

On the advantages of hybrid beam-shell structural finite element models for the efficient analysis of metal wind turbine support towers

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Abstract

Metal wind turbine support towers are very tall and slender shell structures designed to exhibit a stepwise varying distribution of optimised wall thicknesses, with strakes in the upper regions of the tower usually being much thinner than those in the lower regions. Each strake is an individual shell and potentially a critical location for failure, and as the failure location is rarely obvious in advance each strake in theory requires careful meshing in a finite element analysis. It is not unusual for over twenty individual strakes to be present in a design, and the computational cost involved in modelling such a structure with finite elements, particularly in nonlinear analyses, can quickly become prohibitive for execution on a personal workstation. Compromises in mesh resolution must often be made, usually to the detriment of the quality of the global solution.

This paper explores a hybrid beam-shell modelling technique that permits an efficient and insightful analysis of multi-strake wind turbine support towers. It consists of modelling all but a handful of the strakes with beam elements or rigid bodies which have a negligible computational cost compared to shell elements, and to focus the deployment of expensive shell elements only on strakes of interest as part of a resistance assessment. As only strakes meshed with shell elements participate in a failure mechanism, the technique allows the realistic exploration of the relative criticality of *all* tower strakes. The technique is illustrated on a real design of a 1.5 MW 25-strake wind turbine tower.

Keywords

Hybrid finite element models, beam-shell models, wind turbine support towers, buckling, plastic collapse, LBA-MNA, GMNIA, multi-strake shells, optimisation.

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1. Introduction

Political and social efforts to mitigate climate change are ushering in a gradual worldwide migration towards renewable energy sources, chief among them wind energy, with more capacity added each year [1]. This in turn has translated into a huge body of modern scientific literature on wind energy (a search for ‘wind turbine tower’ on sciencedirect.com alone reveals over 9,000 publications as of March 2019). However, most recent advances appear to focus on the wind or seismic-induced loading and associated vibration responses [2,3], as well as blade optimisation [4] and overall risk assessment [5,6]. By contrast, it appears that the structural design of the tower itself is generally considered as a mature topic that has been ‘solved’ by the finite element method and modern design procedures. It is the author’s humble suggestion that the community may benefit from a pause to reflect on whether the powerful and ubiquitous modelling tools now widely available are indeed being used to optimum effect in the analysis and design of these complex structures. To this end, this paper explores the use of hybrid beam-shell structural finite element models for the efficient and insightful analysis of multi-strake metal wind turbine support towers, undertaken within the context of the European Standard on metal shells EN 1993-1-6 [7,8].

In a full shell element model, only the mesh resolution of wall strakes in the vicinity of the buckling or plastic collapse failure mode contributes meaningfully to the accuracy of the overall solution. The true failure mode is often localised, and given the varying levels of compressive and shear membrane stresses and wall thickness along the tower height, rarely obvious in advance. This paper will illustrate how, for this reason, much of the meshing effort in a full shell element model is often misplaced, with the inevitable compromises which must be often made to obtain tolerable computing times degrading the accuracy of the required result. Additionally, a full shell element model offers the analyst virtually no information about failure in locations which are perhaps only slightly less critical but which cannot be ruled out as weak points, given the uncertainty to be found in every structural model. The lack of knowledge of the strength of non-critical strakes also makes it difficult to optimise tower thicknesses under more complex load cases which are not well represented by algebraic expressions [9,10]. In this light,

the technique offers powerful possibilities for improving and optimising the structural design of metal wind turbine support towers.

2. Hybrid beam-shell finite element models

2.1 Principle and modelling detail

The simple principle of a hybrid beam-shell finite element analysis of a metal wind turbine support tower is to model only a select number of ‘strake(s) of interest’ using an appropriately fine mesh of shell elements. A ‘strake of interest’ is any arbitrary strake which the analyst may choose to investigate for criticality in a particular failure mode, regardless of whether it will ultimately be identified as globally critical. The remainder of the tower outside of the ‘strake(s) of interest’ is then modelled using significantly cheaper beam elements approximating the near-cylindrical strakes as piecewise prismatic circular hollow sections or, in special cases, rigid bodies. The use of beam elements in the remaining strakes accounts for their contribution to the global pre-failure elastic stiffness but forbids their participation in a failure mechanism. The savings in computational effort permitted by this simplification allow the use of a finer mesh of shell elements on the ‘strake(s) of interest’ than would otherwise be possible, ultimately providing a more accurate and much faster solution for the global failure mode. Crucially, hybrid models permit an estimate of the conditions of failure in *any* ‘strake(s) of interest’ and give the analyst a realistic insight into the relative criticality of *all* strakes in a tower structure, unachievable using the more expensive full shell models.

The procedure is implemented here using the commercial ABAQUS 2017 [11] finite element software, but it may readily be implemented in any other modelling environment that supports multi-point constraints [12]. Strake connection details adopted in this study are illustrated in Fig. 1. Any two adjacent strakes modelled with shell elements are linked by simple nodal continuity (symbol A). If a shell strake is adjacent to a global boundary, all degrees of freedom (*dofs*) on the boundary edge can be linked via a rigid body kinematic coupling (symbol B) to a centroidal reference point at which boundary conditions may be applied. The connection between a shell and beam strake can similarly be modelled by a rigid body kinematic coupling between the shell

edge *dofs* and the *dofs* of the single beam node at the centroid of the connection (symbol C). Where a shell strake is adjacent to one modelled as a discretised rigid body, the *dofs* on either strake edge may be linked via a tie constraint (symbol D) or simple nodal continuity if the software permits it. In what follows, stiffening flanges will be modelled as beam sections for simplicity, and the use of more complex representations such as the explicit modelling of bolt connections [13], lap joints [14] or door openings [15] etc. in dedicated shell strakes is fully compatible with the hybrid approach encouraged here. The method of implementation of constraints depends on the context, but kinematic couplings in ABAQUS/Standard are usually of the ‘master-slave elimination’ type [11,16].

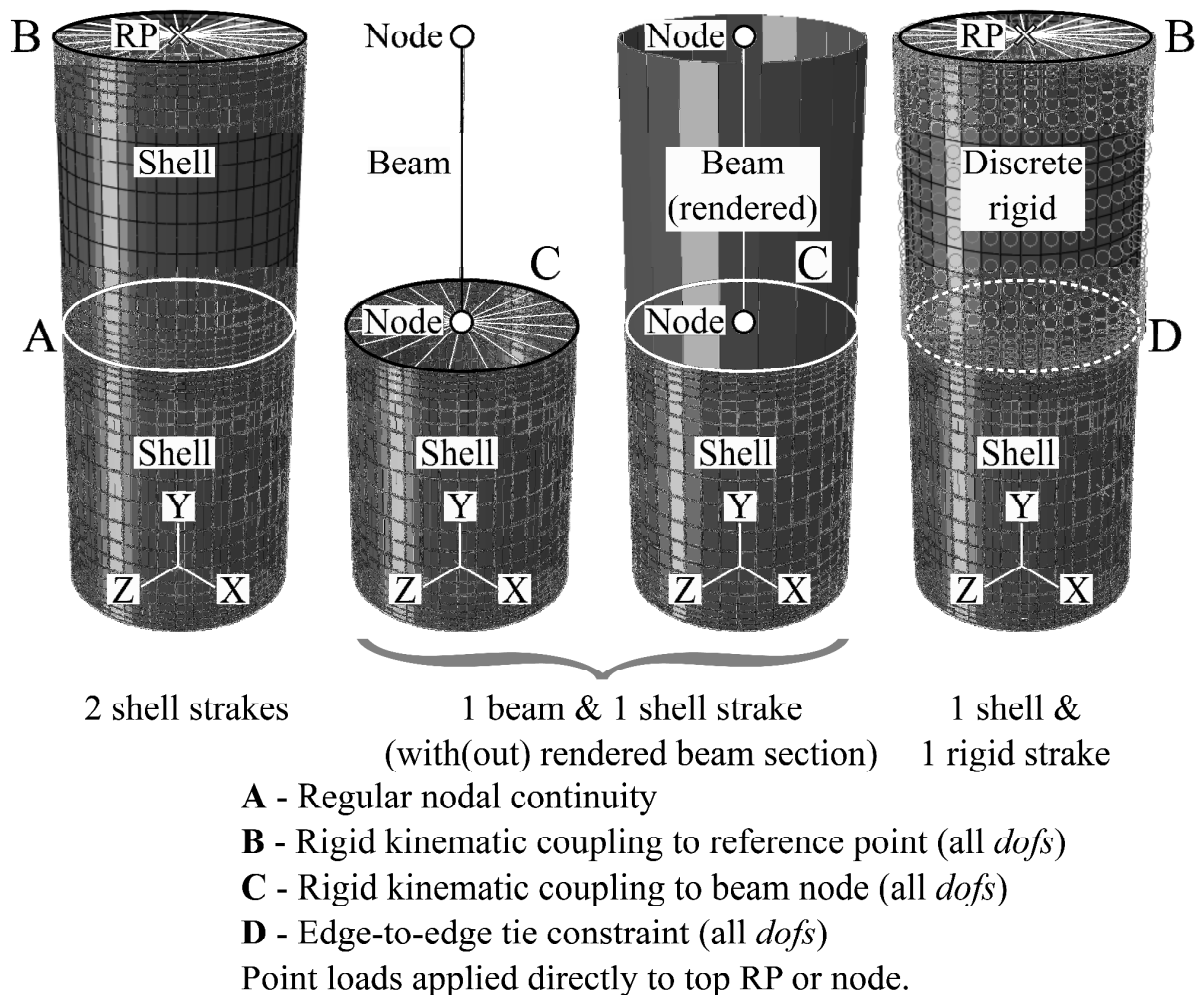


Fig. 1 – Connection details between strakes depending on how they are modelled.

Hybrid beam-shell finite element models are best suited for the analysis of:

- ‘Beam-like’ shell structures with any number of near-cylindrical strakes which can be approximated well by beam elements of prismatic circular hollow cross-section, such as metal wind turbine support towers or chimneys (Fig. 2);
- Long shell structures with a sufficient number of intermediate stiffening rings or flanges to divide the structure into ‘short’ segments so as to preclude any significant pre-buckling cross-sectional ovalisation (which beam elements cannot normally represent) under bending loads;
- Long shell structures with well-defined load cases that may be represented by ‘point’ loads and moments applied at section centroids causing a global stress state dominated by in-plane primary membrane action.

However, they are arguably unsuitable for:

- Multi-strake shells such as containment structures, pressure vessels, silos and tanks that are subject to any sort of detailed distributed surface pressure loading;
- Strongly non-cylindrical shells for which a piecewise-cylindrical approximation is inappropriate (e.g. very shallow conical, spherical or hyperboloid shells);
- Long shells constructed by spiral welding [17] or other technologies where they cannot credibly be treated as shells of revolution.

2.2 *Automated generation of sets of analyses*

It is difficult to know in advance which strake(s) will be critical for failure in a given computational analysis, and thus which strake(s) should be modelled as full shells and which may be idealised as beams using finite elements. It is both necessary and informative to investigate each possibility in turn, as this will identify not just the critical strake(s) but also the relative underutilisation of other strakes, paving the way to an optimisation of the global design. Further, because the computational cost of matrix operations involved in solving systems of linear equations and eigenvalue extraction increases in proportion to the cubic power of the system matrix dimension [18], it is generally computationally faster to perform many smaller analyses rather than one very

large one. It is suggested here to automate the logistics of generating and submitting the full possible set of beam-shell strake configurations using the powerful scripting functionality offered by most commercial software.

To this end, two Python scripts are offered to help generate sets of analyses for any tower design. The distribution of strakes in the tower and their representation as a shell or a beam (or rigid body) is encoded as a bit string that may be manipulated using simple binary arithmetic. Algorithm 1 may be used to generate sets of ‘ n -adjacent’ models, namely all possible configurations of $n \geq 1$ adjacent strakes to be modelled as shells with all others modelled as beams (or rigid bodies). Where n adjacent shell strakes overlap with a flange, the configuration is discarded. This variant is best suited for loading dominated by meridional compression which causes failure modes that are localised and independent of one another, such as local short-wave buckling [10,19] or cross-sectional plastic hinges. Algorithm 2 may be used to generate sets of ‘flange-to-flange’ models, consisting of all possible sets of strakes situated between a flange and either another flange or an end boundary which are the only ones to be modelled as shells. This variant is best suited for loading dominated by global torsion which causes long-wave buckling [10] or a plastic collapse mechanism spanning across multiple strakes, and where the flanges or end boundaries act as stiff cross-sections which constrain the effective length of the failure mechanism. Algorithm 2 leads to larger finite element models than Algorithm 1, but still significantly smaller than a full shell model for towers with multiple flanges. The code presented here is intended for direct use with ABAQUS, to be placed before the code necessary to create and submit a finite element model with strake assignments defined according to the bit string encoding, but it is easily translatable into other programming languages.

```

import copy # Preliminaries
# Function used below to account for flange positions in the ID string
def account_for_flanges(FL_pos,binary):
    binary = list(binary) # Turn input bit string to a list
    for FL in FL_pos: binary[FL] = '2' # Replace entry with flange identifier
    return ''.join(binary) # Turn list to string (no longer a bit string)

## Inputs (example) ##
cons = 2 # No. of consecutive strakes to be modelled as shells (e.g. 2)
tot_strakes = 14 # Number of strakes in the tower, including flanges (e.g. 14)
FL_pos = [4,9] # Identify which strakes are flanges (e.g. 5 and 10)
# Note that the above uses zero-based indexing

## Variant 1 (sets of consecutive strakes to be modelled as shells) ##
IDs, bit = [], int('1'*cons,2) # Create bit string and convert to integer
for I in range(0,tot_strakes-cons+1):
    # For each possible configuration of consecutive shell strakes
    binary = bin(bit)[2:].zfill(tot_strakes) # Convert to bit string
    string = account_for_flanges(FL_pos,binary) # Account for flange IDs
    # Add to list of ID strings only if consecutive shells don't cross a flange
    if string.count('1') == cons: IDs.append(string) # Append to ID string
    bit <= 1 # Left-wise bit shift

## Variant 1 will create the following entries into IDs with the above inputs
## ['00002000020011', '00002000020110', '00002000021100', '00002001120000',
##  '00002011020000', '00002110020000', '00112000020000', '01102000020000',
##  '11002000020000']
## Each '2' should be interpreted as a flange strake
## Each '1' should be interpreted as a shell strake
## Each '0' should be interpreted as a beam / rigid strake

## The code to generate and submit FE models should be placed here, executed
for each entry in the IDs list.

```

Algorithm 1 – Functional Python code with binary arithmetic to create tower ID strings.

This variant treats only n adjacent strakes as shells.

```

## Variant 2 (sets of 'flange-to-flange' consecutive strakes as shells) ##
IDs, FL_pos2 = [], copy.deepcopy(FL_pos) # Temporary deep copy of FL_pos
# If a flange is not at either end of the tower, 'create' one there
FL_pos2.insert(0,-1) # Comment out if not necessary
FL_pos2.insert(len(FL_pos2),tot_strakes) # Comment out if not necessary
for I in range(0,len(FL_pos2)-1):
    # For each possible flange-to-flange segment
    LHS, RHS = FL_pos2[I], FL_pos2[I+1] # Identify bounding flange positions
    bit = int('1'*(RHS-LHS-1),2) # Create bit string and convert to integer
    bit <= tot_strakes-RHS # Left-wise bit shift to move to correct location
    binary = bin(bit)[2:].zfill(tot_strakes) # Convert to bit string
    string = account_for_flanges(FL_pos,binary) # Account for flange IDs
    IDs.append(string) # Append to ID string

## Variant 2 will create the following entries into IDs with the above inputs
## ['11112000020000', '00002111120000', '00002000021111']

## The code to generate and submit FE models should be placed here, executed
for each entry in the IDs list.

```

Algorithm 2 – Functional Python code with binary arithmetic to create tower ID strings.

This variant treats only ‘flange-to-flange’ sets of strakes as shells.

The following three types of analyses were performed (see EN 1993-1-6 [7,8]):

- Linear bifurcation analysis (LBA) to obtain the lowest buckling eigenvalue and eigenmode of the idealised elastic and perfect structure.
- Materially nonlinear analysis (MNA) to obtain the reference plastic collapse load and mechanism of the idealised elastic-rigid plastic and perfect structure.
- Geometrically and materially nonlinear analysis with explicit modelling of geometric imperfections (GMNIA) to obtain a realistic prediction of the nonlinear resistance of the imperfect structure.

Additional details are given in the respective sections in what follows.

2.3 *Meshing of truncated conical shells*

Detailed discussions on the mesh design of cylindrical shells subject to meridional compression were presented in Sadowski *et al.* [20]. Specifically, it was shown how the theory of the Koiter circle [21,22] may be used to design a mesh capable of accurately capturing all theoretically permissible critical non-axisymmetric buckling modes for a particular cylindrical geometry. These considerations are extended here to truncated-conical shells based on the theory of ‘Koiter ellipses’ as presented by Spagnoli [23]. Meridional compression is known to cause localised buckling in thin shells of revolution with buckles that are significantly smaller than those under shear or circumferential compression [10]. A mesh designed to accurately model a short-wave meridional compression buckling mode will thus be able to accurately represent any other buckling mode. The application of such fine meshes everywhere in a multi-strake wind turbine support tower would very computationally expensive, but if this is done in only a select number of ‘strakes of interest’ then the computational cost is tolerable and the solution quality high.

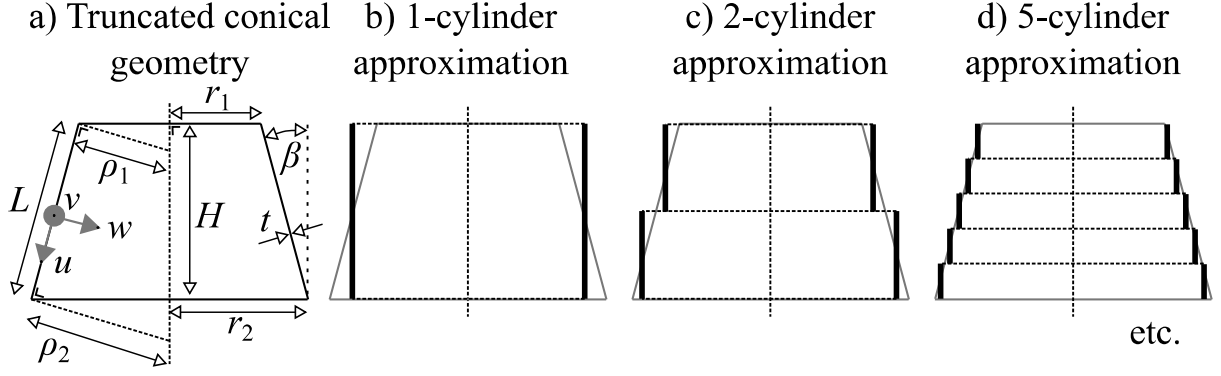


Fig. 2 – Geometry of a truncated conical shell (r are radii in the diametral plane, ρ are circumferential radii of curvature normal to the inclined meridian, and u , v and w are orthogonal displacement components) and cylindrical approximations.

Spagnoli [23] reported that the critical buckling force $[F]$ on a truncated conical shell under uniform meridional compression (Fig. 2a) is given by:

$$P_{cl} = \frac{2\pi Et^2 \cos^2 \beta}{\sqrt{3(1-\mu^2)}} \approx 3.803 Et^2 \cos^2 \beta \text{ for } \mu = 0.3 \quad (1)$$

This may be expressed as a vertical line load $[F.L^{-1}]$ about either the narrow ($j = 1$, with radii r_1 and ρ_1) or wide ($j = 2$, with radii $r_2 > r_1$ and $\rho_2 = r_2 \cdot \sec \beta > \rho_1 = r_1 \cdot \sec \beta$) ends as:

$$N_{cl,j} = \frac{Et^2 \cos^2 \beta}{r_j \sqrt{3(1-\mu^2)}} \approx 0.605 E \frac{t^2}{r_j} \cos^2 \beta \text{ for } \mu = 0.3 \quad (2)$$

By reformulating the equations reported in Spagnoli [23], it may be shown that critical buckling modes corresponding to the P_{cl} load level cluster around a ‘Koiter ellipse’ in $\dot{m} \cdot \dot{\rho}$ vs n space defined by (assuming $\mu = 0.3$):

$$\dot{m}^2 \dot{\rho}^2 + \sec^2 \beta \cdot n^2 - 1.818 \dot{\rho} \sqrt{\frac{\dot{\rho}}{t}} \dot{m} \approx 0 \text{ such that } \dot{m} = m \frac{\pi}{L} \quad (3)$$

where m is the number of buckling half-waves on the inclined meridian of length L , n is the number of buckling full waves around the circumference and $\dot{\rho}$ is the average of the circumferential radii of curvature at both ends (i.e. $\frac{1}{2} \cdot (\rho_1 + \rho_2) = \frac{1}{2} \cdot (r_1 + r_2) \cdot \sec \beta$).

Equation 3 naturally reduces to the ‘Koiter circle’ in $m \cdot r$ vs n space for a cylindrical shell ($\beta \rightarrow 0$, $\sec\beta \rightarrow 1$, $\dot{\rho} \rightarrow r$). An illustration of the linear buckling surface on the m - n plane computed with LBAs using the ‘panel analysis’ technique of Sadowski *et al.* [20] is shown in Fig. 3 for a single truncated conical shell with a purposefully accentuated shallow angle of $\beta = 45^\circ$, unit thickness, $r_1 / t = 500$ (with ‘ring’ conditions: $u = w$ and $v = 0$; Fig. 2), $r_2 / t = 1000$ (with ‘pinned’ conditions: $w = v = u = 0$), a Young’s modulus of $E = 210$ GPa and a Poisson ratio of $\mu = 0.3$. These computations largely confirm that the ‘Koiter ellipse’ expression with continuous m and n in Eq. 3 is indeed close to (though not exactly aligned with) a ‘valley’ in the linear buckling surface identifying the region of critical m - n pairs which minimise the linear buckling load. In particular, Eq. 3 appears to overpredict the critical circumferential wave numbers n , though the mismatch is on the conservative side.

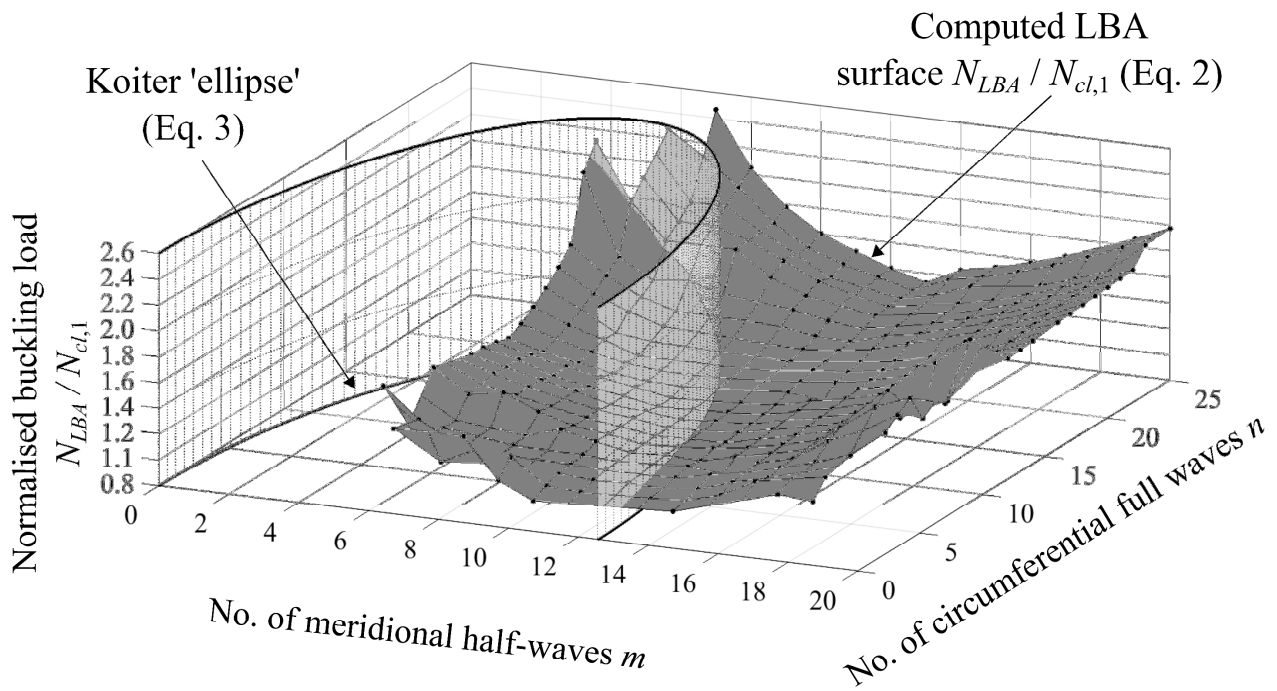


Fig. 3 – Computed linear buckling loads and wave numbers for a truncated conical shell with $t = 1$, $r_1/t = 500$, $r_2/t = 1000$ and $\beta = 45^\circ$ under meridional compression.

For any given truncated conical geometry, Eq. 3 may be drawn upon to help establish upper bounds for the theoretical maximum possible critical wave numbers in both directions:

$$m_{\max} \approx 1.818 \frac{L}{\pi \sqrt{\dot{\rho} t}} \text{ and } n_{\max} \approx 0.909 \cos \beta \sqrt{\frac{\dot{\rho}}{t}} \quad (4)$$

Invoking the ‘rule of thumb’ of at least 10 linear shell elements per buckling half-wave [20,24], this suggests that the minimum resolution of elements meridionally $M_{\min}$ and circumferentially $N_{\min}$ should be:

$$M_{\min} = 10e \cdot m_{\max} \approx \left\lceil 5.79e \frac{L}{\sqrt{\dot{\rho} t}} \right\rceil \text{ and } N_{\min} = 10e \cdot 2n_{\max} \approx \left\lceil 18.18e \cos \beta \sqrt{\frac{\dot{\rho}}{t}} \right\rceil \quad (5)$$

where the special brackets denote rounding upwards to the nearest integer (the *ceil* function), and $e = 1$ or $\frac{1}{2}$ for linear and higher-order elements respectively. The corresponding maximum element sizes are then:

$$\frac{L}{M_{\min}} \approx \frac{0.173}{e} \sqrt{\dot{\rho} t} \text{ meridionally and } \frac{2\pi \dot{\rho} \cos \beta}{N_{\min}} \approx \frac{0.346}{e} \sqrt{\dot{\rho} t} \text{ circumferentially.} \quad (6)$$

Lastly, cross-sectional plastic hinges are supported by in-plane membrane yield which does not place as strong a demand on mesh resolution as local buckling. However, the ‘boundary layer’ of local compatibility bending present at every strake boundary may become amplified during an MNA or GMNIA and contribute to a localised ‘elephant’s foot’ elastic-plastic failure mode [25,26]. For a truncated conical shell, the average half-wavelength of this boundary layer is [27]:

$$\lambda \approx 2.444 \sqrt{\dot{\rho} t} \quad (7)$$

Assuming that a minimum of 10 linear elements per bending half-wave is required for a reasonable discretisation, the maximum element size should be:

$$\frac{\lambda}{10e} \approx \frac{0.244}{e} \sqrt{\dot{\rho} t} \quad (8)$$

As this exceeds the $L/M_{\min}$ upper limit on the element size identified in Eq. 6 above, a mesh designed for local buckling is then also likely to offer an accurate solution for a

response dominated by plasticity in both membrane and local bending action. These bounds will therefore be used in the mesh design of all analyses that follow.

2.4 Geometry of an example 1.5 MW 25-strake tower

The modelling technique will be illustrated on an example 1.5 MW 25-strake wind turbine support tower, with details given in Table 1 (where H is the strake height, r_0 is the outer surface radius of the bottom of each strake and t is the strake thickness). The tower consists of 22 truncated-conical shell strakes that are so steep ($\beta < \sim 1.1^\circ$) as to be nearly cylindrical, with 3 thick intermittent cylindrical flanges for additional cross-sectional stiffness, including at the top of the tower. The base edge of the tower is assumed to be fully clamped. The same tower design has been used elsewhere in computational studies [27-29] and offers a realistic and challenging benchmark for experimenting with advanced finite element modelling techniques.

Table 1 additionally shows approximate dimensionless parameters $\omega \approx L/\sqrt{(r_0 t)}$ and $\Omega \approx (L/r_0)\sqrt{(t/r_0)}$, where $L = H \cdot \sec \beta \approx H$ for $\beta < \sim 1.1^\circ$, established using an averaged outer radius for the given strake. This was done to offer an approximate indication of whether any individual strake or flange-to-flange multi-strake segment may be susceptible to significant cross-sectional ovalisation. Rotter *et al.* [30] and Wang and Sadowski [31] found that ovalisation begins to adversely influence the buckling resistance for elastic cylindrical shells longer than $\Omega \approx 0.5$ and $\Omega \approx 1.6$ for uniform bending and uniform transverse shear respectively (torsion is not known to cause ovalisation [10]). It may be seen that each individual strake is well within these bounds and thus unlikely to ovalise significantly under bending loads. Additionally, multi-strake tower segments bounded by flanges or the base boundary condition (i.e. Strakes 1-10, 11-17 or 18-22) also satisfy the $\Omega \approx 1.6$ boundary, and are thus also too short to develop significant global ovalisation under moment gradients likely to be present. This is a simple check that makes a strong case for using hybrid beam-shell models to analyse this class of structure.

Table 1 – Geometry details of a 1.5 MW 25-segment wind turbine support tower.

Strake [†] ID	H (mm)	r_0 bottom (mm)	t (mm)	average [‡] r_0/t	average [‡] ω	average [‡] Ω
Fla 1	80	1,477.50	96.5	15.3	0.2	0.01
Str 1	2,780	1,530.44	14	107.4	19.2	0.18
Str 2	2,735	1,583.67	10	155.7	21.9	0.14
Str 3	2,730	1,636.80	10	161.0	21.5	0.13
Str 4	2,725	1,689.83	10	166.3	21.1	0.13
Str 5	2,720	1,742.77	10	171.6	20.8	0.12
Str 6	2,715	1,795.61	10	176.9	20.4	0.12
Str 7	2,710	1,848.36	10	182.2	20.1	0.11
Str 8	2,705	1,901.00	10	187.5	19.8	0.11
Str 9	2,700	1,953.55	11	175.2	18.5	0.11
Str 10	2,695	2,006.00	11	180.0	18.3	0.10
Fla 2	220	2,006.00	133.5	15.0	0.4	0.03
Str 11	2,940	2,006.50	12	167.2	18.9	0.11
Str 12	2,940	2,007.00	13	154.4	18.2	0.12
Str 13	2,940	2,007.50	13	154.4	18.2	0.12
Str 14	2,940	2,008.00	14	143.4	17.5	0.12
Str 15	2,940	2,008.50	15	133.9	16.9	0.13
Str 16	2,940	2,009.00	16	125.5	16.4	0.13
Str 17	2,940	2,009.50	17	118.2	15.9	0.13
Fla 3	220	2,009.50	137	14.7	0.4	0.03
Str 18	1,634	2,010.48	17	118.2	8.8	0.07
Str 19	2,940	2,012.23	18	111.7	15.5	0.14
Str 20	2,940	2,013.98	19	106.0	15.0	0.14
Str 21	2,940	2,015.74	25	80.6	13.1	0.16
Str 22	2,940	2,017.50	25	80.7	13.1	0.16
Segments	Σh	average* r_0	average* t	average* r_0/t	$\Sigma^* \omega$	$\Sigma^* \Omega$
Str 1-10	27,215	1,743	10.6	166	201.5	1.24
Str 11-17	20,580	2,008	14.3	142	122.1	0.86
Str 18-22	13,394	2,014	20.8	99	65.5	0.68

[†] ‘Fla’ and ‘Str’ refer to cylindrical ‘flange’ and conical ‘strake’ segments respectively.

[‡] the average of the outer surface radius r_0 at the bottom and top of each shell strake.

* calculated approximately as a simple average/sum of all strakes in the group.

Two example load cases were investigated. First, a single transverse point load of magnitude $Q = 2$ MN was applied through the centroid of the top flange, subjecting the tower to global cantilever action such as that which may arise due to wind or seismic loading. Second, global (tower) torsion was investigated by the additional application of a point torque of magnitude $T = 4$ MNm at the same location. The applied load magnitudes do not constitute normative design loads but are of approximately the right

order of magnitude for this type of structure and were chosen for illustration purposes only, though it will become evident that they are too high for the tower to safely resist. It is stressed that of interest in this study are the potential distributions of strake resistances along the entire tower relative to the globally-critical strake, rather than the absolute values of these resistances. No mass or gravity loading was applied. The bi-quadratic thick-shell reduced-integration S8R element was used in all analyses ($e = 1/2$; Eq. 5), assuming a Young's modulus of 210 GPa, a Poisson ratio of $\mu = 0.3$ and, for MNAs and GMNIAs, a yield stress of 355 MPa. Flange strakes were modelled with a single linear B31 beam element with a circular hollow section, while beam strakes were modelled with only two B31 beam elements assuming a piecewise cylindrical approximation of the truncated conical cross-section (Fig. 2c). Where appropriate, rigid bodies were discretised with the 4-node R3D4 rigid element. Analysts may wish to treat the results presented here as benchmarks against which they may compare their own predictions.

3. LBA calculations

The first LBA was performed using a full shell model of the tower under global shear only and had over 2.5 million degrees of freedom (*dofs*) according to the proposed meshing scheme (Eq. 5). While it may perhaps be tempting to employ classical mesh optimisation methods based on feedback from preliminary analyses with initial 'trial' meshes to reduce the *dof* count in the full shell model, this would only offer a faster solution for the *globally* critical mode. Unfortunately, where an estimate of the load factor corresponding to buckling in any 'strake(s) is of interest' is desired, a high mesh refinement must be maintained throughout the entirety of the structure. The minimum LBA load factor was found to be $LBA_{0Q} \approx 1.822$, with a critical location for local axial compression buckling in Strake 8 in the top tower segment (Fig. 4). Out of 200 LBA eigenmodes extracted at great computational cost from the full shell model and requiring a laborious visual classification, 66% of modes related to buckling in Strakes 7 to 10 just above the second flange, while the remainder related to buckling in Strakes 11 to 14 just below this flange. The LBA load factor of the 200th mode was equal to just $1.212 \cdot LBA_{0Q}$, illustrating the significant clustering of eigenmodes and the great inefficiency in obtaining estimates of buckling in non-critical strakes using this approach.

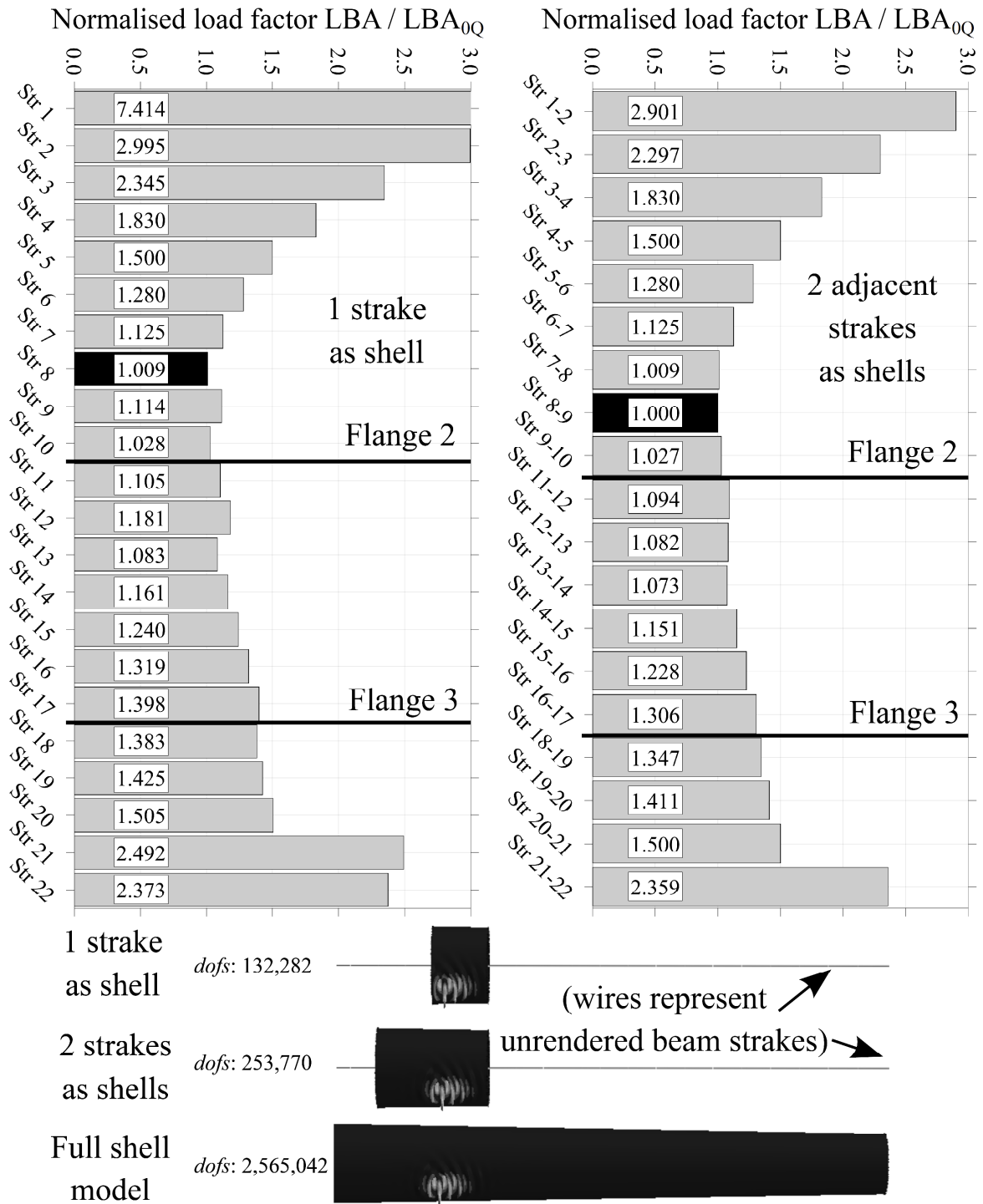


Fig. 4 – Computed LBA critical buckling loads and selected modes for the tower under global transverse shear (‘1- and 2-adjacent’ model sets).

A series of hybrid model analyses was then performed involving ‘1- and 2-adjacent’ sets of models generated using Algorithm 1, with each individual strake modelled as a shell

contributing approximately only 100,000 *dofs* to the computational cost. The complete distribution of estimated critical LBA load factors in all strakes in the tower for ‘1-adjacent’ is shown on the left side of Fig. 4. Artificially restricting buckling to Strake 8 only leads to a mere ~0.9% overestimate of the LBA load factor, suggesting that the real LBA buckling mode would involve slightly more than one strake if permitted. This is confirmed by the ‘2-adjacent’ predictions shown on the right side of Fig. 4, where the LBA load factor for buckling in Strakes 8-9 is now identical to that from a full shell model, at only a fraction of the computational cost. There was no change in the critical LBA load when more than 2 consecutive strakes were modelled as shells, and there were only negligible differences in the global distribution of LBA load factors when non-flange strakes were modelled as beams and when they were modelled as rigid bodies. The distributions shown in Fig. 4 additionally offer a deep insight into the relative criticality of every strake in the tower: the ‘1-adjacent’ and ‘2-adjacent’ sets of analyses confirm that buckling in Strakes 7 to 14 occurs at load factors within only ~20% of the globally critical LBA_{0Q} , a result obtained using significantly less computational and manual effort compared to a full shell model. It also appears that many strakes may be stronger than they need to be under this load case, and the analyst is provided with clear information about strakes that may be candidates for later optimisation.

The tower was next subject to additional global torsion through the top flange centroid and a full shell model revealed a slightly smaller critical LBA load factor of $LBA_{0QT} \approx 1.528 (\approx 0.839 \cdot LBA_{0Q})$ with long-wave shear buckling being critical in Strakes 1 to 10. A ‘1-adjacent’ set of hybrid model analyses treating only individual strakes as shells and the remainder as beams (Algorithm 1) identified Strake 8 as critical with a significantly overestimated load factor of $1.137 \cdot LBA_{0QT}$. Performing a ‘2-adjacent’ set of analyses reduced the critical LBA load factor for buckling in the top tower segment (Strakes 1 to 10) to $1.119 \cdot LBA_{0QT}$, but caused only minimal changes to the critical LBA load factor for buckling in the middle (Strakes 11 to 17) and lower (Strakes 18 to 22) tower segments (Fig. 5). Critical LBA load factors from ‘3-’, ‘4-’ and ‘5-adjacent’ sets of analyses were found to approach LBA_{0QT} progressively (Fig. 6, top). This is because the top tower segment is dominated by a multi-strake torsional buckling response, where the activation of additional strakes as shells relaxes the artificial

buckling constraint imposed by the beam elements and leads to a progressive reduction in the computed load factor. By contrast, the lower tower segments are dominated by localised meridional buckling under a linearly increasing bending moment, and activating additional strakes as shells brought no further benefit.

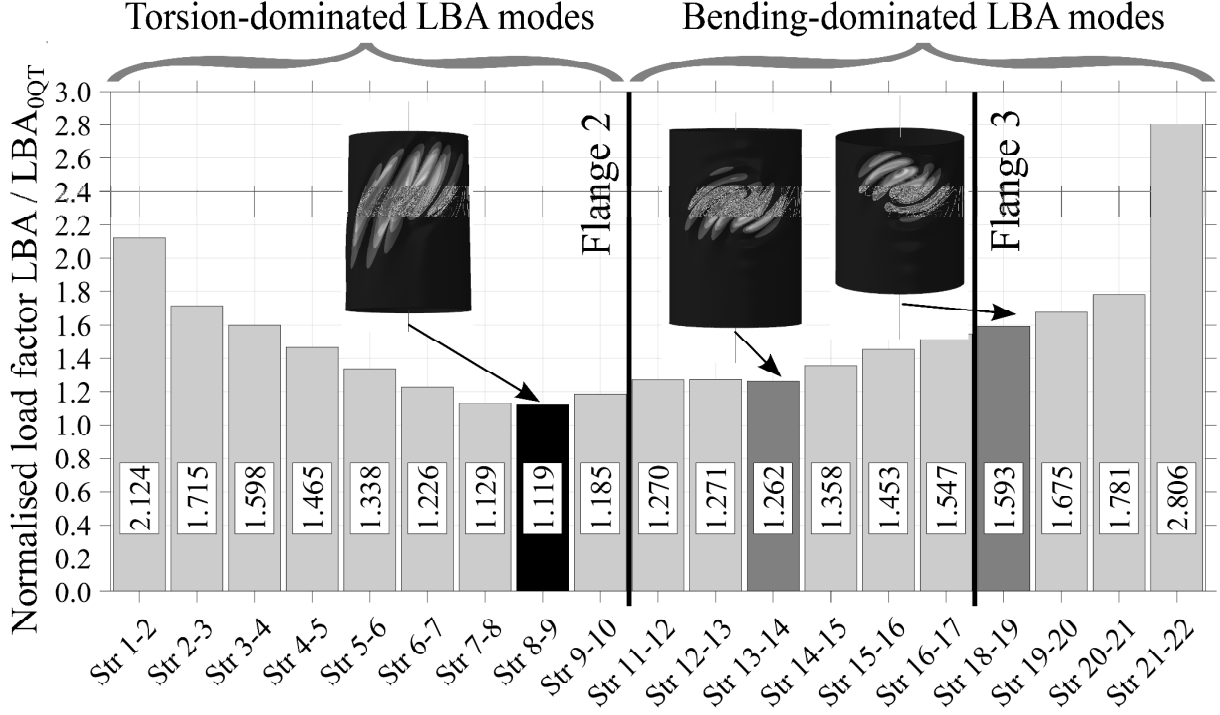


Fig. 5 – Computed LBA critical buckling loads and selected modes for the tower under global transverse shear and global torsion (‘2-adjacent’ model sets).

Hybrid model analyses considering all sets of ‘flange-to-flange’ strakes as shells were performed next using Algorithm 2. Although each individual analysis is more expensive than those in Algorithm 1, there are fewer combinations to consider and such a procedure is advised where critical buckling modes may be non-localised in nature. Algorithm 2 revealed that the critical torsional buckling mode occupying Strakes 1 to 10 occurred at a load factor of exactly LBA_{0QT} (the same as in the full shell model; Fig. 6), while local single-strake meridional buckling occurred within Strakes 11 to 17 and 18 to 22 at load factors of $1.261 \cdot LBA_{0QT}$ and $1.594 \cdot LBA_{0QT}$ respectively (in very close agreement with the predictions of the less expensive ‘2-adjacent’ sets of models; Fig. 5).

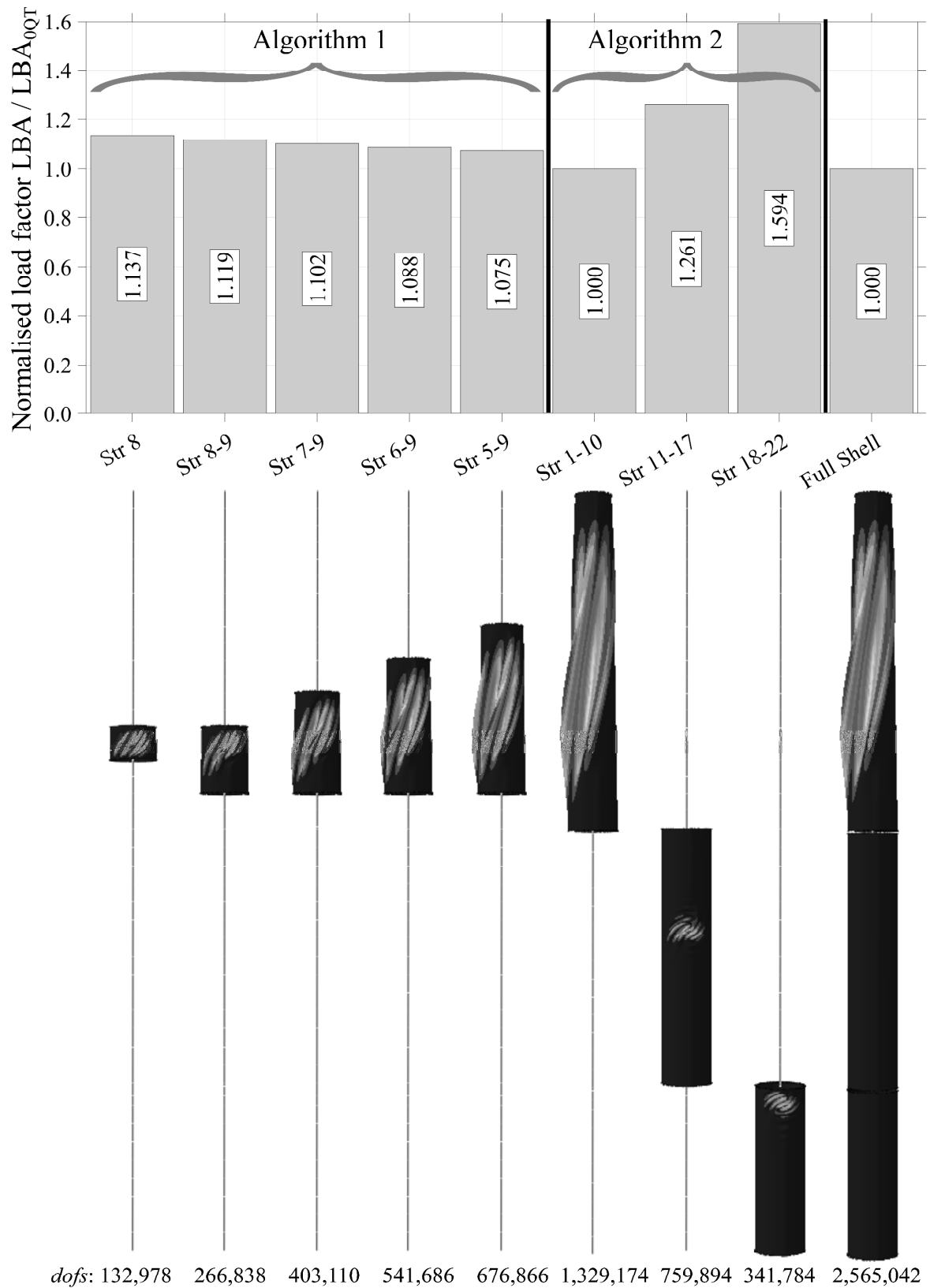


Fig. 6 – Additional computed LBA critical buckling loads and selected modes for the tower under global transverse shear and global torsion (various model sets).

It should be added that a full shell LBA required the extraction of 45 modes to obtain buckling modes in the middle tower segment, and *none* of the first 200 modes corresponded to buckling in the bottom tower segment with the highest load factor attaining only $1.427 \cdot \text{LBA}_{0QT}$. Many more hundreds, and perhaps thousands, of modes may need to be extracted to encounter those corresponding to buckling in each strake or tower segment, making full shell models impractical to confirm anything but the globally critical region.

4. MNA calculations

This section explores the plastic collapse mechanisms of the example tower under the two example load cases using computational MNAs. The material law was assumed to be bilinear elastic-plastic with no hardening, and the equilibrium path was followed by the Riks [32] modified arc-length algorithm. Termination of the analyses was automated using the ‘MN Riks Killer’ `URDFIL` routine of Sadowski *et al.* [33] which implements the ‘Convergence Indicator Plot’ projection of Doerich and Rotter [34] to accurately estimate the plastic collapse load based on only a partially-computed equilibrium path. A full shell model of the tower under global transverse shear only revealed the formation of multiple localised plastic hinges in Strakes 18 to 20 at a critical load factor of $\text{MNA}_{0Q} \approx 0.991$ ($\approx 0.544 \cdot \text{LBA}_{0Q}$), suggesting a stocky global response strongly influenced by plasticity. The full distribution of $\text{MNA} / \text{MNA}_{0Q}$ load factors from ‘1-adjacent’ sets of analyses is presented in Fig. 7, with analyses where non-shell strakes were modelled as beams on the left while those with non-shell strakes modelled as discrete rigid bodies on the right. In a full shell LBA, any number of eigenmodes may in theory be requested to eventually give an estimate of the criticality of buckling in all strakes. However, a full shell MNA can only ever provide the globally critical result, and only a hybrid modelling approach can provide the sort of detailed information shown here.

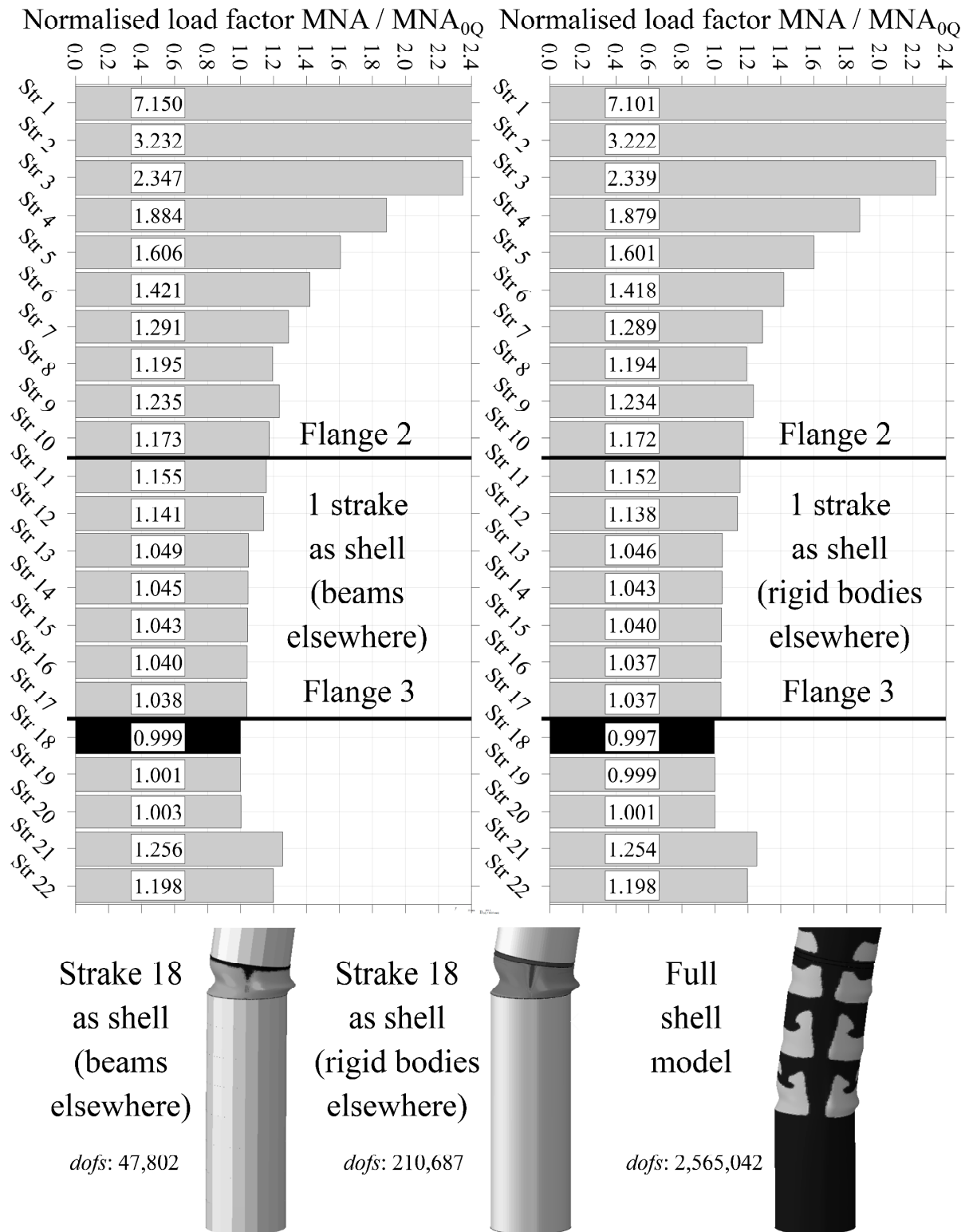


Fig. 7 – Computed MNA plastic collapse loads and mechanisms for the tower under global transverse shear ('1-adjacent' model sets).

Modelling the non-shell strakes as elastic beams and rigid bodies led to very similar distributions of MNA load factors in the tower, unsurprising given that an MNA is intended to provide a classical small-displacement plastic collapse solution where the elastic portion of the response should be negligible. The *dof* count of MNAs with rigid bodies is higher than of those with beams, but many of the *dofs* correspond to ‘elements’ within the discretised rigid bodies which do not require expensive element-level integration, only updating of *dofs* based on the motion of the global structure [11]. Consequently, it was decided to proceed with only beam elements in the remainder of this study. The ‘1-adjacent’ sets of analyses duly identify Strakes 18 to 20 as nearly simultaneously critical for the formation of a plastic hinge, in agreement with the more expensive full shell model. The analyses also identify that strakes in the middle tower segment are quite close to being critical for plastic collapse, but those in the top tower segment are significantly stronger and again potentially candidates for later optimisation. Interestingly, the critical MNA Strake 18 for plastic collapse is in a different location from the critical LBA Strake 8 for linear buckling (Fig. 4), a finding that influences a ‘LBA-MNA’ design assessment [7,8] about which more is said shortly.

A full shell model of the tower under both global shear and torsion reveals an almost unchanged critical load factor of $MNA_{0QT} \approx 0.990 (\approx 0.648 \cdot LBA_{0QT})$, again suggesting a stocky global response influenced by elastic-plastic buckling. The global plastic collapse mechanism (Fig. 8) similarly consists of multiple plastic hinges concentrated at the bottom of several strakes in the middle and bottom parts of the tower, with little contribution from torsion in contrast to the global LBA mode (Fig. 6) and leading to a very similar overall distribution of load factors for both load cases. A set of more economical ‘flange-to-flange’ models were performed next assuming beam elements for non-shell strakes. These revealed MNA / MNA_{0QT} load factors of 1.171, 1.038 and 0.98 in the top, middle and bottom tower segments respectively. The proximity of the latter two load factors with unity is consistent with the result of the much more expensive full shell model, and correctly suggests that several strakes will be nearly simultaneously critical for the formation of multiple and independent plastic hinges. The same information, however, could have been obtained by interpreting the MNA load factors from the more economical still ‘1-adjacent’ set of models (Fig. 8, bottom), which

suggest very similar load factors for plastic hinges in Strakes 13 to 20, consistent with the prediction of the full shell and ‘flange-to-flange’ models. It is of interest to note that the MNA response of the tower under combined global transverse shear and torsion is fundamentally different to its LBA response, being dominated by localised plastic hinges lower down the structure as opposed to torsional buckling in the top part.

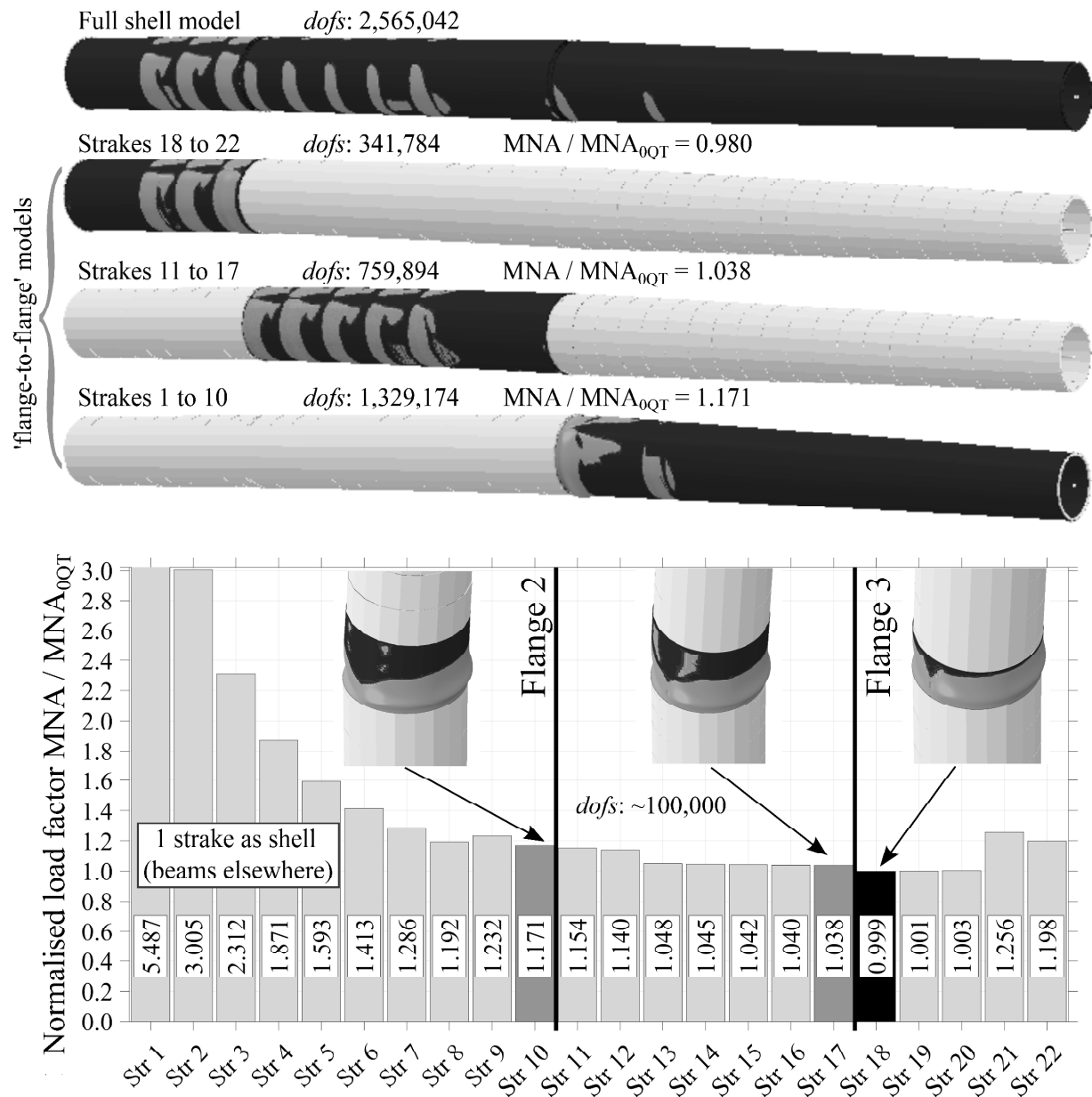


Fig. 8 – Computed MNA plastic collapse loads and mechanisms for the tower under global transverse shear and torsion, with rendered beam elements (various model sets).

5. Example LBA-MNA design assessment

Hybrid beam-shell structural models potentially offer an efficient way to implement the ‘LBA-MNA’ design procedure of EN 1993-1-6 [7,8] for metal wind turbine towers. The assessment procedure requires the analyst to make separate computational predictions for the reference elastic critical buckling load (LBA) and plastic collapse load (MNA), and to use these predictions together with a conservatively assumed buckling curve to obtain the overall resistance to the design loads. Ambiguities of interpretation may arise when different parts of the structure are critical to elastic buckling and plastic collapse [10,35], as has been shown to be the case here, a complication that hybrid models help to counter by allowing the criticality of *every* part of the structure to be examined in isolation.

For illustration, two example LBA-MNA calculations are performed here. As of time of writing, the EN 1993-1-6 Standard is undergoing significant revision and its draft provisions may change before they are finalised, so the well-established rules of the 2007 [7] version were employed. Because the individual conical strakes are so steep as to be nearly cylindrical (apex half-angle of $\beta \leq \sim 1^\circ$ for all strakes), the provisions for cylindrical shells were used assuming average strake radii (Table 1). Meridional buckling parameters from Section D.1.2.2 of EN 1993-1-6 [7] were used for the tower under global transverse shear loading only as these most closely reflect the failure mechanisms (Figs 4 and 7). For the tower under combined global shear and torsion, shear buckling parameters from Section D.1.4.2 were tentatively used for the assessment of the top tower segment (taking LBA and MNA load factors from ‘flange-to-flange’ models; Figs 6 and 8) which exhibited a torsional LBA response, while meridional buckling parameters were used for the assessment of the bending-dominated middle and bottom tower segments (taking LBA and MNA load factors for each strake individually from a ‘1-adjacent’ set of analyses). A Fabrication Tolerance Quality Class C or ‘Normal’ was assumed, corresponding to the highest codified imperfection amplitudes. These buckling parameters account for the additional detrimental influence of geometric nonlinearity, imperfection sensitivity and elastic-plastic interaction in the failure mode [33,36,37], which cannot be assessed using LBAs and MNAs alone.

The distribution of example characteristic resistances R_k for the tower under global transverse shear force only is shown in Fig. 9. Strake 8 was identified as critical (with example intermediate calculation steps involving reference resistances $R_{cr} \equiv R_{LBA} \approx 1.009 \cdot LBA \approx 1.838$ (Fig. 4) and $R_{pl} \equiv R_{MNA} \approx 1.195 \cdot MNA_{0Q} \approx 1.185$ (Fig. 7) for Strake 8, a relative slenderness of $\lambda = \sqrt{(R_{pl} / R_{cr})} \approx 0.803$, a characteristic imperfection amplitude of $\Delta w_k / t \approx 0.856$ assuming an average strake radius of $r \approx 1875$ mm (Table 1), an elastic imperfection reduction factor of $\alpha \approx 0.245$ and a buckling reduction factor of $\chi \approx 0.381$ such that $R_k = \chi \cdot R_{pl} \approx 0.451$), similar to the LBA prediction (Fig. 4) but at odds with the MNA prediction (Fig. 7). The shape of the distribution of resistances closely reflects those from LBA and MNA, and as all $R_k < 1$ the tower was found to be inadequate to resist the example loading applied here. The example characteristic resistances for the tower under combined global shear and torsion is shown in Fig. 10, this time identifying Strake 13 as critical. This is at odds with both the globally-critical LBA and MNA predictions (Figs 6 & 8), although it is consistent with the critical strake for local bending-dominated meridional buckling in the middle tower segment (Fig. 5).

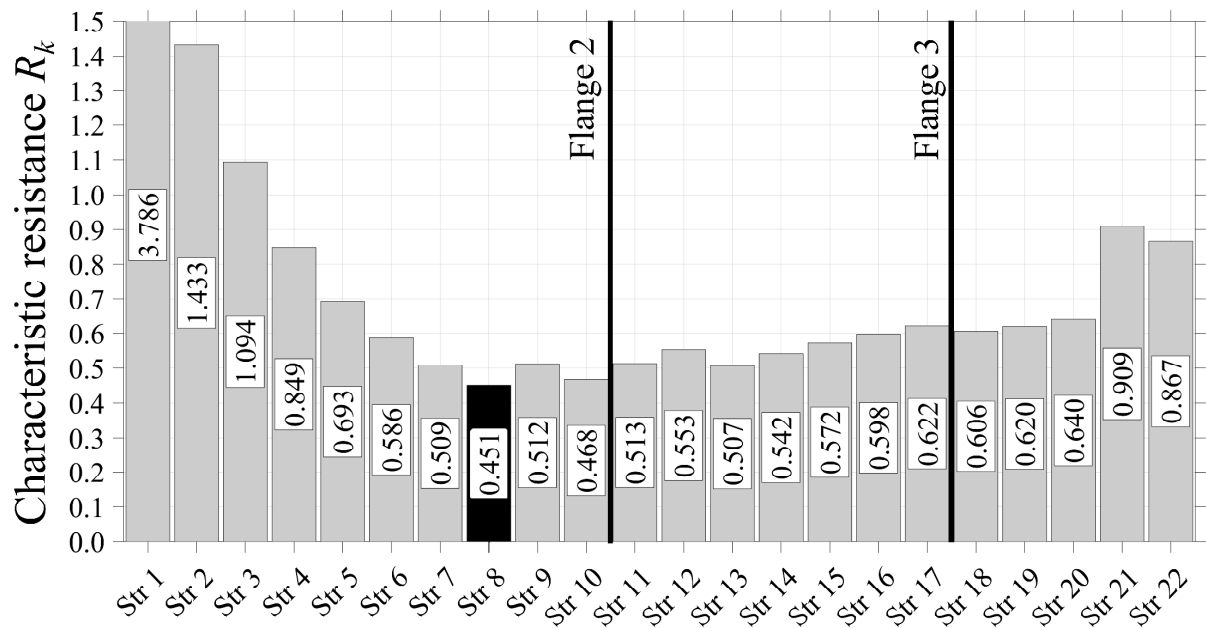


Fig. 9 – Example characteristic resistances for individual strakes under global transverse shear according to the EN 1993-1-6 [7] LBA-MNA design procedure from 2007.

The author stresses that the entirety of hybrid modelling procedure and LBA-MNA design assessment may be automated by appropriate scripting and together offer a powerful tool for the design and optimisation of tower thicknesses. However, optimisation must always be performed on the basis of a full resistance check using the LBA-MNA procedure or a GMNIA, rather than solely on the basis of groups of LBA or MNA load factors. The problem of different strakes being critical for elastic buckling and plastic collapse is largely circumvented by the hybrid beam-shell methodology where the response is dominated by meridional action, since the resistance of each possible combination of strakes is considered in turn to identify the global minimum. However, the choice of shear buckling parameters for the LBA-MNA assessment of the top tower segment (Strakes 1 to 10) is open to interrogation, given the qualitatively different failure modes and locations predicted by the LBA and MNA. This may be addressed in future by the development of more comprehensive codified buckling parameters for the simultaneous presence of a moment gradient and torsion.

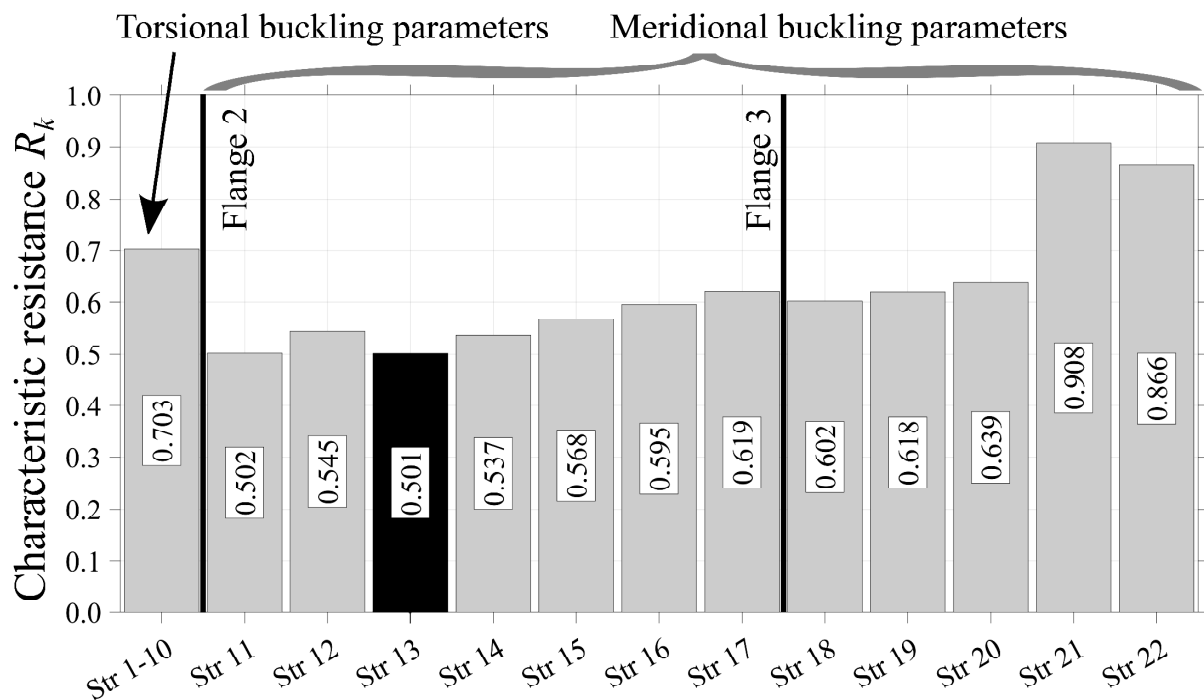


Fig. 10 – Example characteristic resistances for individual strakes under global shear and torsion according to the EN 1993-1-6 [7] LBA-MNA design procedure from 2007.

6. GMNIA calculations

The response of tower was subsequently investigated through a series of GMNIAs which account for both geometric and material nonlinearity, as well as the influence of imperfections. A simple bilinear elastic-plastic law was used with a yield stress of 350 MPa and 0.1% linear strain hardening [38]. Two distinct types of geometric imperfections were investigated, using both hybrid beam-shell and full shell finite element models. The first is the critical LBA eigenmode which, though not particularly realistic, is a staple of algebraic and computational research in shells [21,22,39,40] and the ‘default’ imperfection form prescribed in EN 1993-1-6 [7,8]. It has the additional benefit of being easily importable as a scalable mesh perturbation by most software from one analysis to another. The second is the Type ‘A’ axisymmetric circumferential weld depression of Rotter and Teng [41], an idealised but reasonably realistic representation of the inward curling caused by the cooling of a welded joint and one that has seen widespread use as a computational modelling device [29,42,43]. For this imperfection, the global geometry was generated explicitly using the following adapted formulation which assumes m weld depressions located at each strake-strake boundary (with the exception of strake-flange and strake-base boundaries):

$$\rho(y) = r(y) - \sum_{i=1}^m \delta_{0,i} e^{-\pi|y-y_{w,i}|/\lambda_i} \left[\cos\left(\frac{\pi}{\lambda_i}|y-y_{w,i}|\right) + \sin\left(\frac{\pi}{\lambda_i}|y-y_{w,i}|\right) \right] \quad (9)$$

where $\rho(y)$ and $r(y)$ are the total (imperfect) and nominal (perfect) radial positions of the shell midsurface at a vertical y coordinate respectively, $\delta_{0,i}$ and $y_{w,i}$ are the maximum radial deviation and centreline vertical position of the i -th weld depression respectively, while λ_i is the linear bending half-wavelength associated with the i -th weld depression. For simplicity, and due to the near-cylindrical nature of the strakes, this was taken as:

$$\lambda_i = \frac{\pi \sqrt{r_i t_i}}{\sqrt[4]{3(1-\mu^2)}} \quad (10)$$

where r_i is the local undeformed radius at the weld depression centreline and t_i is the average of the thicknesses of the two strakes on either side of the weld depression.

The nominal imperfection amplitudes were taken from the GMNIA rules in Section 8.7 of EN 1993-1-6 [7] for Fabrication Tolerance Class C ('Normal'), and for the weld depression these were related to the local average thickness t_i . For the full shell model with an eigenmode imperfection, the prescribed amplitude under global transverse shear loading only was related to the thickness of the critical location for LBA buckling in Strake 8 ($t = 10$ mm; Fig. 4), while under combined loading this was conservatively related to the maximum thickness of all strakes participating in the critical global torsional LBA mode (Strake 1; $t = 14$ mm; Fig. 6). Additionally, the scaling factor for the eigenmode imperfection was taken as half of the prescribed value so as to achieve approximately the correct total maximum radial deviation between the 'peak and trough' of the imperfection [19,43]. The Riks [32] algorithm was used to follow the equilibrium path up to the appearance of the first negative eigenvalues in the global tangent stiffness matrix or the first negative load increments, whichever was encountered first, with automatic termination by the 'GN Riks Killer' URDFIL routine of Sadowski *et al.* [33].

Two full shell GMNIA models were first performed for the tower under global shear loading only, one for each imperfection form. The GMNIA load factor for the eigenmode imperfection was found to be 0.539, and the critical location for buckling was in Strakes 8 and 9. The critical location was to be expected, however, since the GMNIA calculation is essentially forced to predict failure at the location corresponding to the location of buckling in the imposed critical LBA eigenmode (Fig. 4). Unless non-critical LBA eigenmodes corresponding to failure in other strakes are imposed as imperfections, a GMNIA is unlikely to predict failure elsewhere, but it is also unlikely that a sufficient number of LBA eigenmodes will have been extracted from the expensive full shell model to permit the full set of GMNIA estimates to be obtained corresponding to failure in all strakes. For the weld depression imperfection, the GMNIA load factor was found to be ~0.48. However, the Riks path-tracing analysis could not be made to proceed beyond the point of onset of instability, and as no converged post-buckling increment was available it was not possible to accurately establish the failure location or the critical strake (although significant local yielding was observed in the vicinity of Strakes 13 and 14). While full shell GMNIA models are

perhaps the final arbiters of the global resistance, they provide unsatisfactorily little auxiliary information to the analyst given their very high computational cost.

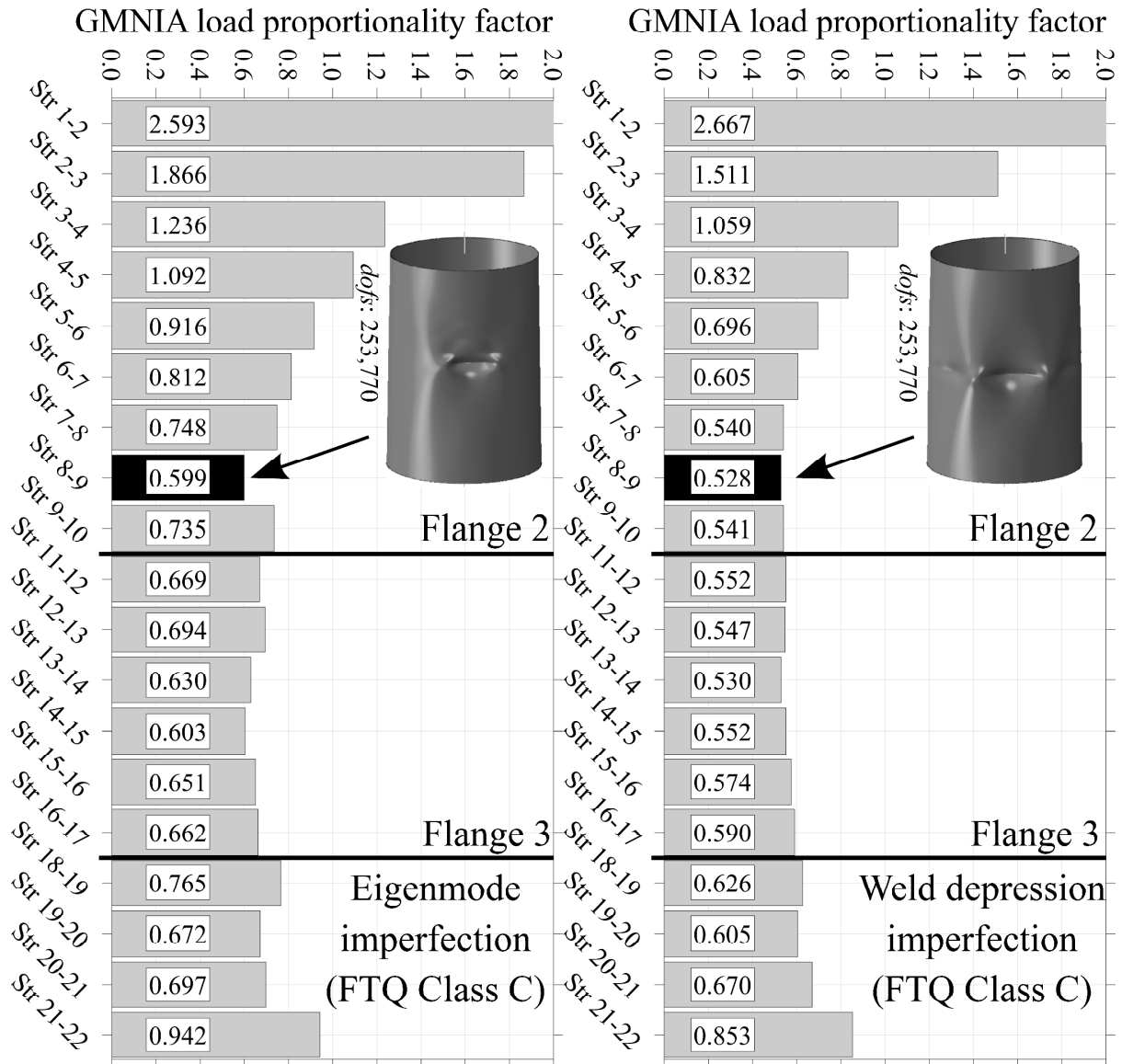


Fig. 11 – Computed GMNIA buckling loads and selected incremental buckling modes for the tower under global transverse shear assuming two imperfections and Fabrication Tolerance Quality Class C ('2-adjacent' model sets).

Two groups of '2-adjacent' sets of GMNIA models were performed next for the tower under global shear only, with distributions of GMNIA load factors shown in Fig. 11. For

the eigenmode imperfection, amplitudes were related to the maximum of the thicknesses of all strakes activated as shells in any analysis. By using elastic beams for all non-shell strakes, failure is forced to occur only in the imperfect geometry of the strakes that have been explicitly modelled as shells. Even in the case of non-convergence of post-buckling increments, the analysis still offers an estimate of the load factor at failure where the location is by design confined to these specific strakes. The ‘2-adjacent’ sets of GMNIA models with either imperfection form identified Strakes 8 to 9 as globally critical for buckling (in both cases with a very similar buckling mode), in agreement with the full shell GMNIA (for the eigenmode imperfection only) as well as the LBAs performed previously (Fig. 4). Interestingly, both sets of GMNIAs suggest that the middle portion of the tower (Strakes 11 to 17) is significantly closer to being as critical as Strakes 8 and 9 and is thus less underutilised than would be concluded on the basis of LBAs alone, with Strake 14 in particular appearing to be almost as critical as Strake 8. The two sets of very different imperfections offer remarkably similar distribution of GMNIA load factors, likely attributable to the fact that a cylindrical shell is similarly sensitive to both of the imperfection forms used here when the amplitudes are defined so as to give the same peak geometric deviation [19,43]. Additionally, the GMNIAs mostly do not produce characteristic resistances that are quite low as those obtained by the LBA-MNA procedure (Figs 9 & 10), even for the same nominal Fabrication Tolerance Quality Class. The likely reasons may be read in Section 7 of Sadowski and Rotter [42] and relate to additional conservatism in the buckling reduction curve in the 2007 version of EN 1993-1-6 [7].

A final series of GMNIAs was performed for the tower under combined global shear and torsion, with full shell models predicting load factors of ~0.79 (higher than under global shear only) and ~0.48 (the same as under global shear only) for the eigenmode and weld depression imperfections respectively (Fig. 12). The incremental buckling modes could not be obtained due to convergence issues in both analyses, making an unambiguous identification of the critical location difficult. However, local yielding was observed on the compressive side in Strakes 18 to 20 and Strakes 13 to 14 for the eigenmode and weld depression imperfections respectively. This is an unsatisfying outcome for an

already very expensive model, and one that it would not necessarily be possible to address using an even finer mesh resolution.

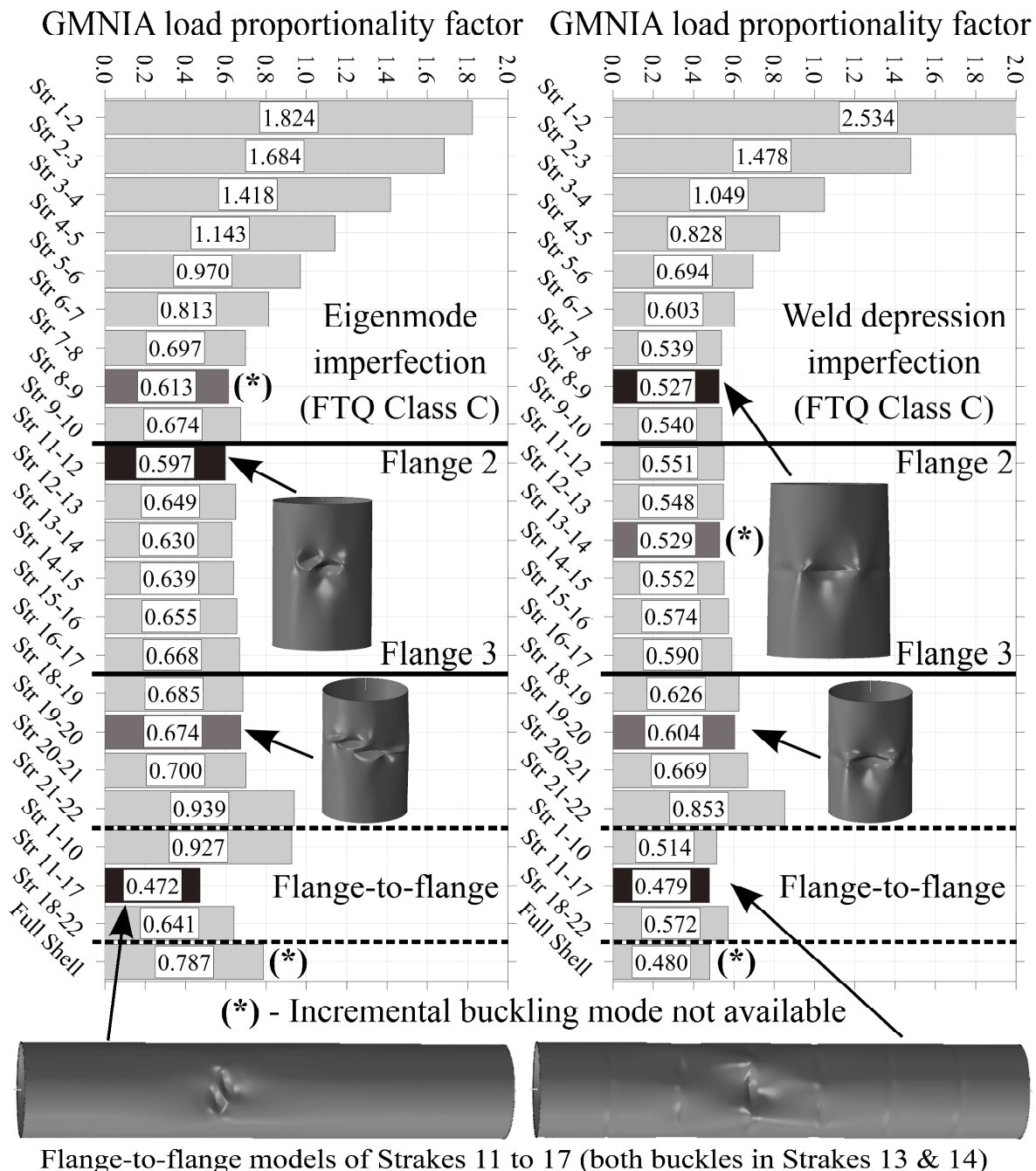


Fig. 12 – Computed GMNIA buckling loads and selected incremental buckling modes for the tower under global transverse shear and global torsion assuming two imperfections and Fabrication Tolerance Quality Class C (various model sets).

Sets of ‘flange-to-flange’ models were performed next, with the amplitudes of the eigenmode imperfection related to the max. thicknesses of only those strakes supporting LBA buckling deformations (Strake 1 with $t = 14$ mm, Strake 14 with $t = 14$ mm and Strake 19 with $t = 19$ mm for the top, middle and bottom tower segments respectively; Fig. 6). These suggested that the middle tower segment was critical under both imperfection forms, with load factors of 0.472 (lower than the full shell model result of ~ 0.79) and 0.479 (in very close agreement with the full shell model result of ~ 0.48) for the eigenmode and weld depression imperfections respectively. The significant discrepancy is due to the qualitatively very different eigenmodes imported as imperfections for the full shell model (a torsional mode covering Strakes 1 to 10; Fig. 6) and the ‘flange-to-flange’ model treating only the middle tower segment as a shell (a local buckling mode in Strakes 13 and 14; Fig. 6), and illustrates how a non-critical LBA eigenmode can be a globally critical imperfection in a GMNIA. The additional discrepancy between the full shell GMNIA of ~ 0.79 and the significantly higher ‘flange-to-flange’ GMNIA of 0.927 for the top tower segment is due to the latter having Strakes 11 to 22 modelled as elastic beams unable to undergo yielding.

Sets of ‘2-adjacent’ analyses revealed that the GMNIA load factors in the middle and bottom tower segments are very similar to their counterparts under global transverse shear only (Figs 11 & 12), particularly for the weld depression, suggesting that the more realistic nonlinear response of the tower is largely unaffected by the presence of global torsion. Indeed, the only significant differences between the two sets of GMNIA load factors are in the top non-critical tower strakes where the effect of global torsion is dominant over that of bending under transverse shear. The minimum GMNIA load factors in the three tower segments predicted by the ‘2-adjacent’ sets of analyses were in reasonably close agreement with those obtained from the ‘flange-to-flange’ analyses, with the exception of the eigenmode imperfection where there is a significant discrepancy in the top tower segment. The GMNIA load factor when Strakes 1 to 10 are all activated as shells is 0.928, while it is 0.613 when only Strakes 9 and 10 are considered as shells. This is partly due to the torsional LBA buckling eigenmode imported as an imperfection being different in shape depending on how many strakes it is permitted to span over (Fig. 6), and partly due to the imperfection amplitude being

related to a different wall thickness for both analyses. In either case, the globally critical torsional LBA eigenmode conflicts with the preferred nonlinear response of the tower, and its use as an imperfection form is unrealistic and at risk of dangerously unconservative predictions of the global resistance.

7. Summary and conclusions

This paper has explored ‘hybrid’ finite element models involving a judicious deployment of both beam and shell elements in the analysis of metal wind turbine support towers. The technique was illustrated on a realistic 1.5 MW 25-strake tower with extensive sets of finite element analyses exploring the linear buckling, plastic collapse and fully nonlinear behaviour. The main conclusions are as follows:

- While a full shell model of a metal wind turbine support tower is arguably the final arbiter of the globally critical behaviour, it provides the analyst with almost no auxiliary information regarding the relative criticality of other locations.
- The extraction of hundreds of linear eigenmodes from an expensive full shell model is a particularly inefficient way of exploring the relative criticality of all strakes in a tower. Many hundreds, and perhaps thousands, of modes would have to be extracted to obtain buckles in all parts of the structure. No such option is even available for nonlinear path-tracing analyses.
- If a series of much smaller analyses is performed treating each individual tower strake as a finely-meshed shell and the remainder as beams, an accurate estimate is obtained of the critical load corresponding to failure in each strake. This offers a powerful insight into the distribution of criticality along the whole structure, and paves the way for a systematic optimisation of the tower design using computational tools which are ideally suited for accurate strength assessments of complex structures under complex load combinations.
- The technique is shown to be particularly useful when used together with the ‘LBA-MNA’ design procedure of EN 1993-1-6, the European Standard on metal shells, helping to address ambiguities which may arise in a complex multi-strake structure if it exhibits different globally critical locations for elastic buckling

(LBA) and plastic collapse (MNA) respectively. By providing accurate estimates of both failure mechanisms at each possible stroke or group of strokes, the technique illustrated here permits a robust and automatable LBA-MNA assessment on a stroke-by-stroke or region-by-region basis.

- It was demonstrated that LBAs, MNAs, an LBA-MNA assessment and GMNIAs with different imperfection forms may all lead to very different estimates of the globally critical failure location.
- Great care should be taken in the implementation of imperfections in a GMNIA, particularly where the assumed imperfection is the globally critical LBA eigenmode. It was illustrated that non-critical LBA eigenmodes may lead to a globally critical GMNIAs, and eigenmode imperfections that are in conflict with the preferred nonlinear response of the structure may lead to an unrealistic and dangerously unconservative GMNIA estimates of the global resistance.

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