

# **Market Making and Dealer Markets**

by

Valentina Lorusso

Submitted to the Business School of Imperial College London  
on June 18, 2017, in partial fulfillment of the  
requirements for the degree of  
Doctor of Philosophy

## **Abstract**

This thesis investigates information and liquidity provision in financial markets. I explore the implications of the strategic behavior of market makers competing with high frequency traders and of dealers involved in long term relationships with clients in the foreign exchange markets. Additionally, I analyse the value of information from the liquidity order flow to market makers and dealers. Further, I reflect on regulatory implications of my findings.

Thesis Supervisor: Gilles Chemla

Title: Professor of Finance

Thesis Supervisor: Katrin Tinn

Title: Assistant Professor of Finance

Thesis Supervisor: Pasquale Della Corte

Title: Associate Professor of Finance

# Contents

|          |                                                                                                        |           |
|----------|--------------------------------------------------------------------------------------------------------|-----------|
| <b>1</b> | <b>Introduction</b>                                                                                    | <b>6</b>  |
| 1.1      | Overview of the thesis . . . . .                                                                       | 6         |
| 1.2      | Market Making . . . . .                                                                                | 8         |
| 1.3      | High Frequency Trading . . . . .                                                                       | 12        |
| 1.4      | The microstructure of the FX market . . . . .                                                          | 17        |
| 1.5      | The debate on OTC markets and benchmark determination, with a special focus<br>on the FX fix . . . . . | 21        |
| <b>2</b> | <b>High Frequency Trading and Market Making</b>                                                        | <b>25</b> |
| 2.1      | Introduction . . . . .                                                                                 | 25        |
| 2.2      | One period model . . . . .                                                                             | 29        |
| 2.2.1    | Setup . . . . .                                                                                        | 29        |
| 2.2.2    | Equilibrium prices . . . . .                                                                           | 31        |
| 2.3      | Two period model . . . . .                                                                             | 34        |
| 2.3.1    | Set up . . . . .                                                                                       | 34        |
| 2.3.2    | Equilibrium prices . . . . .                                                                           | 35        |
| 2.4      | Introducing a cost for Making Markets . . . . .                                                        | 38        |
| 2.4.1    | Set up . . . . .                                                                                       | 40        |
| 2.4.2    | Equilibrium prices . . . . .                                                                           | 40        |
| 2.5      | Discussion and Conclusions . . . . .                                                                   | 42        |
| <b>3</b> | <b>The client-dealer Interaction in dealers' markets</b>                                               | <b>44</b> |
| 3.1      | Introduction . . . . .                                                                                 | 44        |

|          |                                                                            |           |
|----------|----------------------------------------------------------------------------|-----------|
| 3.2      | Benchmark Game . . . . .                                                   | 48        |
| 3.2.1    | Setting . . . . .                                                          | 48        |
| 3.2.2    | Equilibrium in the interdealer market . . . . .                            | 53        |
| 3.2.3    | Equilibria in the full game . . . . .                                      | 60        |
| 3.2.4    | Discussion of the results of the static game . . . . .                     | 63        |
| 3.3      | The Repeated Game . . . . .                                                | 67        |
| 3.3.1    | Equilibrium in the repeated game . . . . .                                 | 67        |
| 3.3.2    | Cooperative equilibrium in the repeated game . . . . .                     | 68        |
| 3.3.3    | Discussion of the results of the dynamic game . . . . .                    | 72        |
| 3.4      | Conclusion . . . . .                                                       | 73        |
| <b>4</b> | <b>The value of information about noise trading in liquidity provision</b> | <b>75</b> |
| 4.1      | Introduction . . . . .                                                     | 75        |
| 4.2      | The Model: Setting and definition of Equilibrium . . . . .                 | 79        |
| 4.3      | Characterization of the equilibrium . . . . .                              | 81        |
| 4.3.1    | Regime 1 . . . . .                                                         | 81        |
| 4.3.2    | Regime 2 . . . . .                                                         | 83        |
| 4.3.3    | Regime 3 and 4 . . . . .                                                   | 85        |
| 4.4      | Comparison of Regimes . . . . .                                            | 86        |
| 4.5      | Discussion and Conclusion . . . . .                                        | 93        |
| 4.6      | Appendices . . . . .                                                       | 103       |
| 4.6.1    | Appendix A . . . . .                                                       | 103       |
| 4.6.2    | Appendix B . . . . .                                                       | 105       |
| 4.6.3    | Appendix C . . . . .                                                       | 112       |
| 4.6.4    | Appendix D . . . . .                                                       | 114       |

|       |                      |     |
|-------|----------------------|-----|
| 4.6.5 | Appendix E . . . . . | 121 |
| 4.6.6 | Appendix F . . . . . | 124 |
| 4.6.7 | Appendix G . . . . . | 127 |

# List of Figures

|     |                                                                                    |    |
|-----|------------------------------------------------------------------------------------|----|
| 1-1 | Direct vs Indirect trades in the FX market. . . . .                                | 19 |
| 2-1 | Utility of the risk averse market maker as a function of the spread level. . . . . | 33 |
| 4-1 | Price impact, Regime 1 . . . . .                                                   | 83 |
| 4-2 | Demand coefficients Regime 1 and Regime 2. . . . .                                 | 85 |
| 4-3 | Difference in profits across the four regimes. . . . .                             | 87 |
| 4-4 | Plot of market depth in the four Regimes. . . . .                                  | 89 |
| 4-5 | Plot of price impact in the four regimes. . . . .                                  | 90 |
| 4-6 | Difference in variance across the four regimes. . . . .                            | 92 |
| 4-7 | Information content of prices in the four Regimes. . . . .                         | 93 |

The copyright of this thesis rests with the author and is made available under a Creative Commons Attribution Non-Commercial No Derivatives licence. Researchers are free to copy, distribute or transmit the thesis on the condition that they attribute it, that they do not use it for commercial purposes and that they do not alter, transform or build upon it. For any reuse or redistribution, researchers must make clear to others the licence terms of this work.

I hereby confirm that the work included in this thesis is my own and all else is appropriately referenced.

# Chapter 1

## Introduction

### 1.1 Overview of the thesis

The thesis investigates information and liquidity provision in financial markets. I explore the implications of the strategic behavior of market makers competing with high frequency traders and of dealers involved in long term relationships with clients in the foreign exchange markets. Additionally, I analyze the value of information from the liquidity order flow to market makers and dealers. Further, I reflect on regulatory implications of my findings.

The first chapter presents a literature review to motivate the following chapters. First, I survey the main findings of the papers on market making most relevant to this thesis. Second, I discuss the regulatory and academic debate on high frequency traders, which are widely viewed as a new type of liquidity providers. Third, I discuss important differences between market makers and FX dealers, including specific features of foreign exchange markets and their informational structure. Lastly, I provide a brief overview of the recent regulatory debate on OTC markets.

The second chapter analyzes the effect of competition between a designated, traditional market maker and a High Frequency Trader providing liquidity. The market maker is risk neutral and the high frequency trader is risk averse, which creates differences in their inventory exposures. The market power of these two participants creates a bid ask spread, but the high frequency trader narrows the spread and improves liquidity. The chapter further investigates the liquidity provision by a monopolistic high frequency trader. I show that having agents with

strong inventory concerns as market makers could hamper liquidity provision. I explain how *ceteris paribus* small changes in the reservation value of liquidity traders can trigger shifts in the equilibrium spread.

The third chapter endogenizes the existence of intermediation in a two-tier market. Specifically, trading takes place sequentially in a client-dealer OTC market and in an interdealer market organized as a limit order book. A privately-informed client chooses between trading through dealers or paying an entry cost to join the interdealer market directly. Dealer rents from intermediation increase in the entry cost. I show that competitive dealers use the bid ask spread strategically to reward the client for the information conveyed by his order flow. Furthermore, I show that the client dealer relationship is affected by a commitment problem: clients who trade *una tantum* execute trades with multiple dealers. Ongoing client dealer relationships viewed as an infinitely repeated game can overcome this problem and the client may benefit from trading exclusively with one dealer.

The fourth chapter analyses information sharing and collusion incentives of strategic liquidity providers and the impact of their cooperation on asset prices. Risk neutral liquidity providers operate in a market with risk-averse informed traders (fundamentalists) and noise traders. I consider four regimes: 1) pure market making; 2) dealership without information sharing; 3) dealership with information sharing but without collusion in trading; and 4) dealership with information sharing and collusion in trading. I show that information sharing substantially increases agents' profits, while colluding in trading has a relatively low additional impact on profits. This suggests that if there are penalties for collusion, dealers may choose to only share information, but not to collude. Furthermore, I investigate the effect of the four regimes on market depth, volatility of prices and information content of prices. I find that dealers sharing information and colluding increase market depth compared to dealership without information sharing. However, the market depth is lower compared to pure market making. Both volatility of prices and the information content of prices increase when liquidity providers act as dealers. The magnitude of these differences depends on the parameters of the model.

## 1.2 Market Making

[Bagehot, 1971] defines the market maker as a liquidity provider who steps in and agevolates trading 'whenever equal and opposite orders fail to arrive in the market at the same time'. The role of the market maker, in Bagehot's view, is not passive. Spread dynamics are related to the nature of market participants: informed traders create pressure towards an increase in the spread to avoid losses, whereas liquidity traders create room for profit for the market maker. A strategic spread mechanism is then vital for the market maker to avoid bankruptcy and relevant to the liquidity of the market.

In the market microstructure literature the bid ask spread is considered a product of two components: an inventory holding cost component and a cost component arising from information asymmetries.

Inventory models explore the problem of uncertainty in the future order flow, which can result in inventory imbalances for the specialist and execution problems for the traders. In line with [?], I consider two main research paradigms that looks at the inventory problem. The first one starts with the work of [Garman, 1976]. [Garman, 1976] focuses on order flow and the role of order flow in determining prices. [Garman, 1976] considers a single monopolistic market maker who maximizes his profit to avoid bankruptcy or failure. The market maker selects a bid and a ask price for a unit quantity and has an infinite horizon, despite setting prices at the beginning of time. The market maker due to the stochastic order flow potentially faces inventory imbalances. For this reason he needs to adjust his level of inventory and cash not to run out of any. The spread, in this context, arises as a necessity for the market maker to protect himself from bankruptcy, but does not rule out a positive probability of failure. [Amihud and Mendelson, 1980], starting from Garman model, focus on the relation between the bid and ask price and the market maker's inventory level over time. They conclude that the market maker actively uses bid and ask quotes to balance his inventory exposure: as inventory rises, both the bid and ask decrease, while if inventory decreases, both the bid and the ask increase. The second research paradigm analyzes explicitly the market maker's optimization problem: in order to accommodate the needs of other traders the market maker is willing to alter his portfolio. In [Stoll, 1978] setting the market maker is risk averse and bid and ask prices represent a compensation for the costs he faces. Such costs depend on several

factors like initial wealth, risk preferences and inventory position. The main finding is that the inventory position of the dealer does not affect the spread size, but rather the placement of the spread: a large inventory decreases bid and ask of the same size and vice versa for a small inventory. [Ho and Stoll, 1981] extends the model to a multi period framework. In this case the spread decreases when the time horizon shrinks: the risk in acting as a market maker is reduced and consequently the spread is set closer the risk neutral case. In the work of [O'Hara and Oldfield, 1986] the horizon of the market maker is infinite and he expects both limit orders and market orders. In this model both the placement and the magnitude of spread depend on the dealer inventory position.

The models with information asymmetries consider the bid ask spread as a pure informational phenomenon: the market maker has to deal with better informed traders, with whom he will always lose in transactions, and noise traders from whom he might earn a profit. [Glosten and Milgrom, 1985a] considers a competitive market maker who faces no transaction costs. He has an unlimited inventory with a zero holding cost and there is no bankruptcy. The market maker sets bid and ask prices for one unit and updates his quotes after observing an order from an investor. There are two types of investors in the market, informed traders and pure liquidity traders. Investors arrive sequentially to the market and either buy, sell or leave based on the comparison of the bid and ask price and the expected value of the security, conditional on his information set and preferences concerning current and future consumption. The specialist sets prices such that they are equal to the expected value of the asset conditional on the order observed. The spread in this model is increasing in the number of informed traders, in the information of the insider and in the elasticity of demand of uninformed traders. If there are too many insiders the market can shut down. [Easley and O'Hara, 1987] extends [Glosten and Milgrom, 1985a] to consider orders of different sizes. Moreover in [Easley and O'Hara, 1987] there is a second level of uncertainty related to the existence of information. In the market there are several risk neutral market makers and an information event occurs with a certain probability. In case there is information, some traders will be informed while others not. Traders are risk neutral and there are no transaction costs. In this market, market makers decides bid and ask prices for two trade size: a large trade and a small one as both traders type can choose to trade one of the two sizes. When

traders arrive to the market decide to trade he chooses the desired price quantity pair. In this setting two possible equilibria arise: a separating equilibrium in which uninformed trade small or large quantities and informed only large quantities and a pooling equilibrium in which both uninformed and informed trade large and small quantities. In the case of separating equilibrium the market maker expectations are not affected when he sees a small trade, while they change if he sees a large trade. For small quantities to trade there is no spread as prices are equivalent to market maker initial expectation, while for large quantities the market maker sets a spread. In case of a pooling equilibrium there is a spread both for large and for small quantities. This paper suggests that block trades execute at a worst price, not only because of inventory imbalance, but also because a large trade might have a signalling effect.

Another stream of literature on market making focuses on the role of the monopolist specialist. Both market maker and monopolist specialist act to increase the liquidity of the market. In the monopolist specialist system there is only one individual (the specialist) who has sole information about the trading process and because of this gains some monopoly power. In [Glosten, 1989] the main finding is that, when trading with better informed traders, the specialist does not reduce the liquidity of the market as the market maker does: if he keeps the market open, he can't learn information from informed traders and this reduces the adverse selection problem. Comparing market outcomes in presence of market makers or a monopolist specialist they find that, in case of severe adverse selection, the liquidity provided by the monopolist specialist is higher. [Kavajecz, 1996] considers a monopolist specialist who chooses jointly prices and depths in order to maximize his profits in a market where one security with two realization values is traded and there are information asymmetries. One finding is that the depth in conjunction with the spread is actively used by the specialist. [Dupont, 2000] adds to this literature suggesting that the monopolist specialist uses more actively the depth rather than the spread to manage asymmetric information risk as he narrows his depth proportionally more than he widens his spread when there is an increase in information asymmetry. Other more recent models focus on the effect of competition between a specialist and limit order suppliers. [Seppi, 1997] analyzes liquidity provision in a model in which a specialist competes against a competitive limit order book. The limit order book contains price contingent orders to sell if the price raises above a pre-specified limit price. In this model there are four

types of investors: one liquidity demander who submits market orders to sell or buy stocks and three liquidity providers. The liquidity providers are liquidity value traders who post limit orders from off the exchange, a single strategic specialist who clears the market considering priority rules, a trading crowd who potentially enters the market to provide liquidity if there is a gross mispricing. In this model the specialist has two advantages: he does not have to pay a cost for orders and he observes the realized volume of the order, rather than trading on expectations as the value traders. In the model first the value traders submit limit orders at time, later the active trader arrives and submits an order to buy or sell with some probability and a distribution over volumes. The specialist then chooses a clearing up price to maximize his expected profit after executing any limit order with priority. An important finding is that the specialist, when increasing the market clearing price, incurs in a trade off: higher price means higher per share profit, but also a lower number of shares left over for him to trade. The optimal clearing price depends on the size of the market order and on the orders in the limit order book. [Bondarenko and Sung, 2003] focus on the direct competition that the specialist suffers from limit order traders. They proceed in a setting that is analogous to [Kyle, 1985a] where the specialist observes aggregate order flow from insider and noise traders and the depth of the limit order book. The specialist sets a price that clears the market and also chooses a quantity that he trades for his own account. In order to get an equilibrium the following should be satisfied: the insider chooses to trade a quantity such that her expected profit is maximized conditionally on the asset value, the specialist chooses a quantity and a market clearing price to maximize his profit conditional on limit order book depth and market order, limit traders get zero expected profits. Starting from this general setting they distinguish two cases. In the first case (basically they are excluding the informational advantage that is the key element of the specialist role). They show that if the depth of the limit order book is known to all market participant (they are excluding the informational advantage that is the key element of the specialist role), the specialist has no incentive to trade, and liquidity will be provided by limit orders. On the other hand if the specialist only observes the realization of the depth. Basic idea is that when depth is too low the specialist clears the market at higher prices, while when liquidity is too high he has an incentive to consume liquidity. Overall the specialist has an advantage that he derives from his private information.

The models presented so far focus on the role of the market maker as a strategic agent. The seminal paper of [Kyle, 1985a] focuses on the trading strategy of an informed trader in a model in which the market maker enforces a market clearing condition. Although in [Kyle, 1985a] model there is no bid ask spread, the model captures the process through which information is embedded into price. In this model the informed trader and noise traders submit simultaneously their orders and, given the aggregate order flow, the market maker sets a market clearing price. The price set by the market maker equals the expectation of the liquidation value of the asset, given the observed order flow. The optimal quantity demanded by the informed trader is increasing in the variance of uninformed traders order flow. The sensitivity of pricing of the market maker is inversely proportional to the variance of the order flow of the noise traders: overall the higher is the proportion of noise trading to the value of insider information the deeper is the market. Various other models have built on the [Kyle, 1985a] setting introducing risk aversion, [Subrahmanyam, 1991], multiple informed, [Holden and Subrahmanyam, 1992], and heterogenous signals, [Admati and Pfleiderer, 1988] and heterogenous signals when all signals can have different precision [Dridi and Germain, 2001].

### 1.3 High Frequency Trading

High frequency trading (HFT) is a subset of algorithmic trading [Chlistalla, 2011]. High frequency trading and algorithmic trading are based on direct market access and automated order submission and execution. While algorithmic trading is used with the purpose of reducing market impact (i.e. execute pre-designated trading decisions when certain conditions are met, with holding periods that vary from days to weeks and months), high frequency trading entails a high-speed advantage<sup>1</sup> that allows to capture small quick profits in a large number of trades. Most of the high frequency trading firms are proprietary traders who submit to the market a large number of orders, typically small in size, at high speed. Additionally they exhibit a high rate of order cancellation as each position is not held open for more than a few seconds. Most likely no position will be carried overnight. According to [Smith, 2010] the start of the high frequency trading trend is the Order Protection Rule passed by the SEC in June 2005 that allows

---

<sup>1</sup>In order to achieve low latency High Frequency traders use co-location services or proximity services.

the automatic execution of trades at the best quote possible. This rule creates an advantage for high speed transaction that are immediately executed without the need of approval by a market maker or specialist.

Although HFT is a natural evolution of the technology used in financial markets<sup>2</sup>, mostly after the flash crash of 6 May 2010<sup>3</sup> regulators, press, industry and academia, developed interest in the way high frequency traders operate, their role as market makers and their impact on market microstructure and price formation process.

Tradeworx<sup>4</sup>, in a document addressed to the SEC [Tradeworx, 2010], explains high frequency trading and its role in US security market. They define high frequency trading as strategy which trades for investment horizons of less than one day and seeks to unwind all positions before the end of each trading day. High frequency traders, according to their view, exhibit a bidirectional flow and do not accumulate large inventory positions. Moreover they emphasize that high frequency traders do not compete with long term investors as they profit from market making and statistical arbitrage. As a traditional market maker, a high frequency trader provides liquidity in the market and does not accumulate shares in his inventory. The short inventory holding time exhibited by these player has, according to [Tradeworx, 2010], a beneficial effect on spreads. In fact the bid ask spread is partly justified by inventory holding cost: having short inventory holding time HFTs reduce the component of the spread related to the holding period. There is indeed empirical evidence of a reduction in the spread after the arrival of HFTs in the market. [Brogaard, 2010], working on a sample of 40 stocks, finds that

---

<sup>2</sup>The rise of high frequency trading is a natural consequence of the evolution of technology. Gomber presents an interesting overview of the phenomenon of HFT and its implication. He observes that the evolution of technology not only reflects on the aspect of order routing, displaying and execution, but also on the order submission process that often is no more carried out by human agents.

<sup>3</sup>Although the flash crash has not been triggered by HFT, some characteristics of their trading strategies might have contributed to esacerbate the sudden drop and price recovery of May the 6th.

<sup>4</sup>In the presentation of their document, addressed to the secretary of SEC, they present themselves as developers of advanced technology solutions for trading US equity securities. These solutions are based on mathematical algos and are used for high performance trading, by Tradeworx for their own account, by their hedge fund and by third parties who purchase their technology.

Moreover from their website: 'Thesys Technologies LLC is the infrastructure affiliate of Tradeworx, serving the high-performance technology needs of all market participants, including institutional investors, professional traders, brokerage firms, exchanges, and regulatory agencies. Thesys offers the fastest and most comprehensive front-to-back trading solution on the market, putting investors and traders on a level playing field with the world's top-tier HFT firms'.

HFT provide the best bid and offer quotes 65.3% of the calendar time. Moreover he suggests that when the spread is unusually high for large stocks there is a significant increase in the times they offer the best quotes. This finding is in line with the idea that HFT attempt to profit when the price of liquidity is high. [Menkveld, 2011], studying the effect of the arrival of an High Frequency trader in the Chi-X market, documents a drop in the bid ask spread of almost 50%. [Castura et al., 2010] focuses on the effect of HFT on market spreads and market liquidity. Overall they suggest there has been an improvement in liquidity and spread in the last couple of years, implying that HFT have a positive effect on the overall health of the market.

Despite the fact that there is consensus on the fact that HFTs reduce the spread, the necessity of liquidating positions very quickly, that differentiate high frequency traders from traditional market makers, might have relevant implication for continuity in liquidity provision especially in case of very volatile market conditions, as during the flash crash of 6 May 2010. A traditional market maker is ([CFTC and SEC, 2010]) a member of an exchange who voluntarily register to act as market makers on a security by security basis. The market maker is subject to the obligation to maintain continuous two sided quotation in the security for which he registers. A firm is classified as traditional market makers as it profits mostly from capturing the bid ask spread. Moreover market makers are supposed to be market neutral and to make use of ETFs or other securities in order to hedge the market exposure they might accumulate because of order flow imbalances. While traditional market makers have the possibility to the day flat only in very few occasion (see [Anand and Venkataraman, 2012] ), [Menkveld, 2011] finds that a HFT acting as a market maker has a null inventory position at the start and end of each trading day (in 69.8% of the trading days he ends his position flat and especially for small stocks he tends to avoid overnight position in 91% of the trading days). Moreover his positions appear to change very frequently. His result further suggests that positions that last less than five seconds generate a profit, while those that last for more than five seconds generate a loss. The aversion to inventory accumulation has been explored to understand the behavior of these traders during volatile market conditions. [Kirilenko et al., ] analyzes what happened during the flash crash and the role high frequency traders played in such event. The flash crash has attracted the attention of several regulators ([CFTC and SEC, 2010],[Barker and Pomeranets, 2011]) as they

question the fact that high frequency traders kept providing liquidity during the price trend, but rather traded aggravating the price trend. [Kirilenko et al., ] find that after HFTs began to accumulate contracts in their inventory on May 6, but after the massive algo sale pressure began, they decided to switch position and get rid of their inventory extremely quickly before experiencing a further decline in price. This is not comparable with what happened in the same market circumstances to intermediaries <sup>5</sup> as they did not offset their position as quickly and their sale strategy began after the price rebounded. They conclude that on May 6, intermediaries were more exposed to losses than HFT, who overall had a high profitability despite the extreme market conditions. According to their results inventory-management considerations at times may lead high frequency trading to aggressively trade in the same direction as the prices are moving, thus, taking liquidity. At other times HFTs passively trade against price movements to revert to their target inventory levels and, thus, provide liquidity. This strategic behavior is not part of the traditional market making liquidity provision. In fact [CFTC and SEC, 2010] documents that in the market conditions that occurred on the 6th of May the main reaction of many market makers was to widen the bid ask spread and to reduce the depth of their quotes. When forced to continue providing two sided orders in extreme market conditions <sup>6</sup> the market makers decided to post stub quotes (extreme high sell and low buy) gaining time to understand what was the best strategy to pursue. [Brogaard, 2010] reaches a different finding regarding the hypothesis that HFTs are around during normal times while flee the market or reduce trading activity during days with high volatility. He finds that HFT activity overall does not seem to increase or decrease substantially depending on the volatility of the market. However they provide almost 10% more liquidity than usual in low volatility days and they provide 10% less liquidity than average in highly volatile days. Overall, as volatility increases, they tend to provide liquidity less often and take liquidity more often. But his study concludes that the change in liquidity provision depending on volatility is not dramatic. Furthermore he finds no

---

<sup>5</sup>They define intermediaries as short horizon investors who follow a strategy of buying and selling a large number of contracts to stay around a relatively low target level of inventory. They use further criteria to ensure the accounts selected are those whose trading strategy is to participate in a large number of transactions, but to rarely accumulate a significant net position. High Frequency Traders are a subset of Intermediaries, who individually participate in a very large number of transactions. To isolate them they order Intermediaries by the number of transactions they participated in during a day, and then designate accounts that rank in the top 7% as High Frequency Traders

<sup>6</sup>many of their automated market making systems were paused by the extreme market conditions

evidence of HFTs exiting the market in case of very high volatility. [Brogaard, 2012] analyzes more in details the relationship between HFTs and volatility. He concludes that HFTs' liquidity provision seems to be more active during short term volatility (less 10 seconds) but declines if volatility affects longer horizons (from 1 minute to 60 minutes or to 120 minutes). The most significant impact of volatility on HFTs liquidity provision activity appears when macro news-induced volatility is measured, as opposed to stock specific volatility. If there is an increase in macro volatility HFTs increase their aggressive trading activity. [Smith, 2010] finds that the arrival of high frequency traders in stock markets creates an increase in the Hurst exponent, hence an higher level of autocorrelation in trades that results in volatility surges. He finds that this increase is mostly related to the increase in trades that are small in number of shares. [Barker and Pomeranets, 2011] findings confirm the decrease in the medium orders' size, which they conclude has dropped of almost one half. [?] expresses doubts about the consistency of HFT liquidity provision: they argue that high frequency traders are most likely to be liquidity provider or liquidity takers depending on their needs. They observe that the liquidity provided by HFTs is less reliable than the one provided by other market participants. The main reason for this is the fact that HFTs do not absorb risk since they carry inventory for milliseconds and then try to end flat. An interesting point of their analysis is concerned with the development of mini-crashes that might be associated with HFTs. The problem is that sudden price declines are reported to be observed more often, but in this study they do not seem to investigate deeper if those price changes are associated with HFT activity.

The academic literature has explored the effect of HFT on markets also within theoretical frameworks. A working paper from [Cartea and Penalva, 2011] (Bank of Spain) develops a market microstructure model in which the HFT acts as an intermediary between the market maker and two liquidity traders <sup>7</sup> Overall they find that the presence of the HFT distorts market conditions through prices. This is because he has higher price impact and this creates an additional cost to other market participants. This effect is increasing in the amount of shares traded. Market makers are unaffected because, in presence of HFTs, they get a higher liquidity discount. They conclude that there is the possibility that market makers costs rise in presence

---

<sup>7</sup>Interesting the fact they notice that [Hendershott et al., 2010] and [Brogaard, 2010] report results based on a database in which is impossible to distinguish between HFT and Algo Trading, so this might be a bias in their conclusions

of high frequency traders, but the question if the liquidity HFTs provide is a good alternative to the institutional is still relevant. [Jarrow and Protter, 2011] consider a setting in which there is no bid ask spread and the market are perfectly liquid. In this model high frequency traders are able to create trends in market prices that they exploit to the disadvantage of ordinary traders. They conclude that despite the empirical literature supports the idea that HFTs reduce bid ask spread and improve liquidity, in a perfectly liquid market without bid ask spread the finding is that high frequency traders increase market volatility and gain abnormal profit to the expenses of the ordinary traders. [Cvitanic and Kirilenko, 2010] studies the distribution of transaction prices generated in an electronic limit order market populated by orders of HFT and low frequency traders. Machines are uninformed, one of the advantages they have is speed. Another advantage of the machine is to know the process that governs the arrival of human orders, and the value of existing orders in the book<sup>8</sup>. They observe that the presence of the machine decreases the inter trade duration and allows for a price distribution that has more mass around the center and thinner tails. When introducing a fast trader who does optimize his profit with a penalty for the accumulated inventory, the machine offers a wider bid ask spread if market is under stress, for example in high volatility conditions.

## 1.4 The microstructure of the FX market

The Forex daily turnover is roughly 36 times larger than the combined exports and imports of the world's 35 largest economies. An estimate of the Bank of International Settlements 2013 [BIS, 2013], quantifies the daily average FX turnover to \$5.3 trillion per day. This market is formally open 24 hours a day, with a period of less intense trading between 7 and 10 pm GMT, when New York traders are gone home and Sydney traders are on their way to work ([Frankel, 2012]). The USD is the world's dominant vehicle currency, with 87% of FX deals with USD on one side of the transaction. The FX market is a highly concentrated market in which the leading banks are reported to have a market share of 45%, after reaching a peak of 60% in 2014 [Euromoney, 2016]. City Bank according to the latest Euromoney survey is the leading

---

<sup>8</sup>They assume that before the trading period the machine has done an estimation procedure through pinging the book.

player in the FX market with an overall market share of 12.91%, followed by JPMorgan with a share of 8.77%, UBS with a share of 8.76%, Deutsche Bank with 7.86%, Bank of America with 6.40% and Barclays at 5.67%. In recent years the turnover in the interdealer market has decreased by almost 25% due to increased concentration and improvement in technology. Increased concentration implies that dealers can match large quantities of customer trades on their own books by internalizing trades. Heavily investment in IT infrastructure implies that the warehousing of inventory risk is facilitated, reducing the need to off-load inventory in the interdealer market.

Forex trading is also becoming more geographically concentrated. [BIS, 2013] finds that the vast majority of global FX trading has occurred via the intermediation of dealer's sales desks in five financial center that account roughly for 75% of global trading in 2013<sup>9</sup>.

The Forex historically is considered as a market that operates at three different levels, involving three kind of market participants: dealers, customers and brokers. Customers trade with dealers in the client-dealer market. Forex dealers do not trade only with customers, but also with other dealers in the direct interdealer market and in the brokered interdealer market<sup>10</sup>. In the past the only possibility for clients to execute their trades was OTC (voice). OTC trades are executed asking to dealers for quotes without revealing the side of the trade, but only the size. After the dealer produces his quotes the client decides if trading or else searching for another dealer. Nowadays financial customers can trade directly on EBS and Reuter matching platforms and other, non interdealer, trading platforms [King et al., 2011]. Moreover clients can trade in a prime brokerage relationship with their dealer or on single bank platforms. In prime brokerage agreements the dealer enables the client to trade directly with a group of predetermined third party banks in the dealer's name. The prime broker is interposed between the third party bank and the client and becomes the counterpart to both legs of the trade. [BIS, 2013] states that 16% of dealers transactions were conducted via prime

---

<sup>9</sup>The Uk accounts for 41%, the USA for 19%, Singapore 5.7% ,Japan 5.6%, Hong Kong 4.1%.

<sup>10</sup>In the direct interdealer market dealers trade with each other without intermediation, while in the brokered interdealer market, dealers trade with the intermediation of a broker. The broker searches quotes to match transactions, keeping the anonymity of the counterpart till the transaction is executed.

Overall the interdealer market accounts for 59\% of total daily market turnover (see [Sager and Taylor, 2006]). Although a large number of banks has a dealer in major spot markets, the top ten banks handle the largest part of the order flow([Lyons, 2001]).

|                 | Direct    | As % by type | Indirect  | As % by type | Undistributed | As % by type |
|-----------------|-----------|--------------|-----------|--------------|---------------|--------------|
| Spot            | 1,280,985 | 62.60%       | 730,526   | 35.70%       | 34,648        | 1.69%        |
| Frwds           | 428,937   | 63.08%       | 229,768   | 33.79%       | 21,289        | 3.13%        |
| FX Swaps        | 1,105,705 | 49.64%       | 1,064,082 | 47.77%       | 57,841        | 2.60%        |
| Curr Swap       | 31,101    | 57.57%       | 19,108    | 35.37%       | 3,813         | 7.06%        |
| FX Options      | 218,006   | 64.74%       | 111,138   | 33.00%       | 7,601         | 2.26%        |
| All instruments | 3,061,731 | 57.31%       | 2,154,622 | 40.31%       | 125,192       | 2.34%        |

Figure 1-1: Direct trades are those executed without the intermediation of a broker. They can be executed through voice directly with a dealing bank or electronically through a single bank trading platform.

brokerage. In case of single bank platforms clients can trade through a single bank limit order book, where the dealer knows the identity of the client. Despite all the technological innovations the vast majority of global FX instruments, including spot, continues to be predominantly traded OTC. In fact OTC turnover by far exceeds the trading volume of standardized FX products on organized exchanges. This feature, combined with the lack of disclosure requirements for dealers, leads to an uncommon information structure ([Lyons, 2001]): customers' orders are private knowledge of the dealing bank executing and the information they receive is comprehensive, not only of side and size of the transaction, but also of the identity of the customer, as an effect of direct trade negotiation.

The Forex microstructure literature highlights the importance of clients' order flow in conveying information that is relevant to exchange rate. This idea is controversial for many macro-economists<sup>11</sup>. As [Lyons, 2001] highlights there are two assumptions that traditionally disconnect order flow from prices. The first assumption is that all the information relevant for exchange rates is publicly known. The second one is that the mapping from information into prices is also publicly known. In the FX market either of the two assumptions can be relaxed, allowing for order flow to convey information that is relevant for prices.

The seminal paper of [Meese and Rogoff, 1983], which concludes that structural macro mod-

<sup>11</sup>[King et al., 2011] includes a comprehensive list of references on scholars work of the predictive power of order flow for exchange rate returns. [Cheung and Wong, 2000] in his survey of market practitioners reports that the majority of Dealers interviewed select "large customer base" and "better information" as the two main sources of large players' competitive advantage.

els cannot outperform a random walk in forecasting exchange rates, "has proven robust and remains a benchmark against which exchange rate models are judged" , [Evans and Lyons, 2005]. On one hand the robustness of this finding substantiates in the lack of consensus on the "correct" macroeconomic model. On the other hand this finding has lead leading scholars (see [Evans, 2010]) to the conclusion that the concept that all the information relevant for exchange rate is publicly known, can be relaxed. This idea leads to the application of the microstructure approach to currency markets: the focus is on the non trivial process of price formation and the flow of information from clients to dealers and in finally into currency prices. Dealers' customer base is composed both by leveraged investors and corporations. It can be argued, in the spirit of [Evans, 2010], that dealers acquire information about the slowly evolving state of the macroeconomy through their customer's order flow. This information is more up to date than macroeconomic releases<sup>12</sup> and is, more importantly, not public to the market yet.

The foreign exchange empirical microstructure literature provides ample evidence on the forecasting power of clients' order flow. [Evans and Lyons, 2005] compares the out of sample forecasting performance of traditional macro models and a micro based model that uses order flow to forecast exchange rate returns. They show that a micro based model that uses order flow that is disaggregated by end users has statistically significant forecasting power for spot rate changes up to an interval of 20 days horizon. [Evans, 2010], suggests that the reason for the forecasting power of order flow is that allocative trades from customers give microeconomic information to dealers who learn about the evolving state of the macroeconomy. The forecasting power of order flow originates from the fact that there is a lag between the time when information generates transactions and the time when this information is widely recognized by the market. This lag provides dealers with an information advantage. The work of [King et al., 2010] enriches the explanation for the forecasting power of order flow. The authors show that dividing end customers order flow into two segments, commercial clients and financial customers, the order flow from financial customers is positively related to exchange rate returns. [Osler and Vandrovych, 2009] conclude that the information embedded in customer order flow is not passively acquired, but rather actively acquired through the conscious effort of specu-

---

<sup>12</sup>The release of macroeconomic information is often lagged. Check for MAIN MACROECONOMIC INDICATORS

lative agents to anticipate market returns. They measure the informational content of trades as the signed return up to one week after execution and compare the results for six groups of bank customers. They find evidence that leveraged investors and specifically hedge funds are a critical source of private information in currency markets.[Bjønnes et al., 2011] furthermore suggest that the information dealers gather from informed investors trades is complemented by the dealing bank own analysis and the dealing bank itself takes position based on informed customers order flow: after trading with informed customers dealers are more likely to trade aggressively in the same direction of the customer. Moreover their work finds evidence that dealers with a larger network of financial customers tend to trade more aggressively on average, while this is not influenced by the extent of trading with corporations or governments.

## **1.5 The debate on OTC markets and benchmark determination, with a special focus on the FX fix**

The Fair and Effective Market Review [BoE, 2015] offers an analysis of OTC markets and policy recommendation related to this market structure. [BoE, 2015] highlights three main problems in OTC markets: concentration, conflicts of interest and lack of transparency. Concentration is typical of such markets as market makers/dealers specialize in information gathering to offer competitive pricing and market making has increasing returns to scale. Concentration is considered risky as it can facilitate collusion. Conflict of interest substantiates in the fact that dealers/market makers have to trade simultaneously as principal with their clients and for themselves. This role coexistence can give rise to the incentive to trade against the customer interest (vs fiduciary role) and even to market manipulations. Lack of transparency (absence of reporting requirements and consolidated tape) is beneficial to customers who want to undertake large transactions without revealing their strategies, but can lead to different pricing across counterparts and market segments increasing information asymmetries. [BoE, 2015] stress the importance of taking active measures in order to reduce the problems that can potentially arise in such markets. One suggestion is to allow market participants to choose among alternative

trading solutions<sup>13</sup> and find ways to improve transparency if OTC is the preferred market structure. A possibility to improve transparency is the adoption of technology, but in some OTC markets there is reluctance to undertake innovative solutions. As an interesting example, 'all-to-all' trading platforms challenge the informational advantage at the hearth of the market making model. This implies that market makers might try to restrict the access to such platforms. Example of these types of attempts are high access fees to limit access of buy side to inter dealer trading platforms, treats to remove liquidity unless certain platforms were/were not used or more technical issues as imposition of limits on the number of dealers from whom investors can request quotes on platforms. Such measures discourage users from joining automated trading platforms (e.g. when facing high access fees new users are willing to join only if they know others will do so). Despite the spot FX is the most developed electronic market in the landscape analyzed, it is still reported to be highly fragmented and this raises pre trade searching costs for participants and does not guarantee post trade transparency. In fact it can be difficult for market participant to assess how orders have been handled once received, whether matching methodologies favour particular styles of trading, whether algorithms deliver expected outcomes, or whether venues rule books are faithfully enforced. [BoE, 2015] suggest that suitable metrics, publicly or bilaterally disclosed, could be implemented without full post trade transparency in the form of a tape.

[BoE, 2015] highlights how some OTC markets have been under scrutiny due to fixing manipulation, e.g. the gold fixing or the FX fix. Such manipulations have been widely discussed by regulators. Both the International Organization of Securities Commission (hereafter IOSCO) and the Financial Stability Board have published reports to articulate policy guidance and principles for benchmark related activities. [IOSCO, 2013] recognize that a benchmark can be risky for several reasons, including conflicts of interests and incentives to manipulate the determination process<sup>14</sup>. [FSB, 2014] focuses on the FX fix manipulation, and identifies it as the product of a conflict of incentives between market makers acting as agents and principals in trading. This leads to two types of illegal behavior: colluding by sharing information about client orders and artificially manipulating the Fix rate by buying and selling aggressively in the

---

<sup>13</sup>Keep OTC for large transactions and non standardized assets vs All-to-all platforms for liquid or standardized assets with two way trading instrument.

<sup>14</sup>Especially where the submitters are also market participants with stakes in the level of the benchmarks.

Fix window. The collusive behavior of traders at the fix has led 6 major dealing banks to get fines for a total of \$10Bn for FX manipulation.

Trading at the Fix exhibits extreme concentration as small and regional banks pass orders related to fix trading on to the dominant few ([Osler et al., 2016]), and a sharp separation between client-dealer trading and interdealer trading. The FX fix manipulation has received a lot of attention from regulators: as highlighted by [Evans, 2014] "fix rates are important as, once calculated <sup>15</sup>, they are not simply archived but rather used to evaluate other securities and portfolios", .

Spot rates behavior in the FIX window presents some anomalies compared to the rest of the trading day. [Evans, 2014] remarks that Fix benchmarks are very unrepresentative of the prices at which currencies pairs trade in the hour or so around the fix and such benchmarks tend to fall towards the extreme of the daily range of spot rates<sup>16</sup>. The incidence of Fix benchmarks near the edge of the intraday spot range is far higher than would be the incidence of randomly chosen rates. This finding is particularly striking on the last day of the month. [Evans, 2014] documents negative correlation between pre fix and post fix rate changes: after the fix window there is a tendency for rates to revert back towards their pre Fix level. Thereafter the paths are flat. Rates do not revert fully and a new equilibrium rate is established, based on information contained in Fix related trading. At the end of month the changes are larger in absolute value and it takes longer for the new post Fix equilibrium rate to be established: such new equilibrium tends to be further away from the extremum of the rate path.

The work of [Michelberger and Witte, 2016] studies the market microstructure of the FX

---

<sup>15</sup>The procedure to calculate the Fix for traded rates uses data captured in a 5 minutes window, centered on the fix time. The source of data used to calculate the Fix varies depending on the currency ("trade" or "non trade") and the availability of data. The major source of FX spot rates is Thomson Reuter Market Data System (TRMDS), EBS is used for a subset of 3 currencies on which it has the dominant market share: EUR, JPY and CHF. When data sufficiency is limited, the WM company uses its own judgment to determine meaningful market rates. Every 1 second best bid and offer rates, and trades executed are recorded. Best bid and best offer are captured simultaneously and a traded rate is defined as a traded bid (TB) or traded offer (TO) depending on the correspondent order hitting a bid or lifting an offer. A spread is calculated as the difference between ask and bid for each rate captured. The spread is added (subtracted) to the captured TB (TO) rate to calculate the other side of the market. TO and TB, after a validation procedure, are pooled together and the median TB and TO is calculated. From the median TB and the median TO a mean rate (Mid) is calculated. An average spread is calculated, and applied to the Mean. This generates the  $A^{fix}$  and  $B^{fix}$ . When there are insufficient valid traded rates from the pooled data source, WM uses order rates. If neither order rates nor traded rates are available, the quoted rates from Reuters are used.

<sup>16</sup>Rates at the Fix appear to be roughly 15 bps-end of the month-and 7 bps-intra month- higher or lower than 15 minutes earlier. Rate changes of this magnitude are very rare in normal trading conditions.

around 4 pm. Their study shows that the order compression in the fix window, indeed changes the market behavior, resulting in spiking volatility within the fixing window and an increased probability for spot rate extrema in this period. Both effects appear predominantly in anticipation of the fix event. This aspect sets the difference between Fix market dynamics and market dynamics related to information arrival. They evidence that, when spot rate movements are induced by information arrival, the probability of an extreme price movement is lower beforehand and increases only after the release of information. Moreover the increase in spot movements' size is retained for long periods, while the probability spikes at 4 pm decay quickly.

[Melvin and Prins, 2015] focus on the relationship between currency returns and equity returns<sup>17</sup>. The hypothesis they aim to test is that hedging trades generated by outperformance of an equity market over the course of one month lead to the selling of that country currency leading up to the last fix of the month. They are testing the hypothesis that the relationship between equity and currency market is negative. To measure the holding of foreign equity they consider overseas investments only, and allocate them on the basis of foreign market capitalization. To have a proxy of the total return to a manager's foreign equity holding in one country they use the specific country index. This choice is justified by the mechanical calculations manager perform to know hedging needs, which is based on stock returns. Their findings suggest that the outperformance of a country equity market over the month is associated to currency depreciation up to the hours leading to the FX fix. The depreciation partially reverts over the Fix. This channel of exchange rate adjustment is more important for currencies with the largest market cap (US, Eurozone and Japan). They show that while on monthly days the average volatility has a fairly constant profile across currencies, at the end of month there is high volatility one hour before and after the fix. They further show that volatility on the end of the month day is higher than usual when predicted hedging flows are large. An interesting aspect of their paper is that they explore the idea that speculators (read hedge funds) might provide liquidity to the end of month hedging flows, partially arbitraging them away<sup>18</sup>. Speculative flows moderates the impact of hedgers, but do not neutralize it in full.

---

<sup>17</sup>They consider market returns prior to the date and fx returns on the date.

<sup>18</sup>In this setting the speculators buy currency ahead of the fix and then sell it to meet hedgers demand at the fix. Otherwise they sell at the fix and buy after.

## Chapter 2

# High Frequency Trading and Market Making

### 2.1 Introduction

In the last few years there has been a growing interest in high frequency trading strategies and their effects on financial markets. In particular from the flash crash regulators, financial institutions and academics have been comparing the beneficial effects offered by high frequency traders (HFTs) and the potential problems that could arise from their activity<sup>1</sup>. The ongoing debate has major implications for regulators and participants of financial markets, as the active presence of such players on market platforms has been considered a potential source of mistrust for long terms investors.

The empirical literature on high frequency traders stress the fact that their active market participation improves liquidity, having a beneficial effect on spreads.[Brogaard, 2012] working on a sample including 40 US stocks highlights the fact that high frequency traders provide the best bid and offer 65,3% of the calendar time. [Menkveld, 2011] finds strong evidence that the arrival of a high frequency trader acting as a liquidity provider in the Chi-X market reduces the spread of almost 50%. [Hendershott et al., 2010] analyzing algorithmic trading, find that

---

<sup>1</sup>Among documents from regulators see [CFTC and SEC, 2010].From the industry [Barker and Pomeranets, 2011],[Chlistalla, 2011],[Tradeworx, 2010]. For a general review of academic work on HFTs and main findings, see [Gomber et al., 2011].

as algorithmic trading increases, market liquidity<sup>2</sup> improves. additionally, several other works highlight the beneficial effects of high frequency traders providing liquidity to markets, see [Brogaard, 2012],[Cvitanic and Kirilenko, 2010],[Hasbrouck and Saar, 2012] and for a general review on main findings see [Gomber et al., 2011]).

Although there is a wide literature that confirms the beneficial effects of such market participants and the rise of high frequency trading is a phenomenon in line with the technological evolution of financial markets (see [Smith, 2010]), there is an ongoing debate if this new technology could need to be monitored closely or even subject to restrictions<sup>3</sup>.

The core of the debate is around the reliability of high frequency traders in providing liquidity. [Cvitanic and Kirilenko, 2010] analyzing the flash crash event, stress the fact that after providing liquidity such players began to demand aggressively liquidity to keep their inventory within a certain target level. Moreover, the evidence of tighter spreads associated with the participation of high frequency traders to the market, raises questions on the possibility that traditional market makers could leave liquidity provision ([Barker and Pomeranets, 2011]).

This work aims to contribute to the research on this topic in two ways. The first one is to analyze the effect of the presence of a high frequency trader competing with a traditional (slow) market maker on the bid ask spread. The second is to investigate the outcome of the competition in terms of market participation for both liquidity providers.

In this work the bid ask spread is an effect of the market power of the liquidity providers, who offer an immediate counterpart for transactions to traders demanding liquidity via market orders. In order to capture the speed advantage of the high frequency trader one of the two market makers has the possibility to update his quotes more often.

In the model the two market makers have different risk preferences. The slow market maker is risk neutral, while the high frequency trader is risk averse. Modelling market makers as risk averse captures the impact that inventory has on pricing. In a mean variance set up this impact is linear. In inventory models, the spread reflects the risk-bearing cost incurred by

---

<sup>2</sup>They use effective spread as a measure of liquidity.

<sup>3</sup>In a report issued from the LSE as a response to a CESR call for evidence on micro-structural issues, when openly asked if there is in their perspective a need for regulation they stress the fact that regulatory intervention should be adopted only in case of evidence of the fact that HFT activity is detrimental for markets. Nevertheless the question itself suggests that regulation is an open issue.

market makers who build up positions to accommodate public order flow. Assuming a dealer is risk averse is equivalent to assuming aversion to diversifiable risk. This is because inventory risk could be diversified through appropriate hedging ([Biais et al., 2005]). There is evidence that traditional banks offering market-making services, actively manage their inventory<sup>4</sup>. Inventory management includes hedging with related financial instruments ([Duffie, 2012]).

In the recent empirical literature on HFTs acting as market makers (see for example [Menkveld, 2011]), there is evidence of HFTs reducing risk by reverting to null inventory positions within seconds. This aspect distinguishes HFTs from traditional market makers. The former targets a zero inventory level throughout the day (see [Guilbaud and Pham, 2013] as an example of a model of HFTs managing inventory with a target of zero), while the latter actively hedges his inventory position.

In this model the implicit assumption is that the slow market maker actively manages his inventory position, diversifying his holdings. This reduces his aversion to diversifiable risk. On the opposite, the HFT targets a zero inventory position and this is captured by mean variance preferences. This intuition is supported by empirical evidence. [Anand and Venkataraman, 2012] concludes that designated market makers close their day flat only in 1% of the cases<sup>5</sup>, and this percentage increases sharply when considering unofficial market makers [Anand and Venkataraman, 2012] or high frequency traders ([Menkveld, 2011]).

The presence of the high frequency trader limits the market power of the incumbent market maker, which translates in a lower spread. In the two stage model the equilibrium outcome suggests that the high frequency trader has no possibility to push out the other liquidity provider, but this is not true vice versa. In fact in equilibrium there are cases where the slow market maker has an incentive to push out his competitor and serve the whole market with a lower spread.

---

<sup>4</sup>The figure of the designated market maker is common to many stock exchange in US and Europe and usually there is an agreement in place between the market maker and the exchange that includes maximum spread requirement and liquidity provision obligations. Often banks act as designated market makers and as a reward for their services they get fee discounts or other types of benefits. The fact that financial institutions operate as market makers and the fact that they incur in no trading fees ([Benos and Wetherilt, 2012]), makes us conclude that they will diversify more their asset holdings, hedging their positions ([CFTC and SEC, 2010]) and consequently reducing their aversion to inventory imbalances.

<sup>5</sup>They analyze a dataset from the Toronto Stock Exchange that contains transactions on 1286 stocks traded 245 days.

On the other hand a high frequency trader providing liquidity as a monopolist does not ensure that market demand will always be served. This is because the high frequency trader will have an incentive to post stub quotes when the level of the spread is not high enough to incentive him to accumulate shares in his inventory. This effect is attributable to the willingness of this liquidity provider to control his inventory.

Overall the equilibrium outcome suggests that the presence of the high frequency trader is beneficial in lowering spreads and providing liquidity in case of competition with other traditional liquidity providers. On the other hand having a monopolistic high frequency trader that acts as a liquidity provider is not a desirable outcome. In light of this finding it is beneficial to monitor the entry costs that designated market makers bear to access the market, and think about subsidizing designated market makers if such costs become too high.

This paper examines the bid ask spread as a result of the market power of market participants. Traditionally in the market microstructure literature the bid ask spread is a product of two elements: an inventory holding cost component and a cost component arising from information asymmetries. Inventory models explore the problem of uncertainty in the future order flow, which can result in inventory imbalances for the specialist and execution problems for the traders. Works like [Garman, 1976], [Stoll, 1978], [Ho and Stoll, 1981] and [O'Hara and Oldfield, 1986] focus on the effects of inventory imbalances and time horizon in affecting the spread. In my work I consider the effect of the inventory on the spread dynamics, but I focus on how different liquidity providers who coexist in the market can have a different attitude towards inventory exposure and what is the effect of such coexistence on spreads. The literature focusing on information asymmetries includes often risk neutral market makers in perfectly competitive markets that enforce a zero profit condition..[Glosten and Milgrom, 1985a] suggests that the specialist needs to deal with two types of traders: better informed traders, with whom he will always loose in transactions, and noise traders, from whom it is possible to earn a profit. In this model the spread is increasing in the number of informed traders populating the market. [Easley and O'Hara, 1987] suggest the fact that block trades usually execute at worst price is due to the fact that uninformed market makers need to protect themselves from the risk that a block trade signals an information event occurring in the market. In my work I abstract from information asymmetries in spread determination. This choice is motivated by

the extremely short holding time of high frequency traders. Another stream of literature analyzes the presence of a monopolist risk neutral market maker, the former specialist in the NYSE. [Glosten, 1989] points out that in cases of severe adverse selection the liquidity provided by a monopolist specialist is higher than in the case of competing market makers. [Kavajecz, 1996] analyzing the specialist system points out that, as the number of informed traders increases the specialist uses both quotes and depth to protect himself from adverse selection. [Dupont, 2000] suggests that the risk neutral specialist might use more the depth than the spread in order to manage asymmetric information risk. Some more recent works consider the case of competition between the market maker and limit order traders, as they both contribute to liquidity provision. [Seppe, 1997] considers a specialist has two types of advantages over the other liquidity providers: he does not pay a cost for transactions and he observes the realized volume of the order of the liquidity demander. The optimal pricing strategy for the specialist depends on the size of the market order and the orders in the limit order book. [Bondarenko and Sung, 2003] analyze the case of a specialist competing with limit order in cases of asymmetric information: if the specialist has no informational advantage in terms of the depth of the limit order book has no incentive to trade and liquidity will be provided by limit orders. On the contrary if the specialist has private information regarding the realized depth, this provides him an incentive to trade, and depending on the depth of the market he uses a different strategy. My model is different from those mentioned as my main focus is on competition between liquidity providers.

The rest of the work is organized as follows. Section 1 includes the one period model and its results. Section 2 describes the two period model, describes the equilibria and discusses them. Section 3 discusses the case of the high frequency trader acting as a monopolist when the market maker decides to leave liquidity provision. Section 4 concludes.

## **2.2 One period model**

### **2.2.1 Setup**

In this section a one stage model is presented. The aim of the one stage model is to understand how competition between two market makers with different risk preferences affects the bid-ask spread. The idea presented in the stage model apply to the dynamic model.

One stock  $V$  is traded and market participants have perfect information. Three types of agents participate to the market. Two liquidity providers and one liquidity demander.

*Liquidity providers.* Two market makers act as liquidity providers submitting bid and ask quotes. One market maker  $M_1$ , moves first, the other,  $M_2$  follows.  $M_1$  is a strategic agent with risk neutral preferences. At  $t=1$  he submits his bid and ask quotes  $(b_1, a_1)$ .  $M_2$  is a strategic agent with mean variance preferences. He observes  $(b_1, a_1)$  and posts his own quotes  $(b_2^1, a_2^1)$ . His utility is:

$$U_2 = E(\pi) - \frac{\tau}{2} \text{Var}(\pi)$$

*Liquidity demander.* One liquidity trader acts as a liquidity demander, submitting market orders. The liquidity trader trades for exogenous reasons, submitting a market buy order and a market sell order with the same probability  $p_b = p_s = \frac{1}{2}$ . His demand is assumed inelastic to price till a reservation price  $\bar{P}$ , for buy orders, and  $\underline{P}$ , for sell orders. Hence for a buy order his demand will be:

$$x(p) = \begin{cases} n, & \text{if } a_i \leq \bar{P} \\ 0, & \text{if } a_i \geq \bar{P}, \text{ for } i=\{1,2\} \end{cases}$$

The situation is symmetric for a sell order.

*Clearing Rules and Market Timing.* The clearing rules for the market follow. The market maker who has the best ask or bid serves the order and the other one does not trade. Otherwise, if both market makers display the same bid or ask they share the market demand. Formally, on the ask side (market order to buy):

$$\begin{cases} \text{if } a_1 < a_2, x_1 = n \text{ and } x_2 = 0 \\ \text{if } a_2^1 < a_1, x_1 = 0 \text{ and } x_2 = n \\ \text{if } a_1 = a_2^1, x_1 = \frac{n}{2} \text{ and } x_2 = \frac{n}{2} \end{cases}$$

And symmetrically on the bid side (market order to sell).

For the one period model the timing consists of  $M_1$  posting  $(b_1, a_1)$  before the opening of the market, and subsequently  $M_2$ , after observing  $(b_1, a_1)$ , posting  $(b_2^1, a_2^1)$ .

At the opening of the market the liquidity trader observes the prices posted by the two market makers and, if the prices are below his reservation price, he posts a market order for  $n$  shares. If prices are above his reservation price he does not trade. Once the order is posted

(or the liquidity trader has left the market without posting an order), the two market makers realize their payoff.

### 2.2.2 Equilibrium prices

To solve for equilibrium I assume that the problem is symmetric on the bid and ask side. For this reason, without loss of generality, the first order reaching the market is assumed to be a buy order. The solution is symmetric for a sell order reaching the market first.

The next definition highlights the thresholds that are necessary to describe the equilibrium.

**Definition 1** Define  $T1 = E(V) + \frac{\tau}{4}nVar(V)$ , as the level of ask where  $M_2$  is indifferent between trading half market and not trading. Define  $T2 = E(V) + \frac{\tau}{2}nVar(V)$ , as the level of ask where  $M_2$  is indifferent between trading  $n$  and not trading. Define  $T3 = E(V) + \frac{3}{4}\tau nVar(V)$ , as the point where  $M_2$  is indifferent between trading the whole market and trading half of it.

The following proposition presents the equilibrium bid and ask outcome depending on where the reservation price of the liquidity trader lies:

**Proposition 2** Depending on where the reservation price of the liquidity trader lies, different equilibria will arise. More in details, assuming that the first order that reaches the market is a buy order:

- I. If  $\bar{P} \geq T_3$ ,  $M_1$  sets  $a_1 = T_3$  and  $M_2$  sets  $a_2^1 = a_1$ . In this case they share the market at  $a_1 = a_2^1 = T_3$ .
- II. If  $T_2 < \bar{P} < T_3$ ,  $M_1$  sets  $a_1 = \bar{P}$  and  $M_2$  sets  $a_2^1 = a_1$ . Also in this case they share the market at  $a_1 = a_2^1 = \bar{P}$ .
- III. If  $\bar{P} = T_2$  there are two possible equilibria among which  $M_1$  is indifferent. The first equilibrium is  $M_1$  sets  $a_1 = T_2$  and  $M_2$  sets  $a_2^1 = a_1$ , implying they share the market. Another possible equilibrium is setting  $a_1 = T_1$  and serving the market as a monopolist.
- IV. If  $T_1 < \bar{P} < T_2$ ,  $M_1$  sets  $a_1 = T_1$ , and  $M_2$  is out of the market.
- V. If  $\bar{P} < T_1$ ,  $M_1$  sets  $a_1 = \bar{P}$ , and  $M_2$  is out of the market.

The proof is provided in Appendix A.

Overall the previous result provides a few hints on how the spread is affected by the arrival of the risk averse market maker as second liquidity provider.

In this setting the two market makers are sequential in posting their prices. This implies that the risk neutral market maker who moves first determines his bid and ask prices considering the strategic behavior of the second mover.

From this consideration two different incentives arise for the first mover. Widening the spread implies a higher margin of profit per share, but also a higher possibility to be undercut, that translates in lower market demand to serve. In case of extremely high spread levels this implies that the first mover is completely pushed out of the market. Narrowing the spread implies that the profit per share is lower, but there is the possibility to serve a higher portion of market. For extremely low levels of spread, the second liquidity provider leaves the market, incurring in a loss<sup>6</sup>.

Figure 4.1 provides an intuition of the result stated in Proposition 2 .

If the risk neutral market maker is alone in the market he can set the maximum feasible spread, extracting always the reservation price of the liquidity trader. The presence of a second market maker ensures a reduction of the spread and, in some cases, the possibility for the liquidity trader to purchase the stock for a price that is lower than his reservation price.

The risk averse market maker is unable to push the first mover out of the market, while the reverse situation is possible. This is due to the difference in the risk preferences of the two market makers. In fact the first mover, being risk neutral, is able to sustain lower levels of spread than the second market maker, still keeping a positive profit. This is because he does not bear a cost associated to the variance of the stock. This is not true for the other market maker, who needs to compensate the inventory cost, hence needs, in proportion, to set a higher price, to keep earning positive profits.

Moreover, considering the fact that  $M_2$  is risk averse, it follows that if the variance increases, assuming  $\bar{P}$  unaffected, there is a wider interval in which it is not profitable for  $M_2$  to enter the market. In this case, although the first mover would serve the market on his own, the potential entrant limits his power in setting a spread.

---

<sup>6</sup>This is due to the risk neutrality of one player and the risk aversion of the other.

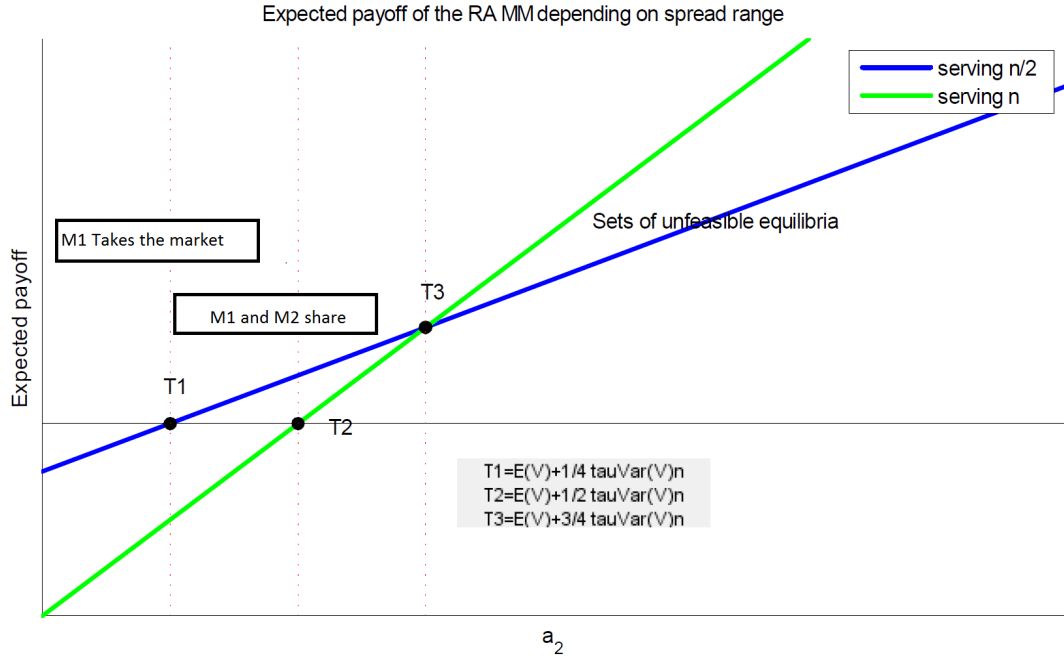


Figure 2-1: The utility of  $M_2$  is plotted as function of the spread level. For spread levels that exceed  $T_3$ ,  $M_2$  has an incentive to undercut  $M_1$  and serve the market on his own. As a result,  $M_1$  never chooses to set such a high spread, as this implies a zero profit. When the reservation price of the liquidity trader is high enough ( $T_2 < \bar{P} < T_3$ ), for some levels of spread there is a sharing equilibrium. This happens in the interval  $(T_2, T_3)$  where, for both market makers, the most convenient option is to set the same spread, share the market and extract from the liquidity trader his reservation price. If the reservation price of the liquidity trader is below  $T_2$ ,  $M_1$  will have an incentive to lower the spread further, as this would result in serving the market alone.

## 2.3 Two period model

### 2.3.1 Set up

This section presents the main model. In the one round setting the two liquidity providers have only different risk preferences, while in the main model they also have different technologies.

*Liquidity providers.* As in the previous case, one market maker  $M_1$ , moves first, the other follows. The second mover,  $H$ , has a technological advantage over the first player. The technology he has adopted allows him to be quicker in updating his quotes. This advantage translates in the fact that  $M_1$  keeps the quotes fixed for the two trading periods, while  $H$  can update them at the end of each trading period. He observes  $(b_1, a_1)$  and posts his own quotes  $(b_2^1, a_2^1)$ . After the first period of trading, he updates his quotes posting  $(b_2^2, a_2^2)$ .

*Liquidity demander.* There is one liquidity trader who demands liquidity submitting market orders. His demand structure is identical to the one round case. The only difference is that in this context the liquidity trader visits the market twice and his demand in the two cases is not correlated.

*Clearing Rules and Market Timing.* The clearing rules for the market are identical to the one round market and analogous for each trading round.

To recall briefly, if both market makers display the same bid or ask, they share the market demand. Otherwise the market maker who has the best ask or bid serves the order and the other one does not trade. As highlighted earlier, the model has two trading period. Before the opening of the market the first mover posts  $(b_1, a_1)$  and subsequently the second mover posts  $(b_2^1, a_2^1)$ . At the opening of the first trading period the liquidity trader arrives to the market. He observes the prices posted by the two market makers and, if the prices are below his reservation price, he posts a market order for  $n$  shares. If the prices are above his reservation price he does not trade. Once the order is posted (or the liquidity trader has left the market without posting an order), the second mover, who differs in the technology he uses, has the opportunity to update his quotes, posting  $(b_2^2, a_2^2)$ . After the quotes are updated a new trading round takes

place and the liquidity trader submits a new market order. Once the second trading period is over the two market makers realize their payoff.

### 2.3.2 Equilibrium prices

In the two period setting the equilibrium outcome depends on the level of the variance of the security and on the level of risk aversion of  $H$ . The following two subsections contain the main results (proofs are provided in Appendix C and D). The third subsection discusses the results.

#### Equilibrium prices, high variance case

This section presents the equilibrium outcome in case of  $\text{Var}(V) > \frac{8}{\tau^2}$ .

**Definition 3** Define  $T4 = E(V) + \frac{5}{4}\tau n\text{Var}(V)$  as the level of spread where  $H$  is willing to serve whole market when two orders of different sign occur, and indifferent if serving whole market and half for the second order when two orders of the same sign occur (given that he has served whole market in the first round). Define  $T3 = E(V) + \frac{\tau}{2}n\text{Var}(V)$ , as the point where  $H$  is indifferent between serving half market and whole market when two orders of different sign occur, and would serve no market for the second order when two orders of the same sign occur. Define  $T2 = E(V) + \frac{2}{15}\tau n\text{Var}(V)$ , as the point where  $M$  is indifferent between sharing the market with  $H$  or lowering the price at a level  $T1 = E(V) + \frac{\tau}{12}\tau n\text{Var}(V)$ , where  $H$  is indifferent between serving half market and no market when two orders of different sign occur and would serve no market for the second order when two orders of the same sign occur.

**Proposition 4** Depending on where the reservation price of the liquidity trader lies, different equilibria will arise. More in details, assuming that the first order that reaches the market is a buy order

I. If  $\bar{P} > T4$ ,  $M$  is indifferent between :

- set  $a_1 = T4$ , where  $H$  undercuts him and set the spread at his maximum (below  $T4$ )
- set  $a_1 = T3$ , where  $H$  shares with him the market.

II. If  $T3 < \bar{P} < T4$ ,  $M$  sets  $a_1 = T3$  and  $H$  shares the market (set  $a_2^1 = a_1$ ,  $a_2^2$  is not posted,  $b_2^1 = b_1$ ).

III. If  $T2 < \bar{P} < T3$   $M$  sets a spread  $c = \bar{P}$  and  $H$  shares the market (sets  $a_2^1 = a_1$ ,  $a_2^2$  is not posted,  $b_2^1 = b_1$ ).

IV. If  $T1 \leq \bar{P} < T2$ ,  $M$  sets  $a_1 = T1$ , and  $H$  stays out of the market.

V. If  $\bar{P} < T1$ ,  $M$  sets  $a_1 = \bar{P}$ , and  $H$  stays out of the market.

### Equilibrium prices, low variance case

This section presents the equilibrium outcome in case of  $\text{Var}(V) < \frac{8}{\tau^2}$ .

As in the previous case, in order to present the equilibrium outcome in terms of bid and ask prices, some thresholds should be defined.

**Definition 5** Define  $T4 = E(V) + \frac{5}{4}\tau n \text{Var}(V)$  as the level of spread where  $H$  is willing to serve whole market when two orders of different sign occur and indifferent if serving no market and half for the second order when two orders of the same sign occur. Define  $T3 = \frac{12 - \sqrt{144 - 18\tau \text{Var}(V)}}{3\tau}$ , as the point where  $H$  is indifferent between serving half market and whole market when two orders of different sign occur and would serve no market for the second order when two orders of the same sign occur. Define  $T2 = E(V) + \frac{2}{15}\tau n \text{Var}(V)$ , as the point where  $M_1$  is indifferent between sharing the market with  $H$  or lowering the price at a level  $T1 = E(V) + \frac{\tau}{12}n \text{Var}(V)$ , where  $H$  will be indifferent between serving half market and no market when two orders of different sign occur and would serve no market for the second order when two orders of the same sign occur.

**Proposition 6** Depending on where the reservation price of the liquidity trader lies, different equilibria will arise. More in details, assuming that the first order that reaches the market is a buy order:

I. If  $\bar{P} \geq T4$ ,  $M$  sets  $a_1 = T3$  and  $H$  sets  $a_2^1 < a_1$ ,  $a_2^2$  will not be posted,  $b_2^1 > b_1$ .

The amount of which  $H$  undercuts  $M_1$  depends on the variance.

- If  $\text{Var}(V) > \frac{24}{5\tau^2}$ , he sets the spread at his maximum (below  $T4$ ).
- If  $\text{Var}(V) < \frac{24}{5\tau^2}$ , he undercuts  $M_1$  of an amount  $\epsilon$ .

- II. If  $T3 < \bar{P} < T4$ ,  $M_1$  sets  $a_1 = \bar{P}$  and  $H$  sets the same spread as before.
- III. If  $T2 < \bar{P} \leq T3$ ,  $M_1$  sets a spread  $c = \bar{P}$  and  $H$  sets  $a_2^1 = a_1$ ,  $a_2^2$  will not be posted,  $b_2^1 = b_1$
- IV. If  $T1 \leq \bar{P} < T2$ ,  $M_1$  sets  $a_1 = T1$ , and  $H$  stays out of the market.
- V. If  $\bar{P} < T1$ ,  $M1$  will set  $a_1 = \bar{P}$ , and  $H$  will stay out of the market.

The best response of the risk averse market maker,  $H$ , changes depending on the spread range<sup>7</sup>. The first thing to notice is that for this market maker it is always profitable to end up offsetting the inventory cost. This means that if he can close the trading day without carrying any inventory exposure he will certainly do so. This preference reflects in the fact that when two orders of different sign (e.g. buy-sell, sell-buy) hit the market, he is willing to serve both, as for him this is always profitable<sup>8</sup>. On the other hand the risk averse market maker decision to serve the market in case of the arrival of two orders of the same sign (e.g. buy-buy, sell-sell) depends on the spread. This is because with two orders of the same sign the inventory cost increases:  $H$  pursues this strategy only for a level of spread that is sufficiently high to cover the cost.

These considerations influence the risk averse market maker in his spread choice. On one hand widening the spread increases the profit per share, but as a trade off induces the other market maker to pursue a higher share of the market.

As mentioned before there are two different equilibrium outcomes depending on variance. For low levels of variance, the risk averse market maker is more willing to take higher exposure in the market. The opposite happens if the variance increases:  $H$  is less willing to hold inventory, and prefers to pursue a strategy that reduces his exposure. This influences the choice of the first mover who knows that  $H$  is willing to grab the whole market till low levels of spreads. The designated market maker, in a low variance scenario serves less market but sets a higher

---

<sup>7</sup>When considering the options for  $H$  in this case there are 27 possible strategies available. This is because for each trading round he has the option to push out  $M(a_2^{\{1,2\}} < a_1, b_2^{\{1,2\}} > b_2)$ , to share the market with him ( $a_2^{\{1,2\}} = a_1, b_2^{\{1,2\}} = b_2$ ) or to leave the market, ( $a_2^{\{1,2\}} > a_1, b_2^{\{1,2\}} < b_2$ ). Assuming a buy order hits the market first, this will give him 3 variable to choose, that are  $a_2^1, a_2^2, b_2^2$ .

<sup>8</sup>In this case the size of the exposure, that is sharing the market or grabbing whole of it, will depend on the spread range, but not the strategy of serving sequential orders of opposite sign.

spread (this implies higher profit per share)<sup>9</sup>. In a different scenario, if variance is high a small reduction in the spread induces H to serve half or no market. For this reason  $M_1$  is more willing to reduce the spread and serve a higher share of market. The presence of H is hence beneficial when the variance is high as M has an incentive to reduce the spread and earn his profit from serving the whole market (increasing his market share) rather than from a higher per share profit with smaller exposure.

In both cases the profit of  $M_1$  is the same. What effectively changes is the price the liquidity trader pays. In a low variance scenario,  $M_1$  has a higher incentive to extract the reservation price from the liquidity trader in order to keep his profit high while in the high variance scenario he can earn a sizable profit not extracting the whole reservation price from the liquidity trader.

In this setting small changes in the reservation price of the liquidity trader induce changes in spread. If the reservation value is lower than the threshold  $T_2$ ,  $M_1$  has an incentive to reduce his spread till a level  $T_1$ , to push the H out of the market. This points out how small changes in the reservation price induce a marked reduction in the spread. This phenomenon can justify changes in spread that are not related to market conditions, for example increases or decreases in volatility, but rather are an effect of competition between liquidity providers.

## 2.4 Introducing a cost for Making Markets

This section considers the equilibrium outcome when the HFT is the only liquidity provider in the market. This can be seen as a case in which the slow Market Maker,  $M_1$ , needs to bear a cost in order to provide liquidity (e.g. a capital requirement that  $M_1$  is asked to fulfill when acting as an official liquidity provider in one or more securities).

As discussed earlier, there is evidence of narrower spreads with the arrival of HFTs. [Jones, 2013] points out that this result is not only a classical product of competition. In fact the narrower spread is also due to the reduction of the inventory holding time and the speed advantage in acquiring and processing information (non necessarily fundamental), as both aspects are relevant in influencing the magnitude of the spread.

---

<sup>9</sup>If  $MM_1$  wants to exclude  $MM_2$  from the market, in this case he should lower the spread till a level that would not give him a higher profit than sharing the market.

A consequence of narrower spreads could be that traditional market makers do not find profitable anymore to engage in liquidity provision. [O’Hara, 2015] outlines how, in a world in which speed advantage entails also informational advantage, all the players (including non-HFTs) needs to adapt to move faster. This insight applies also to traditional market makers.

In the model of [Boco et al., 2017] HFTs are modelled as informed, following the empirical findings of [Biais et al., 2014] and [Biais et al., 2015]. In this type of setting they show how slow traders are adversely affected by the presence of HFTs when trading on information. This is because slow traders are limited in the use of their private information, due to their relative slow speed. In this respect this finding is in line with the conclusion of my model, which shows that slow traders are harmed by the presence of HFTs as they might loose the incentive to perform market making services due to increased risk of adverse selection.

In this perspective, the idea of traditional market makers seems doomed to become obsolete. Despite this, there is regulatory evidence (see [O’Hara, 2015]) that non-HFTs market participants are seeking trading strategies and venues that protect their interests. This separation might allow for the survival of liquidity providers that offer reliable liquidity even in circumstances of demand and supply imbalances and require higher compensation due to longer holding times.

Overall the “survival “of traditional market makers requires two form of adaptation. The first one is technological, in the sense that market makers need to invest in technology to be competitive when compared to other market participants. The second is regulatory, and stems from the pressure of non-HFTs market participants, in asking for separation of venues and protection from predatory strategies.

In this section I aim to analyze the effect of the high frequency trader being the only liquidity provider in the market. My conclusion is that the idea of a market in which liquidity provision is endogenous and motivated only by profitability reasons could lead to a market structure that is fragile. This seems to be especially relevant for small and medium cap stocks, as the liquidity provision of all liquidity providers seems to be sparse and opportunistic, essentially vanishing when there is high inventory risk ([Anand and Venkataraman, 2012] ).

In such case the risk aversion of the profit motivated liquidity provider plays a crucial role, due to the willingness to avoid inventory imbalances that might translate in losses. As stressed

before, an official liquidity provider is someway compensated by the exchange for the losses that might arise due to his activity, while this is not true for an endogenous profit motivated trader. The results of having a risk averse liquidity provider acting alone in the market confirms that not always market demand will be fulfilled, depending on the spread level.

### 2.4.1 Set up

The set up is identical to the two round model except for the fact that H is the only liquidity provider in the market. This implies that the only bid and ask quotes posted in the market are  $(b_2^1, a_2^1)$  and  $(b_2^2, a_2^2)$ . The market timing and market clearing conditions are identical to the previous case.

### 2.4.2 Equilibrium prices

Being the unique liquidity provider in the market H has the option of providing liquidity to the whole market or none. His choice, as in the previous cases depend on where the reservation price  $\bar{P}$  lies.

**Definition 7** Define  $T2 = E(V) + \frac{14 + \sqrt{196 - 4\tau^2 \text{Var}(V)}}{2\tau}$  as the level of spread where H is willing to serve whole market when two orders of same sign occur and indifferent if serving whole market and no market for the second order, when two orders of the same sign occur. Define  $T1 = E(V) + \frac{12 + \sqrt{144 - 8\tau^2 \text{Var}(V)}}{2\tau}$  as the level of spread where H is indifferent if serving only orders of the opposite side, or exiting the market and stopping providing liquidity.

**Proposition 8** Depending on where the reservation price of the liquidity trader lies, different equilibria arise. More in details, assuming that the first order that reaches the market is a buy order:

- I. If  $\bar{P} > T2$ , H sets  $a_2^1 = a_2^2 = \bar{P}$ . In this case serve the whole market, whatever is the sequence of orders.
- II. If  $\bar{P} = T2$ , H is indifferent in setting  $a_2^1 = a_2^2 = \bar{P}$  and serving all market or setting:

- $a_2^1 = \bar{P}$ ,  $a_2^2 > \bar{P}$ , if  $Var(V) < \frac{4}{\tau^2}$ ,
- $a_2^1 = \frac{6}{\tau}$ ,  $a_2^2 > \bar{P}$ , if  $Var(V) > \frac{4}{\tau^2}$ ,

III. If  $T1 \leq \bar{P} < T2$ ,  $H$  sets:

- if  $Var(V) < \frac{4}{\tau^2}$ ,  $a_2^1 = \bar{P}$ ,  $a_2^2 > \bar{P}$ .
- if  $Var(V) > \frac{4}{\tau^2}$ ,  $a_2^1 = \frac{6}{\tau}$ , if  $\bar{P} > \frac{6}{\tau}$  and  $a_2^1 = \bar{P}$  if  $\bar{P} < \frac{6}{\tau}$ ,  $a_2^2 > \bar{P}$

IV. If  $\bar{P} < T1$ , he will be out of the market.

In this setting the  $H$  is a monopolist and for this reason either he decides to serve the whole market or none of it. The choice depends on the reservation price of the  $LT$ . If this price is above  $T2$ , then he sets the maximum price possible, extracts the whole reservation price from the  $LT$  and serves the whole market.

If instead the reservation price drops,  $H$  has the incentive to post stub quotes in case orders of the same sign occur. This means that the demand of the  $LT$  is not served under some circumstance. A key feature of acting as a monopolist liquidity provider is that trading implies committing to serve the whole market. In the competitive case, sharing the market allows sustaining lower spread levels. Moreover the presence of the slow market maker ensures that, even for low levels of spread, liquidity provision continues, while this is not true in the case in which the High Frequency Trader is alone in the market.

Overall the previous considerations imply that a designated traditional market maker competing with a profit motivated liquidity provider offers the best solution. This is because even considering a scenario in which several high frequency traders compete in a Bertrand fashion they are unwilling to serve the market when the spread falls below the minimum level required to cover the inventory cost. Overall competition between the risk neutral and the risk averse market maker ensures a reduction of the spread and, at the same time, enhances the possibility of filling the market demand even in cases of low spreads. This point highlights that liquidity provision is more reliable in a market in which there is a designated market maker.

The reduction of the spread due to the arrival of high frequency traders likely reduces profits from market making. This implies that the costs associated to market making should be closely monitored and eventually policymakers should consider subsidizing traditional market makers.

## 2.5 Discussion and Conclusions

This work aims to capture the effect of competition between a traditional market maker and a high frequency trader providing liquidity in the market. The incumbent is assumed risk neutral<sup>10</sup> while the High frequency trader is assumed risk averse. The aim of this assumption is to capture a higher propensity to avoid inventory imbalances by such players when acting as market makers. This element is confirmed by the fact that often high frequency traders seek to unwind all the inventory positions before the end of each trading day ([Tradeworx, 2010]), while traditional market makers have the possibility to close the day flat only in very few occasion (see [Anand and Venkataraman, 2012] ).

Empirical findings show that high frequency traders have a beneficial effect in terms of spread level, they often provide the best bid and offer and in general improve market liquidity. Despite this evidence the debate on the possibility to adopt regulation measures for such players is still ongoing. The main concern is that their liquidity is not reliable. This consideration and the idea that traditional liquidity provision might suffer a decrease due to the competition from this new technology motivates a closer look to the spread dynamics arising as an effect of the arrival of high frequency traders as liquidity providers.

The main finding suggests that, overall, the arrival of a High Frequency Trader has a beneficial effect on spread, reducing the market power of the traditional liquidity provider. This effect is due to increased competition. Overall it is not possible for the High Frequency Trader to push his competitor out of the market, while vice versa in some cases the designated market maker finds profitable to lower the spread up to a level that does not guarantee to the other player positive profits. This effect arises from the different risk attitude of the two market participants.

In this setting the designated market maker has a trade off when reducing the spread as he reduces the profit per share but increases the market demand he serves. This implies that, depending on the level of the variance, there are different outcomes. In case of a high variance scenario the High Frequency Trader is less willing to have high inventory exposure and this creates an incentive for the risk neutral market maker to reduce the spread. This effect comes

---

<sup>10</sup>Risk neutrality has been justified both because of the compensation system that aims to cover losses due to inventory imbalances and both as an effect of the hedging strategies that MM seem to pursue whenever possible.

from the fact that with small reductions of the spread the market maker can induce the High Frequency Trader to serve lower demand in the market, hence it is profitable for him to reduce a bit his profit per share and increase his market share. Although this is true also in the low variance case the effect is different as the level of spread required to incentive the risk averse player to change his strategy (aiming lower exposure) is often not convenient in terms of profit for the other market participant. The spread level that prevails overall depends on the reservation price of the liquidity trader, but comparing the two cases there is always an interval in which, given the same reservation price, there are different outcomes in setting the spread.

The presence of an High Frequency trader competing with the traditional liquidity provider also induces the possibility to have changes in the spread when the change in the reservation price of the liquidity trader is small. These changes in spread are an effect that arises from the incentive of the traditional market maker to reduce his quotes in order to push the other liquidity provider out of the market. This effect is beneficial for the trader as he pays less than his reservation price. This effect does not arise for every level of the reservation value, but only when such value is close enough to a point where the Market Maker finds it profitable to push out the other liquidity provider. Such finding is interesting in the sense that it can explain changes in spread that are not motivated by market conditions, but rather are an effect of mechanics of competition in liquidity provision and liquidity demand.

An hypothetical situation in which a High Frequency Trader acts as a monopolist liquidity provider is not beneficial. In such scenario the provision of liquidity is strictly entangled to inventory considerations: there are outcomes where the High Frequency Trader stops providing liquidity in case of orders of the same sign as this increases his inventory exposure.

The conclusion of this paper is that the arrival of high frequency traders has a desirable effect for markets in terms of spread and liquidity provision. Despite this, high frequency traders should provide liquidity alongside official liquidity providers to offer continuity in liquidity provision. This paper suggests that policymakers should consider subsidizing traditional market makers.

## Chapter 3

# The client-dealer Interaction in dealers' markets

### 3.1 Introduction

The client dealer relationship plays a central role in dealers' market. In fact, in such markets, final investors do not trade directly with each other but contact a dealer, find out his prices and trade, or else contact another dealer. Examples of dealers' markets are the corporate bond market in the US and Europe, the Foreign Exchange Market, spot commodities and nonstandard derivatives ([Hendershott and Madhavan, 2015], [Foucault et al., 2013]). Dealers' markets, and more in general OTC markets, have been recently under scrutiny from regulators, for various concerns including the lack of transparency in the execution of clients' orders and the delay in the adoption of technology that would allow to offer alternatives in trades execution to clients (OTC vs anonymous automated trading platforms)[BoE, 2015]. Regulators' attention is also motivated by the fact that some of these markets are large: the Forex market has a daily turnover that averages to 3.5 trillion per day ([BIS, 2013]) and the corporate bond market has a daily turnover that averages to 730 billion per day (SIFMA statistics<sup>1</sup>).

This work focuses on two aspects of the client dealer interaction in dealers' markets. The first one is the existence of intermediation in presence of alternative trading venues. The presence

---

<sup>1</sup>Securities Industry and Financial Markets Association:  
<http://www.sifma.org/research/statistics.aspx>

of well established alternative trading venues is not common across all dealers' market. Despite this [BoE, 2015] remarks that the adoption of technology in such market is progressing. My work captures the Forex market structure, but it can encompass other dealers' markets that are similar in structure or that will soon achieve the same level of technology adoption.

The second aspect on which I focus is the mechanism through which information is revealed and disseminated through clients order flow([King et al., 2011]) and indirect sale of information. As in [Lyons, 2001] I relax the assumption that all the information relevant for prices is publicly known. Hence I focus on the indirect mechanism through which clients' order flow reveals information to dealers: this is the first step of information transmission in such markets. Once clients reveal their information to dealers trading with them, information becomes embedded in interdealer prices due to dealers trading activity. This information is then disseminated widely, as quotes in the customer segment of the market are adjusted to reflect the new interdealer prices.

In this paper a two-period model is developed, where one security is traded sequentially in a client-dealer OTC market and a limit order interdealer market (in the spirit of [Glosten, 1994]). One client chooses to pay a cost  $k$  to observe a noisy signal that provides him information about the liquidation value of the security. If the client does not pay  $k$  he does not observe the signal.

The client could be a speculator, e.g. a hedge fund that can produce a forecast of exchange rate returns (or of any macroeconomic parameter of relevance) or a player who has insider information. In this model the client has two possibilities to execute his trade. One is to pay a fee and gain direct access to the interdealer market. Else he can submit his order through one or two dealers, who make him a take it or leave it offer.

In this setting the existence of intermediation is endogenized and my results suggest that it depends on the level of the interdealer market access fees. In fact, if the fee structure is high enough to reduce the client monopolistic profit to a level that is below the dealers' duopoly profit, the client is indifferent if trading directly in the interdealer market or through the dealers. This is because the dealers transfer to the client the same expected profit he would earn trading in the interdealer market net of the fee. Hence the rent from intermediation increases in fees<sup>2</sup>.

---

<sup>2</sup>Up to the point in which fees are too high and prevent information acquisition from the client. In this specific case the two dealers are unwilling to make any offer to the client as he is always better off not observing the signal.

My results also show that, in the absence of competition between dealers, any fee level that allows for information acquisition also supports intermediation. This finding does not hold when dealers compete because of the client's inability to commit to trade exclusively with one dealer. In fact each dealer knows that, in equilibrium, the client, as a profit maximizer, sells his information also to the second dealer, no matter how favorable are the quotes offered to him. Dealers anticipate this behavior and are not willing to offer to the client a total transfer that is higher than his profit as a monopolist net of the access fee. The inability to commit plays a crucial role in this setting.

To overcome the commitment problem and capture the ongoing nature of the client dealer interaction the model is extended to an infinitely repeated game. In the infinitely repeated game I prove that if the interaction between a dealer and his client is ongoing and without a set end, the dealer and the client can engage in a cooperative relationship. This contrasts with the finding that an information motivated client who sells his information to multiple dealers *una tantum* does not have an incentive to sell his information exclusively to one dealer. The client, in exchange for his loyalty, gets privileged quotes. Moreover I find a cost threshold such that, when the cost the client pays to join directly the interdealer market lays below such threshold, the dealers are willing to offer higher transfers in exchange of exclusive sale of information. This in turn affects the minimum discount factor required for the client to sustain a cooperative outcome, as the discount factor is lower when fees are lower. This finding is in line with the finding of [Hendershott and Madhavan, 2015] that insurance companies form few long-standing trading relations with corporate bond dealers and could potentially provide an explanation for the concentrated nature of the FX market <sup>3</sup>.

This work is part of the stream of literature on market making and dealers market. My work is different from the literature on market making as it analyzes the existence information asymmetries from a different angle. The seminal paper of [Kyle, 1985b] has important implications for the process of learning fundamental information from order flow observation, but the market maker in [Kyle, 1985b] observes only the aggregate order flow, which is not an assumption that holds in a dealer market. Other contributions, as [Glosten and Milgrom, 1985b], are relevant to

---

<sup>3</sup>Despite the FX being an extremely liquid market, Euromoney's 2014 survey outlines a concentrated market: the top 5 dealing banks have a market share that reaches 60%.

dealers market dynamics due to the commonality of the process of price discovery: prices adjust slowly to information due to updating in beliefs. Nevertheless the [Glosten and Milgrom, 1985b] model does not capture an essential aspect of the dealer's role: they assume the dealers does not know the identity of their clients. This model, partly motivated by the findings of the existing empirical literature on FX markets (see INTRODUCTION), assumes that a dealer knows the identity of his client and furthermore infers the reasons for his trading activity. Consequently, opposite results are reached with respect to the dealer spread setting behavior.

Part of the literature on interdealer markets focuses on the rationale for interdealer trading as inventory management motivated by customer-dealer trading (see [Werner, 1997]) in the absence of asymmetric information. As highlighted by these papers, inventory considerations are very relevant to interdealer trading. This work voluntarily abstracts from inventory management in order to stress another aspect of interdealer trading: its speculative nature. The work of [Lyons, 1997] captures the essence of simultaneous interdealer trading and stresses important aspects as the informational component of clients order flow and the relevant effect of dealers risk aversion in their price setting strategies. My work, on the other hand, is oriented to capture a niche aspect of client-dealer trading: the dynamics of the customer dealer interaction, the commitment problem and the flow of information. In fact I do not assume each dealer to have a fixed customer base, but I rather aim to look at how the customer base of a dealer develops in the context of a repeated interaction.

Other works, as [Naik et al., 1999], consider the flow of information from clients to dealers and focuses on the role of transparency in trade disclosure in a two stage dealership markets. Their model has important implications in terms of regulatory choices on information disclosure, but does not investigate the possibility of a repeated interaction between clients and dealers.

This paper looks at the bid ask spread differently from the traditional literature on strategic spread setting. Starting with [Bagehot, 1971], the market maker or specialist uses the spread as measure to protect himself from adverse selection. The spread typically arises as a pure information phenomenon and increases in the number of informed traders populating the market ([Glosten and Milgrom, 1985a]). [Kavajecz, 1996] and [Dupont, 2000] include strategic use of depth. [Kavajecz, 1996] suggest that spread increases and depth decreases as the number of informed in the market increases. [Dupont, 2000] suggests that depth is adjusted more actively

than spread to protect the dealer from information risks. [Easley and O'Hara, 1987] suggest that block trades usually execute at worst price as a consequence of the fact that a block trade could signal an information event occurring in the market. Various other works build on this literature and extend the analysis of information asymmetries including models that allow the market maker to have positive profits ([Bernhardt and Hughson, 1997]), the possibility of splitting orders ([Bondarenko, 2001]) and competition between the specialist and limit order book ([Seppi, 1997]). The standpoint I take is different. The fact that in this setting dealers can discriminate between informed and uninformed traders, allows him to use the spread as a tool to transfer profit in exchange of information sharing.

The rest of the paper proceeds as follows. The second section presents the benchmark game: the first subsection presents the setting and defines the equilibrium, the second and third subsections solve for the equilibrium in the client dealer market and in the interdealer market, the fourth subsection discusses the results. The third section presents the infinitely repeated game: setting and equilibrium are defined in the first subsection and results are in the second subsection, the third subsection discusses the results. The fourth section concludes.

## 3.2 Benchmark Game

### 3.2.1 Setting

#### Agents and technologies

In this model one risky asset  $\tilde{\theta}$  is traded, with liquidation value:

$$v = \begin{cases} \bar{\theta} & \text{with } p = \frac{1}{2} \\ \underline{\theta} & \text{with } p = \frac{1}{2} \end{cases}.$$

In the model there are 4 types of risk neutral agents: one client, two dealers, a liquidity trader and risk neutral perfectly competitive limit order submitters (LO traders).

The client,  $C$ , with a cost  $k$  can produce a forecast of the security liquidation value  $v$ , where  $k$  is publicly observable. Assume that the outcome of this forecast is a noisy signal,  $S_H$  or  $S_L$ , according to which he updates his probabilities as:

$$\left\{ \begin{array}{l} \Pr(\theta = \bar{\theta} | S = S_H) = \alpha \\ \Pr(\theta = \underline{\theta} | S = S_H) = 1 - \alpha \end{array} \right. \text{ and } \left\{ \begin{array}{l} \Pr(\theta = \underline{\theta} | S = S_L) = \alpha \\ \Pr(\theta = \bar{\theta} | S = S_L) = 1 - \alpha \end{array} \right., \text{ where } \alpha \in \left(\frac{1}{2}, 1\right]$$

The client decision to invest  $k$  to produce a noisy signal is private and not observable by the two dealers. This assumption aims to capture the fact that the client, who is a speculator, can decide to sustain a cost in order to produce a forecast (for example hiring analysts or gathering data), but this decision cannot be observed from an external third party.

The two dealers,  $D_i$   $i \in \{1, 2\}$ , are identical and uninformed about the liquidation value. They can acquire information trading with  $C$  in the client-dealer market, with a take it or leave it offer  $\gamma_i(q)$  to the client, where  $\gamma_i(q) = (\gamma_{ai}(q), \gamma_{bi}(q))$  and  $\gamma_{ai}(q)$  ( $\gamma_{bi}(q)$ ) is the price at which  $D_i$  offers to sell (buy)  $q$  shares. For ease of notation say  $\gamma_{ai}(q) = \gamma_{ai}$  and  $\gamma_{bi}(q) = \gamma_{bi}$ . The offers  $\gamma_i$  cannot be contingent on the future profits of the dealers as typically quotes are presented for immediate exchange of shares and payment and are binding in nature.

The client can trade with both dealers and the trade between the client and each dealer is not observable by the other dealer. This assumption captures the lack of reporting requirements, in the sense that each dealers' order flow from clients is private information of the dealer himself.

The client can also decide to join the interdealer market for a cost  $K$  publicly observable and the client decision to participate to the interdealer market is observable.

After participating to the client-dealer market, the two dealers, the client, conditional on joining, and a liquidity trader sequentially trade in an interdealer market organized as a limit order book. The order of arrival of traders is randomly selected and assigned by nature. In the interdealer market LO traders fill the book up to a competitive equilibrium. This assumption is mostly motivated by tractability, but is still in line with the high level of turnover of the interdealer market where spreads are extremely tight.

The liquidity trader trades for exogenous reasons. This can be interpreted in the spirit of dealers' trading for hedging needs or reducing the accumulated inventory exposure. He submits demand  $u$ , where  $u$  can take values:

$$u = \begin{cases} X^L, p = \frac{1}{4} \\ X^S, p = \frac{1}{4} \\ -X^S, p = \frac{1}{4} \\ -X^L, p = \frac{1}{4} \end{cases}.$$

LO traders after each trading round update their quotes with zero cost.

## Timeline

The model is divided in three periods as follows:

- At time  $t = 0$ , nature chooses the liquidation value  $v$  of the security. The client chooses whether or not to observe the noisy signal for a cost  $k$ . Denote  $I = 1$  the choice to observe the signal and  $I = 0$  the choice not to observe it.
- At time  $t = 1$ , the client-dealer market opens. The dealers submit a take it or leave it price offer  $\gamma_i$  to trade with the client  $q$  shares. After observing  $\gamma_1 = (\gamma_{a1}, \gamma_{b1})$  and  $\gamma_2 = (\gamma_{a2}, \gamma_{b2})$ ,  $C$  decides whether to accept each offer or to reject it. Additionally he decides whether to pay a cost  $K$  that allows him to trade directly in the inter-dealer market. Denote a strategy for the client as duplet  $\{a, j\}$ , where  $a$  is a vector  $a = (a(\gamma_{a1}), a(\gamma_{b1}), a(\gamma_{a2}), a(\gamma_{b2}))$ . If  $a = 1$  ( $a = 0$ ),  $C$  accepts (rejects) the offer from dealer  $i$ , for any  $i$ . If  $j = 1$  ( $j = 0$ ),  $C$  joins (does not join) the interdealer market. If the offer from  $D_i$  is accepted, a trade between  $D_i$  and  $C$  takes place. A trade consists in an exchange of  $q$  shares for a payment  $\gamma_i$ .
- At time  $t = 2$  the interdealer market opens. In stage 1, LO traders fill the book up to a competitive equilibrium. In stage 2, a trader is selected by nature and he submits his market order  $x$ . In stage 3 LO traders, after observing the incoming market order, revise their quotes. The stages continue until each trader has placed his order. At this point, the liquidation value of the security is realized.

## Definition of equilibria

We start defining the equilibrium in the interdealer market. Define the information set of the client, at time  $t = 2$ , as  $\Omega^C = \{I \cdot E(\theta | S), \gamma, a, j\}$ . Define the information set of dealer  $i$ , at time  $t = 2$ , as  $\Omega^{D_i} = \{\gamma_i, a_i, I \cdot a_i \cdot E(\theta | S), j\}$ , where  $i \in \{1, 2\}$ . Denote the profits of the informed player/players in the interdealer market as  $\pi_2^T = E[(\theta - P(X)) x^T | \Omega^T] \cdot t$ , where  $T \in \{C, D_1, D_2\}$  and  $X$  is the cumulative depth of the book. In case of a buy market order  $t$  is equal to 1 and  $P(X) = A(X)$  and in case of a sell market order  $t$  is equal to  $-1$  and  $P(X) = B(X)$ .

An equilibrium in the interdealer market is defined as follows:

**Definition 9** *An equilibrium in the interdealer market is defined as a triplet  $\{x, A(X), B(X)\}$  such that the following conditions hold:*

- I. Profit maximization: the expected profit of player  $T$ ,  $\pi_2^T = E[(\theta - P(Z)) x^T | \Omega^T] \cdot t$ , where  $T \in \{C, D_1, D_2\}$ , is maximized.*
- II. Market efficiency: the prices in the LOB satisfy the following:*

$$\begin{aligned} A(X) &= E(\theta | x \geq X) \\ B(X) &= E(\theta | x \leq -X) \end{aligned}$$

Now consider the client dealer market. Define  $\Omega'^C = \{I \cdot E(\theta | S), \gamma\}$  the information set of the client.

**Definition 10** *An equilibrium in the client-dealer market is an offer of a pair  $(\gamma_i^*, q^*)$ , from each  $D_i$  to the client  $C$  such that:*

*$(\gamma_i^*, q^*)$  maximizes the dealers' total profits:*

$$E[\Pi^D(\gamma_i, x^{Di}(\gamma_i, I \cdot v \cdot a_i, j))] = E[\pi_2^T(x^{Di}(I \cdot E(\theta | S) \cdot a_i) | a_i)] - E[(\gamma_i - \theta)q_i] \cdot t$$

*where  $t = 1, \gamma_i = \gamma_{ai}$  for a buy order and  $t = -1, \gamma_i = \gamma_{bi}$  for a sell order, subject to the maximization problem for the client:*

$$\{a, j\} \in \arg \max E[\pi^C(a, j) | \Omega'^C] = \left\{ [E(\pi_2^C) - K] j + \sum_i [(\theta - \gamma_i)q_i] \cdot t | \Omega'^C \right\}$$

Now consider the equilibrium in the full game. To simplify notation say  $x^C(I \cdot v, \gamma, a, j) = x^C$ ,  $x^{Di}(\gamma_i, I \cdot v \cdot a_i, j) = x^{Di}$

**Definition 11** *A perfect Bayesian equilibrium of this game is a quadruplet  $\{I^*, a^*, j^*, (x^C)^*\}$  for the client, a triplet  $(\gamma_i^*, q_i, (x^{Di})^*)$  for each dealer, prices  $A(X)$  and  $B(X)$  in the LOB, such that:*

- the expected profit of the client

$$E [\Pi^C(I, a, j, x^C)] = I \left\{ [E(\pi_2^C) - K] j + \sum_i [(\theta - \gamma_i) q_i] \cdot t \right\}$$

is maximized.

- the expected profit of each dealer

$$E [\Pi^D(\gamma_i, x^{Di})] = E [a_i(\gamma_i) \pi_2^T(x^{Di}) I - (\gamma_i - \theta) q_i]$$

is maximized.

- the price in the LOB satisfy

$$A(X) = E(\theta | x \geq X)$$

$$B(X) = E(\theta | x \leq -X)$$

The next section describes the equilibrium.

### 3.2.2 Equilibrium in the interdealer market

This section derives the equilibrium in the interdealer market assuming, and at a later stage proving, that in the equilibrium prices and traded quantities depend on  $k$  and  $K$ . The interdealer market is organized as a limit order book where prices take into account the information contained in market orders. In the market there are various types of players, the two dealers, liquidity traders, the informed client, if he decides to join, and limit order submitters. For tractability LO submitters are assumed to be perfectly competitive and revise costlessly their quotes after a market order arrives. Trades are supposed to be sequential and nature selects the order of traders arrival.

**Proposition 12** *Three possible equilibria arise in the interdealer market, depending on  $k$  and  $K$ :*

*I. No trader is informed. The LOB displays the following offers:*

$$\begin{aligned} A(X) &= E(\theta | S_H) \\ B(X) &= E(\theta | S_L) \end{aligned}$$

*for any quantity  $X$ .*

*II. There is one informed trader. The LOB, , at stage  $s = 1$ , displays the following offers:*

$$\begin{aligned} A(X) &= \begin{cases} \frac{1}{2}E(\theta | S_H) + \frac{1}{2}E(\theta) & \text{for } X \leq X^S \\ \frac{2}{3}E(\theta | S_H) + \frac{1}{3}E(\theta) & \text{for } X^S < X \leq X^L \\ E(\theta | S_H) & \text{for } X > X^L \end{cases} \\ B(X) &= \begin{cases} \frac{1}{2}E(\theta | S_L) + \frac{1}{2}E(\theta) & \text{for } -X \geq -X^S \\ \frac{2}{3}E(\theta | S_L) + \frac{1}{3}E(\theta) & \text{for } -X^L < -X \leq -X^S \\ E(\theta | S_L) & \text{for } -X \leq -X^L \end{cases} \end{aligned}$$

Where  $A(X)$  and  $B(X)$  are updated from LO submitters after observing an incoming

market order. The expected profit of the informed is:

$$E(\pi) = \frac{3}{4} \left[ [E(\theta | S_H) - E(\theta)] \frac{(X_S + 2X_L)}{6} \right]$$

III. There are two informed traders. The LOB, at stage  $s = 1$ , displays the following offers:

$$A(X) = \begin{cases} \frac{2}{3}E(\theta | S_H) + \frac{1}{3}E(\theta) & \text{for } X \leq X^S \\ \frac{4}{5}E(\theta | S_H) + \frac{1}{5}E(\theta) & \text{for } X^S < X \leq X^L \\ E(\theta | S_H) & \text{for } X > X^L \end{cases}$$

$$B(X) = \begin{cases} \frac{2}{3}E(\theta | S_L) + \frac{1}{3}E(\theta) & \text{for } -X \geq -X^S \\ \frac{4}{5}E(\theta | S_L) + \frac{1}{5}E(\theta) & \text{for } -X^L < -X \leq -X^S \\ E(\theta | S_L) & \text{for } -X \leq -X^L \end{cases}$$

Where  $A(X)$  and  $B(X)$  are update from LO submitters after observing an incoming market order. The expected profit of each informed trader is:

$$E(\pi) = E(\pi) = \frac{2}{3} \left[ [E(\theta | S_H) - E(\theta)] \frac{(2X_S + 5X_L)}{15} \right]$$

In this model the spread increases in the number of informed traders, due to adverse selection. This implies that, if only one player is informed, the quotes in the interdealer market are tighter and the spread, as a consequence, is lower. In the next session I consider the possibility of one dealer only acquiring information and trading as a monopolist in the interdealer market and show that this is never the case. In fact I highlight how the client commitment problem rules out such outcome and the only possibility of having an information monopolist in the interdealer market is that the client decides to access directly such market.

In this model liquidity traders are allowed to trade small or large quantities as in [Glosten, 1994]. The effect is that small orders trade for a spread and the marginal order above the small buy/sell order trades for a larger spread. The spread on small orders is motivated by the fact that, although small orders are never submitted by informed traders, orders at the top of the book might execute against small orders with no informational content and large ones, which are instead informative.

Moreover the effect of having liquidity traders placing orders up to an amount  $|X^L|$  is that for higher depth values, the security trades at its expected liquidation value. This feature does not depend on the number of informed traders in the market and it is motivated by the informative nature of the order size. This implies that the profit dealers could make on the units exceeding the maximum liquidity demand are wiped off.

Overall an increase in the number of informed traders reduces the rents from information acquisition. The profit from information is decreasing in the number of informed players. This has an effect on the overall equilibrium of the model and it will be discuss in greater details in the next section.

**Proof.** The proof is divided in two parts. First I derive the equilibrium price and quantity if only one trader is informed. Afterwards I proceed to derive the equilibrium price and quantity if two traders are informed.

- Consider the case of one informed trader. The probability of an informed trader arriving to the market is equal to the probability of an uninformed trader arriving to the market and is  $P(I) = P(U) = \frac{1}{2}$ . If an informed trader arrives to the market his optimal demand  $x_I$  depends on the ask price at each depth. More specifically :

$$\begin{cases} x_I \geq X \text{ if } A(X) \leq E(\theta | S_H) = \alpha \bar{\theta} + (1 - \alpha) \underline{\theta} \\ x_I = 0 \text{ if } A(X) > E(\theta | S_H) = \alpha \bar{\theta} + (1 - \alpha) \underline{\theta} \end{cases}$$

on the ask side, and:

$$\begin{cases} x_I \geq X \text{ if } B(X) \geq E(\theta | S_L) = \alpha \underline{\theta} + (1 - \alpha) \bar{\theta} \\ x_I = 0 \text{ if } B(X) < E(\theta | S_L) = \alpha \underline{\theta} + (1 - \alpha) \bar{\theta} \end{cases}$$

on the bid side.

The liquidity trader is a buyer or a seller with the same probability. Moreover he can submit a large order  $|X_L|$  or a small order  $|X_S|$  with the same probability. Hence:

$$P(U, X^L) = P(U, -X^L) = P(U, X^S) = P(U, -X^S) = \frac{1}{4}$$

While an uninformed trader submits an order that reflects only the desired amount, an unconstrained informed trader matches all the order on the ask (bid) side of the book where the price is lower (higher) than the expected value of the security liquidation value. From the previous consideration the competitive LO submitters estimate the security liquidation value, and consequently the price, according to the following:

$$E(\theta | x \geq X) = E(\theta | U)P(U | x \geq X) + E(\theta | I)P(I | x \geq X) \quad (3.1)$$

for a buy order.

The probability that an order is submitted by an informed trader is given by:

$$P(I | x \geq X) = \frac{P(I)P(\text{buy}, I)P(x \geq X | I)}{P(I)P(\text{buy}, I)P(x \geq X | I) + P(U)P(\text{buy}, U)P(x \geq X | U)} \quad (3.2)$$

From (4.13) it follows that:

$$\begin{aligned} P(I | x \geq X, X \geq X^L) &= 1 \\ P(I | x \geq X, X^S \leq X < X^L) &= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}1} = \frac{2}{3} \\ P(I | x \geq X, X \leq X^S) &= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}1} = \frac{1}{2} \end{aligned}$$

Now from (4.5) it is possible to find:

$$\begin{aligned} E(\theta | x \geq X, X \geq X^L) &= E(\theta | S_H) \\ E(\theta | x \geq X, X^S \leq X < X^L) &= \frac{2}{3}E(\theta | S_H) + \frac{1}{3}E(\theta) \\ E(\theta | x \geq X, X \leq X^S) &= \frac{1}{2}E(\theta | S_H) + \frac{1}{2}E(\theta) \end{aligned}$$

as limit LO traders are perfectly competitive prices will be equal to expectations of the security value conditional on the order size.

On the bid side, the derivation is symmetric and:

$$\begin{aligned}
E(\theta | x \leq -X, -X \leq -X^L) &= E(\theta | S_L) \\
E(\theta | x \leq X, -X^L \leq -X < -X^S) &= \frac{2}{3}E(\theta | S_L) + \frac{1}{3}E(\theta) \\
E(\theta | x \leq X, X \geq -X^S) &= \frac{1}{2}E(\theta | S_L) + \frac{1}{2}E(\theta)
\end{aligned}$$

Overall at time  $t = 0$  the LOB displays the following offers:

$$\begin{aligned}
A(X) &= \begin{cases} \frac{1}{2}E(\theta | S_H) + \frac{1}{2}E(\theta) & \text{for } X \leq X^S \\ \frac{2}{3}E(\theta | S_H) + \frac{1}{3}E(\theta) & \text{for } X^S < X \leq X^L \\ E(\theta | S_H) & \text{for } X > X^L \end{cases} \\
B(X) &= \begin{cases} \frac{1}{2}E(\theta | S_L) + \frac{1}{2}E(\theta) & \text{for } -X \geq -X^S \\ \frac{2}{3}E(\theta | S_L) + \frac{1}{3}E(\theta) & \text{for } -X^L < -X \leq -X^S \\ E(\theta | S_L) & \text{for } -X \leq -X^L \end{cases}
\end{aligned} \tag{3.3}$$

The competitive LO traders revise their expectation after each trade they observe. In order to calculate the expected profits of the insider the order in which trading takes place should be considered as this can inform LO submitters about the informational content of the order.

Assume the informed observes  $S = S_H$ . If he is selected to trade first, with  $p = \frac{1}{2}$ , his expected profit is equal to:

$$E(\pi | S = S_H, f) = \frac{X_S}{2} [E(\theta | S_H) - E(\theta)] + \frac{(X_L - X_S)}{3} [E(\theta | S_H) - E(\theta)] \tag{3.4}$$

Now assume that the informed is selected to trade as second trader, that happens with  $p = \frac{1}{2}$ . In this case his profits depend on the trade submitted by the liquidity trader. In fact if the liquidity trader chooses  $u = X_S$  or  $u = -X_S$ , the limit order submitters update the probability of a trade coming from an informed to 1. Hence the prices will be  $A(q) = E(\theta | S_H)$  for any  $q$  and the profit of the informed will be null. If the uninformed

chooses  $u = X_L$ , LO traders cannot update their quotes and the profit of the informed is the same as in (3.4). Overall the expected profit of the informed is:

$$E(\pi) = \frac{3}{4} \left[ [E(\theta | S_H) - E(\theta)] \frac{(X_S + 2X_L)}{6} \right] \quad (3.5)$$

- Now consider the case in which there are two informed in the market. This changes the implication of (4.13) as:

$$\begin{aligned} P(I | x \geq X, X \geq X^L) &= 1 \\ P(I | x \geq X, X^S \leq X < X^L) &= \frac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{3}\frac{1}{2}} = \frac{4}{5} \\ P(I | x \geq X, X \leq X^S) &= \frac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{3}1} = \frac{2}{3} \end{aligned}$$

Now the expectations in (4.5) are:

$$\begin{aligned} E(\theta | x \geq X, X \geq X^L) &= E(\theta | S_H) \\ E(\theta | x \geq X, X^S \leq X < X^L) &= \frac{4}{5}E(\theta | S_H) + \frac{1}{5}E(\theta) \\ E(\theta | x \geq X, X \leq X^S) &= \frac{2}{3}E(\theta | S_H) + \frac{1}{3}E(\theta) \end{aligned}$$

In this case the prices at time  $t = 0$  are:

$$\begin{aligned} A(X) &= \begin{cases} \frac{2}{3}E(\theta | S_H) + \frac{1}{3}E(\theta) & \text{for } X \leq X^S \\ \frac{4}{5}E(\theta | S_H) + \frac{1}{5}E(\theta) & \text{for } X^S < X \leq X^L \\ E(\theta | S_H) & \text{for } X > X^L \end{cases} \\ B(X) &= \begin{cases} \frac{2}{3}E(\theta | S_L) + \frac{1}{3}E(\theta) & \text{for } -X \geq -X^S \\ \frac{4}{5}E(\theta | S_L) + \frac{1}{5}E(\theta) & \text{for } -X^L < -X \leq -X^S \\ E(\theta | S_L) & \text{for } -X \leq -X^L \end{cases} \end{aligned} \quad (3.6)$$

Also in this case the expected profit of each informed trader depends on the order of

trades. If an informed trader trades first, with  $p = \frac{1}{3}$  he gets:

$$E(\pi | S = S_H, f) = \left[ [E(\theta | S_H) - E(\theta)] \frac{(2X_S + 5X_L)}{15} \right] \quad (3.7)$$

If he trades second, he gets:

- if the other informed is trading first, with  $p = \frac{1}{6}$ , he gets the same profit as in (4.14)
- if the liquidity trader is trading first, with  $p = \frac{1}{12}$ , he gets the same profit as in (4.14)

If he trades third, he gets:

- with probability  $p = \frac{1}{12}$ , he gets the same profit as in (4.14)

Overall the expected profit of an informed trader in this scenario is given by:

$$E(\pi) = \frac{2}{3} \left[ [E(\theta | S_H) - E(\theta)] \frac{(2X_S + 5X_L)}{15} \right] \quad (3.8)$$

Notice that the expected profit in case of one informed trader is higher than the sum of the expected profits of two informed traders, that is  $(3.5) > 2 \cdot (4.16)$ .

■

### 3.2.3 Equilibria in the full game

This section derives the equilibria in the full game. The possibility for dealers to act as intermediaries depends on the cost of acquiring information and on the fees that guarantee the client direct access to the interdealer market.

Denote  $\Pi^T$  the expected total profit of player  $T$ . Denote  $E(\pi^{C,M})$  the expected profit of the client if he trades as an information monopolist in the interdealer market. Denote  $E(\pi^{D,d})$  the expected profit of each dealer if they trade as information duopolist in the interdealer market.

**Proposition 13** *In the full game three possible equilibria arise, depending on  $K$  and  $k$  :*

- If  $\pi_2^C - K > k$  and  $E(\pi^{C,M}) - 2E(\pi^{D,d}) \geq K$ :

$D_1$  and  $D_2$  choose  $\{\gamma^* = (E(\theta|S_L), E(\theta|S_H)), q^* \in R, x_{Di}^* = 0\}$ .

The client chooses  $\{I^* = 1, a^* = 0, j^* = 1, (x^C|S_H)^* = X_L, (x^C|S_L)^* = -X_L\}$ .

The LOB exhibits the prices in (4.15).

The total expected profit of the client is  $\Pi^C = \frac{3}{4} \left[ [E(\theta|S_H) - E(\theta)] \frac{(X_S + 2X_L)}{6} \right] - K - k$ ,  
the total expected profit of the dealers is  $\Pi^{Di} = 0$ .

- If  $\pi_2^C - K > k$  and  $E(\pi^{C,M}) - 2E(\pi^{D,d}) < K$ :

$D_1$  and  $D_2$  choose  $\left\{ \gamma^* = \left[ E(\theta) - \frac{E(\pi^{C,M}) - K}{2q}, E(\theta) + \frac{E(\pi^{C,M}) - K}{2q} \right], q^* \in R \right\}$ .

The client is indifferent between:

- $\{I^* = 1, a^* = 0, j^* = 1, x^C|S_H^* = X_L, x^C|S_L^* = -X_L\}$ . In this case the LOB exhibits the prices in (4.15).

The expected total profit of the client is  $\Pi^C = \frac{3}{4} \left[ [E(\theta|S_H) - E(\theta)] \frac{(X_S + 2X_L)}{6} \right] - K - k$ .

The demand of the dealers in the interdealer market is  $x_{Di}^* = 0$  and the total profit of the dealers is  $\Pi^{Di} = 0$ .

- $\{I^* = 1, a^* = 1, j^* = 0, x^C = 0\}$ . In this case the LOB exhibits the prices in (4.17).

The expected total profit of the client is  $\Pi^C = 2\gamma^* - k$ .

The demand of the dealers in the interdealer market is  $(x^{Di}|S_H)^* = X_L, (x^{Di}|S_L)^* = -X_L$  and the total expected profit of the dealers is  $\Pi^{Di} = E(\pi^{D,d}) - \gamma^*$ .

- if  $\pi_2^C - K < k$ :

$D_1$  and  $D_2$  offer  $\{\gamma^* = (E(\theta|S_L), E(\theta|S_H)), q^* \in R, x_{Di}^* = 0\}$ .

$C$  chooses  $\{I^* = 0, a^* = 0, j^* = 0, x^C = 0\}$ .

The total expected profit of the dealers and the client are  $\Pi^C = \Pi^{Di} = 0$ .

As a first thing notice that, if the cost of acquiring information is higher than the profit of the client net of the direct access fee to the interdealer market, the client does not have an incentive to acquire information. This affects the dealers quotes. Dealers anticipate that the client does not acquire information and, as a consequence, have no incentive to charge a spread below the maximum possible level. This consideration highlights the fact that, increasing the fee above such threshold, prevents information acquisition and reduces to zero the profit of both clients and dealers. Moreover an increase in the fee up to a level that prevents the client from acquiring information, makes prices in the interdealer market less informative.

In the case in which the client has an incentive to invest resources in information acquisition, the existence of intermediation depends on the fees that guarantee access to the interdealer market. The impossibility of a binding commitment for the exclusive sale of securities between the client and one dealer plays a major role. Dealers are prevented to earn monopolistic profits when trading on information as they anticipate that in equilibrium the client always finds profitable to trade with both intermediaries. If the client could ex ante commit to sell information to one dealer only, any level of access fee could support intermediation, guaranteeing to the dealer a level of profit equal to the fee itself. This implies that dealers could act as intermediaries only if the fees are such that they reduce the monopolistic profit below the profit of an information duopoly, earning on aggregate the access fee.

**Proof.** Denote the profit of the client if he is the only informed trader in the interdealer market as  $E(\pi^{C,M})$ . Denote the profit of the dealers if they both have information as  $E(\pi^{D,d})$ .

The total payoff of the client if he trades as a monopolist in the interdealer market is  $\Pi^C = E(\pi^{C,M}) - k - K$ . The total payoff of the client if he accepts the offer of the two dealers is  $\Pi^C = \gamma_1 + \gamma_2 - k$ .

Consider at first the case in which  $k > E(\pi^{C,M}) - K$ . In this case the client does not have an ex ante incentive to observe the signal. The two dealers, as a consequence, do not have

an incentive to offer to the client any positive transfer as his trade would have no information content.

Moreover, any case in which the client accepts the offer of the two dealers and trades in the interdealer market can be ruled out of equilibria: the transfer from the dealers is always smaller than the loss of profit in the interdealer market due to the presence of more informed trading.

Now consider the case in which  $k > E(\pi^{C,M}) - K$ .

If  $E(\pi^{C,M}) - 2E(\pi^{D,d}) \geq K$ ,  $D_1$  and  $D_2$  cannot offer a level of transfer such that  $C$  is indifferent between trading directly in the interdealer market and trading with the dealers. In this case  $C$  is better off joining the interdealer market and  $\gamma^* = (\bar{\theta}, \underline{\theta})$ .

If  $E(\pi^{C,M}) - 2E(\pi^{D,d}) < K$ ,  $D_1$  and  $D_2$  can offer a level of transfer such that  $C$  is indifferent between the two strategies. Assume  $\gamma_1 = \gamma_2 = \gamma$ . In this scenario the two dealers offer a  $\gamma_a^*$  such that  $(E(\theta|S_H) - \gamma_a)q = \frac{E(\pi^{C,M}) - K}{2}$  and a  $\gamma_b^*$  such that  $(\gamma_b - E(\theta|S_L))q = \frac{E(\pi^{C,M}) - K}{2}$ .

Notice that, in this game, any outcome in which one dealer is an information monopolist can be ruled out from equilibria. Prove this first by assuming that both dealers set  $\gamma_{a,i} > \gamma^*$ . In this case the client has incentive to sell information to both dealers as his profit,  $\pi^C = \gamma_1 + \gamma_2 - k$ , increases in the transfers received. Now assume that one of the dealers, say  $D_1$  sets  $\gamma_{a,1} > \gamma_{a,i}^*$  and the other sets  $\gamma_{a,2} = \gamma^*$ . This can never be equilibrium as  $D_2$  by decreasing slightly his transfer  $\gamma_{a,2}$  can obtain an increase in his total payoff  $\Pi^{D2}$ .

Hence, as both dealers have information and set  $\gamma_a^*$  and  $\gamma_b^*$ , the matrix of the game choices reduces to the latter:

|                                      |                                                                                                                                      |                                      |                    |
|--------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------|--------------------|
| Payoff Matrix                        |                                                                                                                                      |                                      |                    |
| Client                               |                                                                                                                                      |                                      |                    |
|                                      | Not Join (q)      Join (1-q)                                                                                                         |                                      |                    |
| $\gamma_i^*$                         | <table border="1"> <tr> <td><math>\pi^{Di} - \gamma_i^*, 2\gamma_i^*</math></td><td><math>0; \pi^C - k - K</math></td></tr> </table> | $\pi^{Di} - \gamma_i^*, 2\gamma_i^*$ | $0; \pi^C - k - K$ |
| $\pi^{Di} - \gamma_i^*, 2\gamma_i^*$ | $0; \pi^C - k - K$                                                                                                                   |                                      |                    |

*Client's payoff from an arbitrary mixed strategy profile (q):*

$$U_C(q) = q2\gamma_i^* + (1 - q)(\pi^C - K)$$

The previous is

$$U_C(q) = \pi^C - K$$

independently of  $q$ . Hence the client plays any  $q \in [0, 1]$ . Now consider  $\pi_2^C - k - K < 0$ . For  $C$  it is not profitable to join the interdealer market. In this case for the dealers it is optimal to offer  $\gamma^* = 0$ . This is because in any case in which  $C$ 's profit depends exclusively on  $\gamma_1$  and  $\gamma_2$ , he is always better off saving  $k$ . Hence I can conclude that whenever  $C$  does not have an incentive to acquire information,  $D_1$  and  $D_2$  offer a pair  $(\gamma_1 = 0, \gamma_2 = 0)$ . ■

### 3.2.4 Discussion of the results of the static game

The results presented highlight how the cost of acquiring information,  $k$ , and the cost of joining the interdealer market,  $K$ , play a crucial role in this model.

I find that if  $k$  is too high, the client does not have the incentive to observe the noisy signal and acquire information about the liquidation value. In such circumstances, dealers do not have an incentive to charge a lower spread than the monopoly one. This is because, if the client order flow does not contain information, dealers cannot take a profitable position accordingly in the interdealer market. This results is motivated by the fact that I focus on the speculative nature of dealers' trading.

Assuming that the client has an incentive to acquire information, my model suggests that the choice of trading with an intermediary depends on the level of trading fees that allow direct access to the interdealer market. The fact that the client does not have the possibility to commit to trade with a single dealer narrows the levels of fees that support intermediation. This is because, if the dealer could trade as a monopolist on the basis of the information contained in the client order flow, any level of access fee would support intermediation. In fact dealers could transfer to the client the profit gained in the interdealer market net of the access fee and make him indifferent if trading with them or directly. This is not possible when trades occur in a static or finitely repeated setting as the problem of commitment arises.

If the level of fee are such that the client has an incentive to trade through dealers, my results suggest that the dealer uses the spread to transfer part of his profit to the client and, as an outcome, the dealer charges lower spread to his informed clients.

My results are strongly motivated by some of the assumptions of my model and in this section I discuss them in more details.

My first assumption is related to the flow of information in an OTC market. According to [Lyons, 2001] information processing in the FX market has two stages. The first stage is the analysis or observation of fundamentals by non-dealer market participants (mutual funds, hedge funds, individuals with special information, etc.). The second stage is the dealer's interpretation of the first-stage analysis. Such interpretation comes from reading the order flow and leads to price setting behavior by dealers. [Menkhoff et al., 2016] shows that dealers have an active role in complementing information received by clients with the information they acquire through their own active research. The process continues as dealers trade in the interdealer market, either directly or through brokers. Quotes in the interdealer market get updated to reflect the information single dealers transmit through order flow. This process continues with the update of the quotes dealers' propose to their customers. Despite the previous considerations, the assumption that only dealers learn from clients and not vice versa is a simplification of the informational structure of OTC markets. In fact it is certainly possible to argue that information transmission is bidirectional. This is because information transmission can take place with indirect sale of information from institutional agents (as in the spirit of [Biais and Germain, 2002]) or with the observation of dealers quotes from uninformed clients. There is evidence of different type of relationships developing between (potentially) informed clients and uninformed clients and dealers ([Hendershott et al., 2016]). Such evidence supports the possibility of different flows of information depending on the reasons motivating the trading process.

The assumption regarding the cost of acquiring information can be interpreted according to various findings. [Osler and Vandrovych, 2009] states that information in the FX market is brought by one specific class of market participants: hedge funds.

According to [Osler and Vandrovych, 2009] such players, through their effort to anticipate market returns, bring information to dealers, who further complement it with their own analysis. They suggest that speculative agents engage in a conscious effort to anticipate market returns: they devote time and skills to generate private information about exchange rates. This is done by focusing intensively on upcoming macro statistical releases, aggregating dealers' summaries of upcoming releases, forecasting key figures and engaging in extensive discussion of related macroeconomic developments. Hence hedge funds generate forecasts that combine existing public information, their currency traders' own observations of the world, and the traders' own

interpretive framework. In this sense the cost paid by the client captures an active effort in gathering the information that needs to be found and aggregated. For this reason I assume that the signal produced and observed is noisy, as it represents a forecast of future exchange rate realization.

Alternatively, such cost can be seen as the 'price' to gather private information before the issuance of debt ([Karpoff and Lee, 1991]), or in the spirit of informed trading in corporate bonds prior to takeover announcements. [Kedia and Zhou, 2014] examines trading activities in the corporate bond market prior to merger and acquisition announcements. Differently from equity holders, bondholders only benefit when their bonds carry greater risk than those of the acquiring company, and lose out otherwise. Hence pre-announcement bond returns are significantly related to acquirer characteristics. This suggests that information about the acquirer and target has a key role in motivating such trading activity, rather than other possible motivations. In fact pre-announcement abnormal bond returns are positive when the target bond stands to gain and negative when the target bond stands to lose. [Kedia and Zhou, 2014] attribute such pattern to informed trading.

I assume that the decision of the client to undertake the cost  $k$  is unobservable<sup>4</sup>, as the process of gathering information ( private or actively aggregated through aggregation of various available information) cannot be directly observed.

In my analysis dealers observe the identity of their client, and anticipate if his order flow contains information in equilibrium. The idea that dealers know the identity of their clients is intuitive in an OTC client dealer market, where trading is done by voice and the identity of the counterpart is revealed. Such assumption, nevertheless, can be extended to the case where dealers allow end customers to trade on single bank trading systems (SBT). As an example [King et al., 2011] states that SBT platforms have been developed by major banks to enable customers to trade electronically with the bank. Dealers, while developing this platforms, have also started to engage in a practice called "customer profiling". Such practice consists of banks using their electronic trading records to profile the trades of each customer in order to classify the type of trades executed by various customers. [King et al., 2011] highlights the fact that

---

<sup>4</sup>This is because the choice of leveraged investors to invest in personnel and data resources cannot be easily observed by a third party and the trades of such investors could be motivated by hedging needs.

trades that are typically associated with subsequent movements of exchange rates are considered informed, and dealers use such information in order to undertake speculative position trading.

My choice to introduce a cost to join the interdealer market is motivated by the fact that, in some type of OTC markets, interdealer trading platforms, that used to be an exclusive dealers' trading venue, has evolved in a different direction.

As an example, in the FX market historically <sup>5</sup> the interdealer market platforms, were accessible only to the major dealing bank. Nowadays financial customers can trade directly on EBS and Reuter matching platforms. Moreover, with the electronic evolution of markets, clients have been allowed to trade directly on other available trading platforms. Despite the availability of such platforms, the number of trades that are intermediated by dealers is still sizable. [Rime and Schrimpf, 2013] study concludes that voice is still a key component of trading in FX, and [BIS, 2013] stress that the Forex market is still heavily based on dealers' intermediation. There are several possibilities to explain the persistence of intermediation in such market. One aspect pertains the fact that the FX market is populated by several players whose demand of currency is motivated by diverse reasons. Corporations demand of currency is motivated by business related reasons. Often corporations do not have trading desks and the quantity of currency demanded is not standard but highly customized. For such players the existence of an intermediary can be intuitively justified by the need of outsourcing activities that are not related to the core business. The situation is different when thinking of Hedge Funds and more generally leveraged investors who have trading skills and actively take bets on the basis of their forecasts on currency returns. Such players would have the skills and the expertise to trade directly in the market without the need of intermediaries to process their orders. Moreover not sharing the information content of their order with other market participant could lead such players, as highlighted in my results, to achieve higher profits. For this reasons the demand of intermediation originating from such market participants is less intuitive. In the model presented I offer an explanation for the existence of intermediation. Moreover the results in my static framework suggest that if the cost to join the interdealer market decreases below the threshold presented in my results, leveraged market participants would find profitable to trade directly on interdealer platforms.

---

<sup>5</sup>(see [King et al., 2011] for a comprehensive and detailed survey of the FX market evolution)

### 3.3 The Repeated Game

In the previous sections the interaction between the dealers and the client has been treated as a static game. This section extends the analysis to an infinitely repeated game that is generated by the repetition of the stage game. The aim of this section is to capture the ongoing nature of the dealer-client interaction and its implication on feasible equilibria.

#### 3.3.1 Equilibrium in the repeated game

Define the transfer each dealer corresponds to the client as  $T_i$ , where  $T_i = E[(\gamma_i - \theta)q_i] \cdot t$  and  $T_i$  simply represents the loss the dealer bears when trading with  $C$  in the client dealer market. Define  $a_{Di} = \{T_i, x^{Di}\}$  the set of possible actions available to dealer  $i$ . Define as  $a_C = \{I, a, j, x^C\}$  the set of actions available to the client in the stage game, where  $a = (a_1, a_2)$  and  $a_i = 1$  means that the client accepts the transfer  $T_i$  and  $a_i = 0$  means the client rejects it. Define the discount factor of the client as  $\delta$ .

Define the action profile played in stage  $t$  as  $a^t = (a_{D_1}^t, a_{D_2}^t, a_C^t)$ .  $a^t$  is the triplet of individuals' stage game actions in each stage  $t$ . Define a history  $h^t$  as a description of all the actions taken up through the stage  $t - 1$  from each player. The history is formally defined as  $h^t = (a^0, a^1, \dots, a^{t-1})$ . The strategy of each player  $s_i^t$  can be written as a function of history:  $a_i^t = s_i^t(h^t)$ . This represents the stage game action the player  $i$  would play in period  $t$  if the previous play has followed the history  $h^t$ .

**Definition 14** *A profile of strategies  $s = (s_{D_1}, s_{D_2}, s_C)$  is a subgame perfect equilibrium in the infinitely repeated game if and only if there is no player  $i$  and no history  $h^t$  such that player  $i$  would gain from deviating from  $s_i(h^t)$ .*

In this specific setting I am interested in the existence of a cooperative equilibrium between one of the dealer and the client, defined as:

**Definition 15** *Define a cooperative equilibrium as a profile of strategies  $s^* = (s_{D_1}^*, s_{D_2}^*, s_C^*)$  such that :*

I.  $D_1$  plays a non forgiving trigger strategy such that

$$\bar{S}_{D_1}^t(h^t) = \begin{cases} (\bar{T}_1, \bar{x}^{D_1}), h^t = ((\bar{T}_1, \bar{x}^{D_1}), \cdot, (I = 1, a_1 = 1, a_2 = 0), j = 0, x^C = 0)^t \\ T'_1 \text{ otherwise} \end{cases} \quad (3.9)$$

Where  $T'_1$  is the stage game transfer defined in Proposition 2.

II.  $C$  finds optimal to conform to

$$(I = 1, a_1 = 1, a_2 = 0), j = 0, x^C = 0) \quad (3.10)$$

for any  $t$ .

### 3.3.2 Cooperative equilibrium in the repeated game

In this section the cooperative equilibrium in the repeated game is derived.

In the repeated game, the conditions that allow for a cooperative equilibrium depend on  $K$ , the cost that the client pays in order to trade directly in the interdealer market. If  $K < E(\pi^{C,M}) - 2E(\pi^{D,d})$ , as each dealer earns zero profit in the stage game, the maximum level of transfer a dealer is willing to offer to obtain cooperation is higher than the transfer he would offer if  $K \geq E(\pi^{C,M}) - 2E(\pi^{D,d})$ , as in the latter case the dealer still earns a positive profit in the stage game. As a consequence, this implies that the cooperative outcome is sustainable for a larger set of discount factors: the requirement on the discount factor of the client to sustain a cooperative outcome decreases in the level of transfer offered by the dealer.

**Proposition 16** *Cooperative equilibrium in the repeated game. In the repeated game there is a cooperative equilibrium, as outlined in Definition 5.*

- If  $K < E(\pi^{C,M}) - 2E(\pi^{D,d})$ , the transfer  $\bar{T}_1$  has to be  $E(\pi^{D_1,M}) - K \leq \bar{T}_1 \leq E(\pi^{D_1,M})$ , for the dealer to be willing to offer  $\bar{T}_1$  to the client and for the client to trade through one dealer. Moreover the discount factor of the client has to be  $\delta \geq \frac{E(\pi^{D,d})}{(T_1 + E(\pi^{D,d}) - E(\pi^{C,M}) + K)}$  for the client not to sell information to both dealers.

- If  $K \geq E(\pi^{C,M}) - 2E(\pi^{D,d})$ , the transfer  $\bar{T}_1$  has to be  $E(\pi^{D_1,M}) - K \leq \bar{T}_1 \leq \frac{3E(\pi^{D_1,M}) - 2E(\pi^{D,d}) - K}{2}$ , for the dealer to be willing to offer  $\bar{T}_1$  to the client and for the client to trade through one dealer. Moreover the discount factor of the client has to be  $\delta \geq \frac{E(\pi^{D,d})}{(T_1 + E(\pi^{D,d}) - E(\pi^{C,M}) + K)}$  for the client not to sell information to both dealers.

**Proof.** According to Definition 4, for  $s^* = (s_{D_1}^*, s_{D_2}^*, s_C^*)$  to be equilibrium, it should be the case that neither the client nor the two dealers have an incentive to deviate from the strategies described in definition 5. This implies that if the client acquires information, the dealer is better off offering to the client a transfer and being an information monopolist in the interdealer market. Moreover it should be the case that if the client deviates from (4.28) the dealer is better off reverting to  $T'_1$ . It should also be the case that if the dealer conforms to (3.9) the client has the incentive to acquire information and to conform to (4.28) and the second dealer is better off offering  $T_2 < \bar{T}_1$ .

The proof is developed in two steps. In the first step I prove that the dealer is better off offering  $C$  a transfer  $\bar{T}_1$  and being an information monopolist. In the second step I prove that if the dealer pays  $\bar{T}_1$ , the client has the incentive to acquire information and sell it to one dealer only.

- I. Define the profit of the dealer if he is an information monopolist as  $E(\pi^{D_1,M})$  Assuming the client conforms to (4.28), the following condition has to be satisfied:

$$E(\pi^{D_1,M}) - \bar{T}_1 \geq E(\pi^{D,d}) - T'_1 \quad (3.11)$$

where  $T'_1$  is the equilibrium transfer in the stage game.

Consider the case in which  $K < E(\pi^{C,M}) - 2E(\pi^{D,d})$ . In this case the profits of the dealers in the stage game are null: this is an implication of the fact that the impossibility of a commitment from the client to trade only with one dealer precludes one dealer from earning monopolistic profit in the interdealer market. In turn this affects the possibility of dealers to compensate the client for the sale of information up to a level that would make him indifferent if using intermediation or trading directly in the interdealer market. In this case, (3.11) implies:

$$E(\pi^{D1,M}) - \bar{T}_1 \geq 0$$

or  $E(\pi^{D1,M}) \geq \bar{T}_1$ . The dealer is better off as long as he gets an arbitrary small profit.

Now consider the case in which  $K \geq E(\pi^{C,M}) - 2E(\pi^{D,d})$ . In this case in the stage game dealers are able to transfer to the client a level of profit such that he is indifferent if trading directly or use intermediation. In this case condition (3.11) implies:

$$E(\pi^{D1,M}) - \bar{T}_1 \geq E(\pi^{D,d}) - \frac{E(\pi^{C,M}) - K}{2}$$

The previous states that for the dealer to offer to the client a transfer for exclusive sale of information it should be the case that  $\bar{T}_1 \leq \frac{3E(\pi^{D1,M}) - 2E(\pi^{D,d}) - K}{2}$ .

Moreover for the dealer it should be optimal not to offer  $\bar{T}_1$  while the client sells information to both dealers (plays  $d$ ). This is:

$$E(\pi^{D,d}) - T'_1 + \sum_{t=1}^{\infty} \delta^t [E(\pi^{D,d}) - T'_1] \geq E(\pi^{D,d}) - \bar{T}_1 + \sum_{t=1}^{\infty} \delta^t [E(\pi^{D,d}) - T'_1] \quad (3.12)$$

The previous simply implies that the cooperative transfer  $\bar{T}_1$  should be greater than the transfer that each dealer corresponds to the client in the stage game ( $\bar{T}_1 > T'_1$ ).

- II. Prove that, if the dealer pays  $\bar{T}_1$ , the client has the incentive to acquire information and sell it to one dealer only. As a first thing notice that if  $k \geq E(\pi^{D1,M}) - K$  the same results of the stage game holds in the repeated game: there is no possibility of information acquisition and Dealers won't be willing to charge any level of spread that is below the maximum possible.

Now consider the case in which the cost of acquiring information does not exceed the client monopolistic profit in the interdealer market net of the access fees, or  $k < E(\pi^{D1,M}) - K$ .

In the stage game, the client earns monopolistic profits net of the access fee  $K$  and of the cost of information acquisition  $k$ . For the client to be willing to use intermediation it should be the case that:

$$\sum_{t=0}^{\infty} \delta^t (\bar{T}_1 - k) \geq \sum_{t=0}^{\infty} \delta^t (E(\pi^{C,M}) - K - k)$$

The previous implies that  $\bar{T}_1 \geq E(\pi^{C,M}) - K$ , otherwise the client is better off trading directly in the interdealer market.

Moreover, for the client to be willing to sell information exclusively to one dealer, rather than both the following condition should hold:

$$\sum_{t=0}^{\infty} \delta^t (\bar{T}_1 - k) \geq \bar{T}_1 + T_2^{Max} - k + \sum_{t=1}^{\infty} \delta^t (E(\pi^{C,M}) - K - k) \quad (3.13)$$

In the previous, the left hand side represents the profit of the client if he cooperates with  $D_1$ . The right hand side represents the proceedings of the client if, after accepting the offer of  $\bar{T}_1$  from  $D_1$ , he accepts a second offer  $T_2^{Max}$  from  $D_2$ . I assume  $T_2^{Max} = E(\pi^{D,d})$ . In equilibrium this level of transfer from  $D_2$  is a credible commitment only for one period. In fact in the next period, as  $D_1$  reverts to  $T'_1$ ,  $D_2$  can consequently reduce his transfer to any  $T'_2 \geq T'_1$ .

The previous condition requires  $\delta \geq \frac{E(\pi^{D,d})}{(T_1 + E(\pi^{D,d}) - E(\pi^{C,M}) + K)}$ .

Hence the resulting conditions to have a cooperative equilibrium can be summarized as follows:

- If  $K < E(\pi^{C,M}) - 2E(\pi^{D,d})$ ,  
then  $\left\{ \begin{array}{l} \bar{T}_1 \leq E(\pi^{D_1,M}), \text{ for the dealer to be willing to offer } \bar{T}_1 \text{ to the client} \\ \bar{T}_1 \geq E(\pi^{D_1,M}) - K, \text{ for the client to trade through one dealer} \\ \delta \geq \frac{E(\pi^{D,d})}{(T_1 + E(\pi^{D,d}) - E(\pi^{C,M}) + K)} \text{ for the client not to sell information to both dealers.} \end{array} \right.$
- If  $K \geq E(\pi^{C,M}) - 2E(\pi^{D,d})$ ,  
then  $\left\{ \begin{array}{l} \bar{T}_1 \leq \frac{3E(\pi^{D_1,M}) - 2E(\pi^{D,d}) - K}{2}, \text{ for the dealer to be willing to offer } \bar{T}_1 \text{ to the client} \\ \bar{T}_1 \geq E(\pi^{D_1,M}) - K, \text{ for the client to trade through one dealer} \\ \delta \geq \frac{E(\pi^{D,d})}{(T_1 + E(\pi^{D,d}) - E(\pi^{C,M}) + K)} \text{ for the client not to sell information to both dealers.} \end{array} \right.$

■

### 3.3.3 Discussion of the results of the dynamic game

I choose a setting with an infinitely repeated game because this type of settings captures the ongoing nature of the client dealer interaction: dealing banks are large and well established institutions and such banks tend to have a roughly constant share of client order flow. The 'loyalty component' of the client-dealer relationship could offer an explanation for such evidence. My model suggest that in the case of an interaction between dealers and clients that does not have a set end (assuming that access fee are not prohibitive for information acquisition) any level of fee can sustain sale of information from clients to dealers. This finding is interesting as it offers an explanation of the fact that, although technological innovations allow clients to trade on new platforms (e.g. Client-dealer trading, single dealer trading platforms, prime brokerage agreements, alternative FX trading platforms, interdealer market) possibly with various access fee levels, intermediation remains a key characteristic of trading in OTC markets, also for leveraged investors.

In this setting I focus on a repeated interaction to solve the commitment problem. A work by [Biais and Germain, 2002] analyses indirect sale of information and shows that in a one shot interaction, optimal contracts can mitigate the conflict of interest arising between the seller of information and his customer. In this case, an informed agent can engage in indirect sales of information to customers, by setting up an investment fund that trades on the basis of his information. At the same time, the informed agent can conduct proprietary trades. They prove the effectiveness of an incentive compatible contract to mitigate the conflict of interest that arises when the intermediary can trade on his own account and on behalf of his client. The contract specifies the trading volume of the fund, and the compensation of the seller of information. Such compensation involves a variable part, depending on gains and losses from the fund, and an upfront payment. With such optimal contract the gains of the informed agents are maximized thanks to the ability to commit ex ante. In my model, if one dealer could sell indirectly information to the other one and at the same time, trade in the interdealer market, stipulating an optimal contract in the same spirit of [Biais and Germain, 2002], the two dealers could share the informational advantage they gain from the client. In such case an optimal outcome for the two dealers would be achieved in a single interaction.

### 3.4 Conclusion

In this work I focus on the client-dealer relationship in an OTC market.

Specifically I look at the existence of intermediation, at the indirect revelation of information from clients to dealers through clients' order flow and at the ongoing nature of the client dealer relationship. First I consider a two stage model in which an informed client has the choice to access directly a LOB interdealer market paying a fee or trade through two dealers. I show that the existence of intermediation depends on the access fee level and that, in a finite horizon interaction, the client has always the incentive to trade with multiple dealers. This highlights the commitment problem that arises due to the impossibility of writing enforceable contracts between clients and dealers. In this paper I also show how the bid ask spread can be used strategically as a tool for profit sharing, rather than as a mechanism to protect the dealer from adverse selection. This finding is motivated by the nature of the client dealer interaction, in which dealers have knowledge of the identity of clients and observe privately their order flow.

I extend my analysis to an infinitely repeated game which aims to capture the ongoing nature of the client dealer relationship. I focus my interest in cooperative equilibria between the client and the dealer. I show that, in the repeated game, there is an equilibrium in which the client always sells information to the same dealer, who acts as an information monopolist in the interdealer market. The commitment problem is solved in the context of a repeated interaction.

This work could be extended enriching the model and complementing it with an empirical analysis. The model could be extended in several ways. On one hand, if focusing on the FX market, it could be interesting to have a model in which trading is motivated by difference in beliefs and show that dealers would offer better pricing to some clients. On the other hand it could be also interesting to enrich the role of the dealing bank as an active party in producing information and show that, still, the bank has the incentive to use customer order flow to achieve accurate predictions.

Moreover, it could be interesting to complement and extend this work with an empirical analysis. As a first thing it could be interesting to test if dealers spread setting behavior is constant or depends on the type of clients they face. It would be also interesting to test if those clients who receive better pricing constitute a constant client base for dealers. Furthermore the

possibility of predictive power in clients order flow and consequently dealers trading strategies could be tested. Such hypotheses could be tested in OTC markets as the FX market or the corporate bond market, subject to availability of data.

Potentially a richer version of this model could be used to generally introduce a new idea of the use of the spread in presence of information asymmetries.

## Chapter 4

# The value of information about noise trading in liquidity provision

### 4.1 Introduction

According to plea agreements to be filed in the District of Connecticut, between December 2007 and January 2013, euro-dollar traders at Citicorp, JPMorgan, Barclays and RBS – self-described members of “The Cartel” – used an exclusive electronic chat room and coded language to manipulate benchmark exchange rates. [...] “The Cartel” traders coordinated their trading of U.S. dollars and euros to manipulate the benchmark rates set at the 1:15 p.m. and 4:00 p.m. fixes in an effort to increase their profits.

The United States Department of Justice

In May 2015, 6 major dealing banks<sup>1</sup> have been fined for a total of \$10Bn for FX manipulation<sup>2</sup>. Since 2008 the attention of the Fed and of UK authorities has been focused on possible

---

<sup>1</sup>The banks fined are Barclays, Citigroup, JPMorgan, RBS, UBS and Bank of America.

[Euromoney, 2016] ranks City as the leading player in the FX market with an overall market share of 12.91% followed by JPMorgan with a share of 8.77%, UBS 8.76%, Deutsche Banks 7.86%, BoA 6.40% and Barclays 5.67%.

<sup>2</sup>Financial Times, 20 May 2015, <https://www.ft.com/content/23fa681c-fe73-11e4-be9f-00144feabdc0>

manipulation of the Benchmark Libor interest rate. While in 2012 the Libor scandal is at its peak, Bloomberg news reports that traders at major dealing banks might be sharing information about customers' order flow and coordinating their trades to profit from another benchmark: the FX 4 pm London Fix. Authorities, at this point, officially took an interest in the FX Fix. In August 2016, Mr Christopher Ashton, a former global head of the FX spot-trading business at the London headquarters of Barclays, has been fined \$1.2m and banned for life from the country banking industry by the Federal Reserve Bank of New York. The Fed alleged that the trader was part of a group self-described as "The Cartel". This group used a chatroom to share confidential information about customers' with traders at other firms in order to manipulate currency pricing benchmarks. One month before Mr Ashton ban, a former trader at UBS, Mr Matthew Gardiner, was banned for life for similar offences.

This chapter presents an exercise to investigate the value that liquidity providers obtain from information about noise order flow. I consider an imperfectly competitive setting in which value informed agents, strategic agents and liquidity traders trade an asset. Value informed agents have fundamental information about the value of the asset and are assumed to be small in size and risk averse. Value informed agents do not take into account the impact of their demand on prices. Strategic agents have diverse information about liquidity order flow. I consider four possible cases: 1) pure market making; 2) dealership without information sharing; 3) dealership with information sharing but without collusion in trading; and 4) dealership with information sharing and collusion in trading. In the first "regime" agents do not possess any private information about liquidity demand. In the second "regime" each agent observes privately a different component of the total liquidity order flow. In the third "regime" agents share information on the liquidity order flow they observe. In the fourth "regime", dealers not only share information, but also collude in the trading strategy.

My analysis of the discussed scenarios is motivated by recent evidence in OTC markets of traders sharing information related to customer order flow, especially in relation to trading around benchmarks determination. [BoE, 2015] highlights how some OTC markets have been under scrutiny due to fixing manipulation, e.g. the gold fixing or the FX fix. [IOSCO, 2013] and [FSB, 2014] highlight the riskiness of benchmarks as they can promote conflicts of interests and incentives to manipulate the determination process. In the context of the FX fix, manipulation

is potentially a product of a conflict of incentives between market makers acting as agents and principals in trading. This leads to two types of illegal behaviors: colluding by sharing information about client orders and artificially manipulating the Fix rate by buying and selling aggressively in the Fix window. Despite the fact that the FX is an extremely liquid market, the evidence of manipulation suggests that traders needed both to share information and to coordinate in trading activities to have an impact on the direction and level of prices.

I investigate the value that information adds to each regime. The difference in profits between regime 2 and 1 provides the value added by dealership. The difference in profits between regime 3 and 2 provides the value added by information sharing. The difference in profits between regime 4 and 3 provides the value added by collusion. My findings suggest that sharing information impacts substantially dealers' profit. Colluding in trading strategy also impacts positively dealers profits, but such impact is moderate. This finding suggest that if dealers face penalties for collusion they might decide to share information but not to collude.

This exercise further looks at the impact that the various information regimes have on some indicators of market quality. I find that markets with market makers offer the highest depth. Dealers markets with no information sharing offer the lowest depth. When dealers collude, market depth is lower than when they share information. Despite this, when dealers observe the same information, colluding or not, market depth is lower than in the no information case, but higher than in the case of partial order flow observation. I find that volatility of prices is always higher than in a setting with market making only. On the other hand informativeness of prices is enhanced in a dealer market, where there is no information sharing and no collusion.

Several market microstructure models consider the effect of one trader, fundamentalist or speculator, having private information on part of the liquidity order flow. My exercise differs from the existing models in its setting, as I consider an imperfectly competitive market with multiple agents informed about supply, and in the fact that I offer a comparison of the outcomes of information sharing in various existing market settings. The models of [Rochet and Vila, 1994], [Röell, 1990] and [Bernhardt and Taub, 2008] consider a setting in the spirit of [Kyle, 1985b]. [Rochet and Vila, 1994] proves that in a static setting, equilibrium outcomes are exactly the same in the case in which a monopolist speculator sees all liquidity trade as in the case in which the speculator sees no liquidity trade at all. The benefits from seeing liquidity trades

are just offset by the corresponding increased equilibrium price sensitivity to order flow. When the speculator offsets half liquidity trade the market makers respond by setting price schedules that are twice as sensitive to order flow. The speculator scales down his trading intensity on private information about the asset proportionally. Such mechanism leaves the informational content of prices and profits unaffected.

[Röell, 1990] models dual capacity trading in a market where dealers are risk neutral and competitive and orders are placed both by uninformed agents and by traders with some measure of inside information. She finds that in an equilibrium with dual trading, the insider trades less vigorously on his information. On the other hand if a customer cannot convince the dealer of the fact that his order contains no information, he suffers worse price impact. Conversely those who can convince the dealer that they have no information, have less price impact as their order moves price less. In this model dual capacity trading reduces the total transaction costs for liquidity traders. The idea is that less noise is brought to the market, as dealers offset part of the noisy order flow, and the insider can hide less behind noise. In this setting Roell identifies two effects as controversial. The first effect is that the trading cost increases for some traders, while decreasing for the overall market. The second effect is that the profit of the informed trader decreases and this can trigger a less intense effort in gathering information. [Bernhardt and Taub, 2008] shows that in a dynamic setting the effects found by [Röell, 1990] and [Rochet and Vila, 1994] are reversed. They develop a model of a monopolist speculator who combines long lived private information and knowledge of current and future liquidity trade. In their model the speculator knows the assets' value at both dates. They find that the speculator is going to trade against the date one liquidity trade at both dates. This is because the market maker, at the last date, confuses past liquidity trades with speculative trade on fundamental. They find that a speculator at first trades in the same direction of future liquidity trade, taking a large offsetting position at date two. This is because the speculator wants to reduce the price impact of his future trade. Through their model they show evidence of positive autocorrelation into net order flow in presence of uncorrelated liquidity order flow. The speculator's profit is higher when he can front run on knowledge of liquidity trade than when he cannot.

[Dumitrescu, 2005] develops a market close in spirit to [Kyle, 1989] where risk neutral agents have 2 types of information: fundamental information and information regarding liquidity. Her

aim is to study how strategic trading on the two types of information affects market liquidity, informational efficiency of prices and other market indicators. Her setting is very close to [Kyle, 1989]. In [Dumitrescu, 2005] model there is only one supply informed agent. Her findings suggest that the strategic supply informed agent decreases the amount of information revealed into prices. Such finding is motivated by the fact that her value informed agents are strategic and take into account the effect that their trading has on prices. In my case this effect is not present. She finds that the supply informed agents obtains positive profits and his presence overall decreases market liquidity. I have a similar finding, but I additionally find that sharing information or colluding increases depth compared to partial information on noisy order flow.

The rest of the chapter proceeds as follows. In the first section I present the model set up and define the equilibrium. In the second section I present the equilibrium in each Regime. In the third section I offer some analysis and a comparison of the results obtained in the various setting. The last section highlights the future developments of such research.

## 4.2 The Model: Setting and definition of Equilibrium

Consider a market where one asset is traded. The asset has liquidation value  $v = \theta + \varepsilon$ , where  $\theta$  is observable and  $\varepsilon$  is not observable. Assume  $\theta \sim N(\bar{\theta}, 1/\tau_\theta)$  and  $\varepsilon \sim N(0, 1/\tau_\varepsilon)$ .

**Market Participants.** In the market there are 3 types of market participants:  $N$  value informed traders, 2 dealers  $d_j$ , and 3 groups of noise traders,  $j = 1, 2$ . Dealers and value informed traders differ in their risk preferences.

Value informed traders have exponential utility with risk aversion parameter  $\rho_I > 0$ , defined as:

$$-\exp\{-\rho_I \pi_I\} \tag{4.1}$$

where  $x$  denotes the number of shares demanded by each trader. Define  $r_I = \frac{1}{\rho_I}$  as the risk tolerance parameter.

Dealers are assumed to be risk neutral profit maximizers.

**Information.** Market participants have heterogeneous information about the liquidation value of the asset. Noise traders do not possess any private information. They trade a quantity

$\varphi_k$  for exogenous reasons where  $\varphi_k \sim N(0, 1/\tau_{\varphi_k})$ ,  $k = 1, 2, 3$ .

Each value informed trader  $i$  observes  $\theta$ . Value informed traders are small and do not take into account the impact that their demand has on prices.

Each dealer,  $d_j$  does not possess any private information about  $\theta$ . Dealers might have some information about contemporaneous noise trading. More specifically there are four possible information regimes,  $r$ , for dealers,  $r = \{1, 2, 3, 4\}$ :

- I. *Regime 1*: Each dealer  $d_j$  does not observe any  $\varphi_k$ , and knows only the prior distribution of noise traders order flow.
- II. *Regime 2*: Each dealer has partial knowledge of  $\varphi_k$ , as each dealer observes only one component of the aggregate noise trading order flow. I assume that  $d_1$  observes  $\varphi_1$  and  $d_2$  observes  $\varphi_2$ . No dealers observe  $\varphi_3$ .
- III. *Regime 3*: Dealers exchange information about the observed  $\varphi_k$ . This means  $d_1$  and  $d_2$  observe both  $\varphi_1$  and  $\varphi_2$  but not  $\varphi_3$ . In this model I am assuming truthful information disclosure.
- IV. *Regime 4*: Dealers exchange information about the observed  $\varphi_k$  as in Regime 3. In this case, though, they trade as a single dealer, hence not only share information but also collude in trading.

Regime 1 is a setting with pure market making. Market makers do not have private information about liquidity order flow. Regime 2 is a dealer market. In a dealer market each dealer observes a portion of the aggregate liquidity order flow. The comparison of Regime 1 and Regime 2 provides the value of dealership. Regime 3 is a dealer market in which dealers share information on liquidity order flow. The difference in Regime 2 and 3 provides the value added by sharing information. Regime 4 represents a dealer market in which dealers share information on liquidity order flow and collude in trading. The difference between Regime 3 and 4 provides the value of collusion.

### **Definition of equilibrium**

Denote the information set of each dealer as  $\Omega_j^r$ , where  $r$  represents the information regime,  $r \in \{1, 2, 3, 4\}$ . For  $d_1$ , and symmetrically for  $d_2$  we have that:

$$\Omega_j^1 = \{p\}, \Omega_j^2 = \{p, \varphi_j\}, \text{ with } j = 1, 2, \text{ and } \Omega_j^3 = \Omega_j^4 = \{p, \varphi_1, \varphi_2\}.$$

**Definition 17** *An Equilibrium is defined as a set of demand schedules  $X_{I,i}(\theta)$  for the  $i = 1, \dots, N$  value informed traders,  $X_{D,j}(p, \Omega_j^r)$ , where  $j = 1, 2$  and  $r \in \{1, 2, 3\}$  and a price function  $P(v, \varphi_A)$ , where  $\varphi_A = \sum_{k=1}^3 \varphi_k$  is the aggregate noise trader order flow, such that:*

*I. The market clears:*

$$\sum_{n=1}^N X_{I,i}(\theta) + \sum_{j=1}^2 X_{D,j}(p, \Omega_j^r) + \sum_{k=1}^3 \varphi_k = 0 \quad (4.2)$$

*II. Each value informed optimizes:*

$$X_{I,n}(\theta) \in \arg \max_{x_i} \{E[-\exp\{\rho_I(v-p)x_i\} | \theta]\} \quad (4.3)$$

*for all  $n = 1, \dots, N$ .*

*III. Each dealer,  $d_j$  optimizes:*

$$X_{D,j}(p, \Omega_j^r) \in \arg \max_x \{E[(v-p)x_D | p, \Omega_j^r]\}$$

*Refinements.* I restrict the attention to linear symmetric equilibria where the demand schedules for Dealers can be written as:

$$X_{D,j}(p, \Omega_j^r) = \alpha_U - \beta_U p - \eta_U \varphi_j \quad (4.4)$$

## 4.3 Characterization of the equilibrium

### 4.3.1 Regime 1

The following proposition characterizes the Bayesian-Nash Equilibrium. In the equilibrium the agents' strategies and the pricing rule are linear.

Derivations are provided in Appendix E.

**Proposition 18** *If  $Nr_I\tau_\varepsilon + \beta_U > 0$ , there exist a unique symmetric linear equilibrium defined as:*

$$\begin{aligned} X_{Dj}(p, \Omega_j^r) &= \alpha_U - \beta_U p \text{ for } j = 1, 2 \\ x_{I,i}(\theta) &= (\theta - p)r_I\tau_\varepsilon \text{ for any } N = 1, \dots, N \end{aligned}$$

where the parameters  $\alpha_U$  and  $\beta_U$  are given by:

$$\begin{aligned} \alpha_U &= \frac{[R\gamma + (R^2\gamma^2 + 8\tau R)^{1/2}] \theta \tau_\theta}{4\tau R + 2R\gamma [R\gamma + (R^2\gamma^2 + 8\tau R)^{1/2}]} \\ \beta_U &= \frac{(R^2\gamma^2 + 8\tau R)^{1/2} - 3R\gamma}{4R} \end{aligned}$$

where  $\gamma = Nr_I\tau_\varepsilon$  and  $R = \frac{(\tau_{\varphi_1}\tau_{\varphi_2}\tau_{\varphi_3})}{(\tau_{\varphi_2}\tau_{\varphi_3} + \tau_{\varphi_1}\tau_{\varphi_3} + \tau_{\varphi_2}\tau_{\varphi_1})}$ .

In this setting there is only one type of private information in the market: the liquidation value of the security. The flow of information is unidirectional as value informed traders do not infer information from prices, as they have an homogeneous signal: they observe  $\theta$  and trade accordingly. Value informed trader demand in equilibrium is given by:

$$x_I = (\theta - p)r_I\tau_\varepsilon \quad (4.5)$$

The value informed trader demand is increasing in the precision of the signal,  $\tau_\varepsilon$ . A higher precision implies more reliable information, and for this reason value informed traders trade more aggressively. Value informed traders demand is increasing in risk tolerance  $r_I$ , or vice versa decreasing in risk aversion,  $\rho = \frac{1}{r_I}$ . Value informed trader demand is positive (negative) if  $\theta > p$  ( $\theta < p$ ), which implies an value informed trader buys (sells) if the price is lower (higher) than the observed value of theta.

Market makers update their expectations according to the price signal and their demand schedule has the form:

$$X_i = \alpha_U - \beta_U p$$

The price impact of market maker  $i$  is given by:

$$\lambda_U = \frac{4R}{R\gamma + (R^2\gamma^2 + 8\tau R)^{1/2}}$$

The  $\lambda_U$  is decreasing in  $N$ , (see Figure 4-1): the higher is the number of value informed traders, the lower the impact that uninformed market makers' orders have on prices, due to the fact that price becomes strongly informative.

For the same reason market makers' price impact is decreasing in  $r_I$  and in  $\tau_\varepsilon$ . If value informed traders are more risk tolerant, their demand is higher and prices are more informative. Analogously as  $\tau_\varepsilon$  increases price is more revealing and this implies that the impact of market makers' trades is lower.

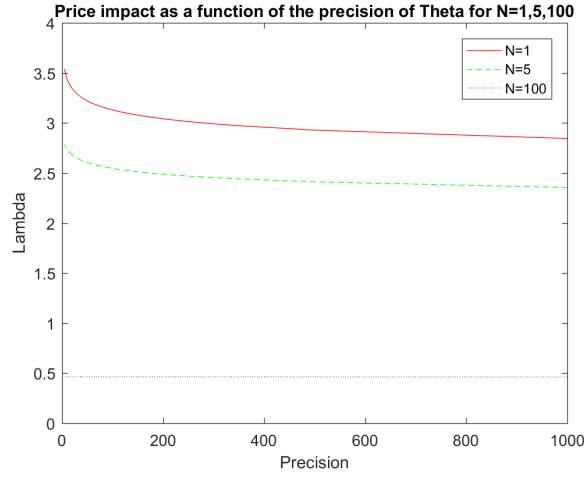


Figure 4-1: Price impact is plotted as a function of  $\tau_\theta$ . Fixing the value of the precision, when the number of informed traders is low, the price impact of dealers is higher. This is because prices are less informative.

### 4.3.2 Regime 2

In this setting each dealer has partial knowledge of  $\varphi_k$  as dealers observe only one component of the aggregate noise trading order flow. I assume that  $d_1$  observes  $\varphi_1$  and  $d_2$  observes  $\varphi_2$ .

The following proposition describes the equilibrium:

**Proposition 19** *If  $Nr_I\tau_\varepsilon + \beta_U > 0$ , there exist a unique symmetric linear equilibrium defined*

as:

$$\begin{aligned} X_i &= \alpha_U - p\beta_U - \eta_U \varphi_i \text{ for } j = 1, 2 \\ x_{I,i}(\theta) &= (\theta - p)r_I \tau_\varepsilon \text{ for any } N = 1, \dots, N \end{aligned}$$

where the parameters  $\alpha_U$ ,  $\beta_U$  and  $\eta_U$  are the solution of the following system of equations:

$$\begin{cases} \alpha_U = \left( \frac{\bar{\theta}\tau_\theta}{\tau} - \frac{2\alpha_U\gamma}{\tau} \left( \frac{\tau_\varphi}{(1-\eta_U)^2} + \tau_\varphi 3 \right) \right) (\gamma + \beta_U) \\ \beta_U = \left( 1 - \frac{(\gamma+2\beta_U)\gamma}{\tau} \left( \frac{\tau_\varphi}{(1-\eta_U)^2} + \tau_\varphi 3 \right) \right) (\gamma + \beta_U) \\ \eta_U = \frac{\gamma(1-\eta_U)}{\tau} \left( \frac{\tau_\varphi}{(1-\eta_U)^2} + \tau_\varphi 3 \right) (\gamma + \beta_U) \\ \tau = \tau_\theta + \gamma^2 \left( \frac{\tau_\varphi}{(1-\eta_U)^2} + \tau_\varphi 3 \right) \end{cases} \quad (4.6)$$

where  $\gamma = Nr_I \tau_\varepsilon$ .

The system of equations 4.6, has been solved numerically.

In figure 4-2 I plot dealers' demand coefficients and their price impact, for Regime 1 and Regime 2 (R1 and R2). In Regime 2 the coefficient  $\beta_U$  is close to zero. This indicates that when dealers have partial information about the order flow, their demand is less responsive to changes in prices. The coefficient  $\eta_U$  indicates that dealers take an opposite position to noise traders order flow trading against roughly one half of the observed order flow from noise traders, when  $N \geq 2$ . Moreover dealers' price impact, if  $N \leq 5$  is significantly higher than in the benchmark case, while for  $N$  that becomes larger the two tend to converge. This is because compared to Regime 1 dealers have more information.

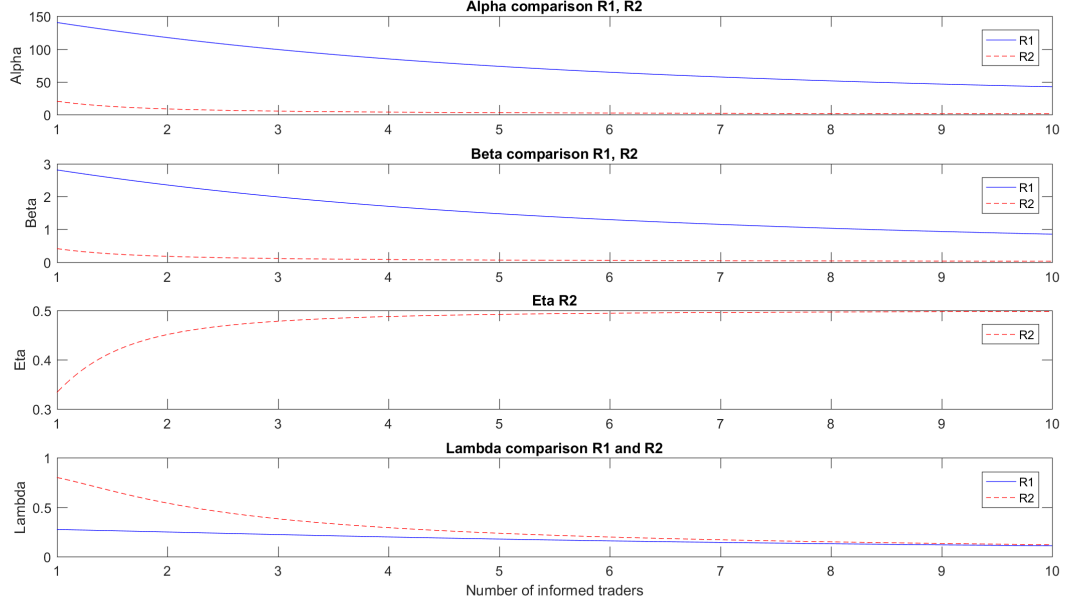


Figure 4-2: Plot of dealers' demand coefficients for Regime 1 and Regime 2, as the number of informed traders increases.

#### 4.3.3 Regime 3 and 4

This section describes the equilibrium in Regime 3 and Regime 4. In both cases the two dealers observe the same information. In Regime 3 the two dealers observe the same information and trade separately, while in Regime 4 they trade as a single dealer.

**Proposition 20** *In Regime 3, if  $Nr_I\tau_\varepsilon + \beta_U > 0$ , there exist a unique symmetric linear equilibrium defined as:*

$$\begin{aligned}
 X_i &= X_U = \alpha_U - p\beta_U - \eta_U(\varphi_1 + \varphi_2), \text{ for } i = 1, 2 \\
 x_{I,i}(\theta) &= (\theta - p)r_I\tau_\varepsilon \text{ for any } N = 1, \dots, N
 \end{aligned}$$

where the parameters  $\alpha_U$ ,  $\beta_U$  and  $\eta_U$  are defined as:

$$\left\{ \begin{array}{l} \alpha_U = \frac{\bar{\theta}\tau_\theta(\gamma+\beta_U)\tau^{-1}}{1+2(\gamma\tau_{\varphi 3})(\gamma+\beta_U)\tau^{-1}} \\ \beta_U = \left[ (\gamma^4\tau_{\varphi 3}^2 + 8\gamma^2\tau_{\varphi 3}\tau)^{\frac{1}{2}} - 3\gamma^2\tau_{\varphi 3} \right] \frac{1}{4\gamma\tau_{\varphi 3}} \\ \eta_U = \frac{\gamma\tau_{\varphi 3}\tau^{-1}(\gamma+\beta_U)}{1+2(\gamma\tau_{\varphi 3})(\gamma+\beta_U)\tau^{-1}} \\ \tau = \tau_\theta + \tau_{\varphi 3}(Nr_I\tau_\varepsilon)^2 \end{array} \right. \quad (4.7)$$

where  $\gamma = Nr_I\tau_\varepsilon$ .

The following proposition describes the equilibrium in Regime 4.

**Proposition 21** *In Regime 4, if  $Nr_I\tau_\varepsilon + \beta_U > 0$ , there exist a unique symmetric linear equilibrium defined as:*

$$\begin{aligned} X_i &= X_U = \alpha_U - p\beta_U - \eta_U(\varphi_1 + \varphi_2), \text{ for } i = 1 \vee i = 2 \\ x_{I,i}(\theta) &= (\theta - p)r_I\tau_\varepsilon \text{ for any } N = 1, \dots, N \end{aligned}$$

where the parameters  $\alpha_U$ ,  $\beta_U$  and  $\eta_U$  are defined as:

$$\begin{aligned} \alpha_U &= \frac{\gamma\bar{\theta}\tau_\theta}{\tau + \gamma^2\tau_{\varphi 3}} \\ \beta_U &= \frac{\gamma\tau - \gamma^3\tau_{\varphi 3}}{\tau + \gamma^2\tau_{\varphi 3}} \\ \eta_U &= \frac{\tau_{\varphi 3}\gamma^2}{\tau + \gamma^2\tau_{\varphi 3}} \\ \tau &= \tau_\theta + \tau_{\varphi 3}\gamma^2 \end{aligned}$$

where  $\gamma = Nr_I\tau_\varepsilon$ .

## 4.4 Comparison of Regimes

In order to understand the value that information from the liquidity order flow creates for market makers and dealers, I start by plotting in figure 4-3 the difference in the expectation of the profit in the 4 regimes, conditional on dealers' information on order flow. I compare the profit in Regime 1 and Regime 2. This provides the value added by dealership. I plot the

difference in Regime 2 and Regime 3, which provides the value added by sharing information and the difference in Regime 3 and Regime 4 which provides the value added by collusion. The difference in dealers' profits in Regime 1 and Regime 2 is small, compared to the increase in profit when dealers share information. The value added by sharing information creates the most sizeable increase in profit. Notice that the amount of profit added by colluding in the trading strategy is small when compared to the one induced by sharing information.

The finding that collusive behavior does not increase profits in a sizable way suggests that dealers might be more inclined to share information rather than to collude in trading. This is motivated by the idea that, if collusive behavior is detected by regulators, dealers could be subject to sizable penalties.

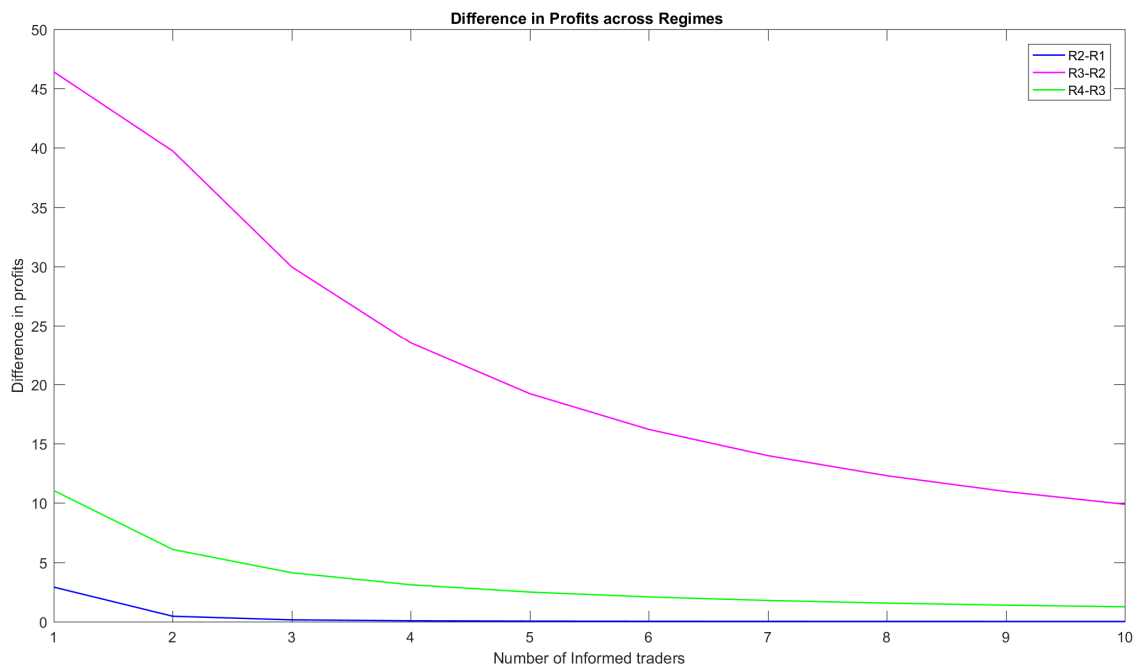


Figure 4-3: In this graph I plot the difference in the expectation of the profit in the 4 regimes, conditional on dealers' information on order flow. I plot the difference in profits between Regime 2 and Regime 1, the difference in Regime 3 and Regime 2, and lastly in Regime 4 and Regime 3.

To understand the effect of the information regimes on market quality I use, as in [Kyle, 1989] and [Dumitrescu, 2005], market depth. Market depth represents the volume of trading needed to move prices by one unit. In fact, an increase (decrease) in supply by  $\Delta$  induces the price to

decrease (increase) by one unit. In my model the market depth is given by

$$\Delta^{-1} = Nr_I\tau_\varepsilon + 2\beta_U$$

for regime 1, 2 and 3 and

$$\Delta^{-1} = Nr_I\tau_\varepsilon + \beta_U$$

for regime 4.

There are two components in market depth. The first component  $\gamma = Nr_I\tau_\varepsilon$  is constant across the models as the value informed traders in my model do not take into account the impact of their demand,

The second component  $\beta_U$  changes depending on the information regimes and consequently on the values that  $\beta_U$  takes in equilibrium. Figure 4-4, presents a plot of market depth as a function of the number of value informed in the market, depending on the information regimes.

The depth of the market is at its maximum when dealers have no information about noisy order flow and at its minimum when dealers observe partial information. As in [Dumitrescu, 2005] agents informed about the liquidity order flow reduce the depth of the market. If dealers do not observe any component of the noisy order flow the coefficient  $\beta_U$ , which indicates the sensitivity of dealers demand to prices, is higher than in the case in which dealers observe one component of the noisy order flow (regime 2). Despite this finding, when dealers observe the same information, colluding or not, market depth is lower than in the no information case, but higher than in the case of partial order flow observation. The depth is higher when dealer share information and lower when they collude.

Now I consider price impact for dealers. Price impact is derived from dealers optimal demand, and it is representative of how responsive price is to a unitary increase in demand

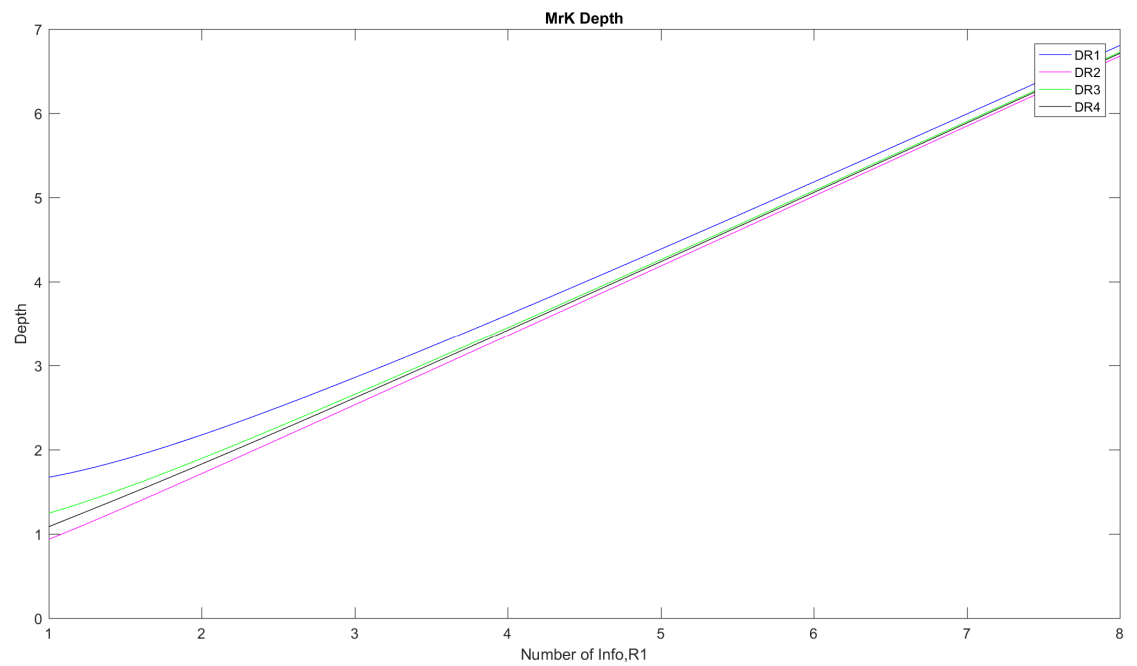


Figure 4-4: In This figures I plot the market depth in the various regime as a function of the number of informed traders. Market depth represents the volume of trading needed to move prices by one unit.

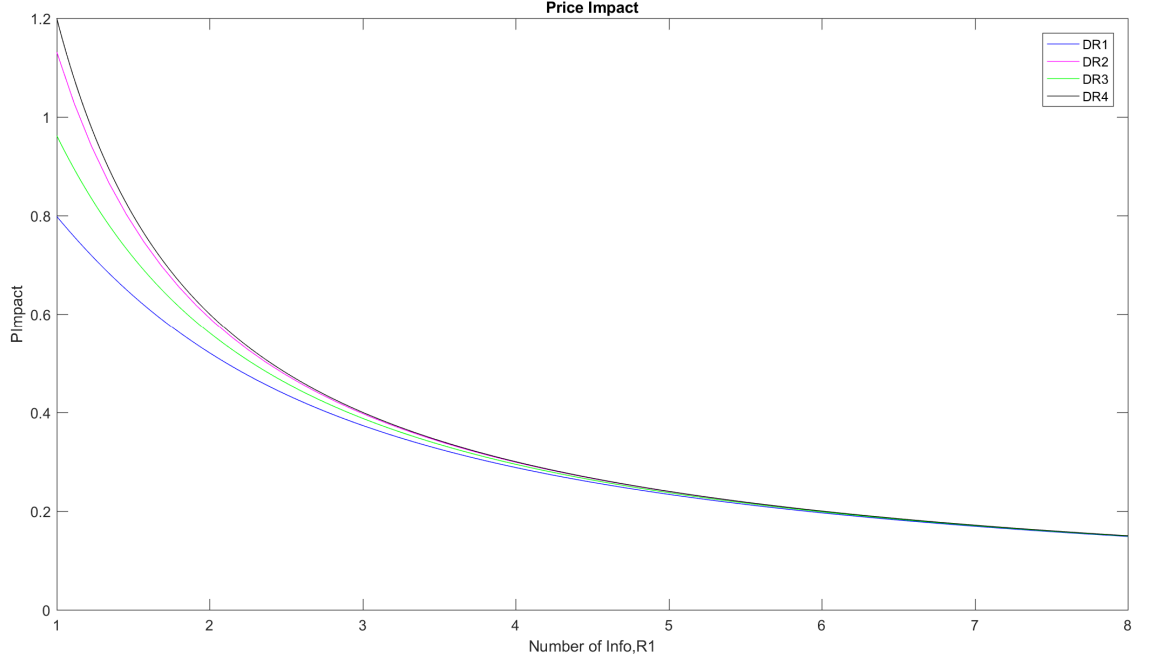


Figure 4-5: In this figure I am plotting price impact. Price impact is representative of how responsive price is to a unitary increase in demand from each dealer. The equation for price impact is  $\lambda_U = (Nr_I\tau_\varepsilon + \beta_U)^{-1}$  for R1,R2,R3 and  $\lambda_U = (Nr_I\tau_\varepsilon)^{-1}$  for R4.

from each dealer. The price impact is equal to:

$$\lambda_U = \frac{1}{Nr_I\tau_\varepsilon + \beta_U}, \text{ for case 1,2 and 3}$$

$$\lambda_U = \frac{1}{Nr_I\tau_\varepsilon}, \text{ for case 4}$$

It can be noticed that the price impact is higher when there is only one dealer, unless the  $\beta_U$  tends to zero. This is the case when the number of value informed trader increases and prices becomes fully revealing.

Figure 4-5, confirms that the price impact of dealers is highest when they collude and trade as a monopolist. In the case in which they observe partial information, as their demand is very reactive to price, they also have a high price impact. The case in which dealers have the lowest price impact is regime 1 and this is because dealers have no private information about the order flow.

To understand how the observation of the noisy order flow affects the volatility of prices I calculate and plot the volatility of prices in the 4 Regimes.

In Regime 1 the volatility of prices is:

$$\begin{aligned}
Var(p) &= E[(p - E(p))^2] = \\
&= E[(\Delta N r_I \tau_\varepsilon (\theta - \bar{\theta}) + \Delta(\varphi_1 + \varphi_2 + \varphi_3))^2] = \\
&= (\Delta N r_I \tau_\varepsilon)^2 \frac{1}{\tau_\theta} + \Delta^2 \left( \frac{1}{\tau_{\varphi_1}} + \frac{1}{\tau_{\varphi_2}} + \frac{1}{\tau_{\varphi_3}} \right)
\end{aligned}$$

In Regime 2 the volatility of price is:

$$\begin{aligned}
Var(p|\varphi_1) &= E[(p - E(p))^2] = \\
&= E[(\Delta N r_I \tau_\varepsilon (\theta - \bar{\theta}) - \Delta \mu_U \varphi_2 + \Delta(\varphi_2 + \varphi_3))^2] = \\
&= (\Delta N r_I \tau_\varepsilon)^2 \frac{1}{\tau_\theta} + \Delta^2 \left( \frac{1}{\tau_{\varphi_2}} + \frac{1}{\tau_{\varphi_3}} \right) + \Delta^2 \mu_U^2 \frac{1}{\tau_{\varphi_2}} - 2\Delta^2 \mu_U \frac{1}{\tau_{\varphi_2}}
\end{aligned}$$

In Regime 3 and Regime 4 the volatility of prices is:

$$\begin{aligned}
Var(p|\varphi_1, \varphi_1) &= E[(p - E(p))^2] = \\
E[(\Delta N r_I \tau_\varepsilon (\theta - \bar{\theta}) + \Delta \varphi_3)^2] &= (\Delta N r_I \tau_\varepsilon)^2 \frac{1}{\tau_\theta} + \Delta^2 \frac{1}{\tau_{\varphi_3}}
\end{aligned}$$

where  $\Delta$  takes a different value in the 2 Regimes.

I proceed plotting in figure 4-6 the difference in volatility of prices in regimes 2, 3 and 4 compared with regime 1:

Partial information about the order flow, in Regime 2, increases volatility of prices. On the other hand sharing information increases the volatility of prices but to a lower extent.

Following [Dumitrescu, 2005] I calculate the information content of prices as

$$\begin{aligned}
I &= Var(\tilde{\theta}) - Var(\tilde{\theta}|p) = \\
&= \left( \frac{1}{\tau_\theta} \right)^{-1} - \left( \frac{1}{\tau} \right)^{-1}.
\end{aligned}$$

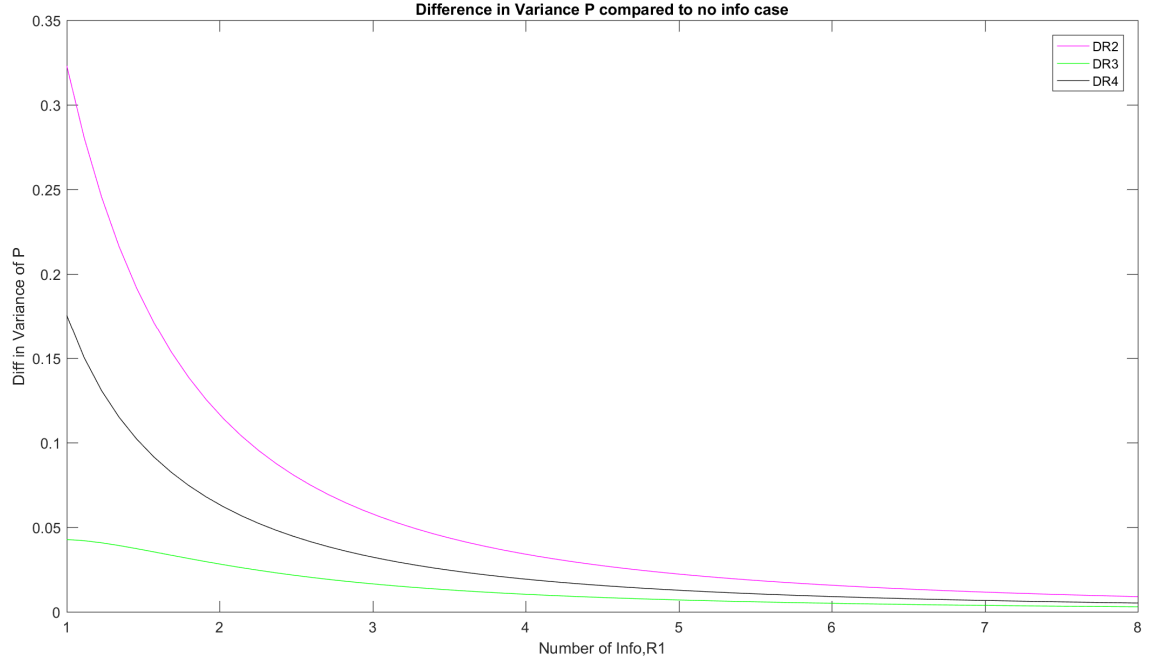


Figure 4-6: In this figure I plot the difference in volatility of prices in regime 2, 3 and 4 compared to regime 1. In all the 3 cases the variance of prices is higher than in the case with no information.

This statistic is calculated as the difference between the variance of the prior distribution and the updated variance. When such difference increases the information content of prices increases.

Figure 4-7 shows that in Regime 1 prices are least informative, while in Regime 2, price is most informative. In regime 3 and 4 the informativeness of prices is equivalent. Overall, in a setting in which value informed traders are not strategic, observation of the liquidity order flow increases informativeness of prices. This finding could likely change if value informed traders took into account the effect of their demand on prices.

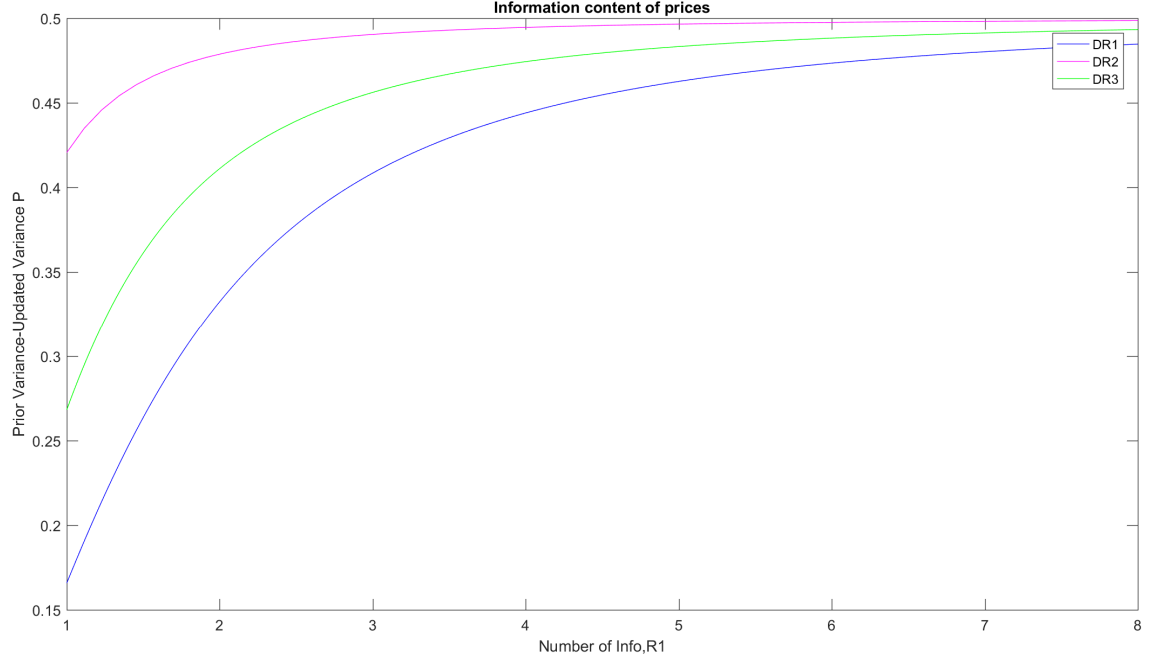


Figure 4-7: In this figure I plot a statistics that measures the information content of prices. calculate it as  $Var(\tilde{\theta}) - Var(\tilde{\theta}|p)$ , for each regime. If the value obtained is small this implies there is low information in prices. This is because the variance of the prior and the posterior are very close. If the value gets larger prices become more and more informative as the variance of the posterior becomes very small and the value converges to the variance of the prior , 0.5 in this specific case.

## 4.5 Discussion and Conclusion

In this chapter I present an exercise where I analyze the value added to liquidity providers by information on noise order flow. I consider four "information regimes" in which agents have diverse information about liquidity order flow. The four regimes aim to capture four market structures: market making, dealership, dealership with information sharing but no collusion and dealership with information sharing and collusion. The choice of such informational structure is motivated by the recent evidence of traders sharing information and potentially colluding in trading in relation to benchmark manipulation.

In this exercise, I calculate the value added by information in the various scenarios and I further show that sharing information has a sizable impact on dealers profits. When analyzing

the effect of the various scenarios on market indicators I find that overall market making offers highest market depth and lower price volatility. Market making, in this exercise, corresponds to the case in which dealers are unable to identify their clients and to observe the components of aggregate order flow. This can be considered as a case of anonymity in trading, compared to a dealer market, in which liquidity providers observe the identity of their clients.

I plan to extend this analysis in various direction as I think this topic is very relevant to understand how institutional features shape financial markets outcomes.

As a first thing I aim to investigate further the ongoing regulatory debate to tailor my research in a direction that offers answers to major regulatory issues. I aim to include in my model fundamentalist traders that are strategic in considering the impact of their order flow to analyze how this impacts my results.

I plan to ponder the idea of sustaining collusion and understand if, in this context, truthtelling is justifiable by an ongoing interaction or if the mechanisms to enforce collusion should be investigated further.

I aim to understand if I should focus my study of collusion on a specific setting, e.g. what happened in the FX market with the fix, or if such problem arises naturally in OTC market environments. In the latter case I would aim to keep my analysis in a more general framework.

Furthermore I think it could be relevant to isolate measures that provide an idea of the value of OTC market, especially considering clients' trading needs. The fact that such market structure persist despite widely recognized problems might indicate that clients still find them beneficial. It could be interesting to understand what is the balance in terms of costs and benefits.

# Bibliography

- [Admati and Pfleiderer, 1988] Admati, A. R. and Pfleiderer, P. (1988). A theory of intraday patterns: Volume and price variability. *Review of Financial studies*, 1(1):3–40.
- [Amihud and Mendelson, 1980] Amihud, Y. and Mendelson, H. (1980). Dealership market: Market-making with inventory. *Journal of Financial Economics*, 8(1):31–53.
- [Anand and Venkataraman, 2012] Anand, A. and Venkataraman, K. (2012). Should exchanges impose market maker obligations? *Available at SSRN 2179259*.
- [Bagehot, 1971] Bagehot, W. (1971). The only game in town. *Financial Analysts Journal*, 27:12–17.
- [Barker and Pomeranets, 2011] Barker, W. and Pomeranets, A. (2011). Financial system review. Technical report, Bank of Canada.
- [Benos and Wetherilt, 2012] Benos, E. and Wetherilt, A. (2012). The role of designated market makers in the new trading landscape. Technical report, Bank of England.
- [Bernhardt and Hughson, 1997] Bernhardt, D. and Hughson, E. (1997). Splitting orders. *The Review of Financial Studies*, 10(1):pp. 69–101.
- [Bernhardt and Taub, 2008] Bernhardt, D. and Taub, B. (2008). Front-running dynamics. *Journal of Economic Theory*, 138(1):288–296.
- [Biais et al., 2014] Biais, B., Foucault, T., et al. (2014). Hft and market quality. *Bankers, Markets & Investors*, 128:5–19.

- [Biais et al., 2015] Biais, B., Foucault, T., and Moinas, S. (2015). Equilibrium fast trading. *Journal of Financial Economics*, 116(2):292–313.
- [Biais and Germain, 2002] Biais, B. and Germain, L. (2002). Incentive-compatible contracts for the sale of information. *Review of Financial Studies*, 15(4):987–1003.
- [Biais et al., 2005] Biais, B., Glosten, L., and Spatt, C. (2005). Market microstructure: A survey of microfoundations, empirical results, and policy implications. *Journal of Financial Markets*, 8(2):217–264.
- [BIS, 2013] BIS (2013). Triennial central bank survey. Technical report, Bank for International Settlement.
- [Bjønnes et al., 2011] Bjønnes, G. H., Osler, C. L., and Rime, D. (2011). Sources of information advantage in the foreign exchange market. *Norges Bank*.
- [Boco et al., 2017] Boco, H., Germain, L., and Rousseau, F. (2017). High frequency trading: Strategic competition between slow and fast traders.
- [BoE, 2015] BoE (2015). Fair and effective market review. Technical report, Bank of England.
- [Bondarenko, 2001] Bondarenko, O. (2001). Competing market makers, liquidity provision, and bid-ask spreads. *Journal of Financial Markets*, 4(3):269–308.
- [Bondarenko and Sung, 2003] Bondarenko, O. and Sung, J. (2003). Specialist participation and limit orders. *Journal of Financial Markets*, 6(4):539–571.
- [Brogaard, 2010] Brogaard, J. (2010). High frequency trading and its impact on market quality. *SSRN Working Paper*.
- [Brogaard, 2012] Brogaard, J. (2012). High frequency trading and volatility. *Available at SSRN: <http://ssrn.com/abstract=1641387> or <http://dx.doi.org/10.2139/ssrn.1641387>*.
- [Cartea and Penalva, 2011] Cartea, Á. and Penalva, J. (May 30, 2011). Where is the value in high frequency trading? *Banco de Espana Working Paper No. 1111. Available at SSRN: <http://ssrn.com/abstract=1855555> or [doi:10.2139/ssrn.1855555](http://dx.doi.org/10.2139/ssrn.1855555)*.

- [Castura et al., 2010] Castura, J., Litzenberger, R., Gorelick, R., and Dwivedi, Y. (2010). Market efficiency and microstructure evolution in u.s. equity markets: a high frequency perspective. *working paper, RGM Advisors, LLC*.
- [CFTC and SEC, 2010] CFTC and SEC (2010). Findings regarding the market events of may 6, 2010. Technical report, SEC, CFTC.
- [Cheung and Wong, 2000] Cheung, Y.-W. and Wong, C. Y.-P. (2000). A survey of market practitioners’s views on exchange rate dynamics. *Journal of International Economics*, 51(2):401–419.
- [Chlistalla, 2011] Chlistalla, M. (2011). Hft, better than its reputation? Technical report, Deutsche Bank Research.
- [Cvitanic and Kirilenko, 2010] Cvitanic, J. and Kirilenko, A. A. (2010). High Frequency Traders and Asset Prices. *SSRN eLibrary*.
- [Dridi and Germain, 2001] Dridi, R. and Germain, L. (2001). Noise and competition in strategic oligopoly.
- [Duffie, 2012] Duffie, D. (2012). Market making under the proposed volcker rule.
- [Dumitrescu, 2005] Dumitrescu, A. (2005). Imperfect competition and market liquidity with a supply informed trader.
- [Dupont, 2000] Dupont, D. (2000). Market making, prices, and quantity limits. *The Review of Financial Studies*, 13(4):pp. 1129–1151.
- [Easley and O’Hara, 1987] Easley, D. and O’Hara, M. (1987). Price, trade size, and information in securities markets. *Journal of Financial Economics*, 19(1):69 – 90.
- [Euromoney, 2016] Euromoney (2016). Foreign exchange survey. Technical report, Eurmeney Institutional Investors.
- [Evans and Lyons, 2005] Evans, M. and Lyons, R. K. (2005). Meese-rogooff redux: Micro-based exchange-rate forecasting. *American Economic Review*, 95(2).

- [Evans, 2010] Evans, M. D. (2010). Order flows and the exchange rate disconnect puzzle. *Journal of International Economics*, 80(1):58–71.
- [Evans, 2014] Evans, M. D. (2014). Forex trading and the wmr fix. *Available at SSRN 2487991*.
- [Foucault et al., 2013] Foucault, T., Pagano, M., Roell, A., and Röell, A. (2013). *Market liquidity: theory, evidence, and policy*. Oxford University Press.
- [Frankel, 2012] Frankel, J. A. (2012). *Handbook of Exchange Rates*.
- [FSB, 2014] FSB (2014). Final report on foreign exchange benchmarks. Technical report, FSB.
- [Garman, 1976] Garman, M. B. (1976). Market microstructure. *Journal of financial Economics*, 3(3):257–275.
- [Glosten, 1989] Glosten, L. R. (1989). Insider trading, liquidity, and the role of the monopolist specialist. *The Journal of Business*, 62(2):pp. 211–235.
- [Glosten, 1994] Glosten, L. R. (1994). Is the electronic open limit order book inevitable? *The Journal of Finance*, 49(4):1127–1161.
- [Glosten and Milgrom, 1985a] Glosten, L. R. and Milgrom, P. R. (1985a). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 14(1):71 – 100.
- [Glosten and Milgrom, 1985b] Glosten, L. R. and Milgrom, P. R. (1985b). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of financial economics*, 14(1):71–100.
- [Gomber et al., 2011] Gomber, P., Arndt, B., Lutat, M., and Uhle, T. (2011). High-Frequency Trading. *SSRN eLibrary*.
- [Guilbaud and Pham, 2013] Guilbaud, F. and Pham, H. (2013). Optimal high-frequency trading with limit and market orders. *Quantitative Finance*, 13(1):79–94.
- [Hasbrouck and Saar, 2012] Hasbrouck, J. and Saar, G. (2012). Low-latency trading. *SSRN eLibrary*.

- [Hendershott et al., 2010] Hendershott, T., Jones, C., and Menkveld, A. (2010). Does algorithmic trading improve liquidity. *The journal of finance* [0022-1082], 66:1.
- [Hendershott et al., 2016] Hendershott, T., Li, D., Livdan, D., and Schürhoff, N. (2016). Relationship trading in otc markets. *Swiss Finance Institute, Working Paper*.
- [Hendershott and Madhavan, 2015] Hendershott, T. and Madhavan, A. (2015). Click or call? auction versus search in the over-the-counter market. *The Journal of Finance*, 70(1):419–447.
- [Ho and Stoll, 1981] Ho, T. and Stoll, H. R. (1981). Optimal dealer pricing under transactions and return uncertainty. *Journal of Financial economics*, 9(1):47–73.
- [Holden and Subrahmanyam, 1992] Holden, C. W. and Subrahmanyam, A. (1992). Long-lived private information and imperfect competition. *The Journal of Finance*, 47(1):247–270.
- [IOSCO, 2013] IOSCO (2013). Principle for financial benchmarks. Technical report, International Organization of Securities Commissions.
- [Jarrow and Protter, 2011] Jarrow, R. A. and Protter, P. (June 29, 2011). A dysfunctional role of high frequency trading in electronic markets. *Johnson School Research Paper Series No. 08-2011*. Available at SSRN: <http://ssrn.com/abstract=1781124> or [doi:10.2139/ssrn.1781124](https://doi.org/10.2139/ssrn.1781124).
- [Jones, 2013] Jones, C. M. (2013). What do we know about high-frequency trading?
- [Karpoff and Lee, 1991] Karpoff, J. M. and Lee, D. (1991). Insider trading before new issue announcements. *Financial Management*, pages 18–26.
- [Kavajecz, 1996] Kavajecz, K. A. (1996). A specialist’s quoted depth as a strategic choice variable. *Working Paper Series Available at SSRN: <http://ssrn.com/abstract=7743>*.
- [Kedia and Zhou, 2014] Kedia, S. and Zhou, X. (2014). Informed trading around acquisitions: Evidence from corporate bonds. *Journal of Financial Markets*, 18:182–205.
- [King et al., 2010] King, M., Sarno, L., and Sojli, E. (2010). Timing exchange rates using order flow: The case of the loonie. *Journal of Banking & Finance*, 34(12):2917–2928.
- [King et al., 2011] King, M. R., Osler, L., and Rime, D. (2011). Foreign exchange market structure, players and evolution. Technical report, Norges Bank.

- [Kirilenko et al., ] Kirilenko, A., Samadi, M., Kyle, A., and Tuzun, T. The flash crash: The impact of high frequency trading on an electronic market. *working paper*. Available at: <http://ssrn.com/abstract=1686004>.
- [Kyle, 1985a] Kyle, A. S. (1985a). Continuous auctions and insider trading. *Econometrica*, 53(6):pp. 1315–1335.
- [Kyle, 1985b] Kyle, A. S. (1985b). Continuous auctions and insider trading. *Econometrica*, 53(6):1315–1335.
- [Kyle, 1989] Kyle, A. S. (1989). Informed speculation with imperfect competition. *The Review of Economic Studies*, 56(3):317–355.
- [Lyons, 1997] Lyons, R. K. (1997). A simultaneous trade model of the foreign exchange hot potato. *Journal of International Economics*, 42(3-4):275–298.
- [Lyons, 2001] Lyons, R. K. (2001). *The microstructure approach to exchange rates*. Citeseer.
- [Meese and Rogoff, 1983] Meese, R. A. and Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of international economics*, 14(1):3–24.
- [Melvin and Prins, 2015] Melvin, M. and Prins, J. (2015). Equity hedging and exchange rates at the london 4p. m. fix. *Journal of Financial Markets*, 22:50–72.
- [Menkhoff et al., 2016] Menkhoff, L., Sarno, L., Schmeling, M., and Schrimpf, A. (2016). Information flows in foreign exchange markets: Dissecting customer currency trades. *The Journal of Finance*, 71(2):601–634.
- [Menkveld, 2011] Menkveld, A. (2011). High frequency trading and the new-market makers.
- [Michelberger and Witte, 2016] Michelberger, P. S. and Witte, J. H. (2016). Foreign exchange market microstructure and the wm/reuters 4 pm fix. *The Journal of Finance and Data Science*.
- [Naik et al., 1999] Naik, N. Y., Neuberger, A., and Viswanathan, S. (1999). Trade disclosure regulation in markets with negotiated trades. *Review of Financial Studies*, 12(4):873–900.

- [O'Hara, 2015] O'Hara, M. (2015). High frequency market microstructure. *Journal of Financial Economics*, 116(2):257–270.
- [O'Hara and Oldfield, 1986] O'Hara, M. and Oldfield, G. (1986). The microeconomics of market making. *Journal of Financial and Quantitative Analysis*, 21(4):361–376.
- [Osler et al., 2016] Osler, C. et al. (2016). Dealer trading at the fix.
- [Osler and Vandrovych, 2009] Osler, C. L. and Vandrovych, V. (2009). Hedge funds and the origins of private information in currency markets.
- [Rime and Schrimpf, 2013] Rime, D. and Schrimpf, A. (2013). The anatomy of the global fx market through the lens of the 2013 triennial survey. *BIS Quarterly Review, December*.
- [Rochet and Vila, 1994] Rochet, J.-C. and Vila, J.-L. (1994). Insider trading without normality. *The Review of Economic Studies*, 61(1):131–152.
- [Röell, 1990] Röell, A. (1990). Dual-capacity trading and the quality of the market. *Journal of Financial intermediation*, 1(2):105–124.
- [Sager and Taylor, 2006] Sager, M. J. and Taylor, M. P. (2006). Under the microscope: the structure of the foreign exchange market. *International Journal of Finance & Economics*, 11(1):81–95.
- [Seppi, 1997] Seppi, D. J. (1997). Liquidity provision with limit orders and strategic specialist. *The Review of Financial Studies*, 10(1):pp. 103–150.
- [Smith, 2010] Smith, R. (2010). Is High-Frequency Trading Inducing Changes in Market Microstructure and Dynamics? *SSRN eLibrary*.
- [Stoll, 1978] Stoll, H. R. (1978). The supply of dealer services in securities markets. *The Journal of Finance*, 33(4):1133–1151.
- [Subrahmanyam, 1991] Subrahmanyam, A. (1991). Risk aversion, market liquidity, and price efficiency. *Review of Financial Studies*, 4(3):417–441.
- [Tradeworx, 2010] Tradeworx (2010). Public commentary on sec market structure concept release. Technical report, Tradeworx, Inc.

[Werner, 1997] Werner, I. M. (1997). A double auction model of interdealer trading.

## 4.6 Appendices

### 4.6.1 Appendix A

#### Proof of Proposition 1.

Before proceeding to the proof notice that the slow market maker is the first one posting his bid and ask quotes, so he has to forecast the strategic response of the other player. The risk averse market maker observes the bid ask quotes of the other market maker, but he has a higher cost due to risk aversion. The risk aversion component has a negative effect on his objective function, decreasing his expected payoff. Since the buy and sell side of the market are mutually exclusive (they lead to different payoffs and they cannot happen simultaneously), the optimal strategy for the bid and for the ask can be treated separately. Moreover since the problem is symmetric, the result will be symmetric too.

As a first step I identify the thresholds that can be adopted as a pricing strategy for the ask side (the problem of the bid is symmetric).

For the high frequency traders, there are three prices that serve as a benchmark. Those are:

- The ask price at which H is indifferent between trading  $\frac{n}{2}$  and not trading:

$$T_1 = E(V) + \frac{\tau}{4}nVar(V),$$

This is because his objective function is:

$$(a_1 - E(V))\frac{n}{2} - \frac{\tau}{8}n^2Var(V) = 0$$

hence solving for  $a_1$  the zero profit ask is found.

- Ask price at which H is indifferent between trading  $n$  and not trading:

$$T_2 = E(V) + \frac{\tau}{2}nVar(V)$$

This is because:

$$(a_2 - E(V))n - \frac{\tau}{2}(n)^2Var(V) = 0$$

and solving for  $a_2$  the zero profit ask is found.

- Ask price at which H is indifferent between trading  $n$  and trading  $\frac{n}{2}$ :

$$T_3 = E(V) + \frac{3}{4}\tau n \text{Var}(V)$$

This is because:

$$(a_1 - E(V))n - \frac{\tau}{2}n^2 \text{Var}(V) = (a_1 - E(V))\frac{n}{2} - \frac{\tau}{8}n^2 \text{Var}(V)$$

That implies his profit is the same in the case in which he trades  $n$  and  $\frac{n}{2}$ .

Clearly  $T_1 < T_2 < T_3$ .

Now the best response of H depends on the spread range and on where the evaluation of the liquidity trader lies. The following are the best strategy depending on both spread range and reservation values:

- If  $a_1 \leq \bar{P}$ :

$$\left\{ \begin{array}{ll} \text{if } a_1 \geq T_3 & \text{H has the incentive to undercut M1} \\ \text{if } T_1 \leq a_1 \leq T_3 & \text{H will set } a_2 = a_1 \\ \text{if } a_1 \leq T_1 & \text{H will be out of the market} \end{array} \right.$$

- If  $a_1 > \bar{P}$ , then:

$$\left\{ \begin{array}{ll} \text{if } \bar{P} \geq T_2 & \text{then H will set } a_2 = \bar{P} \\ \text{if } \bar{P} < T_2 & \text{H will not trade} \end{array} \right.$$

Now finding M1 best strategy is trivial, considering that he has a full information set and he knows what his profit will be depending on the spread range:

- If  $\bar{P} \geq T_3$ , then the most profitable strategy for M1 is to set  $a_1 = T_3$  and the best strategy for H is to set  $a_1 = a_2$ . In this case they will share the market at  $a_1 = a_2 = T_3$ .

- If  $T2 < \bar{P} < T3$ , then the most profitable strategy for M1 is to set  $a_1 = \bar{P}$  and for H the best thing to do is to set  $a_1 = a_2$ . Also in this case the two market makers will share the market at  $a_1 = a_2 = \bar{P}$ .
- If  $\bar{P} = T2$ , there are two possible equilibria, since M1 is indifferent in his expected payoff if he sets  $a_1 = T2$  and shares the market, or if he sets  $a_1 = T1$  and serves the market on his own. In the first case it happens that  $a_1 = a_2 = T2 = \bar{P}$  and they will share the market, in the second case it happens that  $a_1 = T1$  and M1 serves the market alone.
- If  $T1 \leq \bar{P} < T2$ , then M1 sets  $a_1 = T1$  and the best strategy for H is to stay out of the market.
- If  $\bar{P} < T1$ , then M1 sets  $a_1 = \bar{P}$  and H is out of the market.

#### 4.6.2 Appendix B

##### Proof of Proposition 2

##### H, mean variance utility

Overall there are 27 possible strategies available to H. This is because for each trading round he has the option to push out MM1 ( $a_2^{1,2} < a_1, b_2^2 > b_1$ ), to share the market with him ( $a_2^{1,2} = a_1, b_2^2 = b_1$ ) or to leave the market ( $a_2^{1,2} > a_1, b_2^2 < b_1$ ).

Assuming a buy order hits the market first, this will give him 3 variable to choose, that are  $a_2^1, a_2^2, b_2^2$ .

One convenient way to proceed, in order to restrict the domain of strategies, is to compute H mean variance utility (depending on the strategy he pursues) and rule out dominated strategies. H is assumed risk averse. Recall his objective:

$$U^H = E(\pi) - \frac{\tau}{2} Var(\pi) \quad (4.8)$$

To compute his mean variance utility considering all the possible cases, I will assume the first order reaching the market is a buy order (the situation would be symmetric for a sell order).

I will consider first all the possible payoffs he will get if serving the whole market (WM) in the

first round.

Each strategy is denoted by a triplet where the first term indicate the course of action in the first round, the second the course of action in the second round on the same side of the market (e.g. buy-buy) and the third the course of action in the second round on the opposite side of the market (e.g. buy-sell).

**Computation of H payoff assuming the whole market is served in the first round and considering the full strategy space for the second round**

If the HFT decided to serve the whole market in each trading round, on each side of the market his payoff structure is given by:

$$\begin{cases} 2(a - V) & \text{with } P = 1/2 \\ (a - b) & \text{with } P = 1/2 \end{cases}$$

The previous two terms comes from the fact that he can receive both a buy-buy order combination and a buy-sell order combination.

In this case his utility will be:

$$U_{(WM,WM,WM)} = \left( \frac{3}{2}a - \frac{b}{2} - E(V) \right) - \frac{\tau}{2} \left[ 2Var(V) + \left( \frac{a}{2} + \frac{b}{2} - E(V) \right)^2 \right]$$

Assuming that the form of the spread is given by:

$$\begin{cases} a = E(V) + c \\ b = E(V) - c \end{cases}$$

and substituting in the previous:

$$U_{(WM,WM,WM)} = 2c - \tau Var(V)$$

Using exactly the same procedure the utility associated with any possible course of action in round 2 is computed, assuming he has served the whole market in round 1. The following

formulas indicate the utility deriving from the available strategies, considering first the case where he serves the whole market in the first round, afterwards the case in which he serves half of the market in the first round and last the case in which he serves no market in the first round.

- WM,WM,WM:

$$U_{(WM,WM,WM)} = 2c - \tau Var(V) \quad (1.1)$$

- WM,NM,WM:

$$U_{(WM,NM,WM)} = \frac{1}{8} (12c - 2\tau Var(V) - \tau c^2) \quad (1.2)$$

- WM,NM,NM:

$$U_{(WM,NM,NM)} = c - \frac{\tau}{2} Var(V) \quad (1.3)$$

- WM,WM,NM:

$$U_{(WM,WM,NM)} = \frac{1}{8} (12c - 10\tau Var(V) - \tau c^2) \quad (1.4)$$

- WM,WM,HM:

$$U_{(WM,WM,HM)} = \frac{1}{32} (56c - 34\tau Var(V) - \tau c^2) \quad (1.5)$$

- WM,HM,WM:

$$U_{(WM,HM,WM)} = \frac{1}{32} (56c - 18\tau Var(V) - \tau c^2) \quad (1.6)$$

- WM,HM,NM:

$$U_{(WM,HM,NM)} = \frac{5}{4}c - \frac{25\tau}{32} Var(V) \quad (1.7)$$

- WM,HM,HM:

$$U_{(WM,HM,HM)} = \frac{3}{2}c - \frac{5\tau}{8} Var(V) \quad (1.8)$$

- WM,NM,HM:

$$U_{(WM,NM,HM)} = \frac{1}{32} (40c - 10\tau Var(V) - \tau c^2) \quad (1.9)$$

**Computation of HFT payoff assuming half market is served in the first round and considering the full strategy space for the second round**

As in the previous case I can calculate the utility of the HFT if he decides to serve half the market in the first round:

- HM,WM,WM:

$$U_{(HM,WM,WM)} = \frac{3}{2}c - \frac{5}{8}\tau Var(V) \quad (2.1)$$

- HM,NM,WM:

$$U_{(HM,NM,WM)} = \frac{1}{8}(8c - \tau Var(V) - \tau c^2) \quad (2.2)$$

- HM,NM,NM:

$$U_{(HM,NM,NM)} = \frac{c}{2} - \frac{\tau}{8}Var(V) \quad (2.3)$$

- HM,WM,NM:

$$U_{(HM,WM,NM)} = \frac{1}{8}(8c - 5\tau Var(V) - \tau c^2) \quad (2.4)$$

- HM,WM,HM:

$$U_{(HM,WM,HM)} = \frac{1}{32}(40c - 18\tau Var(V) - \tau c^2) \quad (2.5)$$

- HM,HM,WM:

$$U_{(HM,HM,WM)} = \frac{1}{32}(40c - 10\tau Var(V) - \tau c^2) \quad (2.6)$$

- HM,HM,NM:

$$U_{(HM,HM,NM)} = \frac{1}{32}(24c - 10\tau Var(V) - \tau c^2) \quad (2.7)$$

- HM,HM,HM:

$$U_{(HM,HM,HM)} = c - \frac{\tau}{4}Var(V) \quad (2.8)$$

- HM,NM,HM:

$$U_{(HM,NM,HM)} = \frac{1}{32}(24c - 2\tau Var(V) - \tau c^2) \quad (2.9)$$

**Computation of HFT payoff assuming no market is served in the first round and considering the full strategy space for the second round**

As in the previous case I can calculate the utility of the HFT if he decides to serve half the market in the first round:

- NM,WM,WM:

$$U_{(NM,WM,WM)} = c - \frac{\tau}{2}Var(V) \quad (3.1)$$

- NM,NM,WM:

$$U_{(NM,NM,WM)} = \frac{1}{8} (4c - 2\tau Var(V) - \tau c^2) \quad (3.2)$$

- NM,NM,NM:

$$U_{(NM,NM,NM)} = 0 \quad (3.3)$$

- NM,WM,NM:

$$U_{(NM,WM,NM)} = \frac{1}{8} (4c - 2\tau Var(V) - \tau c^2) \quad (3.4)$$

- NM,WM,WM:

$$U_{(NM,WM,WM)} = \frac{1}{32} (24c - 10\tau Var(V) - \tau c^2) \quad (3.5)$$

- NM,WM,WM:

$$U_{(NM,WM,WM)} = \frac{1}{32} (24c - 10\tau Var(V) - \tau c^2) \quad (3.6)$$

- NM,WM,WM:

$$U_{(NM,WM,WM)} = \frac{1}{32} (8c - 2\tau Var(V) - \tau c^2) \quad (3.7)$$

- NM,WM,WM:

$$U_{(NM,WM,WM)} = \frac{c}{2} - \frac{\tau}{8}Var(V) \quad (3.8)$$

- NM,WM,WM:

$$U_{(NM,WM,WM)} = \frac{1}{32} (8c - 2\tau Var(V) - \tau c^2) \quad (3.9)$$

## Iterated deletion of Dominated strategies, HFT

Once all the payoff from the strategy space are defined I can proceed to iterated deletion of dominated strategies. To do so I first group the strategies keeping fixed the choice in the first round, and afterwards allow for comparison between the surviving strategies. Some strategies can immediately be ruled out as they are dominated by other.

- In case he serves the WM in the first round.

The strategy (WM,NM,NM), (1.3), is dominated by (2.8).

The strategy (WM,WM,WM), (1.5) is dominated by (WM,WM,WM) (1.6). The strategy (WM,WM,NM), (1.4), is dominated by (1.2). The strategy (WM,WM,NM), (1.7) is dominated by (NM,NM,NM) for  $c < 0$  and by (HM,WM,WM), (2.1), for  $c > 0$ . Moreover strategy (WM,WM,WM), (1.8), is indifferent to strategy (HM,WM,WM), (2.1), and (WM,NM,WM) is indifferent to the strategy (HM,WM,WM), (2.6).

- In case he serves the HM in the first round.

The strategy (HM,WM,WM), (2.1), is dominated by (HM,WM,WM), (2.8) and by (WM,WM,WM), (1.1). The strategy (HM,NM,NM) is dominated by (NM,NM,NM), (3.3), if  $c < \frac{\tau}{4}$  and by (HM,WM,WM) if  $c > \frac{\tau}{4}$ . The strategy (HM,WM,NM) (2.7) is dominated by (WM,NM,WM) (1.9).

The strategy (HM,WM,WM) (2.5) is dominated by (HM,WM,WM) (2.6).

The strategy (HM,WM,NM) (2.5) is dominated by (HM,NM,WM) (2.2).

- In case he serves no market in the first round it is easy to show that all the strategies are dominated except for the strategy (NM,NM,NM), that guarantees a zero utility even when the other strategies give a negative utility.

Overall there are 8 strategies left over. They are:

$$U_{(WM,WM,WM)} = 2c - \tau Var(V) \quad (1.1)$$

$$U_{(WM,NM,WM)} = \frac{1}{8} (12c - 2\tau Var(V) - \tau c^2) \quad (1.2)$$

$$U_{(WM,HM,WM)} = \frac{1}{16} (28c - 9\tau Var(V) - 2\tau c^2) \quad (1.6)$$

$$U_{(HM,HM,HM)} = c - \frac{\tau}{4} Var(V) \quad (2.8)$$

$$U_{(WM,NM,HM)(HM,HM,WM)} = \frac{1}{32} (40c - 10\tau Var(V) - \tau c^2) \quad (1.9)$$

$$U_{(HM,NM,HM)} = \frac{1}{32} (24c - 2\tau Var(V) - \tau c^2) \quad (2.9)$$

$$U_{(HM,NM,WM)} = \frac{1}{8} (8c - \tau Var(V) - \tau c^2) \quad (2.2)$$

Once dominated strategies are ruled out, the strategies left can be compared in pairs or triplets to obtain some thresholds that are relevant for the equilibria. Defining  $c$  as the half spread<sup>3</sup>, depending on the level of  $c$   $H$  has various possibilities.

Now some additional strategies can be ruled out considering that the  $H$  problem is a two period problem and the terminal values provide some conditions that restrict further the strategy set. The first thing to notice is that  $U_{(HM,HM,HM)}$ , (2.8) is more convenient than  $U_{(HM,NM,HM)}$ , (2.9) only when  $c > \frac{3}{4}\tau Var(V)$ <sup>4</sup>.

It is also easy to notice that the strategy  $U_{(HM,NM,HM)}$ , (2.9) is more profitable than the strategy  $U_{(HM,NM,WM)}$ , (2.2) for  $c < \frac{\tau}{4} Var(V)$ . This consideration implies that the strategy  $U_{(HM,NM,WM)}$  can be ruled out completely.

Now considering  $U_{(WM,NM,WM)}$ , (1.2),  $U_{(WM,WM,WM)}$ , (1.1) and  $U_{(WM,HM,WM)}$ , (1.6) it is possible to show that:

$$\left\{ \begin{array}{ll} U_{(WM,WM,WM)} & \text{is the pref strategy is } c > \frac{7}{4}\tau Var(V); \\ U_{(WM,HM,WM)} & \text{is the pref strategy is } \frac{5}{4}\tau Var(V) < c < \frac{7}{4}\tau Var(V); \\ U_{(WM,NM,WM)} & \text{is the pref strategy is } c < \frac{5}{4}\tau Var(V); \end{array} \right.$$

---

<sup>3</sup>This is because the bid and ask quotes are symmetric and to take the form :

$$\left\{ \begin{array}{l} a = E(V) + c \\ b = E(V) - c \end{array} \right.$$

<sup>4</sup>To prove this is sufficient to check the terminal conditions

Considering the previous points, it is possible to order the strategies left,

There is one interval of spread where  $\frac{\tau}{2}Var(V) < c < \frac{\tau}{2}Var(V)$ , in which H has more than one strategy that is feasible and his choice will depend on the level of the variance of the security.

For  $c = \frac{\tau}{2}Var(V)$ , he will choose to play the strategy  $U_{(WM,NM,WM)}$  instead of  $U_{(HM,NM,HM)}$  if the variance of the stock is below  $\frac{8}{\tau^2}$ . This comes from the fact that for low levels of variance H is more willing to take higher exposure in the market hence, he will prefer to serve the whole market in the first round, while if the variance increases he will be less willing to hold inventory, preferring to share the market. It is trivial to prove that the strategy  $U_{(HM,NM,HM)}$  is always preferred.

### 4.6.3 Appendix C

Consider that if the high frequency traders serves no market he gets zero profit. I have already shown that all the strategies involving serving no market in the first round are dominated except for withdrawing totally from the market. Hence I can exclude the possibility that the high frequency traders will be inactive in the first round and then start to provide liquidity. Moreover it has been shown that the strategy of serving the whole market on the same side is always dominated by serving the whole market on opposite sides, hence I can exclude such strategy too. The strategies I am left with are:  $U_{WM,NM,NM}$  ,  $U_{WM,WM,WM}$  ,  $U_{NM,NM,NM}$  ,  $U_{WM,NM,WM}$ .

If the high frequency traders decides to serve the market only in the first round he gets:

$$U_{WM,NM,NM} = c - \frac{\tau}{2}Var(V)$$

This strategy is preferred to being out of the market ( $U_{WM,NM,NM} > 0$ ) if:

$$c > \frac{\tau}{2}Var(V)$$

If he serves the whole market in each round for any order he gets:

$$U_{WM,WM,WM} = 2c - \tau Var(V)$$

This strategy is preferred to being out of the market ( $U_{WM,WM,WM} > 0$ ) if :

$$c > \frac{\tau}{2} Var(V)$$

Overall  $U_{WM,WM,WM} > U_{WM,NM,NM}$  if

$$c > \frac{\tau}{2} Var(V)$$

and being out of the market is the preferred choice if

$$c < \frac{\tau}{2} Var(V)$$

The previous shows that for  $c > \frac{\tau}{2} Var(V)$ , the strategy of being out of the market and being active only in the first round are dominated by serving the whole market in each round. Vice versa if  $c < \frac{\tau}{2} Var(V)$ , the strategy of being out of the market dominates the rest. Overall if  $c < \frac{\tau}{2} Var(V)$ , he prefers to be out of the market and if  $c > \frac{\tau}{2} Var(V)$  he prefers to serve the whole market. Now I need to compare these two strategies with the strategy of serving only opposite sides of the market.

If he serves the whole market in the first round and no market on the same side and whole market on the opposite side, he gets:

$$\frac{1}{8}(12c - 2\tau Var(V) - \tau c^2)$$

Comparing the last strategy with the two previous ones if find that  $U_{WM,WM,WM} = U_{WM,NM,WM}$  if  $c = +\frac{14+\sqrt{196-4\tau^2 Var(V)}}{2\tau}$  and  $U_{WM,NM,WM} = 0$  if  $c = +\frac{12+\sqrt{144-8\tau^2 Var(V)}}{2\tau}$ .

#### 4.6.4 Appendix D

##### Derivations benchmark case, regime 1

In this subsection I show the complete derivation of the equilibrium described in the previous section.

##### Informed trader demand

Each value informed trader has exponential utility:

$$-\exp\{-\rho_I(v-p)x_i\}$$

with risk aversion parameter  $\rho_I$  and risk tolerance  $r_I$ , assumed constant across traders.

Due to the fact that all the variables are normally distributed the maximization problem of each trader reduces to the maximization of:

Solving the previous I obtain:

$$x_i = \frac{E[v|\theta] - p}{\rho_I[Var v|\theta]}$$

Each trader observes  $\theta$ , but not  $\varepsilon$ . Moreover  $\theta$  is the same across traders, and this implies that value informed traders do not update their signals on price. Knowing that  $v|\theta \sim N(\theta, 1/\tau_\varepsilon)$ , the value informed traders maximization problem can be explicated as:

$$x_i = (\theta - p)r_I\tau_\varepsilon$$

All the value informed traders observe the same signal and their risk aversion is the same, hence  $x_i = x$  and  $\sum_{i=1}^N x_i = Nx = N(\hat{\theta} - p)r_I\tau_\varepsilon$

##### Updating of parameters

Dealers do not observe any information about the fundamental, hence they infer information from price. This occurs updating their expectation according to the variables distribution. In order to derive dealers' updated expectations I proceed as follows.

**Price from market clearing:** Using market clearing and knowing (4.5) I find price explicitly. In this benchmark case dealers do not observe any component of noise order flow, hence  $\eta_U = 0$ . I impose market clearing:

$$\sum_{n=1}^N X_{I,n}(v) + \sum_{j=1}^2 X_{,j}(p, \Omega_j^r) + \sum_{k=1}^3 \varphi_k = 0$$

and proceed substituting the optimal demand for value informed traders derived previously:

$$N(\theta - p)r_I\tau_\varepsilon + \sum_{j=1}^2 X_{,j}(p, \Omega_j^r) + \varphi_1 + \varphi_2 + \varphi_3 = 0$$

I use conjecture (4.4) to rewrite the previous as:

$$N\theta r_I\tau_\varepsilon - pr_I\tau_\varepsilon N + 2\alpha_U - 2\beta_U p + \varphi_1 + \varphi_2 + \varphi_3 = 0$$

After some algebra, the price can be explicitly written as:

$$p = \Delta (N\theta r_I\tau_\varepsilon + 2\alpha_U + \varphi_1 + \varphi_2 + \varphi_3) \quad (4.9)$$

where

$$\Delta = \frac{1}{Nr_I\tau_\varepsilon + 2\beta_U} \quad (4.10)$$

$\Delta^{-1}$  is a measure of market depth (price change if there is an increase in noise traders order flow).

**Updated parameters:** Now, in order to solve dealers' updated expectations, I rearrange separating observables and unobservables, aiming to find the signal that uninformed gather from price. I obtain :

$$\tilde{h} = \frac{1}{Nr_I\tau_\varepsilon} (p\Delta^{-1} - 2\alpha_U) = \theta + \frac{\varphi_1}{Nr_I\tau_\varepsilon} + \frac{\varphi_2}{Nr_I\tau_\varepsilon} + \frac{\varphi_3}{Nr_I\tau_\varepsilon}$$

and, using the fact that variables are normally distributed, I obtain:

$$\tilde{h}|\theta \sim N\left(\theta, \left(\frac{1}{Nr_I\tau_\varepsilon}\right)^2 \left(\frac{1}{\tau_{\varphi_1}} + \frac{1}{\tau_{\varphi_2}} + \frac{1}{\tau_{\varphi_3}}\right)\right)$$

. Hence knowing that  $Var(\theta | \tilde{h})^{-1} = Var(\theta | p)^{-1}$ , the updated precision  $\tau$  can be written as a weighted average of the prior precision and the precision of the signal:

$$\tau = \tau_\theta + \frac{(\tau_{\varphi_1} \tau_{\varphi_2} \tau_{\varphi_3})}{(\tau_{\varphi_2} \tau_{\varphi_3} + \tau_{\varphi_1} \tau_{\varphi_3} + \tau_{\varphi_2} \tau_{\varphi_1})} (Nr_I \tau_\varepsilon)^2$$

Exploiting the fact that  $E(\theta | \tilde{h}) = E(\theta | \tilde{p})$ , I obtain:

$$E(\theta | \tilde{p}) = \frac{\bar{\theta} \tau_\theta}{\tau} + Nr_I \tau_\varepsilon (p \Delta^{-1} - 2\alpha_U) \frac{(\tau_{\varphi_1} \tau_{\varphi_2} \tau_{\varphi_3})}{\tau(\tau_{\varphi_2} \tau_{\varphi_3} + \tau_{\varphi_1} \tau_{\varphi_3} + \tau_{\varphi_2} \tau_{\varphi_1})} \quad (4.11)$$

and, substituting:  $R = \frac{(\tau_{\varphi_1} \tau_{\varphi_2} \tau_{\varphi_3})}{(\tau_{\varphi_2} \tau_{\varphi_3} + \tau_{\varphi_1} \tau_{\varphi_3} + \tau_{\varphi_2} \tau_{\varphi_1})}$  and  $\gamma = Nr_I \tau_\varepsilon$ , the previous is written more compactly as:

$$E(\theta | \tilde{p}) = \frac{\bar{\theta} \tau_\theta - 2\alpha_U R \gamma}{\tau} + \frac{p \Delta^{-1} R \gamma}{\tau} \quad (4.12)$$

## Dealers' demand

In order to derive dealers' demand I proceed in two possible ways, obtaining the same result.

In the first case I derive each dealer's demand, taking the demand of the rest of the market given as for equilibrium, and proceed to solve the dealer maximization problem. I show that I need the conjectured demand is correct in equilibrium.

The second approach is based on the conjecture that each dealer has a self sustaining belief that he is facing a downward sloping demand curve. Starting from conjecturing the price form and the demand form I prove that conjectured and demand are indeed correct in equilibrium.

**Dealers' demand Method 1 Equilibrium demand:** Dealer  $i$ , in equilibrium, assumes that the demand of the other players is given by (4.5) for the value informed traders and (4.4) for the other dealer. This allows me to rewrite the price from market clearing as<sup>5</sup>:

$$p = \underbrace{\frac{Nr_I \tau_\varepsilon + \alpha_U + \varphi_1 + \varphi_2 + \varphi_3}{Nr_I \tau_\varepsilon + \beta_U}}_a + \underbrace{\frac{X_i}{Nr_I \tau_\varepsilon + \beta_U}}_b \quad (4.13)$$

---

<sup>5</sup>Note that such function in the related literature is also defined as residual demand faced by trader  $i$ .

Notice that equation (4.13) distinguishes two components of the price function.  $a$  represents the impact of other traders' demand on price, while  $b$  isolates the impact of trader's  $i$  demand  $X_i$  on price. I define

$$\lambda_U = \frac{1}{Nr_I\tau_\varepsilon + \beta_U}$$

as the price impact of each dealer in this market, that is how responsive price will be to a unitary increase in demand from each of the dealers.

**Maximization problem:** Using (4.13) I write the maximization problem as:

$$\max_{X_j} \left[ E(\theta | \tilde{p}) - \frac{Nr_I\tau_\varepsilon + \alpha_U + \varphi_1 + \varphi_2 + \varphi_3}{Nr_I\tau_\varepsilon + \beta_U} - \frac{X_i}{Nr_I\tau_\varepsilon + \beta_U} \right] X_i$$

that, taking the foc and a few algebraic, manipulations implies:

$$X_i = (Nr_I\tau_\varepsilon + \beta_U)(E(\theta | \tilde{p}) - p) \quad (4.14)$$

where  $Nr_I\tau_\varepsilon + \beta_U \neq 0$  and the soc:

$$-\frac{2}{Nr_I\tau_\varepsilon + \beta_U} < 0$$

implies  $Nr_I\tau_\varepsilon + \beta_U > 0$ .

**Parameters' Derivation:** Once I obtain the dealer demand I need to derive the coefficient to reconcile the conjectured demand with (4.14). To derive the demand coefficients explicitly I plug (4.12) in (4.14) and obtain:

$$X_i = (\gamma + \beta_U) \left[ \left( \frac{\theta\tau_\theta - 2\alpha_U R\gamma}{\tau} \right) - p \left( \frac{\tau - (\gamma + 2\beta_U)R\gamma}{\tau} \right) \right]$$

Hence the coefficients solve the following system of equations:

$$\begin{cases} 2\beta_U^2 R + 3\beta_U R\gamma + R\gamma^2 - \tau = 0 \\ (\tau + 2R\gamma(\gamma + \beta_U))\alpha_U = (\gamma + \beta_U)\theta\tau_\theta \end{cases}$$

Where the explicit solution is calculated as:

$$\begin{aligned}\beta_U &= \frac{(\gamma^2 + 8\tau R^{-1})^{1/2} - 3\gamma}{4} \\ \alpha_U &= \frac{[\gamma + (\gamma^2 + 8\tau R^{-1})^{1/2}] \theta \tau_\theta}{4\tau + 2R\gamma [\gamma + (\gamma^2 + 8\tau R^{-1})^{1/2}]}\end{aligned}$$

And, dealers demand is given by:

$$X_i = \frac{[\gamma + (\gamma^2 + 8\tau R^{-1})^{1/2}] \theta \tau_\theta}{4\tau + 2R\gamma [\gamma + (\gamma^2 + 8\tau R^{-1})^{1/2}]} - p \frac{(\gamma^2 + 8\tau R^{-1})^{1/2} - 3\gamma}{4}$$

Price impact can now be written as:

$$\lambda_U = \frac{4}{Nr_I \tau_\varepsilon + ((Nr_I \tau_\varepsilon)^2 + 8\tau R^{-1})^{1/2}}$$

**Dealers' demand method 2 Conjecture of price form:** Assume that each uninformed trader believes that his demand is related to the price through the expression:

$$p = p_U + \lambda_U X_j \tag{4.15}$$

where  $p_U$  is a term that incorporates all elements of price that are not related to an investor's demand and  $\lambda$  is a nonnegative coefficient.

**Maximization problem:** The problem of the dealer can be written as:

$$\max_{X_j} [E(\theta | \tilde{p}) - p] X_j$$

Using (4.15) the max pb is:

$$\max_{X_j} [E(\theta | \tilde{p}) - p_U + \lambda_U X_j] X_j$$

where the foc and few manipulations exploiting (4.15) provide the following result:

$$X_j = \frac{E(\theta | \tilde{p}) - p_U}{\lambda_U} \tag{4.16}$$

**Reconciliation problem:** To reconcile market clearing with the price assumption in (4.15), it can be shown that  $p_U$  should take the form:

$$p_U = \lambda_U (N\theta r_I \tau_\varepsilon + \alpha_U + \varphi_1 + \varphi_2 + \varphi_3)$$

In fact, from this assumption we find

$$p = \frac{\lambda_U}{1 + \beta_U \lambda_U} (N\theta r_I \tau_\varepsilon + 2\alpha_U + \varphi_1 + \varphi_2 + \varphi_3)$$

and to reconcile such formula with market clearing we should have:

$$\frac{\lambda_U}{1 + \beta_U \lambda_U} = \frac{1}{Nr_I \tau_\varepsilon + 2\beta_U}$$

that leads to

$$\lambda_U = \frac{1}{Nr_I \tau_\varepsilon + \beta_U}$$

and demand can be written as:

$$X_j = (Nr_I \tau_\varepsilon + \beta_U)(E(\theta | \tilde{p}) - p) \quad (4.17)$$

**Parameters' Derivation:** To solve for the demand parameters I substitute 4.12 in the demand 4.17 to obtain:

$$X_i = (\gamma + \beta_U) \left[ \left( \frac{\theta \tau_\theta - 2\alpha_U R \gamma}{\tau} \right) - p \left( \frac{\tau - (\gamma + 2\beta_U) R \gamma}{\tau} \right) \right]$$

The solution is identical to the one obtained with the other method of derivation. Derivation of parameters is identical to the previous section.

#### **Expected profits conditional on the observed order flow**

The expected profits of dealers can be written as:

$$E(\pi) = E[(\theta - p)x] \quad (4.18)$$

In equilibrium price is given by

$$p = \Delta (N\theta r_I \tau_\varepsilon + 2\alpha_U + \varphi_1 + \varphi_2 + \varphi_3) \quad (4.19)$$

and quantity is given by:

$$x = \alpha_U - \beta_U p$$

In order to solve 4.18 I plug the explicit function for quantity and price. I have:

$$\begin{aligned} E(\pi) &= E[(\theta - (\Delta\gamma\theta + 2\alpha_U\Delta + \Delta(\varphi_1 + \varphi_2 + \varphi_3))(\alpha_U - \beta_U(\Delta\gamma\theta + 2\alpha_U\Delta + \Delta(\varphi_1 + \varphi_2 + \varphi_3))))] = \\ &= E[\alpha_U(\theta - (\Delta\gamma\theta + 2\alpha_U\Delta + \Delta(\varphi_1 + \varphi_2 + \varphi_3))) - \\ &\quad E[\beta_U\Delta\gamma\theta(\theta - (\Delta\gamma\theta + 2\alpha_U\Delta + \Delta(\varphi_1 + \varphi_2 + \varphi_3)))] - \\ &\quad E[\beta_U2\alpha_U\Delta(\theta - (\Delta\gamma\theta + 2\alpha_U\Delta + \Delta(\varphi_1 + \varphi_2 + \varphi_3)))] - \\ &\quad E[\beta_U\Delta(\varphi_1 + \varphi_2 + \varphi_3)(\theta - (\Delta\gamma\theta + 2\alpha_U\Delta + \Delta(\varphi_1 + \varphi_2 + \varphi_3)))] \end{aligned}$$

The sum of the previous reduces to:

$$\begin{aligned} E(\pi) &= \alpha_U\bar{\theta} - \alpha_U\Delta\gamma\bar{\theta} - 2\Delta\alpha_U^2 - \beta_U\Delta\gamma\left(\frac{1}{\tau_\theta} + \bar{\theta}^2\right)(1 - \Delta\gamma) + 2\Delta^2\alpha_U\beta_U\gamma\bar{\theta} - 2\beta_U\alpha_U\Delta\bar{\theta} + 2\beta_U\gamma\alpha_U\Delta^2\bar{\theta} \\ &\quad + 4\Delta^2\alpha_U^2\beta_U + \beta_U\Delta^2\left(\frac{1}{\tau_{\varphi_1}} + \frac{1}{\tau_{\varphi_2}} + \frac{1}{\tau_{\varphi_3}}\right) \end{aligned}$$

#### 4.6.5 Appendix E

##### Derivations benchmark case, regime 2

##### Price from market clearing:

Assume that each dealer's strategy takes the form:

$$X_i = \alpha_U - p\beta_U - \eta_U\varphi_i \quad (4.20)$$

Using market clearing and conjecture 4.28 we can find price explicitly.

Substituting the demand derived for the value informed traders and substituting the conjectured demand for uninformed dealers I obtain:

$$N(\theta - p)r_I\tau_\varepsilon + \sum_{j=1}^2 X_{j,j}(p, \Omega_j^r) + \varphi_1 + \varphi_2 + \varphi_3 = 0$$

Now using conjecture (4.4) we can rewrite the previous as:

$$N\theta r_I\tau_\varepsilon - Np r_I\tau_\varepsilon + 2\alpha_U - 2\beta_U p - \eta_U\varphi_1 - \eta_U\varphi_2 + \varphi_1 + \varphi_2 + \varphi_3 = 0$$

After some algebra, the price can be explicitly written as:

$$p = \Delta (N\theta r_I\tau_\varepsilon + 2\alpha_U - \eta_U\varphi_1 - \eta_U\varphi_2 + \varphi_1 + \varphi_2 + \varphi_3) \quad (4.21)$$

where

$$\Delta = \frac{1}{Nr_I\tau_\varepsilon + 2\beta_U} \quad (4.22)$$

**Updated parameters:** Now I solve rearranging observables and unobservables, in order to find the signal that Dealer 1 gathers from price. We have:

$$\tilde{h} = \frac{1}{Nr_I\tau_\varepsilon} (p\Delta^{-1} - 2\alpha_U + \eta_U\varphi_1 - \varphi_1) = \theta + \varphi_2 \frac{(1 - \eta_U)}{Nr_I\tau_\varepsilon} + \frac{\varphi_3}{Nr_I\tau_\varepsilon}$$

and:

$$\tilde{h}|\theta \sim N\left(\theta, \left(\frac{1 - \eta_U}{Nr_I\tau_\varepsilon}\right)^2 \left(\frac{1}{\tau_\varphi}\right) + \left(\frac{1}{Nr_I\tau_\varepsilon}\right)^2 \left(\frac{1}{\tau_{\varphi 3}}\right)\right)$$

. Hence knowing that  $Var(\theta|\tilde{h})^{-1} = Var(\theta|p)^{-1}$ ,  $\tau$  can be written as:

$$\begin{aligned}\tau &= \tau_\theta + \tau_\varphi \frac{(Nr_I \tau_\varepsilon)^2}{(1 - \eta_U)^2} + \tau_{\varphi 3} (Nr_I \tau_\varepsilon)^2 \\ \tau &= \tau_\theta + (Nr_I \tau_\varepsilon)^2 \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right)\end{aligned}$$

Knowing that  $E(\theta|\tilde{h}) = E(\theta|\tilde{p})$ , I can write

$$E(\theta|\tilde{p}) = \frac{\bar{\theta}\tau_\theta}{\tau} + (Nr_I \tau_\varepsilon) \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) \frac{(p\Delta^{-1} - 2\alpha_U + \eta_U \varphi_1 - \varphi_1)}{\tau} \quad (4.23)$$

or, substituting  $\gamma = Nr_I \tau_\varepsilon$ , I get:

$$E(\theta|\tilde{p}) = \frac{\bar{\theta}\tau_\theta}{\tau} - \frac{2\alpha_U \gamma}{\tau} \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) + p \frac{(\gamma + 2\beta_U) \gamma}{\tau} \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) - \varphi_1 \frac{\gamma(1 - \eta_U)}{\tau} \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) \quad (4.24)$$

**Equilibrium demand and Maximization pb:** Following the same procedure used in the previous case, the dealer maximization problem results in the following foc:

$$X_i = (Nr_I \tau_\varepsilon + \beta_U)(E(\theta|\tilde{p}) - p) \quad (4.25)$$

**Solve for parameters:** To define the demand coefficients explicitly I need to plug 4.24 in 4.25 to obtain:

$$X_i = (\gamma + \beta_U) \left[ \left( \frac{\bar{\theta}\tau_\theta}{\tau} - \frac{2\alpha_U \gamma}{\tau} \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) \right) - p \left( 1 - \frac{(\gamma + 2\beta_U) \gamma}{\tau} \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) \right) - \varphi_1 \frac{\gamma(1 - \eta_U)}{\tau} \right]$$

The coefficients solve the following system of equations:

$$\begin{cases} \alpha_U = \left( \frac{\bar{\theta}\tau_\theta}{\tau} - \frac{2\alpha_U \gamma}{\tau} \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) \right) (\gamma + \beta_U) \\ \beta_U = \left( 1 - \frac{(\gamma + 2\beta_U) \gamma}{\tau} \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) \right) (\gamma + \beta_U) \\ \eta_U = \frac{\gamma(1 - \eta_U)}{\tau} \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) (\gamma + \beta_U) \\ \tau = \tau_\theta + \gamma^2 \left( \frac{\tau_\varphi}{(1 - \eta_U)^2} + \tau_{\varphi 3} \right) \end{cases}$$

The previous system has been solved numerically and the results are presented in the main section.

### Expected profits of $\mathbb{D}_1$ conditional on observing $\varphi_1$

The expected profits of dealers can be written as:

$$E(\pi) = E[(\theta - p)x | \varphi_1] \quad (4.26)$$

In equilibrium price is given by

$$p = \Delta(N\theta r_I \tau_\varepsilon + 2\alpha_U - \eta_U \varphi_1 - \eta_U \varphi_2 + \varphi_1 + \varphi_2 + \varphi_3) \quad (4.27)$$

and quantity is given by:

$$x = \alpha_U - \beta_U p - \eta_U \varphi_1$$

In order to solve 4.26 I substitute as in the previous section to obtain the following:

$$\begin{aligned} E(\pi) &= E[(\theta - p)x | \varphi_1] = \\ &\alpha_U \bar{\theta} - 2\alpha_U^2 \Delta + \Delta \eta_U \varphi_1 \alpha_U - \Delta \varphi_1 \alpha_U - \alpha_U \Delta \gamma \bar{\theta} + \\ &-\beta_U \Delta \gamma \left( \bar{\theta}^2 + \frac{1}{\tau_\theta} \right) (1 - \Delta \gamma) + 2\beta_U \alpha_U \Delta^2 \gamma \bar{\theta} - \beta_U \Delta^2 \eta_U \gamma \varphi_1 \bar{\theta} + \beta_U \Delta^2 \gamma \varphi_1 \bar{\theta} + \\ &-2\beta_U \Delta \alpha_U \bar{\theta} + 2\beta_U \Delta^2 \alpha_U \gamma \bar{\theta} + 4\beta_U \alpha_U^2 \Delta^2 - 2\beta_U \Delta^2 \eta_U \alpha_U \varphi_1 + 2\Delta^2 \alpha_U \varphi_1 \beta_U + \\ &+\beta_U \eta_U \varphi_1 \Delta \bar{\theta} - \beta_U \eta_U \varphi_1 \Delta^2 \gamma \bar{\theta} - 2\beta_U \eta_U \alpha_U \varphi_1 \Delta^2 + \Delta^2 \eta_U^2 \varphi_1^2 \beta_U - \beta_U \eta_U \Delta^2 \left( \frac{1}{\tau_{\varphi_2}} \right) + \\ &+\beta_U \Delta^2 \eta_U^2 \left( \frac{1}{\tau_{\varphi_2}} \right) - \beta_U \Delta^2 \eta_U \left( \frac{1}{\tau_{\varphi_2}} \right) + \\ &-\beta_U \varphi_1 \Delta \bar{\theta} + \beta_U \varphi_1 \Delta^2 \bar{\theta} \gamma + 2\alpha_U \beta_U \Delta^2 \varphi_1 - \beta_U \Delta^2 \eta_U \left( \frac{1}{\tau_{\varphi_1}} \right) + \beta_U \Delta^2 \left( \frac{1}{\tau_{\varphi_1}} \right) \\ &-\beta_U \Delta^2 \eta_U \left( \frac{1}{\tau_{\varphi_2}} \right) + \beta_U \Delta^2 \left( \frac{1}{\tau_{\varphi_2}} \right) + \\ &+\beta_U \Delta^2 \left( \frac{1}{\tau_{\varphi_3}} \right) \\ &-\eta_U \varphi_1 \bar{\theta} + \eta_U \varphi_1 \Delta \gamma \bar{\theta} + 2\alpha_U \Delta \eta_U \varphi_1 - \Delta \eta_U^2 \left( \frac{1}{\tau_{\varphi_1}} \right) + \Delta \eta_U \left( \frac{1}{\tau_{\varphi_1}} \right) \end{aligned}$$

#### 4.6.6 Appendix F

**Derivations benchmark case, regime 3.**

**Price from market clearing:**

Assume that each dealer's strategy takes the form:

$$X_i = \alpha_U - p\beta_U - \eta_U(\varphi_1 + \varphi_2) \quad (4.28)$$

Using market clearing and conjecture 4.28 we can find price explicitly.

Substituting the demand derived for the value informed traders and substituting the conjectured demand for uninformed dealers I obtain:

$$N(\theta - p)r_I\tau_\varepsilon + \sum_{j=1}^2 X_{j,j}(p, \Omega_j^r) + \varphi_1 + \varphi_2 + \varphi_3 = 0$$

Now using conjecture (4.4) we can rewrite the previous as:

$$N\theta r_I\tau_\varepsilon - Npr_I\tau_\varepsilon + 2\alpha_U - 2\beta_U p - 2\eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3 = 0$$

After some algebra, the price can be explicitly written as:

$$p = \Delta (N\theta r_I\tau_\varepsilon + 2\alpha_U - 2\eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3) \quad (4.29)$$

where

$$\Delta = \frac{1}{Nr_I\tau_\varepsilon + 2\beta_U} \quad (4.30)$$

**Updated parameters:** Now I solve rearranging observables and unobservables, in order to find the signal that Dealers gather from price. We have:

$$\tilde{h} = \frac{1}{Nr_I\tau_\varepsilon} (p\Delta^{-1} - 2\alpha_U + 2\eta_U(\varphi_1 + \varphi_2) - \varphi_1 - \varphi_2) = \theta + \varphi_3 \frac{1}{Nr_I\tau_\varepsilon}$$

and:

$$\tilde{h}|\theta \sim N\left(\theta, \frac{1}{(Nr_I\tau_\varepsilon)^2} \left(\frac{1}{\tau_{\varphi_3}}\right)\right)$$

. Hence knowing that  $Var(\theta | \tilde{h})^{-1} = Var(\theta | p)^{-1}$ ,  $\tau$  can be written as:

$$\tau = \tau_\theta + \tau_{\varphi 3}(Nr_I \tau_\varepsilon)^2$$

Knowing that  $E(\theta | \tilde{h}) = E(\theta | \tilde{p})$ , I can write

$$E(\theta | \tilde{p}) = \frac{\bar{\theta}\tau_\theta}{\tau} + (Nr_I \tau_\varepsilon)\tau_{\varphi 3} \frac{(p\Delta^{-1} - 2\alpha_U + 2\eta_U(\varphi_1 + \varphi_2) - \varphi_1 - \varphi_2)}{\tau} \quad (4.31)$$

or, substituting  $\gamma = Nr_I \tau_\varepsilon$ , I get:

$$E(\theta | \tilde{p}) = \frac{\bar{\theta}\tau_\theta}{\tau} - \frac{\gamma\tau_{\varphi 3}}{\tau}(2\alpha_U) + p \left( \frac{\gamma\tau_{\varphi 3}}{\tau} \right) (\gamma + 2\beta_U) - (1 - 2\eta_U)(\varphi_1 + \varphi_2) \left( \frac{\gamma\tau_{\varphi 3}}{\tau} \right) \quad (4.32)$$

**Equilibrium demand and Maximization pb:** Following the same procedure used in the previous case, the dealer maximization problem results in the following foc:

$$X_i = (Nr_I \tau_\varepsilon + \beta_U)(E(\theta | \tilde{p}) - p) \quad (4.33)$$

**Solve for parameters:** To define the demand coefficients explicitly I need to plug 4.32 in 4.33 to obtain:

$$X_i = (\gamma + \beta_U) \left[ \left( \frac{\bar{\theta}\tau_\theta}{\tau} - 2\alpha_U \frac{\gamma\tau_{\varphi 3}}{\tau} \right) - p \left( 1 - \left( \frac{\gamma\tau_{\varphi 3}}{\tau} \right) (\gamma + 2\beta_U) \right) - (\varphi_1 + \varphi_2) \left( (1 - 2\eta_U) \left( \frac{\gamma\tau_{\varphi 3}}{\tau} \right) \right) \right]$$

The coefficients solve the following system of equations:

$$\begin{cases} \alpha_U = \left( \frac{\bar{\theta}\tau_\theta}{\tau} - 2\alpha_U \frac{\gamma\tau_{\varphi 3}}{\tau} \right) (\gamma + \beta_U) \\ \beta_U = \left( 1 - \frac{\gamma\tau_{\varphi 3}}{\tau} (\gamma + 2\beta_U) \right) (\gamma + \beta_U) \\ \eta_U = \frac{\gamma\tau_{\varphi 3}}{\tau} (1 - 2\eta_U) (\gamma + \beta_U) \\ \tau = \tau_\theta + \tau_{\varphi 3}(Nr_I \tau_\varepsilon)^2 \end{cases}$$

The coefficients can be explicitated as:

$$\left\{ \begin{array}{l} \alpha_U = \frac{\bar{\theta}\tau_\theta(\gamma+\beta_U)\tau^{-1}}{1+2(\gamma\tau_{\varphi 3})(\gamma+\beta_U)\tau^{-1}} \\ \beta_U = \left[ (\gamma^4\tau_{\varphi 3}^2 + 8\gamma^2\tau_{\varphi 3}\tau)^{\frac{1}{2}} - 3\gamma^2\tau_{\varphi 3} \right] \frac{1}{4\gamma\tau_{\varphi 3}} \\ \eta_U = \frac{\gamma\tau_{\varphi 3}\tau^{-1}(\gamma+\beta_U)}{1+2(\gamma\tau_{\varphi 3})(\gamma+\beta_U)\tau^{-1}} \\ \tau = \tau_\theta + \tau_{\varphi 3}(Nr_I\tau_\varepsilon)^2 \end{array} \right.$$

In proceed to calculate the expected profits of the dealers in this scenario.

$$\begin{aligned} E(\pi | \varphi_1, \varphi_2) &= E[(p - \theta)x | \varphi_1, \varphi_2] = \\ &= E[px | \varphi_1, \varphi_2] - E[x\theta | \varphi_1, \varphi_2]. \end{aligned}$$

I solve as in the previous cases:

$$\begin{aligned} E[\theta x | \varphi_1, \varphi_2] &= E[\theta(\alpha_U - p\beta_U - \eta_U(\varphi_1 + \varphi_2))] = \\ &= \bar{\theta}\alpha_U - \bar{\theta}\eta_U(\varphi_1 + \varphi_2) - \beta_U E(\theta p) \end{aligned}$$

I proceed to make the last term explicit as:

$$\begin{aligned} \beta_U E(\theta p) &= \beta_U [\Delta (N\theta^2 r_I \tau_\varepsilon + \alpha_U \theta - 2\eta_U \theta (\varphi_1 + \varphi_2) + \varphi_1 \theta + \varphi_2 \theta + \varphi_3 \theta)] \\ &= E[\theta^2] \beta_U \Delta N r_I \tau_\varepsilon + \Delta \alpha_U \beta_U \bar{\theta} - 2\Delta \beta_U \eta_U \bar{\theta} (\varphi_1 + \varphi_2) + \Delta \beta_U \bar{\theta} (\varphi_1 + \varphi_2) \end{aligned}$$

Overall I can write:

$$E[\theta x | \varphi_1, \varphi_2] = \bar{\theta}\alpha_U - \bar{\theta}\eta_U(\varphi_1 + \varphi_2) - \beta_U \Delta N r_I \tau_\varepsilon (\bar{\theta}^2 + \frac{1}{\tau_\theta}) - \Delta \alpha_U \beta_U \bar{\theta} + 2\beta_U \Delta \eta_U \bar{\theta} (\varphi_1 + \varphi_2) - \Delta \beta_U \bar{\theta} (\varphi_1 + \varphi_2)$$

Now I need to calculate :

$$\begin{aligned} E[px | \varphi_1, \varphi_2] &= \\ &= E(\alpha_U [\Delta (N\theta r_I \tau_\varepsilon + \alpha_U - 2\eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3)]) - E(\beta_U p^2) \\ &\quad - E(\eta_U(\varphi_1 + \varphi_2) [\Delta (N\theta r_I \tau_\varepsilon + \alpha_U - 2\eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3)]) \end{aligned}$$

From the previous the first term can be written as:

$$\Delta\alpha_U N\bar{\theta}r_I\tau_\varepsilon + \alpha_U^2\Delta - 2\eta_U\alpha_U(\varphi_1 + \varphi_2)\Delta + \alpha_U(\varphi_1 + \varphi_2)\Delta$$

The second term can be written as:

$$\beta_U E(p^2) = Var(p) + [E(p)]^2$$

Then:

$$\begin{aligned} Var(p) &= E[(p - E(p))^2] = \\ E[(\Delta Nr_I\tau_\varepsilon(\theta - \bar{\theta}) + \Delta\varphi_3)^2] &= (\Delta Nr_I\tau_\varepsilon)^2 \frac{1}{\tau_\theta} + \Delta^2 \frac{1}{\tau_{\varphi_3}} \end{aligned}$$

and

$$\begin{aligned} [E(p)]^2 &= E(\Delta^2 (N\theta r_I\tau_\varepsilon + \alpha_U - 2\eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3)^2) = \\ &= \Delta^2 (Nr_I\tau_\varepsilon)^2 (\bar{\theta}^2 + \frac{1}{\tau_\theta}) + \Delta^2 \alpha_U^2 + 4\eta_U^2 (\varphi_1 + \varphi_2)^2 + (\varphi_1 + \varphi_2 + \varphi_3)^2 + \\ &\quad + 2\Delta^2 Nr_I\tau_\varepsilon \theta \alpha_U - 4\Delta^2 \eta_U (\varphi_1 + \varphi_2) N\bar{\theta}r_I\tau_\varepsilon + 2\Delta^2 Nr_I\tau_\varepsilon \theta (\varphi_1 + \varphi_2 + \varphi_3) - 4\eta_U (\varphi_1 + \varphi_2) (\varphi_1 + \varphi_2 + \varphi_3) \end{aligned}$$

The last term left to calculate is

$$\begin{aligned} E(2\eta_U(\varphi_1 + \varphi_2)p) &= E(\eta_U(\varphi_1 + \varphi_2) [\Delta (N\theta r_I\tau_\varepsilon + \alpha_U - 2\eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3)]) = \\ &\quad \eta_U(\varphi_1 + \varphi_2) Nr_I\tau_\varepsilon \Delta \bar{\theta} + \alpha_U \eta_U(\varphi_1 + \varphi_2) \Delta - 2\eta_U^2 (\frac{1}{\tau_{\varphi_1}} + \frac{1}{\tau_{\varphi_2}}) \Delta + \eta_U (\frac{1}{\tau_{\varphi_1}} + \frac{1}{\tau_{\varphi_2}}) \Delta. \end{aligned}$$

#### 4.6.7 Appendix G

**Derivations benchmark case, regime 3.**

**Price from market clearing:**

Assume one dealer trades and his strategy takes the form:

$$X_U = \alpha_U - p\beta_U - \eta_U(\varphi_1 + \varphi_2) \tag{4.34}$$

Using market clearing and conjecture 4.28 we can find price explicitly.

Substituting the demand derived for the value informed traders and substituting the conjectured demand for uninformed dealers I obtain:

$$N(\theta - p)r_I\tau_\varepsilon + X_U(p, \Omega_j^r) + \varphi_1 + \varphi_2 + \varphi_3 = 0$$

Now using conjecture (4.4) we can rewrite the previous as:

$$N\theta r_I\tau_\varepsilon - Npr_I\tau_\varepsilon + \alpha_U - \beta_U p - \eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3 = 0$$

After some algebra, the price can be explicitly written as:

$$p = \Delta (N\theta r_I\tau_\varepsilon + \alpha_U - \eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3) \quad (4.35)$$

where

$$\Delta = \frac{1}{Nr_I\tau_\varepsilon + \beta_U} \quad (4.36)$$

**Updated parameters:** Now I solve rearranging observables and unobservables, in order to find the signal that Dealers gather from price. We have:

$$\tilde{h} = \frac{1}{Nr_I\tau_\varepsilon} (p\Delta^{-1} - \alpha_U + \eta_U(\varphi_1 + \varphi_2) - \varphi_1 - \varphi_2) = \theta + \varphi_3 \frac{1}{Nr_I\tau_\varepsilon}$$

and:

$$\tilde{h} | \theta \sim N \left( \theta, \frac{1}{(Nr_I\tau_\varepsilon)^2} \left( \frac{1}{\tau_{\varphi 3}} \right) \right)$$

. Hence knowing that  $Var(\theta | \tilde{h})^{-1} = Var(\theta | p)^{-1}$ ,  $\tau$  can be written as:

$$\tau = \tau_\theta + \tau_{\varphi 3} (Nr_I\tau_\varepsilon)^2$$

Knowing that  $E(\theta | \tilde{h}) = E(\theta | \tilde{p})$ , I can write

$$E(\theta | \tilde{p}) = \frac{\bar{\theta}\tau_\theta}{\tau} + (Nr_I\tau_\varepsilon)\tau_{\varphi 3} \frac{(p\Delta^{-1} - \alpha_U + \eta_U(\varphi_1 + \varphi_2) - \varphi_1 - \varphi_2)}{\tau} \quad (4.37)$$

or, substituting  $\gamma = Nr_I\tau_\varepsilon$ , I get:

$$E(\theta|\tilde{p}) = \frac{\bar{\theta}\tau_\theta}{\tau} + p \left( \frac{\gamma\tau\varphi_3}{\tau} \right) (\gamma + \beta_U) - \frac{\alpha_U\gamma\tau\varphi_3}{\tau} - (1 - \eta_U)(\varphi_1 + \varphi_2) \left( \frac{\gamma\tau\varphi_3}{\tau} \right) \quad (4.38)$$

**Equilibrium demand and Maximization pb:** Following the same procedure used in the previous case, the dealer maximization problem results in the following foc:

$$X_i = (Nr_I\tau_\varepsilon)(E(\theta|\tilde{p}) - p) \quad (4.39)$$

**Solve for parameters:** To define the demand coefficients explicitly I need to plug 4.32 in 4.33 to obtain:

$$X_i = (Nr_I\tau_\varepsilon) \left[ \left( \frac{\bar{\theta}\tau_\theta}{\tau} - \alpha_U \frac{\gamma\tau\varphi_3}{\tau} \right) - p \left( 1 - \left( \frac{\gamma\tau\varphi_3}{\tau} \right) (\gamma + \beta_U) \right) - (\varphi_1 + \varphi_2) \left( (1 - \eta_U) \left( \frac{\gamma\tau\varphi_3}{\tau} \right) \right) \right]$$

The coefficients solve the following system of equations:

$$\begin{cases} \alpha_U = \left( \frac{\bar{\theta}\tau_\theta}{\tau} - \alpha_U \frac{\gamma\tau\varphi_3}{\tau} \right) (\gamma) \\ \beta_U = \left( 1 - \left( \frac{\gamma\tau\varphi_3}{\tau} \right) (\gamma + \beta_U) \right) (\gamma) \\ \eta_U = \left( (1 - \eta_U) \left( \frac{\gamma\tau\varphi_3}{\tau} \right) \right) (\gamma) \\ \tau = \tau_\theta + \tau_{\varphi_3}\gamma^2 \end{cases}$$

The solution is given by:

$$\begin{aligned} \alpha_U &= \frac{\gamma\bar{\theta}\tau_\theta}{\tau + \gamma^2\tau_{\varphi_3}} \\ \beta_U &= \frac{\gamma\tau - \gamma^3\tau_{\varphi_3}}{\tau + \gamma^2\tau_{\varphi_3}} \\ \eta_U &= \frac{\tau_{\varphi_3}\gamma^2}{\tau + \gamma^2\tau_{\varphi_3}} \end{aligned}$$

In proceed to calculate the expected profits of the dealers in this scenario.

$$\begin{aligned}
E(\pi \mid \varphi_1, \varphi_2) &= E[(\theta - p)x \mid \varphi_1, \varphi_2] = \\
&= E[\theta x \mid \varphi_1, \varphi_2] - E[p x \mid \varphi_1, \varphi_2].
\end{aligned}$$

I solve as in the previous cases:

$$\begin{aligned}
E[\theta x \mid \varphi_1, \varphi_2] &= E[\theta(\alpha_U - p\beta_U - \eta_U(\varphi_1 + \varphi_2))] = \\
&= \bar{\theta}\alpha_U - \bar{\theta}\eta_U(\varphi_1 + \varphi_2) - \beta_U E(\theta p)
\end{aligned}$$

I proceed to make the last term explicit as:

$$\begin{aligned}
\beta_U E(\theta p) &= \beta_U [\Delta (N\theta^2 r_I \tau_\varepsilon + \alpha_U \theta - \eta_U \theta(\varphi_1 + \varphi_2) + \varphi_1 \theta + \varphi_2 \theta + \varphi_3 \theta)] \\
&= E[\theta^2] \beta_U \Delta N r_I \tau_\varepsilon + \Delta \alpha_U \beta_U \bar{\theta} - \Delta \beta_U \eta_U \bar{\theta}(\varphi_1 + \varphi_2) + \Delta \beta_U \bar{\theta}(\varphi_1 + \varphi_2)
\end{aligned}$$

Overall I can write:

$$E[\theta x \mid \varphi_1, \varphi_2] = \bar{\theta}\alpha_U - \bar{\theta}\eta_U(\varphi_1 + \varphi_2) - \beta_U \Delta N r_I \tau_\varepsilon (\bar{\theta}^2 + \frac{1}{\tau_\theta}) - \Delta \alpha_U \beta_U \bar{\theta} + \beta_U \Delta \eta_U \bar{\theta}(\varphi_1 + \varphi_2) - \Delta \beta_U \bar{\theta}(\varphi_1 + \varphi_2)$$

Now I need to calculate :

$$\begin{aligned}
E[p x \mid \varphi_1, \varphi_2] &= \\
&= E(\alpha_U [\Delta (N\theta r_I \tau_\varepsilon + \alpha_U - \eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3)]) - E(\beta_U p^2) \\
&\quad - E(\eta_U(\varphi_1 + \varphi_2) [\Delta (N\theta r_I \tau_\varepsilon + \alpha_U - \eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3)])
\end{aligned}$$

From the previous the first term can be written as:

$$\Delta \alpha_U N \bar{\theta} r_I \tau_\varepsilon + \alpha_U^2 - \eta_U \alpha_U (\varphi_1 + \varphi_2) + \alpha_U (\varphi_1 + \varphi_2)$$

The second term can be written as:

$$\beta_U E(p^2) = Var(p) + [E(p)]^2$$

Then:

$$\begin{aligned} Var(p) &= E[(p - E(p))^2] = \\ E[(\Delta Nr_I \tau_\varepsilon (\theta - \bar{\theta}) + \Delta \varphi_3)^2] &= (\Delta Nr_I \tau_\varepsilon)^2 \frac{1}{\tau_\theta} + \Delta^2 \frac{1}{\tau_{\varphi_3}} \end{aligned}$$

and

$$\begin{aligned} [E(p)]^2 &= E(\Delta^2 (N\theta r_I \tau_\varepsilon + \alpha_U - \eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3)^2) = \\ &= \Delta^2 (Nr_I \tau_\varepsilon)^2 (\bar{\theta}^2 + \frac{1}{\tau_\theta}) + \Delta^2 \alpha_U^2 + \eta_U^2 (\varphi_1 + \varphi_2)^2 + (\varphi_1 + \varphi_2 + \varphi_3)^2 + \\ &\quad + 2\Delta^2 Nr_I \tau_\varepsilon \theta \alpha_U - 2\Delta^2 \eta_U (\varphi_1 + \varphi_2) + 2\Delta^2 Nr_I \tau_\varepsilon \theta (\varphi_1 + \varphi_2 + \varphi_3) - 2\eta_U (\varphi_1 + \varphi_2) (\varphi_1 + \varphi_2 + \varphi_3) \end{aligned}$$

The last term left to calculate is

$$\begin{aligned} E(\eta_U(\varphi_1 + \varphi_2)p) &= E(\eta_U(\varphi_1 + \varphi_2) [\Delta (N\theta r_I \tau_\varepsilon + \alpha_U - \eta_U(\varphi_1 + \varphi_2) + \varphi_1 + \varphi_2 + \varphi_3)]) = \\ &\quad \eta_U(\varphi_1 + \varphi_2) Nr_I \tau_\varepsilon \Delta + \alpha_U \eta_U(\varphi_1 + \varphi_2) - \eta_U^2 (\frac{1}{\tau_{\varphi_1}} + \frac{1}{\tau_{\varphi_2}}) + \eta_U (\frac{1}{\tau_{\varphi_1}} + \frac{1}{\tau_{\varphi_2}}). \end{aligned}$$