

**TEARING TOUGHNESS AND UNSTABLE
DUCTILE BEHAVIOUR OF A TITANIUM
ALLOY**

BY

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ABSTRACT

In this study of titanium alloy (6Al-2Cb-1Ta-1Mo), a useful form of representing its tearing toughness behaviour was sought. The J-Integral was used as the tearing toughness characterising parameter. The presentation of this toughness parameter was made in the form of resistance (R) curves partly because of its usefulness in predicting ductile instability.

To obtain J-R curves, computer-interactive testing procedures were developed for the Unloading Compliance and Key-Curve methods. The Key-Curve method however, appeared unwieldy in comparison to the more time and material-efficient Unloading Compliance method.

Using the Unloading Compliance method, the effect of loading rate, specimen orientation on J-R curves was found to be negligible. Marked dependence of absolute size and ligament length on J-R curves was observed but less so for variations in crack depth ratios and thicknesses, particularly for J_m and J_o -R curves. Sidegrooves of approximately the magnitude of the plane strain plastic zone size for this material were found to give virtually geometry independent J-R curves. Generally, the normalisation of J-R curve's abscissa by parameters such as thickness, B and ligament, b, reduced its dependence on specimen geometry. It is however believed that over the range of specimens tested, the J-R curves appear to be more ligament controlled.

The notion of work rate, presented here as the dissipative work rate, R_{DIS} and defined as:

$$R_{DIS} = (1/B)(dU_{DIS}/da)$$

is put forward as a real toughness parameter. The plot of R_{DIS} against crack extension is presented as a basic resistance curve and therefore challenges J-based resistance curves as being truly representative of a material's toughness behaviour.

Ductile tearing instability occurred in certain tests in this material and was analysed by the I and T (dJ/da) theories. Both incipient and final instabilities occurred, the former for small, deep notch test specimens and the latter for full thickness, shallow notch specimens. The two methods of analysis and types of ductile instability are compared and discussed. The analysis showed that experimentally obtained instability cases could be adequately predicted by both the I and T theories with the former making the predictions with somewhat greater certainty.

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NOTATION

The definitions given below, generally hold true unless otherwise stated for particular cases in the text.

a, a_o, a_7	crack length, initial, seven-point average
a_9, a_{max}, a_f	nine-point average crack length, maximum crack length of a curved crack front, final crack length
B, B_n, B_g	thickness, net, gross
b, b_o, b_i	ligament, original, current
C, C_m, C_n	compliance, machine, notched specimen
$C_o, C_{t.p}, C_u$	overall compliance, test piece compliance, un-notched specimen compliance
COD	crack opening displacement
E, E'	Young's modulus, effective
F_1	normalised load used in the Key-Curve analysis
G	elastic strain energy release rate
I	elastic strain energy release rate in an elastic-plastic system
J	J contour integral
k	stiffness
K	stress intensity factor
K_{IC}	critical plane strain stress intensity factor
l_g	microstructure grain size
L	constraint factor
m	stress intensification factor
M	bending moment
N	work hardening exponent
P, P_L	load, limit
q, q_{pl}, q_{el}	load point displacement, plastic component, elastic component

r	rotational factor
r_{pc}	cyclic plastic zone size
$r_{p\epsilon}$	plane strain plastic zone size
$r_{p\sigma}$	plane stress plastic zone size
R	material resistance, defined here as energy absorbed per crack extension
S	three-point bend specimen span
T	tearing modulus
$T1$	traction vector
U	strain energy or work done, evaluated under a load-displacement trace
U_{pl}, U_{el}	plastic work done, elastic work done
V	displacement vector
V_m	mouth-opening displacement
w	internal energy
W	specimen width
Y	LEFM shape factor
Z	strain energy density

GREEK SYMBOLS

Γ	contour path around a crack tip
δ	crack opening displacement (COD)
Δ	displacement
ν	Poisson's ratio
σ	stress
ϵ	strain
η	factor relating work done to the J integral

ω defined as $(b/J)(dJ/da)$

COMMONLY USED SUFFIXES

app	applied
C	critical
e	effective
el	elastic
DIS	dissipative
i, i+1	$i^{\text{th}}, i^{\text{th}} + 1$ increment
i,j	rectangular coordinates
m	machine
mat	material
max	maximum
min	minimum
pl	plastic
r, θ	polar coordinates
o	flow
() _s	system
TOT	total
x,y,z	directions of mutually perpendicular axes
xx,yy,zz	direction of principal planes
y	yield

ABBREVIATIONS

ASTM	American society for testing and materials
BCC	body centered cubic
BS	British standard
CCT	center-cracked plate
CKS	compact tension specimen
CTOA	crack tip opening angle
CTOD	crack tip opening displacement at a growing crack tip
EPE	elastic-plastic-elastic
EPFM	elastic plastic fracture mechanics
FCG	fatigue crack growth
HCP	hexagonal close packed
HRR	Hutchinson,Rice and Rosengren stress-strain field
J-R	J resistance
LEFM	linear elastic fracture mechanics
NLE	non-linear elastic
SEM	scanning electron microscope
SI	structure insensitive
SS	structure sensitive
Ti(6-2-1-1)	titanium alloy (6Al-2Cb-1Ta-1Mo)
TOM	travelling optical microscope

CHAPTER 1

INTRODUCTION

1.1 GENERAL

In recent years, fracture mechanics has proved itself to be a reliable tool in the assessment of structures for the purpose of useful life predictions and prevention of failures by fracture. Even though fracture is only one of the various known modes of failure, attention here is further restricted to only two mechanisms of fracture, namely:- ductile and brittle/cleavage fracture.

The micromechanism of the brittle type of failure is by cleavage. Cleavage fracture occurs by direct separation along crystallographic planes due to the breaking of inter-atomic bonds. This type of fracture is usually associated with relatively little plastic deformation or energy absorption before and during final separation. This fracture type is essentially elastic and can therefore be studied under a branch of fracture mechanics called linear elastic fracture mechanics (LEFM).

Microscopically, the mechanism of ductile fracture is by micro-void coalescence (MCV), which results in a fibrous-like fracture surface, characterised by dimples. Macroscopically, ductile fracture is synonymous with large plastic deformation and relatively high capacity to absorb energy. Therefore, this mode of fracture falls under the category of fracture mechanics that considers plasticity effects, i.e., elastic plastic fracture mechanics (EPFM). It is, however quite common to have cleavage separation after some ductile crack growth.

Over the past thirty years or so, a theoretically based concept has been developed for the characterisation of defects. The resulting parameter used is the stress intensity factor, K , this is, however restricted to only linear-elastic applications, that is, situations that exhibit relatively little plasticity compared to the overall dimensions of the test specimen or structural component.

Generally accepted standards of fracture toughness testing such as the ASTM Standard E399 (1981), provide critical K values that are used as satisfactory toughness characterisation parameters under plane strain ($\epsilon_z=0$) conditions. Though small amounts of plasticity can be accommodated in linear elasticity by corrections to the

effective crack length (see chapter two), the stress intensity concept ceases to be useful in elastic-plastic type behaviour. The inadequacy of the K concept to describe fracture in ductile materials, where plane stress conditions ($\sigma_z=0$) dominate has led to the development of two leading post yield fracture mechanics parameters.

These are the crack opening displacement, δ (COD), and the J contour integral. The mathematically more rigorous J integral approach, though developed using "non-linear elastic" theories is widely used as a toughness parameter for "real" (elastic - plastic - elastic) materials.

More recently the characterisation of a material's "toughness" in the form of resistance curves (R curves) has proved to be very useful. Resistance curves represent a plot of a materials toughness, be it in J or δ , against crack extension. In post-yield fracture mechanics, J-based resistance curves have been shown to be extremely useful in design and instability predictions. For example, see Turner (1984).

The primary aim of this thesis is to study the J-resistance curves, albeit for titanium alloy (6Al - 2Cb - 1Ta - 1Mo), the study might have implications in the general understanding of resistance curves and its applications. The organisation of this thesis is as follows:

A review of directly relevant topics is given in chapter two. In this chapter, an introduction to some metallurgical properties of titanium is made. This is followed by presentation of pertinent information in linear elastic-fracture mechanics (LEFM) and similar relevant formulations are given under elastic-plastic fracture mechanics (EPFM).

Before J-Resistance curves are mentioned, the (COD) δ , and J integral are introduced. Finally, an application of resistance curves is made in a review of the phenomenon of ductile instability. The literature survey is continued in Chapter three, by reviewing the numerous methods of crack length and J-R curve determination. Virtues and drawbacks of each method are highlighted.

In chapter four, a detailed description of the computer interactive test procedures developed for the Unloading Compliance and Key-Curve analysis is given. The actual computer software developed for numerical processing in these two methods is however described in appendix II and III. Chapter five compares the J-resistance curves

generated by the multiple specimen procedure with the aforementioned computer interactive procedures. Other test method-related parameters are investigated and reported here. Also, crack front curvature studies are made and resulting curvature-corrected compliance predictions are presented in this chapter.

J-Resistance curves are presented in chapter six for a variety of physical alterations to the three-point bend specimen geometry. Variations in absolute size, in-plane constraint, thickness and ligament size were investigated. Normalisation of the J-R curve's abscissa with suitable parameters was also made with the aim of reducing the scatter of these curves and hence be able to represent "toughness" without size and configuration effects. Sidegrooved specimen tests were carried out to investigate the role of shearlip formation in the behaviour of J-R curves. As a direct follow up to these studies an alternative form of representing toughness is presented in chapter eight in the form of work rates, i.e. energy dissipated per area of crack extension.

Before the alternative toughness form is presented, the phenomenon of ductile tearing instability that occurred in this material (Titanium (6-2-1-1)) is analysed. Two methods of analysis and types of instability that occurred in this material are compared and discussed.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION TO TITANIUM

2.1.1 GENERAL PHYSICAL AND MECHANICAL PROPERTIES

Titanium a metallic element, occurs naturally in great abundance but is difficult to reduce from the oxide. Commercially pure titanium contains 0.3% oxygen, 0.1% nitrogen and 0.2% each of carbon and iron, see Brady et al (1977). Its yield strength could be anything between 490 MN/m^2 to 750 MN/m^2 .

Titanium and its alloys have very desirable properties for engineers. Though initial applications of titanium were restricted to exotic uses, more efficient extraction processes have made it cheaper for increased applications elsewhere. Its exceptional combination of fatigue strength, and high strength to weight ratio has enabled titanium to enjoy considerable use in both military and civilian applications. Titanium alloys are generally 40% lighter than steel and 60% heavier than aluminium, see Brady et al (1977). The combination of moderate weight and high strength (yield Strength could be as high as 1350 MN/m^2) have made it particularly attractive to the aerospace industry. Another noteworthy property is its corrosion resistance, if the surface is intact. A tough thin oxide film protects titanium from very harsh atmospheric and sea environments. In this respect, the chemical industry finds titanium very useful. Other general properties are its comparatively high melting point (1670°C), low thermal conductivity, high electrical resistivity and low coefficient of expansion.

2.1.2 METALLURGY OF TITANIUM

Titanium is an allotropic material, that is, it is capable of existing in more than one form. The two known existing forms of titanium, are the alpha (α) and beta (β) phase. The alpha has a Hexagonal Close-Packed (HCP) crystal structure and the beta has a body centered cubic (BCC) structure.

The alpha phase is stable up to about 880°C where it transforms to the beta phase. This temperature is called the beta transus temperature (see fig 2.1). Alloying elements called beta stabilizers stabilize the beta phase below the beta transus temperature so that beta crystals can exist below the transus temperature. Similarly alpha phase stabilizers

stabilize alpha phase crystals at higher temperatures, see the RMI company booklet (1983b). Titanium alloys can exist in three structural types: primary alpha, mixtures of alpha and beta and primary beta. Alloys that have comparable amounts of alpha and beta crystals are called alpha-beta alloys. Alloys which primarily consist of alpha crystals are called alpha alloys and similarly, those with a majority of beta crystals are called beta alloys. More details about titanium alloy classifications and associated physical and mechanical properties can be obtained from the RMI booklet (1983a).

Microstructural examination for titanium reveals a large array of possible morphologies, depending on alloy composition and heat treatment, see the RMI booklet (1983b). The particular titanium alloy studied in this dissertation is titanium alloy(6Al-2Cb-1Ta-1Mo). This alloy, from the RMI Company, is known to have acicular alpha and prior beta grain boundaries (fig 2.2). Since it is primarily alpha with only beta grain boundaries it has an alpha alloy classification.

2.1.3 GRAIN SIZE-FATIGUE CRACK GROWTH RELATIONSHIP

Yoder, Cooley and Crooker (1978, 1979) stated that there is a significant relationship between the grain size, l_g , and fatigue crack growth rate.

From the study of various titanium alloys, Yoder et al (1978, 1979) proposed that a bilinear relationship between the logarithmic plot of crack growth rate da/dN , and stress intensity factor, ΔK exists (fig 2.4). Depending on the heat treatment, the grain size was quantified. An effective grain size had to be quantified for a widmanstatten morphology in the form of a widmanstatten packet size. Such a microstructure is present for a α/β alloy such as Ti 6Al-4V annealed above the beta transus temperature.

Yoder et al (1978, 1984) stated that the fatigue growth rates decrease systematically with increased grain size (fig 2.3). With the effective grain size, relatively large (in comparison to the reversed plastic zone size, r_{pc}), fatigue sensitive mode of crack propagation occurs. This results in crystallographic bifurcations. Thus a lower rate of fatigue crack growth (FCG) occurs. During this mode of crack growth, r_{pc} is less than l_g . However, when r_{pc} exceeds l_g the mode of crack growth becomes microstructurally insensitive (S.I.) therefore bifurcations reduce and the crack growth rate increases as shown in Fig 2.4.

i.e., when:

$$r_{pc} < l_g \quad \text{structure sensitive}$$

and, $r_{pc} > l_g \quad \text{structure insensitive}$

Crystallographic bifurcation is associated with the structure sensitive (SS) mode of FCG results in sizeable "facets" on the fatigue fracture surface. Indeed the resultant path of FCG exhibits a high level of tortuosity, see Yoder (1984).

2.2 LINEAR ELASTIC FRACTURE MECHANICS

2.2.1 INTRODUCTION

Linear Elastic Fracture Mechanics (LEFM) evolved from early studies of stress analysis around materials discontinuities. In more recent years LEFM has proved itself to be very useful in assessing the integrity of cracks in bodies that are essentially elastic.

2.2.2 STRESS INTENSITY FACTOR

Irwin (1957) considered Westergaard's (1939) solutions for crack tip stresses in an elastic tensile stress field.

In terms of polar coordinates, the stresses can be expressed as:

$$\sigma_{x,y} = (K_I / \sqrt{2 \pi r}) f(\theta) \quad (2.1)$$

where K_I is called the stress intensity factor and θ and r are polar coordinates about the crack tip as shown in fig 2.5.

The subscript I is used to refer to the tensile mode of stressing which is considered to be the most severe when compared to the shearing and tearing mode (fig 2.6).

From Westergaard's (1939) solution for a crack in an infinite plate, the stress normal to the crack plane ahead of the tip is:-

$$\sigma_{x,y} = (K_I / \sqrt{2 \pi r}) \quad (2.2)$$

Since, $f(\theta) = 1$ for this stress direction.

Therefore, for the case of a central crack of length $2a$ in an infinite plate, subjected to a tensile stress, σ ,

$$K_I = \sigma \sqrt{(\pi a)} \quad (2.3)$$

For finite width cases, the stress intensity is corrected for in the form of a Y factor, viz:

$$K_I = Y \sigma \sqrt{(\pi a)} \quad (2.4)$$

This is known and calibrated for most standard test pieces. Rooke and Cartwright (1976) and Towers (1981) are easy-to-use sources for Y factors.

2.2.3 PLANE STRAIN AND PLANE STRESS PLASTIC ZONE

For characterisation as a material property, the toughness should be obtained under plane strain conditions, i.e., when the through thickness strain is zero. The associated plastic zone size reduces with increase in stress triaxiality at a crack tip (plane strain). When the plastic zone is large compared to the thickness, yielding can take place freely in the thickness direction, resulting in loss of triaxiality (plane stress).

For estimates of the plastic zone size, in the plane stress condition, the yield strength ,

σ_y is substituted for $\sigma_{x,y}$ in Equ.2.2 and solved for r. This gives:-

$$r_{p\sigma} = 1/2\pi (K_I/\sigma_y)^2 \quad (2.5)$$

In the plane strain region, and usually under LEFM conditions, the elevated stress due to the triaxiality (fig 2.7) causes the plastic zone size to be normally accepted as a third that of plane stress. i.e.,

$$r_{p\varepsilon} = 1/6\pi (K_I/\sigma_y)^2 \quad (2.6)$$

2.2.4 PLANE STRAIN FRACTURE TOUGHNESS TESTING

Strict guidelines for the testing of plane strain (severest, thus lower bound) fracture toughness is given in the ASTM E 399 (1981) standard. Specification on the size requirements to ensure a restricted plastic zone size (thus, plane strain) in comparison to the overall specimen dimension is given in the form:-

$$a, W-a, B > 2.5 (K_{IC}/\sigma_y)^2 \quad (2.7)$$

2.2.5 THE GRIFFITH CRITERION

Griffith (1924) proposed an energy balance criterion for cracking. He stated that crack propagation will occur if the energy released upon crack growth is sufficient to provide all the energy that is required for crack growth. The condition for growth is:

$$dU/da \geq dw/da \quad (2.8)$$

Where U is the strain energy and w is the energy required for fracture. dw/da is usually called R - the material's resistance to fracture.

Griffith (1924) used the work of Inglis (on the stress field solution of a crack of length $2a$ in an infinite plate of unit thickness subjected to a uniform stress) to show that the release of strain energy $-U$, due to the crack is given by:

$$-U = \sigma^2 \pi a^2 / E' \quad (2.9)$$

So the elastic energy release rate (G) per crack tip of unit thickness is:

$$G = -dU/d(2a) = \sigma^2 \pi a / E' \quad (2.10)$$

where $E' = E$ in plane stress and $E' = E/(1-\nu^2)$ in plane strain.

Recalling Equ. (2.3)

$$K_I = \sigma \sqrt{(\pi a)}$$

Equ. (2.10) becomes,

$$G = K_I^2 / E' \quad (2.11)$$

Equ. 2.11 links the elastic energy release rate (G) with the stress intensity factor (K). Though the relationship above (Equ. 2.11) is shown for the infinite plate, it is valid also for finite width cases, within the obvious limits of LEFM.

Garwood (1977) presented a more thorough and rigorous proof of the above stated relationships.

2.2.6 THE LEFM RESISTANCE CURVE

In subsection 2.2.5, a concept of a material's resistance to fracture (R) is introduced. The variation in R with crack extension is presented as a resistance curve.

In metals, plastic deformation occurs during crack growth, and if the energy expended in plastic deformation for every crack increment is the same, then dU/da is a constant where U is energy expended and a is crack length. This is known to be approximately true for plane strain conditions, see Broek (1982). Fig. 2.8 shows that only after a critical value of G (G_C) is exceeded, can crack extension be expected.

This critical value of $G_C = (1 - \nu^2) K_{IC}^2/E$ for plane strain conditions.

When plane stress LEFM conditions occur (i.e. r_{ps} is not negligible compared to B , but $\sigma_{net} < \sigma_y$) and plasticity effects are not negligible, then the value of G_C is no longer a constant. Krafft et al (1961) observed a large variability of fracture toughness with size of laboratory test specimens. Krafft et al (1961) stated that the elastic strain energy release rate is elevated by an increasing load to a position of tangency with the rising resistance curve. At the point of instability, $R = G_C$ and

$$dG/da = dR/da \quad (212)$$

Krafft represented this concept as illustrated in Fig 2.9. The concept of a rising R curve was also rationalised by Krafft et al (1961) when the growth of shear lips (energy absorbed) increases with crack growth. The R curve concept is further studied when the plastic zone size is comparable to the overall specimen dimensions, thus, leading to a consequent invalidation of LEFM.

The desire to establish a toughness parameter in the post-yield regime led to the development of the crack opening displacement COD, δ , and the J-Integral, discussed below. δ and J have, in the recent years emerged as a popular parameter characterising elastic-plastic, (post yield) toughness. The success of these parameters in establishing initiation of critical value of δ or J led to the use of these parameters in forming resistance curves.

2.3 ELASTIC-PLASTIC FRACTURE MECHANICS

2.3.1 GENERAL CONCEPT

When LEFM parameters no longer apply, elastic-plastic fracture mechanics have to be used for crack characterisation purposes. The two parameters that have gained plenty of attention are the crack opening displacement (COD), δ , and the J integral. It should be observed however, that these are single parameters which are used to measure the severity of the crack tip loading conditions. Immediate restrictions appear when using one parameter models for EPFM analysis. Arguments as to the process of separation be it by void coalescence or cleavage can arise. Also McClintock (1965) noted that cases of plane stress (and plane strain where σ_{xx} is not unique) and non-hardenability possess a stress state that is not unique and therefore not capable of being described by a single parameter. If restrictions of near plane strain and describable zones ahead of a crack tip are made (Turner (1984)), then the single parameter model appears more acceptable. But in cases of high constraint and workhardening, the concept of a single EPFM characterising parameter does appear worthwhile and shall be discussed in some detail in the next subsection.

2.3.2 CRACK OPENING DISPLACEMENT, δ

2.3.2.1 INTRODUCTION AND DEFINITION

This strain based characterisation parameter was developed on the premise that after yielding with minimum work hardening, the stress at the crack tip is constant and thus a critical value of strain exists. Wells (1961) proposed such critical strain measurements in the form of δ .

The crack opening displacement is a measure of the crack face separation at the crack tip (see fig 2.10). One parameter characterisations of such post yield crack tip conditions are supported by McMeeking (1977), at least for a given crack tip constraint.

2.3.2.2 EXPERIMENTAL DETERMINATION OF δ

There are two primary means of determining δ . These are:-

- (i) Direct crack tip profile measurement.
- (ii) Extrapolation methods from crack mouth opening measurements.

The subject has been extensively reviewed by Gibson (1986) and Etemad (1983) and so will be introduced only briefly in this section.

Robinson and Tetelman (1974) obtained crack tip profiles by silicone-rubber infiltration. See fig 2.11 a for typical rubber cast profile. Gibson (1986) described in detail, the technique using a syringe device that attempts to force the viscous silicone rubber right up to the crack tip. The uncertainties associated with such a procedure include, possibilities of the silicone rubber not getting to the crack tip and shrinking of the rubber cast after removal from crack face. The somewhat arbitrary position of δ measurement as shown in fig. 2.10 adds to the inconsistency of the measurement technique.

Because of the rather tedious nature of measuring crack profiles, the analytically based extrapolation method appears more attractive. The method simply involves measurement of the crack mouth opening and an assumption of plastic hinge point with which crack tip opening can be extrapolated from. For bending geometries as illustrated in fig. 2.12, the specimen is assumed to rotate about a fraction (rotational factor, r) of the ligament b . Therefore, using similar triangles, fig 2.2 shows the plastic δ to be:

$$\delta_p = V_m r (W-a)/[r(W-a) + a] = V_m r b/[r b + a] \quad (2.13)$$

The overall δ (COD) is given by

$$\delta_p = \delta_e + \delta_p \quad (2.14)$$

where δ_e is the elastic component of δ . This is obtained from Well's proposal of

$$G_C = m \sigma_y \delta_c \quad (2.15)$$

and using Equ. (2.11),

$$\delta_e = K^2/m\sigma_y E \quad (2.15a)$$

$$\delta = (K^2/m\sigma_y E) + V_m r b/[r b + a] \quad (2.16)$$

The British Standard, BS 5762 (1979) has adopted m as 2.0 and r as 0.4. The obvious pitfall in Equ. (2.16) is the uncertainty of the r and m values.

Other less popular methods of δ measurements include transverse optical microscopy and scanning electron microscopy (SEM). The former involves the straightforward observation of the specimen near the crack tip. One disadvantage of this method is that this surface measurement is not representative of the entire crack front opening. SEM analysis involves the measurement of stretch zones and step like features caused by crack tip opening displacements, see Schwalbe (1981, 1974).

2.3.3 THE J-INTEGRAL

2.3.3.1 ORIGINAL CONCEPT

Rice (1968) used the line integral previously and separately developed by Eshelby (1956) to characterise stress and strains around a crack in a non-linear elastic body. He called this two dimensional line integral J and defined it as:

$$J = \int_{\Gamma} \{ Z dy - T_1(\partial V/\partial x) ds \} \quad (2.17)$$

where Z is the strain energy density, T_1 is the traction vector specified over an arbitrary part of the body defined by Γ as shown in Fig 2.13, ds is an increment in the contour path Γ and V is the displacement vector of points on Γ .

Equ. (2.17) was derived for non-linear elastic bodies and the contour path, gives a constant J value if the material ahead of the crack is homogeneous.

For this NLE material, J is also equivalent to the rate of change of potential energy with crack growth, i.e.,

$$J = -1/B (dU/da)|_q \quad (2.18a)$$

where U is the work done with fixed displacement, q .

Similarly for fixed load,

$$J = 1/B [\partial(CE)/\partial a] P \quad (2.18b)$$

Where CE is the complementary energy, $\int qdP$ as shown in fig 2.14. Equations (2.18a) and (2.18b) are consistent with linear elasticity since in linear elasticity U represents recoverable energy. Rewriting Equ. (2.10),

$$dU = - BGda$$

If the elastic component of J is J_{el} , in the limit of LEFM:

$$J = J_{el} = G \quad (2.19)$$

2.3.3.2 HRR STRESS AND STRAIN FIELD SOLUTIONS

In an attempt to use the original non-linear elasticity (NLE) based J for elastic-plastic analysis, Hutchinson (1968) and Rice and Rosengren (1968), determined the stress and strain field around a crack tip of a material obeying a power hardening law:

$$\epsilon_{pl}/\epsilon_y = \alpha (\sigma/\sigma_y)^N \quad (2.20)$$

where $\frac{1}{\epsilon_y/\sigma_y} = E$ and α, N are constants. These solutions are referred to as the HRR solutions, after the authors. The stress and strain equations were rewritten in terms of J by McClintock (1971) as,

$$\sigma_{ij}(r, \theta) = f_1(J) r^{-1/N+1} \sigma_{ij}(\theta) \quad (2.21a)$$

$$\epsilon_{ij}(r, \theta) = f_2(J) r^{-N/N+1} \epsilon_{ij}(\theta) \quad (2.21b)$$

where r and θ are polar co-ordinates. Thus, for total or deformation plasticity, the use of J is justified for monotonic loading only, since it differs for non-linear behaviour when unloading occurs (i.e. it does not return along the loading path as NLE would predict). Various workers such as Hayes (1970) and Sumpter (1974) have shown J to be path independent to within a numerical accuracy of about 5% with incremental laws of plasticity, even though the J integral was derived strictly for NLE or deformation plasticity without unloading.

2.3.3.3 THE η FACTOR

The definition of J as in Eqs. 2.18a, and 2.18b led to the first experimental evaluation of J by Begley and Landes (1972). The procedure involved using specimens of different crack lengths. The J value was then obtained from the gradients of U/B versus crack length for different values of deflection. This lengthy procedure was considered expensive and labour intensive so alternate methods were investigated.

It was later noticed that under considerations of separable variables, J could be expressed for a given stress-strain law as:

$$J = \eta U/Bb \quad (2.22)$$

Where η is a factor that relates the work done, U , to the J integral. For LEFM, Equ. 2.22 transforms to:

$$G = \eta_{el} U_{el}/Bb \quad (2.23)$$

By expressing Equ. 2.23 in terms of Y factors or compliance, C ,

$$\eta_{el} = b/C(dC/da) \quad (2.24a)$$

$$\eta_{el} = bY_a^2 \int (Y_a^2 da) \quad (2.24b)$$

For elastic-plastic η factors, Turner (1984, 1980a) has expressed η_{pl} as:

$$\eta_{pl} = -b/U_{pl}(\partial U_{pl}/\partial a) h_{pl} \quad (2.25a)$$

where U_{pl} is the plastic component of the work done and for perfectly plastic conditions of limit load P_L ;

$$\eta_{pl} = -b/P_L(dP_L/da) \quad (2.25b)$$

Turner (1984) showed the variation of η factors with deformation for various crack depth ratios. For center cracked plates and three-point bend specimens ($S/W = 4$), $\eta_o(\text{overall}) \approx 2.0$ for $0.4 \leq a/W \leq 0.7$ but $\eta_o \approx \eta_{el} \neq 2.0$ for shallow notches.

Therefore with the knowledge of appropriate η factors and a measure of work done, the J integral can be evaluated using Equ. 2.22.

2.3.3.4 RELATIONSHIP BETWEEN J AND δ

Recalling Equ. 2.15,

$$G_C = m \sigma_y \delta_c$$

and using it for elastic-plastic conditions gives:

$$J_c = m \sigma_y \delta_c \quad (2.26)$$

Reported values of m lie between 1 and 3. Inconsistencies are likely to arise because of the rather vague definition of δ , assumptions on the rotational factor and other difficult-to-quantify items such as work hardening and degree of constraint.

2.4 RESISTANCE CURVES (R-CURVES)

2.4.1 INTRODUCTION

In EPFM, the δ or J-based variation of toughness with crack extension is called a δ or J-resistance curve. In subsection 2.2.6, the variation of a material's "resistance" to fracture in LEFM is discussed and representation of such behaviour, in LEFM, is usually made in the form of a resistance curve.

In recent years, the subject of resistance curves has received considerable attention and some properties of such R-curves have been highlighted. Krafft et al (1961) reported that the toughness should increase with shear lip formation and hence crack growth. However, Garwood and Turner (1977a), Etemad (1983) and several other authors have

reported an increase in the measured "toughness" (J , δ) with crack extension even with partial or total absence of shearlips by sidegrooving. Possible explanations are: J , δ formulations do not properly characterise the toughness at a material after crack initiation, crack growth increases the intrinsic toughness of the material, see Gibson (1986). Again, work by Garwood (1977) and Willoughby (1979) have shown that their measure of intrinsic toughness, - crack opening displacement at the growing crack tip (CTOD) remains constant with crack growth. Though, authors like Rice and Sorensen (1978) Shoji (1981), and Chipperfield (1977) have worked on the subject and have suggested reasons for this observation, the general understanding is that CTOD is a measure of local separation toughness whereas J is a measure of the overall structural resistance. In the following pages, a brief review of these properties shall be made.

2.4.2 EXPERIMENTAL DETERMINATION OF J AND δ R CURVES

Though a more detailed review of test techniques is made in chapter three, an introduction to the subject is made here. Certainly, the values of J , δ and crack length are dependent on the means by which these quantities are obtained. Presently, existing guidelines, British standard - BS 5762 (1979) and the American standard - ASTM E813 (1981) are only primarily concerned with critical or initiation toughness values of δ and J respectively.

The numerous options for obtaining δ , J and crack length, (discussed in chapter three) are investigated by different authors with the aim of identifying which method is easiest to implement without loss of accuracy. The variation of techniques is primarily attributed to crack length measurements. The J value is primarily obtained, simply by a measurement of U , the work done. The δ value is a bit more uncertain, with techniques such as, infiltration and crack mouth opening as contenders for the δ measurement. For a schematic of the categories, see section 3.1.

2.4.3 EVALUATION METHODS

2.4.3.1 J FORMULATIONS

The ambiguity associated with measurement is discussed in subsection 2.3.2.2. The rest of this review will be primarily about some properties of the J - R curve which have a direct bearing on the main theme of this dissertation.

The impetus for the study of J-R curves has largely been brought about by the lack of uniqueness in such resistance curves.

Recalling Equ. 2.22,

$$J = \eta U / Bb$$

The above equation evaluates the J integral without correction for crack growth. Thus, to distinguish this formulation from other forms (shown below) a subscript o is added. Therefore Equ. 2.22 becomes

$$J_o = \eta (U + \Delta U) / Bb_o \quad (2.27)$$

where $b_o = W - a$, and ΔU is the additional work done after U. A few of the alternative definitions are mentioned below.

1. The formulation by Ernst et al (1981a) has been accepted by the ASTM Standard E813 (1981) as one form of J-correction for crack growth. It is given as:

$$J_r = J_{i+1} = [J_i + (\eta/b)_i U_{i,i+1}] [1 - (\gamma/b)_i (a_{i+1} - a_i)] \quad (2.28)$$

where $\gamma = 1 + 0.76(b/W)$ for compact tension specimens and $\gamma = 1$ for deep notch three-point bend specimens. The stepwise calculation of J_{i+1} is done with reference to the previous J_i^{th} step. $U_{i,i+1}$ is the work done between the $i+1^{\text{th}}$ and the i^{th} step.

2. Garwood (1977) used NLE implications to account for crack growth. For deep notch, three-point bend specimens, he stated ,

$$J_{\text{TPT}} = J_i [(W - a_{i+1}) / (W - a_i)] + 2\Delta U / [B(W - a_i)] \quad (2.29)$$

The subscript TPT stands for "Three Parameter Technique" - load, displacement and crack growth.

3. Ernst (1983) presented another J formulation based on "deformation theory" J, (J_d) as:

$$J_d = J_m + \int [\partial(J_d - G) / \partial a] |_{q_{pl}} da \quad (2.30)$$

Gibson (1986) expressed Equ. 2.30 in an incremental form for three-point bend and compact tension specimens as shown below in Equ. 2.30a.

$$J_m = (J_m)_i + (\eta/b)_i (U_{i+1} - U_i)/B \quad (2.30a)$$

4. Turner (1984) introduced J_a to model as close as possible, elastic-plastic-elastic (EPE) behaviour.

$$J_a = J_o [1 + N(\Delta a/b_f) g(\eta)_o] \quad (2.31)$$

where N = work hardening exponent in $\sigma = \epsilon^N$ and $g(\eta) = 1 - \eta + (b/\eta)(d\eta/da)$

Equations 2.28 to 2.31 are just a few of the formulations by various authors. They all attempt to model the J-R curve as close as possible to the "real" material behaviour. As seen in Fig 2.15 (Turner (1984)), in spite of these corrections, discrepancies still occur for large amounts of crack growth.

2.4.3.2 SEQUENTIAL AND INCREMENTAL CONSIDERATIONS

To add to the already complex picture of J-R curves, it is also possible to obtain different J-R curves even if test method and formulation are fixed. Different interpretations of Eqs.2.28 through to 2.31 can lead to slightly different J values.

Incremental analysis is performed with values of b and η updated to present stage of evaluation. It appears that J_{i+1} (Equ. 2.28) and J_{TPT} (Equ.2.29) are inherently incremental, but for J_a and J_m the situation is not very clear. If referred to the original step (i.e. b , η are not updated) then the analysis is termed sequential. The difference is shown clearly by Turner (1984) or in fig 2.15.

2.4.4 J CONTROLLED CRACK GROWTH

The variation of J values, especially at large amounts of crack growth led Hutchinson and Paris (1979) to impose certain limitations for the validity of J as a characterising parameter after initiation of crack growth.

Crack growth involves some elastic unloading at the crack tip and non-proportional plastic deformation close to the crack tip. The validity of the J integral is therefore not

fully justified for characterising such crack tip conditions since the integral is modelled from non-linear elastic considerations. Theoretical and numerical analysis by Hutchinson and Paris (1979) and Shih et al (1979) show that for comparatively small amounts of crack growth, a region of proportional loading that is controlled by deformation theory singularity fields exists, see fig 2.16. They stated one condition for crack growth as:-

$$\Delta a \ll R \quad (2.32)$$

where R is the region described as shown in fig.2.16.

Based on the HRR field strain solutions (Equ. 2.21b), another requirement of proportionality of strain increments in the singularity field is given in terms of ω as:-

$$\omega = (b/J) (dJ/da) \gg 1 \quad (2.33)$$

Shih et al (1979) calculated near field values of the J integral for A533B steel, bend geometry using finite element computation. They found a limit for J controlled crack growth to be at:-

$$\Delta a/b_0 = 0.06 \quad (2.34)$$

So, Hutchinson and Paris (1979) claim that for crack growths up to about 6% of the original ligament, the conditions at the crack tip is properly characterised by J and therefore little variation in the J value can be expected.

2.4.5 DROOPING J-R CURVES

Drooping J-R curves have been reported by Etemad et al(1985a) (1985b) and Davies et al (1983). This is considered possible if analysis is based on internal energy, dw, and not applied work dU.

Introducing an energy balance relationship, see Turner (1984)

$$dw = dU - BJda \quad (2.36)$$

From Equ. 2.27, J_0 is always observed to rise once U is always added on. However, J_{i+1} and J_{TPT} values are not unlikely to droop since it allows for crack growth, so does Equ. (2.36) above. That is, if the term BJda is dominant enough to ensure:-

$$dw < 0$$

then such crack-growth-corrected J values can indeed decrease after an initial rise.

Etemad (1985a) and Turner (1980b) have identified the following conditions for a dropping or rising R curve as:-

$$(b/J) (dJ/da) > 1 + (b/\eta)(d\eta/da) : \text{ for a rising curve.}$$

and therefore,

$$(b/J) (dJ/da) < 1 + (b/\eta)(d\eta/da) : \text{ for a drooping curve.}$$

2.4.6 NORMALISATION OF J-R CURVES

Etemad and Turner (1985c) have reported significant reductions in J-R curve discrepancies if the Δa axis is normalised with an appropriate parameter, c . It must be noted that the discrepancy referred to here is not that due to formulations but that due to variation of geometry, size and so on. Etemad (1983) and others have reported significant differences in J-R curves even when a constant J formula is applied to physically different test configurations. The non-dimensionalising term, c , is envisaged to restrict the plastic work done of which J is some measure. Examples of such identified parameters such as thickness, B , and ligament, b , are shown to work satisfactorily in Figs 2.17 a and b (Reproduced from Etemad and Turner (1985c)). It is notionally imagined that if such parameters are identified, chances of an eventual unique R curve are improved.

Garwood et al (1978) suggested that the overall crack growth resistance depends on two components . These are at the edges where the the shearlips are formed and the other is the flat fracture portion at the center. Since the sizes of these two fracture types depend on B and b , a common ground can immediately identified between this study and the analysis by Etemad and Turner (1985c) mentioned above. Green and Knott (1975) modified Garwood's(1976) "split" J formulation to give:-

$$J_r = J_{flat}[(B - 2S)/B] + E_v 2S^2/B \quad (2.36a)$$

where J_{flat} is the J value for flat fracture region (obtained from a sidegrooved specimen test), S is the width of the shearlip and E_v is the energy absorbed per unit volume of shearlip.

2.5 DUCTILE TEARING INSTABILITY

2.5.1 INTRODUCTION

There are two known types of unstable tearing behaviour:

1. Cleavage failure - Brittle classification.
2. Failure by fibrous tearing or microvoid-coalescence - Ductile Classification.

This subsection is concerned with the second type of failure.

2.5.2 DUCTILE INSTABILITY THEORIES

Two methodologies presented here are the T approach and the I approach. The tearing modulus (T) approach proposed by Paris et al (1979) was originally based on the equality of change of plastic displacement and elastic contraction. The instability criterion by this T approach therefore states that instability will occur when:-

$$dJ/da|_{app} > dJ/da|_{mat} \quad (2.37)$$

$$\text{or } T_{app} = (E/\sigma_y^2) (dJ/da)|_{app} > (E/\sigma_y^2) (dJ/da)|_{mat} = T_{mat}$$

The original form of T_{app} put forward by Paris et al (1979) and Hutchinson (1979) was:-

$$T_{app} = 2b^2 S_e / W^3 - JE/\sigma_y^2 b \quad (2.38)$$

where S_e is the effective span to allow for overall system compliance.

$$S_e = S(1 + C_{me}/C_u)$$

$$C_u = \text{Un-notched compliance}$$

$$C_{me} = \text{Effective machine compliance}$$

$$C_m = \text{Machine compliance}$$

Turner (1984) took C_{me} as αC_m where $\alpha = 2$.

Other formulations for T_{app} and T_{mat} proposed by Ernst et al (1981a, 1981b) are:-

$$T_{app} = E/\sigma_y^2 \{ (\partial J/\partial a)|_{\Delta} + (\partial J/\partial \Delta)^2|_a [1/((\partial P/\partial \Delta)|_a + k_{me})] \} \quad (2.39a)$$

$$T_{mat} = E/\sigma_y^2 \{ (\partial J/\partial a)|_{\Delta} + (\partial J/\partial \Delta)^2|_a [1/((\partial P/\partial \Delta)|_a - (dP/d\Delta))] \} \quad (2.39b)$$

where k_{me} is stiffness of system (the exact definition depending on the details of model used). Eqs. 2.39a and 2.39b can be combined to yield:-

$$T_{app} = \frac{E/\sigma_y^2 \left\{ \left(\frac{\partial J}{\partial a} \right) \Big|_{\Delta} + \left(\frac{\partial J}{\partial \Delta} \right)^2 \Big|_a \left[\frac{1}{k_{me} + dP/d\Delta + \left(\frac{\partial J}{\partial \Delta} \right)^2} \right] \right\}}{(\sigma_Y^2/E) T_{mat} - \left(\frac{\partial J}{\partial a} \right) \Big|_{\Delta}} \quad (2.40)$$

All the information in equ. 2.40 can be obtained from a single load (P)-displacement (Δ) plot, all in terms of current values of J, b, dJ/da , $dP/d\Delta$ and P (note that $dP/d\Delta$ is negative after maximum load evaluated per unit thickness). $\left(\frac{\partial J}{\partial a} \right) \Big|_{\Delta}$ and $\left(\frac{\partial J}{\partial \Delta} \right) \Big|_a$ are calibration functions, but may be approximated as follows, see MaCabe and Ernst (1981):-

$$\left(\frac{\partial J}{\partial a} \right) \Big|_{\Delta} = -\gamma J/b \quad \text{and} \quad \left(\frac{\partial J}{\partial \Delta} \right) \Big|_a = \eta P/b$$

MaCabe and Ernst (1981) defined k_{me} as the machine stiffness plus the stiffness of the unnotched specimen. $(\sigma_Y^2/E) T_{mat} = dJ/da$ is obtained from a J-R curve of the material being investigated. $dP/d\Delta$ is taken off the decreasing load portion of the load-displacement plot.

The I approach is based on energy balance considerations. Turner (1984) proposed that ductile instability would occur if the elastic energy rate available for fracture in the elastic plastic system is greater than the energy absorption rate required for fracture. The most recent criterion proposed by Turner (1984) is that ductile instability will occur if:

$$I > (b/\eta)(dJ_I/da) - (J_I/\eta)(f(\eta)) \quad (2.41)$$

where $f(\eta) = 1 + (b/\eta)(dJ_I/da)$. η factors for various crack ratios for three-point bend specimens are given by Turner (1984).

I is the elastic energy release rate under elastic-plastic conditions and is defined as:-

$$I = G[2(\eta_o/\eta_{el})_s - 1] \quad (2.42)$$

where G is the linear elastic energy release rate.

$$\eta_{el,s} = \eta_{el}[1/(1 + \phi)]$$

where $\phi = C_m/C_{t,p}$; subscripts "el" and "s" are used to denote elastic and system respectively. $C_{t,p}$ is the test piece compliance evaluated from elastic slope at the loading line from the load-displacement plot. (corrected for extraneous displacements such as indentations).

And finally,

$$\eta_{o,s} = [\eta_{el}(A) + \eta_{pl}(X)] / (A + X)$$

where $X = U_{pl}/U_{el}$ and $A = \eta_{el}/\eta_{el,s}$

2.5.3 APPLICATION OF THE T AND I THEORIES

By observing the variables in Equ. 2.40 through to Equ. 2.41, it becomes evident that application is only possible in cases that have:

1. Load-displacement trace : For P and Δ and therefore G values with known Y functions, $\partial P/\partial \Delta$ for T_{app} calculations.
2. J-R curve : For J, and dJ/da for both T_{app} and $f(\eta)$ calculations.
3. η factors : For the energy - based I theory.
4. Compliance measurements of testing machines and test specimen.

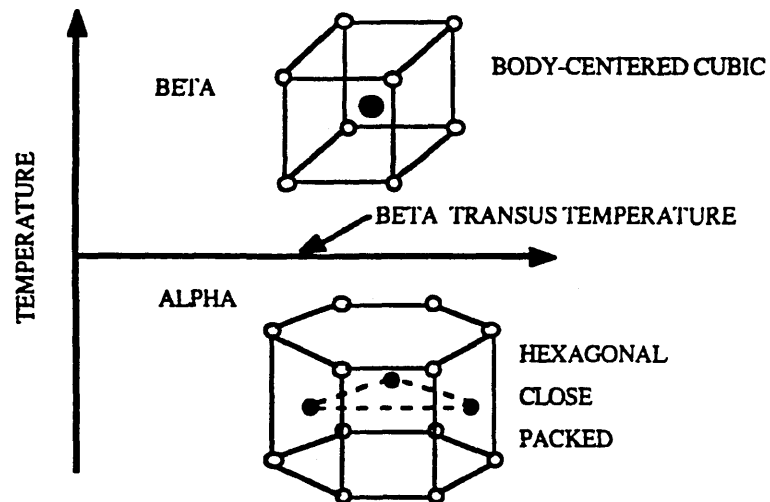


FIG. 2.1 Relationship between temperature and two possible crystallographic forms of titanium.

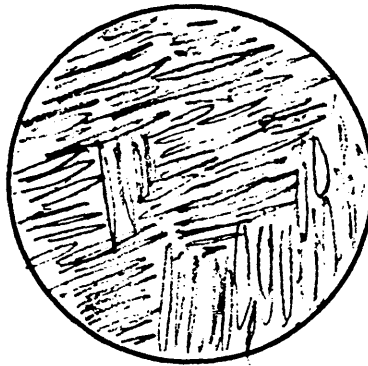


FIG. 2.2 Typical acicular alpha grains

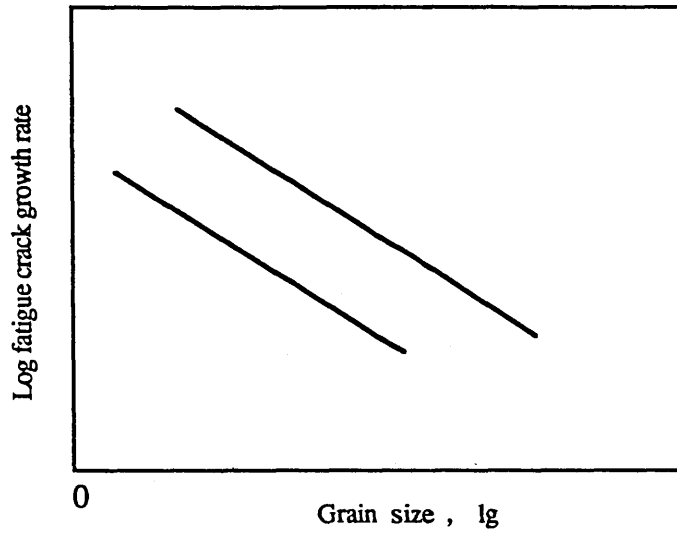


FIG. 2.3 Relationship between grain size and fatigue crack growth rate (Yoder et al (1978))

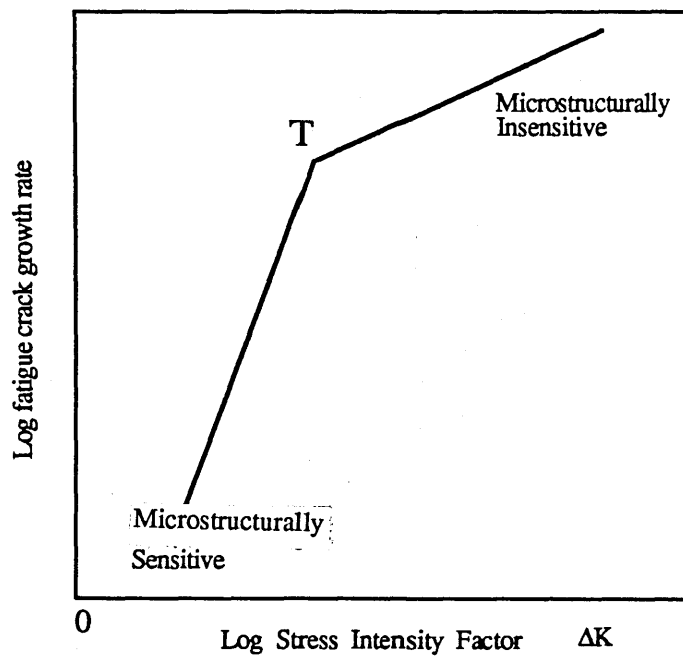


FIG. 2.4 Bilinear mode of fatigue crack growth rate. 'T' represents point of transition from structure sensitive (SS) to structure insensitive (SI) mode of crack growth (Yoder et al (1978)).

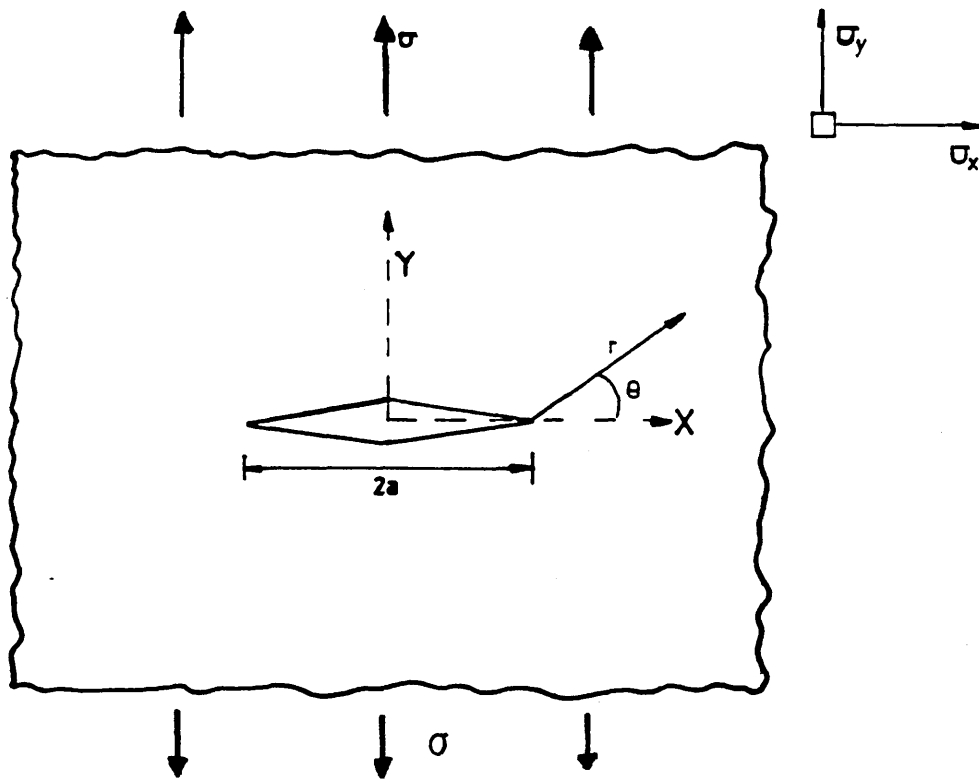


FIG. 2.5 Crack in an infinite plate.

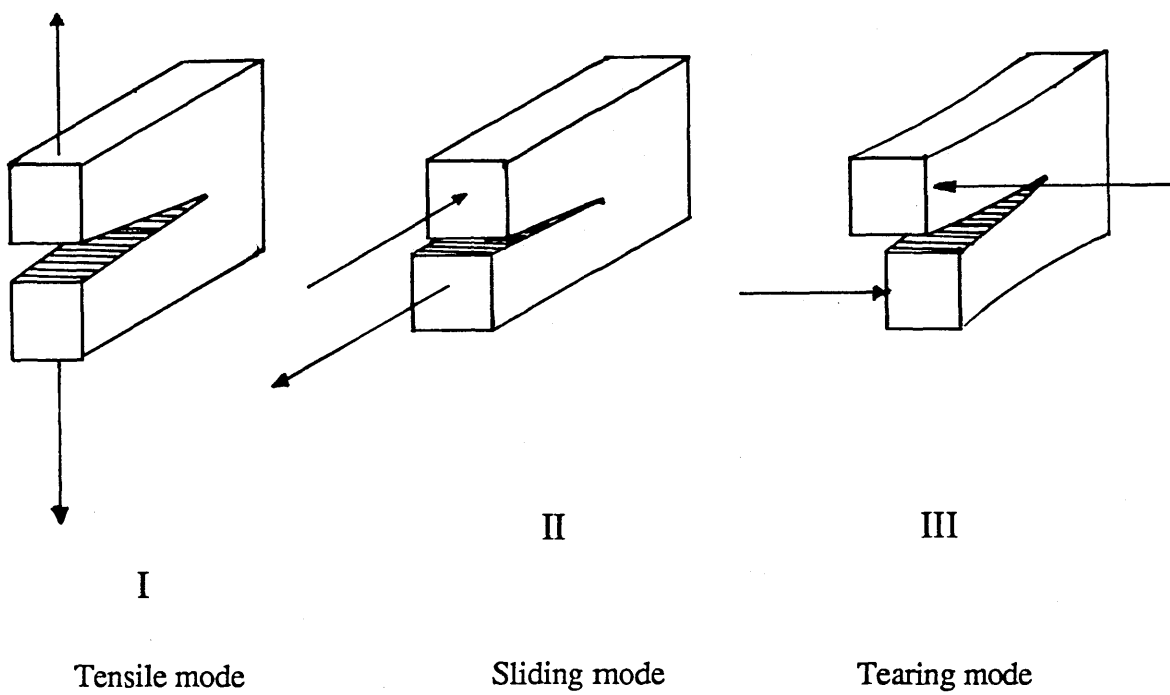


FIG. 2.6 Three modes of crack opening

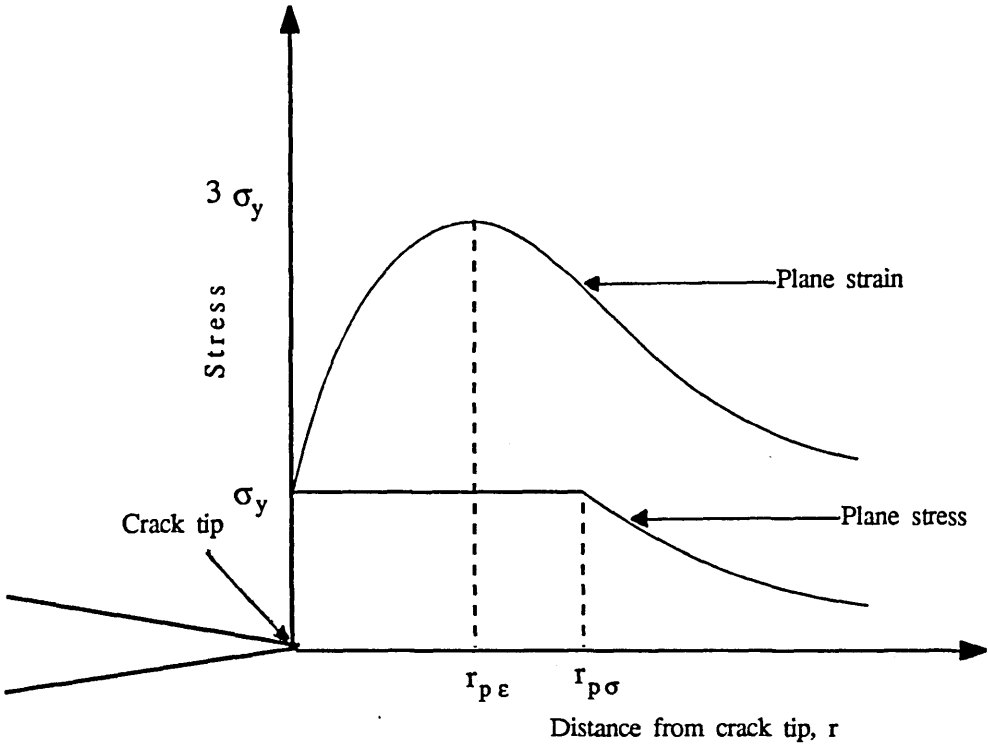


FIG. 2.7 Stress and plastic zone size variations for plane stress and plane strain conditions.

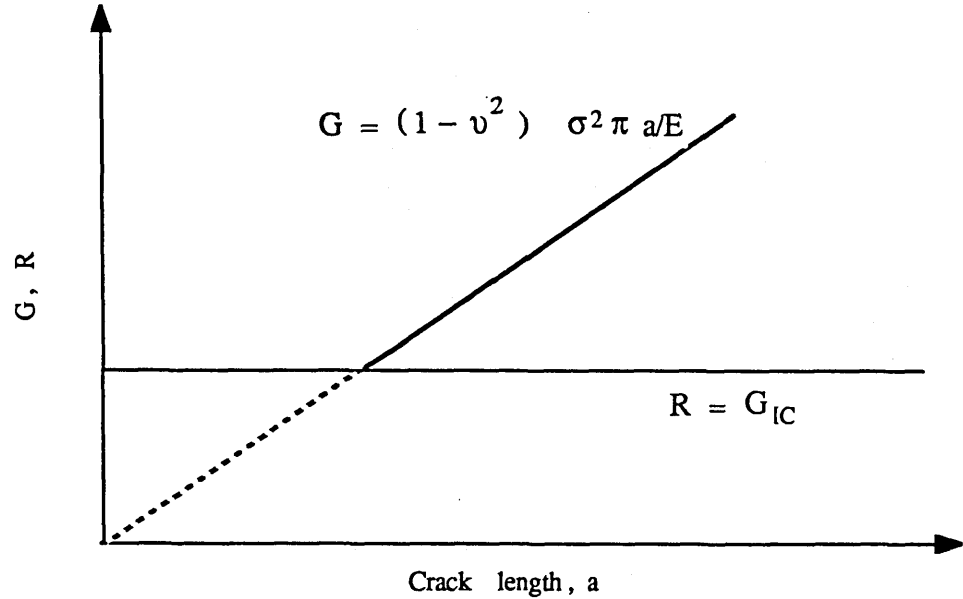


FIG. 2.8 Plane strain resistance curve showing a constant R value.

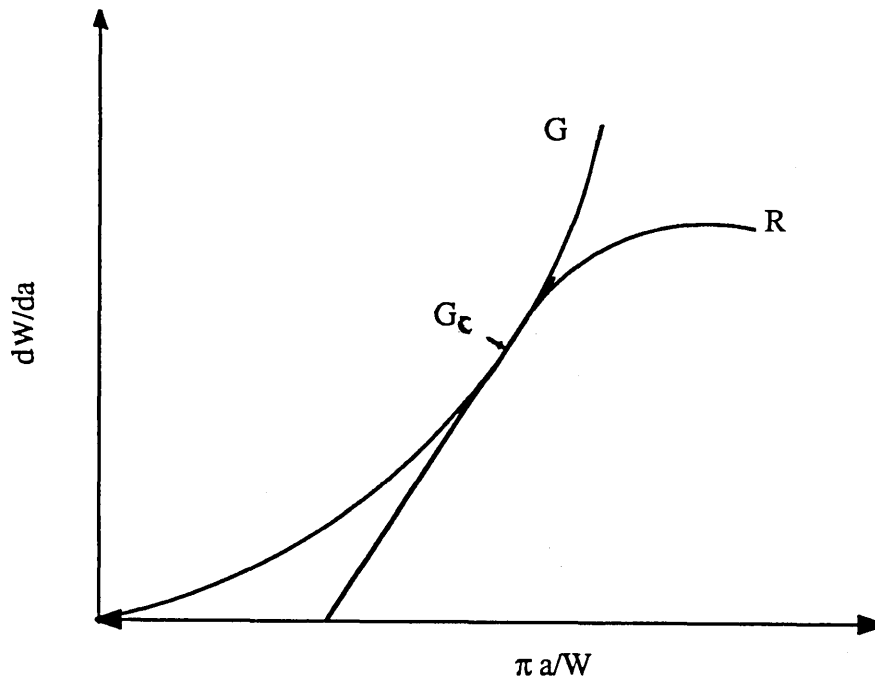


FIG. 2.9 Krafft's original concept of instability occurring at G_c

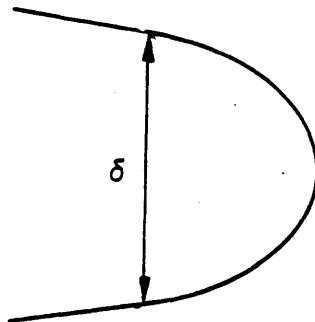


FIG. 2.10 Schematic showing a definition of COD, δ

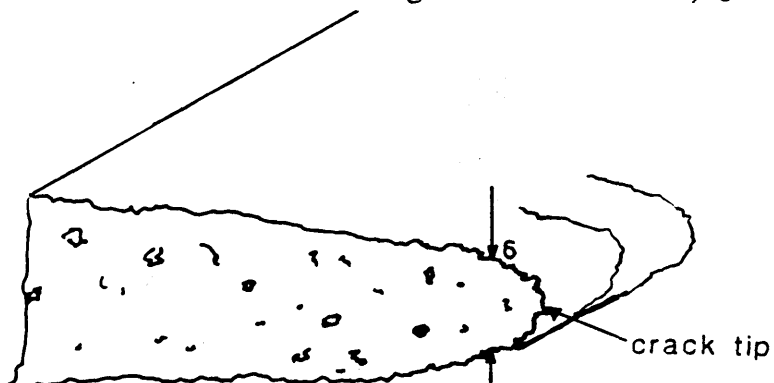


FIG. 2.11 Schematic of a sectioned rubber cast

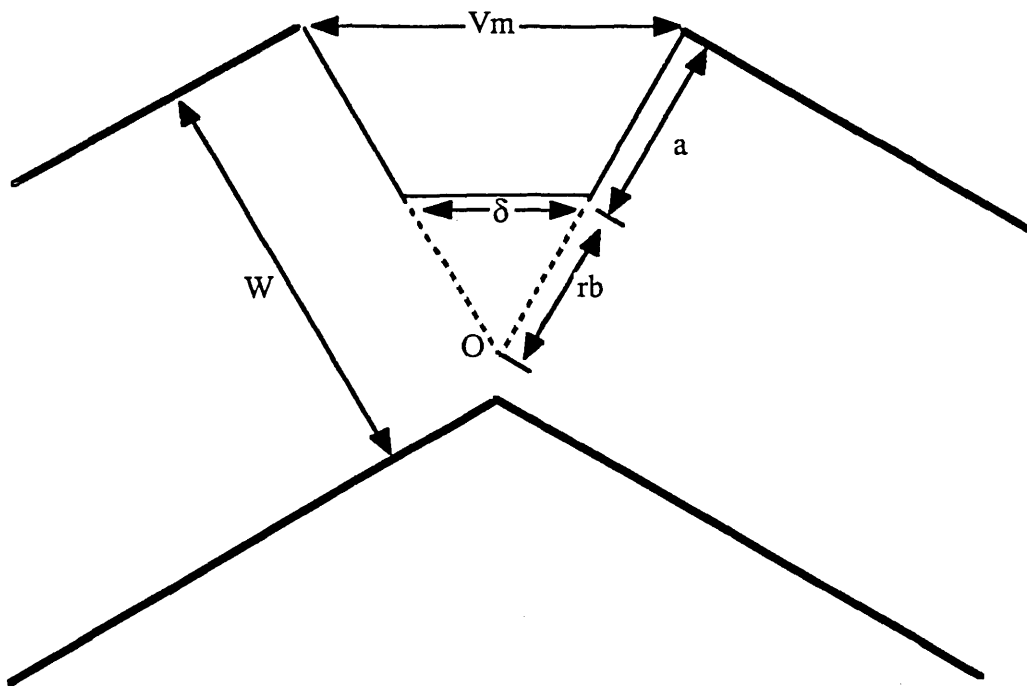


FIG. 2.12 Relationship between crack mouth opening and δ showing position of plastic hinge at O .

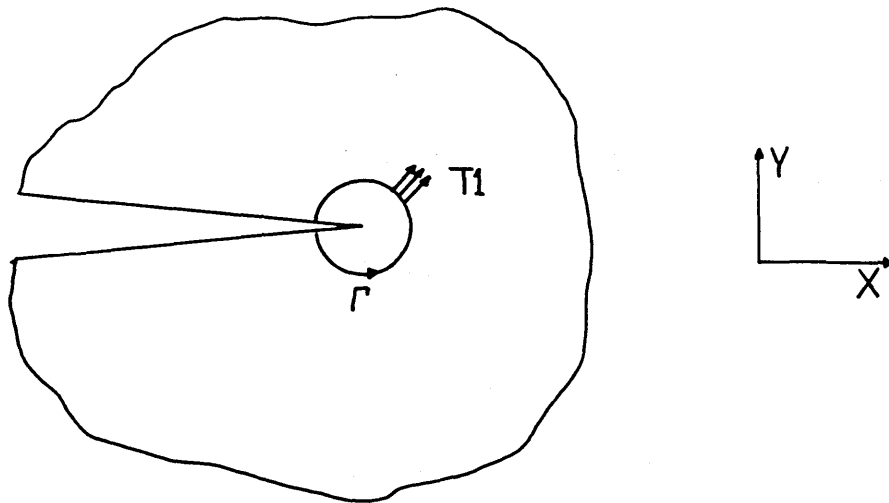


FIG. 2.13 Contour path going anticlockwise from one crack face to another.

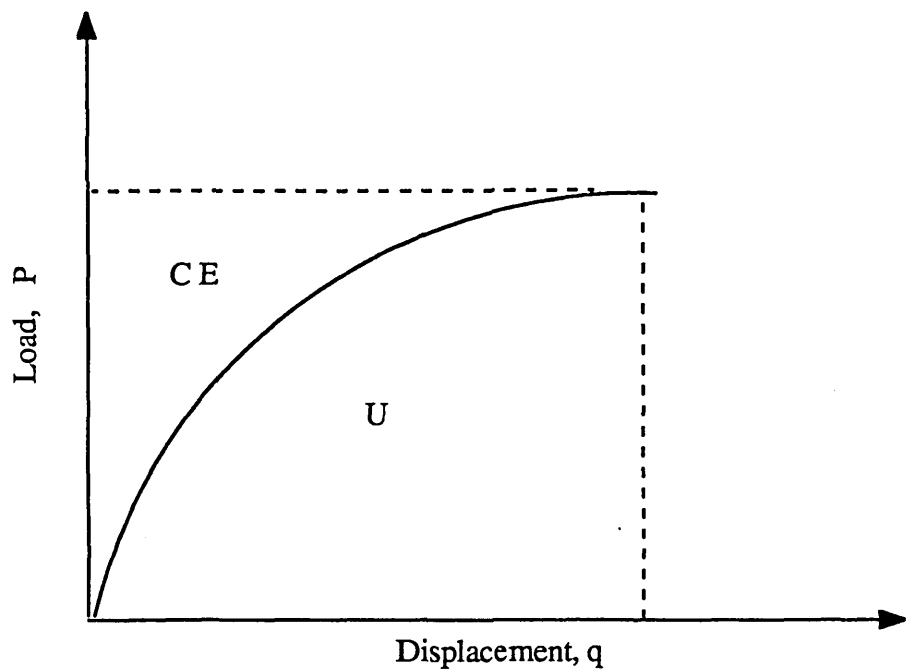


FIG. 2.14 Work, U , and complementary work, CE , measurements for J calculations.

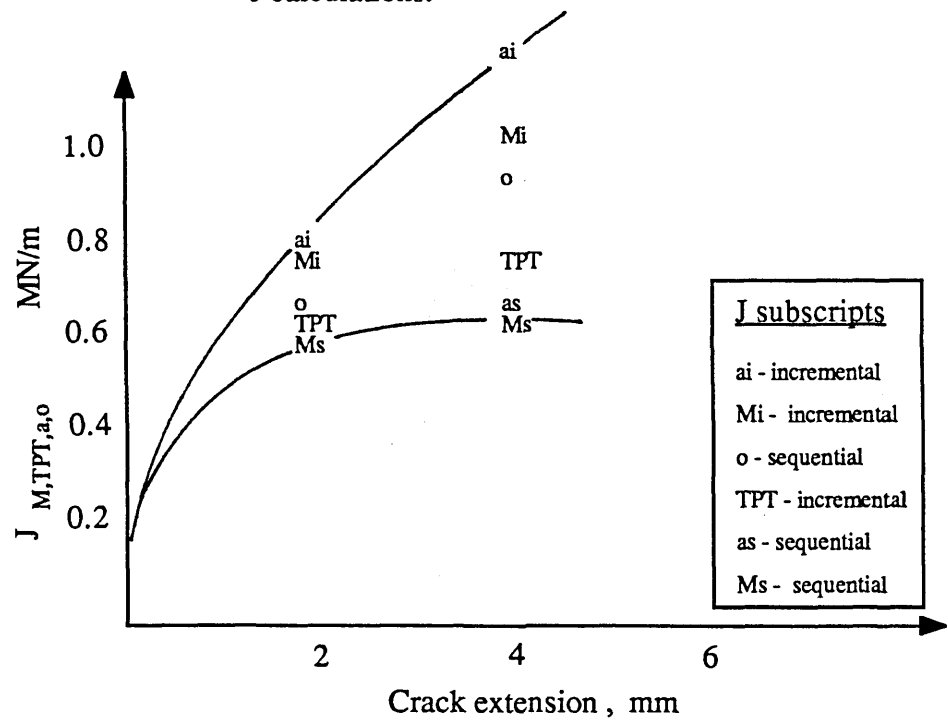


FIG.2.15 J - R curves for three-point bend HY130 specimens, showing differences in J formulation and method of evaluation.

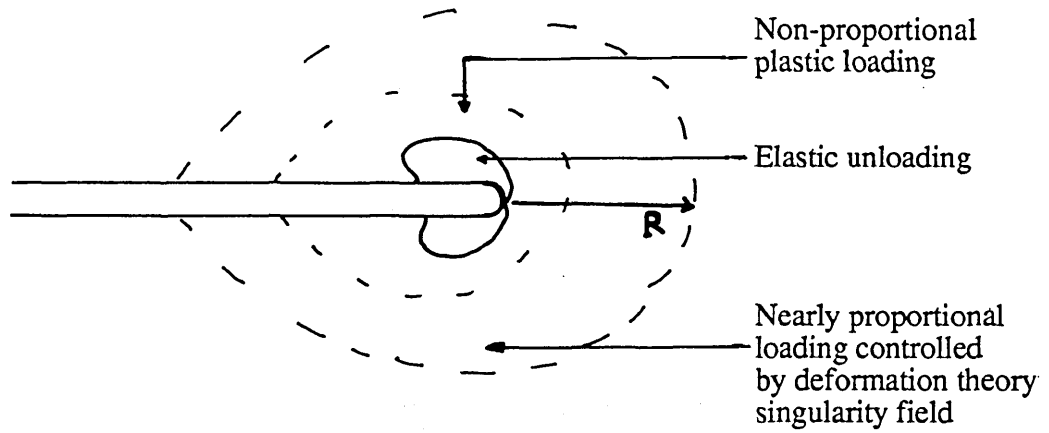


FIG. 2.16 Schematic of crack tip conditions for J-controlled crack growth.

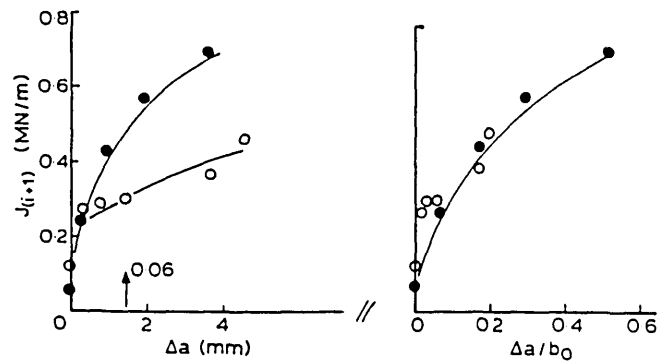


FIG. 2.17a Effect of normalising the abscissa for different widths
 $a/W=0.5$, $W=24\text{mm}$, $S/W=4$, (HY130)
 ○ - 12.5mm ● - 50mm

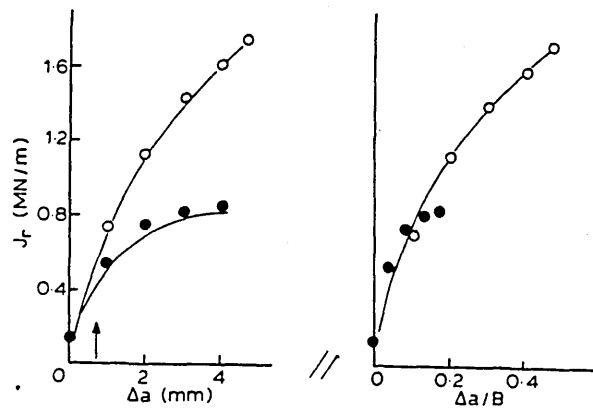


FIG. 2.17b Effect of normalising the abscissa for different thicknesses
 $a/W=0.5$, $W=24\text{mm}$, $S/W=4.0$, (En32)
 ○ - 12.5mm ● - 50mm

CHAPTER 3

REVIEW OF TESTING TECHNIQUES

3.1 CLASSIFICATION OF TECHNIQUES

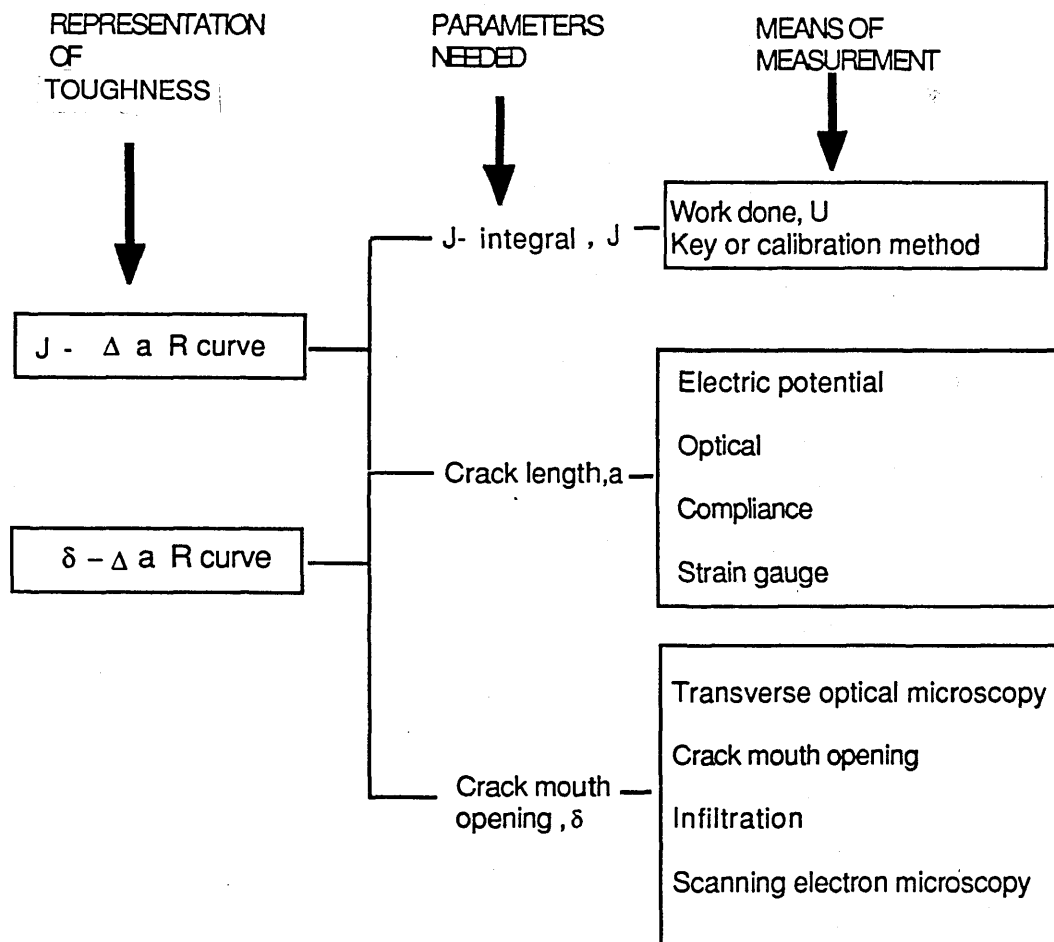


Fig 3.1 Categories of some measuring techniques for J, δ and Δa .

3.2 CRACK LENGTH PREDICTIONS

To obtain a J-R curve, though the J value is evaluated in a relatively straightforward manner (i.e. $\eta U/Bb$), there are however, quite diverse methods for the more tedious task of crack length estimation.

The measurement of δ (COD) is not quite as easy as J. Some of the measurement methods shown in Fig 3.1 are discussed in subsection 2.3.2.2. The rest of this chapter makes a comparative study of the other various means of measurement shown in fig. 3.1

3.2.1 ELECTRIC POTENTIAL METHOD

This method utilises the principle that the voltage applied across a specimen alters as a crack length within the voltage field changes. In principle both direct (DC) and alternating (AC) current are applicable. However, DC methods are preferred to avoid any detrimental effect which may arise due to time-sensitive current gradients.

There are basically two methods of potential drop applications, be it in AC or DC - by calibration or by analytical evaluation of the potential measurements.

Aronson and Ritchie (1979) and Clark and Knott (1975) showed that the sensitivity of the potential method depends strongly on the location of current application points and the potential probe positions. The calibration method entails the calibration of the measured normalised potential, V/V_0 , with the known crack depth ratio, a/w , for different probe positions as shown in a typical calibration plot in fig 3.2. Once a calibration curve has been obtained, a measured potential on an identical specimen and exact probe position can provide an estimate for the crack depth ratio, a/W .

One of the major drawbacks of this method of potential drop application, is the heavy dependence of voltage with probe positions, thus making reproducibility difficult.

The analytical evaluation method by Johnson (1965) is described by Schwalbe (1981) to have significant advantages over the calibration method. These include:

- i. Insensitivity to the position of the potential probes provided the probes are placed symmetrically about the crack.
- ii. Application is convenient as data reduction can be done by means of the analytical

relationship:

$$a = \frac{(2W/\pi) \cos^{-1} \left[\frac{\cosh(\pi Y/2W)}{\cosh \left[V/V_o \cosh^{-1} [\cosh(\pi Y/2W)/\cos(\pi a_o/2W)] \right]} \right]}{\cosh \left[V/V_o \cosh^{-1} [\cosh(\pi Y/2W)/\cos(\pi a_o/2W)] \right]} \quad (3.1)$$

where according to Johnson (1965), V_o is the potential at the initial crack length, a_o , specimen width is $2W$ for a center cracked plate and $2Y$ is potential measurement gauge span as shown in fig 3.3.

iii. Equ. 3.1 can be applied to CCP, CKS, three-point bend specimens and single-edge notch in tension specimens.

iv. Equ. 3.1 provides a calibration which is independent of material, specimen thickness and current.

More recently, Okumura et al (1981) and Gibson (1986a) have reported the reliable use of P.D. systems for determining resistance curves. Gibson's report differed from that of Okumura et al in that he corrected for the variation of the test material's electrical resistance with stress while Okumura et al did not. Also, on the determination of crack initiation, Okumura et al stated it occurred at the minimum P.D. signal. Gibson on the other hand, stated that it occurred at the onset of deviation from linearity of the P.D. - J plot. Gibson reported a good agreement between A.C. and D.C. potential drop methods in determining J-R curves.

Potential drop methods tend to be more popular with tension specimens which are relatively stiff when compared with bend specimens. Because of its reduced stiffness, the cracks in bend specimens are more reliably measured by the compliance method (discussed below).

3.2.2 OPTICAL METHOD

This is the most direct means by which the crack length is measured. Indeed, this method is used for the potential calibration curve discussed earlier and is also used in the ASTM E813 (1981) standard for J_{IC} tests.

For fatigue crack lengths, where the fatigue limit is more or less visible, an average of six to nine measurements across the crack face is made to allow for crack front

curvature, as shown in fig 3.4a. When stable ductile crack growth is measured, it is usually necessary to distinguish between this regime and any other regime. Since it is not clearly visible to the naked eye, the crack face is heat tinted, to distinguish one regime from the other. Heat tinting, as the name implies, means oxidising the crack face by heating, and because of the different surface morphologies, the colouring turns out different after the process. An averaging technique is also used to measure the crack growth as shown in fig. 3.4b. It must also be mentioned that such crack length averaging is done with a travelling optical microscope (TOM). Some of the drawbacks of this method are the time-consuming nature of the method and the measurement across the crack face is only possible when the test specimen is broken apart.

3.2.3 COMPLIANCE METHOD

As the crack grows so does the compliance of the test specimen. Therefore, analytical relationships can link one quantity to the other. Though this method is relatively simple to use with compliant (e.g., bend) specimens, much more accuracy would be required for stiff tension specimens, see Schwalbe (1981).

Saxena and Hudak (1978) expressed compact tension calibration functions by integrating the stress intensity factor expression with respect to a/W . Newman (1974, 1976) used boundary collocation techniques to predict deflections at various locations of bend type specimens. Saxena and Hudak (1978) fitted polynomials to these analyses. The one most common form for compact tension testing is the load-line compliance and it is given in the form below.

COMPACT TENSION (CKS) SPECIMEN

$$a/W = 1.000196 - 4.0632U_x + 11.242U_x^2 - 106.043U_x^3 + 464.33U_x^4 - 650.67U_x^5$$

$$U_x = 1/[\sqrt{(BE'\Delta/P)} + 1] \quad (3.2)$$

where $E' = E$ for plane stress and $E' = E/(1 - \nu^2)$ for plane strain.

And recently, Kapp et al (1985) presented similar compliance calibration functions for three-point bend specimens .

THREE POINT BEND SPECIMEN

Load-Line Compliance

$$a/W = 0.0997 - 0.516U_L + 2.85U_L^2 - 1.43U_L^3 \quad (3.3)$$

where,

$$U_L = \frac{1}{1 + \left[\frac{0.9875}{[(E'B\Delta_L/P(S/W)^2) - S/4(W)]^{1/2}} \right]}$$

Δ_L/P = Load line compliance.

Mouth Opening Compliance

$$a/W = -1.03U_M + 6.00U_M^2 - 6.37U_M^3 + 2.73U_M^4 - 0.321U_M^5 \quad (3.4)$$

where,

$$U_M = \frac{1}{1 + \left[\frac{3.95 (S/W)P/E'B\Delta_M}{\Delta_M} \right]^{1/2}}$$

Δ_M/P = Mouth-opening compliance.

Therefore, by measuring Δ/P , Δ_L/P or Δ_M/P and using the appropriate equation above, the crack length of the CKS or three point bend specimens can be predicted. Further subtleties of this method of crack length prediction are mentioned when the Unloading Compliance J-R curve testing is discussed below, in subsection 3.3.3.

3.2.4 STRAIN GAUGE METHOD

Successes in crack length monitoring using strain gauges have been reported by Dean and Richards (1979) and John (1983). This method can be applied to a wide range of geometries since each geometry has to be calibrated first.

Before calibration is done, the position of highest sensitivity is needed, only then can a strain gauge be attached at this position for calibration. This procedure was illustrated by John (1983) for externally flawed thick-walled cylinders. Fig 3.5 illustrates the strain distribution from which the greatest sensitivity position is determined to be at 22.5° from the crack. Fig 3.6 shows a calibration curve for strain gauges at 22.5° from

the crack and was used reliably for crack monitoring.

While this method was used for fatigue crack growth monitoring, where crack growths were relatively large, reliability for detecting small amounts of growth is questioned. Also, the need to calibrate each specimen type and the painstaking job of attaching the strain gauges rendered this method slightly unpopular.

3.3 EXPERIMENTAL DETERMINATION OF J - R CURVES

Three methods for obtaining J-R curves are introduced here, these are:-

1. Multi-specimen test method.
2. Key or Calibration Curve method.
3. Unloading Compliance method.

A comparative study of these methods is made after the experimental details and some results are presented in chapters four and five respectively.

3.3.1 MULTI-SPECIMEN TEST METHOD

This is the method recommended in the ASTM E813 (1981) for obtaining J_{IC} (a critical J in opening mode I, (see fig 2.6)) for both CKS and three-point bend specimens. Multiple specimens are used (about six to eight). These are precracked to the same crack length, of nominally about $a_0/W = 0.5$. Each specimen is then loaded to different levels of deformation with the expectation that different crack lengths will be obtained. This procedure is illustrated in fig 3.7a. The specimens are then broken open and crack extension measured as described in subsection 3.2.2. The value of J is obtained by any of the formulations given in Equ. 2.27 through to Equ. 2.31, using the value of work done, $U_1, U_2 - U_8$, obtained for each specimen. Equations 2.27 and 2.28 are recommended in the ASTM Standard E813 (1981). Therefore, having measured the crack length and evaluated the J value, a J-R curve can be plotted as shown in fig 3.7b.

3.3.2 KEY-CURVE OR CALIBRATION METHOD

3.3.2.1 INTRODUCTION AND THEORY

The Key-Curve method described here is an experimental technique for developing a continuous J-R curve from load-displacement records alone. Ernst et al (1979), who developed this technique, realised that for simple test specimen geometries, the load-displacement relationship must have the form:-

$$PW/Bb^2 = F_1 (\Delta, a \text{ and material properties.}) \quad (3.5)$$

Using deformation plasticity theory and rewriting Eqs. 2.18a, b,

$$J = -1/W \int_{\Delta}^{\Delta} \partial P / \partial (a/W) \big|_{\Delta} d\Delta = 1/W \int_P^P \partial P / \partial (a/W) \big|_P dP \quad (3.6)$$

Also, assuming J and P to be a function of displacement (Δ) and crack length, a, and that total derivatives of J and P exist, then,

$$dJ = \partial J / \partial \Delta (d\Delta) + \partial J / \partial a (da) \quad (3.7)$$

$$dP = \partial P / \partial \Delta (d\Delta) + \partial P / \partial a (da)$$

Combining Eqs. (3.5) (3.6) and (3.7) it can be shown that,

$$dJ = [(2b/W)F_1 - (b/W)^2(\partial F_1 / \partial (a/W))] d\Delta + [-\int_{\Delta}^{\Delta} 2/W F_1 d\Delta + \int_{\Delta}^{\Delta} 4b/W^2(\partial F_1 / \partial (a/W)) - \int_{\Delta}^{\Delta} (b^2/W^3)(\partial^2 F_1 / \partial (a/W)^2) d\Delta] da \quad (3.8)$$

and

$$da = \frac{[dP - (b/W)^2 \partial F_1 / \partial (\Delta/W) d\Delta] W}{(b^2/W)(\partial F_1 / \partial (a/W)) - 2bF_1} \quad (3.9)$$

Therefore if values for $\partial F_1 / \partial (\Delta/W)$ and $\partial F_1 / \partial (a/W)$ are available, then dJ and da values can be evaluated from Eqs. (3.8) and (3.9) respectively. J and Δa values can then be found by the following operation:-

$$J = \sum_{1}^n dJ$$

$$\Delta a = \sum_{1}^n da \quad (3.10)$$

3.3.2.2 EXPERIMENTAL DETAILS

Ernst et al(1979) attempted this method with small one inch thick compact-tension specimens. About eight of these were precracked to different crack lengths between $a/W=0.5$ and 0.9 . These specimens were then individually loaded to about maximum load. The data points were then digitally recorded for main-frame-computer processing. After the loading test, each specimen was broken open and crack length (assuming no

crack growth) measured as described in subsection 3.2.2. The load was normalised to F_1 and the displacement normalised to Δ/W . Slopes from this plot of (i.e., $F_1 - \Delta/W$) would provide $\partial F_1 / \partial (\Delta/W)$ values and a plot of F_1 against a/W for different levels of Δ/W would give values for $\partial F_1 / \partial (a/W)$. Therefore, having obtained the above parameters, dJ and da from Eqs. 3.8 and 3.9, can in theory, be evaluated.

3.3.3 UNLOADING COMPLIANCE PROCEDURE

The concept of this method, introduced in subsection 3.2.3 provides the groundwork for its application in obtaining a J-R curve with a single specimen. This is done by obtaining crack growth estimates from successive partial unloadings during the loading history of the test specimen. By integrating up to the point of unloading, the work done, U , can be obtained, from which J values can be calculated, hence a J-R curve can be generated. Another advantage of this method is that it is entirely computer-interactive; a computer application has been reported by Joyce and Gudas (1979).

3.3.3.1 APPLICATION OF THE METHOD

In theory, if any test piece has a measurable compliance, analytical expressions like Eqs. 3.2 to 3.4 can be used. The review in this chapter is however restricted to the two specimen types directly relevant to this dissertation, namely compact tension (CKS) and three-point bend specimens. Some possible sources of error inherent with this method are now given below.

- a. Effective Young's modulus: Not knowing the stress state at the crack tip, the value of E as shown in Equ. 3.2 is uncertain, Kapp (1986) proposed an effective E' . This is obtained by simply calculating E from Eqs. 3.2 or 3.3, depending on specimen type, after determining a/W and Δ/P . This ensures an approximate E' value for subsequent crack length predictions during unloading.
- b. Care should be taken when the elastic slope is being evaluated. As seen in the enlargement of fig 3.8, "straight" lines are not exactly straight at the beginning of the unloading and the same is true for large amounts of unloading, where the unloading is no longer completely elastic. Usually about 10% - 25% unloading would provide reasonable linearity for slope evaluation.
- c. Application of this method to the three-point bend specimen appears more difficult than the compact tension specimen because of extraneous (e.g. indentation) displacements that occur primarily in the former specimen. Steenkamp (1985)

attempted to correct for this and other discrepancies in the form:-

$$C_c = C / (FC \cdot FD \cdot FR \cdot FI.) \quad (3.11)$$

where, FC - Crack front curvature correction factor
 FD - Specimen deformation correction factor
 FR - Outer roller correction factor
 FI - Indentation correction factor
 C - Measured compliance
 C_c - Corrected compliance

Steenkamp (1985) based this study on three-point bend specimens using the compliance expression by Tada et al (1973).

- d. Crack tunnelling occurs in non-sidegrooved specimens and though the compliance measured is the "effective" for the given curvature, it usually underestimates the crack length obtained by the eight point average described in ASTM E813 (1981) standard, see Joyce and Gudas(1979).
- e. With both CKS and three-point bend tests, friction should be limited as much as possible. For load-line deflections in CKS specimens and mouth-opening measurements in three-point bend specimen tests, the knife edge should be very sharp. This is essential to facilitate minimum area of contact and hence friction, between the displacement measuring device (usually a clip-gauge) and the knife-edge. As described in the ASTM E(813) (1981) standard, the desired type of contact is shown in fig.3.9. Friction in the CKS specimen is also reduced at the pin-shackle attachment, where a small hardened flat surface is made on the test shackle to allow for small linear movements because of specimen rotation. This is mentioned in the ASTM E(813) (1981) standard and is schematically shown in fig.3.10. Similarly, friction in three-point bend specimens occurs at points of contact, hence these points, called rollers are allowed to move during the test, see fig.3.11.

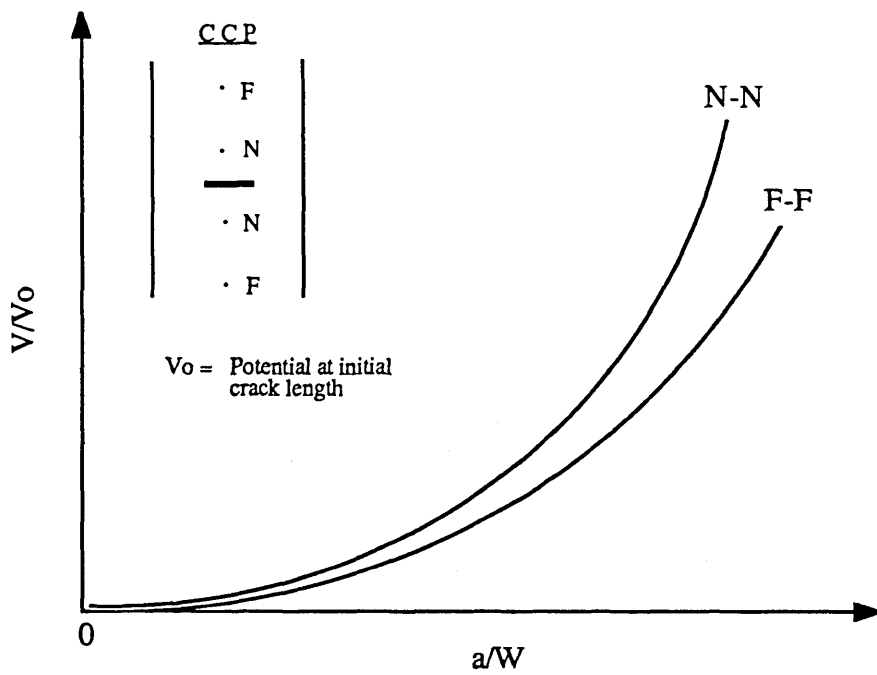


FIG. 3.2 Typical sensitivity variation of probe positions , N-N and F-F on a center-cracked plate, CCP (Schwalbe (1981)).

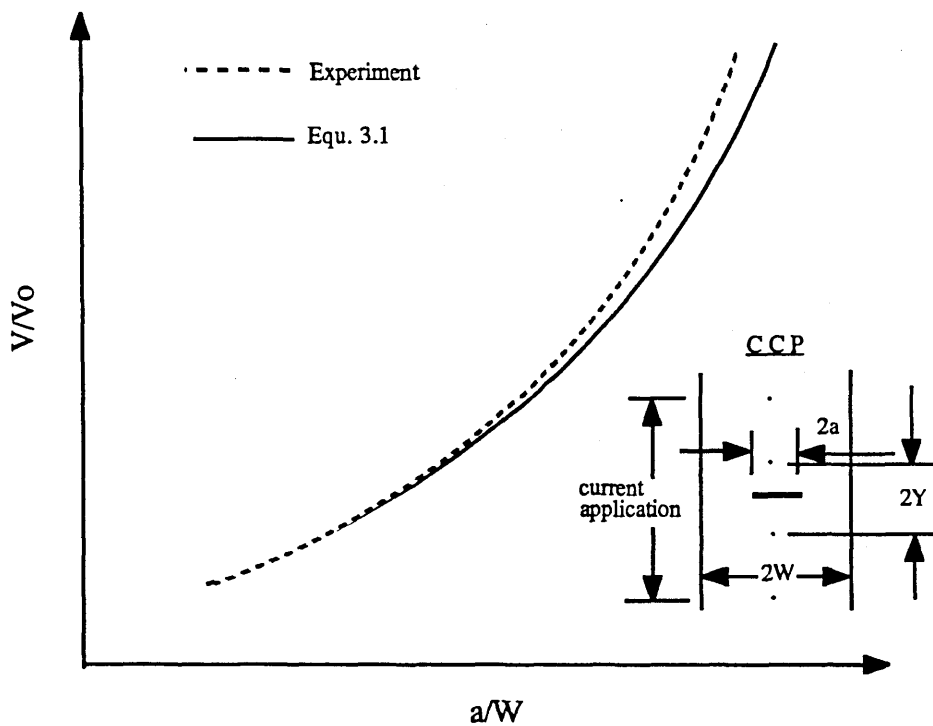


FIG. 3.3 Experimental and theoretical potential calibration curve for a CCP (Schwalbe (1981))

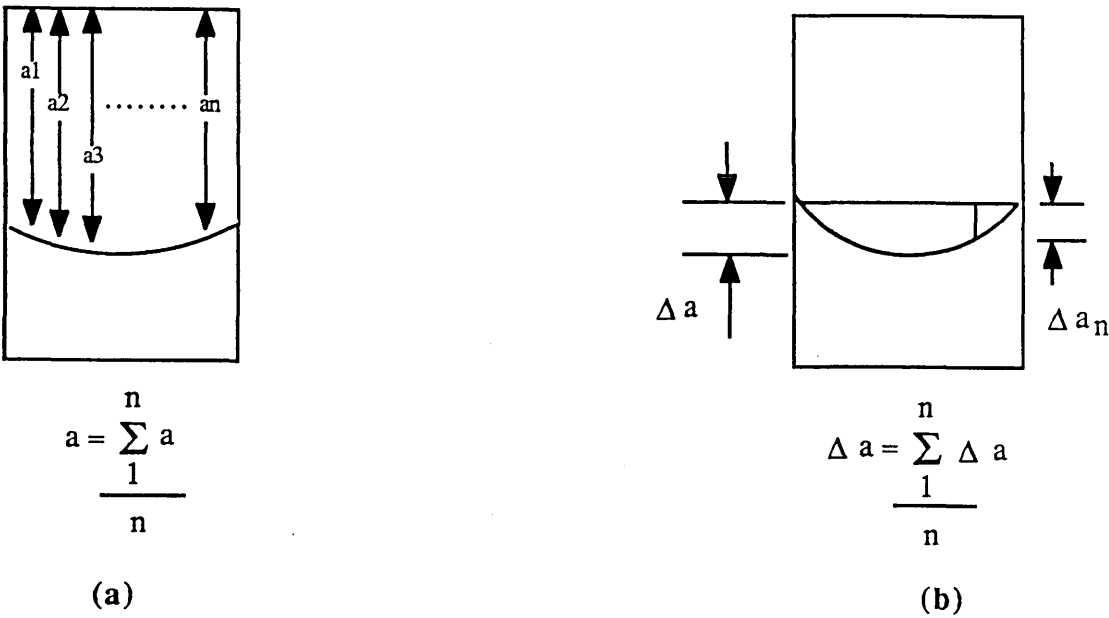


FIG. 3.4 Average crack length determination with the optical measurement technique

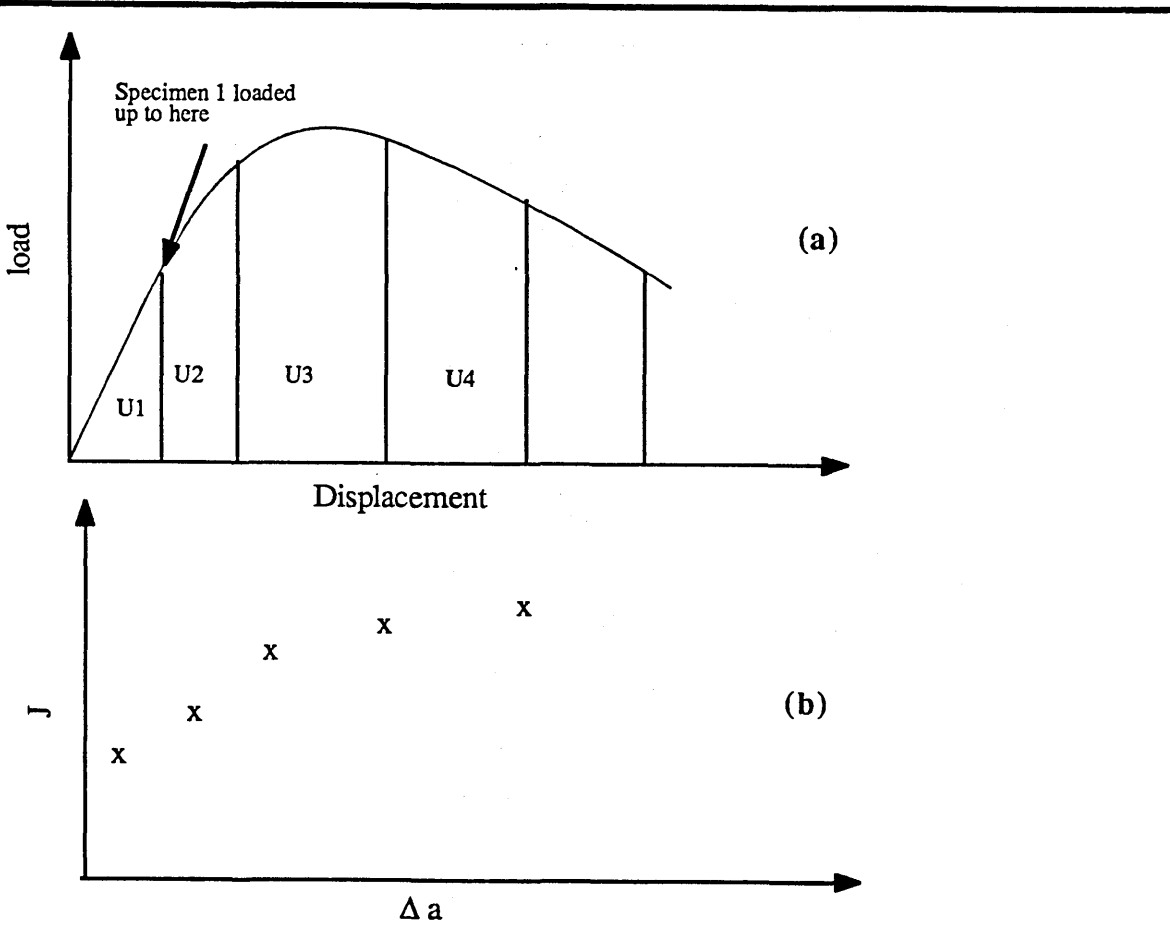


FIG. 3.7 Illustration of the multi-specimen procedure.

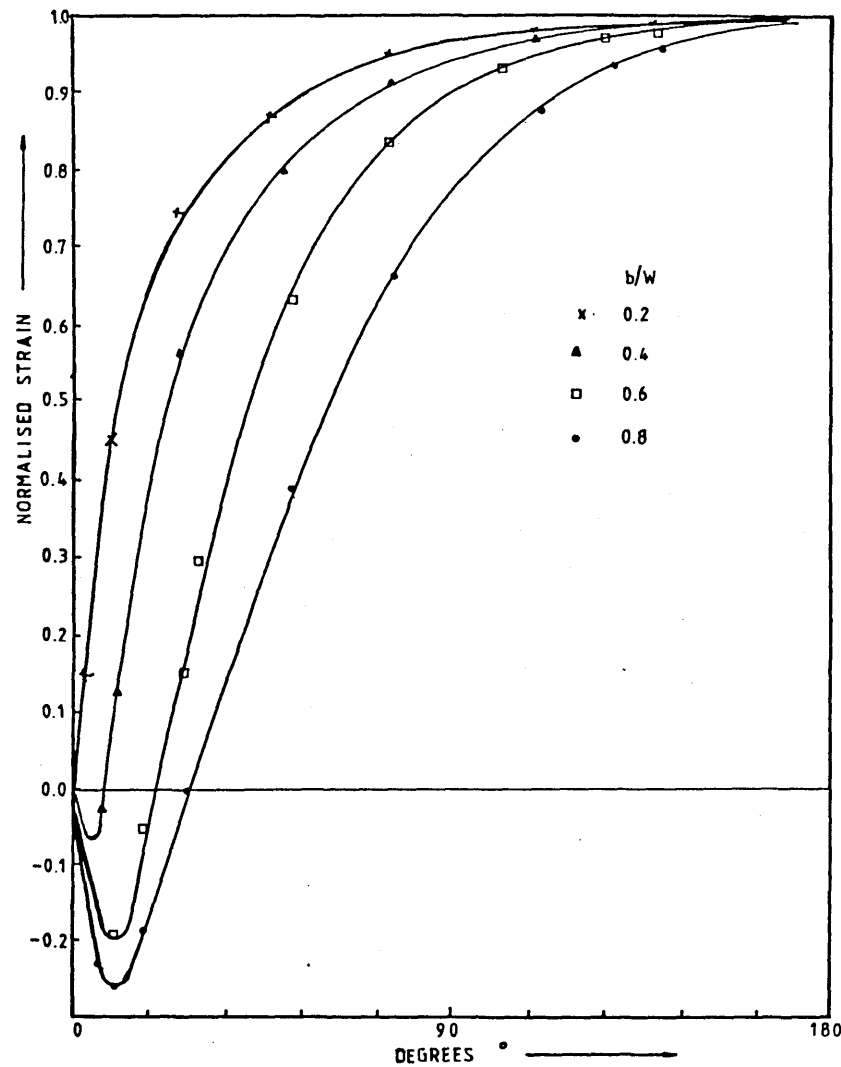


FIG. 3.5 Angular strain distribution determined experimentally for an externally flawed cylinder.(John S.J.(1983)).

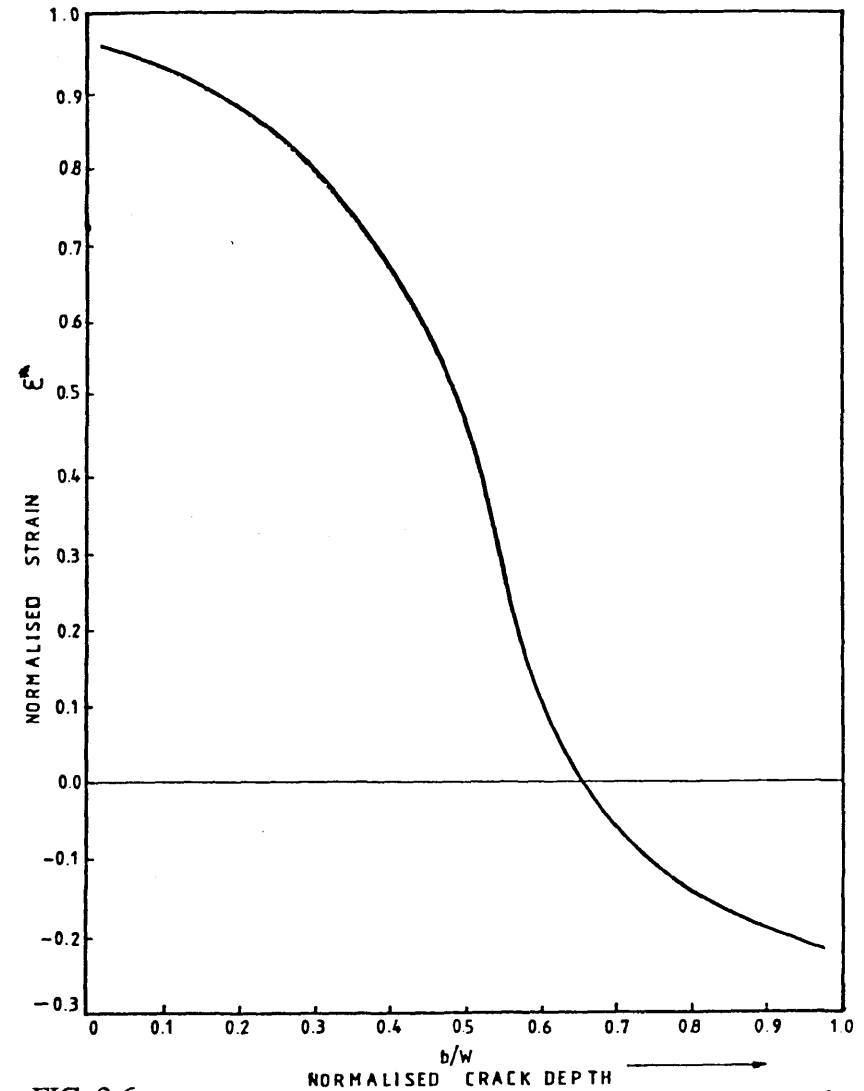


FIG. 3.6 Variation of normalised strain with crack depth (At 22.5°) for an externally flawed cylinder.(John S.J.(1983)).

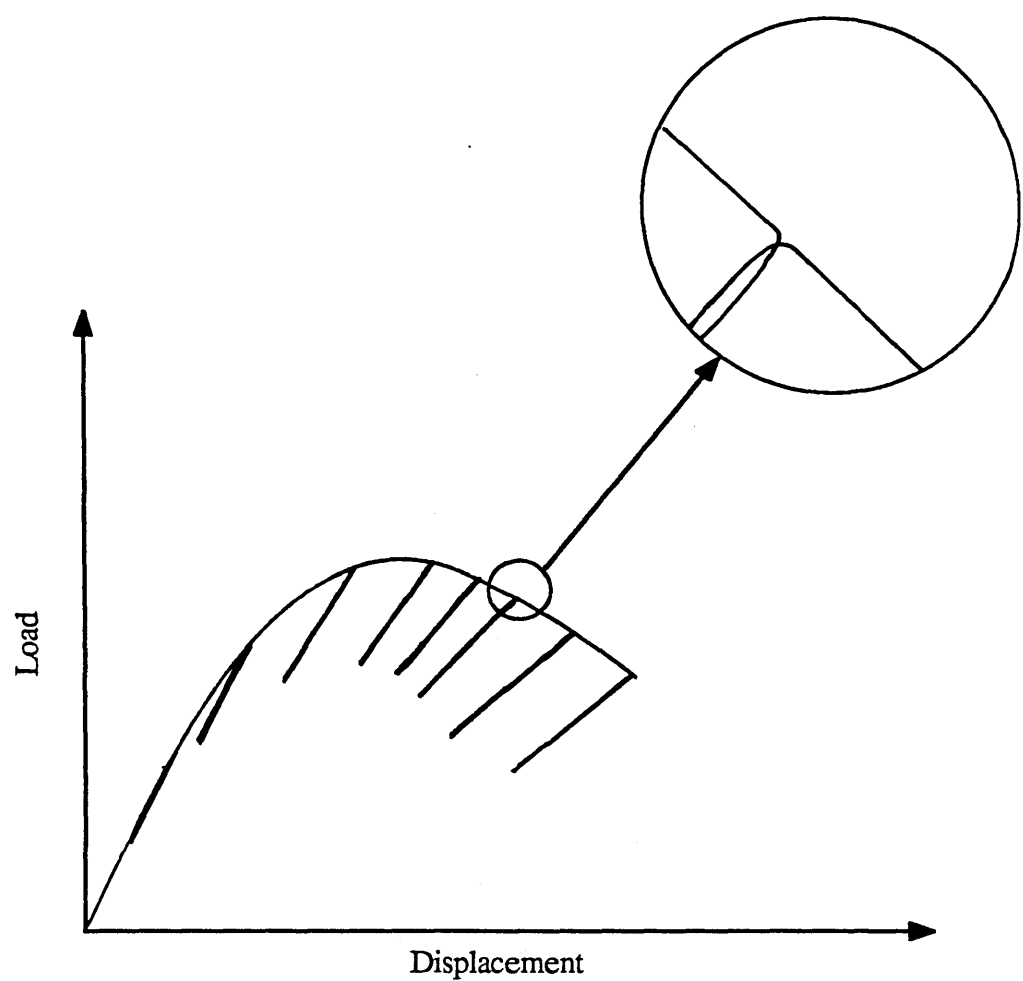


FIG. 3.8 Typical load-displacement trace from an Unloading compliance test

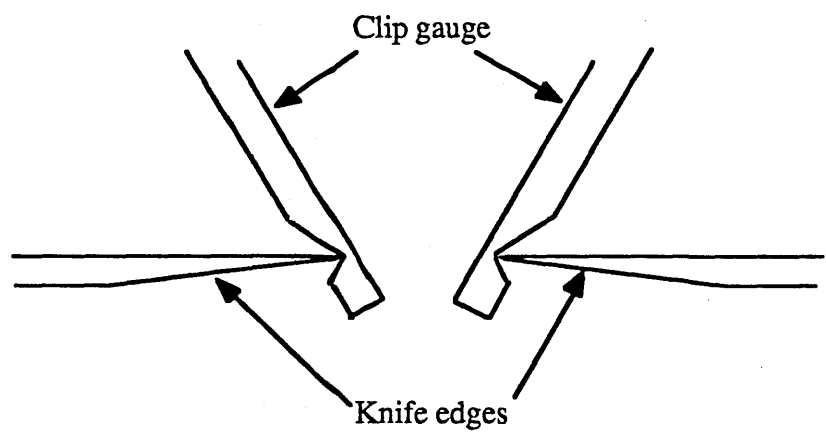


FIG. 3.9 Schematic of how very sharp knife edges reduce area of contact and hence friction, with a clip gauge.

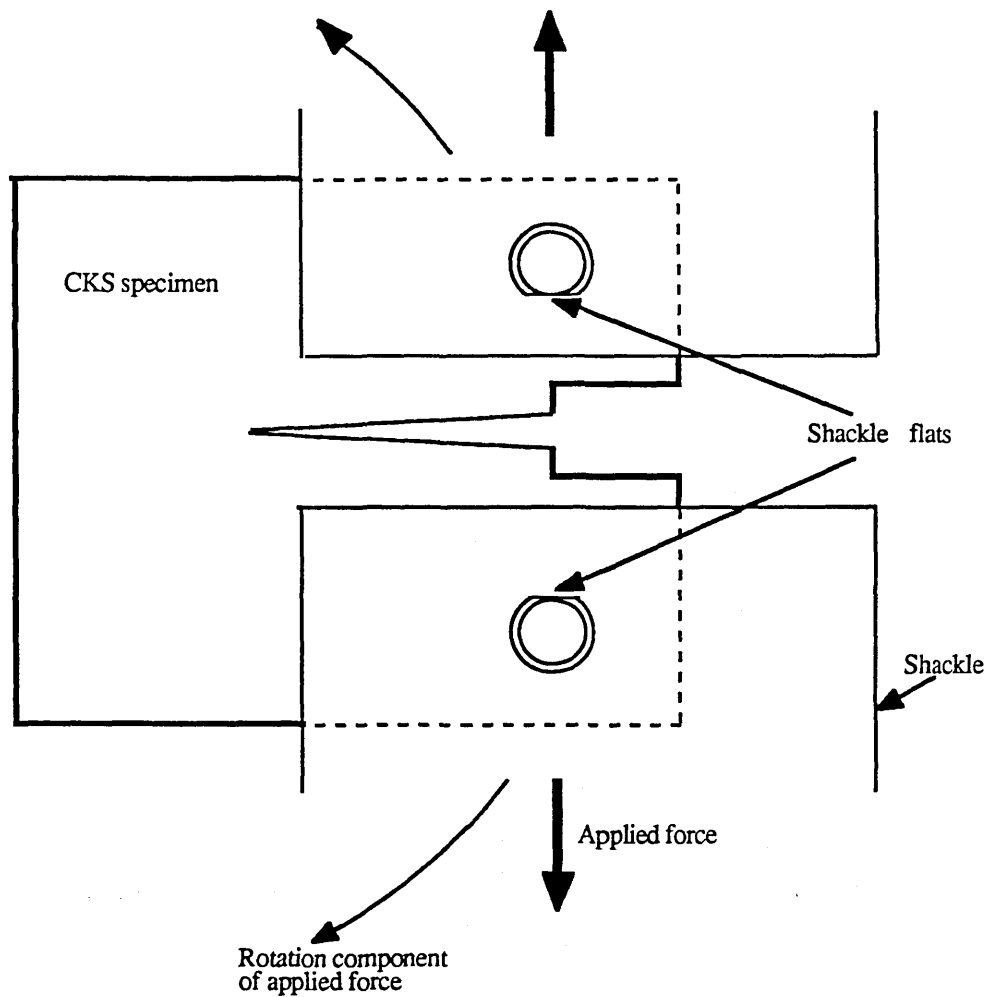


FIG. 3.10 Hole in test shackle with flat base to allow for true linear motion during testing which helps to reduce friction.

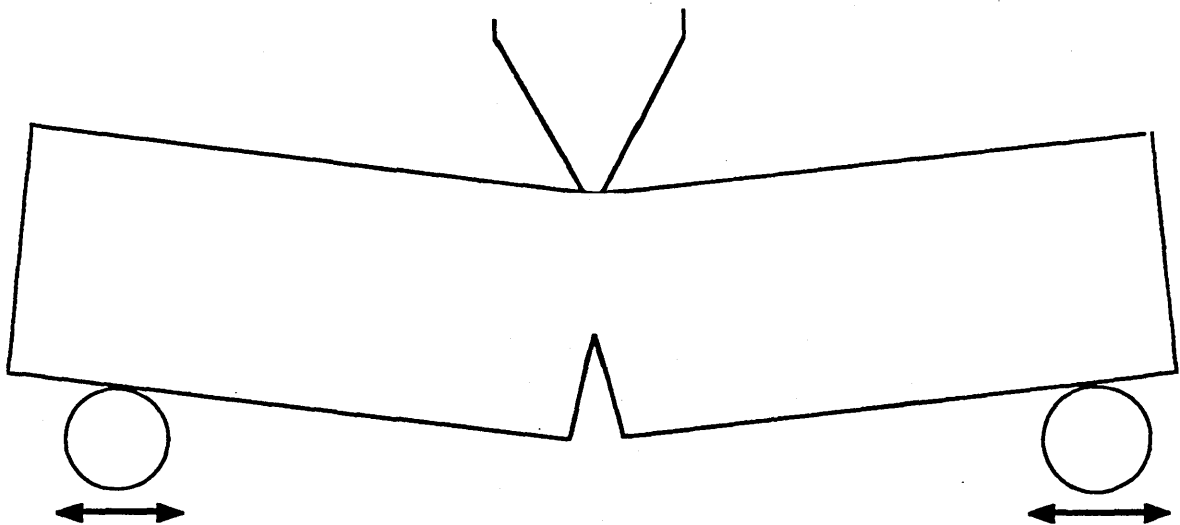


FIG. 3.11 Rollers on the three-point bend rig shown schematically, to have one degree of freedom to reduce contact friction during a bend test.

CHAPTER 4

EXPERIMENTAL AND INTERACTIVE COMPUTATIONAL DETAILS

4.1 INTRODUCTION

The main aim of this chapter is to describe in detail, the testing technique developed for the Unloading Compliance and the Key-Curve method in conjunction with the micro-computer software written for on-line data acquisition and processing. Also involving limited computer interaction, is the description of a fatigue precracking process for titanium (6Al-2Cb-1Ta-1Mo) or Ti (6-2-1-1).

4.2 MATERIAL

All experiments were performed on a titanium alloy that is usually denoted by its major constituents - 6% Aluminium, 2% Columbium, 1% Tantalum and 0.8% Molybdenum. The material was supplied by the RMI Titanium Company and a detailed chemical analysis by this Company is shown in Table 4.1.

Basic mechanical properties as determined by a standard tension test according to BS18 (1971) using a specimen of diameter 11.53mm in the longitudinal direction are shown in Table 4.2. Other properties are given by the RMI Titanium Company in the RMI Company booklet (1983a).

4.3 SPECIMEN GEOMETRY

Compact tension (CKS) and three point bend specimens were used throughout the testing program. Basically the test specimens were required for three test program categories, namely;

1. Development of computer interactive test techniques.
2. Geometry effect on J-R curve analysis and normalisation studies of R-curves (chapter 6).
3. Instability analyses (chapter 7).

Essentially, all the specimens used were of the form shown in Fig 4.1 and 4.2. These dimensions were maintained only for the first category of tests. Size and geometry ratios (i.e. ratios of a/W , b/B , B/W , etc) were altered for the other two test categories. Exact specimen dimensions are given when the respective test categories are introduced.

Most of the compact tension specimens had integral knife edges as part of the specimen

design. See fig 4.1. Simple pencil sharpner blades were used as knife edges for some CKS and all three point bend specimens. This type of knife edge was chosen because of its ready availability and hardness. The knife edges were attached to the three-point bend specimen with size 6BA screws shown in fig 4.2. A spacing of about three to four millimeters between the knife edges was maintained for all specimens.

4.4 TEST EQUIPMENT

The two criteria for the choice of equipment used were machine capacity and capability. For large size specimens the Avery, 1000 KN hydraulic machine (plate 4.1) was used for both CKS and three point bend specimens. The Mayes - 250KN servo-hydraulic machine (plate 4.2) was used for fatigue precracking both specimen types but only CKS specimens were used for J-R curve tests on the machine. The 120,000lb (530KN), screw driven, Tinius Olsen (plate 4.3) testing machine was used exclusively for testing three-point bend specimens.

The instrumentation system consisted of the following units. A clip gauge is supplied by Welwyn strain Measurements Limited. It had a maximum linear output of eight millimeters. The clip gauge was powered and had its signals amplified by a multichannel strain gauge amplifier from Strainstall Limited. This unit amplified signals linearly between -1 volt and +1 volt. A linear variable differential transformer, LVDT, was used for cross-head displacement measurements. The AC signals from a long stroke "RPD" transducer were rectified and amplified into DC signals by the LVDT. Analog load-displacement traces were obtained on Hewlett-Packard 7046A, X-Y plotter. The voltage outputs from the LVDT and the strain gauge amplifier were calibrated for interpretation in millimeters by specially adapted micrometers, see Etemad (1983) The pictures of all the instrumentation system so far mentioned, are shown in plates 4.4 and 4.5.

A KROHN-HITE model 3202 R variable setting signal filter was used to cut out noise (high frequency superimposed signals). This, and a ramp unit designed to control the Mayes machine's ram are shown in plate 4.6.

Data logging and on-line processing was performed by a 32K Commodore Business Machine Computer, Commodore 8296. This was connected to the following systems through a IEEE 488 interface.

1. Commodore 8050 disk drive.
2. JJ Instrument PD4 digital plotter.
3. Commodore 3022 dot-matrix printer.

4. C I L Ltd. Data acquisition system.

Plate 4.7 clearly shows this computer interfaced set up. The computer system and the aforementioned instrumentation components are connected to the test machine and specimen as schematically shown in fig 4.3.

4.5 TEST PROCEDURES

4.5.1 FATIGUE PRECRACKING

4.5.1.1 METALLURGICAL PREVIEW

Ti (6-2-1-1) is normally categorised as an alpha (α) phase alloy and the microstructure is known to have acicular alpha grains with prior beta (β) boundaries, see subsection 2.1.2. The fatigue crack growth would be structurally sensitive (SS), if:-

$$r_{pc} < l_g$$

With a SS mode of fatigue crack growth (FCG) the FCG takes a very tortuous crack growth path. This phenomenon occurred with the initial precracking trials. Because of the suspected SS mode of fatigue crack growth when precracking (as recommended by the ASTM E399 (1981) standard on plane strain toughness testing), the regime of ductile tearing from a J-R curve test was barely distinguishable from that of the fatigue crack. To obtain a smoother, less bifurcated mode of crack growth, the procedure described below was used.

4.5.1.2 FATIGUE PRECRACKING PROCEDURE

Smoother fatigue surfaces were achieved by keeping the K approximately constant during pre-cracking, the reason being that one K value can be associated with one type of fatigue surface. Thus, a significant change in fracture mode (e.g. ductile tearing) could cause a surface morphology to be sufficiently varied such that different modes (thus K) of failure could be identified on the crack face.

Both the CKS and three-point bend specimens were precracked with the Mayes 250 kN servo-hydraulic machine in position control. The entire pre-cracking procedure was controlled manually with the help of a computer program that updates the stress intensity factor for either the CKS and three-point bend geometries. The K values were calculated using the relationship in ASTM E399 (1981) Standard. The surface crack length was measured by a travelling optical microscope, TOM.

With fatigue precracking in position control, any crack growth would result in a drop in load. The new load and the corresponding new crack length are then used to calculate the current stress intensity factor. The short computer programs, "CT CALIB and 3PTB CALIB"(see appendix I) for CKS and three point bend specimens respectively are used for updating the K values. The load is then manually adjusted, if necessary, to maintain a relatively constant level of $K_{\max}$. A $K_{\min}$ value of $2\text{Mpa}\cdot\text{m}^{1/2}$, is maintained while a $K_{\max}$ value at between $31\text{-}37\text{ Mpa}\cdot\text{m}^{1/2}$ is used to sustain a reasonable rate of FCG (four millimeters of crack growth in ninety minutes). An alternative method could have been to get a structure insensitive mode of crack growth (see subsection 2.1.3), i.e.,

$$r_{pc} > l_g$$

This procedure however might have violated the ASTM (1981) Standard which stipulates $K_{\max}$ restrictions during precracking, since a large r_{pc} requires a large $K_{\max}$.

4.5.2 UNLOADING COMPLIANCE PROCEDURE

This procedure was used for CKS and three point bend specimens on the Mayes and Tinius Olsen machines respectively. The three point bend tests used the clip gauge and LVDT(see section 4.4) crack length estimation and work done evaluations respectively. The CKS specimen tests, however, used the clip gauge only for both purposes. This somewhat minor difference and the machine controls are about the only operational differences between the two types of test procedures.

It must be recollected here that the procedure is entirely computer interactive. The computer software, explained in appendix II is developed for the three-point bend testing procedure. The only apparent difference between the three-point bend and CKS specimen tests in the software, is the way the crack lengths are predicted, see subsection 3.2.3. Essentially all the instruments shown in plates 4.4, 4.5, 4.6 and 4.7 were used with the appropriate machine as described above.

A step by step computer interactive procedure is given in appendix II with the occasional note for the CKS use.

4.5.3 DESCRIPTION OF THE KEY-CURVE TECHNIQUE

4.5.3.1 EXPERIMENTAL DETAILS

Ti (6-2-1-1) compact tension specimens (Fig 4.1) were used for developing the Key Curve method described in this section. Specimens of small dimensions ($B = 17.5\text{mm}$, $W = 2B$) were used so that load-displacement values of higher Δ/W could be achieved without proportionately more crack growth. All the specimens were fatigue pre-cracked according to the specification described in subsection 4.5.1 of this chapter. This procedure is however, within the limits of the guidelines given in ASTM E399 (1981). A total of seven specimens were fatigue pre-cracked to crack depth ratios of between $a/W = 0.62$ and 0.92 .

The instrumentation used was the same as the Unloading Compliance method (see fig 4.3) but a different software package was developed for this method; an explicit description of this package is given in appendix III.

Each of the seven specimens was loaded to its limit load and the data points were collected by the computer at the rate of 145 data pairs/minute. A uniform cross-head speed of 0.25 mm/minute was maintained throughout the loading period. An analog plot was obtained on the H-P 7046A X-Y plotter for all the specimens. The data acquired by the microprocessor is plotted by the JJ instruments PD4 plotter and is shown in fig 4.4.

The specimens were then broken by high energy impact, after heat tinting at 400°C for 15 minutes. The fatigue crack length was measured by averaging lengths across the crack face, as described in subsection 3.2.2.

4.5.3.2 COMPUTATION AND THE PROGRAM-DATAFILE-LINK-UP SYSTEM

The load and displacement values are normalised as:-

$$PW/Bb^2 \text{ and } \Delta/W \text{ respectively}$$

These normalised load and displacement functions are then passed through a curve-fitting program, DATFIT to facilitate numerical differentiation for the $\partial F_1/\partial(\Delta/W)$ values in equations (3.8) and (3.9), see chapter three. $\partial F_1/\partial(a/W)$ values are obtained from a plot of F_1 against a/W levels as shown in fig 4.5. From fig

4.5 it is seen that F_1 is a weak function of a/W , therefore $\partial^2 F_1 / \partial (a/W)^2$ was set to zero. Similar trends have also been reported by Joyce et al (1980). A plot of all the F_1 functions against Δ/W after curve fitting is showed in fig 4.6.

Program DATFIT fits a fifth order polynomial to the plots shown in fig 4.4 after normalisation to F_1 and Δ/W . It differentiates the F_1 function numerically with respect to Δ/W and stores all the F_1 function values and its derivatives, $\partial F_1 / \partial (\Delta/W)$. So from primary files created by ATOD PROG, secondary files are created by DATFIT. To obtain the J-resistance curves, discrete versions are written for equations (3.8) and (3.9), namely:-

$$dJ_n = [(2b/W)F_{1n} - (b/W)^2 (\partial F_1 / \partial (a/W))_n] d\Delta_n + [-2/W \sum_{i=1}^n F_{1i} d\Delta_i + 4b/W^2 \sum_{i=1}^n (\partial F_1 / \partial (a/W))_i d\Delta_i] da_n \quad (4.1)$$

and

$$da_n = \frac{[dP_n - (b^2/W^2) \partial F_1 / \partial (\Delta/W) d\Delta_n] W}{(b^2/W) (\partial F_1 / \partial (a/W)) - 2bF_1} \quad (4.2)$$

Therefore,

$$J = \sum_{i=1}^n dJ_n \quad (4.3)$$

and

$$\Delta a = \sum_{i=1}^n da_n \quad (4.4)$$

The final processing program, JRESIST utilises Eqs. 4.1 through to 4.4 for computing J and Δa values. Though a detailed explanation of these programs is given in appendix III, the program-datafile-link-up-system is shown schematically in fig 4.7. Figs 4.4 to 4.6 are discussed, together with the formulations of the Key-Curve analysis, in chapter nine.

Element	C	N	Fe	O	Al	Cb	Ta	Mo	H
% Composition	0.05	0.03	0.25	0.1	6.0	2.0	1.0	0.8	0.025

TABLE 4.1 Chemical composition of Titanium alloy (6Al - 2Cb - 1Ta - 1Mo)

0.2% proof Stress (MN/m ²)	U . T . S (MN/m ²)	Young's Modulus (MN/m ²)	% Elongation
728	828	123200	12

TABLE 4.2 Basic mechanical properties of Ti (6-2-1-1) obtained in the longitudinal direction.

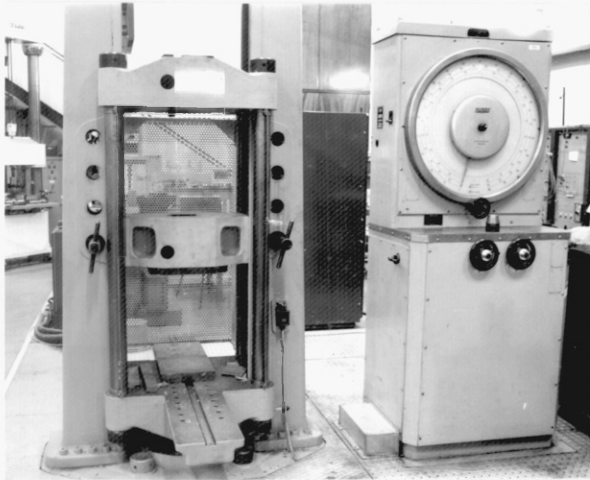


PLATE 4.1 100 ton hydraulic Avery machine

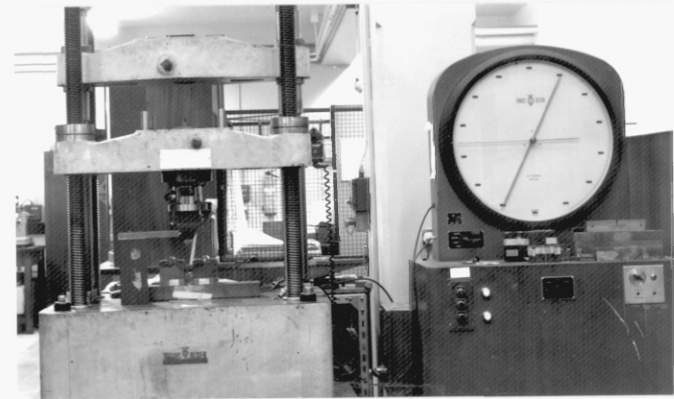


PLATE 4.3 Tinius Olsen screw-driven machine

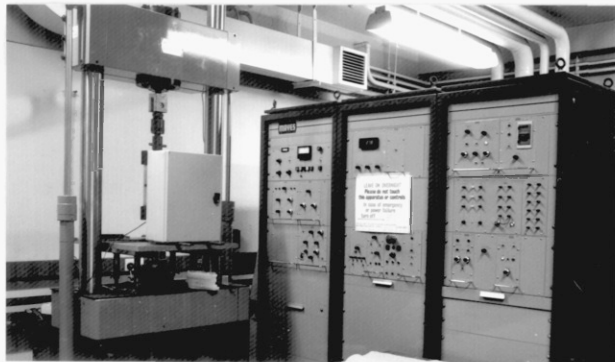


PLATE 4.2 250kN Mayes servo-hydraulic machine

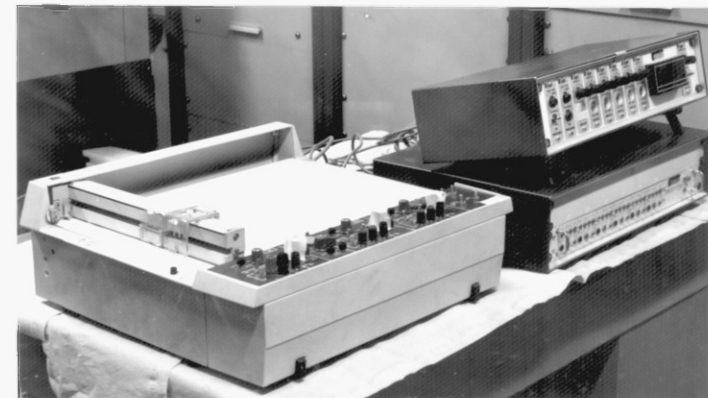


PLATE 4.4 LVDT strain gauge amplifier and the analog Hewlett-Packard plotter

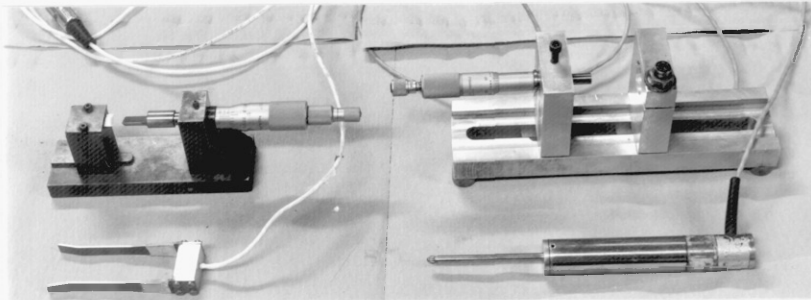


PLATE 4.5 Clip gauge and LVDT transducer with their respective calibrators

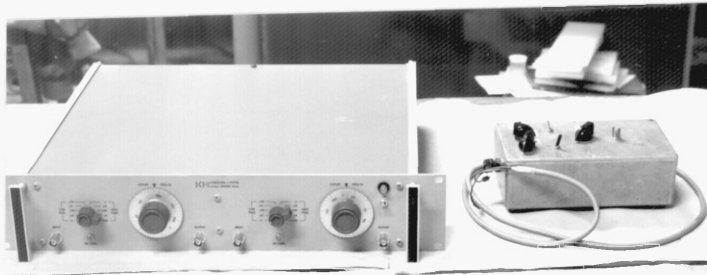


PLATE 4.6 Krohn-Hite variable active filter and the Ramp generator



PLATE 4.7 Computer system with interfaced peripherals

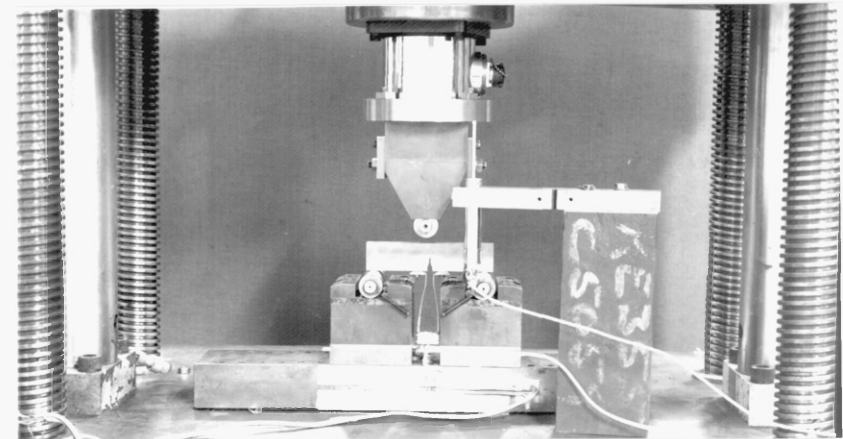


PLATE 4.8 Three-point bend specimen with clip gauge and LVDT transducer in position.

ALL DIMENSIONS IN MM
DRAWING NOT TO SCALE

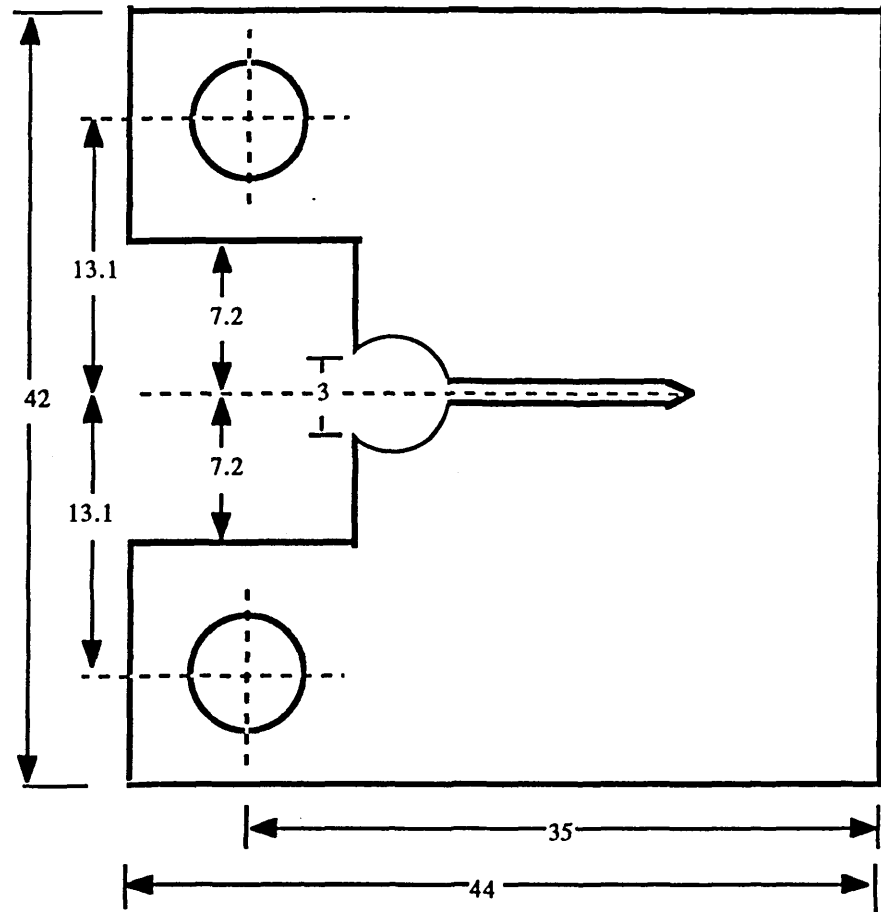
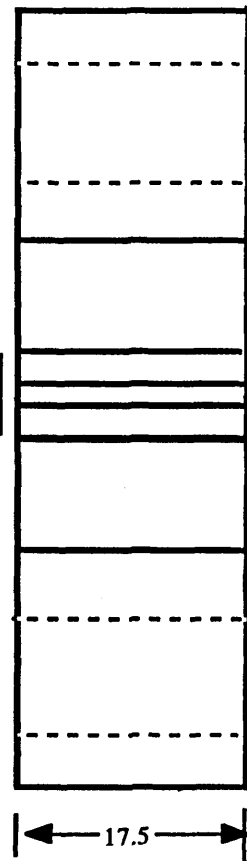


FIG.4.1 Compact tension specimen

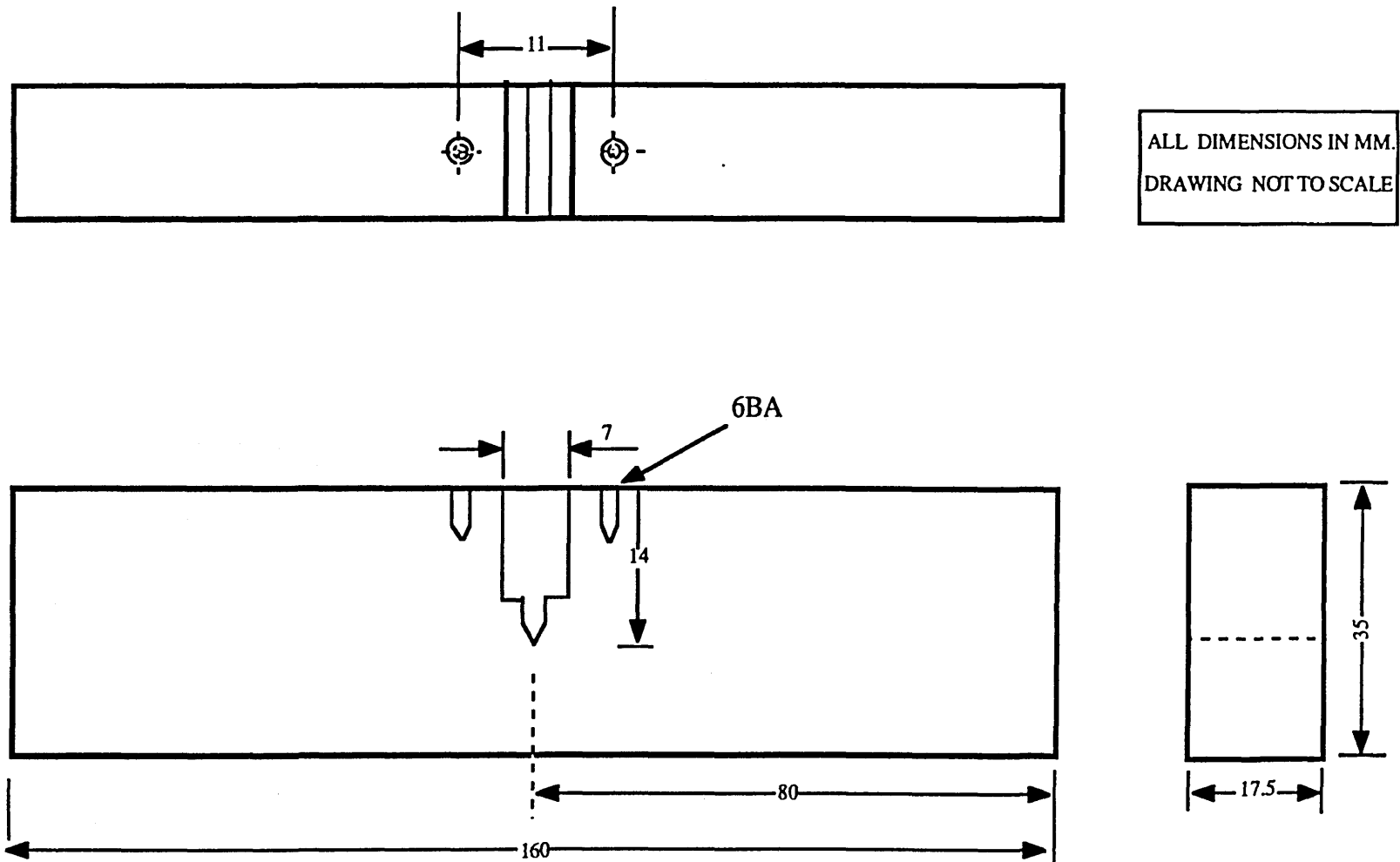


FIG. 4.2 Three-point bend specimen

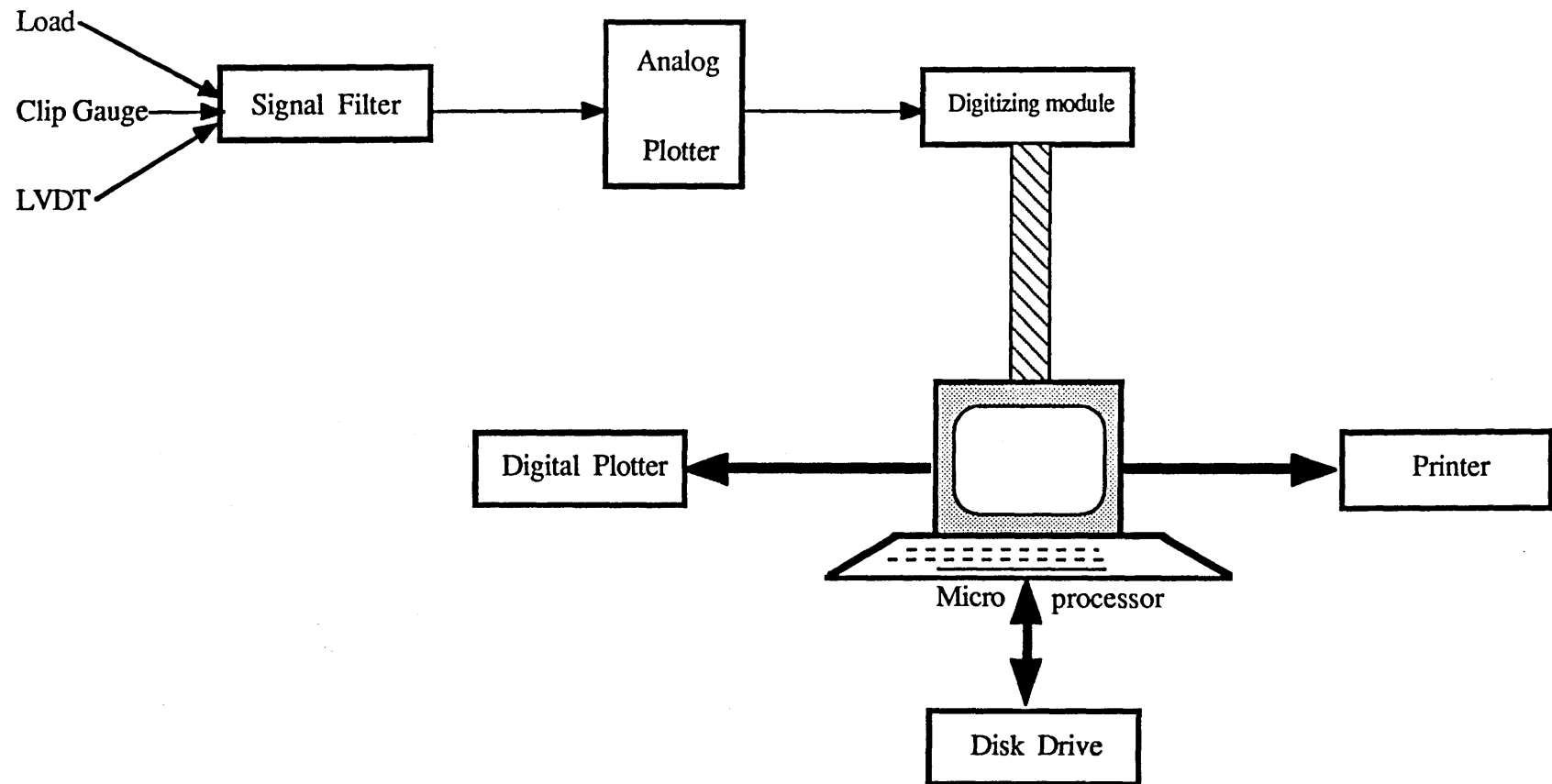
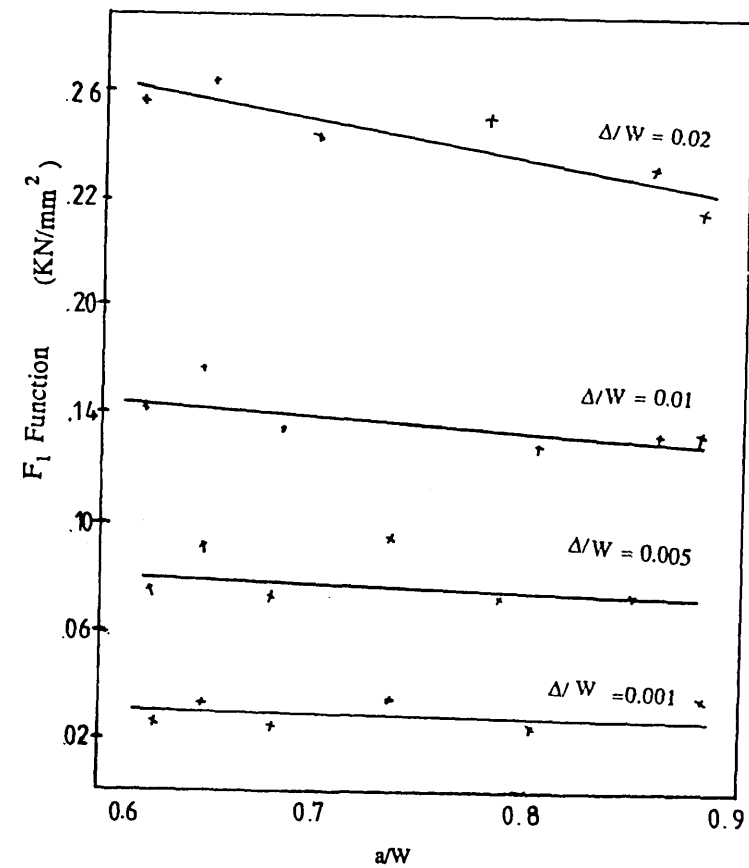
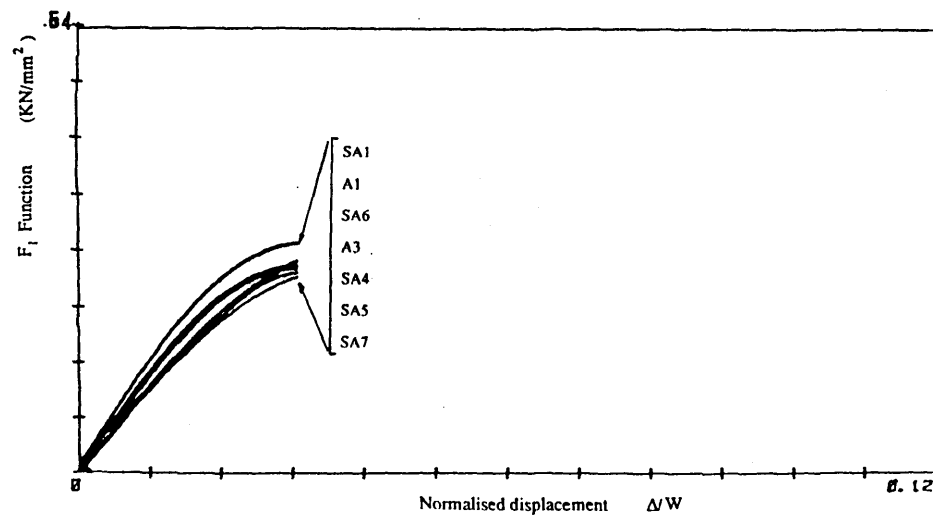
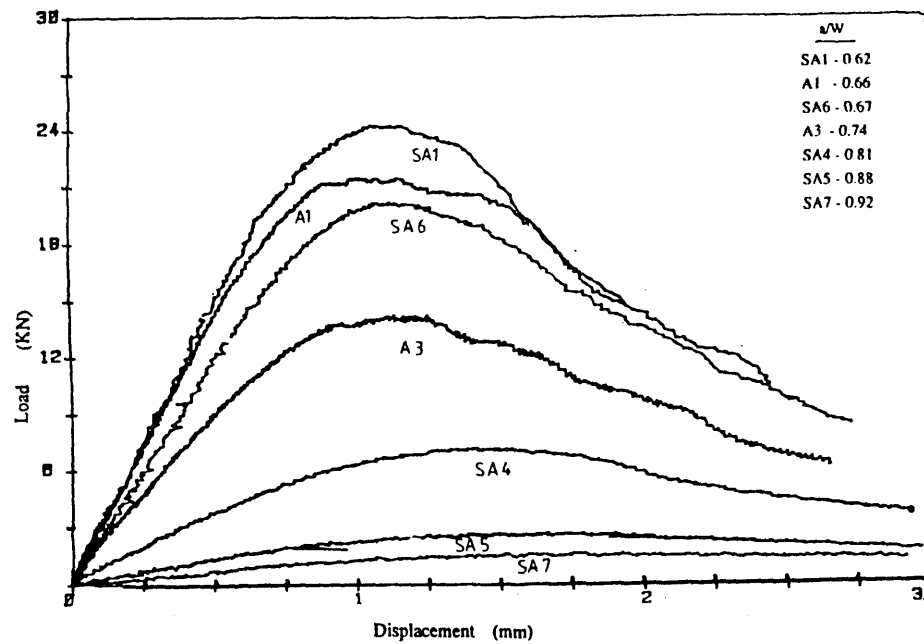


FIG. 4.3 Schematic of the instrumentation system.



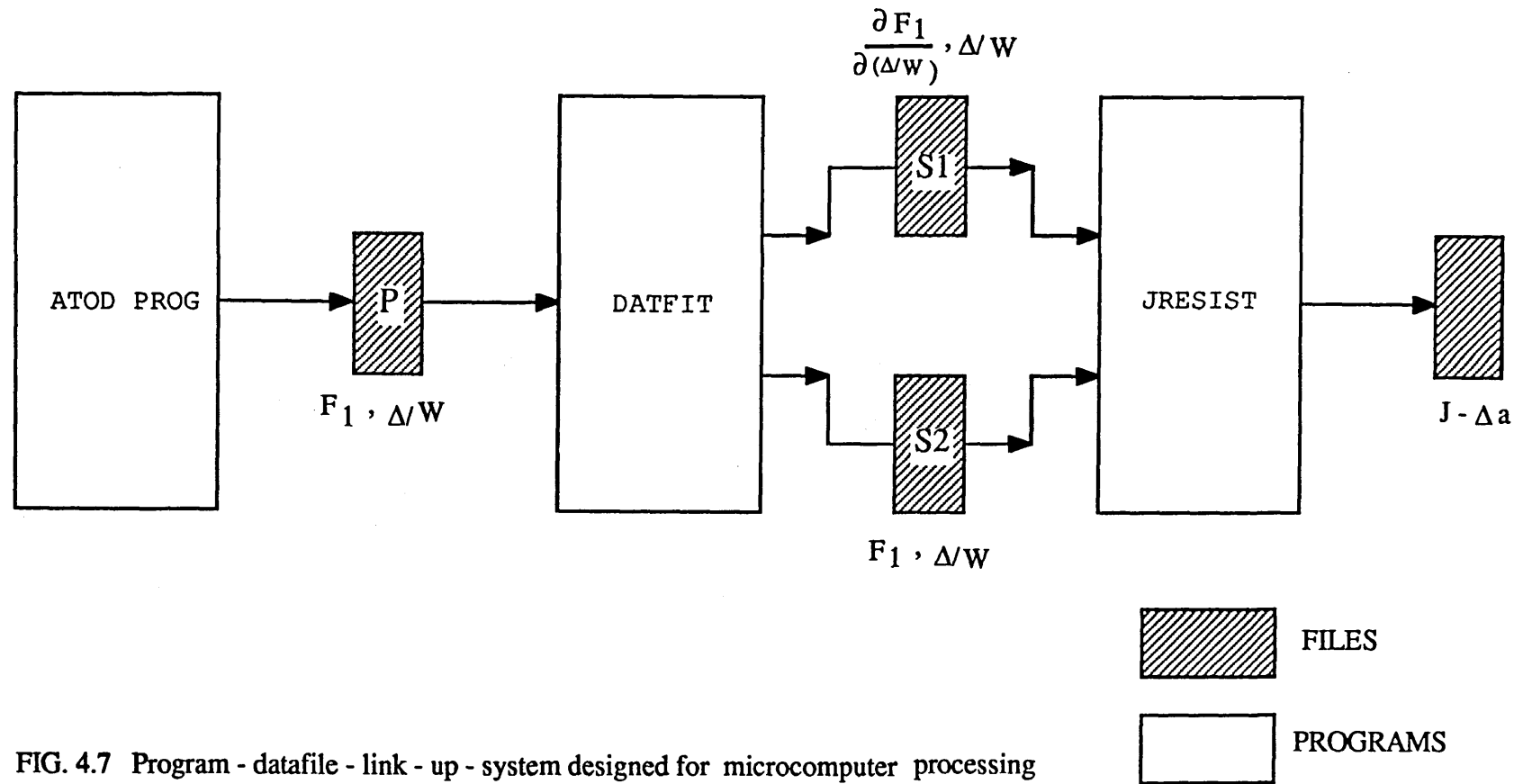


FIG. 4.7 Program - datafile - link - up - system designed for microcomputer processing of the Key - Curve analysis.

CHAPTER 5

TEST TECHNIQUE-RELATED RESULTS

5.1 INTRODUCTION

The initial difficulty in distinguishing fatigue crack growth limits from regions of ductile tearing provided the impetus for developing alternative means of non-visual crack length predictions. Benefits of reduced tortuosity during FCG by maintaining a relatively constant $K_{\max}$ are shown. Several tests were also carried out to check the accuracy of the several test methods mentioned above and below. This was essential so that one technique could be reliably chosen for a detailed experimental study of resistance curves.

5.2 RESULTS FROM THE FCG PRECRACKING PROCEDURE

The results of the recommended precracking procedure described in subsection 4.5.1.2 is shown in plate 5.1a. A comparison is made with results from a variable $K_{\max}$ FCG procedure, which inadvertently occurs for this material when the procedure outlined in ASTM E399 (1981) is followed. A typical crack face from such a procedure is shown in Plate 5.1b.

It can clearly be seen that when the ΔK or $K_{\max}$ is monitored and maintained relatively constant a smoother crack face results. This procedure was consequently used for precracking all specimens mentioned below. It should also be pointed out that the method described in subsection 4.5.1.2. keeps within the $K_{\max}$ restrictions outlined in the ASTM E399 (1981) Standard; the only difference being the additional constraint of sustaining a constant $K_{\max}$ during FCG in the proposed method of precracking (subsection 4.5.1.2.) Ti(6-2-1-1) test specimens.

5.3 ROLLER INDENTATION AND MACHINE COMPLIANCE

ROLLER INDENTATION

To correct for roller indentation in the three-point bend tests, a roller indentation test was set up with a broken half of a test specimen schematically illustrated in fig 5.1a. The load-displacement plot of such a set up is shown in fig 5.1b. From the trace, the inverse slope is obtained and used in correcting for extraneous displacements as shown below.

If C_I is the inverse slope of the indentation trace (fig 5.1b), the correction, in terms of

work done is:-

$$U_I = (1/2) C_I P^2 \quad (5.1)$$

(From work done under linear elasticity ($1/2 Pq$, where $q=C_I P$)).

From Fig 5.1b, C_I is determined to be 3.59×10^{-3} KN/mm for Ti(6-2-1-1) with the three point bend rig on the Tinius Olsen testing machine. This corrective process is incorporated in the computer software, SULCO (for three-point bend Unloading Compliance tests) described in appendix II.

TEST MACHINE COMPLIANCE

Though the test machine compliance is not directly involved in the J-R curve test technique, it is one of the parameters required for ductile instability analyses introduced earlier, in section 2.5.

Machine compliance was determined for the Avery (100 ton) hydraulic machine and Tinius Olsen screw-driven machine. This was done by using a hydraulic jack to further "open" the jaws (top ram and machine base) of the test machine as illustrated in fig. 5.2.

During the jacking process, the load was obtained from the machine's loadcell and the displacement was recorded using a dial gauge for the Avery machine and a LVDT transducer for the screw-driven Tinius Olsen machine. The recorded load-displacement readings of this machine compliance test are shown in table 5.1 and the load displacement traces for the Tinius Olsen machine are shown in figs. 5.4 and 5.5.

The compliance of the Tinius Olsen machine was measured at three positions, as shown in fig. 5.3.

- (i) Base to floor, C1
- (ii) Top ram to floor, C2
- (iii) Top ram to base at center, C3

Inverse slope measurements from fig 5.4 give "full stretch" compliance measurements as:

$$C1 = 17 \times 10^{-4} \text{ mm/KN}$$

$$C2 = 15 \times 10^{-4} \text{ mm/KN}$$

$$C3 = 32 \times 10^{-4} \text{ mm/KN}$$

It should be noted that, not surprisingly $C1 + C2 = C3$ since C1 and C2 can be

modelled as two springs in parallel. All these compliance measurements were done with a gauge length (top ram to base spacing) of 375mm, which was representative of a typical height during most Unloading Compliance J-R curve testing. Surely, the machine compliance is dependent on this spacing.

Also, prior to the "full stretch" (linear portion) of the machine, the non-linear "slack" has to be overcome, see fig 5.5. Therefore during the loading the machine compliance is dependent on the load until non-linearity of the load-displacement trace has ceased, see fig. 5.4.

For the Avery, 100 ton machine (see plate 4.1), load-displacement readings are shown in table 5.1 below, from which an average value of the compliance was obtained as 0.0127 mm/KN.

5.4 ACCURACY OF CRACK LENGTH PREDICTIONS

The specimens used were the CKS and three-point bend, with dimensions as shown in fig 4.1 and 4.2. For the CKS specimen test, the compliance calibration function used was that presented by Saxena and Hudak (1978), see Equ. 3.2 The three-point bend tests employed the equations by Kapp et al (1985) using the mouth-opening compliance measurements, see Equ. 3.4. The actual crack length measurements were done after heat tinting and breaking the test specimen open, using the averaging technique described in the ASTM E 813 (1981) Standard.

When the specimens are not sidegrooved, the Eqs. 3.2 and 3.4 underestimate the value that is obtained by measuring the crack length across the crack face. This is due to crack tunnelling. A comparison of this discrepancy is shown in Fig. 5.6 in the form of J-R curves that were generated using the Unloading Compliance interactive software, SULCO which DID NOT correct for curvature, described in section 5.5. below. The J-R curves shown in fig.5.6 were generated for different degrees of deformation using Ti (6-2-1-1) three-point bend specimens. As seen in figs 5.6a, b , c and d, the discrepancy increases with crack growth due to increased tunnelling.

If however, the test specimens were sidegrooved both compliance calibration equations by Saxena and Hudak (1978) and Kapp et al (1985) predict crack lengths satisfactorily. Accuracies of within 0.25 of a millimeter can be obtained. This is essentially because the crack front is relatively straight and crack tunnelling is virtually absent if sufficient sidegrooves are used. This is clearly shown in fig 5.6c, where a three-point bend specimen is extensively deformed.

5.5 CRACK FRONT CURVATURE CORRECTION

5.5.1 INTRODUCTION

In this section crack front curvature is studied and an empirically-based solution is presented to account for this phenomenon in Ti (6-2-1-1). The final objective being the inclusion in interactive numerical processing of data for on-line, reliable crack length predictions.

5.5.2 PROCEDURE

Three-point bend Ti (6-2-1-1) test specimens of dimensions shown in fig 4.2 were used throughout this study. The steps taken were as follows:-

1. Four test specimens of fatigue pre-crack depth ratio, $a/W = 0.55$ were used to obtain J-R curves for different levels of crack growth; J-R curves of two of these, B10 and B7 are shown in fig 5.6. After the test, the four specimens were heat tinted and broken in two halves. Seven-point and nine-point measurements (see fig. 5.7) were taken across the crack face for both the original ($a_{o,7,9}$) and final crack lengths ($a_{f,7,9}$). Subscripts 7 and 9 represent the seven and nine point average measuring techniques for the crack length across the crack face, see subsection 3.2.2 in chapter three. Crack length averages of these measurements were evaluated. A comprehensive table of these measurements is shown in table 5.2.

2. Based on these measurements, inverse compliance equations by Kapp et al (1985) were used to evaluate what the compliance measurement would have been if $a_{f,7}$, $a_{f,9}$ and $a_{f,max}$ were measured. The inverse mouth-opening compliance equation (by Kapp et al (1985)) used was:-

$$E'B \Delta(1 - a/W)^2 / [3.95 P (S/W)(a/W)] = f_{3P BMC} (a/W) \quad (5.2)$$

$$f_{3P BMC} (a/W) = 2.21 - 6.57(a/W) + 17.9(a/W)^2 - 26.6(a/W)^3 + 19.9(a/W)^4$$

3. These "would be" compliances, $a_{f,7}$, $a_{f,9}$ and $a_{f,max}$, are then normalised with the actual measured compliance $C_{f,comp}$. Table 5.3 displays these "would be" compliances and their respective ratios. A plot of this ratio against $\Delta a/b_0$ is shown in fig 5.8.

4. The values of these curves were then used as data for the curve fitting routine (developed for the Key Curve analysis) described in appendix III.
The result of the computation yielded fifth order polynomials as given below:

$$C_7/C_{\text{comp}} = 1.002 + 3.497(\Delta a/W) - 53.163(\Delta a/W)^2 + 512.9(\Delta a/W)^3 \\ - 1948.8(\Delta a/W)^4 + 2568.8(\Delta a/W)^5 \quad (5.3)$$

$$C_{\text{max}}/C_{\text{comp}} = 1.003 + 3.456(\Delta a/W) + 23.32(\Delta a/W)^2 - 351.52(\Delta a/W)^3 \\ + 1717.05(\Delta a/W)^4 - 2682.3(\Delta a/W)^5 \quad (5.4)$$

$$C_9/C_{\text{comp}} = 1.000 - 0.2335(\Delta a/W) + 8.49(\Delta a/W)^2 - 38.71(\Delta a/W)^3 \\ + 123.72(\Delta a/W)^4 - 136.7(\Delta a/W)^5 \quad (5.5)$$

THESE EQUATIONS ARE VALID ONLY FOR CASES WHERE :

a_0/W IS ABOUT 0.56

AND

$$0 < \Delta a/W < 0.3$$

Therefore for similar tunnelling shapes, see fig 5.7, corrective equations such as Eqs. 5.3 through to 5.5 can be used to "correct" for curvature by evaluating the chosen (seven or nine point) "would be" compliance from crack extensions (Δa) obtained from the measured (uncorrected) compliance.

The new compliance is then evaluated using Eqs. 5.3, 5.4 or 5.5 which is used for evaluating the "corrected" crack length by the usual compliance calibration equations (e.g. Equ.3.4). The form of the "corrective" equations 5.3 to 5.4 was considered convenient for the interactive Unloading Compliance computer program SULCO. Equation 5.3 is included in SULCO and it is used in the calculation routine described above. For further insight into the use of these equations, see appendix II for the listing of the program, SULCO.

A typical crack length corrected J-R curve is shown in fig. 5.9.

5.6 EFFECT OF SPECIMEN TYPES ON J-R CURVES

In this study, only the CKS and three-point bend specimens are used. Both the Multiple-test and Unloading Compliance test procedure were attempted on the CKS and

three-point bend specimen of equal geometry ratios, shown in figs 4.1 and 4.2, the following can be noted:-

1. With a given test method, the J-R curves from the two specimen types compare reasonably well.
2. Both J_0 , Equ (2.27) and J_{i+1} Equ (2.28) formulations are evaluated and both specimen types agree reasonably well for either formulation.

5.7 COMPARISON OF TEST METHODS

To enable a level of confidence be attached to the various test methods described in chapter four, a comparative study was essential. Criteria used for suitability of J-R curve determination are first and foremost - accuracy of parameters that had to be determined, U, work done, and Δa , crack extension. Others include, ease of test execution, number of specimens required and time taken for material preparation and data analysis.

Since the Key Curve method was used only with the CKS specimen, the continuous J-R curve produced by this method is shown alongside the Unloading Compliance and Multi-specimen test methods in fig. 5.11. The Key-Curve was obtained by using parameters in Equ. 4.1 through to Equ. 4.4 which were obtained from figs 4.4, 4.5 and 4.6. All the numerical processing was performed by the process described in subsection 4.5.3.2. The description of the constitutive equations of this numerical package is given in appendix III. The CKS version of SULCO, see appendix II, was used to obtain the Unloading Compliance curve and the Multi-specimen test was obtained as described in subsection 3.3.1. As can be observed from fig 5.11, the curves differ by only 10% - 15%.

Similarly, for the three-point bend specimens a comparison of the test methods is shown in fig 5.12; here also, the curves differ by a similar margin. This margin is further reduced if the loading rate is reduced as discussed in the next section.

5.8 EFFECT OF LOADING RATES ON THE TEST METHOD

Due to the minor discrepancies discussed above an investigation to the loading rates was made. Excessively fast (1.23 mm/min) and slow (0.03 mm/min) tests were performed using the Unloading Compliance and Multi-specimen procedures. For the Unloading Compliance procedure, all the data were completely processed by hand because the computer data acquisition was either too fast or slow for these tests. Fig

5.13 shows clearly that with relatively fast and slow tests (a nominal rate of the cross head movement used was 0.7 mm/min) there is hardly any effect on the J-R curves.

The Multi-specimen test procedure on the other hand, shows a slight response of the J-R curve due to different cross-head movement rates as shown in fig. 5.12. The extremely slow tests produced slightly higher J-R curves and in the same manner, fast ones were either on the "normal" rate curve or slightly lower. A cause of the somewhat higher J-R curves produced by the slow rate tests might be the greater amount of work done, hence higher J, as shown in fig. 5.14.

5.2 EFFECT OF SPECIMEN ORIENTATION ON J-R CURVES

Orientation effects on the J-R curves were investigated before geometry effects were studied in Ti (6-2-1-1). Keeping to one J-formulation, J_0 , a comparative plot of the T-S, T-L and L-T directions in three point bend geometries is shown in fig. 5.15. It can be seen here, that orientation effects of this material on J-R curves are quite minimal.

Load Intervals (KN)	Extensometer dial gauge deflections (mm)	Compliance (mm/kN)
100 - 110	0.127	0.0127
110 - 116	0.0708	0.0118
99 - 112	0.149	0.0122
150 - 159	0.1143	0.013

TABLE 5.1 Load and deflection readings for the Avery 100 ton machine during a machine compliance determination test.

	$C_{f,comp}$ (mm/KN)	$C_{f,7}$ (mm/KN)	$C_{f,max}$ (mm/KN)	$C_{f,9}$ (mm/KN)	C_7/C_{comp}	C_{max}/C_{comp}	C_9/C_{comp}
B9	0.1366	0.1999	0.2706	0.1656	1.46	1.98	1.92
B10	0.0301	0.0333	0.0412	0.0299	1.16	1.37	0.98
B2	0.0269	0.0289	0.0314	0.0267	1.07	1.16	0.99
B7	0.0309	0.0373	0.0761	0.0319	1.2	2.45	1.03

TABLE 5.3 Number-of-crack length-measurement based compliance values for Ti (6-2-1-1) three-point bend specimens.

	$a_{f,comp}$	$a_{f,7}$	$a_{f,9}$	$a_{o,7}$	$a_{o,9}$	$a_{f,max}$ (1)	$a_{f,min}$ (2)	$a_{o,max}$	$a_{o,min}$	(1-2)/2
M I L L I M E T E R S										
B9	27.47	28.68	28.07	20.00	19.76	29.42	26.00	20.03	19.05	0.132
B10	21.08	21.66	21.05	20.03	20.01	22.67	18.30	20.53	18.22	0.239
B2	20.48	20.89	20.45	19.75	19.58	21.28	19.10	19.8	19.05	0.114
B7	21.26	22.19	21.39	18.64	18.54	25.30	18.50	18.87	18.05	0.367

	Δa_7 (mm)	$\Delta a_7/b_0$	$\Delta a_7/W$	Δa_9 (mm)	$\Delta a_9/b_0$	$\Delta a_9/W$	Δa_{comp} (mm)	$\Delta a_{comp}/b_0$	$\Delta a_{comp}/W$	Δa_{max} (mm)
B9	8.67	0.578	0.247	8.31	0.545	0.237	7.54	0.500	0.215	9.39
B10	1.35	0.092	0.038	1.04	0.069	0.029	1.02	0.068	0.029	2.10
B2	1.14	0.074	0.033	0.87	0.056	0.025	0.76	0.050	0.021	1.40
B7	3.55	0.215	0.104	2.85	0.173	0.08	2.66	0.161	0.761	6.43

TABLE 5.2 Crack length measurements across the crack face of the three-point bend specimens ($B=17.5$ mm, $W=2B$) with different levels of ductile crack growth after Unloading Compliance tests.



PLATE 5.1a Titanium alloy (6Al-2Cb-1Ta-1Mo) crack face, fatigue precracked with constant ΔK or K_{max} .

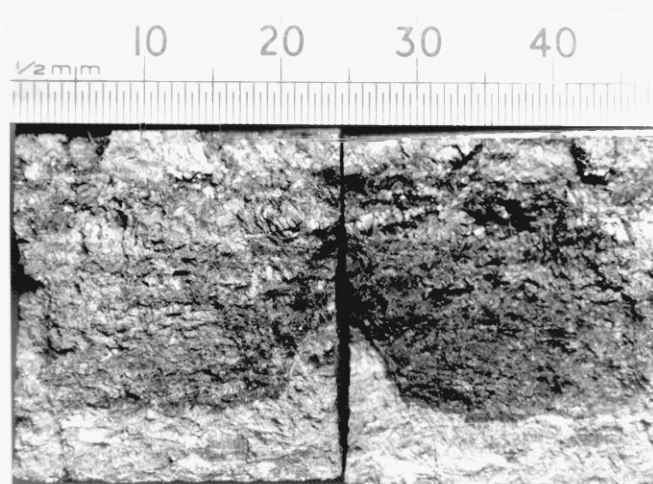


PLATE 5.1b Titanium alloy (6Al-2Cb-1Ta-1Mo) crack face without maintaining a constant K_{max} during fatigue precracking; showing distinct ridges on the fatigue region and no clear demarcation between this region and that of ductile tearing.

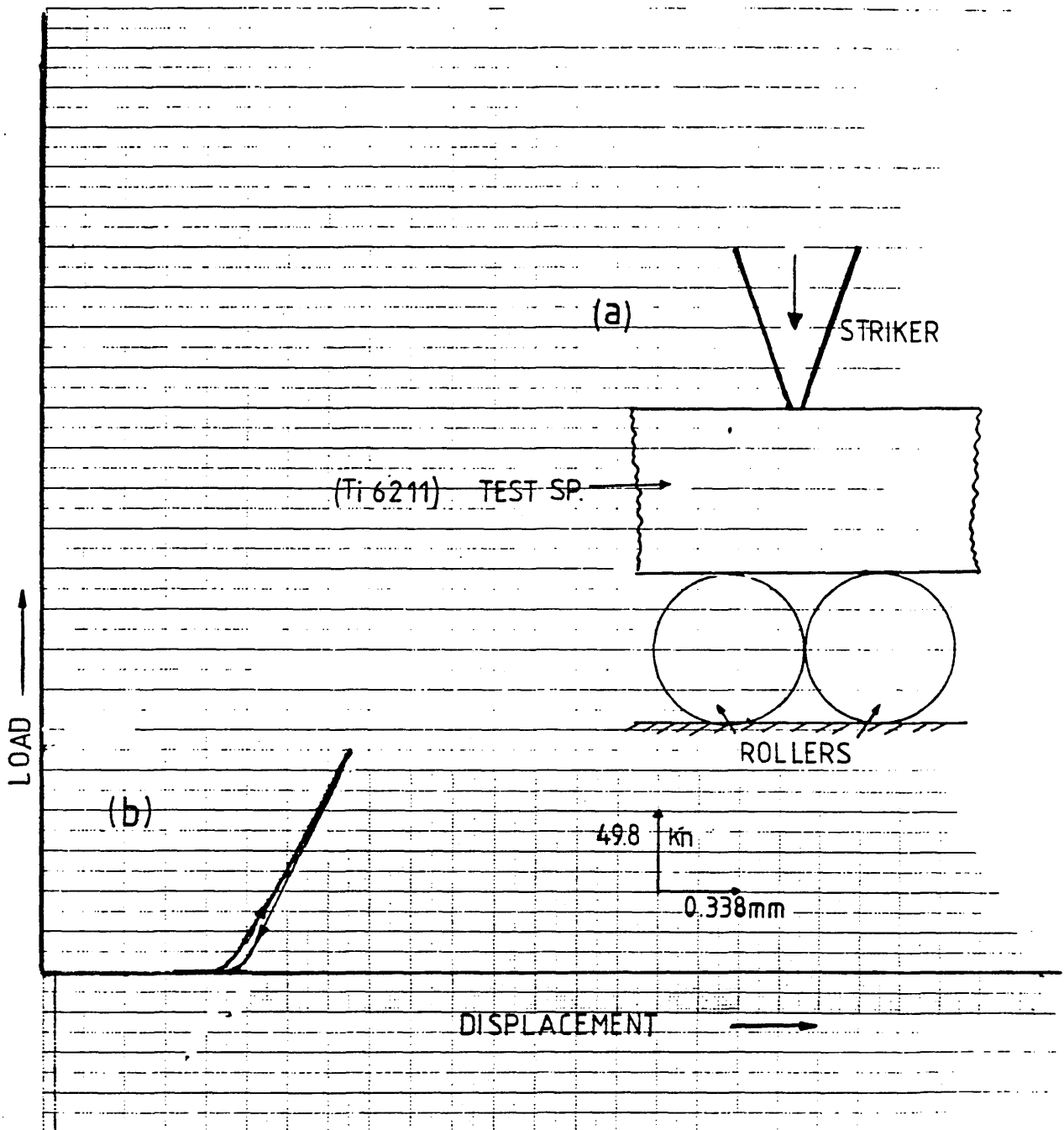


FIG. 5.1a Schematic of the roller indentation setup.

FIG. 5.1b Load-displacement trace of the setup shown in Fig.5.1a.

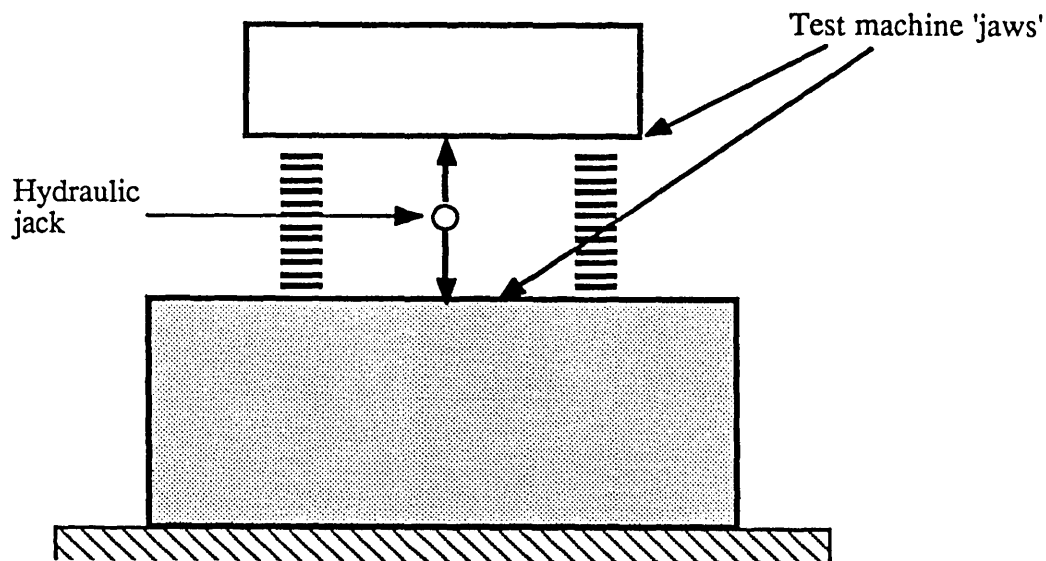


FIG. 5.2 Setup for the determination of machine compliance using a hydraulic jack

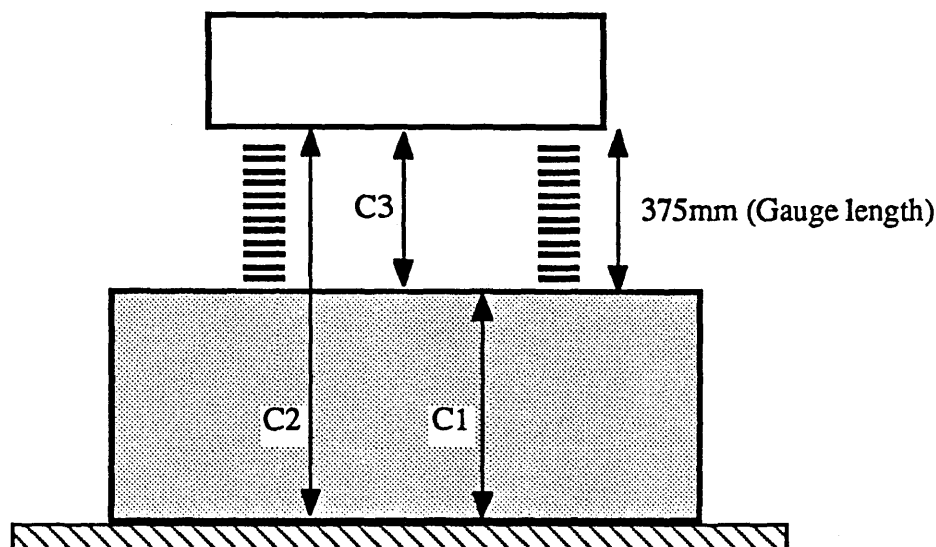


FIG. 5.3 Different compliance measurement positions on the Tinius Olsen machine
(See plate 4.3. for the Tinius Olsen machine)

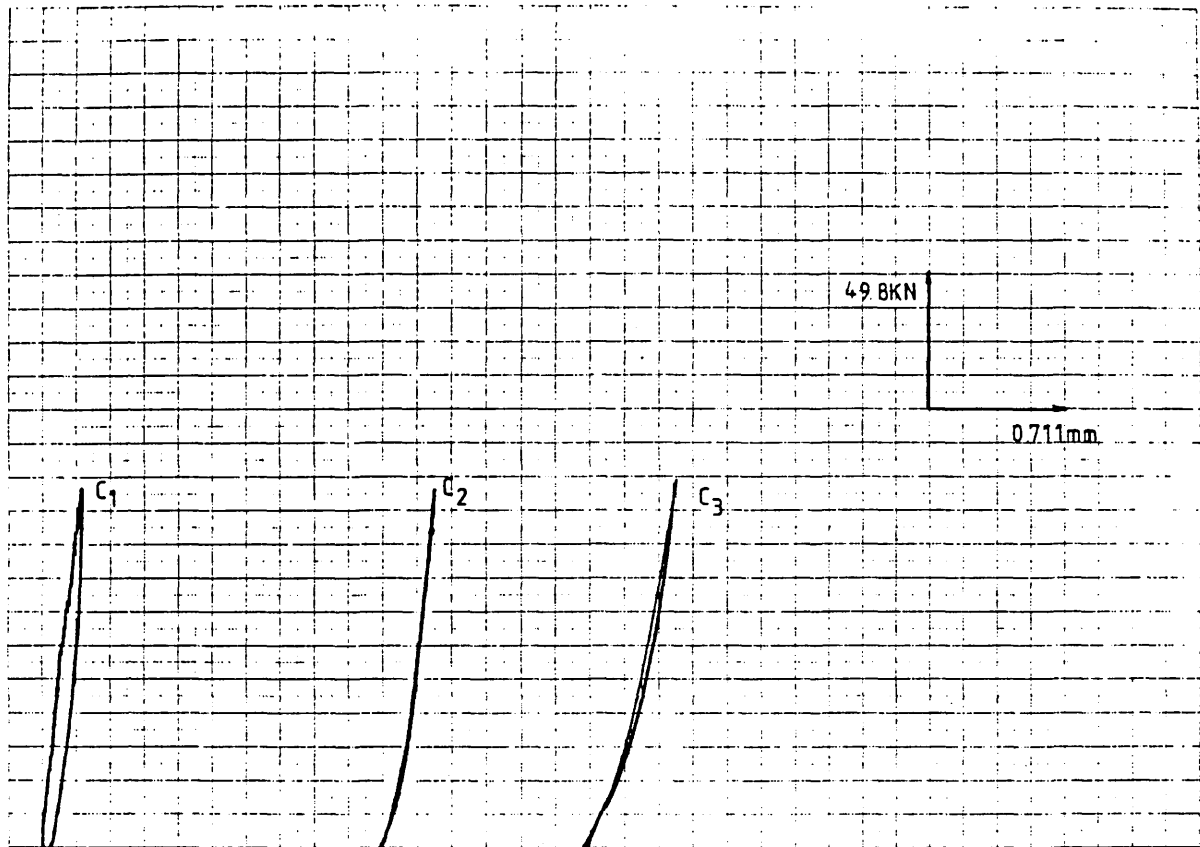


FIG. 5.4 Load-displacement traces for compliance measurement positions C₁, C₂ and C₃ as shown in Fig. 5.3 for a gauge length of 375 mm.

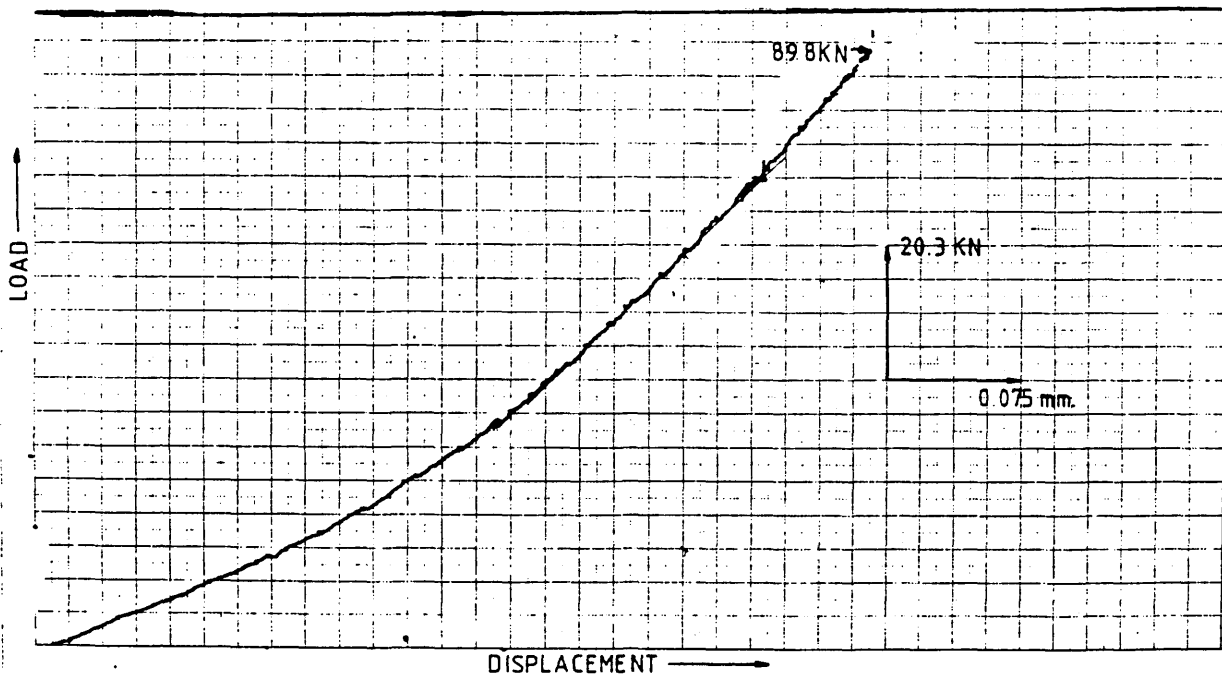


FIG. 5.5 'Slack' region of the Tinius Olsen machine prior to 'full' stretch as shown in Fig. 5.4 (Note scales on respective traces)

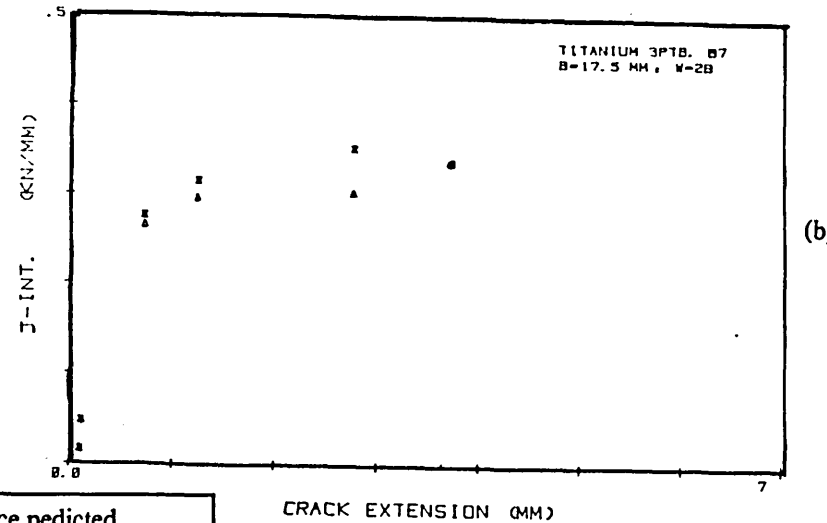
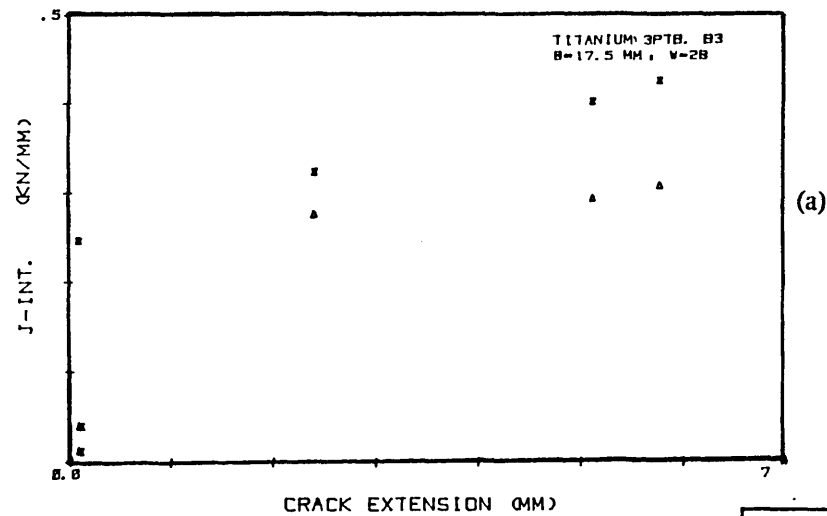
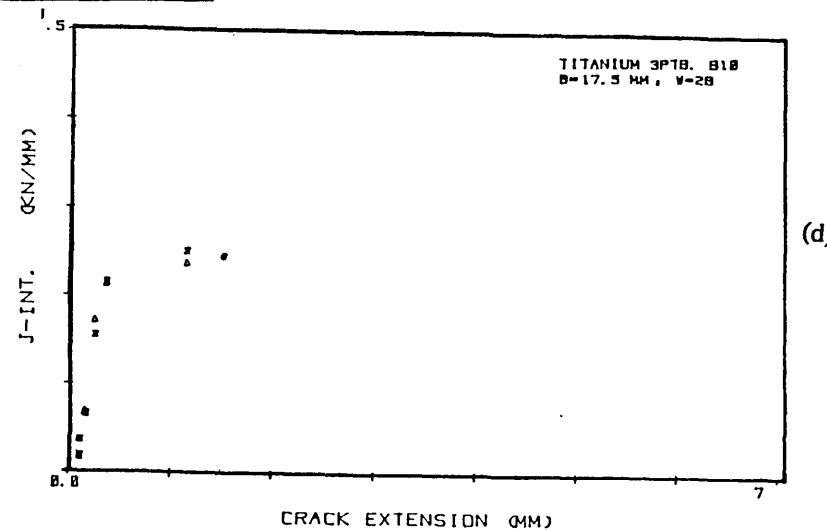
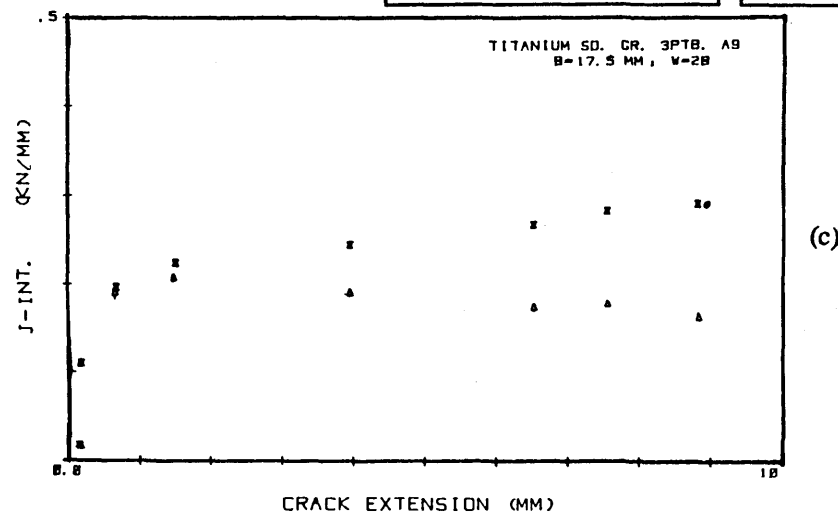


FIG. 5.6 Comparison of actual and compliance-predicted final crack lengths

X - J_0 compliance predicted
 Δ - J_{i+1} compliance predicted
 $\bullet$ - J_0 Manual



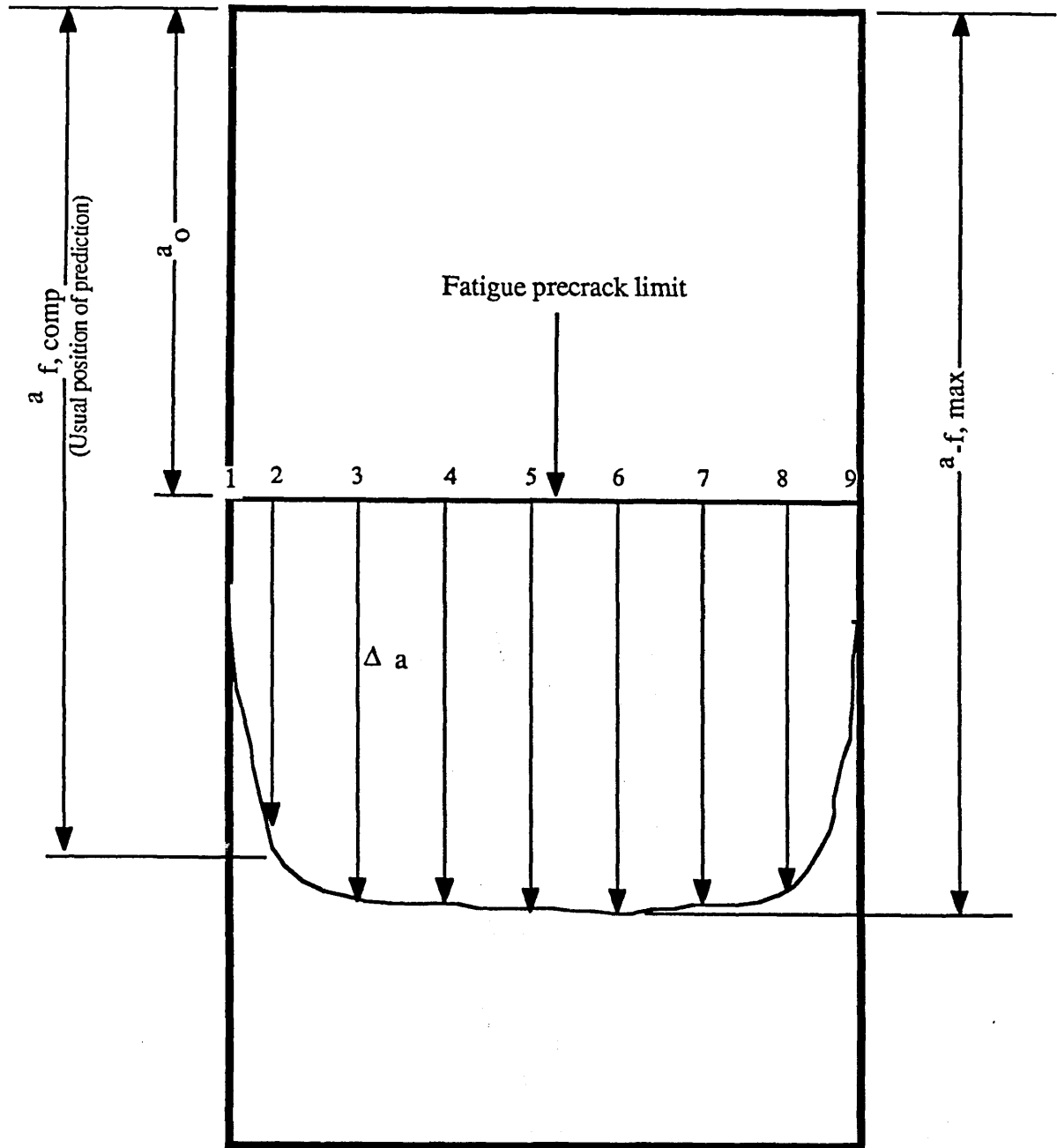


FIG. 5.7 Illustration of crack length measurement taken across crack face for curvature correction analysis

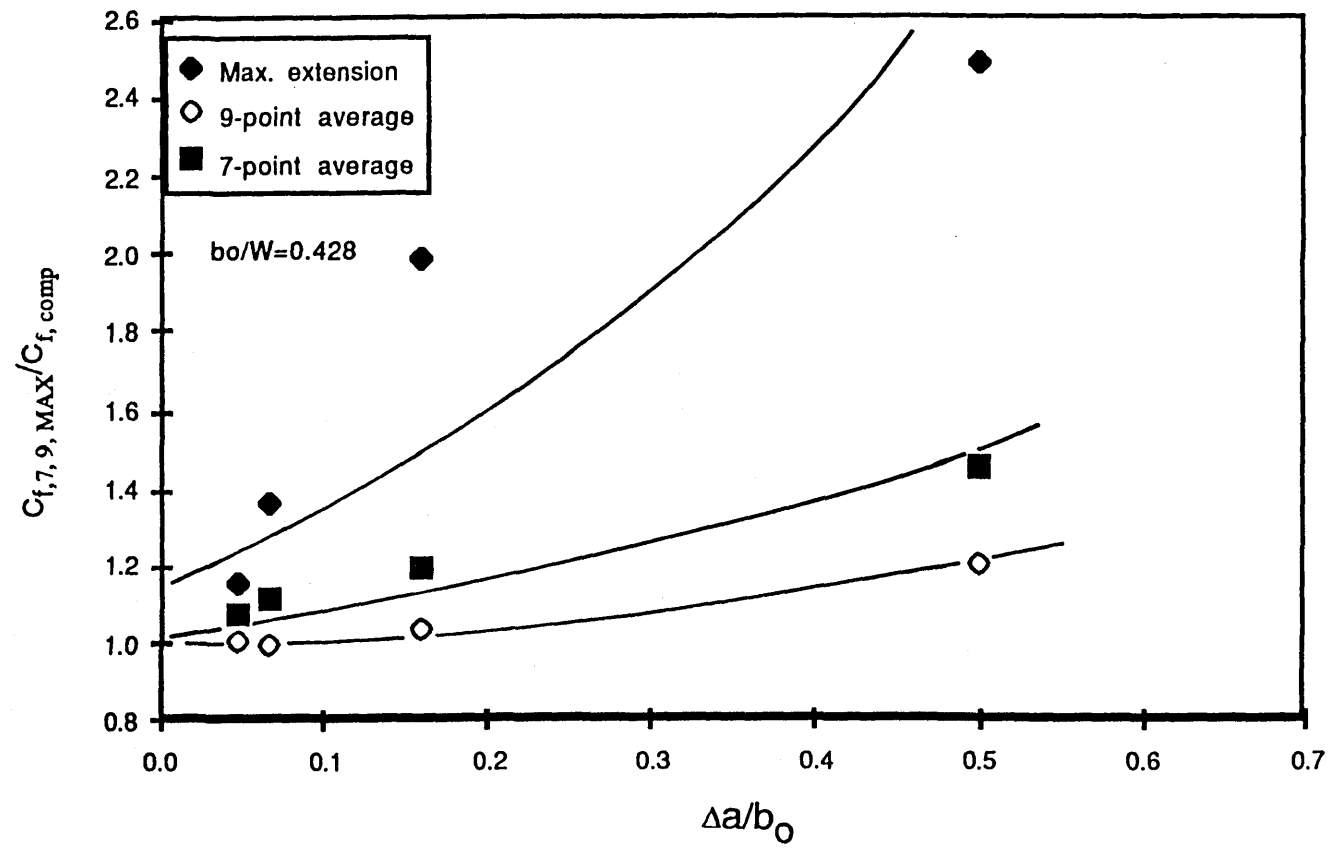


FIG. 5.8 Compliance ratio plot against normalised crack extension Ti(6-2-1-1).

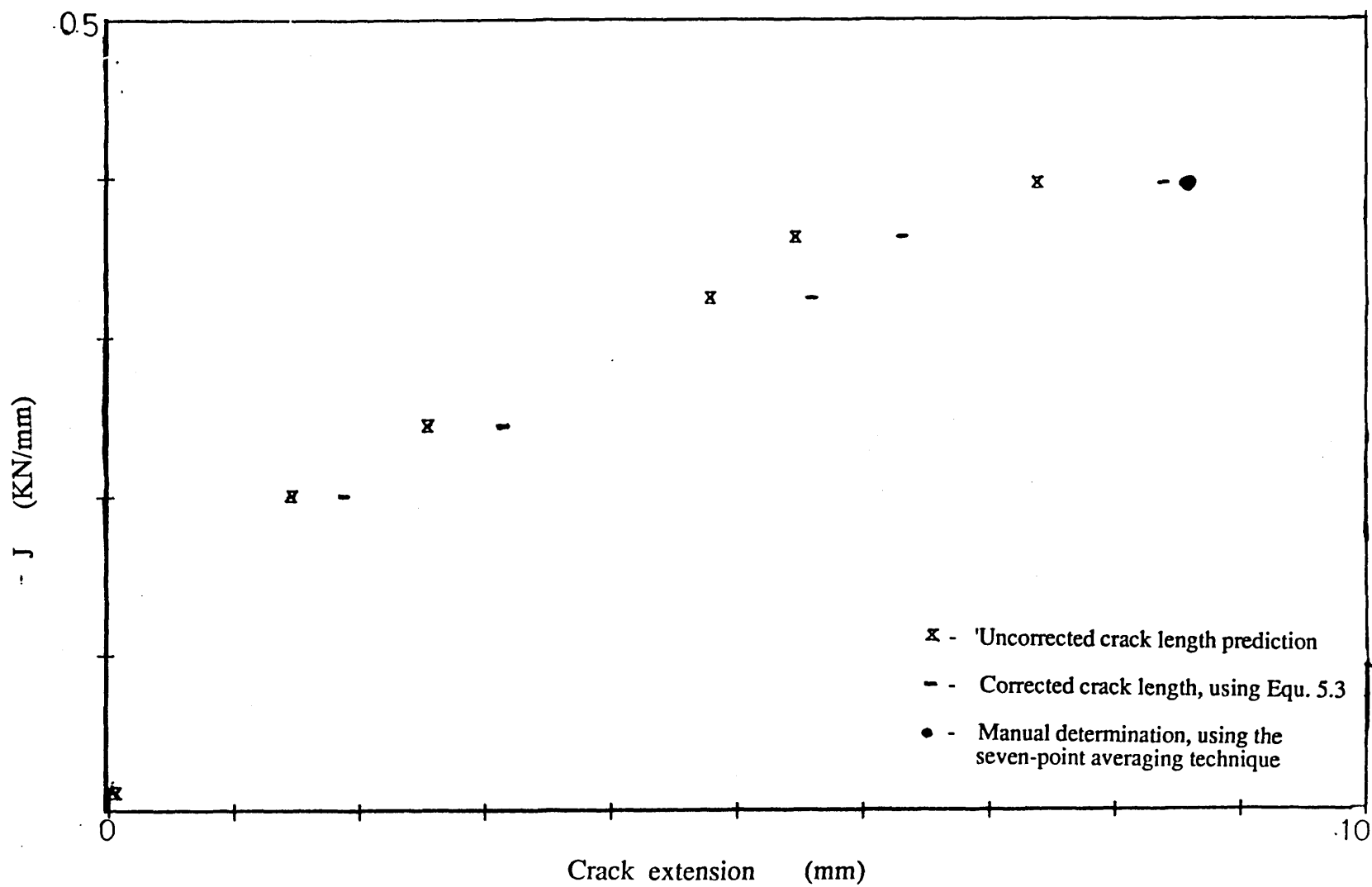


FIG. 5.9 Comparison of crack extension predictions.

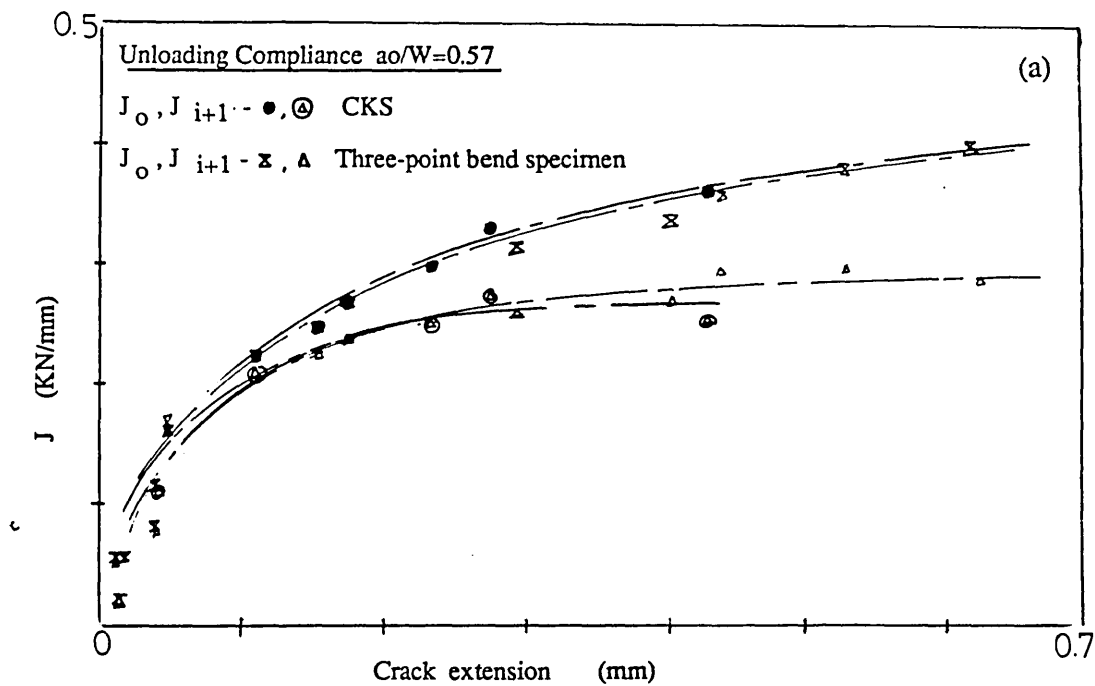
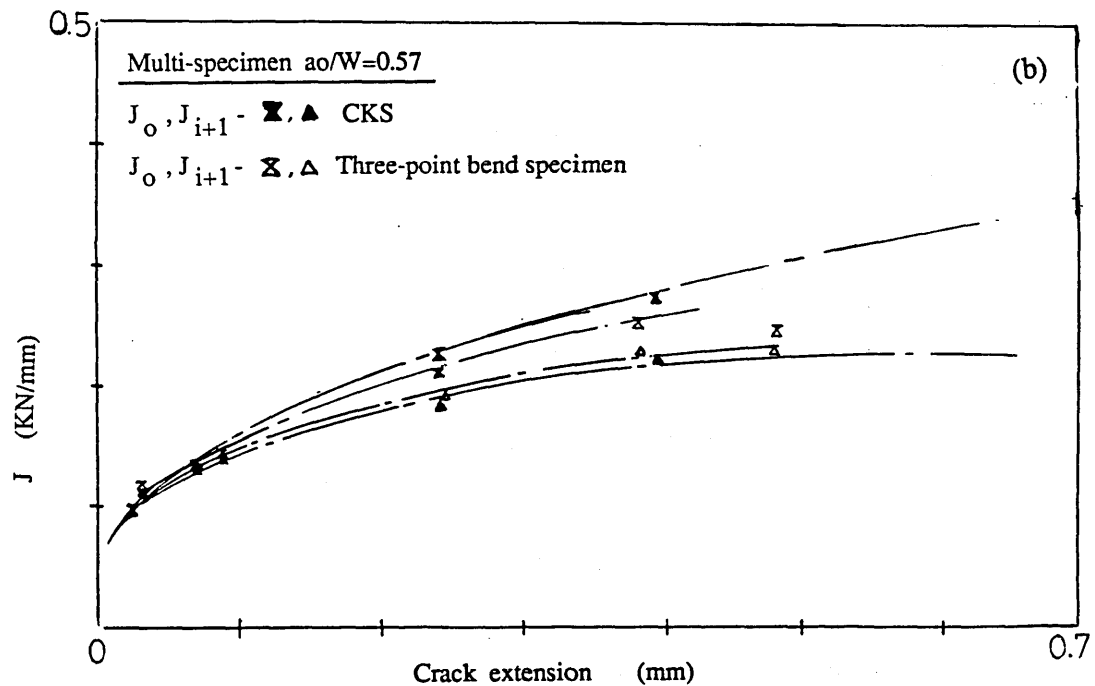


FIG.5.10a Effect of compact tension and three-point bend specimens on Unloading Compliance-obtained J-R curves.

FIG. 5.10b Effect of compact tension and three-point bend specimens on Multi-specimen-obtained J-R curves.



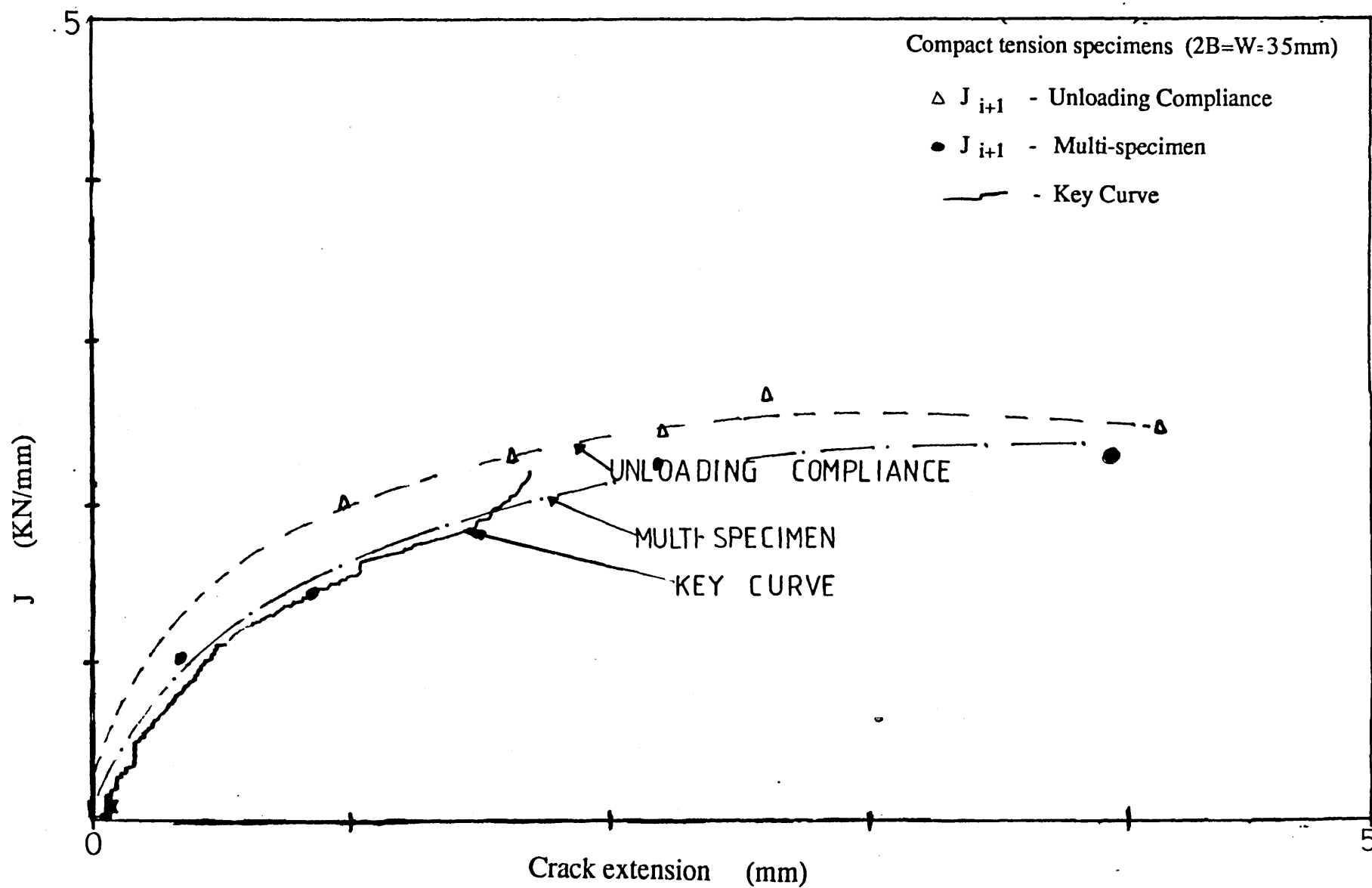


FIG.5.11 Comparison of testing methods used on compact tension specimens

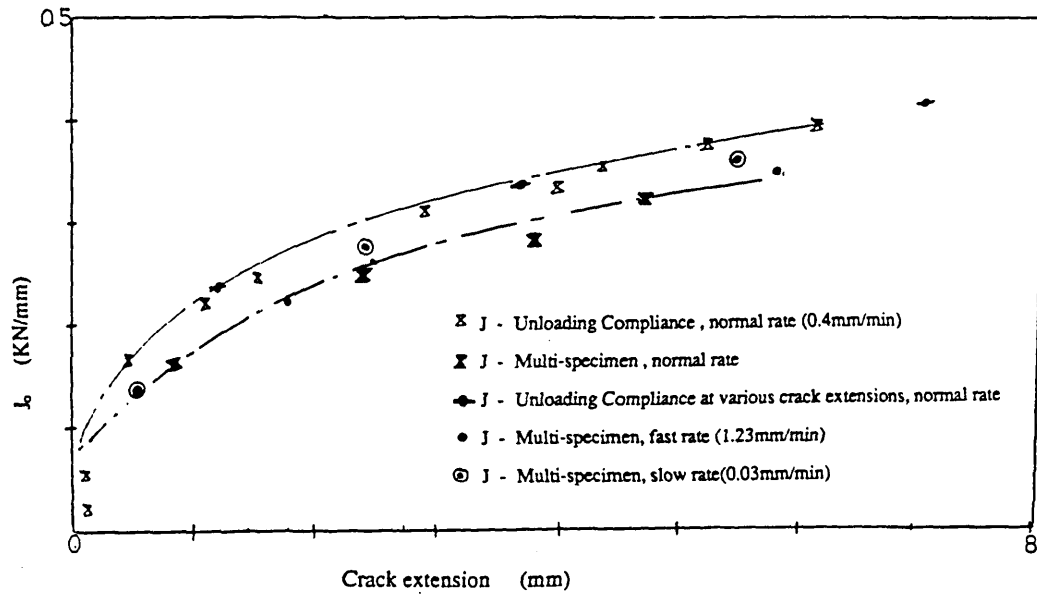


FIG. 5.12 Three-point bend J-R curves using the Unloading Compliance and Multi-specimen technique at different loading rates

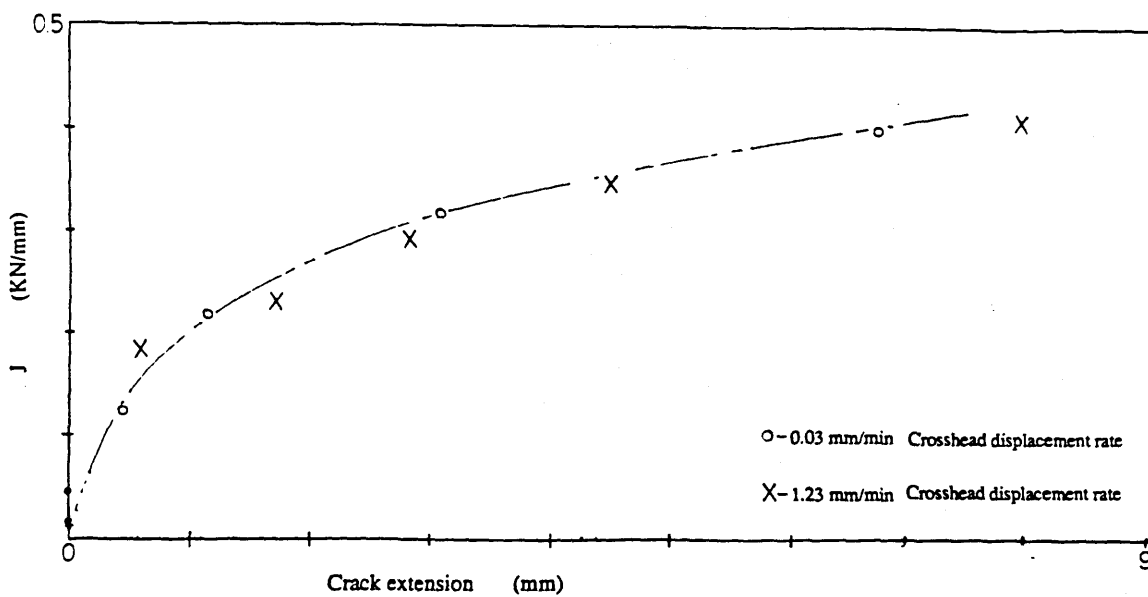


FIG. 5.13 Comparison of crosshead displacement rates for the Unloading Compliance tests using three-point bend specimens.

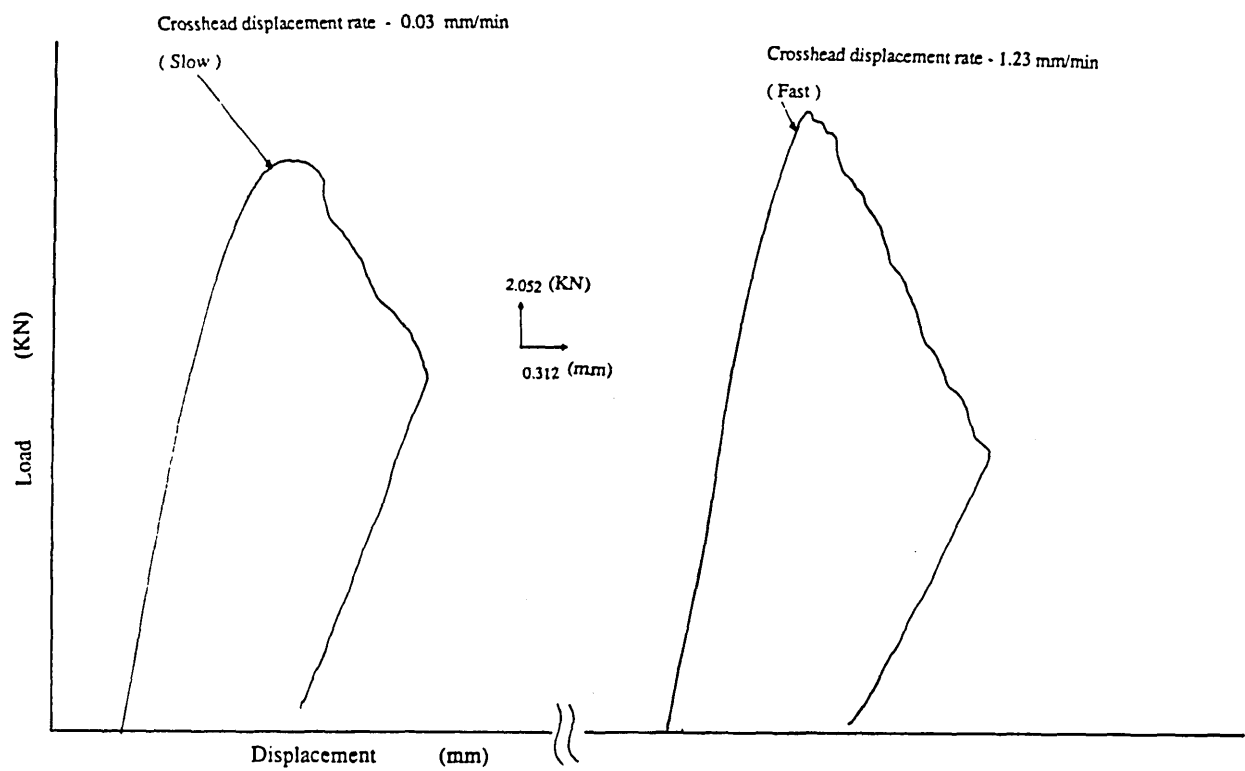


FIG. 5.14 Load-displacement traces for Ti(6-2-1-1) three-point bend specimens with different loading rates.

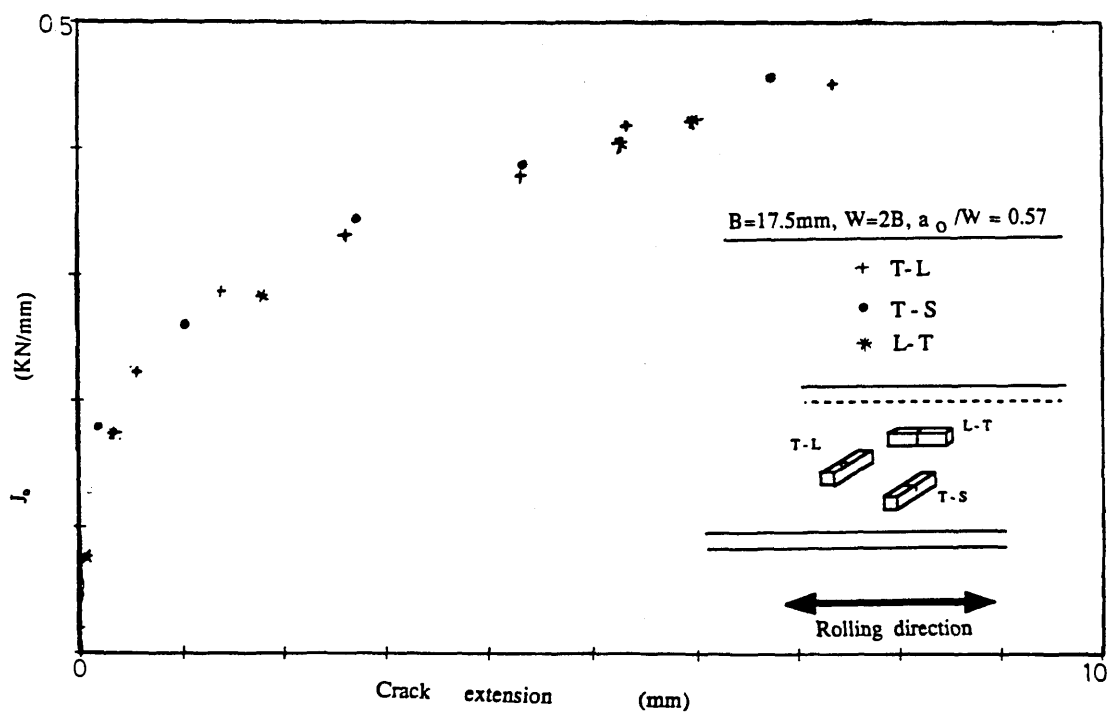


FIG. 5.15 Orientation effects of Ti(6-2-1-1) on J-R curves.

CHAPTER 6

RESULTS ON J-R CURVES

6.1 INTRODUCTION

The Unloading Compliance described in chapter four was used to generate J-R curves for a quantitative study of the effects of configurations within the three-point bend geometry for the T-L orientation. The effects of absolute size, in-plane constraint, ligament, thickness and sidegrooved specimens were investigated. In this chapter a further representation of R curves is made by the normalisation of R curves, by parameters that are suspected to limit plastic deformation, see Etemad and Turner (1985c). It must however be noted that this chapter serves to present the results alone; comments, observations and discussion of the results presented here are appropriately made in chapter nine.

6.2 GENERAL DETAILS OF TESTING AND ANALYSIS

6.2.1 TEST DETAILS

Fatigue precracked bend specimens of the type shown in Fig 4.2 were used throughout this study. A schematic of the geometry variations is shown in fig 6.1. The Unloading Compliance test technique described in subsection 4.5.2. was used to generate all data shown below. The loading rate was kept constant at 0.7mm/min and all tests were performed at room temperature, 20°C. Identical test techniques and conditions were applied to sidegrooved specimens. 1.5mm sidegrooves were inserted on each side of each specimen geometry shown in fig 6.1 except the 35mm x 4mm and 10mm x 10mm specimens.

6.2.2 ANALYSIS

The J integral was evaluated using three formulations, J_O (Equ.2.27), J_{i+1} (Equ. 2.28) and J_m (Equ. 2.30a).

Recalling these equations:

$$J_O = \eta (U + \Delta U) / Bb_O \quad (2.27)$$

$$J_T = J_{i+1} = [J_i + (\eta/b)_i U_{i,i+1}] [1 - (\gamma/b)_i (a_{i+1} - a_i)] \quad (2.28)$$

$$J_m = (J_m)_i + (\eta/b)_i (U_{i+1} - U_i)/B \quad (2.30a)$$

All the unloading compliance slopes and subsequent crack length evaluations were done on-line with the computer interactive program, SULCO, described in appendix II.

6.3 R CURVE RESULTS

6.3.1 VARIATION IN ABSOLUTE SIZE

J-R curves for different absolute sizes of three point-bend specimens ($a_0/W = 0.55$, $S/W = 4$) shown below are presented.

<u>B (mm)</u>	<u>W (mm)</u>
10	10
20	20
30	30
35	35
40	40

These are shown with appropriate abscissa normalisation parameters, b_0 and B for J_0 , J_{i+1} and J_m formulations of the J-Integral, in figs 6.2a to 6.4c.

6.3.2 VARIATION IN IN-PLANE CONSTRAINT (DIFFERENT a_0/W ratios)

For a fixed specimen size ($B=17.5\text{mm}$, $W=35\text{mm}$) the crack depth ratio was varied from $a_0/W = 0.2$ to 0.75 . The J-R curves for this category of investigation are shown in figs 6.5a to 6.7b. The normalisation parameter used here was the original ligament, b_0 .

6.3.3 VARIATION IN THICKNESS

For a width (W) of 35mm and initial crack depth ratio (a_0/W) of 0.55 , specimens of thicknesses that varied as shown in fig. 6.1 were tested. Both the original ligament (b_0) and current ligament (b_i) are used as normalisation parameters in this category of study. The J-R curves generated from this study are shown in figs 6.8a to 6.10b.

6.3.4 VARIATION IN LIGAMENT LENGTH

With a relatively constant crack depth ratio and a fixed thickness, the width was altered and again, the actual specimen dimensions investigated are shown in fig. 6.1. The J-R curves with different initial ligaments are shown in figs 6.11a to 6.13b.

6.4 RESULTS WITH SIDE GROOVED SPECIMENS

The depth of the sidegroove was chosen to coincide with the approximate value of the plane strain plastic zone size, r_{pe} , (Equ. 2.6) for Ti (6-2-1-1). 1.5 mm sidegrooves were introduced on each side of the majority of specimens shown in fig. 6.1.

The resulting J-R curves are presented only for the J_0 and J_{i+1} versions of the J-Integral since J_m appeared to show similar trends to that of J_0 , see fig. 6.2 to 6.13. As can be seen in figs. 6.14a to 6.17b, normalisation of these sidegrooved specimens are arguably not necessary.

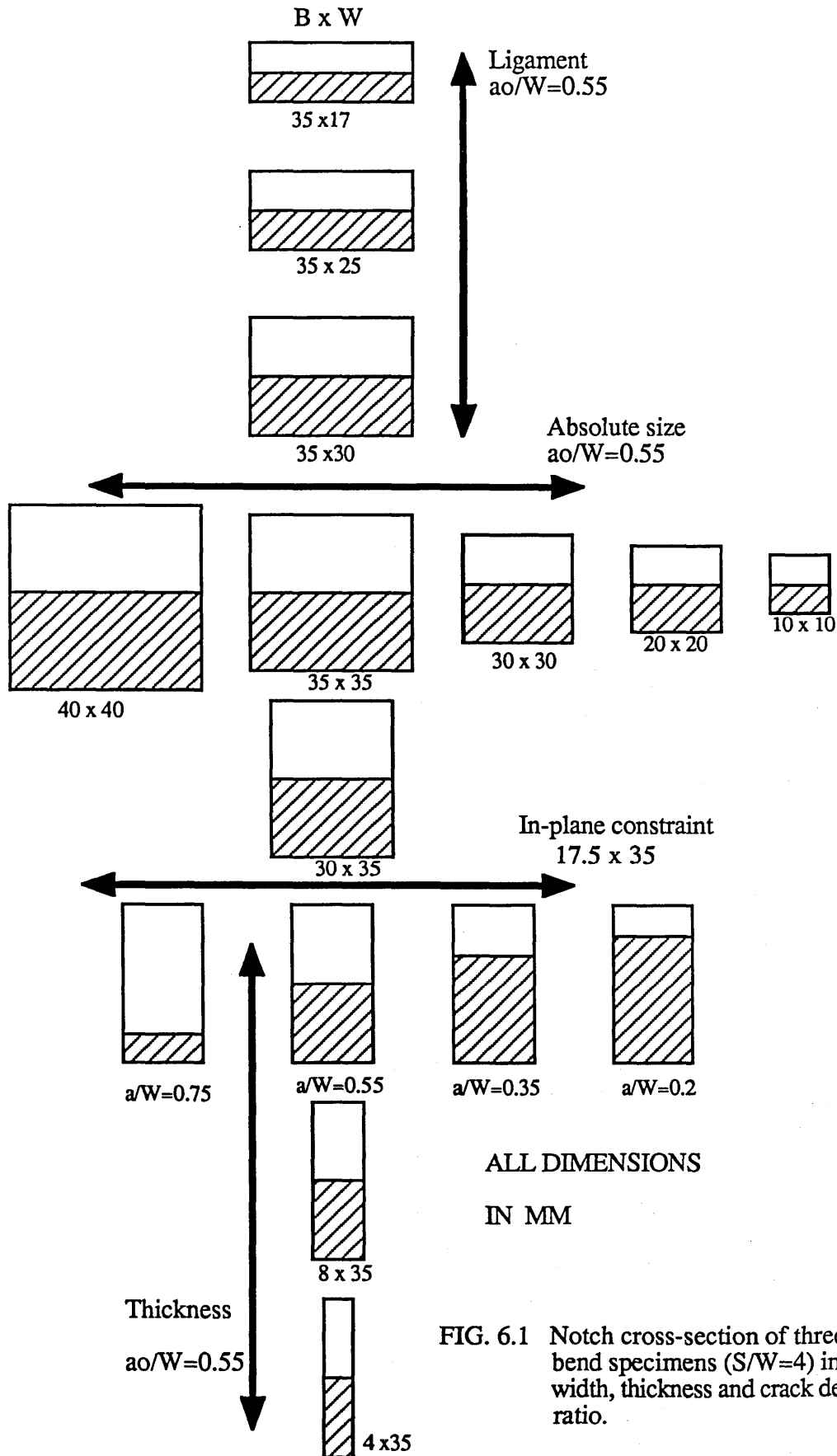


FIG. 6.1 Notch cross-section of three-point bend specimens ($S/W=4$) indicating width, thickness and crack depth ratio.

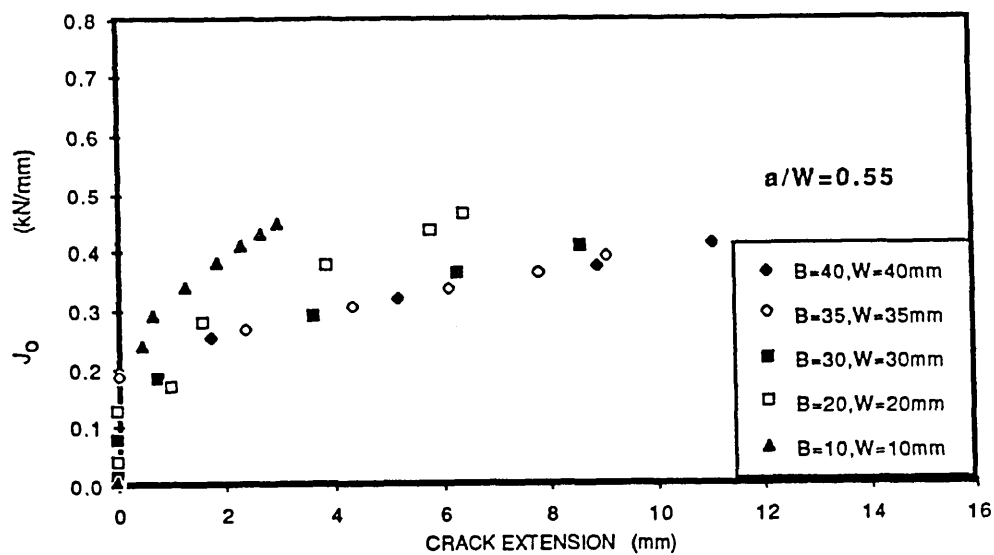


FIG.6.2a J_0 - Δa R curves for various absolute specimen sizes in the three-point bend configuration.

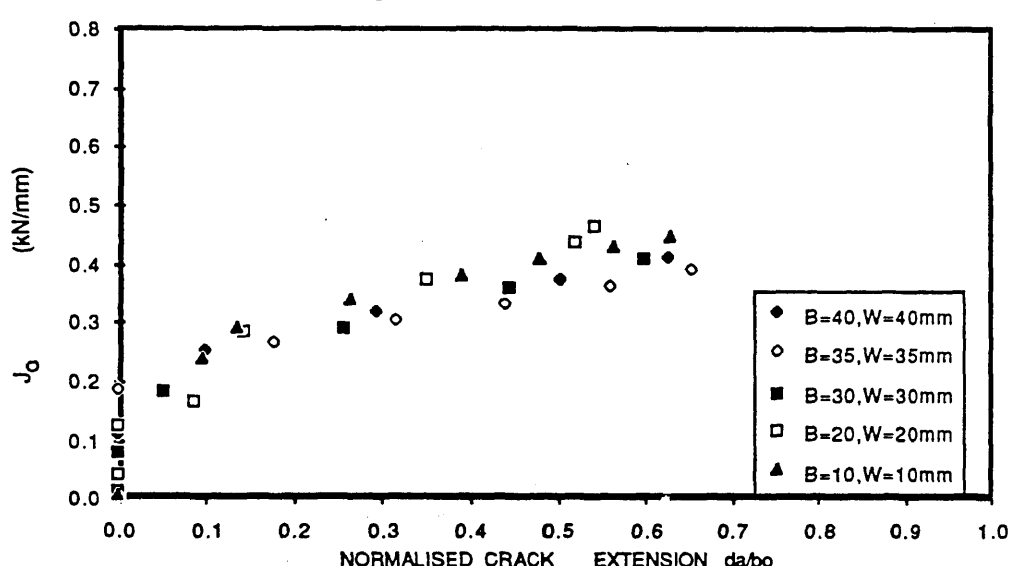


FIG. 6.2b J_0 -R curves with the abscissa normalised with b_0 for various absolute specimen sizes in the three-point bend configuration.

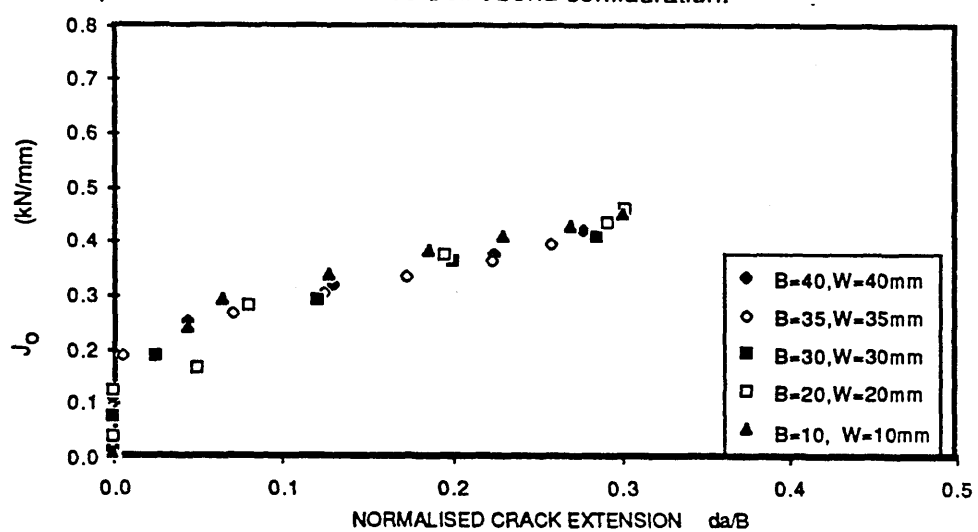


FIG. 6.2c J_0 -R curves with the abscissa normalised with B for various absolute specimen sizes in the three-point bend configuration.

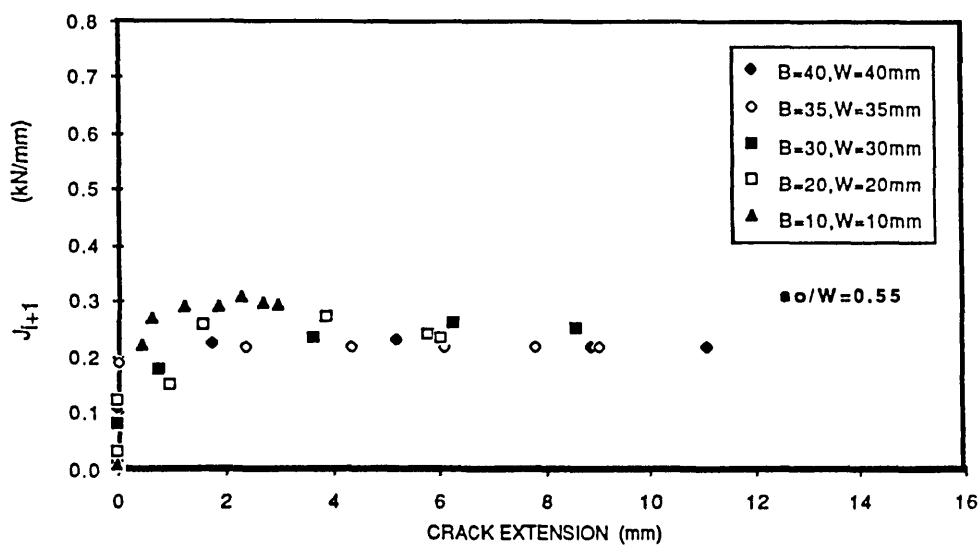


FIG. 6.3a $J_{i+1} - \Delta a$ R curves for various absolute specimen sizes in the three-point bend configuration.

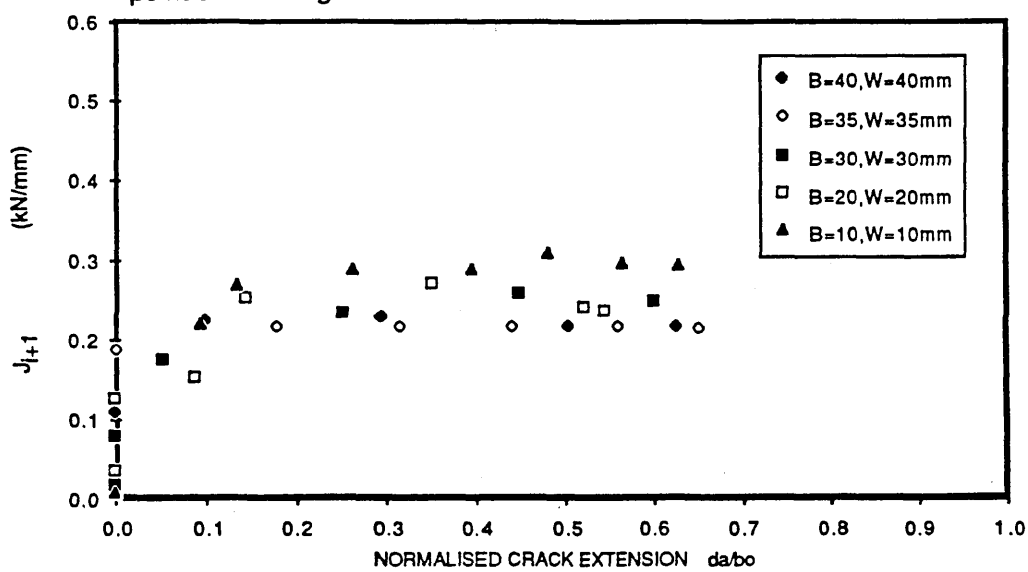


FIG. 6.3b J_{i+1} -R curves with the abscissa normalised with b_0 for various absolute specimen sizes in the three-point bend configuration.

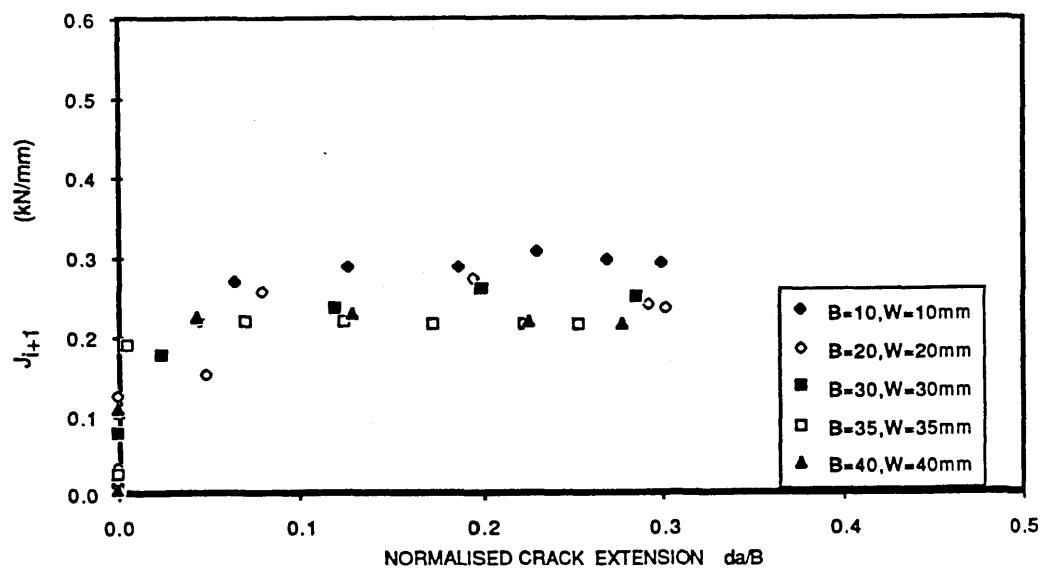
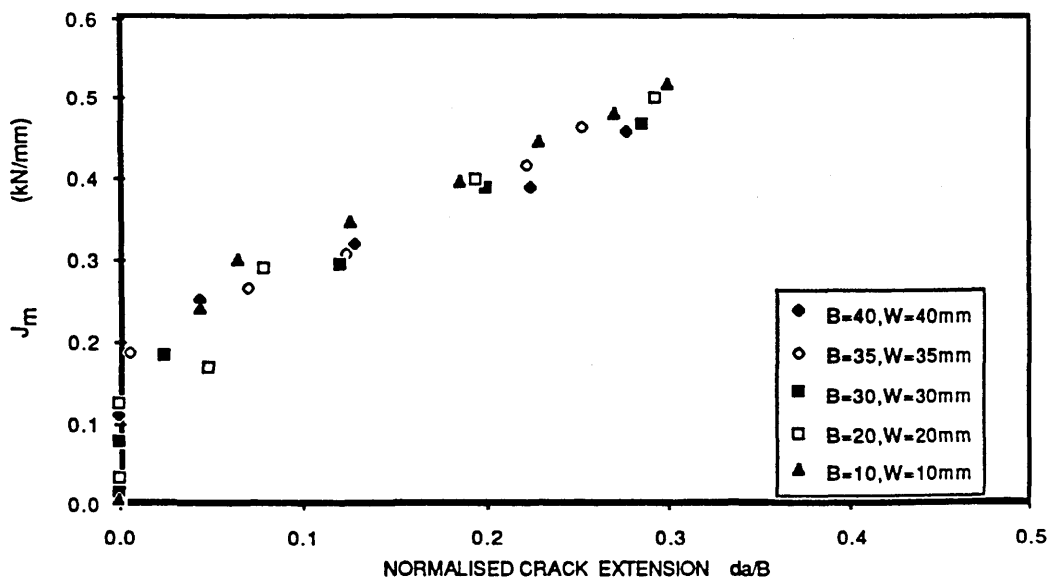
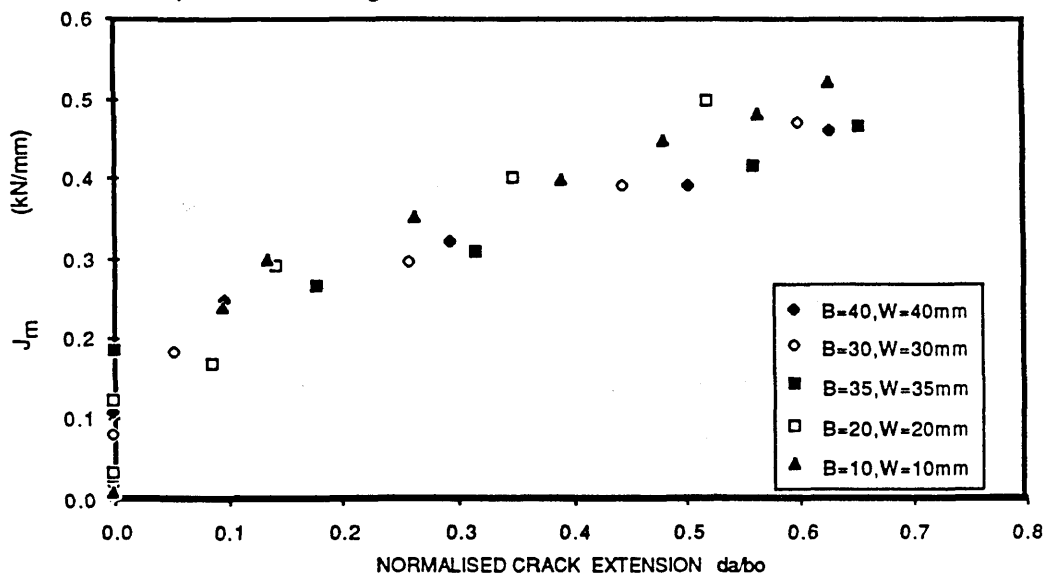
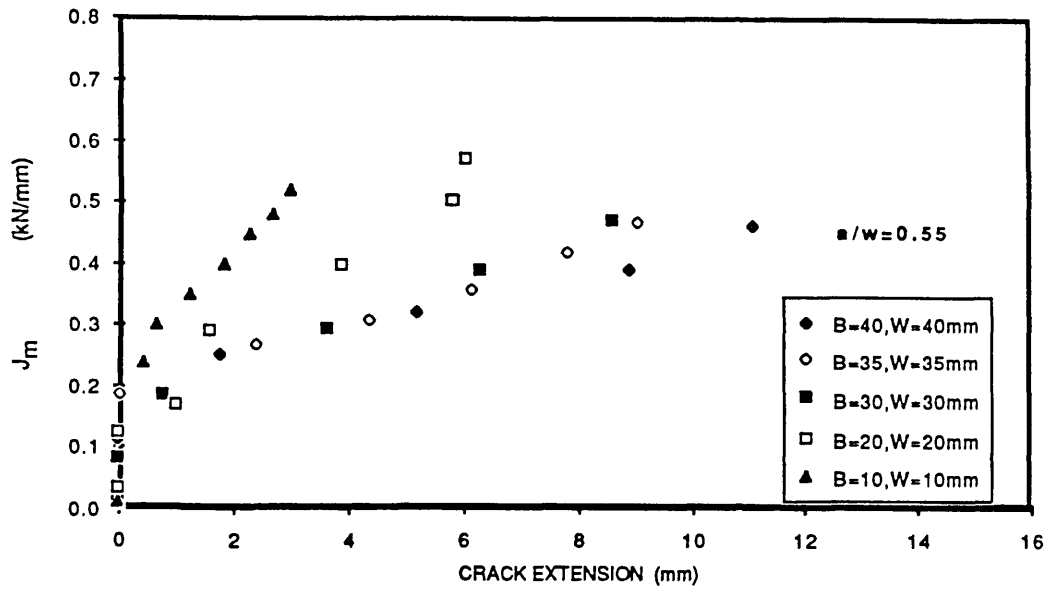


FIG. 6.3c J_{i+1} -R curves with the abscissa normalised with B for various absolute specimen sizes in the three-point bend configuration.



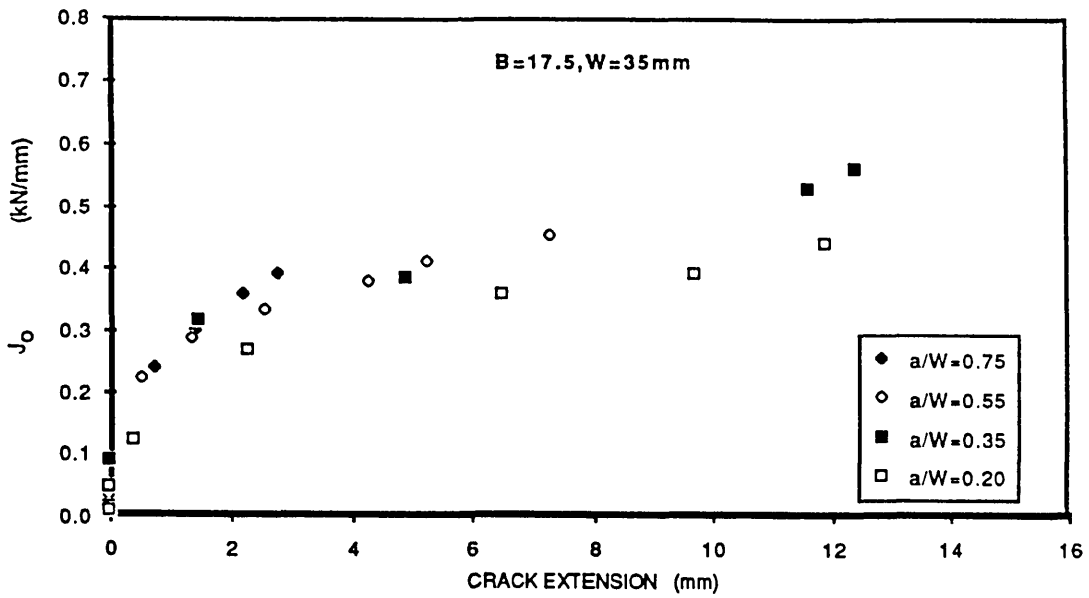


FIG. 6.5a J_0 - Δa R curves for various crack depth ratios in the three-point bend configuration.

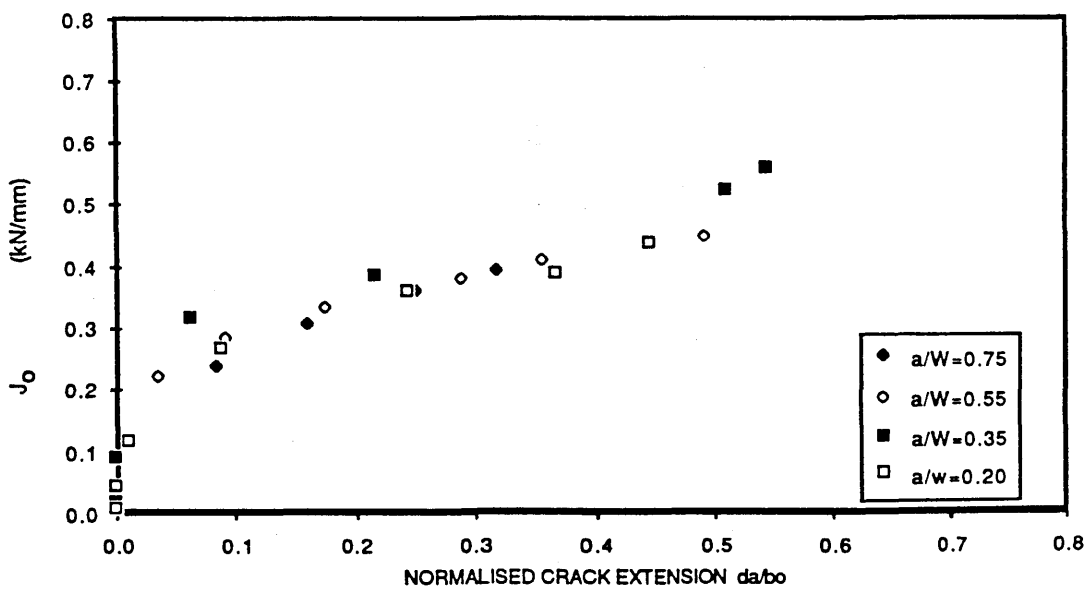
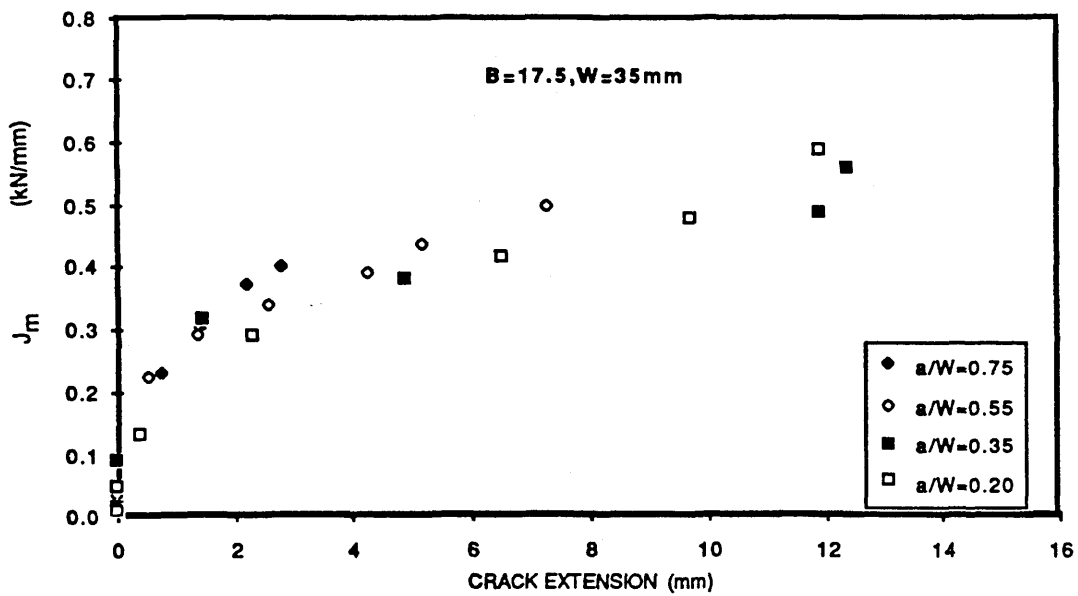
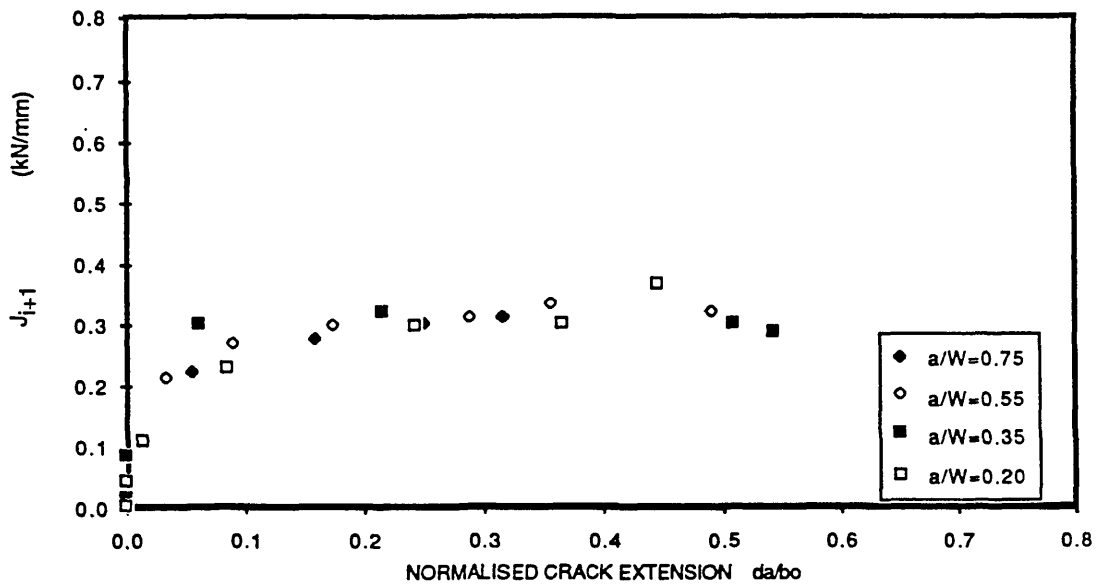
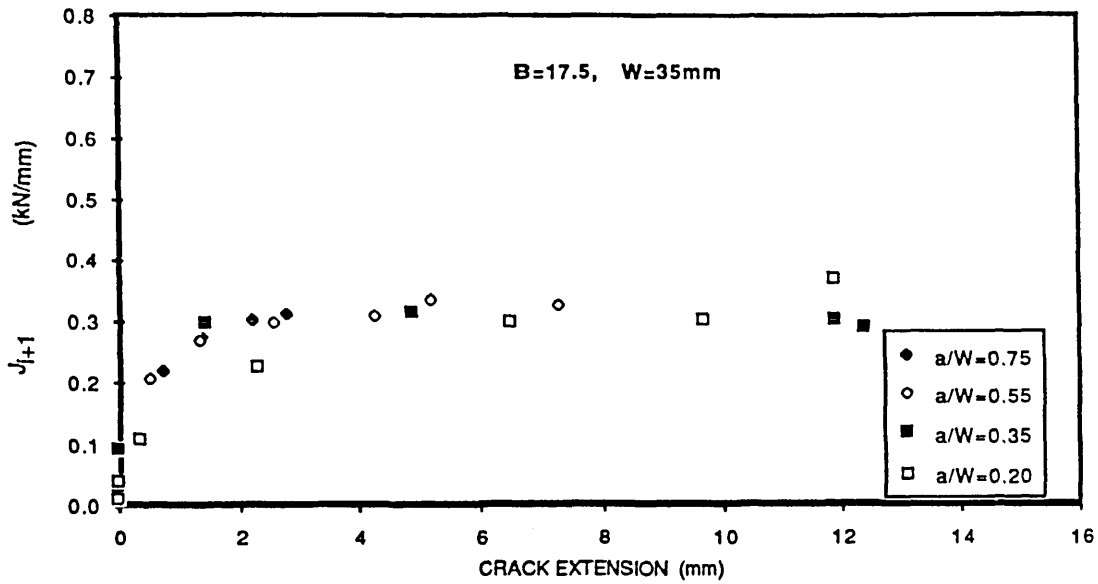


FIG. 6.5b J_0 -R curves with the abscissa normalised with the original ligament, b_0 , for various crack depth ratios in the three-point bend configuration.



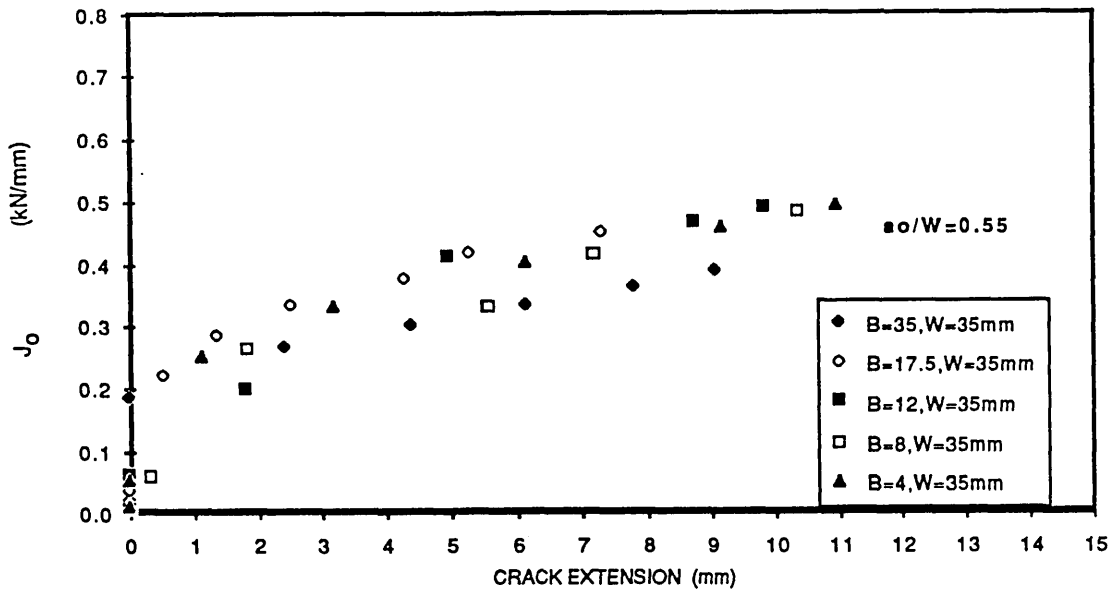


FIG. 6.8a J_0 - Δa (R) curves for various thicknesses in the three-point bend configuration.

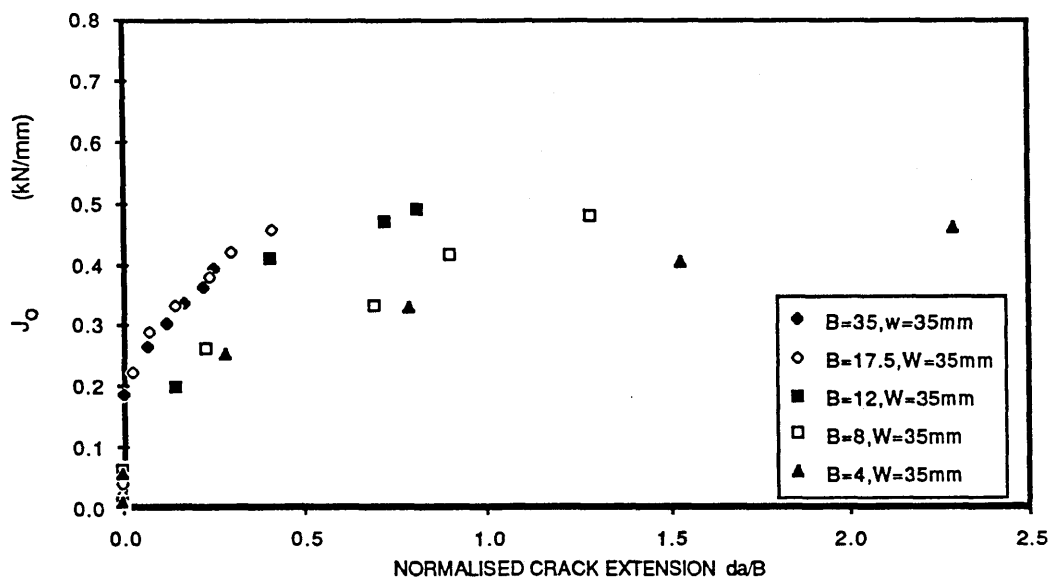


FIG. 6.8b J_0 -R curves with the abscissa normalised with the thickness, B , for various thicknesses in the three-point bend configuration.

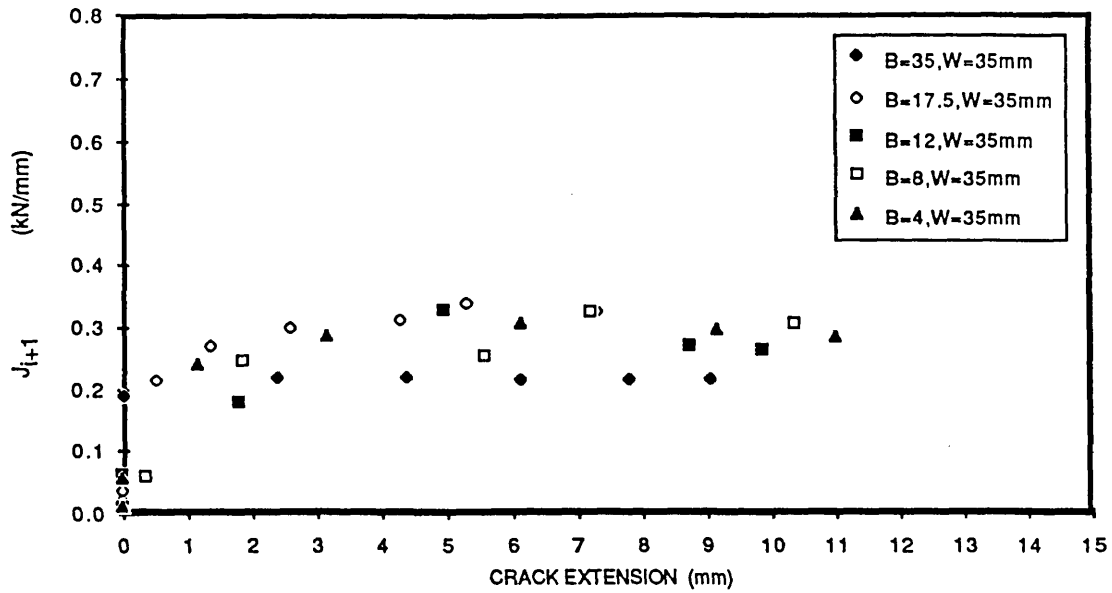


FIG. 6.9a $J_{i+1} - \Delta a(R)$ curves for various thicknesses in the three-point bend configuration.

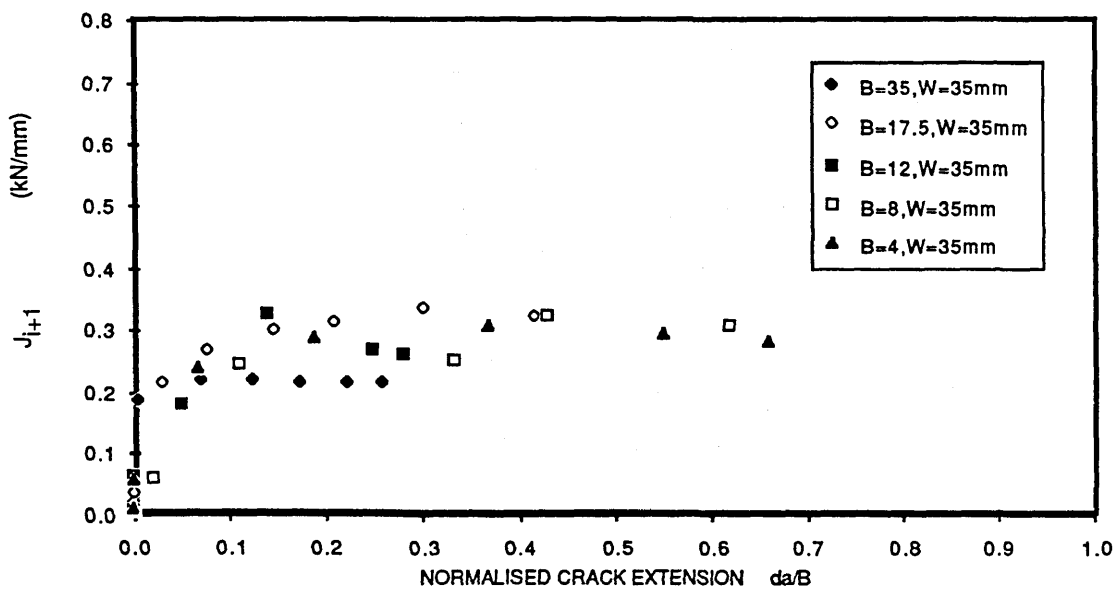


FIG. 6.9b $J_{i+1} - R$ curves with the abscissa normalised with the thickness, B , for various thicknesses in the three-point bend configuration.

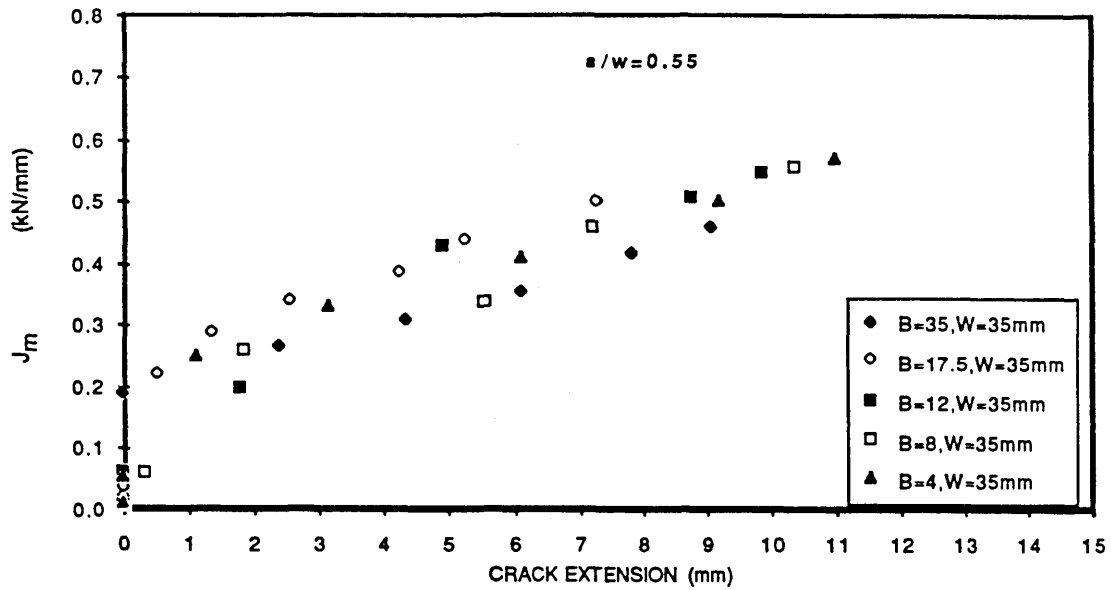


FIG. 6.10a J_m - $\Delta a(R)$ curves for various thicknesses in the three-point bend configuration.

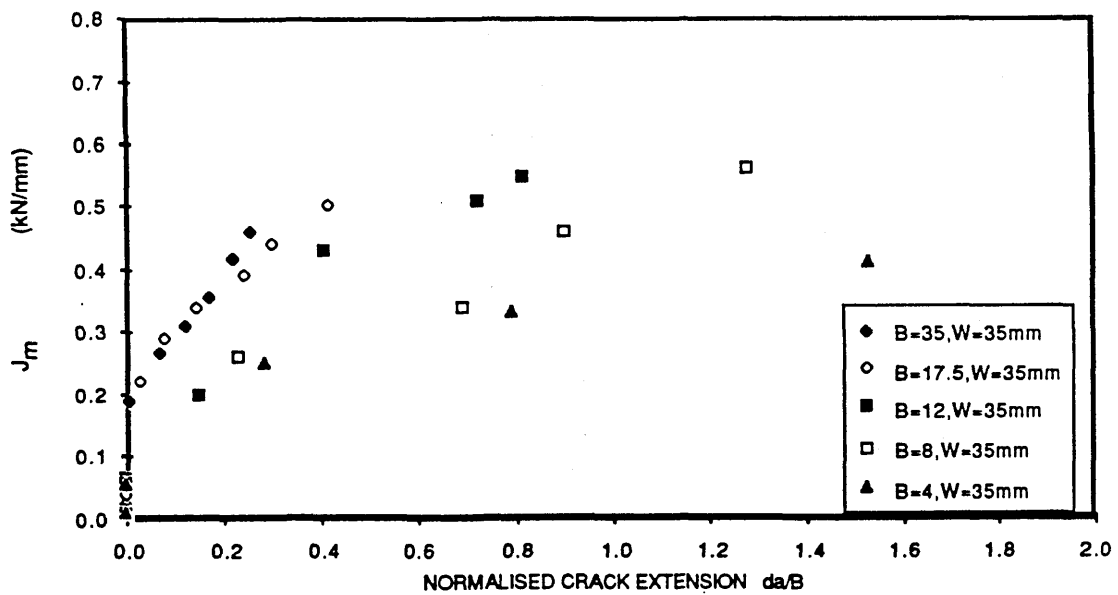


FIG. 6.10b J_m - R curves with the abscissa normalised with the thickness, B , for various thicknesses in the three-point bend configuration.

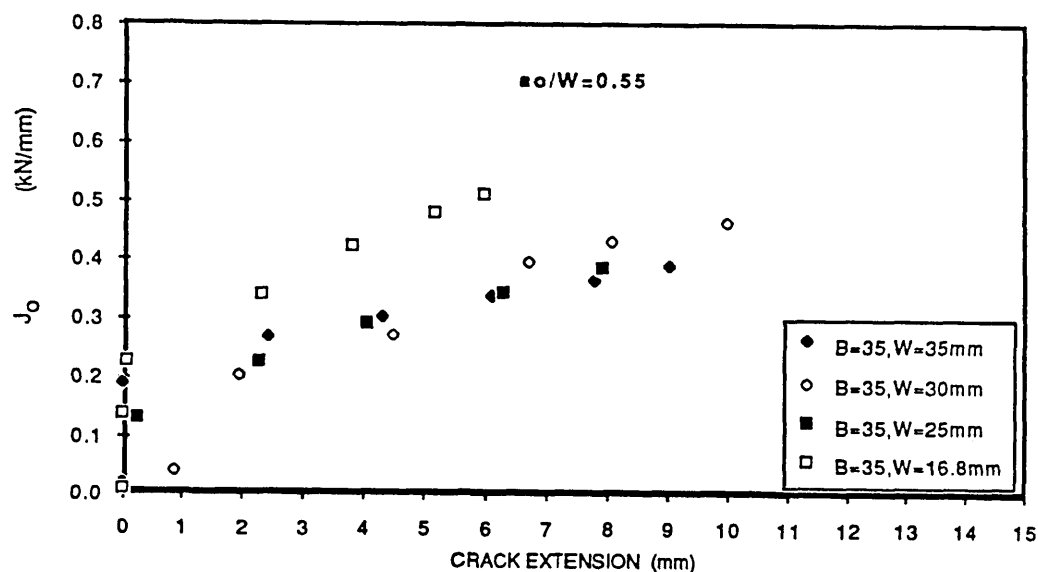


FIG. 6.11a $J_0 - \Delta a(R)$ curves for various ligaments in the three-point bend configuration.

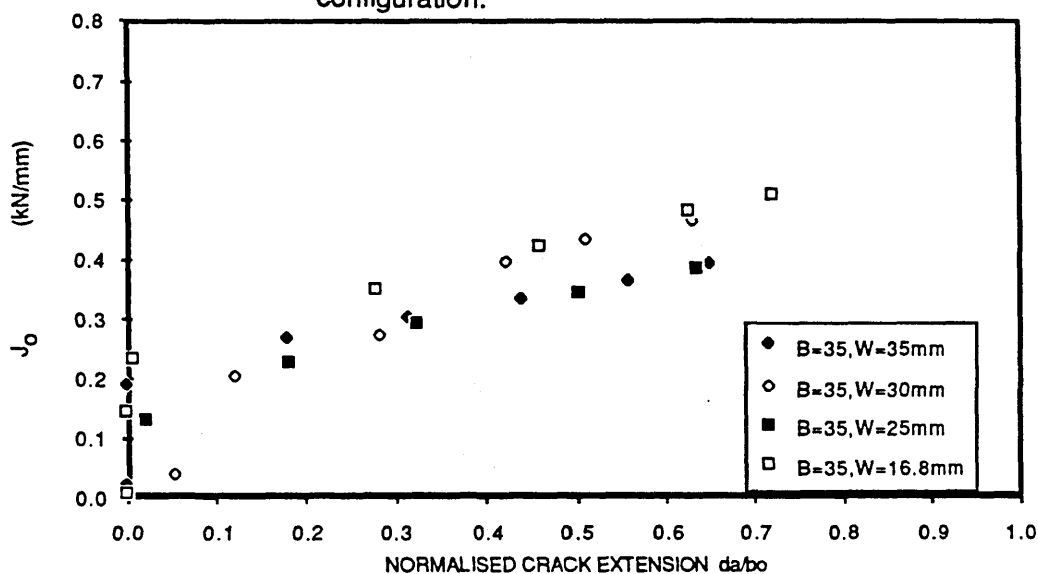


FIG. 6.11b J_0 -R curves with the abscissa normalised with the original ligament, b_0 , for various ligaments in the three-point bend configuration.

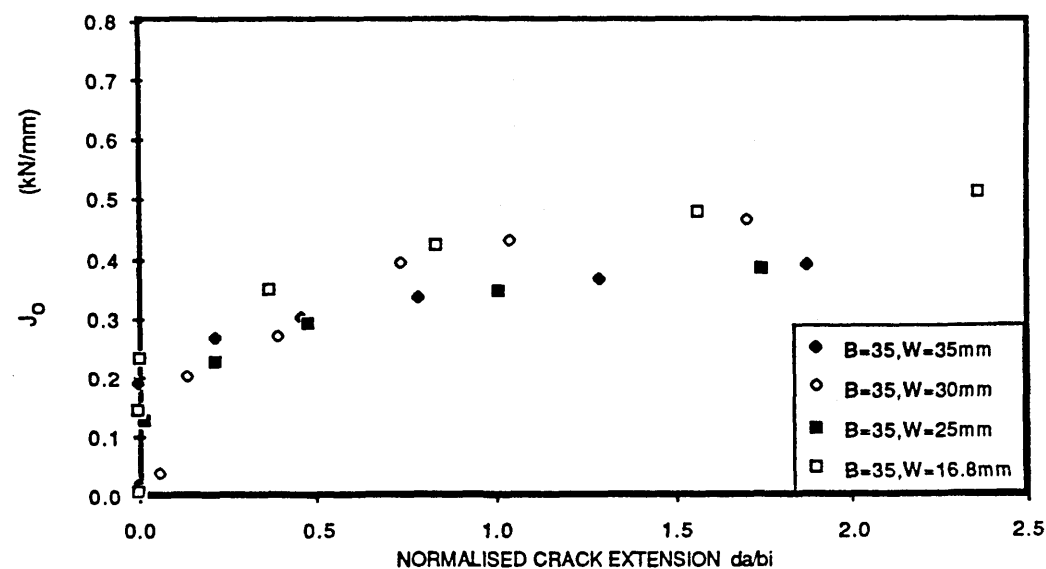


FIG. 6.11c J_0 -R curves with the abscissa normalised with the current ligament, b_i , for various ligaments in the three-point bend configuration.

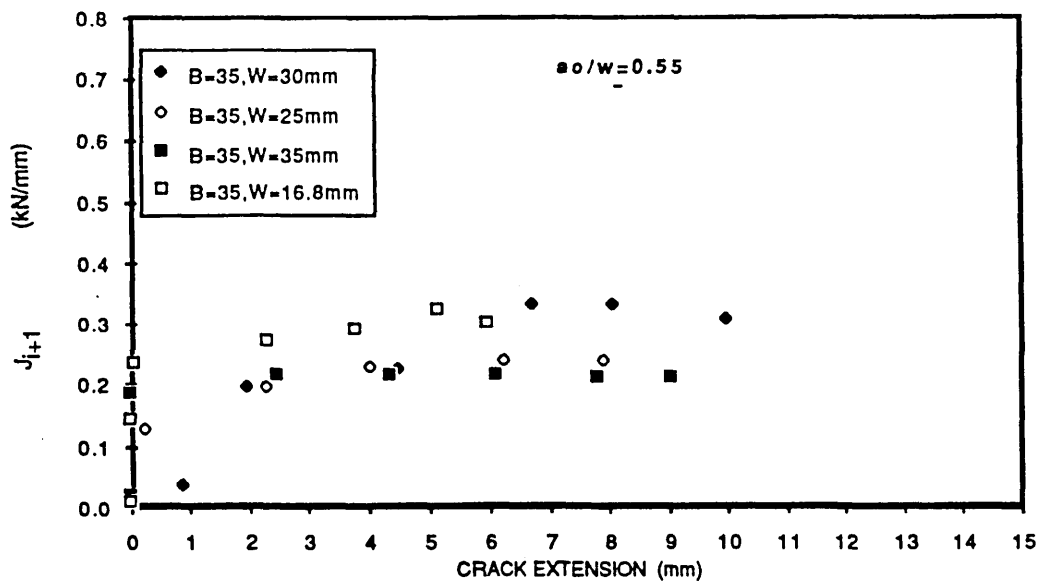


FIG. 6.12a $J_{i+1}-\Delta a(R)$ curves for various ligaments in the three-point bend configuration.

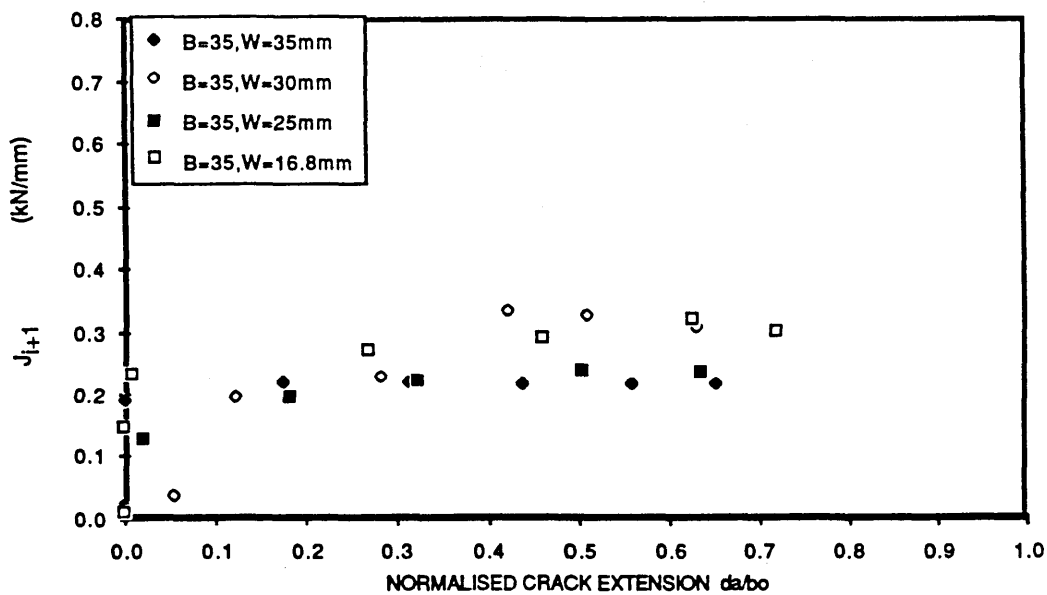


FIG. 6.12b $J_{i+1}-R$ curves with the abscissa normalised with the original ligament, b_o , for various ligaments in the three-point bend configuration.

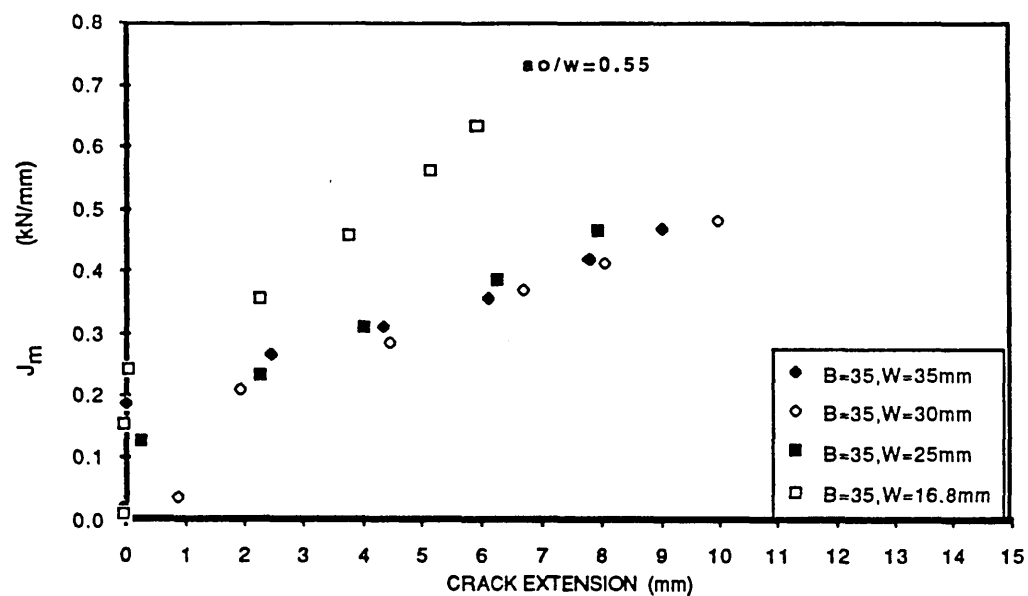


FIG. 6.13a J_m - $\Delta a(R)$ curves for various ligaments in the three-point bend configuration.

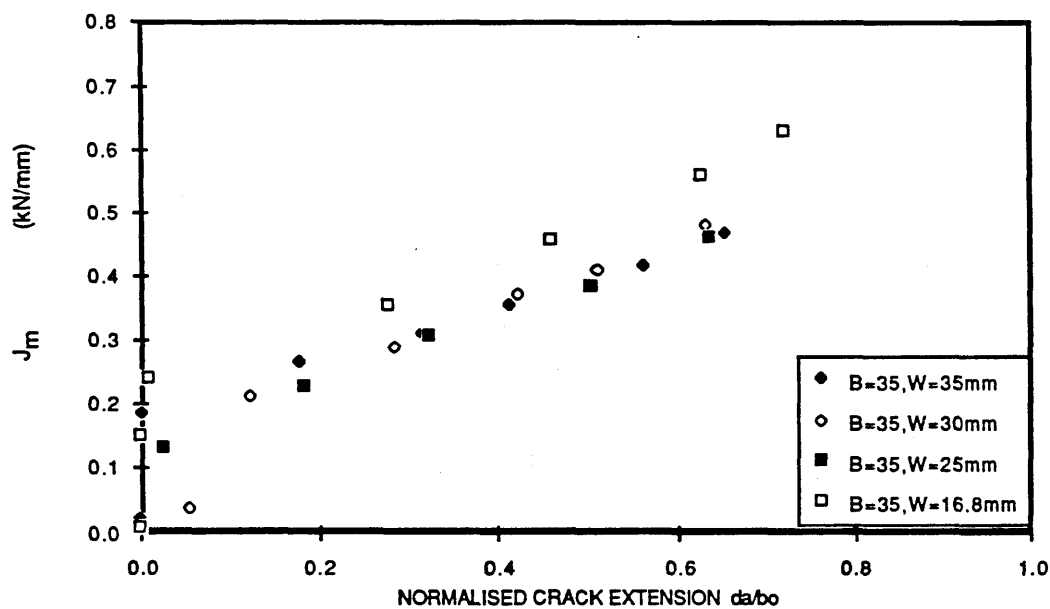


FIG. 6.13b J_m - R curves with the abscissa normalised with the original ligament, b_0 , for various ligaments in the three-point bend configuration.

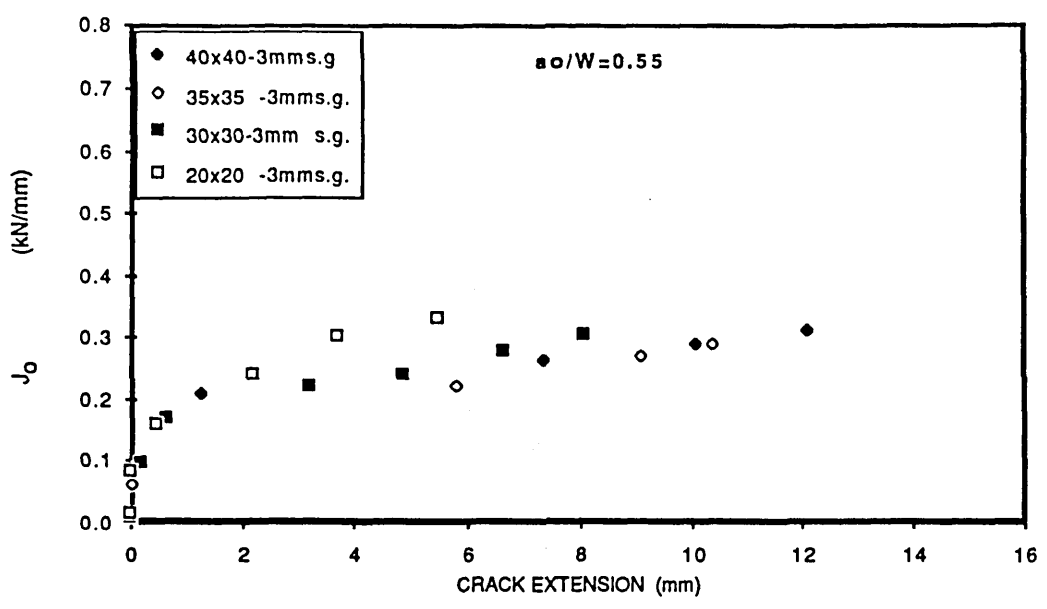


FIG. 6.14a J_0 - Δa (R) curves for various absolute specimen sizes,each with a total sidegroove depth of 3mm.

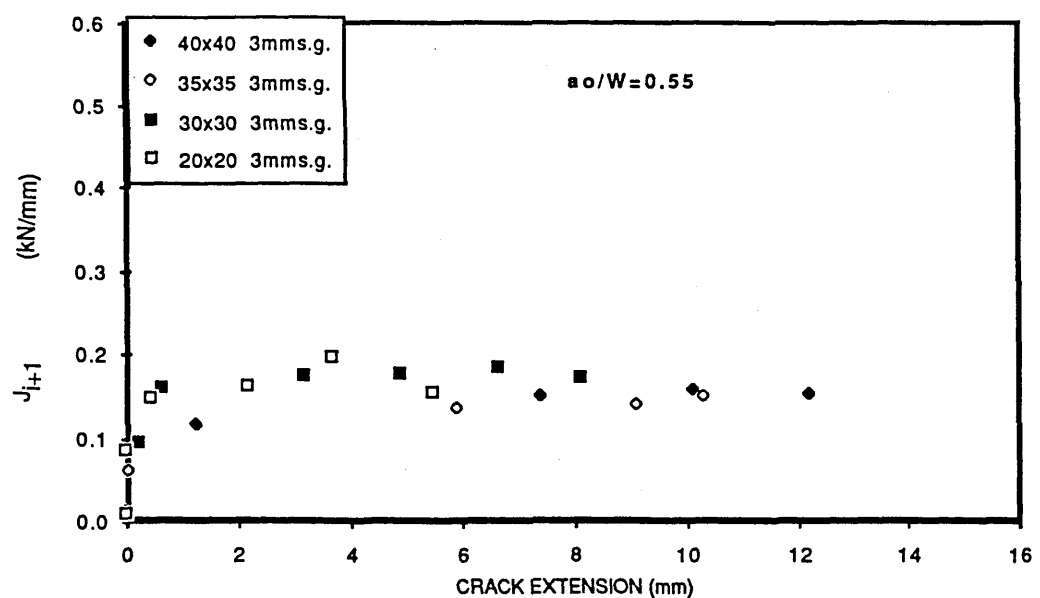


FIG. 6.14b J_{i+1} - Δa (R) curves for various absolute specimen sizes,each with a total sidegroove depth of 3mm.

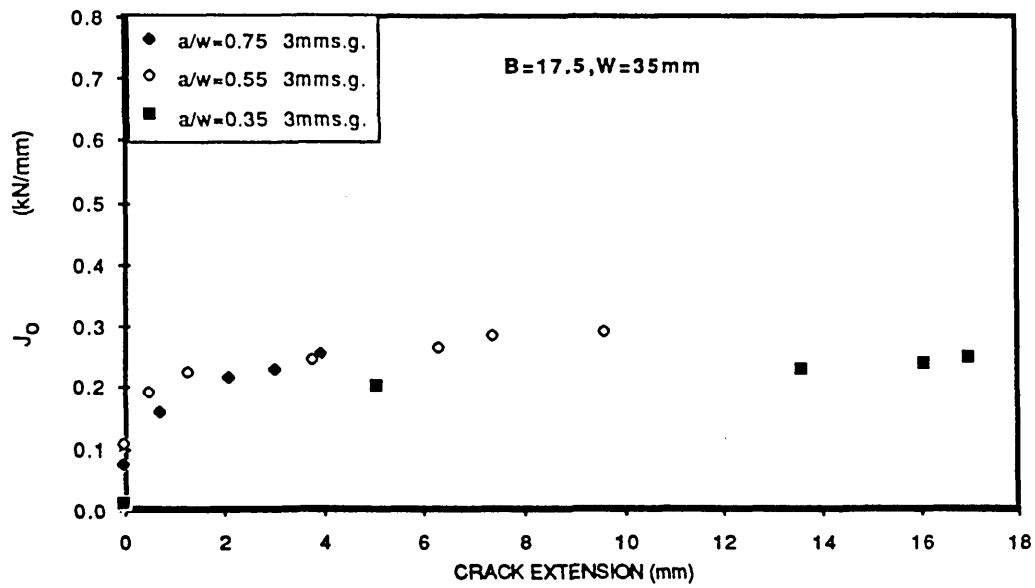


FIG. 6.15a J_0 - Δa (R) curves for various crack depth ratios, each with a total sidegroove depth of 3mm.

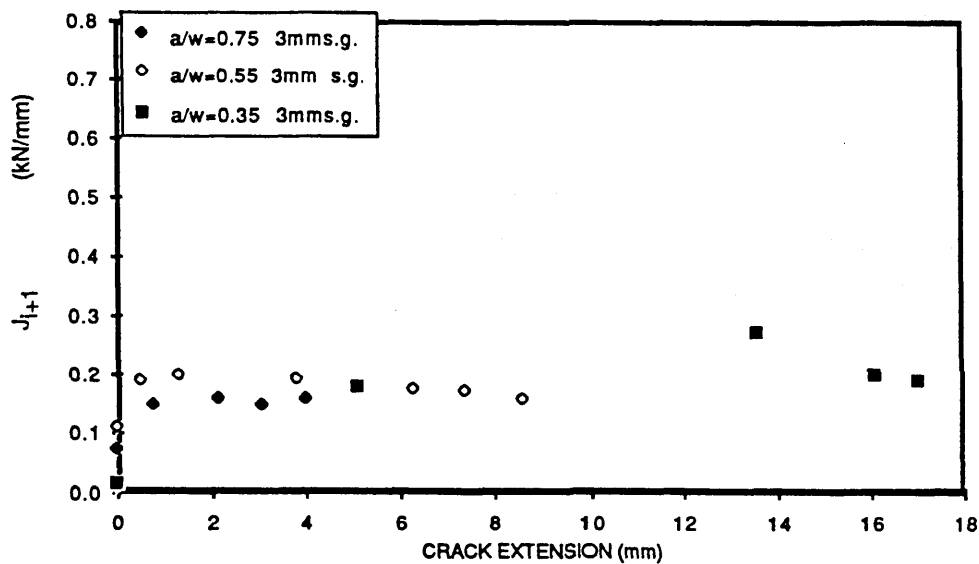


FIG. 6.15b J_{i+1} - Δa (R) curves for various crack depth ratios, each with a total sidegroove depth of 3mm.

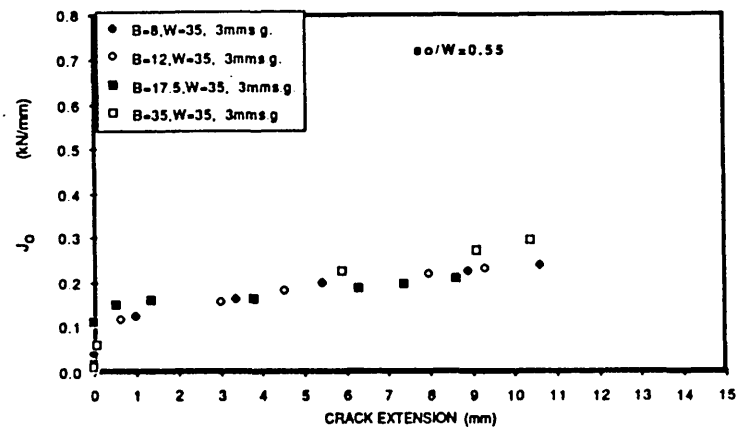


FIG. 6.16a J_0 - Δa (R) curves for various thicknesses, each with a total sidegroove depth of 3mm.

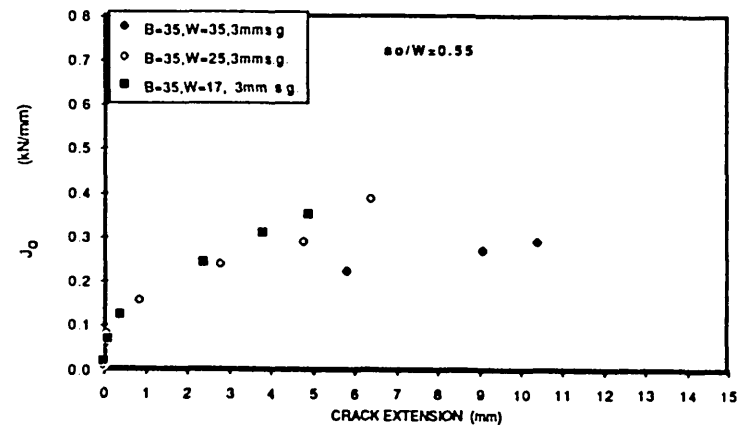


FIG. 6.17a J_0 - Δa (R) curves for various ligament lengths, each with a total sidegroove depth of 3mm.

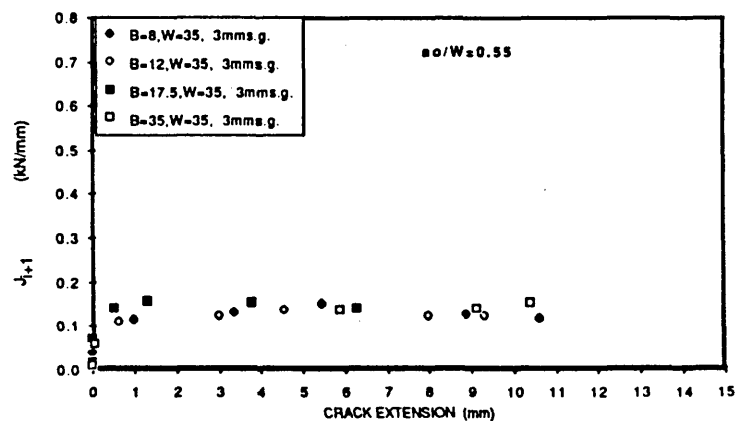


FIG. 6.16b J_{1+1} - Δa (R) curves for various thicknesses, each with a total sidegroove depth of 3mm.

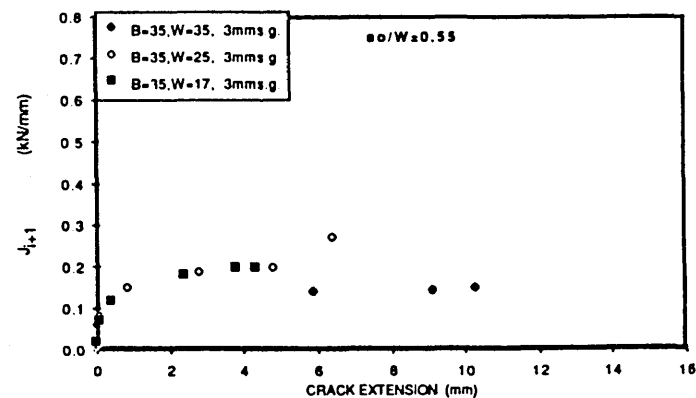


FIG. 6.17b J_{1+1} - Δa (R) curves for various ligament lengths, each with a total sidegroove depth of 3mm.

CHAPTER 7

DUCTILE INSTABILITY ANALYSIS

7.1 INTRODUCTION

Ductile tearing instability occurred in Ti (6-2-1-1) on different specimen configurations. The aim of this chapter is to report the incidences quantitatively, apply existing theories in an attempt to predict the instabilities and compare two types of instability that occurred with this material. The two types of instability that occurred are, for identification purposes, called incipient and final instabilities, the latter, failing in a more catastrophic manner in which the test specimen was almost separated in two halves. The ductile tearing instability theories of Paris et al (1979) and Turner (1984) were applied and compared. The ductile instability cases reported here are those that occurred in Ti(6-2-1-1) CKS and three-point bend specimens.

7.2 LOAD-DISPLACEMENT TRACES OF DUCTILE INSTABILITY INCIDENCES

7.2.1 INCIPIENT INSTABILITY

Ductile instability in general can be detected, ideally, when the load drops at fixed displacement. Usually, however, the response of the instruments might lag slightly behind the actual occurrence of events during a test.

An incipient ductile instability behaviour is, in this study, characterised by miniature audible pop-ins which precede temporary regaining of stability. An overall exhibition of such repetitive behaviour is in the global sense, classified as ductile instability, albeit occurring in a rapid succession of steps. A typical load-displacement trace is shown in fig. 7.1. In Ti(6-2-1-1), this type of behaviour is common with smaller sized specimens. Full thickness specimens (B=50mm) however showed a different and arguably more severe mode of ductile instability discussed below.

7.2.2 FINAL INSTABILITY

Cases of such instability in the ductile mode are shown in the form of load displacement traces in figs. 7.2, 7.3 and 7.4. These tests were done entirely on the 100 ton Avery hydraulic machine (plate 4.1).

It must be noted here that all the incidences of the so called "final" instability occurred in full thickness specimens (B=50mm). It is the original intention of obtaining near LEFM R-curves that initiated these full thickness tests. Indeed, the possibility of these

types of failures not being ductile were considered and investigated with scanning electron microscopy (SEM) studies on the fracture surfaces of these specimens. Evidence showed (see subsection 7.3.1. below) that the fracture modes were certainly ductile, and the possibility of cleavage type failure ruled out, see Jones (1986).

As can be seen in the traces of figs 7.2 to 7.4, the sudden failure of the specimen occurred at about fixed displacement. The concomitant "bang" was also markedly louder than the "pop-ins" of the incipient type. The onset of rapid separation, indicating the start of this mode of instability can be seen, albeit unscientifically, when the trace marker becomes faint. The rough nature of these plots, particularly the step-like advance up the trace is however, a consequence of the test machine controls and NOT the material behaviour. A motor controlled device would have eliminated this problem as it did with the screw-driven Tinius Olsen machine, see trace in fig. 7.1.

7.3 PREDICTION ANALYSIS

7.3.1 STUDY OF FRACTURE SURFACES

7.3.1.1 INTRODUCTION

Electron microscopy studies were carried out on fracture surfaces in order to ascertain the mode of failure in the full thickness, shallow notch ($a_0/W=0.1$) specimen. The load-displacement trace shown in figs 7.2. and 7.3 show fast fracture occurring at about maximum load. Also, a very audible "bang" accompanied this fracture process, thus exhibiting some of the characteristics of cleavage fracture found in some ferric materials.

The procedure of analysis was carried out by observing the fracture surfaces of basically two types of failure that occurred in Ti (6-2-1-1).

- (i) Slow, stable ductile crack growth. Exhibited frequently by three point-bend specimens ($B=17.5$, $B=35\text{mm}$, $a_0/W = 0.55$).
- (ii) Fast fracture, sudden and final instability whose traces are shown in figs. 7.2. and 7.3. ($B= 50\text{mm}$, $W=100\text{mm}$, $a_0/W= 0.1$ and 0.12).

7.3.1.2 OBSERVATIONS OF PHOTOMICROGRAPHS

Plates 7.1. to 7.5 show that even on the shallow notch, full thickness specimen (load-displacement traces shown in figs 7.2 and 7.3), traces of fibrous mode of separation is evident. Similarities between the morphology of the slow, stable fracture and that of fast, unstable can be seen in their respective pictures in plates 7.1. through to 7.4. To corroborate this evidence a photomicrograph of the CKS mode of fracture (load-displacement trace shown in fig. 7.4) is shown in plate 7.5.

7.3.2 DUCTILE INSTABILITY PREDICTIONS FOR THE INCIPIENT CASE : A COMPARISON OF THEORIES

7.3.2.1 INTRODUCTION

In this study the instability analysis is performed on the load-displacement plot shown in fig. 7.1. This type of instability classified here as incipient, is analysed with the T and I theories, whose governing equations are given in section 2.5. Recalling these criteria, and equations for instability:-

THE T APPROACH

CRITERIA FOR INSTABILITY

For ductile instability to occur,

$$dJ/da \big|_{app} > dJ/da \big|_{mat} \quad (2.37)$$

where T_{app} can be expressed also as,

$$T_{app} = 2b^2 S_e / W^3 - JE / \sigma_y^2 b \quad (2.38)$$

Where S_e is the effective span to allow for overall system compliance.

$$S_e = S(1 + C_{me}/C_u)$$

$$C_u = \text{Un-notched compliance}$$

$$C_{me} = \text{Effective machine compliance}$$

$$C_m = \text{Machine compliance}$$

C_{me} here is taken as $C_n + C_m$ where C_n = notched specimen compliance and C_m = machine compliance. In the analysis by Etemad (1983) and Turner (1984), C_n

fortuitously coincided with C_m . Hence in section 2.5, $\alpha = 2$.

Ernst et al (1981a, 1981b) proposed other forms of T_{app} and T_{mat} , viz:

$$T_{app} = E/\sigma_y^2 \{ (\partial J/\partial a) \big|_{\Delta} + (\partial J/\partial \Delta)^2 \big|_a [1/((\partial P/\partial \Delta) \big|_a + k_{me})] \} \quad (2.39a)$$

$$T_{mat} = E/\sigma_y^2 \{ (\partial J/\partial a) \big|_{\Delta} + (\partial J/\partial \Delta)^2 \big|_a [1/((\partial P/\partial \Delta) \big|_a - (dP/d\Delta))] \} \quad (2.39b)$$

$$T_{app} = E/\sigma_y^2 \left\{ (\partial J/\partial a) \big|_{\Delta} + (\partial J/\partial \Delta)^2 \big|_a \left[\frac{1}{k_{me} + dP/d\Delta + \frac{(\partial J/\partial \Delta)^2}{(\sigma_Y^2/E) T_{mat} - (\partial J/\partial a) \big|_{\Delta}}} \right] \right\} \quad (2.40)$$

k_{me} here is taken as the effective stiffness of a series arrangement of the machine stiffness and unnotched specimen stiffness, i.e.,

$$1/k_{me} = 1/k_m + 1/k_u \quad (7.1)$$

where k_m and k_u represent the test machine and unnotched specimen stiffness respectively.

THE I APPROACH

CRITERIA FOR INSTABILITY

Again, recalling the equations in section 2.5,

For ductile instability to occur,

$$I > (b/\eta)(dJ_r/da) - (J_r/\eta)(f(\eta)) \quad (2.41)$$

$$\text{where} \quad I = G[2(\eta_o/\eta_{el})_s - 1] \quad (2.42)$$

G is the linear elastic energy release rate and ,

$$\eta_{el,s} = \eta_{el}[1/(1 + \phi)]$$

$$\eta_{o,s} = [\eta_{el}(A) + \eta_{pl}(X)] / (A + X)$$

where $X = U_{pl}/U_{el}$ and $A = \eta_{el}/\eta_{el,s}$

Numerical values to all the variables shown above can be obtained as described in section 2.5.

7.3.2.2 RESULTS OF INSTABILITY ANALYSIS

At points A,B, and C, in fig 7.1, crack extensions are assumed to be 2mm, 3mm and 4.5mm respectively from an initial crack length of 29.5mm.

T APPROACH

The J-R curve used for the incipient case is shown jointly with other pertinent R curves in fig. 7.5. Using the J_m - R curve and compliance values shown below , table 7.1. can be constructed.

$$C_m = 32 \times 10^{-4} \text{ mm/KN (see section 5.3)}$$

$$C_n = 277 \times 10^{-4} \text{ mm/KN (at } a_o/W = 0.57)$$

$$C_u = 48 \times 10^{-4} \text{ mm/KN (from elastic theory)}$$

Though table 7.1 shows a slightly premature prediction of instability, the trend is correct, i.e., predicting instability after establishing preceding stability.

The other version of the T approach predictions by Equ (2.40) are shown in table 7.2. below: From Equ. 7.1, $k_{me} = 125 \text{ KN/mm}$.

Table 7.2 shows it predicts the onset of instability at all stages of the analysis, while table 7.1 shows a fairly clear 'cross-over' point but with similar trends as that in table 7.1, see fig.7.6 for the plot of these two tables.

I APPROACH

Recalling the instability criterion, Equ. 2.41,

$$I > (b/\eta)(dJ_r/da) - (J_r/\eta)(f(\eta))$$

(R.H.S)

The parameters necessary for the I method of evaluation are shown in table 7.3 , again

for the incipient instability case shown in fig. 7.1.

The plots of the values in tables 7.1 through to 7.3 are shown in figs 7.6 and 7.7 for the T and I theories respectively. From these plots, it is clear to see that both theories predict instabilities, though one formulation of the T approach arguably predicts this occurrence somewhat prematurely and the other shows no affirmative prediction trend. The I theory, on the other hand, predicts instability with greater plausibility and certainty.

7.3.3 THE I THEORY APPLIED TO THE FINAL INSTABILITY CASE

Analyses similar to the one shown in the previous subsection is done for the final instability cases, whose load-displacement plots are shown in figs. 7.2. and 7.3. The CKS behaviour shown in fig. 7.4 did show a fair amount of slow crack growth before fast fracture occurred at F. Using the I theory alone, the tables 7.4 and 7.5 can be produced for the three-point bend shallow notch (fig 7.2) and compact tension deep notch specimens (fig 7.4) respectively.

These tests were done on the 100 ton Avery hydraulic machine, for which the measured compliance, (see section 5.3) is 0.0127 mm/KN.

From both tables 7.4 and 7.5 instability can satisfactorily be predicted by the I theory. Fig. 7.8 attempts to compare what is clearly fast fracture instability (shallow notch, full thickness three point bend specimen shown in figs 7.2 and 7.3) and incipient instability. Fig 7.8 shows, for the incipient instability case, a comparatively rapid fall in the energy absorption rate required for fracture. The significance of this behaviour to resistance curve representation is highlighted in the next chapter.

$a_0 = 29.5\text{mm}$	$2b^2S_e/W^3$	JE/σ_y^2b	T_{app}	T_{mat}	Result
No crack extension	9.8	2.22	7.66	26.6	Stable
A : $\Delta a = 2\text{mm}$	8.14	2.21	5.93	4.8	Unstable
B : $\Delta a = 3\text{mm}$	7.28	2.59	4.60	1.4	Unstable
C : $\Delta a = 4.5\text{mm}$	6.09	2.8	3.29	0.7	Unstable

Table 7.1 Comparison of T_{app} and T_{mat} using Eqs. 2.37 and 2.38 for the incipient instability case shown in fig 7.1.

	$(\partial J/\partial a)_\Delta$ KN/mm ²	$(\partial J/\partial \Delta)_a$ KN/mm ²	$dP/d\Delta$ KN/mm	$(\sigma_Y^2/E) T_{mat}$	T_{app}	T_{mat}	Result
No crack extension	-0.0048	5.2	+ 18	0.113	26.3	26.4	ALL STAGES PREDICTING ONSET OF INSTABILITY
A	- 0.017	56.8	- 54	0.0206	4.82	4.80	
B	- 0.012	5.9	- 211	0.006	1.38	1.39	
C	- 0.013	6.25	- 911	0.003	0.68	0.68	

Table 7.2 Comparison of T_{app} and T_{mat} using Equ. 2.40 for the incipient instability shown in fig 7.1

	a	G	X	$\eta_{o,s}$	$\eta_{el,s}$	R.H.S	I	Result
	mm	KN/mm				KN/mm		
A	31.5	0.154	0.33	1.85	1.81	0.177	0.16	Stable
B	32.5	0.257	0.611	1.87	1.81	0.157	0.276	Unstable
C	34.0	0.329	0.932	1.89	1.81	0.136	0.358	Unstable

Table 7.3 I method predictions for the incipient instability case
(Load-displacement trace shown in fig 7.1)

Shallow notch three point-bend specimen

Resistance curve shown in fig. 7.5, $a_0 = 11\text{mm}$, $B = 50\text{mm}$, $W = 100\text{mm}$

	a	G	X	$\eta_{O,s}$	$\eta_{el,s}$	R.H.S	I	Result
	mm	KN/mm				KN/mm		
A	11.0	0.100	0.296	0.302	0.253	17.9	0.138	Stable
B	12.7	0.224	0.400	0.345	0.270	4.35	0.348	Stable
C	16.5	0.33	0.688	0.441	0.313	0.12	0.600	Unstable

Table 7.4. I analysis for the final instability case(shallow notch, full thickness three point-bend specimen (load-displacement trace shown in fig 7.2).

Deep notch CKS specimen

Resistance curve shown in fig. 7.5, $a_0 = 56\text{mm}$, $B = 50\text{mm}$, $W = 100\text{mm}$

	a	G	X	$\eta_{o,s}$	$\eta_{el,s}$	R.H.S	I	Result
	mm	KN/mm				KN/mm		
A	56.05	0.0673	0.000	0.843	0.843	1.93	0.067	Stable
B	58.14	0.168	0.197	1.007	0.910	0.51	0.200	Stable
C	60.23	0.217	0.46	1.176	0.960	0.24	0.308	Unstable

Table 7.5 I analysis for the final instability case(deep notch, full thickness, CKS ,load-displacement trace shown in fig 7.4).

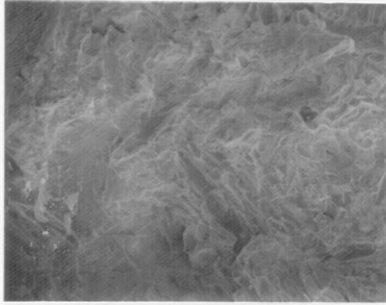


PLATE 7.1 Slow, stable ductile crack growth in titanium alloy (6Al-2Cb-1Ta-1Mo).
(Picture taken from a three-point bend specimen, $B=17.5\text{mm}$, $W=35\text{mm}$, $a_0/W=0.55$). Magnification : X35.

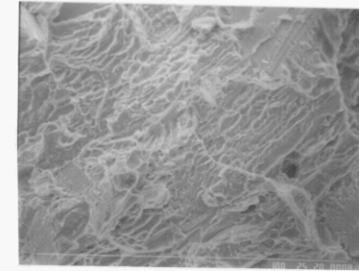


PLATE 7.2 Slow, stable ductile crack growth in titanium alloy (6Al-2Cb-1Ta-1Mo).
(Picture taken from a three-point bend specimen; $B=17.5\text{mm}$, $W=35\text{mm}$, $a_0/W=0.55$). Magnification : X500.
(Higher magnification of plate 7.1).

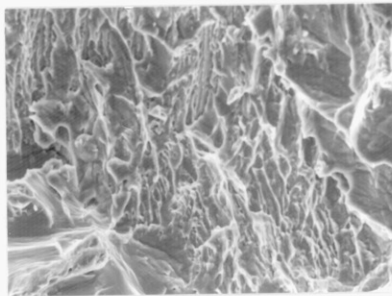


PLATE 7.4 Fast, final unstable fracture in titanium alloy (6Al-2Cb-1Ta-1Mo) 5mm into the fracture zone from the fatigue limit on the crack face of the three-point bend, full thickness, shallow notch specimen ($B=50\text{mm}$, $W=100\text{mm}$, $a_0/W=0.1$).
Magnification : X500.(Higher magnification of plate 7.3).

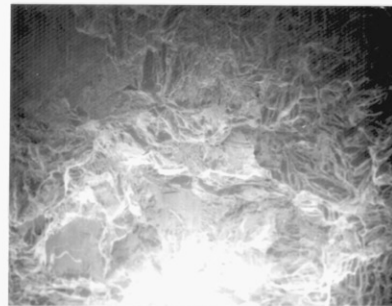


PLATE 7.3 Fast, final unstable fracture in titanium alloy (6Al-2Cb-1Ta-1Mo).
5mm into the fracture zone from the fatigue limit on the crack face of the three-point bend, full thickness, shallow notch specimen ($B=50\text{mm}$, $W=100\text{mm}$, $a_0/W=0.1$).
Magnification : X35.

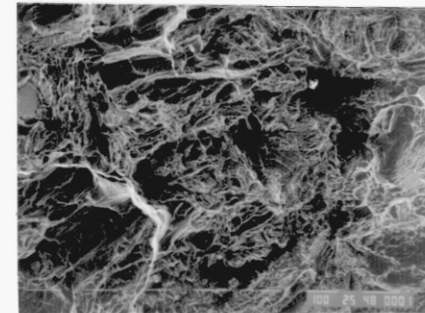


PLATE 7.5 Fast, final unstable fracture in titanium alloy (6Al-2Cb-1Ta-1Mo)(Picture taken from a compact tension specimen, $B=50\text{mm}$, $W=100\text{mm}$; $a_0/W=0.55$, well into the fracture zone). Magnification : X100.

INCIPIENT INSTABILITY

$B=25\text{mm}$, $W=50\text{mm}$, $S/W=4$, $a_0=29.5\text{mm}$

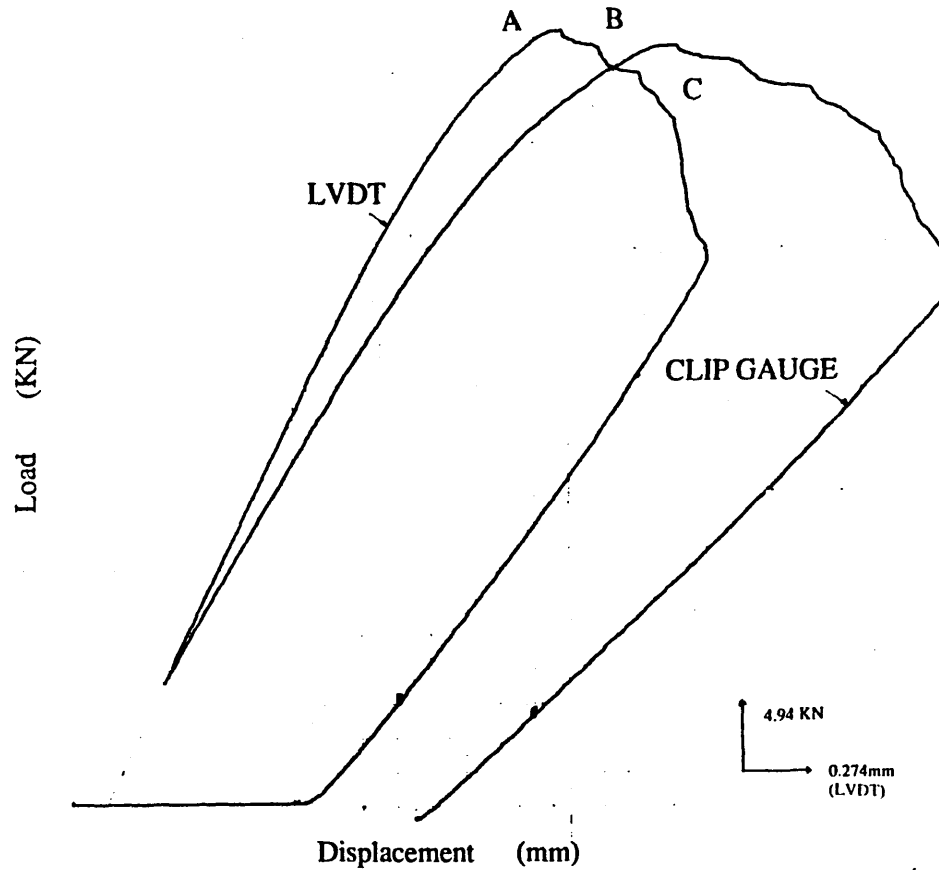


FIG. 7.1 Load-displacement trace for the incipient instability case. (Three-point bend, deep notch, $a_0/W=0.57$).

FINAL INSTABILITY

$B=50\text{mm}$, $W=100\text{mm}$, $a_0/W=0.10$

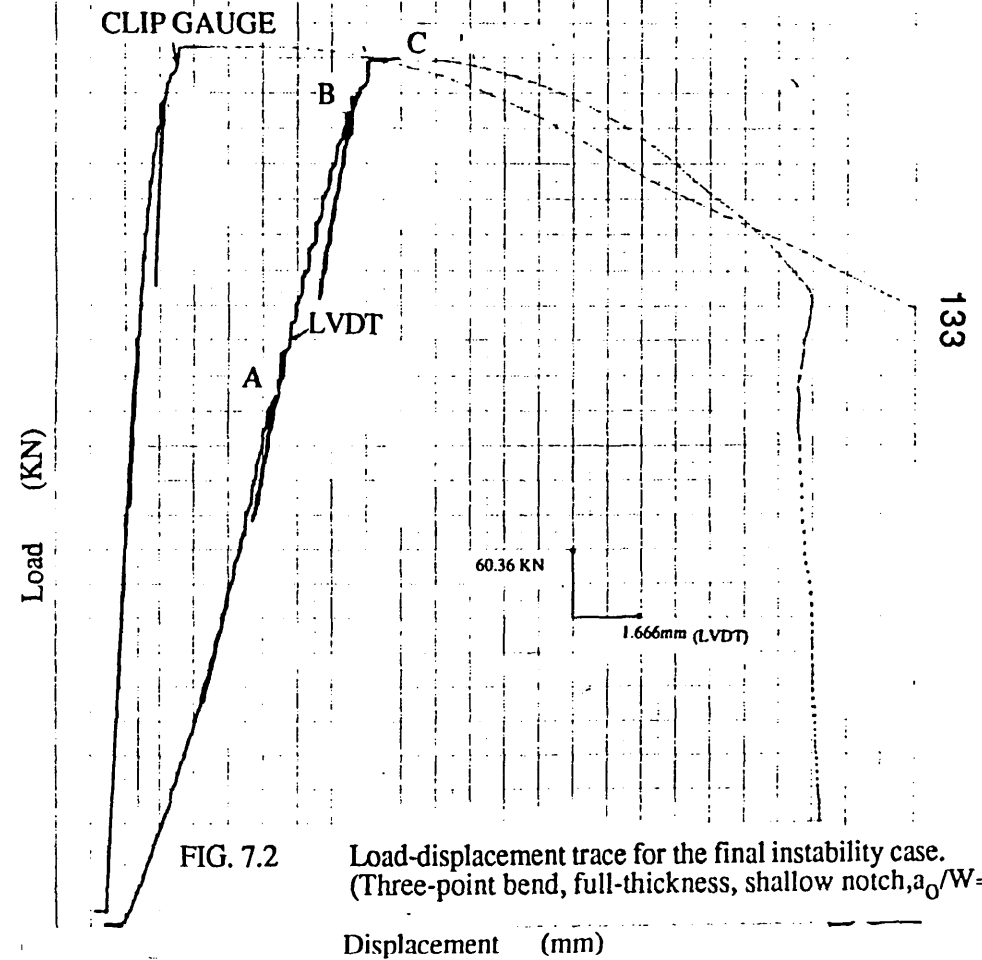
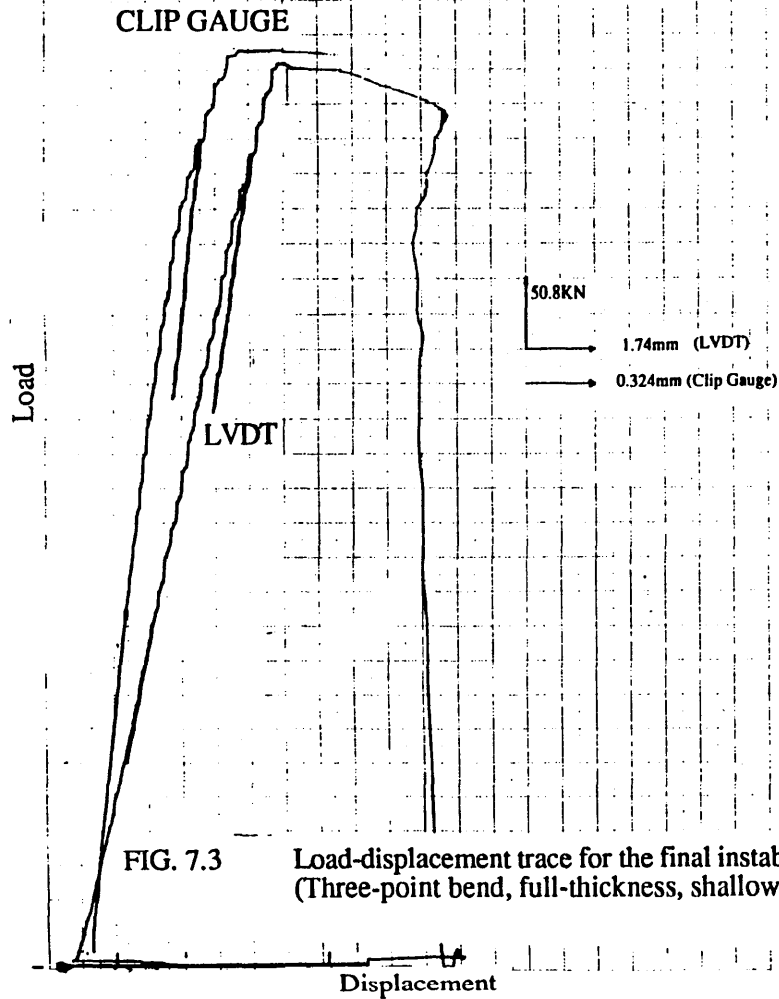


FIG. 7.2 Load-displacement trace for the final instability case. (Three-point bend, full-thickness, shallow notch, $a_0/W=0.1$).

FINAL INSTABILITY

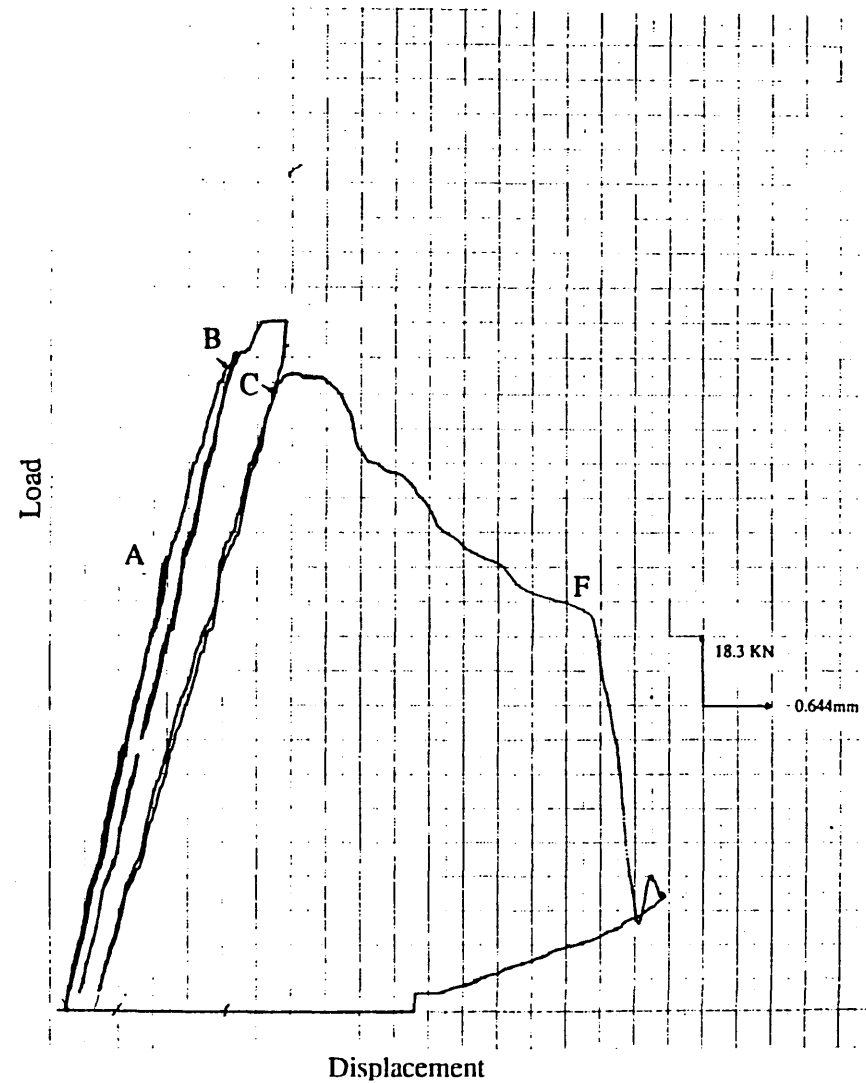
$B=50\text{mm}$, $W=100\text{mm}$, $a_0/W=0.12$



FINAL INSTABILITY

FIG. 7.4

Load-displacement trace for the final instability case. (compact tension, full-thickness, deep notch, $a_0/W=0.55$).



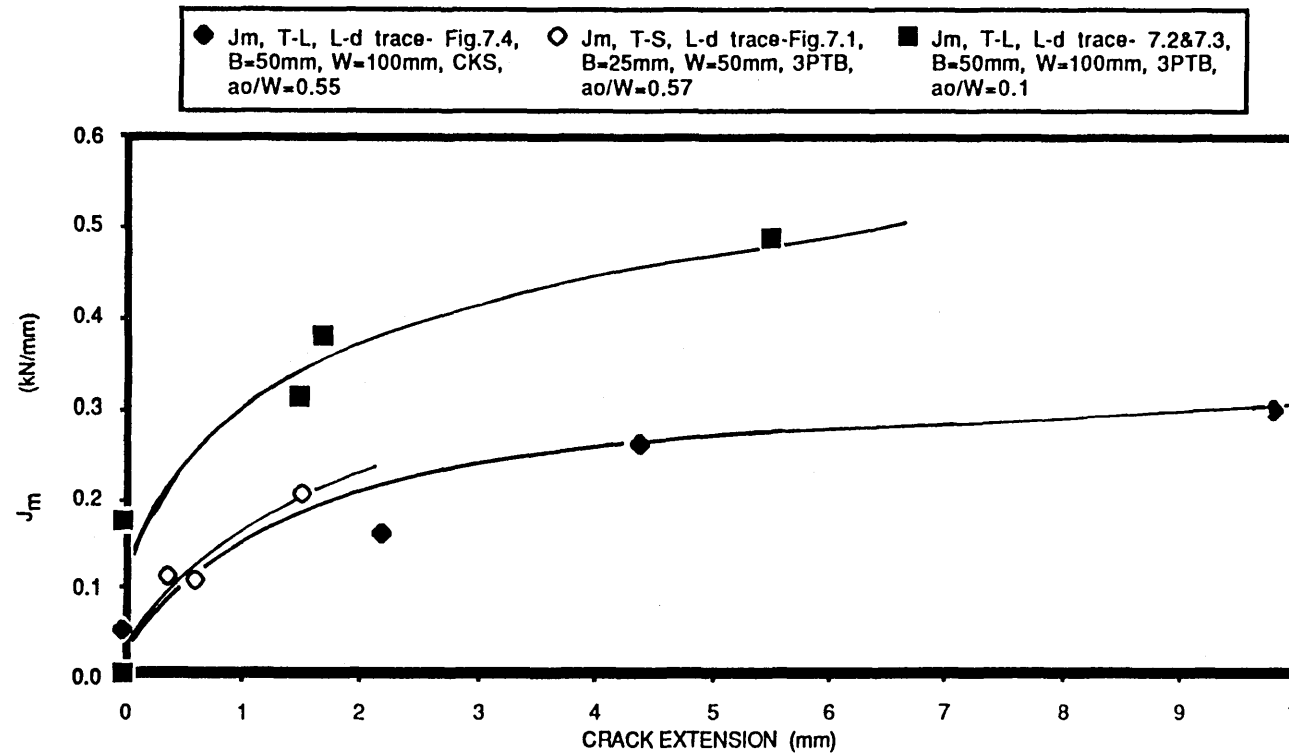


FIG. 7.5 J_m -R curves used for the ductile instability analysis.

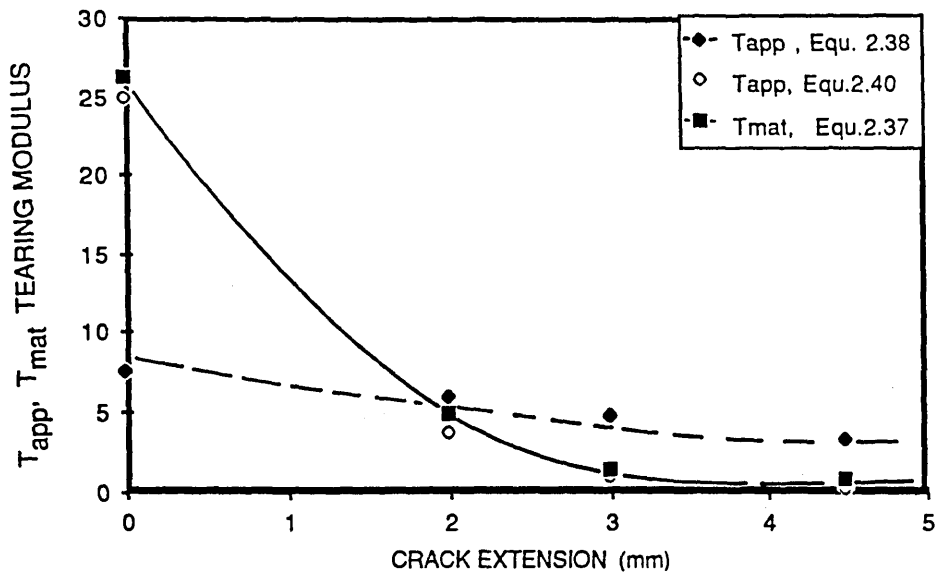


FIG. 7.6 Comparison between T_{app} and T_{mat} for the incipient instability case shown in Fig.7.1.

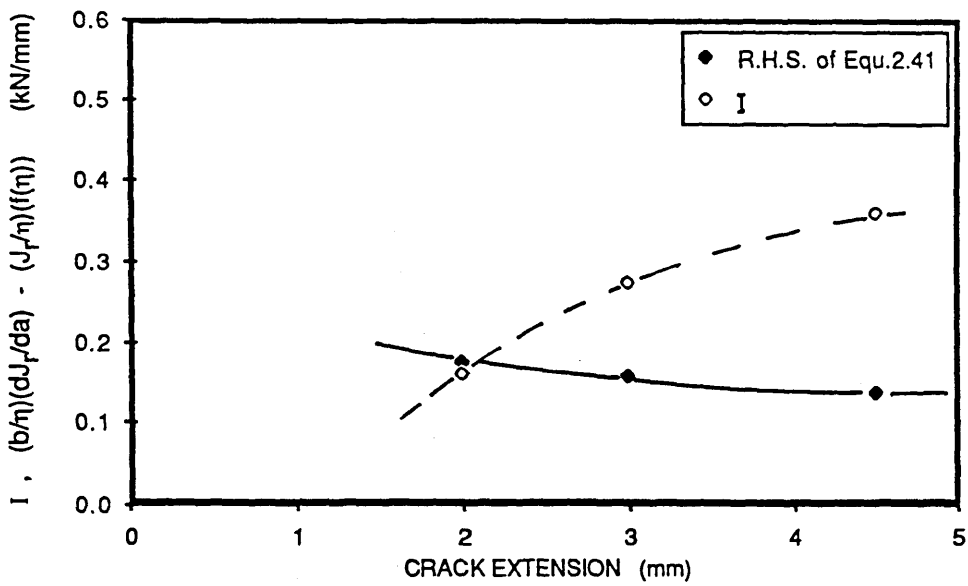


FIG. 7.7 Comparison of controlling parameters for the incipient ductile instability (Fig.7.1) analysis using the I method.

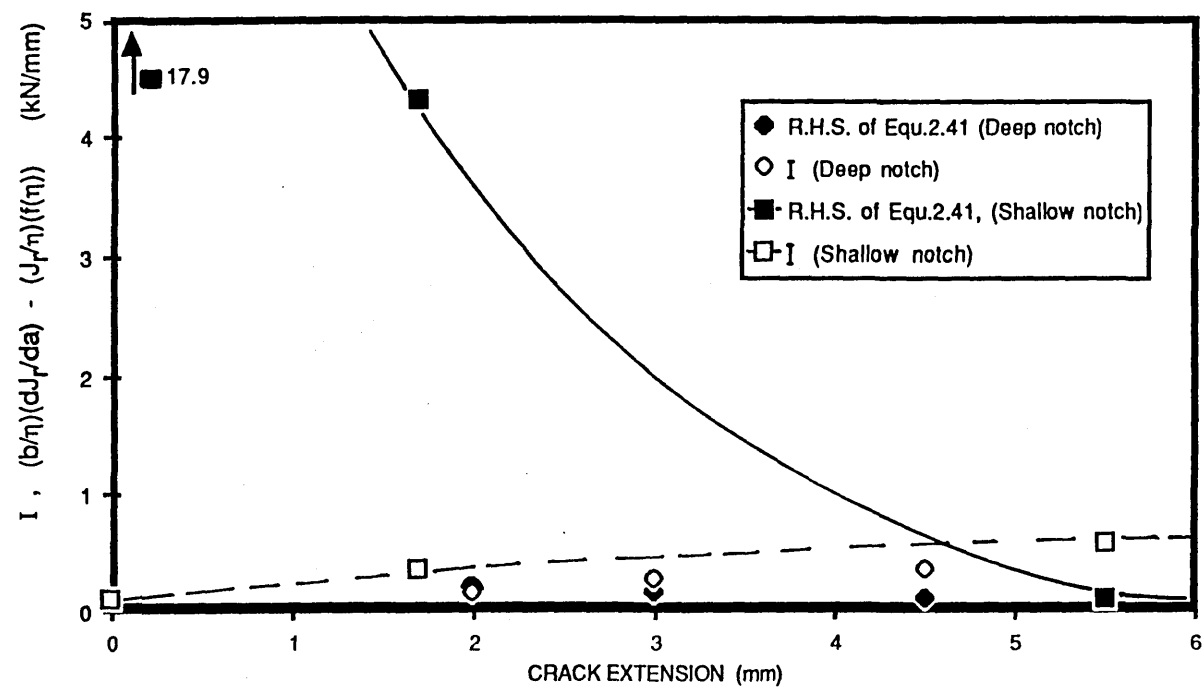


FIG. 7.8 Comparison of the shallow notch(final instability) and the deep notch (incipient instability) using the I method.

CHAPTER 8

WORK RATE - AN ALTERNATIVE FORM OF REPRESENTING TOUGHNESS

8.1 INTRODUCTION

In chapter six, a rationalisation of resistance curves was made in the form of normalisation of the J-R curves. Though some success has been gained by such a procedure, as discussed in the next chapter, an alternative form of toughness representation is introduced here.

The basis of this work questions the very philosophy of a rising resistance curve exhibited by the J-R curves in chapter six. In the post-yield regime, if the toughness of a material is measured by the amount of energy absorbed during crack growth and plastic deformation, it should be represented as such. The form of such a representation chosen in this study is the work rate, i.e. (dU/da) - the rate of energy absorbed per unit area of crack growth, Bda.

8.2 INTRODUCTION TO A BASIC RESISTANCE CURVE

Using the reasoning described above, a "basic" toughness parameter is introduced here:- viz:

$$R = (1/B) (dU_{DIS}/da) \quad (8.1)$$

where dU_{DIS}/da is the rate of energy dissipated during elastic/plastic deformation and crack growth. U_{DIS} can be obtained experimentally by subtracting the elastic component, U_{el} , from the total work dissipation, U_{TOT} .

In this study, a plot of total energy and dissipated energy against crack extension is made, from which the work rate values can be derived.

8.3 WORK RATE RESULTS

The load-displacement plots of the J-R curve study made in chapter six were used to evaluate the values of U_{TOT} and U_{DIS} as schematically illustrated in fig. 8.1a. Fig 8.1b shows how the work rate dU/da is obtained. A typical comparative plot between U_{TOT} and U_{DIS} is shown in fig. 8.5c.

For the same categories investigated in chapter six, U_{TOT} and U_{DIS} are plotted against crack extension in figs. 8.2a through to 8.5b. The corresponding plots of $(1/B)(dU_{TOT}/da)$ and R against crack extension are shown in figs. 8.6 through to 8.9.

From the absolute energy plots (figs. 8.2a - 8.5b), there seems to be an overwhelming evidence to indicate that less energy is required for more crack growth. Therefore, not surprisingly, all the work rate plots show a decreasing representation of "toughness" (as defined by R) with crack extension. A more striking observation is the markedly reduced scatter of these plots, more so with increased crack growth; indicating a reverse trend of toughness representation to that when J is used as a characterising parameter.

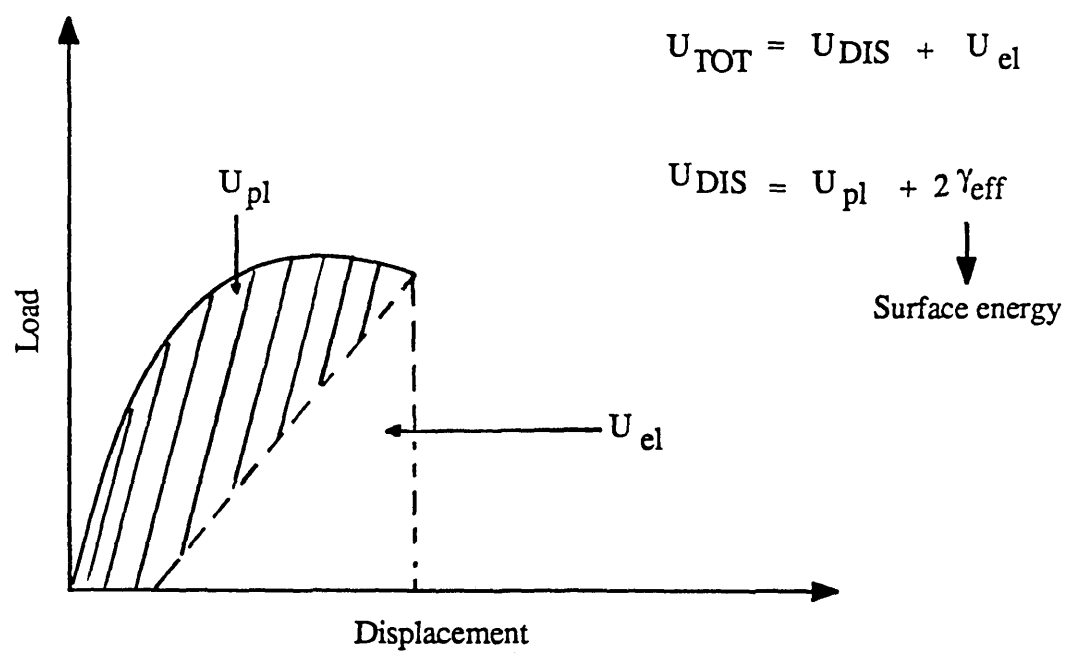


FIG.8.1a Schematic representation of work done and energy dissipated

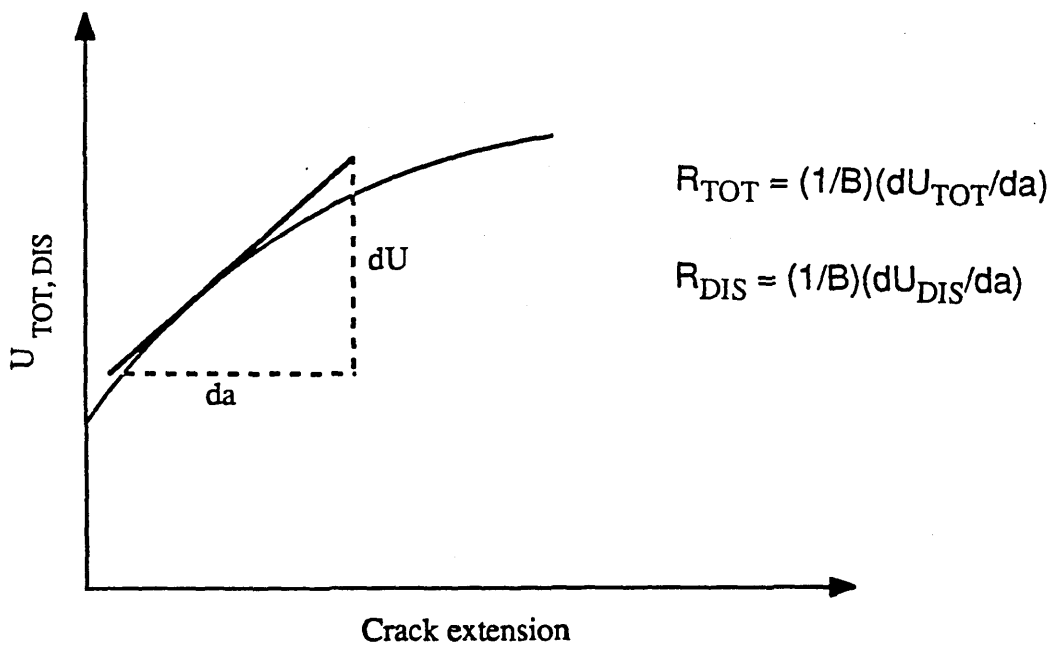


FIG. 8.1b Method of obtaining work rate from a work-crack extension plot.

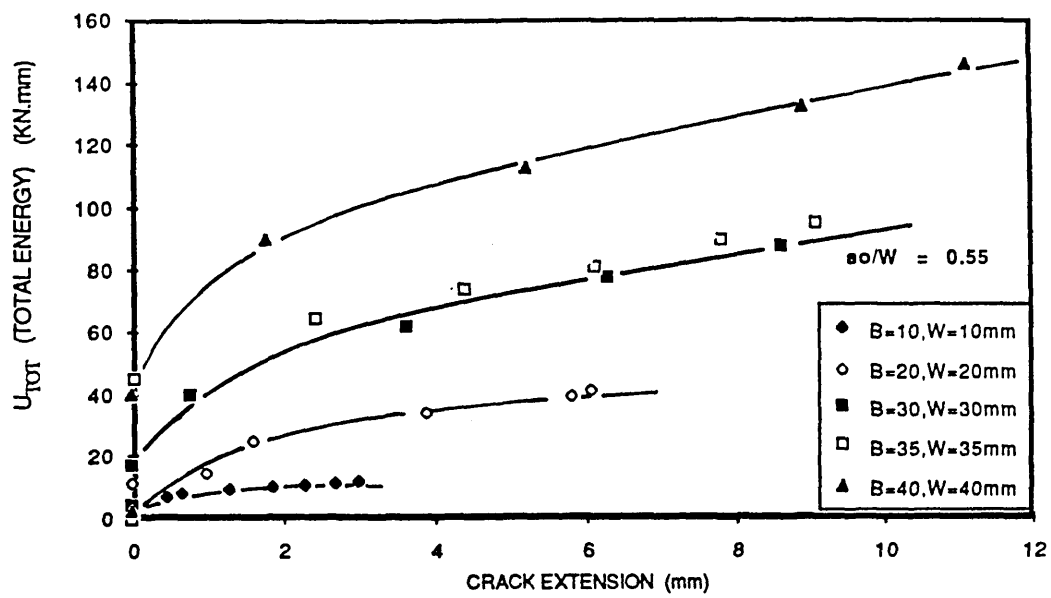


FIG. 8.2a Total work done(U_{TOT}) against crack extension for various geometry sizes.

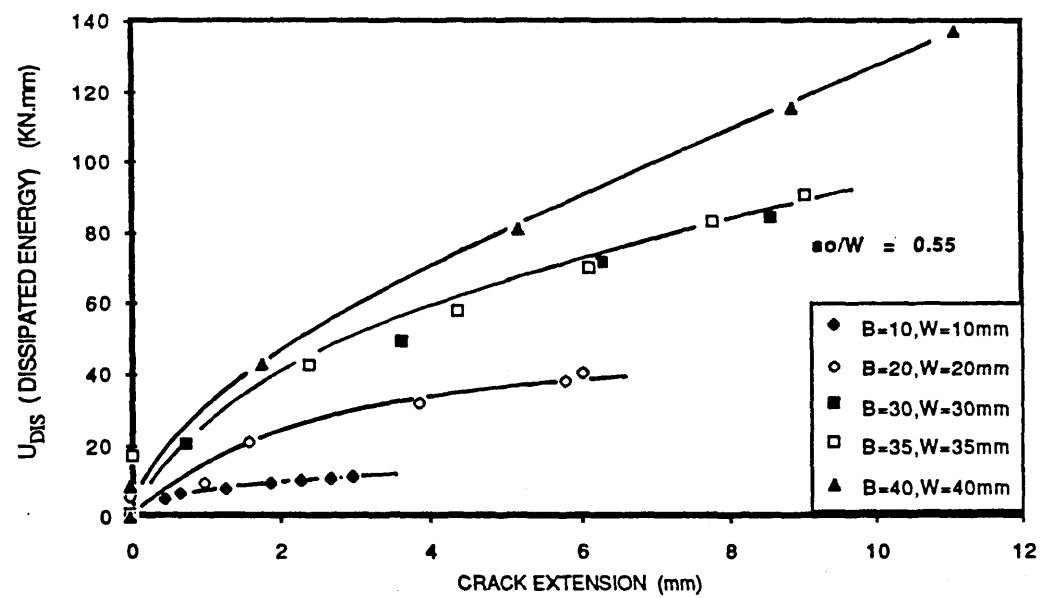


FIG. 8.2b Work dissipated (U_{DIS}) against crack extension for various geometry sizes.

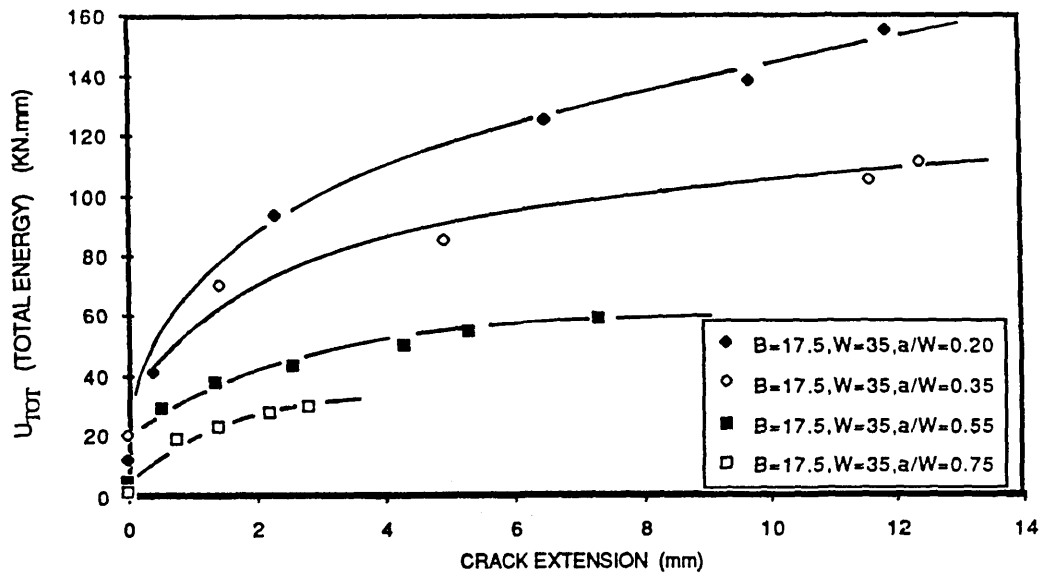


FIG. 8.3a Total work done (U_{TOT}) against crack extension for various crack depth ratios.

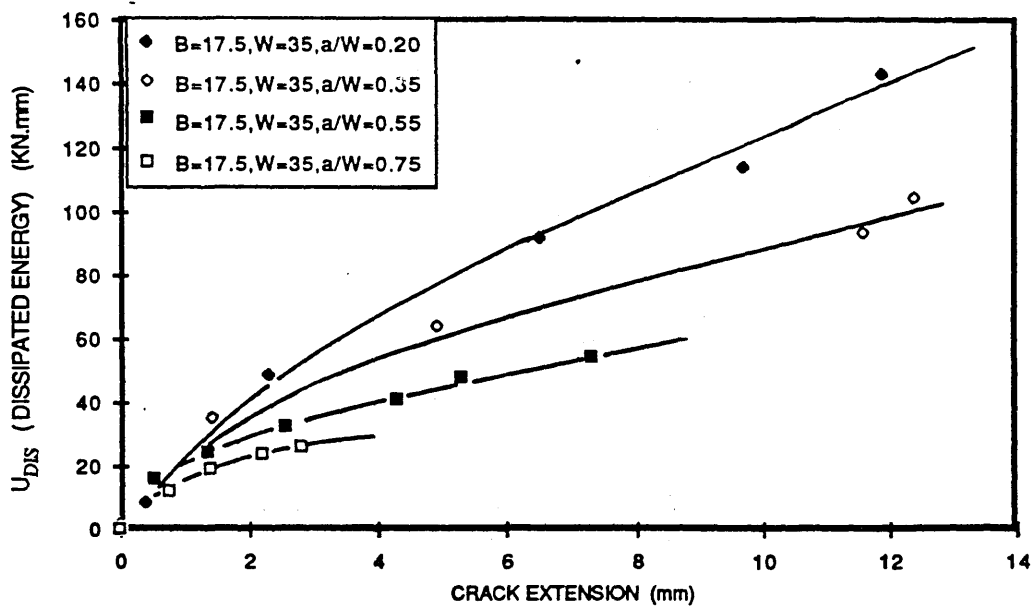


FIG. 8.3b Work dissipated (U_{DIS}) against crack extension for various crack depth ratios.

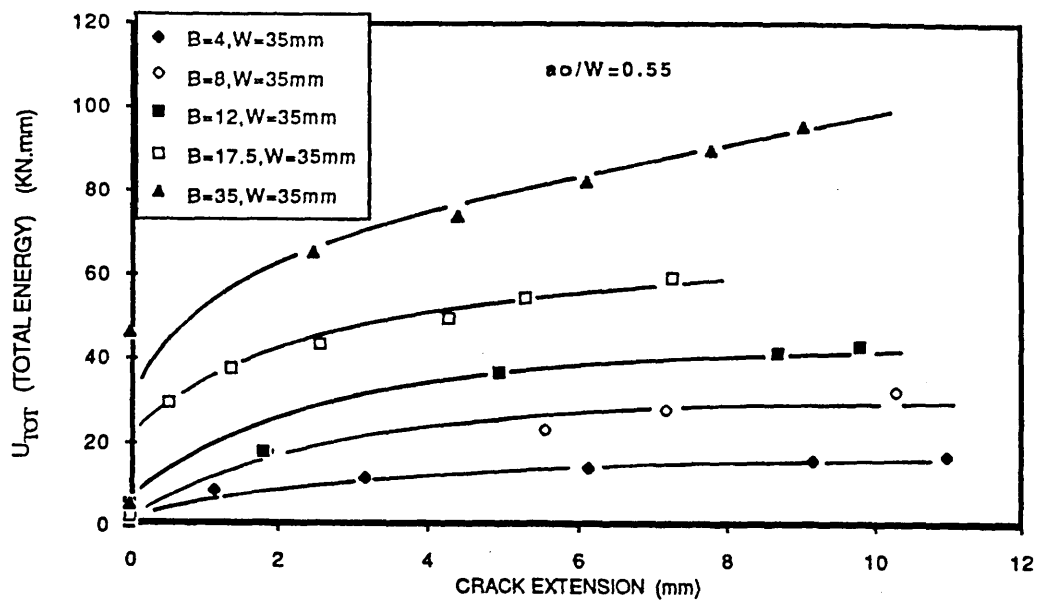


FIG. 8.4a Total work done (U_{TOT}) against crack extension for various thicknesses.

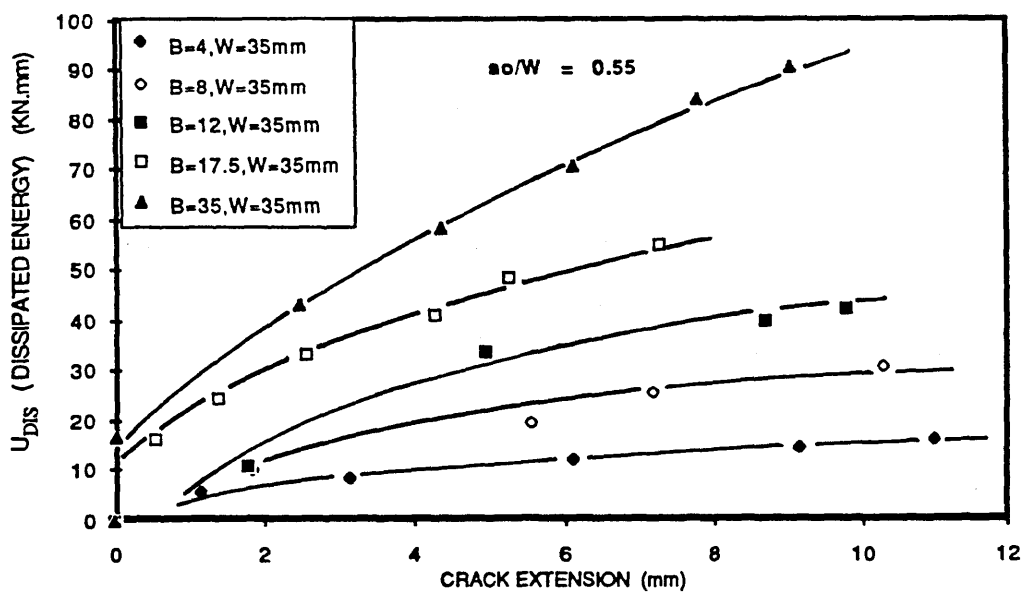


FIG. 8.4b Work dissipated (U_{DIS}) against crack extension for various thicknesses.

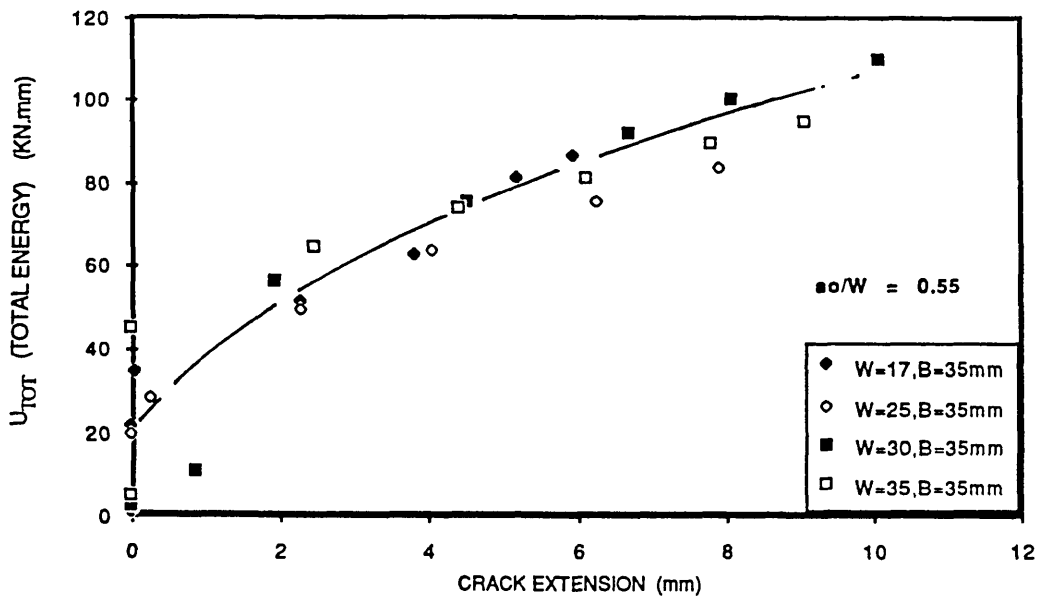


FIG. 8.5a Total work done (U_{TOT}) against crack extension for various ligaments.

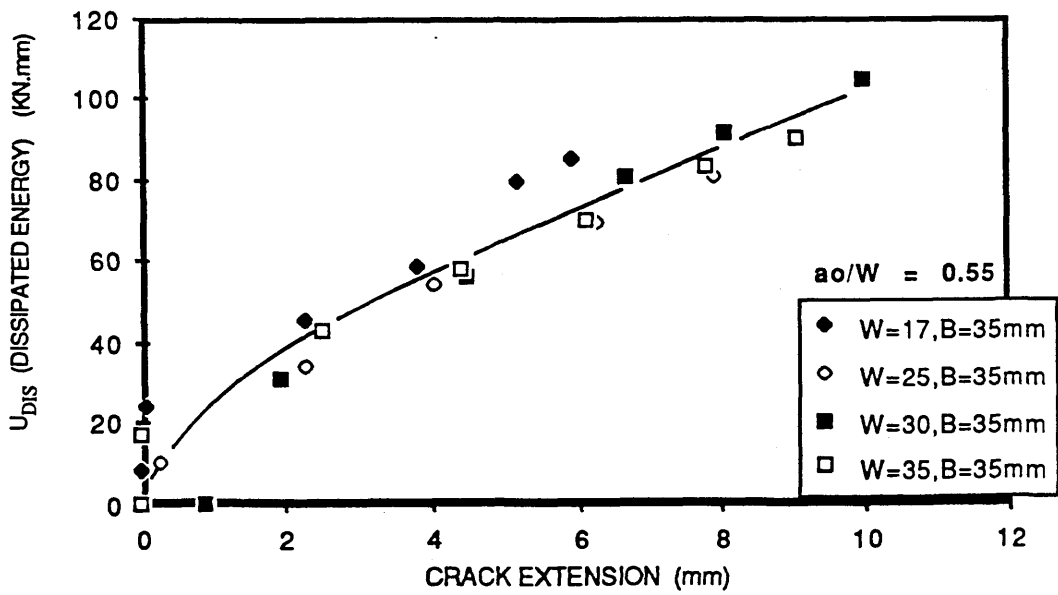


FIG. 8.5b Work dissipated (U_{DIS}) against crack extension for various ligaments.

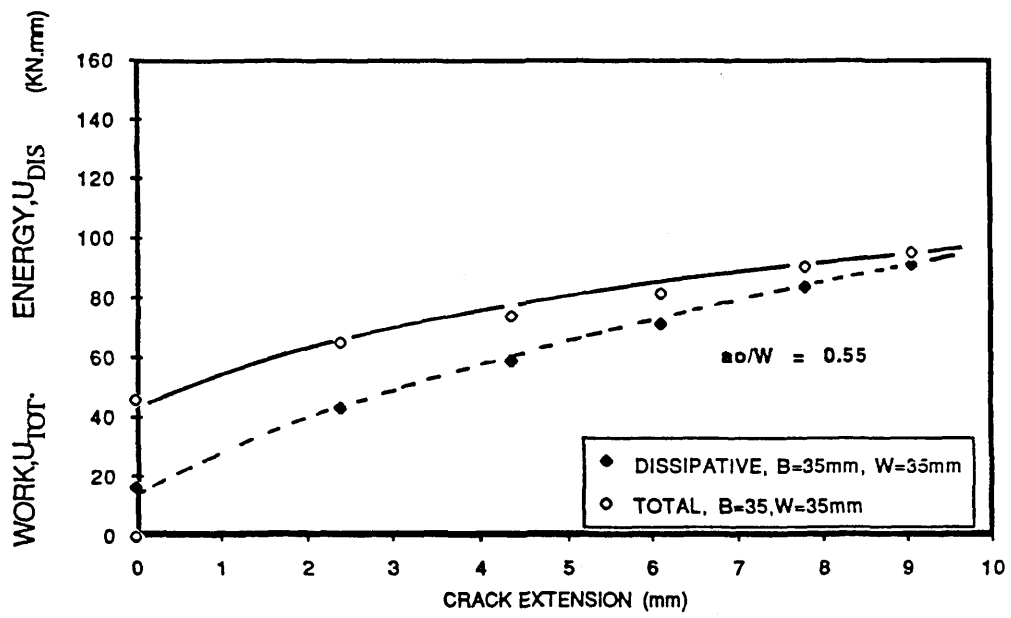


FIG. 8.5c A typical comparison between dissipative work and total work done.

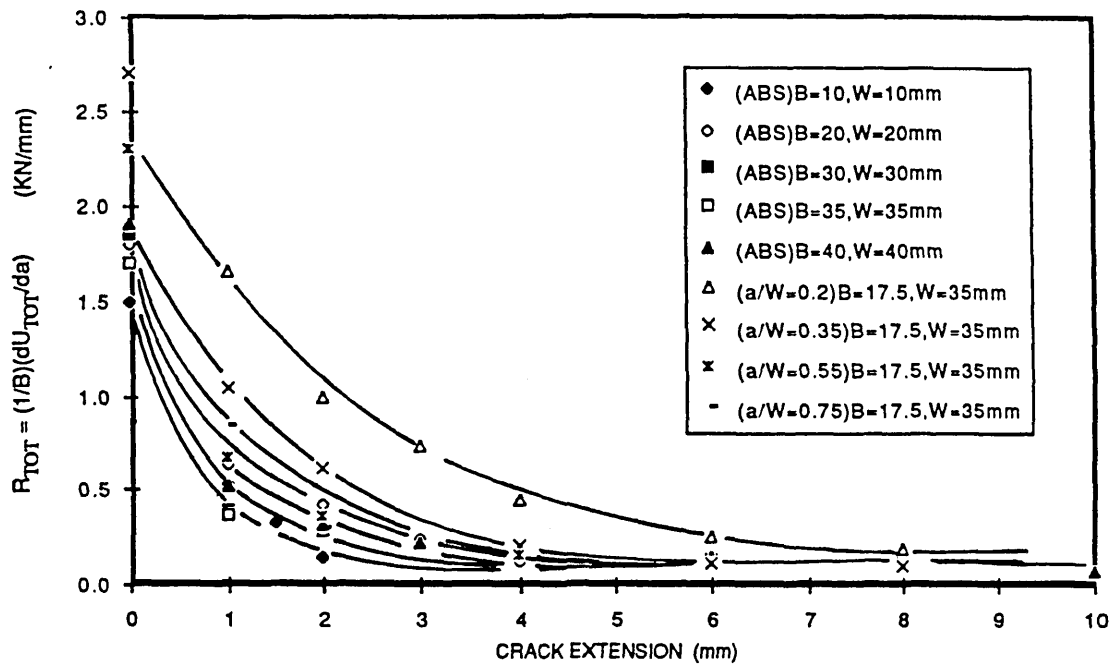


FIG. 8.6 Total work rate, R_{TOT} , against crack extension for various absolute sizes and crack depth ratios.

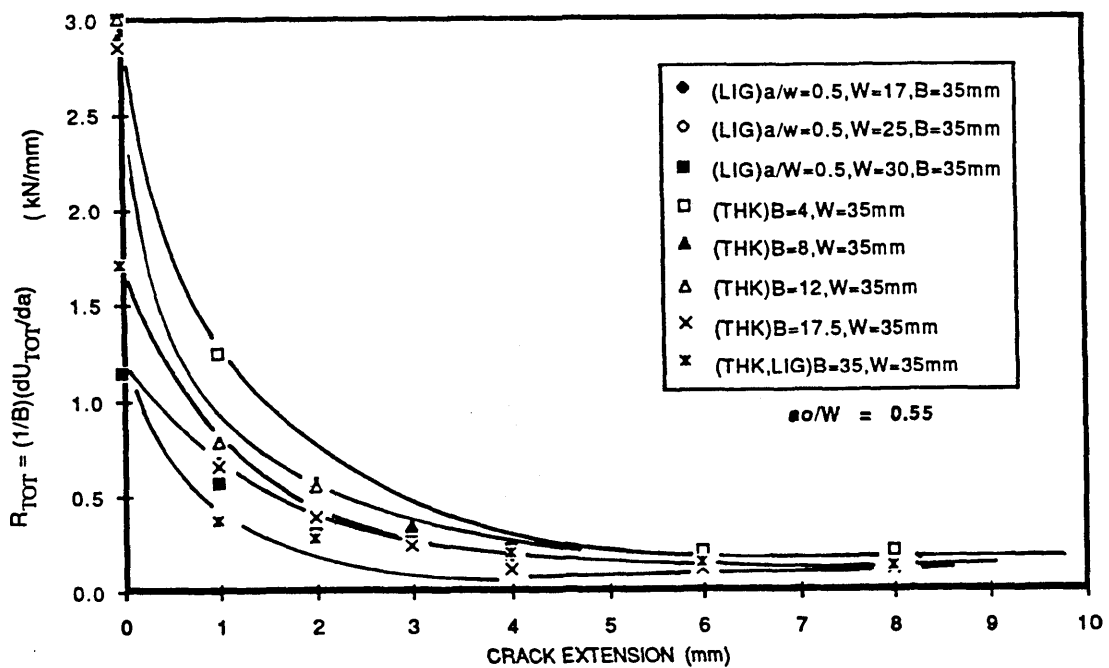


FIG. 8.7 Total work rate, R_{TOT} , against crack extension for various ligaments and thicknesses.

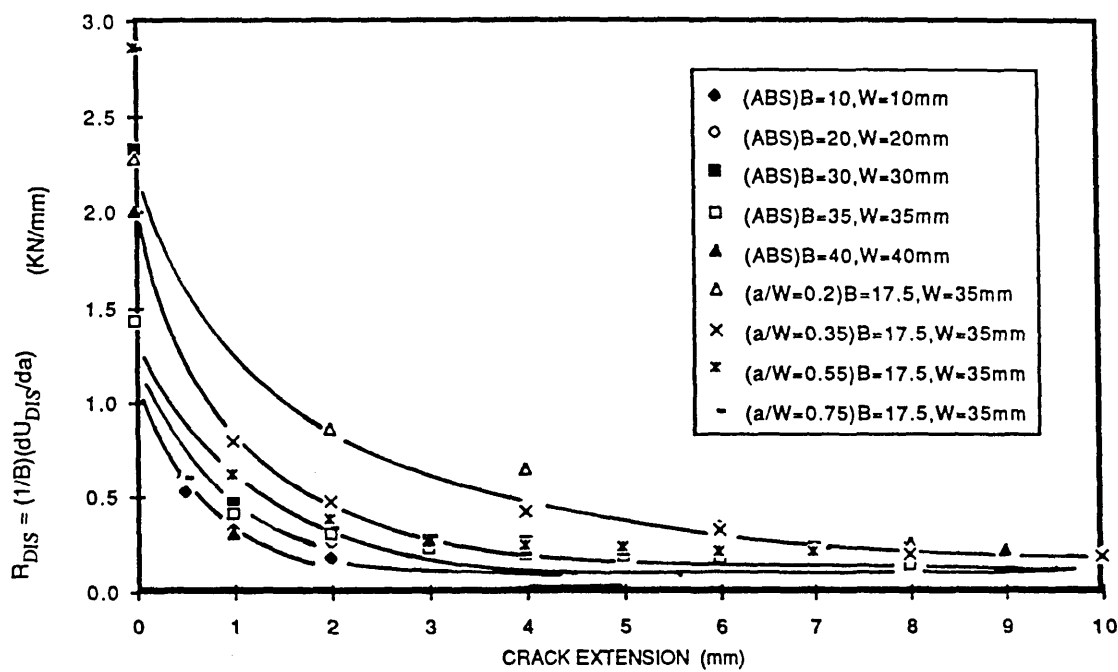


FIG. 8.8 Dissipative work rate, R_{DIS} , against crack extension for various absolute sizes and crack depth ratios.

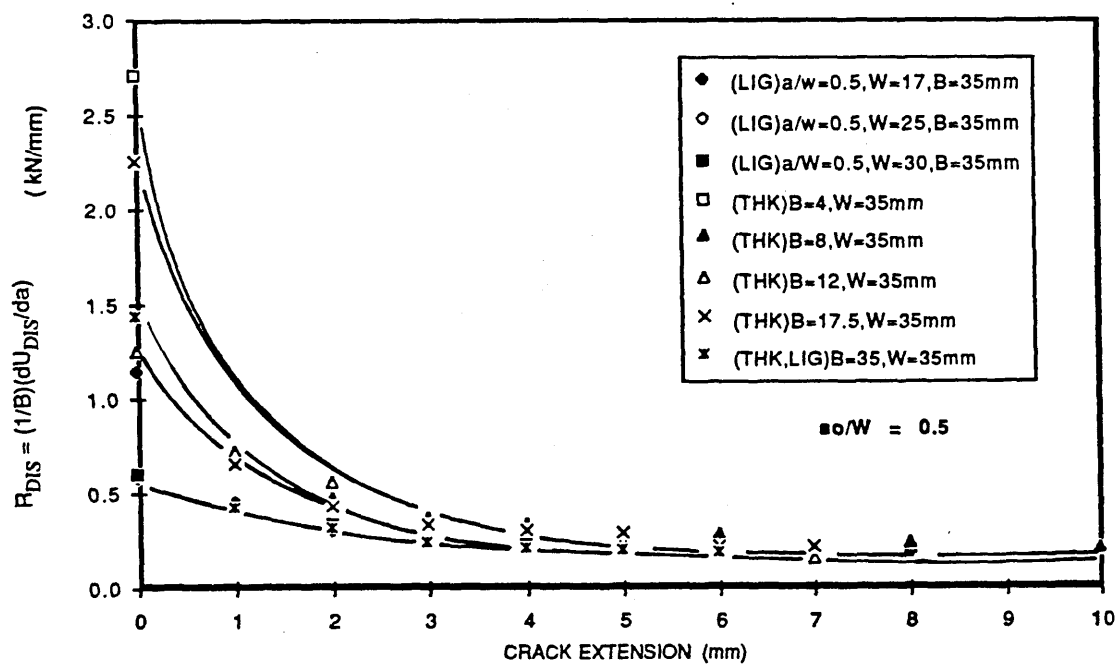


FIG. 8.9 Dissipative work rate, R_{DIS} , against crack extension for various ligaments and thicknesses.

CHAPTER 9

DISCUSSION

9.1 FATIGUE PRECRACKING OF Ti (6Al-2Cb-1Ta-1Mo)

The constant $K_{\max}$ value maintained during fatigue crack growth (FCG) has proved sufficient to ensure a reduced tortuosity on the crack face. It is believed that when the FCG is tortuous, the cyclic plastic zone size, r_{pc} , is less than the grain size, l_g , and hence the FCG is microstructurally sensitive. The reduced bifurcation occurs when,

$$r_{pc} < l_g$$

Though this requires comparatively higher $K_{\max}$ values during FCG, as long as it keeps within the ASTM E399 (1981) standard and maintained at a relatively constant $K_{\max}$ value, the resulting fatigue crack face is considerably a lot smoother than without the above restrictions. Plates 5.1a and 5.1b show the differences in Ti(6-2-1-1). It is suspected that the relatively large grain size of titanium alloys, in general, (see RMI booklet (1983b)), creates the problem of the FCG being microstructurally sensitive.

Therefore, the recommended fatigue precracking procedure for Ti(6-2-1-1) can be regarded as meeting the ASTM E399 (1981) standard, but with the added necessity of a high enough ΔK or $K_{\max}$ to ensure that the cyclic plastic zone size, r_{pc} , is larger than the effective grain size, l_g . This is necessary to establish a microstructure insensitive mode of FCG, see Yoder and Cooley (1978, 1979), and thus result in reduced bifurcation in the crack growth path.

However, the practical difficulty in quantifying r_{pc} and l_g posed a problem. This led to the concept of fatigue precracking at constant ΔK or $K_{\max}$. The premise behind this, is the belief that one degree of FCG bifurcation can be associated with one value of ΔK or $K_{\max}$. Thus, in Ti(6-2-1-1), where the FCG is supposedly prone to microstructure sensitivity, a constant ΔK or $K_{\max}$, might ensure a recognisable pattern on the fatigue

crack face. The morphology of this crack face is generally less convoluted, compared to a procedure that will have numerous degrees of bifurcations for varying values of ΔK or $K_{\max}$, which inadvertently occurs when the test specimens are conventionally fatigue precracked.

Although high values of ΔK or $K_{\max}$ (50-80% K_{IC}) might satisfy the requirement stated above (i.e., $r_{pc} > l_g$), it might violate precracking standards set out by ASTM E399 (1981) standard. Indeed trial fatigue tests that were carried out at excessively high (60-70 Mpa $\sqrt{m}$) $K_{\max}$ values showed comparatively reduced tortuosity on the fatigue crack face.

As a result of observing the ASTM E399 (1981) fatigue precracking restrictions and keeping a relatively constant $K_{\max}$ (or ΔK value since $K_{\min} \approx 1 \text{ Mpa}\sqrt{m}$) value of 31 to 37 Mpa $\sqrt{m}$, a reasonable (four millimeters of crack growth in ninety minutes) rate of FCG can be sustained in Ti(6-2-1-1).

9.2 COMPUTER INTERACTIVE R-CURVE TEST METHODS

9.2.1 THE KEY-CURVE TECHNIQUE

COMPUTATION

The entire computational processing was confined to the 32K microcomputer. This capability was made possible by the program-Datafile-Link-up system, see Fig.4.7, which essentially discretised the numerical processing. Unlike the single specimen-completely computer processed-Unloading Compliance procedure discussed in subsection 9.2.2 below, the Key-Curve method is a Multi-specimen technique and the numerical work is only partially computer processed.

FORMULATIVE EQUATIONS

$$dJ_n = [(2b/W)F_{1n} - (b/W)^2(\partial F_1/\partial(a/W))_n] d\Delta_n + [-2/W \sum_{i=1}^n F_{1i} d\Delta_i + 4b/W^2 \sum_{i=1}^n (\partial F_1/\partial(a/W))_i d\Delta_i] da_n \quad (4.1)$$

and,

$$da_n = \frac{[dP_n - (b^2/W^2) \partial F_1 / \partial (\Delta/W) d\Delta_n] W}{(b^2/W) (\partial F_1 / \partial (a/W)) - 2bF_1} \quad (4.2)$$

Attention here is directed towards the function $\partial F_1 / \partial (a/W)$, values of which are obtained from fig.4.5. From this figure, it can clearly be seen that F_1 is a weak function of a/W . This is however not surprising as seen when F_1 is plotted against normalised displacement for various crack depth ratios in fig.4.6. The strong dependence of the load, P , on displacement and crack length, as shown in fig.4.4, is weakened, especially for crack length when F_1 is introduced since F_1 is PW/Bb^2 ; the b^2 playing a strong normalising role.

Also, for limited crack extension up to about maximum load, F_1 is a particularly weak function of a/W . The weak function of F_1 against a/W , coupled with the extensive scatter shown in fig.4.5 provides suspect values of $\partial F_1 / \partial (a/W)$. Also inherent in the analysis is the assumption that $\Delta a=0$ for all the specimens which could be questioned since crack extension is known to initiate before maximum load in Ti(6-2-1-1). Therefore, the b values in Eqs.4.1 and 4.2 might not entirely be correct, particularly close to maximum load. In the light of the above discussion, the repeatability of the continuous Key-Curve- generated J-R curve shown in fig. 5.11 was difficult to obtain when applied to other a/W ratios. Finally, as seen in the description of the method in subsection 4.5.3.1 and 4.5.3.2., the Key-Curve procedure is definitely much more difficult to apply than the Unloading Compliance method.

2.2.2 THE UNLOADING COMPLIANCE TECHNIQUE

EXPERIMENTATION

This method was considered the most suitable and was consequently used for the geometry effect study of the J-R curves discussed below. The procedure uses a single specimen and thus results in considerable savings in material and time when compared with the Multi-specimen and Key-Curve methods, see section 3.3. Also this procedure is wholly computer interactive, while the other methods are not.

COMPUTATION

The integration under the load-displacement plot is fairly accurate to within 5%, if the proper limits for the integration are obtained, i.e., data sample pairs between successive

unloadings, see appendix II. Extraneous displacements are corrected for by subtracting the indentation work from the actual work done, see section 5.3. The nonlinearity of the unloading lines are limited by the extent of unloading and hence the number of data pairs acquired during unloading. A 10 -15% unloading is usually sufficient to cause substantial linearity of the unloading slope. Also, included in the software, SULCO, described in appendix II, are steps aimed at processing the central portion of the unloading line. These include a small delay in data acquisition when unloading commences and the first and last data pairs are ignored in the least-squares analysis. A measure of linearity is provided by the correlation coefficient for each unloading, see output of SULCO in appendix II.

The crack length predictions by mouth-opening compliance was adopted since the specimen load-line compliance calibration proposed by Tada et al(1973). was inadequate for crosshead compliance measurements. Since practical difficulties were envisaged in measuring specimen load-line compliance, the mouth-opening compliance calibration by Kapp et al (1985) was used and subsequently incorporated in SULCO, the Unloading Compliance interactive program.

Crack predictions (using the mouth-opening compliance calibrations by Kapp et al(1985), see Equ.3.14, and provided the crack front is relatively straight) were generally accurate to within 0.5mm. This is usually the case for sidegrooved specimens. For non-sidegrooved specimens, where crack tunnelling occurs the empirical correction equations 5.3, 5.4, and 5.5 appear to correct the predicted (effective) crack length to the optically measured crack length. Fig.5.9 shows a comparison of predicted, corrected and measured final crack length (based on the seven-point average). This seven-point-average correction equation (Equ.5.3) has also been incorporated in SULCO. The limitations of the use of these equations are given in section 5.5. Therefore, even with non-sidegrooved test specimens, the crack length predictions could be fairly close to the actual measured crack length if the curvature correction equations in section 5.5 are adopted.

9.3 EFFECT OF SPECIMEN TYPE ON J-R CURVES

As shown in fig.5.10, for any formulation of the J integral, the resistance curves for the CKS and three-point bend specimens compare well to within 5%. This is not very surprising since the stress states in these specimen configurations are similar; the CKS approaching the bend situation with increase in crack depth.

2.4 EFFECT OF LOADING RATES AND TEST METHODS ON J-R CURVES

Section 5.8 reports the effect of loading rates on the J-R curve for both the Unloading Compliance and Multi-specimen test techniques for either test method. The effects are minimal, more so for the former technique. The marginally higher J-R curve for the slow test (0.03mm/min) of the Multi-specimen technique is due to the accompanying increase in the measured work after maximum load, see fig. 5.14. This increase might be due to the availability of time, enough to slow down the crack growth rate and hence drop in load during post-maximum load R-curve testing. The Unloading Compliance procedure, on the other hand, hardly showed any change in J-R curves for different testing rates. This is again, not surprising since the rates were monitored only during loading, while the unloading rate was dictated by the digital data acquisition system frequency.

Comparing the effect of test methods, at normal rates (0.4mm/min), on the J-R curve, fig.5.12 shows the Unloading Compliance method marginally higher to within 10% error of that produced by the Multi-specimen method. Also plotted here, are slow (0.03mm/min) and fast (1.23mm/min) points of a Multi-specimen type test. The slow test is slightly higher than the fast and approaches the Unloading-Compliance curve. So, it can be generally stated that within reasonable loading rates (about 0.4mm/min), the J-R curves for Ti(6-2-1-1) are invariant with rates irrespective of the test method used.

2.5 EFFECT OF Ti(6-2-1-1) SPECIMEN ORIENTATION ON J-R CURVES.

In fig.5.15, the J-R curves are shown to compare quite well in the orientations studied (T-S, T-L and L-T).

2.6 GEOMETRY EFFECTS ON J-R CURVES IN THE THREE-POINT BEND CONFIGURATION

2.6.1 J - FORMULATIONS

In this study three versions of the J-integral were used-

J_0 (Equ.2.27), J_{i+1} (Equ. 2.28) and J_m (Equ. 2.30a).

Recalling these equations:

$$J_0 = \eta (U + \Delta U) / Bb_0 \quad (2.27)$$

$$J_r = J_{i+1} = [J_i + (\eta/b)_i U_{i,i+1}] [1 - (\gamma/b)_i (a_{i+1} - a_i)] \quad (2.28)$$

$$J_m = (J_m)_i + (\eta/b)_i (U_{i+1} - U_i)/B \quad (2.30a)$$

The similarity between J_o and J_m is seen here; the difference being that the incremental usage of J_m while J_o is sequentially applied, see subsection 2.4.3.2. Also seen in all the plots of J_o and J_m , both always rise but with J_m slightly higher than J_o . The Ernst's J formulation (J_{i+1}) droops and is considerably lower than J_m and J_o . These observations can clearly be seen in the conventional and normalised J-R curves shown in chapter six.

2.6.2 GEOMETRY EFFECTS ON CONVENTIONAL (J- Δa) J-R CURVES

ABSOLUTE SIZE (Square cross-section, $a_o/W=0.55$)

Here the J_m and J_o show similar trends with a fairly strong dependence on specimen size. J_{i+1} is generally lower, with a weaker dependence on specimen size, see figs. 6.2a, 6.3a and 6.4a. As expected the larger the size, the lower the J-R curve, since triaxiality and hence plane strain conditions are enhanced with specimen size.

IN-PLANE CONSTRAINT ($B=17.5\text{mm}$, $W=35\text{mm}$)

With the initial crack depth ratio varied as described in section 6.3, all the three aforementioned J formulations show a fairly weak dependence on the in-plane constraint, see figs. 6.5a, 6.6a and 6.7a. The shear lips might have attained steady state in all the crack depth ratios investigated, hence the weak dependence.

THICKNESS ($W=35\text{mm}$, $a_o/W=0.55$)

The dependence of thickness on the J_o , J_{i+1} and J_m is weak, though stronger than the correlation of the in-plane study it is weaker than the absolute size study. In general, the thickness effect on Ti(6-2-1-1) is independent of the J -formulation and its effect on J-R curves, as seen in figs. 6.8a, 6.9a and 6.10a, is not significant. However, a usual trend of the thinner specimens having a slightly higher R curve is noticed.

LIGAMENT LENGTH ($B=35\text{mm}$, $a_o/W=0.55$)

The ligament length is particularly dependent on J_m and J_o Rcurves, though the dependence on J_{i+1} R curves is reduced. In this study, as shown in figs. 6.11a and 6.13a, the smallest ligament produces the highest R curve.

2.6.3 EFFECT OF NORMALISING J- Δa CURVES

ABSOLUTE SIZE

The vast discrepancies in the specimen sizes shown for J_0 and J_m in figs. 6.2a and 6.4a are drastically reduced if the abscissa is normalised with the thickness, B , as shown in figs. 6.2c and 6.4c. Scatter is also reduced by normalising the abscissa with the original ligament, b_0 , and the thickness, B . The J_{i+1} -R curve which showed little dependence on absolute size, when normalised with B or b_0 did expectedly, not improve the correlation.

Though all the categories were normalised with a single parameter, the concept of selective normalisation is introduced here. From figs. 6.2a and 6.4a, it is seen that the smaller sized specimens (10x10 and 20x20mm) fall out of line with the larger sized specimens. It could be argued that the smaller sized specimens need normalisation since its size, (particularly its ligament length) is comparable to shearlip sizes found in Ti(6-2-1-1), i.e.,

$$r_{ma} > b, B$$

where r_{ma} is the actual maximum shearlip size found in Ti(6-2-1-1). A value of 8-10mm has been observed in full thickness specimens. For the larger specimens, $r_{ma} < b, B$, therefore do not necessarily require normalisation.

IN-PLANE CONSTRAINT

As discussed in subsection 9.6.2, the variation in J-R curves for this category is minimal and hence does not need normalising. Figs. 6.5b and 6.6b nevertheless show the b_0 -normalised curves and little improvement is shown. Since B is fixed, the shearlip sizes, which are comparatively small in Ti(6-2-1-1), might have attained early steady state sizes in all the categories tested.

THICKNESS

The plane strain plastic zone size, r_{pe} , (Equ. 2.6) for Ti(6-2-1-1) is approximately 1.2mm. Normalising fig. 6.8a and 6.10a for J_0 and J_m respectively, considerable improvement is obtained for $B=12\text{mm}, 17.5\text{mm}$ and 35mm , when the abscissa is normalised with B as shown in figs. 6.8b and 6.10b. It might be argued here that normalisation of the abscissa with B for thicknesses larger than 12mm is good since $B > r_{pe}$ or r_{ma} , see Etemad and Turner (1986). When B approaches r_{pe} or r_{ma} (for

thicknesses of 8mm and 4mm), better correlation is realised when the ordinate is normalised with B , see Etemad and Turner(1986). Hence thickness of 8mm and 4mm fall out of trend when the entire variation-in-thickness category is normalised with B . A fair scatter is shown for the J_{i+1} -R curve (fig.9.6a) with only a marginal improvement when normalised with b_0 as shown in fig.6.9b. This again, is another example of the selective normalisation procedure mentioned above.

LIGAMENT

Better correlation, particularly for J_0 and J_m is made when the abscissas of figs. 6.11a and 6.13a are normalised. This normalisation with the original and current ligament b_0 and b_i , shown in figs.6.11a and 6.11b respectively for the J_0 -R curve. A similar normalisation with b_0 is made for the J_m -R curve in fig.6.13b. Again the normalisation with b_0 improves the correlation for the J_m and J_0 -R curves. The J_{i+1} -R curves are, as expected hardly affected by normalisation with b_0 as shown in fig.6.12b, since the conventional J_{i+1} -R curve showed little variation in the curves for different ligament lengths.

From the above comments and observation of figs.6.2a through to 6.13b, it can be stated that normalisation does, in general, reduce the discrepancy in the J_m and J_0 R curves for Ti(6-2-1-1) since they are work-based formulations. J_{i+1} -R curves can arguably be considered not to need normalisation since J_{i+1} is an energy-based (see subsection 2.4.5). So, generally, normalisation should work better for the work-based J_m and J_0 -R curves. The J_m formulation has more recently been proposed as an improved version of the J integral, see Ernst (1983), since it satisfies the Rice et al (1980) condition.

2.7 EFFECT OF 1.5mm SIDEGROOVES ON EITHER SIDE OF Ti(6-2-1-1) SPECIMENS

The effect of such a small amount of sidegrooving is clear to see when figs.6.14a through to fig.6.17b are observed. The discrepancies in J-R curves, irrespective of the J-formulation is drastically reduced by sidegrooving an amount that is approximately the plane strain plastic zone size, r_{pe} . This work suggests that the removal of the shearlip size, of which r_{pe} is some measure is adequate to provide geometry

independent J-R curves in Ti(6-2-1-1). Though the work reported here is on a titanium alloy, application of this concept to ferric materials such as steel is quite possible. Indeed, work on sidegrooves by Vassilaros et al (1983) show that on a A533B (large r_{pe}), sidegrooves give geometry DEPENDENT J-R curves, while sidegrooved HY130 (small r_{pe}) present J-R curves, INDEPENDENT of geometry.

2.8 $R_{DIS} = (1/B)(dU_{DIS}/da)$ -AN ALTERNATIVE FORM OF REPRESENTING TOUGHNESS

In spite of some rationalisation of J-R curves discussed above, an alternative form of representing toughness, based on the original concept of toughness by Griffith (1922) is discussed here, i.e., work required for crack growth. Since Griffith used fairly brittle materials in his study, the problem of plastic deformation hardly arose.

The analysis here draws a close analogy with that of Griffith but with the added complexity of plasticity. The concept of post-yield toughness introduced here is that which represents a materials energy absorption during (strictly) crack extension and plastic deformation; the latter dominating over the former for most metals. In this study, the toughness parameter is represented by (dU_{DIS}/da) , where U_{DIS} is the energy dissipated during crack extension and plastic deformation. This leads to R_{DIS} defined as (dU_{DIS}/da) , where $(1/B)$ is introduced to account for geometrical variations when applied to a variety of geometries.

It is however the contention of this study, that a falling resistance curve is representative of the true, basic behaviour of a material after crack initiation. The presentation of J and other toughness parameters as a rising resistance curve is hereby challenged and is considered a 'deceptive' form of toughness representation.

Although the shearlip size would grow to an equilibrium size during crack growth, there is overwhelming evidence to suggest (see figs. 8.2a to 8.5b) that whilst it does this, the energy, be it total or dissipated per crack extension, is ever decreasing. Using the slopes shown in figs. 8.2a through to 8.5b to evaluate R_{TOT} (dU_{TOT}/da) and R_{DIS} for the geometry variation categories discussed in section 9.6 above, plots of $R_{TOT, DIS}$ are shown in figs. 8.6 through to 8.9. The trend is clear to see:- Irrespective of geometry, all the curves fall and converge together. This is definitely a desired form of toughness representation where the R-R curves converge rather than diverge as depicted by

conventional J-R curves , particularly beyond J-controlled crack growth.

Comparing R_{TOT} and R_{DIS} resistance curves, although slightly more scatter is shown by the inclusion of the elastic component of the work done, the trend is similar for large amounts of crack growth.

Also, the falling R curve might have some implications in ductile tearing instability instability analysis discussed in section 9.9 below.

2.2 DUCTILE TEARING INSTABILITY IN Ti(6-2-1-1)

2.2.1 COMPARISON OF THE T AND I METHODS OF ANALYSIS FOR THE INCIPIENT INSTABILITY CASE (FIG.7.1)

For the T expression in Equ.2.38:-

$$T_{app} = \frac{2b^2 S_e}{W^3} - \frac{JE}{\sigma_y^2 b} \quad (2.38)$$

In this analysis, the effective span S_e to correct for compliance is taken as,

$$S_e = S (1 + C_{me}/C_u)$$

where $C_{me} = C_n + C_m$ (notched test piece plus machine compliance). This differs from the previous analysis by Etemad (1983) and Turner(1984), where C_{me} was taken as $2C_m$; in that analysis C_n fortuitously coincided with C_m .

The prediction by Equ.2.38 is shown in comparison with T_{mat} and T_{app} from Equ.2.40 in fig.7.6. From this plot, Equ.2.38 predicts affirmative instability whereas the form by Ernst(1981a,1981b) predicts that T_{app} is always close to T_{mat} . Surely, from fig.7.1, this is not the case. However the result is not unexpected when Equ.2.40 is recalled and functions in it, examined.

$$T_{app} = E/\sigma_y^2 \left\{ \left(\frac{\partial J}{\partial a} \right)_{\Delta} + \frac{\left(\frac{\partial J}{\partial \Delta} \right)^2_{\Delta} \left[\frac{1}{k_{me} + dP/d\Delta + \left(\frac{\partial J}{\partial \Delta} \right)^2_{\Delta}} \right]}{\left(\sigma_Y^2/E \right) T_{mat} - \left(\frac{\partial J}{\partial a} \right)_{\Delta}} \right\} \quad (2.40)$$

where $[(\partial J/\partial \Delta)]_a = \eta P/b$ and $[(\partial J/\partial a)]_{\Delta} = -\gamma J/b$. $\gamma=1$ for the compact tension specimen.

To start with, the form proposed by Ernst was fraught with dimensional inconsistencies. k_{me} and $dP/d\Delta$ had to be evaluated per unit thickness to maintain dimensional homogeneity with $[(\partial J/\partial a)]_{\Delta}$. The $[(\partial J/\partial \Delta)^2]_a$ function in the denominator of the second term in Equ.2.40 was put as $[(\partial J/\partial \Delta)^2]_{\Delta}$ by Ernst (1981a,1981b), which is clearly incorrect if Equ.2.40 is derived. Also, the approximations of $[(\partial J/\partial \Delta)]_a$ and $[(\partial J/\partial a)]_{\Delta}$ could be questioned.

In general the form proposed by Equ.2.38 appears more amenable for application to real test cases of ductile instability than the unwieldy form shown in Equ.2.40. The prediction by Equ.2.38 is certainly more plausible and affirmative than that proposed by Equ.2.40.

The energy based I approach predicts instability at about the same extent of deformation and crack growth as Eqn.2.38 of the T method. A simple comparison of the prediction trend of the I (fig.7.7) and the T approach (fig.7.6) shows a even more definitive instability prediction by the former method. The reason for preferring the I method is that the elastic energy available for fracture in this elastic-plastic system, I, continues to increase while the energy absorption rate decreases. Although the T method (Equ.2.38) predicts the instability with close agreement to the actual experimental observation and the I method, both T_{app} and T_{mat} appear to decrease with crack growth and possibilities of these two functions meeting again cannot be ruled out.

An inference from the instability discussion in Ti(6-2-1-1) above, appears to indicate that the energy based I method predicts instability with somewhat greater certainty than

the displacement based T method, with only one (Equ.2.38) of the two T_{app} formulations making plausible predictions.

2.2.2 COMPARISON OF THE INCIPIENT AND FINAL DUCTILE INSTABILITIES USING THE I METHOD

In comparing plates 7.2 and 7.4, electron microscopy studies indicate fibrous modes of failure for both the slow(stable) and fast(unstable) modes of failure in Ti(6-2-1-1). Hence the fast, unstable fracture that occurred in the full thickness, shallow notch specimen did in fact fail in the ductile mode. This qualifies the ductile instability I method for application to this type of failure.

To compare the shallow notch (final) instability to the deep notch (incipient instability), fig.7.7 is re-plotted on a larger scale to accommodate the I analysis values for the final instability case in fig7.8. The one striking difference between the two cases is the very rapid fall in the energy required for fracture in the shallow notch case. A cause for the final instability could be rationalised as follows:-In the full thickness ($B=50\text{mm}$), shallow notch case, the comparatively large elastic energy input into the system is equally resisted against crack growth prior to initiation. Once crack growth has started, this resistance to fracture (energy absorption rate) begins to fall rapidly. Eventually, when I exceeds this ever decreasing energy absorption rate, the enormous initially introduced absolute elastic energy is released with consequences of fast, uncontrolled fracture; powerful enough to sever the 50mm thick specimen in a split second.

Implications of this type of analysis can be linked with the proposed falling R curve-type behaviour exhibited by the R_{DIS} , discussed above. Indeed, the steady state value of R_{DIS} , shown in figs.8.8 and 8.9, coincides with the energy absorption rate term of the I analysis, shown as the (R.H.S) function in figs.7.7 and 7.8. In the light of the I analysis presented here and the benefits of the R_{DIS} -R curve discussed in chapter eight, it is perceived that toughness of a material is better understood and represented when the resistance curves are presented as work-rate-based.

CHAPTER 10

CONCLUSION

THE UNLOADING COMPLIANCE TECHNIQUE

The Unloading Compliance testing procedure for obtaining J-R curves was developed using on-line microcomputer interaction for data acquisition and processing. The capabilities of the software designed specifically for this purpose (details given in appendix II) include evaluation of test specimen crack length estimates (with corrections for crack front curvature), indentation corrections and two versions of the J integral. Plots of the J-R curves generated can be obtained instantly by implementing graphic routines incorporated in the software.

COMPARISON OF THE TESTING TECHNIQUES

The Unloading Compliance testing technique is preferred to the Key-Curve and Multi-specimen methods because it has clear advantages in terms of savings in terms of manual and computational effort required. Data processing for the methods used in this study is wholly computerised for the Unloading Compliance method but only partially so for the Key-Curve method. The Multi-specimen procedure on the other hand, involves no computer interaction and is comparatively labour intensive.

STRUCTURAL REPRESENTATION OF R-CURVES

Resistance curves were obtained for Ti(6-2-1-1) three-point bend specimens and were analysed by the J-based (rising curve concept) and the work rate(R_{DIS})-based (falling curve) concept. In the study of geometry effects on J-R curves within the three-point bend configuration, the role of normalising J-R curves was investigated.

From this study, the following findings have emerged:

Conventional J-R curves (J- Δa plots) represented as J_0 , J_m and J_{i+1} based R curves showed marked dependence of variations in the absolute size and ligament of test specimens. Less dependence was noticed for variations in in-plane constraint and thickness. Toughness behaviour representation in this form is crucial to the prediction of ductile instability, discussed below.

Normalisation of the abscissa with parameters such as B and b appear to reduce the dependence of geometry effects of Ti(6-2-1-1) if resistance

curves are represented in this form. Normalisation works better for the work-based J_O and J_m R curves than it does for the internal energy-based J_{i+1} -R curves, since the latter does not require normalisation by parameters that seemingly, limit the amount of work done.

Sidegrooves of 1.5mm (approximate size of r_{pe} - plane strain plastic zone size) on either side of Ti(6-2-1-1) test specimens result in relatively geometry independent R curves. It is suspected that sidegrooves, as small as the plane strain plastic zone size of any material are sufficient to produce geometry independent J-R curves.

$R_{DIS} = (1/B)(dU_{DIS}/da)$ is presented as an alternative form of representing toughness using resistance curves. A work-rate resistance curve with a toughness parameter such as R_{DIS} , is believed to represent a basic resistance curve for any material.

DUCTILE TEARING INSTABILITY

Incipient and final ductile tearing instability occurred in three-point bend deep and shallow notch specimens respectively.

From the instability analysis done, the following can be stated:

Both types of instability can be predicted satisfactorily, by the I and T methods of analysis with the former method making more affirmative predictions.

The final instability analysis, using the I method, shows a comparatively rapid fall in the work absorption rate. Attention here is focused on the significance of this behaviour in representing resistance curves. It is perceived that a work rate dissipation-based toughness parameter is a true representation of a material's toughness during crack growth. Indeed, the steady state value of R_{DIS} is found to be approximately equal to that of work absorption rate term of the I method of analysis for Ti(6-2-1-1).

APPENDIX I

PROGRAMS FOR UPDATING THE STRESS INTENSITY
FACTORS DURING PRE-CRACKING OF Ti(6-2-1-1)
COMPACT TENSION AND THREE-POINT BEND SPECIMENS

```

100 REM *****
110 REM
120 REM      3PTB  C A L I B
130 REM
140 REM *****
150 REM
160 PRINT"J"
170 REM THIS PROG. CALCULATES S.I.FS FOR THE 3-POINT -BEND SPECIMEN
180 PRINT"PLEASE SUPPLY CRK.DEPTH RATIO"
190 INPUT AW
200 B=3*SQR(AW)*(1.99-AW*(1-AW)*(2.15-3.93*AW+2.7*(AW*AW)))
210 C= 2*(1+2*AW)*(1-AW)^1.5
220 W=100
230 FAW=B/C
240 B1=50
250 S=W*4
260 PRINT"PLS. INPUT MAX. FATGE. LOAD"
270 INPUT P
280 PRINT"J"
290 PRINT"      STRESS INTENSITY UPDATE"
300 PRINT"-----"
310 K=P*S/(B1*W^1.5)*FAW
320 PRINT"K=";K;"KN/MM^1.5";"      F(A/W)=";FAW
330 KM=31.62*K
340 PRINT
350 PRINT
360 PRINT
370 PRINT
380 PRINT"OR K=";KM;"MPA*MM^10.5"
390 CRK=W*AW
400 LIG=W-CRK
410 SIGY=0.728
420 PL=4*B1*(LIG^2)*.728/(S*3)
430 POLL=P/PL*100;LD=224.82*PL
440 PRINT
450 PRINT
460 PRINT"LIMIT LOAD=";PL;"KN";"  OR";LD;"IBF"
470 PRINT
480 PRINT"% OF LIMIT LOAD=";POLL;"%"
490 KIC=91.7
500 POK= KM/KIC*100
510 PRINT
520 PRINT
530 PRINT
540 PRINT
550 PRINT
560 PRINT"% OF KIC=";POK;"%"
570 PRINT
580 PRINT
590 PRINT"USING A/W=";AW;"AND OP. LOAD=";P;"KN"

```

```

100 REM *****
110 REM
120 REM      C T  C A L I B
130 REM
140 REM *****
150 REM
160 PRINT"J"
170 REM THIS PROG. CALCULATES S.I.FS FOR THE COMPACT TENSION
180 PRINT"PLEASE SUPPLY CRK.DEPTH RATIO"
190 INPUT AW
200 B=2+AW
210 C= 0.886+4.64*AW-13.32*AW^2
220 D=14.72*AW^3-5.6*AW^4
230 FAW=B*(C+D)/(1-AW)^1.5
240 B1=49.1
250 W=100
260 PRINT"PLS. INPUT MAX. FATGE. LOAD"
270 INPUT P
280 PRINT"J"
290 PRINTTAB(23)"      STRESS INTENSITY UPDATE"
300 PRINTTAB(23)"-----"
310 K=P/(B1*W^10.5)*FAW
320 PRINTTAB(27)"K=";K;"KN/MM^1.5"
330 KM=31.62*K
340 PRINT
350 PRINT
360 PRINT
370 PRINT
380 PRINTTAB(24)"OR K=";KM;"MPA*MM^10.5"
390 CRK=W*AW
400 LIG=W-CRK
410 SIGY=0.728
420 PL=B1*(LIG^2)*.728/(2*W+CRK)
430 POLL=P/PL*100
440 PRINT
450 PRINT
460 PRINTTAB(25)"LIMIT LOAD=";PL;"KN"
470 PRINT
480 PRINTTAB(25)"% OF LIMIT LOAD=";POLL;"%"
490 KIC=91.7
500 POK= KM/KIC*100
510 PRINT
520 PRINT
530 PRINT
540 PRINT
550 PRINT
560 PRINTTAB(25)"% OF KIC=";POK;"%"
570 PRINT
580 PRINT
590 PRINTTAB(20) "USING A/W=";AW;"AND OP. LOAD=";P;"KN"

```

APPENDIX II

A USER'S GUIDE TO THE UNLOADING COMPLIANCE PROGRAM, SULCO

INTRODUCTION

SULCO, for Specimen UnLoading Compliance, is an interactive computer program developed for data acquisition, numerical processing and fracture mechanics evaluations. It is written entirely in BASIC computer language for Commodore computers. It links the microprocessor to an analog/digital converter, a printer, a plotter and a disk drive. The present version of SULCO is shown for three-point bend specimens, though portions of SULCO that need alterations for conversion to compact tension (CKS) use are pointed out during the sectional description of SULCO, shown below. SULCO uses two formulations of the J-integral and these evaluated values are plotted against estimated crack extension values when a graphics routine in SULCO is activated. This "user's guide" is intended to be used in conjunction with a listing of the program at the end of this appendix. A typical output is shown below, after the listing of SULCO.

STEP-BY-STEP UNLOADING COMPLIANCE PROCEDURE

1. The clip gauge and LVDT (for the three-point bend case alone) were calibrated with the devices shown in Plate 4.5. Calibration factors, in mm/mV, were noted during the calibration for input data for the computer.
2. The load cell is calibrated using a zero-offset knob on the Mayes and Tinius Olsen machines, here also a calibration factor in KN/mV is noted. These calibration steps are essential for the interpretation of the analog plots obtained on the H-P plotter which is constantly on-line.
3. CKS
The clip gauge is attached onto the integral knife edge in line with the loading axis. With the clip gauge in position, the specimen is held in place by shackles on the testing machine.

Three-point bend specimen

The clip gauge is attached to the knife edges which are in line with the specimen's top face. The LVDT transducer is positioned such that the movement of the cross head is centrally measured, see plate 4.8.

4. The calibration factors, as determined above, are typed in with other pertinent test data. Then after a quick check on the load range, ram displacement range and filter settings (set at 2.5k Hz low pass), data acquisition is started simultaneously with starting the test.
5. By observing the analog plot, the specimen is loaded at the rate of 0.7mm/min to about 35% of the estimated limit load. At this point the key "U" is pressed and the computer goes into the "unloading mode". The unloading is manually controlled till about 10% - 20% unloading takes place. During this unloading sequence, 60 - 70 load-displacement pairs are evaluated to obtain the compliance. This compliance value is then used to calculate a predicted initial crack length.
6. After each subsequent unloading, crack growth estimates, J-values and other pertinent information are displayed on the screen. This routine provides the test engineer with the ability to space the unloadings accordingly.
7. When the final unloading sequence is performed all the data evaluated and acquired are immediately stored onto a data disk, as a precautionary measure due to the "volatile" nature of data in the memory banks of a computer.
8. Once all the data have been transferred on disk, computer graphics of J - Δa values can be "safely" implemented on the computer's V.D.U. Since the computer system is interfaced with a plotter and printer, hard copies of all the information can be obtained. See Fig 4.3 for the schematic of the instrumentation system.

PROGRAM ORGANISATION

Figure A1 gives a flow chart of SULCO. It consists of various functions, depending on the stage of the test. The principal subroutines are as follows:

1. Data acquisition loops for the loading and unloading sequences.
2. Least squares analysis of the data points for compliance evaluations.
3. Crack length predictions and curvature corrections using the wide range expressions by Kapp et al (1985) and the curvature correction analysis described in Chapter Five.
4. Integration to find out the area under the load-displacement ((LVDT) plot, using finite strip summations.
5. From the area (work done), J_0 and J_{i+1} are calculated from Eqs. (2.27) and (2.28).
6. Graphics routine for plotting J-R curves on the Visual Display Unit (VDU).
7. Storing and retrieving data from disks in the disk drive.
8. Plotting software for the JJ instruments - PD4 plotter.

PROGRAM VARIABLE NAMES

The BASIC variable names used in SULCO are defined in alphabetical order in the following list.

The variables are categorised by the designations:

as - alpha numeric single
rs - real single
ra - real array
is - integer single

B	rs	Specimen thickness
CAD	ra	Compliance correction for crack curvature
CFC	ra	Crack length correction for crack front curvature
CRK	ra	Measured (effective) crack length
CV	rs	Clip gauge calibration factor
D	ra	LVDT transducer displacement

DT	rs	LVDT calibration factor
DELA	ra	Uncorrected-for-curvature crack extension
D\$	as	Date
E	rs	Young's modulus
EX	is	Sample point number for start of integration
EXTC	ra	Curvature-corrected crack extension
ETA	rs	η factor (three-point bend)
FUL	is	Total number of unloadings
J1T	ra	J-Integral, Equ. (2.28)
JO	ra	J-Integral, Equ. (2.27)
LIC	ra	Measured mouth-opening compliance
LR	rs	Load range calibration
M	rs	Slope on load-displacement plot
P	ra	Applied load
SA	is	Sample number
SAM	is	Sample number total
SG	is	Side grooved or not indication
SN\$	as	Specimen identification
S5	ra	Correlation factor
U	ra	Area under the load-displacement plot (work done)
UNL	is	Unloading number
V	ra	Clip gauge displacement
W	rs	Specimen width
Y	is	Sample point number for end of integration
YS	rs	Yield stress

SECTIONAL DESCRIPTION OF SULCO

LINESFUNCTION

100 - 780	Documentation of SULCO is shown and the input data that is required (lines 400 - 570). Lines 570 - 780 displays input data on screen for checking before proceeding with the program execution. Array sizes are set to a maximum of 1500 because of computer memory space restrictions.
790 - 1500	Acquires data for the CIL A/D unit and it displays the values during the loading stage. This ensures that the microprocessor is actually "in touch" with correct incoming signals. When unloading has commenced, a "manual" control is inserted in 1260 which requires the key "U" be pressed to proceed onto the unloading mode. To proceed to the "OPTIONS" stage, Key "M" needs to be pressed, this instruction is given in line 1270. The data values are converted directly from voltages to real units using some of the input data, RC, CV, DT in lines 1110 - 1250.
1510 - 2000	Data acquisition takes place during unloading. The test engineer then needs to press key "S" to indicate the end of the unloading which usually consists of 60 - 70 data pairs within about 10% - 20% of unloading. After key "S" has been pressed all the necessary data points gathered go for numerical processing. It must be noted also that in line 1690 and 1700 a small delay loop is introduced to avoid the non-linearity problems discussed in subsection 3.3.3b of Chapter three.
2010 - 2250	A least squares analysis is performed for obtaining the slope and correlation factor for the unloading lines of the load-displacement plot. SULCO shown in this appendix uses the mouth-opening measurements for compliance calibration with the crack length. The CKS version would however need only one displacement measurement for both area under the curve and slope evaluation.
2260 - 2530	The mouth-opening compliance, see Kapp et al (1985), is used for crack length predictions and then corrected for curvature (see

LINESFUNCTION

Section 5.5 in Chapter five) if the specimen is not sidegrooved.

In this version of SULCO, the seven point-based curvature correction equation is used, Recalling Equ. (5.3).

$$C_{f,7}/C_{f,comp} = 1.002 + 3.497 (\Delta a/W) - 53.16 (\Delta a/W)^2 + 512.9 (\Delta a/W)^3 - 1948.8 (\Delta a/W)^4 + 2568.8 (\Delta a/W)^5$$

This is included in lines 2410 and 2420.

The CKS version on the other hand, would use the expression by Saxena and Hudak (1978), i.e., Equ. 3.2 in Chapter 3.

2540 - 2740

Integrates the area under the load - (LVDT)displacement using the trapezoidal rule, ie:

$$A_{i-1,i+1} = 1/2 [P(I-1) + P(I)] \times [D(I) - D(I-1)] + 1/2 [P(I) + P(I+1)] \times [D(I+1) - D(I)]$$

Because the summation is carried out every two steps, the number of intervals must be even and lines 2540 - 2600 ensures this.

Also, in lines 2690 - 2720, an indentation correction routine is included. This procedure corrects for extraneous displacements for rising loads as described in Section 5.3 of Chapter five. Due to the configuration of the CKS test set up (Fig 3.10), this procedure is however, not necessary in the CKS Unloading Compliance test.

The limits of the integration are specified by EX and Y in lines 1300 and 1260 respectively. The overall summation is performed by the operation:

$$U_{i+1} = U_i + \int_{EX}^Y U_{i,i+1}$$

<u>LINES</u>	<u>FUNCTION</u>
2750 - 2960	J-integral evaluations are performed with two versions of the J-formulation, i.e., recalling Equ. (2.27) : $J_O (UNL) = U (UNL) + (ETA/SB1) * B$ and Equ. (2.28) : $J_{1T} (UNL) = PTP1 + TPT2$ PTP1 and TPT2 are defined in lines 2860 and 2870 respectively. Numerical processing and calculation ceases after line 2960. The rest of the program activates interfaced peripherals.
2970 - 3240	After each unloading the following acquired and evaluated quantities are displayed on the V.D.U : CRACK LENGTH AREA J (CORRECTED) (Equ. 2.27) J (ERNST) (Equ. 2.28) CRACK EXTENSION DATA POINTS DURING UNLOADING COMPLIANCE CORRELATION FACTOR If the test is to be continued key "L" is pressed after loading has commenced. At the end of the test, key "E" is pressed. This leaves the test engineer with the options given below:
3250 - 3500	At this stage, the test engineer is advised to store all the data before further action is taken. Once this precautionary measure is taken, other capabilities of SULCO can be utilised as described below.
3510 - 3650	Stores all the input data (lines 400 - 520) and evaluated data on disk.
3660 - 3930	Reads all the data from disk and some relevant data is displayed on screen to assure the test engineer that the desired data file has been read.

<u>LINES</u>	<u>FUNCTION</u>
3940 - 4270	Prints specimen/test specification with all the pertinent information for each unloading stage. See a typical output print after the listing of SULCO.
4280 - 5170	Plots J - R curve on VDU and JJ instruments PD4 plotter.
5180 - 5650	Plots load-displacement values on the PD4 plotter.
5700 - 5820	Corrects for negative crack growth by averaging all current crack length values and re-defining crack growth as equal to zero for the current unloading stage.

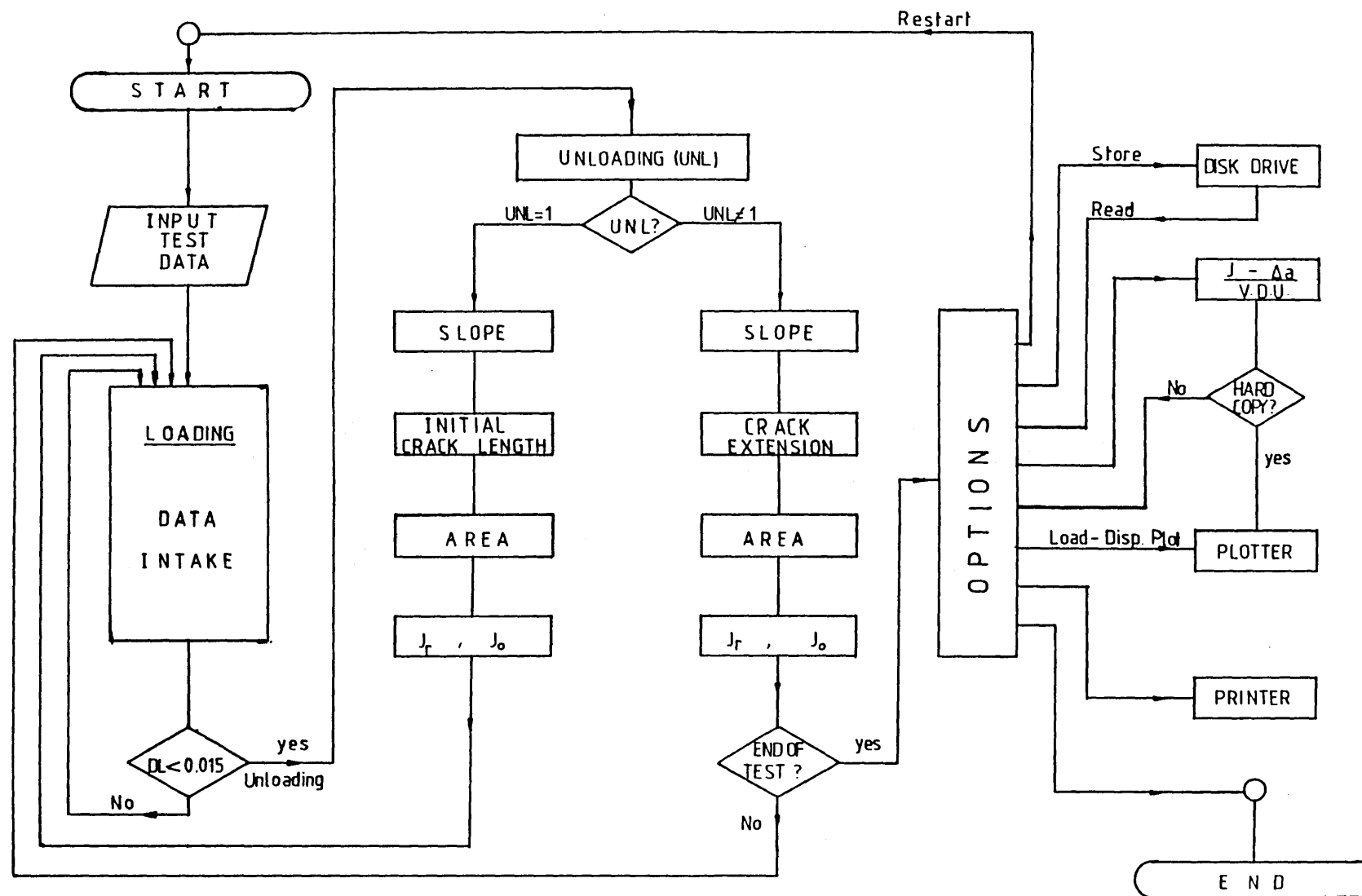


FIG. A1 FLOW CHART FOR SULCO

```

100 REM *****
110 REM
120 REM THIS PROGRAM GENERATES J-RESISTANCE CURVES
130 REM FROM A SINGLE-TEST SPECIMEN USING THE
140 REM UNLOADING COMPLIANCE TEST TECHNIQUE ON
150 REM THREE- POINT BEND SPECIMENS.
160 REM (S,M=4)
170 REM *****
180 REM
190 REM
200 REM
210 REM WRITTEN BY
220 REM S. J. JOHN
230 REM
240 REM
250 REM THIS SOFTWARE IS COMPATIBLE WITH THE FOLLOWING SYSTEMS:-
260 REM
270 REM 1. COMMODORE 4032 AND 3296 COMPUTERS
280 REM 2. COMMODORE 8050 DISK DRIVE
290 REM 3. COMMODORE 3022 PRINTER
300 REM 4. C I L LTD. DATA ACQUISITION SYSTEM
310 REM 5. J J INSTRUMENTS PD4 PLOTTER
320 REM
330 REM
340 DIM P(1500),D(1500),V(1500),JO(20),JIT(20)
350 DIM CRK(20),DELA(20),LIC(20),U(20)
360 DIM SS(20),CV(2),CFC(20),CAD(20),EXTC(20)
370 REM
380 UNL=0:SA=0
390 PRINT "TEST/ SPECIMEN DATA"
400 PRINT " "
410 PRINT "SPECIMEN IDENTIFICATION":INPUTSN$
420 PRINT "IS SPECIMEN SIDE-GROOVED (1 - NO 2 - YES)":INPUTSG
430 PRINT "DATE(D$)":INPUT D$
440 PRINT "THICKNESS(MM)":INPUT B
450 PRINT "WIDTH (MM)":INPUT W
460 PRINT "YOUNGS MODULUS (KN/MM^2)":INPUT E
470 PRINT "FLOW STRESS(KN/MM^2)":INPUT YS
480 PRINT "CLIP GAUGE DISPL. (MM/MV)":INPUTCV(1)
490 PRINT "LVDT CAL. FACTOR (MM/MV)":INPUTDT
500 PRINT "LOAD RANGE (OLSEN (LBS)): TYPE 1 FOR 6000 2 FOR 24000 3 FOR 1200
510 PRINT " [4 FOR 100TON AVERY]"
520 INPUTCH
530 IFCH=1THEN LR=27.1
540 IFCH=2THEN LR=106.65
550 IFCH=3THEN LR=549.1
560 IFCH=4THEN LR=1019.14
570 REM
580 REM ----- SPECIMEN DATA CHECK -----
590 PRINT "SPECIMEN DATA CHECK"
600 PRINTTAB(5) "SPECIMEN NAME" "PRINTTAB(31)SN$
610 PRINTTAB(5) "DATE" "PRINTTAB(31)D$
620 PRINTTAB(5) "THICKNESS (MM)" "PRINTTAB(29)B
630 PRINTTAB(5) "WIDTH (MM)" "PRINTTAB(29)W
640 PRINTTAB(5) "YOUNGS MODULUS (KN/MM^2)" "PRINTTAB(25)E
650 PRINTTAB(5) "CL. GAGE. DISPL. (MM/MV)" "PRINTTAB(25)CV(1)
660 PRINTTAB(5) "LVDT CAL. FACTOR (MM/MV)" "PRINTTAB(25)DT
670 PRINTTAB(5) "FLOW STRESS (KN/MM^2)" "PRINTTAB(30)YS
680 PRINTTAB(5) "LOAD RANGE (KIL/V)" "PRINTTAB(24)LR
690 PRINTTAB(3) "=====

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700 CV=CV(1)
710 PRINTTAB(2) "HIT KEY N IF YOU WISH TO RETYPE !"
720 PRINT " "
730 PRINTTAB(4) "HIT ANY OTHER KEY TO CONTINUE !"
740 PRINTTAB(5) "=====
750 GETA$:IF A$=""THENI750
760 IFA$="N"THENGOTO370
770 REM
780 REM -----DATA ACQUISITION-----
790 REM
800 PRINT "PRESS THE SPACE TAB FOR DATA INTAKE"
810 GETA$:IFA$=""THEN810
820 PRINT " "
830 PRINTTAB(37)"LOADING"
840 PRINT " "
850 PRINT "LOAD VOLTS
860 PRINT "LVDT. DISPLACEMENT VOLTS
870 PRINT "C.GE. DISPLACEMENT VOLTS
880 PRINT "
890 PRINT "
900 PRINT "
910 PRINT "
920 PRINT "
930 PRINT "SAMPLE COUNTER"
940 PRINT " "
950 PRINT " "
960 OPENI,9:PRINT#1,"G1,I0"
970 GET#1,H$(0),L$(0)
980 PRINT#1,"G0,I1"
990 GET#1,H$(1),L$(1)
1000 PRINT#1,"G1,I2"
1010 GET#1,H$(2),L$(2)
1020 CLOSE1:IFX=1THENSAS=SA+1
1030 IFK=1THENK=K+1
1040 PRINT "FORCH=0T02
1050 IFH$(CH)=""THENH$(CH)=0:GOTO1070
1060 H$(CH)=ASC(H$(CH))
1070 IFL$(CH)=""THENL$(CH)=0:GOTO1090
1080 L$(CH)=ASC(L$(CH))
1090 IFH$(CH)>7THENH$(CH)=-(H$(CH)-8):L$(CH)=-(L$(CH))
1100 OP=(INT((256*H$(CH)+L$(CH))/2.049))/1000
1110 PRINTTAB(40) "OP
1120 IFCH=0THEN HK=ABS(MG-(OP*1000)):P(SA)=INT(HK*LR)/1000
1130 IFCH=0THENP(SA)=INT(OP*1000)
1140 IFCH=1THEND(SA)=INT(OP*1000):IFSA=1THENTY=D(SA)
1150 IFCH=2THENV(SA)=INT(OP*1000)
1160 IFA=0THENKL=V(SA)
1170 IFA=1THEN MG=P(SA)
1180 IFCH=0THEN HK=ABS((OP*1000)):P(SA)=INT(HK*LR)/1000
1190 IFCH=1THENVB=ABS(TY-(OP*1000)):D(SA)=INT(VB*DT*1000)/1000
1200 IFCH=2THENN=ABS(KL-(OP*1000)):V(SA)=INT(N*CV*1000)/1000
1210 X=0:GETA$:IFA$=""THENX=1
1220 IFV(SA)>=1.5THENCV=1.0365*CV(1)
1230 IFV(SA)>=2.5THENCV=1.0503*CV(1)
1240 IFV(SA)>=3.5THENCV=1.0651*CV(1)
1250 REM
1260 IFA$="U"THENSAS=SA:CLOSE1:GOTO1490
1270 IFA$="N"THENSAS=SA:FUL=UNL:GOTO3240
1280 IFUNL=0THENGOTO1320
1290 DF=ABS(D(SA)-D(V))
1300 IFDF<.010THEN EX=SA
1310 IFK<6THENGOTO1450
1320 REM
1330 IFCH<160T01450
1340 IFA<3THENGOTO1450
1350 DL=D(SA-2)-D(SA)

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```

UNLOADING";UNL
-----"
LOAD                                VOLTS
LYDT. DISPLACEMENT                 VOLTS
C.GE. DISPLACEMENT                 VOLTS
-----"
UNLOADING S. COUNT                 TOTAL S. COUNT"

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```

2020 R1=Y+3:F1=SA-3
2030 M=0:LD=0:UX=0:A=0:ADW=0:BC=0
2040 N=F1-R1+1
2050 D(0)=0:D(1)=0:D(2)=0:V(0)=0:V(1)=0:V(2)=0:P(0)=0:P(1)=0:P(2)=0
2060 S1=0
2070 S2=0
2080 S3=0
2090 S4=0
2100 S5=0
2110 S6=0
2120 FORI=R1TOF1
2130 S1=S1+V(I)
2140 S2=S2+P(I)
2150 S3=S3+(P(I)*V(I))
2160 S4=S4+(V(I)*V(I))
2170 S6=S6+(P(I)*P(I))
2180 NEXTI
2190 M=((N*S3)-(S1*S2))/((N*S4)-(S1*S1)):REM ! SLOPE
2200 C=(S2-M*S1)/N:REM ! INTERCEPT
2210 Q1=(N*S4-S1*S1)
2220 Q2=(N*S6-S2*S2)
2230 S5(UNL)=(N*S3-(S1*S2))/((SQR(Q1*Q2)):REM ! CORRELATION FACTO
2240 REM
2250 REM
2260 REM----- CRACK LENGTH ESTIMATION-----
2270 LIC(UNL)=ABS(1/M)
2280 PN=0.3:REM ! POISSON'S RATIO
2290 LK=0:ZX1=0
2300 ES=E/(1-FI*PN)
2310 KMO=(1+SQR(15.8/(ES*LIC(UNL)*B)))
2320 UX=1/KMO
2330 A=(-1.03*(UX)+6*(UX^2))
2340 BC=(-6.37*(UX^3)+2.73*(UX^4))
2350 ADW=(A+BC-0.321*(UX^5))
2360 CRK(UNL)=N*ADW
2370 IFUNL<=3THENGOSUB5700
2380 DELA(UNL)=CRK(UNL)-CRK(1)
2390 ZX1=DELA(UNL)/W
2400 IFSG=1THENGOTO2520
2410 LK=1.002+3.497*ZX1-53.163*(ZX1^2)+512.9*(ZX1^3)-1948.8*(ZX1^4)
2420 CAD(UNL)=ABS((LK+2568.8*(ZX1^5))*LIC(UNL))
2430 IFDELA(UNL)>0.3*WTHENCAD(UNL)=LIC(UNL)
2440 KMO1=(1+SQR(15.8/(ES*CAD(UNL)*B)))
2450 UX1=1/KMO1
2460 A1=(-1.03*(UX1)+6*(UX1^2))
2470 BC1=(-6.37*(UX1^3)+2.73*(UX1^4))
2480 ADW1=(A1+BC1-0.321*(UX1^5))
2490 CFC(UNL)=N*ADW1
2500 EXTC(UNL)=CFC(UNL)-CRK(1)
2510 IFDELA(UNL)=0THENCFC(UNL)=CRK(UNL):EXTC(UNL)=0
2520 REM
2530 REM
2540 REM-----EVEN NUMBER CHECK-----
2550 U(0)=0
2560 YD=(Y-EX)/2
2570 YDI=INT(YD)
2580 YR=YDI-YD
2590 IFYR<>0THENY=Y+1
2600 REM :CHECK COMPLETE
2610 REM
2620 REM
2630 REM-----INTEGRATION-----
2640 SUM1=0
2650 FORI=EX+1TOYSTEP2
2660 SUM1=SUM1+((P(I-1)+P(I))*D(I)-D(I-1))+P(I)+P(I+1))*D(I+1)-D(I))/2
2670 NEXTI

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2680 RI=0
2690 IF P(Y)>P(EX) THEN RI=0.5#3.59E-3*(P(Y)^2)-(P(EX)^2)
2700 IF SUM1<0 THEN SUM1=ABS((P(Y)+P(EX))*(D(Y)-D(EX)))/2
2710 IF UNL=1 THEN SUM1=(P(Y)*D(Y)/2)
2720 U(UNL)=U(UNL-1)+(SUM1)-RI
2730 REM
2740 REM
2750 REM -----J-INTEGRAL CALCULATION-----
2760 IF SG=1 THEN CFC(UNL)=CRK(UNL):EXTC(UNL)=DELA(UNL):CRD(UNL)=LIC(UNL)
2770 SB1=W-CRK(1):ETA=0.3
2780 IF CRK(UNL)>(0.2*W) THEN ETA=1.35
2790 IF CRK(UNL)>(0.28*W) THEN ETA=1.80
2800 IF CRK(UNL)>(0.48*W) THEN ETA=2.00
2810 JO(UNL)=U(UNL)*(ETA)/(SB1*B)
2820 IF UNL=1 THEN GOTO 2890
2830 GI1=W-CRK(UNL-1)
2840 DELA(UNL)=CRK(UNL)-CRK(1)
2850 MB=(CRK(UNL)-CRK(UNL-1))
2860 PTP1=(JIT(UNL-1)+((ETA)/GI1)*((U(UNL)-U(UNL-1))/B))
2870 TPT2=(1-(1/GI1)*MB)
2880 JIT(UNL)=(PTP1)*(TPT2)
2890 REM
2900 IF UNL<3 THEN GOSUB 5700
2910 JIT(1)=JO(1)
2920 IF DELA(UNL)=0 THEN EXTC(UNL)=0
2930 IF DELA(UNL)<.0099999 THEN DELA(UNL)=INT(DELA(UNL)*100000)/100000
2940 IF LIC(UNL)<.0099999 THEN LIC(UNL)=INT(LIC(UNL)*100000)/100000
2950 IF EXTC(UNL)<.0099999 THEN EXTC(UNL)=INT(EXTC(UNL)*100000)/100000
2960 REM
2970 REM -----VISUAL DISPLAY OF J-DELTA A VALUES-----
2980 PRINT "J"
2990 PRINT TAB(11)"UNLOADING NUMBER";UNL
3000 PRINT EX:D(EX):P(EX):Y:D(Y):SA:SUM1:RI:ETA
3010 PRINT TAB(11)"=====
3020 PRINT
3030 PRINT "CRACK LENGTH=";CRK(UNL);"MM"
3040 PRINT "AREA =" ;U(UNL);"KN.MM"
3050 PRINT
3060 PRINT "J(UNCORRECTED)=";JO(UNL);"KN/MM"
3070 PRINT
3080 PRINT "J(EARNST)";JIT(UNL);"KN/MM"
3090 PRINT
3100 IF UNL=1 THEN DELA(UNL)=0:EXTC(UNL)=0
3110 PRINT "CRACK EXTENSION(UNCORRECTED) =" ;DELA(UNL);"MM"
3120 PRINT
3130 PRINT "DATA PTS. DURING THIS UNLOADING=";N
3140 PRINT "COMPLIANCE=";LIC(UNL);"MM/KN"
3150 PRINT "CORREL. FACTOR=";S5(UNL)
3160 PRINT
3170 PRINT TAB(5)"ON RESUMPTION OF LOADING, HIT L"
3180 PRINT TAB(6)"AT CONCLUSION OF TEST, HIT E"
3190 PRINT
3200 GETA$:IFA$="" THEN GOTO 3200
3210 IFA$="L" THEN K=1:GOTO 3200
3220 IFA$="E" THEN SAM=SA:FUL=UNL:GOTO 3240
3230 IFA$<>" " THEN GOTO 3200
3240 REM
3250 H$="** YOU HAVE A CHOICE **"
3260 PRINT
3270 PRINT "J"
3280 PRINT TAB((80-LEN(H$))/2)H$
3290 PRINT
3300 PRINT
3310 PRINT TAB(30)"1 ----- 01"
3320 PRINT
3330 PRINT TAB(30)"2 ----- 1 1 1

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3340 PRINT
3350 PRINT TAB(30)"3 ----- 1 1 1 1
3360 PRINT
3370 PRINT TAB(30)"4 ----- 1 1 1 1 1 1
3380 PRINT
3390 PRINT TAB(30)"5 ----- 1 1 1 1 1 1 1
3400 PRINT
3410 PRINT TAB(30)"6 ----- 1 1 1 1 1 1 1 1
3420 PRINT
3430 PRINT TAB(30)"7 ----- 1 1 1 1 1 1 1 1 1
3440 GETA$:IFA$="" THEN 3440
3450 A=VAL(A$)
3460 IFA<00RA>7 THEN 3250
3470 ON AGO(03490,3660,380,4250,5170,5650,3940
3480 GOTO 3250
3490 REM
3500 REM
3510 REM ----- DISK STORAGE -----
3520 PRINT "PLEASE MAKE SURE DISK#1 IS LOADED"
3530 PRINT "PLEASE INPUT NAME OF FILE";INPUT F$
3540 OPEN 1,8,2,"01:"F$+".S.W"
3550 PRINT #1,SN$,"D$","B$","W$","E"
3560 PRINT #1,FUL$,"YS$","LR$","SAM$","CV"
3570 PRINT #1,DT
3580 FOR UNL=1 TO FUL
3590 PRINT #1,JO(UNL),"JIT(UNL)","CRK(UNL)"
3600 PRINT #1,DELA(UNL),"LIC(UNL)","S5(UNL)","U(UNL)","CFC(UNL)","EXTC(UNL)
3610 NEXT UNL
3620 DCLOSE
3630 GOTO 3240
3640 REM
3650 REM
3660 REM ----- DATA ACQUISITION FROM DISK -----
3670 PRINT "TYPE IN NAME FILE YOU WISH TO READ";INPUT F$
3680 PRINT "*****
3690 OPEN 1,8,2,"01:"F$+".S.R"
3700 INPUT #1,SN$,D$,B$,W$,E
3710 INPUT #1,FUL,YS,LR,SAM,CV
3720 INPUT #1,DT
3730 FOR UNL=1 TO FUL
3740 INPUT #1,JO(UNL),JIT(UNL),CRK(UNL)
3750 INPUT #1,DELA(UNL),LIC(UNL),S5(UNL),U(UNL),CFC(UNL),EXTC(UNL)
3760 NEXT UNL:DCLOSE
3770 PRINT "J"
3780 PRINT TAB(5)"SPECMEN NAME " ;PRINT TAB(31)SN$
3790 PRINT TAB(5)"DATE " ;PRINT TAB(31)D$
3800 PRINT TAB(5)"THICKNESS (MM) " ;PRINT TAB(30)B$
3810 PRINT TAB(5)"WIDTH (MM) " ;PRINT TAB(30)W$
3820 PRINT TAB(5)"YOUNGS MODULUS (KN/MM^2)";PRINT TAB(25)E
3830 PRINT TAB(5)"CL. GAGE. DISPLA.(MM/MV)";PRINT TAB(25)CV
3840 PRINT TAB(5)"LVDT CAL FACT. (MM/MV)";PRINT TAB(25)DT
3850 PRINT TAB(6)"FLOW STRESS (KN/MM^2)";PRINT TAB(30)YS
3860 PRINT TAB(5)"LOAD RANGE (KN) " ;PRINT TAB(30)LR
3870 PRINT TAB(6)"TOT. NUMB. OF UNLOADINGS";FUL
3880 PRINT TAB(6)"SAMPLE PT. TOTAL ",SAM
3890 PRINT "*****DO YOU WANT A LISTING (Y/N)"
3900 GETA$:IFA$="" THEN 3900
3910 IFA$="Y" THEN GOTO 3940
3920 IFA$="N" THEN GOTO 3240
3930 GOTO 3900
3940 OPEN 1,4
3950 PRINT #1,"
3960 PRINT #1,"
3970 PRINT #1,"
3980 PRINT #1,"
3990 PRINT #1,"
SPECIMEN NAME ----- " ;SN$
DATE ----- " ;D$
THICKNESS ----- " ;B$,"MM"
WIDTH ----- " ;W$,"MM"
YOUNGS MODULUS ----- " ;E$,"KN/MM^2"

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4000 PRINT#1,"          CL.GAGE. DISPL.  _____  "/CV;"MM/MY"
4010 PRINT#1,"          LVDT CAL. FACT.  _____  "/DT;"MM/MY"
4020 PRINT#1,"          FLOW STRESS  _____  "/VS;"T/INCH2"
4030 PRINT#1,"          LOAD RANGE  _____  "/LR;"T/INCH2"
4040 PRINT#1,"          TOT NUMB. OF UNLOADINGS  _____  "/FUL
4050 PRINT#1,"          SAMPLE PT. TOTAL  _____  "/SAM
4060 PRINT#1,"
4070 PRINT#1,"
4080 FORUNL=1TOFUL
4090 PRINT#1,"          UNLOADING NUMBER"/UNL
4100 PRINT#1,"
4110 PRINT#1,"J(UNCORRECTED)="/JO(UNL); "KN/MM", "J(ERNST)="/JIT(UNL); "KN/MM"
4120 PRINT#1,"
4130 PRINT#1,"CRACK LENGTH="/CRK(UNL); "MM", "CRACK EXTENSION="/DELA(UNL); "MM"
4140 PRINT#1,"
4150 PRINT#1,"CRK.LHT.(C/TED)="/CFC(UNL); "MM", "CRK EXT(C/TED)="/EXTC(UNL); "MM"
4160 PRINT#1,"
4170 PRINT#1,"COMPLIANCE="/LIC(UNL); "MM/KN", "CORRELATION FACTOR="/S5(UNL)
4180 PRINT#1,"
4190 PRINT#1,"          AREA="/U(UNL); "KN.MM"
4200 PRINT#1,"
4210 PRINT#1,"
4220 NEXTUNL
4230 CLOSE1
4240 GOTO3240
4250 REM
4260 REM
4270 REM
4280 REM -----J - DELTA A PLOTTING ROUTINE -----
4290 PRINT#1"J
4300 PRINT#1"PLEASE INPUT J-AXIS LIMIT (KN/MM)"/INPUTY1$;V1=VAL(V1$)
4310 PRINT#1"PLEASE INPUT DELTA-A AXIS LIMIT (MM)"/INPUTX1$;X1=VAL(X1$)
4320 PRINT#1"INPUT SPECIMEN BREATH (MM)"/INPUTR$;B=VAL(R$)
4330 PRINT#1"J
4340 FORQ=0TO1760STEP80
4350 POKE32771+Q,12
4360 NEXT
4370 FORQ=0TO75
4380 POKE34531+Q,45
4390 NEXT
4400 POKE34529,48
4410 FORQ=0TO-1680STEP-80
4420 POKE34526+Q,12
4430 NEXT
4440 FORQ=0TO75
4450 POKE32771+Q,45
4460 NEXT
4470 POKE33569,10
4480 POKE34644,4:POKE34645,5:POKE34646,12:POKE34647,46:POKE34648,1:PRINTY1
4490 V2=22/V1:X2=75/X1
4500 FORQ=1TOFUL
4510 I=INT(EXTC(Q)*X2)
4520 K=INT(JO(Q)*Y2)*80
4530 H=INT(JIT(Q)*Y2)*80
4540 POKE34531+I-H,42
4550 POKE34531+I-K,43
4560 NEXTQ
4570 PRINT#1"*****"TAB(62)"HIT 1 FOR PRINT
4580 PRINTTAB(67)"OR
4590 PRINTTAB(64)"2 FOR MENU
4600 PRINT#1"
4610 PRINT#1"*****"TAB(72);X1
4620 GETA$;IFA$=""THENGOTO4620
4630 A=VAL(A$)
4640 ONAGOTO4660,3240
4650 GOTO3240

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4660 SYS10#4096
4670 OPEN1,7
4680 IMA,20,228
4690 IXA,5,28
4700 IMA,20,228
4710 IYA,7,-32,14
4720 IDR,140,0,0,225
4730 IMA,60,240
4740 IRO,1:ISZ,4
4750 AS="J-INT. (KN/MM)"
4760 IPR,AS
4770 IMA,5,160
4780 IRO,4:ISZ,4
4790 BS="CRACK EXTENSION (MM)"
4800 IPR,BS
4810 IMA,15,10
4820 IRO,4:ISZ,3:IPR,X1$
4830 IMA,15,233
4840 IRO,4:ISZ,3:IPR,"0.0"
4850 IMA,150,75
4860 IRO,4:ISZ,3:IPR,SN$
4870 IMA,145,75:BS="B= MM"
4880 IRO,4:ISZ,3:IPR,BS
4890 IMA,145,68
4900 IRO,4:ISZ,3:IPR,R$
4910 IMA,159,237
4920 IRO,4:ISZ,3:IPR,Y1$
4930 IMA,145,45:C$="":N=2B"
4940 IRO,4:ISZ,3:IPR,C$
4950 REM
4960 REM
4970 Y3=140/Y1:X3=225/X1
4980 FORUNL=1TOFUL
4990 Y=-EXTC(UNL)*X3+225
5000 X=JO(UNL)*Y3+20
5010 IMA,X,Y
5020 ISM,3
5030 NEXTUNL
5040 FORUNL=1TOFUL
5050 Y=-EXTC(UNL)*X3+225
5060 X=JIT(UNL)*Y3+20
5070 IMA,X,Y
5080 ISM,1
5090 NEXTUNL
5100 IPU
5110 CLOSE1
5120 GOTO3240
5130 REM
5140 IFDS>20THENPRINT"ERROR";DS:DCLOSE
5150 RETURN
5160 REM
5170 REM
5180 REM ----- LOAD DISPLACEMENT PLOTTING ROUTINE -----
5190 SYS10#4096
5200 OPEN1,7
5210 PRINT#1"J
5220 PRINT#1"PLEASE INPUT LOAD-AXIS LIMIT (KN)"/INPUTQ1$;Q1=VAL(Q1$)
5230 PRINT#1"PLEASE INPUT DISPLACEMENT-AXIS LIMIT (MM)"/INPUTV1$;V1=VAL(V1$)
5240 V2=22/V1:Q2=140/Q1
5250 PRINT#1"INPUT SPECIMEN BREATH (MM)"/INPUTR$;B=VAL(R$)
5260 IMA,20,228
5270 IMA,5,28
5280 IMA,20,228
5290 IYA,9,-25
5300 IDR,140,0,0,225
5310 IMA,60,240

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5320 !R0,1:ISZ,4
5330 A$="LOAD (KN)"
5340 !PR,A$
5350 !MA,5,160
5360 !R0,4:ISZ,4
5370 B$="DISPLACEMENT (MM)"
5380 !PR,B$
5390 !MA,15,10
5400 !R0,4:ISZ,3: !PR,V1$
5410 !MA,15,233
5420 !R0,4:ISZ,3: !PR,"0.0"
5430 !MA,150,75
5440 !R0,4:ISZ,3: !PR,SN$
5450 !MA,145,75 B$="B= MM"
5460 !R0,4:ISZ,3: !PR,B$
5470 !MA,145,68
5480 !R0,4:ISZ,3: !PR,R$
5490 !MA159,237
5500 !R0,4:ISZ,3: !PR,Q1$
5510 !MA,145,45:C$="": W=2B"
5520 !R0,4:ISZ,3: !PR,$
5530 REM
5540 !MA,20,228
5550 FORI=1TOSAM-2
5560 V=-D(I)*V2+228
5570 X=P(I)*Q2+20
5580 !MA,X,Y
5590 NEXTI
5600 !PU
5610 CLOSE1
5620 GO T03240
5630 REM
5640 REM
5650 REM
5660 PRINT"*****"
5670 PRINT"*****"
5680 PRINT"*****"
5690 END
5700 REM
5710 IFUNL=1THENRETURN
5720 IFLIC(2)<LIC(1)THENK=50
5730 IFK=50THENLIC(1)=(LIC(1)+LIC(2))/2:LIC(2)=LIC(1):J1T(2)=JO(2)
5740 IFK=50THENCRK(1)=(CRK(1)+CRK(2))/2:CRK(2)=CRK(1):DELA(2)=0
5750 IFK=50THENCFC(2)=CFC(1):EXTC(2)=0
5760 IFUNL=2THENGOTO5810
5770 IFLIC(3)<LIC(1)THENC=14
5780 IFC=14THENDELA(3)=0:LIC(3)=LIC(1):J1T(3)=JO(3):CRK(3)=CRK(1):DELA(2)=0
5790 IFC=14THENCRK(2)=CRK(1):J1T(2)=JO(2):LIC(2)=LIC(1)
5800 IFC=14THENEXTC(2)=0:CFC(2)=CFC(1):CFC(3)=CFC(1):EXTC(3)=0
5810 RETURN
5820 END

```

TYPICAL OUPUT FROM SULCO

AREA= 37.4504051 KN.MM

SPECIMEN NAME _____ TI, 3PTR, B3, PL, SD 0.4MM/MMH
DATE _____ SEPT30/85
THICKNESS _____ 17.5 MM
WIDTH _____ 35 MM
YOUNGS MODULUS _____ 123 KN/MM12
CL. GAGE. DISPL. _____ 3.2013144E-03 MM/MV
LVDT CAL. FACT. _____ .01724 MM/MV
FLOW STRESS _____ .728 KN/MM12
LOAD RANGE _____ 106.3 KN/V
TOT NUMB. OF UNLOADINGS 8
SAMPLE PT. TOTAL _____ 1221

UNLOADING NUMBER 1

J(UNCORRECTED)= .0153690808 KN/MM J(ERNST)= .0153690808 KN/MM
CRACK LENGTH= 20.1136516 MM CRACK EXTENSION= 0 MM
CRK.LHT.(C'TED)= 20.1988368 MM CRK EXT(C'TED)= 0 MM
COMPLIANCE= .0242973118 MM/KN CORRELATION FACTOR= .996490995
AREA= 1.99193716 KN.MM

UNLOADING NUMBER 2

J(UNCORRECTED)= .0380980614 KN/MM J(ERNST)= .0380980614 KN/MM
CRACK LENGTH= 20.1136516 MM CRACK EXTENSION= 0 MM
CRK.LHT.(C'TED)= 20.1988368 MM CRK EXT(C'TED)= 0 MM
COMPLIANCE= .0242973118 MM/KN CORRELATION FACTOR= .996712893
AREA= 4.96248389 KN.MM

UNLOADING NUMBER 3

J(UNCORRECTED)= .223161726 KN/MM J(ERNST)= .216854994 KN/MM
CRACK LENGTH= 20.5343519 MM CRACK EXTENSION= .420700334 MM
CRK.LHT.(C'TED)= 20.7325573 MM CRK EXT(C'TED)= .533720471 MM
COMPLIANCE= .0262271865 MM/KN CORRELATION FACTOR= .996663058
AREA= 29.0680531 KN.MM

UNLOADING NUMBER 4

J(UNCORRECTED)= .207514854 KN/MM J(ERNST)= .270023538 KN/MM
CRACK LENGTH= 21.2015338 MM CRACK EXTENSION= 1.08788224 MM
CRK.LHT.(C'TED)= 21.5709998 MM CRK EXT(C'TED)= 1.372163 MM
COMPLIANCE= .0296966361 MM/KN CORRELATION FACTOR= .994041824

UNLOADING NUMBER 5

J(UNCORRECTED)= .332964562 KN/MM J(ERNST)= .301800946 KN/MM
CRACK LENGTH= 22.269367 MM CRACK EXTENSION= 2.15571547 MM
CRK.LHT.(C'TED)= 22.7843665 MM CRK EXT(C'TED)= 2.58552967 MM
COMPLIANCE= .0365336672 MM/KN CORRELATION FACTOR= .990539551
AREA= 43.3776961 KN.MM

UNLOADING NUMBER 6

J(UNCORRECTED)= .379712999 KN/MM J(ERNST)= .313330437 KN/MM
CRACK LENGTH= 23.7842405 MM CRACK EXTENSION= 3.67059692 MM
CRK.LHT.(C'TED)= 24.4958307 MM CRK EXT(C'TED)= 4.29699388 MM
COMPLIANCE= .0500938157 MM/KN CORRELATION FACTOR= .995616067
AREA= 49.4641756 KN.MM

UNLOADING NUMBER 7

J(UNCORRECTED)= .418955325 KN/MM J(ERNST)= .337900346 KN/MM
CRACK LENGTH= 24.6522037 MM CRACK EXTENSION= 4.53855211 MM
CRK.LHT.(C'TED)= 25.4928748 MM CRK EXT(C'TED)= 5.294038 MM
COMPLIANCE= .0608904212 MM/KN CORRELATION FACTOR= .982071011
AREA= 54.5717656 KN.MM

UNLOADING NUMBER 8

J(UNCORRECTED)= .452928944 KN/MM J(ERNST)= .32581423 KN/MM
CRACK LENGTH= 26.4577852 MM CRACK EXTENSION= 6.34413359 MM
CRK.LHT.(C'TED)= 27.4953117 MM CRK EXT(C'TED)= 7.29647494 MM
COMPLIANCE= .095543084 MM/KN CORRELATION FACTOR= .995711786
AREA= 59.0085432 KN.MM

APPENDIX III

DESCRIPTION OF THE CONSTITUTIVE PROGRAMS FOR THE KEY-CURVE ANALYSIS

In this appendix an attempt is made to describe the Key-Curve method in terms of the functions of the following programs which create and use primary and secondary data files for evaluating $J - \Delta a$ values. The programs designed for the processing are:-

1. ATOD PROG
2. DATFIT
3. J RESIST

The processing had to be split because of computation memory space limitations on the microcomputer. The entire computational process can be implemented on the 32K CBM microcomputer 8296 or 4032.

ATOD PROG

This program is very similar to some portions of SULCO described in appendix II. This program simply collects the load-displacement data and stores them in a primary file after the normalisation procedures described in subsection 4.5.3. in chapter four. This program creates file P shown as in Fig. 4.7.

DATFIT

DATFIT fits a fifth order polynomial for data points in file P and gets the derivative of F_1 with respect to Δ/W , $(\partial F_1 / \partial (\Delta/W))$ data points in file P. This is performed using the least squares matrix formulation procedure and the simultaneous equations formed by this method is solved by the so-called "Cholesky's" method. While a brief outline of the mathematical procedure is given here, a detailed study can be obtained from James et al (1977) and Fenner R.T. (1978).

LEAST-SQUARES MATRIX FORMULATION

DATFIT Lines 100-540

Sets up simultaneous equations.

Assume a function $F_1 = f(\Delta/W)$ can be represented by a polynomial of the form:

$$F_1 = C_1 f_1(\Delta/W) + C_2 f_2(\Delta/W) + C_3 f_3(\Delta/W) \dots C_m f_m(\Delta/W)$$

where $C_1, C_2, C_3, \dots, C_m$ are linear undetermined constants. It is desired to fit n data points with m linear algebraic equations ($n > m$). Matrix H , below can hence be formed

$$H = \begin{bmatrix} f_1(\Delta_1/W) & f_2(\Delta_1/W) & \dots & f_m(\Delta_1/W) \\ f_1(\Delta_2/W) & f_2(\Delta_2/W) & \dots & f_m(\Delta_2/W) \\ f_1(\Delta_3/W) & f_2(\Delta_3/W) & \dots & f_m(\Delta_3/W) \\ \vdots & \vdots & & \vdots \\ f_1(\Delta_n/W) & f_2(\Delta_n/W) & \dots & f_m(\Delta_n/W) \end{bmatrix} \quad \text{A.I}$$

It can be shown that,

$$H^T H = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_m \end{bmatrix} = H^T \{F_1\} \quad \text{A.II}$$

H^T is the transpose of H .

In DATFIT, the number of data points (n) is defined by the datafile and m is 6

(i.e., fifth order polynomial). Hence, $f_1()$, $f_2()$, $f_3()$ $f_m()$ represent 1, (Δ/W) ,

$(\Delta/W)^2$, $(\Delta/W)^3$... $(\Delta/W)^5$.

Equ. A.II consists of six simultaneous equations and the following routines briefly describe the Cholesky's method for solving the equations for $C_1, C_2, C_3, \dots, C_6$.

SOLUTION OF SIMULTANEOUS EQUATIONS

DATEFIT Lines 550-1070

Solves simultaneous equation by matrix decomposition, see James et al(1977).

After some mathematical procedures, $H^T H$ can be represented by,

$$\begin{bmatrix} L_{11} & 0 & 0 & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 & 0 & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} & 0 & 0 \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} & 0 \\ L_{61} & L_{62} & L_{63} & L_{64} & L_{65} & L_{66} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} & U_{14} & U_{15} & U_{16} & U_{17} \\ 0 & 1 & U_{23} & U_{24} & U_{25} & U_{26} & U_{27} \\ 0 & 0 & 1 & U_{34} & U_{35} & U_{36} & U_{37} \\ 0 & 0 & 0 & 1 & U_{45} & U_{46} & U_{47} \\ 0 & 0 & 0 & 0 & 1 & U_{56} & U_{57} \\ 0 & 0 & 0 & 0 & 0 & 1 & U_{67} \end{bmatrix} \quad \text{A.III}$$

The extra column in Equ. A.III is the right hand side of Equ. A.II.

The constants $C_1, C_2, C_3, \dots, C_6$ of the polynomial,

$$F_1 = C_1 + C_2 (\Delta/W) + C_3 (\Delta/W)^2 + C_4 (\Delta/W)^3 + C_5 (\Delta/W)^4 + C_6 (\Delta/W)^5.$$

can be obtained from the following operations.

If the form of Equ. A.III, after matrix multiplication is represented by,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{17} \\ a_{21} & a_{22} & a_{23} & \dots & a_{27} \\ a_{31} & a_{32} & a_{33} & \dots & a_{37} \\ a_{41} & a_{42} & a_{43} & \dots & a_{47} \\ a_{51} & a_{52} & a_{53} & \dots & a_{57} \\ a_{61} & a_{62} & a_{63} & \dots & a_{67} \end{bmatrix}$$

the following operation to get $C_1, C_2, C_3, \dots, C_6$ is performed:

$$L_{i,1} = a_{i,1} \text{ for } \{i=1,2,3,\dots,6: j=1\}$$

$$U_{1,j} = a_{1j}/a_{11} \text{ for } \{j=2,3,4,\dots,7: i=1\}$$

$$L_{i,j} = a_{i,j} - \sum_{k=1}^{j-1} L_{ik} U_{kj} \text{ for } \{j=2,3,4,\dots,6: i=j,j+1,\dots,n\}$$

$$U_{i,j} = \frac{a_{i,j} - \sum_{k=1}^{i-1} L_{ik} U_{kj}}{L_{ii}} \quad \text{for } \{i=2,3,\dots,6: j=i+1,i+2,i+3,\dots,n+1\}$$

Back substitution gives,

$$C_n = U_{n,n+1}$$

$$C_i = U_{i,n+1} - \sum_{j=i+1}^n U_{i,j} C_j \quad \text{for } \{i=n-1,n-2,\dots,1\}$$

For a fifth order polynomial, $n=6$, as used in DATEFIT.

DATEFIT Lines 1250-1390

Numerical differentiation is performed here using the central difference expression,

$$F'_i = \frac{F_{i+1} - F_{i-1}}{2h}$$

where $h = (\Delta/W)_{i+1} - (\Delta/W)_i = (\Delta/W)_i - (\Delta/W)_{i-1}$.

DATEFIT Lines 1400-1470

Stores first order derivatives of F_1 with respect to Δ/W , $(\partial F_1 / \partial (\Delta/W))$

J RESIST

J RESIST uses secondary files S_1 and S_2 (Fig.4.7) for obtaining $J-\Delta a$ values using the discretised Eqs.4.1 through to 4.4.

J RESIST Lines 320-460

Calculates crack extensions (Equ.4.2)

J RESIST Lines 470-570

Calculates the j integral values (Eqn.4.1)

$\partial F_1 / \partial (a/W)$ are obtained manually from Fig 4.5 and typed into J RESIST in lines

350-380. These $J-\Delta a$ values are stored and then recalled by a similar program in SULCO

for plotting $J-\Delta a$ values using the PD4 plotter.

ATOD PROG

```

100 REM A TO D
110 SF$=""
120 DIMQ%(2000),V%(2000),D%,P%(35)
130 P%(1)=1
140 REM
150 PRINT"-----"
160 PRINTTAB(9)"  A TO D PROGRAMME"
170 PRINT"-----"
180 PRINT"PLEASE INPUT THE FOLLOWING DATA"
190 PRINT"SPECIMEN NO (SN$)";INPUT SN$
200 PRINT"DATE (D$)";INPUT D$
210 PRINT"THICKNESS (MM)";INPUT B
220 PRINT"WIDTH (MM)";INPUT W
230 PRINT"YOUNGS MODULUS (E)KN/MM12";INPUT E
240 PRINT"YIELD STRESS (YS)KN/MM12";INPUT YS
250 PRINT"CLIP GAUGE DISPLACEMENT (MM/MV)";INPUT CV
260 PRINT"LOAD RANGE(KN)";INPUT LR
270 PRINT"PRESS SPACE TO START AND HOLD SPACE"
280 PRINT"TO STORE DATA. PRESS S TO STOP/RESTART"
290 PRINT"PRESS 'P' TO STORE MARKER. (P%(15))"
300 GETA$:IFA$="" THEN300
310 PRINT"  CHANNEL  VOLTAGE "
320 PRINT"-----"
330 PRINT"
340 PRINT"    LOAD          VOLTS"
350 PRINT"
360 PRINT"
370 PRINT"    CLIP GAUGE     VOLTS"
380 PRINT"
390 PRINT"
400 PRINT"-----"
410 PRINT"MEMORY REMAINING ="
420 PRINT"YOUR NEXT POINTER IS ="
430 OPEN1,9:PRINT#1,"GO,10"
440 GET#1,H$(0),L$(0)
450 PRINT#1,"G1,11"
460 GET#1,H$(1),L$(1)
470 CLOSE1:IFX=1THENSAS=SA+1
480 PRINT"SN";FORCH=0TO1
490 IFA$(CH)="" THENH(CH)=0:GOTO510
500 H(CH)=ASC(H$(CH))
510 IFL$(CH)="" THENL(CH)=0:GOTO530
520 L(CH)=ASC(L$(CH))
530 IFA$(CH)>7 THENH(CH)=-(H(CH)-8):L(CH)=-L(CH)
540 OP=(INT((256*(H(CH)+L(CH))/2.049))/1000)
550 PRINT"MM";CH;TAB(22);"      ";OP
560 IFX=0THENS90
570 IFCH=0THEND%(SA)=INT(OP*1000)
580 IFCH=1THENV%(SA)=INT(OP*1000)
590 NEXTCH
600 X=0:GETA$:IFA$="" THENX=1
610 IFA$="P" THENI=I+1:P%(I)=SA:P=P+1
620 IFA$="S" THEN670
630 PRINTTAB(22)"      ";2000-SA
640 PRINTTAB(22)"      ";P
650 IFA$(2000) THENGOTO430
660 REM
670 PRINTTAB(19)"***** MENU ****"
680 PRINT"NL - RESTART"

```

```

690 PRINT"R2 - PURGE"
700 PRINT"R3 - STORE"
710 PRINT"R4 - READ"
720 GETA$:IFA$="" THEN720
730 A=VAL(A$)
740 IFA$(0000) THEN670
750 ONAGOTO270,760,780,1010
760 SA=0:I=1:P=1:GOTO270
770 PRINT"LOAD RANGE=";LR;KN
780 REM *****DISK ROUTINE*****
790 REM
800 PRINT"STORING DATA TO DISC #1"
810 PRINT"PLEASE MAKE SURE THE DISK IS LOADED"
820 PRINT"IN DISK DRIVE #1"
830 PRINT"SPECIMEN NO =" ;SN$
840 PRINT"DATE =" ;D$
850 PRINT"BREATH =" ;B
860 PRINT"WIDTH =" ;W
870 PRINT"SEMP. TOTAL =" ;SA
880 PRINT"CLIP G. CAL=";CV;"MM/MV"
890 PRINT"PLEASE INPUT NAME OF FILE";INPUTF$
900 OPEN1,8,2,"@1:"+"F$+",S,W
910 PRINT#1,SN$,"D$","B","W","SA","CV","LR"
920 FORI=0TO50
930 PRINT#1,Q%(I),"V%(I)",
940 GOSUB1260
950 PRINT#1,SP$;"T";I,Q%(I),V%(I)
960 NEXTI
970 FORI=0TO30:PRINT#1,P%(I)
980 GOSUB1260:NEXTI
990 DCLOSE
1000 GOTO670
1010 REM
1020 PRINT"PLEASE TYPE THE NAME OF THE FILE"
1030 INPUT"YOU WISH TO READ";F$
1040 OPEN1,8,2,"@1:"+"F$+",S,R
1050 INPUT#1,SN$,D$,B,W,SA,CV,LR
1060 FORI=0TO50
1070 INPUT#1,Q%(I),V%(I)
1080 GOSUB1260
1090 PRINT"      ";SP$;"T";I,Q%(I),V%(I)
1100 NEXTI
1110 FORI=0TO30:INPUT#1,P%(I)
1120 GOSUB1260:NEXTI
1130 DCLOSE
1140 PRINT"SPECIMEN =" ;SN$
1150 PRINT"DATE =" ;D$
1160 PRINT"BREATH =" ;B;"MM"
1170 PRINT"WIDTH =" ;W;"MM"
1180 PRINT"SEMP. TOTAL =" ;SA
1190 PRINT"CLIP G. CAL=";CV;"MM/MV"
1200 PRINT"LOAD RANGE=";LR;"KN"
1210 PRINT"DO YOU WANT A LISTING (Y/N)"
1220 GETA$:IFA$="" THEN1220
1230 IFA$="Y" THENGOTO1290
1240 IFA$="N" THENGOTO670
1250 GOTO1220
1260 PRINT" ";D$,ST
1270 IFD$>20 THENPRINT"ERROR":DCLOSE
1280 RETURN
1290 OPEN1,4
1300 PRINT#1,"SPECIMEN =" ;SN$
1310 PRINT#1,"DATE =" ;D$
1320 PRINT#1,"BREATH =" ;B
1330 PRINT#1,"WIDTH =" ;W
1340 PRINT#1,"CLIP G. CAL=";CV;"MM/MV"

```

```

1350 PRINT#1,"LOAD RANGE=";LR;"KN"
1360 FORI=0T0SASTEP2
1370 PRINT#1,I,Q%(I);V%(I);I+1,Q%(I+1);V%(I+1)
1380 NEXTI
1390 FORI=0T030
1400 PRINT#1,I,P%(I)
1410 NEXTI
1420 CLOSE1
1430 GOT0670

```

DATAFIT

```

100 REM *****
110 REM *
120 REM ***** KEY D I F *****
130 REM *
140 REM *****
150 DIM P(650),D(650),F(50,7)
160 DIM X(6),FT(30,50),A(8,8)
170 DIMH(700),DH(650)
180 PRINT"PLEASE TYPE NAME OF FILE THAT
190 PRINT"
200 PRINT" NEEDS CURVE-FITTING":INPUTF$
210 OPEN1,8,2,"@1:"F$+ ".S,R"
220 INPUT#1,SN$,D$,B,W,SA,CV,LR
230 PRINTSN$,D$,B,W,SA,CV,LR
240 FOR I=0T0SA
250 INPUT#1,P(I),D(I)
260 NEXTI
270 CLOSE1
280 PRINT"J
290 PRINTTAB(8)"*****COMPUTATION IN PROGRESS"
300 PRINTTAB(13)"PLEASE RELAX !"
310 FORI=15T0SASTEP15
320 J=I/15
330 F(J,1)=1:F(J,2)=D(I)
340 F(J,3)=(D(I)^2):F(J,4)=(D(I)^3)
350 F(J,5)=(D(I)^4):F(J,6)=(D(I)^5)
360 F(J,7)=P(I)
370 NEXTI
380 JM=J

```

```

390 DCLOSE
400 FOR I=1T0JM
410 FOR J=1T06
420 FT(J,I)=F(I,J)
430 NEXT
440 NEXT
450 FORI=1T06
460 FORJ=1T07
470 A(I,J)=0
480 FOR K=1T0JM
490 A(I,J)=A(I,J)+FT(I,K)*F(K,J)
500 NEXTK
510 NEXT
520 NEXT
530 PRINT"
540 N=6
550 M=M+1
560 REM CALCULATES 1ST ROW OF UPPER
570 REM
580 REM UNIT TRIANGULAR MATRIX
590 REM *****
600 PRINT"
610 PRINT"
620 PRINT"
630 FORJ=2T0M
640 A(1,J)=A(1,J)/A(1,1)
650 NEXTJ
660 REM
670 REM CAL OTHER ELEMENTS OF U ANDL
680 REM *****
690 FOR I=2T0N
700 J=I
710 FOR II=J TON
720 SUM=0
730 JM=J-1
740 FORK=1T0JM1
750 SUM=SUM+A(II,K)*A(K,J)
760 NEXTK
770 A(II,J)=A(II,J)-SUM
780 NEXTII
790 IP1=I+1
800 FOR JJ=IP1 TO M
810 SUM=0
820 IM1=I-1
830 FOR K=1 TO IM1
840 SUM=SUM+A(I,K)*A(K,JJ)
850 NEXTK
860 A(I,JJ)=(A(I,JJ)-SUM)/A(I,I)
870 NEXT JJ
880 NEXTI
890 REM
900 REM ***** CALCULATES X *****
910 REM
920 X(N)=A(N,N+1)
930 L=N-1
940 FOR MN=1 TO L
950 SUM=0
960 I=N-MN
970 IP1=I+1
980 FOR J=IP1 TO N
990 SUM=SUM+A(I,J)*X(J)
1000 NEXTJ
1010 X(I)=A(I,M)-SUM
1020 NEXT MN
1030 REM
1040 PRINT"J

```

```

1050 FOR I=1 TO N
1060 PRINT TAB(12)"X(I)";X(I)
1070 NEXT I
1080 GOTO 1240
1090 FOR I=1 TO 500
1100 S=S+.0002:D(I)=S
1110 T1=X(I)+X(2)*D(I)
1120 T2=X(3)*D(I)+T1+X(4)*D(I)+T2
1130 T3=X(5)*D(I)+T2+X(6)*D(I)+T3
1140 DH(I)=T1+T2+T3:PRINT DH(I),D(I)
1150 NEXT I
1160 PRINT"PLEASE INPUT FITTED CURVE FILE NAME":INPUT F$
1170 OPEN I,8,2,"@1:"+F$+",S,W"
1180 PRINT#1,SN$,"D$","B","W","SA","CV","LR"
1190 FOR I=0 TO 500
1200 PRINT#1,DH(I),"D(I)"
1210 NEXT I
1220 CLOSE I
1230 END
1240 REM *****
1250 FOR I=1 TO 150
1260 S=S+.0002:D(I)=S
1270 DP=D(I)+0.00005
1280 DM=D(I)-0.00005
1290 H1=X(1)+X(2)*DP
1300 H2=X(3)*DP+T2+X(4)*DP+T3
1310 H3=X(5)*DP+T2+X(6)*DP+T3
1320 H4=H1+H2+H3
1330 E11=X(1)+X(2)*DM
1340 E22=X(3)*DM+T2+X(4)*DM+T3
1350 E33=X(5)*DM+T2+X(6)*DM+T3
1360 E44=E11+E22+E33
1370 H(I)=(H4-E44)/(2*.00005)
1380 PRINT H(I),D(I)
1390 NEXT I
1400 PRINT"PLEASE TYPE FILE NAME "
1410 INPUT F$
1420 OPEN I,8,2,"@1:"+F$+",S,W"
1430 PRINT#1,SN$,"D$","B","W","SA","CV","LR"
1440 FOR I=1 TO 150
1450 PRINT#1,H(I),"D(I)"
1460 NEXT I
1470 CLOSE I

```

JRESIST

```

100 DIM D(650),H(650),Q(650),DH(650),DJ(650),CRD(650),U1(650)
110 PRINT"TYPE IN F1-DISP. FILE NAME":INPUT F$
120 OPEN I,8,2,"@1:"+F$+",S,R"
130 INPUT#1,SN$,D$,B,W,SA,CV,LR
140 FOR I=0 TO 350
150 INPUT#1,DH(I),D(I)
160 PRINT DH(I)
170 NEXT I
180 CLOSE I
190 PRINT"TYPE NAME OF DIFFERENTIATED FILE NAME"
200 INPUT F$
210 OPEN I,8,2,"@1:"+F$+",S,R"
220 INPUT#1,SN$,D$,B,W,SA,CV,LR
230 FOR I=1 TO 350
240 INPUT#1,H(I),D(I)
250 PRINT H(I)
260 NEXT I
270 CLOSE I
280 I=0
290 B=13:W=35
300 REM
310 REM
320 REM ----- CRACK EXTENSIONS -----
330 DH(0)=.0139
340 FOR I=1 TO 350
350 IF D(I)<=.0055 THEN FA=-1.593E-3
360 IF D(I)>.0055 THEN FA=-0.042
370 IF D(I)>.015 THEN FA=-0.071
380 IF D(I)>.025 THEN FA=0.015
390 DQ=INT((DH(I)-DH(I-1))*(17.5*B*B)/(35*17.5)*10000)/10000
400 TA=INT((B*B*H(I)*.007/W/W)*10000)/10000 -DQ
410 BA=(2*B*DH(I)/W/W)-B*B*FA/W
420 CRD(0)=0:U1(I)=ABS(35*INT((TA/BA)*10000)/10000)
430 K1=K1+U1(I)
440 CRD(I)=INT(ABS(K1)*10000)/10000
450 REM
460 REM
470 REM ----- J CALCULATION -----
480 CN=((2*B*DH(I)/W)-(B*B*FA/(W*W)))*.007
490 S1=S1+DH(I)*.007
500 JAS=-2*S1/W
510 S2=S2+FA*.007
520 A3=4*B/W/W*S2
530 DJ(I)=CN+(JAS+A3)*U1(I)
540 G1=G1+DJ(I)
550 DJ(I)=INT(G1*1000)/1000
560 PRINT DJ(I):CRD(I):U1(I);I
570 NEXT I
580 PRINT"PLEASE TYPE IN J-DEL A FILE NAME"
590 INPUT F$
600 OPEN I,8,2,"@1:"+F$+",S,W"
610 FOR I=0 TO 350
620 PRINT#1,DJ(I),"CRD(I)"
630 NEXT I
640 DCLOSE

```

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