

Integral passivity-based control of an underactuated hydraulic soft manipulator with uncertain nonlinear stiffness ^{★★}

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Abstract: This work investigates the integral passivity-based control of an underactuated soft continuum manipulator that bends on a horizontal plane, that is actuated by pressurized fluid, and that is characterized by uncertain nonlinear stiffness. The main contribution of this work is the application of the integral passivity-based control methodology to a system that is not in affine form and that is subject to parameter uncertainty resulting in multiple position-dependent disturbances. The effectiveness of the controller is demonstrated with numerical simulations.

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Keywords: Passivity-based control, integral action, disturbances, soft robotics, nonlinear systems.

1. INTRODUCTION

Model-based control of soft robots has been attracting increasing attention in the research community, see (Della Santina et al., 2023). A variety of classical model-based controllers have been employed to date, including model predictive control (MPC), sliding mode control (SMC), see (Best et al., 2020; Cao et al., 2021), and feedback linearization, see (Wang et al., 2020). However, classical high-gain controllers can increase the closed-loop stiffness of the system, which is undesirable, see (Della Santina et al., 2017). As a result, subsequent works have been combining feedback actions that employ low gains with feed-forward actions or with adaptive algorithms, see (Trumic et al., 2021). Modeling and control of a soft robotic manipulator with an Hamiltonian approach was proposed by Mattioni et al. (2020); Borja et al. (2022); Caasenbrood et al. (2022). In our earlier work (Franco et al., 2021a), we proposed an energy-shaping control law for soft continuum manipulators subject to constant disturbances by employing the *integral Interconnection and damping assignment Passivity based control* (iIDA-PBC) methodology, see (Donaire et al., 2016). This approach builds upon the IDA-PBC methodology, see (Ortega et al., 2002), by introducing an integral action to compensate for the effect of constant matched disturbances. This results in an extended closed-loop dynamics characterized by a port-Hamiltonian structure and by a desired total energy. This approach

offers a range of desirable features, including the physical interpretation of the control action in terms of system energy, which makes it well suited to systems involving energy exchanges between different components, and the fact that the control law does not rely on high gains. The vast majority of research on iIDA-PBC has been focusing on systems in affine form subject to constant matched disturbance, see (Ferguson et al., 2017). Subsequent works have investigated the iIDA-PBC methodology for systems with physical damping, see (Borja et al., 2021), and with parametric uncertainties, see (Teimoorzadeh et al., 2022). In our recent work, we have extended the iIDA-PBC methodology to systems with actuator dynamics subject to matched and unmatched constant disturbances, see (Franco, 2023), and to systems in affine form subject to state-dependent matched disturbances, see (Franco et al., 2023). To the best of our knowledge however, an implementation of the iIDA-PBC methodology for systems with fluidic actuation that are subject to multiple position-dependent disturbances simultaneously is still lacking. This condition is representative of soft manipulators made of hyperelastic materials, such as silicone rubber, which result in nonlinear structural stiffness.

This work presents a new iIDA-PBC controller designed for a soft continuum planar horizontal manipulator with hydraulic actuation that is subject to two simultaneous position-dependent matched disturbances resulting from the uncertain stiffness of the material. The effectiveness of the proposed controller is demonstrated with numerical simulations on a dynamical model of the soft manipulator.

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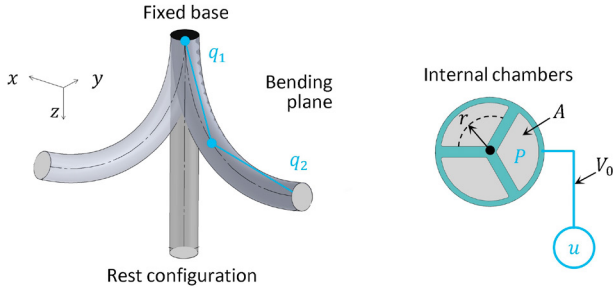


Fig. 1. Schematic of a soft manipulator and of its rigid-link model with $n = 2$ virtual joints; section view of the internal chambers with key model parameters.

2. SYSTEM MODEL

The system considered in this work corresponds to the soft continuum manipulators in (Franco, 2022), which has one or more internal chambers and an inextensible central axis to prevent elongation. The control input is the volumetric flow rate of fluid entering through a pipe in the actuated chamber, which always results in the manipulator bending on a plane, see Figure 1. The orientation of the bending plane depends on which of the three internal chambers is pressurized. At equilibrium and in the absence of external forces, the tip rotation θ depends on the pressure of the fluid P , relative to the atmosphere, according to $\theta = P/k'$, where k' is the structural stiffness of the manipulator, see (Suzumori et al., 1991). The dynamics of the soft continuum manipulator on the bending plane is approximated with a rigid-link model that has n virtual elastic pin joints of stiffness $k > 0$ and damping $D > 0$ in series, see (Godage et al., 2015), as shown in Fig. 1. Indicating with q_i the angle of joint i relative to link $i - 1$ yields n DOFs, where the pressure P acts equally on all joints. The kinetic energy of the rigid-link model is $T(q, p) = \frac{1}{2}p^T M^{-1}p$, where $M(q) = M^T(q) > 0$ is the inertia matrix (i.e., the expression of M for $n = 2$ is provided in Section 4), and $p = M\dot{q}$. The potential energy of the rigid-link model, assuming constant stiffness k , is

$$\Omega(q) = \frac{k}{2} \sum_{i=1}^n q_i^2. \quad (1)$$

The length and the mass of the manipulator are indicated with l and m . The effect of gravity is neglected assuming that the manipulator moves on the horizontal plane.

For inextensible soft manipulators, the volume variation of the chamber caused by bending can be approximated as $V_c = Ar \sum_{i=1}^n |q_i|$, where A is the cross-section area of the internal chamber and r is the distance from the centroid of the chamber to the center of the section, see (Franco, 2022). If all sections of the manipulator bend in the same direction, the volume of the fluid can be expressed as $V = V_0 + Ar\theta$, where V_0 includes the volume of the supply pipes, and $\theta = P/k' = \sum_{i=1}^n q_i$. The internal energy of the fluid with bulk modulus Π_0 , see (Gao et al., 2019), is

$$\Phi = \left(-P + \Pi_0(e^{P/\Pi_0} - 1)\right)(V_0 + Ar\theta). \quad (2)$$

The total mechanical energy of the rigid-link model and of the hydraulic actuation is thus $H = \frac{1}{2}p^T M^{-1}p + \Omega + \Phi$. The full system dynamics in port-Hamiltonian form is

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -I & -D & \frac{\Pi_0 Ar}{V} G \\ 0 & -\frac{\Pi_0 Ar}{V} G^T & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \\ \nabla_P H \end{bmatrix} + \begin{bmatrix} 0 \\ -G\delta \\ \frac{\Pi_0}{V} u \end{bmatrix}, \quad (3)$$

where the system states are the position $q \in \mathbb{R}^n$ of the virtual joints, the momenta $p = M\dot{q} \in \mathbb{R}^n$, and the pressure $P \in \mathbb{R}^a$, where $a \leq n$. The control input $u \in \mathbb{R}^a$ corresponds to the volumetric flow rate in the actuated chambers, see (Acuna-Bravo et al., 2009), while $G \in \mathbb{R}^{n \times a}$. If $a = 1$ the matrix G is a column vector of n elements equal to 1. External forces and model uncertainties, including the effect of nonlinear stiffness, are indicated with $\delta \in \mathbb{R}$. The term I indicates the identity matrix of dimensions $n \times n$, $\nabla_q H$, $\nabla_p H$, and $\nabla_P H$ indicate the vectors of partial derivatives in q , in p , and in P respectively. The system is further qualified by the following assumptions.

Assumption 1: The fluid is isentropic and inviscid, and its bulk modulus Π_0 is constant and known; $a = 1$ and the pressure P of the fluid is uniform in the actuated chamber.

Assumption 2: The position q and the momenta p are known at any instant and are bounded. In addition, $q_i q_j > 0, \forall i, j$, thus all sections of the soft manipulator bend in the same direction. The pressure $P > 0$ is known at any instant and bounded, that is $P \ll \Pi_0$.

Assumption 3: The disturbances δ are parameterized as $\delta = (G^T q)\delta_1 + (G^T q)^s \delta_2$, with known $0 < s < 1$ and $\delta_1, \delta_2 \in \mathbb{R}$ unknown but constant and bounded. The remaining model parameters in (3) are known. The stiffness $k > 0$ and the physical damping $D = D_0 I \succ 0$ are uniform.

The *Assumptions 1* and *2* are similar to those in (Franco, 2022): in particular $q_i q_j > 0, \forall i, j$ if the manipulator does not bend in an ‘S’ shape, which is commonly verified in practice. The *Assumption 3* accounts for uncertain nonlinear stiffness, which is characteristic of the hyperelastic materials commonly employed in soft robotics, see (Franco et al., 2021b). This effect is treated here as a disturbance linearly parameterized in the generalized position q , where the fractional order s captures nonlinear stiffness effects.

3. ENERGY-SHAPING INTEGRAL CONTROL

3.1 Controller design

The control law is designed to achieve the regulation goal $q = q^*$ in the presence of model uncertainties. The closed-loop dynamics becomes then

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{P} \\ \dot{\zeta}_1 \\ \dot{\zeta}_2 \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} & S_{15} \\ -S_{12}^T & -S_{22} & S_{23} & S_{24} & S_{25} \\ -S_{13}^T & -S_{23}^T & -S_{33} & S_{34} & S_{35} \\ -S_{14}^T & -S_{24}^T & -S_{34}^T & -S_{44} & S_{45} \\ -S_{15}^T & -S_{25}^T & -S_{35}^T & -S_{45} & -S_{55} \end{bmatrix} \begin{bmatrix} \nabla_q W_d \\ \nabla_p W_d \\ \nabla_P W_d \\ \nabla_{\zeta_1} W_d \\ \nabla_{\zeta_2} W_d \end{bmatrix}, \quad (4)$$

where the terms S_{ij} are defined in (A.1) so that the open-loop dynamics (3) matches the closed-loop dynamics (4), see Appendix A. The storage function W_d is defined as

$$\begin{aligned}
W_d &= H_d + \frac{K_{I1}}{2} (\zeta_1 - \Psi_1 - \alpha)^2 + \frac{1}{2} \varsigma^T \varsigma \\
&\quad + \frac{K_{I2}}{2} (\zeta_2 - \Psi_2 - \beta)^2, \\
H_d &= \Omega_d + \frac{1}{2} p^T M_d^{-1} p, \\
\Psi_1 &= q^T G G^T p, \quad \Psi_2 = (q^T G)^s G^T p,
\end{aligned} \tag{5}$$

where ζ_1, ζ_2 are the integral states, K_{I1}, K_{I2} are positive constants. The prescribed potential energy Ω_d is

$$\Omega_d = \frac{k}{2k_m} \left(\sum_{i=1}^n q_i^2 + \frac{k_p}{k} (G^T q - nq^*)^2 - \frac{(G^T q)^2}{n} \right), \tag{6}$$

with k_p and k_m positive tuning parameters. The prescribed equilibrium position q^* is a strict minimum of Ω_d (i.e., $q^* = \text{argmin}(\Omega_d)$) and an assignable equilibrium of system (3), that is $G^\perp \nabla_q \Omega = 0$, where $G^\perp$ is the left-annihilator of G , i.e., $G^\perp G = 0$ and $\text{rank}\{G^\perp\} = n - 1$. The prescribed inertia matrix is $M_d = k_m M$. The constants α and β are defined such that

$$\begin{aligned}
G(k_v G^T G + \Gamma_1) \alpha K_{I1} &= G \delta_1, \\
G(k_v G^T G + \Gamma_1) \beta K_{I2} &= G \delta_2,
\end{aligned} \tag{7}$$

with the tuning parameters Γ_1 and k_v . Defining $G^\dagger = (G^T G)^{-1} G^T$, the term ς is given as

$$\begin{aligned}
\varsigma &= G^\dagger \left(-\nabla_q H + M_d M^{-1} \nabla_q H_d + \Pi_0 \frac{Ar}{V} \nabla_P H \right) \\
&+ G^\dagger (D_d - J_d) M_d^{-1} p - (k_v G^T G + \Gamma_1) K_{I1} G^T q (\zeta_1 - \Psi_1) \\
&\quad - (k_v G^T G + \Gamma_1) K_{I2} (G^T q)^s (\zeta_2 - \Psi_2),
\end{aligned} \tag{8}$$

where $D_d = G k_v G^T + D k_m$. Substituting M_d, D_d and $J_d = 0$ in the matching equations

$$\begin{aligned}
G^\perp (\nabla_q \Omega - M_d M^{-1} \nabla_q \Omega_d) &= 0, \\
G^\perp (\nabla_q (p^T M^{-1} p) - M_d M^{-1} \nabla_q (p^T M_d^{-1} p)) \\
&\quad + G^\perp (2J_d M_d^{-1} p) = 0,
\end{aligned} \tag{9}$$

see (Ortega et al., 2002), verifies (9) for all (q, p, P) . The control input is thus

$$\begin{aligned}
u &= Ar G^T \nabla_p H - \frac{V}{\Pi_0} S_{13}^T (\nabla_q H_d + (\nabla_q \varsigma)^T \varsigma) \\
&- \frac{V}{\Pi_0} S_{23}^T (\nabla_p H_d + (\nabla_p \varsigma)^T \varsigma) - \frac{V}{\Pi_0} S_{33} (\nabla_P \varsigma)^T \varsigma \\
&\quad + \frac{V}{\Pi_0} S_{34} (\nabla_{\zeta_1} \varsigma)^T \varsigma + \frac{V}{\Pi_0} S_{35} (\nabla_{\zeta_2} \varsigma)^T \varsigma.
\end{aligned} \tag{10}$$

The time derivatives of the integral states ζ_1 and ζ_2 are

$$\begin{aligned}
\dot{\zeta}_1 &= -S_{14}^T (\nabla_q H_d + (\nabla_q \varsigma)^T \varsigma) \\
&- S_{24}^T (\nabla_p H_d + (\nabla_p \varsigma)^T \varsigma) - S_{34}^T (\nabla_P \varsigma)^T \varsigma \\
&\quad - S_{44} (\nabla_{\zeta_1} \varsigma)^T \varsigma + S_{45} (\nabla_{\zeta_2} \varsigma)^T \varsigma, \\
\dot{\zeta}_2 &= -S_{15}^T (\nabla_q H_d + (\nabla_q \varsigma)^T \varsigma) \\
&- S_{25}^T (\nabla_p H_d + (\nabla_p \varsigma)^T \varsigma) - S_{35}^T (\nabla_P \varsigma)^T \varsigma \\
&\quad - S_{54} (\nabla_{\zeta_1} \varsigma)^T \varsigma - S_{55} (\nabla_{\zeta_2} \varsigma)^T \varsigma.
\end{aligned} \tag{11}$$

Proposition 1: The system (3) in closed-loop with the control law (10) and the integral states defined by their time-derivatives (11) yields (4) with the parameters (A.1) and ς given in (8). The proof is given in Appendix B.

3.2 Stability analysis

Proposition 2: Consider the system (3) with Assumptions 1 to 3 in closed-loop with the control law (10) where the time-derivatives of the integral states are given in (11). Then the equilibrium point $(q, p, P) = (q^*, 0, P^*)$ is asymptotically stable for all positive $k_m, k_p, k_i, k_v, K_{I1}, K_{I2}, \Gamma_1$ such that $\Gamma_1 - k_m D > 0$. In addition, $(\zeta_1 \nabla_p \Psi_1 + \zeta_2 \nabla_p \Psi_2)$ converges to $(\alpha \nabla_p \Psi_1 + \beta \nabla_p \Psi_2)$ asymptotically.

Proof: Consider the storage function W_d defined in (5), which is positive definite by design. Computing the time derivative of W_d along the trajectories of the closed-loop system (4) yields

$$\begin{aligned}
\dot{W}_d &= -\nabla_p W_d^T S_{22} \nabla_p W_d - \nabla_P W_d^T S_{33} \nabla_P W_d \\
&\quad - \nabla_{\zeta_1} W_d^T S_{44} \nabla_{\zeta_1} W_d - \nabla_{\zeta_2} W_d^T S_{55} \nabla_{\zeta_2} W_d \\
&\quad - \nabla_{\zeta_2} W_d^T (S_{54} - S_{45}) \nabla_{\zeta_1} W_d.
\end{aligned} \tag{12}$$

Substituting the matrices $S_{22}, S_{33}, S_{44}, S_{55}, S_{45}$ and S_{54} from (A.1) yields

$$\begin{aligned}
\dot{W}_d &= -\nabla_p W_d^T (k_m D + G k_v G^T) \nabla_p W_d \\
&\quad - \nabla_P W_d^T \frac{k_i}{Ar} \nabla_P W_d \\
&\quad - \nabla_{\zeta_1} W_d^T (\nabla_p \Psi_1^T (\Gamma_1 - k_m D)^T \nabla_p \Psi_1) \nabla_{\zeta_1} W_d \\
&\quad - \nabla_{\zeta_2} W_d^T (\nabla_p \Psi_2^T (\Gamma_1 - k_m D)^T \nabla_p \Psi_2) \nabla_{\zeta_2} W_d \\
&\quad - \nabla_{\zeta_2} W_d^T (\nabla_p \Psi_2^T (\Gamma_1 - k_m D)^T \nabla_p \Psi_1) \nabla_{\zeta_1} W_d \\
&\quad - \nabla_{\zeta_1} W_d^T (\nabla_p \Psi_1^T (\Gamma_1 - k_m D)^T \nabla_p \Psi_2) \nabla_{\zeta_2} W_d
\end{aligned} \tag{13}$$

It follows from (13) that

$$\begin{aligned}
\dot{W}_d &= -\nabla_p W_d^T (k_m D + G k_v G^T) \nabla_p W_d - \frac{k_i}{Ar} |\nabla_P W_d|^2 \\
&\quad - (\Gamma_1 - k_m D) |\nabla_{\zeta_1} W_d \nabla_p \Psi_1 + \nabla_{\zeta_2} W_d \nabla_p \Psi_2|^2 \leq 0.
\end{aligned} \tag{14}$$

for all $k_v \succ 0, k_m > 0$ and $\Gamma_1 - k_m D \succ 0$, thus all states are bounded and the equilibrium is stable. In addition, $\dot{W}_d = 0 \implies \nabla_p W_d = 0 \cap \nabla_P W_d = 0 \cap (\nabla_{\zeta_1} W_d \nabla_p \Psi_1 + \nabla_{\zeta_2} W_d \nabla_p \Psi_2) = 0$. Combining the former expressions and substituting the partial derivatives of W_d (see Appendix B) yields $p = 0$. It follows from LaSalle's theorem (see Theorem 3.4 in (Khalil, 1996)) that the trajectories of the closed-loop system (4) converge asymptotically to the set $p = 0 \cap (\zeta_1 - \Psi_1 - \alpha) \nabla_p \Psi_1 + (\zeta_2 - \Psi_2 - \beta) \nabla_p \Psi_2 = 0 \cap \varsigma = 0$. Computing $\dot{p}$ from (4) in this condition yields $\nabla_q \Omega_d = 0$, thus the equilibrium point $(q, p, P) = (q^*, 0, P^*)$, is asymptotically stable, where P^* is computed from (8). Finally, $(\zeta_1 \nabla_p \Psi_1 + \zeta_2 \nabla_p \Psi_2)$ converges to $(\alpha \nabla_p \Psi_1 + \beta \nabla_p \Psi_2)$ asymptotically. $\square$

Remark 1. Different from prior works, see (Borja et al., 2022; Caasenbrood et al., 2022; Franco et al., 2021a), the new controller employs two integral actions allowing to compensate for multiple simultaneous position-dependent matched disturbances. The proposed design can be extended to account for an arbitrary number N_δ of simultaneous disturbances depending on the generalized position q , including for instance the effect of gravity in case the system is not horizontal. This would result in N_δ integral states with similar stability properties as in Proposition 2. Note that the linear combination of the integral states compensates for the combined effect of the disturbances.

This point is highlighted by the fact that $S_{45} \neq S_{54}$ in (4), and thus (14) ensures the asymptotic convergence to $(\zeta_1 \nabla_p \Psi_1 + \zeta_2 \nabla_p \Psi_2) = (\alpha \nabla_p \Psi_1 + \beta \nabla_p \Psi_2)$ rather than ensuring that each integral state converges to the corresponding parameter. This is a consequence of having multiple matched disturbances acting simultaneously, see (Franco et al., 2023) for details.

4. SIMULATION RESULTS

Simulations have been performed in MATLAB for system (3) employing the model parameters $k = 5$ Nm, $A = 28.3$ mm², $r = 3$ mm, and $D = 0.5I$ for illustrative purposes. The model consists of $n = 2$ virtual links of equal length with mass concentrated at their midpoint and one actuation pressure. The total mass and the total length of the manipulator are $m = 15$ g and $l = 0.1$ m, respectively. Thus $q = (q_1, q_2)$ and the inertia matrix is given by

$$M = \frac{ml^2}{4} \begin{bmatrix} (4 \cos(q_2) + 6) & (2 \cos(q_2) + 1) \\ (2 \cos(q_2) + 1) & 1 \end{bmatrix}.$$

The links at rest are collinear (i.e. $q = 0$). In addition, the structural stiffness k is assumed to be uncertain so that $\delta = \delta_1 \theta + \delta_2 \theta^s$ with $s = \frac{1}{4}$, $\delta_1 = 0.1$, $\delta_2 = 0.2$. The remaining parameters are the pipe volume $V_0 = 28.3$ ml, and the bulk modulus of water, $\Pi_0 = 20000$ bar. The tuning parameters for the controller (10) have been set as $k_p = 1$, $k_m = 5$, $k_i = 100$, $k_v = 2$, $K_{I1} = K_{I2} = 1$, $\Gamma_1 = 2$. The initial conditions are $(q, p, P, \zeta_1, \zeta_2) = (0, 0, 1, 0, 0)$. The prescribed position has been set to $q^* = (\pi/8, \pi/8)$.

The simulation results in Figure 2 show the time histories of the states and the control input for the system (3) in the presence of the matched disturbances δ . For illustrative purposes, the controller (10) with the integral actions (11) has been compared to the IDA-PBC implementation in (Franco, 2022), which accounts for the pressure dynamics and is thus referred to as IDA-PBC+, and to the iIDA-PBC for a soft manipulator with direct actuation and subject to constant disturbances, see (Franco et al., 2021a). The same tuning parameters have been employed for all controllers for consistency. The control input computed with iIDA-PBC has been divided by the scaling factor $K_0 = 500$: this value has been chosen to avoid persistent oscillations. The new controller (10) achieves the regulation goal, and the integral states settle to constant values, while the pressure P varies accordingly. Instead, employing the controller IDA-PBC+, which does not include integral actions, the generalized position does not reach the prescribed value. Employing the iIDA-PBC for constant disturbances with the scaling factor $K_0 = 500$, the control goal is achieved at the cost of noticeable oscillations that degrade the transient performance, which is in agreement with our prior work, see (Franco and Astolfi, 2022). Note that setting $K_0 = 1$ leads to instability in this example. In summary, the new controller yields improved performance compared to the alternatives considered here.

5. CONCLUSIONS

In this work, the iIDA-PBC methodology has been extended by introducing two separate integral states, and it has been implemented for an underactuated soft manipulator with hydraulic actuation and uncertain nonlinear

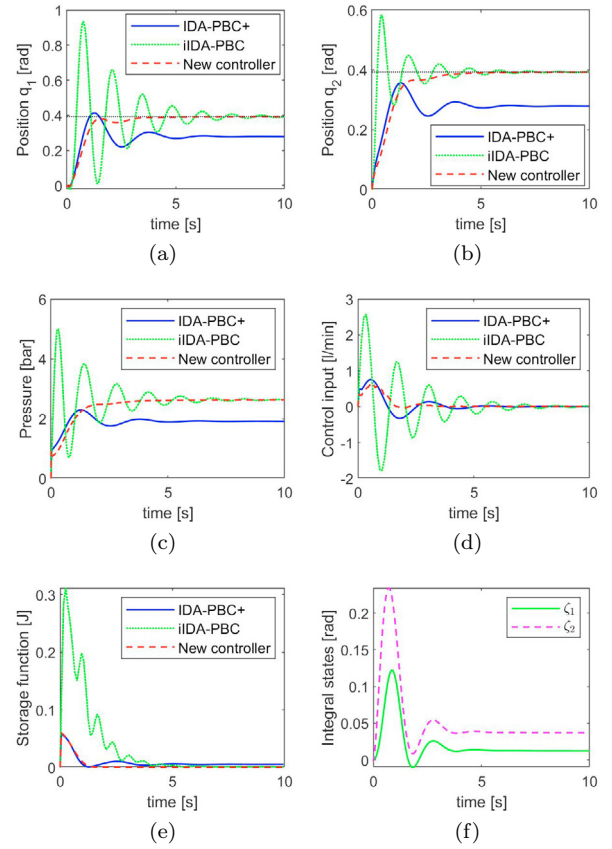


Fig. 2. Simulation results for system (3) with different controllers : (a) position q_1 ; (b) position q_2 ; (c) pressure P ; (d) control input u ; (e) storage function W_d ; (f) integral states ζ_1, ζ_2 computed with (11).

stiffness. The simulation results demonstrate the effectiveness of the new controller and indicate that it yields better performance compared to our previous work. Future work will aim to verify the results with experiments on a soft robotic prototype from our track record.

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Appendix A. MATRIX TERMS IN (4)

The general expressions of the matrix terms in (4) are

$$\begin{aligned}
 S_{12} &= M^{-1}M_d, \quad S_{14} = S_{12}\nabla_p\Psi_1, \quad S_{15} = S_{12}\nabla_p\Psi_2, \\
 S_{13} &= -\frac{(S_{12}\nabla_p\varsigma + S_{14}\nabla_{\zeta_1}\varsigma + S_{15}\nabla_{\zeta_2}\varsigma)}{\nabla_P\varsigma}, \\
 S_{22} &= DM^{-1}M_d + Gk_vG^T - J_d, \quad S_{33} = \frac{k_i}{\nabla_P\varsigma} \\
 S_{24} &= (\Gamma_1 + (J_d - DM^{-1}M_d))\nabla_p\Psi_1 - S_{12}^T\nabla_q\Psi_1, \\
 S_{25} &= (\Gamma_1 + (J_d - DM^{-1}M_d))\nabla_p\Psi_2 - S_{12}^T\nabla_q\Psi_2, \\
 S_{23} &= G\frac{1 + G^\dagger S_{12}^T\nabla_q\varsigma + G^\dagger S_{22}\nabla_p\varsigma}{\nabla_P\varsigma} \\
 &\quad G\frac{-G^\dagger S_{24}\nabla_{\zeta_1}\varsigma - G^\dagger S_{25}\nabla_{\zeta_2}\varsigma}{\nabla_P\varsigma} \\
 &\quad + G^{\perp T}\frac{G^\otimes S_{12}^T\nabla_q\varsigma + G^\otimes S_{22}\nabla_p\varsigma}{\nabla_P\varsigma}, \\
 &\quad + G^{\perp T}\frac{-G^\otimes S_{24}\nabla_{\zeta_1}\varsigma - G^\otimes S_{25}\nabla_{\zeta_2}\varsigma}{\nabla_P\varsigma}, \\
 S_{34} &= -S_{13}^T\nabla_q\Psi_1 - S_{23}^T\nabla_p\Psi_1, \\
 S_{35} &= -S_{13}^T\nabla_q\Psi_2 - S_{23}^T\nabla_p\Psi_2, \\
 S_{44} &= S_{14}^T\nabla_q\Psi_1 + S_{24}^T\nabla_p\Psi_1, \\
 S_{45} &= -S_{14}^T\nabla_q\Psi_2 - S_{24}^T\nabla_p\Psi_2, \\
 S_{54} &= S_{15}^T\nabla_q\Psi_1 + S_{25}^T\nabla_p\Psi_1, \\
 S_{55} &= S_{15}^T\nabla_q\Psi_2 + S_{25}^T\nabla_p\Psi_2,
 \end{aligned} \tag{A.1}$$

where $G^\otimes = (G^\perp G^{\perp T})^{-1}G^\perp$.

Appendix B. PROOF OF PROPOSITION 1

Computing the partial derivatives of W_d yields

$$\begin{aligned}\nabla_q W_d &= \nabla_q H_d + (\nabla_q \varsigma)^T \varsigma - K_{I1} \nabla_q \Psi_1 (\zeta_1 - \Psi_1 - \alpha) \\ &\quad - K_{I2} \nabla_q \Psi_2 (\zeta_2 - \Psi_2 - \beta), \\ \nabla_p W_d &= \nabla_p H_d + (\nabla_p \varsigma)^T \varsigma - K_{I1} \nabla_p \Psi_1 (\zeta_1 - \Psi_1 - \alpha) \\ &\quad - K_{I2} \nabla_p \Psi_2 (\zeta_2 - \Psi_2 - \beta), \\ \nabla_P W_d &= (\nabla_P \varsigma)^T \varsigma, \\ \nabla_{\zeta_1} W_d &= (\nabla_{\zeta_1} \varsigma)^T \varsigma + K_{I1} (\zeta_1 - \Psi_1 - \alpha), \\ \nabla_{\zeta_2} W_d &= (\nabla_{\zeta_2} \varsigma)^T \varsigma + K_{I2} (\zeta_2 - \Psi_2 - \beta).\end{aligned}\tag{B.1}$$

Equating the first row of (3) and of (4) yields

$$M^{-1}p = S_{12} \nabla_p W_d + S_{13} \nabla_P W_d + S_{14} \nabla_{\zeta_1} W_d + S_{15} \nabla_{\zeta_2} W_d.\tag{B.2}$$

Substituting S_{13}, S_{14}, S_{15} from (A.1) and the partial derivatives of W_d from (B.1) into (B.2) yields thus

$$\begin{aligned}M^{-1}p &= S_{12} (\nabla_p H_d + (\nabla_p \varsigma)^T \varsigma) \\ &\quad - S_{12} K_{I1} \nabla_p \Psi_1 (\zeta_1 - \Psi_1 - \alpha) - S_{12} K_{I2} \nabla_p \Psi_2 (\zeta_2 - \Psi_2 - \beta) \\ &\quad - \frac{(S_{12} \nabla_p \varsigma + S_{12} \nabla_p \Psi_1 \nabla_{\zeta_1} \varsigma + S_{12} \nabla_p \Psi_2 \nabla_{\zeta_2} \varsigma)}{\nabla_P \varsigma} (\nabla_P \varsigma)^T \varsigma \\ &\quad + S_{12} \nabla_p \Psi_1 ((\nabla_{\zeta_1} \varsigma)^T \varsigma + K_{I1} (\zeta_1 - \Psi_1 - \alpha)) \\ &\quad + S_{12} \nabla_p \Psi_2 ((\nabla_{\zeta_2} \varsigma)^T \varsigma + K_{I2} (\zeta_2 - \Psi_2 - \beta)).\end{aligned}$$

Refactoring terms yields finally $M^{-1}p = M^{-1}M_d M_d^{-1}p$ which is verified by all $M_d \succ 0$.

Equating the second row of (3) and of (4) yields

$$\begin{aligned}-\nabla_q H - D \nabla_p H + \frac{\Pi_0 A r}{V} G \nabla_P H - G \delta &= -S_{12}^T \nabla_q W_d \\ &\quad - S_{22} \nabla_p W_d + S_{23} \nabla_P W_d + S_{24} \nabla_{\zeta_1} W_d + S_{25} \nabla_{\zeta_2} W_d.\end{aligned}\tag{B.3}$$

Substituting S_{23}, S_{24}, S_{25} from (A.1), equations (7) and (8), and the partial derivatives of W_d from (B.1) into (B.3) yields thus

$$\begin{aligned}-\nabla_q H - D \nabla_p H + \frac{\Pi_0 A r}{V} G \nabla_P H &= \\ -\nabla_p \Psi_1 (G k_v G^T + \Gamma_1) \alpha - \nabla_p \Psi_2 (G k_v G^T + \Gamma_1) \beta &K_{I2} = \\ -S_{12}^T (\nabla_q H_d + (\nabla_q \varsigma)^T \varsigma - K_{I1} \nabla_q \Psi_1 (\zeta_1 - \Psi_1 - \alpha)) & \\ -S_{12}^T K_{I2} \nabla_q \Psi_2 (\zeta_2 - \Psi_2 - \beta) & \\ -S_{22} (\nabla_p H_d + (\nabla_p \varsigma)^T \varsigma - K_{I1} \nabla_p \Psi_1 (\zeta_1 - \Psi_1 - \alpha)) & \\ -S_{22} K_{I2} \nabla_p \Psi_2 (\zeta_2 - \Psi_2 - \beta) & \\ + G \frac{1 + G^\dagger S_{12}^T \nabla_q \varsigma + G^\dagger S_{22} \nabla_p \varsigma}{\nabla_P \varsigma} (\nabla_P \varsigma)^T \varsigma & \\ + G \frac{-G^\dagger S_{24} \nabla_{\zeta_1} \varsigma - G^\dagger S_{25} \nabla_{\zeta_2} \varsigma}{\nabla_P \varsigma} (\nabla_P \varsigma)^T \varsigma & \\ + G^{\perp T} \frac{G^{\otimes} S_{12}^T \nabla_q \varsigma + G^{\otimes} S_{22} \nabla_p \varsigma}{\nabla_P \varsigma} (\nabla_P \varsigma)^T \varsigma & \\ + G^{\perp T} \frac{-G^{\otimes} S_{24} \nabla_{\zeta_1} \varsigma - G^{\otimes} S_{25} \nabla_{\zeta_2} \varsigma}{\nabla_P \varsigma} (\nabla_P \varsigma)^T \varsigma & \\ + ((\Gamma_1 + J_d - D k_m) \nabla_p \Psi_1 - S_{12}^T \nabla_q \Psi_1) (\nabla_{\zeta_1} \varsigma)^T \varsigma & \\ + ((\Gamma_1 + J_d - D k_m) \nabla_p \Psi_1 - S_{12}^T \nabla_q \Psi_1) K_{I1} (\zeta_1 - \Psi_1 - \alpha) & \\ + ((\Gamma_1 + J_d - D k_m) \nabla_p \Psi_2 - S_{12}^T \nabla_q \Psi_2) (\nabla_{\zeta_2} \varsigma)^T \varsigma & \\ + ((\Gamma_1 + J_d - D k_m) \nabla_p \Psi_2 - S_{12}^T \nabla_q \Psi_2) K_{I2} (\zeta_2 - \Psi_2 - \beta).\end{aligned}\tag{B.4}$$

Pre-multiplying (B.4) by $G^\perp$ yields the sum of the PDEs (9) which are verified by (6) with $M_d = k_m M$ and $J_d = 0$. Pre-multiplying (B.4) by $G^\dagger$ yields the identity.

Equating the third row of (3) and of (4) while substituting S_{34}, S_{35} from (A.1) and the partial derivatives of W_d from (B.1) yields

$$\begin{aligned}-\frac{\Pi_0 A r}{V} G^T \nabla_p H + \frac{\Pi_0}{V} u &= \\ -S_{13}^T \nabla_q (\nabla_q H_d + (\nabla_q \varsigma)^T \varsigma - K_{I1} \nabla_q \Psi_1 (\zeta_1 - \Psi_1 - \alpha)) & \\ + S_{13}^T K_{I2} \nabla_q \Psi_2 (\zeta_2 - \Psi_2 - \beta) & \\ -S_{23}^T (\nabla_p H_d + (\nabla_p \varsigma)^T \varsigma - K_{I1} \nabla_p \Psi_1 (\zeta_1 - \Psi_1 - \alpha)) & \\ + S_{23}^T K_{I2} \nabla_p \Psi_2 (\zeta_2 - \Psi_2 - \beta) - S_{33} (\nabla_P \varsigma)^T \varsigma & \\ - (S_{13}^T \nabla_q \Psi_1 + S_{23}^T \nabla_p \Psi_1) ((\nabla_{\zeta_1} \varsigma)^T \varsigma + K_{I1} (\zeta_1 - \Psi_1 - \alpha)) & \\ - (S_{13}^T \nabla_q \Psi_2 + S_{23}^T \nabla_p \Psi_2) ((\nabla_{\zeta_2} \varsigma)^T \varsigma + K_{I2} (\zeta_2 - \Psi_2 - \beta)).\end{aligned}\tag{B.5}$$

Refactoring (B.5) and simplifying common terms cancels α and β yielding the control law (10).

Equating the fourth row of (3) and of (4) while substituting S_{44}, S_{45} from (A.1) and the partial derivatives of W_d from (B.1) yields

$$\begin{aligned}\dot{\zeta}_1 &= -S_{14}^T \nabla_q (\nabla_q H_d + (\nabla_q \varsigma)^T \varsigma - K_{I1} \nabla_q \Psi_1 (\zeta_1 - \Psi_1 - \alpha)) \\ &\quad + S_{14}^T K_{I2} \nabla_q \Psi_2 (\zeta_2 - \Psi_2 - \beta) \\ &\quad - S_{24}^T (\nabla_p H_d + (\nabla_p \varsigma)^T \varsigma - K_{I1} \nabla_p \Psi_1 (\zeta_1 - \Psi_1 - \alpha)) \\ &\quad + S_{24}^T K_{I2} \nabla_p \Psi_2 (\zeta_2 - \Psi_2 - \beta) - S_{34}^T (\nabla_P \varsigma)^T \varsigma \\ &\quad - (S_{14}^T \nabla_q \Psi_1 + S_{24}^T \nabla_p \Psi_1) ((\nabla_{\zeta_1} \varsigma)^T \varsigma + K_{I1} (\zeta_1 - \Psi_1 - \alpha)) \\ &\quad - (S_{14}^T \nabla_q \Psi_2 + S_{24}^T \nabla_p \Psi_2) ((\nabla_{\zeta_2} \varsigma)^T \varsigma + K_{I2} (\zeta_2 - \Psi_2 - \beta)).\end{aligned}\tag{B.6}$$

Refactoring (B.6) cancels α and β yielding $\dot{\zeta}_1$ in (11).

Equating the fifth row of (3) and of (4) while substituting S_{54}, S_{55} from (A.1) and the partial derivatives of W_d from (B.1) yields

$$\begin{aligned}\dot{\zeta}_2 &= -S_{15}^T \nabla_q (\nabla_q H_d + (\nabla_q \varsigma)^T \varsigma - K_{I1} \nabla_q \Psi_1 (\zeta_1 - \Psi_1 - \alpha)) \\ &\quad + S_{15}^T K_{I2} \nabla_q \Psi_2 (\zeta_2 - \Psi_2 - \beta) \\ &\quad - S_{25}^T (\nabla_p H_d + (\nabla_p \varsigma)^T \varsigma - K_{I1} \nabla_p \Psi_1 (\zeta_1 - \Psi_1 - \alpha)) \\ &\quad + S_{25}^T K_{I2} \nabla_p \Psi_2 (\zeta_2 - \Psi_2 - \beta) - S_{35}^T (\nabla_P \varsigma)^T \varsigma \\ &\quad - (S_{15}^T \nabla_q \Psi_1 + S_{25}^T \nabla_p \Psi_1) ((\nabla_{\zeta_1} \varsigma)^T \varsigma + K_{I1} (\zeta_1 - \Psi_1 - \alpha)) \\ &\quad - (S_{15}^T \nabla_q \Psi_2 + S_{25}^T \nabla_p \Psi_2) ((\nabla_{\zeta_2} \varsigma)^T \varsigma + K_{I2} (\zeta_2 - \Psi_2 - \beta)).\end{aligned}\tag{B.7}$$

Refactoring (B.7) yields $\dot{\zeta}_2$ in (11), which concludes the proof. $\square$