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ABSTRACT

The topology of the applied magnetic field is an important design aspect of Hall thrusters. For modern Hall thrusters, the magnetic field topology most often features curved lines with a concave (negative) curvature upstream of the field's peak and a convex (positive) curvature downstream. Additionally, the advent of the magnetic shielding technique has resulted in Hall thruster designs with non-conventional field topologies that exhibit high degrees of concavity upstream of the field's peak. In this article, a detailed study is carried out on the effects that the magnetic field curvature has on the plasma discharge in a 2D configuration representative of a Hall thruster's radial-azimuthal cross section. The analyses are performed for discharges of three propellants of high applied interest: xenon, krypton, and argon. For each propellant, high-fidelity electrostatic reduced-order particle-in-cell (PIC) simulations are performed with various degrees of positive and negative curvatures of the magnetic field. Corresponding 1D radial PIC simulations are also performed for xenon to compare the observations against the 2D results. Most notably, it is observed that the instability spectra in the positive-curvature cases are mostly dominated by electron cyclotron drift instability, whereas the modified two stream instability is dominant in the negative-curvature cases. The distributions of electron and ion temperatures, in particular, as well as the contribution of various mechanisms to electrons' cross-field transport show notable variations between the positive and negative curvature values. Finally, the field curvature is observed to majorly influence the ion beam divergence along the radial and azimuthal coordinates.

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I. INTRODUCTION

Hall thrusters are a plasma propulsion technology that have an extensive heritage of in-space application and are today prime candidates to access the next generation of near-Earth and interplanetary missions. Similar to other electric propulsion (EP) concepts, Hall thrusters provide specific impulse values of at least an order of magnitude higher than the chemical thrusters, thus providing huge mass advantages at the satellite level. Nonetheless, the great applied interest in the Hall thruster technology is mainly driven by the fact that, compared to other EP technologies on the market, Hall thrusters additionally feature a unique combination of high efficiency, high thrust-to-power ratio, and operational versatility.

Hall thrusters are designed to ionize a neutral gas injected into their discharge channel, typically via DC electron bombardment, and to accelerate the resulting plasma ions in order to

generate thrust.¹ The geometry of a Hall thruster is such that the centerline of its discharge channel, which is often parallel the thruster's axis, determines the axial coordinate. The direction along the channel circumference indicates the azimuthal coordinate, and the wall-to-wall direction of the channel is the radial coordinate, which completes a right-handed coordinate system.

The plasma in Hall thrusters is subject to a perpendicular configuration of the electric (E) and magnetic (B) fields, a configuration that is referred to as cross-field or $E \times B$. In an idealized case, the electric field is along the axial direction, whereas the magnetic field is radially oriented. The E -field is self-consistently generated and maintained, whereas the B -field is externally applied. The intensity of the B -field is such that the electrons are fully magnetized, but the ions are mostly non-magnetized and are accelerated under the influence of the electrostatic field. The topology and the

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magnitude of the B -field play a significant role in the performance and the operational physics of the thrusters. As the dynamics of the magnetized electrons is influenced by the B -field, the processes and phenomena that are driven and/or affected by the properties and the behavior of the electron species, from ionization and acceleration to the plasma-wall interactions and instabilities, can be impacted by the applied magnetic field.^{1,2} This emphasizes the critical importance of a proper design of magnetic field topology in terms of its spatial distribution and magnitudes to achieve high-performance and stable operation of Hall thrusters.

In modern Hall thrusters, the magnetic field features a strong gradient along the axial direction of the thruster's discharge channel and a moderate gradient along the radial direction.^{1,3} The field lines have a concave curvature in the axial-radial plane upstream of the axial location of the magnetic field peak intensity, which typically occurs near the exit plane of the channel. Downstream of the magnetic field peak, the field lines have a convex curvature. About the location of the B -field peak, the field lines are mostly radial with no notable curvature. The concave (negative) curvature of the field lines is often referred to as being "ion focusing" in the literature.¹ On the contrary, the field lines with convex (positive) curvatures are known as "ion diverging."¹ In recent years, the technique of magnetic shielding^{4,5} of the channel walls has been applied across the Hall thrusters of various power levels^{6–10} in order to mitigate and/or eliminate the life-limiting issue of the channel erosion.⁴ The so-called "magnetically shielded" thrusters feature B -field topologies with especially large degrees of negative curvature upstream of the magnetic field peak and, hence, strong radial gradients.^{3,4}

A. Objectives and methodology

The overarching objective of the present article is to understand the variations that the curvature of the magnetic field (and the associated radial gradients) causes in the distribution of the plasma properties, in the spectra of the plasma instabilities, in the electron transport characteristics and in the divergence of the ion beam.

Our primary question as posed above is relevant for general $E \times B$ plasma technologies, for which, as the name suggests, the B -field is a main player and can, hence, strongly affect the plasma behavior in the devices, their performance, and operational characteristics. Thus, our question's significance extends beyond the Hall thrusters and also directly encompasses technologies such as the magnetrons used in the manufacturing industry.

This research also aims to address two additional secondary questions: (1) the differences between a 1D radial and a 2D radial-azimuthal study in terms of the observed effects of the magnetic field curvature. Indeed, as will be mentioned in Sec. I B, the B -field curvature effects on the plasma dynamics have been studied in the literature in a 1D radial Hall thruster case.¹¹ It is, therefore, highly important to investigate the significance of the roles likely to be played by the *azimuthal* physics and the changes that resolving azimuthal processes can induce in the reported insights from 1D studies. (2) The extent to which the influences of magnetic field curvature on the global plasma dynamics and the underlying phenomena varies among the plasmas of different ion masses. This is

of both applied and scientific relevance: on the one hand, the cross-field plasma technologies are often operated with different gases in the industry, with the most common being xenon, krypton, and argon. On the other hand, the ion mass is expected to particularly influence the characteristics of the azimuthal instabilities.

In terms of the research methodology, the main body of the studies is carried out in a 2D simulation setup, which is representative of a radial-azimuthal cross section of a Hall thruster, but that also resembles the broader family of the $E \times B$ discharges, including the magnetrons, for example. Corresponding 1D radial simulations are also set up. For both 1D and 2D simulation configurations, inspired by the approach pursued in Refs. 11 and 12, we have varied the curvature of the magnetic field across a set of negative and positive values (see Sec. II). Noting the motivation established above behind the execution of the 1D simulations, these are only carried out for xenon, whereas the simulations in the 2D setup are performed for xenon, krypton, and argon. We also emphasize that all reported simulations in this work are electrostatic.

The setup of the 2D simulations is, overall, similar to that of the radial-azimuthal benchmark case in Ref. 13. In this setup, the development of two particular azimuthal instability modes are reported, namely, the electron cyclotron drift instability (ECDI)^{14,15} and the modified two stream instability (MTSI).^{13,16} Thus, our studies regarding the effects of the magnetic field curvature on the plasma instability spectra are mainly focused on the variations that the field curvature induces in the appearance and the characteristics of these two instabilities. In a broader sense, we intend to demonstrate in this work the variations that different degrees of magnetic field curvature may cause in the plasma phenomena as was observed from the benchmark study of Ref. 13.

Concerning the influences of the propellant ion mass, our intention has been to isolate the direct effects on the studied phenomena in the presence of various degrees of B -field curvature. Accordingly, all simulations are collisionless. The implication that this assumption has on the setup of the simulations are detailed in Sec. II.

B. Review of the prior relevant research

The effects of the axial gradients of the magnetic field on the plasma behavior in Hall thrusters have been the subject of several studies in the past years.^{17–21} However, there has not been as much attention to the effects that the magnetic field curvature (radial gradients) has on the underlying plasma processes, particularly on the azimuthal instabilities and the resulting electron transport. Most high-fidelity studies of these two phenomena, which have been carried out in the setups resolving the radial coordinate, had assumed perfectly radial magnetic field lines.^{13,22,23}

Concerning the impacts of the B -field curvature on the plasma-wall interactions, there have been, nonetheless, a number of previous efforts. These activities were in part motivated by the emergence of the magnetically shielded Hall thrusters and their non-conventional magnetic field topologies. Authors in Refs. 4, 24, and 25 carried out hybrid fluid-PIC simulations in the axial-radial coordinates to investigate the physics underlying the magnetic shielding concept as well as to analyze the variation in the distribution of the plasma properties, the sheath characteristics, and the

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plasma-wall interactions due to the consequent changes in the B -field topology. Authors in Refs. 26 and 27 also performed studies of similar scope on two specific industrial magnetically shielded Hall thrusters using axial-radial hybrid simulations. The focus of these works was on the 2D spatial distribution of the plasma properties, global power and current balances, and the interactions of the plasma with the radial walls.^{26,27}

The effects of the magnetic field curvature have also been assessed using fully kinetic PIC simulations. In this regard, Liu *et al.*²⁸ and Quan *et al.*¹² studied the influence of various degrees of B -field's curvature on the plasma-wall interactions and the erosion phenomenon using 2D axial-radial and limited-domain radial-azimuthal simulations, respectively. Authors in Ref. (12) particularly demonstrated that increasing the concavity of the field lines reduces the energy of the ions reaching the wall and, hence, lowers the erosion rate.¹² On the contrary, in the case of convex field lines, the erosion rate becomes highly significant at large degrees of positive curvature.¹² Marin-Cebrián *et al.* in Ref. 11 carried out 1D radial PIC simulations with various magnetic field curvatures and showed that, for negative curvatures, the electrons' distribution function becomes more isotropic, and the tail of the distribution function is greatly replenished. A concentrating effect in cases with concave field curvatures was observed, with the plasma density in the center of the domain becoming higher than the case with a constant radial magnetic field.¹¹ Taccogna *et al.* assessed, again in a 1D radial setup, the effects of the magnetic field's inclination on the plasma sheath's characteristics in the presence of secondary electron emission and with the plasma properties representative of different axial locations within the discharge channel of a Hall thruster.²⁹

In a previous publication of direct relevance to the present work,³⁰ we performed 2D radial-azimuthal PIC simulations of an $E \times B$ discharge of the xenon gas for two specific B -field configurations representative of magnetic shielding topology. We focused, in that effort, on the variation in the instability spectra between these two configurations and the resulting changes in the dominant mechanism behind the electrons' axial mobility.³⁰ We most notably showed that, in the presence of a strong magnetic field gradient, a long-wavelength wave mode develops due to an inverse energy cascade and induces a significant electron cross-field transport.³⁰ Despite the important insights derived, the setup of the simulations in Ref. 30 was such that the B -field curvature could not be systematically varied in order to fully quantify the correlations between the degree of the field's curvature and the change in the plasma processes. Moreover, both studied B -field configurations served negative concave curvatures.³⁰ Hence, the present effort serves to build upon and extend the studies that we reported in Ref. 30.

II. SIMULATIONS' SETUP AND CONDITIONS

We performed a set of electrostatic quasi-2D (Q2D) simulations using the reduced-order PIC code of the Imperial Plasma Propulsion Laboratory (IPPL), which is named IPPL-Q2D.^{31–33} The domain decomposition associated with the reduced-order PIC scheme^{31–33} was carried out using 50 regions along each of the involved simulation dimensions. More details on the verification results of the 50-region quasi-2D simulation and its computational advantage can be found in Ref. 33, whereas examples of the physics

studies enabled via this specific approximation order of the reduced-order PIC are available in Refs. 30 and 34.

The domain of the Q2D simulations represent a radial-azimuthal cross section of a Hall thruster. Corresponding simulations are performed in a 1D setting along the radial direction of the thruster's domain as well using the IPPL-1D code^{35,36} to enable the comparison of the results with the Q2D simulations. As noted in Sec. I A, such comparison allows distinguishing the processes mainly driven by the azimuthal dynamics from the processes along the radial direction directly associated with the plasma-wall interactions. In the following, we introduce the setup and the conditions of the Q2D simulations. The same setup is adopted for the 1D simulations except for the fact that the azimuthal direction and all the associated conditions are excluded. Maintaining similarity between the 1D and the 2D setups has been necessary to make sure that a direct comparison between the results from the Q2D and the 1D simulations is possible.

The Q2D simulations' domain represents a Cartesian plane whose extents along the radial direction (L_x) and the azimuthal direction (L_z) are equal to 1.28 cm. These domain lengths are chosen for consistency with our baseline reference setup of the radial-azimuthal benchmark.¹³ As the domain is Cartesian, the cylindrical effects, such as the centrifugal force on the charged particle species, are not resolved. Some potential implications of this will be discussed in Sec. IV.

In the simulations, x , y , and z denote, respectively, the radial, the axial, and the azimuthal coordinates. A constant and spatially uniform axial electric field (E_x) with the intensity of 10 kV m^{-1} and a constant radial magnetic field (B_x) with the magnitude of 20 mT are imposed. The electrostatic nature of the simulations means that any potential induced magnetic field due to the azimuthal electron current is not captured. We will return to the possible influences of this assumption on our Q2D results in Sec. IV.

To investigate the impact of the curvature of the magnetic field on the dynamics of the plasma in the adopted simulation configuration, the total applied magnetic field comprises a radially varying axial component (B_y) in addition to the radial component. We chose the same relation adopted in Ref. 11 to prescribe the variation of the B_y along the radial direction,

$$B_y = B_x \tan \theta(r); \quad \tan \theta = \frac{L_x - x}{L_x} \tan \theta_1 + \frac{x}{L_x} \tan \theta_2, \quad (1)$$

where θ_1 and θ_2 are, respectively, the incident angles of the magnetic field lines at the inner and the outer walls, as illustrated in Fig. 1. The magnetic field is assumed to be symmetric with respect to the channel centerline; hence, $\theta_1 = -\theta_2$. We highlight that because the simulations occur on a plane and do not resolve the variations along the axial dimension, the magnetic field does not have an axial gradient.

The Q2D simulations are performed with three different propellants, namely, xenon, krypton, and argon, and with various degrees of magnetic field curvature, as parametrized by $\theta = \theta_1$. The field curvature parameter, θ , ranges from -60° to 60° with a 15° increment. The case with $\theta = 0$ corresponds to purely radial magnetic field topology of the benchmark setup. The counterpart 1D simulations are conducted only for xenon.

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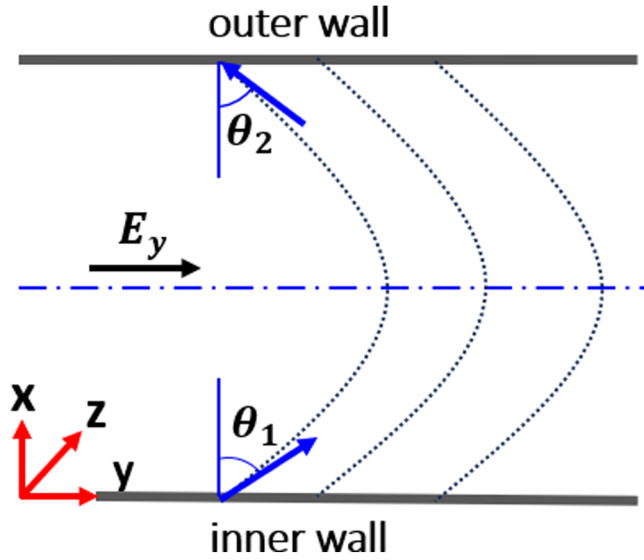


FIG. 1. Schematic of the magnetic field lines in the axial-radial (y - x) plane, with the lines' incidence angle at the inner wall (θ_1) and at the outer wall (θ_2) as used in Eq. (1) illustrated.

The negative curvatures refer to the field lines curving toward the anode ($-y$ direction), which is representative of a typical magnetic field topology upstream of the axial location of the magnetic field peak ($x_{B,\text{peak}}$) within a Hall thruster's discharge channel. Conversely, the positive curvatures signify the field lines curving toward the cathode ($+y$ direction), hence resembling the conditions downstream of the $x_{B,\text{peak}}$ location. The radial variation of B_y and the streamlines of the magnetic field in the simulated cases are illustrated, respectively, in Figs. 2(a) and 2(b).

Regarding the numerical parameters, for the simulations with positive field curvatures and negative field curvatures of -30° and -15° , the cell size (cell number) is $50\,\mu\text{m}$ (256 nodes) along both

the radial and azimuthal directions, and the time step is 1.5×10^{-11} s.

For the negative-curvature simulations with $\theta = -45^\circ$ and -60° , since the steady-state plasma density that establishes in the presence of these large negative curvatures was seen to increase to relatively high values, the cell size and the time step of the corresponding simulations were adapted accordingly to remain below the Debye length and the electron plasma frequency limits, which are associated with the stability of the explicit PIC simulations.^{37,38} Therefore, in the simulations for negative curvatures of -45° and -60° , the cell size (cell number) and the time step are, respectively, $25\,\mu\text{m}$ (512 nodes) and 7.5×10^{-12} s. The total simulated time for all simulations is $30\,\mu\text{s}$.

Table I summarizes the computational and physical parameters used for various simulations in this work.

For the sake of maintaining similarity with the reference benchmark setup,¹³ initialization is performed by loading within the domain uniformly distributed ion-electron pairs at a density of $1.5 \times 10^{16}\,\text{m}^{-3}$. The initial number of macroparticle per cell is 100. The electrons and ions are sampled from Maxwellian distribution functions at the temperatures of 10 and 0.5 eV, respectively.

The simulations are collisionless, and the establishment of a steady-state density in the system is ensured through imposing an ionization source, as per the approach of Ref. 13. The electron-ion pairs are injected at each time step according to this ionization source. The source has an azimuthally uniform distribution and a cosine-shaped profile along the radial direction that spans over $x = 0.09\,\text{cm}$ to $x = 1.19\,\text{cm}$ and features a peak value of $8.9 \times 10^{22}\,\text{m}^{-3}\text{s}^{-1}$.¹³

A periodic and a zero-volt Dirichlet boundary conditions are applied on the plasma potential along the azimuthal and radial directions, respectively. Secondary electron emission is not accounted for, and all particles reaching the walls are removed. Particles crossing the boundaries of domain along the azimuthal direction re-enter the domain from the other end while maintaining their velocities and radial position.

We have assumed a finite extent of $L_y = 1\,\text{cm}$ for the domain along the axial direction in order to prevent the particles' energy to

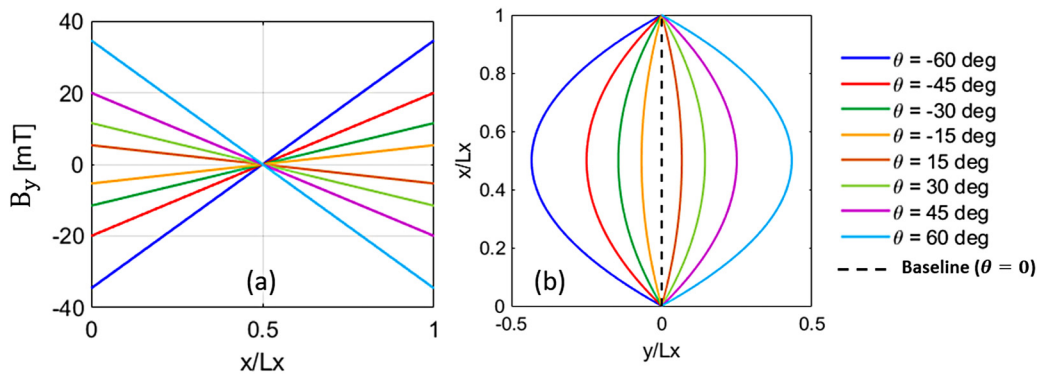


FIG. 2. (a) Radial profile of the axial component of the magnetic field (B_y) for various degrees of field curvature (θ). (b) 2D shape of the magnetic field lines in the axial-radial (y - x) plane for various values of θ .

TABLE I. Summary of the computational and physical parameters for various Q2D radial–azimuthal and 1D radial simulations.

Parameter	Value (unit)	
	Q2D radial–azimuthal (xenon, krypton, argon)	1D radial (xenon)
Computational parameters		
Domain's radial length (L_x)	1.28 (cm)	1.28 (cm)
Domain's azimuthal length (L_z)	1.28 (cm)	...
Domain's virtual axial length (L_y)	1 (cm)	1 (cm)
Radial cell size (Δx) [$\theta = -15, -30, 0, +15$ to $+60$]	50 (μm)	50 (μm)
Radial cell size (Δx) [$\theta = -60, -45$]	25 (μm)	25 (μm)
Azimuthal cell size (Δz) [$\theta = -15, -30, 0, 15$ to $+60$]	50 (μm)	...
Azimuthal cell size (Δz) [$\theta = -60, -45$]	25 (μm)	...
Time step (ts) [$\theta = -15, -30, 0, +15$ to $+60$]	1.5×10^{-11} (s)	1.5×10^{-11} (s)
Time step (ts) [$\theta = -60, -45$]	7.5×10^{-12} (s)	7.5×10^{-12} (s)
Initial number of macroparticles per cell (N_{ppc})	100	100
Total simulated time	30 (μs)	30 (μs)
Physical parameters		
Initial plasma number density ($n_{i,0}$)	1.5×10^{16} (m^{-3})	1.5×10^{16} (m^{-3})
Initial electron temperature ($T_{e,0}$)	10 (eV)	10 (eV)
Initial ion temperature ($T_{i,0}$)	0.5 (eV)	0.5 (eV)
Axial electric field (E_y)	10 000 (Vm^{-1})	10 000 (Vm^{-1})
Radial magnetic field intensity at the mid-radial plane (B_x)	0.02 (T)	0.02 (T)
Electric potential at the walls (ϕ_w)	0 (V)	0 (V)

increase indefinitely.³⁹ The value of this artificial axial extent is chosen in accordance with the benchmark's setup.¹³ The electrons and ions that traverse the artificial axial boundaries of the domain are reintroduced onto the simulation plane, retaining their radial and azimuthal positions, but with new velocities sampled from their respective initial Maxwellian distributions.

III. SIMULATIONS' RESULTS

The results of the simulations on the impact of the magnetic field curvature (θ) on different aspects of the dynamics of the plasma is presented in this section. Further analyses of the results and the discussion of the main findings are subsequently provided in Sec. IV. Here, we start by showing the variation in the radial distributions of the plasma properties. We then present how different degrees of the field's curvature change the characteristics of the developed instabilities, followed by presenting the variations in the electrons' axial mobility. Where applicable, we compare the results of the Q2D simulations with the corresponding ones from the 1D simulations. We also highlight the notable changes in plasma behavior observed among xenon, krypton, and argon propellants.

A. Distribution of the time-averaged plasma properties for different B -field curvatures

The radial profiles of the ion number density (n_i), averaged over the time interval of 20–30 μs , as well as the variation of the peak value of these profiles with θ are presented in Fig. 3. The number density increases notably as θ approaches large negative values. However, it remains nearly identical for cases with $\theta = -30^\circ$, -15° , and 0° . In contrast, n_i exhibits a more moderate variation

across the positive values of θ . The density increases from $\theta = 0^\circ$ to $\theta = 15^\circ$ after which it drops with further increase in θ . The general trend of n_i vs θ is consistent across all propellants, with xenon exhibiting the highest density, argon displaying the lowest, and krypton falling in between. However, there is an exception in the scenario with $\theta = -60^\circ$, where krypton demonstrates the highest density among the three propellants. Explaining this observation requires invoking additional results and analyses. Hence, we will return to this point in the “Discussion” section (Sec. IV), where we will discuss the main variations that the ion mass causes in the observed effects of the magnetic field curvature on the plasma discharge.

The normalized ion density profiles ($n_i/n_{i,\text{mean}}$) from the Q2D simulations are compared against the corresponding profiles from the 1D simulations in Fig. 4. In addition, in Fig. 5, the variation in the peak of these profiles at the domain's centerline vs θ from the 1D and the Q2D simulations is provided.

It is first observed from the plots of Fig. 4 that the impact of a negative curvature of the B -field is more pronounced on the density profile in the 1D simulations compared to the Q2D simulations. In the 1D simulations, the density becomes progressively more confined toward the center as the angle θ changes from 0° to -60° . However, the concentration effect of the negative curvature of the B -field is mitigated in Q2D simulations due to the presence of azimuthal instabilities and their consequences on plasma properties, which will be discussed later.

Second, in the Q2D simulations, the concentration of plasma toward the center is most notable for the cases with $\theta = -60^\circ$ and -45° . The $n_i/n_{i,\text{mean}}$ profiles are rather similar across the rest of the curvatures ($\theta = -30^\circ$ to 60°) and propellants.

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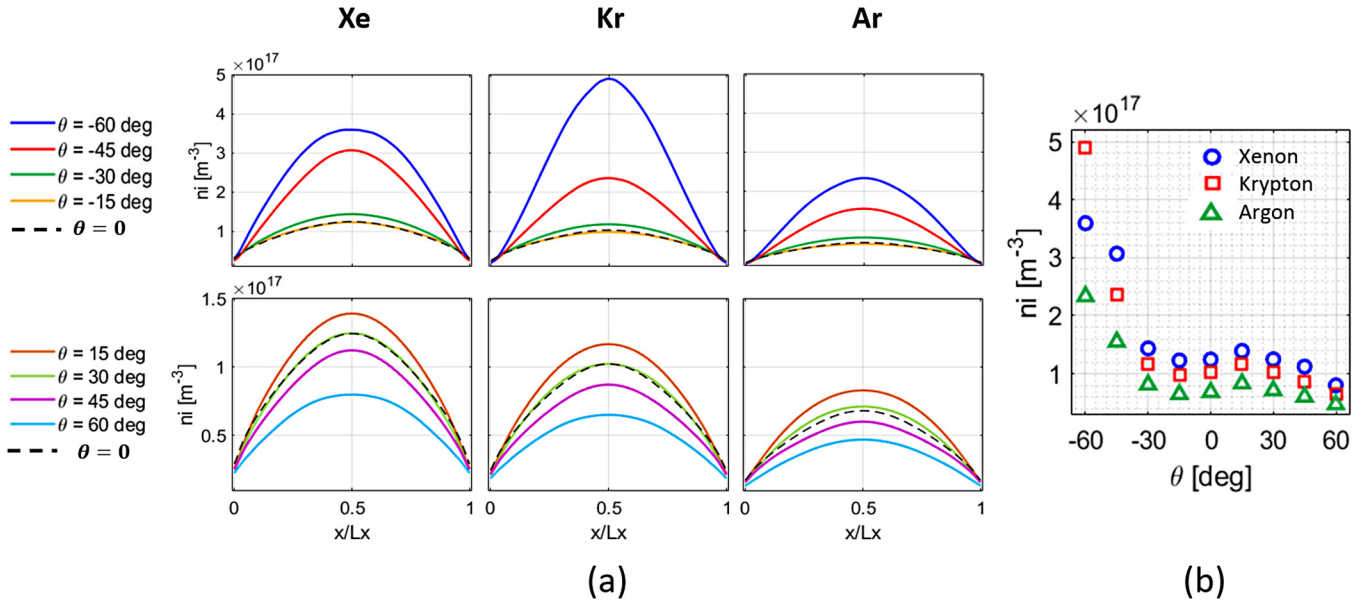


FIG. 3. (a) Time-averaged radial profiles of the ion number density (n_i) from the simulations with various magnetic field curvatures and propellants: (top row) negative curvatures, (bottom row) positive curvatures. (b) Variation in the ion number density peak along the radial direction vs the curvature degree for the three studied propellants.

Referring to Fig. 5 and comparing the 1D vs Q2D simulations, we notice that, unlike for the negative curvatures, the $n_i/n_{i,\text{mean}}$ values at the domain's centerline from the 1D simulations have a good agreement with the Q2D simulations with xenon for the positive curvature values. This can be justified by looking at Fig. 4 where the positive curvatures of the B -field are noticed to have a minor influence on the normalized density profiles in both the 1D and the 2D simulation setups. In addition, for the Q2D simulations at $\theta = -60^\circ$, the extent of the confinement is seen from Fig. 5 to be different among the three propellants.

We now look at the time-averaged profiles of the radial electron temperature (T_{ex}) in Fig. 6 along with the variation of the mean of the T_{ex} profiles vs θ in Fig. 7.

It is noticed from these two figures that the T_{ex} is much higher in the Q2D simulations compared to the 1D simulations. This is due to the electrons' heating effect of the azimuthal instabilities captured in the Q2D simulations. Moreover, in the 1D simulations and in the presence of negative curvatures, the T_{ex} monotonically increases as the θ varies from zero toward more negative values. Contrarily, in the Q2D setting, large degrees of curvature ($\theta = -60^\circ$ and -45°) lead to lower T_{ex} values compared to the baseline case ($\theta = 0$), whereas moderate levels of negative curvature ($\theta = -30^\circ$ and -15°) cause T_{ex} to be higher than that in the baseline case. We will see from the results in Subsec. III B and the discussions in Subsec. IV A that the interplay between the azimuthal instabilities and the relative strength of the developed modes is the root cause behind these trends.

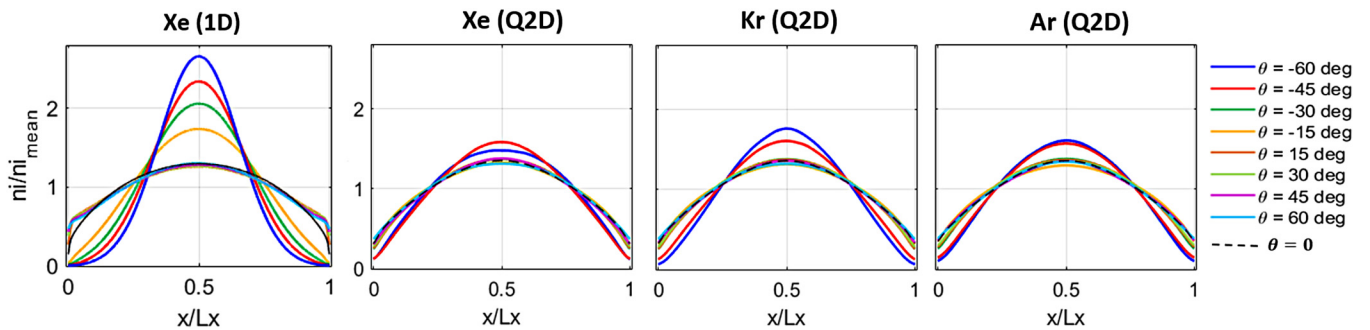


FIG. 4. Time-averaged radial profiles of the ion number density normalized with the radially averaged density value, $n_i/n_{i,\text{mean}}$, from the 1D simulations with xenon propellants and the Q2D simulations with xenon, krypton, and argon. The profiles are shown for various magnetic field curvatures in both the 1D and the Q2D cases.

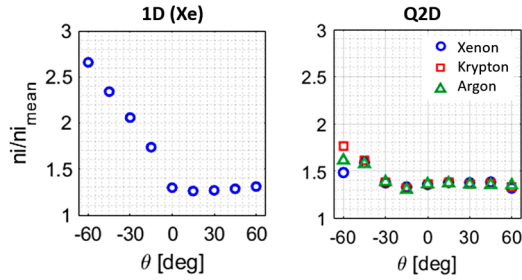


FIG. 5. Variation of the normalized ion number density peak value along the radial direction vs θ from the 1D simulations with xenon (left) and the Q2D simulations with the three propellants (right).

In addition, from Figs. 6 and 7, the disparity in the trend observed in the density profiles for krypton compared to other propellants at $\theta = -60^\circ$ is evident in the behavior of T_{ex} as well. The reason behind this observation is connected to the exception pointed out for krypton from the density profiles and will, hence, be clarified in the Discussion section.

The positive curvatures of the B -field flatten the distribution of the radial electron temperature, which leads to more uniform profiles across the radial extent in both the 1D and the Q2D simulations. The dependency of T_{ex} on the field's curvature is also minor across the positive values of θ .

To summarize the overall trend of the T_{ex} from the Q2D simulations, it is seen in Fig. 7 that the radial electron temperature almost monotonically decreases with θ from -30° to 45° , and it increases slightly at $\theta = 60^\circ$. Across this range of B -field curvatures,

argon consistently demonstrates the lowest T_{ex} , whereas xenon exhibits the highest values.

In Fig. 8, we compare the time-averaged profiles of the ion temperature (T_i) for various θ from the Q2D and the 1D simulations. From the plots corresponding to Q2D simulations, we observe that, at large negative B -field curvatures ($\theta = -45^\circ$ and -60°), the ion temperature reaches up to 35–45 eV. As reported in Refs. 30 and 34, such high T_i values occur as a result of strong interactions between the ion particles and an instability mode characterized by a high frequency and a relatively long wavelength, which we had termed the “long- λ ” mode in Ref. 30. We will demonstrate from the instability analyses’ results in Sec. III B that this mode indeed appears under the large negative curvature conditions.

We also notice from the Q2D simulations’ results in Fig. 8 that, at $\theta = -60^\circ$, T_i is relatively higher for krypton propellant than for xenon and argon. For θ values of -30° to 0° and for all positive curvatures, the ion temperature is lower than the values observed for the $\theta = -60^\circ$ and -45° . As will be clarified from the results in Sec. III B, this can be linked to the relatively mitigated strength of the long- λ mode at the negative curvatures of -30° to -15° and its absence at the positive curvatures.

The radial distribution of the T_i is also seen to vary across the studied θ range. In particular, at $\theta = -60^\circ$, T_i is higher at the centerline. For $\theta = -45^\circ$, the T_i profile is relatively uniform over the domain, and for $\theta = -30^\circ$ to 0° , the T_i peaks near the walls. On the contrary, for all positive curvatures, the shape of the profiles is seen to be similar, with the ion temperature peaking near the wall and dropping toward the center of the domain.

In contrast to the Q2D simulations, in the 1D setting and under negative B -field curvature conditions, the T_i does not substantially vary with θ in the bulk plasma. However, a minor

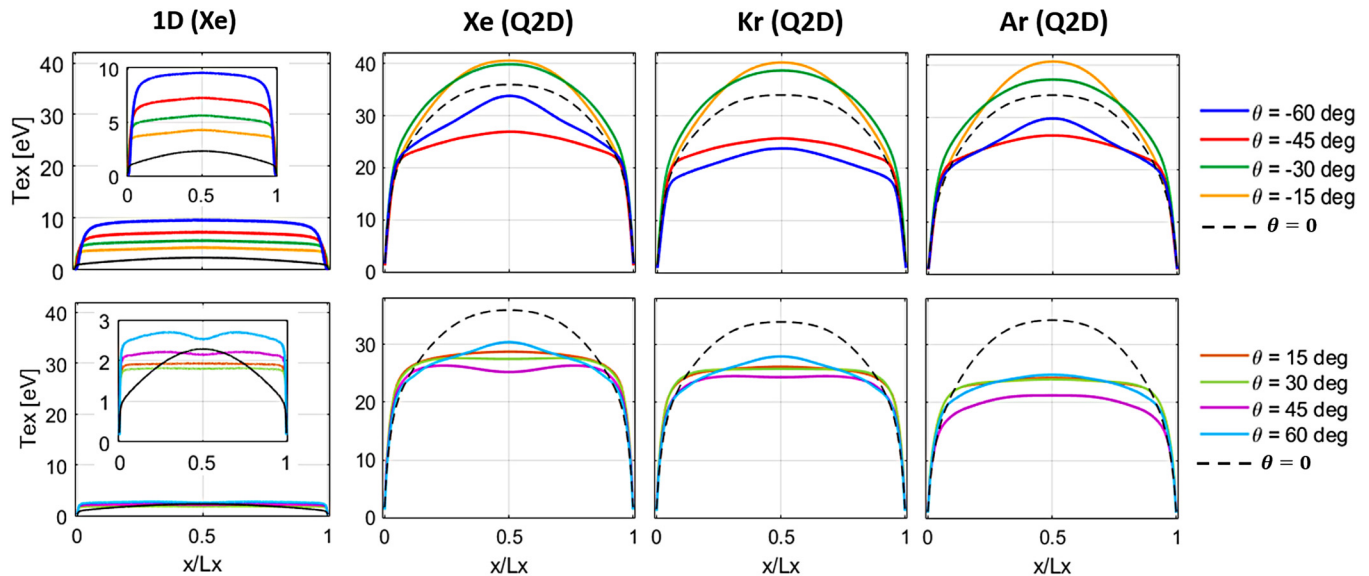


FIG. 6. Profiles of the radial electron temperature (T_{ex}) averaged over 20–30 μ s from the 1D (for xenon) and the Q2D (for xenon, krypton, and argon) simulations with various magnetic field curvatures: (top row) negative curvatures, (bottom row) positive curvatures.

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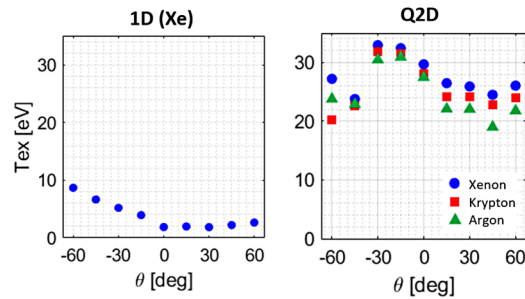


FIG. 7. Variation in the mean radial electron temperature value along the radial direction vs θ from the 1D simulations with xenon (left) and the Q2D simulations with the three propellants (right).

reduction in T_i values is observed adjacent to the walls as θ changes from 0° to -60° . In the cases with positive curvatures, the T_i profiles obtained from the 1D and the Q2D simulations are more consistent. In these cases, as θ increases from 0° to 60° , the T_i profile becomes progressively less uniform with a consistent reduction in values at the centerline.

Before moving to Sec. III B, we point out that, in Appendix 1, we have presented the 2D radial-azimuthal snapshots of several

plasma properties. The plots provide a comparative visual perspective on the dominant azimuthal instabilities across the Q2D simulations with different degrees of B -field curvature and propellant.

B. Variation in the spectra of the azimuthal instabilities

Spectral analyses of the azimuthal fluctuations from the Q2D simulations are carried out using the spatial and temporal Fast Fourier Transform (FFT) to study the instabilities' characteristics, including the wavenumber and the frequency, in addition to the variations of these across different θ values and with various propellants. The analyses are performed on the azimuthal electric field (E_z). For clarity of presentation, we report the results of the spectral analyses separately for the negative- and positive-curvature simulations.

Complementary spectral analyses using the dynamic mode decomposition (DMD) method⁴⁰ have been performed on the spatiotemporal signals of the E_z and for the xenon propellant as an example. The results are presented in Appendix 2. The DMD plots serve to provide additional information into the spatial structure (radial and azimuthal wavelengths) of the dominant instability modes identified below for the studied cases with xenon.

Starting with negative B -field curvature cases, Fig. 9 shows the spatial FFT of the E_z averaged across all radial positions and over the time span of 20–30 μ s. In these plots, the horizontal axis

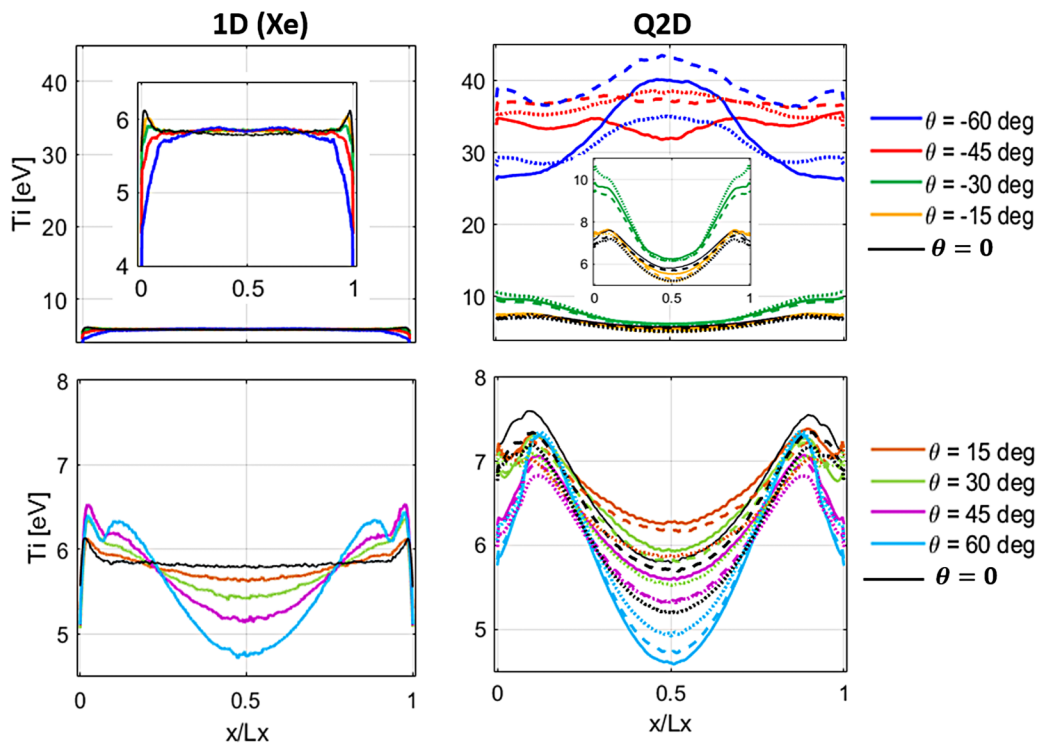


FIG. 8. Radial profiles of the ion temperature (T_i) averaged over 20–30 μ s from the 1D (for xenon) and the Q2D (for xenon, krypton, and argon) simulations with various magnetic field curvatures: (top row) negative curvatures, (bottom row) positive curvatures. Solid lines correspond to xenon, dashed lines to krypton, and dotted lines to argon.

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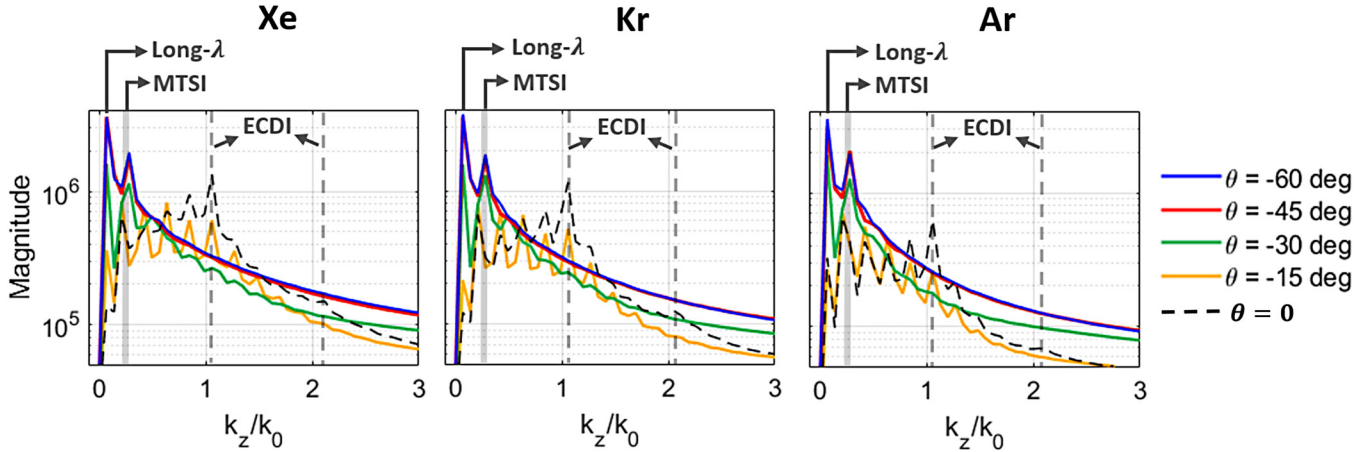


FIG. 9. 1D spatial FFT plots of the azimuthal electric field signal from the Q2D simulations with various negative magnetic field curvatures and different propellants. The FFTs are averaged over all radial positions and over the time interval of 20–30 μ s.

represents the wavenumber that is normalized with respect to the fundamental resonance wavenumber of the ECDI, $k_0 = \Omega_{ce}/V_{de}$, where Ω_{ce} and V_{de} are the electron cyclotron frequency and the electron azimuthal drift velocity, respectively. We notice from this figure that the ECDI is developed only in cases where $\theta = -15^\circ$ or 0° . Interestingly, in these cases, the intensity of the ECDI is lower for argon compared to xenon and krypton. The higher curvatures of the magnetic field suppress the ECDI. Indeed, in the cases with $\theta = -60^\circ$, -45° , and -30° , the ECDI is absent. Moreover, the second harmonics of the ECDI are not observed in any of the cases except for the baseline ($\theta = 0$).

The MTSI and the long- λ instability³⁰ are observed in all negative θ cases, with the amplitudes that are nearly the same across all propellants. At $\theta = -15^\circ$, the intensity of the long- λ instability is lower than that of the MTSI. Furthermore, this instability is

significantly damped at $\theta = 0$ for xenon and krypton, whereas in the case of argon, its intensity is comparable with the mode's intensity at $\theta = -15^\circ$. As pointed out in Sec. I, the long- λ instability has been observed in our previous work³⁰ in the presence of strong magnetic field gradients where it was found to have remarkable impacts on the plasma processes. These included significant electron cross-field transport and strong interaction with the ions leading to large ions' azimuthal drift and heating.³⁰ We will see later in Subsec. III C and IV D, respectively, that these effects are observed in the present simulations as well.

To assess the frequency spectra of the instabilities, the averaged temporal FFTs of the E_z for the negative-curvature simulations of xenon, krypton, and argon are provided in Fig. 10. Looking at these plots, the peaks below 10 MHz in the frequency spectrum consistently shift toward higher frequencies as the

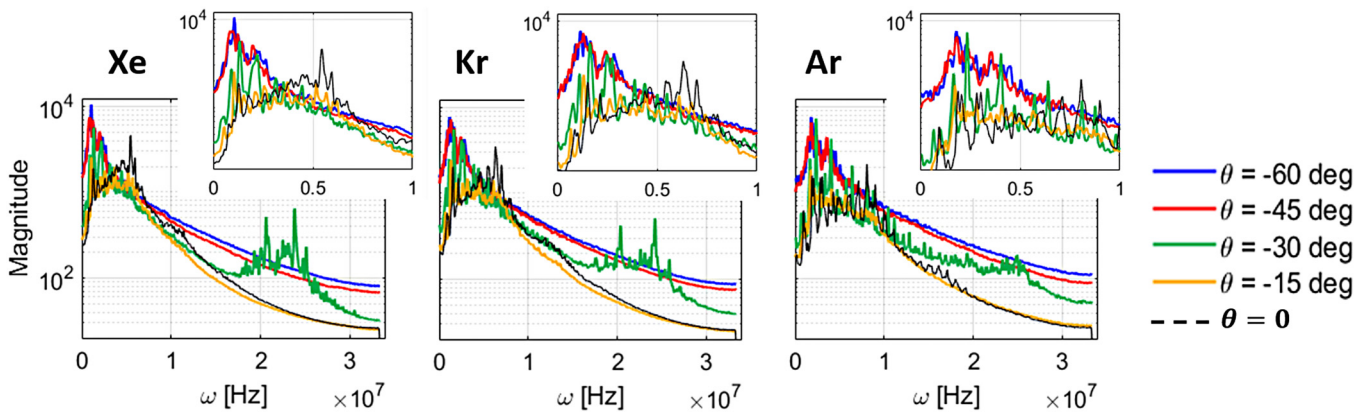


FIG. 10. 1D temporal FFT plots of the azimuthal electric field signal from the Q2D simulations with various negative magnetic field curvatures and different propellants. The FFTs are averaged over all radial positions. The zoomed-in view on each FFT plot in the frequency range of 0–10 MHz is also shown as an embedded subplot.

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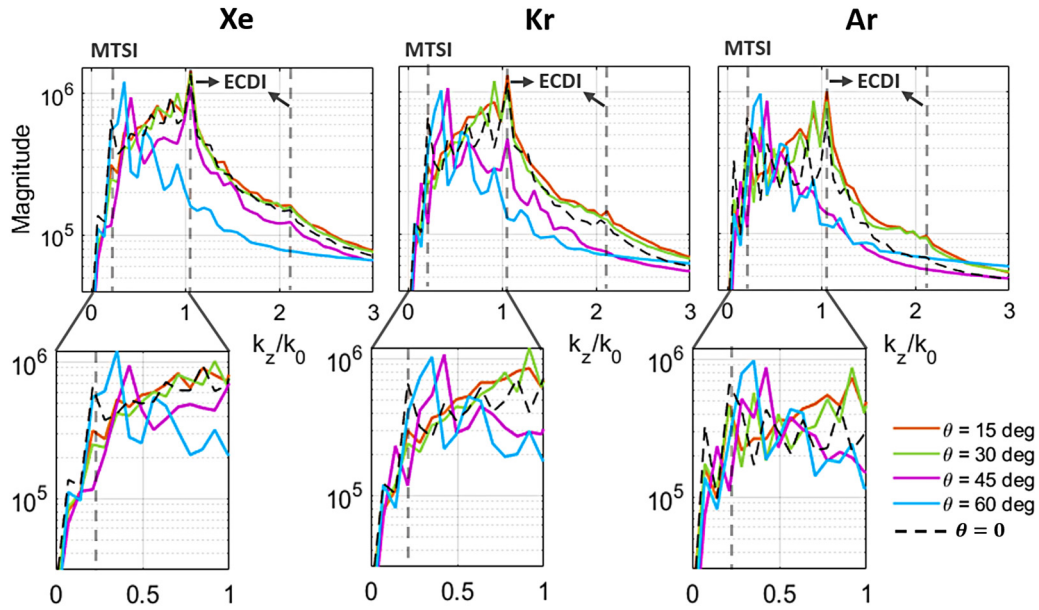


FIG. 11. 1D spatial FFT plots of the azimuthal electric field signal from the Q2D simulations with various positive magnetic field curvatures and different propellants. The FFTs are averaged over all radial positions and over the time interval of 20–30 μ s.

propellant changes from xenon to krypton to argon. Also, the frequency spectrum of argon in cases of $\theta = -30^\circ$, -15° , and 0° exhibit more distinct peaks below 10 MHz compared to the krypton and xenon simulations.

Another noteworthy observation is that, at $\theta = -30^\circ$, the FFT plots reveal the presence of instability modes with frequencies in the range of 20–25 MHz. The amplitude of these modes decreases from xenon to krypton, and they are nearly suppressed in the case of argon. The DMD analyses for xenon in Appendix 2 enabled discovering the spatial structure of these high-frequency modes. It was noticed that, at least for xenon, the mode with the frequency of

~ 20 MHz has the same wavelength as the MTSI mode with the frequency of about 1 MHz.

We now proceed with presenting the same analyses as those presented so far for the cases with positive B -field curvatures. Figure 11 shows the normalized azimuthal wavenumber (k_z/k_0) spectra of the instabilities obtained from the spatiotemporally averaged spatial FFTs of E_z . From this figure, it is evident that the positive curvature of the magnetic field contributes to the suppression of the MTSI, especially for xenon and krypton. In particular, the peaks associated with the MTSI in the normalized k_z spectra of xenon and krypton at $\theta = 15^\circ$ and 30° exhibit notably reduced

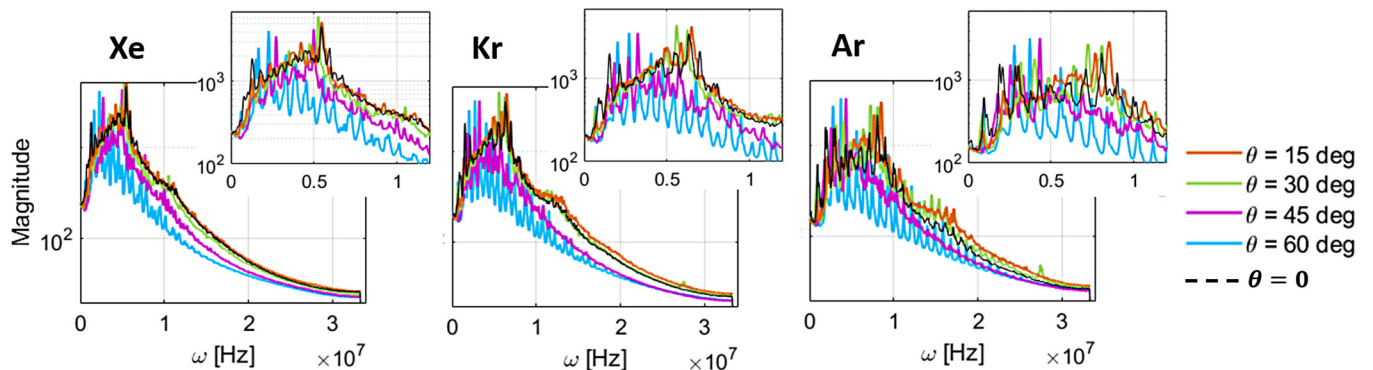


FIG. 12. 1D temporal FFT plots of the azimuthal electric field signal from the Q2D simulations with various positive magnetic field curvatures and different propellants. The FFTs are averaged over all radial positions. The zoomed-in view on each FFT plot in the frequency range of 0–10 MHz is also shown as an embedded subplot.

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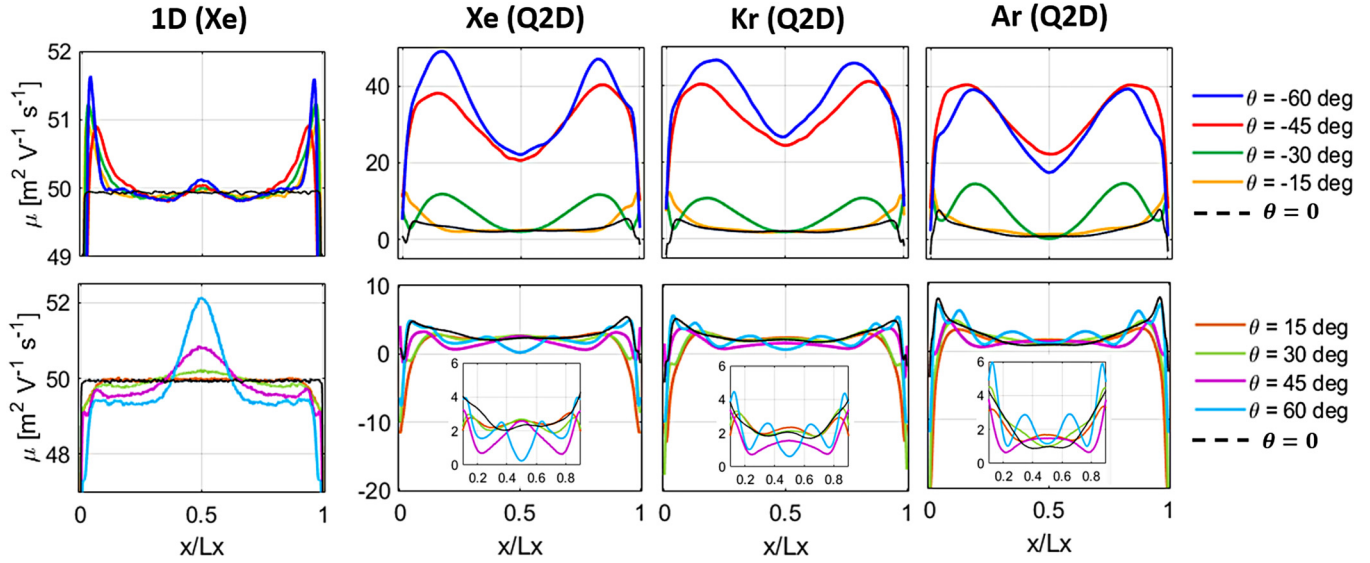


FIG. 13. Time-averaged radial profiles of the electrons' axial mobility (μ) from the 1D (for xenon) and the Q2D (for xenon, krypton, and argon) simulations with various magnetic field curvatures. (top row) negative curvatures, (bottom row) positive curvatures. It is noted that the y-axis scales are different between the plots comparing the 1D and the Q2D results for xenon as well as between the negative- and the positive-curvature plots (the top row and the bottom row, respectively).

amplitudes in comparison to the baseline case ($\theta = 0$), whereas the MTSI's intensity for argon at these curvature values is comparable to the baseline simulation.

Furthermore, the k_z/k_0 spectra of the larger positive curvature cases for all propellants contain peaks at $k_z/k_0 \sim 0.42$ and $k_z/k_0 \sim 0.35$ for the simulations with $\theta = 45^\circ$ and 60° , respectively. However, it is not readily evident if these peaks could correspond to the MTSI since they represent larger azimuthal wavenumbers than those expected for the MTSI from the baseline case ($k_z/k_0 \sim 0.2$). In this respect, the DMD analysis in Appendix 2 provided some further insights into the nature of these modes by characterizing their spatial structures. We particularly understood that, at least for xenon, the peak at $k_z/k_0 \sim 0.35$ and $\theta = 60^\circ$ could be possibly linked to the MTSI as it presents a radial-azimuthal structure reminiscent of this instability.

The first harmonic of the ECDI is the dominant mode in the cases of $\theta = 15^\circ$ and 30° with its amplitude being slightly lower in the case of argon than for xenon and krypton. The $\theta = 45^\circ$ condition leads to the largest discrepancy among different propellants in terms of the intensity of the ECDI. At this curvature, the ECDI manifests its highest intensity for xenon, while it is mitigated for krypton and is absent for argon. In the case of $\theta = 60^\circ$, the nearest peak to the theoretical normalized wavenumber of ECDI's first harmonic ($\frac{k_z}{k_0} = 1$) is shifted toward lower wavenumbers ($k_z/k_0 \sim 0.92$). Additionally, the second harmonic of the ECDI is present in the cases of $\theta = 15^\circ$ and 30° for all the propellants as well as for xenon in the case of $\theta = 45^\circ$.

A comparison between the frequency spectra of E_z from the simulations with the positive B -field curvatures is provided in Fig. 12. Similar to the trend observed in the negative-curvature

cases, the peaks representing the modes below 10 kHz shift toward consistently higher frequencies from xenon to krypton to argon. It is noteworthy that, in the case of $\theta = 60^\circ$, the FFT spectrum exhibits numerous peaks across a wide range of frequencies. The DMD plots of the dominant modes at this curvature value in Appendix 2 show that the modes with frequencies equal to and above 1.63 MHz all have almost purely azimuthal spatial structures that resemble the ECDI.

C. Influence on the electrons' axial transport

The time-averaged (over 20–30 μ s) radial profiles of the electrons' axial mobility ($\mu = \left| \frac{v_{ey}}{E_y} \right|$) for various θ values from the Q2D simulations with different propellants and from the 1D simulations with xenon are presented in Fig. 13. In the relation for μ , v_{ey} is the average electrons' axial drift velocity and E_y is the axial electric field.

Comparing the magnitudes of the electrons' mobility from the Q2D and the 1D simulations, it is evident that across all curvature cases, μ is consistently higher in the 1D simulations. Notably, the disparity in the values of μ is particularly significant in the cases of positive curvature. This is a noteworthy observation considering that the 1D simulations do not resolve the azimuthal instabilities and their induced electrons' transport. Even though the Q2D simulations capture the additional contribution of the instabilities, the resulting electrons' mobility in the Q2D simulations is still lower compared to that observed from the 1D simulations.

In the 1D simulations, the electrons' mobility shows a very small variation across the domain and for various curvature cases. In the negative-curvature cases, the mobility exhibits a slight

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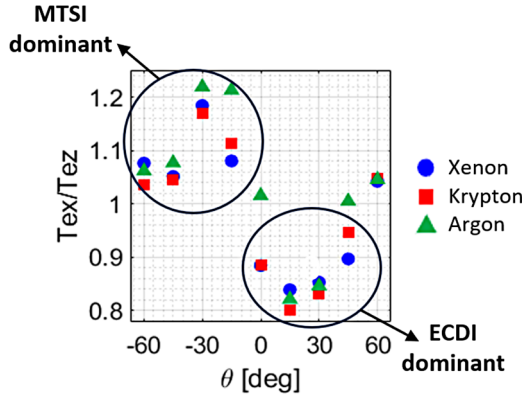


FIG. 14. Variation in the radially averaged T_{ex}/T_{ez} vs θ from the Q2D simulations with various magnetic field curvatures and different propellants.

increase near the walls, whereas, in the positive-curvature cases, it becomes slightly more pronounced toward the center.

In the Q2D simulations, the electrons' mobility has a notable non-uniform distribution across the domain, and its values vary significantly with the field's curvature. The mobility is particularly significant at $\theta = -60^\circ$ and -45° , for which the presence of a strong long- λ instability was observed from the FFT plot in Fig. 9 and from the DMD analysis plot in Appendix 2. Considering the different propellants, the radial distribution of the electrons' mobility is rather similar, whereas there are slight variations in terms of the magnitudes of the mobility across the curvature values. This latter observation can be likely linked to the slight variations in the azimuthal instability spectra among the propellants as was seen in Sec. III B.

IV. ANALYSES AND DISCUSSIONS

We present in this section more detailed analyses of the results of the simulations performed, discuss the main derived insights, and compare our findings against previous relevant numerical and/or experimental observations reported in the literature.

A. Electron temperature anisotropy in the presence of magnetic field curvature and the correlation with the dominant azimuthal instabilities in 2D

To assess potential anisotropy in the electron temperature and its variation with the curvature degree from the Q2D simulations, we have plotted in Fig. 14 the average (over the entire domain and over the time interval of 20–30 μ s) radial-to-azimuthal electron temperature ratio (T_{ex}/T_{ez}) against θ . Based on the results reported in Refs. 41–43 regarding the contributions that the ECDI and the MTSI have to the electrons' heating along the azimuthal and the radial directions, respectively, the T_{ex}/T_{ez} ratio can be an indicative measure of the relative dominance of these two instability modes. Accordingly, two regimes can be defined: (1) $T_{ex}/T_{ez} > 1$, which

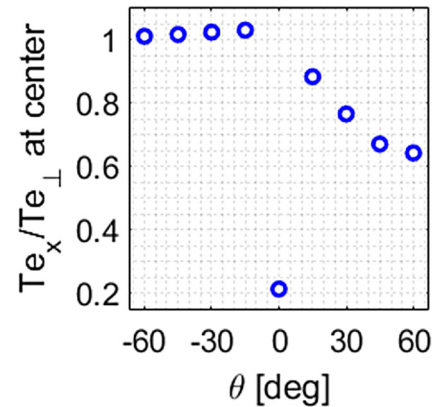


FIG. 15. Variation in the $T_{ex}/T_{e,\perp}$ at the mid-radial point vs θ from the 1D radial simulation with xenon propellant.

signifies the prevalence of the MTSI, and (2) $T_{ex}/T_{ez} < 1$, which indicates that the ECDI is the dominant instability.

Referring to Fig. 14, we can distinctly notice the two regimes mentioned above. Indeed, the presence of negative B -field curvatures results in $T_{ex}/T_{ez} > 1$ and, therefore, the dominance of the MTSI. Conversely, the positive curvatures lead, in general, to $T_{ex}/T_{ez} < 1$, which is attributed to the ECDI modes of relatively high strength.

A particular exception to these general observations are noticed for the case of $\theta = 60^\circ$, where the T_{ex}/T_{ez} ratio is slightly higher than 1 for all propellants. This is, nonetheless, the curvature value for which we identified the presence of a MTSI-representative mode for xenon with the aid of the DMD analysis (Appendix 2). The temperature ratio values at $\theta = 60^\circ$ in Fig. 14 now suggest that this DMD-enabled insight for xenon is highly likely to be also valid for krypton and argon.

For the cases with $\theta = -15^\circ, 0^\circ$, and 45° , argon exhibits a significantly disparate electron temperature anisotropy compared to the simulations with xenon and krypton, which show either similar or rather comparable T_{ex}/T_{ez} ratios at these curvatures. In this regard, at $\theta = -15^\circ$, the T_{ex}/T_{ez} ratio is the highest for argon because of the presence of a noticeably stronger MTSI mode than the ECDI in this case (see Fig. 9). Furthermore, for the cases where $T_{ex}/T_{ez} \sim 1$ for argon, i.e., $\theta = 0^\circ$ and 45° , the results from Figs. 9 and 14 suggest the coexistence of the ECDI and the MTSI modes with comparable intensities at $\theta = 0^\circ$ and the absence of both of these at the 45° curvature.

Following on the above and combined with the instabilities' characteristics assessment presented in Sec. III B, we confirm that, in general, the concavity or convexity of the magnetic field curvature can lead to the establishment of two distinct plasma regimes in a Hall thruster and, by extension, in general $E \times B$ plasma configurations. These regimes are overall characterized by the relative dominance of either the MTSI or the ECDI modes. Accordingly, the negative curvature of the field is seen to suppress the ECDI and the purely azimuthal instabilities except for the azimuthal long- λ mode, which was observed across the negative-curvature cases. In addition,

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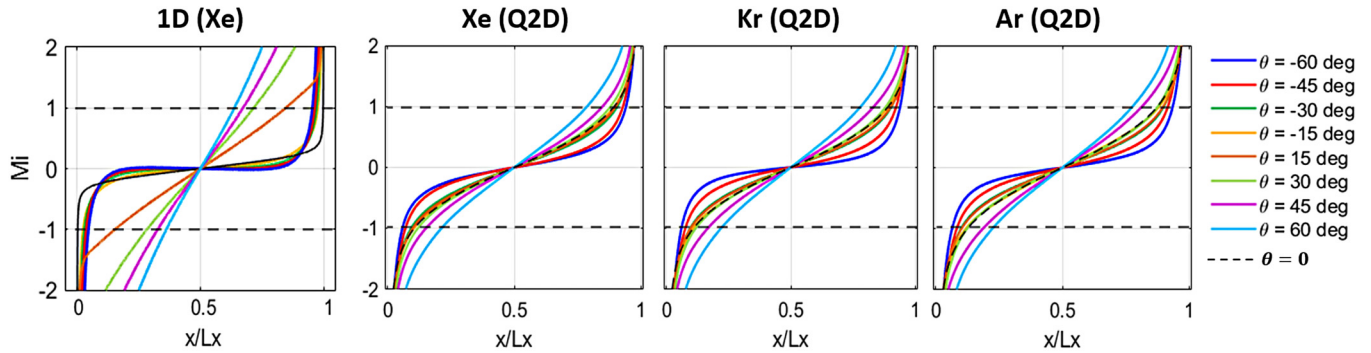


FIG. 16. Radial profiles of the ion Mach number (M_i) averaged over the time interval of 20–30 μ s from the 1D (for xenon) and the Q2D (for xenon, krypton, and argon) simulations with various magnetic field curvatures.

the negative-curvature conditions are noted to promote the development of dominant instabilities in the system similar to the MTSI with wavenumbers along both the radial and the azimuthal directions. The presence of a positive curvature of the magnetic field are observed to hinder the formation of the MTSI and the instabilities with the radial-azimuthal structures in general. Under these circumstances, the modes are seen to primarily consist of purely azimuthal instabilities, such as the ECDI, with different frequencies and wavenumbers. The consistency between these findings and the variations trend of the average electron temperature anisotropy across the simulated B -field curvatures highlights the suitability of the $T_{ex}/T_{e\perp}$ ratio as a general criterion to preliminarily assess the comparative significance of the ECDI and the MTSI modes.

It is further important to note that comparing the above insights on the dominant azimuthal instabilities with the experiments reported in the literature is rather challenging. The experiments have limitations in terms of the spatiotemporal resolution of the measurements of the plasma parameters and have not been able to reliably and non-intrusively survey the rather inaccessible inside of a Hall thruster channel, where the magnetic field curvature is expected to be negative. Furthermore, the MTSI is a relatively newly observed instability mode from the simulations, and hence, a proper effort has not yet been made, to the best of the author's knowledge, on its experimental characterization. Nonetheless, a general experimental evidence that at least partially supports our research findings is that the ECDI is commonly observed in the Hall thrusters' near-plume region,^{44–47} where the magnetic field features positive curvatures. Our numerical results in this work seem to show that the positive curvature of the magnetic field in the thrusters' near-plume region may play a role in why the ECDI has been predominantly observed in the plume and has been suggested to be absent/mitigated inside the channel.⁴⁸

Now, focusing on the electron temperature anisotropy from the 1D simulations, we first point out that the temperature anisotropy in these simulations is defined as the ratio of the parallel-to-perpendicular electron temperature ($T_{e\parallel}/T_{e\perp}$), where the parallel and the perpendicular directions are with respect to the magnetic field. The variation with θ of $T_{e\parallel}/T_{e\perp}$ at the centerline from the 1D simulations is provided in Fig. 15. It is emphasized that, at the centerline, the magnetic field aligns with the radial

direction, resulting in $T_{e\parallel}$ to be equivalent to T_{ex} . We notice from Fig. 15 that, introducing any level of B -field curvature significantly enhances the temperature isotropy in the system, which is in line with the observed behavior in Ref. 11. In fact, for a purely radial B -field ($\theta = 0$), the $T_{ex}/T_{e\perp} \approx 0.2$. However, this ratio shifts to around 1 in the presence of the negative curvature in the magnetic field and stays almost constant across $\theta = -60^\circ$ to -15° . For the positive curvatures, the $T_{ex}/T_{e\perp}$ ratio decreases as θ increases, even though it still maintains significantly higher values when compared to the scenario of no field curvature.

B. Variation in the location of the sheath's edge due to the magnetic field curvature

Figure 16 shows the time-averaged radial distribution of the ion Mach number (M_i) from the Q2D and the 1D simulations for various B -field curvatures. Interestingly, in the Q2D simulations of all propellants, as θ increases from -60° to 60° , the ion sonic point continuously progresses toward the bulk plasma away from the walls. Contrary to the Q2D results, in the 1D simulations of the

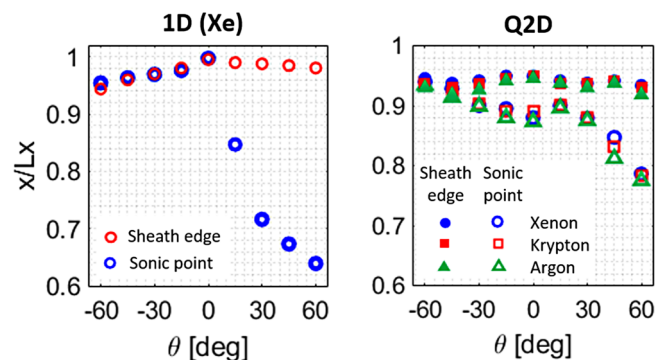
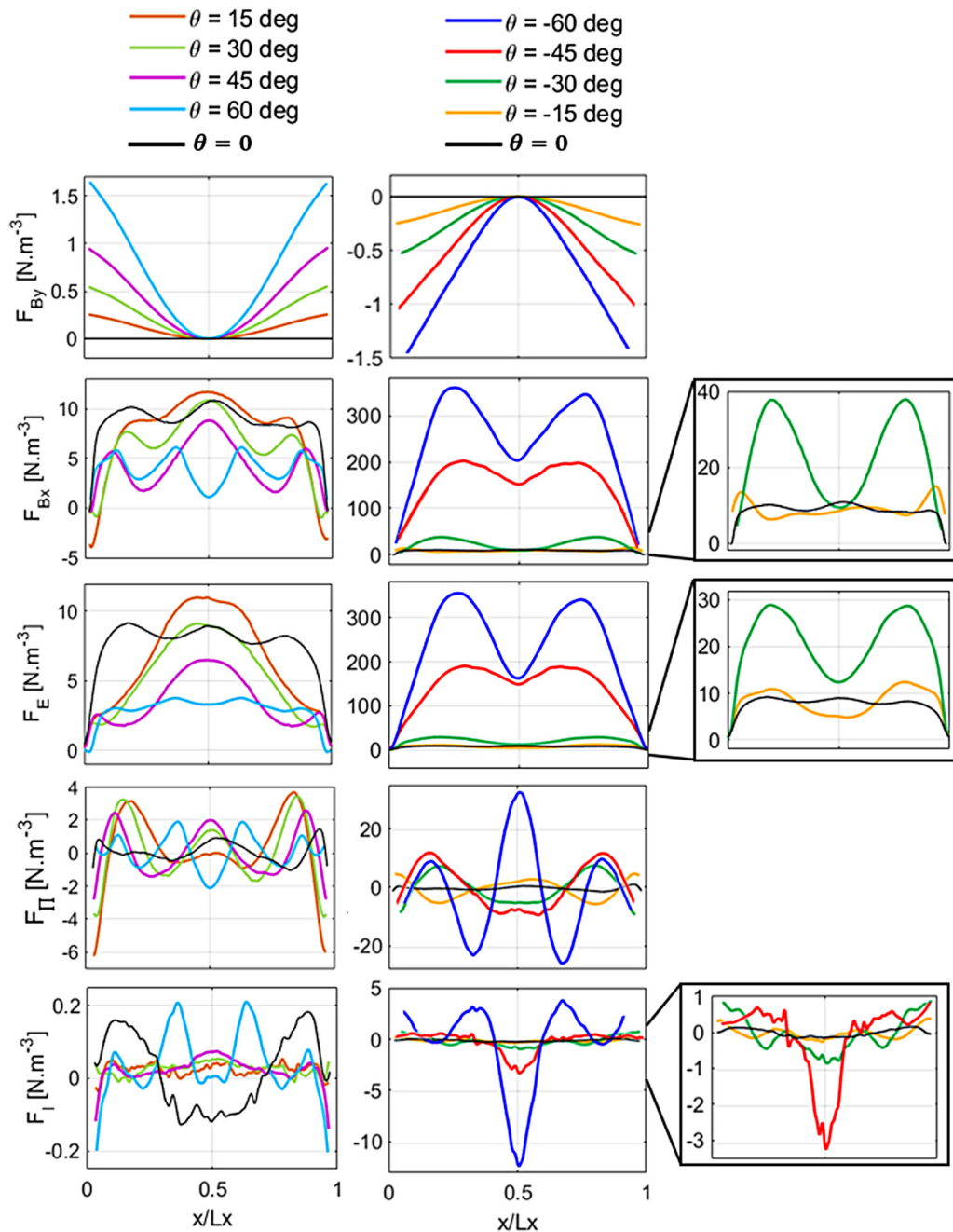


FIG. 17. Variation of the normalized radial location of the ion sonic point and the sheath edge [from Eq. (2)] vs θ from the 1D simulation with xenon (left) and the Q2D simulation with the three propellants (right).

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xenon propellant, the location of the sonic point slowly gets closer to the walls when θ varies from -60° to 0° , whereas it moves rapidly away from the walls as θ increases for the positive curvatures.

Our observation from the 1D simulations has also been reported in Ref. 11, where the authors noticed a similar trend with the field's curvature. The general behavior of the sonic point from Ref. 11 is, however, only in line with the Q2D simulations' results



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FIG. 18. Radial distribution of each of the force terms in Eq. (9) for the xenon propellant and various curvatures of the magnetic field. The momentum terms are averaged over $10 \mu\text{s}$ of the simulations' time ($20\text{--}30 \mu\text{s}$). Left-column plots are for the positive curvatures, and the right-column plots for the negative curvatures. The difference in the y-axis scales among the various plots is noted.

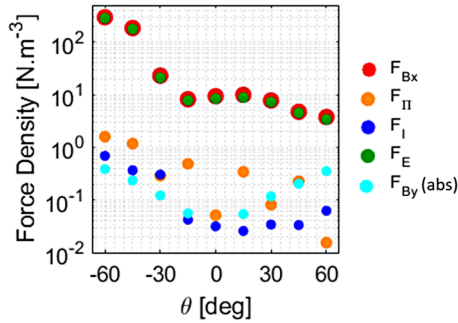


FIG. 19. Variation in the radially averaged value of each force term in the electrons' azimuthal momentum equation [Eq. (9)] vs θ from the Q2D simulations with the xenon propellant.

for the positive B -field curvatures. Even in this case, though, the sensitivity of the position of the ion sonic point to the curvature of the magnetic field is more significant in a 1D setting.

The occurrence of the sonic condition for ions within the plasma bulk, where the quasi-neutrality holds, renders the Bohm's criterion inadequate in establishing the boundary (edge) of the sheath. Thus, as also suggested in Ref. 11, in these situations, the non-quasineutrality serves instead as the appropriate criterion to determine the sheath's edge. According to this criterion,¹¹ the sheath's boundary is defined to coincide with the location where the non-quasineutrality exceeds a certain threshold, typically beyond 5% of the bulk ion density, i.e.,

$$\frac{|n_i - n_e|}{n_i} \approx 0.05. \quad (2)$$

Figure 17 compares for the Q2D and 1D simulations and for different B -field curvatures the locations of the ion sonic point and that of the sheath' edge as described by the non-quasineutrality condition [Eq. (2)] in the vicinity of the right-hand-side wall of the domain. According to Fig. 17 (left), in the 1D simulations and across the negative curvatures, the Bohm's criterion agrees with the non-quasineutrality condition in determining the location of the sheath's edge. However, in the cases of positive curvature, there is a notable disparity between the two criteria in the 1D cases. In the Q2D setting, we see from Fig. 17 (right) that, apart from the positive-curvature cases, the disagreement of the criteria extends to the negative curvatures as well including the cases with $\theta = -15^\circ$ and -30° .

The discrepancy between the criteria for the sheath's edge from the Q2D simulations indeed bears a more significant implication for the negative curvatures. This is because, in a typical Hall thruster design, the magnetic field features negative curvatures within the discharge channel where the formation of a plasma sheath near the walls actually becomes relevant. Hence, the observation that the 1D simulations show a consistency between the sheath's edge criteria for the negative field curvatures serves as an important shortcoming for these simulations.

C. Effect of the magnetic field curvature on the contribution to cross-field transport of the force terms in the electrons' azimuthal momentum equation

For the case of xenon propellant as an example, we assess in this subsection the importance of various contributions to the electrons' axial transport for varying degrees of magnetic field curvature. To this end, the electrons' momentum equation along the azimuthal direction (z) can be thought of as comprising several force terms defined as below:

$$F_{Bx} = -qn_e v_{e,y} B_x, \quad (3)$$

$$F_t = \frac{\partial}{\partial t}(mn_e v_{e,z}), \quad (4)$$

$$F_I = \frac{\partial}{\partial x}(mn_e v_{e,x} v_{e,z}), \quad (5)$$

$$F_{\Pi} = \frac{\partial}{\partial x}(\Pi_{e,xz}), \quad (6)$$

$$F_E = -q\tilde{n}_e \tilde{E}_z, \quad (7)$$

$$F_{By} = -qn_e v_{e,x} B_y. \quad (8)$$

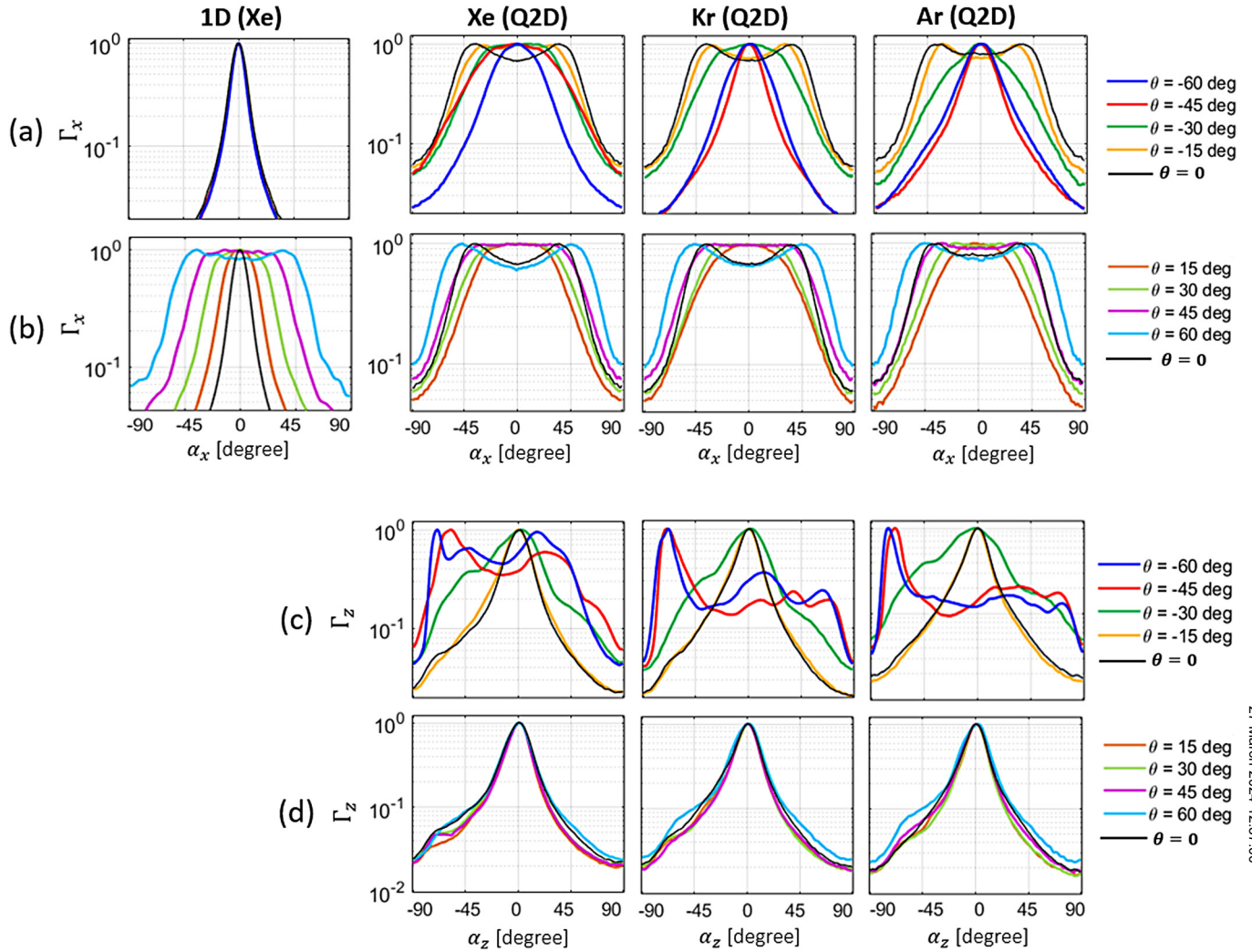
In Eqs. (3)–(8), q is the elementary charge, m is the electron mass, n_e is the electron number density, $v_{e,x}$ and $v_{e,z}$ are the electrons' radial and azimuthal drift velocities, and $\tilde{n}_e$ and $\tilde{E}_z$ are the number density and the azimuthal electric field fluctuations. F_{Bx} is the radial magnetic force term; F_t and F_I represent the temporal inertia and the convective inertia force terms, respectively; F_{Π} corresponds to the viscous effects; and F_E represents the contribution of the azimuthal instabilities. Finally, F_{By} is the axial magnetic force term. The electrons' azimuthal momentum equation [Eq. (9)] can be written as a balance between the above force terms,

$$F_{Bx} = F_t + F_I + F_{\Pi} + F_E + F_{By}. \quad (9)$$

The time-averaged radial distributions of each of the force terms in Eq. (9) for various θ values are compared in Fig. 18. Also, the variation in the field's curvature of the radially averaged terms is plotted in Fig. 19.

In these two figures, the temporal inertia (F_t) is not shown as it was found to have a negligible influence on the electrons' transport in the Q2D simulations. From the plots in Figs. 18 and 19, we notice that the instabilities are the major contributor to the axial transport. In this regard, we refer to Refs. 41 and 42, where we demonstrated in a similar radial-azimuthal configuration to that of this work that the radial profiles of the axial electron current density induced by different instability modes are different. We showed that in the cases where the ECDI is the dominant instability, the axial electron current density peaks at the centerline, whereas the MTSI's dominance leads to the concentration of the electron current near the walls.^{41,42}

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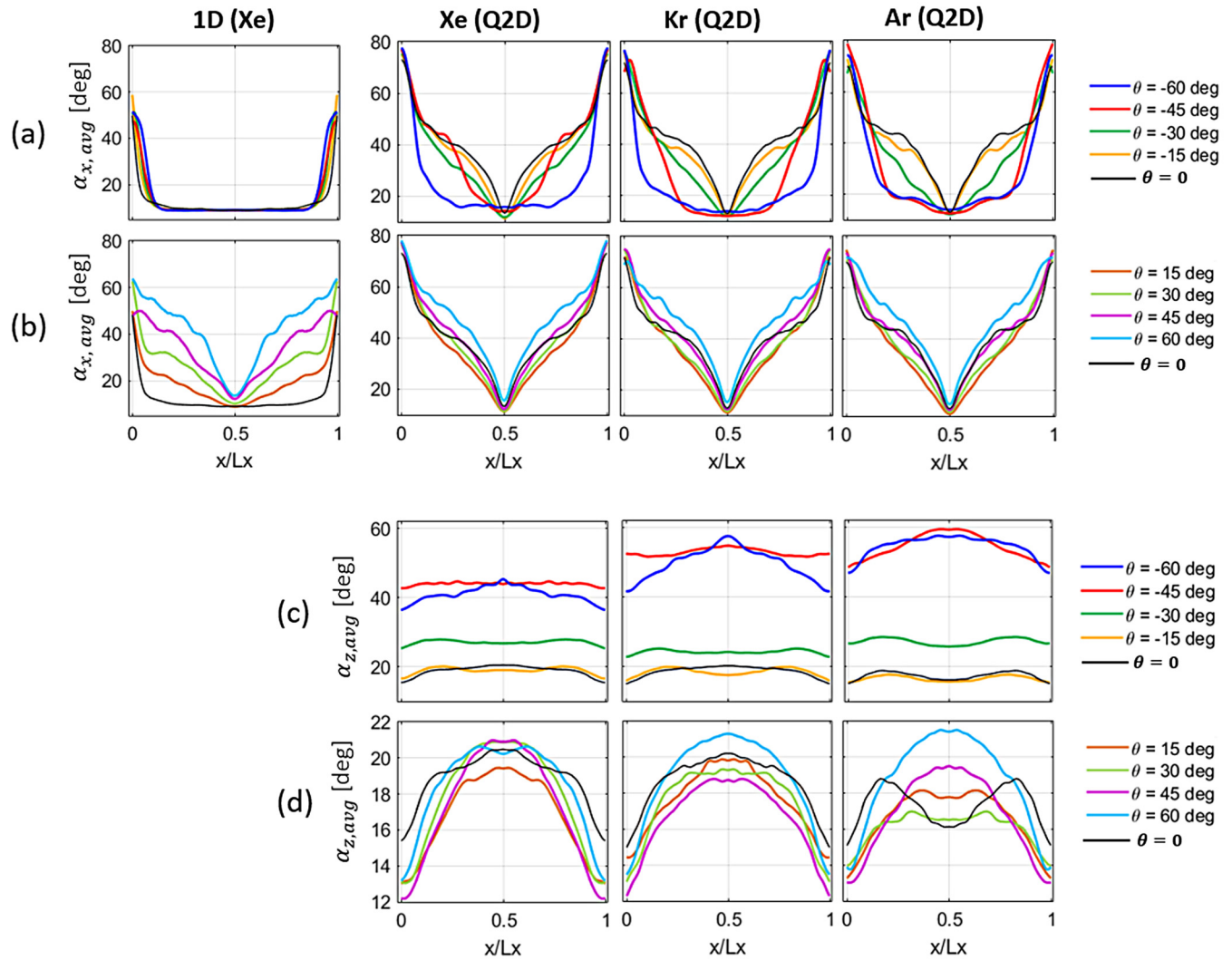
FIG. 20. Radially averaged divergence-angle distribution functions for various magnetic field curvatures due to the radial velocity component of the ions (Γ_x) from the 1D (for xenon) and the Q2D (for xenon, krypton, and argon) simulations (first and second rows) and due to the azimuthal velocity component of the ions (Γ_z) from the Q2D simulations with the three propellants (third and fourth rows): (a) and (c) negative field curvatures, (b) and (d) positive field curvatures.

Consistent with these previous observations, we can see from Fig. 18 that across all negative θ values where the MTSI was seen to be dominant compared to the ECDI from the spectral analyses, F_E is larger near the walls and reduces toward the center of the domain. Moreover, at $\theta = -60^\circ$ and -45° , F_E is remarkably higher than the cases with $\theta = -30^\circ$ and -15° . This behavior had been also observed in Ref. 30, when the long- λ instability mode was seen to be strong within the system.

In the cases with positive θ values of 15° – 45° , where the ECDI is shown in Sec. III B to be the dominant instability, and the MTSI was absent for xenon, the distribution of F_E is mostly concentrated in the center of the domain. However, in the case of $\theta = 60^\circ$ as well

as in the baseline case, the F_E distribution becomes more uniform in line with the coexistence of the ECDI and the MTSI.

Figure 19 shows further that the second significant contributor to the electrons' transport across several curvature degrees is the pressure term (F_{Π}), which from Fig. 18 shows an oscillatory behavior across the radius of the domain for all θ values. Moreover, we also see from Fig. 18 that the axial magnetic field force (F_{B_y}) becomes non-negligible toward the walls for large positive and negative curvatures, which is because, at elevated θ values, the axial component of the magnetic field becomes notable in the regions adjacent to the walls. Indeed, for $\theta = 60^\circ$, Fig. 19 shows that the radial average of the F_{B_y} term is even more significant than the



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FIG. 21. Radial distribution of the average ion beam divergence angle [α_{avg} , Eq. (10)] for various magnetic field curvatures from the 1D (for xenon) and the Q2D (for xenon, krypton, and argon) simulations; (first and second row) divergence angle α_x due to the ions' radial velocity component: (a) negative curvatures, (b) positive curvatures; (third and fourth row) divergence angle α_z due to the ions' azimuthal velocity component: (c) negative curvatures, (d) positive curvatures.

pressure term. Finally, the inertia force term (F_I) plays the least significant role to the axial electron transport in the most cases.

D. Influence of the magnetic field curvature on the ion beam divergence

An interesting impact of the magnetic field curvature, particularly from an applied perspective for a broad family of cross-field plasma technologies, pertains to the ion beam divergence. To quantify the level of divergence from our simulations, we defined an average divergence angle of the ion beam (α_{avg}) as the normalized integral of a novel quantity that we term the “divergence-angle

distribution function,” $\gamma(\alpha, x)$. α_{avg} , calculated as per Our mistake, sorry, should be Eq. (10), is a function of the radial coordinate (x),

$$\alpha_{avg}(x) = \max \left(-\frac{\int_{-\frac{\pi}{2}}^0 \alpha \gamma(\alpha, x) d\alpha}{\int_{-\frac{\pi}{2}}^0 \gamma(\alpha, x) d\alpha}, \frac{\int_0^{\frac{\pi}{2}} \alpha \gamma(\alpha, x) d\alpha}{\int_0^{\frac{\pi}{2}} \gamma(\alpha, x) d\alpha} \right). \quad (10)$$

The parameter α is the angle between the ion particles' trajectory and the axial axis. The ions' “divergence-angle distribution function” or $I\alpha DF$ characterizes the distribution of the ion trajectory angles (α) across the radial extent of the domain in the form of a histogram.

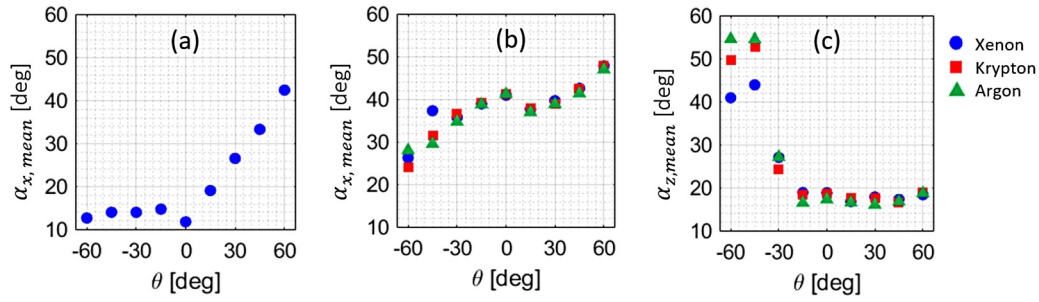


FIG. 22. Variation of the mean divergence angle vs θ ; (a) divergence angle due to the ions' radial velocity component from the 1D simulations with xenon, (b) divergence angle due to the ions' radial velocity component from the Q2D simulations with xenon, krypton, and argon. (c) divergence angle due to the ions' azimuthal velocity component from the Q2D simulations with the three propellants.

According to the ion temperature profiles shown in Fig. 8 and the observed values of the T_i , especially at large negative curvatures, which implied a possible broadening of the ions' velocity distribution function, the radial and azimuthal velocity components of the ions can be expected to contribute to the divergence of the ion beam. Accordingly, we define α_x and α_z as the projections of α on the axial-radial and axial-azimuthal planes, respectively, i.e.,

$$\alpha_x = \tan^{-1} \left(\frac{v_{i,x}}{v_{i,y}} \right), \alpha_z = \tan^{-1} \left(\frac{v_{i,z}}{v_{i,y}} \right). \quad (11)$$

In Eq. (11), $v_{i,x}$, $v_{i,y}$, and $v_{i,z}$ are, respectively, the ion particles' velocity along the radial, axial, and azimuthal directions.

We have integrated the ions' radial $IaDF$ ($\gamma_x(\alpha_x, x)$) and their azimuthal $IaDF$ ($\gamma_z(\alpha_z, x)$) along the radial direction according to the relations in Eq. (12), and the results in terms of Γ_x vs α_x and Γ_z vs α_z are shown in Fig. 20,

$$\Gamma_x(\alpha_x) = \int_0^{L_x} \gamma_x(\alpha_x, x) dx, \quad \Gamma_z(\alpha_z) = \int_0^{L_x} \gamma_z(\alpha_z, x) dx. \quad (12)$$

For the Q2D simulations, we see from the first- and second-row plots of Fig. 20 that, across all curvature cases, the Γ_x demonstrates a spread over a broader range of angles compared to the 1D simulations. For the negative-curvature cases, in addition, as θ varies from 0° to -60° , apart from the decrease in the dispersion of Γ_x across the velocity angles, the two peaks in the Γ_x around $\alpha_x = \pm(30^\circ - 45^\circ)$ merge and form one peak at $\alpha_x = 0^\circ$. For the positive curvatures [second-row plots in Fig. 20], as θ increases from 15° to 45° , the peak of the Γ_x at about $\alpha_x = 0^\circ$ start to further flatten, extending over a wider range of radial velocity angles. At $\theta = 60^\circ$, the Γ_x peaks at about $\alpha_x = \pm 45^\circ$.

Regarding the radially averaged azimuthal $IaDF$ (Γ_z), we observe from the third-row plots in Fig. 20 that the negative curvature of the field increases the population of the ions with non-zero azimuthal velocity angles. In fact, at $\theta = -60^\circ$ and -45° , in particular, for which the presence of a strong long- λ mode had been observed from the spectral analyses in Subsec. III B, a notable ion population having the velocity angle of about $\alpha_z = -55^\circ$ to -70° is noticed. This observation is in line with our previous observations

that the long- λ mode strongly influences the ion population.^{30,34} In contrast, the impact of the positive curvature on the Γ_z is not substantial as seen from the fourth-row plots in Fig. 20.

Now, we use our definition of the average divergence angle (α_{avg}), given by Eq. (10), to obtain the radial distribution of the average ion beam divergence due to the radial and the azimuthal ions' velocity components. The resulting plots for various B -field curvatures and the different propellants from the Q2D simulations are presented in Fig. 21. In addition, in this figure, the profiles of the average radial ion divergence angle from the 1D simulations with xenon are provided.

Starting with the ion divergence angle caused by their radial velocity ($\alpha_{x,avg}$), the plots for the Q2D simulations [Fig. 21(a)] show that, in general, the negative curvature of the magnetic field tends to reduce the ion divergence $\alpha_{x,avg}$ in the central part of the domain. This effect becomes consistently more significant toward higher negative curvatures. It is also observed that the case with $\theta = -45^\circ$ displays a notable disparity in the $\alpha_{x,avg}$ profiles among the different propellants. Specifically, the central ion beam, which can be noticed as a low radial divergence angle, expands over a larger portion of the domain's radial extent going from xenon to argon. In the 1D simulations, however, the ion beam radial divergence does not exhibit significant sensitivity to the degree of negative B -field curvature. Also, contrary to the Q2D simulations, in the 1D setting, the divergence angle remains low in the plasma bulk away from sheaths. These observations from the 1D simulations are, however, majorly influenced by the lack of resolving the important effects of the azimuthal instabilities on the ion population. Thus, a 1D radial analysis can misleadingly point to a low divergence-angle ion beam or, equivalently, an artificially elevated beam divergence efficiency.

Looking at the $\alpha_{x,avg}$ plots for the positive-curvature cases [Fig. 21(b)], we notice that the positive curvature expectedly enhances the ions' radial divergence angle. This effect is, however, notably stronger in the 1D simulations than in the Q2D simulations; an observation that can be again linked to the missing azimuthal physics in the 1D case and the consequent lack of resolving the ions' interactions with the azimuthal fluctuations that affect the particles' energy distribution and the directionality of their velocity vectors.

Referring to Figs. 21(c) and 21(d), we assess the ion beam divergence due to the azimuthal component of the ions' velocity ($\alpha_{z,avg}$) from the Q2D simulations. It is observed that, overall, the

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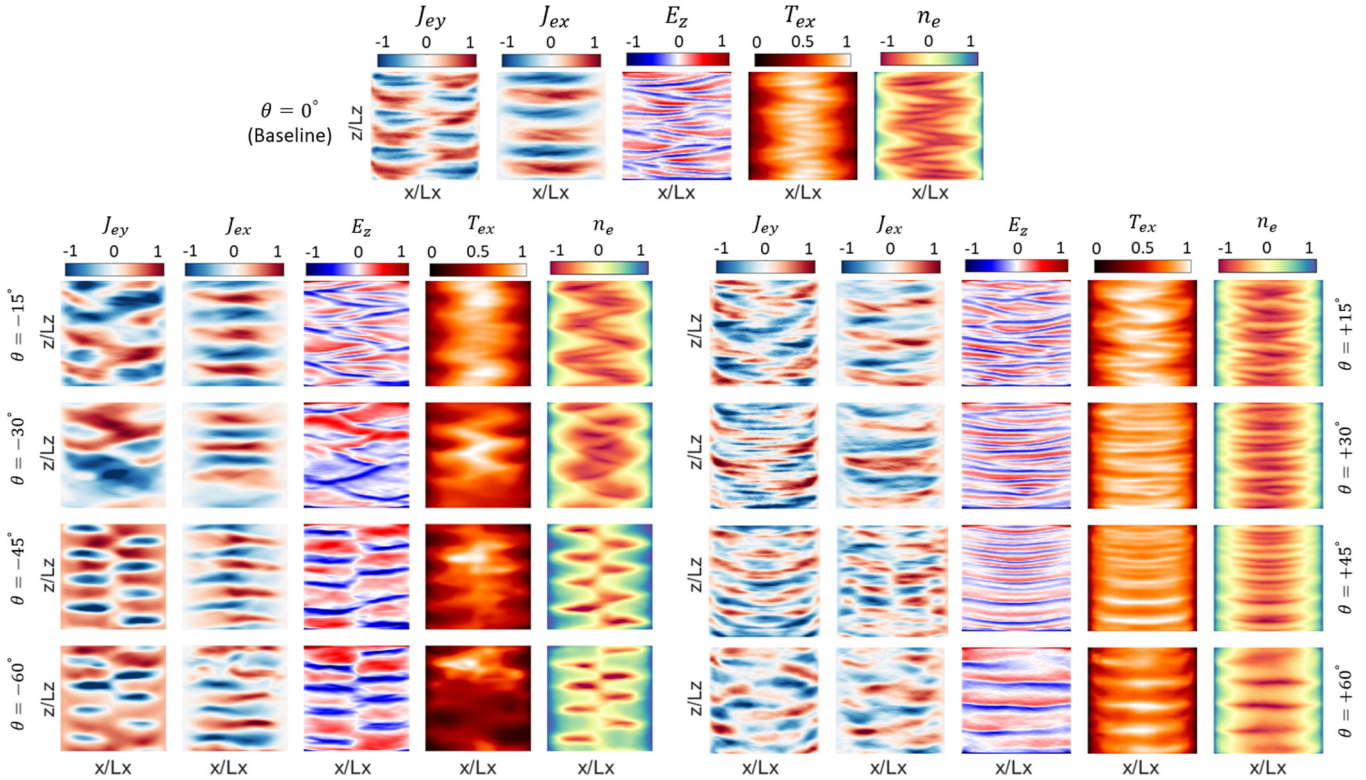


FIG. 23. Comparison of the 2D snapshots of the normalized plasma properties at a time of local maximum of the radial electron temperature for various B -field curvatures and the xenon propellant. The columns, from left to right, represent the axial and radial electron current densities (J_{ey} and J_{ex}), the azimuthal electric field (E_z), the radial electron temperature (T_{ex}), and electron number density (n_e).

negative curvature significantly increases $\alpha_{z, avg}$, which, for the negative θ s, assumes a relatively uniform distribution across the radial extent of the domain. Furthermore, in the cases of $\theta = -60^\circ$ and -45° , there is a consistent rise in $\alpha_{z, avg}$ from xenon to argon. On the contrary, as depicted in Fig. 21(d), the positive curvature of the magnetic field moderately impacts the radial profile of $\alpha_{z, avg}$. In particular, the relationship between $\alpha_{z, avg}$ and the field's curvature displays a non-monotonic trend and exhibits distinct behaviors across various propellants.

To summarize the observed trends, considering the mean of the radial and the azimuthal divergence angles over the entire domain's radius ($\alpha_{x, mean}$ and $\alpha_{z, mean}$) as representative values to quantify the total ion beam divergence, we have plotted the variations against θ of the $\alpha_{x, mean}$ and $\alpha_{z, mean}$ for the Q2D simulations in Fig. 22. The mean radial divergence angle ($\alpha_{x, mean}$) from the 1D simulations is also plotted against θ in Fig. 22.

A main finding to emphasize is that, for the Q2D simulations with high negative B -field's curvatures ($\theta = -60^\circ$ and -45°), the beam divergence is primarily driven by the azimuthal ion velocity. This highlights another significance of resolving the azimuthal direction in the Hall thruster simulations, namely, that in the absence of the azimuthal physics, there is a risk of underestimating the ion beam divergence in the situations of a concave field

topology with large degrees of curvature. Another main observation from the Q2D simulations is that across the moderate negative curvatures as well as the entire positive curvature range, the radial ion velocity is noticed to have the dominant influence on the ion beam divergence.

In the 1D simulations, the ion divergence remains almost invariant across the negative field curvatures, whereas it demonstrates an almost linear increase over the positive curvature values.

To conclude, in terms of the correlation between our numerical results above and the available experimental efforts from the literature, we would mention that the observations of notably increased ion beam divergence for the magnetically shielded Hall thrusters,^{49–51} which feature rather large positive magnetic field curvatures in their near-plume region, are in line with our findings. Moreover, the experimentally measured values of the ion beam divergence for xenon and krypton in Ref. 52 for a Hall thruster with conventional magnetic field topology are also interestingly comparable with the values of $\alpha_{x, mean}$ in Fig. 22 for moderate positive θ values.

Besides, it is also noteworthy to point out that some previous experimental works in cylindrical^{53–55} and annular⁵⁶ Hall thrusters have reported a surprising narrowing of the plume divergence in certain operating conditions of the thrusters, referred to as the “non-self-sustained discharge.” However, these experimental

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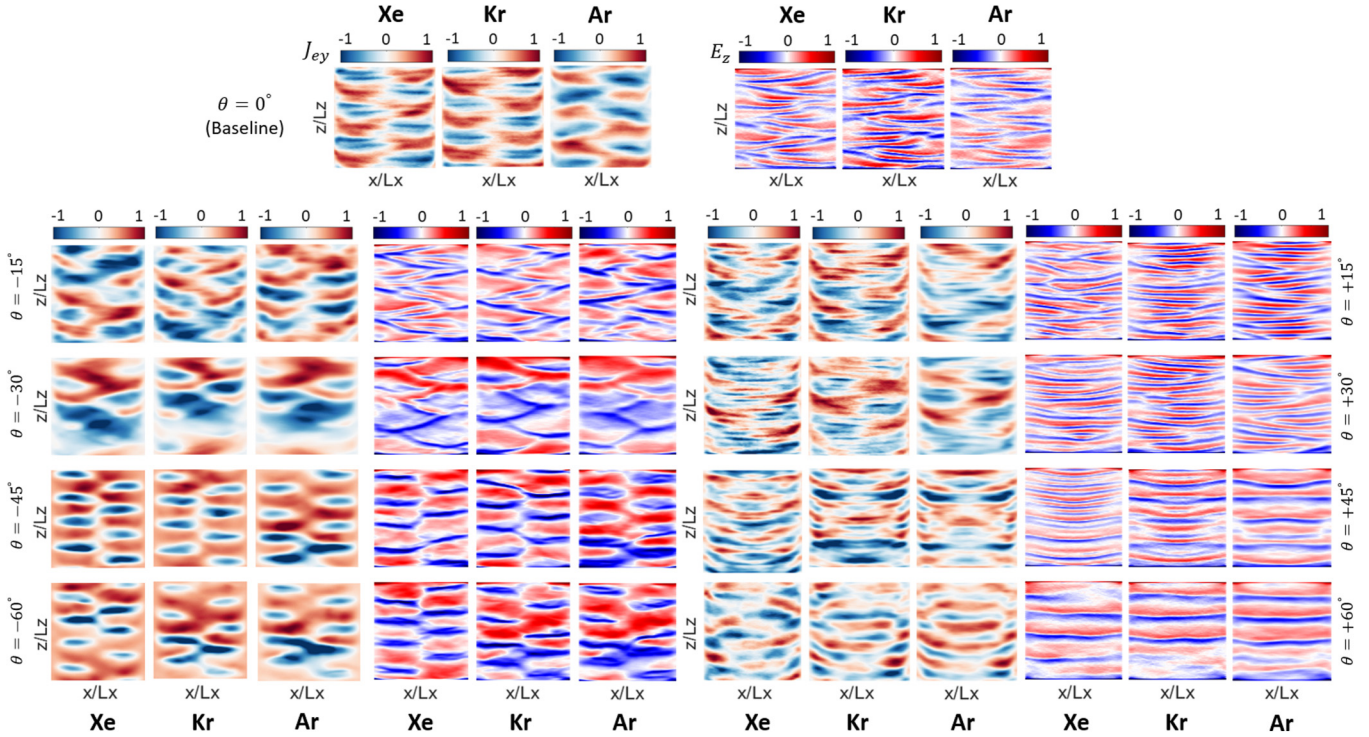


FIG. 24. Comparison of the 2D snapshots of the normalized radial electron current density (J_{ey}) and the normalized azimuthal electric field (E_z) at a time of local maximum of the radial electron temperature for various B -field curvatures and different propellants.

insights cannot be compared against our numerical findings here concerning the ion beam divergence. This is primarily because of the fact that Ref. 57 has theoretically attributed the experimental plume narrowing observations to the combined effect of the centrifugal forces on the plasma species and the induced magnetic field due to the supersonically rotating electron plasma. Our Q2D simulations in this work cannot capture either of these effects due to our Cartesian and electrostatic problem setup. Evaluating the potential impacts of these effects on our results from this work serves as an interesting and important next step.

E. Discussion of the main insights from the 1D vs Q2D simulations comparison and the effects of ion mass

Comparing the 1D radial and the Q2D radial-azimuthal simulations in this work from the perspective of the macroscopic plasma properties highlights that the overall predicted behaviors (trends) for the positive-curvature cases can be rather similar. Contrarily, for the negative-curvature cases, there are specific major differences noted between the 1D and the Q2D simulations.

More specifically, from both the Q2D and the 1D simulations, the peak ion number density (n_i) values drop significantly across the negative curvatures, whereas it shows a very moderate variation with the positive curvatures. Nonetheless, the radial electron temperature (T_{ex}) from the Q2D simulations shows an increase in

values from $\theta = -60^\circ$ to -30° , followed by a monotonic decrease with θ from -30° to 45° and then a slight increase at $\theta = 60^\circ$, whereas the 1D simulations' trend for T_{ex} are very different from the Q2D ones, remaining similar to the n_i variations with θ from the 1D cases. The ion temperature from the Q2D simulations exhibited large values of 35–45 eV for highly negative field's curvature ($\theta = -60^\circ$ and -45°), consistent with the observations reported in Refs. 30 and 34. The T_i 's radial profile is distinctly different in shape and in values for the negative-curvature cases between the 1D and the Q2D simulations while, for the positive curvatures, the distributions and the values are more comparable.

With regard to all the other aspects of the physics investigated in the present article, the 1D and the Q2D results exhibit notable differences. In fact, the values and the profiles of the electrons' mobility for both the negative- and the positive-curvature cases as well as the variations with the curvature degree of the electron temperature anisotropy, the location of the ion sonic point, and the ion beam divergence are all quite different between the 1D and the Q2D studies, with the effects of the field curvature being often more exaggerated in the 1D simulations.

Considering the influence of the ion mass on the plasma processes in the presence of various degrees of magnetic field curvature, the overall observation is that the spectra of the azimuthal instabilities, particularly, the ECDI, the MTSI, and the long- λ mode, exhibit some variations across the different propellants. As most notable

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examples, at $\theta = -15^\circ$ and 45° , where the ECDI mode is observed from the spatial FFT analyses in Subsec. III B, changing to lighter propellants gradually mitigates the amplitude of the ECDI. However, the strength of the long- λ mode, which is noted to develop at the large negative curvatures of -60° and -45° , seems to remain almost invariant across the different propellants.

The variations in the azimuthal instabilities' spectra with the ion mass are reflected in the distribution and the values of the plasma properties. For instance, referring to the exceptions pointed out in Sec. III for krypton regarding its ion (plasma) number density and the radial electron temperature profiles at $\theta = -60^\circ$, we can trace the relatively lower radial temperature and, consequently, the higher density,^{13,33} compared to other propellants to the slightly weaker MTSI mode for this propellant and at this curvature value as evidenced specifically from the slightly lower temperature anisotropy ratio (T_{ex}/T_{ez}) in Fig. 14.

It is also noteworthy that the radial location of the ion sonic point as well as the total ion beam divergence (apart from the highly negative B -field curvature cases) do not show any significant variation with the ion mass.

V. CONCLUSIONS

The variation in the radial-azimuthal dynamics of the plasma in a Hall thruster due to various degrees of negative and positive magnetic field curvature was investigated in detail in this article as the primary objective of the research. We also assessed, as secondary objectives, the differences between the 1D radial and the Q2D radial-azimuthal simulations in terms of the resolved effects of the magnetic field curvature on the plasma discharge in addition to the influence of the ion mass on these resolved effects in a Q2D setting.

We adopted a 2D simulation configuration that is representative of the radial-azimuthal cross section of a Hall thruster, but that also features a rather generic setup definition, which makes our results transferable to other $E \times B$ plasmas similar to the one of the Hall thrusters, such as the magnetrons. The 1D simulations' setup was an adaptation of the 2D setup to the 1D geometry.

We studied the effects of the magnetic field curvature on the spatial distribution of the time-averaged plasma properties, the instability spectra, the electrons' mobility characteristics, and the divergence of the ion beam due to the radial and azimuthal velocity components that can be affected by the underlying instability mechanisms. For xenon, we also compared the contributions of various momentum force terms to the cross-field transport of the electrons.

Regarding the most salient findings, we overall observed that the curvature of the magnetic field induces distinct and significant changes in the characteristics of the dominant instabilities and, in turn, the properties of the plasma. The insights are believed to be novel observations that can have remarkable implications in the basic understanding of the plasma processes within Hall thrusters and similar $E \times B$ technologies. We also noticed the critical significance of the azimuthal physics, which are absent in the 1D simulations, toward studying the magnetic field curvature effects.

The electron temperature anisotropy in terms of the radial-to-azimuthal temperature ratio (T_{ex}/T_{ez}) from the Q2D simulations demonstrated a distinct and insightful variation with the field's curvature, which was also generally shared among all

propellants. We demonstrated that the temperature anisotropy is correlated to the dominance of various instability modes within the discharge. Furthermore, the radial position of the ion sonic point was noticed, from the Q2D simulations, to continuously shift away from the vicinity of the lateral wall with the increasing magnetic field curvature. The discrepancy of the 1D simulations with the Q2D on this aspect and the significance of this observation to determine the correct location of the sheath's edge in the simulations that involve curved magnetic field topologies were highlighted.

Regarding the electrons' mobility, the change in the degree of the field's curvature was noticed to affect the radial distribution and the magnitude of this property in line with the variations in the instability spectra and the dominance of different modes. The results were almost identical for different propellants.

Finally, concerning the divergence of the ion beam with respect to the axial direction, we pursued a novel approach to quantify the divergence induced by the radial and azimuthal components of the ions' velocity. Most notably, we observed from the Q2D simulations that, even though a concave field topology reduces the ion beam's divergence along the radial direction, at highly negative curvature angles, the divergence is dominantly determined by the azimuthal velocity component and is significant in values due to the influence of the azimuthal instabilities. At the moderate negative field's curvatures and for positive curvatures, the ion beam divergence is governed by the radial velocity component and expectedly increases with the curvature degree.

The insights provided in this work from multiple facets into the effects of the magnetic field curvature suggest that, apart from the role of the axial gradients in the B -field, the curvature of the magnetic field can be an additional important reason behind the differences reported in the literature in the Hall thrusters' plasma dynamics before and after the B -field's peak. To further confirm the validity of this point, the numerical future work will entail extending the present study to a 3D PIC simulation setup that resolves self-consistently the axial physics and the collisional processes including the ionization as well as the cylindrical and self-induced magnetic field effects.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors do not have any conflict of interest to declare with regard to the present work.

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Author Contributions

M. Reza: Conceptualization (equal); Formal analysis (lead); Investigation (equal); Methodology (equal); Visualization (lead); Writing – original draft (lead). **F. Faraji:** Conceptualization (equal); Formal analysis (supporting); Investigation (equal); Methodology (equal); Visualization (supporting); Writing – original draft (supporting); Writing – review & editing (equal). **A. Knoll:** Conceptualization (supporting); Project administration (lead); Supervision (lead); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX: SPATIOTEMPORAL CHARACTERISTICS OF THE OBSERVED FLUCTUATIONS

1. 2D spatial distribution maps of various plasma properties

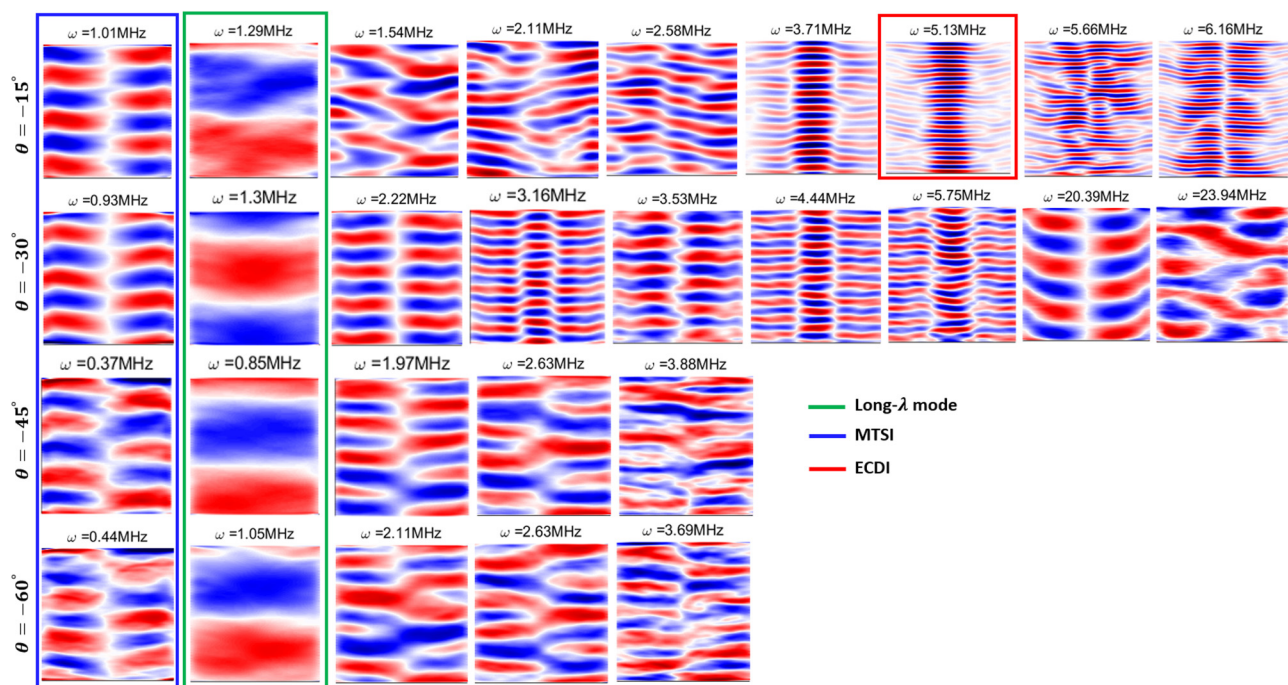
Figure 23 illustrates the snapshots of various plasma properties from the Q2D simulations with varying θ values and for the xenon propellant. The snapshots are captured at a time instant corresponding to a local peak of the radial electron temperature (T_{ex}), during which the MTSI, if present, is expected to be strong.^{13,33} It is evident from this figure that the negative- and the positive-curvature

cases display distinctively different behaviors in terms of the spatial structures of the involved fluctuations. In general, the negative curvature of the B -field tends to reinforce the development of the MTSI-representative patterns^{13,30} across the plasma properties, whereas the positive curvatures seem to suppress the MTSI, leaving the azimuthal waves associated with the ECDI as the primary instability modes in the system. The absence of B -field curvature ($\theta = 0$) represents an intermediate scenario, where the patterns resembling both the ECDI and the MTSI^{13,30} are quite visible in the snapshots. Moreover, the appearance of relatively long-wavelength structures^{30,34} can be noticed in the cases with $\theta = -60^\circ$ and -45° .

Figure 24 depicts a comparison of the snapshots of the axial electron current density (J_{ey}) and the azimuthal electric field (E_z) between xenon, krypton, and argon at their respective instances of the local maximum of T_{ex} and for different magnetic field curvatures. These sample snapshots show that there is an overall similarity between the properties' distributions for the three propellants, when T_{ex} is at its local maximum.

2. Dynamic mode decomposition analysis

To visualize the spatial structure of the instability modes corresponding to the peaks in the frequency spectrum plots (Figs. 9 and 11), we applied the DMD to the time series 2D snapshots of E_z for xenon as a sample application of the approach. DMD enables extracting the coherent spatiotemporal modes from the



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FIG. 25. Visualization of the several dominant DMD modes of the azimuthal electric field data from the radial-azimuthal simulations with different negative θ values and the xenon propellant. The approach pursued to derive these modes are explained in detail in Ref. 58. The modes identified with a box correspond to those with wavelengths equal or close to the wavelengths of the dominant modes visible in the spatial FFT plot in Fig. 9.

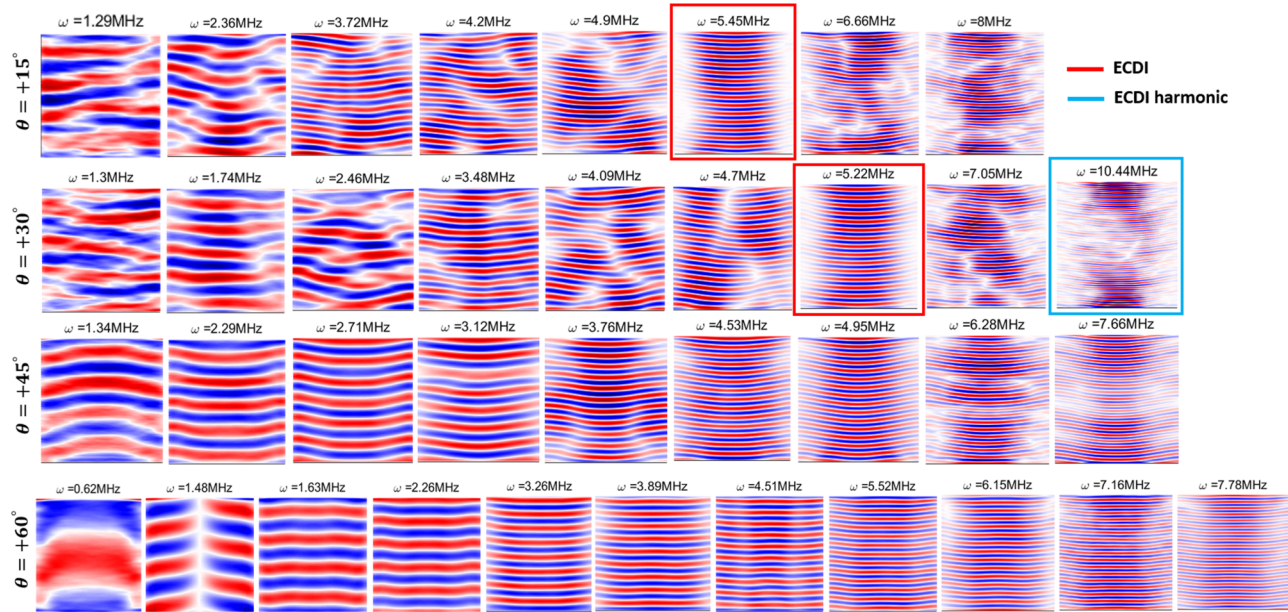


FIG. 26. Visualization of the several consecutive dominant DMD modes of the azimuthal electric field data from the radial–azimuthal simulations with different positive θ values and the xenon propellant. The approach pursued to derive these modes are explained in detail in Ref. 58. The modes identified with a box correspond to those with wavelengths equal to the wavelengths of the dominant modes visible in the spatial FFT plot in Fig. 11.

time-series data. These modes are essentially the distinct spatial patterns (ϕ_i) that capture the dominant structures and behaviors within data. Additionally, these modes are coupled with their respective linear dynamics over time, which are characterized by complex frequencies (ω_i). The E_z time-series snapshots can be reconstructed following the relation in Eq. (A1):

$$E_z(x, z, t) \approx \sum_{i=1}^r b_i \phi_i(x, z) \exp(\omega_i t). \quad (\text{A1})$$

We used optimized DMD (OPT-DMD), which is a significantly improved variant of the DMD method. In Ref. 58, we demonstrated across several test cases how the robustness of the OPT-DMD enables reliable identification and isolation of stable spatiotemporal modes within plasma systems.

The spatial structure and the frequency of several dominant DMD modes for the Q2D xenon simulations with the negative B-field curvatures are presented in Fig. 25.

The modes enclosed within boxes are those whose wavelength matches the instabilities identified from the spatial FFT plots. Consistent with what we observed from the spatial FFT spectra, the DMD discovers azimuthal waves corresponding to the ECDI only in the case of $\theta = -15^\circ$, whereas the MTSI as well as the azimuthal long- λ mode with a wavelength approximately equal to the domain's length and a frequency of about 1 MHz are identified in all cases. All the remaining modes detected in various cases contain both radial and azimuthal wavenumbers.

Finally, the dominant spatiotemporal modes obtained from the OPT-DMD analyses of various positive-curvature cases with xenon propellant as an example are illustrated in Fig. 26. Most of the presented modes comprise purely azimuthal fluctuation patterns with different wavelengths and frequencies.

The DMD analyses as presented above provide an effective route for the cross-validation of our insights against the experiments.⁵⁸ In fact, having an accessible visual representation of the spatial structure (radial and/or azimuthal wavelengths) of various instability modes can inform the design of experimental setups and the involved diagnostics⁵⁸ in order to verify the numerical observations reported concerning the variation in the instability spectra vs the degree of the magnetic field's curvature.

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