/ihs-markit-teardown-reveals-what-higher-apple-iphone-8-plus-cost-actually-buys`. Accessed on 24 September 2018.

Richardson, J. and K. Swan (2003). Examing social presence in online courses in relation to students' perceived learning and satisfaction. *Journal of Asynchronous Learning Networks 7*(1), 68–88.

Richter, F. (2016a). Are smartphone makers overdoing it?. `https://www.statista.com/chart/5497/cell-phone-upgrade-cycles/`. Accessed on 24 September 2018.

Richter, F. (2016b). The "slow iphone" phenomenon. `https://www.statista.com/chart/2514/iphone-releases/`. Accessed on 24 September 2018.

Richter, F. (2018). Eu hits google with record antitrust fine over android. `https://www.statista.com/chart/14752/eu-antitrust-fines-against-tech-companies/`. Accessed on 04 Sep 2018.

Richwine, L. (2017). More tv networks follow netflix to binge-watching model. `http://www.reuters.com/article/us-netflix-binge-idUSKBN1540IZ`. Accessed on 23 August 2017.

Rochet, J.-C. and J. Tirole (2003). Platform competition in two-sided markets. *Journal of the European Economic Association 1*(4), 990–1029.

Rovai, A. P. (2004). A constructivist approach to online college learning. *The internet and higher Education 7*(2), 79–93.

Rovai, A. P. (2007). Facilitating online discussions effectively. *The Internet and Higher Education 10*(1), 77–88.

Salant, S. W. (1989). When is inducing self-selection suboptimal for a monopolist? *The Quarterly Journal of Economics 104*(2), 391–397.

Schifferstein, H. N. and E. P. Zwartkruis-Pelgrim (2008). Consumer-product attachment: Measurement and design implications. *International Journal of Design 2*(3).

Schreiber, M. (2012). Making tv a two-way street: Changing viewer engagement through interaction. *Kellogg on Advertising & Media: The Kellogg School Of Management*, 37–55.

Schweidel, D. A. and W. W. Moe (2016). Binge watching and advertising. *Journal of Marketing 80*(5), 1–19.

Sciretta, P. (2013). Is netflixs full season release strategy a smart business model? http://www.slashfilm.com/is-s-full-season-release-strategy-a-smart-business-model/. Accessed on 24 August 2017.

Seitz, P. (2016). Amazon.com has more u.s. streaming subscribers than netflix. http://www.investors.com/news/amazon-com-has-more-u-s-streaming-subscribers-than-/. Accessed on 25 August 2017.

Shapiro, C., S. Carl, H. R. Varian, et al. (1998). *Information rules: a strategic guide to the network economy*. Harvard Business Press.

Shapiro, C. and H. R. Varian (1998). Versioning: the smart way to. *Harvard Business Review 107*(6), 107.

Sheng, S. and Y. Pan (2009). Bundling as a new product introduction strategy: The role of brand image and bundle features. *Journal of Retailing and Consumer Services 16*(5), 367–376.

Shocker, A. D., B. L. Bayus, and N. Kim (2004). Product complements and substitutes in the real world: The relevance of other products. *Journal of Marketing 68*(1), 28–40.

Slade, G. (2009). *Made to break: Technology and obsolescence in America*. Harvard University Press.

Srinivasan, R., G. L. Lilien, and A. Rangaswamy (2004). First in, first out? the effects of network externalities on pioneer survival. *Journal of Marketing 68*(1), 41–58.

Staley, O. (2016). Don't believe your own hype: How pebble went from being worth $740 million to less than $40 million.

Statista (2018). Digital economy compass 2018. https://www.statista.com/study/52194/digital-economy-compass/. Accessed on 09 Sep 2018.

Stremersch, S., G. J. Tellis, P. H. Franses, and J. L. Binken (2007). Indirect network effects in new product growth. *Journal of Marketing 71*(3), 52–74.

Su, B., C. J. Bonk, R. J. Magjuka, X. Liu, and S.-h. Lee (2005). The importance of interaction in web-based education: A program-level case study of online mba courses. *Journal of Interactive Online Learning 4*(1), 1–19.

Swan, K. (2003). Learning effectiveness online: What the research tells us. *Elements of quality online education, practice and direction 4*(1), 13–47.

Sweney, M. (2018). Netflix puts content above costs but is the policy sustainable? https://www.theguardian.com/media/2018/may/25/netflix-puts-content-above-costs-but-is-policy-sustainable. Accessed on 22 July 2018.

Talluri, K. T. and G. J. Van Ryzin (2006). *The theory and practice of revenue management*, Volume 68. Springer Science & Business Media.

Tang, C. S. (2010). A review of marketing–operations interface models: From co-existence to coordination and collaboration. *International Journal of Production Economics 125*(1), 22–40.

Tapscott, D. (1996). *The digital economy: Promise and peril in the age of networked intelligence*, Volume 1. McGraw-Hill New York.

Taylor, E. and L. Julia (2015). Apple, bmw in courtship with an eye on car collaboration. https://www.reuters.com/article/us-apple-bmw-courtship-insight-idUSKCN0Q52CW20150731. Accessed on 11th Jan 2018.

Tellis, G. J. (2008). Important research questions in technology and innovation. *Industrial Marketing Management 37*(6), 629.

Tellis, G. J., E. Yin, and R. Niraj (2009). Does quality win? network effects versus quality in high-tech markets. *Journal of Marketing Research 46*(2), 135–149.

Torres, J. (2017). ipad pro 10.5 ifixit teardown unmasks an ipad pro 12.9 mini. https://www.slashgear.com/ipad-pro-10-5-ifixit-teardown-unmasks-an-ipad-pro-12-9-mini-14488492/. Accessed on 24 September 2018.

Trainer, D. (2017). Netflix's original content strategy broken along with rest of business. https://www.forbes.com/sites/greatspeculations/2017/06/16/s-original-content-strategy-broken-along-with-rest-of-business/#6dada8175437. Accessed on 25 August 2017.

Trouleau, W., A. Ashkan, W. Ding, and B. Eriksson (2016). Just one more: Modeling binge watching behavior. In *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pp. 1215–1224. ACM.

Tsiotsou, R. (2006). The role of perceived product quality and overall satisfaction on purchase intentions. *International Journal of Consumer Studies 30*(2), 207–217.

Twitter (2017). Here are the most tweeted about tv shows of 2017. https://twitter.com/TwitterData/status/938070936351371265. Accessed on 20 July 2017.

Valletti, T. M. (2000). Minimum quality standards under cournot competition. *Journal of Regulatory Economics 18*(3), 235–245.

Vamosiu, A. (2018). Optimal bundling under imperfect competition. *International Journal of Production Economics 195*, 45–53.

Varian, H. R. (2001a). Economics of information technology. *University of California, Berkeley*.

Varian, H. R. (2001b). High-technology industries and market structure. *University of California, Berkeley 33*.

Varian, H. R. et al. (1999). Market structure in the network age. *Understanding the Digital Economy, MIT Press, Cambridge, MA*, 137–150.

Venkatesh, R. (2017). The design and pricing of bundles: a review of normative guidelines and practical approaches (2009). handbook of pricing research in marketing. 232. amazon ec2 instance types (2017). *Amazon EC2 Instance Types*.

Venkatesh, R. and W. Kamakura (2003). Optimal bundling and pricing under a monopoly: Contrasting complements and substitutes from independently valued products. *The Journal of Business 76*(2), 211–231.

Waldman, M. (1996). Planned obsolescence and the r&d decision. *The RAND Journal of Economics 27*(3), 583–595.

Waldman, M. (2003). Durable goods theory for real world markets. *Journal of Economic Perspectives 17*(1), 131–154.

Ward, C. and V. Ramachandran (2010). Crowdfunding the next hit: Microfunding online experience goods. In *Workshop on Computational Social Science and the Wisdom of Crowds at NIPS2010*, pp. 1–5.

Wei, X. D. and B. R. Nault (2013). Experience information goods:version-to-upgrade. *Decision Support Systems 56*, 494–501.

Weinmann, M., C. Schneider, and J. vom Brocke (2016). Digital nudging. *Business & Information Systems Engineering 58*(6), 433–436.

Whinston, M. D. (1989). Tying, foreclosure, and exclusion. Technical report, National Bureau of Economic Research.

Wilhelm, W., A. Yankov, and P. Magee (2011). Mobile phone consumption behavior and the need for sustainability innovations. *Journal of Strategic Innovation and Sustainability 7*(2), 20–40.

Wilson, J. (2014). Binge vs. buzz: The release model of house of cards vs. true detective. `http://www.huffingtonpost.com/jeremy-wilson/binge-vs-buzz-_b_4906199.html`. Accessed on 24 August 2017.

Xiong, G. and S. Bharadwaj (2014). Prerelease buzz evolution patterns and new product performance. *Marketing Science 33*(3), 401–421.

Yayla-Küllü, H. M. (2013). Capacity investment and product line decisions of a multiproduct leader and a focus strategy entrant. *Decision Sciences 44*(4), 645–678.

Yu, F., A. Zeng, S. Gillard, and M. Medo (2016). Network-based recommendation algorithms: A review. *Physica A: Statistical Mechanics and its Applications 452*, 192–208.

Yu, M., L. Debo, and R. Kapuscinski (2015). Strategic waiting for consumer-generated quality information: Dynamic pricing of new experience goods. *Management Science 62*(2), 410–435.

Zeithaml, V. A. (1988). Consumer perceptions of price, quality, and value: a means-end model and synthesis of evidence. *The Journal of Marketing*, 2–22.

Zhang, Y., E. T. Bradlow, and D. S. Small (2014). Predicting customer value using clumpiness: From rfm to rfmc. *Marketing Science 34*(2), 195–208.

Zhou, E., J. Zhang, Q. Gou, and L. Liang (2015). A two period pricing model for new fashion style launching strategy. *International Journal of Production Economics 160*, 144–156.

Zhu, F. and N. Furr (2016). Products to platforms: Making the leap. *Harvard Business Review 94*(4), 72–78.

Zhu, F. and X. Zhang (2010). Impact of online consumer reviews on sales: The moderating role of product and consumer characteristics. *Journal of Marketing 74*(2), 133–148.

Zhu, Y., B. Xu, X. Shi, and Y. Wang (2013). A survey of social-based routing in delay tolerant networks: Positive and negative social effects. *IEEE Communications Surveys & Tutorials 15*(1), 387–401.

Appendix A

Chapter 2

A.1 Proofs

Proof of Lemma 2.1

For $\bar{\beta} = \min\left\{\frac{p_2\alpha^{t_2}}{p_1}, 1\right\} = \frac{p_2\alpha^{t_2}}{p_1}$, under the assumption of the lemma, we have:

$$\theta_D = \min\left\{\frac{\beta p_1}{\alpha^{t_2}u}, 1\right\} \geq \min\left\{\frac{p_2}{u}, 1\right\} \geq \min\left\{\frac{p_2 - \beta p_1}{(1-\alpha^{t_2})u}, 1\right\} = \theta_{DL},$$

where the identities follow from the definition of θ_D and θ_{DL}, while the inequalities follow from the fact that $\beta \geq \bar{\beta} = \frac{p_2\alpha^{t_2}}{p_1}$.

Hence, the discount buyers segment $(\min\{\theta_E, \theta_{DL}\} - \theta_D)^+$ vanishes in equation (2.1) because of the following reasons: 1) if $\min\{\theta_E, \theta_{DL}\} = \theta_E$, then $\theta_D \geq \theta_{DL} \geq \theta_E$ and $(\theta_E - \theta_D)^+ = 0$; 2) if $\min\{\theta_E, \theta_{DL}\} = \theta_{DL}$, then $(\theta_{DL} - \theta_D)^+ = 0$. Moreover, $(\theta_E - \max\{\theta_L, \theta_{DL}\})^+$ from the late buyers segment simplifies to $(\theta_E - \theta_L)^+$ since the above chain of inequalities show that $\theta_L = \frac{p_2}{u} \geq \theta_{DL}$. Therefore, the profit functions for the solo and dual rollover strategies become equivalent.

For $\min\left\{\frac{p_2\alpha^{t_2}}{p_1}, 1\right\} = 1$, under the assumption of the lemma, we have $\alpha^{t_2}p_2 \geq p_1$ and $\beta = 1$.

For $\beta = 1$, we have:

$$\theta_D = \min\left\{\frac{\beta p_1}{\alpha^{t_2} u}, 1\right\} \geq \min\left\{\frac{p_1}{u}, 1\right\} = \theta_E,$$

where the identities follow from the definition of θ_D and θ_E, while the inequalities follows from the fact that $\beta = 1$. Hence, the discount buyers segment $(\min\{\theta_E, \theta_{DL}\} - \theta_D)^+$ vanishes. Moreover, $(\theta_E - \max\{\theta_L, \theta_{DL}\})^+$ from the late buyers segment also vanishes since $\alpha^{t_2} p_2 \geq p_1$ implies that $\theta_{DL} \geq \theta_L > \theta_E$. ∎

Proof of Proposition 2.2:

By definition of θ_L and θ_E, we have $p_2 \geq p_1$ if and only if $\theta_L = p_2/u \geq \theta_E = p_1/u$, which in turn holds if and only if there are no late buyers. We conclude that there are late buyers if and only if $p_1 > p_2$.

Similarly, by definition of θ_E and θ_B, we have $p_2 \leq p_1(1 - \alpha^{t_2})$ if and only if $\theta_E = p_1/u \geq \theta_B = p_2/[u(1 - \alpha^{t_2})]$, which in turn holds if and only if there are no early buyers. Thus, we conclude that there are early buyers if and only if $p_2 > p_1(1 - \alpha^{t_2})$.

Finally, by definition of θ_B, we have $\theta_B = \min\{1, p_2/[u(1 - \alpha^{t_2})]\} = 1$ if and only if $p_2 \geq u(1 - \alpha^{t_2})$, which in turn holds if and only if there are no buyers that purchase both products. We thus conclude that there are both product buyers if and only if $p_2 < u(1 - \alpha^{t_2})$.

The market segmentation from the statement of the proposition follows from considering the various combinations of the previously established regions, as well as the fact that $p_1, p_2 \in [0, u]$ and $(1 - \alpha^{t_2}) \in (0, 1)$. ∎

Proof of Corollary 2.3: Follows from Proposition 2.2 noting that $(1 - \alpha^{t_2}) \to 1$ when $\alpha \to 0$ and/or $t_2 \to \infty$ and $(1 - \alpha^{t_2}) \to 0$ when $\alpha \to 1$ and/or $t_2 \to 0$. ∎

Proof of Lemma 2.4:

We prove the statement by contradiction. Assume first that the optimal prices would satisfy $p_2^* > p_1^*$. For prices $p_2 \geq p_1$, Proposition 2.5 implies that there are no late buyers, and the

expected profit (2.2) simplifies to

$$\begin{aligned}\Pi_{EB}(p_1,p_2) &= p_1(\theta_B-\theta_E)+(p_1+\delta_s^{t_2}p_2)(1-\theta_B) \\ &= p_1(p_2/u(1-\alpha^{t_2})-p_1/u)+(p_1+\delta_s^{t_2}p_2)(1-p_2/u(1-\alpha^{t_2})).\end{aligned} \tag{A.1}$$

For (p_1^*,p_2^*) to optimize $\Pi_{EB}(p_1,p_2)$, it has to satisfy the KKT conditions. The unique KKT point for the profit maximization problem $\max\{\Pi_{EB}(p_1,p_2): p_2\geq p_1\}$ is $p_1^*=p_2^*=\frac{u\left(1+\delta_s^{t_2}\right)\left(1-\alpha^{t_2}\right)}{2\left(1-\alpha^{t_2}+\delta_s^{t_2}\right)}$, implying that the optimal prices never satisfy $p_2^*>p_1^*$.

Assume now that the optimal prices would satisfy $p_2^*<p_1^*(1-\alpha^{t_2})$. For prices $p_2\leq p_1(1-\alpha^{t_2})$, Proposition 2.5 implies that there are no early buyers, and the expected profit (2.2) simplifies to

$$\begin{aligned}\Pi_{LB}(p_1,p_2) &= \delta_s^{t_2}p_2(\theta_E-\theta_L)+(p_1+\delta_s^{t_2}p_2)(1-\theta_E) \\ &= \delta_s^{t_2}p_2(p_1/u-p_2/u)+(p_1+\delta_s^{t_2}p_2)(1-p_1/u).\end{aligned} \tag{A.2}$$

For (p_1^*,p_2^*) to optimize $\Pi_{LB}(p_1,p_2)$, it has to satisfy the KKT conditions. The unique KKT point for the profit maximization problem $\max\{\Pi_{LB}(p_1,p_2): p_2\leq p_1(1-\alpha^{t_2})\}$ is $p_2^*=p_1^*(1-\alpha^{t_2})=\frac{u(1-\alpha^{t_2})(1+(1-\alpha^{t_2})\delta_s^{t_2})}{2(1+(1-\alpha^{t_2})^2\delta_s^{t_2})}$, implying that the optimal prices never satisfy $p_2^*<p_1^*(1-\alpha^{t_2})$.

We thus conclude that the optimal prices satisfy $p_1^*\geq p_2^*\geq p_1^*(1-\alpha^{t_2})$. ∎

Proof of Proposition 2.5

Lemma 2.4 shows that the optimal prices satisfy $p_1^*\geq p_2^*\geq p_1^*(1-\alpha^{t_2})$. Although the profit function is *not* jointly concave in p_1, p_2 and t_2, one can verify that it is jointly concave in p_1 and p_2 if t_2 is fixed. Therefore, we first determine the optimal prices for fixed t_2, and we subsequently find the optimal introduction time t_2^* of the profit function with optimal prices.

Distinguishing the two cases $p_2\leq u(1-\alpha^{t_2})$ and $p_2\geq u(1-\alpha^{t_2})$, the profit function Π^* can be written as

$$\Pi^*(t_2,\alpha) = \max\left\{\Pi^1(t_2,\alpha), \Pi^2(t_2,\alpha)\right\}$$

where

$$\Pi^1(t_2,\alpha) = \max\left\{\Pi(p_1,p_2,t_2,\alpha) : \min\left\{p_1, u(1-\alpha^{t_2})\right\} \geq p_2 \geq p_1(1-\alpha^{t_2}), p_1, p_2 \in [0,u]\right\}$$

$$\text{and } \Pi^2(t_2,\alpha) = \max\left\{\Pi(p_1,p_2,t_2,\alpha) : p_1 \geq p_2 \geq u(1-\alpha^{t_2}), p_1, p_2 \in [0,u]\right\}.$$

First, we discuss the profit maximization problem $\Pi^1(t_2,\alpha)$ and derive the KKT points. We define functions $\Pi^{11}(t_2,\alpha)$ and $\Pi^{12}(t_2,\alpha)$ over the domain $t_2 \in \mathbb{R}_+$ and $\alpha \in (0,1)$ as:

$$\Pi^{11}(t_2,\alpha) = \frac{u[2(1+\delta_s^{t_2}) - \alpha^{t_2}(1+2\delta_s^{t_2})]}{8-(4-\delta_s^{t_2})\alpha^{t_2} - \delta_s^{t_2}} \tag{A.3}$$

$$\Pi^{12}(t_2,\alpha) = \frac{u\left(\delta_s^{t_2}\left(1-\alpha^{t_2}\right)+1\right)^2}{4\delta_s^{t_2}\left(1-\alpha^{t_2}\right)^2+4} \tag{A.4}$$

Then, for $\Pi^1(t_2,\alpha)$, the unique KKT point and the expression for $\Pi^1(t_2,\alpha)$ are as follows:

$$\begin{cases} p_2^*(t_2,\alpha) = (2p_1^*(t_2)-u)\delta_s^{-t_2} = \frac{3u(1-\alpha^{t_2})}{8-(4-\delta_s^{t_2})\alpha^{t_2}-\delta_s^{t_2}} & \text{if } t_2 \in \mathcal{T}_{11}(\alpha) \\ p_2^*(t_2,\alpha) = p_1^*(t_2)(1-\alpha^{t_2}) = \frac{u(1-\alpha^{t_2})(1+(1-\alpha^{t_2})\delta_s^{t_2})}{2(1+(1-\alpha^{t_2})^2\delta_s^{t_2})} & \text{if } t_2 \in \mathcal{T}_{12}(\alpha) \end{cases} \tag{A.5}$$

$$\text{and } \Pi^1(t_2,\alpha) = \begin{cases} \Pi^{11}(t_2,\alpha) & \text{if } t_2 \in \mathcal{T}_{11}(\alpha) \\ \Pi^{12}(t_2,\alpha) & \text{if } t_2 \in \mathcal{T}_{12}(\alpha) \end{cases} \tag{A.6}$$

where

$$\mathcal{T}_{11}(\alpha) = \left\{t_2 : \left(\frac{1+\delta_s^{t_2}}{2+\delta_s^{t_2}}\right)^{1/t_2} < \alpha\right\} \quad \text{and} \quad \mathcal{T}_{12}(\alpha) = \left\{t_2 : \left(\frac{1+\delta_s^{t_2}}{2+\delta_s^{t_2}}\right)^{1/t_2} \geq \alpha\right\}.$$

One can show that there is a $\tau(\alpha) \in (0,\infty)$ such that $\mathcal{T}_{11}(\alpha) = [0,\tau(\alpha))$ and $\mathcal{T}_{12}(\alpha) = [\tau(\alpha),\infty)$. We note that $\left(\frac{1+\delta_s^{t_2}}{2+\delta_s^{t_2}}\right)^{1/t_2}$ is continuous and strictly increasing in t_2 and maps $\mathbb{R}_+$ into $(0,1)$. Therefore, there is a $\tau(\alpha)$ in $(0,\infty)$ such that $\mathcal{T}_{11}(\alpha) = [0,\tau(\alpha))$ and $\mathcal{T}_{12}(\alpha) = [\tau(\alpha),\infty)$, and thus the intervals $\mathcal{T}_{11}(\alpha)$ and $\mathcal{T}_{12}(\alpha)$ are well defined. It follows that $\Pi^{11}(t_2,\alpha)$ and $\Pi^{12}(t_2,\alpha)$ are continuous because they are quotients of polynomials with strictly positive denominators. We also verify that $\Pi^{11}(\tau(\alpha),\alpha) = \Pi^{12}(\tau(\alpha),\alpha)$. Then, the continuity of $\Pi^1(t_2,\alpha)$ over its entire domain $t_2 \in \mathbb{R}_+$ for every $\alpha \in (0,1)$ follows directly.

A similar discussion applies to the profit maximization problem $\Pi^2(t_2,\alpha)$ and we reach into the conclusion that there is a unique KKT point defined over the entire region $t_2 \in \mathbb{R}_+$.

We now discuss the optimal introduction time $t_2^*(\alpha)$ for $\Pi^*(t_2,\alpha)$. To this end, we first determine the optimal introduction times, $t_{21}^*(\alpha)$ and $t_{22}^*(\alpha)$ for the auxiliary profit functions $\Pi^1(t_2,\alpha)$ and $\Pi^2(t_2,\alpha)$.

In view of $\Pi^1(t_2,\alpha)$, we show that there is $\alpha' \in (0,1)$ such that *(i)* for $\alpha \in (0,\alpha']$, the unique maximizer t_{21}^* of $\max\{\Pi^1(t_2,\alpha) : t_2 \in \mathbb{R}_+\}$ satisfies $t_{21}^* \in \mathcal{T}_{12}(\alpha)$; *(ii)* for $\alpha \in (\alpha',1)$, the unique maximizer t_{21}^* of $\max\{\Pi^1(t_2,\alpha) : t_2 \in \mathbb{R}_+\}$ satisfies $t_{21}^* \in \mathcal{T}_{11}(\alpha)$ as in the following discussion.

$\Pi^{11}(t_2,\alpha)$ and $\Pi^{12}(t_2,\alpha)$ are quotients of polynomials with strictly positive denominators over the entire line $\mathbb{R}_+$ as such both functions are continuous and do not vanish to ∞ at any finite t_2. In addition, one can verify that $\Pi^{11}(t_2,\alpha)$ and $\Pi^{12}(t_2,\alpha)$ are continuously differentiable. We also observe that $\Pi^{11}(t_2,\alpha), \Pi^{12}(t_2,\alpha) \to \frac{u}{4}$ for $t_2 \to 0$ and $\Pi^{11}(t_2,\alpha), \Pi^{12}(t_2,\alpha) \to \frac{u}{4}$ for $t_2 \to \infty$. On the other hand, we have $\Pi^{11}(\tau(\alpha)) = \Pi^{12}(\tau(\alpha)) = \frac{u(1+\delta_s^\tau)}{4+\delta_s^\tau} > \frac{u}{4}$, and we conclude that $\Pi^{11}(t_2,\alpha)$ and $\Pi^{12}(t_2,\alpha)$ are optimized at a finite $t_2 \in (0,\infty)$ over the entire $\mathbb{R}_+$. In addition, for a fixed $\alpha \in (0,1)$ one can verify that the first order optimality condition is satisfied at a single point $\Gamma^1(\alpha) \in (0,\infty)$ for $\Pi^{11}(t_2,\alpha)$ and $\Gamma^2(\alpha) \in (0,\infty)$ for $\Pi^{12}(t_2,\alpha)$ over the entire domain $\mathbb{R}_+$. Thus, we show that for any $\alpha \in (0,1)$, $\Pi^{11}(t_2,\alpha)$ and $\Pi^{12}(t_2,\alpha)$ are globally optimized (i.e., over $\mathbb{R}_+$) at unique points $\Gamma^1(\alpha) \in (0,\infty)$ and $\Gamma^2(\alpha) \in (0,\infty)$.

Next, we show that the functions $f(\alpha) := -\alpha + \left(\frac{1+\delta_s^{\Gamma^1(\alpha)}}{2+\delta_s^{\Gamma^1(\alpha)}}\right)^{1/\Gamma^1(\alpha)}$ and $g(\alpha) := -\alpha + \left(\frac{1+\delta_s^{\Gamma^2(\alpha)}}{2+\delta_s^{\Gamma^2(\alpha)}}\right)^{1/\Gamma^2(\alpha)}$ vanish at unique points $\alpha_1' \in (\delta_s,1)$ and $\alpha_2' \in (\delta_s,1)$. One can show that $\Gamma^1(\alpha), \Gamma^2(\alpha) \in \mathcal{C}^1$, and

it follows that $f(\alpha), g(\alpha) \in \mathcal{C}^1$. As another note, one can verify that $f'(\alpha)$ and $g'(\alpha)$ vanish at unique values of $\alpha^* \in (0,1)$ and $\alpha^{**} \in (0,1)$, respectively. We also confirm that $\lim_{\alpha \to 0} f(\alpha) = 0$ and $\lim_{\alpha \to 0} g(\alpha) = 0$. In addition, it follows that $f(\delta_s) > 0$ and $\lim_{\alpha \to 1} f(\alpha) < 0$ as well as $g(\delta_s) > 0$ and $\lim_{\alpha \to 1} g(\alpha) < 0$. Moreover, one can show that $\lim_{\alpha \to 0} \frac{df(\alpha)}{d\alpha} = \infty$ and $\lim_{\alpha \to 0} \frac{dg(\alpha)}{d\alpha} = \infty$. Thus, it follows that $\alpha^*, \alpha^{**} \in (0, \delta_s)$, and both $f(\alpha)$ and $g(\alpha)$ vanish only once and this happen on the interval $(\delta_s, 1)$. We denote these unique solutions as α_1' and α_2', respectively.

We define $h(t_2) := \left(\frac{1+\delta_s^{t_2}}{2+\delta_s^{t_2}}\right)^{1/t_2}$ over the domain $\mathbb{R}_+$ and we notice that $h(\tau(\alpha)) = \alpha$, which in turns follows by definition of the regions $\mathcal{T}_{11}(\alpha), \mathcal{T}_{12}(\alpha)$. We note that $h(t_2)$ is continuous and strictly increasing. When $\alpha = \alpha_1'$, since $\alpha = h(\tau(\alpha)) = \alpha_1' = h(\Gamma^1(\alpha_1'))$ and $h(t_2)$ is invertible, it follows that $\Gamma^1(\alpha_1') = \tau(\alpha)$. When $\alpha < \alpha_1'$, it follows that $\frac{d\Pi_{11}}{dt_2}\Big|_{t_2=\tau(\alpha)} > 0$ and the global maximizer of $\Pi^{11}(t_2, \alpha)$, which is $\Gamma^1(\alpha)$, is bigger than $\tau(\alpha)$. Since $\Gamma^1(\alpha) > \tau(\alpha)$, it follows, by definition, that $\Gamma^1(\alpha) \in \mathcal{T}_{12}(\alpha)$. When $\alpha > \alpha_1'$, it follows that $\frac{d\Pi_{11}}{dt_2}\Big|_{t_2=\tau(\alpha)} < 0$. Thus, we conclude that $\Gamma^1(\alpha) < \tau(\alpha)$, which in turn implies that $\Gamma^1(\alpha) \in \mathcal{T}_{11}(\alpha)$. Similar reasoning applies to $\Gamma^2(\alpha)$ and it follows that if $\alpha \leq \alpha_2'$ then $\Gamma^2(\alpha) \in \mathcal{T}_{12}(\alpha)$ and when $\alpha > \alpha_2'$ then $\Gamma^2(\alpha) \in \mathcal{T}_{11}(\alpha)$. Thus, we conclude that:

1. If $\alpha \leq \alpha_1'$, then $\Gamma^1(\alpha) \in \mathcal{T}_{12}(\alpha)$ and if $\alpha > \alpha_1'$, then $\Gamma^1(\alpha) \in \mathcal{T}_{11}(\alpha)$.

2. If $\alpha \leq \alpha_2'$, then $\Gamma^2(\alpha) \in \mathcal{T}_{12}(\alpha)$ and if $\alpha > \alpha_2'$, then $\Gamma^2(\alpha) \in \mathcal{T}_{11}(\alpha)$.

Next, we show that $\alpha_1' = \alpha_2'$ by contradiction. First, by definitions of the functions f, g, h, we note that $f(\alpha) = -\alpha + h(\Gamma^1(\alpha))$ and $g(\alpha) = -\alpha + h(\Gamma^2(\alpha))$. If $\alpha_1' \neq \alpha_2'$, it follows that $h(\Gamma^1(\alpha_1')) \neq h(\Gamma^2(\alpha_2'))$, which, in turn, implies that $\Gamma^1(\alpha_1') \neq \Gamma^2(\alpha_2')$. However, we verify that $\frac{d\Pi^2}{dt_2}\Big|_{t_2=\Gamma^1(\alpha_1')} = 0$, and thereby it follows that $\Gamma^1(\alpha_1') = \Gamma^2(\alpha_2')$. Thus, we conclude $\alpha_1' = \alpha_2'$ and we use α' as the unique solution to $f(\alpha) = 0$ and $g(\alpha) = 0$. Hence:

1. If $\alpha \leq \alpha'$, then $\Gamma^1(\alpha), \Gamma^2(\alpha) \in \mathcal{T}_{12}(\alpha)$.

2. If $\alpha > \alpha'$, then $\Gamma^1(\alpha), \Gamma^2(\alpha) \in \mathcal{T}_{11}(\alpha)$.

Next, we show that *(i)* for $\alpha \in (0, \alpha']$, the unique maximizer t_{21}^* of $\max\{\Pi^1(t_2, \alpha) : t_2 \in \mathbb{R}_+\}$ satisfies $t_{21}^* \in \mathcal{T}_{12}(\alpha)$; *(ii)* for $\alpha \in (\alpha', 1)$, the unique maximizer t_{21}^* of $\max\{\Pi^1(t_2, \alpha) : t_2 \in \mathbb{R}_+\}$ satisfies $t_{21}^* \in \mathcal{T}_{11}(\alpha)$; through the following discussion. Consider the case $\alpha \in (0, \alpha']$, and assume to the contrary that $\Pi^1(t_2, \alpha)$ is maximized at some point $t_2^* \in \mathcal{T}_{11}(\alpha)$. Since $\Pi^{11}(t_2, \alpha)$ is continuously differentiable with a unique first order optimality point $\Gamma^1(\alpha) \in \mathcal{T}_{12}(\alpha)$ and $\Pi^1(t_2, \alpha)$ coincides with $\Pi^{11}(t_2, \alpha)$ for $t_2 \in \mathcal{T}_{11}(\alpha)$, we must have $t_2^* = \tau(\alpha)$. Note, however, that $\Pi^{12}(t_2, \alpha)$ coincides with $\Pi^1(t_2, \alpha)$ for $t_2 \in \mathcal{T}_{12}(\alpha)$. Since $\Pi^1(t_2, \alpha)$ is maximized at $t_2^* = \tau(\alpha)$, $\Pi^{12}(t_2, \alpha)$ is continuous and coincides with $\Pi^{11}(t_2, \alpha)$ at $\tau(\alpha)$, we thus conclude that $\Pi^{12}(t_2, \alpha)$ must be maximized at $\tau(\alpha)$ as well. This, however, contradicts the assumption that $\Gamma^2(\alpha) \in \mathcal{T}_{12}(\alpha)$, which follows from the assumption that $\alpha \in (0, \alpha']$. Therefore, we conclude that for $\alpha \in (0, \alpha']$, it follows that the unique maximizer t_{21}^* of $\max\{\Pi^1(t_2, \alpha) : t_2 \in \mathbb{R}_+\}$ satisfies $t_{21}^* \in \mathcal{T}_{12}(\alpha)$. With similar lines, for $\alpha \in (\alpha', 1)$, assume to the contrary that $\Pi^1(t_2, \alpha)$ is maximized at some point $t_2^* \in \mathcal{T}_{12}(\alpha)$. Since $\Pi^{12}(t_2, \alpha)$ is continuously differentiable with a unique first order optimality point $\Gamma^2(\alpha) \in \mathcal{T}_{12}(\alpha)$, we must have $t_2^* = \tau(\alpha)$. Since $\Pi^{11}(t_2, \alpha)$ is continuous and coincides with $\Pi^{12}(t_2, \alpha)$ at $\tau(\alpha)$. This would imply that $\Pi^{11}(t_2, \alpha)$ must be maximized at $\tau(\alpha)$ as well. This, however, contradicts the assumption that $\Gamma^1(\alpha) \in \mathcal{T}_{11}(\alpha)$, which follows from the assumption that $\alpha \in (\alpha', 1)$. Therefore, we conclude that for $\alpha \in (\alpha', 1)$, it follows that the unique maximizer $t_{21}^* \in \mathcal{T}_{11}(\alpha)$.

In view of $\Pi^2(t_2, \alpha)$, with similar lines of the discussion $\Pi^1(t_2, \alpha)$, first we verify that for any $\alpha \in (0, 1)$, the unique maximizer t_{22}^* of $\max\{\Pi^2(t_2, \alpha) : t_2 \in \mathbb{R}_+\}$ satisfies $t_{22}^* \to 0$ since $\Pi^2(t_2, \alpha)$ is decreasing in t_2. Therefore, we conclude that $\Pi^2(t_{22}^*, \alpha) \to \frac{u}{3}$ for any fixed $\alpha \in (0, 1)$. Next, we compare $\Pi^2(t_{22}^*, \alpha) = \frac{u}{3}^-$ with $\Pi^{11}(\Gamma^1(\alpha), \alpha)$ over the entire region $\alpha \in (0, 1)$. First we verify that $k(\alpha) = -\alpha + (1 + \frac{1}{1-7\delta_s^{\Gamma^1(\alpha)}})^{1/\Gamma^1(\alpha)}$ vanish at unique $\alpha \in (\alpha', 1)$ and we denote this unique point as α''. Next, we verify that $\Pi^{11}(\Gamma^1(\alpha), \alpha) > \Pi^2(t_{22}^*, \alpha) = \frac{u}{3}^-$ for $\alpha \in (0, \alpha'')$.

We arrive at the conclusion that there are unique points $\alpha' \in (0, 1)$ and $\alpha'' \in (0, 1)$ such that $\alpha' < \alpha''$ and *(i)* for $\alpha \in (0, \alpha']$, the unique maximizer t_{21}^* of $\max\{\Pi(t_2, \alpha) : t_2 \in \mathbb{R}_+\}$ satisfies $t_2^* \in \mathcal{T}_{12}(\alpha)$; *(ii)* for $\alpha \in (\alpha', \alpha'')$, the unique maximizer t_2^* of $\max\{\Pi(t_2, \alpha) : t_2 \in \mathbb{R}_+\}$ satisfies $t_{21}^* \in \mathcal{T}_{11}(\alpha)$; and *(iii)* for $\alpha \in [\alpha'', 1)$, the unique maximizer t_2^* of $\max\{\Pi(t_2, \alpha) : t_2 \in \mathbb{R}_+\}$ satisfies $t_2^* \to 0$. ∎

Proof of Corollary 2.6: Follows from Lemma 2.4 and Proposition 2.5. ∎

Proof of Corollary 2.7:

1. Proposition 2.5 shows that for $\alpha \in (0, \alpha')$, it follows that $t_2^*(\alpha) = \Gamma^2(\alpha) \in (0, \infty)$; for $\alpha \in (\alpha'\alpha'')$ it follows that $t_2^* = \Gamma^1(\alpha) \in (0, \infty)$; and for $\alpha \in (\alpha'', 1)$ it follows that $t_2^*(\alpha) \to 0$. First, one can derive $\frac{d\Gamma^1}{d\alpha}$ directly from the closed form expression $\Gamma^1(\alpha)$ and verify that $\frac{d\Gamma^1}{d\alpha}$ is continuous and positive over the interval $\alpha \in (0, 1)$ and $\delta_s \in (0, 1)$. Thus, $\Gamma^1(\alpha)$ is increasing in α over the entire interval $\alpha \in (0, 1)$. Next, we note that $\frac{d\Gamma^2}{d\alpha}$ is derived using the implicit differentiation since the complexity of deriving a closed form expression for $\Gamma^2(\alpha)$. Then, we confirm that $\frac{d\Gamma^2}{d\alpha}$ is continuous and positive over the interval $\alpha \in (0, 1)$ and $\delta_s \in (0, 1)$. Thus, $\Gamma^2(\alpha)$ is increasing in α over the entire interval $\alpha \in (0, 1)$. In addition, one can show that $\lim_{\alpha \to 0^+} \frac{d\Gamma^2}{d\alpha} = -\infty$. Thus, $\lim_{\alpha \to 0^+} \Gamma^2(\alpha) = 0$. In addition, Proposition 2.5 shows that for $\alpha = \alpha'$, it follows that $\Gamma^1(\alpha) = \Gamma^2(\alpha)$. Thus, we show that $t_2^*(\alpha)$ continuously increases with α over the region $\alpha \in (0, \alpha'')$, and when $\alpha \to 0$ or $\alpha \in (\alpha'', 1)$, it follows that $t_2^*(\alpha) \to 0$.

2. Proposition 2.5 shows that for $\alpha \in (0, \alpha')$, it follows that $\Pi^*(t_2^*(\alpha)) = \Pi^{12}(\Gamma^2(\alpha))$; for $\alpha \in (\alpha'\alpha'')$ it follows that $\Pi^*(t_2^*(\alpha)) = \Pi^{11}(\Gamma^1(\alpha))$; and for $\alpha \in (\alpha'', 1)$ it follows that $\Pi^*(t_2^*(\alpha)) = \frac{u}{3}^-$. In addition, one can show that $\lim_{\alpha to 0} \Pi^{12}(\Gamma^2(\alpha)) = \frac{u}{2}$. First, we derive the first order derivative of $\Pi^{11}(\Gamma^1(\alpha))$ with respect to α as:

$$\frac{d\Pi^{11}(\Gamma^1(\alpha))}{d\alpha} = -\frac{u9\Gamma^1(\alpha)\delta_s^t \alpha^{\Gamma^1(\alpha)-1}}{\left(\left(\delta_s^{\Gamma^1(\alpha)} - 4\right)\alpha^{\Gamma^1(\alpha)} - \delta_s^{\Gamma^1(\alpha)} + 8\right)^2}$$

Along with our assumptions and previous results, we verify that $\frac{d\Pi^{11}(\Gamma^1(\alpha))}{d\alpha} > 0$ over the entire domain $\alpha \in (0, 1)$. Then, we conclude that $\Pi^{11}(\Gamma^1(\alpha))$ is decreasing over the entire interval $\alpha \in (0, 1)$. Similarly, we derive the first order derivative of $\Pi^{12}(\Gamma^2(\alpha))$ with respect to α and we verify that $\frac{d\Pi^{12}(\Gamma^2(\alpha))}{d\alpha} > 0$ over the entire domain $\alpha \in (0, 1)$. Then, we conclude that $\Pi^{12}(\Gamma^2(\alpha))$ is decreasing over the entire interval $\alpha \in (0, 1)$. In addition, Proposition 2.5 shows that for $\alpha = \alpha'$, it follows that $\Pi^{11}(\Gamma^1(\alpha)) = \Pi^{12}(\Gamma^2(\alpha))$. Thus, we

show that $\Pi^*(t_2^*(\alpha)) \to \frac{u}{2}$ when $\alpha \to 0$ and $\Pi^*(t_2^*(\alpha))$ is continuously decreasing with α for $\alpha \in (0, \alpha'')$ and remains constant at $\frac{u}{3}^-$ for $\alpha \in (\alpha'', 1)$.

3. Proposition 2.5 and its proof specify the optimal price for the first product $p_1^*(t_2^*(\alpha))$ over the entire domain $\alpha \in (0, 1)$. For $\alpha \in (0, \alpha')$, it follows that $p_1^*(t_2^*(\alpha)) = p_1^*(\Gamma^2(\alpha))$; for $\alpha \in (\alpha'\alpha'')$ it follows that $p_1^*(t_2^*(\alpha)) = p_1^*(\Gamma^1(\alpha))$; and for $\alpha \in (\alpha'', 1)$ it follows that $p_1^*(t_2^*(\alpha)) = \frac{2u}{3}^-$. First, we derive the first order derivative of $p_1^*(\Gamma^1(\alpha))$ with respect to α as:

$$\frac{dp_1^*(\Gamma^1(\alpha))}{d\alpha} = \frac{6\delta_s^{\Gamma^1(\alpha)} u \left(-\Gamma^1(\alpha)\alpha^{\Gamma^1(\alpha)}\right)}{\alpha\left(\left(\delta_s^{\Gamma^1(\alpha)} - 4\right)\alpha^{\Gamma^1(\alpha)} - \delta_s^{\Gamma^1(\alpha)} + 8\right)^2}$$

Therefore, we verify that $\frac{dp_1^*(\Gamma^1(\alpha))}{d\alpha} < 0$ for the entire domain $\alpha \in (0, 1)$. Then, we conclude that $p_1^*(\Gamma^1(\alpha))$ is decreasing over the entire interval $\alpha \in (0, 1)$.

In view of $p_1^*(\Gamma^2(\alpha))$, we first show that $\lim_{\alpha\to 0} p_1^*(\Gamma^2(\alpha)) = \frac{u}{2}$ and $\lim_{\alpha\to 1} p_1^*(\Gamma^2(\alpha)) = \frac{u}{2}$. In addition, we also verify that $\lim_{\alpha\to\delta_s} p_1^*(\Gamma^2(\alpha)) > \frac{u}{2}$. Then, using the mean value theorem, we conclude that there exists $\bar{\alpha} \in (0, 1)$ such that $p_1^*(\Gamma^2(\alpha))$ reaches its maximum value. We also confirm that $\lim_{\alpha\to\delta_s} \frac{dp_1^*(\Gamma^2(\alpha))}{d\alpha} < 0$. Thus, we conclude that $\bar{\alpha} \in (0, \delta_s)$. We also confirm that the first order optimality condition is satisfied at a single point $\bar{\alpha}$. Thus, it follows that $\bar{\alpha}$ is a unique point in the interval $(0, \delta_s)$ and it follows that $\bar{\alpha} < \alpha'$ since $\alpha' > \delta_s$ as shown in the proof of Proposition 2.5. Therefore, we conclude that $p_1^*(\Gamma^2(\alpha))$ is continuously increasing for $(0, \bar{\alpha})$, reaches its maximum value when $\alpha = \bar{\alpha}$ and continuously decreases for $(\bar{\alpha}, 1)$. In addition, Proposition 2.5 shows that for $\alpha = \alpha'$, it follows that $p_1^*(\Gamma^1(\alpha)) = p_1^*(\Gamma^2(\alpha))$. Thus, we show that $p_1^*(t_2^*(\alpha)) \to \frac{u}{2}$ when $\alpha \to \frac{u}{2}$; $p_1^*(t_2^*(\alpha)) \to \frac{u}{2}$ continuously increasing with α for $\alpha \in (0, \bar{\alpha})$, $p_1^*(t_2^*(\alpha)) \to \frac{u}{2}$ continuously decreasing with α for $\alpha \in (\bar{\alpha}, \alpha'')$, and remains constant at $\frac{2u}{3}^-$ for $\alpha \in (\alpha'', 1)$.

4. Proposition 2.5 and its proof specify the optimal price for the second product $p_2^*(t_2^*(\alpha))$ over the entire domain $\alpha \in (0, 1)$. For $\alpha \in (0, \alpha')$, it follows that $p_2^*(t_2^*(\alpha)) = p_2^*(\Gamma^2(\alpha))$; for $\alpha \in (\alpha'\alpha'')$ it follows that $p_2^*(t_2^*(\alpha)) = p_2^*(\Gamma^1(\alpha))$; and for $\alpha \in (\alpha'', 1)$ it follows that $p_2^*(t_2^*(\alpha)) = \frac{u}{3}^-$. In addition, one can verify that $\lim_{\alpha\to 0} p_2^*(\Gamma^2(\alpha)) = \frac{u}{2}$. First, we derive

the first order derivative of $p_2^*(\Gamma^1(\alpha))$ with respect to α as:

$$\frac{dp_2^*(\Gamma^1(\alpha))}{d\alpha} = \frac{3u\left(\frac{d\Gamma^1(\alpha)}{d\alpha}\alpha\log(\delta_s)\left(\alpha^{\Gamma^1(\alpha)}-1\right)\left(\left(\delta_s^{\Gamma^1(\alpha)}-4\right)\alpha^{\Gamma^1(\alpha)}-\delta_s^{\Gamma^1(\alpha)}+8\right)-4\Gamma^1(\alpha)\alpha^{\Gamma^1(\alpha)}\right)}{\alpha\left(\left(\delta_s^{\Gamma^1(\alpha)}-4\right)\alpha^{\Gamma^1(\alpha)}-\delta_s^{\Gamma^1(\alpha)}+8\right)^2}$$

We verify that $\frac{dp_2^*(\Gamma^1(\alpha))}{d\alpha} < 0$ for the entire domain $\alpha \in (0,1)$ since $\frac{d\Gamma^1(\alpha)}{d\alpha} > 0$. Then, we conclude that $p_2^*(\Gamma^1(\alpha))$ is decreasing over the entire interval $\alpha \in (0,1)$. Likewise, we derive the first order derivative $p_2^*(\Gamma^2(\alpha))$ with respect to α, and we verify that $\frac{dp_2^*(\Gamma^2(\alpha))}{d\alpha} < 0$ for the entire domain $\alpha \in (0,1)$ since $\frac{d\Gamma^2(\alpha)}{d\alpha} > 0$. Then, we conclude that $p_2^*(\Gamma^2(\alpha))$ is decreasing over the entire interval $\alpha \in (0,1)$. In addition, Proposition 2.5 shows that for $\alpha = \alpha'$, it follows that $p_2^*(\Gamma^1(\alpha)) = p_2^*(\Gamma^2(\alpha))$. Thus, we show that $p_2^*(t_2^*(\alpha)) \to \frac{u}{2}$ when $\alpha \to 0$ and $p_2^*(t_2^*(\alpha))$ is continuously decreasing with α for $\alpha \in (0,\alpha'')$ and remains constant at $\frac{u}{3}^-$ for $\alpha \in (\alpha'',1)$. ∎

Proof of Proposition 2.8:

When $\beta < \bar{\beta}$, by definition of θ_{DL}, θ_L and θ_D, we have $\theta_{DL} = \max\left\{\frac{p_2-\beta p_1}{u(1-\alpha^{t_2})}, 1\right\} > \theta_L = \frac{p_2}{u} > \theta_D = \frac{\beta p_1}{u\alpha^{t_2}}$ and it follows that $\max\{\theta_{DL}, \theta_L\} = \theta_{DL}$.

By definition of θ_E and θ_{DL}, we have $p_2 \geq p_1(1-\alpha^{t_2}+\beta)$ if and only if $\theta_E = p_1/u \leq \theta_{DL} = \max\left\{\frac{p_2-\beta p_1}{u(1-\alpha^{t_2}}, 1\right\}$, which holds if and only if there are no late buyers. We thus conclude that there are late buyers if and only if $p_2 < p_1(1-\alpha^{t_2}+\beta)$.

Similarly, by definition of θ_E and θ_B, we have $p_2 \leq p_1(1-\alpha^{t_2})$ if and only if $\theta_E = p_1/u \geq \theta_B = p_2/[u(1-\alpha^{t_2})]$, which holds if and only if there are no early buyers. Thus, we conclude that there are early buyers if and only if $p_2 > p_1(1-\alpha^{t_2})$.

Moreover, by definition of θ_B, we have $\theta_B = \min\{1, p_2/[u(1-\alpha^{t_2})]\} = 1$ if and only if $p_2 \geq u(1-\alpha^{t_2})$, which holds if and only if there are no buyers that purchase both products. We thus conclude that there are both product buyers if and only if $p_2 < u(1-\alpha^{t_2})$.

Finally, by definition of θ_E, θ_D, θ_{DL} as well as the condition that $\theta_D \leq \theta_{DL}$, we have $\min\{\theta_E, \theta_{DL}\} > \theta_D$ when $\theta_E > \theta_{DL}$, which in turn holds if and only if $p_2 < p_1(1-\alpha^{t_2}+\beta)$ where there are late buyers. Hence, we show that the discount buyers segment always exist

when $p_2 < p_1(1-\alpha^{t_2}+\beta)$. On the other hand, when $p_2 \leq p_1(1-\alpha^{t_2}+\beta)$, by definition, we have $\min\{\theta_E, \theta_{DL}\} - \theta_D = \theta_E - \theta_D$. When $p_2 \leq p_1(1-\alpha^{t_2}+\beta)$ where there is no late buyers, we have $\beta \geq \alpha^{t_2}$ if and only if $\theta_E = \frac{p_1}{u} \leq \theta_D = \frac{\beta p_1}{u\alpha^{t_2}}$, which in turn holds if and only if there are no discount buyers. We thus conclude that there are discount buyers when $p_2 < p_1(1-\alpha^{t_2}+\beta)$ or when $\beta < \alpha^{t_2}$ and $p_2 \geq p_1(1-\alpha^{t_2}+\beta)$.

The market segmentation from the statement of the proposition follows from considering the various combinations of the previously established regions, as well as the fact that $p_1, p_2 \in [0,u]$, $(1-\alpha^{t_2}) \in (0,1)$ and $\beta < \bar{\beta}$. ∎

Proof of Corollary 2.9:

Follows from Lemma 2.1 and Proposition 2.8, with the fact that $\bar{\beta} \to 0$ and $(1-\alpha^{t_2}) \to 1$ when $\alpha \to 0$ and/or $t_2 \to \infty$; as well as $\bar{\beta} \to \min\left\{\frac{p_2}{p_1}, 1\right\}$ and $(1-\alpha^{t_2}) \to 0$ when $\alpha \to 1$ and/or $t_2 \to 0$. ∎

Proof of Lemma 2.10:

We prove the statement by contradiction. Assume first that the optimal prices would satisfy $p_2^* > p_1^*(1-\alpha^{t_2}+\beta)$. For prices $p_2 \geq p_1^*(1-\alpha^{t_2}+\beta)$, Proposition 2.8 implies that there are no late buyers, and the expected profit (2.1) simplifies to

$$\begin{aligned}\Pi_{EBD}(p_1,p_2,\beta) &= p_1(\theta_B-\theta_E)+(p_1+\delta_s^{t_2}p_2)(1-\theta_B)+\delta_s^{t_2}\beta p_1(\theta_E-\theta_D)^+ \qquad (A.7)\\ &= p_1\left(\frac{p_2}{u(1-\alpha^{t_2})}-\frac{p_1}{u}\right)+(p_1+\delta_s^{t_2}p_2)\left(1-\frac{p_2}{u(1-\alpha^{t_2})}\right)+\delta_s^{t_2}\beta p_1\left(\frac{p_1}{u}-\frac{\beta p_1}{u\alpha^{t_2}}\right)^+.\end{aligned}$$

For (p_1^*, p_2^*, β^*) to optimize $\Pi_{EBD}(p_1,p_2,\beta)$, it has to satisfy the KKT conditions. The unique KKT point for the profit maximization problem $\max\{\Pi_{EBD}(p_1,p_2) : p_2^* \geq p_1^*(1-\alpha^{t_2}+\beta), \beta \leq \alpha^{t_2}\}$ gives $p_1^*(1-\alpha^{t_2}+\beta^*) = p_2^*$, implying that the optimal prices never satisfy $p_2^* > p_1^*(1-\alpha^{t_2}+\beta)$.

Assume now that the optimal prices would satisfy $p_2^* < p_1^*(1-\alpha^{t_2})$. For prices $p_2 \leq p_1(1-\alpha^{t_2})$, Proposition 2.8 implies that there are no early buyers, and the expected profit (2.1) simplifies

to

$$\begin{aligned}\Pi_{LBD}(p_1,p_2) &= \delta_s^{t_2}p_2(\theta_E-\theta_{DL})+(p_1+\delta_s^{t_2}p_2)(1-\theta_E)+\delta_s^{t_2}\beta p_1(\theta_{DL}-\theta_D) \\ &= \delta_s^{t_2}p_2(\frac{p1}{u}-\frac{p_2-\beta p_1}{u(1-\alpha^{t_2})})+(p_1+\delta_s^{t_2}p_2)(1-\frac{p_1}{u})+\delta_s^{t_2}\beta p_1(\frac{p_2-\beta p_1}{u(1-\alpha^{t_2})}-\frac{\beta p_1}{u\alpha^{t_2}}).\end{aligned} \tag{A.8}$$

For (p_1^*, p_2^*, β^*) to optimize $\Pi_{LBD}(p_1, p_2, \beta^*)$, it has to satisfy the KKT conditions. The unique KKT point for the profit maximization problem $\max\{\Pi_{LB}(p_1, p_2, \beta) : p_2 \leq p_1(1-\alpha^{t_2})\}$ gives $p_2^* = p_1^*(1-\alpha^{t_2})$, implying that the optimal prices never satisfy $p_2^* < p_1^*(1-\alpha^{t_2})$.

We thus conclude that the optimal prices satisfy $p_1^*(1-\alpha^{t_2}+\beta^*) \geq p_2^* \geq p_1^*(1-\alpha^{t_2})$. ∎

Proof of Proposition 2.11:

Lemma 2.10 shows that the optimal prices satisfy $p_1^*(1-\alpha^{t_2}+\beta) \geq p_2^* \geq p_1^*(1-\alpha^{t_2})$. Proposition 2.8 implies that for this condition the expected profit (2.1) simplifies to

$$\Pi(p_1,p_2,\beta,t_2) = p_1(\theta_B-\theta_E)+\delta_s^{t_2}p_2(\theta_E-\theta_{DL})+(p_1+\delta_s^{t_2}p_2)(1-\theta_B)+\delta_s^{t_2}\beta p_1(\theta_{DL}-\theta_D) \tag{A.9}$$

One can verify that (A.9) is concave in β. We first determine the optimal discount rate β^* for fixed values of p_1, p_2 and t_2. Distinguishing the two cases $p_2 \leq u(1-\alpha^{t_2})$ and $p_2 \geq u(1-\alpha^{t_2})$, the profit function Π^* can be written as

$$\Pi^*(p_1,p_2,t_2) = \max\left\{\Pi^1(p_1,p_2,t_2), \Pi^2(p_1,p_2,t_2)\right\}$$

where

$$\Pi^1(p_1,p_2,t_2) = \max\left\{\Pi(p_1,p_2,t_2,\beta) : \min\left\{p_1(1-\alpha^{t_2}+\beta), u(1-\alpha^{t_2})\right\} \geq p_2 \geq p_1(1-\alpha^{t_2}), \beta \leq \bar{\beta}, p_1, p_2 \in [0,u]\right\}$$

$$\text{and } \Pi^2(p_1,p_2,t_2) = \max\left\{\Pi(p_1,p_2,t_2,\beta) : p_1(1-\alpha^{t_2}+\beta) \geq p_2 \geq u(1-\alpha^{t_2}), \beta \leq \bar{\beta}, p_1, p_2 \in [0,u]\right\}.$$

Optimal solutions to the optimization problems of $\Pi^1(p_1, p_2, t_2)$ and $\Pi^2(p_1, p_2, t_2)$ satisfy $\beta^* = \bar{\beta}$. When $\beta = \bar{\beta}$, Lemma 2.1 implies that the discount buyers segment $(\theta_{DL} - \theta_D)$ vanishes. The

expected profit (A.9) simplifies to the expected profit of the solo rollover (2.2) case. The optimal solution shown by Proposition 2.5 holds for the dual rollover case with $\beta^* = \bar{\beta}$. Thus, we conclude that the optimal rollover strategy for myopic consumers is solo rollover strategy with the optimal pricing and timing strategy proposed by Proposition 2.5. ∎

The proof of Proposition 2.12 relies on the following auxiliary result, which we prove first.

Lemma A.1 *For $\alpha, \delta_c \in (0,1)$ fixed, the scalar function $f(t_2; \alpha, \delta_c) = 1 - \delta_c^{t_2}(2 - \alpha^{t_2})$ is continuously differentiable, and it satisfies the following two properties.*

1. *There is $\widehat{t}_2 \in (0, \infty)$ such that f is strictly decreasing on the interval $(-\infty, \widehat{t}_2)$ and strictly increasing on the interval $(\widehat{t}_2, \infty)$.*
2. *The function f has the roots $\{0\}$ if $\alpha = \delta_c$, $\{t_2^+, 0\}$, $t_2^+ > 0$, if $\alpha < \delta_c$, and $\{0, t_2^-\}$, $t_2^- < 0$, if $\alpha > \delta_c$.*

Proof: One readily verifies that f is continuously differentiable.

In view of the first statement, we note that $t_2^\star = \log\left(\frac{2\log(\delta_c)}{\log(\alpha\delta_c)}\right) / \log(\alpha)$ is the unique point satisfying the first-order optimality conditions of f, and that $f(t_2; \alpha, \delta_c) \to \infty$ for $t_2 \to -\infty$ and $f(t_2; \alpha, \delta_c) \to 1$ for $t_2 \to \infty$. We thus conclude that $t_2^\star$ is the global minimizer of f. Moreover, f must be strictly decreasing (increasing) before (after) $t_2^\star$, as otherwise the continuous differentiability of f would imply the existence of other points satisfying the first-order optimality conditions of f.

As for the second statement, one readily verifies that $f(0; \alpha, \delta_c) = 0$. Moreover, the first-order point $t_2^\star$ satisfies $t_2^\star = 0$ if $\alpha = \delta_c$, $t_2^\star > 0$ if $\alpha < \delta_c$ and $t_2^\star < 0$ if $\alpha > \delta_c$. The second statement then follows from the monotonicity properties of f. ∎

Proof of Proposition 2.12: We first note that the difference between the discounted utilities from early and late buying amounts to $(1 - \delta_c^{t_2}(1 - \alpha^{t_2}))\theta u - \delta_c^{t_2}\theta u = \theta u \cdot f(t_2, \alpha, \delta_c)$, where the function f is defined in Lemma A.1. We thus conclude that early buying is preferred to

late buying whenever

$$(1-\delta_c^{t_2}(1-\alpha^{t_2})\theta u - p_1 \;>\; \delta_c^{t_2}\theta u - \delta_c^{t_2}p_2 \quad\Longleftrightarrow\quad \theta u \cdot f(t_2,\alpha,\delta_c) \;>\; p_1 - \delta_c^{t_2}p_2$$

$$\Longleftrightarrow \quad \begin{cases} \theta > \theta_{LE} & \text{if } f(t_2,\alpha,\delta_c) > 0, \\ \theta < \theta_{LE} & \text{if } f(t_2,\alpha,\delta_c) < 0, \end{cases}$$

where the second equivalence follows from the fact that $\theta_{LE} = \left[\frac{p_1-\delta_c^{t_2}p_2}{u\cdot f(t_2;\alpha,\delta_c)}\right]_0^1$.

If $\alpha < \delta_c$,then Lemma A.1 implies that f has the two roots 0 and $t_2^+ > 0$, and there is $\widehat{t}_2 \in (0, t_2^+)$ such that f is strictly decreasing on the interval $(-\infty, \widehat{t}_2)$ and strictly increasing on the interval $(\widehat{t}_2, \infty)$. We thus conclude that $f(t_2;\alpha,\delta_c) > 0$ for all $t_2 > t_2^+$ and $f(t_2;\alpha,\delta_c) > 0$ for all $t_2 \in (0, t_2^+)$, that is, early buying is preferred for introduction times $t_2 > t_2^+$, whereas late buying is preferred for introduction times $t_2 \in (0, t_2^+)$.

If $\alpha > \delta_c$, on the other hand, then Lemma A.1 implies that f has the two roots $t_2^- < 0$ and 0, and there is $\widehat{t}_2 \in (t_2^-, 0)$ such that f is strictly decreasing on the interval $(-\infty, \widehat{t}_2)$ and strictly increasing on the interval $(\widehat{t}_2, \infty)$. We thus conclude that $f(t_2;\alpha,\delta_c) > 0$ for all $t_2 > 0$, that is, early buying is preferred to late buying for any introduction time $t_2 > 0$. ∎

Proof of Proposition 2.13:

For the first statement, there are both product buyers if and only if the interval $[\max\{\theta_{EB}, \theta_{LB}\}, 1]$ is non-degenerate, which holds if and only if $\theta_{EB} < 1, \theta_{LB} < 1$. The statement now follows from the fact that $\theta_{EB} < 1$ and $\theta_{LB} < 1$ hold if and only if $p_2 < u(1-\alpha^{t_2})$ and $p_1 < u(1-\delta_c^{t_2})$, respectively.

As for the second statement, we first show that there are early buyers if and only if $p_1(1-\alpha^{t_2}) < p_2(1-\delta_c^{t_2})$ for a regular preference ordering where $t_2 > \tau(\alpha,\delta_c)$. We remind the reader that for regular preference ordering, a consumer type of θ prefers early buying over late buying if and only if $\theta \geq \theta_{LE}$ (see Proposition 2.12). There are early buyers if and only if there exists some θ such that $\theta < \theta_{EB}$, $\theta \geq \theta_E$ and $\theta \geq \theta_{LE}$. This, in turn, holds if and only if $\max\{\theta_E, \theta_{LE}\} < \theta_{EB}$. Now, we show that $\theta_E = \left[\frac{p_1}{u(1-\delta_c^{t_2}(1-\alpha^{t_2}))}\right]_0^1 < \theta_{EB} = \left[\frac{p_2}{u(1-\alpha^{t_2})}\right]_0^1$ if and

only if $p_1(1-\alpha^{t_2}) < p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. For the first direction of the iff statement, assume to the contrary that $p_1(1-\alpha^{t_2}) \geq p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. Then, it follows that $\left[\frac{p_1}{u(1-\delta_c^{t_2}(1-\alpha^{t_2}))}\right]_0^1 \geq \left[\frac{p_2}{u(1-\alpha^{t_2})}\right]_0^1$. Now, assume to the contrary of the iff statement (second direction) that $\theta_E = \left[\frac{p_1}{u(1-\delta_c^{t_2}(1-\alpha^{t_2}))}\right]_0^1 \geq \theta_{EB} = \left[\frac{p_2}{u(1-\alpha^{t_2})}\right]_0^1$, which in turn implies that $\frac{p_1}{u(1-\delta_c^{t_2}(1-\alpha^{t_2}))} \geq \frac{p_2}{u(1-\alpha^{t_2})}$. It follows that $p_1(1-\alpha^{t_2}) \geq p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. Thus, we conclude that $\theta_E < \theta_{EB}$ if and only if $p_1(1-\alpha^{t_2}) < p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. Within similar lines, we have $\theta_{LE} < \theta_{EB}$ if and only if $p_1(1-\alpha^{t_2}) < p_2(1-\delta_c^{t_2})$. We note that $p_2(1-\delta_c^{t_2}(1-\alpha^{t_2})) > p_2(1-\delta_c^{t_2})$, under the assumption that $\alpha \in (0,1)$. Thus, we conclude that for $t_2 > \tau(\alpha, \delta_c)$, there exists early buyers if and only if $p_1(1-\alpha^{t_2}) < p_2(1-\delta_c^{t_2})$. A similar argument follows for the irregular case showing that there exists early buyers if and only if $p_1 < p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$ for $t_2 < \tau(\alpha, \delta_c)$.

As for the third statement, we first show that there are late buyers if and only if $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$ for a regular preference ordering. Remember that under a regular preference ordering, a consumer type of θ prefers late buying over early buying if and only if $\theta < \theta_{LE}$ (see Proposition 2.12). Therefore, there are late buyers if and only if there exists some θ such that $\theta < \theta_{LB}$, $\theta \geq \theta_L$ and $\theta < \theta_{LE}$. This in turn, holds if and only if $\min\{\theta_{LE}, \theta_{LB}\} > \theta_L$. Now, we show that $\theta_{LE} = \left[\frac{p_1-\delta_c^{t_2}p_2}{u(1-\delta_c^{t_2}(2-\alpha^{t_2}))}\right]_0^1 > \theta_L = \frac{p_2}{u}$ if and only if $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. For the first direction of the statement, assume to the contrary that $p_1 \leq p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. Then, it follows that $0 < \theta_{LE} \leq \frac{p_2}{u} < 1$, which in turn implies $\theta_{LE} \leq \theta_L$. Now, assume to the contrary of the statement (second direction) that $\theta_{LE} = \left[\frac{p_1-\delta_c^{t_2}p_2}{u(1-\delta_c^{t_2}(2-\alpha^{t_2}))}\right]_0^1 \leq \theta_L = \frac{p_2}{u}$. Then it follows that $\frac{p_1-\delta_c^{t_2}p_2}{u(1-\delta_c^{t_2}(2-\alpha^{t_2}))} \geq \frac{p_2}{u}$, which in turn implies that $p_1 \leq p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. Thus, we conclude that $\theta_{LE} > \theta_L$ if and only if $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. Along similar lines, we have $\theta_{LB} > \theta_L$ if and only if $p_1 > p_2(1-\delta_c^{t_2})$. We note that $p_2(1-\delta_c^{t_2}(1-\alpha^{t_2})) > p_2(1-\delta_c^{t_2})$, under the assumption $\alpha \in (0,1)$. Thus, we conclude that for $t_2 > \tau(\alpha, \delta_c)$, there exist late buyers if and only if $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. A similar argument follows for the irregular case that shows us that there exist late buyers if and only if $p_1 > p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})$. ∎

Proof of Corollary 2.14: Follows from Proposition 2.13 noting that (1) $\alpha^{t_2} \to 1$ if $\alpha \to 1$ and/or $t_2 \to 0$ whereas $\alpha^{t_2} \to 0$ if $\alpha \to 0$ and/or $t_2 \to \infty$; (2) $\delta_c^{t_2} \to 1$ if $\delta_c \to 1$ and/or $t_2 \to 0$ whereas $\delta_c^{t_2} \to 0$ if $\delta_c \to 0$ and/or $t_2 \to \infty$; (3) $\tau(\alpha, \delta_c) \to 0$ if $\alpha \to 0$ and/or $\delta_c \to 1$ ∎

Proof of Lemma 2.15: We prove the statement by contradiction. In view of the regular preference ordering, we first assume that the optimal prices would satisfy $p_2^*(1-\delta_c^{t_2}(1-\alpha^{t_2})) > p_1^*$. For prices $p_2(1-\delta_c^{t_2}(1-\alpha^{t_2})) \geq p_1$, Proposition 2.13 implies that there are no late buyers, and the expected profit $\Pi_S^1(p_1,p_2,t_2)$ simplifies to

$$\Pi_{EB}(p_1,p_2) \;=\; p_1(\theta_{EB}-\theta_E)+(p_1+\delta_s^{t_2}p_2)(1-\theta_{EB}) \tag{A.10}$$

For (p_1^*,p_2^*) to optimize $\Pi_{EB}(p_1,p_2)$, it has to satisfies the KKT conditions. The unique KKT point for the profit maximization problem $\max\{\Pi_{EB}(p_1,p_2) : p_2(1-\delta_c^{t_2}(1-\alpha^{t_2})) \geq p_1\}$ satisfies $p_2^*(1-\delta_c^{t_2}(1-\alpha^{t_2})) = p_1^*$, implying that the optimal prices never satisfy $p_2^*(1-\delta_c^{t_2}(1-\alpha^{t_2})) > p_1^*$. Now we assume that the optimal prices would satisfy $p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}) < p_1^*$. For prices $p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}) \leq p_1$, Proposition 2.13 implies that there are no early buyers, and the expected profit $(\Pi_S^1(p_1,p_2,t_2))$ simplifies to

$$\Pi_{LB}(p_1,p_2) \;=\; \delta_s^{t_2}p_2(\theta_{LB}-\theta_L)+(p_1+\delta_s^{t_2}p_2)(1-\theta_{LB}). \tag{A.11}$$

For (p_1^*,p_2^*) to optimize $\Pi_{LB}(p_1,p_2)$, it has to satisfy the KKT conditions. The unique KKT point for the profit maximization problem $\max\left\{\Pi_{LB}(p_1,p_2) : p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}) \leq p_1\right\}$ satisfies $p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}) = p_1^*$, implying that the optimal prices never satisfy $p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}) < p_1^*$.

We thus conclude that the optimal prices satisfy $p_2^*(1-\delta_c^{t_2}(1-\alpha^{t_2})) \leq p_1^* \leq p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})$ for a regular preference ordering.

In view of the irregular preference ordering, we first assume that the prices would satisfy $p_1^* < p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})$. For prices $p_1 \leq p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})$, Proposition 2.13 implies that there are no late buyers, and the expected profit $\Pi_S^2(p_1,p_2,t_2)$ simplifies to

$$\Pi_{EB}(p_1,p_2) \;=\; p_1(\theta_{EB}-\theta_E)+(p_1+\delta_s^{t_2}p_2)(1-\theta_{EB}) \tag{A.12}$$

For (p_1^*,p_2^*) to optimize $\Pi_{EB}(p_1,p_2)$, it has to satisfies the KKT conditions. The unique KKT point for the profit maximization problem $\max\left\{\Pi_{EB}(p_1,p_2) : p_1 \leq p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})\right\}$ satisfies

$p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}) = p_1^*$, implying that the optimal prices never satisfy $p_1 < p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})$. Now assume that the optimal prices would satisfy $p_1^* > p_2^*(1-\delta_c^{t_2}(1-\alpha^{t_2}))$. For prices $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$, Proposition 2.13 implies that there are no early buyers, and the expected profit $\Pi_S^2(p_1, p_2, t_2)$ simplifies to

$$\Pi_{LB}(p_1, p_2) \quad = \quad \delta_s^{t_2} p_2(\theta_{LB} - \theta_L) + (p_1 + \delta_s^{t_2} p_2)(1 - \theta_{LB}). \tag{A.13}$$

For (p_1^*, p_2^*) to optimize $\Pi_{LB}(p_1, p_2)$, it has to satisfies the KKT conditions. The unique KKT point for the profit maximization problem $\max\{\Pi_{LB}(p_1, p_2) : p_1 \geq p_2 p_2(1-\delta_c^{t_2}(1-\alpha^{t_2})\}$ satisfies $p_1^* = p_2^*(1-\delta_c^{t_2}(1-\alpha^{t_2}))$, implying that the optimal prices never satisfy $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$.

We thus conclude that the optimal prices satisfy $p_2^*(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}) \leq p_1^* \leq p_2^*(1-\delta_c^{t_2}(1-\alpha^{t_2}))$ for an irregular preference ordering. ∎

Proof of Proposition 2.16: The proof follows in analogous steps to those Proposition 2.5, where we first derive the optimal prices p_1, p_2 for fixed t_2 that solves $\Pi_S(p_1, p_2; t_2)$, and following we determine the optimal release time t_2. The difference, however, stems from the two cases of ordering: regular and irregular ordering as well as the parameter δ_c that also plays a role in characterizing the pricing and release timing regimes.

To remind the reader, we define the integral of flow utility v over an infinite time period as $u(\delta_c) := \int_0^\infty v\mathrm{e}^{-rt}\,\mathrm{d}t = -v/\log(\delta_c)$. Therefore, by Lemma 2.15, we have

$$\Pi_S^*(\alpha, \delta_s) = \max\{\Pi_S(p_1, p_2; t_2) : p_1, p_2 \in [0, u(\delta_c)], t_2 \in (0, \infty)\}$$

equals to

$$\Pi_S^*(\alpha, \delta_c) = \max\left\{\Pi_S^{*i}(\alpha, \delta_c), \Pi_S^{*R}(\alpha, \delta_c)\right\}$$

where

$$\Pi_S^{*i}(\alpha, \delta_c) = \max\left\{\Pi_S^i(p_1, p_2; t_2) : p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}) \leq p_1 \leq p_2(1-\delta_c^{t_2}(1-\alpha^{t_2})), t_2 \in (0, \tau(\alpha, \delta_c)]\right\},$$

$$\Pi_S^{*R}(\alpha,\delta_c)=\max\left\{\Pi_S^R(p_1,p_2;t_2):p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))\leq p_1\leq p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}}),t_2\in(\tau(\alpha,\delta_c),\infty)\right\}$$

and

$$\Pi_S^i(p_1,p_2;t_2)=p_1(\theta_{LE}-\theta_E)+\delta_s^{t_2}p_2(\theta_{LB}-\theta_{LE})+(p_1+\delta_s^{t_2}p_2)(1-\theta_{LB}),$$

$$\Pi_S^R(p_1,p_2;t_2)=p_1(\theta_{EB}-\theta_{LE})+\delta_s^{t_2}p_2(\theta_{LE}-\theta_L)+(p_1+\delta_s^{t_2}p_2)(1-\theta_{EB}).$$

Although the profit functions $\Pi_S^i(p_1,p_2;t_2)$ and $\Pi_S^R(p_1,p_2;t_2)$ are *not* jointly concave in p_1, p_2 and t_2, one can verify that they are jointly concave in p_1 and p_2 if t_2 is fixed. Therefore, we first determine the optimal prices for fixed t_2, and we subsequently find the optimal introduction time t_2^* of the profit functions with optimal prices. To ease the notation, we denote the feasible region associated with prices of each problem for fixed t_2 as

$$D_p^i:=\left\{p_1,p_2\in[0,u(\delta_c)]\mid p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})\leq p_1\leq p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))\right\}$$

and

$$D_p^R=\left\{p_1,p_2\in[0,u(\delta_c)]\mid p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))\leq p_1\leq p_2(\frac{1-\delta_c^{t_2}}{1-\alpha^{t_2}})\right\}.$$

In addition, we define $a:=\alpha^{t_2}$, $b:=\delta_c^{t_2}$ and $c:=\delta_s^{t_2}$. In the first part of the proof where we discuss optimal prices for fixed t_2 and $\delta_s\in(0,1)$, we note that c is fixed as well whereas a is a function of α while b is a function of δ_c.

(I) Optimal Prices $(p_1^*(t_2),p_2^*(t_2))$ for fixed t_2:

(a) Irregular Preference Ordering, $t_2\in(0,\tau(\alpha,\delta_c)]$: First, we note that $(0,\tau(\alpha,\delta_c)]$ is non-empty if and only if $\alpha<\delta_c$ (Reader is referred to Proposition 2.12). Distinguishing the two cases $\quad p_1\leq u(\delta_c)(1-b)$ and $\quad p_1\geq u(\delta_c)(1-b)$ where $u(\delta_c):=-v/\log(\delta_c)$, $\Pi_S^i(p_1^*(t_2),p_2^*(t_2);t_2)$ equals to

$$\Pi_S^{*i}(a,b)=\max\left\{\Pi_S^{i1}(a,b),\Pi_S^{i2}(a,b)\right\}$$

where

$$\Pi_S^{i1}(a,b)=\max\left\{\Pi_S^i(p_1,p_2):(p_1,p_2)\in D_p^i,p_1\in[0,u(\delta_c)(1-b)]\right\},$$

$$\Pi_S^{i2}(a,b) = \max\left\{\Pi_S^i(p_1,p_2) : (p_1,p_2) \in D_p^i, p_1 \in [u(\delta_c)(1-b), u(\delta_c)]\right\}.$$

(i) First, we discuss the profit maximization problem $\Pi_S^{i1}(a,b)$ and derive the KKT points. We define functions $\Pi_{EB}^{i1}(a,b)$, $\Pi_{ELB}^{i1}(a,b)$ and $\Pi_{LB}^{i1}(a,b)$ over the domain $t_2 \leq \tau(\alpha,\delta_c)$ and $\alpha, \delta_c \in (0,1)$ as:

$$\Pi_{ELB}^{i1}(a,b) = \frac{u(\delta_c)bc((a-2)b+1)(c(((a-2)a+2)b+a-2)+2(b-1)((a-1)b+1))}{-2bc\left((a(2a-7)+7)b^2+5(a-2)b+3\right)+(b-1)b^2((a-1)b+1)+(b-1)c^2((a-1)b+1)} \tag{A.14}$$

$$\Pi_{LB}^{i1}(a,b) = \frac{u(\delta_c)(1-b)(1-(1-a)b+c)^2}{4(1-(1-a)b)^2+4(1-b)c} \tag{A.15}$$

$$\Pi_{EB}^{i1}(a,b) = \frac{u(\delta_c)(1-(1-a)b)((1-a)c+1-b)^2}{4(1-b)^2+4(1-a)c(1-(1-a)b)} \tag{A.16}$$

We find that the unique KKT point leads to the expression for $\Pi_S^{i1}(a,b)$ as follows:

$$\Pi_S^{i1}(a,b) = \begin{cases} \Pi_{ELB}^{i1}(a,b) & \text{if } t_2 \in \mathcal{T}_{i1}^{1c}(\alpha,\delta_c) \cap \mathcal{T}_{i1}^{2c}(\alpha,\delta_c), \\ \Pi_{EB}^{i1}(a,b) & \text{if } t_2 \in \mathcal{T}_{i1}^{1}(\alpha,\delta_c) \\ \Pi_{LB}^{i1}(a,b) & \text{if } t_2 \in \mathcal{T}_{i1}^{2}(\alpha,\delta_c) \end{cases} \tag{A.17}$$

where

$$\lambda_1^{i1}(a,b) = (-1+a)b^3 + c(1+c) + (-2+a)b^2(-1+(-1+2a)c) - b(1-3(-1+a)c+c^2);$$

$$\lambda_2^{i1}(a,b) = -((-1+a)(-(-1+a)b^3+c(-1+(-1+a)c)+(-2+a)b^2(1+c+ac)+b(1+3c+(-1+a)^2c^2))),$$

$$\mathcal{T}_{i1}^{1}(\alpha,\delta_c) = \left\{t_2 : \lambda_1^{i1}(a,b) \geq 0\right\} \quad \text{and} \quad \mathcal{T}_{i1}^{1c}(\alpha,\delta_c) = \left\{t_2 : \lambda_1^{i1}(a,b) < 0\right\};$$

$$\mathcal{T}_{i1}^{2}(\alpha,\delta_c)=\left\{t_2:\lambda_2^{i1}(a,b)\geq 0\right\}\quad\text{and}\quad\mathcal{T}_{i1}^{2c}(\alpha,\delta_c)=\left\{t_2:\lambda_2^{i1}(a,b)<0\right\}.$$

Next, we show that $\Pi_S^{i1}(a,b)$ is continuous over its entire domain $\alpha<\delta_c$ and $t_2\leq\tau(\alpha,\delta_c)$. First, we show that the domains $\mathcal{T}_{i1}^{1c}(\alpha,\delta_c)\cap\mathcal{T}_{i1}^{2c}(\alpha,\delta_c)$, $\mathcal{T}_{i1}^{1}(\alpha,\delta_c)$ and $\mathcal{T}_{i1}^{2}(\alpha,\delta_c)$ are mutually exclusive and collectively exhaustive. We verify that for $\delta_c\leq\delta_s$ (that is, if $b\leq c$), it follows that $\mathcal{T}_{i1}^{1}(\alpha,\delta_c)=\emptyset$, and thus, $\mathcal{T}_{i1}^{1c}(\alpha,\delta_c)=(0,\tau(\alpha,\delta_c)]$. Within similar lines, we have $\mathcal{T}_{i1}^{2}(\alpha,\delta_c)=\emptyset$ when $\delta_c>\delta_s$ (that is, if $b>c$), and thus $\mathcal{T}_{i1}^{2c}(\alpha,\delta_c)=(0,\tau(\alpha,\delta_c)]$. Secondly, we verify that the functions $\Pi_j^{i1}(a,b)\in\mathcal{C}^1$ in associated domains $\mathcal{T}_{i1}^{1c}(\alpha,\delta_c)\cap\mathcal{T}_{i1}^{2c}(\alpha,\delta_c)$, $\mathcal{T}_{i1}^{1}(\alpha,\delta_c)$, $\mathcal{T}_{i1}^{2}(\alpha,\delta_c)$ for $j=\{ELB,EB,LB\}$, respectively. In addition, we have $\Pi_{ELB}^{i1}(a,b)=\Pi_{EB}^{i1}(a,b)$ at $\lambda_1^{i1}(a,b)=0$, and $\Pi_{ELB}^{i1}(a,b)=\Pi_{LB}^{i1}(a,b)$ at $\lambda_2^{i1}(a,b)=0$. Then, the continuity of $\Pi_S^{i1}(a,b)$ over its entire domain $\alpha<\delta_c$ and $t_2\leq\tau(\alpha,\delta_c)$ follows directly.

We note that for $\delta_c\leq\delta_s$ (or $b\leq c$), it follows that $\mathcal{T}_{i1}^{1}=\emptyset$. In this case, we have

$$\Pi_S^{i1}(a,b)=\begin{cases}\Pi_{ELB}^{i1}(a,b) & \text{if } t_2\in\mathcal{T}_{i1}^{2c}(\alpha,\delta_c),\\ \Pi_{LB}^{i1}(a,b) & \text{if } t_2\in\mathcal{T}_{i1}^{2}(\alpha,\delta_c).\end{cases}\tag{A.18}$$

Within similar lines, we have $\mathcal{T}_{i1}^{2}=\emptyset$ when $\delta_c>\delta_s$ (or $b>c$). Then, it follows that

$$\Pi_S^{i1}(a,b)=\begin{cases}\Pi_{ELB}^{i1}(a,b) & \text{if } t_2\in\mathcal{T}_{i1}^{1c}(\alpha,\delta_c),\\ \Pi_{EB}^{i1}(a,b) & \text{if } t_2\in\mathcal{T}_{i1}^{1}(\alpha,\delta_c).\end{cases}\tag{A.19}$$

(ii) Second, we discuss the profit maximization problem $\Pi_S^{i2}(a,b)$ and derive the KKT points within similar lines. We define the following profit functions:

$$\Pi_{EL}^{i2}(a,b)=\frac{u(\delta_c)bc^2((a-2)b+1)}{(a-1)b^3+b^2(2(a-3)c+1)+bc((a-1)c+2)+c^2}\tag{A.20}$$

$$\Pi_E^{i2}(a,b) = \frac{u(\delta_c)(a-1)^2c^2((a-1)b+1)}{4((1-a)c((a-1)b+1)-a(1-b)b)} \tag{A.21}$$

$$\Pi_L^{i2}(a,b) = (cu(\delta_c))/4 \tag{A.22}$$

We find that the unique KKT point leads to the expression for $\Pi_S^{i2}(a,b)$ as follows:

$$\Pi_S^{i2}(a,b) = \begin{cases} \Pi_{EL}^{i2}(a,b) & \text{if } t_2 \in \mathcal{T}_{i2}^{1c}\alpha,\delta_c) \cap \mathcal{T}_{i2}^{2c}(\alpha,\delta_c), \\ \Pi_E^{i2}(a,b) & \text{if } t_2 \in \mathcal{T}_{i2}^{1}(\alpha,\delta_c) \\ \Pi_L^{i2}(a,b) & \text{if } t_2 \in \mathcal{T}_{i2}^{2}(\alpha,\delta_c) \end{cases} \tag{A.23}$$

where

$$\lambda_1^{i2}(a,b) = (1-a)c\left(\left(a^2-2a-1\right)b^2 + b\left(a^2c-2ac+a+c+1\right) - (1-a)c\right);$$

$$\lambda_2^{i2}(a,b) = \frac{b-c}{2(a-2)b+2},$$

$$\mathcal{T}_{i2}^{1}(\alpha,\delta_c) = \left\{t_2 : \lambda_1^{i2}(a,b) \geq 0\right\} \quad \text{and} \quad \mathcal{T}_{i2}^{1c}(\alpha,\delta_c) = \left\{t_2 : \lambda_1^{i2}(a,b) < 0\right\};$$

$$\mathcal{T}_{i2}^{2}(\alpha,\delta_c) = \left\{t_2 : \lambda_2^{i2}(a,b) \geq 0\right\} \quad \text{and} \quad \mathcal{T}_{i2}^{2c}(\alpha,\delta_c) = \left\{t_2 : \lambda_2^{i2}(a,b) < 0\right\};$$

Next, we show that $\Pi_S^{i2}(a,b)$ is continuous over its entire domain $\alpha < \delta_c$ and $t_2 \leq \tau(\alpha,\delta_c)$. First, we show that the domains $\mathcal{T}_{i2}^{1c}(\alpha,\delta_c) \cap \mathcal{T}_{i2}^{2c}(\alpha,\delta_c)$, $\mathcal{T}_{i2}^{1}(\alpha,\delta_c)$ and $\mathcal{T}_{i2}^{2}(\alpha,\delta_c)$ are mutually exclusive and collectively exhaustive. We verify that for $\delta_c \leq \delta_s$ (that is, if $b \leq c$), it follows that $\mathcal{T}_{i2}^{1}(\alpha,\delta_c) = \emptyset$, and thus, $\mathcal{T}_{i2}^{1c}(\alpha,\delta_c) = (0,\tau(\alpha,\delta_c)]$. Within similar lines, we have $\mathcal{T}_{i2}^{2}(\alpha,\delta_c) = \emptyset$ when $\delta_c > \delta_s$ (that is, if $b > c$), and thus $\mathcal{T}_{i2}^{2c}(\alpha,\delta_c) = (0,\tau(\alpha,\delta_c)]$. Secondly, we verify that the functions $\Pi_j^{i2}(a,b) \in \mathcal{C}^1$ in their associated domains defined by Equation A.23. In addition, we have $\Pi_{EL}^{i2}(a,b) = \Pi_E^{i2}(a,b)$ at $\lambda_1^{i2}(a,b) = 0$, and $\Pi_{EL}^{i2}(a,b) = \Pi_L^{i2}(a,b)$ at $\lambda_2^{i2}(a,b) = 0$. Then, the

continuity of $\Pi_S^{i2}(a,b)$ over its entire domain $\alpha < \delta_c$ and $t_2 \leq \tau(\alpha, \delta_c)$ follows directly.

We note that for $\delta_c \leq \delta_s$ (or $b \leq c$), it follows that $\mathcal{T}_{i2}^1 = \emptyset$. In this case, we have

$$\Pi_S^{i2}(a,b) = \begin{cases} \Pi_{EL}^{i2}(a,b) & \text{if } t_2 \in \mathcal{T}_{i2}^{2c}(\alpha, \delta_c), \\ \Pi_L^{i2}(a,b) & \text{if } t_2 \in \mathcal{T}_{i2}^{2}(\alpha, \delta_c). \end{cases} \tag{A.24}$$

Within similar lines, we have $\mathcal{T}_{i2}^2 = \emptyset$ when $\delta_c > \delta_s$ (or $b > c$). Then, it follows that

$$\Pi_S^{i2}(a,b) = \begin{cases} \Pi_{EL}^{i2}(a,b) & \text{if } t_2 \in \mathcal{T}_{i2}^{1c}(\alpha, \delta_c), \\ \Pi_E^{i2}(a,b) & \text{if } t_2 \in \mathcal{T}_{i2}^{1}(\alpha, \delta_c). \end{cases} \tag{A.25}$$

(b) Regular Preference Ordering, $t_2 \in (\tau(\alpha, \delta_c), \infty)$:

Distinguishing the two cases $p_2 \leq u(\delta_c)(1-a)$ and $p_2 \geq u(\delta_c)(1-a)$ where $u(\delta_c) := -v/\log(\delta_c)$, $\Pi_S^{R*}(a,b)$ can be written as

$$\Pi_S^{*R}(a,b) = \max\left\{\Pi_S^{R1}(a,b), \Pi_S^{R2}(a,b)\right\}$$

where

$$\Pi_S^{R1}(a,b) = \max\left\{\Pi_S^{R}(p) : p \in D_p^R, p_2 \in [0, u(\delta_c)(1-a)]\right\},$$

$$\Pi_S^{R2}(a,b) = \max\left\{\Pi_S^{R}(p) : p \in D_p^R, p_2 \in [u(\delta_c)(1-a), u(\delta_c)]\right\}.$$

(i) First, we discuss the profit maximization problem $\Pi_S^{R1}(a,b)$ and derive the KKT points. We define functions $\Pi_{EB}^{R1}(a,b)$, $\Pi_{ELB}^{R1}(a,b)$ and $\Pi_{LB}^{R1}(a,b)$ over the domain $t_2 > \tau(\alpha, \delta_c)$ and $\alpha, \delta_c \in (0,1)$ as:

$$\Pi_{ELB}^{R1}(a,b) = \frac{u(\delta_c)c((a-2)b+1)\left(a^2b - 2ab + 2(a-1)c + a + 2b - 2\right)}{-(a-1)b^2 + 2(a(2a-7)+7)bc + c(-a(c-4)+c-8)} \tag{A.26}$$

$$\Pi_{LB}^{R1}(a,b) = \frac{u(\delta_c)((a-1)c+b-1)^2}{4\left((a-1)^2c-b+1\right)} \tag{A.27}$$

$$\Pi_{EB}^{R1}(a,b) = \frac{u(\delta_c)(1-a)((a-1)b+c+1)^2}{4(1-a)((a-1)b+1)+4c} \tag{A.28}$$

We find that the unique KKT point leads to the expression for $\Pi_S^{R1}(a,b)$ as follows:

$$\Pi_S^{R1}(a,b) = \begin{cases} \Pi_{ELB}^{R1}(a,b) & \text{if } t_2 \in \mathcal{T}_{R1}^{1c}(\alpha,\delta_c) \cap \mathcal{T}_{R1}^{2c}(\alpha,\delta_c), \\ \Pi_{EB}^{R1}(a,b) & \text{if } t_2 \in \mathcal{T}_{R1}^{1}(\alpha,\delta_c) \\ \Pi_{LB}^{R1}(a,b) & \text{if } t_2 \in \mathcal{T}_{R1}^{2}(\alpha,\delta_c) \end{cases} \tag{A.29}$$

where

$$\lambda_1^{R1}(a,b) = (1-2a)c((a-2)b+1) + (1-a)c^2 - b(1-b); \tag{A.30}$$

$$\lambda_2^{R1}(a,b) = (1-a)c^2 - b(1-b) + (1-2a)c(1+(-2+a)b), \tag{A.31}$$

$$\mathcal{T}_{R1}^{1}(\alpha,\delta_c) = \left\{t_2 : \lambda_1^{R1}(a,b) \geq 0\right\} \quad \text{and} \quad \mathcal{T}_{R1}^{1c}(\alpha,\delta_c) = \left\{t_2 : \lambda_1^{R1}(a,b) < 0\right\};$$

$$\mathcal{T}_{R1}^{2}(\alpha,\delta_c) = \left\{t_2 : \lambda_2^{R1}(a,b) \geq 0\right\} \quad \text{and} \quad \mathcal{T}_{R1}^{2c}(\alpha,\delta_c) = \left\{t_2 : \lambda_2^{R1}(a,b) < 0\right\};$$

Next, we show that $\Pi_S^{R1}(a,b)$ is continuous over its entire domain $t_2 > \tau(\alpha,\delta_c)$ and $\alpha,\delta_c \in (0,1)$. First, we show that the domains $\mathcal{T}_{R1}^{1c}(\alpha,\delta_c) \cap \mathcal{T}_{R1}^{2c}(\alpha,\delta_c)$, $\mathcal{T}_{R1}^{1}(\alpha,\delta_c)$ and $\mathcal{T}_{R1}^{2}(\alpha,\delta_c)$ are mutually exclusive and collectively exhaustive. We verify that for $\delta_c \leq \delta_s$ (that is, if $b \leq c$), it follows that $\mathcal{T}_{R1}^{1}(\alpha,\delta_c) = \emptyset$, and thus, $\mathcal{T}_{R1}^{1c}(\alpha,\delta_c) = (\tau(\alpha,\delta_c),\infty)$. Within similar lines, we have $\mathcal{T}_{R1}^{2}(\alpha,\delta_c) = \emptyset$ when $\delta_c > \delta_s$ (that is, if $b > c$), and thus $\mathcal{T}_{R1}^{2c}(\alpha,\delta_c) = (\tau(\alpha,\delta_c),\infty)$. Secondly, we verify that the functions $\Pi_j^{R1}(a,b) \in \mathcal{C}^1$ in associated domains $\mathcal{T}_{R1}^{1c}(\alpha,\delta_c) \cap \mathcal{T}_{R1}^{2c}(\alpha,\delta_c)$, $\mathcal{T}_{R1}^{1}(\alpha,\delta_c)$, $\mathcal{T}_{R1}^{2}(\alpha,\delta_c)$ for $j = \{ELB, EB, LB\}$, respectively. In addition, we have $\Pi_{ELB}^{R1}(a,b) = \Pi_{EB}^{R1}(a,b)$

at $\lambda_1^{R1}(a,b) = 0$, and $\Pi_{ELB}^{R1}(a,b) = \Pi_{LB}^{R1}(a,b)$ at $\lambda_2^{R1}(a,b) = 0$. Then, the continuity of $\Pi_S^{R1}(a,b)$ over its entire domain $t_2 > \tau(\alpha, \delta_c)$ and $\alpha, \delta_c \in (0,1)$ follows directly.

We note that for $\delta_c \leq \delta_s$ (or $b \leq c$), it follows that $\mathcal{T}_{R1}^1 = \emptyset$. In this case, we have

$$\Pi_S^{R1}(a,b) = \begin{cases} \Pi_{ELB}^{R1}(a,b) & \text{if } t_2 \in \mathcal{T}_{R1}^{2c}(\alpha, \delta_c), \\ \Pi_{LB}^{R1}(a,b) & \text{if } t_2 \in \mathcal{T}_{R1}^{2}(\alpha, \delta_c). \end{cases} \tag{A.32}$$

Within similar lines, we have $\mathcal{T}_{R1}^2 = \emptyset$ when $\delta_c > \delta_s$ (or $b > c$). Then, it follows that

$$\Pi_S^{R1}(a,b) = \begin{cases} \Pi_{ELB}^{R1}(a,b) & \text{if } t_2 \in \mathcal{T}_{R1}^{1c}(\alpha, \delta_c), \\ \Pi_{EB}^{R1}(a,b) & \text{if } t_2 \in \mathcal{T}_{R1}^{1}(\alpha, \delta_c). \end{cases} \tag{A.33}$$

(ii) Second, we discuss the profit maximization problem $\Pi_S^{R2}(a,b)$ and derive the KKT points within similar lines. We define the following profit functions:

$$\Pi_{EL}^{R2}(a,b) = -\frac{u(\delta_c)c((a-2)b+1)((a-1)b+1)}{(6-4a)bc + b^2 + (c-4)c}, \tag{A.34}$$

$$\Pi_E^{R2}(a,b) = \frac{1}{4}u(\delta_c)[1 - b(1-a)], \tag{A.35}$$

$$\Pi_L^{R2}(a,b) = \frac{u(\delta_c)(1-b)^2}{4(1-b-ac(1-a))}. \tag{A.36}$$

We find that the unique KKT point leads to the expression for $\Pi_S^{R2}(a,b)$ as follows:

$$\Pi_S^{R2}(a,b) = \begin{cases} \Pi_{EL}^{R2}(a,b) & \text{if } t_2 \in \mathcal{T}_{R2}^{1c}\alpha,\delta_c) \cap \mathcal{T}_{R2}^{2c}(\alpha,\delta_c), \\ \Pi_{E}^{R2}(a,b) & \text{if } t_2 \in \mathcal{T}_{R2}^{1}(\alpha,\delta_c) \\ \Pi_{L}^{R2}(a,b) & \text{if } t_2 \in \mathcal{T}_{R2}^{2}(\alpha,\delta_c) \end{cases} \tag{A.37}$$

where

$$\lambda_1^{R2}(a,b) = \frac{b-c}{2(a-2)b+2};$$

$$\lambda_2^{R2}(a,b) = \left(-b\left(2a^2c - 4ac + c + 1\right) - 2ac + b^2 + c\right.;$$

$$\mathcal{T}_{R2}^{1}(\alpha,\delta_c) = \left\{t_2 : \lambda_1^{R2}(a,b) \geq 0\right\} \quad \text{and} \quad \mathcal{T}_{R2}^{1c}(\alpha,\delta_c) = \left\{t_2 : \lambda_1^{R2}(a,b) < 0\right\};$$

$$\mathcal{T}_{R2}^{2}(\alpha,\delta_c) = \left\{t_2 : \lambda_2^{R2}(a,b) \geq 0\right\} \quad \text{and} \quad \mathcal{T}_{R2}^{2c}(\alpha,\delta_c) = \left\{t_2 : \lambda_2^{R2}(a,b) < 0\right\};$$

Next, we show that $\Pi_S^{R2}(a,b)$ is continuous over its entire domain $t_2 > \tau(\alpha,\delta_c)$ and $\alpha,\delta_c \in (0,1)$. First, we show that the domains $\mathcal{T}_{R2}^{1c}(\alpha,\delta_c) \cap \mathcal{T}_{R2}^{2c}(\alpha,\delta_c)$, $\mathcal{T}_{R2}^{1}(\alpha,\delta_c)$ and $\mathcal{T}_{R2}^{2}(\alpha,\delta_c)$ are mutually exclusive and collectively exhaustive. We verify that for $\delta_c \leq \delta_s$ (that is, if $b \leq c$), it follows that $\mathcal{T}_{R2}^{1}(\alpha,\delta_c) = \emptyset$, and thus, $\mathcal{T}_{R2}^{1c}(\alpha,\delta_c) = (\tau(\alpha,\delta_c),\infty)$. Within similar lines, we have $\mathcal{T}_{R2}^{2}(\alpha,\delta_c) = \emptyset$ when $\delta_c > \delta_s$ (that is, if $b > c$), and thus $\mathcal{T}_{R2}^{2c}(\alpha,\delta_c) = (\tau(\alpha,\delta_c),\infty)$. Secondly, we verify that the functions $\Pi_j^{R2}(a,b) \in \mathcal{C}^1$ in associated domains $\mathcal{T}_{R2}^{1c}(\alpha,\delta_c) \cap \mathcal{T}_{R2}^{2c}(\alpha,\delta_c)$, $\mathcal{T}_{R2}^{1}(\alpha,\delta_c)$, $\mathcal{T}_{R2}^{2}(\alpha,\delta_c)$ for $j = \{EL, E, L\}$, respectively. In addition, we have $\Pi_{EL}^{R2}(a,b) = \Pi_{E}^{R2}(a,b)$ at $\lambda_1^{R2}(a,b) = 0$, and $\Pi_{EL}^{R2}(a,b) = \Pi_{L}^{R2}(a,b)$ at $\lambda_2^{R2}(a,b) = 0$. Then, the continuity of $\Pi_S^{R2}(a,b)$ over its entire domain $t_2 > \tau(\alpha,\delta_c)$ and $\alpha,\delta_c \in (0,1)$ follows directly.

We note that for $\delta_c \leq \delta_s$ (or $b \leq c$), it follows that $\mathcal{T}_{R2}^{1} = \emptyset$. In this case, we have

$$\Pi_S^{R2}(a,b) = \begin{cases} \Pi_{EL}^{R2}(a,b) & \text{if } t_2 \in \mathcal{T}_{R2}^{2c}(\alpha,\delta_c), \\ \Pi_L^{R2}(a,b) & \text{if } t_2 \in \mathcal{T}_{R2}^{2}(\alpha,\delta_c). \end{cases} \tag{A.38}$$

Within similar lines, we have $\mathcal{T}_{R2}^{2} = \emptyset$ when $\delta_c > \delta_s$ (or $b > c$). Then, it follows that

$$\Pi_S^{R2}(a,b) = \begin{cases} \Pi_{EL}^{R2}(a,b) & \text{if } t_2 \in \mathcal{T}_{R2}^{1c}(\alpha,\delta_c), \\ \Pi_E^{R2}(a,b) & \text{if } t_2 \in \mathcal{T}_{R2}^{1}(\alpha,\delta_c). \end{cases} \tag{A.39}$$

Thus, we obtain the optimal pricing regime for fized t_2. Next, we discuss the optimal release time t_2^*.

II) Optimal Time (t_2^*):

We use the following notation for this subsection:

$$\Pi_S^*(\alpha,\delta_c) = \max\left\{\Pi_S^{*R}(\alpha,\delta_c), \Pi_S^{*i}(\alpha,\delta_c)\right\}$$

where

$$\Pi_S^{*R}(\alpha,\delta_c) = \max\left\{\Pi_S^{*R1}(\alpha,\delta_c), \Pi_S^{*R2}(\alpha,\delta_c)\right\}$$

and

$$\Pi_S^{*i}(\alpha,\delta_c) = \max\left\{\Pi_S^{*i1}(\alpha,\delta_c), \Pi_S^{*i2}(\alpha,\delta_c)\right\}$$

with

$$\Pi_S^{*R1}(\alpha,\delta_c) = \max\left\{\Pi_S^{R1}(a,b) : t_2 > \tau(\alpha,\delta_c)\right\},$$

$$\Pi_S^{*R2}(\alpha,\delta_c) = \max\left\{\Pi_S^{R2}(a,b) : t_2 > \tau(\alpha,\delta_c))\right\},$$

$$\Pi_S^{*i1}(\alpha,\delta_c) = \max\left\{\Pi_S^{i1}(a,b) : t_2 \leq \tau(\alpha,\delta_c)\right\},$$

$$\Pi_S^{*i2}(\alpha,\delta_c)=\max\left\{\Pi_S^{i2}(a,b):t_2\leq\tau(\alpha,\delta_c))\right\}.$$

(i) First, we show that $\Pi_S^*(\alpha,\delta_c)\leq\Pi_E^{*M}(\delta_c)=u(\delta_c)/4$ when we have $\delta_c\geq\delta_s$ (or $b\geq c$). Here, $\Pi_E^{*M}(\delta_c)$ is the optimal profit of the single product case or alternatively the case, where the late product is introduced at time $t_2\to\infty$.

We distinguish the two cases of regular and irregular case. For the regular preference ordering $\Pi_S^*(\alpha,\delta_c)=\Pi_S^{*R}(\alpha,\delta_c)$, we remind the reader that $\Pi_S^*(\alpha,\delta_c)=\max\left\{\Pi_S^{*R1}(\alpha,\delta_c),\Pi_S^{*R2}(\alpha,\delta_c)\right\}$. In view of $\Pi_S^{*R1}(\alpha,\delta_c)$, Equation (19) shows that $\Pi_S^{R1}(a,b)=\Pi_{ELB}^{R1}(a,b)$ if $t_2\in\mathcal{T}_{R1}^{1c}(\alpha,\delta_c)$ while $\Pi_S^{R1}(a,b)=\Pi_{EB}^{R1}(a,b)$ for $t_2\in\mathcal{T}_{R1}^{1}(\alpha,\delta_c)$. That is why, we distinguish the two cases $\Pi_{EB}^{R1}(a,b)$ and $\Pi_{ELB}^{R1}(a,b)$.

In view of $\Pi_{EB}^{R1}(a,b)$, we first define $t_2^*(\delta_s)$ as the point where $\Pi_{EB}^{R1}(a,b)$ attains its maximum. First, we verify that $\lim_{t_2\to\infty}\Pi_{EB}^{R1}(a,b)=\frac{u(\delta_c)}{4}$ and $\lim_{t_2\to0}\Pi_{EB}^{R1}(a,b)=0$. Therefore, if the profit function attains its maximum when $t_2\to\infty$, then $\Pi_{EB}^{*R1}=\frac{u(\delta_c)}{4}$. Rest of the argument discusses the case where $t_2^*(\delta_s)\in(0,\infty)$. To prove this, we first show that $\frac{d\Pi_S^{*R1}}{d\delta_s}>0$. Using the envelope theorem, it follows that $\frac{d\Pi_S^{*R1}}{d\delta_s}=\frac{\partial\Pi_S^{R1}}{\partial\delta_s}(p^*(\delta_s)),t_2^*(\delta_s),\delta_s)$. We have $\frac{\partial\Pi_S^{R1}(p*(\delta_s)),t_2^*(\delta_s),\delta_s)}{\partial\delta_s}=t_2^*\delta_s^{t_2^*-1}(1-\theta_{EB}^*(\delta_s))$. In addition, we have $t_2^*\delta_s^{t_2^*-1}(1-\theta_{EB}^*(\delta_s))>0$ through the assumption $\delta_s,\delta_c,\alpha\in(0,1)$, $t_2>0$, and the definition of θ_{EB}. Therefore, we conclude that $\frac{d\Pi_{EB}^{*R1}}{d\delta_s}>0$ if $t_2^*(\delta_s)\in(0,\infty)$. Secondly, we show that $\lim_{\delta_s\to\delta_c}\Pi_S^{R1}(a,b)\leq\frac{u(\delta_c)}{4}$. We have $\lim_{\delta_s\to\delta_c}\Pi_S^{R1}(a,b)=\frac{u(\delta_c)(1-a)(1+ab)^2}{4(1-a[1-(1-2a)b])}$ and as $a^2b[1-(1-a)b]\geq0$, it follows that $(1-a)(1+ab)^2\leq(1-a[1-(1-2a)b])$. Therefore, we have $\lim_{\delta_s\to\delta_c}\Pi_S^{R1}(a,b)\leq\frac{u(\delta_c)}{4}$. Thus, it follows that $\Pi_{EB}^{*R1}(\alpha,\delta_c)\leq\frac{u(\delta_c)}{4}$ for $\delta_s\leq\delta_c$. Within similar lines, one can show that $\Pi_{ELB}^{*R1}(\alpha,\delta_c)\leq\frac{u(\delta_c)}{4}$ due to the limiting case $\lim_{\delta_s\to\delta_c}\Pi_{ELB}^{R1}(a,b)=\frac{u(\delta_c)(2-a-a^2b)}{4(2-a)}\leq\frac{u(\delta_c)}{4}$ and profit function increasing with δ_s. Therefore, we have $\Pi_S^{*R1}(\alpha,\delta_c)\leq\frac{u(\delta_c)}{4}$. A similar argument applies to $\Pi_S^{*R2}(\alpha,\delta_c)$, where we conclude that $\Pi_S^{*R2}(\alpha,\delta_c)\leq\frac{u(\delta_c)}{4}$ for $\delta_c\geq\delta_c$. Thus, it follows that $\Pi_S^{*R}(\alpha,\delta_c)\leq\frac{u(\delta_c)}{4}$ for $\delta_c\geq\delta_s$.

In view of irregular case, we prove that $\Pi_S^{*i}(\alpha,\delta_c)\leq u(\delta_c)/4$ within similar lines. First, it follows that$\frac{d\Pi_S^{*i}}{d\delta_s}>0$ for $\delta_c\geq\delta_s$. Next, we verify that $\lim_{\delta_s\to\delta_c}\Pi_S^{i}(a,b)\leq\frac{u(\delta_c)}{4}$. Therefore, we have $\Pi_S^{i*}(\alpha,\delta_c)\leq\frac{u(\delta_c)}{4}$. Thus, we conclude that $\Pi_S^{*}(\alpha,\delta_c)\leq\frac{u(\delta_c)}{4}$ for $\delta_c\geq\delta_s$.

(ii) Second, we prove that $\Pi_S^{*i}(\alpha,\delta_c)$ is dominated by $\Pi_S^{*R}(\alpha,\delta_c)$ for $\delta_c<\delta_s$. To prove this

statement, we will show that $\Pi_S^{*i1}(\alpha,\delta_c)$ and $\Pi_S^{*i2}(\alpha,\delta_c)$ are dominated by $\Pi_S^{*R1}(\alpha,\delta_c)$, and thereby $\Pi_S^{*R}(\alpha,\delta_c)$. We remind the reader that $\tau(\alpha,\delta_c)=0$ when $\alpha\geq\delta_c$. Therefore, it suffices to consider the case $\alpha<\delta_c$ to prove the statement.

In view of $\Pi_S^{*i1}(\alpha,\delta_c)$, we show that $\Pi_S^{i1}(a,b)$ is maximized at $t_2=\tau(\alpha,\delta_c)$ and $\Pi_S^{R1}(a,b)$ is maximized when $t_2>\tau(\alpha,\delta_c)$, while it follows that $\Pi_S^{i1}(a,b)=\Pi_{S1}^{R1}(a,b)$ at $t_2=\tau(\alpha,\delta_c)$. First, we verify that $\Pi_S^{i1}(a,b)$ is increasing in t_2 over the domain $t_2\leq\tau(\alpha,\delta_c)$. Following, we verify that $\Pi_S^{i1}(a,b)=\Pi_{LB}^{i1}(a,b)$ for all t_2 sufficiently close to $\tau(\alpha,\delta_c)$ since $\lambda_2^{i1}(a,b)>0$ in that region. We thus conclude that $\Pi_S^{i1}(a,b)$ is maximized at $t_2=\tau(\alpha,\delta_c)$ and that $\Pi_S^{*i1}(\alpha,\delta_c)=\Pi_{LB}^{i1}(a,b)|_{t_2=\tau(\alpha,\delta_c)}$. In view of $\Pi_S^{*R1}(\alpha,\delta_c)$, it follows that $\Pi_S^{R1}(a,b)=\Pi_{LB}^{R1}(a,b)$ for all t_2 sufficiently close to $\tau(\alpha,\delta_c)$ since $\lambda_2^{R1}(a,b)=\frac{(b-1)\left(b^2-c^2\right)}{b}>0$ for $b<c$. In addition, it follows that $\Pi_S^{R1}(a,b)$ is maximized when $t_2>\tau(\alpha,\delta_c)$ since $\Pi_{LB}^{R1}(a,b)$ is increasing at $\tau(\alpha,\delta_c)$, i.e., $\lim_{t_2\to\tau(\alpha,\delta_c)}\frac{d\Pi_{LB}^{R1}}{dt_2}>0$. Finally, we verify that $\Pi_{LB}^{i1}(a,b)=\Pi_{LB}^{R1}(a,b)$ at $t_2=\tau(\alpha,\delta_c)$. This concludes that $\Pi_S^{*R1}(\alpha,\delta_c)>\Pi_S^{*i1}(\alpha,\delta_c)$.

In view of $\Pi_S^{*i2}(\alpha,\delta_c)$, we show that $\Pi_S^{*i2}(\alpha,\delta_c)\leq\max\left\{\Pi_{LB}^{R1}(a,b):t_2>\tau(\alpha,\delta_c)\right\}$ and $\Pi_S^{*i2}(\alpha,\delta_c)\leq\max\left\{\Pi_{ELB}^{R1}(a,b):t_2>\tau(\alpha,\delta_c)\right\}$, which in turn implies $\Pi_S^{*i2}(\alpha,\delta_c)\leq\Pi_S^{*R1}(\alpha,\delta_c)$. First, we verify that $\Pi_S^{i2}(a,b)\leq\frac{u(\delta_c)}{4}$ over its domain $t_2\in(0,\tau(\alpha,\delta_c)]$, which implies that $\Pi_S^{*i2}(\alpha,\delta_c)\leq\frac{u(\delta_c)}{4}$. Next, we show that $\Pi_S^{*R1}(\alpha,\delta_c)\geq\frac{u(\delta_c)}{4}$. To this end, we note that both $\lim_{t_2\to\infty}\Pi_{LB}^{R1}(a,b)=\frac{u(\delta_c)}{4}$ and $\lim_{t_2\to\infty}\Pi_{ELB}^{R1}(a,b)=\frac{u(\delta_c)}{4}$ as well as $\lim_{t_2\to\infty}\frac{d\Pi_{LB}^{R1}}{dt_2}=0^-$ and $\lim_{t_2\to\infty}\frac{d\Pi_{ELB}^{R1}}{dt_2}=0^-$ for $\delta_s>\delta_c$. This implies that $\left\{\Pi_{LB}^{R1}(a,b):t_2>\tau(\alpha,\delta_c)\right\}\geq\frac{u(\delta_c)}{4}$ and $\left\{\Pi_{ELB}^{R1}(a,b):t_2>\tau(\alpha,\delta_c)\right\}\geq\frac{u(\delta_c)}{4}$. Hence, $\Pi_S^{*i2}(\alpha,\delta_c)$ is dominated by $\Pi_S^{*R1}(\alpha,\delta_c)$. Thus, we reach at the conclusion that $\Pi_S^{*i}(\alpha,\delta_c)<\Pi_S^{*R}(\alpha,\delta_c)$ for $\delta_c<\delta_s$.

(iii) So far, we have shown that $\Pi_S^{*}(\alpha,\delta_c)=\begin{cases}\frac{u(\delta_c)}{4} & \text{if } \delta_c\geq\delta_s\\ \Pi_S^{*R}(\alpha,\delta_c) & \text{if } \delta_c<\delta_s\end{cases}$. Next, we show that for $\delta_c<\delta_s$, it follows that

$$\Pi_S^{*R}(\alpha,\delta_c)=\begin{cases}\Pi_S^{*R1}(\alpha,\delta_c) & \text{if } \alpha\in(0,\alpha''(\delta_c))\\ \Pi_S^{*R2}(\alpha,\delta_c) & \text{if } \alpha\in[\alpha''(\delta_c),1)\end{cases}, \tag{A.40}$$

$$\Pi_S^{*R1}(\alpha,\delta_c)=\begin{cases}\max\left\{\Pi_{LB}^{R1}(a,b),t_2>\tau(\alpha,\delta_c)\right\} & \text{if } \alpha\in(0,\alpha'(\delta_c)]\\ \max\left\{\Pi_{ELB}^{R1}(a,b),t_2>\tau(\alpha,\delta_c)\right\} & \text{if } \alpha\in(\alpha'(\delta_c),1)\end{cases}, \tag{A.41}$$

and $\Pi_S^{*R}(\alpha,\delta_c)\geq\Pi_S^{*M}(\delta_c)=\frac{u(\delta_c)}{4}$.

In view of $\Pi_S^{*R1}(\alpha,\delta_c)$, we first show that Equation (A.41) holds in three steps:

1. We show that unique maximizer of $\Pi_S^{*R1}(a,b)$ denoted by $t_S^{*R1}(\alpha,\delta_c)$ has the following properties,

$$t_S^{*R1}(\alpha,\delta_c)=\begin{cases}t_{ELB}^*(\alpha,\delta_c) & \text{if } t_{LB}^*(\alpha,\delta_c)<t_2^{R1}(\alpha,\delta_c)\\ t_{LB}^*(\alpha,\delta_c) & \text{if } t_{LB}^*(\alpha,\delta_c)\geq t_2^{R1}(\alpha,\delta_c),\end{cases} \tag{A.42}$$

   where $t_{LB}^*(\alpha,\delta_c)$, $t_{ELB}^*(\alpha,\delta_c)$ are unique maximizers of $\Pi_{LB}^{R1}(a,b)$ and $\Pi_{ELB}^{R1}(a,b)$ over $t_2\in(\tau(\alpha,\delta_c),\infty)$, respectively, while $t_2^{R1}(\alpha,\delta_c):=\max\left\{t_2\in(\tau(\alpha,\delta_c),\infty):\lambda_2^{R1}(a,b)=0\right\}$ and $t_2^{R1}(\alpha,\delta_c):=0$ if $\lambda_2^{R1}(a,b)\neq 0$ over the domain $(\tau(\alpha,\delta_c),\infty)$.

   (a) We remind the reader that $\Pi_S^{R1}(a,b)=\Pi_{ELB}^{R1}(a,b)$ if $t_2\in\mathcal{T}_{R1}^{2c}(\alpha,\delta_c)$ while $\Pi_S^{R1}(a,b)=\Pi_{LB}^{R1}(a,b)$ for $t_2\in\mathcal{T}_{R1}^{2}(\alpha,\delta_c)$ (Equation (18)).

   (b) One can show that $\Pi_{LB}^{R1}(a,b)$ and $\Pi_{ELB}^{R1}(a,b)$ are continuously differentiable functions with unique finite optimizers, which we denote as $t_{LB}^*(\alpha,\delta_c)>\tau(\alpha,\delta_c)$ and $t_{ELB}^*(\alpha,\delta_c)>\tau(\alpha,\delta_c)$, respectively. In fact, we verify that; (1) $\lim_{t_2\to\infty}\Pi_{LB}^{R1}(a,b)=\frac{u(\delta_c)}{4}$ and $\lim_{t_2\to\infty}\Pi_{ELB}^{R1}(a,b)=\frac{u(\delta_c)}{4}$, (2) $\lim_{t_2\to\infty}\frac{d\Pi_{LB}}{dt_2}=0^-$ and $\lim_{t_2\to\infty}\frac{\partial\Pi_{ELB}}{\partial t_2}=0^-$ (3) $\frac{\partial\Pi_{LB}}{\partial t_2}$ and $\frac{\partial\Pi_{ELB}}{\partial t_2}$ vanish only once.

   (c) In addition, we have $\Pi_{ELB}^{R1}(a,b)\geq\Pi_{LB}^{R1}(a,b)$ for $t_2\in(\tau(\alpha,\delta_c),\infty)$ since $\Pi_{LB}^{R1}(a,b)=\max\left\{\Pi_S^R(p):p_1=p_2(\frac{1-b}{1-a}),p\in D_p^R,p_2\in[0,u(\delta_c)(1-a)]\right\}$ while $\Pi_{ELB}^{R1}(a,b)=\max\left\{\Pi_S^R(p):p\in D_p^R,p_2\in[0,u(\delta_c)(1-a)]\right\}$. In addition, we verify that $\Pi_{ELB}^{R1}(a,b)=\Pi_{LB}^{R1}(a,b)$ when $\lambda_2^{R1}(a,b)=0$ (one can verify this by evaluating equations (13) and (14) at $\lambda_2^{R1}(a,b)=0$).

   (d) We verify that $t_{LB}^*(\alpha,\delta_c)=t_2^{R1}(\alpha,\delta_c)$ if and only if $t_{ELB}^*(\alpha,\delta_c)=t_2^{R1}(\alpha,\delta_c)$. First, we verify that for $t_{LB}^*(\alpha,\delta_c)=t_2^{R1}(\alpha,\delta_c)$, we have $t_{ELB}^*(\alpha,\delta_c)=t_2^{R1}(\alpha,\delta_c)$. Indeed, if

$t^*_{LB}(\alpha,\delta_c) = t^{R1}_2(\alpha,\delta_c)$, then $\lambda^{R1}_2(a,b) = 0$ at $t_2 = t^*_{LB}(\alpha,\delta_c)$ by definition of $t^{R1}_2(\alpha,\delta_c)$. This implies that $\Pi^{R1}_{ELB}(a,b) = \Pi^{R1}_{LB}(a,b)$ at $t^*_{LB}(\alpha,\delta_c)$ (see point (C)). For $\Pi^{R1}_{ELB}(a,b)$ to be weakly above $\Pi^{R1}_{LB}(a,b)$ around the unique maximizer $t^*_{LB}(\alpha,\delta_c)$ of $\Pi^{R1}_{LB}(a,b)$ as claimed by point (C), $\Pi^{R1}_{ELB}(a,b)$ must satisfy first order condition at $t^*_{LB}(\alpha,\delta_c)$ as well. Thus, $t^*_{ELB}(\alpha,\delta_c)$ is at the same point, as $\Pi^{R1}_{ELB}(a,b)$ has a unique first order optimality point. Within similar lines, one can show that $t^*_{ELB}(\alpha,\delta_c) = t^{R1}_2(\alpha,\delta_c)$ implies $t^*_{LB}(\alpha,\delta_c) = t^{R1}_2(\alpha,\delta_c)$.

(e) With the uniqueness of the optimizer $t^*_{LB}(\alpha,\delta_c)$ and the threshold time $t^{R1}_2(\alpha,\delta_c)$, there exist two possible scenarios (1) $t^*_{LB}(\alpha,\delta_c) < t^{R1}_2(\alpha,\delta_c)$ or (2) $t^*_{LB}(\alpha,\delta_c) \geq t^{R1}_2(\alpha,\delta_c)$.

(f) If $t^*_{LB}(\alpha,\delta_c) < t^{R1}_2(\alpha,\delta_c)$, we have $t^*_{ELB}(\alpha,\delta_c) < t^{R1}_2(\alpha,\delta_c)$. First, at $t_2 = t^{R1}_2(\alpha,\delta_c)$, $\Pi^{R1}_{LB}(a,b)$ is decreasing as t^*_{LB} is the unique first order optimality point. Now, let us assume to the contrary that $t^*_{ELB}(\alpha,\delta_c) \geq t^{R1}_2(\alpha,\delta_c)$. Then at $t_2 = t^{R1}_2(\alpha,\delta_c)$, $\Pi^{R1}_{ELB}(a,b)$ should be non-decreasing as it has a unique first order optimality point. Point (C) shows that $\Pi^{R1}_{ELB}(a,b) = \Pi^{R1}_{LB}(a,b)$ at $t_2 = t^{R1}_2(\alpha,\delta_c)$. Then, in the neighbourhood of $t_2 < t^{R1}_2(\alpha,\delta_c)$, we have $\Pi^{R1}_{ELB}(a,b) < \Pi^{R1}_{LB}(a,b)$. This contradicts the statement that Π^{R1}_{ELB} is weakly above $\Pi^{R1}_{LB}(a,b)$.

(g) Within similar lines, for the second case $t^*_{LB}(\alpha,\delta_c) \geq t^{R1}_2(\alpha,\delta_c)$, we verify that that $t^*_{ELB}(\alpha,\delta_c) \geq t^{R1}_2(\alpha,\delta_c)$.

(h) We show that when $t^*_{LB}(\alpha,\delta_c) < t^{R1}_2(a,b)$, it follows that $t^{*R1}_S(\alpha,\delta_c) = t^*_{ELB}(\alpha,\delta_c)$. In fact, when $t^*_{LB}(\alpha,\delta_c) < t^{R1}_2(\alpha,\delta_c)$, then also $t^*_{ELB}(\alpha,\delta_c) < t^{R1}_2(\alpha,\delta_c)$ (see point F). Since, $\Pi_{LB}(a,b) \leq \Pi_{ELB}(a,b)$ (see point C), we thus conclude that $\Pi^{R1}_{ELB}(a,b)$ and $\Pi^{R1}_{LB}(a,b)$ over $t_2 \geq t^{R1}_2(\alpha,\delta_c)$ are lower than $\Pi^{R1}_{ELB}(a,b)$ evaluated at $t^*_{ELB}(\alpha,\delta_c)$. Next, we show that $\Pi^{R1}_S(a,b)$ evaluated at $t_2 < t^{R1}_2(\alpha,\delta_c)$ can be at most as much as $\Pi^{R1}_{ELB}(a,b)$ evaluated at $t^*_{ELB}(\alpha,\delta_c)$. Indeed, either $\Pi^{R1}_S(a,b) = \Pi^{R1}_{ELB}(a,b)$ or $\Pi^{R1}_S(a,b) = \Pi^{R1}_{LB}(a,b)$, depending on t_2, and we have $\Pi_{LB}(a,b) \leq \Pi_{ELB}(a,b)$ over the entire domain $t_2 \geq \tau(\alpha,\delta_c)$ (see point B and C). Next, we verify that $t^*_{ELB}(\alpha,\delta_c) \in \mathcal{T}^{2c}_{R1}(\alpha,\delta_c)$ (see Equation (18)) since $\lambda^{R1}_2(a,b)|_{t_2=t^*_{ELB}(\alpha,\delta_c)} < 0$, and thereby $\Pi^{R1}_S(a,b) = \Pi^{R1}_{ELB}(a,b)$ at $t^*_{ELB}(\alpha,\delta_c)$. Since, $t^*_{ELB}(\alpha,\delta_c) \in \mathcal{T}^{2c}_{R1}(\alpha,\delta_c)$ is the maximizer of $\Pi_{ELB}(a,b)$

and $\Pi_S^{R1}(a,b) \leq \Pi_{ELB}(a,b)$ over the entire domain (by the previous argument), we thus conclude that $\Pi_S^{R1}(a,b)$ evaluated at $t_2 < t_2^{R1}(\alpha,\delta_c)$ can be at most as much as $\Pi_{ELB}^{R1}(a,b)$ evaluated at $t_{ELB}^*(\alpha,\delta_c)$. Thus, $\Pi_S^{R1}(a,b)$ is maximized by $t_{ELB}^*(\alpha,\delta_c)$.

(i) Next, we show that when $t_{LB}^*(\alpha,\delta_c) \geq t_2^{R1}(\alpha,\delta_c)$, it follows that $t_S^{*R1}(\alpha,\delta_c) = t_{LB}^*(\alpha,\delta_c)$. In fact, when $t_{LB}^*(\alpha,\delta_c) \geq t_2^{R1}(\alpha,\delta_c)$, then also $t_{ELB}^*(\alpha,\delta_c) \geq t_2^{R1}(\alpha,\delta_c)$ (see point G). Thus both $\Pi_{ELB}^{R1}(a,b)$ and $\Pi_{LB}^{R1}(a,b)$ are maximized at some points $t_2, t_2' \geq t_2^{R1}(\alpha,\delta_c)$. Moreover, the derivatives of $\Pi_{LB}^{R1}(a,b)$ and $\Pi_{ELB}^{R1}(a,b)$ are non-negative at $t_2^{R1}(\alpha,\delta_c)$. Since we have $\Pi_{ELB}^{R1}(a,b) = \Pi_{LB}^{R1}(a,b)$ at $t_2 = t_2^{R1}(\alpha,\delta_c)$, we thus conclude that $\Pi_{ELB}^{R1}(a,b)$ and $\Pi_{LB}^{R1}(a,b)$ over $t_2 \leq t_2^{R1}(\alpha,\delta_c)$ are lower than or equal to $\Pi_{LB}^{R1}(a,b)$ evaluated at $t_2 = t_2^{R1}(\alpha,\delta_c)$ and thus in particular at $t_{LB}^*(\alpha,\delta_c)$. In other words, $\Pi_S^{R1}(a,b)$ evaluated at $t_2 \leq t_2^{R1}(\alpha,\delta_c)$ can be at most as much as $\Pi_{LB}^{R1}(a,b)$ evaluated at $t_{LB}^*(\alpha,\delta_c)$. Since $\Pi_S^{R1}(a,b) = \Pi_{LB}^{R1}(a,b)$ at $t_2 = t_{LB}^*(\alpha,\delta_c)$, we conclude that $\Pi_S^{R1}(a,b)$ is optimized at some point $t_2 \geq t_2^{R1}(\alpha,\delta_c)$. Since $\Pi_S^{R1}(a,b) = \Pi_{LB}^{R1}(a,b)$ over that region, we conclude that $\Pi_S^{R1}(a,b)$ is optimized at $t_2 = t_{LB}^*(\alpha,\delta_c)$.

Hence, for (1) $t_{LB}^*(\alpha,\delta_c) < t_2^{R1}(\alpha,\delta_c)$, it follows that $t_S^{*R1}(\alpha,\delta_c) = t_{ELB}^*(\alpha,\delta_c)$ whereas for (2) $t_{LB}^*(\alpha,\delta_c) \geq t_2^{R1}(\alpha,\delta_c)$, we have $t_S^{*R1}(\alpha,\delta_c) = t_{LB}^*(\alpha,\delta_c)$. Now, we discuss when, i.e., for which values of parameter α, either one of the two scenarios occur.

2. We show that there exists a unique $\alpha'(\delta_c) \in (0,1)$ such that for $\alpha = \alpha'(\delta_c)$, it follows that $t_{LB}^*(\alpha,\delta_c) = t_2^{R1}(\alpha,\delta_c)$. Further, for $\alpha > \alpha'(\delta_c)$, it follows that (1) $t_{LB}^*(\alpha,\delta_c) < t_2^{R1}(\alpha,\delta_c)$ whereas for $\alpha < \alpha'(\delta_c)$, we have (2) $t_{LB}^*(\alpha,\delta_c) > t_2^{R1}(\alpha,\delta_c)$.

For $\alpha \in (0,1)$, we prove this statement first for $\delta_c < \delta_s$ and $\delta_c \ll \delta_s$ (see point a), and next we complete our proof with the case $\delta_c < \delta_s$ and $\delta_c \not\ll \delta_s$ (see point b). We note that the case $\delta_c \geq \delta_s$ is covered in step (2-i).

(a) For $\delta_c \ll \delta_s$ and $\alpha \in (0,1)$, Lemma A.2 proves that there exists a unique $\alpha'(\delta_c) \in (\delta_c, 1)$ such that for $\alpha = \alpha'(\delta_c)$, it follows that $t_{LB}^*(\alpha,\delta_c) = t_2^{R1}(\alpha,\delta_c)$. Next, Lemma A.8 proves that for $\alpha > \alpha'(\delta_c)$, it follows $t_{LB}^*(\alpha,\delta_c) < t_2^{R1}(\alpha,\delta_c)$, while Lemma A.9 shows that for $\alpha < \alpha'(\delta_c)$, we have $t_{LB}^*(\alpha,\delta_c) \geq t_2^{R1}(\alpha,\delta_c)$.

Lemma A.2 *For $\delta_c \ll \delta_s$, there exists a unique $\alpha'(\delta_c) \in (0,1)$ such that for $\alpha = \alpha'(\delta_c)$, it follows that $t^*_{LB}(\alpha, \delta_c) = t_2^{R1}(\alpha, \delta_c)$, and this $\alpha'(\delta_c)$ satisfies $\alpha'(\delta_c) \in (\delta_c, 1)$.*

Proof of this statement is based on auxiliary results shown by Lemma A.3 and Lemma A.4, which we prove first.

Lemma A.3 *For $\delta_c \ll \delta_s$, if there is $\alpha'(\delta_c)$ such that for $\alpha = \alpha'(\delta_c)$, it follows that $t^*_{LB}(\alpha, \delta_c) = t_2^{R1}(\alpha, \delta_c)$, then we have $\alpha'(\delta_c) \in (\delta_c, 1)$.*

Proof: Assume to the contrary that $\alpha \leq \delta_c$. We then verify that $\lambda^2_{R1}(a,b) > 0$ (Equation (17)) for all $t_2 \in (\tau(\alpha, \delta_c), \infty)$, implying that $t_2^{R1}(\alpha, \delta_c) = 0$ by definition (see point 1(a)-i) This, in turn, implies that $t^*_{LB}(\alpha, \delta_c) > t_2^{R1}(\alpha, \delta_c)$ since $t^*_{LB}(\alpha, \delta_c) > 0$, by its definition (see point 1(a)-i). Since this is a contradiction, we conclude that if there exists $\alpha'(\delta_c)$ such that for $\alpha = \alpha'(\delta_c)$ implying $t^*_{LB}(\alpha, \delta_c) = t_2^{R1}(\alpha, \delta_c)$, we have $\alpha'(\delta_c) \in (\delta_c, 1)$. ∎

Lemma A.4 *For $\delta_c \ll \delta_s$, there exists unique points $t'(\alpha, \delta_c) \in (0, \infty)$, $\hat{\alpha}(\delta_c) \in (\delta_c, 1)$ such that the mappings $t_2 \mapsto \lambda^2(a,b)$ and $t_2 \mapsto \frac{\partial \Pi^{R1}_{LB}}{\partial t_2}$ coincide at $t'(\alpha, \delta_c)$, and the mappings $\alpha \mapsto \frac{\partial \Pi^{R1}_{LB}}{\partial t_2}|_{t_2 = t'(\alpha, \delta_c)}$ and $\alpha \mapsto \lambda^2(a,b)|_{t_2 = t'(\alpha, \delta_c)}$ vanish jointly at $\hat{\alpha}(\delta_c)$.*

The proof of Lemma A.4 relies on the following three auxiliary results, namely Lemma A.5, Lemma A.6 and Lemma A.7, which we prove first.

Lemma A.5 *For every $\delta_c \ll \delta_s$ and $\alpha \in (0,1)$, there exists a unique $t'(\alpha, \delta_c)$ at which the mappings $t_2 \mapsto \lambda^2(a,b)$ and $t_2 \mapsto \frac{\partial \Pi^{R1}_{LB}}{\partial t_2}$ coincide. Moreover, for every $\delta_c \ll \delta_s$, the mappings $\alpha \mapsto t'(\alpha, \delta_c)$ is continuous and strictly increasing.*

Proof: First, we show the existence of $t'(\alpha, \delta_c)$ outlined in the statement. We define $f(t_2, \alpha, \delta_c) := (\frac{\partial \Pi^{R1}_{LB}}{\partial t_2} - \lambda_2^{R1}(a,b))$. One can readily verify that $f(t_2, \alpha, \delta_c)$ is continuously differentiable and that $f(t_2, \alpha, \delta_c)|_{t_2 = 0} = \frac{(\log(\alpha) + \log(\delta_c))^2}{-4 \log(\delta_c)} > 0$ with $\frac{\partial f}{\partial t_2}|_{t_2 = 0} < 0$ and $f(t_2, \alpha)|_{t_2 \to \infty} = 0$ with $\frac{\partial f}{\partial t_2}|_{t_2 \to \infty} = 0^+$. This concludes that $f(t_2, \alpha, \delta_c)$ vanishes at least once, and hence the mappings $t_2 \mapsto \lambda^2(a,b)$ and $t_2 \mapsto \frac{\partial \Pi^{R1}_{LB}}{\partial t_2}$ coincide at least once.

Next, we prove the uniqueness of $t'(\alpha, \delta_c)$. We note that $\frac{\partial f}{\partial t_2}|_{t_2:f(t_2,\alpha)=0} < 0$, that is, the derivative of $f(t_2, \alpha, \delta_c)$ is negative whenever it vanishes, implying that it can only vanish once. Therefore, for every $\delta_c \ll \delta_s$ and $\alpha \in (0, 1)$, there exists a unique $t'(\alpha, \delta_c)$ where $f(t_2, \alpha, \delta_c)$ vanishes. Hence we arrive at the conclusion that for every $\delta_c \ll \delta_s$ and $\alpha \in (0, 1)$, there exists a unique $t'(\alpha, \delta_c)$ as outlined in the statement.

Second, we verify that for every $\delta_c \ll \delta_s$, the mappings $\alpha \mapsto t'(\alpha, \delta_c)$ is continuous and strictly increasing. In view of continuity, we have proved existence and uniqueness of $t'(\alpha, \delta_c)$. We have also verified previously that $f(t_2, \alpha, \delta_c) := (\frac{\partial \Pi^{R1}_{LB}}{\partial t_2} - \lambda^{R1}_2(a, b))$ is continuously differentiable. Therefore, it follows that $t'(\alpha, \delta_c)$, the unique point at which $f(t_2, \alpha, \delta_c)$ vanishes, is continuous in α. One can also show that $t'(\alpha, \delta_c)$ is strictly increasing in α since the implicit function $f(t_2, \alpha) := (\frac{\partial \Pi^{R1}_{LB}}{\partial t_2} - \lambda^{R1}_2(a, b))$ satisfies $\frac{\partial f/\partial \alpha}{\partial f/\partial t} > 0$. Therefore, the mappings $\alpha \mapsto t'(\alpha, \delta_c)$ is continuous and strictly increasing for every $\delta_c \ll \delta_s$. ∎

Lemma A.6 *For every $\delta_c \ll \delta_s$, the mapping $\alpha \mapsto \frac{\partial \Pi^{R1}_{LB}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)}$ is strictly decreasing.*

Proof: We prove this statement by showing for every $\delta_c \ll \delta_s$ that $\frac{\partial^2 \Pi^{R1}_{LB}}{\partial t_2^2}|_{t_2=t'(\alpha,\delta_c)} \frac{\partial t'}{\partial \alpha} + \frac{\partial^2 \Pi^{R1}_{LB}}{\partial t_2 \partial \alpha}|_{t_2=t'(\alpha,\delta_c)} < 0$ for every $\delta_c \ll \delta_s$.

To this end, we first verify that $\frac{\partial^2 \Pi^{R1}_{LB}}{\partial t_2^2}|_{t_2=t'(\alpha,\delta_c)} \frac{\partial t'}{\partial \alpha} < 0$ for every $\delta_c \ll \delta_s$. We remind the reader that $\alpha \mapsto t'(\alpha, \delta_c)$ is continuous and strictly increasing for every $\delta_c \ll \delta_s$ (See Lemma A.5). In addition, one can verify for every $\delta_c \ll \delta_s$ that $t_2 \mapsto \frac{\partial \Pi^{R1}_{LB}}{\partial t_2}$ is decreasing over the domain $t_2 \in (0, \hat{t}(\alpha, \delta_c))$, where $\hat{t}(\alpha, \delta_c)$ is the unique point at which $\frac{\partial^2 \Pi^{R1}_{LB}}{\partial t_2^2}|_{t_2=t'(\alpha,\delta_c)}$ vanishes. We note that $\hat{t}(\alpha, \delta_c)$ is sufficiently larger than $t'(\alpha, \delta_c)$ as $\frac{\partial^2 \Pi^{R1}_{LB}}{\partial t_2^2}|_{t_2=t'(\alpha,\delta_c)} < 0$. Therefore, we conclude that $\frac{\partial^2 \Pi^{R1}_{LB}}{\partial t_2^2}|_{t_2=t'(\alpha,\delta_c)} \frac{\partial t'}{\partial \alpha} < 0$ for every $\delta_c \ll \delta_s$. Further, we verify that $-\frac{\partial^2 \Pi^{R1}_{LB}}{\partial t_2^2}|_{t_2=t'(\alpha,\delta_c)} \frac{\partial t'}{\partial \alpha} > \frac{\partial t^2 \Pi^{R1}_{LB}}{\partial t_2 \partial \alpha}|_{t_2=t'(\alpha,\delta_c)}$ since $-\frac{\partial \Pi^{R1}_{LB}}{\partial t_2} > \frac{\partial \Pi^{R1}_{LB}}{\partial \alpha}$ for every $\delta_c \ll \delta_s$. Hence, we conclude that $\frac{\partial^2 \Pi^{R1}_{LB}}{\partial t_2^2}|_{t_2=t'(\alpha,\delta_c)} \frac{\partial t'}{\partial \alpha} + \frac{\partial^2 \Pi^{R1}_{LB}}{\partial t_2 \partial \alpha}|_{t_2=t'(\alpha,\delta_c)} < 0$ for every $\delta_c \ll \delta_s$. Thus, we prove that the mapping $\alpha \mapsto \frac{\partial \Pi^{R1}_{LB}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)}$ is strictly decreasing. ∎.

Lemma A.7 *For every $\delta_c \ll \delta_s$, we have $\frac{\partial \Pi^{R1}_{LB}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)} > 0$ when $\alpha \downarrow \delta_c$ and*

$\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)} < 0$ *when* $\alpha \uparrow 1$

Proof: For every $\delta_c \ll \delta_s$, we have $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)} = (1-\delta_c^{t'(\alpha,\delta_c)})(-\delta_c^{t'(\alpha,\delta_c)}+\delta_s^{t'(\alpha,\delta_c)}(\delta_s^{t'(\alpha,\delta_c)}+ 2\delta_c^{2t'(\alpha,\delta_c)} - 3\delta_c^{t'(\alpha,\delta_c)} + 1))$ when $\alpha \downarrow \delta_c$. We verify that $= (1 - \delta_c^{t'(\alpha,\delta_c)})(-\delta_c^{t'(\alpha,\delta_c)} + \delta_s^{t'(\alpha,\delta_c)}(\delta_s^{t'(\alpha,\delta_c)} + 2\delta_c^{2t'(\alpha,\delta_c)} - 3\delta_c^{t'(\alpha,\delta_c)} + 1)) > 0$ when $\delta_c \ll \delta_s$. Within similar lines, we have $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)} = (\delta_c^{t'(\alpha,\delta_c)} - 1)(\delta_s^{t'(\alpha,\delta_c)} + \delta_c^{t'(\alpha,\delta_c)}) < 0$ when $\alpha \uparrow 1$ since $(\delta_c^{t'(\alpha,\delta_c)} - 1)(\delta_s^{t'(\alpha,\delta_c)} + \delta_c^{t'(\alpha,\delta_c)}) < 0$ over the domain $\delta_c, \delta_s, \alpha \in (0,1)$ and $t_2 \in (0,\infty)$. ∎

Next, we prove Lemma A.4 based on the axillary results stated above.

Proof of Lemma A.4. We have shown that the mapping $\alpha \mapsto \frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)}$ is strictly decreasing for every $\delta_c \ll \delta_s$ by Lemma A.6. We have also shown that $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)} > 0$ when $\alpha \downarrow \delta_c$ and $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)} < 0$ when $\alpha \uparrow 1$ by Lemma A.7. This implies that there exists a unique point $\hat{\alpha}(\delta_c)$ over the domain such $(\delta_c, 1)$ that for $\alpha = \hat{\alpha}(\delta_c)$, the mapping $\alpha \mapsto \frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)}$ vanishes. ∎

Proof of Lemma A.2. Proof follows directly from auxiliary result stated in Lemma A.3 and Lemma A.4. Lemma A.4 shows that there exists a unique $\hat{\alpha}(\delta_c)$ at which the mappings $\alpha \mapsto \frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)}$ and $\alpha \mapsto \lambda^2(a,b)|_{t_2=t'(\alpha,\delta_c)}$ vanish simultaneously. We remind the reader that $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t'(\alpha,\delta_c)}$ equals $\lambda^2(a,b)|_{t_2=t'(\alpha,\delta_c)}$ by definition of $t'(\alpha,\delta_c)$ (see Lemma A.4). We remind the reader that $t_{LB}^*(\hat{\alpha}(\delta_c),\delta_c)$ is the unique point at which the mapping $t_2 \mapsto \frac{\partial \Pi_{LB}^{R1}}{\partial t_2}$ (see point 1(a)-i-B). This implies that at the unique point $\hat{\alpha}(\delta_c)$, it follows that $t_{LB}^*(\hat{\alpha}(\delta_c),\delta_c) = t_2^{R1}(\hat{\alpha}(\delta_c),\delta_c)$ (see point 1(a)-i for the definition of $t_2^{R1}(\hat{\alpha}(\delta_c),\delta_c)$). Thus, we conclude that $\alpha'(\delta_c)$ is unique over the domain $(\delta_c, 1)$. Lemma A.4 further clarifies that indeed the unique point $\alpha'(\delta_c) \in (\delta_c, 1)$. Thus, we conclude that there exists a unique point $\alpha'(\delta_c) \in (\delta_c, 1)$ as outlined in the main statement (ii). ∎

Lemma A.8 *When* $\alpha > \alpha'(\delta_c)$*, where* $\alpha(\delta_c) \in (\delta_c, 1)$ *is defined on Lemma A.2, we have* $t_{LB}^*(\alpha,\delta_c) < t_2^{R1}(\alpha,\delta_c)$.

Proof: We prove this statement by showing that $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2=t_2^{R1}(\alpha,\delta_c)} < 0$ for $\alpha > \alpha'(\delta_c) \in (\delta_c, 1)$. To this end, it is important to note that $\frac{\partial f}{\partial \alpha} < 0$ for fixed $t_2 \in (0,\infty)$, $\delta_c \ll \delta_s$,

where $f(t_2, \alpha, \delta_c) := (\frac{\partial \Pi_{LB}^{R1}}{\partial t_2} - \lambda_2^{R1}(a,b))$ is verified to be continuously differentiable. In addition, we remind the reader that $t_{LB}^*(\alpha, \delta_c)$ is the unique point at which $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}$ vanishes (see point 1(a)-B). Next, we fix t_2 as $t_2 = t_2^{R1}(\alpha, \delta_c)$. Then, for $\alpha = \alpha'(\delta_c)$, it follows that $f(t_2^{R1}(\alpha, \delta_c), \alpha, \delta_c) = 0$ by Lemma A.2 and by definition of $f(t_2, \alpha, \delta_c)$. This, in turn, implies that $f(t_2^{R1}(\alpha, \delta_c), \alpha, \delta_c) < 0$ for $\alpha > \alpha'(\delta_c)$ since $\frac{\partial f}{\partial \alpha} < 0$ as noted earlier. Therefore, for $\alpha > \alpha'(\delta_c)$, we have $f(t_2^{R1}(\alpha, \delta_c), \alpha, \delta_c) = \frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2 = t_2^{R1}(\alpha, \delta_c)} - \lambda_2^{R1}(a,b))|_{t_2 = t_2^{R1}(\alpha, \delta_c)} < 0$. We have $\lambda_2^{R1}(a,b))|_{t_2 = t_2^{R1}(\alpha, \delta_c)} = 0$ by definition of $t_2^{R1}(\alpha, \delta_c)$ (see point (see point 1(a)-i). Hence, we have $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2 = t_2^{R1}(\alpha, \delta_c)} < 0$ for $\alpha > \alpha'(\delta_c) \in (\delta_c, 1)$. This implies that $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}$ vanishes before the point $t_2^{R1}(\alpha, \delta_c)$. This, in turn, implies that $t_{LB}^*(\alpha, \delta_c) < t_2^{R1}(\alpha, \delta_c)$ by definition (see point 1(a)-i) and uniqueness of $t_{LB}^*(\alpha, \delta_c)$ (see point 1(a)-B). Therefore, we arrive at the conclusion that $t_{LB}^*(\alpha, \delta_c) < t_2^{R1}(\alpha, \delta_c)$ when $\alpha > \alpha'(\delta_c) \in (\delta_c, 1)$. ∎

Lemma A.9 *When $\alpha < \alpha'(\delta_c) \in (\delta_c, 1)$, where $\alpha(\delta_c) \in (\delta_c, 1)$ is defined on Lemma A.2, we have $t_{LB}^*(\alpha, \delta_c) > t_2^{R1}(\alpha, \delta_c)$.*

Proof: Within similar lines with Lemma A.8, we verify that for $\alpha < \alpha'(\delta_c) \in (\delta_c, 1)$, it follows that $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}|_{t_2 = t_2^{R1}(\alpha, \delta_c)} > 0$. This implies that $\frac{\partial \Pi_{LB}^{R1}}{\partial t_2}$ vanishes after the point $t_2^{R1}(\alpha, \delta_c)$, which in turn, implies that $t_{LB}^*(\alpha, \delta_c) > t_2^{R1}(\alpha, \delta_c)$ by definition (see point 1(a)-i) and uniqueness of $t_{LB}^*(\alpha, \delta_c)$ (see point 1(a)-B). Therefore, we arrive at the conclusion that $t_{LB}^*(\alpha, \delta_c) > t_2^{R1}(\alpha, \delta_c)$ when $\alpha < \alpha'(\delta_c) \in (\delta_c, 1)$. ∎

Thus, we arrive at the conclusion that for $\alpha > \alpha'(\delta_c) \in (\delta_c, 1)$, it follows that (1) $t_{LB}^*(\alpha, \delta_c) < t_2^{R1}(\alpha, \delta_c)$ whereas for $\alpha \leq \alpha'(\delta_c)$, we have (2) $t_{LB}^*(\alpha, \delta_c) \geq t_2^{R1}(\alpha, \delta_c)$.

(b) A similar argument follows for $\delta_c < \delta_s$ and $\delta_c \not\ll \delta_s$ except in this case (1) we have two points at which $f(t_2, \alpha) := (\frac{\partial \Pi_{LB}^{R1}}{\partial t_2} - \lambda_2^{R1}(a,b))$ vanishes and we define $t'(\alpha, \delta_c) := \max\left\{t_2 \in (\tau(\alpha, \delta_c), \infty) : \lambda_2^{R1}(a,b) = \frac{\partial \Pi_{LB}^{R1}}{\partial t_2}\right\}$, and (2) $\alpha'(\delta_c) \in (0,1)$ but not necessarily in $(\delta_c, 1)$.

Therefore, for $\alpha > \alpha'(\delta_c)$, it follows that (1) $t_{LB}^*(\alpha, \delta_c) < t_2^{R1}(\alpha, \delta_c)$ whereas for $\alpha \leq \alpha'(\delta_c)$, we have (2) $t_{LB}^*(\alpha, \delta_c) \geq t_2^{R1}(\alpha, \delta_c)$. Thus, we arrive at the conclusion that for (1) $\alpha >$

$\alpha'(\delta_c)$, it follows that $t_S^{R1*}(\alpha,\delta_c) = t_{ELB}^{*}(\alpha,\delta_c)$ while for (2) $\alpha \leq \alpha'(\delta_c)$, it follows that $t_S^{R1*}(\alpha,\delta_c) = t_{LB}^{*}(\alpha,\delta_c)$.

3. It follows that $\Pi_S^{*R1}(\alpha,\delta_c) \geq \frac{u(\delta_c)}{4}$. To this end, we first verify that both $\lim_{t_2\to\infty}\Pi_{LB}^{R1}(a,b) = \frac{u(\delta_c)}{4}$ and $\lim_{t_2\to\infty}\Pi_{ELB}^{R1}(a,b) = \frac{u(\delta_c)}{4}$ as well as $\lim_{t_2\to\infty}\frac{\partial\Pi_{LB}^{R1}}{\partial t_2} = 0^-$ and $\lim_{t_2\to\infty}\frac{\partial\Pi_{ELB}^{R1}}{\partial t_2} = 0^-$ for $\delta_s > \delta_c$.

   This implies that $\left\{\Pi_{LB}^{R1}(a,b) : t_2 > \tau(\alpha,\delta_c)\right\} \geq \frac{u(\delta_c)}{4}$ and $\left\{\Pi_{ELB}^{R1}(a,b) : t_2 > \tau(\alpha,\delta_c)\right\} \geq \frac{u(\delta_c)}{4}$. Thus, we have $\Pi_S^{*R1}(\alpha,\delta_c) \geq \frac{u(\delta_c)}{4}$.

Thus, we prove that Equation (A.41) holds for $\delta_c < \delta_s$.

Next, we show that Equation (A.40) holds. To this end, we cite Equation (A.38) which shows that $\Pi_S^{R2}(a,b) = \Pi_L^{R2}(a,b)$ if $t_2 \in \mathcal{T}_{R2}^{2}(\alpha,\delta_c)$ and $\Pi_S^{R2}(a,b) = \Pi_{EL}^{R2}(a,b)$ if $t_2 \in \mathcal{T}_{R2}^{2c}(\alpha,\delta_c)$.

In view of $\Pi_L^{R2}(a,b)$, we show that $\max\left\{\Pi_L^{R2}(a,b) : t_2 \in \mathcal{T}_{R2}^{2}(\alpha,\delta_c)\right\} \leq \Pi_S^{*R1}(\alpha,\delta_c)$. We verify that $\Pi_L^{R2}(a,b) \leq \frac{u(\delta_c)}{4}$ for any $t_2 \in (0,\infty)$. We remind the reader that $\Pi_S^{*R1}(\alpha,\delta_c) \geq \frac{u(\delta_c)}{4}$ (please see part 2-b). Thus, we conclude that $\Pi_S^{*R1}(\alpha,\delta_c) \geq \Pi_L^{R2}(a,b)$, and hence $\Pi_S^{*R1}(a,b) \neq \Pi_L^{R2}(a,b)$. Next, we investigate $\Pi_{EL}^{R2}(a,b)$ as in the following. First, we confirm that for $\alpha \leq \alpha'(\delta_c)$, it follows that $\mathcal{T}_{R2}^{2c}(\alpha,\delta_c) = \emptyset$. Besides, for $\alpha \to 1$, it follows that $\max\left\{\Pi_{EL}^{R2}(a,b) : t_2 \in \mathcal{T}_{R2}^{2c}(\alpha,\delta_c)\right\} > \max\left\{\Pi_{ELB}^{R1}(a,b) : t_2 > \tau(\alpha,\delta_c)\right\} \approx \frac{u(\delta_c)}{4}$. Further, we show that

$$\Pi_{EL}^{*}(\alpha,\delta_c) := \max\left\{\Pi_{EL}^{R2}(a,b) : t_2 \in \mathcal{T}_{R2}^{2c}(\alpha,\delta_c)\right\} \leq \Pi_{ELB}^{*}(\alpha,\delta_c) := \max\left\{\Pi_{ELB}^{R1}(a,b) : t_2 \in \mathcal{T}_{R1}^{2c}(\alpha,\delta_c)\right\}$$

for $\alpha \leq \delta_s$ as in the following. First, we verify that $\Pi_{ELB}^{R1}(a,b) > \Pi_{EL}^{R2}(a,b)$ for $t_2 \in \mathcal{T}_{R2}^{2}(\alpha\delta_c)$. This implies that $\max\left\{\Pi_{EL}^{R2}(a,b) : t_2 \in \mathcal{T}_{R2}^{2c}(\alpha,\delta_c)\right\} \leq \max\left\{\Pi_{ELB}^{R2}(a,b) : t_2 \in \mathcal{T}_{R2}^{2c}(\alpha,\delta_c)\right\}$. Secondly, it follows that $\max\left\{\Pi_{ELB}^{R2}(a,b) : t_2 \in \mathcal{T}_{R2}^{2c}(\alpha,\delta_c)\right\} = \Pi_{ELB}^{*}(\alpha,\delta_c)$ since $\alpha'(\delta_c) \in (0,\delta_s)$. Finally, we confirm that $\frac{\partial\Pi_{EL}^{*R2}(\alpha,\delta_c)}{\partial\alpha} > 0$ while $\frac{\partial\Pi_{ELB}^{*R1}(\alpha,\delta_c)}{\partial\alpha} < 0$. These, in turn, imply that exists a unique $\alpha''(\delta_c) \in (\alpha'(\delta_c),1)$ such that $\Pi_{EL}^{*}(\alpha,\delta_c) := \max\left\{\Pi_{EL}^{R2}(a,b) : t_2 \in \mathcal{T}_{R2}^{2c}(\alpha,\delta_c)\right\} = \Pi_{ELB}^{*}(\alpha,\delta_c) := \max\left\{\Pi_{ELB}^{R1}(a,b) : t_2 \in \mathcal{T}_{R1}^{2c}(\alpha,\delta_c)\right\}$.

Thus, we prove that Equation (A.40) holds, and this concludes our proof. ∎

Proof of Lemma 2.17: For $\alpha,\delta_c \in (0,1)$, $p_1,p_2 \in (0,u)$, and $t_2 \in \mathbb{R}^+$, there are discount buyers

if and only if there exist some θ such that $\theta < \theta_{DL}$, $\theta < \theta_{DE}$, $\theta < \theta_{DB}$ and $\theta \geq \theta_D$. This, in turn, holds if and only if $\min\{\theta_{DL}, \theta_{DE}, \theta_{DB}\} > \theta_D$.

First, we provide the conditions for β in order for inequalities $\theta_{DL} > \theta_D$, $\theta_{DE} > \theta_D$, and $\theta_{DB} > \theta_D$ hold, respectively. We show that $\theta_{DL} = \left[\frac{p_2-\beta p_1}{u(1-\alpha^{t_2})}\right]_0^1 > \theta_D = \left[\frac{\beta p_1}{u\alpha^{t_2}}\right]_0^1$ if and only if $\beta < \frac{\alpha^{t_2} p_2}{p_1}$. For the first direction of the iff statement, assume to the contrary that $\beta \geq \frac{\alpha^{t_2} p_2}{p_1}$. Then, it follows that $\left[\frac{p_2-\beta p_1}{u(1-\alpha^{t_2})}\right]_0^1 \leq \frac{p_2-\alpha^{t_2} p_2}{u(1-\alpha^{t_2})}$ and $\left[\frac{\beta p_1}{u\alpha^{t_2}}\right]_0^1 \geq \frac{\alpha^{t_2} p_2}{u\alpha^{t_2}}$. This implies that $\left[\frac{p_2-\beta p_1}{u(1-\alpha^{t_2})}\right]_0^1 \leq \left[\frac{\beta p_1}{u\alpha^{t_2}}\right]_0^1$, and hence we arrive at a contradiction. Now, assume to the contrary of the iff statement (second direction) that $\theta_{DL} = \left[\frac{p_2-\beta p_1}{u(1-\alpha^{t_2})}\right]_0^1 \leq \theta_D = \left[\frac{\beta p_1}{u\alpha^{t_2}}\right]_0^1$, which in turn implies that $\beta \geq \frac{\alpha^{t_2} p_2}{p_1}$. Therefore, it follows that $\beta < \frac{\alpha^{t_2} p_2}{p_1}$, a contradiction. Within similar lines, one can show that $\theta_{DE} = \left[\frac{p_1(1-\beta\delta_c^{t_2})}{u(1-\delta_c^{t_2})}\right]_0^1 > \theta_D = \left[\frac{\beta p_1}{u\alpha^{t_2}}\right]_0^1$ if and only if $\beta < \frac{\alpha^{t_2}}{1-\delta_c^{t_2}(1-\alpha^{t_2})}$. In addition, we verify that $\theta_{DB} = \left[\frac{p_2+(1-\beta\delta_c^{t_2})p_1}{u(1-\alpha^{t_2}\delta_c^{t_2})}\right]_0^1 > \theta_D = \left[\frac{\beta p_1}{u\alpha^{t_2}}\right]_0^1$ for any $\beta \in [0,1]$ since $\theta_{DB}|_{\beta=1} = \left[\frac{p_2+(1-\delta_c^{t_2})p_1}{u(1-\alpha^{t_2}\delta_c^{t_2})}\right]_0^1 > \theta_D|_{\beta=1} = \left[\frac{p_1}{u\alpha^{t_2}}\right]_0^1$.

Secondly, we show that $\min\{\theta_{DL}, \theta_{DE}, \theta_{DB}\} = \theta_{DE}$ if and only if $p_1 \leq \frac{p_2(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$ and $\min\{\theta_{DL}, \theta_{DE}, \theta_{DB}\} = \theta_{DL}$ if and only if $p_1 \geq \frac{p_2(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$. First, we verify that (1) $\theta_{DB} > \theta_{DL}$ for $p_1 \geq \frac{p_2(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$ since $\theta_{DB} - \theta_{DL} = \frac{p_1\left(1-\alpha^{t_2}+\beta(1-\delta^{t_2})\right)+p_2\alpha^{t_2}\left(\delta^{t_2}-1\right)}{u(1-\alpha^{t_2})(1-\alpha^{t_2}\delta^{t_2})} > 0$ and (2) $\theta_{DB} > \theta_{DE}$ for $p_1 \leq \frac{p_2(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$ since $\theta_{DB} - \theta_{DE} = \frac{p_2(1-\delta^{t_2})-p_1\left(1-\alpha^{t_2}\right)\delta^{t_2}\left(1-\beta\delta^{t_2}\right)}{u(1-\delta^{t_2})(1-\alpha^{t_2}\delta^{t_2})} > 0$ for $\alpha, \delta_c \in (0,1)$, $p_1, p_2 \in (0,u)$, and $t_2 \in \mathbb{R}^+$. Likewise, we show that $\theta_{DE} \geq \theta_{DL}$ if and only if $p_1 \leq \frac{p_2(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$. Therefore, we conclude that $\min\{\theta_{DL}, \theta_{DE}, \theta_{DB}\} = \theta_{DE}$ if and only if $p_1 \leq \frac{p_2(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$ and $\min\{\theta_{DL}, \theta_{DE}, \theta_{DB}\} = \theta_{DL}$ if and only if $p_1 > \frac{p_2(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$.

Finally, we note that $\frac{p_2(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$ equals to $p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$ when $\beta = \frac{\alpha^{t_2}}{1-\delta_c^{t_2}(1-\alpha^{t_2})}$. For $\beta < \frac{\alpha^{t_2}}{1-\delta_c^{t_2}(1-\alpha^{t_2})}$ and a regular ordering where $(1-\delta_c^{t_2}(2-\alpha^{t_2})) > 0$, we have $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$ implying the condition $p_1 > \frac{p_2(1-\delta_c^{t_2})}{1-\alpha^{t_2}+\beta(1-\delta_c^{t_2}(2-\alpha^{t_2}))}$. This implies that for $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$, a regular preference order has late buyers and thereby the late and discount buyers as adjacent segments. Hence, for $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$ in a regular preference ordering discount buyers segment exists, that is $\theta_{DL} > \theta_D$, if and only if $\beta < \frac{\alpha^{t_2} p_2}{p_1}$. Otherwise, when $p_1 \leq p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$, the early buyers and the discount buyers are adjacent in a regular ordering. In that case, discount buyers segment exists, that is $\theta_{DE} > \theta_D$, if and only if $\beta < \frac{\alpha^{t_2}}{1-\delta_c^{t_2}(1-\alpha^{t_2})}$. Along similar lines, we verify that the discount buyers exists for an irregular

case if $\beta < \frac{\alpha^{t_2} p_2}{p_1}$ for $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$, and if $\beta < \frac{\alpha^{t_2}}{1-\delta_c^{t_2}(1-\alpha^{t_2})}$ for $p_1 \leq p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$.

Thus, we arrive at the conclusion that there exist discount buyers (1) for $p_1 > p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$ if and only if $\beta < \frac{\alpha^{t_2} p_2}{p_1}$, and (2) for $p_1 \leq p_2(1-\delta_c^{t_2}(1-\alpha^{t_2}))$ if and only if $\frac{\alpha^{t_2} p_2}{p_1}$. ∎

Proof of Proposition 2.18: Follows along similar lines to Proposition 2.13.

Proof of Lemma 2.19: Follows along the similar lines with the Lemma 2.15.

Proof of Lemma 2.20: When $t_2 > \tau(\alpha, \delta_c)$, discount buyers and the late buyers are competing segments as implied by the Lemma 2.19. This further indicates that $\Pi_D^R(\beta|p_1, p_2, t_2) = \Pi_S^R(p_1, p_2, t_2) + \delta_s^{t_2}\beta p_1(\theta_L - \theta_D)^+ - \delta_s^{t_2}(p_2 - \beta p_1)(\theta_{DL} - \theta_L)^+$, and therefore $\Pi_D^R(\beta|p_1, p_2, t_2) - \Pi_S^R(p_1, p_2, t_2) = -\frac{\delta_s^{t_2}[(\alpha^{t_2} p_2 - \beta p_1)^+]^2}{u(1-\alpha^{t_2})\alpha^{t_2}}$. One can readily observe that $\Pi_D^R(\beta|p_1, p_2, t_2) - \Pi_S^R(p_1, p_2, t_2)$ is non-positive and thus, the firm sets it to zero to minimise its loss. This implies that, $\beta^* = \min\left\{\frac{\alpha^{t_2} p_2}{p_1}, 1\right\}$. Thus, $\Pi_D^R(\beta^*|p_1, p_2, t_2) = \Pi_S^R(p_1, p_2, t_2)$. ∎

Proof of Lemma 2.21: Follows within similar lines to the Lemma 2.20.

Proof of Proposition 2.22: Follows analogous steps to those of Proposition 2.5 and Proposition 2.16, and therefore omitted.

Mixed Case, where $\gamma \in (0, 1)$ portion of the market consists of strategic consumers:

Proposition A.10 *For $\delta_s, \delta_c, \lambda \in (0, 1)$, there exist $\alpha^1(\lambda)$, $\alpha^2(\lambda)$, $\alpha^3(\lambda)$, $\alpha^4(\lambda) \in (0, 1)$, such that:*

1. *For $\delta_c \in (0, \delta_s)$,*
   - *(a) If $\alpha \in (0, \alpha^2(\lambda)]$, it is optimal to focus on the "myopic consumers" and*
     - *i. if $\alpha \in (0, \alpha^1(\lambda))$, to serve the myopic, late, and both product buyers (and thereby strategic late and both product buyers) (Region LB_M),*
     - *ii. if $\alpha \in [\alpha^1(\lambda), \alpha^2(\lambda)]$, to serve the myopic early, late, and both product buyers (and thereby the strategic late and both product buyers) (Region ELB_M);*

(b) *If* $\alpha \in (\alpha^2(\lambda), \alpha^3(\lambda))$, *it is optimal to focus on the "strategic consumers", and serve the strategic late and both product buyers (and thereby the myopic early, late, and both product buyers)(Region* LB_S*);*

(c) *If* $\alpha \in [\alpha^3(\lambda), 1)$, *it is optimal to focus on both the " myopic and strategic consumers", and*

i. *if* $\alpha \in (\alpha^3(\lambda), \alpha^4(\lambda)]$, *the firm serves the early, late, and both product buyers (Region* ELB_T*).*

ii. *If* $\alpha \in [\alpha^4(\lambda), 1)$, *the firm serves the early and late buyers (Region* EL_T*) with the optimal release time* $t_2^* > \tau(\alpha, \delta_c)$.

2. *For* $\delta_c \in [\delta_s, 1]$, *if* $\alpha \leq \alpha^p$, *it is optimal for the company to target the myopic consumers with the strategy outlined in (1-a) with* $t_2^* > \tau(\alpha, \delta_c)$, *and for otherwise to target the strategic consumers by delaying the release time for the new version infinitely* ($t_2 \to \infty$), *and thus having only the early buyers from both segments.*

Here, $\alpha^1(\lambda) < \alpha^2(\lambda) < \alpha^3(\lambda) < \alpha^4(\lambda)$, *for each value of* $\lambda \in (0, 1)$.

Proof: The proof follows analogous steps to those of Proposition 2.5 and Proposition 2.16, and therefore omitted.

A.2 Extensions

A.2.1 Marginal Costs and Fixed Release Times

In this section, we provide examples in support of Section 2.6.1 that discusses the case where the two critical aspects of our main model are not valid anymore: endogenous release time and pure information goods.

A.2.1.1 Fixed Release Times

One crucial dimension of the rollover strategy that we have discussed so far is release timing, and we observe many industry examples where the firms strategically utilize the time lever in the design of the rollovers. However, one may also find cases where the firm can not change the release time due to earlier commitments or pre-announcements, industry clock speed of the complementary goods, disruptions in the R&D process or the market's expectation due to previous release practices of the firm. To account for this situation, we assume that the release time is exogenous to the firm in this section.

First, we remind the reader the case where the release time is fixed as "long", that is $t_2 > \tau(\alpha, \delta_c)$. Intuitively, this is the situation where the perceived utility $\theta u(1 - \delta_c^{t_2}(1 - \alpha^{t_2}))$ that a strategic consumer derives from the early version is always superior over the net present value of the utility $\delta_c^{t_2}\theta u$ she can derive by waiting for the new version. Therefore, if carried to the second period with a discount, the early version can posit a significant threat to the new version as the consumers highly value it. Instead, the firm would prefer selling the early version in the first period. Since the release time is sufficiently long, the firm can always convince the early buyers to upgrade to the new version with its pricing regime. Therefore, intuitively one would expect a solo rollover strategy to dominate the dual rollover, provided that the release time is fixed as $t_2 > \tau(\alpha, \delta_c)$. In fact, Lemma 2.20 formally shows that dual rollover is inferior when the release time is fixed as long.

If instead, the release time is short, i.e., $t_2 < \tau(\alpha, \delta_c)$, then the net value of the late version is superior over the early release. Therefore, hence carrying the initial version to the second period may not be a bad idea as only the low-value consumers would be attracted to the price discount on early version while the high-value types would be interested in the late version. Lemma 2.20 formally suggests that this is indeed true, that is the firm would prefer dual rollover over the solo rollover if the release time is, for some reason, is fixed as short:

Lemma A.11 *If the firm executes an irregular preference ordering $t_2 < \tau(\alpha, \delta_c)$ with the prices outlined in Lemma 2.19, then the optimal rollover strategy is dual rollover.*

Proof: Lemma 2.19 implies that the early buyers always exist in an irregular preference ordering. Further, Lemma 2.21 suggests that the firm prefers the dual rollover if the early buyers exist in an irregular ordering. Thus, we conclude that the optimal rollover strategy is dual rollover provided that there exists an irregular preference ordering, that is $t_2 < \tau(\alpha, \delta_c)$. ∎

A.2.1.2 Non-Zero Marginal Cost

In this section, we consider semi-digital goods such as smartphones or tablets that have non-zero marginal costs of reproduction due to the hardware components. We denote the marginal costs of the two versions as c_1 and kc_1, respectively for the first and the second version. We note that if $k > 1$ then it's costlier to produce the second version while when $k < 1$ marginal costs are decreasing due to the reasons such as economies of scope.

Reformulating the problem with non-zero costs, we obtain the profit function as:

$$\begin{aligned}\Pi_M(p_1, p_2, t_2, \beta) &= (p_1 - c_1)(\theta_B - \theta_E)^+ + \delta_s^{t_2}(p_2 - kc_1)(\theta_E - \max\{\theta_L, \theta_{DL}\})^+ \\ &+ (p_1 + \delta_s^{t_2} p_2 - c_1 - kc_1)(1 - \max\{\theta_E, \theta_B\}) \\ &+ \delta_s^{t_2}(\beta p_1 - c_1)(\min\{\theta_E, \theta_{DL}\} - \theta_D)^+. \end{aligned} \tag{A.43}$$

We start the discussion with the myopic consumers. We remind the reader that when the firms have zero marginal costs, Lemma 2.10 implies that there always exists late buyers in the market, and hence, by Proposition 2.11, the firm never carries the early version to the second period. We investigate whether or not the Lemma 2.10 still holds when the costs of reproduction are non-zero. To this end, we assume that the first period prices would satisfy $p_2^* > p_1^*(1 - \alpha^{t_2} + \beta)$.

Then, the profit function simplifies to

$$\begin{aligned}\Pi_{EBD}(p_1,p_2,\beta) &= (p_1-c_1)(\theta_B-\theta_E)+(p_1-c_1+\delta_s^{t_2}(p_2-kc_1))(1-\theta_B)+\delta_s^{t_2}(\beta p_1-c_1)(\theta_E-\theta_D)^+\\ &= (p_1-c_1)(\frac{p_2}{u(1-\alpha^{t_2})}-\frac{p_1}{u})+(p_1-c_1+\delta_s^{t_2}(p_2-kc_1))(1-\frac{p_2}{u(1-\alpha^{t_2})})\\ &+ \delta_s^{t_2}(\beta p_1-c_1)(\frac{p_1}{u}-\frac{\beta p_1}{u\alpha^{t_2}})^+. \end{aligned} \tag{A.44}$$

The optimal prices that solve the firm's profit maximization problem are

$$p_1^*=\frac{2(c_1+u)-c_1\delta_s^{t_2}}{4-\alpha^{t_2}\delta_s^{t_2}} \quad , \quad p_2^*=\frac{1}{2}\left(c_1k-u\alpha^{t_2}+u\right) \quad \text{and} \quad \beta^*=\frac{(2+\alpha^{t_2}(1-\delta_s^{t_2}))+\alpha^{t_2}u}{2u+c_1(2-\delta_s^{t_2})}, \tag{A.45}$$

which, in turn,implies that $p_2^*>p_1^*(1-\alpha^{t_2}+\beta)$ holds if and only if $k\le\bar{k}=\frac{\alpha^{t_2}u((\alpha^{t_2}-1)\delta_s^{t_2}-4)+2(\delta_s^{t_2}-2)c_1}{c_1(\alpha^{t_2}\delta_s^{t_2}-4)}$. We also note that iff $c_1\le\frac{2u}{2+\delta_s^{t_2}(1-\alpha^{t_2})}$ and $k\le\frac{u(1-\alpha^{t_2})}{c_1}$ firms generate positive margins, that is $p_1^*>c_1$ and $p_2^*>kc_1$. Thus, we conclude that the firm execute the pricing regime $p_2^*>p_1^*(1-\alpha^{t_2}+\beta)$ only if $k\le\min\{\bar{k},\frac{u(1-\alpha^{t_2})}{c_1}\}$ and $c_1\le\frac{2u}{2+\delta_s^{t_2}(1-\alpha^{t_2})}$. If instead $k>\bar{k}$, the condition $p_2^*>p_1^*(1-\alpha^{t_2}+\beta)$ does not hold and we conclude that the market never segments early buyers. Therefore, by Proposition 2.11, firm never deploys a dual rollover. Therefore, we conclude that even if the goods are semi-digital firm may prefer solo rollover unless the second version is not much costlier than the early version. Figure A.1 illustrates the discussion with a numerical example.

In analogy to the discussion where the market consists solely of the myopic consumers, one can also verify by Lemma 2.19 and Lemma 2.20, that the firm prefers solo rollover over the dual rollover if k is sufficiently low.

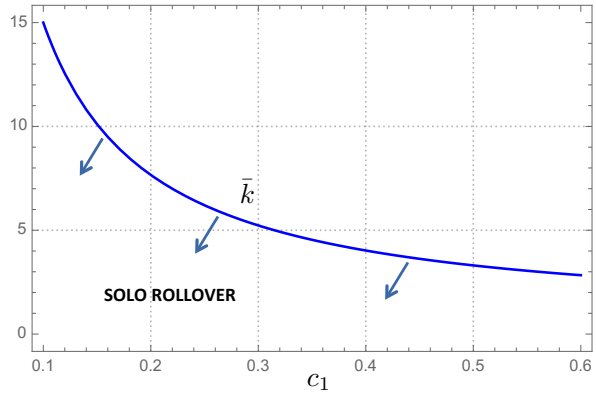


Figure A.1: Non-Zero Marginal Costs for $v = 1, \delta_s = 0.37, \delta_s = 0.79$

A.2.2 Welfare Analysis

In this section, we discuss the welfare implications of the rollover strategy, and start with the inattentive consumers. The welfare they experience in the first period is different than what they have been contemplating at time $t = 0$. Formally, they would think that their net utility is $(\theta u - p_1)$ when they obtain the primary version in the first period, while in reality, their net utility would be $(\theta u(1 - \delta^{t_2}(1 - \alpha^{t_2}) - p_1))$ if they do not upgrade and $(\theta u - p_1 - \delta_c^{t_2} p_2)$ if they upgrade in the second period.

As a result of myopia, the firm obtains an advantage by locking the inattentive consumers into the product category with the prices they would never accept if they had taken the future into account. The firm's welfare increases further with the perceived obsolescence rate $(1-\alpha)$, that is the discrepancy between the expectation and actual perception of the market. The inattentive consumers, on the other hand, is worse off. In addition to the inattentiveness, consumer patience plays a crucial role in the negative consumer welfare *ex-post* the perception of obsolescence. When the consumers are significantly patient, their emphasis on the discrepancy between the contemplated and actualized experience would be higher. Therefore, they experience negative utility even if the market is not eager to the obsolescence.

Conflicting interests of the consumers and the firm, in turn, would lead to an intermediate

obsolescence level ($\alpha_w^* \in (0,1)$) at which the total surplus is maximized. When the consumers are patient, consumer-driven "no-obsolescence" seeking motive becomes dominant. In this case, α_w^* would be closer to 1, that is low-level obsolescence will make the society better off. Along the similar lines, when the consumers are impatient, the welfare is better off when the perceived obsolescence is high. Figure A.2 illustrates the welfare discussion when the consumers are myopic for the case where $v = 1$, $\delta_c = 0.37$ and $\delta_s = 0.79$ while Proposition A.12 summarizes the general results as:

Proposition A.12 *For a fixed $\delta_s \in (0,1)$,*

1. *Consumer surplus is maximized when there is no inclination to perceive obsolescence, i.e., when $\alpha_c^* = 1$, and there exists a unique $\alpha_c^1(\delta_c) \in (0, \alpha')$, strictly increasing with δ_c, such that for $\alpha \in (0, \alpha_c^1(\delta_c))$ the consumers experience negative surplus,*

2. *Producer surplus is maximized when there is full inclination to perceive obsolescence, i.e., when $\alpha_s^* = 0$,*

3. *Total surplus is maximized at unique $\alpha_w^*(\delta_c) \in (0,1)$, and $\alpha_w^*(\delta_c)$ strictly increases with the consumer patience δ_c.*

Here, $\alpha_c^1(\delta_c)$ is depicted in Figure A.2.

Proof Outline: In view of (1), by Proposition 2.5, we have the consumer surplus $CS(\alpha, \delta_c)$ as in the following:

$$\begin{cases} CS_{LB}(\alpha,\delta_c) = \int_{\theta_L^*}^{\theta_B^*} \delta_c^{t_2^*}(\theta u - p_2^*)d\theta + \int_{\theta_B^*}^{1}(\theta u - p_1^* - \delta_c^{t_2^*} p_2^*)d\theta & \text{if } \alpha \in (0, \alpha'(\delta_c)], \\ CS_{ELB}(\alpha,\delta_c) = \int_{\theta_L^*}^{\theta_E^*} \delta_c^{t_2^*}(\theta u - p_2^*)d\theta + \int_{\theta_E^*}^{\theta_B^*}\left(\theta u(1-\delta_c^{t_2{}^*}(1-\alpha^{t_2{}^*})) - p_1^*\right)d\theta + \int_{\theta_B^*}^{1}(\theta u - p_1^* - \delta_c^{t_2^*} p_2^*)d\theta & \text{if } \alpha \in (\alpha'(\delta_c), \alpha''(\delta_c)), \\ CS_{EL}(\alpha,\delta_c) = \int_{\theta_L^*}^{\theta_E^*} \delta_c^{t_2^*}(\theta u - p_2^*)d\theta + \int_{\theta_E^*}^{1}\left(\theta u(1-\delta_c^{t_2{}^*}(1-\alpha^{t_2{}^*})) - p_1^*\right)d\theta & \text{if } \alpha \in [\alpha''(\delta_c), 1]. \end{cases}$$

As an immediate consequence of Proposition 2.5, we have $CS_{LB}(\alpha, \delta_c) = CS_{ELB}(\alpha, \delta_c)$ at $\alpha = \alpha'(\delta_c)$. Besides, we confirm that $\frac{\partial CS_{LB}}{\alpha} > 0$ and $\frac{\partial CS_{ELB}}{\alpha} > 0$. Therefore, we conclude that $CS(\alpha, \delta_c)$ attains it maximum at $\alpha = 0$ over the range $\alpha \in (0, \alpha''(\delta_c))$. In addition, we

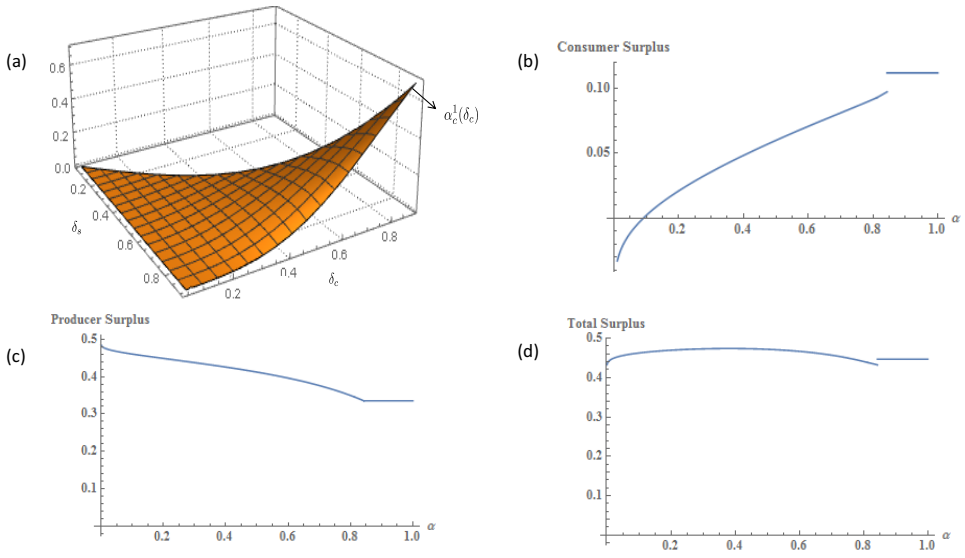


Figure A.2: Welfare Implications in the Presence of the Myopic Consumers for $v = 1, \delta_s = 0.37, \delta_s = 0.79$

confirm that $CS_{EL}(\alpha, \delta_c) = -\frac{v}{9\log(\delta_c)} > CS_{ELB}(\alpha, \delta_c)$. Thus, we conclude that $CS(\alpha, \delta_c)$ attains it maximum when $\alpha = 0$. In view of $\alpha_c^1(\delta_c)$, we first confirm that it $CS_{ELB}(\alpha, \delta_c) > 0$ and $CS_{EL}(\alpha, \delta_c) > 0$, and hence $\alpha_c^1(\delta_c) < \alpha'$. We remind the reader that $\frac{\partial CS_{LB}}{\alpha} > 0$. We also have $\partial CS_{LB}|\alpha \to 0 = \frac{v\left(2\delta_c^{t_2}-1\right)}{8\log(\delta_c)} < 0$ since $\delta_c^{t_2} > 1/2$ (see Proof of Proposition 2.5). Further, we have $CS_{LB}(\alpha, \delta_c) = CS_{ELB}(\alpha, \delta_c) > 0$ at $\alpha = \alpha'(\delta_c)$. Thus, there exists a unique $\alpha_c^1(\delta_c) \in (0, \alpha')$ at which $CS(\alpha, \delta_c)$ vanishes. Next, we note that part (2) follows from Corollary 2.7, and part (3) is a result of the statements (1) and (2), in combination. That concludes the outline of the proof. ∎

Next, we discuss the attentive consumers. Our focus is on the case where the consumers are more impatient than the firm ($\delta_c < \delta_s$) since otherwise ($\delta_c \geq \delta_s$) the firm does not release the second version, and the perceived obsolescence is not applicable. The attentive consumers consider to buy the early version at time $t = 0$ if and only if the price is reasonably low in comparison to the value provided in the first period $u(1 - \delta_c^{t_2})$ versus the future obsolescence ($\delta_c^{t_2} u(1 - \alpha^{t_2})$, and thus limit the firm's pricing power. The more the market is eager to the obsolescence (α low), the more the price leverage the attentive consumers have against the firm. At the extreme, the full perceived obsolescence ($\alpha = 0$) leads the consumers to buy the early version if and only if the first period price is extremely low, and this motivates the firm

to open the release window. As a result of the low prices and reasonable release windows, the attentive consumers are hooked in as both product buyers. They enjoy a constant superior quality experience in return of the low prices. That said, if the consumers are very patient, their valuation of the obsolescence would be significant and dominate over the low first period price benefit attached to the full obsolescence. As a result, the consumer surplus would be maximum when the obsolescence is low ($\alpha = \alpha''$), but not necessarily zero[1]. The firm, on the other hand, is still better off with the full obsolescence and by locking-in the consumers with the low first period prices. The total surplus would be maximized when the obsolescence is full:

Proposition A.13 *For a fixed $\delta_s \in (0,1)$ and $\delta_c \in (0, \delta_s)$, there exists $\hat{\delta}_c \in (0, \delta_s)$ such that*

1. *For $\delta_c \in (0, \hat{\delta}_c]$, consumer surplus is maximized when there is full perceived obsolescence ($\alpha_c^* = 0$), and for $\delta_c \in (\hat{\delta}_c, \delta_s)$ the consumer surplus is maximized when perceived obsolescence is low ($\alpha_c^* = \alpha''$).*

2. *Producer surplus is maximized when there is full inclination to perceive obsolescence, i.e., when $\alpha_s^* = 0$,*

3. *Total surplus is maximized when there is full inclination to perceive obsolescence, i.e., when $\alpha_w^* = 0$.*

Proof Outline: The proof follows in analogous steps to those in Proposition A.12. First, we note that statement in (2) directly follows from the proof of Proposition 2.16, where we show that the profit is decreasing with α, that is $\frac{\partial \Pi_S^*}{\partial \alpha} < 0$ and hence $\Pi_S^*(\alpha, \delta_c)$ attains its maximum when $\alpha = 0$. In view of the first statement, we first remind the reader that the consumer surplus is computed based on the optimal strategy of the firm and the market's reaction for differnet values of α and δ_c as characterized by Proposition 2.16. One insight one can gather from this analysis is that $CS_{LB}(\alpha, \delta_c)$ decreases with α, and hence we have $CS_{LB}(\alpha, \delta_c)$ attaining its maximum when $\alpha_c^* = 0$. Besides, we verify that $\frac{\partial CS_{ELB}(\alpha, \delta_c)}{\partial \alpha} > 0$ while $\frac{\partial CS_{EL}(\alpha, \delta_c)}{\partial \alpha} < 0$

[1] At $\alpha = \alpha''$, the market segments as the early and late buyers as outlined in Proposition 2.16.

and $CS_{EL}(\alpha, \delta_c) < CS_{ELB}(\alpha, \delta_c)$ for $\alpha \in (\alpha''(\delta_c), 1)$. This, in turn, implies that $CS(\alpha, \delta_c)$ attain its maximum at $\alpha_c^* = 0 = \alpha''(\delta_c)$. Thus, the consumer surplus $CS(\alpha, \delta_c)$ attaining its maximum at $\alpha = 0$ if $CS_{LB}(0, \delta_c) > CS_{ELB}(\alpha''(\delta_c), \delta_c)$ while the maximum is at $\alpha_c^* = \alpha''(\delta_c)$ for otherwise. We define a function $f(\delta_c) := CS_{LB}(0, \delta_c) - CS_{ELB}(\alpha''(v\delta_c), \delta_c)$ and show that $f(\delta_c)$ is decreasing with δ_c since we have $\frac{\partial CS_{LB}}{\partial \delta_c} < 0$ and $\frac{\partial CS_{ELB}}{\partial \delta_c} > 0$. We also show that $f(\delta_c)|_{\delta_c \to 0} > 0$ and $f(\delta_c)|_{\delta_c \to \delta_s} < 0$. Thus, we conclude that there exists a $\hat{\delta}_c \in (0, \delta_s)$ at which $f(\delta_c)$ vanishes and when $\delta_c < \hat{\delta}_c$ it follows that $f(\delta_c) > 0$ and vice versa. Thus, we have $\alpha_c^* = 0$ for $\delta_c \in (\hat{\delta}_c, \delta_s)$ and $\alpha_c^* = \alpha''$ for otherwise. Statement in (3) follows from the first two statements. ∎

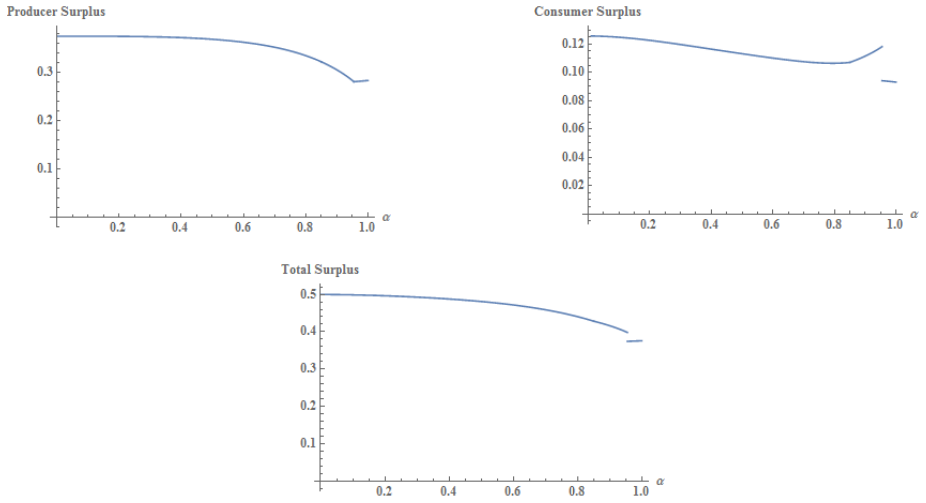


Figure A.3: Welfare Implications in the Presence of the Strategic Consumers for $v = 1, \delta_s = 0.37, \delta_s = 0.79$

Therefore, the key difference between the inattentive (myopic) and the attentive (strategic) settings is due to both product buyers. In the attentive case, the inter-temporal price discriminating firm aims to convince the high-value consumers to buy in the first period with the low price and the second period with the extended release window, especially when the market is eager to the obsolescence. As a result, the consumer welfare is maximum when the market perceives full obsolescence ($\alpha = 0$). In the inattentive setting, however, the consumers end up paying the high total price (present value of p_1 and p_2) as a result of to their myopia for

a future version and obsolescence. Even when they regretted their early buying decision (α low) and given that the second period price is comparably high, they would prefer to upgrade since the alternative (being an early buyer) is even a worse option. Therefore, the high obsolescence creates unhappy consumers, and the consumer welfare is maximized when there exists no obsolescence ($\alpha = 1$).

To wrap up, we conclude that myopic consumers prefer no obsolescence and they may even experience negative surplus when obsolescence is high. Strategic consumers, however, seek obsolescence as they can control when and at which prices they can experience it. Our analysis also suggests that the monopolists also prefer the market to perceive full obsolescence at the time of the release of the new version. Thus, potentially the monopolist would be building high obsolescence if it can do so. A possible policy discussion that may follow from this is that to protect the consumers, and the policymaker may ensure that consumers are educated about the upcoming releases.

A.2.3 Planned (Forced) Obsolescence

Up to this section, we covered obsolescence as a consumer perception exogenous to the firm while the release time enables the firm to profit from it. In this section, we turn our attention to the planned obsolescence, where the firm directly builds in quality deterioration ($1 - \alpha$) at the release of the new version and this time the obsolescence is exogenous to the consumers.

To this end, we first start with technologically myopic consumers who are not attentive to the firm's future decisions. The first period is similar to the perceived obsolescence setting, that is the myopic consumers with marginal utility $\theta > \theta_E = p_1/u$ buy the first version while the rest of the market remains uncovered. If consumers remain myopic ex-post the obsolescence, the firm would be willing to build full obsolescence. That would lead myopic consumers to experience regret. An interesting question here is: would the consumers remain myopic once they are forced to the unexpected obsolescence? In fact, research suggests that the consumers become sensitive to obsolescence as they experience it (Cooper, 2004; Guiltinan, 2009). Therefore, even

if the firm has not yet planned for another future obsolescence, consumers may be inclined to think that there will be another obsolescence in the future.

Formally, consumers who bought in the first period ($\theta > \theta_E = p_1/u$), obtain a utility of $\theta u(1 - \delta_c^t(1 - \alpha) - \alpha)$ when upgrading to the second version. Specifically, $\theta u(1 - \alpha)$ is the additional utility obtained by upgrading while $\theta u \delta_c^t(1 - \alpha)$ represents the expectation of a planned obsolescence. Along similar lines, the consumers who did not buy in the first period obtain a utility of $\theta u(1 - \delta_c^t(1 - \alpha))$ if they decide to obtain the second version at time t. When the release time and the obsolescence is fixed, the pricing policy is:

Proposition A.14 *For $\alpha \in (0, 1)$ and $t \in (0, \infty)$,*

1. *If $p_2 \geq u(1 - \alpha)(1 - \delta_c^t)$, none of the consumers buy both products and*
   - *(a) if $p_2 \geq p_1(1 - \delta_c^t(1 - \alpha))$, then the market consists solely of the early buyers (region E);*
   - *(b) if $p_2 < p_1(1 - \delta_c^t(1 - \alpha))$, then the market consists of the early buyers and late buyers (region EL).*
2. *If $p_2 < u(1 - \alpha)(1 - \delta_c^t)$, there are consumers that buy both products and*
   - *(a) if $p_2 \geq p_1(1 - \delta_c^t(1 - \alpha))$, then the market consists of the early buyers and consumers that purchase both versions (region EB);*
   - *(b) if $p_1(1 - \alpha)(1 - \delta_c^t) < p_2 < p_1(1 - \delta_c^t(1 - \alpha))$, then the market consists of the early buyers, late buyers and consumers that purchase both versions (region ELB);*
   - *(c) if $p_2 \leq p_1(1 - \alpha)(1 - \delta_c^t)$, then the market consists of the late buyers and consumers that purchase both versions (region LB).*

Proof: Follows along the similar lines with Proposition 2.2.

This implies that the expected profit function is

$$\Pi_M(p_1, p_1, t, \alpha) \;=\; p_1(\theta_B - \theta_E)^+ + \delta_s^t p_2(\theta_E - \theta_L)^+ + (p_1 + \delta_s^t p_2)(1 - \max\{\theta_E, \theta_B\}), \tag{A.46}$$

where

$$\theta_B = \frac{p_2}{u(1-\alpha)(1-\delta_c^t)}, \quad \theta_E = \frac{p_1}{u}, \quad \text{and} \quad \theta_L = \frac{p_2}{u(1-\delta_c^t(1-\alpha))}^2.$$

Next, we endogenize the release while still fixing the obsolescence to $(1-\alpha)$ by Proposition A.15.

Proposition A.15 *For fixed $\delta_s, \delta_c \in (0,1)$, there exist α_p', $\alpha_p'' \in (0,1)$, $\alpha_p' < \alpha_p''$ and $\delta_c' \in (0, \delta_s)$ such that:*

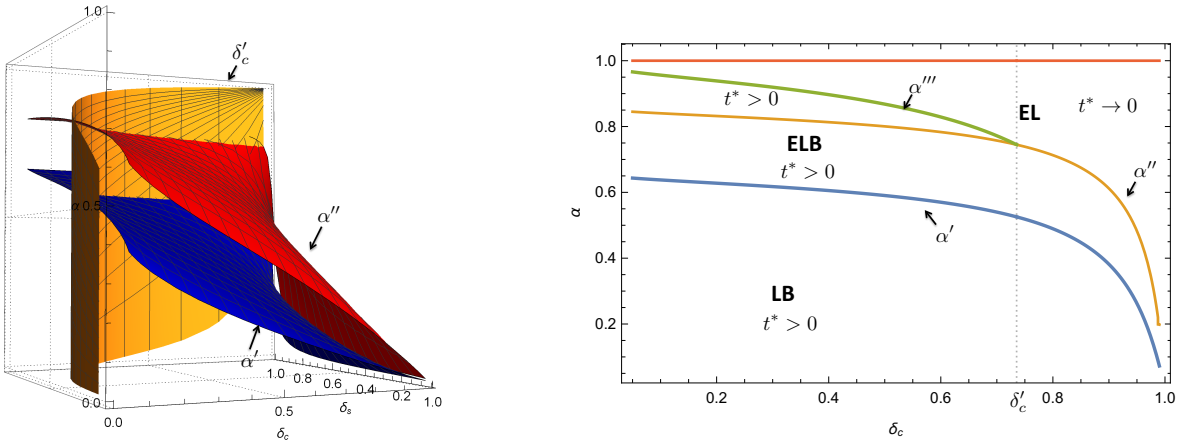
1. *If $\alpha \in (0, \alpha_p']$, it is optimal for the firm to serve the late buyers and consumers that purchase both products (Region LB) with sequential release ($t^* > 0$).*
2. *If $\alpha \in (\alpha_p', \alpha_p'')$, the firm serves the early buyers, late buyers and consumers that purchase both products (Region ELB) with sequential release ($t^* > 0$).*
3. *If $\alpha \in [\alpha_p'', 1)$, the firm serves the early buyers and late buyers (Region EL), and*
   (a) *when $\delta_c \in (0, \delta_c')$, it executes a sequential release policy ($t^* > 0$) for $\alpha \in (\alpha_p'', \frac{\log(\delta_c)}{\log(\delta_s)+\log(\delta_c)})$, and an immediate release of the second product ($t^* \to 0$) for $\alpha \in (\frac{\log(\delta_c)}{\log(\delta_s)+\log(\delta_c)}, 1)$;*
   (b) *when $\delta_c \in [\delta_c', 1)$, with immediate release of the second version ($t^* \to 0$).*

Here, α_p', α_p'', δ_c' are depicted in Figure A.4a.

Proof: Follows along similar lines with Proposition 2.5.

[2]To remind the reader, θ_i is the threshold consumer who is indifferent between choice $i = \{B$ (upgrading to late version), E (obtaining the early version), and L (obtaining the late version for the first time)$\}$ and no action.

Figure A.4: Optimal Segmentation Strategy



(a) for $v = 1$ (b) for $v = 1$ and $\delta_s = 0.9$

Corollary A.16 *For fixed $\delta_s, \delta_c \in (0,1)$, thresholds $\alpha', \alpha'' \in (0,1)$ of perceived obsolescence case are upper bounds for the thresholds α_p', α_p'' of planned obsolescence.*

Proof: One can show that α_p' and α_p'' are decreasing in δ_c and $\lim_{\delta_c \to 0} \alpha_p' = \alpha'$ and $\lim_{\delta_c \to 0} \alpha_p'' = \alpha''$. Thus, it follows that $\alpha_p' \leq \alpha'$ and $\alpha_p'' \leq \alpha''$. ∎

Next, we compare the market segmentation strategies summarised by Proposition A.15 and Corollary A.16 with the perceived obsolescence case. When obsolescence derives from consumer perception, we had immediate release optimal for low, $(\alpha \in [\alpha'', 1))$, or full obsolescence $(\alpha = 0)$ cases. As a departure from the perceived obsolescence case, immediate release of the second version is optimal if and only if the firm builds-in little obsolescence. This is intuitive because knowing that its consumers will become forward-looking, the firm delays the obsolescence unless quality deterioration is insignificant. In the case of the low obsolescence, the negative effect of planned obsolescence could be overturned if the consumers are patient enough $(\delta_c \in [\delta_c', 1))$. In that case, they pay enough attention to the little obsolescence built and develop a positive future outlook. Otherwise, building a high level of obsolescence, $\alpha \in (0, \alpha_p'')$, would come at the cost of delaying the release of the new version $t^* > 0$.

Another departure from the perceived obsolescence, the firm would not be able to convince its existing consumers to upgrade unless the built obsolescence is not high. In the perceived obsolescence case, however, the consumers would be willing to upgrade even if the obsolescence

is not high since the firm is able to control obsolescence indirectly with release time. Secondly, a firm would be more willing to employ an inter-temporal price discrimination strategy, that is, segmenting as the early and late buyers when it can directly manage obsolescence. Interestingly, the firm would overturn the negative consequence of forced obsolescence by ensuring that the quality is maintained and as a return would benefit from the price discrimination its loyal consumers have been exposed to.

Next, we turn our attention to the planned obsolescence rate $(1-\alpha)$.

Proposition A.17 *For a fixed $\delta_s \in (0,1)$, in the presence of "once bitten twice shy consumers", the firm refrains from building full obsolescence, and the optimal level of obsolescence $1-\alpha_s^*(\delta_c)$ has the following properties:*

1. *If the consumers are moderately attentive, $\delta_c \leq \delta_c''$, the optimal level of obsolescence is $1-\alpha_s^*(\delta_c) \in (1-\alpha_p'(\delta_c), 1)$, and it decreases with the discount factor δ_c of consumers,*
2. *If the consumers are very attentive, $\delta_c > \delta_c''$, the firm should build no-obsolescence, that is, the optimal level of obsolescence is $1-\alpha_s^*(\delta_c) \to 0$.*

Here, δ_c'' is depicted in Figure A.5a.

Proof: Follows along similar lines with Proposition A.13.

The optimal level of obsolescence for the firm crucially depends on how sensitive consumers become once they experience it at time t. The more sensitive the consumers (large δ_c), the less the obsolescence the firm should build. That said, the firm still continues to benefit from the partial obsolescence by delaying planned obsolescence experience with low prices and long release windows. When consumers are very sensitive ($\delta_c > \delta_c''$), the firm builds no-obsolescence and instead benefits from "limit inter-temporal price discrimination". In particular, the firm is able to charge higher prices while ensuring full quality to its consumers. In addition, through the release time the firm has partial control over the obsolescence sensitiveness. Formally, the release time mitigates the sensitiveness to the obsolescence, which in turn, reinforces the

mechanics explained above. Next, we turn our attention to consumer welfare with Proposition A.18.

Proposition A.18 *For a fixed $\delta_s \in (0,1)$, surplus of "once bitten twice shy consumers" attains maximum at $\alpha_c^*(\delta_c)$ with the following properties:*

1. *If the consumers are extremely inattentive ($\delta_c \leq \delta_c^1$), the consumer surplus is maximized at $\alpha_c^*(\delta_c) = 0$*
   (full obsolescence),

2. *If the consumers are moderately inattentive ($\delta_c^1 < \delta_c \leq \delta_c^2$), the consumer surplus is maximized at $\alpha_c^*(\delta_c) = 1$ (no-obsolescence),*

3. *If the consumers are moderately attentive ($\delta_c^2 < \delta_c \leq \delta_c^3$), the consumer surplus is maximized at $\alpha_c^*(\delta_c) = \alpha''(\delta_c)$ (low obsolescence),*

4. *If the consumers are extremely attentive ($\delta_c > \delta_c^3$), the consumer surplus is maximized at $\alpha_c^*(\delta_c) = \alpha'(\delta_c)$ (high obsolescence).*

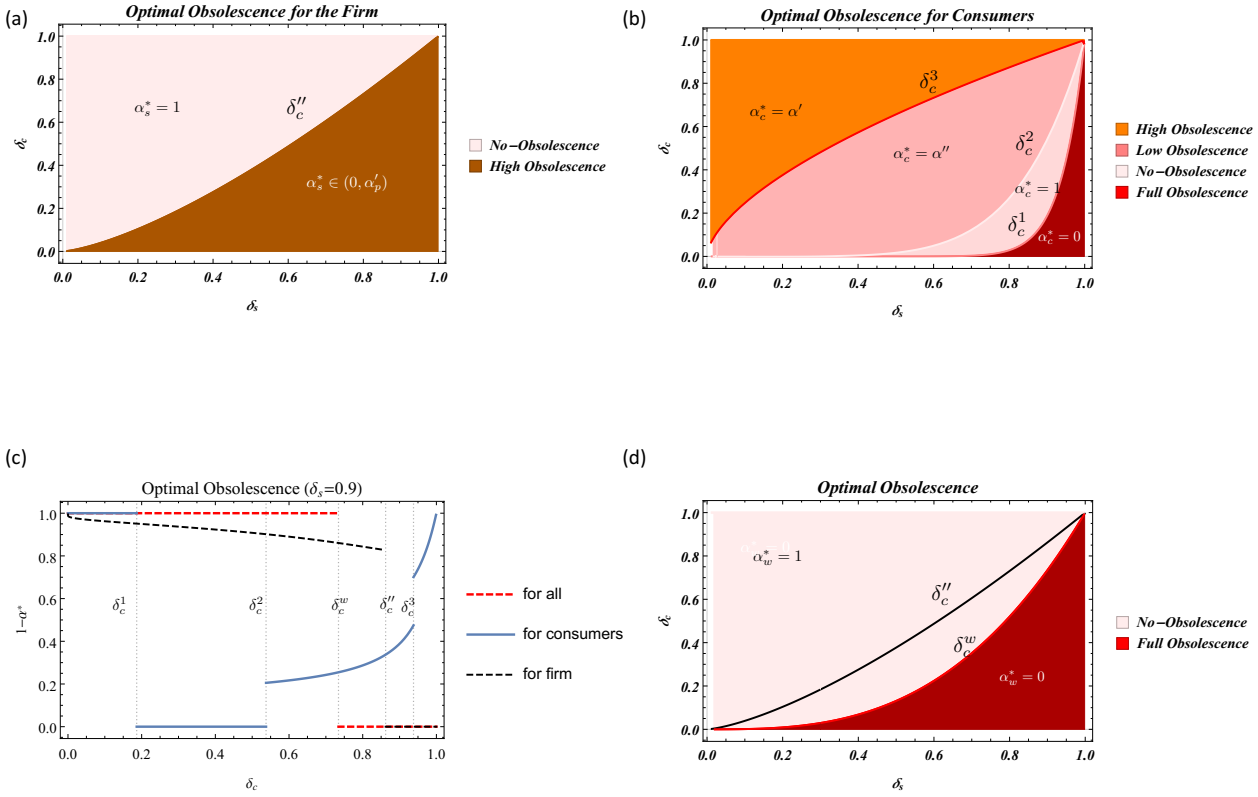
Here, $\delta_c^1, \delta_c^2, \delta_c^3$ are depicted in Figure A.5b.

Proof: One may prove this statement using the proof strategy outline in Proposition A.13.

As the consumers become more sensitive to the obsolescence, ironically, they benefit more from it because of the price leverage they obtain. In particular, both product buyers obtain price benefits that account for their pessimistic outlook. The firm decreases total price paid by both product buyers through the price of the first version and thereby locks-in the consumers in the first period, and convince them to upgrade later.

If instead the consumers are not sensitive to the planned obsolescence, the price leverage is not enough to overturn the negative effect of the quality deterioration. Interestingly, however, when the consumers are extremely insensitive to the obsolescence, they benefit most when

Figure A.5: Welfare Implications of Planned Obsolescence ($v = 1$)



obsolescence is full ($\alpha = 0$). This follows because of the high weight they give to the first-period price relative to the low quality deterioration. In addition, through the release time, the firm has partial control over the obsolescence sensitiveness. Formally, the release time mitigates the sensitiveness to the obsolescence, which in turn, reinforces the mechanics explained above. Next, we discuss the total welfare.

Proposition A.19 *For a fixed $\delta_s \in (0,1)$, there exists $\delta_c^w \in (0, \delta_c'')$, and the total welfare attains maximum at $\alpha_w^*(\delta_c)$ with the following properties:*

1. *If the consumers are extremely sensitive ($\delta_c > \delta_c^w$), the total surplus is maximized at $\alpha_w^*(\delta_c) = 1$ (no-obsolescence),*
2. *Otherwise when ($\delta_c \leq \delta_c^w$), the consumer surplus is maximized at $\alpha_w^*(\delta_c) = 0$ (full obsolescence).*

Here, δ_c^w is depicted in Figure A.5d.

Proof: Follows directly from Proposition A.17 and Proposition A.18.

To illustrate the discussion above, Figure A.5*c* shows the welfare maximising obsolescence levels when the firm has discount factor $\delta_c = 0.9$. The consumers would be more willing to experience obsolescence compared to the firm when consumers are either extremely insensitive or sensitive to the obsolescence. In the extreme insensitive region, it is best for the firm to lower the second period price *ex-post* obsolescence while mitigating consumer sensitiveness directly by lowering the obsolescence. The consumers would be better off when the first period prices are lowered *ex-ante* obsolescence. The sensitiveness of the locked in consumers is reduced by the delayed release of the obsolescence experience along with the new version. We verify that locking-in consumers with low first period prices are the second best strategy for the firm while consumers' second-best alternative would be inter-temporal price discrimination with no-obsolescence. Thereby, total welfare is maximized with consumer locking-in strategy with full obsolescence. When the

consumers are extremely sensitive after being bitten by obsolescence, the firm would be better off with no-obsolescence and use this as a leverage to deploy inter-temporal price discrimination. The consumers, on the other hand, are willing to experience high obsolescence by obtaining a first-period price advantage with the leverage obtained due to the sensitiveness. The no-obsolescence policy is the third best alternative following low obsolescence policy. On account of the firm, the consumer surplus maximising obsolescence is an inferior strategy and the firm would be better off by either lowering or increasing the obsolescence further. These conflicting preferences between two parties lead non-obsolescence policy as the best policy to maximize welfare.

When the consumer is moderately sensitive to obsolescence, the firm would be more willing to build high obsolescence with low second-period price and lowering obsolescence as consumers become more sensitive. The consumers, on the other hand, would be better off with no-obsolescence policy when they are relatively insensitive and with locking-in policy when they are relatively more sensitive. Combining firm's willingness to build high obsolescence with the consumer's, counter-intuitively, being better off when locked-in *ex-post* obsolescence with low first period prices, full obsolescence becomes welfare maximising obsolescence policy. It is important to note that as the consumers become significantly sensitive ($\delta \geq \delta_c^w$), full obsolescence becomes an inferior policy relative to inter-temporal price discrimination with no-obsolescence. This, in turn, leads welfare being better off when obsolescence is never introduced. Interestingly, this result is not driven by consumer welfare but instead is forced by firm's welfare. This concludes our discussion for *ex-ante* myopic *ex-post* strategic consumers.

Next, we cover the *ex-ante* strategic consumers case. We start first with the basic setting where the consumers are only taking into account the planned obsolescence t but not considering further obsolescence that could be built-in after t. To this end, we first discuss the optimal rollover strategy of the firm with a pre-committed planned obsolescence level α (Proposition A.20), and further discuss the profit, consumer surplus, and welfare maximising α (Proposition A.21).

Proposition A.20 *For $\delta_s, \delta_c, \alpha \in (0,1)$,*

1. *If $\delta_c \geq \delta_s$, it is optimal for the company to introduce only the first version with $t_2^* \to \infty$ (Region E).*

2. *If $\delta_c < \delta_s$, it is optimal for the company to sequentially introduce the versions with a regular preference ordering ($t_2^* > -\frac{\log(2-\alpha)}{\log(\delta_c)}$) and there exist $\alpha_p'(\delta_c) \in (0,1)$ and $\alpha_p''(\delta_c) \in (\alpha'(\delta_c), 1)$ such that:*

   (a) *If $\alpha \in (0, \alpha_p'(\delta_c)]$, it is optimal for the company to serve the late buyers and consumers that purchase both products (Region LB),*

   (b) *If $\alpha \in (\alpha_p'(\delta_c), \alpha_p''(\delta_c))$, the company serves the early buyers, late buyers, and consumers that purchase both products (Region ELB),*

   (c) *If $\alpha_p \in [\alpha_p''(\delta_c), 1]$, the company serves the early buyers and late buyers (Region EL).*

Proof: The proof follows analogous steps to those of Proposition 2.5 and Proposition 2.16, and therefore omitted.

Similar to the perceived obsolescence case, release time is a lever for the firm that has committed to a planned obsolescence. As a departure from the perceived obsolescence, however, we find that in the presence of the forced obsolescence inducing both product buyers is suboptimal in a wider range of α. Next, we discuss the welfare implications.

Proposition A.21 *In the presence of the strategic consumers, for a fixed $\delta_s \in (0,1)$ and $\delta_c \in (0, \delta_s)$, there exists $\tilde{\delta}_c^1 \in (0, \delta_s)$ such that*

1. *For $\delta_c \in (0, \tilde{\delta}_c^1]$, the consumer surplus is maximized when there is full obsolescence ($\alpha_c^* = 0$) and for $\delta_c \in (\tilde{\delta}_c^1, \delta_s)$ the consumer surplus is maximized when the obsolescence is medium ($\alpha_p' < \alpha_c^* < \alpha''$).*

2. *The producer surplus is maximized when there is full obsolescence, $\alpha_s^* = 0$,*

3. *The total surplus is maximized when there is high obsolescence, $\alpha_w^* = 0$.*

Proof: One may prove this statement using the proof strategies outline in Proposition A.12 and Proposition A.13.

The welfare implications of the forced obsolescence in the presence of the ex-ante strategic consumers' case is similar to the perceived obsolescence case. The producer, consumer, and hence the total welfare is maximum when the obsolescence is full while the exception for the consumers is when they are extremely patient.

Appendix B

Chapter 3

B.1 Proofs

Proof of Lemma 3.1. We assume that $u_1 > u_2 > 0$, $c_1 > c \in (0, 2)$, and $p_1, p_2 > 0$ are given. We start with $\underline{Y}_E, \underline{Y}_M$ and note that $\underline{Y}_E := \frac{p_2}{cu_2}$ and $\underline{Y}_M := \frac{p_2}{u_2}$ for $u_1 > u_2$. Then it follows that $\frac{p_2}{cu_2} > \frac{p_2}{u_2}$ if $c > 1$, and vice versa. Thus, we have $\underline{Y}_E > \underline{Y}_M$ iff $c > 1$, which implies tautologically that if $c > 1$, then $\underline{Y}_E > \underline{Y}_M$ and if $c \leq 1$ then $\underline{Y}_E \leq \underline{Y}_M$. In view of $\bar{Y}_E, \bar{Y}_M$, we have $\bar{Y}_E := [\frac{p_1-p_2}{c_1u_1-cu_2}]_0^1$ and $\bar{Y}_M := [\frac{p_1-p_2}{u_1-u_2}]_0^1$ for $u_1 > u_2$. If $c > 1$, we also have $c_1 > 1$. This implies that $c_1u_1 - cu_2 > u_1 - u_2$, and hence if $p_1 > p_2$ it follows that $\frac{p_1-p_2}{c_1u_1-cu_2} > \frac{p_1-p_2}{u_1-u_2}$. We note that the firms never price as $p_1 \leq p_2$ when $u_1 > u_2$ as implied by Lemma 3.2-(1). Thus, if $c > 1$, we have $\bar{Y}_E > \bar{Y}_M$. If instead, we have $c \leq 1$, one can guarantee that $c_1 > 1$. In this case, we have $\bar{Y}_E \geq \bar{Y}_M$ if only if $c_1u_1 - cu_2 < u_1 - u_2$, or equivalently $(1 - c_1)u_1 > (1 - c)u_2$, by definition of the thresholds $(\bar{Y}_E, \bar{Y}_M)$. This concludes our proof. ∎

Proof of Lemma 3.2. We prove the statements by contradiction:

1. We assume to the contrary that $p_2u_1 \geq p_1u_2$ is the pricing regime of the category firm, which also implies that $p_2c_1u_1 > p_1cu_2$ since we have $c_1 > c$. Then, by definition, we have $\bar{Y}_M < \underline{Y}_M$ and $\bar{Y}_E < \underline{Y}_E$. This, in turn, implies that $D_{2,M} = (1 - \lambda)\big(\bar{Y}_M - \underline{Y}_M\big)^+ = 0$ and $D_{2,E} = \lambda\big(\bar{Y}_E - \underline{Y}_E\big)^+ = 0$, and hence the category firm serves none of the market.

Thus, it leapfrogs and sets the price as $p_2u_1 < p_1u_2$.

2. We assume to the contrary that $c_1u_1 - cu_2 \leq p_1 - p_2$ is the pricing regime of the ecosystem firm. If $c_1 > 1$, then the statement $c_1u_1 - cu_2 < p_1 - p_2$ implies that $u_1 - u_1 < p_1 - p_2$ since $c_1 > c$ implies that $c_1u_1 - cu_2 > c_1(u_1 - u_2) > u_1 - u_2$. Then, by definition, for $c_1 > 1$ we have $\bar{Y}_M > 1$ and $\bar{Y}_E > 1$, and hence $D_{1,M} = (1-\lambda)\big(1-\bar{Y}_M\big)^+ = 0$ and $D_{1,E} = \lambda\big(1-\bar{Y}_E\big)^+ = 0$ and that the ecosystem firm serves none of the markets. Thus, the ecosystem firm deviates from the pricing regime and sets the price as $c_1u_1 - cu_2 > p_1 - p_2$.

3. We assume to the contrary that $u_1 - u_2 \leq p_1 - p_2$ is the pricing regime of the ecosystem firm. We note that if $c_1 < 1$ and $(1-c_1)u_1 > (1-c)u_2$ (defensive complementarity), then the statement $u_1 - u_2 < p_1 - p_2$ implies that $c_1u_1 - cu_2 < p_1 - p_2$. This, in turn, implies that $c_1u_1 - cu_2 < p_1 - p_2$ and hence the ecosystem firm serves none of the sub-markets. Thus, it sets the price as $u_1 - u_2 \geq p_1 - p_2$ if $c_1 < 1$ and $(1-c_1)u_1 > (1-c)u_2$. This, in turn, implies that $D_{1,M} = (1-\lambda)\big(1-\bar{Y}_M\big)^+ = 0$ and $D_{1,E} = \lambda\big(1-\bar{Y}_E\big)^+ = 0$. Thus, the ecosystem firm sets the price as $u_1 - u_2 > p_1 - p_2$.

Furthermore, we note that when $c_1 < 1$ and $(1-c_1)u_1 < (1-c)u_2$ (aggressive built-in complementarity), then the ecosystem may prefer deploying the pricing policy $u_1 - u_2 < p_1 - p_2$ as it would not imply $D_{1,M} = (1-\lambda)\big(1-\bar{Y}_M\big)^+ = 0$ and $D_{1,E} = \lambda\big(1-\bar{Y}_E\big)^+ = 0$ at the same time. ∎

Proof of Lemma 3.3. The proof follows from Lemma 3.2:

1. Lemma 3.2-(1) states that the category firm sets the prices as $p_2u_1 < p_1u_2$, which, in turn, implies that $\bar{Y}_M > \underline{Y}_M$ and hence $D_{2,M} = (1-\lambda)\big(\bar{Y}_M - \underline{Y}_M\big)^+ > 0$. Thus, we conclude that the category firm always exists in Motta'a market.

2. Lemma 3.2-(2) states that the ecosystem firm sets the prices as $c_1u_1 - cu_2 > p_1 - p_2$ when $c > 1$, which, in turn, implies that $\bar{Y}_E < 1$, and hence $D_{1,E} = \lambda\big(1-\bar{Y}_E\big)^+ > 0$. Thus, we conclude that the ecosystem firm always exist in the ecosystem market if $c \geq 1$. In view of the Motta's market, we note that $D_{1,M} = (1-\lambda)\big(1-\bar{Y}_M\big)^+ > 0$ if and only if $\bar{Y}_M < 1$ which holds if and only if $u_1 - u_2 > p_1 - p_2$ by definition of $\bar{Y}_M$.

3. Lemma 3.2-(3) states that the ecosystem firm sets the prices as $u_1 - u_2 > p_1 - p_2$ when $c_1 < 1$ and $(1-c_1)u_1 > (1-c)u_2$, which, in turn, implies that $\bar{Y}_M < 1$, and hence $D_{1,M} = (1-\lambda)\big(1-\bar{Y}_M\big)^+ > 0$. Thus, we conclude that the ecosystem firm always exist in the Motta's market if $c_1 < 1$ and $(1-c_1)u_1 > (1-c)u_2$. If instead $(1-c_1)u_1 < (1-c)u_2$, then one can not guarantee that the ecosystem firm sets the prices as $u_1 - u_2 > p_1 - p_2$ due to the intrinsic dependency (c) of the product categories, and hence ecosystem firm exists in Motta's market as long as it sets the prices as $u_1 - u_2 > p_1 - p_2$. As for the ecosystem market, we have $D_{1,E} = \lambda\big(1-\bar{Y}_E\big)^+ > 0$ if and only if $\bar{Y}_E < 1$, which is the case if and only if $c_1u_1 - cu_2 > p_1 - p_2$ bu definition of $\bar{Y}_E$.

Proof of Lemma 3.4. We assume that $u_2 > u_1 > 0$, $cu_2 > c_1u_1$, $c_1 > c \in (0,2)$, and $p_1, p_2 > 0$ are given. We start with $\underline{Y}_E, \underline{Y}_M$ and note that $\underline{Y}_E := \frac{p_1}{c_1u_1}$ and $\underline{Y}_M := \frac{p_1}{u_1}$ for $u_1 > u_2$. Then it follows that $\frac{p_1}{c_1u_1} > \frac{p_1}{u_1}$ if $c_1 > 1$, and vice versa. Thus, we have $\underline{Y}_E > \underline{Y}_M$ iff $c_1 > 1$ and if $c_1 \leq 1$ then $\underline{Y}_E \leq \underline{Y}_M$. In view of $\bar{Y}_E, \bar{Y}_M$, we have $\bar{Y}_E := [\frac{p_2-p_1}{cu_2-c_1u_1}]_0^1$ and $\bar{Y}_M := [\frac{p_2-p_1}{u_2-u_1}]_0^1$ for $u_2 > u_1$ and $cu_2 > c_1u_1$. If $c > 1$, we also have $c_1 > 1$. We note that the firms never price as $p_2 \leq p_1$ when $u_2 > u_1$ as implied by Lemma 3.5-(1). Therefore, we have $\bar{Y}_E > \bar{Y}_M$ if and only if $c_1u_1 - cu_2 > u_1 - u_2$, or equivalently $(1-c_1)u_1 < (1-c)u_2$, by definition of the thresholds $(\bar{Y}_E, \bar{Y}_M)$. This concludes our proof. ∎

Proof of Lemma 3.5. We prove the statements by contradiction:

1. We assume to the contrary that $c_1p_2u_1 \leq cp_1u_2$ is the pricing regime of the ecosystem firm, which also implies that$p_2u_1 < p_1u_2$ since we have $c_1 > c$ and $u_2 > u_1$. Then, by definition, we have $\bar{Y}_M < \underline{Y}_M$ and $\bar{Y}_E < \underline{Y}_E$. This, in turn, implies that $D_{1,M} = (1-\lambda)\big(\bar{Y}_M - \underline{Y}_M\big)^+ = 0$ and $D_{1,E} = \lambda\big(\bar{Y}_E - \underline{Y}_E\big)^+ = 0$, and hence the ecosystem firm serves none of the market. Thus, it leapfrogs and sets the price as $c_1p_2u_1 > cp_1u_2$.

2. We assume to the contrary that $u_2 - u_1 \leq p_2 - p_1$ is the pricing regime of the category firm. If $c < 1$, then it follows that $u_2 - u_1 > cu_2 - cu_1 > cu_2 - c_1u_1$ since $c_1 > c$. Therefore, $u_2 - u_1 \leq p_2 - p_1$ implies that $cu_2 - c_1u_1 \leq p_2 - p_1$, and it follows that $\bar{Y}_M > 1$ and $\bar{Y}_E > 1$. This, in turn, implies that $D_{2,M} = (1-\lambda)\big(1-\bar{Y}_M\big)^+ = 0$ and $D_{2,E} = \lambda\big(1-\bar{Y}_E\big)^+ = 0$

and that the category firm serves none of the markets. Thus, the category firm deviates and sets the price as $u_2 - u_1 > p_2 - p_1$.

Proof of Lemma 3.6. The proof follows from Lemma 3.5:

1. Lemma 3.5-(1) states that the ecosystem firm sets the prices as $c_1 p_2 u_1 > c p_1 u_2$, which, in turn, implies that $\bar{Y}_E > \underline{Y}_E$ and hence $D_{1,E} = \lambda\big(\bar{Y}_E - \underline{Y}_E\big)^+ > 0$. Thus, we conclude that the ecosystem firm always exists in ecosystem market. In view of the Motta's market, if and only if $p_2 u_1 > p_1 u_2$ we have $\bar{Y}_M > \underline{Y}_M$, and hence $D_{1,M} = (1-\lambda)\big(\bar{Y}_M - \underline{Y}_M\big)^+ > 0$.

2. Lemma 3.5-(2) states that the category firm sets the prices as $u_2 - u_1 > p_2 - p_1$ when $c < 1$, which, in turn, implies that $\bar{Y}_M < 1$, and hence $D_{2,M} = (1-\lambda)\big(1 - \bar{Y}_M\big)^+ > 0$. Thus, we conclude that the category firm always exist in the Motta's market if $c < 1$. In view of the ecosystems market, we note that $D_{2,E} = \lambda\big(1 - \bar{Y}_E\big)^+ > 0$ if and only if $\bar{Y}_E < 1$ which holds if and only if $cu_2 - c_1 u_1 > p_2 - p_1$ by definition of $\bar{Y}_E$.

3. We note that Lemma 3.5 does not provide dominant pricing regime for the category firm for the values of $c > 1$. In this case, the category firms exists in Motta's market iff $D_{2,M} = \lambda\big(1 - \bar{Y}_M\big)^+ > 0$, which holds if and only if $\bar{Y}_M < 1$ and hence when $p_2 - p_1 < u_2 - u_1$. Along the similar lines, when $p_2 - p_1 < cu_2 - c_1 u_1$ we have $\bar{Y}_E < 1$ and this implies that $D_{2,E} = \lambda\big(1 - \bar{Y}_E\big)^+ > 0$.

Proof of Lemma 3.7. We have fixed quality levels such that $u_2 > u_1$, $c_1 u_1 > c u_2$ and $c_1 > c$.

1. Let us assume that $p_1 > p_2$. Then, we have $\bar{Y}_M := [\frac{p_2 - p_1}{u_2 - u_1}]_0^1 = 0$ and hence it follows that there exists no consumers buying from the ecosystem firm, that is $\{y : \frac{p_1}{u_1} \leq y \leq \bar{Y}_M\} = \emptyset$ while the consumer types buying from the category firm is shown as $\{y : y \geq \frac{p_2}{u_2}, y \geq \bar{Y}_M\} = 1 - \frac{p_2}{u_2}$. Thus, we have only the category firm in Motta's market. In view of the ecosystem market, the proof follows as in Lemma 3.2.

2. Let us assume that $p_2 > p_1$. Then, we have $\bar{Y}_E := [\frac{p_1 - p_2}{c_1 u_1 - c u_2}]_0^1 = 0$ and hence it follows that there exists no consumers buying from the category firm, that is $\{y : \frac{p_2}{u_2} \leq y \leq \bar{Y}_E\} = \emptyset$

while the consumer types for the ecosystem firm is shown as $\{y : y \geq \frac{p_1}{u_1}, y \geq \bar{Y}_E\} = 1 - \frac{p_1}{u_1}$. The proof continues as in Lemma 3.5 as for Motta's market.

Proof of Proposition 3.8. We have u_1, u_2, c, c_1 fixed such that $u_1 > u_2 > 0$ and $c_1 > c \in (0, 2)$. Firms $(i = 1, 2)$ sets prices p_i^* to maximize their profits Π_i (see Equation 1 and 2) simultaneously. If both firms target both sub-markets, then their profits can be written as

$$\Pi_1^* = \max\{\Pi_1 : p_1 - p_2 < u_1 - u_2, p_1 - p_2 < c_1 u_1 - c u_2, p_1 \in [0, \max\{u_1, c_1 u_1\}], p_2 \in [0, \max\{u_2, c u_2\}]\}, \text{and}$$

$$\Pi_2^* = \max\{\Pi_2 : p_1 c u_2 > p_1 c_1 u_1, p_1 \in [0, \max\{u_1, c_1 u_1\}], p_2 \in [0, \max\{u_2, c u_2\}]\}.$$

Solving this problem yields the following prices

$$p_1^* = -\frac{2u_1(u_1 - u_2)(c_1 u_1 - c u_2)\left(c^2(u_2 - \lambda u_2) + c c_1(\lambda - 1)u_1 + c_1\lambda(u_2 - u_1)\right)}{(-\lambda((c-1)u_2 + u_1) + c u_2 + c_1(\lambda - 1)u_1)(c_1\lambda u_1(4(c-1)u_1 - (c-4)u_2) + c(4u_1 - u_2)(c u_2 - c_1 u_1) + c\lambda u_2(-4c u_1 + (c-1)u_2 + u_1))}$$

$$\text{and } p_2^* = \frac{c u_2(u_1 - u_2)(c_1 u_1 - c u_2)}{c^2(\lambda - 1)u_2(4u_1 - u_2) - c c_1(\lambda - 1)u_1(4u_1 - u_2) + c\lambda u_2(u_2 - u_1) + 4c_1\lambda u_1(u_1 - u_2)}, \tag{B.1}$$

as long as $\lambda > \lambda_1$, where λ_1 is the unique value where $f_1(\lambda) := (c_1 u_1 - c u_2) - (p_1^* - p_2^*)$ vanishes. To prove the uniqueness of λ_1, we note that $\lim_{\lambda \to 1} f_1(\lambda) = \frac{2c_1 u_1}{4c_1 u_1 - c u_2} > 0$ and $\frac{\partial f_1(\lambda)}{\partial \lambda} < 0$. We do not provide the closed form expression for λ_1 due to the space limitations. Next, we note that if $c > c' = \frac{2u_1 - u_2}{4u_1 - u_2}$ and $c_1/c > m = \frac{\frac{6u_1}{4u_1 - u_2} - 1 - \frac{(1-c)u_2}{u_1}}{c}$, then it follows that $\lim_{\lambda \to 0} f_1(\lambda) = \frac{2c_1 u_1}{4c_1 u_1 - c u_2} > 0$, and hence $\lambda_1 < 0$. Thus, the firms compete with the optimal prices (p_1^*, p_2^*) in Equation B.1 which leads to the relative prices $p_1^*/p_2^* = \frac{c c_1(1-\lambda)u_1 + c_1\lambda(u_1 - u_2) - 2u_1\left(c^2(1-\lambda)u_2\right)}{c_1 u_1(1-\lambda) + \lambda(u_1 - u_2)) - c u_2(c(1-\lambda)u_2}$.

If instead, the $\lambda < \lambda_1$, then the firms do not compete with the prices in Equation B.1 since these prices lead to $f(\lambda) < 0$, and hence one of the constraints the ecosystem firm faces in order to be in both of the markets is violated. Specifically, the ecosystem does not compete in the ecosystem market when $\lambda < \lambda_1$. In this case, the profits of the two firms can be written as

$$\Pi_1^* = \max\{\Pi_1 : p_1 - p_2 < u_1 - u_2, p_1 \in [0, \max\{u_1, c_1 u_1\}], p_2 \in [0, \max\{u_2, c u_2\}]\}, \text{and}$$

$$\Pi_2^* = \max\{\Pi_2 : p_1 c u_2 > p_2 c_1 u_1, p_1 \in [0, \max\{u_1, c_1 u_1\}], p_2 \in [0, \max\{u_2, c u_2\}]\}.$$

The prices that solve this problem are

$$p_1^* = \frac{(u_1-u_2)(2(c-1)\lambda u_1 - 2cu_1 - (c-2)\lambda u_2)}{c(\lambda-1)(4u_1-u_2)+4\lambda(u_2-u_1)} \quad \text{and} \quad p_2^* = -\frac{c(\lambda+1)u_2(u_1-u_2)}{c(\lambda-1)(4u_1-u_2)+4\lambda(u_2-u_1)}, \tag{B.2}$$

provided that $\lambda_1 > \lambda > \lambda_2$, where $\lambda_2 := \frac{-4cu_1+(c-1)u_2+u_1}{(3-4c)u_1+(c-3)u_2}$ is the unique value where $g(\lambda) := (p_1^* c u_2 - p_2^* c_1 u_1)$ vanishes. Thus, the firms compete with the optimal prices (p_1^*, p_2^*) in Equation B.2 which leads to the relative prices $p_1^*/p_2^* = \frac{2c(1+\lambda)u_1+c\lambda u_2+2\lambda(u_1-u_2)}{c(1+\lambda)u_2}$.

Finally when $\lambda < \lambda_1 >$ and $\lambda < \lambda_2$, then none of the firms exist in the ecosystem market, and the firms consider the following profit functions:

$$\Pi_1^* = \max\left\{\Pi_1 : p_1 - p_2 < u_1 - u_2, p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\}, \text{and}$$

$$\Pi_2^* = \max\left\{\Pi_2 : p_1u_2 > p_2u_1, p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\},$$

and the prices that solve this problem are

$$p_1^* = 2u_1\left(\frac{3u_1}{u_2-4u_1}+1\right) \quad \text{and} \quad p_2^* = \frac{u_2(u_1-u_2)}{4u_1-u_2}, \tag{B.3}$$

which implies the relative prices $p_1^*/p_2^* = \frac{2u_1}{u_2}$. This concludes our proof. ∎

Proof of Proposition 3.9. We prove the statement along the similar lines to those in Proposition 3.8. We have u_1, u_2, c, c_1 fixed such that $u_2 > u_1 > 0$ and it follows that $c_1 > c \in (0, 2)$. Firms $(i = 1, 2)$ sets prices p_i^* to maximize their profits Π_i (see Equation 3.3 and Equation 3.4) simultaneously. If both firms target both submarkets, then their profits can be written as

$$\Pi_1^* = \max\left\{\Pi_1 : p_2u_1 > p_1u_2, p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\}, \text{and}$$

$$\Pi_2^* = \max\left\{\Pi_2 : p_2 - p_1 < u_2 - u_1, p_2 - p_1 < cu_2 - c_1u_1, p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\}.$$

Solving this problem yields the following prices

$$p_1^* = -\frac{c_1u_1(u_1-u_2)(c_1u_1-cu_2)}{-cc_1(\lambda-1)u_2(u_1-4u_2)+4c\lambda u_2(u_1-u_2)+c_1^2(\lambda-1)u_1(u_1-4u_2)+c_1\lambda u_1(u_2-u_1)} \quad \text{and}$$

$$p_2^* = -\frac{2u_2(u_1-u_2)(c_1u_1-cu_2)\left(-cc_1(\lambda-1)u_2+c\lambda(u_2-u_1)+c_1^2(\lambda-1)u_1\right)}{\left(-cc_1(\lambda-1)u_2(u_1-4u_2)+4c\lambda u_2(u_1-u_2)+c_1^2(\lambda-1)u_1(u_1-4u_2)+c_1\lambda u_1(u_2-u_1)\right)\left(-\lambda((c-1)u_2+u_1)+cu_2+c_1(\lambda-1)u_1\right)}, \tag{B.4}$$

as long as $\lambda > \lambda_1$ and $\lambda < \lambda_2$, where λ_1 and λ_2 are the unique values where $f_2(\lambda) := (cu_2 - c_1u_1) - (p_2^* - p_1^*)$ and $g_2(\lambda) := p_2^*u_1 - p_1^*u_2$ vanish, respectively. To prove the uniqueness of λ_1, we note that $\lim_{\lambda\to 1} f_2(\lambda) = \frac{2cu_2}{4cu_2-c_1u_1} > 0$ and $\frac{\partial f_2(\lambda)}{\partial\lambda} < 0$. We do not provide the closed form expression for λ_1 due to the space limitations. In view of λ_2, it follows that $\frac{\partial g_2(\lambda)}{\partial\lambda} < 0$. We note that $\lim_{\lambda\to 1} f_2(\lambda) = \frac{u_2(2c-c_1)(cu_2-c_1u_1)}{(u_1-u_2)(c_1u_1-4cu_2)} < 0$ if and only if $c_1 < 2c$, and we have $\lim_{\lambda\to 0} f_2(\lambda) = \frac{u_2}{4u_2-u_1} > 0$. Therefore, for $c_1 > 2c$ we have $\lambda_2 > 1$, which, in turn, implies that the constraint $g_2(\lambda) := p_2^*u_1 - p_2^*u_1$ is never violated. Thus, for $c_1/c < 2$ and $\lambda > \lambda_1$, the firms compete with the optimal prices (p_1^*, p_2^*) in Equation B.4 which leads to the relative prices $p_1^*/p_2^* = \frac{c_1u_1(-\lambda((c-1)u_2+u_1)+cu_2+c_1(\lambda-1)u_1)}{2u_2\left(-cc_1(\lambda-1)u_2+c\lambda(u_2-u_1)+c_1^2(\lambda-1)u_1\right)}$.

If instead, we have $\lambda < \lambda_1$ for $c_1/c < 2$, then the firms do not compete with the prices in Equation B.4 since these prices lead to $f_2(\lambda) < 0$, and the category does not compete in the ecosystem market when $\lambda < \lambda_1$. In this case, the profits of the two firms can be written as

$$\Pi_1^* = \max\left\{\Pi_1 : p_2u_1 > p_1u_2, p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\},$$

$$\Pi_2^* = \max\left\{\Pi_2 : p_2 - p_1 < u_2 - u_1, p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\}; \tag{B.5}$$

and the prices that solve this problem are

$$p_1^* = -\frac{c_1(\lambda+1)u_1(u_1-u_2)}{c_1(\lambda-1)(u_1-4u_2)+4\lambda(u_2-u_1)} \quad \text{and} \quad p_2^* = -\frac{(u_1-u_2)(c_1\lambda u_1-2c_1(\lambda-1)u_2+2\lambda(u_2-u_1))}{c_1(\lambda-1)(u_1-4u_2)+4\lambda(u_2-u_1)}, \tag{B.6}$$

and the relative prices $p_1^*/p_2^* = \frac{c_1(\lambda+1)u_1}{c_1\lambda u_1-2c_1(\lambda-1)u_2+2\lambda(u_2-u_1)}$. That completes the proof for Proposition 3.9-(1). If, on the other hand, we have $c_1/c > 2$, then there exists $0 < \lambda_2 < 1$, and hence the firms set the prices (p_1^*, p_2^*) in Equation B.4 as long as $\lambda_2 > \lambda > \lambda_1$. Otherwise when $\lambda > \lambda_2$ or $\lambda < \lambda_1$, the constraints $f_2(\lambda) := (cu_2 - c_1u_1) - (p_2^* - p_1^*)$ and $g_2(\lambda) := p_2^*u_1 - p_1^*u_2$ are violated if firms use the prices in Equation B.4, respectively. In particular, if we have $\lambda < \lambda_1$ but $\lambda > \lambda_2$, then it follows that $f_2(\lambda) < 0$ while $g_2(\lambda) > 0$ and hence the firms solve the problem specified in (B.5). Therefore, they compete with the prices in Equation B.6. If instead we have $\lambda > \lambda_2$ but $\lambda > \lambda_1$, then we have $g_2(\lambda) < 0$, and hence the firms have the profits as

$$\Pi_1^* = \max\left\{\Pi_1 : p_2c_1u_1 > p_1cu_2, p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\},$$

$$\Pi_2^* = \max\left\{\Pi_2 : p_2 - p_1 < cu_2 - c_1u_1, p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\}, \tag{B.7}$$

and hence the prices are set as

$$p_1^* = \frac{c_1u_1(cu_2 - c_1u_1)}{-4c^2(\lambda-1)u_2 + 4c(c_1(\lambda-1)u_1 + \lambda u_2) - c_1\lambda u_1} \quad \text{and} \quad p_2^* = \frac{2cu_2(c_1u_1 - cu_2)}{4c^2(\lambda-1)u_2 - 4c\lambda(c_1u_1 + u_2) + 4cc_1u_1 + c_1\lambda u_1}. \tag{B.8}$$

Thus, the firms compete with the relative prices $p_1^*/p_2^* = \frac{c_1u_1}{2cu_2}$.

Finally when $\lambda < \lambda_1$ and $\lambda < \lambda_2$, then we have both $f_2(\lambda) < 0$ and $g_2(\lambda)$. Therefore, the ecosystem firm serves solely the ecosystem market and the category firms focuses on Motta's market. Thus, the firms consider the following profit functions:

$$\Pi_1^* = \max\left\{\Pi_1 : p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\},$$

$$\Pi_2^* = \max\left\{\Pi_2 : p_1 \in [0, \max\{u_1, c_1u_1\}], p_2 \in [0, \max\{u_2, cu_2\}]\right\}, \text{and}$$

and the prices that solve this problem are

$$p_1^* = \frac{c_1u_1}{2} \quad \text{and} \quad p_2^* = \frac{u_2}{2}, \tag{B.9}$$

which implies the relative prices $p_1^*/p_2^* = \frac{c_1u_1}{u_2}$. This concludes our proof. ∎

Proof of Proposition 3.10. The statement may be proved along the similar steps to those in Proposition 3.8 and Proposition 3.9 using Equation 3.5 and Equation 3.6. ∎

Proof of Lemma 3.11. The statement follows from Proposition 1 in Motta (1993).

Proof of Lemma 3.12. We prove the statement along similar steps to those in Proposition 1 of in Motta (1993). In particular, we first assume that the firms are either the quality leader or the follower, and solve the two-stage game with sub-game perfection.

We start with the case where quality leader is the ecosystem firm ($u_1 > u_2$). Proposition 3.8 implies that both firms compete in both markets when $\lambda = 1$, and hence the firms set prices in Equation B.1, which simplify to $p_1^* = \frac{2c_1u_1(cu_2 - c_1u_1)}{cu_2 - 4c_1u_1}, p_2^* = \frac{cu_2(cu_2 - c_1u_1)}{cu_2 - 4c_1u_1}$ and hence $(p_1^*/p_2^* = \frac{2c_1u_1}{cu_2})$ for $\lambda = 1$. From these equilibrium prices, we can derive the profits as $\Pi_1^* = \frac{4c_1^2u_1^2(c_1u_1 - cu_2)}{(cu_2 - 4c_1u_1)^2}$ and

$\Pi_2^* = \frac{cc_1u_1u_2(c_1u_1-cu_2)}{(cu_2-4c_1u_1)^2}$. Next, we solve for the quality game, where the firms choose u_i and the ecosystem firm also determines c_1 to optimize their profits $\pi_1 = \Pi_1^* - {u_1}^2/2 - k(c_1-c)^2/2$ and $\pi_2 = \Pi_2^* - {u_2}^2/2$. One can immediately verify that π_1 is strictly jointly concave in u_1 and c_1 and π_2 is strictly concave in u_2. Following, we derive the first order conditions of this problem as

$$\frac{\partial \pi_1}{\partial u_1} = u_1\left(\frac{4c_1^2\left(2c^2u_2^2 - 3cc_1u_1u_2 + 4c_1^2u_1^2\right)}{(4c_1u_1 - cu_2)^3} - 1\right),$$

$$\frac{\partial \pi_1}{\partial c_1} = c_1\left(\frac{4c_1u_1^3(4c_1u_1 - 3cu_2)}{(4c_1u_1 - cu_2)^3} - k\right) + c\left(\frac{8cc_1u_1^2u_2^2}{(4c_1u_1 - cu_2)^3} + k\right),$$

$$\frac{\partial \pi_2}{\partial u_2} = \frac{cc_1^2u_1^2(4c_1u_1 - 7cu_2)}{(4c_1u_1 - cu_2)^3} - u_2.$$

There exists a unique real solution $u_1^{l*}(c,k) > u_2^{f*}(c,k), c_1^*(k)$ that solves this problem, and we do not provide the closed form of the unique solution due to the space limitations while Figure 3.8 shows numerically the quality equilibria for fixed values of k, c.

Within similar lines, we derive the price and quality equilibria for the case where $u_2 > u_1$. Proposition 3.9 implies that both firms compete in both markets when $\lambda = 1$, and hence the firms set prices in Equation B.1, which simplify to $p_1^* = \frac{c_1u_1(c_1u_1-cu_2)}{c_1u_1-4cu_2}, p_2^* = \frac{2cu_2(cu_2-c_1u_1)}{4cu_2-c_1u_1}$, and hence we have $p_1^*/p_2^* = \frac{c_1u_1}{2cu_2}$ for $\lambda = 1$. From these equilibrium prices, we can derive the profits as $\Pi_1^* = \frac{cc_1u_1u_2(cu_2-c_1u_1)}{(c_1u_1-4cu_2)^2}$ and $\Pi_2^* = \frac{4c^2u_2^2(cu_2-c_1u_1)}{(c_1u_1-4cu_2)^2}$. Next, we solve for the quality game, where the firms choose u_i and the ecosystem firm also determines c_1 to optimize their profits $\pi_1^{'} = \Pi_1^* - {u_1}^2/2 - k(c_1-c)^2/2$ and $\pi_2^{'} = \Pi_2^* - {u_2}^2/2$. One can immediately verify that π_1 is strictly jointly concave in u_1 and c_1 and π_2 is strictly concave in u_2. Following, we derive the first order conditions of this problem as

$$\frac{\partial \pi_1'}{\partial u_1} = \frac{-4c^3u_2^3(c_1 - 16u_1) + c^2c_1u_1u_2^2(7c_1 - 48u_1) + 12cc_1^2u_1^3u_2 - c_1^3u_1^4}{(c_1u_1 - 4cu_2)^3},$$

$$\frac{\partial \pi_1'}{\partial c_1} = -\frac{4c^3 u_1 u_2^3}{(c_1 u_1 - 4c u_2)^3} + \frac{7c^2 c_1 u_1^2 u_2^2}{(c_1 u_1 - 4c u_2)^3} + ck - c_1 k,$$

$$\frac{\partial \pi_2'}{\partial u_2} = u_2 \left(-\frac{4c^2 \left(4c^2 u_2^2 - 3c c_1 u_1 u_2 + 2c_1^2 u_1^2\right)}{(c_1 u_1 - 4c u_2)^3} - 1 \right).$$

As in the previous case, we confirm that there exists a unique real solution $u_2^{l*}(c,k) > u_1^{f*}(c,k), c_1^*(k)$ that solves this problem, and Figure 3.8 shows numerically the quality equilibria for given values of k, c. Our discussion so far has shown that there exists unique local optimum points $(u_1^{l*}(c,k) > u_2^{f*}(c,k), c_1^*(k)$ and $u_2^{l*}(c,k) > u_1^{f*}(c,k), c_1^*(k))$ for each case. To prove that these points are indeed the Nash Equilibria, one needs to verify that none of the players has an incentive to deviate from their quality position. To this end, one needs to confirm that:

$$\pi_1(u_1^{l*}(c,k), c_1^*(k), u_2^{f*}(c,k)) \geq \pi_1'(u_1, c_1, u_2^{f*}(c,k)) \quad \forall \quad u_1 < u_2^{f*}(c,k), \tag{B.10}$$

$$\pi_2(u_1^{l*}(c,k), c_1^*(k), u_2^{f*}(c,k)) \geq \pi_2'(u_1^{l*}(c,k), c_1^*(k), u_2) \quad \forall \quad u_2 > u_1^{l*}(c,k), \tag{B.11}$$

$$\pi_1'(u_1^{f*}(c,k), c_1^*(k), u_2^{l*}(c,k)) \geq \pi_1(u_1, c_1, u_2^{l*}(c,k)) \quad \forall \quad u_1 > u_2^{l*}(c,k), \tag{B.12}$$

$$\pi_1'(u_1^{f*}(c,k), c_1^*(k), u_2^{l*}(c,k)) \geq \pi_1(u_1^{f*}(c,k), c_1^*(k), u_2) \quad \forall \quad u_2 < u_1^{f*}(c,k), \tag{B.13}$$

that is neither of the firms has an incentive to leapfrog the quality position (leader or follower) of its rival (Motta, 1993). We start first with the first statement in (B.10), where we have $\pi_1'(u_1, c_1, u_2^{f*}(c,k)) = \frac{2cu_2(cu_2 - c_1 u_1)}{4cu_2 - c_1 u_1} - {u_1}^2/2 - k(c_1 - c)^2/2$. We follow same steps in Motta (1993), and confirm that $\frac{2cu_2(cu_2 - c_1 u_1)}{4cu_2 - c_1 u_1}$ attains its maximum when $u_1^* = \frac{4cu^{f*}}{7c_1}$, which in turn implies that $\pi_1'(u_1^*, c_1, u_2^{f*}(c,k)) < \frac{cu_2^{f*}(c,k)}{48}$. This, in turn, implies that $\pi_1'(u_1^*, c_1, u_2^{f*}(c,k)) \leq \pi_1(u_1^{l*}(c,k), c_1^*(k), u_2^{f*}(c,k))$ for all $c_1 > c$ since we have $\pi_1(u_1^{l*}(c,k), c_1^*(k), u_2^{f*}(c,k)) \geq \frac{cu_2^{f*}(c,k)}{48}$. Thus, we conclude that $\pi_1(u_1^{l*}(c,k), c_1^*(k), u_2^{f*}(c,k)) \geq \pi_1'(u_1, c_1, u_2^{f*}(c,k))$ for all $u_1 < u_2^{f*}(c,k)$.

For the statement in (B.11), we rewrite $\pi_2'(u_1^{l*}(c,k), c_1^*(k), u_2)$ as

$$\pi_2'(u_1^{l*}(c,k), c_1^*(k), u_2) = \frac{1}{2} m^2 u_1^{l*} \left(\frac{8c^2(cm - c_1)}{(c_1 - 4cm)^2} - u_1 \right)$$

for $u_2 = mu_1^{l*}$, and $m > 1$ as in Motta (1993).

One can readily verify that $\left(\frac{8c^2(cm-c_1)}{(c_1-4cm)^2} - u_1\right) < 0$ since $c < c_1$ and $m > 1$. Thus, we confirm that $\pi_2'(u_1^{l*}(c,k), c_1^*(k), u_2) < 0 < \pi_2(u_1^{l*}(c,k), c_1^*(k), u_2^{f*}(c,k))$. Thus, we conclude that firms do not deviate from $u_1^{l*}(c,k) > u_2^{f*}(c,k)$, and hence $u_1^{l*}(c,k), c_1^*(c,k), u_2^{f*}(c,k)$ is a Nash Equilibrium. We also verify that the firms do not deviate from $u_2^{l*}(c,k) > u_1^{f*}(c,k)$ by showing that the statements in (B.12) and (B.13) hold along to similar steps shown for (B.10) and (B.11). This concludes our proof. ∎

Proof of Corollary 3.13. We start with the case where the ecosystem firm is the quality leader, that is when $(u_1 > u_2)$. Lemma 3.1 states that the ecosystem firm deploys an aggressive built-in complementarity if and only if $(1-c)u_1^{l*} > (1-c_1^*)u_2^{*f}$, and hence we have $\bar{Y}_E < \bar{Y}_M$ when $c \geq c' = \frac{c_1^* u_1^{l*} - u_1^{l*} + u_2^{*f}}{u_2^{*f}}$. We confirm that $c' < 1$, which is also implied by Lemma 3.1. This proves the statement in (1-a). Within similar lines, we prove the statement in (1-d) using Lemma 3.4 that states that the ecosystem firm implements an aggressive built-in complementarity if and only if $(1-c)u_1^{f*} > (1-c_1^*)u_2^{*l}$, and hence the category firm deploy an aggressive built-in strategy if and only if $c \geq c' = \frac{c_1^* u_1^{f*} - u_1^{f*} + u_2^{*l}}{u_2^{*l}}$. We verify that $c - c'$ vanish at the equilibrium quality levels is and only if $c = c' = 1$. Thus, when $c > c' = 1$, we have $\bar{Y}_E > \bar{Y}_M$ and hence an aggressive built-in complementarity while we have $\bar{Y}_E < \bar{Y}_M$ when $c < c' = 1$. That proves the first statements in (1-c) and (1-d).

In view of (1-b), we remind the reader that the equilibrium quality levels for $\lambda = 0$ are $u_1^{*l} = 0.253$ and $u_2^{*f} = 0.048$, by Lemma 3.11. Once we insert $u_1^{*l} = 0.253$ and $u_2^{*f} = 0.048$ in the profit functions π_1 and π_2 (see proof of Lemma 3.12 for the functions), we obtain $\pi_1 =$ and $\pi_2 =$. Following, we verify that π_1 attains its maximum at a unique value $c_1 = c = c''$ such that $c'' < c'$ (the closed form expression of $= c''$ is not provided due to the space considerations). This, in turn, implies that when $c = c''$, we have $u_1^{*l} = 0.253$ and $u_2^{*f} = 0.048$ as the equilibrium stand-alone qualities for $\lambda = 1$. We also confirm that when $\frac{\partial u_1^{*l}}{\partial c} > 0$ and $\frac{\partial u_2^{*l}}{\partial c} > 0$ with implicit differentiation. In particular, we confirm that $\frac{\partial u_1^{*l}}{\partial c} = -\frac{\partial R/\partial c}{\partial R/\partial u_1} > 0$ where $R(u_1, c) := \frac{\partial \pi_1}{\partial u_1} = 0$ due to the firs-order optimality condition. Thus, we prove that $u_1^{*l} < 0.253$ and $u_2^{*f} < 0.048$ iff $c < c''$. We omit the proof for second statements for (1-c) and (1-d) which follows along the

similar steps in (1-b). ∎

Proof of Proposition 3.14. The proof is based on Proposition 3.8 and follows along the similar lines to those in Lemma 3.12 for $u_1 > u_2$. Likewise, the proof follows from Proposition 3.9 and with steps similar to those in Lemma 3.12 for the case where $u_2 > u_1$.

Proof of Lemma 3.15. The statement can be proved along the similar steps to those in Lemma 3.12 and Proposition 1 of Motta (1993), for fixed levels of $\lambda \in (0,1)$ and corresponding market segmentation mentioned in Proposition 3.14. ∎

Proof of Lemma 3.16. We provide the sketch of the proof as in the following. We assume that $k > 0$ is fixed, and start first with the case where $c > 1$. Then, the firms compete in both sub-markets (see Proposition 3.14) and the optimal profits are derived as $\pi_1(u_1, u_2, c_1)$ and $\pi_2(u_1, u_2, c_1)$ using the optimal prices specified in (B.1). Solving this problem, one yields two equilibrium points as u_1^{*l}, u_2^{*f} and c_1^* or u_1^{*f}, u_2^{*l} and c_1^*. In view of u_1^{*l}, we verify that $\frac{\partial u_1^{*l}}{\partial \lambda} = -\frac{\partial R/\partial \lambda}{\partial R/\partial u_1} > 0$ where $R(u_1, \lambda) := \frac{\partial \pi_1}{\partial u_1} = 0$ due to the first-order optimality condition. Likewise, we use implicit differentiation to verify that $\frac{\partial u_2^{*f}}{\partial \lambda} > 0$, $\frac{\partial u_1^{*f}}{\partial \lambda} > 0$ and $\frac{\partial c_1^*}{\partial \lambda} > 0$. If $\max\{\bar{c}_1(k), \bar{c}_2(k) < c < 1\} < c < 1$, then still both firms compete in both sub-markets as specified by Proposition 3.14. In this case, firms set the prices in (B.1) in the second-stage of the game which leads to two equilibrium points as u_1^{*l}, u_2^{*f} and c_1^* or u_1^{*f}, u_2^{*l} and c_1^*. As in the previous case, we use implicit differentiation to compute the following $\frac{\partial u_1^{*l}}{\partial \lambda}$, $\frac{\partial u_2^{*f}}{\partial \lambda}$, $\frac{\partial u_1^{*f}}{\partial \lambda}$ and $\frac{\partial c_1^*}{\partial \lambda}$. We note that for $\max\{\bar{c}_1(k), \bar{c}_2(k) < c < 1\} < c < 1$ it follows that $\frac{\partial u_1^{*l}}{\partial \lambda} > 0$, $\frac{\partial u_2^{*f}}{\partial \lambda} > 0$ and $\frac{\partial c_1^*}{\partial \lambda} > 0$ as in the case where $c > 1$. However, we have $\frac{\partial u_1^{*l}}{\partial \lambda} < 0$ and $\frac{\partial u_2^{*f}}{\partial \lambda} < 0$ for sufficiently small λ. Otherwise when $c < \bar{c}_1(k)$ and $c < \bar{c}_2(k)$, the firms do not necessarily focus on both sub-markets as outlined by Proposition 3.14 and hence they deploy various pricing regimes as specified in detail in Proposition 3.8 and Proposition 3.9. The rest of the discussion follows in similar steps to the cases of $c > 1$ and $\max\{\bar{c}_1(k), \bar{c}_2(k) < c < 1\} < c < 1$. This concludes the outline of the proof. ∎

Proof of Lemma 3.17. We provide the sketch of the proof as in the following. We assume that $k_2, k > 0$ are fixed. We remind the reader that when $\lambda > \lambda_1^*$ for the case where the ecosystem firm is the quality leader and when $\lambda > \lambda_2^*$ for the case where the category firm is the quality

leader, both firms compete in both sub-markets by Proposition 3.14. In view of the case where the ecosystem firm is the quality leader and $\lambda > \lambda_1^*$, the profits of the firms can be written as

$$\Pi_1(p_1,p_2,u_1,u_2,c_1,c_2) = p_1\bigg((1-\lambda)\Big(1-\frac{p_1-p_2}{u_1-u_2}\Big)+\lambda\Big(1-\max\big\{\frac{p_1-p_2}{c_1u_1-cu_2},\frac{p_1}{c_1u_1}\big\}\Big)\bigg)-u_1^2/2-k(c_1-c)^2/2+k_2(c_2-c)^2/2$$

$$\Pi_2(p_1,p_2,u_1,u_2,c_1,c_2) = p_2\bigg((1-\lambda)\Big(\frac{p_1-p_2}{u_1-u_2}-\frac{p_2}{u_2}\Big)+\lambda\Big(\frac{p_1-p_2}{c_1u_1-cu_2)}-\frac{p_2}{cu_2}\Big)\bigg)-u_2^2/2-k_2(c_2-c)^2/2. \tag{B.14}$$

Within similar lines to Proposition 3.8 and Lemma 3.15, we solve the two stage sub-game perfection and obtain the equilibrium values as $u_1^* > u_2^*$ and c_1^*, c_2^*. To this end, we first solve the second stage of the game as

$$p_1^* = -\frac{2u_1(u_1-u_2)(c_1u_1-c_2u_2)\left(c_1\lambda((c_2-1)u_1+u_2)-c_1c_2u_1+c_2^2(\lambda-1)(-u_2)\right)}{(c_1(\lambda-1)u_1-\lambda((c_2-1)u_2+u_1)+c_2u_2)(c_1u_1(c_2(\lambda-1)(4u_1-u_2)+4\lambda(u_2-u_1))+c_2u_2(\lambda(u_1-u_2)-c_2(\lambda-1)(4u_1-u_2)))},$$

$$\text{and } p_2^* = \frac{c_2u_2(u_1-u_2)(c_1u_1-c_2u_2)}{c_1u_1(4\lambda(u_1-u_2)-c_2(\lambda-1)(4u_1-u_2))+c_2u_2(c_2(\lambda-1)(4u_1-u_2)+\lambda(u_2-u_1))}, \tag{B.15}$$

which yield the first-stage profits $\Pi_1^*(u_1,u_2,c_1,c_2)$ and $\Pi_2^*(u_1,u_2,c_1,c_2)$. We do not provide the expression for the profits due to the space considerations. Following the steps in Lemma 3.12, we derive the equilibrium quality levels as $u_1^* > u_2^*$ and $c_1^* > c_2^*$. The discussion for the case where the category firm is the quality leader and $\lambda > \lambda_2^*$ follows along the similar lines where we derive the equilibrium as $u_2^* > u_1^*$ and $c_2^* > c_1^*$. For the second half of the proof, we confirm that $\frac{\partial c_1^*-c_2^*}{\partial\lambda} > 0$ for $\lambda > \lambda_1^*$ and $\frac{\partial c_2^*-c_1^*}{\partial\lambda} > 0$ for $\lambda > \lambda_1^*$ for $\lambda > \lambda_2^*$ as long as c is sufficiently large using implicit differentiation. The threshold that defines whether or not c is sufficiently large is denoted by $\hat{c}(\lambda)$. While obtaining the closed form expression for $\hat{c}(\lambda)$ is complex, one can immediately derive $\hat{c}(\lambda)$ for fixed values of λ, k, k_2. Therefore, we have $\frac{\partial|c_1^*-c_2^*|}{\partial\lambda} > 0$ for $\lambda > \min\{\lambda_1^*, \lambda_2^*\}$, the range of λ assumed in the statement. If instead we have c is small, then we have $\frac{\partial|c_1^*-c_2^*|}{\partial\lambda} > 0$ if and only if λ is sufficiently large, $\lambda > \lambda'$. We note that λ' can be easily computed for given values of c, k and k_2. ∎

Proof of Lemma 3.18. We provide the sketch of the proof as in the following. We assume that k is fixed. We remind the reader that when $\lambda > \lambda_1^*$ for the case where the ecosystem firm is the

quality leader and when $\lambda > \lambda_2^*$ for the case where the category firm is the quality leader, both firms compete in both sub-markets by Proposition 3.14. We first assume that the ecosystem firm is the quality leader, and hence the firms have the profit functions $\Pi_1^*(u_1, u_2, c_1, c_2)$ and $\Pi_2^*(u_1, u_2, c_1, c_2)$ specified in the proof of Lemma 3.17, except now the ecosystem firm has one extra decision variable, k_2. Using the solution we obtained in Lemma 3.17 for fixed k, that leads to $\Pi_1^*(u_1, u_2, c_1, c_2)$, we verify that $\frac{\partial \Pi_1^*(u_1,u_2,c_1,c_2)}{\partial k_2} > 0$. One can verify this in different ways such as using the envelope theorem or the implicit differentiation. Therefore, we conclude that $k_2^* \to \infty$ if $u_1 > u_2$. The discussion for the opposite case is also similar, where we obtain $\frac{\partial \Pi_1^*(u_1,u_2,c_1,c_2)}{\partial k_2} < 0$, and hence $k_2^* \to 0$ if $u_2 > u_1$. ∎

B.2 Supplementary Discussion for the Consumer Welfare

To derive the consumer welfare, we revisit the *net additional surplus* $V(y;i)$ from Section 3.2, where where $i = \{1, 2, 0\}$ denotes the set of actions (buying from firm 1, 2 or not buying) and a characterization function (I) represents if a consumer has bought the base category ($I = 1$) or not ($I = 0$):

$$V(y;i) = \left\{ \begin{array}{ll} yu_1(1 - I(1 - c_1)) - p_1 & \text{if } i = 1 \\ yu_2(1 - I(1 - c)) - p_2 & \text{if } i = 2 \\ 0 & \text{if } i = 0 \end{array} \right\}$$

We remind the reader that there exists four groups of consumers contributing to the welfare: Motta's consumers buying from the ecosystem firm (firm 1) and the category firm (firm 2), and the ecosystem consumers buying from the ecosystem firm (firm 1) and the category firm (firm 2). To obtain the total consumer welfare for each group of consumers, we aggregate the net additional surpluses of the consumers that belongs to the group of interest, which we discuss next.

First, we focus on the case where the ecosystem firm is the quality leader ($u_1 > u_2$). The total consumer welfare $CW(i,j)$, in turn, associated with each group of consumers that belongs to the sub-market j and obtaining the new good from the firm (i) is:

$$CW(i,j)|u_1 > u_2 = \left\{ \begin{array}{ll} (1-\lambda)\big(\int_{y=\bar{Y}_M}^{1} V(y;i)\,dy\big) & \text{if } i = 1, j = M \\ (1-\lambda)\big(\int_{y=\underline{Y}_M}^{\bar{Y}_M} V(y;i)\,dy\big) & \text{if } i = 2, j = M \\ \lambda\big(\int_{y=\bar{Y}_E}^{1} V(y;i)\,dy\big) & \text{if } i = 1, j = E \\ \lambda\big(\int_{y=\underline{Y}_E}^{\bar{Y}_E} V(y;i)\,dy\big) & \text{if } i = 2, j = E \end{array} \right\}.$$

Secondly, we provide the consumer welfare for the case where the category firm is the quality leader ($u_2 > u_1$):

$$CW(i,j)|u_2 > u_1 = \left\{ \begin{array}{ll} (1-\lambda)\big(\int_{y=\underline{Y}_M}^{\bar{Y}_M} V(y;i)\,dy\big) & \text{if } i=1, j=M \\ (1-\lambda)\big(\int_{y=\bar{Y}_M}^{1} V(y;i)\,dy\big) & \text{if } i=2, j=M \\ \lambda\big(\int_{y=\underline{Y}_E}^{\bar{Y}_E} V(y;i)\,dy\big) & \text{if } i=1, j=E \\ \lambda\big(\int_{y=\bar{Y}_E}^{1} V(y;i)\,dy\big) & \text{if } i=2, j=E \end{array} \right\}.$$

Appendix C

Chapter 4

Column1	Column2	Column3	Viewer 1	Column4	Viewer 2	Column5	Viewer 3	Column6	Viewer 4	Column7	Viewer 5	Column8
Soap Opera	Episodes	Epsiode Name	Start Time	End Time	Start Time	End Time	Start Time	End Time	Start Time	End Time	Start Time	End Time
A	1	A1	27/06/2018 20:55	27/06/2018 21:55	27/06/2018 21:36	27/06/2018 22:55	27/06/2018 20:00	27/06/2018 20:50	27/06/2018 20:55	27/06/2018 21:55	01/07/2018 21:36	01/07/2018 22:55
A	2	A2	27/06/2018 22:05	28/06/2018 22:55	01/07/2018 22:13	01/07/2018 23:15	27/06/2018 20:51	27/06/2018 21:50	27/06/2018 22:05	28/06/2018 22:55	01/07/2018 22:57	01/07/2018 23:57
A	3	A3	18/07/2018 22:05	18/07/2018 22:52	12/07/2018 12:05	12/07/2018 13:12	27/06/2018 21:50	27/06/2018 22:50	NAN	NAN	02/07/2018 12:05	02/07/2018 13:12
A	4	A4	19/07/2018 00:51	19/07/2018 01:51	15/07/2018 00:51	15/07/2018 01:51	27/06/2018 22:52	27/06/2018 23:50	NAN	NAN	03/07/2018 00:51	03/07/2018 01:51
A	5	A5	19/07/2018 01:53	19/07/2018 02:51	15/07/2018 01:53	15/07/2018 02:51	27/06/2018 23:52	28/06/2018 00:51	NAN	NAN	03/07/2018 01:53	03/07/2018 02:51
A	6	A6	27/07/2018 09:14	27/07/2018 10:14	15/07/2018 09:14	15/07/2018 10:14	15/07/2018 20:00	15/07/2018 20:50	NAN	NAN	03/07/2018 09:14	03/07/2018 10:14
A	7	A7	27/07/2018 11:04	27/07/2018 11:54	22/07/2018 11:04	22/07/2018 11:54	15/07/2018 20:51	15/07/2018 21:50	NAN	NAN	03/07/2018 11:04	03/07/2018 11:54
A	8	A8	27/07/2018 14:31	27/07/2018 15:19	22/07/2018 14:31	22/07/2018 15:19	15/07/2018 21:50	15/07/2018 22:50	NAN	NAN	NAN	NAN
A	9	A9	28/07/2018 06:13	28/07/2018 07:03	23/07/2018 06:13	23/07/2018 07:03	27/07/2018 22:52	27/07/2018 23:50	NAN	NAN	20/07/2018 06:13	20/07/2018 07:03
A	10	A10	28/07/2018 10:45	28/07/2018 11:41	25/07/2018 10:45	25/07/2018 11:41	27/07/2018 23:52	28/07/2018 00:51	NAN	NAN	20/07/2018 10:40	20/07/2018 11:41
B	1	B1	29/06/2018 20:55	29/06/2018 21:43	13/07/2018 12:05	13/07/2018 12:53	02/07/2018 21:50	02/07/2018 22:38	NAN	NAN	21/07/2018 11:41	21/07/2018 12:29
B	2	B2	30/06/2018 21:43	30/06/2018 22:31	15/07/2018 12:53	15/07/2018 13:41	04/07/2018 22:38	04/07/2018 23:26	NAN	NAN	22/07/2018 12:29	22/07/2018 13:17
B	3	B3	30/06/2018 23:26	01/07/2018 00:14	20/07/2018 13:41	20/07/2018 14:29	09/07/2018 23:26	10/07/2018 00:14	NAN	NAN	23/07/2018 13:17	23/07/2018 14:05
B	4	B4	01/07/2018 01:10	01/07/2018 01:58	21/07/2018 14:29	21/07/2018 15:17	11/07/2018 00:14	11/07/2018 01:02	NAN	NAN	24/07/2018 14:05	24/07/2018 14:53
B	5	B5	01/07/2018 02:53	01/07/2018 03:41	22/07/2018 12:53	22/07/2018 13:41	11/07/2018 22:38	11/07/2018 23:26	NAN	NAN	25/07/2018 14:53	25/07/2018 15:41
B	6	B6	01/07/2018 04:37	01/07/2018 05:25	22/07/2018 14:36	22/07/2018 15:24	12/07/2018 00:21	12/07/2018 01:09	NAN	NAN	26/07/2018 15:41	26/07/2018 16:29
B	7	B7	01/07/2018 06:20	01/07/2018 07:08	22/07/2018 16:20	22/07/2018 17:08	12/07/2018 02:04	12/07/2018 02:52	NAN	NAN	27/07/2018 16:29	27/07/2018 17:17
B	8	B8	11/07/2018 07:08	11/07/2018 07:56	22/07/2018 18:03	22/07/2018 18:51	12/07/2018 03:48	12/07/2018 04:36	NAN	NAN	28/07/2018 17:17	28/07/2018 18:05
B	9	B9	13/07/2018 07:08	13/07/2018 07:56	22/07/2018 19:47	22/07/2018 20:35	12/07/2018 05:31	12/07/2018 06:19	12/07/2018 05:31	12/07/2018 06:19	29/07/2018 18:05	29/07/2018 18:53
B	10	B10	14/07/2018 05:32	14/07/2018 06:20	22/07/2018 21:30	22/07/2018 22:18	12/07/2018 07:15	12/07/2018 08:03	12/07/2018 07:15	12/07/2018 08:03	30/07/2018 18:53	30/07/2018 19:41
C	1	C1	NAN	NAN	NAN	NAN	25/06/2018 20:55	25/06/2018 21:43	29/06/2018 03:12	29/06/2018 04:00	01/07/2018 06:31	01/07/2018 07:19
C	2	C2	NAN	NAN	NAN	NAN	26/06/2018 21:43	26/06/2018 22:31	01/07/2018 04:00	01/07/2018 04:48	01/07/2018 07:41	01/07/2018 08:29
C	3	C3	NAN	NAN	NAN	NAN	26/06/2018 23:26	27/06/2018 00:14	06/07/2018 04:48	06/07/2018 05:36	22/07/2018 07:41	22/07/2018 08:29
C	4	C4	NAN	NAN	NAN	NAN	27/06/2018 01:10	27/06/2018 01:58	07/07/2018 05:36	07/07/2018 06:24	22/07/2018 10:27	22/07/2018 11:15
C	5	C5	NAN	NAN	NAN	NAN	27/06/2018 02:53	27/06/2018 03:41	08/07/2018 04:00	08/07/2018 04:48	22/07/2018 11:29	22/07/2018 12:17
C	6	C6	NAN	NAN	NAN	NAN	27/06/2018 04:37	27/06/2018 05:25	08/07/2018 05:43	08/07/2018 06:31	30/07/2018 18:50	30/07/2018 19:38
C	7	C7	NAN	NAN	NAN	NAN	27/06/2018 06:20	27/06/2018 07:08	08/07/2018 07:26	08/07/2018 08:14	30/07/2018 20:40	30/07/2018 21:28
C	8	C8	NAN	NAN	NAN	NAN	07/07/2018 07:08	07/07/2018 07:56	08/07/2018 09:10	08/07/2018 09:58	31/07/2018 00:07	31/07/2018 00:55
C	9	C9	NAN	NAN	NAN	NAN	09/07/2018 07:08	09/07/2018 07:56	08/07/2018 10:53	08/07/2018 11:41	31/07/2018 15:49	31/07/2018 16:37
C	10	C10	NAN	NAN	NAN	NAN	10/07/2018 05:32	10/07/2018 06:20	08/07/2018 12:37	08/07/2018 13:25	31/07/2018 20:21	31/07/2018 21:09
D	1	D1	23/07/2018 14:29	23/07/2018 15:17	28/06/2018 16:07	28/06/2018 16:55	30/06/2018 22:24	30/06/2018 23:12	01/07/2018 16:00	01/07/2018 16:48	03/07/2018 06:31	03/07/2018 07:19
D	2	D2	25/07/2018 15:17	25/07/2018 16:05	29/06/2018 16:55	29/06/2018 17:43	02/07/2018 23:12	03/07/2018 00:00	03/07/2018 16:48	03/07/2018 17:36	04/07/2018 07:19	04/07/2018 08:07
D	3	D3	30/07/2018 16:05	30/07/2018 16:53	29/06/2018 18:38	29/06/2018 19:26	08/07/2018 00:00	08/07/2018 00:48	08/07/2018 17:36	08/07/2018 18:24	04/07/2018 09:02	04/07/2018 09:50
D	4	D4	31/07/2018 16:53	31/07/2018 17:41	29/06/2018 20:22	29/06/2018 21:10	09/07/2018 00:48	09/07/2018 01:36	09/07/2018 18:24	09/07/2018 19:12	04/07/2018 10:46	04/07/2018 11:34
D	5	D5	01/08/2018 15:17	01/08/2018 16:05	29/06/2018 22:05	29/06/2018 22:53	09/07/2018 23:12	10/07/2018 00:00	10/07/2018 16:48	10/07/2018 17:36	04/07/2018 12:29	04/07/2018 13:17
D	6	D6	01/08/2018 17:00	01/08/2018 17:48	29/06/2018 23:49	30/06/2018 00:37	10/07/2018 00:55	10/07/2018 01:43	10/07/2018 18:31	10/07/2018 19:19	04/07/2018 14:13	04/07/2018 15:01
D	7	D7	01/08/2018 18:44	01/08/2018 19:32	30/06/2018 01:32	30/06/2018 02:20	10/07/2018 02:38	10/07/2018 03:26	10/07/2018 20:14	10/07/2018 21:02	04/07/2018 15:56	04/07/2018 16:44
D	8	D8	01/08/2018 20:27	01/08/2018 21:15	10/07/2018 02:20	10/07/2018 03:08	10/07/2018 04:22	10/07/2018 05:10	10/07/2018 21:58	10/07/2018 22:46	14/07/2018 16:44	14/07/2018 17:32
D	9	D9	01/08/2018 22:11	01/08/2018 22:59	12/07/2018 02:20	12/07/2018 03:08	10/07/2018 06:05	10/07/2018 06:53	10/07/2018 23:41	11/07/2018 00:29	16/07/2018 16:44	16/07/2018 17:32
D	10	D10	01/08/2018 23:54	02/08/2018 00:42	13/07/2018 00:44	13/07/2018 01:32	10/07/2018 07:49	10/07/2018 08:37	11/07/2018 01:25	11/07/2018 02:13	17/07/2018 15:08	17/07/2018 15:56
E	1	E1	31/07/2018 23:17	01/08/2018 00:05	03/07/2018 17:36	03/07/2018 18:24	03/07/2018 11:12	03/07/2018 12:00	NAN	NAN	05/07/2018 00:00	05/07/2018 00:48
E	2	E2	03/08/2018 00:05	03/08/2018 00:53	05/07/2018 18:24	05/07/2018 19:12	05/07/2018 12:00	05/07/2018 12:48	NAN	NAN	07/07/2018 00:48	07/07/2018 01:36
E	3	E3	08/08/2018 00:53	08/08/2018 01:41	10/07/2018 19:12	10/07/2018 20:00	10/07/2018 12:48	10/07/2018 13:36	NAN	NAN	12/07/2018 01:36	12/07/2018 02:24
E	4	E4	09/08/2018 01:41	09/08/2018 02:29	11/07/2018 20:00	11/07/2018 20:48	11/07/2018 13:36	11/07/2018 14:24	NAN	NAN	13/07/2018 02:24	13/07/2018 03:12
E	5	E5	10/08/2018 00:05	10/08/2018 00:53	12/07/2018 18:24	12/07/2018 19:12	12/07/2018 12:00	12/07/2018 12:48	NAN	NAN	14/07/2018 00:48	14/07/2018 01:36
E	6	E6	10/08/2018 01:48	10/08/2018 02:36	12/07/2018 20:07	12/07/2018 20:55	12/07/2018 13:43	12/07/2018 14:31	NAN	NAN	14/07/2018 02:31	14/07/2018 03:19
E	7	E7	10/08/2018 03:32	10/08/2018 04:20	12/07/2018 21:50	12/07/2018 22:38	12/07/2018 15:26	12/07/2018 16:14	NAN	NAN	14/07/2018 04:14	14/07/2018 05:02
E	8	E8	10/08/2018 05:15	10/08/2018 06:03	12/07/2018 23:34	13/07/2018 00:22	12/07/2018 17:10	12/07/2018 17:58	NAN	NAN	14/07/2018 05:58	14/07/2018 06:46
E	9	E9	10/08/2018 06:59	10/08/2018 07:47	13/07/2018 01:17	13/07/2018 02:05	12/07/2018 18:53	12/07/2018 19:41	NAN	NAN	14/07/2018 07:41	14/07/2018 08:29
E	10	E10	10/08/2018 08:42	10/08/2018 09:30	13/07/2018 03:01	13/07/2018 03:49	12/07/2018 21:25	12/07/2018 22:25	NAN	NAN	14/07/2018 10:13	14/07/2018 11:02

Table C.1: Running Example