

University of London
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Technology

COLLAPSE OF RING STIFFENED
CYLINDRICAL SHELLS UNDER
COMBINED EXTERNAL PRESSURE AND
AXIAL COMPRESSION

by

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ABSTRACT

This thesis investigates the collapse behaviour of ring stiffened cylinders under combinations of axial loading and external pressure.

Eight strain gauged cylindrical models carefully fabricated at 1/20th scale and with geometries typical of main leg members used in the construction of floating offshore platforms have been tested to collapse under various loading ratios. The initial geometrical imperfections of all the models were monitored in detail to study their effects on the collapse behaviour.

The models were designed to fail in the following two distinctly different modes:

- a) collapse of the shell panel between adjacent ring stiffeners
- b) general instability involving the simultaneous collapse of the entire shell and stiffeners

The experimental results have been correlated against an elasto-plastic finite element program. Good agreement has been obtained in most cases. Comparisons with design codes have also been carried out to highlight areas where re-appraisals are needed.

Theoretical formulations based on a rigid plastic mechanism approach have been developed for each of the two collapse modes observed. The validity of the formulations has been confirmed by comparisons with experimental results from various sources. Based on these formulations, a method for designing ring stiffeners under pressure dominated loadings has been derived and verified by finite element correlations for various geometries. Comparisons with the DnV design code have shown the present method to be particularly economical for hydrostatic pressure loading.

Parameters outside the range of validity of the plastic mechanism formulations have been studied using the finite element program. These results have been combined with the mechanism results to provide design guidance.

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NOTATION

A	area
A_{mn}, B_{mn}	Fourier coefficients
D	diameter
E	Young's modulus
f	Mises yield function
H	length of mechanism leg
H_f, H_w	depth of stiffener flange and stiffener web respectively
I	inertia
K	loading ratio ($=\bar{N}/\bar{p}$)
L	length
M	moment
m	number of axial half-waves
N	axial load
N_s	number of ring stiffeners
n	number of circumferential waves
p	pressure
R	radius
t, t_f, t_w	thickness of shell plating, stiffener flange and stiffener web respectively
u	stress distribution factor
V	volume
W	work
w	radial displacement
x, y, z	cartesian coordinates
Δ	end displacement
ϵ	strain
θ	angle of rotation
λ	constant of proportionality
ν	Poisson's ratio
σ	stress

Subscripts

a	axial
B	non-deflecting diaphragms

c	general collapse
cAX	critical axisymmetric mode
cr	critical
DnV	specified or predicted by Det norske Veritas design code
E	external
EQ	equivalent
e	elastic
ex	experiment
f	local panel failure
I	internal
imp	imperfection
Lef	effective shell plating
m	number of axial half-waves
n	number of circumferential waves
o	yield
p	shell panel
reqd	required
s	stiffener
T	shell-stiffener assembly
t	tension
u	ultimate
w	web
θ	hoop
0	initial

Superscript

- non-dimensionalised with respect to yield

Other symbols will be defined as they appear.

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CHAPTER 1

INTRODUCTION

Cylindrical shells are efficient structures for resisting axial compression and pressure loading. Their resistance to pressure loading has been utilized for many years for the construction of pressure vessels and storage containers. In the past few decades, in view of their effectiveness in reducing drag forces, the application of cylindrical shells has extended into the aircraft, aerospace and submarine industries. They are now also being extensively used as primary members for offshore construction.

Many offshore structures have been designed as gravity or fixed platforms. Due to the relative insignificance of structural self-weight considerations, these structures are generally constructed from stocky tubulars, the ultimate strengths of which are closely related to the material yield stress. However, the development of floating platforms, such as the one recently completed for the Hutton oil field (see fig.1.1) and indeed many exploratory semi-submersibles, has necessitated the use of more slender tubulars to satisfy buoyancy requirements. The geometries of these members (radius to thickness ratios approximately in the range of 100 to 500), lie between those of conventional fixed platforms and aerospace shells and are in a range where failures are expected to occur elasto-plastically. Only limited research has been conducted on these intermediate geometries.

The need to reduce structural self-weight and fabrication costs also results in the orthogonal stiffening used commonly for fixed platforms being replaced by a simpler arrangement involving only ring stiffening. Apart

from reducing fabrication and design costs, this is expected to lead to a more efficient use of material. Ring stiffening is an exceptionally effective means of increasing the strength of cylinders under pressure loading and numerous studies related to the design of pressure vessels and submarine structures have confirmed this.

Data from these studies are, however, inadequate for the design of offshore platforms since in addition to resisting external pressure, offshore tubulars are also expected to simultaneously carry the axial compressive loading arising from the dead weight of the superstructure as well as other live and environmental loads. The behaviour of ring stiffened cylinders under this combined loading situation can be very complex, particularly when the effects of initial geometric imperfections and different boundary conditions are taken into consideration. Little experimental and theoretical data are available and as a result design codes are generally ill-founded for this combined load case.

In order to overcome the lack of design information in this and other areas of offshore construction, the Science and Engineering Research Council of Great Britain funded Imperial College and other centres to set up a general research programme to investigate the behaviour of offshore cylindrical shell structures. This thesis describes the research effort related to the collapse behaviour of ring stiffened cylindrical shells under combined axial compression and external pressure. The geometries investigated are related to those of floating platforms. The major aim of this research is to provide experimental, finite element analytical and theoretical data to enable design methods to be formulated on more rational bases.

1.1 Layout of thesis

A literature review concentrating on research related to the offshore industry is presented in chapter 2. An outline of the research activities from other fields of engineering applications is also included.

The testing of eight small scale ring stiffened models is described in chapter 3. Two different collapse modes are studied. The results are compared with various European and American design codes.

In chapter 4, these experimental results are used to provide correlation with an elasto-plastic finite element package to develop confidence in the program for investigating parameters not covered by the experiments.

Simplified theoretical models based on rigid plastic mechanism formulations are developed in chapter 5 to analyse the experimental collapse modes obtained in chapter 3. Based on these formulations a method is derived for designing ring stiffeners under pressure loadings.

Collapse behaviour under axial compression dominated loadings is studied in chapter 6 using the finite element program. The whole range of geometries relevant to floating platforms is considered.

Pressure loaded cases not covered by the simplified theoretical models of chapter 5 are studied in chapter 7 using the finite element program. The results are combined with those from chapters 5 and 6 and presented in the form of design charts. Comparisons with various design codes are also given. The main results relate to stiffener requirements which optimise on the buckling resistance of the stiffened shells and the resulting strengths that can be achieved under interactive loading.

General conclusions and suggestions for future work are presented in chapter 8.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The stability of the cylindrical shell is a subject on which a vast number of papers have been published. For nearly a century, theoreticians have been challenged by the complexity involved in predicting failure loads, while experimentalists have been fascinated by the wide variation of results from seemingly similar test specimens. This intellectual challenge combined with the wide application of the cylindrical shell as a structural element has led to more than one thousand six hundred references being published under the title of cylindrical shell buckling [141].

The present author does not intend to present a complete survey of the subject since a number of thorough reviews on research into various aspects of the collapse behaviour of cylindrical shells have already been published by other investigators. Many of these reviews are concerned with shell geometries and fabrication techniques relevant to the aircraft and aerospace industry [14, 24, 61, 125, 149, 152, 186, 236, 275, 291, 292, 312]. Instead, the present work intends to concentrate on research related to the offshore industry, although an outline of the research activities from the other fields of engineering application is also given since these activities form the foundation on which current knowledge is being built. However, it should be pointed out that the behaviour of offshore members does possess important differences to that found in these other areas of engineering application and any extrapolation of

results should only be attempted with caution.

The following review is confined to isotropic and ring stiffened cylinders. Reviews including stringer stiffened cylinders have been presented by Agelidis [1], Fahy [116] and Tsang and Harding [333].

In view of the large number of references involved, it has been found necessary to classify these according to the loading type. Such a division is not entirely rational since some of the references are relevant to more than one loading category. The adoption of this artificial and somewhat arbitrary classification is nevertheless necessary to enable the review to be presented in a systematic way. References on axial compression are reviewed first, followed respectively by those on hydrostatic pressure, lateral pressure and combined loading. Within each category, the research related to offshore applications is presented separately.

2.2 Axial compression

2.2.1 Unstiffened cylinders

2.2.1.1 Background

The theory of thin shells may be traced back to the end of the 19th century to the work of Love [199]. The buckling of axially compressed cylindrical members was analysed approximately by Lorenz [198] in 1908 and then in succeeding years by Timoshenko [326], Southwell [306] and Von Mises [350].

These approaches were essentially based on the classical small deflection theory. Uncertainty about the validity of these approaches occurred when the experimental work of Wilson and Newmark [361], Donnell [89] and Fluegge [120] yielded buckling loads as low as 20%, 40% and 52% respectively of the theoretical predictions. Fluegge

attempted to explain this discrepancy in terms of the radial restraint at the ends of the experimental models, but later tests by Lundquist [204] on thin aluminium cylinders produced failure loads too low for Fluegge's explanation to be adequate. Nevertheless, these and other test results were collated by Batdorf et al [32] and Schilling [273] for the formulation of design procedures.

2.2.1.2 Initial imperfections

In 1934, Donnell [89] attempted to account for the discrepancy between theory and experiment by including the effect of initial geometric imperfections in his approximate large deflection theory. Although the uncoupled characteristics of the resulting equations became widely used by many subsequent researchers, Donnell's analysis was over-simplified and the results were unsatisfactory.

By adopting an energy minimisation approach, Donnell and Wan [92] demonstrated the sensitivity of the failure load to initial imperfections. Their work has been extended by Sobey [302] to include the effects of arbitrary imperfection modes.

In 1963, Tennyson [317,318] developed an experimental technique to produce high quality nominally perfect cylinders from Hysol plastics. His results indicated that the attainment of the classical buckling load was possible provided the initial imperfections were small enough.

Experimental investigations have been carried out on imperfect cylinders to assess the effects of different imperfection modes [7,16,17,25,72,158,164]. Based on these experimental results, Arbocz and Babcock [18] proposed procedures to relate design loads to 'knockdown' factors. There was, however, still a need to set up an initial imperfection data bank to enable design loads to be specified on a more rational basis [15]. In particular, it was important to be able to associate characteristic initial

imperfection distributions with different fabrication techniques [145].

2.2.1.3 Postbuckling behaviour

In 1945, Von Karman and Tsien [349] investigated the postbuckling behaviour of the geometrically perfect cylindrical shell by using Donnell's large deflection equations and assuming a buckling mode with two free parameters in the radial displacement expression. Their results showed, that for a large range of applied loading, there were equilibrium positions with buckled modes involving loads much lower than the value predicted by the classical small deflection theory. They interpreted the behaviour of the shells as a 'jump' from the prebuckling path to the postbuckling path, prior to the attainment of the bifurcation load, induced by small unavoidable external disturbances. Tsien [340] attempted to predict the transition by equating the strain energy of the different equilibrium states.

In attempts to quantify reliable lower bounds to the load carrying capacity of practical cylinders, Legget and Jones [195], Michielsen [210], Kempner [170] and Almroth [3] adopted approaches similar to that of Von Karman and Tsien [349] but took a greater number of free parameters to define the radial displacement in the assumed buckling mode. Their work was extended by Tsao [339] whose analysis also included terms to define displacements in the axial and circumferential directions. The minimum postbuckling loads obtained in all cases were lower than the results of [349], but subsequent computations by Hoff et al [153] based on a rigorous formulation involving no less than 1100 terms in the potential energy expression yielded a trivial solution for the case of an infinitely slender cylinder with the predicted buckling mode taking the form of the so-called 'Yoshimura' pattern.

Despite this finding, Esslinger and Geier [110] continued to use the concept of minimum postbuckling load.

Their tests on Mylar cylinders indicated that the lowest postbuckling load provided a reasonable lower limit to the scatter of experimental results.

Pedersen [250] studied the advanced postbuckling behaviour of unstiffened cylinders and discovered that in some cases, the cylinders were theoretically capable of supporting higher loads than the bifurcation load.

The postbuckling behaviour associated with the 'Yoshimura' buckling mode was studied by Korol and Sherbourne [187] by using a geometrical model based on energy formulations. Fairly good correlations were obtained with their own test results [282].

A general postbuckling theory was developed by Koiter [182] in 1945, but it remained relatively unknown until 1963 when an English translation became available. The general nature of the theory is useful for assessing the imperfection sensitivity of different types of structures and has been adopted by many investigators [5, 153, 162, 208] for studies on the cylindrical shell.

In subsequent work, Koiter [183,184] further developed his theory, arriving in the early 1960s at his Special Theory [185] which could handle the effect of small axisymmetric imperfections on the collapse load. Encouraging correlation with this theory has been obtained by Tennyson and Muggeridge [322] from tests on cylindrical shell specimens with carefully machined axisymmetric imperfections [78].

Excellent introductions to Koiter's work have been presented by Brush and Almroth [54] and Budiansky [58]. Reviews on the subject of postbuckling theory have been given by Hutchinson and Koiter [163], Langhaar and Boresi [190] and Thielemann and Esslinger [325].

Recently, Wittek [364] has developed a postbuckling theory to monitor the changes in the membrane energy during the transition between different equilibrium states, while extensive experimental data on the postbuckling behaviour of cylindrical shells over a wide range of geometries have been obtained by Yamaki et al [368] from tests on polyester film models.

2.2.1.4 Boundary conditions

Another major reason which led to discrepancies between theory and experiment was found to be the effect of boundary conditions. This problem was first noticed by Fluegge [120] from tests on celluloid cylinders and has been studied by Ohira [239,240], Stein [310,311], Fischer [119] and Pflueger [252] in subsequent years.

The importance of boundary conditions for finite and semi-infinite length cylinders was examined by Hoff [151] and Hoff and Rehfield [154]. From an analysis based on Donnell's equations, Hoff and Soong [155] discovered that the critical load was almost independent of the length of the cylinder except when the shell was very short, in which case restraints in both the radial and tangential direction can have drastic effects in reducing the critical load [4,228]. Hoff and Soong's analysis has been confirmed by Thielemann and Essingler's [325] calculations.

2.2.1.5 Plastic buckling

The earliest contribution with regard to the plastic buckling of cylindrical shells under axial compression was made by Bijlaard [39] who demonstrated that the collapse load obtained from the deformation theory was more accurate than the apparently more rigorous flow theory.

Gerard [134,135] obtained similar results by using Donnell's equations and assuming an axisymmetric collapse mode. Lee [192] attempted to bring flow theory predictions into closer agreement with test results by including the effects of initial imperfections, but discovered that the deformation theory again provided better estimates. Lee attributed this discrepancy to the initial imperfections used, but Gerard [138] argued that for the group of cylinders Lee tested, the effect of initial imperfections on

the inelastic buckling strength should have been completely insignificant.

Batterman and Lee [36] corrected an error in Lee's [192] earlier analysis and concluded that the correct assumption of the buckling mode was crucial for the accuracy of the flow theory. Prior to this, Batterman's [35] experimental tests on thirty cylinders had produced results which brought the flow theory to close agreement with the deformation theory.

A discussion of this paradox between the flow and deformation theories of plasticity is given in a detailed review on plastic buckling by Hutchinson [161].

Murphy and Lee [224] used a modified form of the flow theory to include the effects of prebuckling deformation caused by edge restraints. End effects were also accounted for in the numerical analyses of Bushnell [59] and Sobel and Newman [301]. The inclusion of end radial restraints in these studies essentially eliminated the discrepancy between test and theory and revealed that it is not necessary to resort to the use of bifurcation buckling analyses with deformation theory or flow theory involving a singularity in the loading surface in order to bring test and theory into agreement [61].

The effect of initial imperfections on the plastic buckling of axially compressed cylinders was studied by Gellin [133] who showed that the collapse loads were not as sensitive to axisymmetric imperfections as for buckling in the elastic range.

Hutchinson [160] and Onat et al [241] have shown that in the presence of initial imperfections, the discrepancy between deformation theory and flow theory largely disappears.

2.2.2 Ring stiffened cylinders

It is generally recognised that under static loadings, a ring stiffened cylinder may fail in one of three modes:

- a) collapse of the shell panel between adjacent ring stiffeners
- b) general instability involving the simultaneous collapse of the entire shell and stiffeners
- c) torsional buckling of the ring stiffeners

For ring stiffened cylinders under axial compression, mode (a) is the most common form of failure. The development of mode (b) is possible only if the stiffeners are small and closely spaced. Mode (c) is more relevant to pressure loading and is considered in a later section. In this section, only mode (a) and mode (b) types of failure are reviewed.

2.2.2.1 Panel collapse mode

Ring stiffened cylinders have been found to be very inefficient for carrying axial compressive loads compared to their unstiffened equivalent weight counterparts [50,139]. This is because rings are required at very close spacings to enable them to participate in the resistance to buckling. When rings are placed at spacings greater than the critical wavelength, they have no significant favourable effects on the load carrying capacity of the cylinder [111], and the shell panels between the stiffeners may be effectively considered as an unstiffened cylinder of length equal to the ring spacing. For this reason, only limited research has been conducted specifically on the collapse of ring stiffened cylindrical shells under axial compression.

The inclusion of ring stiffeners can, however, affect the shape of the initial imperfections. This phenomenon has been widely studied both theoretically and experimentally [17, 21, 162, 294, 322]. Based on experimental data, Arbocz and Sechler [20] have proposed a

simple design formula to take account of initial imperfections as well as the effect of the boundary conditions provided by the rings. This effect had previously been investigated by Weller et al [357] by using stability equations developed by Baruch and Singer [30] and also by Cohen [70] who adopted an energy approach.

The postbuckling behaviour of the ring stiffened cylindrical shell has been studied experimentally by Esslinger and Geier [109]. Their results showed that ring stiffened shells buckle in much the same way as unstiffened ones with very similar postbuckling loads.

Tenereli and Horton [323] carried out a series of tests on machined steel ring stiffened cylindrical models to determine the inter-ring panel strengths under axial compression. Critical loads deduced by the Southwell plot technique provided good correlation with the test results. The radial deflections of the shell were carefully monitored and were observed to change directions during loading.

Work related to the behaviour of inter-ring panels connected with recent offshore developments is discussed in section 2.2.3.

2.2.2.2 General collapse mode

The earliest theoretical investigations were carried out by Fluegge [120] who derived a set of coupled equations to describe the equilibrium of the ring stiffened cylinder and by Taylor [316] whose single uncoupled equilibrium equation for orthogonally stiffened cylinders was found to reduce to the Donnell equations for isotropic shells. Other early research on the general instability mode of collapse is given in a review compiled by NACA [227].

Ring stiffened cylinders cannot buckle in the general collapse mode unless the stiffeners are closely spaced in which case the orthotropic shell approach becomes a very convenient method of analysis. This approach which assumes the stiffeners to be continuously distributed (or

smeared out) over the shell reference surface has been adopted by many investigators [37,281,345]. Such an approach, however, does not permit taking into account the eccentricity of the stiffeners [290], the effect of which has been studied by a number of aerospace researchers [148,295]. Singer and Haftka [296] have attempted to study the discreteness effect of the ring stiffeners by a linear 'discrete' stiffener theory in which the stiffeners are considered as linear discontinuities represented by a Dirac delta function instead of being smeared. However, conclusive experimental verification for this theory is still required [293].

Becker et al [38] performed tests on twenty ring stiffened cylinders and obtained good correlation with a linear orthotropic theory derived in ref. [37]. The general collapse modes were found to develop asymmetrically.

Weller and Singer [358] tested twenty nine aluminium ring stiffened models and discovered that the onset of general instability was primarily governed by the area of the stiffeners.

2.2.3 Offshore related research

2.2.3.1 Theoretical approaches

With the advent of digital computers, various numerical techniques have been adopted for the analyses of cylindrical shells used as offshore structural elements. Both finite element [330,353] and finite difference [6,60,143] computer packages are now available to provide design data for offshore tubular members.

However, the high computing costs involved in using these packages mean that they alone cannot provide all the required design information over the geometric, material and loading parameters at present being adopted in offshore

practice. It is important that they should be supplemented by simplified theoretical models.

Such a model has been proposed by Croll and Batista [77] who used the reduced stiffness method to provide explicit lower bound estimates for unstiffened cylinders under axial compression. The reduced stiffness method is so called because it neglects the membrane energy in the minimum critical mode. This method has been extended to include the effects of plasticity [74] and to the analysis of the general collapse mode [102]. Reliable lower bounds to experimental results have been obtained [104].

The plastic mechanism method is another useful simplified theoretical approach and has been used by Alexander [2], Andronicou and Walker [13], and Tsang and Harding [334] to study the axisymmetric mode of collapse. A review of other plastic methods of analysis applied to offshore tubulars has been given by Sherman [285].

2.2.3.2 Experimental investigations

The frequent use of cylindrical members as components in offshore structures has made the sound understanding of their behaviour essential in marine structural design. Although a wealth of experimental data has been accumulated over the past few decades from tests carried out in the aeronautical and aerospace industries, the important differences which exist between the behaviour of shells used for offshore construction and those used in these other fields of engineering makes the extrapolation of these data to offshore design very difficult. This has been realised by the authorities and companies involved and since the late 1970s experimental programmes have been set up in the U.S.A., Europe and Japan.

Recent experimental studies in the U.S.A. were initiated on unstiffened tubulars at Lehigh University where tests on thick-walled columns [68] and short cylinders [20,244] were carried out.

A total of forty one large and small scale tests on ring stiffened models were conducted by Chicago Bridge and Iron Co. (CBI) [211]. The models tested had radius to thickness ratios in the range of 242 to 500 and failures in both local and general collapse modes were obtained. Different grades of steel were used to fabricate the models. Those fabricated from the lowest grade steel developed plasticity before collapse. A summary of these and other tests conducted in the U.S.A. has been given by Miller [214].

At Det norske Veritas (DnV), Norway, an experimental investigation on the axial buckling strength of ring stiffened cylinders has been carried out by Odland [238]. Twelve welded and nine machined models were tested with failure in all cases occurring in the shell panel between the stiffeners. Both axisymmetric and asymmetric buckling patterns were obtained, but the collapse loads were found to be relatively independent of the buckling pattern. The initial imperfections present in the welded models resulted in the buckling strengths dropping by more than 30% compared to their machined counterparts. These tests were part of an experimental shell buckling project conducted by DnV. Other activities involved in this project have been reviewed by Valsgard and Foss [342].

In Britain, a nationally coordinated programme of laboratory tests has been sponsored by the Department of Energy and the Science and Engineering Research Council. Under this programme, ring stiffened cylinders have been tested at Imperial College [96,97] and University College [307,352].

The Imperial College series involved testing three large scale models, each consisting of three bays. All models were extensively scanned for imperfections prior to testing. The stockiest model having ring spacings less than the elastic critical wavelength had a failure mode dominated by the Poisson effect resulting in outward plastic bulging between the rings. The two more slender models failed very suddenly near the rings at lower collapse loads.

In the University College series, ten small scale models were tested. Both axisymmetric and asymmetric buckling patterns were obtained. Failure in all cases occurred in the ends of the shell panel. The steep slope of

the unloading paths suggested a high degree of imperfection sensitivity.

Reviews on the research activities in Britain have been given by Dowling and Harding [94,95].

Full scale imperfection studies on actual offshore tubulars with three axial welds have been made by Dwight [99]. He discovered the dominant imperfection mode to consist of three waves in the circumferential direction. Methods for measuring initial imperfections have been suggested by Kendrick [178].

In Japan, a series of tests consisting of eight fabricated ring stiffened models ($R/t=312$) have been completed by Mitsubishi Heavy Industries [220]. Unfortunately, the results have been kept confidential.

2.2.3.3 Design considerations

In current codes [11,86,114], the design strength of cylindrical members is generally obtained by applying appropriate 'knockdown' factors to the elastic buckling stress to allow for the effect of initial imperfections and plasticity. These factors are often derived empirically from tests and no distinction is made between the different collapse modes which occur in practice. A design procedure which avoids the use of these factors has been proposed by Batista and Croll [34]. The components of the membrane stiffness most likely to be eroded during nonlinear mode interaction are identified to enable a simplified method for calculating the critical load to be derived. Comparisons with experimental data have shown the method to provide reliable and safe lower bounds.

A design routine adopted for desk top computers has been described by Richards [262]. It utilises the linear buckling theory of Timoshenko [327] to determine the axial and circumferential wavelength corresponding to the elastic critical stress for the perfect shell. Factors are then applied to account for initial imperfections and plasticity.

Correlation with the test results reported above [96,352] has shown the routine to provide good comparisons with a tendency towards conservatism.

General design philosophies adopted by American codes for fabricated metal structures [10,11,311] have been reviewed by Miller [212]. Comparisons between the API [11] and the DnV [86] predictions have been made to assess the performance of these codes for the specific purpose of offshore design [313]. Significant differences were obtained for the buckling strength predictions.

The background of the ECCS code [114] has been described by Vandepitte [346]. Comparisons with results of axial compression tests on both stiffened and unstiffened cylinders have produced safe lower bounds [347]. However, correlations made by Bornscheuer [49] have indicated the ECCS code to be nonconservative in some cases, particularly when compared with the experimental results of Schulz [274]. Nonconservatism demonstrated by other European codes [87,242,287] was also highlighted and a method has been proposed to improve the correlation with the DAST code [87].

Recently, a major review involving the collation of experimental data from worldwide sources has been completed by Ellinas et al [107]. Comparisons with a number of European and American codes have been made. Various problems related to offshore design have been discussed and areas where future research should be orientated have also been highlighted.

2.3 Hydrostatic pressure

Circumferential stiffening is an efficient method of increasing the buckling resistance of cylindrical shells under pressure loading. As in the case of axial compression, ring stiffened cylinders under hydrostatic pressure may also fail in various modes. The local panel collapse mode is closely related to the collapse mode associated with

unstiffened cylinders. For this reason, these have been reviewed in parallel.

In this thesis, the term hydrostatic pressure has been defined as uniform pressure acting simultaneously in both the axial and radial directions. Research on non-uniform pressure has not been reviewed. The interested reader may consult ref. [12, 82, 167, 278] for further information.

2.3.1 Local panel and unstiffened cylinder collapse

2.3.1.1 Background

The problem of elastic buckling of a cylindrical shell subjected to hydrostatic pressure was first solved by Bresse [51] in 1859. In 1888, Bryan [55] used the energy criterion for stability to arrive at the same expression as Bresse. The same expression was also derived by Southwell [305] in 1913. In 1929, Von Mises [351] extended his earlier analyses for lateral pressure to cover the case of hydrostatic pressure and allowed the buckled mode to be determined for the first time.

The earliest experimental investigations to obtain the hydrostatic collapse pressure of cylindrical shells were conducted by Fairbairn [117], Carmen [66], Stewart [313], Carmen and Carr [67], Southwell [315], Cook [71], Sanders and Windenburg [272] and Jasper and Williams [165].

In 1934, Windenburg and Trilling [363] correlated the results from some of the theoretical work mentioned above with experimental tests on thirty six cylinders and proposed a simplified form of Von Mises' formula for design purposes. Other simplified forms of Von Mises' formula have subsequently been produced by Bleich [42] and Batdorf [31].

Donnell [88], basing his analysis on thin shell theory, derived uncoupled differential equations to describe the equilibrium of a cylindrical shell under hydrostatic

pressure. The effect of transverse shearing forces was neglected for simplification. Donnell's analysis has subsequently been extended and improved by Loo [197], Vlasov [348] and Meck [209]. Results from these analyses together with that of Von Mises [351] have been compared by Provost [253] who found Vlasov's [348] analysis to provide the best correlation with experimental data.

Soong [304] derived the uncoupled Sanders-type [271] equations and obtained closed-form solutions for finite shells with simply supported ends. His equations yielded the same number of circumferential buckle waves as those of Donnell.

Stein [310] used a large deflection theory to overcome the shortcomings of the classical buckling theories which neglected the importance of prebuckling deformations. His analysis showed excellent correlation with the experimental results of Batdorf [31] over the whole range of geometries considered.

Nash [230] used an energy approach to obtain the analytical solution for the elastic critical pressure of a ring stiffened cylinder. However, the labour involved in setting up the stability determinant was considerable and simplified formulations involving the introduction of approximations for the boundary conditions provided by the ring stiffeners were obtained [4,231].

For many years, experimental investigations into the stability of ring stiffened cylindrical shells were carried out at the David Taylor Model Basin in the U.S.A. [166]. Some of the test results produced poor correlations with the theories of Von Mises [351] and Salerno and Levine [269] and the reason was attributed to the effect of initial imperfections [297,298].

2.3.1.2 Initial imperfections

Early theoretical and experimental investigations into the effect of initial imperfections on the collapse

behaviour of hydrostatically loaded cylindrical shells were conducted by Sturm [315] on both unstiffened and ring stiffened specimens. Based on this work, Holt [156] developed a formula for calculating the allowable design stress taking into account the effect of out-of-roundness in the initial shape of the cylinder. The results from this formula, however, provided unsatisfactory correlations with the test results reported by Windenburg and Trilling [363]. Early design recommendations on permissible out-of-roundness had also been given by Windenburg [362].

Donnell [90] used a finite deflection theory which he had developed earlier [89] to analyse the effect of imperfections for the case of the buckling of a simply supported thin cylinder. The imperfections were described by an 'unevenness' factor which was related to the material and manufacturing process. Correlation with the test results of [363] was not completely satisfactory. Donnell attributed this to the incorrect choice of boundary conditions. Clamped boundaries were assumed in a later analysis [91] and improved agreements with test results were obtained.

A theoretical investigation into the effect of initial imperfections was also carried out by Galletly and Bart [128]. Based on classical small deflection theory, equations were derived for clamped boundaries while a slightly modified form of the equations developed by Bodner and Berks [44] was used for the case of simple supports. The results obtained from these equations were compared with test results on nine cylinders. The authors noted that good correlation could be obtained by using the procedure suggested by Holt [156] to consider the initial imperfections.

Zyczkowski and Sobiesiak [372,374] adopted the large deflection approach developed by Nash [233] to determine the influence of initial imperfections on the critical pressures of long cylindrical shells.

The effect of axisymmetric imperfections was studied by Esslinger and Geier [111] who based their analysis on Flügge's [122] theory. The computation of the buckling loads, performed by using Kalnín's [168] procedure for transfer matrices, showed that the buckling pressures were scarcely influenced by the prebuckling stresses.

2.3.1.3 Postbuckling behaviour

Early analyses concentrated on using the postbuckling behaviour to provide expressions to predict the critical pressure [171,191]. However, comparisons with test results were not always satisfactory.

Theoretical studies have subsequently been carried out by Yamaki and Tani [369] who through detailed calculations were able to obtain the end shortening, radial deflections and volume changes with increasing pressure for a wide range of shell geometries.

Experimental studies carried out by Yamaki and Otomo [367] and Dow [93] have indicated that cylinders loaded by hydrostatic pressure can retain much of their strength after buckling.

In the Soviet Union, theoretical investigations into the postbuckling behaviour of hydrostatically loaded cylinders have been undertaken by Babenko [26,27]. Asymptotic expressions describing the equilibrium path in the initial postbuckling region were obtained for thin medium length cylinders with simply supported ends. The load deflection relations he obtained were in qualitative agreement with the theory of Koiter [184].

A number of theoretical investigators have used Koiter's postbuckling theory to assess the effect of different types of initial imperfections on the critical pressures. Axisymmetric [162], dimpled shaped [9] and random [8] imperfections have all been studied. Budiansky and Amazigo [58] have expressed the imperfection sensitivity of both unstiffened and ring stiffened cylinders in terms of a postbuckling coefficient, the value of which was calculated for a wide range of shell geometries. Their results have been adopted for current design practice [86].

2.3.1.4 Boundary conditions

The effect of boundary conditions on critical load is much more important for hydrostatic pressure than for axial compression [324]. Solutions for simply supported boundary conditions were obtained by Von Mises [351] while attempts to solve clamped boundary cases were made by Nash [231], Galletly and Bart [128] and Palyi [245].

General differential equations developed by Donnell [88] and Fluegge [122] have been solved by Yamaki [365,366] for a number of boundary conditions. The clamping effect provided at the ends of the cylinder was found to be significant only for stocky cylinders (Batdorf parameter Z less than 20). This observation has been confirmed by the numerical calculations of Sobel [300], who also discovered the importance of axial restraints at the boundaries in increasing the collapse pressure of moderately stocky cylinders. On the other hand, such an effect was found to be negligible by Singer [289] from calculations based on very slender cylinders (Z up to 1000).

Seibel [276] tested thirty six nominally perfect cylinders with Z values ranging from 59 to 1817 to assess the effect of boundary conditions. The models tested with clamped ends were found to have buckling strengths approximately 30% stronger than those tested with simply supported ends. However, it has been pointed out by Galletly and Bart [128] that the differences between the collapse pressures for different boundary conditions will be smaller in the presence of initial imperfections of practical magnitudes.

Recently, the design method proposed by Windenburg and Trilling [363] for simply supported cylinders has been extended by Malik et al [205] to include other boundary conditions such as free ends and mixed conditions at each end. The case of the free end was particularly interesting since it represented the first analytical solution to the problem [205].

2.3.1.5 Plastic buckling

Two basic inter-ring collapse modes can occur when cylinders fail plastically under pressure loading. These are axisymmetric collapse in which radial deflections around the circumference are uniform, and asymmetric collapse in which alternate inward and outward lobes develop around the circumference of the shell. Empirical limits defining these two types of collapse have been obtained from tests carried out by Montague [221].

Axisymmetric collapse was initially investigated by Lunchick [200] whose experimental results showed good correlation with the rigid plastic theory of Hodge [150]. Deformation theory was used in comparisons with the experimental strain values. Good agreement was obtained in the circumferential direction [201].

The axisymmetric mode was also studied by Lee and Murphy [194] who considered the effects of prebuckling deformation caused by constraints provided by the rings. By using Drucker's [98] postulate of positive work, they developed a modified flow theory which produced good agreement with the test results of Boichot and Reynolds [46].

The asymmetric mode of collapse was considered by Reynolds [260] who derived expressions for the collapse pressure based on the differential equations of Gerard [135]. Good correlation with test results was obtained.

Graphical methods to determine the failure pressure associated with both collapse modes have been produced by Lunchick [202] while the effects of various boundary conditions on elasto-plastic collapse have been investigated by Paul [248].

2.3.2 General collapse

2.3.2.1 Elastic buckling

The ring stiffened cylindrical shell failing by elastic general instability was first considered by Tokugawa [329] in 1925. Unfortunately, his formulation did not take into account the interaction between the shell panel and the stiffeners and was generally regarded as being inadequate since it was independent of the stiffener number and ring spacings.

Salerno and Levine [269] attempted a more rational analysis with Rayleigh-Ritz energy methods involving a buckling configuration extending over the entire shell-stiffener assembly. Nash [231] and Kendrick [172] working independently discovered errors in their derivation and both have subsequently published acceptable but different solutions derived from energy principles. Kendrick's equations were rather difficult to solve and graphical solutions have been produced by Reynolds [259] for pressures in the range of 1.4 N/sq.mm to 17 N/sq.mm. Experimental verification of Kendrick's work has been produced by Reynolds and Blumenburg [261].

Analytical solutions based on energy methods were also produced by Bryant [56], but they were considered to be unconservative by some investigators [29,147]. Empirical design formulae based on extensive test results were suggested by Blumenburg [43].

Kendrick's equations were derived by assuming simply supported boundary conditions at the shell-stiffener intersection. Solutions for the clamped cases were obtained by Kaminsky [169] and Yamamoto [370]. However, tests conducted on machined cylinders by Galletly et al [132] showed better correlation with Kendrick [172] than with Kaminsky [169] and Nash [231]. The assumption of simple supports has been proposed for design considerations [175].

Kendrick's [172] work has subsequently been extended by Ross [265] to allow for different degrees of rotational restraint at the shell-stiffener boundary. Good agreement with experimental results has been obtained [266].

The effect of initial imperfections on the general

instability mode of collapse was first considered by Kendrick [173]. Hom [157] using a different out-of-roundness function obtained an expression for the bending stress introduced in the stiffener flange as a result of stiffener eccentricity.

Kinra and Miller [179,217] have performed tests on twenty fabricated cylinders to study the effect of imperfections. Six of these failed by general instability, but the results showed poor agreement with the theories of Kendrick [173], Hom [157] and Greimann [140].

The strain energy methods developed by Kendrick [172], Nash [231] and others described earlier were by no means the only theoretical approaches adopted. A differential equation approach was adopted by Radok [255], while Fluegge [120] attempted to analyse the ring stiffened configuration by an orthotropic shell theory. Bodner and Providence [45] also used the orthotropic shell approach, but their equations were derived by variational methods from strain energy considerations. Their results showed good correlation with conventional strain energy approaches as well as the experimental results of Wenk et al [360].

2.3.2.2 Plastic buckling

Notable early contributions on the plastic general instability of ring stiffened cylinders were made by Gerard [135], Lunchick [203], and Bijlaard [41]. Gerard's work which was originally based on Donnell's assumptions for elastic shells was subsequently improved by Li and Han [196] who avoided Donnell's assumptions.

Lee [193] by applying the variational principle to a flow theory formulation has obtained very good correlation with the test results of Boichot and Reynolds [46] for machined aluminium cylinders. Other test results for aluminium models have also been obtained by Nott [235] and Ross et al [267].

The effects of large deflections and initial

imperfections have been considered by Shao and Chen [279] by using orthotropic shell theory. Good correlation has been obtained with the test result from a fabricated steel cylinder.

All the experimental results reported in this section will be discussed in greater detail in chapter 5 where they will be used to provide correlation with the theoretical results obtained from a rigid plastic mechanism approach.

2.3.3 Offshore related research

In papers related to offshore research, the present hydrostatic pressure load case is often studied for the purpose of providing a theoretical or an experimental point on the combined loading interaction diagram. In view of this, it is considered more appropriate to review these papers in a later section under the title of combined loading. In this section, only the work specifically related to hydrostatic pressure has been discussed.

2.3.3.1 Theoretical approaches

The importance of initial imperfections and residual stresses on the collapse of ring stiffened cylinders with wide ring spacings was highlighted by Smith and Kirkwood [299] from calculations based on a simplified beam-column approach.

The collapse of imperfect unstiffened cylinders has been studied by Batista and Croll [34]. Reliable lower bounds to experimental results have been obtained by using the reduced stiffness method of analysis described earlier.

Reis and Walker [256] have presented a nonlinear buckling analysis for local collapse of ring stiffened cylinders. Failure is considered to have occurred when the stress state defined by Ilyushin's criterion reaches yield. The influence of postbuckling behaviour on local collapse strength has also been considered.

Both local panel collapse and general collapse have been studied by Creswell and Galletly [73]. An analytical method involving energy minimisation was developed. Comparisons with the results from the BOSOR 4 program [60] have shown the method to be efficient for estimating the general collapse of cylinders with regular ring spacings. The results are, however, confined to elastic collapse.

Theoretical information on plastic general instability has been very limited [106]. Recently, however, methods of analysis have been proposed by Ellinas and Croll [105] and by Tsang and Harding [336].

2.3.3.2 Experimental investigations

The test series conducted by Kinra and Miller [179,217] reported earlier produced plastic local and elastic general collapses. The radius to thickness ratios ranging from 16 to 64 are significantly less than those being adopted at present for the newer generation of floating platforms. Based on these test results, new equations relevant to inelastic buckling were proposed for the API design code [10].

The local panel mode of collapse has recently been obtained from a model tested by Tsang and Harding [335]. Further details regarding the fabrication technique and test data will be given in the next chapter.

2.4 Lateral pressure

The case of lateral pressure loading is generally treated in parallel with hydrostatic pressure loading. In view of this, only a brief review has been presented in this section since many of the papers have already been discussed in the previous section under hydrostatic pressure. A more detailed review has been presented by Tsang and Harding [333].

Early solutions to the problem of the stability of a cylindrical shell under lateral pressure were obtained by Lorenz [198], Southwell [305] and Von Mises [350]. These solutions expressed the critical pressure in terms of the buckled mode. Certain attempts were made to express the critical pressure only in terms of the geometry and the material properties of the shell [100,363]. Such expressions, however, generally produced results of poor accuracy [23].

Donnell [88] by neglecting the effect of transverse shearing stresses succeeded in obtaining simplified governing differential equations from considerations of equilibrium. His simplification, however, led to inaccurate results in certain cases and were modified by Loo [197]. The modification made by Loo has subsequently been criticised by Armenakas and Hermann [22] since it made the resultant stiffness matrix unsymmetrical. More accurate analyses based on Sander's [271] equations have since been made by Soong [304]. Comparison with Fluegge's [122] more rigorous equations have also demonstrated Donnell's formulation to be inaccurate when the number of circumferential buckle waves is small [365].

Other analytical solutions based respectively on minimisation techniques and vibration analyses have been obtained by Zyczkowski [372] and Shirakawa and Tamada [286].

The effect of initial imperfections on the buckling pressure had initially been studied by Sturm [315]. However, since the development of Koiter's [184] postbuckling theory, a number of attempts have been made to assess the imperfection sensitivity of cylinders under lateral pressure by monitoring the postbuckling behaviour [8,9,58].

Simites et al [288] using an analysis based on the

smeared technique with the Karman-Donnell nonlinear kinematic relations, have shown that cylinders under lateral pressure can be imperfection sensitive, but not to the same extent as for the hydrostatic pressure or the axial compression load cases.

Experimental investigations carried out by Montague [221,222] have indicated the effect of initial imperfections to be important only for certain shell geometries.

The effect of plasticity was considered by Baker et al [28]. A design formula based on the use of 'knockdown' factors to account for inelastic behaviour was proposed. These factors were dependent on a number of parameters. Formulae for a wider range of geometries have been proposed by Gerard and Becker [139] and at NASA [229].

Little information related to offshore design for this load case is available. The only experimental data with regard to the general collapse mode was produced by Tsang and Harding [335]. However, this load case has been included in a number of papers on combined loading. These will be reported in the next section.

2.5 Combined axial compression and external pressure

2.5.1 Theoretical background

Early theoretical solutions to the elastic buckling of a simply supported perfect cylindrical shell subjected to combined axial compression and external pressure were obtained by Von Mises [351], Fluegge [122], Donnell [88] and Timoshenko [327]. A solution to the clamped case was obtained by Kirshnamoorthy [180] who based his analysis on displacement functions assumed by Lackshmikantham and Gerard [189].

Using Timoshenko's and Donnell's equations linear interaction formulae have been proposed respectively by Sharman [280] and Brush and Almroth [54] for design. A

simplified design procedure using a linear interaction equation has been suggested by Dambski [80].

Tennyson et al [320,321] presented an elastic analysis based on large deflection shell equations originally set up for the buckling of an anisotropic shell [47]. They found that contrary to the prediction of small deflection theories, the interaction curve for the stockier cylinders changed from a concave locus to a convex one, making the simple linear interaction formula often used in design unconservative. The tests they performed on Mylar cylinders displayed a similar but less marked trend. However, it is unlikely that normal structural steel will display the same behaviour since the collapse of equivalent steel models would not occur elastically [129]. The authors also studied the effect of boundary conditions, particularly the restraint in the axial direction. The importance of this restraint had previously been noted by Sobel [300].

The effect of initial axisymmetric imperfections was investigated by Hutchinson [159] whose theoretical model based on Donnell's equations yielded very simple solutions. His analysis produced good correlation with the experimental results of Weingarten and Seide [356]. Comparison with the analysis of Booton [47] has also produced good overall agreement [130].

Analyses incorporating plasticity were initially presented by Radhakrishnan [254]. Shell slenderness parameters defining the onset of plastic collapse have been obtained by Zyczkowski and Bucko [373].

2.5.2 Experimental background

Weingarten and Seide [356] conducted an extensive experimental programme on Mylar polyester cylinders. The results correlated poorly with theory [277], but the linear interaction curve provided an acceptable lower bound. Their results have also been correlated against a design procedure proposed by Pedersen and Jensen [251]. Good agreement was

obtained in most cases.

Other experiments performed on Plexiglass, Melinex and brass [123,223] models have all indicated the linear interaction curve to be safe but conservative.

2.5.3 Offshore related studies

2.5.3.1 Theoretical approaches

In the last few years, both finite difference [6,19,144] and finite element [330] numerical techniques have been employed by a number of investigators [113,131,146,331] for the provision of additional analytical data for the elasto-plastic collapse of offshore tubulars under combined axial compression and external pressure. All these investigations have demonstrated the importance of initial imperfections and boundary conditions in affecting the collapse behaviour.

A simplified theoretical approach using the reduced stiffness method was developed by Croll [75] to predict lower bound elasto-plastic collapse loads for unstiffened cylinders under combined axial compression and external pressure. The method has been extended to analyse ring stiffened cylinders failing in the general [105] and axisymmetric [76] modes of collapse. Axisymmetric collapse has also been studied by Reis et al [257] and Tsang and Harding [334] using simplified theoretical models.

2.5.3.2 Experimental investigations

Early experimental research into the collapse

behaviour of offshore tubulars under combined axial compression and external pressure was initiated in 1977 at Det norske Veritas (DnV) with the testing of large scale aluminium models [142]. The geometries of the cylinders were rather slender and elastic failures were obtained in all cases.

Walker et al [355] conducted a series of tests on unstiffened steel models, but the results obtained for the collapse loads did not cover the whole range of the loading interaction.

Shell geometries relevant to the new generation of floating platforms were investigated by Tsang et al [331] through a series of tests on small scale ring stiffened models. The imperfections and fabrication techniques were representative of the prototype structures. The results demonstrated a general conservatism in design code predictions.

The work reported above represents early attempts to provide experimental data for offshore tubulars under combined loads. These and some of the earlier experimental background have been summarised by Miller and Grove [216] and Galletly and Pemsing [131].

Recently, more experimental data relevant to floating platforms have been obtained by Frieze et al [124] and Miller et al [215]. All the models tested were ring stiffened with realistic imperfections. Unfortunately, not all the experimental data have been released.

At the time of writing, a series of tests has just been completed at the University of Liverpool [131]. Both welded and machined models were tested over the whole range of the loading interaction. The results from these and some of the above tests will be discussed in greater detail in chapter 5 when they will be used to provide correlation with theoretical analyses.

Tests on thirty models with geometries relevant to fixed platforms have been performed by CBI industries [101]. The ring spacings on these models were considerably larger than those representative of floating platforms, with typical panel length to radius ratios greater than 2. Good agreement has been obtained between the API code [10] and the test results [218].

2.5.3.3 Design considerations

The lack of experimental data on this combined load case has led to the use of semi-empirical or empirical interactive buckling equations in the ASME [11], DnV [86], DAST [87] and ECCS [114] design codes. The BS5500 code [52] is also considering this problem [127]. A brief review of the general design procedures adopted by these codes as well as discussions on the choice of interactive equations has been given by Galletly [126].

Nonlinear interactive equations have been proposed by Miller [214] and Odland [237] and more recently by Faulkner et al [118] for this present biaxial compression load case. Results from [214] and [237] together with those from a linear interaction equation have been presented in [130] where comparisons with available test data have shown all three equations to be conservative.

Although the recent test series reported above [124,141,215] will provide some data to establish a more rational basis for design, more tests over a wider range of geometries are needed before any definite proposals can be made.

Despite the importance attached to experimentation, it is most unlikely that test data alone will be able to provide all the required design information over the geometric, material and loading parameters at present being adopted in practice. In view of this, some authors have chosen to base design formulations on simplified theoretical approaches. Design schemes based on the reduced stiffness method of analysis have been proposed by Croll and Ellinas [79,103] for various modes of collapse.

2.6 Torsional buckling of ring stiffeners

Many attempts have been made to determine minimum stiffener sizes for optimum designs, either to meet the criterion of minimum weight [53,136,247] or to satisfy the

requirement of avoiding general instability [337]. Whilst sophisticated optimisation techniques [246] have made it possible to design ring stiffened cylinders to the maximum efficiency, the importance of avoiding torsional buckling failures of ring stiffeners should not be overlooked. In general, design codes specify rules on stiffener geometries to prevent this type of failure from occurring. Although stiffener tripping is not one of the topics examined in this thesis, some of the related literature has, nevertheless, been briefly reviewed since this type of failure can seriously weaken the resistance of the cylinder to general collapse. This in turn may lead to catastrophic results in practice.

Early theoretical investigations into the torsional buckling of ring stiffeners were based on differential equation [359] and strain energy [174] approaches. From the results of [174], a simplified design procedure forming the basis of the current BS5500 code [52] was proposed [177].

Based on the recommendations of BS5500 [52], Creswell and Galletly [73] have developed an approximate matrix method to estimate ring stiffener sizes and stresses. A matrix approach has also been adopted in the Soviet Union by Manevich [206]. His analysis was based on the governing equations derived by Darevskiy [81].

In recent years, intensive research has been performed at Det norske Veritas where experimental testing [65,181,341], computer simulation studies [64,344] and simplified theoretical analysis [108] have all indicated the recommendations given in DnV [86] to be safe [107].

At present, experimental and analytical investigations using dynamic relaxation finite difference techniques are being carried out at Glasgow University [63]. Results from these investigations should be available in the near future.

CHAPTER 3

EXPERIMENTAL TESTS

3.1 Introduction

The experimental work described in this chapter was part of a research programme set up by the Science and Engineering Research Council of Great Britain with the aim of providing design information for stiffened cylindrical offshore members. This chapter describes two series of tests conducted by the present author to continue the experimental research effort on the collapse behaviour of ring stiffened cylindrical shells.

A total of eight 1/20th scale models fabricated with narrow ring spacings to represent the geometry of the new generation of tension leg platforms were tested under various combinations of external pressure and axial loading. During the fabrication process, detailed attention was focused on boundary conditions and initial geometric imperfections to ensure that the test models produced were accurate representations of the prototype structure.

In the first series five models with nominally identical shell panel geometries were tested under combined loads extending over the whole range of the loading interaction. The failure mode of interest was confined to inter-ring panel buckling.

The three cylinders comprising the second series were designed to fail by general instability. To allow the development of this mode of collapse, these cylinders were fitted with lighter stiffeners and were tested under pressure dominated loadings.

In the later sections of this chapter, comparisons

with various design codes both in terms of stiffener size specifications and general strength predictions are presented.

3.2 Model geometries

3.2.1. Cylinders 1-5

These five cylinders comprised the first series of tests. Cylinders 1 to 4 had nominally identical geometries, each consisting of five bays and each with a R/t ratio of 267 and a ring spacing of $0.15R$ for the centre panel. Ring stiffeners for these models had a thickness 2.67 times that of the centre bay shell plate and a depth equal to 8 times the thickness. The second moment of area including an effective length of shell plating, I_{Lef} , was 569.34 mm^4 . These four cylinders were tested to failure under different ratios of axial compression and external pressure loading so that an interaction curve describing the ultimate strength of the models over the entire range of the interaction was obtained. Cylinder 5 was identical to the other four models except that it had more slender ring stiffeners ($H_w/t_w=16$, $I_{Lef}=149.16 \text{ mm}^4$). This fifth model was tested under hydrostatic pressure. The object of this test was to observe any premature torsional buckling of the stiffeners due to the high slenderness.

The models were designed so that the centre panel would fail first under all the loading combinations envisaged. To ensure this, the centre bay of each model was designed to be longer than the other bays, the differences between the length of these bays being determined from previous analyses [144]. By maintaining the thickness of the adjacent bays the same as that of the centre bay, the boundary condition at each end of the centre panel was kept representative of the prototype structure. The end bays were made from thicker panels to ensure that local boundary

effects and increased imperfections did not cause premature failures in these bays. The use of five bays provided more uniform stress conditions for the centre panel. The geometry of all five models is given in table 3.1.

3.2.2 Cylinders 6-8

These three models each consisted of five equal bays and were designed to fail by general instability. The shell panel geometry of each bay with a R/t ratio of 267 and a length of $0.15R$ was identical to that of the centre panel of the models tested in the first series.

Cylinders 6 and 7 were reinforced by internal T-stiffeners with a web depth to web thickness ratio of 8.33 and a flange depth to flange thickness ratio of 10. Investigations carried out by Ellinas et al [107] have indicated that these stiffener aspect ratios are typical of offshore design. Cylinder 8 was fitted with internal flatbar stiffeners with a web depth to web thickness ratio of 8.33. The geometry of these models is given in table 3.2.

Cylinder 6 was tested under hydrostatic pressure and cylinders 7 and 8 under lateral pressure with no axial loading.

3.3 Fabrication

Manufacture of the test models was carried out by a firm of specialist fabricators (Research Models and Equipment Ltd., Reading). Despite the small scale of the models, it was still possible to connect the stiffeners and shell panels by TIG welding. Previous experimental work [97] had indicated that the initial geometric imperfections

resulting from this welding technique were within design rule tolerances.

Due to the small dimensions involved, the flatbar stiffeners were cut from flat plates while the T-stiffeners were machined directly from a thick circular tube to avoid the necessity of flange to web welding. The shell panels between the stiffeners were rolled from flat plates, the length of which included an allowance for weld shrinkage. Cylindrical panels were formed by applying the closing longitudinal butt welds. Two adjacent panels were then put around copper formers and inserts, together with the intermediate ring stiffener. All three components were rigidly held in a jig. A butt weld was then laid to connect the stiffener and the cylindrical shell panels together. Next, the formers were removed and the same procedure repeated for the other stiffeners.

After all the welding was completed, the models were heat treated to remove any unrealistically high residual stresses. Before this stress relieving was performed, two of the cylinders in the first series (cylinders 1 and 2) were sent to Imperial College to enable the initial imperfections to be measured. This was to monitor the changes to the shape and magnitude of the initial imperfections as a result of stress relieving. Results of these measurements are presented in a later section. The stress relieving process itself was carried out using appropriate jigs to maintain tolerance levels.

In the final stage of fabrication, end rings were attached to provide effectively clamped end conditions for the whole model. Two types of end rings were used. These are shown in fig.3.1. For the combined loading tests, the type shown in fig.3.1a was used. This type of end ring contained small accurately machined grooves to which the end of the cylinders were fitted. A sand-araldite mixture was then cast into these grooves to hold the cylinder in position and hence provide the required clamping effect. The type of end ring used for the axial compression test is shown in fig.3.1b. From previous experience [97], this type of end ring was found to provide a better and more uniform loading condition since the cylinders could protrude through the sand-araldite mixture to come into direct contact with the loading platen.

3.4 Material testing

The cylindrical shell panels were fabricated from sheets of 0.6mm steel. From these sheets, test coupons were also cut and subjected to heat treatment at the same time as the models. Tensile tests were carried out from these coupons to determine the material properties of the shell panels. Testing was conducted in an servo-controlled testing machine at an approximately constant rate of elongation.

In the first series, eight such specimens were tested, the dimensions of which are shown in fig.3.2. Half of these specimens were cut parallel and half transverse to the direction of rolling. Two of the specimens tested were cut from pre-heat treatment coupons to assess the effect of the stress relieving process on the material properties. A summary of the test results is presented in table 3.3.

In the second series, the failure mode of interest was general instability. In addition to measuring the material properties of the shell panels, test coupons cut from the circular tube used to produce the ring stiffeners were also tested to determine the material properties of the stiffeners. All specimens tested in this series were heat treated at the same time as the test models. The results are also given in table 3.3.

The load-extension curves for both series were very similar. Typical examples for both pre-heat treatment and post-heat treatment specimens from the first series are shown in fig.3.3. It is apparent that the heat treatment process did not change the material properties significantly. This suggested that the models did not contain high residual stresses. As a result, pre-heat treatment specimens were not examined in the second series.

Table 3.3 shows that for specimens transverse to the direction of rolling, values for both the tensile yield stress and Young's Modulus are higher compared to those for the specimens cut in the longitudinal direction. However,

the maximum difference, less than 6.5%, is tolerably small and isotropic material behaviour has been assumed in all subsequent analytical correlations. For all specimens, work hardening occurred at about 3.2% strain.

Different approaches exist for the estimation of compressive yield stress. For the present study, a value of lower tensile yield + 10 N/sq.mm has been adopted.

3.5 Test rigs

Two rigs were used for the present experimental tests. The first, at Imperial College, was used to measure the initial geometric imperfections of all eight models and also to perform the axial compression test for cylinder 1. The second rig, at Surrey University, was used to perform the combined loading tests.

The rig at Imperial College is shown in fig.3.4a. It consists of a circular frame of 2.5 m high and 1.0 m in diameter. This frame is made up of four 125 mm thick plates acting as a multiple sandwich for the test model, load cells and screw jack. The screw jack is located between the two bottom plates. During the axial compression test, the load was applied by reaction against the upper plate through six 50 mm diameter tension rods. The load was transferred through three load cells which provided the only means of support for the plate carrying the model and transducer frame (shown in fig.3.4b). Although it was self-reacting, the entire rig was anchored to the laboratory floor by means of a 200 ton capacity cross frame. This considerably increased the stiffness of the rig and allowed the post-collapse path to be followed with greater precision.

The rig at Surrey University is shown in fig.3.5. Basically, it consists of a self-reacting frame with a heavy flange to which a hyperbaric chamber capable of sustaining a working pressure of 14 N/sq.mm is fitted. Axial load is applied by a servo-hydraulic actuator which has a capacity

of 500 kN and a reading accuracy of ± 0.025 kN. The ram displacement on the actuator is controlled using a signal from a transducer mounted on the frame of the rig. During the tests, the models were placed on the mechanical mounting ring fitted to the heavy actuator via a spherical seating which could be inverted to provide either axial tensile or compressive loading. A fuller description of this rig is given in ref.[355].

3.6 Instrumentation

For the axial compression test conducted at Imperial College, readings from the load cells, linear voltage displacement transducers (L.V.D.T.) and strain gauges were stored automatically on floppy discs to facilitate subsequent processing. A Solartron System 35 data logging facility, comprising data scanners, microprocessor, visual display unit and line printer, was used throughout the test.

For the combined loading tests at Surrey University, transducers were not available to measure the deflections of the models. The output from the load cell was recorded on a chart running at 1 mm/min. The power for the actuator was supplied by a hydraulic power pack which had a 40 l/min capacity. This was well in excess of the requirement of the present tests. Two pumps were used to drive water into the test chamber; a low pressure pump with a reversible valve arrangement and a small positive displacement pump which had a variable stroke to permit the adjustment of flow rates. Outputs from these pumps were recorded on a x-y plotter which was adjusted to run at 10 mm/min to synchronise with the load cell outputs. For each load increment, strain gauge readings were recorded on paper tape outputs via an Orion 3530 data logging system.

3.7 Strain gauging

In the first test series, the centre panels of the five cylinders were strain gauged at the four quarter points around the circumference to monitor the uniformity of loading and to determine the areas of first yield. In addition, some strain gauges were placed on the ring outstands to obtain the stress levels in the stiffeners.

For economic reasons, strain gauging of the cylinders tested in the second series was confined to the section diametrically opposite the longitudinal weld line. This section was selected because it represented the location likely to be least affected by residual stresses and initial imperfections resulting from welding. For this reason, strain gauging of the stiffener outstands was also chosen at this circumferential location. For cylinder 6, the three bottom stiffeners and shell panels were all strain gauged. Cylinders 7 and 8 were only strain gauged at the centre panel and at the stiffener defining the bottom boundary of this panel.

The type of strain gauges chosen was dictated by their location on the test models. On inside (dry) surfaces, T.M.L. (FLE 1 and FCA 1) gauges were used because of their relative cheapness whilst for outside (wet) surfaces, Micromasurement (CEA-06-062) gauges were chosen in view of their better reputation for water resistance. The products used for waterproofing described below were also manufactured by Micromasurement.

The strain gauges on the outside surfaces of the models tested in the hyperbaric facility at Surrey University had to be waterproofed in a manner capable of sustaining the high external pressure envisaged. Each of these gauges was first bonded to the model surface using M Bond 200 adhesive. After the connection leads were soldered on, the gauge was covered by a thin polyurethane shield. The surrounding area was then coated with a sealing agent (M Coat D) and a thick mastic coating (M Coat F) was applied to give further protection. Finally, the whole area was covered by an adhesive foil to ensure a watertight seal with the mastic.

3.8 First test series

3.8.1 Measurement of imperfections

The five cylinders in the first series were all scanned to determine the initial geometric imperfections of the cylinder surface. Each model was measured at forty circumferential and fifteen longitudinal locations over the centre three bays.

The readings were obtained using a measurement frame consisting of a ring supporting forty transducers equally spaced around the circumference and in contact with the shell surface (fig.3.4b). Each transducer was carefully calibrated and it is estimated that the readings obtained were accurate to within ± 0.05 mm. The ring supporting the transducers could be raised or lowered by means of threaded supporting columns. The datum for the imperfection measurements was provided by an accurately machined solid dummy cylinder.

The results obtained were analysed by a program described in ref. [1] which uses a least square fit to determine the coordinates of the cylinder which best fits the imperfection data and then calculates the deviation of all points from this perfect cylinder. It also calculates the coefficients of the following Fourier series to represent the imperfections.

$$w_0 = \sum_{m=1,2,\dots} \sum_{n=0,1,\dots} (A_{mn} \cos n\theta + B_{mn} \sin n\theta) \sin \frac{m\pi x}{L} \quad (3.1)$$

It should be pointed out that this series is only approximate since it generates zero imperfections at the ring stiffener boundaries. However, this series has been chosen for subsequent finite element analyses to conform

with the work of a previous investigation [97]. Data input as individual mesh values rather than as Fourier coefficients was also used in the analytical correlation to assess the accuracy of the above approximation. Results from these analyses are described in the next chapter.

3.8.2 Description of testing

Cylinder 1 was tested at Imperial College where with the availability of more extensive data logging equipment and without the restriction imposed by the hyperbaric chamber, it was possible to make use of more extensive gauging. The results from these gauges are shown in figs.3.41 to 3.61. The model failed 'softly' at 69% of the squash load. Actual values of all failure loads are shown in table 3.4. The buckling mode shown in fig.3.111 consisted essentially of a single half-wave outward bulge of the centre panel. No significant visual buckles occurred prior to the attainment of peak load.

Cylinders 2 to 5 were tested in the hyperbaric facility at Surrey University under various combinations of axial compression and external pressure. Cylinders 2 and 5 were tested under hydrostatic pressure. The behaviour of these two cylinders was very similar, both failing with a loud audible sound at very near the squash pressure. This was slightly unexpected since cylinder 5 was fitted with much lighter stiffeners, the inertia of which was significantly outside the requirements of DnV. Cylinder 3, under pure lateral pressure, provided extremely high buckling resistance and did not fail until the pressure had reached 1.87 p . The failure was gradual in this instance. The only indication of failure was a drop in the pressure gauge reading. Cylinder 4, under an axial to hoop stress ratio (K) of 1.5, failed at an unexpectedly low load which fell below the interaction curve established by the other cylinders. Failure was accompanied by a very slight 'pop' sound.

During the testing of these cylinders, strain gauge readings were taken at each load increment. After failure, each cylinder was brought back to Imperial College to have the buckling deflections measured. The strain gauge readings, buckling loads and buckling modes are shown respectively in figs.3.62 to 3.100, table 3.4 and figs.3.112 to 3.115. Detailed discussion of these results is given in the next section.

3.8.3 Discussion of experimental results

3.8.3.1 Imperfection results

From the imperfection results obtained, plots have been produced showing the vertical and plan sections of the centre panel of each of the models. These are shown in figs.3.20 to 3.33. To give overall impressions of the shapes of the imperfections, the centre panel of each model has been plotted as a three dimensional surface. These are shown in figs.3.34 to 3.40. The Fourier coefficients A_{mn} and B_{mn} as defined by eqn.3.1 are shown in figs.3.6 to 3.19. Comparisons with DnV tolerances are given in table 3.5.

For cylinder 1, the imperfections were measured twice, once before and once after heat treatment. Figs.3.27 and 3.29 indicate that the heat treatment process did not significantly change the shape or the magnitude of the imperfections. This can be confirmed by the plots of the Fourier coefficients in figs.3.6, 3.7, 3.10 and 3.11 which show the dominance of the $n=2$ mode both before and after stress relieving. In general, the imperfections were quite small. All values were well within DnV tolerance limits.

The imperfections of cylinder 2 were measured twice in a similar manner to cylinder 1. In this case, the heat treatment process has increased the magnitude of the imperfections although the mode has remained comparatively unaffected. The average value of the post-heat treatment

Fourier coefficients is higher than that for pre-heat treatment with the $m=1$ mode playing a more dominant role (figs.3.8, 3.9, 3.12 and 3.13). Although the average magnitude of the imperfections was higher than that of cylinder 1, DnV tolerance limits were still satisfied.

Cylinder 3 exhibited a dominance of the $n=2$ mode as shown in figs.3.14 and 3.15. The magnitude of the imperfections was larger than that of cylinders 1 and 2 and the DnV tolerance for out-of-straightness was violated by about 30% in local areas.

The magnitude of the imperfections in cylinder 4 was, in general terms, similar to that of cylinder 3. The shape was, however, significantly different in that no distinctive mode could be identified. From fig.3.16, it can be seen that the distribution of the Fourier coefficients has a large number of small peaks with no definite dominance of any particular mode. Physically, such a distribution may be interpreted as a series of circumferential ripples, some of which may have very short wavelengths. Like cylinder 3, this cylinder also violated the DnV out-of-straightness tolerance requirement by some 30% in local areas.

For cylinder 5, the $n=2$ component is particularly dominant as shown in fig.3.18 (note difference in vertical scale). Higher modes are also present although not to the same extent as for cylinder 4. Like cylinders 1 and 2, this cylinder satisfied all DnV tolerance limits.

3.8.3.2 Strain gauge results

The strain gauge results from the five cylinders are presented in figs.3.41 to 3.100. The stresses and strains shown are the membrane components being an average of the two surface values. In these plots, compressive quantities are shown as positive. All stresses were computed from the constitutive relations of elasticity and are therefore invalid once surface yielding has occurred.

Cylinder 1

Except at the weld line (location E), the longitudinal distribution of the axial stresses and strains was quite uniform as shown in fig.3.53. The circumferential distribution of these quantities was also quite uniform as can be seen from fig.3.41 to 3.46. However, when the applied load reached about 90 kN, location W showed only a marginal increase in axial strain. This was accompanied by rapid increases in the axial strain at the other locations, especially at the mid-panel section of location E. In general, the axial stresses did not deviate greatly from the values predicted by simple membrane considerations ($\sigma_a = N/2\pi Rt$). Yielding was initiated by the axial strain in the middle of the centre panel at the longitudinal weld.

The circumferential distributions of hoop strains (figs.3.47, 3.49 and 3.51) were very similar to those of the axial strains except that in these cases, the Poisson expansion relative to the rings resulted in the strains being tensile. Again at about 90 kN, strains at location W began to stop increasing and large hoop strain increases at the other locations were recorded. Except at the mid-panel of location E, hoop stresses were generally tensile and, as expected, were much lower than the corresponding axial stresses. The distributions are illustrated in figs.3.48, 3.50 and 3.52. Comparison between these figures indicates that the average hoop stress in the bottom half of the centre panel was much higher than that in the top half.

For the ring stiffeners, stresses were calculated assuming negligible radial stresses since the geometry of these stiffeners would lead to insignificant radial components. Figs.3.54 to 3.57 show the circumferential distributions of the stresses and strains in the two ring stiffener outstands. These distributions were not as uniform as those for the shell panel. As expected, all these quantities were tensile because of the effect of the

relative Poisson expansion of the shell panel.

Figs.3.59 and 3.61 show the radial variation of the stiffener outstand hoop stresses. It may be seen that the stiffeners were primarily subjected to bending action in the circumferential direction. The amount of bending depended on the location around the circumference, but the general magnitude of the stresses was very low indicating that the design of these stiffeners was very conservative. For the present axial compression load case, the ring stiffeners had an inertia very substantially larger than the DnV requirement.

Cylinder 2

For this cylinder, a few of the gauges did not function properly and some of the data had to be interpolated to complete the plots in figs.3.62 to 3.65. The circumferential distribution of hoop strains and stresses at the mid-panel section (figs.3.64 and 3.65) indicates that the applied pressure was quite evenly resisted although the strains at location S were slightly less than those at the other locations. Yielding was initiated by the mid-panel hoop strains. For all load increments, the measured hoop stresses were considerably lower than the nominal hoop stress value of pR/t which does not take the effect of the axial component of the pressure loading into account.

The distribution of the axial stress and strain (figs.3.62 and 3.63) was very similar to the corresponding hoop stress and strain distribution. As expected, the axial stresses were smaller than the hoop stresses, but the differences were considerably smaller than the factor of 2 predicted by simple theory ignoring the presence of the rings.

Figs.3.66 and 3.67 show the circumferential distributions of the hoop strains and stresses in the stiffener outstand at section 11. Both distributions were uniform, the magnitudes being much lower than the equivalent

values recorded on the shell panel. The radial variation of the hoop stresses and strains in the stiffener outstand (fig.3.68) indicates that the stiffener was subjected to some bending. The general stress level was well below yield. This was expected since under the present hydrostatic pressure loading, the stiffeners were designed with an inertia of 2.8 times the DnV requirement.

Plots of nominal stresses versus experimental average strains shown in figs.3.69 and 3.70 are basically linear. The equivalent stress versus equivalent strain plot (fig.3.71) is of a similar form with failure occurring when the equivalent stress reached a value of 0.91 of the nominal yield stress.

Cylinder 3

As anticipated, the highest recorded strains in the cylinder were the shell panel hoop components, the circumferential distribution of which is shown in fig.3.74. The increases were very uniform in the lower pressure range. However, when the lateral pressure loading reached 0.86 N/sq.mm, the increase at location E became more rapid than at the other locations and eventually it was at this location that plasticity was first recorded. The circumferential distribution of the shell panel hoop stresses (fig.3.75) has a similar shape to that of the hoop strains. The magnitude of these stresses was generally lower than the nominal average value ignoring the effect of the rings.

Although very small, some shell panel axial strains were recorded (fig.3.72). They were tensile due the Poisson effect. Some small axial stresses were also present as shown in fig.3.73. This could be due to the fact that the axial component of the pressure loading had not been completely offset by the actuator supplying the axial tensile load.

The circumferential distribution of the hoop strains and stresses in the stiffener outstand was very uniform as

shown in figs.3.76 and 3.77. It had been anticipated that this lateral pressure load case would induce the highest stresses in the ring stiffeners. However, no sign of yielding was detected from the strain gauge data. In fact, all the stiffener stresses remained quite small even at failure, thus demonstrating an apparent conservatism in the design of these ring stiffeners. Under the present lateral pressure load case, the stiffeners had an inertia of 1.6 times the DnV requirement. From the radial variation of the hoop stresses and strains in the stiffener outstand shown in fig.3.78, it is clear that the stiffeners were not subjected to a high degree of bending.

Figs.3.79 and 3.80 show respectively the plots of nominal hoop stress versus average recorded hoop strain and equivalent stress versus equivalent strain. These graphs remained linear even when the yield stress was well exceeded. Failure did not occur until the pressure had reached over 1.8 times the nominal squash pressure ignoring the presence of the rings. This indicates that for cylinders with the present short ring spacing, boundary restraints provided by the ring stiffeners must be taken into consideration for any theoretical prediction of the failure pressure. For this cylinder, there is little doubt that failure was caused by yielding rather than by loss of stability.

Cylinder 4

The plots of the circumferential variation of the mid-panel axial strains and stresses in figs.3.81 and 3.82 indicate that the axial loading was evenly applied. The axial stresses were generally above the theoretical average values. The circumferential distribution of the mid-panel hoop strains and stresses were irregular (figs.3.83 and 3.84). The hoop stresses at location N remained essentially constant as the loads were increased whereas those at the

diametrically opposite point (location S) increased disproportionately fast compared with the other locations. This was probably caused by the imperfection shape which could have hindered hoop expansion at location S causing a build up of local stresses there. This explanation is reinforced by fig.3.84. The general shape of these stress and strain distributions suggests that the prebuckling deformation consists of circumferential ripples.

The circumferential distribution of the hoop stresses in the stiffener outstand in fig.3.86 shows that the stiffeners were quite evenly stressed except at location E where the stresses could have been influenced by the presence of the longitudinal weld. The distribution of the stiffener outstand hoop strains (fig.3.85) has a similar shape to the corresponding stress distribution. Both stiffener stresses and stiffener strains were very low compared to the values recorded in the shell panel, but the degree of bending is quite high (fig.3.87). Such low stresses are not unexpected since for the present load case of $K=1.5$, the stiffeners had an inertia of 3.6 times the DnV requirement.

The plots of nominal stresses versus average recorded strains in the axial and hoop directions are shown in figs.3.88 and 3.89. Even at failure, all stresses were below yield. The membrane stress-strain relationship in the axial direction is much more linear than that in the hoop direction. Prebuckling deformation in the form of circumferential ripples probably induced significant bending stresses in the hoop direction. The plot of equivalent stress versus equivalent strain in fig.3.74 is virtually linear, indicating that plasticity did not play an important role in controlling failure.

Cylinder 5

The mid-panel circumferential variation of hoop strains and stresses illustrated in figs.3.93 and 3.94

indicates that both these quantities assumed very even distributions. First yield was detected in the hoop direction in the middle of the centre panel. The hoop stresses were near the nominal theoretical average values although they were generally on the low side. The circumferential distribution of the axial strains and stresses in the mid-panel section was uniform indicating that the axial component of the loading was evenly applied (figs.3.91 and 3.92). The axial stresses were also quite near the nominal values.

The distribution and magnitude of the stiffener outstand hoop stresses were of particular interest since this cylinder was designed with lighter stiffeners compared with the other four cylinders in this series; the inertia was only 0.61 of the DnV requirement. Before the test, it was conjectured that such a slender geometry might lead to general collapse of the whole model induced by local stiffener tripping. However, the test result has shown that not only did the stiffeners not fail, but the stresses remained well below yield (fig.3.97). This indicates that the stiffener size requirements specified in DnV may be very conservative.

Figs.3.98 to 3.100 illustrate respectively the plots of the nominal axial stress versus average axial strain, nominal hoop stress versus average hoop strain and equivalent stress versus equivalent strain. These relationships remained essentially linear up to the failure pressure which was very near the squash value.

3.8.3.3 Failure load and collapse mode

The failure load of all five cylinders is shown in table 3.4. Comparisons with design codes are discussed later. After each test was completed, the buckled cylinder was placed back into the imperfection measuring rig at Imperial College to enable to buckled deflections to be scanned. The results are shown in figs.3.101 to 3.105. The

magnitude of the deflections has been magnified by ten times to enhance the buckling characteristics. Since the tests were stopped at different points on the postbuckling path, the relative magnitude of the deformations for different cylinders has no significance. Three dimensional surface plots and photographs of the buckled cylinders are shown respectively in figs.3.106 to 3.110 and figs.3.111 to 3.115.

The buckling mode of cylinder 1 consisted basically of a single half-wave outward bulge around most of the circumference as shown in fig.3.111. Fig.3.101 indicates that the cylinder buckled with small ripples extending over local areas. In general, the buckled deflections were quite small except in these local areas, e.g. locations scanned by transducer no. 2 and 22.

Cylinder 2 failed at just above the squash pressure with a buckling mode which comprised a well defined longitudinal half-wave inward bulge accompanied by some local ripples. This is clearly illustrated in fig.3.102. The most prominent circumferential mode is that corresponding to $n=2$, but an axisymmetric component is also evident. The deflections of the ring stiffeners (vertical section 5 and 11) were significant although the original circular form was maintained even after failure. For this cylinder, the relationship between the failure mode and the imperfection mode is not well defined.

Cylinder 3 failed at over 1.8 times the nominal squash pressure, indicating that for the present short ring spacing, boundary conditions provided by the ring stiffeners must be taken into consideration to make the concept of squash pressure meaningful. For such short panels, the boundary conditions must be expected to play a very important role in controlling collapse. If the squash pressure was calculated by including full moment restraints at the rings in the consideration of equilibrium, the ratio between failure pressure to squash pressure would drop to 1.2. This shows that for the present lateral pressure loading and stiffener geometry, the assumption of clamped boundaries is reasonable. Moreover, in view of the fact that panels adjacent to the centre bay were simultaneously loaded by the same pressure, prebuckling rotations about the plane of the stiffeners must be expected to be minimal.

At failure, general deflections were not large and

the buckling mode was a shallow inward half-wave. Figs.3.103 illustrate that at each of the seven vertical sections where buckling deflections were measured, the radial deformations were quite uniform around the entire circumference. The formation of such an axisymmetric pattern indicates the importance of yielding in controlling collapse. It is generally accepted that this type of collapse is not very imperfection sensitive.

Cylinder 4 generated the failure mode of the greatest interest with buckling deformations which were partially inward and partially outward in the form of circumferential waves as illustrated in fig.3.104. It may be noted that this failure mode was obtained from a model which had an imperfection shape lacking any dominant single mode. Comparison of the shape of the initial imperfections (fig.3.32) with the shape of the failure mode (fig.3.104) shows that for some of the vertical sections, the shape of the failure mode was, in simplistic terms, a direct magnification of the shape of the initial imperfections. This indicates that the initial imperfections must have had a marked influence on the buckling mode. The failure load of this cylinder fell below the interaction curve formed by the other cylinders. This suggests that for this ratio of combined loading ($K=1.5$), cylinders of the present geometry may become very imperfection sensitive since outward bulging caused by axial compression is counteracted by the presence of external pressure and results in very small prebuckling deformations.

Like cylinder 2, cylinder 5 was also tested under hydrostatic pressure. Its failure pressure was very similar to that of cylinder 2 being very near the nominal squash pressure.

The two ring stiffeners at the boundaries of the centre panel of cylinder 5 behaved rather differently. At the top stiffener (vertical section no.11 of fig.3.105), the buckling deformations were very small as confirmed by the low strain gauge readings obtained. The bottom stiffener (vertical section no. 5), however, deformed significantly with a large number of local ripples. From visual examination of the buckled model, it appears that these large ring deformations were caused by large deflections in the bottom half of the centre panel. It is unfortunate that

due to limited data logging facilities, strain gauging of this stiffener was not possible. Additional strain data would have provided useful information for assessing the conservatism in the design of these smaller stiffeners. Elsewhere, in the lower half of the centre panel, deformations consisted mainly of large local ripples. In the top half of the centre panel, deformations were, in general, considerably smaller and much more uniform. It is difficult to relate the buckling mode to the initial imperfection shape, but from the fact that this cylinder failed at a pressure virtually identical to that of cylinder 2, it would be reasonable to suppose that for the hydrostatic pressure load case, cylinders with the present geometry are not very imperfection sensitive.

3.8.3.4 General comparison of behaviour of cylinders 1-5

The strain gauge data obtained from the present series of tests revealed that the failure stress states of the centre panels of the cylinders were very different. Cylinder 4 remained virtually elastic throughout the test with only a very small amount of plasticity occurring at failure. In contrast, for the case of cylinder 3, plasticity had spread considerably in the centre panel even at 60% of the ultimate pressure. These results demonstrate the importance of loading type in affecting the stress state of cylinders with the present geometry.

For cylinder 1, a number of strain gauges had stopped functioning before the failure load was reached making it very difficult to assess the exact extent of plasticity at failure. However, extrapolation of the available data suggests that at failure, the centre panel state must have been at least partially plastic. Cylinders 2 and 5 were subjected to identical loading and it is not surprising to find that the failure stress state of these cylinders was very similar in that there was extensive

yielding.

In general, for the present cylinders plasticity, when it occurred, was first detected in the middle of the centre panels since this location often produced the highest bending strains. The direction in which initial yielding occurred was governed by the ratio of the combined loading.

A phenomenon common to the cylinders tested in this series was the fact that all stresses in the stiffener outstands remained well below the elastic limit even after failure. However, the magnitude and direction of bending did vary considerably between the cylinders as illustrated in fig.3.116. The pure axial compression load on cylinder 1 produced only a very low level of stress in the ring stiffener, with tensile hoop stresses causing the bending. On the other hand, the stiffeners of cylinder 3 were subjected to very little bending, but as expected, the general stress level was much higher than that of cylinder 1. Cylinders 2 and 4 had a very similar stress distribution in the ring stiffeners in that the stresses were rather low, the direction of bending being one which tended to reduce the curvature of the ring. These two cylinders, under a combination of axial and lateral pressure loading produced stiffener stress distributions intermediate in shape between those of cylinders 1 and 3. Cylinder 5 was fitted with smaller stiffeners (section modulus= 18.81mm^3) and it is not unexpected to find that the stiffener outstand stress level was higher than that of the other cylinders (section modulus= 63.97mm^3). Like cylinder 2, cylinder 5 was also subjected to hydrostatic pressure, but since the section modulus was smaller, the degree of bending was also smaller for a given stress level.

3.8.4 Comparison with design codes

Comparisons were made between the failure loads obtained from the experiments and those predicted by the ASME [11], DnV [86], DAST [87] and ECCS [114] design codes.

These comparisons are presented in fig.3.118 in the form of an interaction diagram.

For pressure loading, comparisons are difficult since predictions from codes are generally quoted in terms of an allowable hoop stress which is not a quantity conveniently obtainable from the experiments due to the effects of plasticity and the model scale. Hence, an assumption regarding the degree of restraint provided by the rings must be made in order to assess the influence on the load carrying capacity of the centre panel. The curves presented in fig.3.118 ignore such an interaction between the rings and the panel. Pressure is related to the hoop stress by the simple relationship: $\sigma_{\theta} = pR/t$ which is applicable for an unstiffened cylinder or for the panel stress of a cylinder with widely spaced rings. If this value is used, all the design code predictions are very conservative in the pressure dominated area. Of all the codes quoted, only the DnV code gives guidance to calculate the hoop stress from the pressure loading at the same time allowing for panel-stiffener interaction. Calculations from the DnV code produce an equivalent experimental hoop stress of $\sigma_{\theta} = 0.31pR/t$ at the ring position. Using the experimental failure pressure of $1.87p_0$, σ_{θ} becomes $0.58\sigma_0$. This compares with a DnV allowance of $0.78\sigma_0$ which is clearly unconservative by some 35%. DnV does not give any guidance on how the hoop stress varies along the panel. Clearly some average value between the two extremes of pR/t (or a slightly reduced value) and $0.31pR/t$ should be used. The factor of 3 on strength which is to be covered means that this stress needs to be known accurately if a reasonable prediction is to be made. A plot of the experimental applied pressure versus the recorded hoop stress in the middle of the centre panel of cylinder 3 as shown in fig.3.117 gives a relationship of $\sigma_{\theta} = 0.97pR/t$ (c.f. $\sigma_{\theta} = 0.31pR/t$ for a wide ring spacing). Considerable variation therefore does exist over the panel length. This value was, however, derived by considering the stresses in the elastic range only. Its validity must be questioned at higher pressures, particularly in the plastic range, when the greater prebuckling rotations are likely to induce considerable bending stresses. Moment reaction at the rings would then play a more important role in resisting the applied pressure

although once plasticity has occurred at these locations, resisting moments will inevitably reduce. It should also be noted that the hoop stress prediction for the ring location in the DnV code is independent of the ring spacing which seems inconsistent with the effect being considered.

The DAST code presents allowable stress in hoop stress terms but defines the pressure-hoop stress relationship as $\sigma_{\theta} = pR/t$. The ECCS code quotes allowable load capacity in terms of pressure. Both these codes are very conservative in the pressure dominated area.

In the ASME code, cylindrical panel design is treated by assuming the boundary conditions at the ring stiffeners to be simply supported. Hence, the predictions obtained from this code are also very conservative, though not to the same extent as those of the DAST and ECCS codes. However, Miller [214], upon whose work the relevant section of the ASME code was originally based, suggests that the DnV code should be used to give a more accurate assessment of the panel hoop stress under external pressure.

In the axial load dominated area of the interaction, all the code predictions are less conservative, particularly bearing in mind that the imperfections of the test models were generally less than the design code tolerances. It is interesting to note that although the failure mode of cylinder 4 was lower than expected, it was, nevertheless, still above all design code predictions.

3.9 Second test series

3.9.1 Measurement of imperfections

The initial imperfections of the three cylinders comprising this series were measured on both the shell panels and the ring stiffeners. Those on the shell panels were measured and analysed in the same way as for cylinder 1

to 5. However, in the present series, all five bays were scanned in detail since failures were expected to occur in the overall collapse mode. For each cylinder, 31 longitudinal and 40 circumferential sections were measured resulting in readings at 1240 locations around the shell surface.

The ring stiffeners were also intensively scanned to measure their imperfections. The readings were recorded by rotating the cylinder on a turntable relative to a fixed transducer. For the T-stiffened cylinders, 6 readings (3 at the flange and 3 at the web) were taken at each of the 40 circumferential sections. For the flatbar stiffened cylinder, only 2 readings were possible at each circumferential section. The raw data for each stiffener were processed by a least squares analysis using the stiffener/shell intersection point at the longitudinal weld line as the reference datum to obtain the 'best-fit' stiffener. Deviations of the raw data from this 'best-fit' stiffener provided the imperfection values.

3.9.2 Description of testing

Cylinders 6 to 8 were designed to fail by general instability and were tested in the hyperbaric facility at Surrey University. Cylinder 6 was tested under hydrostatic pressure. It failed unexpectedly by local collapse of the bottom panel (see fig.3.155) at 91% of the nominal squash pressure. Cylinders 7 and 8 were subjected to lateral pressure and failed in the intended general collapse mode. The failure load and collapse mode of these two cylinders are shown in table 3.4 and figs.3.157 to 3.160 respectively. Failure of all three cylinders in this test series was accompanied by a slight audible sound.

After failure, the cylinders were transported back to Imperial College to have the buckled deflections measured. These measurements together with the initial imperfection and strain gauge results are discussed in the

following sections.

3.9.3 Discussion of experimental results

3.9.3.1 Imperfection results

From the initial imperfection measurements on the shell panels, plots have been produced showing the vertical sections as well as the plan section of the centre of each of the five panels and the four ring stiffener positions (figs.3.131 to 3.136). Plan sections showing the intermediate positions and other detailed results are presented in ref.[338]. The major Fourier coefficients defined by eqn.3.1 are plotted in figs.3.119 to 3.130. Comparisons with DnV tolerance limits for the shell panels are presented in table 3.5. Corresponding comparisons for the ring stiffeners are given in table 3.6. Three dimensional plots of these stiffeners are shown in figs.3.137 to 3.139.

Figs.3.119 to 3.130 show that the $n=2$ mode was dominant for all three cylinders, although for cylinders 7 and 8 higher modes were also present to a significant extent. The general level of shell imperfections was similar for all three cylinders. DnV tolerances were exceeded in cylinder 6 in local areas (table 3.5). Figs.3.131 to 3.133 indicate that the shell imperfections at the ring stiffener boundaries were smaller than those in the inter-ring panels. The welding of these stiffeners has, in some cases, produced slight kinks in the imperfection shapes.

The ring stiffener imperfections shown in figs.3.137 to 3.139 exceeded the DnV tolerance values in virtually all cases (table 3.6). Such large imperfections were somewhat unexpected since the ring stiffeners were machined as an integral unit. They were probably caused during welding of the stiffeners to the shell panels.

3.9.3.2 Strain gauge results

The strain gauge data are presented in figs.3.140 and 3.145. All stresses and strains shown are membrane components.

The longitudinal stress and strain distributions in the bottom three panels of cylinder 6 are shown in fig.3.140 and 3.141. These indicate that the bottom end panel was the most highly strained and yielded well before the failure pressure was attained. The general shapes of these distributions suggest the presence of considerable bending stresses, caused probably by large prebuckling rotations. The stress and strain distributions at the stiffener flanges show substantial bending at the bottom stiffener (fig.3.142). This is the stiffener dividing the two shell panels to which failure buckles were eventually confined.

The shell panel stress and strain distributions of cylinders 7 and 8 were very similar as can be seen from figs.3.144 and 3.145. The shape of the axial stress distributions indicates considerable bending in the axial direction. The shapes of the stress and strain distributions in the web of the stiffener outstands are also very similar (fig.3.143), but as expected, the outstand stress level for cylinder 8 was higher due to the use of lighter stiffeners. For both cylinders, the stiffener outstand strains were measured at the circumferential location diametrically opposite the longitudinal weld line. The results in fig.3.143 show that at this location, the degree of circumferential bending was small.

3.9.3.3 Failure load and collapse mode

Fig.3.146 shows that the failure of cylinder 6

occurred in the panel collapse mode. The buckles were almost exclusively confined to the bottom end panel. The fact that failure occurred in an end panel was particularly interesting since intuitively one would expect instability to be initiated at one of the centre panels where the boundary conditions were weaker especially since the stiffeners were very light. This would have been the case had the failure occurred elastically. However, for the present geometric parameters and loading, the mode of collapse is affected by the extent of yielding. Yielding which initiates collapse begins in the vicinity of the end boundary and is caused by localised bending stresses arising from the restraint to the Poisson expansion at the ends; the degree of expansion being related to the amount of axial load present.

Collapse of cylinder 6 could equally have been expected to occur in the top end panel, but comparison between the initial imperfections and buckled shape (fig.3.152) indicate the bottom end panel to have larger sympathetic imperfections. The plan sections of the failure mode shown in fig.3.149 demonstrate a remarkable resemblance to those of cylinder 5 of the first series (see fig.3.105). The inertia of the stiffeners of cylinder 6 was 0.46 times the DnV requirement (c.f. 0.61 for cylinder 5) and as a result the failure load obtained for this cylinder was slightly lower than that of cylinder 5 (table 3.4).

Under lateral pressure, cylinder 7 failed at a load of 1.3 times the nominal squash pressure. General buckling was obtained with all the rings failing inwards in the form of a simple inward lobe. The width of the lobe was approximately 1/9th of the circumference; the centre of the lobe being coincident with the longitudinal weld (fig.3.150).

Cylinder 8 was also tested under lateral pressure and failed at a pressure of 1.09 times the squash pressure. As for cylinder 7, the stiffeners failed together with the shell over the entire length of the cylinder except that in this case, the lighter stiffening has led to larger prebuckling deformations which resulted in the formation of two inward lobes. This collapse mode is shown in fig.3.151. One lobe was slightly larger than the other. The width of

the main lobe was approximately 1/10th of the circumference. The centre of the lobe was again coincident with the longitudinal weld.

For cylinders 7 and 8, no obvious relationship exists between the shape of the initial imperfections and the collapse mode (fig.3.153 and 3.154). The fact that both these cylinders produced failure along the longitudinal weld was possibly due to the effect of residual stresses remaining after heat treatment. It is unlikely that the location of failure was controlled by initial imperfections as the imperfection shape (figs.3.135 and 3.136) did not reveal any significant sympathetic imperfections at the weld. Unevenness in the applied loading is also an unlikely explanation for this coincidence since the weld location of cylinder 8 was deliberately positioned differently to the test rig by 90 degrees compared with cylinder 7.

3.9.4 Comparison with design codes

The cylinders tested in the present series were all lightly stiffened and did not satisfy the requirements regarding ring stiffener sizes specified in design codes being used in current practice [11,52,86,114]. The inertia requirements of the BS5500 [52] and ECCS [114] codes are identical for hydrostatic and lateral pressure loading. Comparisons between the experimental collapse pressure and the pressure predicted by the various codes for panel collapse are presented in table 3.7. All codes were found to be conservative for panel collapse. The experimental result for cylinder 6, in particular, has demonstrated the conservatism of both the DnV and the ECCS codes regarding ring stiffener specifications. This cylinder, despite having stiffener inertias well outside the requirement of these codes, has not suffered any stiffener failure or sign of significant stiffener deflection.

Only the ASME [11] code gives guidance for general

collapse and comparisons for this are also presented in table 3.7. The predicted collapse pressure was reasonably accurate for both cylinders 7 and 8, even though the stiffeners on the experimental models did not satisfy the ASME inertia requirements.

In comparing the experimental results with the design codes, it should be remembered that design codes, in general, do not take adequate account of the effect of diaphragm proximity in deciding stiffener strength. When collapse occurs in an overall mode, ignoring the presence of diaphragms may lead to a severe underestimate of the ultimate load carrying capacity of the cylinder.

The inclusion of the diaphragms is of prime importance in establishing a reasonable estimate of the collapse pressure. Conversely, the stiffener inertia required to prevent overall buckling is very appreciably reduced even by the presence of relatively widely spaced diaphragms falling within the normal configurations of floating structures. The DnV code makes no allowance for this effect and base ring stiffener requirements on the classical $n=2$ mode which is unlikely to occur in the presence of diaphragms. It acknowledges this to be conservative and indicates that more precise methods can be used, but the apparent degree of conservatism means that existing specifications undoubtedly lead to very heavy stiffeners when pressure loading is significant. These stiffeners are often a significant part of the structural weight.

There is no doubt that the failure pressures of cylinders 7 and 8 were very significantly affected by the proximity of the end rings and this must be borne in mind when interpreting any experimental results in this area, especially when drawing conclusions relating to design rules. The conservatism or otherwise of existing rules is very much a function of the overall structural configuration.

3.10 Summary

The testing of eight small scale models has been described in this chapter. The five cylinders tested in the first series all failed by panel collapse. These cylinders were tested under various ratios of combined loading over the whole range of the pressure-axial compression interaction. The three cylinders in the second series were subjected to pressure dominated loadings and produced one local panel failure and two overall failures.

General difficulties involved in comparing the test results with strength predictions and stiffener inertia requirements given in design codes have been discussed in relation to the complexity associated with defining pressure-hoop stress relationships and boundary restraints.

The present experiments have demonstrated the convenience of small scale modelling and have provided useful data for assessing the accuracy of theoretical models. Correlations with a nonlinear finite element program and simplified theoretical formulations will be presented in the following two chapters.

CHAPTER 4

FINITE ELEMENT ANALYSES OF EXPERIMENTAL MODELS

4.1 Introduction

In this chapter, the test results presented in the previous chapter are used to provide data for correlation with results from finite element analyses. The aim of this exercise is to develop confidence in a finite element program to enable it to be used as a research tool for investigating parameters not covered by the experimental tests.

4.2 Description of finite element program

The finite element program used in the analyses (FINAS) was developed in the Department of Civil Engineering of Imperial College by Trueb [330]. The shell and stiffener elements incorporated in the program were developed by Bates [33]. Only a brief description of the program is given here, the interested reader may consult the above references for further information.

In the analyses, the cylinder surfaces were modelled by doubly curved, eight-noded isoparametric shell elements. Each node has six degrees of freedom. The element is capable of accounting for transverse shear stresses. The element formulation is based on a nonlinear strain-displacement

relationship and includes large displacement and rotation effects. Plasticity is allowed for using a multi-layer technique incorporating the Prandtl-Reuss flow rule and Mises yield criterion. Gauss numerical integration is employed to generate the element stiffness matrices.

The ring stiffeners were modelled by three-noded isoparametric space beam elements. The formulation of these elements is based on thin-walled beam theory and includes transverse shear deformation. They are fully compatible with the shell elements in that their inclusion does not increase the bandwidth of the structural stiffness matrix. Each element is capable of representing any open section of up to twenty linear segments.

The nonlinear solution strategies used in the program are based on various schemes of the Newton-Raphson technique. Should any chosen scheme fail to attain convergence within the specified number of iterations, the program automatically reduces the step size, move to a more powerful scheme and retakes the step. This feature makes the program very convenient for crossing limit points.

The program also contains an option to allow the node displacing at the fastest rate to be selected for the control of load increments. This option improves the numerical stability of the solution and was used for all the analyses.

4.3 Convergence study

Before any numerical analyses could be carried out, it was first necessary to perform a convergence study to ensure that the chosen mesh configuration would converge to solutions of acceptable accuracy. In view of the large number of variables present, it was important to identify the structural parameters which could most affect mesh convergence and to concentrate the study around such parameters.

It is widely known that in the finite element

solution of buckling problems, mesh convergence is highly dependent on the buckle mode of the structure. The formation of short wavelength buckles is generally the most critical and requires the greatest number of elements for accurate modelling. Depending on the shell geometry and the direction of loading, such short wavelength buckles may occur in either the circumferential or the axial direction. The following study was confined to mesh variation in the circumferential direction. Mesh variation in the axial direction is investigated in chapter 6 as part of a parametric study related to axial compression dominated loadings.

The wavelength of circumferential buckles is determined to a large extent by the ratio of combined loading, K . It is difficult to pinpoint the value of K that will always make n highest for any given geometry. It was decided to base this study on the $K=0$ and $K=\infty$ load cases, since these corresponded to the loading ratios selected for the experimental tests and would provide useful points for interpolation of solution accuracy over the whole range of K .

The finite element model used for the present study was a simply supported unstiffened cylinder. Initially, the L/R ratio of 0.1 was investigated. The shortness of this geometry meant that fewer elements would be required and only one element was used to model the axial direction. In the circumferential direction, studies were carried out for 'coarse', 'medium' and 'fine' meshes consisting of 15, 20 and 30 elements respectively to represent the entire circumference. R/t 's of 100 and 500 were examined for each load case. To avoid the difficulties of crossing the limit point encountered by Agelidis [1] in analyses involving perfect cylinders, nominal imperfections in the form of an outward sine wave with peak amplitude defined by DnV tolerance values were incorporated into all models.

The results of the analyses are presented in table 4.1. For all loadings and geometries examined, failures were axisymmetric and as a result the various meshes produced very similar failure loads. In particular, for the $K=0$ load case, deviations in the ultimate pressure were very small with the maximum difference being only 1.59%. In view of this, it was decided to confine subsequent investigations to

the $K=\infty$ load case which produced slightly larger variations.

The next geometry considered was $L/R=0.3$. The meshes adopted were identical to the $L/R=0.1$ cylinders except that in this case, 3 elements were used to model the axial direction. This resulted in some very large runs, particularly for the finer meshes.

The results are presented in table 4.2. The deviations in the ultimate load carrying capacities at up to 7.5% were generally larger than those for the $L/R=0.1$ geometry. The 'medium' mesh comprising 20 elements provided the best compromise between economy and accuracy and was chosen for all subsequent analyses.

4.4 Analyses of cylinders 1-5

4.4.1 Description of finite element models

Cylinders 1 to 5 all failed by collapse of the centre panel. In the finite element analyses, attention was focused on this centre panel to see whether the experimental behaviour could be reproduced by the numerical results. For each cylinder, eight analyses were performed with two different finite element models to assess the effect of initial imperfections, boundary conditions and stiffening on the collapse behaviour of the cylinder.

Initially, the centre panel of each cylinder was analysed as an unstiffened cylinder by using a model which consisted of 20 shell elements around the entire circumference and 3 shell elements along the length. This model is shown in fig.4.1.

The fact that the whole centre panel was modelled meant that it was possible to incorporate the measured initial imperfections as nodal values into the finite element analyses. This type of imperfection is denoted as mesh B in fig.4.1. Of the cases attempted, this mesh should have provided the best correlation with the experimental data.

The imperfection sensitivity of each cylinder was

assessed by comparing the failure load obtained from mesh B with that of the geometrically perfect cylinder (denoted by mesh A). In addition, imperfections obtained from equation 3.1 were used to generate a further set of coordinates (mesh C) to examine the accuracy of the Fourier series representation.

For each mesh, the effect of the ring stiffeners was approximated by both simply supported and clamped boundaries to determine the type of support which could best represent the experimental behaviour. The definition of these boundary conditions is illustrated in fig.4.1.

The actual boundary conditions provided by the rings were subsequently analysed by a discretely stiffened model. In this model, the three centre panels and the two intermediate ring stiffeners were represented. This model was considerably larger than the previous unstiffened one and as a result only half of the circumference could be represented, the other half being simulated by symmetry. 10 shell elements were used to model this portion of the circumference while 7 shell elements were used longitudinally to represent the three centre panels; 3 for the centre and 2 for each of the adjacent bays. As before, the boundary conditions at the ends of this stiffened configuration were analysed both as simple and clamped supports.

The initial imperfection mesh used for this model is denoted by mesh D and is generated by equation 3.1 for all three panels as experimental data were not available for the adjacent bays. It was theoretically possible to use the measured imperfections for all three panels, but the separate use of the imperfection analysis program for each panel would introduce artificial and unrealistic discontinuities at the ring boundaries where the two imperfections met. Details of this model are also given in fig.4.1.

4.4.2 Cylinder 1

The failure loads from the finite element analyses are shown in table 4.3 where the experimental failure load is also included for comparison. For the unstiffened analyses, the maximum deviation in the collapse loads between the perfect and imperfect cases was about 25% indicating that the experimental model was quite imperfection sensitive under axial compression.

The corresponding load-deflection curves are shown in fig.4.2. The maximum loads obtained from the different boundary conditions of the unstiffened models suggest that the ring stiffeners provided partial but not complete restraint against out-of-plane rotation at the ends of the centre panel. In general, the clamped models led to closer but unconservative estimates of the failure load. As expected, the stiffened model with clamped supports produced the best correlation with the experimental results since it was the most realistic representation of the experimental model. The corresponding simply supported model resulted in a comparatively low failure load. In the latter case, collapse was actually initiated at one of the outer panels, caused probably by the rather unrealistic assumption of simply supported boundary conditions bearing in mind that the experimental model was fitted with thicker panels at the end bays to restrain rotation.

The postbuckling behaviour is difficult to categorize since only short postbuckling paths were obtained in some cases. The large number of elements involved in the analyses meant that it was impractical to extend the unloading paths into the advanced postbuckling region due to restrictions imposed by computer time limits. Nevertheless, it is still possible to identify the imperfection dependent nature of the behaviour from the steepness of some of the unloading paths. In all cases, axial displacements were selected by the program to control the loading increments.

Plasticity was found to be most widespread in the perfect models. The absence of initial imperfections enabled plasticity to develop evenly around the surface of the shell and collapse was initiated by yielding rather than buckling. The development of such an even stress distribution was not possible with the imperfect meshes. As a result, failure

occurred with varying extents of plasticity. In all cases, yielding was initiated in the vicinity of the end supports. This was caused by localised bending stresses arising from the restraint to Poisson expansion at the ends. This effect also resulted in plasticity being initiated in the outer panels of the stiffened models. It is not possible to correlate this observation with the experimental behaviour as strain gauges were not fitted to the outer panels of the test models.

The collapse modes for all eight analyses are shown in fig.4.3. The experimental collapse mode involving outward bulging of the centre panel was reproduced by the finite element models. In the discretely stiffened analyses, the ring stiffeners remained undistorted, but the deflections of the outer panels were larger than those observed experimentally. However, direct comparison with the experimental behaviour is difficult since the analytical initial imperfections for all three panels were generated by Fourier coefficients based on the the centre panel only.

4.4.3. Cylinder 2

For this cylinder, the analyses with clamped supports produced much better correlation with the experimental failure load than those with simple supports as shown in fig.4.4. The results from this hydrostatic pressure load case are not as imperfection sensitive as for cylinder 1.

For the unstiffened models with clamped supports, the rotational restraint provided by the end fixity enabled the cylinder to remain stable until the entire shell panel had yielded. But most of these clamped cases still produced failure loads lower than that of the experiment. The reason for this inconsistency may be attributed to strain hardening which is an effect not incorporated into the finite element formulation. The distribution of plasticity in the stiffened models was also dependent on the boundary conditions. Due to

the weaker nature of the end supports, considerably larger deflections were produced in the end panels of the simply supported model. This resulted in a wider overall spread of plasticity than for the clamped model. However, examination of the stress state of both stiffened models revealed that the extent of plasticity in the clamped model was actually 50% higher when only the centre panels were considered.

The collapse modes shown in fig.4.5 demonstrate the dominance of the axisymmetric pattern observed experimentally. The similarity between meshes B and C both in terms of buckling pattern and load-deflection behaviour (see fig.4.4) indicates that the Fourier series representation is a satisfactory analytical means of reproducing the experimental initial imperfections.

4.4.4 Cylinder 3

The only factor which significantly affected the analytical collapse strengths of this cylinder was boundary conditions. Under lateral pressure loading, the out-of-plane rotation of the shell plating at the ring stiffener boundaries must be expected to be minimal. Hence, it was not surprising to find that the analyses with clamped boundaries provided much better correlation with the experimental result as illustrated in fig.4.6. But rather unexpectedly, these analyses have turned out to be slightly conservative. Strain hardening could have again been the reason for this inconsistency. It is worth noting that prior to the failure of the experimental model, one of the strain gauges attached to the centre panel had already recorded a strain of 0.1 which is 3 times higher than the value required for strain hardening to occur.

The analyses have confirmed that for lateral pressure, the experimental model was not imperfection sensitive as variations to the imperfection data made very little difference to the collapse pressure. This conclusion is reinforced by the flatness of the unloading paths.

During the analyses, the number of load steps had to be limited for economic reasons and the ultimate loads could not always be reached. Many load steps were invariably required to overcome the apparent increase in resistance provided by the analytical models, even at deflections of the order of several times the shell thickness.

In all the analyses, plasticity had spread over the entire centre panel prior to collapse. For the discretely stiffened models, about 50% of the elements representing the outer panels had yielded. There is little doubt that yielding was the primary cause of failure. Despite the high level of stresses present in the shell elements, the stiffener outstands, as in the experimental test, remained completely elastic.

The experimental collapse mode has also been reproduced by the finite element modelling (see fig.4.7). Axisymmetric failures were obtained in all cases with buckling deflections considerably larger than those of the cylinders tested under other loading ratios.

4.4.5 Cylinder 4

Under the loading ratio of $K=1.5$, simply supported unstiffened imperfect models provided good correlation with the experimental result and the analyses demonstrate that in this axial compression dominated area of the loading interaction, the assumption of simply supported boundary conditions is more realistic. Comparison of these imperfect cases with the perfect mesh reveals a high degree of imperfection sensitivity. This is due to the fact that outward barrelling produced by axial compressive load is counteracted by the external pressure and only small prebuckling deformations occur.

Analyses on the perfect mesh show that it is theoretically possible to load the cylinder to complete yield without the onset of instability. In fact, the

presence of external pressure was found to have an ameliorating effect on the strength of the models resulting in an increase in the axial load carrying capacity compared with cylinder 1. However, this increase in strength cannot be relied upon in design situations where the influence of initial imperfections is likely to play a much more dominant role in controlling collapse. The presence of initial imperfections can also drastically change the stress distribution in the cylinder. This is clearly illustrated by the result of one of the analyses on mesh B which buckled virtually elastically.

The two discretely stiffened analyses produced upper and lower bounds to the experimental collapse load. However, failures in both cases were predicted in the outer panels as illustrated in fig.4.9. The reason for this discrepancy is not clear, but it is possible that the outer panel imperfection data used in the analyses were larger than, or of a different form to, the actual imperfections in the experimental model.

The experimental collapse mode involving a combination of inward and outward buckles in the form of circumferential waves was reproduced by the unstiffened imperfect analyses. This buckling pattern is a further confirmation of the high imperfection sensitivity that is characteristic of the present load case.

4.4.6 Cylinder 5

This cylinder, like cylinder 2, was also subjected to hydrostatic pressure. As expected, the analytical collapse pressures for the unstiffened cases were very similar to those of cylinder 2 as shown in table 4.3. However, the collapse pressure for the simply supported stiffened case was considerably lower than the corresponding analysis of cylinder 2. This can be attributed to the difference in ring stiffener sizes between the two cylinders. The smaller stiffeners used for cylinder 5

produced ring stresses of the order of 100% higher than the corresponding stresses in cylinder 2. The stiffener, however, remained completely elastic even after the peak load was reached. The stiffened model with clamped ends was a more realistic representation of the experimental model and produced a failure load identical to that obtained experimentally as shown in fig.4.10.

The similarity between the collapse loads obtained from the unstiffened analyses of cylinders 2 and 5 is reflected by the similarity in collapse modes as shown in fig.4.5 and 4.11.

4.4.7 Interaction curves

The collapse loads from all the finite element analyses are presented in fig.4.12 in the form of interaction curves. As expected, the unstiffened perfect models denoted by mesh A provided unconservative estimates of the collapse strength across most of the loading interaction. Meshes B and C predicted similar failure loads to each other in most cases. In axial load dominated situations, the assumption of simply supported boundary conditions provided reasonably conservative estimates of the collapse loads while for pressure loading, the choice of clamped supports appears to be more realistic.

The most accurate representation of the experimental configuration was the stiffened model with clamped supports (mesh D). As expected, the resulting interaction curve has produced very good overall correlation with the experimental results. The inclusion of ring stiffeners has also reduced the difference between the predictions from clamped and simply supported boundaries.

4.5 Analyses of cylinders 6 - 8

4.5.1 Description of finite element model

Cylinders 6 to 8 were designed to fail by general instability. It was therefore important to devise an analytical model that was capable of reproducing this mode of collapse.

The finite element model adopted consisted of half of the circumference and half of the length between end boundaries. A total of 50 shell elements were used for each analysis (10 elements for the circumferential direction and 5 for the axial direction). This was the maximum size of model considered to be economically feasible. 10 beam elements were used to represent each of the two ring stiffeners present in the finite element model.

Initially, two runs were performed for each cylinder; one with zero imperfections and one with imperfections for the shell generated by equation 3.1. It was not possible to use the measured imperfections for the stiffeners due to restrictions on the data input format. DnV tolerance values were used instead.

4.5.2 Cylinder 6

This cylinder, in spite of a severe reduction in stiffener size, failed by panel collapse of the end bay during the experiment.

The failure loads in table 4.4 and the load deflection behaviour in fig.4.14 show that the difference in the ultimate load attained by the perfect and imperfect models is very small, confirming that for the present geometric parameters panel failures are not very imperfection sensitive under pressure dominated loadings. The flatness of the postbuckling path shows that collapse in

the present mode is not catastrophic with the cylinder retaining much of its load carrying capacity in the post-collapse range.

The experimental collapse load was about 11% lower than the finite element predictions. The reason for this difference is not obvious, but the effect of localised imperfections could have been a significant factor since the Fourier series representation could not reproduce such imperfections.

At collapse, about 50% of the shell elements comprising the imperfect model had yielded. The corresponding figure for the perfect model was about 60%. The stiffener elements in both models remained completely elastic, the maximum ring stresses being only about 60% of yield.

The analytical collapse mode of the imperfect model is shown in fig.4.5. As in the experiment, an end panel collapse was obtained in the half of the cylinder analysed. A separate finite element analysis using 2 elements to represent $1/20$ of the circumference and 10 elements to represent the whole length of the cylinder was conducted to determine which end of the cylinder would collapse first. The collapse load obtained was very similar to the other cases as shown in fig.4.14, but this finite element model produced a failure mode completely consistent with the experimental result with collapse occurring at the bottom end panel as shown in fig.4.16.

4.5.3 Cylinder 7

The finite element load-deflection curves for this cylinder (fig.4.17) indicate that the present mode of imperfections did not substantially reduce the strength of the cylinder. However, the steepness of the postbuckling paths suggests that considerable reduction in strength is possible for other modes of imperfection. In particular, the

unloading path for the perfect model demonstrates a potentially high imperfection sensitivity. This type of behaviour has been encountered in other finite element analyses of perfect cylinders and is most pronounced when the effects of mode interaction are important.

Both finite element models failed with a high degree of yielding. At collapse all elements in the perfect model, including stiffeners, had yielded. The imperfect model also developed extensive plasticity in the shell elements, but yielding in the stiffener elements was confined to the region of the buckle.

A plan view of the finite element collapse mode is shown in fig.4.18. The buckling pattern is very similar to the one obtained experimentally in that failure consisted of the simultaneous collapse of the shell and the stiffeners in the form of single inward lobe. The circumferential location of this lobe was different to that obtained in the experiment. This difference could possibly be due to residual stresses arising from welding not being fully removed by heat treatment and causing the experimental model to fail along the longitudinal weld line.

4.5.4 Cylinder 8

Much of the discussion on cylinder 7 also applies to this cylinder. Due to the use of lighter stiffeners, the radial stiffness of this cylinder is slightly less than that of cylinder 7 as shown in fig.4.17. Deflections in general were larger and this resulted in collapse extending over a greater portion of the circumference. The analytical collapse mode of the imperfect model shown in fig.4.19 is very similar to that of the experimental model with the formation of two inward lobes. The location of the buckles is again different to that of the experimental model, which like cylinder 7, also failed at the longitudinal weld. During the test, the position of the longitudinal weld of cylinder 8 relative to the test rig was deliberately changed

by rotating the cylinder through 90 degrees about the axis.

The present finite element analyses together with the experimental results of cylinders 7 and 8 have shown that for the present geometric parameters, the classical $n=2$ mode is unlikely to occur because of the presence of the rigid boundaries prevents large overall radial deflections in this mode. The short wavelength localised buckling mode can more readily occur within the boundary limits and will be affected by the presence of initial imperfections which induce localised plasticity in restricted areas of the shell.

4.5.5 Summary

Both the local panel and the overall modes of collapse obtained from the experimental tests described in chapter 3 have been successfully reproduced by finite element analyses. The importance of boundary conditions and initial imperfections across the whole range of loading interaction has been examined.

For cylinders 1 to 5, unstiffened models with clamped boundaries produced better correlations with the experimental results for pressure dominated loadings.

In the axial load dominated region of the loading interaction, the assumption of simple supports provided more reasonable lower bound estimates. The most accurate predictions were obtained by modelling the ring stiffeners using a discretely stiffened model. In this region of the loading interaction, the collapse behaviour was found to be more imperfection sensitive.

The finite element analyses of these five cylinders together with those of cylinder 6 have demonstrated the conservatism associated with the design of the ring stiffeners. Stiffener outstand stresses were well below yield for all these cylinders. Stiffener deflections were also very small.

The collapse behaviour of cylinders 7 and 8 has also

been accurately reproduced by the finite element analyses. The results have indicated that the general collapse load of ring stiffened cylinders can be imperfection sensitive, even under lateral pressure loading.

The extensive correlation carried out in this chapter has demonstrated the finite element program FINAS to be an accurate and versatile tool for analysing the collapse of ring stiffened cylinders under combined axial compression and external pressure loading. In subsequent chapters, this program is used to investigate structural parameters outside those covered by the test series as well as to provide data for comparison with a theoretical approach.

CHAPTER 5

THEORETICAL FORMULATIONS

5.1 Introduction

In the design of ring stiffened cylinders, finite element and other numerical methods are generally considered to be unsuitable in view of the high computing costs involved. Experimental testing is unlikely to provide all the design information over the geometric, material and loading parameters adopted in current practice. It is therefore important that simplified theoretical models are available to supplement and verify existing design codes.

The experimental and finite element analytical results reported in chapters 3 and 4 have produced two distinct modes of collapse. These are :

- a) Inter-ring panel collapse
- b) General instability involving the simultaneous collapse of the entire shell and stiffeners

In this chapter, theoretical models based on rigid plastic mechanism approaches are developed to analyse each of these modes. Results from these formulations are correlated with experimental data from chapter 3 as well as various other sources.

Later in the chapter, the plastic mechanism models are combined to form an optimisation process for evaluating stiffener sizes using avoidance of general collapse as the criterion.

5.2 Background

The plastic mechanism method has been adopted by many investigators for the analysis of structures which undergo significant plasticity before collapse. In recent years, the method has been applied to a variety of structural components including channel columns [226], submarine pipelines [84,309], damaged tubulars [264], stiffened panels [225], and plates under compression [83,354] and shear [62,115] loadings.

The use of plastic mechanisms as a means of estimating collapse strengths was initially recommended by Sherbourne and Korol [187,283]. They proposed that an upper bound to the collapse strength may be obtained by determining the point of intersection between an elastic loading curve and a mechanism unloading curve. Their analysis produced encouraging correlation with experimental results [284]. This approach was extended by Mitra [219] to the analysis of diaphragms and acceptable agreement with test results was also obtained.

Recently, a rigorous collapse mechanism theory which involves the discretisation of the structure into rigid elements has been developed by Fahy [116]. The theory is general and may be applied to a variety of structural forms. Use is made of constitutive equations derived by Dean [85] and the effect of elastic strain energy can also be considered. The resultant equations are, however, difficult to solve and require the use of complex solution techniques.

The plastic mechanism method has also been used by some researchers for the analysis of cylindrical shells. A mechanism model was developed by Nimmer and Mayers [234] to study the Yoshimura mode of collapse. Although the model was capable of predicting the axial collapse load for various asymmetric imperfection modes, scepticism on the grounds of theoretical justification must be expressed since it is well known that the Yoshimura collapse mode can only be associated with the elastic bifurcation of very slender

cylinders.

A more realistic approach is to assume that the plastic deformation occurs in an axisymmetric shape as experimental tests [308] have shown that the elasto-plastic collapse of cylinders under axial compression is more likely to occur in this mode. Such an approach was adopted by Alexander [2], but his model was rather over-simplified. His work has subsequently been improved by Andronicou and Walker [13] who incorporated the Mises yield criterion and Prandtl-Reuss flow rule to define stress and strain in the plastic range. This improved model was capable of representing the postbuckling behaviour of unstiffened cylinders under axial compression. Correlations with experimental results were generally very good.

Andronicou and Walker's model has been extended by Tsang and Harding [334] to predict the collapse load of ring stiffened cylinders under combined axial compression and external pressure. More recently, a model to represent general collapse under pressure loadings has also been developed by the same authors [336]. These models are discussed in greater details later in this chapter.

5.3 Inter-ring panel collapse

5.3.1 Theory

When a ring stiffened cylindrical shell is subjected to the simultaneous action of axial compression and external pressure, the shell panel between the rings may buckle either inward or outward. Which mode occurs is determined to a large extent by the ratio of the two types of loading (K), but the shape of the initial imperfections may also exert a significant influence. In the following derivation, an axisymmetric mechanism is assumed and the governing equations for both outward and inward directions are

developed to ensure that the correct failure mechanism is obtained.

The assumption of an axisymmetric failure limits the applicability of the method to cylinders which have ring spacings of the order of the elastic critical buckling wavelength and this should be borne in mind in its use.

The formulation assumes the ring stiffeners provide clamped boundary conditions at the ends of the shell panel.

Let M_o be the moment of resistance per unit length at each plastic hinge. In fig.5.1, the work done dW_{I1} in bending at the three plastic hinges during a virtual change of deformation is:

$$\begin{aligned} dW_{I1} &= 2(\pi D M_o d\theta) + 2\pi M_o (D - 2H \sin\theta) d\theta \\ &= 4\pi M_o (D - H \sin\theta) d\theta \end{aligned} \quad (5.1)$$

For a wide strip of material, M_o is given by:

$$M_o = \frac{2}{\sqrt{3}} \frac{\sigma_o t^2}{4} [1 - (N/N_o)^2] \quad (5.2)$$

The factor of $2/\sqrt{3}$ has been introduced because the material has been assumed to behave under conditions of plane strain. The full derivation is given in appendix A.

Combining equations (5.1) and (5.2), dW_{I1} becomes:

$$dW_{I1} = (2/\sqrt{3}) \pi \sigma_o t^2 (1 - \bar{N}^2) (D - H \sin\theta) d\theta \quad (5.3)$$

where $\bar{N} = N/N_o$

In any virtual change of geometry, circumferential and longitudinal deformations must be compatible. Consider an annulus of material of length dh as shown in fig.5.2.

Initial length (before virtual deformation) = $2\pi(R - h\sin\theta)$

Final length (after virtual deformation) = $2\pi[R - h\sin(\theta + d\theta)]$

Hence, shortening in length = $2\pi h[\sin(\theta + d\theta) - \sin\theta]$

$$= 2\pi h \cos\theta \quad (5.4)$$

(since $\sin(\theta + d\theta) - \sin\theta \doteq \cos\theta d\theta$)

Taking σ_θ as the average value for the panel, i.e. effectively constant, the work done dW_{I2} during this shortening in length for the whole mechanism is:

$$dW_{I2} = 2\pi\sigma_\theta t \cos\theta d\theta \left(2 \int_0^H h dh\right) = 2\pi\sigma_\theta t H^2 \cos\theta d\theta \quad (5.5)$$

The Mises yield function f may be expressed as:

$$f \equiv \sigma_a^2 - \sigma_a \sigma_\theta + \sigma_\theta^2 - \sigma_o^2 = 0 \quad (5.6)$$

Solving for σ_θ gives:

$$\sigma_\theta = [\bar{\sigma}_a \pm \sqrt{(4 - 3\bar{\sigma}_a^2)}] / 2 \quad (5.7)$$

where $\bar{\sigma}_\theta = \sigma_\theta / \sigma_o$ and $\bar{\sigma}_a = \sigma_a / \sigma_o$

Compatibility of strains implies that deformation must occur in the longitudinal direction. Since such deformations take place under yield conditions, a flow rule is adopted to determine the change in the plastic strain increment.

$$\begin{aligned}
 d\epsilon_a &= \lambda(df/d\sigma_a) \\
 &= \lambda(2\sigma_a - \sigma_\theta)
 \end{aligned} \tag{5.8}$$

and

$$\begin{aligned}
 d\epsilon_\theta &= \lambda(df/d\sigma_\theta) \\
 &= \lambda(2\sigma_\theta - \sigma_a)
 \end{aligned} \tag{5.9}$$

λ is a constant of proportionality.

Combining equations (5.8) and (5.9) gives:

$$d\epsilon_a = \frac{2\bar{\sigma}_a - \bar{\sigma}_\theta}{2\bar{\sigma}_\theta - \bar{\sigma}_a} d\epsilon_\theta \tag{5.10}$$

The strain in the circumferential direction during the virtual change is:

$$d\epsilon_\theta = \frac{h\cos\theta}{R - h\sin\theta} d\theta$$

This may be approximated to:

$$d\epsilon_\theta \approx \frac{h\cos\theta}{R} d\theta \tag{5.11}$$

The change in length of both segments of the mechanism Δ_{II} is:

$$\Delta_{II} = \frac{2\bar{\sigma}_a - \bar{\sigma}_\theta}{2\bar{\sigma}_\theta - \bar{\sigma}_a} 2 \int_0^H \frac{h\cos\theta}{R} d\theta dH$$

$$\Delta_{I1} = \frac{2\bar{\sigma}_a - \bar{\sigma}_\theta}{2\bar{\sigma}_\theta - \bar{\sigma}_a} \frac{H^2}{R} \cos\theta d\theta \quad (5.12)$$

Assuming the axial stress σ_a to be uniform ($=N/t$) along the length of the mechanism, the corresponding virtual work done dW_{I3} is:

$$dW_{I3} = \pi DN \frac{(2\bar{\sigma}_a - \bar{\sigma}_\theta)H^2}{(2\bar{\sigma}_\theta - \bar{\sigma}_a)R} \cos\theta d\theta \quad (5.13)$$

The total external work done comprises two parts; the work done by the axial loading, dW_{E1} and the work done by the external pressure loading dW_{E2} .

During a virtual change in θ angle, neglecting any strain deformation, the axial displacement Δ_{E1} and the radial displacement Δ_{E2} are given by (see fig.5.3):

$$\begin{aligned} \Delta_{E1} &= H\cos(\theta + d\theta) - H\cos\theta \\ &= H\sin\theta d\theta \end{aligned} \quad (5.14)$$

and

$$\begin{aligned} \Delta_{E2} &= H\sin(\theta + d\theta) - H\sin\theta \\ &= H\cos\theta d\theta \end{aligned} \quad (5.15)$$

Hence dW_{E1} may be written as:

$$\begin{aligned} dW_{E1} &= \pi DN (2\Delta_{E1} + \Delta_{I1}) \\ &= \pi DNH \left[2\sin\theta + \frac{(2\bar{\sigma}_a - \bar{\sigma}_\theta)}{(2\bar{\sigma}_\theta - \bar{\sigma}_a)} \frac{H}{R} \cos\theta \right] d\theta \end{aligned} \quad (5.16)$$

and dW_{E2} may be written as:

$$\begin{aligned} dW_{E2} &= p\pi D^2 H \Delta_{E2} / 2 \\ &= p\pi D H^2 \cos\theta d\theta \end{aligned} \quad (5.17)$$

To satisfy the principle of virtual work, the total internal work must equal the total external work, i.e.

$$dW_{I1} + dW_{I2} + dW_{I3} = dW_{E1} + dW_{E2}$$

Therefore,

$$\begin{aligned} (2/\sqrt{3})\pi\sigma_o t^2(1 - \bar{N}^2)(D - H\sin\theta)d\theta + 2\pi\sigma_o t H^2 \cos\theta d\theta &= 2\pi D N H \sin\theta d\theta \\ &+ p H^2 \pi D \cos\theta d\theta \end{aligned} \quad (5.18)$$

By writing $K = \bar{N}/\bar{p}$, and $\sigma_o = p_o D/2t$, equation (5.18) may be expressed as:

$$\begin{aligned} (2/\sqrt{3})t[1 - (K\bar{p})^2][D - H\sin\theta]d\theta + 2\bar{\sigma}_\theta H^2 \cos\theta d\theta &= 2DK\bar{p}H\sin\theta d\theta \\ &+ 2\bar{p}H^2 \cos\theta d\theta \end{aligned} \quad (5.19)$$

Noting that $\bar{\sigma}_\theta = [\bar{\sigma}_a + \sqrt{(4-3\bar{\sigma}_a^2)}]/2$ for an inward mechanism and $\bar{\sigma}_\theta = [\bar{\sigma}_a - \sqrt{(4-3\bar{\sigma}_a^2)}]/2$ for an outward mechanism, the equation of equilibrium for each mechanism may be obtained. It should be remembered that for the case of an outward mechanism, the dW_{I1} term must be preceded by a -ve sign to take account of the fact that the direction of $d\theta$ is opposite to the direction of $+M_o$.

Hence for an inward mechanism,

$$\begin{aligned} (2/\sqrt{3})t[1 - (K\bar{p})^2][D - H\sin\theta] + H^2 \cos\theta[K\bar{p} + \sqrt{(4 - 3K^2\bar{p}^2)}] \\ = 2DK\bar{p}H\sin\theta + 2\bar{p}H^2 \cos\theta \end{aligned} \quad (5.20)$$

and for an outward mechanism,

$$\begin{aligned}
 & -(2/\sqrt{3})t[1 - (K\bar{p})^2][D - H\sin\theta] + H^2\cos\theta[K\bar{p} - \sqrt{(4 - 3K^2\bar{p}^2)}] \\
 & = 2DK\bar{p}H\sin\theta + 2\bar{p}H^2\cos\theta
 \end{aligned} \tag{5.21}$$

The length of the buckling mechanism H may be obtained by minimisation of equations (5.20) and (5.21) with respect to H .

Hence for a inward mechanism,

$$H = \frac{\tan\theta[\sqrt{3}(K^2\bar{p}^2 - 1) - 2DK\bar{p}]}{4\bar{p} - 2[K\bar{p} + \sqrt{(4 - 3K^2\bar{p}^2)}]} \tag{5.22}$$

and for an outward mechanism,

$$H = \frac{\tan\theta[\sqrt{3}(K^2\bar{p}^2 - 1) + 2DK\bar{p}]}{2[K\bar{p} - \sqrt{(4 - 3K^2\bar{p}^2)}] - 4\bar{p}} \tag{5.23}$$

Substitution of H from (5.22) into (5.20) and from (5.23) into (5.21) gives the required relationship between $\bar{p}$ and θ . If $2H$ is greater than the ring spacing L_p , the value of $L/2$ should be used instead of H .

It is interesting to note that although H is dependent on θ (see eqn.5.22 and 5.23), its value remains fairly constant except in the very early stages of deformation. This zone of behaviour is removed by the presence of a significant imperfection. In the pressure dominated region of the loading interaction, H generally takes the fixed value of the ring spacing.

The radial displacement w at the middle hinge may be obtained from:

$$w = H\sin\theta \tag{5.24}$$

and the total end shortening Δ is given by:

$$\Delta = 2H(1 - \cos\theta) + \frac{2\bar{\sigma}_a - \bar{\sigma}_\theta}{2\bar{\sigma}_\theta - \bar{\sigma}_a} \frac{H^2}{R} \sin\theta \tag{5.25}$$

For the case of simply supported boundary conditions, the formulation is identical to the clamped boundary conditions except that in this case, fewer hinges are required to form the mechanism. If $H < L_p/2$ two hinges are needed and dW_{II} should be modified to $4M_0\pi(0.75D-H\sin\theta)d\theta$. If H is equal to $L_p/2$, only one plastic hinge is required and the magnitude of dW_{II} should be further modified to $4M_0\pi(0.5D-H\sin\theta)d\theta$.

The effects of initial geometrical imperfections may be incorporated into the analysis by introducing an initial angle θ_0 . θ should then be measured from the initially distorted configuration. The analysis is modified by replacing θ by $(\theta + \theta_0)$. The expressions for w and Δ now become:

$$W = H(\sin\theta - \sin\theta_0) \quad (5.26)$$

$$\Delta = 2H[\cos\theta_0 - \cos(\theta + \theta_0)] + \frac{2\bar{\sigma}_a - \bar{\sigma}_\theta}{2\bar{\sigma}_\theta - \bar{\sigma}_a} \frac{H^2}{R} [\sin(\theta + \theta_0) - \sin\theta_0] \quad (5.27)$$

An upper bound estimate to the failure load may be obtained by determining the point of intersection of the plastic unloading curve for a particular value of K and the elastic loading curve for the same value of K . To simplify the construction of the elastic loading curve, all geometrical nonlinear effects are neglected.

Let p_e and N_e be respectively the pressure and axial loadings in the elastic range. σ_θ and σ_a may be written as:

$$\sigma_\theta = u p_e R/t \quad (5.28)$$

$$\sigma_a = N_e/t \quad (5.29)$$

where u is a factor introduced to account for the uneven distribution of hoop stress in the panel.

By defining $\theta' = \theta_0 + \theta$ and $w' = w_0 + w$, the hoop strain ϵ_θ may be written as:

$$\epsilon_\theta = \frac{W}{R} = \frac{H}{R} (\sin\theta' - \sin\theta_0) \quad (5.30)$$

But since ϵ_θ may also be expressed as:

$$\epsilon_\theta = (\sigma_\theta - \nu\sigma_a)/E \quad (5.31)$$

expressions for $\bar{p}_e$ and $\bar{N}_e$ may be obtained by combining equations (5.28), (5.29), (5.30) and (5.31) :

$$\bar{p}_e = \frac{H(\sin\theta' - \sin\theta_0)Et}{R(up_o R - \nu KN_o)} \quad (5.32)$$

and

$$\bar{N}_e = \frac{1}{\nu N_o} [up_e p_o R - \frac{Et}{R} H(\sin\theta' - \sin\theta_0)] \quad (5.33)$$

where $\bar{p}_e = p_e / p_o$ and $\bar{N}_e = N_e / N_o$

For an inward mechanism, combining equations (5.20) and (5.32) and assuming $\tan\theta = \sin\theta'$ gives:

$$Et \left[\frac{2t}{\sqrt{3}} (K^2 \bar{p}^2 - 1) - 2DK\bar{p} \right] \sin^2\theta' - Et \sin\theta_0 \left[\frac{2t}{\sqrt{3}} (K^2 \bar{p}^2 - 1) - 2DK\bar{p} \right] \sin\theta'$$

$$- \bar{p}_e R (up_o R - \nu KN_o) [4\bar{p} - 2\{K\bar{p} + \sqrt{4 - 3K^2 \bar{p}^2}\}] = 0 \quad (5.34)$$

This is a quadratic equation in $\sin\theta'$. The positive root of this equation gives the angle θ_f at which $\bar{p}_e = \bar{p} = \bar{p}_f$ where $\bar{p}_f$

denotes the failure pressure divided by p_o .

By substituting the above value of θ_f into equation (5.20), the following implicit expression for $\bar{p}_f$ may be obtained:

$$\begin{aligned} \frac{2t}{\sqrt{3}}[1 - (K\bar{p}_f)^2][D - H\sin\theta_f] + H^2\cos\theta_f[K\bar{p}_f + \sqrt{(4 - 3K^2\bar{p}_f^2)}] \\ = 2DK\bar{p}_fH\sin\theta_f + 2\bar{p}_fH^2\cos\theta_f \end{aligned} \quad (5.35)$$

Solution for $\bar{p}_f$ may be obtained by an iterative procedure.

In a similar manner, an equation describing the outward mechanism may be obtained by replacing θ and $\bar{p}$ in equation (5.21) by θ_f and $\bar{p}_f$. θ_f in this case is the negative root of the following quadratic in $\sin\theta$:

$$Et[\frac{2t}{\sqrt{3}}(K^2\bar{p}^2 - 1) + 2DK\bar{p}]\sin^2\theta' - Et\sin\theta_0[\frac{2t}{\sqrt{3}}(K^2\bar{p}^2 - 1) + 2DK\bar{p}]\sin\theta'$$

$$-\bar{p}_e R(u_{p_o} R - vKN_o)[2\{K\bar{p} - \sqrt{(4 - 3K^2\bar{p}^2)}\} - 4\bar{p}] = 0 \quad (5.36)$$

5.3.2 Experimental data

Until recently, experimental data on the elasto-plastic collapse of cylinders under combined axial compression and external pressure has been very limited. This situation has been somewhat improved by the test results presented in chapter 3. Correlation between these results and the present theory is given in the next section.

Recently, more results have become available by the completion of test programs at Liverpool [131] and Glasgow [124] Universities. Results from these tests have also been used to correlate with the present theory. A brief description of the cylinders tested is first given.

Experimental results of seventeen machined and eighteen welded models from the test program conducted at Liverpool University were used to correlate with the present theory. All models tested had a nominal diameter of 0.15m with a R/t value of approximately 100. Three different lengths were investigated. These were approximately 0.33, 0.74 and 1.45 times the radius. The models were tested under combined loading ratios extending over the whole range of the loading interaction. The machined models were stress relieved and were regarded as having nominally perfect geometries. The welded models which were not stress relieved contained quite large initial imperfections, some of which exceeded the tolerance value specified by ECCS [114]. All models were unstiffened with end supports provided by nominally fully clamped boundaries. Details of these models are given in tables 5.1 and 5.2.

The experimental program conducted at Glasgow University consisted of four test series with R/t 's ranging from 155 to 290 and L_p/R 's from 0.1 to 0.4. These geometries were chosen to represent a range of possibilities similar to the first North Sea tension leg platform. All models were ring stiffened and were stress relieved before testing. Initial imperfection results were not available, but were expected to satisfy DnV tolerance limits [124]. For pressure loadings, experimental failure loads were quoted in terms of ultimate hoop stress values. In table 5.3, these failure loads have been re-expressed in terms of ultimate pressure by using the relationship $p = u \sigma_{\theta} t / R$. A procedure for determining u is described in the next section. Details of geometries and loadings are also given in table 5.3. Information on model behaviour and collapse modes have not yet been published.

5.3.3 Discussion and experimental correlation

By first assuming a value of 1.0 for u , the effect of initial imperfections on the load carrying capacity was

studied for the shell geometries presented in chapter 3. The mode of imperfection incorporated into the analysis is a single longitudinal half-wave. Both outward and inward modes were investigated. For the lateral pressure load case ($K=0$), fig.5.4 shows that the collapse behaviour is not imperfection sensitive and confirms the conclusion reached in chapter 3. For the $K=0.5$ and 1.5 cases, the inward imperfection mode has the more adverse effect since the failure modes for these load cases, being dominated by external pressure, are also inward. In the case of pure axial compression ($K=\infty$) failure occurs by outward bulging and, as expected, the outward imperfection mode is the more critical. In general, analyses for simply supported boundaries lie below those of clamped boundaries.

Before any comparisons with the experimental results can be made, it is first necessary to relate the level of hoop stress present in the experimental models to the pressure applied. A stress distribution factor (u) has been introduced into the theoretical formulation so that the effect of the rings in reducing the hoop stress level in their vicinity can be allowed for. The average hoop stress level in the shell panel is represented by $u p R / t$.

The difficulties involved in quantifying u have already been discussed in chapter 3. In this section, a procedure based on the recommendation of DnV [86] is developed to enable u to be determined consistently over the whole range of geometries and loading ratios considered.

In the present procedure, the hoop stress in the shell panel is assumed to increase in the form of a half sine wave from a value of $(p R / t) [(1 - \nu K) \frac{1.56 t / R t}{A_s + (1.56 t / R t)} + \nu K]$ (suggested by DnV) at the ring stiffener position to a value of $p R / t$ at the panel centre line. Since the average value for a sine distribution is $2/\pi$, u may be expressed as:

$$u = \frac{2}{\pi} + \left\{ \frac{(\pi - 2)}{\pi} \left[(1 - \nu K) \frac{1.56 t / R t}{A_s + (1.56 t / R t)} + \nu K \right] \right\} \quad (5.37)$$

A_s is the cross sectional area of each ring stiffener.

Equation (5.37) is used for determining u in all subsequent correlations.

To illustrate the procedure involved in the present formulation for predicting failure loads, the plastic mechanism unloading curves and the elastic loading curves for two of the cylinders tested in the first series are shown in figs. 5.5 and 5.6. Load-deflection curves from the finite element analyses reported in chapter 4 are also included for comparison since experimental data were not available. For cylinder 3 (fig.5.5), the predicted failure load assuming clamped boundaries is very near the experimental value. Despite the simplistic approach adopted in the present formulation, the elastic loading curve when combined with the mechanism unloading curve provides a reasonably close estimate of the finite element behaviour over the whole range of prebuckling deformations. The reason for this good correlation is due to the dominant axisymmetric component in the collapse mode of the finite element model.

The corresponding correlation for cylinder 2 is not so good. The finite element model contained realistic imperfections whereas the imperfection mode incorporated into the present theoretical analysis was a single longitudinal inward lobe with peak value defined by the DnV tolerance limit. More importantly, the finite element radial deflections for this cylinder were far from uniform around the circumference and the deflection which increased at the fastest rate was used in the construction of the load-deflection curve since the complex behaviour of the finite element model precludes any other approach. This explains why the stiffness of the finite element model appears to be considerably less than that indicated by the theoretical elastic loading curve even allowing for nonlinear prebuckling deformations. Both the plastic mechanism and finite element failure load predictions are, however, very near the experimental value.

Correlation between the test results from the first experimental series and the present formulation is shown in fig.5.7. The imperfection levels present in the experimental models were similar to the DnV tolerance limit. Two different approaches to the use of the DnV imperfections were adopted in the analyses. One approach involved factoring down DnV tolerance values to take account of the difference between the DnV gauge length and the ring spacing

(or the length of the mechanism whichever is less). The other approach involved using actual (unfactored) DnV tolerance values.

Fig.5.7 shows the the use of imperfection values based on actual DnV tolerance limits in the critical mode gives remarkably good agreement with the experimental results and much better than the use of factored DnV values. For these models, the use of DnV imperfections in general seems justified as measured peak values were comparable to the tolerance level given in the code.

In the range $K=0$ to $K=0.5$, the plastic mechanism method appears to be slightly conservative even with clamped boundaries. This could have been caused by strain hardening which is not accounted for in the present formulation or possibly some modal interaction between the shell panel and the ring stiffeners.

For $K < 0.5$, therefore, the analysis for clamped boundaries gives a good conservative estimate of the failure load. For $K \geq 0.5$, the analyses with simply supported boundaries result in moderately conservative predictions of the failure loads. Curves are presented later in this chapter combining the above two limits for presentation for design formulations.

Comparison with the strength predictions of DnV (also shown in fig.5.7) indicates that the plastic mechanism results provide much closer estimates of the collapse loads over the entire range of the combined loading interaction.

5.3.3.1 Liverpool models

Comparison between the present analysis and the test results of the machined models is shown in fig.5.8. For each geometry, the elastic buckling strengths of the corresponding unstiffened perfect cylinder are also included to assess the role of elastic bifurcation in the collapse of the experimental models and to provide an approximate limit

to the validity of the present plastic mechanism approach for any particular geometry. The elastic buckling strengths were determined from equations given by Brush and Almroth [54] based on simply supported boundary conditions.

The shortest models ($L/R \approx 0.3$) provided the best correlation with the theoretical results since yielding was most widespread in these models. In the pressure dominated region of the loading interaction, the upper bound nature of the plastic mechanism approach has produced unconservative estimates of the collapse strengths. In this region of the loading interaction, failure pressures are generally closely related to the boundary conditions. It is possible that the fully clamped conditions assumed in the analyses were overestimates of the actual fixity provided for the experimental models which, although defined as clamped, could nevertheless have allowed a certain degree of rotation in reality. The present formulation cannot account for partial boundary restraints, but an additional analysis assuming simply supported boundary conditions was carried out for the lateral pressure load case. It produced a failure pressure of $1.21p_0$ (c.f. $1.11p_0$ from experiment).

For the longer models, correlation is reasonable in the axial compression dominated region of the loading interaction. For pressure loadings, however, correlation is very poor. This is not surprising since for these longer geometries, failure under pressure loadings was expected to occur by elastic bifurcation. The L/R ratios of these models are significantly larger than those being currently adopted in the design of floating offshore structures. Their inclusion in the present comparison is merely to illustrate the limitation of the plastic mechanism approach.

For the comparison with the welded models (see fig.5.9) the two imperfection levels based on DnV tolerance values described previously were incorporated into the analyses as exact maximum magnitudes of experimentally measured values were not available. In any case, in view of the rather simplistic approach adopted by the present formulation in the treatment of imperfections, it would have been difficult to represent the actual imperfections in the experimental models by a single value. The levels of imperfections assumed in the present analyses, however, should not be too unrealistic since some of the experimental

models were reported to contain maximum imperfections of the order of the ECCS tolerance limit. Tolerance values given in ECCS and DnV are similar.

Many of the comments relating to the previous case of machined models also apply to the present welded models. However, direct correlation between theoretical and experimental results is complicated by the presence of residual stresses arising from welding.

A noticeable feature of the theoretical interaction curves shown in fig.5.9 is that when the actual (unfactored) DnV tolerance in the critical mechanism mode was incorporated into the analysis, the load carrying capacity in the zone where axial compression is combined with a small level of pressure showed significant drops for all geometries considered. The reason for such sharp strength reductions is due to the lengths of the buckling mechanisms being considerably shorter than those for other load combinations and larger initial angles had to be included in the analysis to produce the equivalent imperfection. Although such sharp strength reductions have not been reproduced by the present test models, experimental and finite element analytical results presented in chapters 3 and 4 have indicated this zone of the loading interaction to be potentially very imperfection sensitive.

5.3.3.2 Glasgow models

Correlation between the present formulation and the results of the four experimental series conducted at Glasgow University is shown in figs. 5.10 and 5.11. Both actual and factored DnV tolerance values were incorporated into the analyses to represent the imperfections in the experimental models. The use of these imperfection levels may not be entirely justified, but in view of the absence of experimental data, this was considered to be the only available approach.

The models in Series 1.1 all failed by plastic

collapse and as a result very good correlation with the present formulation is obtained over the entire range of the loading interaction. In particular, the analyses with clamped and simply supported boundaries have produced upper and lower bounds to the experimental results for most values of K when factored DnV tolerances are used as peak imperfection values.

The results of Series 1.2 (fig.5.10) also provided good correlation with the present formulation although the theoretical predictions had a tendency to become unconservative, particularly for pressure loadings. The larger ring spacing in this series ($L_p/R=0.4$) has significantly lowered the elastic critical pressures and failures under pressure loadings were more of an elastic rather than a plastic phenomenon. It is unfortunate that no detailed experimental data is available to confirm this.

In fig.5.11, results from Series 1.3 have produced very good correlations with the theoretical predictions using factored DnV tolerance values as peak imperfections. The present models had the shortest ring spacing ($L_p/R=0.1$) and it is unlikely that failures were much affected by elastic bifurcation.

Series 1.4 provided the least accurate correlations with the present formulation. Theoretical and experimental failure loads, however, were still of comparable magnitudes. With such a slender geometry ($R/t=290$, $L_p/R=0.3$), there is little doubt that failures were much more influenced, if not entirely dictated, by elastic bifurcation.

5.3.4 Summary

In the present section, a concise plastic mechanism approach for the prediction of ultimate loads corresponding to the axisymmetric local panel failure mode has been developed. Test results from various sources have been used to provide comparisons with the present formulation. These comparisons have demonstrated that provided failure is not

initiated by elastic instability, the present formulation is capable of producing accurate predictions over the entire range of the loading interaction. The difficulties involved in selecting representative imperfection levels have been discussed, but the use of factored DNV tolerance values as peak imperfection magnitudes has produced acceptable correlations in most cases.

5.4 General collapse

5.4.1 Theory

When a cylindrical shell collapses in the general instability mode, the deflected shape in the axial direction is affected by the restraints provided by the presence of intermediate heavy rings or diaphragms. The longitudinal distribution of buckled deflections may be approximated by a half sine wave between such diaphragms. This shape may be complicated by local shell imperfections. In the circumferential direction, experimental results [235,279,335] have shown that for a certain range of geometry, the combined effects of initial imperfections, plasticity, and boundary conditions will restrict collapse to a local form over a limited part of the circumference. Such a collapse mode is illustrated in fig.5.12. The present formulation assumes the formation of three longitudinal plastic hinges since this is the minimum number required for a mechanism to form. In fig.5.12, the location of the hinges are marked by A,B and C. It is assumed that all rotations in the circumferential direction are restricted to these positions. Hinges A and C are restrained against all translational movements along the entire length while the radial deflection of hinge B is defined by the axial sine curve. In the derivation of the governing equations, a rigid plastic model has been used to avoid the inclusion of elastic strains.

The mechanism involves stretching as well as hinge rotation in order to provide a reasonably compatible kinematic model.

Assume that during a virtual deformation, the mid-span section M-M undergoes a rotation $d\theta$ at hinges A and C (see fig.5.13). The total work done in the three hinges dW_{II} is given by:

$$dW_{II} = (2/\pi)N_s M_o 4d\theta \quad (5.38)$$

where $(2/\pi)$ is the mean value for the sine distribution of the radial deflection

N_s is the number of stiffeners present

M_o is the reduced plastic moment of resistance of each stiffener with a shell plating on $L_p/2$ to each side

M_o is a function of the cross section as well as the level of hoop stress present in the cylinder. The calculation procedure for the case of a T-section stiffener is given in appendix B.

During a virtual rotation $d\theta$, the length of each leg of the mechanism at section M-M is shortened from L_m to L'_m . From fig.5.13, it may be seen that:

$$L_u = R \sin \alpha$$

$$L_m = L_u \sec \theta'$$

$$L'_m = L_u \sec(\theta' - d\theta)$$

where

$$\theta' = \gamma - \theta$$

$$\gamma = \alpha/2$$

θ is the total angle of rotation.

Hence,

$$\begin{aligned}
 L_m - L'_m &= L_u \left[\frac{1}{\cos \theta'} - \frac{1}{\cos(\theta' - d\theta)} \right] \\
 &= L_u \left[\frac{\sin \theta' d\theta}{\cos \theta' (\cos \theta' + \sin \theta' d\theta)} \right] \\
 &= \frac{L_u \tan \theta'}{\cos \theta'} (1 + \tan \theta' d\theta) d\theta \\
 &\approx R \sin \alpha \tan \theta' d\theta / \cos \theta'
 \end{aligned} \tag{5.39}$$

(since $\tan \theta' d\theta \ll 1$)

In the above derivation, the initial length of the mechanism leg represented by arc AB has been replaced by chord AB. (For $\alpha < 30$, the error $< 1.14\%$).

The work done by the two legs of the mechanism dW_{I2} due to hoop compression is:

$$dW_{I2} = 2(\sigma_{\theta p} A_p + \sigma_o A_s) N_s (2/\pi) [R \sin \alpha (\tan \theta' / \cos \theta')] d\theta \tag{5.40}$$

where $\sigma_{\theta p} A_p$ is the hoop force in the shell panel and $\sigma_o A_s$ is the hoop force in the stiffener, both associated with yield. The term $(2/\pi)[R \sin \alpha (\tan \theta' / \cos \theta') d\theta]$ is the average shortening of each leg.

In equation (5.40), the level of hoop stress in the stiffener has been assumed to be independent of σ_a , the axial stress in the shell panel. The hoop stress in the shell panel, $\sigma_{\theta p}$ must, however, be related to σ_a by the Mises yield function f in the following way:

$$f \equiv \sigma_a^2 - \sigma_a \sigma_{\theta p} + \sigma_{\theta p}^2 - \sigma_o^2 = 0 \tag{5.41}$$

By writing $\bar{\sigma}_a = \sigma_a / \sigma_o$ and $\bar{\sigma}_{\theta p} = \sigma_{\theta p} / \sigma_o$, equation (5.41)

may be expressed in the following dimensionless form:

$$\bar{\sigma}_{\theta p} = [\bar{\sigma}_a \pm \sqrt{(4 - 3\bar{\sigma}_a^2)}]/2 \quad (5.42)$$

Since $\sigma_{\theta p}$ is always positive for a mechanism which deforms inward, the negative root of equation (5.42) will not need to be considered.

$\bar{\sigma}_a$ is equal to $\bar{N}$ which is related to the $\bar{p}_T$ by the ratio K_T in the following way:

$$\bar{N} = K_T \bar{p}_T \quad (5.43)$$

$$\text{where } \bar{p}_T = p/p_{oT} \quad (5.44)$$

p_{oT} is the squash pressure of each stiffener-shell assembly and is given by:

$$p_{oT} = \frac{\sigma_o A_T}{RL_p} \quad (5.45)$$

where A_T is the cross sectional area of the stiffener-shell assembly

Combining equations (5.42), (5.43) and (5.44) gives:

$$\bar{\sigma}_{\theta p} = [K_T \bar{p}_T + \sqrt{(4 - 3K_T^2 \bar{p}_T^2)}] \quad (5.46)$$

Hence equation (5.40) may be re-expressed as:

$$dW_{I2} = 2 \left[\left\{ \frac{1}{2} [K_T \bar{p}_T + \sqrt{(4 - 3K_T^2 \bar{p}_T^2)}] A_p \right\} + A_s \right] \sigma_o N_s (2/\pi) [R \sin \alpha \tan \theta' / \cos \theta'] d\theta \quad (5.47)$$

Compatibility of strains implies that deformation

must occur in the longitudinal direction. Since this deformation takes place under yield conditions, a flow rule is adopted to determine the change in plastic strain. The incremental change in axial strain $d\varepsilon_a$ is:

$$\begin{aligned} d\varepsilon_a &= \lambda(df/d\sigma_a) \\ &= \lambda(2\sigma_a - \sigma_{\theta p}) \end{aligned} \quad (5.48)$$

Similarly, the incremental change in hoop strain $d\varepsilon_{\theta p}$ is:

$$\begin{aligned} d\varepsilon_{\theta p} &= \lambda(df/d\sigma_{\theta p}) \\ &= \lambda(2\sigma_{\theta p} - \sigma_a) \end{aligned} \quad (5.49)$$

Combining equations (5.48) and (5.49) gives:

$$d\varepsilon_a = \frac{2\bar{\sigma}_a - \bar{\sigma}_{\theta p}}{2\bar{\sigma}_{\theta p} - \bar{\sigma}_a} d\varepsilon_{\theta p} \quad (5.50)$$

The average value of $d\varepsilon_{\theta p}$ is $(2/\pi)(L_m - L'_m)/L_m$. From equation (5.39), $(L_m - L'_m)/L_m$ is $\tan\theta d\theta$.

Hence the virtual work done in the axial direction dW_{I3} is:

$$dW_{I3} = 2NL_m(2/\pi) \left[\frac{2\bar{\sigma}_a - \bar{\sigma}_{\theta p}}{2\bar{\sigma}_{\theta p} - \bar{\sigma}_a} \right] \tan\theta' d\theta L_B \quad (5.51)$$

where L_B is the total length of the cylinder.

The work contribution due to axial bending should strictly be included in the above formulation, but for the

present mode of collapse, such contribution is likely to be unimportant compared to bending in the hoop direction.

The total external work contribution comprises two parts; the work done by the axial loading, dW_{E1} and the work done by the external pressure loading dW_{E2} .

During a virtual rotation $d\theta$, neglecting any strain deformation, the radial deflection at mid-span is increased from w_m to $(w_m + \delta w)$ and the end deflection is increased from Δ to $(\Delta + \delta\Delta)$ as shown in fig.5.14. Assuming that all end deflections are concentrated at one end, the relationship between w_m and Δ is given by:

$$\Delta + \delta\Delta = \frac{1}{2} \int_0^{L_B} (w_m + \delta w)^2 \sin^2(\pi z/L_B) dz \quad (5.52)$$

The derivation of equation (5.52) is given in ref.[69]. The standard result for Δ is:

$$\Delta = \pi^2 w_m^2 / (2L_B) \quad (5.53)$$

From equation (5.52),

$$\Delta + \delta\Delta = \pi^2 (w_m^2 + 2w_m \delta w) / (2L_B) \quad (5.54)$$

Hence

$$\delta\Delta = w_m \pi^2 \delta w / L_B \quad (5.55)$$

In fig.5.13 w_m is represented by the length DB, and may be expressed as:

$$w_m = L_0 \sin \gamma - L_u \tan \theta' \quad (5.56)$$

L_0 is the initial length of each mechanism leg and is equal to $2R\sin\gamma$.

Also in fig.5.13, δw is represented by the length ED and may be expressed as:

$$\delta w = L_m \sin\theta' - L_u \tan(\theta' - d\theta) \quad (5.57)$$

Since $L_m = L_u \sec\theta'$ and $\tan(\theta' - d\theta) \approx \tan\theta' - d\tan\theta$, δw may be simplified to (by assuming $d\tan\theta = d\theta$):

$$dw \approx L_u d\theta \quad (5.58)$$

Hence the expression for dW_{El} for both legs of the mechanism is:

$$dW_{El} = 2L_m N \frac{1}{2} \delta\Delta + 2NL_m (2/\pi) \left[\frac{2\bar{\sigma}_a - \bar{\sigma}_{\theta p}}{2\bar{\sigma}_{\theta p} - \bar{\sigma}_a} \right] \tan\theta' d\theta L_B \quad (5.59)$$

The second term in equation (5.59) accounts for the work done due to strain deformation. By combining equations (5.55), (5.56) and (5.58), the first term of equation (5.59) may be written as: $R \sin^3 \alpha \sec^2 \theta' k_T \bar{p}_T \sigma_o t [2 \sin^2(\alpha/2) - \sin \alpha \tan \theta'] \pi^2 / L_B$.

To calculate the work done due to the external pressure loading, it is necessary to determine the change of volume caused by the virtual rotation.

At section M-M, the change in enclosed area δA is given by:

$$\begin{aligned} \delta A &= \frac{1}{2} L_m L_u \sin\theta' - \frac{1}{2} L_m' L_u \sin(\theta' - d\theta) \\ &\approx \frac{1}{2} L_m L_u \cos\theta' d\theta \end{aligned} \quad (5.60)$$

The corresponding change in volume δV is therefore:

$$\delta V = L_m L_u \cos \theta' d\theta (2/\pi) L_B \quad (5.61)$$

Hence dW_{E2} may be written as:

$$\begin{aligned} dW_{E2} &= (2/\pi) \bar{p}_T p_{OT} L_B L_m L_u \cos \theta' d\theta \\ &= (2/\pi) \bar{p}_T p_{OT} L_B R^2 \sin^2 \alpha d\theta \end{aligned} \quad (5.62)$$

To satisfy the principle of virtual work, the total internal work done must equal the total external work done. Hence, by summing and equating the appropriate equations (5.38, 5.47, 5.51, 5.59 and 5.62), the following expression is obtained.

$$\begin{aligned} &4(2/\pi) N_s M_o + 2 \left[\left\{ \frac{1}{2} [K_T \bar{p}_T + \sqrt{(4 - 3K_T^2 \bar{p}_T^2)}] A_p + A_s \right\} \sigma_o N_s (2/\pi) [R \sin \alpha \tan \theta' / \cos \theta'] \right. \\ &- R^3 \sin^2 \alpha \sec \theta' K_T \bar{p}_T \sigma_o t (2 \sin^2 \frac{\alpha}{2} - \sin \alpha \tan \theta') \pi^2 / L_B \\ &\left. - (2/\pi) \bar{p}_T p_{OT} L_B R^2 \sin^2 \alpha \right] = 0 \end{aligned} \quad (5.63)$$

It should be noted that K_T may be related to the nominal loading ratio K in the following way:

$$K_T = K[A_p + A_s] / A_p \quad (5.64)$$

A_p and A_s are the cross sectional area of the shell panel and the stiffener respectively.

The solution in principle is to determine the value of α (i.e. the hinge position) that minimises the collapse pressure. Due to the complex nature of equation (5.63), the rigorous minimisation of α with respect to $\bar{p}_T$ cannot be conveniently obtained. Hence a range of values must be used to determine the lowest solution for $\bar{p}_T$. The locus of such

values of $\bar{p}_T$ for different values of α determines the unloading characteristic.

As described in section 5.3.1, an upper bound estimate of the collapse load may be obtained by determining the point of intersection of the plastic unloading curve represented by equation (5.63) with an elastic loading curve obtained for the corresponding value of α . This procedure is illustrated in figs.5.5 and 5.6 for the local panel collapse mechanism developed earlier.

For the present collapse mode, no definitive elastic approach can be considered which would allow a solution to be obtained. By ignoring elastic deflections in other areas of the shell, however, the corresponding elastic behaviour comparable with the plastic deformation would approximate that of a pinned arch. By considering the collapsed portion of the circumference in this way, a standard solution relating the angle of hinge rotation θ to the external loading is available [263] (see also appendix C). Clearly, this elastic deflection would be a poor representation of the true behaviour in the early stages of loading. It is being used, however, to define the point of collapse at which stage the deflections are approximately those of the plastic mechanism. The final assessment of this assumption can only be made in terms of experimental correlation.

The relationship between θ and the external loading is:

$$\theta = \frac{2R^2}{\pi EN_s I} [(\sin\alpha - \alpha\cos\alpha)H_A - \bar{p}_{eT} p_{oT} N_s L_p R(\alpha\cos^2\alpha - \sin\alpha\cos\alpha)] \quad (5.65)$$

$$\text{where } H_A = \frac{\bar{p}_{eT} p_{oT} N_s L_p R[3\sin\alpha\cos^2\alpha - \alpha\cos\alpha - 2\alpha\cos^3\alpha]}{\alpha + 2\alpha\cos^2\alpha - 3\sin\alpha\cos\alpha} \quad (5.66)$$

I is the moment of inertia of stiffener-shell assembly about the centroid and $\bar{p}_{eT}$ is p_e non-dimensionalised with respect to p_{oT} .

Referring back to fig.5.13, the relationship $\theta' = \gamma - \theta$ may be used in conjunction with an iterative procedure to determine the value of θ at which $\bar{p}_e$ given by equation (5.65) is equal to $\bar{p}_T$ from equation (5.63). This value of θ

corresponds to $\bar{p}_{Tc}$, the collapse pressure p_c non-dimensionalised with respect to p_{oT} . Since it is more conventional to express p_c in terms of p_o , $\bar{p}_{Tc}$ may be factored by $(A_p + A_s)/A_p$ to obtain $\bar{p}_c$.

The effects of initial geometric imperfections may be incorporated into the analysis by introducing an initial angle θ_0 . The relationship between θ_0 and the initial radial imperfection w_0 is illustrated in fig.5.15. Elementary trigonometry gives:

$$w_0/\sin\theta_0 = L_0/\sin(\frac{\alpha}{2} - \theta_0 + 90^\circ) \quad (5.67)$$

This gives:

$$\theta_0 = \tan^{-1} \left[\frac{\cos(\alpha/2)}{(L_0/w_0) - \sin(\alpha/2)} \right] \quad (5.68)$$

If the assumed mode of imperfection is such that it forms n_{imp} waves around the circumference, the angle subtended at the centre α_0 is given by:

$$\alpha_0 = \pi/n_{imp} \text{ rad.} \quad (5.69)$$

If $\alpha_0 > \alpha$, then α should be replaced by α_0 in equation (5.68). The analysis is identical to the perfect cylinder case except that the relationship between θ' and θ is now given by:

$$\theta' = (\alpha/2) - \theta_0 - \theta$$

since γ has been replaced by $(\alpha/2) - \theta_0$ in the analysis.

In the present analysis, the maximum value considered for α is approximately 30° since for larger values the errors introduced in replacing the initially curved

configuration of the mechanism leg by straight elements may become unacceptably large. Hence $n_{imp} < 6$ is considered to be outside the scope of the present approach.

5.4.2 Discussion and experimental correlation

Experimental data on the general instability mode of collapse has been obtained from cylinders 7 and 8 reported in chapter 3. This data has been used to provide correlation with the results from the present theory. Details of these experimental models are summarised in table 5.4.

Both these cylinders were tested under lateral pressure and failed in V-shaped mechanisms around a relatively small portion of the circumference. The failure mechanism of cylinder 7 is clearly illustrated by the plan section taken at mid-span (fig.5.16). It may be seen that deformations were concentrated within the V-shaped lobe. Elsewhere, the deformations were relatively small, thus justifying the assumption of negligible elastic strain implied by the present rigid plastic model. The outside hinges have shown significant translational movements. This is inconsistent with the assumption of fixed hinge positions adopted in the analysis. Such an assumption is, however, necessary since there exist no simple rule to define the locus of these hinges. Experimental measurements on the collapsed model have shown that the sinusoidal wave assumed in the analysis for the axial variation of radial deflections is very realistic (see fig.3.147). The failure mode of cylinder 8 was similar to that of cylinder 7 except that the formation of the V-shaped mechanism was accompanied by a smaller lobe adjacent to it. Further details regarding the experimental behaviour are given in chapter 3.

Before any comparisons with the experimental results can be made, it is first necessary to determine the magnitude (w_0) and mode (n_{imp}) of initial imperfections to be included in the analytical correlation. Due to the rather simplified approach which the present formulation adopts

towards the treatment of initial imperfections, both w_0 and n_{imp} can only be assigned a single value in each analysis.

The decision on the appropriate value to use for n_{imp} is particularly difficult since the experimental data revealed that many imperfection modes were simultaneously present. The relationship between the non-dimensionalised collapse pressure $\bar{p}_c$ and n_{imp} obtained by the mechanism formulation is shown in fig.5.17. DnV tolerance values were used for w_0 . Comparisons with the collapse pressure for the perfect cases indicate that the strengths of the cylinders were not strongly dependent on n_{imp} . Hence for the purpose of correlating with the experimental results, it was decided to include imperfections in the critical mode since this represented the most systematic approach available. Critical mode in the present context is defined as the mode into which the cylinder would buckle in the absence of any imperfections.

By using the critical mode of imperfections, the variations of $\bar{p}_c$ with w_0 have been plotted for the range $-2 \leq w_0/t \leq 2$ as shown in fig.5.18. For the inward mode of imperfection, the predicted strength of the cylinders decreased approximately linearly with increasing imperfections. The slopes of the graphs are gentle, indicating that for the lateral pressure load case, the cylinders were not particularly imperfection sensitive for the range of geometric parameters being investigated. Tolerance values from DnV were adopted for w_0 in the correlation with the experimental results since these were found to be comparable with the values measured experimentally. For outward imperfections, as expected, the strength increased. The third curve in fig.5.18 is discussed later.

Fig.5.19 shows the correlation between the theoretical and experimental results. $\bar{p}_c$ for both cylinders has been obtained for α in the range $0 < \alpha < 32^\circ$. For imperfections in the critical mode, it may be seen that p_c is significantly α -dependent. The value of α corresponding to the lowest value of $\bar{p}_c$ represents the predicted mode of collapse since least energy is required to cause the cylinder to buckle into this mode. The corresponding value of $\bar{p}_c$ is therefore the predicted collapse pressure. Solutions for the perfect cases have also been included for

comparison.

For cylinder 7, the correlation between the theoretical and experimental collapse loads is very good, the difference being less than 5%. The theoretical value of α corresponding to collapse is 22° which is 2° larger than the value obtained experimentally. This difference is well within the margin of error associated with the subjective nature by which collapse modes from experiments are generally estimated.

For cylinder 8, the difference between the theoretical and experimental collapse loads is just over 8%. Again the prediction for the collapse mode is very accurate as illustrated in fig.5.19.

The unloading characteristics of the mechanism model are shown in fig.5.20 where $\bar{p}_c$ has been plotted against w_0/t . Data obtained from the elasto-plastic finite element program FINAS are also shown. The levels of imperfections used for the finite element analysis were based on experimental measurements, while DnV tolerance values in the critical mode were used for the plastic mechanism calculations. The conservative nature of this approach is reflected by the fact that the plastic mechanism unloading curves are slightly lower than the corresponding finite element curves. The slopes of both sets of curves are, however, comparable indicating that the present formulation is a reasonably accurate representation of the actual structural response.

In addition to correlating with the experimental results of chapter 3, experimental data obtained by Boichot and Reynolds [46], Nott [235], Ross et al [267] and, Shao and Chen [279] have also been used to correlate with the present analysis. Details of these experimental models are summarised in table 5.4. Comparisons between theoretical and experimental results are presented in fig.5.21. For the machined models, w_0 was assumed to be zero in the analyses, while for the fabricated model of [279], DnV tolerance values in the critical mode were used.

For Boichot and Reynolds' models, fig.5.21 shows that the plastic mechanism analyses slightly over-predicts the collapse loads in all cases. The reason may be attributed to the fact that the collapse loads in the experiments were slightly lowered by the inevitable presence

of geometric imperfections. More importantly, it must be doubtful whether the assumption of negligible elastic strains inherent in the present rigid plastic formulation can be strictly applied to a material such as aluminium which has a much lower value of E than steel and a nonlinear stress strain characteristic. It is unfortunate that information on the experimental collapse modes is not available.

The experimental data of Ross et al [267] represented the worst correlation with the present analysis with the maximum difference in the collapse loads reaching about 17%. Examination of the theoretical results revealed that $\bar{p}_c$ for all models were independent of α , indicating that collapse was affected more by material yielding than by loss of stability. Not much information regarding collapse modes was given in ref.[267], but a plan section of model P1 indicated that collapse occurred in the $n=3$ mode. Due to unequal stiffener spacings, the inward lobes were not formed along the entire length of the models. In some cases, only half the length had buckled. Also, it is conceivable that the models had undergone strain hardening prior to collapse. The combined effect of these factors is difficult to assess. In spite of this, the correlation with the theoretical results is still generally very satisfactory.

For Nott's [235] model, the correlation between the collapse loads was good, but the analysis somewhat over-estimated the value of n . The same conclusions may be drawn regarding the correlation with Shao and Chen's [279] model. It should be pointed out that the predicted collapse mode depends to a great extent on the value of n_{imp} chosen. This behaviour is illustrated in fig.5.22 which shows the relationship between n and n_{imp} for cylinders 7 and 8. For values of n_{imp} less than 12, n was equal to n_{imp} . Hence, it is clear that in order to predict the experimental collapse modes, more information on the imperfection modes contained in the test models is required.

It was stated earlier that for cylinders 7 and 8, the values of $\bar{p}_c$ were not particularly imperfection sensitive. However, for the hydrostatic pressure load case, imperfection sensitivity is increased as illustrated in fig.5.18 for Shao and Chen's model. This increase in sensitivity is due to the axial component of the external

loading and is consistent with the behaviour of the local panel collapse mode discussed previously.

5.4.3 Summary

In this section, a systematic rigid plastic formulation has been derived to predict the collapse load corresponding to the general instability mode of collapse under pressure dominated loadings. The assumed mode of collapse for a certain range of geometries has been reproduced by experiments.

Available experimental data from various sources have been used to provide comparison with the theoretical results. Good correlations have been obtained for a wide range of collapse pressures. The influence of initial imperfections have also been discussed in detail.

The present formulation is only valid for the general collapse mode. Depending on the structural configuration, it is very possible for the panel between the ring stiffeners to collapse before this load is attained as demonstrated in section 5.3. Interaction between panel collapse and general instability may also affect any predictions of collapse load obtained from the individual analyses.

5.5 Combined mechanism formulation

In this section, a method is presented for the optimisation of stiffener sizes under pressure loading. The approach is based on combining the rigid plastic mechanisms for local and general collapse modes developed in sections 5.3.1 and 5.4.1. The criterion used for the optimisation

process involves the determination of stiffener geometries which produce concurrent collapse in the two modes. Later, in this section, results are presented in the form of design charts which enable the ultimate load carrying capacity and the required stiffener size to be evaluated for a range of shell geometries.

5.5.1 Finite element validation

Numerical results from the finite element program FINAS were used to correlate with the results from the plastic mechanism analyses. The correlations form part of subsequent sections in this chapter and for this reason a brief description of the finite element model is included here. It consists of a quarter of the circumference and half of the total length. A total of twenty five shell and ten beam elements were used for each analysis. The magnitude of initial imperfections incorporated into the analyses was the DnV tolerance value factored to take account of the difference between the ring spacings and the gauge lengths specified in DnV. The imperfection mode consisted of two circumferential waves and one axial half wave since this combination is the most critical for elastic buckling. Experimental data presented in chapter 3 have indicated this mode to be a major component of the imperfection form found in welded cylinders.

5.5.2 Relationship between local and general collapse modes

The formulation for general collapse considers a certain geometrical failure mode which is appropriate for relatively stocky cylinders loaded by predominantly pressure

loading.

Fig.5.23 shows the general collapse behaviour prediction for cylinder 7, illustrating that for the given stiffener size, the lateral pressure load case results in the lowest general collapse pressure.

For this particular geometry, the curve for local panel collapse is also shown. From this, it can be seen that for the pressure dominated region of the interaction, the predicted mode of failure is general collapse while for the axial loaded area local collapse would occur.

In this instance, the critical case where local and general collapse occur simultaneously is predicted to be at a K value of about 0.26. This means that local collapse will occur for example under hydrostatic pressure loading ($K=0.5$). This prediction has been verified by the experimental and finite element results of cylinder 6 presented in chapters 3 and 4.

In spite of the fact that the general collapse mechanism assumed is not applicable to the axial compression region of the interaction, it can readily be assumed that with this stiffener size (appropriate to a pressure dominated environment) stiffener failure will not occur in that area. In mechanism terms, if K is greater than about 1.0 no solution is possible for the governing equation since the cylinder is likely to fail by an outward mode. Allowing an outward mechanism of a similar type would be inappropriate because of the likely different mode that would occur in the axial load dominated region.

For typical offshore structures, it is normal for the stiffeners to be proportioned so that failure in tension never occurs and only limited experimental evidence relevant to offshore geometries is available for load cases corresponding to $K>0.5$. It seems likely that the stiffeners would have to be so small to produce general collapse that fabrication considerations would dictate their design in practice. This area of the loading interaction is investigated in chapter 6 using the finite element program FINAS.

Referring back to fig.5.23, and ignoring possible interaction between the two modes of collapse, it can be seen that for $K=0.26$ the stiffener size is such that in this instance general and local collapse occur together and that

the stiffener is the minimum necessary for this ratio and level of loading. If the stiffeners were to be larger, local, but not general, collapse would occur at the same load level and if smaller, general collapse would occur at a lower load level.

5.5.3 Optimisation of stiffener size

The minimum required stiffener size for a given stiffener geometry and loading ratio is achieved from the mechanism approaches by calculating and equating the collapse loads $\bar{p}_c$ and $\bar{p}_f$ corresponding to general and local collapse respectively for a range of stiffener sizes. When the stiffener size is such that $\bar{p}_c = \bar{p}_f$ an ultimate load of the stiffened panel equal to the inter-ring collapse load is obtained.

The method for determining the required stiffener size is illustrated in figs.5.24 and 5.25. The geometrical parameters given are based on the second test series reported in chapter 3. The area of the stiffener A_s is plotted on the abscissa non-dimensionalised with respect to the area equivalent to the inertia requirement of DnV. The reason for choosing the stiffener area as a parameter cannot be fully explained at this stage but will be discussed later. It is interesting to note that for the practical range of stiffener forms, the inertia for a given area ratio is practically constant for a stiffener including an effective shell plating.

For the T-stiffened cylinder under lateral pressure shown in fig.5.24, the intersection of local and general collapse predicted by the mechanisms occurs at a value of area fractionally less than the DnV requirement. The corresponding load carrying capacity is, however, $1.76p_0$ which is well in excess of the collapse pressure predicted by DnV. For values of A_s less than this limit point predicted failure will be by general collapse and this is confirmed by the experimental result lying very close to the

mechanism curve.

Also shown on the graph are the predictions of a series of finite element analyses for various stiffener sizes and while no definite transition is apparent, the results model closely the trends of the mechanism curves. The absence of a transition point is not unexpected because in reality the transition between the collapse modes corresponds to a gradual increase in stiffener deflections. There is also little indication of interaction between the two modes producing destabilising actions and unexpected losses in strength and this is also not unexpected because pressure failures of this type do not exhibit marked instability. The imperfection insensitivity of this zone reflects the relatively stable behaviour.

For the hydrostatic pressure case also shown in fig.5.24, the mechanism predictions and the finite element curve exhibit very similar shapes. The predicted ultimate load carrying capacities are also very similar. Both approaches predict a definite change of collapse mode at a stiffener area of only approximately 20% of the DnV requirement.

Many of the observations made about the T-stiffened cylinder apply equally to the flatbar stiffened case illustrated in fig.5.25. However for this case, the mechanism results for hydrostatic pressure are significantly higher than the finite element results for small stiffeners. The upper bound nature of the mechanism approaches could in part explain the discrepancy in the prediction of the collapse loads but in fact a more defineable reason is apparent. For this loading type, the critical elastic buckling stress for the present geometry σ_{cr} is close to the material yield stress and buckling occurs partially as an elastic phenomenon. The basic assumptions of the mechanism approach are therefore reaching the limit of their applicability in this instance. It is shown later that these limits can be readily defined and do not impose practical limitations for a wide range of structural forms.

The DnV requirement is again a reliable guideline for stiffener sizing for lateral pressure but is very conservative for hydrostatic pressure. This is examined later for a wider range of geometrical parameters. It is important, however, to point out that the collapse load

predictions of DnV are very conservative in all cases. General comparisons with this and other codes are presented in chapter 7.

5.5.4 Limits of applicability

Equations given by Brush and Almroth [54] for the elastic buckling of a simply supported unstiffened cylinder under combined axial compression and external pressure were used for calculating σ_{cr} . The results are summarised in fig.5.26 for the range of R/t and L_p/R ratios typical of offshore fabrication. Values of σ_o for mild and normal high strength steel are used to show the effect of variation in yield stress.

Above these curves the cylinder would be expected to fail largely by elastic buckling and below largely by plastic buckling. These limits are, however, only approximate as the effects of initial imperfections and stiffening are not included. The value of 500 for R/t has been chosen for a cut-off value because it represents a normal limit to offshore design and the buckling behaviour above this limit would not be accurately reflected by the plastic mechanism assumptions.

The curves show that the plastic mechanism analyses, subject to the assumption regarding buckling form, should be applicable for the majority of geometries used in offshore practice, particularly with the lower value of yield stress.

It is of interest that the geometry of fig.5.24 ($R/t=267$, $L_p/R=0.15$) lies almost exactly on the limit line for the hydrostatic load case but significantly below the line for lateral pressure loading. The results of figs.5.24 and 5.25 and subsequent finite element correlations for certain geometries show conclusively that this factor is responsible for the relatively poor agreement between mechanism and numerical analysis predictions.

5.5.5 Effect of yield stress

Fig.5.27 shows the influence of yield stress on the collapse of a T-stiffened cylinder under lateral and hydrostatic pressure. For both local and general collapse, the lateral pressure strength exhibits a linear dependence on yield stress, while for the hydrostatic pressure load case the panel collapse strength is slightly nonlinear. For design purposes, however, the dependence could be regarded as linear in all cases, thus greatly simplifying the design formulation.

5.5.6 Design curves

By using the concepts developed in the preceding sections, the optimum stiffener sizes for equating local and general collapse loads have been evaluated using the mechanism approaches for a range of R/t and L_p/R ratios falling generally within the feasible region of fig.5.26. A yield stress value of 375N/sq.mm has been chosen as this is more restrictive from the geometrical viewpoint and provides a sounder base for interpolation for other steel strengths.

Required stiffener areas are presented for both lateral pressure and hydrostatic pressure in fig.5.28 and 5.29.

In each graph, the ultimate pressure, non-dimensionalised with respect to the yield pressure is plotted against the required stiffener area and a range of geometrical shell parameters with flatbar and T-bar stiffener sections. One major point emerging from the graph is that these two significantly different stiffener configurations have produced almost exactly identical design requirements establishing that area rather than inertia

appears to be the critical parameter in determining design strengths. For design purposes the T-stiffener curves could be used to cover the normal configurations encountered in practice. It is, however, interesting to note that while the actual stiffener inertias for a selection of cases (see table 5.5) are significantly different for the two configurations, the inertias including an effective width of shell plating (taken as the DnV effective width in this instance) are not very significantly different. If required for a particular design formulation effective inertia could also be taken as the design parameter.

This dependence of strength and behaviour on stiffener area is not altogether surprising as the predominantly pressure loading has been shown by the experiments of chapter 3 to produce essentially circumferential membrane rather than bending stresses in the ring stiffeners. In particular, the circumferential contraction of the stiffeners was noted to play an important role in controlling behaviour. Local out-of-roundness effect will of course induce some bending stresses, but these would not seem to be of prime importance.

The other major point is that the abscissa $\sqrt{A_{\text{reqd}}}/t$ (A_{reqd} is the required stiffener area) has been found to be a true non-dimensionalising parameter and to be independent of the actual shell thickness.

5.5.7 Validation of design curves

In order to check the curves presented over a larger range of geometries than has been accomplished by the experimental correlations, a series of finite element analyses covering the extremes of figs.5.28 and 5.29. have been carried out.

All the analyses included four stiffeners between diaphragms and contained imperfections equal to the DnV tolerance values factored to take account of the ring spacing relationships with the code gauge length.

For the lateral pressure load case (fig.5.28) excellent agreement has been obtained for three of the four cases analysed. The fourth case ($R/t=150$, $L_p/R=0.3$) is in error by less than 20% on collapse load. This is again because of the proximity of the geometry to the critical stress line of fig.5.26 and emphasises the need to restrict the present method to outside a small band of such values. The general agreement with the mechanism predictions can only be regarded as conclusive.

For the hydrostatic pressure case (fig.5.29) agreement is again very good as long as the parameters fall outside the critical stress range. The dangers of using the plastic mechanism formulations for more slender geometries can be seen to the bottom left of the figure. The $R/t=250$, $L_p/R=0.1$ geometry produces a tolerably good comparison, the finite element collapse load being only 11% below the mechanism prediction, while for the $R/t=350$ and $L_p/R=0.1$ case, a large discrepancy is produced (the difference in scales between fig.5.28 and 5.29 should be noted).

5.6 Conclusions

Two simplified theoretical models based on rigid plastic mechanism approaches have been developed to investigate two different modes of collapse which can occur in ring stiffened cylinders under combined axial compression and external pressure loading. The first model represents the inter-ring axisymmetric collapse of the shell panel while the second describes general collapse involving both shell and stiffeners. The latter is applicable only to pressure dominated loadings. Good correlations between these models and experimental results from various sources have been obtained for shell geometries in which instability is preceded by yielding.

By combining these two models, a method has been derived to determine the most economic stiffener sizes as well as the corresponding load carrying capacities for a

range of geometries. This was accomplished by establishing the limiting stiffener sizes at which general and inter-ring local collapses occurred simultaneously. Direct interaction between these modes was ignored but correlation with finite element results covering the full range of geometries indicates that the method provides more than acceptable accuracy for design purposes.

Guidelines for selecting shell geometries to avoid elastic collapse which invalidates the plastic mechanism assumptions have also been given. This type of collapse is investigated in chapter 7 where general comparisons between the present plastic mechanism results and the predictions of various design codes will be presented.

CHAPTER 6

FINITE ELEMENT PARAMETRIC STUDY

6.1 Introduction

The collapse behaviour of ring stiffened cylinders under pressure dominated loadings over a wide range of parameters has been studied in the previous chapter by using the rigid plastic mechanism approach. In the present chapter, the structural response under loadings dominated by axial compression is investigated using the finite element program FINAS described earlier. No geometrical limitations are in this instance imposed as FINAS, being a nonlinear package, is capable of modelling both elastic and plastic collapse. The range of geometries examined was: $100 \leq R/t \leq 500$ and $0.1 \leq L_p/R \leq 0.3$. A survey carried out by Valsgard et al [343] indicated that these limits were realistic for common offshore tubular members.

In all the analyses which follow, shell elements were used to represent the cylinder while ring stiffeners were modelled by beam elements. Both types of elements have been described in chapter 4.

6.2 Convergence study

The effect of mesh variation in the circumferential direction has been studied in chapter 4. In the present chapter, only the variation of mesh size in the axial direction is studied. The finite element model used for the convergence study consisted of a five bay cylinder with simply supported boundary conditions and with ring

stiffeners designed to DnV requirements. Only a thin strip of the circumference was analysed and was represented by a single element subtending an angle of 4 degrees at the cylinder axis. The model was, therefore, only capable of reproducing axisymmetric collapse. The limitations associated with this model will be discussed more fully in later sections.

As discussed in chapter 4, the wavelength of the buckles is a critical parameter in determining the required mesh sizes and is expected to be shortest when the cylinder is subjected to predominantly axial loadings. It was decided to confine the present convergence study to the two loading ratios on which subsequent investigations in this parametric study were to be based, namely, pure axial compression ($K=\infty$) and a loading ratio corresponding to $K=1.5$. R/t ratios of 100 and 500 were investigated under each loading while the L_p/R ratio chosen was 0.3 since this represented the cylinders with the longest length and hence most sensitivity to mesh size variation in the axial direction.

The convergence study involved analysing each geometry with three different meshes. These meshes will be described as 'coarse', 'medium' and 'fine' and were represented by 8, 4 and 2 elements per panel respectively. The results of these analyses are summarised in table 6.1. To avoid the numerical difficulties associated with the analyses on perfect cylinders encountered in chapter 4, nominal imperfections in the form of a continuous sine wave with magnitudes defined by DnV tolerance limits were incorporated in the present analyses.

For the $K=1.5$ load case, table 6.1 shows that the 'medium' mesh is adequate for all geometries since the difference in the values obtained for P_u were no larger than 0.2% compared to the 'fine' mesh model. For the pure axial compression load case, the 'medium' mesh is also very accurate for the stocky geometry of $R/t=100$ while for the more slender geometry of $R/t=500$, an analysis with an extra mesh consisting of 6 elements per panel was used to verify the validity of the solution from the 'fine' mesh model. The deviation of σ_u between the 'medium' mesh model and the 'fine' mesh model is 4.29%. This was considered to be acceptable and the 'medium' mesh was adopted for the subsequent investigations.

In this study, the ring stiffeners designed to DNV requirements were found to have adequate strength to enforce collapse in the more critical local mode. This mode is more critical for a mesh convergence study as overall collapse involves significantly longer buckling wavelengths.

6.3 Choice of initial imperfections

It is well known that the strength of cylinders under axial compression is strongly dependent on the shape and magnitudes of the initial geometric imperfections. Since such imperfections are inevitably present in all structures, any analytical or theoretical model must be able to allow for their effect on the collapse behaviour. In particular, it is important to be able to identify the types of imperfection which will lead to the most significant reduction in the collapse load.

Harding [143,144] and Estefen [112] have shown with numerical results from finite difference programs that for axially loaded cylinders, imperfections in the axial direction are much more important than imperfections in the circumferential direction as far as their effect on axial load carrying capacity is concerned. This phenomenon was observed for both unstiffened and ring stiffened cylinders over a wide range of shell geometries.

It is commonly accepted that the distribution of axial imperfections is not entirely random, but is generally related to the particular fabrication process involved. For ring stiffened cylinders employed as offshore tubulars, imperfections induced by circumferential welding normally assume predominantly axisymmetric modes as discovered by Odland [238] from measurements on fabricated models. Ellinas et al [107] reported that in measurements on prototype tubulars carried out by Dwight [99], such modes had been observed. Theoretical studies by Sridharan et al [308], and Ellinas and Croll [103] have shown that these axisymmetric modes can have a very strong deleterious effect on the load carrying capacity of cylinders prone to elasto-plastic collapse in axisymmetric modes. Experimental evidence provided by Tennyson [319] indicates that axisymmetric

components of the imperfections are much more critical than the asymmetric components in reducing strength.

The adoption of only axisymmetric modes in analytical formulations does not, however, constrain collapse to occur in an axisymmetric form. Analyses carried out by Koiter [185], and Almroth [3] have shown that although axially loaded cylinders with axisymmetric imperfections may have axisymmetric configurations of equilibrium, they can also buckle into periodic asymmetric modes. This has been confirmed by the experiments of Arbocz and Babcock [16].

Based on these concepts, it was decided to incorporate axisymmetric imperfections into the present finite element models. Measurements reported in chapter 3 and those taken by Onoufriou [243] and Dowling et al [97] on ring stiffened cylinders of various geometries provided the experimental data to aid the selection of imperfection modes. All these measurements have been processed so that the relative magnitude of each mode could be represented in terms of Fourier coefficients. Since only axisymmetric imperfections are being considered, the actual magnitude of the imperfections w_0 may be expressed as:

$$w_0 = \sum_{m=1, 2 \dots} A_m \sin \frac{m\pi x}{L_p} \quad (6.1)$$

where A_m is the Fourier coefficient corresponding to the number of axial half waves m .

The experimental data have been summarised in table 6.2 where it may be seen that for nearly all the models measured, there was a strong dominance of the $m=1$ mode. The higher modes were progressively smaller. The overall average values indicate that the amplitude of each mode was approximately inversely proportional to m . The same phenomenon has been repeatedly observed by Agelidis [1] from measurements on welded stringer stiffened cylinders.

In table 6.2, the signs of the Fourier coefficients have been omitted since their inclusions would have caused considerable confusion in the calculation of the relative magnitude of each mode. Representative combined maximum magnitudes of the imperfections are difficult to envisage, but comparisons with tolerance values in DnV (see chapter 3)

have indicated that the latter values are reliable guidelines. Such tolerance values were used for the peak imperfection magnitudes in the later analyses. In situations where the ring spacing was less than the gauge length specified in DnV, the magnitudes were factored down linearly.

6.4 Description of finite element models

The inclusion of only axisymmetric imperfections suggests the possibility of basing the parametric study on an axisymmetric model. This model would be much more economical than a corresponding asymmetric one and would enable a much greater number of investigations to be carried out. It is, however, first necessary to justify and examine the limitations of such a model so that its suitability for the present purpose may be assessed. In particular, it is important to be able to define the boundary of its validity so that in areas where the assumption of axisymmetry becomes unrealistic it may be replaced by a more accurate but more complex asymmetric model.

In axial compressive tests carried out on ring stiffened cylinders by Sridharan et al [308] and Singer et al [294], it was reported that regardless of the final mode of collapse, the prebuckling deformations were always axisymmetric. The same phenomenon was repeatedly observed over a wide range of shell geometries. Similar tests conducted by Odland [238] on cylinders with geometries in the range of $163 \leq R/t \leq 279$ and $0.106 \leq L_p/R \leq 0.917$ revealed that the failure loads corresponding to the axisymmetric collapse mode were not significantly different to those corresponding to the asymmetric mode.

Despite these encouraging results in favour of the axisymmetric model, it was felt necessary to examine further any inherent approximations. It is of particular interest to consider the differences between the critical stress corresponding to axisymmetric collapse σ_{CAX} and that corresponding to the natural collapse mode σ_{Cr} over the whole range of geometries relevant to offshore design. The use of Timoshenko's [327] theory produces values of σ_{CAX} and

σ_{Cr} which are valid for elastic collapse of perfect unstiffened cylinders. This was considered to be satisfactory for the present purpose since only preliminary estimates were required.

The results of these calculations are presented in table 6.3. The maximum difference between σ_{CAX} and σ_{Cr} is 13.5% ($R/t=500$, $L/R=0.1$). For the majority of the other geometries, the differences are considerably smaller. These findings together with the experimental results of Odland [238] suggested that an axisymmetric finite element model was adequate for at least parts of the parametric study. However, the maximum discrepancy of 13.5% between σ_{CAX} and σ_{Cr} meant that an asymmetric model had to be used for certain geometries. Other reasons for choosing this more complex modelling will be discussed later.

Despite the limitations inherent in axisymmetric approaches, a number of analytical and theoretical models based on this assumption have been proposed by many researchers in view of the inherent simplicity. Notable examples are the work of Reis et al [257], Galletly and Pemsing [129], Pedersen [249], Esslinger and Geier [111], Croll [76] and Walker et al [353]. Most of this work was related to geometries relevant to offshore tubulars.

The axisymmetric finite element model adopted consisted of five equal panels with each panel represented by four shell elements. The angle which the elements subtended at the axis of the cylinder was 4 degrees. This angle was selected to ensure that the aspect ratio of the elements over the whole range of geometries to be examined was within the applicability limits given by Bates [33]. Each of the four ring stiffeners was modelled by a beam element. The entire configuration together with the boundary conditions is illustrated in fig.6.1.

So far the validity of the axisymmetric model has been discussed only in terms of local panel buckling. The situation with regard to overall buckling involving the simultaneous collapse of the stiffeners and shell plating needs consideration.

Unlike local panel buckling, experimental evidence for the overall buckling of ring stiffened cylinders under axial compression is much more difficult to obtain, particularly for geometries relevant to offshore design. The

only experimental data available to the author were from tests performed by Weller and Singer [357], Becker et al [38] and Miller [211]. These tests show that overall buckling always occurs in an asymmetric mode involving considerable bending in the circumferential direction, particularly at the ring stiffeners. Such bending action is very important in the assessment of stiffener sizes to avoid overall collapse. Since the axisymmetric model described above is incapable of representing this action, an asymmetric model has been used for investigating stiffener size requirements. The description of this model is given below.

In the asymmetric model, five shell elements were used to represent a quarter of the circumference. Since this number of elements was considerably larger than for the axisymmetric model, limitations imposed by computing costs meant that only three panels could be incorporated. These were modelled by four elements per panel. Five beam elements were used to represent each of the two ring stiffeners. This model is illustrated in fig.6.2. In view of the high cost involved, the use of this model was kept to a minimum.

6.5 Discussion of results

6.5.1 Effect of imperfection modes and magnitudes on collapse

Based on the relationship that A_m is inversely proportional to m (see section 6.3), three different imperfection modes denoted by A, B and C have been generated. These modes were obtained by superimposing the axial waves in the range $1 \leq m \leq 5$. In mode A, the component corresponding to $m=1$ was considered to vary in the form of a continuous sine wave. In mode B, only positive values (outward imperfections) of this component were considered to generate what is sometimes referred to as the 'hungry horse' mode. Mode C is the superposition of mode A on a simple outward half wave of amplitude $A_1/4$ spanning over the entire

length of the cylinder. This mode was introduced to assess the effect of overall imperfections.

The imperfection sensitivity of each of these modes was studied by using the axisymmetric model with flatbar stiffeners designed to DnV. Studies were carried out for the most stocky ($R/t=100$, $L_p/R=0.1$) and the most slender ($R/t=500$, $L_p/R=0.3$) geometries under axial compression. The relationships between strength and imperfection magnitude are plotted in figs.6.3 and 6.4. In these graphs, the imperfection magnitudes are represented by the commonly used parameter w_0/t . The choice of this parameter, however, can sometimes give misleading impressions since neither the radius nor the length of the cylinder is taken into consideration. A more logical approach including the effect of these other dimensions is given in DnV. Imperfection limits based on the tolerance values specified in the DnV code are also shown in figs.6.3 and 6.4. and are seen to provide much better guidance for the assessment of imperfection sensitivity. As explained previously, these limits have been factored to take account of the difference between the ring spacing and the DnV gauge length. It may be seen that for the imperfection magnitude equivalent to this factored tolerance value, the strength of the more slender cylinder drops appreciably more than that of the stockier cylinder from the perfect cylinder strength. This would be expected.

The imperfection sensitivity for both geometries is very similar for all three modes of imperfection. For any specific imperfection magnitude, the strengths from the different modes are also very similar with mode A providing a safe lower bound over the whole range of imperfections. For this reason, imperfections in this mode were adopted for all subsequent investigations.

The spread of plasticity was found to be dependent on the imperfection magnitude. For the $R/t=100$ geometry, the perfect model collapsed with 100% of its shell elements undergoing plastic deformation. The corresponding figure for the models with the largest imperfection was only approximately 40%. For the $R/t=500$ geometry, the effect of plasticity on collapse was, as expected, much less with the extent of plasticity varying from 30% to 10%. In general, plasticity was initiated at the shell elements adjacent to

the ring stiffeners and in all cases the ring stiffeners remained elastic with stress levels much lower than those of the shell panels.

For both geometries, it was discovered that the collapse modes were determined by the imperfection modes and not by imperfection magnitudes. Typical examples of the collapse configuration from each imperfection mode are presented in fig.6.5. In all cases, the ring deflections remained relatively small compared to those of the shell panels indicating that stiffeners designed to DnV were adequate to prevent general collapse. It is noticeable that the buckles in the $R/t=500$ models had much higher curvatures than those of the $R/t=100$ models and this accompanied the formation of a greater number of axial waves. This observation confirms the prediction of Timoshenko [327].

The load-deflection paths have been classified schematically in appendix D. The shapes of these paths again depended on the imperfection modes with imperfection magnitudes having very little effect. The perfect $R/t=100$ model falls into category D while the corresponding imperfect models all fall into category B. For the more slender $R/t=500$ geometry, the perfect and imperfect models fall respectively into categories A and C. The common features of these curves are the well defined peak loads and the very unstable postbuckling characteristics. As the geometries become more slender, the unloading behaviour becomes more violent.

6.5.2 Effect of yield stress on collapse

The effect of variations in yield stress σ_0 on collapse was investigated for cylinders with the extreme geometries ($R/t=100$, $L_p/R=0.1$ and $R/t=500$, $L_p/R=0.3$) for both $K=\infty$ and $K=1.5$.

The 5 panel axisymmetric model with flatbar stiffeners designed to DnV was again used throughout. Five values of stresses were considered, ranging from 100 N/sq.mm to 375 N/sq.mm. All the analyses incorporated initial imperfections in mode A with maximum total amplitudes defined by factored DnV tolerance values.

The result of this investigation is summarised in fig.6.6 where the load carrying capacity normalised with respect to the squash load ($\bar{\sigma}_u$ or $\bar{p}_u$) has been plotted against σ_o . For the cylinder with the slender geometry, both $\bar{\sigma}_u$ and $\bar{p}_u$ drop approximately linearly with increasing σ_o . The value obtained for $\bar{\sigma}_u$ or $\bar{p}_u$ from the highest value of σ_o provides a lower bound for cylinder strength for other values of yield stress. For the more stocky cylinder, $\bar{\sigma}_u$ is essentially independent of σ_o until σ_o increases to very high values. This is because for cylinders with such stocky geometry collapse is more influenced by yielding than by loss of stability. The curve corresponding to the K=1.5 load case is slightly more complex with $\bar{p}_u$ rising initially and then dropping as σ_o is increased. The strengths corresponding to $\sigma_o = 375$ N/sq.mm again provide a safe lower bound. Hence this value of σ_o has been adopted for the remainder of this parametric study. This approach has been taken because of the unacceptable level of cost associated with taking the yield stress as a parameter in the general study. The difference in behaviour between the two loading cases precludes an idealised approach. The degree of conservatism introduced is significant for slender geometries, but any other approach would be considerably more complex.

In this investigation, the stress state of the cylinders was carefully monitored. Plasticity was discovered to be confined to the shell panels with the ring stiffener outstands remaining completely elastic in all cases. First yield always occurred at either the shell/stiffener junctions or at the end supports. The extent of plastic flow was found to depend on the shell geometry as well as the loading type, but the value of σ_o also had an important effect. The extent of plasticity is quantified in table 6.4 together with information on the load-deflection behaviour. The shapes of the load-deflection curves were not affected by changes in σ_o since the loading has been nondimensionalised with respect to the squash load. As expected, plasticity was generally more widespread for lower yield stresses.

6.5.3 Effect of stiffener shapes on collapse

As in the above investigations, attention has again been concentrated on the cylinders with the extreme geometries. Both $K=\infty$ and $K=1.5$ loading ratios have been studied. However, the range of stiffener size involved in the present investigation meant that the more expensive asymmetric mesh (see fig. 6.2) had to be used to allow for stiffener bending as well as to enable the general collapse mode to be modelled.

The analyses in chapter 5 indicated that under pressure dominated loadings, the stiffener area A_s was the critical parameter in determining the strength of a shell for a given geometry. This is in contrary to the formulation of most design codes [52,86,114] which regard stiffener inertias as the controlling parameter for design. In the investigations which follow, this inconsistency was examined by analysing stiffener shapes with identical areas but different inertias over a wide range of stiffener sizes to see how the collapse strengths varied. The shapes examined were a flatbar and a T section. The depth to thickness ratio for the web and flange (applicable only to T sections) was maintained at 10 in all cases to ensure that it was well within the limits specified by DnV to prevent stiffener tripping. A survey carried out by Ellinas et al [107] has indicated this ratio to be typical for offshore members.

The DnV code with required inertias I_{DnV} were converted to required areas A_{DnV} for the sizing of the stiffeners. Three stiffener sizes were examined for each geometry and loading. These corresponded to areas of 1, 0.5 and less than 0.1 times A_{DnV} .

The choice of initial imperfections was identical to mode A (see section 6.5.1) in the longitudinal direction. The validity of using this type of imperfection, particularly with very small stiffeners will be discussed in section 6.5.4. A $n_{imp}=2$ mode was also superimposed on this in the circumferential direction since experimental data [332] has indicated this to be realistic.

The results of this investigation are summarised in table 6.5. For any specified area of stiffener, the collapse loads obtained from the flatbar and T stiffeners were very similar, the maximum difference being only 2.3%. The vast

majority of cases in fact produced differences much less than this. The collapse modes and load-deflection behaviour corresponding to each stiffener shape were also very similar for each geometry and loading. These aspects of the structural behaviour will be discussed in greater details in the next section when the effect of variation in stiffener sizes will be examined.

From the observations described above, it seems equally relevant to use stiffener area as a critical parameter for controlling collapse strengths under predominantly axial loading. This phenomenon was also observed in tests carried out by Weller and Singer [358].

The similarity between the collapse loads for the different stiffener shapes is only to be expected for the panel collapse mode since for this mode of collapse ring stiffeners in general are not highly stressed by the axial component of the applied loading and their only role in resisting collapse is in the control of the buckle wavelength in the axial direction. For the general collapse mode, it is very interesting and somewhat surprising to discover that the collapse loads were also unaffected by changes in stiffener shape. In this mode of collapse, ring stiffeners are normally subjected to considerable bending and one would expect collapse behaviour to be related to stiffener inertia. However, for the shell geometries that are employed in offshore construction, general collapse does not occur unless the stiffeners are very small, in which case their contribution to the resistance against collapse compared with that of the associated shell is negligible whether expressed in terms of inertia or area. To remain consistent with the work on pressure loaded cylinders it seems reasonable and appropriate to use stiffener area as the design parameter in this context. Quantitative assessments of stiffener sizes involved in this mode of collapse will be given in the next section.

6.5.4 Effect of stiffener size on collapse

As in section 6.5.3, the 3 panel asymmetric model

was used to conduct the present investigations. The use of this model involves high computing costs and without the versatility, economy and other advantages offered by the rigid plastic mechanism approach developed in the previous chapter, only a limited number of cases could be examined. However, using a finite element package means that a greater range of geometrical parameters could be studied since the numerical results obtained are equally valid for both the elastic and plastic behaviour. The geometries examined were L_p/R ratios of 0.1, 0.2, 0.3 and R/t ratios of 100, 300, 500. It was hoped that these ratios would be spaced at sufficiently close intervals to enable the structural behaviour of the intermediate geometries to be interpolated. For each geometry, a range of stiffener sizes was analysed to determine the collapse strength corresponding to each stiffener geometry as well as to assess the limiting stiffener size to prevent the onset of general instability. Due to the relative unimportance of stiffener shapes in influencing the collapse strength flatbar stiffeners were chosen for reasons of simplicity and economy. As before, the depth to thickness ratio of the stiffeners was maintained at 10.

The initial imperfections used for the present analyses were identical to those of section 6.5.3. Imperfections in the shape of mode A have been justified in section 6.3 for relatively large stiffeners. For the small stiffeners that were involved in the present investigations, the choice of such an imperfection mode may not be representative of the imperfections found in actual structures. Moreover, the use of such an imperfection mode becomes unrealistic as the stiffener size approaches the limit of the unstiffened case. But this type of imperfections was nevertheless adopted in order to facilitate the direct comparison of collapse strength with that obtained from analyses involving the larger stiffeners. As the main objective of the study was to obtain the stiffener size which just prevented overall collapse the imperfections for this particular stiffener size should not be too unrealistic as the stiffeners still effectively formed nodal lines.

Additional runs with a $n_{imp}=5$ mode were performed

for the $R/t=500$ geometry to assess the importance of circumferential imperfections for cylinders that are likely to fail in non-axisymmetric modes. The results from these analyses are discussed in a later section.

The variation of collapse strength with stiffener size under loadings with $K=\infty$ and $K=1.5$ is plotted in figs.6.7 and 6.8 respectively. For the pure axial compression load case, once the stiffeners were large enough to prevent general collapse, further increases in stiffener size did not result in any improvement in the load carrying capacity regardless of the geometry of the cylinders. This type of behaviour had been encountered by Esslinger and Geier [111], Harding [144] and Odland [238]. In design terms, it is therefore important to assess stiffener size accurately to avoid the inclusion of unnecessarily large stiffeners. The stiffener size in fig.6.7 is plotted in terms of A_{DnV} and the conservatism of the DnV code in the design of ring stiffeners under axial compression is clearly illustrated. However, it is important to point out that whilst reduced stiffener size will lead to savings in material, the difficulty in maintaining fabrication tolerances involved in the welding of very small stiffeners must be taken into consideration in assessing overall production economy.

In examining the structural behaviour of the various geometries shown in fig.6.7, it is clear that apart from possessing many common characteristics, some important differences are also evident. Data on the stress states at collapse shown in table 6.6 reveal that the cylinders with stockier geometries fail with varying extents of plasticity. Some completely elastic failures were obtained for the more slender geometries, but none of these correspond with local panel collapse. For plastic collapse, yielding is always initiated in the shell panel at the junction with the bottom ring stiffener. This is believed to be caused by the high stresses resulting from the restraints to Poisson expansion provided by the rings. However, the position at which yielding is initiated also corresponds with the position of the sharpest curvature in the imperfection mode and results of analyses on unstiffened cylinders using the same imperfection mode have indicated yielding to occur in exactly the same way. As discussed above, the present

imperfection mode is only appropriate for cylinders reinforced by stiffeners of realistic proportions since it is the welding of these stiffeners to the shell panels that causes this form of imperfection. Results obtained for the unstiffened cases are therefore subject to some interpretation, but nevertheless can be used to illustrate the importance of imperfection mode in relation to initiation of plasticity.

The collapse modes for all cylinder geometries are shown in figs.6.9 to 6.11. For each geometry, the collapse mode corresponding to the largest and smallest stiffeners has been plotted. All the unstiffened cases have also been included for comparison. For the shortest geometry (see fig.6.9) collapse of the $R/t=100$ cylinders occurred in the expected axisymmetric mode. The addition of ring stiffeners has increased the strength slightly compared to the unstiffened case since the presence of the rings has forced the cylinder to collapse with a buckle wavelength shorter than that corresponding with the unstiffened case. Data on the critical buckling wavelength L_{Cr} based on the elastic collapse of unstiffened cylinders are given in table 6.3. For the $R/t=300$ and 500 cylinders, the collapse behaviour is similar for all stiffener sizes. These geometries have buckle wavelengths shorter than L_{Cr} and as a result the addition of rings does not significantly affect collapse.

For cylinders of intermediate length (see fig.6.10), collapse modes for the $R/t=100$ and 300 geometries are identical for all stiffener sizes and this is reflected by the constant collapse strength shown in fig.6.7. However, for the $R/t=500$ case, there is a drop in strength as the stiffeners became smaller. Theoretically, this geometry can buckle with many circumferential waves leading to considerable bending in the hoop direction. It is conceivable that some of the stiffener sizes used were not strong enough to restrict the circumferential buckles to the shell panels between the stiffeners. Instead, the buckles were allowed to form in the position which required the least energy, i.e. the position at which they would have appeared in the absence of any stiffening. As a result, the cylinder with the smallest stiffener collapsed elastically while the high bending stiffness provided by the largest stiffeners led to the development of some plasticity in the shell panel.

The above explanation is better illustrated in fig.6.11 which shows the collapse modes for the longest cylinders. The $R/t=500$ and 300 geometries changed collapse mode as the stiffeners became smaller. Collapse eventually involved a high degree of circumferential bending. At collapse, plasticity had spread to 20% of the stiffeners for the $R/t=500$ geometry and represented the only case of stiffener outstand yielding under the present pure axial compression load case. It is interesting to note from the curve for $R/t=500$ (see fig.6.7) that there is a characteristic collapse load for each collapse mode, i.e. the collapse load will remain constant provided the collapse mode is the same regardless of changes to stiffener size.

Comparison between the nine curves in fig.6.7 reveals that the strengths of ring stiffened cylinders are not directly dependent on the length of the shell panels provided the longitudinal buckle wavelength is shorter than the ring spacing. However, ring spacings greater than the critical wavelength may still have a deleterious effect on the collapse strength if the resulting imperfection mode happens to be dominated by components near the critical wavelength.

For the stiffener sizes defining the limit between panel and general collapse of the $R/t=500$ and $L_p/R=0.1$ and 0.3 geometries, additional runs performed with a $n_{imp}=5$ mode produced identical collapse loads compared with the $n_{imp}=2$ cases. Results for stresses and deflections were also very similar. These findings demonstrate the relative insignificance of circumferential imperfection modes on the on the induction of general collapse.

So far, discussion has been limited to the pure axial compression load case. For the $K=1.5$ load case, fig.6.8 shows that the stiffener sizes specified by DnV are again very conservative though not to the same extent as for axial compression. When stiffener size is decreased, changes in collapse modes do not occur until the stiffeners are relatively small. Limits between panel and general collapse are defined by kinks on the curves. Such kinks were not always present for the previous load case.

The collapse modes (figs.6.12 to 6.14) are similar to those in figs.6.9 to 6.11 except that for the present

case, the pressure component of the applied loading has produced more widespread buckling with better definition of the general collapse mode. For the $L_p/R=0.3$ geometries, cylinders with the smallest stiffeners all collapse with plasticity in the stiffener outstands. The extent of plasticity ranges from 30% for the $R/t=500$ case to 70% for the $R/t=100$ case.

Load deflection behaviour is summarised in table 6.6 for both types of loading. The presence of external pressure in the $K=1.5$ load case produces equilibrium paths with a more stable postbuckling characteristic. The bifurcation point behaviour denoted by types A,B and C was more common for the pure axial compression load case and presented considerable numerical difficulties to the attainment of peak load.

6.6 Summary

The collapse behaviour of ring stiffened cylinders under predominantly axial loading has been studied over a wide range of parameters. A comparatively economical axisymmetric finite element model was used to the maximum extent, but, in situations where the assumption of axisymmetry became unrealistic, it was supplemented by a more appropriate asymmetric model. In this way, it was possible to generate a large quantity of useful data at a reasonable cost. The initial imperfections incorporated into the finite element models were based on experimental measurements from a number of sources and the geometries considered were within the range adopted in common offshore design.

The results have demonstrated the conservatism of the DnV code in the design of ring stiffeners under predominantly axial loadings. The relative unimportance of stiffener shapes in controlling collapse has also been highlighted. By combining the results with those from the rigid plastic mechanisms given in chapter 5, stiffener size

requirements over the whole range of the loading interaction may be assessed. This will be presented in the next chapter where the results will be discussed in terms of general design relevance.

CHAPTER 7

DESIGN CONSIDERATIONS

7.1 Introduction

In the design of ring stiffened cylindrical shells, most design codes used in offshore practice aim to specify stiffener sizes to ensure that general instability is always preceded by inter-ring panel failure. This is to avoid any serious reduction in strength resulting from the interaction of the two modes of collapse. In fixed offshore structures, stiffener weight is not of the greatest importance. However, in the design of the new generation of floating structures, ignorance of the precise requirements for avoiding general collapse may lead to very heavy stiffeners and result in an unnecessary increase in the self-weight of the structure. This is especially true as the narrow panels used in such structures result in the stiffener weight being a significant proportion of the total structural weight.

Design codes in general specify stiffener size requirements in terms of inertia values. However, the finite element results from chapter 6 have shown that the stiffener area is an equally valid parameter in controlling collapse in the axial compression dominated region of the loading interaction. The rigid plastic mechanism formulation presented in chapter 5 produced similar results for collapse under pressure dominated loadings. The latter results were, however, only valid for the stockier shell geometries which fail by elasto-plastic collapse.

In the present chapter, finite element analyses are presented for the shell geometries beyond the validity of the plastic mechanism formulation. These results when

combined with those from chapters 5 and 6 will provide useful and extensive data on the design of ring stiffeners over the whole range of combined loading and shell geometries used in floating structures. In the later part of this chapter, general comparisons with various design codes are carried out in the context of stiffener size requirements and predicted strengths.

7.2 Determination of stiffener sizes

In chapter 6, stiffener size requirements to prevent general instability were investigated using the finite element program FINAS for loadings dominated by axial compression. In this section, a similar investigation is presented for slender shells subjected to hydrostatic and lateral pressure loadings.

The initial imperfections, material properties and the aspect ratio of 10 for the flatbar stiffeners used in the present analyses are the same as those used in the corresponding investigation carried out in chapter 6. This value of the aspect ratio is well within the limit specified by DnV for preventing torsional buckling, and surveys carried out by Ellinas et al [107] have indicated it to be realistic for common offshore members. The finite element model used was identical to the one illustrated in fig.6.2 except that for the lateral pressure load case, the out-of-plane rotation degree of freedom at both the top and bottom boundaries was restrained to represent clamped conditions. Experimental and analytical evidence from chapters 3 and 4 indicated these boundary conditions to be more realistic for the lateral pressure load case.

The major aim of the present investigation was to obtain the ultimate pressure for a range of stiffener sizes. As the number of analyses involved was quite large, it was necessary to reduce the computing cost of each run to a minimum. This was achieved by reducing the number of displacement steps so that each analysis was terminated in the early postbuckling region.

Results for the lateral pressure load case are presented in fig.7.1. As in chapter 6, the DnV inertia requirement expressed in terms of stiffener area A_{DnV} is used as a non-dimensionalising parameter. These curves do not indicate a definite transition in collapse modes. The absence of such a transition point, as discussed in chapter 5, is not unexpected since in reality the transition between local panel collapse and general collapse corresponds to a gradual increase in stiffener deflections. As a result of this behaviour, it is difficult to obtain a criterion for specifying stiffener sizes to prevent general collapse. In the present investigation, stiffener deflections corresponding to the stiffener areas of approximately A_{DnV} were found to be acceptably small for all geometries. It was, therefore, decided to use these areas as the limits for establishing stiffener sizes to prevent general collapse and to predict collapse loads. The stiffener stresses corresponding to these stiffener sizes were found to remain completely elastic for all shell geometries considered.

The stresses in the shell elements were, in general, higher than those of the stiffeners. The extent of plasticity ranged from none for the $L_p/R=0.3$, $R/t=500$ geometry to 93% for the $L_p/R=0.2$, $R/t=300$ case. This latter geometry was on the border of validity of the plastic mechanism approach established in chapter 5. Subsequent analyses using the mechanism formulation have produced a stiffener area 11% lower and an ultimate pressure 15% higher than the present finite element results. These comparisons, although demonstrating the upper bound nature of the plastic mechanism approach, indicate the present criterion for establishing stiffener sizes to be reasonably consistent with the approach adopted in chapter 5.

For the hydrostatic pressure load case (see fig.7.2), the transition points between local panel and general collapse are better defined for most geometries compared with the above lateral pressure case. However, for some of the more slender geometries, the behaviour for these two load cases was found to be very similar. Results for the present hydrostatic pressure case illustrate that the stiffener specification given in DnV is very conservative for most shell geometries. The finite element analyses have shown that the stiffener areas to prevent general collapse

under hydrostatic pressure are substantially smaller than for lateral pressure and demonstrate the importance of differentiating between these two types of loading.

7.2.1 Comparison with ECCS

The ECCS code [114] does not take into account the difference between hydrostatic and lateral pressure and bases its stiffener size specification and strength predictions on the BS5500 code [52]. The predicted strength is quoted in terms of pressure and is related to the size of the stiffeners. In the present comparison, stiffener sizes determined from the finite element and plastic mechanism analyses were substituted into the equations given within ECCS to calculate the allowable pressures. The safety factor was taken as 1 so that the calculated pressures could be directly related to the theoretical results. Comparisons for both lateral and hydrostatic pressure are shown in fig.7.3.

As expected, the ECCS approach of directly applying the BS5500 equations for hydrostatic pressure to the lateral pressure load case has made the code predictions very conservative for lateral pressure, particularly for the more slender geometries. Although predictions for hydrostatic pressure are generally less conservative, it would still be desirable to reduce the degree of conservatism for the higher L_p/R ratios.

7.3 Stiffener sizes for design

The stiffener areas obtained in section 7.2 have been combined with those from chapters 5 and 6 and presented in the form of design charts which enable the required stiffener sizes for various ratios of combined loading and

shell geometries to be determined. These are shown in fig.7.4. The figure does not include any safety factors. The ultimate strengths corresponding to these stiffener sizes are presented in a later section in the form of loading interaction curves.

As expected, the stiffener area requirements for pressure loadings are considerably larger than for axial compression. In fact, for most cases, stiffener areas required for axial compression are so small that fabrication requirements and not strength are likely to dictate their design, and the curves for $K=$ case would then be asymptotic to this practical limit in certain cases. Despite the number of assumptions introduced into the plastic mechanism formulation, the resulting stiffener areas have fitted in very well with the finite element predictions. The curves would, of course, require some apparent "smoothing" and a little adjustment to cope with slight apparent "illogicalities" in the axial area before this could be used in the design situation. The latter "illogicalities" are due to the interaction of critical buckling wavelength and ring spacing and are in fact quite realistic but basically unacceptable in practical design terms.

7.4 Comparison with DnV

The DnV code [86] gives stiffener inertia requirements over the whole range of the combined loading interaction. Comparisons have been made between these requirements re-expressed in terms of stiffener areas and the results of the plastic mechanism and finite element analyses. Fig.7.5 shows that the stiffener areas determined by these theoretical analyses are larger than the DnV values for most lateral pressure cases, especially for the stockier shell geometries. This does not, however, imply that DnV is non-conservative for this load case. Reference to fig.7.6 will show that the theoretical collapse pressures are substantially greater than those predicted by DnV. It should be noted that for the purpose of the present comparison, the

allowable hoop stress quoted within DnV has been re-evaluated in terms of the DnV pressure-hoop stress relationship to produce allowable pressure values. These are, however, subject to some interpolation approximations as discussed in chapter 5.

For the other load cases shown in fig.7.5, the DnV code is clearly very conservative for most geometries since compared to the theoretical results, it requires larger stiffeners to resist lower levels of applied loadings.

This conservatism is not unexpected because in establishing stiffener sizes to prevent general collapse, DnV ignores the restraints provided by intermediate non-deflecting diaphragms and assumes collapse to occur in the classical $n=2$ mode. A finite element investigation was carried out to examine the effect of ignoring such restraints and to quantify the degree of conservatism involved. This investigation is described in the next section.

7.4.1 Effect of boundary restraints

The importance of non-deflecting intermediate heavy ring stiffeners or diaphragms in affecting the collapse behaviour of ring stiffened cylinders has been discussed in chapter 3. The effect of diaphragm presence is most significant for the pure lateral pressure load case.

Finite element analyses using an axisymmetric (fig.7.7) and an asymmetric model (fig.7.8) were carried out to assess the effect of such diaphragms. The number of stiffeners N_s used in the analyses was a variable quantity and is not necessarily the same as shown in figs.7.7 and 7.8. These finite element models are similar to those described in chapter 6. However, the number of elements per panel has been reduced to 2 as the convergence study reported in chapter 4 indicated this number to be adequate for the present lateral pressure load case.

The cylinder examined had an identical shell panel geometry to the experimental models described in chapter 3

with a R/t of 267 and a L_p/R of 0.15. Analyses were performed with tee and flatbar stiffeners, the geometries of which were identical to those of cylinders 7 and 8 in the second experimental series. The initial imperfections were in the form of two circumferential waves and one longitudinal half-wave with peak magnitudes defined by DnV tolerance levels. The fact that only one longitudinal half-wave was considered meant that it was possible to analyse only half of the cylinder in the longitudinal direction and to simulate the other half by symmetry.

For each stiffener shape, analyses using both the axisymmetric and asymmetric model were carried out for three, five and sixteen stiffeners to monitor the variation in the collapse pressure p_c due to increases in N_s . Despite the considerable savings resulting from the use of longitudinal symmetry, some extremely expensive runs were still produced and it was not economically viable to extend the analyses into the advanced post-collapse range.

Additional analyses simulating the behaviour of the case with an infinite number of stiffeners were also performed for each stiffener shape. Such infinite cases were represented by models consisting of a single stiffener with the imposition of appropriate symmetric boundary conditions as shown in figs.7.7 and 7.8.

The results of the present investigation are presented in fig.7.9 where the collapse pressure non-dimensionalised with respect to the squash pressure $\bar{p}_c$ is plotted against N_s . For the T-stiffener case, the load carrying capacity from the asymmetric model had reduced by less than 12% for the $N_s=16$ case, but eventually dropped to $0.4p_o$ as N_s approached infinity. The behaviour of the flatbar stiffener case was similar to that of the T-stiffener case except that slight reductions in the load carrying capacity occurred at a lower value of N_s . As expected, the load carrying capacity of the flatbar stiffener model was less than that of the corresponding T-stiffener model over the whole range of N_s .

For lower values of N_s , the collapse modes produced by the asymmetric finite element model were similar to those of cylinders 7 and 8 with failures being confined to local regions around the circumference and extending over the entire length between diaphragms. As N_s increased, the local

buckles became less pronounced and for N_s equal to infinity, failures were in the form of the $n=2$ mode predicted by classical theories. For this mode of buckling, all stresses remained elastic. The collapse pressures for these infinite cases were very similar to the predictions obtained by Boresi [48] for the elastic buckling of rings under uniform lateral pressure. Fig.7.9 also shows that using the axisymmetric model produces no drop in load carrying capacity with increases in N_s , thus confirming the nature of the transition in buckling modes.

The above results have shown that both the load carrying capacities and the collapse modes are a function of the proximity of the boundary restraints provided by non-deflecting diaphragms. Although such diaphragms have considerable beneficial effect on the load carrying capacity under pressure loading, their presence is neglected in the ring stiffener requirement given in the DnV code which assumes failure to occur in the $n=2$ mode. This is the basic reason why the code is so conservative in its ring stiffener requirement. In practice, the diaphragm spacing required to produce the increase in strength will to some extent dependent on cylinder geometry and caution should be exercised if the present results are to be extrapolated to other geometries. The results do strongly suggest, however, that for the type of geometry considered here diaphragms placed at, say, ten ring spaces apart will ensure collapse loads of at least 90% of the predictions of the limited length analyses.

7.5 Strength comparisons with various codes

Of the codes that are used in common practice for the design of cylinders under combined loading, only two [10,86] are especially drafted to meet the needs of offshore design. The API code [10] is primarily intended for the design of fixed structures with a range of applicability restricted to R/t ratios much lower than those being adopted

for the new generation of floating structures. For this reason, this code has not been included in the present comparisons.

In addition to the DnV code [86], this section also considers the ECCS [114] and the ASME [11] codes since these codes are capable of predicting allowable strengths under combined loadings. Neither of these codes are, however, directly intended for offshore use.

The procedure adopted by all these codes for assessing the allowable strengths is very similar. It involves applying empirical knockdown factors to the elastic critical stress to account for the effect of initial imperfections and plasticity. A detailed description of this procedure and discussions on the derivation and choice of these knockdown factors have been given by Galletly [126].

Comparisons between the theoretical results and the design code predictions are presented in figs. 7.10 to 7.12 in the form of interaction curves. No safety factor has been incorporated in the design code comparisons. Before these results are discussed, some clarification of the derivation of these curves is necessary.

For pressure loadings, the DnV code quotes predicted strengths in terms of allowable hoop stress values. For the present comparisons, these values were converted to allowable pressures by using the expression $p = u \sigma_{\theta} t / R$. The factor u was determined, as in chapter 5, by using the pressure-hoop stress relationships suggested within DnV.

The ASME code also quotes strengths in hoop stress terms but does not give any guidance on how to convert hoop stress to pressure. In view of this, the factor u described above was also used to re-express strengths in pressure terms.

The ECCS code quotes strengths in pressure terms, but as mentioned earlier, does not differentiate between hydrostatic and lateral pressure and bases both load cases on the hydrostatic pressure case given in BS5500. The interactive curve it proposes is a straight line joining the allowable pressure to the allowable axial stress. The present author feels that using the allowable hydrostatic pressure to construct this straight line may lead to unconservative estimates for the lateral pressure case. On the other hand, basing this straight line on the lateral

pressure case may result in unnecessarily conservative predictions for the hydrostatic pressure case. As a compromise, it was decided to construct each interaction curve by joining the predicted strengths for the axial compression, hydrostatic pressure and lateral pressure load cases with two separate straight lines.

The interaction curves shown in fig.7.10 to 7.12 demonstrate that all design codes are very conservative for the lateral pressure load case. For axial compression, all three codes, particularly DnV and ASME have a tendency to become slightly unconservative for the lower R/t geometries. It is possible that the imperfection mode used in the finite element analyses was too critical in terms of strength reductions. However, it may be recalled that the imperfection mode was based on measurements of experimental models fabricated with realistic techniques. It appears that the knockdown factors assumed in the various codes for the axial compression load case may need to be reassessed. It is certainly an area which requires further investigation.

In the combined loading region of the interaction, code predictions are generally satisfactory for most geometries. The single straight line interaction curve recommended by ECCS would have given very conservative predictions. The double straight line adopted here is a much better approximation. The parabolic interaction relationships assumed by DnV and by ASME for elastic buckling have produced curves which follow the general trend of the theoretical results. The rectangular curves recommended by ASME for plastic buckling are satisfactory across most of the loading interaction. They are, however, exceptionally conservative for the lateral pressure case. For hydrostatic pressure, code predictions are generally also very conservative.

7.6 Summary

In the present chapter, finite element analyses have been carried out to determine stiffener sizes which will

just prevent general collapse. Attention has been confined to pressure loadings and to the more slender shell geometries. The resulting stiffener sizes have been combined with those obtained in chapters 5 and 6 and presented in the form of design charts. Comparisons with the DnV and ECCS design codes have indicated that the present results produce more economical stiffener sizes. Differences in stiffener size requirements compared with codes are very significant in all but the pure lateral pressure part of the interaction. This formulation, however, is dependent on the existence of intermediate non-deflecting diaphragms.

A separate finite element investigation has been carried out to examine the effect of the non-deflecting diaphragms on the collapse behaviour under lateral pressure loading. The investigation, although confined to limited geometries, has confirmed that ignoring such restraints can lead to gross over-estimates of stiffener sizes.

The ultimate strengths obtained from the plastic mechanism and finite element analyses have been compared with the DnV, ECCS and ASME codes. All these codes were found to be very conservative for the lateral pressure load case. For axial compression, code predictions were found to be slightly non-conservative for certain geometries. The straightline interaction curve recommended by ECCS was found to be very conservative for combined loading. Modifying the ECCS code by the use of two straight lines has been suggested in the present chapter and produces better correlation with the results.

CHAPTER 8

GENERAL CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

8.1 General conclusions

One of the major motivations behind the present research programme was to provide design data for the ring stiffened main leg members of floating offshore platforms. In particular, design information was required for members under combined axial compression and external pressure as only limited previous research had been conducted in this area. The experimental, finite element and plastic mechanism results presented in this thesis have produced a useful data bank which could be used directly for design as well as for the validation of existing design codes.

The experimental work involving two test series totalling eight ring stiffened models has demonstrated the convenience and accuracy of small scale modelling in reproducing initial imperfections and boundary conditions representative of prototype structures. The detailed monitoring of initial imperfections provided an extensive data base for subsequent numerical studies.

The models in the first series all failed by local panel collapse and produced experimental failure loads over the whole range of the combined loading interaction. Correlation with design codes has highlighted the important need of obtaining a definitive relationship between hoop stress and external pressure before any rational comparison can be made. By neglecting the effects of ring stiffeners and defining hoop stress as pR/t , design codes were found to be very conservative for pressure loadings as well as in the specifications of ring stiffener sizes.

The second test series concentrated on the general collapse mode. The test results for lateral pressure, to the best knowledge of the present author, represented the first experimental data for this mode of collapse as no other experimental work has been conducted for this load case. The collapse modes, although extending over the entire length of the cylinders, were confined to a relatively small region of the circumference and demonstrated that in the presence of rigid boundaries, the $n=2$ mode predicted by classical theories cannot readily occur. The experimental and subsequent finite element analytical results have shown that neglecting these rigid boundaries and assuming collapse to occur in the $n=2$ mode can lead to very conservative requirements for stiffener sizes. Such an approach is adopted by the DnV code.

The collapse loads and modes obtained from the experiments were accurately reproduced by finite element modelling. Behaviour under axial compression dominated loadings was found to be much more imperfection sensitive than under pressure loadings. The numerical analyses have also demonstrated the importance of boundary conditions in affecting the panel collapse loads. For lateral pressure loading, the assumption of clamped boundaries for the shell/stiffener junctions provided close estimates of the experimental behaviour while for other loading ratios, the use of simple supports produced safe lower bounds. Analyses using a discretely stiffened model provided the best overall correlation with the experimental results.

Simplified theoretical formulations based on rigid plastic mechanism approaches were developed to analyse each of the two collapse modes observed experimentally. Correlations with experimental data from available sources produced good agreements in situations where instability was preceded by material yielding. A convenient method for designing ring stiffeners to avoid general collapse has been derived by combining the plastic mechanism for each mode of collapse. Design charts based on this method have been produced for evaluating the optimum stiffener areas under pressure dominated loadings. Good agreement with finite element results has been obtained. Comparison with the DnV code has indicated substantial savings in both stiffener and shell plating sizes for hydrostatic pressure loading. For

lateral pressure loading, the results were less conclusive with larger stiffeners producing higher allowable pressures.

In situations where the plastic mechanism formulations were inappropriate, design data have been generated by the finite element program. Results for axial compression dominated loadings have demonstrated the DnV code to be again very conservative for stiffener sizing. The axisymmetric finite element model used for some of the analyses illustrated the convenience and economy of this type of modelling, but in situations where the assumption of axial symmetry was unrealistic, a larger and more complex asymmetric model was used to give better accuracy.

The strength predictions from the plastic mechanism and finite element analyses have been compared with the DnV, ECCS and ASME codes. All these codes were found to be very conservative for the lateral pressure load case. In the axial compression dominated area, code predictions were found to be non-conservative for certain geometries. The straight line interaction curve recommended by ECCS was very conservative for combined loading. An alternative approach for constructing this straight line has been suggested in this thesis and has produced improved correlations with the theoretical results.

8.2 Suggestions for future work

More small scale experiments might be carried out to investigate other geometric and material parameters for both the local and general collapse modes. It would be useful to perform a small number of large scale tests to verify the accuracy of small scale modelling. In particular, large scale testing appears to be the only viable experimental approach for studying general collapse under axial compression dominated loadings as the smallness of the stiffeners required to produce this type of collapse invariably leads to fabrication difficulties for small scale modelling.

Additional finite element parametric studies might

be conducted to provide design data for intermediate geometries, especially with regard to the evaluation of stiffener sizes. The finite element program could also be used to investigate other loading types and combinations. In particular, it may become necessary to study non-axisymmetric pressure such as that caused by surge waves. This is a common type of loading encountered in practice and theoretical studies have already been initiated in the Soviet Union.

Torsional buckling of ring stiffeners has not been considered in this thesis. This type of instability can substantially weaken the resistance of a shell to general collapse. Both experimental and numerical studies should be carried out to provide safer and more rational limits for determining stiffener slendernesses. The shape of the collapse mode involved in this type of instability also suggests the possibility of using a rigid plastic mechanism approach to obtain simplified theoretical solutions.

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APPENDIX A

YIELDING UNDER PLANE STRAIN CONDITION

Consider an element of a solid body subject to principal stresses σ_x , σ_y and σ_z . Reuss [258] assumed that the plastic strain increment in direction i $d\epsilon_i$ is proportional to the instantaneous stress deviators σ'_i . Therefore in general,

$$d\epsilon_i = \sigma'_i \lambda \quad (A1)$$

where λ is a constant of proportionality.

If the body is infinitely long in the z direction (plane strain),

$$\text{equation (A1)} \Rightarrow 0 = \sigma'_z \lambda \quad (A2)$$

σ'_z may be written as:

$$\sigma'_z = \sigma_z - \sigma_m \quad (A3)$$

where σ_m is the hydrostatic stress and is given by:

$$\sigma_m = (\sigma_x + \sigma_y + \sigma_z)/3 \quad (A4)$$

Substituting (A3) and (A4) into (A2) gives:

$$\sigma_z = (\sigma_x + \sigma_y)/2 \quad (A5)$$

Mises yield criterion for a general state may be written as:

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 = 2\sigma_0^2 \quad (A6)$$

Substituting (A5) into (A6) and simplifying gives:

$$(\sigma_x - \sigma_y)^2 = 4\sigma_0^2/3 \quad (A7)$$

For the special condition of a uniaxial stress state in the x direction ($\sigma_y = 0$), equation (A7) gives:

$$\sigma_x = 2\sigma_o / \sqrt{3} \quad (A8)$$

APPENDIX B

METHOD OF CALCULATING YIELD MOMENT FOR

T-STIFFENER WITH EFFECTIVE SHELL PLATING

The following is a concise method for calculating M_o , the plastic moment capacity about the centroid in the presence of hoop stress. It has been assumed that the resultant hoop force acts through the centroid of the cross section. Without such an assumption, it is impossible to give a unique value for the moment capacity of the section in the presence of axial loading as any other load position produces its own moment contribution about this axis.

The calculation procedure is as follows:

1. Calculate A_p, A_s, A_f, A_T

$$\begin{aligned} \text{where } A_p &= L_p t_p \\ A_f &= H_f t_f \\ A_s &= (H_w t_w) + A_T \\ A_T &= A_s + A_p \end{aligned}$$

2. Calculate A_I

$$\begin{aligned} \text{where } A_I &= A_T [1 - (\sigma_{\theta T} / \sigma_o)] / 2 \\ \sigma_{\theta T} &= p R L_p / A_T \end{aligned}$$

3. Calculate x_1 from table B1.

4. Calculate x_2 from table B2.

5. $M_o = A_I \sigma_o [(H_w + t + t_f) - (x_1 + x_2)]$

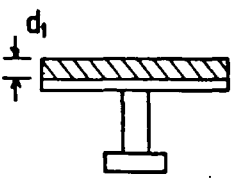
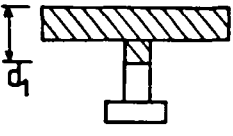
	case	d_1	x_1
	$A_I \leq A_p$	A_I / L_p	$d_1 / 2$
	$A_I > A_p$	$\frac{A_I - A_p}{t_w} + t$	$\left(1 - \frac{A_p}{A_I}\right) \left(\frac{d_1 - t}{2} + t\right) + \frac{A_p t}{2 A_I}$

Table B1

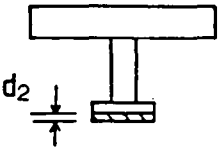
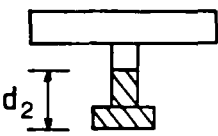
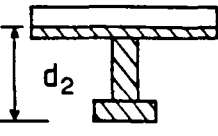
	case	d_2	x_2
	$A_I < A_f$	A_I / H_f	$d_2 / 2$
	$A_f < A_I < A_s$	$\frac{A_I - A_f}{t_w} + t_f$	$\left(1 - \frac{A_f}{A_I}\right) \left(\frac{d_2 - t_f}{2} + t_f\right) + \frac{A_f t_f}{2 A_I}$
	$A_I \geq A_s$	$\frac{A_I - A_s}{L_p} + t_f + H_w$	$\frac{A_f t_f}{2 A_I} + \frac{A_w}{A_I} \left(\frac{H_w}{2} + t_f\right) + \frac{(d_2 - H_w - t_f)(d_2 + H_w + t_f)L_p}{2 A_I}$

Table B2

APPENDIX C

ELASTIC DEFLECTIONS OF PINNED CIRCULAR ARCH

Consider a two hinge circular arch under a uniform radial load of intensity q . The primary forces causing deformation will be bending action since the contributions of axial and shear forces are expected to be negligible for geometries where $R \gg t$ [328]. Numerical calculations by Roark and Young [263] have also confirmed this.

From fig.C1, vertical equilibrium $\Rightarrow V_A = V_B = qR \sin \alpha$.

The strain energy due to bending u_b is given by:

$$u_b = \int_0^{2\alpha} \frac{M^2 R}{2EI} d\phi \quad (C1)$$

where the bending moment M is given by:

$$M = 2qR^2 \sin^2(\phi/2) + H_A 2R \sin(\phi/2) \sin(\alpha - \frac{\phi}{2}) - 2qR^2 \sin \alpha \sin(\phi/2) \cos(\alpha - \frac{\phi}{2}) \quad (C2)$$

Since the hinges are restrained from translational displacements, Castigliano's Second Theorem implies that:

$$\partial u_b / \partial H_A = 0 \quad (C3)$$

Therefore,

$$\frac{\partial}{\partial H_A} \left[\int_0^{2\alpha} \frac{M^2 R}{2EI} d\phi \right] = 0 \quad (C4)$$

$$\Rightarrow \frac{R}{2EI} \int_0^{2\alpha} M \frac{\partial M}{\partial H_A} d\phi = 0 \quad (C5)$$

$$\text{From (C2), } \partial M / \partial H_A = 2R \sin(\phi/2) \sin[\alpha - (\phi/2)] \quad (C6)$$

Substituting (C6) into (C5) and re-arranging gives:

$$H_A = \frac{qR \left[\int_0^{2\alpha} \sin^3 \frac{\phi}{2} \sin(\alpha - \frac{\phi}{2}) d\phi - \int_0^{2\alpha} \sin \alpha \sin^2 \frac{\phi}{2} \sin(\alpha - \frac{\phi}{2}) \cos(\alpha - \frac{\phi}{2}) d\phi \right]}{\int_0^{2\alpha} \sin^2 \frac{\phi}{2} \sin^2(\alpha - \frac{\phi}{2}) d\phi} \quad (C7)$$

Integrating and simplifying gives:

$$H_A = \frac{qR[3\sin\alpha\cos^2\alpha - \alpha\cos\alpha - 2\alpha\cos^3\alpha]}{\alpha + 2\alpha\cos^2\alpha - 3\sin\alpha\cos\alpha} \quad (C8)$$

To find rotation θ , an imaginary moment M_1 is applied. Applying Castigliano's Second Theorem again gives:

$$\theta = \frac{\partial u_b}{\partial M_1} = \int_0^{2\alpha} \frac{M}{2EI} \frac{\partial M}{\partial M_1} R d\phi \quad (C9)$$

Since $\partial M / \partial M_1 = 1$, substituting (C2) into (C9) gives:

$$\theta = \frac{R^2}{EI} \left[qR \int_0^{2\alpha} \sin^2 \frac{\phi}{2} d\phi + H_A \int_0^{2\alpha} \sin^2 \frac{\phi}{2} \sin(\alpha - \frac{\phi}{2}) d\phi - qR \int_0^{2\alpha} \sin \alpha \sin^2 \frac{\phi}{2} \cos(\alpha - \frac{\phi}{2}) d\phi \right] \quad (C10)$$

Integrating and simplifying gives:

$$\theta = \frac{R^2}{EI} [(\sin\alpha - \cos\alpha) H_A - qR(\alpha\cos^2\alpha - \sin\alpha\cos\alpha)] \quad (C11)$$

Since H_A is known from (C8), θ may be determined from (C11).

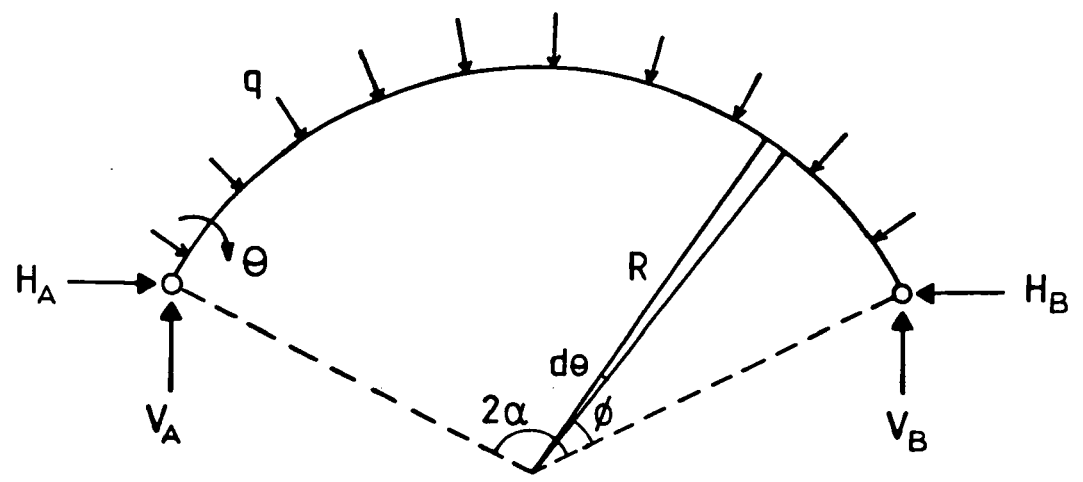
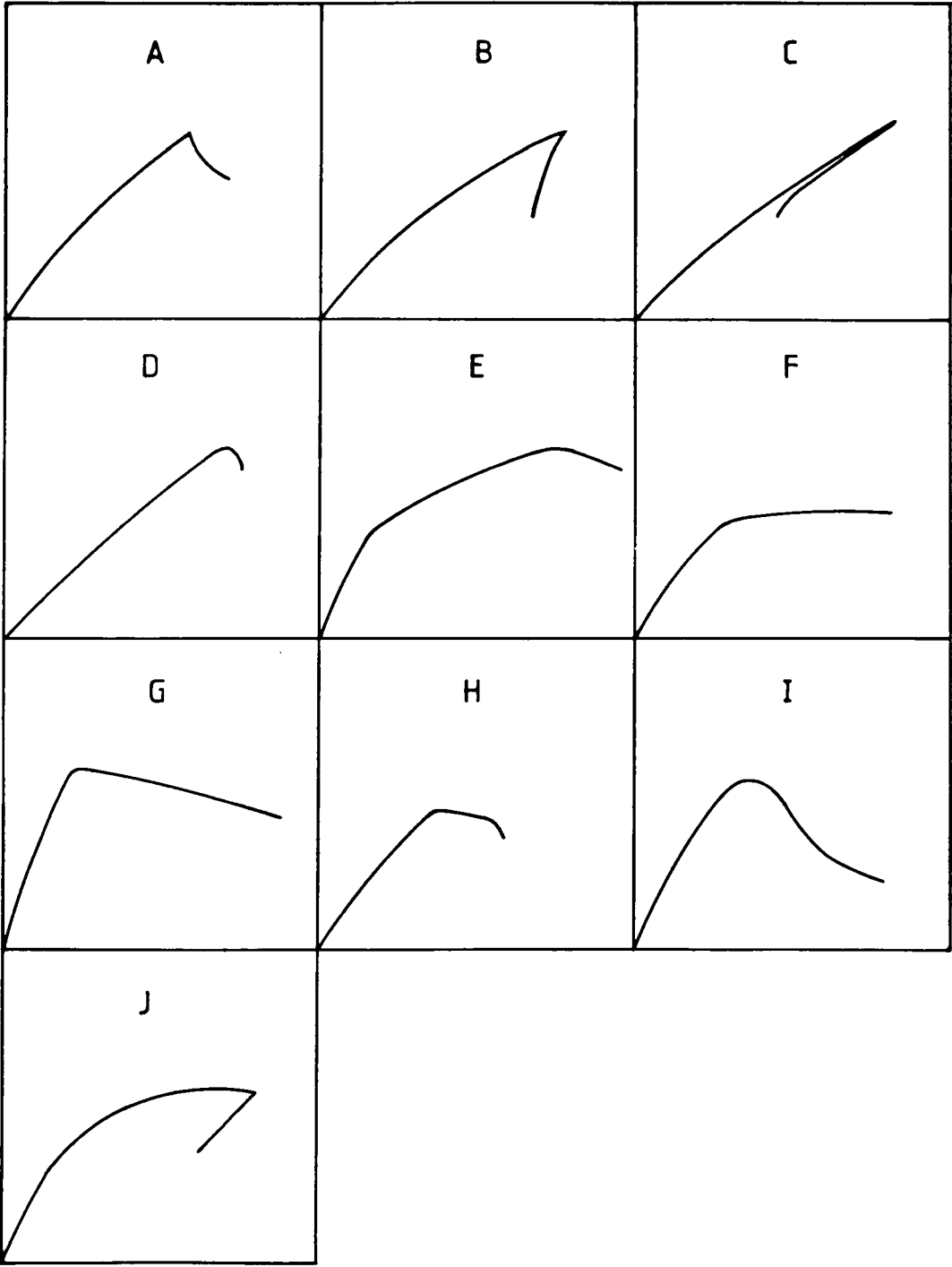
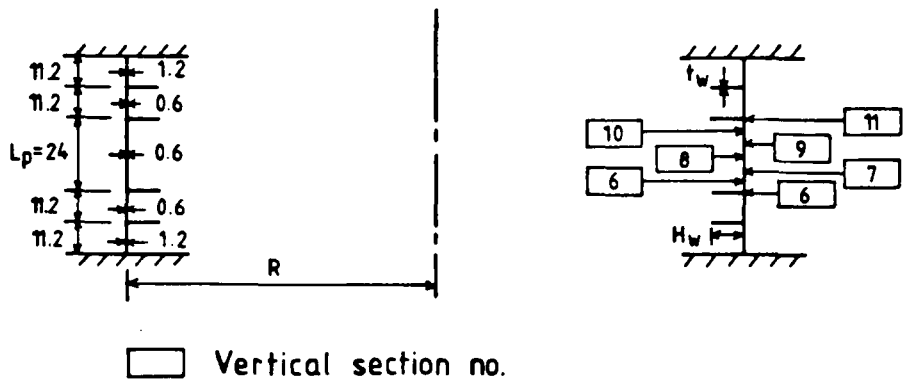


Fig. C1 Hinged circular arch

APPENDIX D SCHEMATIC LOAD - DEFLECTION CURVES

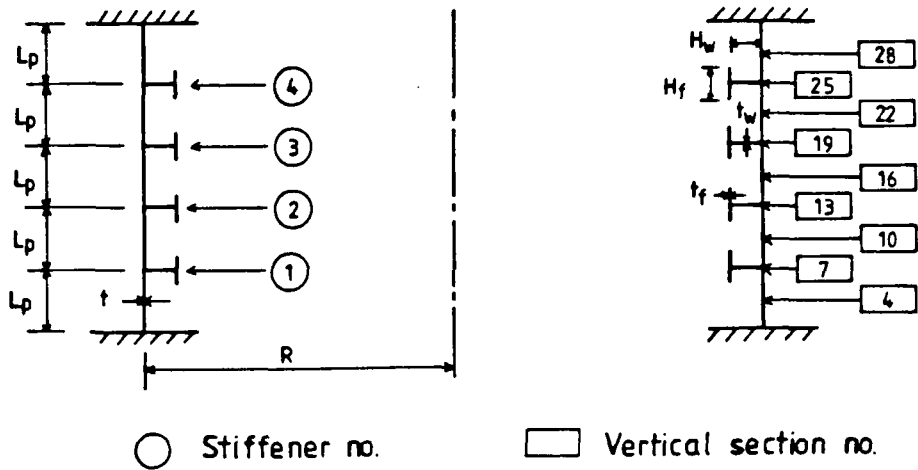




Cyl. no.	L_p	R	t	H_w	t_w	LOADING TYPE
1	24	160	0.6	12.8	1.6	AXIAL COMP.
2	24	160	0.6	12.8	1.6	HYDROSTATIC
3	24	160	0.6	12.8	1.6	LATERAL PRESSURE
4	24	160	0.6	12.8	1.6	$\sigma_o/\sigma_\theta = 1.5$
5	24	160	0.6	9.6	0.6	HYDROSTATIC

All dimensions in mm

Table 3.1 Details of cylinders 1-5



Cyl. no.	L_p	R	t	H_f	t_f	H_w	t_w	LOADING TYPE
6	24	160	0.6	6	0.6	5	0.6	HYDROSTATIC
7	24	160	0.6	6	0.6	5	0.6	LATERAL PRESSURE
8	24	160	0.6	0	0	5	0.6	LATERAL PRESSURE

All dimensions in mm

Table 3.2 Details of cylinders 6-8

Cylinder no.	Specimen identification [†]	σ_{ot} (N/mm ²)			E (kN/mm ²)			σ_o (N/mm ²) σ_u (N/mm ²)	
		Measured value	Group ave.	Overall ave.	Measured value	Group ave.	Overall ave.		
1-5	A1	363.3	362.2	365.2	200.0	205.93	209.88	375.0	432.0
	A2	363.3			209.5				
	A3	360.0			208.3				
	C1	371.3	368.2		220.0	213.83			
	C2	370.0			211.1				
	C3	363.3			210.4				
	PA1*	365.0	374.2	374.2	219.0	219.5	219.5	384.0	428.0
	PC1*	383.3			220.0				
6-8	A4	362	359.7	371.9	223	228.3	229.0	382.0	420.0
	A5	362			223				
	A6	355			239				
	C4	384	384		214	229.0			
	C5	391			239				
	C6	377			234				
Stiffeners{	ST1	368	366	366	204	205	205	376	519
	ST2	364			206				

[†] 'A' denote specimens parallel to rolling, 'C' denote specimens transverse to rolling.

* Pre-heat treatment specimen

Strain hardening occurred at approximately 3.2% strain

$$\sigma_o \approx \sigma_{ot} + 10 \text{ N/mm}^2$$

Table 3.3 Details of material properties

Cylinder no.	Loading Ratio (σ_a/σ_o)	Failure Load	Squash Load	<u>Failure Load</u> <u>Squash Load</u>
1	∞	156.07 kN	226.19 kN	0.69
2	0.5	1.48 N/mm ²	1.41 N/mm ²	1.05
3	0	2.64 N/mm ²	1.41 N/mm ²	1.87
4	1.5	0.54 N/mm ²	1.41 N/mm ²	0.38
5	0.5	1.45 N/mm ²	1.41 N/mm ²	1.03
6	0.5	1.30 N/mm ²	1.43 N/mm ²	0.91
7	0	1.86 N/mm ²	1.43 N/mm ²	1.30
8	0	1.56 N/mm ²	1.43 N/mm ²	1.09

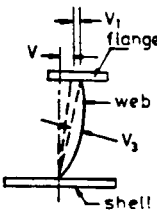
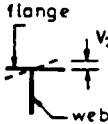
Table 3.4 Failure loads for cylinders 1-8.

Cylinder no.	Perfect radius	Max. outward imperfection	Max. inward imperfection	Out of straightness δg	Deviation from circle e
1	159.55	0.23	-0.18	0.15 (0.33)	0.07 (0.48)
2	159.69	0.26	-0.25	0.21 (0.33)	0.12 (0.48)
3	159.68	0.47	-0.42	0.44 (0.33)	0.14 (0.48)
4	159.61	0.26	-0.34	0.40 (0.33)	0.15 (0.48)
5	159.63	0.33	-0.35	0.22 (0.33)	0.14 (0.48)
6	159.56	0.69	-0.63	0.54 (0.33)	0.27 (0.48)
7	159.61	0.52	-0.32	0.31 (0.33)	0.20 (0.48)
8	159.64	0.41	-0.47	0.23 (0.33)	0.17 (0.48)

Values in mms
DnV tolerances in brackets

Model out of straightness values have been measured relative to ring stiffener positions.
Ring spacing is less than the DnV gauge length.

Table 3.5 Comparison of initial imperfections with DnV tolerances.

Cylinder no.	Stiffener no.	Value of maximum imperfection (DnV tolerances in brackets)				Definition of v, v_1, v_2, v_3
		$v(0.04)$	$v_1(0.09)$	$v_2(0.075)$	$v_3(0.05)$	
6	1	-	0.17	0.11	0.20	
	2	-	0.04	0.08	0.11	
	3	-	0.34	0.21	0.24	
	4	-	0.31	0.11	0.12	
7	1	-	0.06	0.10	0.23	
	2	-	0.27	0.14	0.36	
	3	-	0.07	0.07	0.44	
	4	-	0.48	0.09	0.16	
8	1	0.21	-	-	-	
	2	0.24	-	-	-	
	3	0.19	-	-	-	
	4	0.35	-	-	-	

Values in mm

Table 3.6 Summary of initial imperfection data for stiffeners of cylinders 6-8.

Cylinder no.	P_{ex} N/mm^2	P_{ex}/P_{pr} (Panel collapse)			P_{ex}/P_{pr} (Overall collapse)	I_{ex}/I_{rq}		
		DnV	ECCS	ASME	ASME	DnV	ECCS	ASME
6	1.3	1.38	1.3	1.28	0.9	0.46	0.18	0.94
7	1.86	1.45	1.9	1.62	1.3	0.27	0.18	0.68
8	1.56	1.30	4.5	1.35	1.09	0.054	0.24	0.22

P_{pr} is collapse pressure predicted by codes
 I_{rq} is second moment of area specified by codes
 Experimental quantities are indicated by subscript ex

Table 3.7 Comparison of experimental results and design code predictions for cylinders 6-8.

K	R/t	number of elements around circumference	σ_u/σ_o or p_u/p_o	% difference in σ_u or p_u
∞	100	30	0.864	-
		20	0.859	-0.58
		15	0.923	+6.83
	500	30	0.374	-
		20	0.370	-1.07
		15	0.364	-2.67
0	100	30	10.80	-
		20	10.78	-0.19
		15	10.71	-0.83
	500	30	1.26	-
		20	1.27	+0.79
		15	1.28	+1.59

$$\sigma_o = 375 \text{ N/mm}^2$$

All cylinders unstiffened

Table 4.1 Results of mesh size variation in circumferential direction for $L/R = 0.1$.

K	R/t	number of elements around circumference	σ_u/σ_o	% difference in σ_u
∞	100	30	0.712	-
		20	0.710	-0.28
		15	0.766	+7.58
	500	30	0.280	-
		20	0.276	-1.42
		15	0.269	-3.93

$$\sigma_o = 375 \text{ N/mm}^2$$

All cylinders unstiffened.

Table 4.2 Results of mesh size variation in circumferential direction for $L/R = 0.3$.

	FINITE ELEMENT ANALYSES								Experiment
	Simply Supported				Clamped				
	A	B	C	D	A	B	C	D	
Cyl. 1 $\bar{\sigma}_f$	0.75	0.59	0.56	0.52	0.84	0.78	0.74	0.63	0.69
Cyl. 2 $\bar{p}_f$	0.82	0.71	0.63	0.90	1.06	0.98	0.95	1.0	1.05
Cyl. 3 $\bar{p}_f$	1.14	1.03	1.03	1.71	1.79	1.73	1.74	1.71	1.87
Cyl. 4 $\bar{p}_f$	0.54	0.43	0.38	0.30	0.58	0.54	0.50	0.40	0.38
Cyl. 5 $\bar{p}_f$	0.82	0.75	0.64	0.68	1.06	1.0	0.94	1.03	1.03

Table 4.3 Finite element and experimental failure loads for cylinders 1-5 (see Fig. 4.1 for definition of A, B, C, D).

	Finite Element Analyses		Experiment
	Perfect Model	Imperfect Model	
Cyl. 6 $\bar{p}_f$	1.09	1.02 (0.99*)	0.91
Cyl. 7 $\bar{p}_f$	1.43	1.38	1.30
Cyl. 8 $\bar{p}_f$	1.19	1.07	1.09

* 2 x 10 model

Table 4.4 Finite element and experimental failure loads for cylinders 6-8.

Model No.	K	L mm	t mm	R/t	L/R	σ_o N/mm ²	p_f/p_o (σ_f/σ_o for $K=\infty$)	
							Experiment	Theory
A1-2	∞	24.375	0.74	101	0.325	425	0.87 (0)	0.974
A6	4	24.75	0.735	102	0.33	425	0.27 (0)	0.249
A3	2	24.75	0.735	102	0.33	425	0.51 (10 + 0)	0.490
A5	1	24.375	0.74	101	0.325	425	0.82 (12)	0.942
A2	0.5	24.375	0.74	101	0.325	425	0.97 (16)	1.29
A4	0	24.375	0.74	101	0.325	425	1.11 (14)	1.43
C1	∞	54	0.769	97.5	0.72	425	0.90 (0)	0.975
C5	4	54.75	0.768	97.6	0.73	425	0.25 (8)	0.249
C6	2	54	0.768	97.6	0.72	425	0.45 (9)	0.49
C4	1	54	0.769	97.5	0.72	425	0.67 (9)	0.94
C2	0.5	54	0.767	97.8	0.72	425	0.69 (10)	1.19
C3	0	54	0.766	97.9	0.72	425	0.73 (10)	1.09
F1	∞	108.75	0.789	95	1.45	340	0.85 (0)	0.98
F5	4	108.75	0.765	98	1.45	340	0.25 (7)	0.25
F4	2	108.75	0.798	94	1.45	340	0.43 (6)	0.49
F2	0.5	108.75	0.798	94	1.45	340	0.49 (7)	1.17
F3	0	108.75	0.798	94	1.45	340	0.51 (7)	1.02

R = 75 mm, E = 207000 N/mm².
Number of circumferential waves given in brackets.

Table 5.1 Details of Liverpool University machined models.

Model No.	K	L mm	t mm	R/t	L/R	σ_o N/mm ²	p_f/p_o (σ_f/σ_o for $K=\infty$)	
							Experiment	Theory
B1	∞	25.125	0.748	100.3	0.335	270	0.83 (0)	0.91
B5	2.0	24.75	0.747	100.4	0.33	270	0.42 (10)	0.48
B3	1.0	24.375	0.747	100.4	0.325	270	0.78 (14)	0.89
B2	0.5	24.75	0.747	100.4	0.33	270	0.92 (20)	1.23
B6	0.3	24.75	0.748	100.3	0.33	270	0.96 (18)	1.36
B4	0	24.75	0.747	100.4	0.33	270	1.05 (14)	1.42
E1	∞	55.5	0.752	99.8	0.74	270	0.81 (0)	0.91
E6	4.0	55.5	0.749	100.2	0.74	270	0.22 (8)	0.24
E5	2.0	55.5	0.75	100.0	0.74	270	0.40 (10)	0.48
E4	1.0	55.5	0.75	100.0	0.74	270	0.55 (10)	0.89
E2	0.5	55.5	0.75	99.9	0.74	270	0.62 (12)	1.15
E3	0	55.5	0.75	100.4	0.74	270	0.67 (12)	1.08
G1-2	∞	108.75	0.75	100.0	1.45	225	0.81 (0)	0.91
G4	4.0	108	0.728	103.0	1.44	225	0.2 (2 + 0)	0.24
G3	2.0	108.75	0.728	103.0	1.45	225	0.27 (7)	0.48
G2	1.0	108.75	0.728	103.0	1.45	225	0.33 (8)	0.89
G6	0.5	108	0.728	103.0	1.44	225	0.36 (7)	1.14
G5	0	108.75	0.728	103.0	1.45	225	0.38 (8)	1.02

R = 75 mm, E = 207000 N/mm².
Number of circumferential waves given in brackets.

Table 5.2 Details of Liverpool University welded models.

Model No.	K	L _p mm	R mm	t mm	H _w mm	t _w mm	R/t	L _p /R	$\frac{\sigma_{ex}}{\sigma_o}$	u	$\frac{p_{ex}}{p_o}$ ($\frac{\sigma_{ex}}{\sigma_o}$ for K = ∞)
1.1a	∞	93.1	465.5	3.0	48.0	3.0	155.2	0.2	0.935	1.0	0.935
1.1b	0.35	93.1	465.5	3.0	48.0	3.0	155.2	0.2	1.167	0.853	1.37
1.1c	0	93.1	465.5	3.0	48.0	3.0	155.2	0.2	1.175	0.836	1.41
1.1d	1.14	93.1	465.5	3.0	48.0	3.0	155.2	0.2	0.674	0.892	0.76
1.2a	∞	186.2	465.5	3.0	48.0	3.0	155.2	0.4	0.96	1.0	0.96
1.2b	0.35	186.2	465.5	3.0	48.0	3.0	155.2	0.4	0.797	0.853	0.93
1.2c	0.02	186.2	465.5	3.0	48.0	3.0	155.2	0.4	0.935	0.837	1.12
1.2d	1.25	186.2	465.5	3.0	48.0	3.0	155.2	0.4	0.584	0.839	0.65
1.3a	∞	46.5	464.8	1.6	25.6	1.6	290.5	0.1	0.734	1.0	0.734
1.3b	0.69	46.5	464.8	1.6	25.6	1.6	290.5	0.1	0.868	0.892	0.97
1.3c	1.38	46.5	464.8	1.6	25.6	1.6	290.5	0.1	0.466	0.920	0.51
1.3d	2.08	46.5	464.8	1.6	25.6	1.6	290.5	0.1	0.363	0.949	0.38
1.3dR	0	46.5	464.8	1.6	25.6	1.6	290.5	0.1	1.19	0.863	1.38
1.4a	∞	93.0	464.8	1.6	25.6	1.6	290.5	0.2	0.582	1.0	0.582
1.4b	0.56	93.0	464.8	1.6	25.6	1.6	290.5	0.2	0.728	0.886	0.82
1.4c	0	93.0	464.8	1.6	25.6	1.6	290.5	0.2	0.986	0.863	1.14
1.4d	1.25	93.0	464.8	1.6	25.6	1.6	290.5	0.2	0.435	0.915	0.48

$$\sigma_o = 350 \text{ N/mm}^2$$

E assumed to be 207000 N/mm² in analysis.

$$p_{ex} = u \sigma_{ex} t / R$$

Table 5.3 Details of Glasgow University models.

REF. NO.	MODEL NO.	MATERIAL	METHOD OF MANUFACTURE	GEOMETRIC PARAMETERS										K	σ_b N/mm ²	E kN/mm	P ₀ N/mm ²	P _{ex} /P ₀ (n)	P _c /P ₀ (n)	
				L _B	L _P	R	t	N _s	H _w	t _w	H _f	t _f	R/t							L _P /R
335	7	STEEL	FABRICATED	120	24	160	0.6	4	5	0.6	6	0.6	267.7	0.15	0	379	217	1.43	1.30 (9)	1.24 (8)
	8			120	24	160	0.6	4	5	0.6	0	0				379	217	1.43	1.09 (10)	1.0 (9)
46	10-52	ALUMINIUM	MACHINED	33.3	4.8	26.1	1.32	6	1.98	0.69	0	0	19.8	0.18	0.5	580.3	71	29.3	1.11 [*]	1.21 (13)
	10-54			35.3	5.0	26.1	1.31	6	2.90	1.02	0	0	19.9	0.19		578.2	71	29.1	1.33 [*]	1.43 (11)
	10-82			43.6	6.2	26.5	2.11	6	2.92	0.99	0	0	12.5	0.24		574.1	71	45.9	1.15 [*]	1.21 (13)
	10-88			50.9	7.3	26.5	2.11	6	6.30	2.13	0	0	12.5	0.28		576.2	71	46.1	1.81 [*]	1.85 (8)
	15-82			63.6	9.1	26.5	2.11	6	3.56	1.17	0	0	12.6	0.34		574.8	71	45.8	1.15 [*]	1.21 (13)
	15-84			67.0	9.6	26.5	2.11	6	5.13	1.73	0	0	12.6	0.36		572.7	71	45.6	1.38 [*]	1.43 (11)
	15-88			72.2	10.3	26.5	2.11	6	7.54	2.54	0	0	12.6	0.39		570.6	71	45.5	1.76 [*]	1.85 (8)
	20-82			83.3	11.9	26.5	2.11	6	4.01	1.37	0	0	12.5	0.45		565.8	71	45.2	1.13 [*]	1.22 (13)
	20-84			87.1	12.4	26.5	2.11	6	5.84	2.01	0	0	12.6	0.47		561.7	71	44.8	1.36 [*]	1.44 (11)
	267			P1	STEEL	MACHINED	204.6	20.5	50.8	1.56	9	1.65	1.57	0		0	32.6	0.4	0.5	162
P2		204.4	20.4	50.8			1.54	9	2.37	1.54	0	0	33	162	200	4.91	1.32 [*]			1.11 [*]
P1/A		204.2	20.4	50.8			1.54	9	1.65	1.55	0	0	33	184	207	5.58	1.24 [*]			1.08 [*]
P2/A		204.1	20.4	50.8			1.52	9	2.37	1.55	0	0	33.4	188	200	5.63	1.35 [*]			1.12 [*]
235	1	ALUMINIUM	MACHINED	62.0	13.0	50.4	0.78	4	2.04	0.76	1.88	0.76	65	0.26	0.5	527.5	76.6	8.12	1.1 (8)	1.14 (13)
279	—	STEEL	FABRICATED	220	44	165	1.4	4	4.0	4.1	0	0	117.9	0.27	0.5	395	204	3.35	1.24 (6)	1.13 (11)

ALL DIMENSIONS IN MM * VALUE OF n NOT AVAILABLE

Table 5.4 Comparison between experimental and general collapse mechanism results

L_p/R	R/t	FLATBAR			T- BAR		
		A_{reqd} mm ²	I_s mm ⁴	I_{Lef} mm ⁴	A_{reqd} mm ²	I_s mm ⁴	I_{Lef} mm ⁴
0.1	200	47520	1.88×10^9	3.69×10^9	48420	1.37×10^9	3.59×10^9
0.1	500	54360	2.46×10^9	5.39×10^9	56250	1.86×10^9	5.51×10^9
0.3	100	7290	4.43×10^7	1.52×10^8	7560	3.35×10^7	1.68×10^8
0.3	150	9360	7.30×10^7	2.44×10^8	9540	5.34×10^7	2.62×10^8

Table 5.5 Tabulation of stiffener properties for sample cases (t = 30 mm)

K	R/t	H _w	t _w	number of elements per panel	σ_u/σ_o	% difference in σ_u
∞	100	3.0	0.3	8	0.736	-
				4	0.735	- 0.14
				2	0.804	+ 9.23
	500	8.0	0.8	8	0.373	-
				6	0.374	+ 0.27
				4	0.389	+ 4.29
1.5	100	11.0	1.1	8	0.794	-
				4	0.794	0
				2	0.843	+ 6.17
	500	22	2.2	8	0.511	-
				4	0.512	+ 0.20
				2	0.493	- 3.52

$t = 1.0 \text{ mm}$, $L_P/R = 0.3$, number of panels = 5, $\sigma_o = 375 \text{ N/mm}^2$.
All stiffeners designed to DnV.

Table 6.1 Results of mesh size variation in axial direction.

Ref.	Model no.	t mm	L _P /R	R/t	$\frac{ A_m }{\sum_{m=1,2,..} A_m }$				
					m = 1	m = 2	m = 3	m = 4	m = 5
332	1	0.6	0.15	267	0.285	0.612	0.103	-	-
	2	0.6	0.15	267	0.292	0.381	0.327	-	-
	3	0.6	0.15	267	0.543	0.333	0.124	-	-
	4	0.6	0.15	267	0.632	0.039	0.329	-	-
	5	0.6	0.15	267	0.240	0.585	0.175	-	-
335	6	0.6	0.15	267	0.41	0.59	-	-	-
	7	0.6	0.15	267	0.86	0.14	-	-	-
	8	0.6	0.15	267	1.0	0	-	-	-
97	IC1	3.52	0.18	213	0.618	0.049	0.214	0.015	0.104
	IC2	3.52	0.18	213	0.825	0.087	0.084	0.003	0.001
	IC3	3.52	0.15	128	0.454	0.054	0.303	0.008	0.182
243	1	0.6	0.105	267	0.403	0.242	0.295	0.0596	-
	2	0.6	0.105	267	0.483	0.220	0.161	0.136	-
	3	1.2	0.15	133	0.468	0.308	0.118	0.105	-
	4	1.2	0.15	133	0.378	0.362	0.115	0.144	-
	5	0.6	0.105	267	0.616	0.226	0.084	0.074	-
Overall average values:					0.531	0.264	0.187	0.068	0.096
Normalised values:					1	0.497	0.352	0.128	0.181
Nominal values:					1	0.5	0.33	0.25	0.2

$$w_o = \sum_{m=1,2,..} A_m \sin \frac{m\pi x}{L_P}$$

Table 6.2 Summary of axisymmetric initial imperfections measured experimentally.

L/R	R/t	NATURAL COLLAPSE MODE			AXISYMMETRIC COLLAPSE MODE		$\frac{L_{cr}}{L}$	$\left[\frac{\sigma_{cr} - \sigma_{cAX}}{\sigma_{cr}}\right] \times 100$
		m	n	σ_{cr}	m	σ_{cAX}		
0.1	100	1	0	2057.0	1	2057.0	1.72	0
	300	1	1	421.5	1	421.5	1.0	0
	500	1	17	253.4	1	287.7	0.77	-13.5
0.2	100	1	6	1251.3	1	1304.2	0.86	- 4.2
	300	1	13	631.1	2	681.5	0.50	- 8.0
	500	1	20	253.1	3	264.5	0.38	- 4.5
0.3	100	1	9	1248.4	2	1301.0	0.57	- 4.2
	300	1	15	420.3	3	421.5	0.33	- 0.3
	500	1	18	252.2	4	253.9	0.26	- 0.4

All stresses in N/mm^2 .

Table 6.3 Collapse loads and modes using Timoshenko's theory.

K	L_P/R	R/t	σ_o N/mm ²	% of shell elements undergone plasticity	Type of load- deflection curve*
∞	0.1	100	375	90	B
			300	90	
			250	100	
			200	100	
			100	100	
	0.3	500	375	10	C
			300	25	
			250	30	
			200	30	
			100	40	
1.5	0.1	100	375	90	F
			300	100	
			250	95	
			200	100	
			100	100	
	0.3	500	375	10	D
			300	50	
			250	50	
			200	50	
			100	50	

* See appendix D

Table 6.4 Summary of yield stress investigation.

$L_{P/R}$	R/t	K	A_s mm ²	A_s/A_{DnV} (approx)	Flatbar stiff.		T-stiff.		% difference in collapse load
					σ_u/σ_o or P_u/P_o	Collapse Mode	σ_u/σ_o or P_u/P_o	Collapse Mode	
0.1	100	∞	2.5	1.0	0.881	P	0.883	P	0.2
			0.9	0.5	0.884	P	0.884	P	0
			0.1	<0.1	0.854	G	0.854	G	0
		1.5	10.0	1.0	0.629	P	0.637	P	1.3
			5.0	0.5	0.615	P	0.616	P	0.2
			0.4	<0.1	0.582	G	0.582	G	0
0.3	500	∞	6.4	1.0	0.293	P	0.293	P	0
			3.2	0.5	0.290	P	0.290	P	0
			0.23	<0.1	0.240	G	0.241	G	0.4
		1.5	48.4	1.0	0.211	P	0.216	P	2.3
			24.1	0.5	0.210	P	0.210	P	0
			4.9	<0.1	0.113	G	0.113	G	0

$t = 1.0 \text{ mm}$, $H_w/t_w = 10.0$, $H_f/t_f = 10.0$

P denotes local panel collapse

G denotes general collapse

Table 6.5 Comparisons between collapse strengths of flatbar and T-stiffeners.

L_P/R	R/t	$K = \infty$			$K = 1.5$		
		A/A_{DnV}	X	Y	A/A_{DnV}	X	Y
0.1	100	4.72	83	A	3.64	62	H
		0.045	75	C	0.045	45	I
		0	75	G	0	37	I
	300	3.67	38	B	3.70	42	I
		0.01	38	D	0.037	25	I
		0	38	D	0	10	G
	500	3.22	28	C	3.52	25	J
		0.026	25	C	0.044	6	I
		0	25	C	0	6	F
0.2	100	3.80	42	B	3.92	72	G
		0.012	45	B	0.088	20	I
		0	45	B	0	33	I
	300	3.90	25	C	3.75	25	G
		0.01	25	C	0.066	3	C
		0	25	C	0	0	C
	500	3.76	12	C	3.91	25	G
		0.054	0	C	0.061	0	C
		0	0	C	0	0	C
0.3	100	3.91	25	B	3.43	57	G
		0.013	33	B	0.078	70	G
		0	33	B	0	22	I
	300	3.64	20	C	3.89	25	G
		0.025	25	F	0.061	60	G
		0	25	F	0	0	F
	500	3.88	12	C	3.45	25	G
		0.038	3	E	0.115	30	G
		0	3	E	0	0	B

Column X denote % of shell element gone plastic

Column Y denote type of load-deflection curve (see appendix D)

Table 6.6 Summary of stiffener size investigation.

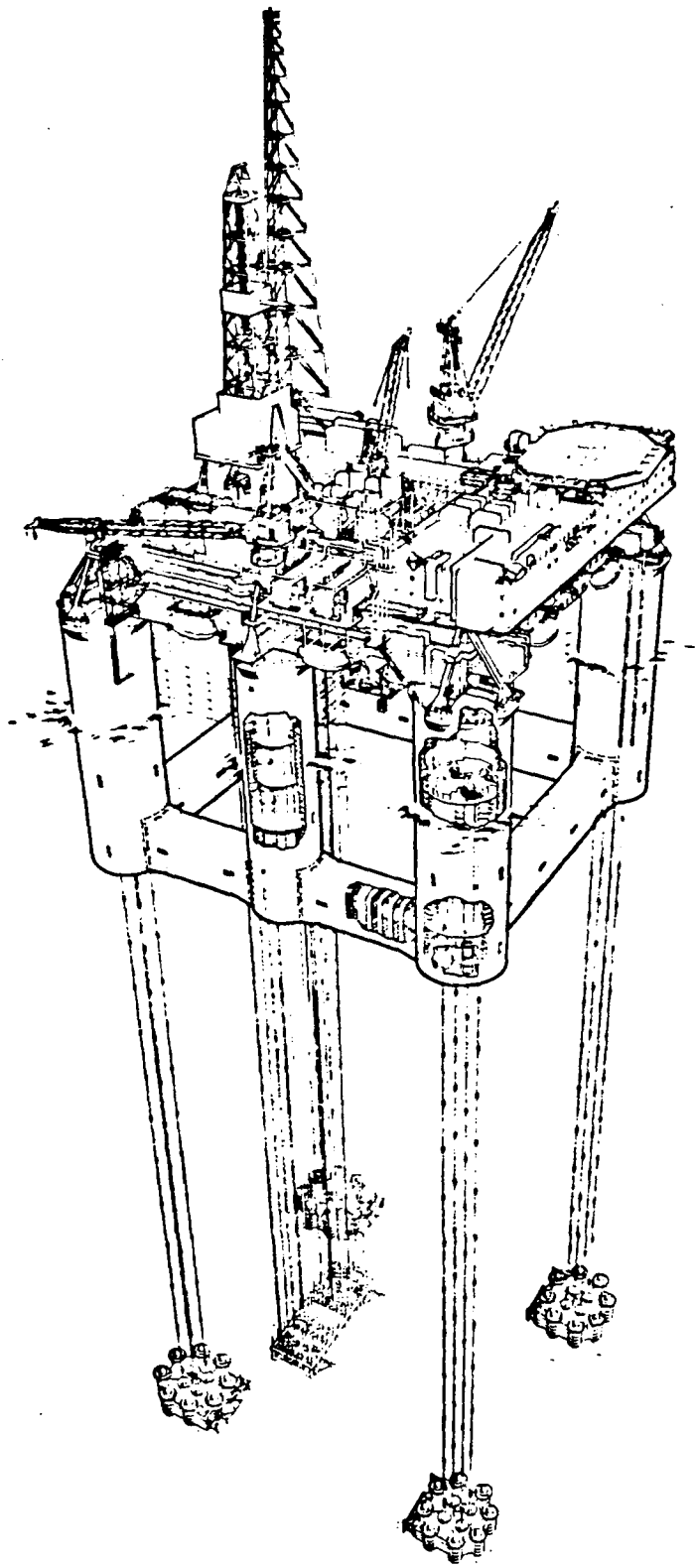


Fig. 1.1 Hutton field tension leg platform

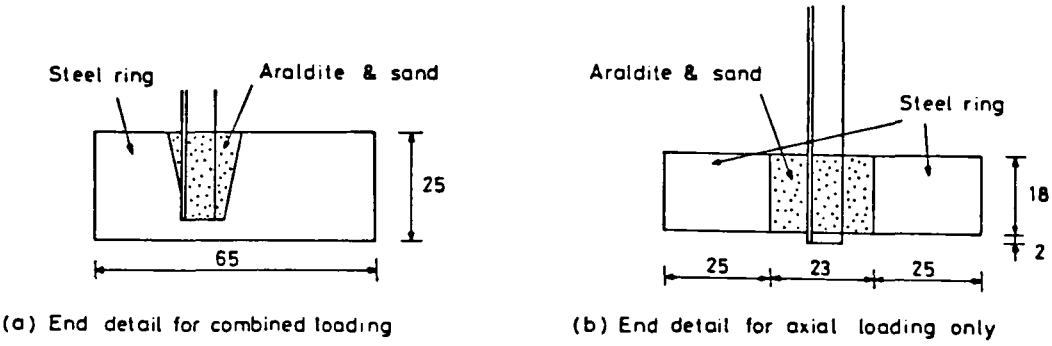


Fig 3.1 Cylinder end conditions

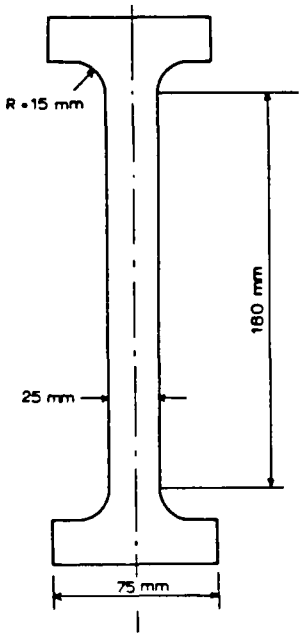


Fig 3.2 Dimensions of tensile test specimens

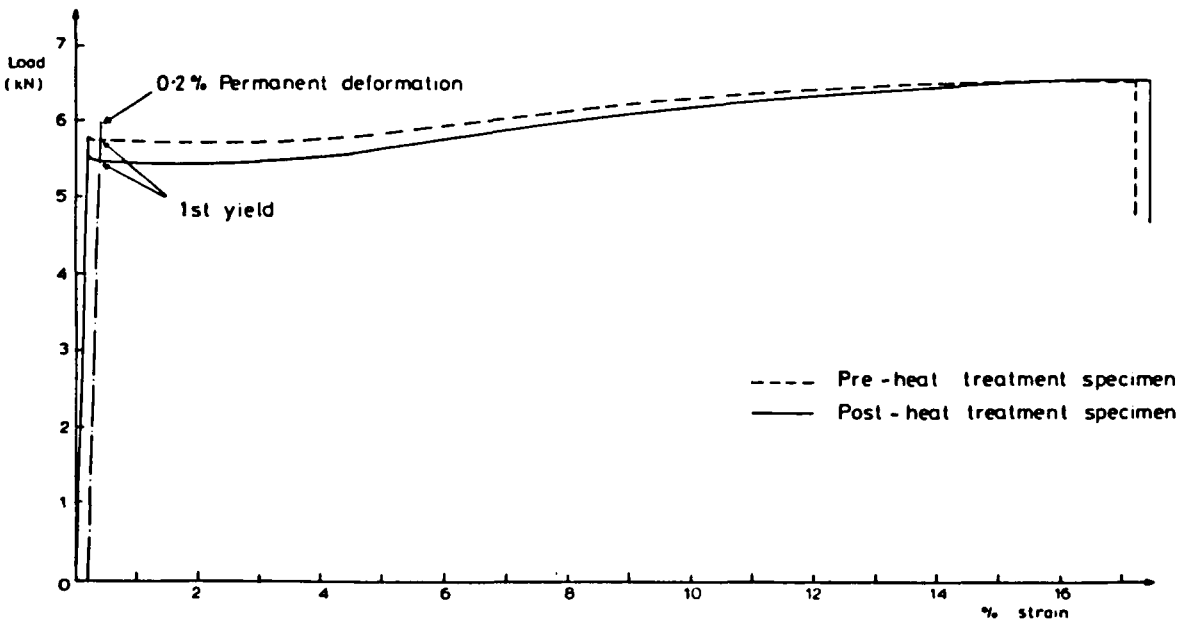


Fig 3.3 Typical load - strain curves for tensile test specimens

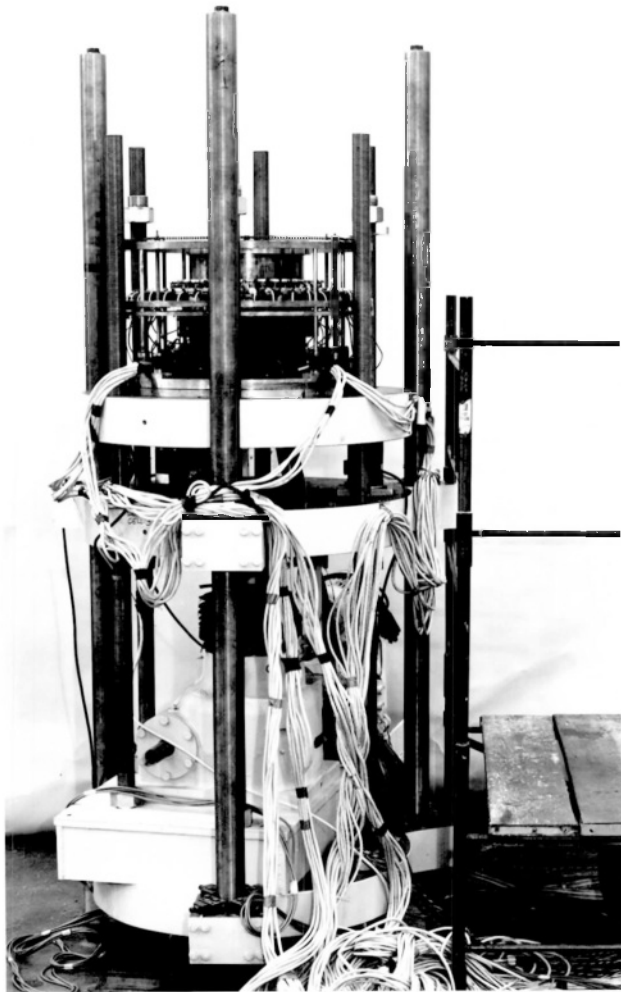


Fig. 3.4a Axial loading rig.

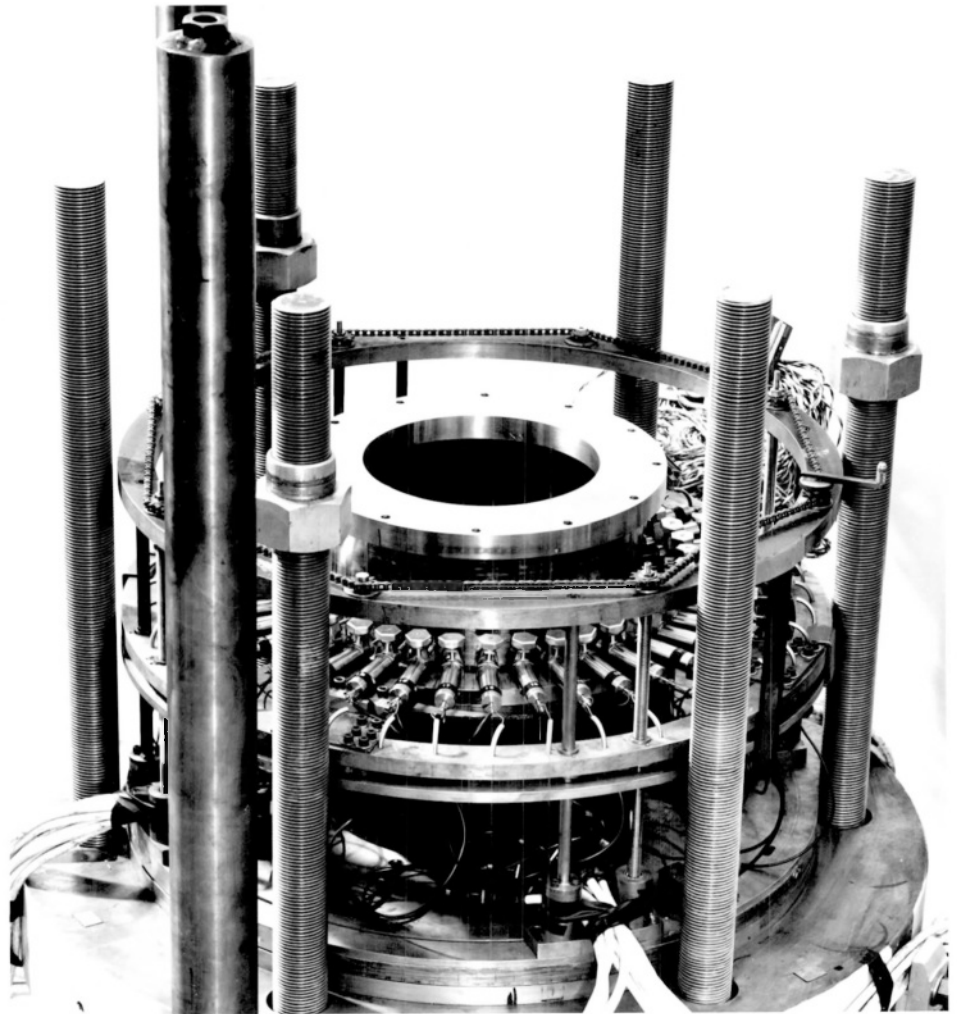


Fig. 3.4b Imperfection measuring frame.

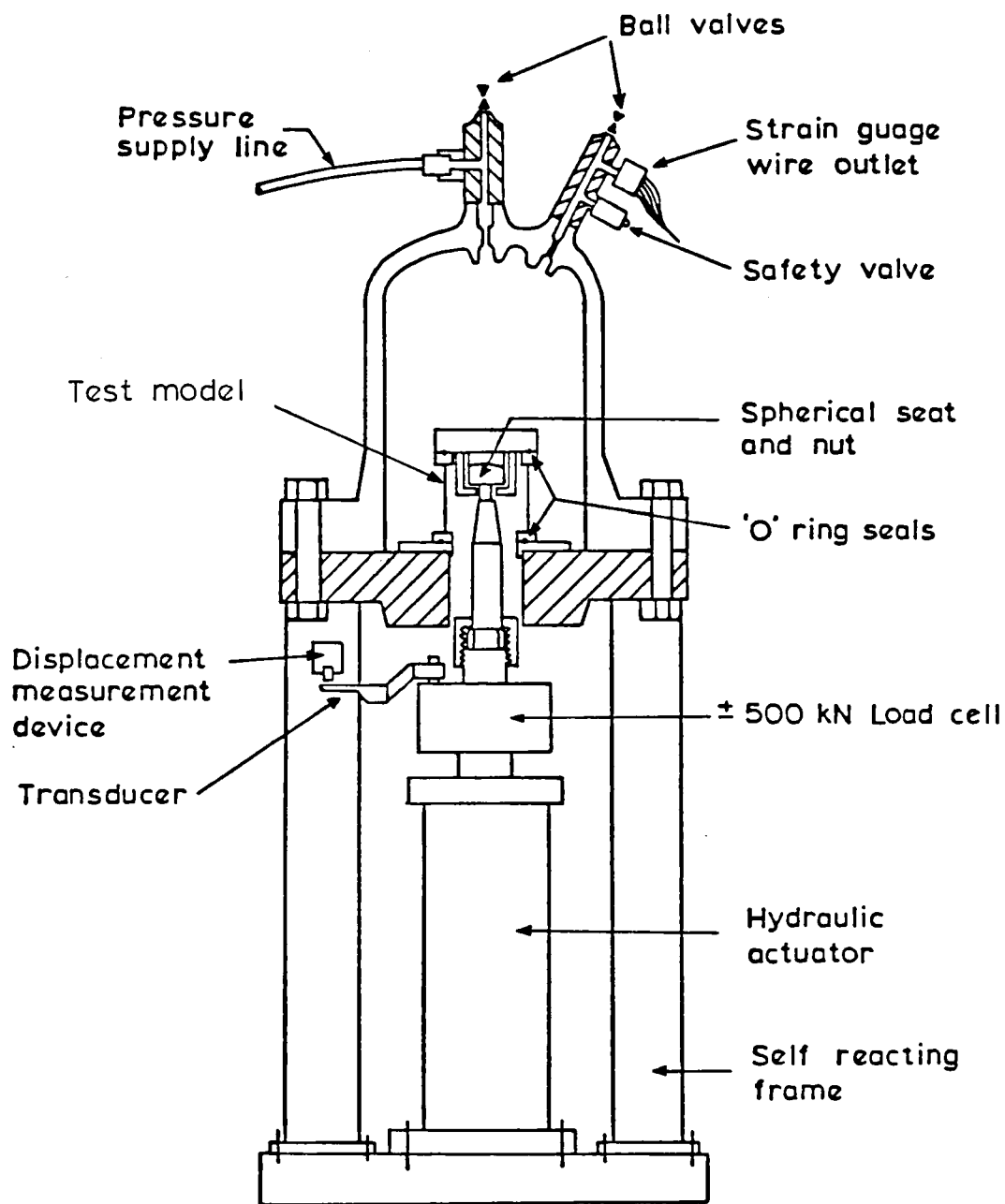


Fig 3.5 Schematic diagram of hyperbaric chamber

CYLINDER 1 – FOURIER COEFFICIENTS (PRE-HEAT TREATMENT)

$$W_0 = \sum_{M=1,2,..} \sum_{N=0,1,..} (A_{MN} \cos N\vartheta + B_{MN} \sin N\vartheta) \sin \frac{M\pi x}{L}$$

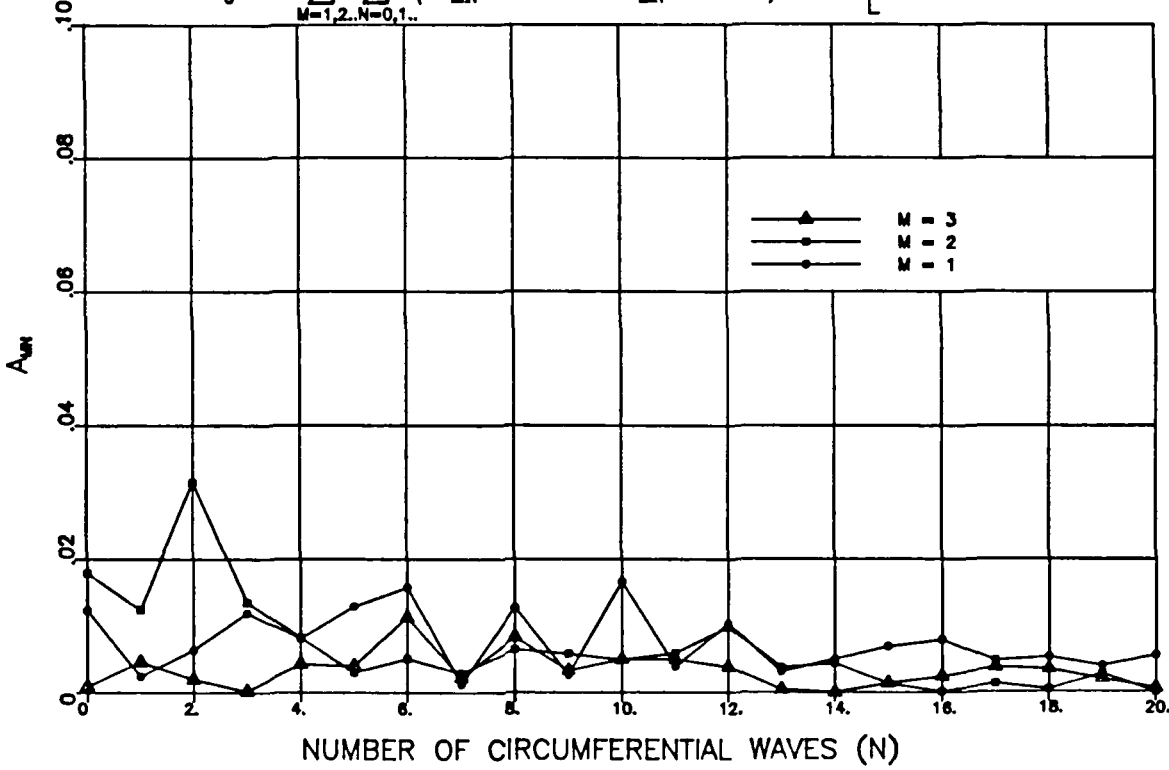


Fig. 3.6

CYLINDER 1 – FOURIER COEFFICIENTS (PRE-HEAT TREATMENT)

$$W_0 = \sum_{M=1,2,..} \sum_{N=0,1,..} (A_{MN} \cos N\vartheta + B_{MN} \sin N\vartheta) \sin \frac{M\pi x}{L}$$

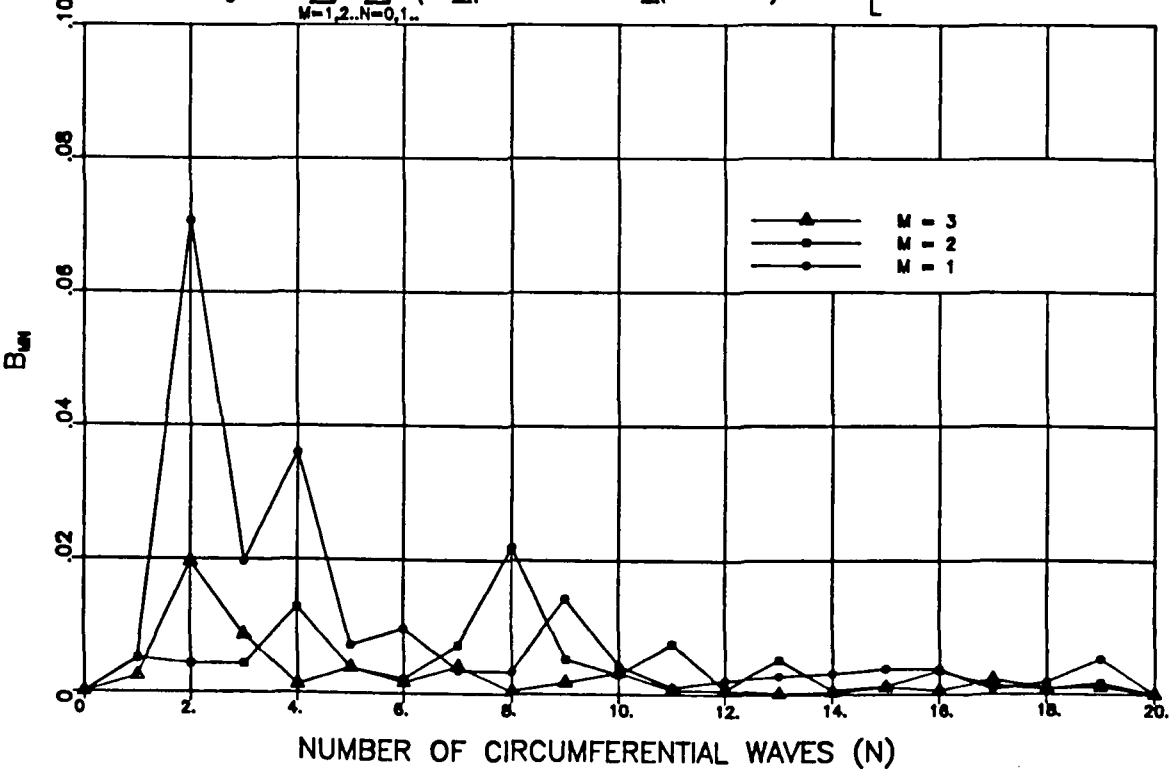


Fig. 3.7

CYLINDER 2 – FOURIER COEFFICIENTS (PRE-HEAT TREATMENT)

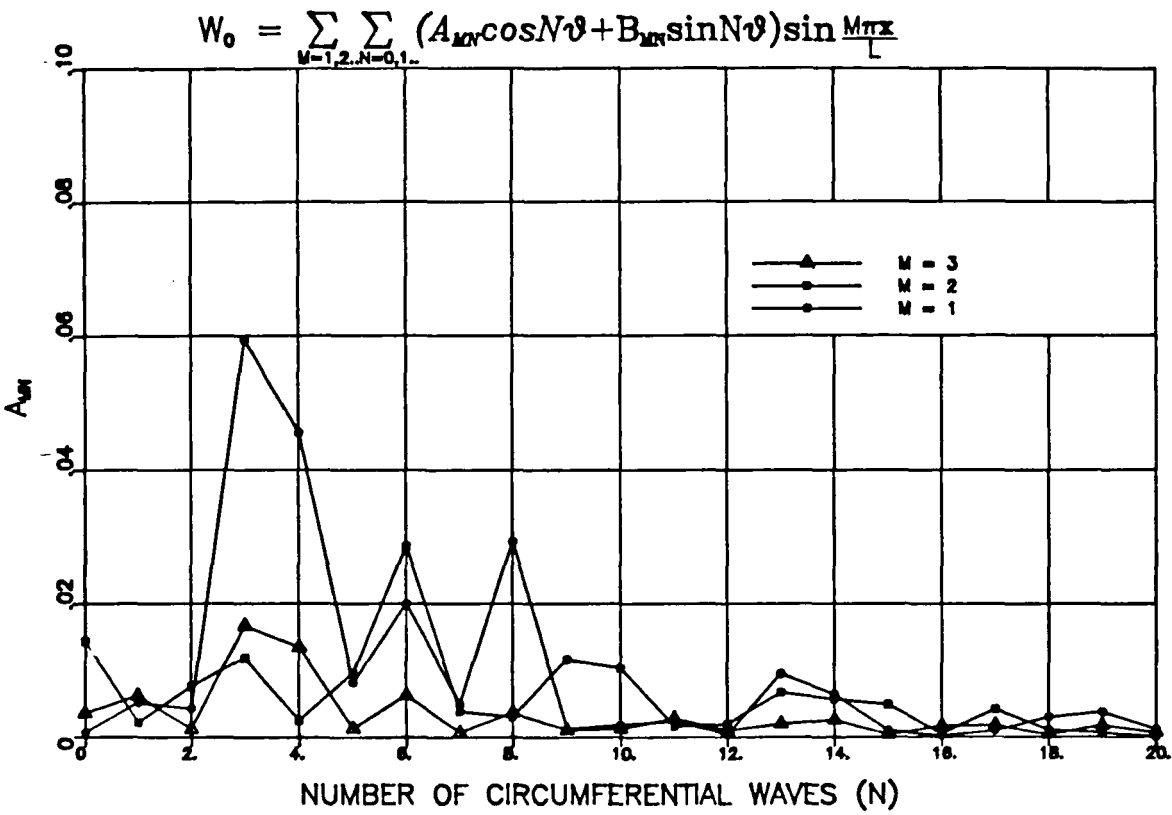


Fig. 3.8

CYLINDER 2 – FOURIER COEFFICIENTS (PRE-HEAT TREATMENT)

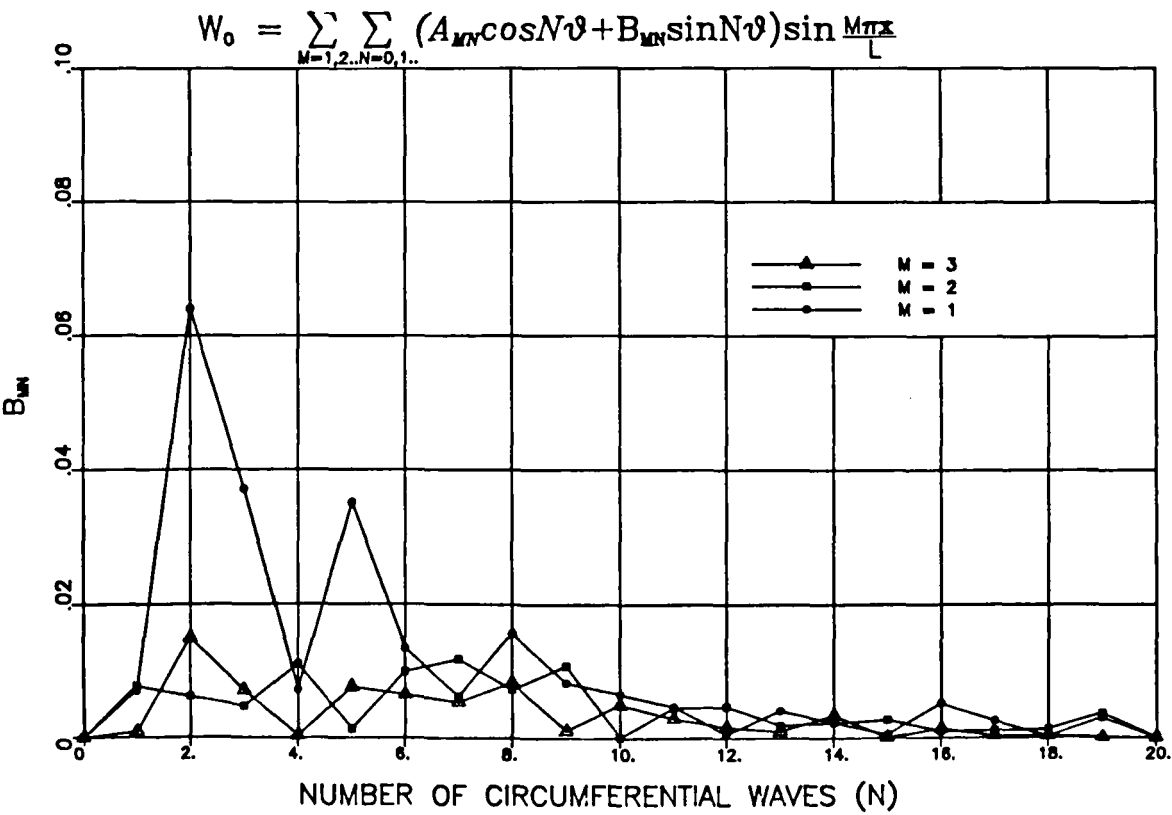
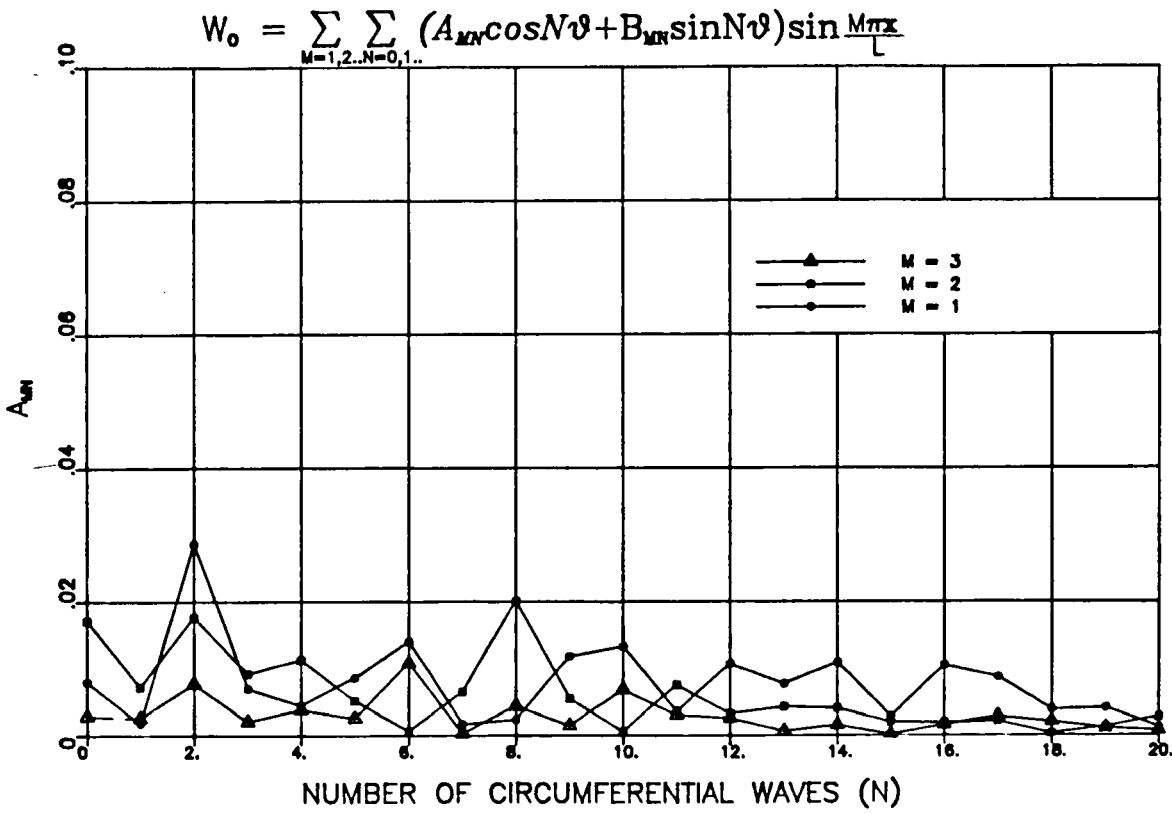
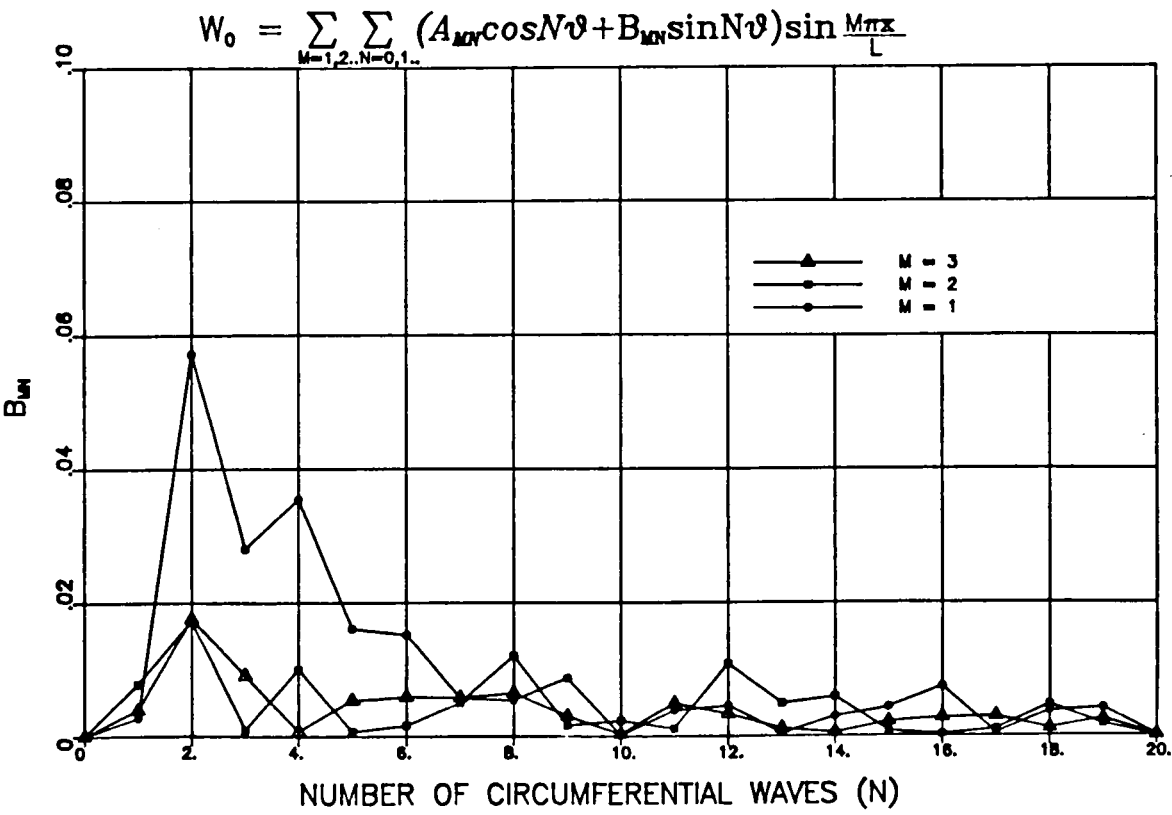


Fig. 3.9

CYLINDER 1 – FOURIER COEFFICIENTS



CYLINDER 1 – FOURIER COEFFICIENTS



CYLINDER 2 – FOURIER COEFFICIENTS

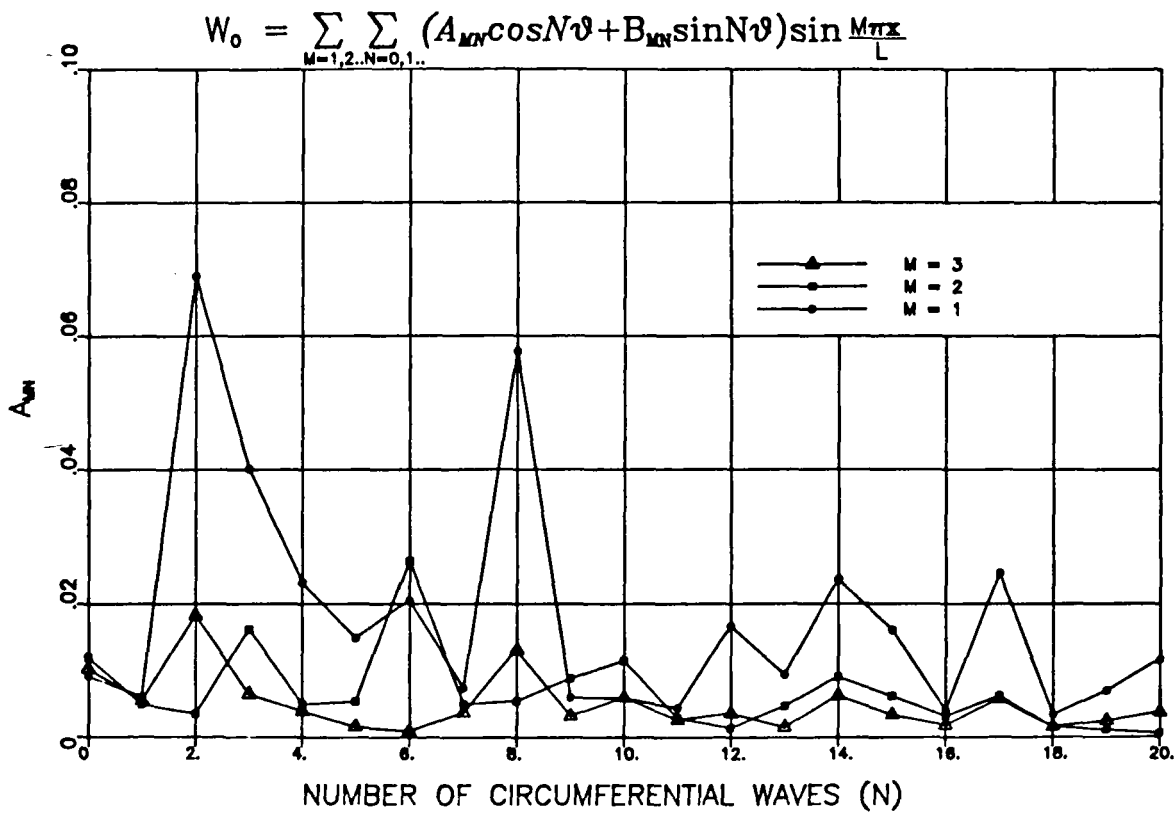


Fig. 3.12

CYLINDER 2 – FOURIER COEFFICIENTS

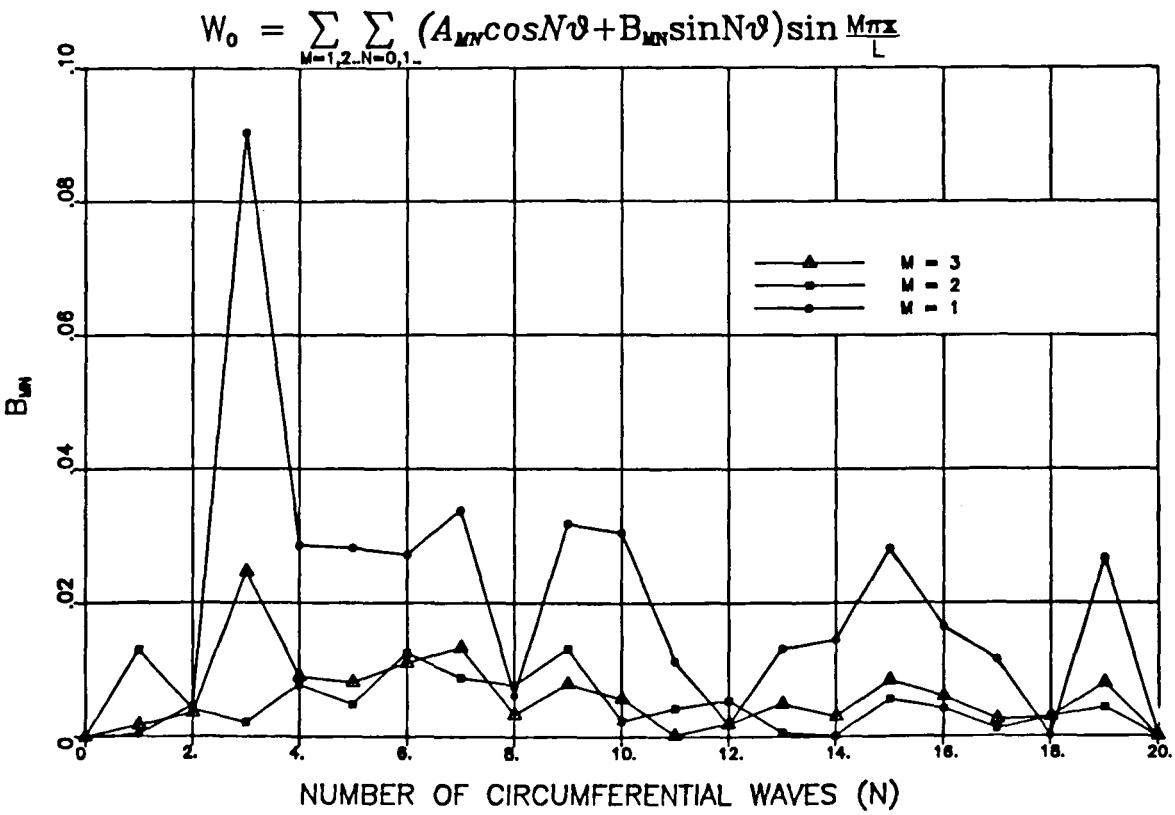


Fig. 3.13

CYLINDER 3 – FOURIER COEFFICIENTS

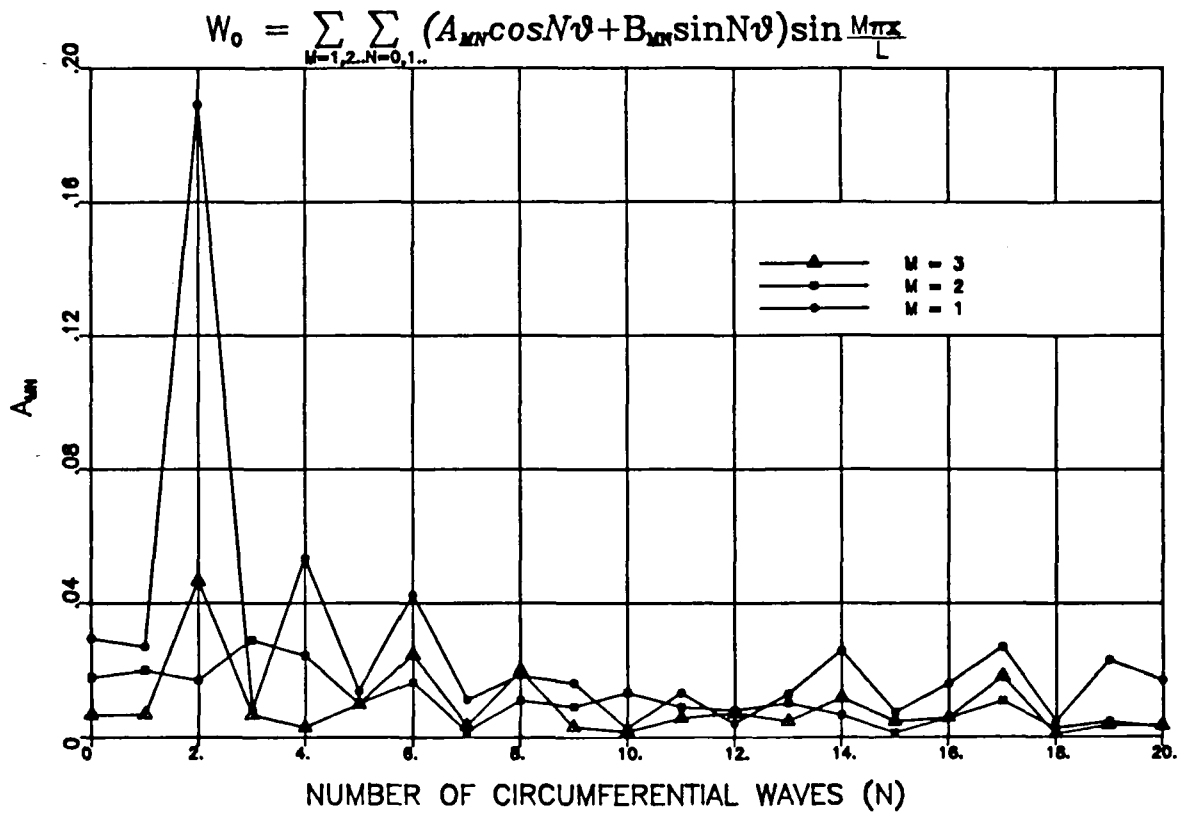


Fig. 3.14

CYLINDER 3 – FOURIER COEFFICIENTS

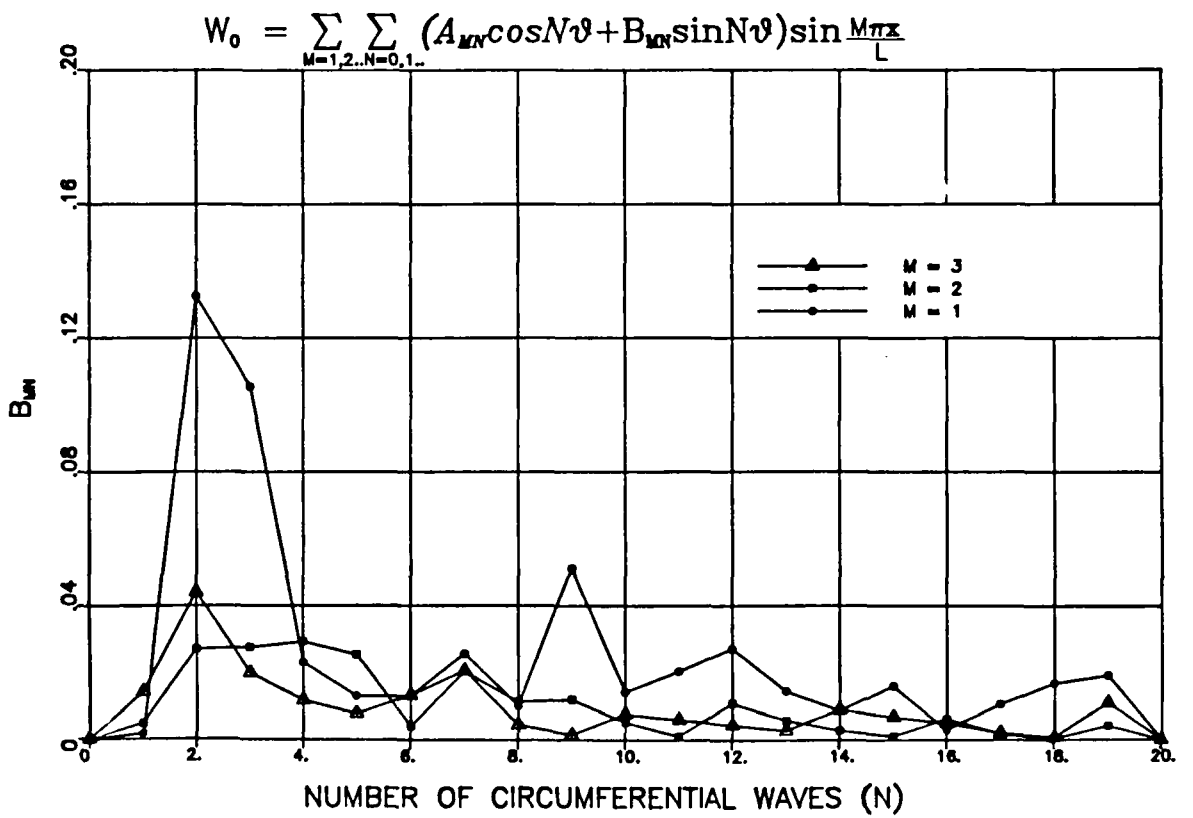


Fig. 3.15

CYLINDER 4 - FOURIER COEFFICIENTS

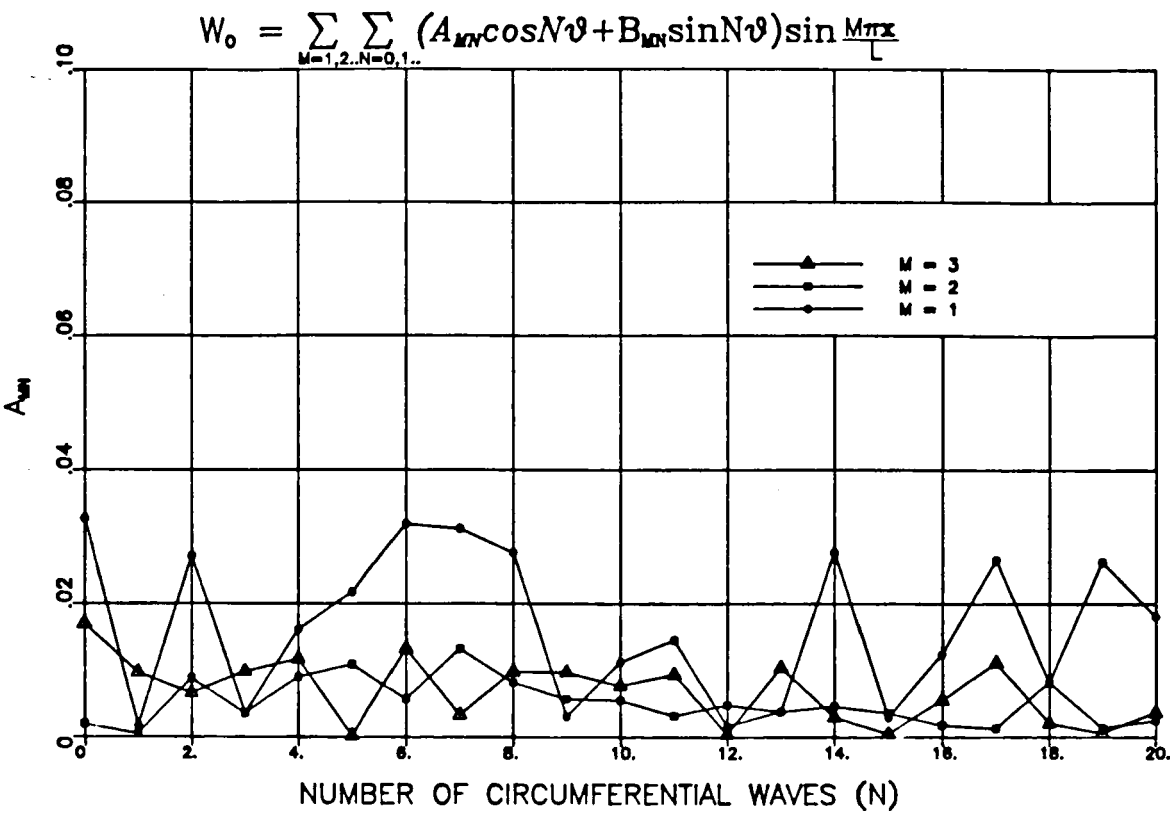


Fig. 3.16

CYLINDER 4 - FOURIER COEFFICIENTS

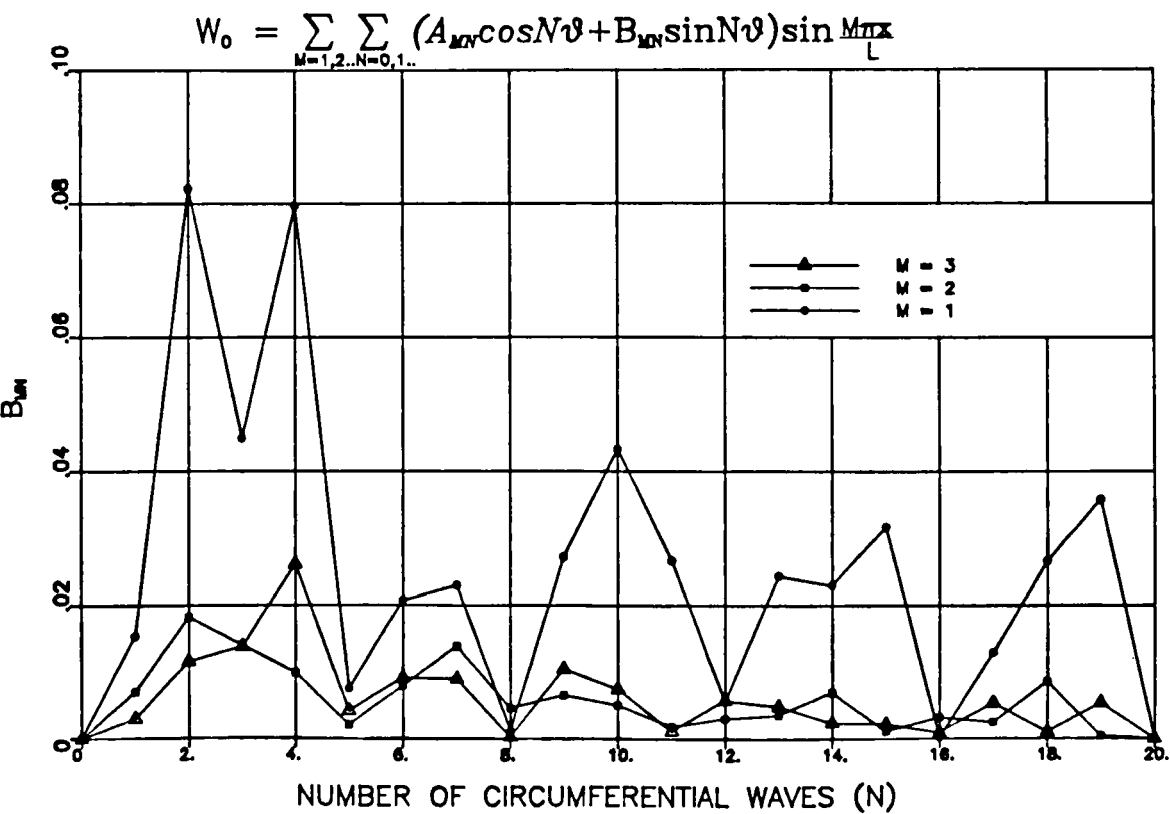


Fig. 3.17

CYLINDER 5 – FOURIER COEFFICIENTS

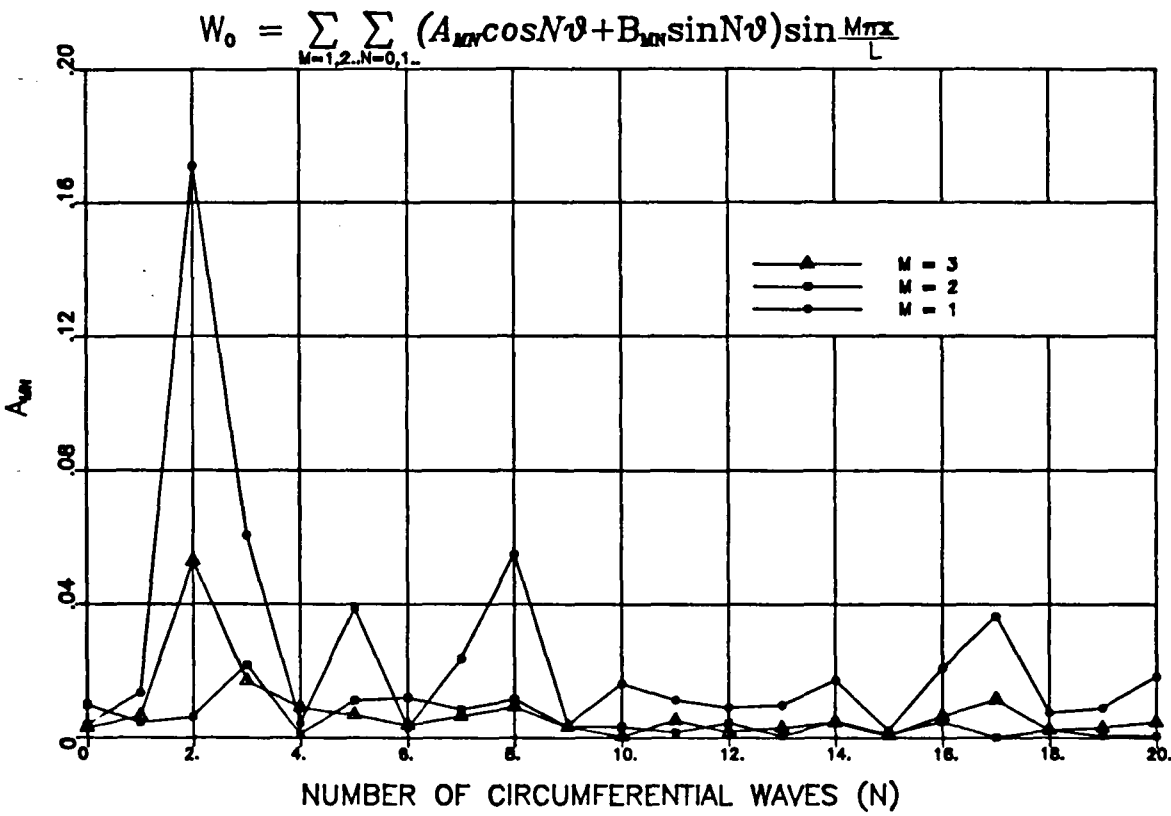


Fig. 3.18

CYLINDER 5 – FOURIER COEFFICIENTS

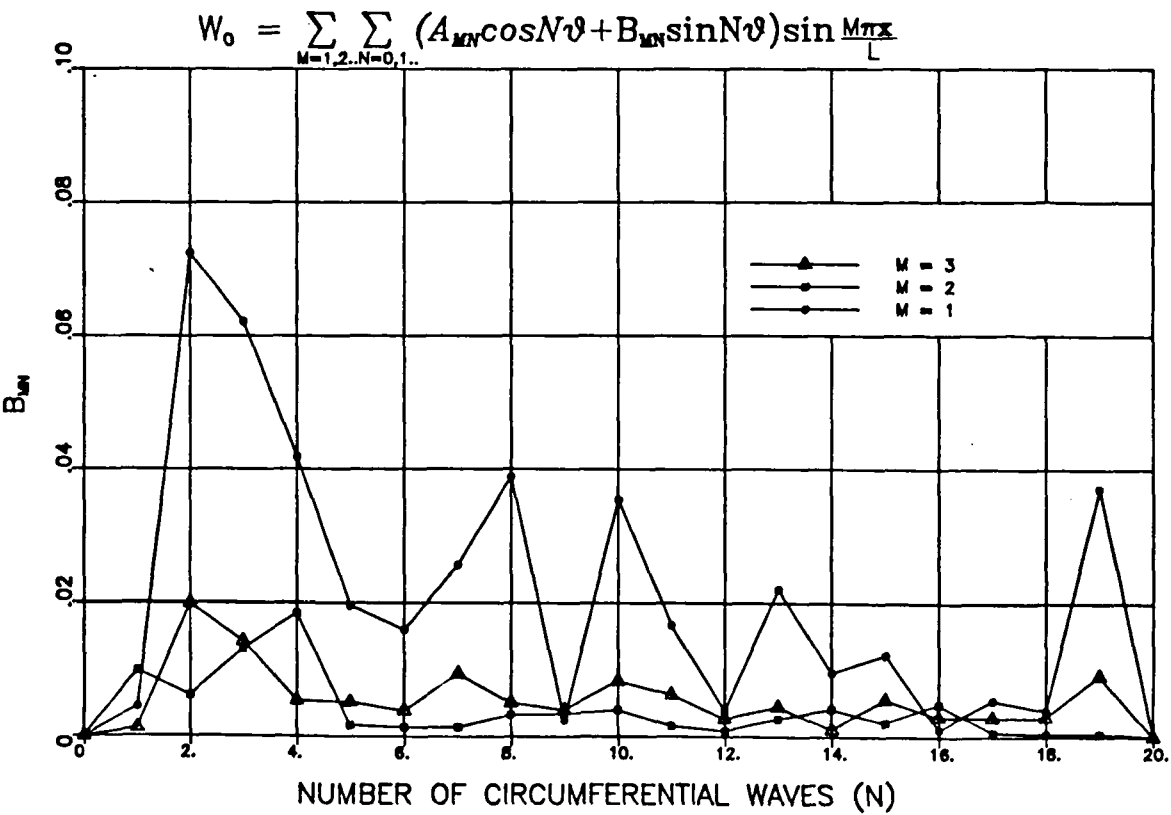


Fig. 3.19

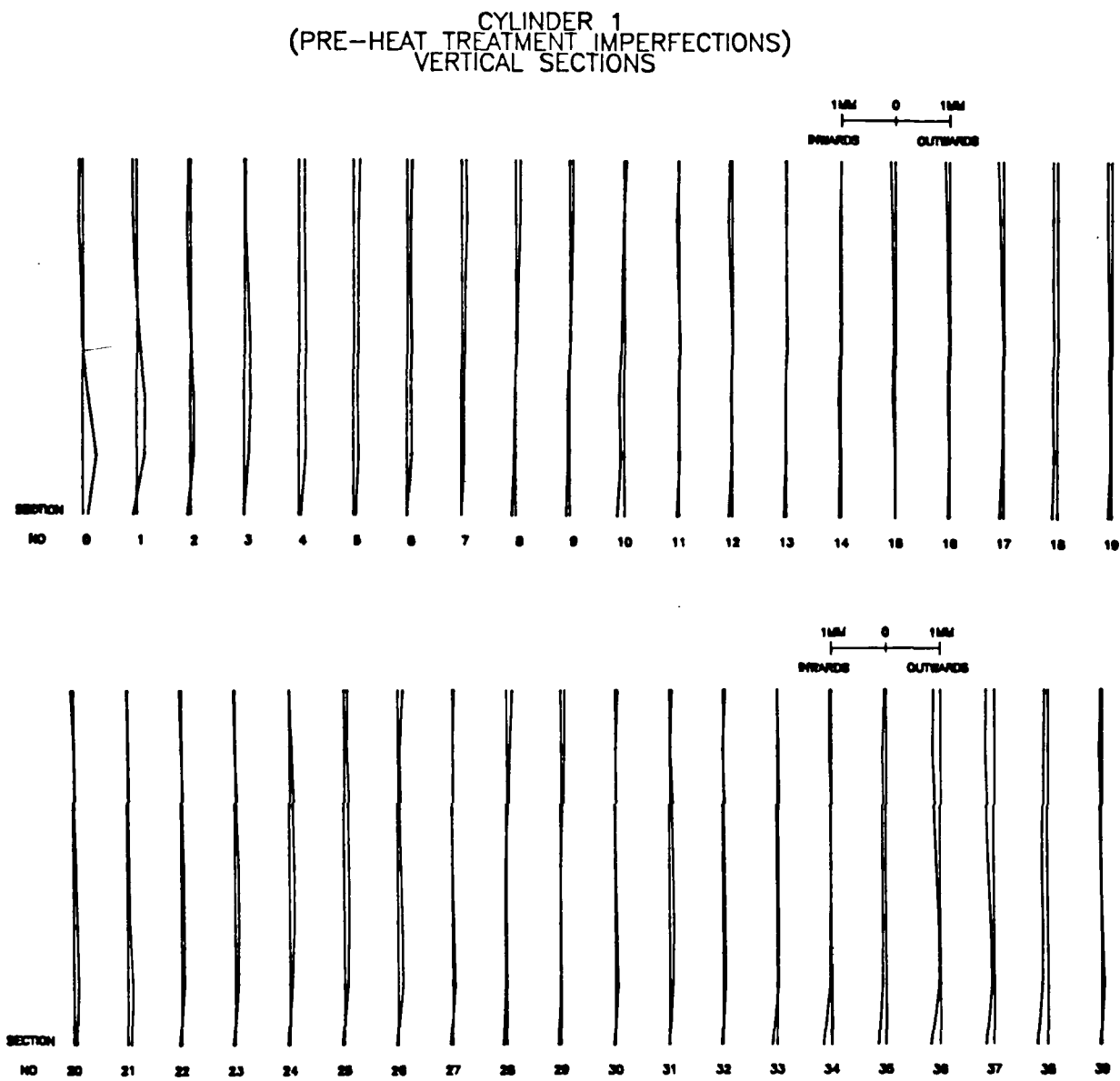


Fig. 3.20

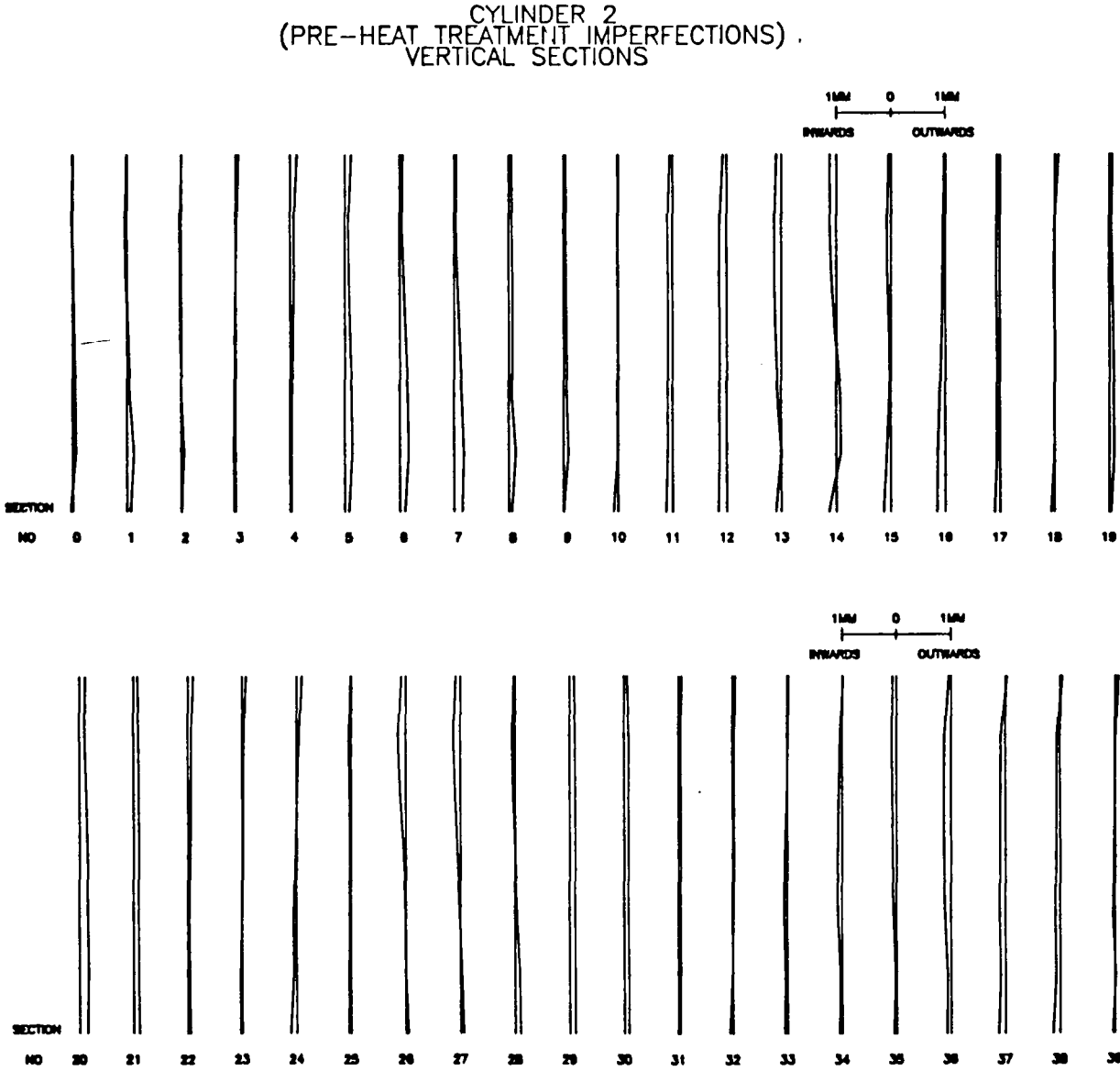


Fig. 3.21

CYLINDER 1 – INITIAL IMPERFECTIONS
VERTICAL SECTIONS

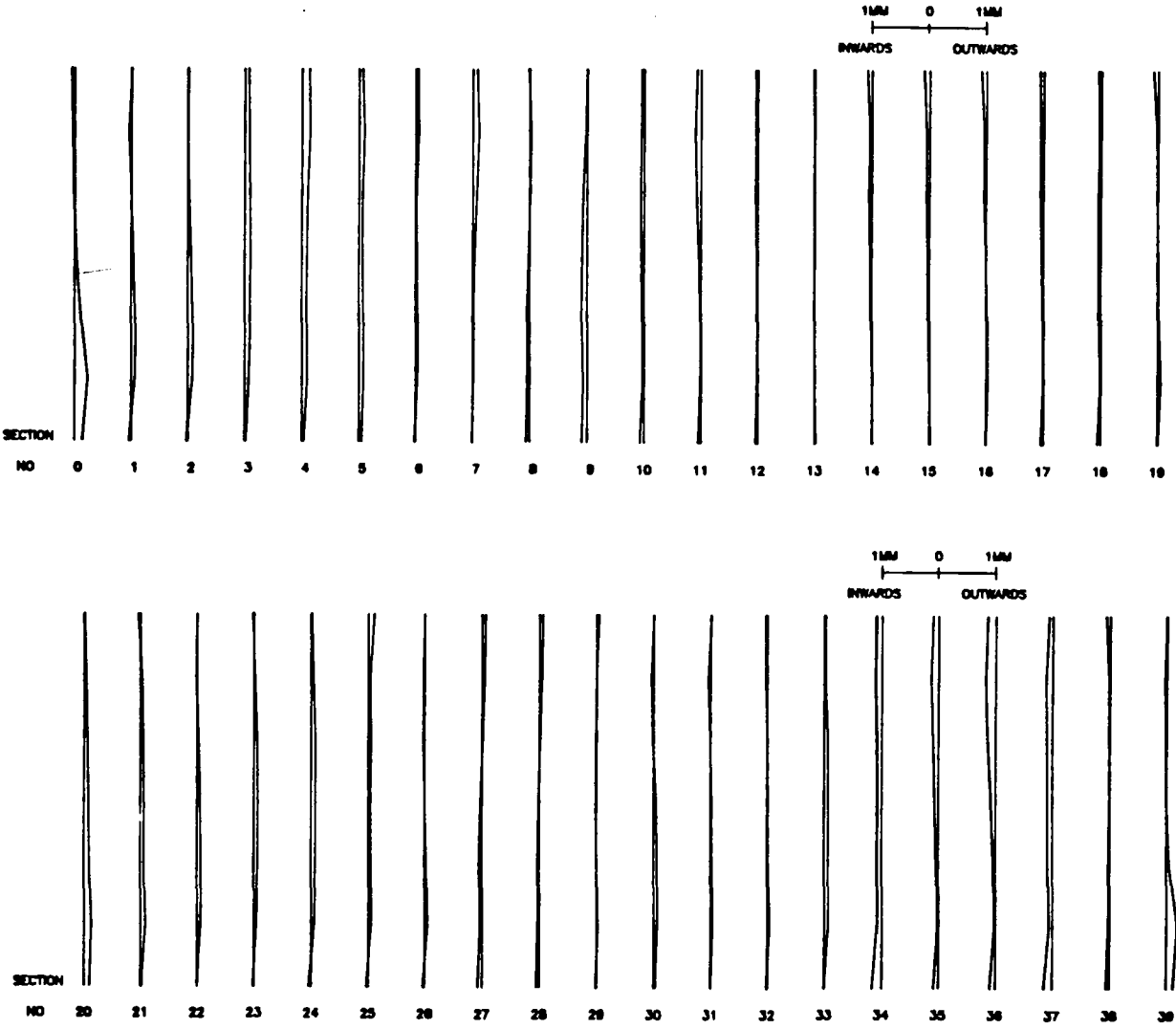


Fig. 3.22

CYLINDER 2 – INITIAL IMPERFECTIONS
VERTICAL SECTIONS

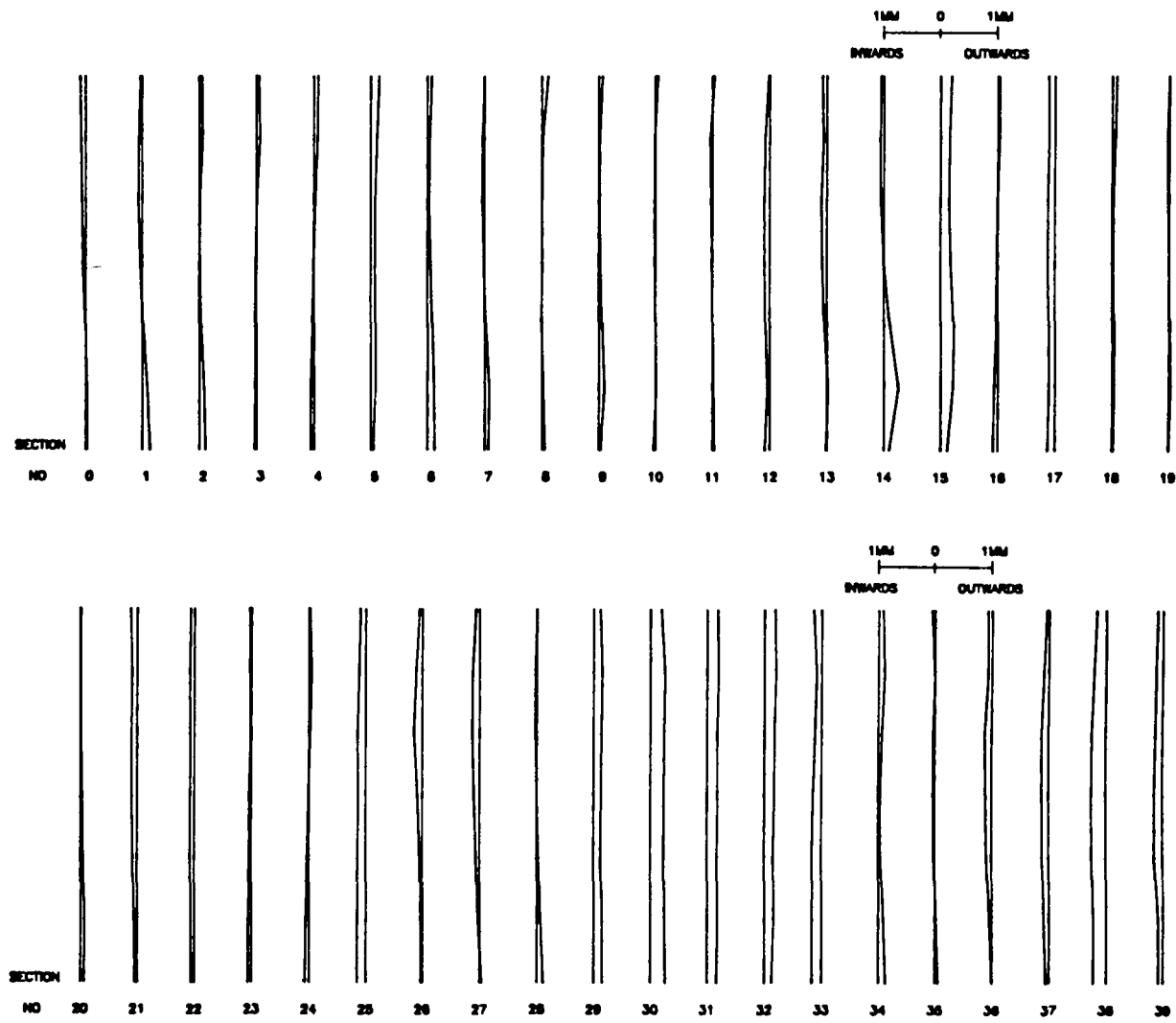


Fig. 3.23

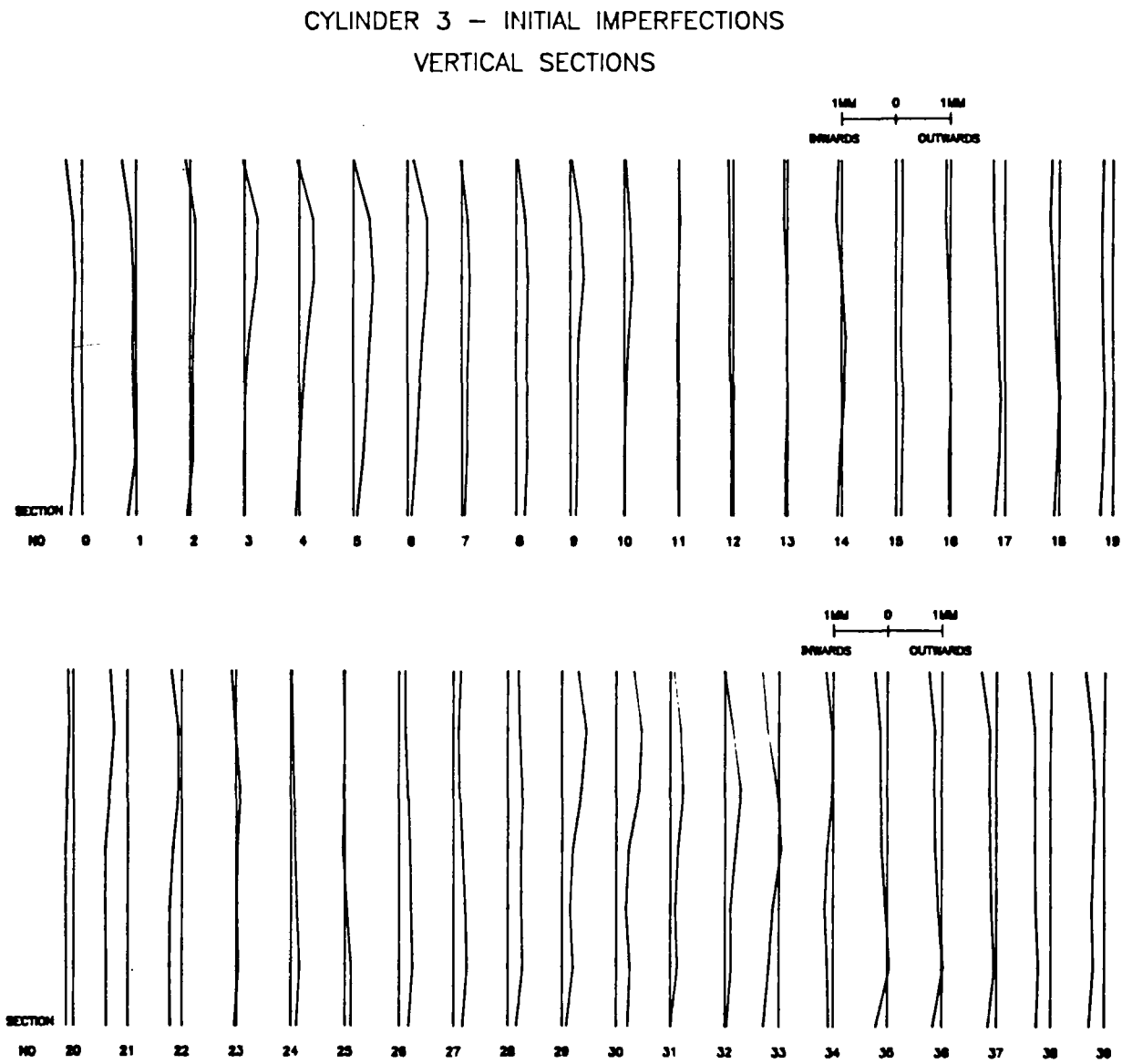


Fig. 3.24

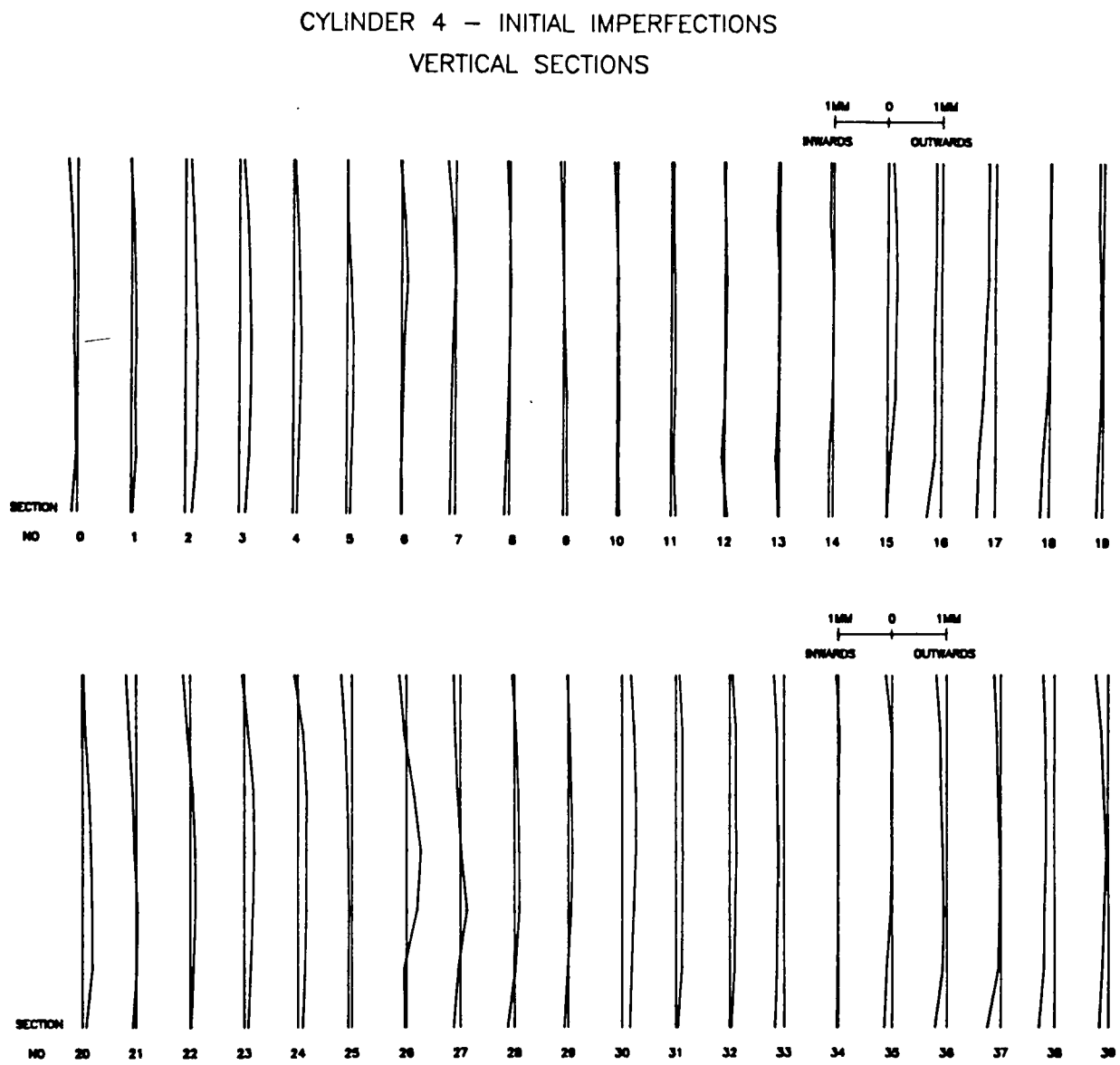


Fig. 3.25

CYLINDER 5 – INITIAL IMPERFECTIONS
VERTICAL SECTIONS

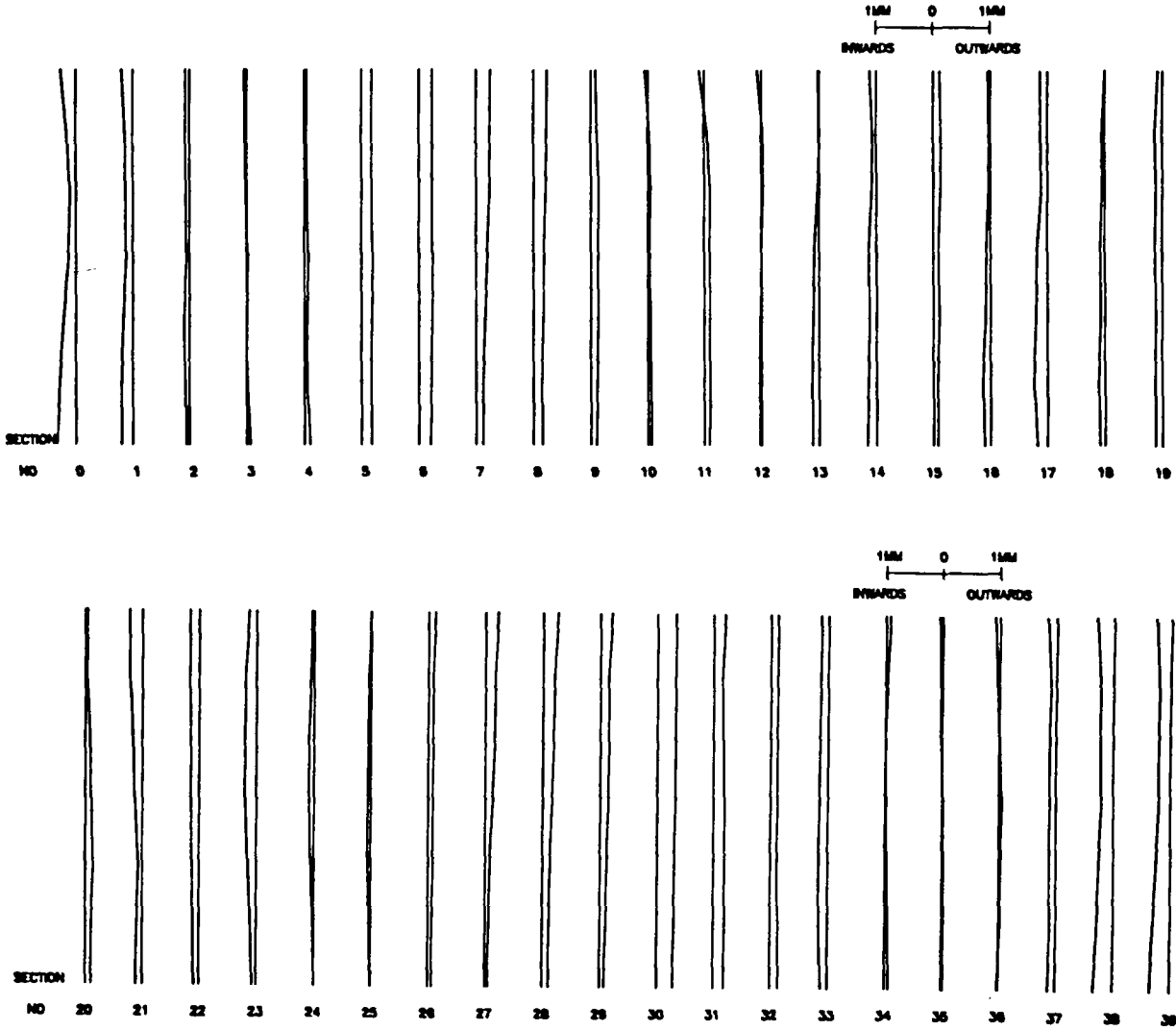
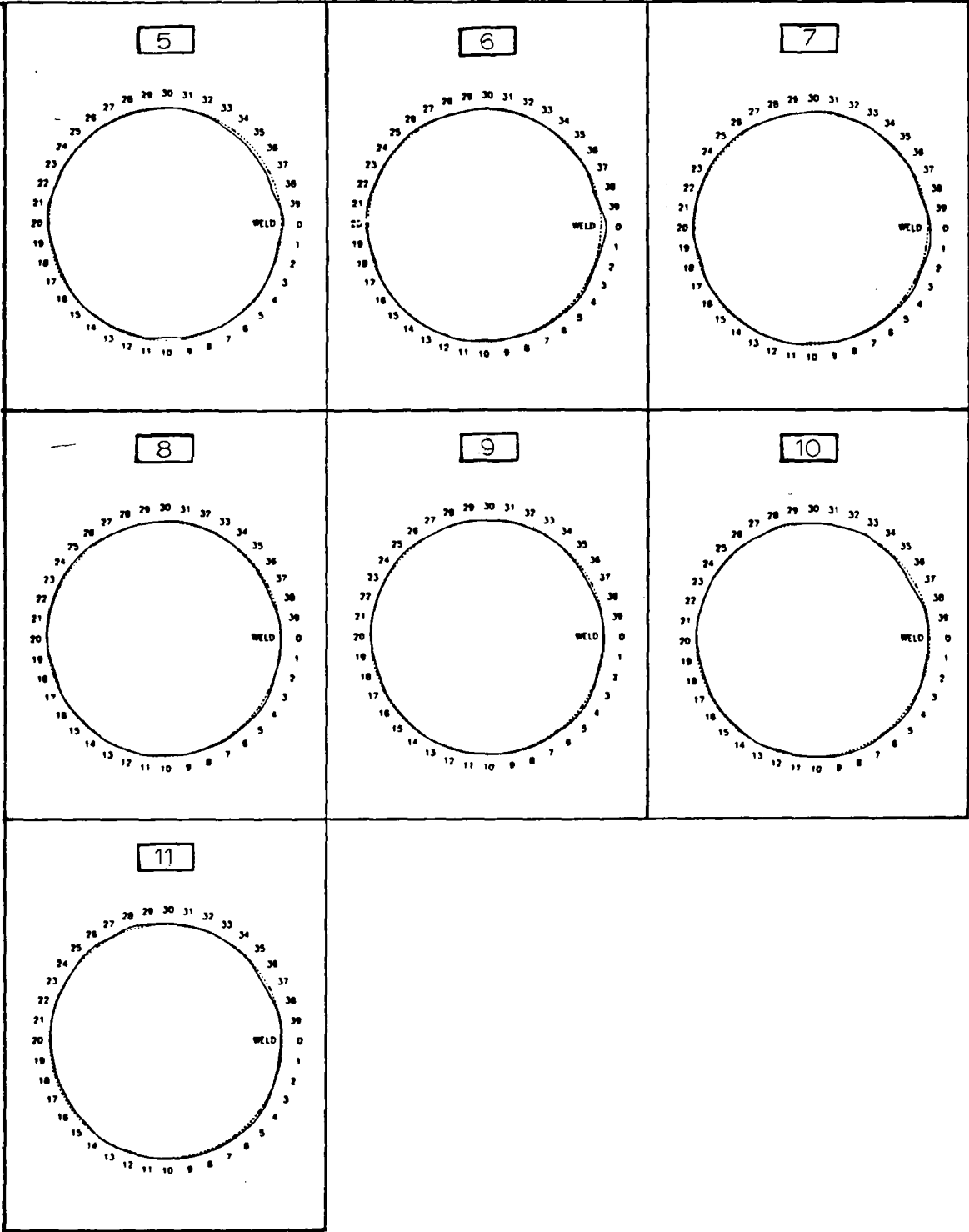


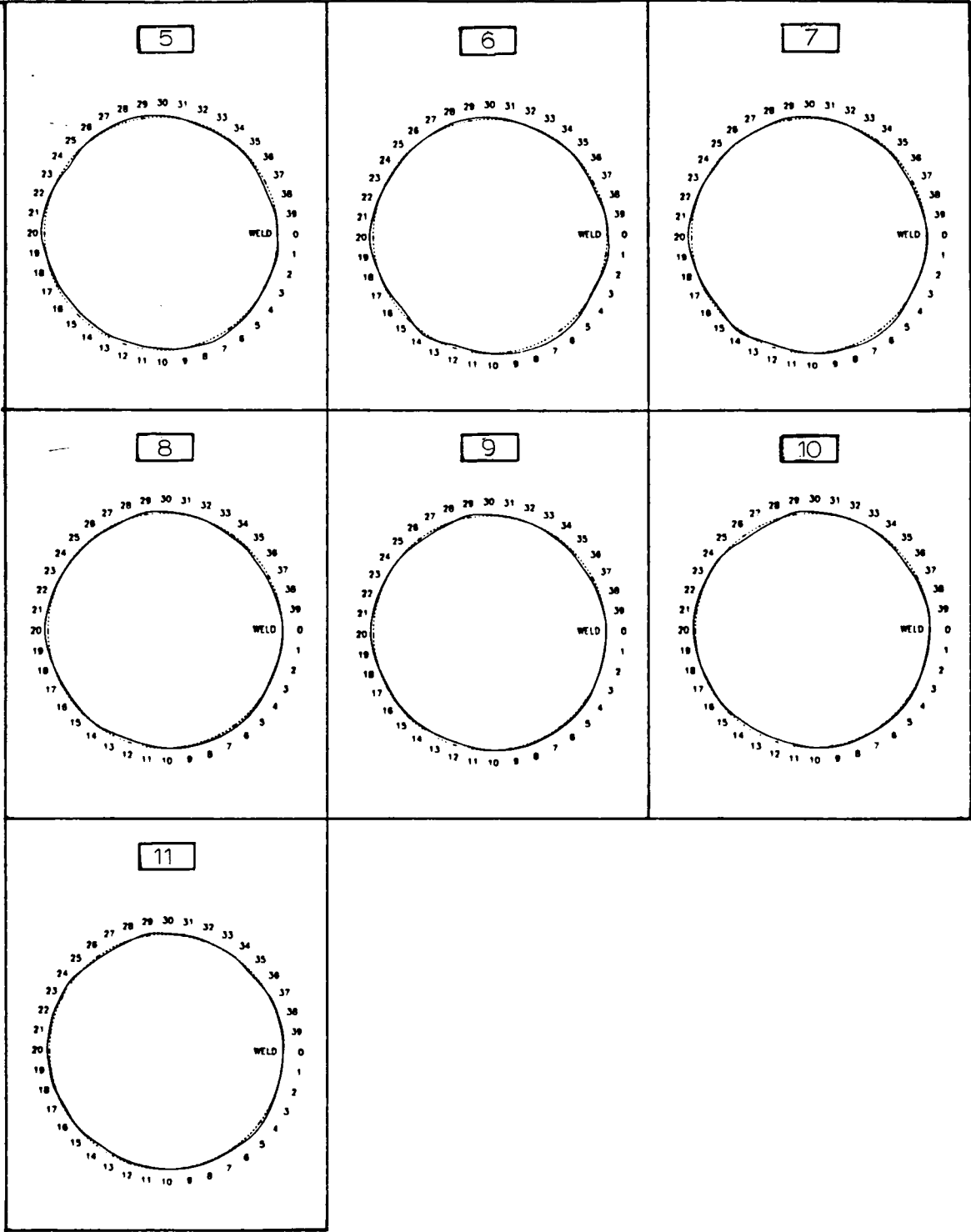
Fig. 3.26



Vertical section

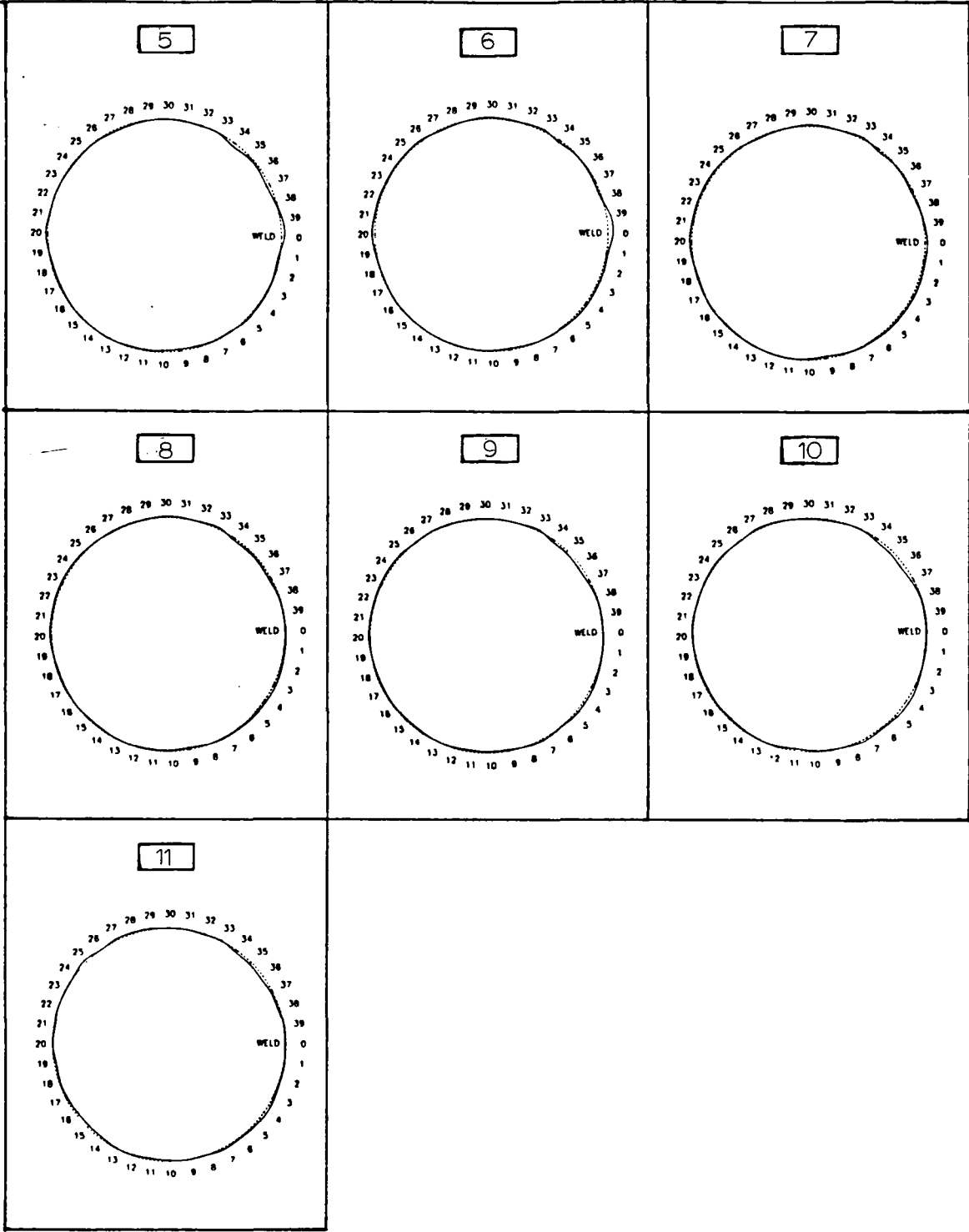
Imperfection magnification = 30

Fig.3.27 Imperfections of cylinder 1 (pre - heat treatment)



Vertical section
Imperfection magnification = 30

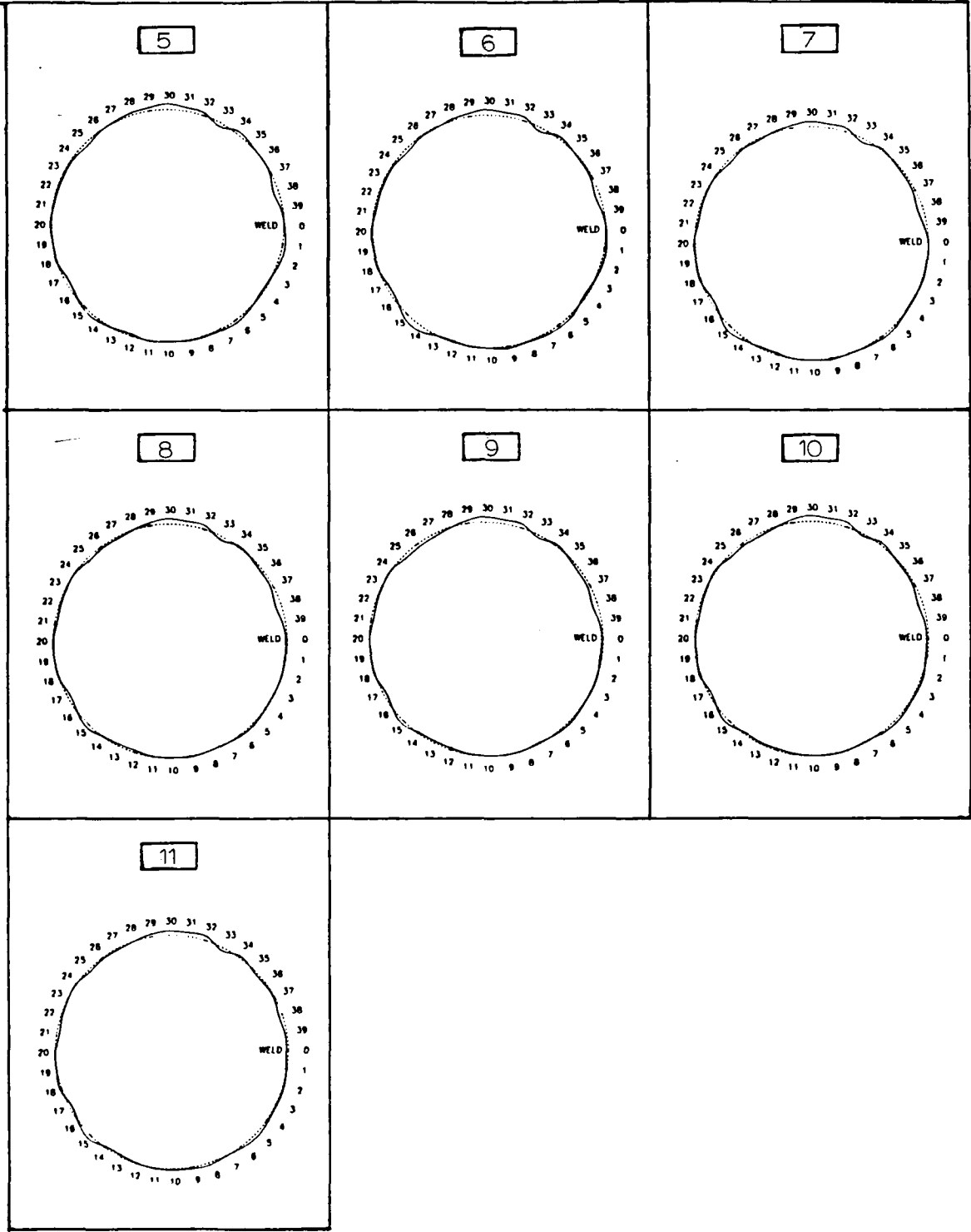
Fig. 3.28 Imperfections of cylinder 2 (pre - heat treatment)



 Vertical section

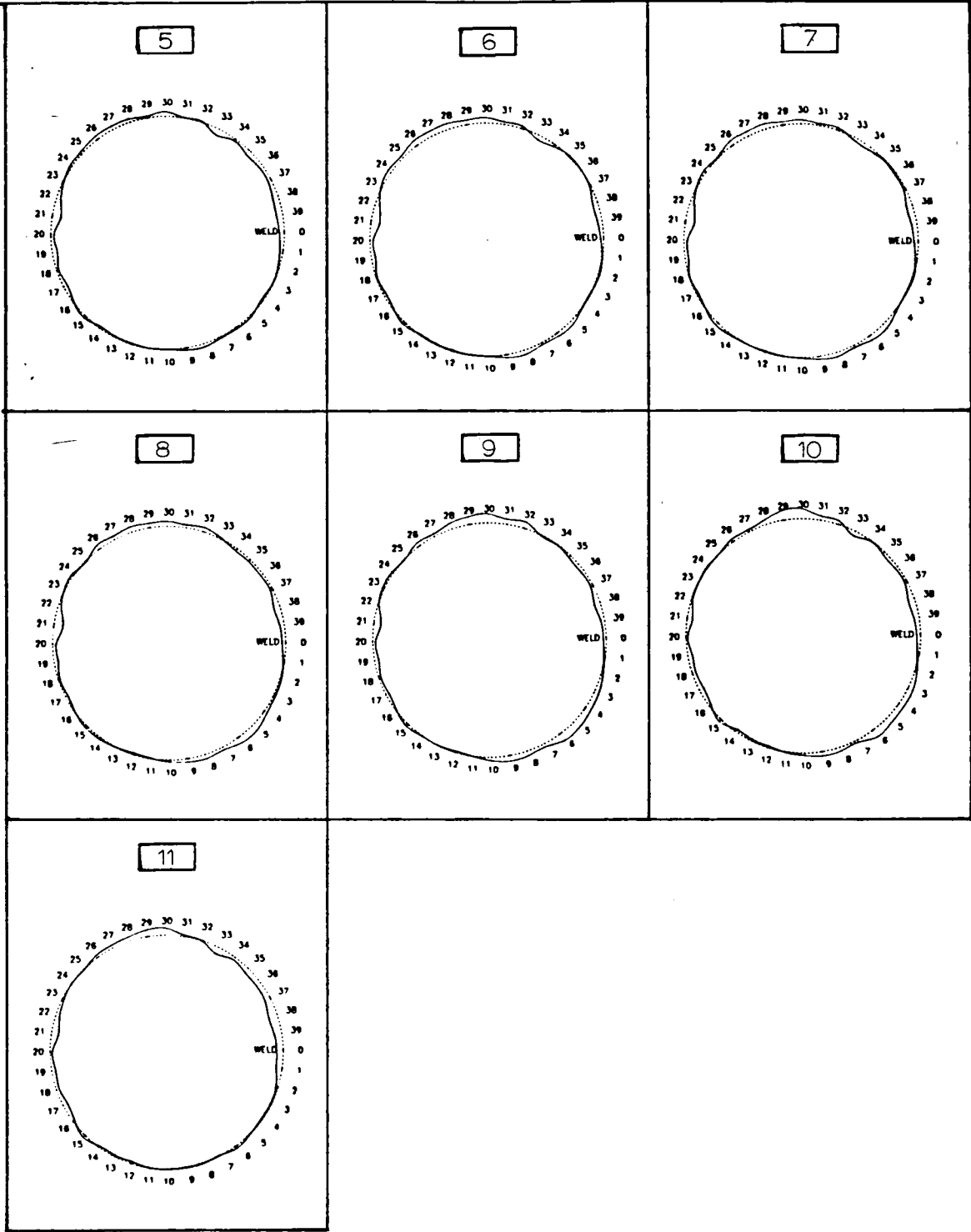
Imperfection magnification = 30

Fig. 3.29 Imperfections of cylinder 1



 Vertical section
Imperfection magnification = 30

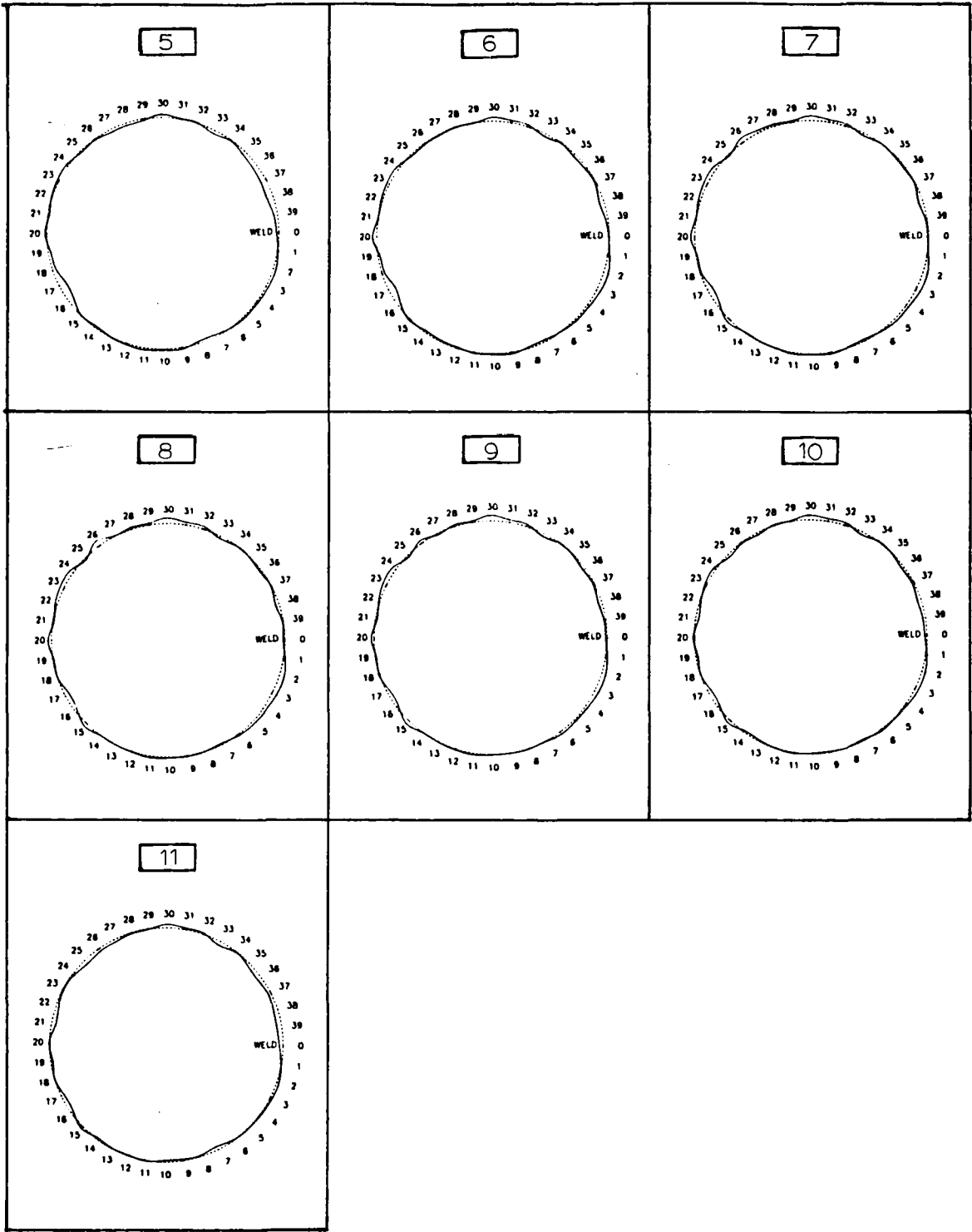
Fig 3.30 Imperfections of cylinder 2



 Vertical section

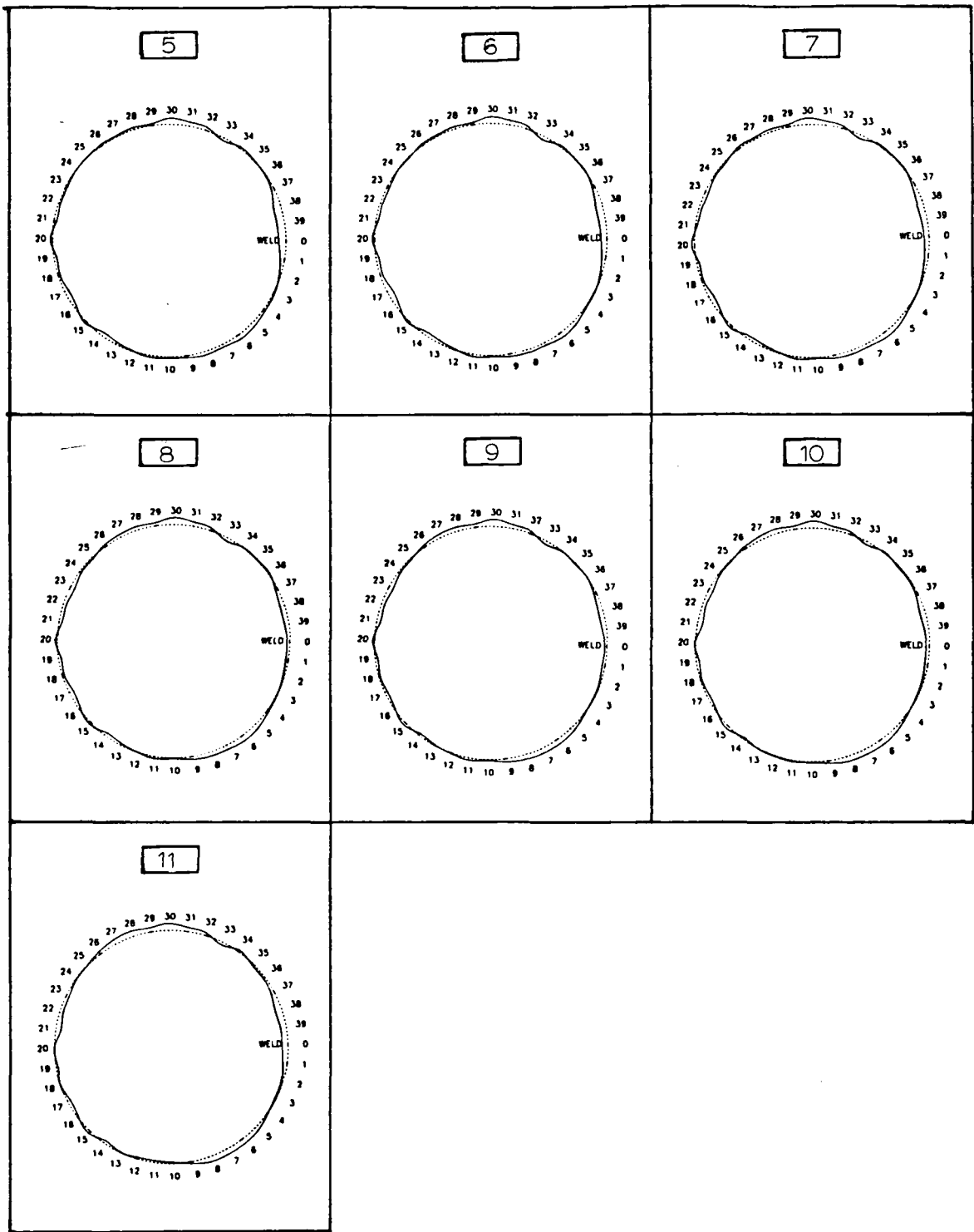
Imperfection magnification = 30

Fig. 3.31 Imperfections of cylinder 3



Vertical section
Imperfection magnification = 30

Fig. 3.32 Imperfections of cylinder 4



Vertical section

Imperfection magnification = 30

Fig. 3.33 Imperfections of cylinder 5

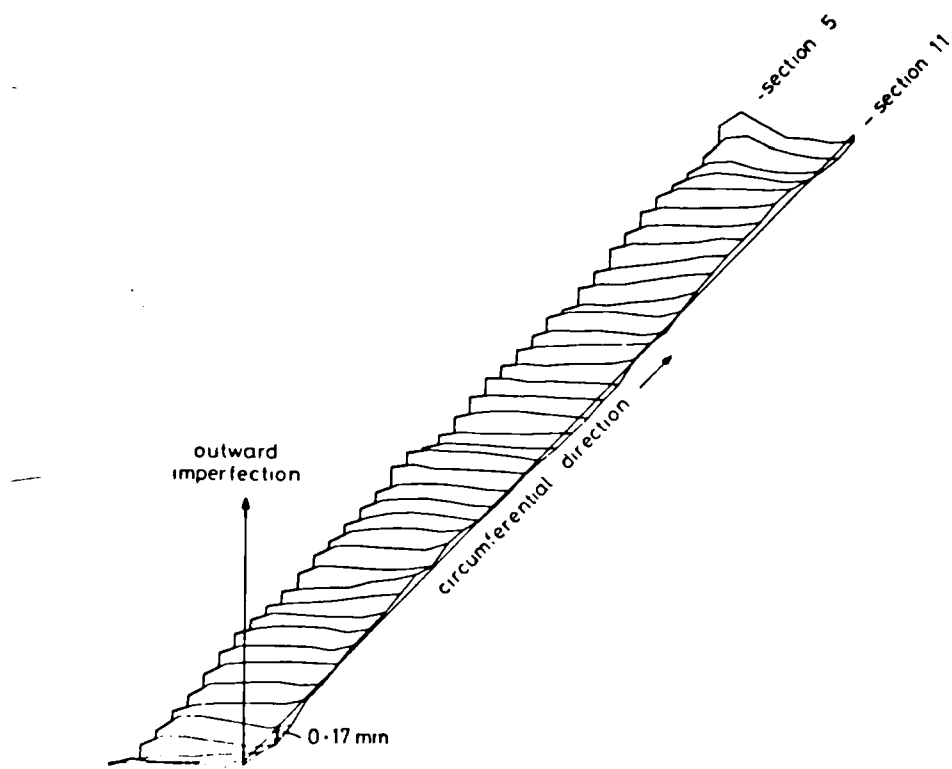


Fig. 3.34 Pre - heat treatment imperfections of cylinder 1

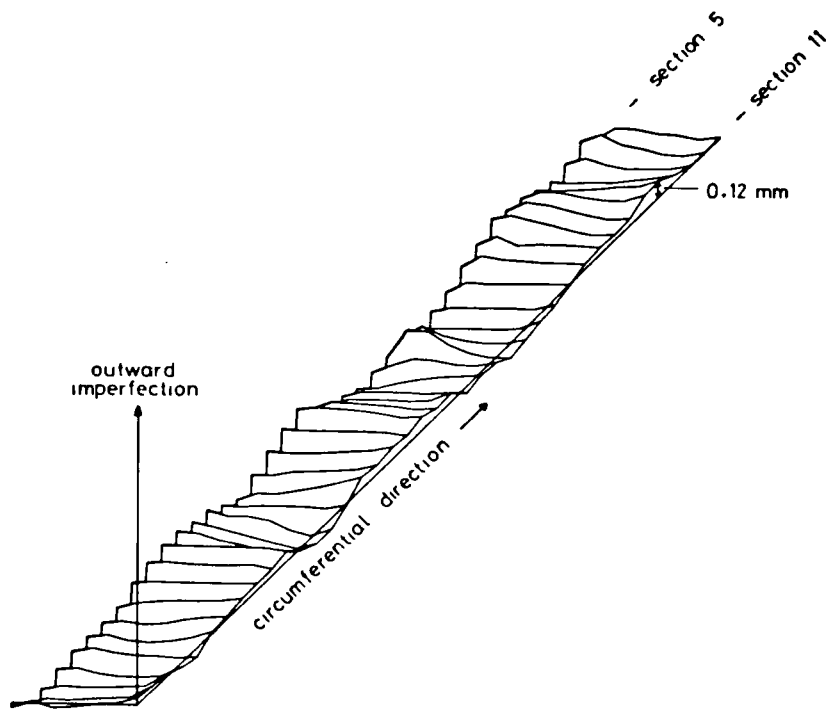


Fig. 3.35 Pre - heat treatment imperfections of cylinder 2

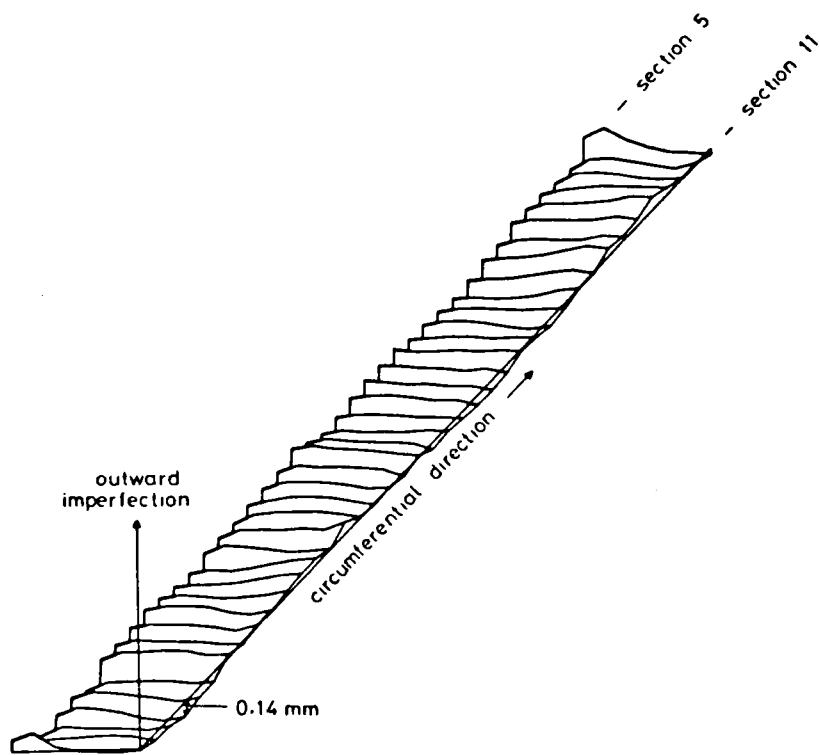


Fig. 3.36 Imperfections of cylinder 1

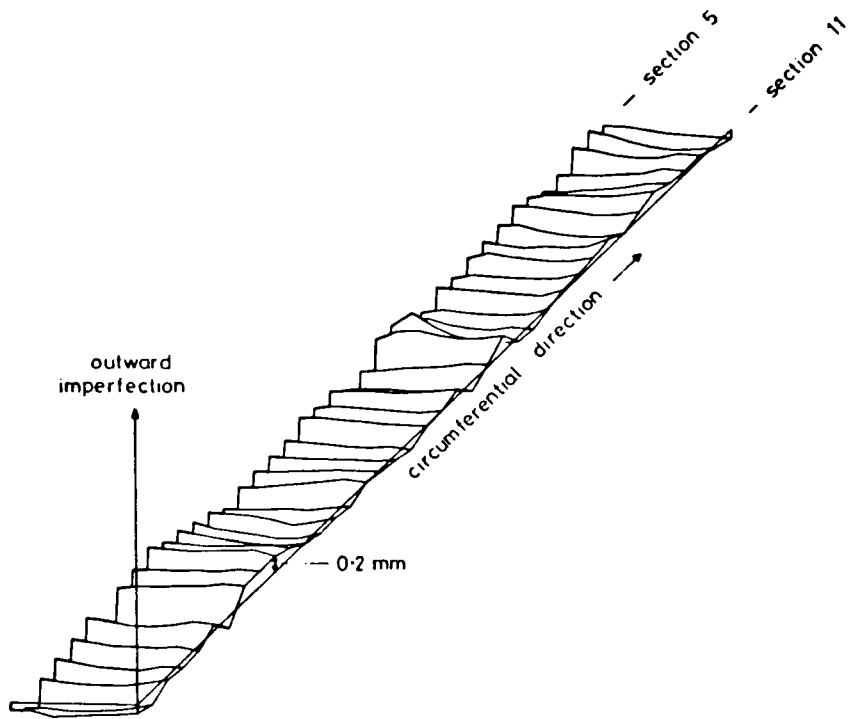


Fig. 3.37 Imperfections of cylinder 2

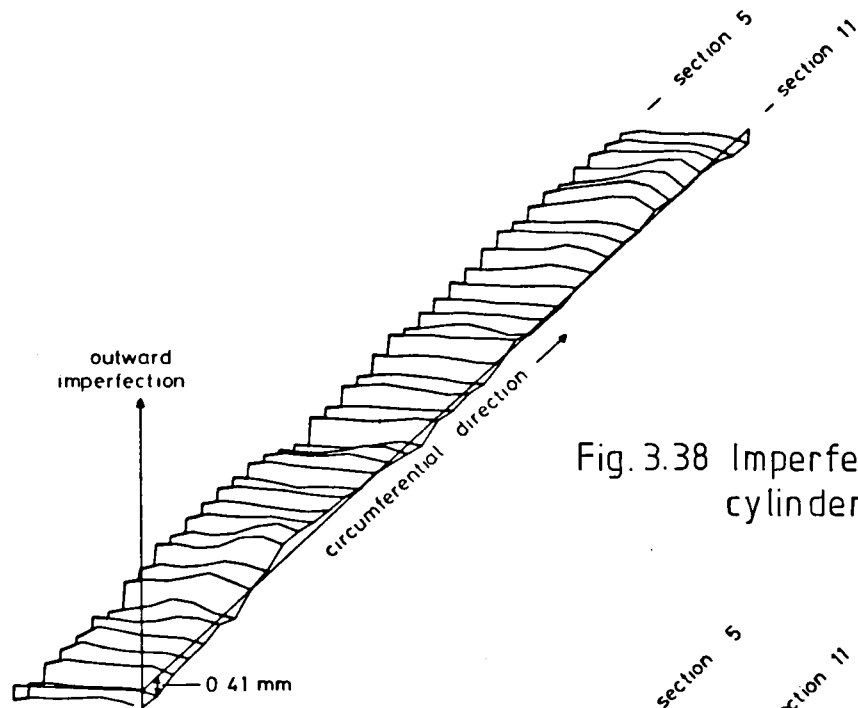


Fig. 3.38 Imperfections of cylinder 3

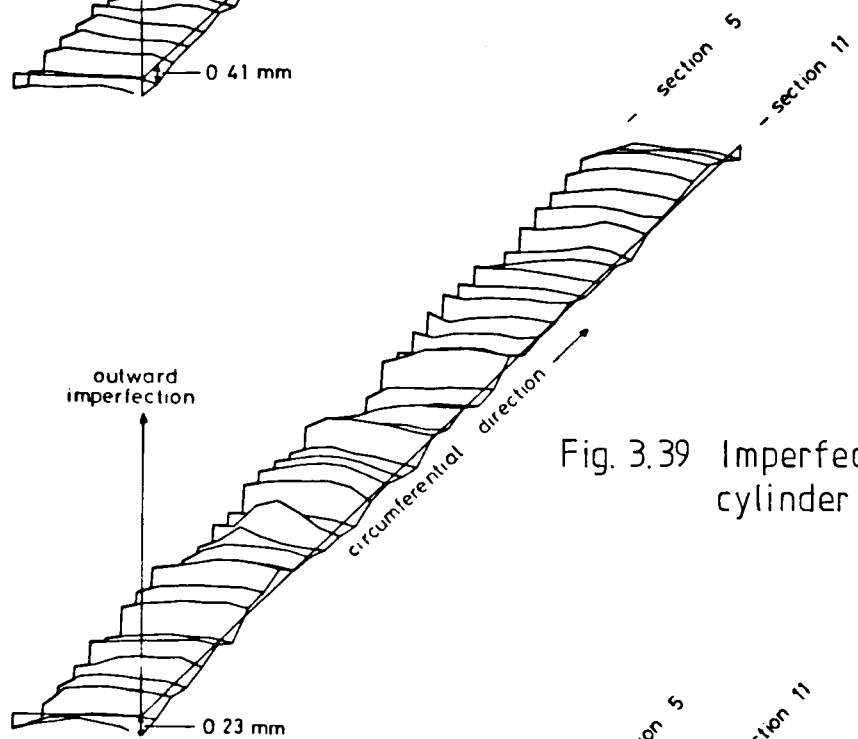


Fig. 3.39 Imperfections of cylinder 4

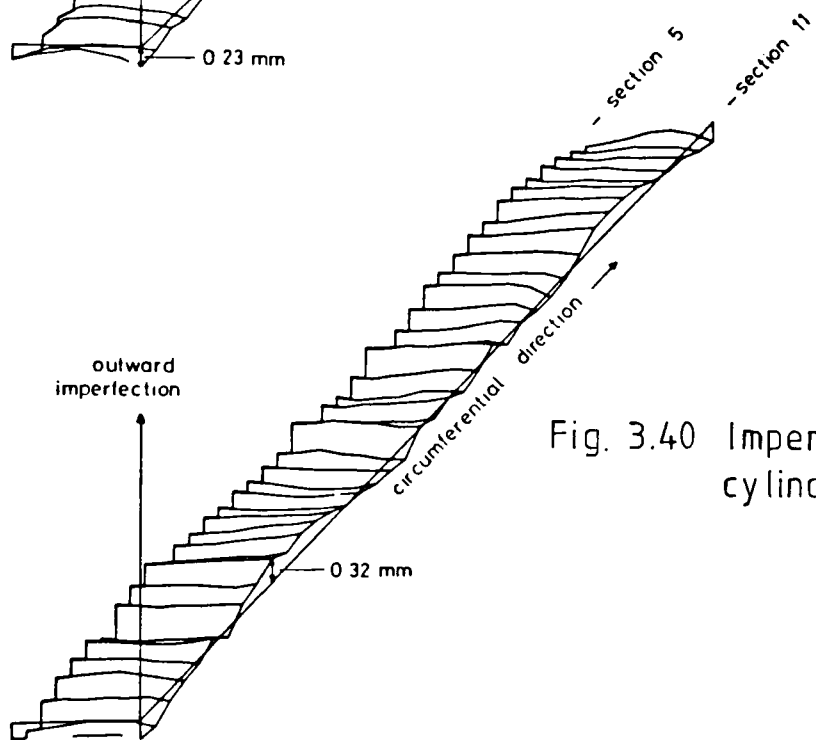


Fig. 3.40 Imperfections of cylinder 5

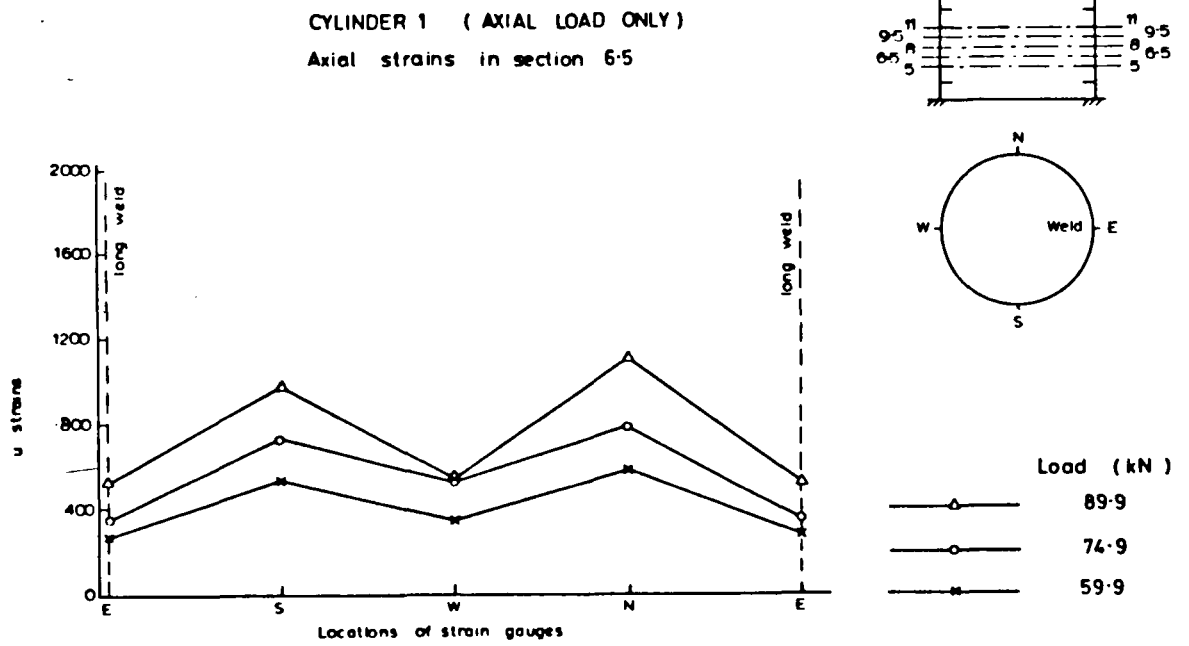


Fig. 3.41

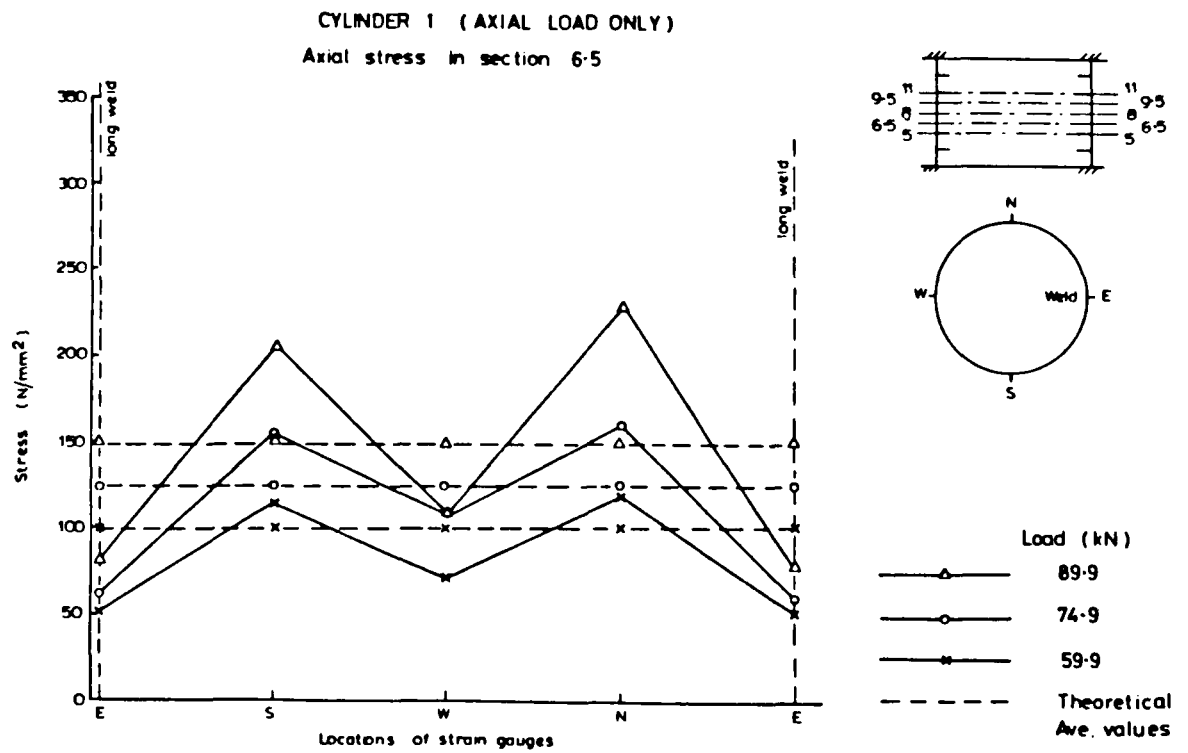


Fig. 3.42

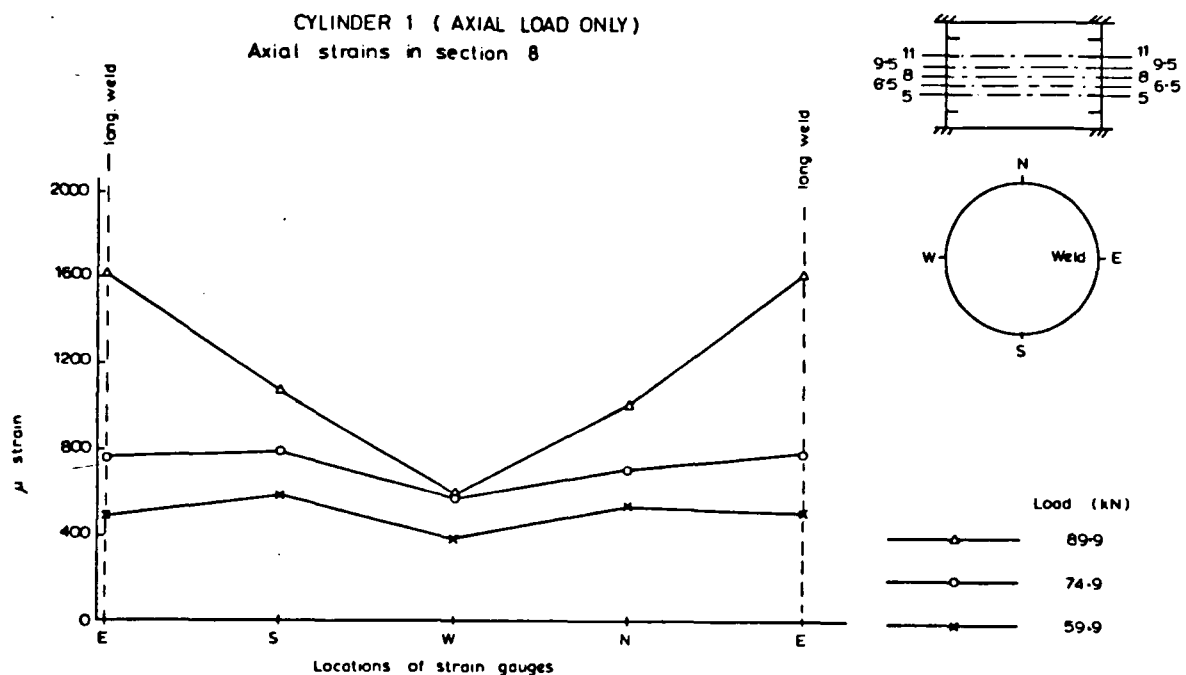


Fig. 3.43

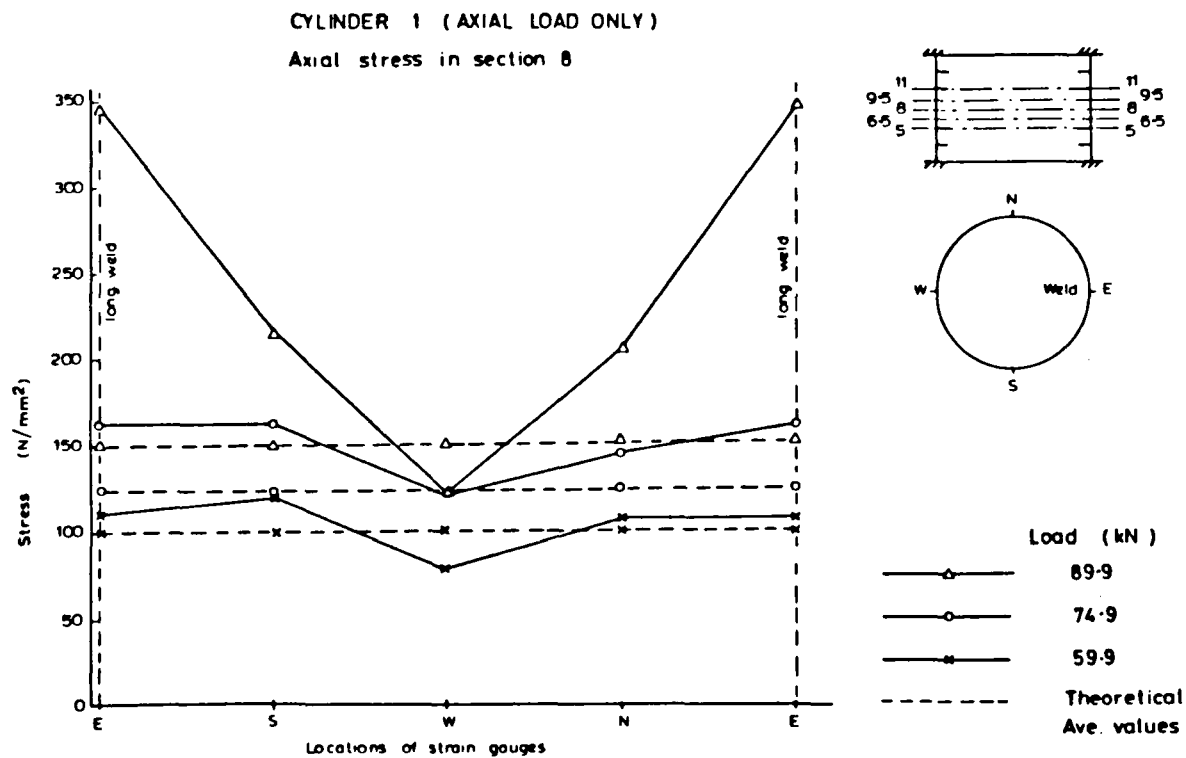


Fig. 3.44

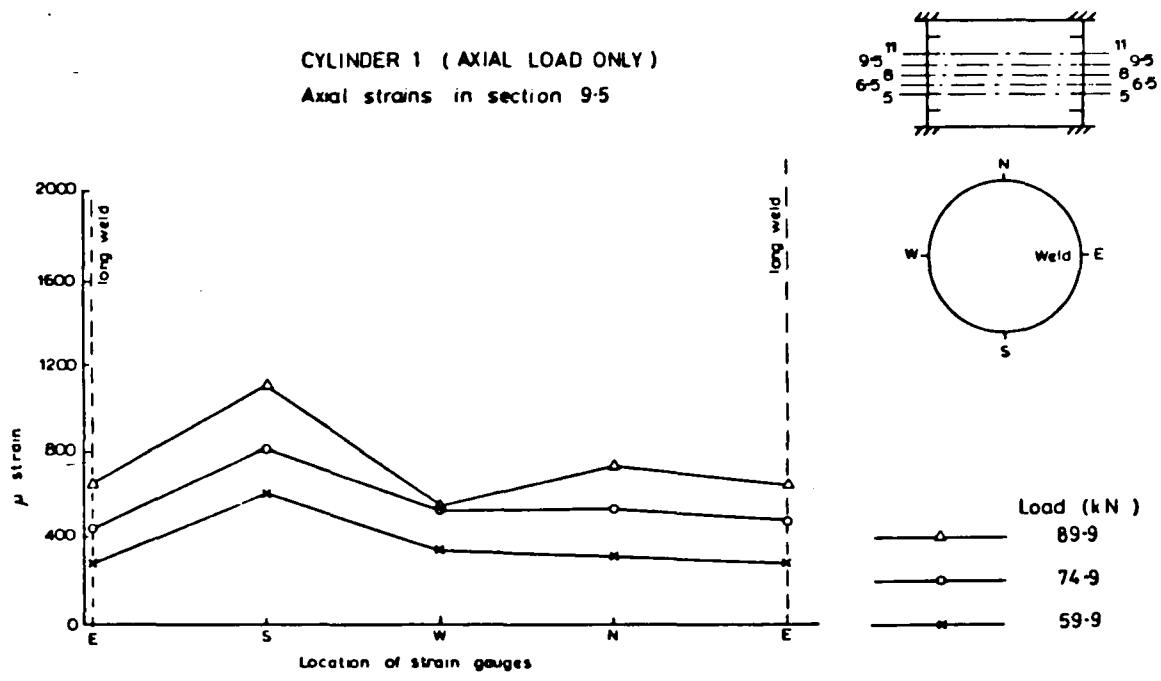


Fig. 3.45

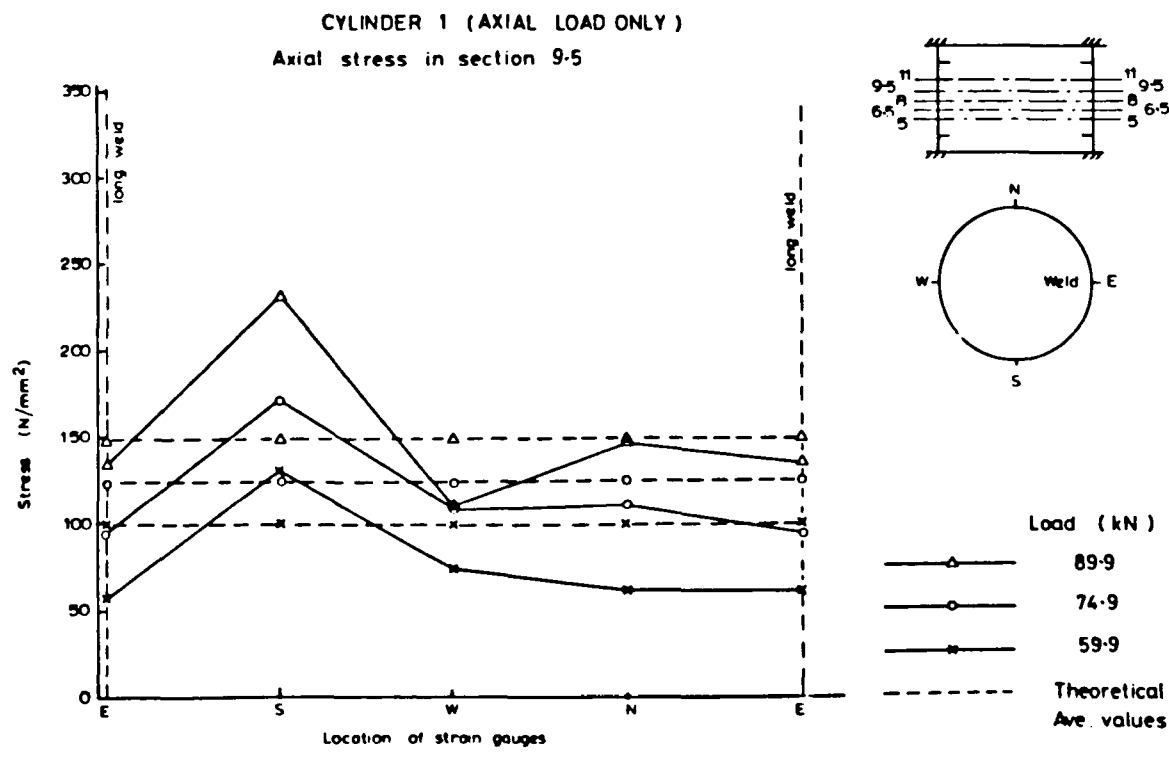


Fig. 3.46

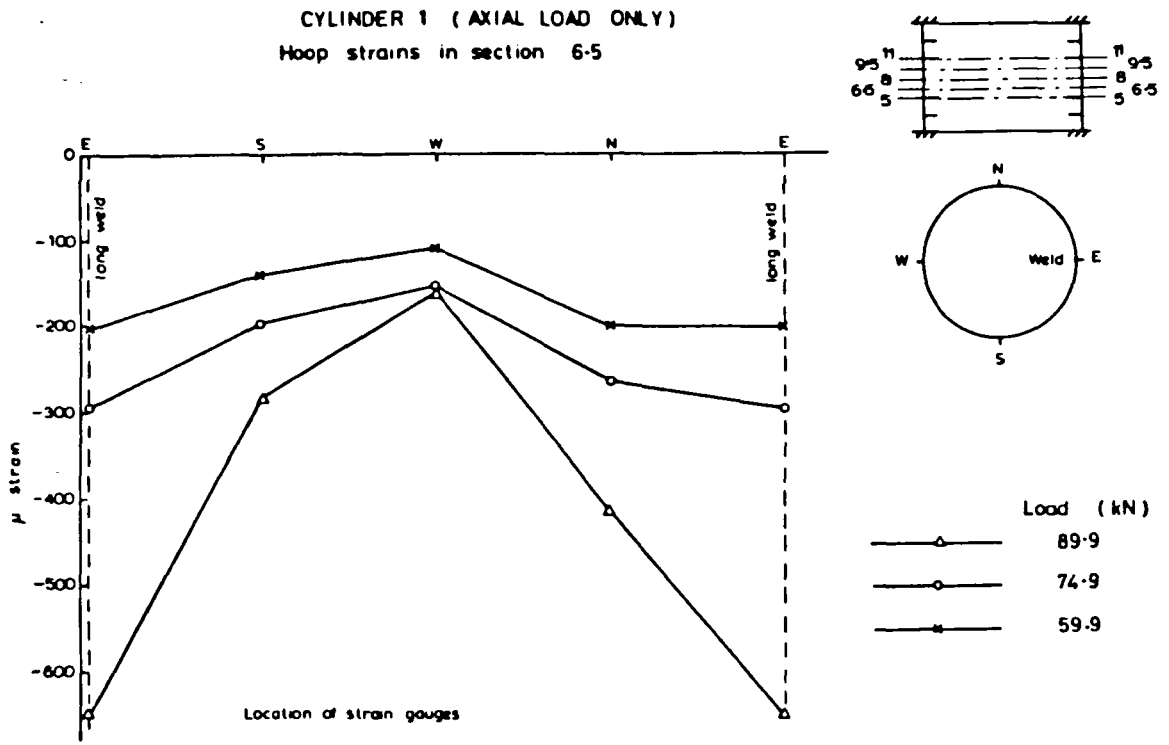


Fig. 3.47

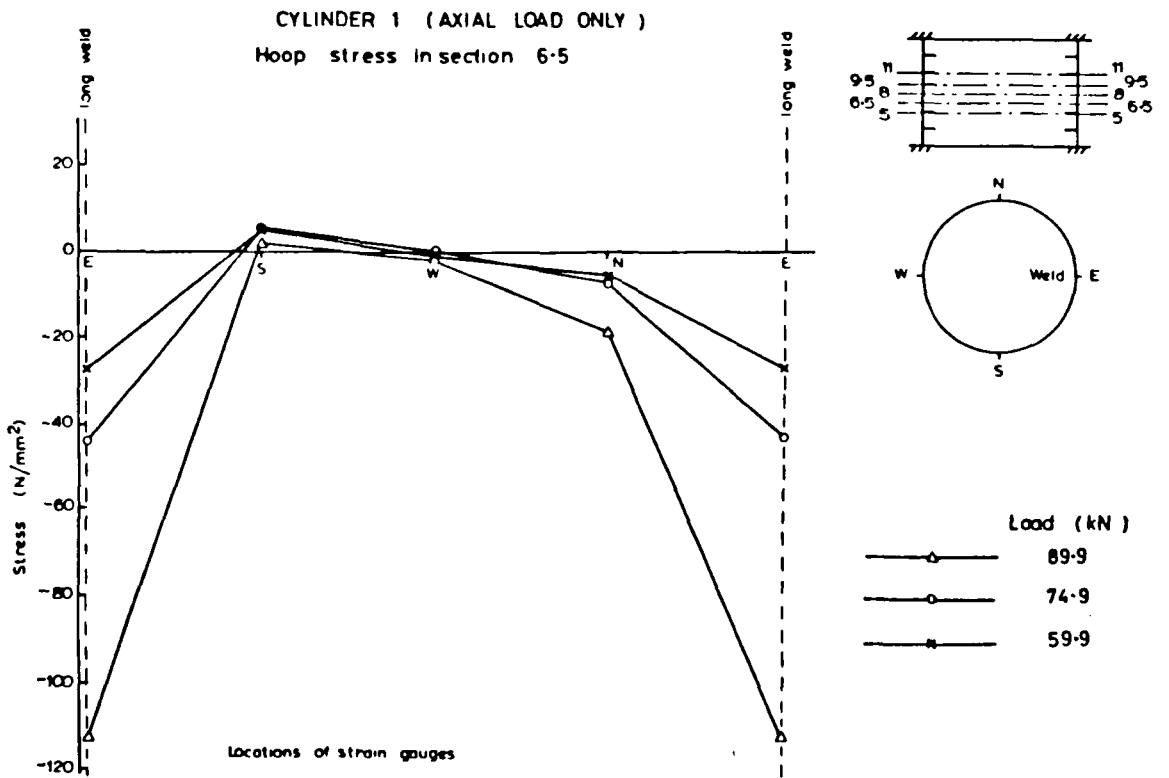


Fig. 3.48

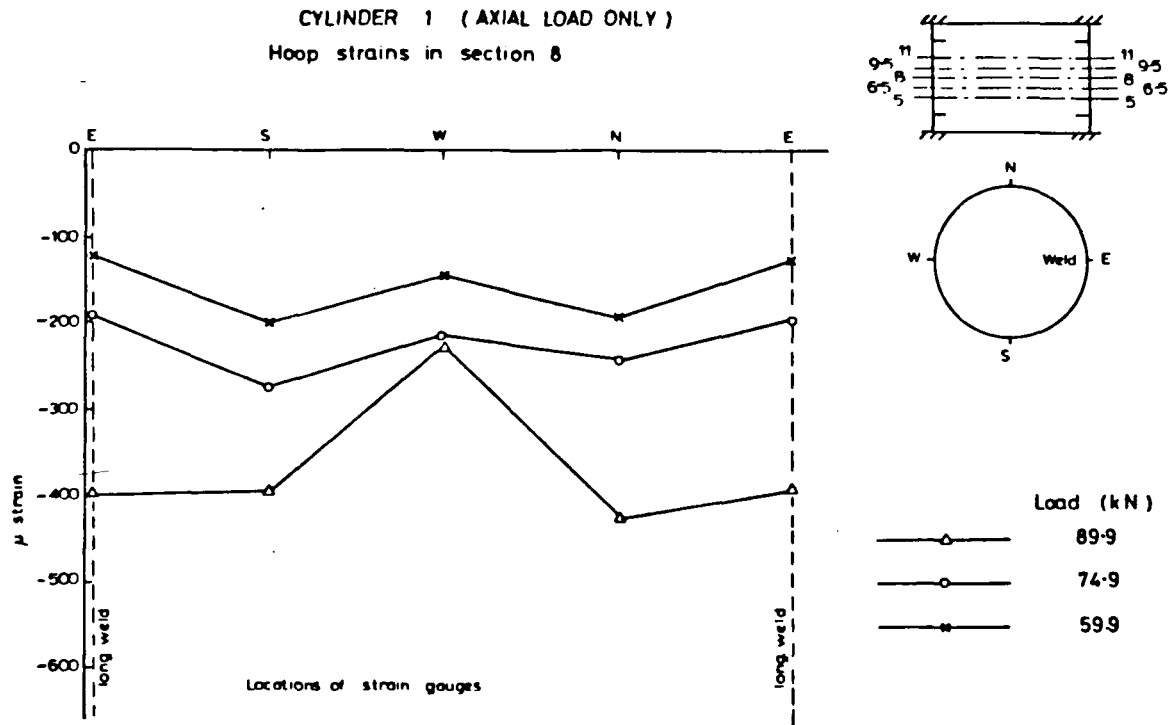


Fig. 3.49

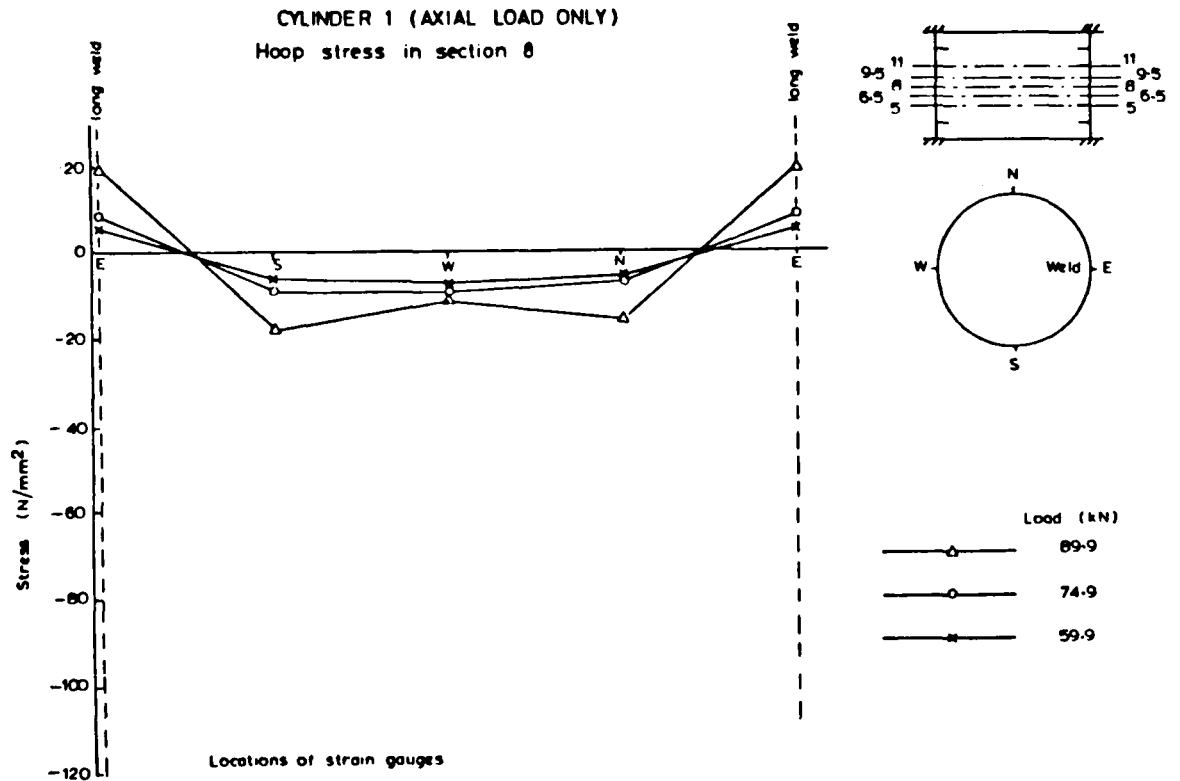


Fig. 3.50

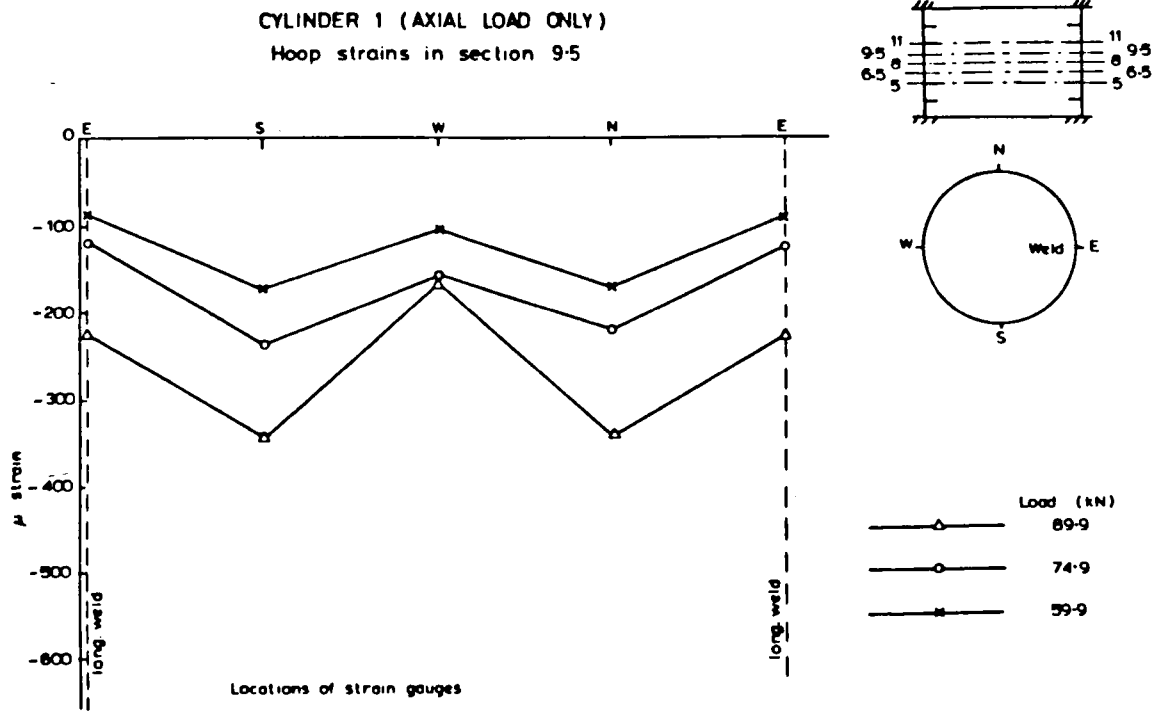


Fig. 3.51

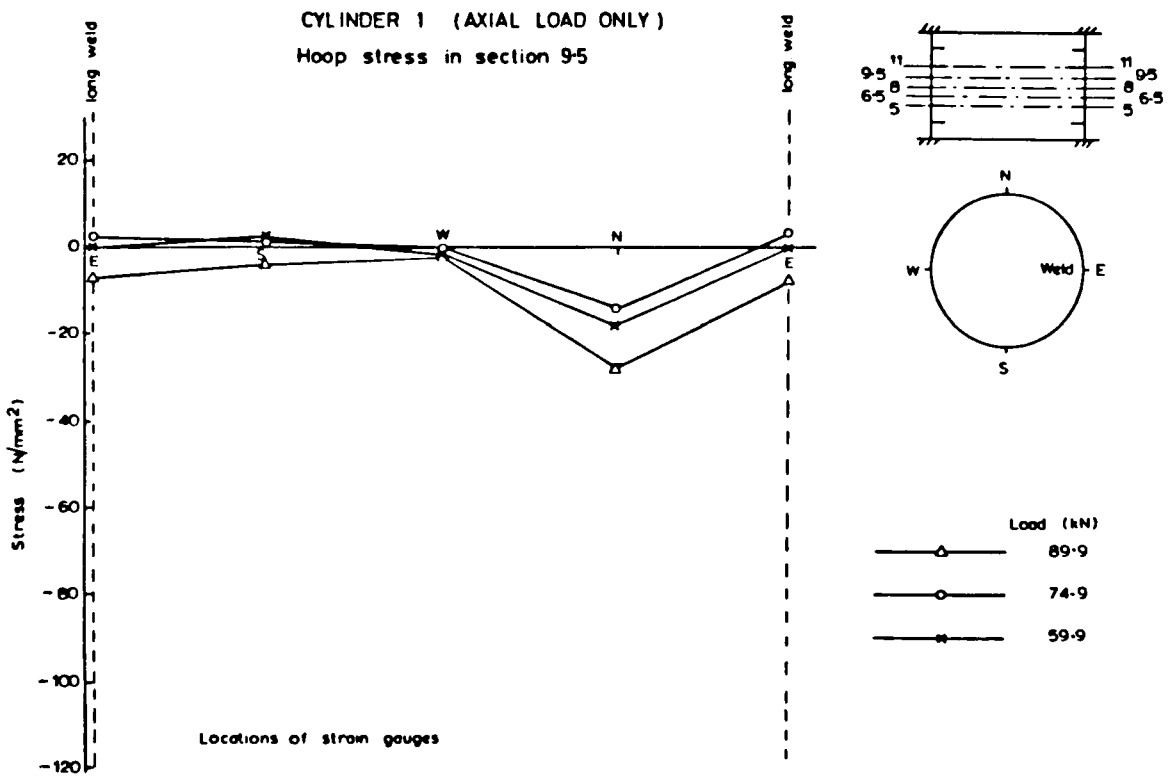


Fig. 3.52

CYLINDER 1 (AXIAL LOAD ONLY)
Longitudinal distribution of axial stress and strain

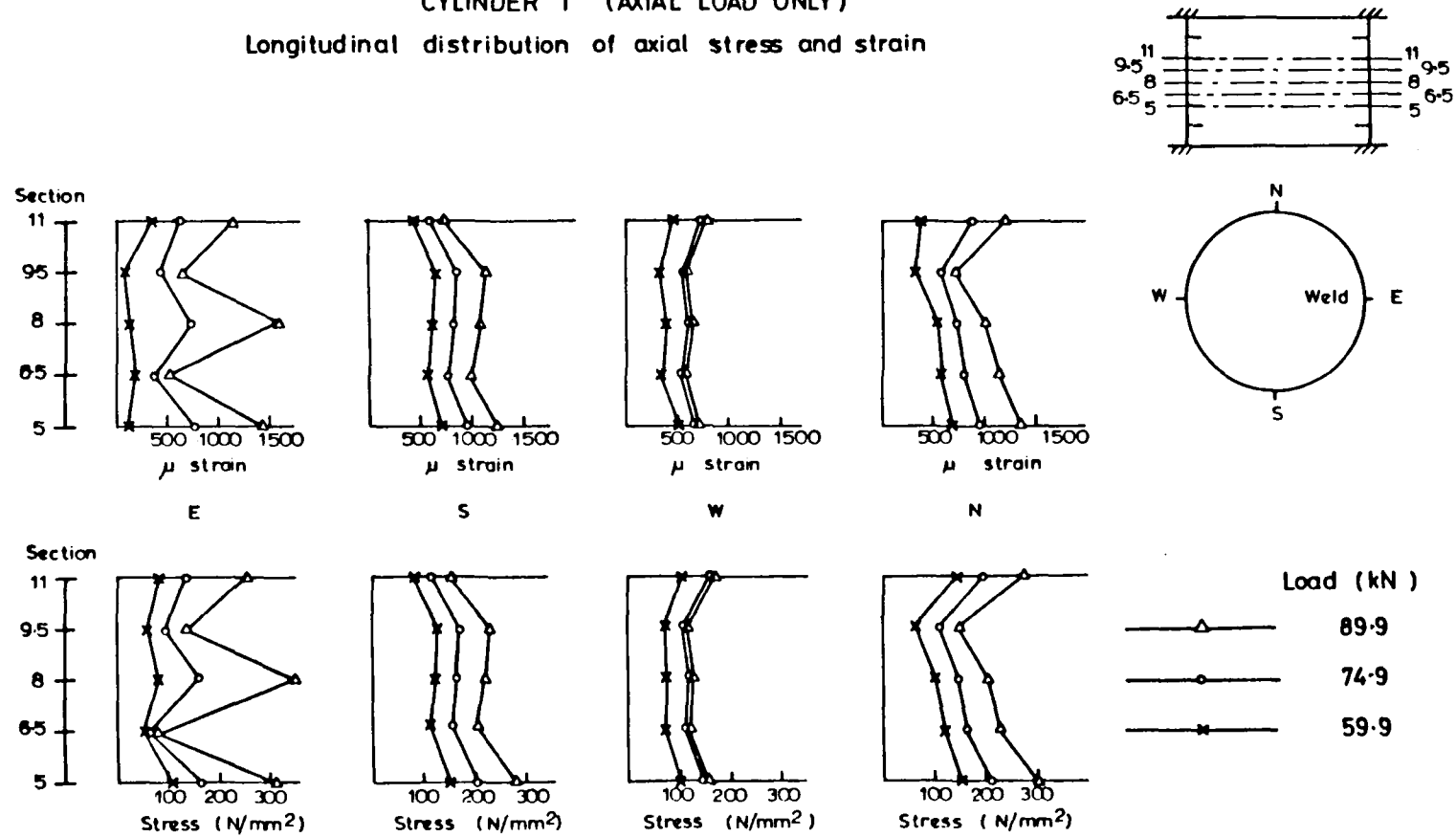


Fig. 3.53

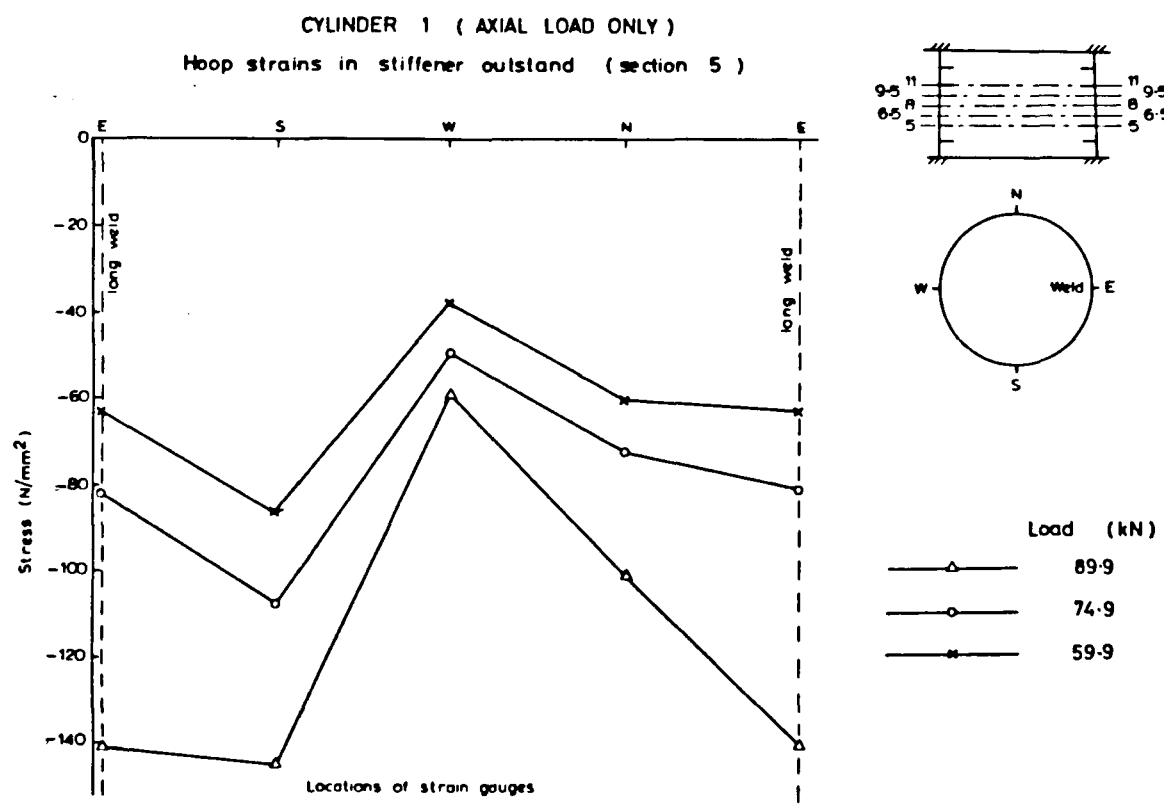


Fig. 3.54

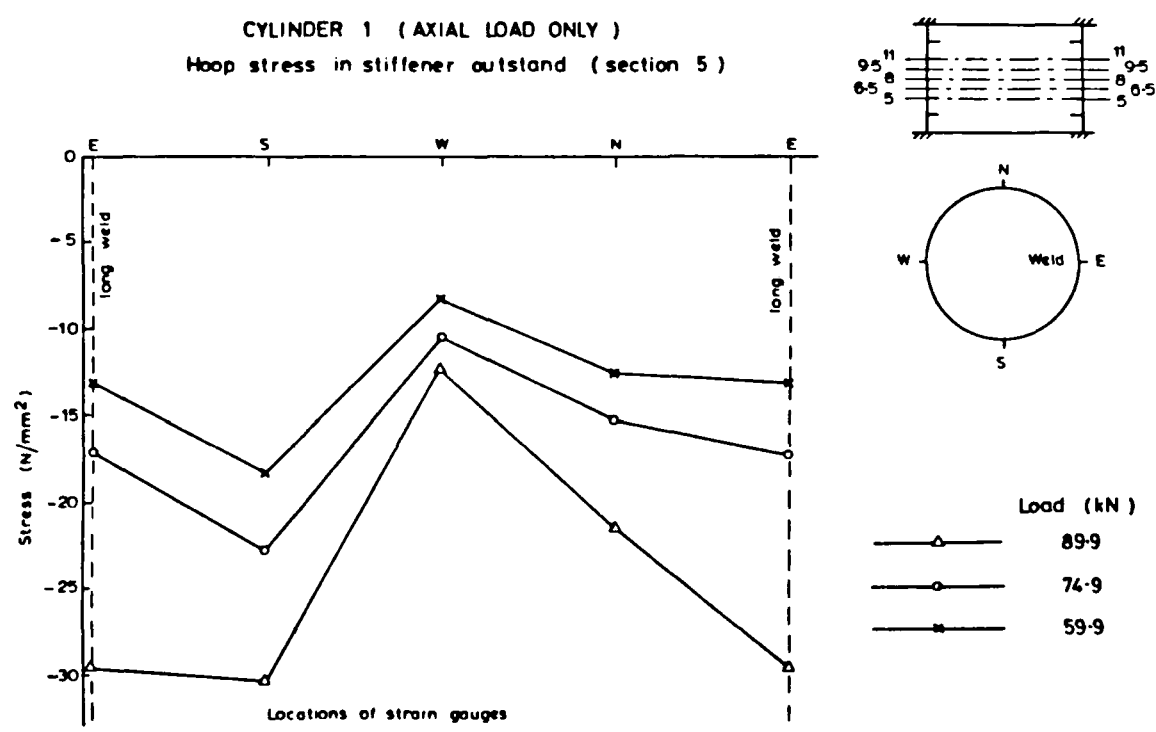


Fig. 3.55

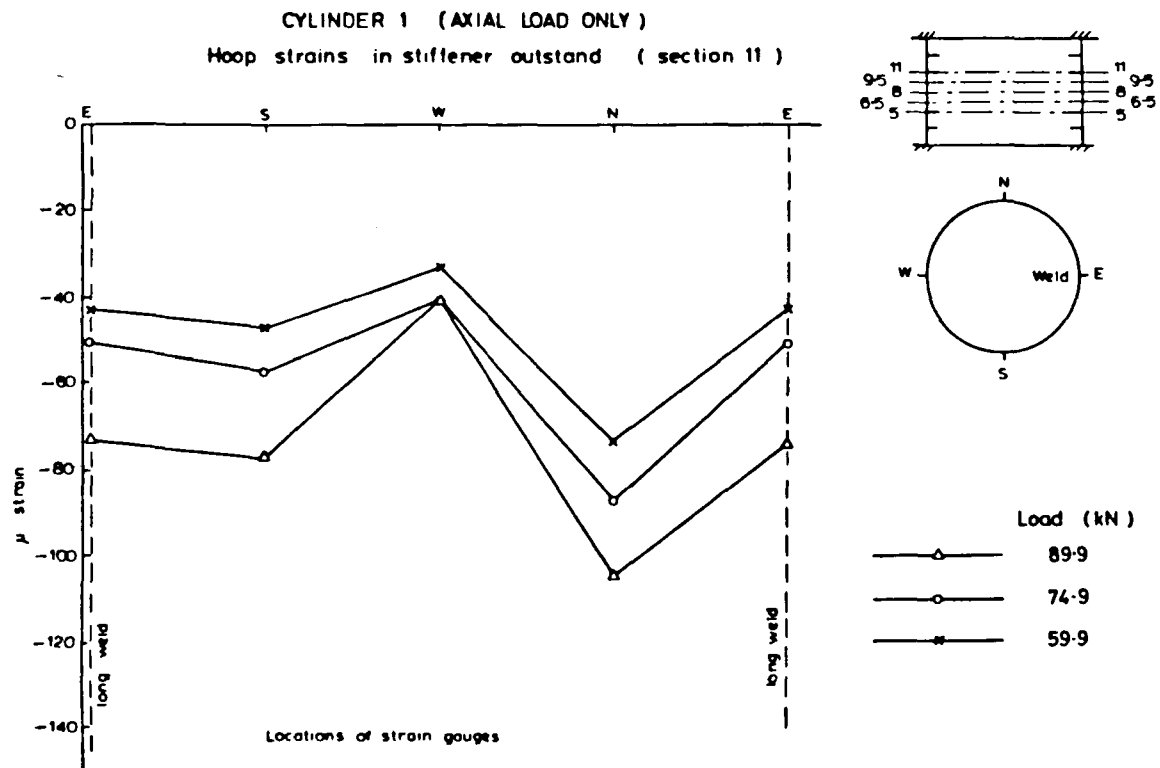


Fig. 3.56

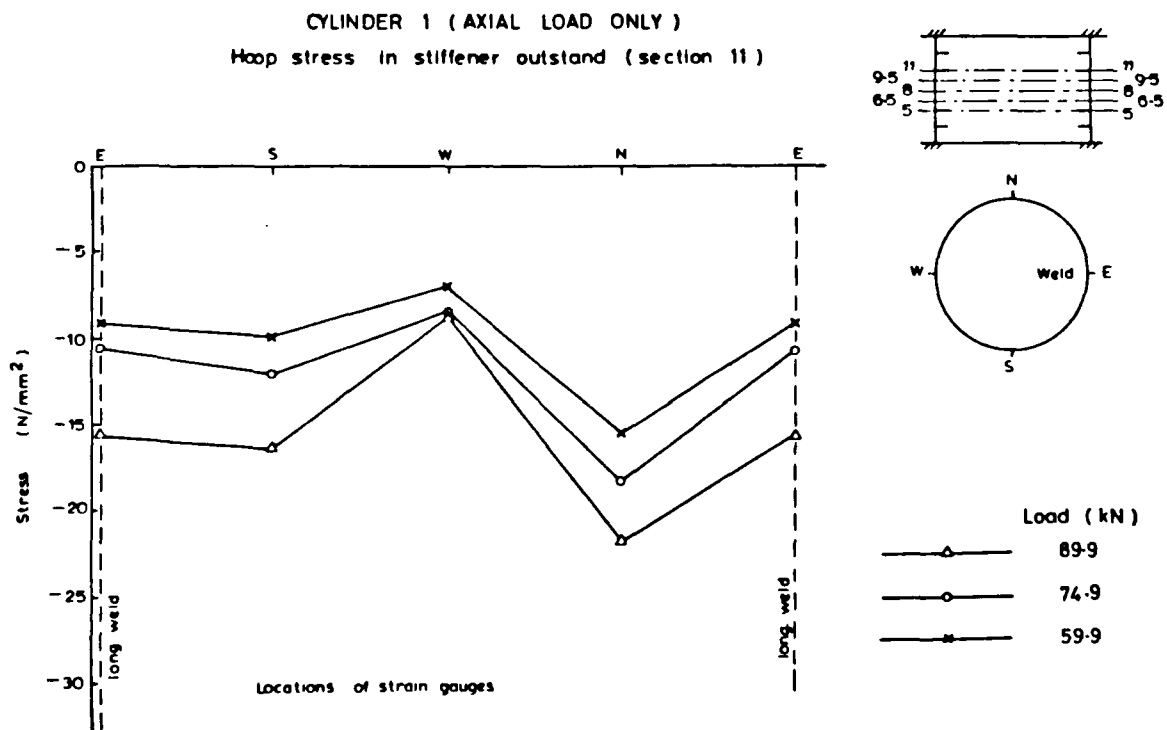


Fig. 3.57

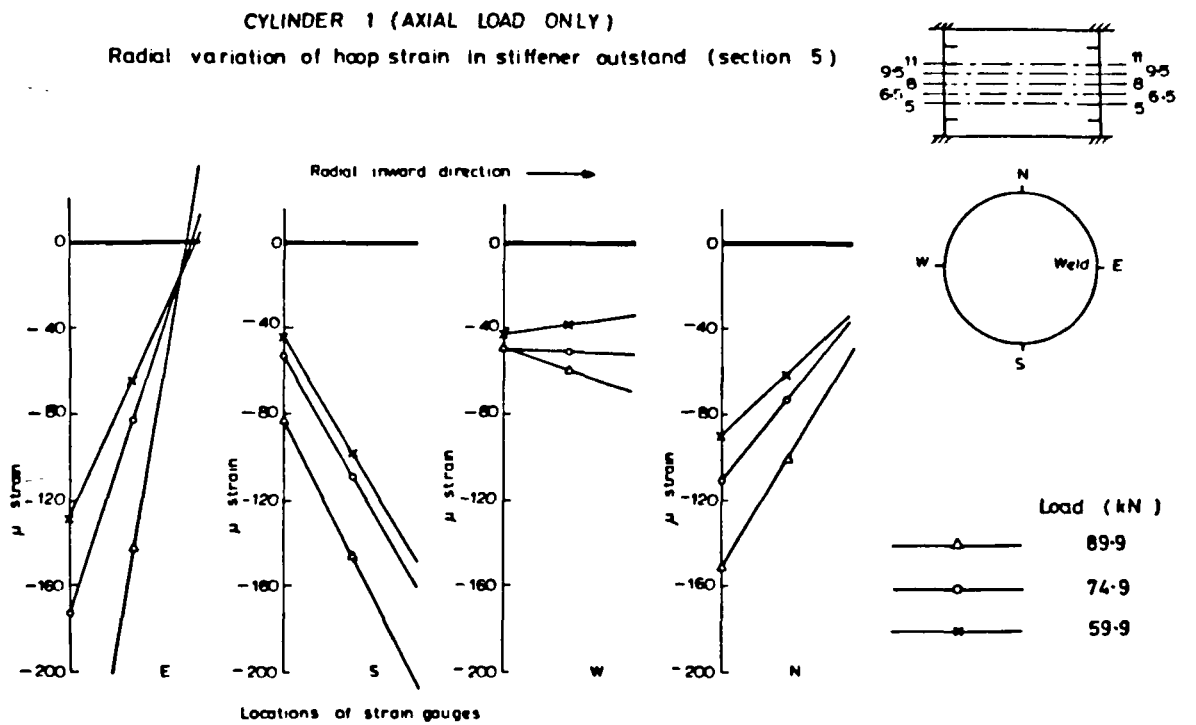


Fig. 3.58

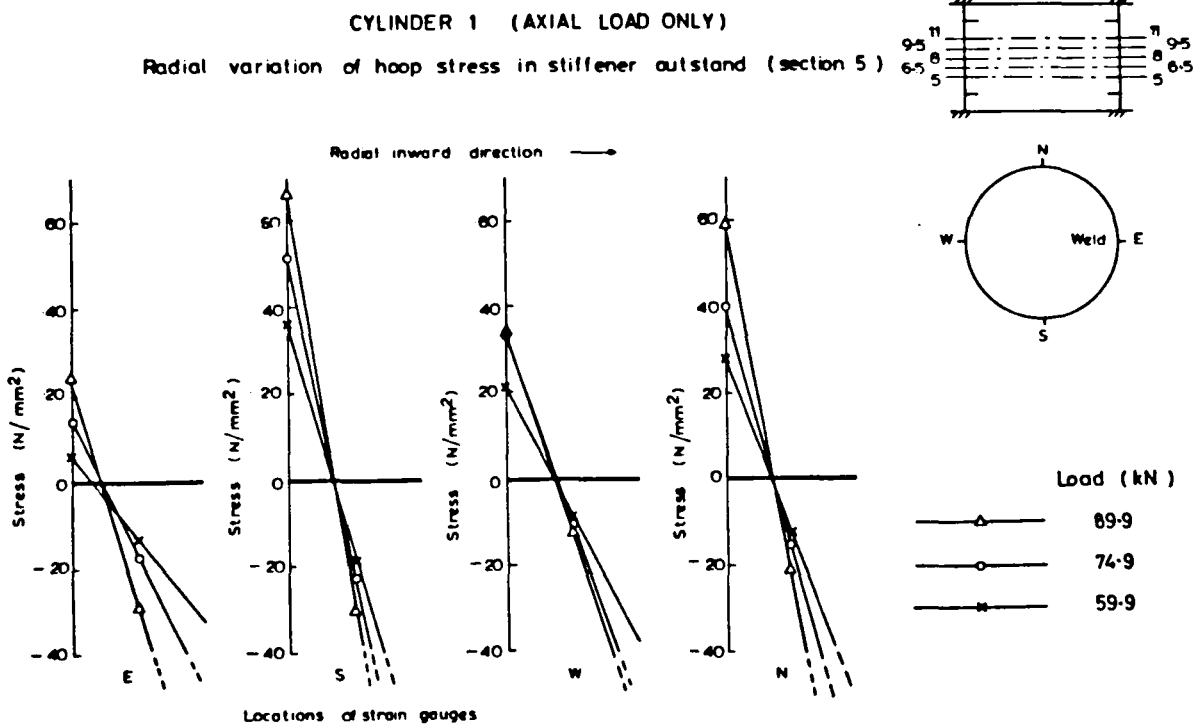


Fig. 3.59

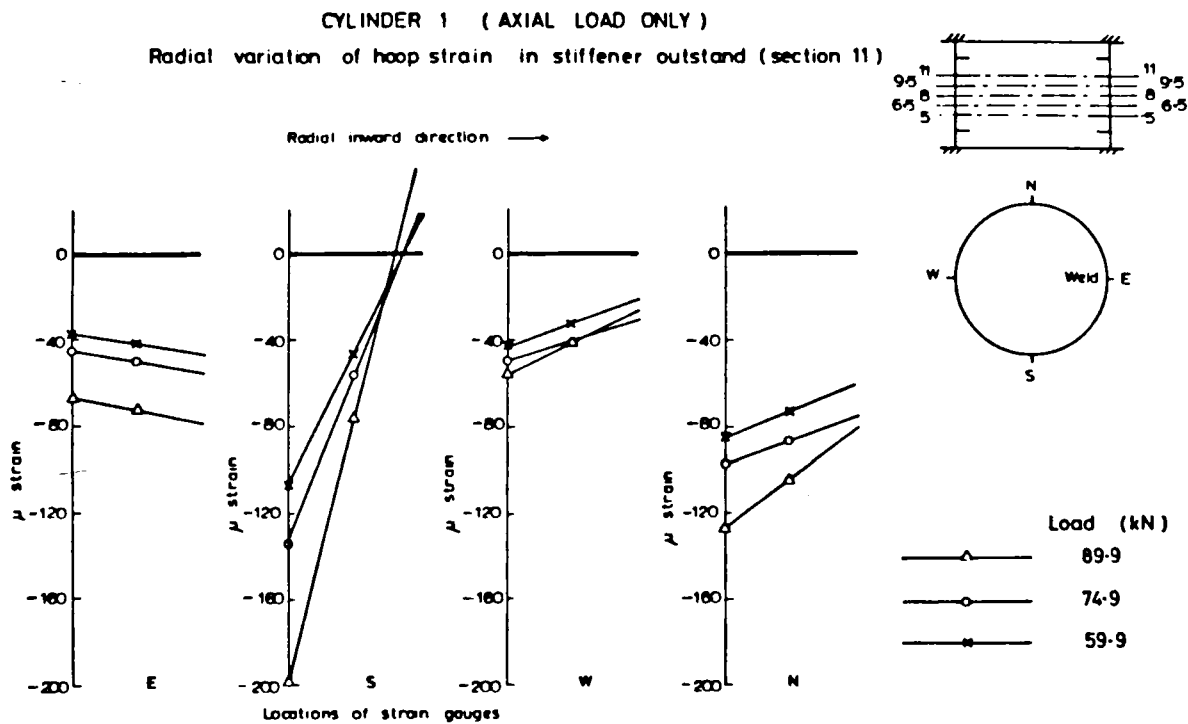


Fig. 3.60

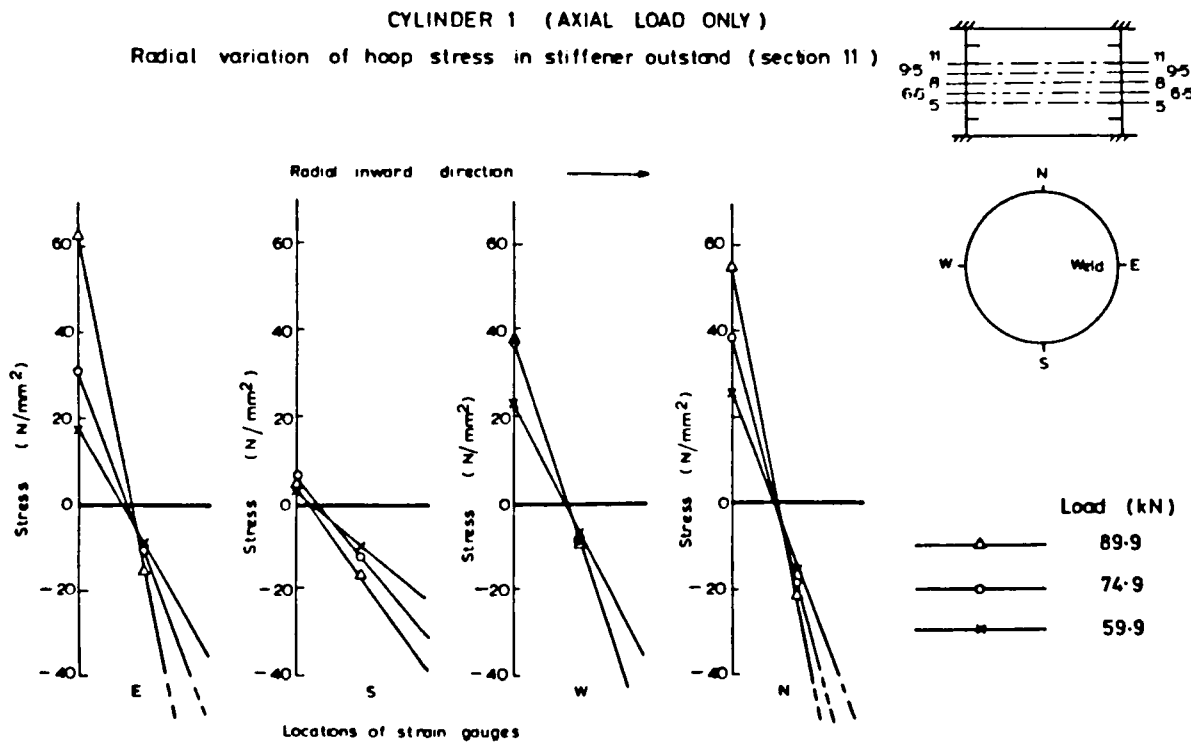


Fig. 3.61

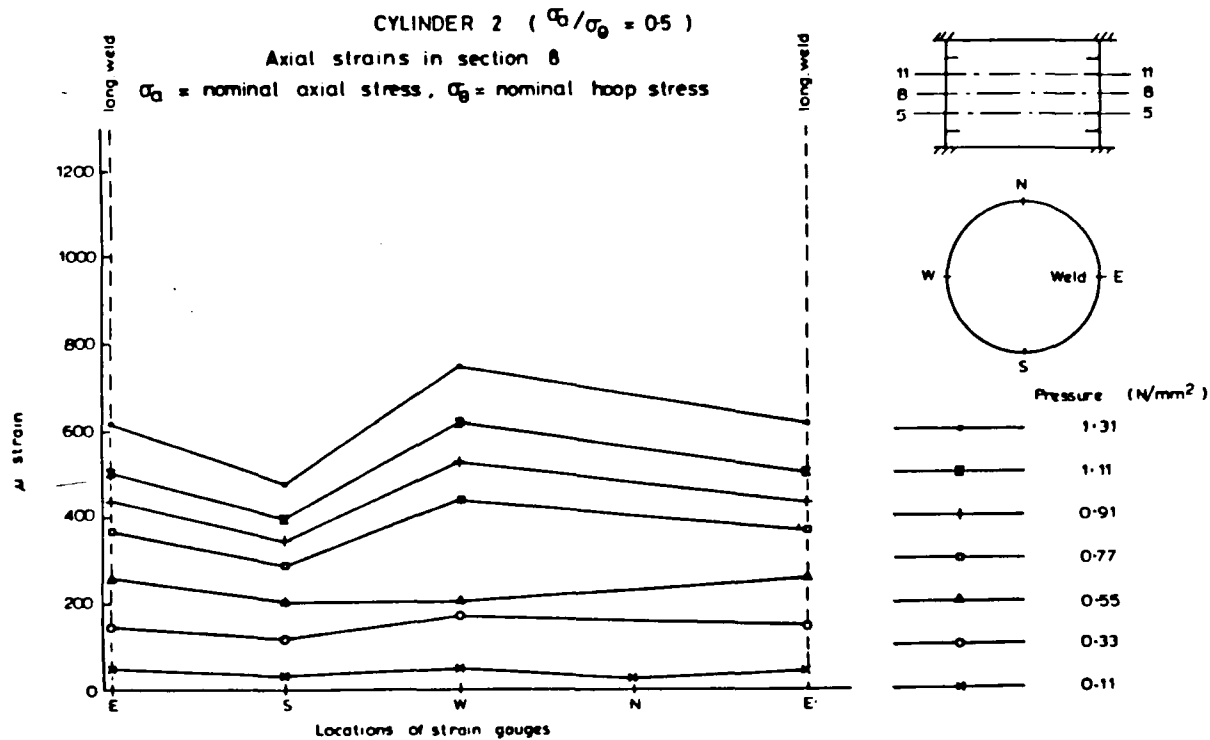


Fig. 3.62

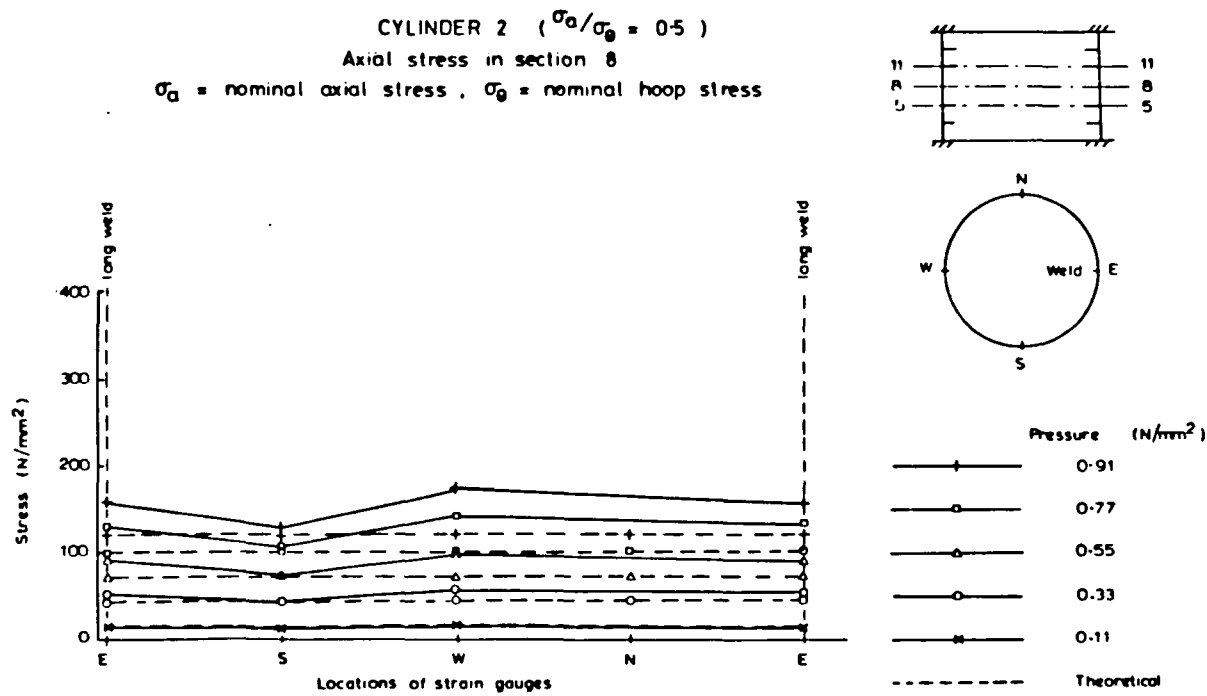


Fig. 3.63

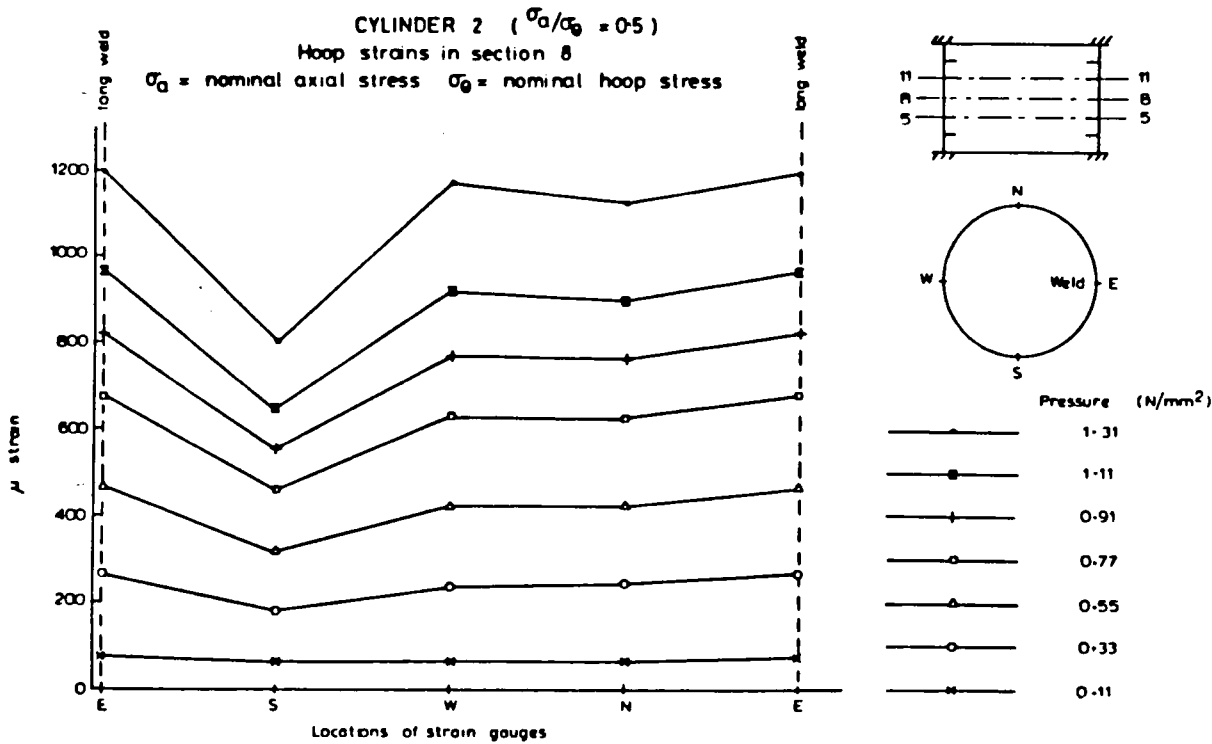


Fig. 3.64

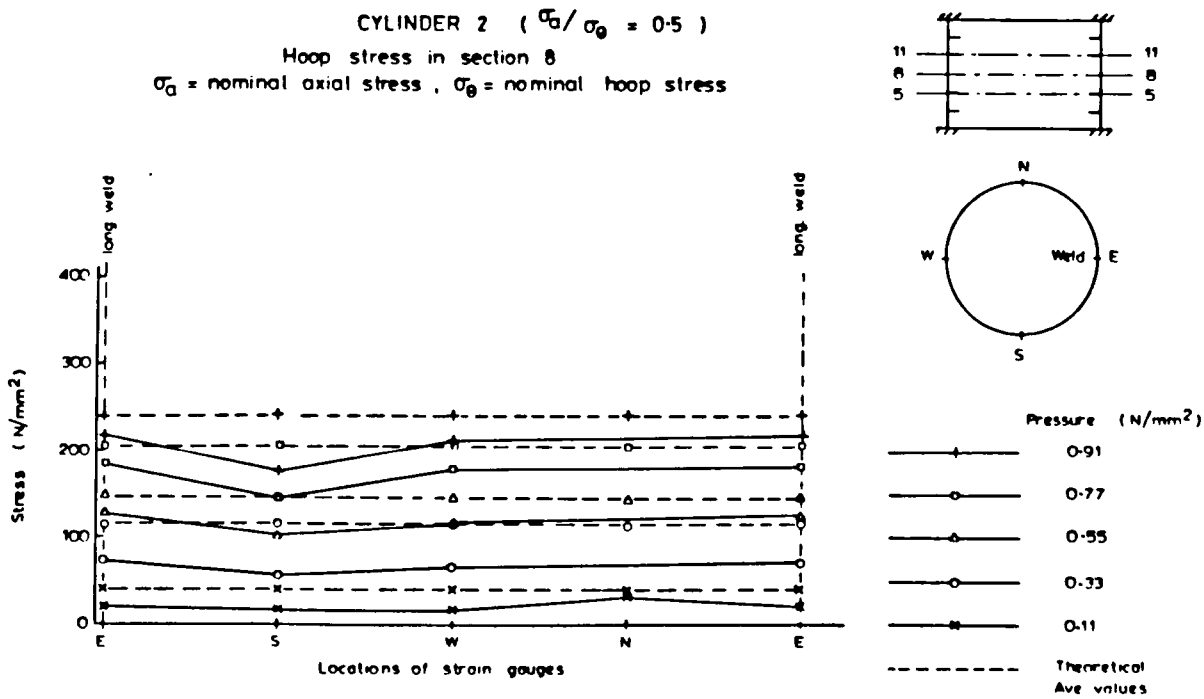


Fig. 3.65

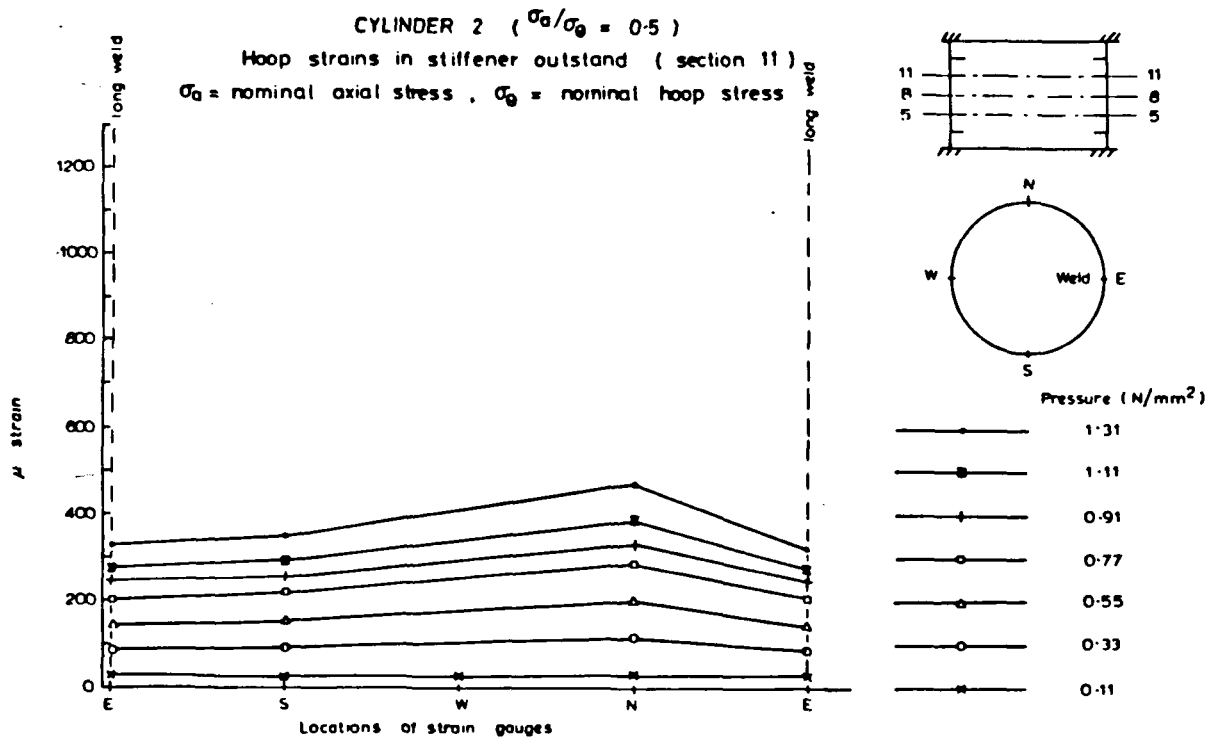


Fig. 3.66

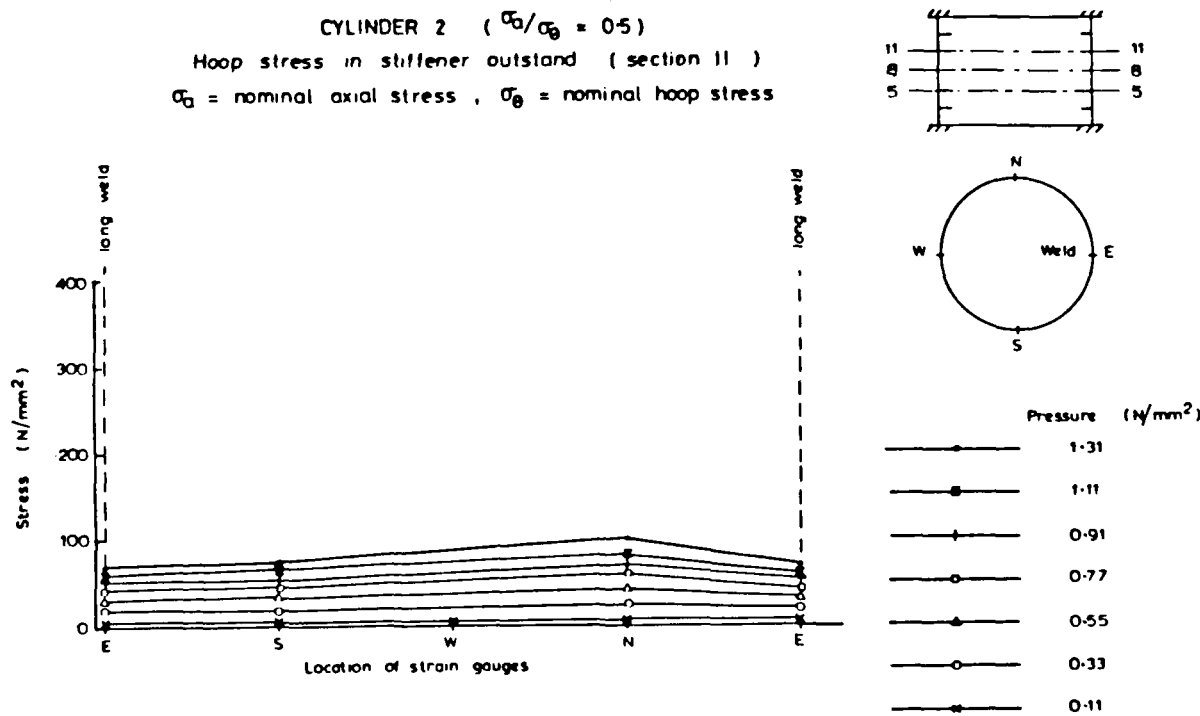


Fig. 3.67

CYLINDER 2 ($\sigma_a/\sigma_\theta = 0.5$)
 Radial variation of hoop stress and hoop strain in stiffener outstand
 (section 11)
 σ_a = nominal axial stress , σ_θ = nominal hoop stress

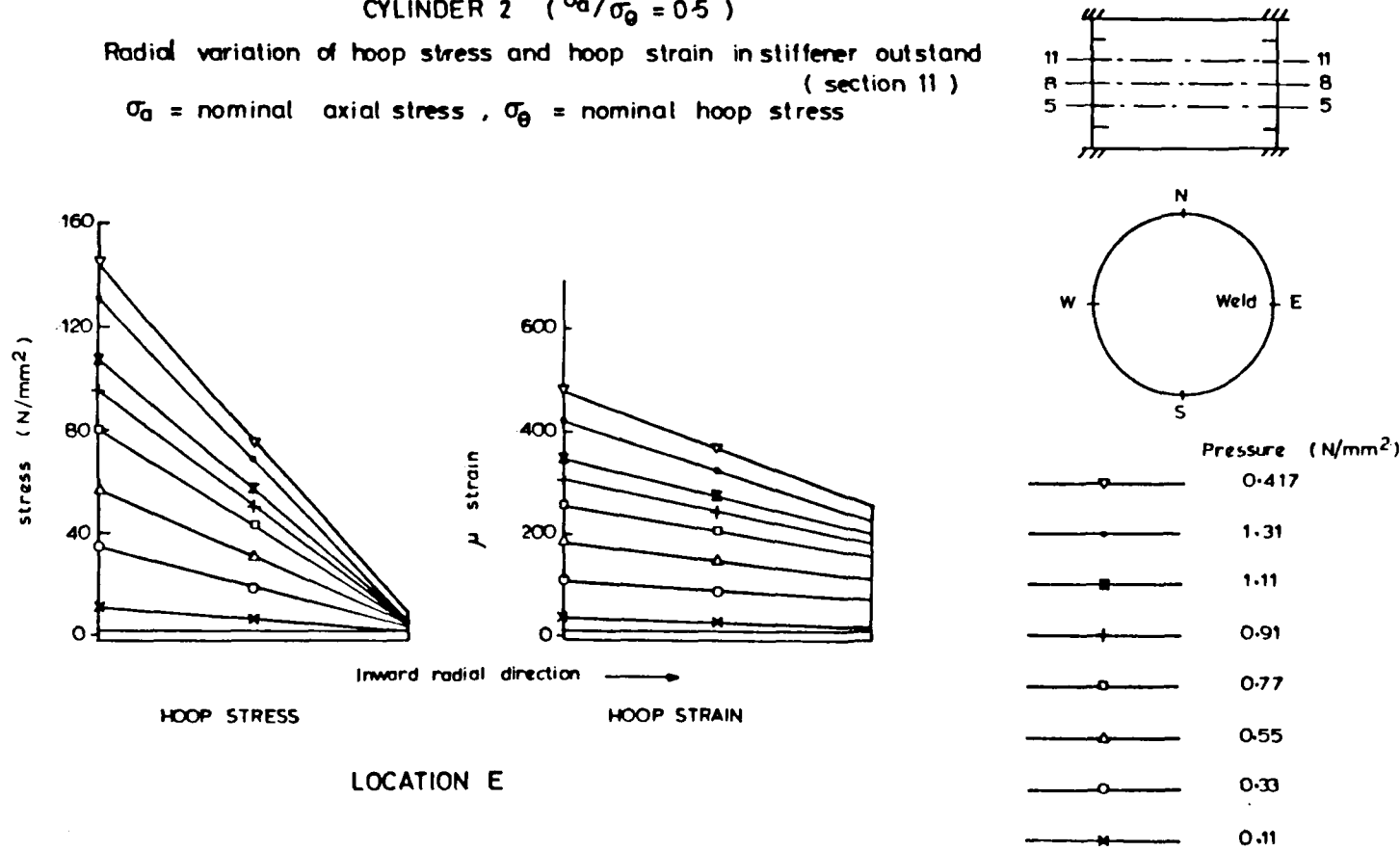


Fig. 3.68

CYLINDER 2

Plot of nominal axial stress vs. average axial strain

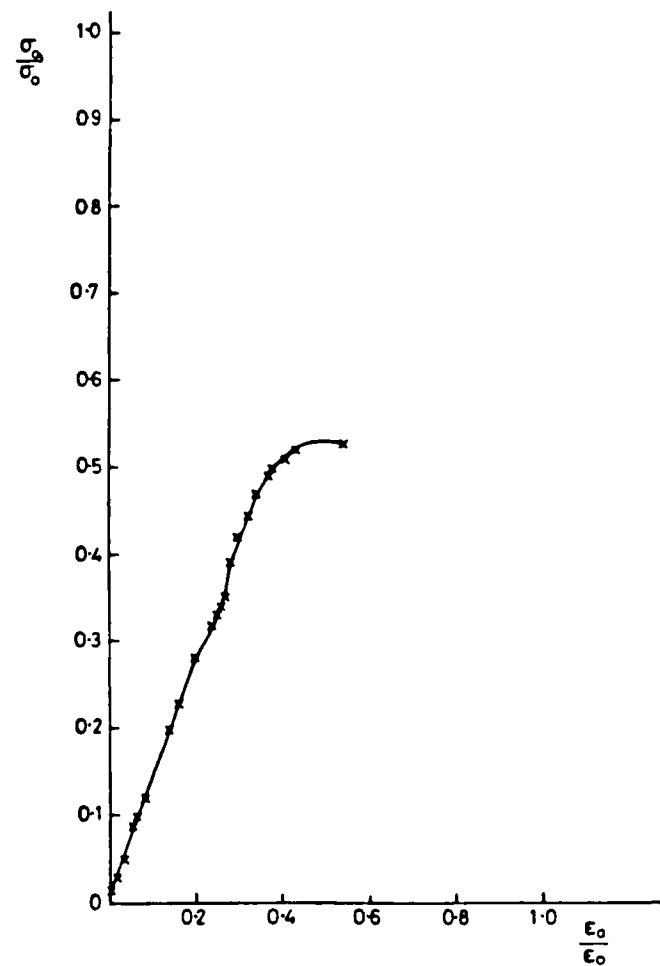


Fig. 3.69

CYLINDER 2

Plot of nominal hoop stress vs. average hoop strain

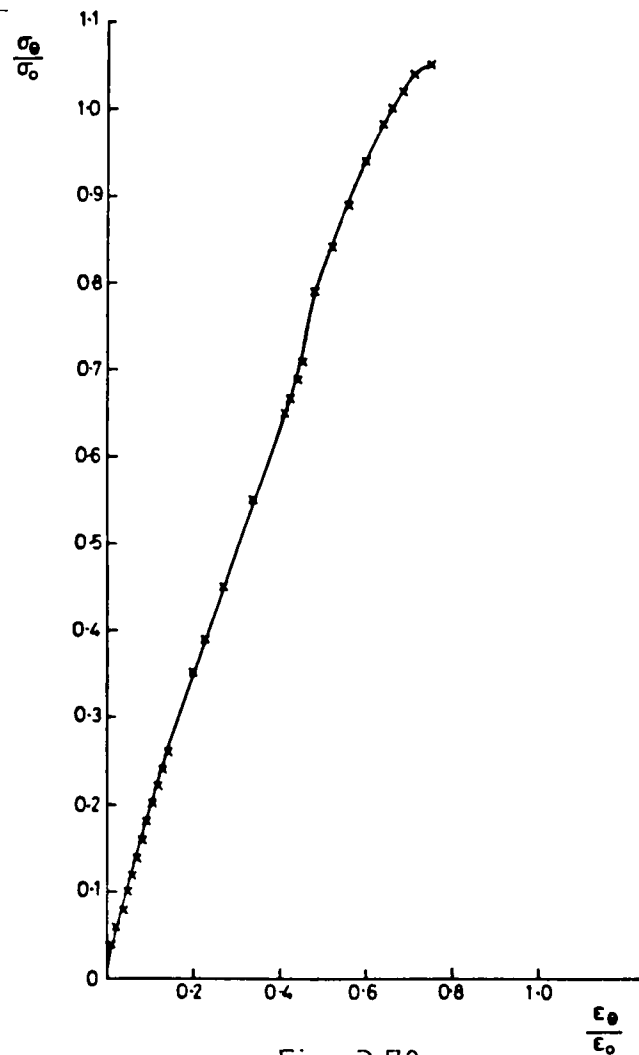


Fig. 3.70

CYLINDER 2

Plot of equivalent stress vs. equivalent strain

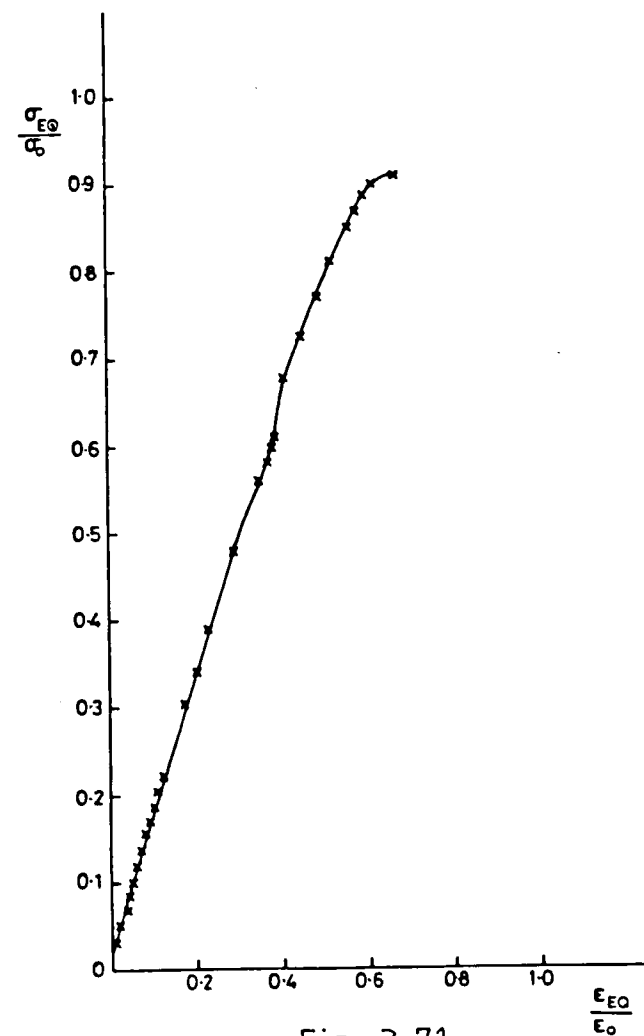


Fig. 3.71

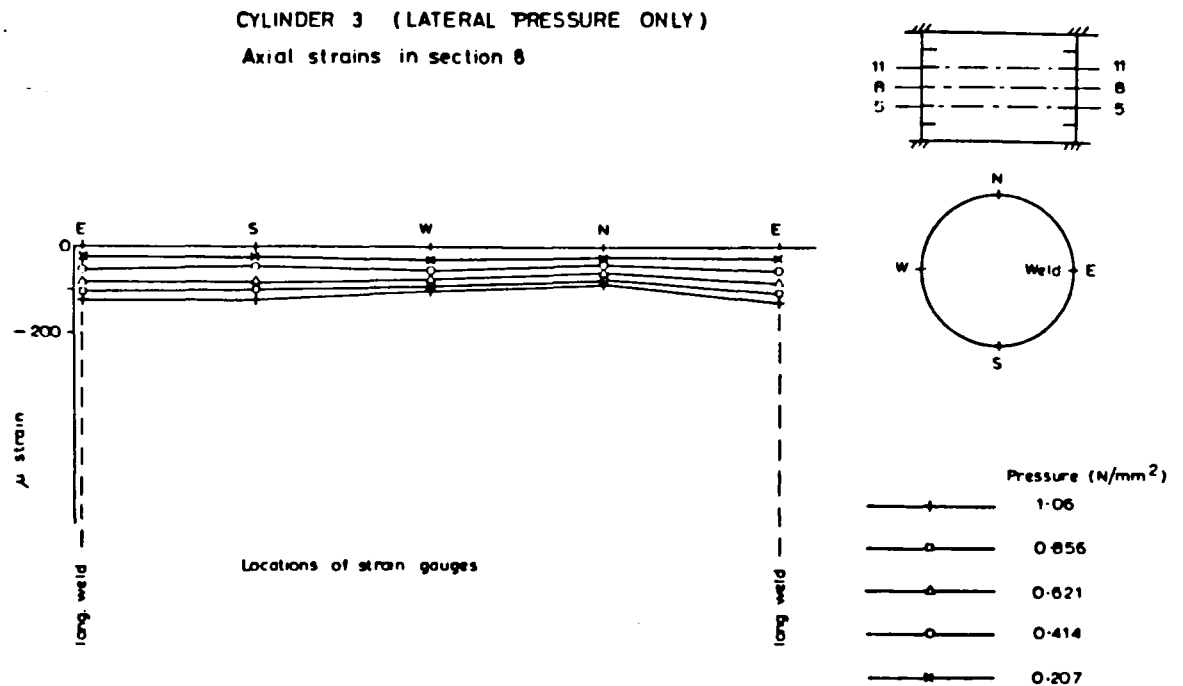


Fig. 3.72

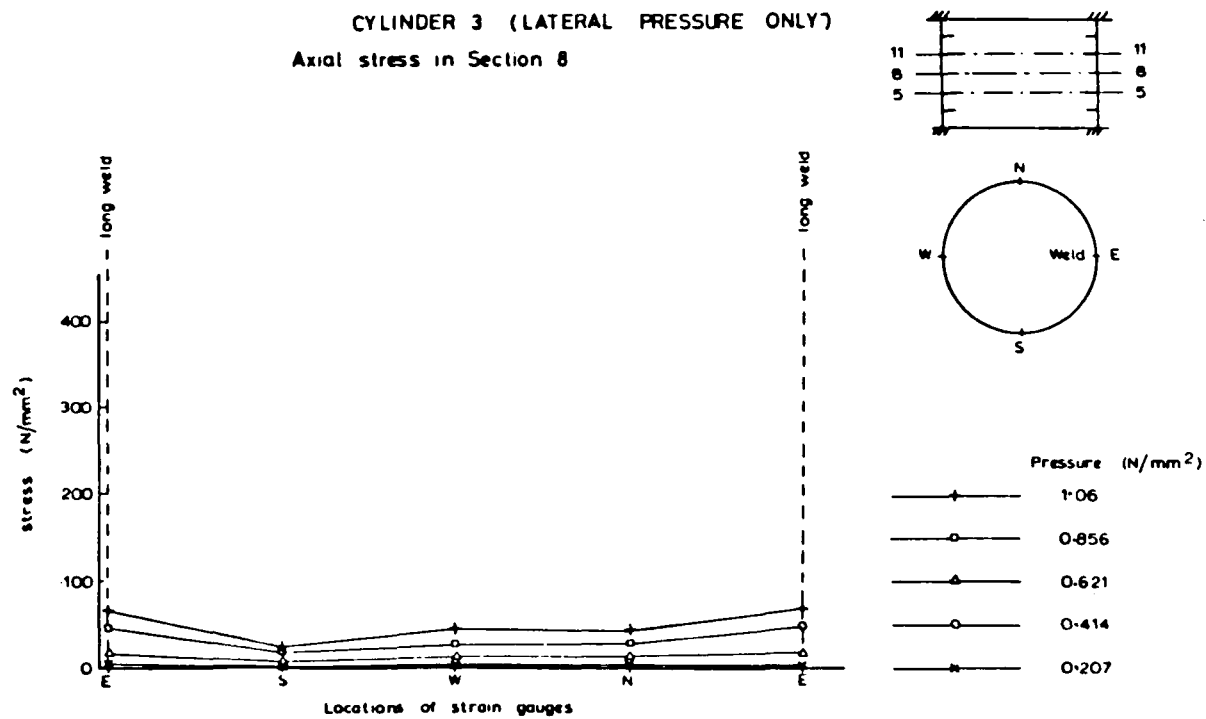


Fig. 3.73

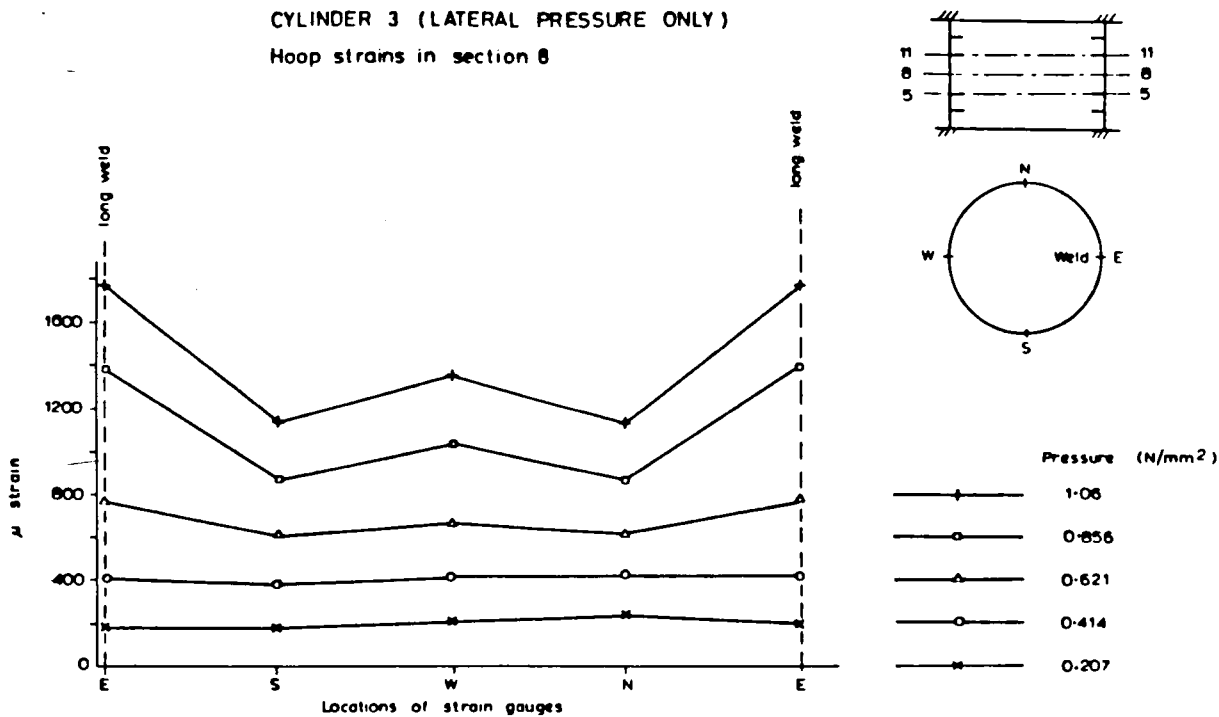


Fig. 3.74

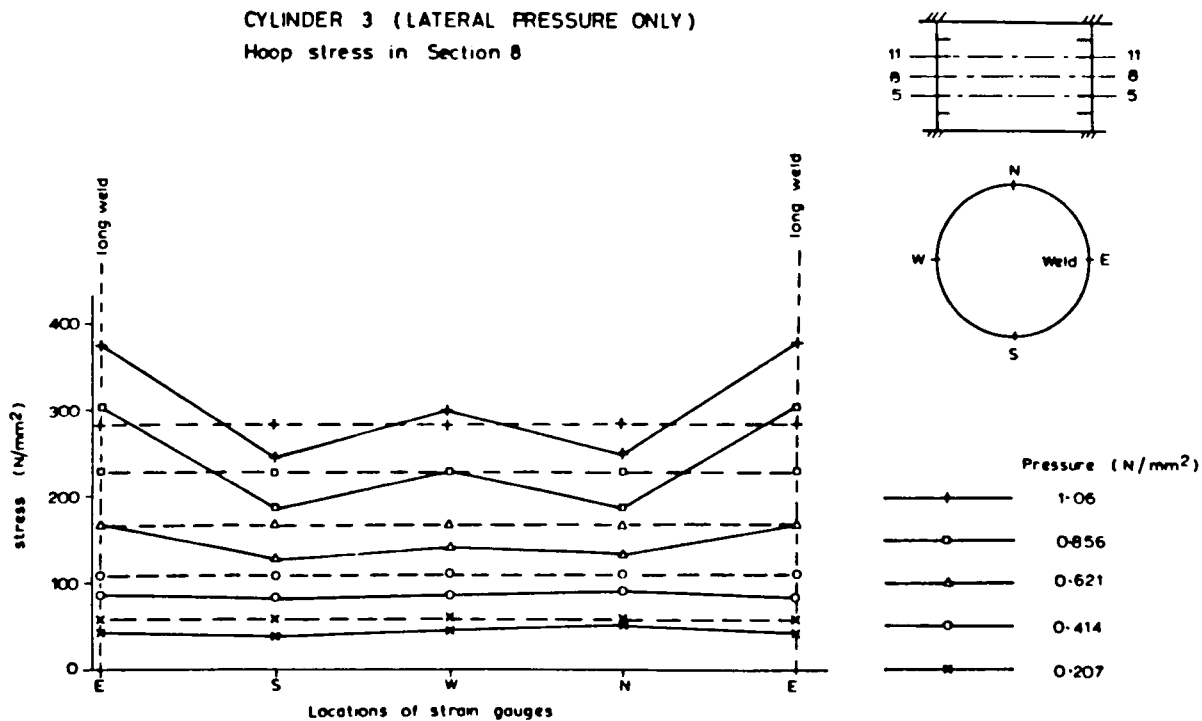


Fig. 3.75

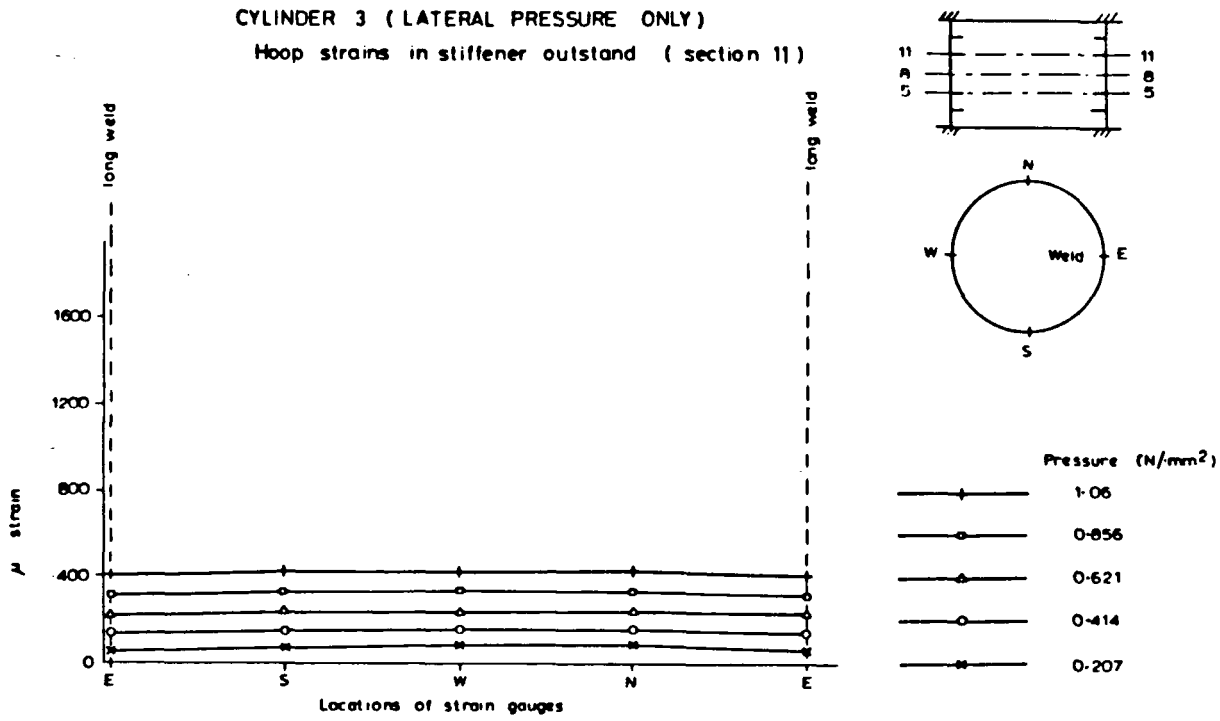


Fig. 3.76

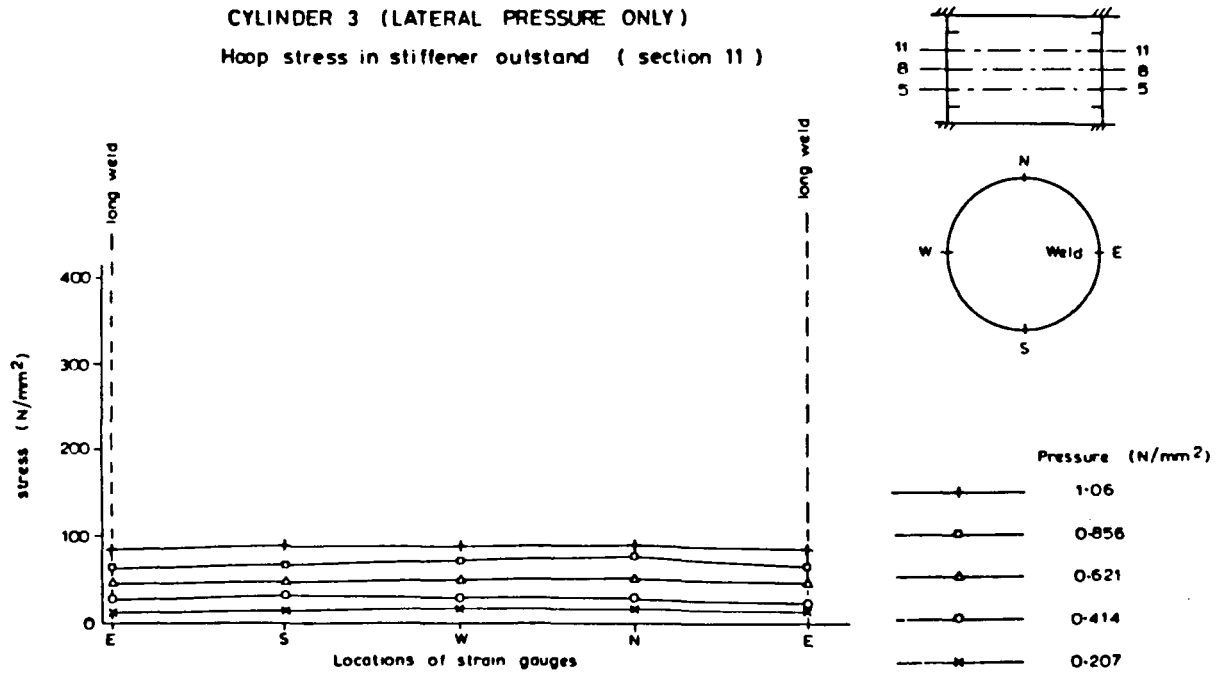


Fig. 3.77

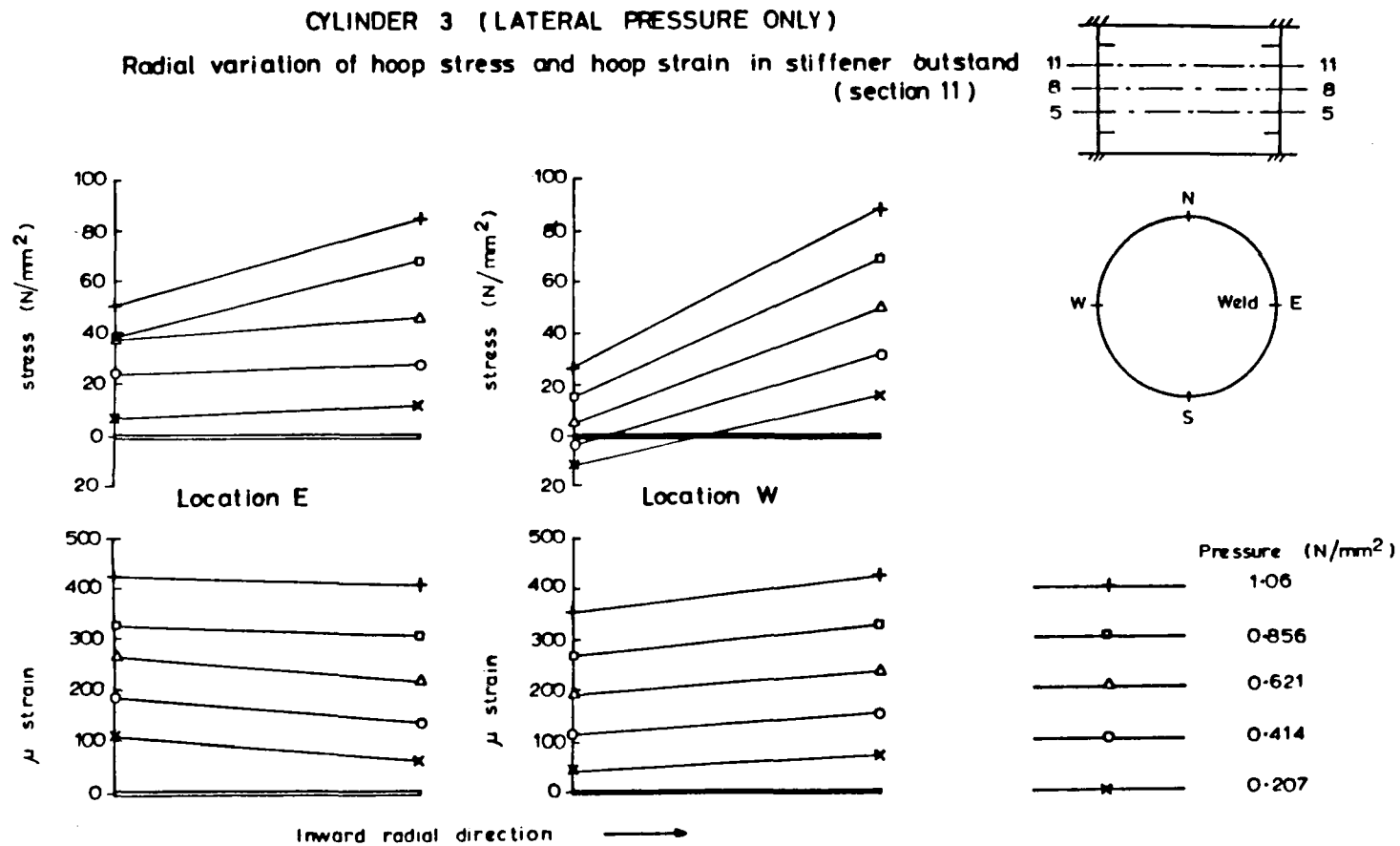


Fig. 3.78

CYLINDER 3

Plot of nominal hoop stress vs. average hoop strain

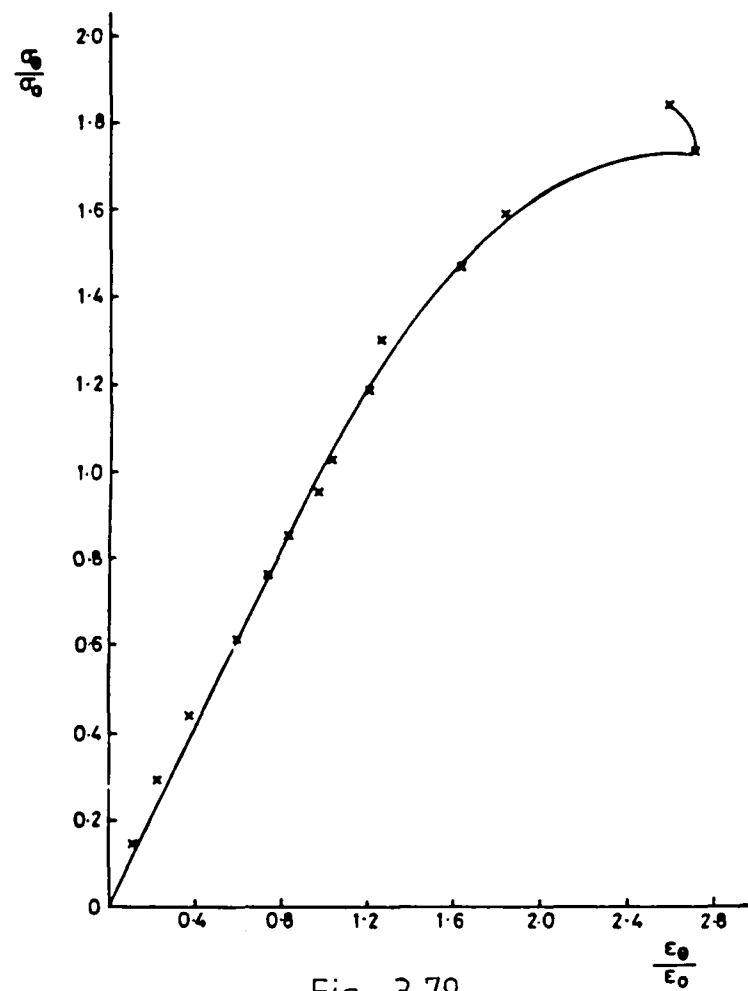


Fig. 3.79

CYLINDER 3

Plot of equivalent stress vs. equivalent strain

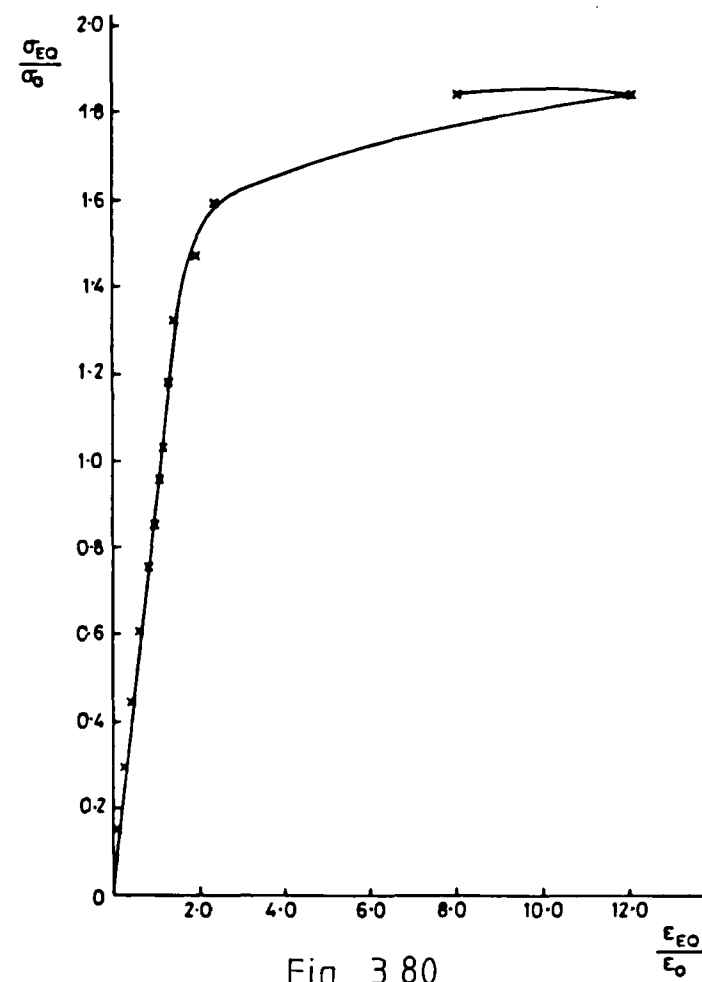


Fig. 3.80

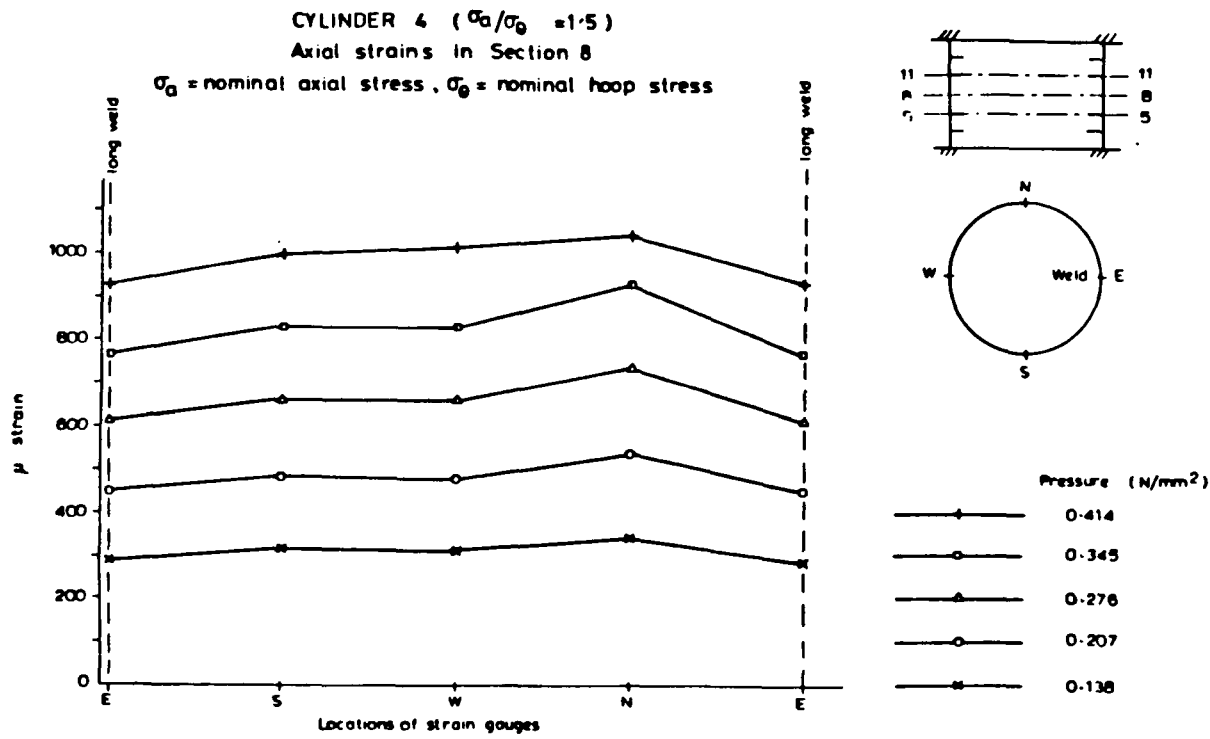


Fig. 3.81

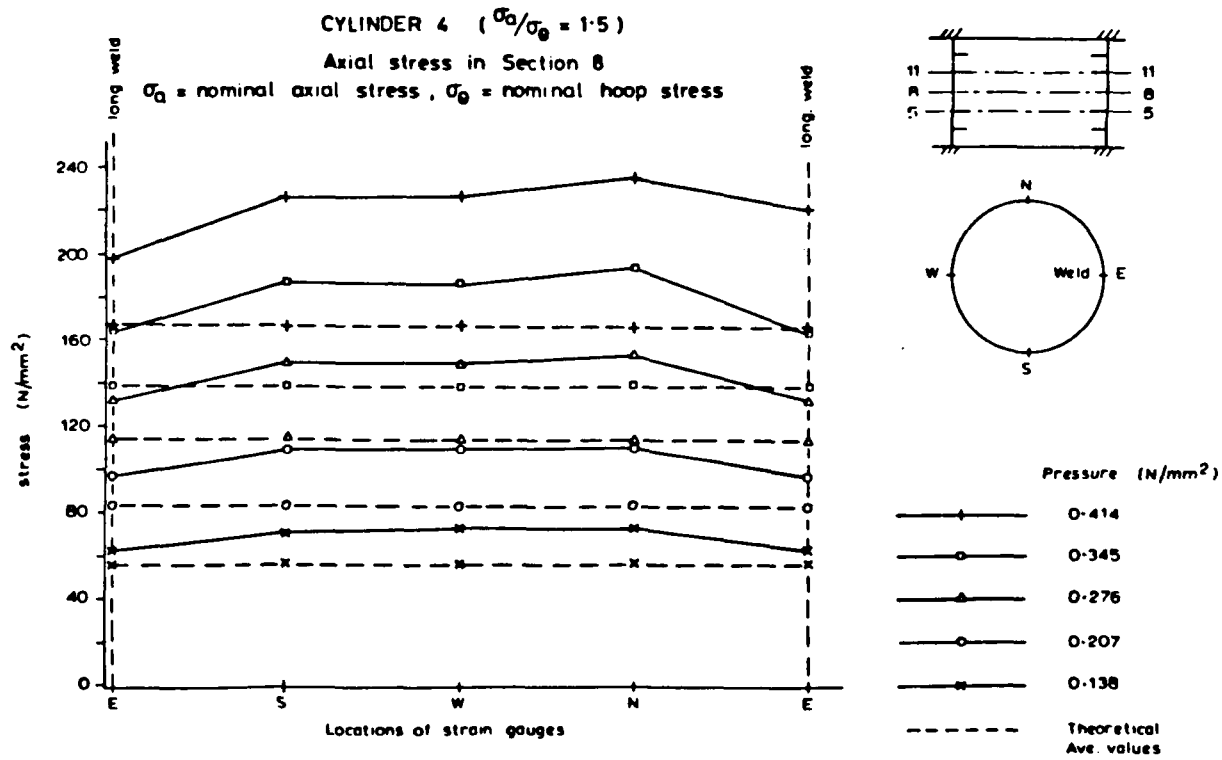


Fig. 3.82

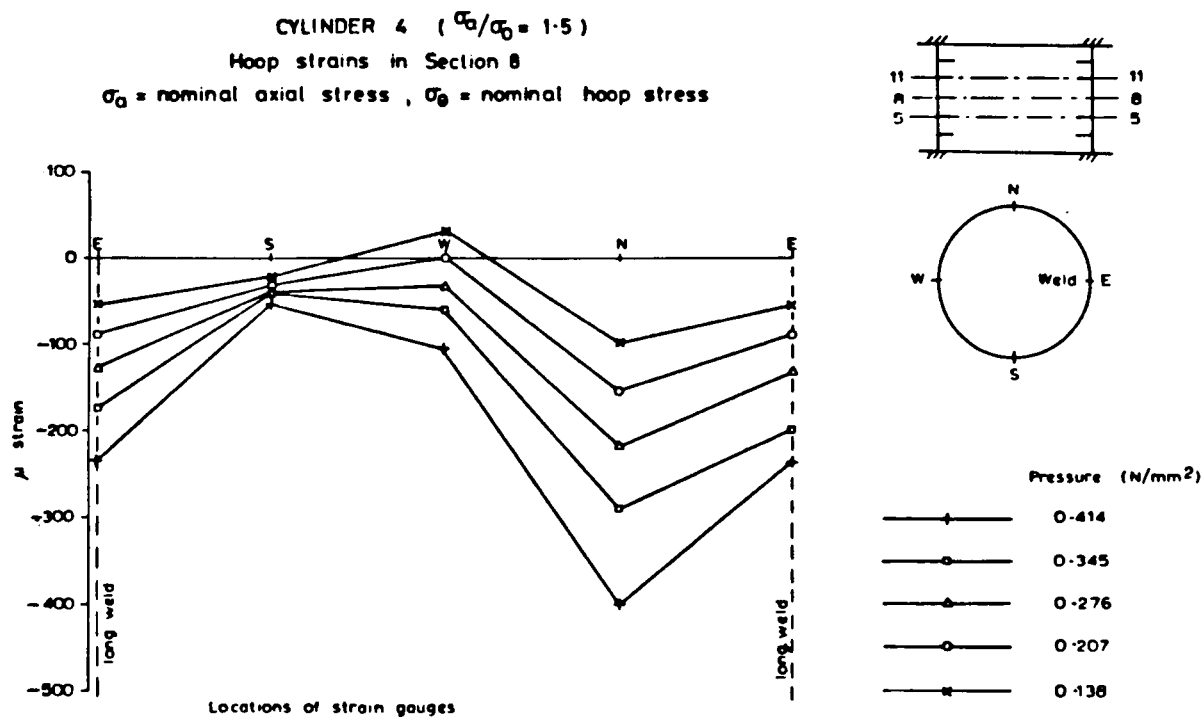


Fig. 3.83

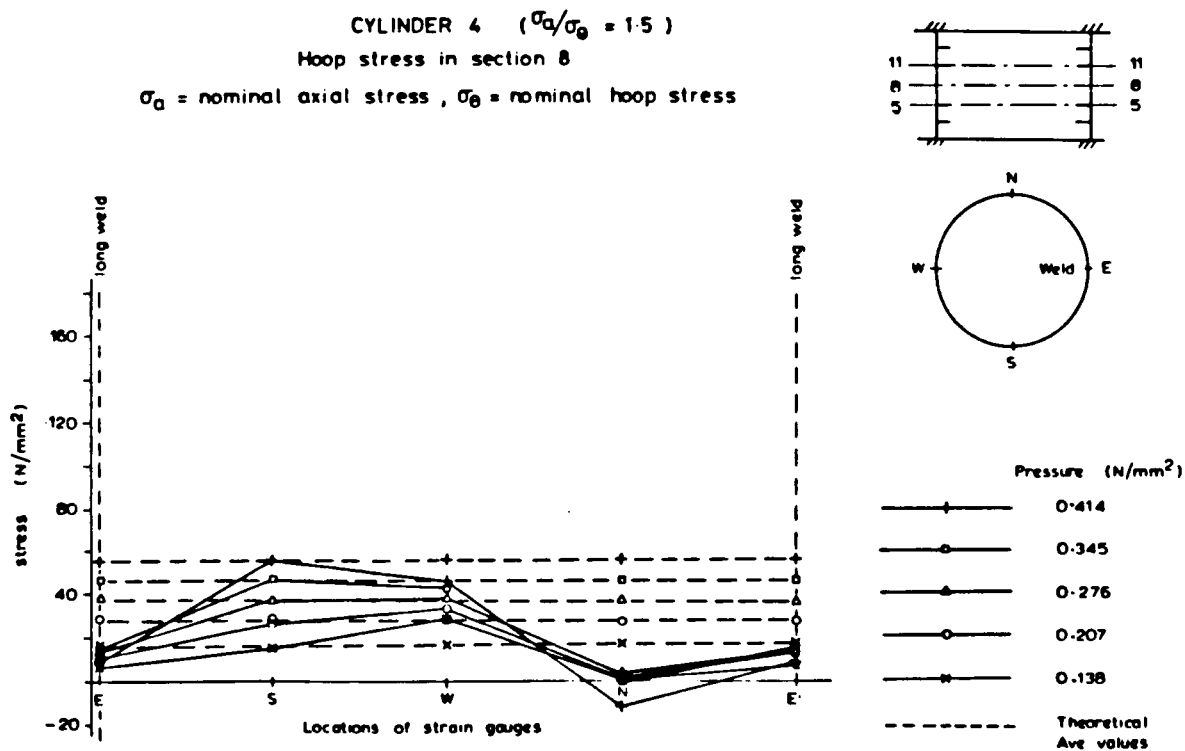


Fig. 3.84

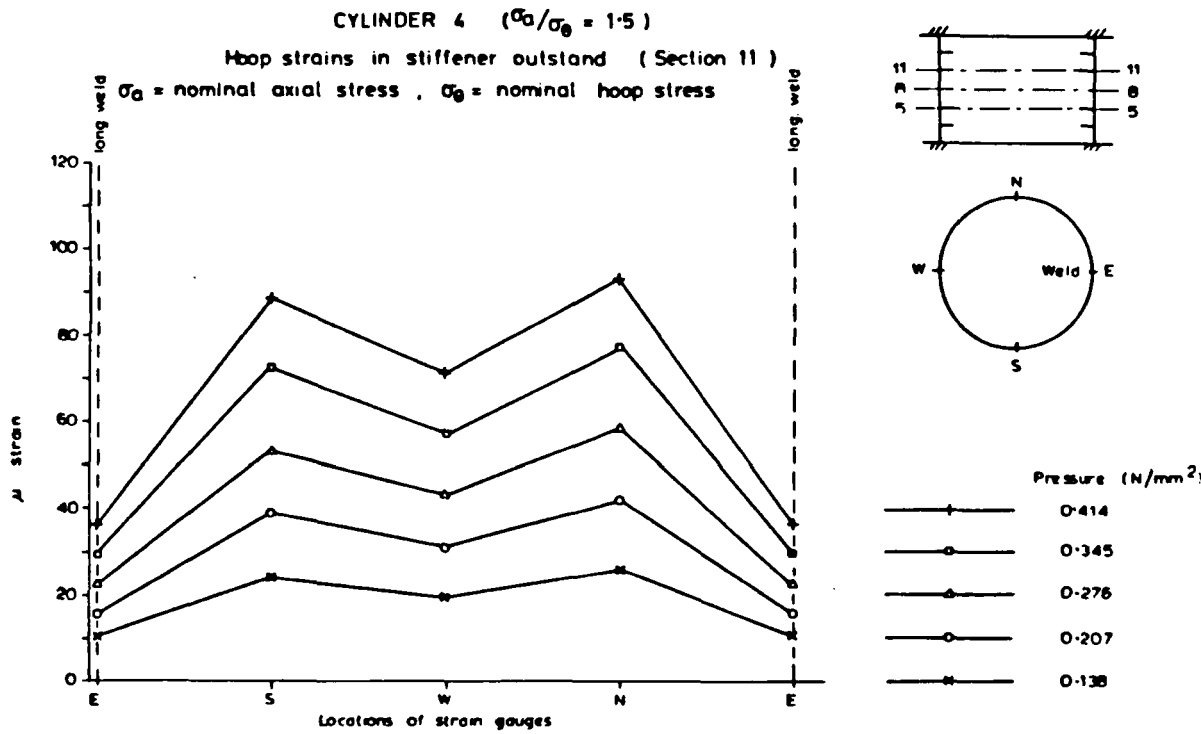


Fig. 3.85

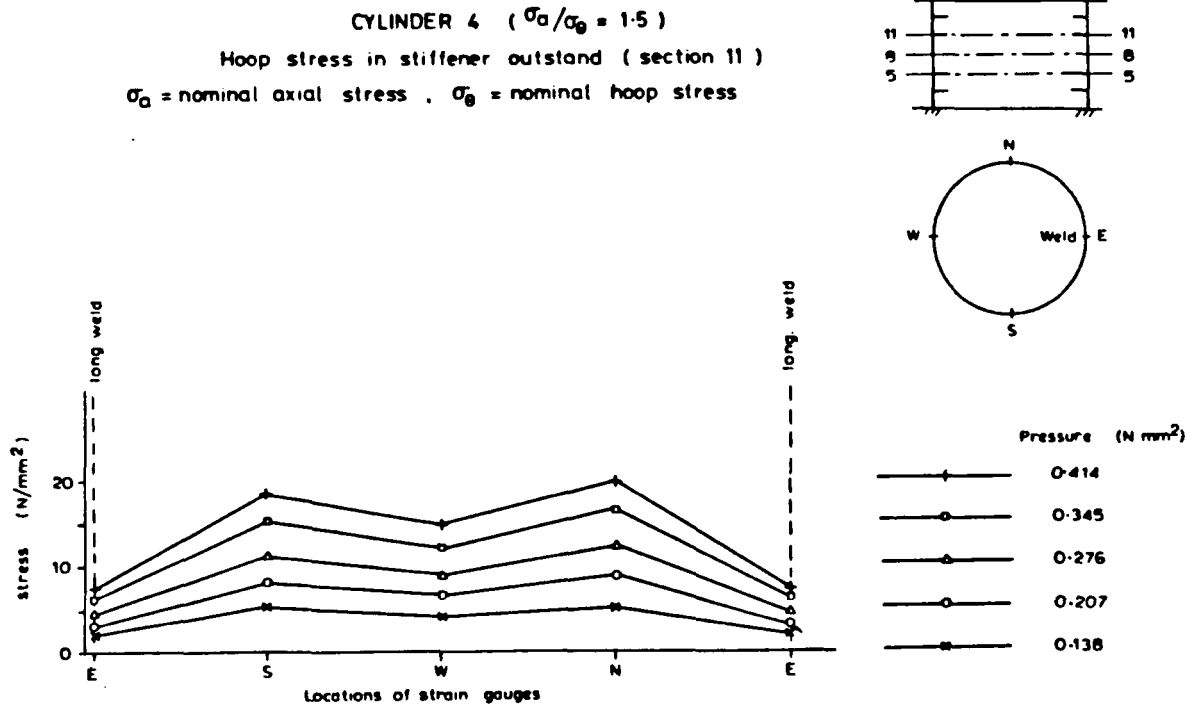
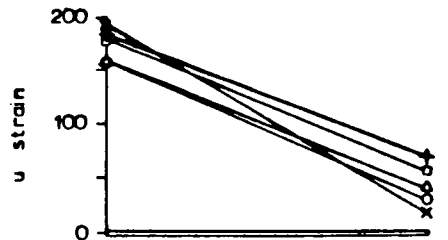
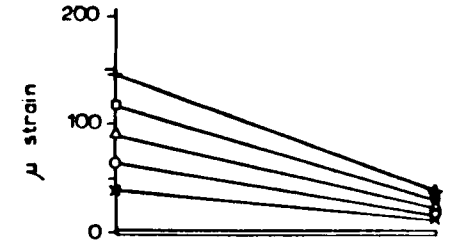
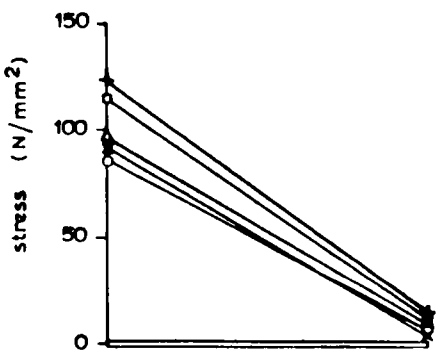
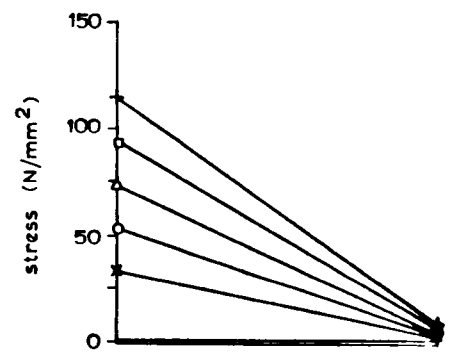
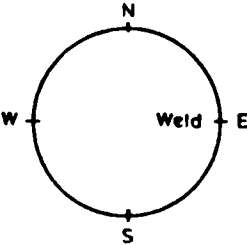
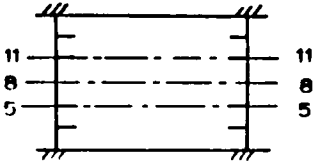


Fig. 3.86

CYLINDER 4 ($\sigma_a/\sigma_\theta = 1.5$)

Radial variation of hoop stress and hoop strain in stiffener outstand
(Section 11)
 σ_a = nominal axial stress , σ_θ = nominal hoop stress



	Pressure (N/mm ²)
— + —	0.414
— o —	0.345
— Δ —	0.276
— □ —	0.207
— x —	0.138

Fig. 3.87

CYLINDER 4

Plot of nominal axial stress vs. average axial strain

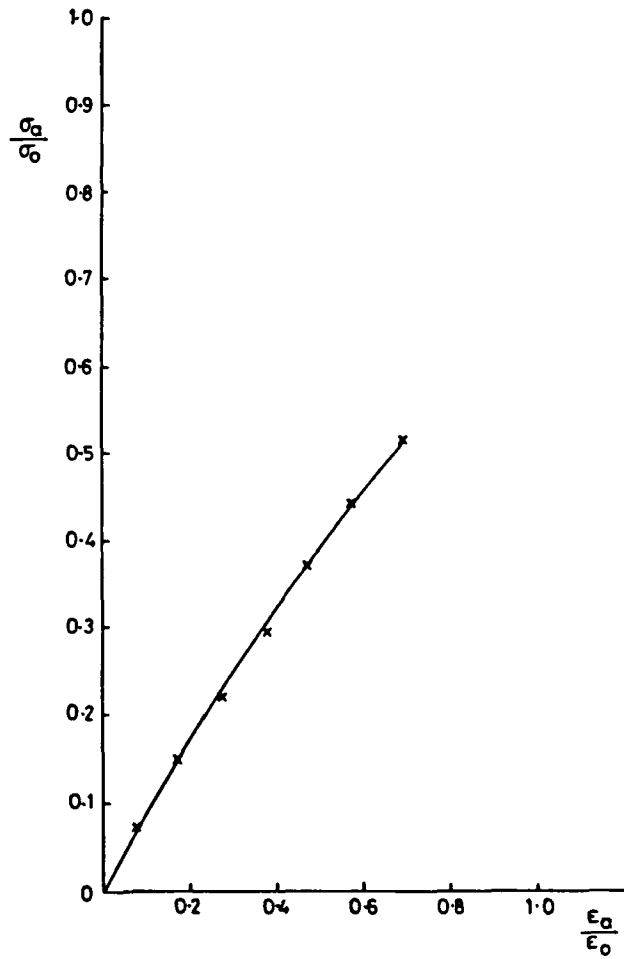


Fig. 3.88

CYLINDER 4

Plot of nominal hoop stress vs average hoop strain

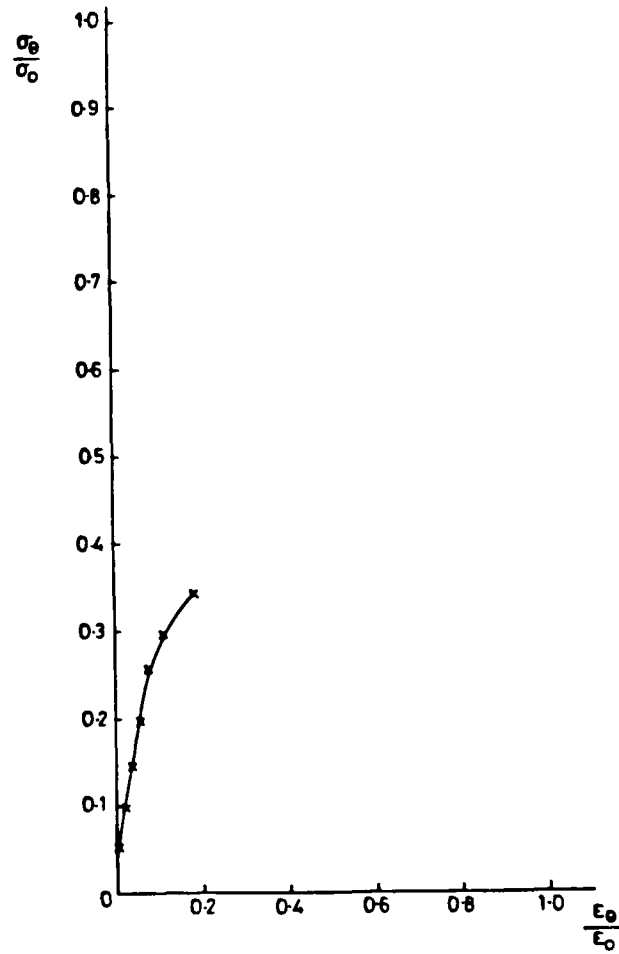


Fig. 3.89

CYLINDER 4

Plot of equivalent stress vs equivalent strain

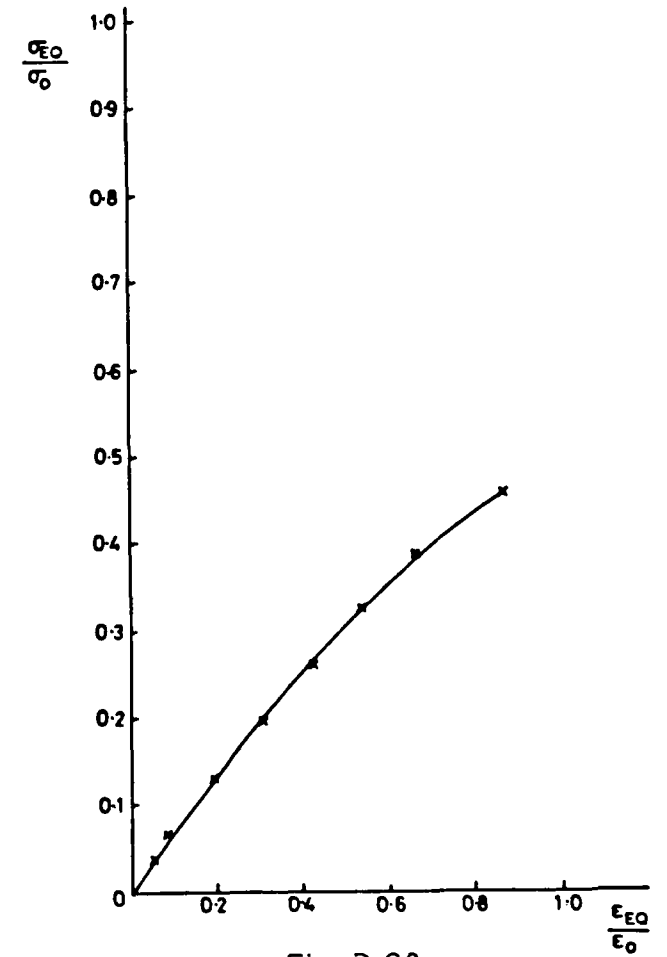


Fig. 3.90

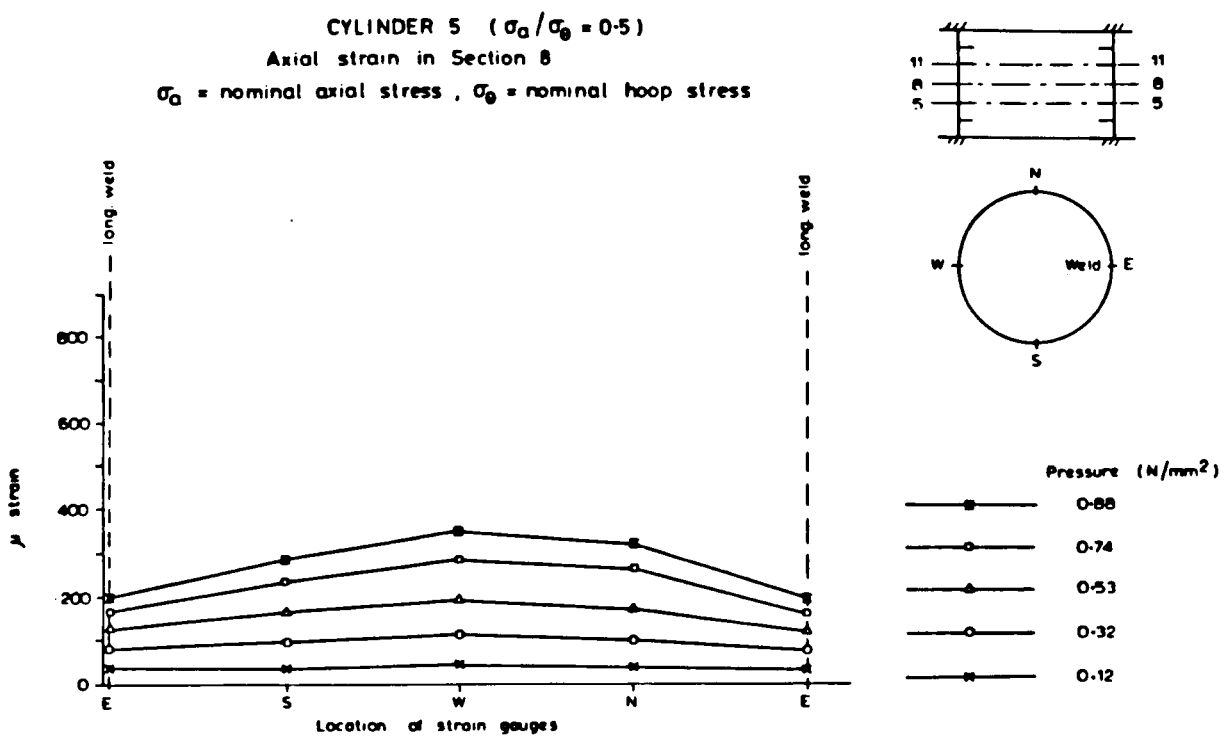


Fig. 3.91

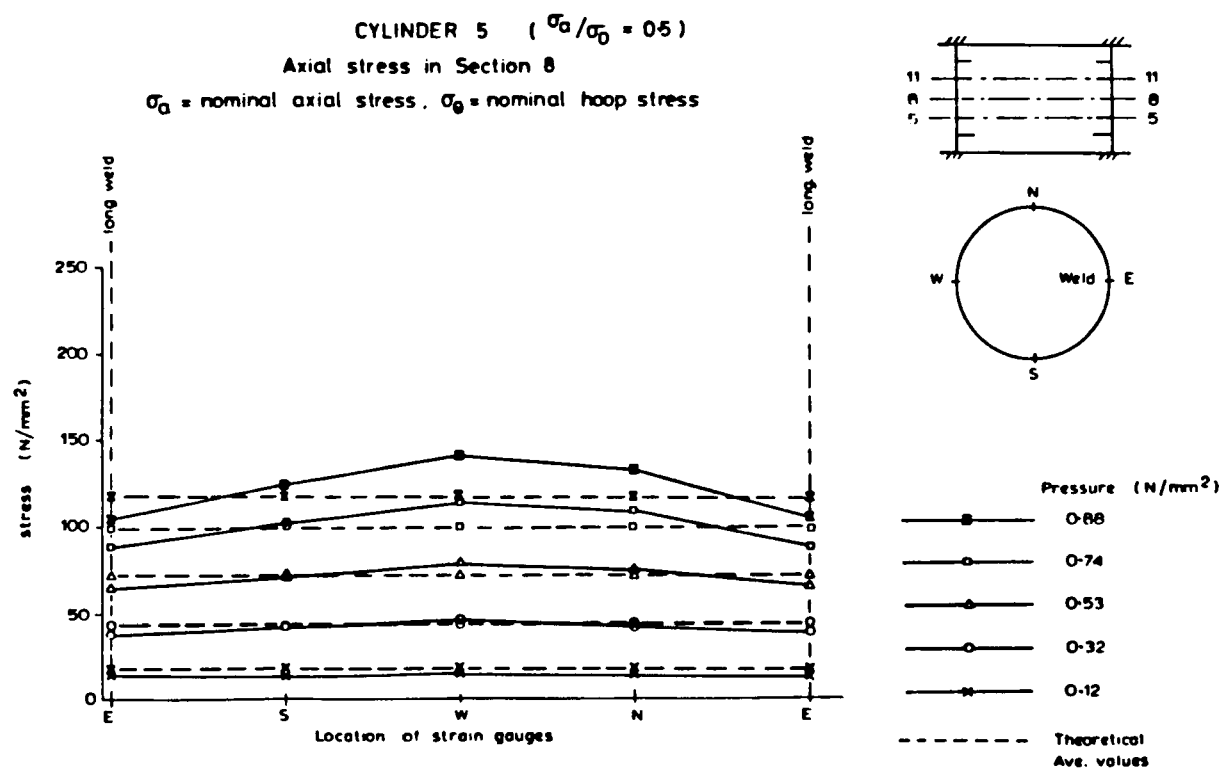


Fig. 3.92

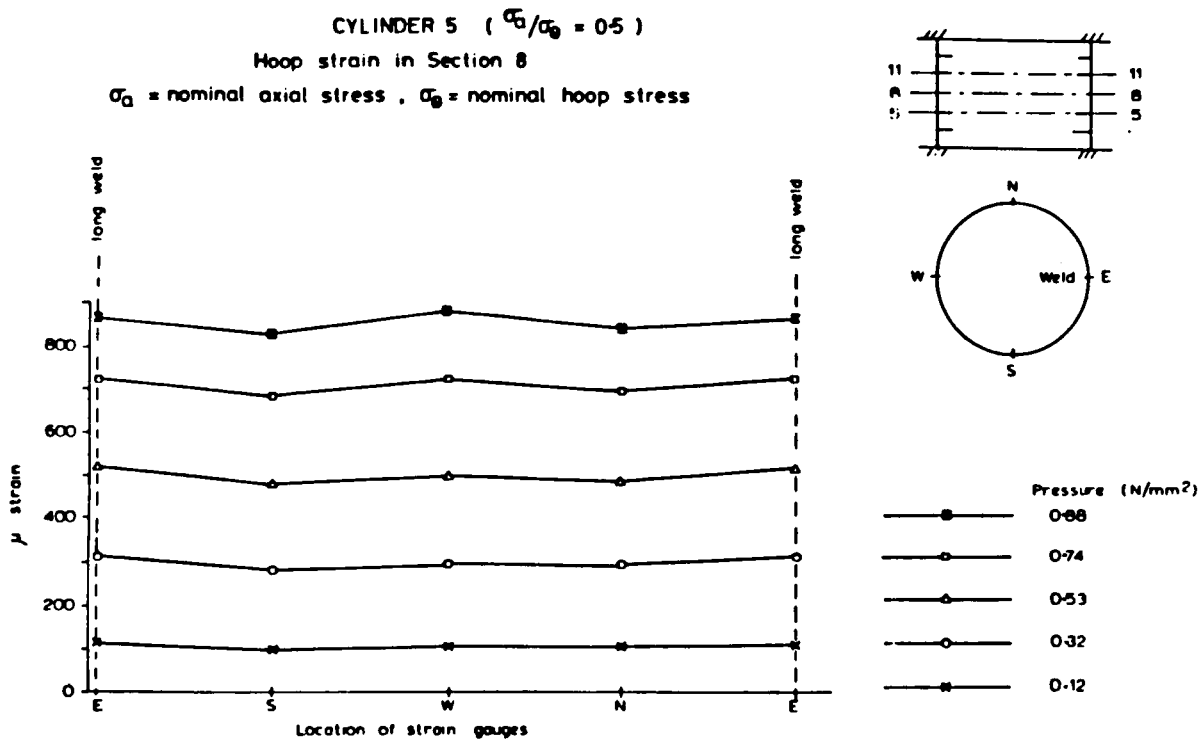


Fig. 3.93

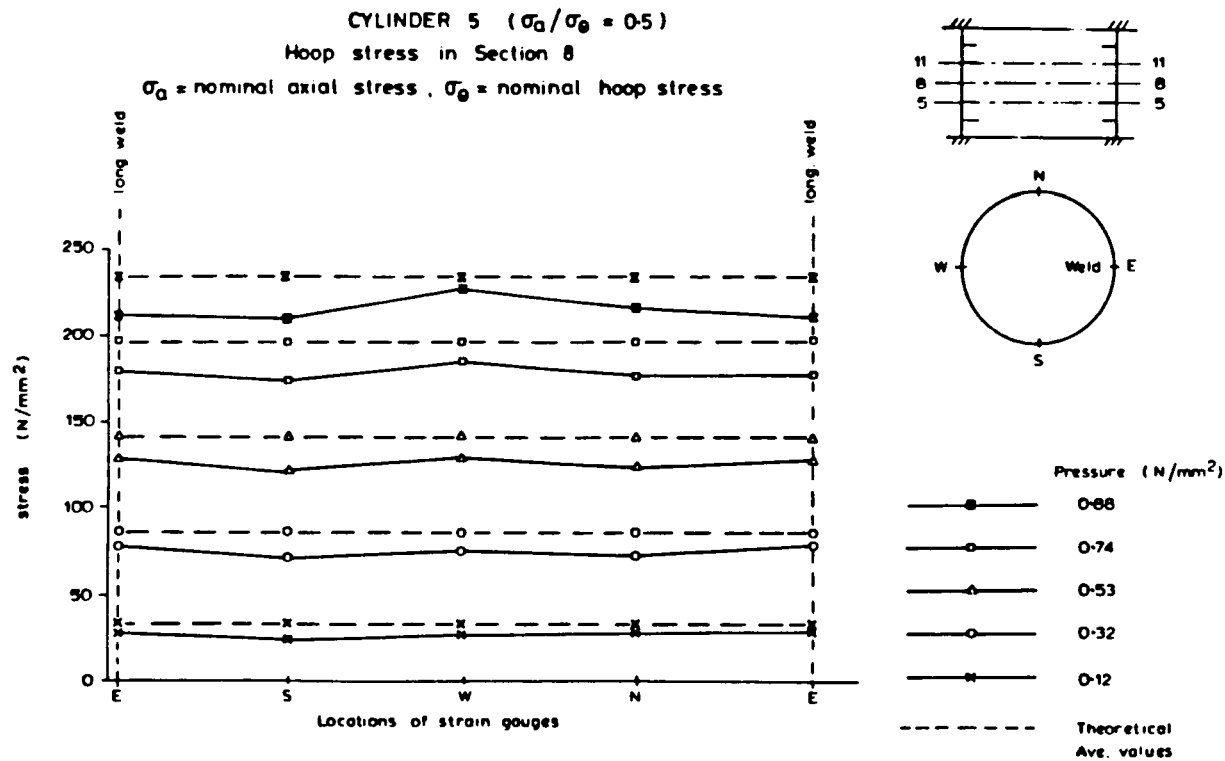


Fig. 3.94

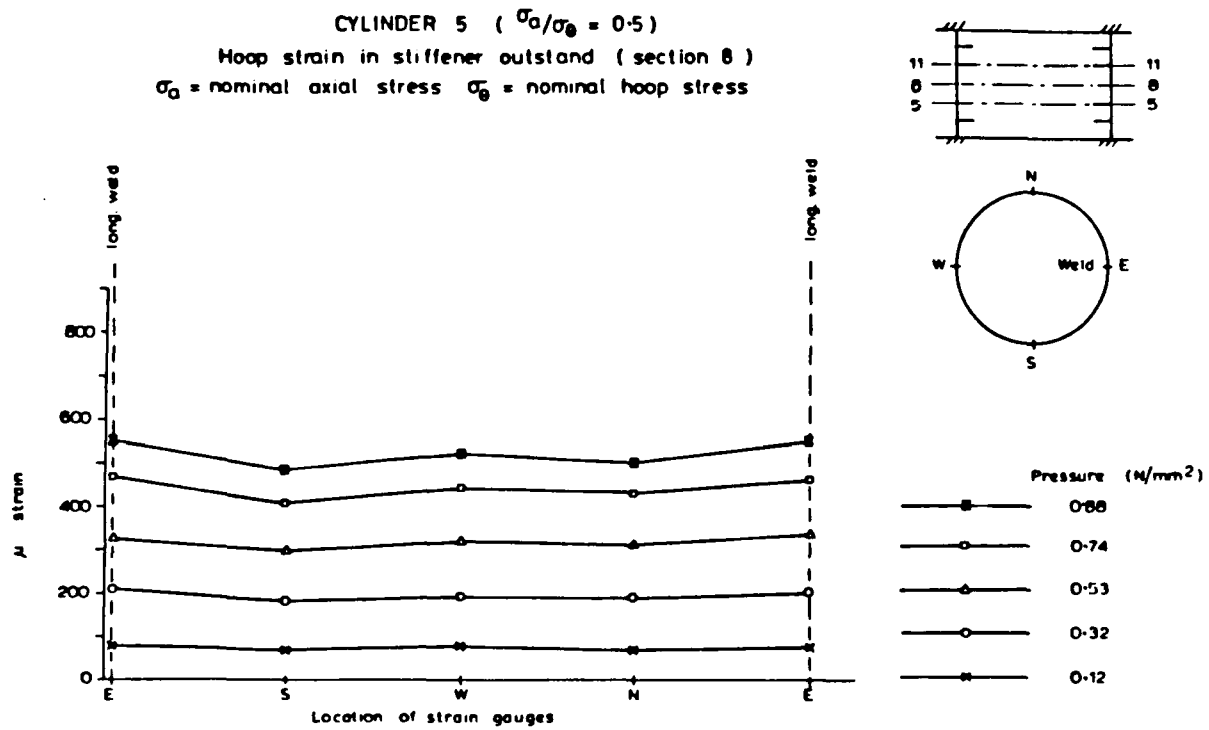


Fig. 3.95

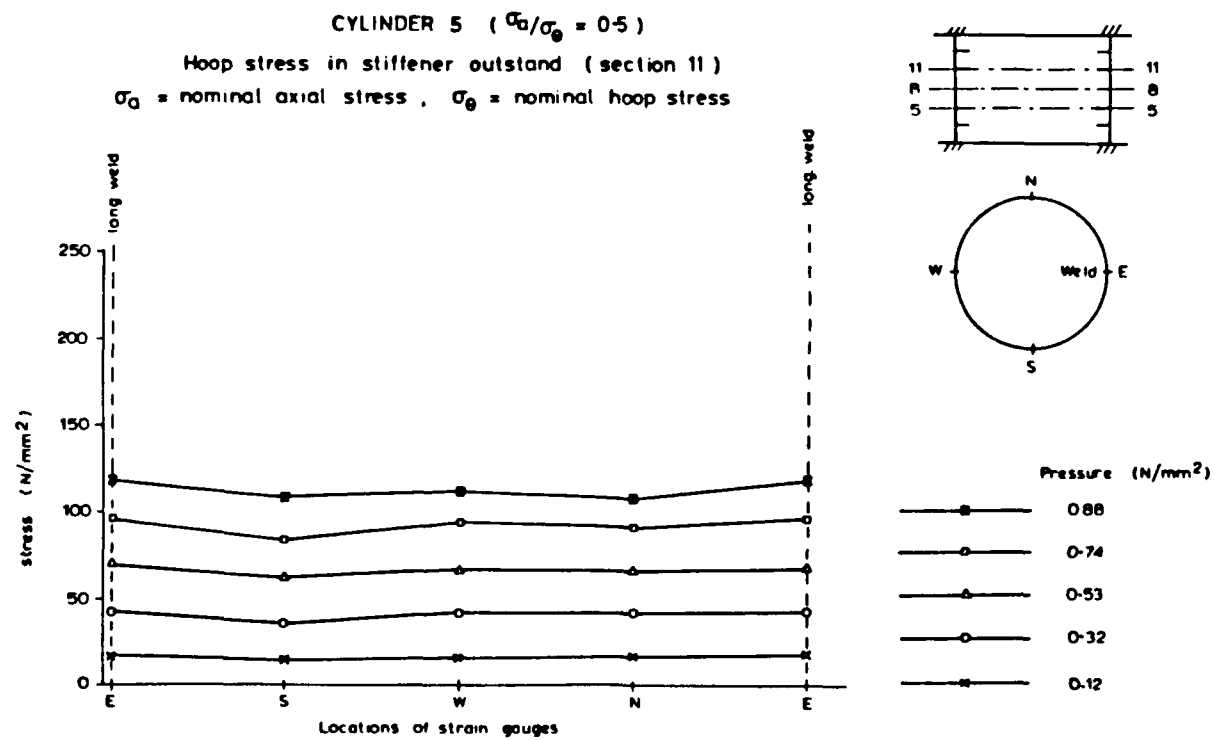
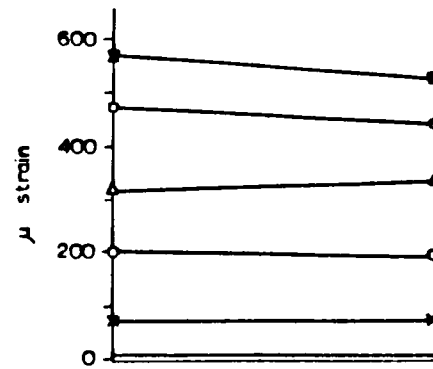
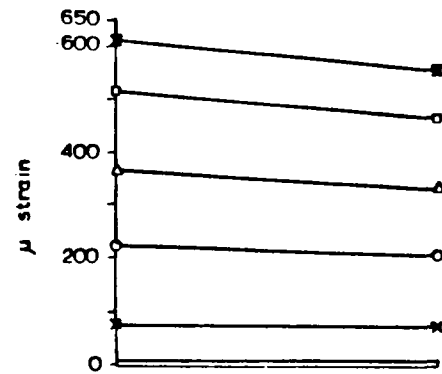
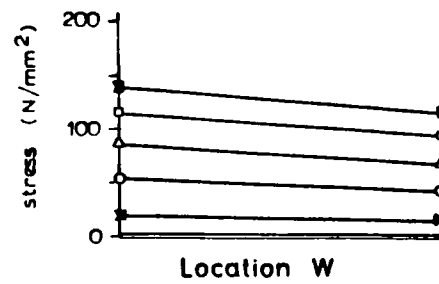
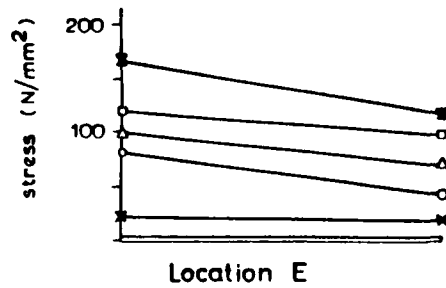
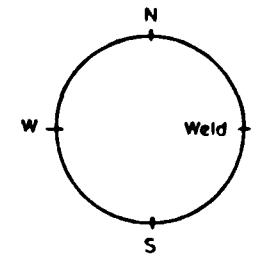
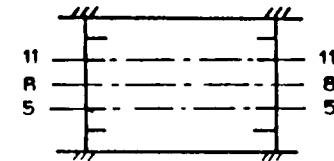


Fig. 3.96

CYLINDER 5 ($\sigma_a/\sigma_\theta = 0.5$)
 Radial variation of hoop stress and hoop strain in stiffener outstand
 (Section 11)
 σ_a = nominal axial stress , σ_θ = nominal hoop stress



Inward radial direction →

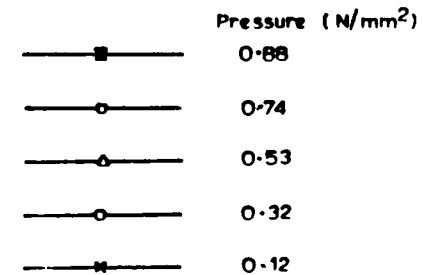


Fig. 3.97

CYLINDER 5

Plot of nominal axial stress vs. average axial strain

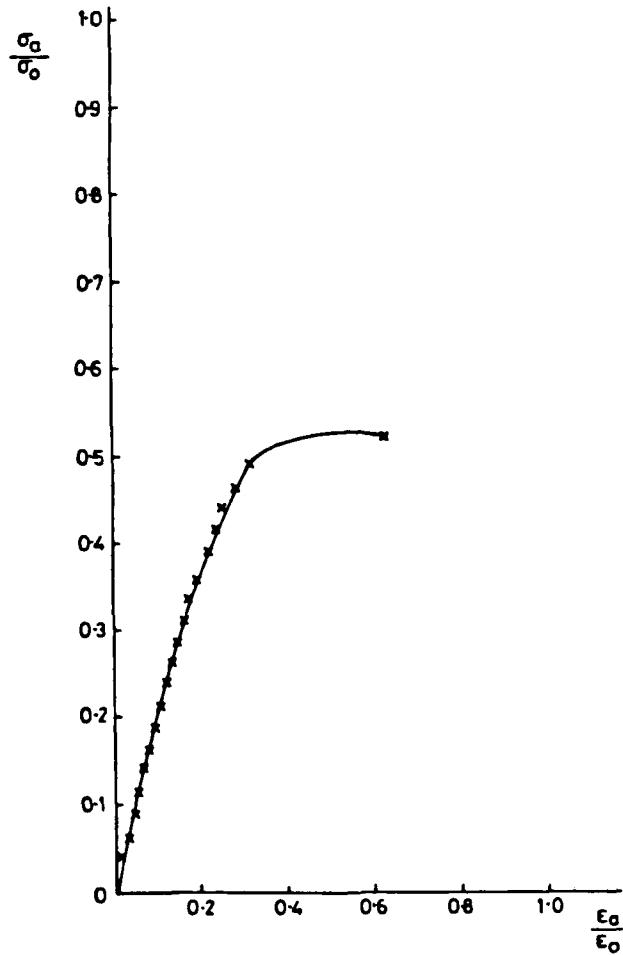


Fig. 3.98

CYLINDER 5

Plot of nominal hoop stress vs. average hoop strain

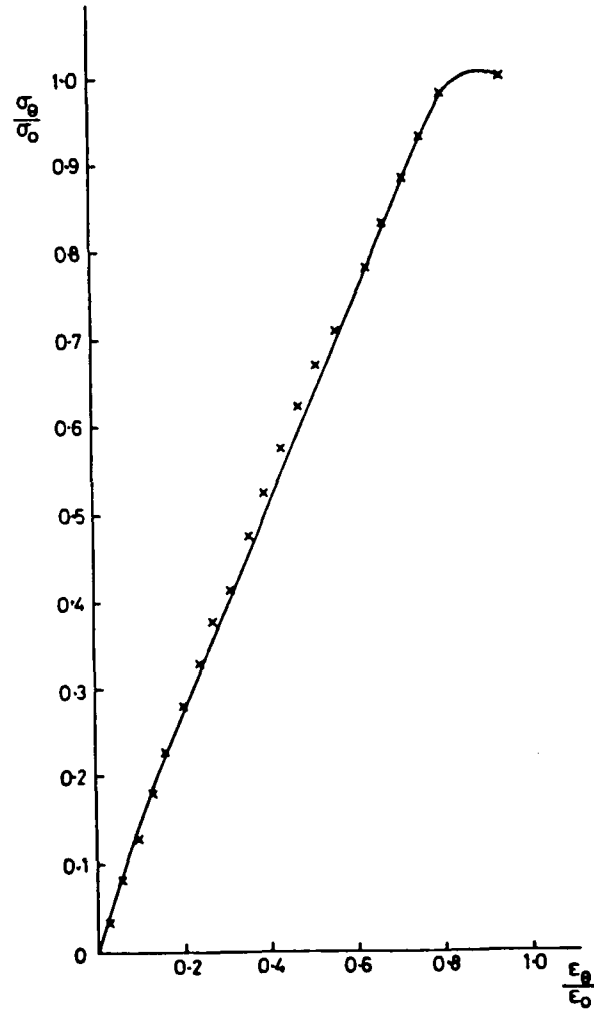


Fig. 3.99

CYLINDER 5

Plot of equivalent stress vs equivalent strain

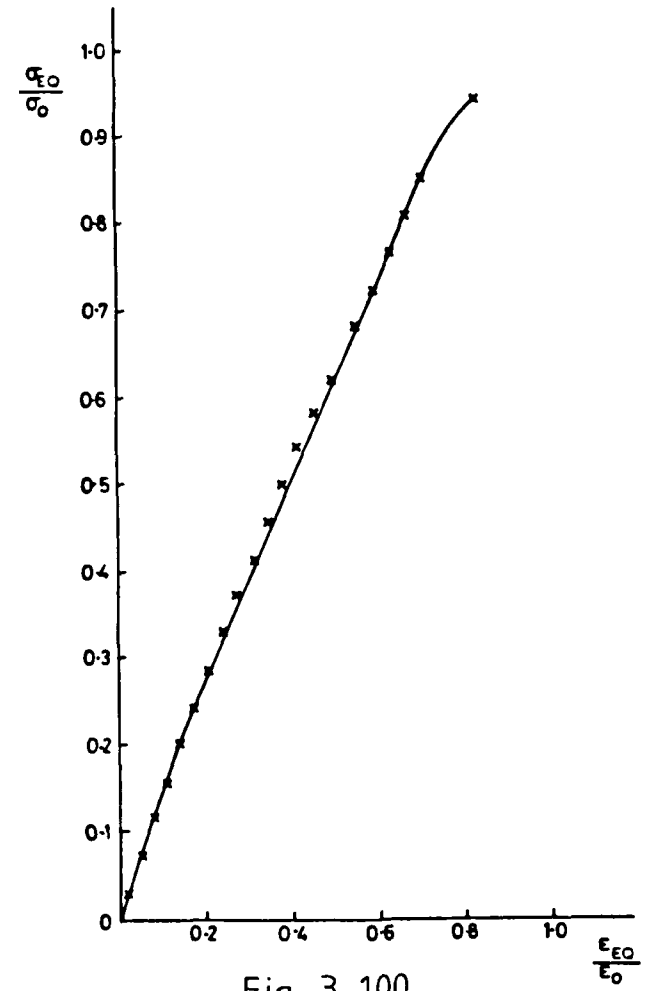
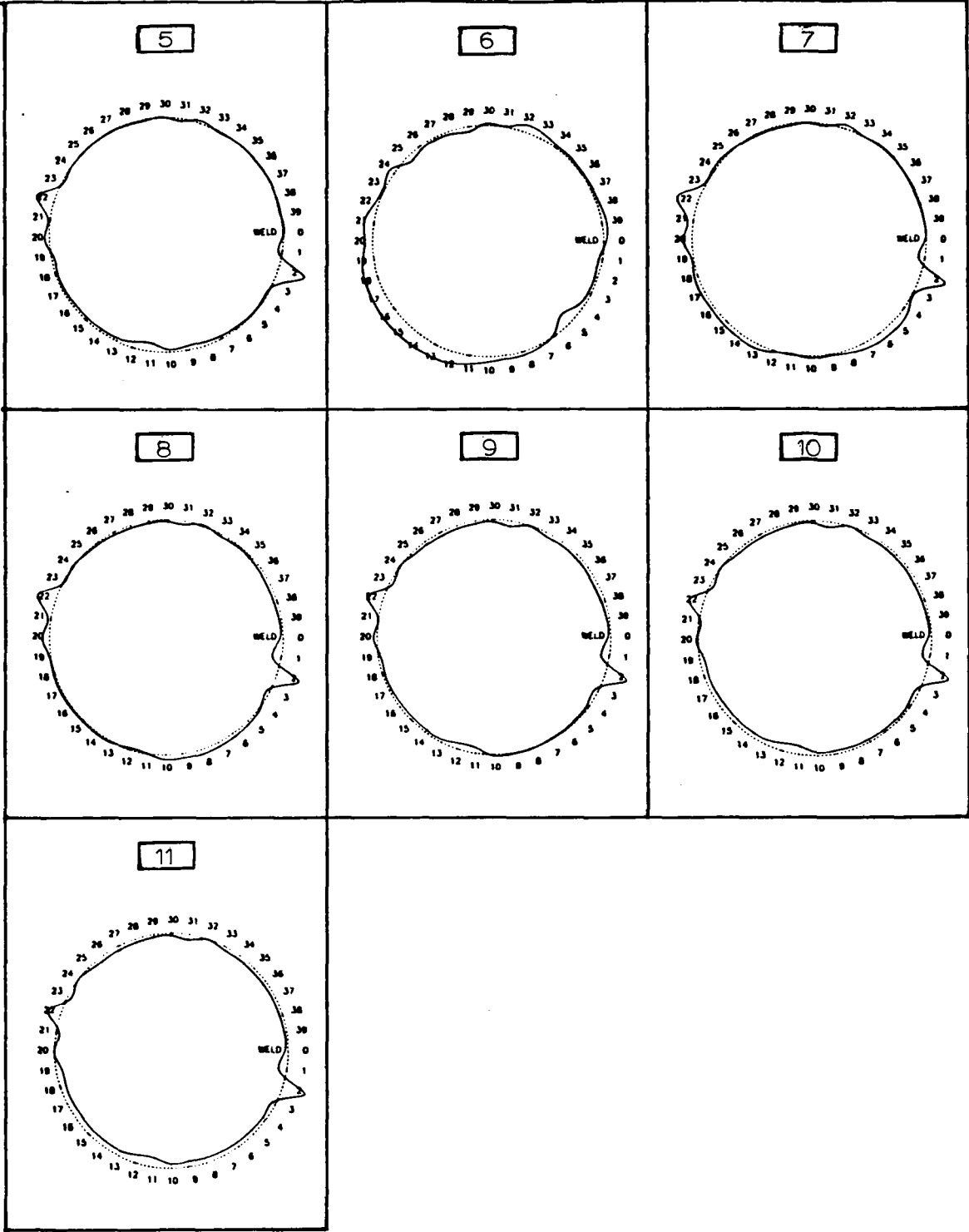


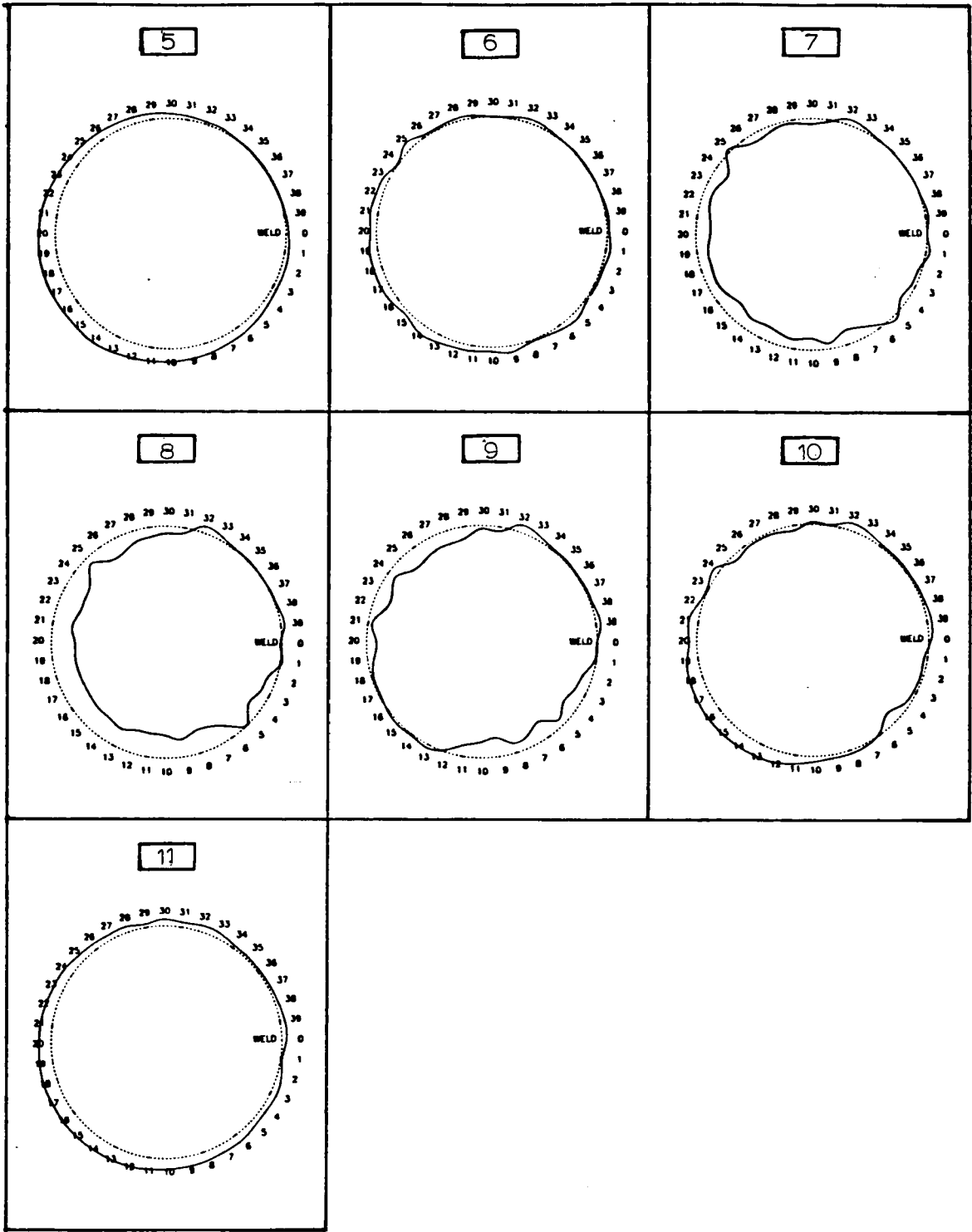
Fig. 3.100



 Vertical section

Deflection magnification = 10

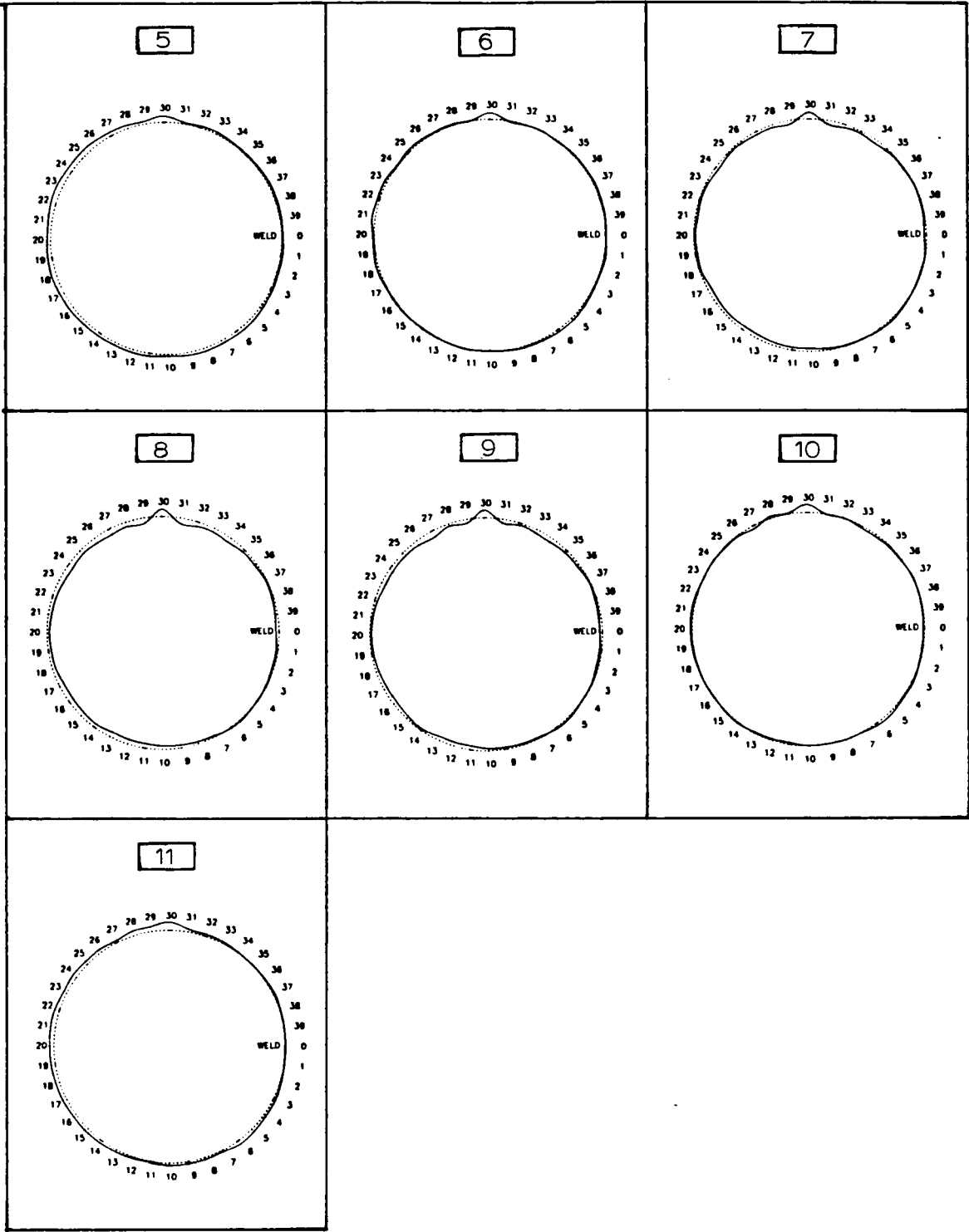
Fig 3.101 Buckled shape of cylinder 1



 Vertical section

Deflection magnification = 10

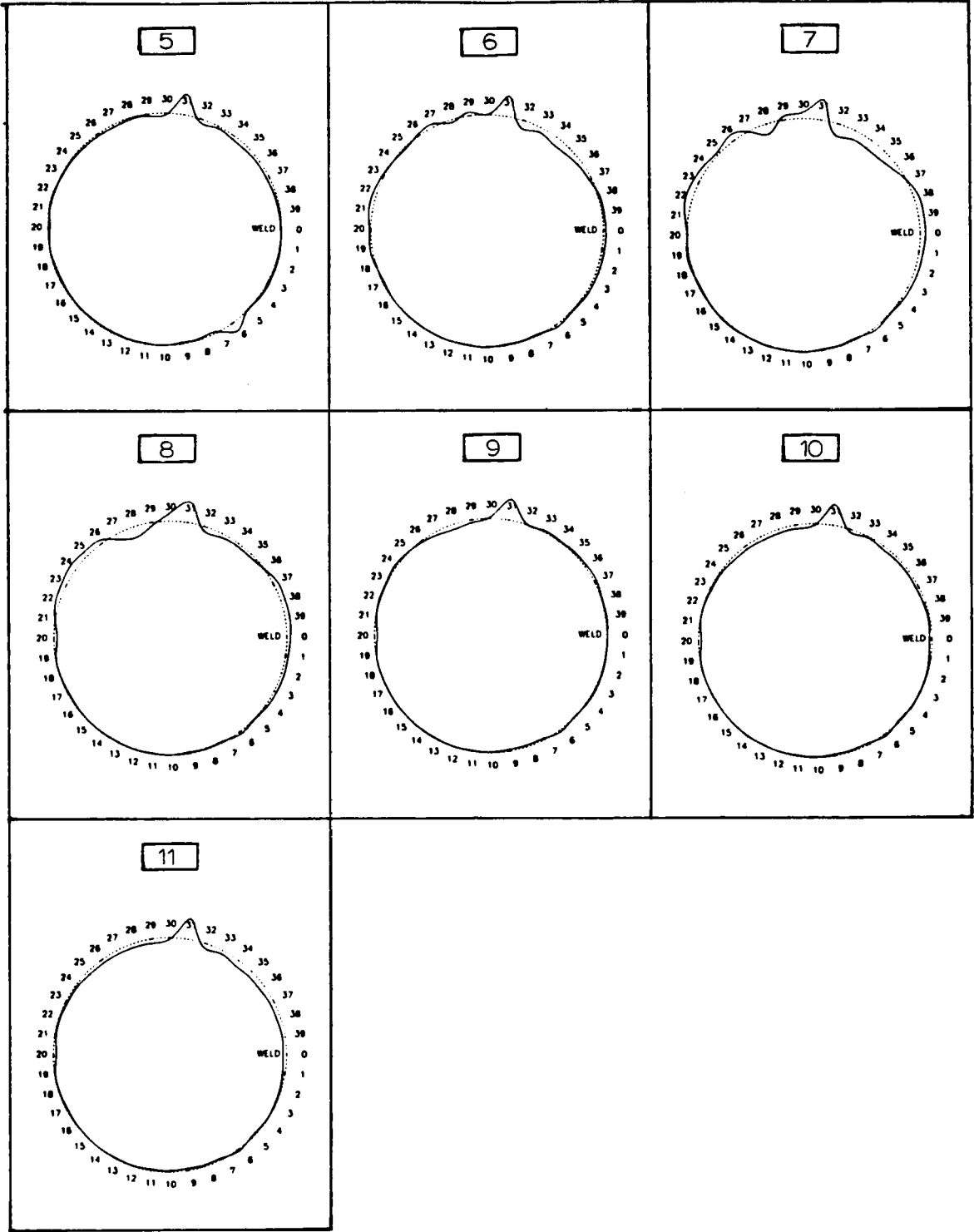
Fig. 3.102 Buckled shape of cylinder 2



 Vertical section

Deflection magnification = 10

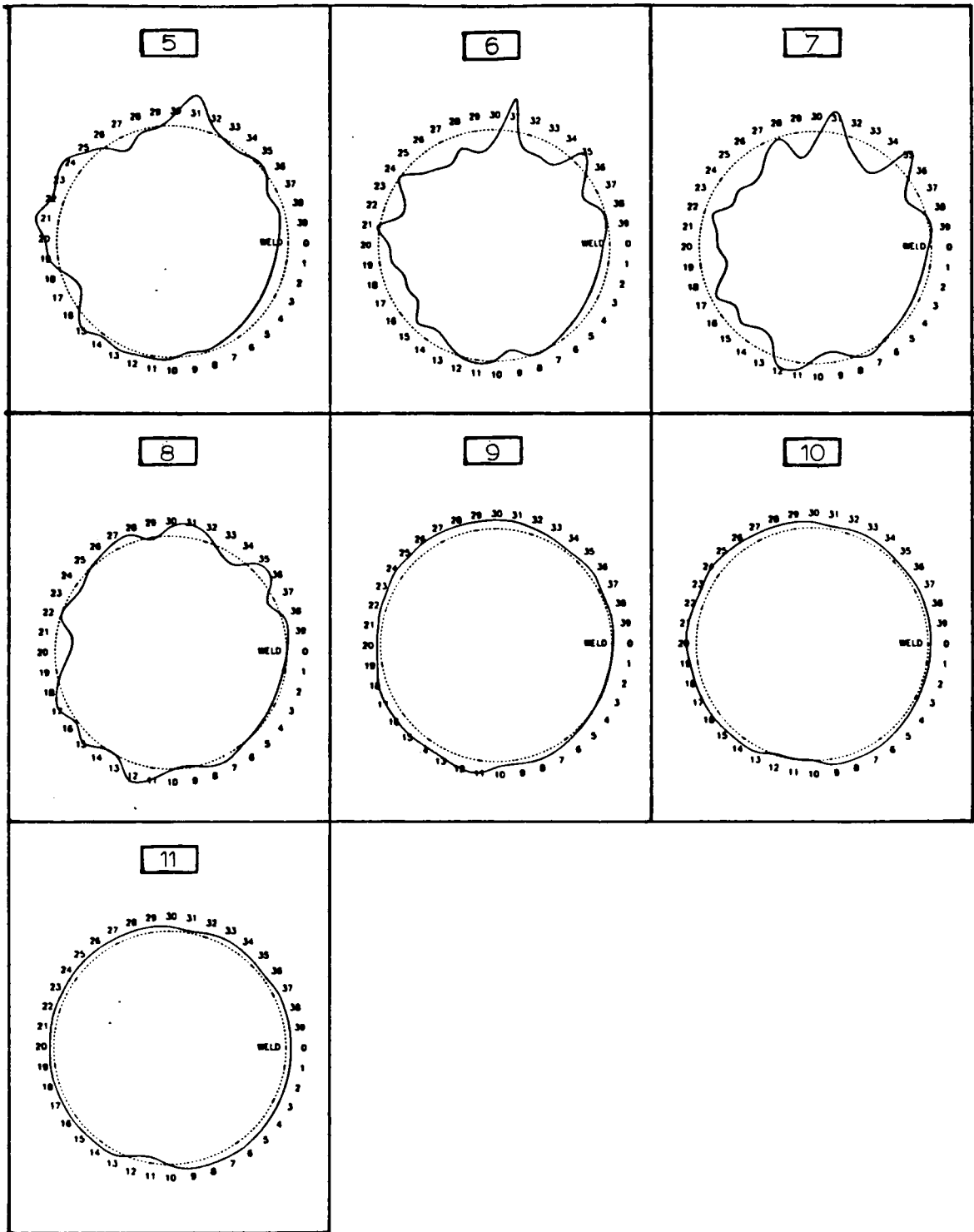
Fig. 3.103 Buckled shape of cylinder 3



 Vertical section

Deflection magnification = 10

Fig. 3.104 Buckled shape of cylinder 4



 Vertical section

Deflection magnification = 10

Fig 3.105 Buckled shape of cylinder 5

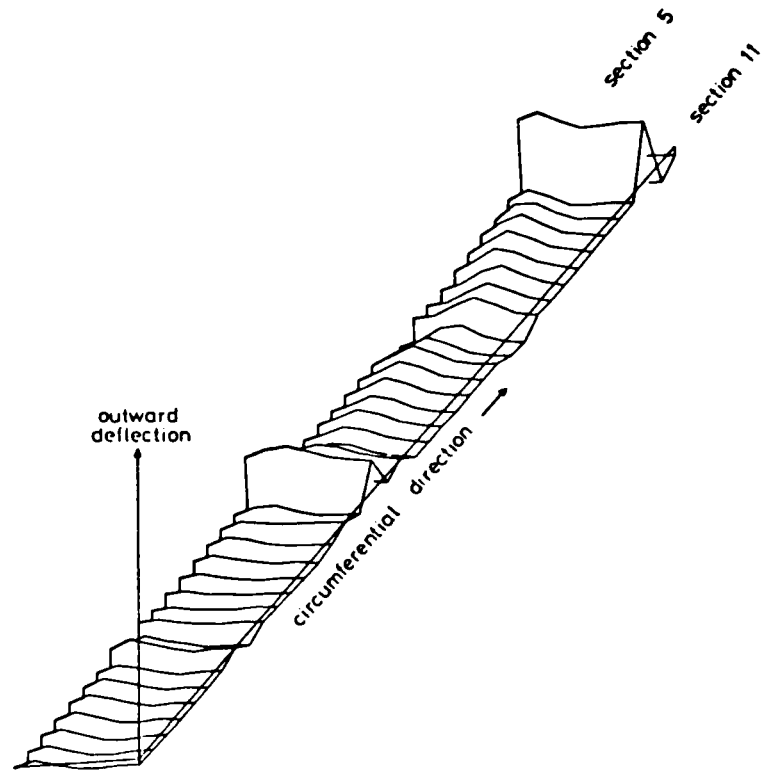


Fig. 3.106 Buckled shape of cylinder 1

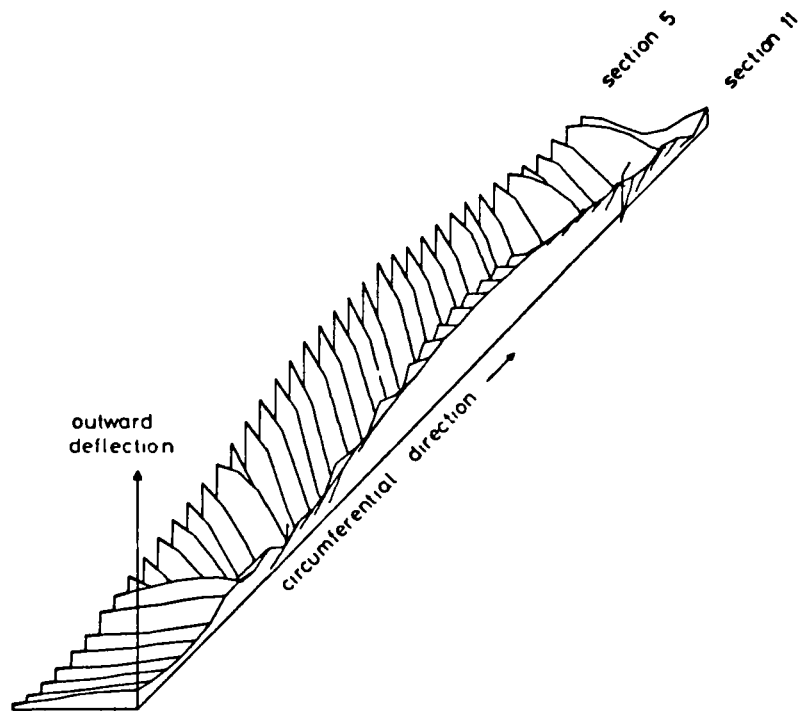


Fig. 3.107 Buckled shape of cylinder 2

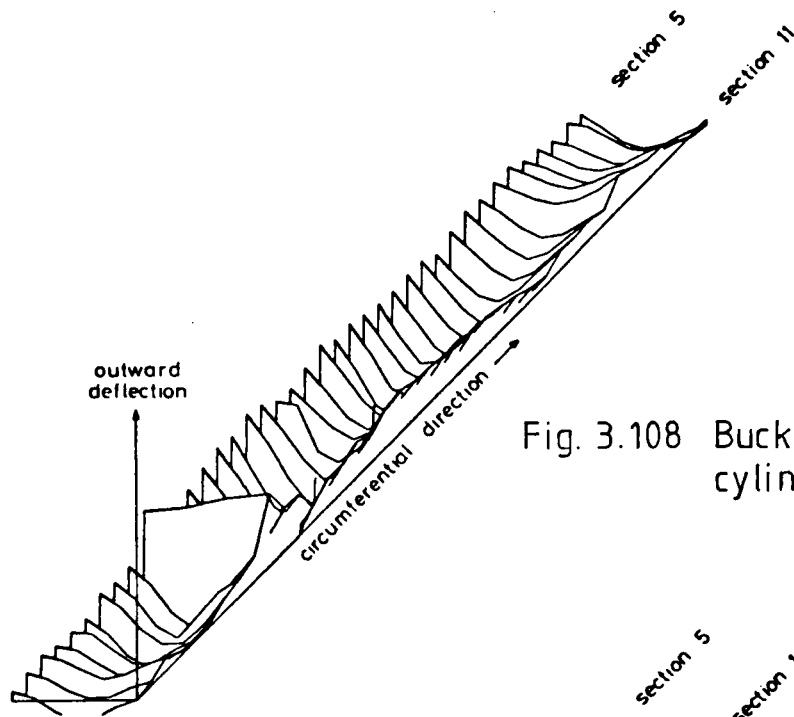


Fig. 3.108 Buckled shape of cylinder 3

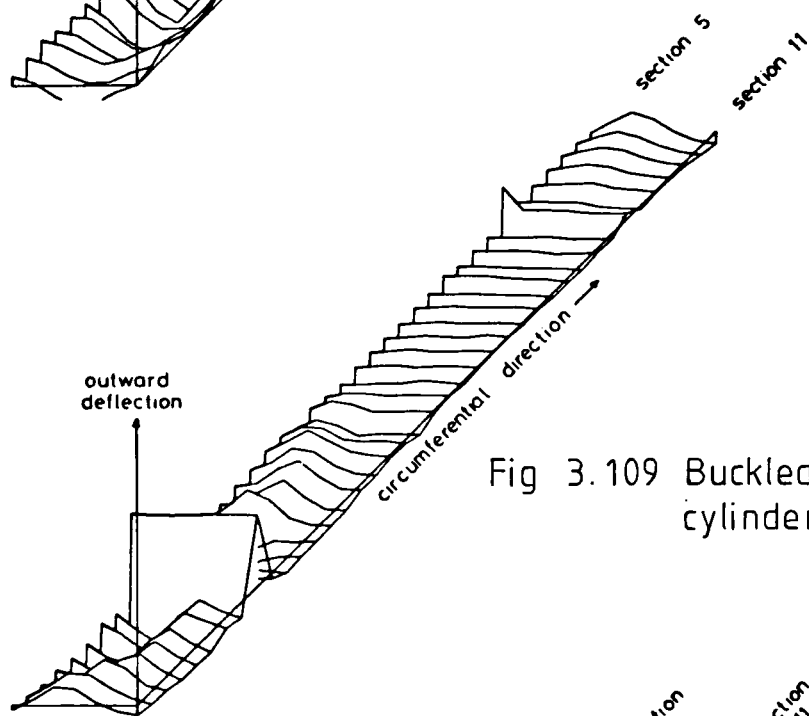


Fig 3.109 Buckled shape of cylinder 4

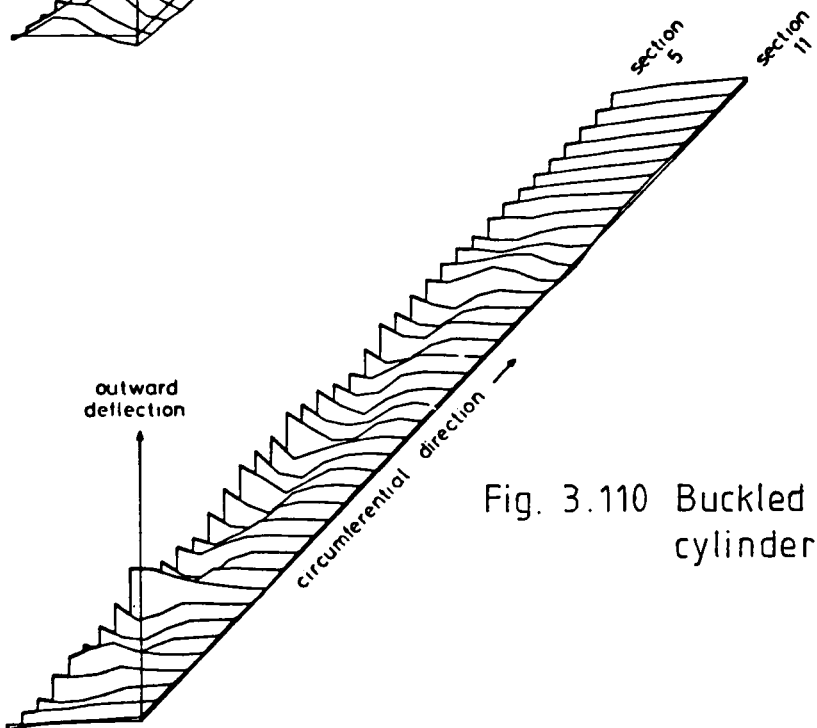


Fig. 3.110 Buckled shape of cylinder 5



Fig. 3.111 Failure of cylinder 1.

Fig. 3.112 Failure of cylinder 2.



Fig. 3.113 Failure of cylinder 3.

Fig. 3.114 Failure of cylinder 4.



Fig. 3.115 Failure of cylinder 5.

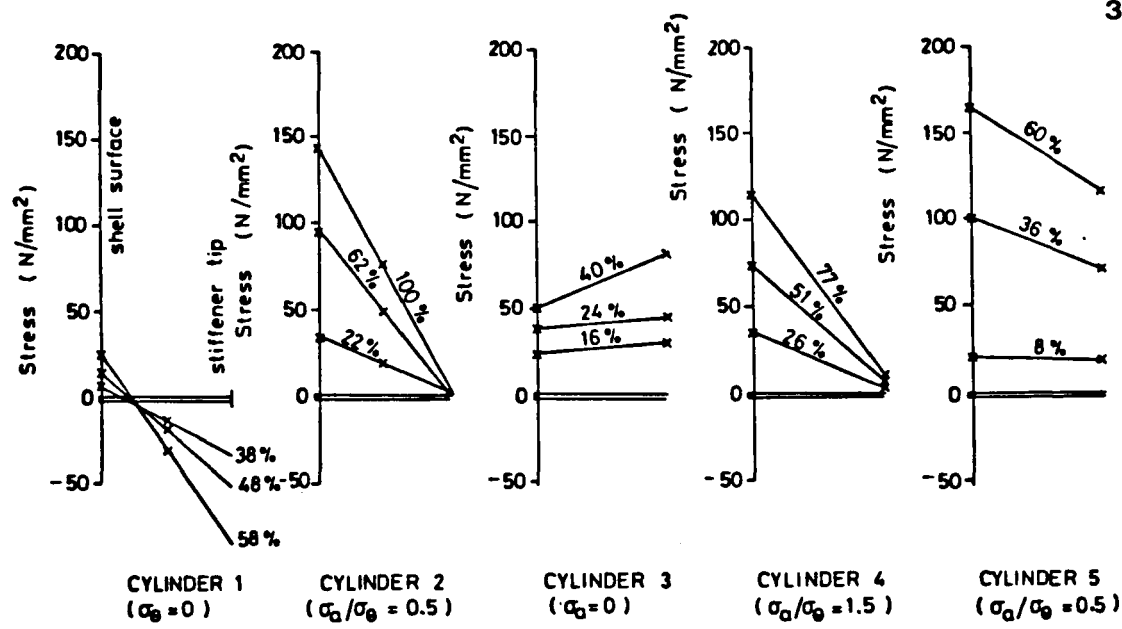


Fig. 3.116 Stiffener outstand stresses for cylinders 1 – 5
(% figures expressed in terms of failure load)

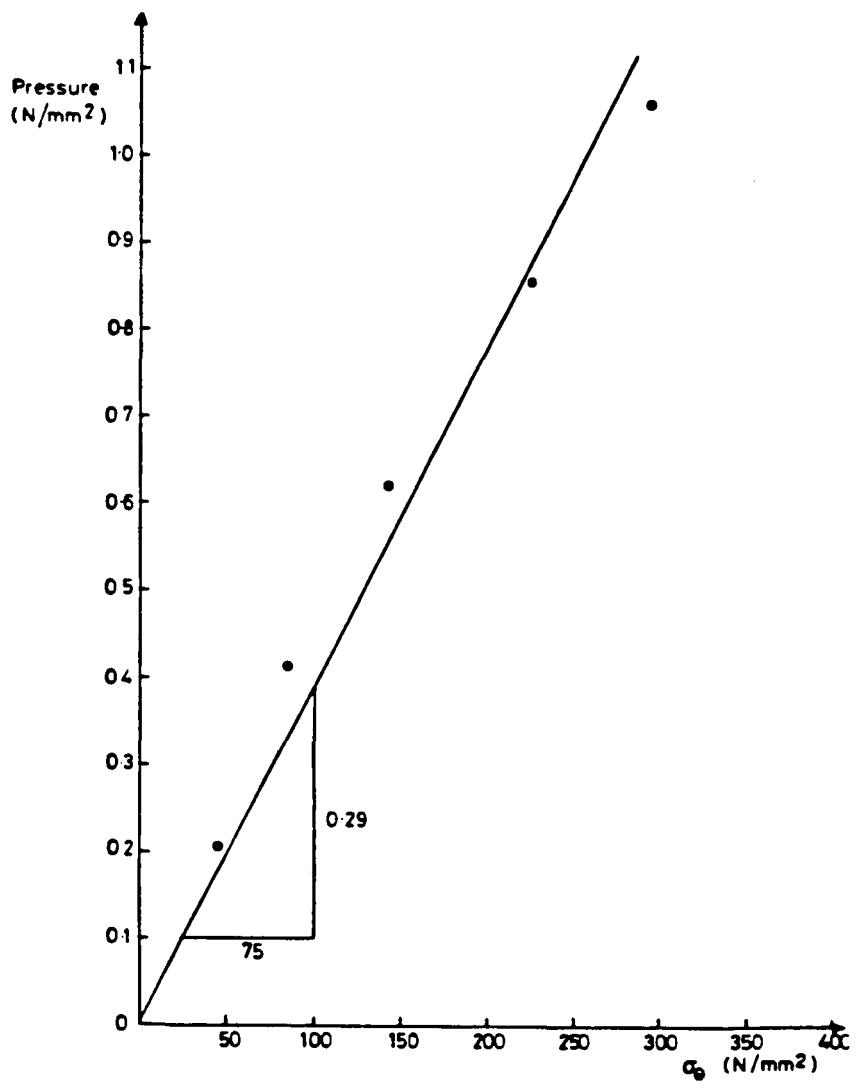


Fig. 3.117 Relationship between applied pressure and centre panel hoop stress of cylinder 3

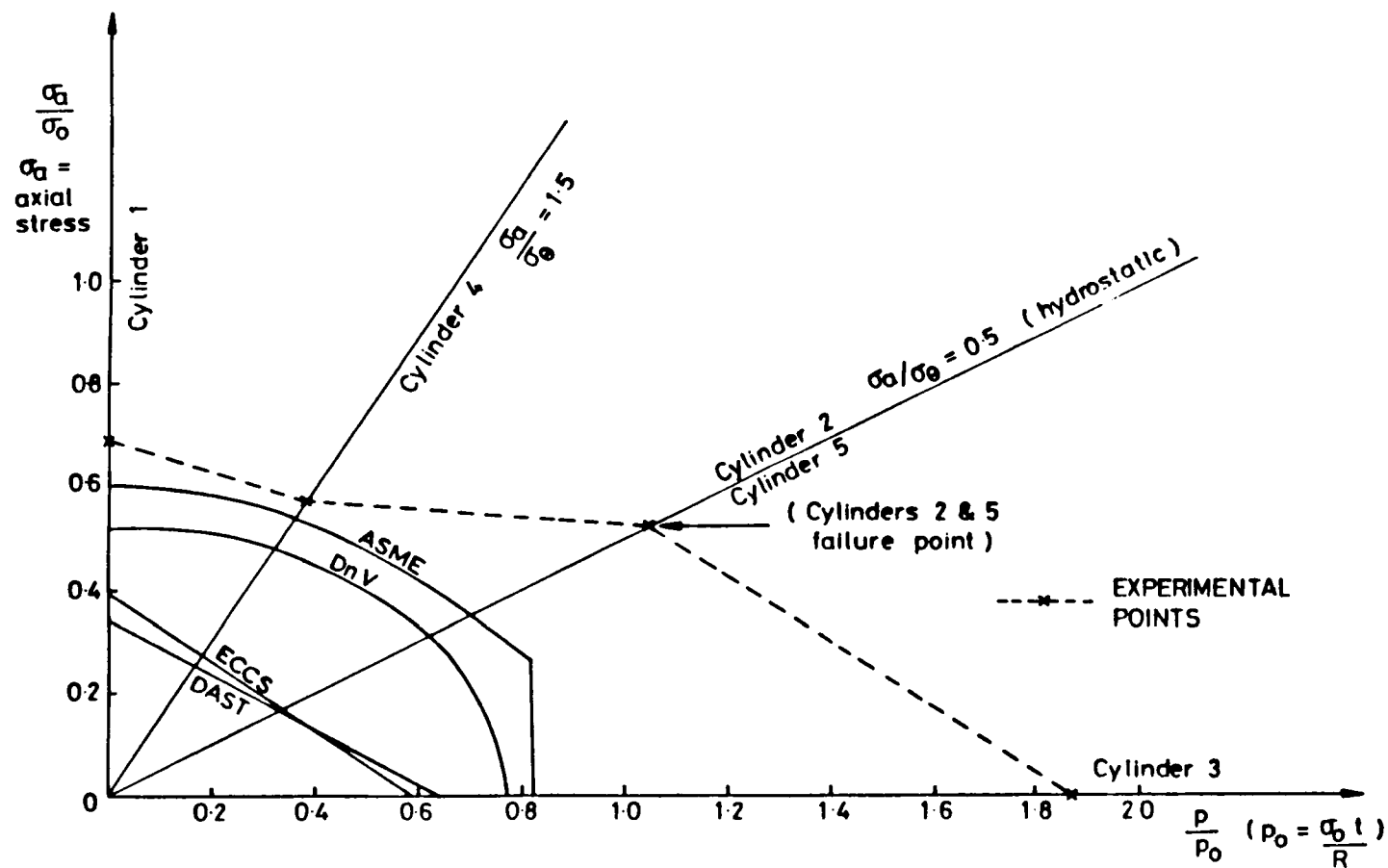
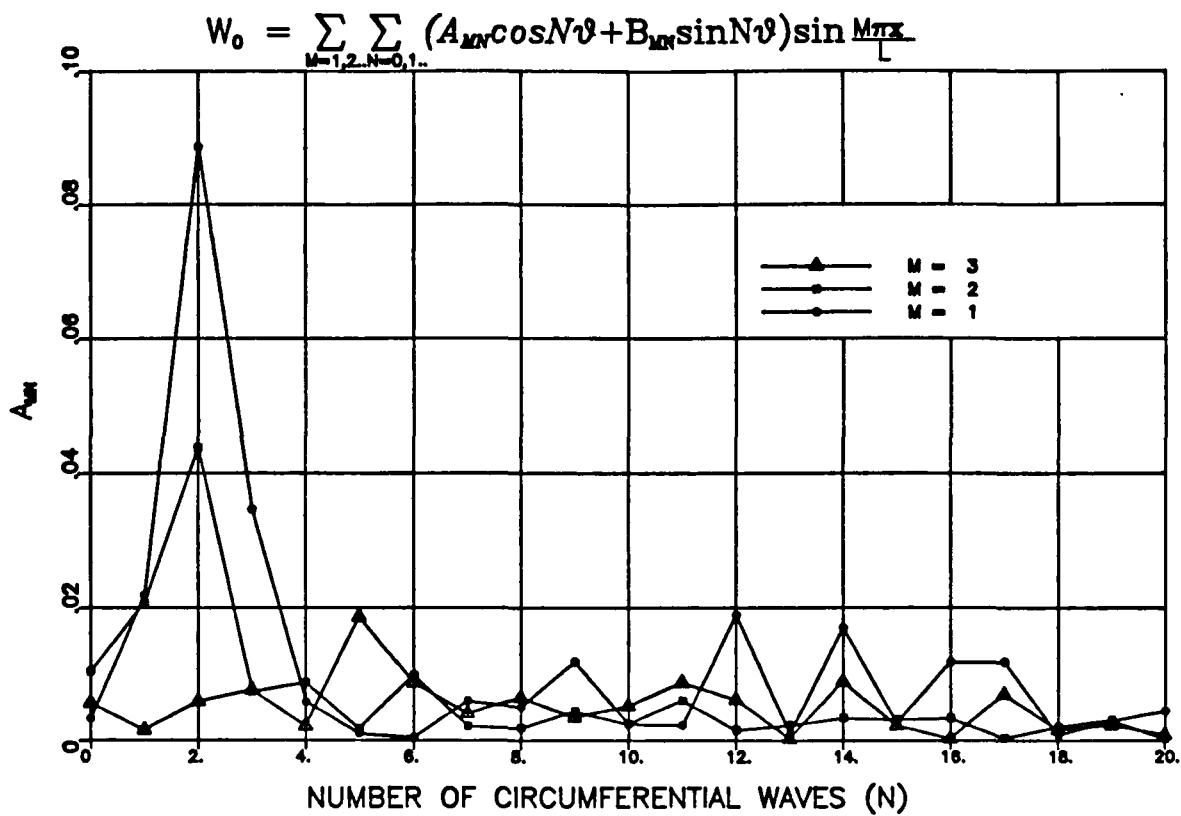
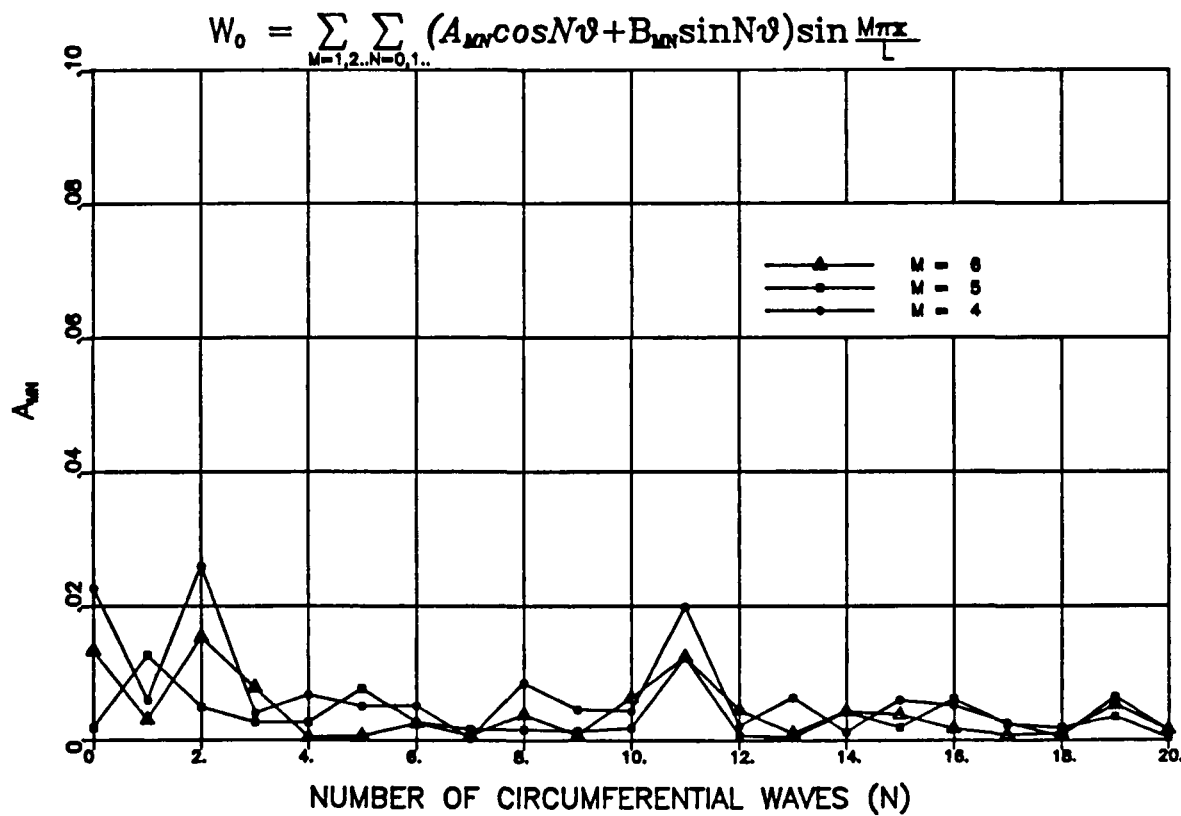


Fig. 3.118 Comparison of Experimental Results and Design Rule Predictions

CYLINDER 6 – FOURIER COEFFICIENTS



CYLINDER 6 – FOURIER COEFFICIENTS



CYLINDER 6 - FOURIER COEFFICIENTS

$$W_0 = \sum_{M=1,2} \sum_{N=0,1,\dots} (A_{MN} \cos N\vartheta + B_{MN} \sin N\vartheta) \sin \frac{M\pi x}{L}$$

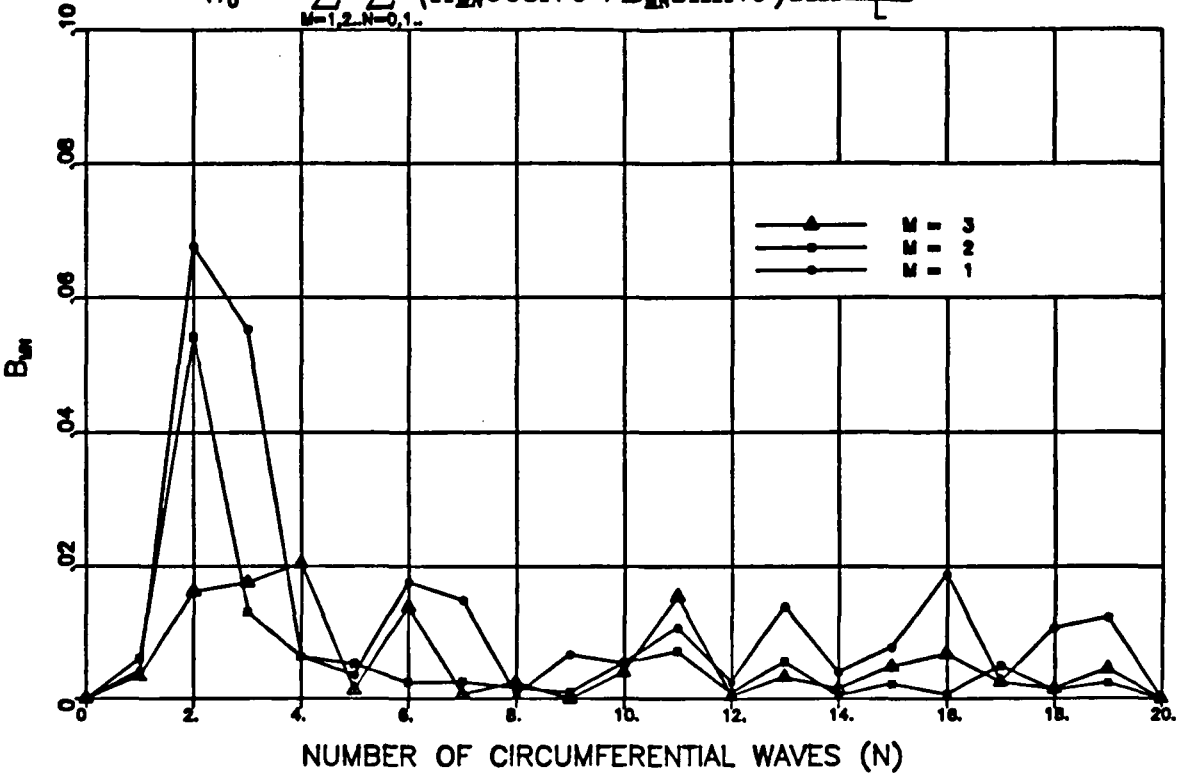


Fig. 3.121

CYLINDER 6 - FOURIER COEFFICIENTS

$$W_0 = \sum_{M=1,2} \sum_{N=0,1,\dots} (A_{MN} \cos N\vartheta + B_{MN} \sin N\vartheta) \sin \frac{M\pi x}{L}$$

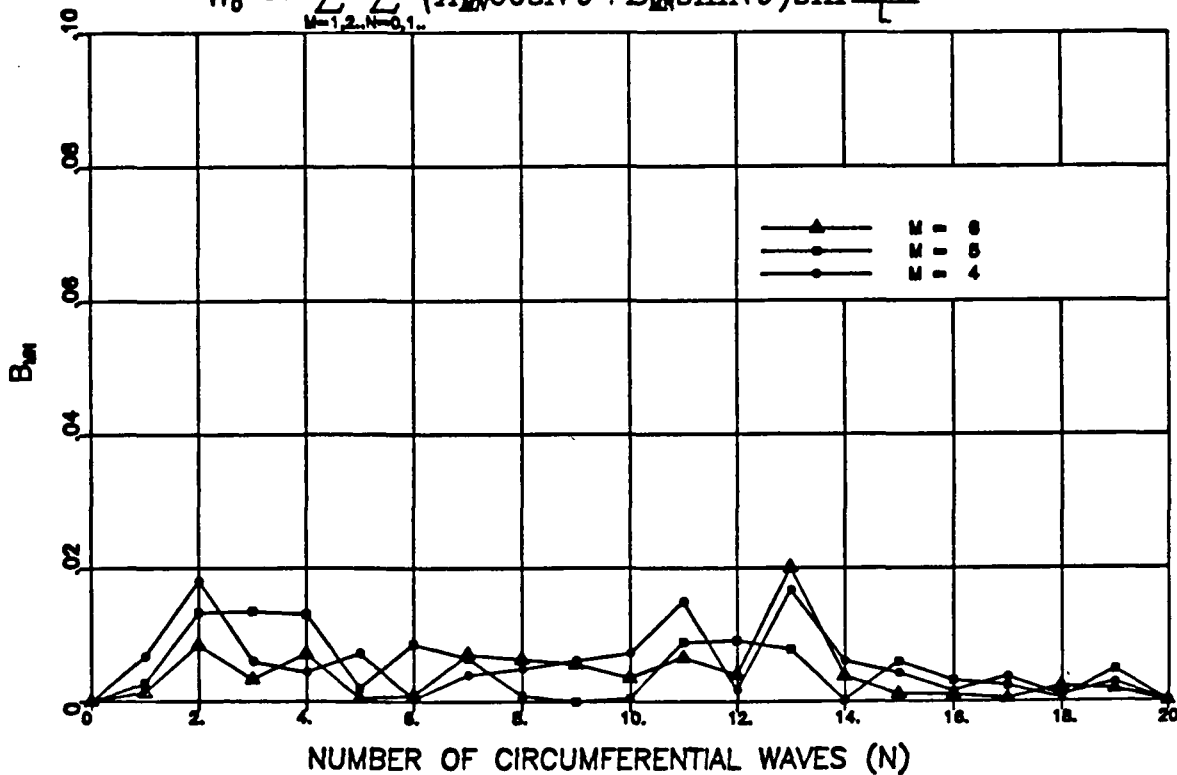


Fig. 3.122

CYLINDER 7 – FOURIER COEFFICIENTS

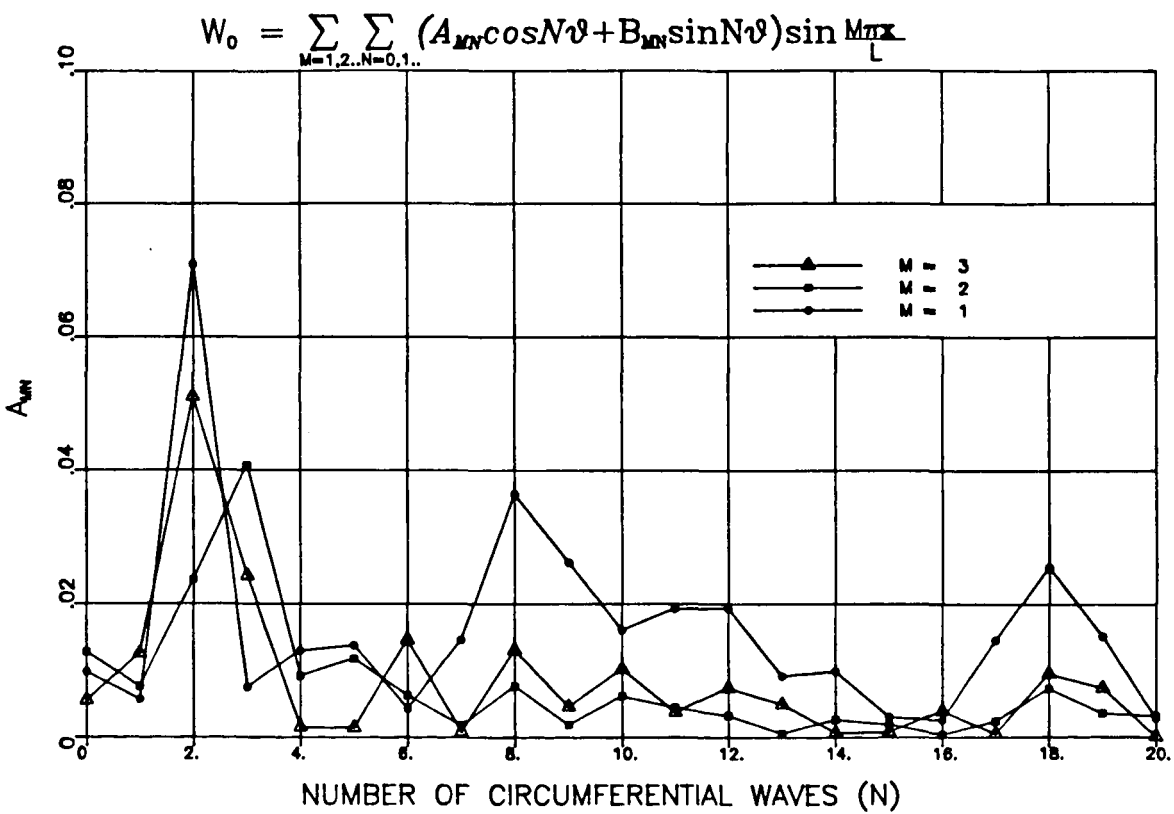


Fig. 3.123

CYLINDER 7 – FOURIER COEFFICIENTS

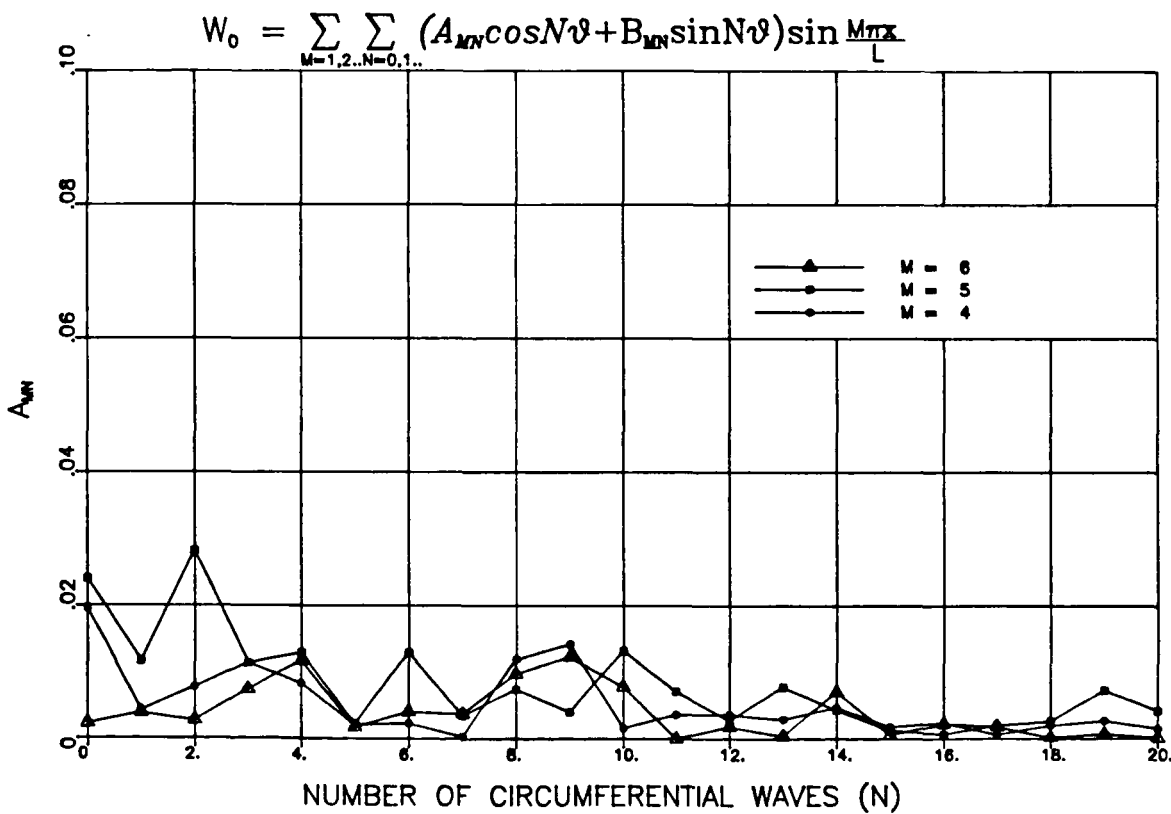


Fig. 3.124

CYLINDER 7 - FOURIER COEFFICIENTS

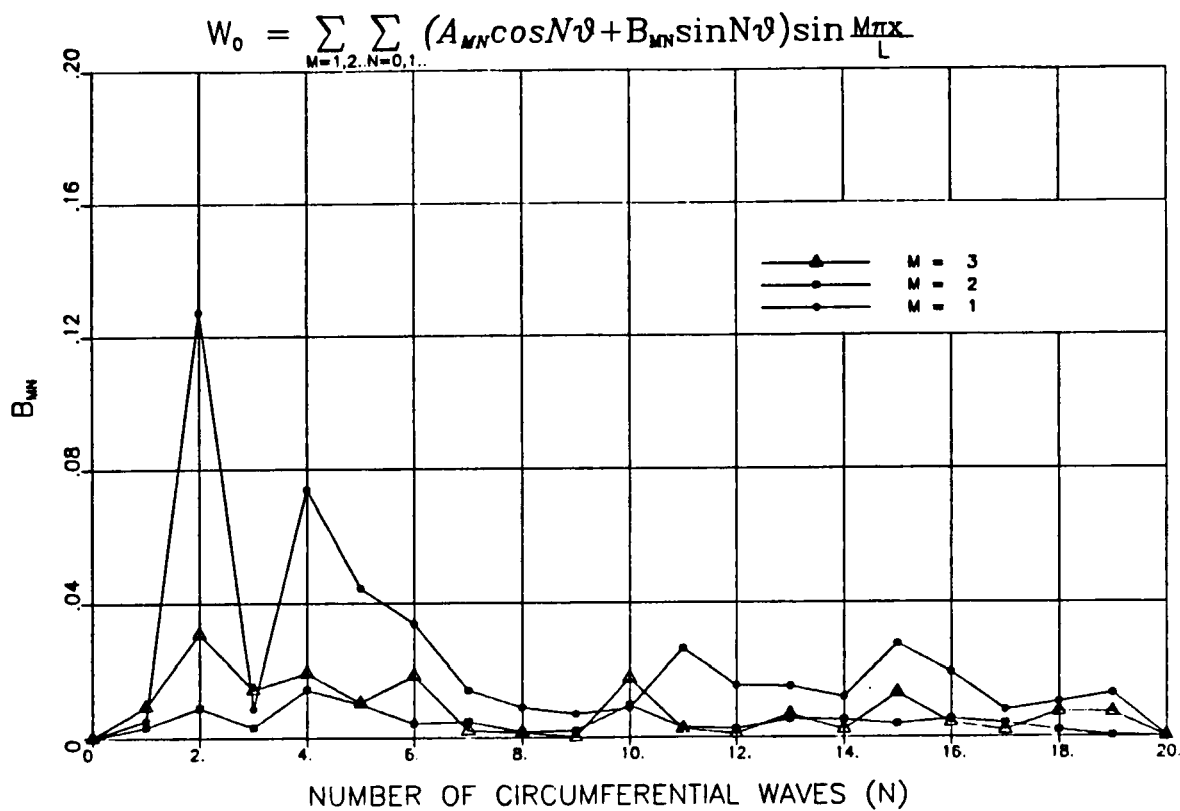


Fig. 3.125

CYLINDER 7 - FOURIER COEFFICIENTS

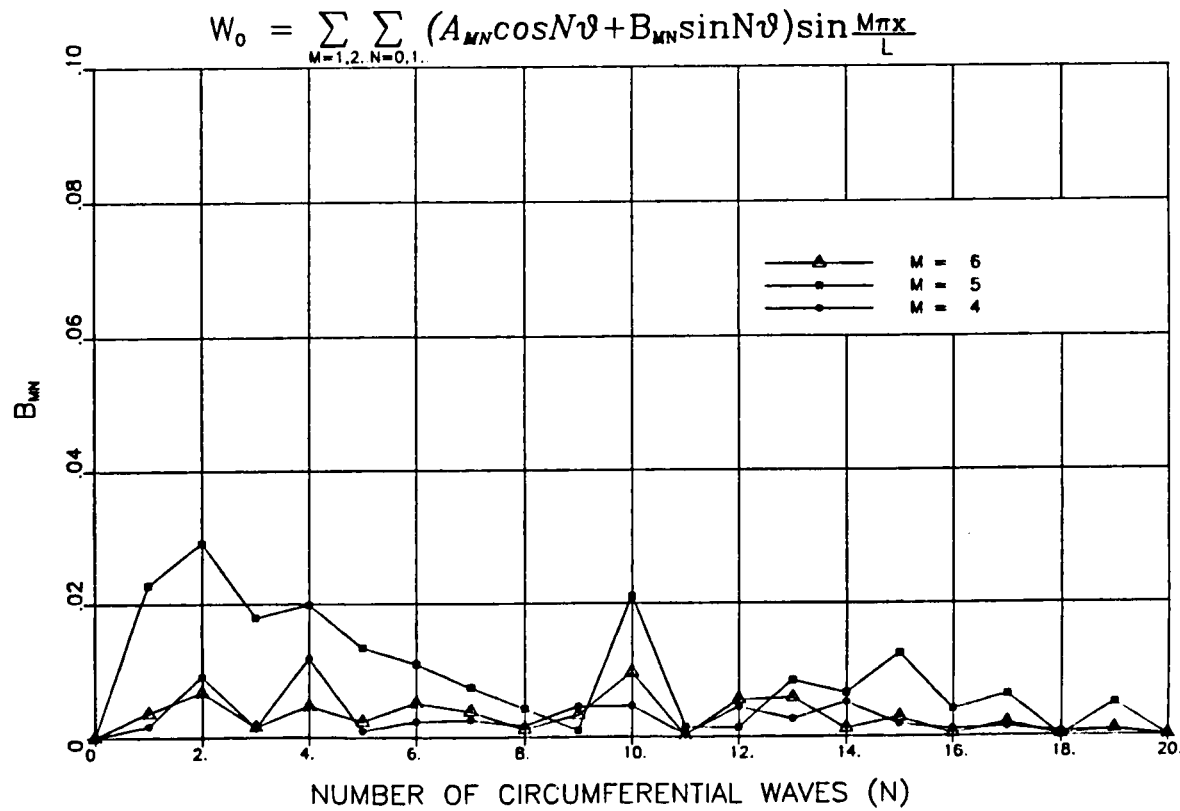
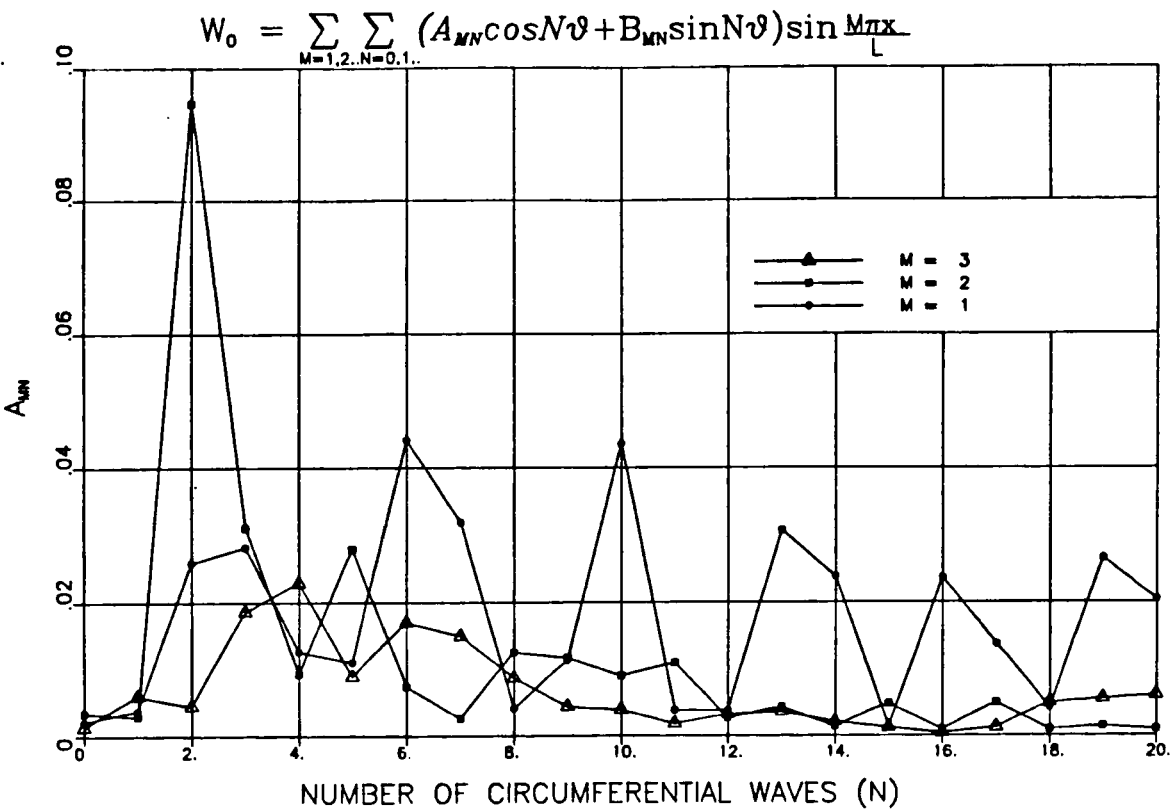
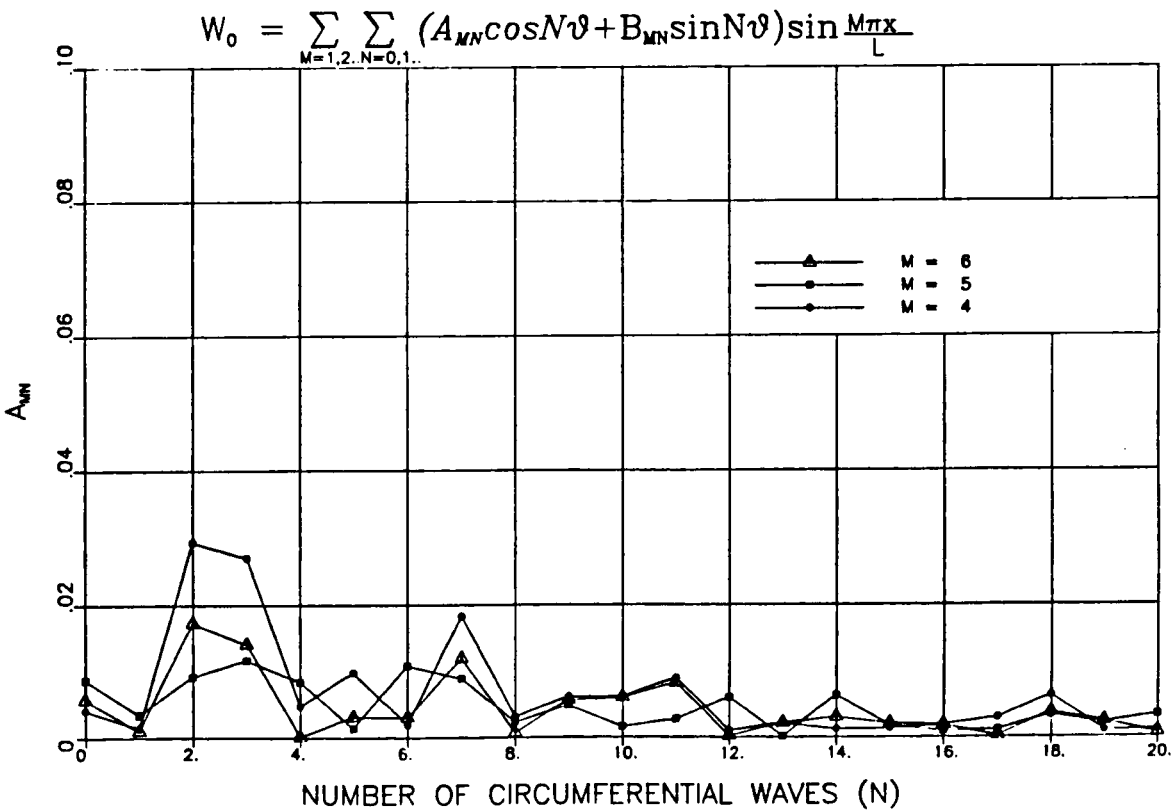


Fig. 3.126

CYLINDER 8 – FOURIER COEFFICIENTS



CYLINDER 8 – FOURIER COEFFICIENTS



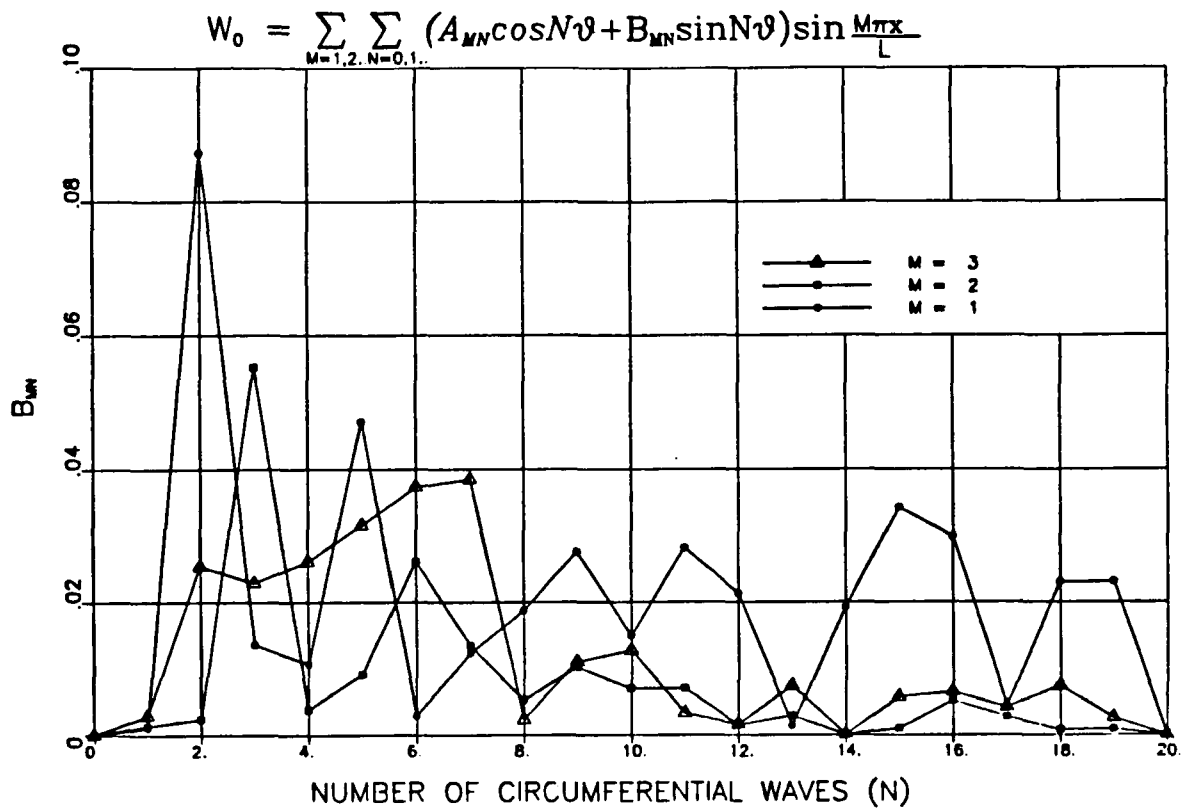


Fig. 3.129

CYLINDER 8 – FOURIER COEFFICIENTS

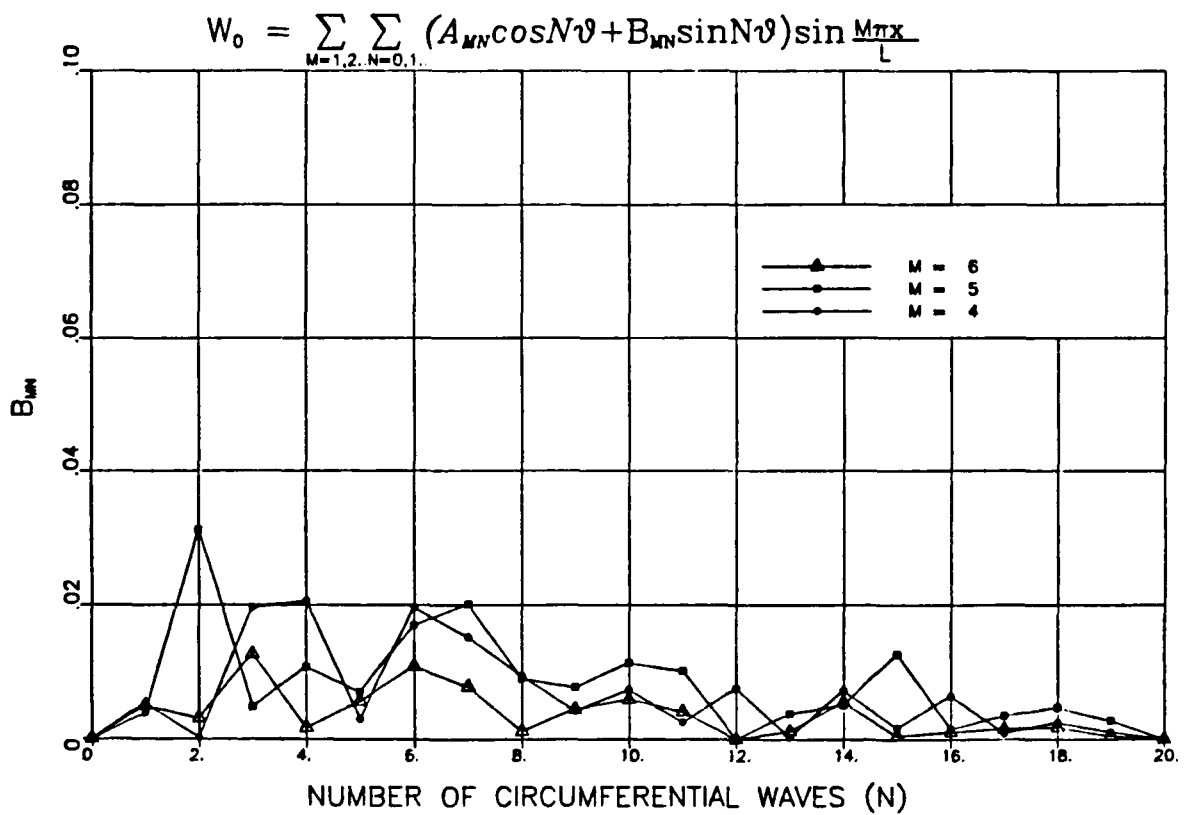


Fig. 3.130

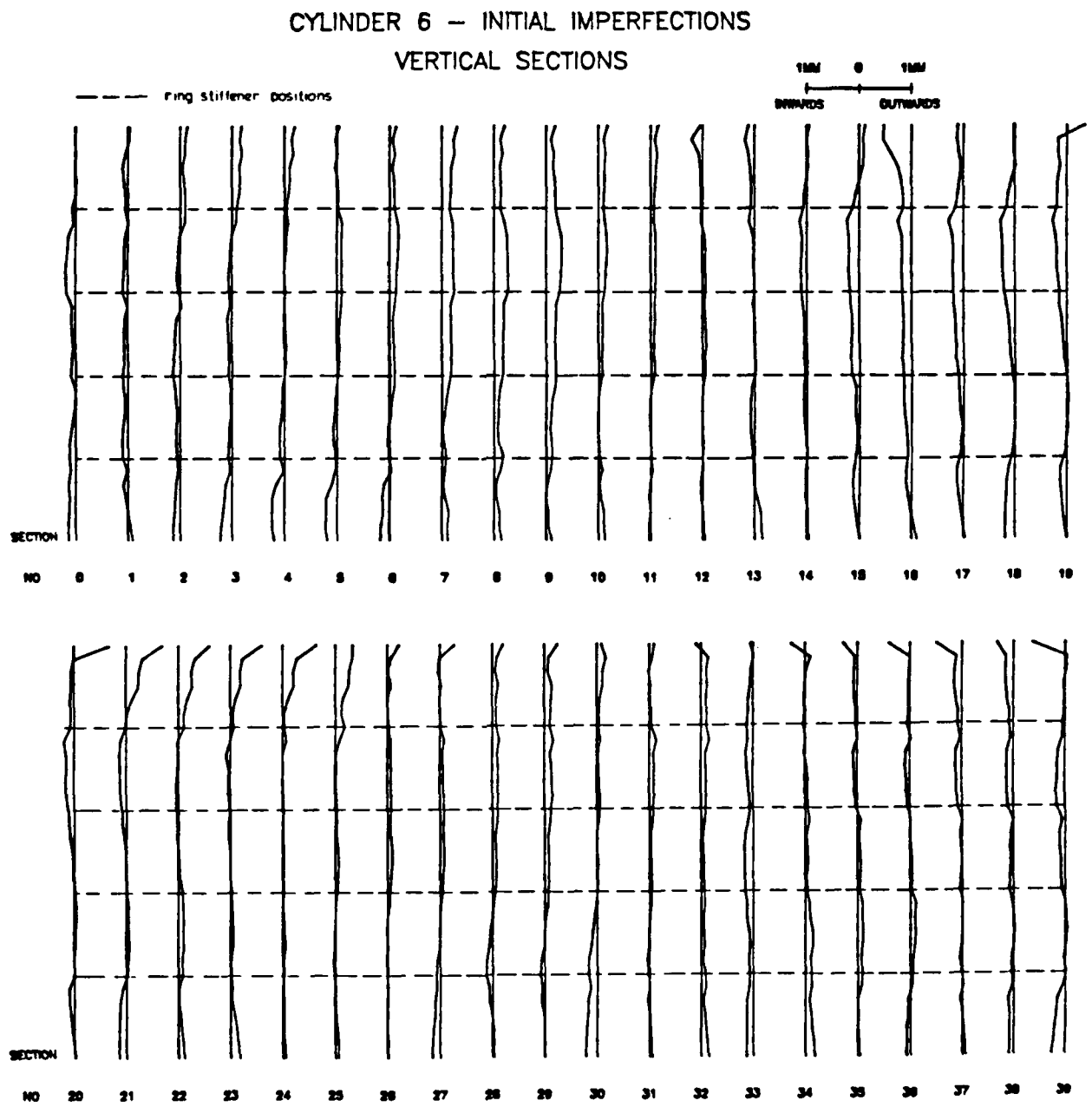


Fig. 3.131

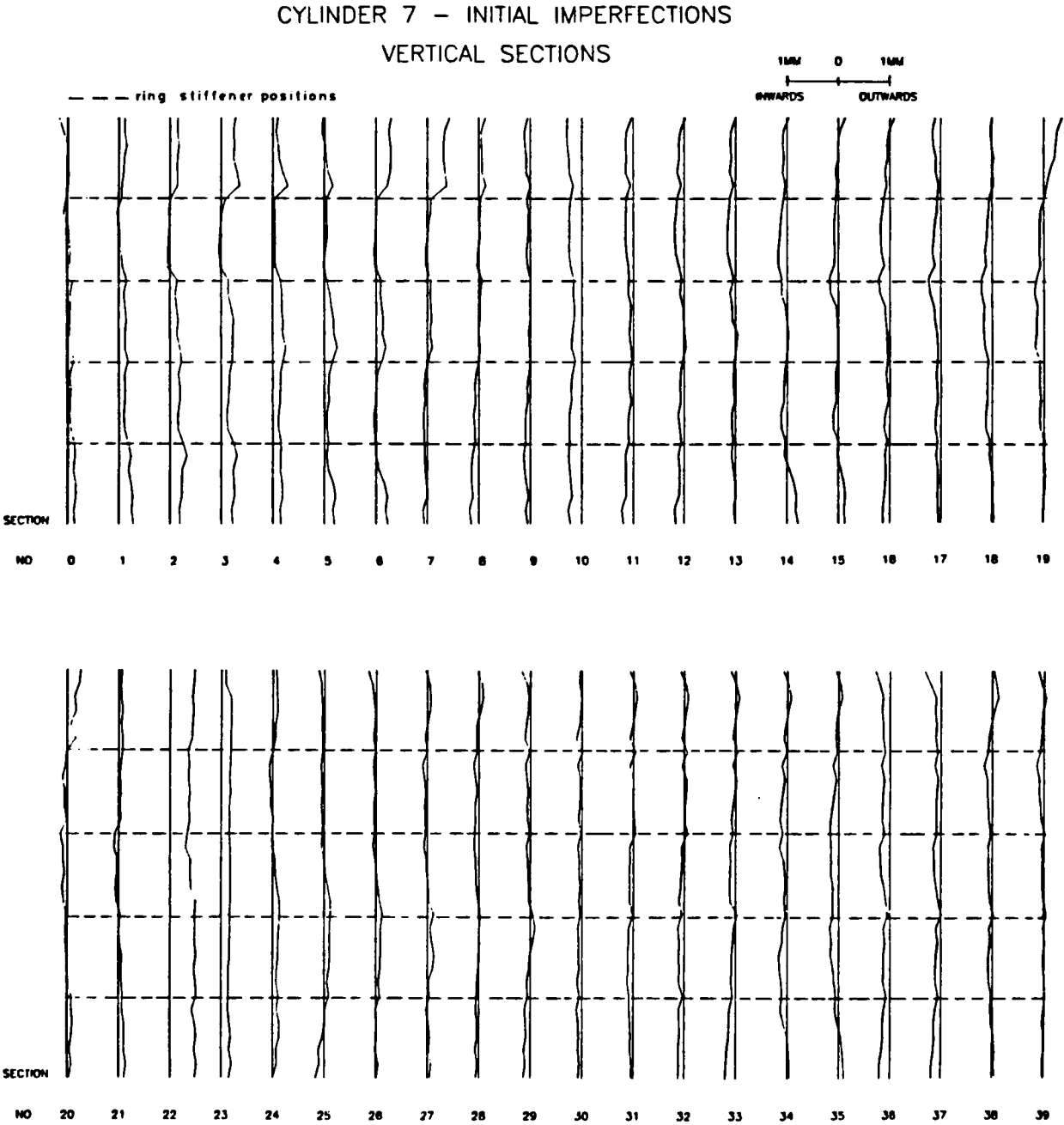


Fig.3.132

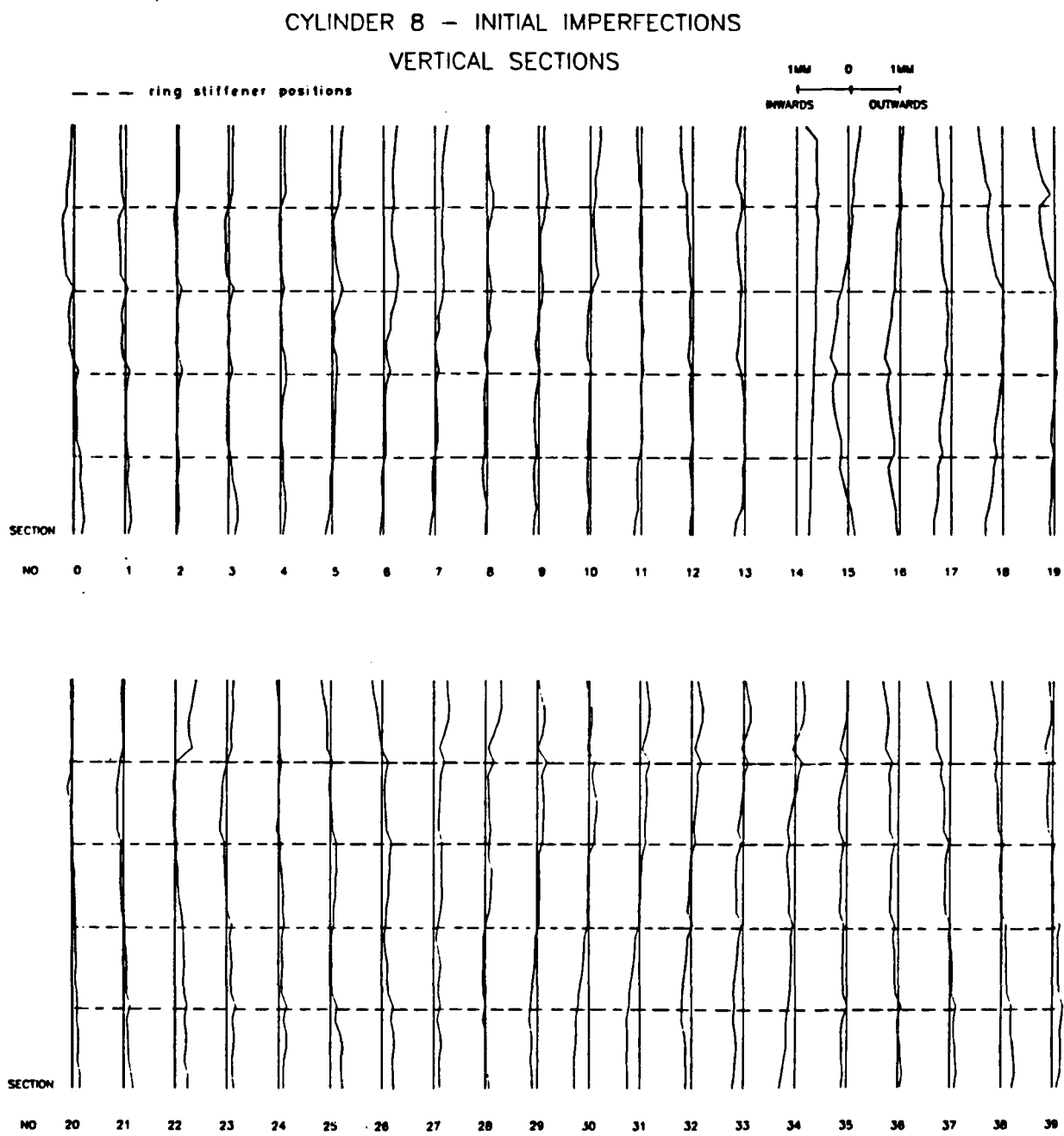
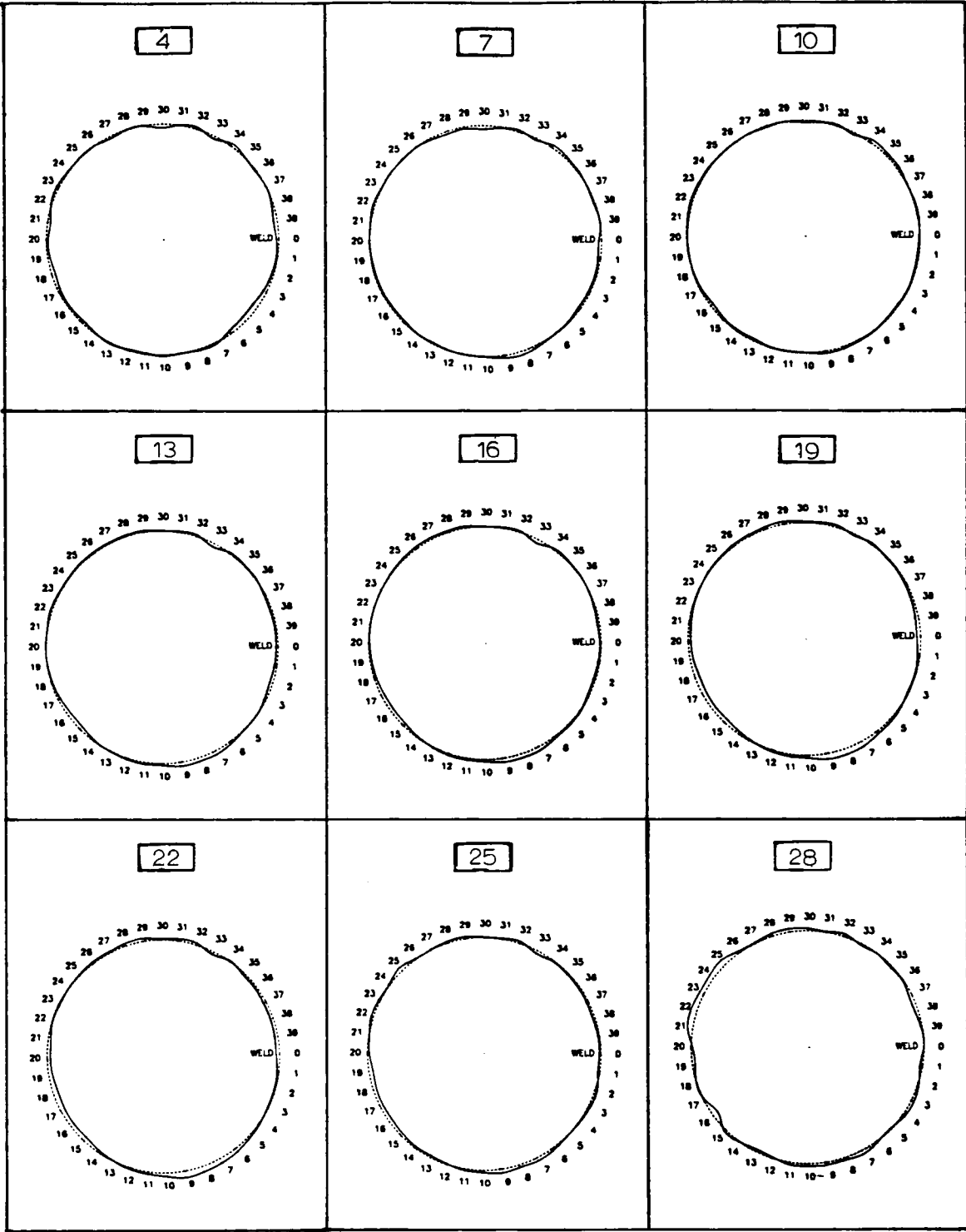


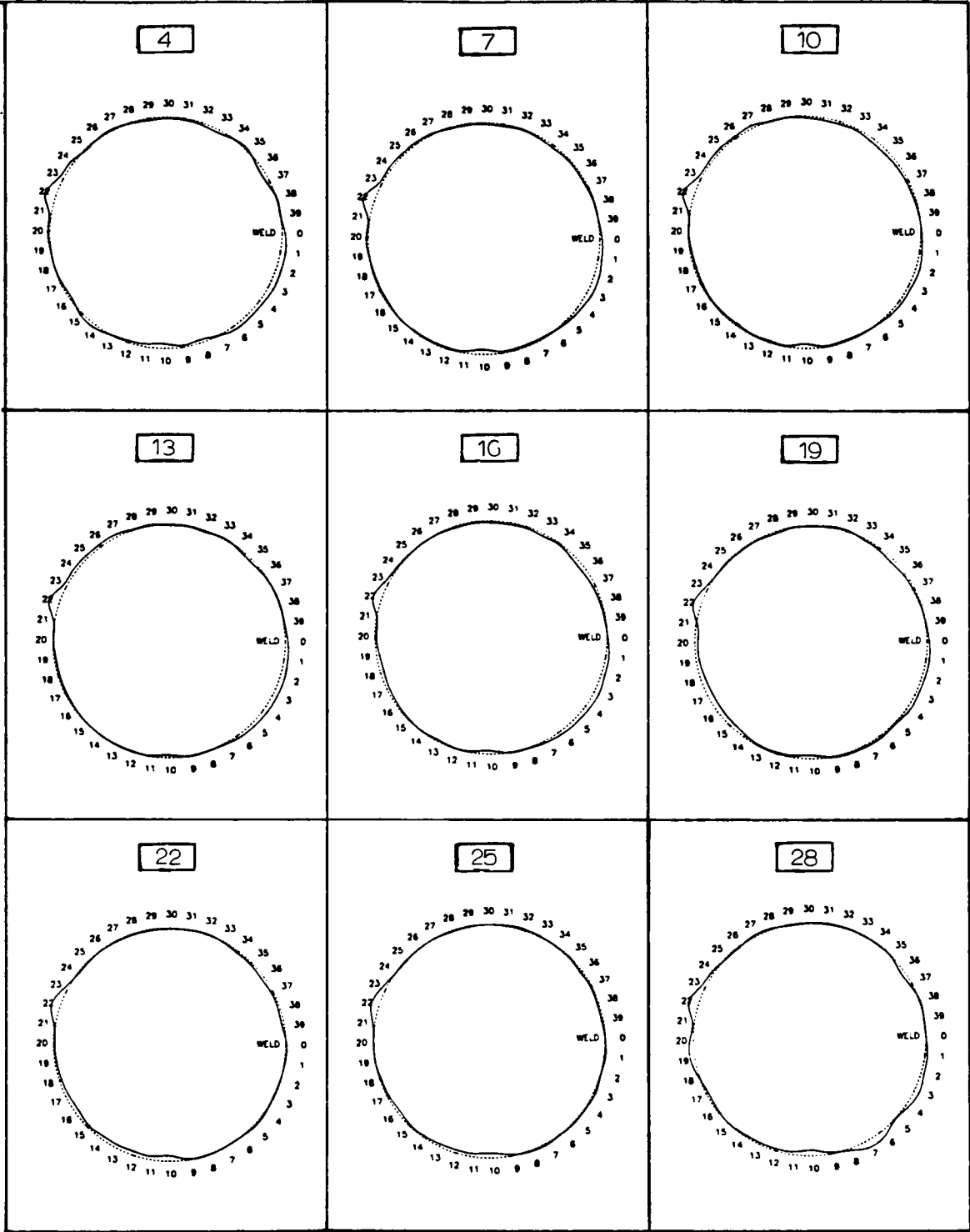
Fig. 3.133



 Vertical section

Imperfection magnification = 30

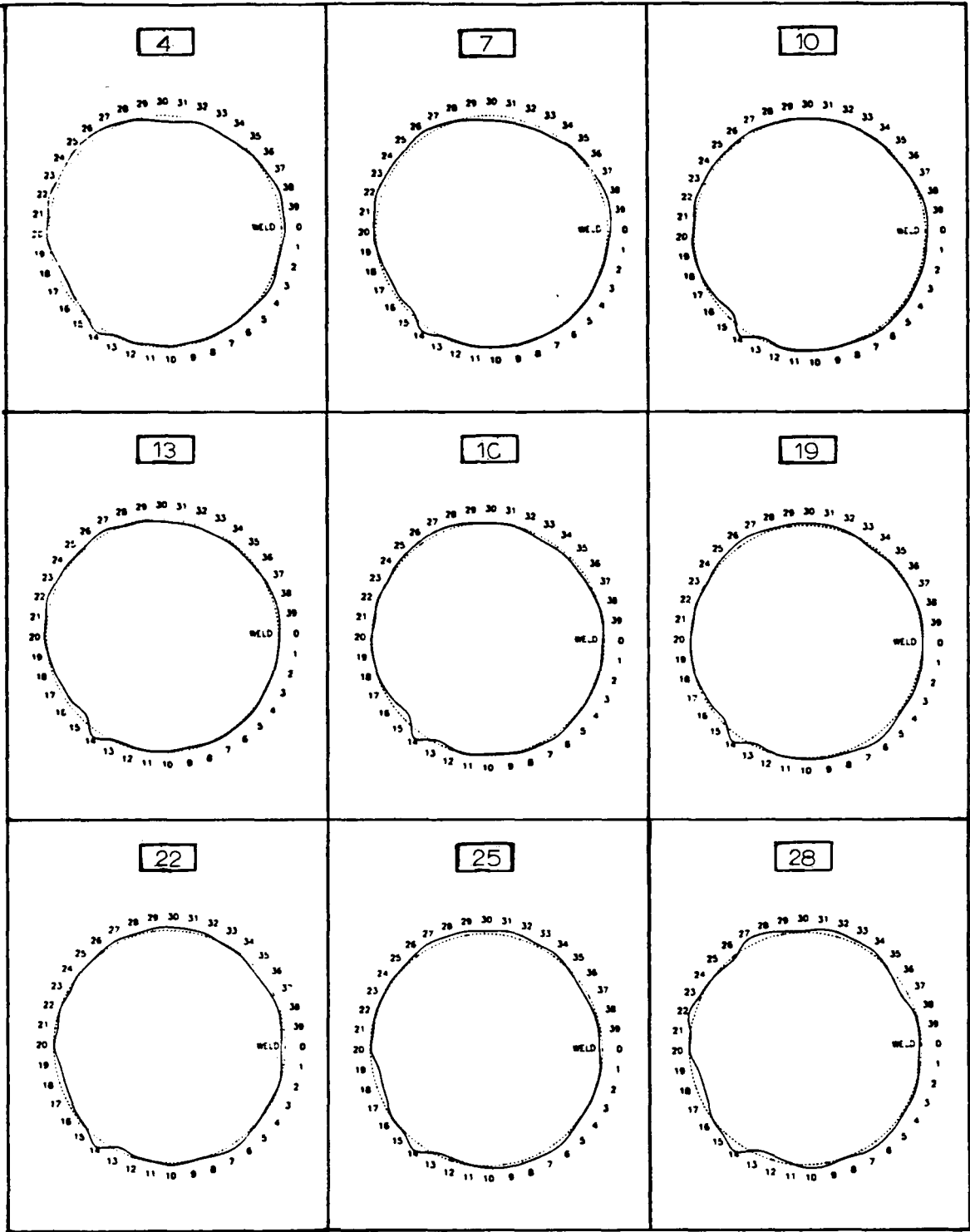
Fig 3.134 Imperfections of cylinder 6



 Vertical section

Imperfection magnification = 30

Fig. 3.135 Imperfections of cylinder 7



 Vertical section

Imperfection magnification = 30

Fig. 3.136 Imperfections of cylinder 8

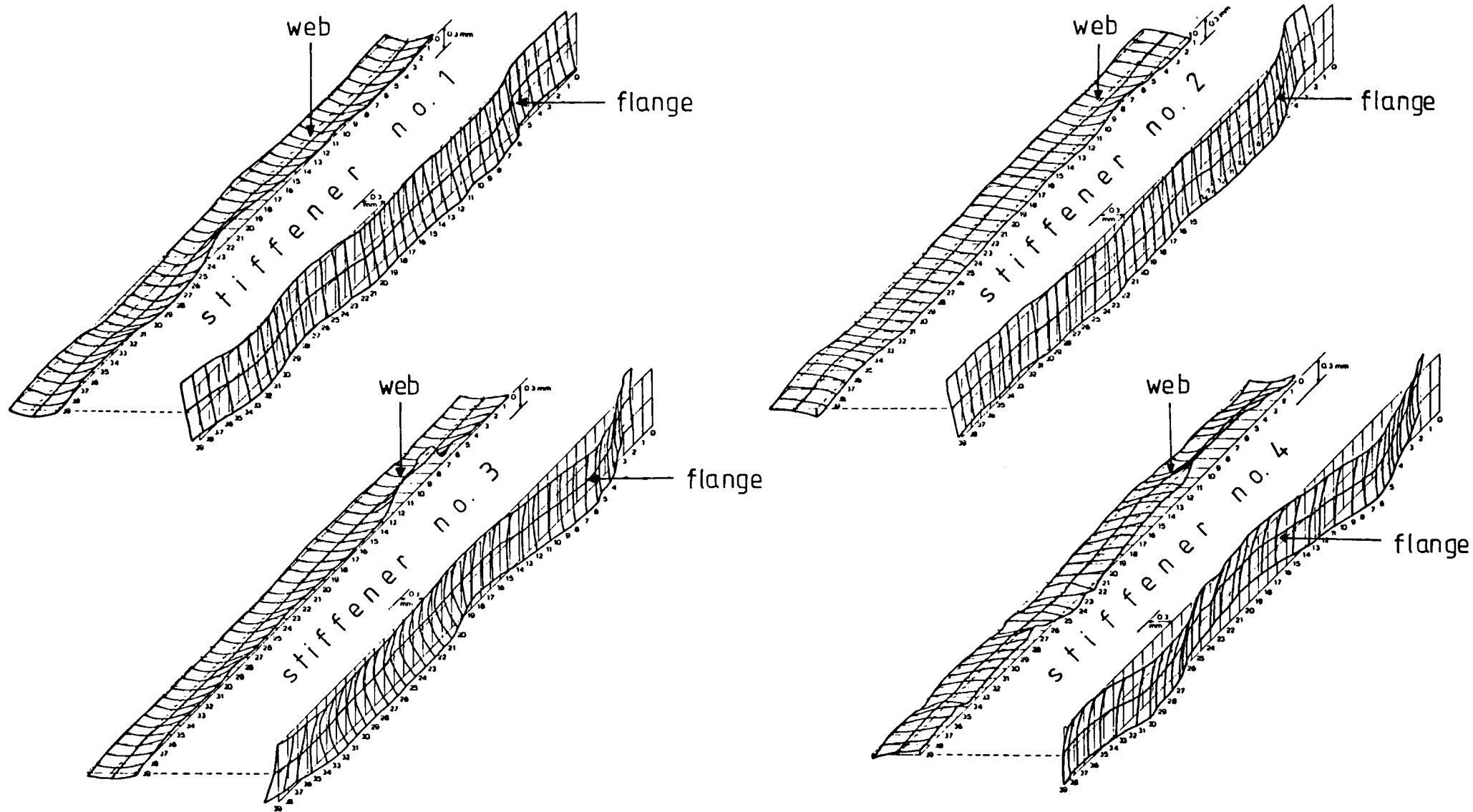


Fig. 3.137 Stiffener imperfections of cylinder 6

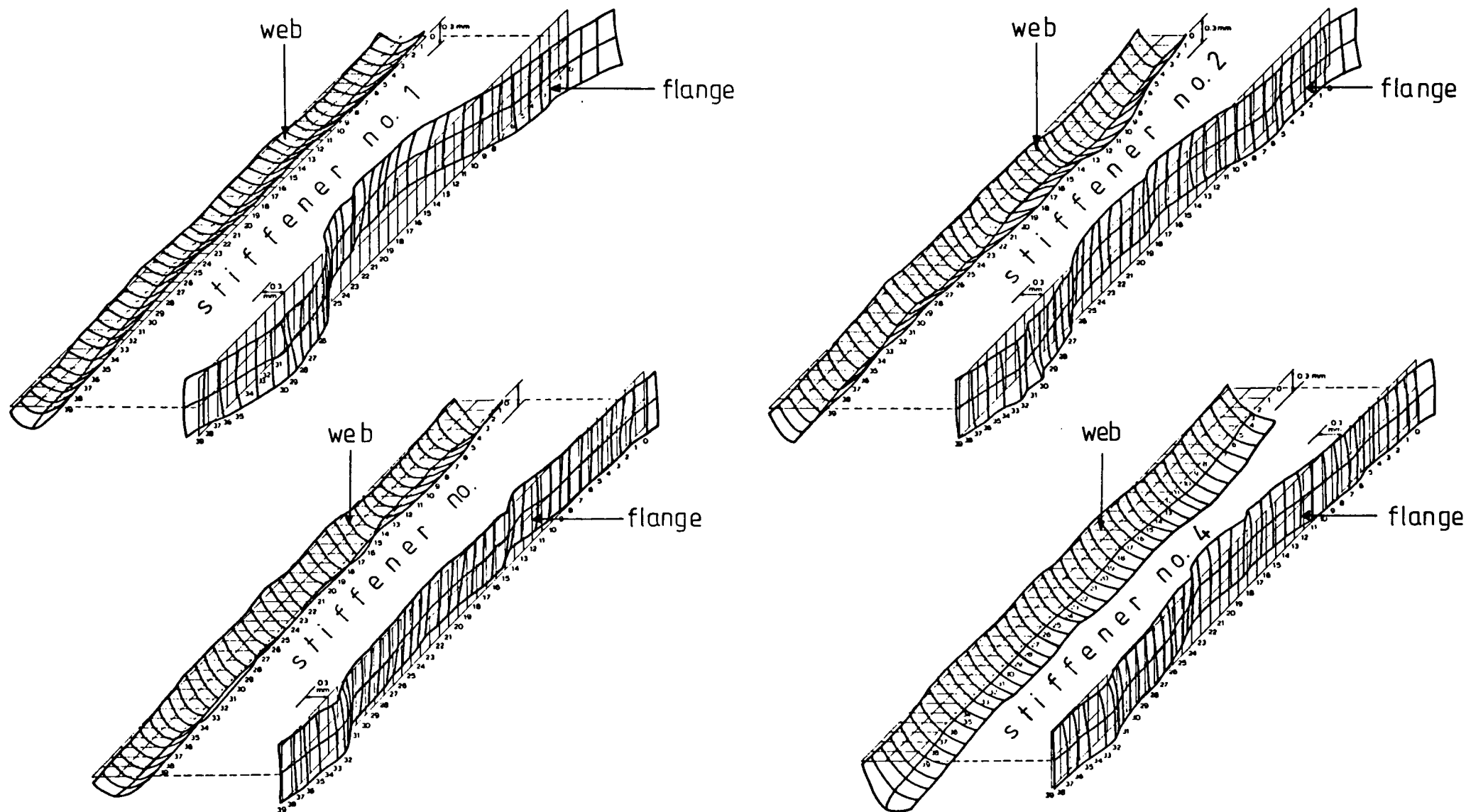


Fig. 3.138 Stiffener imperfections of cylinder 7

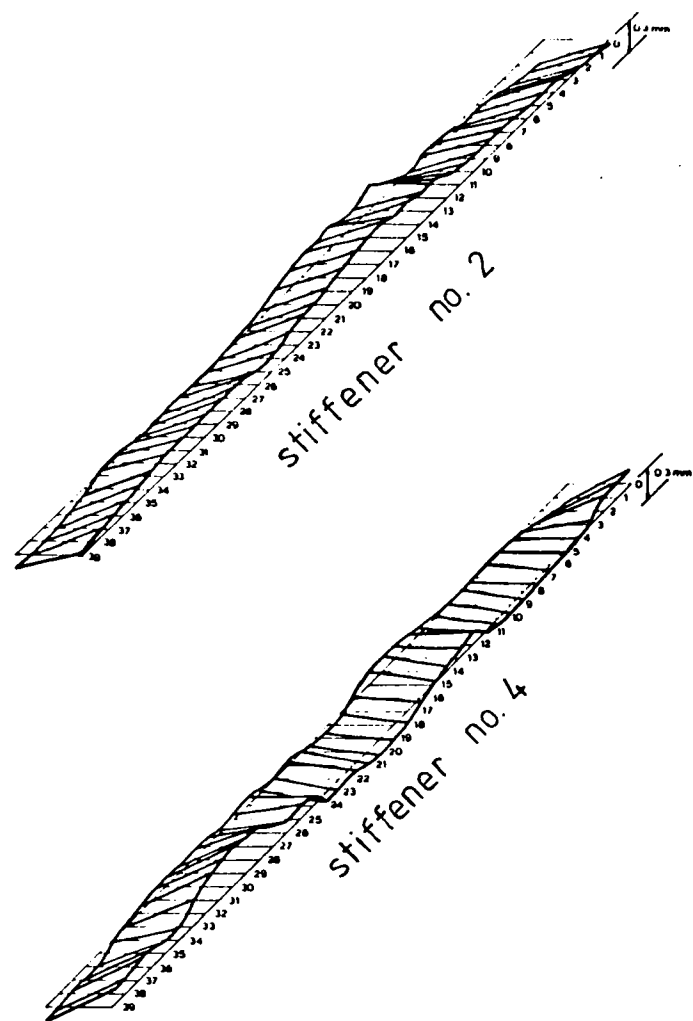
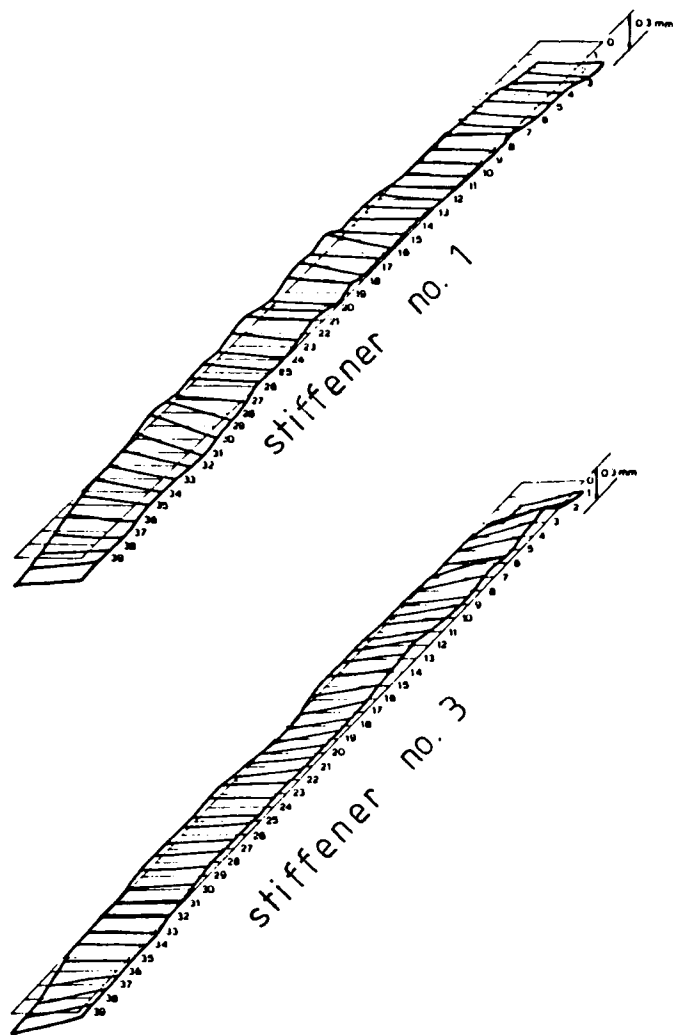


Fig. 3.139 Stiffener imperfections of cylinder 8

CYLINDER 6 (HYDROSTATIC PRESSURE)
Longitudinal distribution of axial stress and strain

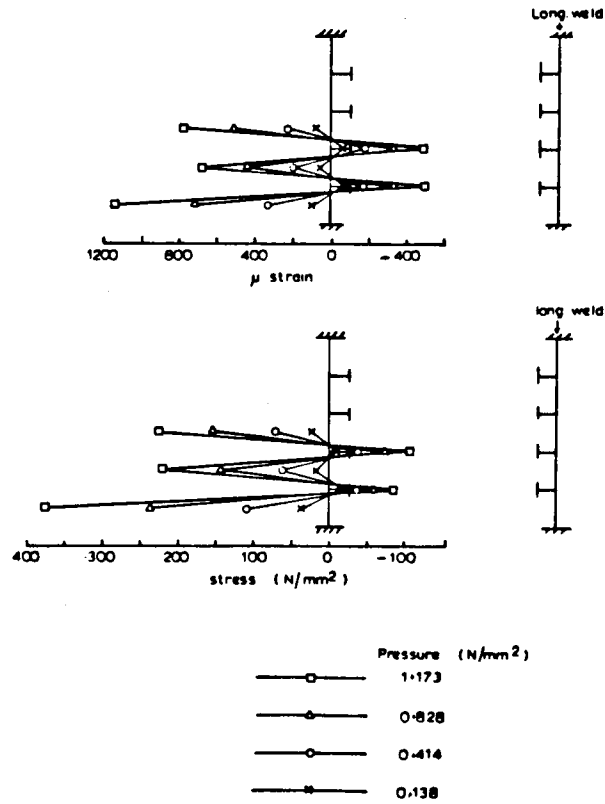


Fig. 3.140

CYLINDER 6 (HYDROSTATIC PRESSURE)
Longitudinal distribution of hoop stress and strain

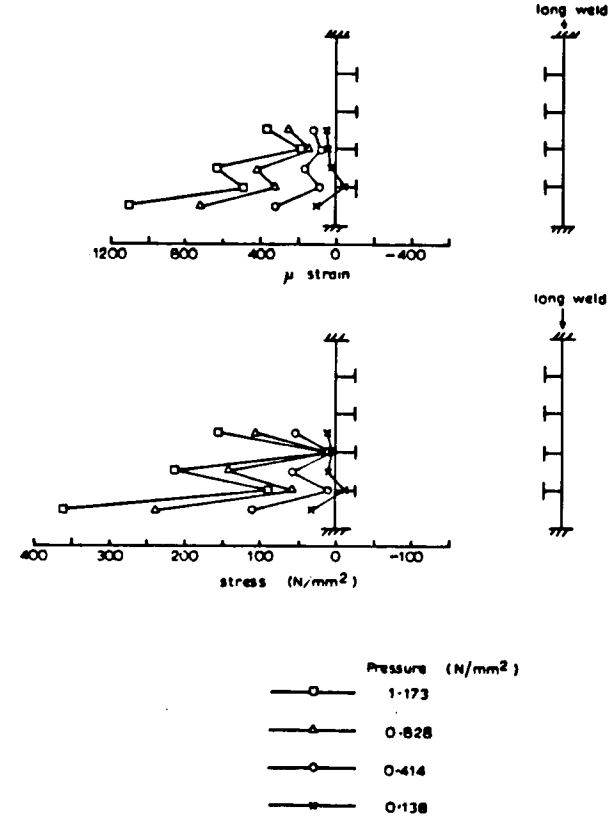


Fig 3.141

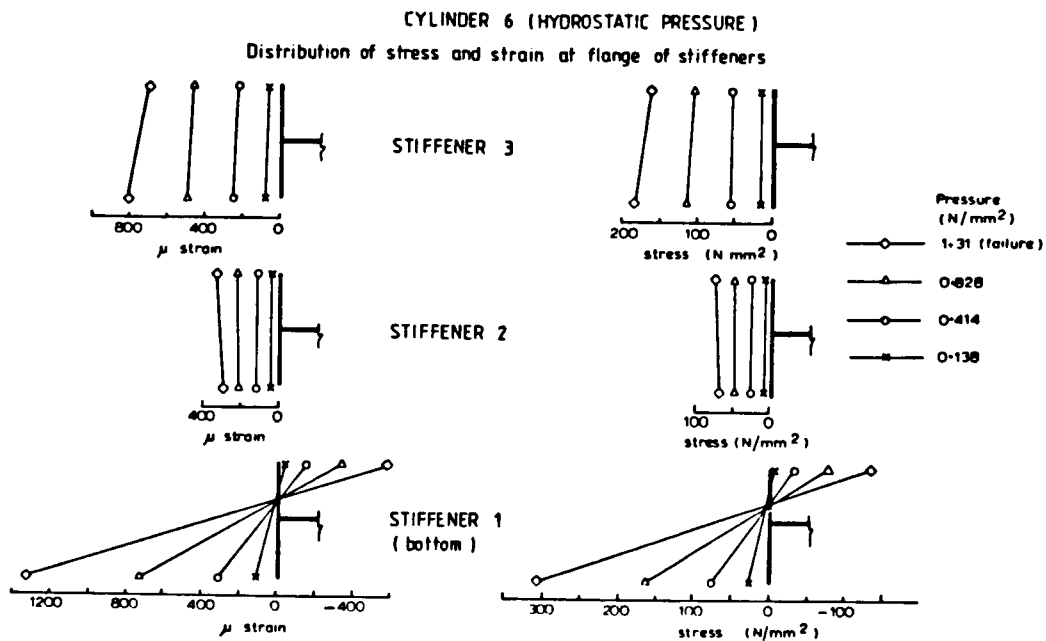


Fig. 3.142

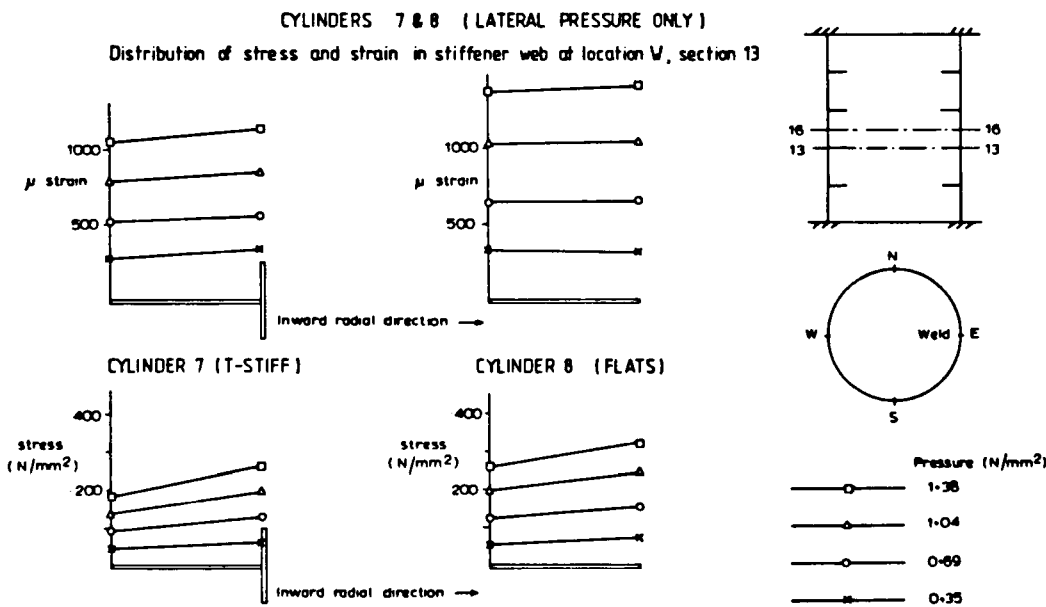


Fig 3.143

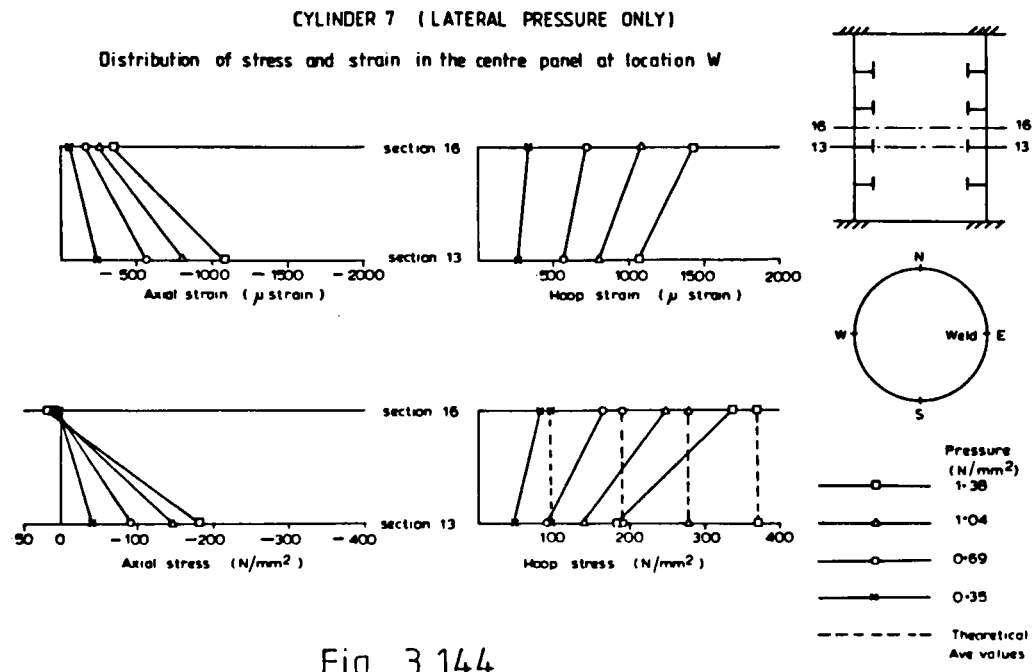


Fig. 3.144

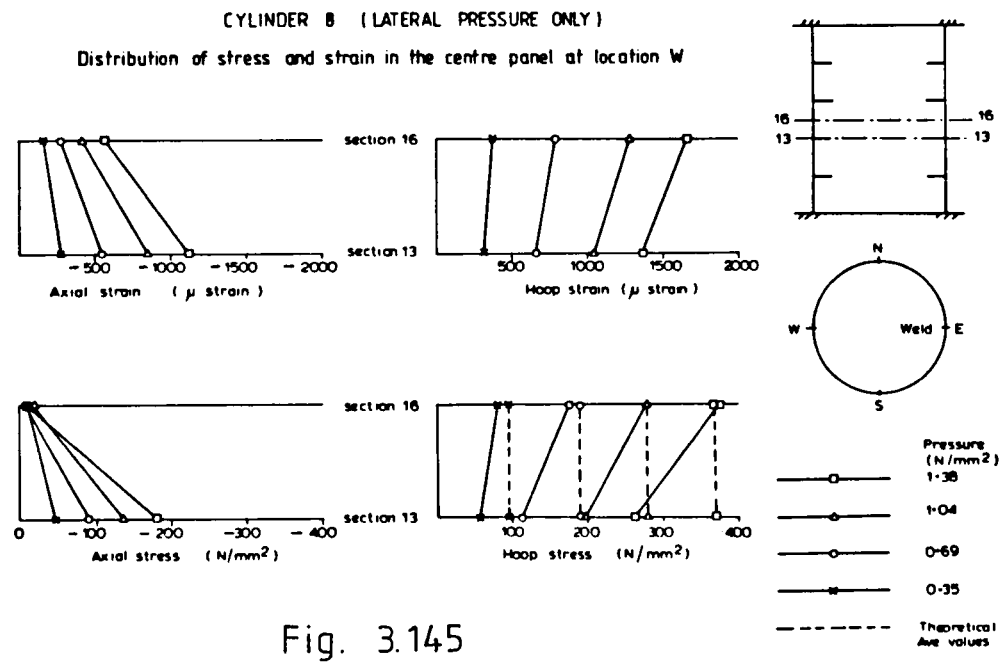


Fig. 3.145

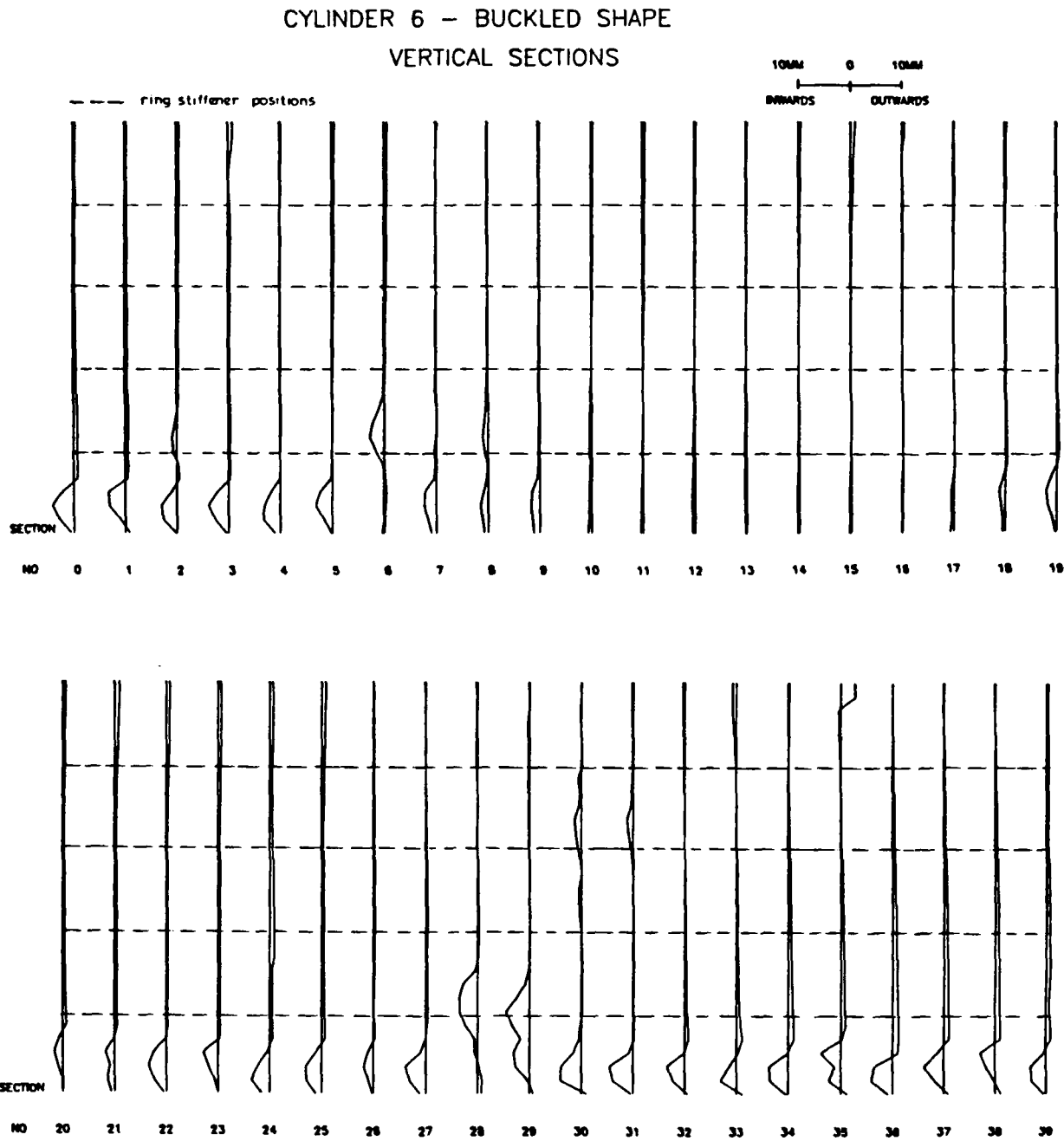


Fig. 3.146

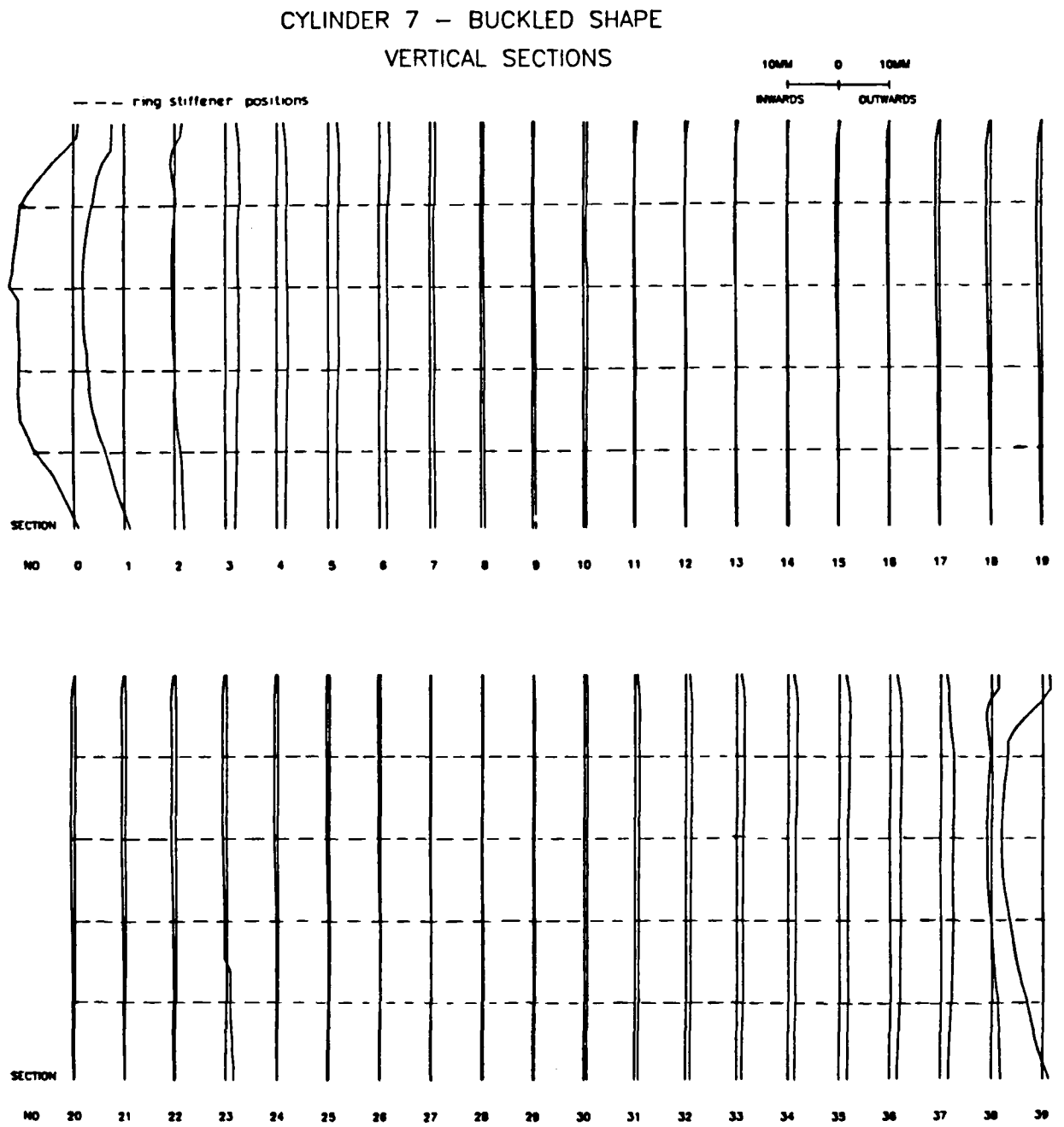


Fig. 3.147

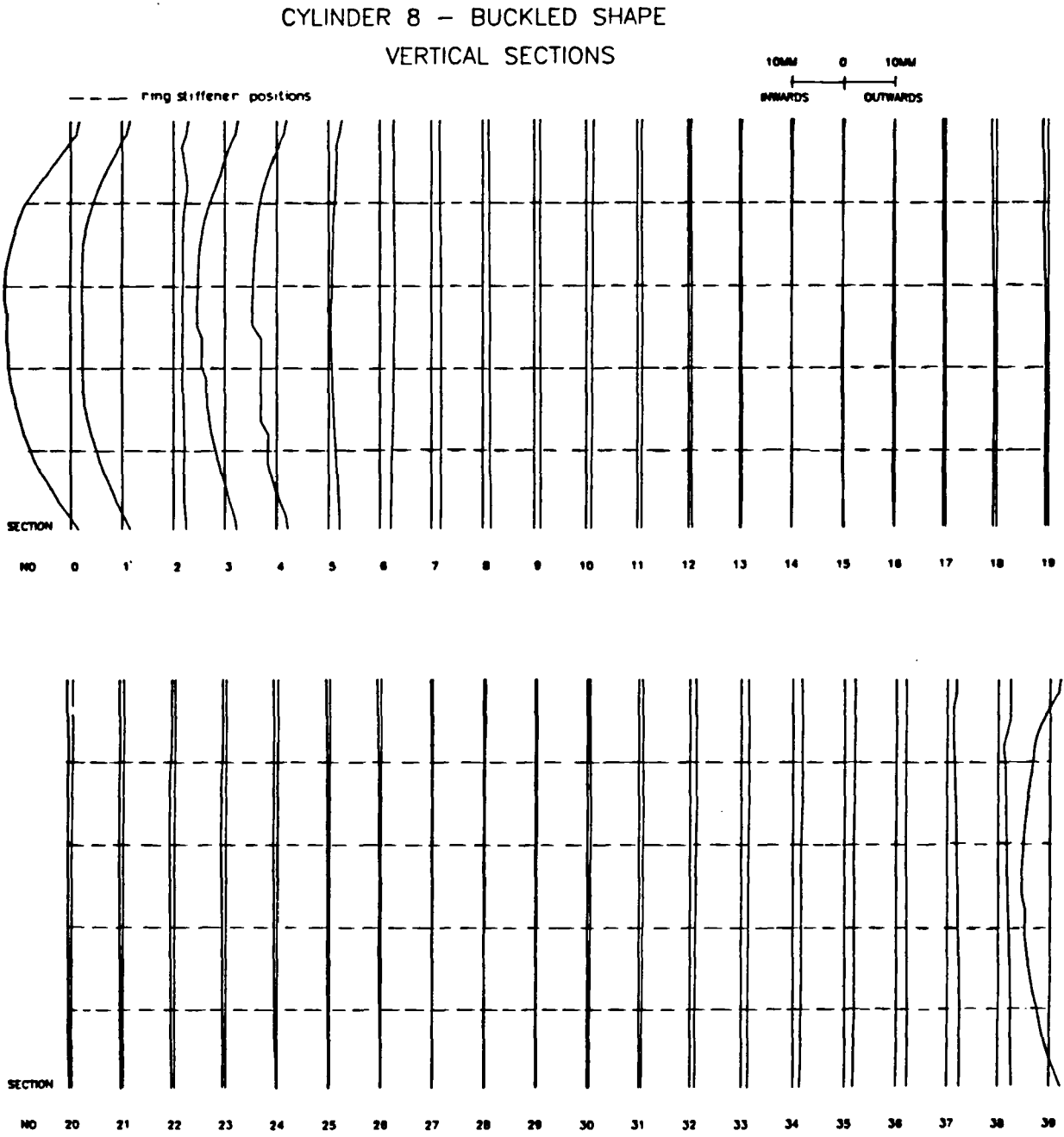
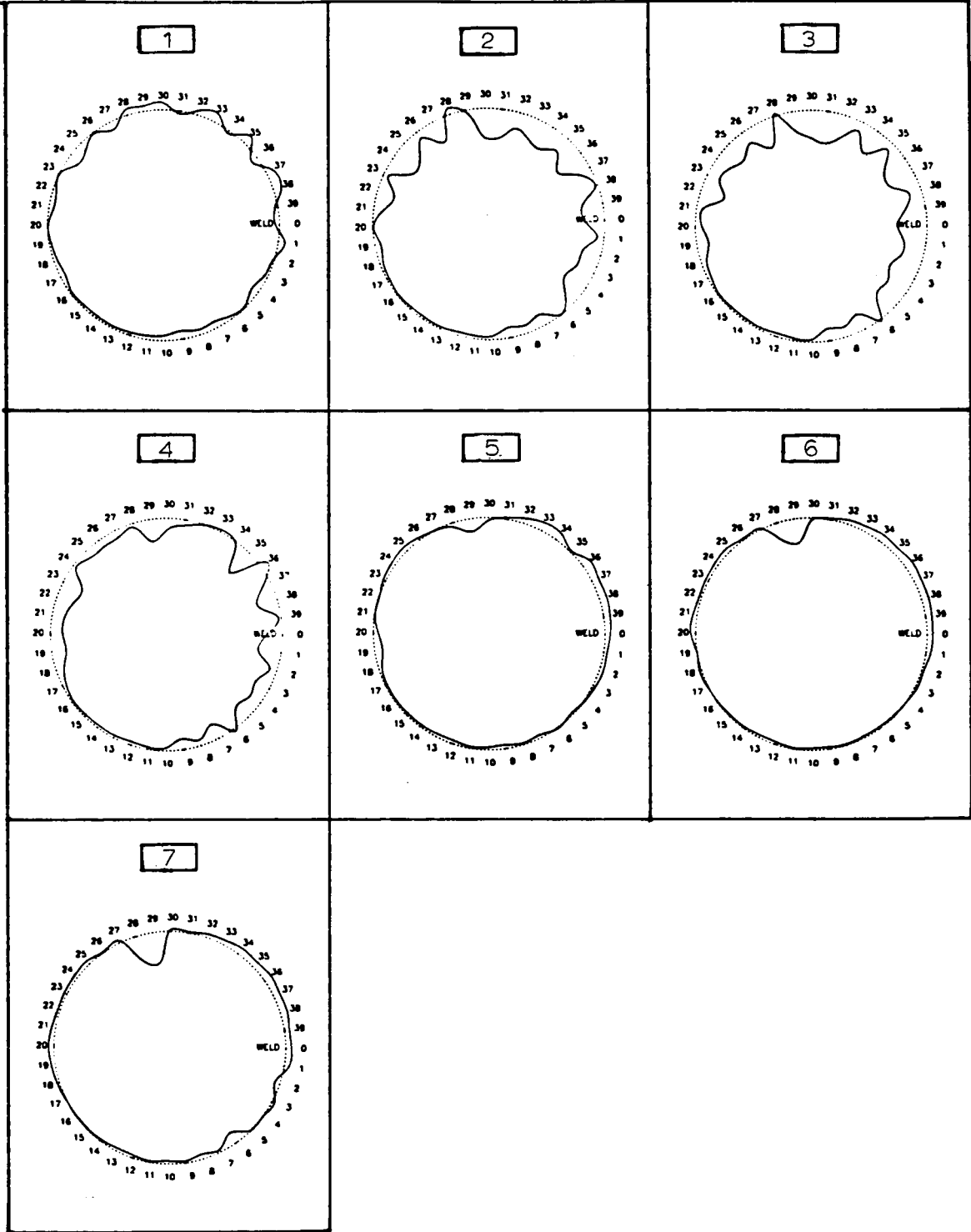


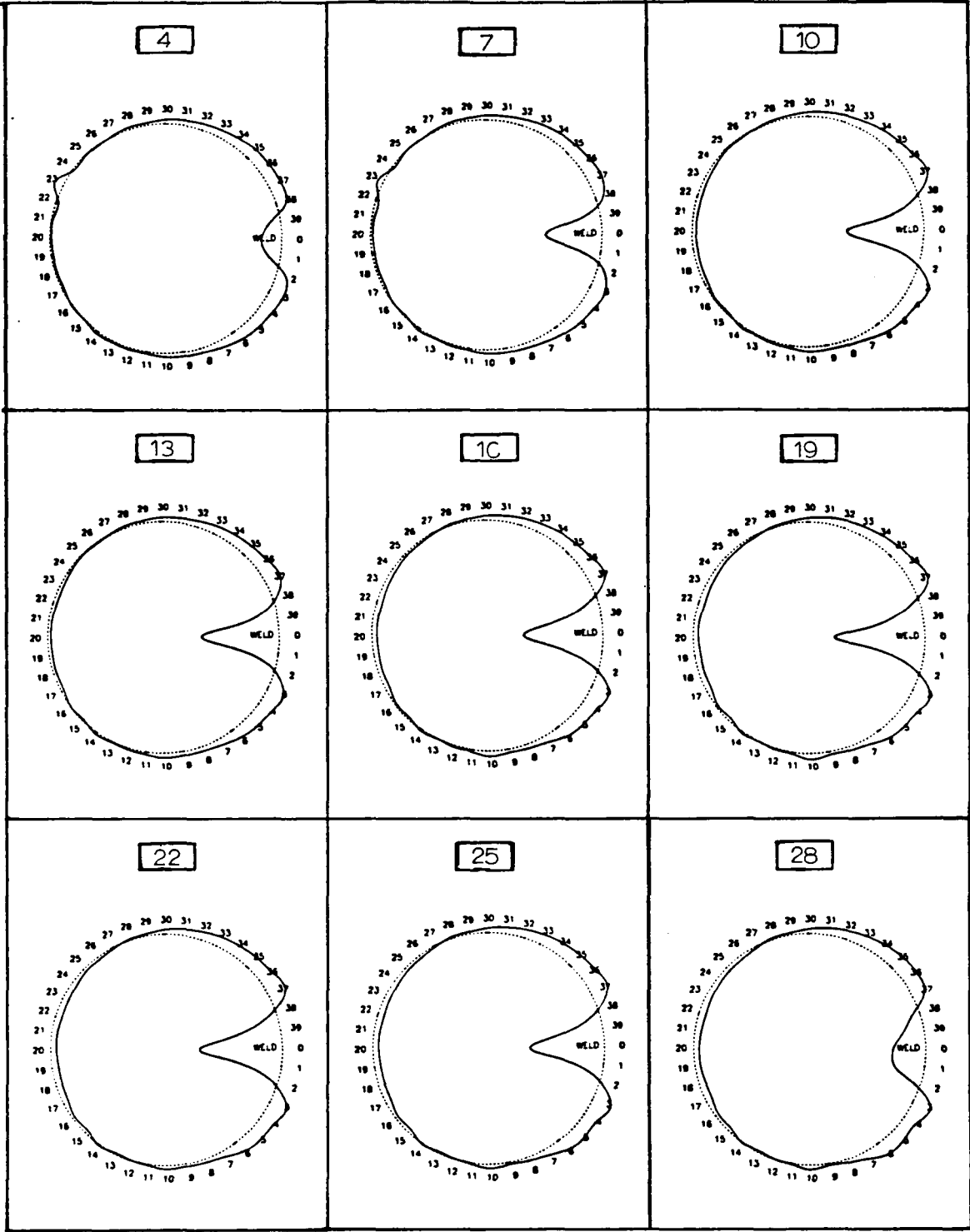
Fig. 3. 148



Vertical section

Deflection magnification = 10

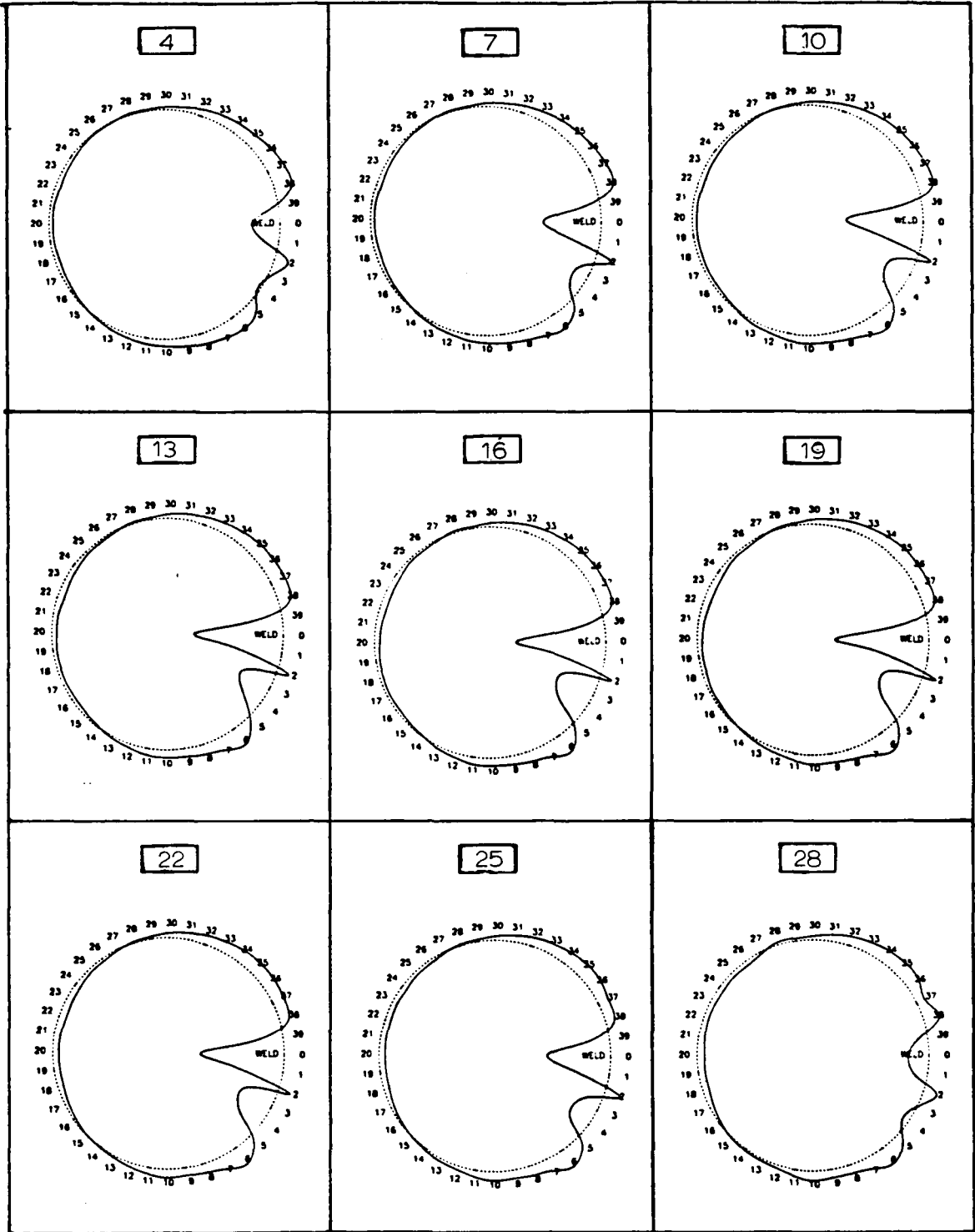
Fig. 3.149 Buckled shape of cylinder 6



 Vertical section

Deflection magnification = 10

Fig. 3.150 Buckled shape of cylinder 7



 Vertical section

Deflection magnification = 10

Fig. 3.151 Buckled shape of cylinder 8

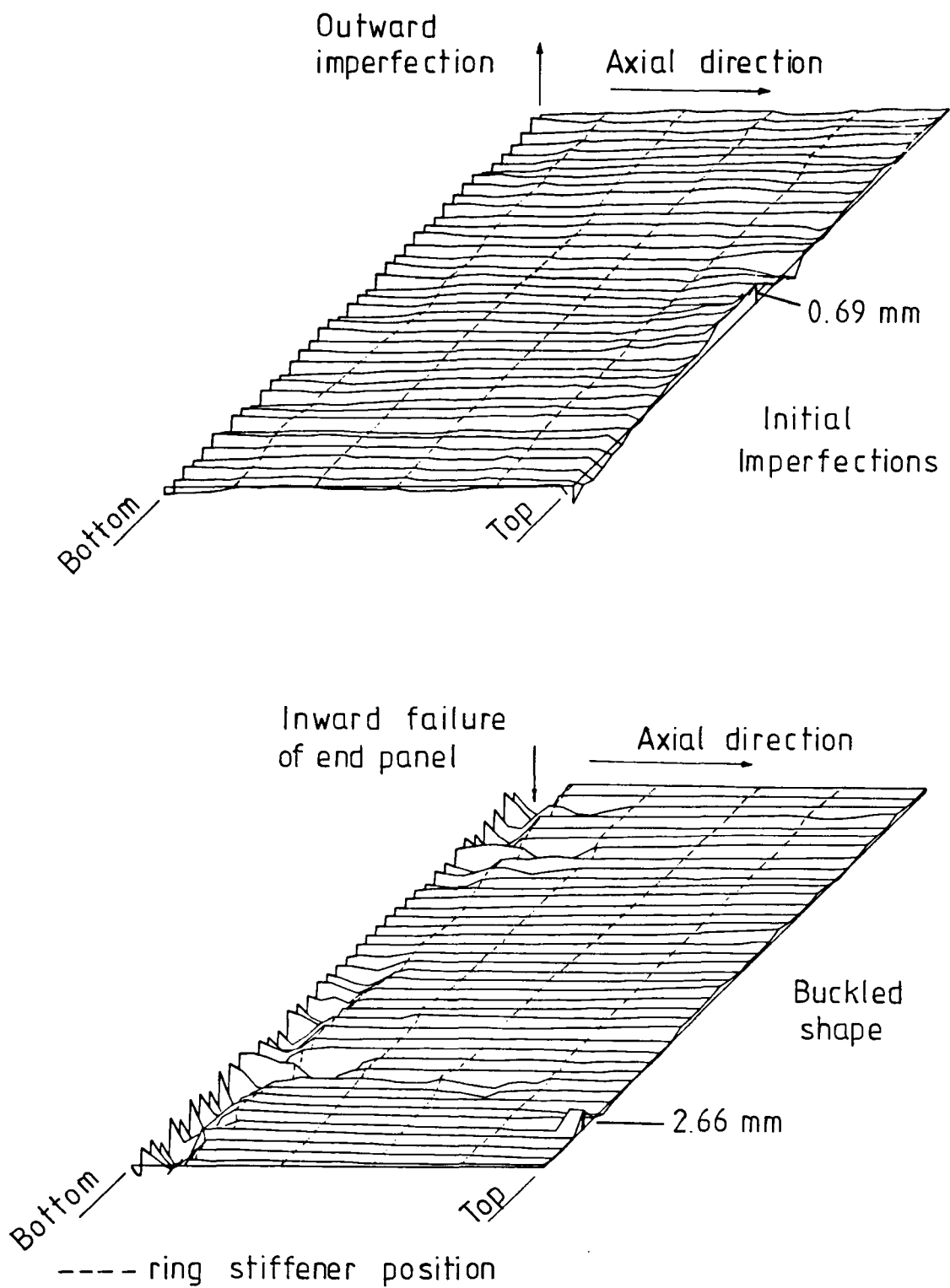


Fig 3.152 Comparison between initial imperfections and buckled shape of cylinder 6

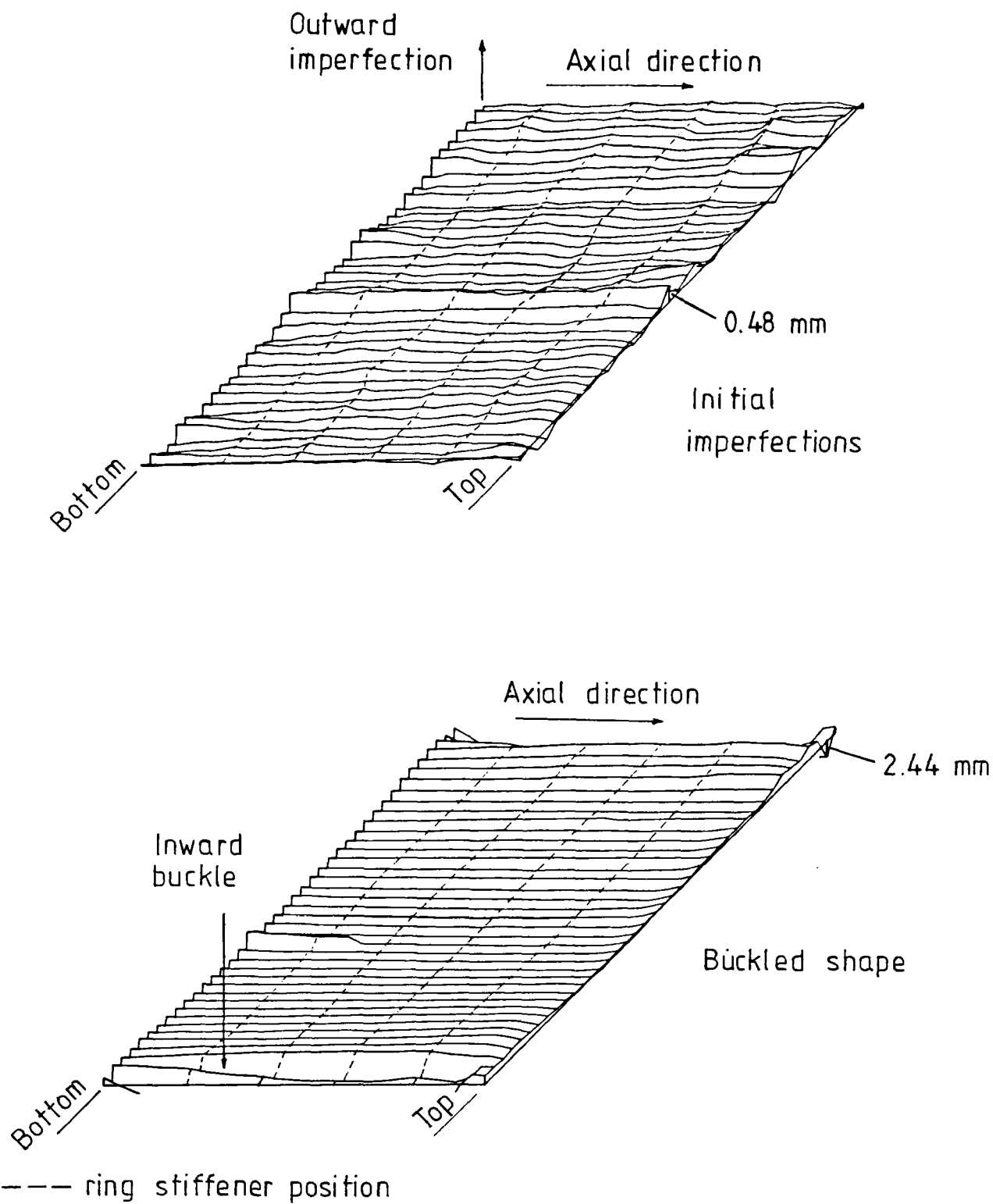


Fig. 3.153 Comparison between initial imperfections and buckled shape of cylinder 7

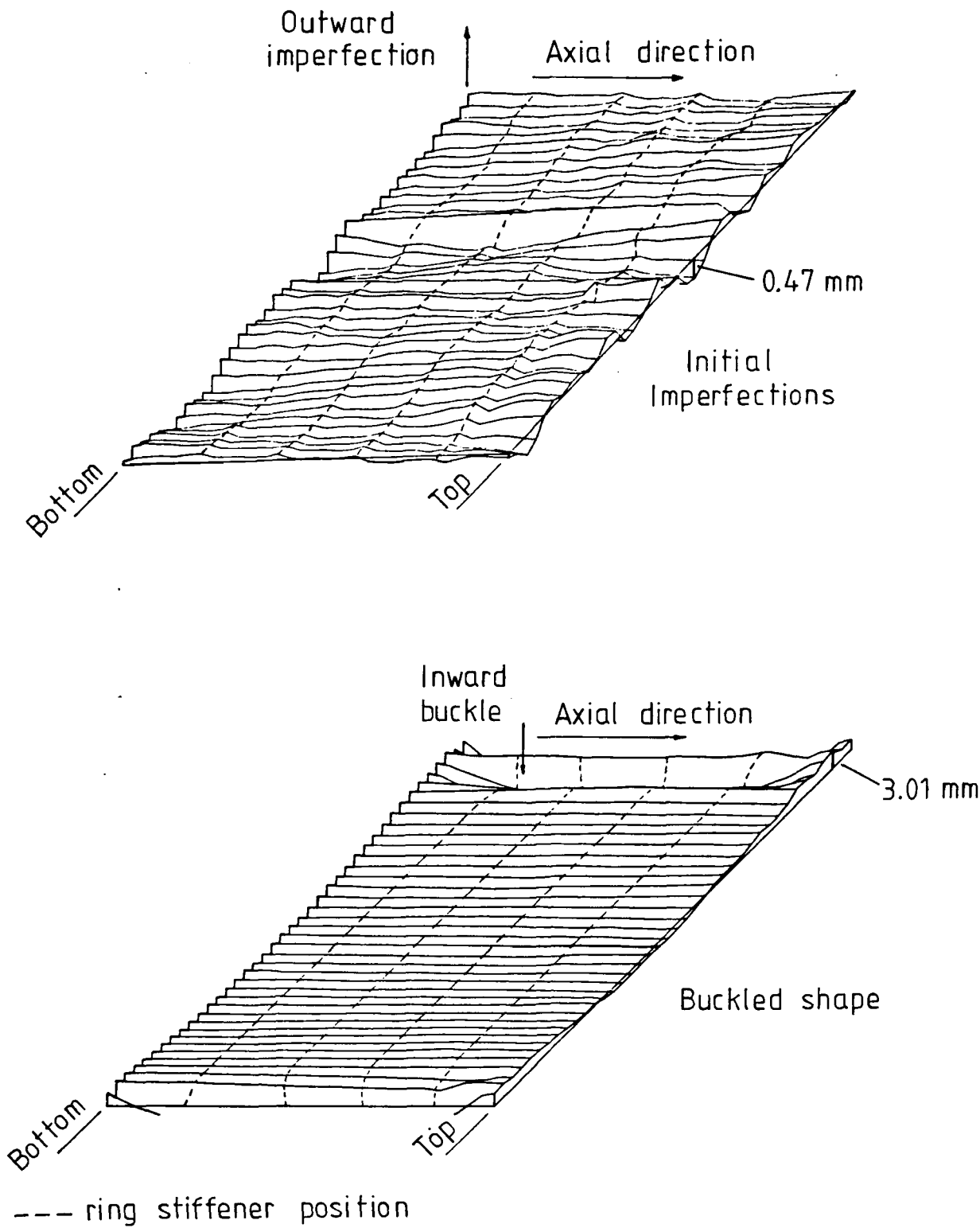


Fig. 3.154 Comparison between initial imperfections and buckled shape of cylinder 8

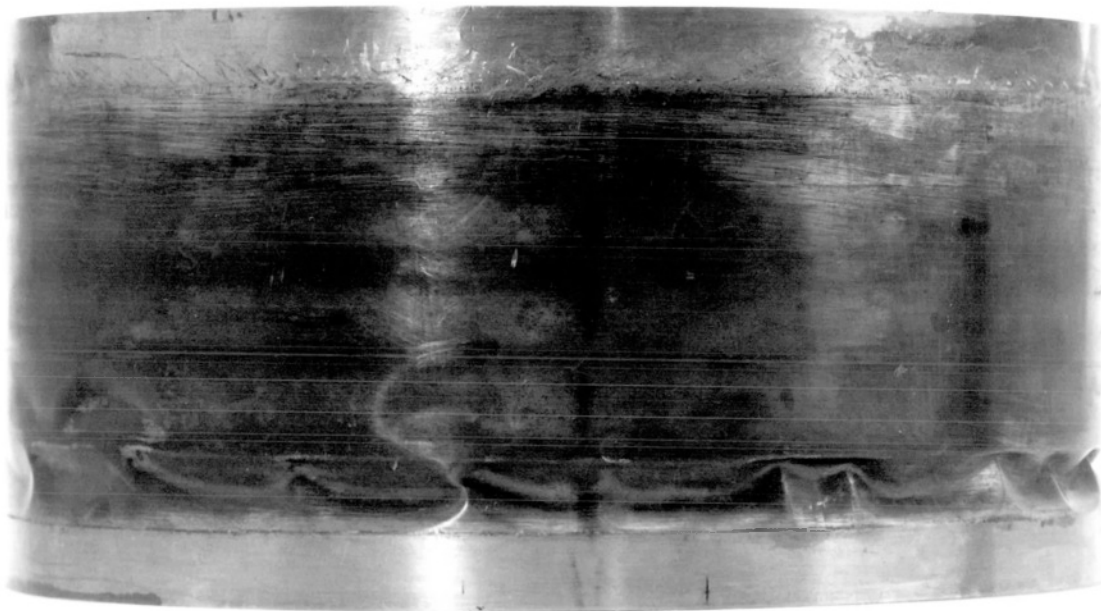


Fig. 3.155 Outside view of failure of cylinder 6.

Fig. 3.156 Inside view of failure of cylinder 6.

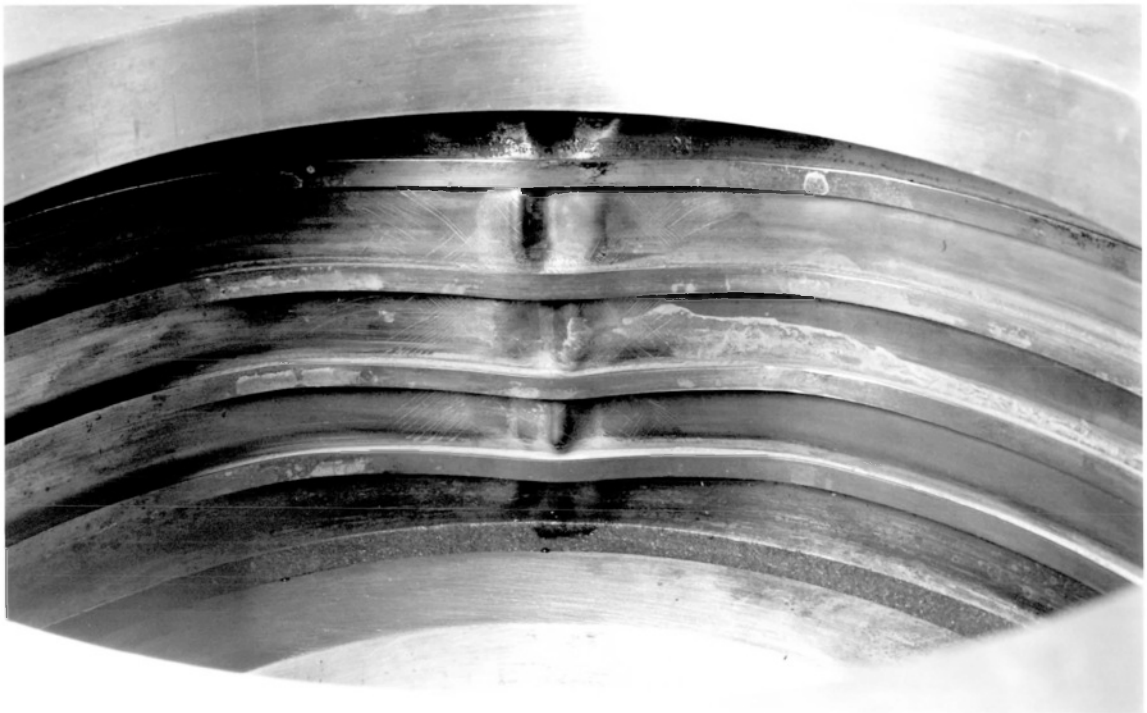
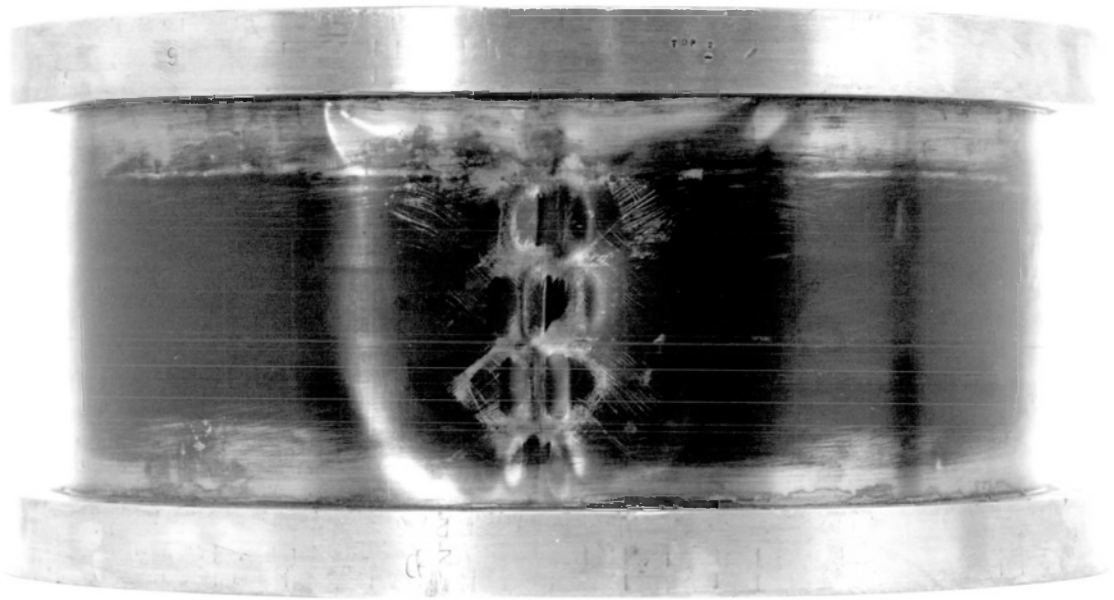


Fig. 3.157 Outside view of failure of cylinder 7.

Fig. 3.158 Inside view of failure of cylinder 7.

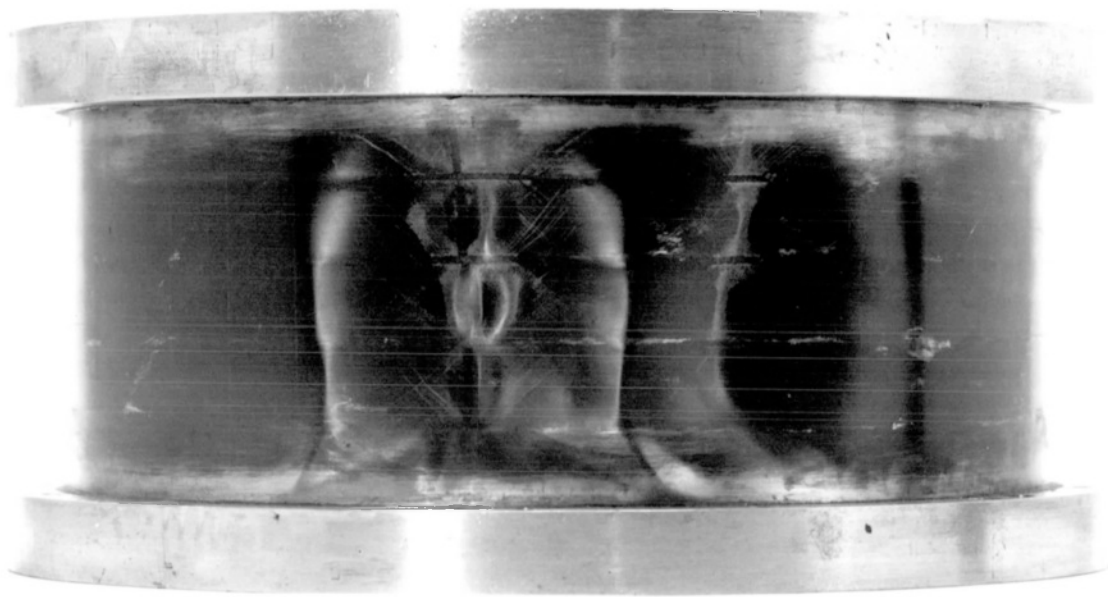
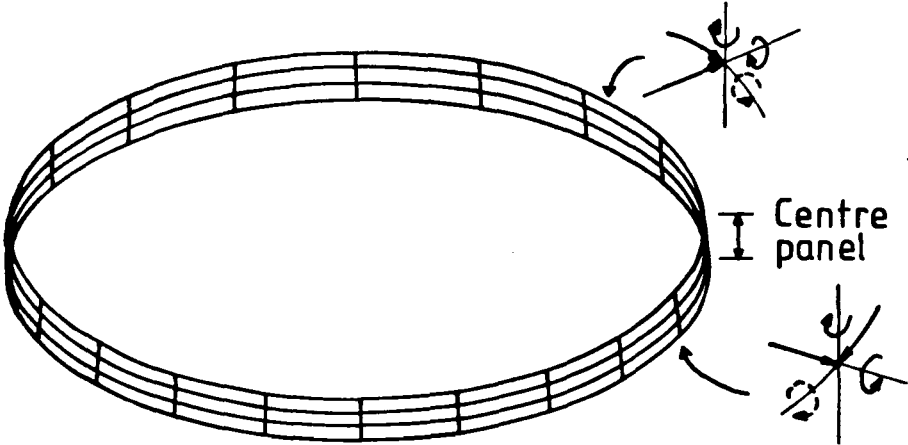
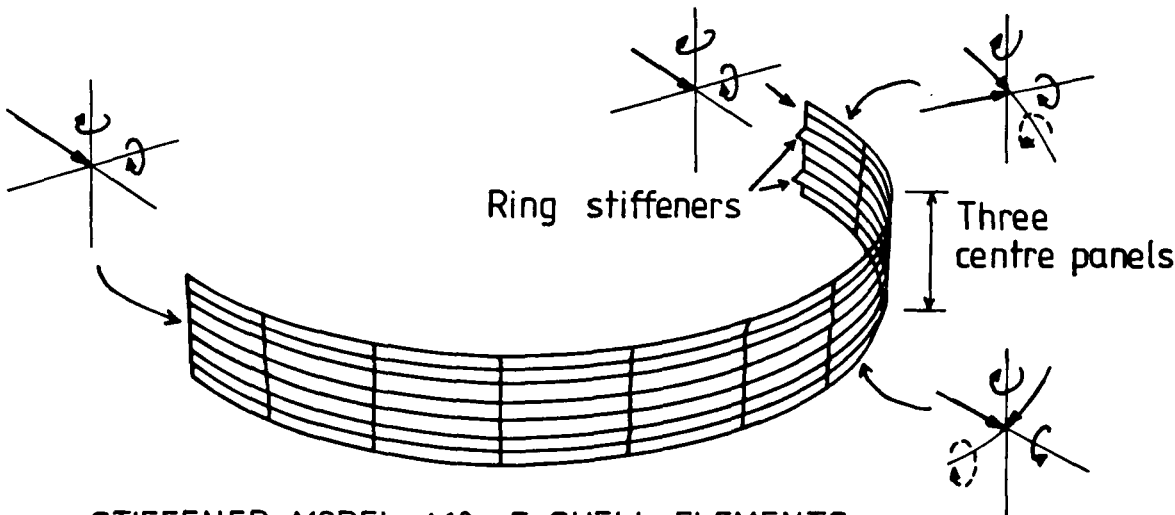


Fig. 3.159 Outside view of failure of cylinder 8.

Fig. 3.160 Inside view of failure of cylinder 8.



UNSTIFFENED MODEL (20 × 3 SHELL ELEMENTS)

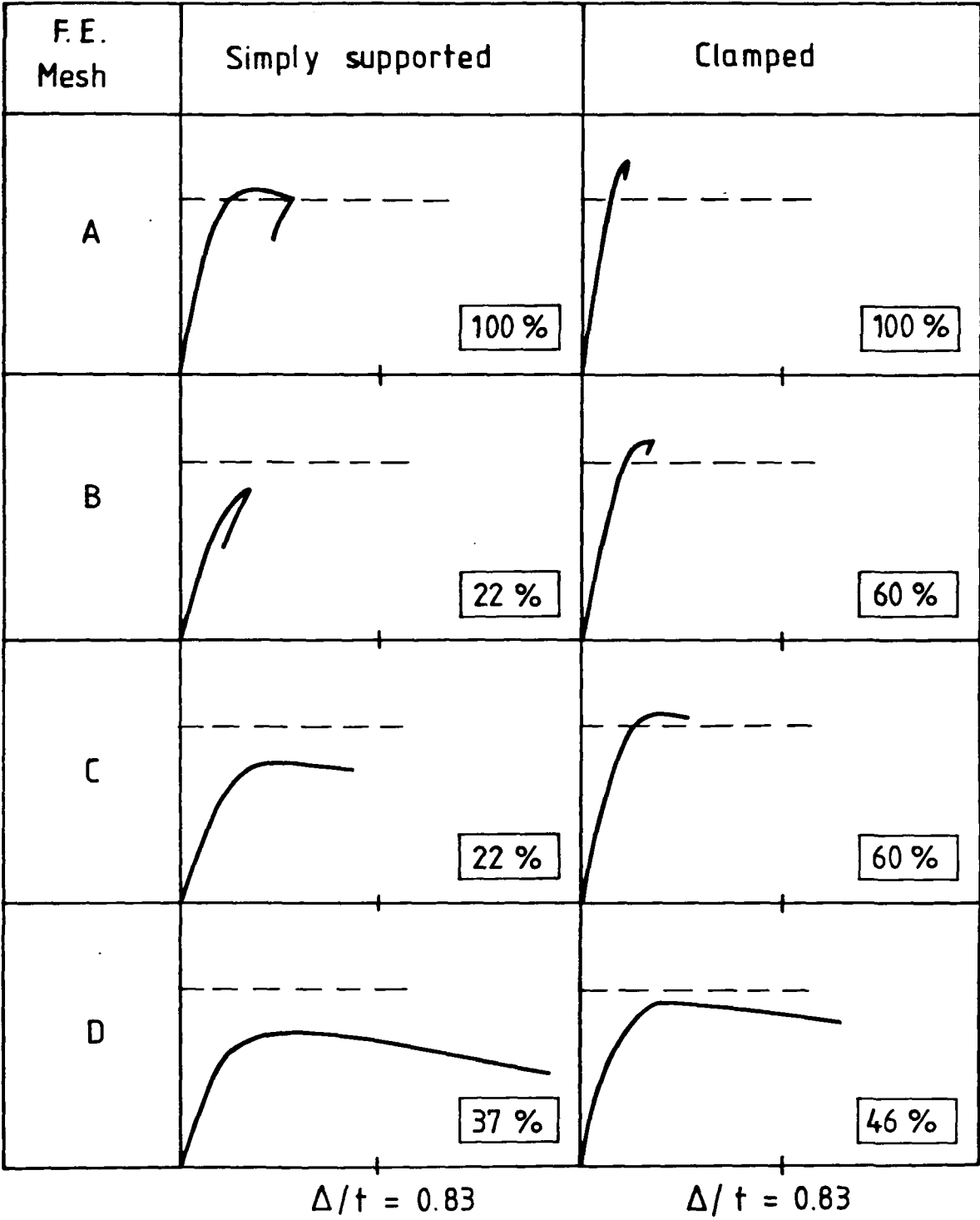


STIFFENED MODEL (10 × 7 SHELL ELEMENTS,
2 × 10 BEAM ELEMENTS)

---> denote restrained degree of freedom for clamped models
—> denote restrained degrees of freedom for all models

	MESH	INITIAL IMPERFECTIONS
UN - STIFFENED	A	Zero imperfections
	B	Actual imperfections
	C	Imperfections generated by Fourier Series
STIFFENED	D	As Mesh C

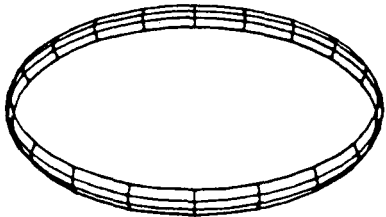
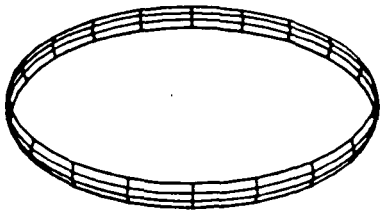
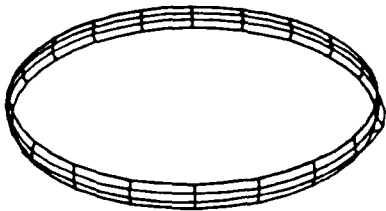
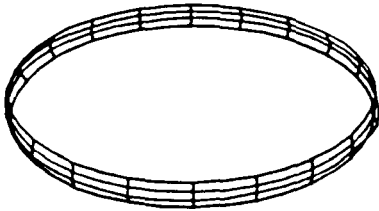
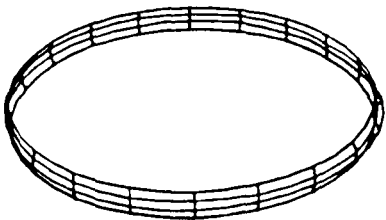
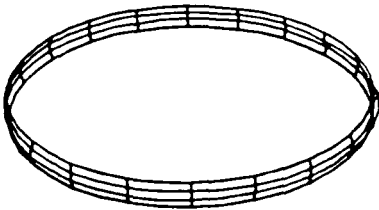
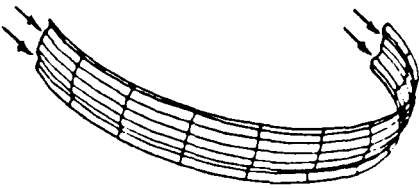
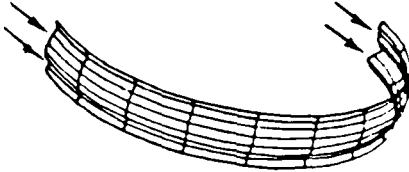
Fig 4.1 Finite element models used to analyse cylinders 1 - 5



----- Experimental failure load (= $0.69 \sigma_0$)

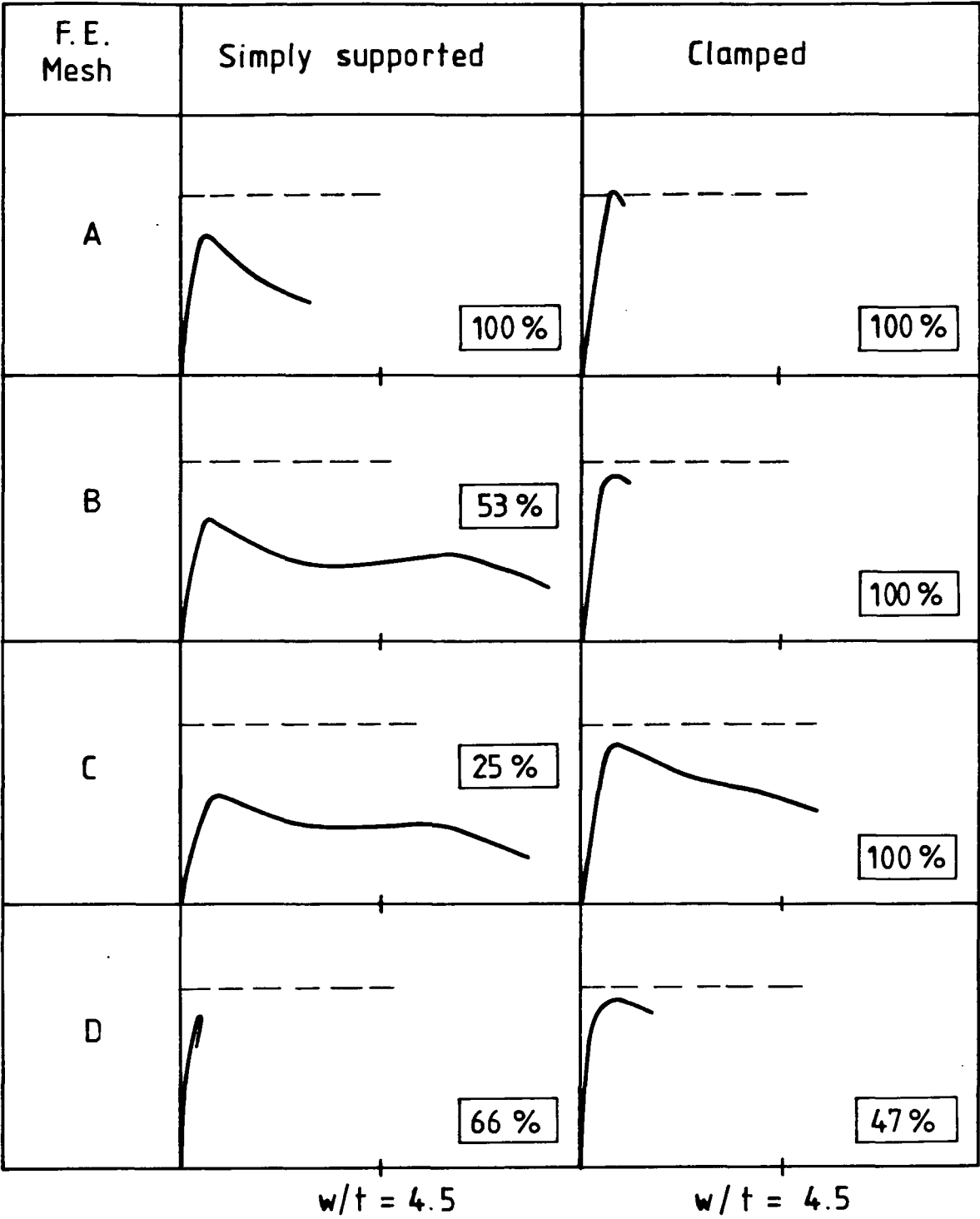
% of shell elements undergone plasticity at failure

Fig.4.2 Analytical load deflection curves of cylinder 1

F. E. Mesh	Simply supported	Clamped
A		
B		
C		
D		

Deflection magnification = 30

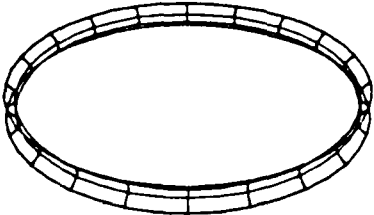
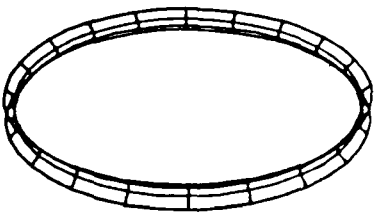
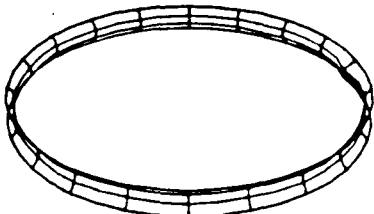
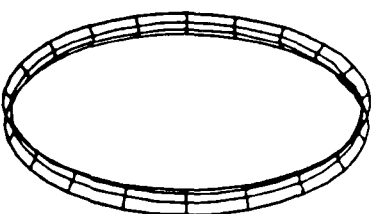
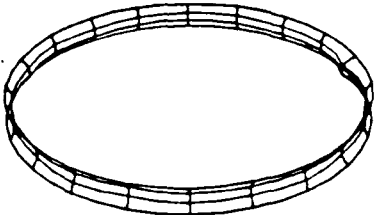
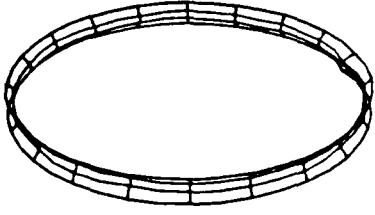
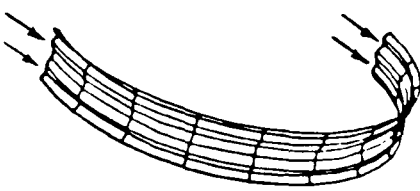
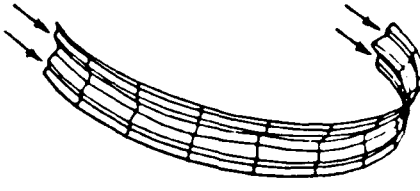
Fig 4.3 Analytical collapse modes of cylinder 1



— — — Experimental failure load (= 1.05 p_0)

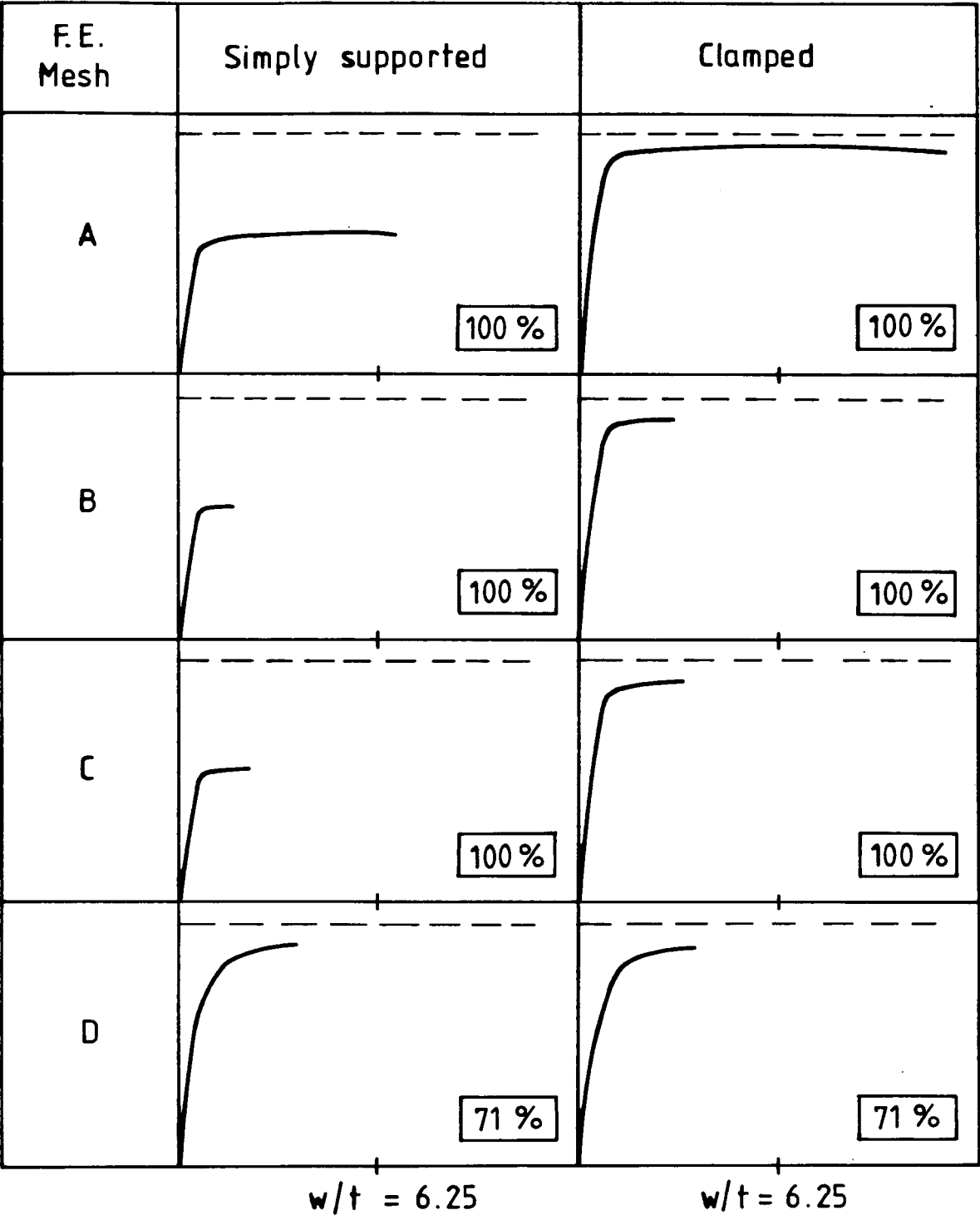
% of shell elements undergone plasticity at failure

Fig.4.4 Analytical load deflection curves of cylinder 2

F. E. Mesh	Simply supported	Clamped
A		
B		
C		
D		

Deflection magnification = 30

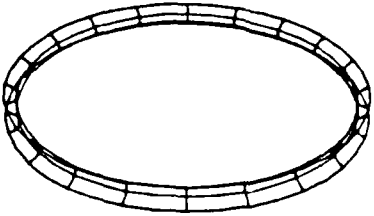
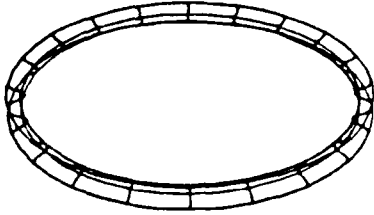
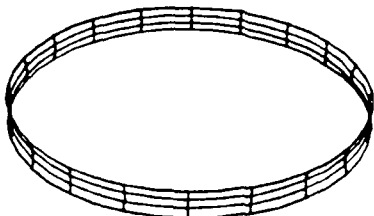
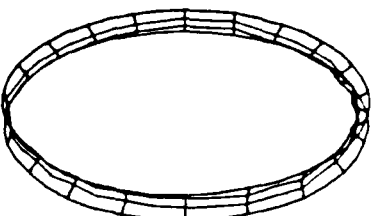
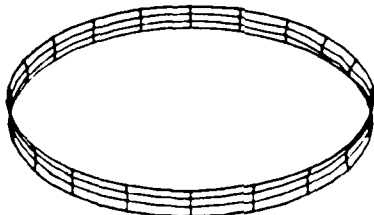
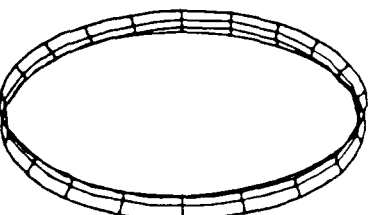
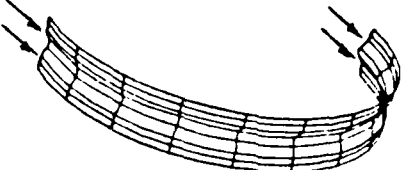
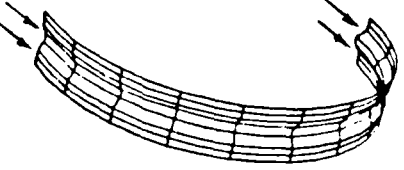
Fig 4.5 Analytical collapse modes of cylinder 2



— — — Experimental failure load ($= 1.87 p_0$)

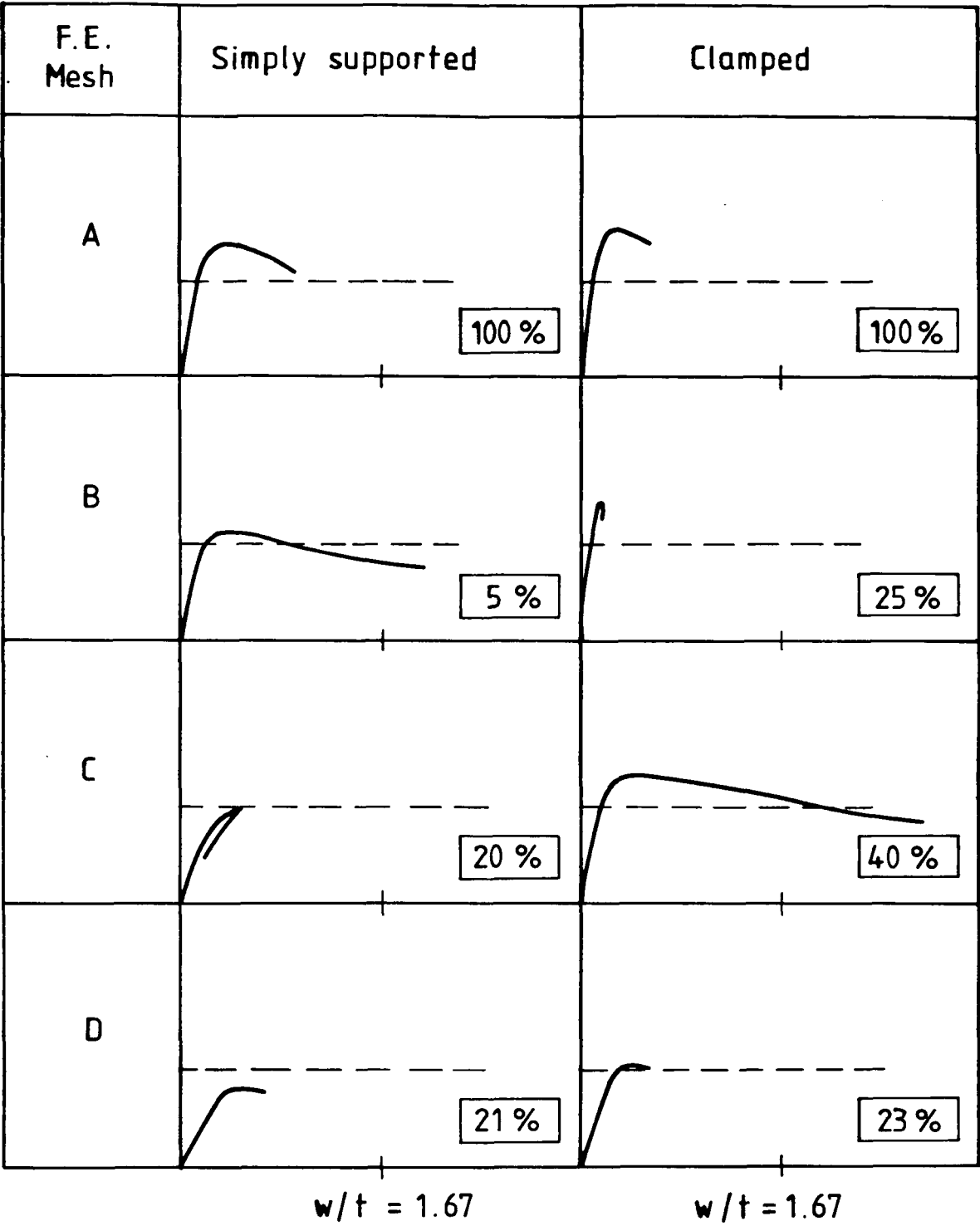
% of shell elements undergone plasticity at failure

Fig 4.6 Analytical load deflection curves of cylinder 3

F. E. Mesh	Simply supported	Clamped
A		
B		
C		
D		

Deflection magnification = 5

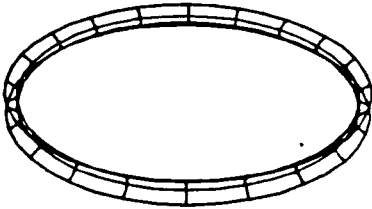
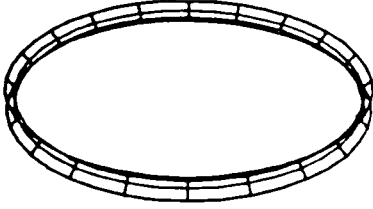
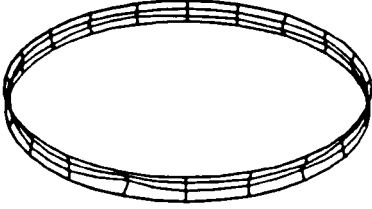
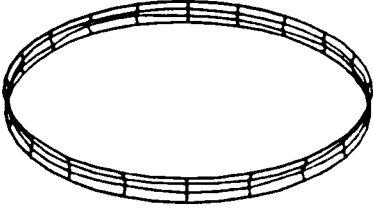
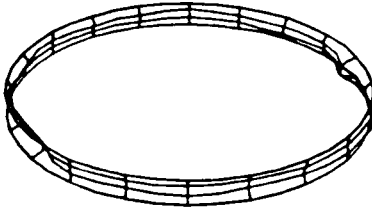
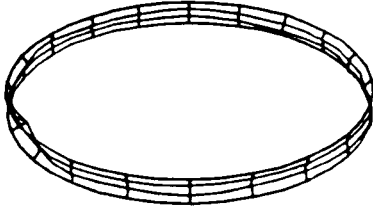
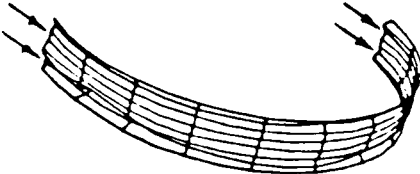
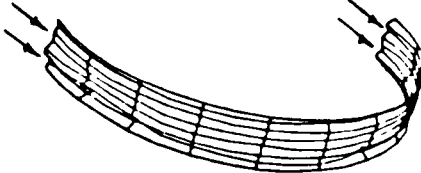
Fig 4.7 Analytical collapse modes of cylinder 3



— — — Experimental failure load ($= 0.38 p_0$)

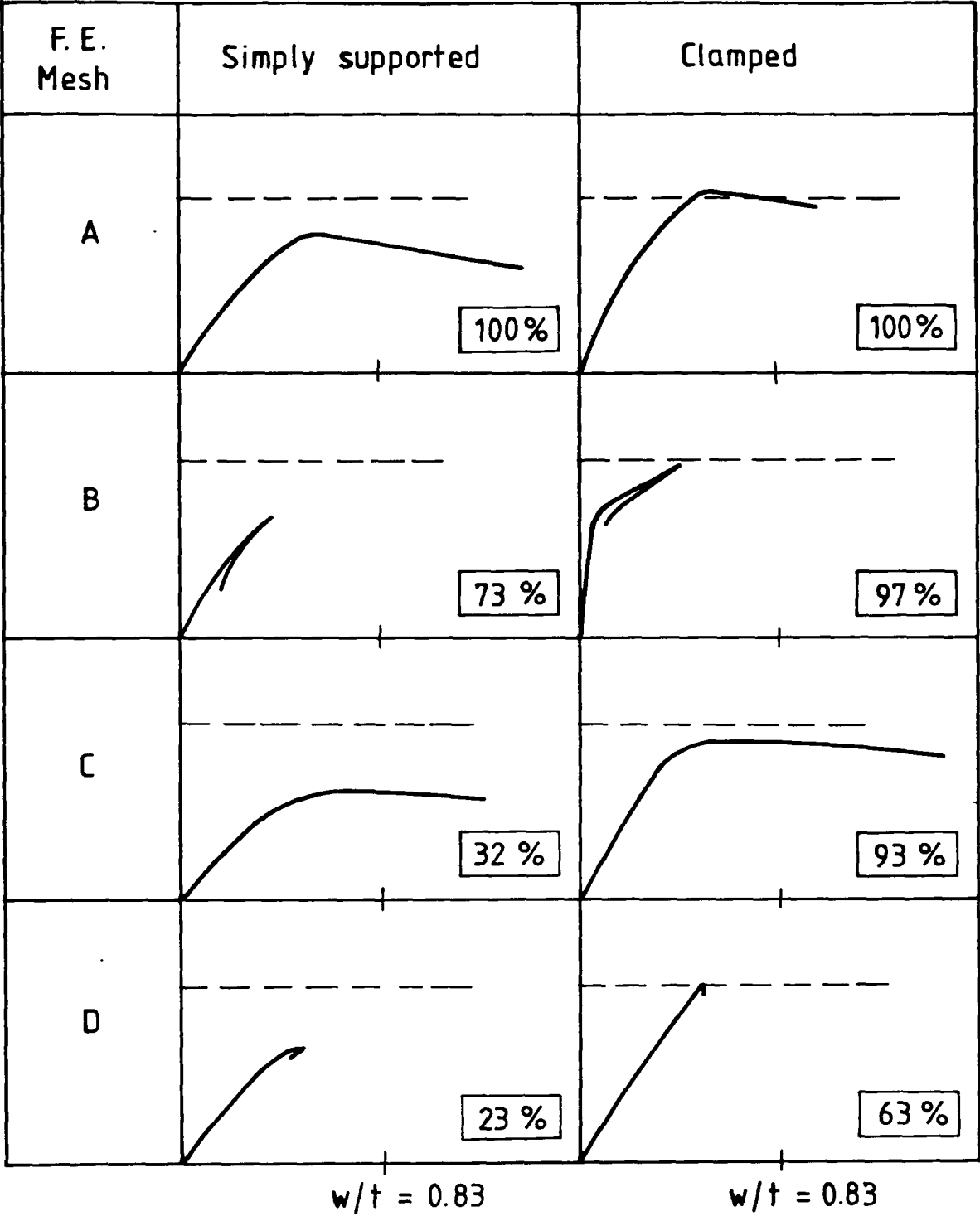
% of shell elements undergone plasticity at failure

Fig. 4.8 Analytical load deflection curves of cylinder 4

F.E. Mesh	Simply supported	Clamped
A		
B		
C		
D		

Deflection magnification = 50

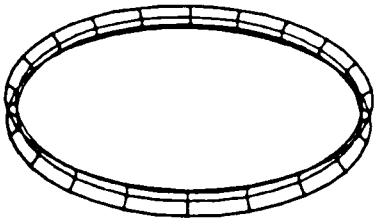
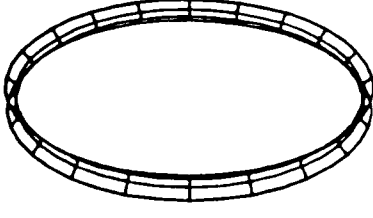
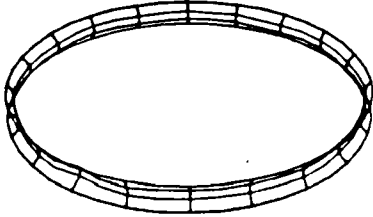
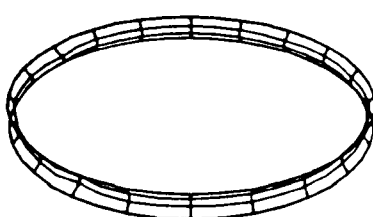
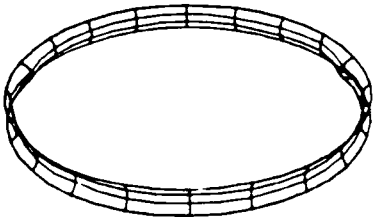
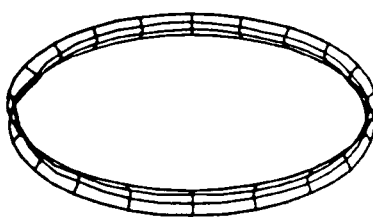
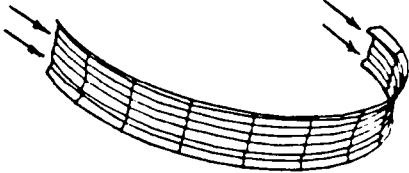
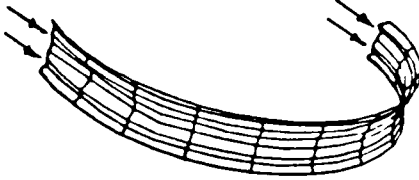
Fig 4.9 Analytical collapse modes of cylinder 4



— — — Experimental failure load ($= 1.03 p_0$)

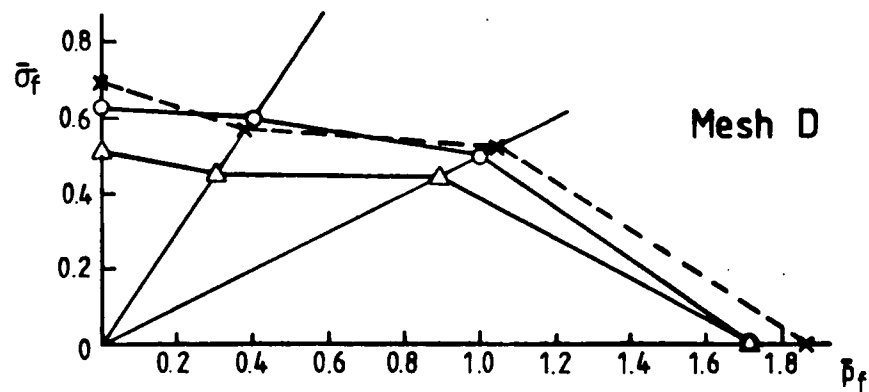
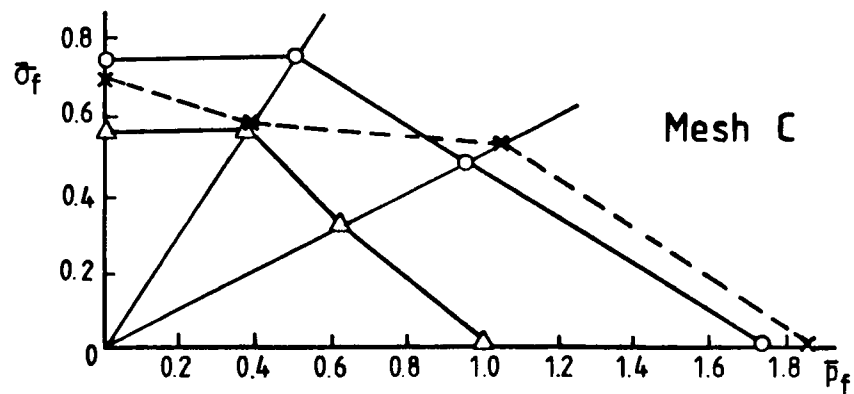
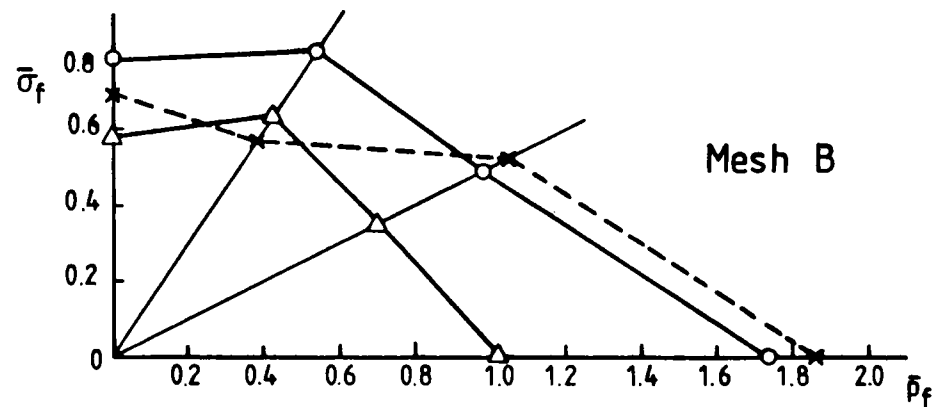
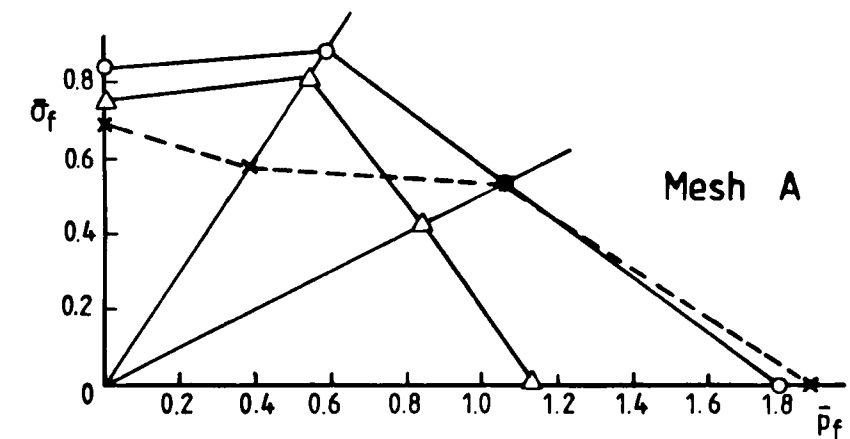
% of shell elements undergone plasticity at failure

Fig. 4.10 Analytical load deflection curves of cylinder 5

F.E. Mesh	Simply supported	Clamped
A		
B		
C		
D		

Deflection magnification = 30

Fig 4.11 Analytical collapse modes of cylinder 5



—○— clamped
—△— simply supported

--x-- experiment

Fig. 4.12 Comparison between experimental and analytical interaction curves

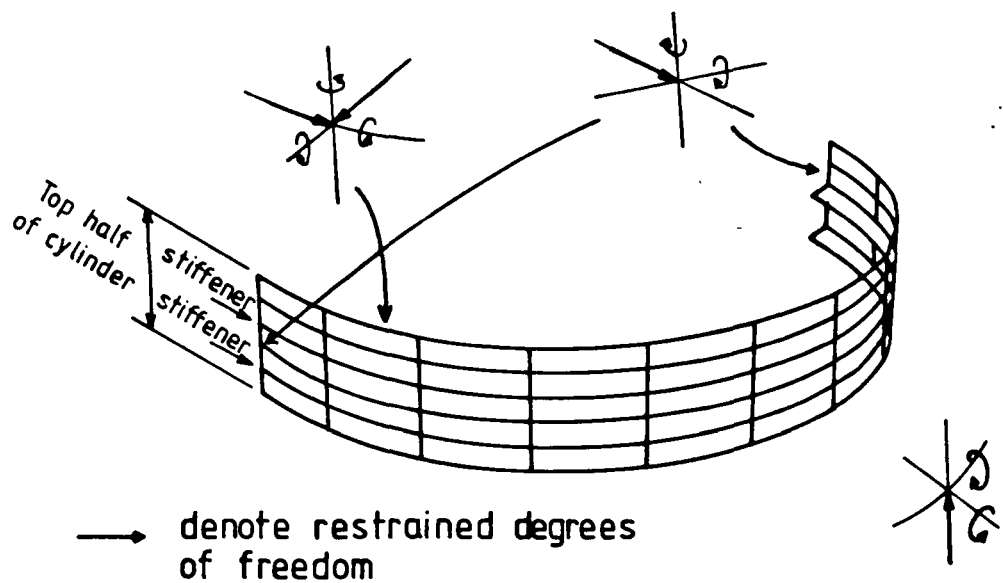


Fig 4.13 Finite element model used to analyse cylinders 6 – 8

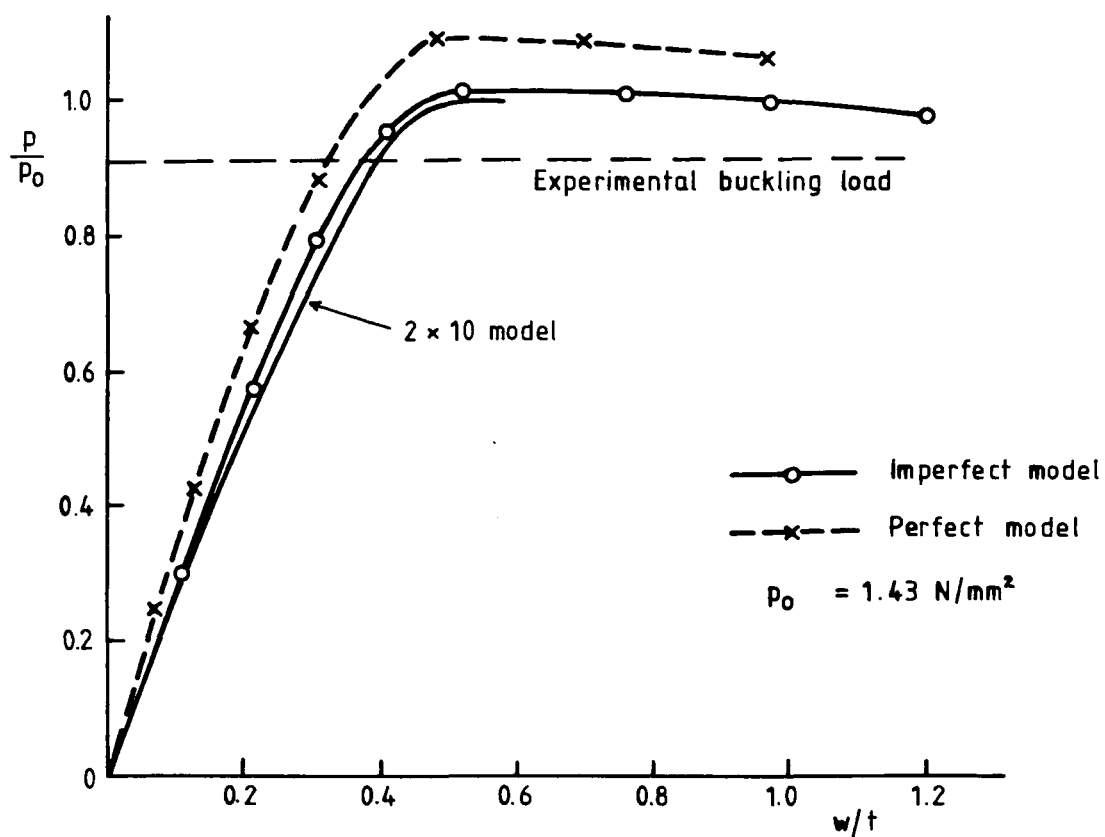


Fig. 4.14 Analytical load – deflection curves of cylinder 6

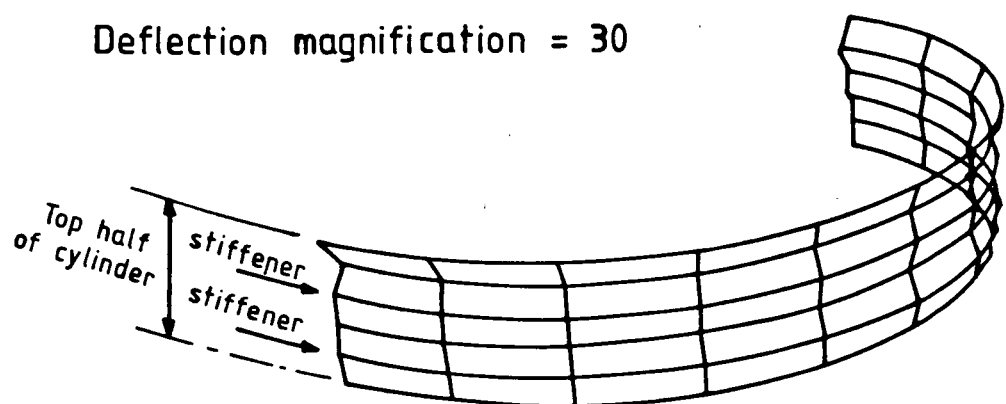


Fig. 4.15 Analytical collapse mode of imperfect model of cylinder 6 (10×5 shell elements)

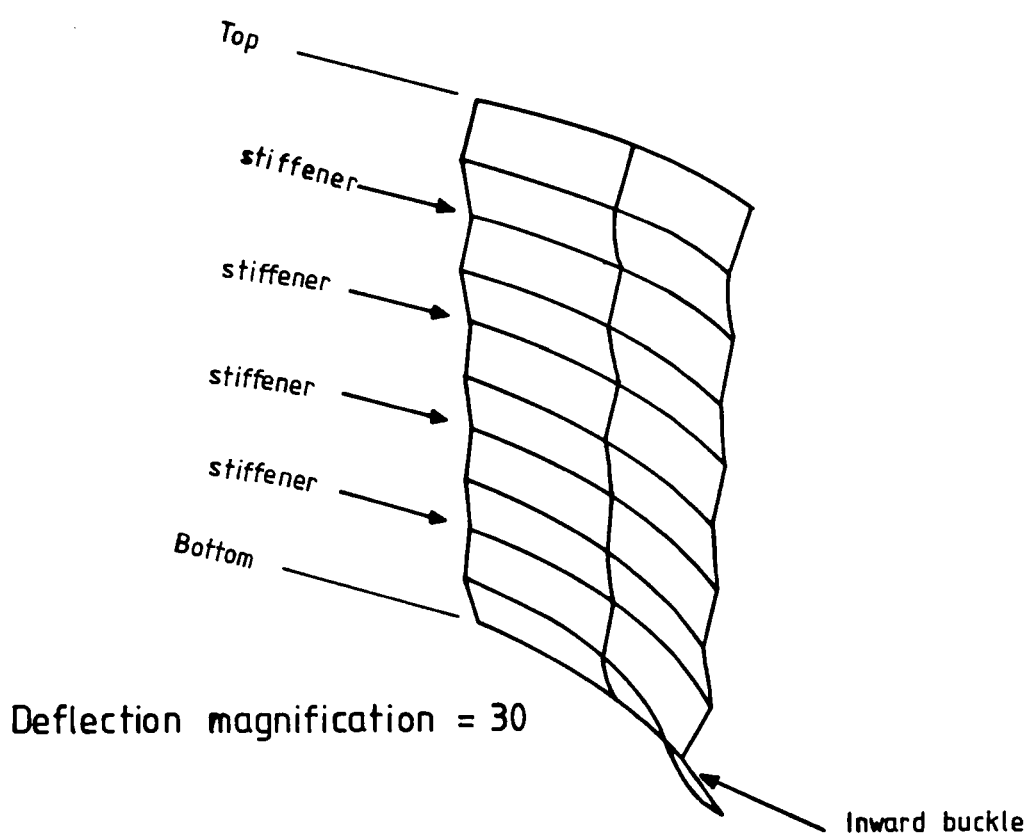


Fig.4.16 Analytical collapse mode of imperfect model of cylinder 6 (2 × 10 shell elements)

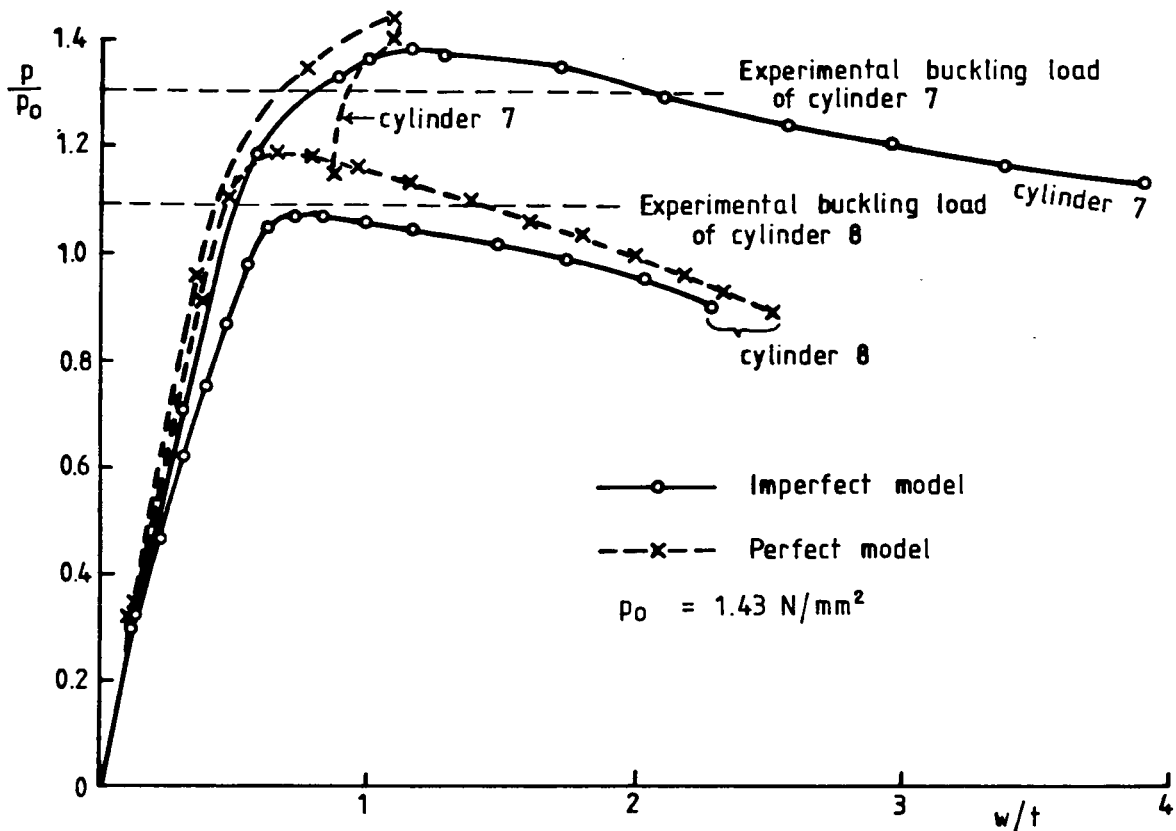


Fig 4.17 Analytical load deflection curves of cylinders 7 and 8

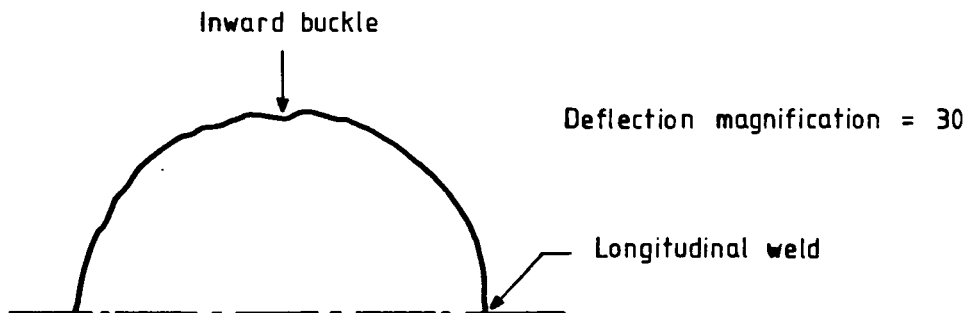


Fig 4.18 Plan section of analytical collapse mode of imperfect model of cylinder 7 taken at middle of centre panel

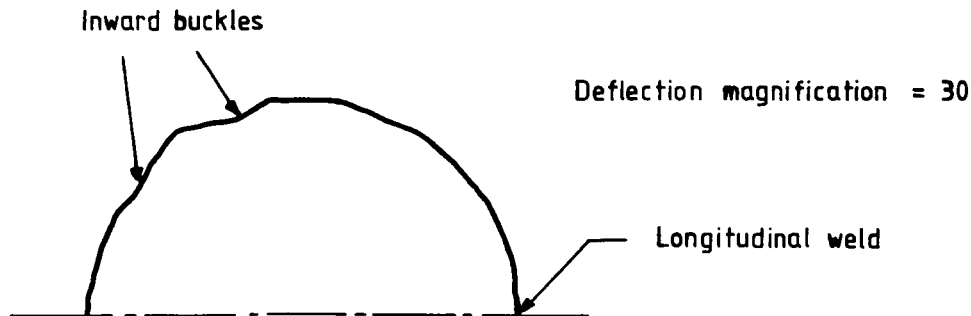


Fig 4.19 Plan section of analytical collapse mode of imperfect model of cylinder 8 taken at middle of centre panel

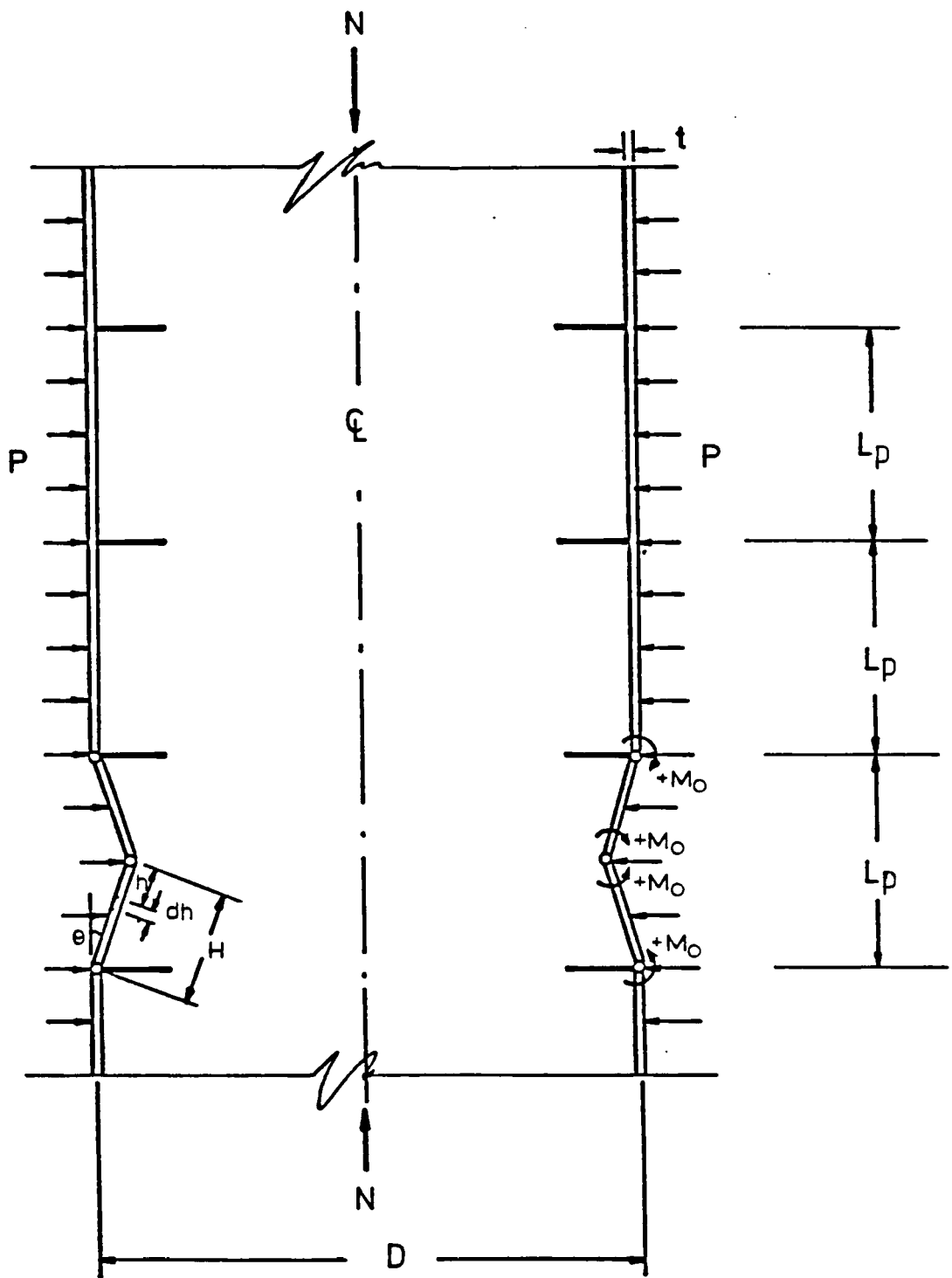


Fig. 5.1 Kinematics of inter - ring collapse mechanism

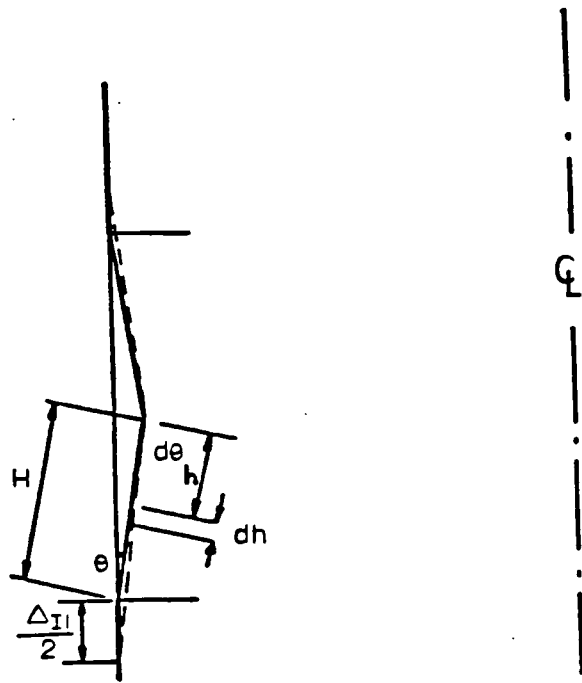


Fig. 5.2 Material stretching component of deformation

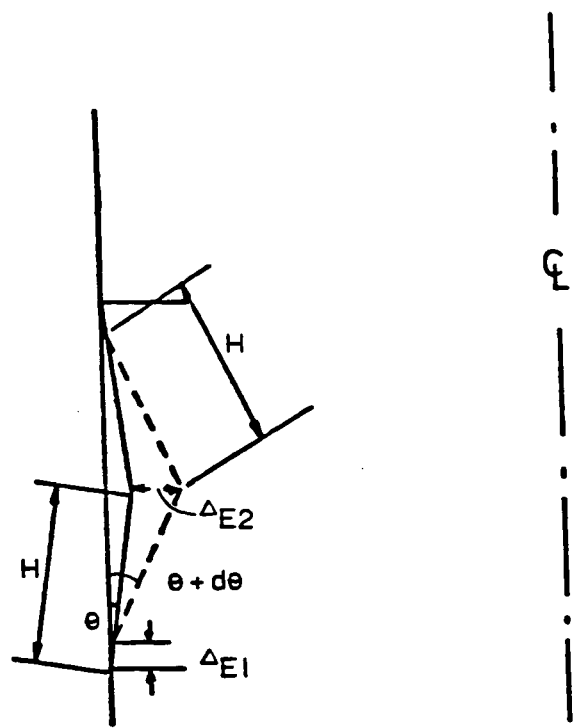


Fig. 5.3 Rotational component of deformation

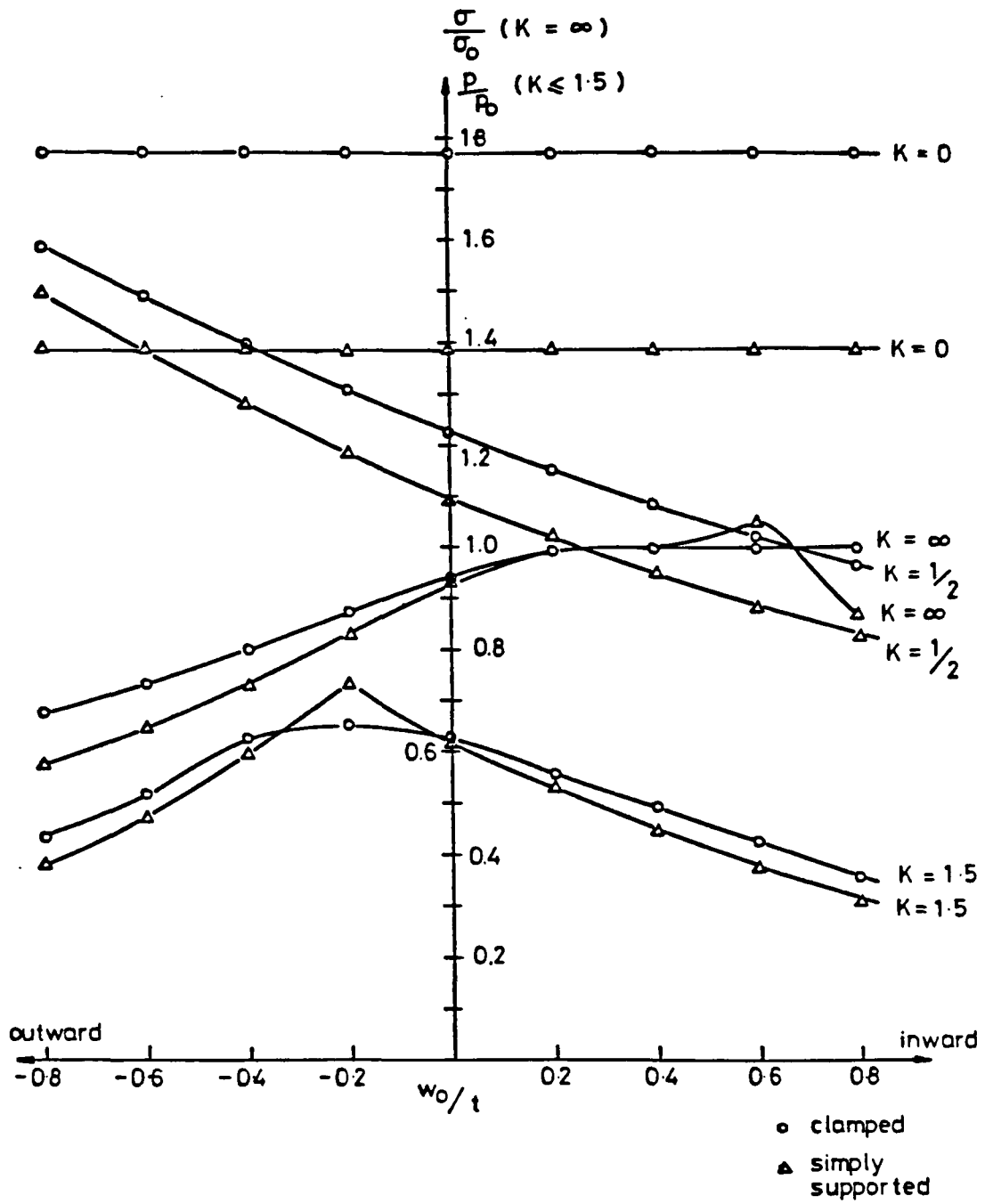


Fig. 5.4 Variation of load carrying capacity with initial imperfection level ($u = 1.0$)

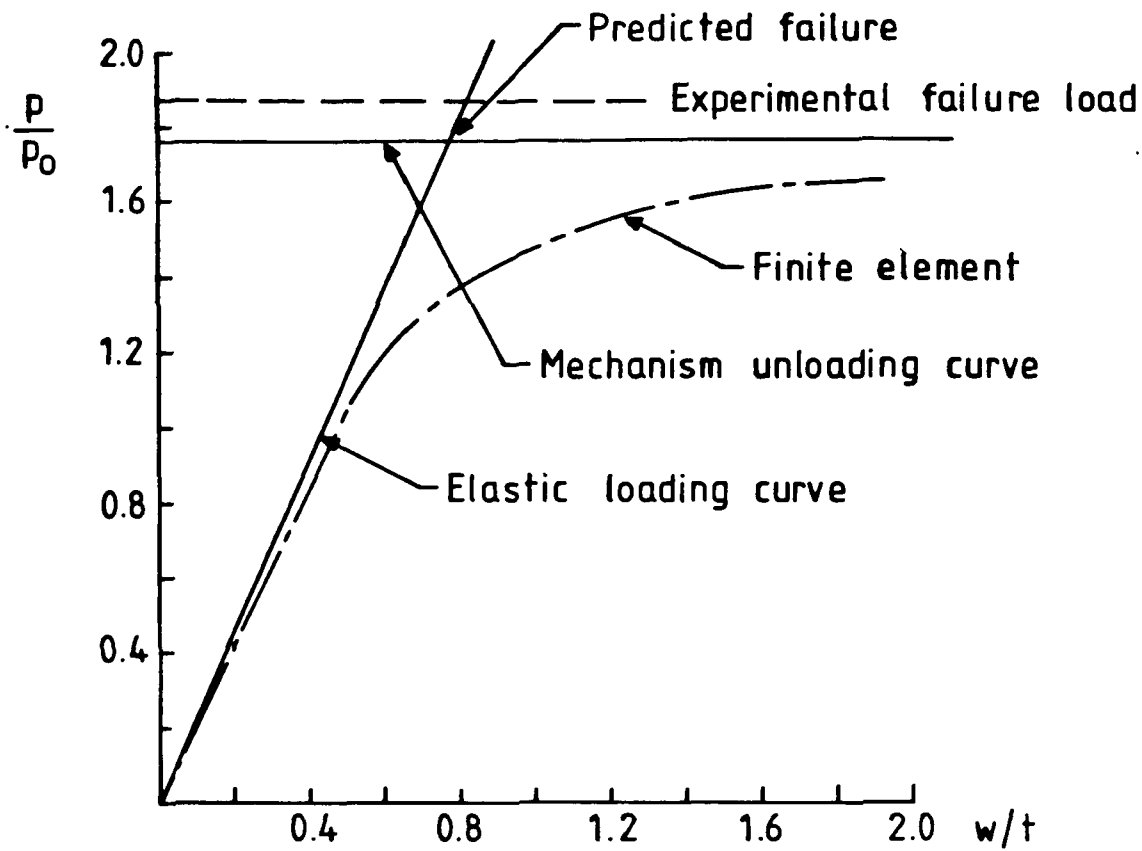


Fig. 5.5 Failure predictions for cylinder 3 assuming clamped boundaries

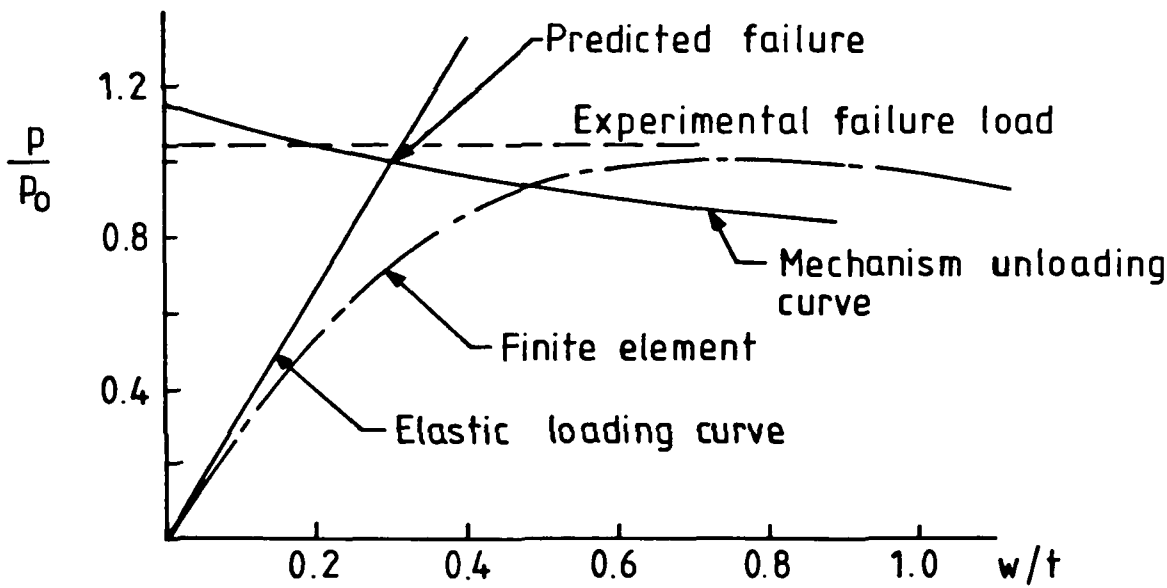


Fig. 5.6 Failure predictions for cylinder 2 assuming simply supported boundaries

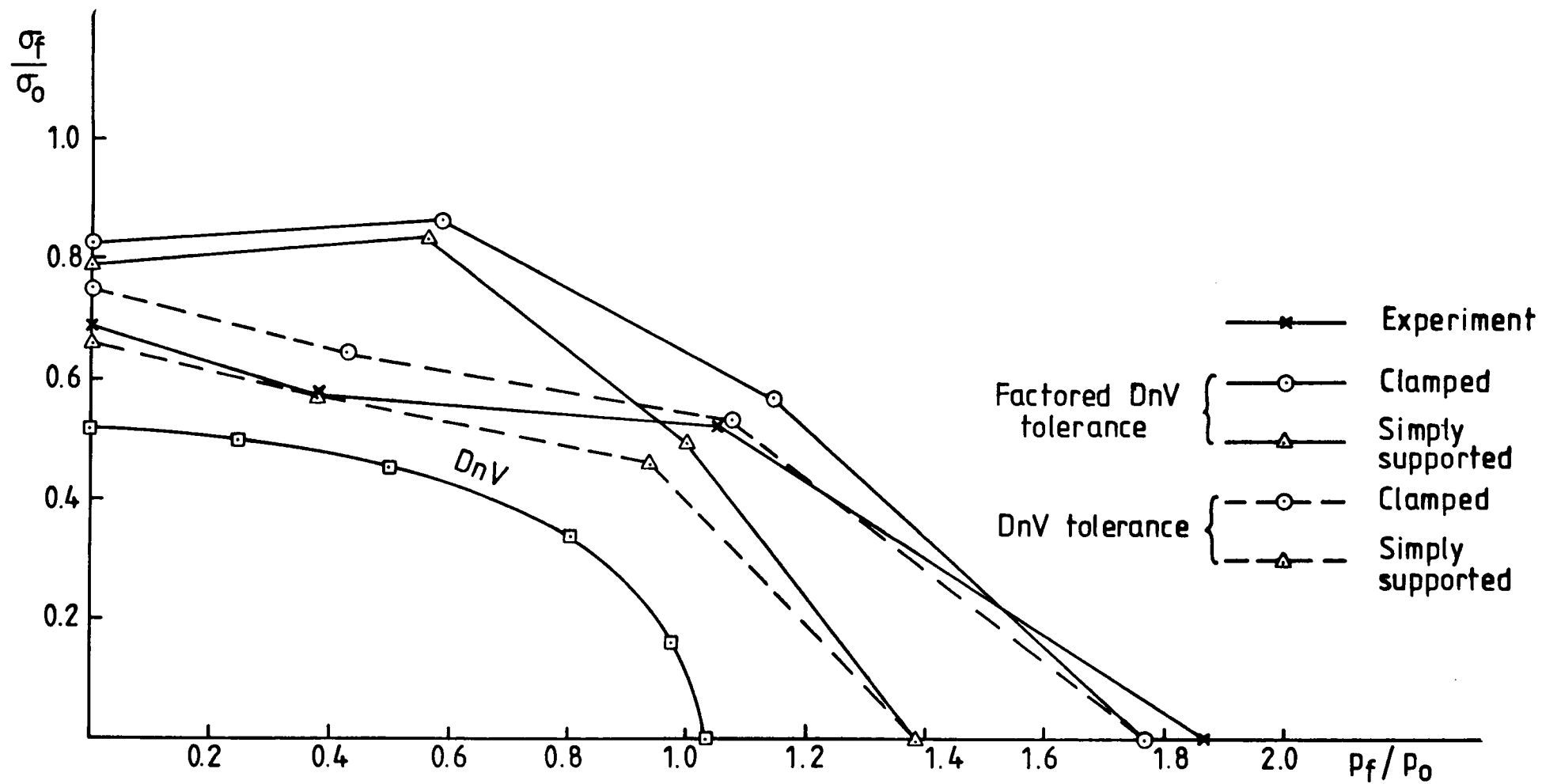


Fig. 5.7 Correlation between Series I experiments and local collapse mechanism

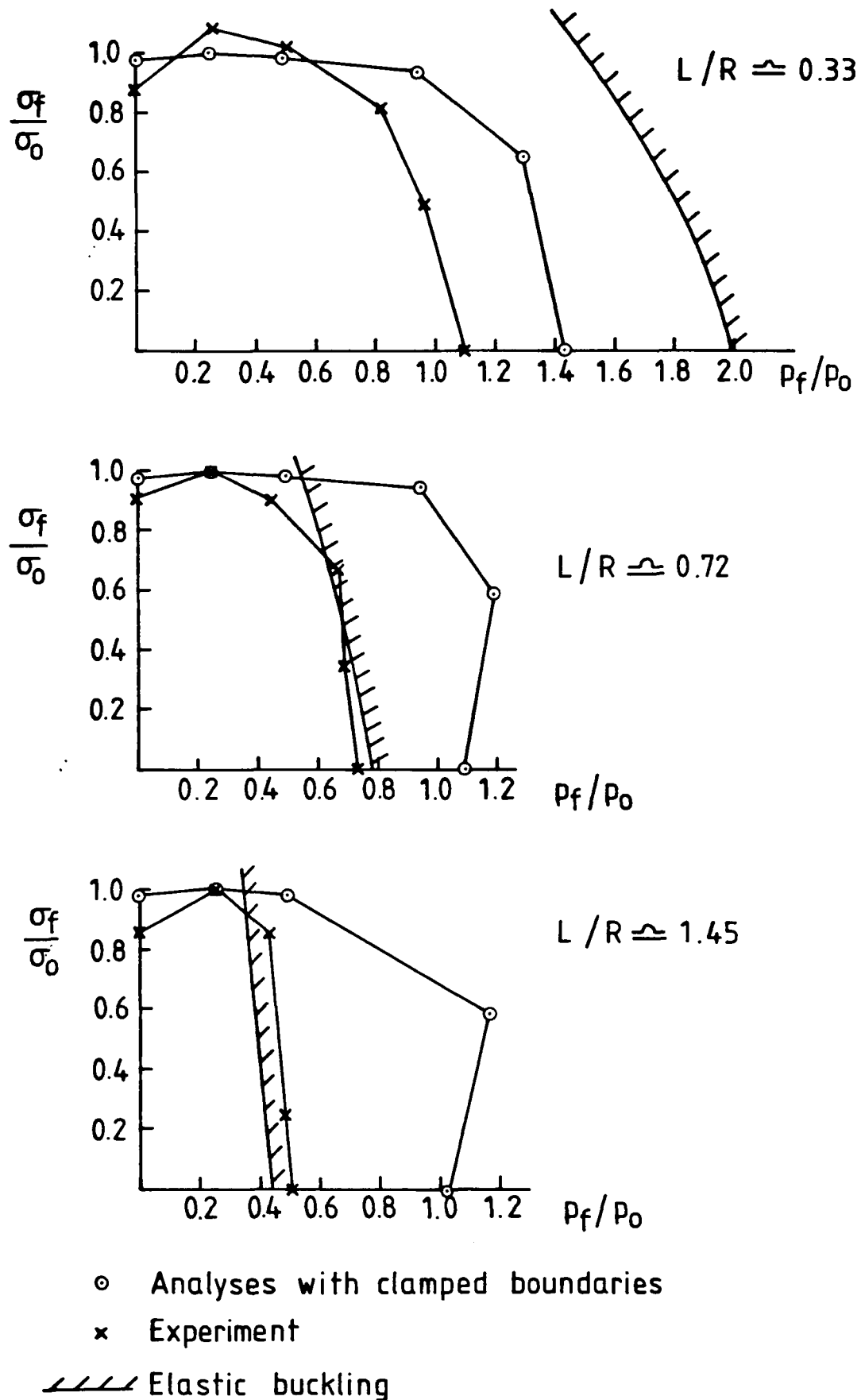


Fig 5.8 Correlation between Liverpool machined models and local collapse mechanism ($R/t \approx 100$)

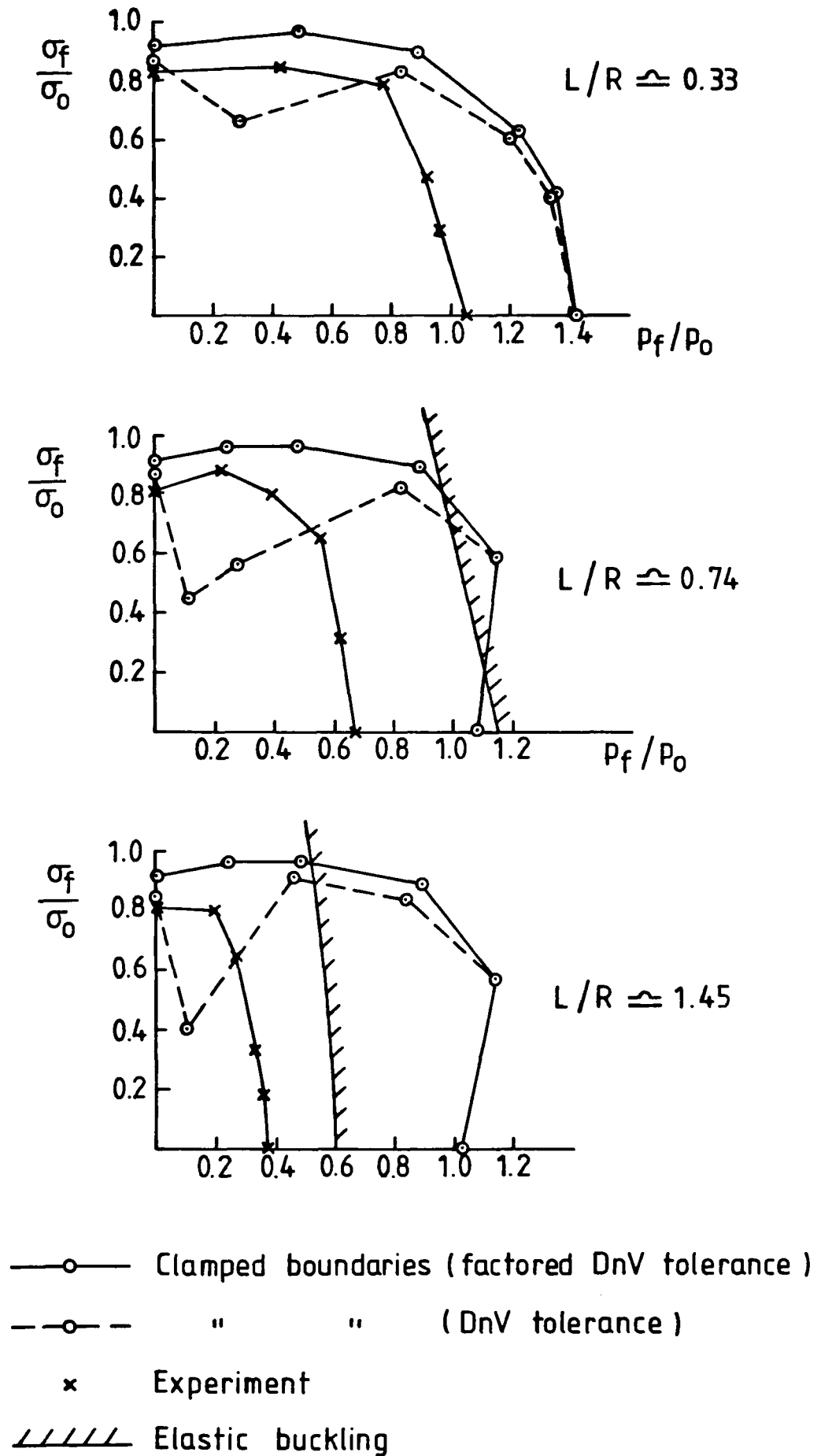


Fig. 5.9 Correlation between Liverpool welded models and local collapse mechanism ($R/t \approx 100$)

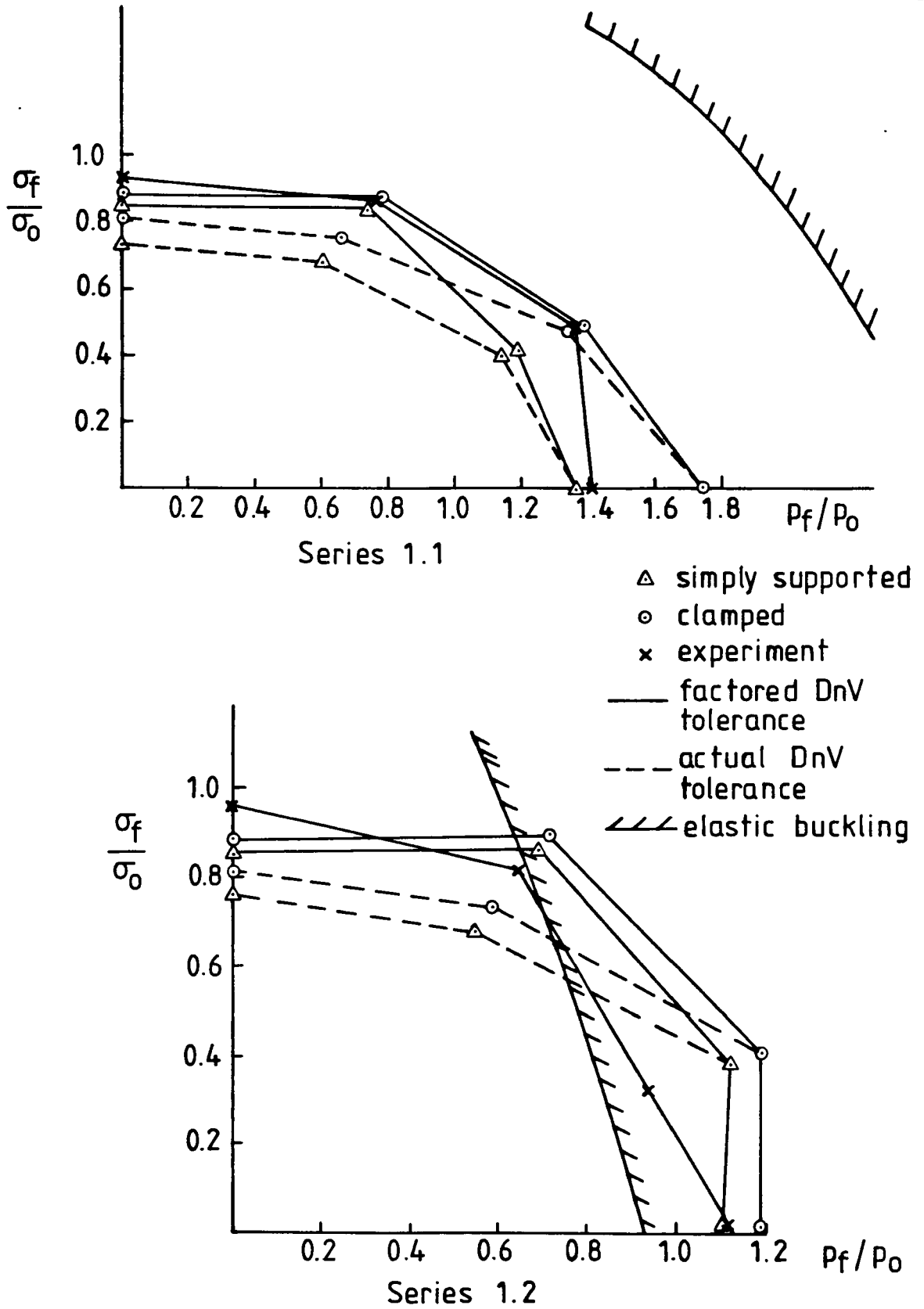


Fig. 5.10 Correlation between Glasgow models and local collapse mechanism

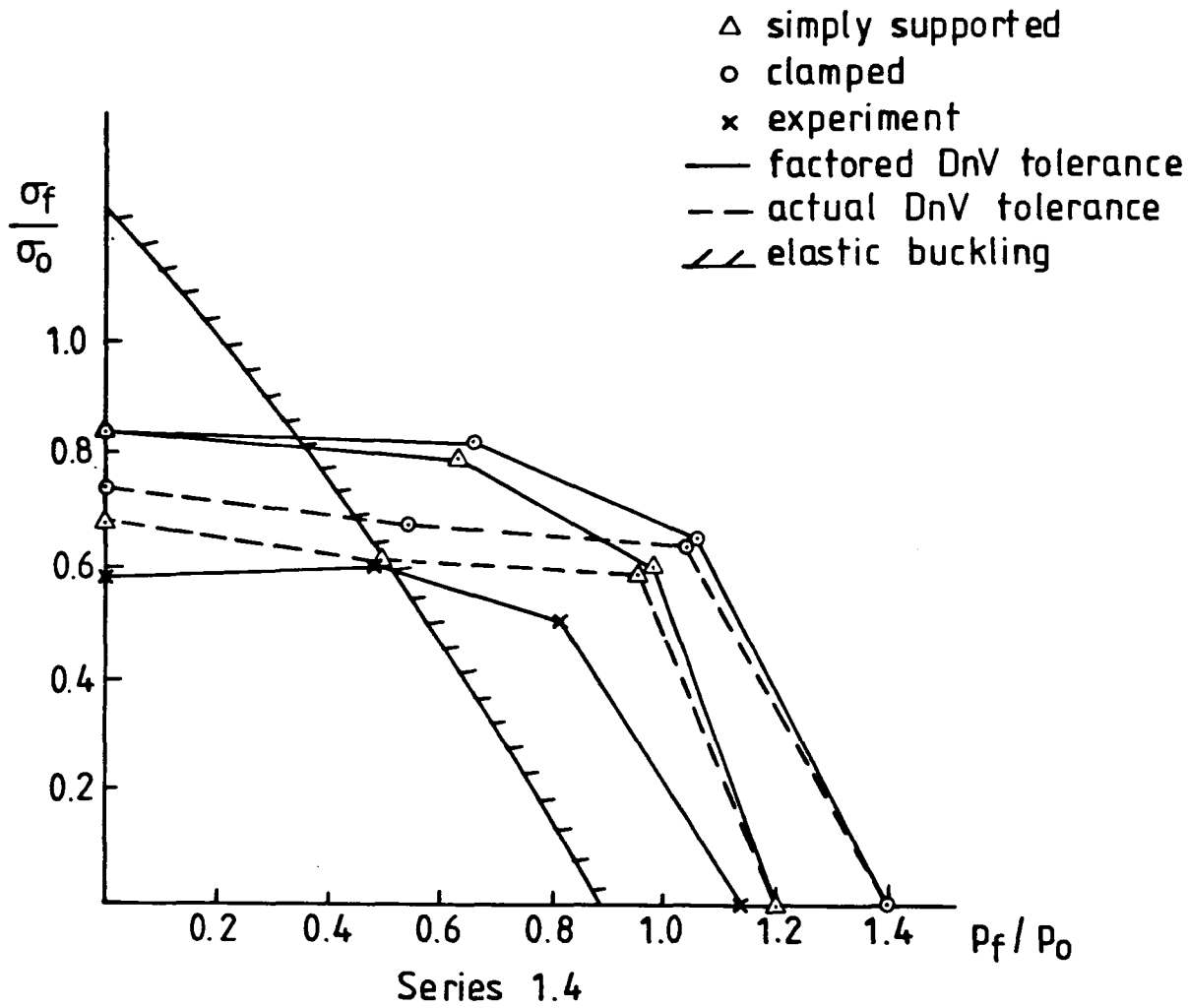
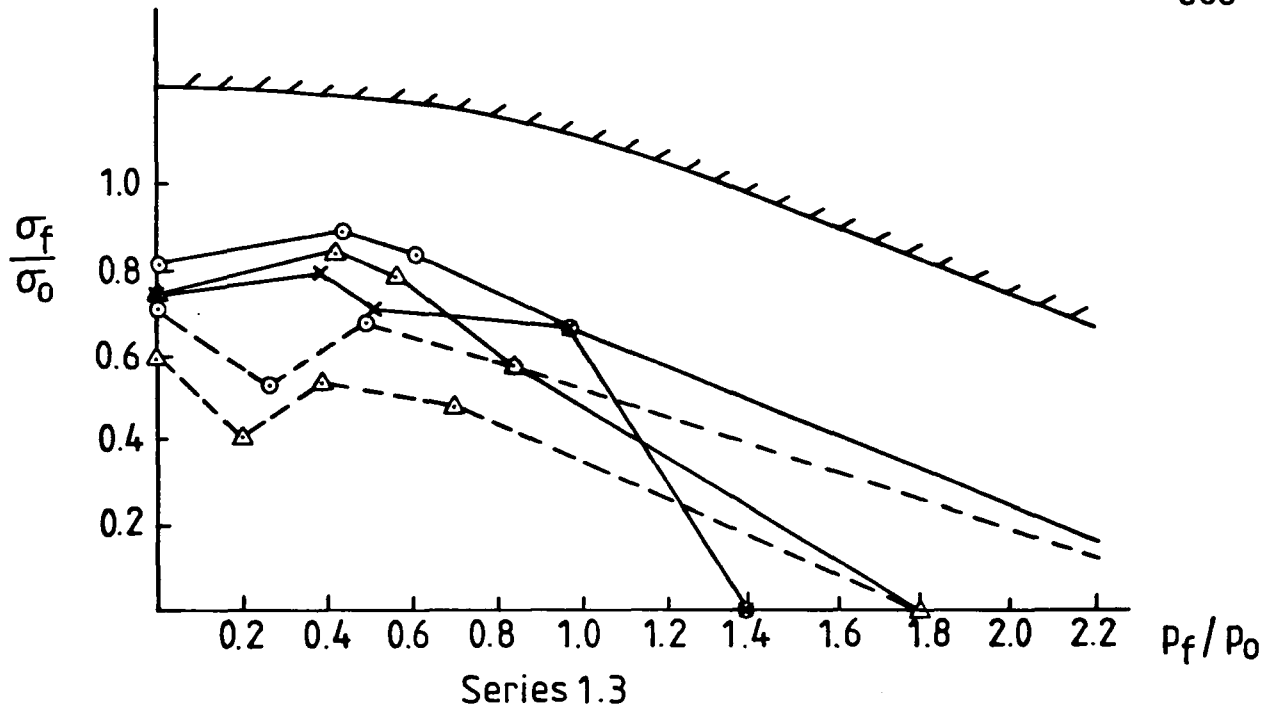


Fig. 5.11 Correlation between Glasgow models and local collapse mechanism

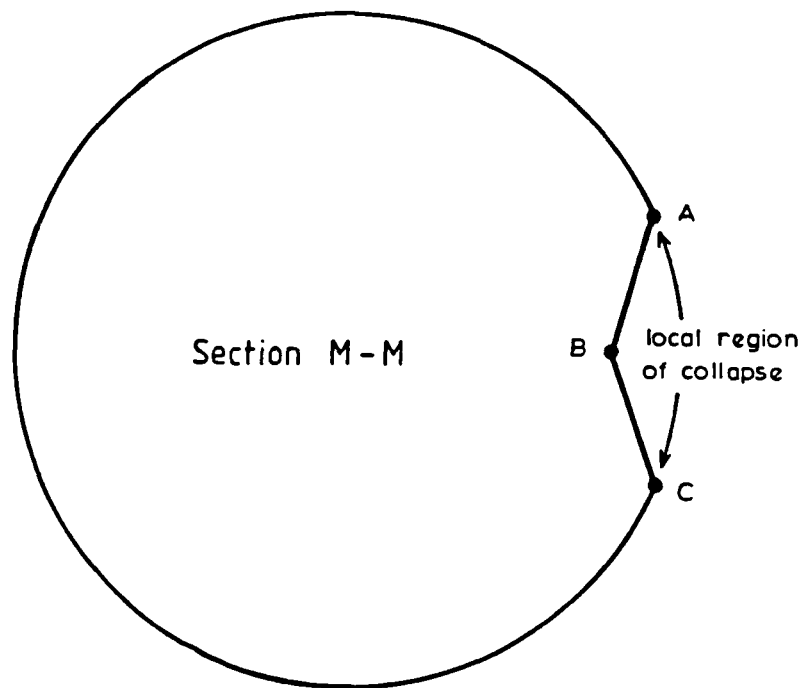
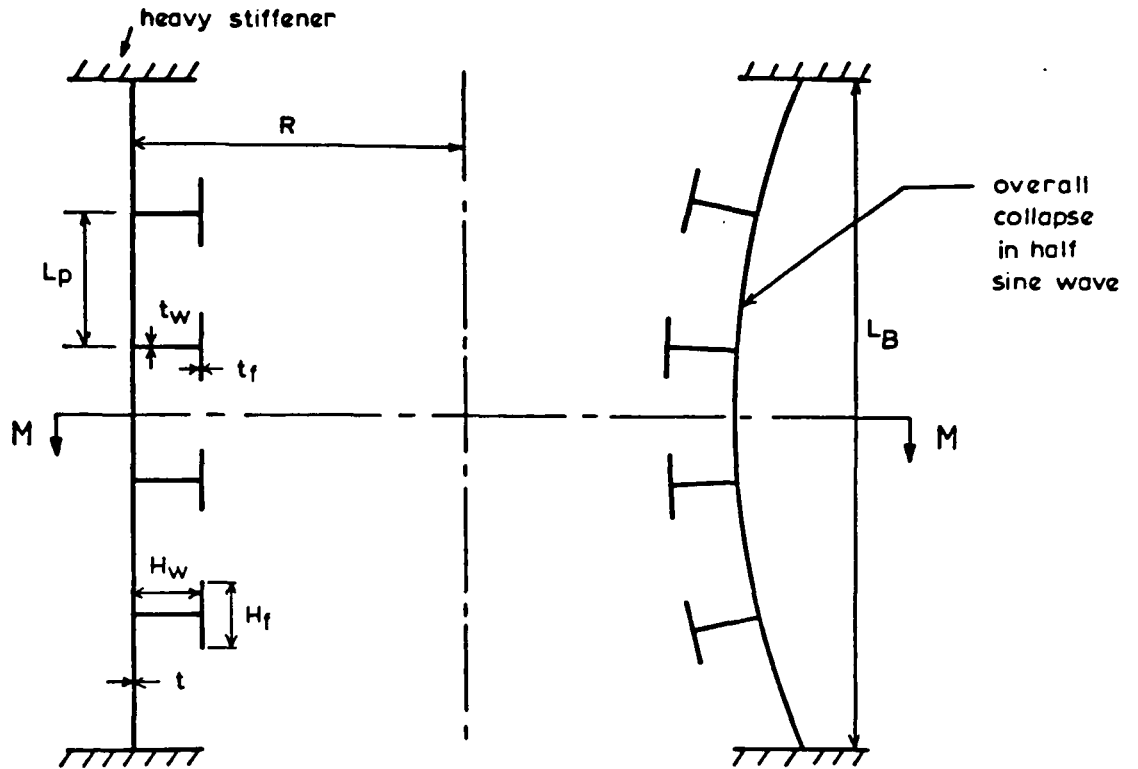


Fig. 5.12 Failure assumed by general collapse mechanism

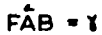
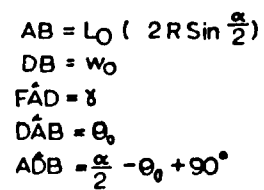


Fig. 5.14 Geometry of axial deformation



Deflection magnification = 10

367

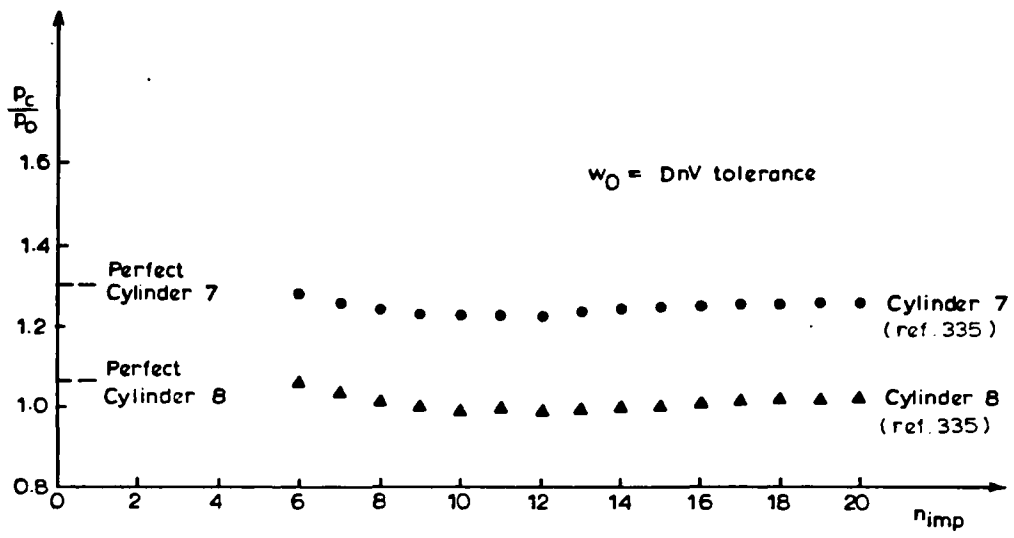


Fig.5.17 Effect of n_{imp} on cylinder strength

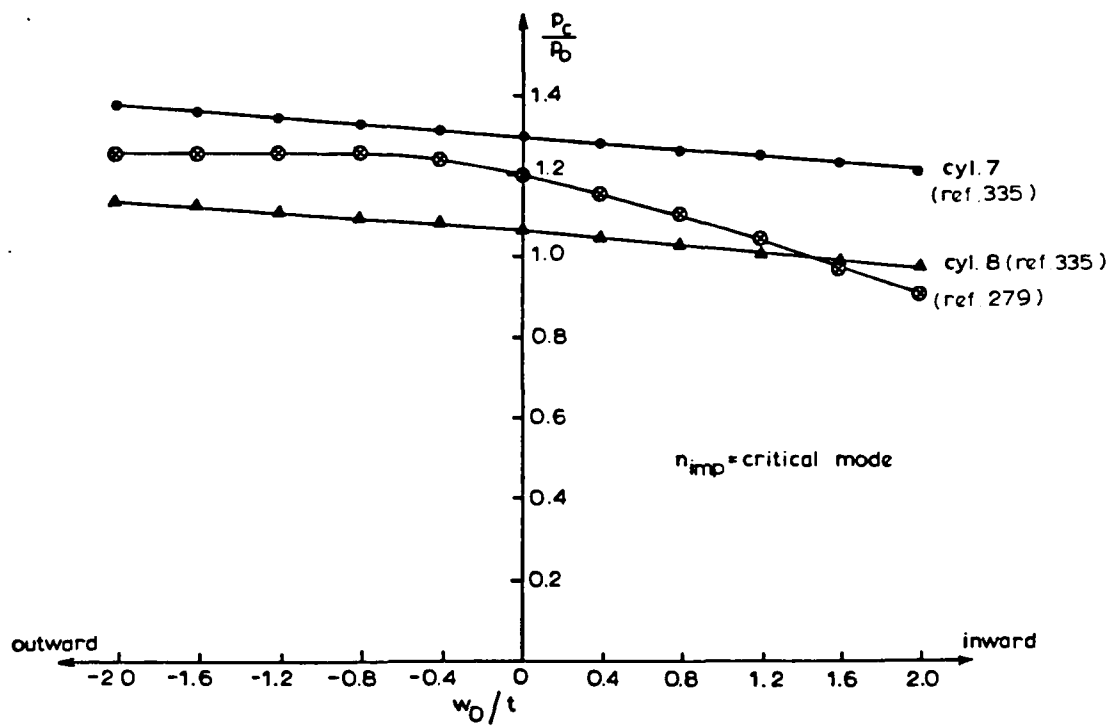


Fig.5.18 Influence of imperfections on cylinder strength

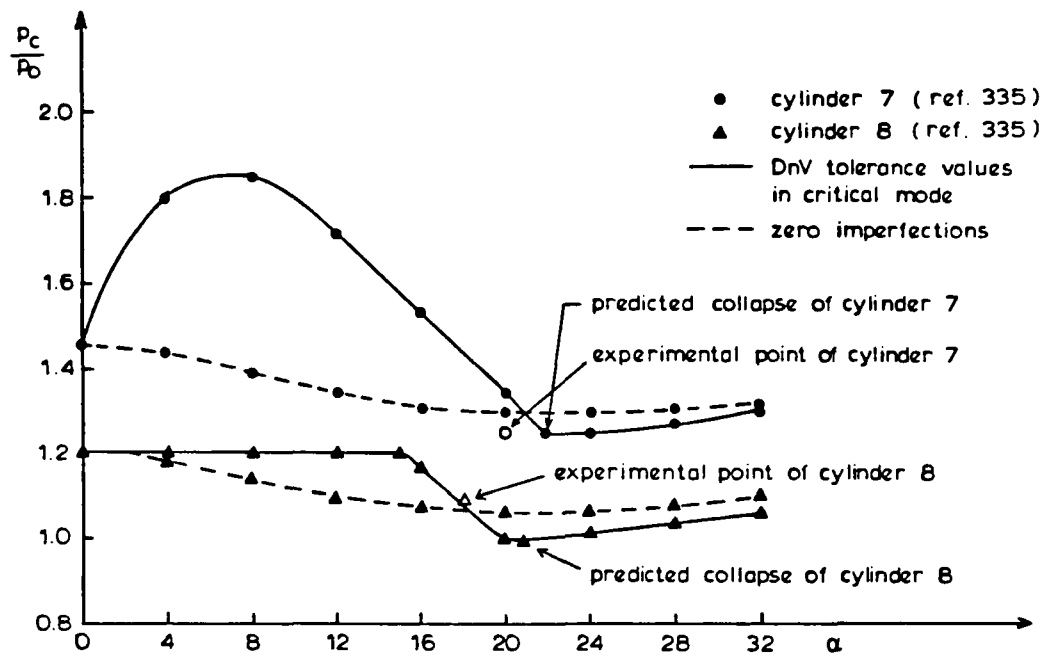


Fig.5.19 Comparison with experimental results of ref.(335) for various α

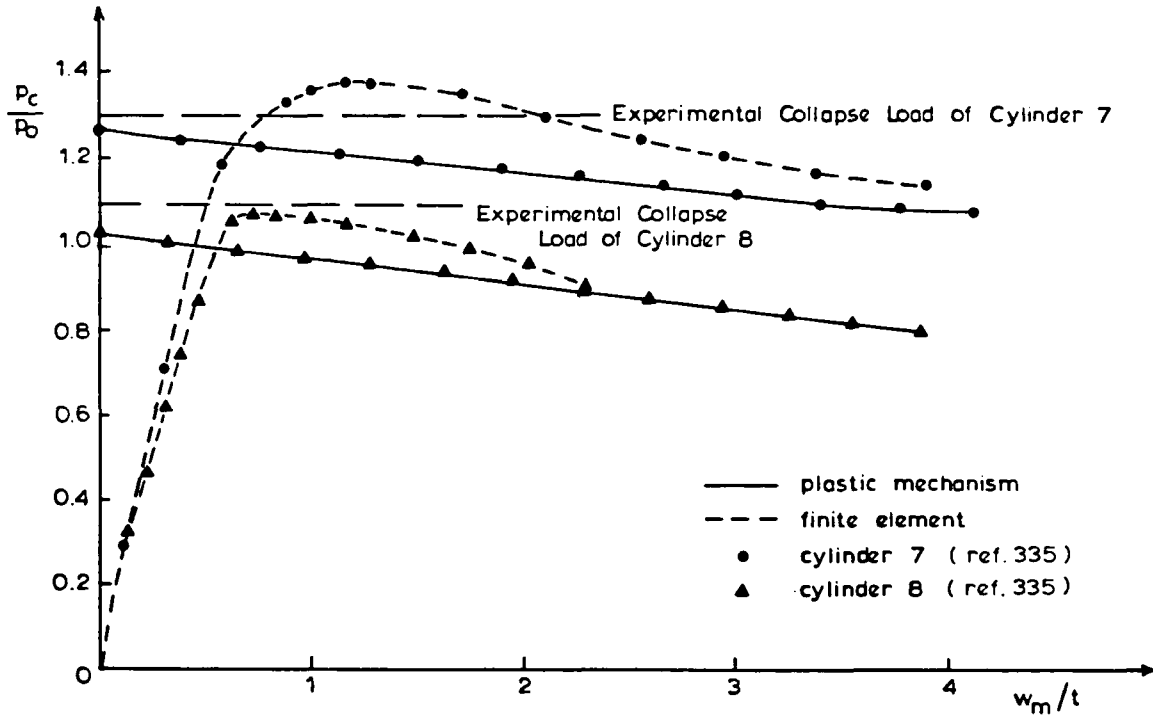


Fig.5.20 Comparison of unloading characteristics with finite element analysis

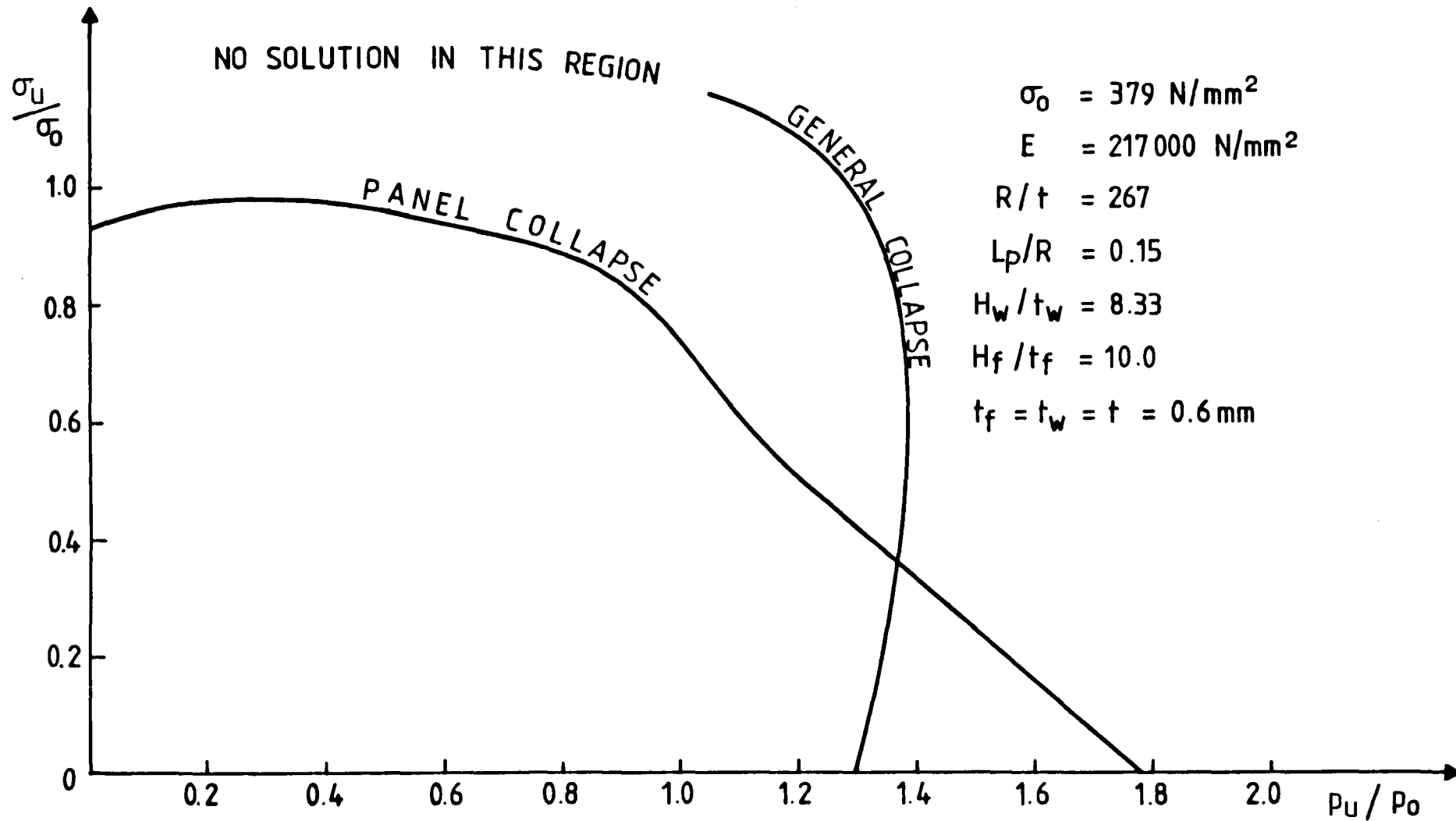


Fig. 5.23 Effect of loading ratio (K) on collapse behaviour of perfect cylinder

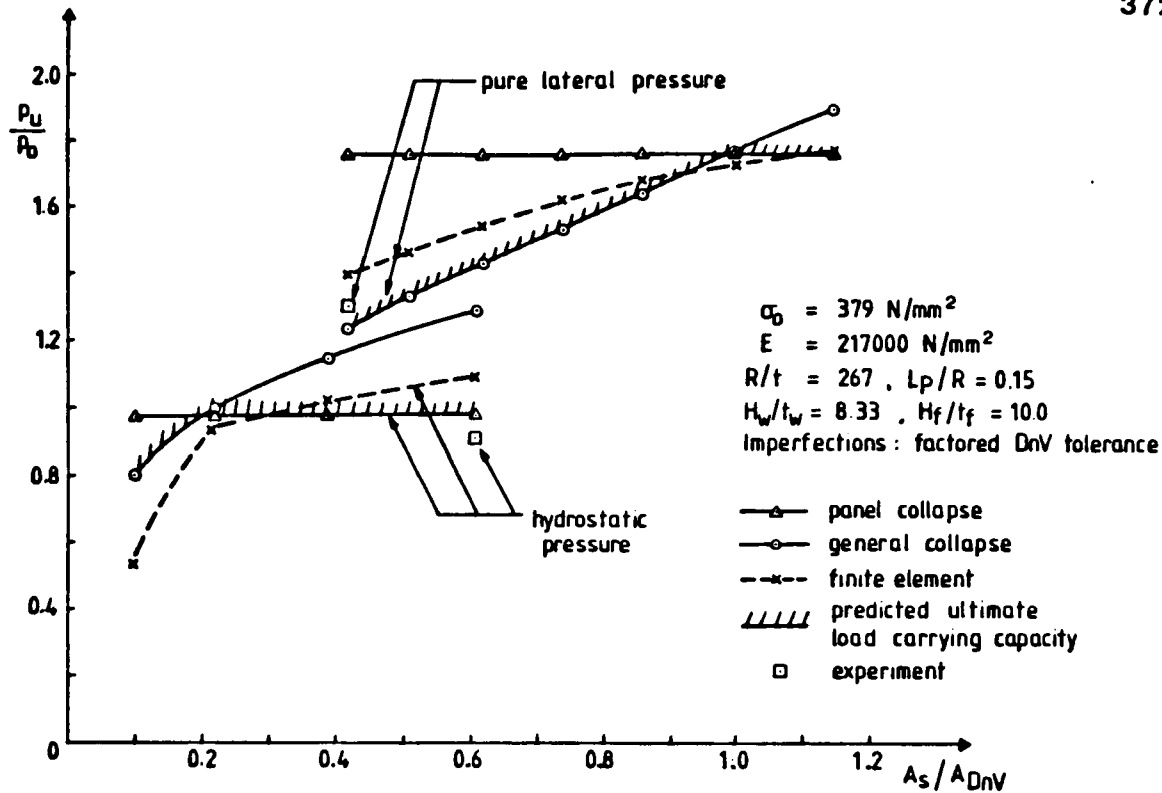


Fig.5.24 Effect of stiffener size on load carrying capacity (T-stiffener)

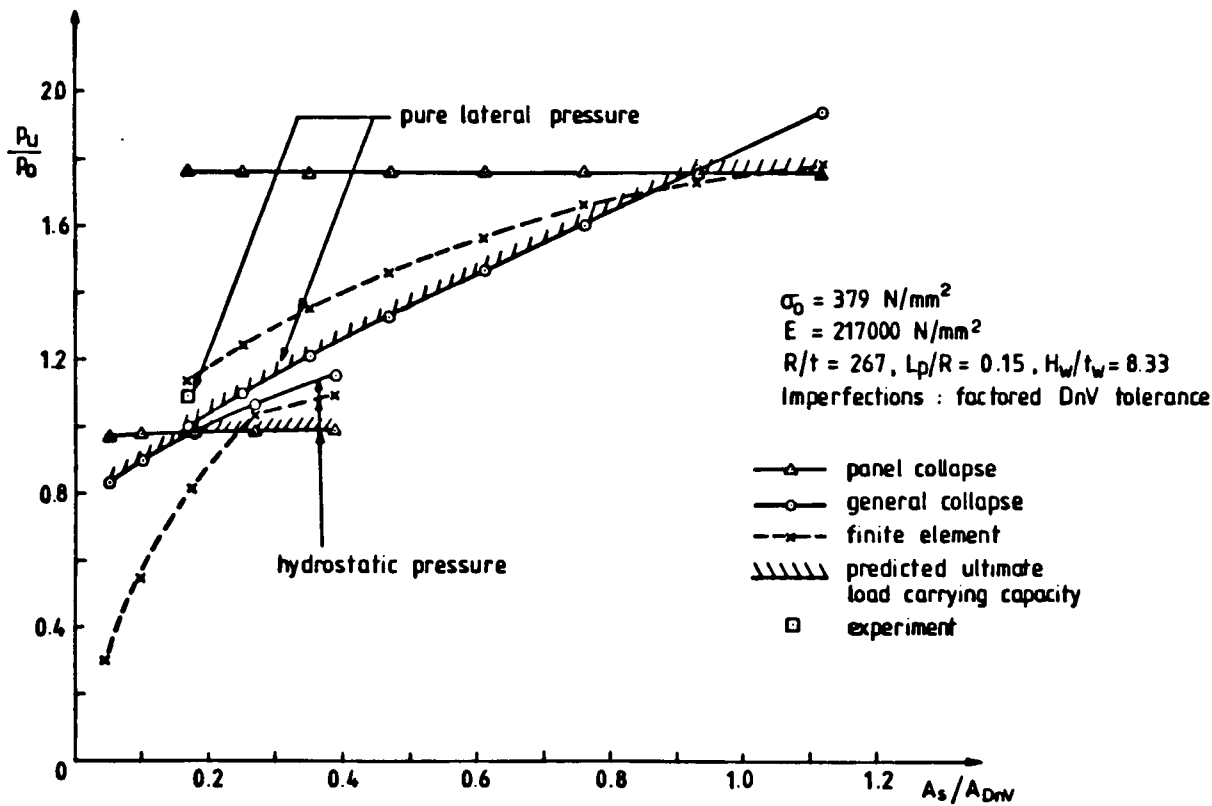


Fig.5.25 Effect of stiffener size on load carrying capacity (flat bar stiffener)

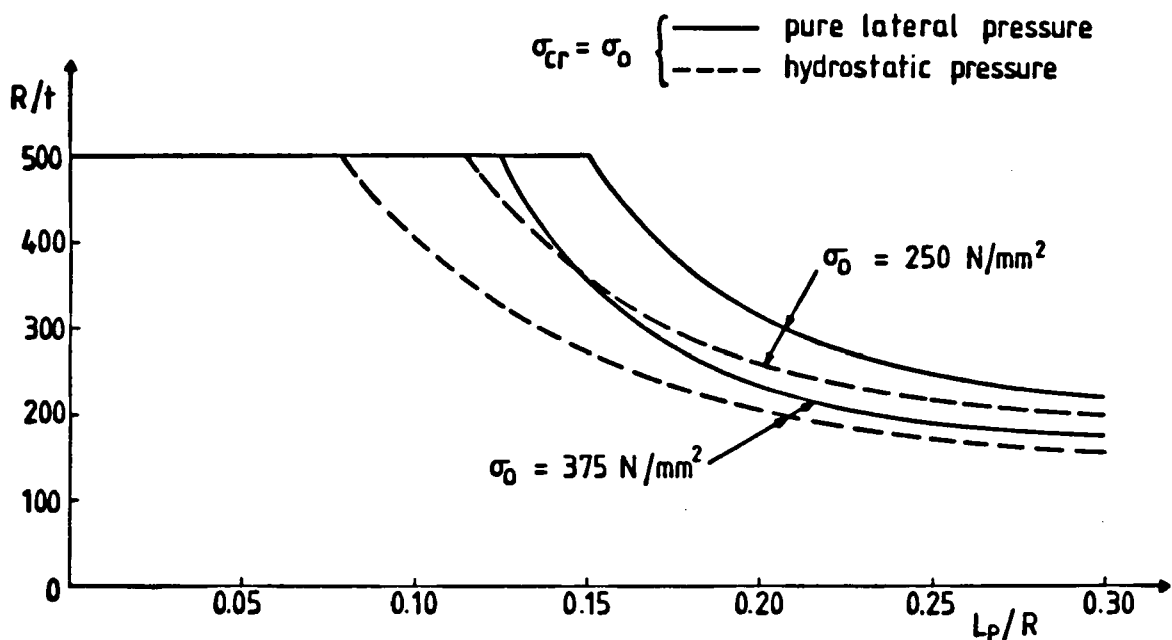


Fig. 5.26 Variation of σ_{cr} for different geometries

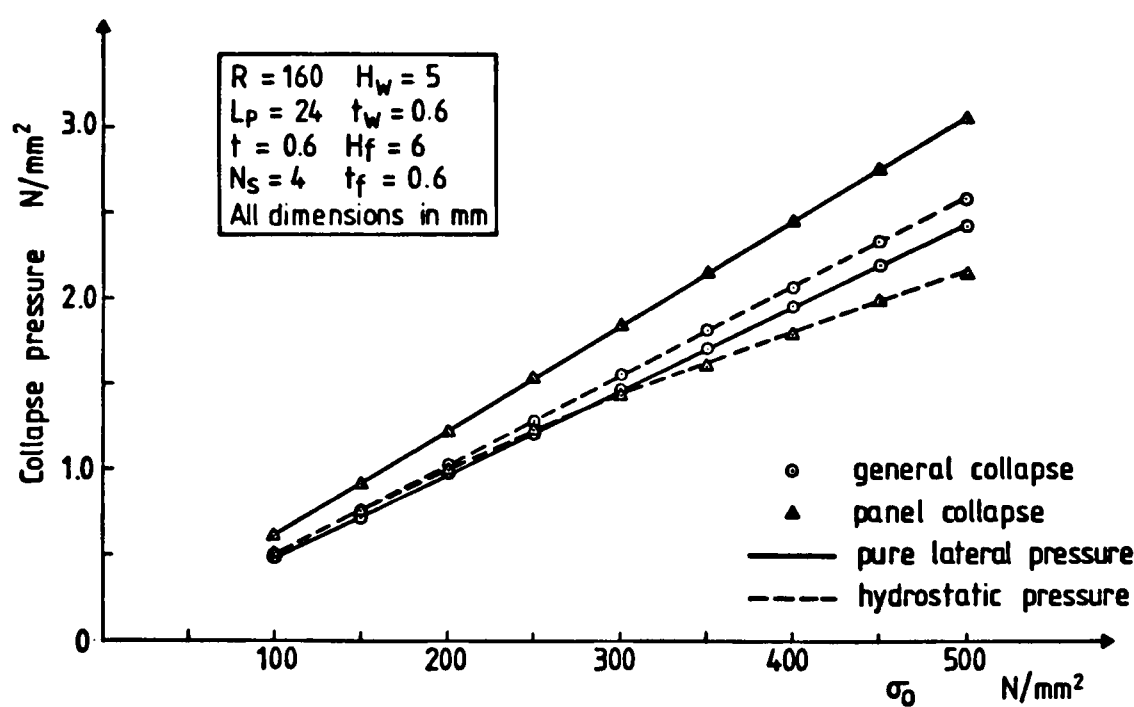


Fig. 5.27 Influence of σ_0 on collapse pressure of perfect T-stiffened cylinder

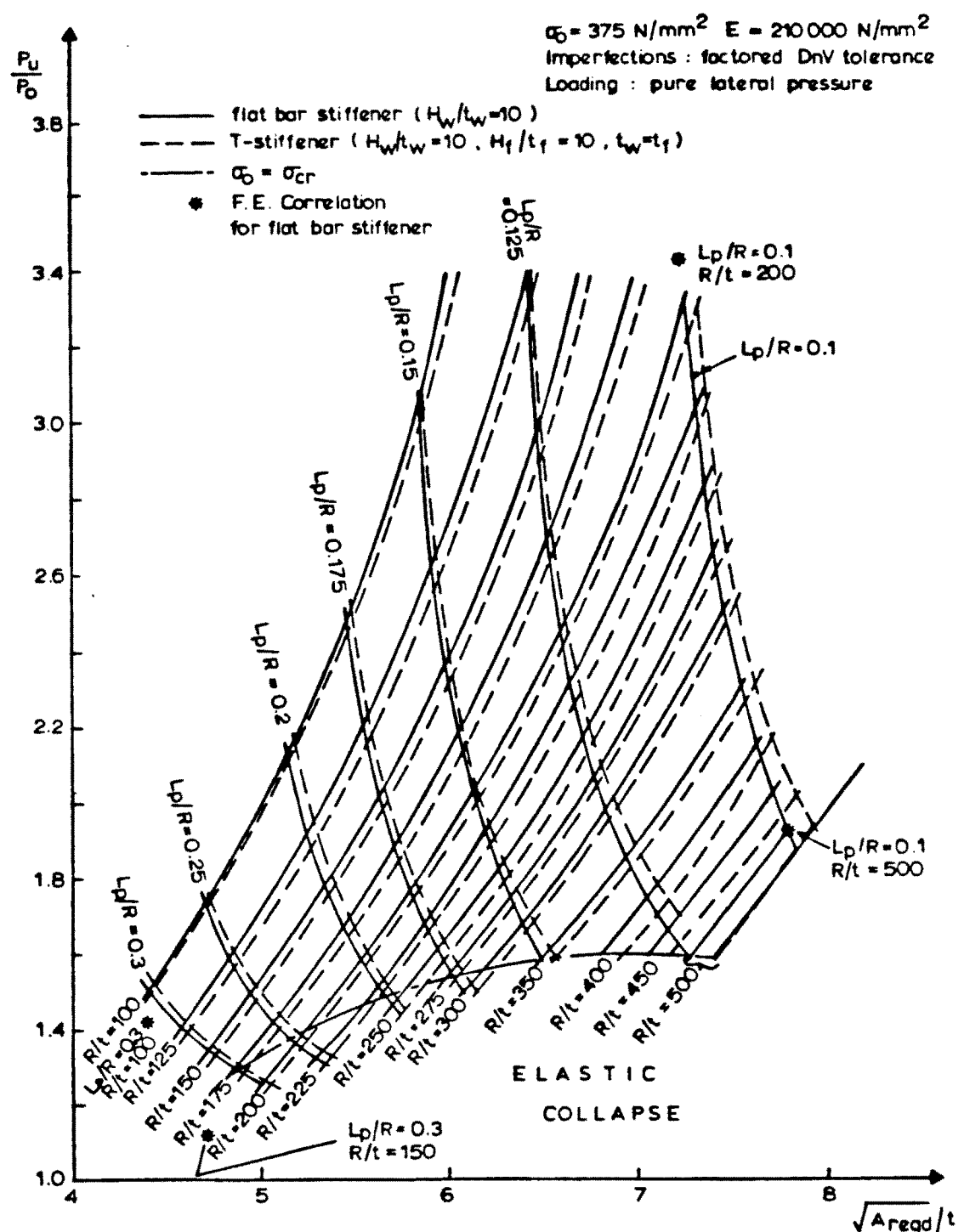


Fig. 5.28 Effect of stiffener size on load carrying capacity under pure lateral pressure

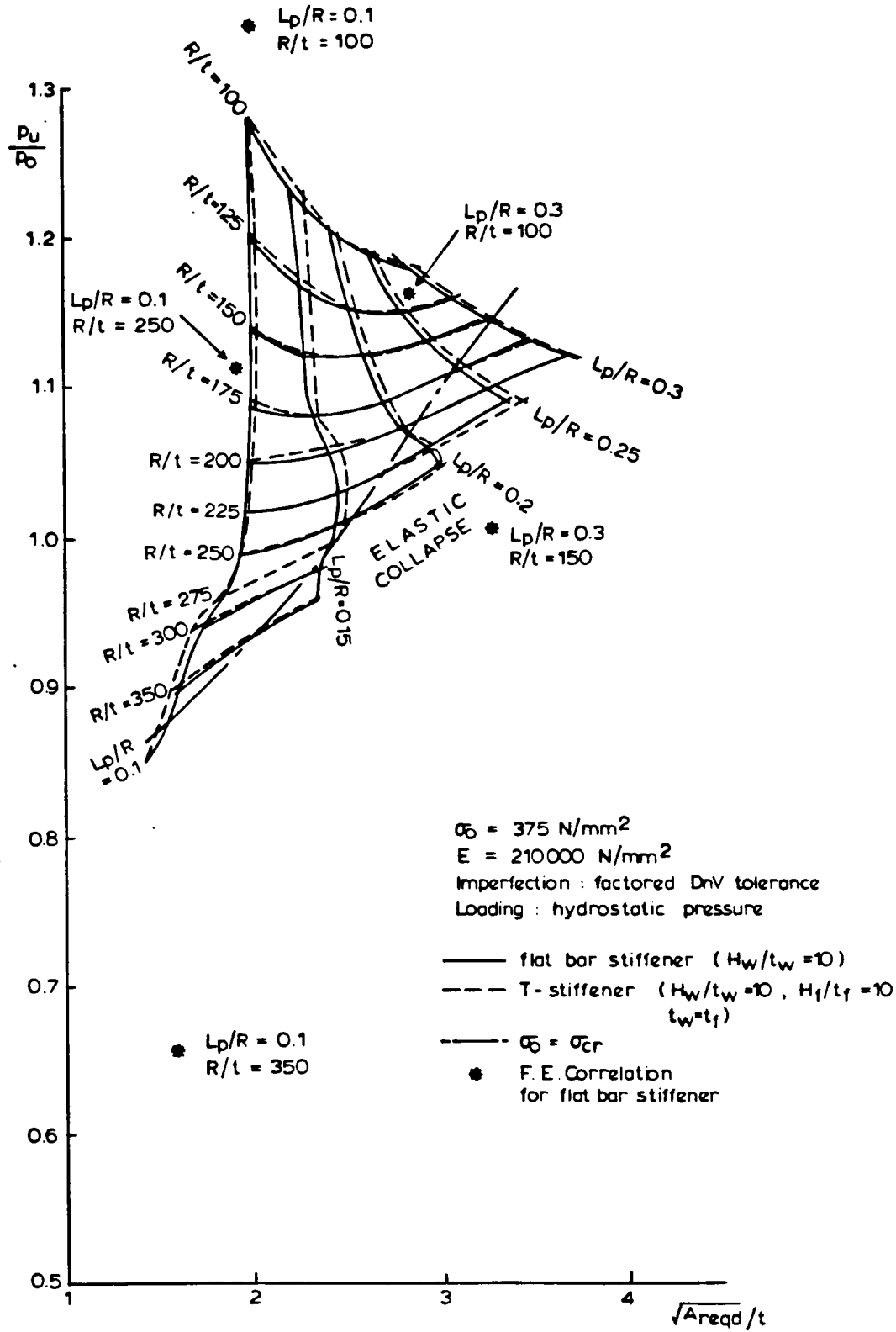


Fig. 5.29 Effect of stiffener size on load carrying capacity under hydrostatic pressure

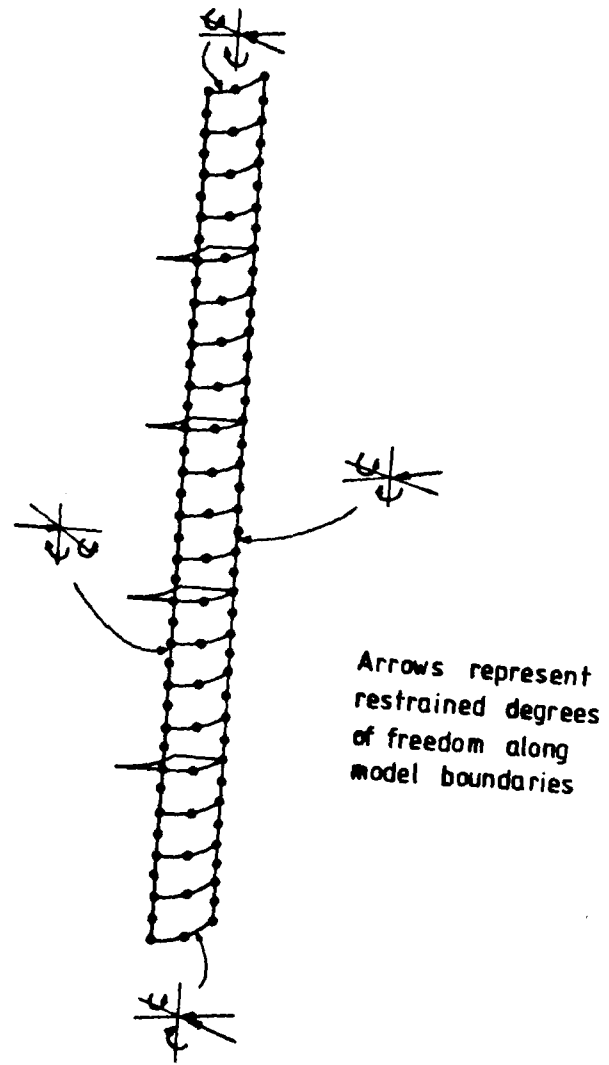


Fig.6.1 Axisymmetric finite element model

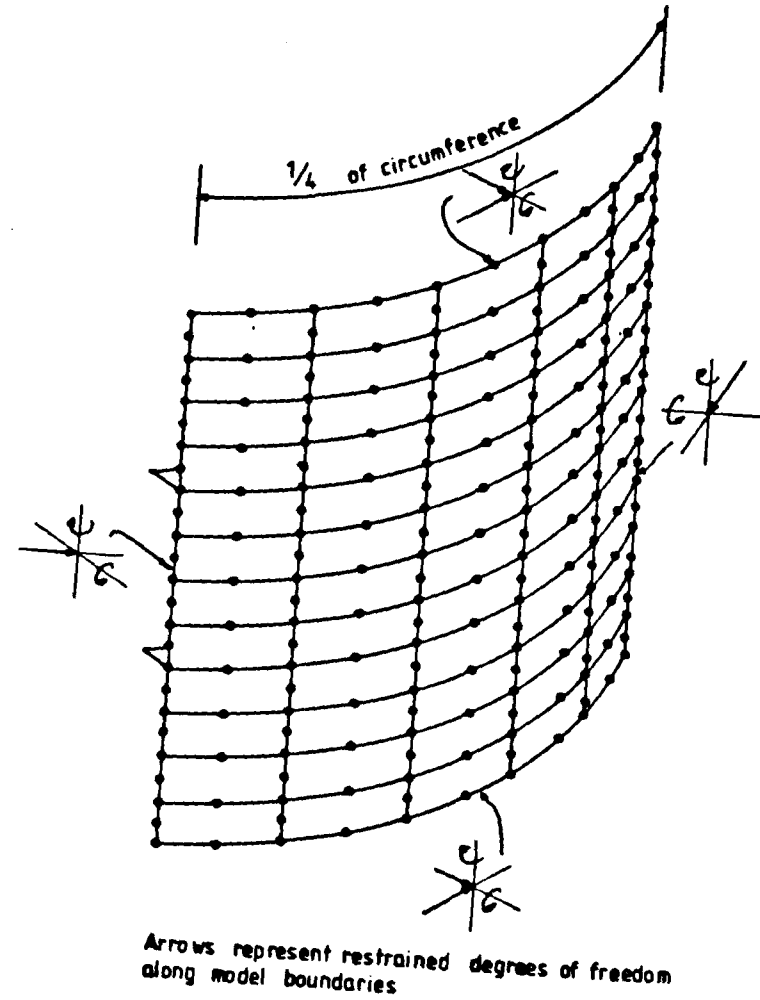


Fig. 6.2 Asymmetric finite element model

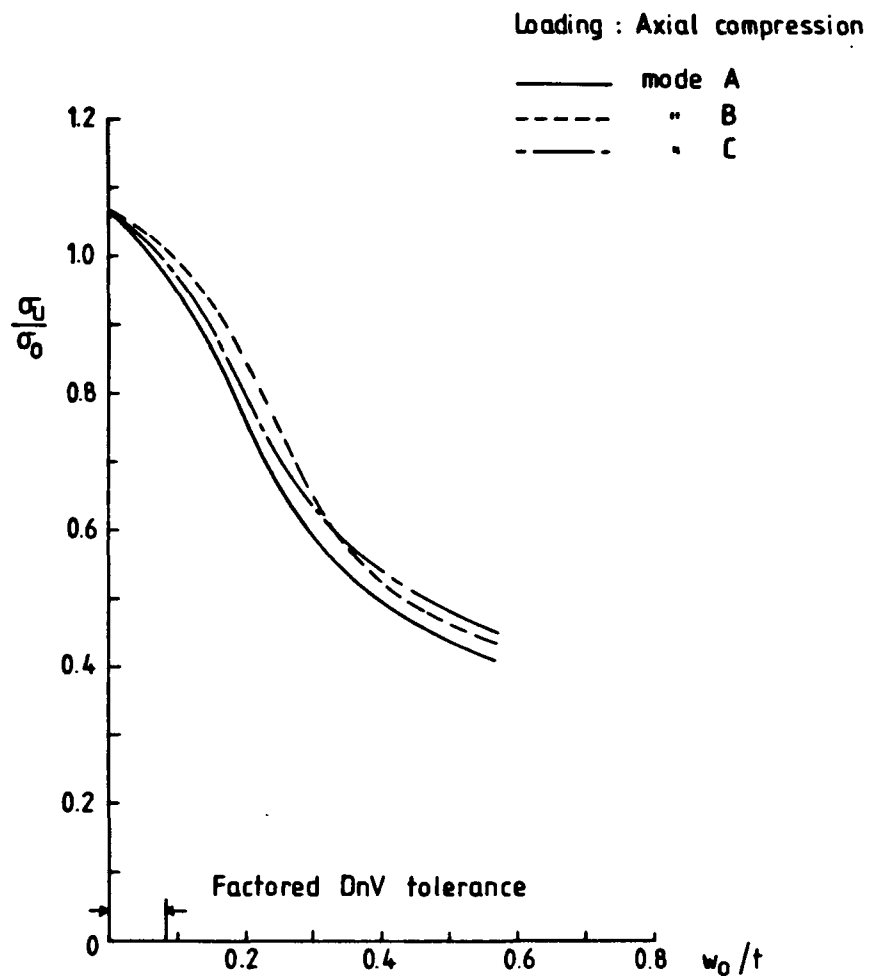


Fig. 6.3 Imperfection sensitivity of various modes for $R/t = 100$, $L_p/R = 0.1$

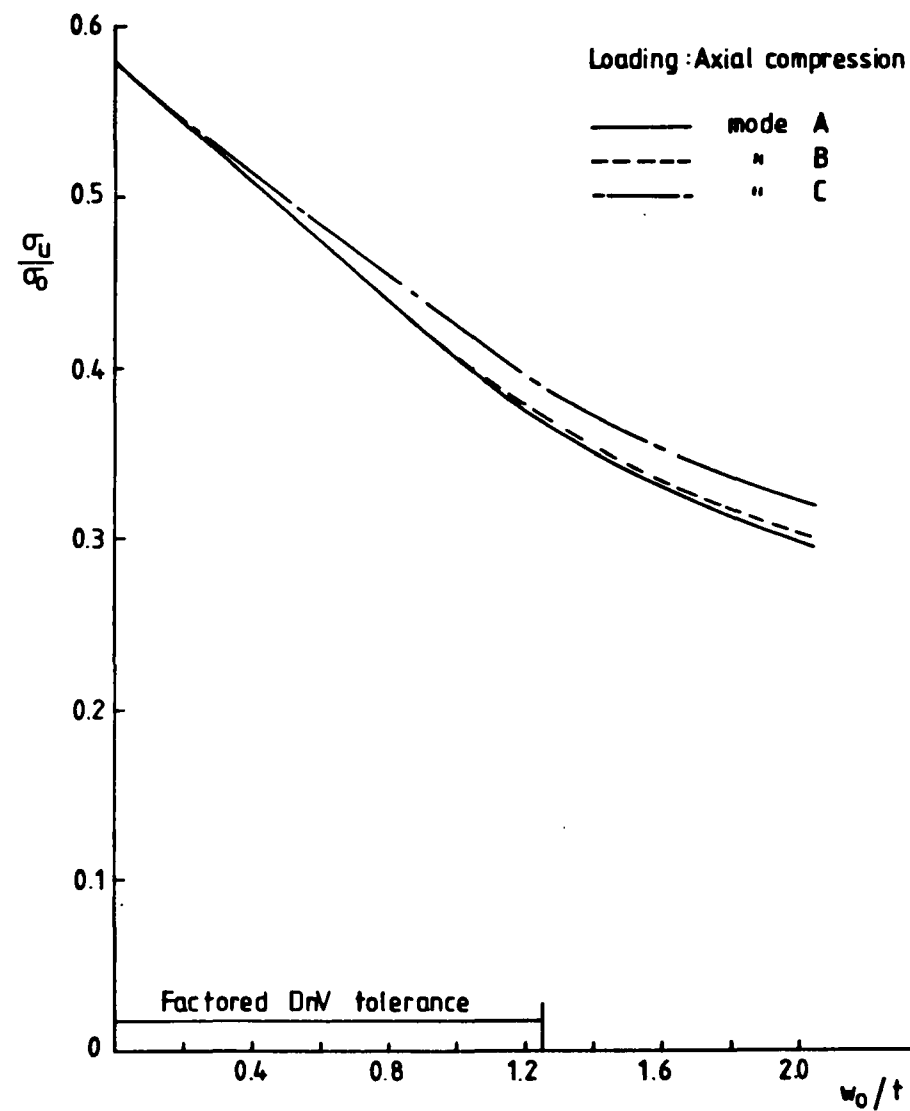


Fig. 6.4 Imperfection sensitivity of various modes for $R/t = 500$, $L_p/R = 0.3$

Imperfection Mode	Cylinder Geometry	
	$L_p/R = 0.1, R/t = 100$	$L_p/R = 0.3, R/t = 500$
perfect		
A		
B		
C		

Arrows denote position of ring stiffeners

Fig 6.5 Buckled shapes of cylinders with various imperfection modes

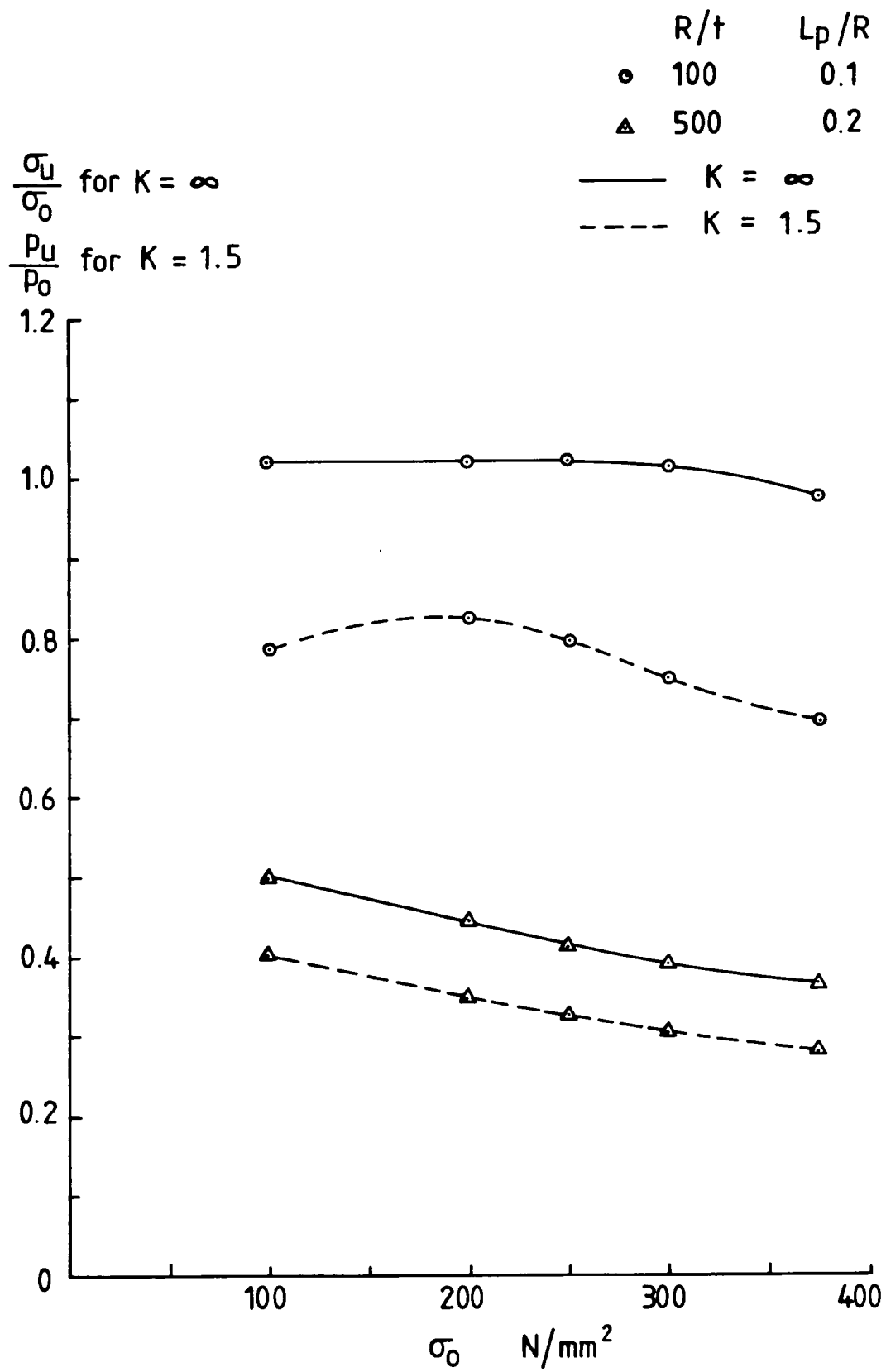


Fig. 6.6 Effect of σ_0 on loading carrying capacity

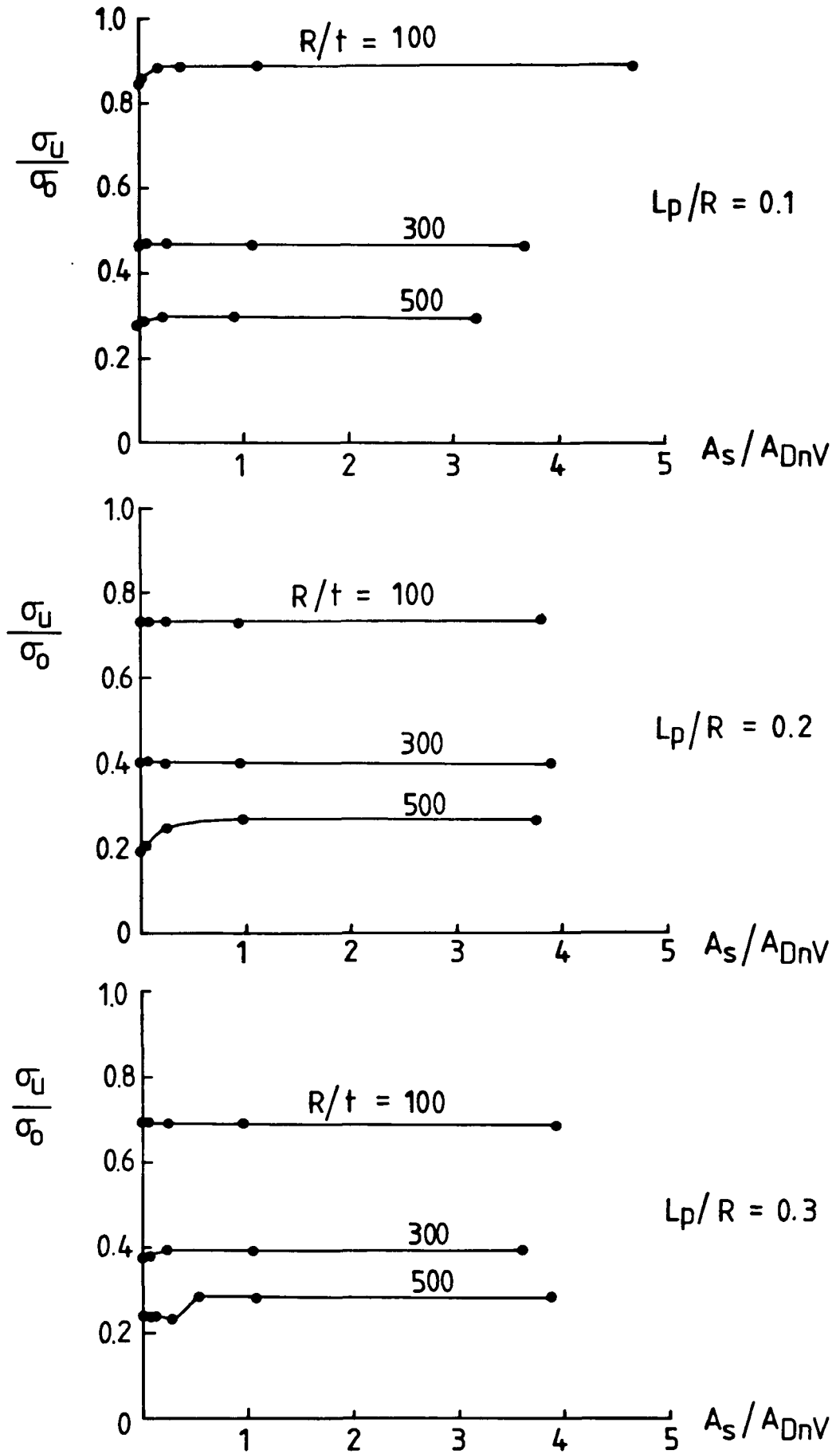


Fig 6.7 Effect of stiffener area on load carrying capacity under axial compression

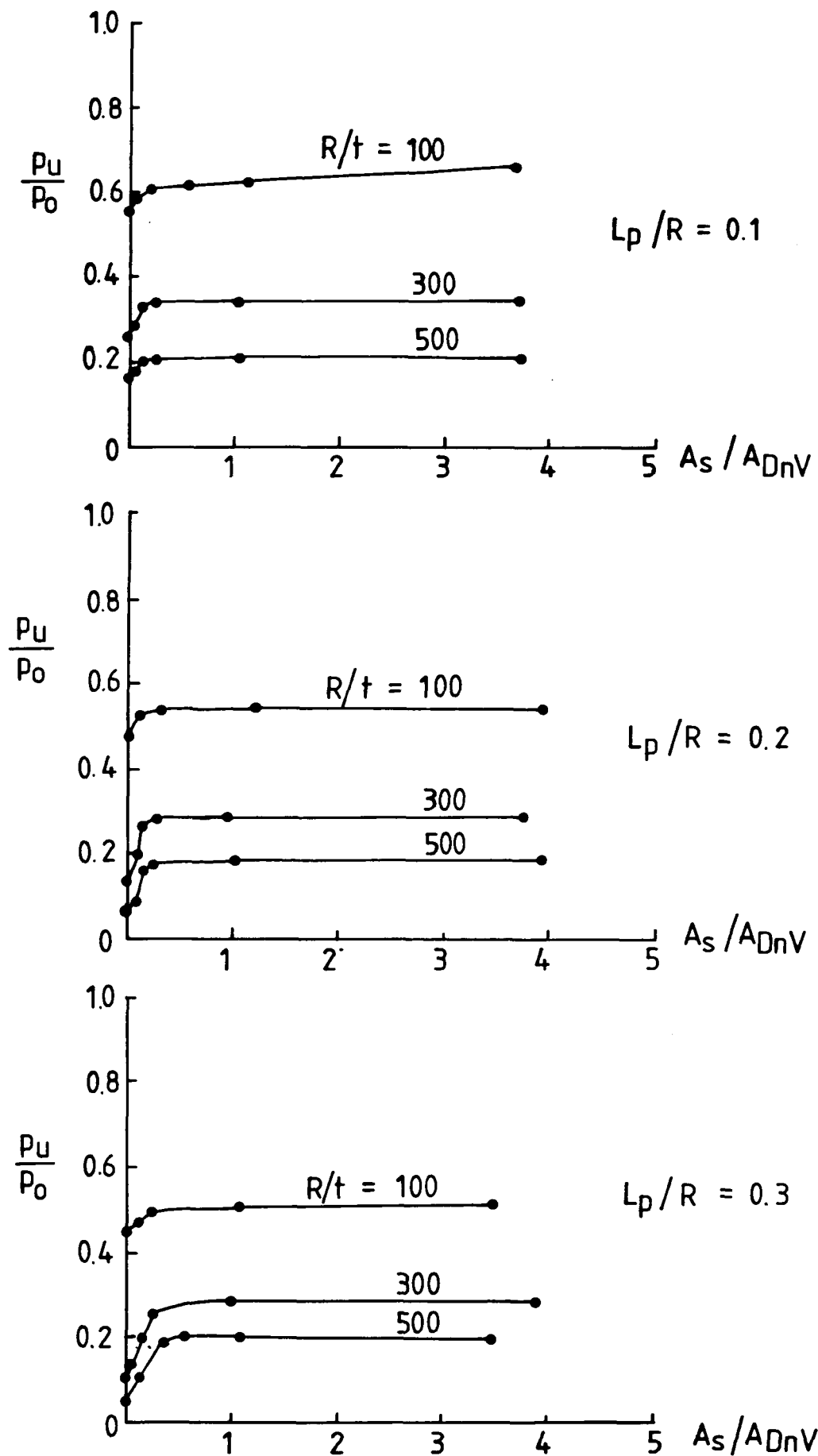
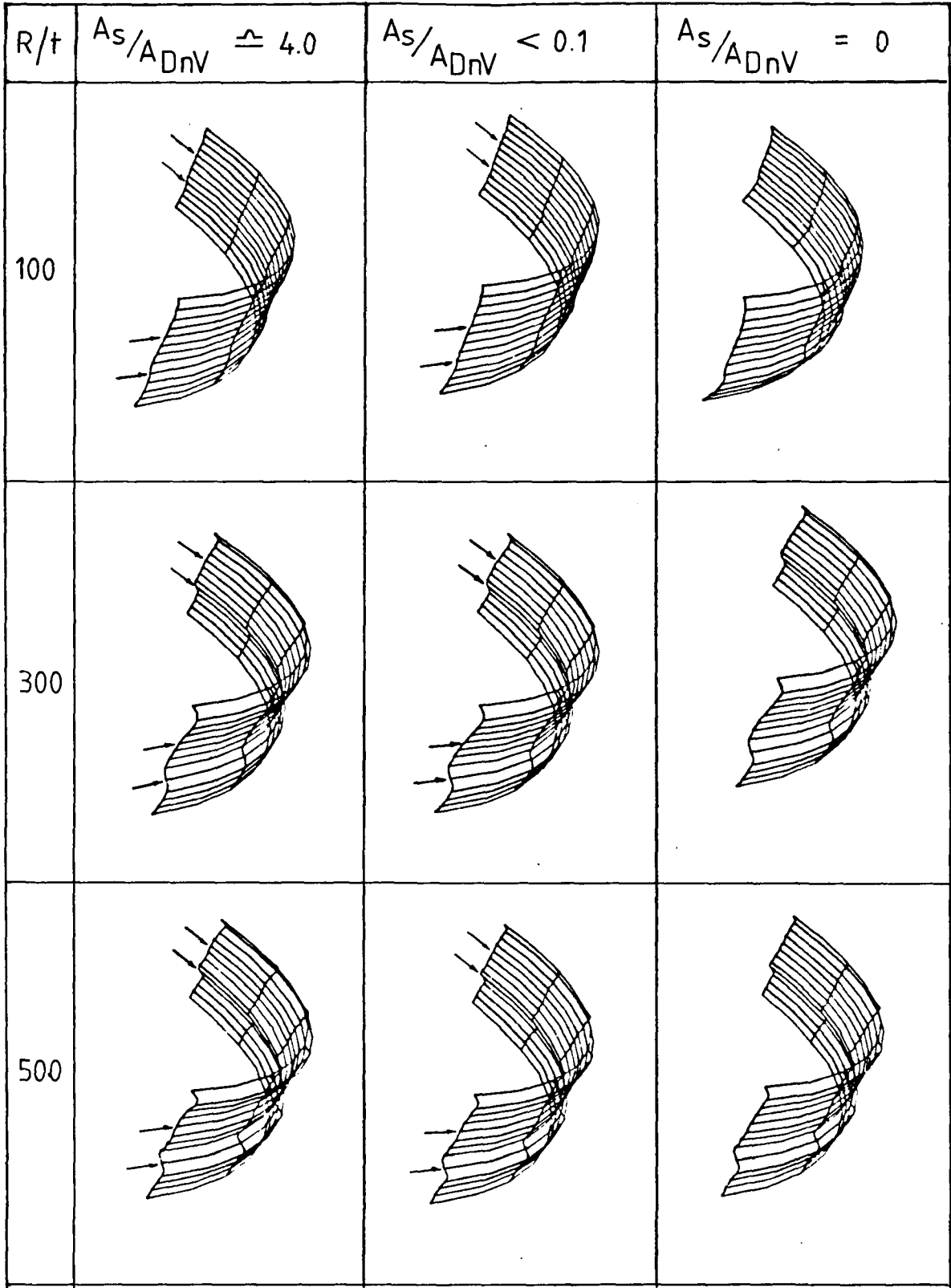
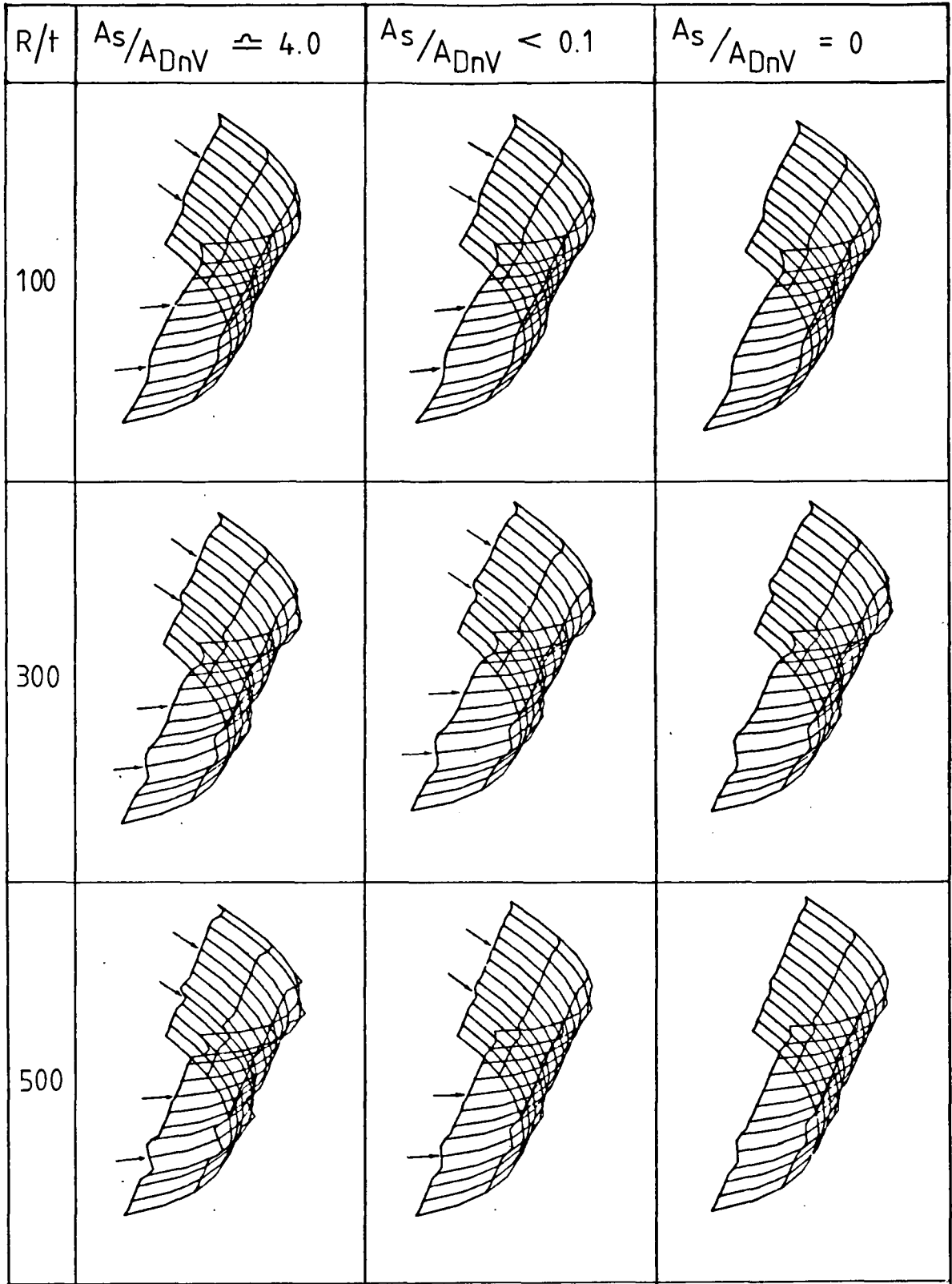


Fig 6.8 Effect of stiffner area on load carrying capacity for $K = 1.5$



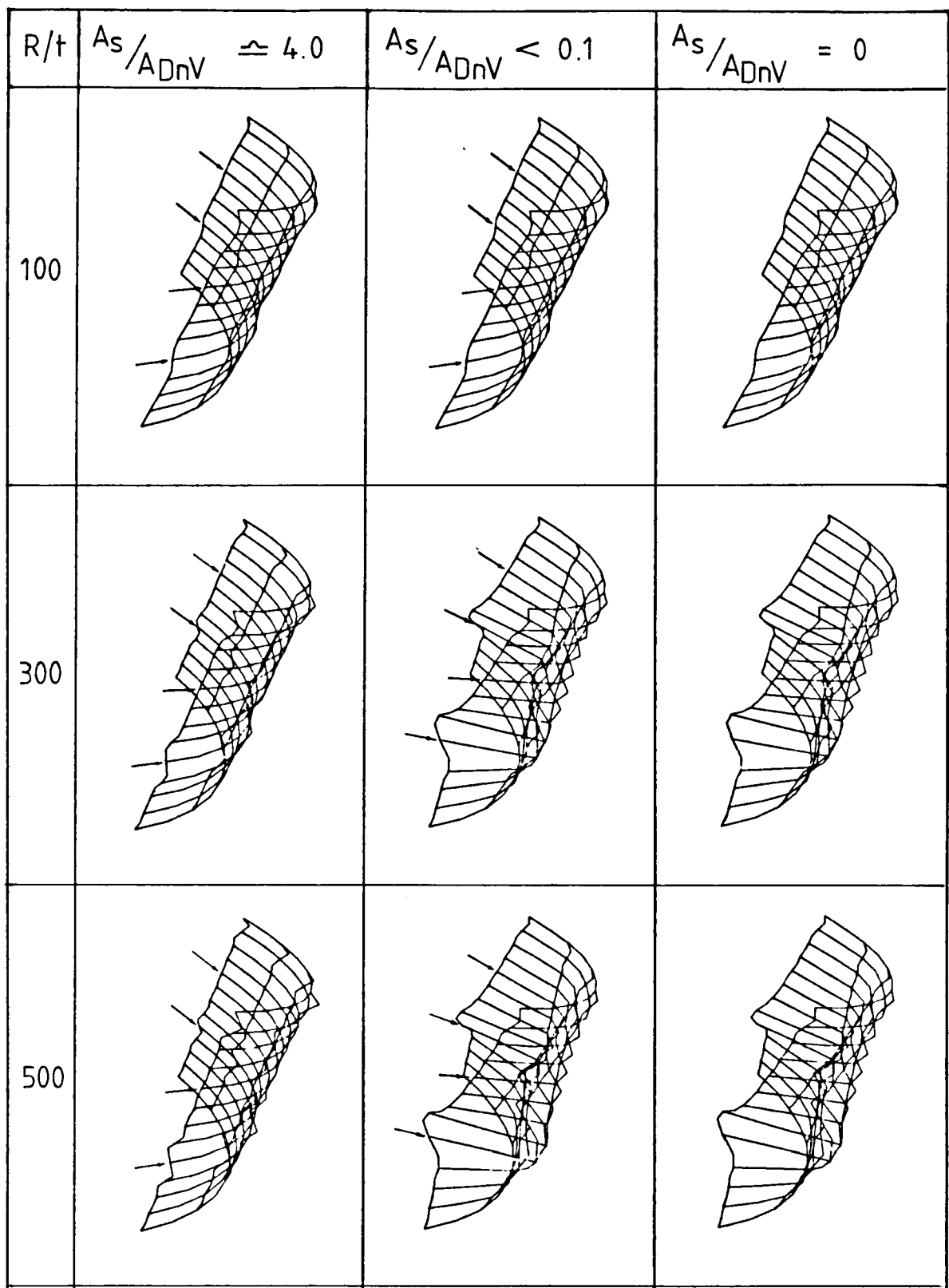
Arrows denote stiffener positions

Fig. 6.9 Collapse modes for $K = \infty$ ($L_p/R = 0.1$)



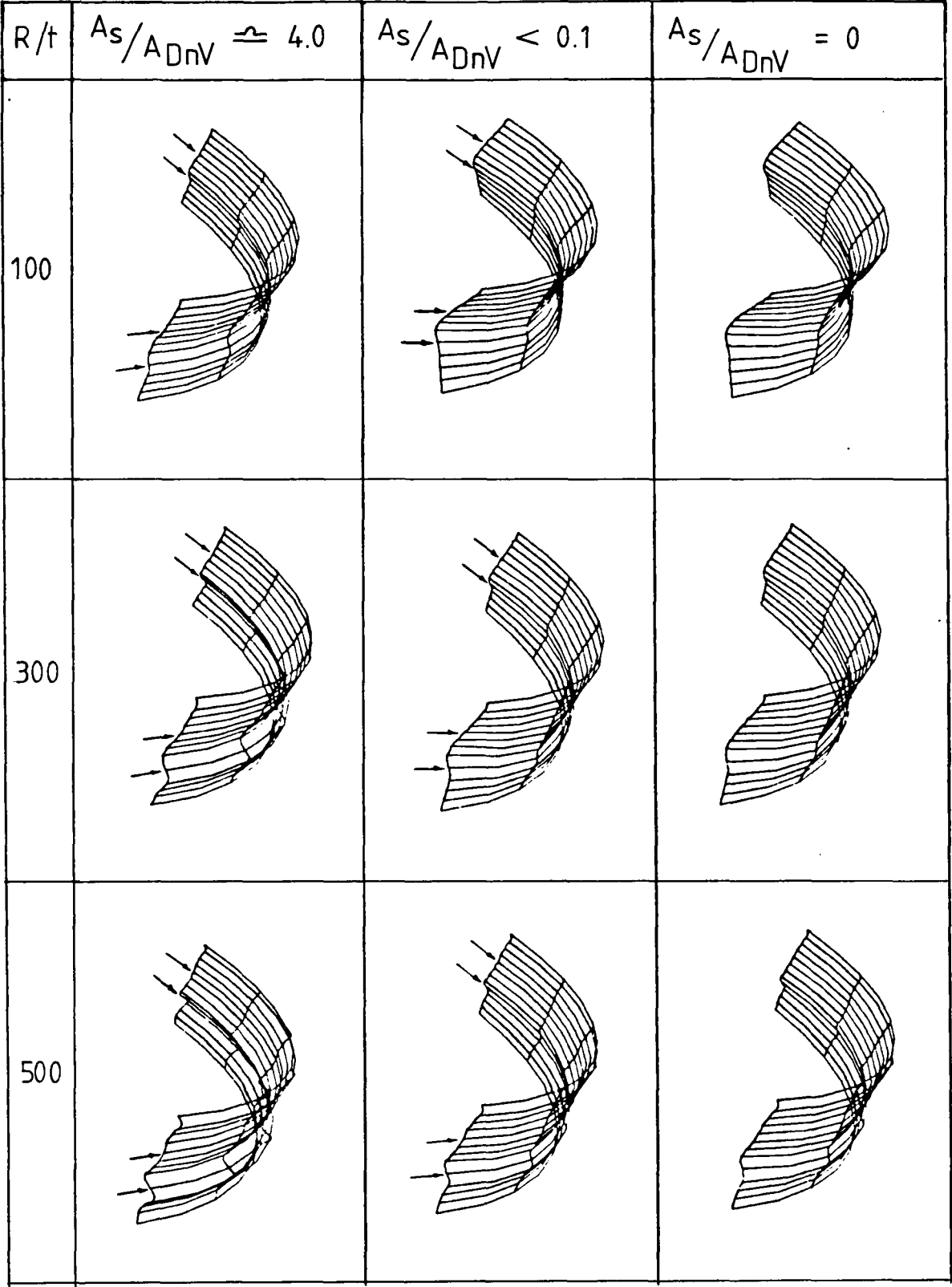
Arrows denote stiffener positions

Fig. 6.10 Collapse modes for $K = \infty$ ($L_p/R = 0.2$)



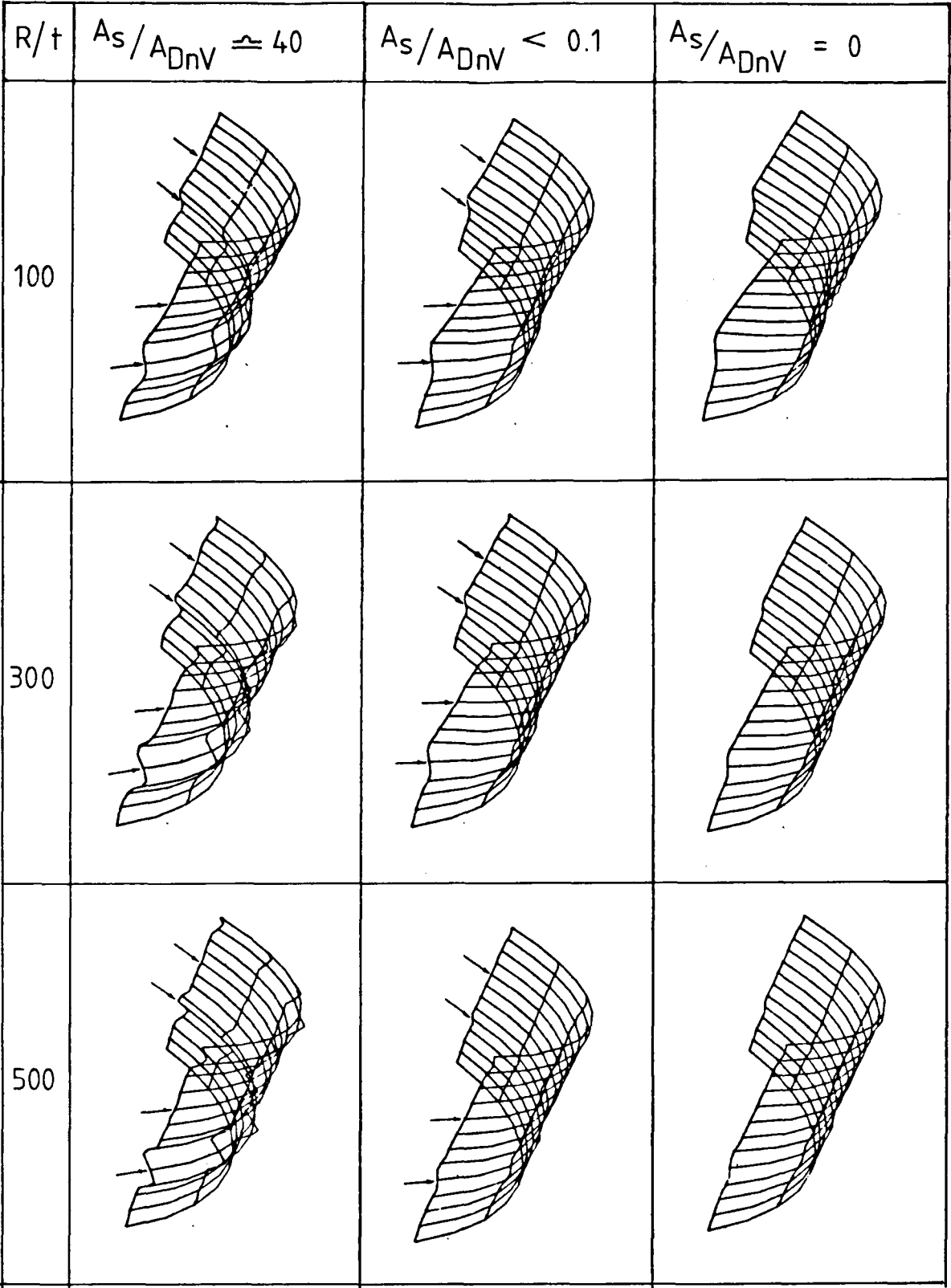
Arrows denote stiffener positions

Fig. 6.11 Collapse modes for $K = \infty$ ($L_p/R = 0.3$)



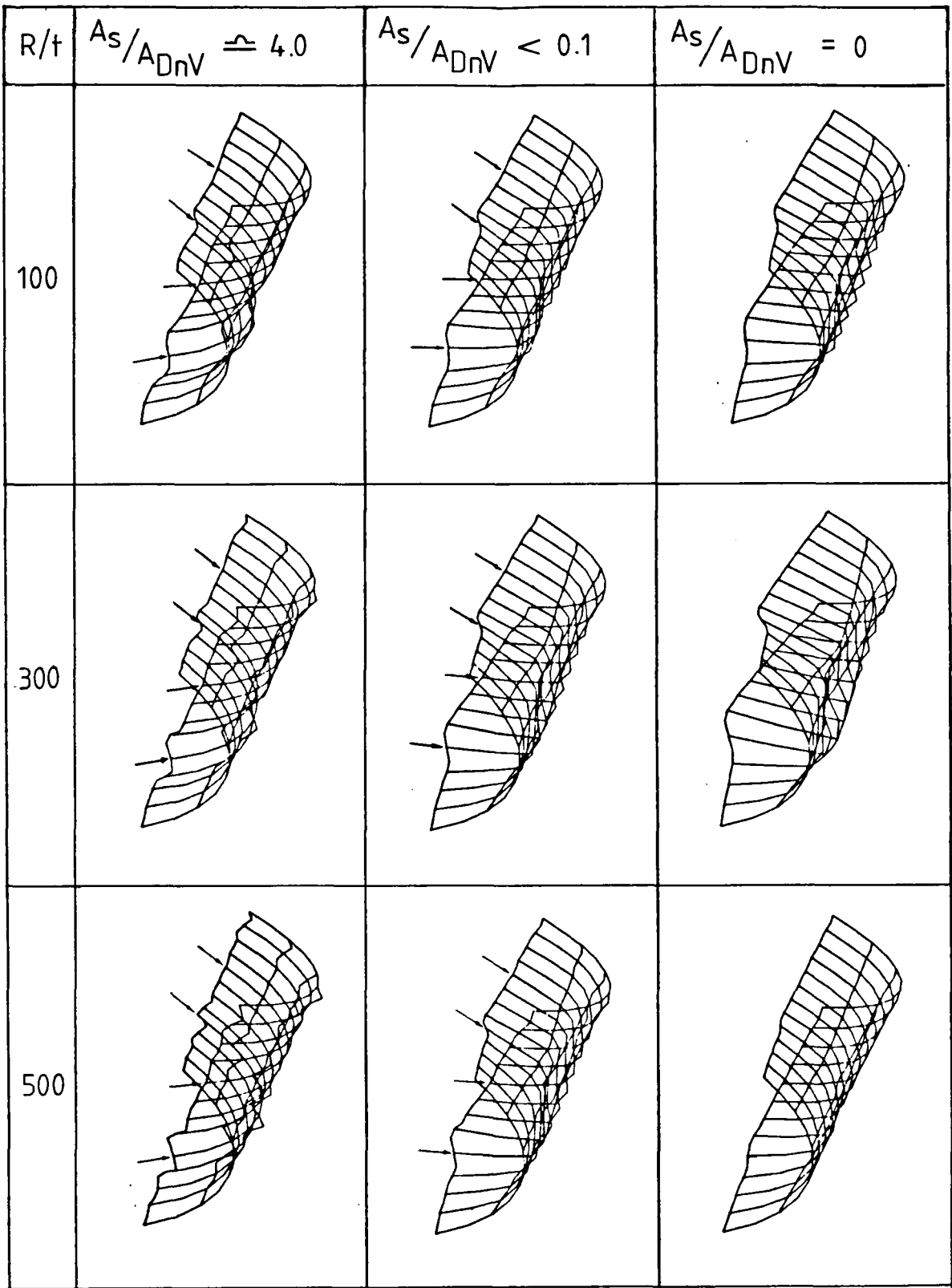
Arrows denote stiffener positions

Fig. 6.12 Collapse modes for $K = 1.5$ ($L_p/R = 0.1$)



Arrows denote stiffener positions

Fig 6.13 Collapse modes for $K = 1.5$ ($L_p/R = 0.2$)



Arrows denote stiffener positions

Fig 6.14 Collapse modes for $K = 1.5$ ($L_p/R = 0.3$)

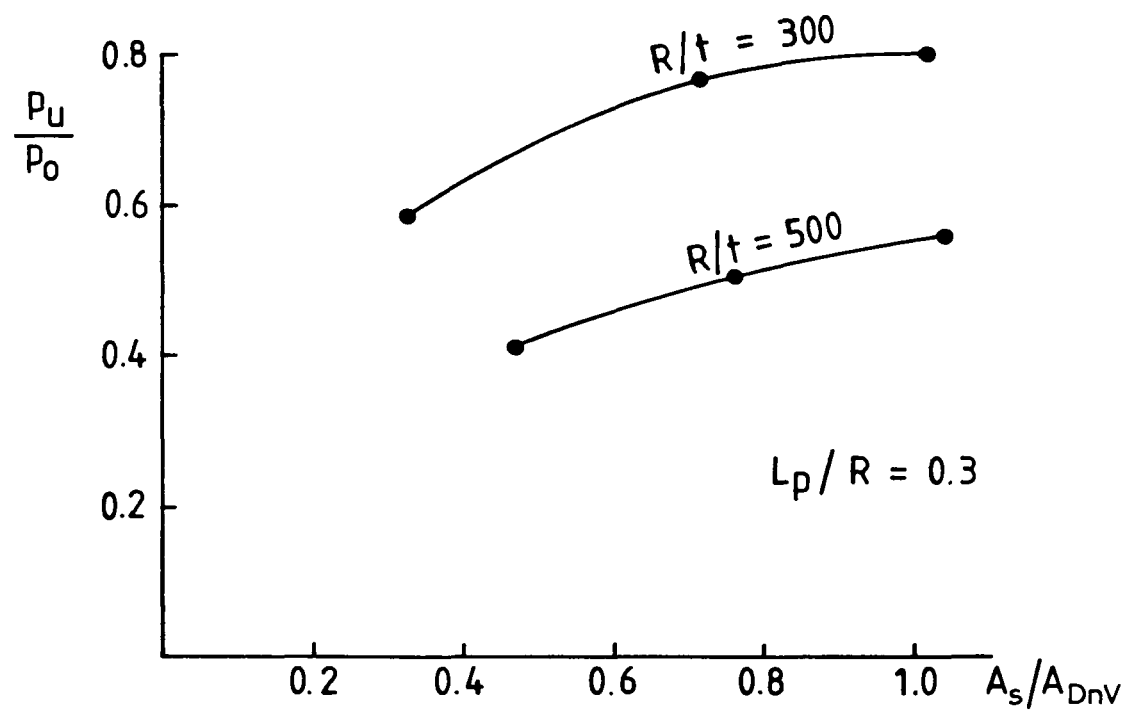
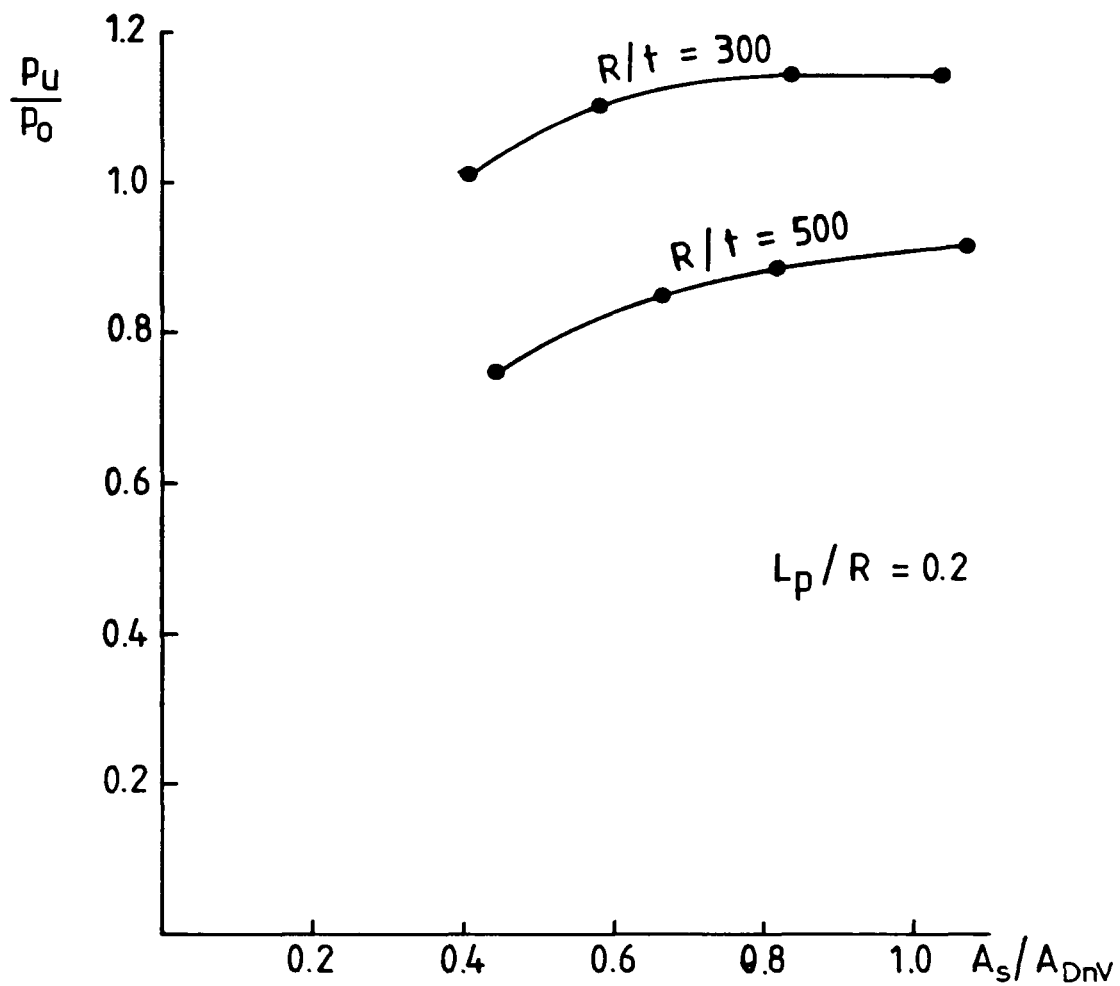


Fig 7.1 Effect of stiffener area on load carrying capacity under lateral pressure

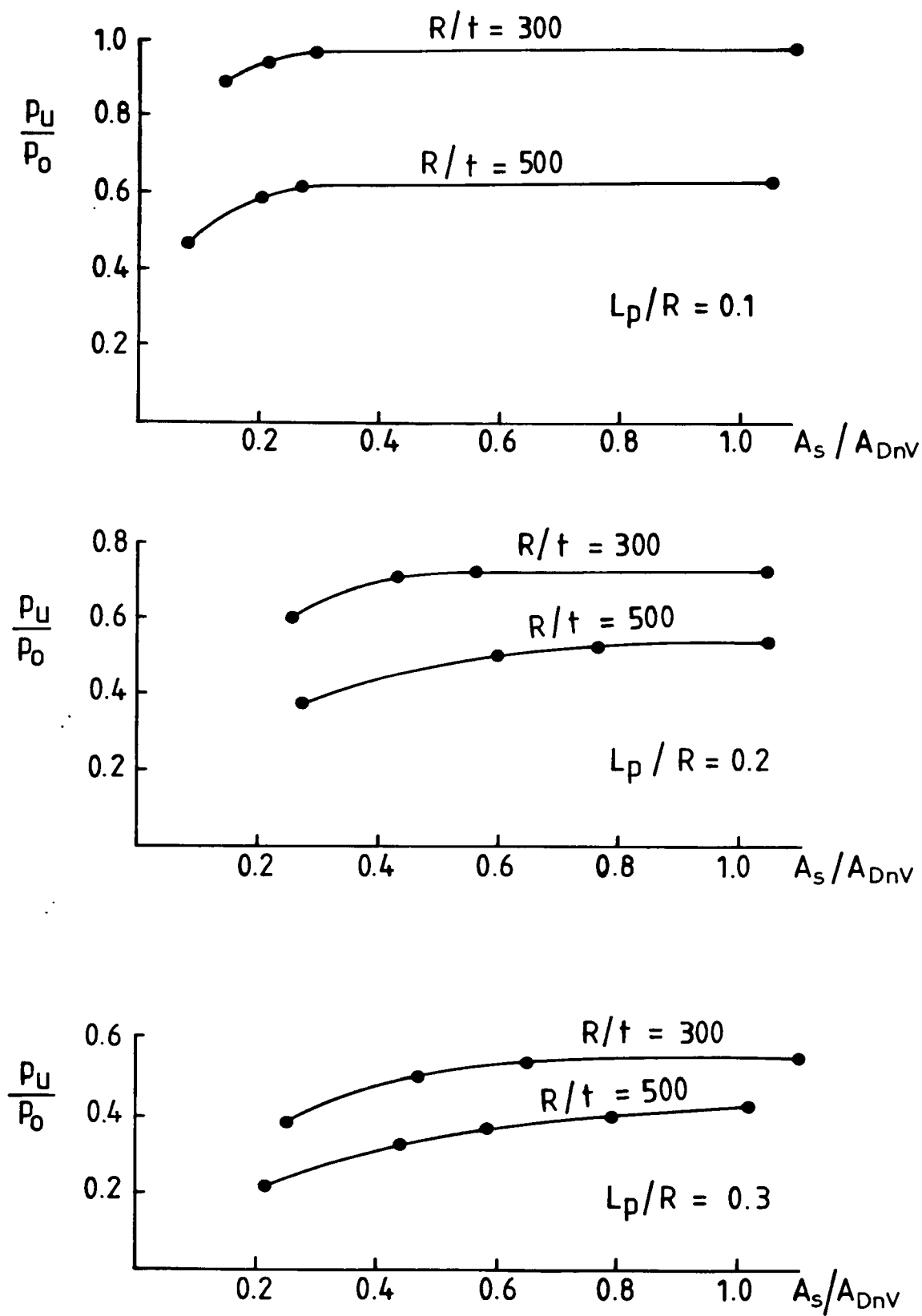
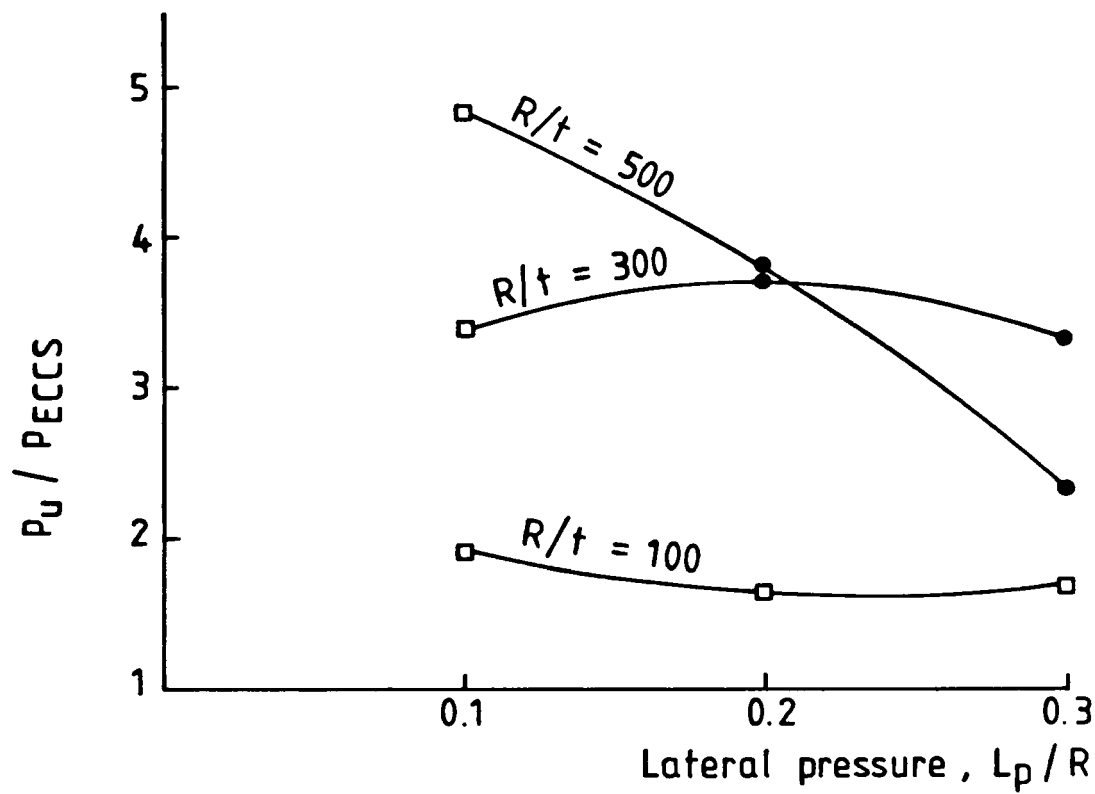


Fig. 7.2 Effect of stiffener area on load carrying capacity under hydrostatic pressure



- plastic mechanism
- finite element

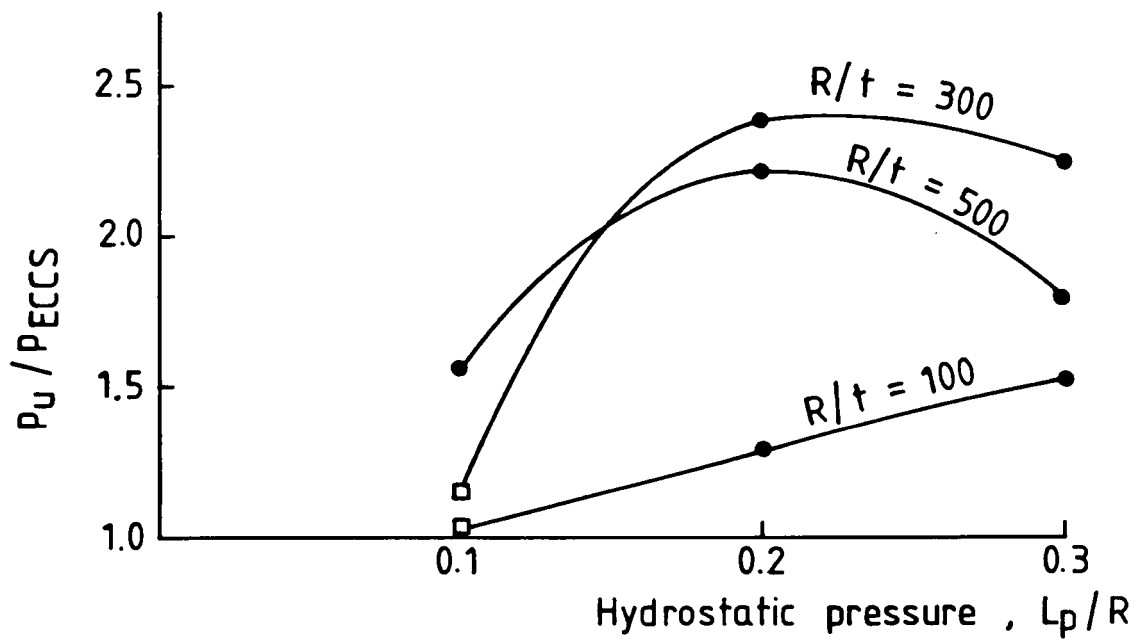


Fig. 7.3 Comparison of panel strength with ECCS predictions

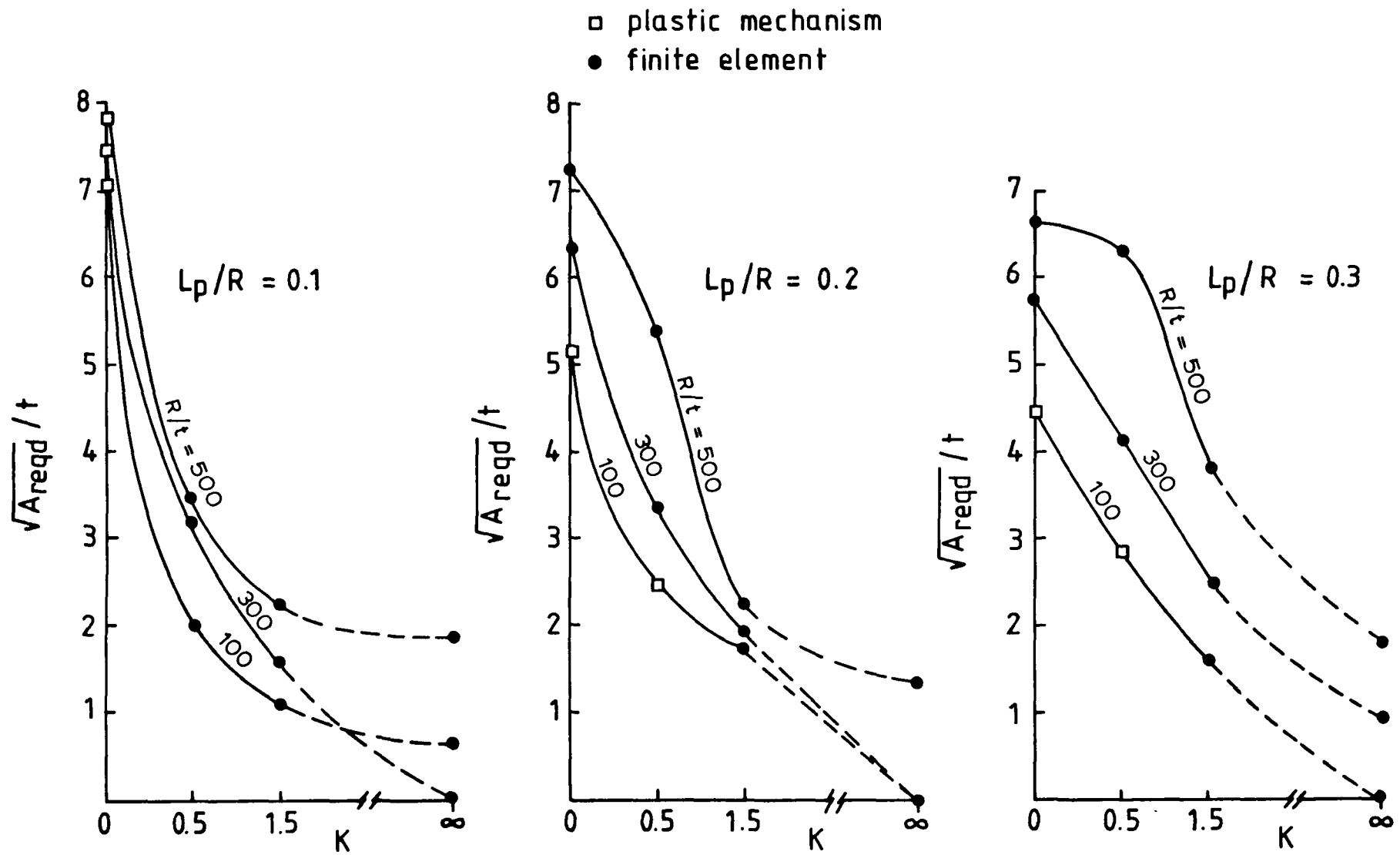


Fig. 7.4 Stiffener size requirement for various geometries

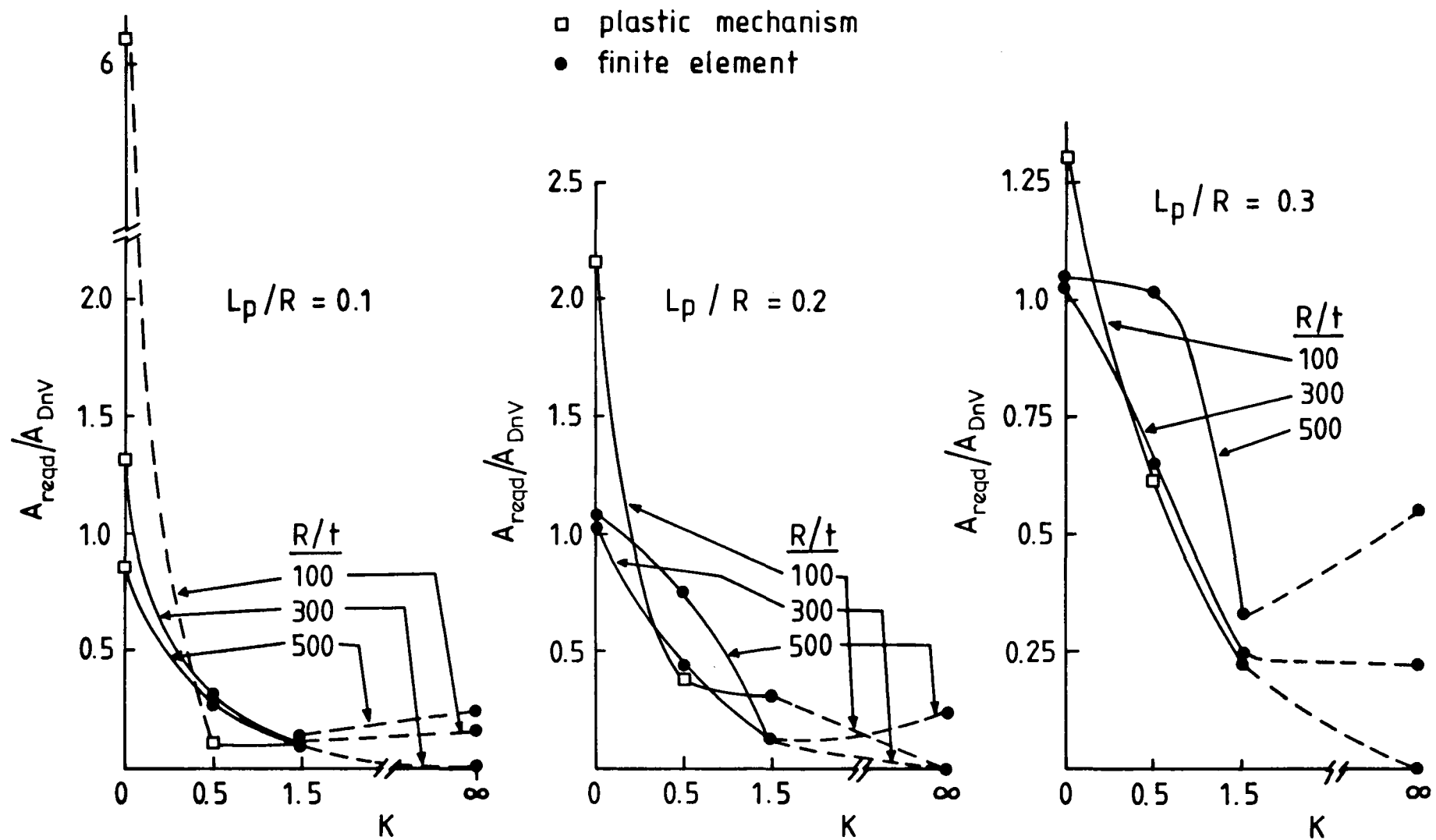


Fig. 7.5 Comparison of required stiffener areas with DnV predictions

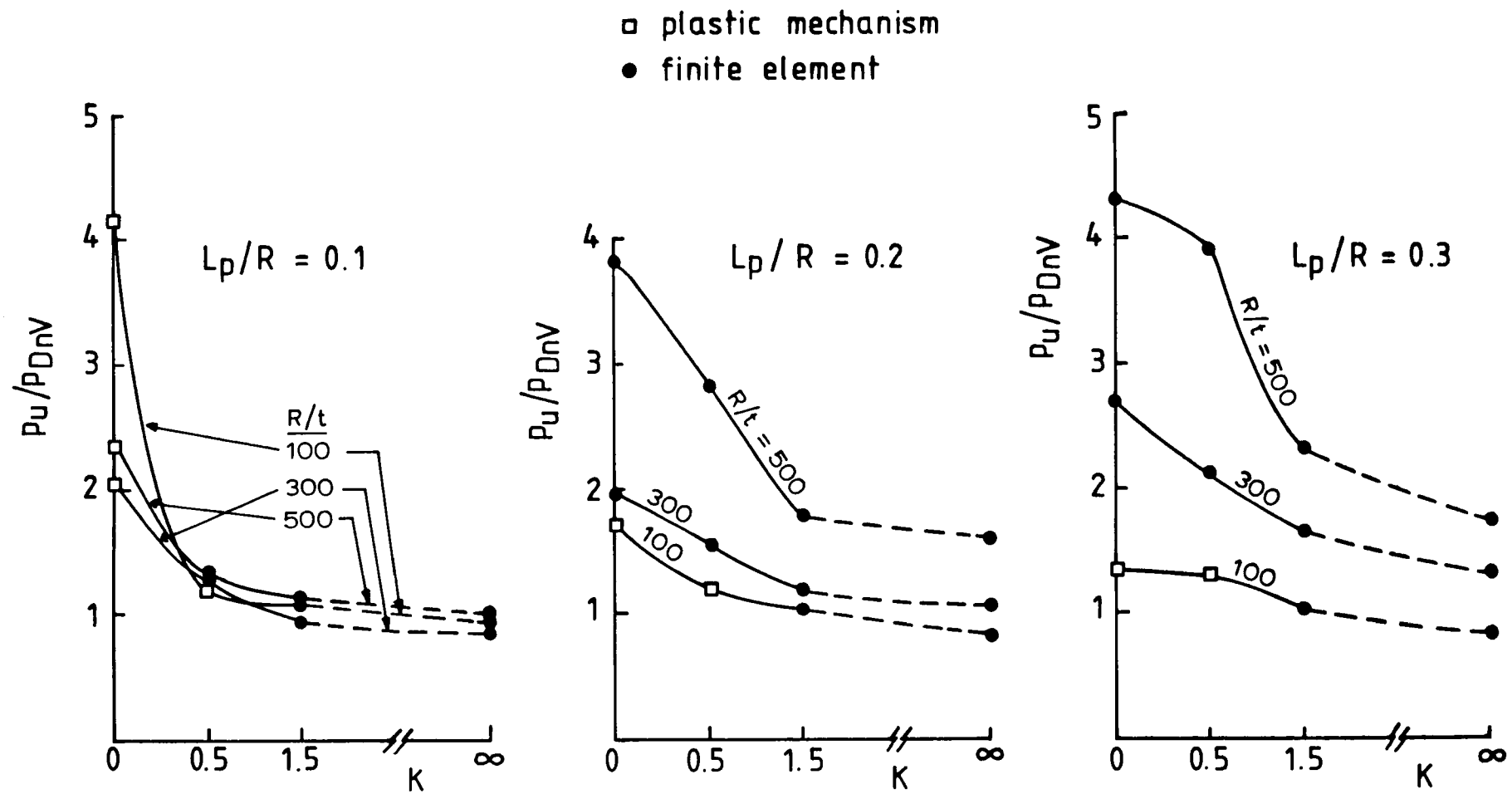


Fig.7.6 Comparison of strengths with DnV panel predictions

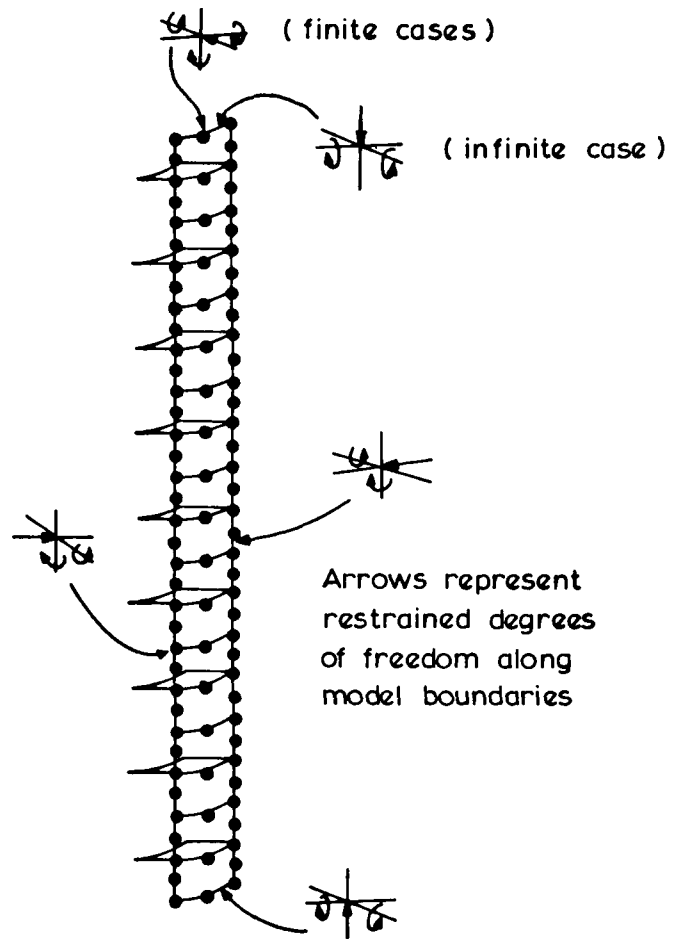


Fig. 7.7 Axisymmetric finite element model

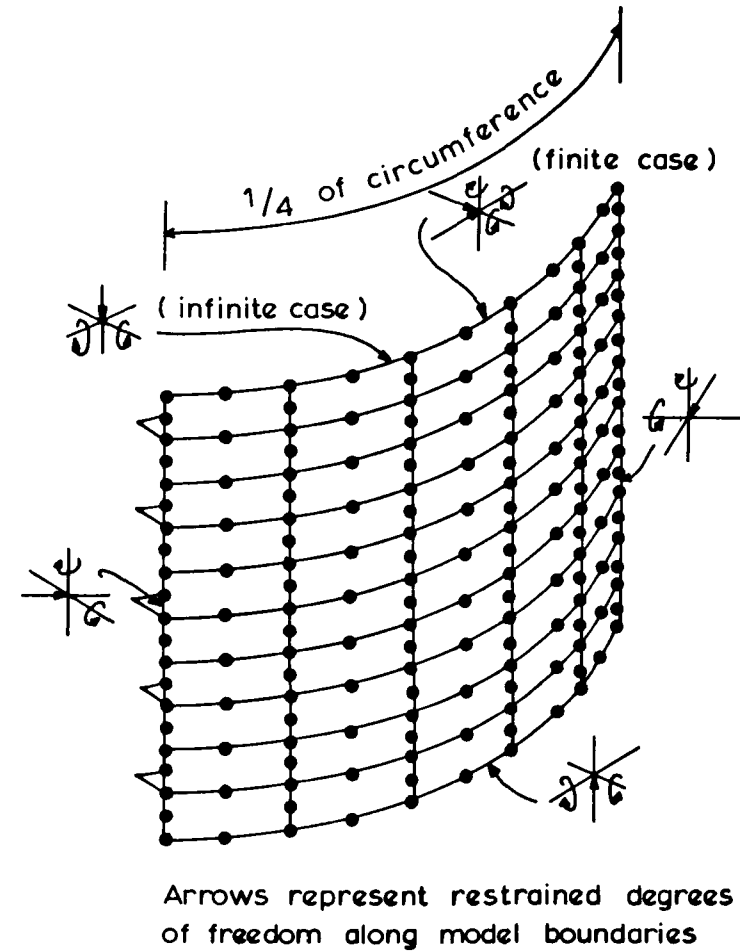


Fig. 7.8 Asymmetric finite element model

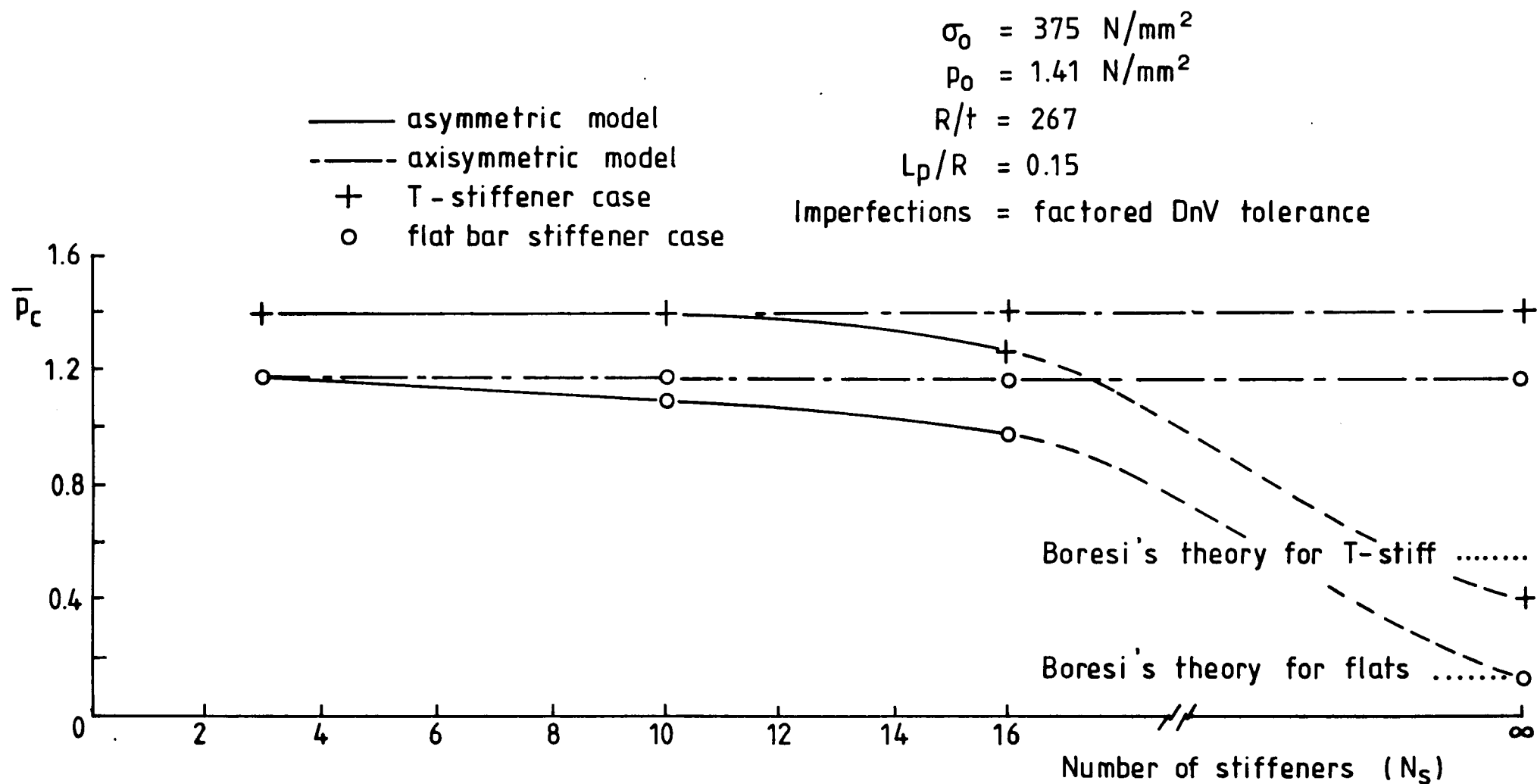


Fig.7.9 Dependence of collapse pressure on stiffener number

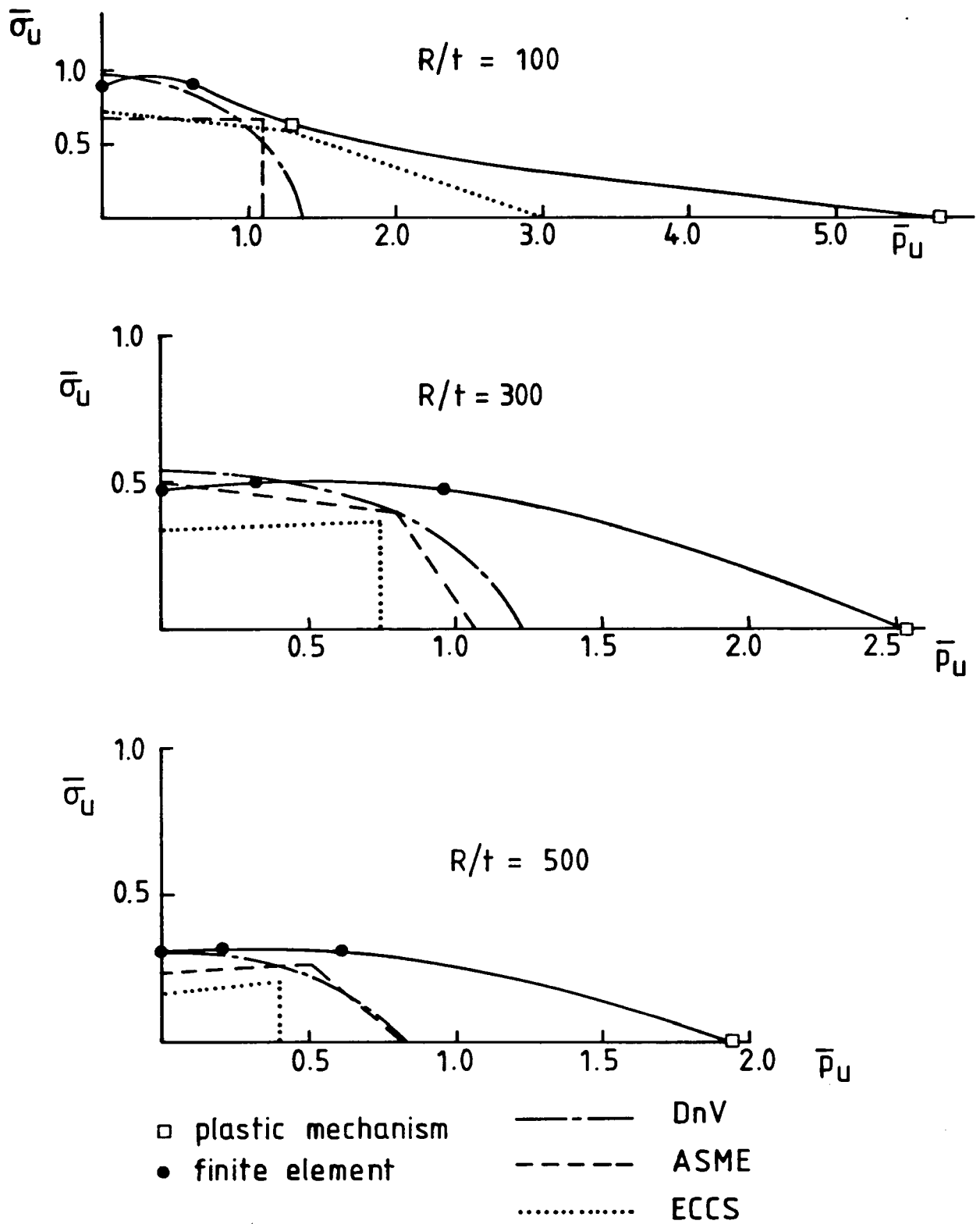


Fig. 7.10 Comparison with design Code predictions for $L_p / R = 0.1$

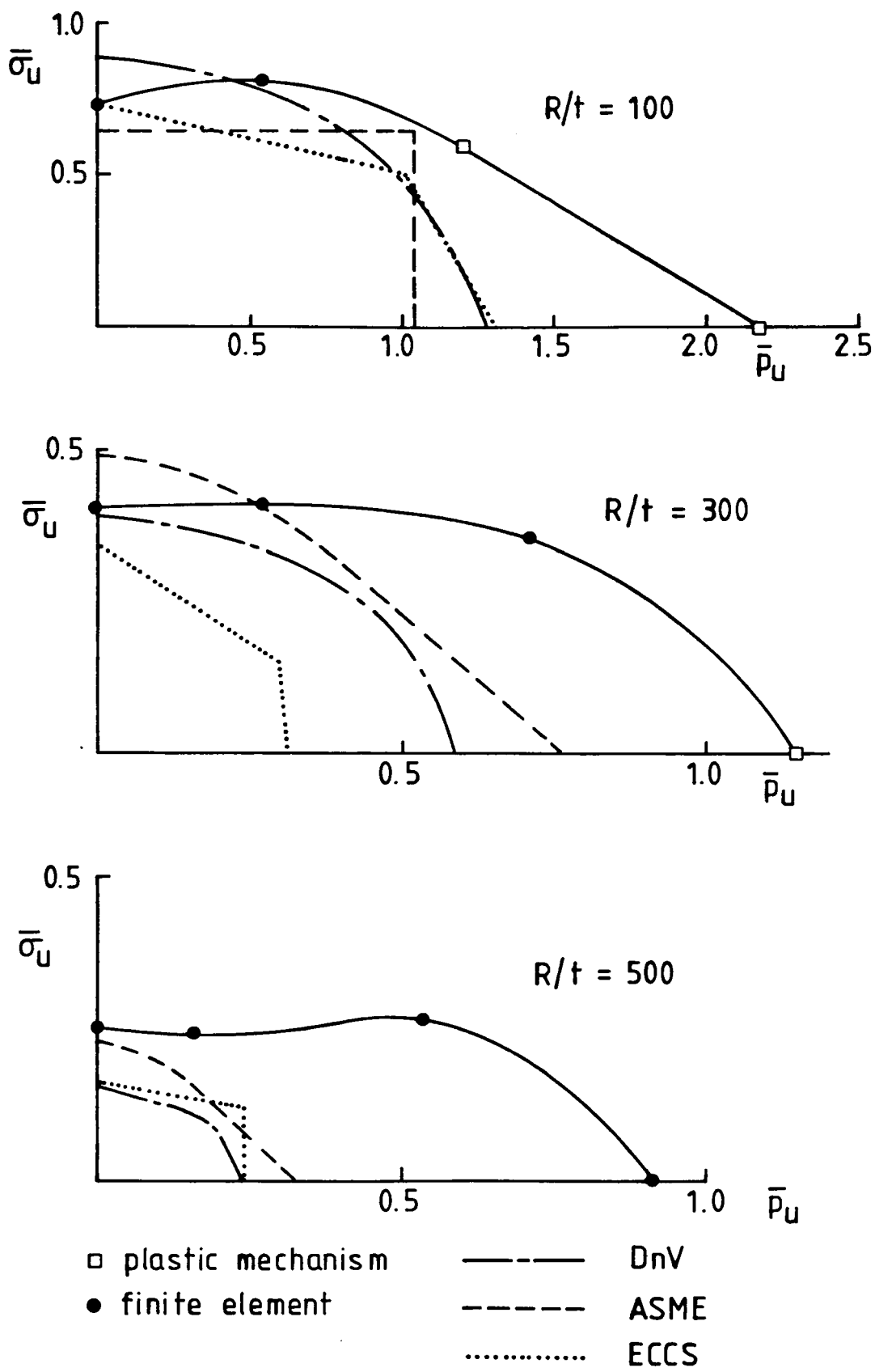


Fig. 7.11 Comparison with design code predictions for $L_p / R = 0.2$

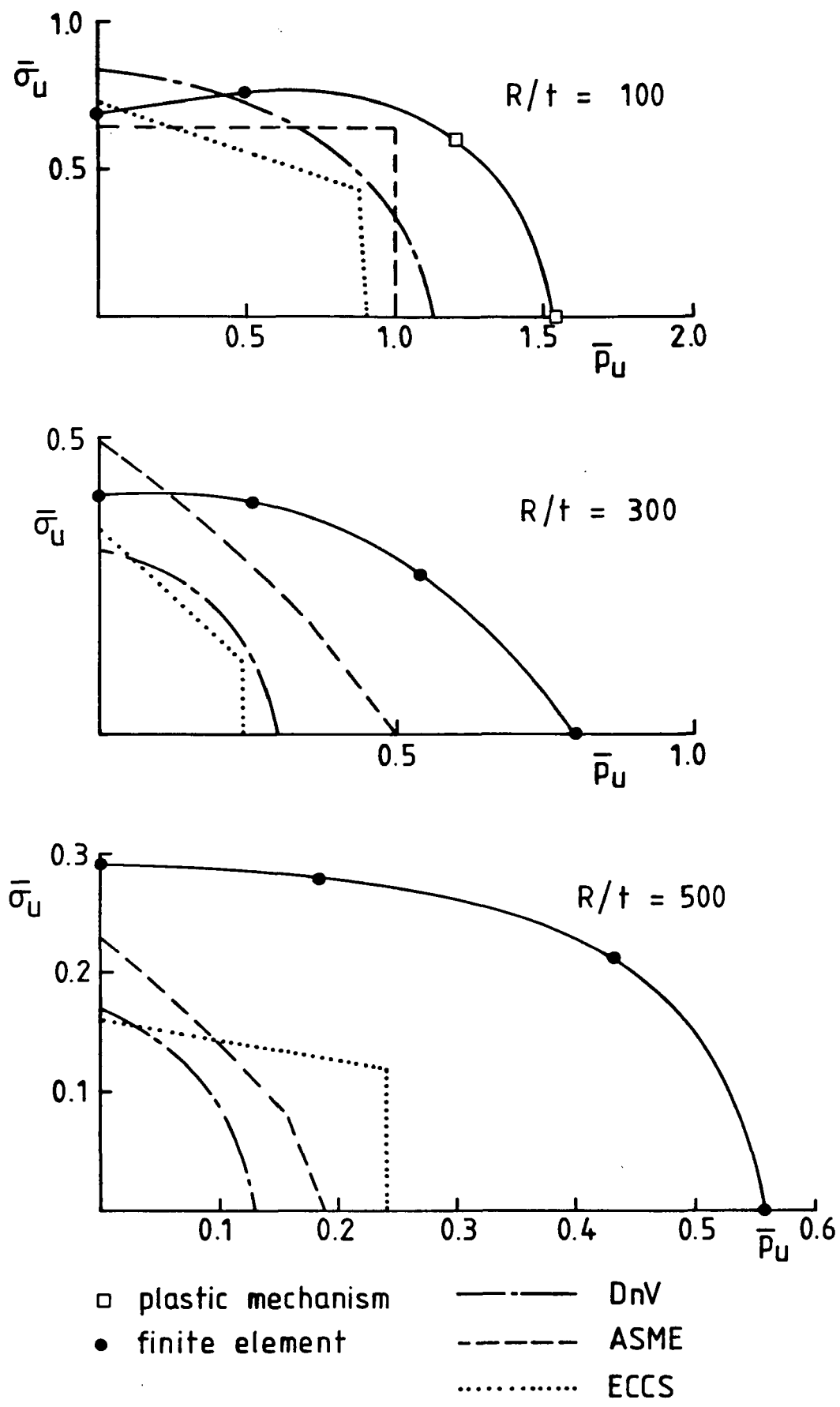


Fig. 7.12 Comparison with design code predictions for $L_p/R = 0.3$