
Thermodynamic investigation of a Carnot battery based multi-energy system with cascaded latent thermal (heat and cold) energy stores

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Abstract

Carnot batteries store electricity in thermal form, allowing for power balancing and also multi-vector energy management as a unique asset. Cascaded thermal energy storage therefore has a vital role in Carnot battery, particularly multi-energy systems delivering electricity and thermal energy at various temperatures. Here, we propose a Carnot battery multi-energy system with cascaded latent thermal energy stores. The effects of compressor pressure ratio, total stage number, stage area and fluid velocity in tubes, on system-level coefficient of performance and total exergy efficiency are investigated. The results show that both the coefficient of performance and exergy efficiency increase significantly and then level off with the total stage number and stage area increasing, while they only increase slightly before a significant decrease with the increase of fluid velocity in tubes. The selection of compressor pressure ratios relates to the demands of cold, heat and electricity. The multi-energy system achieves a maximum coefficient of performance of 95.0% in combined cooling, heating and power mode. Although the thermodynamic performance yields comparable with Carnot battery systems integrated with packed-bed or liquid-based sensible heat stores, the Carnot battery multi-energy system can deliver electricity and multi-grade heat and cold simultaneously, thus increasing flexibility in future energy systems.

Keywords

Carnot battery; Pumped thermal electricity/energy storage; Joule-Brayton cycle; Cascaded latent heat and cold store; Thermodynamic analysis; Combined cooling, heating and power

Denotes shared first-authorship reflecting equal contributions

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Nomenclature

Abbreviations			
CB	Carnot battery	PTES	pumped-thermal electricity/energy storage
PBSHCS	packed-bed sensible heat and cold store	LBSHCS	liquid-based sensible heat and cold store
CLHCS	cascaded latent heat and cold store	CLHS	cascaded latent heat store
CLCS	cascaded latent cold store	CCHP	combined cooling, heating and power
HTF	heat transfer fluid	PCM	phase change material
HX	heat exchanger	HHX	heat exchanger for heating
CHX	heat exchanger for cooling	CHP	combined heating and power
PES	pure electricity storage	COP	coefficient of performance
CCP	combined cooling and power	NTU	number of transfer units
RTE	roundtrip efficiency		
Greek symbols			
η	efficiency	γ	specific heat ratio
π	pressure ratio	ψ	temperature ratio
τ	charging/discharging time, s	ε	heat transfer effectiveness
Symbols			
N	total stage number	A	heat transfer area (stage), m ²
T	temperature, K	D	diameter (shell side), m
d	diameter (tube side) or distance, m	W	width, m
P	pressure, kPa	$\dot{m}$	mass flow rate, kg/s
c_p	specific heat capacity, J/(kg·K)	k	thermal conductivity, W/(m·K)
Pr	Prandtl number	Δh	specific latent heat, J/kg
U	heat transfer coefficient, W/(m ² ·K)	$\dot{Q}$	heat transfer rate, W
C	parameter related to NTU	$\dot{E}_x$	exergy transfer rate, W
t	time, s	n	tube number
v	velocity, m/s	L	length of each stage, m
f	friction factor	h	convective heat transfer coefficient, W/(m ² ·K)
Re	Reynolds number	Nu	Nusselt number
Superscripts and subscripts			
i	the i -th stage	m	melting
c	charging	d	discharging
i	inner	o	outer
in	inlet	out	outlet
f	fluid	st	storage
re	release	COM	compressor
EXP	expander	elec	electric
Ex	exergy		

1. Introduction

The share of power generated from renewable sources, such as solar and wind energy, has been growing year by year with total global renewable energy installations amounted to 3,382 GW by 2022 [1]. However, the intermittency and fluctuation of these renewable sources have to be tackled to ensure stability, reliability and resilience of the grid with the increasing renewable penetration. Grid-scale energy storage technologies are indispensable for efficiently incorporating renewable energy sources into the grid and ensuring operational flexibility in addressing discrepancies between generation and demand. Currently, pumped-hydro storage (PHS) and compressed-air energy storage (CAES) are two widely-deployed grid-scale electricity storage technologies, but they suffer from drawbacks including geographical and geological constraints, high initial capital costs, and long construction times [2-4]. Moreover, electrochemical energy storage (EES) is being promoted, but the scale is still restricted by the lifetime and high costs [5, 6]. As a novel energy storage technology, Carnot batteries (CBs) have attracted increasing attention thanks to large-scale capacities, low costs and geographically independent characteristics [7]. Liquid-air energy storage (LAES) belongs to generalised CBs and has already moved into the early stages of commercialisation [8], while in a narrow sense, CBs refer to pumped-thermal electricity/energy storage (PTES) specifically and are still in the conceptualisation and laboratory stage. These CBs are typically classified into three categories based on the thermodynamic cycles: (i) Joule-Brayton cycle-based CBs, (ii) Rankine cycle-based CBs, and (iii) transcritical cycle-based CBs, among which Joule-Brayton cycle-based CBs have wide operating temperature ranges and the potential to achieve the storage and release of heat, cold and electricity and hence are promising to further develop into multi-energy management systems [9, 10].

Joule-Brayton cycle-based CBs convert electricity into heat and cold, and store electricity in the form of heat and cold, respectively, so heat and cold stores are important components of the CBs. So far, packed-bed sensible heat and cold stores (PBSHCSs), liquid-based sensible heat and cold stores (LBSHCSs), packed-bed latent heat and cold stores (PBLHCSs), and packed-bed thermochemical heat stores (PBTHSs) have been proposed for Joule-Brayton cycle-based CBs.

PBSHCSs have the advantages of large heat transfer surfaces, broad applicability and low costs, and extensive relevant research can be found in the literature. By incorporating an unsteady packed-bed model, Desrues et al. [11] determined the overall theoretical efficiency of CB systems and obtained systematic roundtrip efficiency of 67%, along with investigation into the transient performance of CB systems. White et al. [12-16] modified Schumann-type thermal store model and analysed the impacts of operating temperature, operating mode, storage material, cycle duration and store configuration on the thermodynamic and hydrodynamic performance of the packed-bed sensible stores. Moreover, thermodynamic optimisation based on systematic roundtrip efficiency of CB systems was further carried out, showing that the overall roundtrip efficiency was 70% [17, 18]. Chen et al. [19] improved the roundtrip efficiency to 47% by integrating CB systems with PBSHCSs with organic Rankine cycle units. Wang et al. [20, 21] developed an unsteady model of the Joule-Brayton cycle-based CB system, investigated the effects of pressure ratio, particle diameter and store structure on the systematic thermodynamic performance, and found that the roundtrip efficiency reached 57%. In addition, the authors studied the effect of unbalanced flow rate within the packed-bed sensible stores on the systematic roundtrip efficiency, and found that there were 0.62%, 0.26% and 0.36% unbalanced mass flow for the hot store, cold store and PTES close loop respectively. Zhang et al. [22] conducted thermo-economic optimisation and analyses for CB systems, considering charging-discharging duration ratios ranging from 1:3 to 3:1. The 60 MWh CB system demonstrated the highest roundtrip efficiency at 71% and the lowest LCOS of 0.190 ± 0.043 \$/kWh. Albert et al. [23] studied the effects of aspect ratio of the thermal stores and operational segmentation of the packed bed on the roundtrip efficiency of CB systems, and found that longer and thinner stores could enhance efficiency, and the length of the hot store exerted a more substantial influence on system performance compared to that of the cold store. Although PBSHCSs have shown possible

advantages to achieve preferable thermodynamic performance, the high cost, which is associated with the operating pressure, is the main drawback for practical applications. In order to eliminate the effect of operating pressure, Zhang et al. [24] proposed a CB system with indirect packed-bed sensible thermal energy storage (I-CB), and found that the I-CB presents advantages, particularly in terms of low installation costs, when the electricity storage duration exceeded six hours.

LBSHCS is another effective way to replace PBSHCS in terms of operating pressure. Laughlin [25] proposed a CB system with LBSHCSs, of which the heat/cold stores consisted of a pair of unpressurised containers and a heat exchanger, which offered cost advantages at high-pressure operating conditions. Zhao et al. [26] investigated detailed conventional and advanced exergy analyses of CB systems with LBSHCSs, and found that the cold heat exchanger during discharge has the highest share (95%) of the avoidable exergy loss rate. Gonzalez-Ayala et al. [27] conducted a multi-objective optimisation of a CB system with liquid media based on Brayton cycles, and found that the roundtrip efficiency exhibited noticeably larger values and achieved 49% over the endoreversible case in the most ideal scenario. Yang et al. [28, 29] established a dynamic model of a 5 MW CB system with LBSHCSs and obtained the disturbance characteristics of the electric power input, finding that the thermal energy storage temperature dropped by 16.52 K when the electric power input ramped down by 7%. McTigue et al. [30] studied a thermo-economic model for CB systems utilising recuperated Joule-Brayton cycles and two-tank configuration, and found that the lifetime costs were competitive with lithium-ion batteries for discharging durations exceeding six hours. Shi et al. [31] studied the dynamic characteristics of the closed Brayton cycle-based CB systems with LBSHCSs for five variable load methods, and found that the rotor speed fluctuation rate ranged from -9.83% to +2.51% during the charge and discharge process.

To increase the energy density of CB systems, CB systems with PBLHCSs and PBTHSs have been explored [32]. Ge et al. [33] explored the effects of compression ratio, porosity, isentropic efficiency and inlet velocity on the thermodynamic performance of CB systems with PBLHCSs, and found that the systematic power density reached 216.5 kW/m³. Xue et al. [34] proposed a transient model of the CB system with PBLHCSs, and found that the systematic roundtrip efficiency could reach 55%, and the systematic energy density could be improved from 202 kWh/m³ to 267 kWh/m³ by deploying PBLHCSs instead of PBSHCSs. Saghaififar et al. [35, 36] proposed a CB system incorporating PBTHSs that utilise a redox reaction for heat charging and discharging, and found that the systematic roundtrip efficiency ranged from 40% to 55% at a capacity of 250-350 kWh/m³. Pang et al. [37] developed the thermodynamic models of CB systems based on thermochemical energy storage considering the temperature difference between charging and discharging, and found that the system utilising the Ca(OH)₂/CaO working pair could attain roundtrip efficiency ranging from 43% to 61%. However, similar to PBSHCSs, PBLHCSs and PBTHSs are also considered pressure vessels whose costs are sensitive to the operating pressure. Moreover, the CB systems with PBTHSs are still under conceptualisation, primarily owing to the technical complexity and lack of readiness.

The Joule-Brayton cycle-based CB systems mentioned above all focus on the performance of pure electricity storage (PES) mode, while the systems can also achieve combined cooling, heating, and power (CCHP) mode and extend to multi-energy management systems [38-41]. Zhang et al. [42] proposed a CCHP system based on a Joule-Brayton cycle-based CB system with packed-bed sensible heat stores, and explored the system characteristics under different modes. The results showed that the coefficient of performance (COP) of PES and CCHP modes reached 64% and 188%, respectively. However, under the CCHP mode, this system could not deliver electricity, heat and cold simultaneously unless the packed-bed arrays were adopted, and heat or cold could only be supplied at a single low grade.

In fact, the Joule-Brayton cycle-based CB systems can further classify the stored heat and cold into different grades and coordinate electricity and multiple grades of heat and cold simultaneously (e.g., through cascaded latent heat stores), thus becoming highly flexible multi-energy management systems [43]. Zhao et al. [44] have previously investigated the thermodynamic properties of cascaded latent heat stores (CLHSs), on the basis of

which the highly flexible CB-based multi-energy system with cascaded latent heat and cold stores (CLHCSs) is proposed to overcome the limitations in the above literature. In continuity with Zhao et al.'s component-level investigation [44], a system-level research focusing on the flexible CB-based multi-energy system is urgently needed to find out how electricity and multi-grade heat and cold interact with each other and achieve maximum overall efficiency, which has not been studied yet and is still the knowledge gap of CB systems.

In this work, a thermodynamic model of CB-based multi-energy system with CLHCSs is developed. The effects of key design parameters such as compressor pressure ratio, total stage number, stage area and fluid velocity in store tubes, on the COPs and the exergy efficiencies of PES mode, combined heating and power (CHP) mode, combined cooling and power (CCP) mode and CCHP mode are explored. Moreover, an optimisation of CB systems with CLHCSs, LBSHCS and PBSHCS is conducted, and the performance is also analysed and compared in PES, CHP, CCP and CCHP modes, respectively. Finally, the flexibility of the CB-based multi-energy management systems to meet complex multi-energy scenarios is explored.

2. Modelling methodology

2.1. System configuration

A schematic diagram of a Joule-Brayton cycle-based CB system with CLHCSs is shown in Figure 1, which is also considered as a multi-energy management system. The system consists of a cascaded latent heat store (CLHS), a cascaded latent cold store (CLCS), two pairs of compressors/expanders and two sets of heat exchangers (HX1 and HX2). A heating heat exchanger (HHX) and a cooling heat exchanger (CHX) are also deployed to deliver heat and cold, respectively.

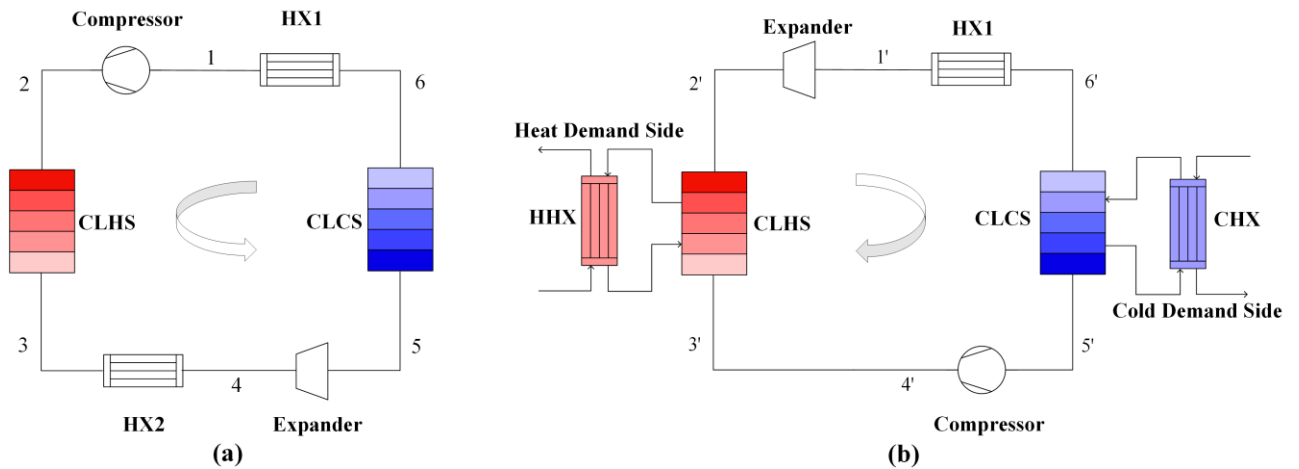


Figure 1. Joule-Brayton cycle-based Carnot battery multi-energy management system with cascaded latent heat and cold stores: (a) charge, (b) discharge.

More details of the CLHS and CLCS are presented in Figure 2. In the shell-and-tube type CLHCSs, a total of N stages is deployed, each of which is replete with one kind of PCM. Taking the CLHS as an example, as shown in Figure 2 (a), $T_{m,i}$ denotes the PCM melting temperature of the i -th stage, while $T_{c,i}$ and $T_{d,N-i+1}$ represent the outlet temperatures of the heat transfer fluid (HTF) for the i -th stage during charging and discharging, respectively. The PCMs in all stages of CLHCSs are organised in descending order of the melting temperatures along with HTF flow direction during charging. During charging (heat storage), the hot HTF (red colour in Figure 2 (a)) flows in tubes in a sequence aligned with the decreasing direction of PCM melting temperatures and transfers heat to the PCMs of each stage in turn, thus decreasing its own temperature stage by stage and storing latent heat in different grades; during discharge (heat release), the cold HTF (blue colour in Figure 2 (a)) flows in tubes along the increasing direction of PCM melting temperatures and absorbs the heat stored in the PCM of each stage in turn with its

temperature increasing stage by stage, thus turning into a high-temperature HTF at the outlet of CLHS. Meanwhile, CLCS, as shown in Figure 2 (b), operates in a similar manner, except that cold HTF enters during charging and hot HTF enters during discharging.

Figure 2 (c) shows the cross section of the shell-and-tube type CLHCSs, where the store is divided into multiple uniform cubic elements, each of which comprises an individual tube, PCMs and HTFs. HTFs flow inside the tubes, while PCMs wrap around the outside of the tubes. It is feasible for the HTFs to absorb heat/cold from the lowest stage to the highest stage to generate electricity, and it is also feasible to absorb heat/cold from a certain stage or stages to deliver different grades of heat (supplied by CLHS)/cold (supplied by CLCS) after charging the CLHCSs.

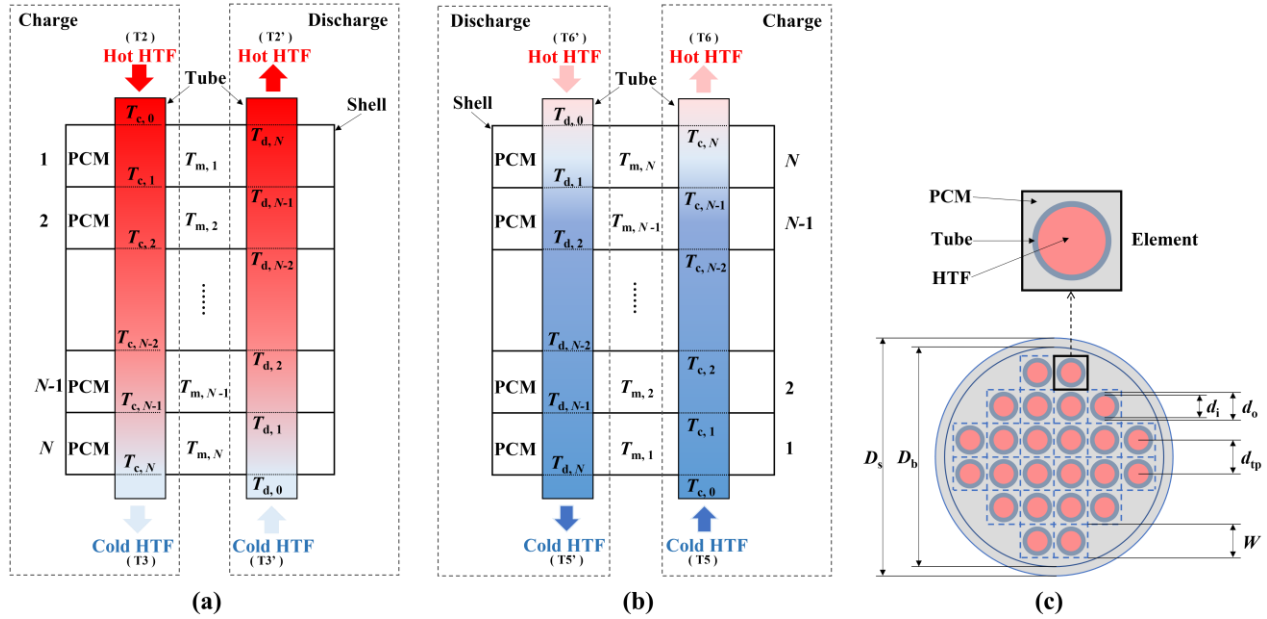


Figure 2. Shell-and-tube type cascaded latent heat and cold stores concept: (a) cascaded latent heat stores, (b) cascaded latent cold stores, (c) cross section.

Charging process can be explained as follows: as shown in Figure 1 (a), the HTF flows into the CLHS to release heat stage by stage after being compressed to a high-temperature and high-pressure state by the compressor, and flows into the CLCS to release cold stage by stage after entering the expander to expand to a low-temperature and low-pressure state. The HTF releases excess cold and heat in HX1 and HX2, respectively, to improve heating and cooling effect. The operation of the CLHCSs-based Joule-Brayton cycle-based CB multi-energy management system can be divided into four modes, which is mainly distinguished by the discharging process (see Figure 1 (b)): (a) PES mode: the HTF is compressed by the compressor to the high-pressure state after flowing through the CLCS to absorb the stored cold stage by stage. Then, the HTF enters the CLHS to absorb the stored heat stage by stage, and directly flows into the expander to do work to generate electricity; (b) CCP mode: the cold quantity can be selected in CLCS for supply according to the different grades of cold demand. HTF absorbs the stored cold in the CLCS which meets the cold demand and delivers it through the CHX, absorbing other cold into the compressor for power generation process; (c) CHP mode: the heat quantity can also be selected in CLHS for supply according to the different grades of heat demand. HTF absorbs the stored heat in the CLHS which meets the heat demand and delivers it through the HHX, absorbing other heat into the expander for power generation process; (d) CCHP mode: combining the CCP and CHP modes, HTF absorbs the stored cold in the CLCS and the stored heat in the CLHS that meet the demand, and delivers them through the HHX and CHX, respectively, absorbing other cold and heat for power generation process.

It is worth noting that this work exclusively focuses on operating conditions that deliver high-grade heat and cold, because the low-grade heat can be provided by the waste heat discharged from the heat exchanger HX1 and

HX2; the low-grade cold can be supplied by the waste cold discharged from HX1, and these waste heat and cold does not affect the performance of power generation.

The following assumptions are made to simplify the physical problem:

- (1) The melting and solidifying temperatures of each stage of PCMs are constant, and only the latent heat is considered during charging and discharging.
- (2) The thermophysical properties of both PCMs and HTFs in the stores are considered constant.
- (3) Each stage of the stores has the same geometry and size.
- (4) The operation durations and HTF mass flow rates during charging and discharging are the same.
- (5) Heat leakage losses from all components to the ambient are ignored.
- (6) The pressure losses and heat leakage losses of the pipelines connecting the main components are ignored.
- (7) The HTF is considered as ideal gas and its temperature variation normal to the flow direction is negligible.

2.2. Mathematical models

2.2.1 Compressor and expander

Polytropic efficiency is widely used as an indicator to the performance of turbomachineries [11, 45]. The polytropic efficiency of the compression process η_{COM} is defined as the ratio of the ideal work in a reversible isentropic process to the work in an adiabatic process in each infinitesimal step:

$$\eta_{\text{COM}} = \frac{v dp}{dh} \quad (1)$$

The polytropic efficiency η_{EXP} of the expansion process is defined as the ratio of the actual work in an adiabatic process to the ideal work in a reversible isentropic process in each infinitesimal step:

$$\eta_{\text{EXP}} = \frac{dh}{v dp} \quad (2)$$

The specific heat ratio of the ideal gas $\gamma = c_p/c_v$, which is substituted into Eqs. (1) and (2), and the relations between the temperature ratio and pressure ratio are obtained:

$$\psi_{\text{COM}} = \pi_{\text{COM}}^{\frac{\gamma-1}{\gamma} \eta_{\text{COM}}} \quad (3)$$

$$\psi_{\text{EXP}} = \pi_{\text{EXP}}^{\frac{\gamma-1}{\gamma} \eta_{\text{EXP}}} \quad (4)$$

where π_{COM} and ψ_{COM} are the pressure ratio and temperature ratio across the compressor, and π_{EXP} and ψ_{EXP} are the pressure ratio and temperature ratio across the expander.

The power consumed by the compressor $\dot{w}_{\text{COM}}$ is determined by the mass flow rate and enthalpy change of the HTF, which can be calculated as follows:

$$\dot{w}_{\text{COM}} = \dot{m}_f (h_{f,\text{COM,out}} - h_{f,\text{COM,in}}) \quad (5)$$

where $\dot{m}_f$ is the mass flow rate of the HTF, and $h_{f,\text{COM,out}}$ and $h_{f,\text{COM,in}}$ are the specific enthalpies at the inlet and outlet of the compressor, respectively.

The output power of the expander $\dot{w}_{\text{EXP}}$ is calculated as:

$$\dot{w}_{\text{EXP}} = \dot{m}_f (h_{f,\text{EXP,in}} - h_{f,\text{EXP,out}}) \quad (6)$$

where $h_{f,\text{EXP,in}}$ and $h_{f,\text{EXP,out}}$ are the enthalpy of the inflow specific enthalpy and outflow specific enthalpy of the expander, respectively.

2.2.2 Cascaded latent heat and cold stores

During the charging process, the heat/cold storage rate at the i -th stage is determined by the following expression:

$$\dot{Q}_{st,c,i} = \dot{m}_f c_{p,f} |T_{c,i-1} - T_{c,i}| = U_i A_i \left\{ (T_{c,i-1} - T_{c,i}) / \ln \left[(T_{c,i-1} - T_{m,i}) / (T_{c,i} - T_{m,i}) \right] \right\} \quad (7)$$

where $\dot{m}_f$ and $c_{p,f}$ are the mass flow rate and specific heat capacity of HTF, respectively. $T_{c,i-1}$ and $T_{c,i}$ denote the inlet and outlet temperatures of the HTF in the i -th stage, respectively. U_i and A_i represent the heat transfer coefficient and heat transfer area of the i -th stage, respectively. $T_{m,i}$ is PCM melting temperature in the i -th stage. The vertical bars in the equation represent absolute value signs, and the item $T_{c,i-1} - T_{c,i}$ is positive for heat storage rate while it is negative for cold storage rate.

The number of transfer units (NTU) in the i -th stage is expressed as:

$$NTU_i = (U_i A_i) / (\dot{m}_f c_{p,f}) = \ln \left[(T_{c,i-1} - T_{m,i}) / (T_{c,i} - T_{m,i}) \right] \quad (8)$$

To simplify the formula, define $C = e^{NTU}$, then the PCM melting temperature $T_{m,i}$ in the i -th stage is:

$$T_{m,i} = (CT_{c,i} - T_{c,i-1}) / (C - 1) \quad (9)$$

The exergy storage rate at the i -th stage and the total exergy storage rate of the whole stores are:

$$\dot{E}x_{st,c,i} = \dot{Q}_{st,c,i} |1 - T_e / T_{m,i}| \quad (10)$$

$$\dot{E}x_{st,c,tot} = \sum_{i=1}^N \dot{E}x_{st,c,i} = \sum_{i=1}^N \left[\dot{Q}_{st,c,i} |1 - T_e / T_{m,i}| \right] \quad (11)$$

where T_e is the ambient temperature, and N is the total stage number in the store.

During discharging, the ideal HTF outlet temperature in the i -th stage is defined as the upper limit:

$$T_{d,N-i+1}^{ideal} = \frac{C-1}{C} T_{m,i} + \frac{1}{C} T_{d,N-i} \quad (12)$$

Therefore, the ideal heat/cold release rate of the HTF in the i -th stage can be calculated as:

$$\dot{Q}_{re,d,i}^{ideal} = \dot{m}_f c_{p,f} |T_{d,N-i+1}^{ideal} - T_{d,N-i}| \quad (13)$$

According to the assumption of equal duration for the charging and discharging, the heat/cold release rate $\dot{Q}_{re,d,i}$ during discharging can only be less than or equal to the heat/cold storage rate $\dot{Q}_{st,c,i}$ during charging in the i -th stage, so HTF outlet temperature in the i -th stage is finally expressed as:

$$T_{d,N-i+1} = \begin{cases} \frac{C-1}{C} T_{m,i} + \frac{1}{C} T_{d,N-i}, & \dot{Q}_{re,d,i}^{ideal} \leq \dot{Q}_{st,c,i} \\ T_{c,i-1} - T_{c,i} + T_{d,N-i}, & \dot{Q}_{re,d,i} > \dot{Q}_{st,c,i} \end{cases} \quad (14)$$

where $T_{d,N-i}$ is the HTF inlet temperature in the i -th stage during discharging.

Therefore, the actual heat/cold release rate of the HTF in the i -th stage is expressed as follows:

$$\dot{Q}_{re,d,i} = \dot{m}_f c_{p,f} |T_{d,N-i+1} - T_{d,N-i}| \quad (15)$$

The melting temperature distribution of PCMs in CLHCSs directly determines the HTF outlet temperatures, so that it affects the COP of the whole system. Based on the charge-discharge cycle characteristics of CB systems, it can be deduced that a CB system achieves its highest COP when all the heat/cold stored in each stage can be released completely during discharging, i.e., the heat/cold released from each stage during discharge equals the heat/cold stored in each stage during charging, as shown in Eq. (16):

$$\dot{Q}_{st,c,i} = \dot{Q}_{re,d,i} \quad (16)$$

Substituting Eqs. (7) and (15) into Eq. (16), the following expressions can be obtained:

$$T_{c,i-1} - T_{c,i} = T_{d,N-i+1} - T_{d,N-i} \quad (17)$$

$$T_{c,i} - T_{c,i+1} = T_{d,N-i} - T_{d,N-i-1} \quad (18)$$

It can be assumed that $\Delta T_i = T_{c,i-1} - T_{c,i}$, $\Delta T_{i+1} = T_{c,i} - T_{c,i+1}$, where ΔT_i and ΔT_{i+1} are the HTF temperature differences between the inlet and outlet in the i -th stage and $i+1$ -th stage, respectively.

Substituting Eq. (9) into Eq. (14), the following expressions can be obtained:

$$T_{d,N-i+1} = T_{c,i} - \frac{1}{C} T_{c,i-1} + \frac{1}{C} T_{d,N-i} \quad (19)$$

$$T_{d,N-i} = T_{c,i+1} - \frac{1}{C} T_{c,i} + \frac{1}{C} T_{d,N-i-1} \quad (20)$$

By subtracting Eq. (19) from Eq. (20), the following can be obtained:

$$T_{d,N-i+1} - T_{d,N-i} = T_{c,i} - T_{c,i+1} - \frac{1}{C} (T_{c,i-1} - T_{c,i}) + \frac{1}{C} (T_{d,N-i} - T_{d,N-i-1}) \quad (21)$$

Substituting Eqs. (17) and (18) into Eq. (21), the following expressions are obtained:

$$\Delta T_i = \Delta T_{i+1} - \frac{1}{C} \Delta T_i + \frac{1}{C} \Delta T_{i+1} \quad (22)$$

Therefore,

$$\Delta T_i = \Delta T_{i+1} \quad (23)$$

In other words, the inlet and outlet temperatures T_c and T_d of each stage in CLHCSs during the charging and discharging follow an arithmetic distribution. Meanwhile, it can be inferred that the melting temperatures T_m of PCMs in CLHCSs also follow an arithmetic distribution according to Eq. (9).

The difference between the i -th stage and $i+1$ -th stage can be calculated using Eqs. (9), (14) and (23):

$$\Delta T_i = \frac{(T_{c,0} - T_{d,0}) \left(1 - \frac{1}{C}\right)}{2 + (N-1) \left(1 - \frac{1}{C}\right)} \quad (24)$$

$$T_{c,i} = T_{c,0} - i \cdot \Delta T_i \quad (25)$$

$$T_{d,i} = T_{d,0} + i \cdot \Delta T_i \quad (26)$$

$$T_{m,i} = T_{c,0} - \left(i + \frac{1}{C-1}\right) \cdot \Delta T_i \quad (27)$$

where $T_{c,0}$ is the HTF inlet temperature of CLHCSs during charging, and $T_{d,0}$ is the HTF inlet temperature of CLHCSs during discharging.

The geometric relationship of CLHCSs can be referred to the previous work [44]. Tube pitch d_{tp} is equal to the width of tube-and-PCM element W and can be obtained from:

$$d_{tp} = \sqrt{\frac{0.78}{n_{tube}}} D_b \quad (28)$$

where D_b and n_{tube} are the bundle diameter and tube number of CLHCSs, respectively, and can be calculated from:

$$D_b = 0.995 D_s - 0.012 - d_o \quad (29)$$

$$n_{\text{tube}} = \left\lceil \frac{4\dot{m}_f}{\pi\rho_f v_{\text{tube}} d_i^2} \right\rceil \quad (30)$$

where D_s is the inner shell diameter of CLHCSSs, d_o is the outer tube diameter, $\dot{m}_f$ is the total HTF mass flow rate, ρ_f is the HTF density, v_{tube} is the HTF velocity in the tubes of the heat/cold store and d_i is the inner tube diameter.

The length of each stage L can be estimated from:

$$L = \frac{A}{n_{\text{tube}}\pi d_o} \quad (31)$$

where A denote the stage area.

To ensure that PCMs can be properly deployed within each stage, the length of each stage L should satisfy:

$$\frac{\int_0^\tau \dot{Q}_{\text{st},c,i} dt}{\Delta h_{\text{PCM},1} \rho_{\text{PCM},1} (\pi D_s^2 / 4 - \pi d_o^2 n_{\text{tube}} / 4)} \leq L \leq \frac{\int_0^\tau \dot{Q}_{\text{st},c,i} dt}{\Delta h_{\text{PCM},2} \rho_{\text{PCM},2} (\pi D_s^2 / 4 - \pi d_o^2 n_{\text{tube}} / 4)} \quad (32)$$

where $\Delta h_{\text{PCM},1}$ and $\Delta h_{\text{PCM},2}$ denote the upper and lower limits of PCM specific latent heat, $\rho_{\text{PCM},1}$ and $\rho_{\text{PCM},2}$ represent the upper and lower limits of PCM density, and τ is the charging time.

The pressure loss Δp in the heat/cold stores can be obtained [46]:

$$\Delta p = \frac{f \rho_f v_{\text{tube}}^2 NL}{2d_i} \quad (33)$$

where f is the friction factor [46]:

$$f = \frac{1}{(0.79 \ln Re - 1.64)^2} = \frac{1}{\left(0.79 \ln \frac{\rho_f v_{\text{tube}} d_i}{\mu_f} - 1.64\right)^2} \quad (34)$$

where Re and μ_f are the Reynolds number and the HTF dynamic viscosity.

The heat transfer coefficient U in each element can be determined from [47]:

$$U = \frac{1}{d_o / (h_i d_i) + d_o \ln(d_o / d_i) / (2k_{\text{tube}}) + d_o \ln(1.08W / d_o) / (2k_{\text{PCM}})} \quad (35)$$

where k_{tube} and k_{PCM} are the thermal conductivities of the tube shell and PCMs, and h_i is the inner convective heat transfer coefficient inside tubes that satisfies Gnielinski's correlation [47]:

$$h_i = \frac{k_f}{d_i} Nu = \frac{k_f}{d_i} \frac{(f/8)(Re - 1000)Pr_f}{1 + 12.7(f/8)^{1/2}(Pr_f^{2/3} - 1)} \quad (36)$$

where Nu is the Nusselt number, k_f is the HTF thermal conductivity and Pr_f is the Prandtl number.

2.2.3 Performance indicators

The input electric energy during charging E_{input} is the difference between the energy consumed by the compressor and the energy produced by the expander, which can be expressed as:

$$E_{\text{input}} = (\dot{w}_{\text{c,COM}} - \dot{w}_{\text{c,EXP}})t_c \quad (37)$$

where $\dot{w}_{\text{c,COM}}$ and $\dot{w}_{\text{c,EXP}}$ are the specific compression power by the compressor and the specific expansion work output of the expander during the charging process, respectively, and t_c is the charging time.

The electric energy output during discharge is the difference between the energy produced by the expander and the energy consumed by the compressor, which can be expressed as:

$$E_{\text{output}} = (\dot{w}_{\text{d,EXP}} - \dot{w}_{\text{d,COM}})t_{\text{d}} \quad (38)$$

where $\dot{w}_{\text{d,EXP}}$ and $\dot{w}_{\text{d,COM}}$ are the expander output work and compressor consumption work during discharge, respectively, and t_{d} is the charging time.

The CB system with CLHCSs can simultaneously output electricity, heat and cold. The electric efficiency η_{elec} is represented as:

$$\eta_{\text{elec}} = \frac{E_{\text{output}}}{E_{\text{input}}} \quad (39)$$

The heat efficiency η_{heat} and cold efficiency η_{cold} are as follows:

$$\eta_{\text{heat}} = \frac{Q_{\text{heat}}}{E_{\text{input}}} \quad (40)$$

$$\eta_{\text{cold}} = \frac{Q_{\text{cold}}}{E_{\text{input}}} \quad (41)$$

where Q_{heat} and Q_{cold} are the output heat and cold supplied by CB systems, respectively.

The coefficient of performance (COP) of the CB multi-energy management system can be calculated using:

$$\text{COP} = \frac{E_{\text{output}} + Q_{\text{heat}} + Q_{\text{cold}}}{E_{\text{input}}} \quad (42)$$

The heat exergy efficiency $\eta_{\text{Ex, heat}}$ and cold exergy efficiency $\eta_{\text{Ex, cold}}$ are as follows:

$$\eta_{\text{Ex, heat}} = \frac{Ex_{\text{heat}}}{E_{\text{input}}} \quad (43)$$

$$\eta_{\text{Ex, cold}} = \frac{Ex_{\text{cold}}}{E_{\text{input}}} \quad (44)$$

where Ex_{heat} and Ex_{cold} are the output heat exergy and cold exergy supplied by CB systems, respectively.

The total exergy efficiency of the CB multi-energy management system is calculated as:

$$\eta_{\text{Ex}} = \frac{E_{\text{output}} + Ex_{\text{heat}} + Ex_{\text{cold}}}{E_{\text{input}}} \quad (45)$$

2.3. Case study description

In this paper, argon is chosen as HTF thanks to its suitable thermodynamic properties. The polytropic efficiencies of the compressor and expander are set at 90% according to previous studies [11, 20, 21, 45]. Organics and inorganic salts are usually used as PCMs for heat and cold storage, with density mainly distributed at 800-2500 kg/m³ and latent heat mainly distributed at 100-400 kJ/kg [32]. Considering that 2 W/(m·K) can be easily achieved for the thermal conductivity of PCMs by the modern enhanced heat transfer method, the thermal conductivity of PCM is set to be 2 W/(m·K) in this work [48]. The operating conditions and CLHCSs design parameters are summarised in Table 1. The tubes used for the flow of argon are 3/4-in. O.D., 16 BWG (Birmingham Wire Gage) type, which is commonly used in heat exchange equipments, and the shell diameters of CLHS and CLCS are taken as 2 m and 3 m, respectively, to balance their lengths and pressure losses. The four operating modes of the CB multi-energy management systems are listed in Table 2.

Table 1. Operating conditions and CLHCSs design parameters

<i>Conditions and parameters</i>	<i>Unit</i>	<i>Value</i>
Ambient temperature, T_e	K	298
Argon inlet temperature of the compressor during charging, T_1	K	298
Argon inlet pressure of the compressor during charging, P_1	kPa	105
Maximum system operating temperature	K	850
Minimum system operating temperature	K	123
Argon mass flow rate, $\dot{m}_i$	kg/s	12.5
Charge/discharge time, τ	s	21600
Compressor polytropic efficiency, η_c	%	90
Expander polytropic efficiency, η_e	%	90
Shell diameter of CLHS, D_s	m	2
Shell diameter of CLCS, D_s	m	3
Tube outer diameter, d_o	m	1.9×10^{-2}
Tube inner diameter, d_i	m	1.57×10^{-2}

Table 2. Operating modes of the CB multi-energy management systems

<i>Mode</i>	<i>Case</i>	<i>Description</i>
PES	1	CB systems output electricity only.
CHP	2	CB systems heat air with a mass flow rate of 6.2 kg/s from 700 K to 800 K while outputting electricity.
	3	CB systems heat air with a mass flow rate of 6.2 kg/s from 500 K to 600 K while outputting electricity.
CCP	4	CB systems cool air with a mass flow rate of 6.5 kg/s from 200 K to 150 K while outputting electricity.
	5	CB systems cool air with a mass flow rate of 6.5 kg/s from 230 K to 180 K while outputting electricity.
	6	CB systems cool air with a mass flow rate of 6.5 kg/s from 250 K to 200 K while outputting electricity.
CCHP	7	CB systems heat air with a mass flow rate of 6.2 kg/s from 700 K to 800 K and cool air with a mass flow rate of 6.5 kg/s from 200 K to 150 K while outputting electricity.
	8	CB systems heat air with a mass flow rate of 6.2 kg/s from 500 K to 600 K and cool air with a mass flow rate of 6.5 kg/s from 230 K to 180 K while outputting electricity.
	9	CB systems heat air with a mass flow rate of 6.2 kg/s from 500 K to 600 K and cool air with a mass flow rate of 6.5 kg/s from 250 K to 200 K while outputting electricity.

3.Results and discussion

3.1 Parametric analysis

The compressors and expanders are the components responsible for the conversion between electricity and thermal energy, while the CLHCSs are the components responsible for energy storage and release, so the key parameters of these two parts have an important impact on the operational performance of the CB multi-energy management system. Therefore, a parametric analysis is conducted, focusing on the following specific parameters: compressor pressure ratio during charging π_{c1} , compressor pressure ratio during discharging π_{c2} ; total stage

number N_1 , stage area A_1 and HTF velocity in store tubes v_{tube1} for CLHS; total stage number N_2 , stage area A_2 and HTF velocity in store tubes v_{tube2} for CLCS. Considering the limitations of technical maturity and manufacturing cost, π_{c1} and π_{c2} are set in the range of 1.1-15, N_1 and N_2 are set in the range of 1-100, A_1 and A_2 are set in the range of 10-1000 m², v_{tube1} and v_{tube2} are set in the range of 0.8-20 m/s, respectively. In the parameter analysis, some parameters are preset that are easy to implement and can bring performance benefits according to the previous research [44, 45]: $\pi_{c1} = 10.5$, $\pi_{c2} = 7.0$, $N_1 = N_2 = 30$, $A_1 = 300$ m², $A_2 = 400$ m², $v_{\text{tube1}} = 2.0$ m/s, and $v_{\text{tube2}} = 3.0$ m/s. In the parametric analysis of the above parameters, Case 1, Case 3, Case 5 and Case 8 in Table 2 are selected as representatives of PES, CHP, CCP and CCHP modes, respectively.

3.1.1 Parameters of cascaded latent heat and cold stores

The total stage number determines the abundance of heat and cold storage grades. Figure 3 shows the effect of the total stage number of CLHS N_1 and that of CLCS N_2 on the COP and exergy efficiency when $\pi_{c1} = 10.5$, $\pi_{c2} = 7.0$, $A_1 = 300$ m², $A_2 = 400$ m², $v_{\text{tube1}} = 2.0$ m/s, and $v_{\text{tube2}} = 3.0$ m/s. For the analysis of the effect of N_1 , N_2 is kept constant at 30. On the other hand, for the analysis of the effect of N_2 , N_1 is fixed at 30.

It can be seen from Figure 3 (a) that both COP and exergy efficiency increase significantly first and tend to be stable with the increase of N_1 in PES, CHP, CCP and CCHP modes. In PES mode, the COP of the CB system, which equals the exergy efficiency, is only 31.8% when $N_1 = 2$, and as N_1 reaches 30, the COP improves to 57.8%. However, further increasing the CLHS total stage numbers N_1 does not lead to a significant enhancement in the COP, and the COP reaches its peak at 59.7% when $N_1 = 100$. The highest COP occurs in CHP mode, where the COP improves from 44.9% to 71.1% when N_1 grows from 2 to 30, reaching its peak at 73.0% when $N_1 = 100$. However, the exergy efficiency only grows from 56.6% to 58.6% when N_1 grows from 30 to 100. The lowest COP appears in CCP mode, where the COP improves from 33.2% to 53.1% when N_1 grows from 2 to 30, reaching its peak at 54.5% when $N_1 = 100$. Besides, the exergy efficiency improves from 46.0% to 47.4% when N_1 grows from 30 to 100. The CCHP mode has the lowest exergy efficiency because the CB system delivers the most heat and cold in this mode, resulting in the most exergy losses. The exergy efficiency improves from 44.8% to 46.3% when N_1 increases from 30 to 100.

Figure 3 (b) shows a similar trend to Figure 3 (a), but N_2 cannot be less than 6 in CCP mode and CCHP mode, otherwise, the minimum temperature of cold that the CB system provides is higher than 180 K. The N_1 and N_2 can be taken around 30, which is the same as the total number of stages in the CB system. Excessive stage number causes an increase in cost, so it is recommended that N_1 and N_2 are around 30 to balance the thermodynamic performance and the size of the CB system.

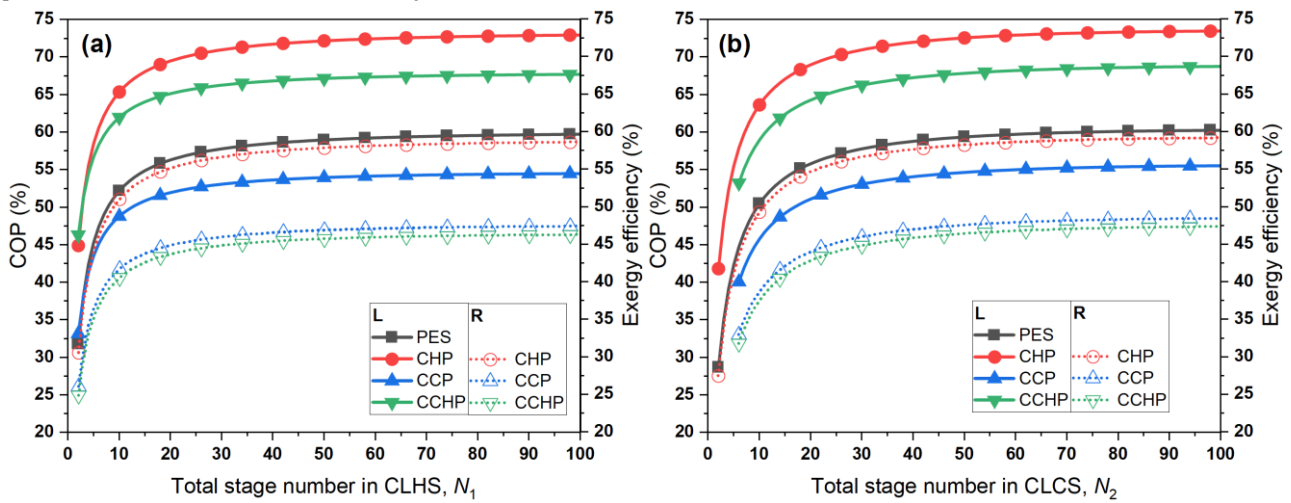


Figure 3. Effect of total stage number of CLHS N_1 and that of CLCS N_2 on the COP and exergy efficiency: (a) total stage number of CLHS N_1 , and (b) total stage number of CLCS N_2 . $\pi_{c1} = 10.5$, $\pi_{c2} = 7.0$, $A_1 = 300$ m², $A_2 = 400$ m², $v_{\text{tube1}} = 2.0$ m/s, $v_{\text{tube2}} = 3.0$ m/s.

The stage areas affect the CLHS/CLCS size and the heat transfer performance between argon and PCM. Figure 4 illustrates the effect of CLHS stage area A_1 and CLCS stage area A_2 on the COP and exergy efficiency. For the analysis of the effect of A_1 , A_2 is kept constant at 400 m². On the other hand, for the analysis of the effect of the A_2 , A_1 is fixed at 300 m². It can be seen from Figure 4 (a) that both COP and exergy efficiency increase significantly first and then tend to be stable with the increase of A_1 and A_2 in PES, CHP, CCP and CCHP modes. When $A_1 = 300$ m² and $A_2 = 400$ m², the COP is 57.8%, 71.0%, 53.1% and 66.3% in PES, CHP, CCP and CCHP modes, respectively, while the exergy efficiency is 56.7%, 46.0% and 44.8% in CHP, CCP and CCHP modes, respectively. Considering that a large stage area increases the size of CB system, while a small stage area damages the thermodynamic performance of CB system, it is recommended to be around 300 m² for A_1 and 400 m² for A_2 , respectively.

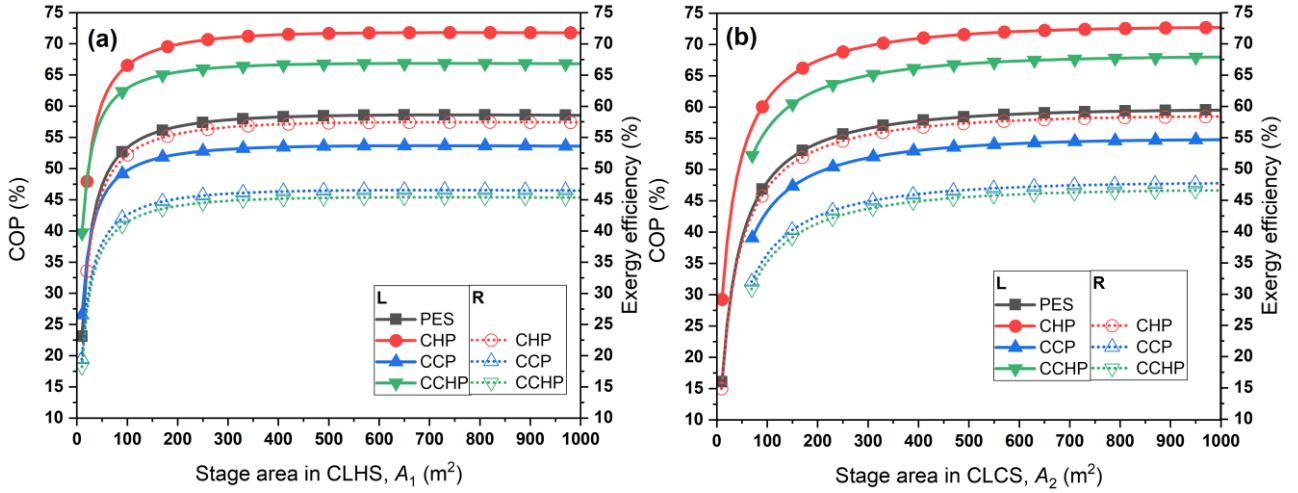


Figure 4. Effect of CLHS stage area A_1 and CLCS stage area A_2 on the COP and exergy efficiency: (a) CLHS stage area A_1 , and (b) CLCS stage area A_2 . $\pi_{c1} = 10.5$, $\pi_{c2} = 7.0$, $N_1 = N_2 = 30$, $v_{tube1} = 2.0$ m/s, $v_{tube2} = 3.0$ m/s.

HTF velocity in store tubes affects the number of HTF tubes inside the heat and cold stores. Figure 5 shows the effect of HTF velocity v_{tube1} in CLHS tubes and HTF velocity v_{tube2} in CLCS tubes on the COP and the exergy efficiency. For the analysis of the effect of v_{tube1} , v_{tube2} is kept constant at 3.0 m/s. On the other hand, for the analysis of the effect of the v_{tube2} , v_{tube1} is fixed at 2.0 m/s. It can be seen that the COP and exergy efficiency show a weak increase and then a rapid decrease with the increase of v_{tube1} and v_{tube2} in PES, CHP, CCP and CCHP modes. It is recommended that v_{tube1} should not exceed 5 m/s and v_{tube2} should not exceed 8 m/s.

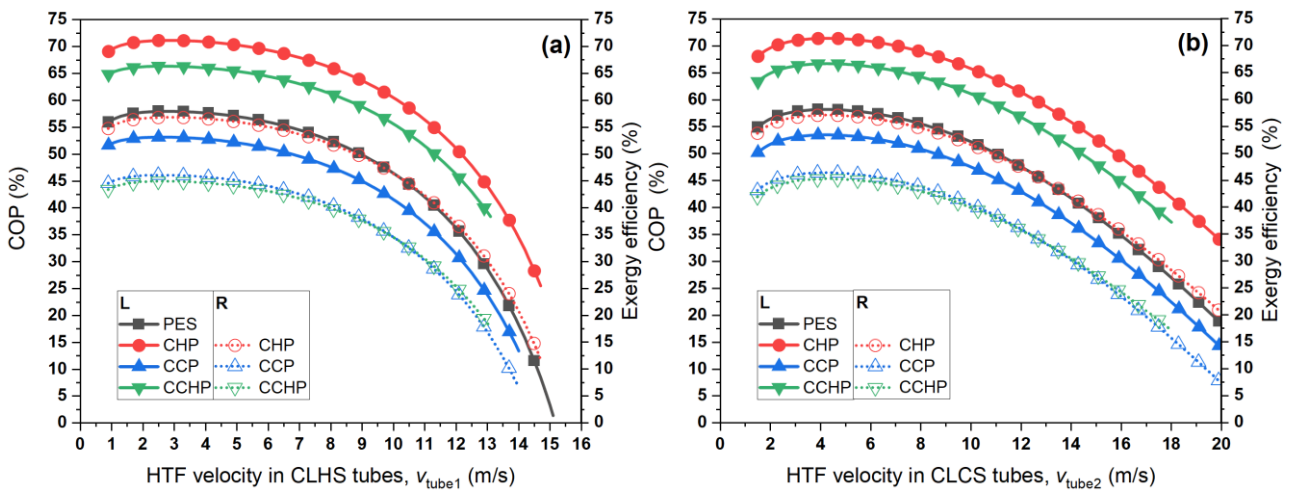


Figure 5. Effect of HTF velocity in CLHS tubes v_{tube1} and HTF velocity in CLCS tubes v_{tube2} on the COP and exergy efficiency: (a) HTF velocity in CLHS tubes v_{tube1} , and (b) HTF velocity in CLCS tubes v_{tube2} . $\pi_{c1} = 10.5$, $\pi_{c2} = 7.0$, $N_1 = N_2 = 30$, $A_1 = 300$ m², $A_2 = 400$ m².

3.1.2 Parameters of turbomachineries

Figure 6 illustrates the distribution of the COP with the compressor pressure ratio π_{c1} during charging and π_{c2} during discharging in PES, CHP, CCP and CCHP modes when $N_1 = N_2 = 30$, $A_1 = 300 \text{ m}^2$, $A_2 = 400 \text{ m}^2$, $v_{\text{tube1}} = 2.0 \text{ m/s}$, $v_{\text{tube2}} = 3.0 \text{ m/s}$. The COP peaks at 57.8% when $\pi_{c1} = 10.5$ and $\pi_{c2} = 7.1$ in PES mode, peaks at 81.5% when $\pi_{c1} = 5.2$ and $\pi_{c2} = 2.9$ in CHP mode, peaks at 56.5% when $\pi_{c1} = 4.8$ and $\pi_{c2} = 2.5$ in CCP mode and achieves a maximum value of 95.0% when $\pi_{c1} = 5.2$ and $\pi_{c2} = 2.1$ in CCHP mode, respectively. It can be seen that small π_{c1} and π_{c2} can ensure a high COP when the CB multi-energy management system is delivering heat and cold to the outside.

Figure 7 shows the T - s diagram of CB systems corresponding to the maximum COPs in the four modes shown in Figure 6. It can be noticed that the maximum temperature of heat stored during charging is slightly above the temperature range for heating, and similarly the minimum temperature of cold stored during charging is slightly below the temperature range for cooling in CHP, CCP and CCHP modes. In other words, the COPs achieve maximum in CHP, CCP and CCHP modes when the CB systems are on the edge of meeting the heat and cold demands. However, such conditions would sacrifice power generation, because a small pressure ratio leads to a small charging power, and the generating power is reduced under the same heating and cooling conditions.

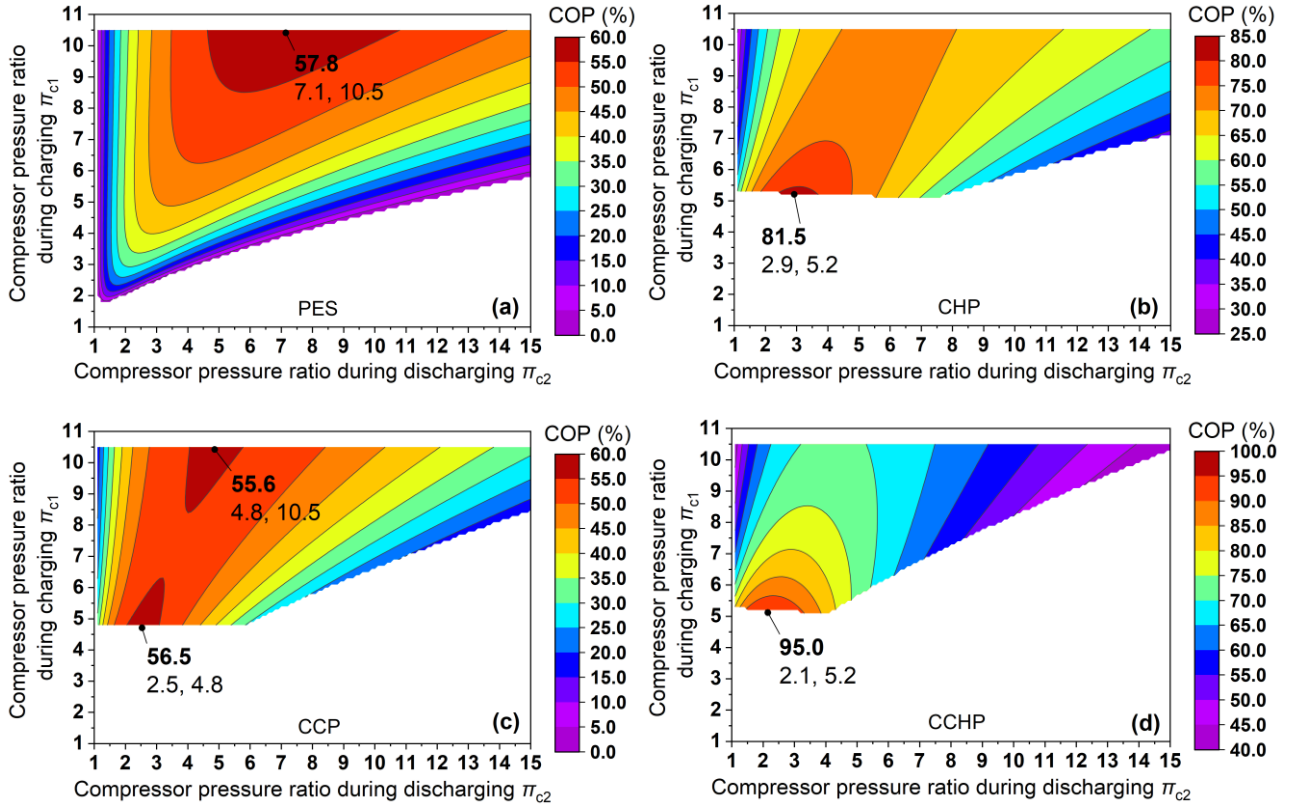


Figure 6. Variation of COP with compressor pressure ratio during charging π_{c1} and during discharging π_{c2} in different modes: (a) PES mode, (b) CHP mode, (c) CCP mode, and (d) CCHP mode. $N_1 = N_2 = 30$, $A_1 = 300 \text{ m}^2$, $A_2 = 400 \text{ m}^2$, $v_{\text{tube1}} = 2.0 \text{ m/s}$, $v_{\text{tube2}} = 3.0 \text{ m/s}$.

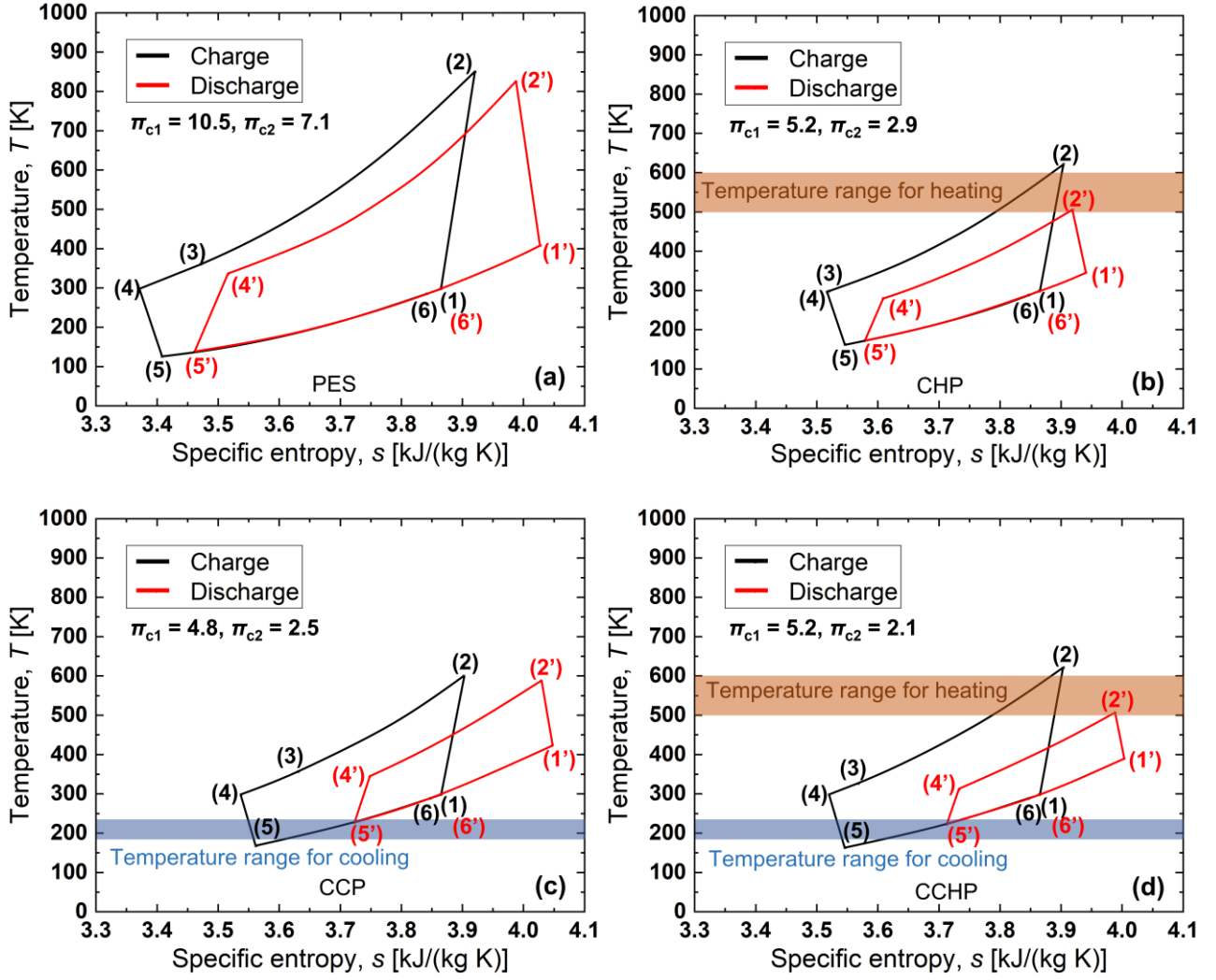


Figure 7. T - s diagrams of CB systems with highest COPs in different modes: (a) PES mode, (b) CHP mode, (c) CCP mode, and (d) CCHP mode.

Figure 8 shows the distribution of the exergy efficiency with the compressor pressure ratio π_{c1} during charging and π_{c2} during discharging in CHP, CCP and CCHP mode when $N_1 = N_2 = 30$, $A_1 = 300 \text{ m}^2$, $A_2 = 400 \text{ m}^2$, $v_{\text{tube}1} = 2.0 \text{ m/s}$, $v_{\text{tube}2} = 3.0 \text{ m/s}$. The exergy efficiency peaks at 56.9% when $\pi_{c1} = 10.5$ and $\pi_{c2} = 6.1$ in CHP mode, peaks at 48.5% when $\pi_{c1} = 10.5$ and $\pi_{c2} = 4.8$ in CCP mode and peaks at 51.6% when $\pi_{c1} = 5.2$ and $\pi_{c2} = 2.1$ in CCHP mode, respectively. It can be found that the exergy efficiency achieves its maximum value when $\pi_{c1} = 10.5$ in CHP and CCP modes, while it obtains its maximum when $\pi_{c1} = 5.2$ in CCHP mode, which implies that π_{c1} and π_{c2} should be selected according to the demand of cold, heat and electricity. In addition, the maximum values of COP and exergy efficiency do not always occur at the same position, which means that the COP and the exergy efficiency should be weighed when determining π_{c1} and π_{c2} in different operating modes.

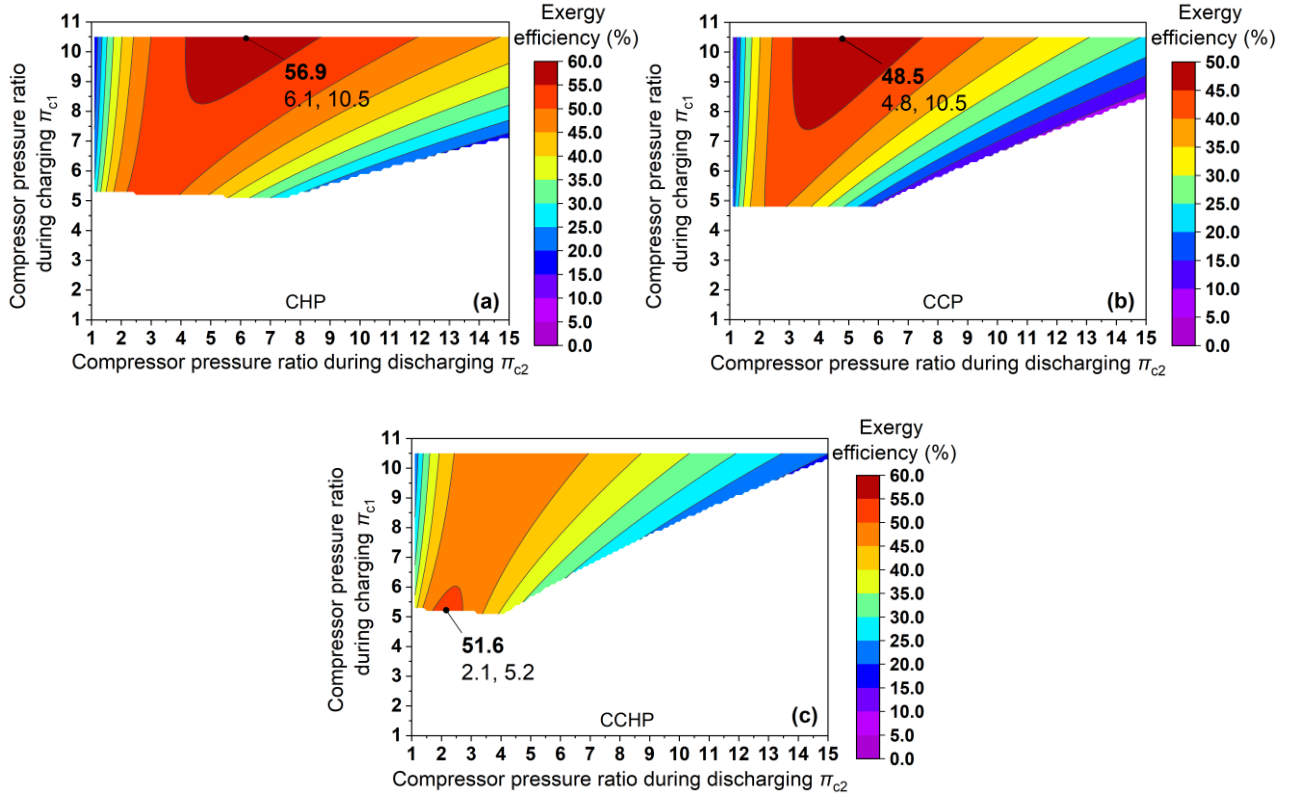


Figure 8. Variation of exergy efficiency with compressor pressure ratio during charging π_{c1} and during discharging π_{c2} in different modes: (a) CHP mode, (b) CCP mode, and (c) CCHP mode. $N_1 = N_2 = 30$, $A_1 = 300 \text{ m}^2$, $A_2 = 400 \text{ m}^2$, $v_{\text{tube1}} = 2.0 \text{ m/s}$, $v_{\text{tube2}} = 3.0 \text{ m/s}$.

3.2 Optimisation and comparison

In this section, an optimisation of CB systems with CLHCSs, LBSHCS and PBSHCS is conducted with COP as the objective function, and the performance is analysed and compared in PES, CHP, CCP and CCHP modes. The lower and upper bounds of the design variables are shown in Table 3, and the variables optimisation ranges of CB systems with LBSHCS and PBSHCS are in Appendix A.

Table 3. Ranges of design variables

Variables	π_{c1}	π_{c2}	v_{tube1} (m/s)	v_{tube2} (m/s)	A_1 (m ²)	A_2 (m ²)
Upper bounds	15	15	20	20	1000	1000
Lower bounds	1	1	0.81	1.44	10	10

In this section, Case 1, Case 2, Case 3, Case 4, Case 6, Case 7 and Case 9 in Table 2 are selected as representatives of PES, CHP, CCP and CCHP modes, respectively.

3.2.1 PES mode

Figure 9 shows the COPs and electric efficiencies of CB systems with CLHCSs (10-100 stages), LBSHCS, and PBSHCS in PES mode (Case 1), which is the optimal value after optimisation.

The COPs and electric efficiencies of CB system with CLHCSs range between 53.0% and 64.0%, with the minimum and maximum values obtained at the total stage number $N = 10$ and $N = 100$ ($N_1 = N_2 = N$), respectively. CB system with LBSHCS demonstrates a COP of 59.4%, which is similar to the performance of CB system with CLHCSs with a total stage number of $N = 20$. On the other hand, CB system with PBSHCS exhibits a COP of 63.7%, which is comparable to CB system with CLHCSs with a total stage number of $N = 80$. Overall, CB system with CLHCSs

($N = 100$) exhibits higher COP than CB systems with LBSHCS and PBSHCS. Further, it is necessary to analyse the exergy losses of these three CB systems.

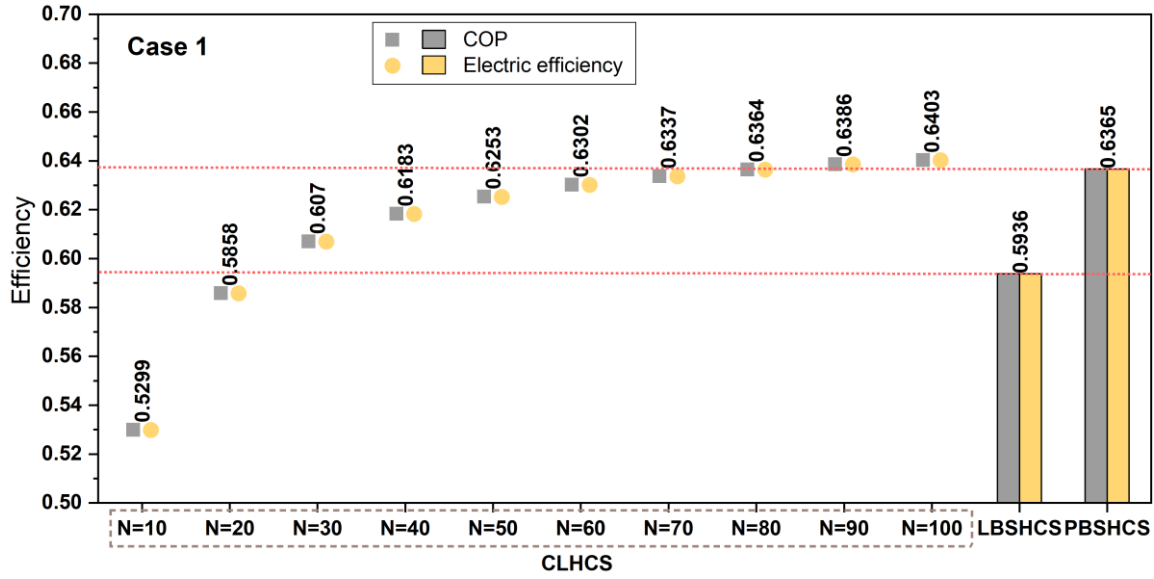


Figure 9. COPs and electric efficiencies of CB systems with CLHCSs (10-100 stages), LBSHCS, and PBSHCS in PES mode. Case 1: CB systems output electricity only.

In Figure 10, the system exergy loss rates of CB systems and each component are compared. The total exergy loss rates of the three CB systems are 896.8 kW, 1015.7 kW and 908.2 kW, respectively, and the CLHCS-based CB system has the lowest exergy loss rates and the LBSHCS-based CB system has the highest exergy loss rates. Within the CB system, the two sets of compressors and expanders account for the largest share of exergy loss rates, which is mainly influenced by their polytropic efficiencies. The third largest exergy loss comes from HX1, because the exhausted heat from the expander is discharged directly to the environment through HX1, resulting in exergy loss. Both CLHS and CLCS have lower exergy loss rates compared to the compressors/expanders and HX1. Among the three kinds of TES units, LBSHCS has the highest exergy loss rates, while CLHCSs have the lowest exergy loss rates. In PES mode, the CB system with CLHCSs ($N = 100$) outperforms CB systems with LBSHCS and PBSHCS in terms of both COPs and exergy loss rates.

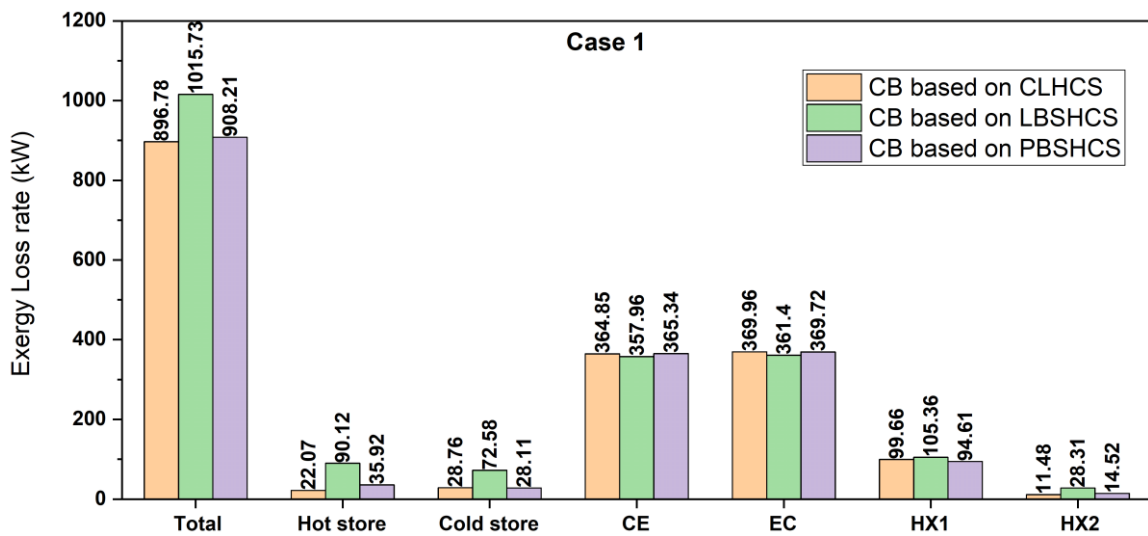


Figure 10. System exergy loss rates of CB systems with CLHCSs ($N = 100$), LBSHCS and PBSHCS in PES mode. Case 1: CB systems output electricity only. CE: compressor in charging and expander in discharging, EC: expander in charging and compressor in discharging, HX1: heat exchanger 1, HX2: heat exchanger 2.

3.2.2 CHP mode

Two CHP mode cases (Case 2 and Case 3) are discussed in this section to compare the impact of the delivered thermal energy grade on the performance of the CB systems.

Figure 11 shows the system performance of CB systems with CLHCSs, LBSHCS, and PBSHCS in CHP mode (Case 2 and Case 3). For CB systems with CLHCSs, the COPs range between 66.4% and 76.7%, the electric efficiencies range from 40.0% to 50.3%, and the heat efficiencies range from 26.4% to 26.4%; For the CB system with LBSHCS, the COP is 72.5%, the electric efficiency is 46.1%, and the heat efficiency is 26.4%, which is similar to CB system with CLHCSs with a total stage number of $N = 20$; On the other hand, CB system with PBSHCS has a COP of 76.3%, an electric efficiency of 49.9%, and a heat efficiency of 26.4%, which is comparable to CB system with CLHCSs with a total stage number of $N = 80$. The electric efficiencies of the three CB systems significantly decrease when they deliver high-grade thermal energy externally, but the COPs improve because the quality of thermal energy is lower than that of electric energy. CB systems with CLHCSs ($N = 100$) have higher COP and electric efficiency compared to CB systems with LBSHCS and PBSHCS, and the heat efficiency is the same as that of the latter two because they deliver the same amount of heat.

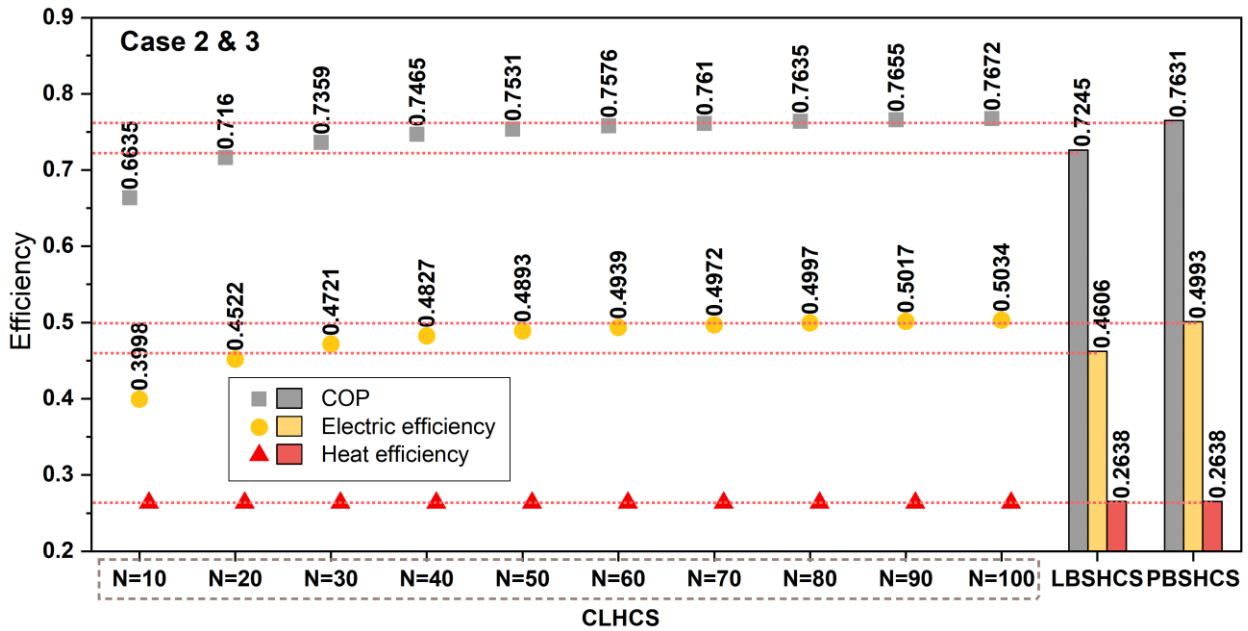


Figure 11. System performance of CB systems with CLHCSs, LBSHCS, and PBSHCS in CHP mode. Case 2: CB systems heat air with a mass flow rate of 6.2 kg/s from 700 K to 800 K while outputting electricity; Case 3: CB systems heat air with a mass flow rate of 6.2 kg/s from 500 K to 600 K while outputting electricity.

Figure 12 (a) and Figure 12 (b) compare the system exergy loss rates of CB systems with CLHCSs ($N = 100$), LBSHCS and PBSHCS in CHP mode for Case 2 and Case 3, respectively. The total exergy loss rates of these three in Case 2 are 844.2 kW, 953.1 kW and 856.4 kW respectively, which are smaller than those in PES mode (Case 1). The primary reason for this reduction is the decrease of exergy loss rates within HX1, which is attributed to the fact that the temperature of HTF entering the expander becomes lower after heating, and the temperature becomes even lower at the outlet of the expander, resulting in a reduction in the heat released by HX1 to the environment. The total exergy loss rates of these three in Case 3 are 939.5 kW, 1048.4 kW and 951.7 kW, respectively, which are larger than those in Case 2. The increase of exergy loss rates is caused by the decrease of heating temperature, marking as heating-induced increase, i.e., the exergy loss rates of the CB systems are greater when they deliver lower grade heat. For CB systems with CLHCSs and LBSHCS, the most significant increase in exergy loss rates occurs in the heat stores, from 35.2 kW to 130.1 kW, and from 98.1 kW to 193.4 kW, with a total increase of 94.8

kW and 95.3 kW, respectively; However, for CB system with PBSHCS, the most prominent increase in exergy loss rates occurs in the heating heat exchanger and the hot gas-mixing tank, with a total increase of 95.3 kW. In the CHP mode, the CB system with CLHCSs ($N = 100$) has higher COPs and lower exergy loss rates than the CB systems with LBSHCS and PBSHCS.

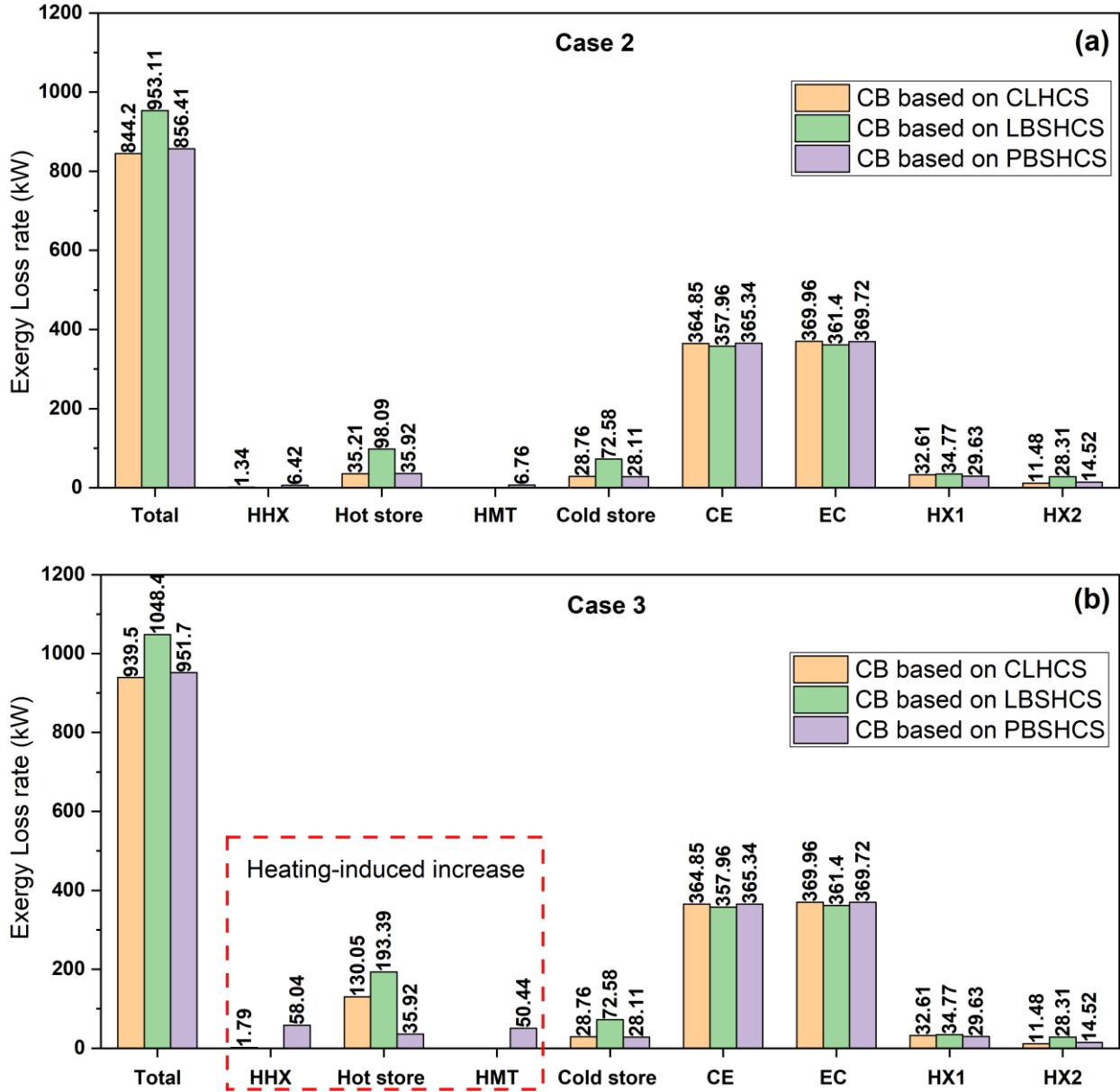


Figure 12. System exergy loss rates of CB systems with CLHCSs ($N = 100$), LBSHCS and PBSHCS in CHP mode: (a) Case 2, and (b) Case 3. Case 2: CB systems heat air with a mass flow rate of 6.2 kg/s from 700 K to 800 K while outputting electricity; Case 3: CB systems heat air with a mass flow rate of 6.2 kg/s from 500 K to 600 K while outputting electricity. HHX: heating heat exchanger, HMT: hot mixing tank.

3.2.3 CCP mode

Two CCP mode cases (Case 4 and Case 6) are discussed in this section to compare the impact of the delivered cold thermal energy grade on the performance of the CB systems. Figure 13 shows the system performance of CB systems with CLHCSs, LBSHCS, and PBSHCS in CCP mode (Case 4 and Case 6). For the CB system with CLHCSs, the COPs range between 49.5% and 57.2%, the electric efficiencies range from 36.3% to 44.0%, and the cold efficiencies are 13.2%; For the CB system with LBSHCS, the COP is 54.5%, the electric efficiency is 41.3%, and the cold efficiency is 13.2%,

which is similar to CB system with CLHCSs with a total stage number of $N = 30$; On the other hand, CB system with PBSHCS has a COP of 56.6%, an electric efficiency of 43.4%, and a cold efficiency of 13.2%, which is comparable to CB system with CLHCSs with a total stage number of $N = 70$. The COPs and electric efficiencies of the three CB systems decrease when they deliver high-grade cold thermal energy externally. CB system with CLHCSs ($N = 100$) have higher COP and electric efficiency compared to CB systems with LBSHCS and PBSHCS, and the cold efficiency is approximately the same.

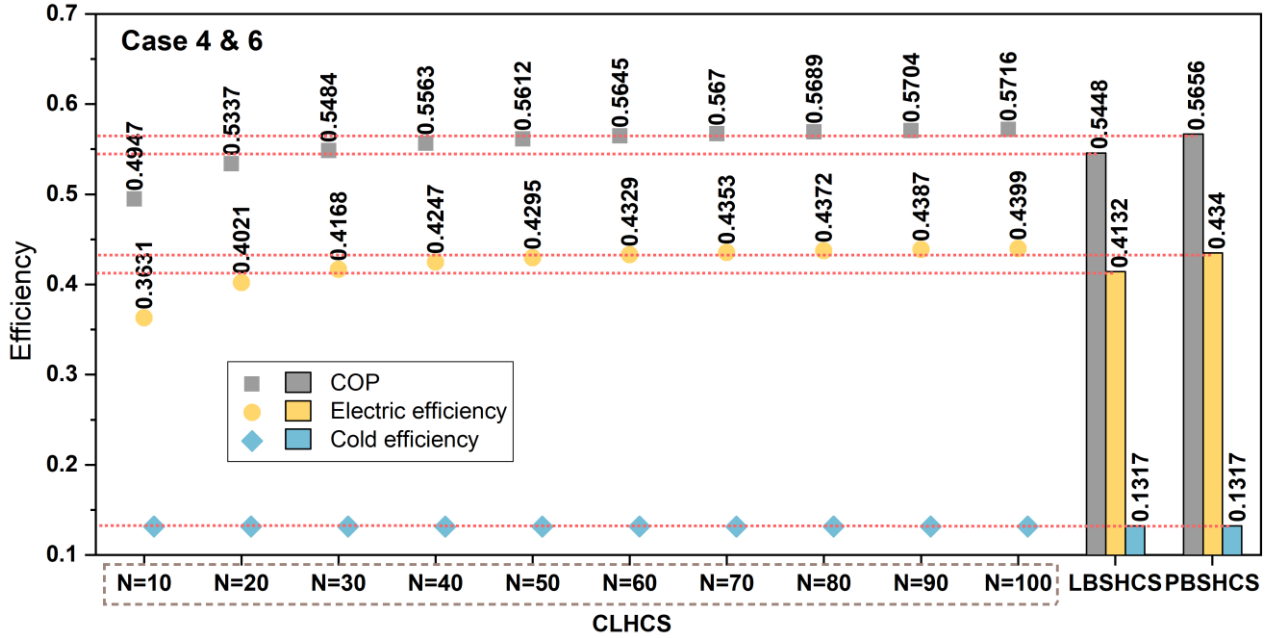


Figure 13. System performance of CB systems with CLHCSs, LBSHCS, and PBSHCS in CCP mode. Case 4: CB systems cool air with a mass flow rate of 6.5 kg/s from 200 K to 150 K while outputting electricity; Case 6: CB systems cool air with a mass flow rate of 6.5 kg/s from 250 K to 200 K while outputting electricity.

Figure 14 (a) and Figure 14 (b) show the system exergy loss rates of CB systems with CLHCSs ($N = 100$), LBSHCS and PBSHCS in CCP mode for Case 4 and Case 6, respectively. The total exergy loss rates of these three in Case 4 are 1160.3 kW, 1229.8 kW and 1177.0 kW respectively, which are greater than the total exergy loss rates in PES mode (Case 1). The primary reason for this increase is the increase in exergy loss rates within HX2, which is attributed to the fact that the temperature of HTF entering the compressor becomes higher after cooling, resulting in an increase in the waste heat discharged in HX2. The total exergy loss rates of these three in Case 6 are 1286.1 kW, 1355.6 kW and 1302.8 kW, respectively, which are larger than those in Case 4, that is, the exergy loss rates of the CB systems are greater when they deliver lower grade cold. For CB systems with CLHCSs and LBSHCS, the most significant increase in exergy loss rates occurs in the cold stores, from 103.0 kW to 232.2 kW, and from 130.1 kW to 255.9 kW, with a total increase of 129.2 kW and 125.8 kW, respectively; However, for CB system with PBSHCS, the most prominent increase in exergy loss rates occurs in the cooling heat exchanger and the cold gas-mixing tank, with a total increase of 125.8 kW. In the CCP mode, the CB system with CLHCSs ($N = 100$) has higher COPs and lower exergy loss rates than the CB systems with LBSHCS and PBSHCS.

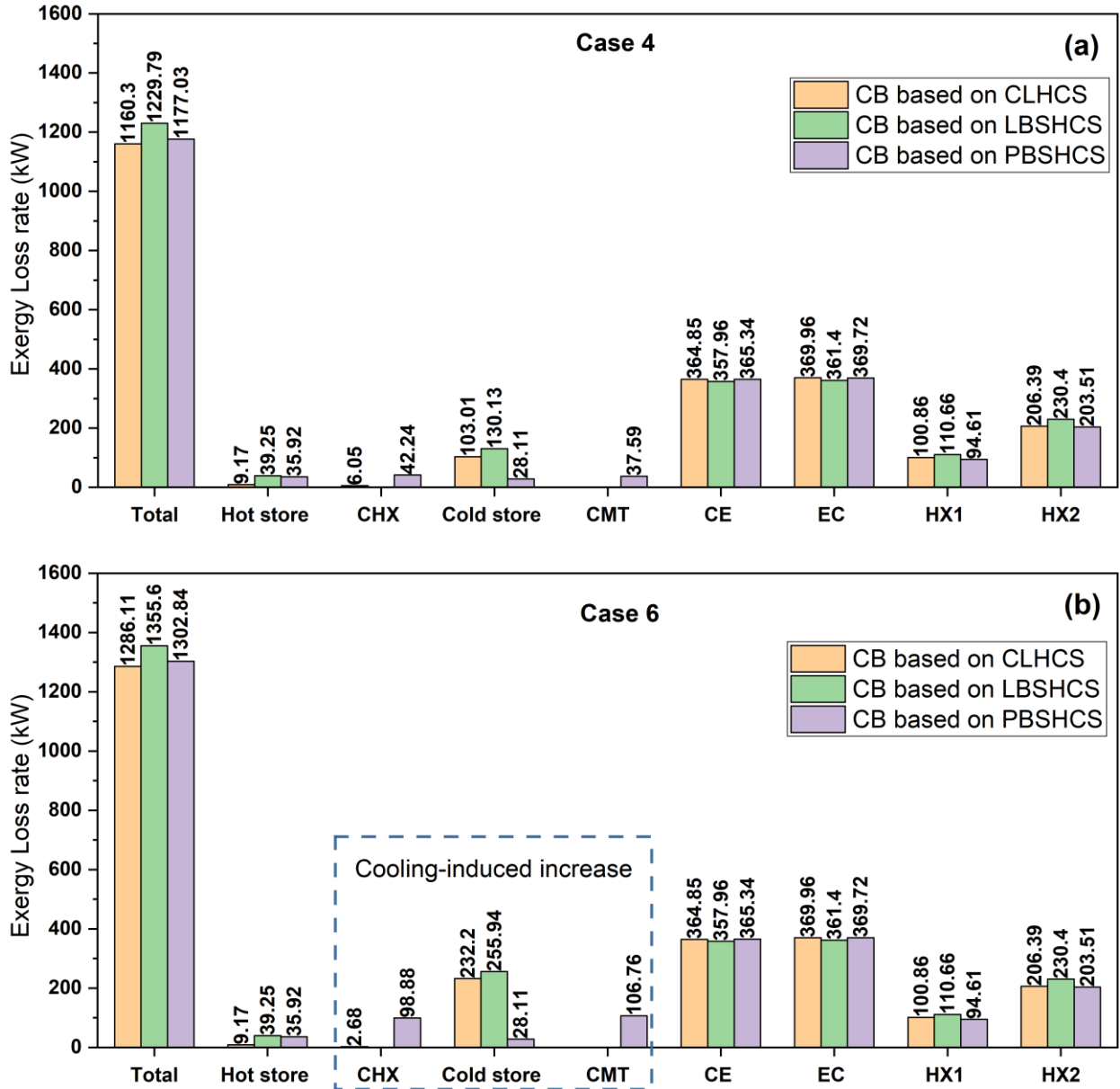


Figure 14. System exergy loss rates of CB systems with CLHCSs ($N = 100$), LBSHCS and PBSHCS in CHP mode: (a) Case 4, and (b) Case 6. Case 4: CB systems cool air with a mass flow rate of 6.5 kg/s from 200 K to 150 K while outputting electricity; Case 6: CB systems cool air with a mass flow rate of 6.5 kg/s from 250 K to 200 K while outputting electricity. CHX: cooling heat exchanger, CMT: cold mixing tank.

3.2.4 CCHP mode

Two CCP mode cases (Case 7 and Case 9) are discussed in this section to compare the impact of the delivered heat and cold thermal energy grade on the performance of the CB systems.

Figure 15 shows the system performance of CB systems with CLHCSs, LBSHCS, and PBSHCS in CCHP mode (Case 7 and Case 9). For CB system with CLHCSs, the COPs range between 62.8% and 69.8%, the electric efficiencies range from 23.3% to 30.2%, the heat efficiencies are 26.4% and the cold efficiencies are 13.2%; For CB system with LBSHCS, the COP is 67.6%, the electric efficiency is 28.0%, the heat efficiency is 26.4%, and the cold efficiency is 13.2%, which is similar to CB system with CLHCSs with a total stage number of $N = 30$; On the other hand, CB system

with PBSHCS has a COP of 69.2%, an electric efficiency of 29.7%, the heat efficiency is 26.4%, and a cold efficiency of 13.2%, which is comparable to CB system with CLHCSs with a total stage number of $N = 60$. CB system with CLHCSs ($N = 100$) have higher COP and electric efficiency compared to CB systems with LBSHCS and PBSHCS when they deliver the same amount of heat and cold.

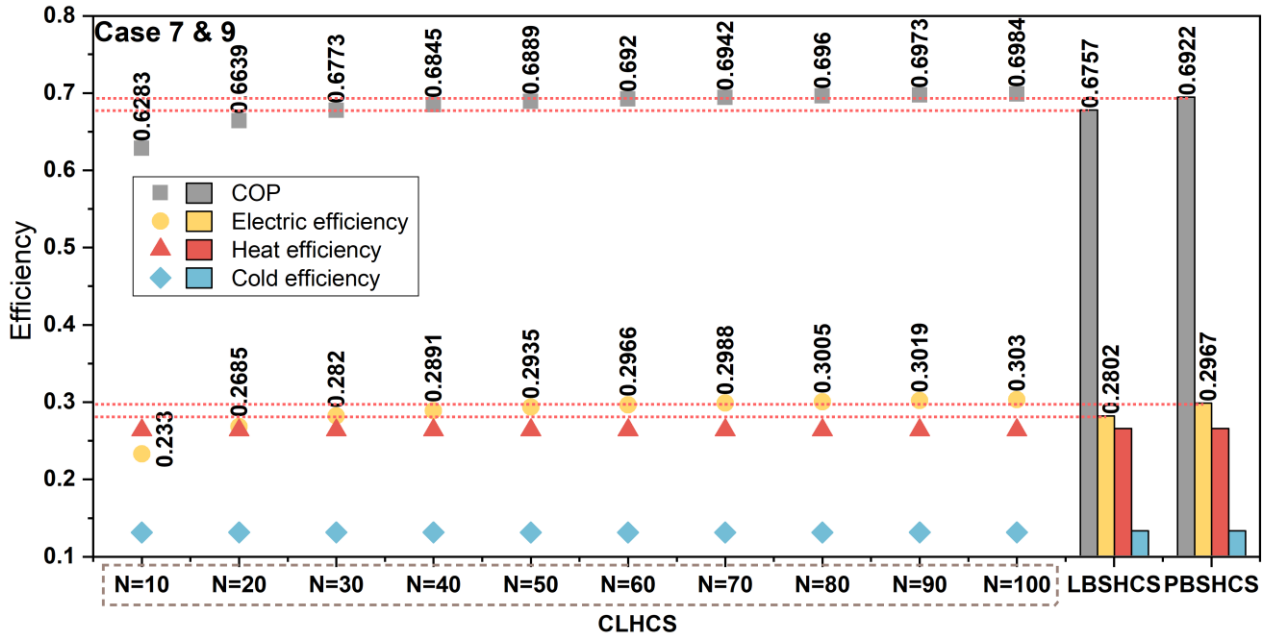


Figure 15. System performance of CB systems with CLHCSs, LBSHCS, and PBSHCS in CCHP mode. Case 7: CB systems heat air with a mass flow rate of 6.2 kg/s from 700 K to 800 K and cool air with a mass flow rate of 6.5 kg/s from 200 K to 150 K while outputting electricity; Case 9: CB systems heat air with a mass flow rate of 6.2 kg/s from 500 K to 600 K and cool air with a mass flow rate of 6.5 kg/s from 250 K to 200 K while outputting electricity.

Figure 16 (a) and Figure 16 (b) show the system exergy loss rates of CB systems with CLHCSs ($N = 100$), LBSHCS and PBSHCS in CCP mode for Case 7 and Case 9, respectively. The total exergy loss rates of these three in Case 7 are 1107.7 kW, 1167.2 kW and 1125.2 kW, respectively, which are greater than the total exergy loss rates in PES mode (Case 1). Compared to Case 1, the exergy loss rates of all components in CB systems, except for HX1, have increased, which is caused by the combined effect of both cooling and heating. The total exergy loss rates of these three in Case 9 are 1328.8 kW, 1388.3 kW and 1346.3 kW, respectively, which are larger than those in Case 7, that is, the exergy loss rates of the CB systems are greater when they deliver lower grade cold and heat.

For CB systems with CLHCSs and LBSHCS, the most significant increase in exergy loss rates occurs in the heat and cold stores, from 22.8 kW to 117.2 kW, and from 103.1 kW to 232.3 kW, respectively; However, for the CB system with PBSHCS, the most prominent increase in exergy loss rates occurs in the heating and cooling heat exchangers and the hot and cold gas-mixing tanks, with a total increase of 221.1 kW. In the CCHP mode, the CB system with CLHCSs ($N = 100$) outperforms CB systems with LBSHCS and PBSHCS in terms of both COPs and exergy loss rates.

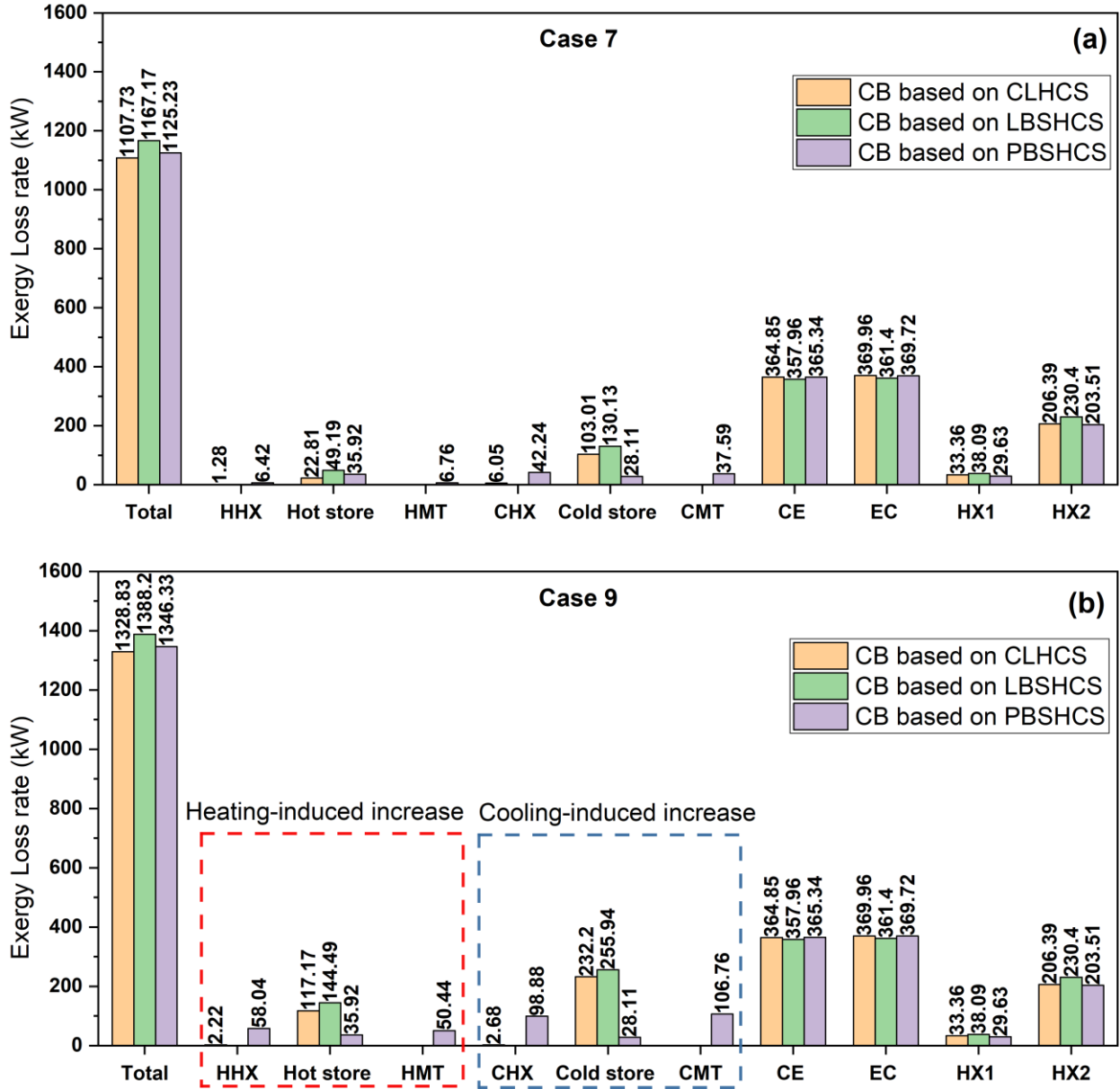


Figure 16. System exergy loss rates of CB systems with CLHCSs ($N = 100$), LBSHCS and PBSHCS in CHP mode: (a) Case 7, and (b) Case 9. Case 7: CB systems heat air with a mass flow rate of 6.2 kg/s from 700 K to 800 K and cool air with a mass flow rate of 6.5 kg/s from 200 K to 150 K while outputting electricity; Case 9: CB systems heat air with a mass flow rate of 6.2 kg/s from 500 K to 600 K and cool air with a mass flow rate of 6.5 kg/s from 250 K to 200 K while outputting electricity. CHX: cooling heat exchanger, CMT: cold mixing tank.

The analysis of the seven cases reveals that CB system with CLHCSs ($N = 100$) exhibits superior performance compared to CB systems with LBSHCS and PBSHCS. Despite the CB system with PBSHCS being able to provide comparable COP and exergy loss rates to the CB system with CLHCSs, it requires additional gas-mixing tanks to maintain the stability of the expander and compressor during cooling and heating. Additionally, the high pressure within PBSHCS may lead to higher costs.

3.3 Flexibility to meet multiple energy demands

Compared with LBSHCS and PBSHCS, CLHCSs split the electro-transformed heat and cold into multiple grades for storage, resulting in a more flexible release of cold and heat. Therefore, the CLHCS-based CB multi-energy management system can deliver multiple types of grades of heat and cold simultaneously, and the cases of the

system delivering two, three and four types of grades of heat or/and cold simultaneously are listed in Table 4, respectively. Taking the CCHP mode as an example, a grade number of two represents that the CB multi-energy management system outputs electricity while heating two streams of hot air from 780 K to 800 K and from 680 K to 700 K, respectively, and cooling two streams of cold air from 160 K to 140 K and from 200 K to 180 K, respectively.

Table 4. The cases of CB system delivering two, three and four grades of heat or/and cold simultaneously

Grade number	Temperature variation of hot air (K)				Temperature variation of cold air (K)			
2	780-800	680-700			160-140	200-180		
3	780-800	680-700	580-600		160-140	200-180	240-220	
4	780-800	680-700	580-600	480-500	160-140	200-180	240-220	280-260

Figure 17 shows the three cases of delivering multi-grade heat or/and cold simultaneously of the CB multi-energy management system shown in Table 4. As can be seen from Figure 17 (a), the COP reaches 66.7% and the total exergy efficiency reaches 56.7% in CHP mode when CB system delivers four types of grades of heat simultaneously. In CCP mode, as shown in Figure 17 (b), the COP of the system is only 47.8% and the total exergy efficiency is 37.3% when CB system delivers four types of grades of cold simultaneously. So, the thermodynamic performance of the system is worse when the CB system delivers cold. In CCHP mode, as shown in Figure 17 (c), the COP of the system is 58.1% and the total exergy efficiency is 37.7% when CB system delivers four types of grades of heat and cold simultaneously.

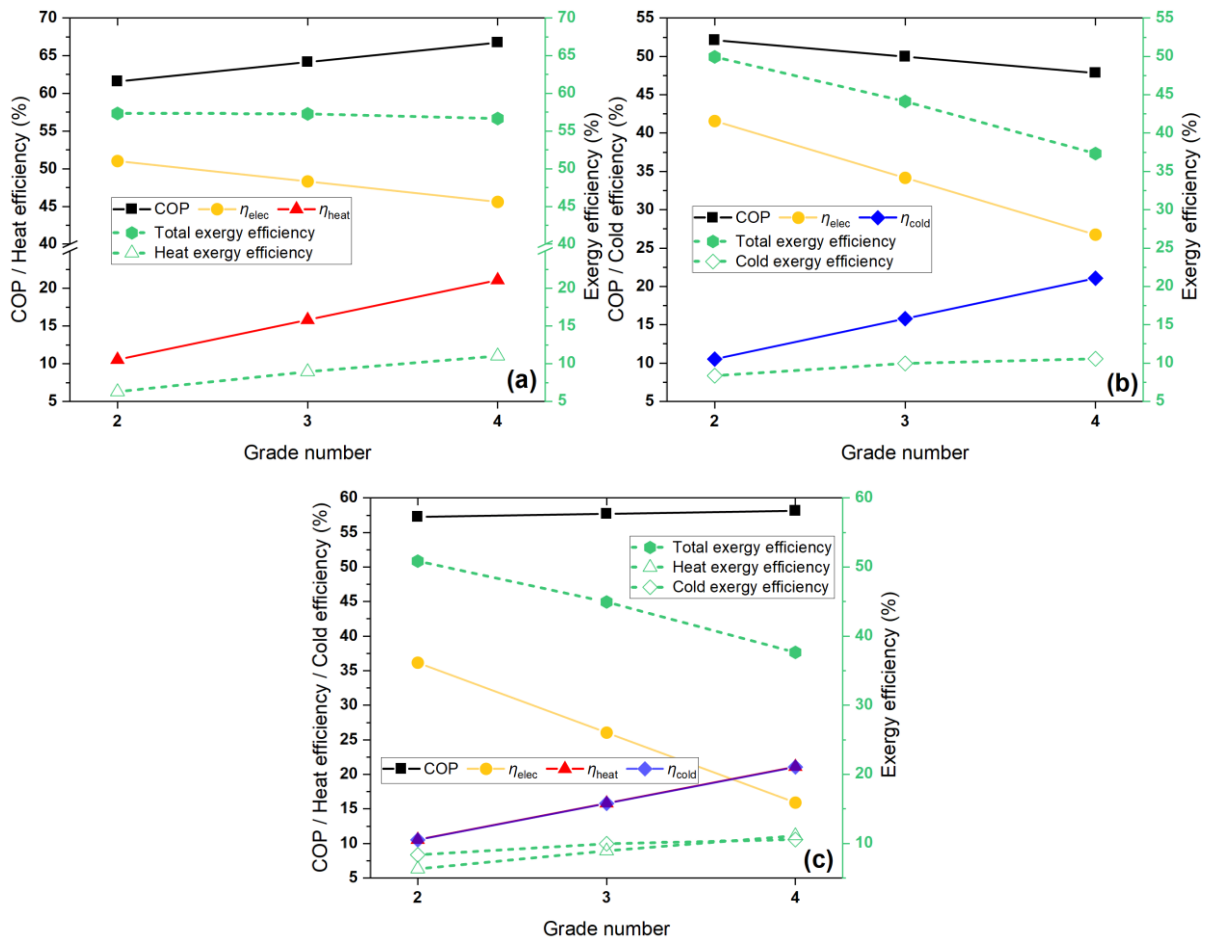


Figure 17. The thermodynamic performance of the CB multi-energy management system in the cases of delivering multi-grade heat or/and cold simultaneously: (a) CHP mode, (b) CCP mode, and (c) CCHP mode.

It can be found that the COP increases from 61.6% to 66.7% when the grade number grows from two to four in CHP mode, while the COP decreases from 52.1% to 47.8% when the grade number grows from two to four in CCP mode. Besides, the total exergy efficiency decreases with the increase of the grade number in CHP, CCP and CCHP mode. The result shows that it is feasible for the CB systems with CLHCSs to deliver multi-grade heat and cold simultaneously, but the quantity and quality of heat and cold should be reasonably matched to ensure the thermodynamic performance of the CB multi-energy management system.

4. Conclusions

In this paper, we propose a concept of Joule-Brayton cycle-based Carnot battery (CB) multi-energy management systems with cascaded latent heat and cold stores (CLHCSs). The effects of key design parameters such as compressor pressure ratio, total stage number, stage area and fluid velocity in store tubes, on the COPs and the exergy efficiencies in different operating modes are explored. The performance of CB systems with CLHCSs, LBSHCS and PBSHCS is compared in different modes and the flexibility of the CB-based multi-energy system is explored. The main conclusions are drawn in detail:

According to the parametric analysis, both coefficient of performance (COP) and exergy efficiency increase significantly first and then tend to be stable with the increase of the total stage number and stage area of the CLHCSs, and increase slightly and then decrease significantly with the increase of the HTF velocity in store tubes in PES, CHP, CCP and CCHP modes. Considering the thermodynamic performance and size of the CB systems, it is recommended that the total stage numbers N_1 and N_2 are around 30, the stage area A_1 in CLHS is around 300 m², the stage area A_2 in CLCS is around 400 m², the HTF velocity in CLHS tubes v_{tube1} should not exceed 5 m/s and the HTF velocity in CLCS tubes v_{tube2} should not exceed 8 m/s, when the CB multi-energy management system work in these four modes. In addition, a small compressor pressure ratio during charging π_{c1} and during discharging π_{c2} can ensure a high COP when the CB multi-energy system is supplying heat and cold to the outside, and it achieves a maximum value of 95.0% when π_{c1} is 5.2 and π_{c2} is 2.1 in CCHP mode. Therefore, π_{c1} and π_{c2} should be selected according to the demand of cold, heat and electricity.

The CB systems with CLHCSs is comparable to CB systems with LBSHCS and PBSHCS in terms of thermodynamic properties. The COPs of CB system with CLHCSs range between 53.0% and 64.0%, the COP of the CB system with LBSHCS is 59.4%, and the CB system with PBSHCS exhibits a COP of 63.7% in PES mode. In addition, for CB system with CLHCSs, the COPs range between 62.8% and 69.8%, the COP of the CB system with LBSHCS is 67.6%, and the CB system with PBSHCS has a COP of 69.2% in CCHP mode.

In terms of flexibility, it is feasible for the CB multi-energy management systems with CLHCSs to deliver electricity and multi-grade heat and cold simultaneously, and the COP is 66.7%, 47.8% and 58.1% in CHP, CCP and CCHP mode, respectively; and the total exergy efficiency is 56.7%, 37.3% and 37.7% in CHP, CCP and CCHP mode, respectively, when CB systems delivers four types of grades of heat and cold simultaneously. The COP of CCHP mode is determined by the quantity and quality of heat and cold, which should be reasonably matched to ensure the thermodynamic performance of the CB multi-energy management system.

The proposed CB multi-energy management system has demonstrated sufficient flexibility in addressing complex energy application scenarios, particularly in situations requiring electricity as well as different grades of heat and cold, which holds great promise for further development into a future smart energy management system for cities or districts.

CRedit authorship contribution statement

Yao Zhao: Writing – review & editing, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Jiaying Huang:**

Writing – original draft, Software, Methodology, Investigation, Formal analysis, Data curation. **Jian Song:** Writing – review & editing, Supervision, Investigation, Formal analysis. **Yulong Ding:** Writing – review & editing, Supervision, Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

A.1. Carnot battery system with liquid-based sensible heat and cold stores

A.1.1. Physical description

A Carnot battery (CB) system with liquid-based sensible heat and cold stores (LBSHCSs) is shown in Figure A1. Liquid store consists of a heat exchanger and two stores with heat storage liquid media at different temperatures. Working fluid (WF) flows between two stores with different temperatures, and absorbs or releases heat through the heat exchanger. In CCHP mode, the stores are selected and matched according to the temperature of the heat/cold demand, and the fluid on the heat/cold demand side is heated/cooled directly in the corresponding store.

Two kinds of WFs for heat storage and cold storage are deployed to cover the wide operating temperature range. Therefore, the liquid-based sensible heat store (LBSHS) consists of two high-temperature stores (S1, S3), two low-temperature stores (S2, S4), and two sets of heat exchangers (HX3, HX4); The liquid-based sensible cold store (LBSCS) consists of two high-temperature stores (S6, S8), two low-temperature stores (S5, S7), and two sets of heat exchangers (HX5, HX6). During the operation, the temperature of WF within S2 is equal to that of WF within S3, and the temperature of WF within S6 is equal to that of WF within S7.

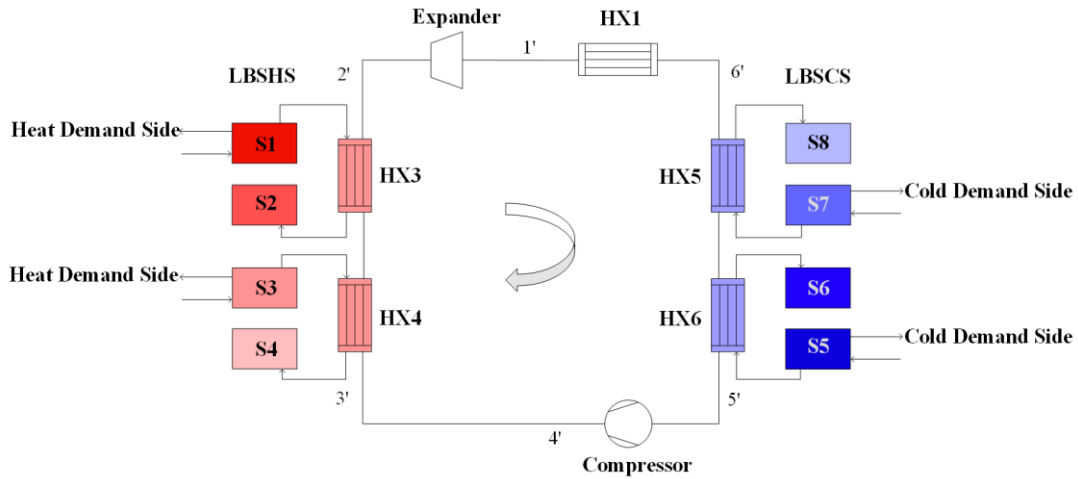


Figure A1. Schematic diagram of a Carnot battery system with liquid-based sensible heat and cold stores

Optimisation of the WF temperatures at the outlets of the stores S2, S4, S6 and S8 during charging is required to find the maximum roundtrip efficiency. The optimisation is carried out with COP as the objective function by using GA optimisation algorithm, and the lower and upper limits of the optimisation parameters are listed in Table A1.

Table A1. Values of optimised parameters in CB system with liquid-based sensible heat and cold stores.

<i>Optimised parameters</i>	<i>Lower bound</i>	<i>Upper bound</i>
Pressure ratio during charge, π_{chg}	1.0	15.0
Pressure ratio during discharge, π_{dis}	1.0	15.0
Temperature of S2 storage material at the outlet of the S2 during charge [K]	400	800
Temperature of S4 storage material at the outlet of the S4 during charge [K]	298	500
Temperature of S6 storage material at the outlet of the S6 during charge [K]	123	250
Temperature of S8 storage material at the outlet of the S8 during charge [K]	200	298

A.1.2. Mathematical models

Heat transfer effectiveness ε is [49]:

$$\varepsilon_{HX} = \frac{(\dot{m}c_p)_h (T_{h, in} - T_{h, out})}{(\dot{m}c_p)_{\min} (T_{h, in} - T_{c, out})} = \frac{(\dot{m}c_p)_c (T_{c, out} - T_{c, in})}{(\dot{m}c_p)_{\min} (T_{h, in} - T_{c, in})} \quad (A1)$$

where $\dot{m}$ and c_p are the mass flow rate and the specific heat of the WF, respectively. $T_{h, in}$ and $T_{c, in}$ are the inlet temperatures of the hot-side and cold-side fluids, respectively, and $T_{h, out}$ and $T_{c, out}$ are the outlet temperatures of the hot-side and cold-side fluids, respectively. All heat transfer effectiveness ε is taken as 0.95 to simplify the model.

Then the heat transfer rate of the LBSHCS $\dot{Q}_{LB}$ is:

$$\dot{Q}_{LB} = \dot{m}c_p (T_{HX, in} - T_{HX, out}) \quad (A2)$$

where $T_{HX, in}$ and $T_{HX, out}$ are the WF temperatures at the inlet and outlet of the LBSHCS, respectively.

The exergy changing rate $\dot{E}_{LB}$ within the LBSHCS when WF changes from state 1 to state 2:

$$\dot{E}_{LB} = \dot{m}c_p [T_1 - T_2 - T_0 \ln(T_1 / T_2)] \quad (A3)$$

Where T_1 and T_2 are the WF temperatures at state 1 and state 2, respectively, and T_0 represents the environment temperature.

A.2. Carnot battery system with packed-bed sensible heat and cold stores

A.2.1. Physical description

Figure A2 shows schematic diagram of a CB system with packed-bed sensible heat and cold stores. The particles are uniformly arranged in the heat and cold stores and exchange heat directly with the HTF to storage and release heat/cold.

When there is cold demand, HTF flows out from PBSCS, part of HTF flows into cooling heat exchanger CHX for cold supply, another part of HTF flows into cold mix tank to mix with the HTF after cold supply, and then enter into the compressor; When there is heat demand, HTF flows out from PBSHS, part of HTF flows into heating heat exchanger HHX for heat supply, another part of HTF flows into hot mix tank to mix with HTF after heat supply, and then enters into the expander.

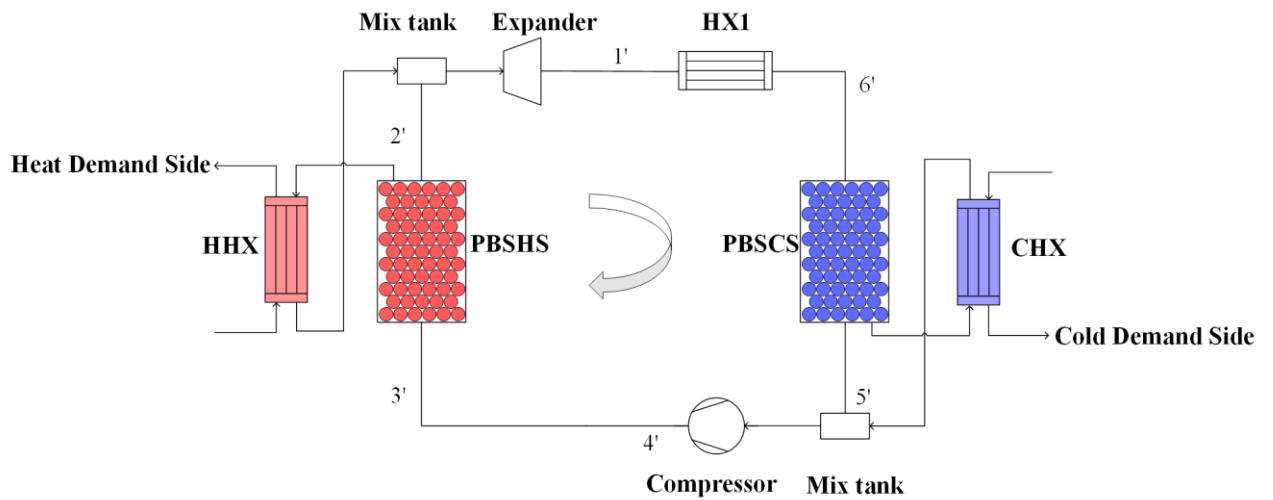


Figure A2. Schematic diagram of CB system with packed-bed sensible heat and cold stores

The compressor pressure ratios during charging and discharging need to be optimised to obtain the maximum roundtrip efficiency. The optimisation is carried out with COP as the objective function by using GA optimisation

algorithm, and the lower and upper limits of the optimisation parameters are displayed in Table A2.

Table A2. Values of optimised parameters in CB system with packed-bed sensible heat and cold stores.

<i>Optimised parameters</i>	<i>Lower bound</i>	<i>Upper bound</i>
Pressure ratio during charge, π_{chg}	1.0	15.0
Pressure ratio during discharge, π_{dis}	1.0	15.0

A.2.2. Mathematical models

In this study, the steady heat transfer model for the PBSHCS proposed by Guo et al.[50] is used. It is assumed that the hot HTF and cold HTF exchange heat directly with each other through a simple heat transfer model, in which the thermal storage material is ignored.

The heat transfer rate of the PBSHCS $\dot{Q}_{\text{PB}}$ is:

$$\dot{Q}_{\text{PB}} = \dot{m}c_p (T_{\text{PB, in}} - T_{\text{PB, out}}) \quad (\text{A4})$$

where $\dot{m}$ and c_p are the mass flow rate and the specific heat of the HTF, respectively. $T_{\text{PB, in}}$ and $T_{\text{PB, out}}$ are the HTF temperatures at the inlet and outlet of the PBSHCS, respectively.

The exergy loss rate $\dot{I}_{\text{PB}}$ is:

$$\dot{I}_{\text{PB}} = \dot{m}c_p T_0 \ln \frac{(T_{\text{hot}} - \delta T)(T_{\text{cold}} + \delta T)}{T_{\text{hot}} T_{\text{cold}}} \quad (\text{A5})$$

where T_0 represents the environment temperature, T_{hot} is the hot HTF temperature at the inlet of the PBSHCS during charging, T_{cold} is the cold HTF temperature at the inlet of the PBSHCS during discharging, and δT is the average temperature difference between hot HTF (during charging) and cold HTF (during discharging) at the axial local position of the PBSHCS, which is 10 K [50].

References:

- [1] Agency, I.R.E., Renewable Energy Statistics 2023. 2023, International Renewable Energy Agency.
- [2] Hameer, S. and J.L. van Niekerk, A review of large-scale electrical energy storage. International Journal of Energy Research, 2015. 39(9): p. 1179-1195.
- [3] He, W. and J. Wang, Optimal selection of air expansion machine in Compressed Air Energy Storage: A review. Renewable and Sustainable Energy Reviews, 2018. 87: p. 77-95.
- [4] Mouli-Castillo, J., et al., Inter-seasonal compressed-air energy storage using saline aquifers. Nature Energy, 2019. 4(2): p. 131-139.
- [5] Holland-Cunz, M.V., et al., Three-dimensional composite Li metal anode by simple mechanical modification for high-energy batteries. Frontiers in Energy, 2023. 17: p. 569-584.
- [6] Mennel, J.A. and D. Chidambaram, A review on the development of electrolytes for lithium-based batteries for low temperature applications. Frontiers in Energy, 2023. 17(1): p. 43-71.
- [7] Vecchi, A., et al., Carnot Battery development: A review on system performance, applications and commercial state-of-the-art. Journal of Energy Storage, 2022. 55: p. 105782.
- [8] She, X., et al., Flexible integration of liquid air energy storage with liquefied natural gas regasification for power generation enhancement. Applied energy, 2019. 251: p. 113355.
- [9] Sharma, S. and M. Mortazavi, Pumped thermal energy storage: A review. International Journal of Heat and Mass Transfer, 2023. 213: p. 124286.

-
- [10] Steinmann, W., Thermo-mechanical concepts for bulk energy storage. *Renewable and Sustainable Energy Reviews*, 2017. 75: p. 205-219.
- [11] Desrues, T., et al., A thermal energy storage process for large scale electric applications. *Applied Thermal Engineering*, 2010. 30(5): p. 425-432.
- [12] McTigue, J.D. and A.J. White, A comparison of radial-flow and axial-flow packed beds for thermal energy storage. *Applied Energy*, 2018. 227: p. 533-541.
- [13] White, A.J., J.D. McTigue and C.N. Markides, Analysis and optimisation of packed-bed thermal reservoirs for electricity storage applications. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 2016. 230(7): p. 739-754.
- [14] White, A.J., Loss analysis of thermal reservoirs for electrical energy storage schemes. *Applied Energy*, 2011. 88(11): p. 4150-4159.
- [15] McTigue, J.D., C.N. Markides and A.J. White, Performance response of packed-bed thermal storage to cycle duration perturbations. *Journal of Energy Storage*, 2018. 19: p. 379-392.
- [16] White, A., J. McTigue and C. Markides, Wave propagation and thermodynamic losses in packed-bed thermal reservoirs for energy storage. *Applied Energy*, 2014. 130: p. 648-657.
- [17] McTigue, J.D., A.J. White and C.N. Markides, Parametric studies and optimisation of pumped thermal electricity storage. *Applied Energy*, 2015. 137: p. 800-811.
- [18] White, A., G. Parks and C.N. Markides, Thermodynamic analysis of pumped thermal electricity storage. *Applied Thermal Engineering*, 2013. 53(2): p. 291-298.
- [19] Chen, L.X., et al., Thermodynamic analysis of a High Temperature Pumped Thermal Electricity Storage (HT-PTES) integrated with a parallel organic Rankine cycle (ORC). *Energy Conversion and Management*, 2018. 177: p. 150-160.
- [20] Wang, L., et al., Cyclic transient behavior of the Joule–Brayton based pumped heat electricity storage: Modeling and analysis. *Renewable and Sustainable Energy Reviews*, 2019. 111: p. 523-534.
- [21] Wang, L., et al., Unbalanced mass flow rate of packed bed thermal energy storage and its influence on the Joule-Brayton based Pumped Thermal Electricity Storage. *Energy Conversion and Management*, 2019. 185: p. 593-602.
- [22] Zhang, H., et al., Parametric optimisation and thermo-economic analysis of Joule–Brayton cycle-based pumped thermal electricity storage system under various charging–discharging periods. *Energy*, 2023. 263: p. 125908.
- [23] Albert, M., et al., Operation and performance of Brayton Pumped Thermal Energy Storage with additional latent storage. *Applied Energy*, 2022. 312: p. 118700.
- [24] Zhang, H., et al., Technical and economic analysis of Brayton-cycle-based pumped thermal electricity storage systems with direct and indirect thermal energy storage. *Energy*, 2022. 239: p. 121966.
- [25] Laughlin, R.B., Pumped thermal grid storage with heat exchange. *Journal of renewable and sustainable energy*, 2017. 9(4).
- [26] Zhao, Y., et al., Advanced exergy analysis of a Joule-Brayton pumped thermal electricity storage system with liquid-phase storage. *Energy Conversion and Management*, 2021. 231: p. 113867.
- [27] Gonzalez-Ayala, J., et al., Multicriteria optimization of Brayton-like pumped thermal electricity storage with liquid media. *Journal of Energy Storage*, 2021. 44: p. 103242.
- [28] Yang, H., J. Wu and X. Du, Thermo-economic analysis and multi-objective optimization of solar aided pumped thermal electricity storage system. *Journal of Energy Storage*, 2023. 70: p. 107994.
- [29] Yang, H., et al., Dynamic performance for discharging process of pumped thermal electricity storage with reversible Brayton cycle. *Energy*, 2023. 263: p. 125930.
- [30] McTigue, J.D., et al., Techno-economic analysis of recuperated Joule-Brayton pumped thermal energy storage.

Energy Conversion and Management, 2022. 252: p. 115016.

- [31] Shi, X., et al., Variable load modes and operation characteristics of closed Brayton cycle pumped thermal electricity storage system with liquid-phase storage. *Renewable Energy*, 2023. 203: p. 715-730.
- [32] Zhao, Y., et al., Medium- and high-temperature latent and thermochemical heat storage using metals and metallic compounds as heat storage media: A technical review. *Applied Energy*, 2020. 280: p. 115950.
- [33] Ge, Y.Q., Y. Zhao and C.Y. Zhao, Transient simulation and thermodynamic analysis of pumped thermal electricity storage based on packed-bed latent heat/cold stores. *Renewable Energy*, 2021. 174: p. 939-951.
- [34] Xue, X.J. and C.Y. Zhao, Transient behavior and thermodynamic analysis of Brayton-like pumped-thermal electricity storage based on packed-bed latent heat/cold stores. *Applied Energy*, 2023. 329: p. 120274.
- [35] Saghafifar, M., M.A. Schnellmann and S.A. Scott, Chemical looping electricity storage. *Applied Energy*, 2020. 279: p. 115553.
- [36] Saghafifar, M. and S.A. Scott, The use of high decomposition temperature materials for chemical looping electricity storage. *Chemical Engineering Journal*, 2021. 423: p. 128789.
- [37] Pang, J., et al., Thermodynamic analysis and parameter optimization of a Chemical Looping Electricity Storage system. *Journal of Energy Storage*, 2022. 55: p. 105832.
- [38] Zhao, Y., et al., Carnot battery for energy storage: Advancements and challenges. *Green Energy and Resources*, 2023. 1(4): p. 100048.
- [39] Mancarella, P., MES (multi-energy systems): An overview of concepts and evaluation models. *Energy*, 2014. 65: p. 1-17.
- [40] Mohammadi, M., et al., Energy hub: From a model to a concept – A review. *Renewable and Sustainable Energy Reviews*, 2017. 80: p. 1512-1527.
- [41] Frate, G.F., et al., Thermally integrated pumped thermal energy storage for multi-energy districts: Integrated modelling, assessment and comparison with batteries. *Journal of energy storage*, 2023. 61: p. 106734.
- [42] Zhang, H., et al., Combined cooling, heating, and power generation performance of pumped thermal electricity storage system based on Brayton cycle. *Applied Energy*, 2020. 278: p. 115607.
- [43] Zhao, Y., et al., Experimental study on the thermodynamic performance of cascaded latent heat storage in the heat charging process. *Energy*, 2018. 157: p. 690-706.
- [44] Zhao, Y., et al., Thermodynamic investigation of latent-heat stores for pumped-thermal energy storage. *Journal of Energy Storage*, 2022. 55: p. 105802.
- [45] Zhao, Y., et al., Thermo-economic assessments of pumped-thermal electricity storage systems employing sensible heat storage materials. *Renewable Energy*, 2022. 186: p. 431-456.
- [46] Incropera, F.P., *Fundamentals of heat and mass transfer*. Wiley, 2006.
- [47] Tay, N.H.S., M. Belusko and F. Bruno, An effectiveness-NTU technique for characterising tube-in-tank phase change thermal energy storage systems. *Applied Energy*, 2012. 91(1): p. 309-319.
- [48] Michels, H. and R. Pitz-Paal, Cascaded latent heat storage for parabolic trough solar power plants. *Solar Energy*, 2007. 81(6): p. 829-837.
- [49] Shah, R.K. and D.P. Sekulic, *Fundamentals of Heat Exchanger Design*. 2002: Fundamentals of Heat Exchanger Design.
- [50] Guo, H., et al., Thermodynamic Analysis of Packed Bed Thermal Energy Storage System. *Journal of Thermal Science*, 2020. 29(2): p. 445-456.

Highlights

- A concept of multi-energy management systems based on Carnot battery is proposed.

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- Effects of key parameters on system performance are evaluated.
 - A coefficient of performance of 95% can be achieved in CCHP mode.
 - Cascaded latent stores promote flexibility in provision of cooling, heating and electricity.