

THE INFLUENCE OF MICROSTRUCTURE ON RESISTANCE
TO SLOW CRACK GROWTH IN STRUCTURAL STEEL

by

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ABSTRACT

The resistance to initiation and propagation of stable, fibrous fracture is investigated in terms of the J-integral and COD, for two structural steels. It is found that the initiation toughness is reasonably independent of specimen geometry, but is greatly influenced by the orientation of elongated manganese sulphide inclusions, and by prestrain. It is postulated that the probability of crack extension depends on the strain intensification effects of elongated voids ahead of the crack tip, and may be expressed in terms of the inclusion dimensions in the direction of fracture. Experimental studies yield semi-quantitative estimates of this effect, which is used as the basis of a model to explain the variation of initiation COD with specimen orientation and prestrain.

The R-curves have steep positive slopes even in the absence of shear lips. The indications are that the initial slope of the "plane strain" R-curve is reasonably independent of specimen size. Prestrain and orientation, however, have a large influence, and a good explanation for the dependence of the initial slopes on these factors is provided by the strain intensification model. For bend specimens an analytical relationship is developed between J and COD during crack growth, and it agrees well with experiment for materials of low work-hardening. This treatment is contrasted with the observation, using recrystallisation, that the local strains at the propagating crack tip are constant during crack growth, and that the apparent rise in toughness is caused by remote plastic work. Finally it is shown that the rotational factor in bending, the elastic unloading compliance and the shear lip contribution to toughness all depend on the extent of slow crack growth.

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Chapter One

INTRODUCTION

At one time the important concern when designing a structure or component was to ensure that the material specified had adequate tensile strength, compared to the loads encountered. There are, however, several ways in which failure can occur; for instance, by buckling, plastic collapse, tensile instability and cracking (or fracture). This last possibility was not considered sufficiently, and some spectacular failures resulted, amongst which were the King's Bridge, Melbourne and some of the Second World War Liberty ships. Defects, which act as stress concentrators, are always present in a structure, particularly as a result of poor fabrication or design. It was soon recognised that fracture could occur at such defects, even in materials which appeared ductile in a standard tensile test.

Cracks may initiate and grow from flaws in various ways. Slow crack extension may take place by fatigue, under oscillating loads, or by stable, ductile tearing at gradually increasing displacement. Fast, unstable crack propagation may occur by cleavage along crystallographic planes, or by rapid ductile tearing. Both initiation and propagation of cracks are influenced by material properties and chemical environment.

The first attempt to assess the resistance of a material to fracture was by means of tests such as the Charpy impact test. Measurements are made of the energy absorbed when small notched bars of the material are broken under impact loading. A diagram is plotted of the impact energy absorbed versus testing temperature, as in Fig. 1.1. In steels of body centred cubic crystal structure, this impact energy is small at low temperatures and fracture takes place by cleavage. A transition to a much higher impact energy occurs over a small temperature range above which ductile,

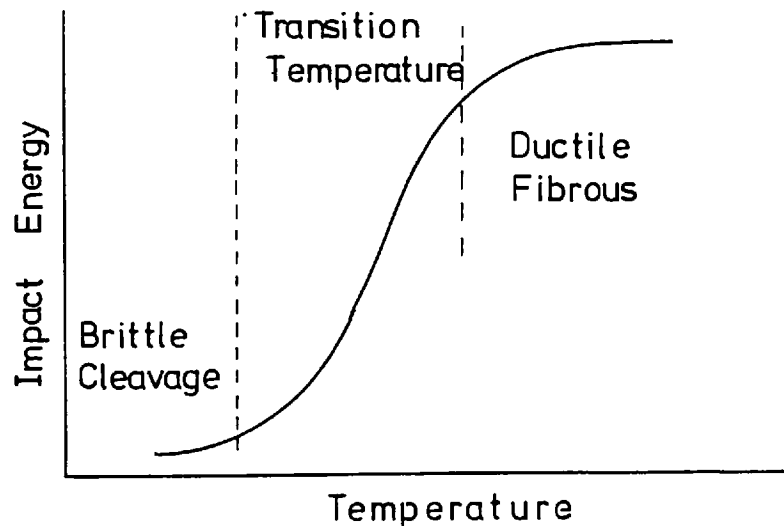


Fig. 1.1 Relationship between impact energy absorbed and temperature, for a typical structural steel.

fibrous fracture prevails.

When using impact tests to select materials, it is ensured that the transition temperature range is below the service temperature, and that the ductile "upper shelf" impact energy is sufficiently high to give the required fracture resistance. Unfortunately, no quantitative information is obtainable concerning the critical combination of flaw size and stress above which fracture may be expected, so that impact tests are not a great help in engineering design. They are, however, useful for material characterization and in quality control.

It is the aim of fracture mechanics to provide the engineer with a quantitative means of designing against fracture. The material is treated as a homogeneous continuum. By analyzing the stress and strain concentration at a sharp crack tip in a loaded body, or by considering energy changes in the body as a whole, it is possible to deduce the critical condition of load and crack length at which crack extension will occur. The mathematics is rigorous for an ideal, linear elastic continuum. However, steels have a discrete microstructure, on a scale large compared to the atomic

dimensions, and deform plastically. The latter property is being tackled with the development of yielding fracture mechanics.

Structural steels may fracture under monotonically increasing loads, by one of two mechanisms. One is cleavage, and is accompanied by negligible plastic deformation. The fracture surface is composed of characteristic cleavage facets. Modern metallurgical practice in steel making has ensured that the impact transition temperature in structural steels is well below normal service temperatures. The result is that cleavage fracture has largely been overcome, except where residual stresses or embrittlement by welding increase the severity of the service conditions. The second mechanism is ductile tearing, and is accompanied by large plastic deformation. Voids initiate at microstructural inhomogeneities such as inclusions, and subsequently grow to the point of coalescence. The fracture surface has the characteristic "dimpled" appearance, with cusps left by expanded voids.

Normally, catastrophic fracture does not immediately follow the initiation of ductile crack extension, unless cleavage fracture intervenes. Crack advance occurs gradually, and requires an increase in the boundary displacement, and also possibly in the load. Hence the apparent resistance of a ductile material to slow crack growth commonly increases as the crack extends. At a certain point unstable fracture may occur, the onset of which is determined by the resistance of the material, the instantaneous values of load, crack length and system compliance, and the configuration of the cracked body.

It is the purpose of the work described herein to investigate resistance to slow crack growth in structural steels, with particular emphasis on the role of metallurgical factors, such as work-hardening capacity and anisotropy (caused by deformed inclusions). Chapters 2-4 describe the theory behind yielding fracture mechanics and relevant models of ductile fracture. In Chapters 5-9, experiments

are discussed in which these theories have been applied to describe resistance to slow crack growth. The applicability of the methods and the implication of the results are considered.

Chapter Two

LINEAR ELASTIC FRACTURE MECHANICS

2.1 The Elastic Stress Intensity Factor

The theoretical cohesive strength of a solid is about $E/10$, where E is Young's Modulus, but in practice the strength of elastic materials such as glass is several orders of magnitude lower. Griffith (1920) showed that the discrepancy was due to the presence of small flaws which acted as stress concentrators, enabling the fracture stress of the material to be reached locally, at small remote stresses.

The stress concentrating effects of notches were first noted by Inglis (1913), who obtained an expression for the maximum stress, σ_{11} , perpendicular to the plane of the notch, at the tip of an ellipse of major axis a , minor axis b under a remote tensile stress σ :

$$(\sigma_{11})_{\max} = \sigma(1 + 2a/b). \quad (2.1)$$

As the ellipse becomes long and narrow, σ_{11} becomes approximately

$$(\sigma_{11})_{\max} = 2\sigma(a/\rho)^{\frac{1}{2}} \quad (2.2)$$

where ρ is the radius of curvature at the tip. For an infinitely sharp crack ($\rho=0$), σ_{11} becomes infinite.

For the general solution of the stress field at the tip of a sharp crack, Irwin (1957) considered the Westergaard (1939) solutions for cracks in tensile stress fields, fig. 2.1. He obtained for σ_{11} , the normal stress in the plane of the crack:

$$\sigma_{11} = \frac{\sigma\sqrt{a}}{\sqrt{(2r)}} = \frac{K_I}{\sqrt{(2\pi r)}} \quad (2.3)$$

where r is the displacement from the crack tip, and K_I ($= \sigma\sqrt{(\pi a)}$) is known as the stress intensity factor. K_I is independent of r , being determined only by the crack length

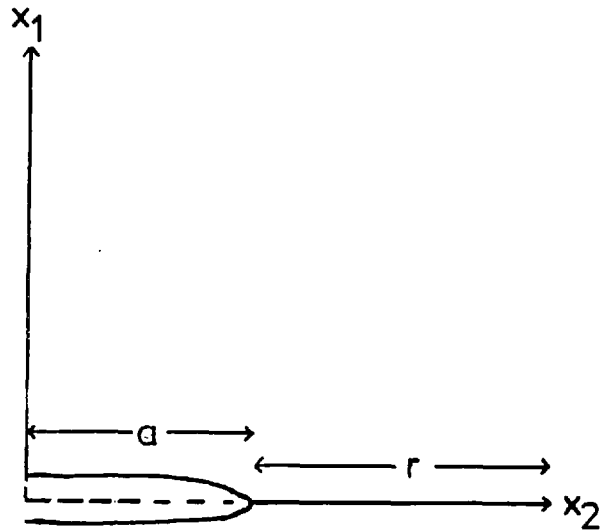


Fig. 2.1 Coordinate system for a crack in an elastic material.

and remote stress. Similar expressions involving K_I can be deduced for the other stresses σ_{22} and σ_{33} . It thus determines the magnitude of the stress, and is a single parameter describing the crack tip environment.

These remarks have been confined to the case of a crack in a tensile stress field (called mode I). The other cases of in-plane shear (mode II) and out of plane shear (mode III) are depicted in fig. 2.2, and similar results are obtained, with the elastic stress intensity factors denoted by K_{II} and K_{III} respectively.

The relationship, $K_I = \sigma\sqrt{\pi a}$, was derived for a crack in an infinite body. For more complex cases, such as an edge crack in a finite body under tensile or bending

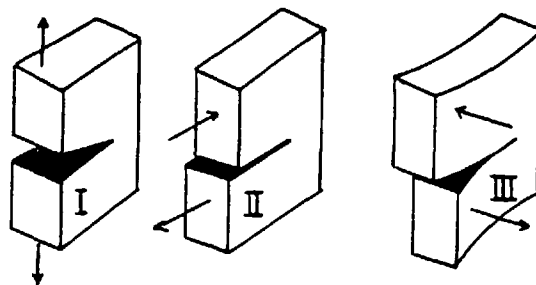


Fig. 2.2 The three loading modes for a cracked body.

loads, this is modified to:

$$K_I = \sigma\sqrt{(\pi a)}.\lambda \quad (2.4)$$

where λ is a geometrical correction factor.

The stress intensity factor K completely specifies the stress field around the crack tip in a purely elastic body. If fracture is governed by conditions at the crack tip, the criterion for fracture becomes the attainment of a critical stress intensity factor, K_c . The exact cause of fracture, whether it requires a critical stress, strain or energy release rate, need not be known. The fracture condition can be written from eqn. (2.4)

$$K_c = \sigma_c\sqrt{(\pi a_c)}.\lambda. \quad (2.5)$$

K_c is expected to be a material constant, and may be determined in the laboratory. Then the critical combination of stress and crack length may be determined for any geometry from eqn. (2.5).

This analysis is based on the material being ideally linear elastic, a property however which is not realised in practice. Eqn. (2.3) indicates that as r approaches zero, σ_{11} approaches infinity. In materials of interest to the engineer, yielding intervenes, causing a plastically deformed zone around the crack tip.

2.2 The Elastic Energy Release Rate

A physical explanation of fracture is provided by the energy balance approach (Griffith, 1920). This is based on the assumption that a crack will only extend if by so doing it allows the total energy of the body to decrease. By considering energy changes in the body as a whole it is possible to direct attention from the highly strained crack tip region and still derive an expression for the fracture stress.

Using Inglis' (1913) results, Griffith considered the elastic energy released when a centre crack of length $2a$ was

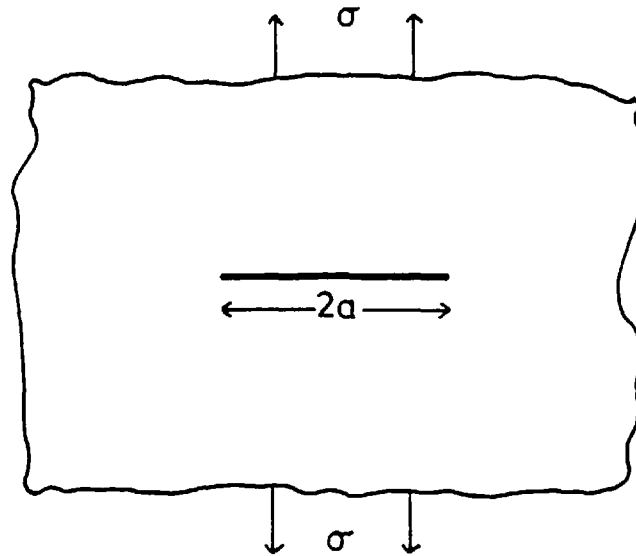


Fig. 2.3 A centre crack in an infinite elastic body of unit thickness (Griffith, 1920).

placed in a body of unit thickness, subject to a remote tensile stress σ while keeping the boundaries fixed (fig. 2.3). The first analysis contained an error which was later amended (Griffith, 1924). The correct change in energy of the body, ΔU , is

$$\Delta U = \pi \sigma^2 a^2 / E' \quad (2.6)$$

where

$$E' = \text{Young's modulus } E, \text{ in plane stress} \\ = E / (1 - \nu^2) \text{ in plane strain}$$

and ν is Poisson's ratio.

If the surface energy per unit area is γ the work done in creating new surfaces is $4a\gamma$. Unstable fracture occurs at a critical stress σ_c and crack length a_c , when a small increase in crack length causes an energy release greater than or equal to the increase in surface energy.

$$\text{i.e. when} \quad \frac{d}{da} \left(\frac{\pi \sigma^2 a^2}{E'} \right) \geq \frac{d}{da} (4a\gamma)$$

$$\text{Hence the critical stress } \sigma_c = \left(\frac{2E'\gamma}{\pi a} \right)^{\frac{1}{2}} \quad (2.7)$$

Results of experiments on glass demonstrated the validity of the theory for an elastic material. However, its application to metals is less clear. In addition to creating new surfaces, fracture in more ductile materials necessitates some degree of plastic work. Irwin (1948) and Orowan (1949)

independently proposed that the surface energy γ be modified to γ' , the effective work of fracture per unit area, including local plastic work. Provided that the plastic zone at the crack tip does not disturb the elastic stress field significantly, eqn. (2.7) may be used with the substitution of γ' .

In practice the plastic work term dominates γ' , and since the former cannot be measured directly, unlike the true surface energy, an indirect estimate must be used. Irwin and Kies (1952) suggested an experimental method for measuring the elastic energy release rate which involved the compliance relationship for a cracked body. For linear elasticity the strain energy of a cracked body is

$$U = \frac{1}{2} Q^2 C \quad (2.8)$$

where Q is the load and C is the elastic compliance (= displacement per unit load). Then at constant load, for instance,

$$- \frac{dU}{da} = \frac{1}{2} Q^2 \cdot \frac{dC}{da} \quad (2.9)$$

The negative sign is used since U decreases and C increases as the crack length increases.

Following Griffith the elastic energy release rate at fracture, $(dU/da)_c$, is equated to the rate of increase of surface energy, $2\gamma' \cdot B$, where B is the thickness of the body:

$$- \left(\frac{dU}{da} \right)_c = 2\gamma' \cdot B \quad (2.10)$$

Thus, from eqn. (2.9)

$$\gamma' = \frac{Q_c^2}{4B} \cdot \frac{dC}{da} \quad (2.11)$$

dC/da may be found from a series of load-displacement curves on specimens of different crack length, and substitution in eqn. (2.11), together with the fracture load Q_c , enables γ' to be found.

Irwin (1964) introduced the parameter G , which he defined as the crack extension force, or the energy released

by the system per unit crack extension, per unit thickness.

$$\text{i.e. } G = -\frac{1}{B} \frac{dU}{da} \quad (2.12)$$

at constant displacement. The same result is obtained at constant load. The critical value of G at fracture is G_c . Hence, from eqn. (2.10)

$$G_c = 2\gamma' \quad (2.13)$$

G may be interpreted as the area between the load/displacement plots for two bodies of unit thickness, with adjacent crack lengths a and $a + \Delta a$, taken to the same final load or displacement. The difference between the areas under constant load or constant displacement conditions is second order and vanishes in the limit as $\Delta a \rightarrow 0$.

2.3 The Relationship between the Stress Intensity and Energetic Approaches.

It has been shown that fracture will occur in a cracked body when a critical value of either the stress intensity, K_c , or the crack extension force, G_c , is attained. The two concepts are related in a simple manner:-

Consider an infinite plate of unit thickness containing a central crack of length $2a$. Differentiating eqn. (2.6) gives the crack extension force.

$$\text{i.e. } G = -\frac{dU}{d(2a)} = \frac{\pi\sigma^2 a}{E'} \quad (2.14)$$

But from eqn. (2.4) with $\lambda = 1$, $K_I = \sigma\sqrt{(\pi a)}$

$$\text{Thus } G = \frac{K_I^2}{E'} \quad (2.15)$$

Irwin (1957) proved the same result formally for an arbitrary geometry and configuration. Similar expressions may be derived for mode II and mode III loading. In general,

$$G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu}$$

where μ is the shear modulus.

2.4 The Development of Crack Tip Plasticity

2.4.1 The Irwin Plastic Zone Correction Factor.

It was noted above that in an elastic material the stress intensity approach predicts singularities in the stresses at the tip of a sharp crack. This is of course impossible, and in practice yielding intervenes. Following Irwin (1964), the tensile stress σ_{11} equals the yield stress σ_y inside a zone of length R_o , in plane stress (fig. 2.4). If r_y is the distance ahead of the tip at which $\sigma_{11} = \sigma_y$ in the elastic material, from eqn. (2.3)

$$\sigma_{11} = \sigma_y = K/(2\pi r_y)^{\frac{1}{2}}$$

i.e.
$$r_y = \frac{K^2}{2\pi \sigma_y^2} \quad (2.16)$$

and the material in this region would carry a load

$$\int_0^{r_y} \frac{K}{(2\pi r_y)^{\frac{1}{2}}} \cdot dr = \left(\frac{2r_y}{\pi}\right)^{\frac{1}{2}} \cdot K$$

This load must be borne by the plastic region, which must therefore extend further to R_o . Hence

$$R_o \sigma_y = \left(\frac{2r_y}{\pi}\right)^{\frac{1}{2}} \cdot K$$

So, using eqn. (2.16), $R_o = 2r_y \quad (2.17)$

Thus the plastic zone extends to twice the distance at which the yield stress would be reached if the material had remained elastic. The elastic stress field beyond the plastic zone is equivalent to that for a crack of length $a + r_y$. This result led Irwin (1964) to introduce the concept of a "notional" crack of length $a + r_y$, with its tip in the middle of the plastic zone. The new elastic stress distribution is given by

$$\sigma_{11} = \frac{K^*}{(2\pi r)^{\frac{1}{2}}}$$

where
$$K^* = \sigma \cdot \{\pi(a+r_y)\}^{\frac{1}{2}} \cdot \lambda \quad (2.18)$$

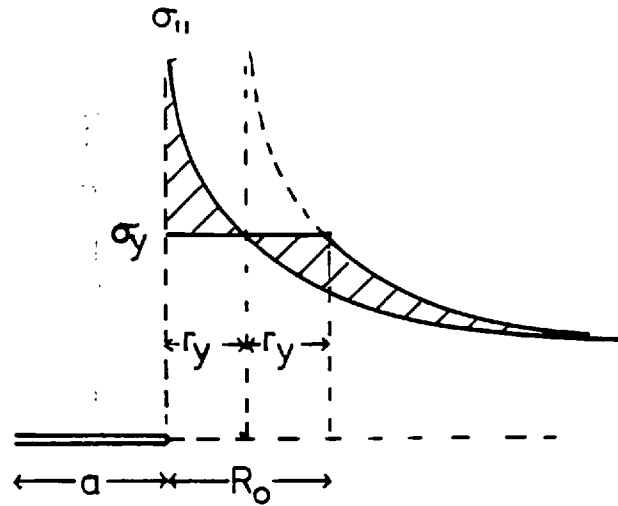


Fig. 2.4 Illustration of the Irwin (1964) plastic zone correction.

r_y is known as the plastic zone correction factor, and its use in eqn. (2.18) was an attempt to modify the linear elastic stress analysis for the case of small scale yielding around the crack tip.

2.4.2 Plastic Zones in Plane Stress and Plane Strain.

A free surface cannot support a force perpendicular to itself and so is always in a state of plane stress. Thus for a thin sheet with a crack in it the through-thickness stress, σ_{33} , is zero. Tresca's yielding criterion gives

$$\sigma_{11} - 0 = 2\tau_y = \sigma_y \quad (2.19)$$

where τ_y and σ_y are the yield stresses in shear and tension respectively. Throughout the plastic zone, $\sigma_{11} = \sigma_y$, as in fig. 2.5(a). The plastic zone size is given approximately by eqn. (2.16)

$$\text{i.e. } r_y = \frac{1}{2\pi} \frac{K^2}{\sigma_y} \quad (2.16)$$

In the centre of a thick body the material is prevented from deforming in a direction parallel to the crack front (i.e. along x_3) by the surrounding regions, and so the

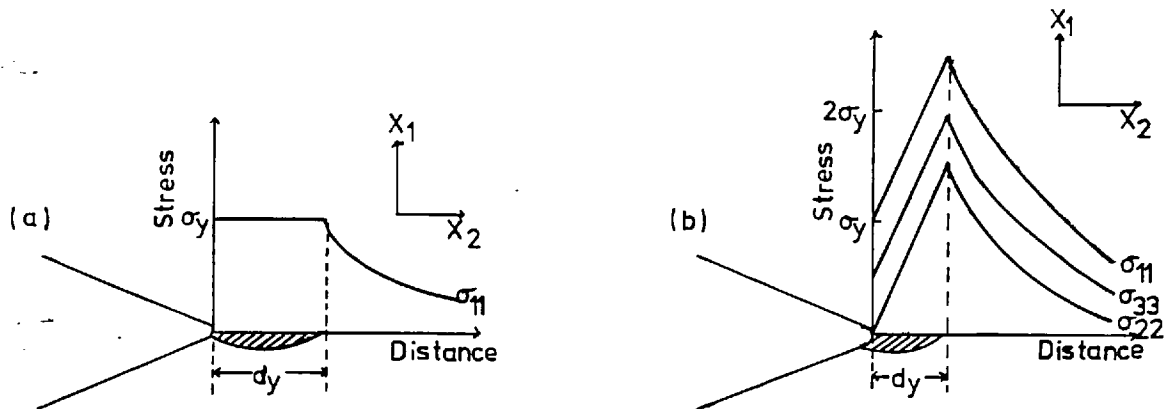


Fig. 2.5 Tensile stress distributions and plastic zones in (a) plane stress and (b) plane strain (after Knott, 1973).

strain, ϵ_{33} , in this direction is zero. The centre is thus in plane strain. The smallest principle stress is now σ_{22} (zero at the free surface of the notch root) and the yield criterion becomes

$$\sigma_{11} = \sigma_y + \sigma_{22} \quad (2.20)$$

The through thickness stress σ_{33} is

$$\sigma_{33} = \nu(\sigma_{11} + \sigma_{22}) \quad (2.21)$$

where ν is Poisson's ratio. σ_{22} is zero at the notch root and rises through the plastic zone. The maximum stress σ_{11} rises to $2.57 \sigma_y$ or $2.96 \sigma_y$, depending on whether the Tresca or von Mises yield criterion is assumed (fig. 2.5b).

In both these cases elastic/perfectly plastic conditions are assumed. Work-hardening can raise the stress considerably above the values given here (Rice and Johnson, 1970, and Section 4.2).

By analogy with the previous section, a smaller plastic zone is developed in plane strain than in plane stress, since the load sustained there is correspondingly higher. As $\sigma_{11} \approx 3 \sigma_y$ in plane strain, the plastic zone size is normally taken as one third its plane stress size. Thus from eqn. (2.16)

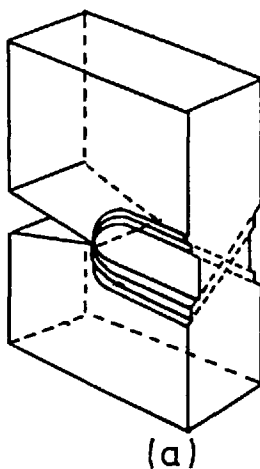
$$r_{Iy} = \frac{1}{6\pi} \frac{K^2}{\sigma_y^2} \quad (2.22)$$

Here the subscript I refers to plane strain conditions.

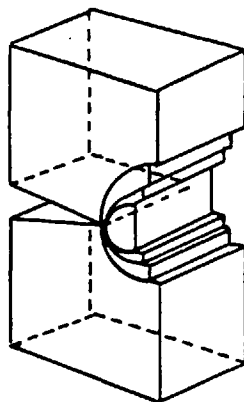
The pattern of yielding differs in the two stress states. In plane stress $\sigma_{33} = 0$, so 45° shear lines are developed between x_1 and x_3 (fig. 2.6(a)). In plane strain σ_{22} is the smallest stress so a hinge type of deformation arises between x_1 and x_2 , as in fig. 2.6(b). In practice, thin sheets are generally assumed to be in plane stress and display full slant fracture. In thicker bodies plane stress occurs on the surface, gradually giving way to plane strain in the middle. Crack extension results in a central region of flat fracture (plane strain) bordered by 45° shear lips (plane stress), as in fig. 2.7.

2.5 Linear Elastic Fracture Toughness Testing.

The basis of linear elastic fracture mechanics (LEFM) is the assumption that the parameter K has a critical value at fracture which is a property of the material alone. The results of many experiments on high strength materials (eg. ASTM STP numbers 381, 410 and 463) show that this is the case provided that the critical values are measured in plane strain,



(a)



(b)

Fig. 2.6

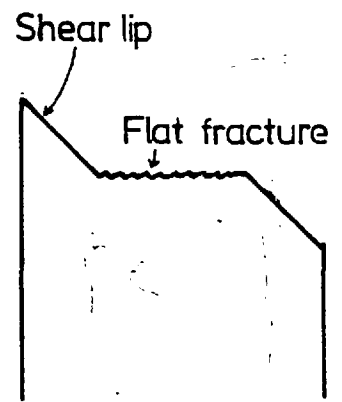


Fig. 2.7

Fig. 2.6 Yielding pattern in (a) plane stress and (b) plane strain

Fig. 2.7 Shear lip formation in mixed mode fracture.

as defined by certain empirically derived criteria. These are set out in DD3 (1971) and ASTM E399-70T.

The standard procedure is to use three point bend or compact tension test-pieces and load them up to instability. A clip gauge monitors the opening at the mouth of the crack, which is plotted against load. Briefly, the requirements for a valid test are as follows:-

(i) The maximum allowable non-linearity in the load-displacement trace is 5%, which restricts crack extension to 2% (Broek, 1974). In more brittle materials no slow crack growth precedes instability, and the measured K_{IC} corresponds to K at initiation, but this need not always be the case.

(ii) The size of the crack tip plastic zone must be small compared to the thickness (B), crack length (a) and ligament depth ($W-a$), so that plane strain conditions predominate. The requirements are

$$a, W-a, B \geq 2.5 \left(\frac{K_{IC}}{\sigma_y} \right)^2 \quad (2.23)$$

Test-pieces smaller than the ASTM requirements give invalid fracture toughness results. K_c has its minimum value, K_{IC} , in plane strain, but then increases with decreasing thickness, as the shear lip contribution increases, to a maximum in plane stress, at 100% slant fracture.

For a high strength maraging steel of yield stress 2000 MN m^{-2} and toughness K_{IC} of $55 \text{ MN m}^{-3/2}$, the minimum thickness required to derive a valid K_{IC} is, from eqn. (2.23), about 2mm. Specimens of this size are easily tested in the laboratory. However, as strength decreases and toughness increases, larger specimens are required. For A533B steel at room temperature, with $\sigma_y = 500 \text{ MN m}^{-2}$ and $K_{IC} = 150 \text{ MN m}^{-3/2}$, the minimum thickness is approximately 25 cms. For a structural mild steel this is increased by an order of magnitude. Thus it becomes impossible to derive valid K_{IC} results in the laboratory as the ductility increases.

2.6 Resistance to Crack Growth in Linear Elasticity

2.6.1 Definition and use of the R-curve

In a valid K_{IC} test crack extension is limited to about 2%. In many situations, however, such as with more ductile materials or thin sheets, much larger crack extensions under rising load may be encountered. The resistance curve concept may be used in such cases to predict the onset of final unstable crack extension.

Irwin and Kies (1954) defined the resistance, R , of a material to crack propagation as the rate of change, with respect to crack length, of the energy, v , required to extend the crack. For unit thickness,

$$R = \frac{dv}{da} \quad (2.24)$$

The energy v is assumed to be absorbed in the surface work of fracture and in the associated local plasticity.

The energy available for crack advance per unit extension is G , the elastic energy release rate or crack extension force. For the crack to extend this must be equal to the resistance of the material to crack advance.

$$\text{i.e.} \quad G = R \quad (2.25)$$

It was originally assumed that the absorbed energy, v , was constant with crack growth, so that R was constant at $2\gamma'$, or G_{IC} . However, this was found to be insufficient to describe fracture in more ductile materials. Boyd (1953) found that the development of shear lips greatly increased the fracture resistance as the crack extended. Stable crack growth could occur under rising load before the point of catastrophic failure.

Therefore, Irwin (1960) suggested that R increases with crack extension. A typical R curve is drawn in fig. 2.8, where R is plotted against crack extension, Δa . The initial crack size is plotted to the left. R has a finite value at initiation of crack growth, and rises thereafter. The G

loading lines are plotted on the same diagram. For constant load conditions these have the form

$$G = \lambda^2 \cdot \pi \sigma^2 a / E'$$

The compliance function, λ^2 , causes them to be curved, concave upwards. As σ increases, the slope of the G loading lines increases, and the crack extends. Eventually the critical combination of stress and original crack length is reached, and catastrophic fracture results. At this point, the G curve is a tangent to the R curve (fig. 2.8). Thus, for instability

$$\frac{dG}{da} \geq \frac{dR}{da} \quad (2.26)$$

The amount of crack extension prior to instability, Δa_c , and the critical crack extension force G_c , may be read off from the diagram. If the body a had different initial crack length the corresponding values of G_c and Δa_c would be different.

If constant displacement conditions obtain, the G loading lines have a negative slope and, since dR/da is always positive, it is impossible to get instability. This gives a means of measuring the complete R curve (detailed further in Section 3.6) and also demonstrates that instability is governed by the loading conditions as well as crack length.

Krafft, Sullivan and Boyle (1961) postulated that for a given thickness, the R -curve is an invariant material property, independent of initial crack length and other specimen dimensions. The validity of this hypothesis has not been firmly established, either analytically or empirically. They also modelled the R -curve in terms of shear lip development, as described in Section 9.2.1. Attempts have been made by Raju (1969) and Wnuk (1971) to derive theoretical expressions in terms of the rate of dissipation of plastic energy. The vast majority of the work in this area, however, has been experimental.

2.6.2 Measurement of Linear Elastic R-Curves

Linear elastic resistance curves have usually been derived from experiments on thin sheets of high strength alloys used in aircraft construction. Of course the specimen thickness does not satisfy the ASTM conditions for valid fracture toughness testing, but the plastic zone size should remain small compared to the specimen width and crack length. Experimental techniques are described in ASTM STP 527 (1973) and a recommended standard (1973, revised 1974) proposes the use of three specimen configurations. The most popular is the double cantilever beam (DCB) with wedge loading, as it allows large crack extensions under displacement controlled conditions.

The effective crack length, with plastic zone correction, is usually used in the construction of the R-curve. This may be measured optically and corrected appropriately, or estimated by specimen compliance measurements. Of the latter method the most common is the double compliance technique, pioneered by Boyle (1962), in which gauges record crack opening at two points outside the plastic zone, and the effective crack length is deduced with the help of elastic compliance data.

Resistance curves were first obtained in terms of G , but K is now a more common parameter. The two are simply related, since $K^2 = E'G$ in small scale yielding. The terms G_R or K_R are used to indicate the parameter employed.

The Krafft et al (1961) hypothesis, concerning the geometry independence of the R-curve, has not been proven experimentally. For instance, Heyer and McCabe (1972), Wang and McCabe (1974) and Broek and Vlieger (1974) found the R-curve to be independent of initial crack length and testing configuration, whereas Bradshaw and Wheeler (1974) and Adams and Munro (1976) found slight variations with configuration.

In these thin sheet materials, the rise in the R-curve may be attributed to the growth of shear lips, and an

accompanying rise in plane stress plastic zone size. This latter feature tends to invalidate the linear elastic analysis on which the technique is based. In addition, interest has grown recently in the application of the R-curve concept to thick sections, where the "plane strain" resistance may change significantly with crack extension. Current work centres on the adaptation of yielding fracture mechanics to describe slow crack growth.

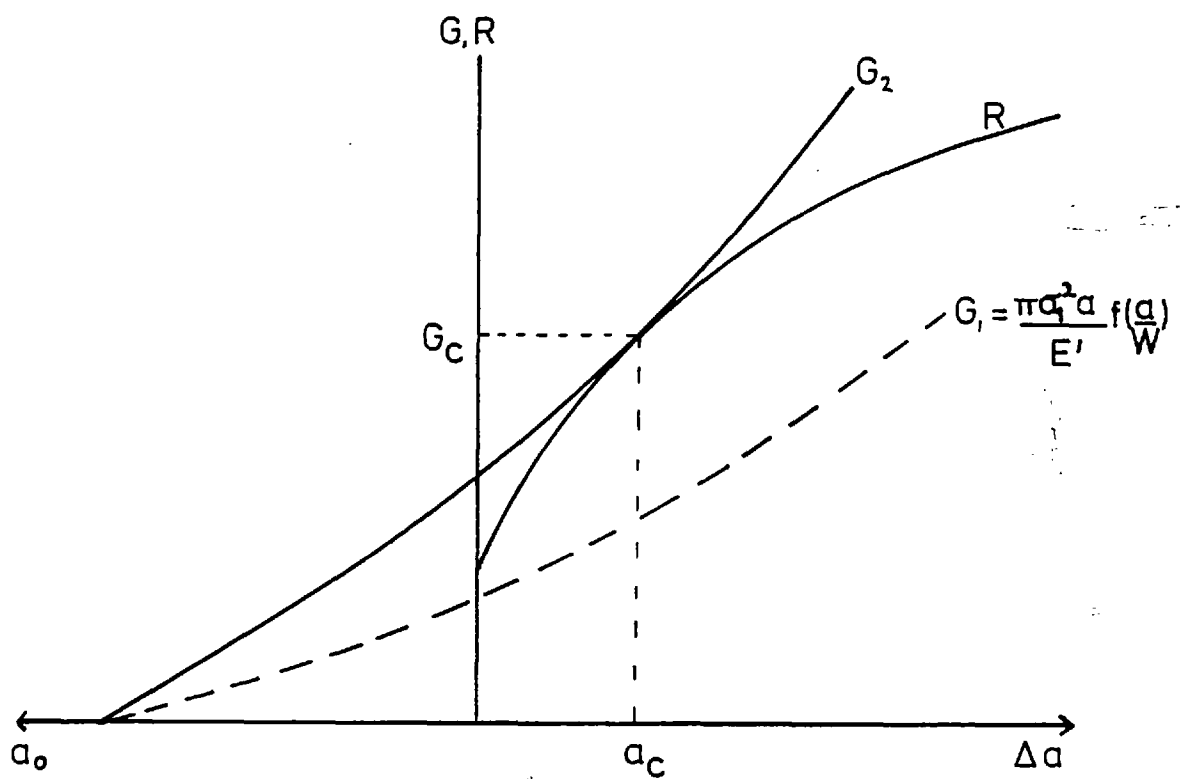


Fig. 2.8 Prediction of instability using R-curves.

Chapter Three

YIELDING FRACTURE MECHANICS

In an elastic material the stress and strain fields are completely determined by the stress intensity factor, K . A consequence of the elastic theory, however, is that the stress at an infinitely sharp crack tip goes to infinity. Yielding ahead of the crack always intervenes in metals. Since a complete elastic-plastic analysis of the stress and strain fields around a crack tip is not available, it is not possible to establish the existence of a one-parameter description of the tip environment in the presence of plasticity. If the extent of local plasticity is very small compared to the crack length and specimen dimensions, the deviations from linear elastic behaviour are correspondingly small and may be ignored. For slightly more extensive yielding, various correction factors have been proposed by, for instance, Irwin (section 2.4.1) or Dugdale (see below). When the plastic zone size becomes comparable with the specimen dimensions the errors occasioned by an essentially linear elastic approach increase. Other methods based on crack opening displacement (COD) or on the J contour integral hold more promise and are described below.

3.1 The Dugdale Model

Dugdale (1960) approximated the plastic zone to a thin strip along the crack line, ahead of the tip. Appropriate analysis, for an infinite plate containing a central crack of length $2a$ under a remote tensile stress σ , gave, for the plane stress plastic zone size, R_0 ,

$$R_0 = a \left\{ \sec \frac{\pi \sigma}{2\sigma_y} - 1 \right\}$$

Expanding for small σ/σ_y ,

$$R_0 = \frac{\sigma^2 \pi^2 a}{8\sigma_y^2} \quad (3.1)$$

Hence, taking the plastic zone correction factor r_y as $R_0/2$,

the stress intensity factor becomes

$$K = \sigma \left\{ \pi a \left(1 + \frac{\sigma_y^2 \pi^2}{16 \sigma^2} \right) \right\}^{\frac{1}{2}} \quad (3.2)$$

The correction is slightly larger than in the Irwin treatment.

Bilby, Cottrell and Swinden (1963) obtained similar results by modelling the crack and plastic zone as a linear array of dislocations. On both models the total displacement δ of the crack faces at the tip was derived as

$$\delta = \frac{8\sigma_y a}{\pi E} \ln \sec \left(\frac{\pi \sigma}{2\sigma_y} \right) \quad (3.3)$$

These results, evaluated from a strip yielding model, are of limited use, since firstly their validity becomes questionable as the extent of yielding increases, and secondly they only attempt to describe plane stress conditions. The treatment of plasticity in plane strain is much more difficult due to the effects of constraint upon the yielded zone.

3.2 Crack Opening Displacement

3.2.1 Definition and Use

As ductility increases, or specimen size decreases, the extent of yielding increases at a given remote stress, until it spreads across the entire remaining section. In linear elastic fracture mechanics crack extension occurs when a critical stress or strain is reached at the crack tip. When the material at the crack tip yields, however, the stress there remains constant (assuming no work-hardening), and it may then be expected that fracture initiates at a critical strain. The displacement at the crack tip was proposed independently by Wells (1961) and Cottrell (1961) as a measure of the local strain, and is known as the Crack Opening Displacement, or COD.

When a sharp crack opens, the tip blunts and extends slightly, making the measurement point for the COD arbitrary. Wells (1961) chose the position of the original crack tip.

Using Irwin's plastic zone expression, Section 2.4.1, this is the point where the elastic-plastic boundary intersects the crack faces (fig 3.1). Assuming small scale yielding and plane stress, the COD was expressed in terms of the linear elastic crack tip parameters, K or G:-

$$\delta = \frac{4K^2}{\pi E \sigma_y} = \frac{4G}{\pi \sigma_y} \quad (3.4)$$

This may be rewritten,

$$G = M \sigma_y \delta \quad (3.5)$$

where in this case $M = \pi/4 = 0.78$. Other models give essentially the same relationship with slightly different values of M. For the Dugdale model (plane stress), $M = 1$ is obtained by expanding eqn. (3.3). In plane strain, finite element analyses (eg. Rice, 1973) indicate that $M \approx 2$.

The exact value of M is of no great importance in practice, provided that it is a constant for a given material and geometry. Then it can be postulated that the critical COD, δ_i , at which crack extension begins is a constant, and that this same value obtains in both small laboratory - sized specimens and a cracked structure which is too large to be tested conveniently. In designing against fracture the critical COD, δ_i , for the material in use may be measured in the laboratory, and then the critical crack length and stress in a structural member derived from a relationship such as eqn. (3.3). See Burdekin and Dawes (1971) for a full description.

A drawback concerning the use of COD in design is that it is the critical value of COD at crack initiation which is commonly employed. In a large member slow crack growth under rising load may precede instability, in which case design on the basis of δ_i will be conservative.

3.2.2 The Measurement of COD

The first measurements of COD were made on specimens with machined notches (Burdekin and Stone, 1966). A rotating

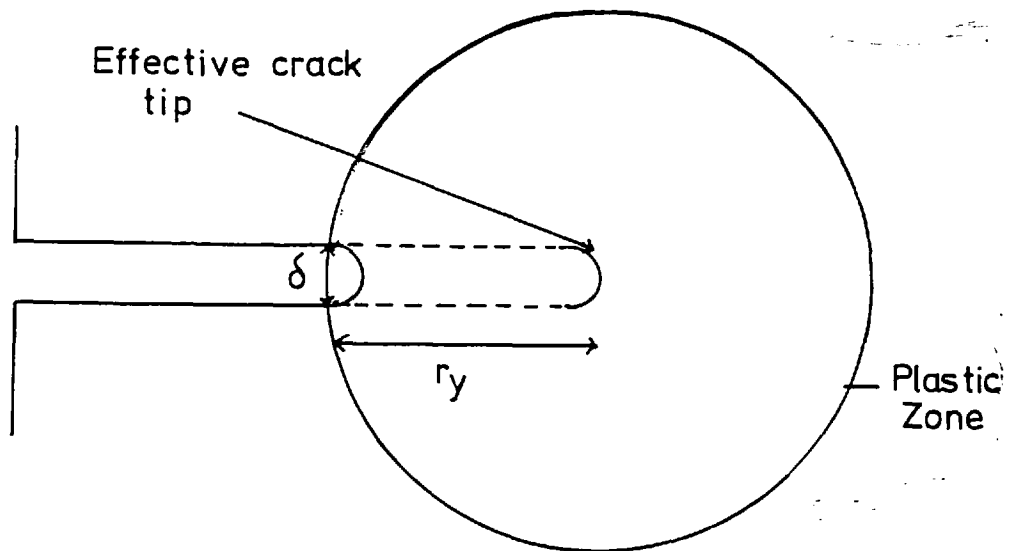


Fig. 3.1 The definition of COD (Wells, 1961)

paddle meter was placed at the bottom of the slot, which had a practical minimum width of 0.1 mm. This had the advantage of measuring COD directly at the root of the notch, but could not be used for specimens precracked in fatigue. In this case the easiest method, for a bend test-piece, is to measure the opening at the mouth of the crack, and infer the COD by means of a geometrical construction.

A proposed British standard, DD19 (1972), recommends the use of precracked, three point bend test-pieces. The opening of the crack at standard knife edges is measured using a clip gauge (fig. 3.2). Two alternative formulae are proposed for converting the clip gauge displacement, V_g , to COD: one, due to Ingham et al. (1971), assumes that the two halves of the specimen rotate about a pivot a distance $r(W-a)$ ahead of the crack tip, where r is known as the rotational factor. Then

$$\delta = \frac{V_g}{1+(a+z)/r(W-a)} \quad (3.6)$$

where z is the height of the knife edges, W is the specimen width and a is the crack length. Ingham suggested that r be taken as $1/3$.

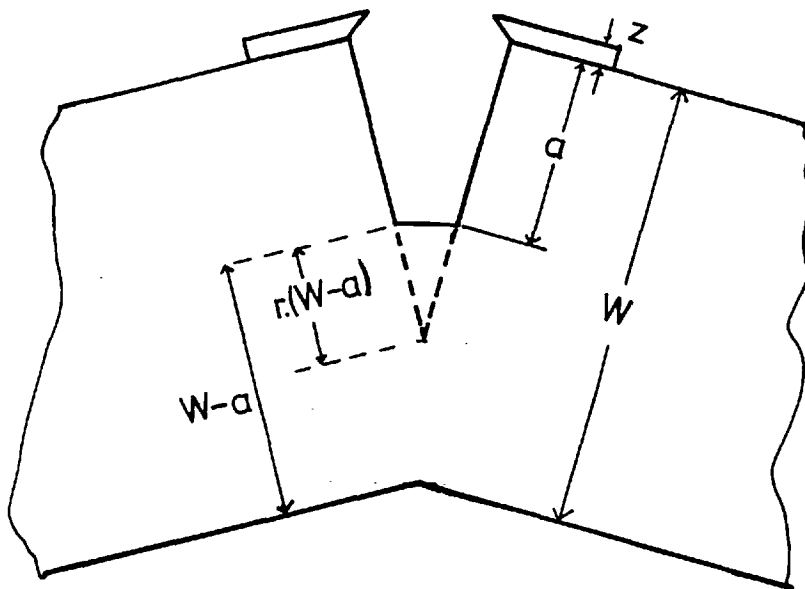


Fig. 3.2 Geometrical configuration of a cracked, rigid-plastic beam in three-point bending.

The other formula (Wells, 1971) is based on slip line theory in plane strain, and assumes $r = 0.45$. Some allowance is made for the observed parabolic relationship between COD and the applied moment in the elastic regime:-

$$\delta = \frac{0.45(W-a)}{0.45W+0.55a+z} \left[V_g - \beta \frac{\sigma_y W}{E'} \right], \quad V_g > \frac{2\beta \sigma_y W}{E'}$$

$$\delta = \frac{0.45(W-a)}{0.45W+0.55a+z} \left[\frac{V_g^2 E'}{4\beta \sigma_y W} \right], \quad V_g \leq \frac{2\beta \sigma_y W}{E'} \quad (3.7)$$

$\beta = V_g E' / \sigma_y W$ is the non-dimensional clip gauge displacement at elastic limit load. The variation of β with a/W is given in DD19 (1972).

The derivation of COD from V_g is a simple technique and is by far the most common method. However, it can only be employed on bend specimens, where a rotational factor is measured or assumed. Other, more direct techniques, have been developed for studying the crack tip. Robinson and Tetelman (1974) infiltrated the crack tip with a catalytically hardening silicone rubber, and then broke the specimen open in the cleavage mode. Measurements of COD could be made on sections of the rubber cast. Garwood (1977) used this technique to measure the COD at a propagating crack tip.

The COD in plane stress can be measured directly on the surface of the specimen. For mid-section, plane strain values, the specimen must be unloaded and sectioned. This gives off-load values of COD, but the crack can be held open partially by filling it with a hardening resin under load (Lewis et al., 1975).

Another possible method is the measurement of the blunted profile at the crack tip, known as the "stretch zone". In theory the COD can be deduced from this method after the crack has propagated, raising possible applications in failure analysis. Stretch zone formation is discussed in more detail later.

Many experiments have been performed to determine how COD varies with test-piece geometry. The details are summarised in section 7.2.1, but the general conclusion is that the COD at initiation of crack extension, δ_i , is independent of geometry provided the test-piece is sufficiently large to allow plane strain conditions in the centre. The same is not true of the COD after initiation (for instance, at maximum load), where variations occur with specimen size (eg. Green and Knott, 1975b; Griffis 1975).

3.3 The J Contour Integral

3.3.1 Theoretical Basis

A convenient means of evaluating the energy release rate G in elasticity is provided by a line integral surrounding the crack tip. Eshelby (1956) proposed the use of such an integral, but its application to fracture is usually credited to Rice (1968a). He defined the J contour integral in two dimensions as

$$J = \int_{\Gamma} (w dx_1 - T_i \frac{du_i}{dx_2} ds) \quad (3.8)$$

for a crack aligned in the x_2 direction (fig. 3.3), where Γ is a contour surrounding the tip starting from the lower surface and finishing on the upper, s is arc length, T_i is the surface traction on Γ and u_i the surface displacement

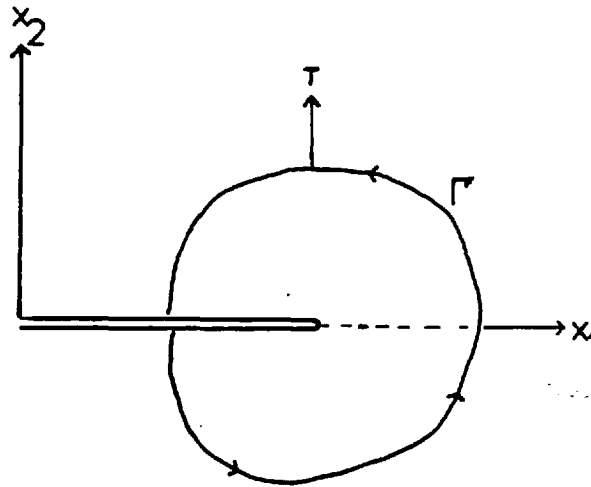


Fig. 3.3 The definition of the J-integral (Rice, 1968a)

in Cartesian coordinates, and w is the strain energy density.

Rice (1968b) then showed that in a material exhibiting linear or non-linear elasticity, J is independent of Γ , the contour chosen. Thus J may be evaluated along any convenient contour in the body which begins on one crack surface and ends on the other. It was also shown that J was equal to the rate of change, with respect to crack length, of potential energy P , for a cracked body of unit thickness:

$$J = - \frac{dP}{da} \quad (3.9)$$

The potential energy of a cracked body is defined as the difference between the internal strain energy and the potential energy of the external forces:

$$P = \int_V w dV - \int_{A_T} T_i u_i dA$$

where V is the volume of the body, and A_T the surface area on which the tractions T_i act. For two similar bodies with crack lengths a and $a + \Delta a$, loaded to either the same load or displacement, it can be shown that the difference in potential energy ΔP is given by the area, ΔU , between the two load/deflection diagrams (fig 3.4). The difference in ΔU between the two cases of fixed load or fixed displacement is

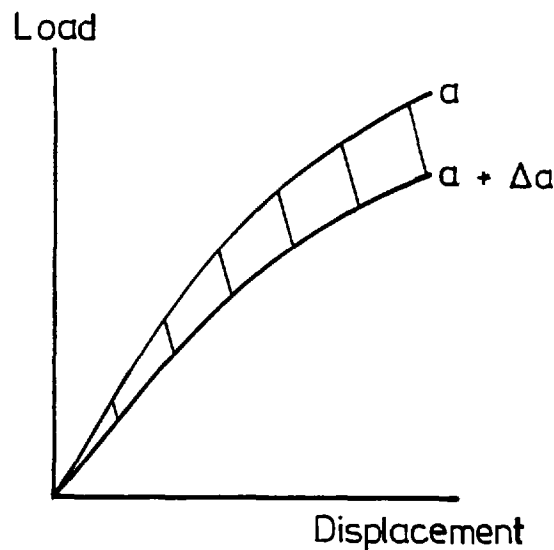


Fig. 3.4 The difference in strain energy between two adjacently-cracked, non-linear elastic bodies.

second order and thus negligible, for small Δa .

For linear elasticity,

$$\begin{aligned} J &= - \frac{dP}{da} \\ &= G = K^2/E' \end{aligned} \quad (3.10)$$

using eqn. (2.12) and (2.15). Hence J is identical to the strain energy release rate, and so the energy for a crack advance of Δa may be expressed as $J.B.\Delta a$.

This same result is true for non-linear elasticity. Let the crack tip and contour move forward by a distance da . From eqn. (3.8) the quantity $J.da$ comprises the sum of two terms:

$$\begin{aligned} w dx_1 da &= \text{the strain energy gained or lost on moving} \\ &\quad \text{to the new contour} \\ T_i \frac{du_i}{dx_2} ds da &= \text{work done by tractions } T_i \text{ on the contour.} \end{aligned}$$

Thus $J.da$, given by the sum, is the total energy passing through the contour as it moves. Since J is path independent, the same result is achieved for all contours, including one just surrounding the crack tip. Thus the quantity $J.B.da$ is the energy available at the tip for crack extension.

In addition, J can be shown to have a characterising role similar to that of K in linear elasticity. The contour integral, eqn. (3.8), is path independent, so that as the contour is shrunk onto the crack tip, only the singular stresses and strains there are important.

3.3.2 The Role of the J-Integral as a Crack Tip Characterising Parameter in Elastic-Plastic Materials.

It was seen in the previous section that, in non-linear elasticity, the J -integral is a measure both of the energy available for crack extension, and of the stress and strain singularities at the crack tip. It may be used as a characteristic parameter in the same way that G is used for linear elasticity.

The extension of the concept to elastic-plastic materials rests on the assumption that, during loading, their stress-strain behaviour is identical to that of non-linear elastic ones. The load of displacement curves for two adjacently cracked non-linear elastic bodies are shown in fig. 3.5(a), and the corresponding curve for elastic-plastic bodies in fig. 3.5(b). The difference is that in the elastic bodies the loading and unloading paths are identical, whereas in the elastic-plastic bodies they are not, due to irreversible plastic deformation. However, the loading paths are the same in the two cases, so that J , as calculated from the shaded areas using eqn. (3.9), is not affected. Thus J can be used in the elastic-plastic regime provided that the loading curves are assumed to be non-linear elastic. This is the same as assuming a "deformation" theory of plasticity, where strains are independent of load path, in contrast to the more realistic "incremental" theory.

$$\text{Then} \quad J = - \frac{dP_s}{da} \quad (3.11)$$

where P_s is now the "psuedo" potential energy, termed such because the plastic part of the absorbed energy is not recoverable.

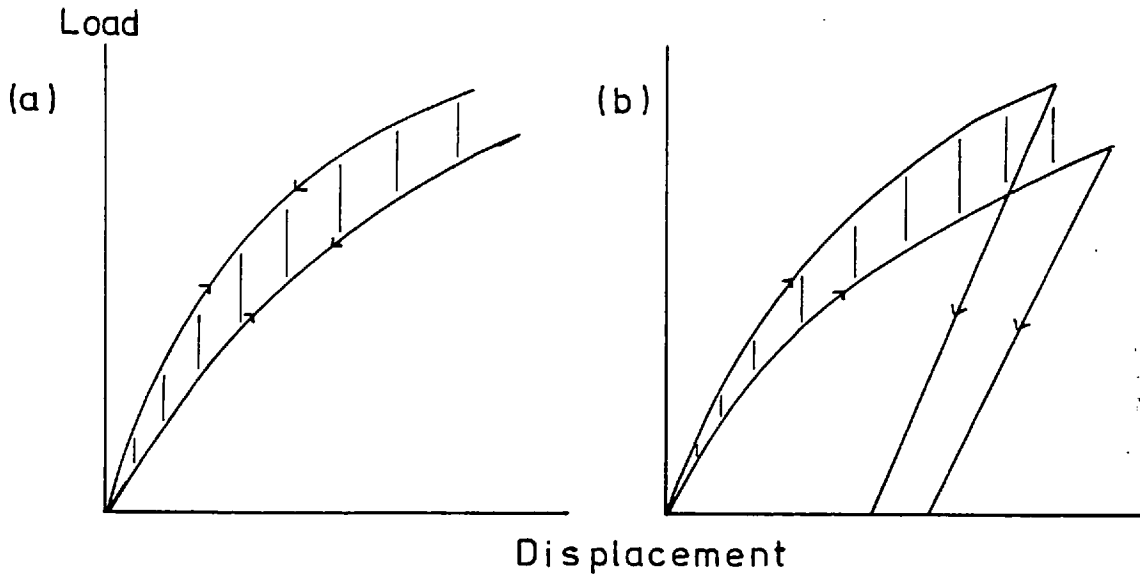


Fig. 3.5 Load/displacement curve for two adjacently cracked bodies which are (a) non-linear elastic, and (b) elastic-plastic.

Although J may be calculated in the elastic-plastic regime, its relevance to fracture is questionable. Some of the pseudo potential energy is lost in irreversible plastic deformation, so J becomes an energy absorption rate, rather than the energy release rate available for crack extension. Thus J loses its energetic meaning in the presence of plasticity.

However, Rice and Rosengren (1968) and Hutchinson (1968) showed that a stress or strain singularity still exists at the tip of a crack, for a material governed by a deformation theory of plasticity. McClintock (1971) expressed these singularities in terms of the J -integral, in the form:-

$$\begin{aligned} \text{stress } \sigma &= f_1(J) r^{-n/(n+1)} \sigma(\theta) \\ \text{plastic strain } \epsilon_p &= f_2(J) r^{-1/(n+1)} \epsilon(\theta) \end{aligned} \quad (3.12)$$

where f_1 and f_2 are functions of J , r is the polar distance from the crack tip, n is the strain hardening exponent, and $\sigma(\theta)$ and $\epsilon(\theta)$ are angular functions. Thus in deformation plasticity J has a characterizing role similar to that of K in linear elasticity.

In practice real materials are governed by an incremental theory of plasticity. The deformation theory only applies for monotonic loading in tension. The theory does not allow complex changes in loading history, or unloading. In particular, the relevance of the J-integral during slow crack growth is questionable and will be examined later.

Finite element analyses indicate that the conditions relating to the validity of J can be relaxed somewhat. Hayes (1970) and Sumpter (1974) confirmed that the J-integral is approximately path independent to beyond general yield in an incremental plasticity material. After allowing for finite geometry changes at the crack tip, McMeeking (1977a and b) found the J-integral path independent everywhere except in the near tip region, and concluded that deformation here was controlled by the remote value of J. In the same vein, Broberg (1971) postulates the existence of a small, "autonomous" region at the crack tip, where continuum mechanics descriptions break down. However, J should be path independent outside this end region, and it is concluded that the J-integral or COD are both valid one-parameter fracture criteria for ductile materials. Similar arguments are used by Paris (1977).

In summary, the concept of the J contour integral is based on sound theoretical grounds but its application to real ductile materials and its relevance as a fracture parameter, though promising, is less clear. Experimental evidence is required to determine the limits of its validity.

3.3.3 The Measurement of the J-Integral

Landes and Begley (1972) and Begley and Landes (1972) were the first to measure J from monotonic load/deflection curves on specimens of slightly dissimilar crack lengths, by means of the formula

$$J = - \frac{1}{B} \frac{\Delta U}{\Delta a} \quad (3.13)$$

where B is specimen thickness, and U the area under the load deflection diagram. By measuring these areas a family of curves of U/B vs crack length was developed for different

displacements, allowing J vs. displacement curves to be derived from the gradients. No allowance is made for stable crack extension in this method, and the original assumption was that the cracks initiated at maximum load.

This method required many calibration specimens. Rice et al. (1973) derived a formula whereby J could be estimated from the load/deflection curve of a single specimen. In three point bending,

$$J = \frac{2 U_{\text{crack}}}{B(W-a)} \quad (3.14)$$

where U_{crack} is the absorbed energy due to the presence of the crack. For deeply cracked test-pieces, this is approximately the same as the total absorbed energy, U_{tot} , so

$$J \approx \frac{2 U_{\text{tot}}}{B(W-a)} \quad (3.15)$$

Sumpter (1974) suggested that the absorbed energy be split into elastic and plastic components. Then

$$U_{\text{tot}} = U_e + U_p \quad (3.16)$$

$$J_{\text{tot}} = J_e + J_p \quad (3.17)$$

where the subscripts e and p refer to the elastic and plastic components respectively. J could then be split into separate terms:

$$J = \frac{\kappa_e U_e}{B(2-a)} + \frac{\kappa_p U_p}{B(W-a)} \quad (3.18)$$

κ_e and κ_p are factors dependent upon geometry and configuration, and their derivations for several configurations are given in Appendix I. In three point bend the result is that

$\kappa_e = \kappa_p = 2$, for $0.4 \leq a/W \leq 0.7$, so that

$$\begin{aligned} J &= \frac{2 U_e}{B(W-a)} + \frac{2 U_p}{B(W-a)} \\ &= \frac{2 U_{\text{tot}}}{B(W-a)} \end{aligned} \quad \begin{array}{l} \text{Three point bend} \\ 0.4 \leq a/W \leq 0.7 \end{array} \quad (3.19)$$

The Sumpter formulae employ the total absorbed energy ($U_e + U_p$), whereas Rice et al. recommend the subtraction of the uncracked body energy. The correct treatment is still under discussion. Some theoretical (Srawley, 1976) and experimental (Chipperfield, 1976) studies support the use of total energy, but experimental work by Underwood (1975) suggests the converse.

Other methods of estimating the J-integral have been developed. One such, by Bucci et al (1972), uses an approximate relationship between J and displacement, derived from linear elastic and rigid-plastic analyses. Another, by Sumpter and Turner (1976), evaluates J from the clip gauge displacement, linear elastic mechanics and the plastic limit load.

All methods are based on the energy absorbed by the cracked body. Therefore, extraneous displacements, caused for instance by roller indentation or jig deflection, must be subtracted from the overall displacements, as recommended by Robinson and Tetelman (1976) and Garwood (1977).

Until recently, interest was centred almost exclusively on the measurement of J at initiation of crack extension, J_i (also called J_{IC} , although the term is ambiguous). As with COD, the R-curve technique is most commonly employed (Landes and Begley, 1974), with J being plotted against crack extension, Δa . By extrapolating back to zero crack extension, J_i can be deduced. If measurements of J_i were made from the start of ductile tearing, this point is at $\Delta a = 0$. If they were taken from the original fatigue crack tip, they will include the stretch zone, and a line, of assumed slope $\approx 2\sigma_{flow}$ (where σ_{flow} is the average of the yield and ultimate stresses), must be constructed (fig. 3.6).

Published experimental results on J_i determinations are reviewed in section 7.2.1, but the bulk of the evidence indicates that J_i is a material constant, independent of geometry and configuration, provided certain dimensional criteria are met. These are

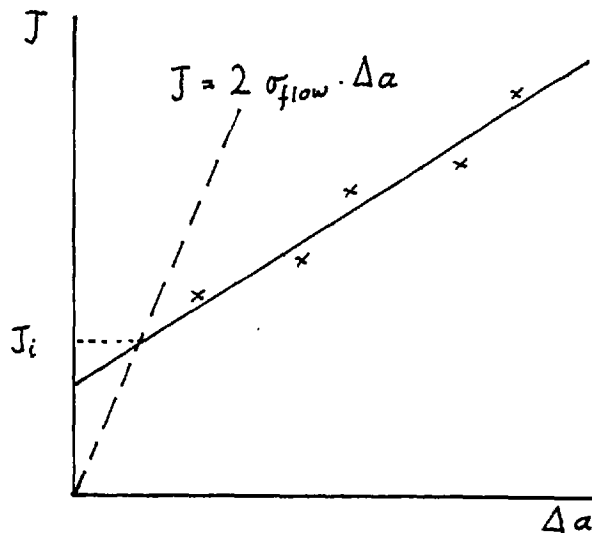


Fig. 3.6 The R-curve construction for determining J at initiation.

$$B, a, W-a \geq \alpha \frac{J_i}{\sigma_{flow}} \quad (3.20)$$

where the recommended value of α is 50 (Paris, 1972) or 25 (Landes and Begley, 1972). As with COD, the critical value of J at maximum load is geometry dependent, due to the effects of slow crack growth.

3.4 Other Methods of Assessing Fracture Toughness

3.4.1 The Equivalent Energy Method (Witt, 1971)

Witt postulated that the energy absorbed in fracture of an elastic-plastic body is the same as if the body had remained linear elastic. From the experimental load/displacement plot, a linear elastic plot can be constructed with the same initial slope and equivalent absorbed energy given by the area under the lines. The elastic fracture load is thus inferred and substituted into the appropriate formula for K_{IC} .

The explanation for the apparent success of the empirical approach would seem to be that, for bend bars with $0.4 < a/W < 0.7$, the elastic and plastic κ factors in eqn. (3.18) are equal, i.e. that elastic and plastic energies are equivalent (if J is a valid fracture parameter). However, this is not true for other geometries or configurations, where the κ

factors may be very different (see Appendix I), and in any case the effects of slow crack growth are uncertain.

3.4.2 The Non-Linear Energy Release Rate, $\tilde{G}$

Liebowitz and Eftis (1971) introduced the parameter $\tilde{G}$ as a means of estimating the surface work of fracture, $2\gamma'$, from an energy balance. For a non-linear elastic material, $\tilde{G}$ at initiation will equal J_i . For a real material the calculation of $\tilde{G}$ is more complex than that of J , since an analytical representation of the load/displacement curve is required for each specimen. A more fundamental drawback is that, unlike J , $\tilde{G}$ has no characterizing meaning attached to it. Its derivation is based solely on energetic considerations and as will be seen in section 3.6, this leads to many uncertainties.

3.5 The Application of Yielding Fracture Mechanics to Slow Crack Growth

In ductile materials parameters concerned solely with the initiation of fracture are inadequate for describing material characteristics, and tend to give highly conservative estimates of toughness. There is a need, therefore, to describe resistance to fibrous crack extension. Through the R-curve concept there is also the possibility of predicting the onset of catastrophic fracture in structural members. The two most promising parameters to describe slow crack growth are the J-integral and COD.

There are theoretical drawbacks to the use of J . Strictly speaking, it can only be applied to elastic-plastic materials if no unloading occurs. This is not the case for material near the crack borders during crack extension. Some of the energy absorbed goes into irreversible plastic work and may not contribute to further crack advance, so use of total energy in calculating J is likely to prove an over-estimate. Another uncertainty is the correct value of crack length to be used for calculation.

Difficulties with COD are more practical. The method is restricted to bend configurations for ease of measurement, and it is not clear at what point in the specimen the COD should be calculated. Possibilities are at the original fatigue crack tip, or at the propagating tip.

3.5.1 Application of the J-integral

J may be estimated by means of one of the estimation formulae described in Section 3.3.3. Landes and Begley (1974) used eqn. (3.15) for bend bars, viz

$$J_o = \frac{2 U}{B(W-a_o)} \quad (3.21)$$

The subscript o signifies that the original crack length, a_o , is used in the expression. Clarke et al. (1976), Logsdon (1976) and Shih et al. (1977) have used the same method for compact tension specimens, and Begley and Landes (1976) for bend bars and centre cracked panels.

Another way of estimating J is via calibration specimens (McCabe, 1974 and 1976; Griffis and Yoder, 1974). The value of J for a specimen which had undergone crack growth is assumed to be the same as that of a calibration specimen, blunt-notched to suppress crack extension, at the same load and final crack length. Plastic deformation accompanying crack growth is ignored, and in fact a smaller displacement is observed in the test-piece than in the calibration specimen at the same final load (Garwood, 1977).

Calibration methods utilise two parameters, final load and crack length (or equally, final displacement and crack length) to derive J. The three parameter method of Garwood, Robinson and Turner (1975) estimates J from all three parameters, and is an attempt to provide a correction for the effects of slow crack growth. J is derived for a postulated non-linear elastic specimen with the same final values of load, displacement and crack length as the real specimen which had undergone crack extension. The details of the method are described in Appendix II; the result, however, is that after a small increment of crack extension

Δa ,

$$J = J_i \frac{(W-a_o-\Delta a)}{(W-a_o)} + \frac{2 \Delta U}{B(W-a_o)} \quad (3.22)$$

where J_i is the initiation value of J , a_o is the initial crack length, W is the specimen width and ΔU the energy absorbed during crack extension.

Recently attempts have been made to define J during slow crack growth in a more rigorous manner. Burns and McMeeking (1978) deduce that, for bend bars

$$J = \frac{2}{b_1} \int_0^\theta M \cdot d\theta + \frac{2}{b_1} \int_{b_o}^{b_1} J \cdot db \quad (3.23)$$

where M is the bending moment per unit thickness, θ the angle of bend due to the presence of the crack, and b_o and b_1 are the initial and final lengths of the uncracked ligament. Curry (1978) has shown that this equation reduces to a form very similar to the Garwood et al. expression. Hutchinson and Paris (1977), derive another form:

$$J = 2 \int_0^\theta \frac{M}{b} d\theta - \int_{a_o}^a \frac{J}{b} da \quad (3.24)$$

Garwood (1977) showed that the difference in calculated J values between eqns. (3.21) and (3.22) was zero at initiation and increased thereafter, and that at $\Delta a = 0$ the R-curves as calculated by either method were tangential.

3.5.2 Application of Crack Opening Displacement

COD is usually determined in bending by measuring the crack mouth opening and assuming a value of rotational factor. The centre of rotation is either taken as remaining fixed in the test-piece, or assumed to move, remaining at a constant fraction of the ligament depth. The errors occasioned by either assumption are small, for small crack extensions.

In many cases COD resistance curves have been used as a means of determining the initiation COD unambiguously (eg Smith and Knott, 1971; Green and Knott, 1975a; Robinson and Tetelman, 1976). Elliott et al. (1971) proposed that

the COD, δ_o , at the original crack tip could be used to characterize the propagation toughness, a suggestion also made by Green and Knott (1975b). It is likely that the δ_o vs Δa curve will have a positive slope even for brittle materials, as a geometrical consequence of crack extension.

The COD, δ_f , extrapolated to the position of the final, growing crack tip has been employed to portray the R-curve (Tanaka and Harrison, 1976; Kamath and Harrison, 1977; Garwood 1977). It is shown in section 8.3 that this parameter has little physical significance. A similar technique has been developed by McCabe (1976) for use on wedge loaded, DCB specimens in plane stress. A double compliance procedure is used to derive the COD at the propagating crack tip, and the toughness calculated from the formulae $G_R = \sigma_y \cdot \delta_f$, and $K_R^2 = EG_R$. Again, the physical basis of the method is questionable.

3.6 Energetic Considerations in Fracture

The general energy balance for fracture is usually modelled on the Griffith theory. The approach outlined here follows that of Rice (1965). The detailed mathematics are complex and may be found there: the principles and conclusions are discussed below.

3.6.1 The Energy Balance for Crack Extension

Consider a cracked continuum (Fig. 3.7), loaded with surface tractions $\underline{T}$ on the portion of the bounding surface A_T , body forces $\underline{F}$ per unit volume through-out the volume V , and imposed displacement $\underline{u}$ on the portion of the bounding surface A_u . Let the crack extend under constant tractions $\underline{T}$ and displacements $\underline{u}$ from an initial state (1) to a state (2), so that the crack surface (assumed traction-free) increases in area by A' . The work of the applied forces is

$$\Delta W = \int_{A_T} T_i (u_{i1} - u_{i2}) dA + \int_V F_i (u_{i1} - u_{i2}) dV$$

Subscripts i denote the Cartesian component, and 1 and 2 the initial and final states respectively. This work can be

absorbed in three ways, assuming no temperature change:-

- i) As stored elastic energy and energy dissipated in the material, of amount $\Delta U = \int_V \left\{ \int_1^2 \sigma_{ij} d\epsilon_{ij} \right\} dV$, where σ_{ij} and ϵ_{ij} are the stress and strain components respectively
- ii) As changes in kinetic energy
- iii) As the energy required to create the new surfaces, of amount $\gamma A'$, where γ is the surface energy per unit area.

Assuming no changes in kinetic energy, the energy balance simplifies to

$$\Delta W = \Delta U + \gamma A' \quad (3.25)$$

3.6.2 Quasi-Static Fracture in the Elastic Regime

Fracture begins when the crack extends by an infinitesimal amount. Therefore, rearranging eqn. (3.25) and taking the limit as $A' \rightarrow 0$, the fracture criterion becomes

$$\lim_{A' \rightarrow 0} \frac{1}{A'} [\Delta W - \Delta U] = \gamma \quad (3.26)$$

This was the result obtained in section 2.2 for the Griffith fracture criterion in brittle, elastic materials, with $\Delta W = 0$ (fixed grips). The left hand side of eqn. (3.26) is the rate of change of potential energy per unit thickness, $-dP/da = G$. If the material is non-linear elastic, this quantity becomes J .

Substituting for ΔW and ΔU , eqn. (3.26) may be transformed, and assuming the stress and strain fields to be continuous, and non-singular everywhere except at the tip, it becomes (omitting several steps)

$$\lim_{A' \rightarrow 0} \int_{V_0} \left\{ \int_1^2 (\sigma_{ij1} - \sigma_{ij}) d\epsilon_{ij} \right\} dV = \gamma \quad (3.27)$$

where V_0 is now any arbitrary volume surrounding the tip.

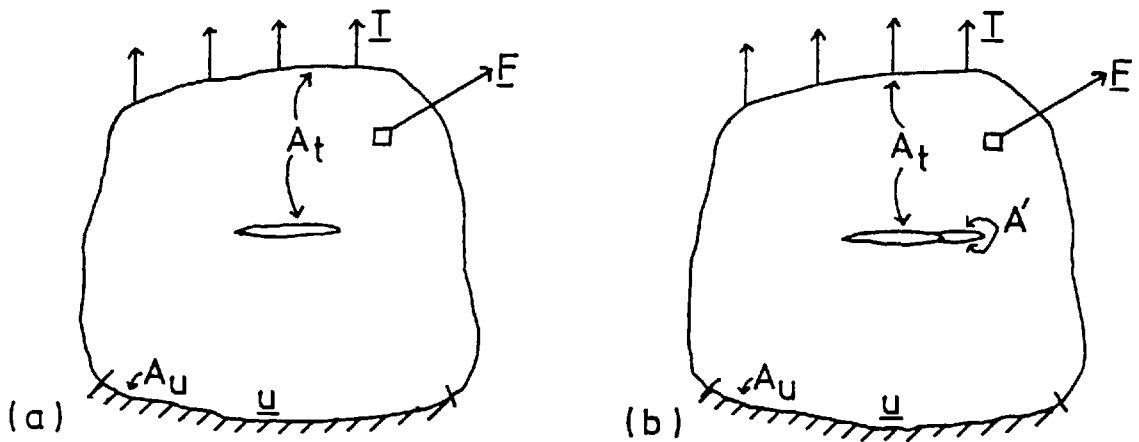


Fig. 3.7 Central crack in a body, (a) with crack extension imminent, (b) after crack surface extended by area A' (Rice, 1965).

Since V_0 may be taken as small as desired, any terms in eqn. (3.27) which are non-singular at the tip make no contribution, and the fracture criterion is determined solely by the stress and strain singularities at the crack tip. This result supports the notion that fracture occurs when the crack tip stress intensity factor reaches a critical value.

Eqn. (3.26) may be further transformed into a surface integral. Since the new crack surfaces are free of tractions, the fracture criterion becomes

$$-\lim_{A' \rightarrow 0} \frac{1}{A'} \int_{A'} \left\{ \int_1^2 T_i du_i \right\} dA = \gamma \quad (3.28)$$

The left hand side represents the work done per unit area by the new crack surfaces against the forces tending to hold them together, and is derived by means of continuum mechanics. The surface energy, γ , is calculated from microstructural considerations. The two are derived independently, and Griffith type fracture occurs when they become equal.

In brittle materials such as glass, γ is the true surface energy. There are drawbacks, however, to applying this concept to ductile fracture. Crack extension occurs by micro-void coalescence, and the dimensions of the fracture

process are large compared to atomic dimensions; coupling between surface work and local deformation may then be expected, and continuum estimates of energy will be affected. Secondly, the ductile fracture mechanism is the attainment of a critical strain for void coalescence, rather than a critical stress.

Irwin (1948) and Orowan (1949) suggested the modification of γ to γ' , which includes the local plastic work. If yielding is on a small scale it will include all the plastic work, and may be assumed to be a material constant. It does, however, reflect the inadequacy of the continuum estimates of plastic work near the crack tip. Even so, provided the extent of yielding is small compared to the crack length, the linear elastic stress field will be but little disturbed, and the strength of the singularities there will control deformation in the yielded zone. Hence the fracture criterion given by eqn. (3.27) should still apply.

3.6.3 Implications for Ductile Fracture Criteria

For a linear elastic material the strain energy has a unique value at a given stress or strain, and the energy balance may be expected to provide a fracture criterion. In the elastic-plastic regime the plastic strain energy is no longer unique, so the general results outlined above cannot be rigorously proven. Nevertheless, the energy balance approach is of interest.

An increment in strain, de_{ij} , may be split into an elastic, recoverable part de_{ij}^e and a plastic, permanent part de_{ij}^p . Then the increment in strain energy is

$$\begin{aligned}\Delta U &= \int_V \{ \int \sigma_{ij} de_{ij}^e + \int \sigma_{ij} de_{ij}^p \} dV \\ &= \Delta U_e + \Delta U_p\end{aligned}$$

where U_e and U_p are the elastic and plastic parts, respectively, of the strain energy. Let an edge crack of length a extend by Δa in a body of thickness B . The change in (elastic)

potential energy is

$$\Delta P = -(\Delta W - \Delta U_e) \quad (3.29)$$

The strain energy release rate, G , is the negative of the rate of change of potential energy with respect to crack length (Irwin, 1948).

$$\text{i.e. } G = \frac{1}{B} \frac{d}{da} (\Delta W - \Delta U_e) \quad (3.30)$$

Defining p as the energy dissipated in irreversible plastic deformation per unit change in crack area,

$$p = \frac{1}{B} \frac{d}{da} (\Delta U_p) \quad (3.31)$$

the energy balance becomes,

$$G - p = 2\gamma' \quad (3.32)$$

(The factor of two arises because A' in eqn. (3.26) is $2B\gamma$, there being two crack surfaces). Here γ' is the surface work modified by local plasticity, and p is the dissipation rate more remote from the crack, which can be estimated from continuum mechanics. γ' is generally taken as a material constant, and is small compared to G or p at fracture. However, these two can be calculated whether or not the body fractures, and it is only when the difference between them reaches 2γ that the crack extends. Thus the surface energy, though small, determines the fracture load. The Irwin-Orowan modification to γ assumes that $2\gamma \ll p_c$, where p_c is the critical value of p , so that the fracture criterion becomes

$$G_c = p_c. \quad (3.33)$$

p_c is taken as a material property. However, this approximation is incorrect for large scale plasticity, since p depends on geometrical factors.

An elastic-plastic fracture criterion could thus be based on the attainment of a critical value of $G - p$, where

$$G_c - p_c = \frac{1}{B} \frac{d}{da} (\Delta W - \Delta U_e - \Delta U_p) \quad (3.34)$$

This is the basis of $\tilde{G}$ (Liebowitz and Eftis, 1971), and is also an energetic interpretation of the J-integral in non-linear elasticity, with the quantity $\Delta U_e + \Delta U_p$ equivalent to the non-linear elastic strain energy. In the presence of plasticity Sumpter (1974) introduces a parameter J^* , where

$$J^* = -\frac{1}{B} \frac{dP_s}{da} = \frac{1}{B} \frac{d}{da} (\Delta W - \Delta U_e - \Delta U_p) \quad (3.35)$$

$$\text{Thus} \quad J^* = G - p = 2\gamma \quad (3.36)$$

P_s in eqn. (3.35) is now the "psuedo" potential energy, since some of it has been absorbed in irreversible plastic work. J^* may be calculated from the area between the load/displacement plots of two adjacently cracked bodies, as described in section 3.3.

J^* is not necessarily the same as J , the contour integral, except for a deformation theory of plasticity. However, Sumpter (1974) and Boyle (1972) obtained reasonable agreement between J^* and J from finite element calculations in incremental plasticity, with no unloading.

The energy balance becomes more complicated during crack propagation. The resistance curve concept of Srawley and Brown (1965) equates R , the resistance to crack extension per unit thickness, to the elastic potential energy release rate.

$$R = -\frac{dP}{da} = \frac{d}{da} (\Delta W - \Delta U_e)$$

The energy balance is $\frac{d}{da} (\Delta W - \Delta U_e) - p = 2\gamma'$

$$\text{Hence} \quad R = 2\gamma' + p \quad (3.37)$$

Since p is probably geometry dependent, the R -curve as defined here will be too. However, if linear elastic conditions predominate, so that yielding only occurs around the crack tip, the modified surface energy γ' will encompass all the plastic deformation associated with the plane strain

portion of the crack front. The only contribution to p then comes from the shear lips, and may be reasonably independent of geometry and configuration, within certain limits. Eqn. (3.37) is the same as the Irwin and Kies (1954) definition of R , (eqn. 2.24).

Throughout slow crack growth

$$\frac{d}{da} (\Delta W - \Delta U_e) = 2\gamma' + p$$

i.e.

$$G = R$$

Instability is given by the condition of tangency between the R curves, described in section 2.6

$$\frac{dG}{da} \geq \frac{dR}{da}$$

For larger scale yield, Garwood (1977) postulated that, provided the correct measure of J , allowing for the effects of unloading during slow crack growth, is taken (called J_r), the energy balance is still analogous to that for a non-linear elastic material, and may be written,

$$J_r = 2\gamma' \quad (3.38)$$

$$\text{with } J_r = \frac{1}{B} \frac{d}{da} (\Delta W - \Delta U_e - \Delta U_p) \quad (3.39)$$

The J_r curve would thus be a measure of the modified surface energy of the material, and should be independent of geometry and configuration.

The argument is not convincing, however, since the separation of continuum and microstructural energies in the balance breaks down with large scale yield. It is likely that some remote plastic work is included in the term $2\gamma'$, which would be expected to increase with deformation and crack extension. The dependence of J_r on remote plastic work is indicated by experimental results, since the value of J_r is known to depend directly on the fraction of shear lips on the fracture surface (Garwood, 1977; Garwood et al., 1978). If all remote plastic work were incorporated in the term

ΔU_p in eqn. (3.39), then this should include deformation in the shear lips, and J_r should be independent of shear lip size. Hence, on energetic considerations, the J_r curve is likely to depend to some extent on geometry.

Chapter Four

MICROSTRUCTURAL ASPECTS OF DUCTILE FRACTURE

4.1 Introduction

At normal temperatures fracture in steels may occur by one of two mechanisms; cleavage, or ductile tearing. The former occurs without gross plastic deformation, although some local yield is required to initiate a cleavage crack (Low 1954). All tests described herein were conducted at temperatures above the ductile/cleavage transition temperature, so that fully ductile fracture occurred.

Second phase particles are of prime importance in ductile tearing. Small voids are formed at particles, such as non-metallic inclusions and carbides in steel, and crack extension takes place by their growth and coalescence. Significant plastic flow is required, and so for real materials the process is more complicated and less well understood than cleavage fracture.

Ductile tearing thus involves two stages; the nucleation of voids, and their subsequent growth to coalescence. Theoretical models for both processes are described below.

4.2 Nucleation of Voids

Voids may initiate at micro-cracks caused by dislocation interactions, but they are more usually observed at second phase particles, formed either by failure of the particle-matrix interface, or by cracking of the particle itself. Gurland and Plateau (1963) suggest that the stress concentrating effects of particles allow void nucleation to occur even under elastic loading. In ductile materials, however, local yielding is likely to intervene. The model of Ashby (1966) is based on displacement incompatibilities which develop between a homogeneously sheared continuum and a rigid inclusion. The resulting shear stresses are reduced by the formation of dislocation loops around the particle, which

cause interfacial normal stresses, reaching a maximum at the poles. A cavity is formed when the interfacial strength is reached.

Argon, Im and Safoglu (1975) have developed Ashby's model by considering the dislocation loops as a continuously distributed dislocation density. They conclude that cavities can only open up at particles above a certain diameter (about 7-10 nm). An expression is derived for the interfacial stress in terms of shear yield stress and strain, and work hardening characteristics. Particle size above the critical diameter has no effect on the ease of void nucleation, and the observed dependence is explained on the basis of local variations of volume fraction.

From observations of aluminium alloys, in which void nucleation appeared to be the critical event in fracture, Broek (1973) proposed a qualitative dislocation model for the system. Dislocation pile-ups on 45° shear bands cause void formation at the junction between the particle and the matrix. The pile-up then empties itself into the void, resulting in spontaneous growth and coalescence.

A feature of all the models is the requirement of both high stresses and strains for void nucleation. Strong second phase particles firmly bonded to the matrix are assumed, where the nucleation barrier of cavities must first be overcome. In low strength steels, void formation occurs principally at non-metallic inclusions. It is thought that there is no nucleation barrier, since these particles are very weakly bonded, and may already be cracked before deformation begins. Rupture is then simply a process of hole growth and coalescence.

4.3 Growth and Coalescence of Voids

The linking of voids may occur on planes which are perpendicular to the applied stress (normal rupture), or are parallel to it (delamination), or along shear bands at an angle to it (shear fracture). Most attempts at understanding

ductile fracture have considered only the first type.

Gurland and Plateau (1963) proposed that the strain concentration around elliptical cavities depends on their frontal radius of curvature and on their length perpendicular to the maximum stress. In developing the model, an expression for the fracture strain is derived on the assumption that the mean centre-to-centre spacing of voids varies with plastic strain. Baker (1971) noted that this assumption is in error and that the spacing remain constant. With this correction, the true strain to fracture, ϵ_f , becomes

$$\epsilon_f = \frac{1}{2} \ln \left\{ 1 + \frac{2k_2}{3k_1} \left(\frac{1-V_f}{V_f} \right) \right\}$$

where k_1 and k_2 are constants, and V_f is the volume fraction of inclusions.

The more refined analysis of McClintock (1968) considers the growth of elliptical prismatic holes stressed normal to their longitudinal direction. As with the previous model, coalescence occurs when the holes touch. An approximate expression is obtained for the equivalent strain to fracture, $\bar{\epsilon}_f$, in terms of initial void dimensions, stress triaxiality and work hardening:

$$\bar{\epsilon}_f = \frac{(1-n) \ln (d/2b)}{\sinh[(1-n)(\sigma_x + \sigma_y) \sqrt{3/2\sigma}]}$$

where d is the initial void spacing, b is the initial radius, n is the strain hardening exponent (from $\sigma = k \epsilon^n$) and $(\sigma_x + \sigma_y)/\bar{\sigma}$ is the ratio of transverse to equivalent stresses. The model predicts a large dependence of fracture strain on triaxiality. It is in general qualitative agreement with experimental results. Ductility increases as the strain hardening exponent increases, or as the volume fraction of voids (proportional to $(b/d)^3$) and the transverse stress diminish. However, as Thomason (1974) points out, the model gives a smaller dependence on volume fraction of voids than observed, and considerably overestimates the fracture strains in the simple tensile tests of Edelson and Baldwin (1962). One explanation is that interactions between voids are

completely neglected, fracture occurring when they have grown sufficiently to touch. As the initial volume fraction of voids increases, and as they expand, such interactions become increasingly important. On the other hand, Perra and Finnie (1977) found, from measurements on the growth of isolated voids in plane strain, that the McClintock analysis underestimates the observed rate of expansion by a factor of two, which would lead to an overestimation of fracture strain.

The Rice and Tracey (1969) model considers the growth of an isolated spherical void in a state of remotely uniform stress and strain rate. The sphere has an initial void radius r_0 , and the remote strain field is one of uniaxial extension at a strain rate $\dot{\epsilon}$. The relative rate of void expansion, $D = \dot{r}_0 / \dot{\epsilon} r_0$, is given approximately, for high triaxiality, by

$$D = \frac{\dot{r}_0}{\dot{\epsilon} r_0} = 0.283 \exp \left(\sqrt{3} \frac{\sigma_\infty}{2\tau_y} \right)$$

where σ_∞ is the remote mean normal stress and τ_y the shear yield stress. As the triaxiality, σ_∞ / τ_y , increases, the rate of void expansion increases rapidly.

Rice and Johnson (1970) used the results of Rice and Tracey's void growth model to predict the onset of crack extension. The first stage was to obtain an expression for the stresses and strains around the crack tip in an elastic-plastic material. The slip line field solution was modified

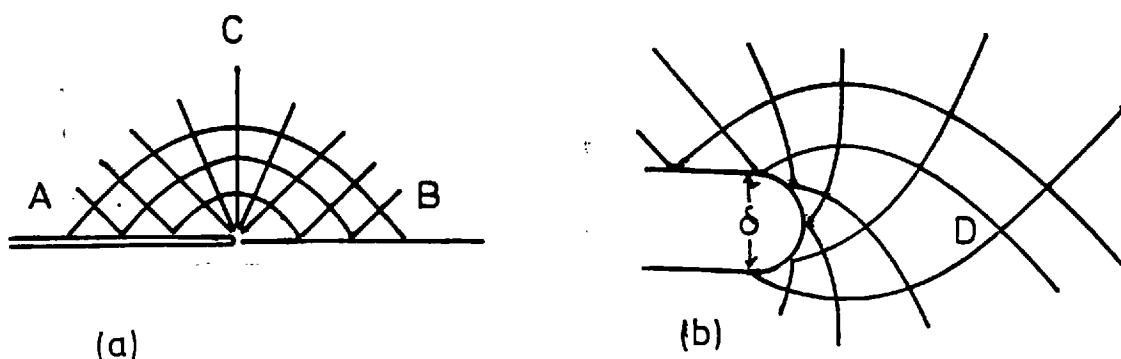


Fig. 4.1 Slip line fields around a crack (a) for a sharp tip, (b) after the tip has blunted (Rice and Johnson, 1970).

to allow for the large geometry changes occurring at the tip. The solution for a sharp crack is depicted in Fig. 4.1(a). There are no strain concentrations except at the tip. This then blunts to a semi-circle with an opening, δ , Fig. 4.1(b). The fans, which were previously centred at the tip, now centre within a region of size D immediately ahead, giving intense strain concentrations there. The size of the zone D is given by

$$D \approx 1.9 \delta$$

The calculated tensile strain distribution ahead of the tip is given in Fig. 4.2(a), with origin at the tip before deformation.

For a non-hardening material, the maximum tensile stress, σ_{11} , reaches $\approx 3\sigma_y$ at the end of region D . If the material hardens the stress distribution close to the tip is given by Fig. 4.2(b). There is a singularity, but this is weak and may be ignored. The result is to confirm that the maximum stress achievable over any reasonable size scale is finite, and is reached slightly ahead of the tip. Also shown in the figure are the Rice and Rosengren (1968) predictions of the stresses outside the region affected by blunting.

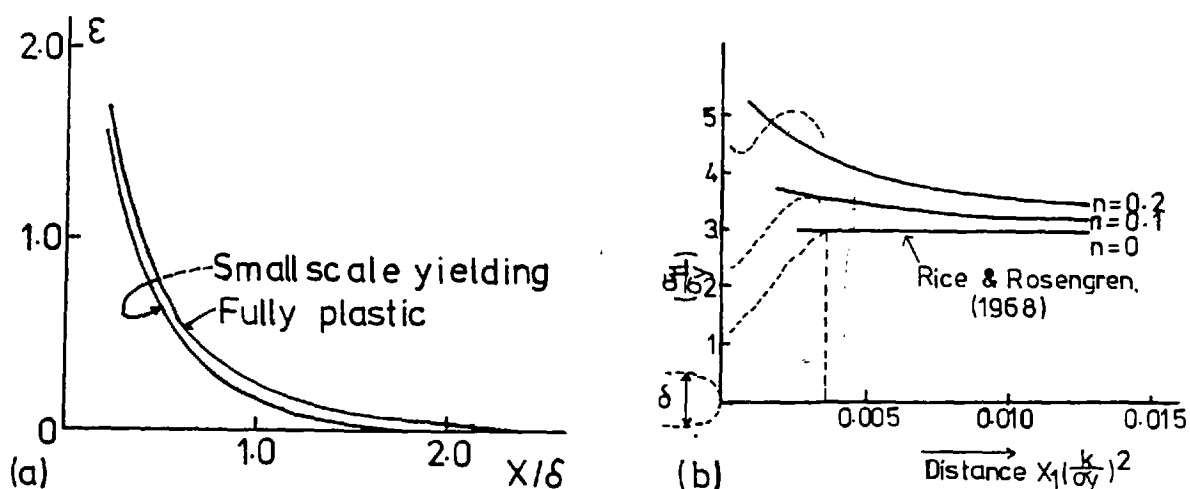


Fig. 4.2 Distribution ahead of a blunted crack tip of (a) tensile true strain, ϵ , (b) tensile stress σ_{11} (Rice and Johnson, 1970).

Rice and Johnson then consider a void of radius R_0 spaced a distance X_0 from an initially sharp crack tip, in the plane of the crack. As the COD increases the tip blunts. The void is enveloped by the large strain region and starts to enlarge, finally coalescing with the crack tip. The model can then be used to predict the COD at fracture initiation, from the ratio of particle separation to radius, X_0/R_0 , which is related to the volume fraction for a uniform distribution. The predictions are shown in Fig. 4.3.

Experimental verification of the model is difficult, firstly because it takes no quantitative account of work-hardening, and secondly because in most metals the inclusions are not of uniform size and spacing, and may not be equiaxed. Green and Knott (1976) had limited success for transverse specimens, with high work-hardening capacity, a paradoxical result in view of the perfectly plastic assumptions of the model. Heavily prestrained material gave much lower fracture ductilities than predicted. McMeeking (1977b) compares this and other published data with the predictions, and concludes that the model consistently overestimates fracture strains, except where the void nucleating particles are strongly bonded. This is a feature of most ductile fracture models, particularly at large volume fractions of particles, because relatively uniform void expansion is assumed. In fact, it is known that highly-localised growth can occur on shear bands within the material.

A somewhat different approach is used by Thomason (1968). A square array of prismatic holes, of height $2a$ and separation $2b$, is assumed. Under the remote stresses and strain the voids deform with the matrix, elongating and coming close together. Eventually the ligament between them is sufficiently small for an internal neck to form, and void coalescence rapidly ensues. The condition for the onset of necking is given by

$$\sigma_n(1 - V_f) + P < \sigma_{22} + 2\tau_y$$

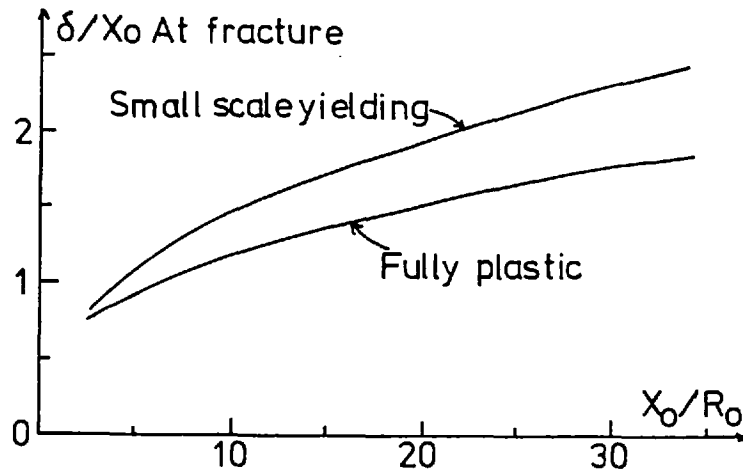


Fig. 4.3 Relationship between the critical COD, δ_i , and initial void parameters (Rice and Johnson, 1970).

where σ_n is the mean stress in the neck, σ_{22} is the external transverse stress, τ_y is the shear yield stress, V_f is the volume fraction of voids and P is the external hydrostatic pressure. Up to the onset of coalescence, the change in void height and spacing is given by

$$\frac{a}{b} = \frac{\sqrt{V_f}}{1 - \sqrt{V_f}} \exp(2\epsilon_{11})$$

where ϵ_{11} is the uniform longitudinal strain. This strain increases until the ligament geometry, a/b , and corresponding constraint factor, $\sigma_n/2\tau_y$, satisfy the conditions for coalescence. The model predicts a large effect of volume fraction on fracture strain, and agreement with Edelson and Baldwin's (1962) results is encouraging.

In extending the model to describe the fracture process at a crack tip (Thomason, 1971), the real stress-strain curve is replaced by a non-hardening approximation, and the COD in plane strain is obtained by modifying Dugdale's expression (eqn. 3.3). Assuming a triangular plastic zone ahead of the crack tip, filled with prismatic cavities, an expression for K_{IC} is deduced:-

$$\frac{K_{IC}}{\sqrt{3} \cdot \tau_y} = \left\{ \rho \left[4.62 \frac{E}{\sqrt{3} \cdot \tau_y} \cdot \frac{(\exp \epsilon_I - 1)}{1 - \nu^2} + 8.81 \tan \left(\frac{\pi}{2} \left(1 - \frac{V_f}{0.09} \right) \right) \right] \right\}^{\frac{1}{2}}$$

where ρ is the root radius of the crack tip, and ϵ_I is the instability strain in uniaxial tension. The point of fracture initiation moves closer to the crack tip for increasing volume fractions of holes, until at $V_f \geq 0.09$, fracture occurs at the tip. Difficulties with the model are the assumptions made concerning the stress and strain fields ahead of the crack, and the fact that the crack tip radius, ρ , must be known. Thomason suggests that for a sharp crack the minimum value of ρ should be taken as a grain diameter.

The treatment of Krafft (1964), though not concerned with the details of microvoid coalescence, will nevertheless be described here since it attempts to relate ductile fracture toughness to inclusion and matrix characteristics. It is assumed that if the extent of yielding is small the elastic strain fields will dominate the crack tip region. The fracture processes occur within a region, known as the process zone, of size d_T . At the boundary of this zone the strain is taken as being equal to the uniaxial instability strain, $\epsilon = n$, where n is the strain hardening exponent. From the linear elastic analysis of strain distributions, the strain n at distance d_T from the tip is related to K_{IC} via

$$K_{IC} = E \cdot n (2\pi d_T)^{\frac{1}{2}}$$

which implies that K_{IC} is proportional to the work hardening exponent. By matching values of K_{IC} and n for various strain rates Krafft obtained consistent estimates of the process zone size, d_T . However, for mild steel he obtained $d_T \approx 0.2 \mu\text{m}$, which is far too small to be the process zone size as determined by the spacing of non-metallic inclusions. Doubtless the application of an essentially linear elastic model to a situation involving gross plasticity causes errors.

There have been few systematic attempts to relate matrix flow properties to fracture toughness in the ductile mode. This is due partly to the difficulties encountered when analysing ductile fracture, and partly to the inter-

dependence of the various flow properties. It might be expected that an increase in yield stress would increase toughness by reducing plastic strains, but this invariably lowers the work-hardening capacity, which itself has a strong effect. The relative importance of the two factors is indicated by the experiments of Garrett (1973) on aluminium alloys. The empirical relationship

$$K_{IC} \propto n(\sigma_y)^{\frac{1}{2}}$$

was obtained, where the dependence of toughness on work hardening is the same as predicted by the Krafft model.

Chapter Five

EXPERIMENTAL DETAILS

5.1 Materials

The main purpose of the work to be described in the following chapters was to investigate the influence of micro-structure on the R-curve, and also to note any effects of test-piece geometry. Two structural steels, En32B and En24, had previously been the subject of research in this laboratory, (Robinson, 1976; Garwood, 1977). The En32B was a low-strength steel containing elongated inclusions which could be expected to dominate void formation, and was chosen for the investigation of microstructural effects. The En24 was a higher-strength, heat treatable steel and was used to study geometry dependence.

The chemical composition of En32B is given in Table 5.1. It was supplied in rolled bar form, of cross-section 62.5 mm x 25 mm, and contained non-metallic inclusions elongated in the rolling direction, as in Plate 5.1. These were rod-like, but elliptical in cross-section. The characteristics of the inclusion population are described in section 6.2. Two heat treatments were employed. In one case the steel was austenitised at 880°C and then normalised by cooling in air, and in the other it was austenitised at 1000°C and then left to cool slowly in the unheated furnace. Ferrite-pearlite structures, with grain sizes of 20 and 40 μm and Vickers hardnesses of 131 and 112 respectively, were obtained, as in Plates 5.2(a) and (b). Tensile flow properties were determined using Hounsfield no. 12 tensile specimens, with axes in the longitudinal (L) and transverse (T) directions (as defined in section 5.3), and these results are summarised in Table 5.2.

In addition, some of the normalised En32B was pre-strained in compression by approximately 10 or 20%. This was accomplished by cutting oversized specimen blanks of a calculated thickness, and compressing them, down to the

thickness of machined spacer blocks, between the lubricated platens of a 50 tonne hydraulic press. The tensile flow properties of the two prestrained conditions are included in Table 5.2. Because of the proximity of the proof stress to ultimate stress in tension for the prestrained material the work-hardening exponent, n , could not be determined with any accuracy and is not shown.

The En24 was a medium carbon low alloy steel of chemical composition given also in Table 5.1. Like the En328, it was supplied in rolled bar of cross-section 62.5 x 25 mm. It was heat treated by quenching in water from 850°C and tempering at 650°C for 16 hours, since in this condition it was of a sufficiently low strength to give good shear lips. A tempered martensite structure resulted, as illustrated in Plate 5.3. The tensile properties in the longitudinal direction are shown in Table 5.2.

In addition, a 60/40 brass (60% Cu, 40% Zn), supplied as rolled plate 16 mm thick, was used for model studies of fracture initiation. This material was chosen because it had fairly similar flow properties to mild steel, and was easy to machine and to join by soldering. This was used in the as-received condition.

5.2 Specimen Design

5.2.1 Specimen Types

The majority of the tests were conducted on three point bend fracture toughness specimens, as depicted in Fig. 5.1(a) (at the end of the chapter). In most cases the thickness B and width W were equal and either 10 or 24mm, and the span S was equal to $4W$. Machining was carried out after heat treatment.

Centre cracked tension (CCT) specimens, illustrated in Fig. 5.1(b), were designed so that 24 mm square bend test-pieces could be machined from the broken halves, with similar geometry to one arm of the CCT specimen. Both bend and tension test-pieces had notches and side-grooves which were

machined with a sharp-pointed, 3.2 mm wide cutter, although in a few cases round-bottomed side-grooves of the same width were used.

Compact tension (CT) specimens, Fig. 5.1(c) and Plate 5.4, were made from the 60/40 brass. They contained blunt, "key hole" notches, and holes or slots drilled ahead of the notch tip. Two thicknesses were used, 16 or 32 mm, the latter being made by silver soldering two 16 mm plates together, as described in section 5.4.2.

5.2.2 Specimen Thickness and Side-Groove Depth

As mentioned in section 3.3.3, the recommended minimum dimensions for valid J_{IC} testing are

$$a, B, W-a > \alpha \frac{J}{\sigma_{flow}} \quad (3.20)$$

where $\alpha = 25$ or 50 , and $\sigma_{flow} = (\sigma_y + \sigma_{U.T.S.})/2$. For En32B, taking from section 7 a value of J_i of 100 kJ/m^2 and a σ_{flow} of 365 MN/m^2 , the right hand side of eqn. (3.20) becomes 7 or 14 mm. For En24 the equivalent dimensions are smaller at 4 or 8 mm (taking $J_i = 100 \text{ kJ/m}^2$, and $\sigma_{flow} = 585 \text{ MN/m}^2$).

For a valid δ_i or J_i test, however, it is only necessary to achieve plane strain in the centre of the specimen, where the crack initiates. Robinson and Tetelman (1975) found, from measurements of transverse stress ϵ_{33} at the notch root in a low strength steel, that even for a 10 mm thick specimen $\epsilon_{33} = 0$ over the central third of the thickness. They suggested that for a valid COD test,

$$a, B, W-a > 25 \delta_i$$

for En32B, taking δ_i as 0.12 mm, this would give a minimum thickness of 3 mm.

Side-grooves were used by Freed and Krafft (1966) to allow the estimation of K_{IC} from undersized specimens. In this case a small amount of crack extension may occur ($\approx 2\%$). Green and Knott (1975a) found that, even with

substantial crack growth, it was only necessary to eliminate the shear lips in a thin specimen by side-grooving to obtain a similar COD as that of a thick specimen. Garwood (1977) also observed, in experiments on the same En32B steel as used here, that provided sufficient side-groove depth was employed the same R-curve was obtained on 10mm thick specimens as on larger ones.

When testing bend specimens of En32B in the transverse orientation the maximum obtainable length of the specimen was about 60mm, and the requirement of $S = 4W$, with an allowance at each end, meant that the maximum obtainable width was about 12-14mm. In fact the smallest specimens used had $B = W = 10\text{mm}$., in order to avoid wastage of material (two specimens could then be cut from a bar of thickness 25 mm). The recommendations of Robinson and Tetelman allow reasonable confidence to be placed in the initiation toughness results. The specimen dimensions also appear adequate for the valid derivation of R-curves, and this point will be considered further in section 8.2.1.

5.3 Specimen Orientation

All the materials tested were supplied in the form of rolled bar or plate, and thus the steels contained elongated inclusions. Some banding of the pearlite was also visible and remained after normalising. In accordance with DD3 (1971), the three principal directions with respect to rolling plane and direction in the bars will be designated by the letters L, T and S, for the longitudinal, transverse and short transverse direction respectively, as illustrated in Fig. 5.2. There are thus six principal orientations of a crack, and these will be identified by two letters, the first corresponding to the normal to the crack plane and the second to the direction of crack propagation as in Fig. 5.2. All six orientations were tested for En32B, but for En24 only the LS orientation was studied.

5.4 Preparation of Specimens

5.4.1 Welding of Short Transverse Test-Pieces

The bar dimensions of En32B were ample for three point bend test-pieces to be extracted in both longitudinal and transverse orientations, but for the ST and SL orientations the bar thickness of 25 mm necessitated the welding of extension arms. A mild steel of similar composition was used for the arms, with a "K type" weld preparation, (Fig. 5.3). The specimens were normalised after welding, and then the notch cut centrally. Measurements of hardness on the central regions of specimens prepared in this way revealed no difference between them and the standard, unwelded test-pieces of normalised En32B.

5.4.2 Preparation and Soldering of Brass Specimens

The purpose of the brass CT specimens was to examine the effect of an inclusion (represented by a hole or slot) upon the deformation around a crack tip (represented by a blunt notch) in both plane stress and plane strain. A large root radius (2.4 mm) was chosen, since the characteristic dimensions of the strain field from the notch needed to be fairly large for accurate measurement. The theoretical analysis of Rice and Johnson (1970) on initially sharp cracks, and the finite element work on blunt ones of McMeeking (1977a and 1977b) predict that the deformation pattern should scale with crack tip opening. Experimental work also shows that at initiation the COD (Smith and Knott, 1971) and the J-integral (Begley et al., 1977) are linearly dependent on the root radius. Thus a blunt notch should allow a large COD before crack initiation, which in turn should result in a widely spaced deformation pattern.

Surface (plane stress) strains could be measured easily by scribing a grid on the side of the specimens, Plate 5.5(a). For "plane strain" values, two similar specimens were soldered together, with a grid already scribed on the inside face of one. Since the grids would fill with solder they had to be fairly coarse and deep,

in order to be distinguished afterwards. They were cut to a depth of 0.25 mm on the bed of a milling machine, using a sharpened tool in the stationary chuck, and then photographed, Plate 5.5(b).

It was found easiest to prepare one half of the sandwich specimens with a hole for the notch, the machined slot, and the internal scribed lines, but leave the other half blank. Four screw holes were drilled through both halves for alignment purposes, two at the back corners and two where the loading pin holes would later pass (see Fig. 5.1(c)).

The specimen was now ready for joining. A silver solder, "Easy-flo", was used because it wetted brass well, had a low melting point (about 620°C) and had similar mechanical properties to those of brass. Because of the difficulty of heating a large specimen evenly and rapidly with a torch, furnace brazing in an inert atmosphere (to preserve the flux) was chosen. The inner faces of the two pieces were degreased and coated with Easy-flo flux paste. Half a dozen sticks of solder were then placed in the middle of one inner face, leaving a solder-free border of about 10 mm. A large excess of solder was used so that as it melted and ran outwards it would sweep the flux with it. The other piece was then bolted on top and the compact placed horizontally, scribed half uppermost, in the furnace. This was running at 700°C with an atmosphere of forming gas (75% N₂, 25% H₂). Previous experiments on dummy brass specimens had determined the time required for the specimen to reach the melting point of the solder (about 12 minutes). It was then removed from the furnace, the two halves squeezed gently to expel excess solder from the joint, and left to cool. In most cases a good joint was achieved which remained intact during subsequent straining. Occasionally the joint failed during a test at the notch root, due to poor wetting. In this case the internal grid was not used.

Finally, the alignment bolts were drilled out, the notch and slot extended into the other half of the specimen,

the "crack" saw-cut to the notch and the surface grid scribed. The single thickness, "plane stress" test-pieces were given an identical heat treatment to preserve uniformity of flow properties.

5.4.3 Precracking

The steel fracture toughness test-pieces were all given fatigue precracks. A stroke with a razor blade along the bottom of the machined notch facilitated initiation. The small bend specimens were cycled on a modified Avery eccentric wheel, constant displacement machine, at small deflections ($\approx 1^\circ$) to prevent large scale yielding. For the larger bend and CCT specimens a 10 tonne, servohydraulic Losenhausen was used, under displacement control, at loads of less than half the calculated load for general yield. The load was reduced accordingly as the crack propagated. Cracks generally grew to the required length of 1-3 mm in about 50,000 cycles. They could be monitored by the naked eye on the plane-sided specimens, or with the aid of a microscope on side-grooved ones.

It was found that in the short transverse specimens the fatigue crack grew rapidly, and since it appeared to lie in one plane, easier control and observation was obtained by cycling plane-sided specimens and side-grooving afterwards. In bending, the fatigue cracks led slightly from the edges in side-grooved test-pieces, and from the centre in plane-sided ones, but were otherwise fairly straight. The CCT specimens had all been deeply side-grooved before precracking, and here the curvature of the crack was initially pronounced. Some amelioration was achieved by blunting the corners between the notch and side-grooves with a file, but a better solution would be to cut the side-groove after precracking. For all configurations a fatigue crack length-to-width ratio, a_0/W , of 0.45 was intended, and in all cases this ratio lay between 0.4 and 0.6, unless otherwise stated.

Where the crack plane was normal to the rolling direction (ie., longitudinal orientations) the precrack

was readily distinguished from subsequent slow growth on the fracture surface. For other orientations, however, the demarcation was less obvious. Marking the fatigue crack by heat tinting at about 150°C for 15 minutes overcame the difficulty.

5.5 Test Equipment

All the test specimens were loaded on a 10 tonne screw-driven Instron. To eliminate strain-rate effects a constant displacement rate of 0.5 mm/min. was used for the steel specimens, and 2 mm/min. for the brass CT specimens.

In the case of the bend geometry the testing configuration shown in Fig. 5.4 was used. Knife edges 1.6 mm thick were attached to the specimen with methyl cyanoacrylate adhesive, and were easily removed with a sharp tap. A clip gauge, of design similar to that specified by Fisher et al. (1966), was mounted across them. It was powered by a 12V accumulator. A linear variable differential transformer (LVDT) type of transducer measured displacement between loading points. The outputs from the clip gauge, LVDT and Instron load cell were fed into an X-2Y flat bed recorder to give simultaneous plots of mouth opening, and load-point displacement, vs. load.

After allowing at least an hour for the equipment to stabilise, the readings of load, clip gauge displacement and LVDT displacement were calibrated, and rechecked at hourly intervals. The Instron chart recording of load vs. cross-head displacement served as a check on the other plots.

CCT specimens were loaded in tension, and the displacement between the crack faces was measured with the clip gauge, to give a single recording of displacement vs. load.

For the brass CT specimens, a trace of load vs. displacement was not required. They were merely loaded on the Instron to a fixed final displacement, achieved by applying vernier calipers between knife edges mounted across the crack mouth.

5.6 Experimental Techniques

5.6.1 R-Curve Determination

R-Curves were measured on the steel precracked test-pieces at room temperature ($\approx 20^{\circ}\text{C}$), using a multi-specimen technique. It is likely that this is the only method possible for use on small, generally yielded test-pieces, as the standard means of determining crack lengths become innaccurate as plastic deformation increases (see Section 9.4).

The bend specimens were loaded to different displacements so that varying amounts of ductile crack extension occurred. They were then unloaded and cooled in liquid nitrogen (at -196°C), where upon they could be held in a vice and broken open with a hammer, a cleavage fracture resulting. Immersion of the broken halves in methanol prevented them rusting, and allowed the fracture surfaces to be examined.

For the CCT test-pieces the procedure was similar. However, in order to break them open after ductile crack growth, saw-cuts about 5 mm long were made from the back faces to lower the fracture load. They were then cooled in liquid nitrogen, rapidly replaced in the Instron grips, and loaded at a high displacement rate (50 mm/min) in order to promote cleavage.

The crack lengths, a_0 and a_f , before and after slow crack growth, and the width of the fracture plane, B' , in side-grooved specimens, were then measured on the fracture surfaces. A travelling microscope, accurate to 0.01 mm, was used and measurements of crack lengths were made at nine equally-spaced intervals along the crack front. The CCT specimens each contained two crack fronts; both were measured at 5 different points each and all the results averaged.

In the bend configuration cracks were always stable, and the amount of stable, fibrous fracture achievable was limited only by the ligament depth. However, for the CCT

test-pieces the combination of specimen design and machine compliance ensured that after about 1 mm of slow crack growth, a ductile instability was reached and one side of the specimen broke. Thus the extent of stable ductile tearing was limited to about 1 mm for this particular geometry and configuration.

When estimating the J-integral in bend, Robinson and Tetelman (1976) and Garwood (1977) both recommend making an allowance for extraneous displacement of the loading points (see section 3.3.3). The two upper rollers were brought together and a broken specimen half compressed to the final load during the test, as shown in Fig. 5.5. The measured displacements were subtracted from the test-piece load/displacement record to give a corrected plot. The area under this plot was measured with a planimeter, and it was found that for En32B test-pieces the correction for extraneous displacement amounted to less than 5%. Accordingly, since the correction was time-consuming, it was discontinued in favour of adjusting the "elastic" loading line for non-linearity, as illustrated in Fig. 5.6. It was felt that this correction was of equal or, particularly in the case of the welded short transverse specimens, greater importance. Considerable initial non-linearity, not present in the clip gauge displacement plot, resulted in some of these, due to incomplete fusion at the weld and bending of the arms. For En24 specimens the extraneous displacement correction was more marked, amounting to as much as 17% difference in area at small displacements. Consequently all the load-displacement plots for this material were corrected in the manner suggested by Robinson and Tetelman.

5.6.2 Crack Tip Studies in En32B

A number of specimens of En32B steel were not broken open after slow crack growth, but instead were sectioned in a plane normal to the line of the crack front so that the profile of the crack tip could be studied. They were first unloaded, sectioned and mounted in resin. Evacuation and re-pressurisation of a chamber containing the unset mount aided penetration of the resin to the crack tip. However,

such specimens were easily bevelled during polishing and many were nickel plated prior to sectioning. A standard plating solution was used, and it was found that the nickel plate could penetrate right to the crack tip, but did not always adhere well.

After some R-curves had been measured, it was considered desirable to measure strains in the vicinity of a growing crack tip. The first method tried was that of micro-hardness. As a work-hardening material is strained, the stress required for further plastic flow increases, and this stress, and thus the prior strain, may be estimated from the hardness. In order to obtain a calibration of strain vs. microhardness for normalised En32B, Hounsfield No. 12 tensile specimens were machined in the longitudinal and transverse orientations and pulled to failure. The broken halves were then mounted, axes vertical, in bakelite, ground and polished. Microhardness readings (at 300gm load) were taken on the cross-sections.

Assuming, following Bridgman (1952), that the strain across a necked tensile specimen is a constant, and that necking occurs at constant volume, the axial strain may be estimated as follows. Consider a small cylinder of material of constant volume V , radius b and height h with its axis in the tensile direction as in Fig. 5.7. The volume is

$$V = \pi b^2 h = \text{constant}$$

As the neck is extended the dimensions of the cylinder change by small amounts. Hence

$$\frac{dV}{dh} = \pi b^2 + 2\pi b h \frac{db}{dh} = 0$$

$$\text{i.e.} \quad \frac{dh}{h} = - 2 \frac{db}{b}$$

$$\text{Integrating} \quad \epsilon_{yy} = - 2 \epsilon_{rr} \quad (5.1)$$

where ϵ_{yy} and ϵ_{rr} are the true tensile strains in the axial and radial directions, respectively. By grinding down

layers of the mounted specimen, polishing and measuring the mean diameter of the section through the neck with a travelling microscope, the radial strain at any point was found from

$$\epsilon_{rr} = \ln(d/d_0) \quad (5.2)$$

where d_0 and d are the initial and final diameters, respectively, of the section.

Six readings of microhardness at each section were taken, enabling a calibration curve of microhardness vs. tensile true strain, ϵ_{yy} , to be deduced for normalised En32B, Fig. 5.8(a). There was little difference in the hardness/strain relationship between longitudinal and transverse specimens.

Possible errors in this method of calibration are the variability of microhardness with position of indentation, and the formation of internal voids in the material which would invalidate the assumption of constant volume. The first point was overcome by measuring many hardness indentations across the section. The second would only become significant at higher deformations and would lead to an underestimation of strain. Fig 5.8(a) shows that the hardness appeared to saturate at strains above about 0.6, and in any case the method could not be used to determine strains above about 0.4 with any accuracy.

The plastic strains around a crack tip during fibrous fracture are usually much higher, of the order of unity, so hardness measurements could not be used to determine their magnitude. Instead, a recrystallisation technique was developed, similar to that described by Shoji (1976). When a cold-worked material is held at an elevated temperature, new grains nucleate and grow in place of the deformed ones. The number of nucleation sites, and hence the final recrystallised grain size, depends on the amount of prior plastic strain. There is also a critical strain below which recrystallisation does not occur at an appreciable rate. Shoji used this latter feature, coupled with etching, to

delineate the highly strained region around notches in a medium strength, bainitic steel. The En32B steel used here, however, was found to have a critical strain for recrystallisation of about 0.12, which is too low to be considered as a high strain.

The method chosen here, therefore, was based on the dependence of recrystallised grain size on prior plastic strain. Tensile specimens were pulled to failure and then given various heat treatments in a vacuum furnace, at temperatures between 550°C and 700°C, for times between 1 hour and 18 hours. It was found that 3 hours at 650°C gave the best results. Plate 5.6(a) shows a section through a recrystallised tensile specimen, which had been polished and deep-etched in nital. Plate 5.6(b) shows a similar section through a precracked bend specimen which had undergone slow crack growth.

The calibration of grain size vs. plastic strain was performed on necked tensile specimens, in the same manner as the microhardness calibration. Plate 5.7(a) and (b) show typical grain sizes at different strains. Unlike microhardness, grain size cannot be determined at a particular point in the material, but as an average over a discrete area. The method chosen was to project the microstructure onto a screen at a known magnification, and to count the number of grains within a particular area (50 or 100 μm square). From this the average grain diameter was calculated. Obviously, for strains little larger than the critical strain for recrystallisation, the large grain size would render such a method inaccurate. However, interest was focussed more on the regions of higher strains; and where the strain is changing rapidly with distance averages over as small an area of possible are required. The final calibration curve of true strain vs. average, recrystallised grain diameter, for normalised En32B, is shown in Fig. 5.8(b).

5.6.3 Strains between a Crack Tip and a Void in the Brass Specimens.

The brass CT specimens were made in two thicknesses,

16 or 32 mm. The thicker ones were prepared by soldering together two 16 mm plates as described in section 5.4.2, and were used for measurements of strains in both plane stress and plane strain. The thinner ones were machined simply from one thickness of plate, and only surface strains (in plane stress) determined. By photographing the grids, pulling the specimen to a predetermined mount opening, and then rephotographing the grid, or measuring with a travelling microscope, the strain distribution could be determined. The average tensile true strain between two grid intersections, ϵ_{ii} , is given by

$$\epsilon_{ii} = \ln(l_i/l_o) \quad (5.3)$$

where l_o and l_i are the initial and final separation, respectively, of the intersections.

In order to measure the mid-section deformation on the compound specimens, they had first to be unsoldered. They were re-heated in the inert atmosphere furnace, and if balanced in a near-vertical position with a block under one half, the other half often slid away when the solder melted. However, a surer means of separation was to saw-cut a small portion of the interface away, leaving a slot down which a wedge could be driven on removal of the heated specimen from the furnace. When cool the appropriate face was given a careful skim to remove the solder layer and reveal the deformed grid. The brass was a very similar colour to the solder filling the grid lines, but etching in alcoholic ferric chloride solution distinguished them.

The surface grids were fairly fine, so that the centre of the grid lines was relatively easy to pinpoint. It is estimated that the error in surface strain determination, allowing for errors in the measurement apparatus, was about 5-10%. The interior grids were of necessity coarser, and the presence of the solder made finding the centres more difficult, so that the estimated errors in strains are correspondingly larger, at 15-20%. In fact the percentage error would tend to be largest at very

small strains.

Table 5.1 Chemical compositions of En32B and En24, in weight %.

	C	Si	Mn	S	Ni	Cr	Mo	Cu	V	P
En32B	0.17	0.23	0.74	0.033	0.02	0.01	0.01	0.08	-	-
En24	0.38	0.26	0.66	0.028	1.43	1.21	0.27	0.19	0.01	0.02

Table 5.2 Tensile properties of En32B and En24, as determined on Hounsfield No. 12 tensile specimens.

Material, Condition and orientation of tensile axis	Lower yield stress, σ_y , MN/m ²	U.T.S σ_u , MN/m ²	Reduction in area %	Strain-hardening exponent, n *
<u>En32B</u>				
Normalised, L	290	440	70	} ≈ 0.28
Normalised, T	290	440	51	
Slow cooled L	220	390	65	} ≈ 0.28
Slow cooled T	215	390	52	
10% Prestrain L	480 ⁺	530	65	
10% Prestrain T	500 ⁺	570	44	
20% Prestrain L	590 ⁺	640	60	
20% Prestrain T	610 ⁺	660	33	
<u>En24</u>				
Quenched and tempered, L	540	630	65	≈ 0.11

* n obtained from $\sigma_{\text{true}} = A(\epsilon_{\text{true}})^n$

+ 0.1% offset yield stress.



Plate 5.1 Inclusions in En 32B, x 1900

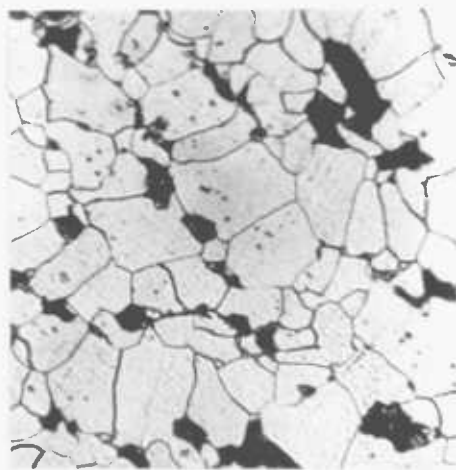
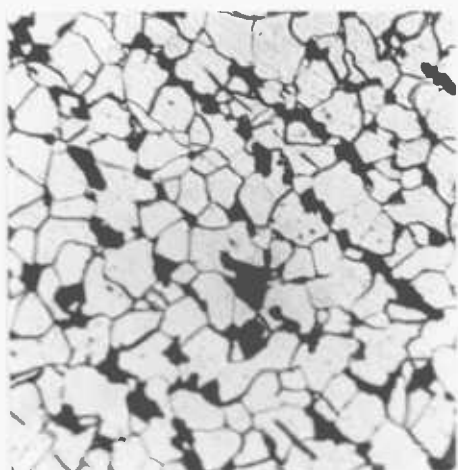


Plate 5.2 Microstructure of En 32B, etched nital x 200

(a) Normalised

(b) Slow cooled

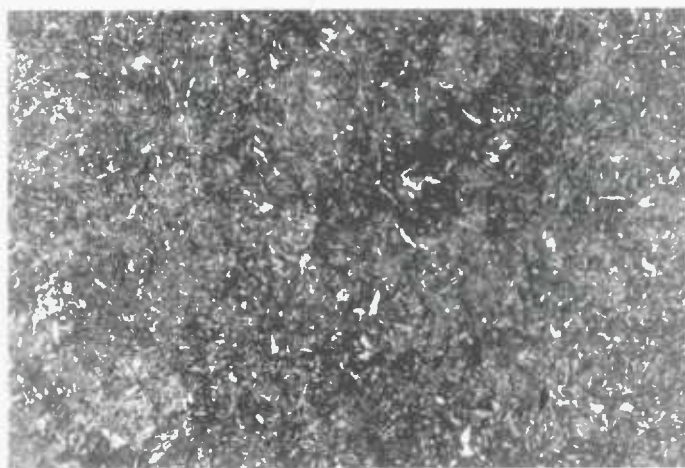


Plate 5.3 Microstructure of En 24, etched nital, x 350

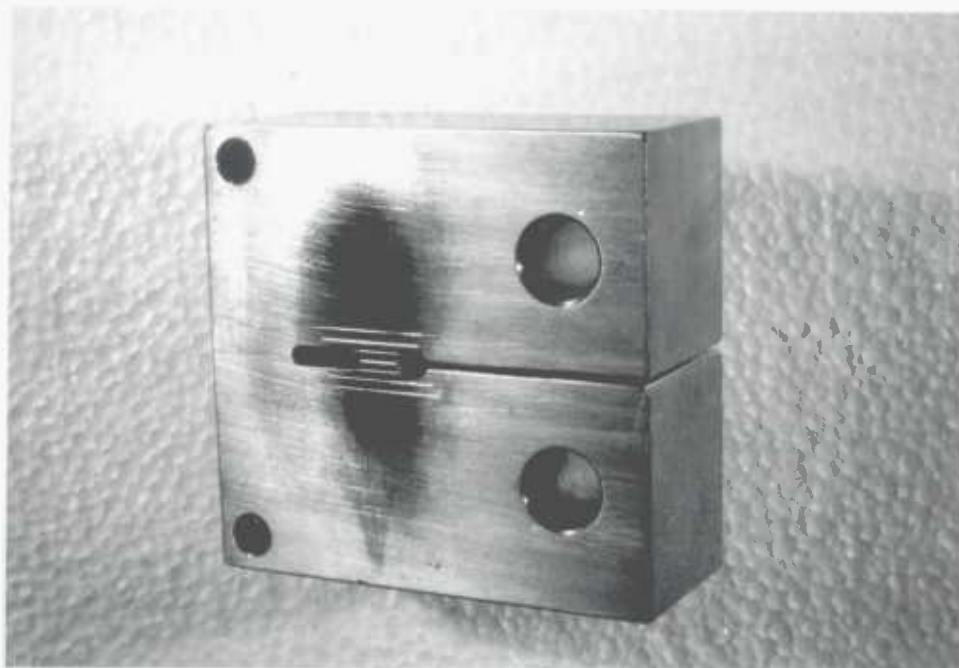


Plate 5.4. Double thickness brass CT test-piece, x 0.75

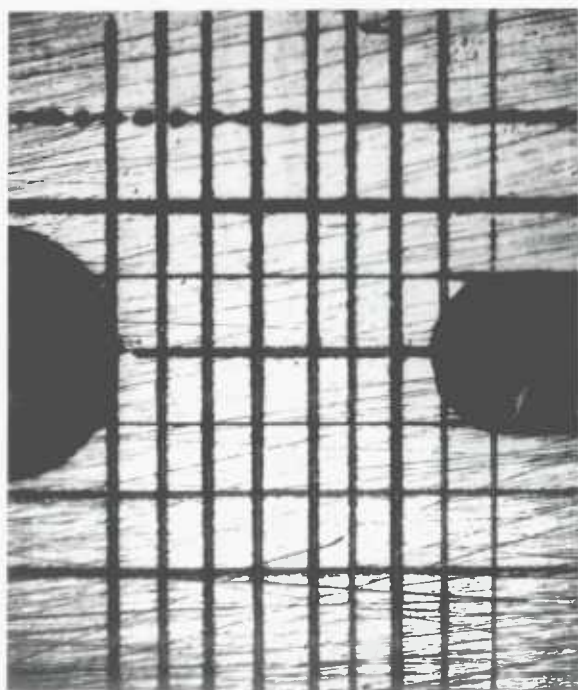
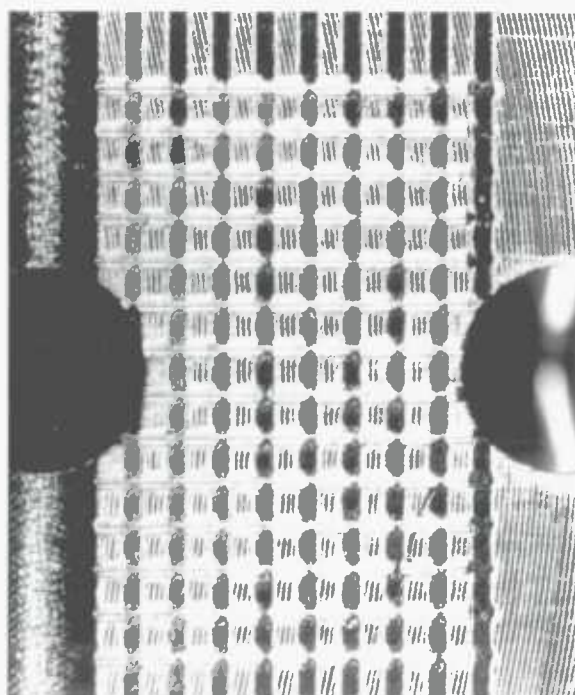


Plate 5.5. Grids between notch and slot in brass CT test pieces, x 6

(a) On surface



(b) In centre

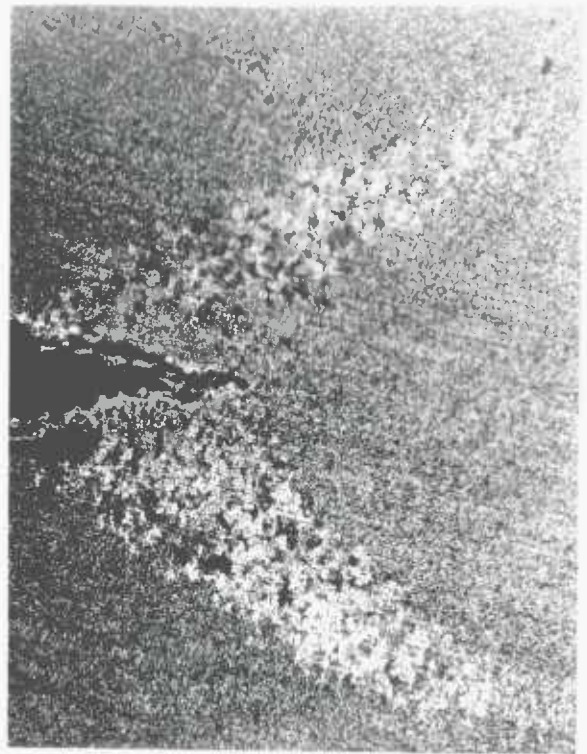
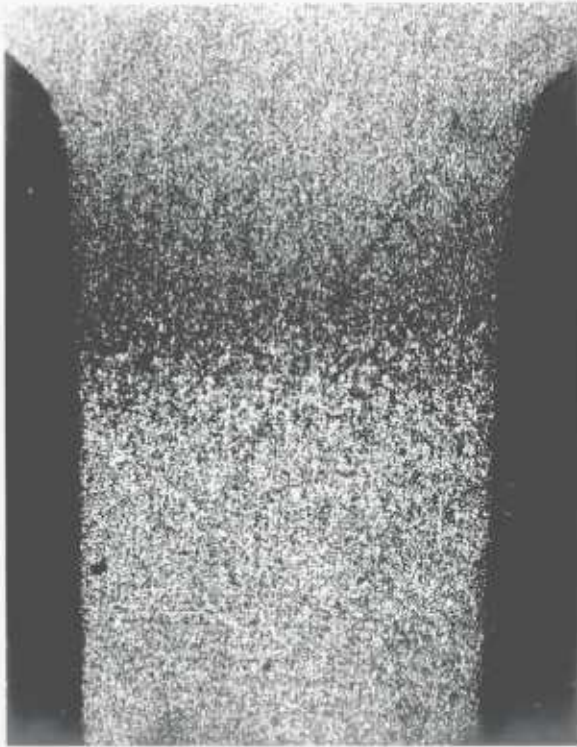


Plate 5.6 Recrystallised zones in En 32B, etched nital

(a) Tensile specimen x 60

(b) Cracked bend specimen, x 40

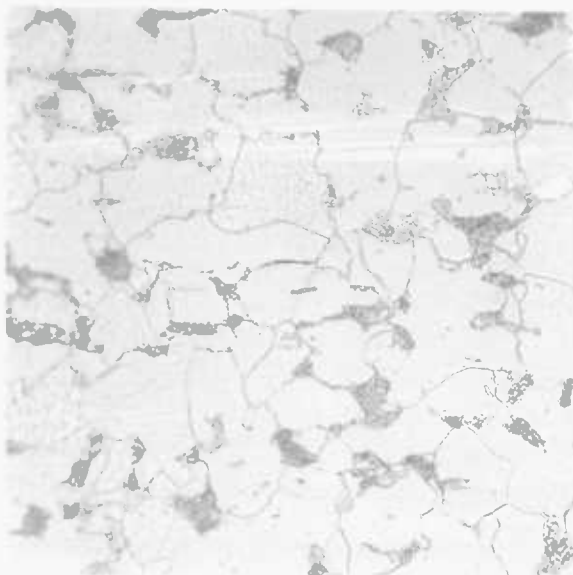


Plate 5.7 Recrystallised grains in En 32B, etched nital, x 500

(a) At true strain ≈ 0.2

(b) At true strain ≈ 1.0

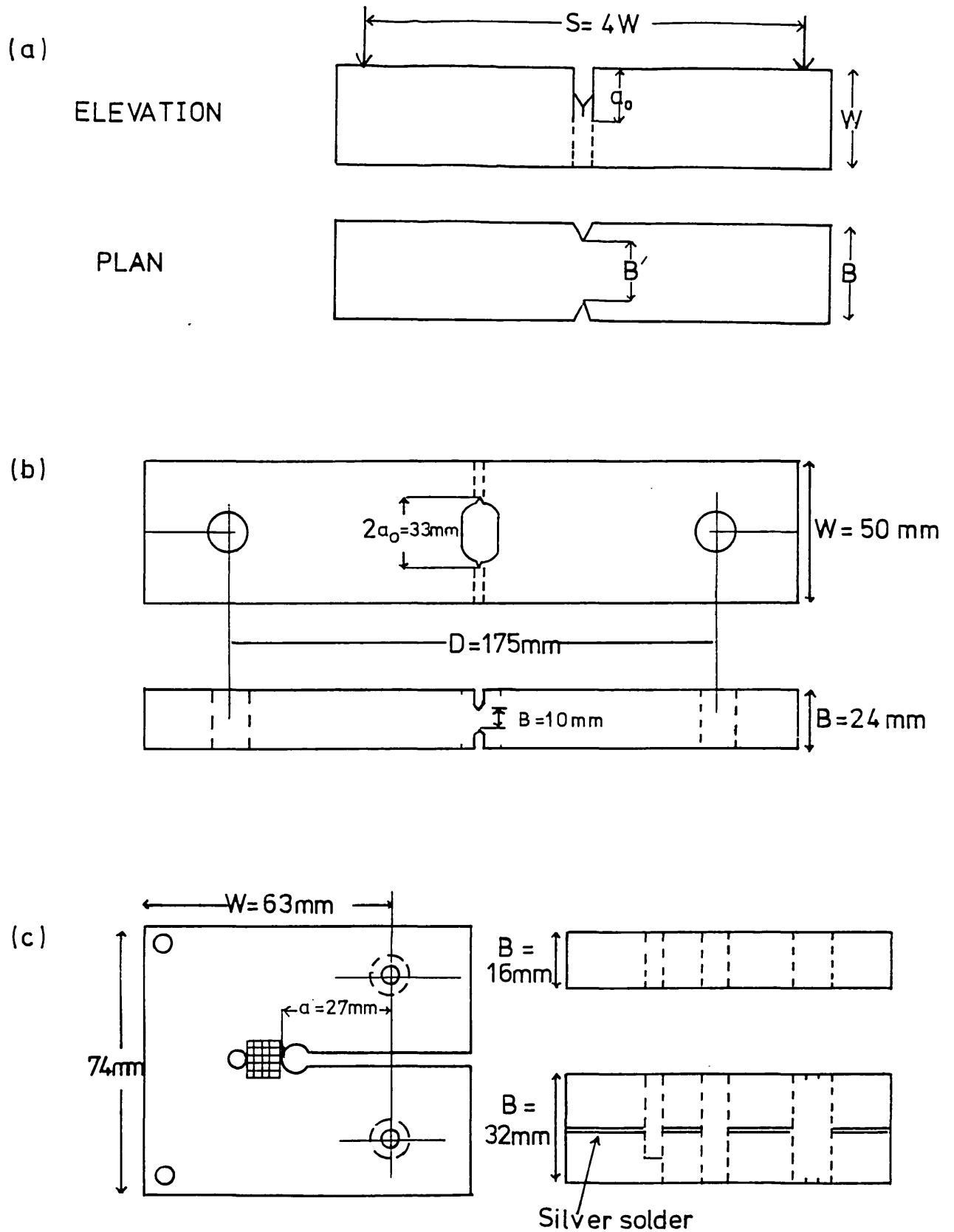


Fig. 5.1 Specimen types (a) Three point bend, (b) Centre cracked tension (CCT) (c) Compact tension (CT).

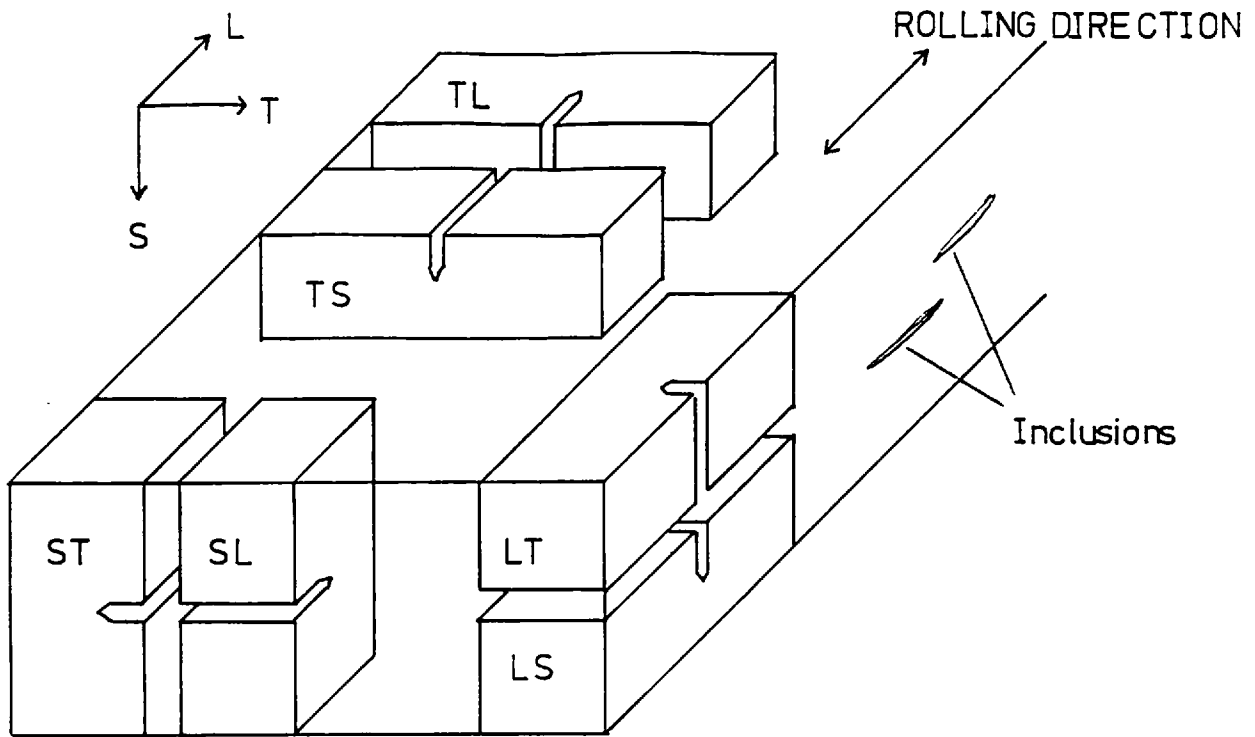


Fig. 5.2 Orientation of test-pieces.

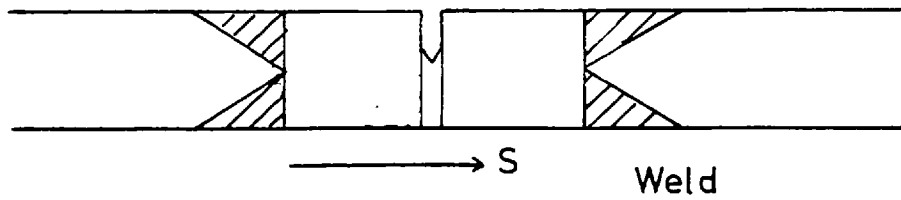


Fig. 5.3 Design of weld for short transverse test-pieces.

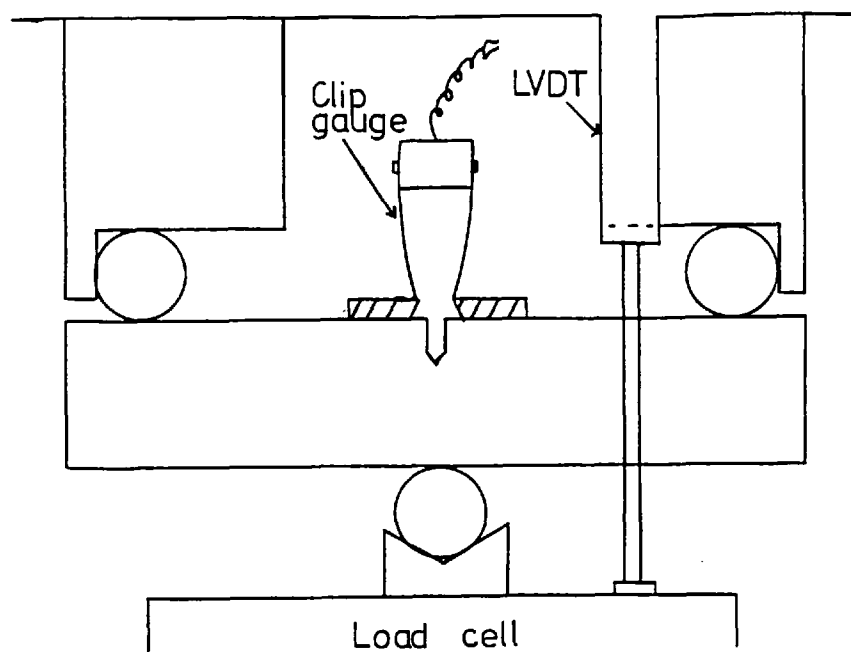


Fig. 5.4 Testing configuration for three point bend test-pieces.

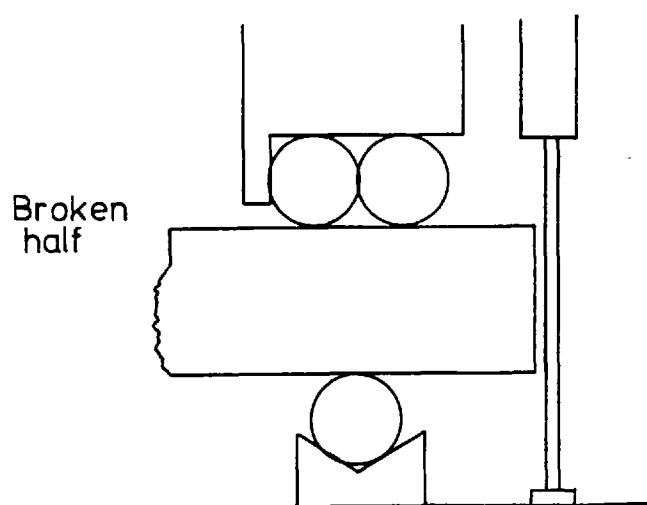


Fig. 5.5 Measurement of extraneous displacement.

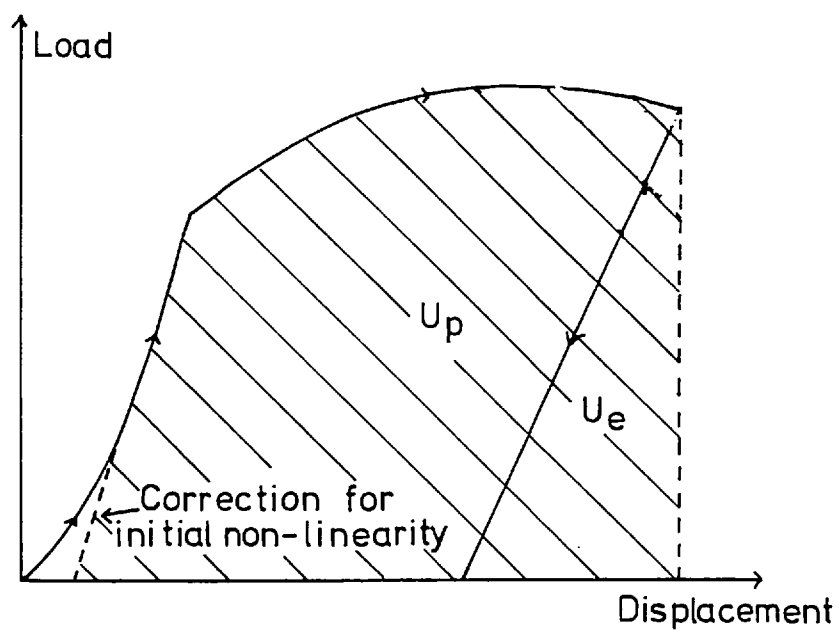


Fig. 5.6 A typical load/displacement plot, showing the correction for initial non-linearity, and the measurement of area U .

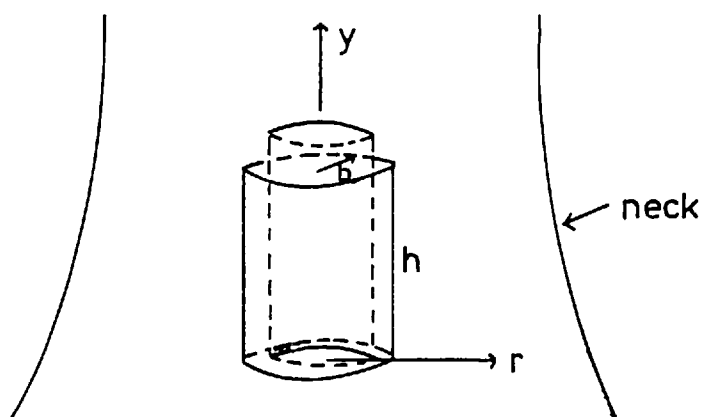


Fig. 5.7 Deformation in the neck of a tensile specimen.

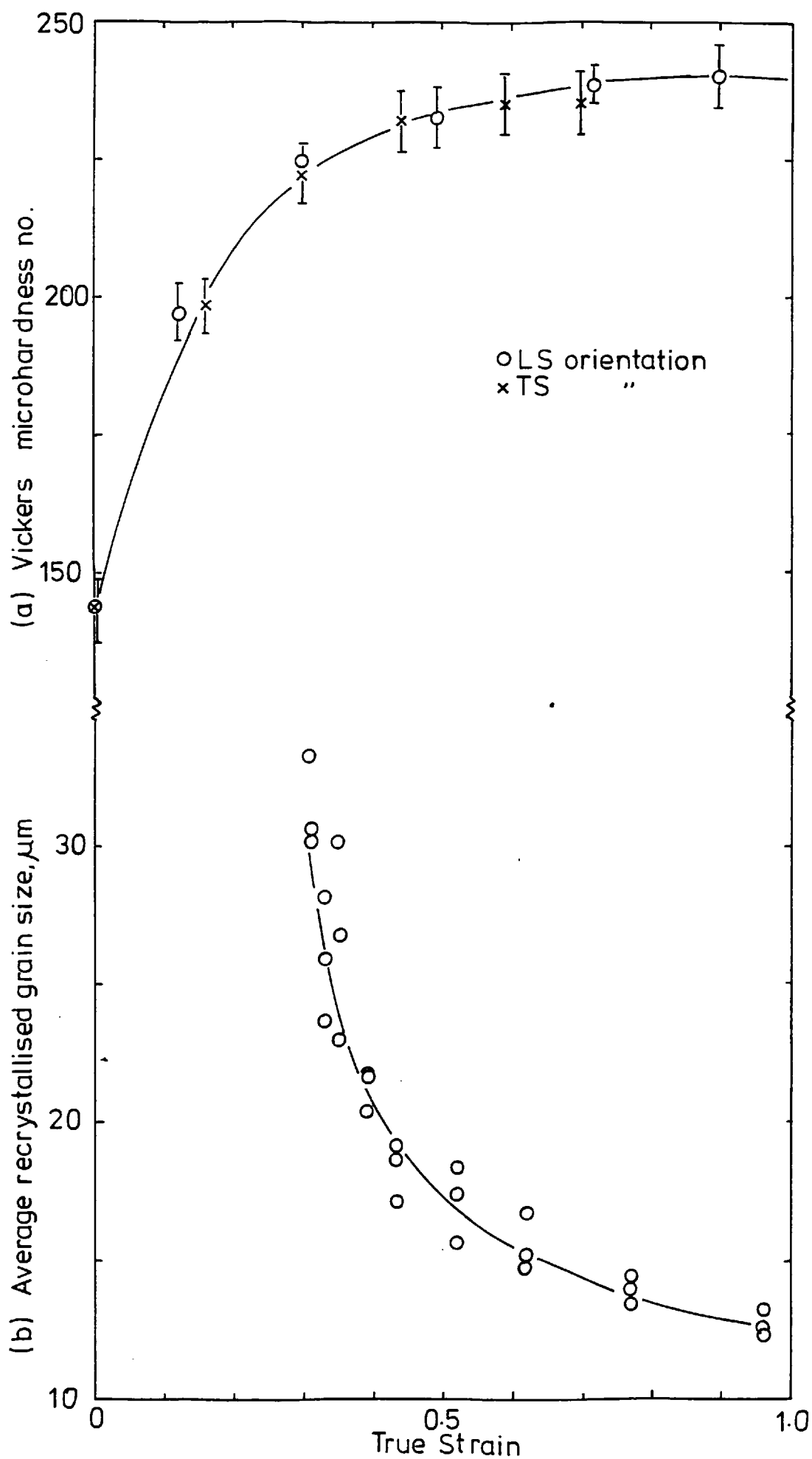


Fig 5.8 Calibration curves for normalised En32B of true tensile strains vs. (a) Vickers microhardness (b) Recrystallised grain size.

Chapter Six

INCLUSIONS AND FRACTOGRAPHY

6.1 Introduction

It is well known that non-metallic inclusions have a profound effect on ductile fracture in low strength steels, since they are generally the principal sites for void formation. This is shown, for instance, by their presence on fracture surfaces. Plate 6.1(a) shows the fracture surface of a three point bend specimen of En32B, in the LS orientation. Crack extension began with the formation of a "stretch zone" as the fatigue crack blunted, followed by a region of fibrous tear. Final fracture occurred by cleavage at low temperature.

Closer examination of the fibrous fracture surface revealed that it was composed of dimples, formed by the coalescence of voids, as in Plate 6.1(b). Inclusions could be observed in some of the dimples, but many of the holes were too deep, and in any case there was a tendency for exposed inclusions to drop out. They were more obvious in the short transverse fractures, where the dimples were much shallower (Plate 6.2).

The process of ductile crack extension is one of initiation of voids at second phase particles in the strain field of the crack, followed by growth and coalescence. For specimens where the final fracture was by cleavage, voids which formed ahead of the propagating crack tip should be revealed on the cleavage facets. Plate 6.3 shows some examples of dimples which were found 300-400 μm ahead of the fibrous crack tip. Inclusions are visible in some of them. Since the strains should be relatively small this far ahead of the tip (the initiation COD $\approx 100\mu\text{m}$), this observation supports the notion that the strain required for void nucleation at inclusions is small.

In En32B, the non-metallic inclusions appeared to dominate void formation. Therefore, before examining the fractographic features, it is pertinent to describe the composition and distribution of these inclusions. This will be preceded by a short review.

6.2 Review of Non-Metallic Inclusions in Steel.

6.2.1 Origin and Morphology

Comprehensive surveys of non-metallic inclusions in steel have been written (e.g. Pickering, 1974), and the details will not be repeated here. Briefly, however, the most common inclusions may be divided into two chemical categories - oxides and sulphides. The first comprises oxides of aluminium, manganese and silicon, which are formed when these elements are added to the molten steel to de-oxidise it. The second consists, in simple steels, of manganese sulphide, MnS. Considerably more manganese is added than is required by stoichiometry to ensure that no free iron sulphide is formed, with its consequent problems of hot-shortness. An estimate of the volume fraction of MnS present may be obtained by assuming that all the sulphur is tied up, and from the relative densities of the two constituents (Baker, 1971):-

$$\text{Percent volume fraction of MnS} = 5.35 \times (\text{wt.\%S}) \quad (6.1)$$

Manganese sulphides are of particular importance in ductile fracture due to the deformation they can undergo on rolling. They may occur in any of three forms, known as types I, II and III, depending on the chemical composition of the steel. The deformation of inclusions at rolling temperatures is explained by their greater plasticity relative to the matrix, which may be expressed in terms of the hardness ratio of matrix and inclusion (Charles, 1974). Hard inclusions do not deform, and this is the basis of the rare earth modified steels, in which non-deformable rare earth sulphides replace the MnS, to give better transverse properties.

Void formation may be facilitated by the production, on rolling, of small cavities at the ends of the inclusion (Charles, 1974). In any case void nucleation at inclusions is thought to be easy. Experimental evidence (Wahlster et al, 1969) suggests that the inclusion/matrix interface is weak and readily separated and the greater thermal contraction of the inclusion on cooling may mean that separation has occurred before straining begins (Brooksbank and Andrews, 1968). Voids may also nucleate by the cracking of inclusions, as observed by Butcher and Allen (1977), but whatever the mechanism they are frequently observed first at low matrix strains.

6.2.2 Effect on Ductility and Toughness

Toughness and, to a lesser extent, ductility are found to be affected by inclusions, whereas yield and ultimate strengths in simple tension are scarcely altered. The volume fraction of inclusions bears an inverse relationship to tensile ductility (eg. Edelson and Baldwin, 1962; Gurland and Plateau, 1963; Gladman et al., 1971) and to fracture toughness (eg. Franklin and Tegart, 1964; Birkle et al., 1966). The Thomason (1968) model outlined in section 4.3 appears to describe tensile ductility fairly well (Thomason, 1974) and the Rice and Johnson (1970) model predicts the right trend for fracture toughness and volume fractions (McMeeking, 1977b). For equi-axed inclusions all the available evidence indicates that low volume fractions are needed for good ductility and toughness.

However, this is not a sufficient condition, due to the powerful anisotropic effects of inclusion deformation. The result is that toughness and ductility are not much altered in the longitudinal direction, but may be much reduced in the transverse and short transverse directions, where inclusions are elongated normal to the applied tensile stress (Smith and Knott, 1971). Baker and Charles (1971) found that the COD in the short transverse direction had a strong inverse relationship to prior rolling reduction. This explains the phenomenon of lamellar tearing in welded

structures, in which cracks propagate easily in the short transverse plane under the influence of through-thickness stresses (Watkinson et al., (1974).

Inclusion size may also be expected to affect toughness through its relationship to spacing, since at a given volume fraction the ratio of size to spacing is a constant, for uniform particles. Assuming that the ease of void formation does not vary with size, smaller particles, which are more closely spaced, will provide more void sites, and thus produce a lower toughness, a result found by Baker and Charles (1974) and Pellisier (1968).

6.3 Inclusion Analysis

6.3.1 Chemical Composition

Fracture surfaces from several orientations of specimen were examined in the scanning electron microscope (SEM). This had an energy dispersive X-ray analysis system (EDAX) as an attachment which allowed the identification of chemical constituents.

Several large elongated inclusions on one of the transverse fracture surfaces (as in Plates 6.7 and 6.8) were analysed. The principal elements encountered were sulphur, manganese and iron, indicating that the inclusions were mainly manganese sulphide (MnS), as commonly found. The iron observed was probably due to traces in the inclusion, and to the background. Other inclusions, which were smaller and equi-axed, were also found occasionally. The EDAX showed that these contained manganese but no sulphur, so it is likely that they were manganese oxide, MnO.

Not all of the large dimples on the fracture surfaces contained particles, but were probably formed around inclusions which had remained on the other fracture surface, or had dropped out. Between the large dimples were smaller ones which also did not contain particles. These were particularly prevalent in the longitudinal orientations, Plates 6.1(b) and 6.6. No traces of manganese or sulphur could be

detected inside them, and it is probable that they were small linking voids, nucleated by cracking of cementite, in the final stages of void coalescence.

6.3.2 Content and Distribution

Plane polished sections of the steels were examined optically using a "Quantimet" Image Analysing Computer. The inclusions visible were analysed in terms of area fraction, maximum feret diameter and mean spacing. Sections perpendicular to the T and S directions were used, with the longest inclusion axes perpendicular to the scan lines of the Quantimet: this orientation gave the greatest accuracy for inclusion length measurements (Baker, 1971).

Examination of fracture surfaces indicated that voids tended to form around the larger inclusions. In addition, the resolution of the optical system made measurements of less than about 1 μm inaccurate. Thus it was decided to count inclusions of diameter greater than 2 μm only. For En32B a total of 1500 fields, and for En24 800 fields, of size 0.0157 mm^2 each, were examined. The size distribution for En32B is given in Fig. 6.1, and the remaining results tabulated below.

	En32B	En24
Area fraction (%)	0.20 $\pm$ 0.05	0.18 $\pm$ 0.03
Centre-to-centre spacing (μm)	58 $\pm$ 4	82 $\pm$ 7
Max. feret diameter (μm)	6.2 $\pm$ 0.9	7.3 $\pm$ 1.5

The amount of sulphur present was 0.033 and 0.028 wt.% respectively for En32B and En24, from which eqn. (6.1) gives volume fractions of MnS of 0.18% and 0.15%, in reasonable agreement with the area fractions measured on the Quantimet. The centre to centre spacings are calculated by the computer as the reciprocal of the square root of the number of inclusions per unit area. The errors calculated here are the approximate standard deviations, derived from the means of sets of 100 fields.

All measurements of inclusion size are likely to be underestimates, for two reasons:

i) Sections through inclusions (assumed ellipsoidal) perpendicular to one of the major axes will not in general pass through the centre. On average the apparent area of each inclusion will be less than the maximum. As shown in Appendix III, the true semi-axis l is related to the apparent semi-axis l' by

$$l' = \sqrt{(2/3)} l \quad (\text{III.4})$$

(ii) More importantly, if the inclusions are not straight, but contain kinks, a single one may appear as several on a plane section, causing an underestimate of true length. In addition, inclusions which are separated but very close together may behave during fracture like a single inclusion and form one large void. This was probably a common occurrence, since inclusions were often observed arrayed in lines.

If the centres of the inclusions form a random dispersion then subsequent uniform deformation cannot order the array or change the dispersion parameters. This was confirmed by analysing sections of prestrained En32B, and no significant difference in any of the inclusion parameters was found.

In a separate analysis of 200 fields of the En32B on the S plane, the inclusion dispersion was graded into separate categories, in terms of maximum feret diameter F_L , as follows:

	$F_L = 2-5\mu\text{m}$	$5-10\mu\text{m}$	$10-20\mu\text{m}$	$20+\mu\text{m}$
Area fraction (%)	0.042	0.046	0.055	0.046
Total no. counted	490	266	133	42
Mean spacing (μm)	80	109	153	273
Projected length (mm/mm^2)	0.52	0.58	0.57	0.40

Before quantitative predictions concerning the effect of inclusions on fracture can be evaluated, it is necessary to deduce some representative inclusion dimensions. It will be assumed that only those inclusions of length $5\mu\text{m}$ and greater have a primary effect on void formation. As shown in Appendix III, average values of the semi-axes of

the inclusions (assumed ellipsoidal) may be derived from the total projected length per unit area, P_L , the area analysed A , the number counted N and the area fraction A_f , viz.

$$\begin{aligned} \text{Rearranging eqn. (III.5)} \quad l' &= \frac{P_L \cdot A}{2 \cdot N} \\ m' &= \frac{A_f \cdot 2}{\pi \cdot P_L} \end{aligned} \quad (\text{III.6})$$

where l' and m' are the average apparent semi-axes of the ellipse. For inclusions of length $F_L > 5 \mu\text{m}$, taking $P_L = 1.55 \text{ mm/mm}^2$, $A = 3.14 \text{ mm}^2$, $N = 441$ and $A_f = 0.147$, gives $l' = 5.52 \mu\text{m}$, and $m' = 0.60 \mu\text{m}$.

The apparent semi-axis in the S direction, n' , was probably too small to be measured accurately by the Quantimet, since it was near the limit of resolution, and in any case particles of dust on the specimen and lens would also be counted. However, n was smaller than m , as the transverse and short transverse fracture surfaces show (Plates 6.2, 6.7, 6.8 and 6.9). Therefore, n' will be estimated at $0.4 \mu\text{m}$. The apparent semi-axes, for inclusion of apparent length greater than $5 \mu\text{m}$, are thus on average,

$$\begin{aligned} l' &= 5.52 \mu\text{m} \\ m' &= 0.60 \mu\text{m} \\ n' &\approx 0.4 \mu\text{m} \end{aligned}$$

6.4 Fractography and Initiation of Crack Extension

When the faces of a sharp crack are loaded, the tip blunts. Green and Knott (1976) and Garwood and Turner (1978) have noted that the COD at the tip during propagation is smaller than its value at initiation. The result is that a ridge, known as the "stretch zone" remains at the position of the blunted fatigue crack tip (Fig. 6.2). Attempts have been made to deduce the initiation COD from the height, SZ, of the stretch zone (e.g. May and Smith, 1973; Harris and Elliott, 1975). This would have obvious applications in

the analysis of failed components. If SZ is measured at an angle of 45° to the crack plane, as in Fig. 6.2(c), and initiation is assumed to occur in the centre of the blunted tip profile, the relationship with COD becomes

$$\delta_i = \frac{2}{\sqrt{2}} \cdot (SZ) \quad (6.2)$$

Plate 6.4 shows stretch zones formed in specimens of En32B, LT orientation. It is obvious that the height is not readily determined, since it varies along the crack front. This is probably due to initiation occurring off-centre on the tip profile, as in Fig. 6.2(d), giving different heights for the same COD. In general,

$$0 \leq \frac{SZ}{\sqrt{2}} < \delta_i$$

Thus the use of stretch zone to determine COD is likely to prove unreliable unless many measurements are made.

The Landes and Begley (1974) method of determining J_i , described in Section 3.3.3 and Fig. 3.6, uses measurements of crack extension including the stretch zone. A line of slope $2\sigma_{flow}$ is then constructed on the R-curve to define the start of fibrous tearing. The slope is based on the assumption that the blunted profile is a semi-circle, so that the stretch zone length $\Delta a'$ is $\delta/2$. However, assuming that in plane strain,

$$J \approx 2 \sigma_{flow} \delta$$

the equation for the blunting line becomes

$$J = 4\sigma_{flow} \Delta a'$$

Thus the slope should be $4\sigma_{flow}$ in plane strain.

6.5 Fractography and Crack Propagation

6.5.1 Effect of Side Grooves

In plane-sided test-pieces the surface is in a state

of plane stress. This leads to the formation of shear lips during propagation, as the crack front tunnels forwards in the centre. It was observed in the plane-sided bend specimens of En24, and is dealt with in Section 9.2.

The majority of the En32B test-pieces contained side-grooves to eliminate shear lips. The edges of the fracture surfaces were examined optically and in the SEM for signs of shear. Narrow bands could be observed next to the root of the side-grooves but these were small in comparison to breadth of the fracture surface. Plate 6.5 show this band for the TS orientation, where it was 20-40 μ m wide. In the LS orientation it had a width of 100-150 μ m, compared to a minimum fracture surface breadth B' of 6mm in the side-grooved, 10mm square specimens.

6.5.2 Effect of Specimen Orientation

SEM photographs of the fracture surfaces for the LS and ST orientations of En32B are shown in Plate 6.1 and 6.2, and for the other orientations in Plates 6.6-6.9. The most notable feature is the effect of the elongated inclusions. In the transverse and short transverse fractures, voids were formed almost exclusively around them. In the LS and LT orientations the larger dimples formed at inclusions were linked by small, shallow ones, nucleated possibly at carbides in the final stages of void coalescence.

In the TL and SL orientations there were obvious signs of the tendency towards crack propagation down elongated inclusions. Plate 6.9(b) shows the ductile/cleavage boundary (SL orientation), where the dimples around the elongated inclusions can be seen extending well ahead of the crack front on either side. Since crack extension occurs by the linking of voids around inclusions, it is likely that this feature would also occur in the TS and ST orientations. Such linkage is most likely to occur between the tips of elongated voids, due to stress concentration effects, and the crack could immediately propagate down the length of an inclusion. The micro-mechanism of crack

advance should be the same in the TS and TL, or the ST and SL, orientations, but the macroscopic direction, as determined by the configuration of the specimen, is different.

Crack profiles were also examined by taking polished sections, through the crack tips, of specimens which had not been broken in cleavage. Plate 6.10 shows such sections for the LS, SL and ST orientations. For the longitudinal orientations the crack surfaces were broken by the deep fissures of elongated voids, which tended to "blunt" the crack tip. Nevertheless propagation tended to occur straight ahead, rather than by a "zig-zag" mechanism down the inclusions. In the transverse and short transverse orientations, on the other hand, void linkage took place by shear, at an angle to the macroscopic crack plane, particularly where the inclusions were elongated in the direction of crack advance (TL and SL). A model for void linkage is discussed later, and it is shown that coalescence is unlikely to occur in the rolling (L) direction, where the inter-inclusion distances are greatest.

Another notable feature is the difference in angle between the faces of the extending crack (the "flank angle") between the different orientations. This angle was much greater in the longitudinal orientations than in the transverse or short transverse ones.

6.5.3 Effect of Prestrain

The occurrence of "zig-zag" fracture, where void linkage takes place at an angle to the crack plane, has been noted in certain orientations. Clayton and Knott (1976) observed an increased tendency towards this type of fracture mechanism as work-hardening capacity was reduced. This localisation of deformation into shear bands is a probable explanation for the tendency of most models of void growth to overestimate fracture strain. Theoretical work by Asaro and Rice (1977) indicates that this strain localisation can occur when the work-hardening exponent has been lowered to a small, but non-zero, value. The

fracture surfaces and profiles show this effect most clearly for prestrained specimens of En32B in the TS orientation (Plate 6.11), where fine shear cracks can be seen linking the voids. The fracture surface has a stepped appearance. The distance between the dimples on the fracture surface appears smaller than in the unstrained case (Plate 6.7), but this is probably due to the increased angle of shear, rather than to a change in inclusion spacing.

This zig-zag type of fracture was less prevalent in the LS orientation at 20% prestrain (Plate 6.12). Presumably strain localisation is harder in this orientation since the inclusions, lying normal to the crack plane, would tend to spread the deformation more uniformly. The angle between the crack flanks was found to be much smaller in the cold-worked material, as may be seen by comparing Plates 6.12(b) and 6.10(a).

6.5.4 Delamination and Splitting

Considerable variation was encountered in the toughness of the En32B specimen in the LT and ST orientations, as described in Section 8.2. Examination of the LT fracture surfaces showed that lower toughnesses were accompanied by a series of vertical splits.

Similar splits were observed by Green (1975), and void formation at grain boundary carbides was proposed as an explanation. In the present case a more likely cause seemed to be local variations in inclusion characteristics. A sulphur print of the cross-section of the rolled En32B bar revealed bands of inclusions parallel to the edges (Plate 6.13). The fracture surface of a test-piece of 25mm square cross-section, cut in the LS orientation, showed splits parallel to the rolling plane (Plate 6.14). Incidentally, these splits were not observed in the smaller LS specimens used for R-curve measurement, which may be due fortuitously to the location of the crack tip with respect to the inclusion bands, or may suggest that

delamination is influenced by constraint.

The fracture surfaces of two of the LT test-pieces showing different behaviour were examined. Both had fibrous crack extensions of about 1.5mm, but the one whose toughness lay on the steeper R-curve had a relatively uniform fracture surface (Plate 6.15(a)), whereas the one on the shallower curve exhibited large splits, which again lay approximately in the rolling plane (Plate 6.15(b)). These splits appeared to begin around bands of inclusions (Plate 6.16(a)), and could advance several hundred microns ahead of the crack tip, as indicated by observation of the cleavage fracture surface (Plate 6.16(b)). Sections from the two specimens were polished and the inclusions parameters analysed on the Quantimet; however, no significant overall difference could be detected between the two cases, although there were large variations between sets of fields. This supports the notion that if banding occurred, it was a localised phenomenon.

It might be expected that splits on a fracture surface would enhance toughness since they act as crack dividers, relaxing the lateral constraint and allowing plane stress conditions to develop on either side. While some evidence of elongated shear dimples bordering the splits was found here, Plate 6.16(a), this was obviously a minor effect. By tilting the fracture surfaces an indication of the depth of dimples could be obtained. Comparison of Plates 6.17(a) and (b) shows that the less tough specimens appeared to have a less "rough" fracture surface between the splits, with shallower dimples.

It is tentatively proposed that the lower toughnesses were caused by banding of large inclusions in the rolling plane, with regions of shorter inclusions in between. These short inclusions could be responsible for the lower toughnesses in the LT orientation. In the ST orientation reduced toughness would arise when the crack propagated along one of these bands. The reason why the other orientations were unaffected is not clear.

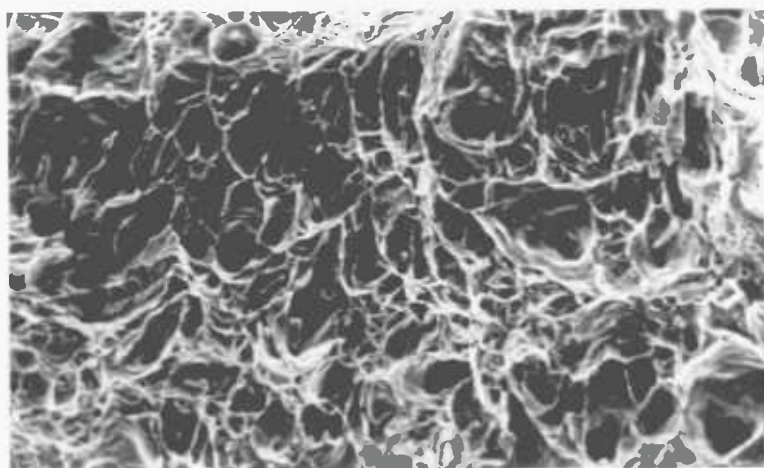
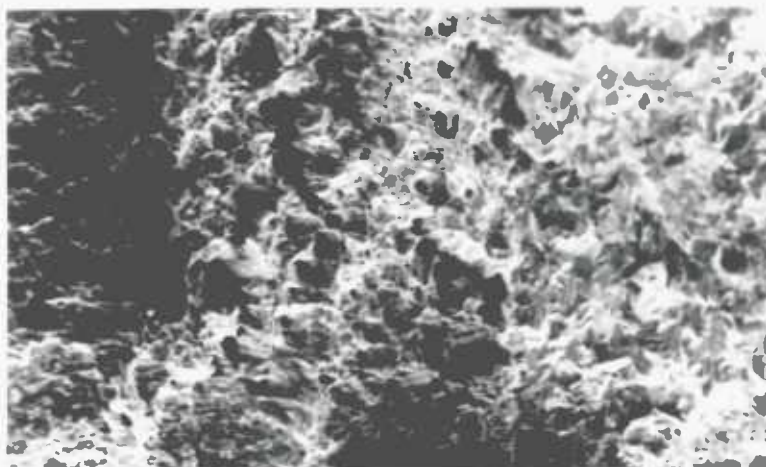


Plate 6.1. S.E.M. view of fracture surface in LS orientation

- (a) General view, showing from left to right, regions of fatigue, ductile rupture and cleavage, x 90
- (b) Details of dimples, x 550

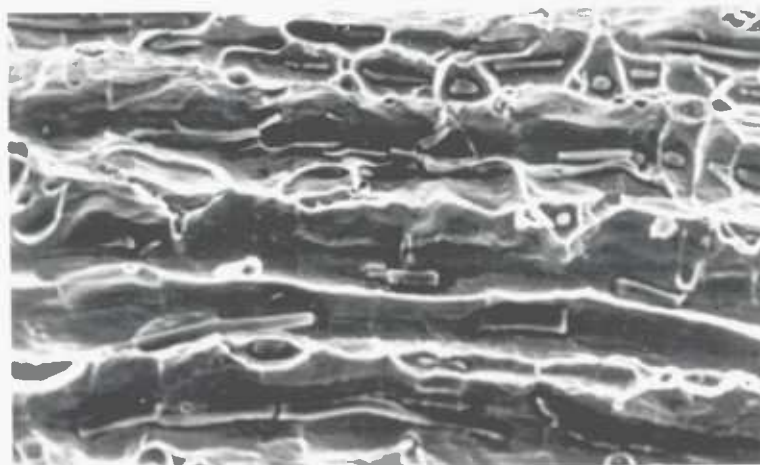


Plate 6.2. Dimples on fracture surface of En 32B, ST orientation, x 1100, SEM.

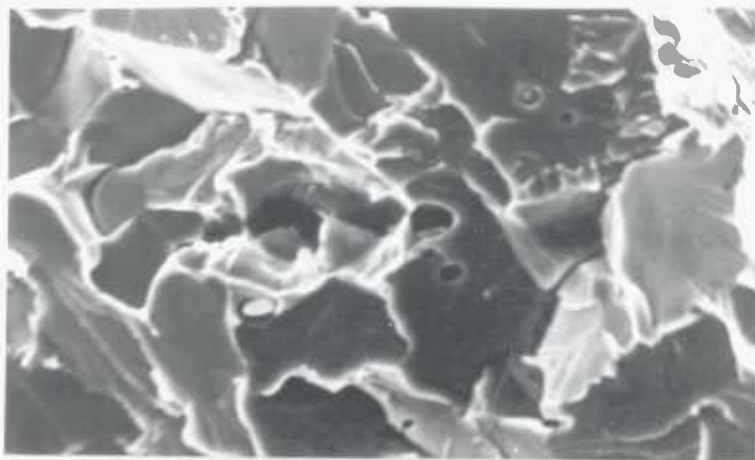


Plate 6.3. Dimples on cleavage fracture surface, 300-400 μm ahead of crack tip, in En 32B, LT orientation. SEM, x 1500

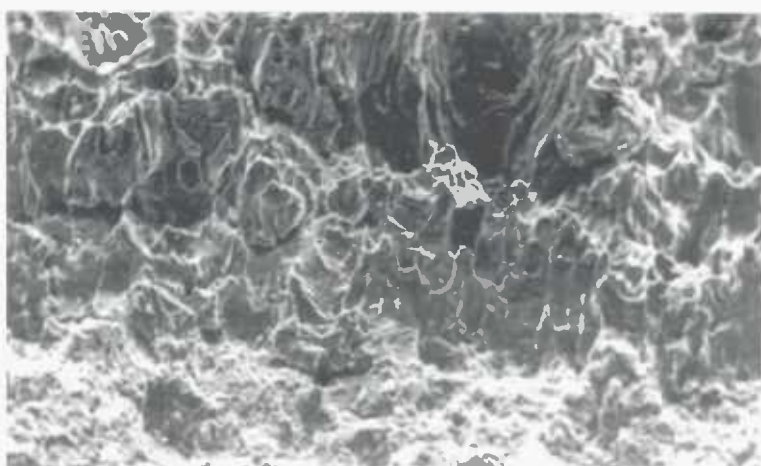
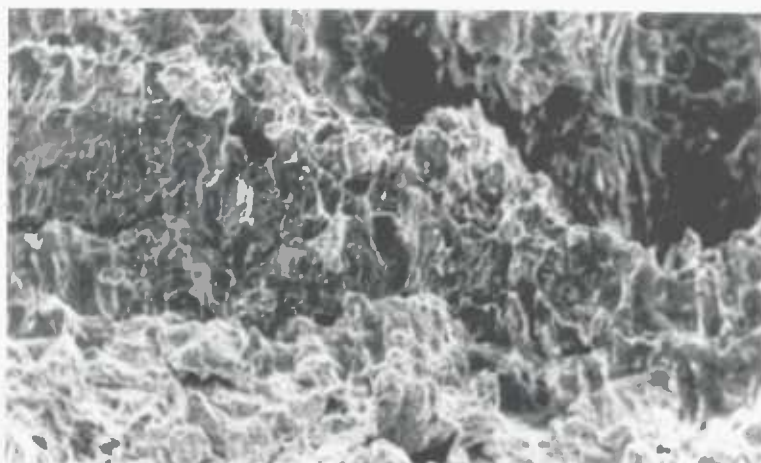


Plate 6.4. Stretch zones in En 32B, LT orientation, at 60° tilt.
SEM

(a) In three point bend, x 300

(b) In CCT, x 340

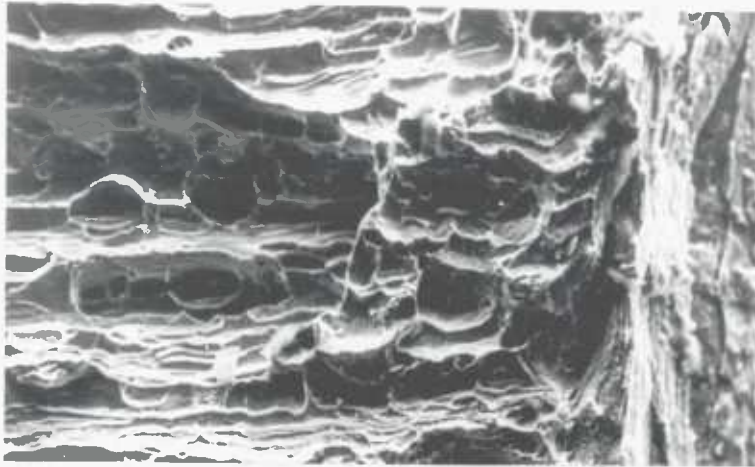


Plate 6.5. Edge of fibrous fracture surface adjacent to side-groove in En 32B, TS orientation. SEM, x 330

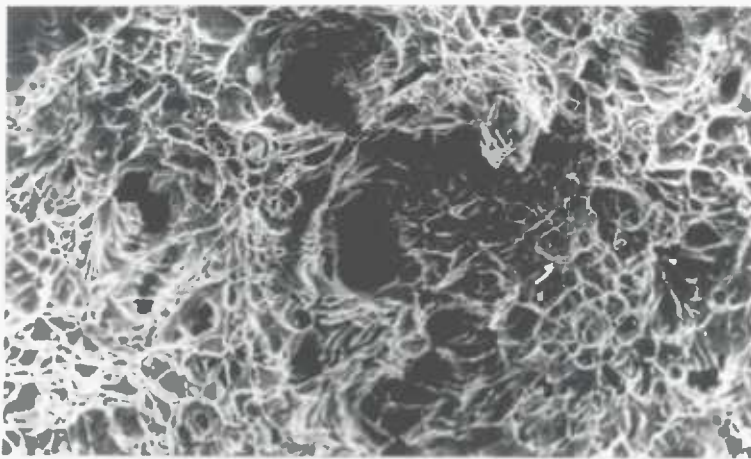


Plate 6.6. Fibrous fracture surface of En 32B, LT orientation. SEM, x 380

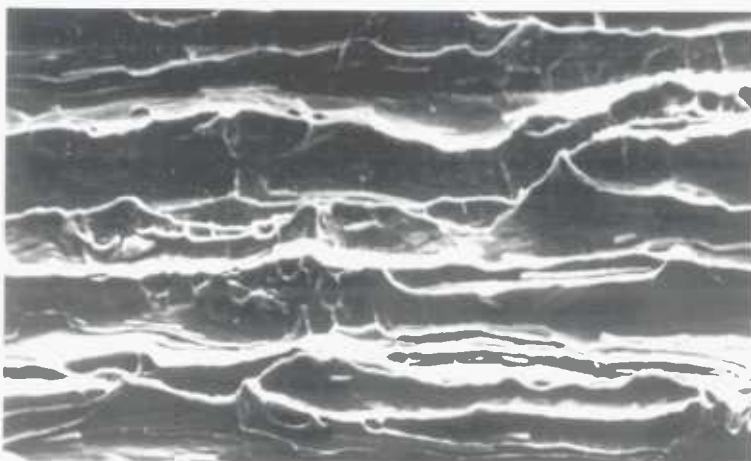


Plate 6.7. Fibrous fracture surface of En 32B, TS orientation S.E.M, x 650

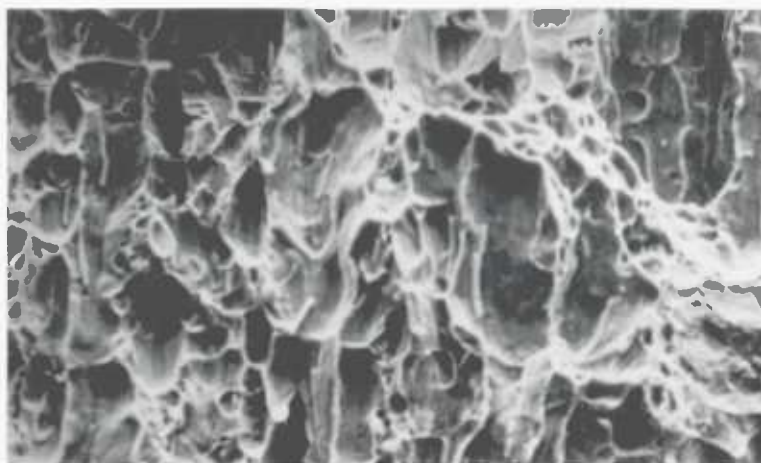


Plate 6.8. Fibrous fracture surface of En32B, TL orientation.
S.E.M, x 750

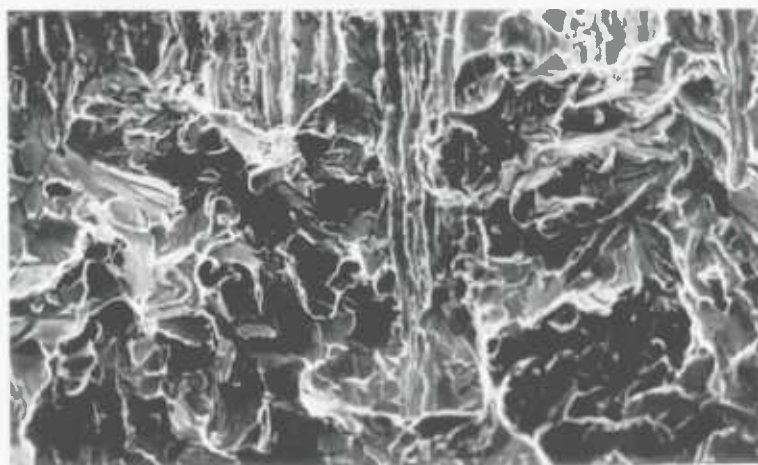
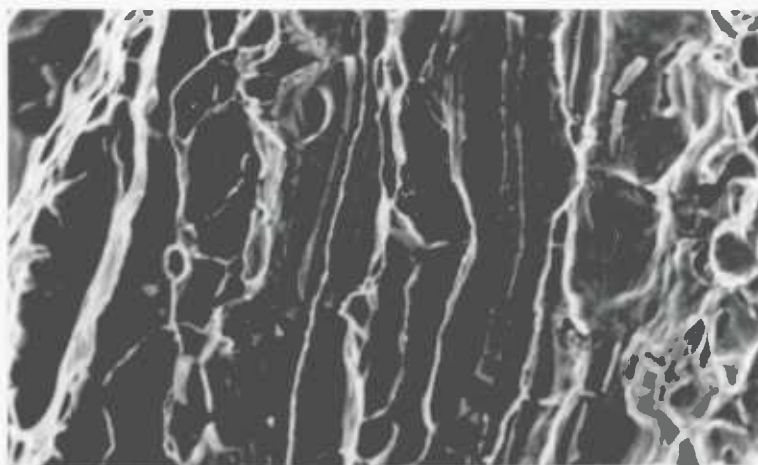


Plate 6.9. Fracture surface in En 32B, SL orientation, S.E.M.

(a) Fibrous fracture, x 1100

(b) Fibrous/cleavage boundary, x 280

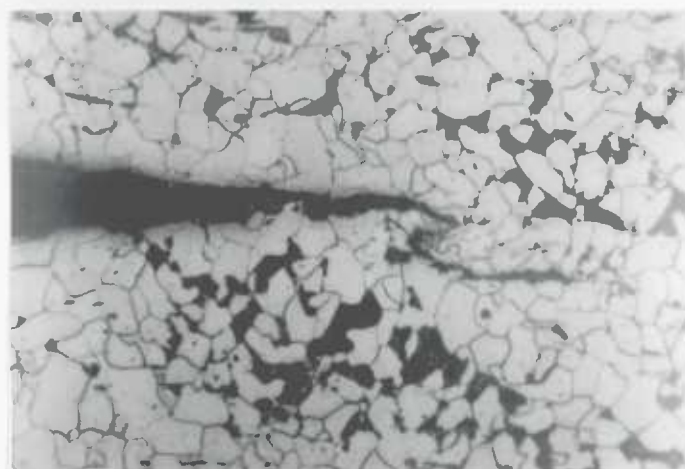
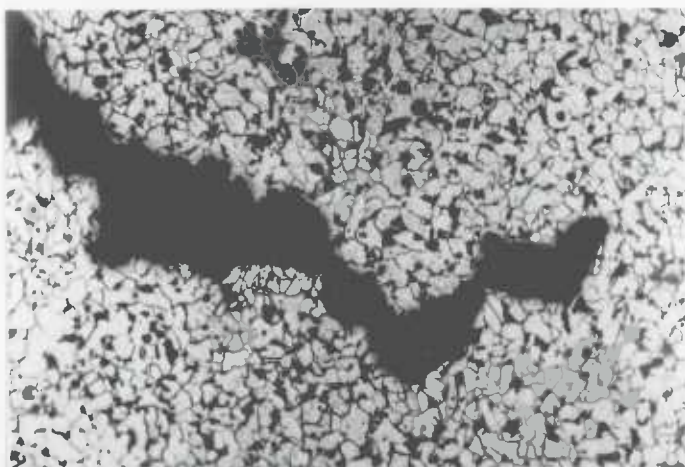
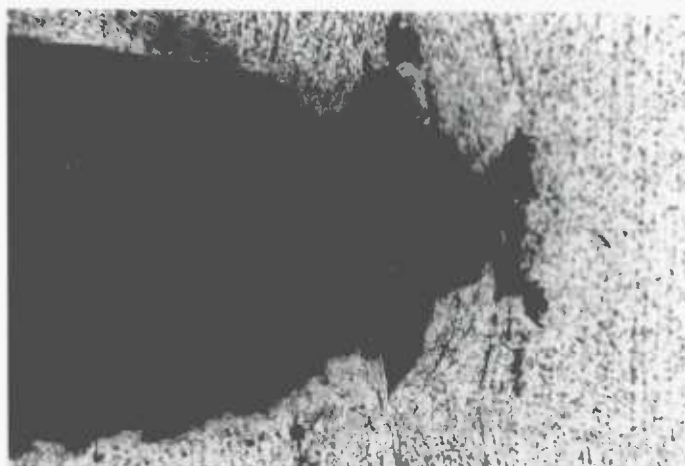


Plate 6.10 Crack tip profiles in En 32B, etched nital

(a) LS orientation x 50

(b) ST orientation x 150

(c) SL orientation x 300

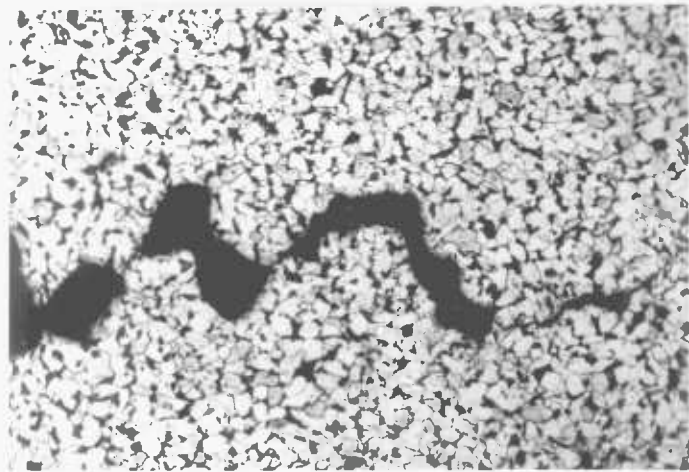
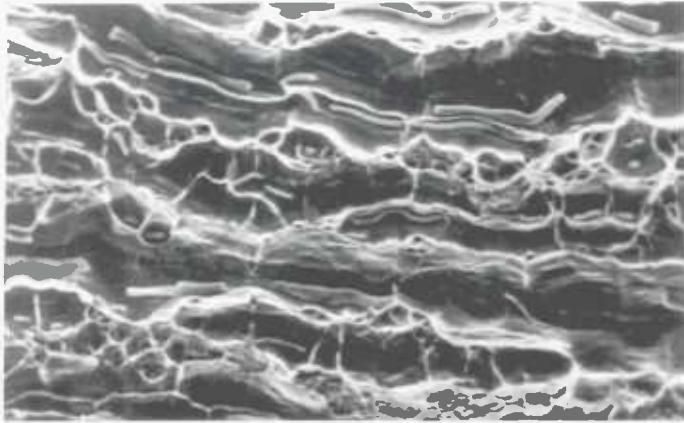


Plate 6.11. Fibrous fracture in En 32B,
prestrained 20%, TS orientation
(a) Fracture surface, SEM, x 950
(b) Crack tip profile, etched nital, x 120

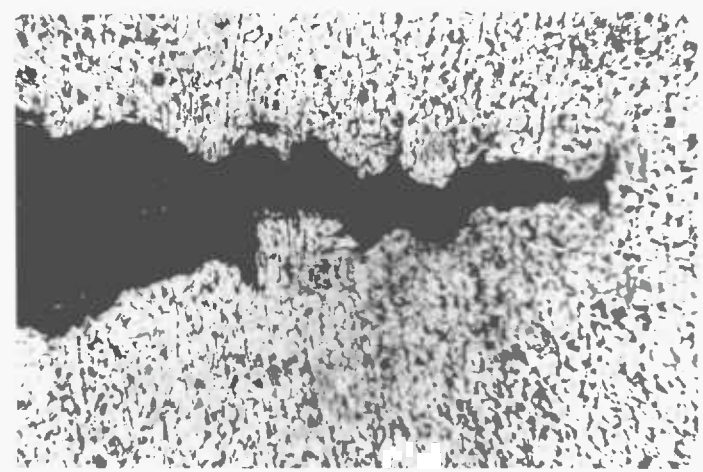
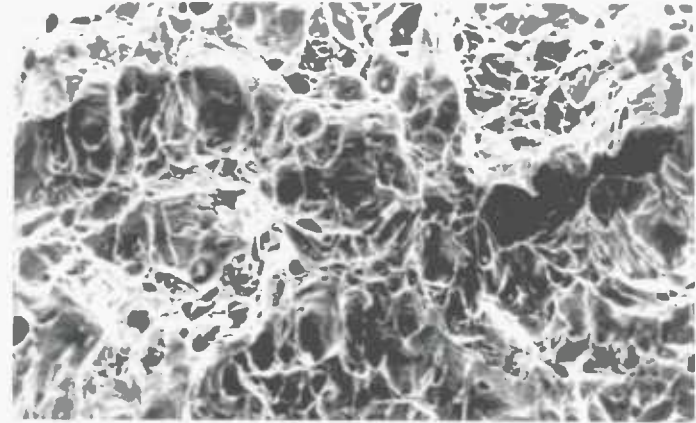


Plate 6.12. Fibrous fracture in En 32B, prestrained 20%,
LS orientation
(a) Fracture surface SEM x 460
(b) Crack tip profile, etched nital, x 60



Plate 6.13. Sulphur print of cross-section of En 32B bar, showing banding of inclusions x 1

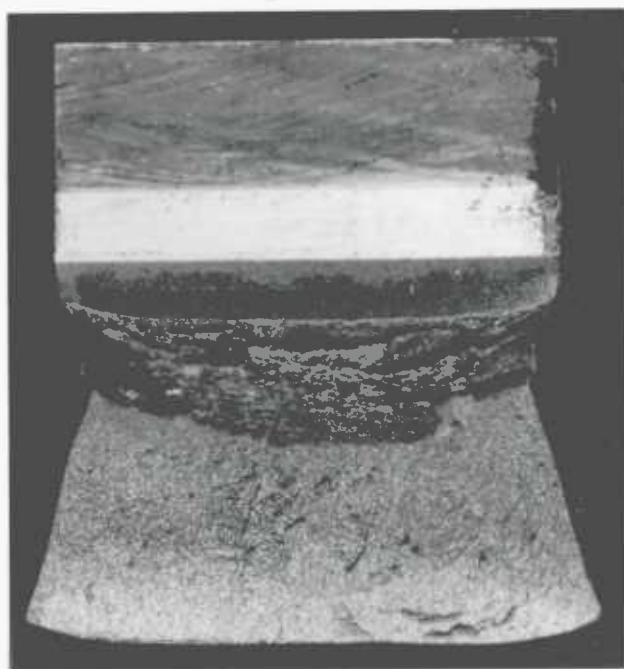


Plate 6.14. Delamination on fibrous fracture surface of En 32B bend bar, LS orientation, x 3

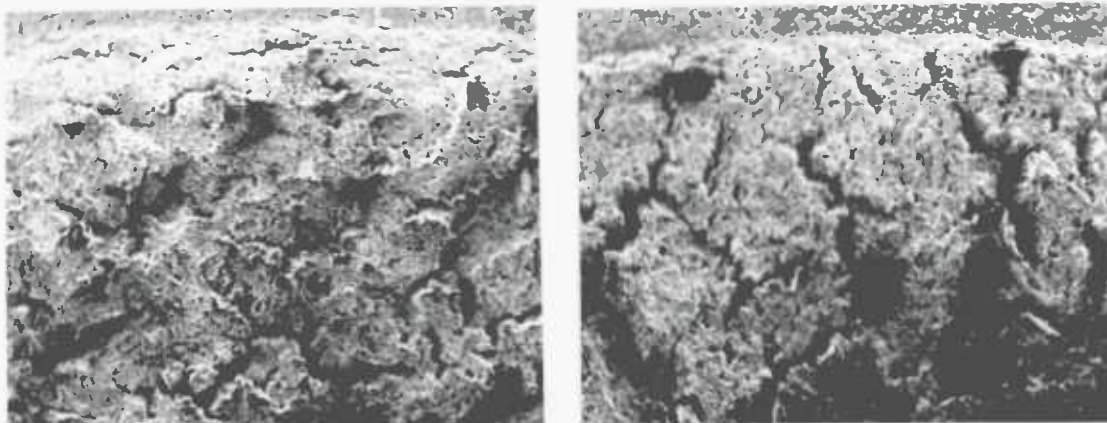


Plate 6.15. Fibrous fracture surfaces of En 32B, LT orientation
SEM, x 25

(a) Tougher specimen

(b) Less tough specimen, showing splits

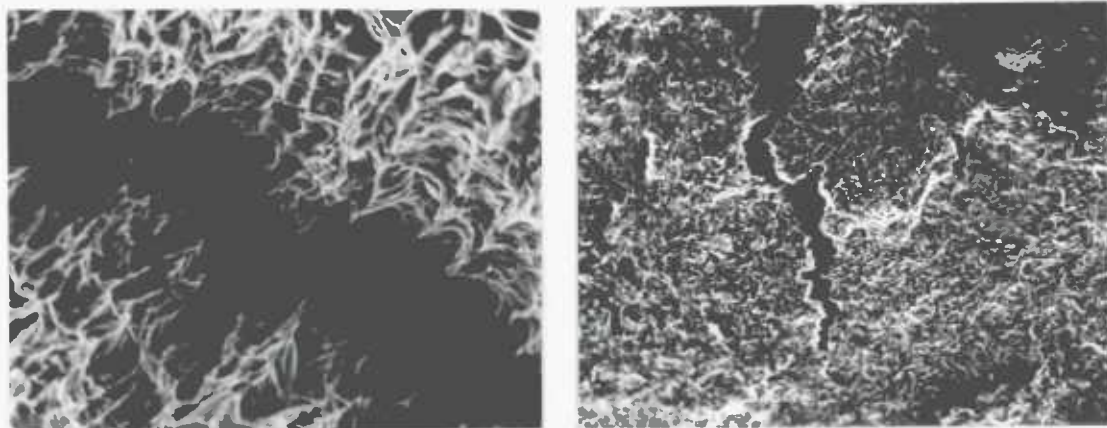


Plate 6.16. Splits in LT orientation, En 32B, SEM

(a) On fibrous fracture surface
x 675

(b) Penetrating ahead of fibrous
fracture, x 65

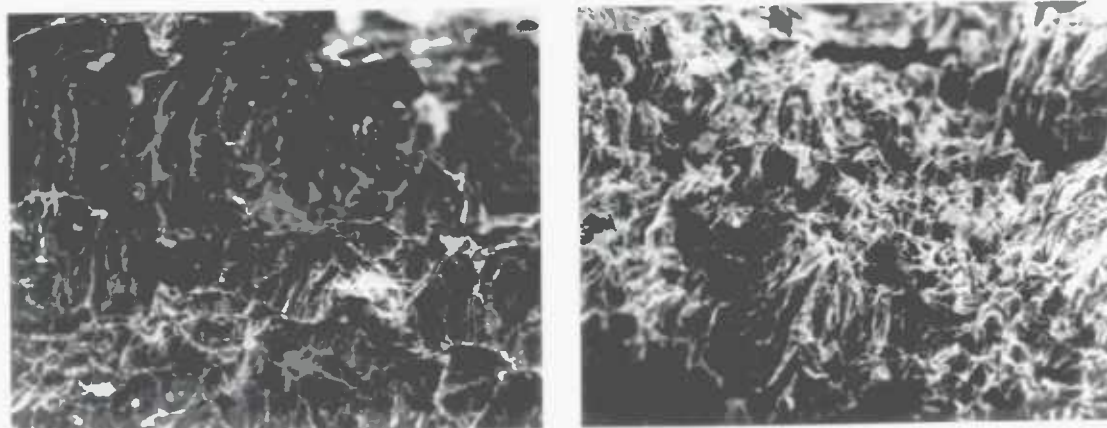


Plate 6.17. "Roughness" of LT fibrous fracture surfaces in En 32B at 70° tilt, S.E.M.

(a) High toughness specimen, x 220

(b) Low toughness specimen, x 270

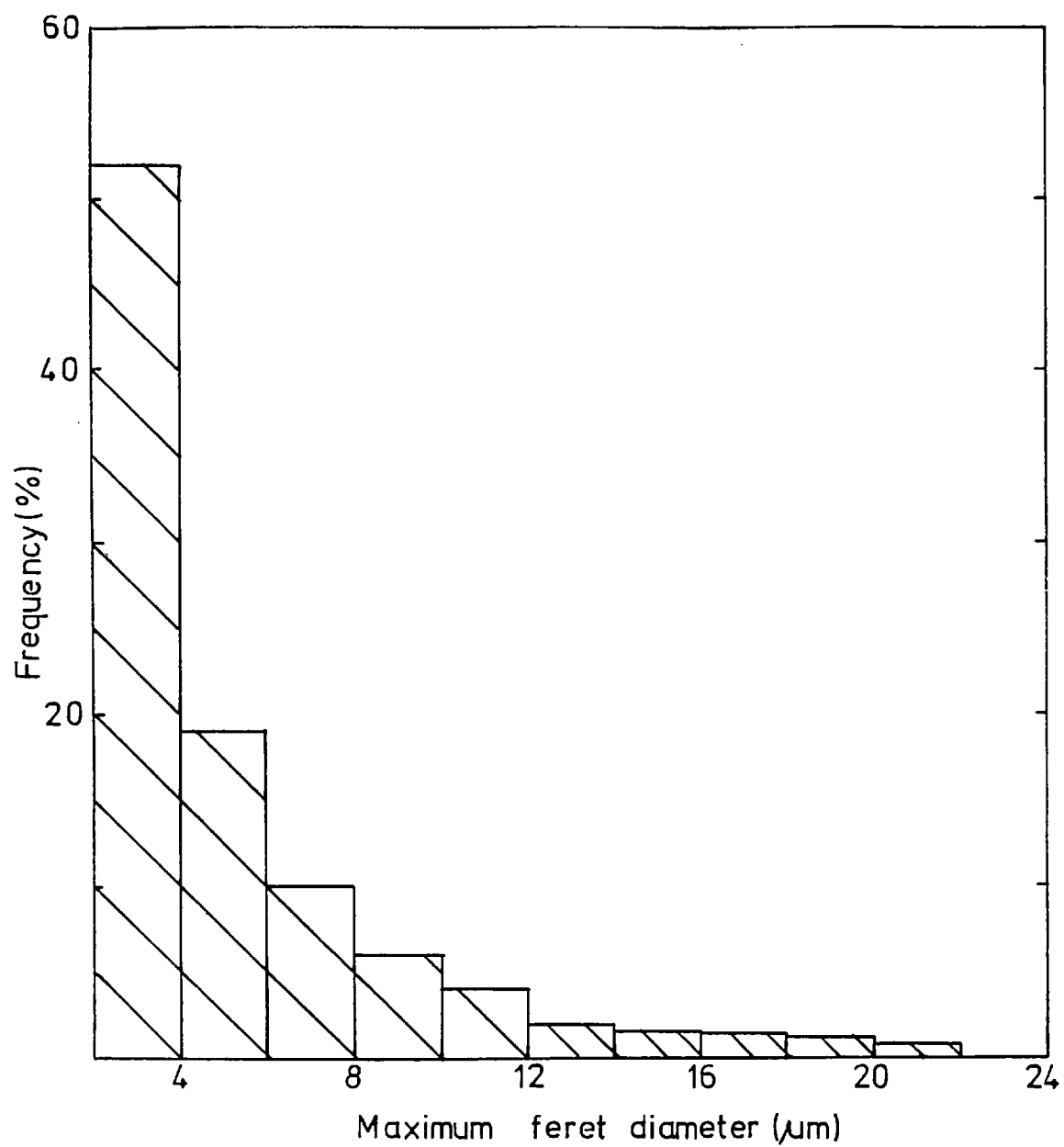


Fig. 6.1 Size distribution of inclusions in En32B.

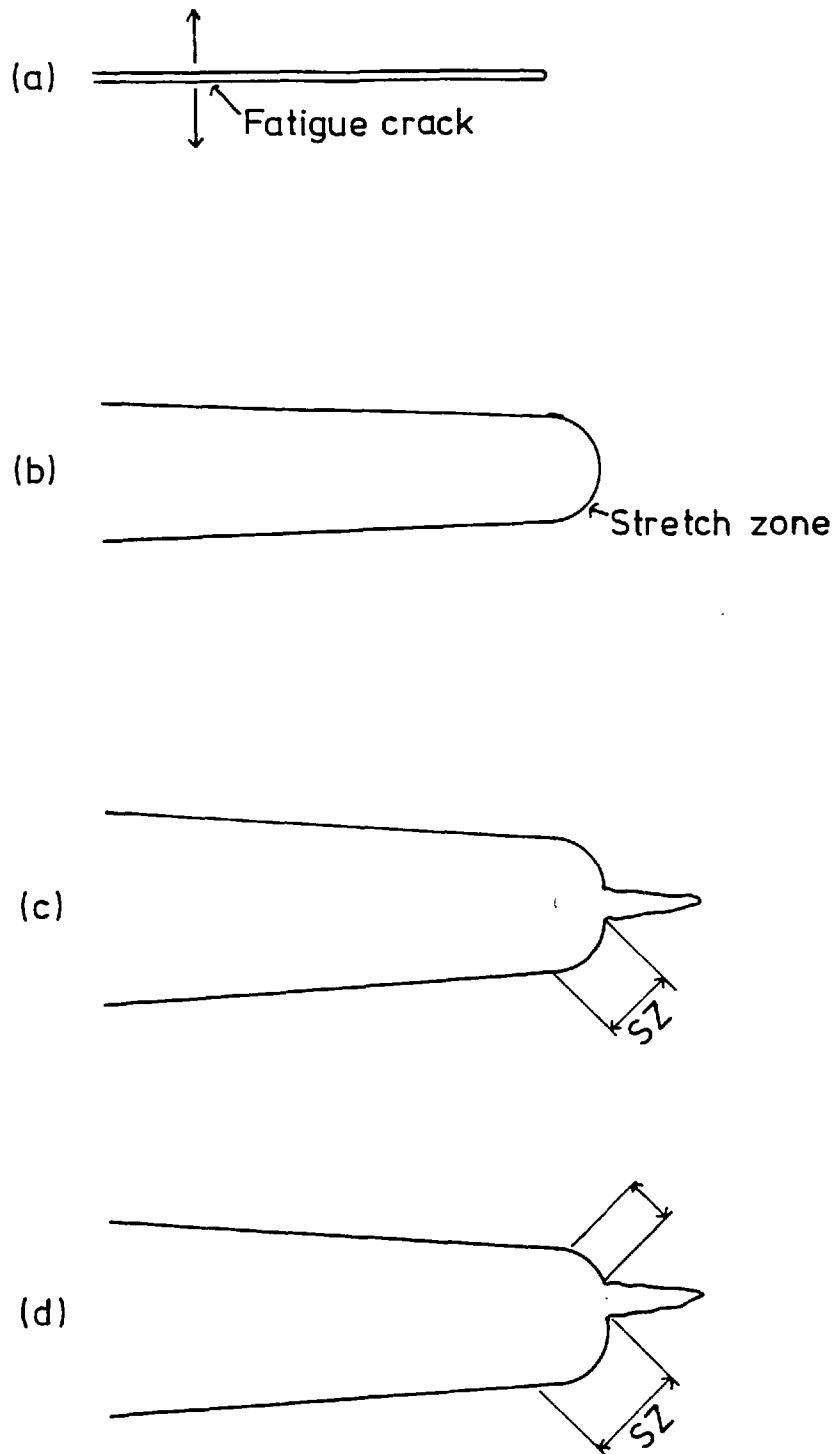


Fig. 6.2 Crack lip blunting and formation of stretch zones.

Chapter Seven

INITIATION OF DUCTILE FRACTURE

7.1 Introduction

During tensile loading of a body containing an initially sharp crack, plastic flow at the tip causes blunting and a small amount of crack advance. At the same time voids form at suitable sites ahead of the crack. With further loading the blunting of the tip increases, and the ligaments separating the voids from the tip neck down. At a critical strain one ruptures and a small amount of crack extension occurs. The crack tip really starts to advance as soon as the load is applied, but the region of crack growth is conventionally taken to be that resulting from microvoid coalescence.

This convention is followed in the work described here. The J-integral and COD were determined at varying amounts of crack growth, Δa , so that by extrapolating back to $\Delta a = 0$ the initiation values, J_i and δ_i , could be deduced. In all cases Δa was measured from the start of ductile rupture, excluding the stretch zone. In practice the stretch zone width was very small ($\approx \delta_i/2$), but it could have a significant effect where the initial slopes of the R-curves were large.

The experimental technique for measuring the R-curves was described in Section 5.6.1. The COD, δ_0 , referred to the position of the original crack tip, was calculated from a version of eqn. (3.7), modified for slow crack growth by assuming that the centre of rotation remains a constant fraction of the remaining ligament ahead of the crack tip.

$$\delta_0 = \frac{r(W-a_0) + \Delta a}{rW + (1-r)(a_0 + \Delta a) + z} \left[V_g - \frac{\beta \sigma_y W}{E'} \right] \quad (7.1)$$

r was taken as 0.45. The J-integral was estimated by means of eqn. (3.19) for three point bend specimens (all with

$$0.4 \leq a_0/W \leq 0.7)$$

$$J_0 = \frac{2 U_{\text{tot}}}{B'(W-a_0)} \quad (3.21)$$

The subscript o shows that the original crack length a_0 was used in the calculation. B' is the breadth of the fracture surface, allowing for side-grooves (if present). The Sumpter formula (eqn. 3.18) was used for CCT specimens, the precise version of which is shown in Appendix I. In both formulae the absorbed energy U was not corrected for the energy due to the uncracked body.

The effect of the following variables on J_i and δ_i was investigated, using the En32B steel:- prestrain, specimen orientation, geometry and configuration. Most specimens had side-grooves. The effect of thickness was investigated using plane-sided, bend test-pieces of En24 cut in the LS orientation. The results for the steels are summarised in Table 7.1. Finally, as a means of assessing the effect of inclusion orientation on initiation of fracture, strains were measured between notch tips and voids, of varying morphology, using the model brass system described in Section 5.6.3.

7.2 Experimental Results for Steels

7.2.1 Effect of Specimen Geometry and Configuration

Smith and Knott (1971) showed that δ_i varies with width of starter notch, and Begley et al. (1977) observed a similar relationship for J_i . Hence test-pieces with sharp fatigue precracks are now standard. Smith and Knott (1971), Fearneough et al. (1971), Ritchie et al. (1975), Green and Knott (1975b) and Garwood (1977) found that δ_i was independent of specimen dimensions, provided a minimum thickness was employed. However, Griffis (1975) found a slight variation of δ_i with specimen thickness. Almost all the published work on COD refers to the bend configuration, because of the difficulty otherwise encountered in relating COD to clip gauge displacement, V_g . Using the rubber

infiltration technique, Robinson (1976) found δ_i to be the same in both bend and CCT, and suggested the following relationship between COD and V_g in the CCT configuration:

$$\delta_o = \frac{1}{2} V_g \quad (7.2)$$

Cowling et al. (1978), however, achieved poor correspondence between δ_i values for different configurations, but they do not specify their means of inferring COD from V_g in tension.

The early work on J was confused because no allowance was made for slow crack growth (Landes and Begley 1972; Begley and Landes 1972). However, in those cases where the R-curve method was used to define initiation unambiguously, J_i has been found to be reasonably independent of geometry and configuration (eg. Landes and Begley, 1974; Griffis and Yoder, 1974; Griffis, 1975; Underwood, 1975; Robinson, 1976; Begley and Landes, 1976; Clarke et al., 1976; Garwood, 1977). Reasonable agreement also was obtained between G_{IC} evaluated from a large specimen and J_i from a small one, by Begley and Landes, (1972), Kobayashi et al. (1973), Liebowitz et al. (1974), Robinson and Tetelman (1974) and Underwood (1975). In fact a large ductile test-piece is likely to experience some slow crack growth before the K_{IC} measurement point, and after allowing for this good agreement was also obtained by Logsdon (1976) and Garwood (1977).

Figs. 7.1 and 7.2 show the effect of thickness on the R-curves for plane-sided bend specimens of En24. Two thicknesses were used, 12 and 24mm, both in the LS orientation with $W = 24$ mm. The R-curves for the thinner specimens showed considerable scatter; this is dealt with further in Section 9.2. However, the initiation values were fairly unambiguous within about $\pm 10\%$. They are in reasonable agreement for the two thickness at $J_i = 90 \text{ kJ/m}^2$, $\delta_i = 0.09 \text{ mm}$ for the 12mm thickness, and $J_i = 110 \text{ kJ/m}^2$, $\delta_i = 0.11 \text{ mm}$ for the 24mm thickness. If they were used to obtain estimates of K_{IC} , the 20% difference would give a

10% difference in K_{IC} .

Similar results were obtained on En32B in the LT orientation. Figs. 7.3 and 7.4 show the R-curves for bend specimens with $W = B = 10\text{mm}$, and either 2 or 3 mm deep side-grooves. The resulting initiation values were $J_i = 96 \text{ kJ/m}^2$ and $\delta_i = 0.13\text{mm}$. Figs. 7.5 and 7.6 show the equivalent curves for larger test-pieces, with $W = B = 24\text{mm}$ and side-groove depths of 7mm, which gave $J_i = 98 \text{ kJ/m}^2$ and $\delta_i = 0.12\text{mm}$. Thus good agreement was obtained with different sized bend specimens.

It is noticeable that despite the scatter in all of the R-curves, the initiation points could be fairly closely defined. The specimen thickness should make no difference to initiation, as long as it is sufficiently large to give plane strain conditions in the centre. The crack then initiates there as a fibrous "thumbnail" in plane sided test-pieces, and the outer, plane stress ligaments only exert their influence as the crack grows. Thus the presence of side-grooves, and their depth and profile, should make no difference to initiation toughness, as found by Green and Knott (1975a) and Garwood (1977).

Centre cracked tension specimens of En32B were also tested in the LT orientation. Slip line field patterns for bend and CCT are very different and so rigid-plastic specimens may not experience the same conditions at the crack tip. Landes and Begley (1972) suggested that work-hardening and crack tip blunting may over-ride the effect of the slip line field in determining J_{IC} .

The specimens tested here had $W = 50\text{mm}$, $B = 24\text{mm}$ and contained 7mm deep side-grooves. The crack length to width ratio, a_0/W , was approximately 0.6-0.7. The difficulties encountered in fatigue precracking and the rounded cracks obtained have been mentioned in Section 5.4.3. The R-curves are shown in Fig. 7.7. J_0 was calculated from the formula given in Appendix I, and δ_0 was estimated from the Robinson formula, eqn. (7.2), which

was derived empirically for the same steel in this orientation and configuration. Both curves show toughness decreasing rapidly at small crack extensions, and if this region were extrapolated to the axis would imply zero toughness at finite Δa . The agreement with the bend results at large crack extensions is, however, better (worse agreement is expected for δ_o , since this is bound to increase faster in bend as the test-piece halves rotate).

As a check on these results, fracture surfaces were examined using the S.E.M. Stretch zones were visible in both bend and tension, as in Plates 6.4(a) and (b), which implies that δ_o and J_o must have finite values at $\Delta a = 0$. In both cases the height of the stretch zone varied between 40 and 120 μm , which suggests that there was in fact little difference between the bend and CCT configurations at initiation. Due to large variations in stretch zone height, no precise estimate of δ_i was possible. However, these results suggest that the trend in the R-curves for $\Delta a < 0.25\text{mm}$ was incorrect. It could perhaps have been caused by the large curvature in the precrack, which tended to straighten out with crack advance, or to small amounts of ductile crack extension occurring while the test-pieces were finally being broken open at low temperature.

7.2.2 Effect of Grain Size

Figs. 7.8 and 7.9 show the variation of J_o and δ_o with Δa for the two different heat treatments of En32B, in two orientations of specimen, LS and TS. Bend specimens were used with $W = B = 10\text{mm}$ and a side-groove depth of 2mm each side (i.e. $SG = 2\text{mm}$, $B' = 6\text{mm}$). The two heat treatments, normalising and slow cooling, resulted in approximate grain sizes of 20 and 40 μm and yield stresses of 290 MN m^{-2} and 220 MN m^{-2} respectively (see Table 5.2). In both orientations no significant effect of heat treatment was discernable on the J_o R-curves (Fig. 7.8), whereas there is a variation in the δ_o R-curves. The values of J_i and δ_i , obtained by extrapolation, are summarised below (and in Table 7.1)

	J_i kJ/m ²	δ_i mm
Normalised En32B, LS orientation	80	0.10
Normalised En32B, TS orientation	80	0.09
Slow Cooled En32B, LS orientation	80	0.17
Slow Cooled En32B, TS orientation	80	0.16

If J_i is taken as the measure of toughness, these results indicate that a small change in matrix flow stress has little effect. The increase in δ_i with decrease in flow stress is consistent with a relationship of the form

$$J_i = M \sigma_y \delta_i$$

where σ_y is either the yield stress, or the flow stress to allow for work-hardening. As shown in Table 7.1, however, the observed increases in δ_i are somewhat larger than would be expected if M were to remain constant. Smith and Knott (1971) found a similar change in δ_i with change in grain size for a free-cutting mild steel.

7.2.3 Effect of Prestrain

Blank specimens of En32B were given nominal compressive prestrains of 10% and 20% in the manner described in Section 5.1. Before considering the toughness of such specimens, the general effects of the prestraining treatment will be discussed. Some barreling occurred, and after machining to size two specimens were sectioned and the variation of microhardness through the thickness was measured (Fig. 7.10). Hardness was lower at the edges, but since all the prestrained specimens had 2mm side-grooves, only the central 6mm need be considered. Here the hardness was more uniform, and using the calibration described earlier (Fig. 5.8(a)) the average true strains in this region were deduced to be 0.13 ± 0.03 and 0.24 ± 0.08 at 10% and 20% nominal strains, respectively.

All tests were in three point bend with $W = B = 10\text{mm}$ and 2mm deep side-grooves. The resulting R-curves in terms of J_0 and δ_0 are shown in Figs. 7.11 and 7.12, and the values of J_i and δ_i tabulated below.

	Prestrain %	J_i kJ/m ²	δ_i mm
LS orientation	0	80	0.10
	10	50	0.05
	20	25	0.02
TS orientation	0	80	0.09
	10	40	0.05
	20	19	0.02

There is a consistent decrease in initiation toughness with increasing prestrain. Similar results have been reported by Green, Smith and Knott (1973), Clayton and Knott (1976) and Chipperfield (1976) for a variety of steels. The relative independence of toughness on yield strength alone, described in the previous section, suggests that the cause is a reduction in the work-hardening capacity, and hence in the fracture strain. If fracture initiates when a critical strain is reached in a small region ahead of the crack tip, prior cold work will allow this strain to be reached at a lower COD or J . The tensile ductilities given in Table 5.2 will not precisely reflect this critical strain, since in uniaxial tension triaxiality of stresses is only attained after the onset of necking. In addition, the amount of cold work is greater in the centre of the bend test-pieces than the tensile data would indicate, because of the non-uniformity of prestrain (Fig. 7.10). The maximum microhardness of approximately 200 at 10% nominal prestrain would imply a true strain, from Fig. 5.8(a), of about 0.15. For 20% nominal prestrain, the equivalent figures in the centre of the bend test-pieces are a microhardness of ≈ 235 , and a true strain in excess of 0.4. Crack extension is likely to take place at the easiest, most work-hardened place first, so it is these strains which are likely to be the important ones rather than the average values across the test-piece thickness.

7.2.4 Effect of Specimen Orientation

The influence of anisotropy on fracture toughness in rolled, low strength steel has been noted by several workers, among them Smith and Knott (1971) and Green and Knott (1976). In particular, very low toughnesses have been observed in the short transverse direction, with the

consequent danger of lamellar tearing in welded structures. The cause is attributed to the elongation of inclusions by rolling. If these have different dimensions in all three directions there are six possible orientations of fracture plane and direction.

The R-curves in terms of J_0 and δ_0 were measured in all six orientations on the En32B, and are presented in Figs. 7.3, 7.4, 7.8, 7.9 and 7.13. Bend specimens with $W = B = 10\text{mm}$ and 2mm deep side-grooves were used. The short transverse test-pieces had extension arms welded on. The resulting J_i and δ_i values appear in Table 7.1. Considerable scatter was found in the R-curves for the LT and ST orientations, but this decreased at small crack extensions, so that fairly unambiguous initiation values could be deduced. The cause of this scatter was discussed in Section 6.5.4.

Ranking the different orientations in order of initiation toughness gives

$$LT > LS \gtrsim TS > ST > TL \approx SL \quad (7.3)$$

A surprising result is the similarity between the LS and TS orientations, in view of the larger differences between these two found by Smith and Knott (1971) and Green and Knott (1976). The close similarity between the two orientations in En32B was confirmed, however, by the results for the prestrained material, Table 7.1. One possible explanation is the difficulty of obtaining J_i or δ_i accurately when the R-curve rises steeply, as in the LS and LT orientations. When measurements are made on several different specimens there is inevitably some experimental scatter, and it is a matter of judgement where to draw the best line. For instance, an error of 0.05mm in Δa would change the initiation values by about 30% in the LS orientation, but by only 10% in the TS orientation.

Assuming that the initiation of ductile fracture requires the attainment of a critical strain over a certain

region, the most likely explanation for the variation of initiation toughness with orientation may be obtained by considering how the inclusion shape influences the strain ahead of the crack tip. To this end, further experiments were carried out on brass compact tension specimens, and the results are described in Section 7.3.

7.2.5 The Relationship between J_i and δ_i

In Section 3.2.1, mention was made of the relationship between G and δ . This can be expressed as

$$G = M \sigma_y \delta \quad (3.5)$$

as suggested by Wells (1969). The results of several analyses indicated that $M \approx 1$ in plane stress and $M \approx 2$ in plane strain (e.g. Thornton, 1970; Rice, 1973). Levy et al. (1971) suggested that, in an analogous manner

$$J = M \sigma_y \delta \quad (7.4)$$

However, the appropriate value to be assigned to M is now less certain. Boyle (1972), Rice (1973) and Hayes and Turner (1974) all deduced that $M \approx 2$ in plane strain. In contrast, Sumpter (1974) calculated that, in plane strain, $M \approx 2$ in bending and $M \approx 1$ for the centre cracked tension configuration. Work-hardening may be taken into account by writing

$$J = M' \sigma_{flow} \delta \quad (7.5)$$

where σ_{flow} is the average of the yield and ultimate strengths.

Values of M and M' at initiation are given in Table 7.1 for all the test data. M' varies between 2.4 and 1.5, being lower in the less tough orientations, in the prestrained material and in the En24, which had a lower work-hardening exponent. These results are in qualitative agreement with those of Robinson (1976) for the same steels. He found that M was about 2.2-2.6 for En32B in both the bend and CCT configurations, and nearer unity for En24 (in a higher

strength condition than used here). He concluded that M depended more on work-hardening than on testing configuration. On this basis the lower M values for the less tough orientations are to be expected, despite the correspondence in yield and ultimate strengths between the different directions. As seen in the next section, voids can augment strain concentration ahead of cracks. Unfavourable inclusion orientation increases this effect, causing greater localisation of strains, whereas work-hardening has the opposite effect of spreading out the deformation. Thus M and M' decrease with decreases in toughness, whether the latter is caused by unfavourable inclusion orientation or by prior cold work.

7.3 Experiments to Study the Strain Distribution between a Crack Tip and an Inclusion.

The experimental results given in Section 7.2.4 showed the effects of inclusion orientation on J_i and δ_i . Consider a weakly-bonded inclusion or a void situated a short distance ahead of a crack tip. It is assumed that, at any instant, the strain distribution due to the crack tip and that due to the void are additive. The latter distribution will depend on inclusion parameters such as length, radius of curvature and separation. It is postulated that crack extension will occur when the average strain in the ligament between the crack tip and the void reaches a critical value. Before the fracture toughness results in different orientations can be interpreted, it is necessary to understand how inclusion parameters affect this average strain, for the case of an elongated inclusion or void ahead of a crack tip in an elastic plastic, work-hardening matrix.

There have been several investigations into the effect of inclusion or voids on stresses and strains, but these are not easily applied to the present situation. The elastic case has been well studied, from Inglis' (1913) solution for the stress at the tip of an elliptical crack to Atkinson's (1971) treatment for a sharp crack and a spherical void or inclusion. A similar analysis by

Tirosh and Tetelman (1976) emphasises the detrimental effects on toughness of voids ahead of a crack tip. However, while the qualitative effects are likely to be similar in elasticity or gross plasticity, quantitative estimates may be very different. Calculations for an array of coplanar cracks with small scale yielding have been made by Smith (1964) using dislocation models. Estimates of the stresses and strains around a rigid, circular inclusion, strongly bonded to the matrix, have been obtained for larger scale plasticity by Orr and Brown (1974), but there have been few investigations into the effects of non-equiaxed flaws. The experimental work by Butcher and Allen (1977) used small remote strains and strongly bonded inclusions. The finite element studies of Harkegaard (1973) indicated the effects of aspect ratio for an elliptical void in a perfectly plastic matrix, but did not show the distribution of strains.

The strain in the ligament between a crack tip and an elongated void may be influenced by several of the void parameters. For instance:-

- (i) The separation, X_0 , between the crack tip and the tip of the void
- (ii) The frontal radius of curvature, ρ , of the void
- (iii) The length, L , of the void in the direction of crack propagation
- (iv) The angle, θ , between the line connecting the tips of the crack and void, and the crack line.

The distance X_0 will depend on volume fraction of inclusion, orientation and direction of linking. The theoretical model of Rice and Johnson (1970), described in Section 4.3, predicts a direct dependence of δ_i on X_0 for spherical voids. The effect of frontal radius of curvature and inclusion length was postulated by Gurland and Plateau (1963) on the basis of a modified elastic solution, and was later incorporated into the model of Gladman et al. (1971).

In order to investigate further these suggestions, the brass compact tension specimens were machined and tested as described in Chapter 5. The notch and slot configurations are shown schematically in Fig. 7.14 and are given in detail in Table 7.2. Unfortunately it was not possible to obtain surface and centre measurements at the same mouth opening in all cases, as specimens II and VI started cracking in the centre, at the root of the notch, at small deformations. Some cracking also occurred in specimens IV and V. In these cases the halves were separated, the grids photographed, and then one half deformed further to allow the surface grid to be measured at the required mouth opening.

7.3.1 Effect of Stress State

Fig. 7.15 shows the variation of tensile strain, ϵ_{yy} , with distance along the crack plane for specimen I, which had no slot. At a mouth opening (M.O.) of 9.1mm the strains in the centre are considerably higher than on the surface. The same pattern was found in all cases, Figs. 7.17-7.20. The distribution of strains, however in the centre and on the surface, appear similar, with surface strains lower by a factor of between 2 and 6. The difference in strains between the two stress states is most marked in specimen II, Fig. 7.16, where the slot shape results in greater strain concentrations, and least in specimen III, Fig. 7.17, where the effect of the slot shape is to reduce the local strains. This may be explained by the fact that, with the slot perpendicular to the crack plane, as in specimens III and VIII, considerable contraction in the X direction of the intervening ligament could occur. (This was allowed for on the graphs, the X co-ordinate representing the final position of the grid intersections). As a result, plane strain conditions could not be achieved even in the centre, so the difference between centre and surface strains was lower.

The general similarity in strain distribution at the surface and in the centre is important, because it means

that trends in the variation of strain pattern with different void shapes in plane strain may be determined from plane stress measurements. The experiments performed here may be repeated and extended at lower cost and with greater simplicity by using thin sheets scribed with surface lines. The effects of different configurations, such as centre cracks or double edge cracks, could also be investigated. It is not certain of course that plane strain was attained in the centre of the double specimens. However, the thickness required for a valid COD or J_{IC} test in this material should be around 10mm. The actual thickness of 32mm should be sufficient, even allowing for the blunt notch, which would allow a larger plastic zone size and hence greater relaxation of through-thickness stresses.

The greater magnitude of the strains in the centre of the specimens may be explained in terms of the plastic zone size. This is larger in plane stress than in plane strain, by a factor of about 3. The boundary displacements are the same in each case, so the necessary plastic displacements should be more diffusely spread in the surface, plane stress region.

Rice and Johnson's (1970) theoretical model predicts the strain distribution, scaled in terms of the tip opening. As a comparison, these predictions are also shown on Fig. 7.15 and were calculated by using the measured COD for specimen I of 2.5mm at M.O. = 9.1mm. The measured strain distribution shows a "weaker singularity". Of course in a real material failure at the crack tip would occur before the strains reached the levels predicted. The discrepancy may be due to work-hardening of the brass, which would tend to distribute the strains more evenly, and also to the effect of the blunt notch.

7.3.2 Effect of Void Spacing and Morphology

The effects of the various void parameters on the strains in the intervening ligament may be divided into three categories.

i) The ligament width, X_0 .

The effect of decreasing X_0 may be seen by comparing specimens II and VI (Fig. 7.16), and IV and V (Figs. 7.18 and 7.19). The former two had elongated slots, and decreasing X_0 from 7 to 4mm increased the surface strain from 0.07 to 0.17 (averaged over the ligament), at M.O. = 5.2mm. The latter two had circular voids, and both suffered some cracking in the centre, rendering the plane strain values at the notch root inaccurate. Again, however, decreasing X_0 from 7 to 4mm raised the average strain at M.O. $\approx$ 14mm from 0.13 to 0.32. The average strain in the un-voided case (specimen I) were 0.12 and 0.17 over 7mm and 4mm respectively. For specimen V the average surface strain was little affected by the presence of the hole, which was possibly too small and remote to influence the strain field from the notch. However, the overall trend, of increasing strain with decreasing ligament, is clear, and agrees with the observations of Tirosh and Tetelman (1976).

ii) The void length, L .

This had a large effect on strain. Specimens II and V (Figs. 7.20 and 7.19) show that, at M.O. $\approx$ 14.5, increasing L by a factor of 3.9 while maintaining the same frontal radius of curvature changed the average surface strain from 0.13 to 0.29, an increase by a factor of 2.2. The same trend is shown by specimens IV and VI (Figs. 7.18 and 7.16), if it assumed that the strains scale with mouth opening, and caused the latter specimen to start cracking at M.O. = 5.2mm. The Inglis (1913) solution for the stress concentration caused by an elliptical void in an elastic material gives the increase in stress as proportional to $(L/\rho)^{1/2}$, where ρ is the frontal radius of curvature. The variation of plastic strains with L found here is similar, although the results indicate, if anything, a slightly stronger dependence.

iii) Frontal radius of curvature, ρ .

The elastic solution predicts that the increase in stress or strain varies with $(\rho)^{-1/2}$. Comparison of specimens V and VII, which had a factor of 2 difference in

ρ , with other dimensions the same, show that a small increase in strain, from 0.13 to 0.17, results (Fig. 7.19). This is reasonably consistent with the elastic solution. However, specimen VIII, where $\rho \approx \infty$, had a higher average strain of 0.18 (Fig. 7.20), whereas the elastic solution would predict no increase in strain above that for the un-voided body. The unexpectedly low strain in specimen V has already been mentioned, and it is possible that these results were in error. It is also possible that strains were elevated in specimen VIII as a result of the large amount of material removed near the notch. No firm conclusion concerning the effect of frontal radius of curvature of the void is possible, but it is likely that the ligament strain has a weak, inverse dependence on that factor.

This model has many limitations. The errors in strain measurement could be as high as 20% in the centre, as noted in Section 5.6.3. The choice of notch and slot dimensions was arbitrary, and may not be a good representation of a void ahead of a sharp crack tip. Variation in material properties would increase the errors. In addition linkage by shear at an angle to the crack plane was not considered, due to the difficulties of measuring shear strains. Nevertheless, some reasonable conclusions may be drawn.

The dependence of ligament strain on void parameters may be written,

$$\epsilon' = \epsilon(1 + F) \quad (7.6)$$

where ϵ' is the average true strain in the intervening ligament, ϵ is the true strain in the absence of the void, averaged over the same distance, and F is the "strain intensification factor" of the void. In general

$$F = F(\rho, \theta, X_0, L)$$

If it is assumed that F varies with each of these factors separately, then

$$F = f_1(\rho) \cdot f_2(\theta) \cdot f_3(X_0) \cdot f_4(L)$$

and eqn. (7.6) becomes

$$\epsilon'/\epsilon - 1 = f_1 f_2 f_3 f_4 \quad (7.7)$$

The left hand side of eqn. (7.7) could, in theory, be calculated for the various notch/void configurations studied, thus enabling the precise form of the functions f_1 - f_4 to be deduced. In practice, however, this proved impossible. Too few results were obtained at the same mouth opening, and the errors incurred were too great.

Therefore, certain assumptions will be made which are reasonably consistent with the experimental results. Firstly, it will be assumed that the plastic strain intensification factor varies inversely as the ligament width,

$$f_3(X_0) = 1/X_0$$

Secondly, that it varies directly as the square root of the projected void length in the direction of crack propagation (i.e. normal to the applied tensile stress)

$$f_4(L) = \sqrt{L}$$

The experimental results show that these assumptions are reasonable, although both may be a slight underestimate of the strength of the dependence on the void parameters. Equation (7.6) then becomes

$$\epsilon' = \epsilon \left(1 + \kappa \frac{\sqrt{L}}{X_0} \cdot f_1 \cdot f_2 \right) \quad (7.8)$$

where κ is a dimensional constant, and $f_1(\rho)$ has a weak, inverse dependence on ρ .

Since κ has dimensions, quantitative estimates which might be deduced from the model could not be applied to the real situation, where the inclusion dimensions are several orders of magnitude smaller, without the use of some scaling factor. However, for materials where the

volume fraction of inclusions is large enough for them to exert a strong influence on the crack tip strains, the first bracketed term of eqn. (7.8) may be ignored. The dependence of the average strain on inclusion parameters then becomes approximately

$$\epsilon' \propto \frac{\sqrt{L}}{\chi_0} \cdot f_1(\rho) \cdot f_2(\theta) \quad (7.9)$$

7.4 A Model for Crack Initiation in Ductile Materials

It is assumed that for crack extension to begin, a critical strain ϵ_c must be reached in a region ahead of the crack tip. This critical strain is a function of the work-hardening and flow properties of the matrix. The region over which this strain must be attained is determined by the spacing of void forming particles, and will be taken as the ligament between the crack tip and adjacent void, on average.

The initiation point, as defined by the extrapolation of the R-curves to $\Delta a = 0$, does not represent any crack advance (apart from the formation of a stretch zone), but is reached at the instant before the crack moves, when conditions ahead of the tip allow it to do so. Initiation is a two dimensional process, as illustrated by Fig. 7.21. The initial crack front is a straight line on the crack plane, and as it blunts strains are developed between it and nearby inclusions. The crack front remains linear, however. Presumably those points on the line normal to the crack front and connecting the latter to the incipient voids experience the greatest strains, and thus these lines define the points on the crack front which initiate first. There will be no tendency for points on the crack front to extend in directions other than normal to the front, in plane strain. Initiation may thus be represented in two dimensions on a plane normal to the crack line, as in Fig. 7.22. The inclusions will not in general lie in the crack plane, but will subtend angles θ with this plane at the crack tip.

Considering the ligament between the crack tip and an inclusion, Fig. 7.22, the probability of fracture, P_i , will be proportional to the average strain ϵ' in the ligament, inversely proportional to the critical strain ϵ_c , and also vary with the angle of joining, θ . The average strain, ϵ' , will depend on the COD, δ , and on the strain intensification factor, F , due to the inclusion. Assuming, therefore, that $\epsilon' \propto \delta(1+F)$, the probability of fracture becomes

$$P_i \propto \frac{\delta}{\epsilon_c} (1 + F) \cdot f(\theta) \quad (7.10)$$

where $f(\theta)$ is some function of the angle of joining. When $P_i = 1$, $\delta = \delta_i$ and fracture initiates. Rearranging eqn. (7.10)

$$\delta_i \propto \frac{\epsilon_c}{(1+F) \cdot f(\theta)} \quad (7.11)$$

which becomes approximately $\delta_i \propto \frac{\epsilon_c}{F \cdot f}$ if the inclusions are assumed to exert a large influence on the fracture behaviour. Using the expression for F in the previous section,

$$\delta_i \approx \kappa \frac{X_0}{\sqrt{L \cdot f_1(\rho)} \cdot f'(\theta)} \quad (7.12)$$

where κ is a factor which varies directly with the fracture strain, and $f'(\theta)$ is another function of θ . Following Baker (1971), it will be assumed that $f' = \cos\theta$, since fracture is most likely to occur in the straight ahead direction, ($\theta = 0$), and least likely in the direction of maximum tensile stress ($\theta = \pi/2$).

The interpretation of X_0 is not obvious. Fig. 7.23 shows typical inclusion patterns around the crack tip in the three orientation groups (a) LS, LT, (b) TS, ST and (c) TL, SL. On the plane considered the centre-to-centre spacing of inclusions in groups (a) and (c) is the same, whereas it is much less in group (b) since the cross-sectional area of each inclusion is smaller. However, linkage between the

crack tip and the inclusion may occur in a variety of directions, and for elongated inclusion the distance X_0 will depend on this direction.

To describe this situation, Baker (1971) defined the inter-inclusion distance in any direction as the reciprocal of the number of inclusions intersecting unit length of a zone of cross-sectional area equal to the projected area of the inclusion in that direction. The details are set out in Appendix III, but the principles and results will be summarised here.

Assuming a random dispersion of uniform ellipsoidal inclusions, the inter-inclusion distance D in two dimensions becomes

$$D = \frac{lm}{3 V_f} (l^2 \sin^2 \alpha + m^2 \cos^2 \alpha)^{-\frac{1}{2}}$$

where V_f is the volume fraction of inclusions, l and m are the major and minor semi-axes of inclusion, and α is the angle the major axis makes with the direction considered. This defines an ellipse, of the same shape as the inclusion but with axes $1/3 V_f$ times as big. For a volume fraction of 0.1%, the spacing is about 330 times the inclusion semi-axis in the direction chosen. Optical examination of plane polished surfaces indicates this to be a reasonable result (e.g. Plate 5.1).

Baker also postulated that the ease of inclusion linking was i) inversely proportional to the separation
ii) directly proportional to $\cos \theta$.

As shown in Appendix III, this defined a circle of equal probability of joining immediately ahead of the crack tip. As the circle was expanded (with increasing COD) it would meet the lines defining the inter-inclusion distance (centred at the crack tip) and linkage would occur. The linkage distance and direction are then given by the position, relative to the crack tip, of the point of tangency of the two loci.

Baker's postulates concerning the ease of linkage are consistent with those used here. There is, however, one difference to be taken into account when applying it to initiation. Baker was considering linkage between inclusions, when it was assumed that there was one already positioned at the crack tip. However, the fatigue crack tip, prior to extension, need not be positioned at an inclusion, but is likely to be somewhere in between. It will be assumed that on average it is half-way between inclusions - i.e. $X_0 = D/2$. The size of the crack tip, however, is assumed to be equivalent to the inclusion size. This is likely to be an underestimate, since the tip in fact blunts to a size several times larger.

This treatment, therefore, accounts for the function $X_0/f'(\theta)$. As shown in Appendix III, it predicts very similar joining distances which are proportional to one of the inclusion minor axes in all six orientations. The angle of joining, θ , is also predicted as 0 in the LT, LS and TS orientation, 45° in the TL and SL orientations, and between 0 and 45° in the ST orientation. These directions agree well with those observed, except perhaps in the TS and ST orientations, where "zig-zag" fracture was normally found.

The other inclusion parameter of interest, the projected inclusion length L , is given by

$$L = 2(l^2 \sin^2 \alpha + m^2 \cos^2 \alpha)^{\frac{1}{2}}$$

as a general expression. In fact, for the six orientations considered, $\alpha = 0$ or 90° , so that L becomes $2l$, $2m$ or $2n$, depending on orientation.

Using eqn. (7.12), the predicted initiation CODs are evaluated in Table III.1 in Appendix III. Substituting the typical values for the inclusion parameters suggested in Section 6.3.2 ($l = 6\mu\text{m}$, $m = 0.6\mu\text{m}$, $n = 0.4\mu\text{m}$) enables ratios of δ_i to be calculated in the different orientations, and these are compared with the observed ratios, taken from

Table 7.1.

	LT	LS	TS	ST	TL	SL
δ_i predicted	1	0.82	0.82	0.98	0.62	0.42
δ_i observed	1	0.77	0.69	0.46	0.31	0.31

The predictions give an indication of the trend in δ_i in the various orientations, except in the ST orientation, where the prediction is much too high. Part of the reason for this must lie with the approximate nature of the assumptions made in the original strain intensification model. In particular, the contribution of frontal radius of curvature to strain intensification could not be determined quantitatively, and so has been ignored. Although it appeared to have only a small effect on the brass model, for the actual inclusions the range of ρ in the different orientations was large - several orders of magnitude - so it is probable that in reality it would be significant. This could have the effect of reducing the predicted CODs in the transverse and short transverse orientations, giving better agreement. Another drawback of the model is its inability to predict the correct angle of joining in the transverse orientation, when the two minor inclusion axes are of the same order (as was the case here). Zig-zag fracture was found in both cases (to a greater extent in the ST orientation), but the model gives relatively small angles of joining. The localisation of strain would be expected to reduce δ_i in the transverse fractures.

The effect of prestrain may be deduced from the model also. The factor k in eqn. (7.12) varies directly with the critical fracture strain, and prior cold work will decrease its magnitude. Hence δ_i will decrease also. However, quantitative predictions will be inaccurate since the increased tendency towards localisation of strains with reduction in work-hardening is not built into the model.

Another error occasioned by this treatment is the necessity to assume a dispersion of completely uniform inclusions. In reality the dispersion is far from uniform,

and voids will tend to form first at the larger inclusions. This may have the effect of "destabilising" the deformation ahead of the crack tip and promoting strain localisation. In addition, it was shown in Section 6.3.2 that measurements on plane surfaces always underestimate inclusion length. This error would cause high values of COD to be predicted in the SL and TL orientations.

7.5 Summary

The crack opening displacement and the J-integral are used to characterise toughness in ductile steels. Initiation values, δ_i and J_i , were derived by the R-curve method. The results confirmed the postulated invariance of δ_i and J_i with dimensions of specimen.

The mechanism of fracture was microvoid coalescence, the voids being formed principally at non-metallic inclusions. Work-hardening capacity and anisotropy of inclusions influenced the toughness at initiation. It is argued that initiation is a two-dimensional process, and occurs on the attainment of a critical strain over some characteristic distance ahead of the crack tip. Based on the results of an experimental model, the effect of inclusion parameters on initiation is explained in terms of the strain intensification they cause in the characteristic region. The model predicts in a satisfactory manner the observed trends in δ_i with specimen orientation and prior cold work.

Table 7.1 J_i , δ_i , M and M' for three point bend specimens.
Estimated error in J_i and δ_i is $\pm 10\%$.

Material, Condition and orientation	Geometry (1)	J_i kJ/m ²	δ_i mm	M (2)	M' (3)
<u>En24</u>	<u>Plane-sided</u>				
Quenched and } tempered. LS }	W=B=24mm	110	0.11	1.8	1.7
	W=24, B=12mm	90	0.09	1.8	1.7
<u>En32B</u>	<u>Side-grooved</u>				
Normalised LT	W=B=24mm	98	0.12	2.8	2.2
LT	(1)	96	0.13	2.5	2.0
LS		80	0.10	2.8	2.2
TS		80	0.09	3.1	2.4
ST		~42	~0.06	2.4	1.9
TL		25	0.04	2.2	1.7
SL		25	0.04	2.2	1.7
Slow-cooled LS		80	0.17	2.1	1.5
TS		80	0.16	2.2	1.6
Prestrained } LS		50	0.05	2.1	2.0
10% } TS		40	0.05	1.6	1.5
Prestrained } LS		25	0.02	2.1	2.0
20% } TS		19	0.02	1.6	1.5

(1) Unless otherwise stated, $W=B=10\text{mm}$

(2) From $J_i = M \sigma_y \delta_i$

(3) From $J_i = M' \sigma_{\text{flow}} \delta_i$, $\sigma_{\text{flow}} = (\sigma_y + \sigma_{\text{UTS}})/2$

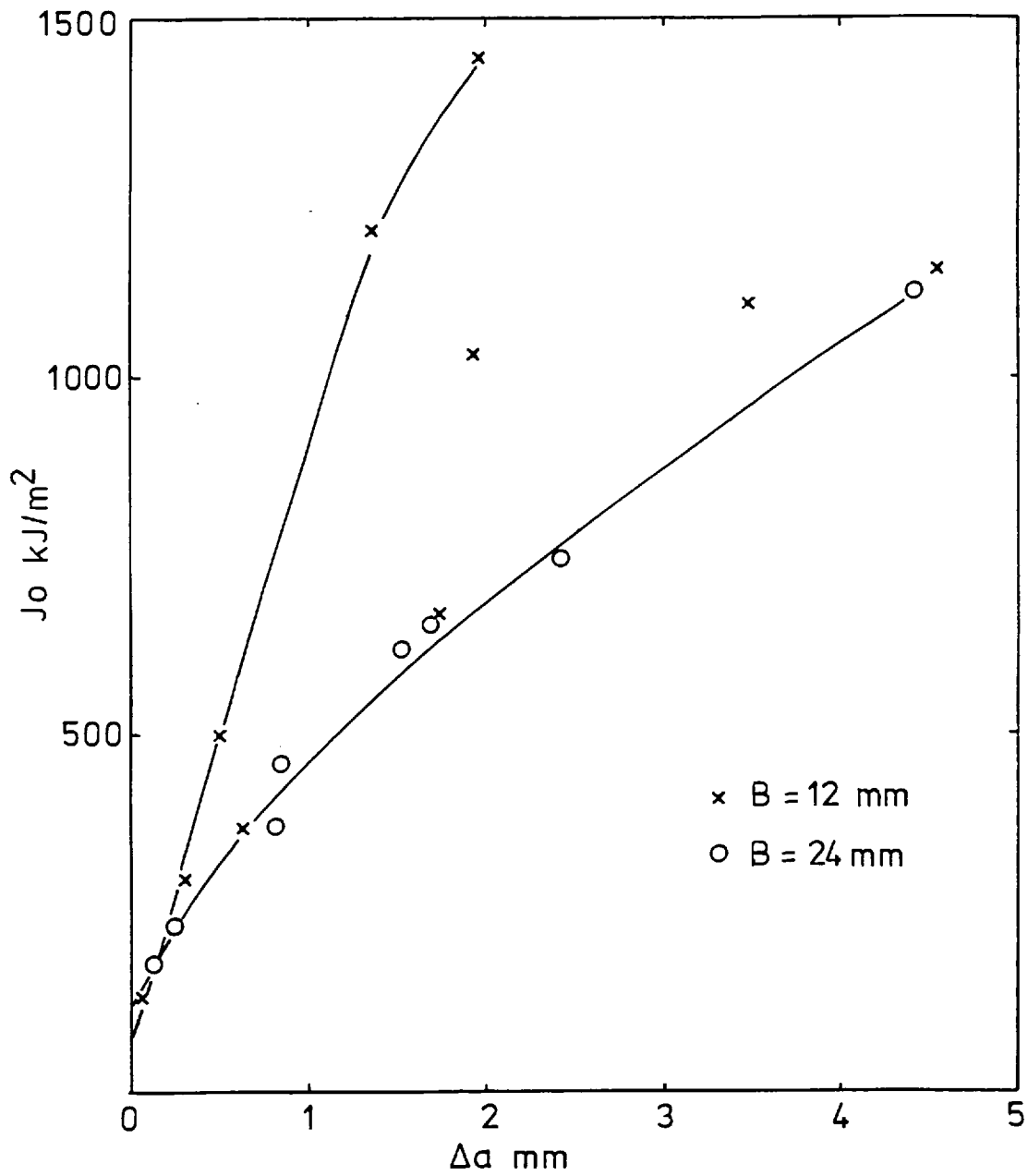


Fig. 7.1 Variation of J_0 with Δa for two thicknesses of En24, in bending.

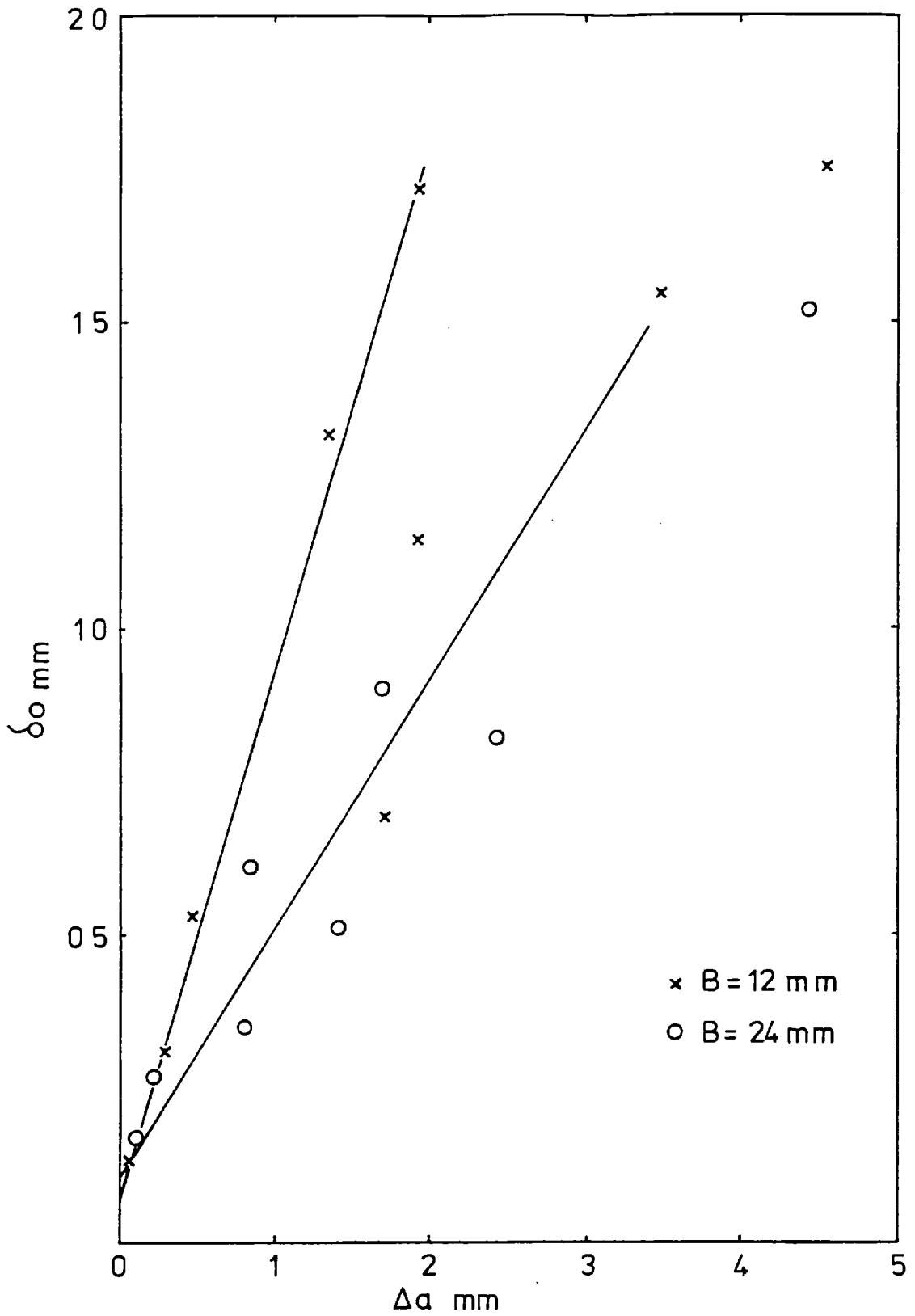


Fig. 7.2 Variation of δ_0 with Δa for two thicknesses of En24, in bending.

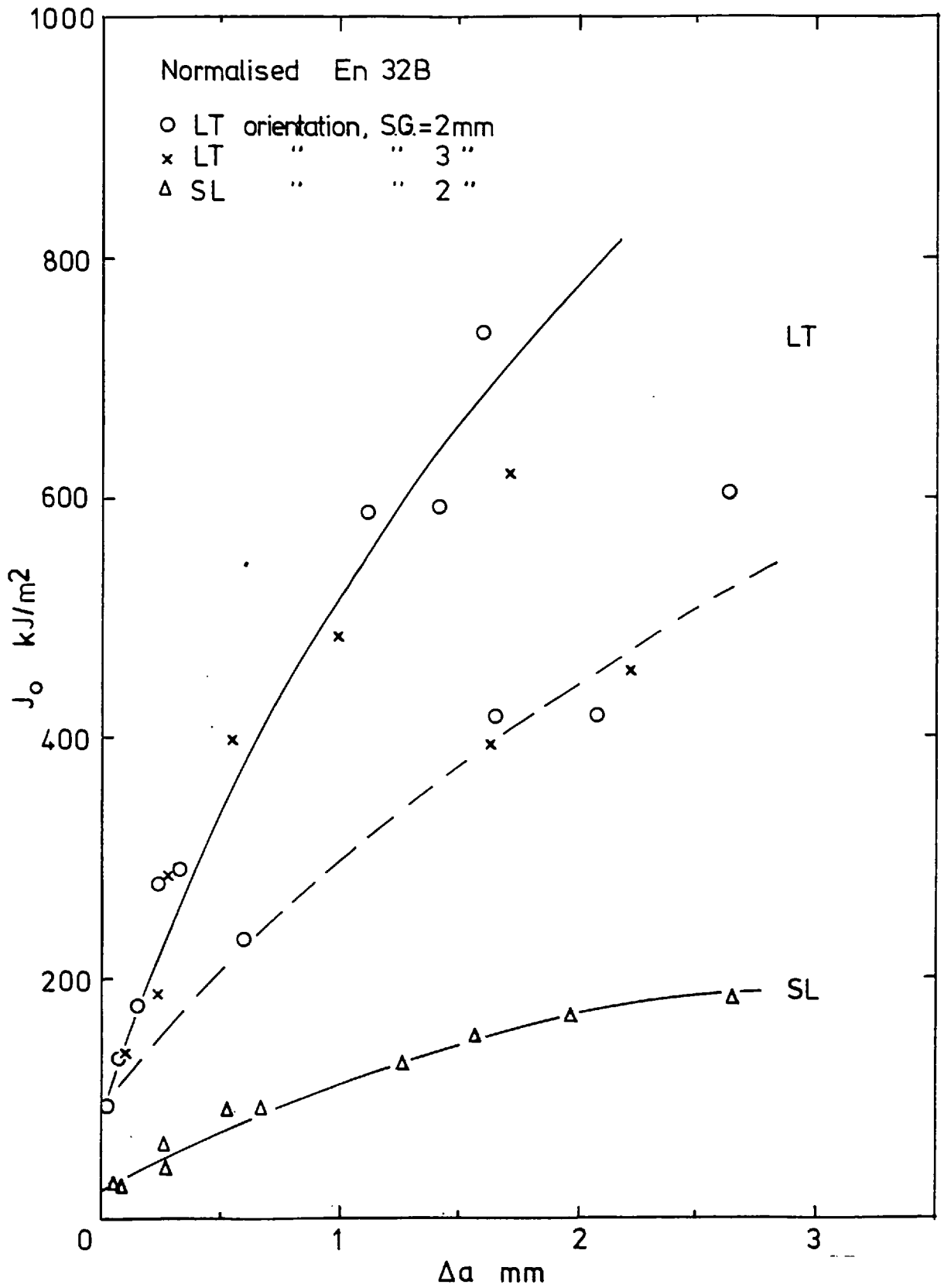


Fig. 7.3 Variation of J_0 with Δa for En32B, in bending,
 $W = B = 10\text{mm}$.

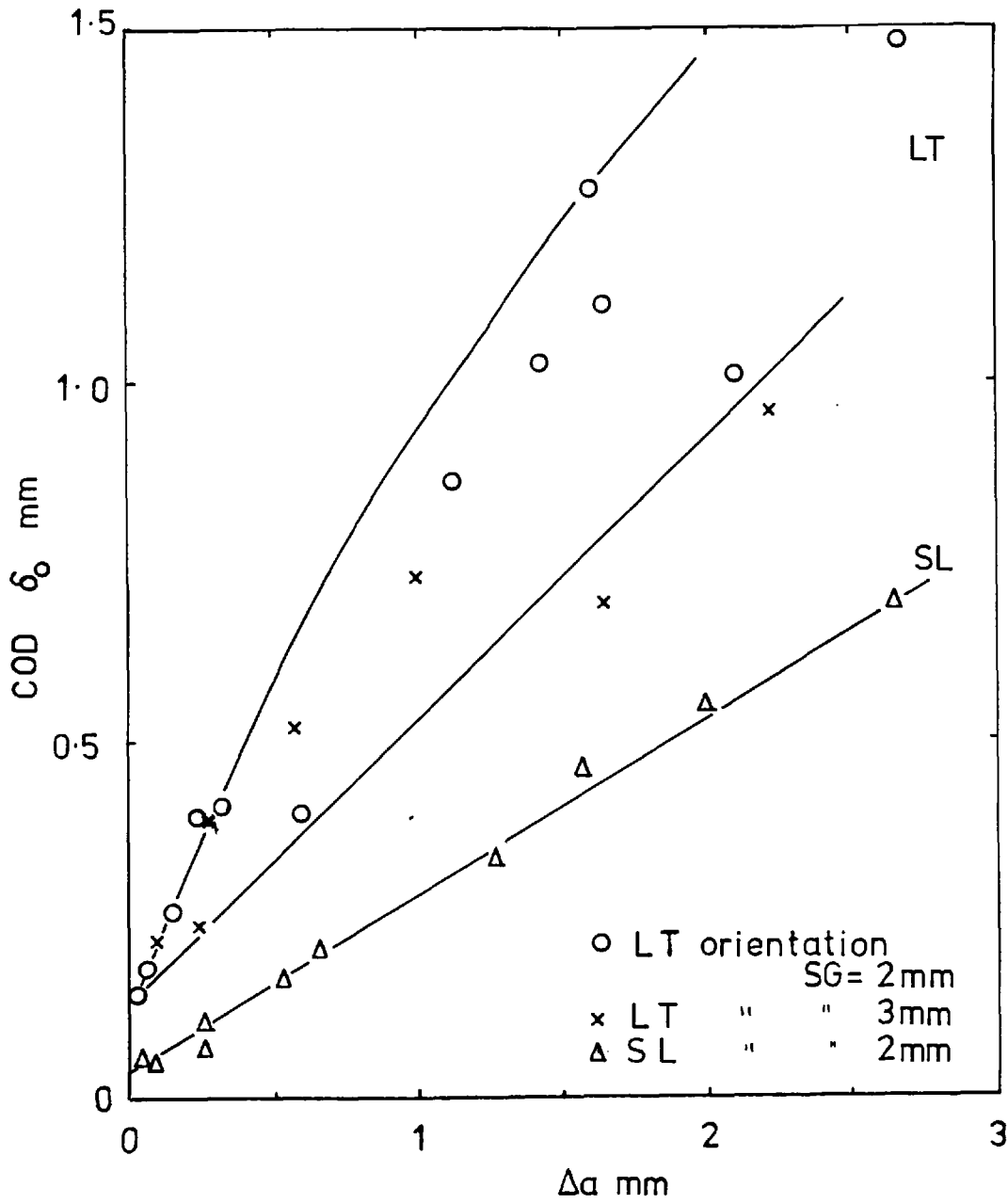


Fig. 7.4 Variation of δ_0 with Δa for En32B, in bending, $W = B = 10\text{mm}$.

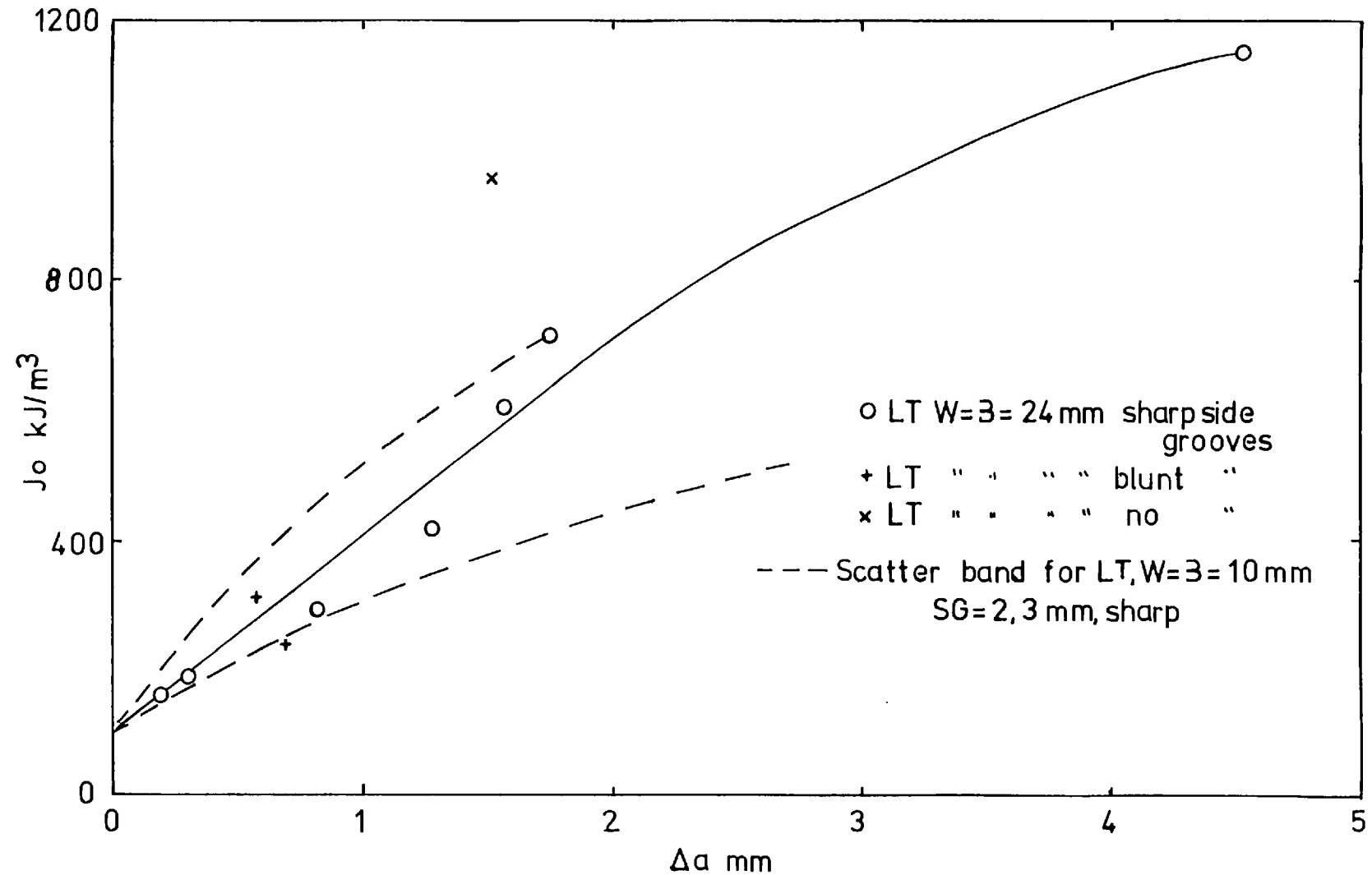


Fig. 7.5 Effect of thickness on the J_0 R-curve for En32B, in bending.

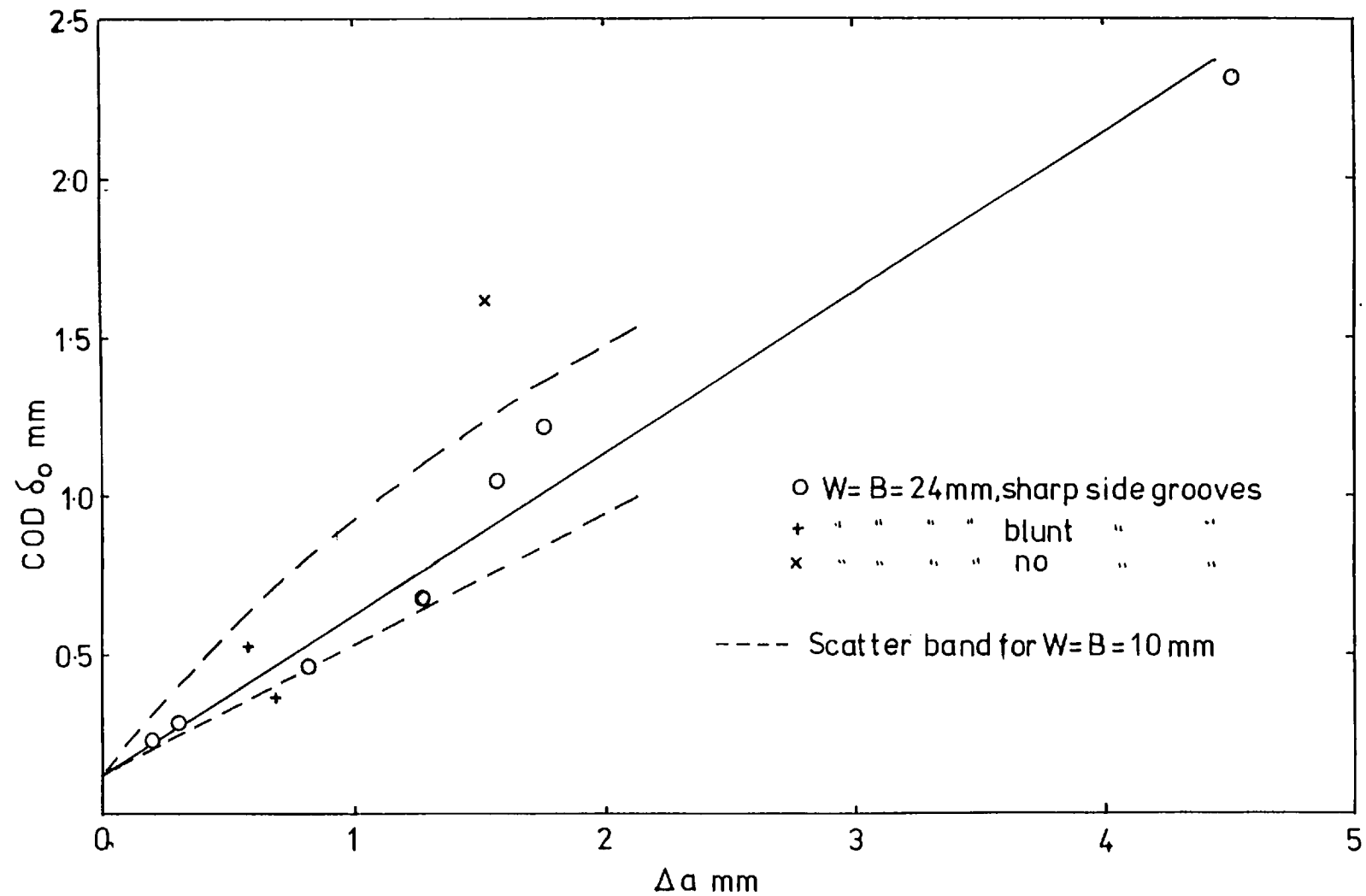


Fig. 7.6 Effect of thickness on the δ_0 R-curve for En32B, in bending.

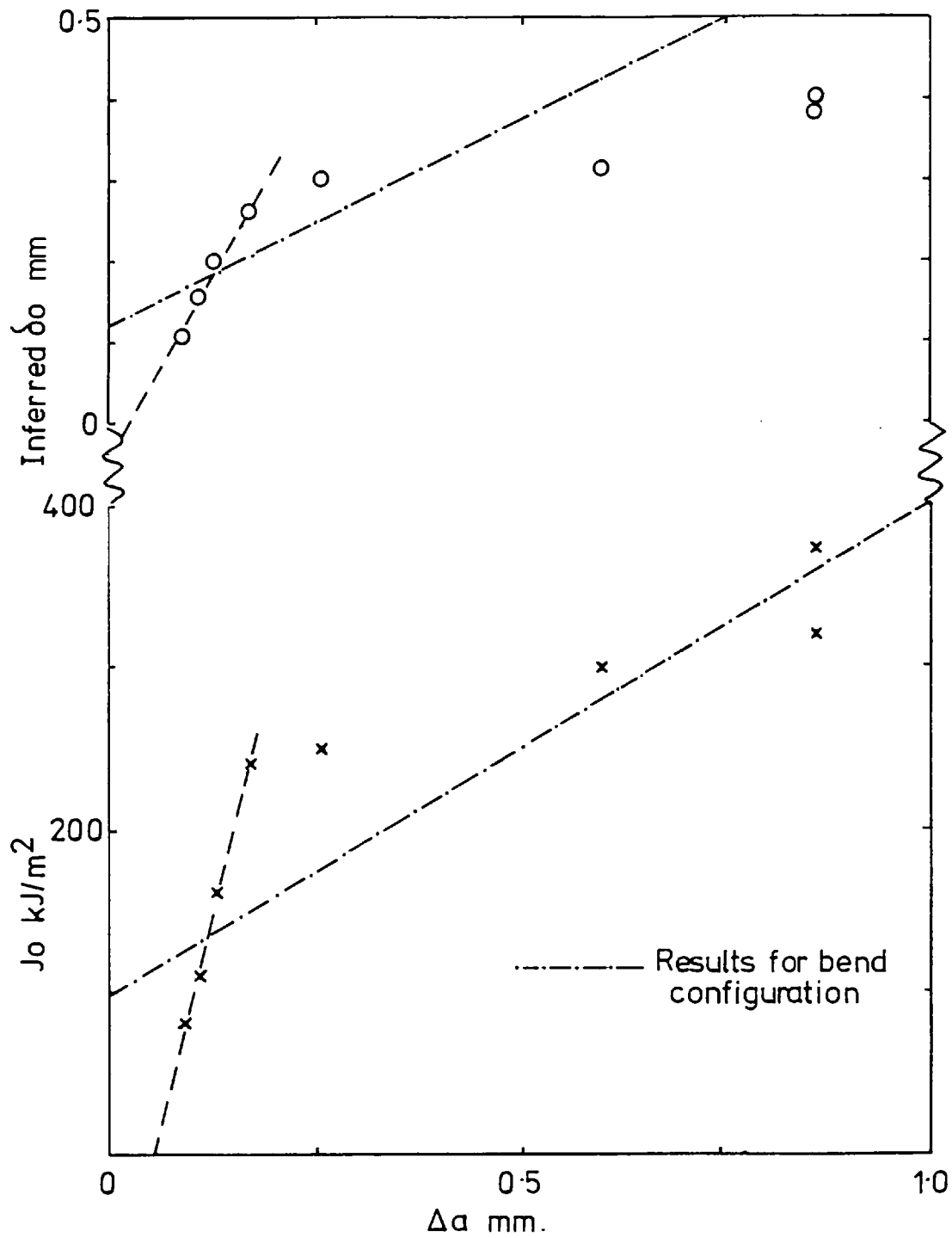


Fig. 7.7 Variation of δ_0 (inferred from eqn. 7.2) and J_0 with Δa for En32B, CCT configuration.

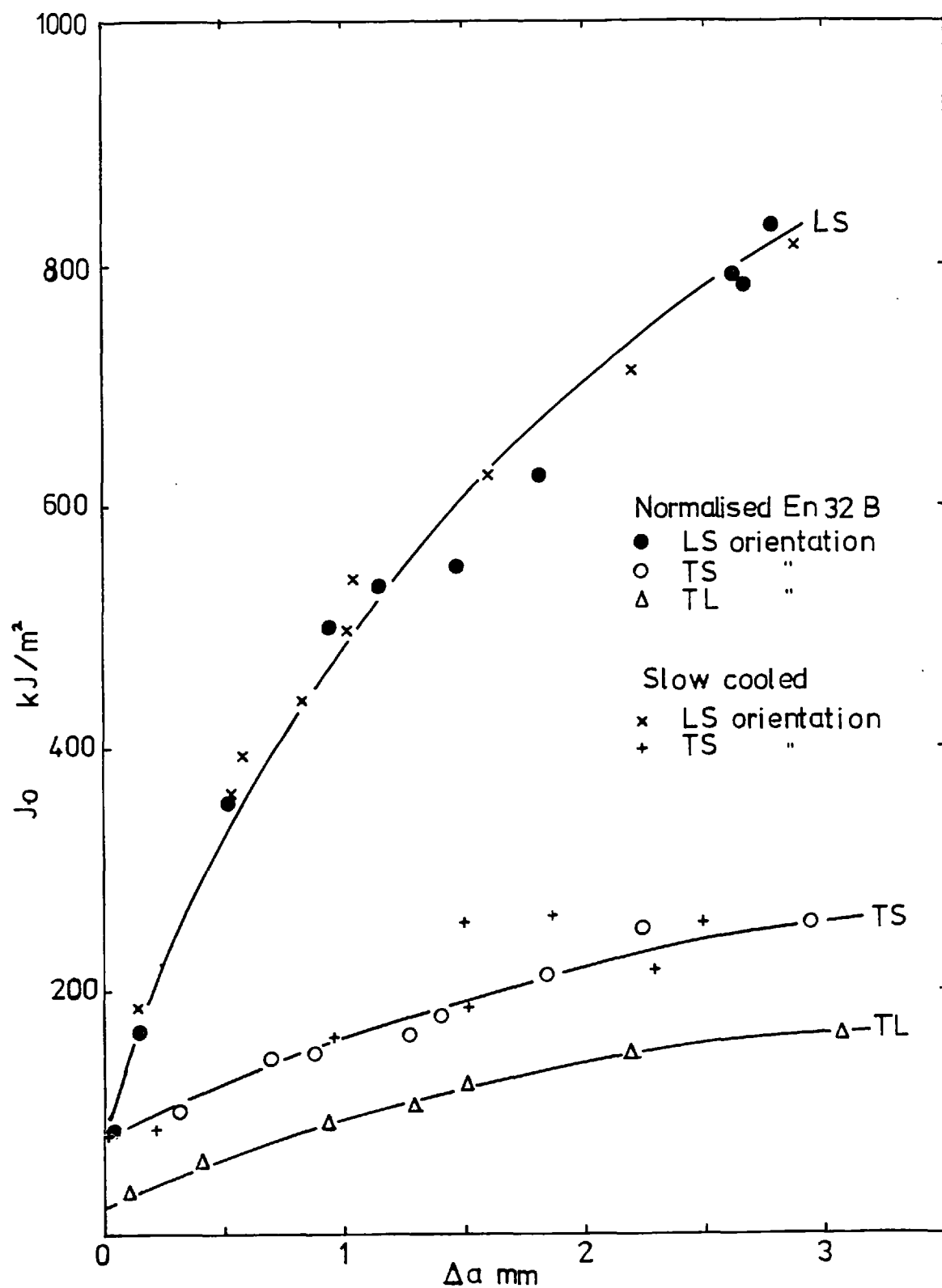


Fig. 7.8 Effect of heat treatment and orientation on the J_0 R-curve, En32B.

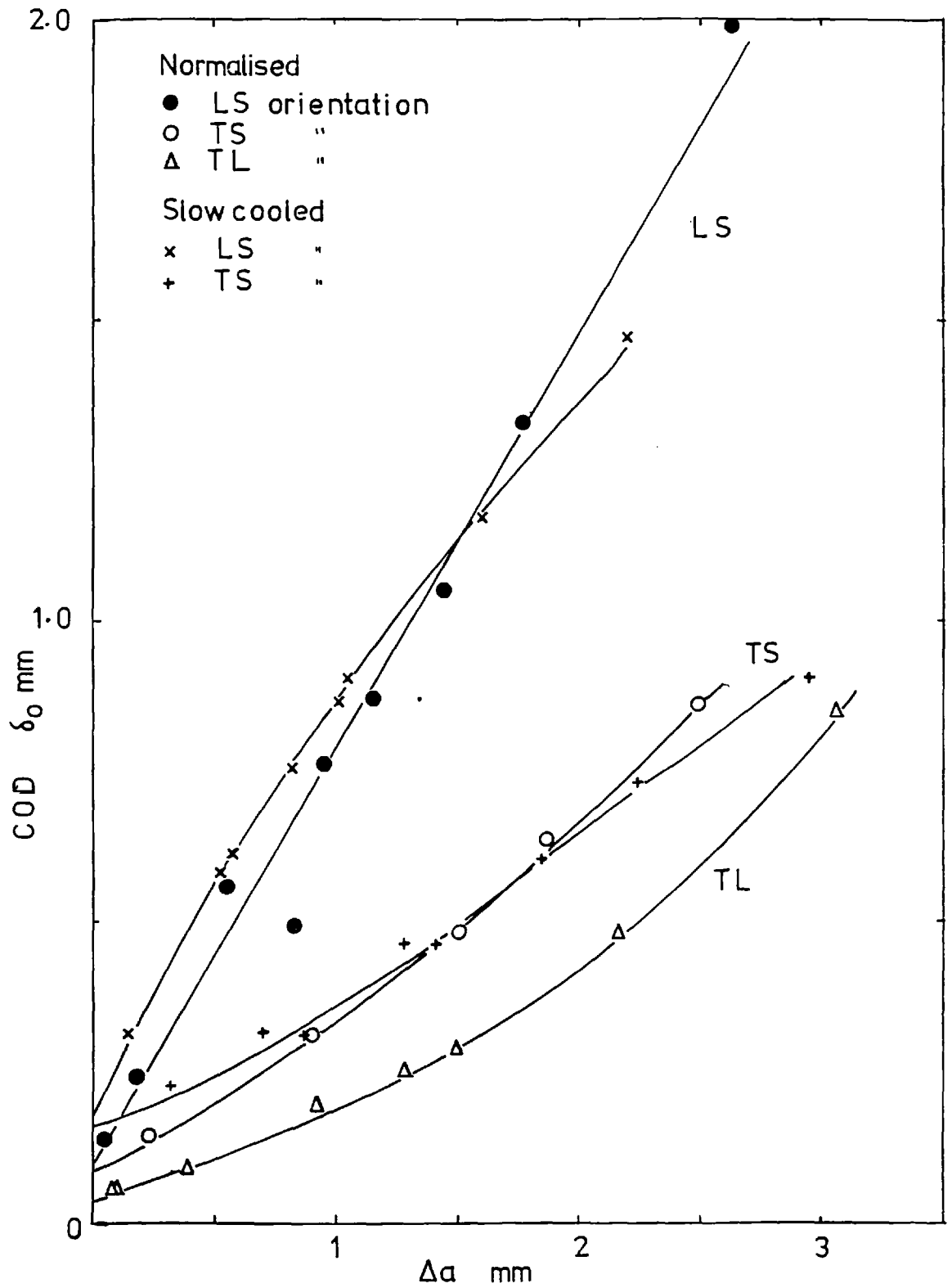


Fig. 7.9 Effect of heat treatment and orientation on the δ_0 R-curve, En32B.

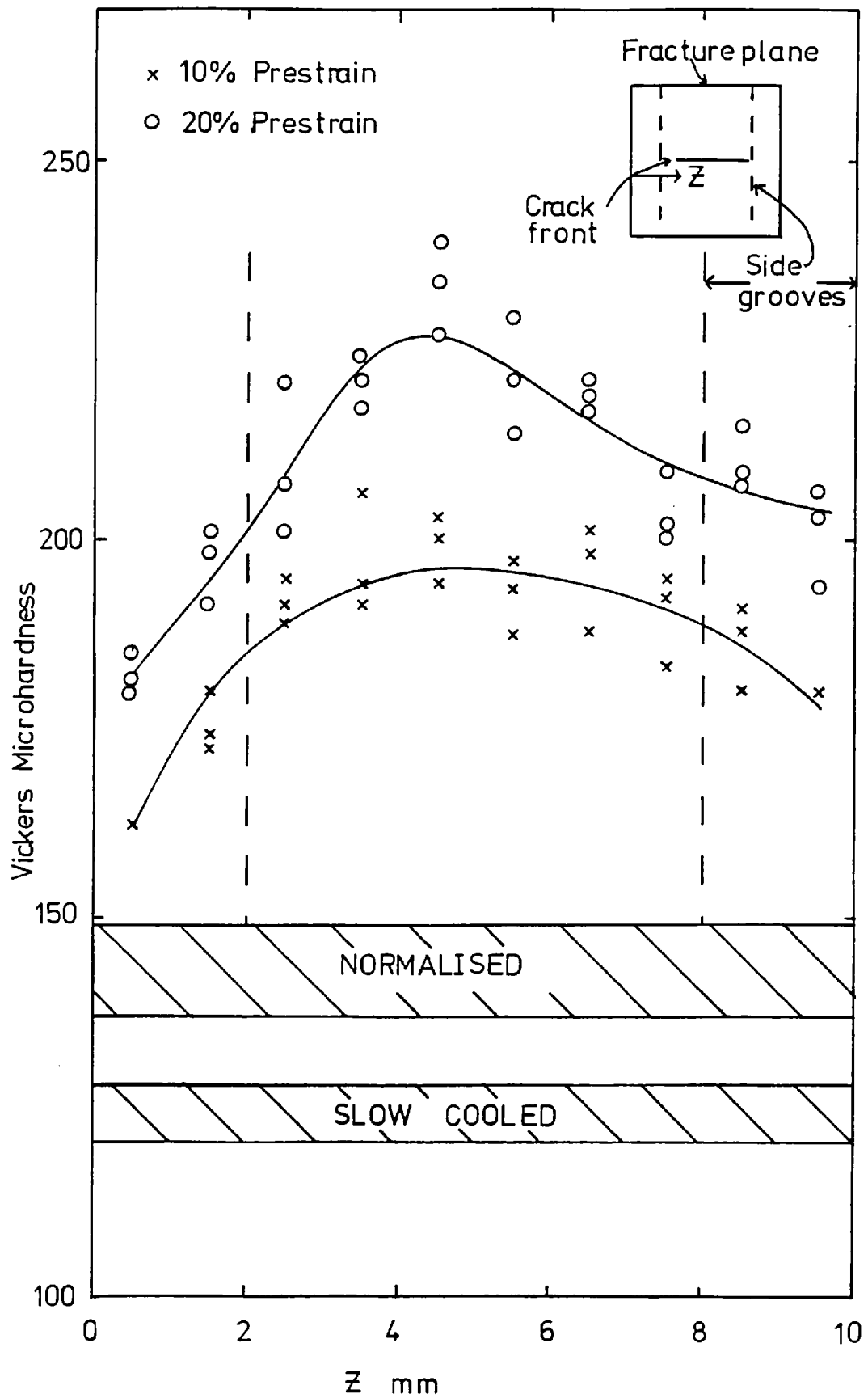


Fig. 7.10 Variation of microhardness across the thickness of prestrained test-pieces of En32B.

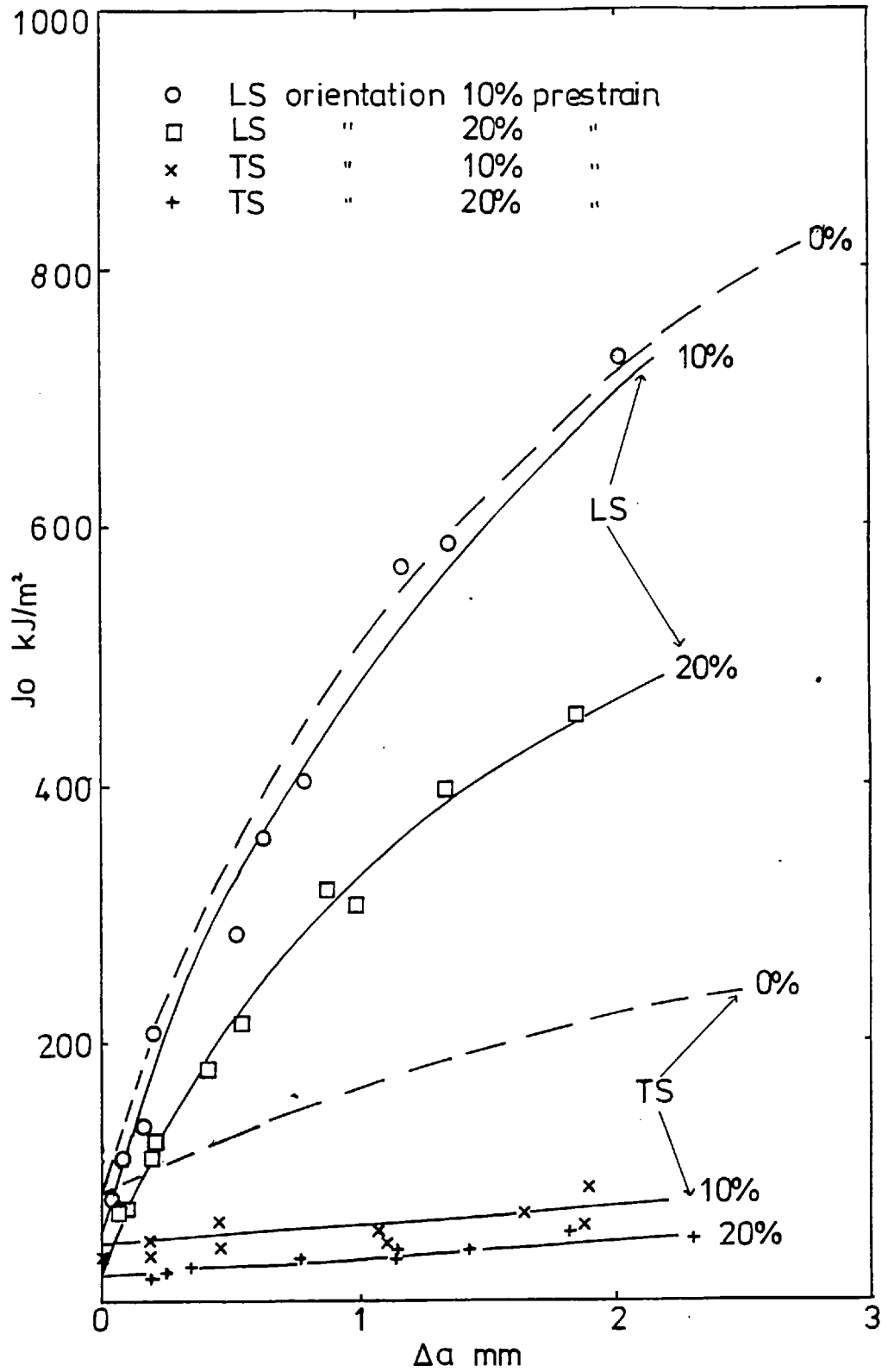


Fig. 7.11 Effect of prestrain on the J_0 R-curves for En32B.

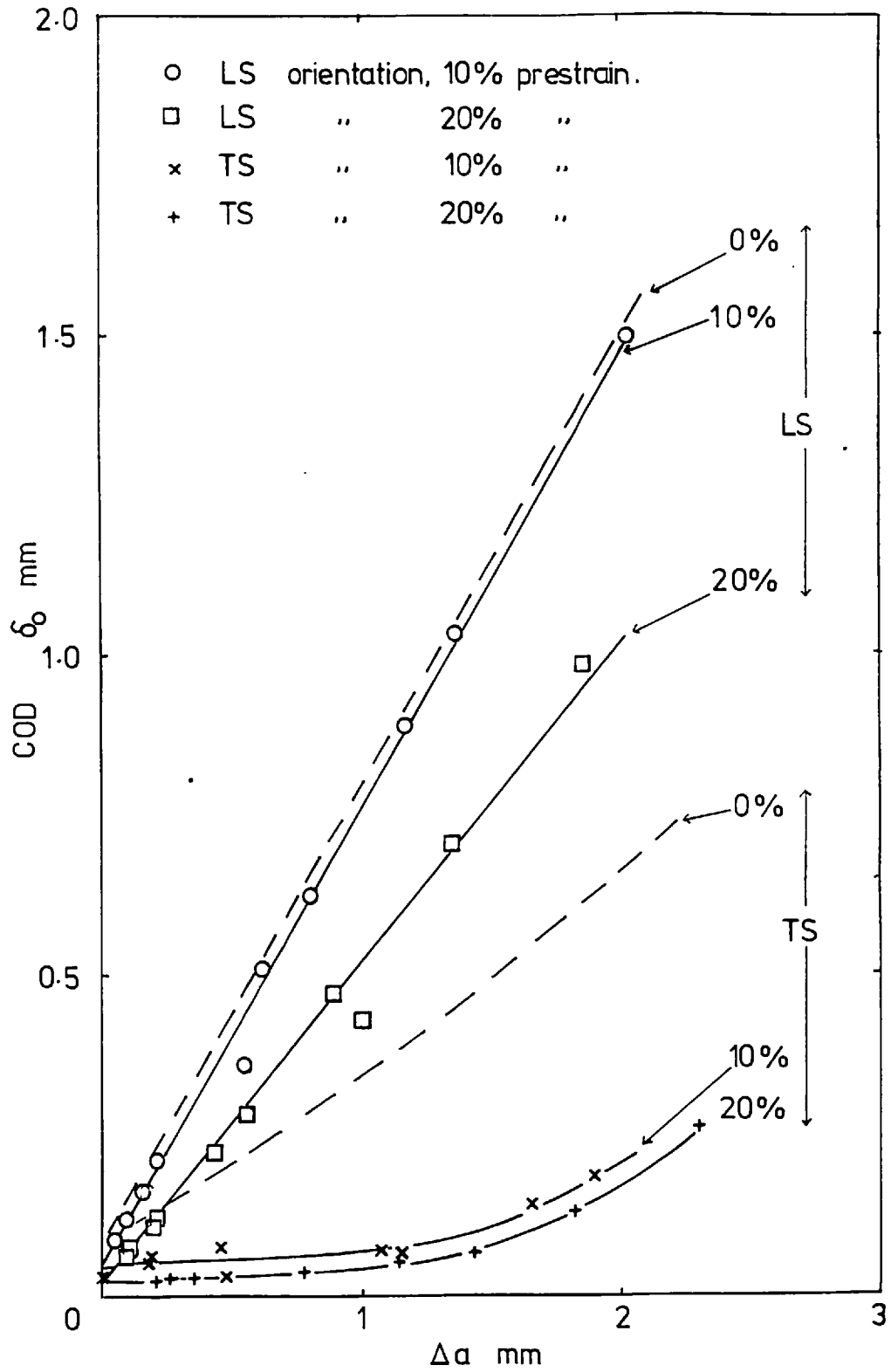


Fig. 7.12 Effect of prestrain on the δ_0 R-curves for En32B.

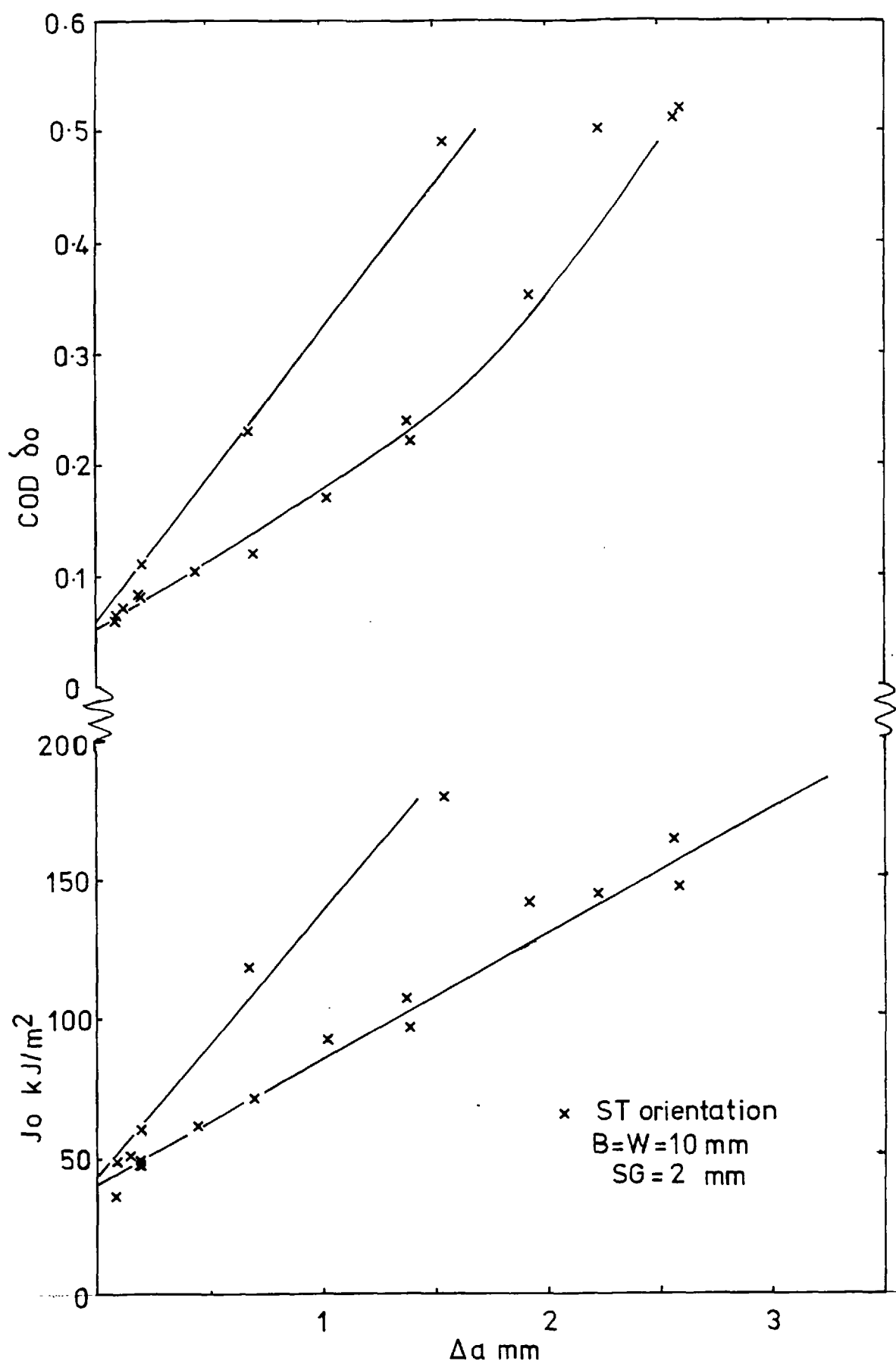


Fig. 7.13 R-curves for the ST orientation, En32B.

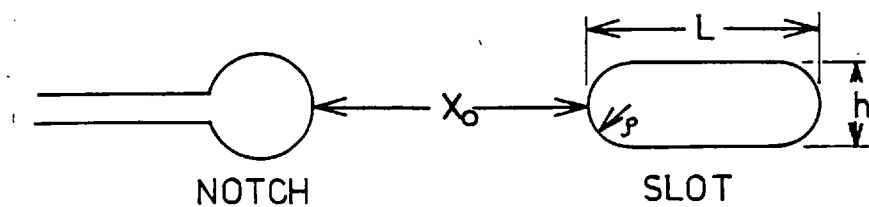


Fig. 7.14 Notch and slot configurations in brass CT specimens.

Table 7.2 Slot parameters in brass CT specimens. Notch root radius = 2.4mm. Symbols as Fig. 7.14.

Specimen No.	X_0 mm	L mm	h mm	p mm
I *	No slot	-	-	-
II *	7	12.5	3.2	1.6
III *	4	3.2	12.5	∞
IV *	4	3.2	3.2	1.6
V *	7	3.2	3.2	1.6
VI	4	12.5	3.2	1.6
VII	7	3.2	1.6	0.8
VIII	7	3.2	12.5	∞

* Double thickness specimens with centre grid.

Figs. 7.15 - 7.20 Distribution of tensile strains ahead of a blunt notch, for various void configurations and mouth openings.

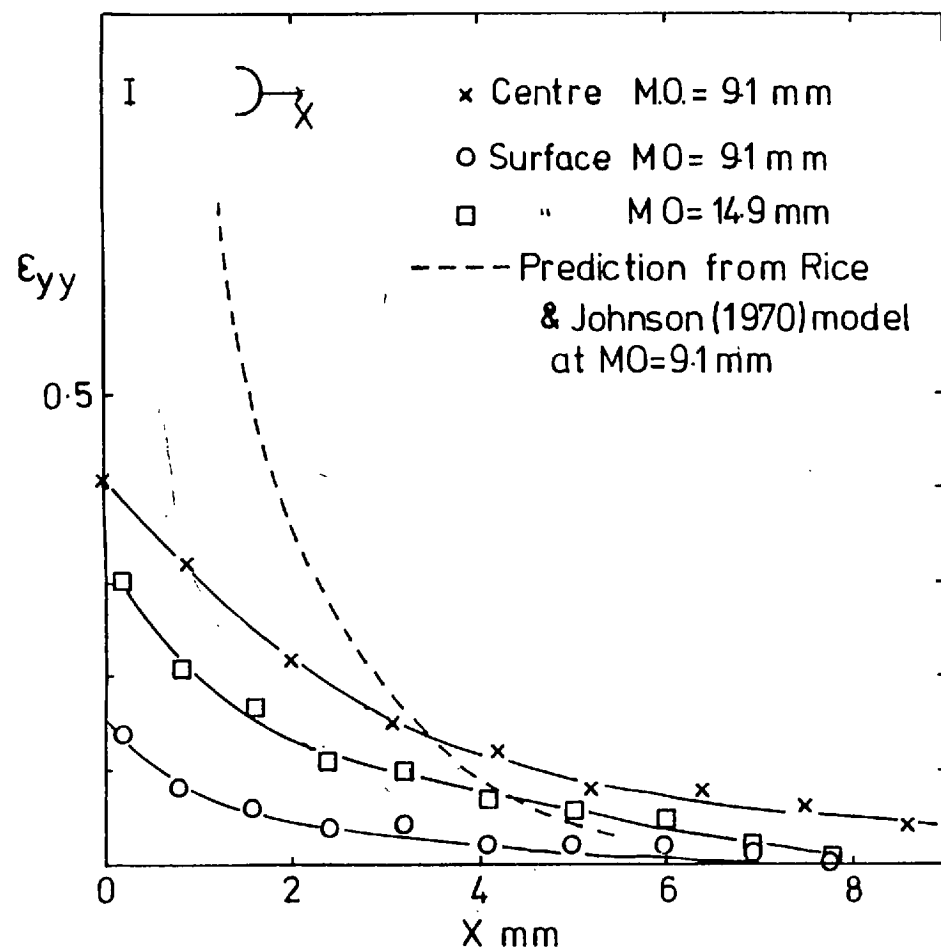


Fig. 7.15

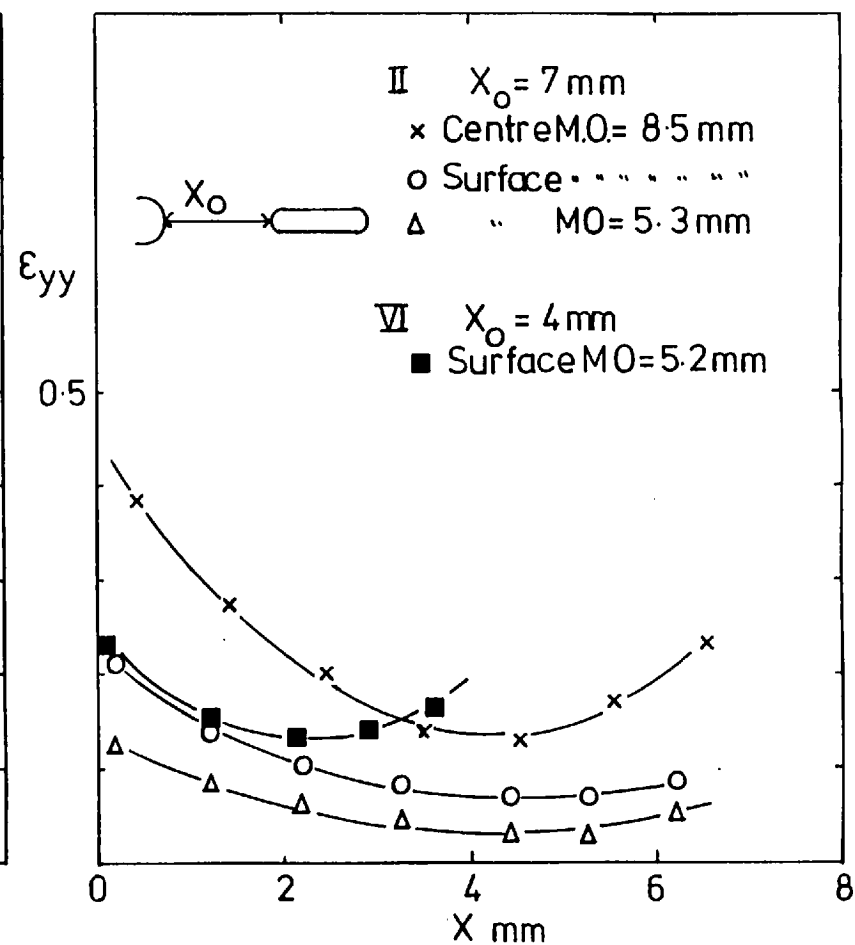


Fig. 7.16

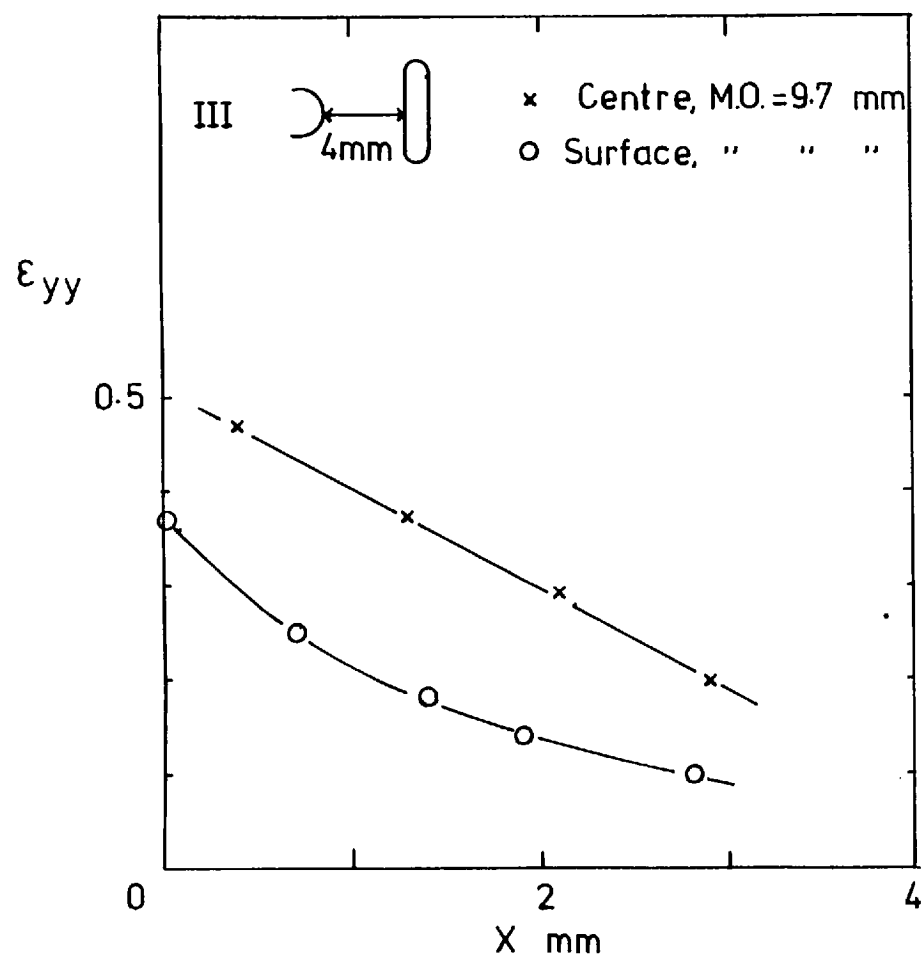


Fig. 7.17

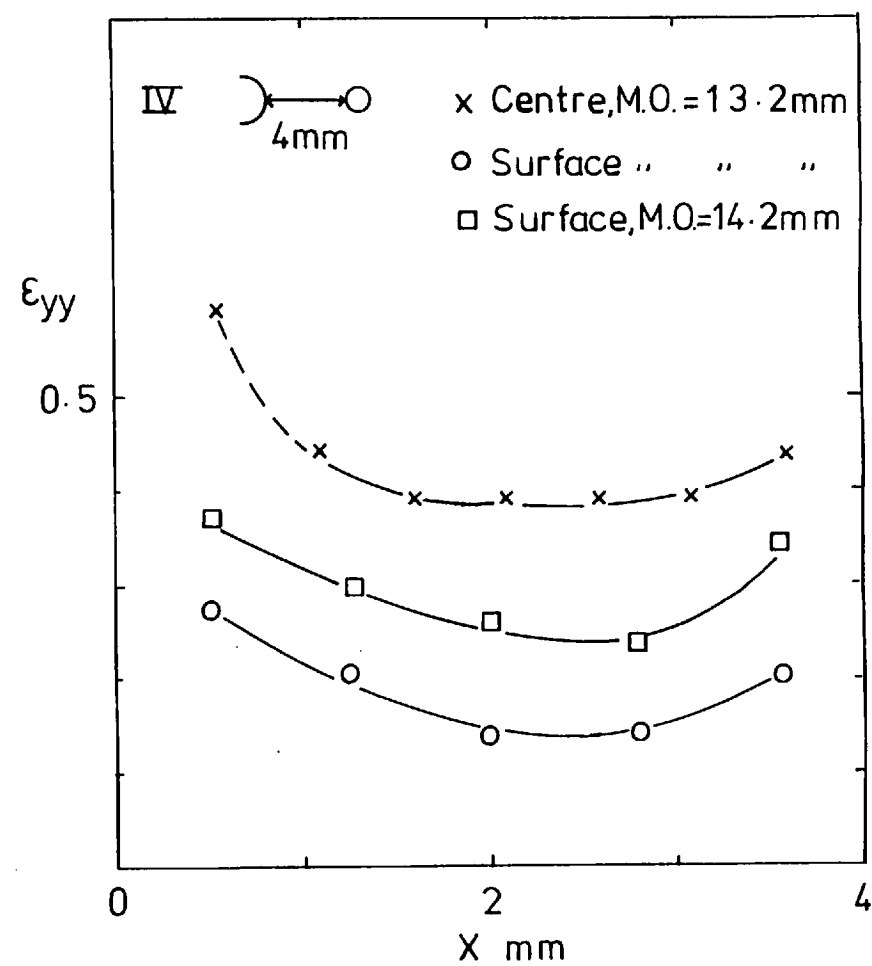


Fig. 7.18

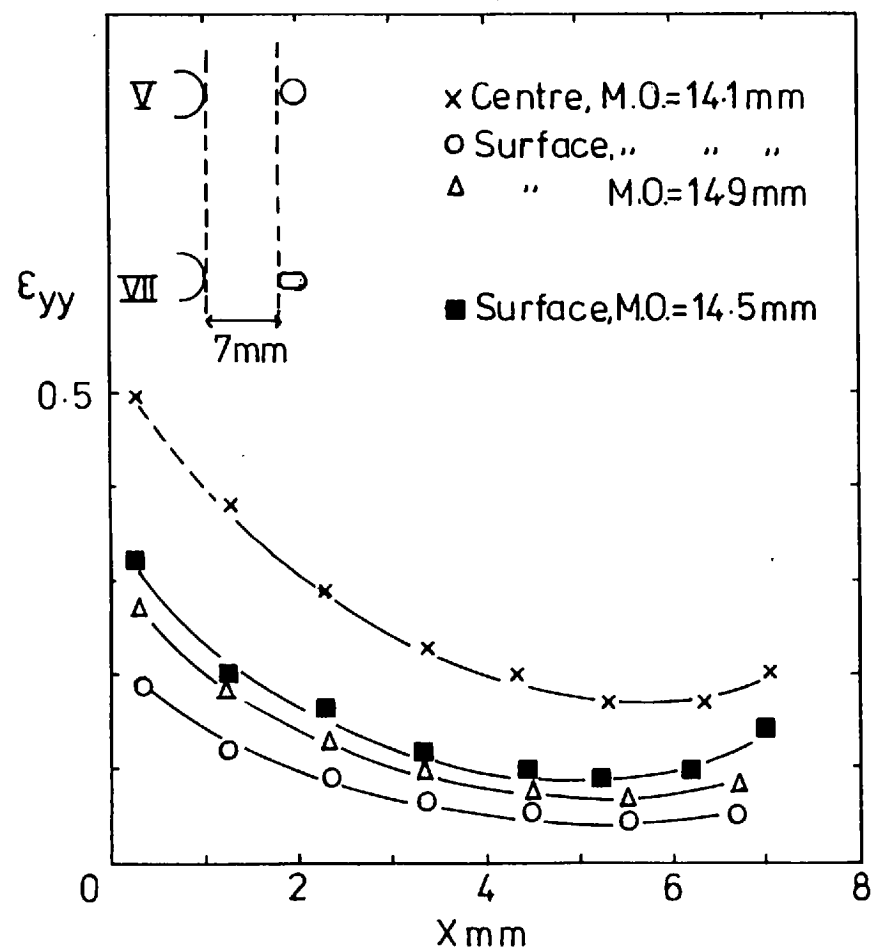


Fig. 7.19

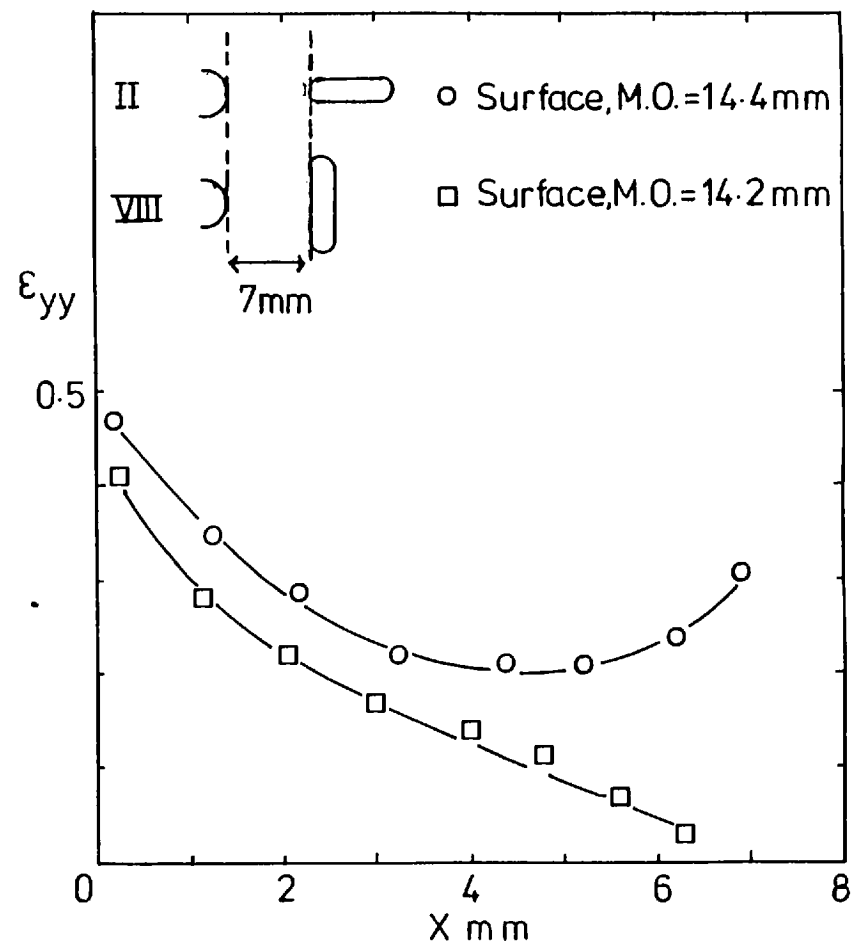


Fig. 7.20

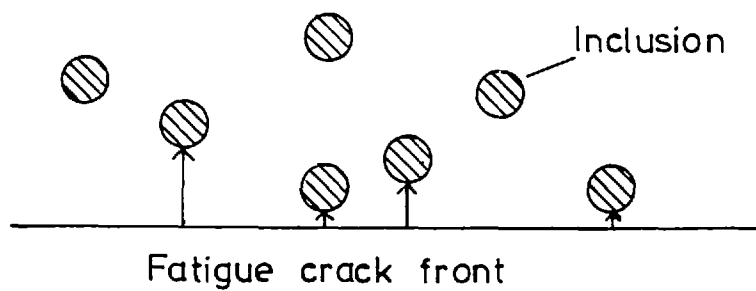


Fig. 7.21 Sketch of the crack plane prior to initiation.

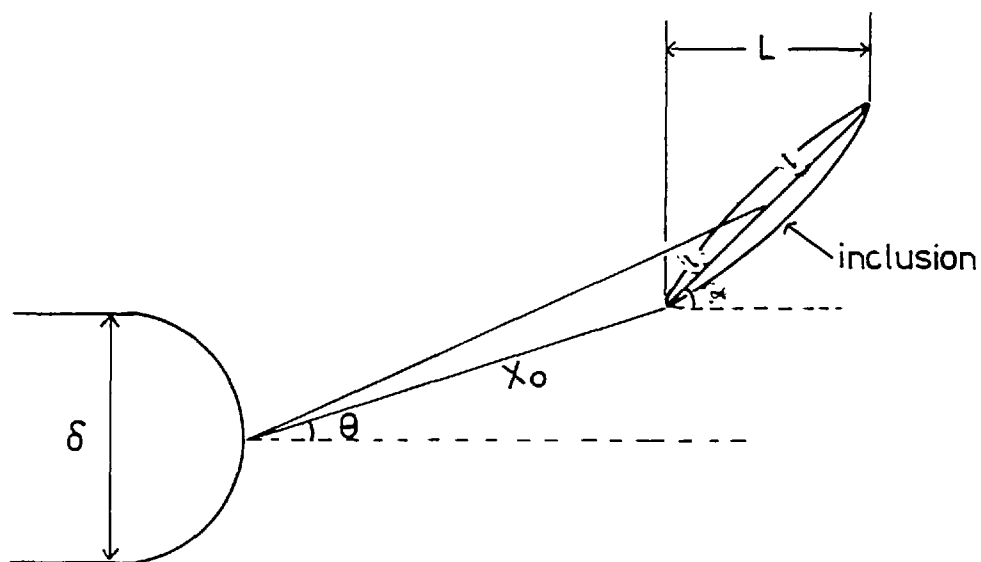


Fig. 7.22 2-D representation of an inclusion ahead of a crack tip.

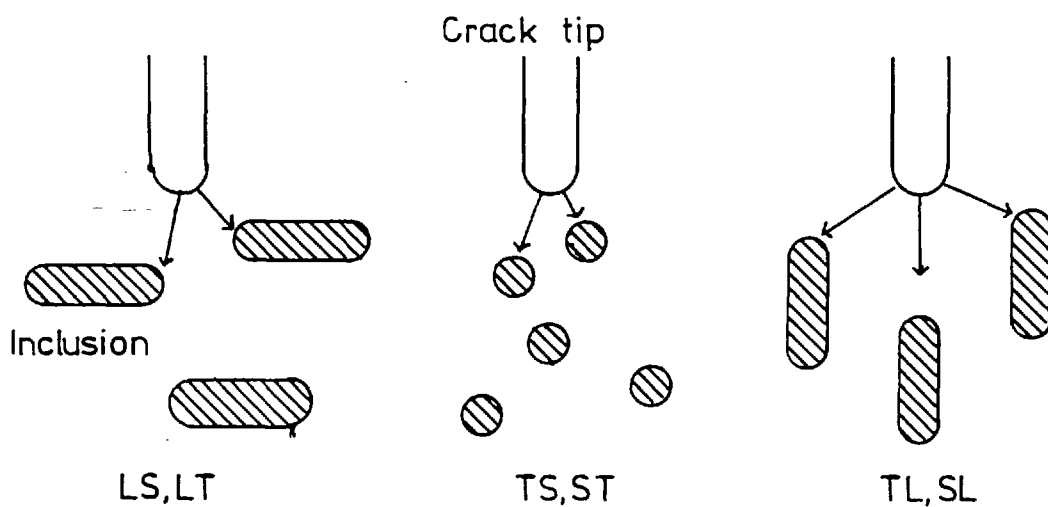


Fig. 7.23 Interaction between inclusions and the crack tip, in the different orientations.

Chapter Eight

PROPAGATION OF DUCTILE FRACTURE

8.1 Introduction

In this chapter, resistance to slow stable growth of a crack extending by microvoid coalescence is investigated. The R-curve is described in terms of J_0 and δ_0 , and the effects of specimen geometry and microstructure are considered. Since most of the specimens were small (mostly with $W=B=10\text{mm}$), the amount of crack growth that could be accommodated before a/W exceeded 0.7 was limited to a maximum of a few millimetres; there was thus little possibility of reaching steady state conditions. Therefore discussion is mainly confined to resistance during the initial stages of crack extension, and this is best described by the initial slopes of the R-curves.

Figs. 7.1-7.13 in the previous chapter show that the early stages of the δ_0 R-curves are linear. They may be described by

$$\delta_0 = \delta_i + A \Delta a \quad (8.1)$$

where A is the initial slope. The linearity is maintained for substantial crack extensions (2-3mm) in the longitudinal orientations. The same result has been reported by Elliott et al. (1971), Green and Knott (1976), Tanaka and Harrison (1976), Garwood (1977) and Shih et al. (1977). In contrast the J_0 vs. Δa curves have a slope which decreases continuously with crack extension. The symbol $J_0'(0)$ will be used for the initial slope, where

$$J_0'(0) = \frac{dJ_0}{d(\Delta a)} \quad \text{at } \Delta a = 0$$

The initial slopes of all the R-curves presented in Chapter 7 were measured, and are set out in Table 8.1, in the first five columns.

Other measures of the R-curve in terms of the J-integral and the COD were described in Section 3.5. A relationship is developed between the initial slopes as described by some of those different parameters.

8.2 Experimental Results

8.2.1 The Effects of Geometry and Configuration

The effect of thickness on the R-curve was investigated using plane-sided bend specimens of En24. The 24mm thick specimens had a slope of $A = 0.41$ ($J_0'(0) = 470 \text{ MJ/m}^2$). The thinner (12mm) specimens showed scatter which increased with crack extension; however, the range of $A = 0.85-0.4$ shows that the tendency was for the slope to increase as the thickness decreased. This was explained by Krafft et al (1961) on the basis of shear lip development, and is considered further in Section 9.2.

All the remaining specimens had side-grooves to prevent the formation of shear lips. Comparing the different specimen geometries in the LT orientation showed that while there was considerable scatter in the small specimens, with $W=B=10\text{mm}$, (as described in Section 6.5.4) the slope for the larger specimens, with $W=B=24\text{mm}$, fell within this scatter. This is shown on the graphs, Fig. 7.5 and 7.6. R-curve slopes independent of specimen size were also found by Clarke et al. (1976), Garwood (1977) and Shih et al. (1977).

The smaller LT test-pieces had side-groove depths of either 2 or 3mm, and this did not appear to alter the slope (Figs. 7.3 and 7.4). Green and Knott (1975a) concluded that the side-groove depth required to give thickness - independent R-curves was slightly more than the size of the shear lips that would be present in plane-sided specimens, of the same thickness. Garwood (1977) observed that thickness - independent R-curves were obtained if the combined side-groove depth exceeded a certain proportion of the gross thickness (about 0.4 for a mild steel). This gave a

"lower bound" R-curve. The tentative conclusion that may be drawn from these two observations is that the shear lips grow to form a certain proportion of the specimen thickness, rather than being independent of size. Incidentally, the larger test-pieces had either sharp or round bottoms to the side-grooves, and this appeared to have no effect, as also found by Freed and Krafft (1966).

An important point is whether the side-grooves are able to eliminate the plane stress element at the edges of the fracture surface. As shown in Section 6.5.1, the shear zone adjacent to the side-groove is small, so that the vast majority of the surface consists of flat fracture. Green and Knott (1975a) pointed out, however, that the exclusive presence of a flat fracture surface is no guarantee of true plane strain. Nevertheless, the evidence of a lower bound R-curve, at a sufficient side-groove depth, supports this possibility.

Comparison of the slopes for the bend and CCT configurations, Fig. 7.7 and Table 8.1, shows that they are smaller in the latter case. Considering the difficulties encountered with the CCT specimens, described in Section 7.2.1, no firm conclusion can be drawn. The result, however, is surprising in view of the effect of configuration reported elsewhere (Begley and Landes, 1976; Garwood, 1978), where the CCT configuration is found to give a much steeper slope. The explanation may be that the published data refers to plane-sided specimens, whereas these results were all obtained in flat fracture, with heavy side-grooves. The lower constraint in the CCT configuration could allow fuller development of shear lips, which would increase the slope. In fact, such a variation in shear lip thickness between the two configurations was noted by Garwood.

8.2.2 The Effect of Microstructure

The change in grain size from 20 to 40 μ m in the En32B had little effect on the slopes of the R-curves. The J_0 vs. Δa curves for the two grain sizes were indistin-

guishable (Fig. 7.8), as previously noted. It is therefore concluded that small changes in grain size had a negligible effect on resistance to slow crack growth.

Prestrain had a similar influence on the slopes of the R-curves as it had on initiation toughness. In most cases, the slopes decreased, with increasing prestrain (Table 8.1). The exception is $J_0'(0)$ for the LS orientation at 10% prestrain, which is greater than $J_0'(0)$ for the unstrained material. The slopes of the δ_0 R-curves, however, follow the expected order. If an equation of the form $J_0 = M' \sigma_{flow} \delta_0$ were to apply during the early stages of slow crack growth, it is possible that the elevation of σ_{flow} by prestraining would outweigh the reduction in M' , so raising J_0 disproportionately. This anomaly was not present in the TS orientation, and reference to Table 7.1 shows that here M' was depressed more, from 2.4 to 1.5, by a 10% prestrain. The reduction in work-hardening capacity had a much greater effect in this orientation, reducing the slope to almost zero. It is probable that here the greater tendency to strain localisation, with the consequent "zig-zag" fracture path mentioned in Section 6.5.3, was the principal cause of the lower slopes.

The specimen orientation had a dramatic influence on the initial slopes, in a manner that did not necessarily correspond to its effect on initiation. For instance, the LS orientation had both high initiation toughness and slope, whereas the TS orientation, with a similar J_i , had a much smaller slope. The result was that, after 1mm of crack growth, the apparent toughness in the LS orientation was three times as large as in the TS orientation. The initial slopes for the various orientations may be divided approximately into two categories:-

LS, LT >> TS, ST, TL, SL

The large differences in appearance of the fracture surface between these two, with the tendency for the elongated inclusions to blunt the propagating crack tip in the first

category, were described in Section 6.5.2.

8.2.3 The Propagating Crack Tip

Garwood and Turner (1978) report that the COD of the propagating crack tip, δ_p , is constant and smaller than δ_i . This was confirmed here, within rather large experimental error, by measuring δ_p directly on sectioned specimens (Fig. 8.1). The error was caused by the fact that δ_p is dependent upon the position of the crack tip with respect to void with which it is coalescing, and also upon the size and shape of the inclusion forming the void. The crack profile in the LS and LT orientations contained many fissures, formed around elongated inclusions normal to the crack plane. If one of these was positioned at the crack tip, the separation of the flanks immediately behind it were measured. With these microstructural irregularities it is not surprising that there was a large amount of scatter in the data. Average values obtained for δ_p were $30\mu\text{m}$ (LS and LT), $17\mu\text{m}$ (TS) and $15\mu\text{m}$ (TL), with an estimated error of $\pm 50\%$. They were all measured with the specimen off-load.

If the COD at incipient crack extension is a measure of the critical strain required in the material immediately ahead of the crack tip for separation to occur, the approximate constancy of δ_p suggests that this critical strain is a constant during crack growth. The relative magnitudes of δ_p and δ_i are to be expected. At initiation, the strain at the crack tip must be raised from zero to the critical value, and the COD is thus relatively large. At the same time material at points well ahead of the crack tip experiences a sub-critical strain. As the propagating crack approaches these points the net strain required to reach the critical value is thus lower, and a lower propagation COD, δ_p , is required.

Despite the observed approximate invariance of δ_p with Δa , it is not feasible to use δ_p as a parameter to characterise the conditions at the propagating crack tip, since it is affected far too much by microstructural

irregularities, and is too awkward to measure.

8.3 The Relationship between Parameters used to Express The R-Curve.

8.3.1 Common Parameters

Several parameters are in common use during large scale yielding, and were described in Section 3.5. The COD is usually derived by means of either the Ingham et al. (1971) formula, or the Wells (1971) formula, and may be referred to either the position of the original, fatigue crack tip (δ_o), or to that of the final, growing tip (δ_f). Both formulae are based on the assumption that a bend specimen rotates about a pivot, the centre of rotation.

Various estimates of the J-integral are also in general use. The one considered here is that derived by eqn. (3.21), termed J_o . The three parameter estimate of Garwood, Robinson and Turner (1975), J_r , was found to be identical for the early stages of crack growth (Garwood, 1977), and the calculations of J_r for the LS, TS and TL orientations, in En32B, confirm this (Fig. 8.2). The J_o and J_r curves have the same initial slopes.

8.3.2 Relationship between δ_o and δ_f

A section through a cracked bend specimen is shown in Fig. 8.3, where a stable crack of length Δa has initiated at the blunted tip of a fatigue crack. The centre of rotation is assumed to move ahead of the growing crack tip. It is always removed from the tip by a distance that is a constant fraction, r , of the remaining ligament. Thus δ_o and δ_f are related by similar triangles:

$$\frac{\delta_f}{r(W-a_o-\Delta a)} = \frac{\delta_o}{\Delta a + r(W-a_o-\Delta a)} \quad (8.2)$$

$$\text{From (8.1)} \quad \delta_o = A\Delta a + \delta_i$$

$$\text{Substituting} \quad \frac{\delta_f}{rb} = \frac{A\Delta a + \delta_i}{rb + \Delta a} \quad \text{where } b = W-a_o-\Delta a$$

$$\text{i.e.} \quad \delta_f = \frac{(A\Delta a + \delta_i)rb}{rb + \Delta a} \quad (8.3)$$

The usual means of generating δ_f R-curves is by application of Wells formula, eqn. (3.7), with $a = a_0 + \Delta a$. The points calculated by this means are shown in Fig. 8.4, for the LS, TS and TL orientations, En32B. Good agreement is obtained with the solid lines, which were derived through eqn. (8.3), using the average values of A and δ_i for each orientation taken from Tables 8.1 and 7.1, respectively. Also shown are the δ_o R-curves from Fig. 7.9. This agreement between the two methods of calculating δ_f is to be expected, since the same geometrical construction is used in the derivation of the two equations. It shows, however, that the assumption concerning the centre of rotation was reasonable.

Differentiating eqn. (8.3) with respect to Δa ,

$$\frac{d(\delta_f)}{d(\Delta a)} = \delta_f' = \frac{(rb + \Delta a)\{rbA - r(A\Delta a + \delta_i)\} - (A\Delta a + \delta_i)rb(1-r)}{(rb + \Delta a)^2}$$

$$\text{since} \quad db/d(\Delta a) = -1$$

$$\begin{aligned} \text{At } \Delta a = 0, \quad b &= (W - a_0) \\ \therefore \delta_f'(0) &= \frac{rA(W - a_0) - \delta_i}{r(W - a_0)} \end{aligned} \quad (8.4)$$

For a large specimen, if $rA(W - a_0) \gg \delta_i$

$$\delta_f'(0) \approx A = \delta_o'(0) \quad (8.5)$$

Thus the initial slopes of the δ_f and δ_o R-curves are approximately the same, as is confirmed by Fig. 8.4. Thereafter the two curves diverge, the slope of the δ_f curve decreasing with crack extension.

As Fig. 8.3 illustrates, δ_f is not a measure of the COD at the true, propagating crack tip (here termed δ_p), but rather a geometrical consequence of the mouth opening and centre of rotation of the test-piece. The true COD, δ_p , is much smaller and approximately constant. Thus δ_f

has no physical meaning as a parameter relating to the crack tip, and since it can be derived from δ_o via eqns. (8.1) and (8.3), it seems superfluous.

8.3.3 Relationship between δ_o and J_o

Eqn. (3.21) for J_o in bending may be re-written, using the original definition of Rice et al. (1973):

$$J_o = \frac{2 \int M d\theta}{B(W-a_o)} \quad (8.6)$$

where M is the bending moment, and θ the angle of bend. Neglecting elastic deflections of the arms, θ may be obtained approximately from δ_o and the geometry of bending (Fig. 8.3)

$$\theta = \frac{\delta_o}{rb + \Delta a} \quad \text{if } \theta \text{ small} \quad (8.7)$$

Combining with eqn. (8.1) and differentiating gives

$$\frac{d\theta}{d(\Delta a)} = \frac{Ar(W-a_o) - \delta_i(1-r)}{(rb + \Delta a)^2} \quad (8.8)$$

$$\text{From eqn. (8.6)} \quad J_o = \frac{2}{B(W-a_o)} \int M \frac{d\theta}{d(\Delta a)} \cdot d(\Delta a) \quad (8.9)$$

$$\text{Differentiating } J_o' = \frac{2}{B(W-a_o)} \cdot M \frac{d\theta}{d(\Delta a)}$$

Substituting for $d\theta/d(\Delta a)$, eqn. (8.8)

$$J_o' = \frac{2M}{B(W-a_o)} \cdot \frac{Ar(W-a_o) - \delta_i(1-r)}{(rb + \Delta a)^2} \quad (8.10)$$

Taking the special case of a rigid/perfectly plastic material, M may be expressed through the formula of Wilson (1970)

$$M = 0.31 \sigma_y B \cdot b^2 \quad (8.11)$$

Some allowance may be made for work-hardening by using σ_{flow} in place of σ_y . Eqn. (8.10) becomes

$$J_o' = \frac{0.62 \sigma_{flow} b^2}{(W-a_o)} \cdot \frac{Ar(W-a_o) - \delta_i(1-r)}{(rb + \Delta a)^2} \quad (8.12)$$

$$\text{At } \Delta a = 0, J_o'(0) = \frac{0.62 \sigma_{flow}}{(W-a_o)r^2} \{Ar(W-a_o) - \delta_i(1-r)\} \quad (8.13)$$

Thus the initial slope of the J_o R-curve may be obtained from the slope of the δ_o R-curve and the initiation COD. Predictions of $J_o'(0)$ using eqn. (8.13) with $r = 0.45$ are shown in the last column of Table 8.1. Agreement with measured values is quite good, particularly in the less tough orientations and in the prestrained material. The model is essentially a rigid/plastic one, and better agreement is expected in cases where the effect of work-hardening is smaller. The En24 bend bars also had low work-hardening exponents, but since they were plane-sided the growth of shear lips should render the model less accurate.

To obtain the whole J_o R-curve, eqn. (8.12) may be integrated, with the final result:

$$J_o = J_i + 0.62 \sigma_{flow} \left(Ar - \frac{R\delta_i}{b_o} \right) \left[\frac{1}{R^3} (\mu - 2rb_o \cdot \ln \mu - \frac{r^2 b_o^2}{\mu}) - \frac{2rb_o}{R} \cdot \left(\ln \mu + \frac{rb_o}{\mu} \right) - \frac{b_o^2}{R\mu} \right]_0^{\Delta a} \quad (8.14)$$

where $b_o = W-a_o$, $R = 1-r$ and $\mu = rb + R\Delta a$

This equation gives quite good predictions for the J_o R-curve in those cases where work-hardening was less influential, as shown in Fig. 8.5. As expected, agreement in the tougher orientations is worse. Thus the possibility exists that either one of the parameters, J or COD, may be used to express the R-curve, and the other one derived from it, at least for materials of low work-hardening capacity. The measurement of COD is less time consuming than that of J , but the latter may be more universally applicable since it does not assume a rotational factor (which may vary).

For large specimens, with $A(W-a_0) \gg \delta_i$, eqn. (8.13) becomes

$$J_o'(0) \approx \frac{0.62}{r} \sigma_{flow} A \quad (8.15)$$

Two points are of interest. Firstly, this equation is analogous to $J = M' \sigma_{flow} \delta$, which is used up to initiation. Taking $r = 0.45$, the factor corresponding to M' is $0.62/0.45 = 1.4$, which is close to the values of M' obtained on the prestrained material. Secondly, eqn. (8.15) predicts that, for specimens of sufficient size, the geometry dependence of the initial slope $J_o'(0)$ is determined solely by the geometry dependence of A . For a large test-piece, it may be assumed that the dimensions of the crack tip region immediately before and after initiation (i.e. δ_i and δ_p) are independent of test-piece width and initial crack length, within certain limits. Thus A is likely to be independent of these factors, at least initially, and so the initial slope of the J_o R-curve should be too. This result has been confirmed by the experimental work of Chipperfield (1976) and of Shih et al. (1977), and coupled with the observed invariance of J_i and δ_i , it suggests that the R-curve is geometry independent in bending, at least for the early stages of crack extension.

8.3.4 The Crack Tip Flank Angle

The angle Φ between the flanks of the propagating crack may be derived with reference to Fig. 8.3

$$\tan \frac{\Phi}{2} = \frac{(\delta_o - \delta_i) - \delta_p}{2\Delta a}$$

$$\text{If } \Phi \text{ small} \quad \Phi \approx \frac{\delta_o - \delta_i - \delta_p}{\Delta a} \quad (8.16)$$

$$\text{Substituting eqn. (8.1)} \quad \Phi \approx A - \frac{\delta_p}{\Delta a} \quad (8.17)$$

This predicts that Φ is zero until $\Delta a = \delta_p/A$, and then increases rapidly to the asymptotic value of A radians at $\Delta a = \infty$. Eqn. (8.17) is plotted as the solid lines on

Fig. 8.6 using values of A and δ_p for the LS, LT, TS and TL orientations. This is compared with the flank angles measured "off-load" on sections through crack tips, by recording $(\delta_o - \delta_i)$, δ_p and Δa for each specimen, and substituting into eqn. (8.16). Despite the large amount of scatter, caused by microstructural irregularities, agreement is quite good in the LS and LT orientations. In the TS and TL orientations, however, eqn. (8.17) consistently underestimates the measured flank angles. This may be due to the non-linearity of the δ_o vs Δa plot in these orientations. The initial values of A were used in eqn. (8.17), whereas the slopes did in fact increase after some crack extension.

The initial part of the Φ vs. Δa curves, dotted in Fig. 8.6, goes to zero at small Δa . This is just a consequence of the continuum nature of the model. In reality crack extension will occur in a series of steps, the size of which is given by the inclusion spacing. If this step-size is large compared with δ_p/A , the initial part will not be observed. These findings lend support to the suggestion that the slope A of the δ_o R-curve (if this is linear) is a good measure of propagation toughness, since it is directly related to a crack tip parameter, the flank angle.

8.4 The Cause of the R-Curve

8.4.1 Introduction

Some possible causes of the increase in resistance as the crack extends will now be examined. It is possible that the observed effect is purely a consequence of the measurement technique employed and has no bearing on the actual specimen toughness. However, this may be discounted for several reasons. Firstly, both δ_o and J_o increase with crack extension, so that if this were the case both would be measuring artefacts. Secondly, crack growth in a small test-piece begins before maximum load is attained, and the increasing load required for the early stages of crack propagation implies an increasing toughness. Thirdly, the different slopes of the R-curves in different orientations

shows that they are influenced by microstructure. Fourthly, the use of eqn. (3.21), where the total absorbed energy U is employed, means that $J_0 \propto U$, so that the absorbed energy must increase as the crack extends.

Therefore the R-curve is a measure of some real effect. The question remains as to whether it is really characteristic of the crack tip, as K or G is in a brittle fracture, and as J or COD are thought to be prior to crack advance. The rise in the R-curve could be ascribed to two possible causes:

i) An increase in "micro toughness". As discussed at the end of Section 3.6.3, the J R-curve, when properly calculated to allow for crack extension, could be a direct measure of the effective surface energy, $2\gamma'$. In this case an increase in the R-curve would correspond to an increase in the work done against forces trying to hold the crack faces together at the tip.

ii) An increase in the energy dissipated remote from the crack. Broberg (1971; 1975) postulates that the micro-toughness of the crack tip remains constant. For small scale yielding, the J -integral prior to initiation is a measure of the energy flow to the "end region" surrounding the crack tip. As the crack advances the plastic zone size increases and "screens" the tip, so that only a small proportion of the energy absorbed reaches it. J evaluated along a remote contour increases, whereas J evaluated along a small contour around the end region is a constant. Thus Broberg argues that the observed rise in the R-curve is due solely to an increase in remote plastic work.

As already discussed, it is unlikely that the first possibility can account wholly for the observed increase in apparent toughness with crack extension. However, in order to determine whether it makes any contribution during flat, "plane strain" fracture, strains around the crack tip were estimated using recrystallisation and microhardness measurements.

8.4.2 The Strain at the Propagating Crack Tip

The recrystallisation technique for estimating strains was described in Section 5.6.2. It was applied to test-pieces of En32B which had been precracked in fatigue, bent to different displacements, nickel plated to help edge-retention, and sectioned. Plate 8.1 shows some typical crack tip profiles after recrystallisation. The small size of the recrystallised grains at the crack tips are indicative of the high strains there. By counting the number of grains in a known area, the average grain size, and hence the average strain there, was determined. Fig. 8.7 shows the strains in a region 50 μ m square plotted against Δa , for the LS, LT, TS and TL orientations. As the crack extends the average strain remains constant, at about 0.75 in the LS and LT orientations, and 0.43 in the TS and TL orientations. The two data points in square brackets were obtained on specimens where the crack tip was on the point of coalescing with a void immediately in front. Consequently the strain in the intervening ligament was higher.

The lower strains observed at $\Delta a = 0$ referred to specimens in which crack growth had not initiated. The strain ahead of the tip was then less than the critical value required for crack extension.

As a check on the technique, the strains in a larger region, 100 μ m square, were also estimated. The results were similar, with average strains of 0.42 (LS and LT) and 0.34 (TS and TL), with no discernable trend as the crack extended. This suggests that the region of intense strain is not increasing in size. A fatigue precracked specimen was also given the same annealing treatment, and no signs of recrystallisation around the fatigue crack were found. This showed that the strains during fatigue were too low or on too small a scale to give grain growth.

These results provide strong support for the argument that a critical strain is required over a small region at the tip for crack extension to occur. The lower strains observed here in the TS and TL orientations do not

necessarily imply that the critical strain is lower in these cases. The measured strains are an average over an arbitrary size, which was chosen as 50 μ m as a first approximation to a "process zone" size. A smaller region would have too few grains in it to give a consistent count. It is likely that the critical strain required for material separation is a constant, depending on matrix flow properties. The tendency for strain localisation to occur in the transverse orientations has already been mentioned. The average strain over a larger area than the critical region would then appear smaller in these orientations. Also, it is possible that the amount of recrystallisation depends on the size of the strained region, if this is small.

The similarity in local crack tip strains in the LS and LT orientations, and in the TS and TL orientations, suggests that the crack tip conditions are similar. This is not surprising in the first two cases, where the longest axes of the inclusions lie normal to the crack plane. However, in the second two cases the inclusion orientation with respect to direction of propagation is very different. Since the R-curves for the TS and TL orientation also have similar slopes, the evidence all indicates that it is the inclusion dimension either normal to, or parallel to, the applied tensile stress which is important. The direction of propagation within the crack plane is of little significance to the toughness.

8.4.3 The Plastic Zone Extent.

In a rigid/perfectly plastic solid, the size and shape of the plastic zone is given by slip line field theory. In a fully plastic, three point bend, cracked test-piece "plastic hinges" are predicted, similar to the shape of the recrystallised zone shown in Plate 5.6(b). It is likely however that the microstructural processes occurring at the crack tip as it extends, together with work-hardening, could modify the pattern of yielding.

To investigate this possibility, polished sections normal to the crack plane were taken through fractured specimens, and the lateral spread of plasticity outward from the crack plane was determined by microhardness measurements. Bend specimens in the LS, TS and TL orientations were chosen, and all had ductile crack growth of 1mm. The measured increments in J_o (i.e. $J_o - J_i$) for the three specimens were 400 kJ/m^2 (LS), 60 kJ/m^2 (TS) and 65 kJ/m^2 (TL).

Fig. 8.8 shows the hardness profiles measured laterally from the end of the slow crack growth, as indicated. In the LS orientation, the lateral extent of the work-hardened material, as indicated by the knee of the curve, was 3-4mm, whereas in the TL and TS orientations it was 2-2.5mm. The microhardness number of the unstrained material was about 145 for the LS and TS orientations. The slightly higher value of ≈ 150 for the TL specimens probably reflects the fact that it was given the initial normalising treatment in a different batch from the other two. The microhardness values near the crack tips confirm, on a coarser scale, the findings of the previous section - that the strains there are higher in the longitudinal orientations than in the transverse ones. The overall implication of these results is that the magnitude and spatial extent of plasticity is related to the toughness recorded during slow crack growth.

Finally, the size of the recrystallised zone was measured on the annealed specimens. The boundary between the recrystallised and the unchanged regions was not easy to detect in most cases, and where found it was diffuse. However, some success was achieved in the LS orientation by deep-etching the surface with nital and viewing under crossed polars. Fig. 8.9 is a plot of the change in lateral spread of the recrystallised zone vs. the crack extension, and clearly shows a sharp increase in the early stages of crack growth. The critical strain for recrystallisation was approximately 0.12.

8.4.4 Implications for the R-Curve

R-Curves as measured here, and elsewhere, on test-pieces with large scale yielding may suggest dramatic increases in toughness with work extension. Experimental evidence shows that the "micro-toughness" at the propagating crack tip remains constant, as shown by the constancy of (i) the propagating tip COD, (ii) the size and shape of microvoids on the fracture surface, and (iii) the strains in the immediate vicinity of the crack tip. However, the extent of the plastic zone is related to the toughness in the different orientations. It may be concluded that the apparent increase in toughness is caused by the spread of plasticity remote from the crack tip.

The error in the hypothesis that the J resistance curve is a direct measure of micro-toughness is presumably caused by the attempt to give J an energetic interpretation during slow crack growth. This is invalid because of the coupling between microstructural and continuum estimates during ductile fracture, as discussed in Section 3.6.

One thing so far ignored is the change in stress and strain singularities at the crack tip. Prior to initiation, they are given by terms of the form $1/r$ (eqns. 3.12, where r is distance from the tip), but during propagation they are replaced by weaker singularities. Rice (1968b and 1973) suggests that there are of the form $\log(1/r)$, and Amazigo and Hutchinson (1977) obtain a $1/r^n$ singularity, with $n \approx 0.2$ for moderate work hardening. Although the strain at the crack tip remains a constant on average, the changing stress singularities there may alter the effective surface energy absorbed in fracture. It is not possible to say what effect such changes will have on the R-curve. However; they ought to be independent of orientation, which is not the case for the R-curve.

The increases in the R-curves are caused by the spread of plasticity away from the crack tip during propagation. Slip line field theory predicts that the size and shape of

plastic zones in generally yielded bars are dependent upon the geometry and loading configuration. This implies that the R-curve should be dependent upon these factors too. In contrast, the considerations of Section 8.3.3 suggest that the early stages of the R-curve should be independent of specimen size, if sufficiently large. This apparent contradiction may possibly be reconciled in terms of the factors controlling the energy dissipation rate. Turner (1978) points out that two geometry dependent factors, η and U , are used when estimating J from formulae such as eqn. (3.18), so that the geometry effect cancels. Hutchinson and Paris (1977) argue that "J-dominated" growth gives unique R-curves. Before crack advance unique stress and strain fields, characterized by J , dominate a region around the crack tip. Some degree of work-hardening is required. For small amounts of crack growth well within this region, the dissipation of energy is controlled by these J-dominated fields. Paris et al. (1977) introduce an ω factor where

$$\omega = \frac{W-a}{J} \cdot \frac{dJ}{da} \quad (8.18)$$

For J-controlled growth, $\omega \gg 1$. If this condition is not met, the R-curve is likely to be geometry dependent.

For the small specimens of En32B in the LT orientation, taking $(W-a_0) = 5\text{mm}$, $J_1 = 100 \text{ kJ/m}^2$, and $dJ/da = 500 \text{ MJ/m}^3$ gives $\omega = 25$ at initiation. On this criterion the initial part of the R-curve should be geometry independent. On the other hand, for the TS orientation at initiation, the equivalent calculation gives $\omega \approx 5$. This indicates that J-controlled growth is less likely without using a larger specimen, and this may be the explanation for the non-linearity observed in the δ_0 R-curves in the transverse and short transverse orientations.

The influence of the microstructure on the initial slope becomes clearer. Plastic dissipation spreads from the tip until the critical strain is reached there, whereupon void coalescence and crack extension occur. The micro-

structure, however, will determine the ease with which this critical strain is reached (through the strain intensification factor of the inclusions), and hence controls the dissipation of energy from the crack tip. This then determines the initial slope of the R-curve.

8.5 Discussion

8.5.1 Theoretical treatments

Various theoretical models have been proposed to describe slow crack growth, although the phenomenon is still not well understood. Wnuk (1971 and 1977) derives expressions for the slopes of J_0 and δ_0 R-curves, for rate dependent materials, by assuming a critical local dissipation of energy by material within the process zone. The analysis only applies to small scale yield. Andersson (1973 and 1974) describes a finite element approach in which the criterion for crack extension is the attainment of a critical crack flank angle. He predicts that the plastic zone size increases initially with crack advance, until steady state conditions are reached.

Broberg (1975) develops his model of the crack tip autonomous region. If the energy flow to this end region is I , then during propagation

$$I = \alpha J,$$

where $\alpha(<1)$ is a screening factor. The R-curve rises during crack growth because of increasing remote plastic work described by decreasing α . This provides a qualitative explanation for the results found here. However, it would predict geometry-dependent R-curves, since the parameter α depends on remote plasticity. Indeed on dimensional grounds α must be $\sim(\text{length})$, which itself implies some size dependence.

Rice (1973) uses a small geometry change analysis to predict the variation of δ_0 with crack growth, assuming that the COD at the propagating crack tip is zero. In a refinement of the analysis, using finite geometry changes,

Rice (1978) gives the opening, δ , between the flanks of the growing crack in terms of distance x back from the tip, assuming steady state conditions:

$$\delta = \frac{\alpha}{\sigma_y} \frac{dJ}{da} \cdot x + \beta \frac{\sigma_y}{E} \cdot x \left\{ \ln\left(\frac{R}{x}\right) + 1 \right\} \quad (8.19)$$

where α is a number, not necessarily constant, probably equivalent at small crack extensions to $1/M$, where M is given by $J = M \sigma_y \delta$;

β is a dimensionless constant ≈ 4 ;

R is a length which scales with plastic zone size - $R \approx 0.16 EJ/\sigma_y^2$ is suggested.

At small x , the second term in eqn. (8.19) dominates and gives a blunted profile to the growing tip. As Garwood et al. (1978) remark, further from the tip the second term may be neglected, so that

$$\delta \approx \frac{\alpha}{\sigma_y} \frac{dJ}{da} \cdot x \quad (8.20)$$

This gives linear crack flanks, in agreement with the experimentally observed profiles. The crack flank angle, Φ is given by

$$\Phi = \frac{\delta}{x} = \frac{\alpha}{\sigma_y} \frac{dJ}{da} \quad (8.21)$$

Substituting the result derived earlier, for crack extensions small compared to specimen dimensions, viz

$$\frac{dJ}{da} = \frac{0.62}{r} \sigma_{flow} A \quad (8.15)$$

$$\Phi = \frac{\alpha \cdot 0.62}{r} \cdot A$$

Taking $\alpha = 0.5$, and $r = 0.45$, gives

$$\Phi = 0.7 A.$$

This is in good qualitative agreement with eqn. (8.17), which gave $\Phi \rightarrow A$ after a small amount of crack growth. The simple treatment of Section 8.3.4 gave Φ approximately constant during crack extension. Rice's treatment implies the same, provided that dJ/da and α are constant. The predicted magnitude of Φ is different in the two approaches, but the

extent of agreement is encouraging.

8.5.2 A Microstructural Model for Ductile Crack Propagation

The major influences on the slope of the R-curves in mild steel appeared to be work-hardening capacity (i.e. prestrain) and inclusion orientation. The effect of prestrain may be explained in a qualitative manner through its effect on ductility and fracture strain. The influence of inclusion orientation will be explained in terms of the strain intensification factor.

It has been suggested here, and elsewhere, that the flank angle, or opening angle, of the propagating crack would be a suitable parameter with which to characterise resistance to crack extension. It is easily measured, since it corresponds to the slope of the COD R-curve, and is a constant over a significant amount of crack extension, at least if the slope is sufficiently large. Rice (1973) points out that on dimensional considerations some length must be included in the critical condition, and suggests (Rice, 1978) that the crack opening, δ , at a finite distance back from the tip should be measured (since his model gives $\delta = 0$ at the tip). This amounts effectively to the same quantity as the flank angle Φ used here, where Φ was derived from the opening at the original, fatigue crack tip. Therefore $\Phi = \Phi_c$ will be taken as the critical condition for crack extension, with the assumption that $\Phi_c = A$ (where Φ_c is the critical flank angle, and A is the slope of the δ_0 R-curve).

The strains ahead of the growing crack tip will be influenced by the shape and spacing of inclusions. The situation during propagation is analogous to that at initiation, but with the critical quantity δ_i replaced by Φ_c . In the same manner as in Section 7.4, the strain, ϵ' , ahead of the crack tip may be written

$$\begin{aligned}\epsilon' &\propto (1 + F) f(\theta) \\ &\propto F.f(\theta)\end{aligned}$$

if the inclusion have a large influence over the strains. The crack growth criterion then becomes

$$\epsilon'_c \propto \bar{\epsilon}_c F \cdot f(\theta) .$$

Employing the same approximations for the strain intensification factor, F , as previously, this may be rearranged

$$\bar{\epsilon}_c = A \propto \epsilon_c \frac{X_0}{\sqrt{L}} \frac{f'(\theta)}{f_1(\rho)} \quad (8.22)$$

where the symbols have the same meaning.

The interpretation of the linkage distance X_0 , however, differs from that at initiation. The propagating crack front is composed of voids or inclusions that have just coalesced, and so will not be straight on a microscopic scale. Consider such an inclusion, as in Fig. 8.10. In general, the inclusion with which it will link next will not be positioned directly ahead. It may also lie out of the crack plane. Void coalescence thus occurs in three dimensions, rather than in the two dimensions which were important at initiation. It is shown in Appendix III that the interinclusion separation is proportional to the semi-axis in the direction under consideration. Thus, with the extra degree of freedom available to the joining direction once the crack has initiated, this direction is likely to be along the line of the smallest inter-inclusion separation available. This will lie in the plane defined by the two semi-minor axes of the inclusions. Since these are of similar order, it will be assumed, as a first approximation, that the factor $X_0 f'(\theta)$ is the same in all six orientations. There may in reality be a factor of $\sqrt{2}$ difference depending on whether linkage occurs in the fracture plane, or at an angle to it, but predictions of this precision are not possible, due to the 3-D nature of the linkage process.

By the same token the projected inclusion length L now refers to the length in the crack plane, perpendicular to the applied tensile stress. This is the same in both the transverse and short transverse orientations, independent of the orientation of the inclusion with respect to the

direction of propagation. This is supported by the re-crystallisation results, Section 8.4.2, where the strains around the crack tip were found to be the same in both the TS and TL orientations. The test-piece orientations may be divided into two categories, in which (i) the inclusion major axes lie normal to the crack plane (LS,LT) and (ii) the major axes lie in the crack plane (TS, ST, TL, SL). Taking the average inclusion semi-axes as $l = 6\mu\text{m}$, $m = 0.6\mu\text{m}$ and $n = 0.4\mu\text{m}$ as before, and applying eqn. (8.22)

$$A \propto [f_1(\rho) \cdot \sqrt{1.0}]^{-1} = 1.0/f_1(\rho) \quad \text{LS,LT}$$

$$A \propto [f_1(\rho) \cdot \sqrt{12}]^{-1} = 0.29/f_1(\rho) \quad \text{TS,ST,TL,SL}$$

In the longitudinal orientations the inclusion length L is taken here as the average of $2m$ and $2n$. The factor $X_0 f'(\theta)$ is assumed to be the same in all orientations. Neglecting the $f_1(\rho)$ factor, the ratio of the predicted slopes, A , in the six orientations becomes

$$(LS,LT):(TS,ST,TL,SL) \approx 3.4:1$$

This compares with the observed ratios of (3.3-4.6):1 taken from Table 8.1 (using the upper bound curves for the LT and ST orientations).

Allowing some effect of frontal radius of curvature would decrease the slopes in the transverse and short transverse orientations. The model is subject to the same assumptions and approximations as was its counterpart for initiation, Section 7.4. Nevertheless, the correct trend in the variation of propagation toughness with inclusion orientation is predicted.

8.6 Summary

R-curves are described in terms of δ_0 and J_0 , $dJ/d(\Delta a)$ decreased with crack extension, whereas $d\delta_0/d(\Delta a)$ remained constant over significant crack extension, provided it had a relatively large magnitude. In mild steel, the R-curves were strongly influenced by the orientation of elongated

inclusions, and by prior cold work, but small changes in grain size had a negligible effect. Relationships are developed between the different parameters δ_o , δ_f and J_o during crack growth, and predictions arising from these relationships show good agreement with observations, particularly in cases where work-hardening was less important.

It is shown that the apparent increase in toughness during crack extension is due to plastic deformation remote from the crack tip. Despite this, the R-curve could still be geometry independent, with certain restrictions, but further investigation is needed.

An approximate model is developed which describes the influence of inclusion orientation on the crack flank angle, and hence on $d\delta_o/d(\Delta a)$. It is emphasised that void linkage during propagation may take place in one of three dimensions. The important inclusion parameters reduce to those in the crack plane, perpendicular to the applied tensile stress.

Table 8.1 Initial slopes, A and $J_0'(0)$, of the R-curves.

Material, condition and orientation	Geometry and configuration (1)	A measured	$J_0'(0)$ MJ/m ³	
			measured	predicted (3)
<u>En24</u>	<u>Plane-sided</u>			
Quenched and tempered, LS	W=8=24mm W=24, B=12mm	0.4 0.4-0.85	470 470-810	320 320-680
<u>En328</u>	<u>Side-grooved</u>			
Normalised LT	CCT (2) W=8=24mm (1)	0.2-0.3	150-200	-
LT		0.50	310	250
LT		0.40-0.70	270-540	190-340
LS		0.69	590	340
TS		0.21	83	94
ST		0.12-0.18	44-78	53-83
TL		0.15	98	71
SL		0.17	92	81
Slow-cooled LS		0.8	590	320
TS		0.18	83	60
Prestrained } LS		0.50	670	350
10% } TS		0.02	12	6
Prestrained } LS		0.50	407	457
20% }		0.02	13	13

- (1) Unless otherwise stated, three point bend, $W=B=10\text{mm}$
 (2) Centre cracked tension, $W = 50\text{mm}$, $B = 24\text{mm}$.
 (3) From eqn. (8.13).



Plate 8.1. Crack tip profiles in En 32B, recrystallised, etched nital.

(a) LS orientation, x 110

(b) TS orientation, x 150

(c) TL orientation, x 370

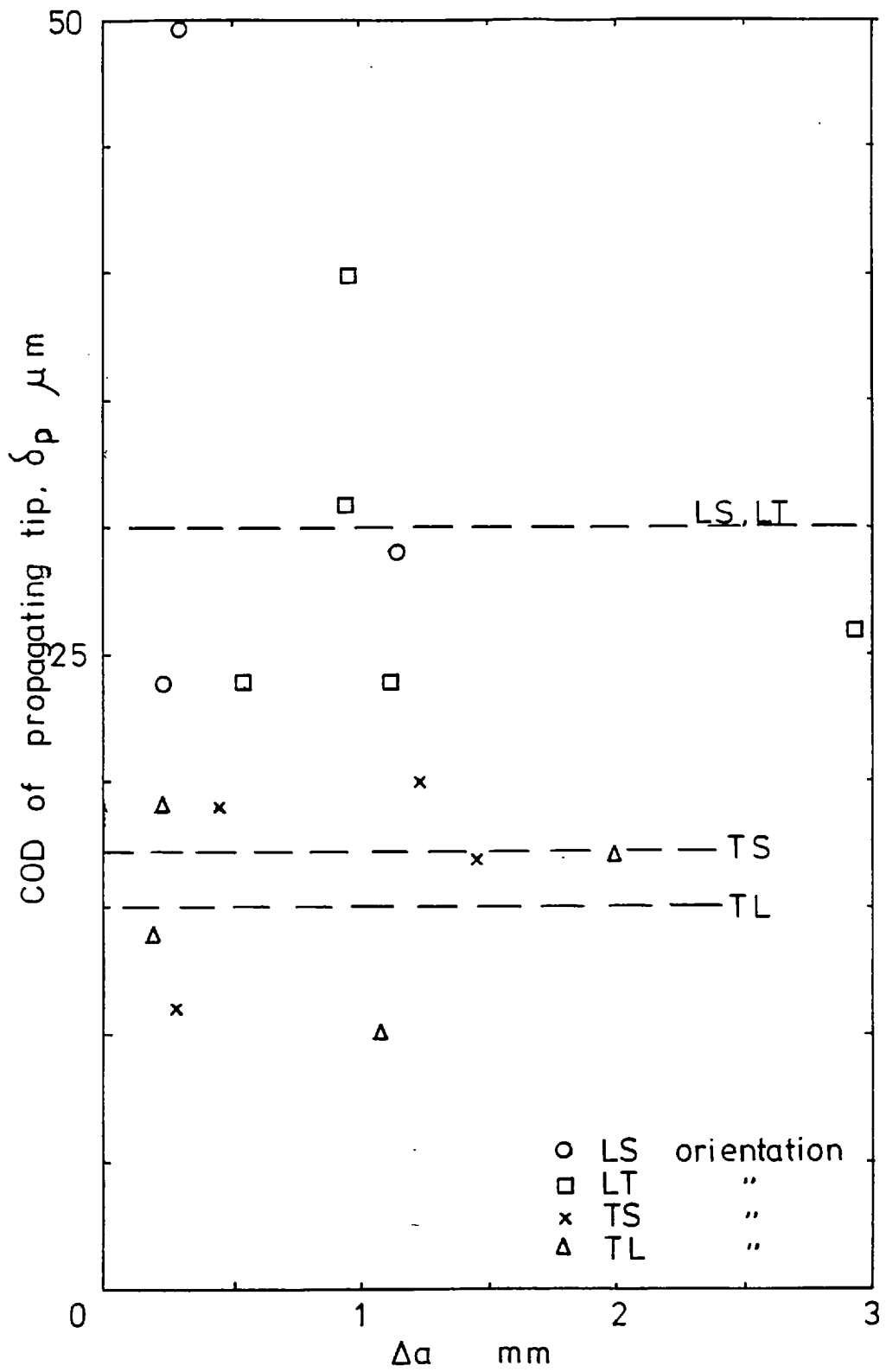


Fig. 8.1 Variation of δ_p with Δa , En32B.

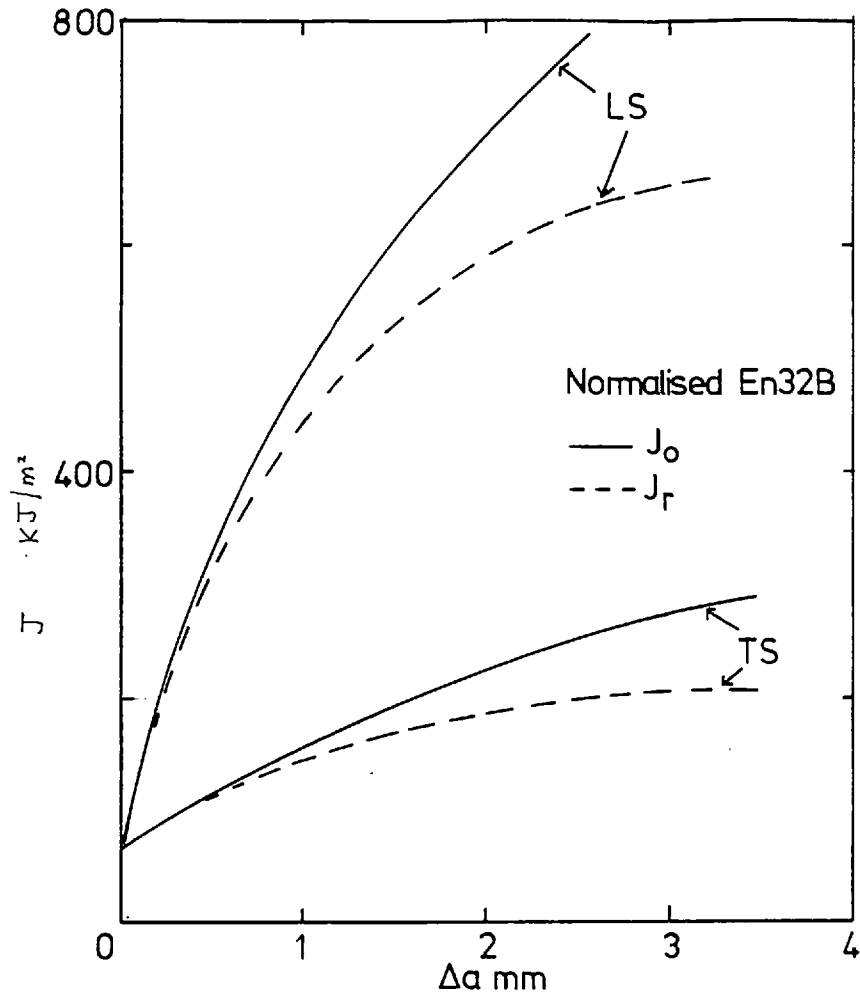


Fig. 8.2 Comparison between J_0 and J_r for En32B. Data points omitted for clarity.

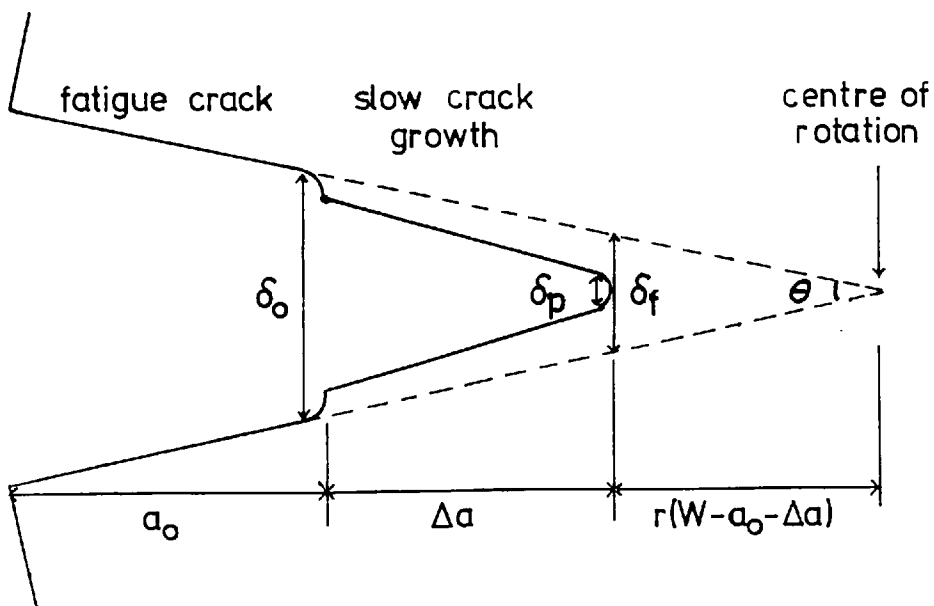


Fig. 8.3 The geometrical relationship between different CODs in three point bend.

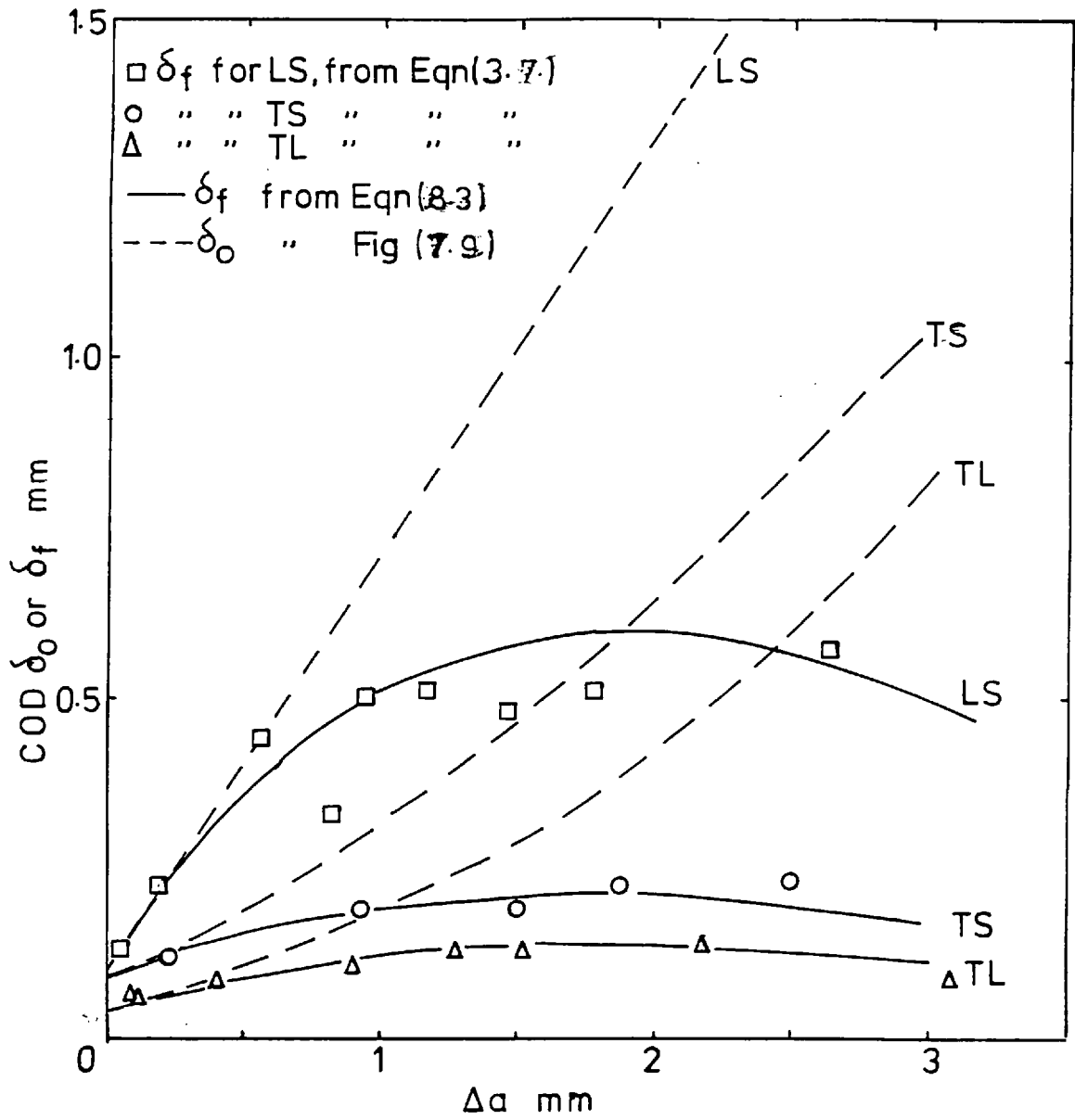


Fig. 8.4 Comparison between measured and predicted values of δ_f in En32B.

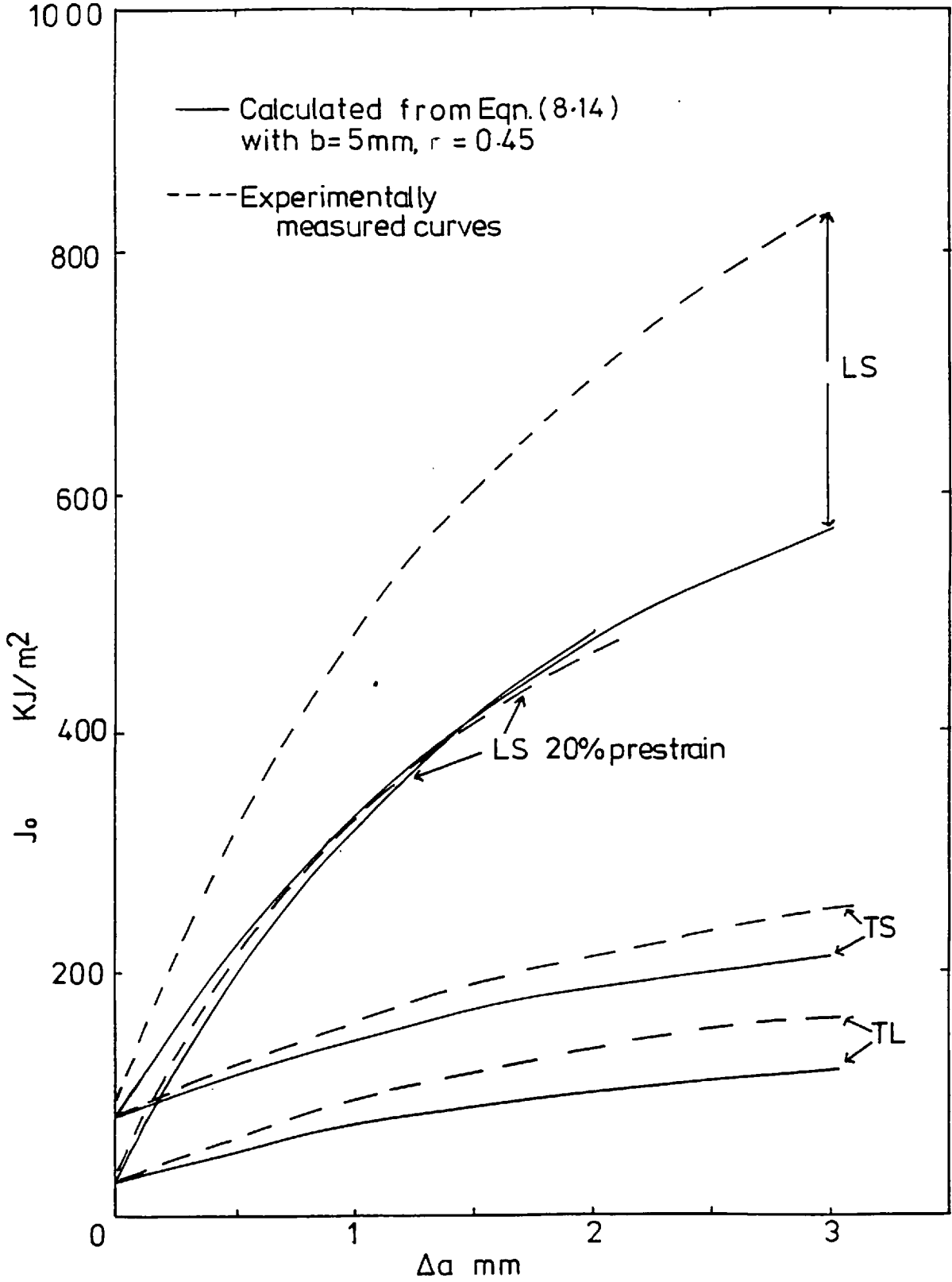


Fig. 8.5 Comparison between measured and predicted values of J_0 , for En32B.

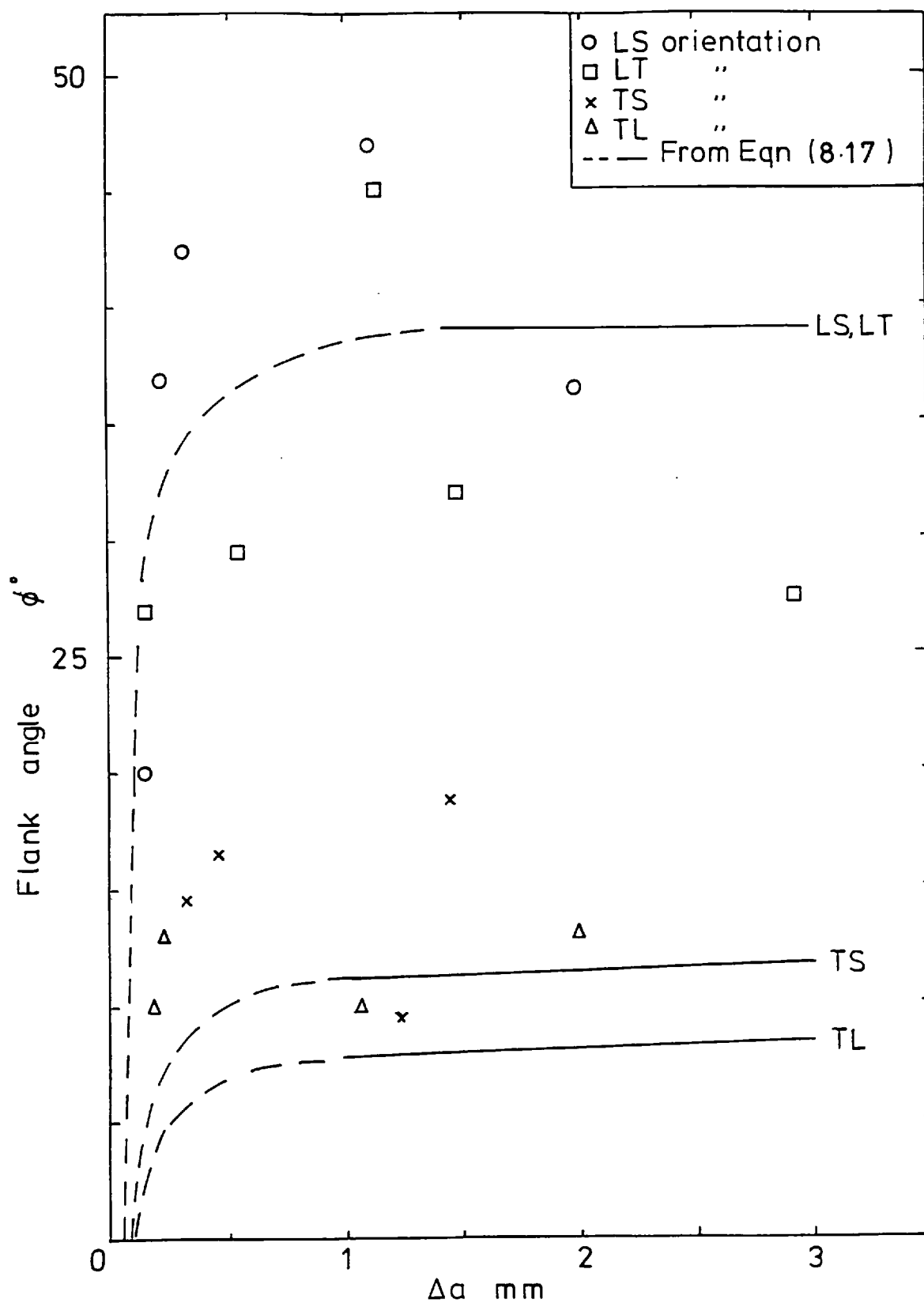


Fig. 8.6 Variation of flank angle with Δa , En32B.

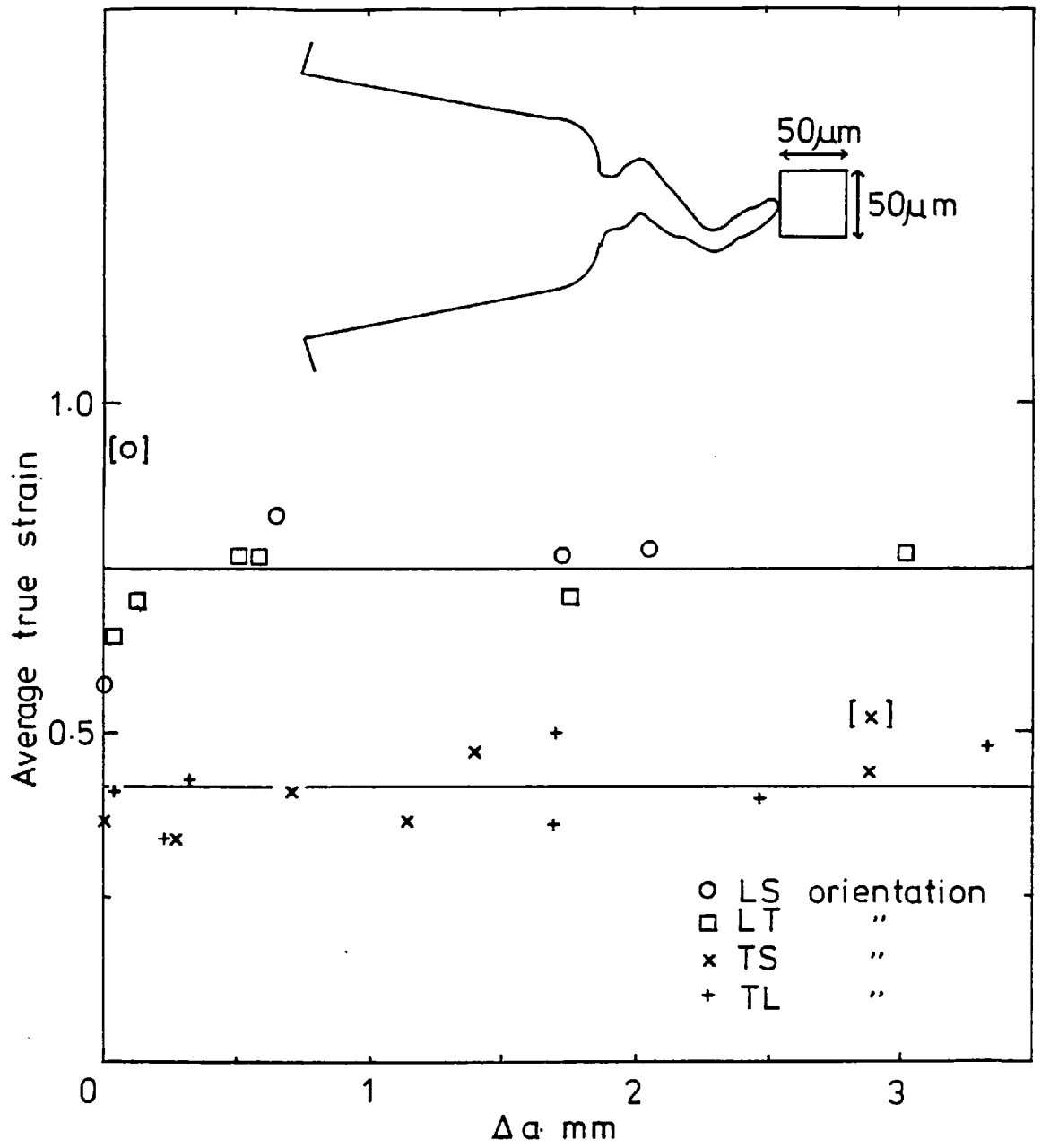


Fig. 8.7 Variation with Δa of true strain in a region 50 μm square at the crack tip, as estimated by recrystallisation.

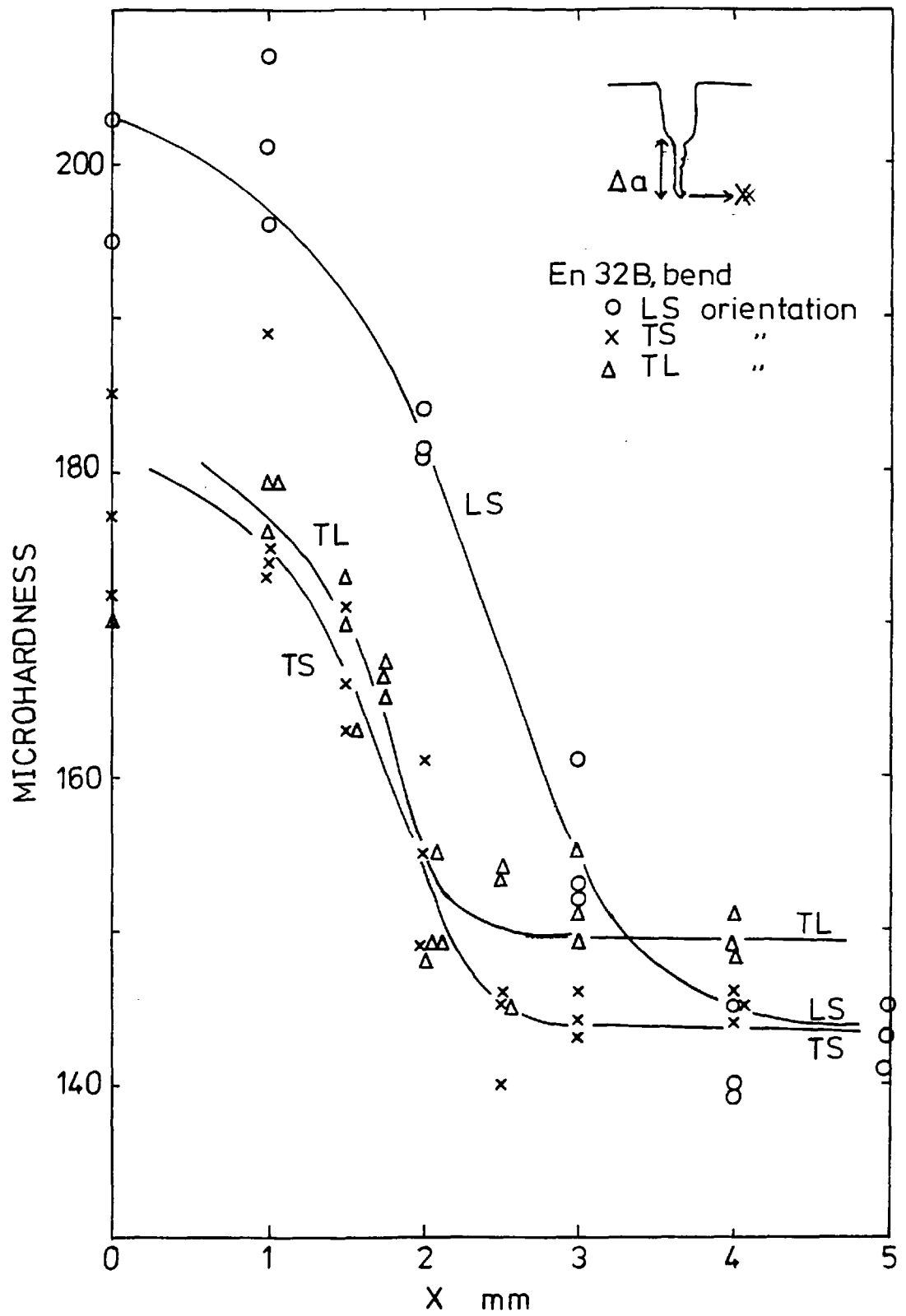


Fig. 8.8 Microhardness profile around crack tips in En32B, showing lateral extent of the plastic zone. $\Delta a = 1\text{mm}$.

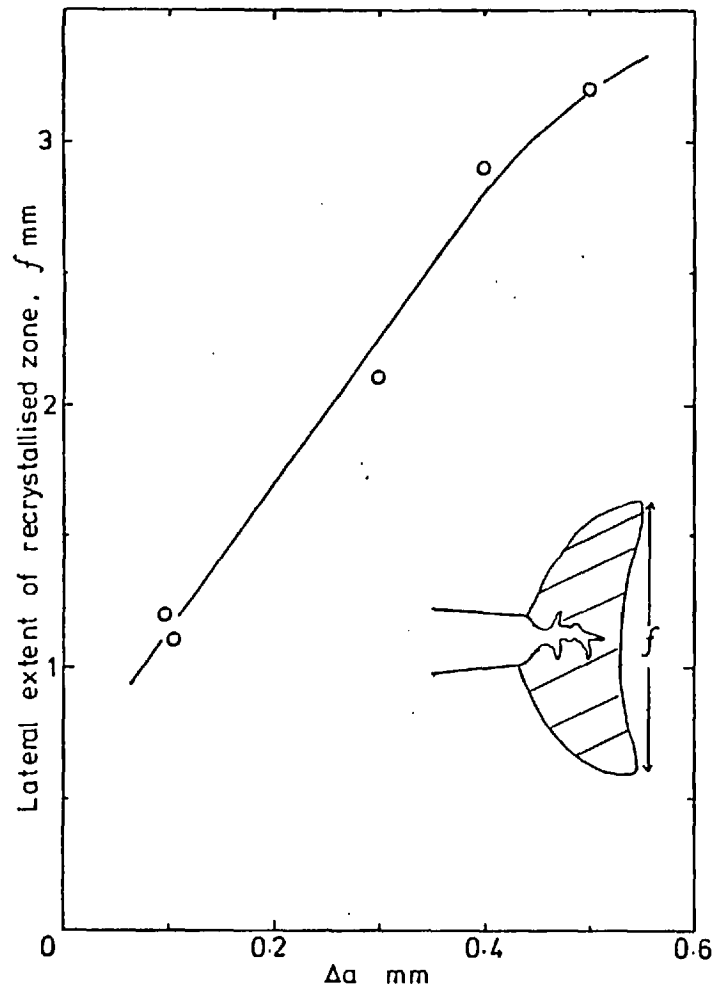


Fig. 8.9 Growth of the recrystallised zone in En328.

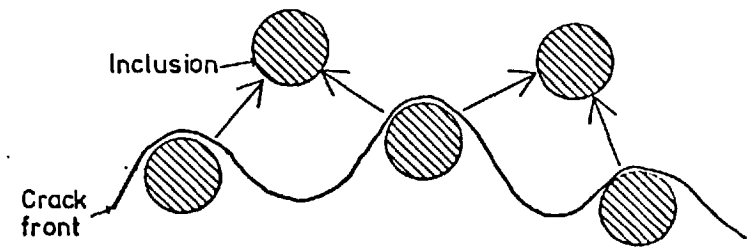


Fig. 8.10 Sketch of the crack front during propagation.

Chapter Nine

FURTHER ASPECTS OF DUCTILE CRACK PROPAGATION

9.1 Introduction

This chapter is concerned with miscellaneous factors such as specimen thickness, shear lip development and side-grooves. The effect of these on toughness, rotational factors and overall specimen compliance is investigated.

9.2 The Growth of Shear Lips

9.2.1 The Krafft et al. (1961) Model

The presence of shear lips has long been recognised as a cause of high fracture resistance, and explains variation of toughness with specimen width. Krafft, Sullivan and Boyle (1961) proposed an explanation for this phenomenon. Fig. 9.1 shows a typical section through a fracture surface, where the central region of flat fracture is bordered by 45° shear lips. Krafft et al. assumed that the work of fracture is composed of two terms:

(i) The surface work associated with extending the crack, taken to be a constant per unit area of crack advance. This is equivalent to assuming that the R-curve is horizontal in flat fracture.

(ii) The work needed to deform the volume of the shear lips.

Then for crack advance Δa , the work done is

$$\Delta W = \frac{dW}{dA} \cdot B\Delta a + \frac{dW}{dV} \frac{(B-f)^2}{2} \cdot \Delta a \quad (9.1)$$

where dW/da is the surface work rate, dW/dV is the energy density per unit volume of the shear lips (both assumed constant), B is the specimen width and f the width of flat fracture surface. $(B-f)^2 \cdot \Delta a / 2$ represents the volume of both shear lips (including any contraction of specimen width), assuming they are bordered by 45° shear lines.

From eqn. (9.1), the fracture resistance is

$$R = \frac{\Delta W}{B\Delta a} = \frac{dW}{dA} + \frac{dW}{dV} \cdot \frac{(B-f)^2}{2B} \quad (9.2)$$

By plotting R vs. $(B-f)^2/2B$ a straight line of slope dW/dV and intercept dW/dA should result. Quite good agreement was obtained for two thicknesses of aluminium sheet.

In extending the model to more ductile materials, Garwood (1977) points out that the R -curve for flat fracture is not constant but increases with Δa . This implies that dW/dA increases. In terms of the J -resistance curve, eqn. (9.2) is rewritten,

$$J_r = (J_r)_I + \frac{dW}{dV} \cdot \frac{(B-f)^2}{2B} \quad (9.3)$$

where J_r is the total resistance with shear lips, and $(J_r)_I$ is the resistance in flat fracture. The possibility then exists of adding the shear lip contribution to the R -curve for flat fracture, to obtain the R -curve for mixed mode fracture. Reasonable agreement was obtained for different thicknesses of bend specimen.

In a further modification, Green and Knott (1975b) assume that the "plane strain" resistance only refers to the flat fracture region, of width f . Eqn. (9.3) would then become

$$J_r = (J_r)_I \cdot \frac{f}{B} + \frac{dW}{dV} \cdot \frac{(B-f)^2}{2B} \quad (9.4)$$

This correction becomes significant as the shear lips become large.

9.2.2 Experimental Results

The R -curves for the plane-sided bend specimens of En24, in two thicknesses (12 and 24mm), were shown in Figs. 7.1 and 7.2, and were discussed in Sections 7.2.1 and 8.2.1. The J_0 R -curves were not corrected for the effects of crack growth upon the J -integral, and applying the three parameter formula (described in Appendix II) yielded the curves shown

in Fig. 9.2. This formula only gave a significant correction for $\Delta a > 3\text{mm}$.

It was noted that the thicker specimens had a single R-curve, whereas the thinner ones exhibited considerable scatter, falling principally onto two curves; the lower curve coincided approximately with that for the thicker test-pieces. Plate 9.1(a) shows the fracture surface of a 12mm thick specimen in the tougher category. Comparing it with a specimen in the less tough category, Plate 9.1(b), shows that the tougher one had horizontal splits on the fracture surface, together with a more fibrous surface, and also showed greater development of shear lips at a given crack extension. The splits are similar to those observed in the En32B, except that here they coincided with a "rougher" surface and caused an increase in toughness. The latter is reasonable since they would act as "crack arrestors". The larger test-pieces had a single resistance curve, and splits were not found on their fracture surfaces, Plate 9.1(c). The explanation for the splits is again uncertain, but may be non-uniformity of the inclusion distribution.

Differences in shear lip development are likely to have more influence than splitting. To assess these differences, the width of flat fracture, f , was measured on each specimen, and a plot of the shear lip width, $B-f$ (including lateral contraction) against Δa revealed that there was a trend to smaller shear lips in the less tough test-pieces (Fig. 9.3). Re-arranging eqn. (9.4)

$$J_r \cdot \frac{B}{f} = (J_r)_I + \frac{dW}{dV} \frac{(B-f)^2}{2f} \quad (9.5)$$

In order to fit the results to this equation, a constant value of $(J_r)_I$ must be assumed (i.e. a horizontal R-curve for flat fracture). Plotting $J_r \cdot B/f$ against $(B-f)^2/2f$ should then give a straight line. This is shown in fig. 9.4, and a least squares fit gave the intercept $(J_r)_I$ as 370 kJ/m^2 , and the slope dW/dV as 350 MJ/m^3 , with a correlation coefficient of 0.988.

Garwood (1977) showed that in fact $(J_r)_I$ for this steel in a higher strength condition is not constant, but rises with crack extension. Nevertheless, the Krafft model can still give good results. The reason must be the dominant effect of the shear lips on fracture resistance. However, the value of $(J_r)_I$ inferred from the model is, at 370 kJ/m^2 , much higher than J_i ($\approx 100 \text{ kJ/m}^2$) and reflects the fact that $(J_r)_I$ increases during crack extension.

9.2.3 Discussion

The larger plastic zone size in plane stress than in plane strain implies that at a given overall displacement, the local strains in the former state will be lower. This was confirmed by the results from the brass model, Section 7.3, and explains the observed tunnelling of the flat fracture region in the centre of a test-piece. If it is assumed that the intense deformation ahead of the crack tip is accommodated over some region, the size of which depends on plastic zone size, the larger region in plane stress will result in a smaller strain, and hence a larger critical COD. The difference in intense strain between the two stress states is indicated as a factor of 2.5 from Fig. 7.16. This explains why the crack initiates first in the plane strain, mid-section, and later at the edges, giving the characteristic "thumb-nail" appearance.

The difference in the appearance of the fracture surface in the 12mm thick bars, together with the difference in shear lip development, suggests that the growth of shear lips is linked to the variation of toughness in plane strain. To test this, two 10mm square bars of En32B, prestrained 20%, were notched and fractured in slow bend. One specimen was cut in the LS orientation, and the shear lips grew from negligible size, to occupy finally 30% of the width of the fracture surface. The other was cut in the TS orientation, and the shear lips grew to a maximum of 5% of the fracture surface. It was shown in Chapters 7 and 8 that these two orientations had closely similar toughnesses at initiation, but different R-curves for flat fracture. The implication

is that the plane strain R-curve is linked to the (plane stress) shear lip development, a reasonable result since both depend on remote plastic deformation.

9.3 The Rotational Factor in Three Point Bending

9.3.1 Introduction

In order to infer a value of COD from clip gauge displacement in bend specimens, a centre of rotation must be assumed. Harrison (quoted by Wells, 1971), Ingham et al. (1971) and Archer (1975) all found that, for three point bend specimens slowly loaded, the rotational factor, r , rises from about 0.1 to 0.45 as the extent of yielding increases. A value of 0.45 for fully plastic bars in plane strain was suggested by Wells (1963) on the basis of a slip line field analysis, and was incorporated into his formula for COD. Ingham recommended using $r = 1/3$ as this gave conservative estimates of COD for all but the smallest deflections. All these measurements were surface values, determined from the displacements of points on the notch flanks. Mokka et al. (1974) obtained r from microhardness readings on polished sections and found it lay between 0.38 and 0.42, with no detectable difference between surface and mid-section values.

Relatively few investigations have been undertaken to determine the rotational factor during slow crack growth. Kamath and Harrison (1977) deduced r from clip gauge and load point displacement, obtaining ≈ 0.3 for small crack extensions in a high strength steel. Garwood (1977), using rubber infiltration of the crack, found it decreased from 0.45 to 0.4 as the crack extended, for fully plastic side-grooved specimens of mild steel.

If COD is to be used to characterise slow crack growth it is important to know how the rotational factor varies, both in plane strain and in the presence of shear lips.

9.3.2 Experimental Methods and Results

In theory it is possible to derive the rotational factor from clip gauge and load-point displacement. The angle of bend is (referring to Fig. 3.2)

$$\theta = V_g / \{z + a + r (W-a)\} \quad (9.6)$$

where V_g is the clip gauge displacement. θ may also be obtained from the load-point displacement q , via

$$\theta = 4q/S = q/W$$

where $S (=4W)$ is the span of the rollers. Combining these gives

$$r = \frac{WV_g/q - (z + a)}{W-a} \quad (9.7)$$

This approach, however, is inherently inaccurate, as may be seen by assuming that, for instance, $r = 0.45$, $a = 0.6W$, $z = 0.1W$. From eqn. (9.7), this gives

$$V_g/q = 0.78$$

and the numerator of eqn. (9.7) becomes $\{0.78W - 0.7W\}$. Thus this equation requires the subtraction of two closely similar numbers, and any small inaccuracies in V_g , q , z or a are magnified. Nonsensical results were obtained by applying the equation to small specimens.

Thus the technique of scribing lines on the side faces of the specimens was employing. This has the disadvantage of giving surface results only, but provided measurements are made in the elastic portion, they should apply to the specimen as a whole. Photographs were taken of the scribed grid before and after bending, under load in the latter case. Plates 9.2(a) and (b) show the deformed grids on a plane-sided and on a side-grooved specimen, respectively. By measuring the initial position of the grid intersections, and their subsequent displacements, plots such as those in Fig. 9.5 could be drawn. The centre of rotation could then

be located as the point on the axis which underwent zero displacement. If the distance between this point and the crack tip is x , the rotational factor r is, by definition,

$$r = x/(w-a) \quad (9.8)$$

where the crack length, a , includes any slow crack growth.

This method was used on the bend specimens of En32B, in the LT orientation, which had $W=B=24\text{mm}$. The majority contained deep side-grooves. It was also applied to some plane-sided test-pieces, also of En32B, but in the LS orientation and with $W=24\text{mm}$ and $B = 12$ or 24mm . R-curves were not derived for these specimens.

The variation of rotational factor with crack extension is shown in Fig. 9.6. For the side-grooved specimens it falls fairly regularly from 0.5 at initiation to 0.35 after 4mm of crack extension. This trend is confirmed by the plane-sided test-pieces, but the magnitude of r is generally somewhat smaller, and the scatter is considerable. The main cause of this scatter was probably errors in crack length determination. Measurements were made at only three points along the crack front. The fatigue cracks were bowed, and the curvature increased with crack extension as the centre tunnelled forward. This would tend to increase the errors in crack length measurement. The trend to lower values of r in the presence of shear lips is confirmed, however, by the plane-sided LT specimen (closed circle, Fig. 9.6), where the crack length was measured at nine places along the front.

Since COD is derived by means of the rotational factor, it is important to allow for significant variations in r . A comparison between δ_o using $r = 0.45$, and δ_o using the measured r , is presented in Fig. 9.7, for the LT, 24mm thick specimens. For side-grooved specimens the correction is slight, except at large Δa , and would raise the value of δ_i by 10-15%, and decrease the slope by 5-10%. The correction is much greater for the plane-sided test-piece, as would be expected from the low rotational factor

obtained there. On the basis of these results it is recommended that a rotational factor of 0.45 be used when determining the COD at initiation. This would yield slightly conservative values for δ_i .

9.3.3 Discussion

Previous work, supported by the results of Chapter 7, has shown that either COD or the J-integral may be used as a toughness parameter at initiation. In Section 8.3 the equivalence of the two parameters during slow crack growth was demonstrated. In both cases a constant rotational factor was assumed, and this appears to be fairly closely maintained for crack extension by flat fracture. Thus when measuring plane strain R-curves either of the two parameters may be measured and should give consistent results, independent of specimen thickness.

This is not so in the presence of shear lips. There is considerable variation in rotational factors between specimens, with a tendency for r to decrease with crack extension. The assumption of constant r would then yield δ_o R-curves with too steep a slope, suggesting a higher resistance to crack propagation than the reality. This implies that the COD, as usually estimated in bending, is unsuitable for characterising resistance to slow crack growth in the presence of significant plane stress fracture, except at initiation. In this situation the J-integral approach seems more promising, particularly in view of the possibility of adding flat fracture and shear lips contributions.

9.4 The Measurements of Crack Length

9.4.1 Methods Available

The sole method used here for measuring crack lengths was that of breaking the specimen open in the cleavage mode and examining the fracture surface. The region of fibrous crack growth could also be marked by ink-staining, heat-tinting or fatigue. This direct method is unambiguous, but many specimens are required to develop an R-curve. In some

cases, too, cleavage did not initiate from the crack tip, but at some point on the flank, rendering impossible the location of the final crack tip. It would be advantageous to monitor crack lengths while the test was in progress, enabling an R-curve to be obtained from a single specimen.

Several methods are in current use. Potential drop (P.D.) techniques have been used with apparent success on high strength steels. Ritchie (1972), however, showed that direct current P.D. could not be used to detect initiation in mild steels due to the effects of plasticity and void formation. Chipperfield (1976) found that the alternating current method gave even worse results, for 316 stainless steel. Acoustic emission is a promising technique for detecting crack initiation, but is still in the development stage, and in any case could not give crack lengths. Ultrasonic techniques also show promise, but are extremely expensive and specimen geometries on which they may be used are limited. Unloading compliance was used successfully on large test-pieces by Heyer and McCabe (1972) and Kamath and Harrison (1977), but Chipperfield (1976) found it underestimated crack extensions in smaller test-pieces. However, since it is a very simple and inexpensive technique, the effect of crack extension on compliance is examined further below.

9.4.2 Experimental Results

R-curve measurements yielded traces of load vs. clip gauge displacement and load vs. load-point displacement for each test-piece. The elastic unloading lines of the latter traces exhibited pronounced curvature, due in part to extraneous displacement effects. The curvature of the equivalent clip gauge trace was considerably less, and was probably caused by reversed plasticity. These traces were thus considered more suitable for the measurement of compliance. This was obtained from the slope of the tangent to the unloading curve, as shown in Fig. 9.8, using the initial (upper) part where the effect of reversed plasticity should be least.

Fig. 9.9(a) shows the variation of clip gauge compliance, C_V , with final crack length, a_f , for 24mm square bend bars of En32B, all but one of which had side-grooves. The error in the measured compliances was estimated at $\pm 5\%$, but despite this there is a significant trend to lower compliances with larger crack extensions, at constant a_f . The scatter caused by crack extension would prevent the compliance being of use for monitoring crack growth, in low strength materials of this specimen size: the largest crack extension ($\Delta a \approx 4.5\text{mm}$) would be underestimated by some 20%.

In Section 9.3 it was shown that the rotational factor decreased with crack extension for these specimens. The effect of this would be to decrease the mouth opening, and hence the clip gauge compliance. To remove this effect, the load-point compliance, C_q , was calculated using the measured value of rotational factor. Rearranging eqn. (9.7) and substituting $C_q = q/Q$ and $C_V = V_g/Q$ (where Q is the load)

$$C_q = C_V \frac{W}{z + a_f + r(W - a_f)} \quad (9.8)$$

C_q is plotted against a_f in Fig. 9.9(b). This shows that the correction had little effect. The trend of decreasing compliance with increasing crack extension remains.

The plane-sided specimen (closed squares in Fig. 9.9) shows a greater compliance than the side-grooved ones at the same final crack length, and the difference is accentuated when the rotational factor is taken into account, Fig. 9.9(b). To examine further the effect of shear lips, the clip gauge compliances for the En24, plane-sided, bend test-pieces were also calculated and plotted in Fig. 9.10. The 24mm thick specimens show a slight trend of decreasing compliance with increasing crack growth, but the scatter in compliance is small, presumably because the plastic zone is smaller in En24 than in En32B and the shear lip fraction is small. For the 12mm thick specimens, however, the variation is much larger. High compliances are associated with high shear lip fractions, particularly in specimens whose toughness lay on the higher

R-curve, referring to Figs. 9.2 and 9.3.

A larger compliance may be expected in plane stress than in plane strain, since in the former state the stiffness is proportional to Young's modulus, E , whereas in the latter state it is proportional to $E/(1-\nu^2)$, which is some 10% higher for steel. Thus if larger shear lip fractions imply that a greater proportion of the thickness is in plane stress a higher compliance would be expected.

9.4.3 Discussion

The unloading compliance method for monitoring crack extension was found to be unsuitable for grossly yielded test-pieces of highly ductile materials, or where shear lips occupy a significant proportion of the fracture surface. It is more promising, however, for higher strength materials and specimens with smaller shear lips.

The clip gauge compliance has the advantage that there are no extraneous displacement effects due to roller indentations, jig deflections and elastic bending of the specimen arms, but since it varies with rotational factor, r , an allowance should be made for the variation of r with crack extension. The double compliance technique developed by Heyer and McCabe (1972) has the correction built in. Alternatively, if the true load-point displacement, corrected for extraneous effects, can be obtained, using for instance the experimental arrangement of Dawes (1976), the load-point compliance could be used equally well, and with greater facility.

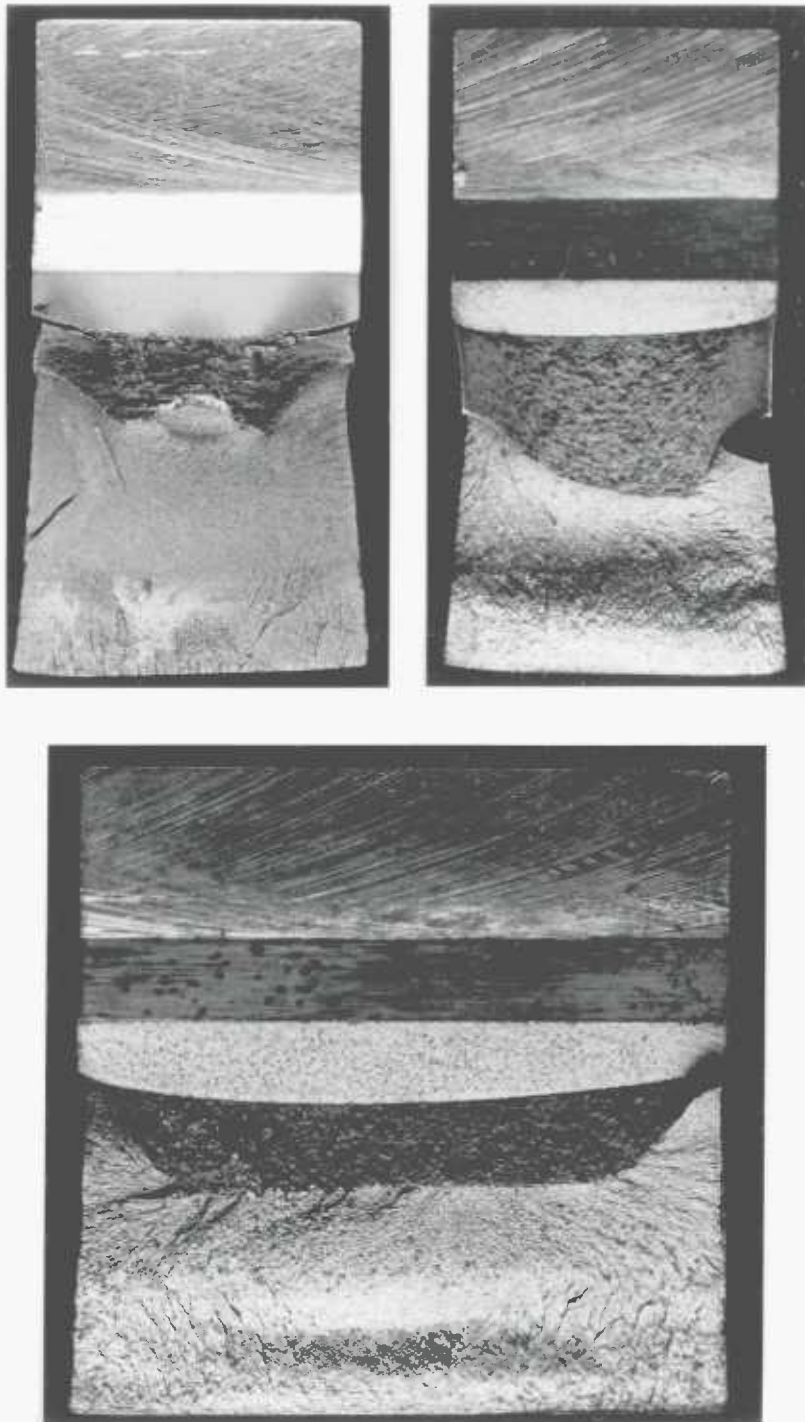


Plate 9.1. Fracture surfaces in En 24, showing development of shear lips, x 3.8

(a) 12mm thick, tough specimen

(b) 12mm thick, less tough specimen

(c) 24mm thick specimen

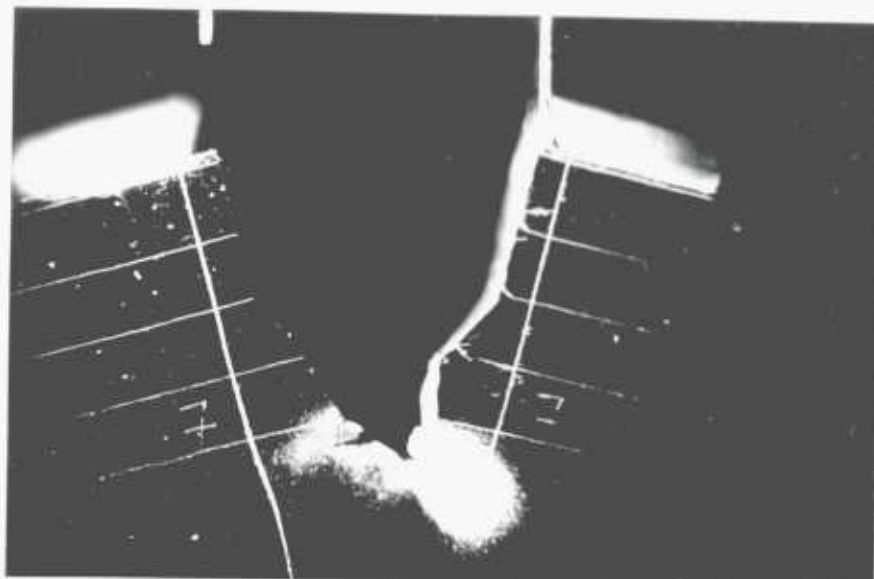


Plate 9.2. Grids on surface of bend specimens of En 32B, used for deriving rotational factors.

(a) Plane sided, x 3.5

(b) Side-grooved, x 3.3

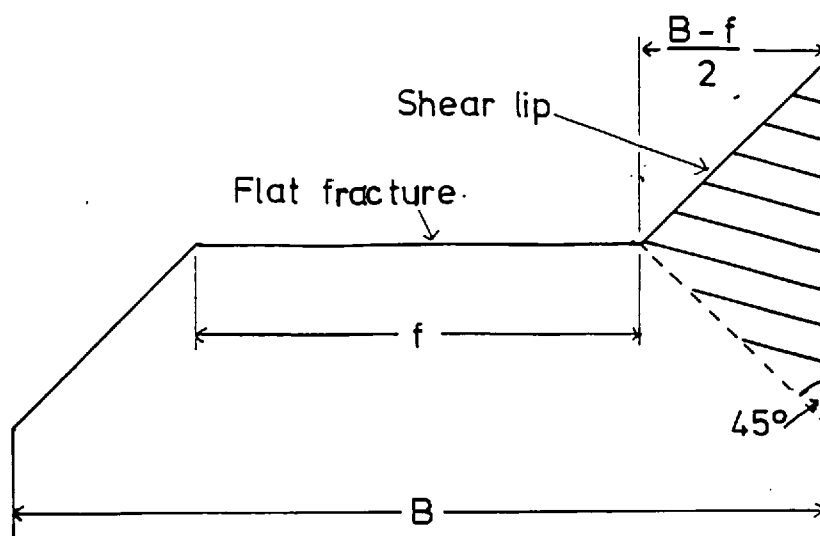


Fig. 9.1 Section through the crack plane showing shear lips, (Krafft et al., 1961).

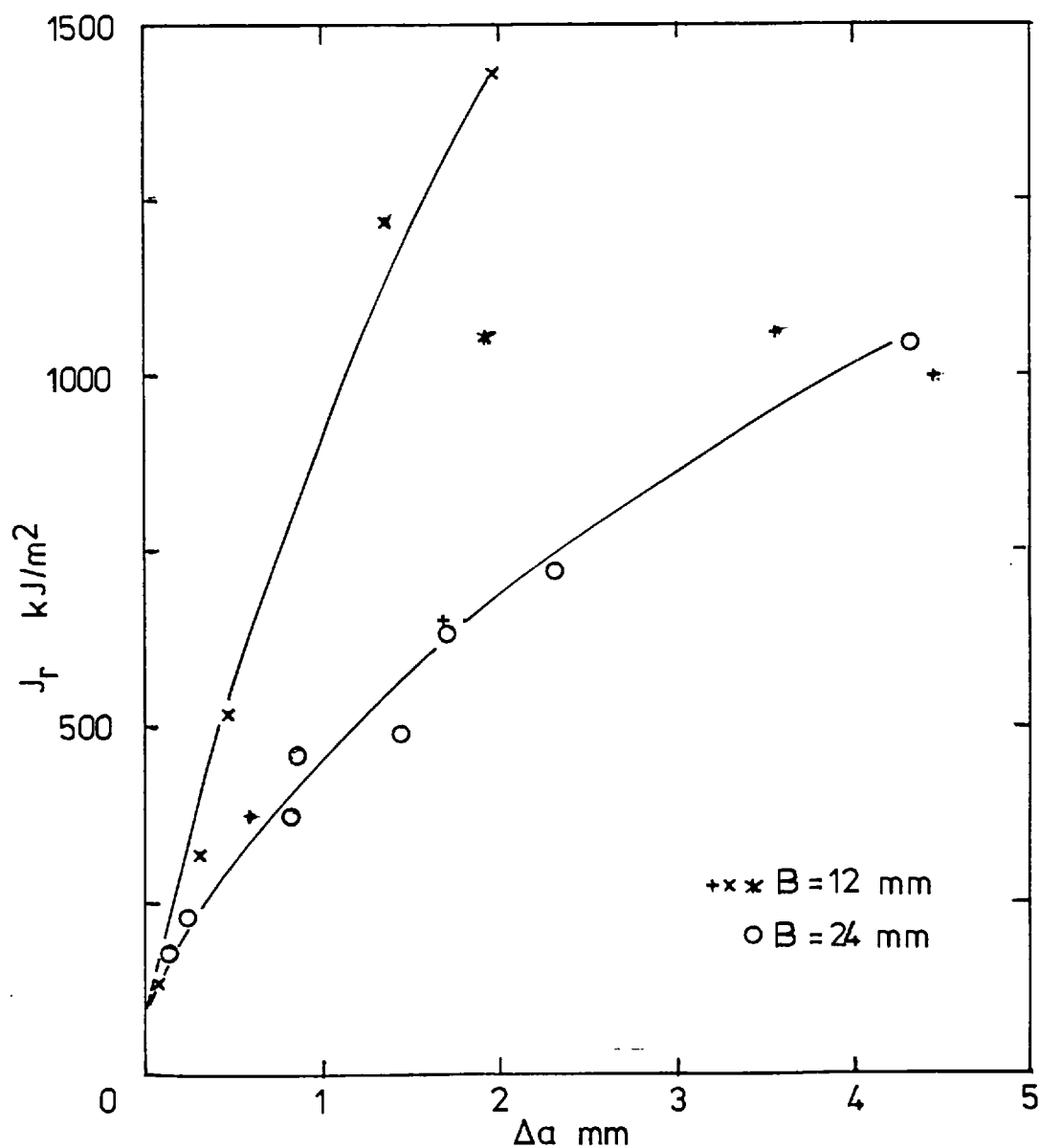


Fig. 9.2 Variation of J_r with Δa , for En24, in bending.

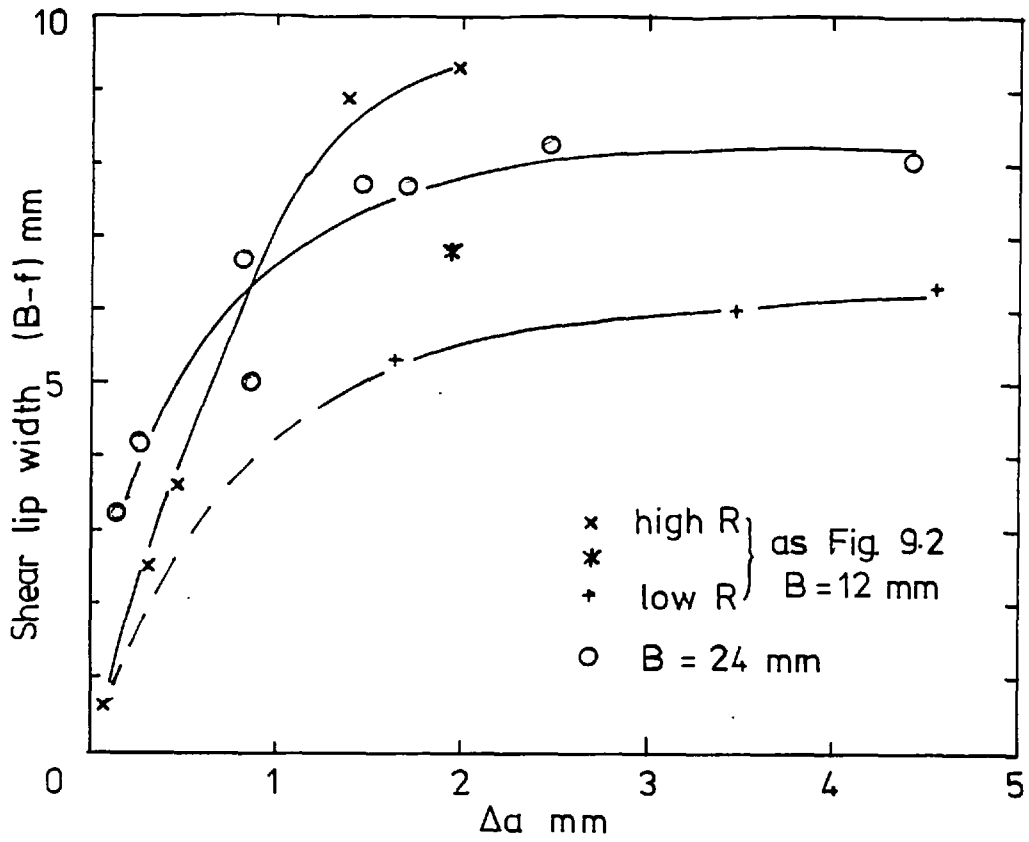


Fig. 9.3 Variation of shear lip width with Δa , for En24.

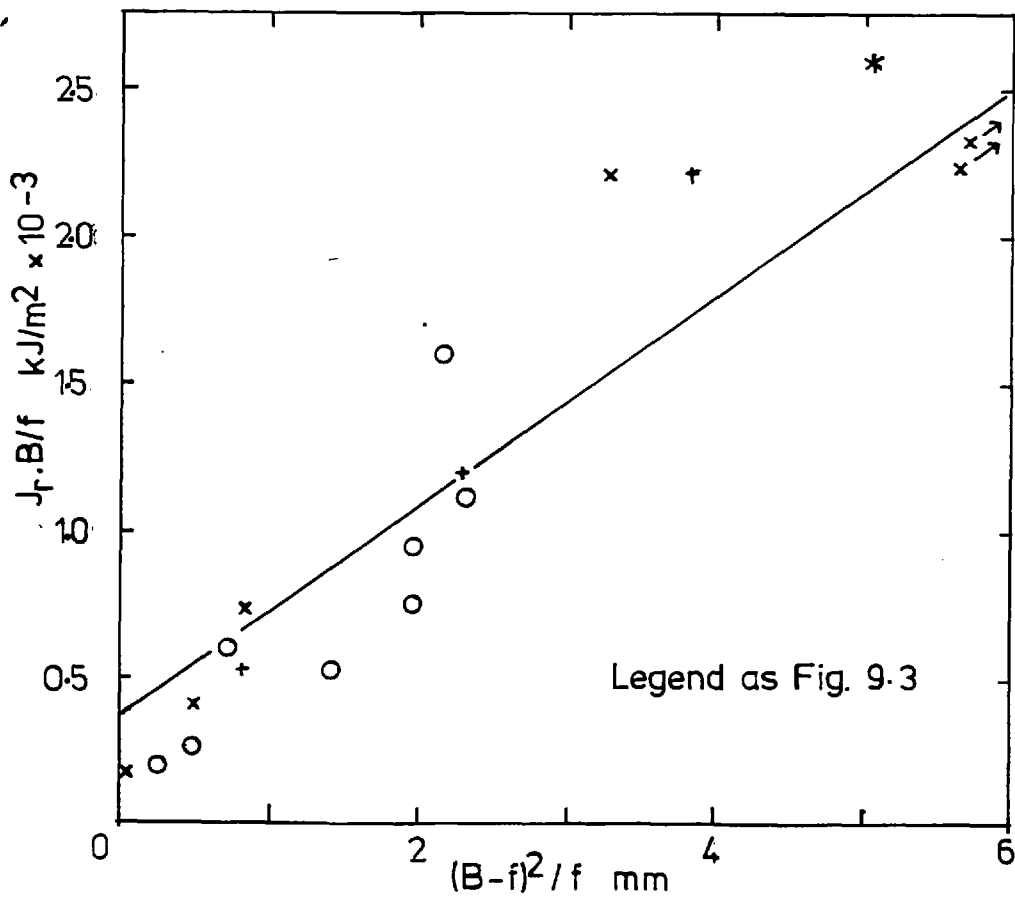


Fig. 9.4 Variation of J_r with shear lip fraction, En24.

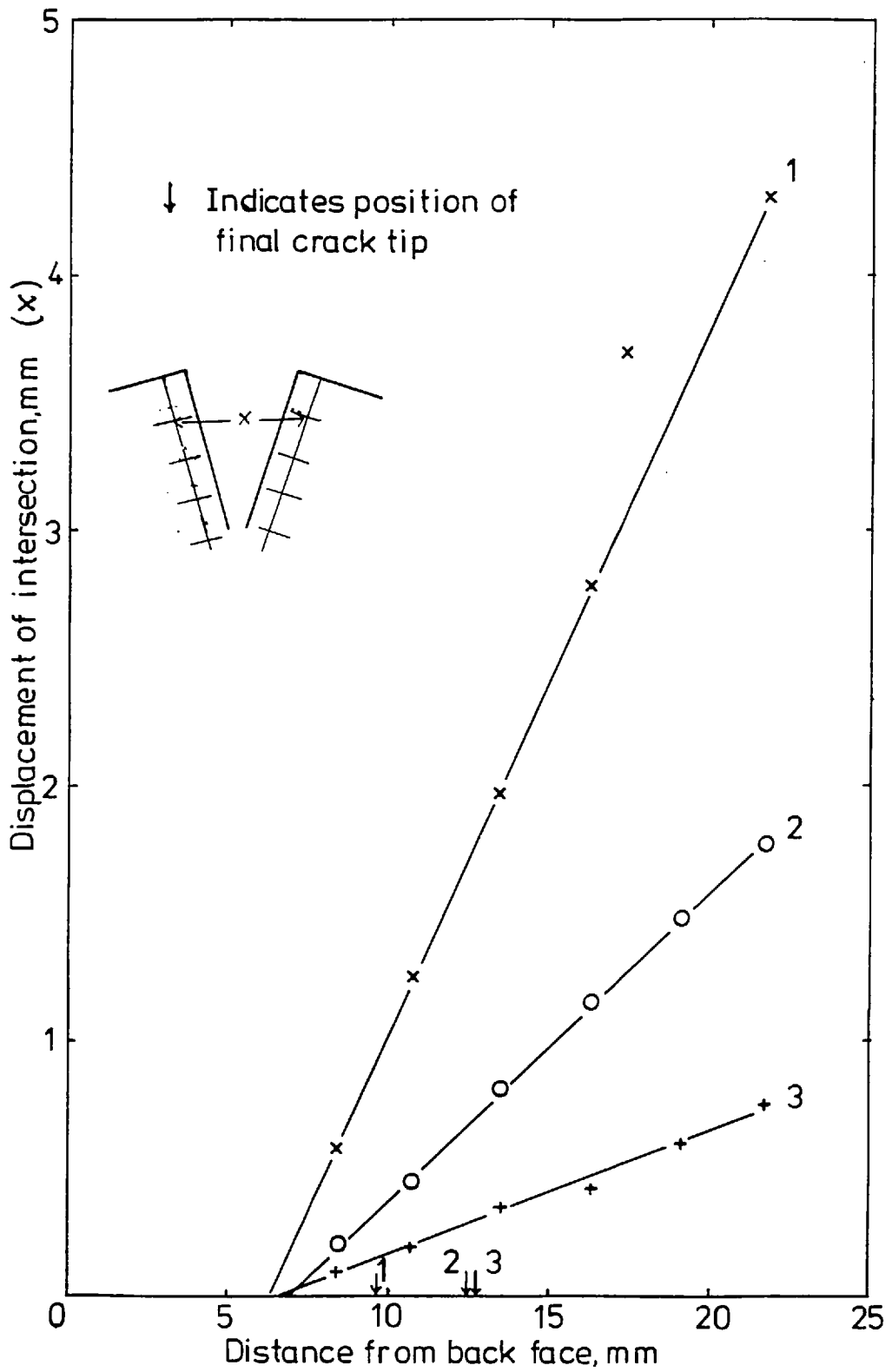


Fig. 9.5 The experimental derivation of rotational factor,

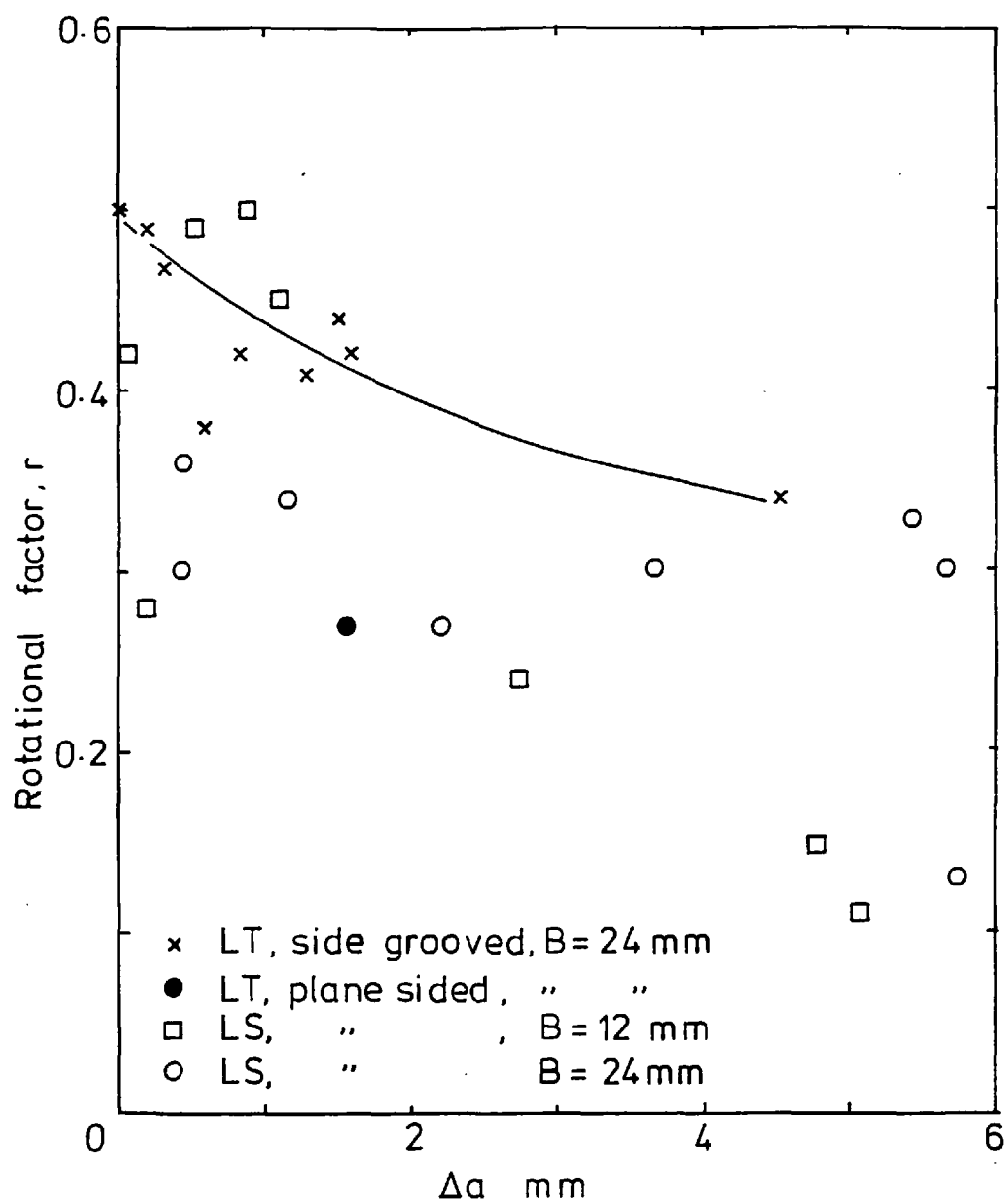


Fig. 9.6 Variation of rotational factor with Δa .

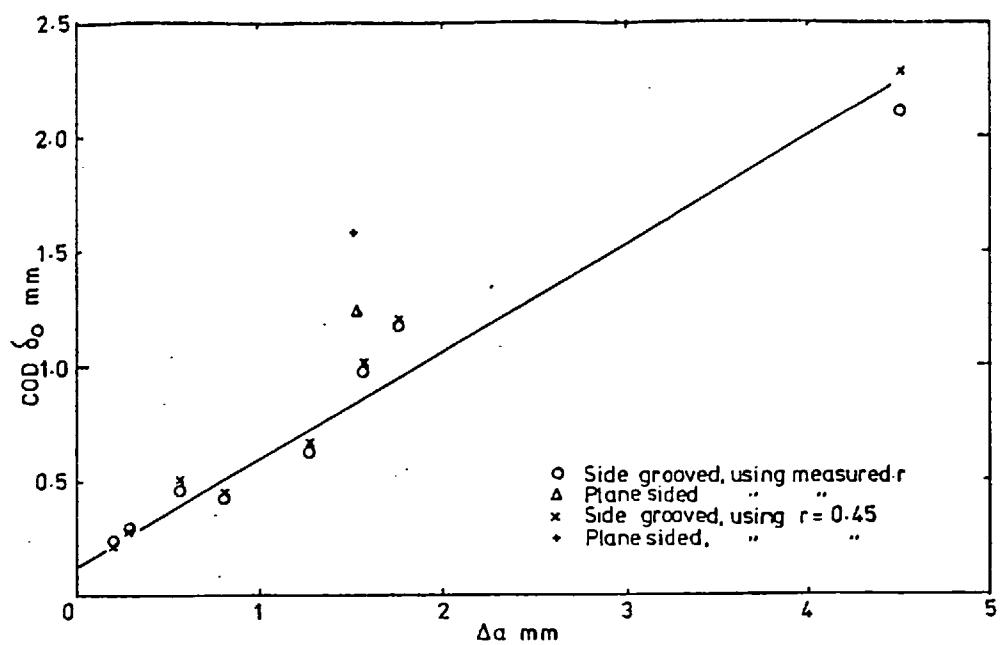


Fig. 9.7 δ_0 R-curve for En32B, LT orientation, corrected for variation of rotational factor.

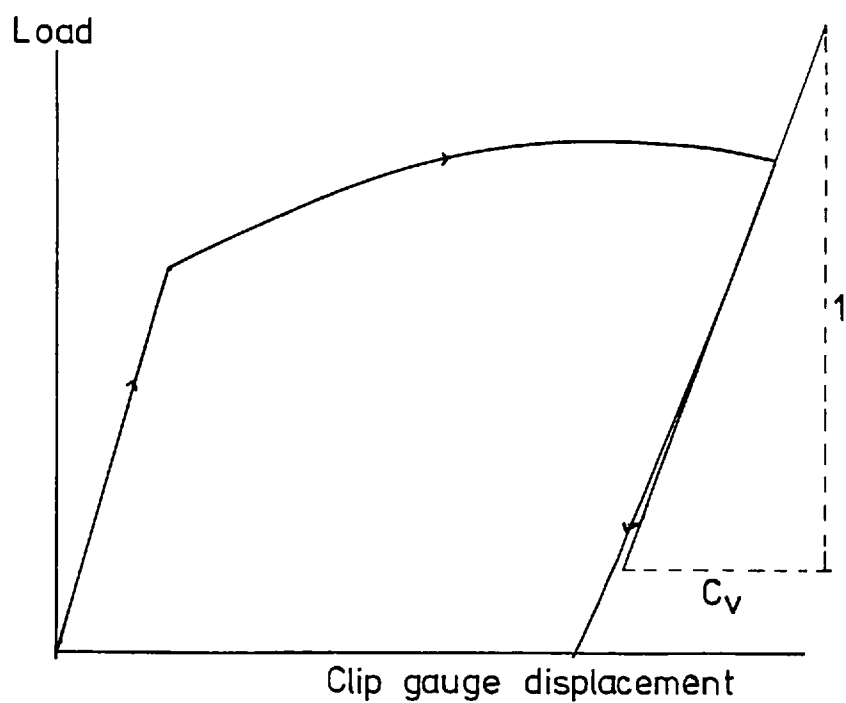


Fig. 9.8 The measurement of elastic unloading compliance from the clip gauge trace.

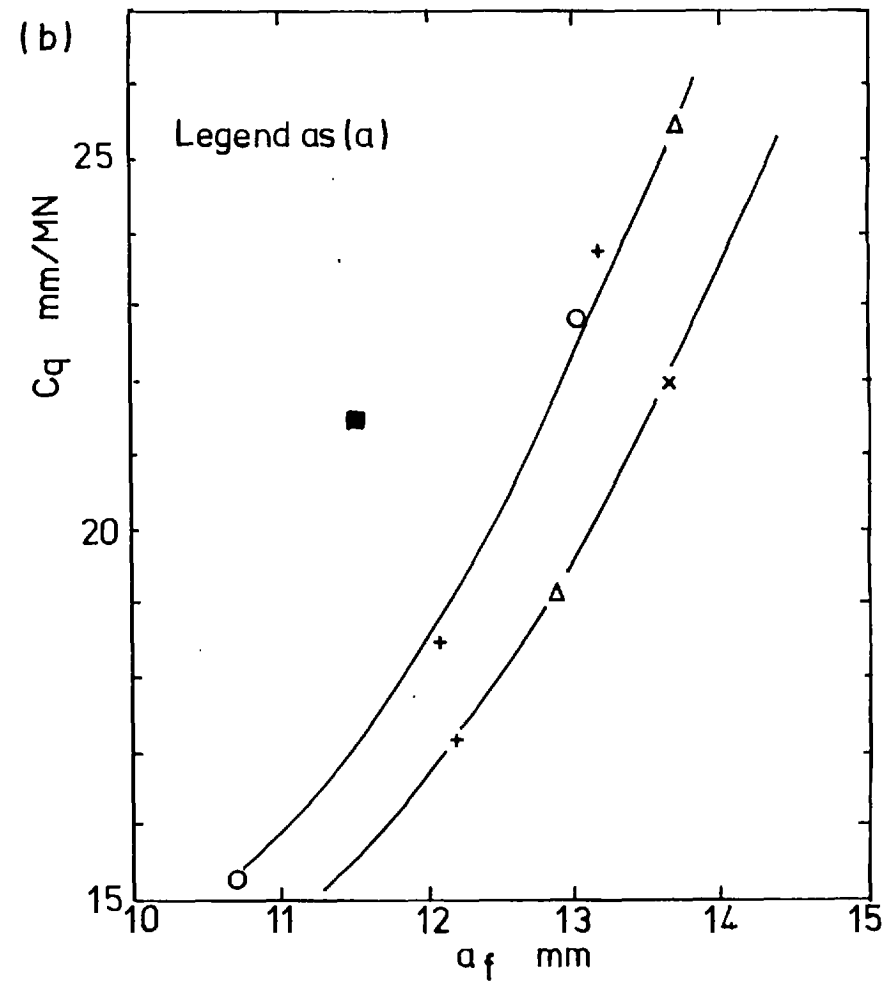
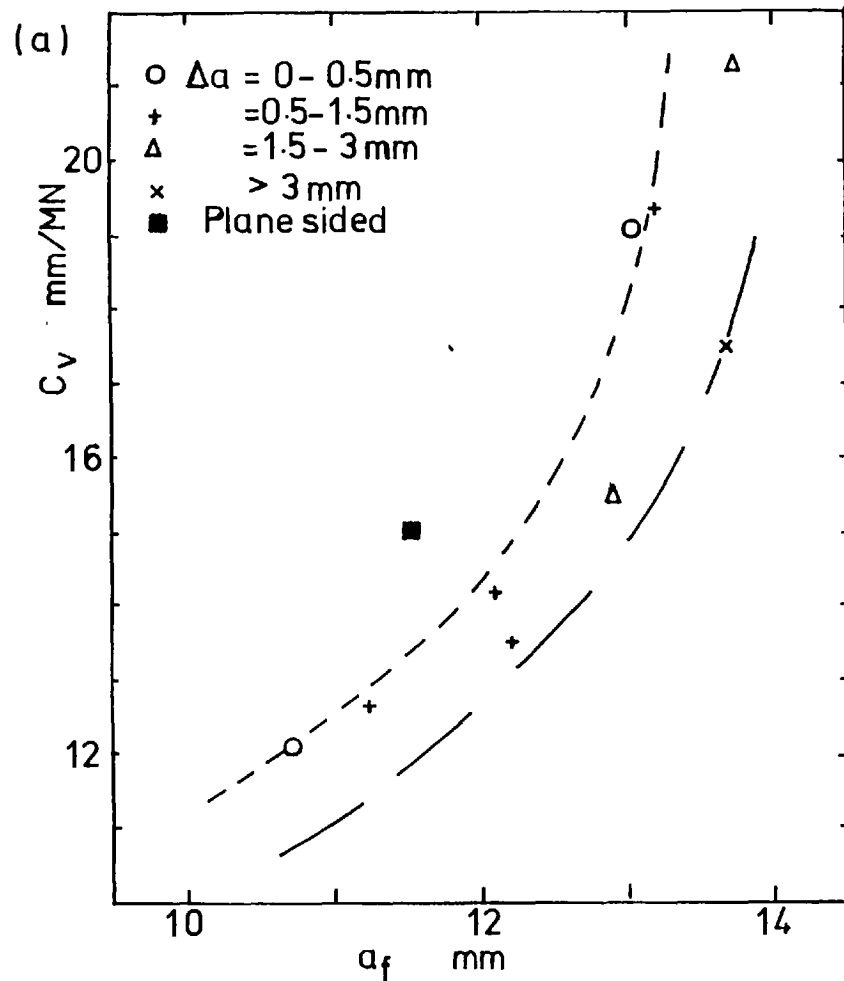


Fig. 9.9 Variation of compliance with crack length for En32B

(a) clip gauge compliance C_v
 (b) load point compliance C_q

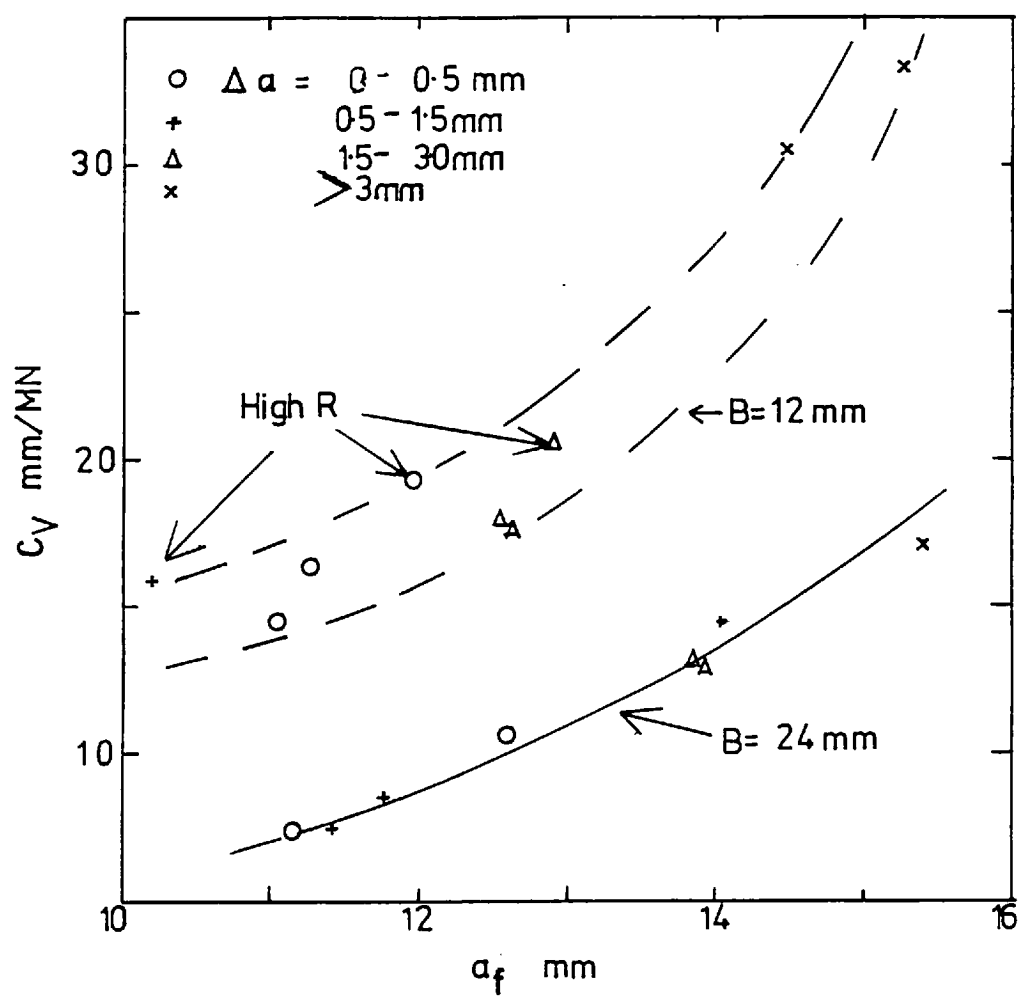


Fig. 9.10 Variation of C_V with crack length, En24.

Chapter Ten

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

10.1 Conclusions

The main conclusions arising from the work described in the previous chapters may be summarised as follows:

10.1.1 Initiation

(i) Both J_i and δ_i are independent of specimen size, for three point bending, within 20%.

(ii) J_i and δ_i may be related through the expression

$$J_i = M' \sigma_{\text{flow}} \delta_i$$

where M' varies between 1.5 and 2.4 in three point bend, depending on work-hardening capacity and inclusion orientation.

(iii) Initiation toughness is reduced by prestrain. The influence of grain size in mild steel is relatively unimportant.

(iv) Inclusion orientation has a large effect on initiation toughness, as a result of the strain intensification due to the inclusion shape.

(v) The plastic strain intensification between a crack tip and a void is greater in plane strain than in plane stress, but the trends occasioned by changing the void parameters are similar in the two cases.

(vi) The plastic strain intensification increases with decreasing void separation and with increasing void elongation perpendicular to the applied tensile stress.

10.1.2 Propagation of Flat Fracture

(i) The toughness during small amounts of crack extension may be described by the initial slopes of the COD or J R-curves.

(ii) The slopes $d\delta/da$ and dJ/da may be related analytically, in three point bending and perfect plasticity. Agreement with experiment is good where work-hardening capacities are small.

(iii) Prestrain tends to reduce propagation toughness and increases the likelihood of localised shear between voids ("zig-zag" fracture).

(iv) Elongation of inclusions perpendicular to the applied tensile stress also reduces toughness and promotes localised shear.

(v) The strains around the propagating crack tip remain constant during crack extension. The rising R-curve is caused by increasing plastic energy absorption in regions remote from the tip.

10.1.3 Further Aspects of Crack Propagation

(i) Shear lips, if present, dominate resistance to ductile fracture. Their growth is linked to resistance in flat fracture.

(ii) In three point bending, a rotational factor of 0.45 is recommended for determining δ_i . This should give slightly conservative values.

(iii) r decreases slightly during flat fracture propagation. In the presence of shear lips the decrease is more marked, and this casts doubts on the suitability of COD as a parameter to measure propagation toughness in this case.

(iv) In low strength steel the elastic unloading compliance is influenced by both the amount of crack extension and by the growth of shear lips. Monitoring of crack lengths by means of this method is thus limited to the higher strength materials (or larger test-pieces) and to small shear lip fractions.

10.2 Suggestions for Further Work

There are many questions to be answered before ductile fracture may be characterised, and the influence of micro-structure predicted. The work described here indicates the following as fruitful lines of research:

(i) The influence of loading configuration on plane strain R-curves. All published work in this area refers to mixed mode fracture.

(ii) The limits with respect to specimen dimensions within which the R-curve in flat fracture may be geometry independent.

(iii) The influence in quantitative terms of void orientation and dimensions on the intensification of plastic strain. This should aid the understanding of the role of inclusions in ductile fracture.

(iv) The mechanism of strain localisation.

Appendix I

ESTIMATION FORMULAE FOR THE J-INTEGRAL

Using the Sumpter (1974) formula, eqn. (3.18)

$$J = \frac{\kappa_e U_e}{B(W-a)} + \frac{\kappa_p U_p}{B(W-a)}$$

κ_e and κ_p are factors which depend on geometry and configuration.

Calculation of κ_e

$$\begin{aligned} J_e &= G = K^2/E' \\ &= \frac{\kappa_e U_e}{B(W-a)} \end{aligned}$$

$$\text{and } U_e = \frac{1}{2} Q q$$

$$\text{Hence } \kappa_e = \frac{2 K^2 \cdot B \cdot (W-a)}{E' \cdot Q \cdot q} \quad (I.1)$$

K and q may be obtained from elastic calibration formulae (e.g. Sumpter, 1974). The variation of κ_e with a/W is given in Table I.1 for the three point bend and CCT configurations.

Calculation of κ_p

Assuming rigid-plastic, non work-hardening materials, the plastic limit load, Q_L , is given by

$$Q_L = C \sigma_y B(W-a)^2/S \quad \text{3 pt. bend} \quad (I.2)$$

$$Q_L = C' \sigma_y B(W-2a) \quad \text{CCT} \quad (I.3)$$

where C and C' are numerical constants.

From eqn. (3.13) $J = -\frac{1}{B} \frac{dU}{da}$

For three point bend

$$\begin{aligned} U_p &= Q_L q \\ &= C \sigma_y B(W-a^2)q/S \end{aligned} \quad (I.4)$$

Differentiating, and substituting in eqn. (3.13)

$$J_p = C \sigma_y \cdot 2(W-a)q/S \quad (I.5)$$

Combining eqns. (I.4) and (I.5) gives

$$J_p = \frac{2 U_p}{B(W-a)}$$

Hence $\kappa_p = 2$, regardless of a/W ratio. For $0.4 \leq a/W \leq 0.7$, $\kappa_e = 2 = \kappa_p$. Thus for this range of a/W ,

$$J = \frac{2 U_{tot}}{B(W-a)}$$

The same calculation for the CCT configuration gives

$$J_p = \frac{U_p}{B(W-2a)} = \frac{0.5 U_p}{B(W/2-a)}$$

Hence $\kappa_p = 0.5$

Table I.1 Variation of κ_e with a/W or $2a/W$ (CCT)

a/W or $2a/W$	0.4	0.5	0.6	0.7	0.8
3 point bend, $S/W = 4$	1.88	1.98	2.02	1.90	-
CCT, $D/W = 3.6$	-	0.14	0.15	0.16	0.16

D is the distance between the loading pin centres in CCT.

Appendix II

THE THREE PARAMETER J_r ESTIMATION

Garwood et al. (1975) derived an estimate of the J -integral which was designed to allow for slow crack growth. Referring to Fig. II.1, OPQ is the loading line for the real body of initial crack length a_0 . Crack extension begins at P, and at Q the crack is of length $a_1 = a_0 + \Delta a$. QRO is the unloading curve of a postulated non-linear elastic body of crack length a_1 , taken to the same final load and displacement as the real body. If areas OPS = U_0 , ORQT = U_1 , OPQRO = U_2 and PQTS = ΔU , then

$$U_0 + \Delta U = U_1 + U_2 \quad (\text{II.1})$$

If J_1 is the value of J for the postulated body, using eqn. (3.15) for three point bend,

$$J_1 = \frac{2U_1}{B(W-a_1)}$$

$$\text{i.e. } U_1 = J_1 B(W-a_1)/2 \quad (\text{II.2})$$

$$\text{similarly } U_0 = J_0 B(W-a_0)/2 \quad (\text{II.3})$$

U_2 is approximately the area between two non-linear elastic loading curves for crack lengths a_0 and a_1 . Using eqn. (3.13), and taking J as the average of J_0 and J_1 , gives

$$\frac{J_0 + J_1}{2} = \frac{U_2}{B \Delta a}$$

$$\text{i.e. } U_2 = B \Delta a (J_0 + J_1)/2 \quad (\text{II.4})$$

Substituting eqns. II.2, 3 and 4 in II.1,

$$\begin{aligned} J_0 B(W-a_0)/2 + \Delta U &= J_1 B(W-a_1)/2 + (J_0 + J_1)B \Delta a/2 \\ \text{or } J_1 &= J_0 \frac{(W-a_1)}{(W-a_0)} + \frac{2 \Delta U}{B(W-a_0)} \end{aligned} \quad (\text{II.5})$$

ΔU can be found from the load/displacement traces. The point P in Fig. II.1 need not be at initiation, and the formula may be generalised, for small increments of crack growth;

$$J_n = J_{n-1} \frac{(W-a_n)}{(W-a_{n-1})} + \frac{2 \Delta U_n}{B(W-a_{n-1})} \quad (\text{II.6})$$

Ideally the J_r curve should be determined on one specimen, but owing to the difficulty of monitoring crack lengths, a multi-specimen technique was employed. If the specimens had varying initial crack lengths a correction to ΔU is required. This is given by Garwood (1977), for the n^{th} test-piece in a series, as

$$\Delta U_n = U_n - U'_n$$

where U_n is the area under the load/deflection curve, and U'_n is given by

$$U'_n = B \sum_{m=2}^n \left[\frac{J_{m-1} + J_{m-2}}{2} (\Delta a_{m-1} - \Delta a_{m-2}) \right] + \frac{J_{n-1}}{2} \{ (W-a_0)_n - \Delta a_{n-1} \}$$

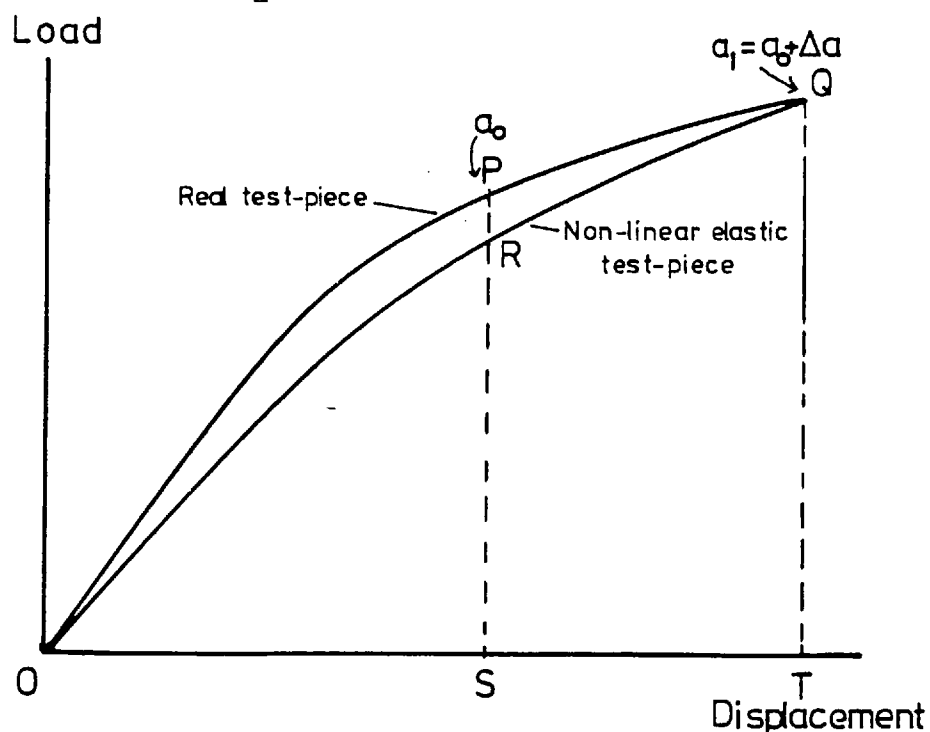


Fig II.1 Load/displacement curves for real, and hypothetical, non-linear elastic test-pieces.

Appendix III

CHARACTERISTICS OF INCLUSION DISPERSIONS

It is assumed that:

- (i) The centres of the inclusions form a completely random array
- (ii) Inclusions are ellipsoids of uniform size, shape and orientation.

Symbols

The following symbols will be used:

- A_f = The area fraction of inclusions on a plane surface
- D = inter-inclusion separation in any direction
- L = projected inclusion length
- P_L = projected inclusion length per unit area on a plane surface
- N_A = number of inclusions per unit area of plane surface
- N_V = number of inclusions per unit volume
- V_f = volume fraction of inclusions.

l , m and n are the inclusion semi-axes in the longitudinal, transverse and short transverse directions, respectively.

III.1 Inclusions on Plane Surfaces

- (i) Number per unit volume, N_V

An inclusion will only be intersected by a plane if its centre is located within a distance from the plane of half its length in the direction normal to the plane. On the short transverse plane, inclusions whose centres are within a band of width n will be intersected. Hence

$$N_A = N_V \cdot 2n$$

$$\text{or } N_V = N_A / 2n \quad (\text{III.1})$$

Similarly for the other planes.

(ii) Apparent inclusion size

The volume fraction of inclusions is given by the product of the number per unit volume and the volume of each one

$$\text{i.e. } V_f = N_v \cdot \frac{4}{3} \pi l m n \quad (\text{III.2})$$

On a plane surface the area fraction must be equal to the volume fraction. However, the inclusion dimensions will appear smaller on average, since the plane of intersection will not, in general, intersect them through their centres. The area fraction is thus (for the short transverse plane)

$$A_f = V_f = N_A \cdot \pi l' m' \quad (\text{III.3})$$

where l' and m' are the apparent semi axes. Comparing with (III.2), and substituting for N_A from (III.1),

$$l' m' = (2/3) l m$$

Since all parallel sections through an ellipsoid have the same ratio of semi-axes,

$$l' = \sqrt{(2/3)} l \quad (\text{III.4})$$

The projected inclusion length per unit area is equal to the product of the number per unit area and the apparent length of each. Thus, on the rolling plane in the rolling direction

$$P_L = N_A \cdot 2l' \quad (\text{III.5})$$

Substituting for l' from (III.3)

$$m' = \frac{2 A_f}{\pi P_L} \quad (\text{III.6})$$

III.2 Projected Inclusion Length

Let the inclusion major semi-axis be inclined at an angle α to the direction considered (Fig. III.1). In two dimensions, on the short transverse plane, it can be shown

(Baker, 1971) that the projected inclusion length, L , normal to this direction, is

$$L = 2(l^2 \sin^2 \alpha + m^2 \cos^2 \alpha)^{\frac{1}{2}} \quad (\text{III.7})$$

III.3 Inter-Inclusion Spacing.

Baker (1971) defined the inter-inclusion spacing, D , in any direction as the reciprocal of the number of inclusions which intercept unit length of a zone of cross-sectional area equal to the projected area of the inclusion in that direction. Since all the inclusions have a projected length, L , in this direction, they will be intersected by the zone if their centres are within a band of width L . In two dimensions, on the short transverse plane, L is given by (III.7). The projected area is thus an ellipse of area πnL . Hence the number of inclusions intersected per unit length of zone is

$$\pi \cdot 2nL \cdot N_v$$

$$\text{Hence } D = (\pi \cdot 2nL \cdot N_v)^{-1}$$

$$\begin{aligned} \text{From (III.2) } D &= \frac{4}{3} \frac{\pi \cdot lmn}{\pi \cdot 2nL \cdot V_f} \\ &= \frac{2lm}{3V_f L} \\ &= \frac{lm}{3V_f} (l^2 \sin^2 \alpha + m^2 \cos^2 \alpha)^{-\frac{1}{2}} \quad (\text{III.8}) \end{aligned}$$

This defines an ellipse of the same shape as the inclusion, but with axes which are $1/3V_f$ times as big.

III.4 Inclusion Separation in the Direction of Linkage.

Baker (1971) postulated that the probability of a void linking with a crack tip was

- (i) inversely proportional to the separation,
- (ii) directly proportional to $\cos \theta$, where θ is the angle of deviation from the direction of macroscopic crack propagation.

The locus of equal joining probability is a circle of radius b , with centre a distance b ahead of the crack tip. This circle is expanded (as the COD increases) until it touches the locus of interinclusion separation. The point of contact then defines the most probable direction for void linkage and the most probable separation. This is shown, on the short transverse plane, in Fig. III.2.

The direction of joining is given by

$$\begin{aligned}\theta &= \tan^{-1} \left\{ 1 - \frac{2m^2}{l^2} \right\}^{\frac{1}{2}} & m < \sqrt{2}l \\ &= 0 & m > \sqrt{2}l\end{aligned}\tag{III.10}$$

III.5 Initiation of Fracture

It is argued in Section 7.4 that initiation is a 2-D process, and only dimensions in the plane normal to the line of the crack front are important. The situation depicted in Fig. III.2 refers to fracture in the TL orientation. For other orientations the appropriate pair of inclusion dimensions must be chosen.

Fracture in the LT, LS and TS orientations is depicted in Figs. III.3(a) and (b). The larger inclusion dimension in the plane of the diagram is normal to the crack plane. Therefore the most likely direction of joining is straight ahead, since this gives the shortest inter-inclusion distance, and so $\theta = 0^\circ$. In the TL and SL orientations, the smallest inter-inclusion spacing is perpendicular to the crack plane, so fracture occurs at $\theta = 45^\circ$ (assuming $l \gg m, n$), Fig. III.3(c). In the ST orientation this is still the case, Fig. III.3(d), but since m and n are of the same order, eqn. (III.10) must be used with the appropriate inclusion dimensions,

$$\text{i.e. } \theta = \tan^{-1} \left(1 - \frac{2n^2}{m^2} \right)^{\frac{1}{2}} \tag{ST}$$

Taking $m = 0.6\mu\text{m}$ and $n = 0.4\mu\text{m}$ (i.e. $m/n = 1.5$) gives $\theta = 18^\circ$.

The separation, X_0 , between the crack tip and the inclusion at initiation is assumed to be $D/2$, since on average the fatigue crack tip will be positioned between inclusions. Table III.1 summarizes the parameters X_0 , θ and L for the six orientations.

Table III.1 Predicted inclusion parameters at initiation.

Orientation	$X_0(x6V_f)$	θ	L	δ_i/K^*
LT	m	0°	$2m$	$\sqrt{(m/2)}$
LS	n	0°	$2n$	$\sqrt{(n/2)}$
TS	n	0°	$2n$	$\sqrt{(n/2)}$
ST	$\sqrt{2} \cdot n$	$\approx 18^\circ$	$2m$	$1.05 \cdot n / \sqrt{m}$
TL	$\sqrt{2} \cdot m$	45°	$2l$	$m \cdot \sqrt{(2/1)}$
SL	$\sqrt{2} \cdot n$	45°	$2l$	$n \cdot \sqrt{(2/1)}$

- * Using eqn. (7.12) with $f'(\theta) = \cos\theta$, but ignoring any effect of strain intensification due to the frontal radius of curvature of the inclusion.

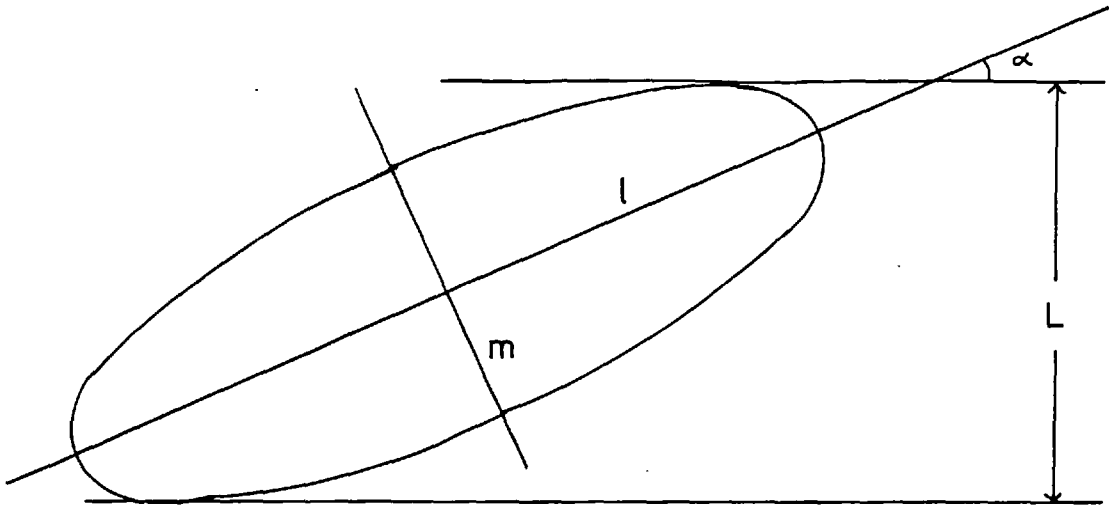


Fig. III.1 Projected width of an ellipsoidal inclusion.

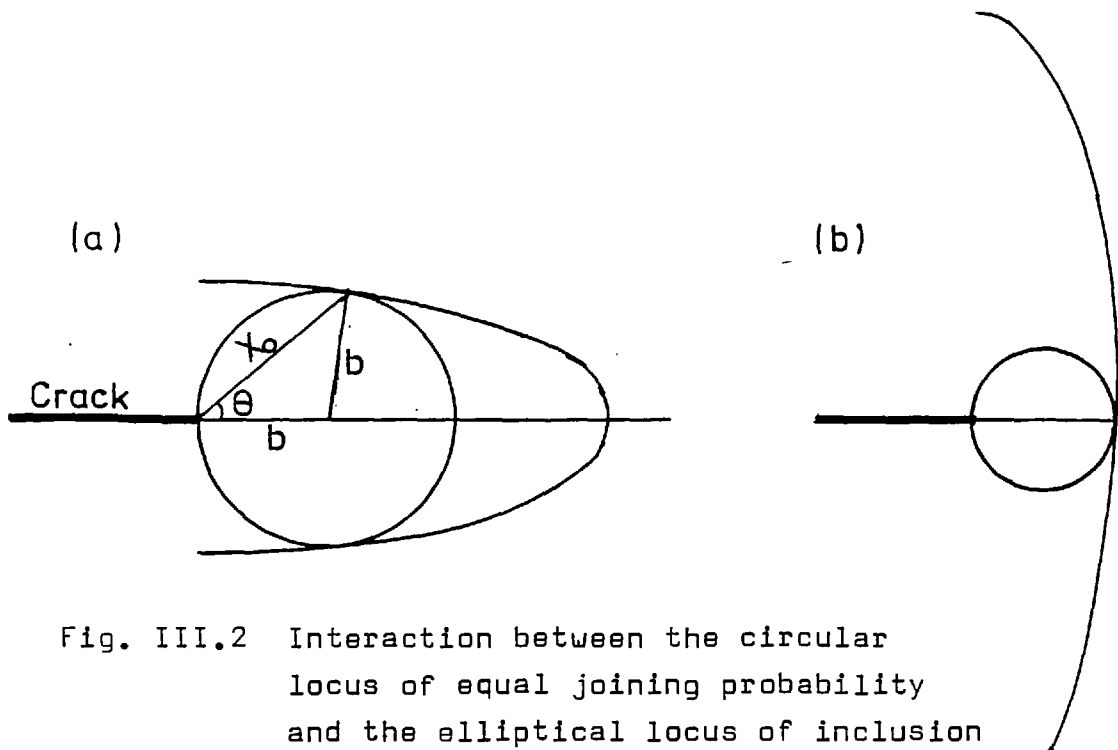


Fig. III.2 Interaction between the circular locus of equal joining probability and the elliptical locus of inclusion separation.

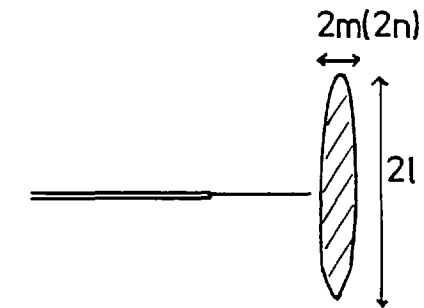
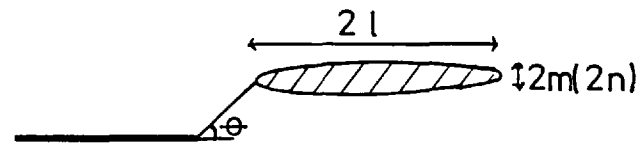
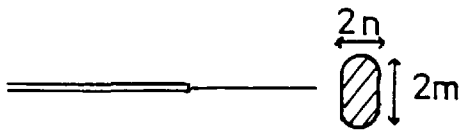
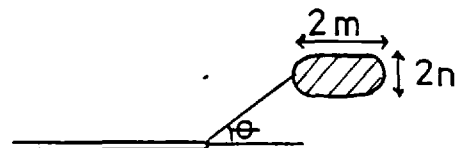
(a) LT (LS) , $\theta = 0^\circ$ (c) TL (SL) , $\theta = 45^\circ$ (b) TS , $\theta = 0^\circ$ (d) ST , $\theta = 0 - 45^\circ$

Fig. III.3 Most likely directions of joining between a fatigue crack tip and an inclusion, in the different orientations.

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