

**The Aerodynamics of an Isolated Wheel
Rotating in Contact with the ground**

by
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Summary

The problems associated with modelling the wheel flow in the wind tunnel are discussed and it is shown that rotation of the wheel and contact between the wheel and floor are both necessary if the simulation is to include the important features of the flow.

Six wheels (three different widths times two different edge profiles) were tested in the wind tunnel, using a moving floor rig. The drag and lift of the wheels, obtained by integrating surface pressure measurements, were found to increase with width, but a drag coefficient based on an area given by multiplying the tread width in contact with the ground times the diameter, remains fairly constant. Of the two edge shapes, the sharper edged wheels had the better aerodynamic characteristics.

The rotation of the wheel is shown to produce high pressures near the ground and a forward movement of separation from the top of the wheel. Tyre tread patterns, simulated by grooves in the wheel's surface, were found to decrease the lift of the wheel, but to have little effect on the drag, because of the localised nature of the changes they produced.

The special features associated with the rotation of the wheel and the presence of the ground are discussed in detail, and some theoretical models are used to examine these features. A free streamline solution for a cylinder on the ground is derived and compared with experimental results.

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(1) INTRODUCTION

Although there have been many investigations into the aerodynamics of cars in recent years, both of specific vehicles and of more generalized shapes, (e.g. see City University (1969)), the flow field associated with the wheels has received very little attention; mainly due to the difficulties in obtaining an adequate simulation of this complex flow. Yet, in many cases where aerodynamic design is important, the wheels can play an important role in the behaviour of the vehicle. The extreme example is the "Grand Prix" racing car, which travels at high speeds with totally exposed wheels and, because the body is relatively well "streamlined", these produce the dominant effects on the drag force and also greatly influence the lift characteristics. It is therefore desirable to have some understanding of the wheel's flow field, in order to develop realistic simulation techniques for wind tunnel testing of vehicles, where having rotating wheels in contact with the ground is not always feasible. This understanding may also help in dealing with the entrainment and dispersal of spray by wheels on a wet road, which can form a hazard to following vehicles.

We can expect the aerodynamic effects on an automobile wheel to be influenced by the following factors; the

wheel is a bluff body and, in particular, is basically a circular cylinder of low aspect ratio; it is rotating; and it is on the ground. We shall therefore need to consider all these factors in attempting to understand the aerodynamics of the wheel. In the following sections a review of relevant work on the above effects will be presented. First, by considering aspects of flow common to all bluff bodies, then by looking in detail at the two dimensional flow past circular cylinders , followed by a consideration of the effects of lowering aspect ratio, of rotation, and of the ground on these flows.

(1.1) Features of bluff body flows

A bluff body can be defined as a body of any shape, which experiences complete boundary layer separation before the trailing edge due to the large adverse pressure gradient set up over that part of the body behind the position of maximum thickness. This pressure gradient decelerates the slow moving fluid within the boundary layer near to the surface and eventually causes a reversed flow and hence separation (Fig.(1-1)). Bearman (1972) has given a description of the present knowledge regarding the behaviour of the separated flow. After separating, the shear layers entrain fluid from within the separated region, lowering the pressure within this region and hence causing themselves to curve inwards. When the shear layers meet, fluid is returned to the base region to balance that removed by entrainment. The downstream position

of this meeting point is stabilised by, and depends on, the entrainment of fluid from the base region. Thus, in the mean, we have the flow configuration shown in Fig.(1-1).

The vast difference between this flow and the fully attached potential model of the flow leads to corresponding changes in the pressure acting on the body. In particular, the pressure over the rear of the body does not attain the stagnation value but is usually constant and negative within the separated region, thus leading to a large pressure drag on the body. Typical results for bluff body flows, including the recirculating bubble, can be found in the work of Fail, Lawford and Eyre (1959), who studied the wakes of flat plates normal to the free stream.

So far we have described only the mean flow about the body. In practice, except at very low Reynolds numbers, the detached shear layers are unstable, create highly unsteady flow downstream of the body and have a tendency to roll up into discrete vortices. The latter is especially true of high aspect ratio bodies. In fact, in two dimensional flow, at most Reynolds numbers, it is the entrainment of fluid from the base region by the formation of discrete vortices, that dominates the wake mechanics (see Gerrard (1966)), whereas in three dimensional flow, although discrete vortices are formed (e.g. see Fail, Lawford and Eyre (1959)), they are relatively weak and entrainment is mainly by the shear layers.

However, on a bluff body with finite curvature the attached boundary layer can have as important an effect on the wake mechanics as the free shear layer interaction, in particular, in establishing the Reynolds number dependence of the flow. Thus a study of the two dimensional flow about a circular cylinder can be expected to yield information relevant to the three dimensional flow about a wheel.

(1.2) Flow about circular cylinders

Morkovin (1964) has given an excellent review of the flow features of circular cylinders, giving many references on the work carried out in this field up to 1964. For our purposes, the important point to emerge from this work is that the flow exhibits distinct flow regimes dependent on Reynolds number, Fig. (1-2). It is the higher Reynolds number which are of interest in the present study; for automobile wheels, it will normally be around 10^5 - 10^6 , spanning the subcritical, critical and postcritical regions and at very high speeds entering the transcritical region.

The factor governing the flow alterations at these higher Reynolds numbers, is the variation in the position of transition to turbulence within the shear layers. The Von Karman vortex street at $Re. < 150-300$ is set up by a purely laminar instability of the two shear layers due to mutual interaction (see Abernathy and Kronauer (1962)). At Reynolds

numbers above 300, transition occurs in the separated shear layers which then roll up to form turbulent vortices (Sub-critical Regime). However, this does not alter the basic vortex wake structure, the cylinder still suffering fluctuating lift, due to the alternate shedding of vortices, and the Strouhal number ($f d/U_\infty$) remains fairly constant throughout the range.

The point of transition to turbulence moves upstream with increasing Reynolds number and at the same time the individual shear layers can undergo local laminar instabilities ahead of the turbulence. When these instabilities reach the surface of the cylinder, they can radically alter the flow producing a closed separation bubble (Critical Regime).

As the Reynolds number increases further, the separation bubbles disappear and the boundary layers become fully turbulent before separation (Supercritical Regime).

In the critical and supercritical regimes, the separation line is highly three dimensional, being affected by local transitions to turbulence. This random three-dimensionality destroys the regular shedding of vortices by greatly decreasing any spanwise correlation of the vortex shedding (i.e. reduces correlation length). However, once the transition

to turbulence within the boundary layer occurs sufficiently far upstream of the separation position to prevent local transitions affecting the separation line, regular vortex shedding recommences. (Transcritical Regime). The boundary layers then separate in a fully turbulent state, but the separation position occurs at about the same angular position across the span of the cylinder, restoring the two dimensionality of the flow.

The effect of the above changes in the flow on the mean pressure distribution around the cylinder is shown in Fig. (1-3). In those regimes, with regular vortex shedding, the pressures fluctuate producing an oscillating lift force. The effect on the drag coefficient is shown in Fig. (1-4). Note that the flow around sharp edged objects is not so susceptible to Reynolds number changes, because the sharp edges maintain straight separation lines, which are little affected by transitions to turbulence.

(1.3) Lower Aspect Ratio Bodies

In deciding how much of the above is relevant to the flow around the wheel, we must take into account that the wheel has an aspect ratio (width/diameter) of less than one. The effect of aspect ratio on the drag coefficients of the circular cylinder and flat plate is given in Fig.(1-5). The variation of the three curves between $AR = 1$ and 10 is very similar, the drag ratio increasing fairly slowly. Above

10 the drag of the flat plate appears to increase more rapidly than that of the cylinder. Fail, Lawford and Eyre (1959) give detailed results for the effects of aspect ratio on flat plates and in view of the above similarity in drag behaviour, these may have some relevance to the circular cylinder. It was found that the base pressure became more negative with increasing aspect ratio, as might be expected since there is always a close correspondence in the behaviour of the negative base pressure and the drag of bluff bodies. The recirculating bubble length divided by the square root of the plate area decreases with aspect ratio, although the same length divided by the chord increases to a maximum at $AR=10$ before decreasing. Sharply defined shedding frequencies were detected down to an aspect ratio of .5 and the Strouhal number based on the chord varied slightly, having a minimum at $AR = 10$. This vortex shedding will not play as dominant a part in the wake mechanics as in two dimensional flow, however, as mentioned previously.

The above results cannot tell us whether the Reynolds number dependence of the circular cylinder, associated with transition to turbulence, still continues at lower aspect ratios. We might suspect that it does since this is basically a curved surface phenomenon; the downstream movement of the separation position with transition is due to the turbulent boundary layer being more efficient at obtaining energy from the free stream and so able to sustain a greater adverse pressure gradient. To examine this we can turn to results for the flow past spheres (which do bear some resemblance to

our round edged wheels). Fig.(1-4) shows the variation of drag coefficient with Reynolds number given by Schlichting (1968), and Fig. (1-6) the variation in pressure distribution found by Fage (1937). The great similarity between these figures and the corresponding ones for the circular cylinder suggests that the same processes are taking place, including the formation of local separation bubbles and the transition to turbulence.

Both cylinder and sphere are very susceptible to small disturbances when in the critical regime. Such agents as free stream turbulence, support interference and trip wires can promote an earlier transition to turbulence, with a corresponding change in the pressure distribution and drag.(For example see the photographs of Weisselberger (1914) in Schlichting (1968) for the effect of a trip wire on the flow around a sphere). Because of this there is much scatter in the data from different experiments in this range of Reynolds number depending on how adequately the above agents have been eliminated.

The effect of surface roughness on both is also similar (ref. Goldstein (1938),vol.2) . With increase in roughness the fall in drag occurs at a lower Reynolds number and is of smaller extent. When the roughness is very coarse, the critical drop in C_D tends to disappear and the drag to become independent of Reynolds number. From our discussion of the circular cylinder flow we see that, because the coarse surface is provoking an early transition, the critical and

supercritical regimes are being suppressed and the transcritical regime encouraged. These effects may be significant when considering the flow over a treaded tyre, which may exhibit characteristics of the transcritical regime at Reynolds numbers lower than would be necessary for a smooth wheel.

However, the wheels used in the experiments described herein are smooth, as are the tyres used on racing cars, so it becomes very necessary to choose the Reynolds number to at least fall within the same flow regime as that of the real flow that we wish to model. An average racing car wheel of 20" dia. has a Reynolds number of 5×10^5 at 30 m.p.h. and 2.5×10^6 at 150 m.p.h. both of which are in the supercritical range. A choice of Reynolds number within this range would therefore seem to cover most real situations of interest.

(1.4) Rotating Bodies

Apart from any direct applicability to the wheel, it is desirable to have an understanding of the flow around circular cylinder and spheres because these shapes have featured in all experiments on the effects of rotating bluff bodies. Simple potential theory yields a solution for flow past a rotating cylinder, which indicates a lift force on the cylinder proportional to the circulation about the cylinder. Most of the early experimental work on rotating cylinders was restricted to measuring the lift and drag forces in order to compare the performance with that of aerofoil sections. When it was found that the aerofoils were generally more efficient for

aircraft application, (having less drag for a given lift) interest in rotating cylinders waned. There is, therefore, a fair amount of data on the forces experienced by the cylinder but little regarding those details of the flow field which might be of use in understanding the wheel flow. In fact the only work containing detailed measurements in the external flow field is that of Thom (1931), whilst Prandtl and Tietjens (1934) have photographed the flow at various values of v/U_∞ , the ratio of the circumferential velocity to the free stream velocity.

Thom's work was mainly at a Reynolds number of 2×10^4 and that of Prandtl would appear to be at a much lower number. That the flow is still sensitive to Reynolds number is shown in Figs. (1-7) and (1-8), where the lift and drag coefficients are plotted against v/U_∞ and the variation with Reynolds number indicated. The lift curves incorporate the results of several experiments and yet the results agree fairly well over the range of v/U_∞ shown. However, only one set of results has been used for the drag curves, as the value of the drag of even the stationary cylinder varies considerably between investigators, seeming to depend on the aspect ratio used. The variation of the drag with v/U_∞ is similar in all cases, however, although the absolute values are different. At higher values of v/U_∞ than are shown there is considerable difference in the results of different experiments in both lift and drag. The range of aspect ratio covered by these results is quite extensive and it is doubtful if even approximate two dimensional flow would have been

achieved in many cases so that end effects would be important. For example the results of Griffiths and Ma (1969) given in Fig (1-8) were obtained with a cylinder of aspect ratio of only 2 and although end plates were fitted the C_D of 0.8 for the stationary case is lower than would be expected for two dimensional flow at the Reynolds number indicated. Because of these discrepancies and also because the detailed results near the critical range will be influenced by the conditions of the various experiments only broad trends with Reynolds number are indicated in Figs. (1-7) and (1-8).

To see what effect a lower aspect ratio will have on the above results we can compare them with some results obtained for rotating spheres. The lift and drag coefficients for spheres are presented on Fig. (1-9), together with the results of Reid (1924) for a cylinder. The results of Davies (1949) are for smooth and dimpled golf balls, which were set spinning at a constant rotational speed and then allowed to fall through an air stream, the forces being obtained from measurements of the trajectory. The results of Maccoll (1928) were obtained using a conventional balance arrangement.

Davies' method avoided complications due to support interference, but was subject to other errors, principally the fact that the ball has an appreciable velocity downwind near the end of its trajectory and was also influenced by the walls of the tunnel. Despite these differences the variation in the coefficients with v/U_∞ is of a similar form for all smooth spheres, although the drag coefficient found by Maccoll

for $v/U_\infty = 0$ seems a little high. The cylinder also shows a similar but more extreme variation.

An interesting feature of both cylinder and sphere flows is the negative lift, (opposite to that indicated by potential theory), which is obtained at Reynolds numbers approaching the critical number and at small values of v/U_∞ . Griffiths and Ma (1969) suggest that for the cylinder this is due to the upstream moving surface provoking transition and hence a downstream movement of the separation point on that side of the cylinder. This gives rise to a pressure distribution resembling A in Fig. (1-3) on the downstream moving side of the cylinder and one resembling C in Fig. (1-3) on the upstream moving side. The net result is a negative lift. For subcritical Reynolds number, Prandtl's photographs (Prandtl and Tietjens (1934)) and Thom's pressure distributions (Fig. (1-10)) show that the downstream moving surface suppresses separation, whilst the flow separates closer to the stagnation point on the other side. Apparently, for v/U_∞ greater than .5, this influence of rotation overcomes the effects of transition since a positive lift is found for all Reynolds number up to 4.6×10^5 , the maximum at which data is available.

The similar results for the lift force on smooth spheres suggest that the same transition effects are occurring in the three dimensional flow.

(1.5) Separation From Rotating Bodies

The concept of separation over a rotating body needs some consideration. For the present, we shall define the separation point in terms of the pressure distribution as the position at which the mean static pressure on the surface becomes constant. On a stationary body this definition would give a position only just downstream of that at which $\partial u / \partial y|_w$ becomes zero, the normal definition of separation point. On a rotating body, the distinction between these two definitions becomes more necessary because $\partial u / \partial y|_w$ can become zero even in a favourable pressure gradient. To see this, consider Fig.(1-11) which presents Thom's results for the pressure distribution at $v/U_\infty = 2$, together with his measurements of velocity profiles and static pressure at selected points around the cylinder. The velocity profile at $\theta = 20^\circ$ has $\partial u / \partial y|_w$ negative, whilst at $\theta = 40^\circ$ it is positive, therefore, between these two, $\partial u / \partial y|_w$ must be zero, without having any noticeable effect on the measured pressures. Similarly $\partial u / \partial y|_w$ must be zero between $\theta = 60-120^\circ$ but between points where the pressure distribution shows significant behaviour, i.e. $\theta = 160^\circ-280^\circ$, $\partial u / \partial y|_w$ is always non zero. However, $\partial p / \partial y|_w$, the static pressure gradient at the wall, does become zero and even negative within this region. These profiles were obtained with Pitot and static tubes, held tangential to the surface, which are likely to have errors near a rapidly rotating curved body, so we must beware of drawing

too many conclusions from them. However, a $\partial p / \partial y < 0$ does violate the boundary layer assumptions which require $\partial p / \partial y = K \rho u^2$ (K = curvature, positive for a convex body). The obvious conclusion is that the normal velocity is no longer small in the neighbourhood of points where $\left. \partial p / \partial y \right|_w$ is negative.

Some idea of the errors involved in the use of static tubes can be gained from Fig.(1-11), where the results from the static tube readings are compared with the pressures obtained from a static hole in the surface of the cylinder. The pressures were measured through this hole by opening a valve at a preset angle on each revolution of the cylinder, the pressure being taken from the cylinder through a mercury seal. This method was not without error since the valve was open for a finite angle and tended to have a pumping action which raised the measured pressure. However, the error is likely to be less than that associated with the static tube. The major difference between the two occurs at $\theta = 160^\circ$ where reading of static pressure closest to the surface (at .5 mm.) seems to indicate an increasing pressure inwards. If this were so, the appreciable normal velocities would probably make the readings invalid, so the results for this position must be viewed with suspicion. Agreement over the rest of the cylinder is much better, suggesting that the profiles at the other points may be more accurate.

Another remarkable feature of Fig. (1-11) is that the pressure distribution given is the difference in the

pressures measured with the wind on and with it off. That is, in making the correction for centrifugal effects, no account has been taken of the thin layer of air which is set in rotation by the cylinder in still air and so lowers the centrifugal pressure below the theoretical value. If allowance is made for this layer, then the values of c_p given in Fig. (1-11) are too high and the maximum value would be less than one. The conclusion Thom draws is that the cylinder induces the same rotation in the layer of air next to it independent of any external flow. The pressures shown therefore are those over the surface, not of the cylinder, but of an annulus corresponding to this thin layer of air.

From the above measurements of Thom, the corresponding photographs of Prandtl and our own measurements and theoretical work in section (4), we can conjecture that the mean flow about a rotating cylinder with $v/U_\infty = 2$ is of the form shown in Fig. (1-12). Behind the cylinder is the usual recirculating bubble associated with bluff bodies, although now offset to one side and asymmetric due to the rotation. Next to the cylinder is a layer of air which remains attached to it and rotates in the same direction as the cylinder's surface. This means that separation must occur from this layer of air as shown not from the cylinder's surface and that each separation point will involve the meeting of four streamlines. The separation from the upstream moving surface is that relevant to the wheel, as indicated in Fig. (1-19). Here the profiles of tangential velocity are basically similar before and

after separation, although after separation there would be a greater normal velocity. (c.f. Fig.(1-1) and velocity profiles at $\theta = 280$ and 300° Fig. (1-11).)

Separation in three dimensional flow is more complex than in two dimensions so one must be careful of applying two dimensional conclusions . Maskell (1955) has identified two basic forms of separation from a general curved surface (Fig. (1-13)). He uses the concept of limiting streamlines in the surface to indicate the direction of the streamlines as the normal distance tends to zero. These streamlines can be observed by painting paraffin oil onto the body. In the bubble type of separation the fluid enclosed by the separation surface is the recirculating bubble already mentioned. When the 'free shear layer' is formed the fluid either side is main stream fluid. Separation over the top of the wheel will be of the first type. In the formation of the bubble there is a singular point S, at which the behaviour of the flow is the same as in two dimensional flow. At all other points of separation the limiting streamlines run tangentially into the line of separation, from which a surface of separation originates. The behaviour in the neighbourhood of separation on a rotating body is obviously going to be still more complex. However, if we can apply our two dimensional ideas near the singular point, we might expect that the line of separation lies in a surface above the cylinder and that there is a thin layer of fluid next to the cylinder, moving with the cylinder's surface.

(1-6) The Effect of the Ground

The last of the features of the flow around the wheel mentioned at the beginning of the introduction is the effect of the ground.

The real situation which we wish to model in the wind tunnel is that of a wheel rolling forward in contact with the ground through still air. In the wind tunnel, we fix the position of the wheel and move the air past it, but we must also move the floor with the same velocity as the air, in order to correctly simulate the boundary conditions at the ground. The wheel should then rotate in contact with the floor, with, for no slipping, the circumferential velocity of the wheel equal to the velocity of the floor. A subsidiary problem is the elimination of the tunnel boundary layer ahead of the moving floor in order to prevent the slower fluid in the layer from going above the floor and creating the wrong conditions there. This required simulation is shown in Fig. (2-4).

Previous investigators of the flow about the wheel have not attempted this complete simulation, but have mounted the wheel with a clearance between itself and the floor to enable balance measurements of the forces to be obtained. If the wheel is run in contact with the floor the varying reaction forces between the two can not be adequately allowed for in

taking balance measurements. In order to appreciate the implications of this incorrect simulation it is necessary to look at the differences in the flow about bodies close to the ground and those actually on it.

Potential theory indicates that the venturi effect of air rushing through the gap beneath a body placed near the ground generates large suction on the undersurface and so causes a lift force towards the ground. (see section (4-2) for cylinder and Milne Thompson (1968) p.562 for sphere). This force increases with proximity to the ground and is also increased by rotation (Fig. (4-5)). A rotating wheel is a lifting body and can be expected to generate trailing vortices, as was shown by Maccoll (1928) for a rotating sphere. Stollery and Burns (1969) used the concept of an image horseshoe vortex to simulate ground effect on a three dimensional wing (see H. Glauert (1948) for details). They showed that the main effect of the image system was to increase the lift at any given incidence, α and to reduce the induced drag. Thus $dC_L/d\alpha$ increases as the wing approaches the ground. This effect for lifting bodies is additional to the venturi effect discussed earlier and, for example, can mean that a wing at large positive incidence can have its upward lift increased by the proximity of the ground.

However, whether either of these potential theories will have much significance for a bluff body, such as the wheel, is extremely doubtful. For, in addition to these two

inviscid effects, we find in practice that viscosity can play an important part in determining the overall flow, either through a change in the separation position of the boundary layers on the body, or, with a small clearance, by directly modifying the flow through the gap. Balance measurements on bodies close to the ground show that the above effects can have a decisive influence on lift and drag forces, but this type of data can only give a limited understanding of the flow and no detailed work has been done on the interaction of viscous and inviscid effects in this situation.

The interrelation between these three effects, the two inviscid and one viscous, for a more slender body than the wheel, can be examined in the results of Stollery and Burns (1969), who measured the forces on an idealized car-like body, mounted above a stationary ground plane. As shown in Fig. (1-14), $dC_L/d\alpha$ increases when the body approaches the ground but the lift curve as a whole is moved downwards due to the venturi effect. As the incidence is decreased the curves show a levelling off and reversal of slope, the value of α at which this occurs being dependant on the height above the ground. The corresponding increase in the drag coefficients, at the same incidence, suggests that the boundary layer on the undersurface has separated upstream of the trailing edge. This would inhibit the flow beneath and thereby reduce the venturi effects, decreasing the negative lift.

That separation effects can have a crucial influence on the flow past a body near the ground is emphasised

by the results of Wadcock (1970), who studied the two dimensional flow past two circular cylinders in close proximity. It might be thought that the potential solution for this flow, given in section (4.2.1), would at least indicate the direction and relative magnitude of the lift forces on the cylinders as the two approach, yet, Wadcock's results show that the measured mean forces are in the opposite direction to that indicated by the theory, being repulsive not attractive. (Fig. (4-5)). Mean pressure measurements around the cylinders indicate that this is due to a difference in the separation positions on the two sides of the cylinder, which acts to oppose and overcome the venturi effect.

Interactions between the vortices shed in the two wakes of the cylinders have a significant effect on this two dimensional flow, so that we need not necessarily have the same situation in three dimensions, although the results of Hyde (1972), for a circular cylinder of aspect ratio 1.7 mounted above a stationary ground plane, do show the same repulsive forces as Wadcock's results. This resemblance is superficial, however, since in Hyde's case the separation of the ground plane boundary layer ahead of the circular cylinder was probably the most significant effect in producing the upward lift. This again illustrates the danger of incorrect simulation of the ground boundary conditions for curved bluff bodies like the wheel.

(1.7) Effect of Ground Clearance on Wheel Flow Simulation

The above discussion on the effect of the ground can be related to the wheel through the work of the Motor Industry Research Association (MIRA) (1970) who carried out tests on model wheels, both rotating and stationary, mounted at various ground clearances.

A complete idealized Formula One car model was used, the forces due to the wheels being obtained by deducting those due to the body alone. The model was mounted above a fixed ground (i.e. the working section floor) with a boundary layer thickness, $\delta_{.99}$, of about 2 ins. The wheels, made of polystyrene were 6 ins in diameter and 2,3 and 4 ins. in width all with square edge profiles. Obviously therefore the boundary layer on the floor would have a large effect on the flow under the wheels at the small ground clearances which were used. Some tests were carried out with a moving floor but with a gap under the wheels. This meant that the movement of the floor was of no advantage since it merely increased the flow through the gap and produced greater error. At the test speed of 61 ft/sec the Reynolds number based on wheel diameter was 2×10^5 which is in the critical range of circular cylinder and sphere flows.

The results MIRA designate as zero ground clearance were obtained by sealing the gap between the wheel and the

stationary floor with a strip of paper, not by actually having contact between the two (Fig (1-18)). Although this will prevent flow under the wheel, it is still an incomplete simulation of the true flow and will be subject to errors. For one, the boundary layer on the stationary floor is going to produce effects unrepresentative of the real flow. Also, the incorrect boundary conditions at the junction of the wheel and floor will mean that a unique and significant feature of the flow will not be modelled correctly. In the correct simulation, shown in Fig. (2-4), the wheel rotates in contact with the moving floor and this means that both the surface of the wheel and that of the floor are moving together at the line of contact. This movement draws air in the layers next to the surfaces into the corner between them. Because the contact between wheel and floor prevents flow under the wheel, this air must return back upstream as indicated in Fig. (1-19). The above effects occur only in a very small region under the wheel, but because of the large pressures and appreciable velocities produced, they can have a wider influence on the flow field. None of these effects would be correctly simulated in the MIRA experiments, so that their results at zero ground clearance must not be thought to accurately represent the true picture when the wheel is on the ground.

Despite these deficiencies it is worth considering the MIRA results, as they do give some insight into the real behaviour of three dimensional bluff body flows with small gaps, although it must be borne in mind that the detailed

results will only apply to the particular conditions of the experiment.

The lift and drag coefficients for the 3ins wide wheel found at various heights above the ground are given in Fig. (1-15), the coefficients being based on the projected frontal area of the wheel. The most important feature of Fig. (1-15) is the behaviour of the lift coefficient of both the rotating and stationary wheels. This becomes more negative as the gap is reduced, but at some stage this trend reverses and the coefficient ends up positive when the gap is sealed. In the rotating case, the change from negative to positive occurs at a very small gap, although the exact position is doubtful, because of the wide spacing between experimental points. This late changeover is despite the fact that the floor boundary layer must have helped to inhibit the flow beneath the wheel.

MIRA also measured the static pressure distribution around the centre-line of the wheels. However, this was done with a static tube held as shown in Fig. (1-16), which was probably subject to some error in the thin highly curved shear layer and mounted in the way shown must have considerably disturbed the boundary layer on the wheel. At the Reynolds number of the experiment (2×10^5) this might have led to unknown transition effects, so altering separation and thus the overall pressure distribution. Another point is that measurements were only made at every 30° around the wheel, so that the position of separation points and other features cannot

be fixed with any great certainty. Thus the results of these tests are unlikely to have any great accuracy.

If the results are considered in a general way, however, ignoring the detailed values they can be of use, if only in demonstrating the striking differences in the pressure distribution when the gap is sealed to when it is not. Figs. (1-16) and (1-17) have been drawn with the above consideration in mind. The details of the MIRA results have not been included but the general picture of the distributions is shown and the different distribution with the gap sealed is emphasised. The large suction generated on the underside of the stationary and, especially, the rotating wheel with open gaps, agreeing to some extent with potential theory, contrast markedly with the more positive pressure over the same region which occurs with the gap sealed. (The solid line in Figs (1-16) and (1-17) shows the interpolation at $\theta = 90$ given by MIRA but if the gap were truly sealed it would seem more reasonable to have the pressure tending to stagnation value in front of the seal as indicated by the dotted line, although it may be that the thick floor boundary layer has some influence). The differences in the pressures on the undersurface at the various ground clearances is mostly responsible for the changes in the lift coefficients of Fig. (1-15).

The other main point to note from the MIRA distributions is the difference in separation positions on the stationary and rotating wheels, at all ground clearances. If we

examine the separation position on the upper and lower (except $g/d = 0$) surfaces, then we find that the effect of rotation is to suppress separation on the undersurface, which is moving in the downstream direction, from $\theta \pm 120^\circ$ to $\theta \pm 200^\circ$, but to promote early separation on the upper surface, from $\theta \pm 210^\circ$ to $\pm 270^\circ$. This is in agreement with Thom's results (Fig. (1-10)) and also Prandtl's photographs for rotating cylinders.

The above work of MIRA has important implications for the results presented by Morelli (1969) for the aerodynamic actions on an automobile wheel. A full sized racing car wheel was used for these tests, at speeds giving a Reynolds number range from 5×10^5 to 2×10^6 . Because the forces on the wheel were to be measured by a balance, the wheel did not rotate in contact with the floor, but was mounted so that the bottom projected 15mm. into a slot cut in a fixed ground board (see Fig. 1-18)). Measurements with a yawed wheel and various fairings were obtained, but the basic result for an unyawed wheel in free air was that the C_L was about -0.1 and the C_D about 0.5. The negative lift force is especially significant as it is completely at odds with the results presented in this thesis. The great weakness of Morelli's system lies in the gap of 10mm allowed under the wheel. This gives a gap/diameter ratio of .016 and we see from MIRA's results (Fig. (1-15)) that, for a rotating wheel, this can mean a negative lift instead of the positive value obtained without the gap. Morelli does give a correction for the suction experienced by the wheel due to the gap as,

$$\Delta p = \rho U^2 \log_e \left(\frac{R+\Delta R}{R} \right) \text{ ————— (1)}$$

where

Δp is the suction on that part of the wheel in the slot

R is the radius of the wheel 31.5 cm

ΔR is the gap i.e. 1 cm

U , the velocity of the wheel which is assumed to be the velocity of the air in the slot between the wheel and the ground.

The above expression will be small since $\Delta R/R$ is small and Morelli therefore assumed that the effect of the gap on the lift is small.

All that this is saying however, is that the static pressure gradient due to centrifugal effects is small in a thin layer under the wheel; the normal boundary layer assumption. It ignores completely the fact that there will be a pressure gradient between the front and rear of the slot which tends to force air through, this tendency being helped by the rotation of the wheel. A better approximate correction would be obtained by ignoring these small curvature effects and assuming that the transverse velocity components v, w are zero in the slot. The problem then reduces to that of unidirectional flow between parallel plates whose solution is given by Batchelor (1967). It is worthwhile examining this solution here, as it

illustrates the interaction of the flow through the gap with the external stream. The continuity equation reduces to

$$\frac{\partial u}{\partial x} = 0$$

so u is independent of x and we assume independent of y . Then the full Navier-Stokes equations reduce to

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2}, \quad \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = 0$$

As the right hand side of the 1st equation is independent of x , we must have

$$\frac{\partial p}{\partial x} = \text{const} = -C, \text{ say, through the slot.}$$

Thus, if the pressure is known near the two ends of the slot a linear pressure gradient could be assumed through the slot. This would have been a far better approximate correction for Morelli to have used than the one he proposes.

A solution of the above equation, with boundary conditions $u = U_{\infty}$ when $z = g$ and $u = 0$ when $z = 0$, is

$$u = \frac{-C}{2\mu} z(g-z) + U_{\infty} z/g$$

$$\text{so } Q = \frac{\text{volume flux}}{\text{unit span}} = \int_0^g u dy = \frac{Cg^3}{12\mu} + \frac{U_{\infty}g}{2}$$

Now the volume flux depends on C , the pressure

gradient determined by the external flow, but the external flow itself will be influenced by the volume of fluid allowed through the gap. It is this influence which makes any results obtained with a gap under the wheel very doubtful, even if some correction could be applied to the lift, for the drag, as well as the lift, may be altered.

(1.8) Conclusions

We feel that the features discussed in the introduction have shown the importance of having a correct simulation of the wheel flow. The Reynolds number of the experiment must be, at least, in the same range as the real flow, since otherwise the separation position and hence the pressure distribution may be wrong. For similar reasons, the wheel must be rotating at the correct velocity, since we have shown that rotation can have significant effects on boundary layer separation. The results of MIRA and Morelli, in particular, have shown that the effects of even a small gap under the wheel are such as to invalidate any application of the results to the real flow. Furthermore, because the boundary conditions near the wheel-floor junction give rise to important viscous effects, it is necessary that these same effects should be produced in the simulation. We therefore require a wheel rotating in contact with the ground, which, for a wind tunnel, implies a moving floor. Because the varying reaction forces which are then generated preclude balance measurements, the

forces on the wheel must be obtained from integrating static pressure measurements over the wheel surface. This is the arrangement adopted in the experiments described herein.

Obviously, there are a large number of variable factors which it would be possible to model in the experiment, e.g. the tyre tread patterns, the deformation of the wheel and the effect of various fairings around the wheel. However, initially it was decided to look at the simplest case of an isolated, non-deformable, smooth surfaced wheel, in order to provide a basic starting point from which the effects of the different factors could then be examined as required.

(2) SUMMARY OF EXPERIMENTAL APPARATUS AND PROCEDURE

A detailed discussion of experimental techniques is given in the Appendix, here it is merely intended to give an outline description in order that the results should be understandable.

The initial interest in the flow around a wheel arose out of a study of the aerodynamics of 'Grand Prix' cars, which, because of the racing rules, have totally exposed wheels. As a consequence the shapes tested are representative of the tyres and rims currently used for 'Formula One' racing cars. The diameter of all the model wheels is 16.375 ins and, at the test speed of 61 ft/sec, this gives a Reynolds number of 5.3×10^5 , which is considered sufficiently high for the tests to be accurately representative of the real flow (i.e. in the supercritical range of cylinders and spheres). Six wheel shapes were tested in all; three different tread widths (designated A, B, and C with increasing width) and two edge profiles (designated 1, 2). Details of the profiles and the distribution of the pressure holes are shown in Fig. (2-1) and (2-2). After being tested with a smooth rim, wheel C2 had three grooves cut into the surface as indicated in Fig. (2-2) and was then retested. The wheels, which were machined from light alloy, rotate freely in bearings and are supported rigidly in contact with a moving floor by a system of adjustable

rods. Photographs of a wheel mounted in the tunnel are given in Fig. (2-3). Checks with a stroboscope indicate that the peripheral speed of the wheel is the same as the floor, so no slipping occurs between them.

The wind tunnel boundary layer is bled off ahead of the moving floor by a fairing and suction is then applied through a 'Vyon' porous plastic sheet to prevent a new layer forming (Fig. (2-4)). Originally the speed in the thin detached shear layer shed from the trailing edge of the fairing fell to about 80% of the free stream value at the wheel centre position, but in its final form the fairing prevents the speed falling below 95% (see Fig. (2-5)). As noted in the introduction, the flow in the corner region between wheel and floor is likely to be significant, so it is desirable that the above shear layer should be reduced, as much as possible, to decrease any effect on this flow.

As the wheel rotates, the pressure sensing holes on its surface experience a cyclic variation in signal, corresponding to the distribution of mean pressure around the perimeter. Superimposed on this are random fluctuations arising from the unsteady flow in the wake. The measuring system must be able to respond sufficiently rapidly to resolve the transient details of these fluctuations, and the data acquisition equipment has to be capable of separating the random signals from the mean cyclic variation. The pressure sensing instrumentation is shown in Fig. (2-6). A condenser microphone (B.&K. 1/4"), with its associated oscillator (DISA No. 51E32),

is placed on the axis of rotation, to eliminate centrifugal effects on it, and connected to one pressure port at a time. The relatively high level signal is then taken from the rotating assembly, through slip rings (using low noise silver graphite brushes), to a DISA Reactance Converter (no. 51E01), the output of which is proportional to the pressure acting on the microphone.

Mounting the microphone on the axis meant that the connecting tube between it and the sensing port had to be at least 12 ins. long and was chosen to have $3/32$ ins internal diameter to lessen the effects of attenuation. As a consequence resonance was experienced at about 200 Hz (and at 3,5,7 etc. times this), but by placing wire wool in the connecting tube to act as a damping agent, the resonance was minimised and a frequency response such as that of Fig. (2-8) obtained. Note for comparison that the frequency of rotation of the wheel is 14.25Hz. The same length tube was used for all the connections to the sensing ports to ensure a unique calibration. A correction is applied to all the resulting pressure measurements to allow for the centrifugal pressure difference between the microphone on the axis, and the surface of the wheel.

A problem, that was not anticipated, was the unacceptable, large long term drifting of the transducer output. This difficulty was overcome by making a branch in the tube connecting the microphone to the sensing hole and leading this pressure through narrow bore tubing and a rotating seal to a

manometer. This instrument therefore indicates the mean level of the signal and the microphone the fluctuating component. The output signal from the microphone is processed by a Hewlett-Packard Correlator (Type 3721A) which operates in the 'signal recovery' mode. This instrument is able to digitally sample and average several thousand pressure histories; thus accurately eliminating the random components and retaining only the true cyclic mean distribution. A trigger signal to initiate each sampling cycle is derived every revolution of the wheel by an inductive pick up. The mean pressure distributions are stored within the correlator, from which they are recorded on an X-Y plotter and on punched paper tape for subsequent computing of the forces on the wheel.

These arrangements could not be used for measuring the pressures acting on the stationary wheel. These distributions were obtained graphically on an X-Y plotter using signals from the pressure transducer and a potentiometer connected to the wheel to serve as an angular displacement transducer. By slowly rotating the wheel by hand the distribution for each hole was plotted automatically.

(3) RESULTS

Note on Errors

A discussion of possible errors and where they arise is included in the appendix but it was thought necessary to give some indication here of the likely error limits of the results given in this section. The general conclusion reached in the appendix is that the results obtained for the lift and drag coefficients of the wheels should be within 5% of the true value (i.e. the true value of the forces on the experimental wheels; the forces on the real road wheel will be affected by such factor as tread pattern and type of road surface. This is discussed later). However, because the B wheels were the first tested and measuring techniques were progressively improved they may be subject to greater errors although within about 10%. In particular the procedure of examining the rapidly changing pressures near the ground at smaller intervals, of $1/2^\circ$ between points instead of the normal 5° , was not adopted until later, which will have some effect on the results for the lift. The B2 wheel also shows some variation in the base pressure measured by different holes across the wheel, which is difficult to account for, and the results for this wheel must therefore be viewed with some suspicion. It is unfortunate that there was insufficient time to repeat the measurements for this wheel.

(3.1) Lift and Drag Coefficients

Two sets of lift and drag coefficients were calculated, the first (denoted by subscript 1) are based on the projected frontal area of the wheel (i.e. the 'X-sect. area') and the second (subscript 2) based on an area given by multiplying the wheel diameter with the width of tread in contact with the ground. This tread width is 4.25", 6.75" and 9.25" for the A, B and C wheels respectively. Thus, although the cross sectional area of the shape 1 and 2 wheels at any given width will differ because of the change in edge shape, the area based on tread x diameter will be the same. This second area was felt to be of more practical interest, since it is the tread width in contact with the ground which is the basic length parameter in determining the road holding qualities of the wheel.

The lift and drag coefficients for the six wheel shapes tested are given in Table 1 and graphically in Fig. (3-1). These coefficients were obtained by integrating the static pressure over the external surface of the wheel. They do not therefore include any skin friction contribution to the drag. However, this contribution is usually a small part of the drag of bluff bodies because the separated region produces such a large pressure drag. The coefficients do not include any contribution from inside the hub either. These are given in a separate column of table 1.

Considering the subscript 1 coefficients first, we see that both the lift and drag coefficients increase with width. The coefficients for the A and C wheels are almost the same for the two different edge shapes, whilst those for the B wheels differ by about 10%, although this may be due to the greater error associated with these wheels. In changing to the second set of coefficients, these values are multiplied in varying degree, depending on the ratio of the cross-sectional area to the 'tread x diameter' area, to yield the results shown. Surprisingly, the drag coefficients no longer increase with width and, in fact, are almost constant for the edge shape 1 wheels.

The above coefficients can be taken to represent the forces on flush sided wheels, since they do not include any contributions from the indented hub. These were obtained by integrating over the hub surface using the pressures from the holes shown in Fig. (2-2). The contribution to the drag is greatest for the narrowest A wheels, whilst that to the lift is negligible for all the wheels. For this reason a column of values for C_{L1} is not included in the table. We thus have the surprising result that, when the hub contribution is taken into account, C_{D2} is actually greatest for the narrowest wheels.

(3.2) Comparison with Previous Work

If we now compare our results with those of previous

investigators as given in table 2, we will find that the MIRA values are on the whole compatible with the present results, but that Morelli's results are entirely different. The reasons for the latter were given in the introduction and a difference in values was only to be expected. We suggest that Morelli's results are not at all representative of the real flow about a wheel on the ground due to the gap under his wheel and, when interpreted as such, are considerably in error, even to the extent of producing a lift force of the wrong sign.

In comparing with the work of MIRA, it should be remembered that these results were obtained using a complete idealised 'Formula One' racing car model, the wheels of which were mounted above a stationary floor with the gaps underneath sealed by paper strips, and, as explained in the introduction, this arrangement is subject to a number of errors. Note also that the MIRA wheels are square edged and flat sided. This means that the coefficients based on actual x-sect. area and those based on tread x diameter will be the same (i.e. C_{D2} , C_{L2} , and C_{D1} , C_{L1} , are the same) and there is no additional contribution from the hub.

In table 2 the coefficients with subscript 1 are for the forces due to the front wheels only, but those with subscript 4 are for the forces due to all four wheels. Surprisingly, the MIRA results for the front wheels only are quite close to the results for the B wheels in the present experiment, despite the differences in simulation. It is unfortunate, from our point of view, that the effects of wheel

width were only studied for the four wheels together, since it makes comparison with the present experiment difficult. It is not possible to separate the effect of the width on the individual wheels from its effect on the interference between the front and rear wheels. However, the variation with width is fairly similar to that of the subscript 2 coefficients of the sharper edged shape 2 wheels.

It appears as though the MIRA results do give the right order of values for the coefficients, despite the inadequate simulation in their experiments, although the coefficients for the wheel shape they tested may not be entirely correct. It is impossible to quantify any errors, unless the complete simulation is done for their wheel shape, and this is really the weakness in their results.

(3.3) Variation of C_L and C_D across the Wheels

One advantage of finding the force coefficients from pressure measurements is that much additional information is also obtained. Thus, one not only finds the total forces, but also the contribution to these forces from the different parts of the wheel. This ability has been utilised to find the variation in the lift and drag coefficients across the wheels given in Figs. (3-2) to (3-5). In all these figures, the distance from the centreline of the wheel, w , was non-dimensionalized by dividing by half the same tread width, t , as was used in calculating the subscript 2 coefficients.

This does mean that the maximum value of w/t will be different for each wheel, which would not be the case if the total width of the wheel had been used. However, dividing by the tread width does have certain advantages when interpreting the figures in relation to the C_{D2} , C_{L2} results of table 1.

This is demonstrated by Fig.(3-2) which gives the variation in C_D across the wheels for edge shape 1. It is immediately obvious from the figure how we obtain the rather surprising result that C_{D2} is almost constant for the three widths. The coefficients for the wider wheels are greater near the centre-line, as might be expected, but this is balanced by the more rapid fall near the edge of the wheel. This latter fact is reasonable, when one considers that the tread width forms a much smaller proportion of the total wheel width of the narrow wheels than it does of the wide wheels. By using the tread width as our basic length therefore we are, in effect, causing the wider wheels to have sharper edges than the narrow wheels.

Note that the results were only taken for one half of the widest C wheels. This was because the length of tubing, between the measuring holes and microphone, which would have been required to obtain measurements over the whole wheel, would have resulted in a resonance in the microphone response at too low a frequency to have given satisfactory results. In calculating the total C_D and C_L for these wheels, therefore, it was assumed that the distribution

was symmetric about the centreline.

Fig. (3-3) gives the corresponding variation in drag coefficients for edge shape 2 wheels. The most striking feature is the behaviour of the B2 wheel, in particular the large wavy variation near the centreline. This, and an equivalent waviness of the lift coefficient (Fig. (3-5)), are consequences of the variation of pressure in the base region of the wheel (illustrated in Fig. (3-17)). None of the other wheels show such large changes in base pressure across the wheel and, as suggested before, there must therefore be some suspicion that the measuring system was in error in this case.

The behaviour of the lift coefficients given in Figs. (3-4) and (3-5) is very similar to that of the drag already discussed, including the negative contributions from the edges of the wheels, with the exception that the B wheels have value of C_L very close to or exceeding (for B2) those of the C wheels, even near the centre-line of the wheels. This may well be a consequence of not examining the pressures near the ground at smaller intervals, since the result of doing this for the other wheels was a reduction of about 3% in the lift coefficients near the centre-line.

(3.4) Pressure Distributions for A Wheels

It is not intended to present all the pressure

distributions obtained for all the wheels since there is an essential similarity between wheels, rather we shall give detailed results for the A-wheels only and then concentrate on pointing out any significant differences for the other wheels.

Figure (3-6) presents the pressures measured around the centre-line of the A-wheels. The most remarkable feature of both curves is the sharp rise to a positive pressure coefficient greater than 2 near the floor ($\theta = 90^\circ$). Now, the c_p cannot rise above the stagnation value unless extra energy is being injected into the flow and this is produced by the viscous actions referred to in the introduction and associated with the moving boundaries. The pressure does not fall immediately to a low value after $\theta = 90^\circ$ but takes about 4° to reach a c_p of zero and then oscillates briefly before settling down at the base pressure. This might be an effect connected with the response of the measuring system, but the frequency of oscillation, about 800 Hz, does not correspond to any of the theoretical resonance peaks of the system (nearest are 600, 1000 Hz). Instead we feel that this may well be caused by the moving floor lifting in the low pressure region behind the wheel, to touch the wheel's surface, and then falling back. This process would occur in the small region under the wheel, behind the line of contact with the floor, and would create the fluctuating pressure picked up by the measuring hole.

It would also account for the lack of the negative pressure peak predicted by the theoretical solution for this region given in section (4-2-2).

The positive pressure peak, which was found, will contribute a significant amount to the lift coefficient of the wheel. In fact the range of θ from 75° to 105° contributes almost 30% of the lift coefficient for this hole (.64).

In connection with the lift, the other important feature is the position of separation from the top of the wheel. Experiments to be described later, using flow visualization and Pitot-static traverses of the wheel boundary layer, all suggest that separation occurs about $\theta = 280-290^\circ$, i.e. $10^\circ-20^\circ$ in front of the top of the wheel. The pressure distribution shows a slight adverse pressure gradient before this position ($\theta \approx 300^\circ$ to 280°) and then a reversal of slope at around 280° . We shall take this as indicating the separation position, although the mean pressures over the rear of the wheel behind this position are not particularly constant.

However, the important point is the absence of any large negative pressure over the top of the wheel, which would occur if the boundary layer remained attached. This early separation therefore acts to lower the lift coefficient of the wheel, opposing the action of the viscous effects at the bottom of the wheel.

Both these effects are associated with the moving

boundaries of the rotating wheel and the moving floor, (it is shown later that the early separation does not occur on the stationary wheel), and so would not be produced in any simulation which did not involve these features. Such simulation would therefore be liable to errors in estimating the lift coefficient and, since these special features will presumably have some influence on the recirculating flow behind the wheel and hence the base pressure, the drag coefficient might also be in error.

Note the great similarity between the curves in Fig. (3-6), suggesting that changing the edge shape appears to produce little change at the centre-line even on the narrowest wheel.

Fig. (3-7) and (3-8) give the pressure distributions at holes 8 and 7 respectively, both of which are on that part of the wheel which comes into contact with the ground. The form of the curves for these holes and hole 9 is very similar; all show the sharp rise in pressure near the ground and have the separation position around $275-280^{\circ}$. The base pressure is also similar and shows the same variation. The most noticeable change is that for hole 7 the pressure no longer rises to near stagnation value at the front, but this might be expected since this hole is displaced by 1.6 ins from the centre-line.

Hole 6 (Fig.(3-9)) is the first position where the surface does not actually contact the floor at the bottom, although the gap in this case is very small (about .05 ins).

However, at $\theta = 90^\circ$ there is still a sharp rise in pressure, although not to the same extent as before, and now immediately followed by a similar sudden decrease in pressure. Other features of the distribution correspond to those of previous holes.

The distribution shown for hole 5 is of a different form to those already described. The pressure does not become positive at any point on the curve and indeed becomes highly negative between $\theta = 50^\circ$ and 80° , for both wheels. For wheel A2 the pressure is more negative over most of the front than it is over the rear. Note that, for this hole and following ones up to hole 1, the positions of the holes on the two wheels no longer correspond (see Fig. (2-1)), so that agreement between the two cannot be expected. Because of the more negative pressure acting in the lower front region, this hole contributes a negative drag and lift to the total forces, for both wheels.

The distribution for hole 4 (Fig. (3-10)) is similar to that for hole 5, but with less variation near the ground.

Holes 3, 2, 1 at the side of the wheel give similar results, with the variation in pressure over the rear half growing larger from hole 3 to hole 1.

Fig. (3-12) shows the pressures measured by two holes placed inside the hub at positions indicated in Fig.

(2-2). The positive pressures at $\theta = 140^\circ$ to 200° from hole H1, suggests that the air is entering the hub in this region. The contribution to the lift from these holes is not very significant, but there is quite a large contribution to the drag. In fact integrating over the surface of the hub using just these two holes gives an addition of .12 and .14 to the C_{D2} of .97 and .90 for A1 and A2 respectively. There is little point in obtaining this value to greater accuracy by using more holes, since it will depend on the shape of the hub which is likely to vary from car to car. However, since our hub shapes are similar for all the wheels tested, we can directly compare between the results. In the present case, the addition to the drag is the same for both wheels and significant enough to suggest that a cover over the hub well, flush with the wheel edge, would greatly reduce the drag of the wheel, although there may be other considerations, such as brake cooling, which would make this undesirable.

Only mean pressures were given in the above figures, as obtained from the correlator. However, the fluctuating part of the signal can contain some useful information. For example, the separation position could be deduced as the point at which large scale pressure fluctuations begin, due to the unsteady nature of flow in the separation bubble. An example of the non-averaged signals is given in Fig. (3-13) where the signals for wheel A1 are presented. Note that for holes 9, 6 and 5 the major variations in the signal are over the rear, beginning at about the point previously referred to as

the separation point, and also near the ground at the front (cf. Figs. (3-6) and (3-9)). A noticeable feature of the signals from the three edge holes, is the negative pulse-like fluctuations that occur, of about 1 to $1\frac{1}{2}$ times the dynamic pressure in magnitude, although these are not as frequent as might be suggested by the photographs, which were chosen to include this phenomenon in order to illustrate the very large scale of the turbulent pressure fluctuation around the edge.

(3.5) Pressure Distributions for other Wheels

We shall now compare the above results with those for the other wheel widths, doing this first for the shape 1 wheels. Fig. (3-14) presents the mean pressures given by the holes on the centre-lines of the three wheels.

At the ground, $\theta = 90^\circ$, the B curve does not rise quite as high as the other and then falls more slowly. Again, this is, most likely, a consequence of using 5° intervals between measurements for this wheel, as opposed to $1/2^\circ$ for the other, since the other curves would also be more spread out if a higher resolution had not been used. It is this spread which suggests that the contribution to the lift from this region may be too high for the B wheel, although this will be mitigated, somewhat, by the lower maximum value at $\theta=90^\circ$. The oscillation in the pressure after $\theta = 90$ will not show up with the 5° intervals and, in interpolating between points in

this region, we have drawn a smooth curve, as this is the shape assumed in the integration of the pressure distributions.

Major differences in the pressure occur over the lower front face, where the narrow A-wheel has a much lower pressure than the other two wheels, and over the base region where the widest wheel has the lowest pressure, as might be expected. Near the separation position, $\theta = 280^\circ$, the B and C wheels behave in a similar way and both appear to have separation slightly further forward than does the A wheel. Over the upper front of the wheel all three wheels behave alike.

The pressure distributions at holes 6 and 5 are presented in Fig. (3-15). The most obvious features of the figure is that the complete change in appearance of the distributions between these two holes is common to all three wheels. For hole 5, the pressure distribution of the B wheel is much closer to that of the C wheel than to that of the A wheel, although for hole 6 this is not so noticeable.

If we look at the pressure distributions given by hole 1 and by the two holes inside the hub (Fig. (3-16)), we again find that the B and C wheels behave alike, while the A wheel is noticeably different. There are no results for H1 from the B-wheel as no hole was provided in this position for either of the B-wheels. The greater variation in pressure inside the hub of the A-wheel suggests that there is more interaction with the external flow for this wheel and this

is reflected in the respective lift and drag coefficients from the hub given in table 1.

The pressure distributions near the centre-line of the shape 2 wheels show similar features to those for shape 1 given above. The only exception is the behaviour of the pressures in the base region of the B2-wheel, which has been mentioned previously as being anomalous. The large variation in base pressure across the wheel is illustrated in Fig. (3-17) which gives the distributions at two adjacent holes. The pressure distributions near the edge of the wheels are given in Fig. (3-18). There is less evidence of any difference in the behaviour of the A-wheel compared to the others, especially at hole 1. Inside the hub the variation in pressure is much as before, although with a slightly greater variation for all the shape 2 wheels.

(3.6) Effects of Rotation

In order to isolate the specific effects of wheel rotation on the flow field, a comparison was made between a stationary and a rotating wheel, including measurements of total head. The wheel chosen for this comparison was B2. It is unfortunate that the B2 rotating results are not very consistent with measurements taken of the other wheels, at a later date, but it is felt that these inconsistencies are of such a nature as to in no way invalidate any general conclusions

drawn from the comparison with the stationary wheel.

The two sets of results are given in table 3, which suggests that the effects of rotation are beneficial in reducing both the drag and the lift of the wheel. If we turn to fig. (3-19), which gives the mean pressure distribution around the centre-line, we can see the reasons for these reductions. The lift produced by the sharp pulse of high pressure at the bottom of the rotating wheel is more than balanced by the large region of low pressure over the top of the stationary wheel, whilst the lower pressure acting over the rear of the stationary wheel contributes towards the greater drag. The figure shows that the early separation from the wheel noted previously is a product of the rotation of the wheel and not an intrinsic quality of the shape. This early separation probably also plays a large part in determining the different base pressure, although the action of the rotation on the separated region itself may be significant, since one would normally expect a later separation to yield a higher base pressure.

If we compare the pressures in Fig. (3-19) with those in Fig. (1-3) and (1-6) for a stationary sphere and cylinder respectively, we see that the pressures over the top of the stationary wheel bear a close resemblance to those of the sphere and cylinder at an equivalent Reynolds number (5×10^5), but the pressures over the top of the rotating wheel resemble those of the sphere and cylinder at a lower Reynolds

number, with laminar separation. Despite this resemblance, it is shown later (section (4-1-1)) that the separating boundary layer above the rotating wheel is indeed turbulent before separation, which is in agreement with the results of Griffiths and Ma (1969), discussed in the introduction, who showed that the rotating surface moving against the free stream provokes transition and does not suppress it.

It is illuminating to consider a direct comparison between the pressure distributions obtained by MIRA for the wheel with a sealed gap and the present results, for both the stationary and rotating cases. This is given in Figs. (3-20) and (3-21). Dealing with the rotating case first, we see that the base pressures are in fair agreement, but, near the ground and near separation, there are wide discrepancies between the two results.

The low pressures given by MIRA at $\theta = 90^\circ$ are quite unrealistic for zero ground clearance, being either due to poor interpolation, or poor simulation, as mentioned previously. Near separation, we feel that the measuring technique used by MIRA would have been in error, if only by disturbing the boundary layer flow sufficiently to affect the separation position. Note that the MIRA results imply a very steep adverse pressure gradient before separation.

The two stationary wheels show much the same

differences. The agreement in base pressure is not so good, but both results give the stationary wheel a lower base pressure than the rotating one. The same remarks as before can be made regarding the differences at the ground and at separation. Note that although the MIRA results do not agree with the present ones near separation, they do show a greater region of low pressure over the top of the stationary wheel, than over the rotating wheel.

The great differences between our results for the rotating and stationary pressure distributions occur right across the wheel as is shown by Fig. (3-22), which compares the pressures at hole 6 and hole 5, and also by Fig. (3-23), which gives the variation in C_D and C_L across the wheel. Note that the stationary wheel exhibits the same form of wavy variation in C_L and C_D near the centre-line as the rotating wheel, although not to quite such a large extent. It appears from the pressure distributions for the stationary wheel that at $w/t = 3$, (hole 9) the boundary layer remains attached further around the wheel (Fig. (3-24)) giving the greater lift and lower drag at this point.

As well as the above results for the pressures and forces acting on the wheel, the external flow field was examined by measuring total head with a Kiel tube (for description see Appendix). A total head measurement below the free stream value indicates that the fluid concerned has undergone viscous action. The position of shear layers and

wake regions can therefore be identified by a survey of total head. The total head was measured in four planes at right angles to the flow direction at positions, shown in Fig.(3-25), the results obtained for these planes being plotted in the next four figures. These results are presented in the form of contour lines joining points of equal total head value. Only the contours for values of $\frac{H - p_{\infty}}{1/2\rho U_{\infty}^2} = 0.9$ are shown, partly for clarity and partly because it was felt that further in the wake the total head measurements would be affected by the high intensity of the turbulence. The 0.9 contour can loosely be thought of as a map of the edge of the wake. Only the half of the flow field opposite to the supporting rods is shown, since their wakes would only confuse the picture, and for the same reason measurements were not taken very close to the ground due to the wake of the fairing at the front of the floor.

Fig. (3-26) shows the .9 total head contour for plane 1, 1.7 ins. downstream of the wheel axis. Contours for the rotating and stationary cases are compared (designated R and S respectively). Note that the R-contour is well away from the top surface of the wheel, suggesting that the boundary layer has already separated upstream of plane 1. On the other hand, the S-contour shows the boundary layer still attached to the stationary wheel, thus tending to confirm the pressure measurements described earlier.

At the side of the wheel, the R-contour is well away

from the wheel outside the upper part of the recessed hub, but comes quite close further down. Measurements, using wool tufts to give the flow direction, suggested that the air inside the hub is on the whole rotating in the same way as the wheel. Thus, the air in the upper part is moving against the external flow, whilst that in the bottom part is moving with the flow. We would therefore expect there to be more viscous interaction between the air in the upper part of the hub and the free stream. This effect does not occur with the stationary wheel, of course; the contour suggesting that there is little interaction.

Lower down, the contours in both cases move right away from the wheel. This is the turbulent region produced by air coming out from under the front of the wheel. Rather surprisingly, at first sight, the S-contour lies outside the R-contour. One might have expected the jet produced under the rotating wheel would have created a greater disturbance. However, the wider spread around the stationary wheel is probably due to the boundary layer on the stationary floor separating ahead of the wheel and rolling up to form a horse-shoe shaped vortex around the wheel.

Figs. (3-27), (3-28) and (3-29) show the development of the wake as it moves downstream. The main points of note are the differences between the two contours at the top of the wheel and the spread of the large disturbed region at the bottom. The R-contours at the top on the rotating case

remain at a high level, just above wheel height, but the S-contours suggest that the air is swept down close behind the wheel, only breaking into a disturbed wake at just about half wheel height. One interesting feature of the stationary case is that the contour at about 1 ins from the centre-line appears to remain closer to the wheel than it does at neighbouring points. This is the position corresponding to hole 9 and, as noted previously, the pressure distribution measured by this hole also suggests the above. However, the total head measurements for the rotating wheel do not show any such non-uniformity in this region, even though the lift and drag coefficients have the same wavy variation as the stationary ones. The disturbed region at the bottom of the wheel spreads both outward and upward, the R-contour remaining inside the S contour at each station, and this region extends halfway up the wheel and out to the edge of the moving floor by plane 3.

As a further examination of the separation region and the flow around the bottom of the wheel, some flow visualization experiments were conducted using wool tufts and smoke. Some of the photographs obtained are given in Figs. (3-30) and (3-31). The top smoke photographs clearly show the earlier separation and higher wake region of the rotating wheel. The wool tufts confirm this and also show the reversed flow back over the top of the wheel. The spread of the flow around the bottom of the wheel is shown in the two lower photographs of Fig. (3-30). Note the greater spread in the

non-rotating case confirming the total head measurements.

The above results have enabled us to distinguish those features of the flow which are specifically due to the rotation of the wheel. The early separation over the top of the wheel is seen to be caused by the rotation and not to be an inherent quality of the wheel at the Reynolds number used. The rotation also produces viscous actions near the ground and these, together with the early separation, are the basic agents in determining the pressures and hence the forces acting on the wheel. The wake region is also to a large extent determined by these effects, together with any extra rotation induced into the wake by the moving surface of the wheel.

(3.7) The Effect of Grooves in the Wheel Surface

Three, $1/4$ " wide, $1/8$ " deep, grooves were machined into the surface of the C2 wheel, at the centre-line and 2" either side of it. It was hoped that these grooves would approximately simulate the effects of tyre tread pattern on the wheel flow. In fact, the grooves were deliberately made larger than most tread patterns to exaggerate any such effects.

Table 4 presents the results for the two configurations of C2. From what has been said previously regarding the consequences of leaving even a small gap under the wheel, it might have been expected that the grooves would have a more

dramatic effect on the wheel but the decrease in the lift coefficient C_{L1} is only about 10%, whilst the drag coefficient C_{D1} , remains the same. In generating the second set of coefficients, the tread width in contact with the ground of the grooved wheel is taken to be less than that of the normal wheel by $3/4$ " (i.e. 8.5" instead of 9.25"), so the area on which the coefficients are based will be less. Thus C_{D2} becomes greater for the grooved wheel, while C_{L2} is almost the same for the two wheels. When viewed in terms of the amount of wheel actually contacting the ground, therefore, the grooved wheel is less efficient than its normal counterpart.

To seek the reason for this low effectiveness, we must consider the individual pressure distributions. Fig.(3-32) gives a comparison at hole 11 on the centre-line of the two wheels and, therefore, within the central groove of the grooved wheel. The first point of note is that, although the pressures differ markedly in the vicinity of the ground, over the rest of the wheel there is virtually no change at all. Our discussion of the effect of gap under the wheel leads us to expect a low pressure in the groove at the bottom, but note that the c_p rises to a value close to 1 at about 80° before falling to its lowest value of -1.2 at 95° . Thus we see that the effect of the groove is confined to a fairly small region at the bottom of the wheel, although this is an important region for the lift, because of the very high pressures normally found there. Thus the lift for this particular hole is reduced considerably, but because of the localized nature of the effect

the drag is not altered.

To see if the groove effect is less localized across the wheel, we compare the pressures at hole 10, midway between two grooves (Fig.(3-33)). It is immediately obvious that the grooves have no effect on this hole, in particular, the high pressure at the bottom of the wheel is as strong as ever. This is not too surprising, if we remember that it is generated by viscous forces which are themselves very localized in action. If we examine the pressures measured by the other holes, we find that only those holes actually in a groove are at all affected. This is best seen by looking at the variation of C_L and C_D across the wheel for the two cases, given in Fig. (3-34). Here we have used the same value of $t = 9.25/2$ for both wheels, as we wish to make a direct comparison of the same position across the wheel.

It is surprising that the air passing through the groove at the bottom does not have more effect on base pressure, since even quite low rates of base bleed can reduce the negative base pressure and hence drag by a significant amount. It is possible that the viscous actions, still generated by the surface which contacts the ground, act to reduce the effect of the air passing through the grooves, since the action in the rear corner will tend to induce any air to flow back into the corner, as discussed in section (4-23).

It has been argued (e.g. Morelli (1969)) that, because the grooved tread pattern of tyres allows air to pass

under the wheel, this is equivalent to having a slight gap between wheel and floor. The major criticism of this practice is that any errors introduced will not be quantifiable unless the full simulation is carried out. Our results show that the effect of the grooves is to decrease the lift in their immediate neighbourhood only and that the viscous actions associated with the contacting surfaces of the wheel and ground will still occur and be important. This being so, allowing any gap under the wheel, which will eliminate these effects, will not give a realistic representation.

(3.8) Variation in Wind Speed

In order to accurately model the wheel flow in the wind tunnel it is necessary to equate the wind speed with the speed of the floor. However, since the wind speed is likely to drift slightly with changes in temperature and other atmospheric conditions, it was decided to investigate alteration of the wind speed from the optimum equal value in order to discover whether the flow was very susceptible to such changes. If this was so, it would be a source of error difficult to account for.

Fortunately, our results for wind speed values either side of the desired equal value shows that there is little deviation from the results already described. Fig. (3-35) compares the three pressure distributions at the centre-line of the C2 wheel for $v/U_{\infty} = .87, 1.0, 1.07$ (v = rotational

speed). The pressures given are uncorrected for blockage effects, since the correction used (due to Maskell, see Appendix) depends on a knowledge of the mean base pressure across the whole of the base region of the wheel and this was not available for the different wind speeds, only the pressures on the centre-line being obtained. The pressures were measured at 5.1° intervals around the wheel only, so the interpolation near $\theta = 90^\circ$ may be inaccurate. Bearing this in mind the agreement, within 2%, for the C_D and C_L is quite acceptable. The most significant difference in the pressure distribution occurs near the separation position, $\theta \approx 280^\circ$, where it would appear from the lower pressure that the separation occurs further downstream at higher wind speed (i.e. lower v/U_∞). If, as has been suggested, the early separation from the wheel is produced by the rotation, it would seem reasonable that a lower v/U_∞ would decrease this effect.

To investigate this further, some more pressure distributions were obtained at much lower wind speeds. It was not possible to obtain any at higher speeds because of excessive belt lift. Also, the wind speed was chosen to be varied, because changing the floor speed involves the fitting of new gear ratios in the drive, for every speed change. The results, uncorrected for blockage as above, are presented in Fig. (3-36). The pulse of high pressure at $\theta = 90^\circ$ appears greater at the lower speeds although in absolute magnitude this is not so. In fact this phenomenon is more closely related to the speed of the boundaries, which is kept constant, than to the wind

speed and, as the pressures at each wind speed are normalized by the appropriate dynamic pressure, the pulse magnitude, in coefficient terms, will appear greater at the lower speeds.

The position of separation at the higher values of v/U_∞ is difficult to interpret, since the base pressures are far from constant. It is probably best merely to say that the low pressure over the top of the wheel, is progressively reduced by the rotation, although the indentations in the pressure distributions, around $\theta = 300^\circ$, do suggest that the phenomenon we have called separation is moving upstream as the rotational speed increases, except that it no longer gives rise to even approximately constant pressure. Despite this reduction in pressure at the top of the wheel the C_L of the lower v/U_∞ values is the greatest because of the increase in importance of the high pressure at the bottom. The C_D remains much the same for all the values of v/U_∞ tested. This applies only to the uncorrected values, of course, if a correction for blockage is applied the above conclusion may be different.

It is felt that the findings of section (3-6), where the rotating and stationary wheels were compared, have been confirmed in the present section. It appears that the rotation is responsible for reducing the pressure over the top of the wheel (i.e. provoking early separation) and for reducing the negative base pressure, the amount of reduction in both cases being proportional to the speed of rotation.

(3.9) Two Dimensional Flow

It was felt that the two dimensional flow past the wheel (or, in two dimensions, therefore, a cylinder) would prove of interest in a number of ways; it would provide a bridge between the theoretical work which applies to the two dimensional case and the experimental results for the wheel; it would give an extreme value for the aspect ratio results; and should enable those special features associated with rotation to be more easily observed, since in a sense, these are diluted in the three dimensional flow.

Unfortunately, in the time available, it was not possible to use a cylinder of very high aspect ratio as this would have meant major changes to the apparatus, so a compromise had to be reached by using the widest wheel available with square edge profiles, giving a width of 13 1/4 ins., and fitting it between endplates. The arrangement in the tunnel is given in Fig. (3-37). Flat plates were fitted normal to the air stream outside the end plates and flaps fitted to the rear of the endplates, in order to balance the flow through the three sections. The ends of the wheel were covered in felt cloth and the bottom of the endplates with draught excluding strip in order to prevent undesired flow through any gaps.

Although care was taken to achieve as uniform a flow through the central section as possible, clearly, with an

aspect ratio less than one, true two dimensional flow will not be attained and significant three dimensional effects were found. Our results must therefore be considered as not truly representative of the two dimensional case, especially in such features as separation and base pressure. The realisation that we were unlikely to attain very satisfactory answers decided us against pursuing this project in detail, instead measurements were restricted to obtaining the mean pressure distribution near the centreline of the cylinder and taking some smoke photographs of the flow.

Another source of possible error is contained in the calculation of the correction which must be applied to the dynamic head to allow for blockage. The blockage corrections of the flat plates in the outer sections of the tunnel were calculated using Maskell's method (which can be applied to flat plates near the ground according to Gould (1970)) and the results averaged. It was then assumed that, since the flow in all three sections had been balanced, the blockage correction of the wheel in the central section would be the same as the average of the plates. The addition to the dynamic head, given by this process, amounted to almost 30%. Such a large degree of blockage would almost certainly affect the flow in other ways, besides increasing the effective free stream velocity; in particular, the separation position may well be altered. However, despite all the foregoing, the results obtained are considered sufficiently interesting to be included here.

Fig. (3-39) gives the average pressure distribution around the wheel, the average being taken over 5 holes spaced at 1 ins. intervals with the middle hole on the centre-line. Some idea of the non-uniformity of the flow is given by comparing the results from each hole as in Fig. (3-38). The rotating distribution shows much the same features as with the ordinary wheel, but the base pressure is lower and the separation slightly further forward. There is now no stagnation region near the centre of the wheel at the front, but the pressure at the ground still rises to a high value, although no higher than in the three dimensional case, again demonstrating the local nature of this feature. Note that average pressures were only taken at every 5.1° , so any oscillation in pressure after $\theta = 90^\circ$ will not show up.

The stationary results are considerably different from those presented for the B2 wheel in section (3.3). One major reason is that the boundary layer on the stationary floor in front of the two dimensional wheel separates ahead of the wheel and thus effectively 'streamlines' the flow. Thus the pressure on the lower face is that in this separated region. Note that separation from the wheel now occurs much closer to the top, about $\theta = 260^\circ$, which is probably the reason why the low pressure over the top is no more than in the three dimensional flow (see Fig. (3-19)). The base pressure is more constant in the two dimensional case and is also remarkably similar to the rotating value, although this may well be an effect of the upstream separation from the floor,

rather than an indication that the rotation no longer affects the base pressure. The results of Wadcock (1970) for two stationary cylinders in contact, indicate a base pressure around -1.5, but in our results the stationary base pressure is little lower than the three dimensional stationary case, at about -.7. However, Wadcock's figure was obtained at a much lower Reynolds number, which might explain the difference (2.5×10^4 against 5.2×10^5).

Smoke photographs for the stationary and rotating cases are given in Fig. (3-40). The earlier separation and higher wake region of the rotating wheel is demonstrated in the figure, with the difference from the three dimensional flow that the separation from the stationary wheel now occurs near the top, agreeing with the pressure distributions. The bottom photographs are of the flow in front of the wheel. Unfortunately, because of the separation in front of the stationary wheel, the two pictures do not indicate a very clear difference between the flows. However, one can distinguish that the flow leaving the floor in the rotating case does so at a sharper angle. This flow above the moving floor is almost certainly caused by the upstream moving jet emerging from the corner between the wheel and floor and is not a boundary layer separation under adverse pressure gradient as in the stationary case.

For comparison with the above, some more photographs were obtained in a small smoke tunnel (26" x 2 1/4" section)

of the flow around a simple image-system model, consisting of two 2 ins. diameter cylinders in contact, with the wakes being separated by a splitter plate, 20" length. Fig. (3-41) presents photographs of the flow, with the cylinders rotating in the same way as a wheel and in the opposite direction, and with them stationary. The influence of the rotation on the separation position can clearly be seen. Note that the separation appears further forward in the stationary case than it does in Fig. (3-40), which is probably connected with the lower Reynolds number of the image system (2×10^4), but may also be influenced by the lack of separation in front of the cylinder. When the cylinder is rotating as a wheel, there is evidence of a large disturbed region at the front. The solution for this flow in the corner suggests that the jet emerges midway between the bounding surfaces, which would indicate that, with the image system, the jet would point directly upstream, whereas, with the moving floor, it would be angled upwards. In this way, the image system is not a complete analogy to the wheel flow.

From the above observations of the flow, together with the low Reynolds number solutions for the flow in the corners, plus what has been conjectured about separation over the rotating body, the likely mean streamlines of the various two dimensional configurations have been drawn (Fig. (3-42)). It must be emphasised that these figures have been constructed from the above information and are not the results of specific measurements in the flow. They do, however, account for the features shown in the smoke photographs.

The figure of the cylinder on the moving floor illustrates the unique properties of the flow, including the jetting at the front, the free separation point with its associated complex boundary layer and the way in which the flow near the ground at the rear affects the recirculating bubble. Note that, in the mean, the fluid in the layer next to the cylinder recirculates about the cylinder and separation region. In this, it is comparable to the layer shown in Fig. (1-12), which circulates around the rotating cylinder in a free stream. In the three dimensional flow, the fluid which comes over the top of the wheel is lost around the side in the forward going jet and this mechanism must therefore extract fluid from the base region. On the other hand, the action in the rear corner should tend to induce fluid back into the base region and would also contribute to the general increase in circulation in the separated region behind the rotating wheel.

(3.10) Conclusions

We have usually drawn conclusions from the results as they were presented and here we merely intend to summarize the more important ones. Two sets of coefficients were derived in analysing the results, in order to study them from different viewpoints. By basing the coefficients on an area equal to the tread width in contact with the ground $\times$ the diameter, we were able to show that, for a given tread width, the sharper edged wheels were better as regards drag and, except for the

B wheels, as regards lift also. The same set of coefficients showed no obvious increasing trend with increase in width, but coefficients based on the projected frontal area of the wheels, as is more usual, did show an increase with width in both lift and drag. In all cases the lift force was directed upwards contrary to the results of Morelli (1969).

Rotation of the wheel was found to have a beneficial effect, in the sense that both the drag and the lift were decreased. The pressure distributions showed that this was due to a reduction in base pressure, combined with a forward movement of the position of separation from the top of the wheel. This movement of separation and also the high pressure found under the wheel are connected with the movement of the boundaries of the wheel and floor and would only be produced in a complete simulation of the real flow. Because of their effect on the overall forces acting on the wheel, these features are examined in greater detail in the next chapter.

A smooth surfaced, non deformable wheel was chosen for these tests, in order to present a basic result upon which the effects of any other factors could be added. The smooth surfaced wheel has direct application to one practical case where the aerodynamics are of great importance, that of Grand Prix cars which run with treadless tyres, but it was felt that some attempt should be made to study the effects of tread pattern and this was done by simulating the pattern with slots in the wheel surface. It was found that the slots decreased

the lift on the wheel, but had little effect on the drag. The results also indicate that it is not possible to accurately simulate the effects of tyre pattern by leaving a small gap under the wheel, since the flow near the ground, produced by the raised part of the surface, is still important.

No tests were made on the effects of deformation of the wheel under an axle load, but it seems likely that the same flow features will still occur. However, there will probably be a small reduction in the value of the lift and drag coefficients, because the effective height of the wheel above the floor will be decreased and also because there will be a finite area in contact with the ground, on which no ^{air} pressures act.

An attempt was made to obtain some two dimensional results, but, because of the low aspect ratio and high blockage of the cylinder used, these are probably not very accurate. The two dimensional flow may well repay further study using a better experimental arrangement, since some of the more interesting features of the flow, such as the special ^{form of} separation, should be easier to study in two dimensions.

(4) DISCUSSION OF FLOW FEATURES ASSOCIATED WITH THE WHEEL

The results of the last chapter have established that the major factors in determining the wheel flow are the rotation of the wheel and the presence of the ground. In this chapter, it is intended to give some consideration to the special features produced by these agents. Inevitably, a great many simplifying assumptions have to be made in applying any theory to such a complex flow and we do not claim that any of the theoretical models can accurately predict the real flow about the wheel. Rather, the theory is used solely as an aid in understanding the experimental work and in this sense bad agreement with experiment can be as illuminating as good agreement.

We shall first consider those features associated with the rotation only, especially the early separation from the top of the wheel. Then, the effects of the ground are discussed, including those features which are due to a combination of the rotation and the ground. Finally, a free streamline theory for a cylinder on the ground is presented, which takes into account the effects of separation on the pressure distribution over the body.

(4.1) Features Associated with Rotation

(4.1.1) The Boundary layer on the wheel near separation

Because the early separation from the rotating wheel has such an important effect on the lift, it was decided to investigate the boundary layer near the separation position. Unfortunately, the present experimental set-up does not lend itself to this type of investigation. The main difficulty is that the wheel is subject to a certain amount of vibration, which appears to be initiated by the joint in the endless belt of the floor passing between the wheel and the roller. This occurs every $3\frac{1}{2}$ revolutions of the wheel. The thickness of the belt at the joint is greater by about .1 mm and the amplitude of vibration of the wheel is somewhat more than this, whilst, at the Reynolds number of the experiment, the boundary layer thickness, $\delta_{.99}$, on the front face of the wheel, is about 3 mm. This vibration introduces an amount of uncertainty into the measurement of distance from the wheel surface and also means that the very lowest portion of the boundary layer cannot be observed. Since our main interest was in seeing whether the velocity profile of the layer, near separation on the centreline, resembled that conjectured for the two dimensional flow in the introduction, it was felt sufficient to use pitot and static tubes to measure this, rather than hot wires, which would be easily damaged by contact with the wheel. A flattened

pitot tube was used, Fig. (2-16), because of the thinness of the boundary layer. A correction was applied, to allow for the displacement effect on the measurements in the shear layer, of .2 times the external normal dimension of the tube (i.e. .1mm). The above form of the correction really applies to round tubes only, but it was felt that, because of the uncertainty in normal distance measurement due to the vibration, there was little point in trying to obtain a more accurate correction. Measurements in the free stream indicated that the Pitot ^{-static} tube read high at low speeds, although no more than 5%, and another correction was applied for this.

The results for the velocity profiles of the layer on the centreline of the wheel A 1 are presented in Fig.(4-1). The important point is the overall shape of the profiles, rather than the actual values of the velocity, which may be in error for the reasons given above. It is unfortunate that the reversed part of the layer on the rotating wheel is so thin that we were unable to make measurements of reversed velocity except at $\theta = 270^\circ$, at which position the layer has definitely separated. Note the inflexion in the profile at this position, whereas at the other positions the shape is basically similar, although with some alteration and a definite thickening at $\theta = 280^\circ$. The rotating layer, as a whole, is thicker than the stationary one and the thickness increases with movement downstream. The stationary layer, on the other hand, remains much the same throughout the region measured, confirming that separation does not occur on the stationary

wheel until later.

Because the pressure distribution around the centre-line of the rotating wheel resembled that about a sphere with laminar separation, as has been noted earlier, the question arose of whether the wheel layer was laminar or turbulent. All that was required was some indication of the relative turbulent intensities in different parts of the layer. Thus, comparative measurements of pressure fluctuation, such as those given by a stethoscope, would prove sufficient. To obtain permanent recordings of the results, the B & K $1/4$ " microphone, used previously in the wheel, was now connected via flexible tubing to a probe, consisting simply of a straight metal tube $1/16$ " inside diameter, which could be placed in the boundary layer. Photographs were then taken of the signal from the microphone displayed on an oscilloscope screen.

The results presented in Fig. (4-2) therefore are representative of pressure fluctuations in the boundary layer. Note that the first photograph does not indicate a high turbulence level in the free stream, merely that the measuring apparatus is somewhat 'noisy'. Near the front stagnation position, there is no great increase above the free stream fluctuation, suggesting a laminar region. This region extends only about 10° above the stagnation point and about 20° below; downstream of these positions, a small higher frequency pressure fluctuation becomes noticeable, indicating a turbulent layer. The amplitude of these fluctuations grows gradually over the

top of the wheel, until at about $\theta = 280^\circ$ a larger lower frequency oscillation is also picked up. This is probably associated with the pressure fluctuations in the wake and thus suggests that the layer has separated at about this position. Investigations on the stationary wheel indicate a laminar layer until about 10° behind the top, then a short separation bubble, with turbulent transition in the shear layer, which reattaches about $20-25^\circ$ behind the top. The transition to turbulence in this case is quite abrupt and at reattachment the pressure fluctuations are as large as those of the rotating layer just before separation. On the rotating wheel no such abrupt change takes place, the pressure fluctuations just increase slowly with movement downstream.

Although the wheel apparatus is not well suited for investigations of the boundary layer, we feel that the above results do indicate that the boundary layer near the centreline of the rotating wheel does have the form at separation that was conjectured for the two dimensional flow past a rotating circular cylinder. It would appear to agree in basic form with the work of Thom (Fig.(1-11)) and suggests that near the centreline two dimensional ideas might be applied. (Note that by two dimensional here we mean the rotating cylinder in free air, as the boundary layer for the cylinder on the ground is more complex and less representative of the flow near the centreline of the wheel). Moreover, although the layer separates far forward on the wheel, it would appear to be

turbulent before separation. This agrees with the work of Griffiths and Ma (1969) on rotating cylinders and that of Maccoll (1928) and Davies (1949) on rotating spheres, all of which indicated that the upstream moving surfaces provoked early transition to turbulence within the boundary layer. This early transition implies that there is likely to be little change at higher Reynolds numbers than that of the present experiment, since the transition is already so far forward and, hence, the experiment will be representative of wheel flow over a wide range of car speeds.

As discussed in the introduction, it appears that separation from a rotating body is of unique form, actually being separation from a thin layer of air which remains attached to the surface and moves in the same direction as the surface. This form of separation does not imply that the shear stress at the wall becomes zero at separation, but does appear to be more easily provoked than ordinary separation, with hardly any adverse pressure gradient before separation. In the next section we shall discuss some theories connected with the special features of this type of separation.

(4.1.2) The two-dimensional boundary layer on a rotating cylinder

Although, as discussed in the introduction, separation in three dimensions is in general of a different form to

that in two, near the critical position (see Fig. (1-13)), the flow does show some two dimensional behaviour and our results indicate that this also applies in the rotating case. Also, the movement of the separation points with rotation seems to occur whether the boundary layer is laminar or turbulent. It would therefore seem reasonable to use some of the two-dimensional laminar boundary layer theory available, as a starting point in examining the boundary layer flow on the wheel, in particular near separation.

Glauert, M.B.(1957), has given a general boundary layer theory which has applications to rotating cylinders. The theory is based on the assertion that if $(u_1, v_1, w_1, \rho_1, T_1)$, which are known functions of (x, y, z, t) , are a solution of the general boundary layer equations, with given boundary conditions, and there exists a second set of expressions $(u_2, v_2, w_2, \rho_2, T_2)$, such that

$$u_2(x, y, z, t) = u_1(x, y, z + \zeta, t)$$

$$v_2(x, y, z, t) = v_1(x, y, z + \zeta, t)$$

$$\rho_2(x, y, z, t) = \rho_1(x, y, z + \zeta, t) \quad (1)$$

$$T_2(x, y, z, t) = T_1(x, y, z + \zeta, t)$$

$$w_2(x, y, z, t) = \left(w_1 - \frac{\partial \zeta}{\partial t} - \frac{u_1}{h_1} \frac{\partial \zeta}{\partial x} - \frac{v_1}{h_2} \frac{\partial \zeta}{\partial y} \right)_{x, y, z + \zeta, t}$$

(where $\zeta = \zeta(x, y, t)$)

and $h_1 dx, h_2 dy$ are elements of length in the coordinate directions)

then the quantities with subscript 2 also satisfy the boundary layer equations and so give a second solution, which will satisfy different conditions at $z = 0$ to the first solution. Glauert proves the theorem by direct substitution of quantities (2) into the boundary layer equations, proving that they satisfy the equations, provided ζ does not change so rapidly that additional curvature terms must be introduced.

In two dimensional, steady, incompressible flow equations (1) reduce to

$$\begin{aligned} u_2(x, z) &= u_1(x, z + \zeta) \\ w_2(x, z) &= w_1(x, z + \zeta) - u_1(x, z + \zeta) \frac{d\zeta}{dx} \end{aligned} \quad (2)$$

where $\zeta = \zeta(x)$

or, in terms of the stream function ψ

$$\psi_2(x, z) = \psi_1(x, z + \zeta)$$

using Taylor's expression on the above gives

$$\begin{aligned} \psi_2(x, z) &= \psi_1 + \zeta \frac{\partial \psi_1}{\partial z} + \frac{1}{2} \zeta^2 \frac{\partial^2 \psi_1}{\partial z^2} + \dots \\ &= \psi_1 + \zeta u_1 + \frac{1}{2} \zeta^2 \frac{\partial u_1}{\partial z} + \dots \end{aligned} \quad (3)$$

$$u_2 = u_1 + \zeta \frac{\partial u_1}{\partial z} + 1/2 \cdot \zeta^2 \frac{\partial^2 u_1}{\partial z^2} + \dots \quad (3)$$

$$\frac{\partial u_2}{\partial z} = \frac{\partial u_1}{\partial z} + \zeta \frac{\partial^2 u_1}{\partial z^2} + \dots$$

At $z = 0$, solution 1 satisfies $\psi_1 = u_1 = 0$,

$$\left(\frac{\partial u_1}{\partial z} \right)_{z=0} = \frac{\tau_1}{\mu} \quad (4)$$

where τ is the skin friction

and

$$\left(\frac{\partial^2 u_1}{\partial z^2} \right)_{z=0} = \frac{1}{\mu} \frac{dp}{dx} = - \frac{\rho U}{\mu} \frac{dU}{dx} \quad (5)$$

from the boundary layer equations, where $U(x)$ is the velocity outside the boundary layer.

Glauert now assumes that $(\zeta/\delta)^2 \ll 1$, i.e. the distance ζ is small compared to the boundary layer thickness δ . Then at $z = 0$

$$\psi_2 = 0, \quad u_2 = \frac{\zeta \tau_1}{\mu}, \quad \tau_2 = \tau_1 - \zeta \rho U \left(\frac{dU}{dx} \right)$$

from (3), (4), (5).

It is because u_2 is not necessarily zero at the wall that the theorem can be applied to a rotating cylinder. Thus, if the cylinder has a small tangential velocity, U_w at

the surface and solution 1 gives the flow with zero velocity at the surface, choosing

$$\zeta = Uw\mu/\tau_1 \quad (6)$$

gives the required solution 2, with

$$\tau_2 = \tau_1 - Uw \frac{\rho U}{\tau_1} \frac{dU}{dx} \quad (7)$$

Note that the external velocity U , used for solution 1, must be that for the rotating cylinder. Also that (ζ/δ) small implies Uw/U small, so the solution will only apply to slowly rotating cylinders, but, since we are basically interested in the qualitative effect of rotation on the layer, this is not a great disadvantage, since our results on wind speed variation suggest that effects which are connected with the rotation, such as the forward movement of separation, are amplified by an increase in the rotational speed, without undergoing any basic change in character.

When Uw is negative, i.e. when the surface is moving in the opposite direction to the external stream, the second solution will not be defined for $\zeta \leq z \leq 0$, but Glauert overcomes this difficulty by postulating an analytic continuation of the original solution across $z = 0$ for the small distance ζ .

We are most interested in the solution near the separation points, where, for the non-rotating cylinder, the shear

stress τ_1 becomes small. Thus ζ becomes large and the solution ceases to be valid. However if the terms in ζ^2 are retained in (3), the boundary condition on u_2 is satisfied by

$$U_w \mu = \zeta \tau_1 - 1/2 \cdot \zeta^2 \rho U \left(\frac{dU}{dx} \right) \quad (8)$$

Glauert now substitutes the value of ζ satisfying (8) into (7) and so derives

$$\tau_2 = \left(\tau_1^2 - 2U_w \mu \rho U \left(\frac{dU}{dx} \right) \right)^{1/2} \quad (9)$$

which is equivalent to (7) when τ_1 is not small, but does not become large near separation.

Using (9), Glauert considers the values of τ_2 at the two separation points. If U_w is positive, so that the surface is moving in the same direction as the external stream, then τ_2 has a finite value when $\tau_1 = 0$, since $\frac{dU}{dx}$ is negative here.

The validity of the solution ceases at this point, so no further deductions can be made, although Glauert supposes that τ_2 would fall steadily to zero as x increases. When U_w is negative, $\tau_2 = 0$ when

$$\tau_1^2 = 2U_w \mu \rho U \left(\frac{dU}{dx} \right) \quad (10)$$

i.e. when $\tau_1 > 0$, and the solution cannot be extended further, because there is no point in solution 1, or its continuation

below $z = 0$, which could be taken as the new position of the surface. Glauert thus assumes that τ_2 remains zero beyond this point.

It would be natural to assume that what Glauert shows above is that separation moves forward on the upstream moving surface, especially as this would agree with the experimental results given previously. However, this is not true in the sense in which we have been using separation, since a value of the shear stress τ_2 equal to 0 need not produce any apparent effect on the pressure distribution, as is true for a rotating body. This is demonstrated by the work of Thom (1926 and 1931), already discussed in the introduction (see Fig. (1-11)), although his results are for a faster rotation rate than the theory. Glauert, himself, is careful not to equate the shear stress being zero with separation and this independence of the two can be shown by drawing a further conclusion, which Glauert fails to make. This is that, with τ_1 positive, U_w positive, and (dU/dx) positive, τ_2 can still be zero from (9). Thus τ_2 can be zero in a favourable pressure gradient and, in fact, would be zero in the same regions as that indicated by the results of Thom mentioned above.

On the surface moving against the external stream, our picture of separation would suggest that τ_2 does not become zero at all, whereas the theory suggests that it might. However, at the lower rotational speeds to which the theory is applicable, it may be that the speed of the recirculating

fluid in the separated region will be as great as that of the rotating surface, in which case τ_2 will become zero. Also note that both the results of Thom and the present ones indicate that at the higher rotational speeds (dU/dx) hardly becomes negative at all, so that τ_2 is unlikely to become zero by (10).

To summarise, the theory of Glauert is applied to a cylinder rotating such that $U_w/U \ll 1$, but does agree with the independence of the pressure distribution from the wall shear stress, deduced from Thom's results for a higher rotational velocity. This independence does mean that one cannot make the assumptions, which Glauert does make, that τ_2 will fall to zero on the U_w -positive surface, or that, if τ_2 once becomes zero on the U_w -negative surface, it will remain zero.

We can take this examination a stage further, by considering the flow near separation in a manner suggested by Milne-Thompson (1968) for a stationary surface.

If

$$\Omega_w = \Omega(x, 0) = \text{vorticity at the wall, } z = 0,$$

then

$$\Omega_w = - \left. \frac{\partial u}{\partial z} \right|_w$$

Therefore near to the wall, we assume

$$u = -\Omega_w z$$

From the equation of continuity

$$\frac{\partial w}{\partial z} = - \frac{\partial u}{\partial x} = \Omega'_w z$$

where the dash refers to differentiation with respect to x .

Thus

$$w = 1/2 \Omega'_w z^2$$

since $w = 0$ when $z = 0$

$$\therefore \frac{w}{u} = -1/2 (\Omega'_w / \Omega_w) z \quad (1)$$

which is small for z small, provided $(\frac{\Omega'_w}{\Omega_w}) = O(1)$, but if $\Omega_w = 0, \Omega'_w \neq 0$ it becomes infinite and we shall take this to be an indication of separation. Thus, separation occurs when $\partial u / \partial z \big|_w = 0$.

Now, if we try and apply the same analysis to a moving boundary, with wall velocity U_w , then we must assume

$$u = -\Omega_w z + U_w$$

hence

$$\frac{\partial w}{\partial z} = - \frac{\partial u}{\partial x} = \Omega'_w z$$

$$w = \frac{1}{2} \Omega'_w z^2 \text{ as before, since } w = 0 \text{ when } z = 0$$

$$\text{so } \frac{w}{u} = \frac{1/2 \Omega'_w z^2}{-\Omega_w z + U_w} \quad (2) \quad \text{which remains small for } z \text{ small,} \\ \text{even if } \Omega_w = 0$$

Thus, again demonstrating that $\partial u / \partial z|_w$ can be zero on a moving boundary, without indicating any equivalent separation to that shown above.

However, if instead, we consider the point in the boundary layer above the moving wall, where the tangential velocity u is zero, at height z_0 say. Then if Ω_0 = vorticity at this point $= \frac{\partial u}{\partial z}|_0$, assuming $\frac{\partial w}{\partial x}$ small, since it is of $O(\delta)$

in the boundary layer,

we can let

$$u = -\Omega_0 (z - z_0) \quad \text{near } z_0 \\ = -\Omega_0 h \text{ say.}$$

Then from the equation of continuity

$$w = \int_0^z -\frac{\partial u}{\partial x} dz = \Omega'_0 \int_0^z (z - z_0) dz = \Omega'_0 (z^2/2 - z z_0)$$

$$w = 1/2 \cdot \Omega'_0 (z^2 - 2z z_0 + z_0^2) - 1/2 \Omega'_0 z_0^2 = 1/2 \Omega'_0 (h^2 - z_0^2)$$

$$\text{so } \frac{w}{u} = \frac{1}{2} \frac{\Omega'_0 (z_0^2 - h^2)}{\Omega_0 h} \quad (3)$$

This will, of course, tend to infinity as h tends

to zero, since u has been specified to be zero at this point whilst w need not be, but it will also tend to infinity for $\Omega_0 = 0$ and $h \neq 0$.

This suggests that a new condition for separation, which could be applied above moving boundaries, is that $\partial u / \partial z$ should become zero at the value of z at which u is zero. In fact, this could be taken as a more general definition of separation, since the stationary case is included, and would be compatible with the idea of a free separation point above the solid surface.

(4.1.3) The movement of separation position

The above theories, although indicating why the separation positions might be different on a rotating cylinder, do not indicate what sort of changes would occur. The theoretical determination of the separation position is obviously a complex issue, involving a continuous interaction between the external stream, the wake and the boundary layer and, in fact, the problem has not been entirely solved for even a stationary cylinder. All that we can do here is to offer some suggestions for the movement of separation, concentrating on the case of most interest at present, that when the surface is moving opposite to the external stream.

On a rotating body, the flow near separation is complicated by the presence of a reversed sublayer of fluid next

to the body's surface. In our time mean picture of the flow past a rotating cylinder (Fig. (1-12)), this sub-layer is completely separate from the recirculating fluid in the wake region and also separate from the external stream. The separation point marks the limit of the upstream movement of air in the recirculating region behind the cylinder. In a separation from a stationary body, the adverse pressure gradient has first to decelerate the fluid near the wall against the influence of the faster moving fluid in the outer part of the boundary layer, but in the present case, which already has a reversed sublayer, this should not be necessary and it would seem likely that even a slight adverse pressure gradient would produce an upstream flow of the fluid in the separated region, especially as any greater circulation induced into the wake region by the rotation of the cylinder would aid in this. It is not surprising, therefore, that the experimental pressure distributions obtained in the present experiment, and by Thom, show little or no adverse pressure before separation.

Another physical effect, which is ignored by boundary layer theory but which may be of importance near separation, has been discussed by Kuchemann (1967). This is that the vorticity in the shear layer, when it is turned through some non-zero angle, such as at separation, has an induced velocity field with a component parallel to the body's surface which acts to retard the flow before separation. If this effect is strong enough, therefore, it may provoke separation further upstream. Kuchemann uses a simple model of flow with constant vorticity to examine the above effect in separation from the

trailing edge of an aerofoil, but the same model can be thought of as a general separation from a plane surface with the flow in the separated region ignored, as shown in Fig. (4-3(ii)).

Far away from the separation point, at some arbitrary distance l , the constant vorticity model produces a straight velocity profile with a non zero velocity, u_0 , at the wall (Fig. (4-3(i))), which may be regarded as an approximation to a turbulent boundary layer profile without the laminar sublayer. In this model, the condition for separation is taken as being when the velocity at the wall falls to zero. This will always occur at $r = 0$, the original separation point, but can occur upstream of this point, to form a separation bubble, as shown in Fig. (4-3 (iii)) and this is interpreted as a forward movement of separation. The necessary condition for the bubble is found to be

$$U_0 < \omega l \tan \theta_0$$

where ω is the vorticity and θ_0 is the angle the flow is turned through at separation

U_0 is simply the difference between the true wall velocity and the constant vorticity model velocity at the wall. If we assign a velocity to the solid boundary and hold U_0 constant, then we shall have a greater value of vorticity, ω , if the boundary moves against the external stream and less if it moves with it. An increase in the value of ω acts to increase the chance of the above condition being fulfilled. Thus, if we think of the solution with no enclosed bubble as

being the conditions at separation on the stationary surface, movement of the surface against the flow will tend to move the separation position forward, which agrees with the experimental results. It is not suggested here that the vorticity is necessarily a major agent in the movement of separation, the special form of the separation when acted upon by the changed external pressure distribution is probably more important, but it is one possible cause,

(4.1.4) Flow near the stagnation point on a moving surface

Another singular point in the boundary layer, which is likely to have a different form when the surface is moving, is the stagnation point.

The solution for the two dimensional flow near stagnation with a moving surface of any velocity has been given by Glauert, M.B. (1956), as a special case of an oscillating surface. The solution is derived from that when the surface is stationary, given e.g. in Schlichting (1968) and shown below.

The potential flow solution for flow against an infinite flat plate at $z = 0$ is given by

$$u = cx, \quad w = -cz \quad (1)$$

For a bluff body of any shape, the above is true in the neighbourhood of the stagnation point, $x = z = 0$, (i.e. x, z small cf. the radius of curvature of body) To solve the flow with viscosity included, we assume that in this neighbourhood

$$u = xf'(z) \text{ and } w = -f(z) \quad (2)$$

which satisfies the equation of continuity. Using non-dimensional variables (2) becomes

$$u = cxf'(\zeta), \quad w = -(\nu c)^{1/2} f(\zeta) \quad \text{where } \zeta = z\left(\frac{c}{\nu}\right)^{1/2}$$

(ν = kinematic viscosity)

If these values are substituted in the boundary layer equation in two dimensions, with boundary conditions

$$u = w = 0 \quad \text{at } z = 0 \quad u \rightarrow U = cx \quad \text{as } z = \infty,$$

(or into the full Navier-Stokes, two dimensional equations) then

$$f'''' + ff'' - f'^2 + 1 = 0 \quad (3)$$

with boundary conditions (for a stationary surface)

$$f(0) = f'(0) = 0, \quad f'(\infty) = 1$$

(which is the Hiemenz equation)

With a boundary moving transverse to the flow with a velocity

U_w we look for solutions of the form

$$u = cx f'(\zeta) + U_w \phi(\zeta), \quad w = - (c\nu)^{1/2} f(\zeta) \quad (4)$$

where f and ζ are as before and the continuity equation is still satisfied.

The new boundary conditions are

$$u = U_w, \quad w = 0 \quad \text{at } \zeta = 0; \quad u \rightarrow U = cx \text{ as } \zeta \rightarrow \infty$$

Substituting in the boundary layer equation now gives

$$c^2 x (f''' + f f'' - f'^2 + 1) + U_w (\phi'' + f \phi' - f' \phi) = 0 \quad (5)$$

the expression in the first bracket is equal to zero by equation (3) so

$$\phi'' + f \phi' - f' \phi = 0 \quad (6)$$

with boundary conditions

$$\phi(0) = 1 \quad \phi(\infty) = 0 \quad (7)$$

If we differentiate equation (3) we have

$$f^{(IV)} + f f''' - f' f'' = 0 \quad (8)$$

therefore $\phi = f''$ is a solution of (6)

Numerical integration of (3) yields $f''(0) = 1.2326$ and $f''(\infty) = 0$. Hence

$$\phi = f''/1.2326 = .8113f'' \text{ satisfies the boundary conditions (8)}$$

$$\begin{aligned} \text{Thus } u &= cx f'(\zeta) + .8113 U_w f''(\zeta) \\ w &= -(cV)^{1/2} f(\zeta) \end{aligned} \quad (9)$$

is the solution derived by Glauert

If ψ is the stream function, such that

$$\frac{\partial \psi}{\partial z} = u \quad \text{and} \quad -\frac{\partial \psi}{\partial x} = w \quad (10) \quad (11)$$

then

$$(10) \text{ implies } \psi = (cV)^{1/2} xf + .8113 U_w \left(\frac{V}{c}\right)^{1/2} f' + g(x)$$

and

$$(11) \text{ implies } \psi = (cV)^{1/2} xf + h(z)$$

choosing $\psi = 0$ when $\zeta = 0$ (i.e. on the boundary) gives

$$\psi = U_w \left(\frac{V}{c}\right)^{1/2} \left(\frac{cx}{U_w} f + .8113 f' \right)$$

In fig. (4-4), we have plotted $xf + .8113f' = \text{constant}$ lines, in order to give an idea of the form of the flow given by the above. The streamlines shown, suggest that the flow is divided into two parts by $\psi = 0$, although the two halves of $\psi = 0$ do not meet. Streamlines to the left of $\psi = 0$

in Fig. (4-4) go to the left, while those to the right eventually go to the right. No streamline actually connects with the surface and there is no singular point in the stagnation region, at which a streamline divides. These features of the flow have been used in proposing the picture of the two dimensional flow past a rotating circular cylinder, given in Fig. (1-12) and discussed in the introduction.

(4.1.5) Conclusion

In the present section we have discussed separation from a rotating body, in particular from the upstream moving surface and have pointed out some of the unique features associated with flow about such a body. Simplified mathematical models have been used to demonstrate some of the features although no attempt has been made to obtain any quantitative theoretical results because of the complexity of the flow. These models have shown the independence of the wall shear stress and separation. A new criterion has been suggested, namely that separation should be deemed to have occurred at the position where both u and $\partial u / \partial z$ are zero. The causes of separation movement were discussed and one possible agent indicated, although other effects will probably be as important.

The experimental results for the wheel indicate that the special form of separation plays an important part in determining the overall forces, especially in reducing the

lift component, so that further study of the conditions near separation would prove useful. The present experimental set-up, which was designed, basically, for measuring the surface pressure distribution on the wheel, is not well suited to a detailed study of the boundary layer, but such results as were obtained do not disagree with the separation picture proposed, at least near the centreline of the wheel. A study of the two dimensional separation from a cylinder rotating in free air would be a suitable starting point for any further investigation, as this would eliminate the difficulties experienced in taking measurements in the boundary layer and provide suitable results for a theoretical treatment. The same forward movement of separation seems to occur over a wide range of Reynolds number provided the rotation rate is high enough to overcome transition effects, so that lower numbers might be used initially to increase the boundary layer thickness and thus make measurements easier.

(4.2) Features Associated With the Ground and Rotation

(4.2.1) Potential flow solutions for cylinder

In this section we shall give some potential flow solutions for a cylinder above and on the ground, and with rotation added. The results will then be compared with available experimental data. Of course, a potential solution for such a bluff body as a cylinder is unlikely to give realistic results for the magnitude of forces but we might expect it to

indicate correct trends with variation in height above the ground.

The potential solution for flow past a cylinder radius, a , in free air at $z = ic$ consists of a uniform stream plus a doublet, thus

$$w = \phi + i\psi = Uz + \frac{Ua^2}{z-ic}$$

To model the ground we must include an image cylinder at $z = -ic$ i.e. an image doublet strength $\mu_1 = Ua^2$ at $z = -ic$. But this in turn must have an image in the first cylinder to keep the cylinder shape as a streamline i.e. a doublet of strength $\mu_2 = \mu_1 a^2/4c^2$ at $z = i(c-d_2)$ where $d_2 = a^2/2c$. We then require an image of this doublet in the image cylinder and so on. Thus we generate an infinite series;

$$w = Uz + \sum_{n=1}^{\infty} \frac{\mu_n}{z \pm i(c-d_n)} \quad (1)$$

where $\mu_1 = Ua^2$

$$\mu_n = \frac{\mu_{n-1} a^2}{(2c-d_{n-1})^2} \quad (2)$$

$$\therefore \mu_n = \mu_{n-1} \frac{d_n^2}{a^2}$$

and $d_1 = 0$

$$d_n = \frac{a^2}{2c-d_{n-1}} \quad (3)$$

The solution is given in the above form by Carpenter (1958).
From (1)

$$-\frac{dw}{dz} = u - iv = U - \sum_1^{\infty} \frac{\mu_n}{(z-i(c-d_n))^2} \quad (4)$$

We are most interested in the pressures on the cylinder, and if we take the radius $a = 1$, so that $x^2 + (y-c)^2 = 1$

$$v = 2Ux \sum_1^{\infty} \frac{\lambda_n (y+(c-d_n))}{(1-2cd_n+d_n^2+2y(2c-d_n))^2} \quad (5)$$

$$\text{where } \lambda_n = \mu_n/U = \lambda_{n-1}d_n^2 \text{ and } \lambda_1 = 1 \quad (6)$$

Terms

$$\frac{\lambda_n (y+(c-d_n))}{(1-2cd_n+d_n^2+2y(2c-d_n))^2} + \frac{\lambda_{n+1} (y-(c-d_{n+1}))}{(1-2cd_{n+1}+d_{n+1}^2+2yd_{n+1})^2}$$

can be combined, using (3) and (6), as

$$\frac{(y-c) \left(\frac{1}{\lambda_{n+1}} + \frac{1}{\lambda_n} \right) + 2\frac{d_{n+1}}{\lambda_{n+1}}}{\left(1 + \frac{2d_{n+1}}{\lambda_{n+1}} y \right)^2} \quad (7)$$

$$\text{Define } A_n = 1/\lambda_n \text{ then } A_{n-1} = d_n^2 A_n \text{ and } A_1 = 1/\lambda_1 = 1 = \lambda_1$$

(9)

(10)

thus (7) becomes

$$(y-c) \frac{(A_{n+1} + d_{n+1}^2 A_{n+1}) + 2d_{n+1} A_{n+1}}{(1 + 2d_n A_n y_n)^2}$$

The uncombined term can be included in the summation in (5) in the same form as the combined terms so that

$$v = 2Ux \sum_1^{\infty} \frac{(y-c)(1+d_n^2)A_n + 2d_n A_n}{(1+2d_n A_n y_n)^2}$$

$$\text{or } \frac{v}{U} = 2xS \quad (11)$$

where S represents the summation.

The other velocity component, u, can be treated in a similar way to yield

$$u/U = -2(y-c)S \quad (12)$$

We can prove that S is a convergent series in the following way:

$$d_1 = 0 \quad \text{and if } d_{n-1} < 1, \text{ then } d_n = \frac{1}{2c-d_{n-1}} < \frac{1}{2c-1} \leq 1$$

since $c \geq 1$. Thus $d_n < 1$, all n

Similarly, we can show by induction that $d_n \geq 0$ and $d_n > d_{n-1}$

for all n .

Also $-1 \leq (y-c) \leq 1$

$$\text{so } \left| \frac{(1+d_n^2) A_n \left[(y-c) + \frac{2d_n}{(1+d_n^2)} \right]}{(1+2d_n A_n y)^2} \right| < \left| \frac{2 \cdot A_n \cdot 3}{(2d_n A_n y)^2} \right| < \frac{3}{2y^2} \cdot \frac{1}{A_{n-1}}$$

Now

$$\sum_{n=1}^{\infty} (1/A_n) = 1 + d_2^2 + d_2^2 d_3^2 + \dots (d_2^2 \dots d_n^2) + \dots$$

$$< \sum_{r=0}^{\infty} (d_{\infty}^2)^r \quad \text{which is a convergent geometric series since}$$

$$|d_{\infty}^2| < 1$$

Therefore $\sum_1^{\infty} (1/A_n)$ is convergent and hence S is absolutely convergent by comparison.

To take rotation into account, we must add vortices inside the cylinders, so as to retain the cylinders as streamlines. Thus, if the vortices are added at $z = \pm ib$, $b \geq c$, the complex potential due to the vortices strength KU , $-KU$ is

$$w_2 = \frac{iKU}{2\pi} \log\left(\frac{z+ib}{z-ib}\right) \quad (13)$$

and

$$= \frac{KU}{2\pi} \log \frac{PA}{PB} \quad \text{where } PA, PB \text{ are defined in Fig. (4-5)}$$

thus, $\frac{PA}{PB} = \text{constant}$, are streamlines and

$$\frac{PA}{PB} = \frac{(c+a)-b}{(c+a)+b} = \frac{b-(c-a)}{b+(c-a)} \quad \text{at C and D respectively}$$

$$\therefore b^2 = (c+a)(c-a) \text{ or } (c+1)(c-1) \text{ if } a = 1$$

$$\text{Then } \frac{dw_2}{dz} = \frac{2bKU}{z^2 + b^2} \quad (14)$$

and therefore on cylinder

$$v_2 = -\frac{UxKb}{y} \quad u_2 = -bK\frac{U(c-y)}{y} \quad (15)$$

Adding these values to the previous solution for a stationary cylinder, gives

$$\begin{aligned} u/U &= -2(y-c)(S - bK/2y) = -2(y-c)S', \text{ say} \\ v/U &= 2x(S - bK/2y) = 2xS' \end{aligned} \quad (16)$$

Let

$$q^2 = \frac{u^2 + v^2}{U^2} = 4S'^2$$

then

$$c_p = 1 - 4S'^2 \quad (17)$$

and the forces on the cylinder can be found by integration.

To obtain the potential for a stationary cylinder actually on the ground, we return to equation (1) and take $c=a$, which gives

$$w = U \left(z + \frac{a^2}{z+ia} + \frac{a^2/4}{z+ia/2} + \frac{a^2/9}{z+ia/3} + \dots \right)$$

Combining $\frac{1}{z+ia}$ terms gives

$$w = U \left(z + \sum_{n=1}^{\infty} \frac{2a^2 z}{a^2 + n^2 z^2} \right)$$

$$= a\pi U \coth \left(\frac{a\pi}{z} \right) = \pi U \coth \left(\frac{\pi}{z} \right), \text{ for } a=1$$

which is the same as the solution given by Milne-Thompson, (1968) p. 180.

Near the ground, the rotational part of the potential, given by (13), as $c \rightarrow a$ or $b \rightarrow 0$ becomes $-\frac{bK}{\pi z}$, by expanding the logarithm in a Taylor series and ignoring terms $O((b/z)^3)$. This term will give rise to a velocity which tends to infinity at the origin, $z=0$. However, near the ground, we might expect viscous forces to modify the flow significantly. Indeed, the low Reynolds number solution for the flow in the corner between cylinder and ground, given later, suggests that when the cylinder is actually touching the ground, the moving boundaries give rise to an upstream moving jet and the experimental results indicate a c_p greater than 1 at the ground, which obviously cannot be obtained in a potential solution. The jet might be modelled by reversing the sense of the vortex doublet in the solution with rotation, but, although this will give a flow picture bearing a closer resemblance to the real one, it will also give an infinite negative pressure on the cylinder near the ground. Since all potential models are likely to be un-

realistic on a bluff body, it would seem that taking the stationary case for the cylinder on the ground will be sufficient for the present purposes, i.e. the effect of touching the ground is taken to be destroy the circulation about the cylinder.

The variation in the lift with height above the ground is given in Fig. (4-5). This suggests that the cylinder will experience a lift force towards the ground, which reaches very high values near to the ground, and that rotation of the cylinder exaggerates this trend. Note that the values of rotation used in the theory were chosen to give a free stream lift force close to the experimental values for rotating two dimensional cylinders with $v/U_{\infty} = 1$ and for the MIRA wheel. Once the cylinder actually touches the ground, there is a complete reversal of lift force and a large upward lift is generated.

The pressure distributions, given in Figs. (4-6) and (4-7) for the stationary and rotating cylinders respectively, at different heights, show how the downward lift force is generated by the large suction on the under-surface of the cylinder, caused by fluid rushing through the gap between the cylinders. They also show how the increase in negative pressure is proportionally greater in the rotating case. When the cylinder is in contact with the ground, the theoretical pressures are of a completely different form (Fig. (4-8)). The negative pressures on the undersurface have been eliminated by the prevention of flow through the gap and have been replaced by an extensive region of high pressure, which together with the large suction over the top of the cylinder,

produces a high upward lift.

Wadcock's (1970) mean pressure results are included in Fig. (4-6) and show how different the experimental distributions are, as well as indicating how the movement of the two separation points produces a positive lift force. It might have been thought that the theoretical model would most resemble Wadcock's two dimensional flow, but in fact the results of MIRA for the wheel (Figs. (1-16) and (1-17)) bear a closer resemblance to the potential solution, probably because of the elimination of wake interference effects. The conclusion to be drawn is that we must be very careful about applying potential solutions to bluff bodies near the ground, for they may give completely wrong answers, even for the direction of the forces.

Potential theory can be extended to three dimensions for lifting bodies by using the Horseshoe Vortex model described by Glauert, H. (1948). Here the body is replaced by a bound vortex and two trailing vortices. Because the theory ignores the body's shape, it obviously cannot take any separation effects into account and so is unlikely to give very good results for bluff bodies near to the ground. Thus, although this theory has been applied to 'streamlined' bodies near the ground (Stollery & Burns (1969)), with fair results at larger clearances, it was thought unlikely to give very satisfactory results for a rounded bluff body such as the wheel, no more so than in two dimensions. Similarly the numerical potential methods of the type described by Hess & Smith (1968), are unlikely to be

of great use for such a body and hence were not pursued. The only way in which potential theory can be applied to bodies with separation, with reasonable success, is in some form of free streamline theory as discussed in section (4-3), possibly extending to three dimensions by combining the numerical methods with free streamline assumptions. Even then those features of the flow determined by viscous effects will have to be obtained empirically or by a separate analysis.

(4.2.2) Viscous flow very near the ground

The flow in the neighbourhood of the line of contact between wheel and floor will be dominated by the viscous forces arising from the movement of the boundaries. We might therefore obtain some understanding of the behaviour in this region by looking at a theoretical solution for the two dimensional viscous flow, induced entirely by the movement of straight boundaries meeting at a finite angle. (see Fig. (4-9)).

The Navier-Stokes equation is

$$\frac{\partial \underline{v}}{\partial t} + \nabla (1/2 \underline{v}^2) - \underline{v} \wedge \underline{\omega} = - \frac{1}{\rho} \nabla p - \nu \nabla \wedge \underline{\omega} \quad (1)$$

$$\text{where } \underline{\omega} = \nabla \wedge \underline{v} \text{ is the vorticity} \quad (2)$$

and the equation of continuity is

$$\nabla \cdot \underline{v} = 0 \quad (3)$$

We can now merely assume that the inertial effects are negligible, in which case the left hand side of (1) disappears, or we can make the approximation more formally, by introducing the Reynolds number Ul/ν , and using non dimensional variables defined thus

$$\hat{t} = ft, \quad \hat{x} = x/l, \quad \hat{v} = v/U, \quad \hat{p} = \frac{R(p-p_\infty)}{\rho U^2} \quad (4)$$

Then (1) becomes

$$R \left(S \frac{\partial \hat{v}}{\partial \hat{t}} + \hat{\nabla} (1/2 \hat{v}^2) - \hat{v}_\Lambda \hat{\omega} \right) = - \hat{\nabla} \hat{p} - \hat{\nabla}_\Lambda \hat{\omega}$$

where $\hat{\nabla}$ is the gradient operator with respect to $\hat{x}$

$$\hat{\omega} = \hat{\nabla}_\Lambda \hat{v}$$

$$S = \text{Strouhal number} = f \cdot \frac{1}{U} \quad (f = \text{frequency})$$

If we now let $R \rightarrow 0$ and assume that all the terms in the bracket remain finite, (5) becomes

$$\hat{\nabla} \hat{p} = - \hat{\nabla}_\Lambda \hat{\omega}$$

or dropping the $\hat{}$ symbol

$$\nabla p = \nabla_\Lambda \omega \quad (6)$$

and this even applies to the unsteady case provided the frequency of variation is not great.

The equation of continuity is unaffected by the approximation.

Since our boundary conditions are expressed in $\underline{v}$ alone, we will eliminate p from (6) by taking the curl of both sides thus

$$\nabla_{\wedge}(\nabla_{\wedge} \underline{\omega}) = 0 \quad (7)$$

Since $\nabla \cdot \underline{\omega} = \nabla \cdot (\nabla_{\wedge} \underline{v}) = 0$, (7) is equivalent to

$$\nabla^2 \underline{\omega} = 0 \quad (8)$$

In two dimensional flow, with polar coordinates (r, θ) , $\underline{\omega}$ has only one component in the direction normal to the plane (r, θ) , the z direction, say

$$\omega_z = \frac{1}{r} \frac{\partial}{\partial r} (r v_{\theta}) - \frac{1}{r} \frac{\partial v_r}{\partial \theta}$$

and defining a stream function such that

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad \text{and} \quad v_{\theta} = - \frac{\partial \psi}{\partial r} \quad (9)$$

$$\omega_z = -\nabla^2 \psi$$

$$\text{where } \nabla^2 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right)$$

so from (8)

$$\nabla^2(\nabla^2 \psi) = 0 \quad (10)$$

We shall look for a separated solution of (10) in the form

$$\psi = r^n f_n(\theta) \quad (11)$$

(where n = arbitrary real or complex number)

satisfying boundary conditions

$$\frac{1}{r} \frac{\partial \psi}{\partial \theta} = -U, \quad \frac{\partial \psi}{\partial r} = 0, \quad \text{on } \theta = 0 \quad (12)$$

$$\frac{1}{r} \frac{\partial \psi}{\partial \theta} = -U, \quad \frac{\partial \psi}{\partial r} = 0, \quad \text{on } \theta = \alpha$$

The condition $\frac{1}{r} \frac{\partial \psi}{\partial \theta} = \text{constant}$ along the walls is such as to suggest that $n = 1$, i.e.

$$\psi = rf(\theta) \quad (13)$$

Then $\nabla^2 \psi = 1/r \cdot (f'' + f)$

$$\text{and } \nabla^2(\nabla^2 \psi) = 1/r^3 (f^{(IV)} + 2f'' + f) = 0 \quad (14)$$

Try $f = e^{\mu\theta} + \theta e^{\mu\theta}$

Then (14) becomes

$$(\mu^2 + 1)^2 \theta e^{\mu\theta} + (\mu^2 + 1) (\mu^2 + 4\mu + 1) e^{\mu\theta} = 0$$

which is true if $\mu = \pm i$, so

$$f = A \sin \theta + B \cos \theta + C \theta \sin \theta + D \theta \cos \theta \quad (15)$$

Now let $\psi = Urf$

so that boundary conditions become

$$f=0, \quad f'=-1, \quad \text{on } \theta=0 \quad (16)$$

$$f=0, \quad f'=-1, \quad \text{on } \theta=\alpha$$

Applying these boundary conditions to (15) gives 4 simultaneous equations for the constants, which can be solved to give

$$A = \frac{-\alpha}{\alpha - \sin\alpha}, \quad B = 0, \quad C = \frac{1 - \cos\alpha}{\alpha - \sin\alpha}, \quad D = \frac{\sin\alpha}{\alpha - \sin\alpha}$$

so that,

$$\psi = \frac{Ur}{\alpha - \sin\alpha} (-\alpha \sin\theta + (1 - \cos\alpha) \cdot \theta \sin\theta + \sin\alpha \cdot \cos\theta)$$

or

$$\psi = \frac{Ur}{\alpha - \sin\alpha} (\theta \sin(\alpha - \theta) - (\alpha - \theta) \sin\theta) \quad (17)$$

and it is immediately apparent that $\psi = 0$ when $\theta = 0, \alpha$ and $\alpha/2$.

From (17)

$$v_r = \frac{U}{\alpha - \sin\alpha} (\sin(\alpha - \theta) - \theta \cos(\alpha - \theta) + \sin\theta - (\alpha - \theta) \cos\theta) \quad (18)$$

$$v_\theta = \frac{-U}{\alpha - \sin\alpha} (\theta \sin(\alpha - \theta) - (\alpha - \theta) \sin\theta) \quad (19)$$

The velocities given by (18) and (19) are independent of r , so will be of the same form at any radius. (e.g. section AB Fig. (4-9)). As an example, they are plotted for $\alpha = 30^\circ$, $U = 1$ in Fig. (4-10) and the form of the flow field indicated. Note that the two boundaries moving into the corner actually produce a jet of air moving outwards at a maximum velocity about $U/2$.

The shear stress on $\theta = 0$ is given by

$$\mu \left(r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \left(\frac{\partial v_r}{\partial \theta} \right) \right) \\ = \frac{\mu U}{r} (f + f'')$$

$$\text{i.e. } \propto 1/r$$

and the pressure from (6) as

$$\frac{\partial p}{\partial r} = -\frac{1}{r^2} \left(\frac{\partial^2 v_r}{\partial \theta^2} - v_r - 2 \frac{\partial v_\theta}{\partial \theta} \right) \\ = \frac{-U}{r^2} (f''' + f')$$

$$\text{i.e. } p - p_\infty \propto 1/r$$

Thus as $r \rightarrow 0$, the solution implies infinite values. In practice, the boundaries do not make contact at a non-zero angle and there is never perfect contact between two bodies, so that the values remain finite.

Now

$$v_r \Big|_{\theta=\alpha/2} = U \left(\frac{2\sin(\alpha/2) - \alpha\cos(\alpha/2)}{(\alpha - \sin\alpha)} \right)$$

and for small α , we can substitute for sines and cosines from the appropriate Taylor expansions, with terms $O(\alpha^4)$ neglected.

Thus

$$v_r \Big|_{\theta=\alpha/2} = \frac{U \left[2(\alpha/2 - \frac{(\alpha/2)^3}{3!}) - \alpha(1 - \frac{(\alpha/2)^2}{2!}) \right]}{(\alpha - \alpha + \alpha^3/3!)}$$

$$= U/2 \quad \left(\text{to } O\left(\frac{\alpha^2}{100}\right) \right) \quad (20)$$

so that the jet emerges with half the velocities of the boundaries.

The motion is reversible, since the governing equations are linear and homogeneous, so that the flow in the rear corner between wheel and floor can also be modelled and this time there would be an inward flow in the central region between the boundaries (Fig. (4-9)).

Now, the assumptions regarding the dominance of the viscous terms in the Navier-Stokes equation will only be true in a very small region near the corner. However, we feel that this form of flow does occur under the wheel and gives rise to the high pressures measured there; not the infinite pressures of the theory, probably because the curvature of the wheel enables the actual contact with the floor to be tangential. Moreover, because the central jet has a significant velocity $(O(U/2))$ in a region of little external flow, the effects of these viscous actions are noticeable over a much greater region than that for which the assumptions are valid. Thus, in the photographs of the two dimensional flow (Figs. (3-40) and (3-41)) the effects of the forward moving jet can clearly be seen, although in the three dimensional flow most of the jetting is lost around the side of the wheel. Conversely, the flow in the corner is relatively little affected by the external flow, because of the localised nature of the actions producing it.

Fig. (4-11) compares the pressures near the ground, with a free stream velocity equal to that of the boundaries and with no external velocity.

The solution in the rear corner indicates a flow, with velocity of $O(U/2)$ into the corner, of fluid in the separated region, which would appear likely to have an important effect on this region and thus in determining the base pressure. Our results do not show a large negative pressure here, which might be expected from the theory, and this cannot be entirely due to the response of the transducer, since the low pressures in the grooves of wheel C2G were followed quite easily. It would seem likely that the moving floor would lift up to touch the wheel, if a large negative pressure were applied above it and this probably accounts for the oscillation of the pressure in this region, which was discussed in section (3.4). There may, therefore, be an error in the results from the experimental modelling with a moving floor, although a real deformable tyre might undergo a similar reaction in this rear corner region; the tyre moving to meet the floor instead of vice versa. In that case, the basic feature of a large pressure near the ground would still occur. One could never model this region very close to floor entirely, anyway, since on this scale the different road surfaces encountered by the wheel will have a significant effect.

This viscous flow, produced in a very localized region under the wheel, can be significant in determining the overall

flow field around the wheel and will be important in studying such features as spray dispersal from a wet road. It is therefore necessary to duplicate these actions in the wind tunnel, as far as possible, by having the wheel in contact with a moving floor, if realistic results are to be obtained.

(4.3) Free Streamline Theory for Cylinder on Ground

(4.3.1) Free Streamline Theory

The basic assumption underlying free streamline theory is that the region of separated flow behind a bluff body can be separated from the main stream by steady streamlines. These streamlines can be thought of as an idealized form of the separated shear layers as the Reynold's number tends to infinity; the shear layers become thinner with rise in Reynolds number and hence could be considered as lines of discontinuity, the free streamlines, on extrapolating to infinity. In real flows, these shears layers soon break down into the turbulent wake region, with entrainment of fluid from the mainstream, violating the assumptions of the theory. However, it might be expected that the free streamline concept will have some validity close to the body and, in particular, in predicting the pressures on the wetted surface of the body. (The surface before separation).

The theory cannot be used to predict base pressure, or the position of the separation point on curved bodies, since

these properties depend on the mechanics of the wake and of the attached boundary layer, both of which are ignored by the theory. Consequently, the values of these items must be fed into the theory from empirical data, or from separate mathematical treatments of the boundary layer and/or wake, which must then be matched with the theory. (for example Roshko(1955) and Bluston & Paulson (1972)).

Having made the above assumption, one is faced with the problem of finding the potential flow of the main stream, given the direction of the velocity on the wetted surface and its magnitude along the free streamline, the position of the free streamline being unknown. This last point, the indeterminacy of the free streamline position in the physical plane (z plane), means that it is more convenient to work with the complex potential, $w = \phi + i\psi$, as the independent variable, since streamlines are then represented by $\psi = \text{constant}$ in the w -plane and, in particular, the streamline which stagnates on the body, forms the wetted surface and then separates to become the unknown free streamlines, can be simply represented as $\psi = 0$. Because of the form of the mixed boundary conditions described above, the most useful function of w to solve for is

$$\begin{aligned}\sigma &= \log_e \left(U_\infty \frac{dw}{dz} \right) = \log_e \left(U_\infty / q e^{-i\theta} \right) \\ &= \log_e (U_\infty / q) + i\theta = \tau + i\theta, \text{ say}\end{aligned}\tag{1}$$

since θ will be given on the wetted surface and τ will be given

on the free streamlines, and this separation of the boundary conditions enables the problem to be solved. Once $\sigma = \sigma(w)$ has been found for a given flow, other properties can be derived immediately, from

$$z = \frac{1}{U_{\infty}} \int e^{\sigma} dw \quad (2)$$

$$q^2 = \left| \frac{dw}{dz} \right|^2 = U_{\infty}^2 e^{-2\tau} \quad (3)$$

and then from Bernoulli's equation

$$p = p_{\infty} + 1/2 \cdot \rho U_{\infty}^2 (1 - e^{-2\tau}) \quad (4)$$

The initial form of the theory, derived by Helmholtz and Kirchhoff, applies only to rectilinear boundaries with the velocity on the free streamlines taken as constant and equal to the free stream velocity, U_{∞} . The boundaries will then form a polygon in the σ plane, since θ is constant on the wetted surface, except for discontinuities, and τ is constant on the free streamlines (Fig.(4-12)). The boundaries can then be transformed to the real axis of an intermediate ζ_1 -plane by a Schwarz-Christoffel transformation. The boundaries in the w -plane are of a similar form and can be transformed to the real axis of another intermediate plane, ζ_2 . The relation between the ζ_1 and ζ_2 planes can then be uniquely determined from three pairs of corresponding points on the real axis, usually the stagnation point and the two separation points. In theory, ζ_1 and ζ_2 can be eliminated and $\sigma = \sigma(w)$ found, but in practice one of the ζ 's is usually retained as a parameter.

The further development of the theory has been concerned with two points; the extension to curved boundaries and allowing non constant velocities along the free streamlines, either of which means that the simple form of the boundaries in the σ -plane will be lost and, in general, the Schwarz-Christoffel transform can no longer be used. (Fig. (4.14)).

Levi-Civita (1907) considered the case of flow past a general bluff body. The transform from the w -plane to the intermediate ζ_2 -plane is

$$w = 4a(\zeta_2 - \sin(\lambda/2))^2 \quad (5)$$

where

$$a = (\phi_1^{1/2} + \phi_2^{1/2})^2 / 16$$

$$\sin(\lambda/2) = \frac{(\phi_2^{1/2} - \phi_1^{1/2})}{(\phi_2^{1/2} + \phi_1^{1/2})}$$

and ϕ_1 and ϕ_2 are the values of ϕ at the separation points (The stagnation point is taken as $\phi = 0$ see Fig. (4-13)).

The ζ_1, ζ_2 relation is defined as

$$\zeta_2 = -\frac{1}{2} \cdot (\zeta_1 + 1/\zeta_1) \quad (6)$$

and so

$$w = a(\zeta_1 + 1/\zeta_1 + 2\sin(\lambda/2))^2 \quad (7)$$

(6) maps the upper half of the ζ_2 -plane onto the

upper half of the unit circle in the ζ_1 -plane, such that the wetted surface lies on the semi-circle and the free streamlines lie on the real axis. It is now only necessary to specify the mapping from σ -plane to the ζ_1 -plane to complete the solution. Levi-Civita assumes a mapping $\sigma(\zeta_1)$, then uses (6), (5) and (2) to obtain the corresponding boundary shape. His theory is thus an inverse method only, since the boundary shape cannot be specified in advance.

However, Brodetsky (1923) was able to obtain a numerical solution for flow past circular and elliptic cylinders by making use of Levi-Civita's method and the method was extended by Villat (1920), to form the basis of a direct method of solution by an iterative process. Villat's method is in principle similar to the method of Woods (1955), which is described later, but suffers from the disadvantage of having to specify a constant velocity along the free streamlines, (as do all the methods mentioned so far).

The severity of this disadvantage can be appreciated from a consideration of the base pressure acting on the surface of a bluff body, down stream of separation. The base pressure coefficient, c_{pb} , is defined as $p_b - p_\infty / \frac{1}{2}\rho U_\infty^2$ and, in real flows, is always negative and virtually constant across the base. A velocity of U_∞ at separation implies, through Bernoulli, that $c_{pb} = 0$, which will therefore lead to a much lower value for the drag than the experimental value. To obtain a negative base pressure, the velocity on the free

streamlines at separation must be given as KU_∞ , where $K > 1$ and must be determined from empirical data, or from a consideration of the wake flow. (The velocity must be the same on the two streamlines because of the constant base pressure). Since the velocity on the free streamlines should tend to U_∞ , far downstream of the body, this implies that the velocity, and hence pressure, must vary along the streamlines.

Another major defect of assuming constant velocity is that, far downstream, the width of the wake tends to infinity. Since, in the mathematical model, the fluid inside the wake is assumed to have zero velocity, the wake width is equivalent to the displacement thickness and, from a consideration of momentum balance, the far downstream wake displacement thickness should be finite and proportional to the drag of the body. Any theory which purports to give a realistic drag value ought, therefore, to lead to a finite value of wake width at infinity.

Attempts to obtain a realistic base pressure have been made by Riabouchinsky (1920) and Gilbarg and Rock (1945), who retained the constant velocity along the free streamline, but with a closed wake cavity, and by Roshko (1954), who allowed the velocity to vary along the free streamline. Unfortunately, although Roshko's method is fairly straightforward for straight edged objects, he adapts Brodetsky's numerical method for the circular cylinder, resulting in a complex procedure which is unlikely to be applicable to a general curved obstacle.

For a general curved body the method of Woods (1955), which is based on the work of Levi-Civita and Villat, is much to be preferred. Here the upper half ζ_2 -plane of Fig. (4-13) is mapped, not into the upper half circle of the ζ_1 -plane, but into an infinite strip by

$$\zeta_2 = \tanh \zeta_1, \quad \zeta_1 = \delta + i\xi \quad (8)$$

(see Fig. (4-15))

The boundaries now transform into $\xi = 0$ for the wetted surface and $\xi = \pi/2$ for the free streamlines. Thus the function

$\sigma = \tau + i\theta$ is such that θ is given on $\xi = 0$ and τ is given on $\xi = \pi/2$. Woods uses the properties of such functions to obtain $\sigma(\xi_1)$ as

$$\sigma(\delta, \xi) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left(\frac{\theta_0(\delta^*)}{\tau_{\pi/2}(\delta^*)} \frac{\operatorname{cosech}(\delta^* - \delta - i\xi)}{\operatorname{sech}(\delta^* - \delta - i\xi)} + \right) d\delta^* \quad (9)$$

where $\theta_0(\delta) = \theta(\delta, 0)$

$$\tau_{\pi/2}(\delta) = (\delta, \pi/2)$$

Because the infinite range of integration makes numerical treatment of the above integral difficult, it is more convenient to transform to $\chi = \eta + i\gamma$ by

$$\zeta_2 = \tanh \zeta_1 = -i \sinh(\chi/2) \quad (10)$$

(see Fig. (4-15))

Here, the wetted surface is $\eta = 0$, $-\pi \leq \gamma \leq \pi$ and the free streamlines are $\eta = \pi$, $\gamma < 0$ and $\eta = -\pi$, $\gamma < 0$. Substituting in (9) gives

$$\sigma(\chi) = \frac{\cosh(\chi/2)}{2\pi} \left[\int_{-\pi}^{\pi} \frac{\theta_0(\gamma^*) d\gamma^*}{\sin(\gamma^*/2) + i \sinh(\chi/2)} + \int_{-\infty}^0 \frac{\tau_1(\eta^*) d\eta^*}{\cosh(\eta^*/2) + i \sinh(\chi/2)} + \int_{-\infty}^0 \frac{\tau_2(\eta^*) d\eta^*}{\cosh(\eta^*/2) - i \sinh(\chi/2)} \right] \quad (11)$$

where suffixes 1 and 2 refer to the upper and lower streamlines respectively, and integrating the 1st term by parts gives

$$\sigma(\chi) = i\theta_0(-\pi) - \frac{1}{\pi} \int_{\gamma^*=-\pi}^{\pi} \ln \left\{ \frac{\sin(\frac{\gamma^* + i\chi}{4})}{\cos(\frac{\gamma^* - i\chi}{4})} \right\} d\theta_0(\gamma^*) + \frac{\cosh(\chi/2)}{2\pi} \int_{-\infty}^0 \left[\frac{\tau_1(\eta^*) d\eta^*}{\cosh(\eta^*/2) + i \sinh(\chi/2)} + \frac{\tau_2(\eta^*) d\eta^*}{\cosh(\eta^*/2) - i \sinh(\chi/2)} \right] \quad (12)$$

$w(\chi)$ is given from (5) and (10) by

$$w = 4a(i \sinh(\chi/2) + \sin(\lambda/2))^2 \quad (13)$$

On the body where $\chi = i\gamma$

$$d\theta_0(\gamma) = \frac{d\theta_0}{ds} \frac{ds}{d\phi} \frac{d\phi}{d\gamma} d\gamma = -\frac{1}{Rq_0} \frac{d\phi}{d\gamma} d\gamma \quad (14)$$

where s is the distance along the surface, $R = -(\frac{ds}{d\theta_0})$ is the radius of curvature of the surface and q_0 is the velocity on the surface.

From (13)

$$\frac{d\phi}{d\gamma} = q \frac{ds}{d\gamma} = 4a(\sin(\gamma/2) - \sin(\lambda/2)) \cos(\gamma/2) \quad (15)$$

so on the body (12) becomes

$$\begin{aligned} \tau_o(\gamma) &= \log_e \left(\frac{U_\infty}{q_o(\gamma)} \right) \\ &= \frac{4a}{\pi} \int_{-\pi}^{\pi} \frac{1}{Rq_o(\gamma)} (\sin(\gamma_x/2) - \sin(\lambda/2)) \cos(\gamma_x/2) \cdot \ln. \left| \frac{\sin(\frac{\gamma_x - \gamma}{4})}{\cos(\frac{\gamma_x + \gamma}{4})} \right| d\gamma_x \\ &\quad - \frac{\theta_T}{\pi} \ln \left| \frac{\sin(\frac{\lambda - \gamma}{4})}{\cos(\frac{\lambda + \gamma}{4})} \right| \end{aligned}$$

(which is the contribution to the 1st integral in (12) from the discontinuity of size θ_T , in θ_o at the stagnation point $\gamma = \lambda$.)

+ the terms in (12) involving $\tau_1(\eta)$, $\tau_2(\eta)$ and independent of q_o

(16)

Now if some distribution of velocity on the free streamlines is specified, so that τ_1 and τ_2 can be assumed known, the above represents a complicated integral equation for q_o . The velocity distribution chosen should be such as to give a finite wake width far downstream, for reasons discussed earlier. By expanding $\sigma(\chi)$ in (12) at infinity, Woods shows that the above condition requires τ_1, τ_2 to satisfy

$$\int_{-\pi}^{\pi} \theta_0(\gamma) \sin(\gamma/2) d\gamma - \int_0^{\infty} (\tau_1 + \tau_2) \cosh(\eta/2) d\eta = 0 \quad (17)$$

Also, since the base pressure is constant, the pressure at the two separation points must be the same, or

$$\tau_1(\eta=0) = \tau_2(\eta=0) = \tau_s, \text{ say} \quad (18)$$

$$\text{so } c_{p_b} = 1 - e^{-2\tau_s}$$

If τ_1 and τ_2 are required to have the correct form near infinity, the wake width there must be set proportional to drag, but since the main use of the theory is for calculating pressure distributions on the body, the exact form at infinity is not necessary and some simple mathematical form can be chosen to satisfy the first two conditions only. Woods adopts

$$\tau_1 = \tau_2 = \frac{\tau_s}{1 + b^2 \sinh^2(\eta/2)} \quad (19)$$

which satisfies (18) and (17) if

$$\frac{1}{b} = \frac{-1}{2\pi\tau_s} \int_{-\pi}^{\pi} \theta_0(\gamma) \sin(\gamma/2) d\gamma \quad (20)$$

If this value for τ_1 and τ_2 is inserted into (16) the equation reduces to

$$\ln\left(\frac{U_\infty}{q_0(\gamma)}\right) = \frac{4a}{\pi} \int_{-\pi}^{\pi} \left[\cos(\gamma/2) \left(\frac{\sin(\gamma/2) - \sin(\lambda/2)}{Rq_0(\gamma)} \right) \ln \left| \frac{\sin(\frac{\gamma-\gamma^*}{4})}{\cos(\frac{\gamma+\gamma^*}{4})} \right| \right] d\gamma^* +$$

$$-\frac{\theta_T}{\pi} \ln \left| \frac{\sin(\frac{\lambda - \gamma}{4})}{\cos(\frac{\lambda + \gamma}{4})} \right| + \frac{\tau_s}{1 + b \cos(\gamma/2)} \quad (21)$$

We can attempt to solve for $q_0(\gamma)$ in the above by numerical iteration. A distribution $q_0(\gamma)$ is assumed and (15) used to obtain $s = s(\gamma)$. Then $R = R(\gamma)$ can be found since $R = R(s)$ is known for any given body. These values of $R(\gamma)$ and $q_0(\gamma)$ are then introduced into (21) and a new $q_0(\gamma)$ is found, completing one iteration. There is no proof of the convergence of this process and, although Woods found that for circular cylinders the convergence was quite rapid, the present author has found cases where there was no convergence. This is discussed more fully in section (4.3.3.). A numerical procedure for solving the equation in the above manner is given by Woods in his paper (1955) and this method was followed in devising a computer program to solve for the flow about a cylinder on the ground (i.e. two cylinders touching). The process is quite suitable for computerizing, so provided convergence can be achieved, the lengthy and complex numerical procedures required are not necessarily a problem. Much more of a problem was the lack of convergence found for certain cases.

Woods theory provides the formal solution to the general problem of flow over curved surfaces, with restrictions placed on symmetry and smoothness only. More recently, Parkinson and Jandali (1970) have used a much simpler approach to the

problem. They make use of the fact that the required wake characteristics resemble those of a 'wake' produced by a source in a uniform stream (The distribution chosen by Woods is also similar to a wake source flow) and simulate the wake effect by sources.

The flow model is set up in a ζ -plane (Fig.(4-17)) and consists of the uniform flow past a doublet at the origin, with the wake effect supplied by two surface sources, of strength $2Q$, symmetrically located at angles $\pm\delta$ on the circumference of the circle. The complex potential for such a flow is

$$w(\zeta) = V\left(\zeta + \frac{R^2}{\zeta}\right) + \frac{Q}{\pi} \left(\ln(\zeta - Re^{i\delta}) + \ln(\zeta - Re^{-i\delta}) - \ln\zeta \right) \quad (22)$$

and the velocity comes from

$$v(\zeta) = \frac{dw}{d\zeta} = V\left(1 - \frac{R^2}{\zeta^2}\right) + \frac{Q}{\pi} \left(\frac{1}{\zeta - Re^{i\delta}} + \frac{1}{\zeta - Re^{-i\delta}} - \frac{1}{\zeta} \right) \quad (23)$$

The circle of radius R is a streamline of the flow and has symmetrical stagnation points located at S_1 and S_2 due to the upstream flow of the two sources.

The aim now is to find a conformal mapping, $z=f(\zeta)$ (24), from the circle to a given body shape in the z -plane. This mapping must preserve the direction of velocity at infinity and have S_1 and S_2 as critical points, at which $f'(\zeta)$ has simple zeros. Hence angles of intersection of curves are doubled

at S_1 and S_2 , when they are mapped into the z -plane. The stagnation streamlines in the ζ -plane then become streamlines tangentially leaving the body surface at S_1 and S_2 in the z -plane and the circle actually maps onto the slit, $S_1AS_2BS_1$, flow inside the free streamlines being ignored. If $f(\zeta)$ can be found then velocity in the z -plane follows from

$$v(z) = \frac{dw}{dz} = \frac{dw}{d\zeta} \frac{d\zeta}{dz} = \frac{v(\zeta)}{f'(\zeta)} \quad (25)$$

and pressures would come from Bernoulli's equation .

The separation points are given in the z -plane and Q and δ must then be chosen to ensure that $v(\zeta) = 0$ at the corresponding points in the ζ -plane, thus providing one relation between them. A second relation is obtained by equating the velocity at separation to its given value, thus

$$\left| \frac{v(z)}{U_\infty} \right| = K = (1 - c_{p_b})^{1/2} \quad (26)$$

(Note that $v(z)$ is finite at separation since both $v(\zeta)$ and $f'(\zeta)$ are zero there).

The two relations enable Q and δ to be specified for a given base pressure and separation point, and the problem solved, provided $f(\zeta)$ can be found for the given body.

Parkinson and Jandali give mappings $f(\zeta)$ for a flat plate, a wedge and a circular cylinder and also deal with

elliptic cylinders by mapping from the circular cylinder solution. The transform for the circular cylinder

$$z = f(\zeta) = \zeta - \cot\alpha - \frac{1}{\zeta - \cot\alpha} \quad (27)$$

where α is the angle of the stagnation points in the ζ -plane and is related to θ_{1s} , the separation angle in the z -plane by

$$\alpha = \theta_{1s}/2 \quad (28)$$

Now $f'(\zeta) = 0$ when

$$\zeta = \pm i + \cot\alpha, \quad z = \pm 2i$$

which fixes the position of the singular points so that

$$R = \operatorname{cosec}\alpha \text{ and } h = 4\operatorname{cosec} \theta_{1s} \quad (29)$$

setting $v(\zeta) = 0$ at $\theta = \alpha$ gives

$$Q = 2\pi U_{\infty} \frac{(\cos\delta - \cos\alpha)}{\sin\alpha} \quad (30)$$

and substituting in (23) for Q , (25) becomes

$$\frac{q_0}{U_{\infty}} = \frac{|v_0(z)|}{U_{\infty}} = \frac{|v(\zeta)|}{|f'(\zeta)| U_{\infty}} = \sin\theta \frac{(1 - 2\cos\alpha\cos\theta + \cos^2\alpha)}{\cos\delta - \cos\theta} \quad (31)$$

on the wetted surface.

If $v(z)$ in the above is set equal to its desired value, $K U_\infty$, at separation, $\theta = \alpha$, then

$$\cos\delta = \cos\alpha + \frac{\sin^3\alpha}{K} \quad (32)$$

which completes the solution.

Thus, the theory leads to a simple analytic form for q_0 on the wetted surface, compared with the complicated integral equation of Woods. When the two theories are compared with the experimental values of pressure distribution for a circular cylinder, the agreement of the Parkinson and Jandali method is at least as good as that of the Woods method, despite the fact that it is only a model of the flow and not a formal solution of the problem. Thus, if one can find $f(\zeta)$ for the given body, or transform from one of the known solutions for a circular cylinder, it would be better to use the simpler method.

A free streamline solution, obtained by any of the above methods, may be subject to the physical unrealities shown in Fig. (4-17), i.e. the free streamlines cross each other producing a doubly covered region, or they intersect with the downstream solid surface (although the former is not applicable to the Parkinson and Jandali model). Woods (1955) has examined both types of physical unreality. He considers the first type by expanding $\sigma(\chi)$ on the free streamlines, far downstream, to show that the distance between the free streamlines remains positive and tends to a finite limit at infinity,

provided the condition given in equation (17) holds. The other type of unreality is considered by examining the radius of curvature near the separation point of the free streamline. The condition that the streamline should have either finite curvature or infinite curvature away from the body is shown by Woods to imply

$$A = \int_{-\pi}^{\pi} \tan\left(\frac{\gamma+\pi}{4}\right) d\theta_0(\gamma) + \int_0^{\infty} \left[\coth\left(\frac{\eta}{4}\right) d\tau_1(\eta) - \tanh\left(\frac{\eta}{4}\right) d\tau_2(\eta) \right] \leq 0 \quad (33)$$

By then considering the pressure gradient just before separation, Woods shows that a value of $A < 0$ implies an infinite adverse gradient, which in turn implies that the boundary layer would have separated earlier. It therefore appears as though $A = 0$ is a necessary condition for physical reality at separation. However, real flows are found for which $A \neq 0$, suggesting that viscous boundary layer effects, neglected by the theory, may be of importance in determining conditions near separation. The conditions, given in equations (17) and (33), must not therefore be thought to necessarily place physical restrictions on real flows, but simply to examine the reality of the free streamline solutions. Even then the condition $A \leq 0$ cannot guarantee that the streamlines would not intersect with the body further downstream, if the body's curvature reduces downstream. It is therefore a necessary condition, but not sufficient and must be checked

by calculating the positions of the free streamlines.

When the Parkinson and Jandali model is applied to a circular cylinder, the conditions on the separation point reduce to a simple form, as they show by considering the pressure gradient on the wetted surface near separation and comparing with Wood's results. The condition is

$$\left. \frac{\partial c_p}{\partial \theta_1} \right|_{\text{sepn.}} \text{ is } \begin{array}{ll} +ve \infty & K < 3/2 \cdot \sin \theta_1 \\ \text{finite if} & K = 3/2 \cdot \sin \theta_1 \\ -ve \infty & K > 3/2 \cdot \sin \theta_1 \end{array} \quad (34)$$

corresponding to $A < 0$, $A = 0$, $A > 0$. Thus, for a given base pressure, the separation angle, θ_1 , is restricted to a range of values, for which $\sin \theta_1 \geq 2K/3$ and, if $\sin \theta_1 = 2K/3$, the streamlines will separate with finite curvature.

(4.3.2) Transform for cylinder on ground

The easiest method of obtaining a free streamline solution for the flow past a cylinder on the ground seemed to be a transform from the single cylinder to two cylinders in contact, the second cylinder being the image of the first in the ground plane. Once found, such a transform can be used with any of the methods discussed above that will give realistic

solutions for a single cylinder, but, because of its simplicity, the Parkinson and Jandali method was preferred. In addition, Woods theory appeared to offer a method of tackling the problem directly and it was decided to apply it to the two cylinder case, both to compare with the results obtained by transforming the cylinder solution and to see if his numerical routine would converge to a solution for the bluffer obstacle presented by two cylinders.

Considerable thought was given to the problem of obtaining a suitable transform and, eventually, the solution was suggested by the 'coaxal' coordinates of Milne-Thompson (1968, p.176). If A and B are two points on the x axis at $(c, 0)$ and $(-c, 0)$ respectively, the coordinates of any point P may be expressed in terms of (r_1, θ_1) , (r_2, θ_2) .

Where (r_1, θ_1) and (r_2, θ_2) are the polar coordinates of P, treating A and B as poles respectively (see Fig.(4-18)). If we define 'coaxal' coordinates by

$$\xi = \theta_1 - \theta_2 \quad \text{and} \quad \eta = \log_e(r_2/r_1) ,$$

then $\chi = \xi + i\eta$ is given by

$$\begin{aligned} \chi &= i \log_e \left(\frac{z+c}{z-c} \right) \\ z &= i c \cot(\chi/2) \end{aligned} \tag{1}$$

The important point, from our point of view, is that all curves, $\xi = \text{constant}$, are circles passing through A and B, whose centres are at $(0, c \cot \xi)$, with radii of $c \operatorname{cosec} \xi$; in particular $\xi = \pi/2$

is a semi-circle, radius c , centre at the origin, (Fig. (4-18)). Thus, if we transform a circular cylinder at the origin in the z_1 -plane to the χ -plane defined above, the wetted surface transforms into two straight lines, which are parts of the lines $\xi = \pi/2$ and $\xi = -\pi/2$, Fig. (4-18). A simple linear transform

$$\chi_2 = a\chi \quad (2)$$

moves the lines to lie on $\xi = a\pi/2$, $-a\pi/2$, but these determine two equal circular segments in the z_2 -plane with the x -axis as the line of symmetry (see Fig.(4-19)). If a tends to zero, the shape in the z_2 -plane tends to the desired two cylinders, whilst $a \rightarrow 2$ produces a straight line on the x_2 axis.

Thus, the required transform from the circular cylinder solution of Parkinson and Jandali consists of

$$z_1 = z - b \quad (3)$$

$$\chi = i \log \left(\frac{z_1 + c}{z_1 - c} \right) \quad (4)$$

$$z_2 = id \cot (a\chi/2) \quad (5)$$

which may be combined to yield

$$z_2 = d \left[\frac{(z - (b - c))^a + (z - (b + c))^a}{(z - (b - c))^a - (z - (b + c))^a} \right] \quad (6)$$

although it is simpler in practice to use the separate transforms. (3) transforms the cylinder, radius c , with centre at $z = b$, into one at the origin. Thus, (28) and (29) of the previous section, determine c and b as

$$c = \frac{h}{2} = 2 \operatorname{cosec} 2\alpha \quad (7)$$

$$\text{and } b = c \cos \theta_{1s} = -c \cos 2\alpha = -2 \cot 2\alpha \quad (8)$$

(The separation points lie on the y axis in the z -plane)

The constant a in (5) must be chosen to give the required shape in the z_2 -plane, in our case $a \rightarrow 0$. The constant d is obtained from the requirement that the velocity should tend to U_∞ as $z_2 \rightarrow \infty$.

We shall first deal with the general solution for any a and then take the limiting case as $a \rightarrow 0$. In both cases, we shall be aiming mainly to find the pressures acting over the wetted surface.

The velocity in the z plane

$v(z) = v(\zeta)/f'(\zeta)$ is known from the Parkinson and Jandali solution for a cylinder, so

$$\left| v(z_2) \right| = \left| \frac{dw}{dz_2} \right| = \frac{\left| v(z) \right|}{\left| \frac{dz_1}{dz} \right| \left| \frac{d\chi}{dz_1} \right| \left| \frac{dz_2}{d\chi} \right|} \quad (9)$$

From (3)

$$\left| \frac{dz_1}{dz} \right| = 1 \quad (10)$$

From (4)

$$\frac{d\chi}{dz_1} = i \left(\frac{1}{z_1+c} - \frac{1}{z_1-c} \right)$$

On the wetted surface $z_1 = ce^{i\theta_1}$, so

$$\frac{d\chi}{dz_1} = \frac{-2ic}{z_1^2 - c^2} = \frac{-2ic}{c^2(e^{2i\theta_1} - 1)} = \frac{-2i}{c(\cos 2\theta_1 - 1 + i\sin 2\theta_1)}$$

so

$$\left| \frac{d\chi}{dz_1} \right| = \left[\frac{4}{2c^2(1 - \cos 2\theta_1)} \right]^{1/2} = \frac{1}{c \sin \theta_1} \quad (11)$$

From (5)

$$\frac{dz_2}{d\chi} = - \frac{ida}{2} \operatorname{cosec}^2 \frac{a\chi}{2} = \frac{-ida}{(1 - \cos a\chi)}$$

$$\left| \frac{dz_2}{d\chi} \right| = \frac{da}{[(1 - \cos a\xi \cosh a\eta)^2 + (\sin a\xi \sinh a\eta)^2]^{1/2}}$$

which reduces to

$$\left| \frac{dz_2}{d\chi} \right| = \frac{da}{(\cosh a\eta - \cos(a\pi/2))} \quad (12)$$

on the wetted surface, where from (4)

$$\xi = \pm \pi/2$$

(13)

$$\eta = -\log(\tan(\theta_1/2))$$

Hence, on the wetted surface

$$|v(z_2)| = |v(z)|.c. \frac{\sin\theta_1(\cosh a\eta - \cos^{a\pi/2})}{ad} \quad (14)$$

At the front stagnation point in the z_2 -plane, which corresponds to $\eta \rightarrow -\infty$, $\theta_1 \rightarrow \pi$, we require $|v(z_2)| \rightarrow 0$.

Now

$$\begin{aligned} \cosh a\eta &= \frac{1}{2} \left(\frac{1}{(\tan \theta_1/2)^a} + (\tan \theta_1/2)^a \right) \\ &= \frac{1}{(\sin \theta_1)^a} \left[\frac{(1+\cos \theta_1)^a + (1-\cos \theta_1)^a}{2} \right] \end{aligned} \quad (15)$$

and

$$|v(z)| \sim \sin \theta_1 \quad \text{from (31) of the last section,}$$

so from (14)

$$|v(z_2)| \sim (\sin \theta_1)^{2-a} \quad \text{as } \theta_1 \rightarrow \pi$$

$$\therefore |v(z_2)| \rightarrow 0 \quad \text{provided } a < 2$$

Another requirement is that $v(z_2)$ should tend to U_∞ as $z_2 \rightarrow \infty$ and this is best examined using $\mathcal{X}$ as the variable, then

$$\frac{dz_1}{d\mathcal{X}} = \frac{-ic}{1-\cos \mathcal{X}} \quad \text{and} \quad \frac{dz_2}{d\mathcal{X}} = \frac{-ida}{(1-\cos a\mathcal{X})} \quad (16)$$

Thus

$$\begin{aligned} |v(z_2)| &= |v(z)| \left| \frac{c(1-\cos a\chi)}{ad(1-\cos \chi)} \right| \\ &= |v(z)| |f(\chi)|, \text{ say} \end{aligned}$$

Now $z_2 \rightarrow \infty$ is equivalent to $\chi \rightarrow 0$ and, using L' hospitals rule, the 2nd derivative of $f(\chi)$ gives $\left. \frac{ca^2 \cos a\chi}{ad \cos \chi} \right|_{\chi=0} = \frac{ac}{d}$

. . We choose $d = ac$ to satisfy our requirement, since

$$\left| \frac{v(z)}{U_\infty} \right| \rightarrow 1 \text{ as } z \rightarrow \infty$$

Hence, the cylinders in the z_2 plane are of radius

$$\frac{d}{\sin^{a\pi/2}} = \frac{ac}{\sin^{a\pi/2}}, \text{ with centres at } \pm ac \cot\left(\frac{a\pi}{2}\right)$$

and as $a \rightarrow 0$, the radius $\rightarrow 2c/\pi$, with centre $\rightarrow \pm \frac{2c}{\pi}$, which gives the two cylinders just touching.

The relation between the base pressures in the z_2 and z_1 planes comes straight from (14) as

$$K_2 = \frac{K_1}{a^2} \left(\sin \theta_1 (\cosh a\eta - \cos^{a\pi/2}) \right) \Big|_{\text{sepn}} \quad (17)$$

where

$$K_2 = (1 - c_{p_{b2}})^{1/2}$$

It merely remains to find the relation between θ_1 and θ_2 , so that θ_1 at separation can be found for a given separation angle, θ_{2s} . From (13)

$$\eta = \log \left(\frac{1}{\tan \theta_{1/2}} \right) = \frac{1}{a} \log \left| \frac{z_2 + d}{z_2 - d} \right|$$

so

$$\tan \theta_{1/2} = \left| \frac{z_2 - d}{z_2 + d} \right|^{1/a}$$

which reduces to

$$\tan \theta_{1/2} = \left[\frac{1 + \sin(\theta_2 - a\pi/2)}{1 + \sin(\theta_2 + a\pi/2)} \right]^{1/2a} \quad (18)$$

A more convenient relation for changing from θ_1 to θ_2 is from (5)

$$\tan \theta_2 = \frac{1 - \cos^{a\pi/2} \cosh a\eta}{\sin^{a\pi/2} \sinh a\eta} \quad (19)$$

which can be written as

$$\tan \theta_2 = \frac{2(\sin \theta_1)^a - \cos^{a\pi/2} ((1 + \cos \theta_1)^a + (1 - \cos \theta_1)^a)}{\sin^{a\pi/2} ((1 + \cos \theta_1)^a - (1 - \cos \theta_1)^a)}$$

to show that at $\theta_1 = \pi$

$$\tan \theta_2 \Big|_{\text{STAG}} = \frac{-\cos^{a\pi/2}}{-\sin^{a\pi/2}} \cdot \frac{2^a}{2^a} = \frac{1}{\tan^{a\pi/2}} \quad (20)$$

which fixes the value of a for a given body shape in the z_2 plane.

To obtain the solution when $a \rightarrow 0$, it is simpler to make an assumption of small a in transform (5), rather than try to take the limits of the previous results. This then becomes

$$z_2 = \frac{2id}{a\chi} = \frac{2d}{a \log\left(\frac{z-b+c}{z-b-c}\right)} \quad (21)$$

Then

$$\frac{dz_2}{d\chi} = \frac{-2id}{a\chi^2}$$

so using (10) and (16)

$$\frac{1}{\left| \frac{dz_1}{dz} \cdot \frac{d\chi}{dz_1} \cdot \frac{dz_2}{d\chi} \right|} = \frac{ca\chi^2}{2d(1-\cos\chi)}$$

$z_2 \rightarrow \infty$ as $\chi \rightarrow 0$, so applying L'hospitals rule, we find that the above expression $\rightarrow \frac{ac}{d}$ as $\chi \rightarrow 0$ and we therefore choose $d = ac$ as before to give $|v(z_2)| \rightarrow U_\infty$.

Then

$$z_2 = \frac{2ic}{\chi} \quad (22)$$

$$\text{and } \left| \frac{dz_2}{dx} \right| = \frac{2c}{|\chi|^2} = \frac{2c}{(\pi^2/4 + \eta^2)} \quad (23)$$

on the wetted surface.

From (16)

$$\left| \frac{d\chi}{dz_1} \right| = c / \cosh \eta \quad (24)$$

Thus, on the wetted surface,

$$\left| v(z_2) \right| = \frac{\left| v(z) \right| \left(\frac{\pi^2}{4} + \eta^2 \right)}{2 \cosh \eta} = \left| v(z) \right| f(\eta) \quad (25)$$

or in terms of θ_1

$$\left| v(z_2) \right| = \left| v(z) \right| \frac{\sin \theta_1}{2} (\pi^2/4 + (\log(\tan \frac{\theta_1}{2}))^2) \quad (26)$$

$$= \left| v(z) \right| f(\theta_1), \text{ say}$$

To examine the behaviour of f at the stagnation point write $\eta_1 = -\eta$ then

$$\frac{\eta^2}{\cosh \eta} = \frac{\eta^2}{\theta_1} = \frac{1}{\left(\frac{1}{\eta_1^2} + \frac{1}{\eta_1} \right) + \frac{1}{2!} \eta_1 + \dots} \rightarrow 0 \text{ as } \eta_1 \rightarrow \infty$$

$$\therefore \left| v(z_2) \right| \rightarrow 0 \text{ as } \eta \rightarrow -\infty$$

The relationship between the base pressure comes from (26) as

$$K_1 = \frac{K_2}{f(\theta_{1s})} \quad (27)$$

and the relationship between θ_1 and θ_2 is

$$\tan \theta_1/2 = e^{-\eta} = \exp \left(-\frac{\pi}{2} \cdot \frac{\cos \theta_2}{1 + \sin \theta_2} \right) \quad (28)$$

or

$$\tan(\theta_2/2) = \frac{(\pi/2) - \eta}{(\pi/2) + \eta} = \frac{(\pi/2) + \log(\tan \theta_1/2)}{(\pi/2) - \log(\tan \theta_1/2)} \quad (29)$$

To find the coordinates of the streamlines in the z_2 -plane we start with the streamline equation in the Parkinson and Jandali ζ -plane.

$$\text{Im } w(\zeta) = \pm Q \quad (30)$$

If we write $\zeta = \mu + i\nu$ and substitute into the equation for $w(\zeta)$ ((32) of section (4-3-1)) the above becomes

$$\frac{\nu(\mu^2 + \nu^2 - R^2)}{(\mu^2 + \nu^2)} + \frac{Q}{\pi U_\infty} \text{atan} \left(\frac{\nu(\mu^2 + \nu^2 - R^2)}{(\mu^2 + \nu^2)(\mu - 2R \cos \delta) + \mu R^2} \right) = \frac{Q}{U_\infty} \quad (31)$$

which can be written in the form

$$\nu = \frac{Q/\pi U_\infty (\mu^2 + \nu^2)}{(\mu^2 + \nu^2 - R^2)} \text{atan} \left(\frac{-\nu(\mu^2 + \nu^2 - R^2)}{(\mu^2 + \nu^2)(\mu - 2R \cos \delta) + \mu R^2} \right) \quad (32)$$

$$\text{and } \frac{Q}{\pi U_\infty} = \frac{2 \sin^2 \alpha}{K_1} ; 2R \cos \delta = 2 \cot \alpha + \frac{Q}{\pi U_\infty} = 2 \cot \alpha + \frac{2 \sin^2 \alpha}{K_1}$$

So once α and K_1 are known the above equation can be solved by iteration to find ν for a given μ . It then remains to transform the coordinates into the z_2 -plane. For the case when $a \rightarrow 0$ suitable transforms are (writing $\cot \alpha$ as ct and $\tan \alpha$ as t)

$$\underline{z_1 = x_1 + iy_1}$$

$$x_1 = (\mu - ct) \left[1 - \frac{1}{(\mu - ct)^2 + v^2} - \frac{(t^2 - 1)}{t} \right] \quad (33)$$

$$y_1 = v \left(1 + \frac{1}{(\mu - ct)^2 + v^2} \right)$$

$$\underline{X = \xi + i\eta}$$

$$\xi = \operatorname{atan} \frac{y_1}{x_1 - c} - \operatorname{atan} \frac{y_1}{x_1 + c} = \operatorname{atan} \left(\frac{2cy_1}{(x_1 - c)(x_1 + c) + y_1^2} \right) \quad (34)$$

$$\eta = \frac{1}{2} \log_e \left[\frac{\left(\frac{x_1 + c}{y_1} \right)^2 + 1}{\left(\frac{x_1 - c}{y_1} \right)^2 + 1} \right]$$

where $c = t/\sin^2 \alpha$

$$\underline{z_2 = x_2 + iy_2}$$

$$\frac{x_2}{r_2} = \frac{\pi}{(\xi^2 + \eta^2)}, \quad \frac{y_2}{r_2} = \frac{\pi}{(\xi^2 + \eta^2)} \quad (35)$$

where $r_2 = \frac{2c}{\pi}$ = radius of cylinders in z_2 -plane

The far downstream width of the wake can be obtained in the Parkinson and Jandali model from continuity considerations as

$$2Q = U_{\infty} H \quad (36)$$

Substitute for Q from (4.3.1), equation (30)

$$H = \frac{2\pi}{K_1} (1 - \cos \theta_{1s})$$

Hence the wake width is finite downstream.

$$\frac{H}{h_2} = \frac{\pi}{4} \frac{H}{h_1} = \frac{\pi^2}{8} \frac{\sin \theta_{1s}}{K_1} (1 - \cos \theta_{1s})$$

where

h_2 = total width of obstruction in z_2 -plane

= 2 x diameter of cylinder

h_1 = diameter of cylinder in z_1 plane

The condition, $\sin \theta_{1s} \geq \frac{2}{3} K_1$, for the free streamlines not to intersect the cylinder in the z_1 -plane, also determines the physical reality at separation in the z_2 -plane. In terms of z_2 variables, this is

$$K_2 \leq 3\pi \frac{\operatorname{sech}^2 \left(\frac{\pi}{2} \left(\frac{\cos \theta_{2s}}{1 + \sin \theta_{2s}} \right) \right)}{8(1 + \sin \theta_{2s})}$$

(4.3.3.) Comparison of results

Computer programs were written for the two transform cases, of $0 < a < 2$ and $a \rightarrow 0$, and also for Woods theory, using the

numerical procedure given in his report (1955). It was found that this procedure would not always converge to a solution for the bluffer body shape presented by the two cylinders in contact. This lack of convergence seemed to be connected with the length of wetted surface, S_{TOT} ; the rate of convergence decreasing with increasing S_{TOT} until finally ceasing altogether i.e. one iteration would yield a set of values for q_0 , which when iterated returned to the previous set and so on. This behaviour was not investigated very extensively, since Woods theory was mainly being used as a comparison with the transform theory and sufficient cases with convergence could be obtained for this requirement.

Woods theory was therefore applied to some of those shapes, with a between 0 and 1 for which the iterations converged and compared with the transform results for the same shape. It was felt that agreement between the two theories, for these shapes, would be a good check on the correctness, or otherwise, of the mathematics and programming involved. Some of the results are shown in Fig. (4-20). The fairly close agreement for $a = .333$ suggests that both programs are working as expected, although as a gets smaller the agreement between the two decreases. This is probably due to the bluffer body shapes emphasizing the differences between the two theories. Our results indicate that, surprisingly, the Parkinson and Jandali transformed model gives closer agreement with available experimental data than does Woods theory.

The results for the transform theory are first compared with the pressure distribution obtained by Wadcock (1970) for the two dimensional flow around two cylinders in contact, since this flow is the closest experimental analogue to our free streamline model. (Fig.(4-21)). The experimental pressure coefficient at separation is -1.35 and the theoretical results are plotted for two separation angles. The experimental results appear to have separation at $\theta = 280^\circ$, but a theoretical value of $\theta_s = 290^\circ$ appears to give the best overall fit. The theoretical curves have an infinite adverse pressure gradient at separation, since A in equation (4.3.2.(33)) is less than zero, and so cannot be expected to give good agreement with real flows in this region; viscous effects not considered in the theory tend to smooth out the experimental curve. However, away from the separation region, the curve for $\theta_s = 290^\circ$ is almost coincident with the experimental curve, in particular, showing the very large stagnation region between $\theta = 30^\circ$ and 90° .

In fig. (4-22), the theory is compared with the pressure distribution obtained for the flow over the stationary wheel with endplates. It should be remembered that the floor boundary layer separated before the stationary wheel and therefore stagnation pressure was not attained near the floor. This also had the effect of 'streamlining' the flow over the wheel and of thus smoothing out the expected pressure variation over the body surface. Since the theory does not take these effects into account the agreement with the experimental pressure is not as

good as before. The same considerations apply to the rotating wheel (Fig.(4-23)), the viscous effects leading to the high pressures and upstream jetting at the bottom of the wheel being ignored by the theory, as are any special effects at separation.

The position of the free streamlines after separation is shown for the three cases in Fig.(4-24). In the stationary wheel case, the solution for $\theta_s = 270^\circ$ is chosen, as this gives a closer agreement for free streamline position with the smoke photograph, Fig.(3-40). The other solutions can also be compared with the appropriate smoke photographs (Figs.(3-40) and (3-41)) when it will be seen that the streamline positions at least show the same trends as the photographs, the rotating solution having the highest streamline.

The main use of free streamline theory in connection with the wheel is that, having established that the theory gives good agreement to the flow around two stationary cylinders in contact, we can point out those areas of the two-dimensional flow around the wheel on the ground which differ significantly from the theory and thus from the simple flow. We can then postulate reasons for the differences and test our conclusions by further experiment and/or by other theoretical considerations. If desired it may then be possible to use the theory as a basis for a more exact theoretical treatment of the flow, bringing in the boundary layer and the viscous effects in the corner. However, even then, the theory in its present form

could not be applied to the three dimensional flow about the wheel, although it is possible that three dimensional bluff bodies could be dealt with by modelling the wake with sources and using some form of numerical potential method to obtain the flow on the wetted surface. However, this was thought to be too difficult to attempt in the time available, so, instead, we have used the theory in its simple form to help in understanding the two dimensional flow, which in turn should lead to a further understanding of the three-dimensional case.

(4.4) Conclusions

Of necessity, the theoretical work in this section has had to be confined to simplified approaches to some of the interesting features of the wheel flow and has, therefore, given rise to a number of isolated models of these features. Because the assumptions used are not the same in each case, it is not generally possible to mathematically join the various solutions together to form a whole. However, one can do this intuitively to some extent and use has been made of the various solutions in postulating the flow pictures of Figs. (1-12), (1-19) and (3-42). For example, in Fig. (3-42 (iii)) , we began with the low Reynolds number solution for flow in the front and rear corners, added the picture of separation that has been built up, and then, joining up the mean streamlines to

to prevent any starting or finishing in the flow, the picture had to be as shown. The same type of process was used to form Fig.(1-12); here, starting with the pictures of separation and stagnation which had been deduced.

The extension from these two dimensional models to the three dimensional flows about the wheel is more difficult, although they provide a suitable basis for this, when allowance is made for flow around the sides of the wheel. For example, we would expect to quickly lose the upstream jetting from the front corner around the side of the wheel, but the high pressure under the wheel can still be explained by assuming a flow similar to the low Reynolds number solution will still take place, sufficiently close to the line of contact. Again, although separation in three dimensions is different from that in two, in general, near the centreline, the resemblance will be close, so the movement of separation above the wheel can be partly explained in terms of the special type of separation, deduced for two dimensional flow.

Where the theoretical work has not been directly applicable to the wheel, as in the case of free streamline theory, it has been of use in comparing with the two dimensional experimental results, as an aid to understanding them, and the theory, as given here, could form a possible starting point for a more extensive treatment. Thus, although the ideas in this section have applied to diverse simplified models, they have all helped to contribute to an understanding of the flow, as revealed by the experimental results.

References

- Abernathy, F.H. 'The formation of vortex streets'.J.F.M.
Kronauer, R.E. Vol. 13, pp 1-20, 1962
- Batchelor, G.K. 'An Introduction to fluid dynamics'
C.U.P. 1967
- Bearman, P.W. 'Some recent measurements of flow around
bluff bodies in smooth and turbulent
streams', Imperial College, Aero dept.
report No 72-09, 1972
- Berry, A.J. 'An experimental investigation of ground
effect using a moving surface simulation'
Univ. of London M.Phil. Thesis, 1968
- Betz, A. 'Zeitschr des Verines deutscher inge-
nieure' 69, 1925
(quoted in Goldstein (1938)II)
- Bluston, M.S. 'A theoretical solution for laminar
Paulson, R.W. flow past a bluff body with a separated
wake' Jnl. de Mecanique, Vol II, No.1,
pp161-180, 1972

- Brodetsky, S. 'Discontinuous fluid motion past circular and elliptic cylinders' Proc. Roy. Soc. A, Vol. 102, p542, 1923
- Carpenter, L.H 'On the motion of two cylinders in an ideal fluid'. Jnl. of Research of the National Bureau of Standards, Vol.61, pp 83-87, 1958
(quoted in Wadcock (1970))
- Chubb, T.W. 'The response of a narrow bore pressure measuring system to step and oscillatory pressures. RAE. Tech. rep. No.68010, 1968.
- City University 'Proceedings of the first symposium on road vehicle aerodynamics'. City University, London, Nov.1969.
- Davies, J.M. 'The aerodynamics of golf balls' J. Appl. Phys. Vol.20, No.9, 1949
- Fage, A. 'Experiments on a sphere at critical Reynolds numbers', A.R.C. R M No.1766, 1937
- Fage, A. 'Further experiments on the flow around a circular cylinder', A.R.C. R M. No. 1369, 1931 (quoted in Goldstein (1938)II)
- Falkner, V.M.

- Fail, R., Lawford, J.A. 'Low speed experiments on the wake characteristics of flat plates normal to an air stream', A.R.C. R. M. No.3120, 1957
- Eyre, R.C.W.
- Fink, P.T. 'Some low speed experiments with 20° Delta wings'. A.R.C. Report No.17854, 1955
- Taylor, J.
- Flaschbart 'Handbuch der experimental physik' Vol.4, part2, 1932 (quoted in Goldstein (1938)I)
- Gerrard, J.H. 'The mechanics of the formation region of vortices behind bluff bodies' J.F.M., Vol. 25, pp 401-413, 1966
- Gilbarg, D. 'On two theories of plane potential flows with finite cavities' Memo U.S. Naval Ordnance Lab. 8718, 1945
- Rock, D.H. (quoted in Thwaites (1960))
- Glauert, H. 'Aerofoil and airscrew theory' C.U.P., 2nd Ed, 1948
- Glauert, M.B. 'The laminar boundary layer on oscillating plates and cylinders' J.F.M., Vol.I, pp97-110, 1956

- Glauert, M.B. 'A boundary layer theorem, with applications to rotating cylinders' J.F.M. Vol.2, p89, 1957.
- Goldstein, S. 'Modern Developments in fluid dynamics' O.U.P. Vols.1 2, 1938
- Gottingen 'Ergebnisse der aerodynamischen Versuchsanstalt zu Gottingen' Vol.2. 1923 (quoted in Goldstein (1938))
- Gould, R.W.F. 'Wake blockage corrections in a closed wind tunnel for one or two wall mounted models subject to separated flow' A.R.C. R M No. 3649, 1970
- Griffiths, R.T. 'Differential boundary layer separation effects in the flow over a rotating cylinder' J.Roy. Aero. Soc., Vol.73, pp524-526 1969.
- Ma, C.Y. 'Calculation of potential flow about arbitrary bodies' Prog. Aero. Sci. Vol.8 1968
- Hess, J.L. 'High speed land vehicles', M.Sc. Project report, Imperial College, 1972.
- Smith, A.M.O.

- Kuchemann, D. 'Inviscid shear flow near the trailing edge of an aerofoil' Z.Flugwiss, Vol.15, p292, 1967
- Levi-Civita, T. 'Scie e leggi de resistenza'
R.C. Circ. Mat. Palermo, Vol.23,1907
(quoted in Milne-Thompson (1968))
- Linke, W. Physik Zeitschr, Vol.32, 1931
(quoted in Goldstein (1938), II)
- Maccoll, J.W. 'Aerodynamics of a spinning sphere'
J.Roy. Aero. Soc. Vol.32, p777, 1928
- Maskell, E.C. 'Flow separation in three dimensions'
A.R.C. Report No. 18063, 1955
- Maskell, E.C. 'A theory of the blockage effect on bluff bodies and stalled wings in a closed wind tunnel' A.R.C. R M No.3400
1963
- Milne-Thompson, L.M. 'Theoretical aerodynamics' 5th Ed.
Macmillan Co, 1968
- M.I.R.A. 'Aerodynamic characteristics of exposed rotating wheels' by W.R. Stapleford and G.W. Carr, MIRA report No. 1970/2, 1970

- Morelli, A 'Aerodynamic actions on an automobile wheel.' 5th paper at the First Symposium on Road Vehicle Aerodynamics, City University, 1969.
- Morkovin, M.V. 'Flow around circular cylinder - a Kaleidoscope of challenging fluid phenomena' presented at the Symposium of fully separated flows, The Am. Soc. of Mech. Eng., Ann Arbor 1964
- Olson, H.F. 'Acoustical engineering' 2nd ed., Van Nostrand, 1957
- Olson, H.F. 'Dynamical analogies' 2nd ed. Van Nostrand, 1958
- Parkinson, G.V.
Jandali, T. 'A wake source model for bluff body potential flow' J.F.M. Vol.40 pp577-594, 1970
- Prandtl, L.
Tietjens, O. 'Applied hydro-and aeromechanics' Dover, 1934
- Ried, E.G. 'Tests of rotating cylinders' NACA T.N. 209, 1924 or Flight, Jan8, 1925

- Riabouchinsky, D. 'On steady fluid motion with free surfaces'
Proc. Lond. Math. Soc. Vol.19, p206,
1920 (quoted in Thwaites (1960))
- Roshko, A. 'A new hodograph for free streamline
theory' NACA TN 3168, 1954
- Roshko, A. 'On the wake and drag of bluff bodies'
J. Aero Sci., Vol. 22, p124, 1955
- Roshko, A. 'Experiments on the flow past a circular
cylinder at very high Reynolds number'
J.F.M., Vol.10, p345, 1961
- Schlichting, H. 'Boundary layer theory' 5th ed, McGraw-
Hill, 1968
- Stollery, J.L.
Burns, W.K. 'Forces on bodies in the presence of
the ground' 1st paper, First Symposium
of Road Vehicle Aerodynamics, City Univ.
1969.
- Thom, A. 'Experiments on the air forces on
rotating cylinders', A.R.C. R M 1018,
1925
- Thom, A. 'The pressures round a cylinder rotating
in an air current' A.R.C. R M 1082,
1926

- Thom, A. 'Experiments on the flow past a rotating cylinder' ARC R M 1410, 1931
- Thwaites, B 'Incompressible aerodynamics' O.U.P. 1960
- Villat, H. 'Aperçus theoriques sur la resistance des fluides' Collection Scientia Gauthiers Villars, Paris 1920
(quoted in Thwaites (1960))
- Wadcock, A.J. 'The interference between a pair of circular cylinders normal to a stream ' MSc project report, Imperial College, 1970
- Weisselberger, C. ' Derluftwiderstand von Kugeln' ZFM, Vol.5, pp140-144, 1914
(quoted in Schlichting (1968))
- Woods, L.C. 'Two dimensional flow of a compressible fluid past given curved obstacles with infinite wakes' Proc. Roy. Soc. Lond, vol. 227, p367, 1955

Foot Note

After this thesis had been written, a report by Telionis and Werle (1972) came to the author's attention. The report discusses a numerical computation of the boundary layer above moving boundaries. Models of the flow near separation, with the boundary moving upstream and downstream, are given, which agree with the models discussed in this thesis. It appears that these were first proposed by Moore, Rott and Sears in connection with unsteady separation from a fixed boundary. The numerical results of Telionis and Werle show features that have been discussed in this thesis. For example, the numerical integration proceeds smoothly through a point of vanishing skin friction without any singularities. When a singularity of the type normally associated with separation does appear, it occurs some perpendicular distance away from the boundary and nonsingular flow continues underneath it. These results lend support to the models of the flow near separation given in this thesis.

Reference

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| Telionis, D.P. | 'Boundary layer separation from moving |
| Werle, M.J. 1972 | boundaries' Virginia Polytechnic Institute and State University Report No. VPI-E-72/13 |

Appendix

Details of experimental apparatus

(A-1) The Wind Tunnel

The tunnel used is the Aeronautics departments 5' x 4' closed return tunnel, with a speed range of 40-150 ft/sec. The working section is 7 ft. long, of rectangular shape, with corner fillets, which reduce the average cross sectional area to 17.5 sq.ft.. The fillets decrease in size along the working section, to allow for boundary layer growth. Breather slots at the downstream end of the working section ensure that the static pressure within the section is close to atmospheric, although slightly less due to leakage around the tunnel circuit. When the moving floor is used, it is necessary to remove the bottom tunnel boundary layer ahead of the floor. This means that for a given speed the static pressure is lower and has more variation along the working section, as given in Fig. (2-7). A honeycomb and gauze placed upstream of the contraction ensure a free-stream turbulence level of about 0.2%.

The tunnel speed was measured from the difference in the static pressure before the contraction and at the beginning of the working section, this difference having first been calibrated against the speed in the empty working section

at the wheel centre position, given by a Pitot-static tube. In calculating the results a correction is applied to the speed measured in this way, to allow for blockage effects.

(A-2) The Moving Floor

A detailed description of the design and development of the moving floor has been given by A.J. Berry (1968). The floor consists, basically, of an endless belt running under tension between two rollers one of which is driven by a constant speed AC motor. (Fig. (2-4)). To control any sideways tracking of the belt, the end rollers are crowned and the bearings of the non-driven roller are movable, to provide unequal tensioning. The belt is 33 ins. wide and has 5 ft. between roller centres, these dimensions being chosen to enable the floor to fit under the bottom corner fillets, replacing the fixed tunnel flooring.

For use with the wheel a support roller is mounted under the floor on bearings (Fig. (2-4)). The wheel is arranged to press down on this roller, sandwiching the floor between them. When the floor moves therefore, it causes the wheel and roller to rotate. Checks of the floor speed and wheel rotation, with a stroboscope show that there is no slipping between them.

When the belt is subjected to a negative overpressure, such as might occur beneath and behind a model mounted above

the belt, it will lift away from its supporting surface into the tunnel. This can have important consequences for tests on models mounted close to the floor, altering the flow on their underside and hence possibly the whole flow field. In the present experiment the wheel pressed down on the middle of the belt, helping to prevent lifting. However, at high tunnel speeds, the large negative static overpressure caused the belt to lift excessively all around the wheel and especially in the wake region. For this reason, it was felt desirable to run the tunnel at as low a speed as possible, within the limitations imposed by Reynolds number requirements.

In order to accurately model the real flow with the moving floor, it is necessary to eliminate the tunnel boundary layer before the floor, as completely as possible. Initially, this was attempted by fitting a fairing in front of the floor to give a 2 ins. duct, through which the boundary layer could bleed away. The duct was fitted with a variable venturi, to ensure a favourable pressure gradient through it. This arrangement eliminated the tunnel boundary layer, but allowed a new layer to grow over the top of the fairing, resulting in a boundary layer over the moving floor of the form shown in Fig. (2-5(i)). To overcome this, a portion of the upper surface of the fairing was removed and replaced with Vyon porous plastic material, through which suction was applied. This improved the situation, but not as much as had been hoped, resulting in the boundary layer profile of Fig. (2-5(ii)). It was felt that this was due to the area through which suction

was applied being too small and possibly by the blockage to the duct caused by the suction chamber beneath the fairing. The whole fairing was therefore redesigned to increase the suction area, reduce the blockage and at the same time increase its strength. This final version is shown in Fig.(2-5 (III)), together with the much improved boundary layer profile.

(A-3) The Wheel

As explained previously, the diameter of the wheel had to be such as to give a Reynolds number in the super-critical range, when used at the maximum tunnel speed at which the moving floor would not lift excessively. The choice was for a wheel of $16\frac{3}{8}$ " diameter, running at a speed of 60.8 ft/sec, giving a Reynolds number of 5.3×10^5 and a maximum blockage ratio of 8%. The static pressure in the working section at this speed is of the order of 0.05 ins. of water, without the wheel present, and the maximum negative over-pressure (in the wake close to the wheel) was less than 0.5 ins. of water, which did not cause the belt to lift excessively.

It was decided to have three different rim widths, $4\frac{1}{4}$ ins., $6\frac{3}{4}$ ins and $9\frac{1}{4}$ ins, all of $16\frac{3}{8}$ ins. diameter, in order to study the effects of varying aspect ratio. These rims were to be combined with 2 edge profiles, representative of tyres used on Grand Prix cars, to give a total of 6 models to be tested. (fig.(2-1)). The different edge shapes give

overall widths which are 4 ins. and $3\frac{1}{4}$ ins. greater than the width of the particular rim used. To accomodate the various overall widths needed, one inner hub (4 ins. wide), two outer hubs ($3\frac{1}{2}$ ins. and 6 ins.) and two spacers ($3\frac{1}{4}$ ins. and $2\frac{1}{2}$ ins.) are used in various combinations. All the above are cast from aluminium alloy.

Pressure measuring holes ($1/16$ ins. dia.) are distributed over the wheels at the positions indicated in Figs. (2-1) and (2-2). Flexible plastic tubing ($3/32$ ins. I/D) connects the holes to brass tubes in the hub well and the transducer on the wheel axis is connected to one tube at a time (Figs. (2-3) and (2-9)). The tubing is arranged to have the same overall length for all the measuring holes, for reasons connected with the frequency response of the system. (section (A-4)).

The wheel rotates freely on bearings, the bearing assembly being supported by $3/8$ ins. diameter mild steel rods each of which is adjustable in length to a small extent. Thus the rods can be adjusted to allow the wheel to press down onto the floor, with sufficient force to prevent any slipping between the two. Outside the tunnel, the rods are connected to a heavy girder framework bolted to the laboratory floor.

Fig. (2-9) shows the mounting arrangement for the pressure transducer, which is placed on the wheel axis to avoid centrifugal effects.

The output from the transducer is led through a shaft of mild steel tube to a DISA oscillator No 51E32, rotating with the wheel. The oscillator output is taken from the rotating assembly through two silver graphite brushes, running on silver slip rings. (Fig.(2-10)).

Because of difficulty experienced with drifting of the D.C. component of the transducer signal, a small diameter tube was teed to the tube leading to the transducer and the mean pressure is led along the rotating shaft through this tube and out via the pressure seals, as shown in Fig.(2-10). Another tube is led through the shaft to connect the air behind the transducer diaphragm to the external atmosphere, to allow for long term variations in atmospheric pressure. The size of this tubing is sufficiently small to prevent short term variations in pressure being transmitted with sufficient power to affect the transducer.

(A-4) The Pressure Measuring System

The transducer, mounted at the centre of the wheel, is a B. & K. 1/4 ins condenser microphone, type 4135. This consists of a thin diaphragm (2μ) mounted close to a rigid back plate, the two forming the electrodes of a capacitor. The chamber behind the diaphragm communicates with the external air as described above. When the diaphragm is subjected to a varying pressure on its front face, it moves and so alters

the distance between itself and the back plate.

The capacity thus varies in step with the pressure. This capacitance forms part of a resonant circuit, with a basic resonant frequency of around 5 MHz. The variation of the microphone capacitance changes this frequency by a few KHz., producing a frequency modulated signal.

The resonant circuit and the driving signal of 5MHz. are provided by the DISA 51E32 oscillator, rotating with the wheel. The oscillator is powered by and the resulting F.M. signal returned to a DISA reactance converter, Type 51E01. (Fig.(2-6)). The reactance converter amplifies and rectifies the F.M. signal, producing a varying D.C. voltage at the output, dependent on the pressure acting on the transducer diaphragm. Over the range of pressure used, this variation was found to be linear (Fig.(2-11)).

The frequency response of the transducer in free air is virtually flat between 6 Hz. and 5 KHz.. However, to measure the pressure at the rim of the wheel, the transducer has to be connected to the measuring holes by tubing around 13 ins. long. This tubing then determines the frequency characteristics of the system, since it both attenuates higher frequencies and suffers from resonances.

The attenuation of pressure waves in narrow tubes has been given by Olson (1957). If A is the amplitude at a

distance x along the tube from where the amplitude is A_0 we can write

$$A = A_0 e^{-\alpha x}$$

Then for a tube length l , diameter d , the attenuation is determined by

$$\alpha x = B \cdot \sqrt{f} \cdot l / d$$

where f = frequency

and B is a constant

In theory, therefore, attenuation can be overcome, for any particular frequency required, by choosing a large enough internal diameter for the tubing (in our case 3/32 ins.) However, if a large diameter is used, there are problems with resonance of the air in the tube. The simple theory for resonance in a tube, with one end closed, is given in many texts (e.g. Milne-Thompson, 1968, p.462) and shows that resonances occur at frequencies of

$$\frac{c}{4l} \text{ (fundamental frequency), } \frac{3c}{4l}, \frac{5c}{4l} \text{ etc.}$$

where c = speed of sound.

In particular, the frequency of the first resonance varies as $1/l$. Thus, to have the fundamental frequency as high as possible and to reduce phase change of the pressure waves, in passing down the tube, we require l to be small. However,

this would necessitate reducing the diameter of the wheel, d , and the frequency of rotation of the wheel, f_w , varies as U_∞/d . From the above, the first resonance, f_r , varies as $1/d$. So

$$f_w/f_r \sim U_\infty \sim \frac{\text{Re. No.}}{d}.$$

Thus, to obtain the best definition in pressure variation around the wheel would require choosing the wheel diameter large, although this would increase the attenuation in the tubing. Obviously some compromise has to be reached between these conflicting requirements and the other requirements, such as tunnel blockage.

For the chosen wheel size and speed, the tube length of around 13 ins. gives a first resonance peak about 200 Hz, compared to the frequency of rotation of the wheel of 14.25 Hz. This resonance caused a pronounced ringing in the signal from the wheel, as the pressure hole passes the floor. Initially, we attempted to eliminate this ringing by electrical filtering, but found it very difficult to counter completely in this way, so eventually a combination of electrical filtering and mechanical damping was used. The mechanical component consists of a tuft of wire wool inserted into the tube leading from the transducer to the connection in the hub. By varying the compactness and amount of wire wool, it was possible to obtain a much flatter frequency response, smoothing out the resonance peaks. (Fig. (2-8)).

However, this does not eliminate all the unfortunate

features of using a long tube. As described previously, when the measuring hole passes the floor, it receives a sharp pulse-like input, due to the rise in pressure in front of the line of contact between wheel and floor, followed by the sharp decrease just after. This pulse contains components at many frequencies, including high frequencies, and, as these travel along the tube, they suffer varying amounts of attenuation and phase change. Thus, on recomposition at the transducer head, the signal will no longer have its initial form. Thus, the sudden rise and fall of pressure at the floor is not followed exactly by the transducer and one cannot know the magnitude of the error involved, since the true form of the signal is unknown. The pulse response of the system was studied using the set up shown in Fig.(2-11(1)). It was found that the results, in the sense of fastest rise and fall times, could be obtained by making the frequency response as flat as possible up to as high as possible.

As mentioned in section (4.2.2), in this region, there will also be some error due to the lifting of the belt in the low pressure region behind the line of contact. In the determination of the lift force on the wheel, however, the movement of the separation position has most effect, since the pressures near the ground act only on a very small area of the wheel's surface (although because of the large values involved they are significant). It is hoped, therefore, that the error in the lift force for the model wheels is not greater than 5%. However, the lift forces on actual wheels, running on different types of road surface, will have some variation from our figures, because of the effect on this region near the

ground. The drag is little affected by the pressures in this region and so should not have such a great variation.

An initial difficulty in operating the system was the high rate of drift of the D.C. level, the rate being variable and influenced by such factors as wind speed and wheel rotation. To overcome this difficulty, it was decided to measure the mean pressure directly with a manometer, as described before, and to use the microphone to measure the fluctuating pressures only. The fitting of the side branch, to do this, will have an effect on the response of the transducer to the pressure fluctuations passing down the main tube, as can be seen by considering the electrical analogy of the acoustic system. (see Olson (1958)). Because, at the frequencies of interest, the wavelengths of the pressure fluctuations are long compared with the tube length, we can use a lumped impedance analogy, where electrical current is analogous to volume current and e.m.f. to pressure. Then the acoustic system can be represented as in Fig. (2-13) and we can apply Kirchoff's Laws, in the normal way, to get four simultaneous equations, which can be solved to give

$$\frac{e_o}{e} = \frac{z_2 z_4}{(z_2(z_1 + z_3 + z_4) + z_1 z_4 + z_1 z_3)}$$

in particular, the dependance on z_2 is in the form

$$\frac{e_o}{e} = \frac{Az_2}{Bz_2 + C}$$

We should like the system to be as independent of z_2 as possible. The form of the above shows two ways of doing this; (1) if z_1 is made small so that $z_1 z_3 \rightarrow 0$ then $\frac{e_0}{e} = \frac{z_4}{z_3 + z_4}$ independent of z_2 . Physically, this means taking the side branch as close to the measuring hole as possible; (2) z_2 is made large or, physically, the side tube diameter is made small

Because, for simplicity, the side branch was taken from the tube joining the transducer to the connecting tubes on the hub, it was not possible to achieve the first alternative above. A small diameter (.1 mm) side tube was therefore used, but the time taken for the manometer to attain the mean pressure was then excessive (being inversely dependent on the fourth power of diameter (Chubb(1968))) and the tube was susceptible to blockage. The simple answer was to use the system, without the side branch, to measure the fluctuating pressures, then to fit a wider side branch (1mm dia.) to measure the mean pressure. This 1mm tubing does affect the fluctuating signal received by the transducer, but this should not affect the value of the mean pressure.

The mean pressure, obtained in this way, has to be corrected for the effects of the centrifugal pressure gradient, since we are taking the pressure out of the rotating system at some small radius, r_m , (fig.(2-10)) and we wish to know the pressure at the radius of the measuring hole, r_s . The correction is obtained by considering the pressure difference across a small element of thickness, dr , i.e. $\rho \omega^2 r dr$, and

integrating to give

$$p_s = p_m + \frac{\rho U_\infty^2}{2} \cdot \left(\frac{r_s^2 - r_m^2}{r_o^2} \right)$$

where $r_o = \overset{\text{max.}}{\text{radius of wheel.}}$

The mean pressure must also be adjusted to allow for the static pressure at the wheel position. This is obtained by measuring the static pressure of the undisturbed flow at the beginning of the working section and then assuming the same variation with distance along the working section as occurs when the wheel is not present.

The calibration of the transducer was performed in the range 15-800Hz using a B & K Pistonphone No.4220, which produces a constant amplitude signal throughout this range of frequencies. Measurements at higher frequencies were performed by comparing the response of the measuring assembly with a transducer placed at the source of the pressure waves (Fig. (2-11)), the same set-up being used to compare the pulse response of the system.

(A-5) Data Reduction

The fluctuating signal, emerging from the pressure measuring system, is fed into a Hewlett-Packard Correlator, No. 3721A (Fig.(2-6)). By taking an average from several thousand samples (usually 8K), the correlator is able to eliminate any

noise present on the signal; the 'noise' in this case being mostly produced by the turbulence in the separated region of the flow, very little electrical noise being present.

The correlator averages in the following way: At each revolution of the wheel, a trigger pulse is fed to the correlator to initiate a sampling cycle, in which the incoming signal is sampled consecutively at 100 points, the time interval between points being adjustable between 1 μ sec and 1 sec. Each sample of 100 points is divided by the total number of samples and added to previous samples. In this way we eventually obtain 100 equally spaced points around the wheel, each of which represents the average of the signal at that point. Any disturbance on the signal which is not correlated with the rotation of the wheel, e.g. noise or hum, is therefore eliminated, provided enough samples are taken. The correlator is used in an AC mode, i.e. with all signal fluctuations below 1Hz. filtered off. This is to eliminate the varying D.C. level, referred to earlier. The correlator output can then be recombined with the reading of mean pressure from the manometer to produce the complete averaged pressure signal.

The electronics for applying the trigger pulse to the correlator, once every revolution of the wheel, are given in schematic form in Fig.(2-12). The inductive transducer head is mounted on the fixed wheel supports, close to the hub (Fig.(2-9)). A small steel insert in the hub passes close to the transducer head, as the wheel rotates, causing a change

in the inductance of the transducer head and hence altering the signal from the bridge network. This signal is processed by the rest of the system, to provide a suitable pulse to the correlator every revolution of the wheel. An adjustable time delay could be added to the system by inserting a second monostable.

This time delay allows any particular portion of the signal to be picked out in greater detail. For example, the basic signal from the wheel is repeated every 70 msec and the correlator is usually set up with 1 msec between points so that every two revolutions of the wheel yield 100 msec of the signal and the distance between points is 5.1° around the wheel. (The second trigger pulse generated during this sampling is ignored). If it is required to look at any portion of this signal in greater detail, the time delay can be set to initiate the sample just before the portion of interest and the time interval between points reduced to give the desired definition. Thus, setting the interval to 0.1 msec one could examine 10 msec of the signal at every 0.5° around the wheel. This ability was particularly useful in providing small intervals between points near the floor. The error from numerical integration in this region of rapid change can be reduced to fall within the error due to the transient response of the system.

The correlator was connected to a Facit 4070 punch, in order to obtain the output on paper tape for computing

purposes. It was also connected to a H-P 7034A X-Y plotter, to give a permanent visual record of the output display, for use in comparing with the processed results, as a check for any computing errors.

A program was developed to provide the pressure, lift and drag coefficients for the wheel. As input, the program requires the averaged pressure signal data for each hole, from the correlator, with a specification of the angular interval between the points of this data, and of the position and direction of each hole. Also required, are an indication of the position, within each set of 100 points, which corresponds to the point of contact with the floor, the dynamic head corrected for blockage, the mean pressure corrected for centrifugal effects and tunnel static pressure, and a zero level signal. The last, obtained from the rotating wheel with the wind tunnel off and the transducer not connected to any of the pressure measuring holes, is necessary to eliminate any effects, such as varying resistance around the slip rings, producing a fluctuating signal which is correlated with the wheel's rotation. The position corresponding to the point of contact with the floor was determined by scrutinizing the detailed X-Y plots of the pressure near the ground and choosing the position of highest pressure as the required point. As a check, the effect on C_L and C_D of displacing the floor position 2° away from this maximum position was computed for a typical signal. This produced a change of less than 4%.

For each hole, the program subtracts the zero level data from the pressure data and then adds the appropriate mean pressure and divides by the dynamic head, to give the pressure coefficients around the wheel for that hole. Because the signal from the wheel repeats every 70 msec only 70 of the 100 points are needed, the others being discarded.

The integrals for the total lift and drag coefficients are

$$C_L = \int_0^w \int_0^{2\pi} c_p \sin\theta \cos\phi \, r d\theta \, dw / \text{Area}$$

$$C_D = \int_0^w \int_0^{2\pi} c_p \cos\theta \cos\phi \, r d\theta \, dw / \text{Area}$$

where the symbols are explained in Fig.(2-14) and the area chosen was either

$$\int_0^w 2r \, dw \quad (\text{projected frontal Area})$$

or tread width x diameter.

In practice, the lift coefficient for each hole is defined as

$$c_L = 1/2 \cos\phi \int_0^{2\pi} \sin\theta \, d\theta$$

and this is calculated using Simpsons rule. The normal interval between points is 5.1° , but near the ground a smaller interval of 0.51° is used and the contribution to the integral from this region is evaluated separately.

The total C_L of the wheel is then

$$\int_0^w rc_L dw$$

$$\int_0^w r dw$$

and these integrals are evaluated by the trapezium rule, to deal more simply with the unequal interval size, occasioned by the unequal spacing of the pressure measuring holes across the wheel.

The drag coefficient is evaluated in a similar way.

The dynamic head, used in the above calculations, must first be corrected for tunnel blockage. This is done using a correction due to Maskell (1963), who uses a model of the separated flow similar to that in the free streamline theory discussed in section (4-3), but applied in three dimensions. That is, he assumes that the separated region is divided from the main stream by a surface of discontinuity, on which the velocity is KU . By considering the momentum balance past the body in the tunnel, Maskell obtains the relation

$$\frac{K^2}{K_c^2} = 1 + \frac{C_D}{K_c^2 - 1} \frac{S}{C} + o\left(\left(\frac{S}{C}\right)^2\right)$$

or

$$K_c^2 = \frac{+B + \sqrt{B^2 - 4K^2}}{2}$$

neglecting terms $o\left(\left(\frac{S}{C}\right)^2\right)$

where

$$B = \left(K^2 + 1 - \frac{C_D S}{C}\right)$$

S = Area of the model used in calculating C_D

C = X-sect. area of wind tunnel working section
and subscript c denotes corrected values.

The correction for the dynamic head $q = 1/2 \cdot \rho U_\infty^2$ comes from

$$\frac{K^2}{K_c^2} = \frac{1 - c_p}{1 - c_{p_c}} = \frac{q_c}{q}$$

Gould (1970) has shown that the above can be applied with success to wall mounted flat plates, which follows from the fact that the theory makes no use of images, but considers only the momentum balance in the tunnel. The theory does assume that the effect of blockage is a simple increase in dynamic head, which is equivalent to saying that the distribution of the static pressure on the model is unaltered by blockage. Maskell shows that this is largely true for flat

plates normal to the stream, but for more three dimensional bodies, with rounded surfaces, the effect on the pressure distribution is likely to be greater. In such circumstances, it is best to keep the blockage ratio, S/C , small so that any errors are minimized. The corrections calculated for the wheels are given in table 5. The base pressure used is the mean taken over the base region of the wheel.

(A-6) Additional apparatus, used in examining the external flow

For the measurements of total head, given in section (3-6), a Kiel tube was used because of its insensitivity to the yawing of the flow. The dimensions of the tube are given by Fink and Taylor (1955), together with a yaw calibration showing the flat response to yaw angles between $\pm 30^\circ$. Despite this, because of the large deviations in the flow directions around the wheel, the tube had to be mounted in the tunnel with the ability to point in any direction and this was achieved using the supporting system shown in Fig. (2-15). The approximate flow direction was found by using a wool tuft and the yaw insensitivity of the tube made use of to eliminate any errors in the direction so obtained.

For measurements of the boundary layer profiles on the wheel, a flattened Pitot tube and a static tube were used, the sizes are given in Fig. (2-16). The special problem connected

with this work, was due to the vibration of the wheel and the supports of the Pitot or static tube had to be fairly stiff and well damped, in case the wheel should contact the tube and set it vibrating. The arrangement shown in Fig. (2-16) was found to be satisfactory. At each station where a profile was required, this arrangement was set up with the Pitot or static tube parallel to the wheel's surface and then the tube and supporting rods traversed normal to the surface.

Wheel No.	$\frac{\text{Total Width}}{\text{Dia.}}$	$\frac{\text{Tread Width}}{\text{Dia.}}$	X-Sect Area sq.ft.	Tread x Dia. sq.ft.	Coefficients Based on Sect. Area		Coefficients Based on Tread x Dia.		Contribution From Hub		
					C_{D1}	C_{L1}	C_{D2}	C_{L2}	ΔC_{D1}	ΔC_{D2}	ΔC_{L2}
A 1	.50	.26	.91	.48	.52	.28	.97	.53	.06	.12	-.01
B 1	.66	.41	1.19	.77	.63	.40	.98	.61	.03	.05	0
C 1	.81	.56	1.48	1.05	.70	.43	.97	.61	.02	.02	0
A 2	.46	.26	.85	.48	.51	.28	.90	.49	.08	.14	-.02
B 2	.61	.41	1.13	.77	.58	.44	.85	.65	.03	.05	0
C 2	.76	.56	1.42	1.05	.70	.43	.94	.57	.01	.01	0

Table 1

Table 2

	Total Width Dia.	Tread Width Dia.	Four Wheels		Front Wheels	
			C_{D_4}	C_{L_4}	C_{D_1}	C_{L_1}
M.I.R.A.	.33	.33	1.11	.50	—	—
	.50	.50	1.00	.60	.64	.40
	.67	.67	1.06	.44	—	—
Morelli	.39	.28	—	—	.45	-.1

Table 3

Wheel No.	C_{D_1}	C_{L_1}	C_{D_2}	C_{L_2}
B2 Rotating	.58	.44	.85	.65
B2 Stationary	.77	.76	1.13	1.12

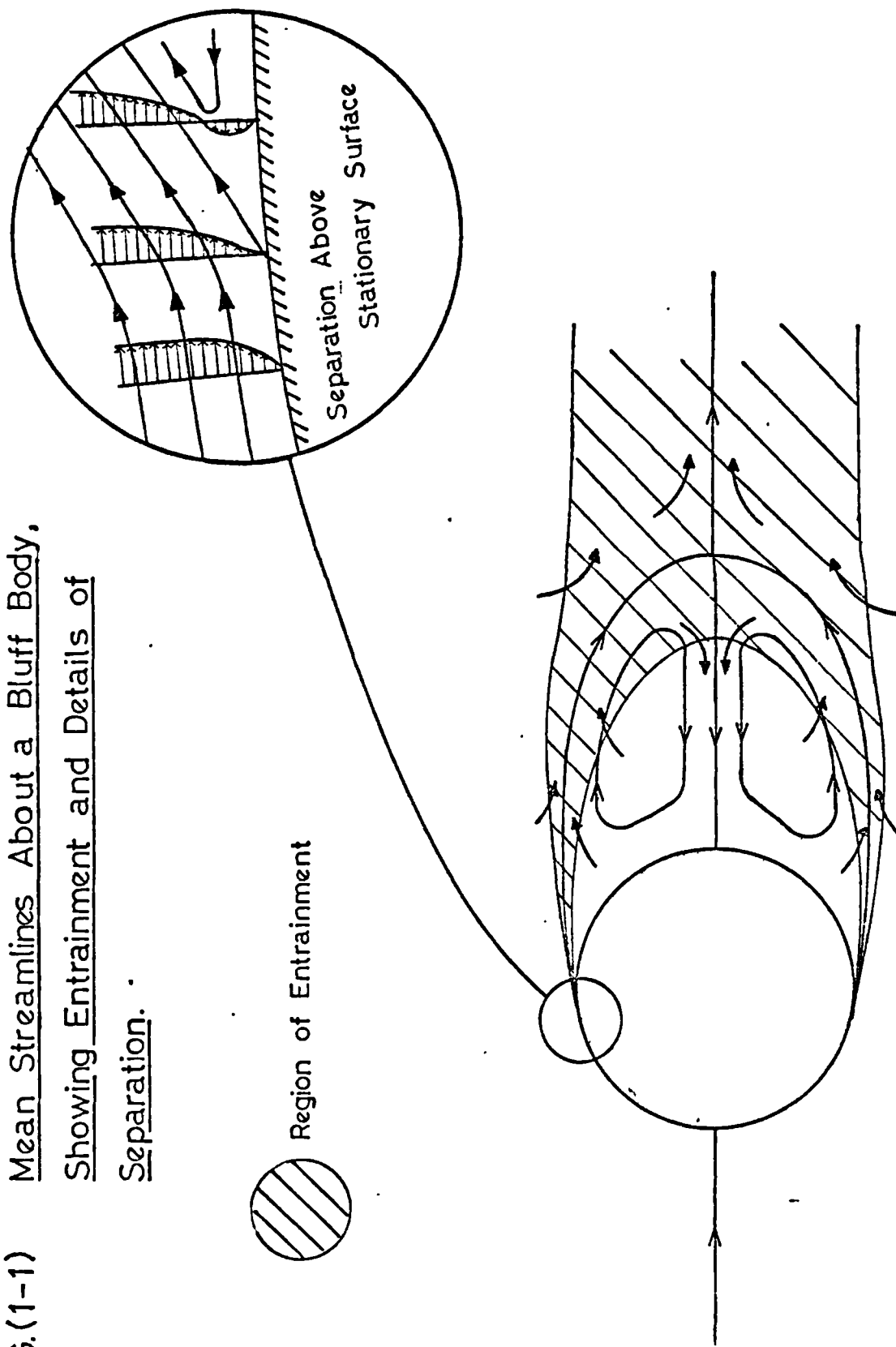
Table 4

Wheel No.	X-Sect Area sq. ft.	Tread x Dia. sq. ft.	C_{D_1}	C_{L_1}	C_{D_2}	C_{L_2}
C2	1.42	1.05	.70	.43	.94	.57
C2G	1.42	.97	.70	.38	1.02	.56

Table 5

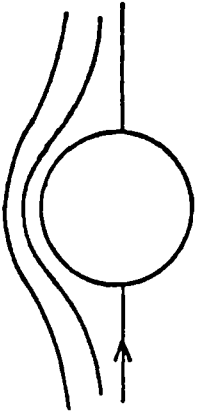
Wheel No.	A1	B1	C1	A2	B2	C2
S/C	.051	.068	.085	.048	.065	.081
Correction K^2/K_c	1.085	1.131	1.162	1.073	1.120	1.155

FIG.(1-1) Mean Streamlines About a Bluff Body,
Showing Entrainment and Details of
Separation.



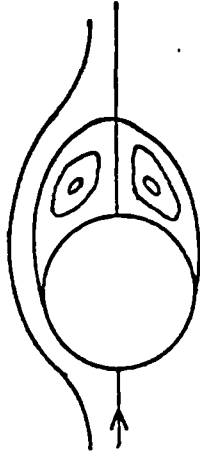
(After Bearman (1972))

(1) $Re < 1$



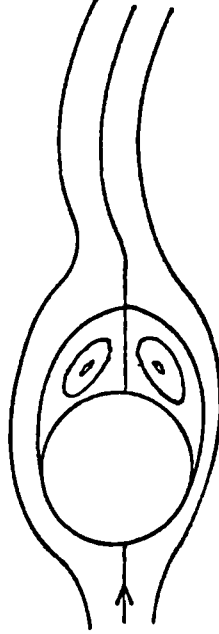
Large lateral vorticity penetration.

(2) $3-5 < Re < 30-40$



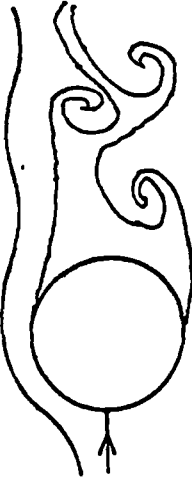
Laminar, steady.

(3) $30-40 < Re < 80-90$



Laminar wake instability. Ultimate slow viscous decay.

(4) $80-90 < Re < 150-300$



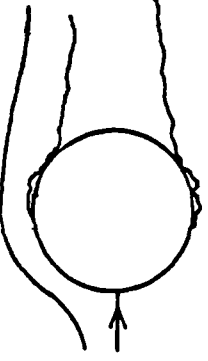
'Pure Karman Range'
Laminar instability of
double vortex sheet.

(5) $150 < Re < 10 \times 10^3$
300



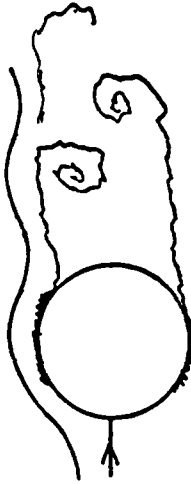
'Subcritical Range'
Forward progression of
turbulence and increasing
3 - dimensionality.
Double sheet instability
still dominant.

(6) $10^3 < Re < 35 \times 10^5$



'Critical and Post-Critical Range'
Rapid narrowing of wake corresponding
to local reattachment with laminar
bubbles and final turbulent reattachment.
Progressive loss of bubble and loss
of dominant periodicity.

(7) $35 \times 10^5 < Re < ?$



'Transcritical Range'
Transition to turbulent layer on front
face of cylinder. Probable instability
of turbulent double sheet. Recovery of
pronounced periodicity corresponding
to medium wake width.

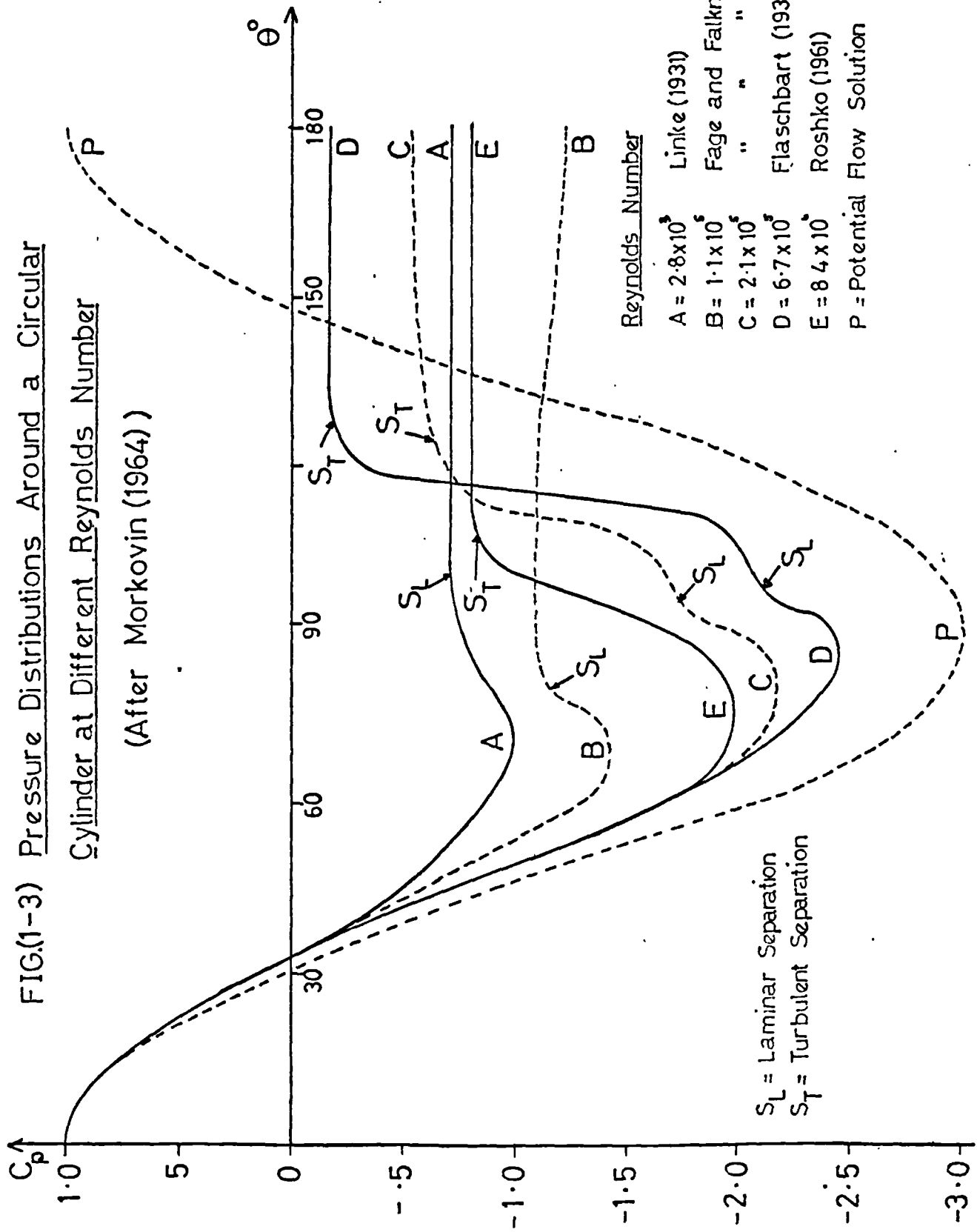
FIG.(1-2)

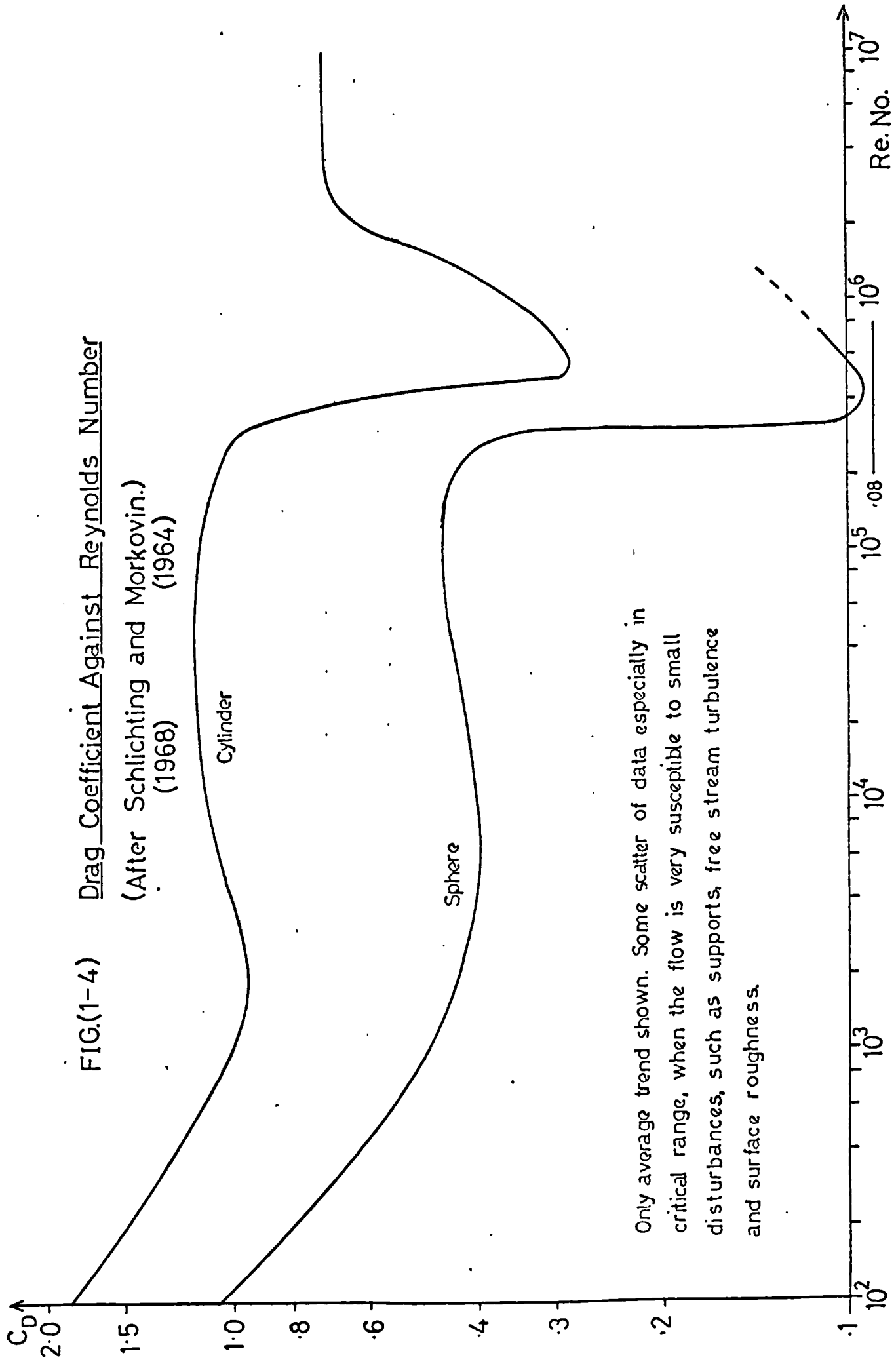
Circular Cylinder Flow Régimes

(After Morkovin (1964))

FIG.(1-3) Pressure Distributions Around a Circular
Cylinder at Different Reynolds Number

(After Morkovin (1964))





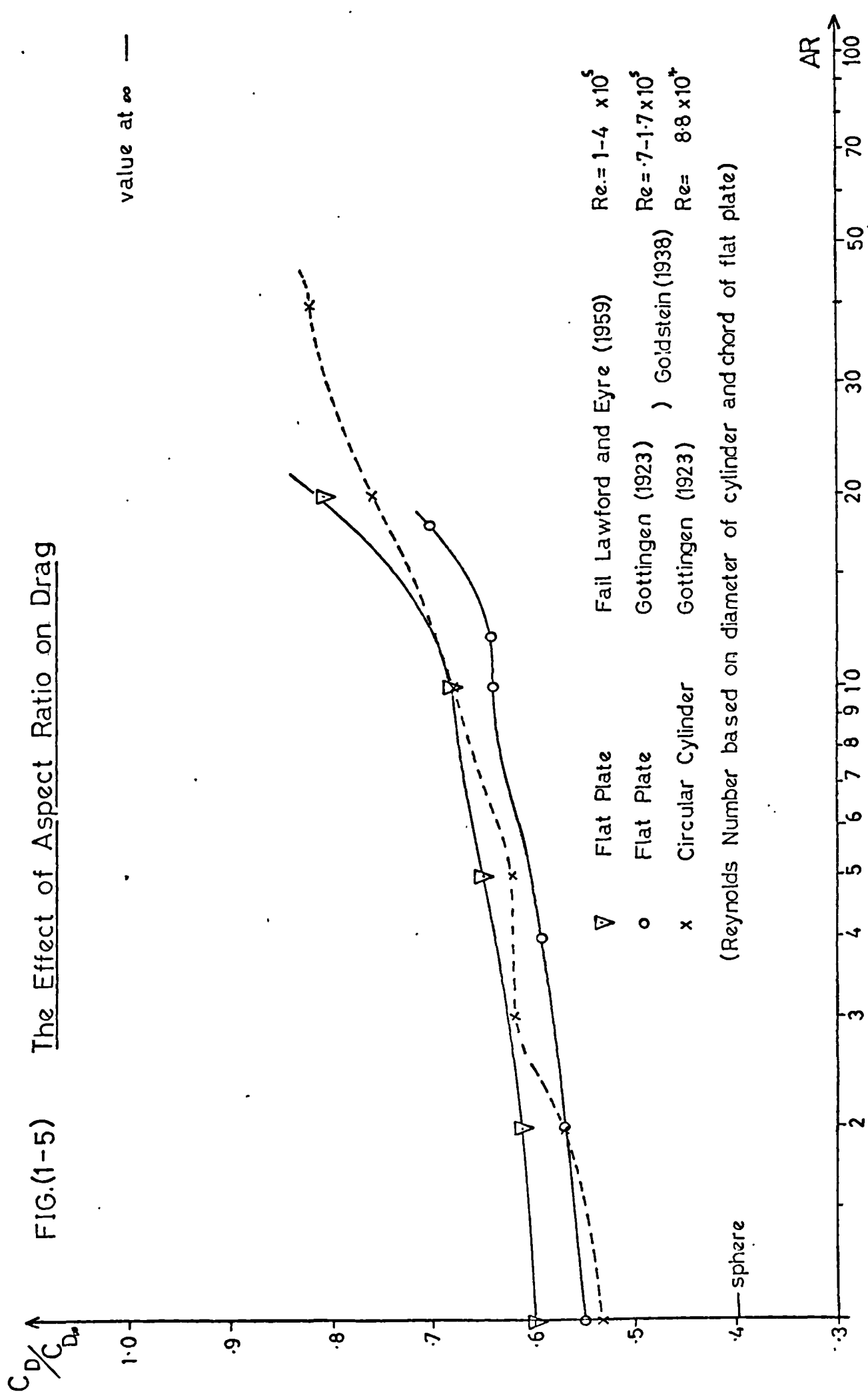
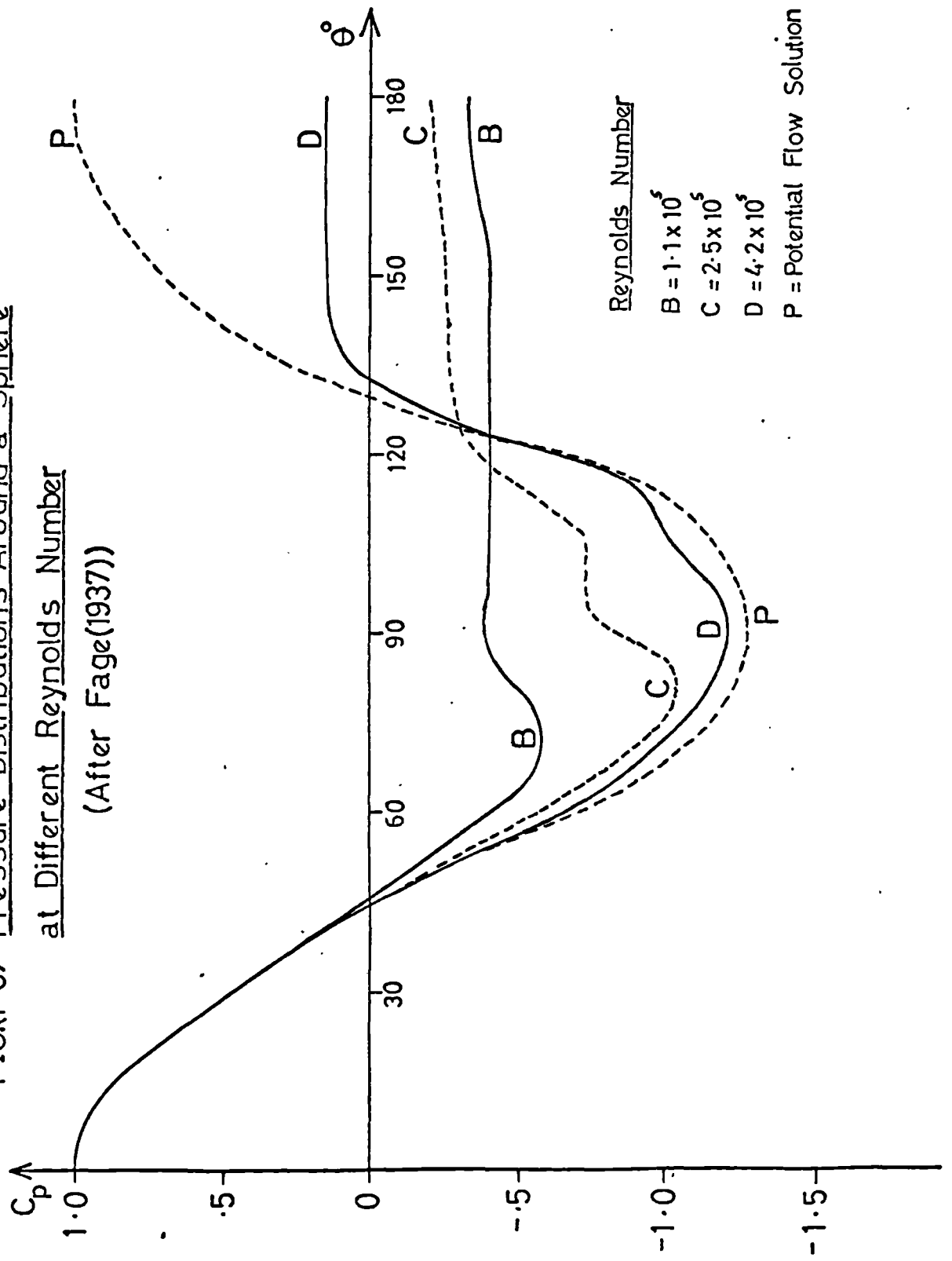


FIG.(1-6) Pressure Distributions Around a Sphere
at Different Reynolds Number
 (After Fage(1937))



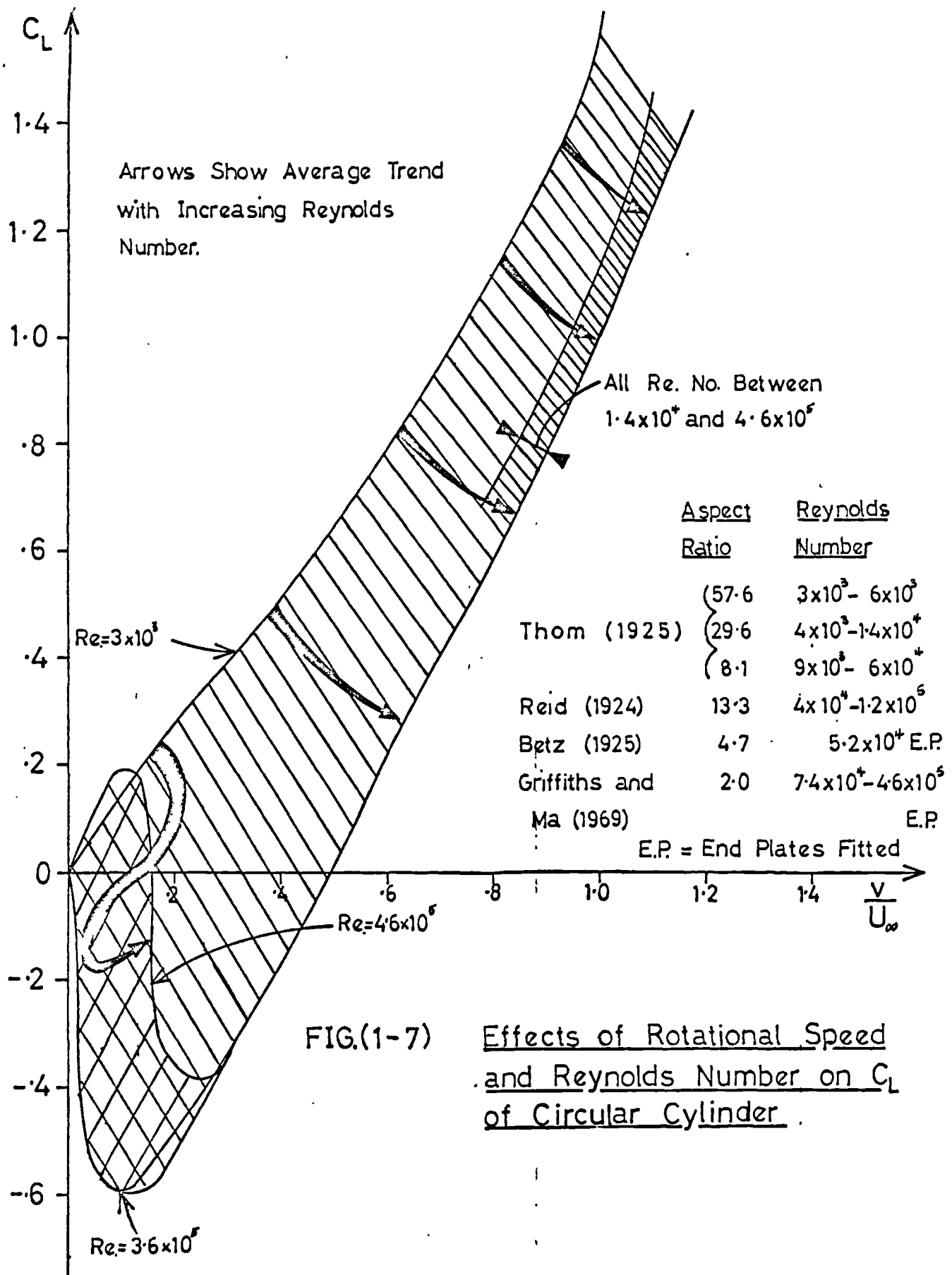
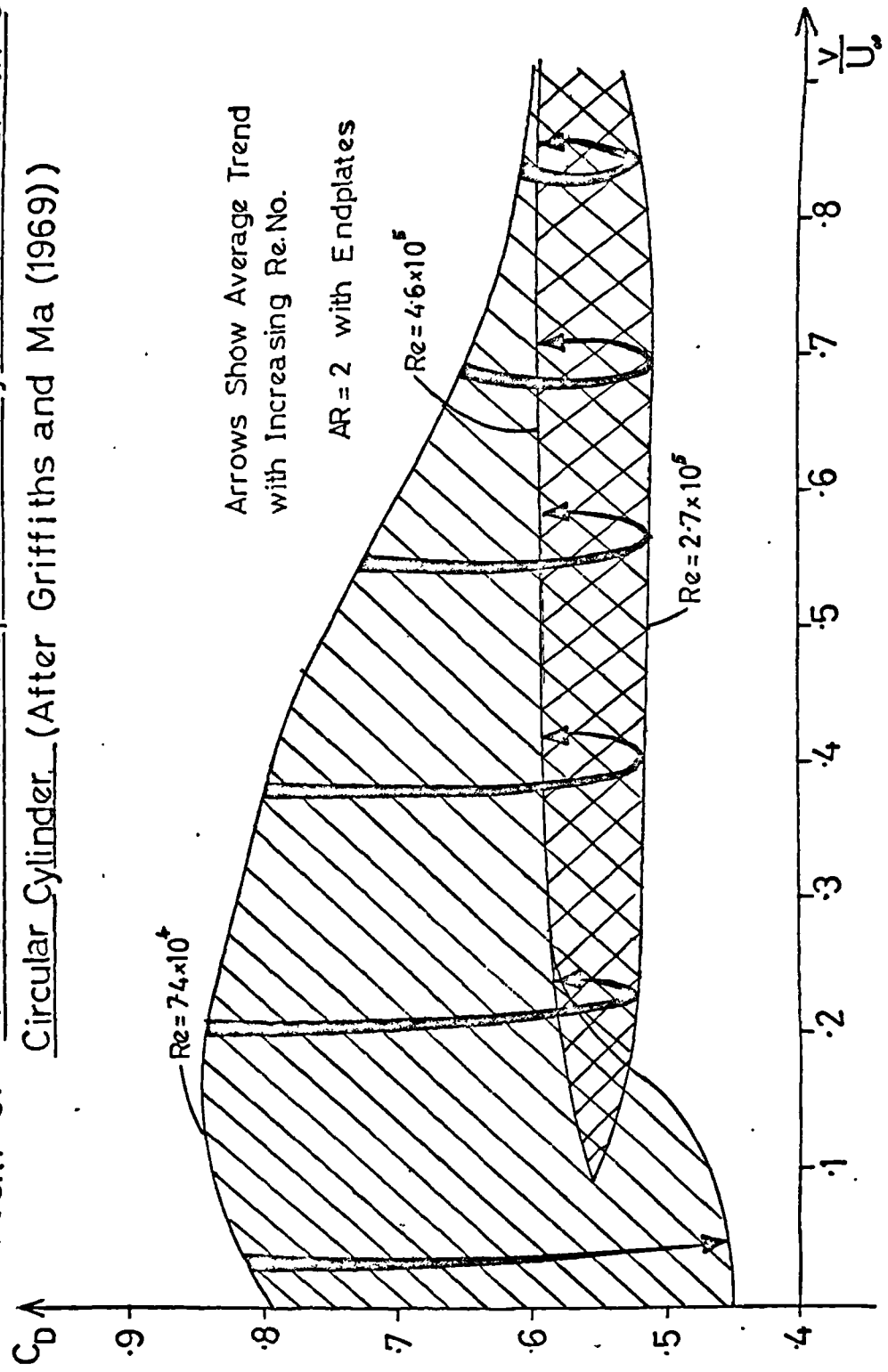


FIG.(1-7) Effects of Rotational Speed and Reynolds Number on C_L of Circular Cylinder.

FIG.(1-8) Effect of Rotational Speed and Reynolds Number on C_D of Circular Cylinder (After Griffiths and Ma (1969))



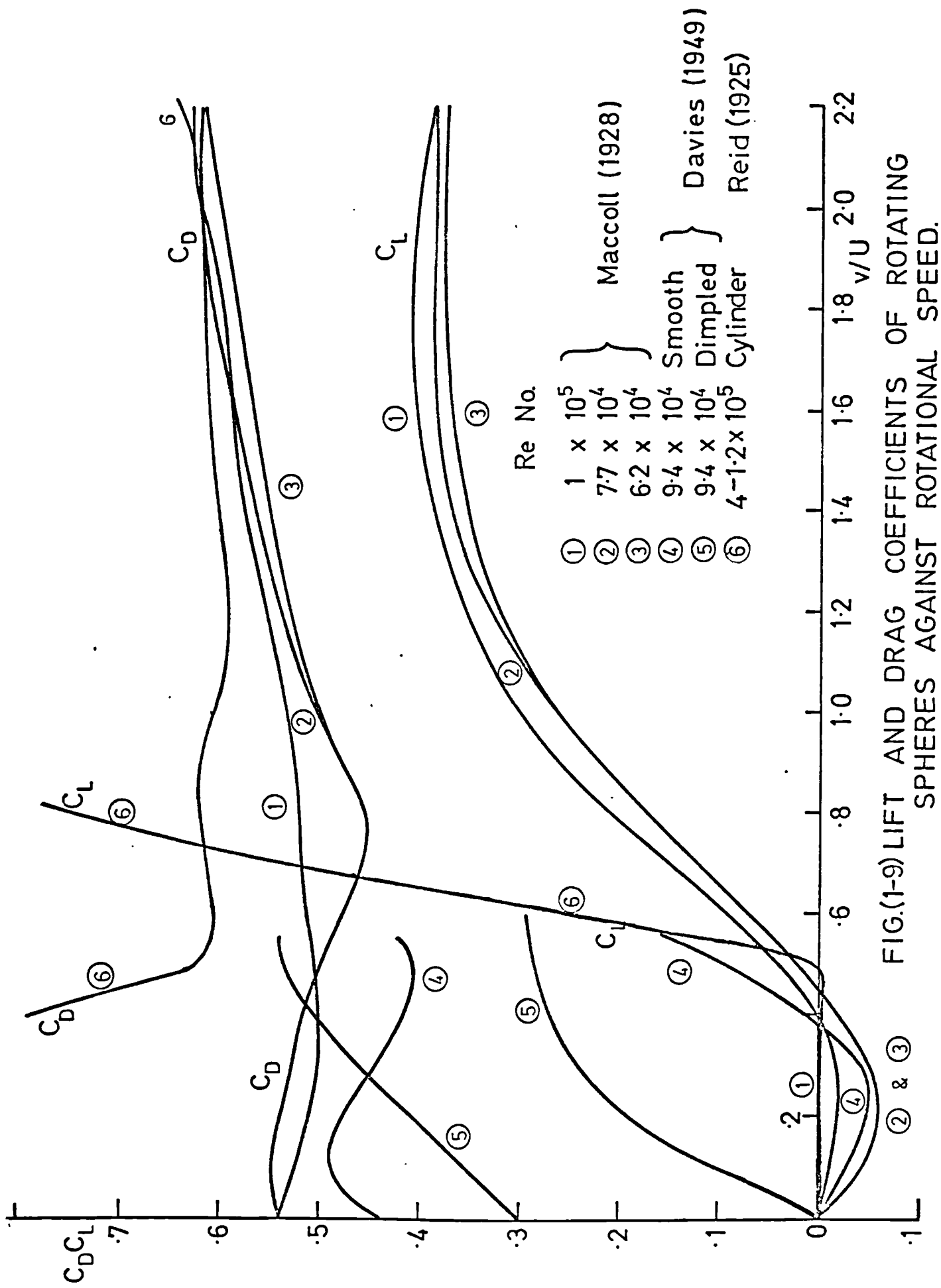


FIG.(1-9) LIFT AND DRAG COEFFICIENTS OF ROTATING SPHERES AGAINST ROTATIONAL SPEED.

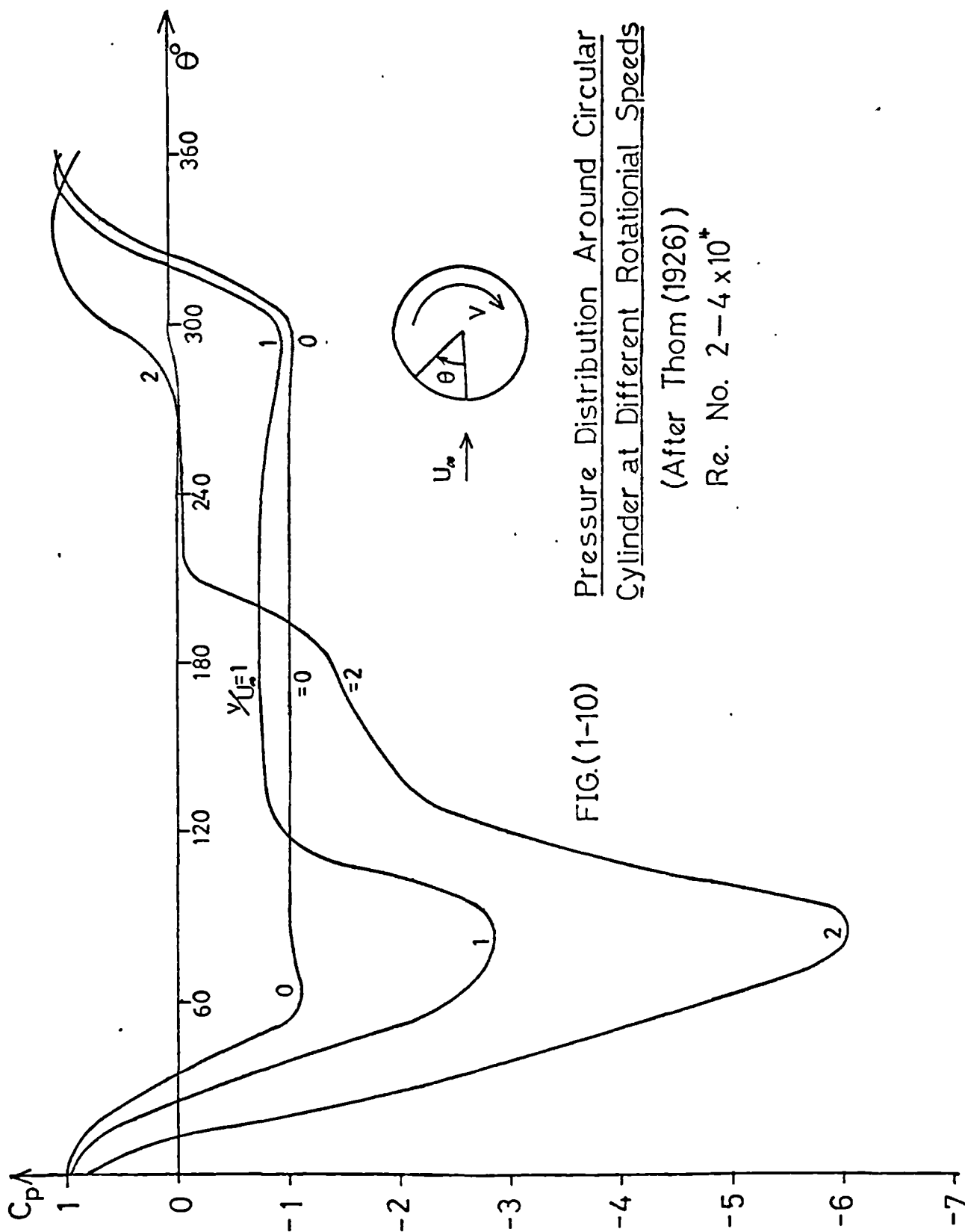


FIG.(1-10)

Pressure Distribution Around Circular
Cylinder at Different Rotational Speeds

(After Thom (1926))

Re. No. $2-4 \times 10^4$

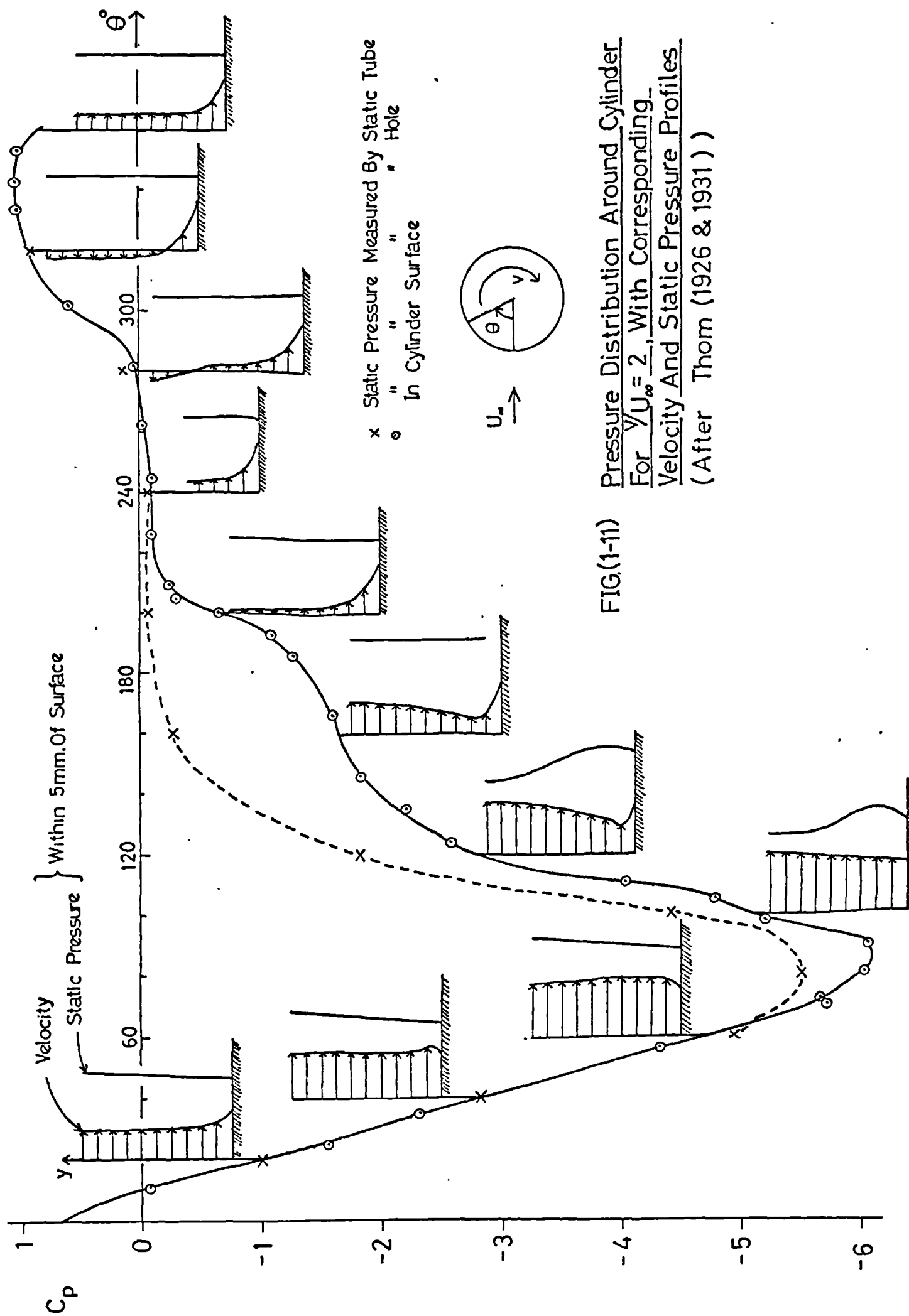


FIG.(1-11)

Pressure Distribution Around Cylinder
 For $V/U_\infty = 2$, With Corresponding
 Velocity And Static Pressure Profiles
 (After Thom (1926 & 1931))

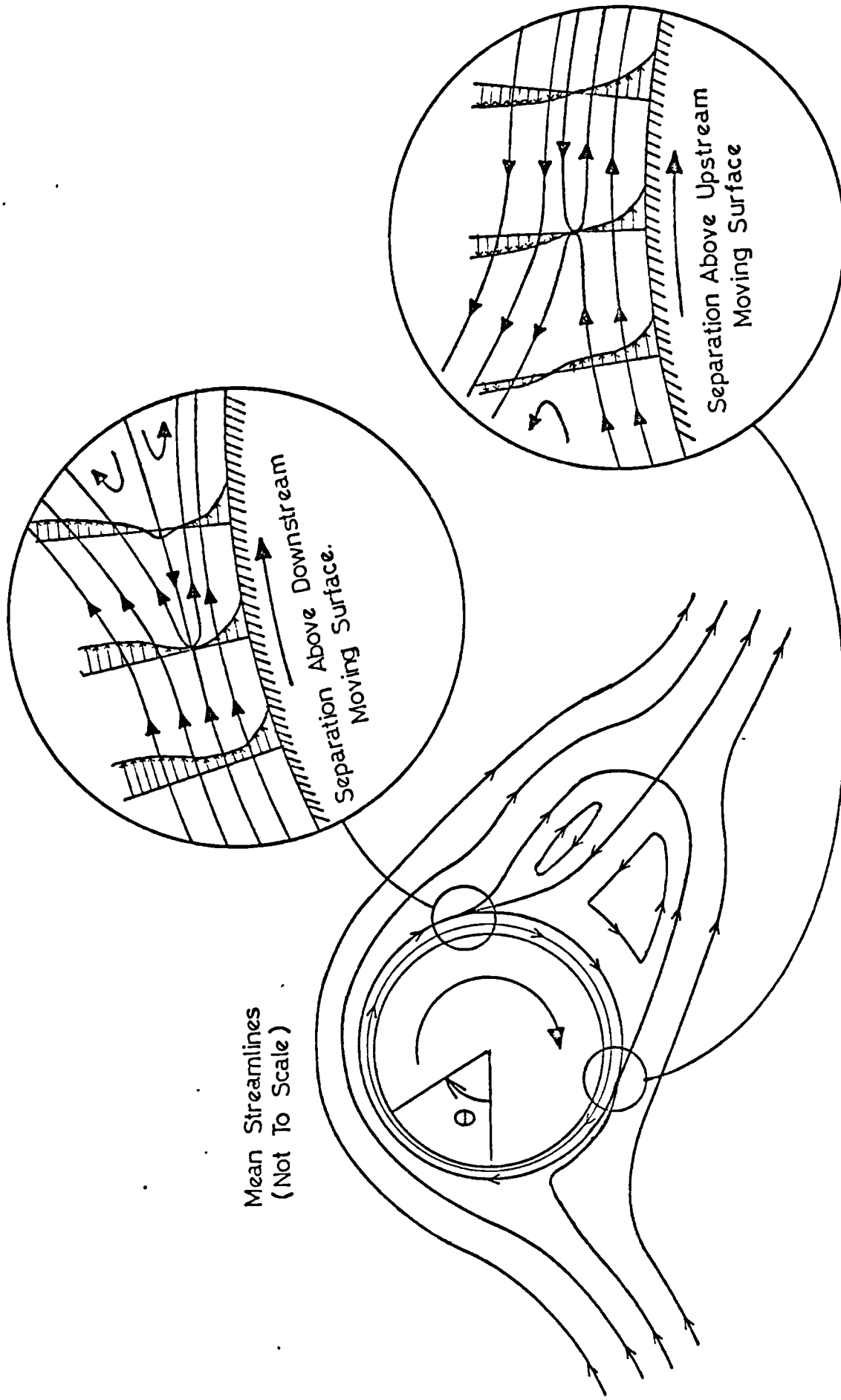


FIG.(1-12) Proposed Mean Streamlines for Flow Over Cylinder Rotating at $\frac{V}{U_\infty} = 2$, with Details of Separation

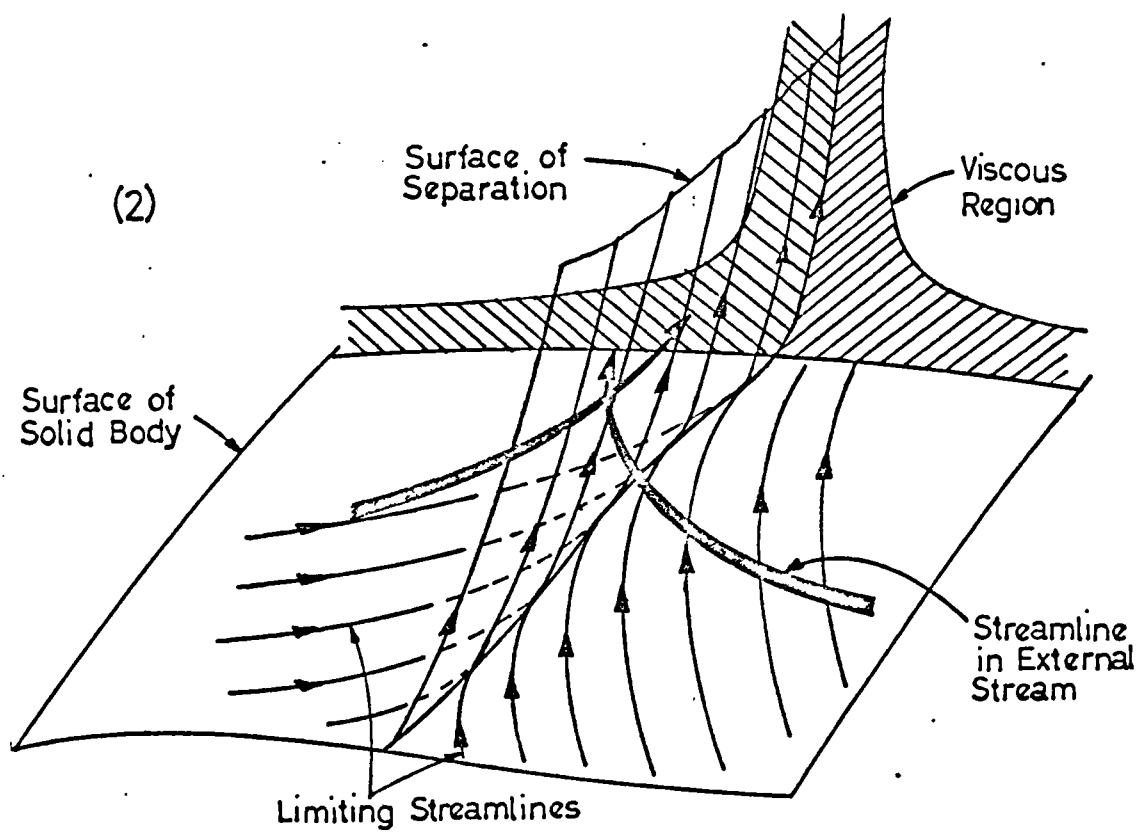
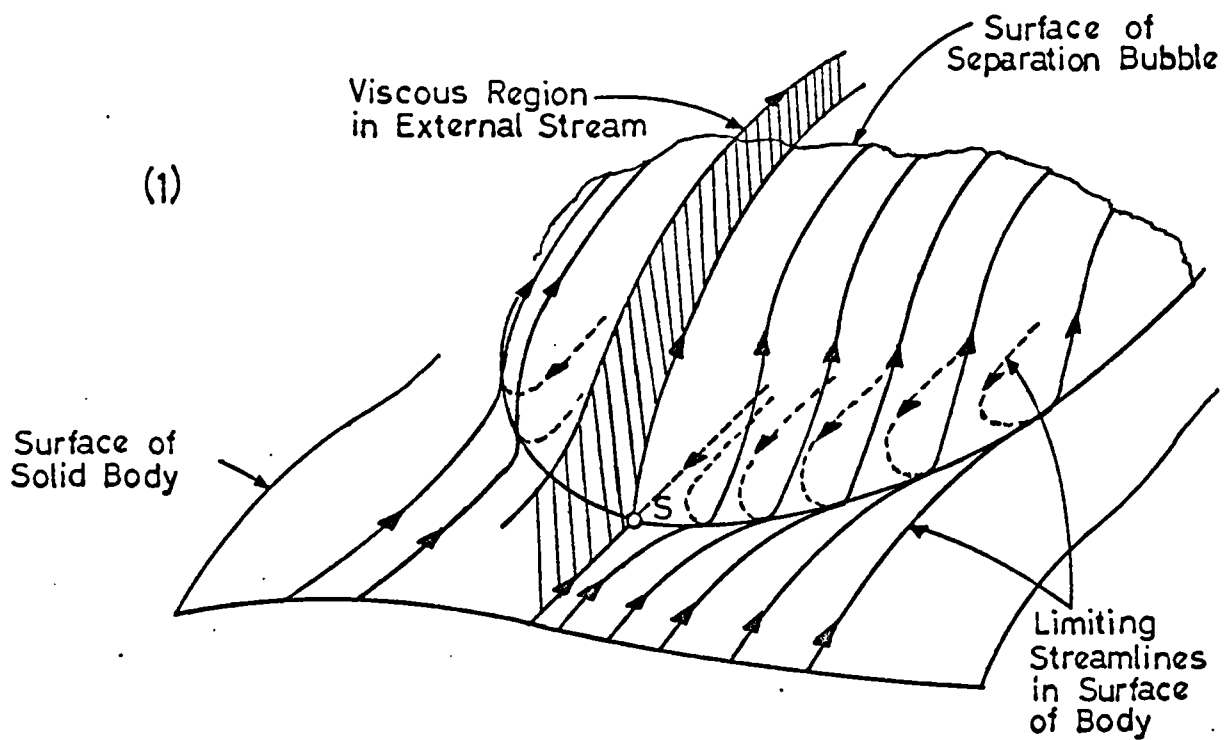
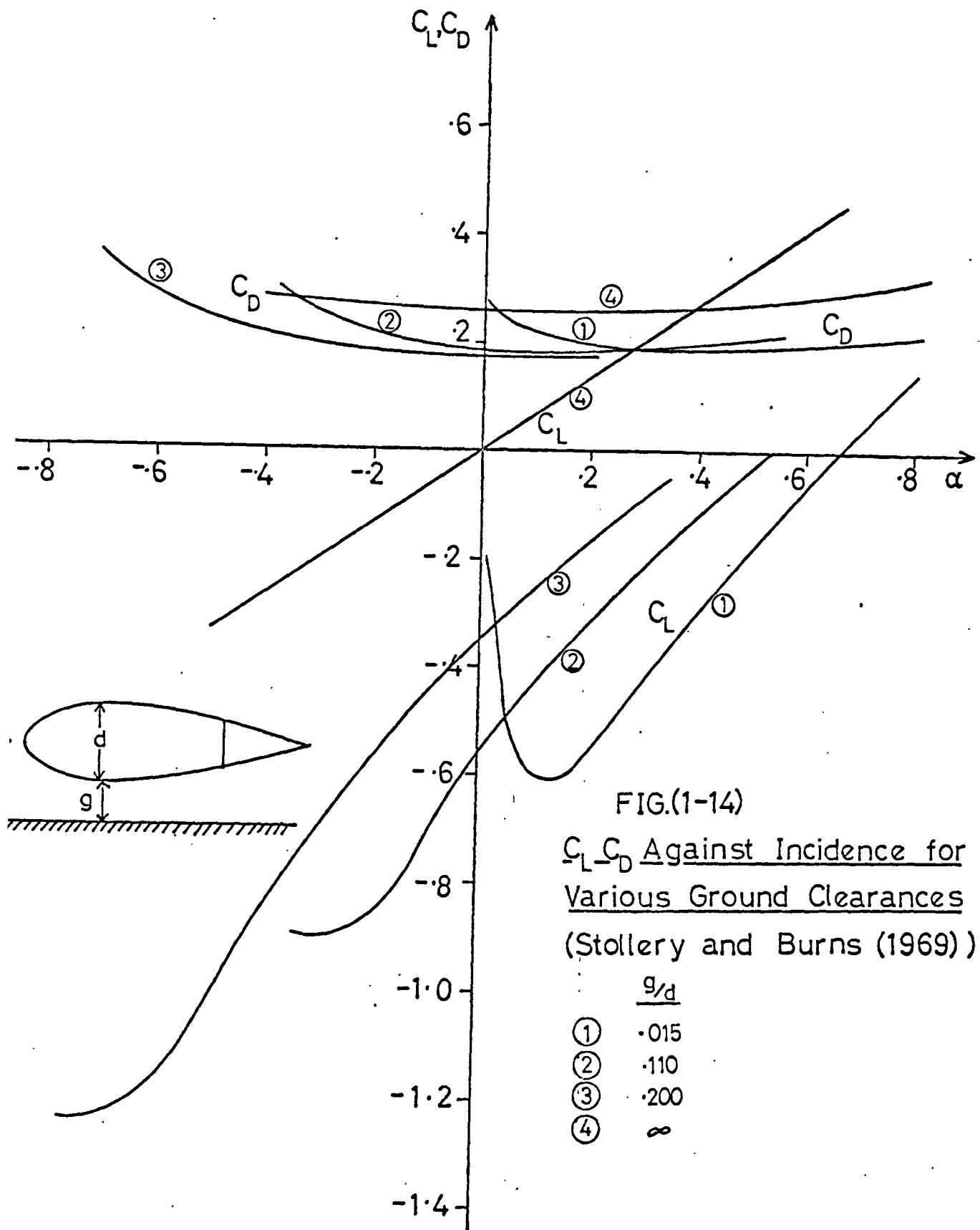
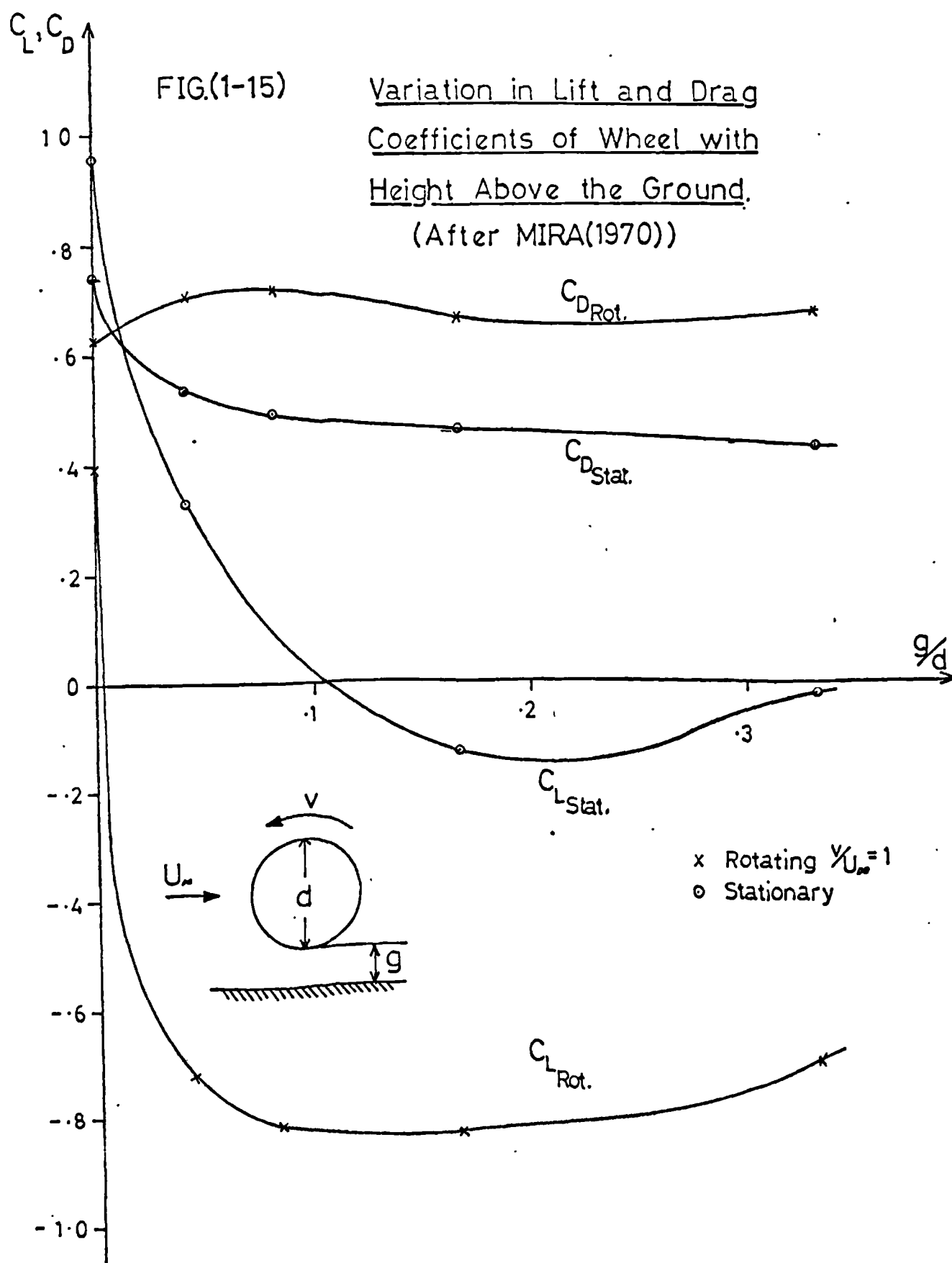
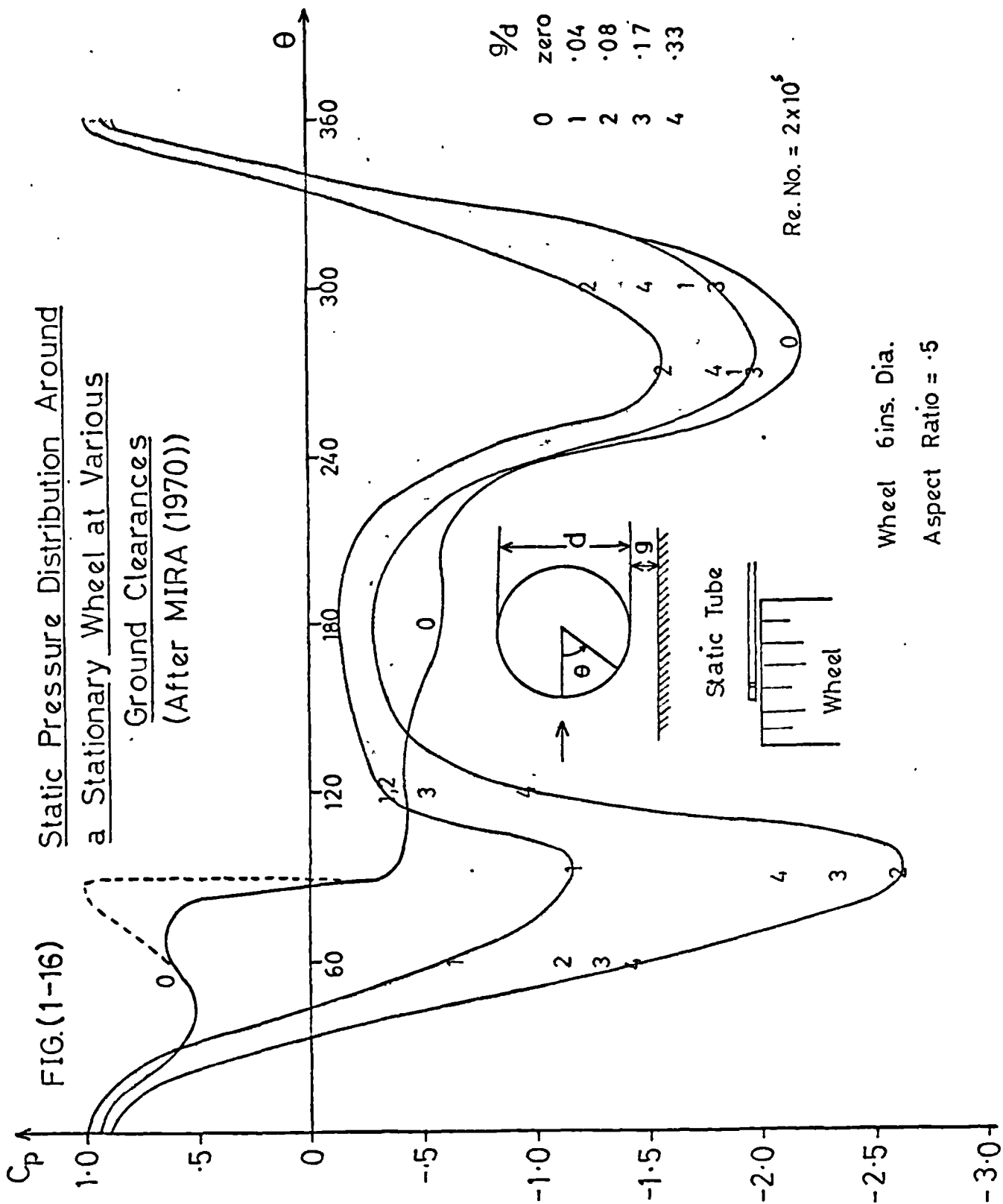
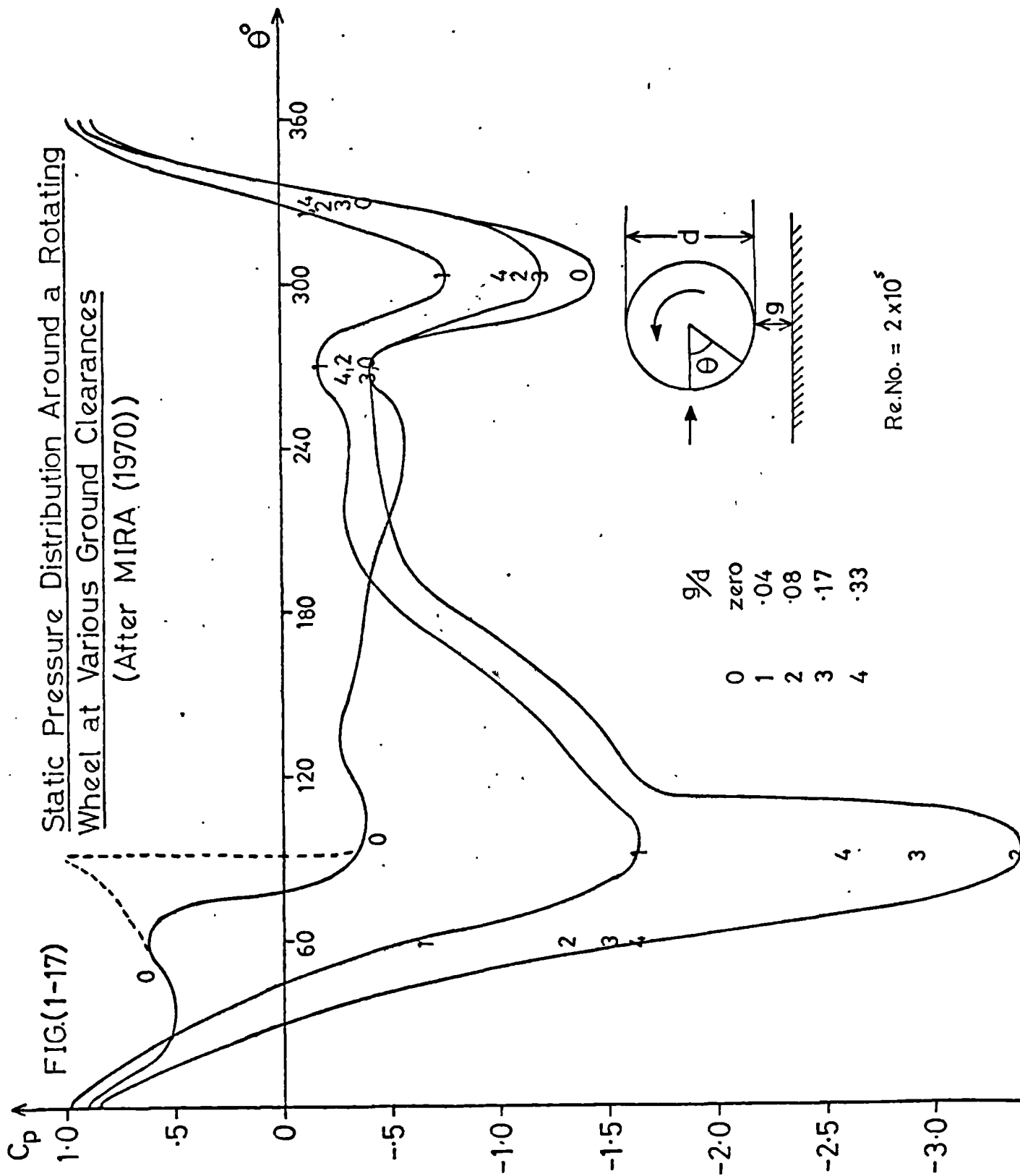


FIG.(1-13) Forms of Separation in Three Dimensional Flow

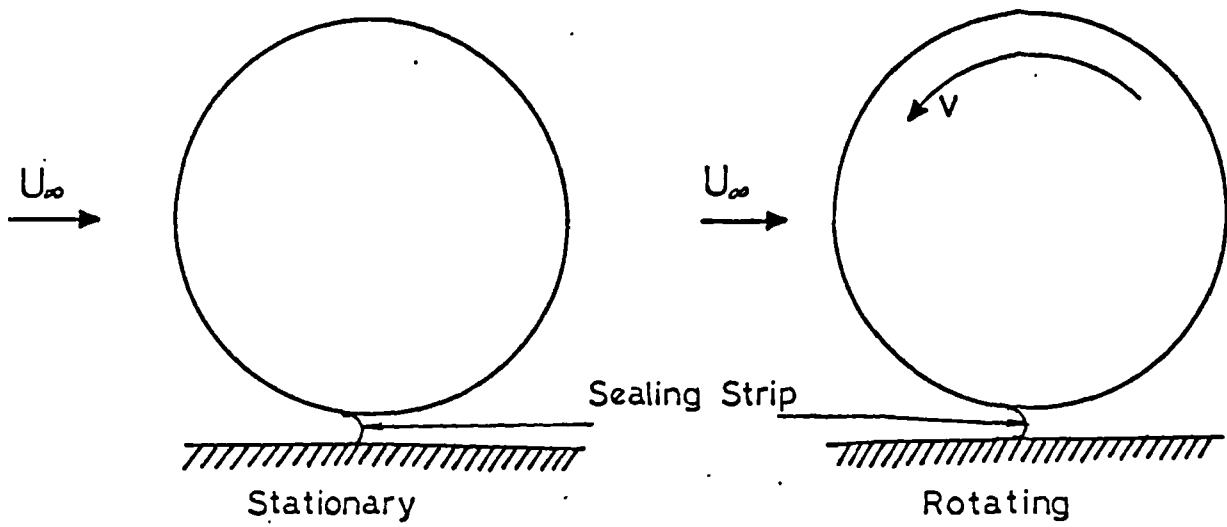








M.I.R.A.



Morelli

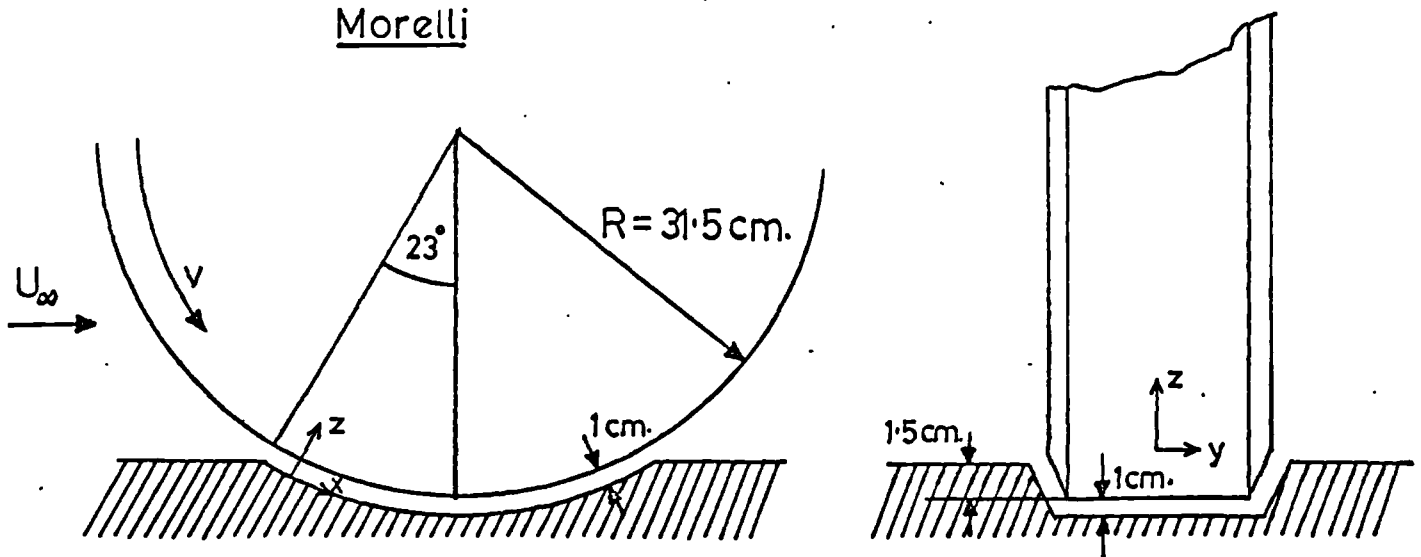
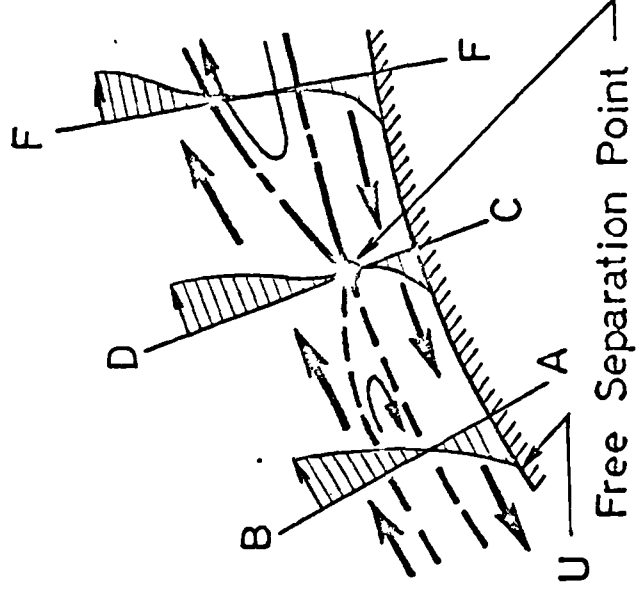
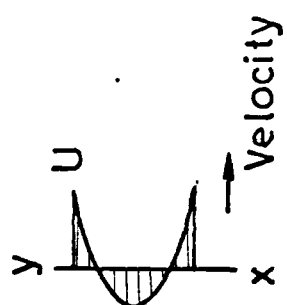
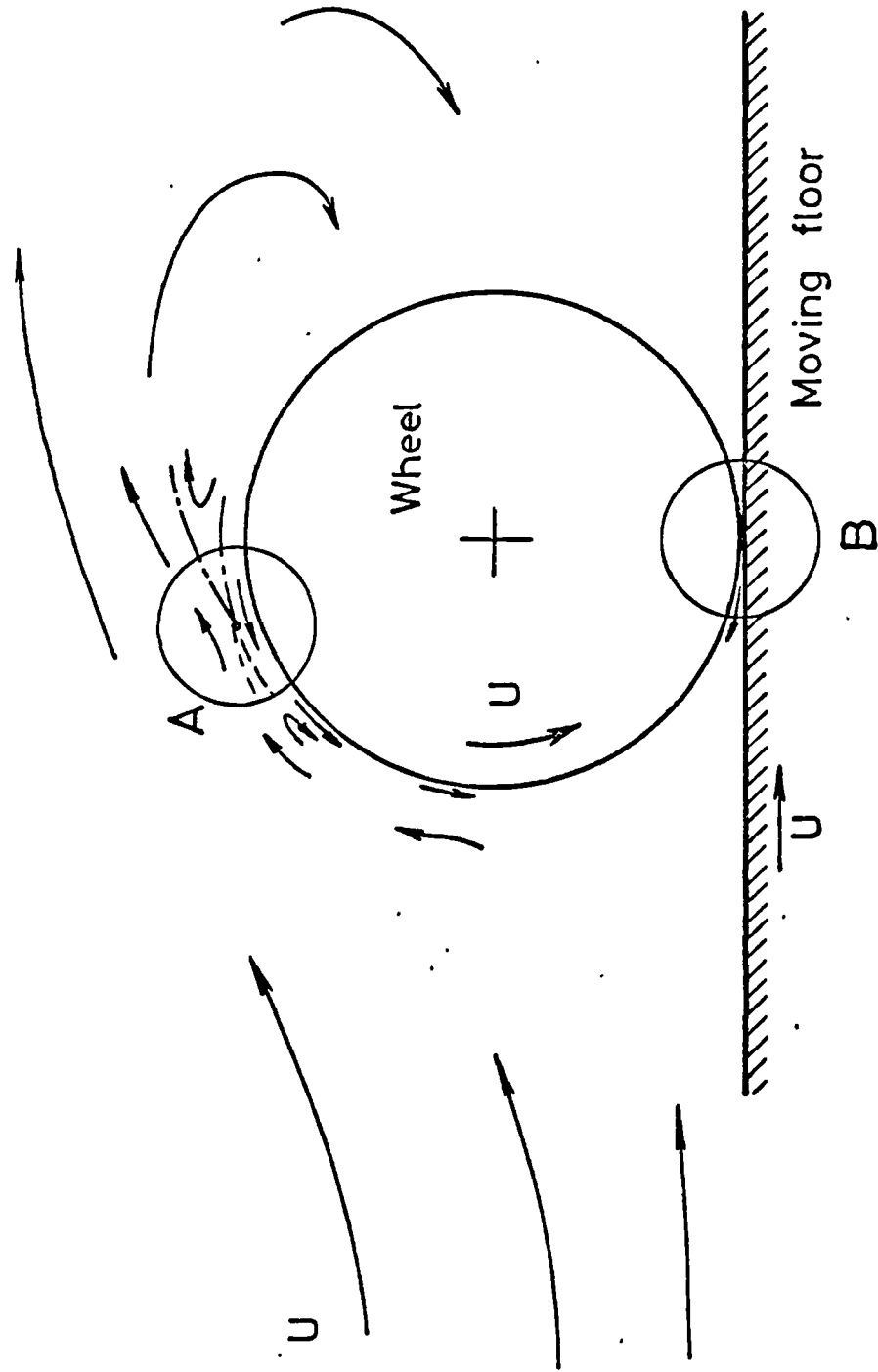
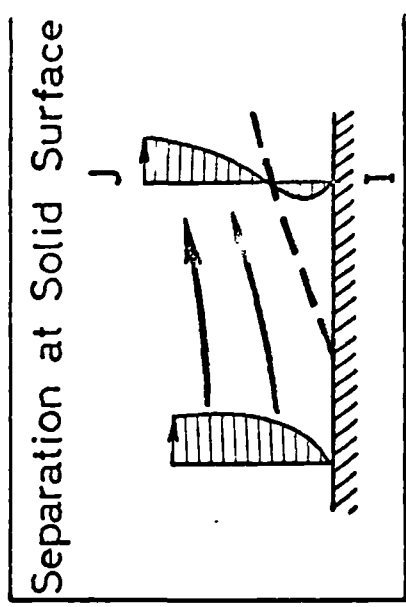


FIG.(1-18) Experimental Configurations



Region A



Region B

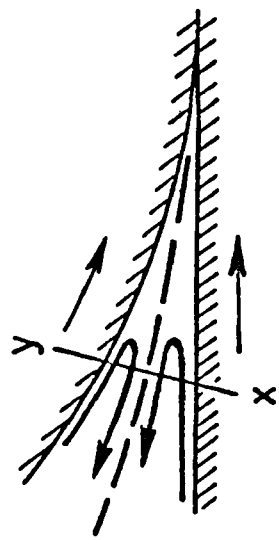
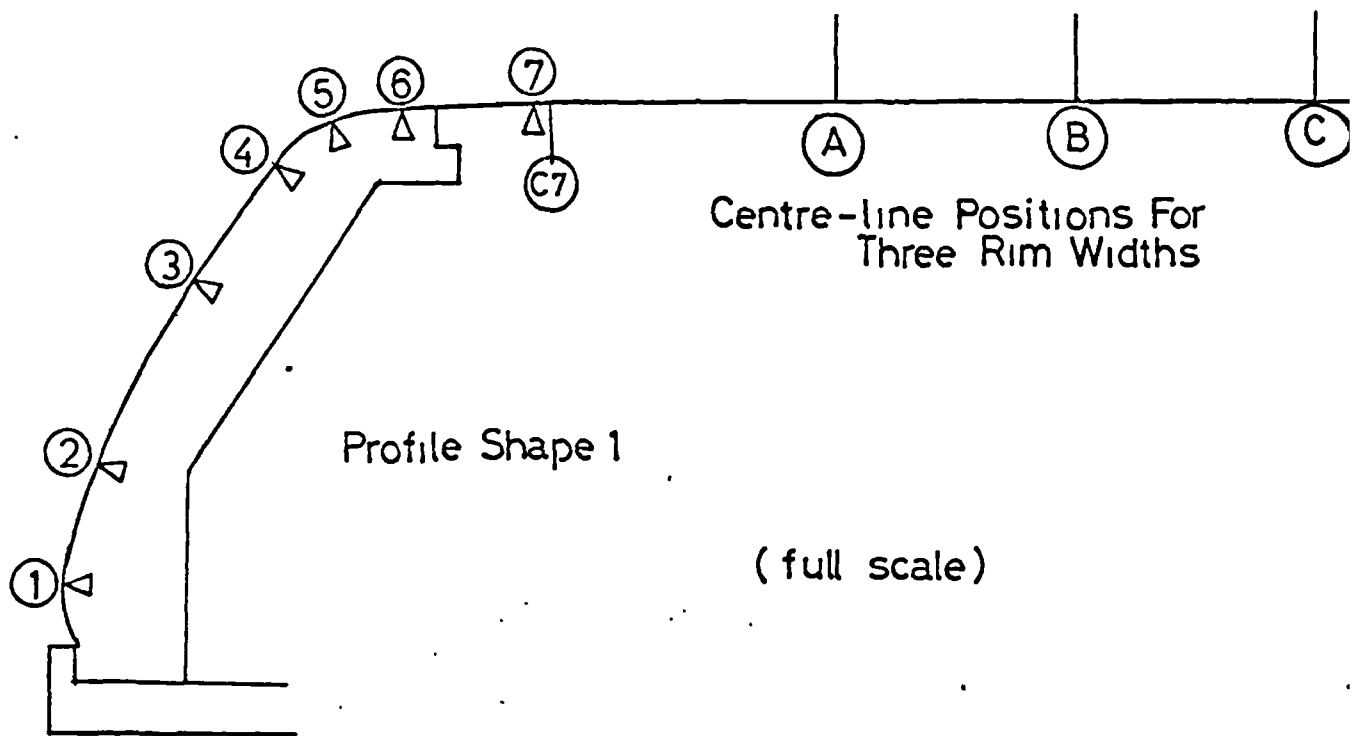


FIG.(1-19) DETAILS OF THE SEPARATION POINT AND JETTING FLOW.



(Pressure Holes Marked Δ Are Common To All Rim Widths,
Hole Positions Near Centre-line Vary From Rim To Rim)

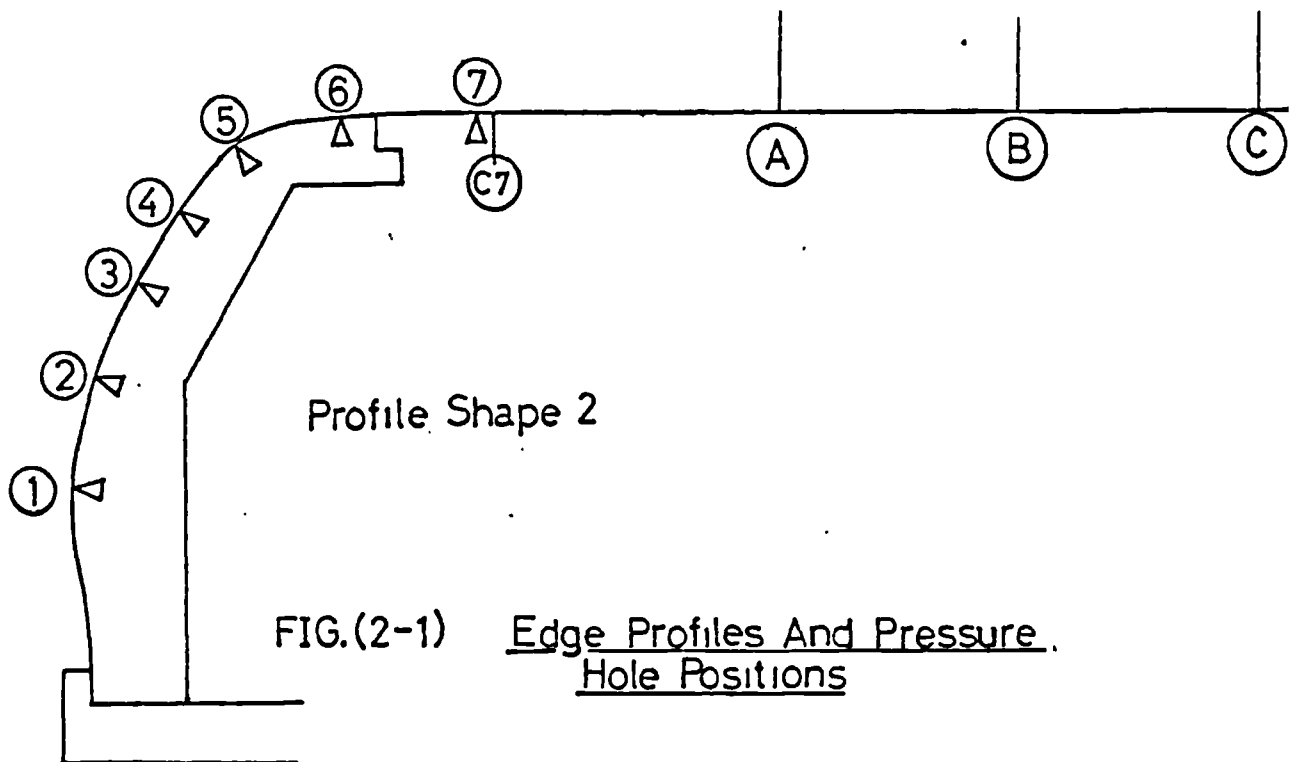


FIG.(2-1) Edge Profiles And Pressure Hole Positions

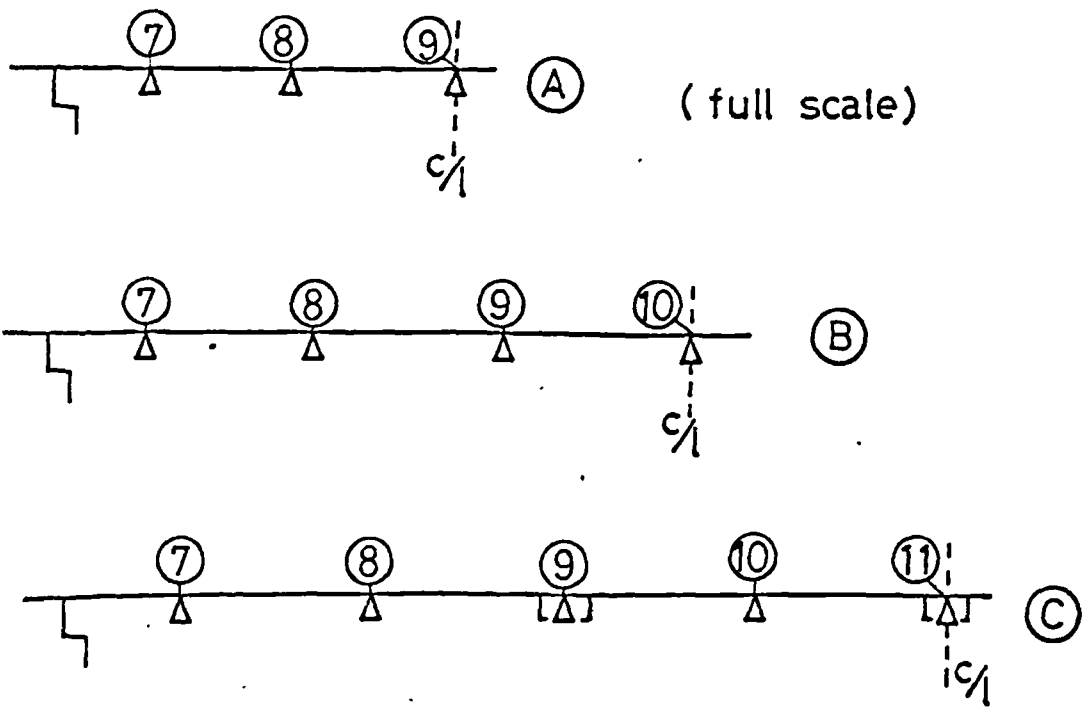
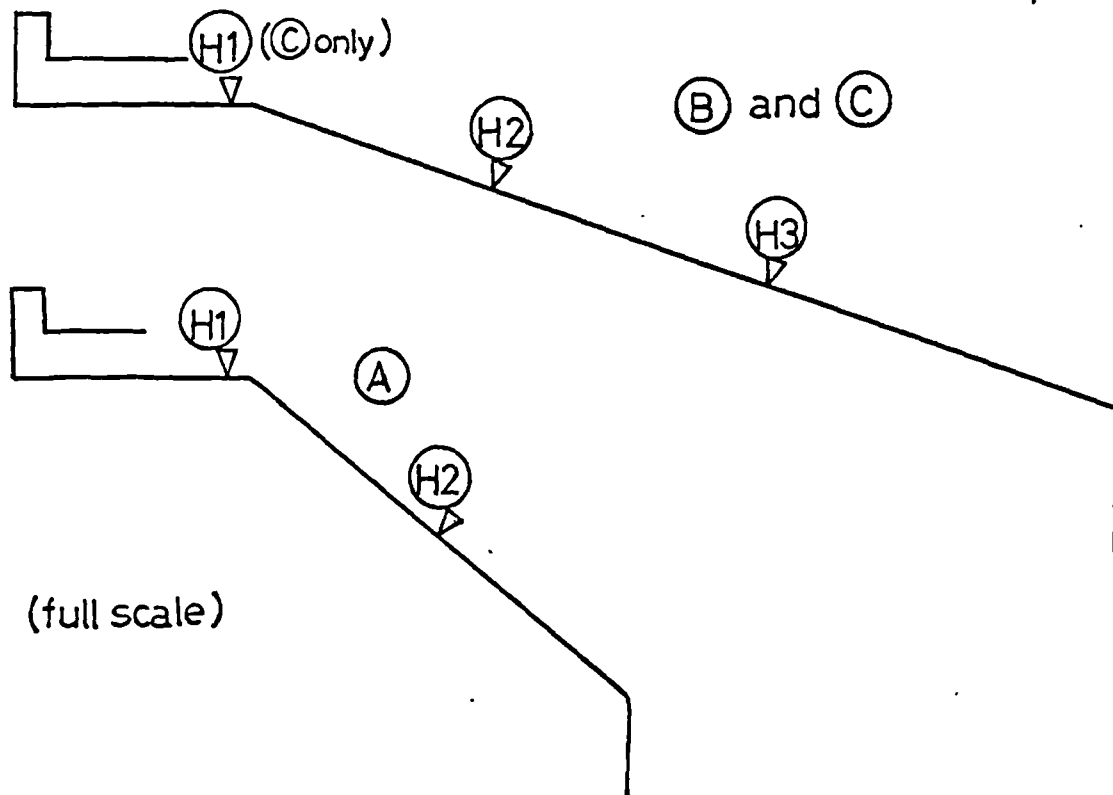
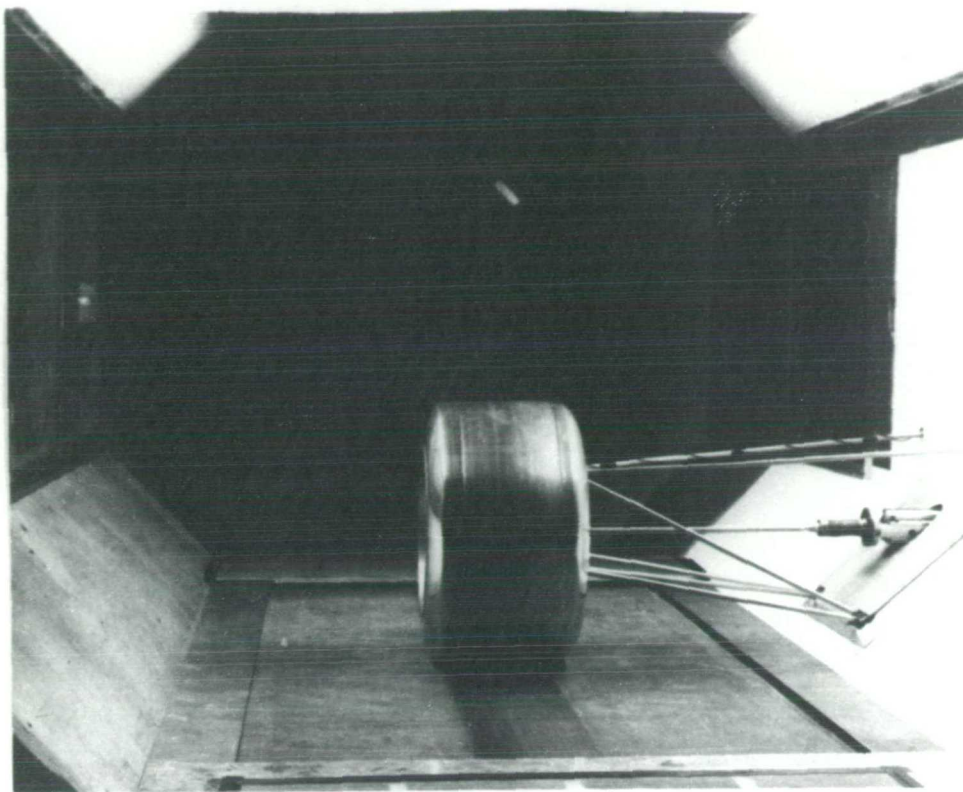
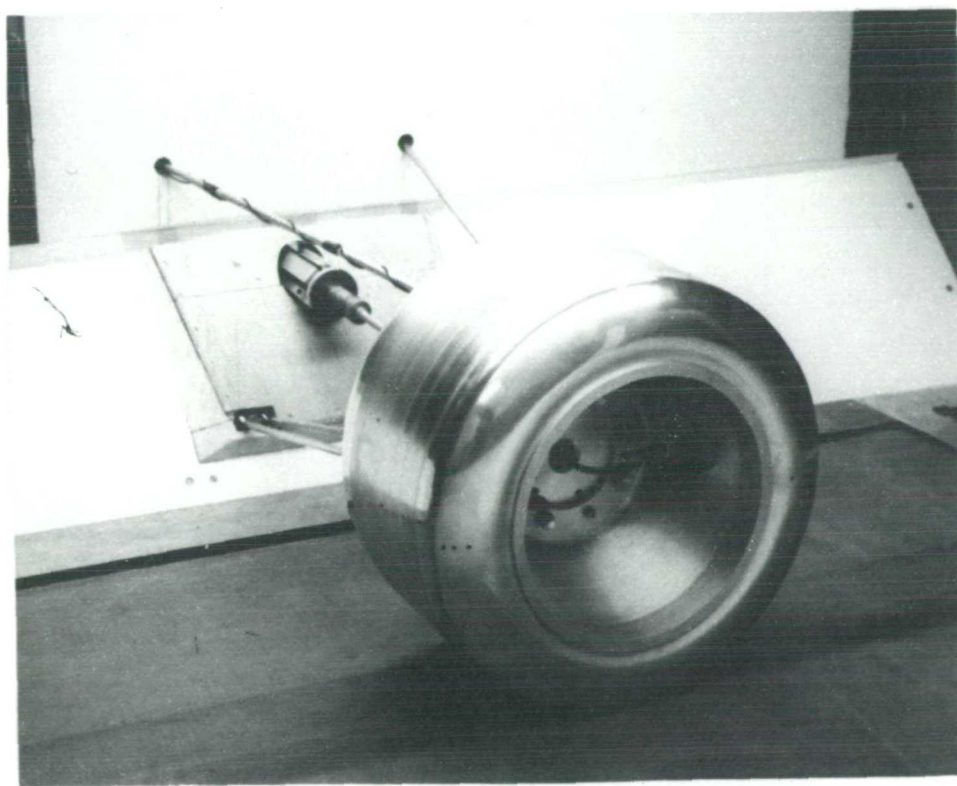


FIG.(2-2) Pressure Hole Positions, Near Centre-line of Rim and Inside Hub





(i) View from Upstream



(ii) Side View

Fig.(2-3) Wheel in Wind Tunnel

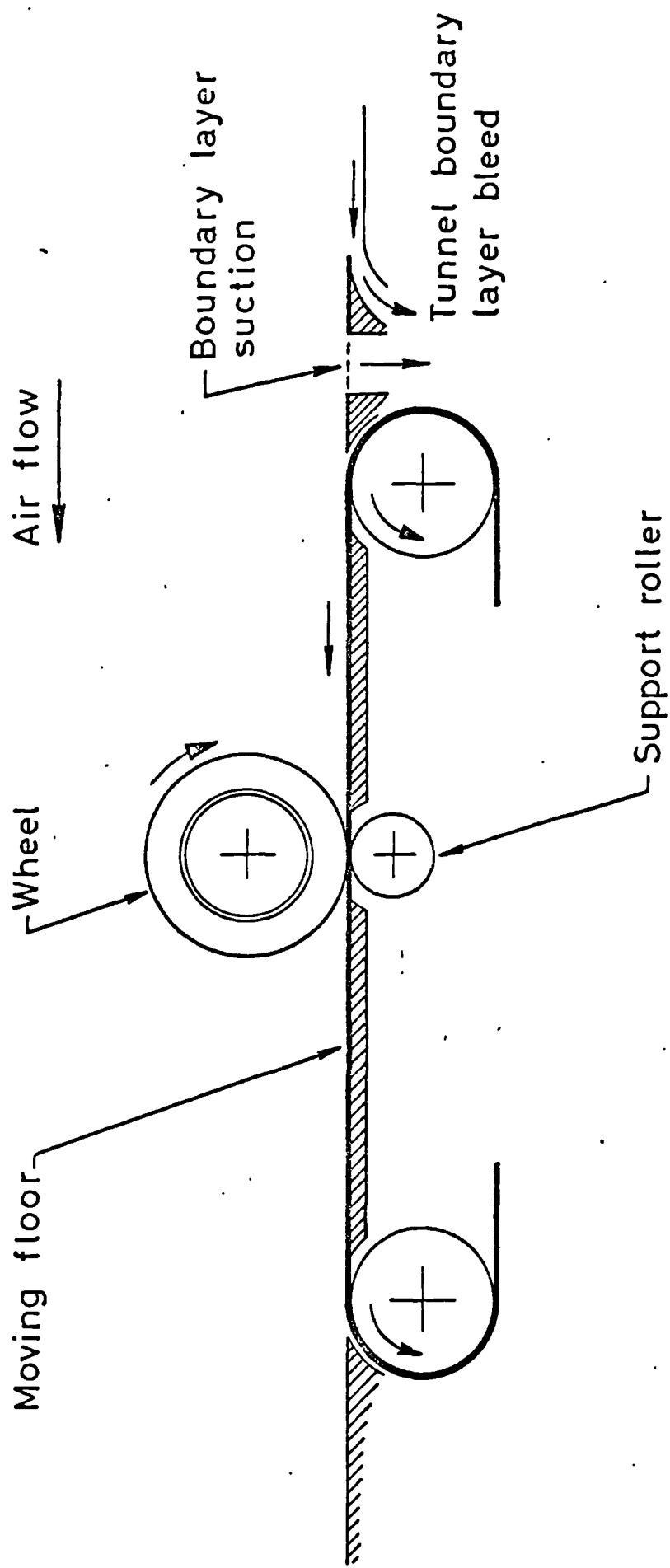
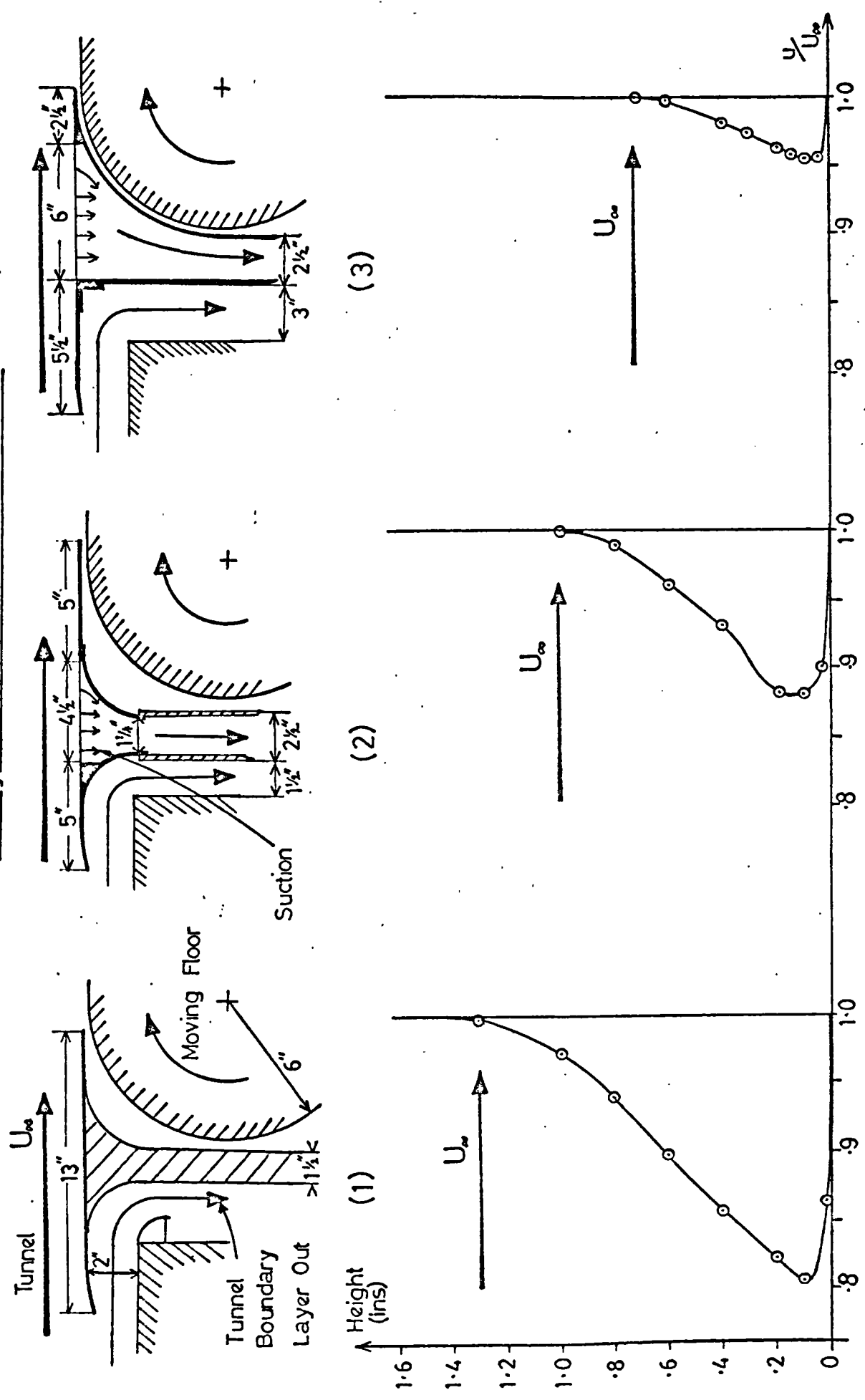


Fig.(2-4) Wheel and Moving Floor

FIG.(2-5) Developement of Fairings to Remove Boundary Layer with Corresponding
Velocity Profiles at Wheel Position



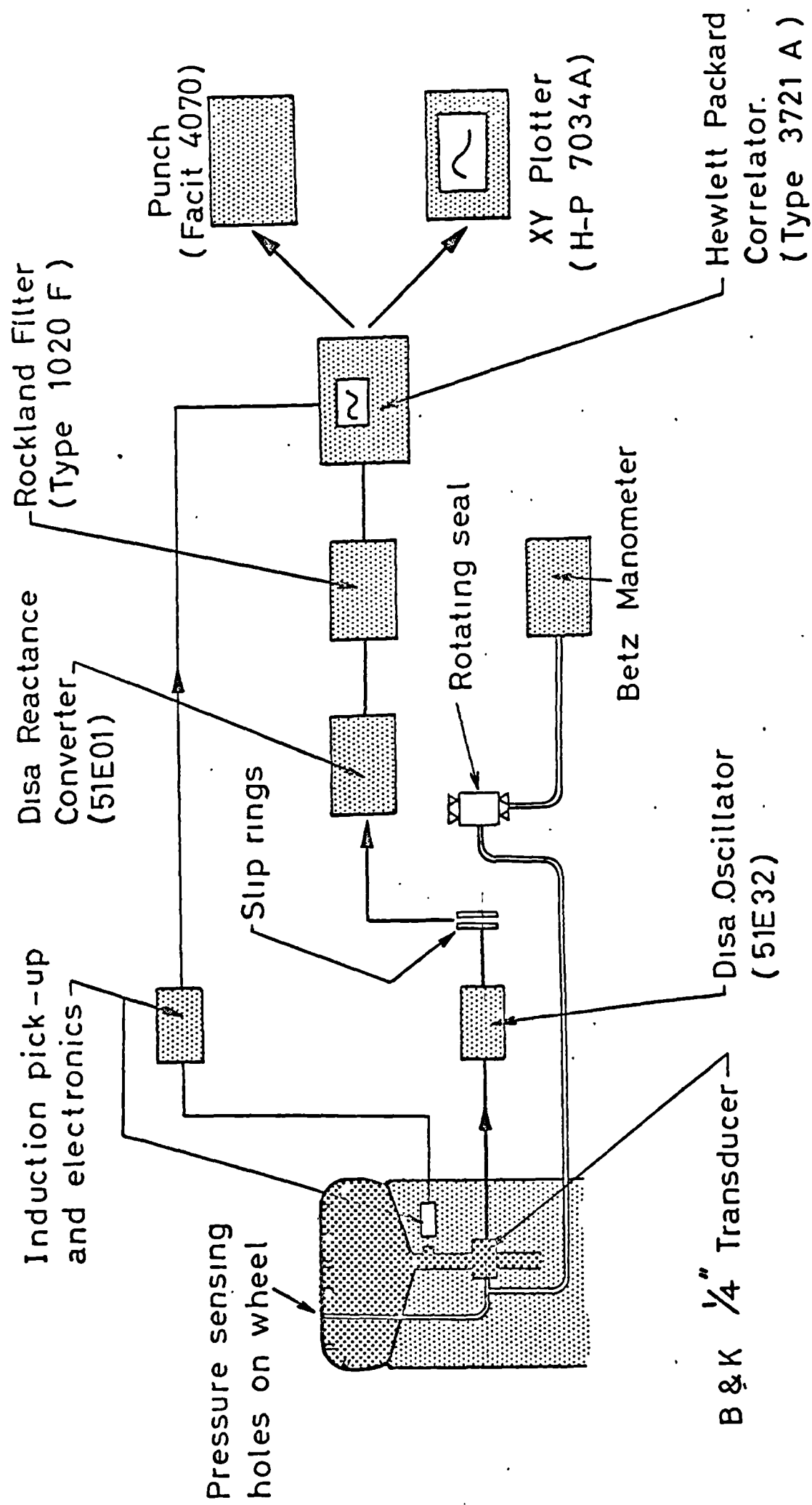


Fig.(2-6) Schematic of Instrumentation System

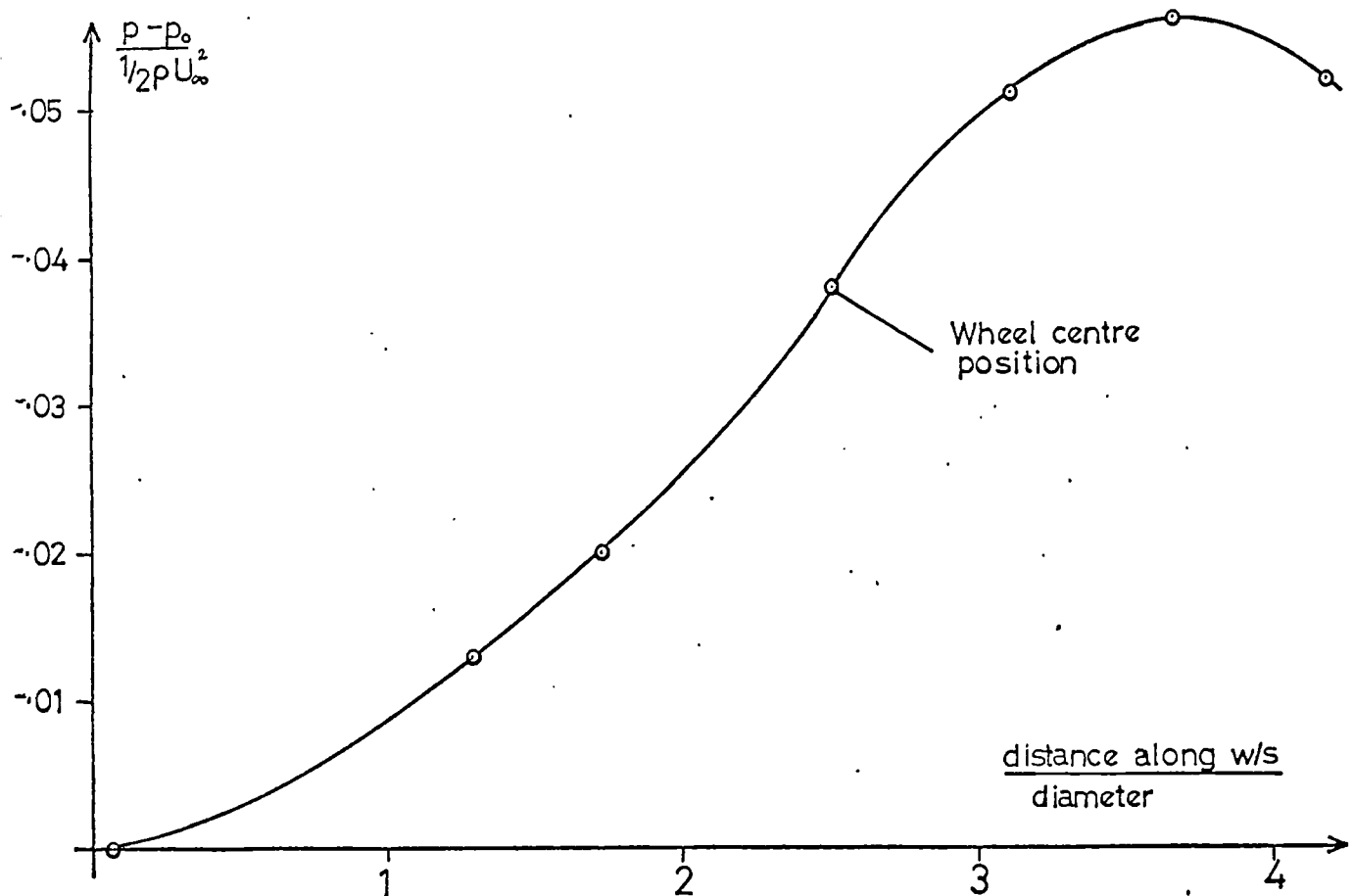


FIG.(2-7) Static Pressure in Working Section

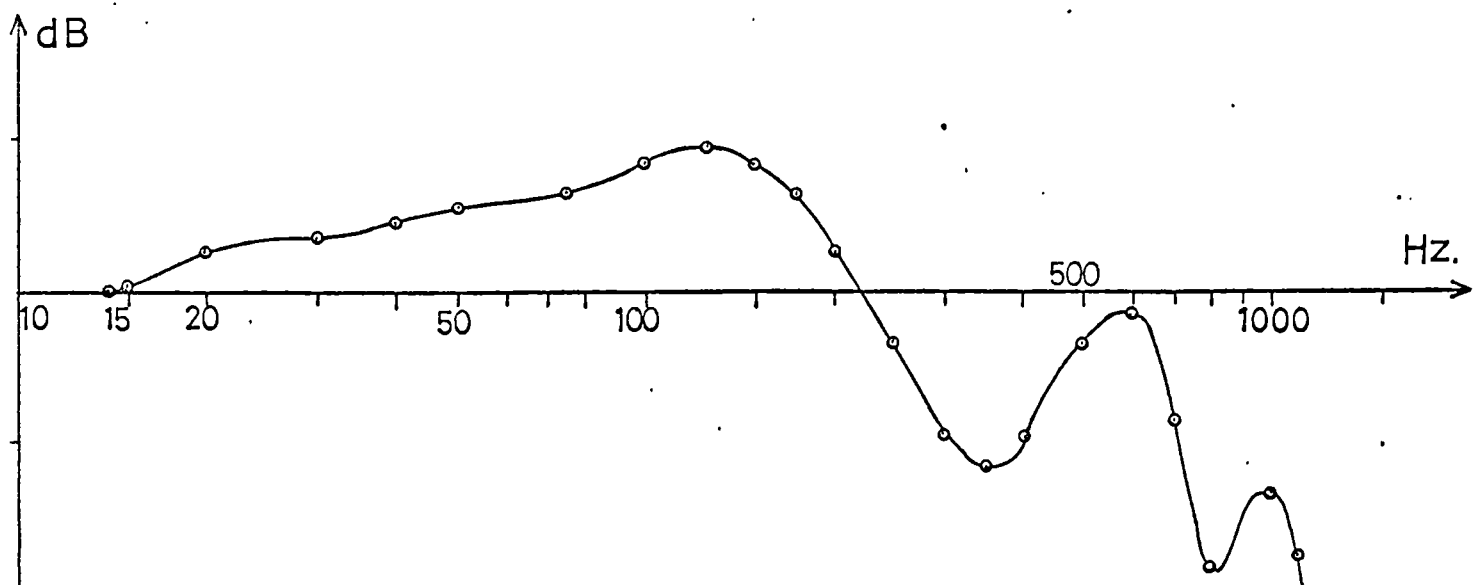
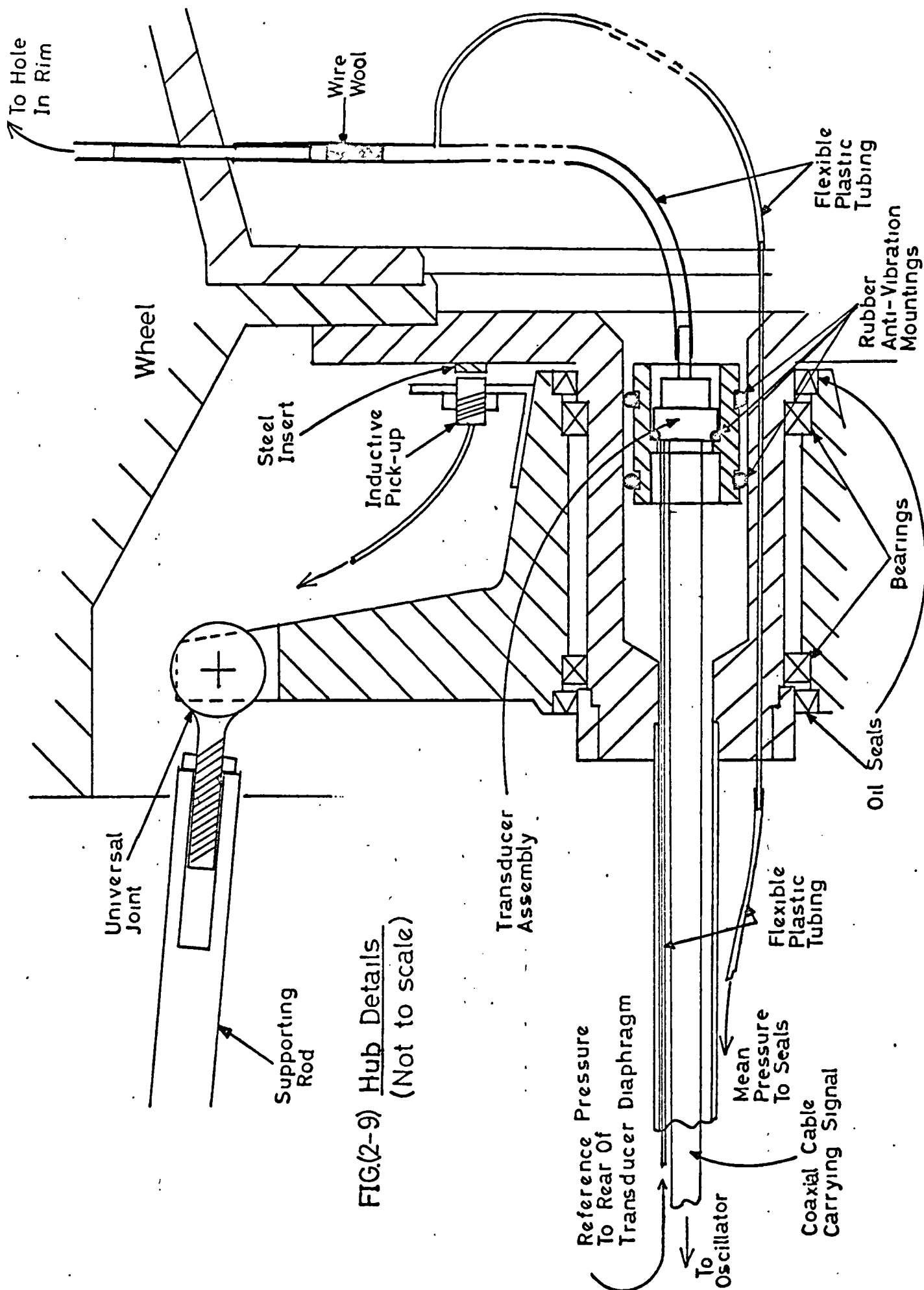
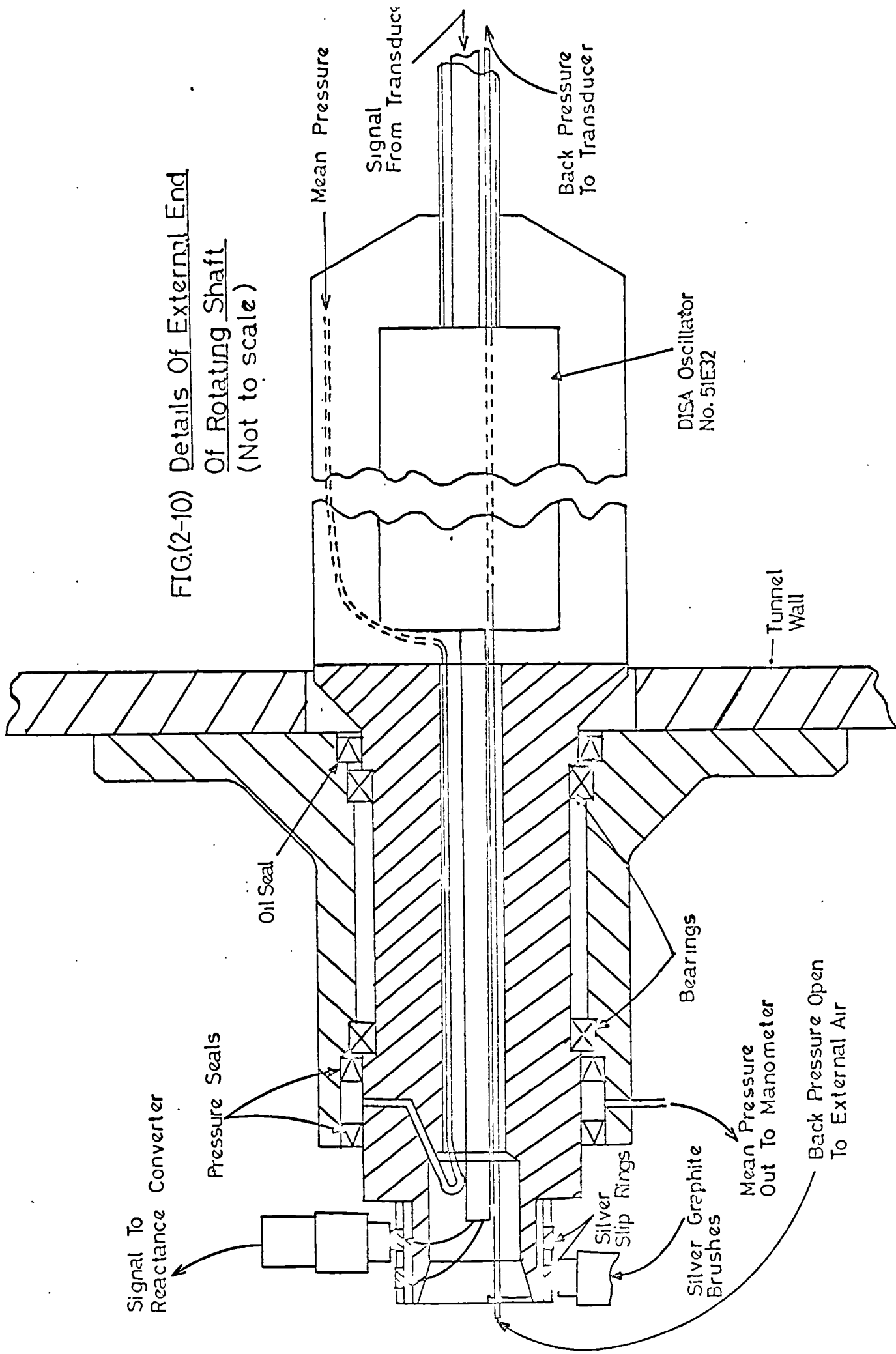


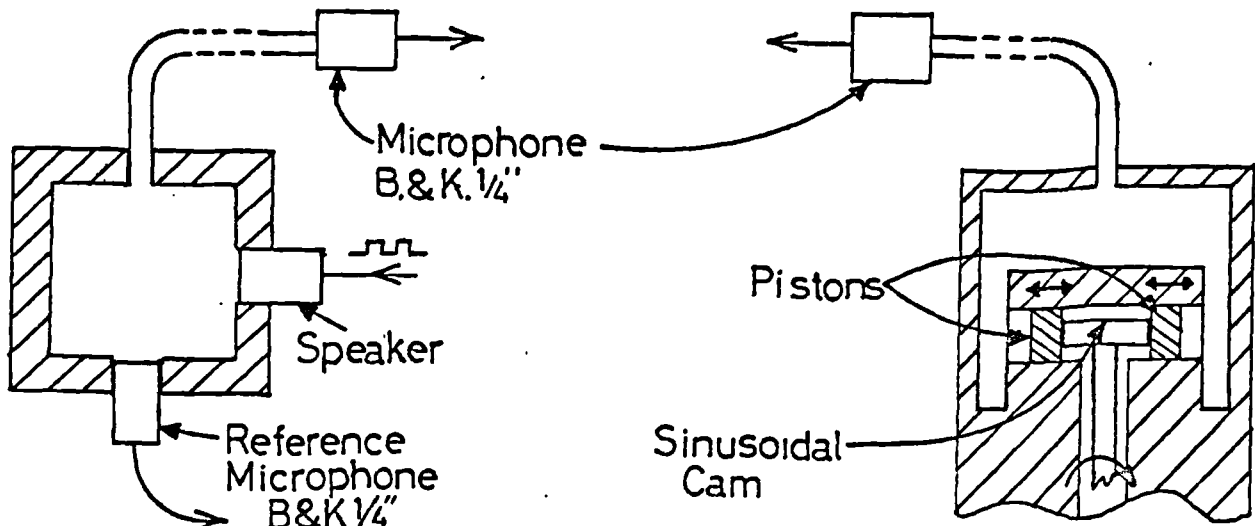
FIG.(2-8) Typical Frequency Response



FIG(2-9) Hub Details
(Not to scale)

FIG(2-10) Details Of External End
Of Rotating Shaft
(Not to scale)





i) Pulse Response And Frequency Response For Frequencies $> 800\text{Hz}$.

(ii) Frequency Response 14-800Hz.
Using B&K Pistonphone

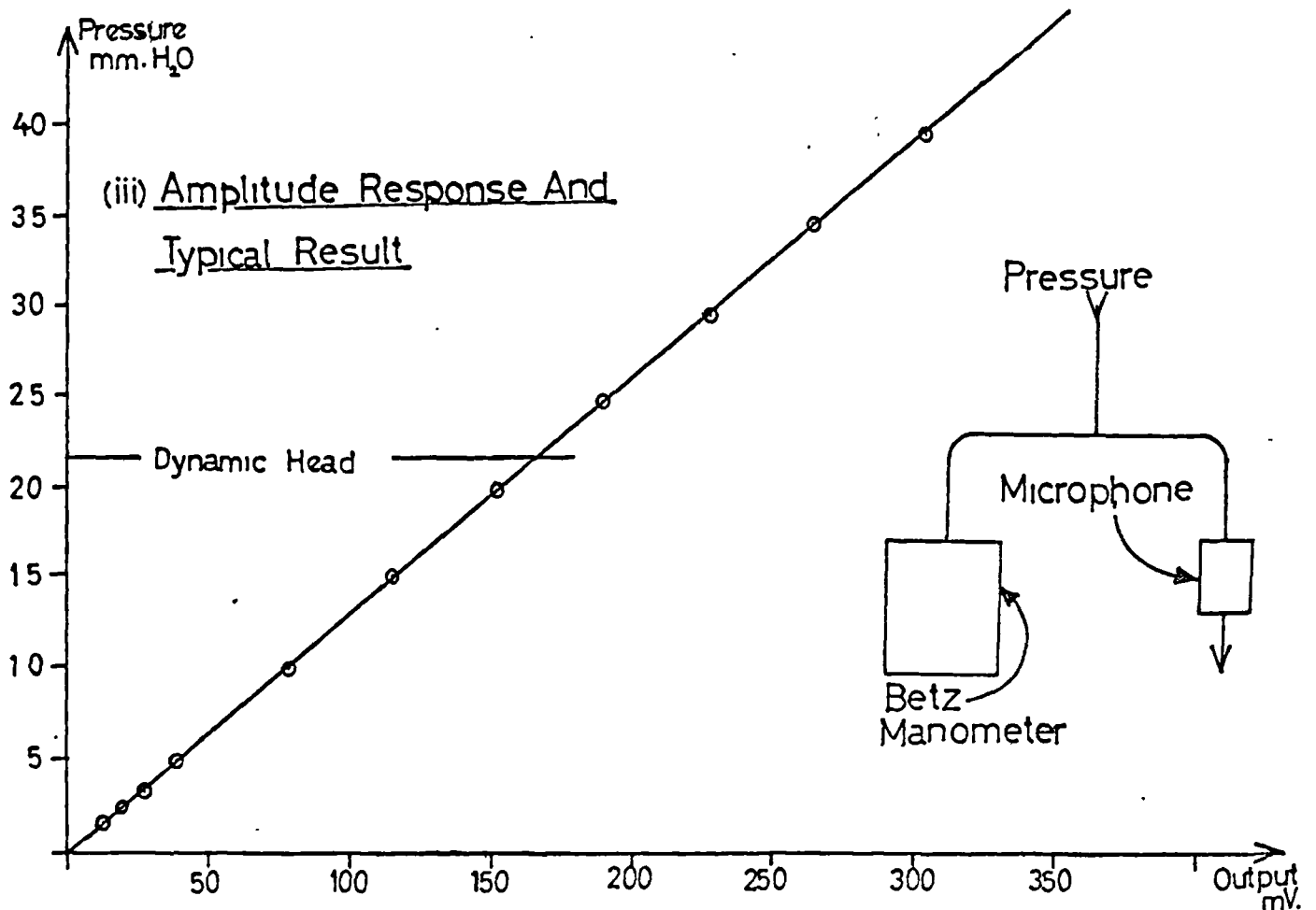


FIG.(2-11) Methods Of Calibrating Transducer

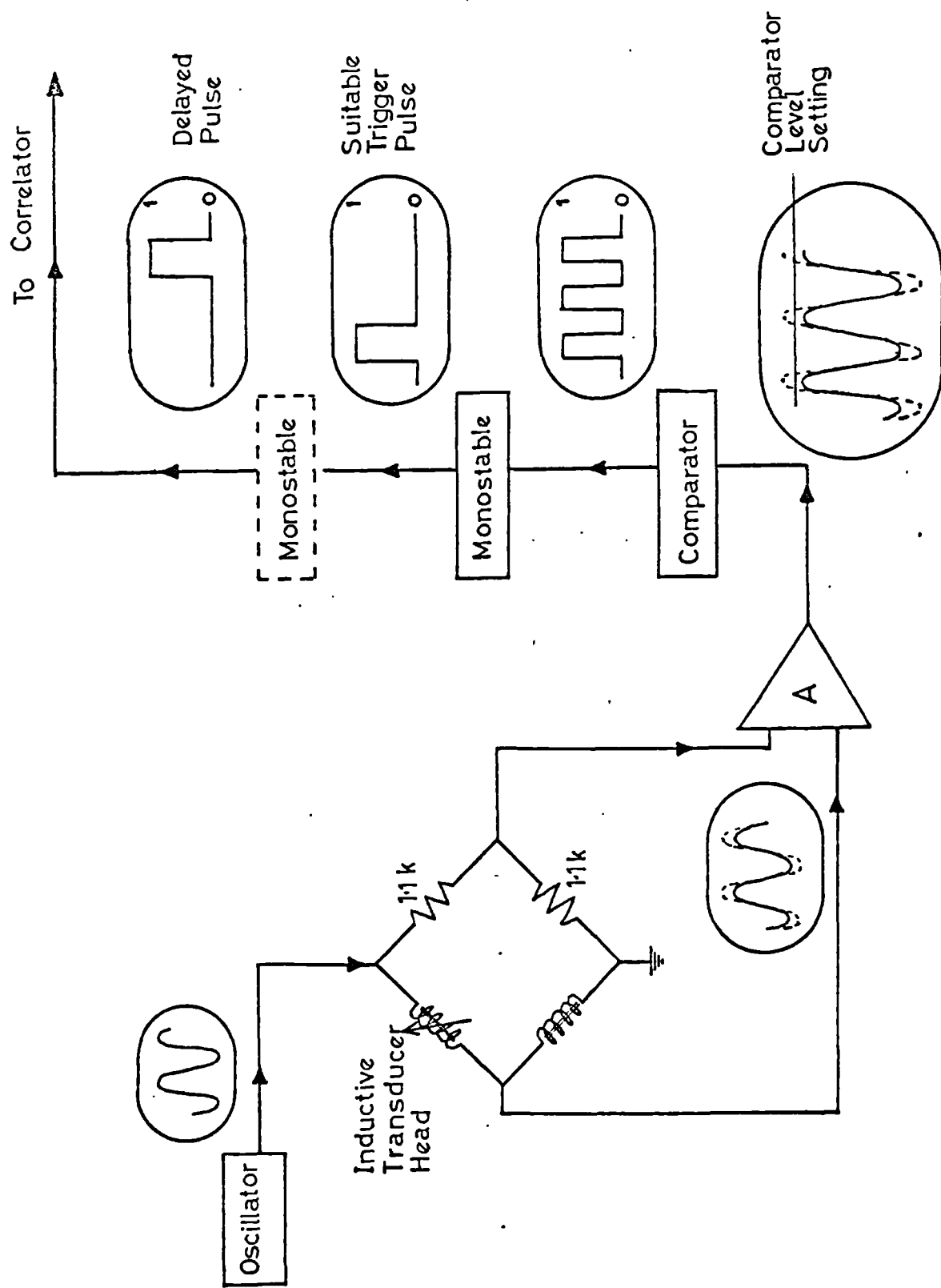


FIG.(2-12) Synchronizing Trigger Electronics

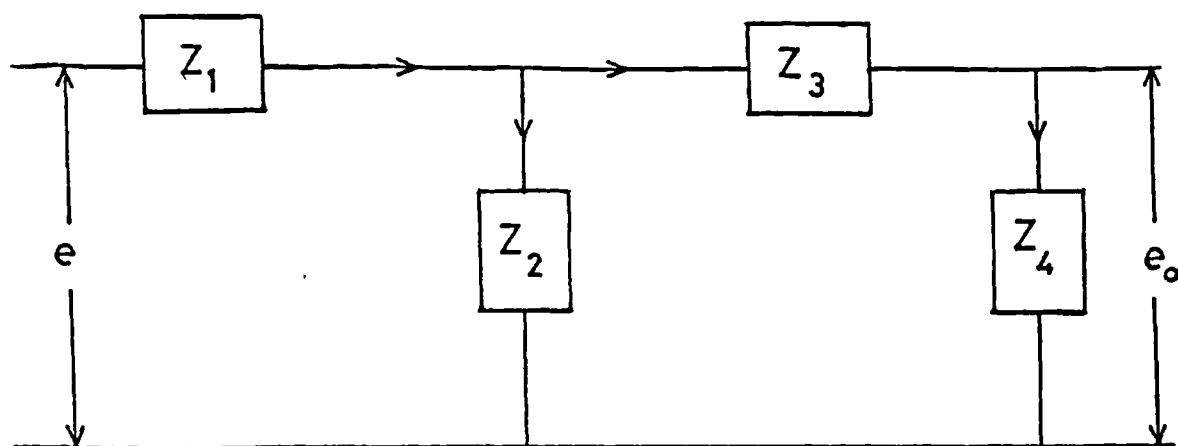


FIG.(2-13) Electric Analogy of Acoustic System

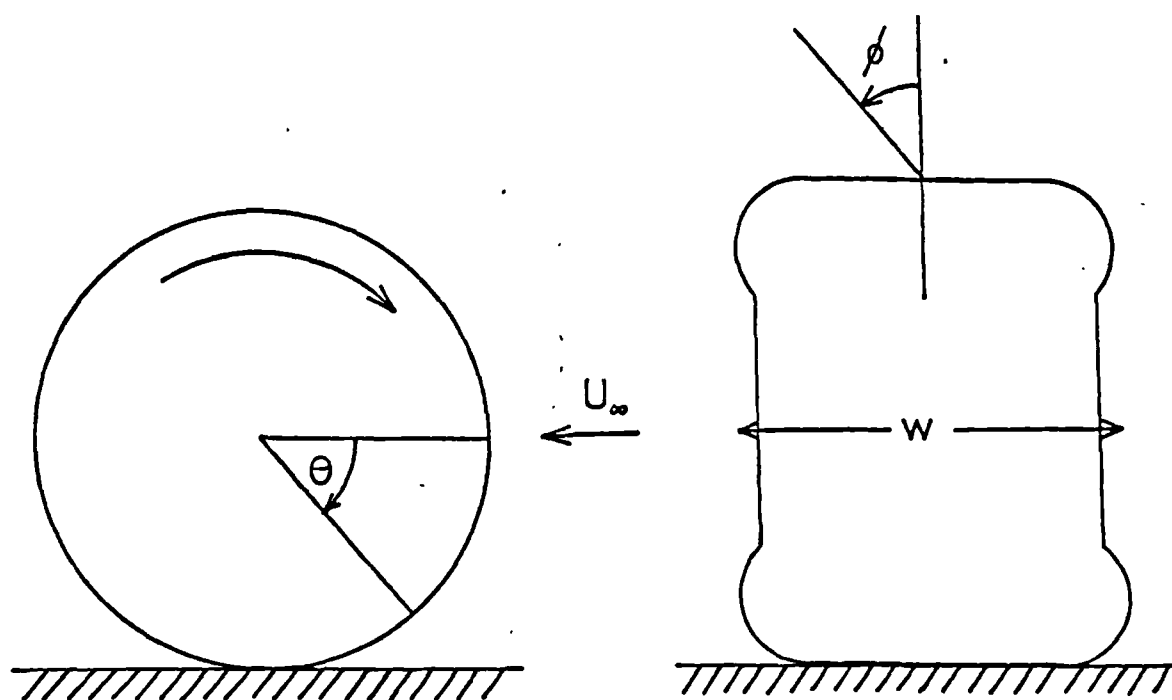
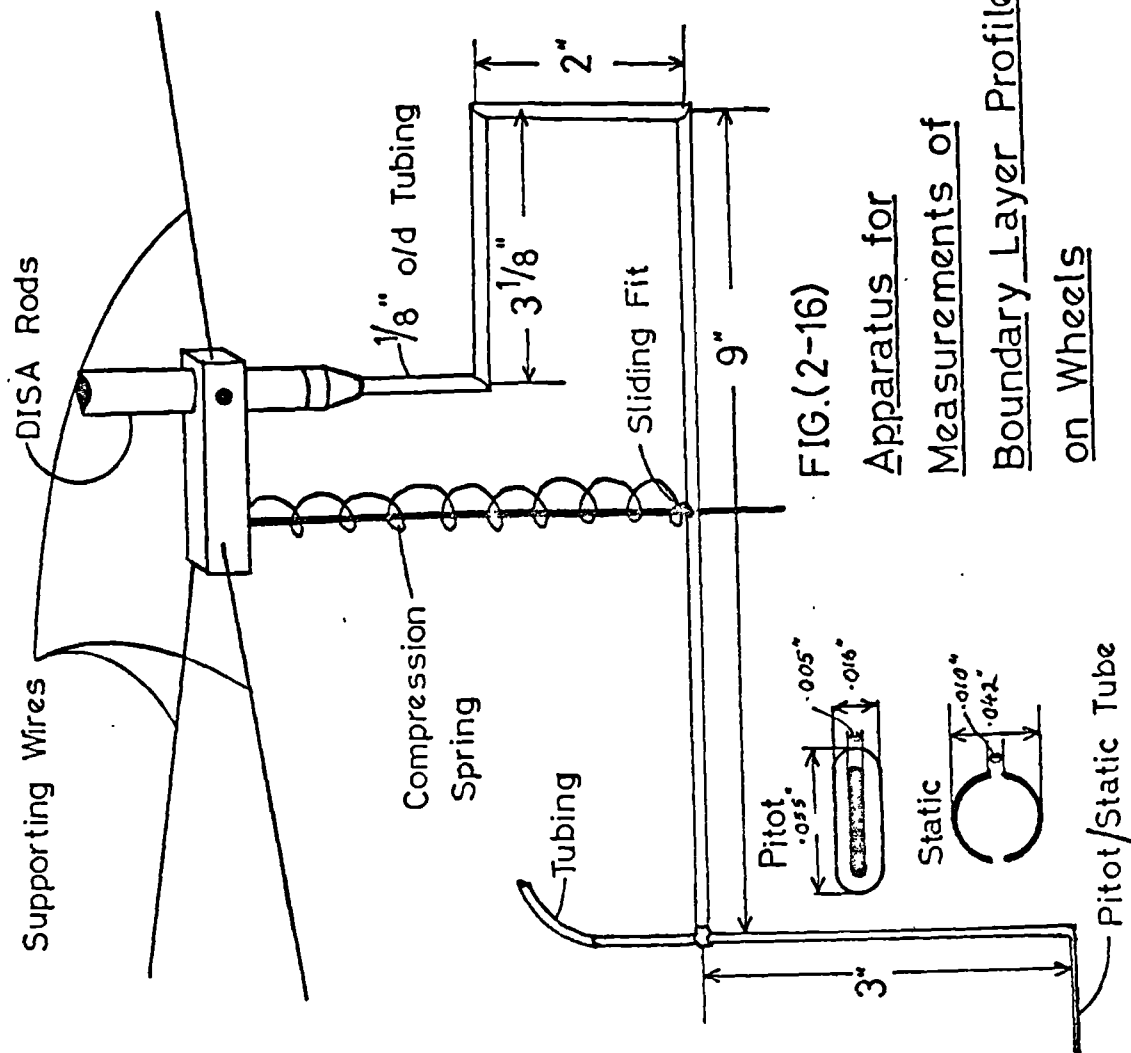
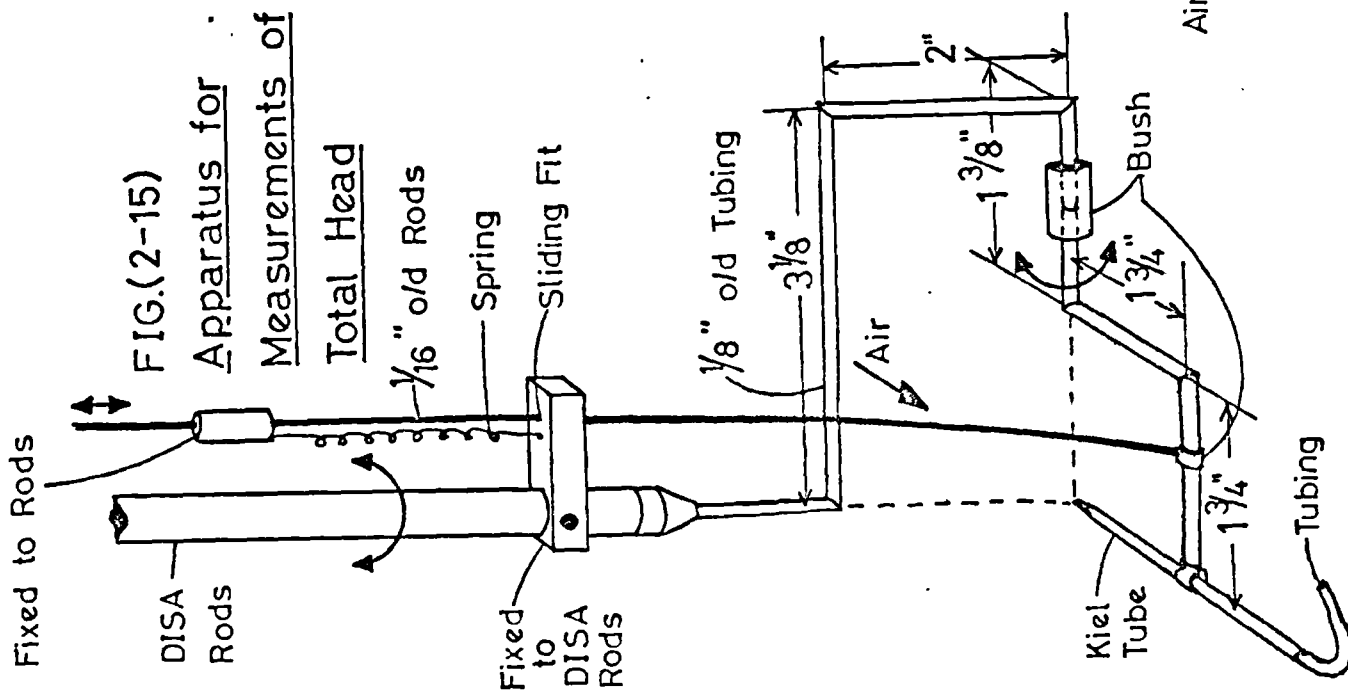
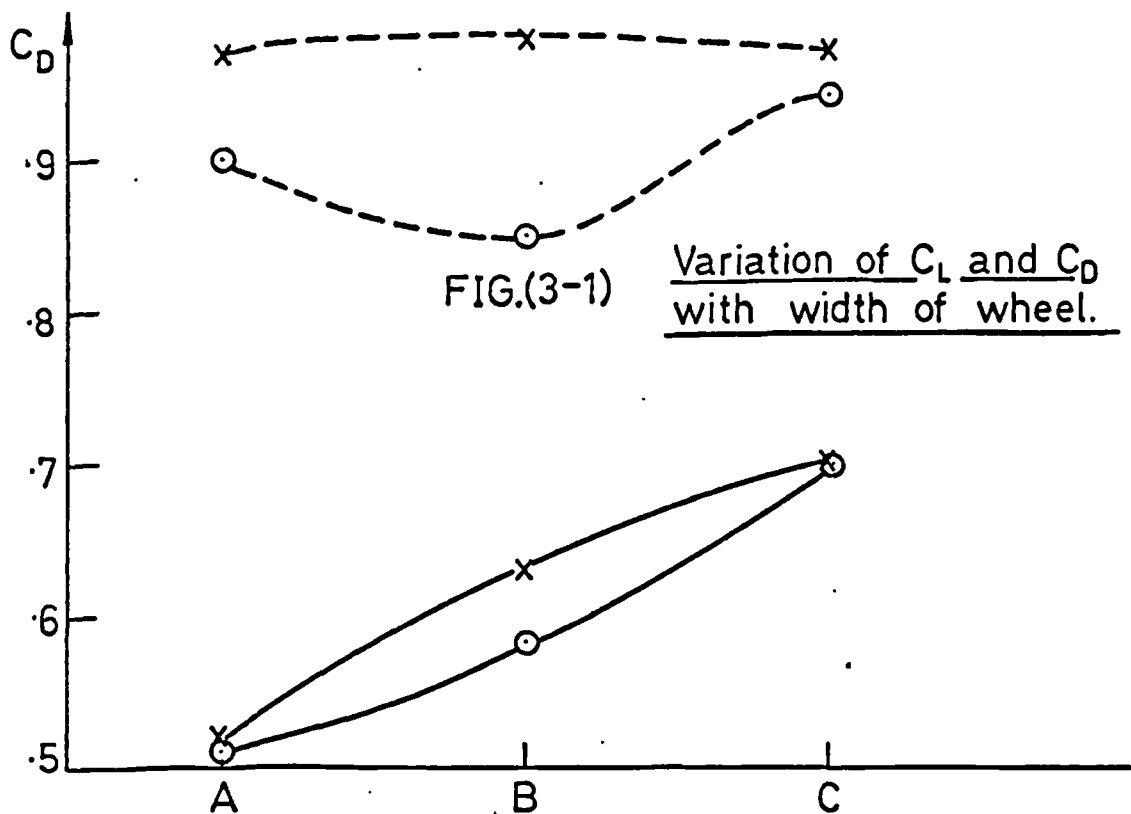


FIG.(2-14) Symbols for Integration over Wheel's Surface





- Coefficients based on X-sect. area.
 --- Coefficients based on tread x dia.
 x Edge shape 1.
 o Edge shape 2

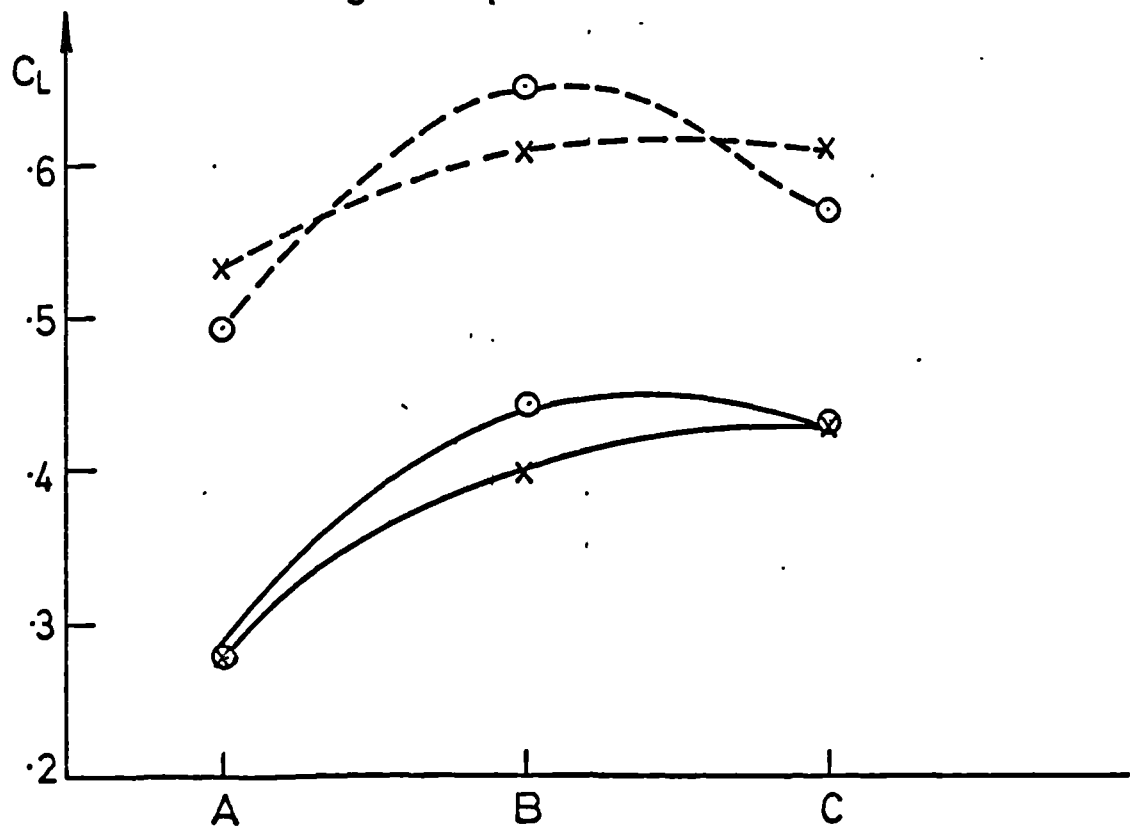
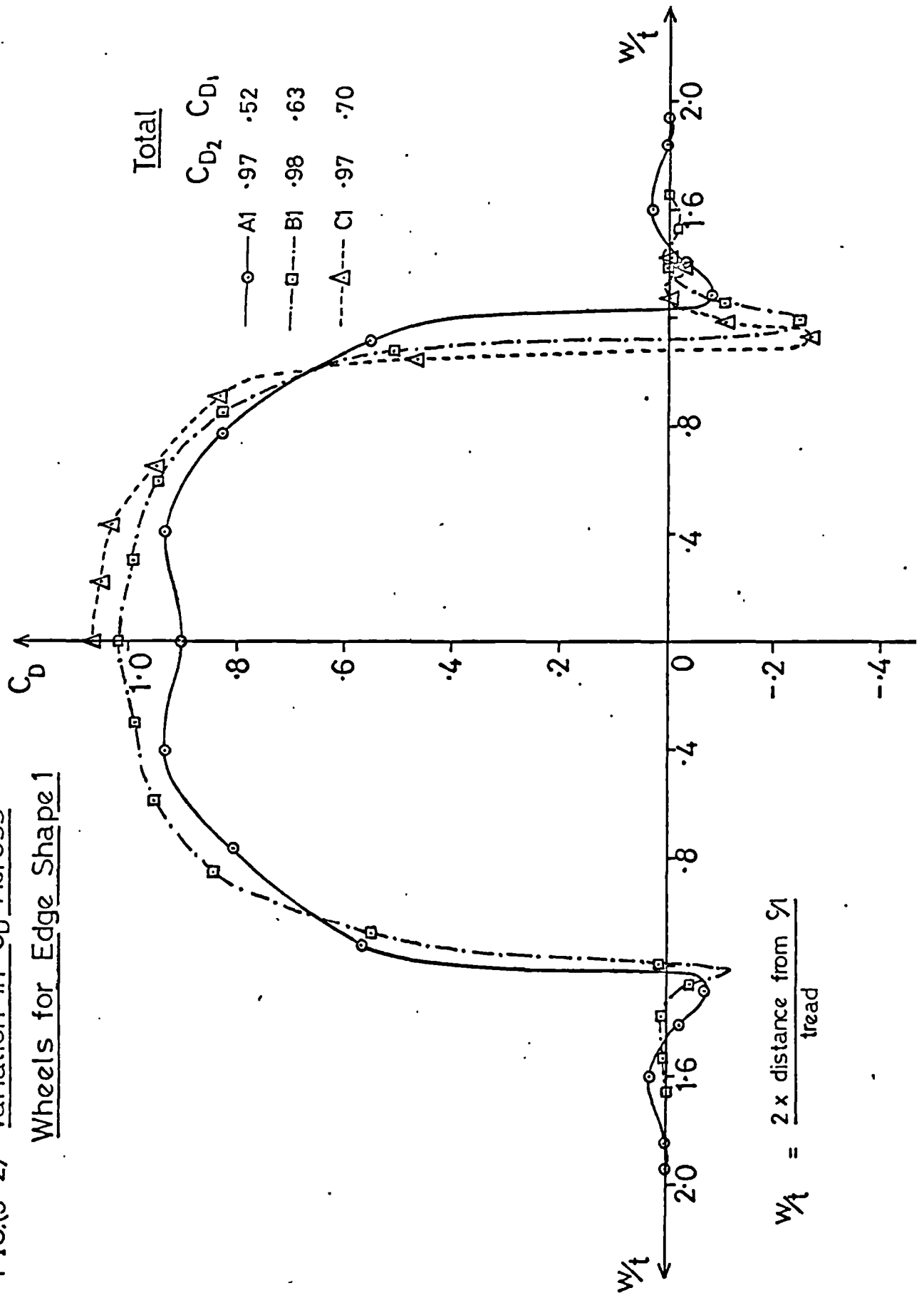


FIG.(3-2) Variation in C_D Across
Wheels for Edge Shape 1



FIG(3-3) Variation in C_D Across

Wheels for Edge Shape 2

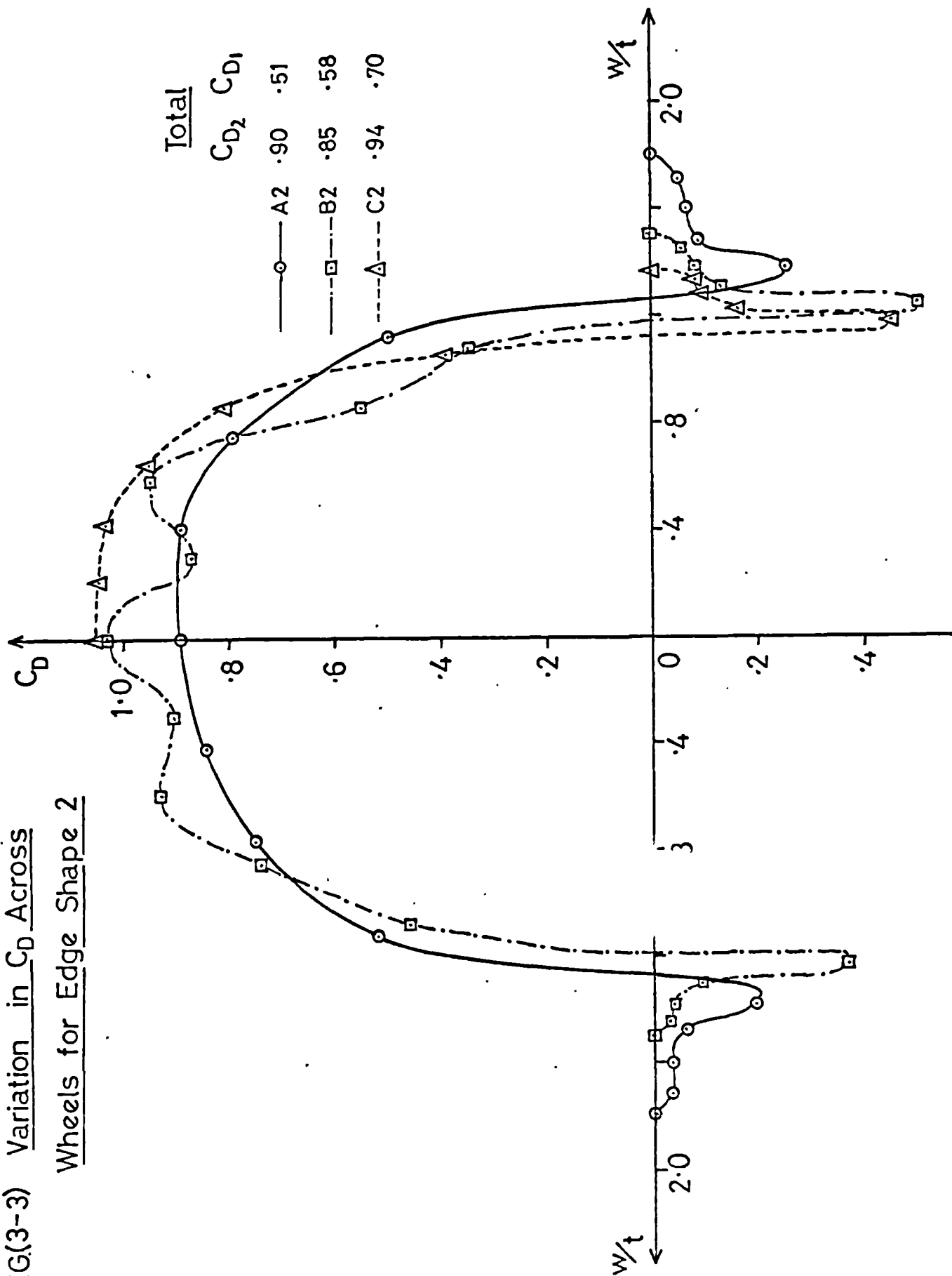


FIG.(3-4) Variation in C_L Across Wheels for Edge Shape 1

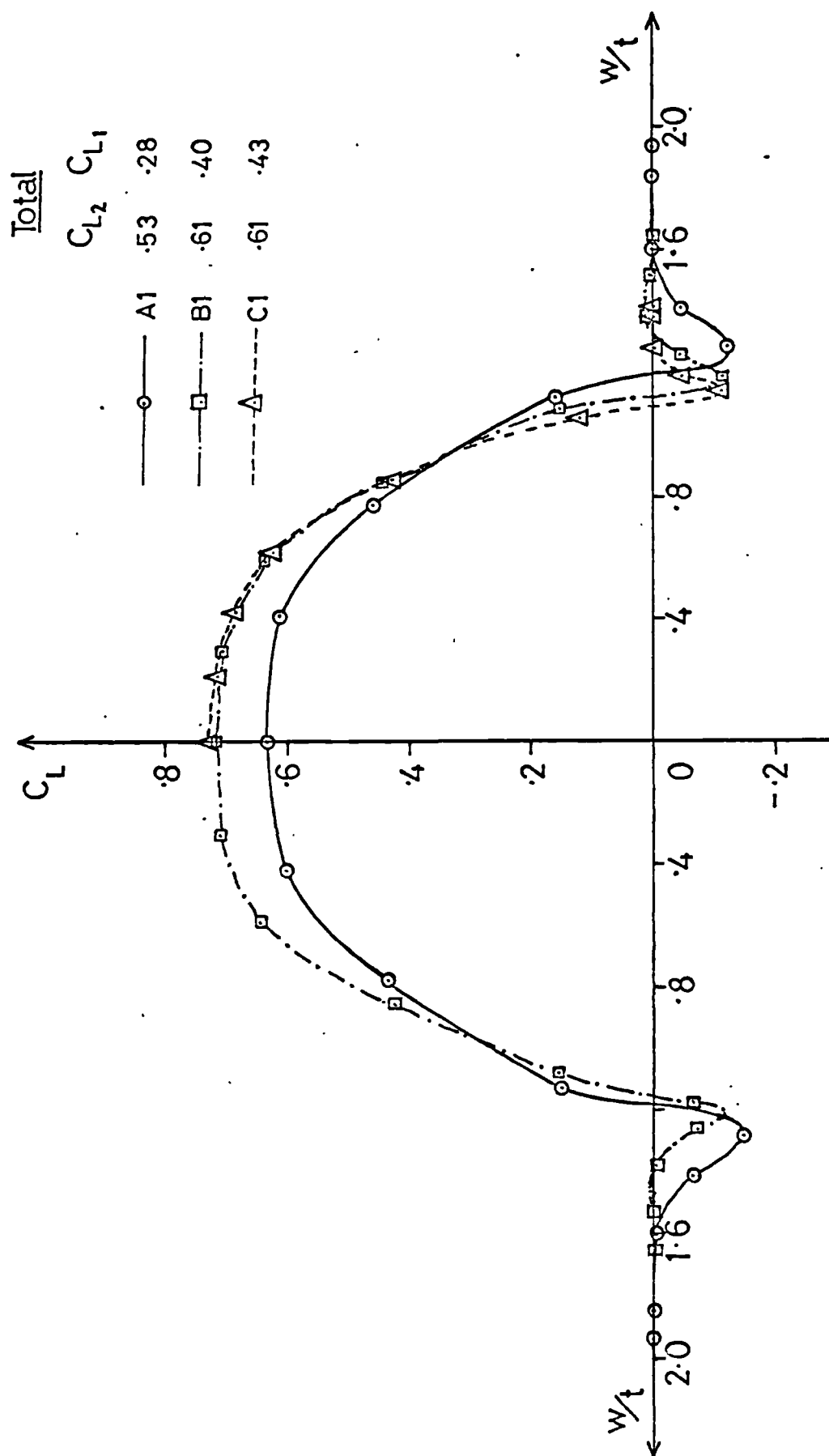


FIG.(3-5) Variation in C_L Across
Wheels for Edge Shape 2

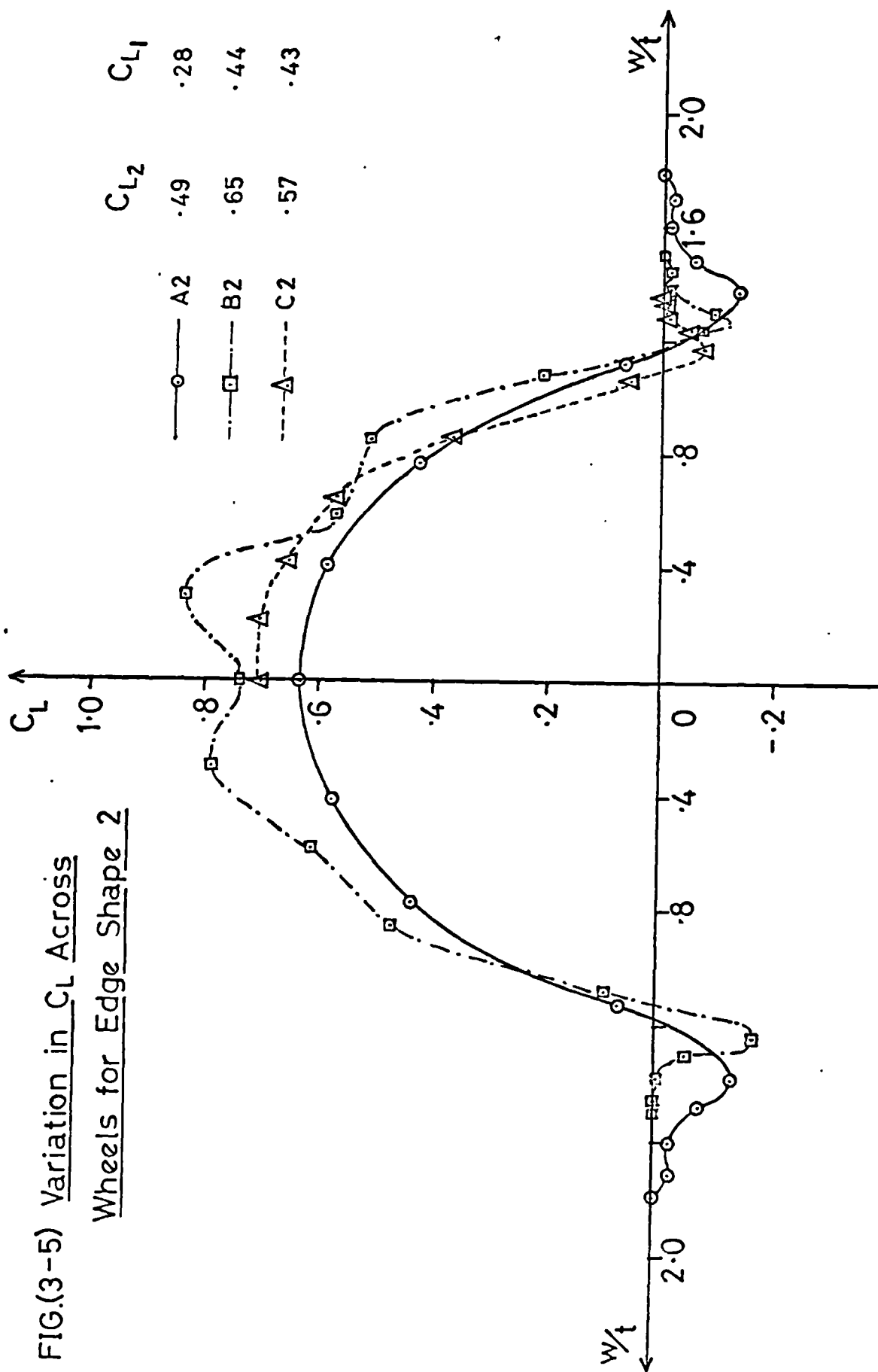


FIG.(3-6) Pressure Distribution at Hole 9

Wheels A1 and A2

	C_L	C_D
— A2	.64	.90
- - - A1	.64	.90

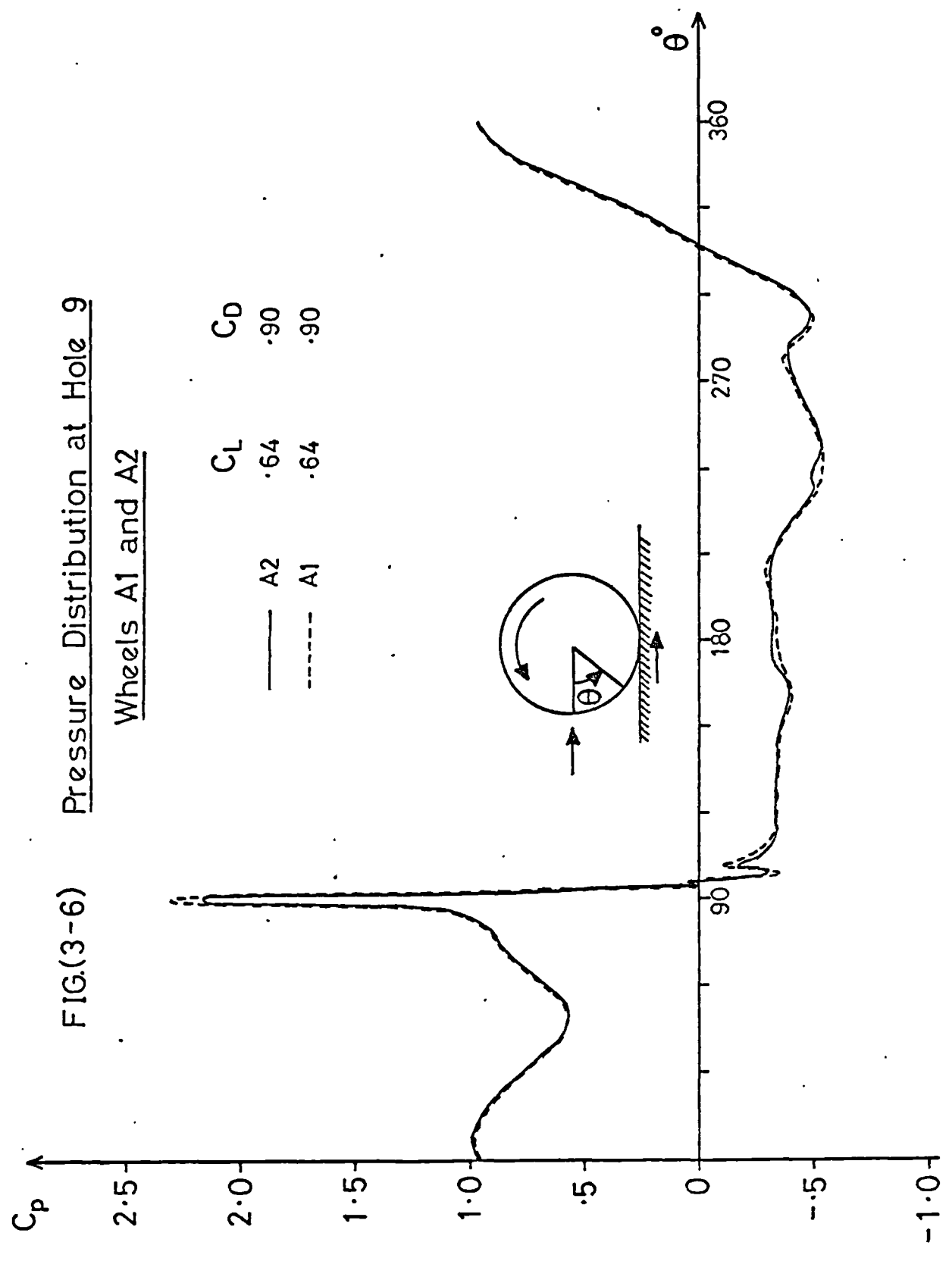
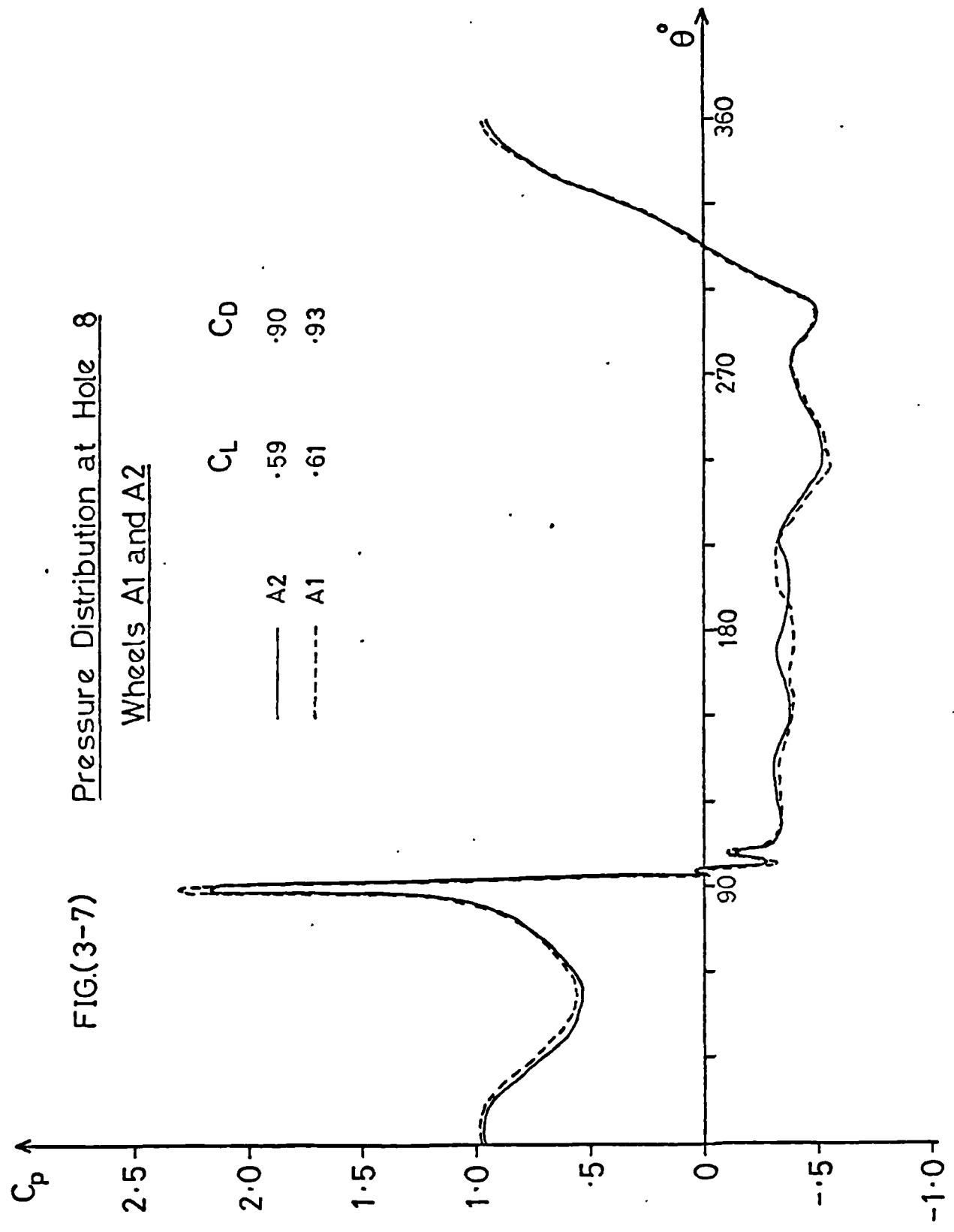
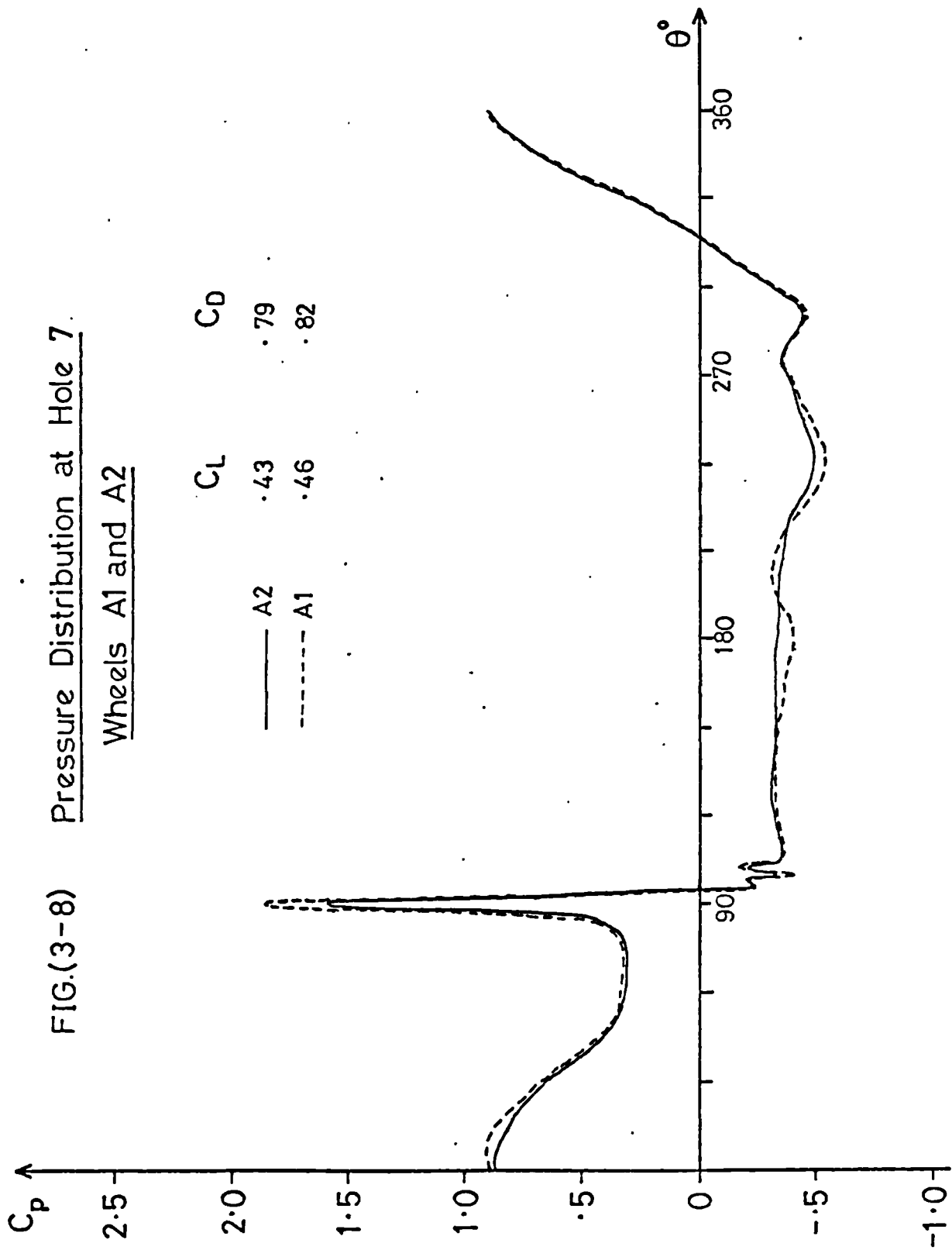


FIG.(3-7) Pressure Distribution at Hole 8





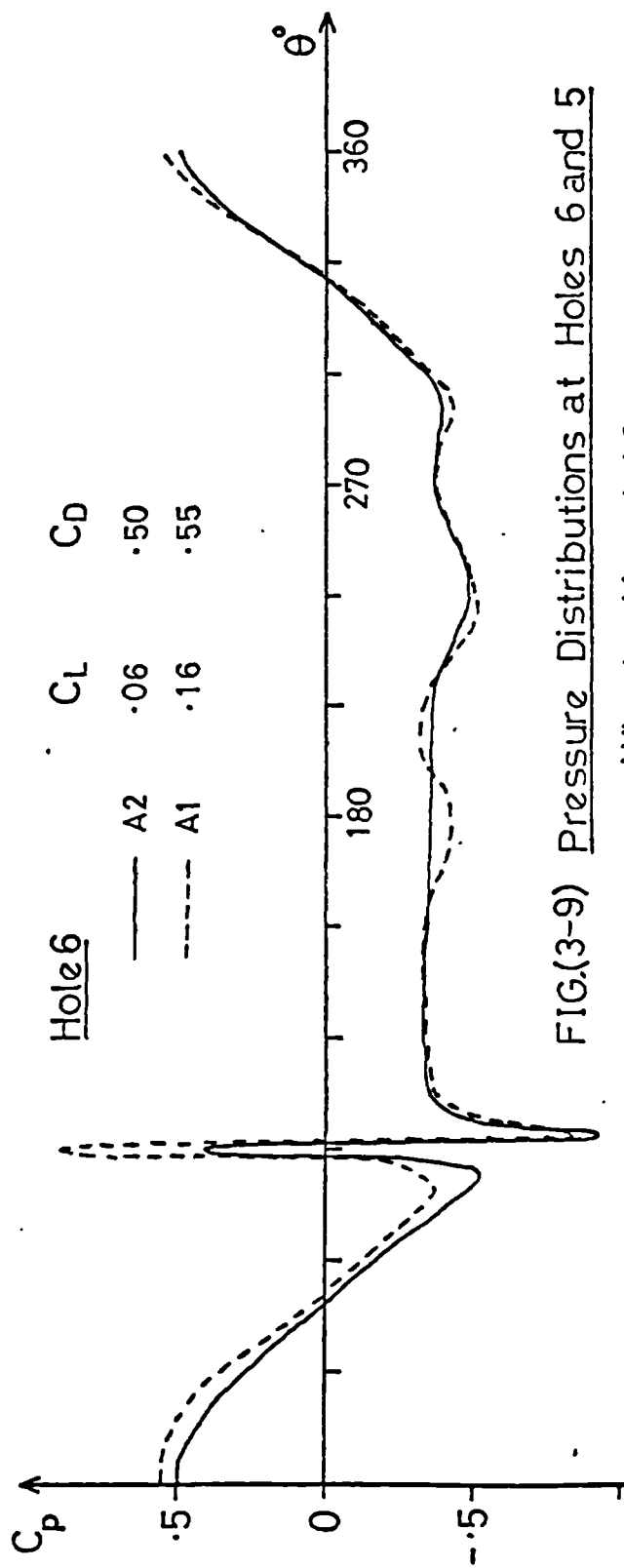
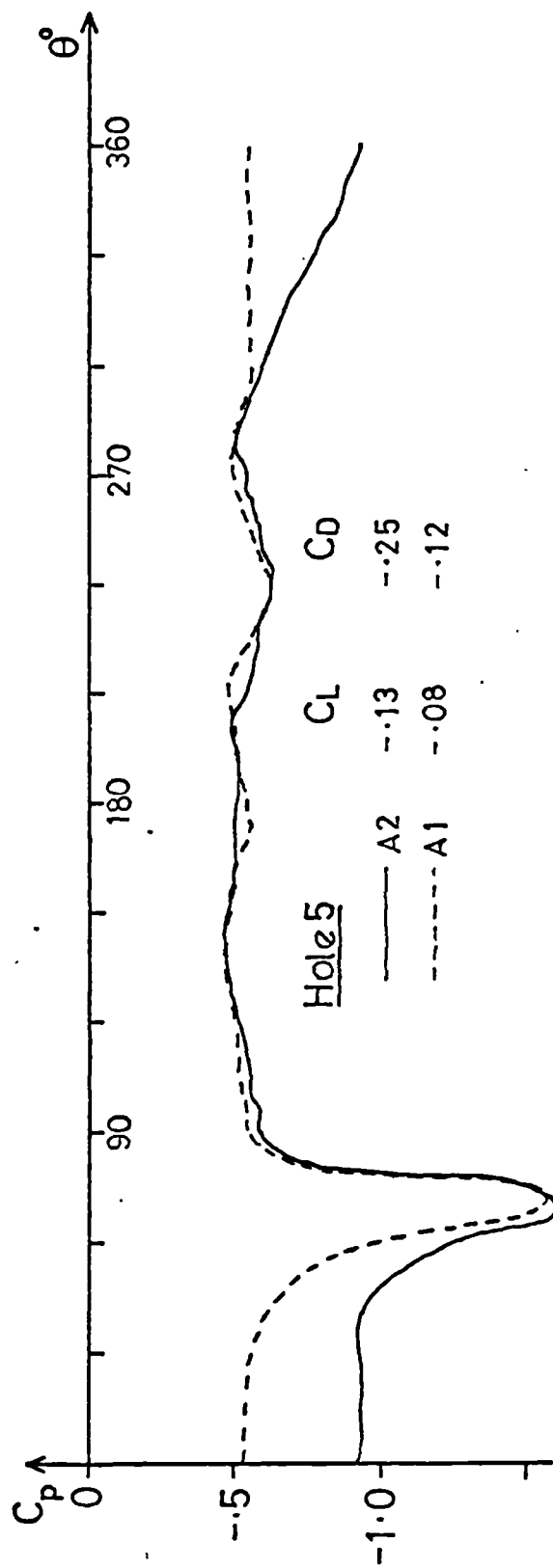


FIG.(3-9) Pressure Distributions at Holes 6 and 5

Wheels A1 and A2



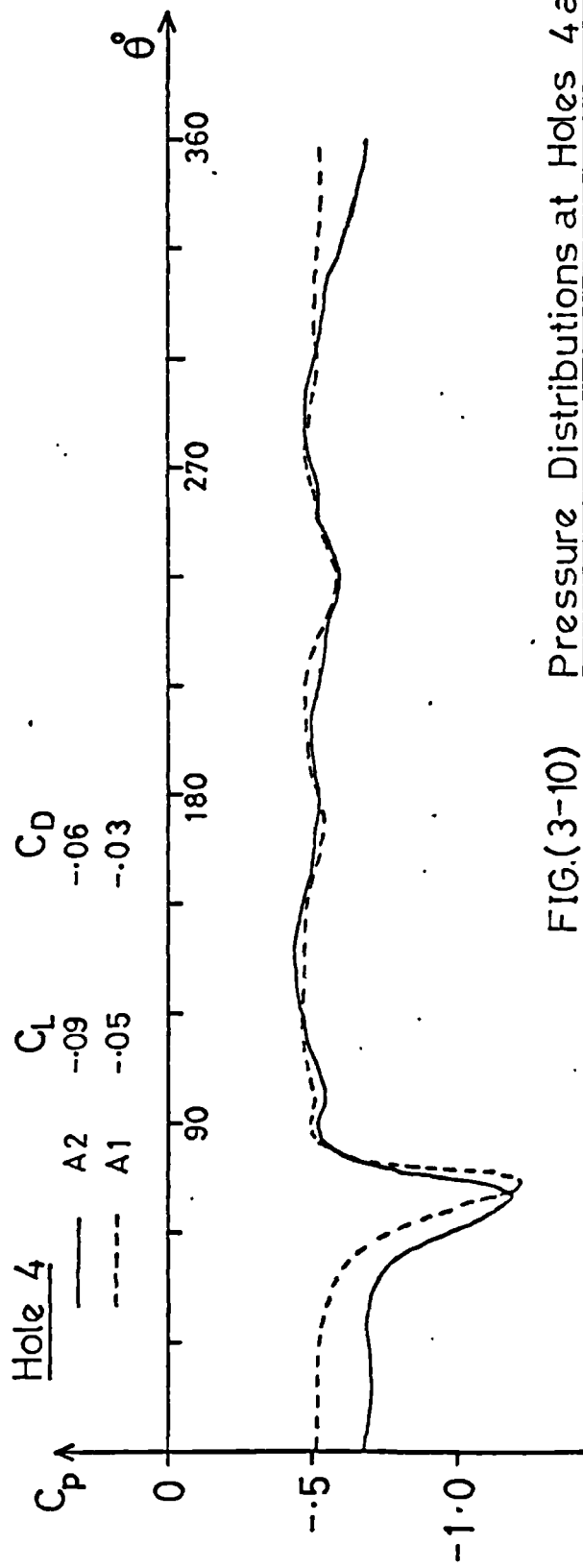
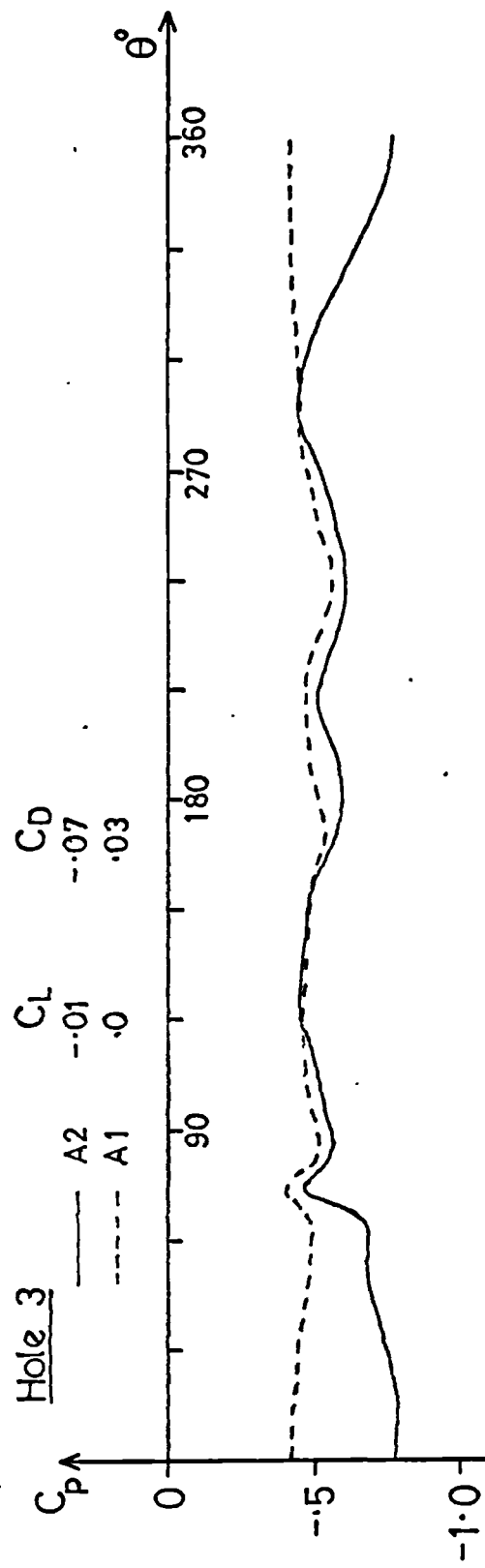


FIG.(3-10) Pressure Distributions at Holes 4 and 3
Wheels A1 and A2



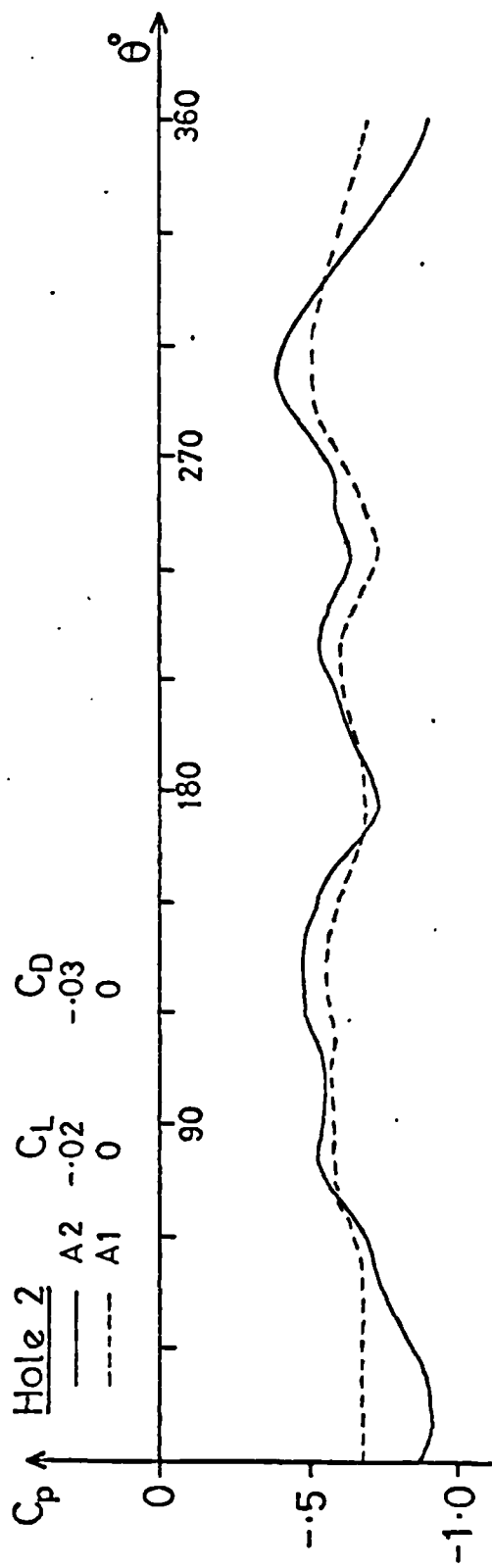
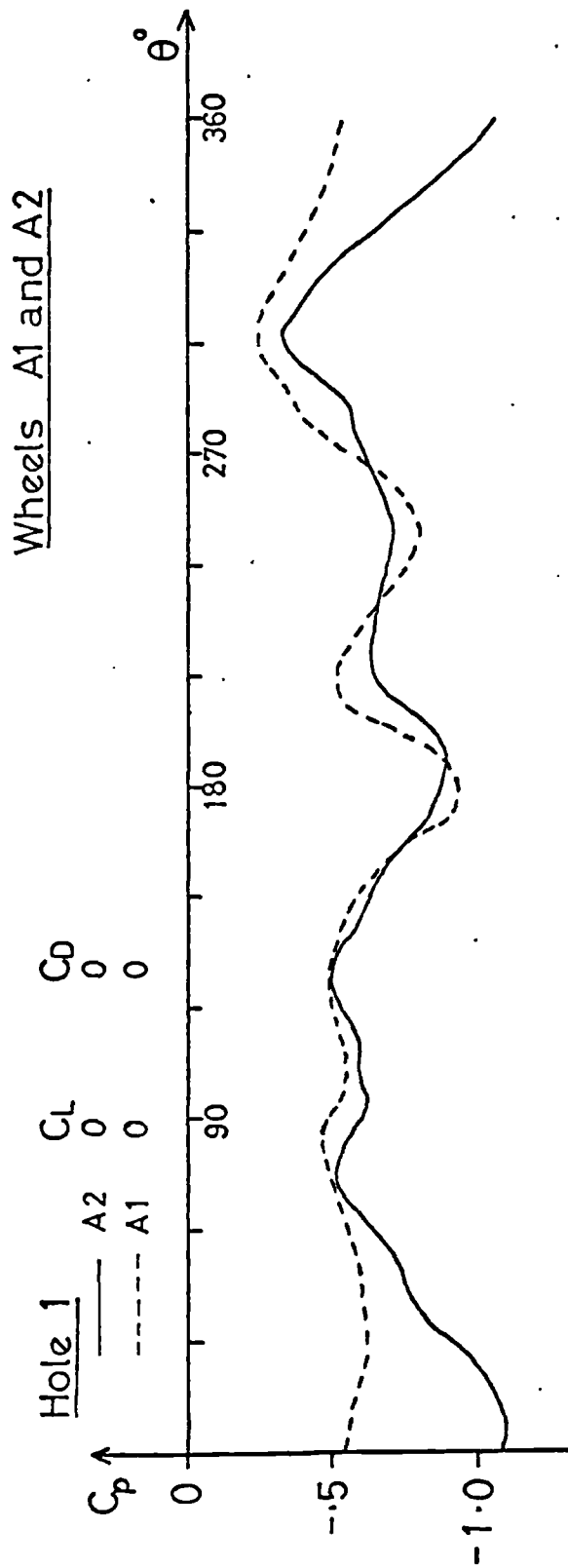


FIG.(3-11) Pressure Distributions at Holes 2 and 1



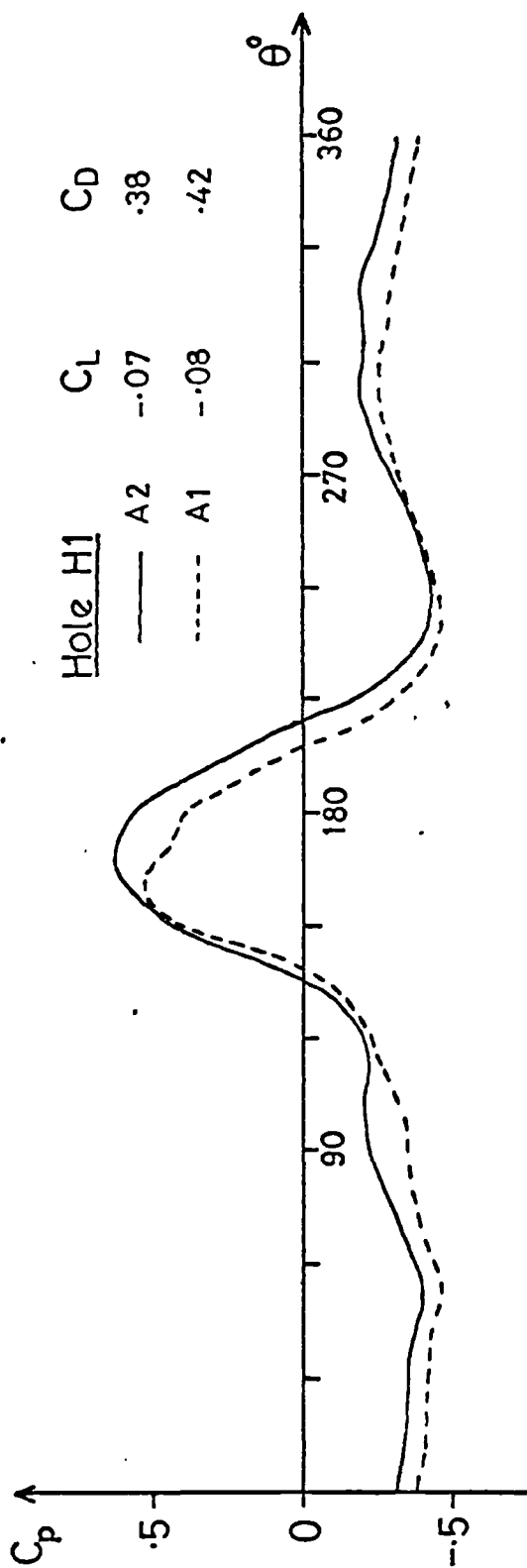
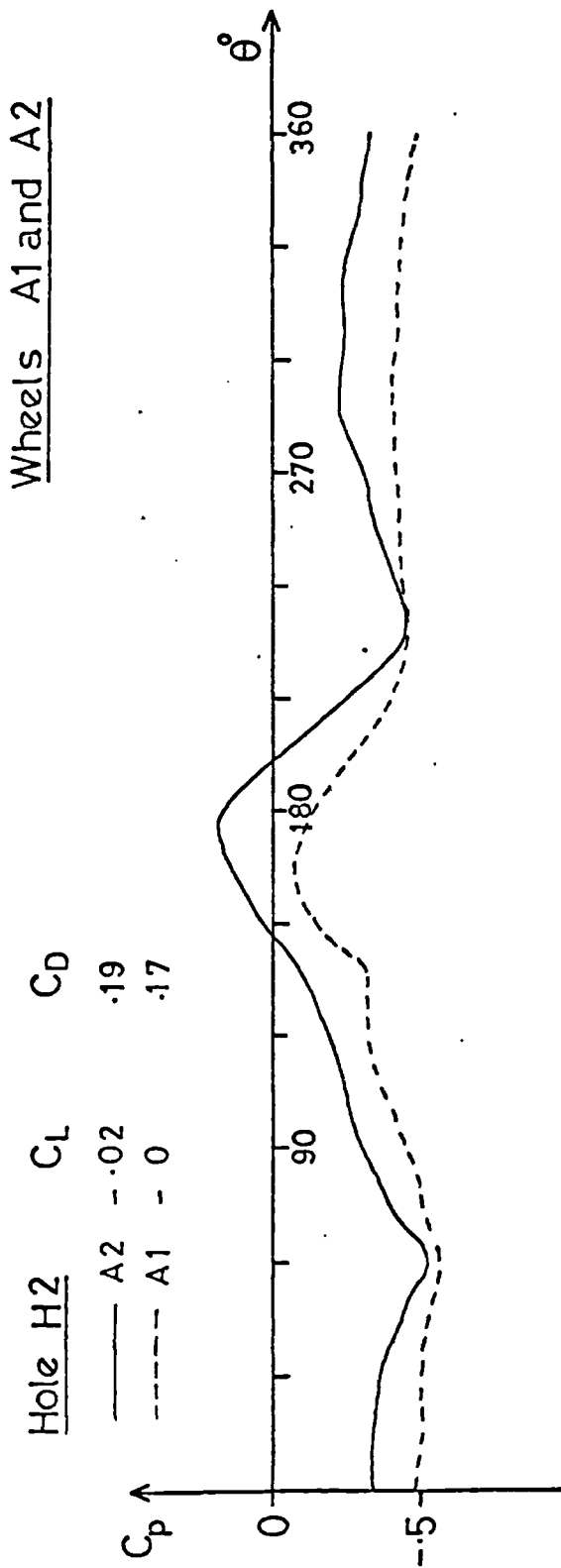
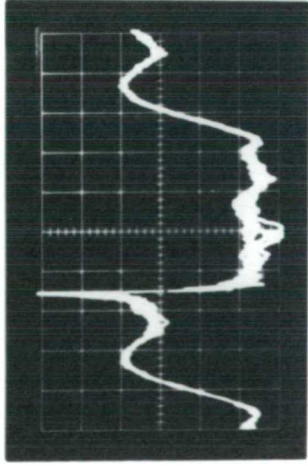


FIG.(3-12) Pressure Distributions at Holes H1 and H2

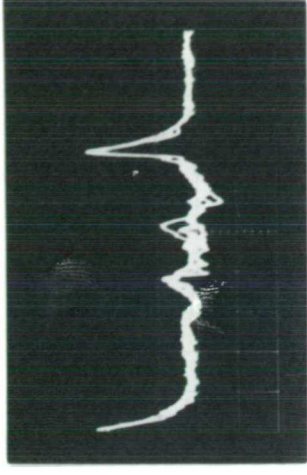




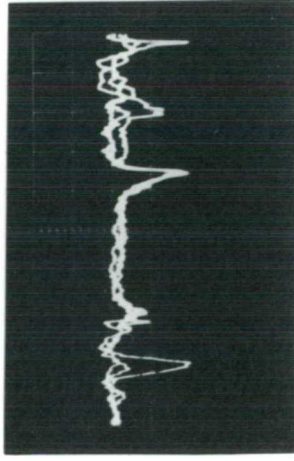
(i) Hole 9 (A1)



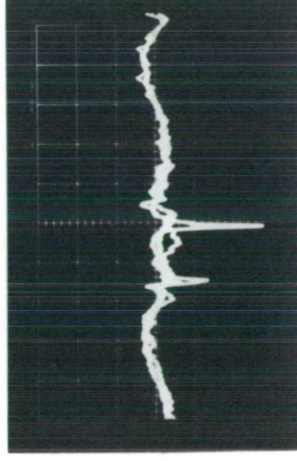
(ii) Hole 6



(iii) Hole 5



(iv) Hole 4



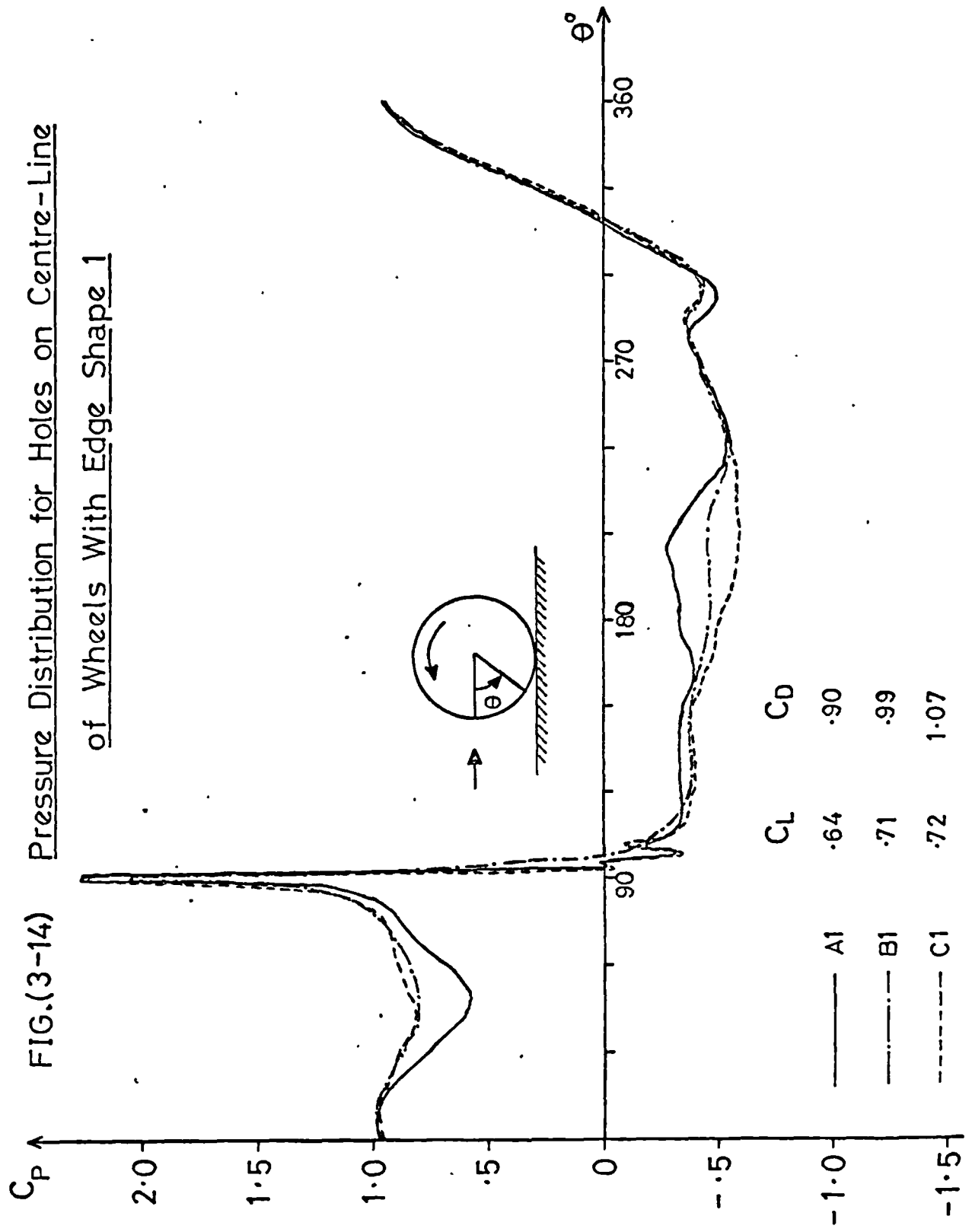
(v) Hole 3



(vi) Hole 2

FIG.(3-13) Oscilloscope Traces Representing Pressures Measured Around Wheel on Several Successive Revolutions (10 msec/div.)

FIG.(3-14) Pressure Distribution for Holes on Centre-Line
of Wheels With Edge Shape 1



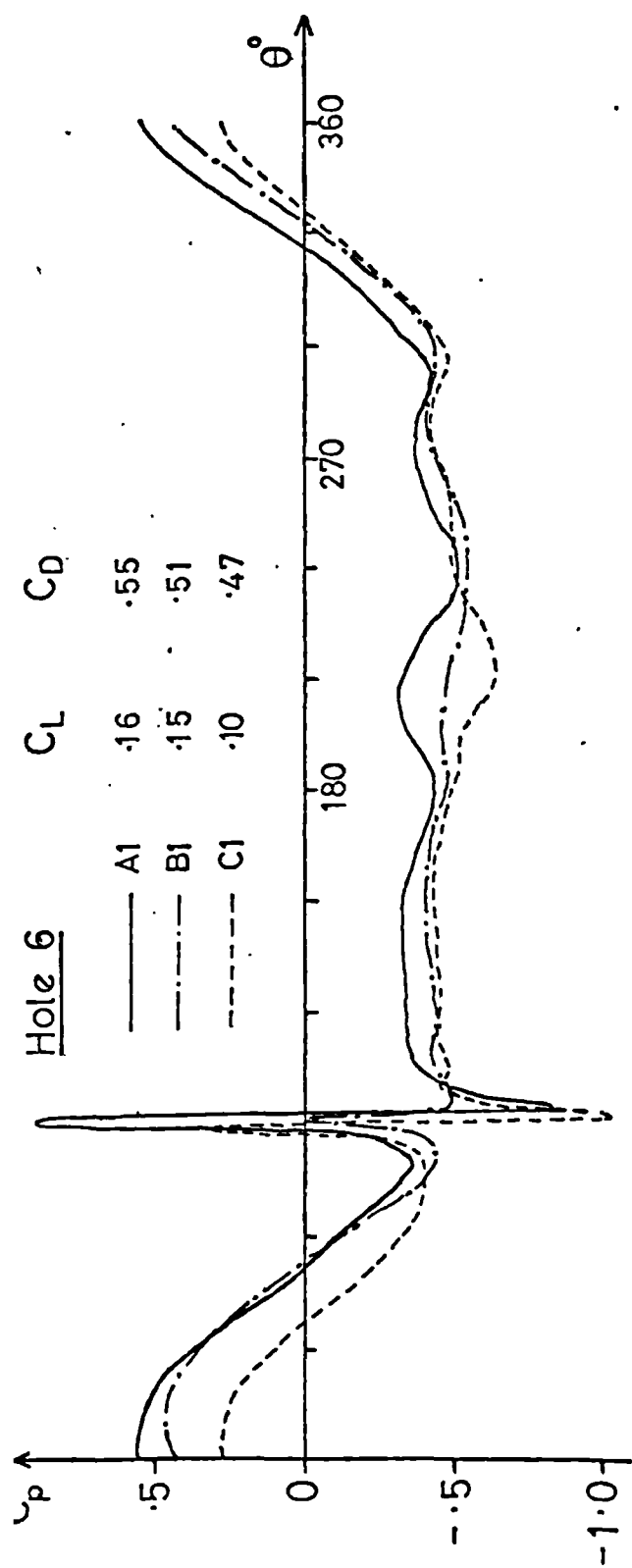
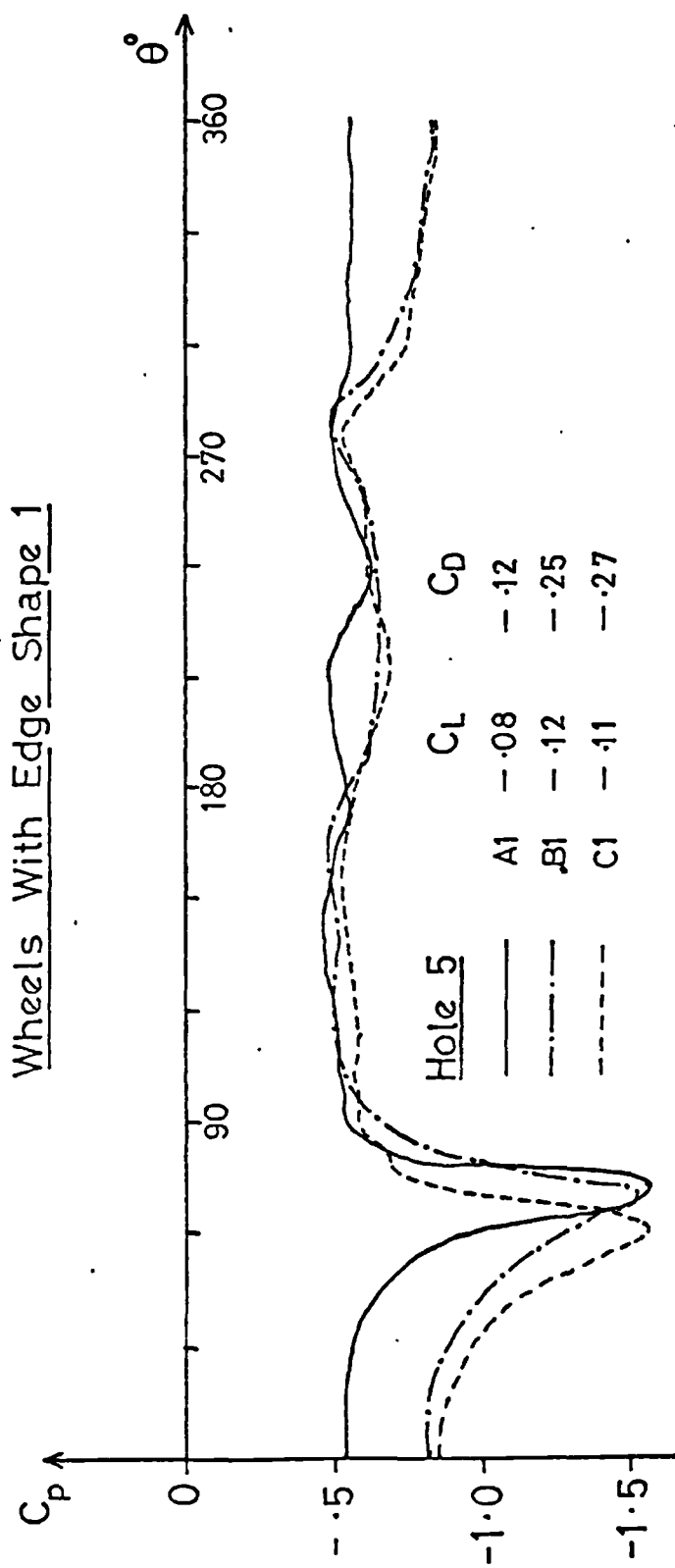


FIG.(3-15) Pressure Distributions at Holes 6 and 5



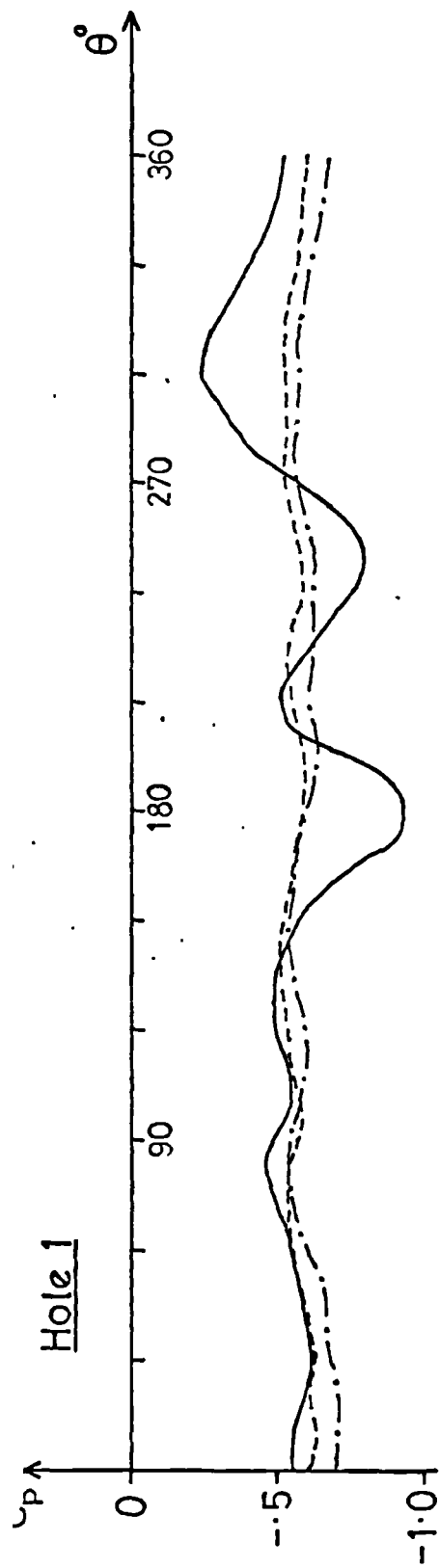
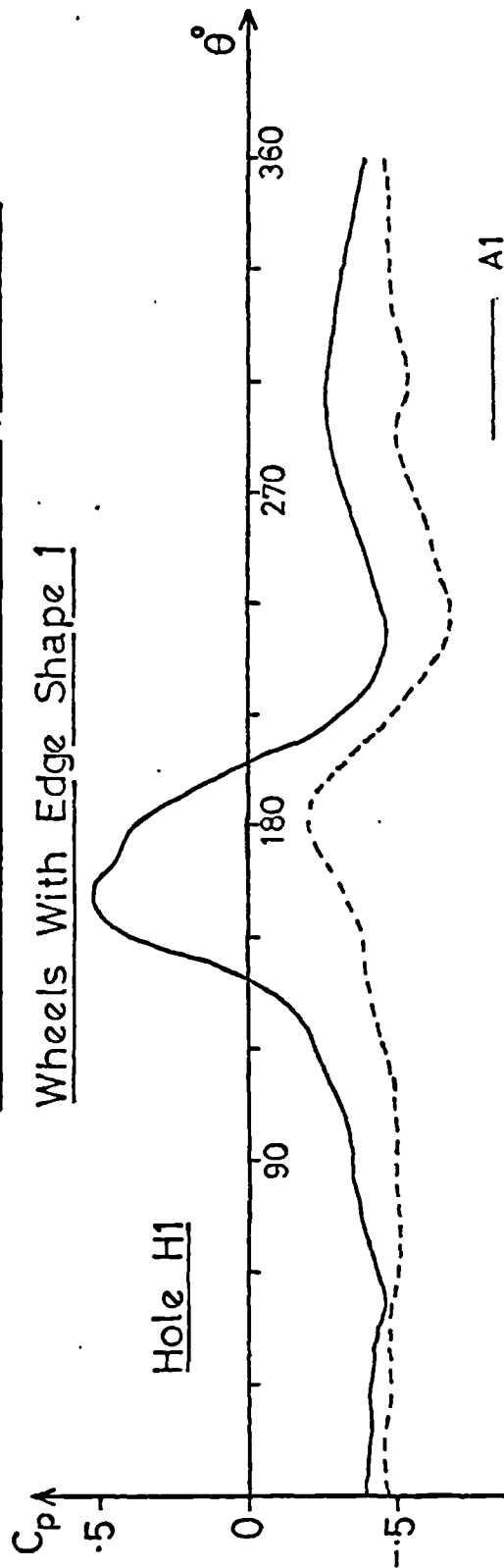


FIG.(3-16) Pressure Distribution for Holes 1, H1 and H2,
Wheels With Edge Shape 1



A1
B1
C1

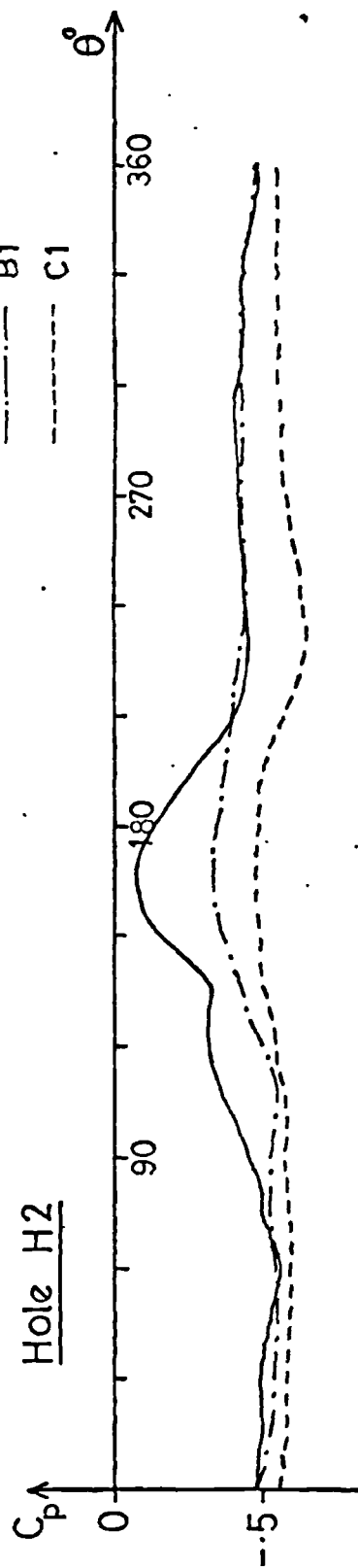
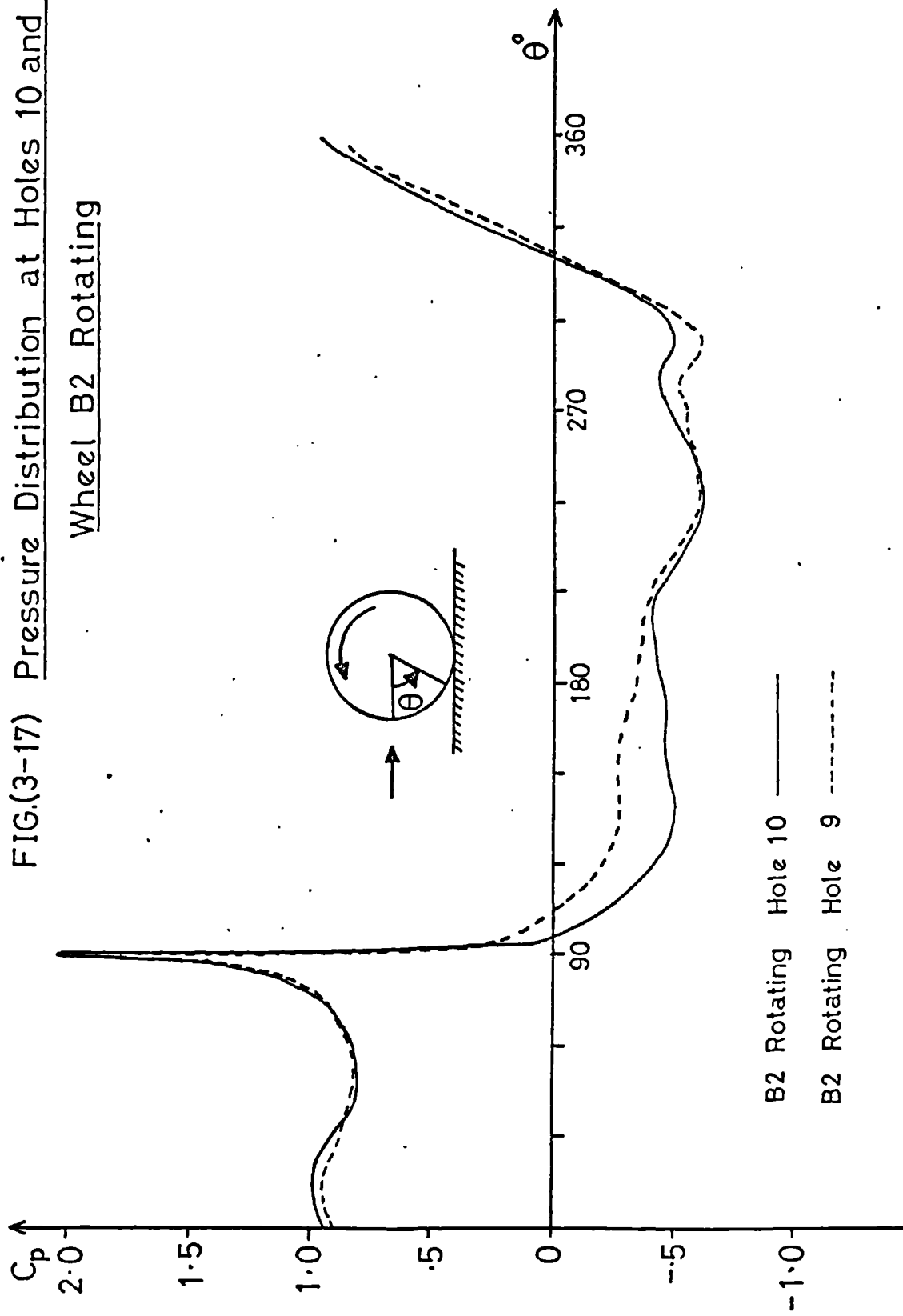
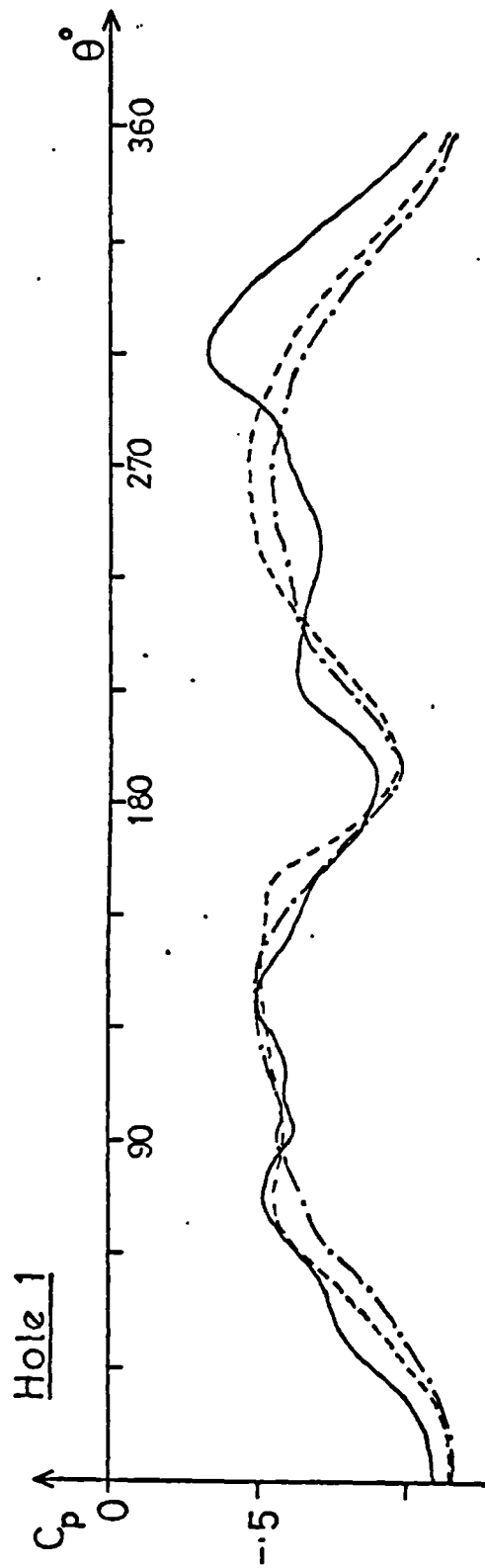


FIG.(3-17) Pressure Distribution at Holes 10 and 9
Wheel B2 Rotating





FIG(3-18) Pressure Distribution for
Holes 1,H1 and H2,
Wheels With Edge Shape 2

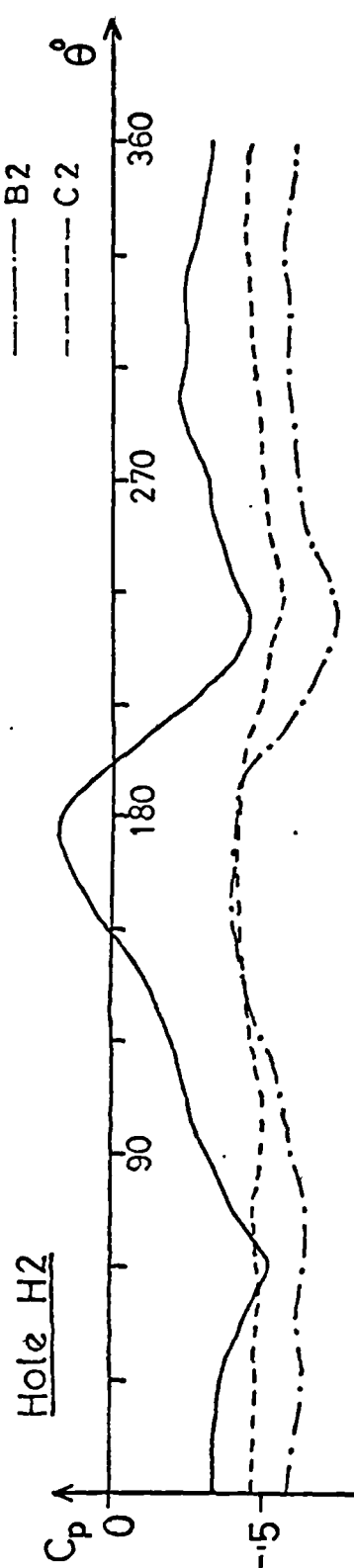
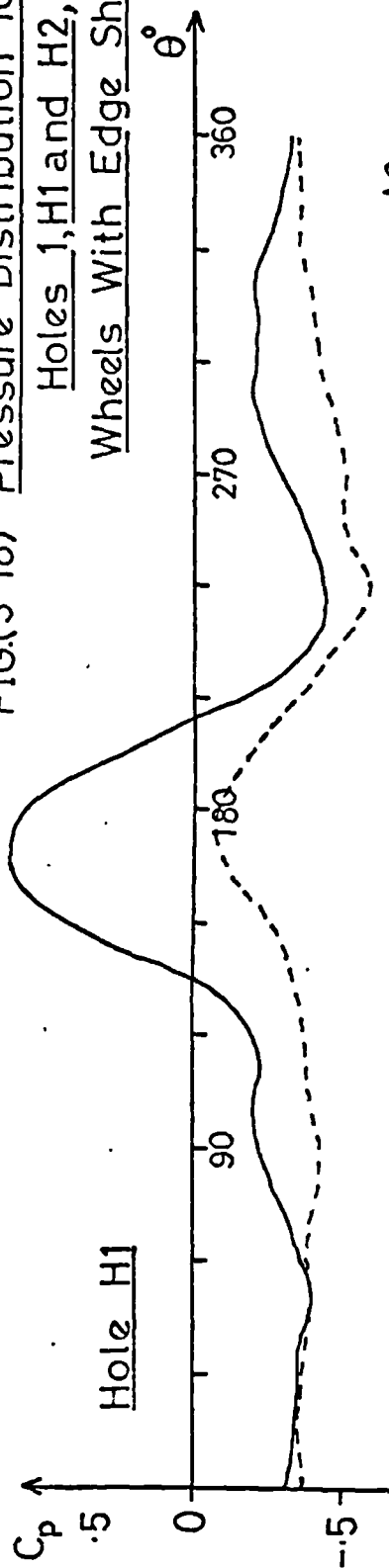


FIG.(3-19) Pressure Distribution at Hole 10
Wheel B2 Rotating and Stationary

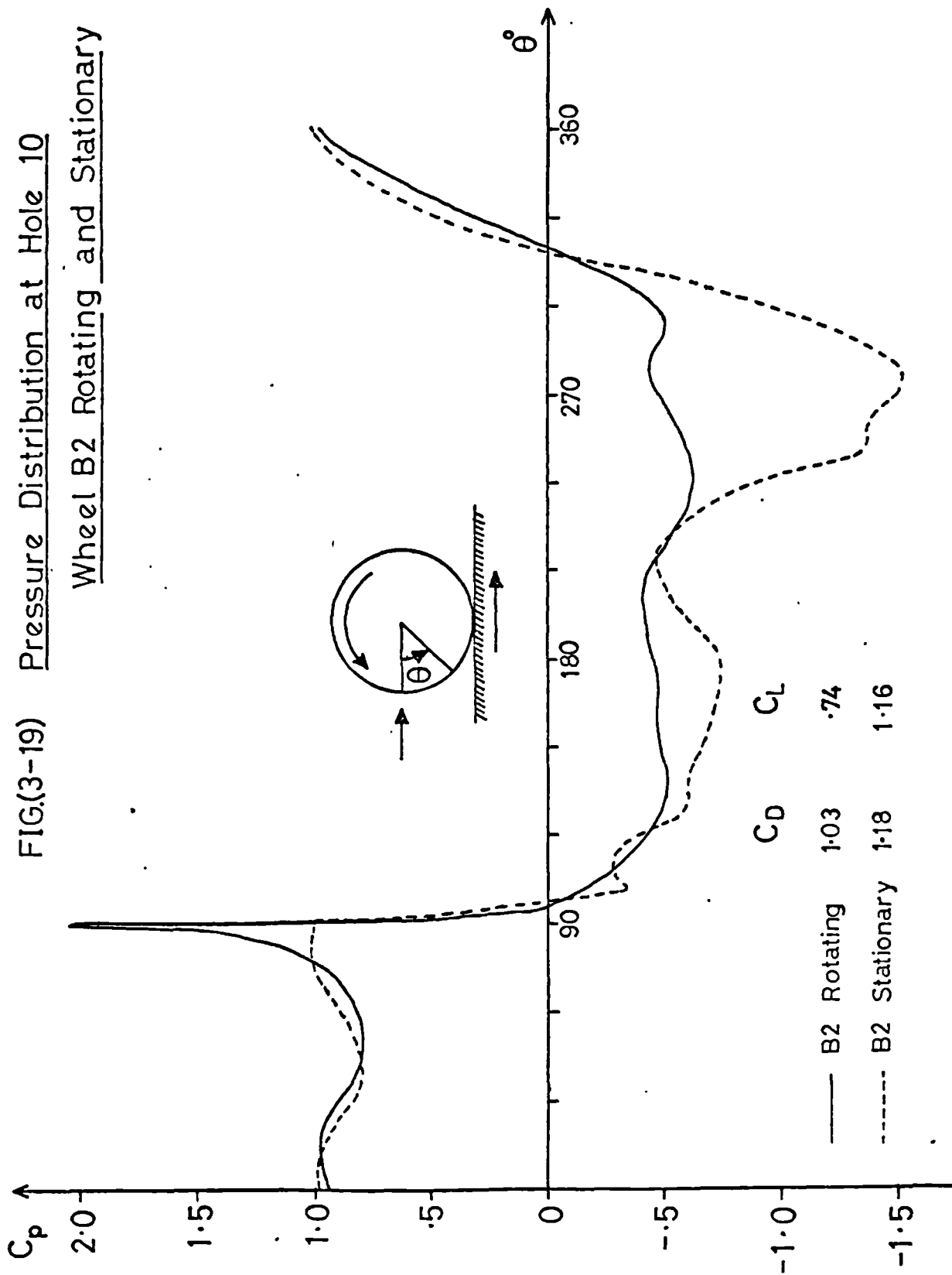


FIG.(3-20) Pressure Distribution Around Centre-Line
of Rotating Wheel

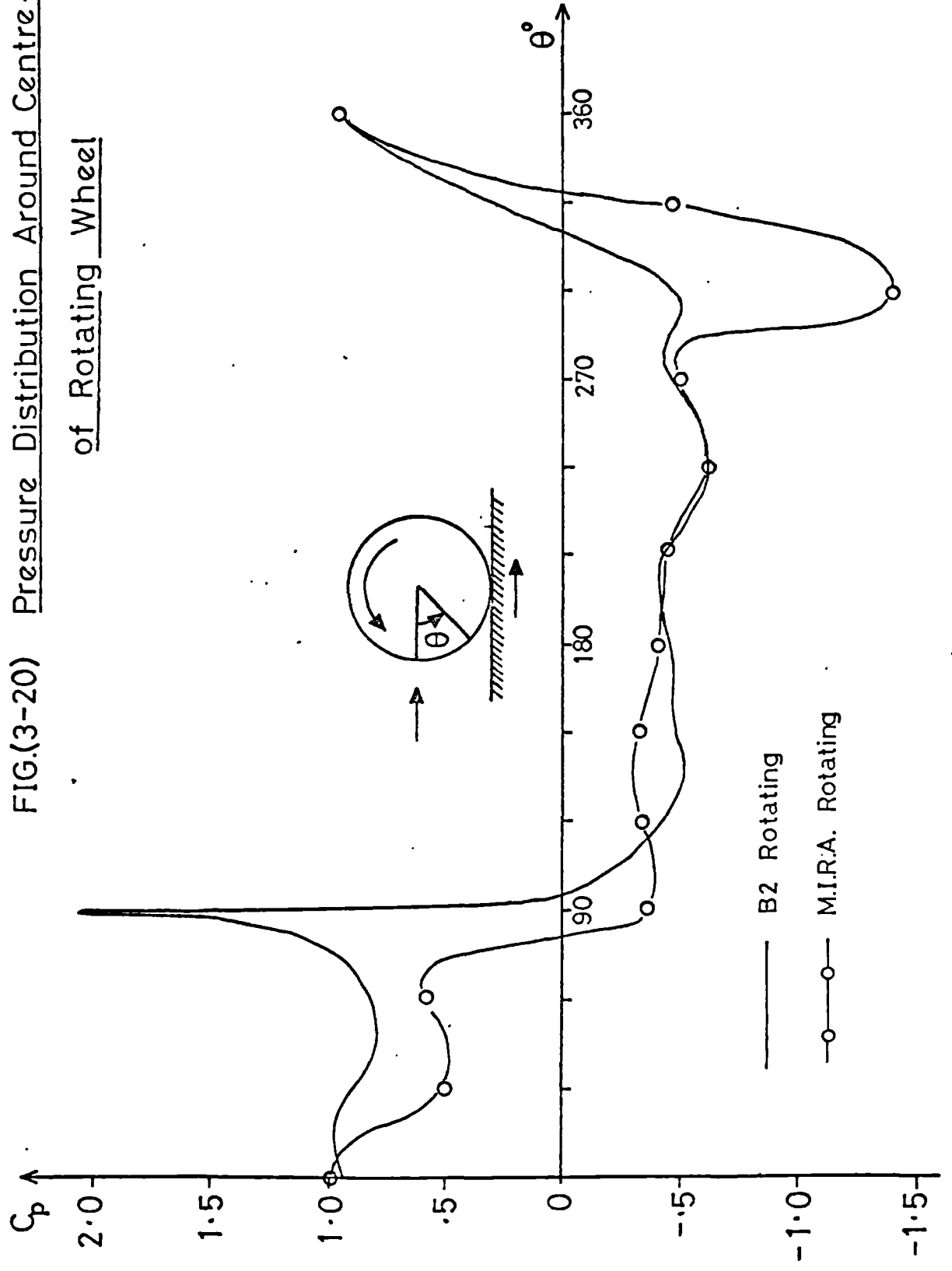
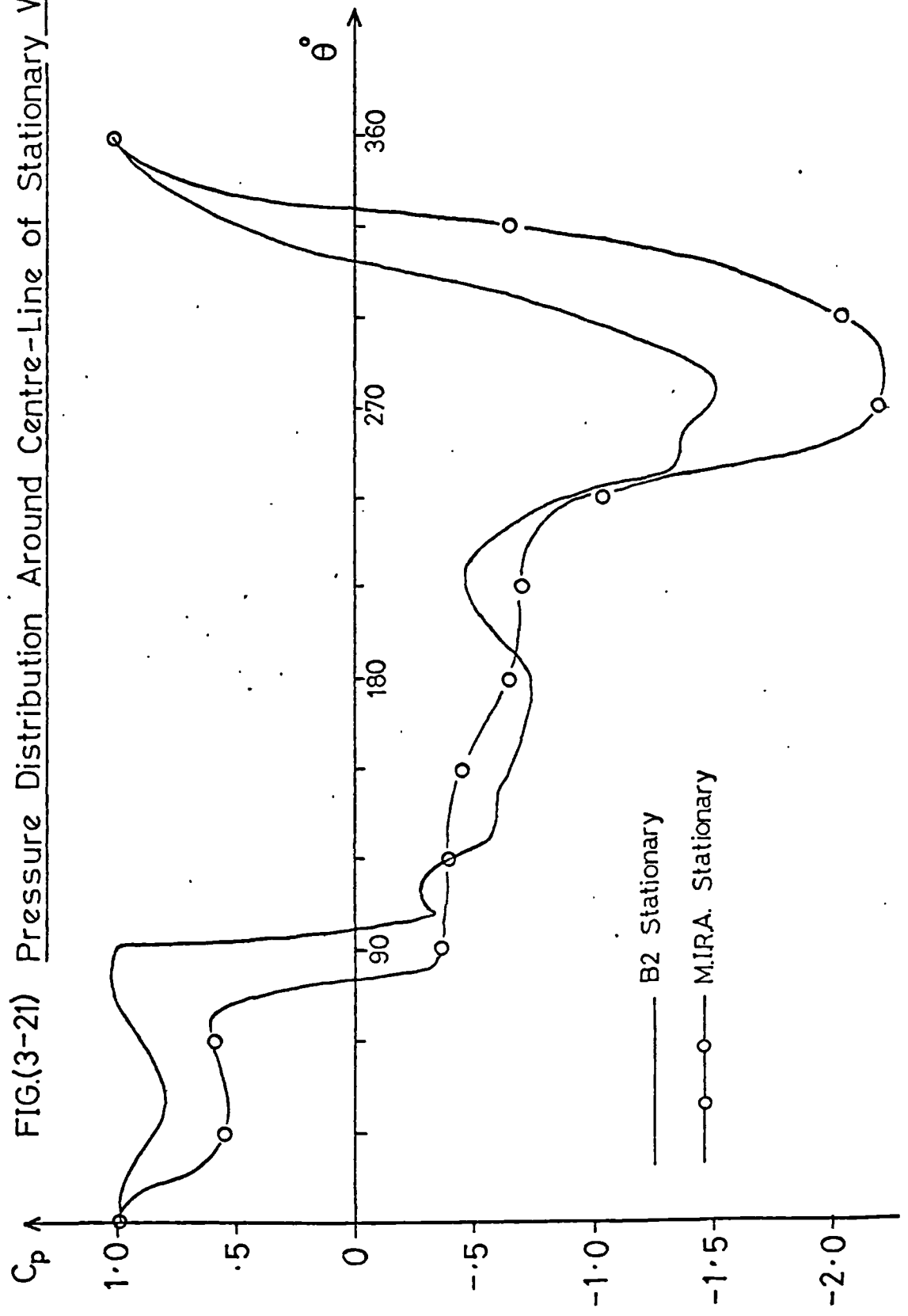


FIG.(3-21) Pressure Distribution Around Centre-Line of Stationary Wheel



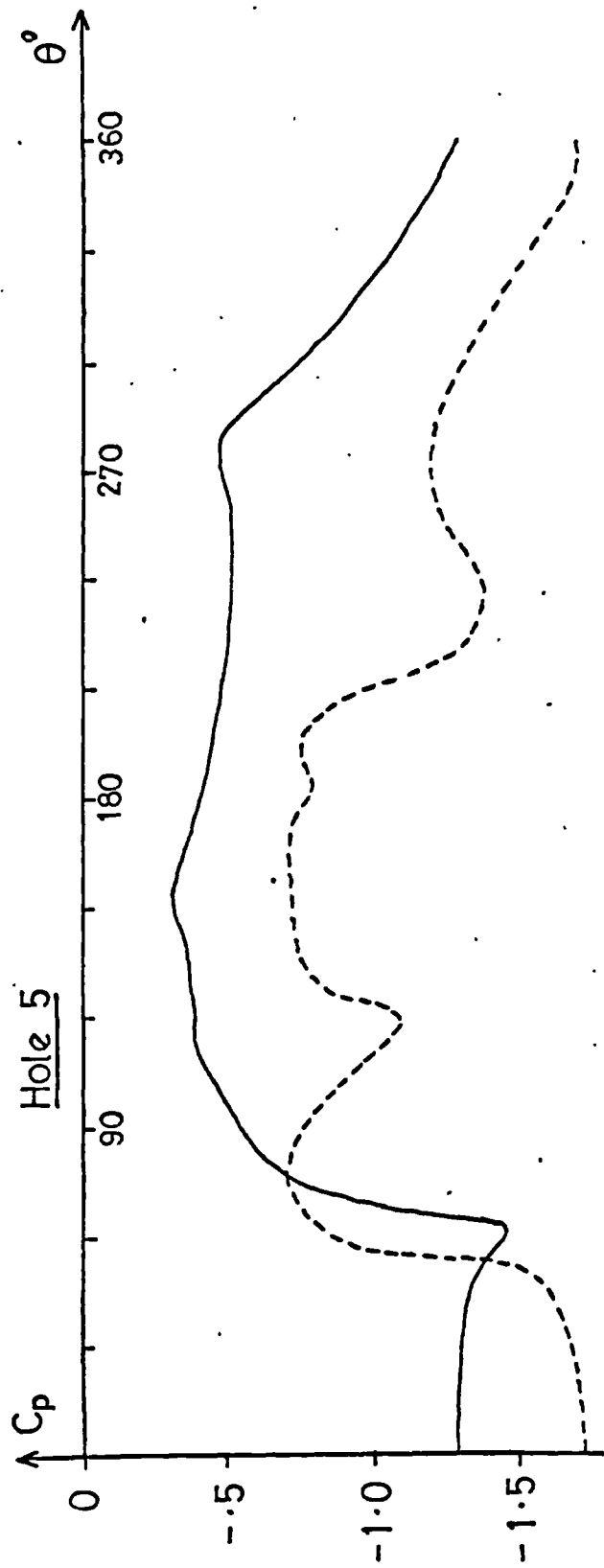


FIG.(3-22) Pressure Distributions at Holes 5 and 6
Wheel B2 Rotating and Stationary

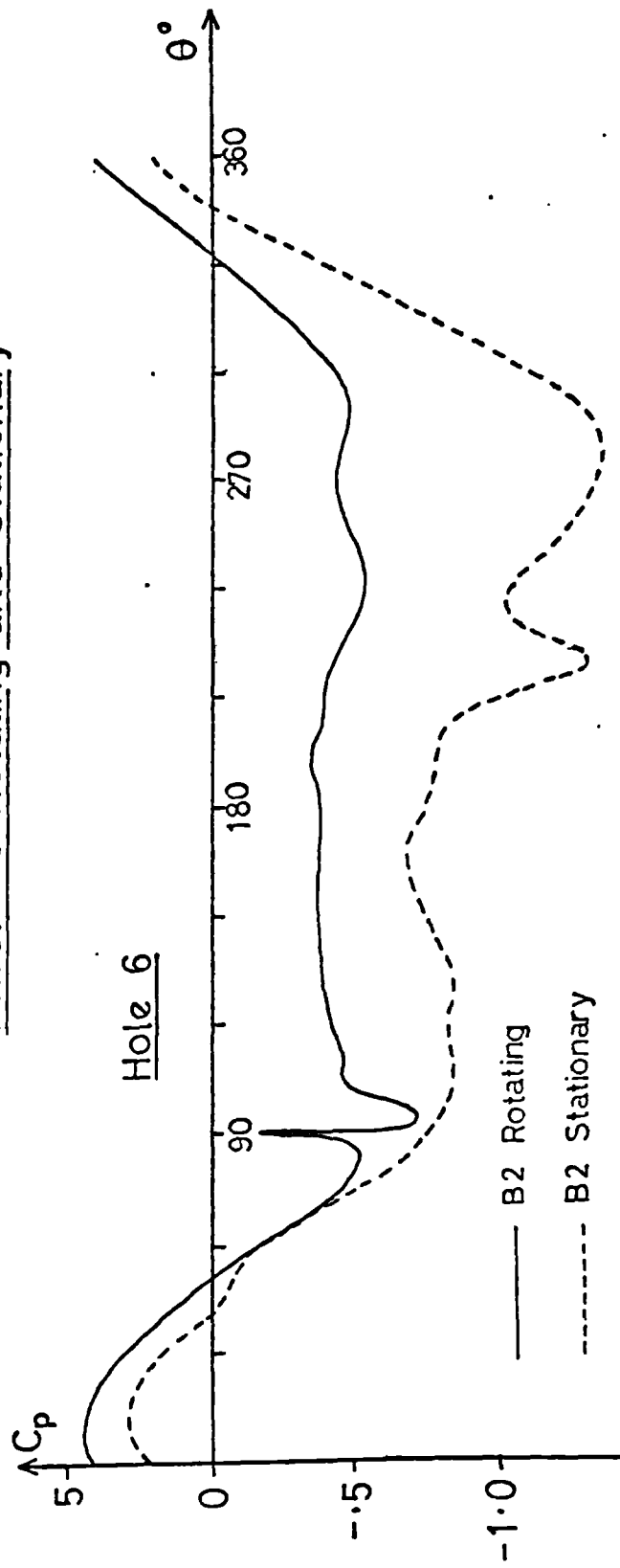


FIG. (3-23)

Variation of C_L, C_D Across

Wheel B2, Rotating

and Stationary

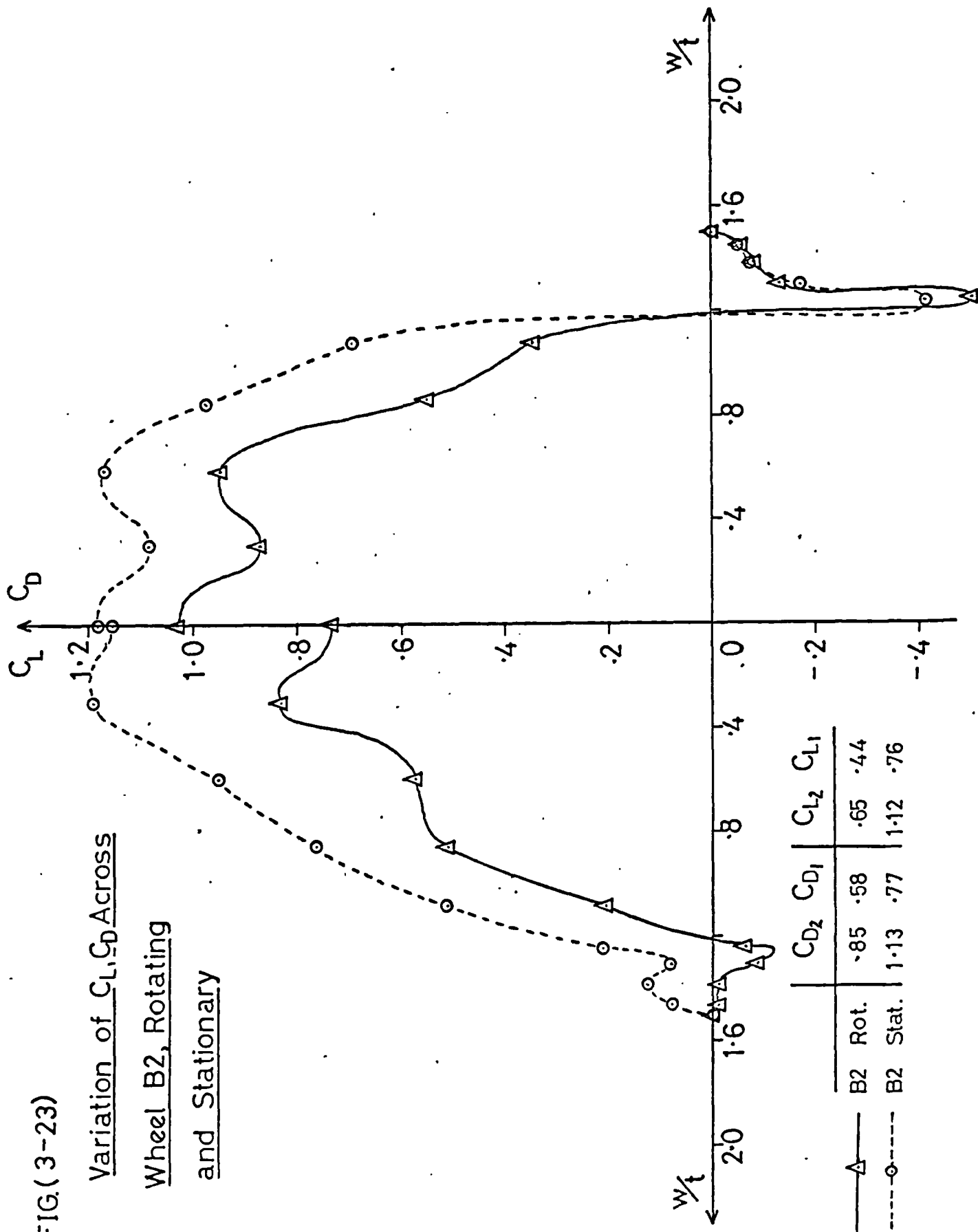
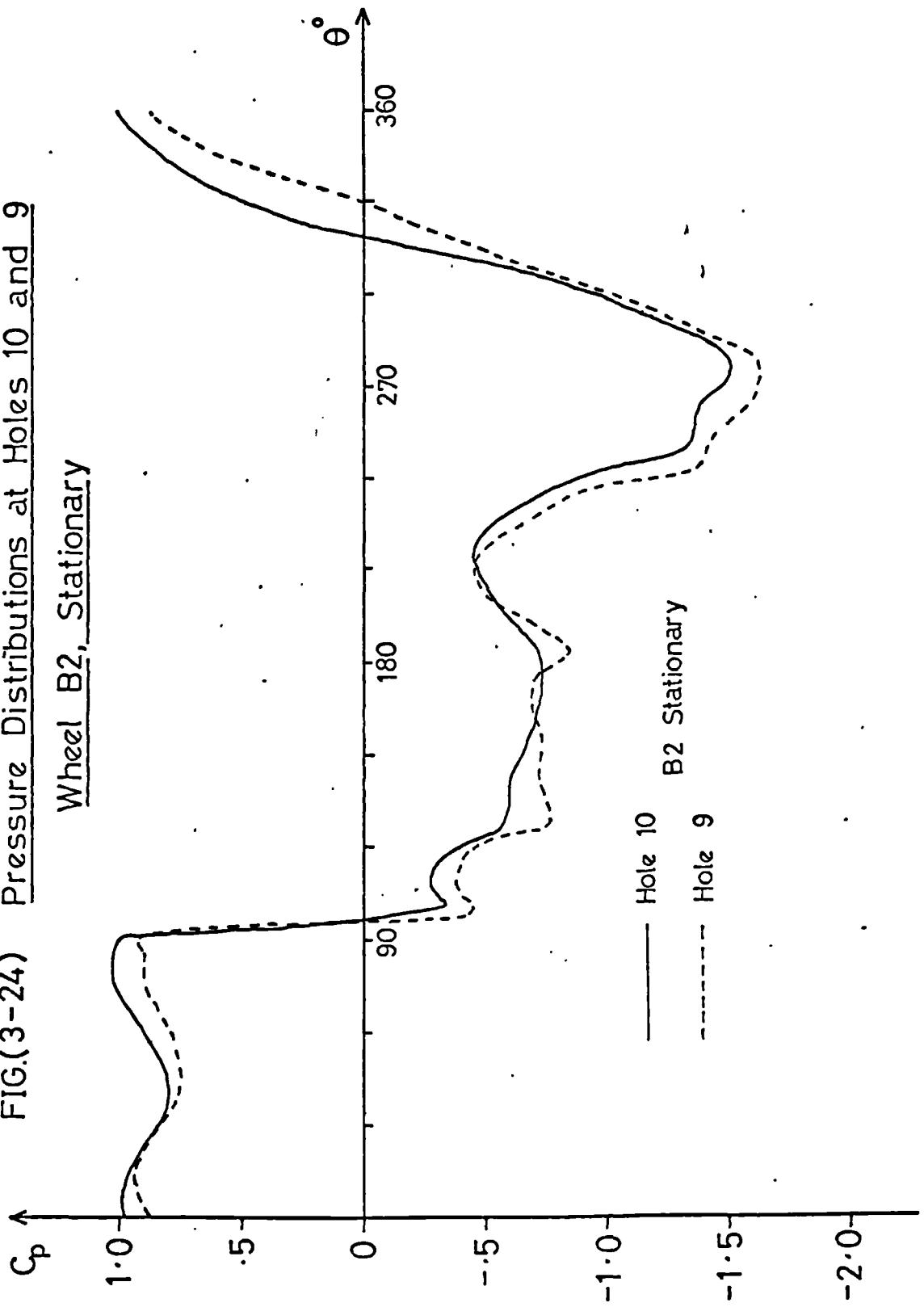


FIG.(3-24) Pressure Distributions at Holes 10 and 9

Wheel B2, Stationary



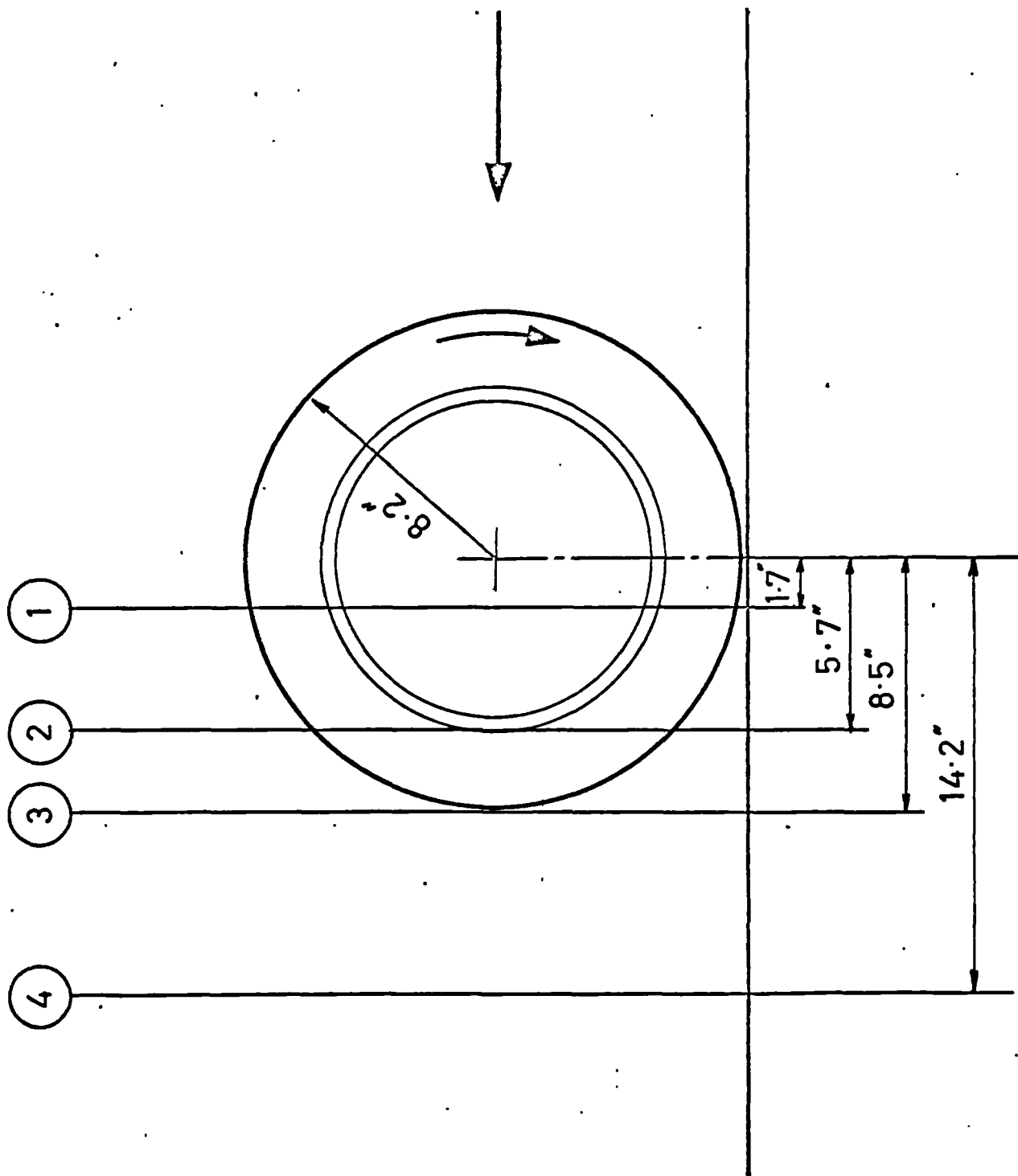


Fig.(3-25) Position of Total Head Contour Planes.

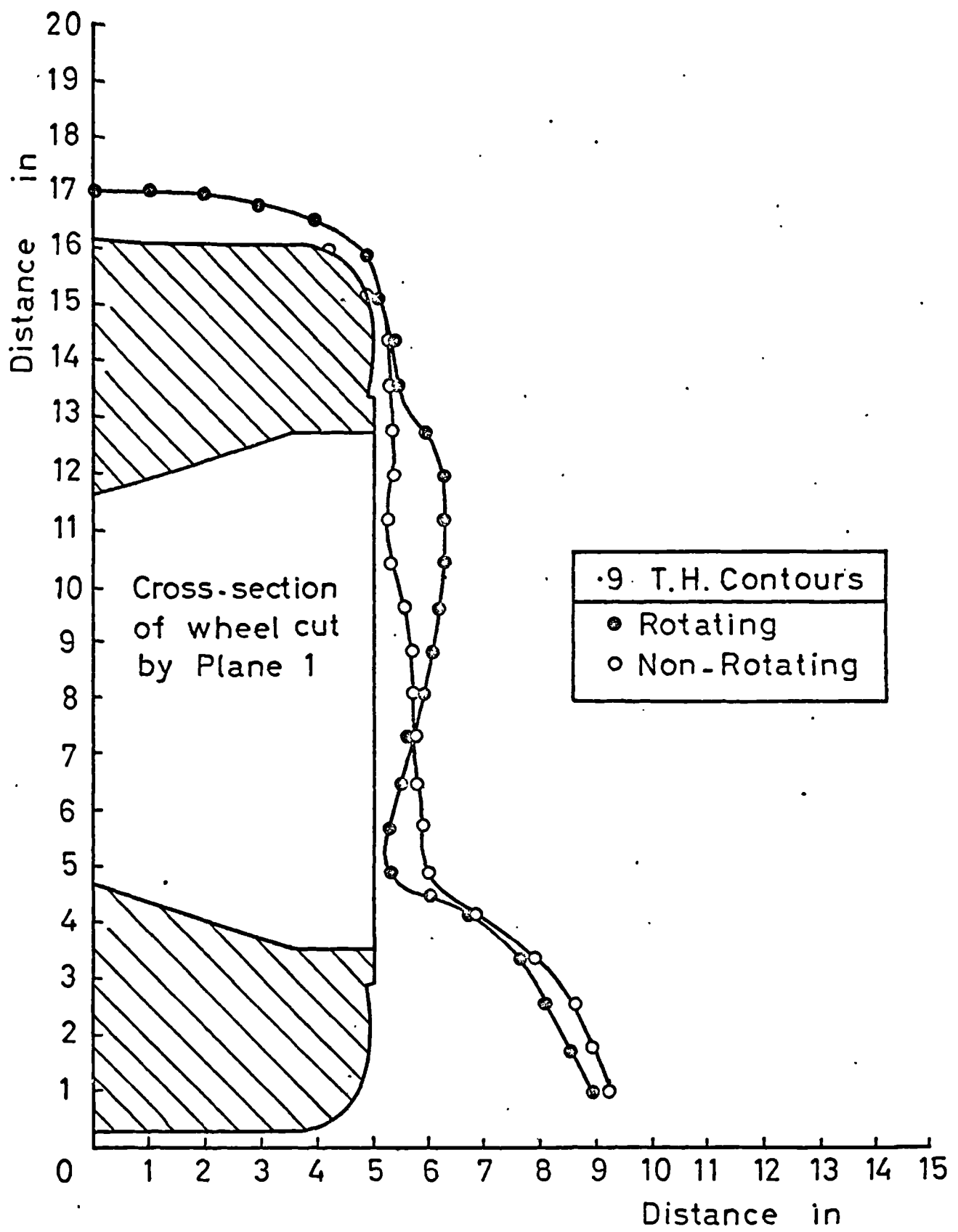
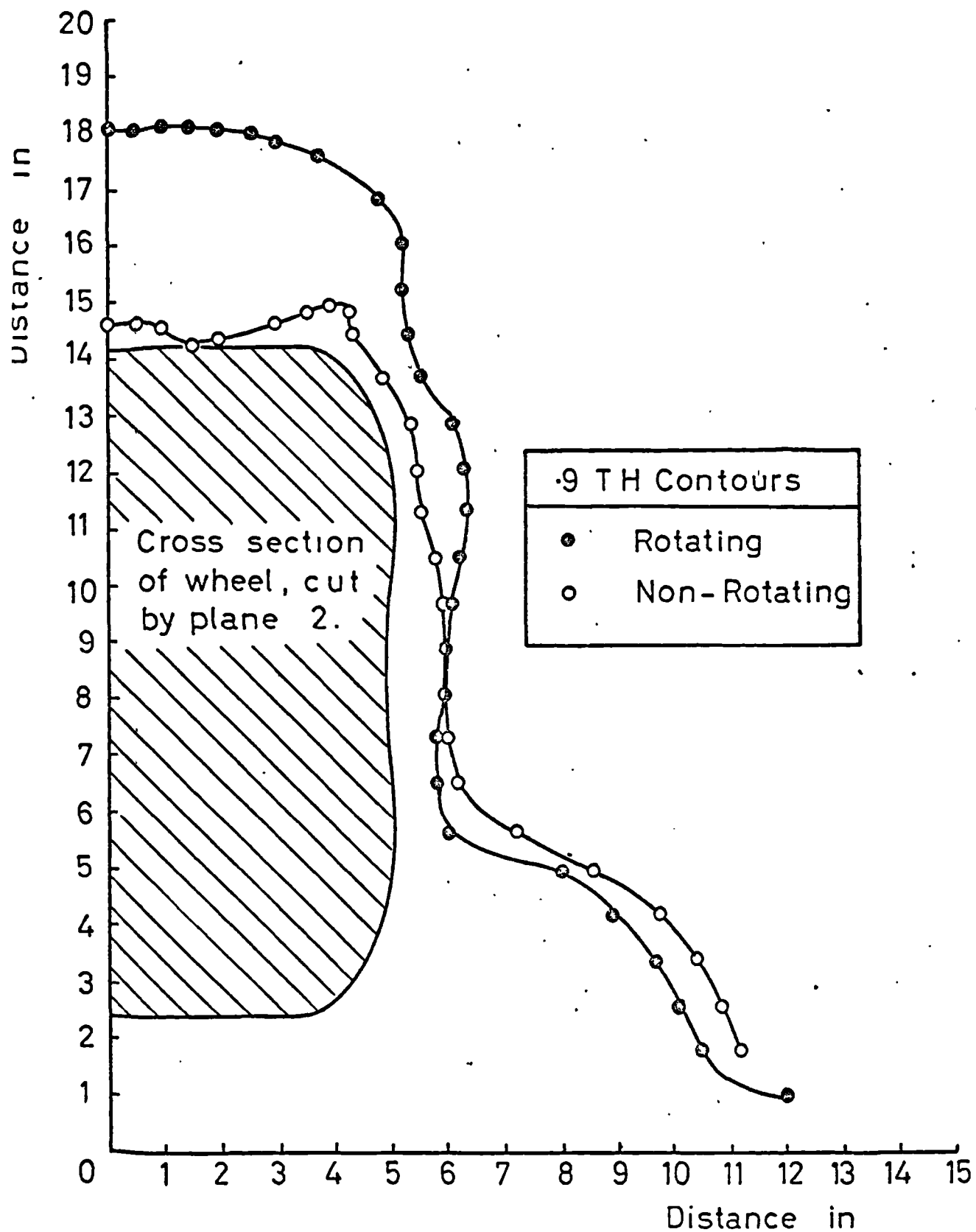


Fig.(3-26) Total Head Contours at Plane 1.
1.7 in from wheel centre.



Fig(3-27) Total Head Contours at Plane 2.
5.7 in from wheel centre.

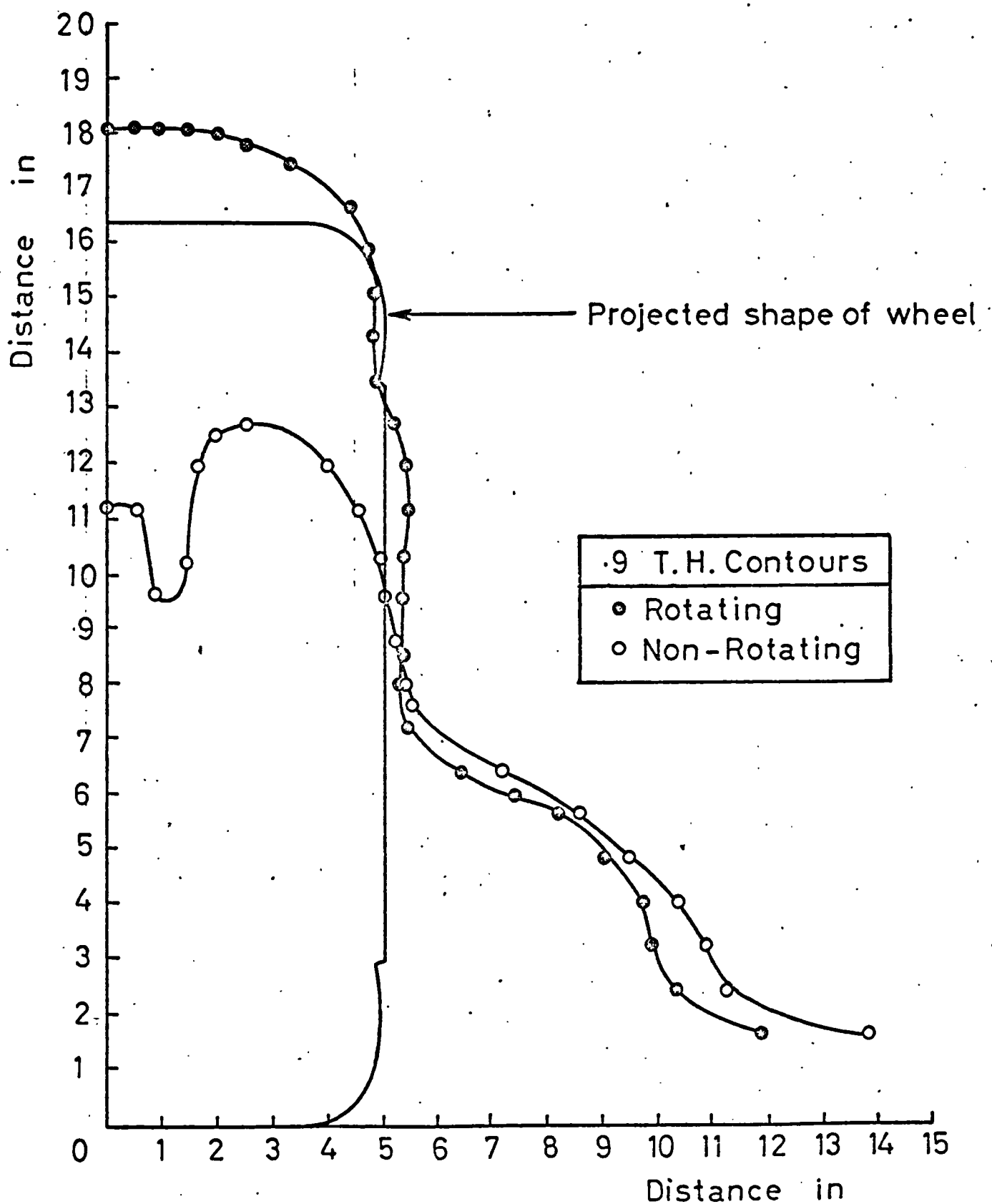


Fig.(3-28) Total Head Contours at Plane 3.
8.5 in from wheel centre.

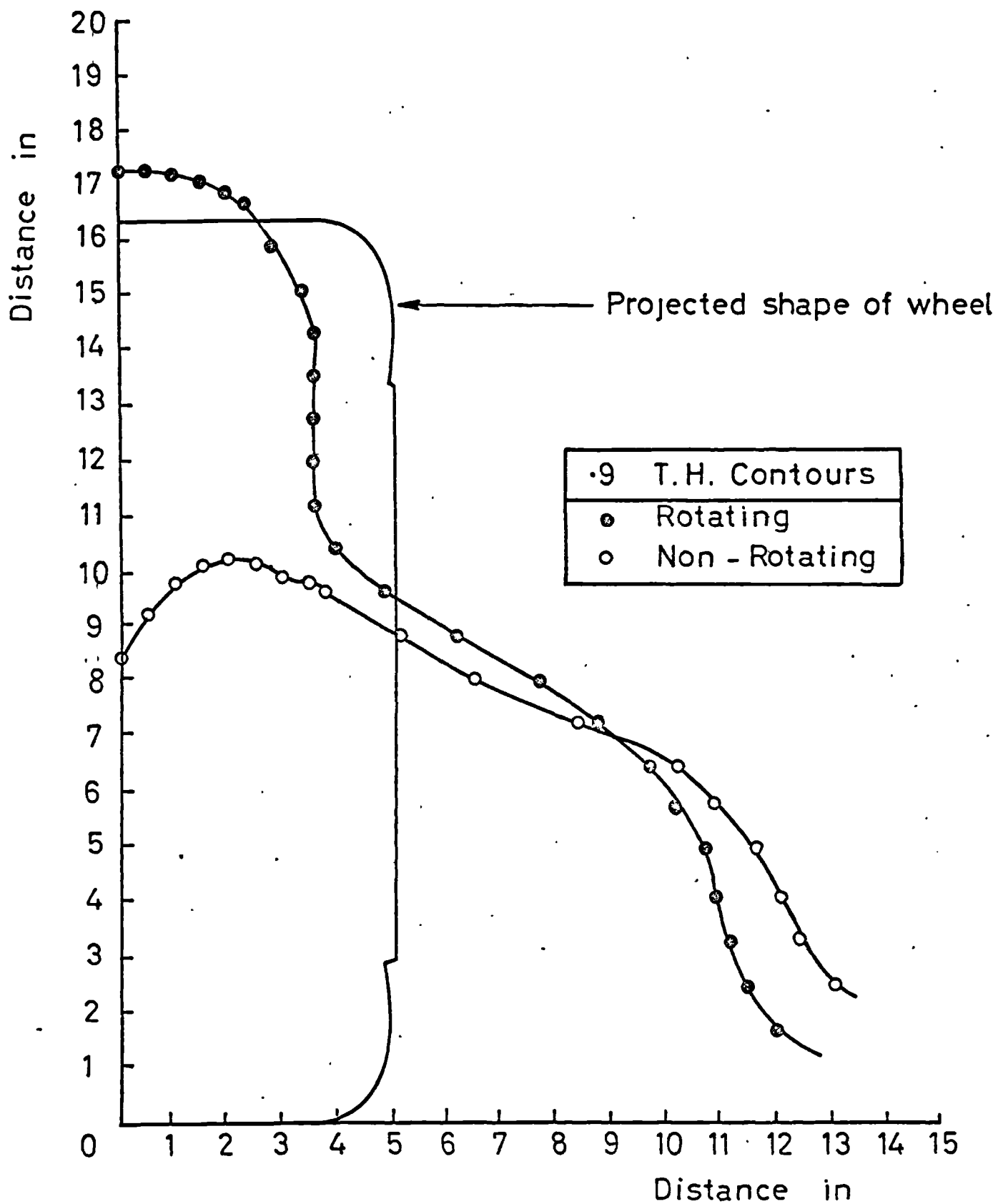
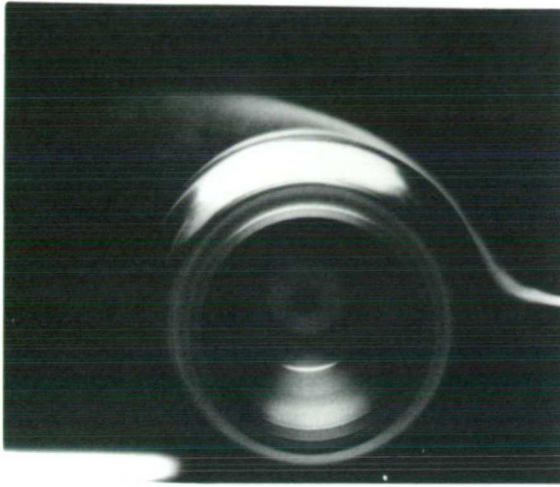
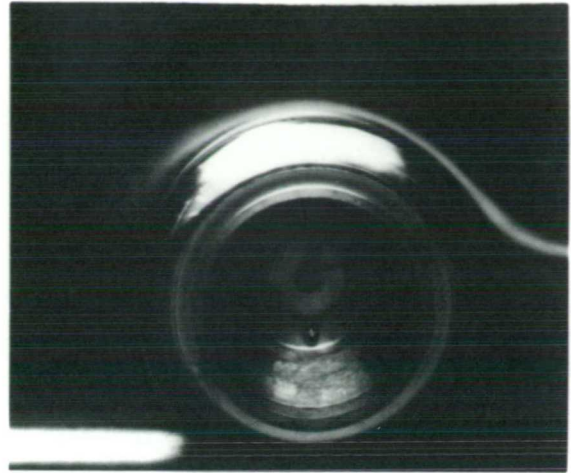


Fig.(3-29) Total Head Contours at Plane 4.
14.2 in from wheel centre



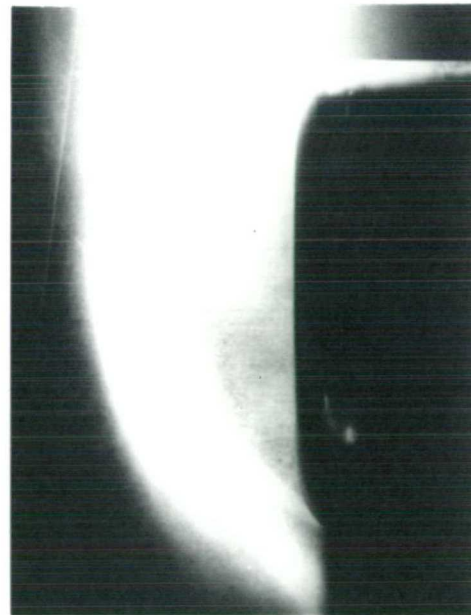
(i) Side View , Rotating



(ii) Side View , Stationary

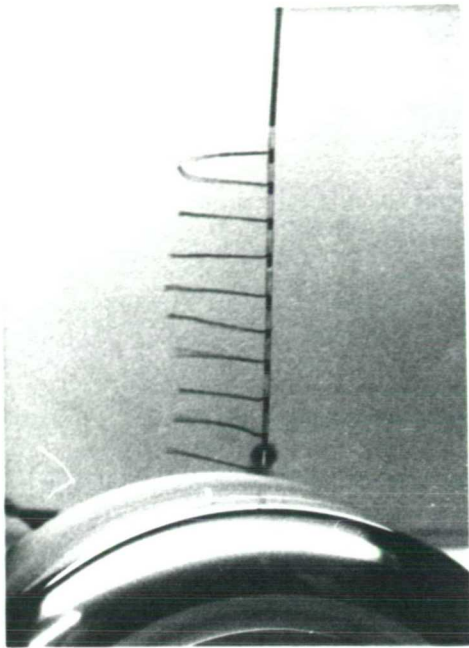


(iii) Top View , Rotating

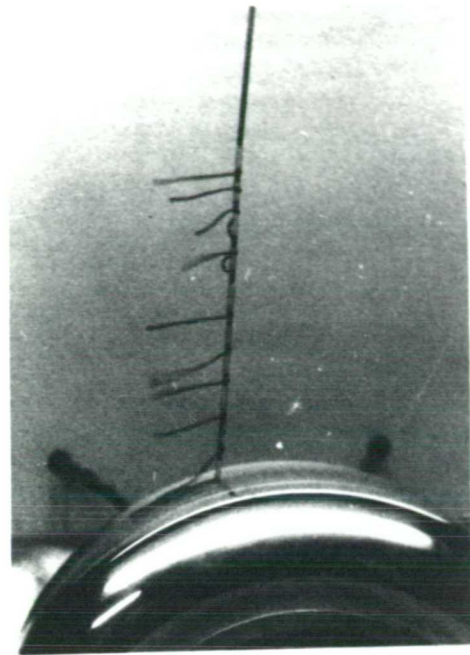


(iv) Top View , Stationary

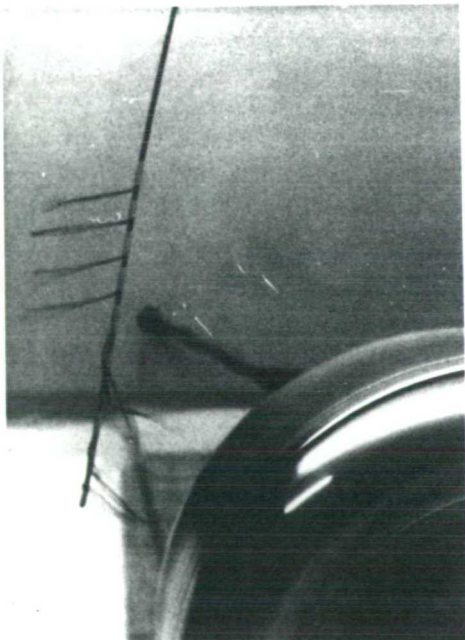
FIG. (3-30) Smoke Visualisation of Flow past Wheel



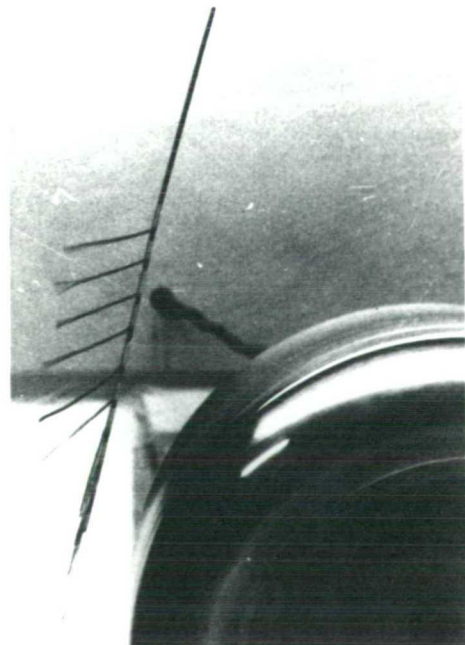
(i) Top of Wheel ,Rotating



(ii) Top of Wheel ,Stationary



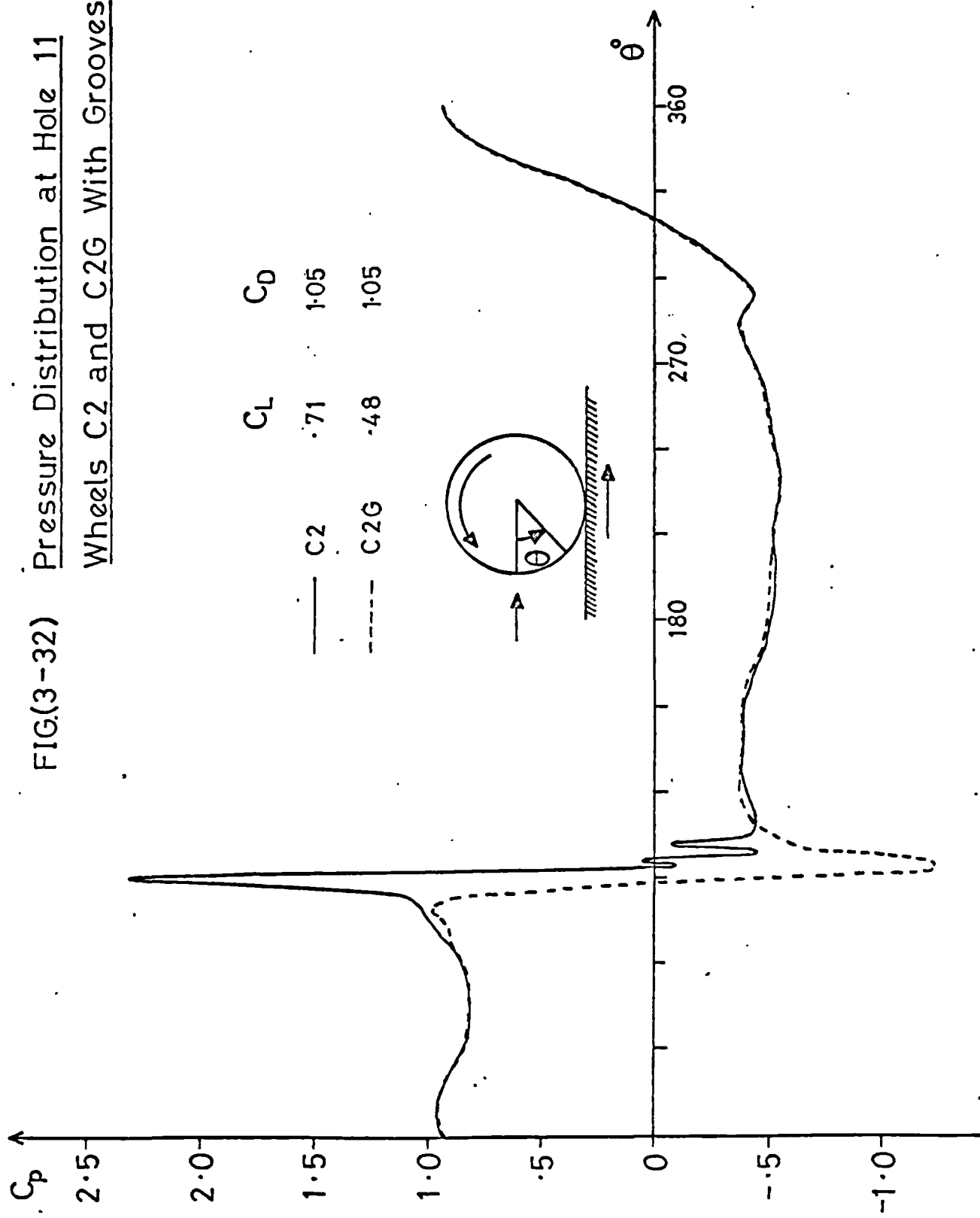
(iii) Behind Top ,Rotating



(iv) Behind Top ,Stationary

FIG.(3-31) Wool Tufts Showing Flow over Wheel

FIG(3-32) Pressure Distribution at Hole 11
Wheels C2 and C2G With Grooves



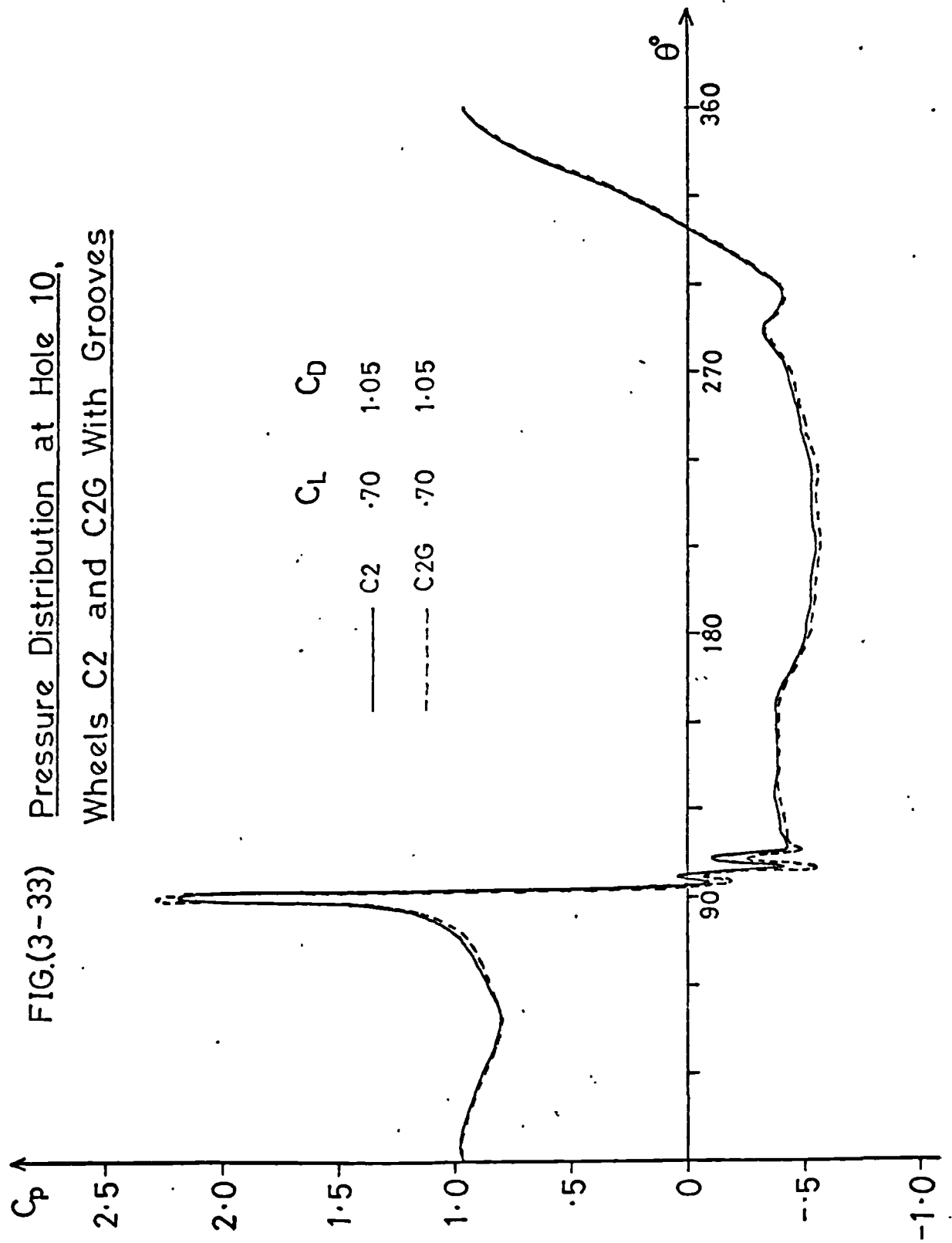


FIG.(3-34) Variation of C_L, C_D Across
Wheels C2 and C2G

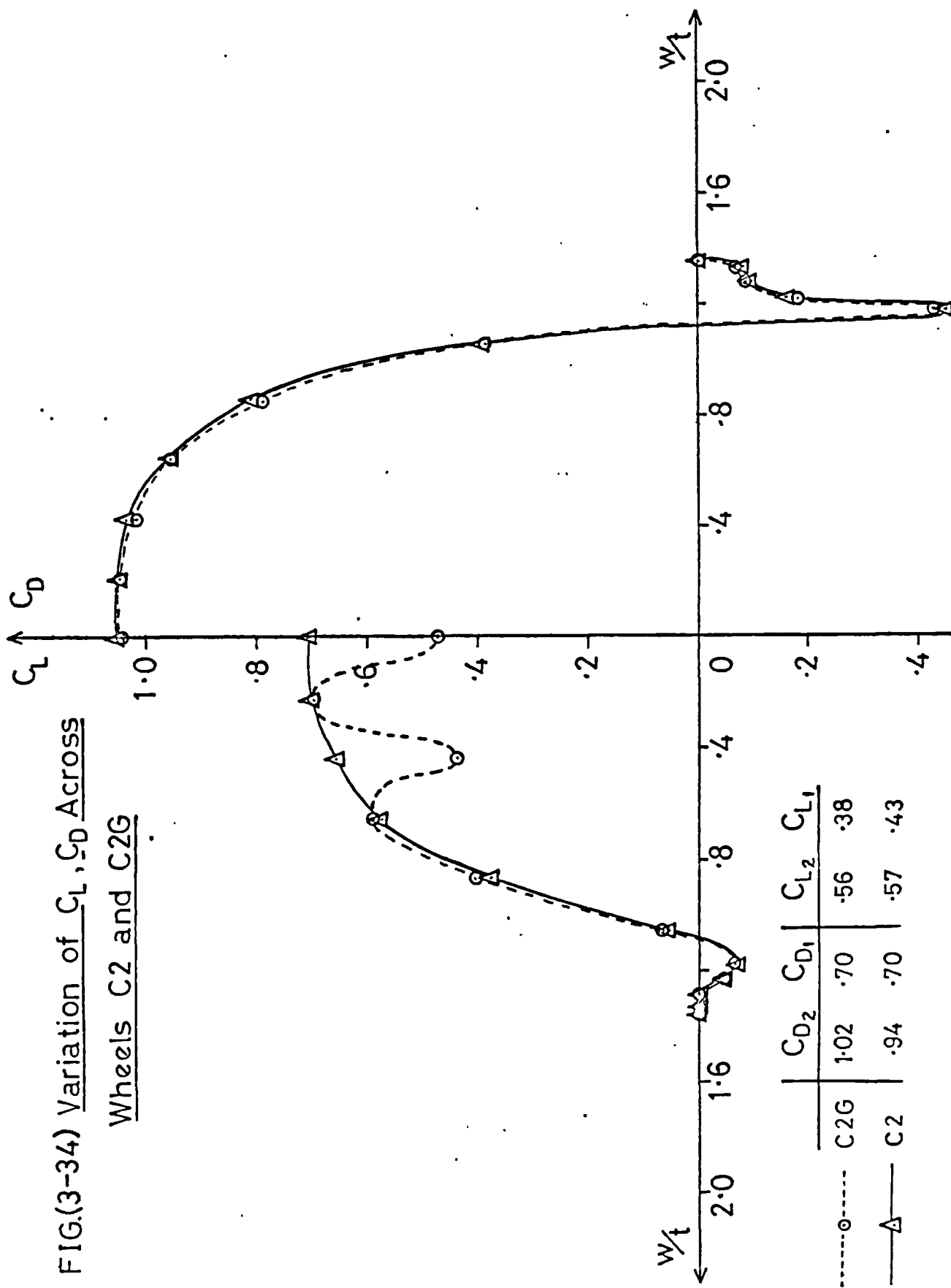


FIG.(3-35) Pressure Distribution Around the Centre-Line of Wheel C2
at Different Values of v/U_∞

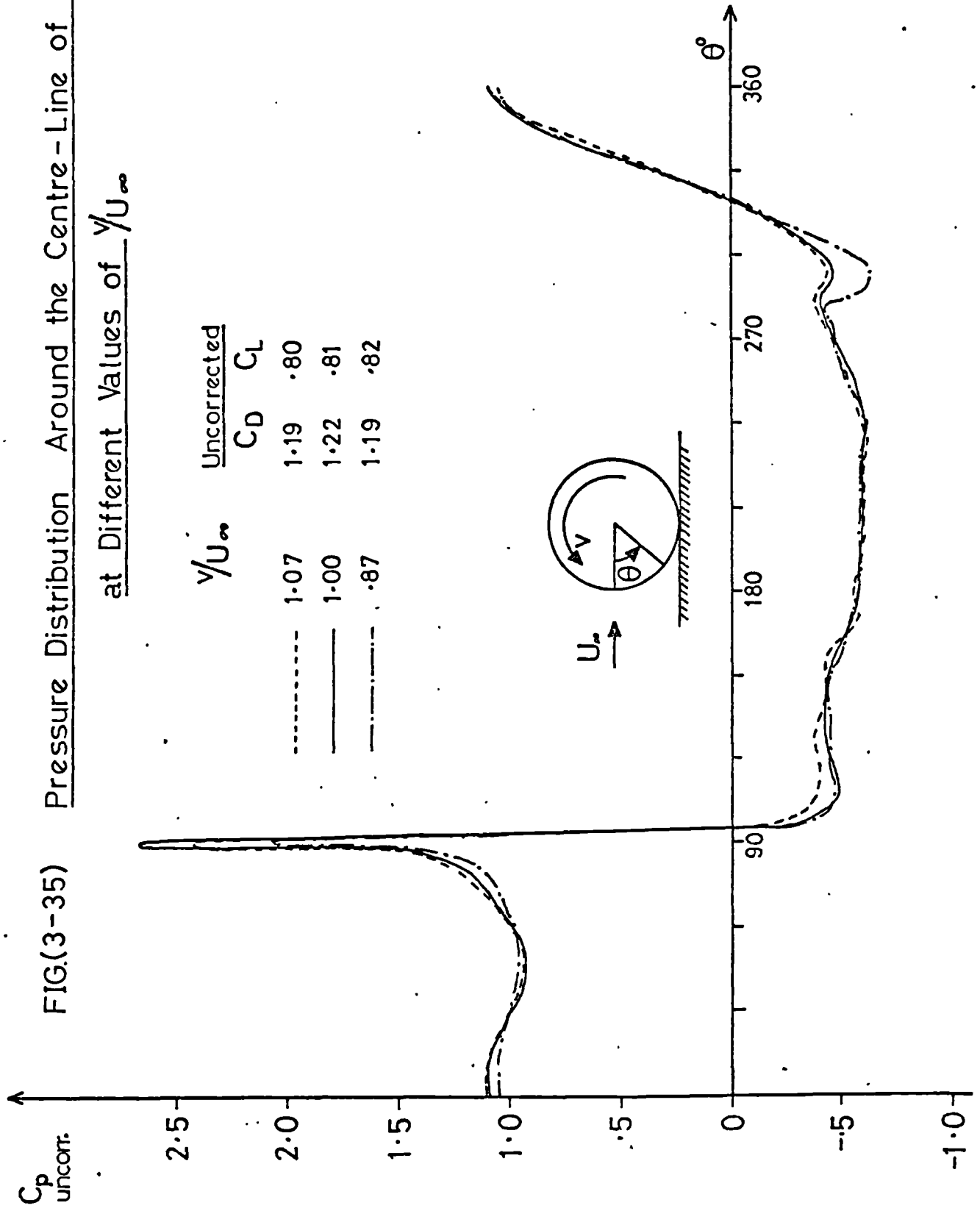


FIG.(3-36) Pressure Distribution Around the Centre-Line
of Wheel C2 at Different Values of v/U_∞

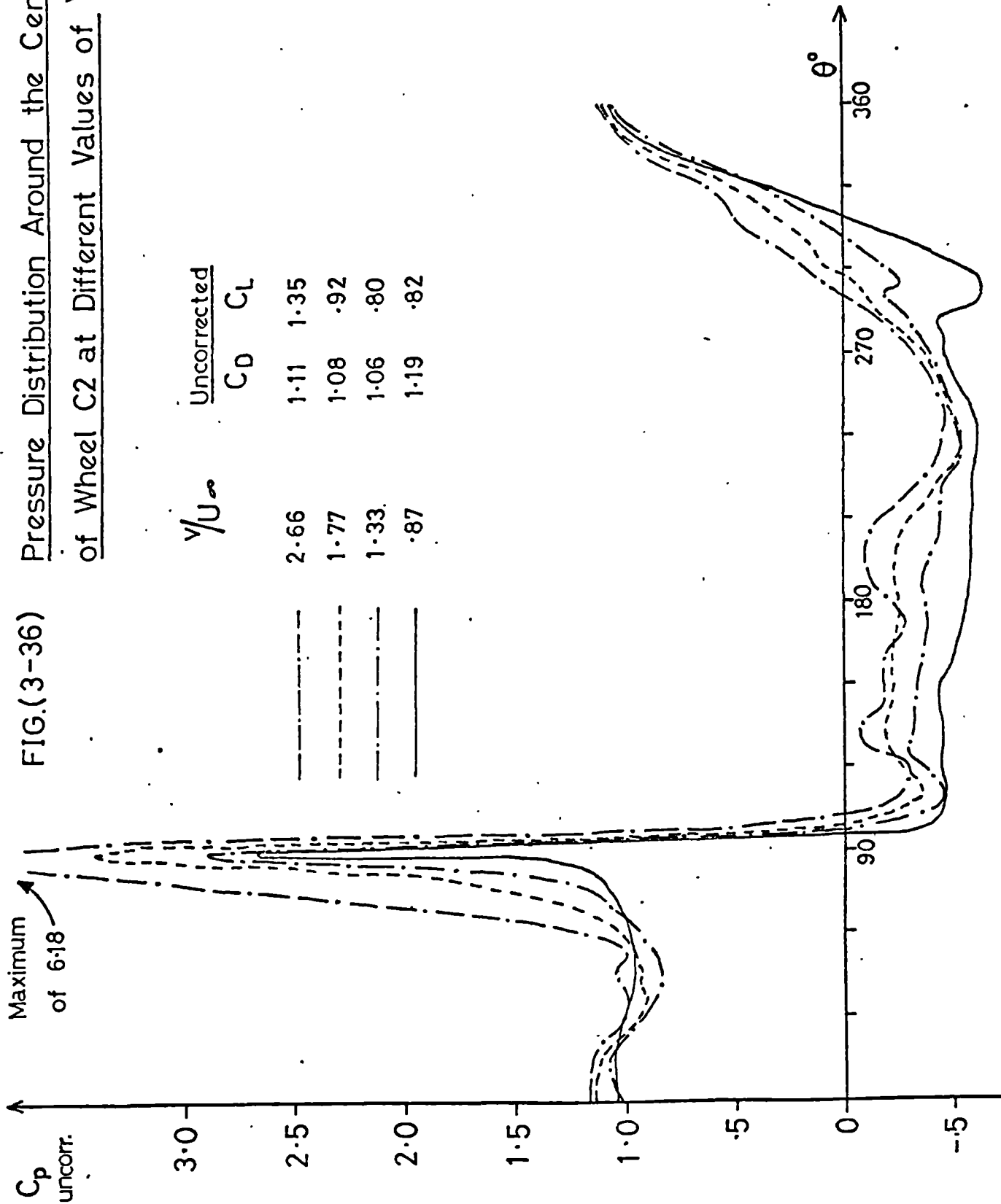


FIG.(3-37)

'Two Dimensional' Tunnel Arrangements

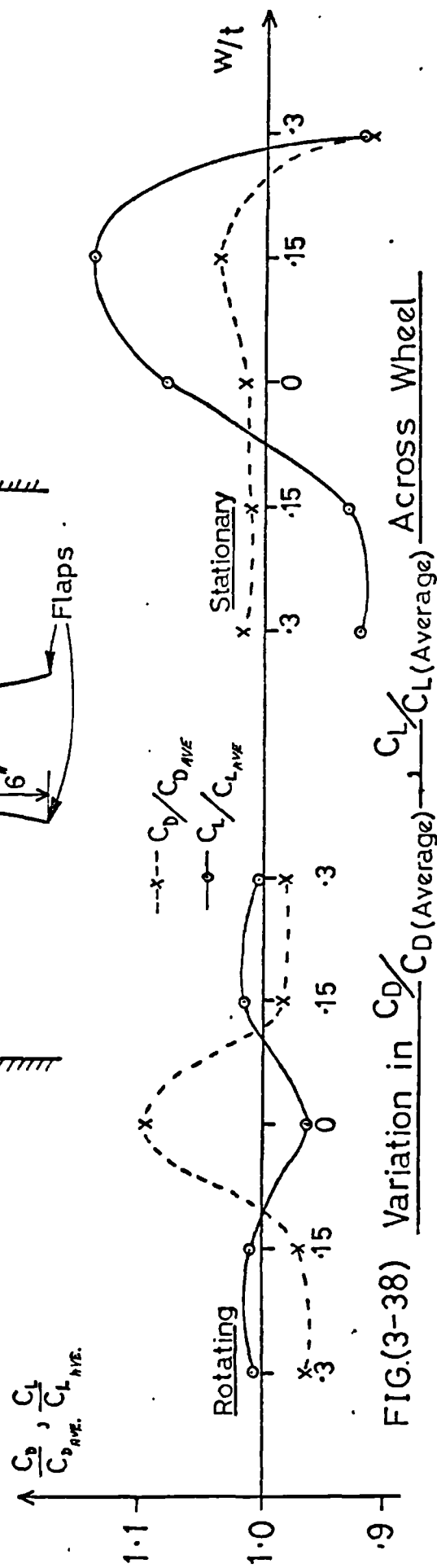
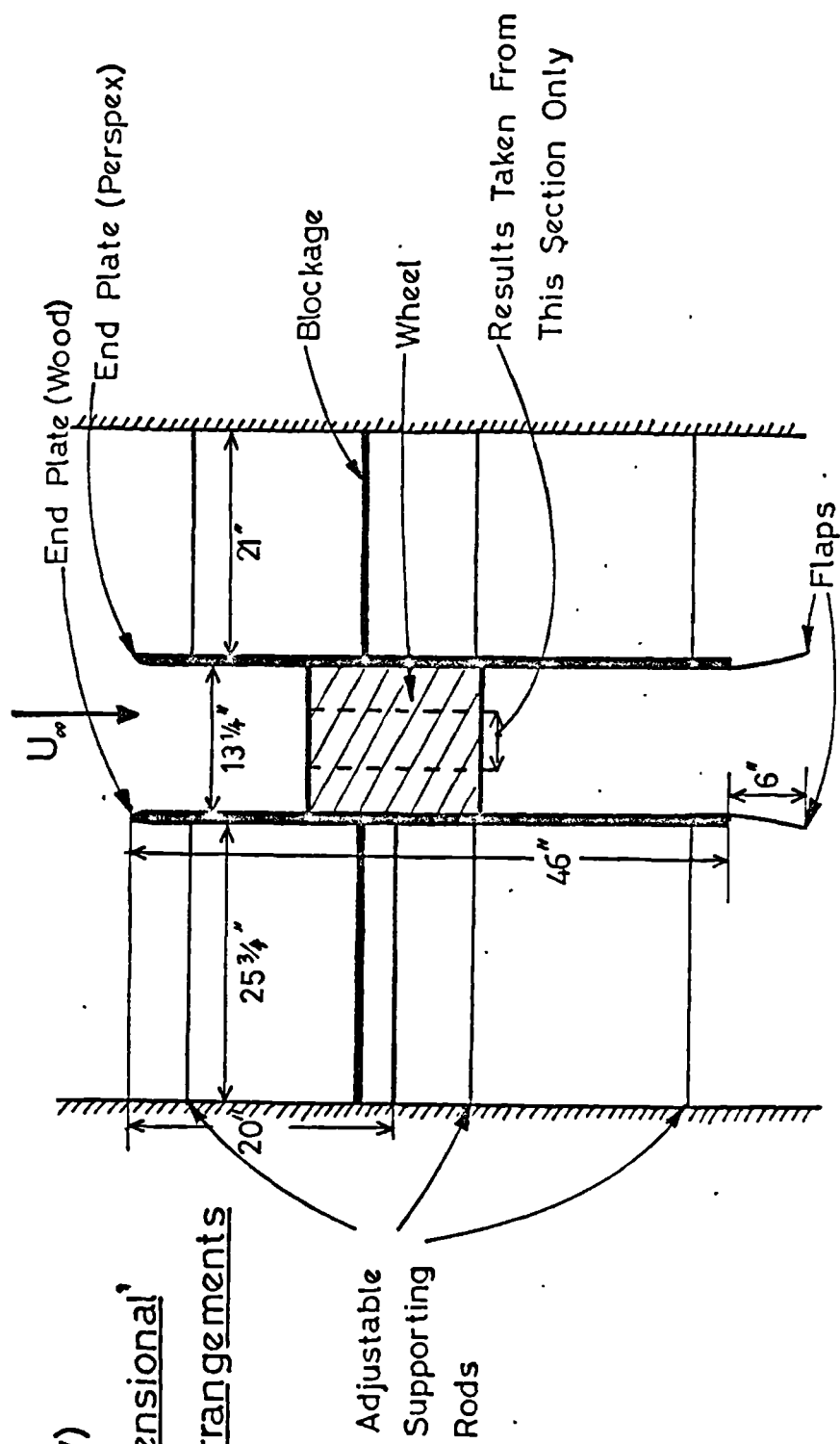
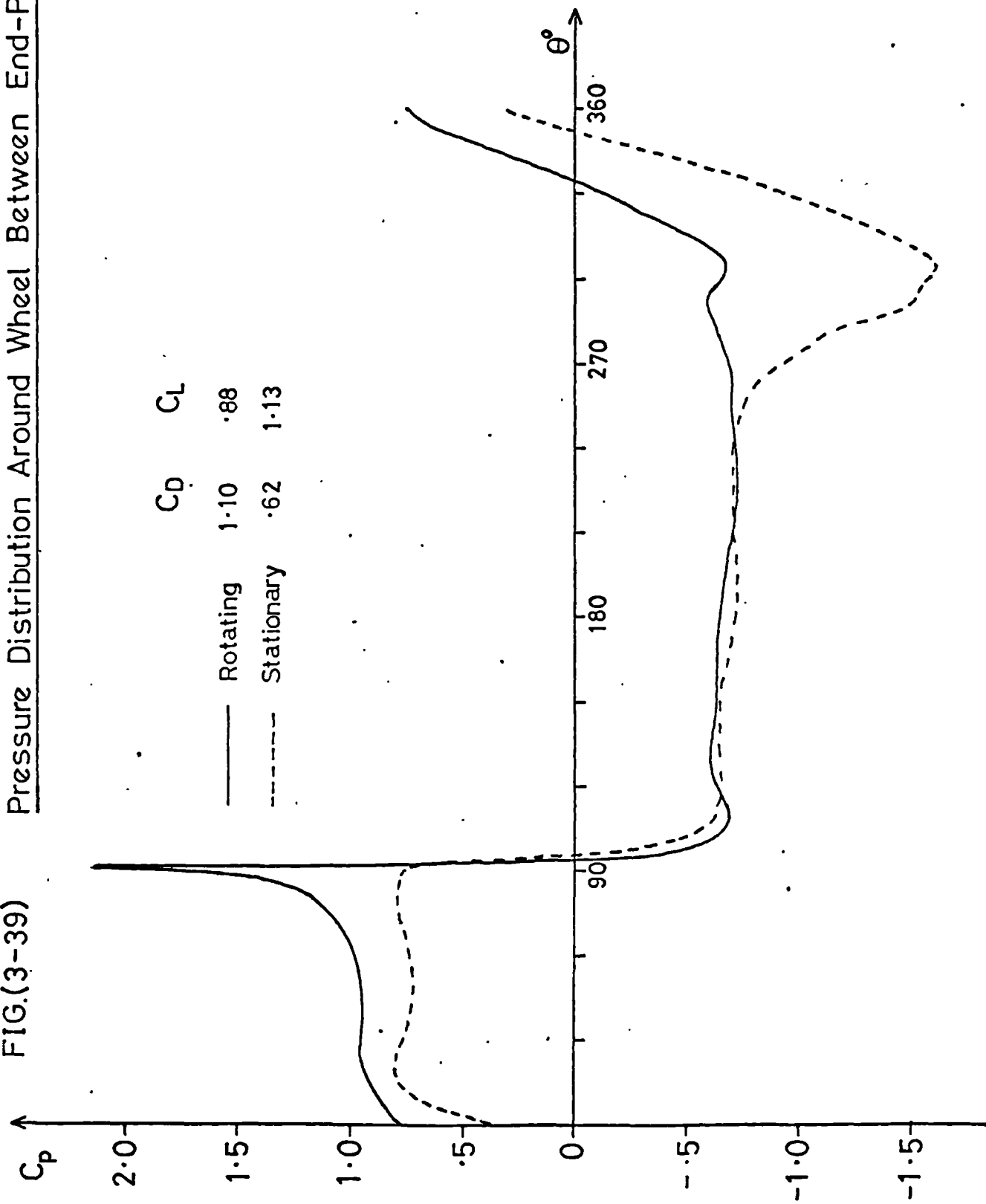
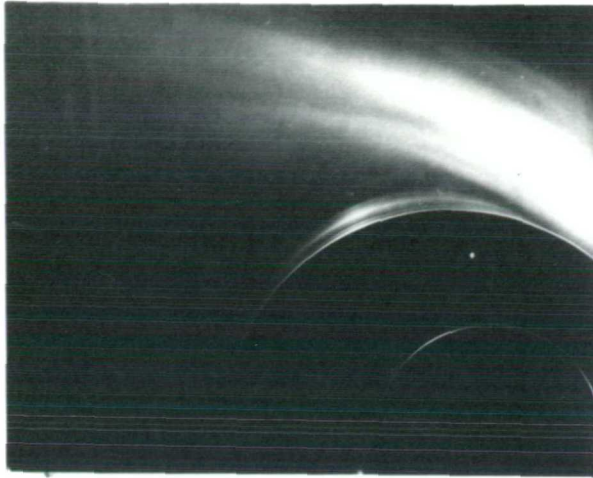


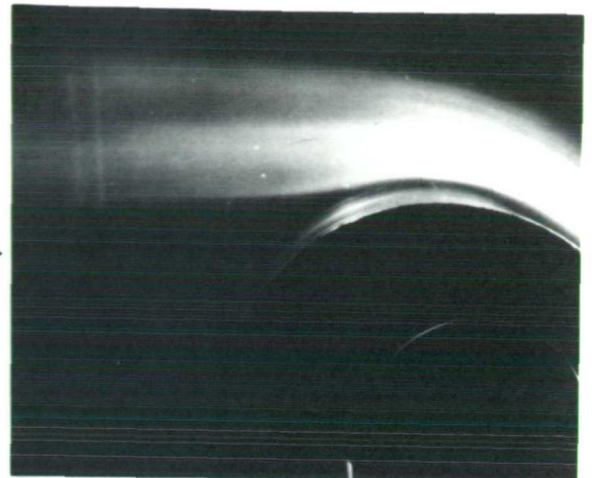
FIG.(3-38) Variation in $\frac{C_D}{C_{D(Average)}}$ and $\frac{C_L}{C_{L(Average)}}$ Across Wheel

FIG.(3-39) Pressure Distribution Around Wheel Between End-Plates

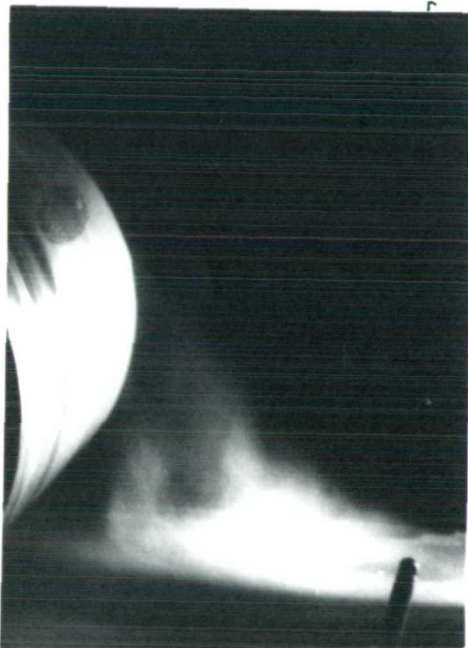




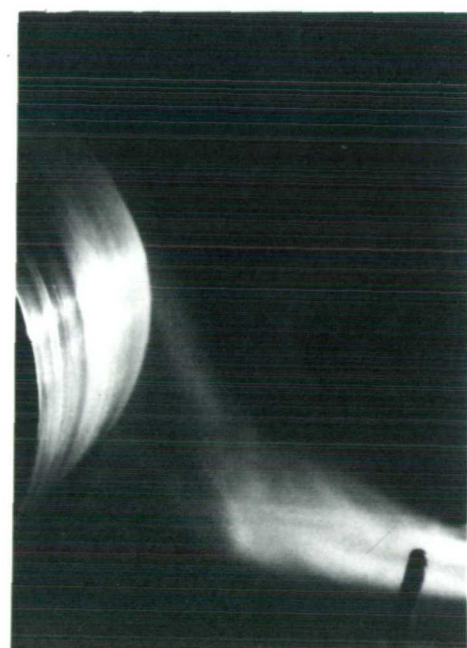
(i) Side View , Rotating



(ii) Side View , Stationary

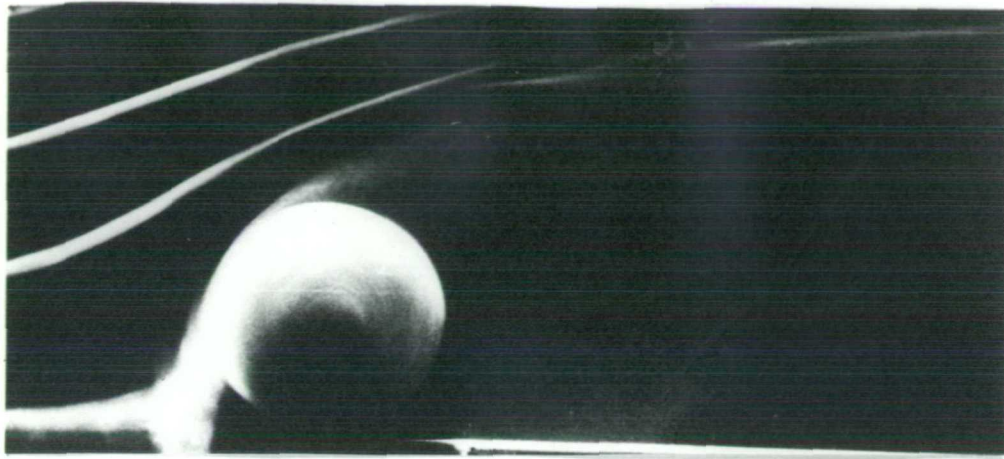


(iii) View of Front , Rotating

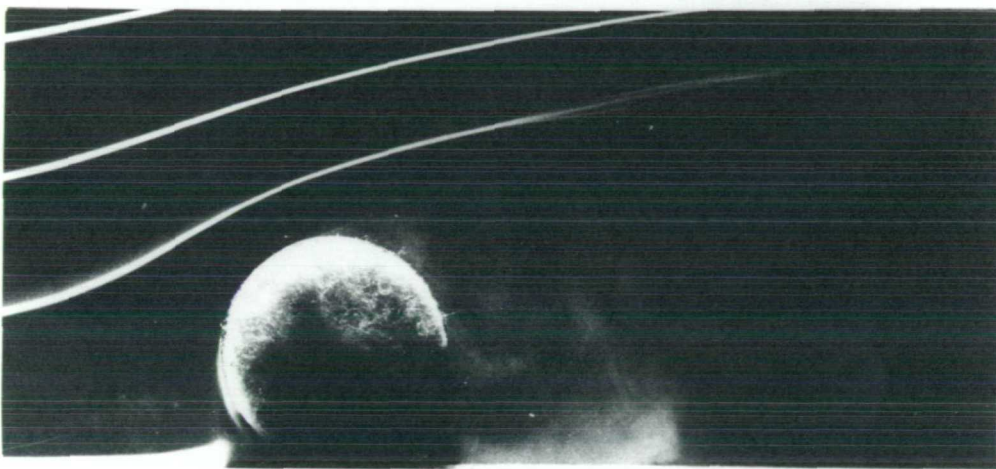


(iv) View of Front , Stationary

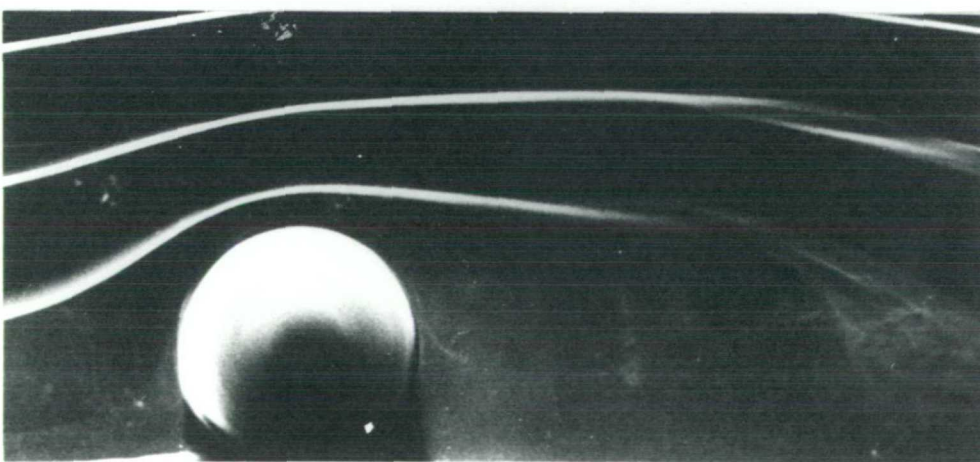
FIG.(3-40) Smoke Visualisation of 'Two-Dimensional'
Flow past Wheel



(i) Cylinder Rotating as Wheel , $v/U_\infty = 1$



(ii) Cylinder Stationary , $v/U_\infty = 0$

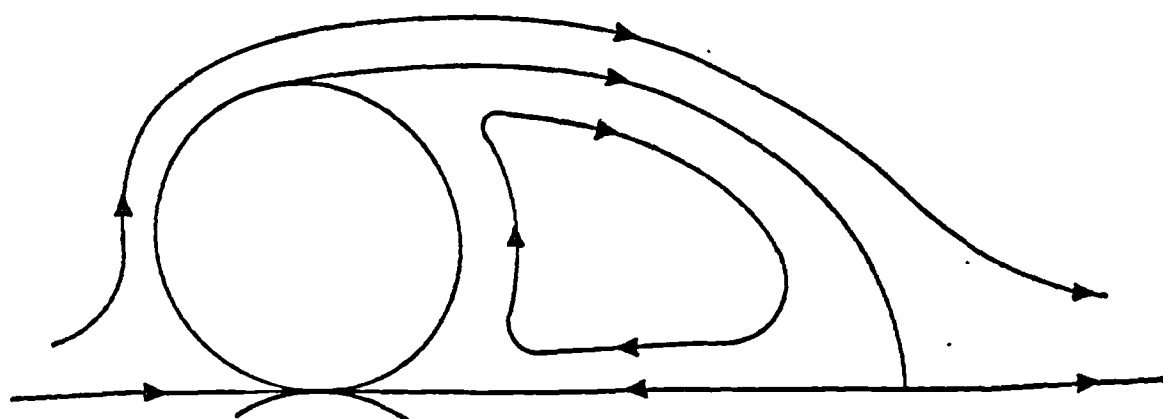


(iii) Cylinder Rotating Opposite to Wheel , $v/U_\infty = -1$

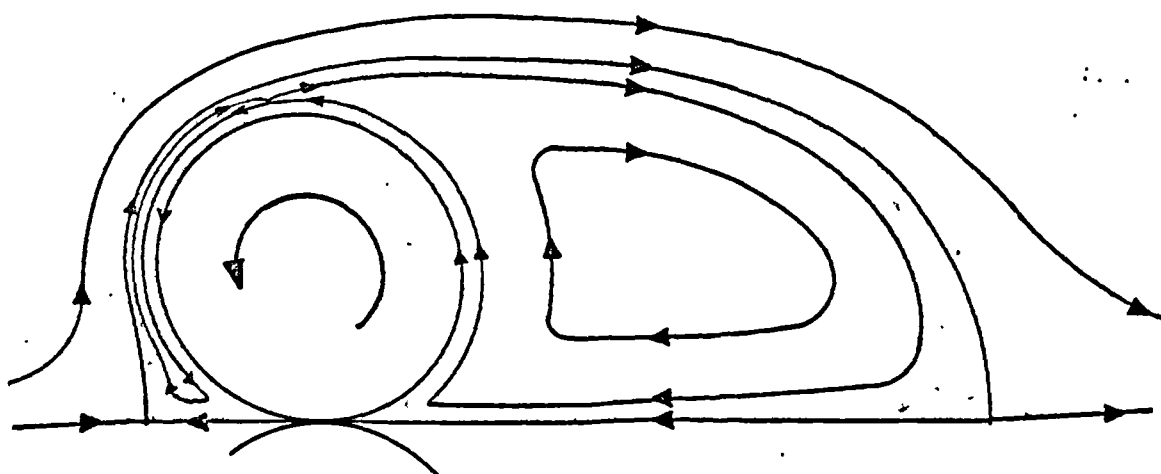
FIG.(3-41) Smoke Visualisation of Two Dimensional Flow past Cylinder , $Re = 2 \times 10^4$

FIG.(3-42)

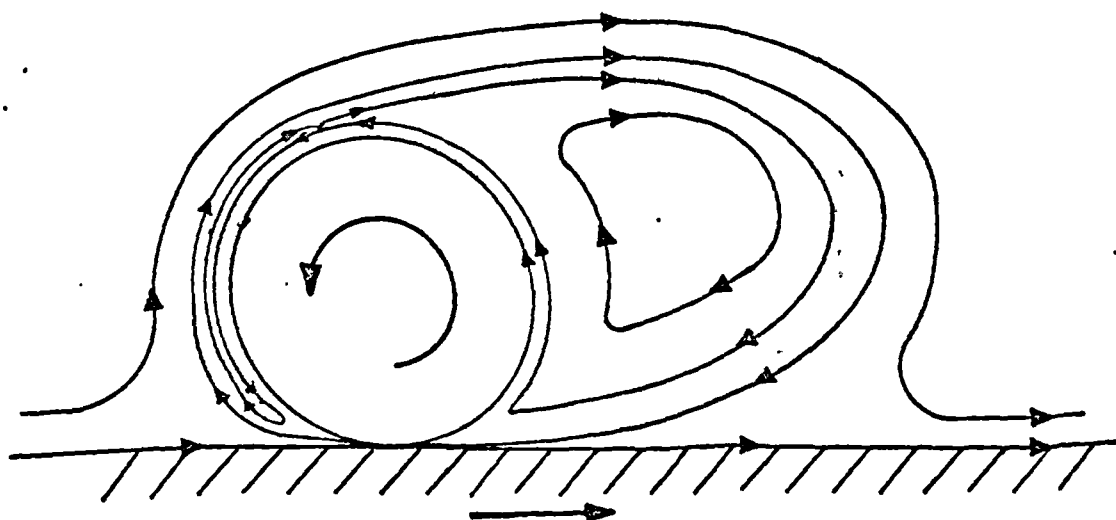
Two Dimensional Mean Streamlines



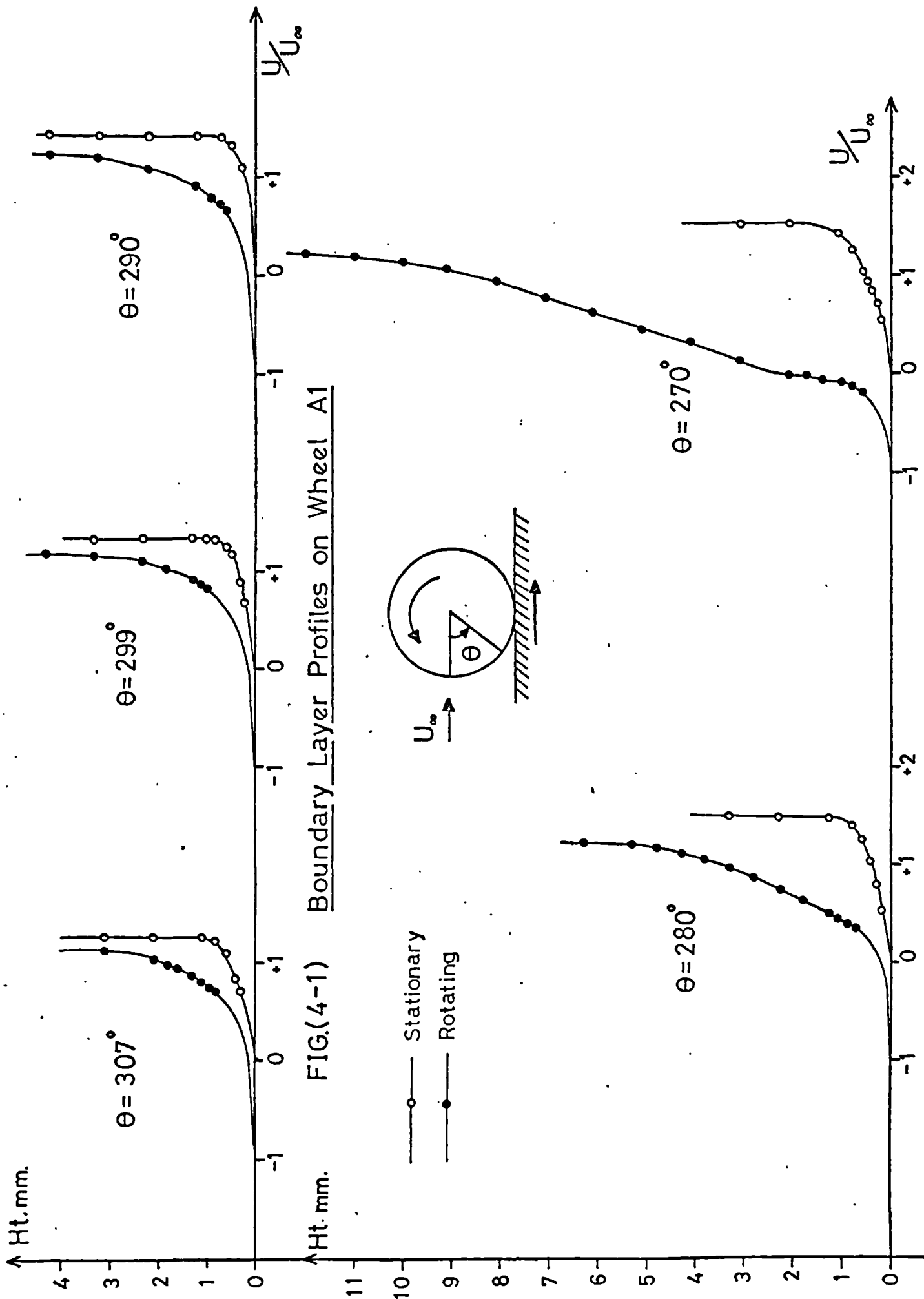
(i) Stationary (Mirror Image)

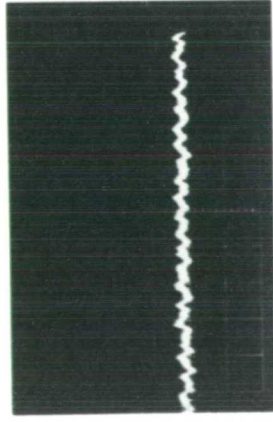


(ii) Rotating (Mirror Image)

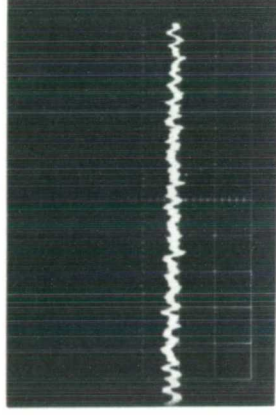


(iii) Rotating (Moving Floor)

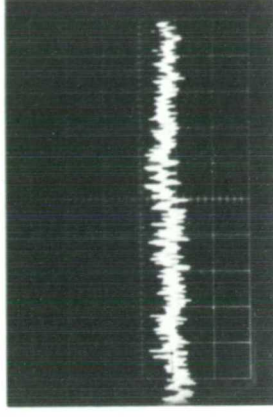




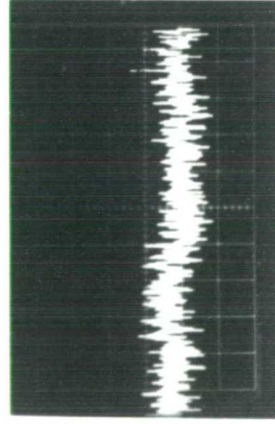
(i) Free Stream



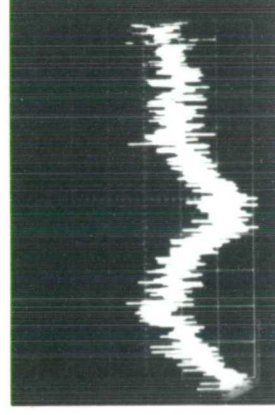
(ii) Within Range of θ from 350° to 20°



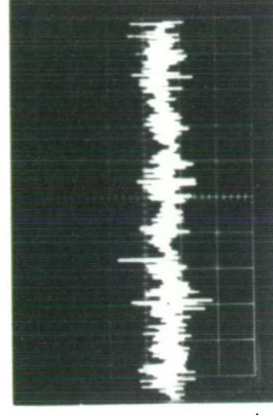
(iii) θ approx. 330°



(iv) θ approx. 305°



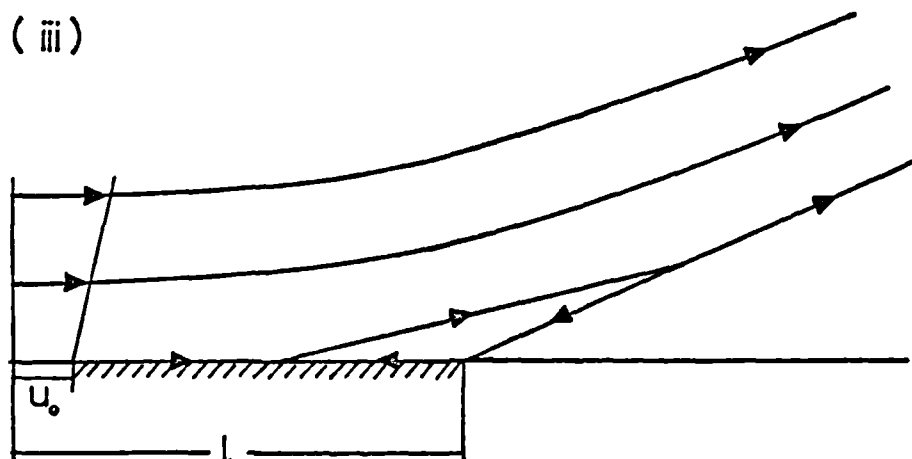
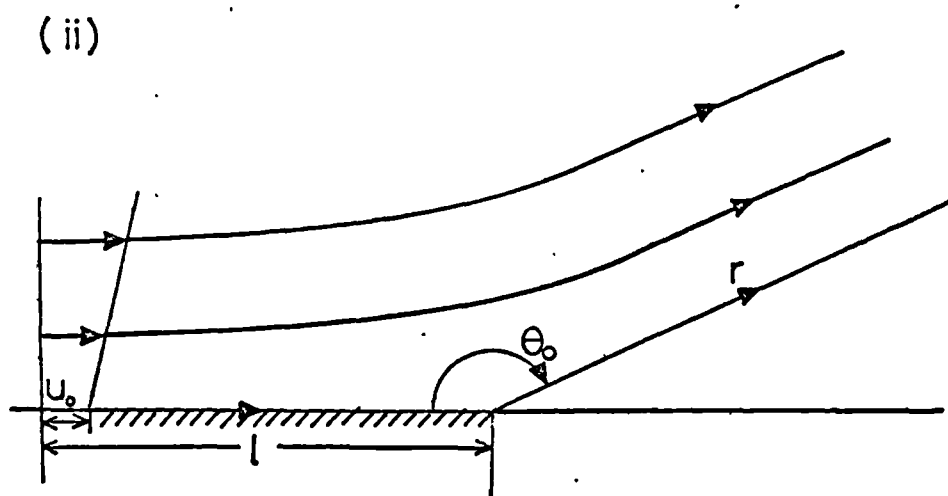
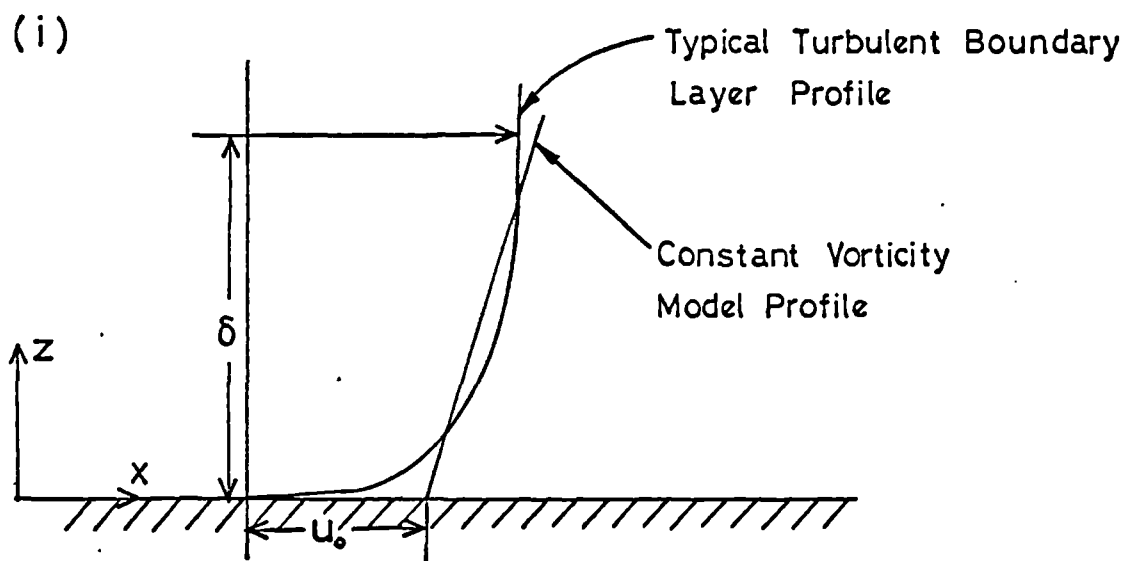
(v) θ approx. 290°



(vi) θ approx. 245° ,
Stationary Wheel

FIG.(4-2) Oscilloscope Traces Representing Pressure Fluctuations in Boundary Layer on Centre-line of WheelA1 (20 msec/div.)

FIG.(4-3) Kuchemann's Model of Separation



$$\psi = x f + 0.8113 f'$$

FIG.(4-4) Streamlines
Near Stagnation Point When
Boundary Has a Uniform
Transverse Velocity

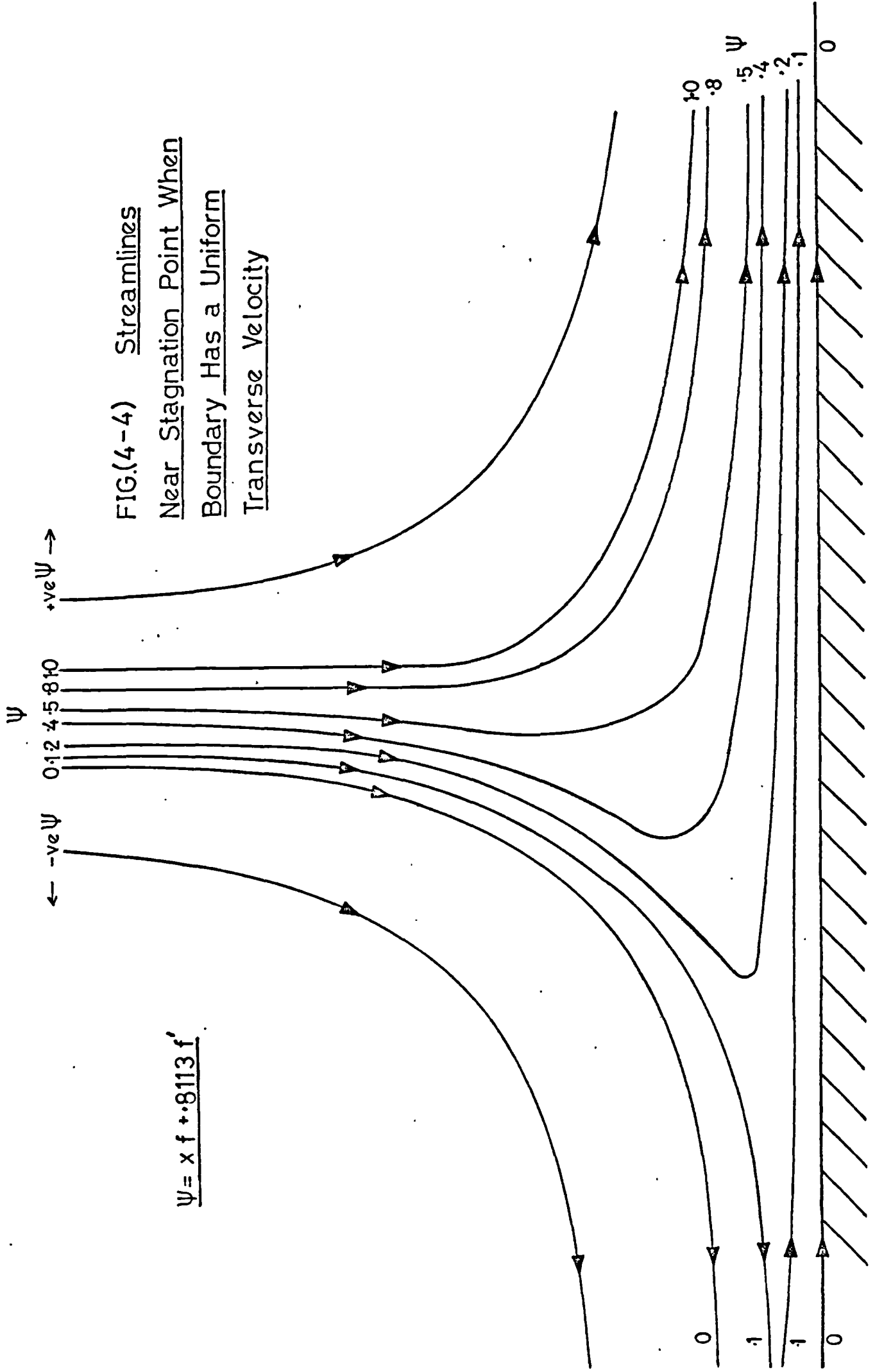
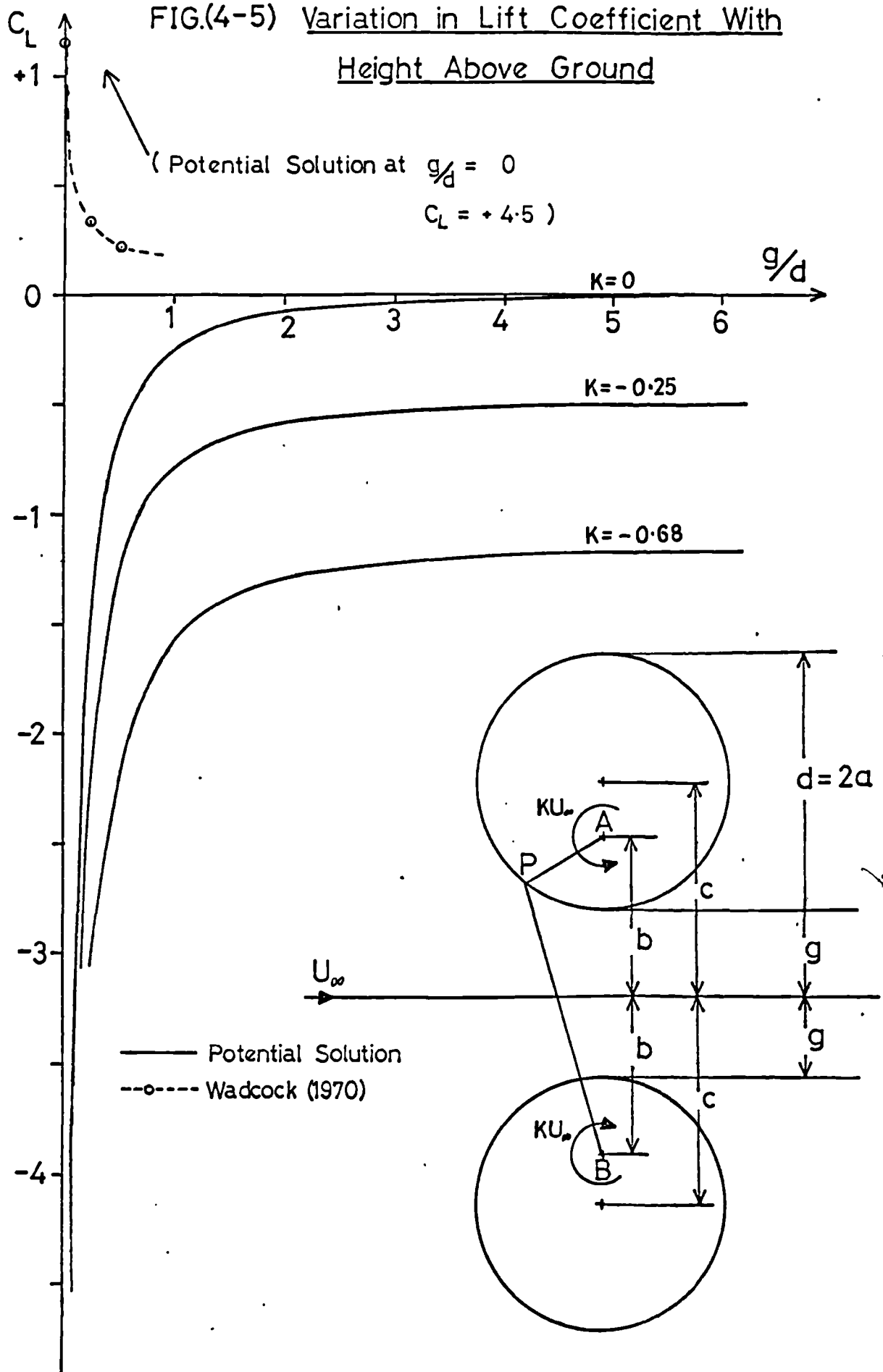


FIG.(4-5) Variation in Lift Coefficient With Height Above Ground



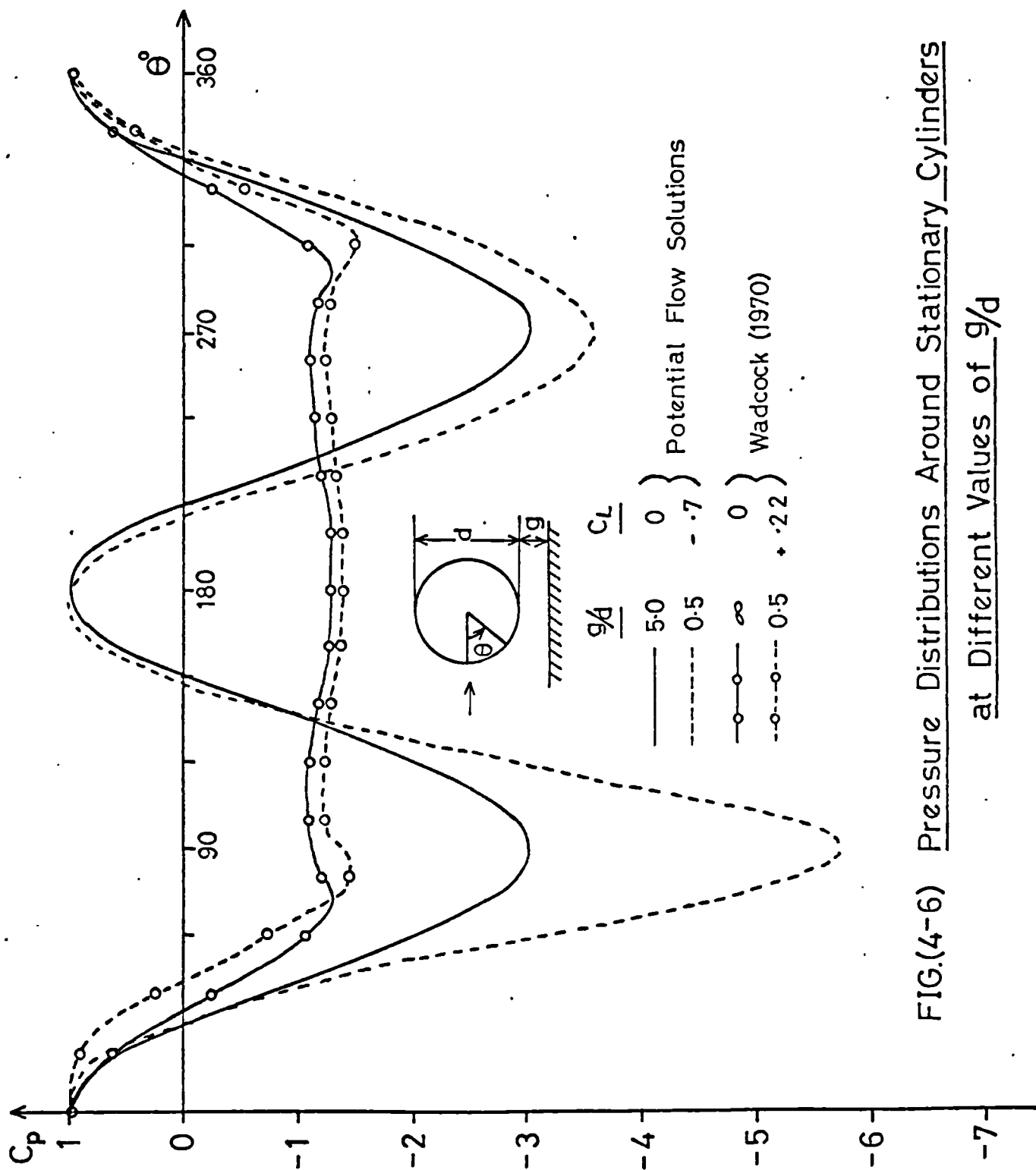


FIG.(4-6) Pressure Distributions Around Stationary Cylinders
at Different Values of g/d

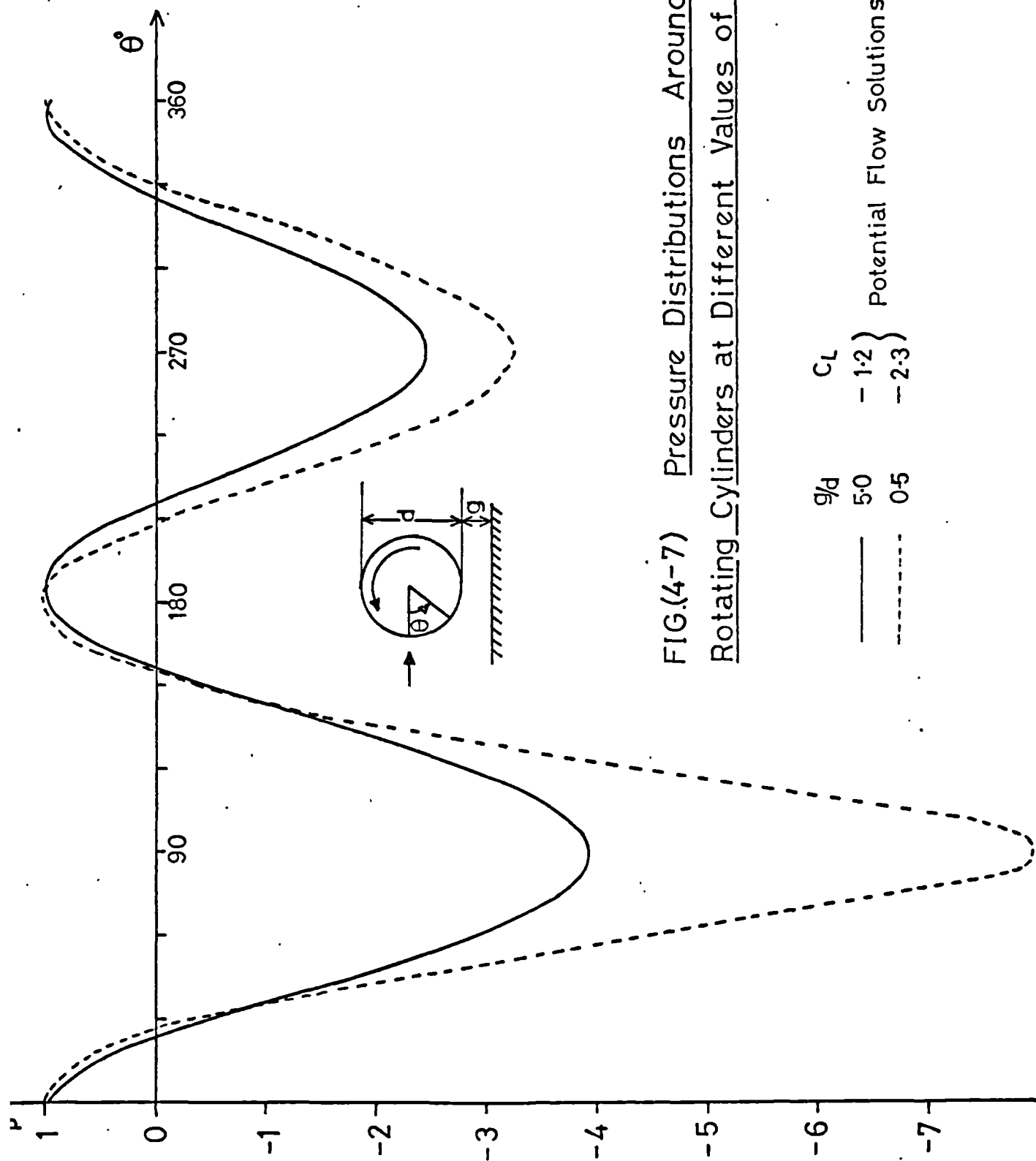
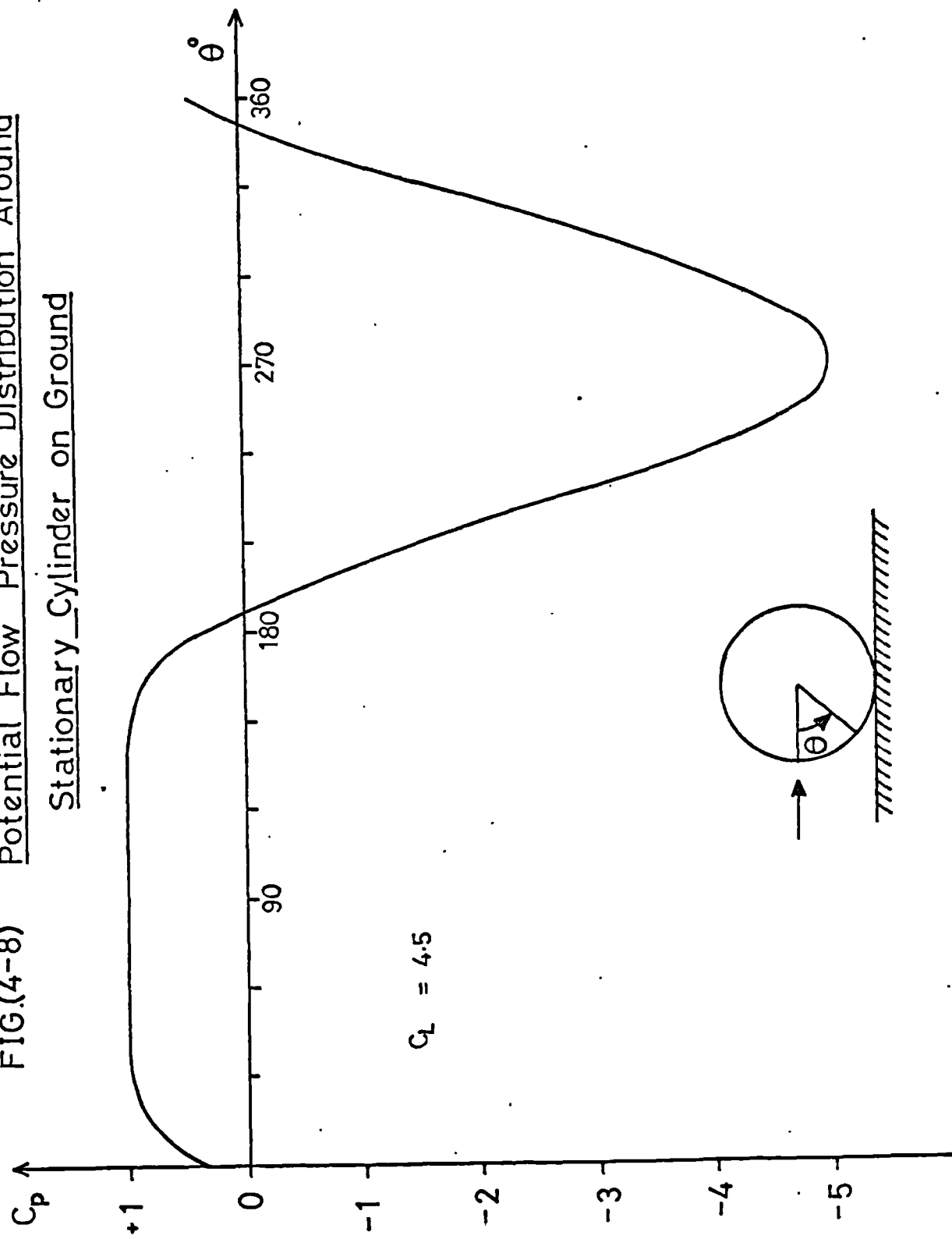


FIG.(4-7) Pressure Distributions Around Rotating Cylinders at Different Values of g/d

FIG.(4-8) Potential Flow Pressure Distribution Around
Stationary Cylinder on Ground



FIG(4-9) Low Reynolds Number Models for Flow in Corners

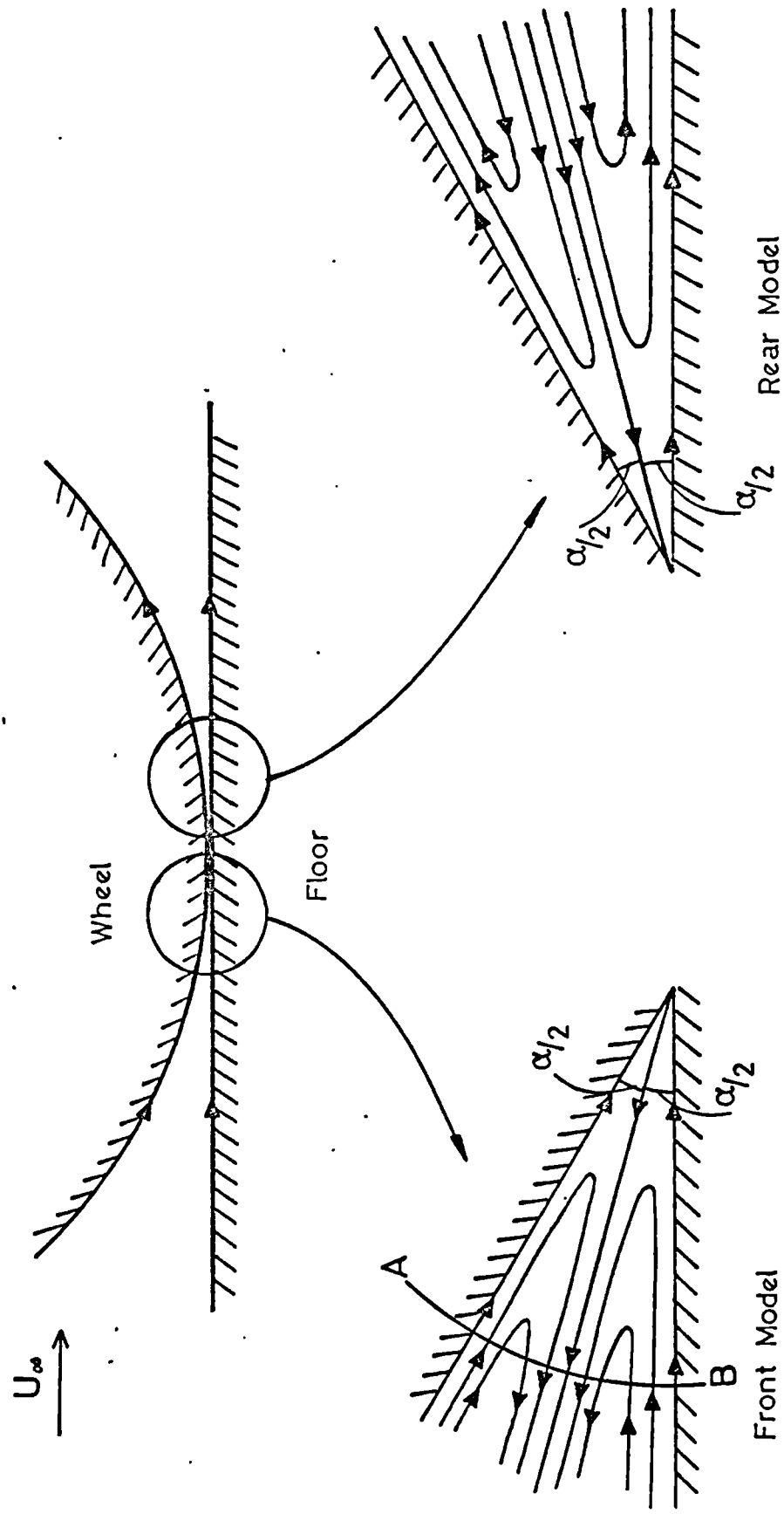
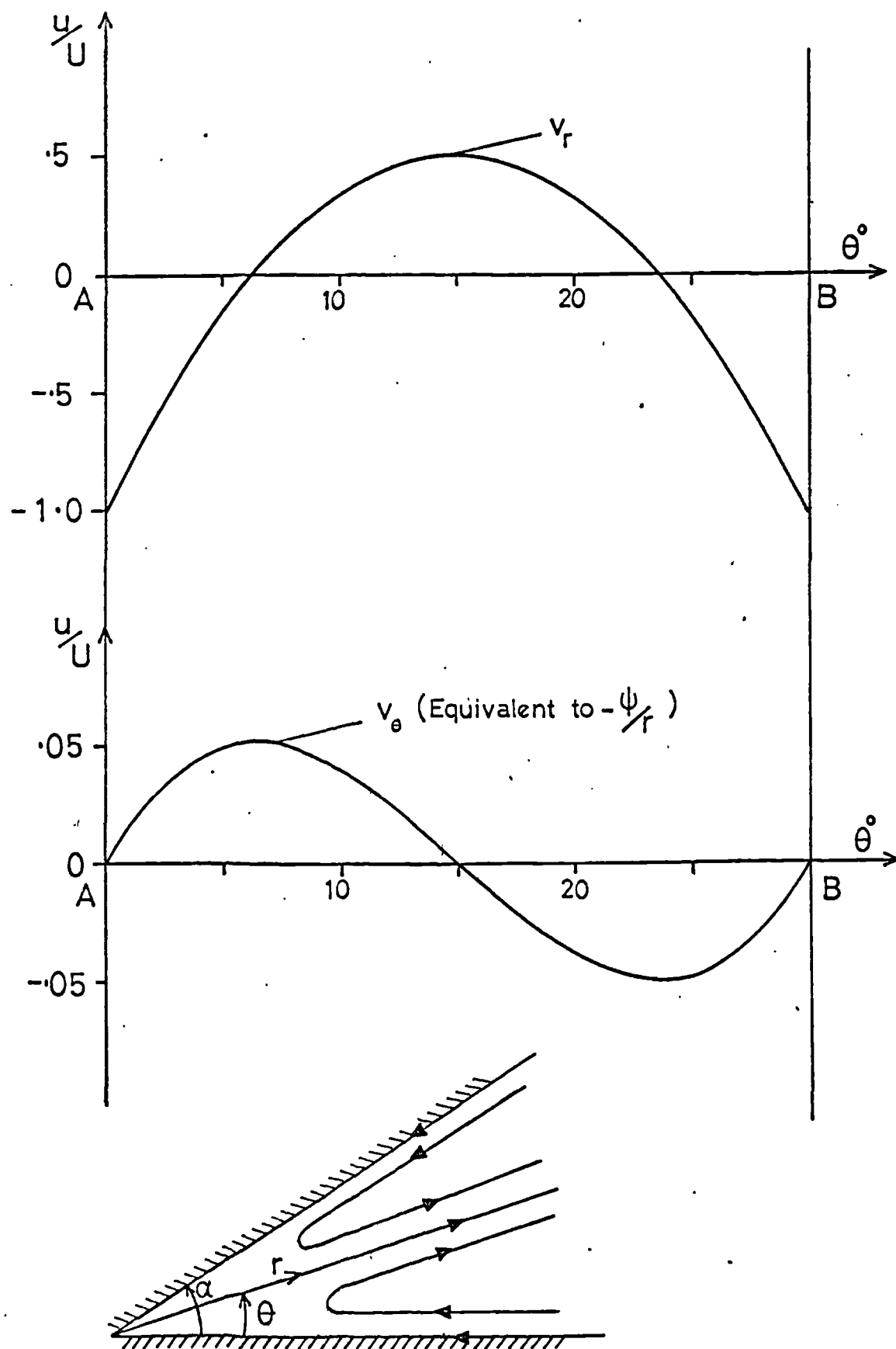


FIG.(4-10) Results for Flow in Corner
With $\alpha = 30^\circ$



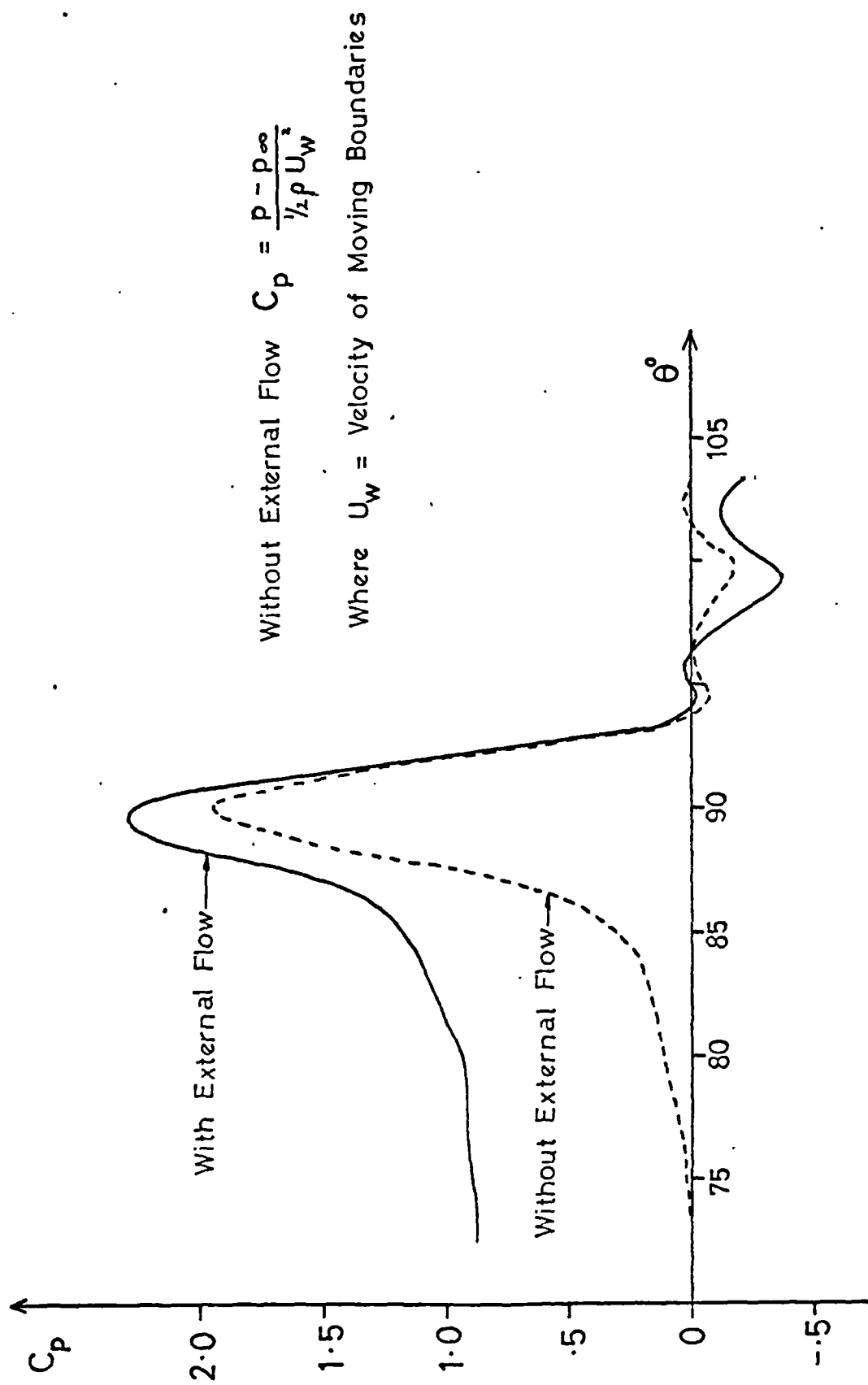
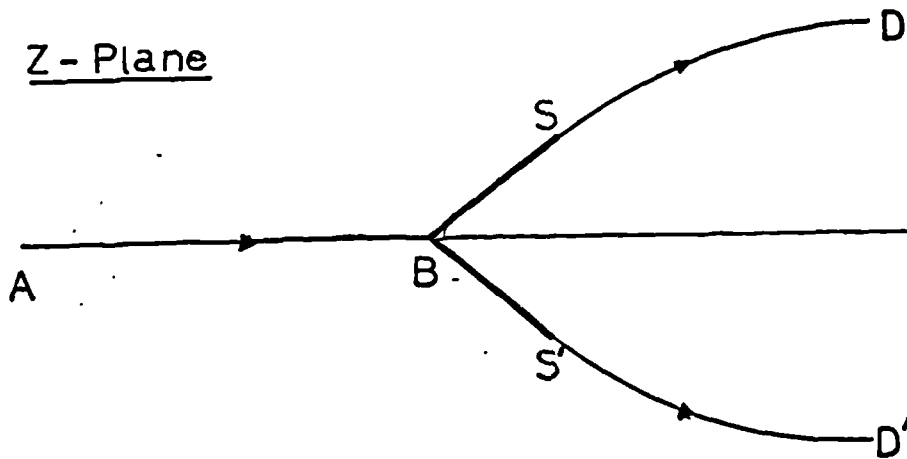
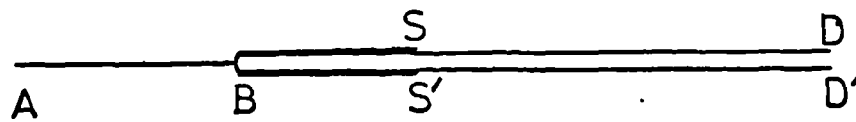


FIG.(4-11) Pressures Near Ground at Hole 9 Wheel A1,
With and Without External Flow

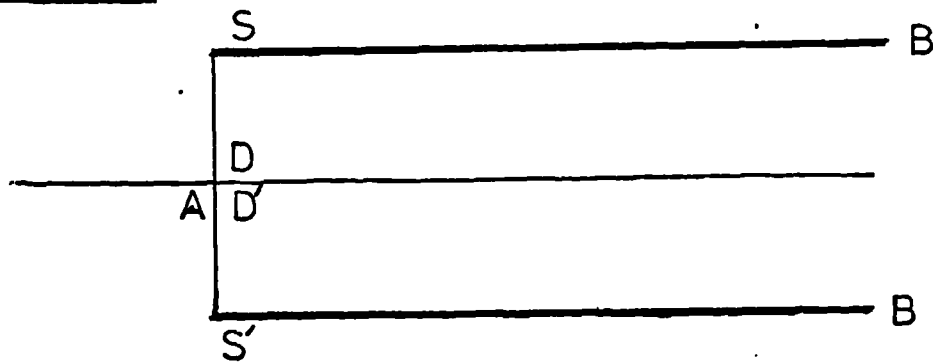
Z - Plane



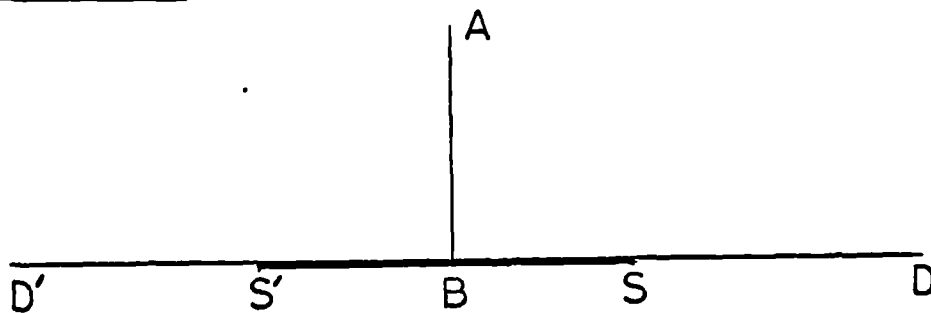
W - Plane



σ - Plane

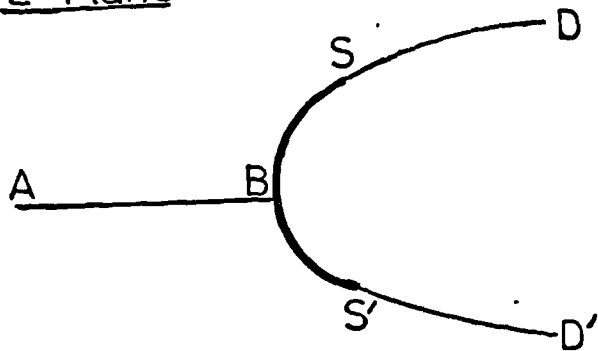


ζ , or ζ_1 Plane

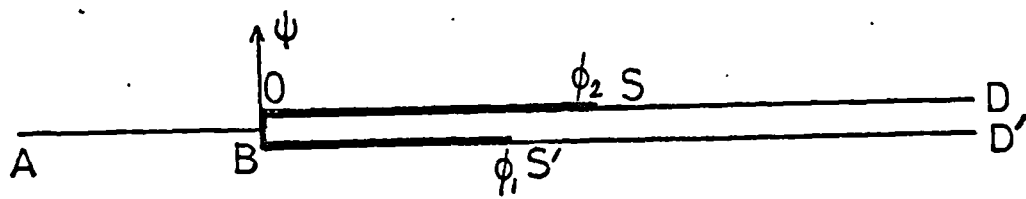


FIG(4-12) Kirchoff Theory Transform Planes

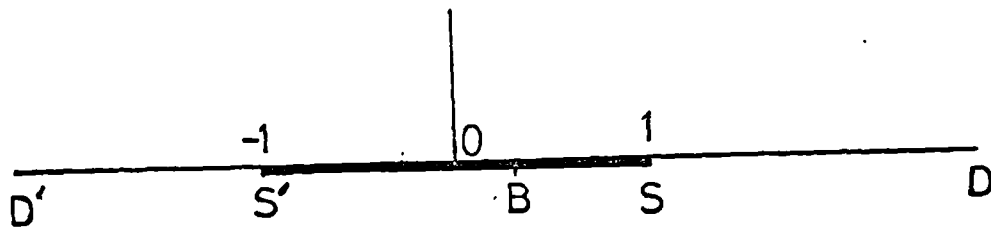
Z-Plane



W-Plane



ζ_2 -Plane



ζ_1 -Plane

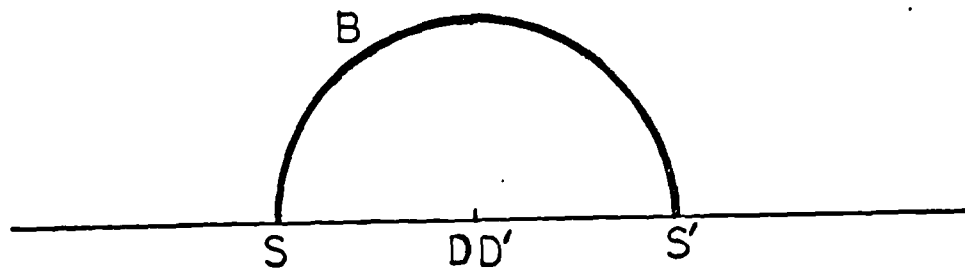


FIG.(4-13) Levi-Civita Transform Planes

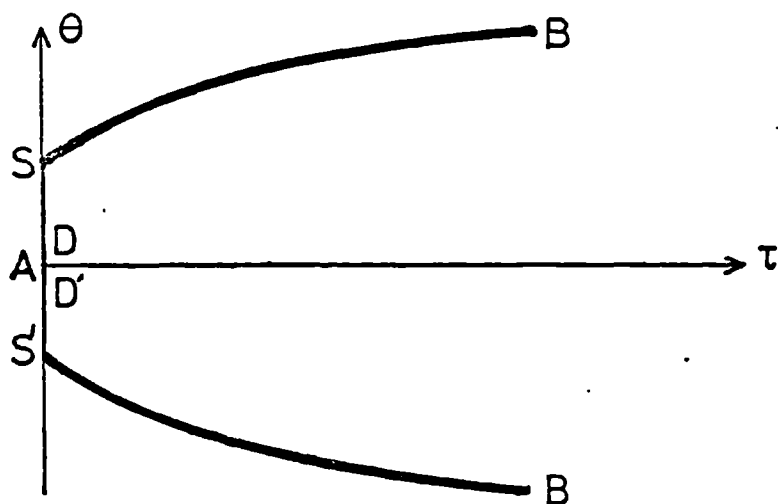
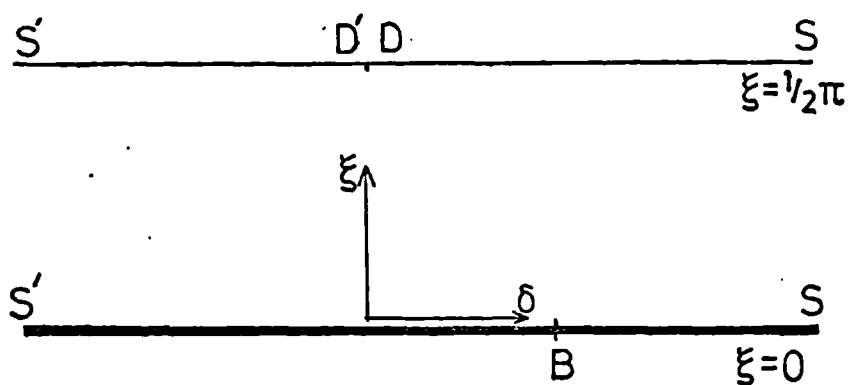


FIG.(4-14) σ -Plane For Curved Obstacles

ζ_1 -Plane



χ -Plane

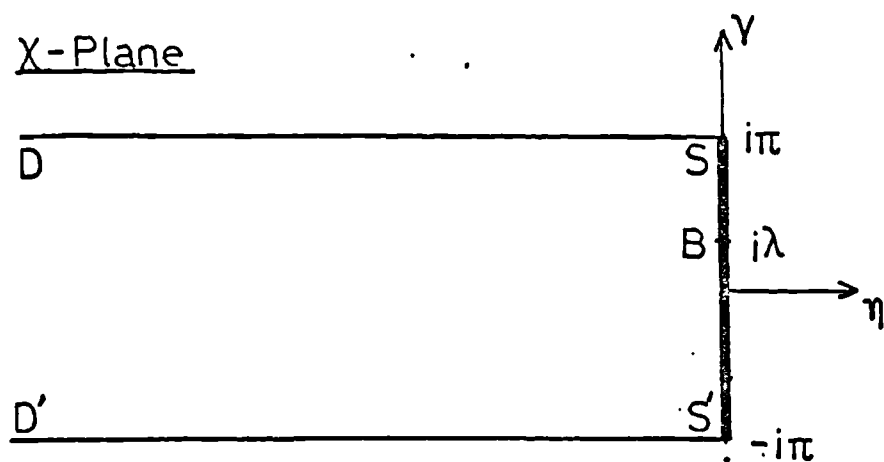


FIG.(4-15) Transform Planes For Wood's Theory

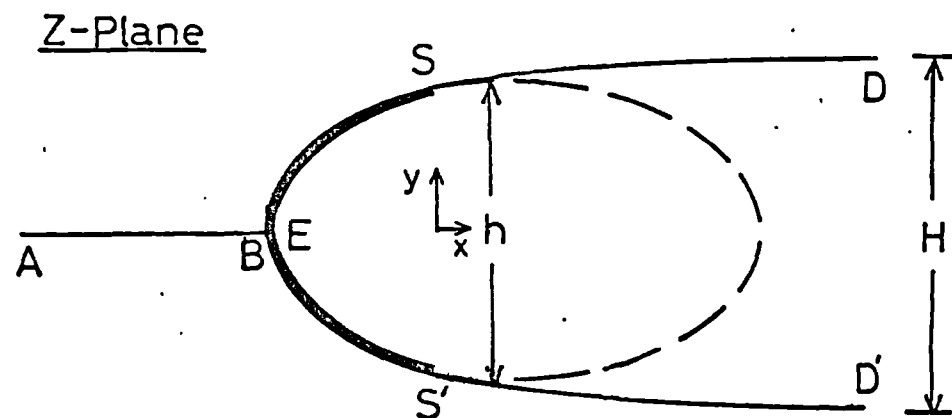
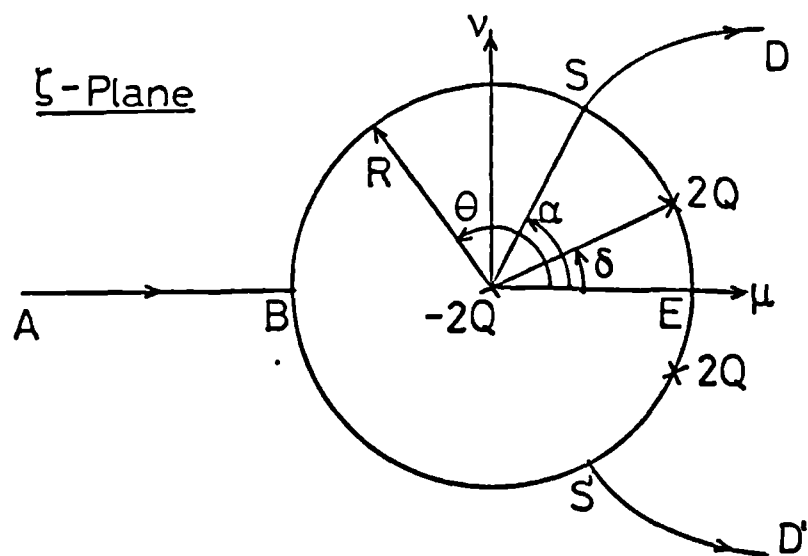


FIG.(4-16) Parkinson & Jandali
Transform Planes

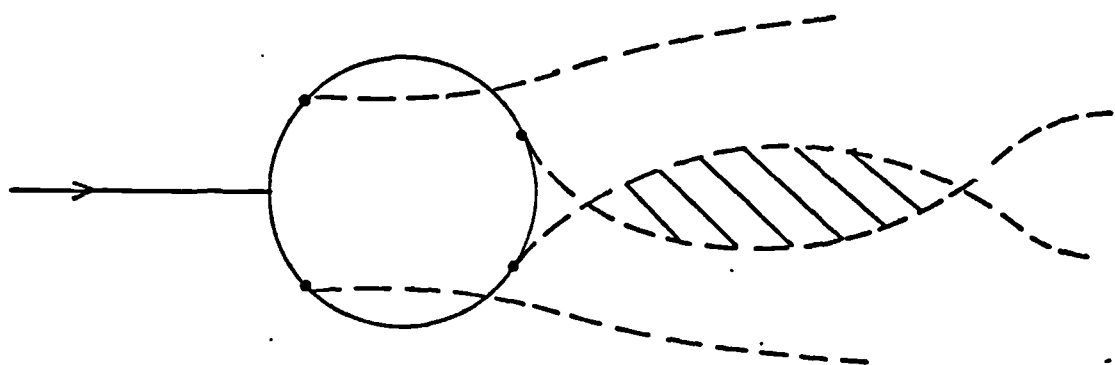
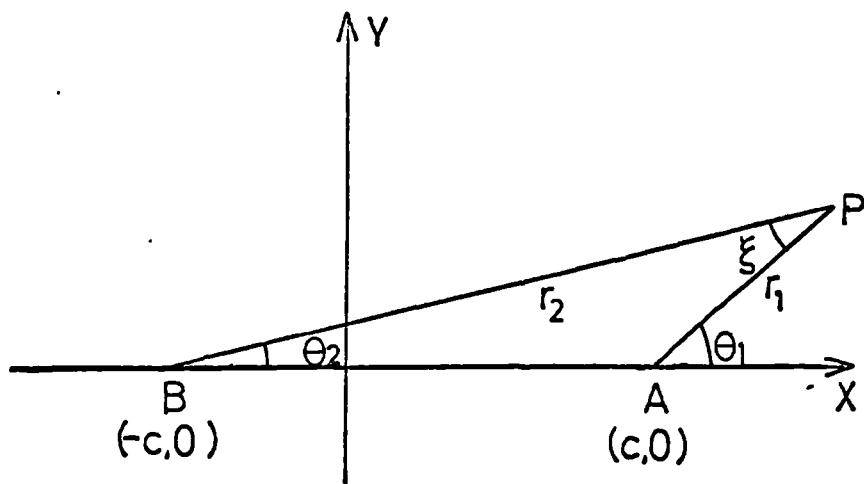
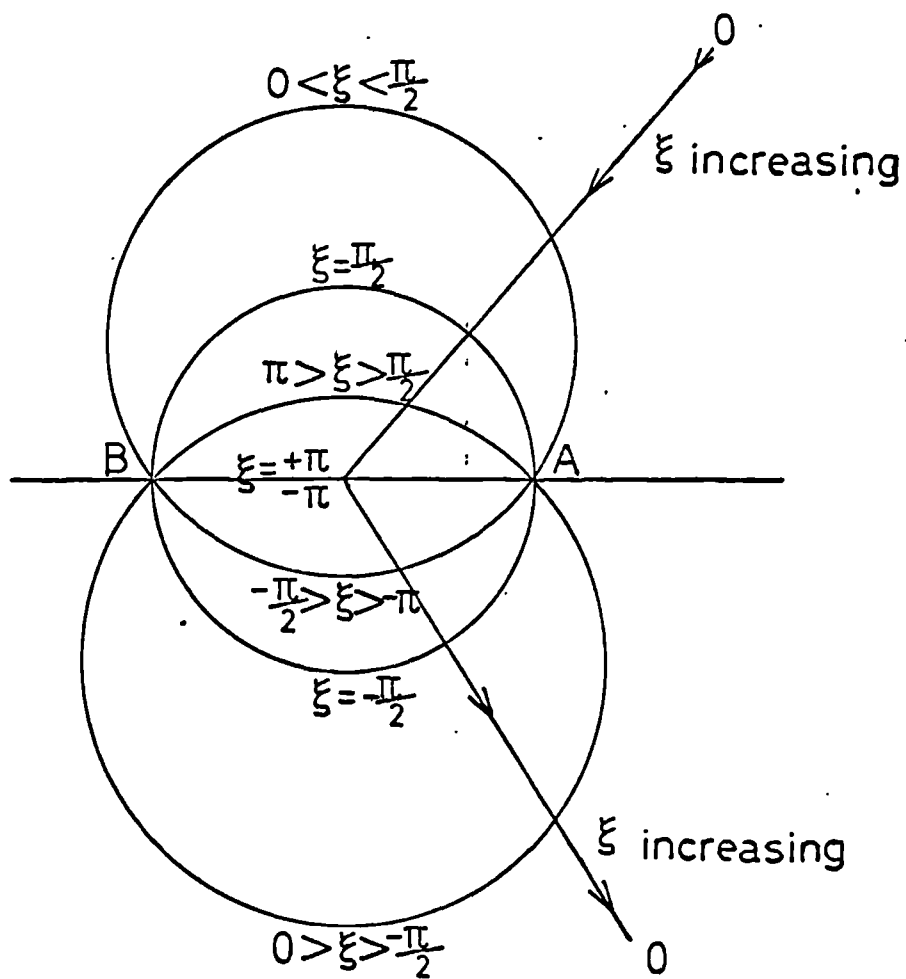


FIG.(4-17) Possible Unrealities Of Free
Streamline Solutions



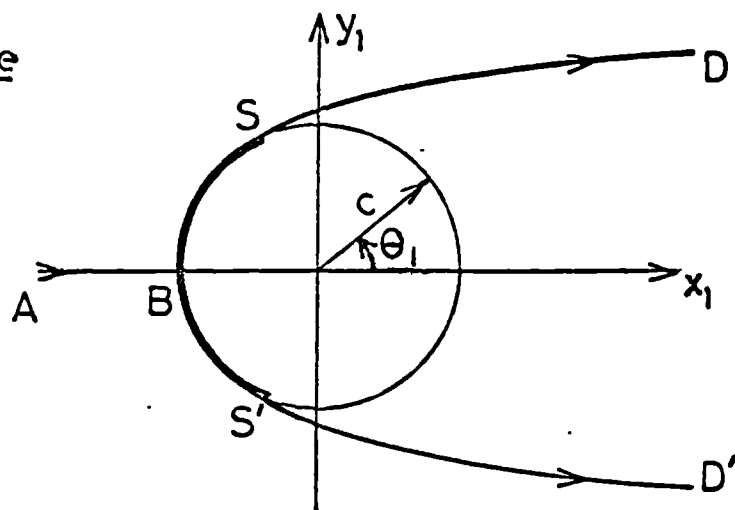
(i) Coordinates in Z-Plane



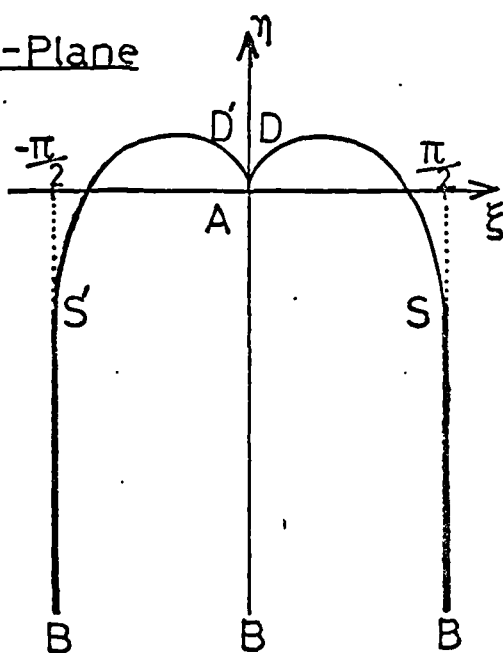
(ii) Lines $\xi = \text{constant} = \xi_0$

FIG.(4-18) Coaxial Coordinates

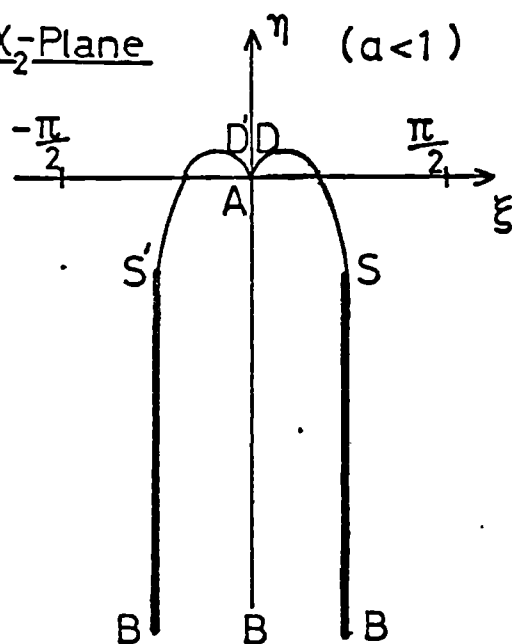
Z_1 -Plane



X -Plane



X_2 -Plane



Z_2 -Plane

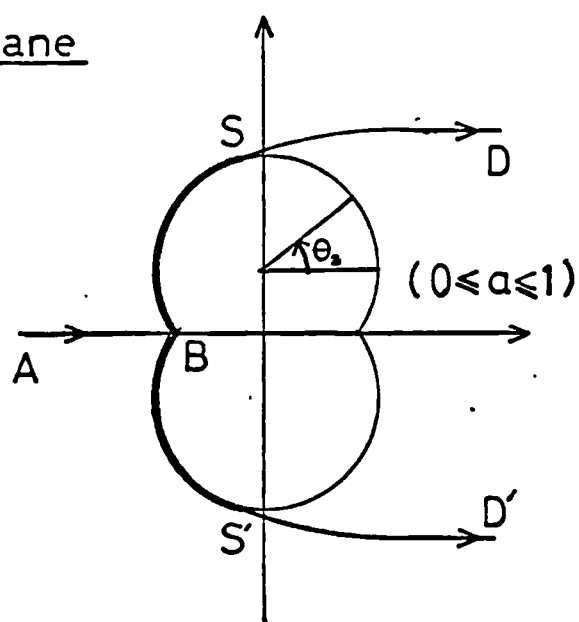
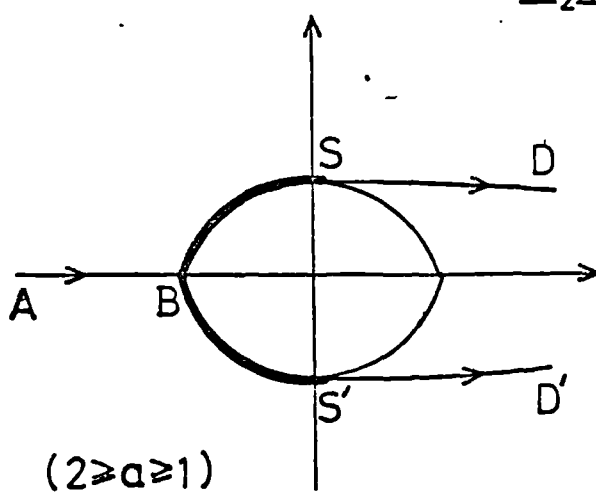
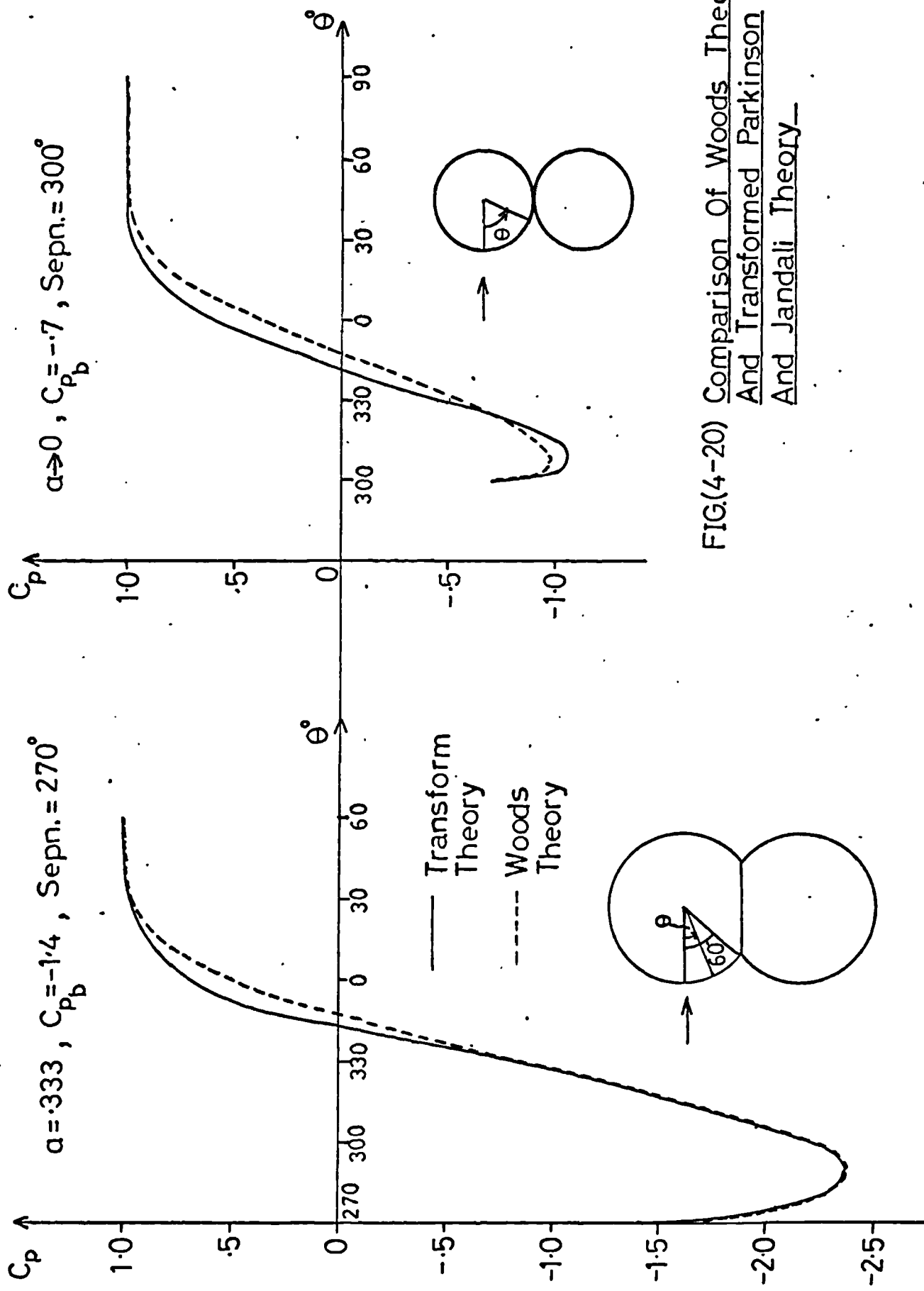


FIG.(4-19) Transforms To Z_2 -Plane



FIG(4-20) Comparison Of Woods Theory-
 And Transformed Parkinson
 And Jandali Theory—

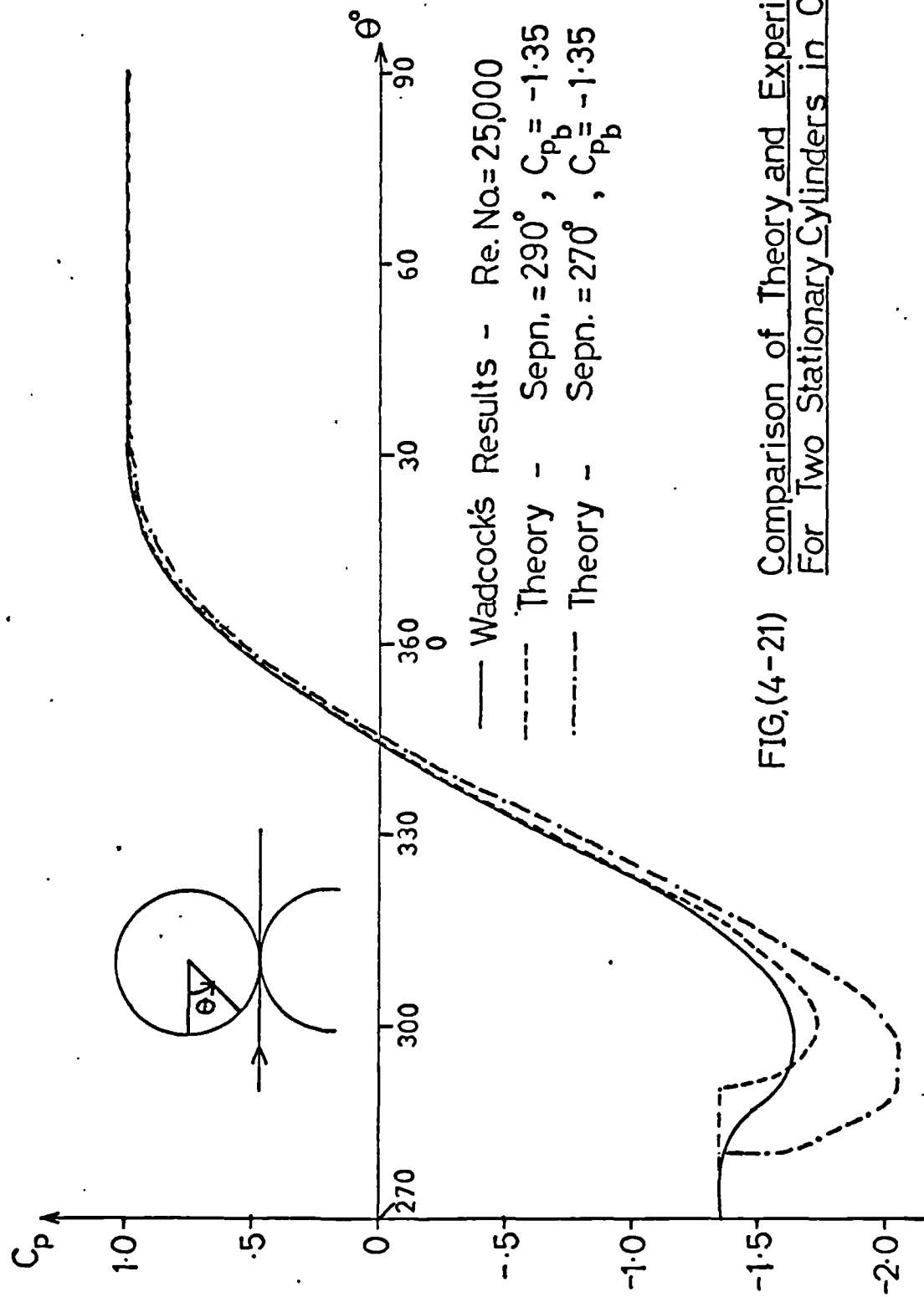
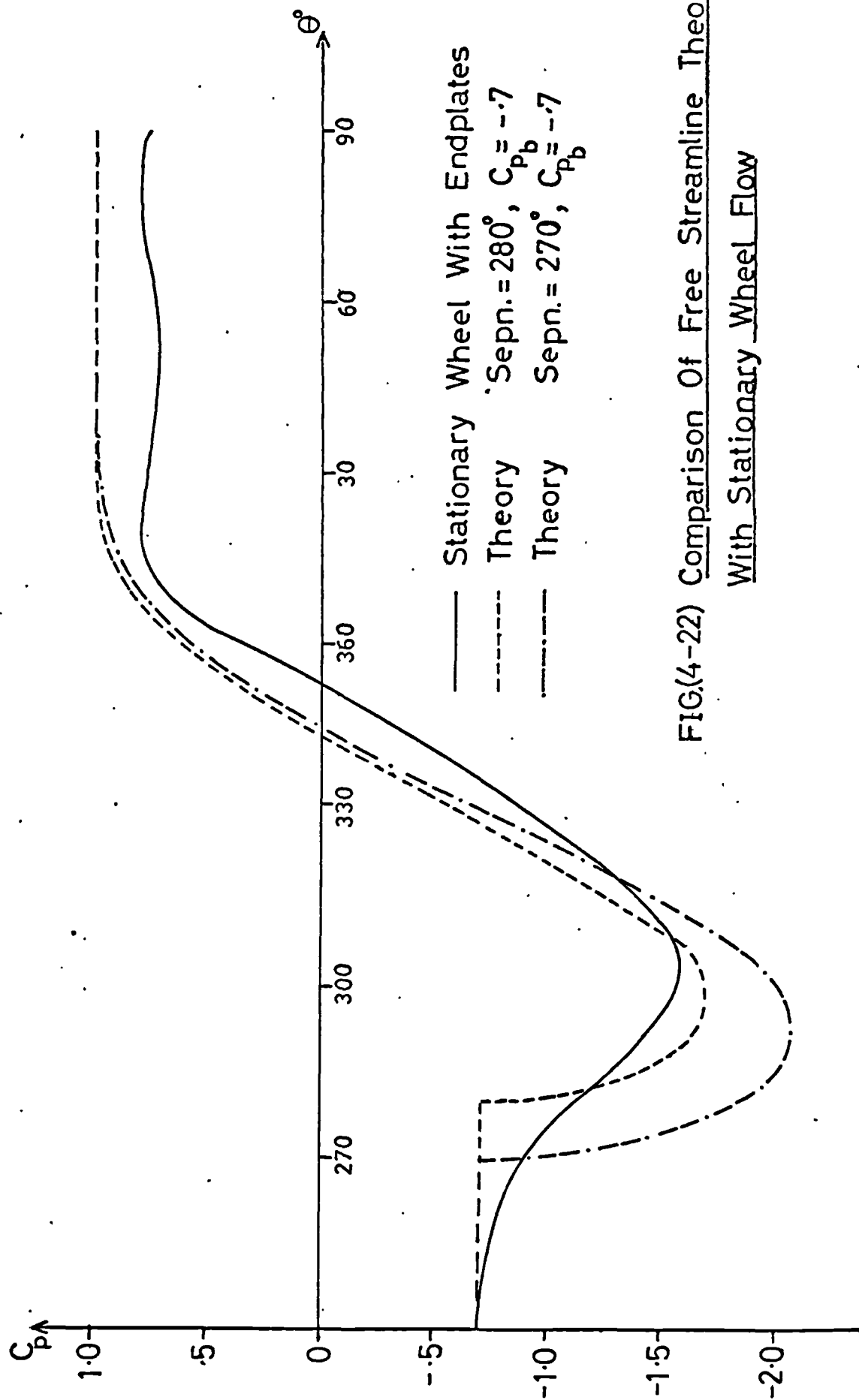


FIG.(4-21) Comparison of Theory and Experiment
For Two Stationary Cylinders in Contact



FIG(4-22) Comparison Of Free Streamline Theory
With Stationary Wheel Flow

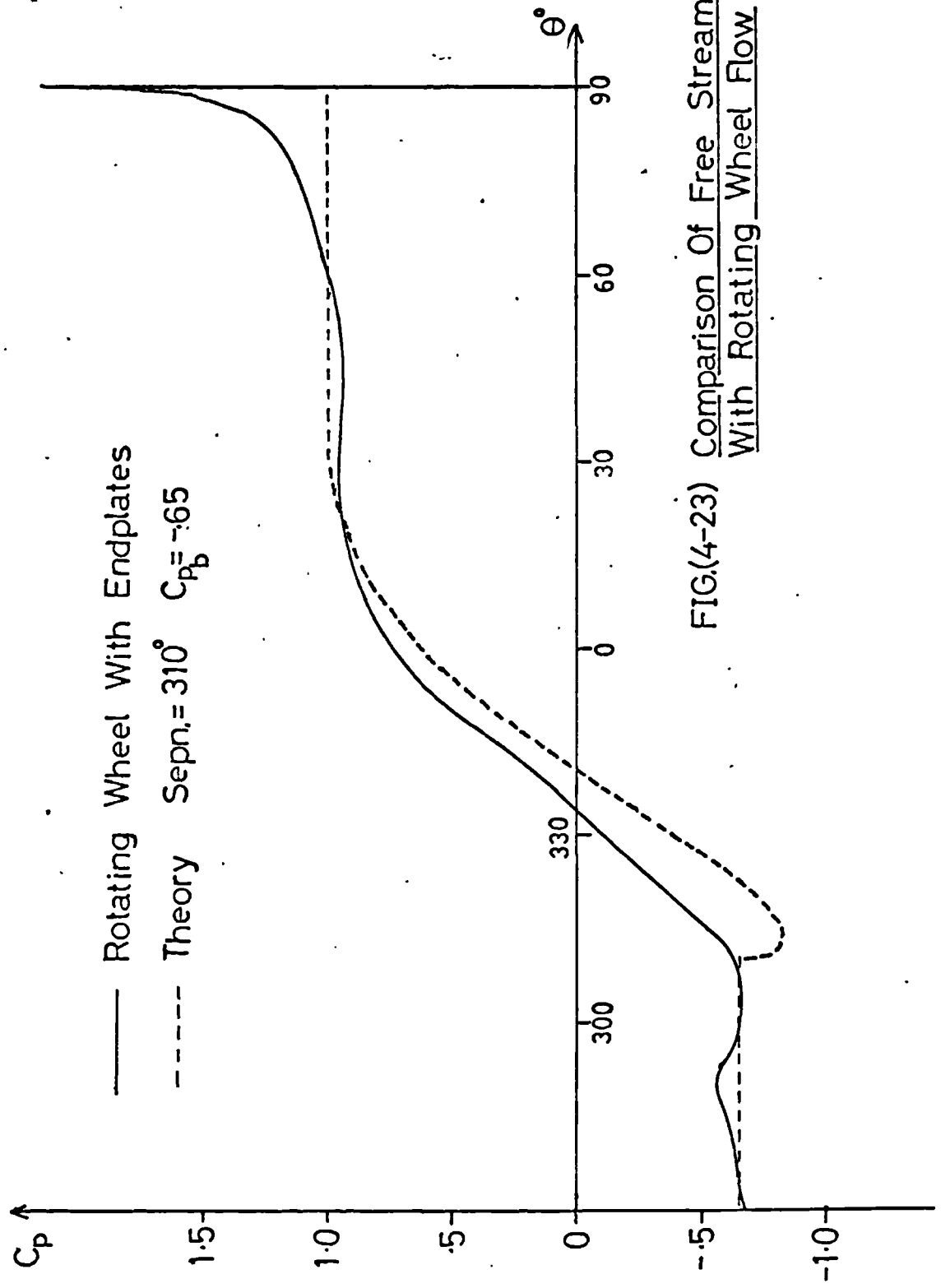


FIG.(4-23) Comparison Of Free Streamline Theory_
With Rotating Wheel Flow

FIG.(4-24) Separating Streamline
Positions In Z_2 -Plane

