

THE FORMATION OF A SPHERICAL SHOCK WAVE
BY AN ACCELERATING CONTACT SURFACE

A Thesis Submitted for the Degree of
Doctor of Philosophy in the
Faculty of Engineering of the
University of London

by

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October 1974

ABSTRACT

An extension of standard to spherically symmetric flow in the analysis of shock tube performance is used to explain the blast wave developments that occur on the release of a gas from a cylindrical vessel into the free atmosphere following the rupture of a bursting disc. The characteristics of such a flow are essentially an atmospheric compression by an accelerating contact surface followed by the formation of a shock wave and such occurrences find their place in industry associated with pressure relief mechanisms and unconfined vapour cloud explosions. The scaling of these explosive forms of energy release is discussed and has been investigated with a view to predicting the hazards of a potentially explosive situation from small scale experimentation by a variation of the main venting parameters. The parameters considered here are the disc bursting pressure, the area available for venting and the type of gas vented.

The process of shock wave formation is examined by both computer simulation and photographic techniques, each showing similar flow developments under the same conditions. Quantitative analysis of the contact surface and shock wave trajectories have allowed a comparison of the scaled results to be made with a point source blast wave solution. In this respect the effectiveness of two scaling lengths is examined, one represented by the analytical expression below,

$$\epsilon_1 = 1.28 \left[\left(\frac{2}{\gamma + 1} \right)^b \frac{c_o}{\bar{c}} \frac{P_o}{\bar{P}} A_2 \right]^{\frac{1}{2}}$$

$$\text{where } b = (3\gamma - 1)/2(\gamma - 1)$$

and the other (ϵ_2) determined empirically from a knowledge of the energy release rate. The ranges over which various parameters have been investigated are given below together with evaluations of scaling lengths.

Gas	Disc Bursting Pressure (psig)	Orifice Diameter (in)	Scaling Lengths	
			$\epsilon_1(\text{ft})$	$\epsilon_2(\text{ft})$
Air	70	1	0.2948	0.2064
	to	to	to	to
	250	4	1.1271	0.8646
Helium	140	1	0.4666	0.2135
	to	to	to	to
	300	4	1.9455	1.0227

Absolute values of blast wave parameters cannot be predicted for all situations using these scaling lengths but they prove to be effective in predicting the relative changes brought about by a variation in the venting conditions, the plots of the shock strength scaled distance dependence merging into a single line after the shock development stages are past. These early stages are associated with the blast wave energy injection time and it is shown from experimental evidence of gas flow within the vessel that the blast wave is fully developed once steady state discharge through the orifice has been achieved. The strength of the wave is found to be related to the energy released from the vessel over this period.

ACKNOWLEDGEMENTS

The author would like to express his gratitude to the following people without whose assistance this work could not have been accomplished.

Professor A.R. Ubbelohde, F.R.S. for the provision of the facilities to pursue this research.

Dr. G. Munday for his personal supervision, guidance and encouragement throughout this work.

Mr. L. Tyley and his staff, particularly Mr. R. Wood, for their assistance and advice on the construction and use of the electronic equipment.

Mr. J. Oakley and the engineering workshop staff for their help in the development and construction of much of the apparatus.

Finally the Science Research Council for the provision of grants for both maintainance and equipment.

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CHAPTER ONE

INTRODUCTION

1.1 Shock Waves

Up to the early 1800's any disturbance propagating through air was generally regarded as an acoustic wave moving with the speed of sound. It was Stokes (1848) who first reported the existence of discontinuities moving with speeds far greater than that of sound. By using Poisson's integral describing the one-dimensional motion of an acoustic wave in an ideal elastic fluid, he showed how the leading edge of the wave gradually became steeper and eventually discontinuous. Stokes, however, doubted whether such a discontinuous motion could exist in a real fluid and argued that dissipative effects, such as viscosity and heat conduction, would prevent its formation and if for an instant, discontinuous motion was present, it would be rendered continuous again almost immediately. Mathematical treatment of the propagation of such disturbances or shock waves was later made by Riemann (1860), Rankine (1870), Hugoniot (1887), Chapman (1899), Jouguet (1902, 1917) and Taylor (1910). In particular Riemann refined the theory of shock transition but made the incorrect assumption that the process was isentropic and it was left to Rankine to point out this defect and show that shock transition complies with the principle of conservation of energy but not of entropy. To confirm this Hugoniot pointed out that, in the absence of viscosity and heat conduction, conservation of energy across a shock wave automatically implies a change in entropy.

A shock wave as opposed to a sound wave was later described as a disturbance which displaced particles along the direction of propagation, the velocity of the particles being such that it cannot be neglected in comparison with the disturbance velocity. The equations which govern the

changes in such a wave are given by the Rankine - Hugoniot relations which, for present purposes, are most conveniently written in the form below and are fully derived in Appendix A.

$$\frac{u_1 - u_0}{c_0} = \frac{2}{\gamma + 1} \left\{ M - \frac{1}{M} \right\} \quad (1.1)$$

$$\left(\frac{c_1}{c_0} \right)^2 = \frac{T_1}{T_0} = 1 + 2 \frac{(\gamma - 1)}{(\gamma + 1)^2} \left\{ \gamma M^2 - \frac{1}{M^2} - (\gamma - 1) \right\} \quad (1.2)$$

$$\frac{\rho_1}{\rho_0} = \left\{ 1 - \frac{2}{\gamma + 1} \left\{ 1 - \frac{1}{M^2} \right\} \right\}^{-1} \quad (1.3)$$

$$\frac{P_1}{P_0} = 1 + \frac{2\gamma}{\gamma + 1} (M^2 - 1) \quad (1.4)$$

$$\frac{s_1 - s_0}{c_v} = \ln \left\{ 1 + \frac{2\gamma}{\gamma + 1} (M^2 - 1) \right\} + \gamma \ln \left\{ 1 - \frac{2}{\gamma + 1} \left\{ 1 - \frac{1}{M^2} \right\} \right\} \quad (1.5)$$

1.2 The Shock Tube

The first practical use of shock waves was made when Vielle (1899) developed the shock tube for studying high speed flows of gases. The shock was produced by the rupturing of a diaphragm which separated a high pressure gas in one section of the tube from a gas at a lower pressure in the other. Velocities were measured by mechanical means up to about twice the speed of sound in a 20 ft. long and 22 mm. diameter tube using a diaphragm pressure ratio of 27.

There was a lapse of 40 years before interest in shock tubes was revived by Payman and Shepherd (1946) who set out to verify the theoretical relationships governing the motion of shock waves. They used a wave-speed

camera to obtain continuous Schlieren records of the disturbances produced when the diaphragm ruptures under a variety of conditions, the main experimental changes being the use of different gases, the pressure ratio across the diaphragm and the length of each section of the shock tube.

The classical description given of the one-dimensional gas motion when a diaphragm ruptures assumes that it is completely and instantaneously removed and a shock wave is formed immediately as shown schematically in Figure 1.1

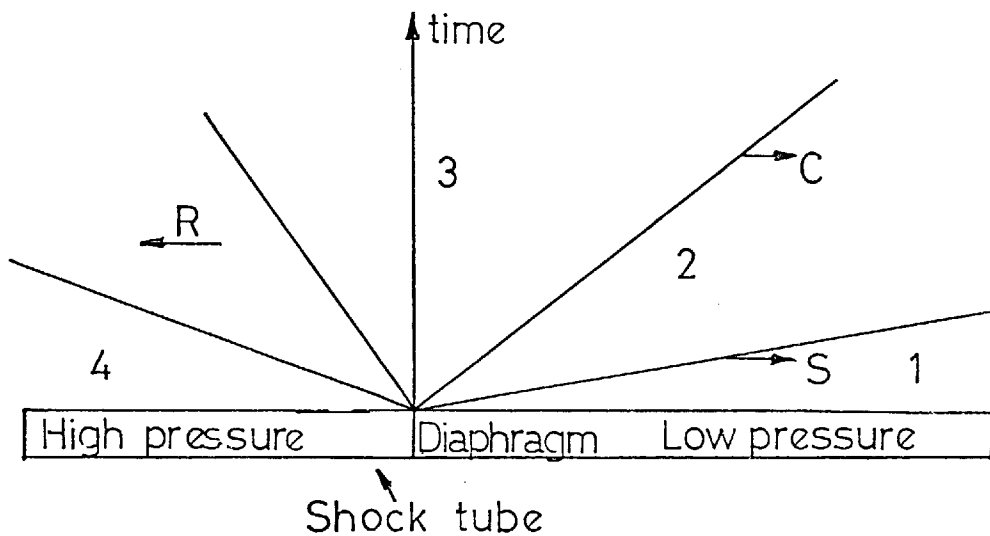


Figure 1.1 Wave form in an ideal shock tube

The shock wave(S) propagates into the undisturbed gas, region (1), with constant supersonic velocity which is determined by the type of gas used in the tube and the pressure ratio across the diaphragm. The gas in region (2) is that which has undergone an irreversible adiabatic compression on passing through the shock wave and is now travelling in the same direction and is assumed to have a uniform velocity and thermodynamic

state. It is followed by the contact surface (C) which separates the shocked gas from the expanded high pressure gas. The gas velocity and pressure either side of the contact surface are equal but there is a temperature discontinuity due to the contrasting thermodynamic processes through which each region has passed. The properties of the gas in region (3) are achieved by the undisturbed high pressure gas, region (4), accelerating through an expansion or rarefaction wave (R) which is centred due to the earlier assumption of instantaneous diaphragm removal.

Ideally the contact surface is completely planar and there is zero mass flux between each region it separates. Because of this property no alterations to the flow pattern will be introduced if the contact surface is replaced with a solid boundary which follows the same path.

The problem of a solid boundary or piston moving within a gas is dealt with by Courant and Friedrichs (1948) for the one-dimensional case by an investigation of the method of characteristics for solving the differential equations of motion for a compressible flow. The problems of a rarefaction wave and a compression wave are treated as separate entities, the first being an example of a piston retreating from a gas which is initially at rest and the second an example of a piston advancing into a gas under the same conditions. The disturbances set up in either case can best be illustrated by a diagram showing the characteristic or disturbance paths.

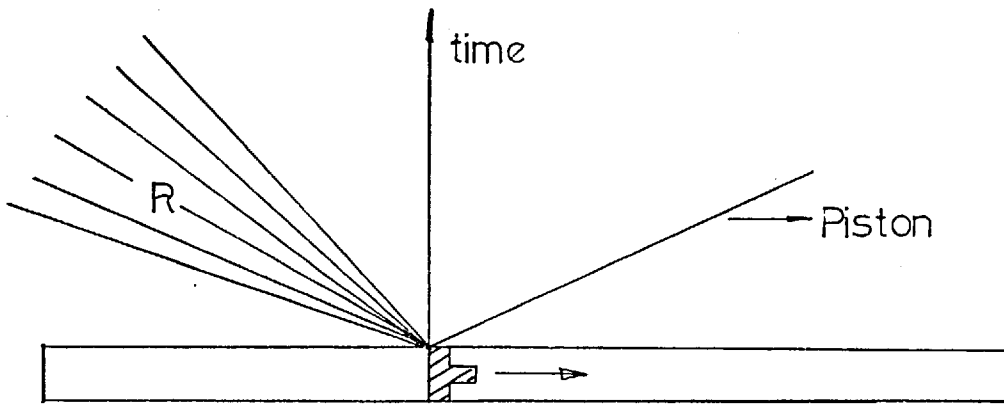


Figure 1.2a Wave form for a piston retreating with constant velocity

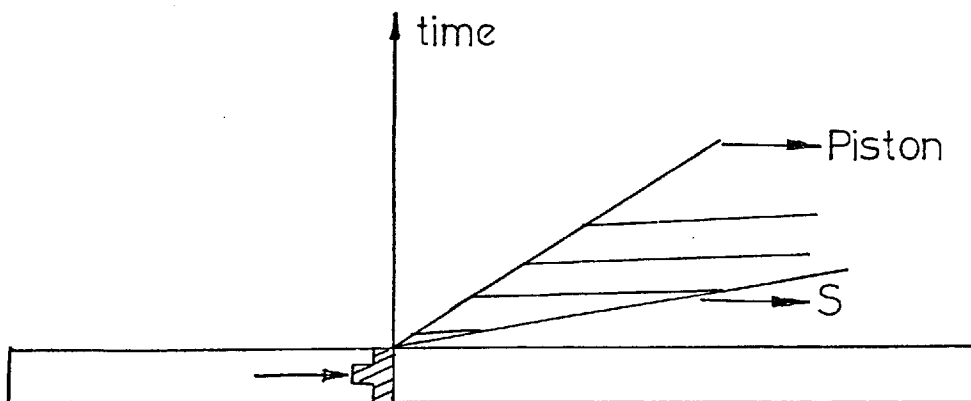


Figure 1.2b Wave form for a piston advancing with constant velocity

Figures 1.2a and 1.2b show both a retreating and advancing piston which have attained a steady velocity instantaneously and set up a network of disturbances travelling at the local speed of sound relative to the gas

motion. In the case of the rarefaction wave the first and last disturbances leave the piston surface at the same time and form what is known as a centred rarefaction wave. The advancing piston forms a shock wave immediately which proceeds with a constant supersonic velocity and is followed by a set of disturbances travelling through a uniform gas. Such a motion cannot exist in real situations as a piston will always accelerate from zero velocity to its maximum over a finite period of time and generate a non-centred rarefaction wave and an initially unsteady, shock free gas motion ahead of the piston as illustrated in figure 1.3 a and b.

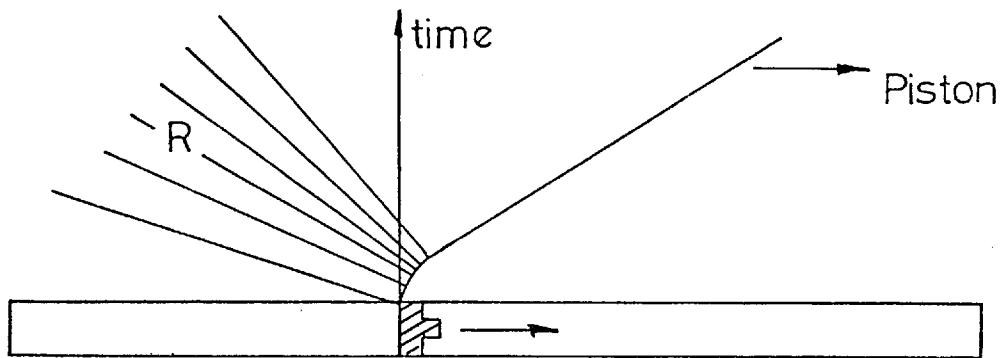


Figure 1.3a Wave form for a piston retreating with an initial acceleration

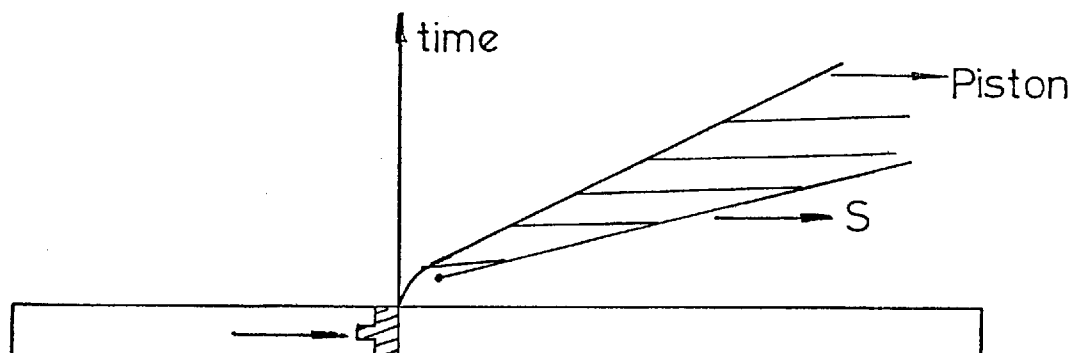


Figure 1.3b Wave form for a piston advancing with an initial acceleration

In both cases of piston motion the rarefaction wave is adjacent to a region of steady state and for one-dimensional flow it is known as a simple wave for which there exists an analytical solution over the whole flow field. Each disturbance or characteristic path has its own particular value of u and c , the particle velocity and sonic velocity respectively, which are maintained throughout the flow subject to it remaining a simple wave. The path of the disturbance, $x = (u + c)t$ is thus represented by a straight line.

The qualitative statements pertaining to rarefaction waves also apply to compression waves, but in this case the disturbance paths converge due to the increasing gas and sonic velocities at the piston surface. The simple wave solution for a compression only exists for a certain time as the converging characteristics will intersect and form an envelope on which the gas properties would conflict as shown in figure 1.4

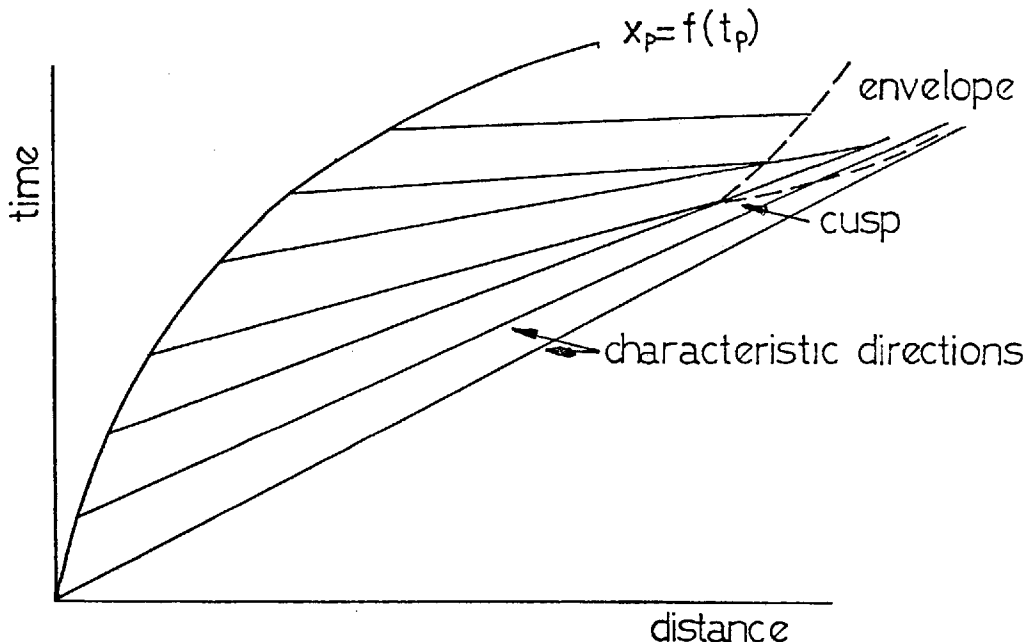


Figure 1.4 Converging characteristic directions for
an accelerating piston

It is at the cusp of this envelope that the solution first begins to be invalid and the problem can only be overcome by introducing a discontinuity or shock wave into the flow.

The solutions of interest made by Courant and Friedrichs (1948) are for the one-dimensional plane flows associated with the two different piston motions given by the following expressions:

$$x_p = a_2 t_p^2 \quad (1.6)$$

$$x_p = a_3 t_p^3 \quad (1.7)$$

The equation for the resulting envelope of characteristics for each case has been derived and a simple extension of this theory gives the coordinates of its cusp, that is the point at which a discontinuity first forms. Equation (1.6) represents a motion with constant acceleration and the corresponding expression for the envelope is:

$$t = \frac{\gamma^2 a_2 t_p + c_o}{(\gamma + 1) a_2} \quad (1.8)$$

The minimum value of t on this envelope is given when $t_p = 0$, and is thus on the line representing the first disturbance emitted by the piston at a value given by:

$$t_s = \frac{c_o}{(\gamma + 1) a_2} \quad (1.9)$$

The second piston motion (eq. 1.7) has a non-uniform acceleration with the corresponding envelope given by:

$$t = \frac{t_p}{2} \frac{(3\gamma + 1)}{(\gamma + 1)} + \frac{c_o}{3a_3 t_p (\gamma + 1)} \quad (1.10)$$

The minimum value of t on this envelope is found by differentiating the above expression with respect to t_p and setting the result equal to zero which gives the equation below:

$$t_s = \frac{2}{\gamma + 1} \sqrt{\frac{c_o (3\gamma + 1)}{6a_3}} \quad (1.11)$$

A similar observation on the shock formation for one-dimensional flows was made by Jones (1958) in a theoretical study of flame acceleration. It is shown that the disturbances produced by a uniformly accelerating flame cause a discontinuity to form on the path of the initial disturbance at a time given by:

$$t_s = \frac{c_o}{(\gamma + 1)a_2 \beta} \quad (1.12)$$

where β is defined by the expression

$$u = \beta U \quad (1.13)$$

relating the gas velocity in front of the flame to the flame velocity. If β is set to unity, in otherwords making the flame a solid boundary, the expression reduces to that obtained for the piston motion. Furthermore for a non-uniformly accelerating flame the discontinuity in the flow is first shown to form between the initial disturbance and the flame front. In this case the time co-ordinate is given by:

$$t_s = \frac{2}{\gamma + 1} \sqrt{\frac{c_o [\beta(\gamma + 1) - 2/\beta]}{6a_3}} \quad (1.14)$$

which again reduces to the expression obtained for the piston motion when β is set to unity.

Figure 1.5 shows each disturbance emitted from an initially accelerating piston coalescing with the shock wave and increasing its velocity up to a maximum value. This coincides with the intersection of the disturbance which left the piston at the time it ceased to accelerate, from then on the shock will propagate with a constant velocity equal to that of the piston. The acceleration of the shock wave introduces a region of non-uniform entropy into the flow because in the initial stages the unshocked gas experiences first of all an adiabatic compression and then is compressed by a shock of gradually increasing strength.

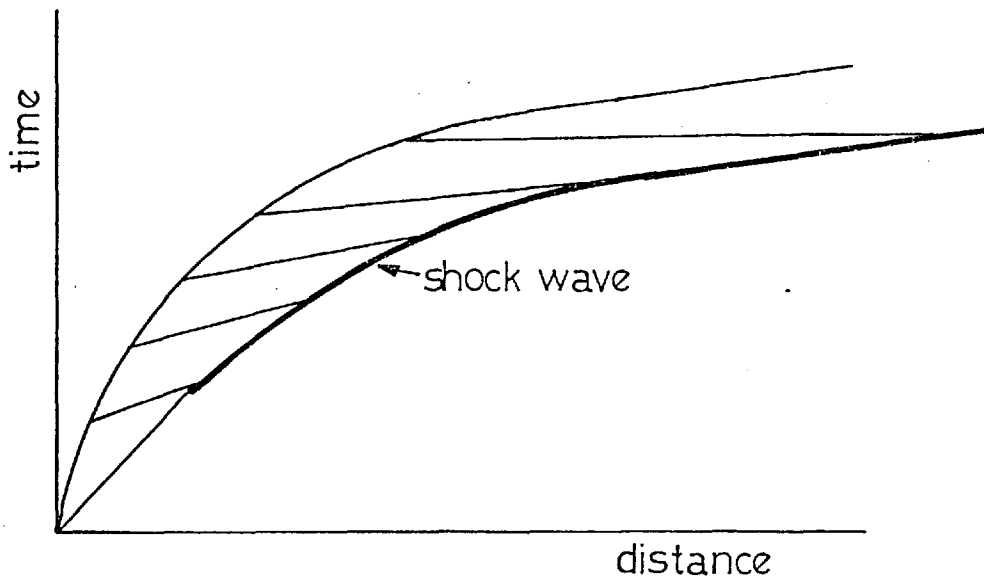


Figure 1.5 Development of a shock wave by
an accelerating piston

Physically a concept like that of a piston within a shock tube cannot exist as a perfectly defined surface but tends to appear as a finite layer due to turbulence, caused by the imperfect rupture of the diaphragm, which grows from the effects of conduction and diffusion. These dissipative processes also influence the propagation of the shock wave and instead of maintaining its constant velocity until it comes under the influence of the reflected

rarefaction wave there is a noticeable attenuation. Glass et al (1953) assume this attenuation is associated with the boundary layer build-up within the shock tube and is most marked for strong shock waves.

1.3 Spherically Symmetric Flows

Taylor (1946), in his paper dealing with the air wave surrounding an expanding sphere, introduces the concept of extending the use of a moving piston to simulate spherically symmetric flows and comparing the results obtained in this way with those for explosions where the contact surface has an identical path to that of the piston. Examples of such motions are the fracturing of a glass sphere containing a high pressure gas, Brode (1955); the explosion of an unconfined vapour cloud, Strehlow et al (1973); the detonation of a solid explosive charge, Dewey (1963); gunfire; the explosion of a thin metal wire under the discharge of a large amount of electrical energy, Brossard (1970); and the focusing of an impulse beam from a laser; all of which in general terms can be represented by the rapid release of a large amount of energy into a finite volume, the resulting motion generally being known as a blast wave.

The solution of this type of problem falls into one of three categories depending on the strength of the explosion. For the case of the shock overpressure maintaining a value greater than ten atmospheres at distances and times which are large compared with the dimensions of the energy source and the energy release time respectively, the point source solution can be applied. The problem was treated about the same time, but independently, by Taylor (1950), Sedov (1946) and Von Neumann (1944), the former being asked to estimate the effects of the release of a large amount of energy into the atmosphere in the form of an atomic bomb. By assuring the finite amount of energy is released in an infinitely concentrated form, and the ambient pressure $\bar{P}$ is negligible compared with the subsequent shock overpressure, it is possible to scale the dependant variables and show that

the shape of the distribution of each new variable does not change with time, the motion being self similar. The parameters which determine the gas motion are the energy of the explosion E_0 , and the ambient gas density $\bar{\rho}$ only, and these can be combined with distance r and time t to form a dimensionless similarity variable η given by:

$$\eta = r \left\{ \frac{\bar{\rho}}{E_0 t^2} \right\}^{1/5} \quad (1.15)$$

The solution can now be expressed in the following form:

$$\frac{P}{P_1} = f(\eta), \quad \frac{u}{u_1} = g(\eta), \quad \frac{\rho}{\rho_1} = h(\eta) \quad (1.16)$$

where subscript 1 refers to the value of the variable immediately behind the shock front, while f , g and h are functions of the similarity variable η . At this point Taylor took the analytical solution no further, but proceeded by substituting these relationships into the flow equations, reducing them to ordinary differential ones which are solved by simple numerical techniques. Sedov and Von Neumann obtained a completely analytical solution as have more recent workers including Latter (1955) and Taylor J.L, (1954).

Analytical solutions also exist for the weak shock region where an approximate acoustical theory can be applied as in the work of Bethe (1944) and Von Neumann and Bethe (1958). Alternatively a different approach was made by Landau (1945), Whitham (1950) and Sedov (1959) who assumed that in this region the shock asymptotically decayed to a sound wave.

The intermediate shock strength region has eluded a suitable analytical approach except for an approximate treatment by Von Neumann and Bethe (1958)

which gives more of a qualitative than quantitative description of a point source explosion. An analytical solution which is valid over all three of these regions is arrived at by an extension of the shock propagation theory of Brinkley and Kirkwood (1947) by Sachdev (1972) and gives excellent agreement with other valid solutions. A simple analytical solution has been made by Chernyi (1957) which assumes that for a strong explosion the entire mass of gas encompassed by the shock wave is concentrated in a thin layer immediately behind the shock and has a density equal to the maximum attainable by shock transition given by:

$$\rho_1 = \frac{\gamma + 1}{\gamma - 1} \rho_0 \quad (1.17)$$

On the basis of this assumption the gas velocity can be taken to be uniform within the layer and on equating the change in momentum of the layer to the work done by compressing the gas an expression is arrived at for the path of the shock as a function of time. A similar technique was adopted by Vlasses and Jones (1966) to predict the path of a cylindrical shock wave, produced in an inverse pinch machine, from the strong shock region down to the acoustic limit. It was expected that a smooth joining of the two limit solutions would give a satisfactory description of the behaviour in the intermediate region. This method has been extended to the case of spherically symmetric flow by Jones (1968) and has given good agreement with other theories for predicting the shock path. Unfortunately this type of solution cannot be used to give the pressure profile behind the shock due to the assumptions made concerning the distribution of the shocked gas. The intermediate strength wave only received suitable treatment on the advent of high-speed digital computers, when it was possible to perform a complete numerical solution. Since the unsteady spherically symmetric flow of an

ideal gas is governed by a set of non-linear hyperbolic partial differential equations the numerical methods of solution are limited to three main techniques, namely the characteristic mesh, finite difference and particle-in-cell. Elliot (1971) performs a critical analysis of each of these methods. The method of characteristics is the most accurate of the three but tends to be rather complex when there are a number of discontinuities in the flow. The finite difference method, which is usually used with an artificial viscosity term to smooth out shock waves and simplify the solution, is prone to inaccuracies if the mesh size is not kept small and generally gives poor shock definition. The artificial viscosity term most widely used in this method is that due to Von Neumann and Richtmyer (1950) who discuss some of the stability problems of the solution. It has been put into practical use by Brode (1955) in an extension of the point source explosion from the strong shock region using the results of the analytical solution as initial conditions. The particle-in-cell method (p.i.c.) was first developed by Evans and Harlow (1957), and for one dimension is somewhat inferior to the characteristic and finite difference techniques as regards accuracy and computing time, but in two or more dimensions it has the advantage it can deal with many more flow discontinuities.

The flow in front of an expanding spherical piston is only expected to develop a single shock wave and therefore ^{to provide} the most accurate solution the method of characteristics is chosen to deal with this problem. It was originally introduced by Massau (1899) and redeveloped by gas dynamicists firstly for the solution of steady two-dimensional and unsteady one-dimensional planar flow. Courant and Friedrichs (1948) describe the application of the method of characteristics to these types of flow and formulate analytical solutions. The solution is based on a linear combination of the unsteady flow equations in which the dependant variables are

differentiated in the same direction. There are two sets of these directions, the positive and negative characteristics, which intersect and form a grid in the $x - t$ plane. Along each direction the partial differential equations become ordinary ones which can be solved either analytically for the types of flow mentioned above or for the general case by using finite difference relations between each characteristic intersection.

For spherically symmetric flow no analytical solutions exist, meaning that in the early applications of this method the network of characteristics had to be built up in small steps which was a very laborious task. The introduction of electronic high-speed computers brought about a significant time saving and an increase in accuracy due to the possible reduction in step size. The application of the method of characteristics with the use of a computer is described by Hartree (1953), Ferri (1954), Glass (1959) and is given in Chapter 2. An extension of this method to the problem of three independent variables sees the characteristic paths replaced by conical surfaces which intersect along bicharacteristics on which the equations are solved in a similar way to the previous case. Various schemes of constructing the computational grid are given by Ferrari (1949), Holt (1956), Butler (1960) and others.

As in the case of a shock tube the energy release in an explosion can be taken, on one hand, as being instantaneous with the immediate formation of a shock wave which in this case due to its spherical nature will attenuate as it progresses from its origin (figure 1.6). On the other hand a more realistic approach would be to consider an energy source of finite extent, for which the transfer of energy to the blast wave occupies a certain time during which both the contact surface and shock, after it has formed, accelerate before spherical attenuation sets in (figure 1.7).

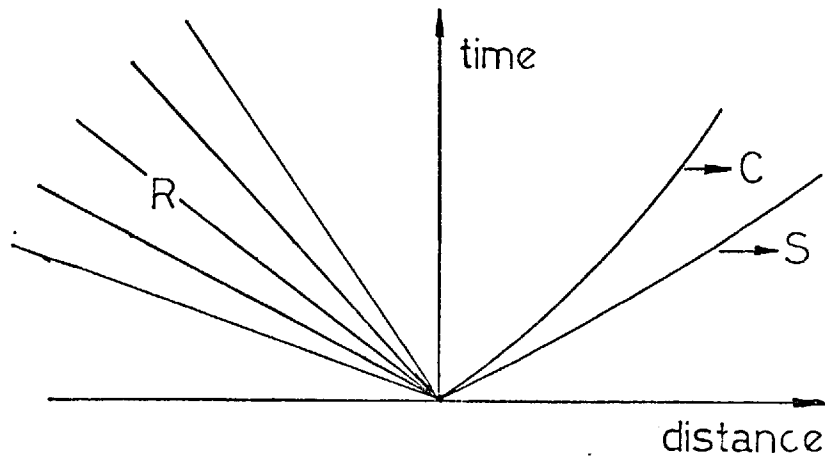


Figure 1.6 Wave form for a point source explosion

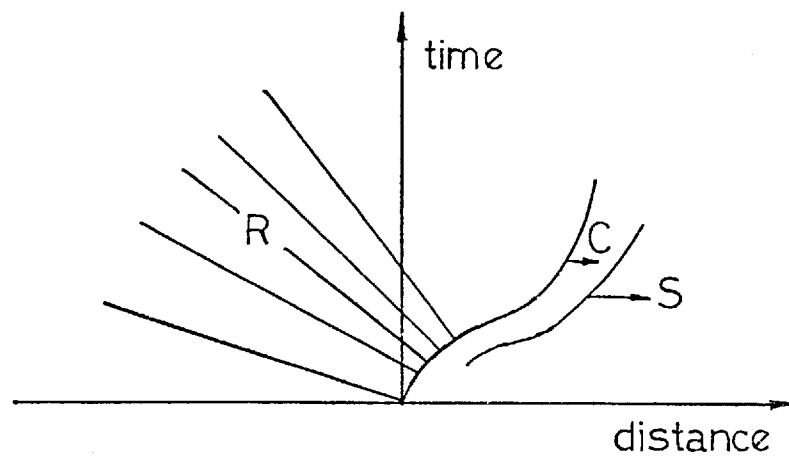


Figure 1.7 Wave form for a real explosion

Brode (1955) compared the plots of the shock overpressure versus shock radius for the point source solution against similar plots for isothermal spheres. Two of them contained hot gases with equal densities inside and

out, one at 2000 atmospheres and the other at 121 atmospheres and a third contained a cold high pressure gas. After suitable scaling of the distance and time to allow for the different energy contents of the spheres it was noticed that the plots coincided with the point source solution at distances which were smaller for the higher temperature spheres. The phenomenon responsible for this effect is the slower rarefaction speed in the cold sphere reducing the acceleration of the contact surface. Consequently the shock formation and the attainment of the overpressures given by the point source solution are delayed (figure 1.8).

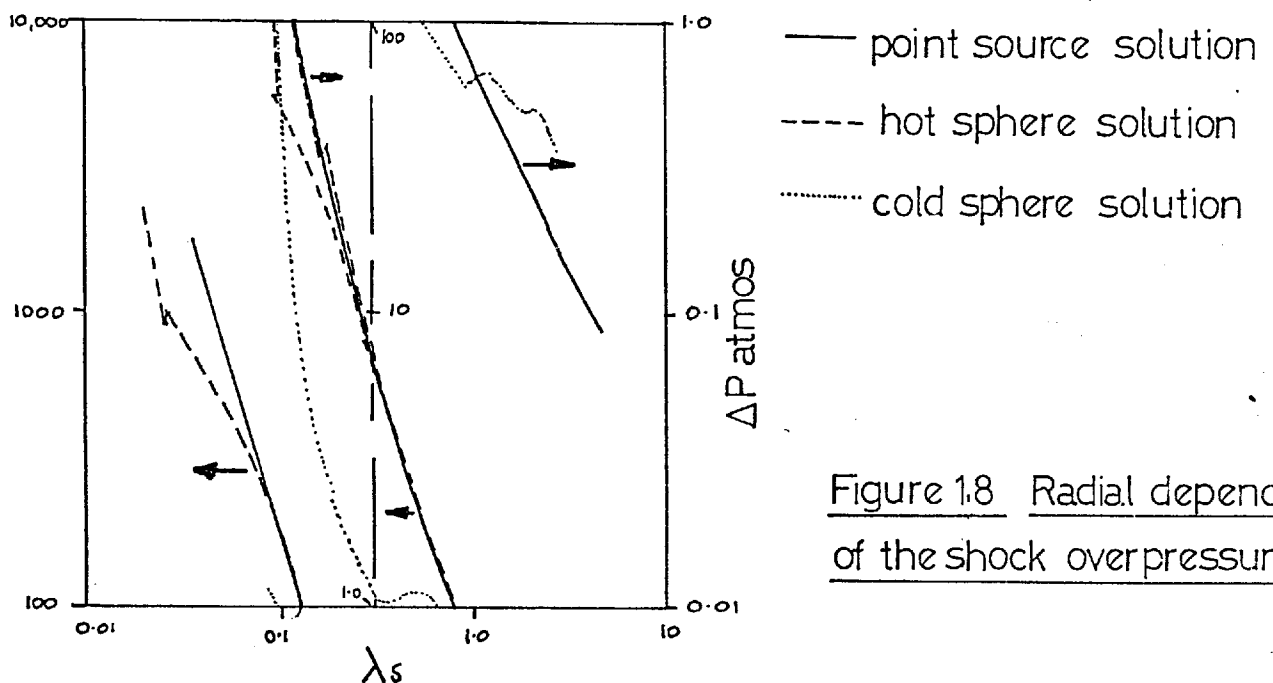


Figure 1.8 Radial dependence
of the shock overpressure

In the case of energy sources of considerable volume the blast wave may remain quite different from the point source solution for much of its life as energy transfer is over a much extended period. On the other hand if the energy release rate is restricted by the partial removal of the retaining boundary it will be unable to maintain the initial acceleration of the contact surface and the blast wave will reach a maximum strength and begin to

attenuate at a much earlier time. It follows, therefore, that the energy in a fully developed blast wave is not necessarily equal to the initial value at the explosive source but is a function of the rate of transfer of energy into the flow by the contact surface and the duration of the period of acceleration. It appears that at increasing distances from finite sources of energy the blast wave approaches the strength of that from the point source. As this can only be true when all the accelerating phases of blast development have expired and the energy has ceased to be transmitted into the shocked gases by the contact surface it can be used as a criterion for examining the energy content of blast waves. The period of flow when the shock development is still under the influence of the piston and the total energy of the blast is still time dependent has been examined using a similarity solution by Rogers (1958), Taylor (1946) and Morgenroth (1962) and experimentally by Brossard (1970). The latter, using an exploding wire, pointed out that the nominal energy liberated in the circuit under the discharge of a capacitor is not sufficient alone to characterise the blast and showed that the effective energy transferred was a function of the time constant of the discharge circuit.

1.4 Hazards of Blast Waves in Industry

The necessity, in this technologically based society, for the expansion of chemical plants, oil refineries and oil storage depots has been paramount. It has, however, brought with it an inevitable increase in potential hazards together with new problems of designing and siting apparatus to minimise explosion risk and effects. Two of the most common hazards present in modern industry are those associated with (i) pressure relief mechanisms and (ii) unconfined vapour cloud explosions.

1.4.1 Pressure Relief Mechanisms

Pressure relief is designed to prevent the total failure of a vessel which is subject to an uncontrollable rising pressure. It has to be effective in such a way that it can reduce or maintain the pressure at a safe value and by doing so a quantity of high pressure gas must be vented in a very short period. Vessels encountered in industry normally have one of two types of pressure relief devices, the relief valve or the bursting disc assembly. The former finds its main application where the rate of pressure increase is small and the surplus gas can be removed by gradual venting. The advantage of the relief valve over its counterpart is that once the pressure has returned to normal the aperture is automatically closed and the contents of the vessel remain intact. The inherent advantages of the bursting disc as a pressure relief mechanism lie in its ability to form an unobstructed opening in a very short time and therefore its most attractive feature is its ability to deal with rapidly rising pressure and even provide explosion protection.

The criteria for design of suitable relief mechanisms for explosions are beyond the scope of this work but a few important points are brought to light which must be taken into account for satisfactory design. An explosion, due to its rapidity, will set up pressure gradients within a vessel and therefore the location of the relief area is critical, being best sited as near to the explosion source as possible to minimise the time before venting occurs. Also bad design may cause the relief mechanism to be ineffective in reducing the vessel pressure, this will occur if the maximum rate of venting is less than the rate at which the explosion is generating new gaseous products. The amount of data available on the venting of explosions is limited and is usually based on past experience rather than obtained by an analytical approach to the problem. The ratio of the venting area to the vessel volume obviously plays an important part in

determining the maximum pressure developed in an explosion. Recommended figures for this ratio have been published by Cousins and Cotton (1951) after experimentation with the ignition of propane/air and hydrogen/air mixtures. A good allowance is between 2 and 10 sq. ft./100 cu. ft. depending on whether the explosion is mild or violent. A similar lack of knowledge on the external effects of pressure relief is evident and again protective measures taken to minimise damage are based on past experience and design procedures which utilise 'worst case' principles. One of these principles is the use of the total energy within a vessel as a measure of shock strength in a blast wave. To design an explosion protection device one must know the energy imparted to such an object from both the blast pressure and high speed missiles torn from the structure. For the former a knowledge of the blast characteristics as a function of distance would provide a basis from which the strength and positioning of protective structures would be determined.

The formation of a blast wave as mentioned earlier for an idealised case depends on the rate of energy release and the duration of the accelerating phase of the blast development. It is therefore necessary to determine the governing factors and to what extent they influence these two parameters in order to establish adequate procedures for current and future design requirements. For a venting pressure vessel it is possible to determine the energy release as a function of time by the measurement of the internal pressure transients. The vented gas does work on its surroundings by the interface acting as an expanding piston. A measurement of the distance-time profile of the interface can be used to determine the energy imparted to the surrounding gas and to calculate the dynamic and thermodynamic changes during the compression and rarefaction stages of the venting process.

1.4.2 Unconfined Vapour Cloud Explosions

The spillage of a large quantity of vapour into the atmosphere followed by accidental ignition and explosion is a major problem for which there is little quantitative data available. A method of determining the vapour cloud explosion energy release rates is proposed by Strehlow et al (1973). By assuming that the flow is shock free and using acoustic techniques an assessment of the energy is made in each case from disturbance arrival time measurements. As for a venting pressure vessel a vapour cloud explosion contains accelerating layers of gas, in this case driven by a flame front. The rate at which the flame imparts energy to the flow can be determined from a knowledge of its trajectory. The flame does not act as a true piston due to the mass flux across it, however, with a knowledge of the thermodynamic changes the gas undergoes on crossing the flame it would be possible to determine the complete flow domain after applying the correct boundary conditions.

Simpson and Brasie (1968) compared the damage done by a confined vapour cloud explosion with the estimated energy release. An analysis showed that the probable initial yield of the explosion was only a fraction of that calculated by assuming the structure contained the explosive vapour as a stoichiometric mixture. Table 1.1 is reproduced and lists the conditions under which elements fail when subject to a blast overpressure.

Structural Element	Type of Failure	Overpressure of incident blast (psig)
Glass windows	Shattering	0.5 - 1
Corrugated asbestos	"	1 - 2
Corrugated steel or Al panels	Connection failure followed by buckling	1 - 2
Wood siding panels and House Structures	Whole panel blown in after failure of main connections	2 - 3
Concrete wall panels 8" or 12" thick (not reinforced)	Shattering	2 - 3
Self framing steel panel buildings	Collapse	3 - 4
Oil storage tanks	Rupture	3 - 4
Wooden utility poles	Snapping	5
Loaded rail cars	Overturn	7
Brick wall panel 8 - 12" thick	Shearing	7 - 8

TABLE 1.1 - Structural damage from blast overpressure

1.5 The Scaling of Spherical Blast Waves

The necessity for scaling blast waves stems from the need to predict the characteristics of any particular explosion using the results obtained from small scale experimentation and from the monitoring of intense explosions under controlled conditions.

The principle of the scaling laws for explosive blasts first formulated by Hopkinson (1915) and described by Ericsson and Edin (1960) and Kinney (1962) is similar to those used elsewhere and is based on geometrical similarity. The approach, further developed by Sachs (1944), has been widely used for determining the distance from the origin for the occurrence of a given overpressure. The variables that govern the profile and position of an ideal spherical blast wave are the energy released, E_0 , the ambient pressure $\bar{P}$, the ambient sonic velocity, $\bar{c}$, and the distance and time

co-ordinates, r and t . A dimensional analysis of these variables yields the following dimensionless groups:

$$\lambda = \frac{r}{(E_o/P)^{1/3}} \quad \text{and} \quad \tau = \frac{t \bar{c}}{(E_o/P)^{1/3}}$$

The term $\left(\frac{E_o}{P}\right)^{1/3}$ is a scaling length which gives a measure of the strength of the blast wave. These groups suggest that for two different explosions identical blast wave intensities would be expected at distances and times which are proportional to the cube root of their respective energy release. Provided all the initial parameters of a real blast scale then it should be possible to represent any strength wave by a single curve in the $\lambda - \tau$ plane. This curve can be drawn for a blast of some standard reference strength and then information on other values can be obtained by either scaling the parameters up or down.

The problem in scaling blast waves is to determine the value of the energy E_o , coupled to the blast. As mentioned earlier this value is not necessarily the total amount of available energy in an explosion but some fraction of it which is determined by the initial rate of energy release and the duration of the accelerating phases. To determine the value of E_o a measurement of the shock trajectory or overpressure for distances and times which are large compared with the dimensions and energy release time of the explosive source would suffice. The technique for determining E_o from such experimental data is reported by Jones (1968) for intermediate strength blasts and by Kinney (1962) for the detonation of solid explosives.

Other workers have investigated the scaling of blast waves, particularly Chou and Huang (1969) and Lutzky and Lehto (1970), who made a numerical analysis of the late stage equivalence of spherical explosions, also Larson (1970) investigated the effect of the ambient conditions on the blast pressure, particularly in the close-in region.

The similarity between the trajectories of scaled blast waves and that of the point source explosion was noticed by Brode (1955) who pointed out that the blast from an initial isothermal sphere of gas assumed that of the point source (to within 10%) after 10 times the initial mass of the gas in the sphere had been engulfed by the shock wave. Use of this was made by Munday (1971) to determine the energy coupled to the blast for the venting of a partially failed pressure vessel. In a novel, but equally valid approach, a relationship was derived which expressed the value of the scaling length as a function of the vessel dimensions and the relevant gas properties and has the form below:

$$\epsilon_1 = 1.28 \left[\left(\frac{2}{\gamma + 1} \right)^b \frac{C_0}{C} \frac{P_0}{P} A_2 \right]^{\frac{1}{2}} \quad (1.18)$$

where $b = (3\gamma - 1)/2(\gamma - 1)$

Owing to a lack of experimental measurements to test the validity of this scaling length it is not known if it is reliable. However, a similarly derived expression for a vented reservoir of a hydraulic analogue suggests that this approach is both effective and sound.

CHAPTER TWO

THE THEORY OF ONE-DIMENSIONAL UNSTEADY COMPRESSIBLE FLOW AND ITS APPLICATION TO GAS EXPANDING INTO THE FREE ATMOSPHERE FROM A VESSEL

The process of venting a high pressure gas from a vessel provides two distinct problems that can be solved independently. These problems, which are the main topics of this chapter, are the solution of the gas flow external to the vessel and the solution of the wave action inside the vessel during the period of venting. To form a complete solution of the whole flow system some means of linking each separate part must be found. The interface between the vessel contents and the atmosphere forms a distinct boundary, the trajectory of which is readily recorded by photographic techniques. It is therefore ideal for use as the boundary condition in the solution of the associated gas flow. With a completely spherical system, i.e. a spherically symmetric release of compressed gas, this boundary condition will suffice for a solution in both regions under review, making use of the equations of unsteady one-dimensional flow. Photographs show that for the release of gas from one end of a cylindrical vessel the interface is essentially spherically symmetric and its use as the boundary condition for the solution of the gas flow downstream with these same equations is perfectly valid. However, in this case the interface trajectory cannot easily be used to provide the boundary condition for the solution of the wave action inside the vessel during venting. The difficulty arises when the type of flow changes in the region of the vessel orifice from one-dimensional spherical to axially symmetric flow and requires the use of the two-dimensional unsteady ^{flow} equations for a solution. Initially this region occupies only a short space, of the order of the orifice diameter, but as the flow-rate in the orifice increases and the interface moves down stream the transition region expands and the axially symmetric flow extends further

out from the vessel, eventually forming a steady state jet. The flow in the orifice remains planar, and being readily determined by experimentation, is more conveniently used as the boundary condition for the vessel internal wave action. Since the histories of both the interface and the flow in the orifice can be determined from the same absolute starting time there is a continuous relationship between their development. It therefore becomes unnecessary to solve for the flow region between orifice and the interface to match up the two boundary conditions. The solution of the flows pertaining to these two types of boundary conditions is discussed thoroughly in the remainder of the chapter along with an analysis of the flow transition region.

2.1 External Gas Movements and The Method of Characteristics

The unsteady one-dimensional flow of an ideal gas is governed by a set of quasilinear partial differential equations of the hyperbolic type.

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + (\eta - 1) \frac{\rho u}{x} = 0 \quad (2.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} = 0 \quad (2.2)$$

$$\frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} = 0 \quad (2.3)$$

$$P = P_0 \left[\frac{\rho}{\rho_0} \right]^\gamma \exp \left[\frac{S - S_0}{C_v} \right] \quad (2.4)$$

where $\eta = 1, 2$ or 3 for planar, cylindrical or spherical flow. The first two equations express the conservation of mass and momentum respectively, the third is the necessary condition for isentropic flow and the fourth is the equation of state for an ideal gas.

The sound speed c is given by the relation:

$$c^2 = \left[\frac{\partial P}{\partial \rho} \right]_S = \text{const.} \quad (2.5)$$

Using equations (2.5) and (2.3) the derivatives of the density in equation (2.1) can be eliminated to give:

$$\frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} + \rho c^2 \frac{\partial u}{\partial x} + (\eta - 1) \rho \frac{u c^2}{x} = 0 \quad (2.6)$$

In these equations the direction of differentiation of each dependant variable is different. However a linear combination of equations (2.2) and (2.6) can be formed such that each variable is differentiated in the same direction. These are known as the characteristic directions and along them the partial derivatives become total.

Multiplying equation (2.2) by λ and adding to equation (2.6) gives:

$$\frac{\partial P}{\partial t} + \frac{\partial P}{\partial x} \left[u + \frac{\lambda}{\rho} \right] + \lambda \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \left[\lambda u + \rho c^2 \right] + (\eta - 1) \rho \frac{u c^2}{x} = 0 \quad (2.7)$$

The condition that P and u are differentiated in the same direction is then:

$$u + \frac{\lambda}{\rho} = u + \frac{\rho c^2}{\lambda}$$

from which:

$$\lambda = \pm \rho c$$

Each value of λ gives a characteristic direction whose path becomes:

$$(\lambda_1) \quad \frac{dx}{dt} = u + c, \quad (\lambda_2) \quad \frac{dx}{dt} = u - c$$

Along each of these directions equation (2.7) reduces to:

$$(\lambda_1) \quad \frac{dP}{dt} + \rho^c \frac{du}{dt} + (\gamma - 1) \rho \frac{uc^2}{x} = 0 \quad (2.8)$$

$$(\lambda_2) \quad \frac{dP}{dt} - \rho^c \frac{du}{dt} + (\gamma - 1) \rho \frac{uc^2}{x} = 0 \quad (2.9)$$

Also along a third direction $(\lambda_3) \quad \frac{dx}{dt} = u$, the entropy equation (2.3) becomes:

$$(\lambda_3) \quad \frac{dS}{dt} = 0 \quad (2.10)$$

The form of the characteristic equations (2.8), (2.9) and (2.10) are ideal for numerical solution by finite difference techniques along a network of characteristic directions. It is convenient to replace u and c by a new set of dependent variables which have a special significance for one-dimensional planar flow that will be explained shortly.

$$r = \frac{u}{2} + \frac{c}{\gamma - 1} \quad (2.11)$$

$$s = \frac{u}{2} - \frac{c}{\gamma - 1} \quad (2.12)$$

The characteristic directions now become:

$$(\lambda_1) \quad \frac{dx}{dt} = \frac{(\gamma + 1)}{2} r - \frac{(\gamma - 3)}{2} s \quad (2.13)$$

$$(\lambda_2) \quad \frac{dx}{dt} = \frac{(3 - \gamma)}{2} r + \frac{(\gamma + 1)}{2} s \quad (2.14)$$

$$(\lambda_3) \quad \frac{dx}{dt} = r + s \quad (2.15)$$

Also from the equation of state (equation 2.4) and the use of the definition of the sound speed (equation 2.5) the following relationship is derived.

$$\frac{dP}{\rho c} = \frac{2}{\gamma - 1} dc - \frac{c}{C_p (\gamma - 1)} dS \quad (2.16)$$

and hence the characteristic equations become:

$$(\lambda_1) \quad \frac{dr}{dt} = \frac{1}{4C_p} (r - s) \frac{dS}{dt} - \frac{(\gamma - 1)}{4} (r^2 - s^2) \frac{(\eta - 1)}{x} \quad (2.17)$$

$$(\lambda_2) \quad \frac{ds}{dt} = \frac{1}{4C_p} (s - r) \frac{dS}{dt} - \frac{(\gamma - 1)}{4} (s^2 - r^2) \frac{(\eta - 1)}{x} \quad (2.18)$$

$$(\lambda_3) \quad \frac{dS}{dt} = 0 \quad (2.19)$$

It can be seen that for one-dimensional planar homentropic flow, as distinct from isentropic flow, that the new dependant variables are constant along their respective characteristics and are generally referred to as the Riemann invariants. This considerably simplifies the calculations and leads to analytic solutions for some special cases.

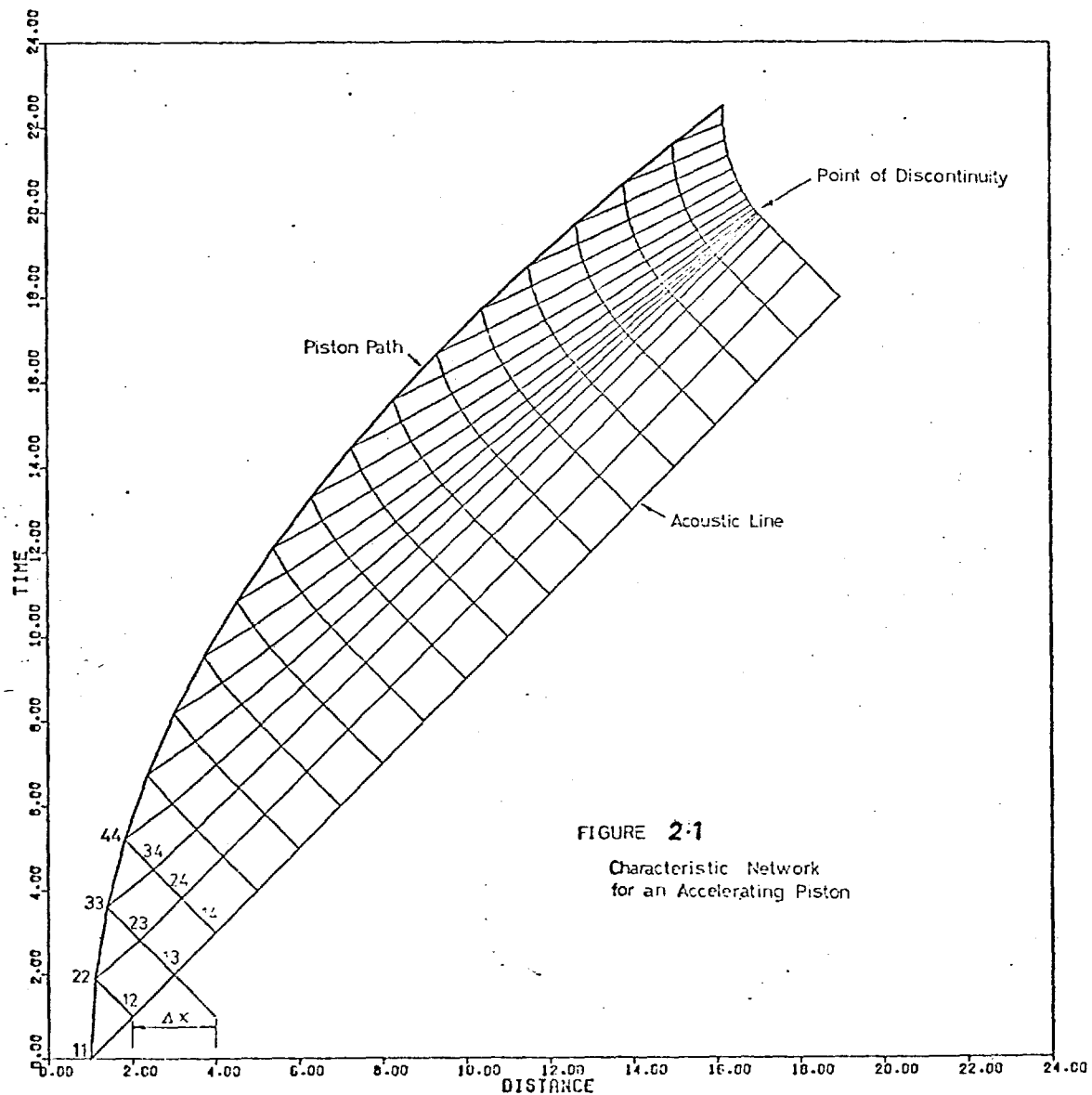
The normal solution of equations (2.17), (2.18) and (2.19) is obtained by replacing the differentials by finite difference relations and assuming the variable coefficients are constant over the step length. The correct value for the coefficient is obtained by iteration procedures.

The method adopted in this work for the solution of these equations makes use of a C.D.C. 6400 computer and the programing methods described below are based on the techniques presented by Courant and Friedrichs (1948), Ferri (1961), Glass (1959), and Hartree (1953).

2.2 A Computer Program To Determine The Motion of a Gas Ahead of an Expanding Spherical Piston

A computer program has been written which solves the flow equations, using the method of characteristics, for a gas moving in front of an expanding spherical piston. It deals with both compression and rarefaction waves and can introduce a single discontinuity into the flow at the appropriate time and position. Use of the program is limited to a continuous piston motion and is easily adapted to deal with one-dimensional cylindrical and planar flows. The program, for which a listing and flow diagram appears in Appendix B, consists of the main routine, PISTON, which sets the boundary and initial conditions, controls the integration steps and builds up the characteristic network and shock profile. Three subroutines, PT3, PATH and SHOCK are called during the integration to determine the gas properties throughout the required distance-time domain.

In the early stages of the integration the execution of the main program is straight forward and it is only when discontinuities have to be introduced into the flow that more complex procedures are adopted. Initially integration is performed over a network of characteristics as in Figure 2.1, the upper left hand boundary representing the path of an accelerating piston and the lower right hand boundary the path of the first disturbance caused by the piston motion which travels at the speed of sound. The conditions at the starting point (1,1) are known, since there is no motion at $t = 0$. The step length Δx determines the starting point (1,2) of the next negative characteristic. The conditions at this point, which is on the acoustic line, are again those of the undisturbed gas and they determine the gradient of the characteristic drawn to intersect the piston path at point (2,2). From this point a new positive characteristic is introduced and its first intersection is with the third negative one at point (2,3). The procedure is continued by integrating up each negative



characteristic to the piston and thus building up the network. It can be seen that there are three types of intersection, (i) at the acoustic line, (ii) in between each of the boundaries, and (iii) at the piston path. The co-ordinates and gas properties at intersections of type (i) are easily determined due to the undisturbed conditions along the acoustic line, those of type (ii) and (iii) are determined by calling the subroutines PT3 and PATH respectively.

SUBROUTINE PT3

In general subroutine PT3 is called from the main program to determine any point on the network (i, j) where $i \neq 1$ or $j \neq 1$. It is supplied with the co-ordinates of and the values of gas velocity, ^{sonic velocity} and entropy at the points $(i-1, j)$ and $(i, j-1)$ and it returns the respective values for point (i, j) . The method is described below with the use of Figure 2.2

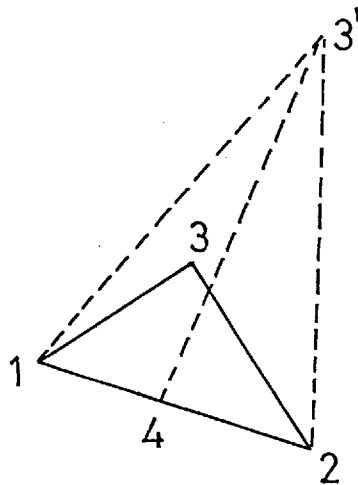


Figure 2.2 Part of the characteristic network

If all the flow parameters and the co-ordinates are known at points 1 and 2 on part of the network and provided the points are sufficiently close

together the assumption that the characteristic directions are straight and the dependant variables change linearly between any two points is quite acceptable. The co-ordinates of point 3, to a first approximation, are those of point 3' and are determined from the solution of equations (2.13) and (2.14) using the values of r and s for points 1 and 2. The entropy at point 3' is found by projecting a path line from point 4 suitably chosen to intersect 3'. Along a path line entropy is conserved and therefore the value of S_3 is found by linear interpolation between S_1 and S_2 . A similar procedure for determining the values of r_3 and s_3 is adopted, this time using equations (2.17) and (2.18). A second approximation to the co-ordinates and the values of r_3 , s_3 and S_3 can be made by repeating the calculations using the mean values for the quantities on the right hand side of the equations instead of the initial values.

The error incurred by approximating the differential equations in this way is of the order $(\Delta t)^3$ for each integration step; it is cumulative and may lead to instabilities of the solution if the integration step is too large or if the calculations are extended over too great a domain.

SUBROUTINE PATH

Subroutine PATH is called from the main program to determine the profile of the piston path together with the co-ordinates and gas properties at the points where the characteristics intersect. It is supplied with these values for the point $(j-1, j)$ and imposes the necessary boundary conditions that exist at the piston. At this surface the gas velocity is equal to that of the piston due to there being zero mass flux across the boundary. The movement of the piston is determined by the values of the co-efficients of a polynomial which are chosen either to fit experimental data or are suitable alternatives. The method used in this subroutine is described below using Figure 2.3.

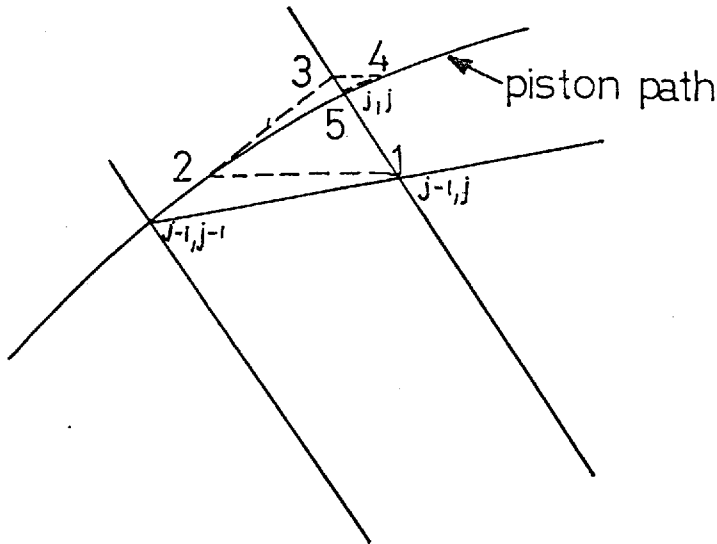


Figure 2.3 Intersection of a characteristic and
the piston path

Knowing $x = f(t)$ for the piston path an initial estimate is made of $t_{j,j}$ by setting $t_{j,j} = t_{j-1,j}$. The value of x at point 2 is now given by $x = f(t_{j,j})$ and the corresponding value of gas velocity or piston speed is $\frac{dx}{dt} = f'(t_{j,j})$. A line is now drawn with this gradient to intersect the negative characteristic projected from point $(j-1, j)$ at point 3. The value of the time co-ordinate at point 3 gives an improved estimate of $t_{j,j}$. An iteration procedure is now implemented, calculating points 4, 5, etc., until convergence is complete. The value of s at the point of intersection is found from equation (2.18) and knowing the gas velocity the sonic velocity can be determined. The entropy, S , is taken as being equal to that of the undisturbed gas because, the piston motion being continuous, the gas at its surface undergoes no irreversible processes. The subroutine also calculates the amount of work done by the expanding piston on the gas that is driven ahead of it.

As the characteristic network is built up a point will be reached at

which two positive characteristics intersect. The values of r and s on each characteristic at this point will differ and lead to a discontinuity in the solution with which the procedure used up to now will not deal. To overcome this problem a shock wave is introduced into the flow and the integration is performed as before on either side with the Rankine-Hugoniot relations used to take the solution from one side to the other. It is at this point that the main program revises its method for fixing the starting co-ordinates of all negative characteristics. The positive characteristics will initially intersect the shock path from both behind and ahead and determine its profile by the values of r and s they carry. It is convenient to start the new negative characteristics at such a position on the acoustic line that they meet the shock and the positive characteristics at their intersection points (figure 2.4).

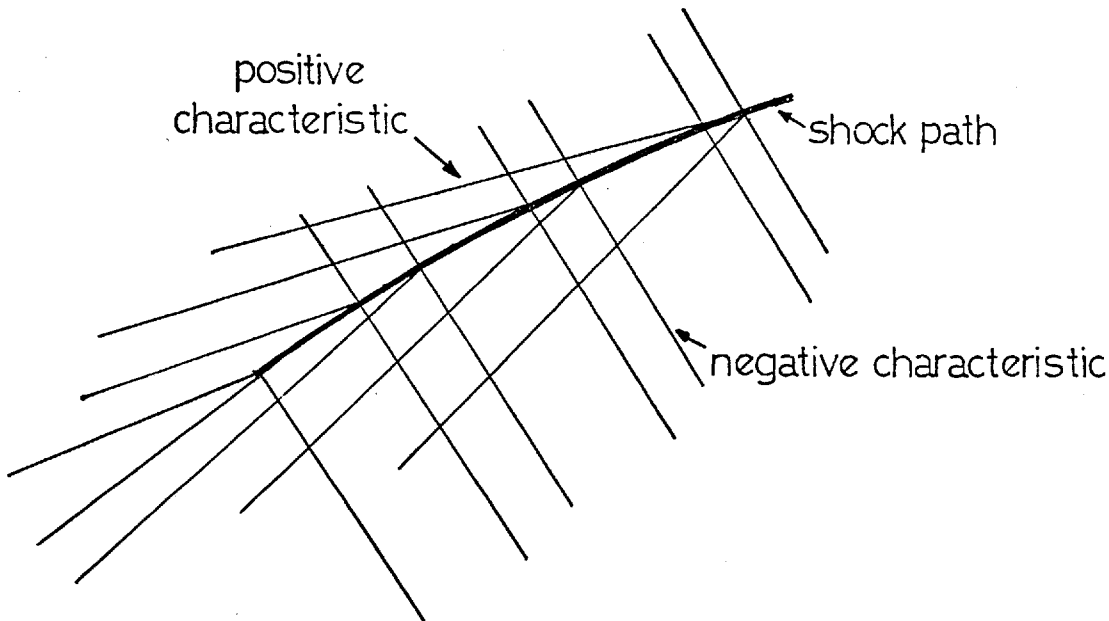


Figure 2.4 Positioning of new negative characteristics

An iterative procedure is adopted for choosing the starting point. There are two types of shock intersection points which have to be dealt with in slightly different ways, (i) the point where the positive characteristic

intersects the shock from ahead and (ii) the point where the positive characteristic intersects the shock from behind. In the first case normal integration is made upto the shock front at point (i,j) (figure 2.5).

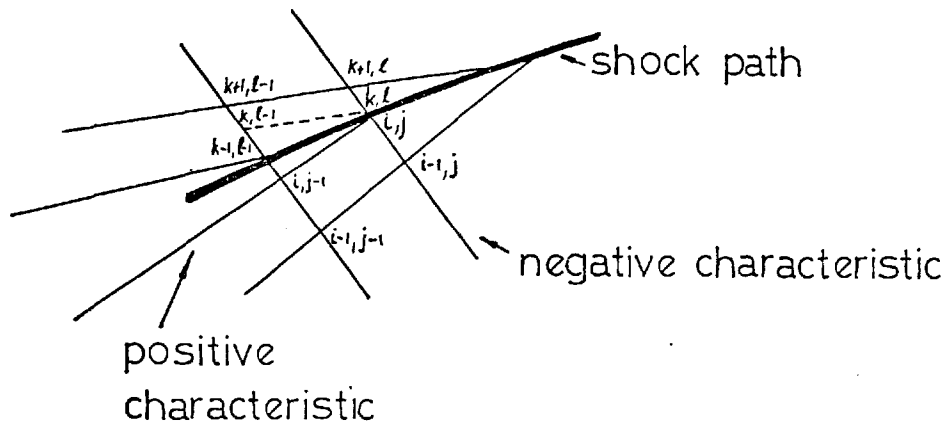


Figure 2.5 Positive characteristic intersection ahead of the shock wave

The shock velocity is chosen such that its transition is compatible with the conditions of flow already existing behind it. The program initially sets the shock velocity equal to $(u + c)_{k+1,l-1}$ and then by calling subroutine SHOCK with the gas properties at point (i,j) , the Rankine-Hugoniot conditions are used to determine the appropriate values at point (k,l) behind the shock. The approximate values of these properties can be determined by introducing a further positive characteristic between points $(k,l-1)$ and (k,l) and using equation (2.17). The flow properties at point $(k,l-1)$ and the gradient of this characteristic are found by interpolation between points $(k+1, l-1)$ and $(k-1, l-1)$. The shock velocity is adjusted and the procedure repeated until the solution for the conditions at point k,l converges. The integration is now continued to the piston path as before.

For the second case integration proceeds upto point $(i-1, j)$ (figure 2.6) and then a new positive characteristic is introduced between points $(i, j-1)$

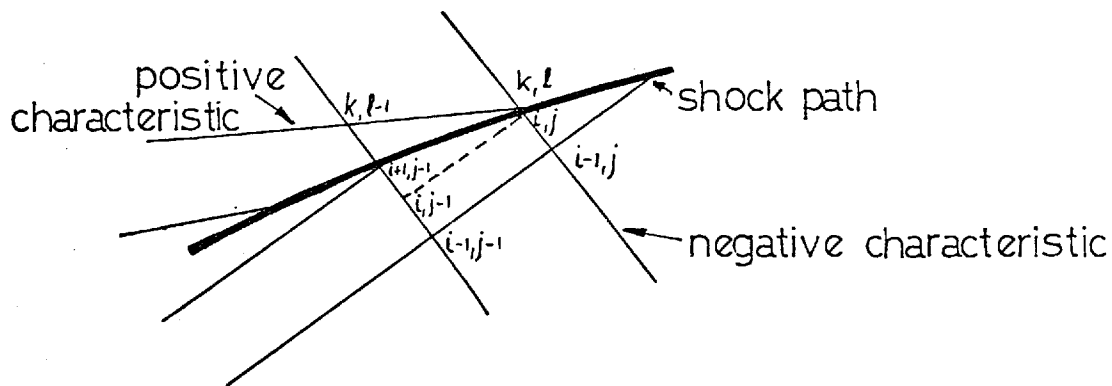


Figure 2.6 Positive characteristic intersection behind
the shock wave

and (i, j) . The flow properties at point $(i, j-1)$ and the gradient of this characteristic are found by interpolation between points $(i+1, j-1)$ and $(i-1, j-1)$. Subroutine PT3 is called to determine the appropriate values at point (i, j) immediately in front of the shock. An initial guess of the shock velocity is made and subroutine SHOCK called for the conditions behind the wave at point (k, l) . As in the previous case the conditions at point (k, l) are also determined from point $(k, l-1)$ using equation (2.17) and an identical procedure is used to find the correct shock velocity. Eventually the shock wave will be intersected from ahead by the final positive characteristic whereupon it emerges into the undisturbed gas. The integration is now solely performed behind the shock wave and is started by calling subroutine SHOCK and continuing as in case one.

The main program ends by making use of the computer graph plotting

facilities. The entire characteristic network can be drawn together with the piston and shock paths. Alternatively, gas velocity and sonic velocity profiles can be plotted for a series of six constant time lines between the piston path and the outermost disturbance.

2.3 Gas Movements Inside the Vessel During Discharge

Giffen (1940) discusses the theory of the rapid discharge of a gas from a vessel which had at the time an important part to play in the exhaust process of an internal-combustion engine. The implications of the term "rapid discharge" are considered and it is pointed out that the rate of release of a gas determines the criterion used for developing the associated theory. One method uses the theory of propagation of waves within the vessel and allows for internal pressure gradients. A second uses a quasi-steady state theory which, being an approximation, considers the gas pressure in the vessel to be uniform and the gas velocity as being negligible. The former theory has obvious advantages when the area, A_2 , of the orifice formed after the rupture of a bursting disc is not small compared with the cross-sectional area of the vessel, A_1 . In this case the rate of fall of pressure at the orifice end of the vessel will be significant before the disturbance can be transmitted throughout the whole volume giving rise to internal pressure gradients. For a relatively small orifice area the pressure drop is considerably less and the disturbance is transmitted throughout the vessel before any further significant pressure drop can take place and therefore the approximation of a uniform vessel pressure is valid. Kestin and Glass (1951) examined the rate of discharge of gas using Giffen's wave theory and compared it with the values obtained from the quasi-steady state theory. It was shown that the latter method gave mass flow rates which were considerably less than those predicted by the wave theory and from experimental data. The discrepancy between the two theories was most apparent

for the smaller area ratios, e.g. when $A_1/A_2 = 1$ the ratio of the two mass flow rates is 1.709, whereas when $A_1/A_2 = 5.8$ the corresponding ratio is only 1.006.

2.31 The Wave Action Within the Vessel during Venting

The assumptions that Giffen makes in developing his wave theory are that the expansion process is adiabatic and the gas motion is in one dimension only, that being the axis of the vessel. The opening of the vent is taken to be instantaneous and the pressure in the orifice assumed to drop immediately to a value which is determined by the initial vessel pressure. If the initial pressure P_0 exceeds the critical value given approximately by:

$$P_c = \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma}{\gamma - 1}} \bar{P} \quad (2.20)$$

where $\bar{P}$ is the surrounding atmospheric pressure, then the gas velocity in the orifice will be sonic and the corresponding pressure, P_2 , will be that reached by adiabatic expansion to these sonic conditions. When P_0 falls below this critical value then the pressure in the orifice, having reached atmospheric, remains constant and the flow becomes subsonic.

At the instance of the orifice opening a disturbance is transmitted with the speed of sound to the rear of the vessel; as the gas accelerates and expands the changes that take place at the orifice are transmitted to the bulk of the gas by a rarefaction wave as shown in Figure (2.7a). The final disturbance, or tail of the wave leaves the front of the orifice when the pressure has dropped to P_2 and the gas velocity has become sonic. As the stationary gas passes through this wave it is set in motion towards the open end of the vessel and achieves a velocity u_1 at pressure P_1 (figure 2.7b). On reaching the rear of the vessel the rarefaction wave is reflected

back into the moving gas. The boundary conditions at the wall require the new value of the gas velocity after passing through the reflected wave to be zero and the pressure falls to P_3 . When the wave reaches the front of the vessel all the gas is momentarily in a stationary state at the new lower pressure. The process is now repeated; the rarefaction wave travelling to the vessel rear reduces the pressure to P_4 leaving a new set of sonic conditions in the orifice provided P_3 is greater than P_c . This is continued until the vessel pressure is reduced to atmospheric in a series of steps as shown in Figure 2.8 for two vessel locations.

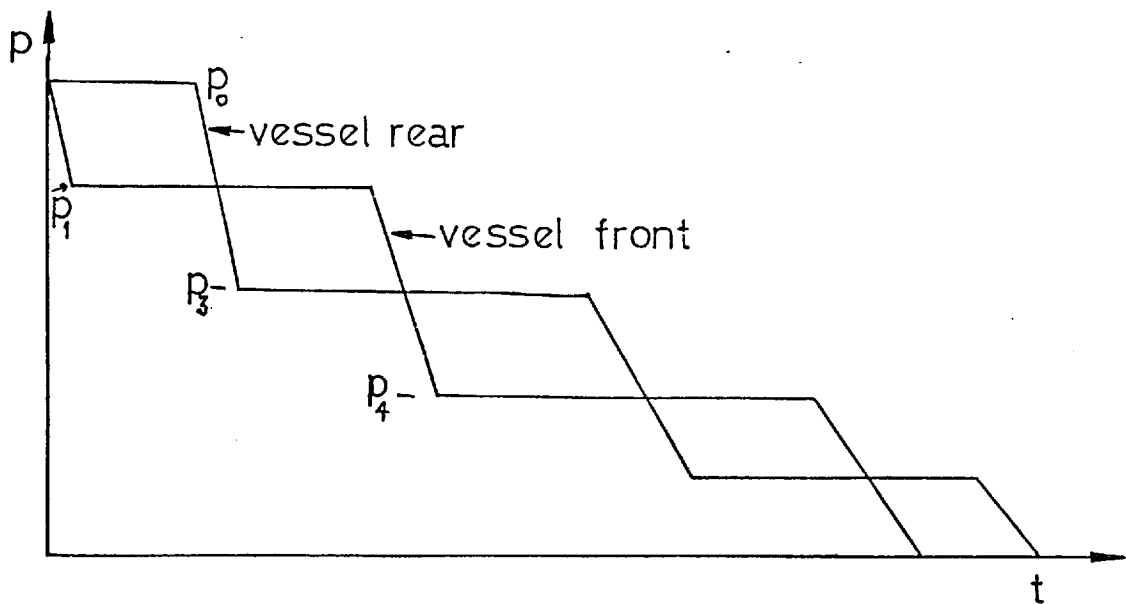


Figure 2.8 Pressure profile at the rear and the front of a vessel during venting

The pressure transient duration of the rarefaction wave increases with time as shown ⁱⁿ figure 2.7c and d. This is because the head of the wave is always travelling at a speed greater than that of the tail and thus the pressure transition is spread out over the length of the vessel. For certain vessel conditions it is possible for the head of the rarefaction wave, after reflection, to catch up with the tail of the wave which has not

been reflected. When this occurs there is no location in the vessel at which stationary conditions exist.

Analytical solutions for the pressure wave formed during the venting of a vessel can be obtained using the method of characteristics. For this type of flow the values of r and s along the positive and negative characteristics respectively are constant (Riemann invariants) and since the part of the rarefaction fan abd (Figure 2.7a) is adjacent to an undisturbed fluid the wave is of a simple type. The positive characteristics that enter the rarefaction zone across the head of the wave ab all carry the same value of r given by:

$$r = \frac{u_0}{2} + \frac{c_0}{\gamma - 1} = \frac{c_0}{\gamma - 1} \quad (2.21)$$

Along any negative characteristic direction in this region

$$s = \frac{u}{2} - \frac{c}{\gamma - 1} = \text{const.} \quad (2.22)$$

From equation (2.21)

$$\frac{u}{2} + \frac{c}{\gamma - 1} = \frac{c_0}{\gamma - 1} \quad (2.23)$$

Substituting into equation (2.22) for u gives

$$s = \frac{1}{\gamma - 1} (c_0 - c) - \frac{c}{\gamma - 1} = \text{const.} \quad (2.24)$$

Therefore along a negative characteristic both c and u are constant and thus the characteristics path is a straight line. For complete transition of the rarefaction wave.

$$\frac{c_0}{\gamma - 1} = \frac{u_1}{2} + \frac{c_1}{\gamma - 1} \quad (2.25)$$

$$\therefore \frac{c_1}{c_0} = \left\{ \frac{P_1}{P_0} \right\}^{\frac{\gamma-1}{2\gamma}} = 1 - \frac{(\gamma-1)}{2} \frac{u_1}{c_0} \quad (2.26)$$

Similar reasoning for the reflected rarefaction wave in the region cdfe, which is adjacent to the stationary region gfc, shows the positive characteristic directions to be straight and gives the following expression for gas properties after expansion through the wave.

$$\frac{c_1}{c_3} = \left\{ \frac{P_1}{P_3} \right\}^{\frac{\gamma-1}{2\gamma}} = 1 + \frac{\gamma-1}{2} \frac{u_1}{c_3} \quad (2.27)$$

In the region cbd where the incident and reflected wave interact there is no part adjacent to a stationary region and the rules of a simple wave cannot be applied. Neither Giffen nor Bannister and Mucklow (1948), in a later paper, took into account the wave interaction but assumed that the characteristic directions remained straight in this region.

To determine the duration of the wave reflection at the rear of the vessel accurately it is necessary to solve for the flow conditions in the interaction region. One way of doing this is described by Moszynski (1952) using the method of characteristics and determining the network by graphical means. Alternatively the co-ordinates and properties at each point on the network can be determined using subroutine PT3 of the program PISTON with a slightly modified form of the main routine to provide the boundary and initial conditions and to control the integration steps.

2.3.2 Discharge Rates for a Non-centred Rarefaction Wave

In real situations the time for both the removal of the bursting disc and the acceleration of the expanding gases gives rise to a non-centred rarefaction wave, which is shown diagrammatically in figure 2.9. The

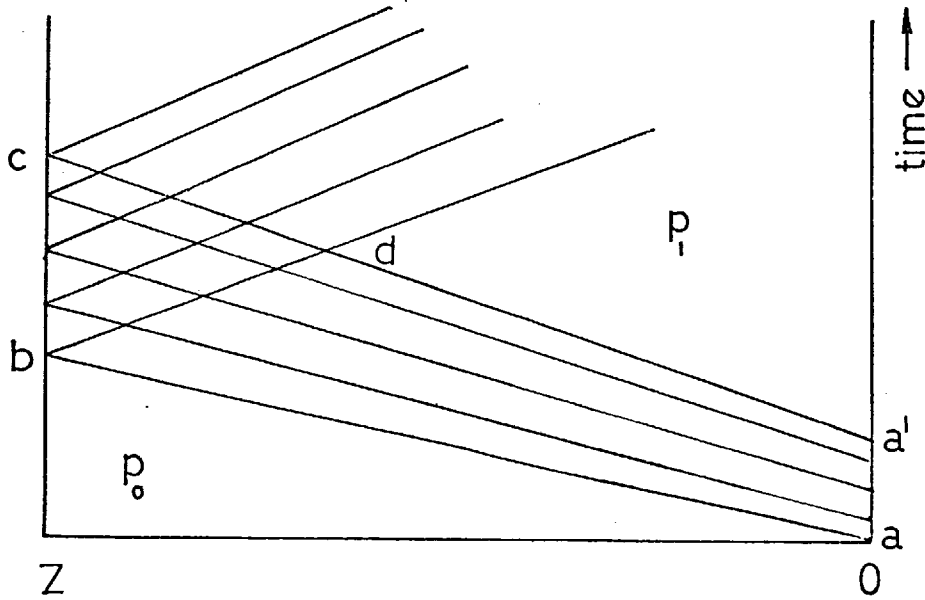


Figure 2.9 Non-centred rarefaction wave

choked conditions in the vessel throat are not obtained instantaneously, but over a finite period, the throat choking time, determined by which ever is the governing factor of these two real effects. The sonic conditions eventually obtained in the throat, however, are identical to those for a centred rarefaction wave because both are reached by isentropic expansion from the same starting conditions. It follows that the non-centred rarefaction wave is also of the simple type and the relations derived for the gas transition properties still hold. The reflection of the head of the wave at the vessel rear will occur at the same time for both types of wave. The tail of the wave is delayed by an increment aa' , which is maintained throughout its life due to the gas properties on each side being identical to those of the centred wave.

2.4 The Development of a Steady State Gas Jet

The initial outflow of gas from a vessel orifice has a relatively small velocity and the sudden expansion process it experiences will take place equally in all directions giving rise to a spherically symmetric

region of moving gas separated by the contact surface from a compressed layer of the atmosphere. As the gas velocity in the orifice increases, the expansion process becomes orientated along the axis of the vessel. The effect of this will be to distort the symmetrical flow in a localised region and initiate the development of a gas jet. At this stage, however, the shape of both the contact surface and any shock wave that has subsequently developed will not be significantly affected by a deviation from spherical symmetry at the point of energy injection. Furthermore, as the flow develops the more advanced disturbances will become less dependant on the changes that occur around the vessel orifice. The type of gas jet which can be formed will depend on the ratio of the vessel gas pressure to that of the surrounding gas and also to the velocity of efflux. Hartmann and Lazarus (1941) describe the properties of jets with pressure ratios in the range 3 to 7 and a velocity of efflux equal to the local velocity of sound. Pack (1948) describes jets with pressure ratios only a little above unity as having a periodic structure to a first approximation. He outlines a method for computing the conditions in a steady two-dimensional supersonic jet using the method of characteristics to solve the hyperbolic second order partial differential equations of flow.

The venting of a gas above its critical pressure through a rapidly opened aperture will cause a steady state jet to be formed only after the orifice has become choked. It is well known that such a jet will not form a blast wave on its own accord and it is reasonable to assume that such a jet will not contribute to the development of a previously formed blast wave. Therefore the time between the start of venting and the throat becoming choked will provide a good indication of the period during which the energy associated with the blast wave is released from the vessel. The method by which this period of time is calculated is directly linked to

a measurement of the changing gas pressure at the vessel rear. The energy vented into the blast wave up to the choking time is then readily obtained and can be used for the purposes of scaling the blast wave and calculating the deviation from the point source solution. The following section gives a detailed description of how these venting characteristics are determined by a combination of empirical and numerical schemes.

2.4.1 The Vent Throat Choking Time

A calculation is performed for a centred rarefaction wave to determine the pressure transition time at the vessel rear for the ideal situation in which the gases in the vent reach critical conditions instantaneously. The discrepancy between this and the experimental values is used to determine the throat choking time.

Figure 2.10 shows part of the characteristic network used in the calculations.

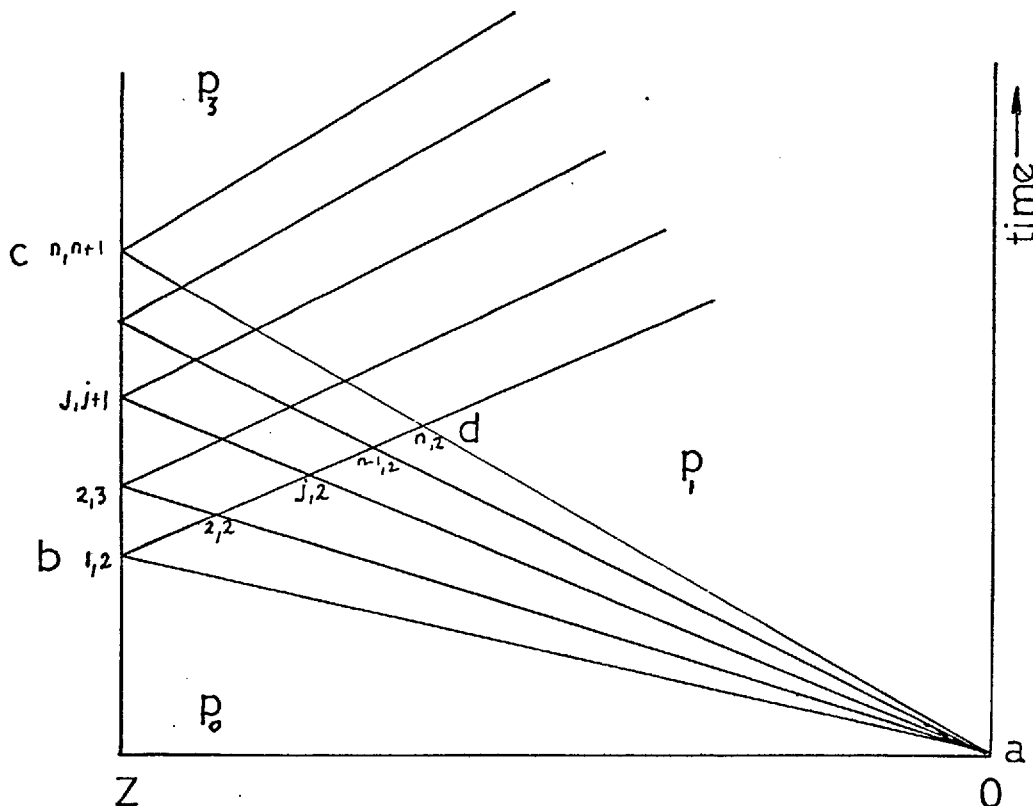


Figure 2.10 Characteristic network for a
centred rarefaction wave

The boundary conditions for the integration over the area of wave interaction are determined from the analytical solution of the simple wave equations. The value of P_1 is determined from P_3 using equations (2.26) and (2.27) and eliminating u_1 giving:

$$c_1 = \frac{1}{2} (c_o + c_3) \quad \text{or} \quad \left\{ \frac{P_1}{P_o} \right\}^{\frac{\gamma-1}{2\gamma}} = \frac{1}{2} \left\{ 1 + \left\{ \frac{P_3}{P_o} \right\}^{\frac{\gamma-1}{2\gamma}} \right\} \quad (2.28)$$

The rarefaction wave is divided into $N - 1$ equal increments each having a pressure step given by $\Delta P = \frac{P_o - P_1}{N - 1}$

This provides the fan of characteristic directions over which the solution of the flow is determined. The conditions of gas velocity and sonic velocity along each of these directions up to the head of the reflected wave, bd, are determined from equation (2.25). For the j^{th} negative characteristic

$$\frac{P_j}{P_o} = \frac{P_o - \Delta P (j - 1)}{P_o} \quad (2.29)$$

and

$$\frac{c_j}{c_o} = \left\{ \frac{P_j}{P_o} \right\}^{\frac{\gamma-1}{2\gamma}} \quad (2.30)$$

thus

$$\frac{u_j}{c_o} = \frac{2}{\gamma - 1} \left\{ \frac{c_o - c_j}{c_o} \right\} \quad (2.31)$$

The gas properties and co-ordinates of each point on the grid are determined using subroutine PT3 from program PISTON. For points on the rear of the vessel the boundary conditions must be imposed, that is next to the wall the gas velocity must be set to zero.

2.4.2 The Rate of Energy Discharge During Venting

The rate at which energy is vented to the atmosphere depends on the effective area of the orifice and also the local gas velocity and density. Thus

$$\frac{dE}{dt} = \frac{1}{\gamma - 1} \frac{P_o}{\rho_o} \dot{M}_o \quad (2.32)$$

where the term $\frac{1}{\gamma - 1} \frac{P_o}{\rho_o}$ is the specific internal energy of the gas in the vessel and $\dot{M}_o$ is its mass flow rate towards the open end. During the initial stages of venting the rate of energy flow from the vessel will increase from zero and become constant at a value corresponding to that at choked conditions. Integrating equation (2.32) gives:

$$E(t) = \frac{1}{\gamma - 1} \frac{P_o}{\rho_o} A_1 \int_0^t \rho u \, dt \quad (2.33)$$

Both ρ and u can be expressed in terms of the pressure P immediately behind the orifice plane

$$\rho = \rho_o \left[\frac{P}{P_o} \right]^{\frac{1}{\gamma}} \quad (2.34)$$

$$\frac{u}{c_o} = \frac{2}{\gamma - 1} \left\{ 1 - \left(\frac{P}{P_o} \right)^{\frac{\gamma - 1}{2\gamma}} \right\} \quad (2.35)$$

To solve equation (2.33) it is required to know the time dependence of the pressure P . The pressure changes that occur at the orifice end of the vessel will be represented by the waveform at the rear of the vessel. Both the theoretical result for a centred rarefaction wave (figure 2.11) and the experimental result for a non-centred wave (figure 2.12) show approximately linear time dependence of pressure at the vessel rear. Any non-

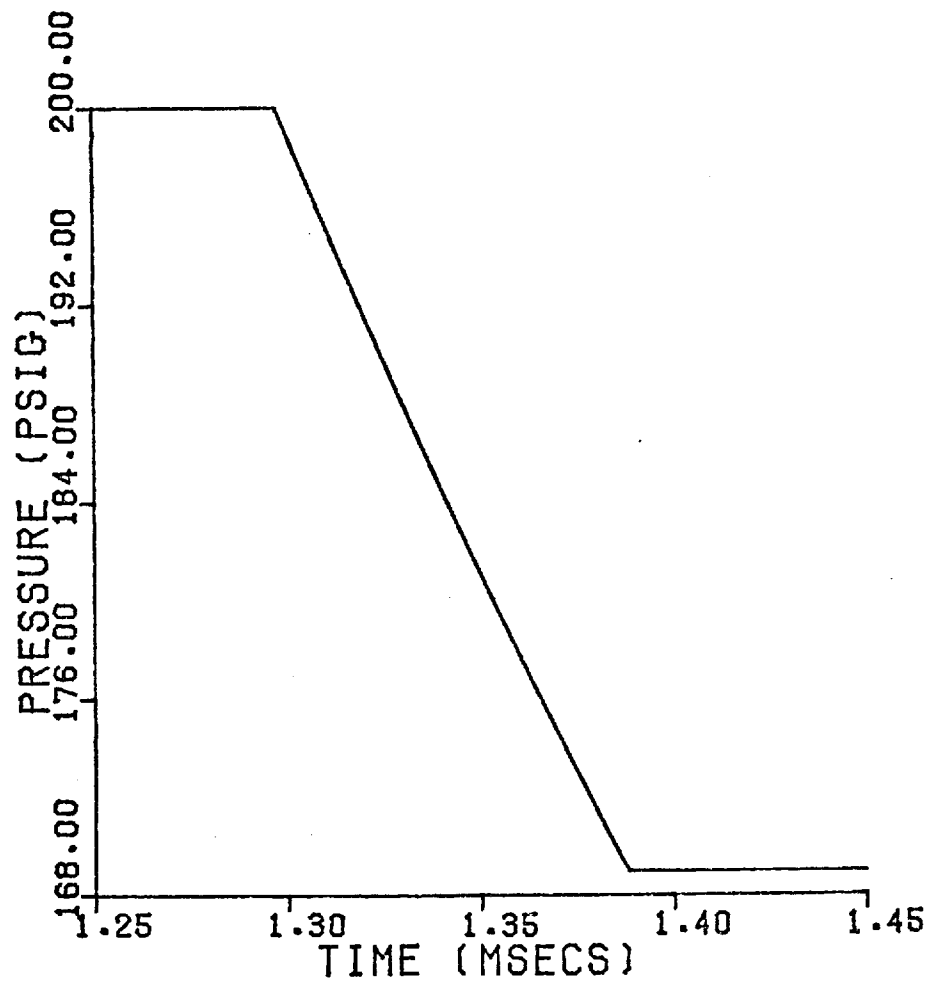


Figure 2.11 Theoretical pressure profile at the rear of
the vessel for a centred rarefaction wave

linear behaviour in the pressure change at the front of the vessel will be transmitted to the rear and will show up in the recordings. It is therefore quite reasonable to express the pressure change at the front of the vessel by a linear equation of the following form:

$$\frac{P}{P_0} = 1 - \frac{P_0 - P_1}{P_0} \frac{t}{t_c} \quad (2.36)$$

where t_c is the throat choking time.

A solution of equation (2.33) is thus obtained by substituting for ρ and u from equations (2.34) and (2.35) and then replacing the unknown pressure term by equation (2.36) leaving the equation in a suitable form for numerical integration.

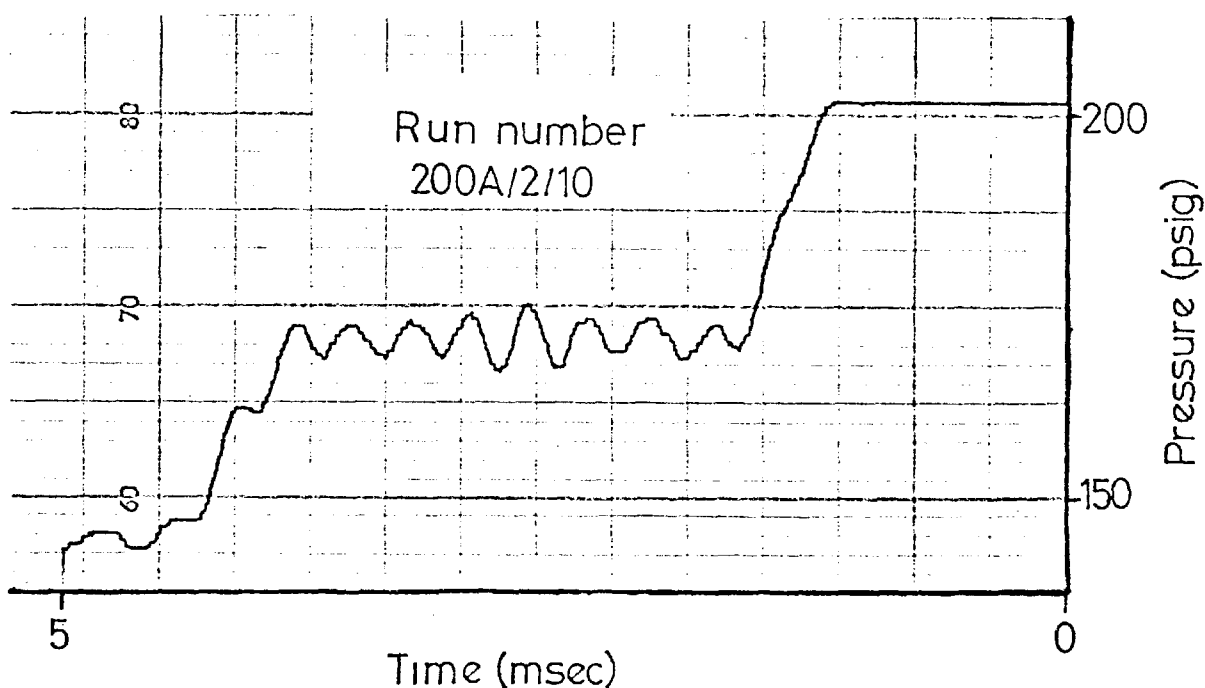


Figure 2.12 Experimental^{*} pressure profile at the rear of the vessel for a non-centred rarefaction wave

* See Chapter Three for the details of the experimental work.

CHAPTER THREE

EXPERIMENTAL INVESTIGATION

The aims of the experimental investigation are to study the formation and propagation of the shock wave resulting from the rapid venting of a high pressure gas from a vessel into the free atmosphere. In order to simplify the gas release mechanism and to maintain some resemblance to the pressure relief devices found in industry, it was decided to employ a cylindrical vessel fitted with a bursting disc assembly at one end.

The measurements made throughout this work fall into two main categories, firstly an analysis of the disturbances set up externally during the venting process and secondly an analysis of the corresponding internal effects. In a little more detail; the external disturbances were investigated in the proximity of the bursting disc by photographic techniques and further downstream by the measurement of the shock wave arrival times at a series of detection points; and a chronological record of the pressure at the vessel rear was made during the venting process, determining the conditions at the orifice and the rate of energy release during the initial stages. It is important that all time measurements are synchronised so that any interdependence between the external and internal effects can be readily ascertained.

Prior to the detailed design of the experimental apparatus some knowledge of the scale of the phenomenon is required to ensure that the changes in blast strength brought about by variation of the experimental conditions are such that they can be detected. Using the point source solution developed by Brode (1955), the total energy originally contained in a vessel can be used as an approximation to E_0 in the scaling length expression without incurring more than an order of magnitude error in the calculated arrival times. Calculations by this method were thus made for estimates of changes

in arrival times of a shock wave, generated by the sudden release of high pressure air from a vessel, for conditions varying over a range of pressures. Using these results and bearing in mind the upper limitations imposed on the experiments by safety considerations it is possible to prescribe the required experimental accuracy for each of the measured parameters for a vessel of ~ 0.3 cu.ft. and an upper limit of bursting pressure for the discs of 20 atmospheres:

- (i) External disturbance photographs over a period of 1.5 msec. with exposure times of $< 1 \mu\text{sec.}$ and a framing rate of 10^5 f.p.s.;
- (ii) Shock arrival times at distances up to 4 ft. from the vessel must be to an accuracy of $\pm 1\%$ over a range from 1 to 4 ms;
- (iii) Bursting pressure up to 20 atm. $\pm 1\%$;
- (iv) Second stage static pressure, $\pm 1\%$; and
- (v) Pressure transition times at the rear of the vessel to within 2% over a time scale of 3 msec.

3.1 The Pressure Vessel and Associated Equipment

A 6 in. bore, 1 ft. 6in. long mild steel tube with $1/4$ in. walls was used to construct a pressure vessel of ~ 0.3 cu.ft. to the specifications laid down in B.S. 1500 and was tested to 30 atmospheres. As shown in figure 3.1 the front flange plate has provision for holding a bursting disc assembly. This assembly is chosen from a series enabling the gas to be vented through orifices with diameters ranging from 1 in. to 6 in. in 1 in. steps. The bursting disc is clamped between two 8 in. diameter plates, the inner one having a 6 in. diameter aperture and the outer a smaller aperture

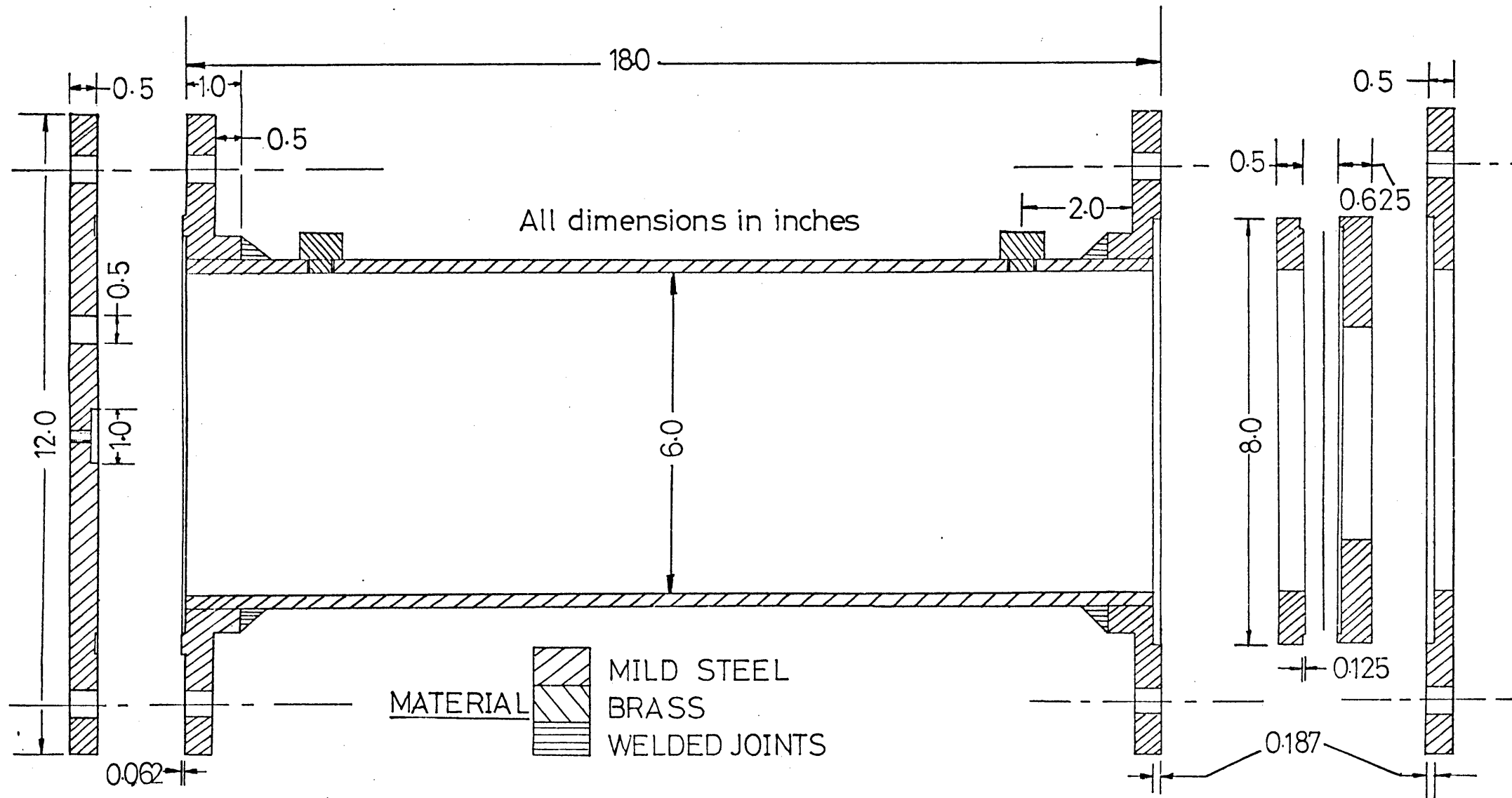


FIGURE 3.1 PRESSURE VESSEL (side elevation)

MATERIAL
MILD STEEL

All dimensions in
inches

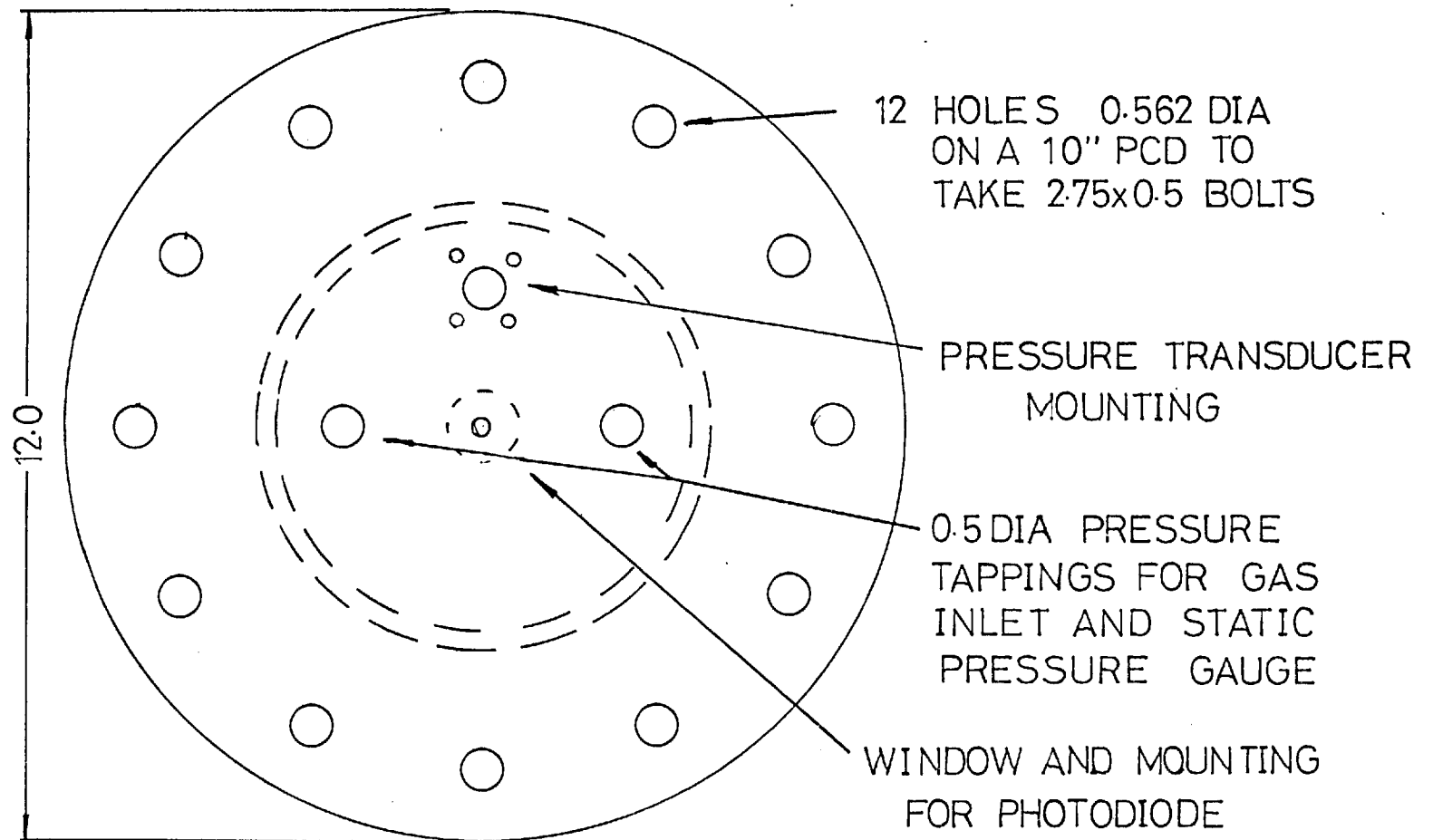


FIGURE 3.2 PRESSURE VESSEL (end elevation)

appropriate to the orifice size required. The plates and disc are held in position by two retaining screws (not shown). The outer plate has a $1/8$ in. radius machined on the inside edge of the orifice to allow the bursting disc to distort freely without shearing around its perimeter.

The flange at the rear of the vessel has three pressure tapings located on a 2 in. radius; two of these are for pressure measurement by a Budenburg gauge and a strain gauge pressure transducer and the remaining one is for connection to a reservoir of high pressure gas. At the centre of this plate is a $1/4$ in. diameter window onto which a photodiode can be mounted. (Figure 3.2). The cylinder itself has provision for fitting a thermocouple at either end which can be used for monitoring the gas temperature during the filling process.

3.2 Photographic Investigation

The number of photographic instruments and techniques developed for the study of high speed motion of fluids cover a wide range of application and are reviewed by Singh (1969). Firstly the three most well known methods of forming an image are those of shadowgraphy, schlieren and interferometry. The type of image required in this work is one that possesses both good spatial and time resolutions and therefore it is desirable to form an image of high contrast. Considering the amount of detail there will be in the area of the bursting disc after rupture, schlieren records are found to be particularly suitable for the quantitative and qualitative study of flows containing shock waves, expansion waves and contact surfaces.

The principle of schlieren photography is based on the change in refractive index of light passing through a disturbance in which there are density gradients. There are several methods of achieving schlieren which are described by North (1959), one of which is by placing a point source of

light at twice the focal length of a concave mirror, (Figure 3.3).

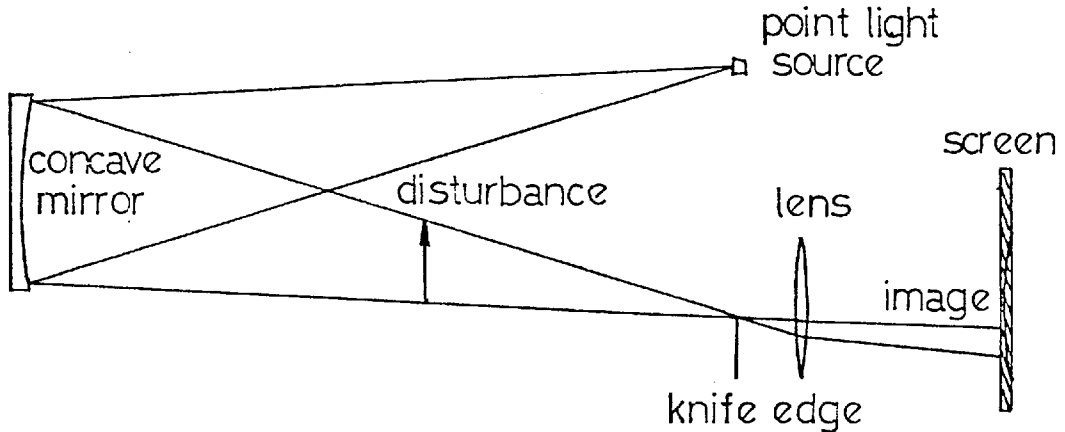


Figure 3.3 Single mirror schlieren arrangement

An image of the light source is formed at the same distance from the mirror on a knife edge which is placed so as to just allow the light beam to pass by. If a disturbance is in the path of the beam refraction occurs and part of the light is intercepted by the knife edge. This has the effect of causing a series of dark and light areas on the photographic record which correspond to regions of different density in the flow field. The image of the disturbance is focused on the screen by a suitable lens.

The cameras used for studying high speed events are usually of two types, those that form streak images and those that have framing facilities. The former provides a high precision in the measurement of time at the expense of spatial resolution and consequently the records are often difficult to interpret. Conventional high-speed framing cameras have maximum rates of about 10^5 f.p.s. with an image size of 10mm square on 35mm roll film.

Latest developments, however, have increased the framing rate to over 10^6 f.p.s. but with a loss of image size and a deterioration in spatial resolution. All the cameras mentioned above use a continuous light source and have high speed moving parts and are therefore prone to mechanical trouble. The camera used in this work is relieved of this potential fault by the use of a multiple spark source with a series of prisms to provide the image sequencing. A detailed description of its operating principles is given below.

3.2.1 The Chronolite Eight Multiple Spark Camera

The principle of the Multiple Spark Camera as supplied by Impulsphysik GmbH. of Hamburg, W. Germany is based on the optical system due to Cranz and Schardin (1929) and obtains separation of each successive image by purely optical means. The essential optical components of this system are the multiple spark head and the recording camera (Figures 3.4 and 3.5), together with the combined power supply and control unit.

The spark head contains eight adjustable spark gaps in a vertical row which can be triggered sequentially from the control unit with a time interval between each flash continuously variable over a range of 1 μ sec. to 1 sec. The spark head can be pressurised with suitable gases or gas mixtures.

The recording camera is composed of a receiving head, the lens, bellows and plate holder. The geometrical arrangement of the camera and light source is similar to that for a single spark schlieren system. The spark head is placed at a distance of twice the focal length from the concave mirror and the eight light beams from the sparks are focused at the same distance from the mirror, after a deflection of 90 degrees by a prism, onto the knife edges in the schlieren head. They are then directed onto the eight adjustable deflecting prisms by means of which the images are

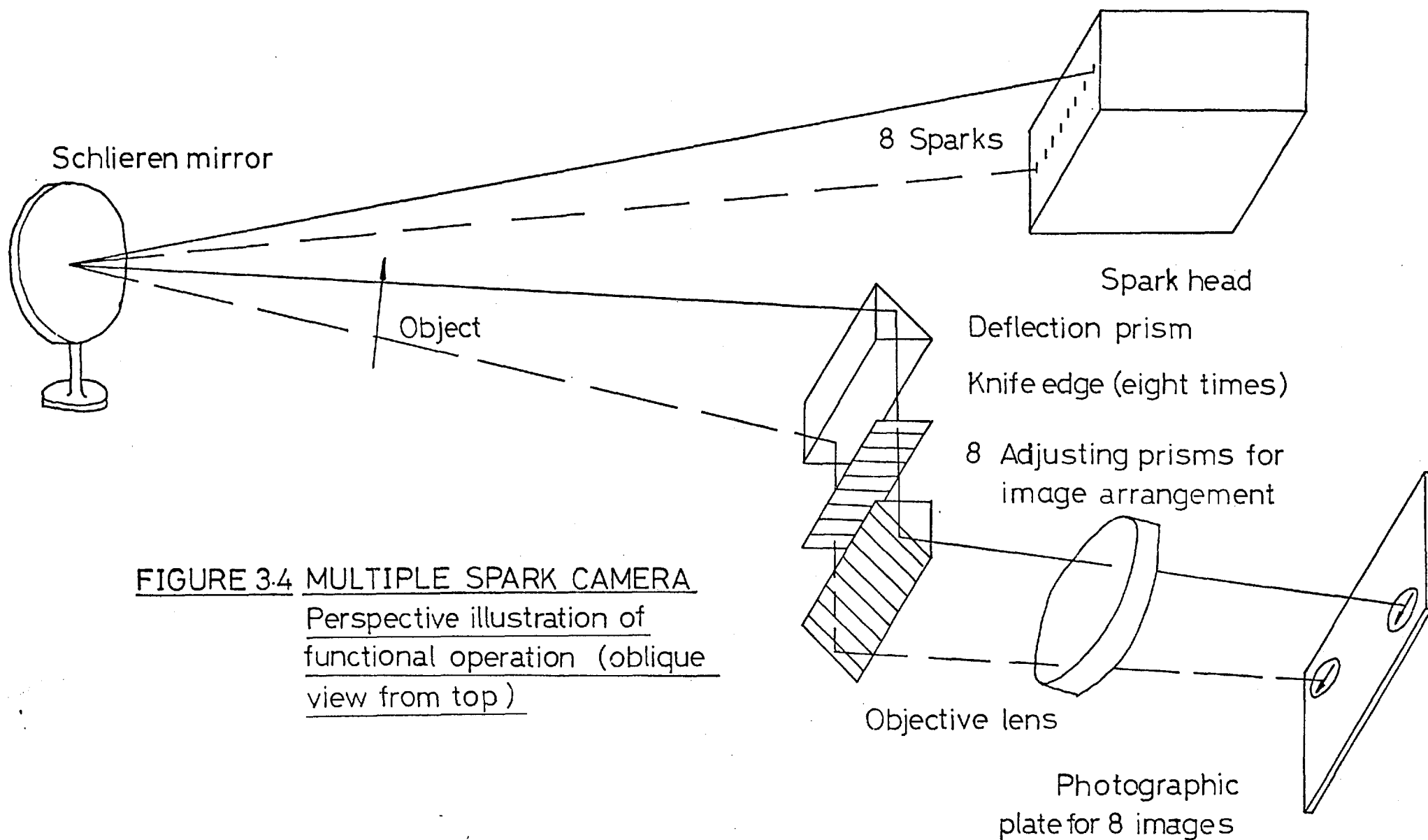


FIGURE 3.4 MULTIPLE SPARK CAMERA
Perspective illustration of
functional operation (oblique
view from top)

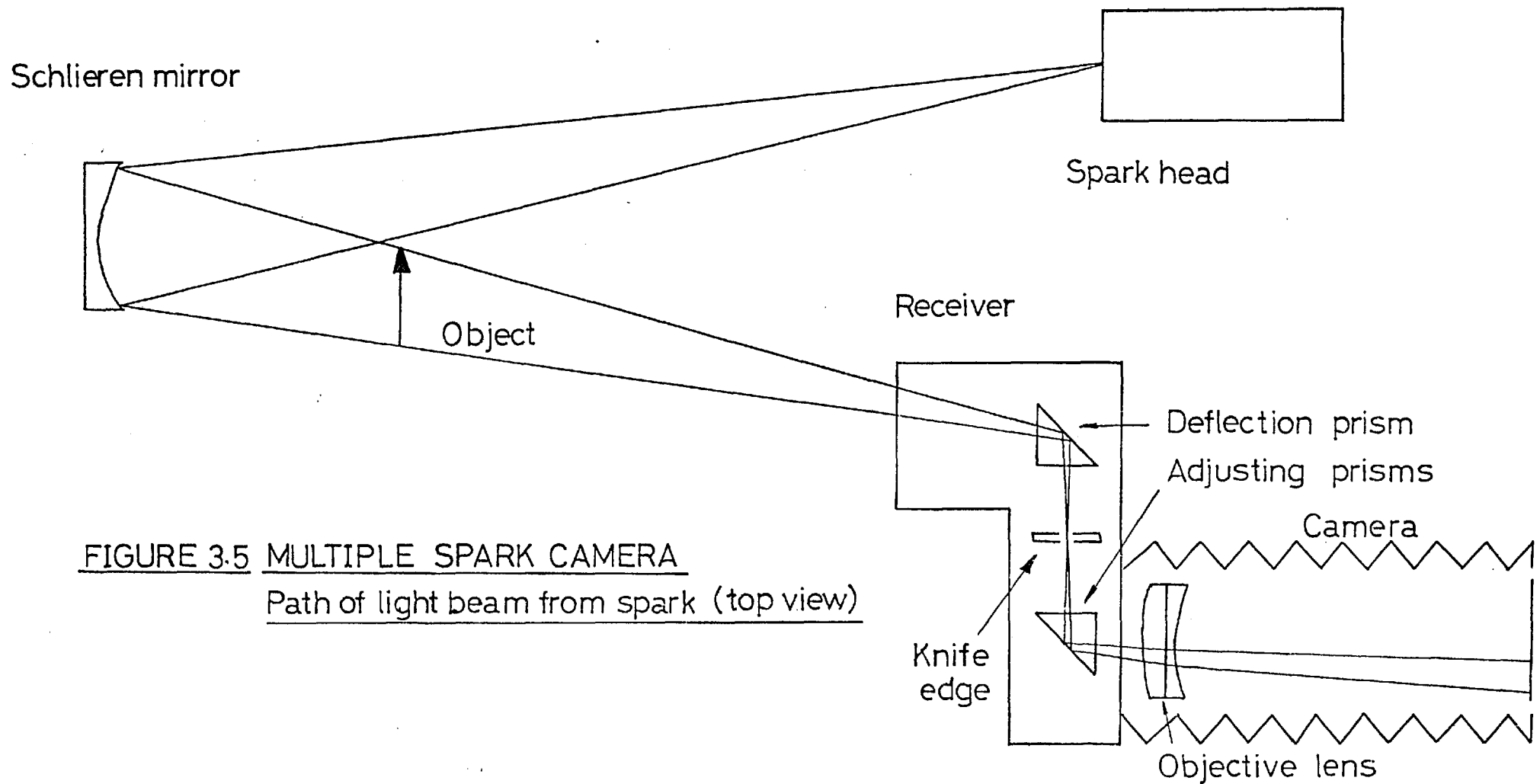


FIGURE 3.5 MULTIPLE SPARK CAMERA
Path of light beam from spark (top view)

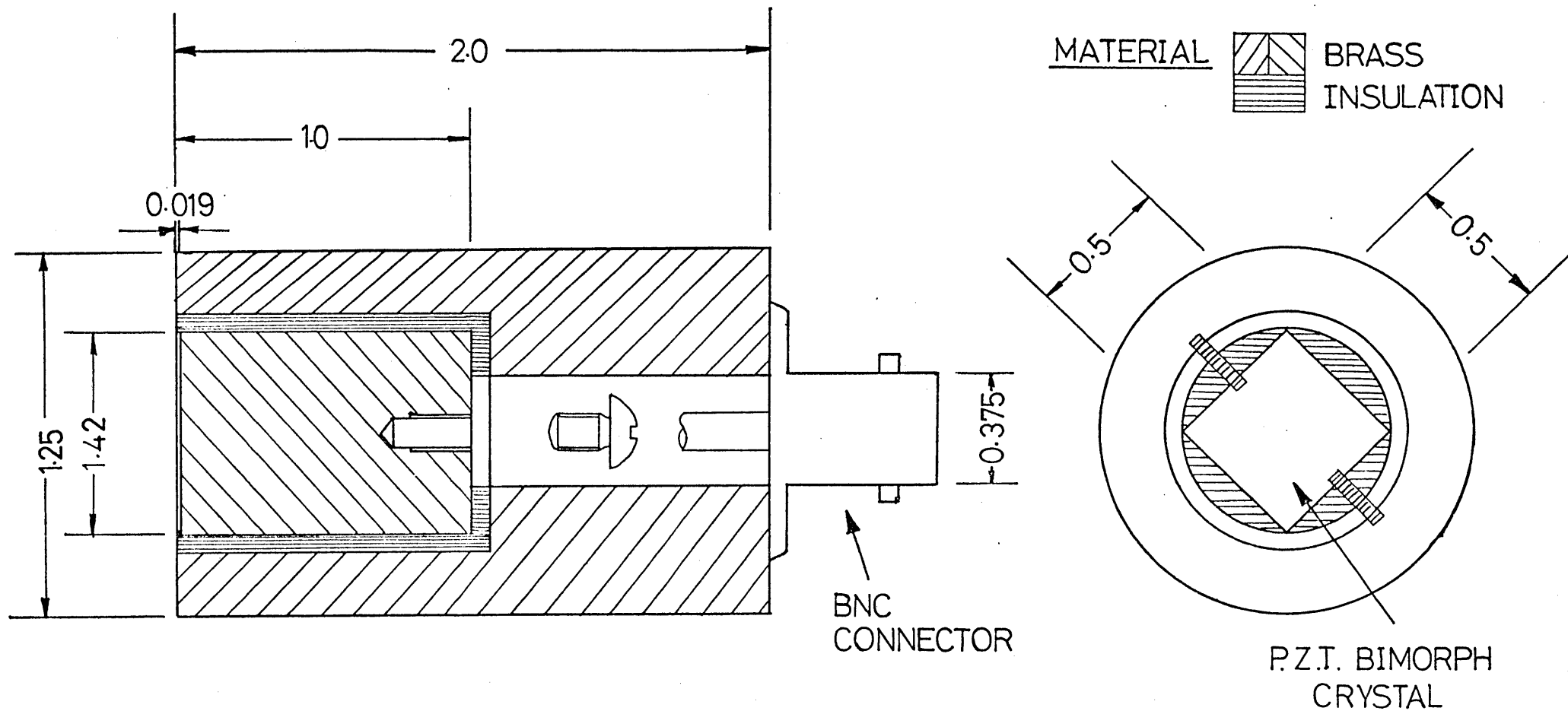
arranged in the desired position on the back of the camera giving a format of eight reproductions of the flow field each approximately 30 mm diameter.

3.3 Shock Detector Gauges and Arrival Time Measuring System

Four piezo-electric gauges have been designed to detect the arrival time of a weak shock wave to an accuracy of one microsecond. To convert a small high frequency pressure transient into a suitable electrical signal it is necessary to have a large area of contact between the sensing element and the medium in which the measurements are to be made. The sensor must be mounted on a solid body which in turn must be held in a rigid manner to minimise undesirable low frequency signals. The gauge, illustrated in figure 3.6, has a sensing element of a $\frac{1}{2}$ in. square crystal of lead zirconate-titanate (PZT Bimorph) as supplied by Vernitron Ltd. Each face of the crystal has a silver coating which acts as the connection to the gauge amplifier. Contact with the underside of the crystal is achieved by attaching it with a conducting adhesive to a central brass plug which is connected to the centre pin of a B.N.C. plug. The earth contact is made to the main body of the gauge with a band of silver paint across the face of the assembly.

The action of a pressure force on a piezoelectric material sets up an internal strain which changes the dipole moments within the molecular structure of the crystal. This induces a surface charge resulting in a potential difference between each crystal face which is detected by a high input impedance pre-amplifier. The one whose circuit is given in Appendix C has an input impedance of 100 M Ω and a rise time of < 0.1 μ sec.

Tests were made on the performance of the gauge with the aid of a shock tube. The resulting signal, illustrated in figure 3.7, was produced by a shock wave of approximately 10 psig overpressure. The rise time, which depends primarily on the transition time of the acoustic wave between each



All dimensions in inches

FIGURE 3.6 SHOCK DETECTOR GAUGE

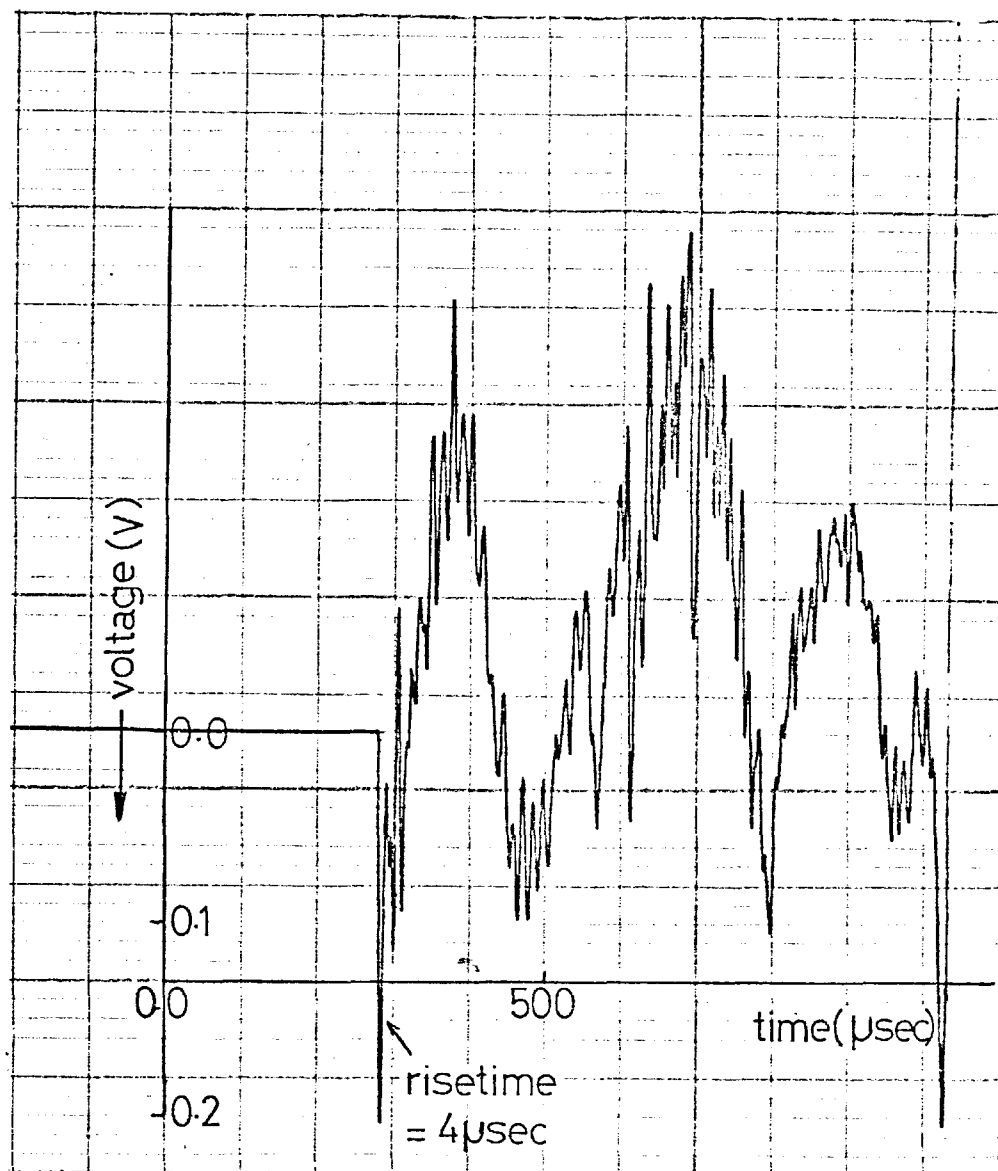


FIGURE 3.7

Output from the crystal of the
shock detector gauge (without amplification)

face of the crystal, is 4 μ sec. for the 0.019 in. element used here and has a peak of 0.2 volts without amplification. The oscillations following the initial rise are due to multiple reflections of the wave off the sides of the gauge; the form of the signal can in no way be used as either a measure of the pressure step behind the shock or the duration of the positive phase of the wave.

The high frequency signal from each gauge is amplified and differentiated with a suitable time constant using the multichannel 10 MHz chronometer described by Way (1971).

3.4 Pressure Transducer

The pressure transducer used for monitoring the step-like wave form at the rear of the vessel during the venting process consists of a strain gauge mounted onto an aluminium diaphragm. The choice of the dimensions and properties of the diaphragm material must be made with a consideration for sensitivity, natural frequency and linear response over the required pressure range. A procedure for compromise among the various transducer qualities is given by Werner (1953). He lists a set of constraints which must be adhered to for a good diaphragm design and then goes on to express these as a set of inequalities. Charts have been constructed for steel, dural and dural metal which show the necessary inequalities and the area of suitable diaphragm dimensions on a log-log plot of pressure against t/a , the ratio of the diaphragm thickness to its radius. The design chart for aluminium is reproduced in figure 3.8 and the details of its construction are given below.

The equations that are used in the design are the usual expressions for tangential and radial strains which only hold for small deflections and for $t/a \ll 0.4$. It is important that the transducer has adequate sensitivity at the minimum pressure for which readings are to be taken. The sensitivity

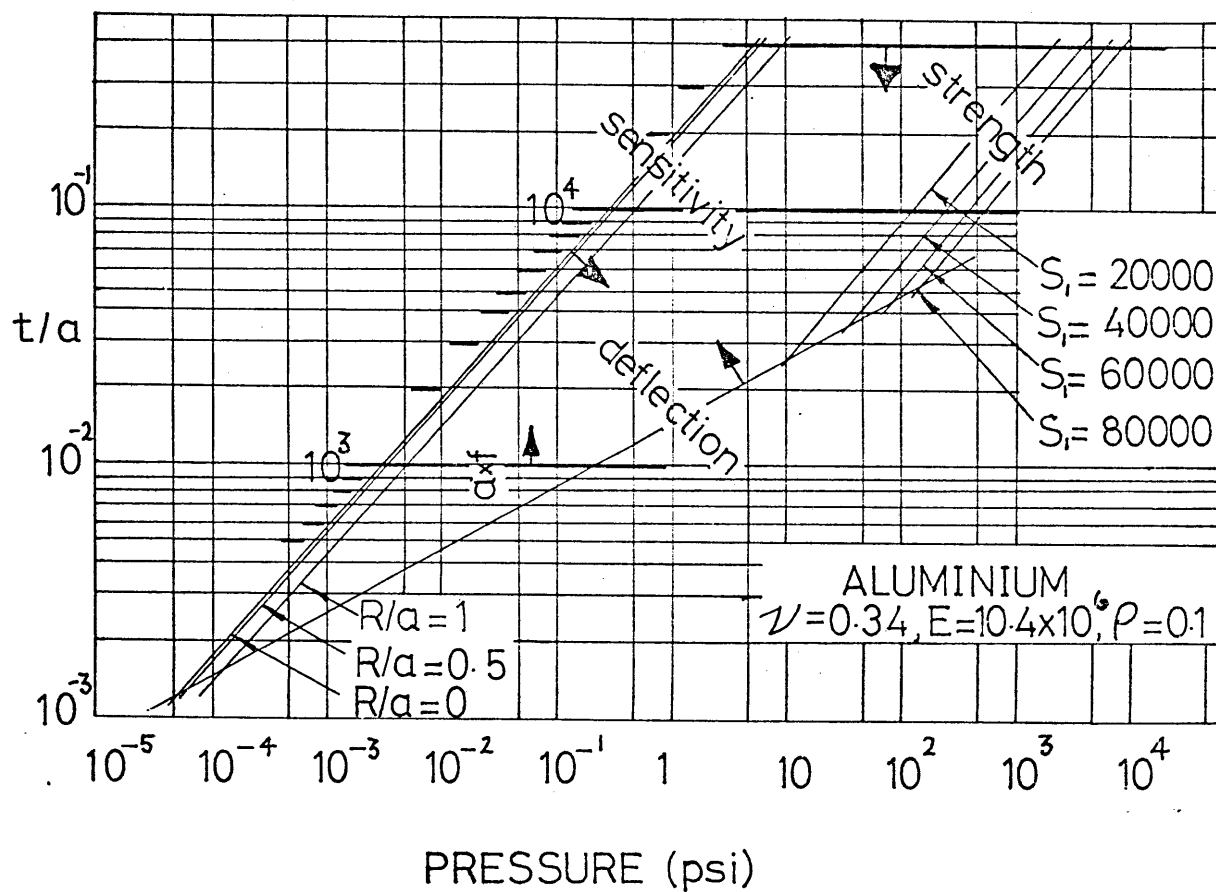


FIGURE 3.8
DESIGN CHART FOR ALUMINIUM DIAPHRAGMS

lines are calculated with a value of strain equal to 10^{-6} at the minimum useful pressure and are plotted for different values of R/a where R is the strain gauge radius. Should the minimum strain requirement differ from 10^{-6} then the curve is used by shifting it along the abscissa by a factor of $\epsilon_{min}/10^{-6}$ or alternatively the pressure scale is divided by this value. The maximum stress (S_{max}) that the diaphragm is subject to during operation must be well within the maximum working stress (S_1) of the material used resulting in the inequality

$$S_{max} \leq S_1/2$$

This constraint on the minimum strength of the material is shown plotted for various values of S_1 is psi. Finally it is important that the natural frequency of the diaphragm is adequate for accurate measurement of the transient pressures. This constraint on the frequency, $f \gg f_{min}$, where f_{min} is several times the maximum frequency of the pressure signal, is plotted as $a \times f$ against t/a on the chart.

A calculation of the minimum electrical signal required which can be suitably amplified suggests that an average minimum strain of 10^{-5} is sufficient in this case. For a useful pressure range from 10 to 300psi the diaphragm dimensions are restricted to the polygon and bounded on the left and right by the 1 psi and 300 psi lines respectively. At high natural frequencies the sensitivity curves begin to restrict the useful pressure range. The maximum frequency that can be obtained while still maintaining the required sensitivity is that corresponding to a value of $t/a = 0.16$. It is convenient to set a to 0.25 in. from which t is found to be 0.04 in., and (from a value of $a \times f = 1.5 \times 10^4$) the diaphragm will have a natural frequency of 60 Khertz. Aluminium has a maximum working stress of 35,000 psi which indicates that the upper pressure limit is approximately 1,000 psi.

The transducer is constructed to these dimensions as illustrated in

figure 3.9. The main body is of mild steel and encloses the diaphragm on all sides except that open to the pressure source. The diaphragm is provided with a thick flange in order to isolate the sensitive area from external strains when the transducer is clamped to the rear of the vessel. The type of strain gauge employed, illustrated in figures 3.10, is a Micro-Measurements 455 JB which is bonded to the diaphragm with the recommended adhesive. The gauge acts as a simple Wheatstone bridge, two arms picking up radial strains and two tangential strains. With this configuration the strain gauge is self-compensating for slowly changing environmental temperatures. The output from the transducer is amplified using the circuit shown in Appendix C and with a suitable combination of gain and excitation voltage a sensitivity of 4 mv/psi is obtained. The calibration was performed with a standard Budenburg Test Gauge and the resulting curve is plotted in figure 3.11 and shows good linearity and undetectable hysteresis. An example of the type of pressure record made by the transducer is shown in figure 2.12 for the first few milliseconds of the venting process for a 200 psig bursting pressure and a 2 in. diameter vent. Calculation of results makes it necessary to have a high degree of accuracy in interpreting this signal for both the pressure levels and the pressure transition times. Therefore the pressure signal is fed to a Data Lab DL 905 transient recorder which has an eight bit digital store giving a pressure resolution of $1/256$ of the full scale setting and a time resolution of 0.1% of the recording time subject to a minimum of 0.5 μ sec. The recorded signal can then be transferred to an oscilloscope or a chart recorder for a detailed analysis of the wave form, or direct time interval measurement can be made by precise location of the stored information.

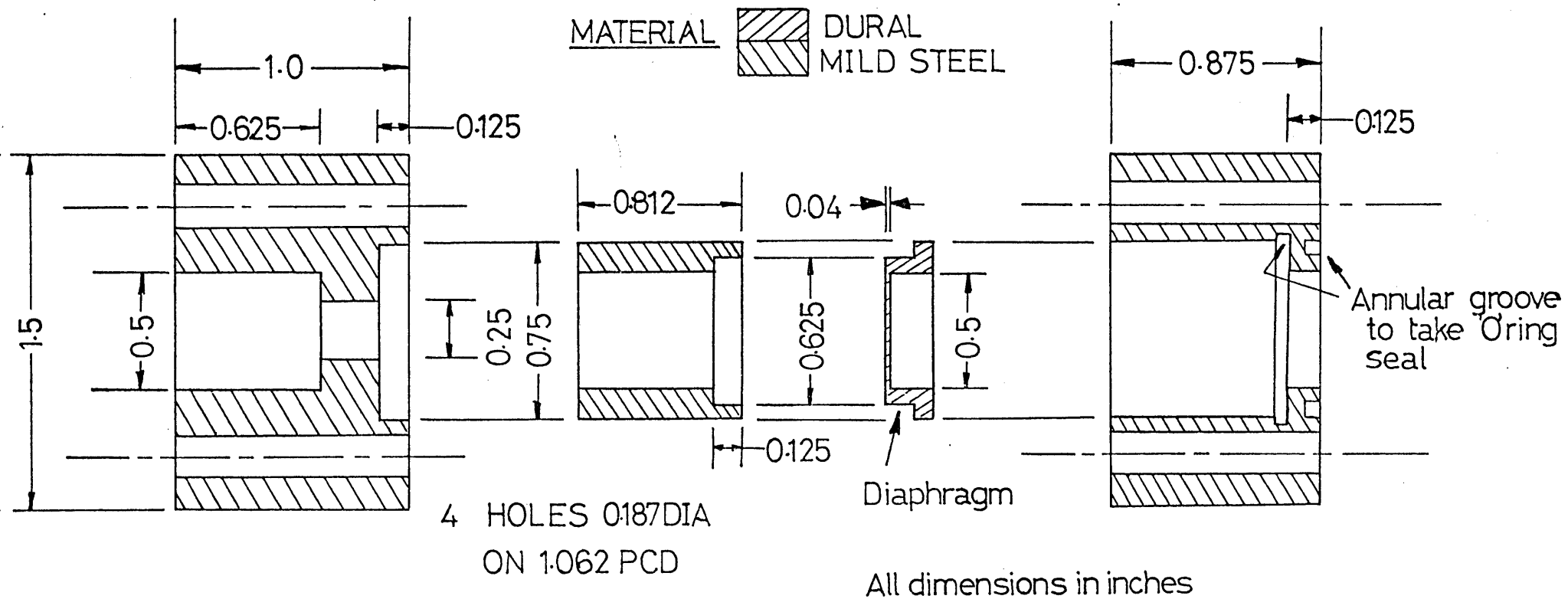


FIGURE 3.9 PRESSURE TRANSDUCER

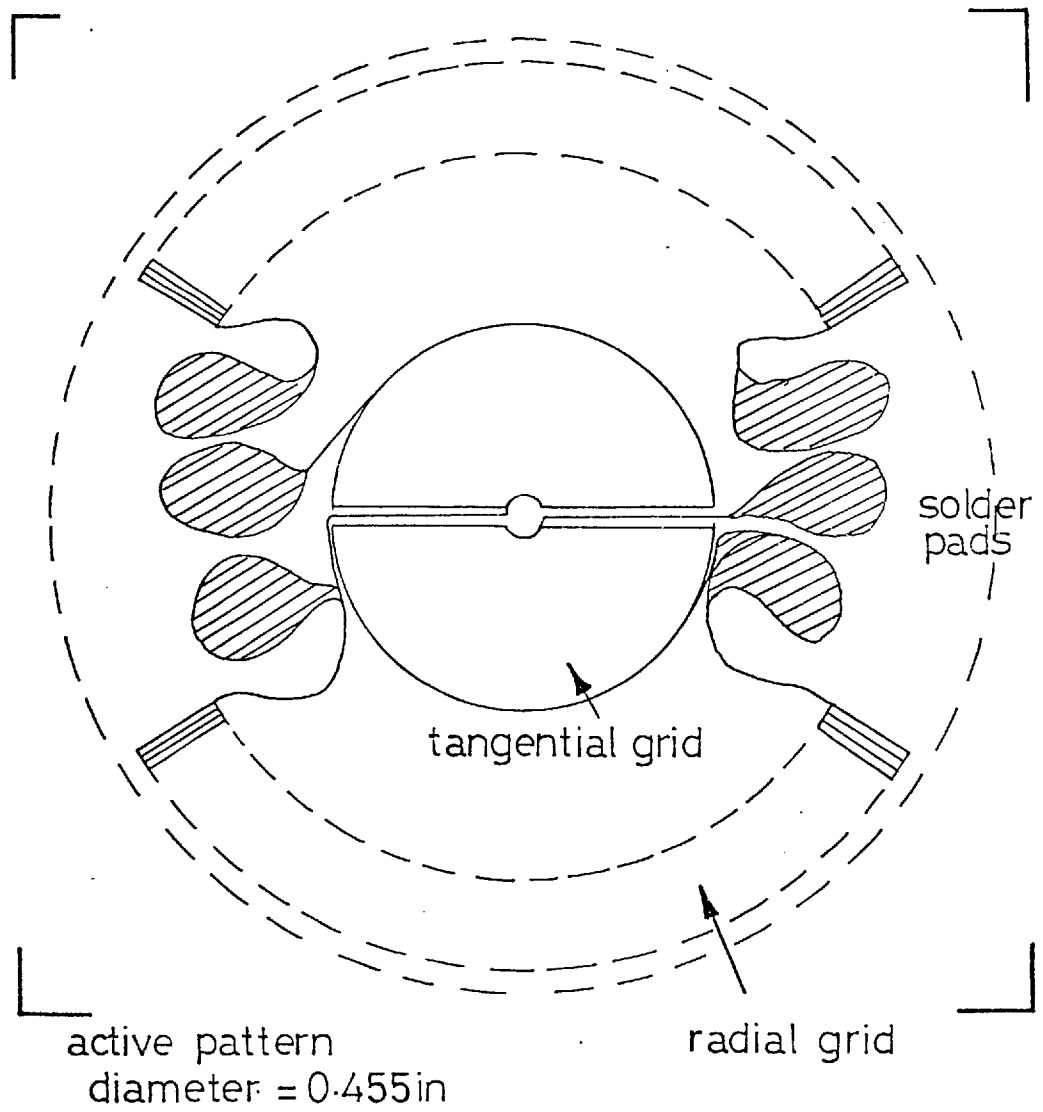


FIGURE 3.10
PRESSURE TRANSDUCER STRAIN GAUGE
CONFIGURATION

PRESSURE TRANSDUCER
CALIBRATION CHART

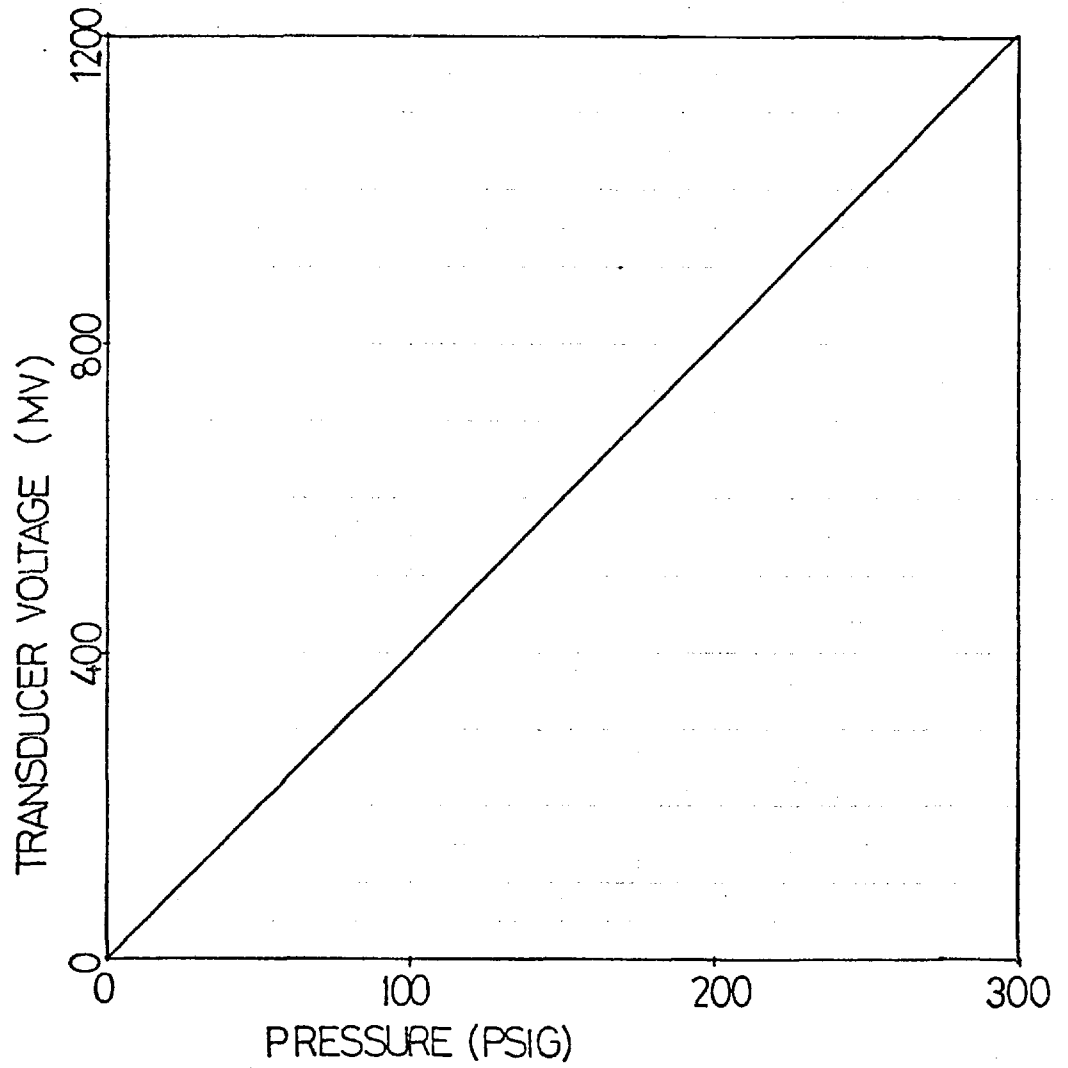


FIGURE 3:11

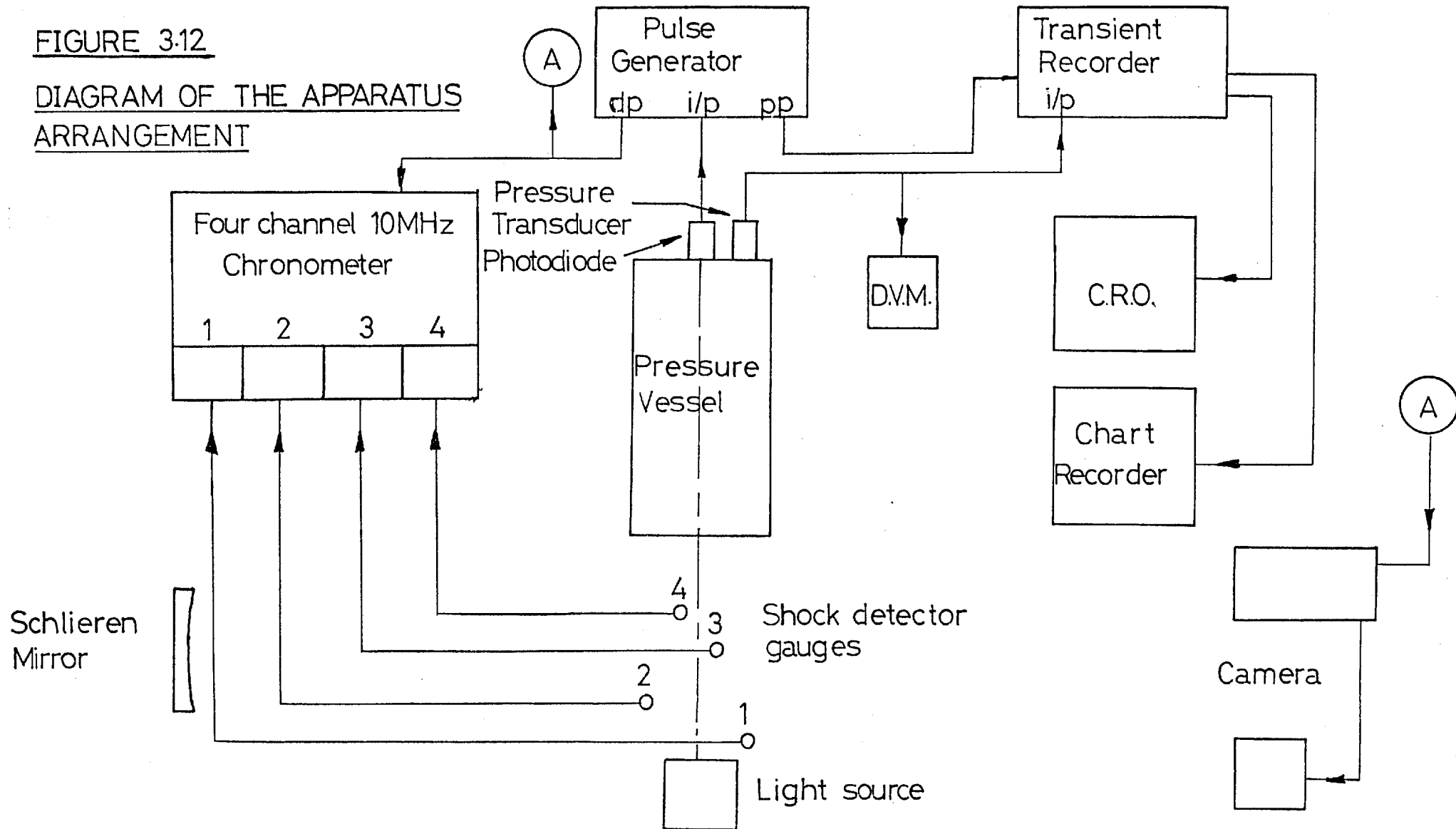
3.5 Experimental Procedure

The arrangement of the apparatus is shown schematically in figure 3.12. The vessel, which is mounted in a rigid manner, has its axis in the horizontal plane and at the same height as the centres of the concave mirror, the spark head and receiving head of the photographic recording system, the latter two being placed about 3 ft. apart to minimise parallax. The shock detector gauges are placed at suitable locations downstream from the vessel and are connected via the preamplifiers to their respective amplifier on each channel of the chronometer. A point light source is situated downstream from the vessel with a lens focusing the beam along the axis onto the photodiode at the rear of the vessel. Next to the photodiode is the strain gauge pressure transducer, from which the amplified output is displayed on the D.V.M. for current pressure readings and fed to the transient recorder for storage. The recorded signal is displayed on a C.R.O. or transferred to a chart recorder when required for further analysis.

Before commencing the experimentation it is necessary to check the alignment and functioning of each part of the apparatus. Firstly to obtain a satisfactory sequential firing of the spark head several critical adjustments have to be made. It is advisable to make the initial adjustment of the spark gaps in air with 4 KV across the discharge capacitors and then to pressurise the spark chamber with a suitable gas or gas mixture. This increases the energy that can be discharged and consequently the light output of each spark. In this work the spark chamber was initially pressurised to 2 atmospheres with nitrogen and the voltage was set to 5 KV; a trial and error procedure was then adopted for the addition of argon until a satisfactory sparking sequence was obtained, the final chamber pressure being approximately 3 atmospheres. Both the plane of the spark gaps and receiving head of the camera must be vertical so the knife edges can be

FIGURE 3.12

DIAGRAM OF THE APPARATUS
ARRANGEMENT



adjusted to give the same density image for each spark. The beam of light from each spark, reflected by the schlieren mirror, is directed to pass in front of the bursting disc such that the vessel is just in the field of view and can be used as a reference plane for distance measurements off the photographs. The camera is loaded with an Ilford HP3 12 cm x 9 cm plate.

To provide a satisfactory triggering pulse the light beam from the point source must travel along the vessel axis and be focused on to the photodiode through the small glass window in the vessel rear. This ensures that there is a minimum of delay between the opening of the bursting disc and the light falling onto the photodiode. The bursting disc thickness and orifice diameter are then chosen to give the required combination of bursting pressure and venting area and the whole assembly is clamped to the front of the vessel. The shock detector gauges are placed at suitable locations in front of the vessel, their centres being within 6 in. and at the same height as the axis. It is important to ensure that the sensitive element of each gauge is normal to the shock wave at the point of impact. This is achieved by aligning each gauge holder in turn with the use of a dummy gauge which has a 0.031 in. diameter capillary bore along its axis. On viewing the centre of the bursting disc through this capillary the front face of the gauge becomes tangential to any spherical disturbance centred at the intersection of the vessel axis and the undistorted disc plane. After re-setting the various instruments the filling process is started by emitting gas from a reservoir to the vessel (after purging for gases other than air). The flow rate is adjusted by a needle valve such that the gas velocity inside the vessel can be taken as zero and the pressure, which is displayed on the D.V.M., as uniform. The whole process takes about 3 to 4 minutes during which time the gas temperature in the vessel is checked, the flashing rate is set, the laboratory is darkened and the cover is removed from the photographic plate. On reaching the bursting pressure of the disc,

rupture occurs and the contents of the vessel are vented through the orifice which is formed. The light beam, previously intercepted by the complete bursting disc, falls on to the photodiode giving a signal which is amplified and used to mark absolute zero in time by feeding to the input of a Venner pulse generator. The output is taken either delayed by a known increment or coincident with the input to start both the flashing sequence of the camera and the arrival time measurements by removing the unit inhibit on the four channels of the chronometer. A prepulse is also taken to trigger the transient recorder. The shock wave which is formed downstream from the vessel strikes each shock detector gauge in sequence, the signal from which inhibits the appropriate channel of the counter. Each scaler then displays the interval between the start signal and the arrival of the shock wave at the gauge locations. The recorded pressure wave form is transferred to the chart recorder and the bursting pressure and the 2nd stage steady pressure are read. For precise determination of the pressure transition times the Z-modulation facility of the recorder is used to locate the points of interest on the signal displayed by the C.R.O. The measurements made are the time between the triggering and the pressure disturbance reaching the rear of the vessel together with the duration of the wave reflection.

It was not found possible to make parallel photographic recordings and arrival time measurements due to the electrical noise from the sparks interfering with the timing system. However, this could be overcome by providing a delayed re-set pulse followed by a start pulse to the chronometer after the photographic record is complete.

The experiments were performed in sets for each combination of bursting pressure and venting area and for two different gases. Approximately 30 runs were made for the first few sets of observations: about 22 arrival time measurements; and 8 to make photographic recordings under the same

nominal conditions of bursting pressure and venting area. Analysis of these results showed that it was only necessary to perform 10 runs for the arrival time measurements in the remaining sets of experiments. Table 3.1 shows the conditions for each set of experiments performed. The nomenclature used to refer to a particular run in any one set of experiments is of the form XXXG/Y/Z where XXX is the nominal bursting pressure (psig), G is the initial letter of the gas used to fill the vessel (air = A, helium = H), Y is the orifice diameter (inches) and Z is the run number in that set. All gases were vented into the atmosphere under normal conditions of temperature and pressure without any obstruction hindering the development of the pressure waves.

TABLE 3.1
NOMINAL CONDITIONS FOR EACH SET OF EXPERIMENTS

Experimental Set	Nominal Bursting Pressure (psig)	Orifice Diameter (inches)	Number of Runs	
			Arrival Times	Photographs
70A/2/-	70	2	21	8
100A/2/-	100	2	21	8
110A/2/-	110	2	20	8
145A/2/-	145	2	21	8
95A/3/-	95	3	25	8
200A/2/-	200	2	10	8
130A/3/-	130	3	10	8
100A/4/-	100	4	10	8
130A/4/-	130	4	10	8
200A/4/-	200	4	10	8
260A/3/-	260	3	10	8
275A/2/-	275	2	10	8
275A/1/-	275	1	10	8
250A/4/-	250	4	10	8
140H/4/-	140	4	10	8
250H/4/-	250	4	10	8
275H/1/-	275	1	10	8
200H/2/-	200	2	10	8
300H/4/-	300	4	10	8

3.6 Experimental Accuracy

3.6.1 Photographic Recordings

The trigger from the photodiode after the bursting disc has ruptured defines the starting point for all time measurements. Ideally this time should be coincident with that at which the disc first ruptures, but due to fragments of the disc intercepting the light beam for extended periods it has to be corrected to obtain absolute values for these measurements. This delay between the initiation of the venting process and the photodiode trigger pulse is readily determined by subtracting the time interval between the trigger pulse and the disturbance reaching the rear of the vessel from the time a sound wave takes to travel this distance. This delay can be determined from the pressure recordings to within $\pm 3 \mu\text{sec}$. Each sequence of photographs is therefore subject to this error of $3 \mu\text{sec}$ and a further error of 1% of the recording interval introduced by the setting of the flashing rate, giving a total absolute error of up to $\pm 10 \mu\text{s}$.

The only significant errors in the distance measurements taken from the photographic plates are those introduced by the lack of definition of the various boundaries. The use of a high contrast and fine grain emulsion keeps this error below 0.03 in.

3.6.2 Shock Distance - Time Measurements

As the chronometer start pulse is provided by the photodiode these time measurements are subject to the same correction factor and error as the photographic recordings i.e. $\pm 3 \mu\text{s}$. The shock detector gauges provide a signal which marks the end of each measurement period and by suitable adjustment the error caused by the triggering levels in the detection circuits can be reduced to less than $1 \mu\text{sec}$. The readings displayed on each channel of the chronometer are also subject to an error of $\pm 0.1 \mu\text{s}$

due to the possibility of the start and stop signals not coinciding with the 10 MHz counting pulse, giving a total absolute error for the arrival time measurements of approximately 4 μ sec. The position of the shock detector gauges is measured to the centre of the undistorted bursting disc plane and can be determined to ± 0.05 in.

3.6.3 Pressure Measurements

Errors arising from the pressure measurements are accumulated at each stage of the processing of the signal from the transducer. Firstly the non-linear response of the diaphragm and hysteresis effects introduce small errors of 0.2% and 0.3% respectively over the 0 - 300 psig. range. To achieve the greatest accuracy in storing the signal in the transient recorder the input gain is adjusted such that the signal covers as much of the range as possible. Errors introduced at this stage can amount to 0.8%. Again it is advantageous to set the controls on the chart recorder so that when the pressure record is plotted it gives near to a full scale deflection and can be read to within 0.5%. A total maximum error of 1.8% is thus incurred for each pressure measurement. A further error of approximately 0.5% is apparent when reading the second stage steady pressure which shows small oscillations due to reflections of the rarefaction wave from the vessel walls.

3.6.4 Pressure Transition Times

The accuracy in obtaining the time duration of pressure transients is determined by the natural frequency of the transducer and the resolution of the transient recorder. The former has a quarter cycle of 4 μ sec and the latter a resolution of 0.1% of the recording time giving a typical error in this measurement of 10 μ sec.

CHAPTER FOUR

RESULTS

In order to analyse the properties of the blast waves generated under different conditions of bursting pressure and venting area the principle readings listed below were made during the experimentation. The average values for each case are given as indicated in the tables that follow.

- (i) Nominal bursting pressure, orifice diameter and vented gas description as per Run Number. (Table 4.1).
- (ii) Nominal bursting disc thickness (Table 4.1).
- (iii) Disc bursting pressure (Table 4.1).
- (iv) Second stage steady pressure (Table 4.1).
- (v) Pressure transition time at vessel rear (Table 4.1)
- (vi) Shock wave arrival time measurements at a number of locations (Table 4.2)

Additionally the coefficients of the polynomial representing the path of the contact surface used in the execution of program PISTON are tabulated in two groups.

- (i) Hypothetical coefficients for a second-order polynomial used in the determination of the part played by each coefficient in the development and propagation of the shock wave in the associated flow (Table 4.3).
- (ii) Coefficients for a fourth-order polynomial which provides a least squares fit to the experimental measurements of the contact surface distance-time relationship (Table 4.4)

The relevance of each reading will become clear from the description of the analytical techniques that are employed to obtain the results in a form which will provide a quantitative assessment of the properties that determine the strength of the blast wave as outlined in Chapter 1.

In Table 4.1 the bursting disc thickness is the nominal value as specified by the manufacturers of the aluminium foil and sheet used for all the experiments. Where two figures are given two discs, one of each thickness, were loaded into the holder. The average bursting pressure is an indication of the total vessel energy content and together with the second stage pressure and the associated transition time provide a means of assessing the rate of energy release during the initial stages of venting. The rate of opening of the disc depends on its thickness and the forces acting upon it, a simple application of Newton's 2nd law to the system provides an estimate of the bursting disc opening time.

Table 4.2 shows the arrival time measurements of the shock wave, averaged over a series of runs, together with the standard deviation for each location of the detector gauge. The distances are measured from the sensitive element on the face of the gauge to the intersection of the vessel axis with the plane of the undistorted bursting disc, these all being at the same height. The results after suitable scaling can be compared with those for a point source explosion.

The polynomial representing the path of the contact surface when running program PISTON on hypothetical data was of the form:

$$x = a_0 + a_2 t^2 \quad (4.1)$$

where x is the contact surface distance at a time t (msec.). The coefficients a_0 and a_2 represent the contact surface initial radius and an acceleration term respectively. The values of the coefficients come in four groups and are chosen to be of the same order of magnitude as that obtained from the experimental work. Program PISTON was run for all the

TABLE 4.1

AVERAGE EXPERIMENTAL PRESSURE READINGS

Run Number	Nominal Disc Thickness (thou.)	Average Bursting Pressure (psig)	Average 2nd Stage Steady Pressure (psig)	Pressure transition time at vessel rear (μsec)
70A/2	4	69.3	57.9	473
100A/2	5	98.1	82.2	417
110A/2	6	109.6	92.0	405
145A/2	8	143.9	122.2	401
95A/3	8	96.3	63.4	740
200A/2	10	197.5	167.3	427
130A/3	10	131.4	88.7	769
100A/4	10	98.4	42.8	1196
130A/4	10 + 5	128.3	61.1	1052
200A/4	10 + 10	197.3	99.8	1084
260A/3	10 + 10	262.2	182.4	624
275A/2	10 + 6	273.6	236.5	432
275A/1	8	274.1	263.8	142
250A/4	26	248.2	124.2	1126
140H/4	16	139.3	59.6	641
250H/4	26	245.2	107.2	731
275H/1	8	286.2	274.0	87
200H/2	10	201.0	167.3	336
300H/4	32	297.6	134.1	756

data for one-dimensional spherical co-ordinates and for one-dimensional planar co-ordinates where indicated in Table 4.3.

The experimental measurements of the distance-time relationships for the contact surface are taken from the photographic images recorded for each set of conditions, and example of which is shown in Plate 2 . The constraints that are imposed on the form of the polynomial fitted to this data arise from two considerations both at the initiation of venting. When

TABLE 4.2

ARRIVAL TIME MEASUREMENTS (MSEC) AND THEIR STANDARD DEVIATION (μ SEC)

Distance Run Number	1.5313		2.0313		2.5313		3.0313		3.5313		4.0313	
(ft)												
70A/2	-		-		2.2318	28	2.6629	27	3.1035	29	3.5331	29
100A/2	1.3330	15	1.7720	24	2.2100	26	2.6429	25	3.0807	20	3.5137	20
110A/2	1.3340	23	1.7686	25	2.2077	27	2.6387	26	-			
145A/2	1.2968	25	1.7226	29	2.1590	25	2.5895	28	3.0206	20	3.4560	21
95A/3	1.3425	23	1.7730	22	2.2120	21	2.6465	30	-			
200A/2	1.2648	29	1.6994	30	2.1197	28	-		-			
130A/3	1.3216	28	1.7753	45	2.1649	30	-		-			
100A/4	1.3967	47	1.8383	48	2.2780	62	2.5650	52	-			
130A/4	1.2260	88	1.6525	91	2.0708	89	2.5055	90	-			
200A/4	1.2007	107	1.6121	109	2.0152	110	2.4433	112	-			
260A/3	1.1717	58	1.5842	59	1.9851	57	2.4102	57	-			
275A/2	1.1731	27	1.5944	29	2.0112	29	2.4448	29	-			
275A/1	1.2244	16	1.6562	16	2.0852	17	2.5233	18	-			
250A/4	-		1.6383	25	2.0369	30	2.4530	27	2.8814	31		
140H/4	-		1.2943	85	1.6545	81	2.0606	86	2.4650	84		
250H/4	-		1.2703	50	1.6265	56	2.0014	61	2.3987	72		
275H/1	-		1.4454	16	1.8531	14	2.2744	15	2.7054	16		
200H/2	-		1.4231	23	1.8249	25	2.2325	23	2.6613	29		
300H/4	-		1.2448	50	1.5787	50	1.9547	55	2.3499	61		

TABLE 4.3
COEFFICIENTS OF THE SECOND -ORDER POLYNOMIAL REPRESENTING
A CONTACT SURFACE TRAJECTORY

Run Number	a_0	a_2	Co-ordinates
A. 1/1	0.1	0.4	spherical
1/2	0.1	0.5	"
1/3	0.1	0.6	"
1/4	0.1	0.7	"
1/5	0.1	0.8	"
1/6	0.1	0.9	"
1/7	0.1	1.0	"
1/8	0.1	1.2	"
1/9	0.1	1.4	"
B. 2/1	0.1	1.0	"
2/2	0.2	1.0	"
2/3	0.3	1.0	"
2/4	0.4	1.0	"
2/5	0.6	1.0	"
2/6	0.8	1.0	"
2/7	1.2	1.0	"
2/9	2.4	1.0	"
2/10	3.2	1.0	"
2/11	6.4	1.0	"
2/12	0.5	1.0	Planar
2/13	0.1	1.0	"
2/8	1.6	1.0	spherical
C. 3/1	0.1	5.0	spherical
3/2	0.2	5.0	"
3/3	0.3	5.0	"
3/4	0.4	5.0	"
3/5	0.5	5.0	"
3/6	0.6	5.0	"
3/7	0.8	5.0	"
3/8	1.2	5.0	"
3/9	2.4	5.0	"
3/10	0.5	5.0	Planar
D. 4/1	0.1	0.5	spherical
4/2	0.2	0.5	"
4/3	0.3	0.5	"
4/4	0.4	0.5	"
4/5	0.5	0.5	"
4/6	0.6	0.5	"
4/7	0.8	0.5	"
4/8	1.2	0.5	"
4/9	1.6	0.5	"
4/10	3.2	0.5	"
4/11	0.1	0.5	Planar

the time is zero, firstly the position of the contact surface is zero; and secondly, the contact surface is stationary. The latter assumption rules out the concept of infinite acceleration and the polynomial takes the form below,

$$x = t^2 (a_0 + a_1 t + a_2 t^2) \quad (4.2)$$

where x is the position of the contact surface (ft) at time t (msec), the values of the coefficients being given in Table 4.4 for each set of experimental conditions.

TABLE 4.4

COEFFICIENTS OF THE FOURTH-ORDER POLYNOMIAL TO FIT THE EXPERIMENTAL MEASUREMENTS OF THE CONTACT SURFACE TRAJECTORY

Run Number	a_0	a_1	a_2
70A/2	4.0611	- 7.0127	3.6143
100A/2	4.2445	- 7.2534	3.6591
110A/2	5.3574	- 10.960	6.6888
145A/2	5.3036	- 10.584	6.3810
95A/3	5.2793	- 10.287	5.9565
200A/2	6.4506	- 13.569	8.3217
130A/3	9.2691	- 28.072	24.836
100A/4	7.3519	- 18.300	13.428
130A/4	10.309	- 26.373	18.164
200A/4	11.899	- 36.548	32.585
260A/3	13.155	- 45.763	47.563
275A/2	10.600	- 30.535	26.019
275A/1	12.376	- 41.778	40.455
250A/4	15.687	- 55.529	56.071
140H/4	45.637	-256.58	391.04
250H/4	39.441	-210.56	322.72
275H/1	41.002	-258.79	476.04
200H/2	31.343	-170.94	275.35
300H/4	32.121	-152.54	209.70

CHAPTER FIVE

DISCUSSION OF RESULTS

It is convenient to relate all the properties of blast waves to those developed under ideal conditions and then to discuss how real effects lead to discrepancies between the ideal situations and actual occurrences. The outcome of the computer studies of blast wave development has been given priority in this chapter, as a consequence of the results being derived from what are the simplest of boundary and initial conditions dealt with throughout this work. A comparison of the computer results with any individual blast wave is made difficult by a lack of reproducibility in the way each blast wave develops, thus requiring average measurements to be used as typical for each case. A comparison of the blast wave strength for the range of experimental conditions present in this work is made with those measured by other workers under different geometrical conditions, but where the amounts of energy available for blast wave development are comparable. These results are then related to those for a point source explosion from which a means of scaling the non-ideal blast is put forward.

5.1 Computer Studies of Shock Formation and Development

Computer studies have been made of the flow produced by an accelerating piston in both one-dimensional planar and spherically symmetric systems. The examination deals with both uniform and non-uniform accelerations in the former co-ordinate system and uniform accelerations in the latter. An extension to non-uniform acceleration for the spherically symmetric system is discussed. The results were produced in both numerical and graphical formats, the former being far too extensive for presentation in full here. However a substantial amount of information can be obtained from the graphical presentations and, for some cases, they provide a clearer picture

of the effects being investigated. Where necessary the principle numerical results have been extracted and presented in tabular form.

5.1.1 One-Dimensional Plane Flow with Uniform Acceleration

The type of wave-form produced for the case of a one-dimensional plane piston motion with uniform acceleration is shown schematically in Figure 5.1. Both axes have dimensions of feet, although the ordinate is reduced time TC_0 , where C_0 is the velocity of sound in the undisturbed gas. This has the advantage of making the gradient of the first disturbance emitted by the piston, the so called acoustic line, take a value of unity and also simplifies the task of determining whether a particular set of co-ordinates is ahead of or trailing this initial disturbance. For a particular point on the graph with co-ordinates $(X_1, T_1 C_0)$; if $X_1 > T_1 C_0$ then the point is ahead of, and if $X_1 < T_1 C_0$, it trails the initial disturbance.

In accordance with the hypothesis put forward in Chapter 1, Figure 5.1 shows no discontinuity in the flow during the initial stages of gas motion. A shock wave first forms on the acoustic line and increases in strength as the pressure waves from behind catch up and coalesce. This shock development process is clearly shown in Figure 5.2 for the same piston motion, where the profiles of both the gas velocity and the sonic velocity are plotted along constant TC_0 lines from Figure 5.1. The distance co-ordinate is scaled such that the piston boundary is always defined by zero, and the position of the outermost disturbance, whether it be the acoustic-wave or the shock wave takes the value of unity. In this case the discontinuity in the flow is clearly first shown at $TC_0 = 0.6$. The development of the shock wave here has no real interest due to the piston being continuously accelerated, resulting in an infinite amount of energy eventually being transferred to the flow.

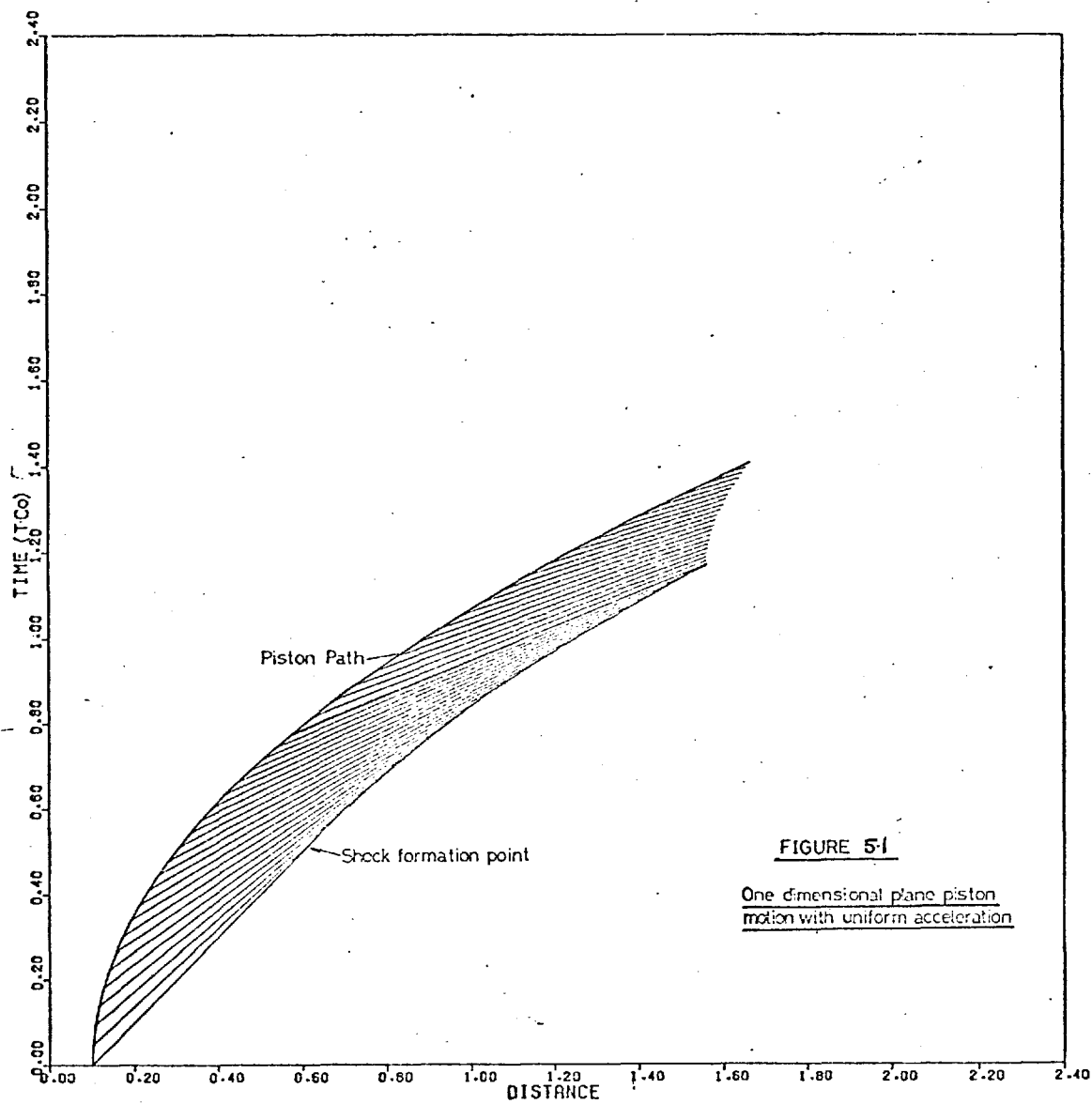
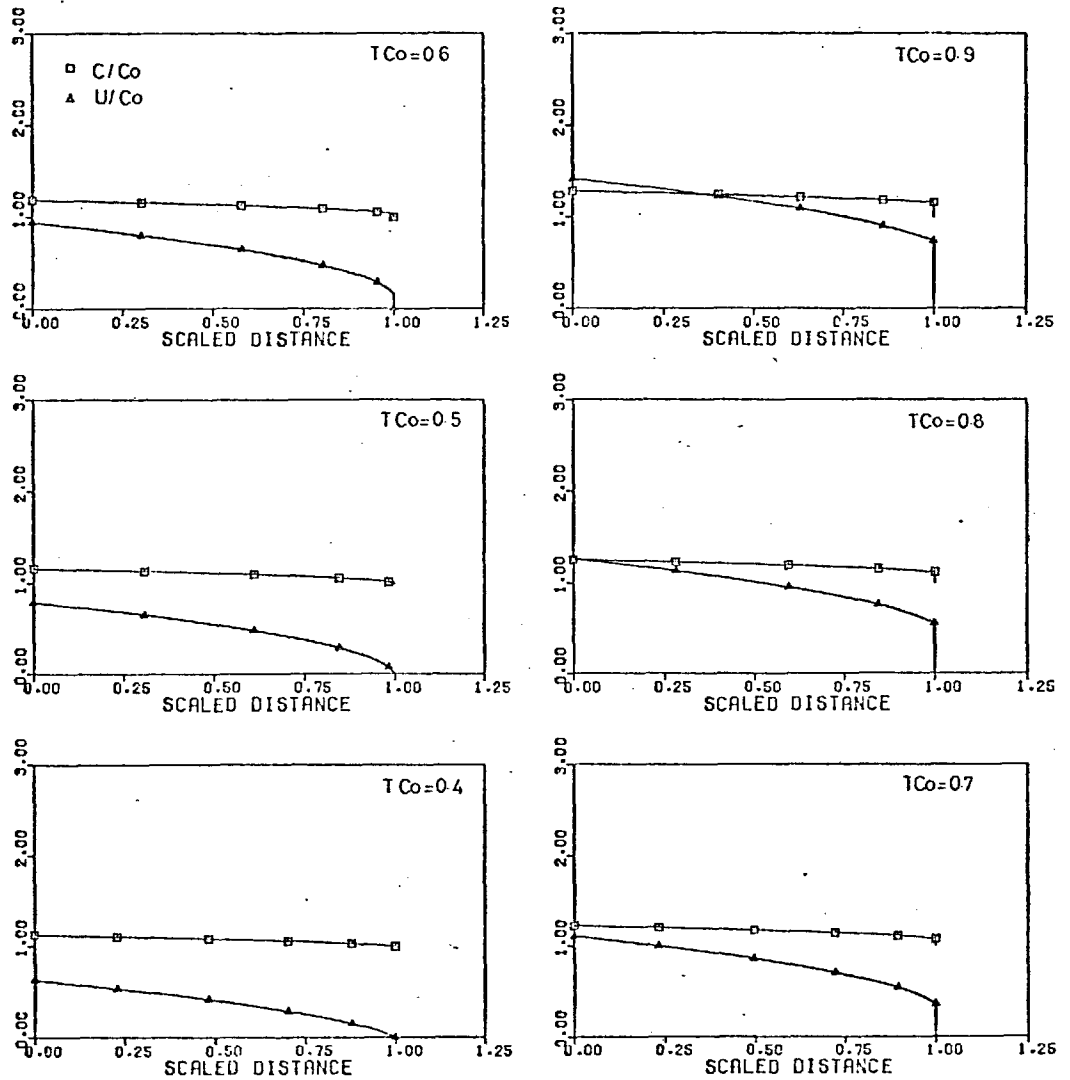


FIGURE 5.2

One dimensional plane piston motion with uniform acceleration
Sonic and Gas Velocity Profiles



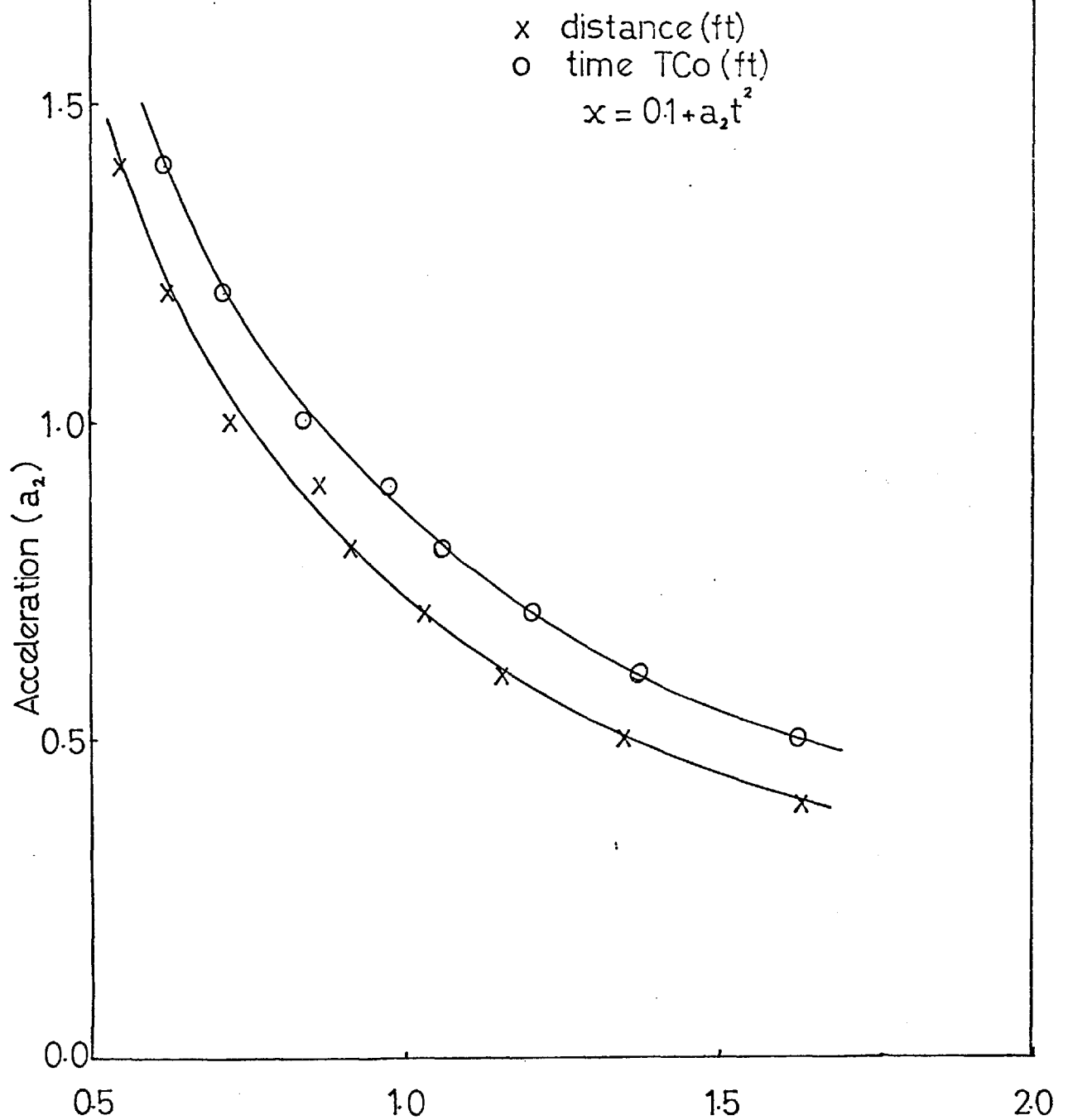
5.1.2 One-Dimensional Spherically Symmetric Flow With Uniform Acceleration

A one-dimensional spherical flow presents a different picture of shock formation as illustrated in Figure 2.1 (following page 30) for a general case. The shock formation point is now behind the acoustic line and as the shock wave increases in strength it will overtake the disturbances ahead of it and eventually enter the region of undisturbed gas. As this example is typical of the shock development in the experimental work it will be dealt with in greater detail at a later stage.

In each case the position of the point of shock wave formation is related to the motion of the piston. For the piston that has the larger acceleration the convergence of the characteristic directions, which depends on the rate of change of $(u + c)$ at the piston boundary, will be greater and thus give an earlier discontinuity. Additionally an identical acceleration, but coupled with a larger piston area will increase the value of c at this boundary, also giving rise to an earlier discontinuity. These two effects of acceleration and initial piston radius are shown in Figures 5.3 and 5.4 using the piston motions listed in Table 4.3. Both the distance and time co-ordinates are plotted so as to indicate the position of the shock formation relative to the acoustic line, the latter being measured from the initial piston radius. In Figure 5.3 the relationship is continuous within the region of experimentation and for all values of a_2 , X is less than TC_0 indicating the shock formation is within the acoustic line.

At low values of acceleration the shock formation is furthest away from the initial piston position and also at the greatest distance behind the acoustic wave. An increase in piston acceleration brings the shock formation point closer to both the initial position and the acoustic line. Figure 5.4 illustrates the effect of changing the initial piston radius for three

Figure 5.3 Shock formation co-ordinates
as a function of piston acceleration



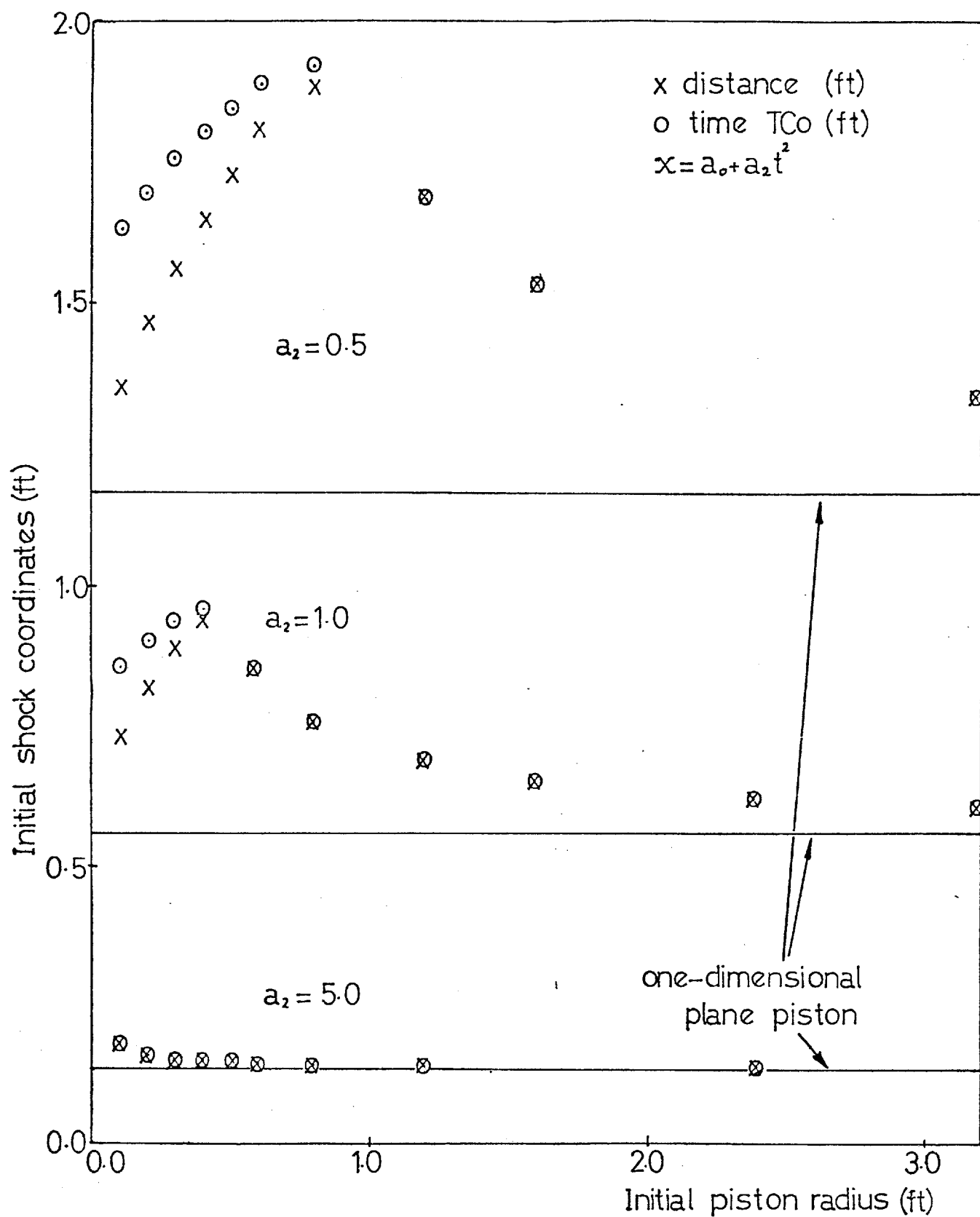


Figure 5.4 Shock formation coordinates as a function of initial piston radius

different accelerations using data of Table 4.3 B, C, and D. These curves all show a maximum value where the shock formation point first coincides with the acoustic line. As the initial piston radius increases from zero the shock formation co-ordinates, initially trailing the acoustic line, increase in value. On reaching the acoustic line this tendency is reversed and further increases in initial piston radius give shock formation points on the acoustic line at decreasing values. The curve becomes asymptotic to the shock formation point for a plane piston motion as would be expected for a_0 , the initial piston radius, approaching infinity.

5.1.3 One-Dimensional Plane Flow With Non-Uniform Acceleration

An extension to a non-uniform acceleration for a one-dimensional plane flow yields a similar result to that for one-dimensional spherical flow, i.e. shock formation behind the acoustic wave. The program PISTON provides the results for this particular case in Figures 5.5 and 5.6, the former showing the piston trajectory and positive characteristic directions and the latter the gas velocity and sonic velocity profiles between the piston boundary and the outermost disturbance for a number of constant TC_0 lines. At $TC_0 = 1.0$ the shock wave is well defined, yet inside the acoustic wave, whereas at $TC_0 = 1.1$ the shock is travelling into an undisturbed gas, replacing the acoustic wave as the outermost disturbance.

5.1.4 One-Dimensional Spherically Symmetric Flow With Non-Uniform Acceleration

The gas motion associated with an expanding spherical piston with a non-uniform acceleration is similar to that of a symmetric explosive source. In this case the movement of the contact surface is governed by both the forces present and the mass of gas being accelerated. Initially these forces will be large and the amount of gas under the influence of the contact surface at a minimum, resulting in a high acceleration. As more gas is set in motion

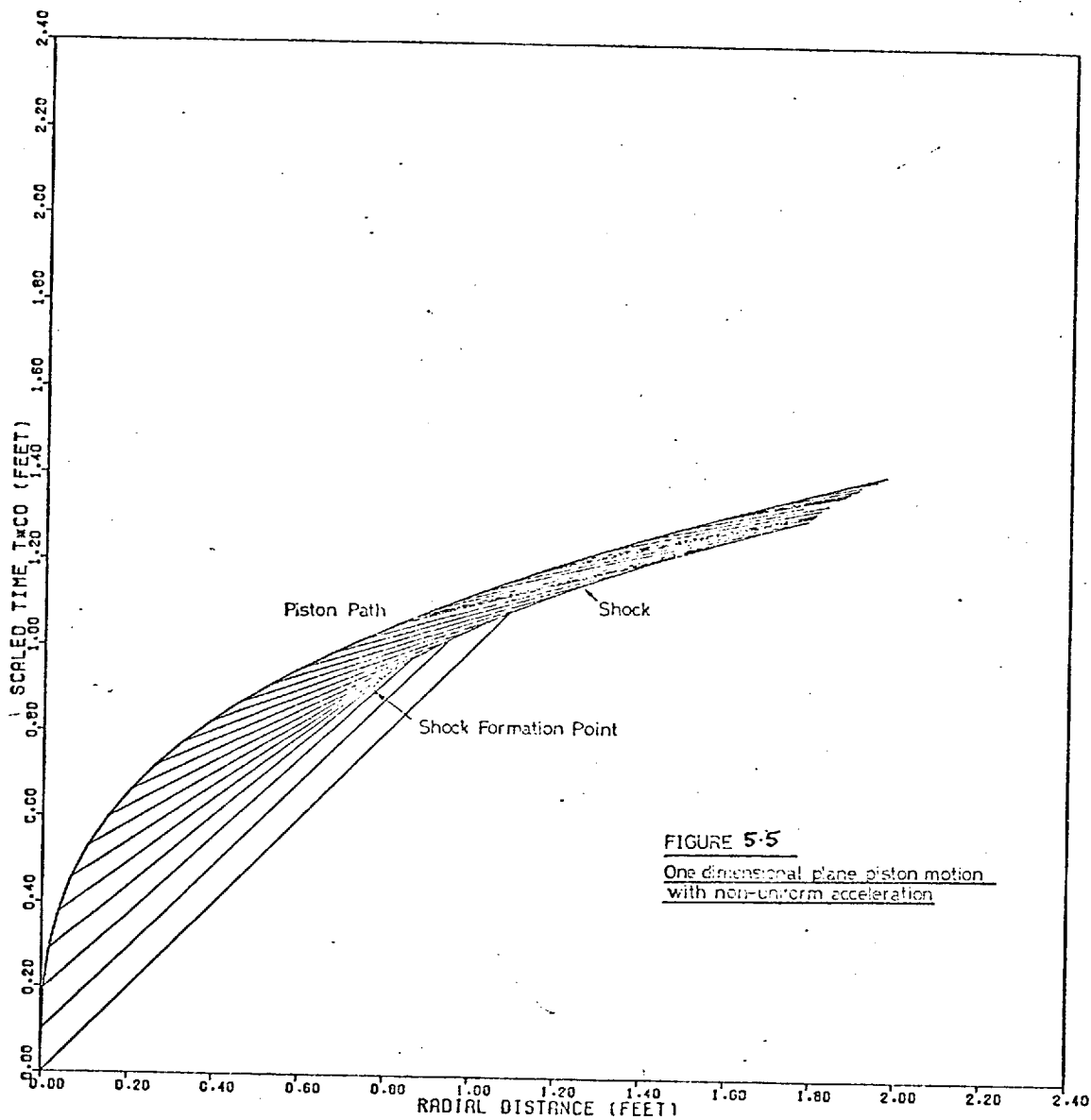
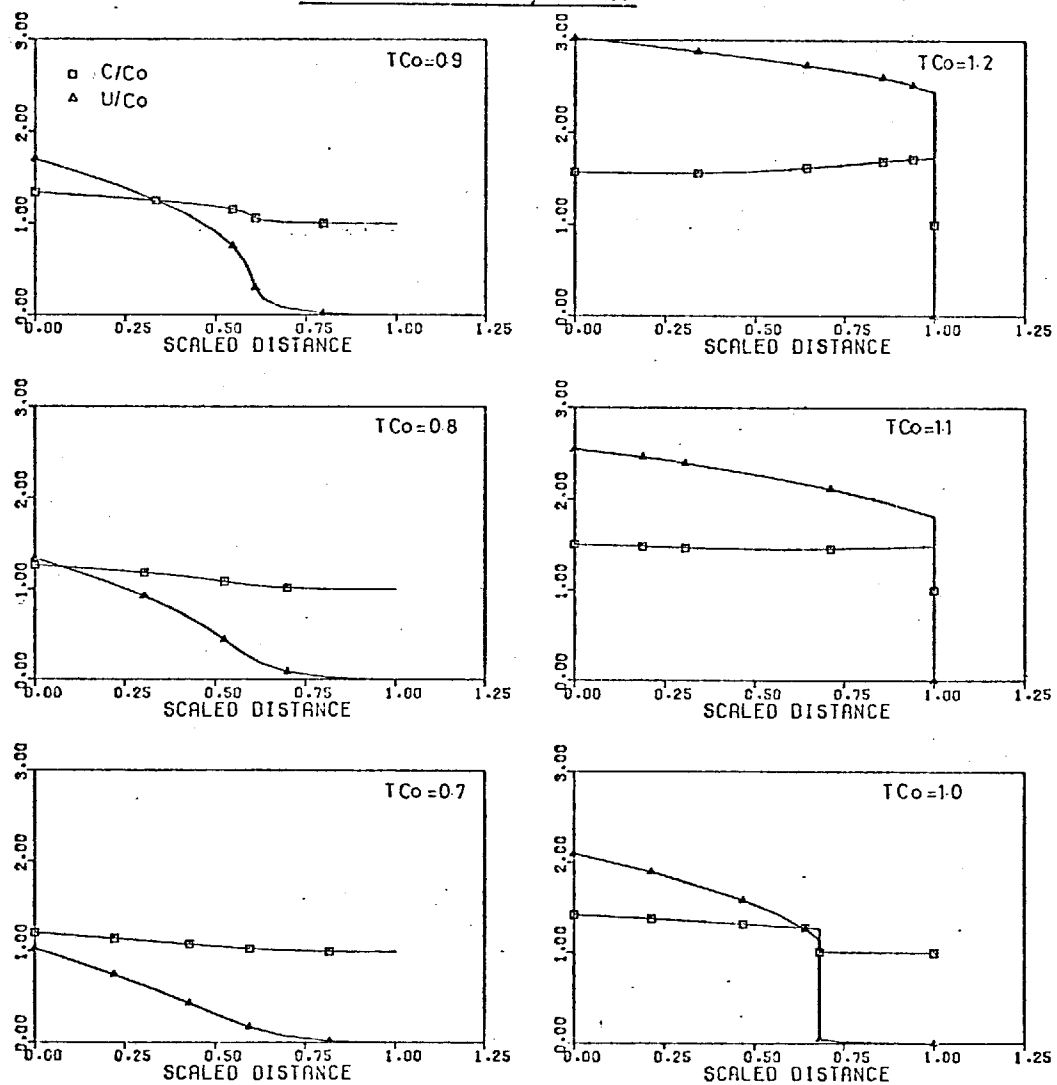


FIGURE 5-5

One dimensional plane piston motion
with non-uniform acceleration

FIGURE 5-6

One dimensional plane piston motion with non-uniform acceleration
Sonic and Gas Velocity Profiles



the driving force of the explosion will diminish and the acceleration of the contact surface will decrease and eventually become negative as all the energy is dispersed. The characteristics of such a flow will be dominated by two regions, one of compression and one of expansion or rarefaction. The former is subtended by an accelerating piston where the characteristic directions converge making it possible for a shock wave to form. The adjacent rarefaction region is subtended by a decelerating piston where the flow is expanding and the motion of the gas decreasing. A more detailed analysis of these two regions is made in the following section which deals with the computer simulation of a real explosion by an expanding spherical piston.

This method of describing the flow and shock wave development associated with an expanding piston is only suitable for an ideal explosion and could not be expected to deal rigorously with the complicated flow patterns that are set up in a real situation. Its validity, however, for the former application is confirmed by a comparison with the work of others, particularly that of Courant and Friedrichs (1948) and Jones (1958) as mentioned in Chapter 1 (pages 8 and 9). A direct comparison of the $T_s C_o$ co-ordinates determined by program PISTON for a one-dimensional plane flow with those obtained from equation (1.9) is made in Table 5.1 below for the three accelerations considered.

TABLE 5.1

COMPARISON OF THE SHOCK FORMATION CO-ORDINATES DETERMINED BY
PROGRAM PISTON AND BY ANALYTICAL MEANS

Acceleration (a_2)	$T_s C_o$ (Program PISTON)	$T_s C_o$ [Equation (1.9)]
0.5	1.165	1.06
1.0	0.556	0.530
5.0	0.131	0.106

A similar comparison for a non-uniform acceleration with a value of $a_3 = 1.0$ substituted into equation (1.11) gives a figure of 0.925 for $T_{s0} C$ whilst the corresponding value from Figure 5.5 is 0.910.

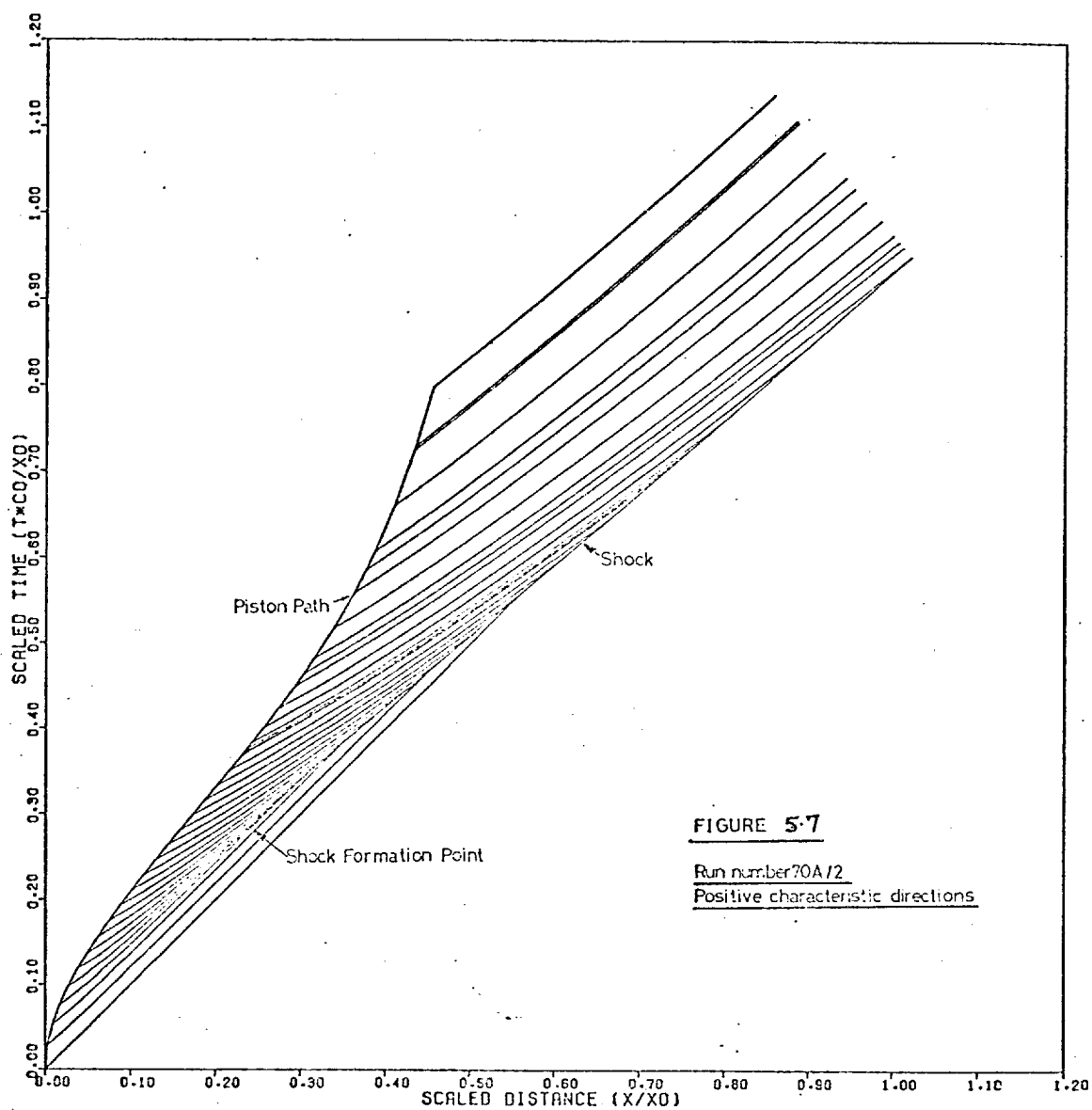
5.2 Computer Studies of The Gas Flow Associated With a Spherical Piston Movement Representing the Experimentally Determined Motion of a Contact Surface in an Explosive System

The flow of gas has been determined for the case of a spherical piston moving into the atmosphere in such a way that its trajectory represents the motion of the contact surface present when a high pressure gas is released from a cylindrical vessel. The trajectory of the piston is defined by using the method of least squares to fit a polynomial to the experimentally determined distance-time data. The constraints on the equation that have to be satisfied are that initially the piston velocity and its distance co-ordinate are both zero. The distance-time measurements are made off photographic glass-plate negatives from one of which Plate 2 is printed. The co-efficients of the polynomial, which has the form:

$$x = t^2 (a_0 + a_1 t + a_2 t^2) \quad (5.1)$$

were given for each set of experimental conditions in Table 4.4. It is evident that in each case the motion of the piston is represented by an initial acceleration followed by a period of attenuation. The range over which the polynomial forms a good fit to the experimental data is well covered by these two periods and any subsequent motion is not representative and has to be ignored.

The solution of the flow for such a piston motion is performed by program PISTON by developing the network of characteristic directions between the piston path and the outermost disturbance. The form of graphical output is shown in Figure 5.7 for the results obtained from a 70 psig air blast over



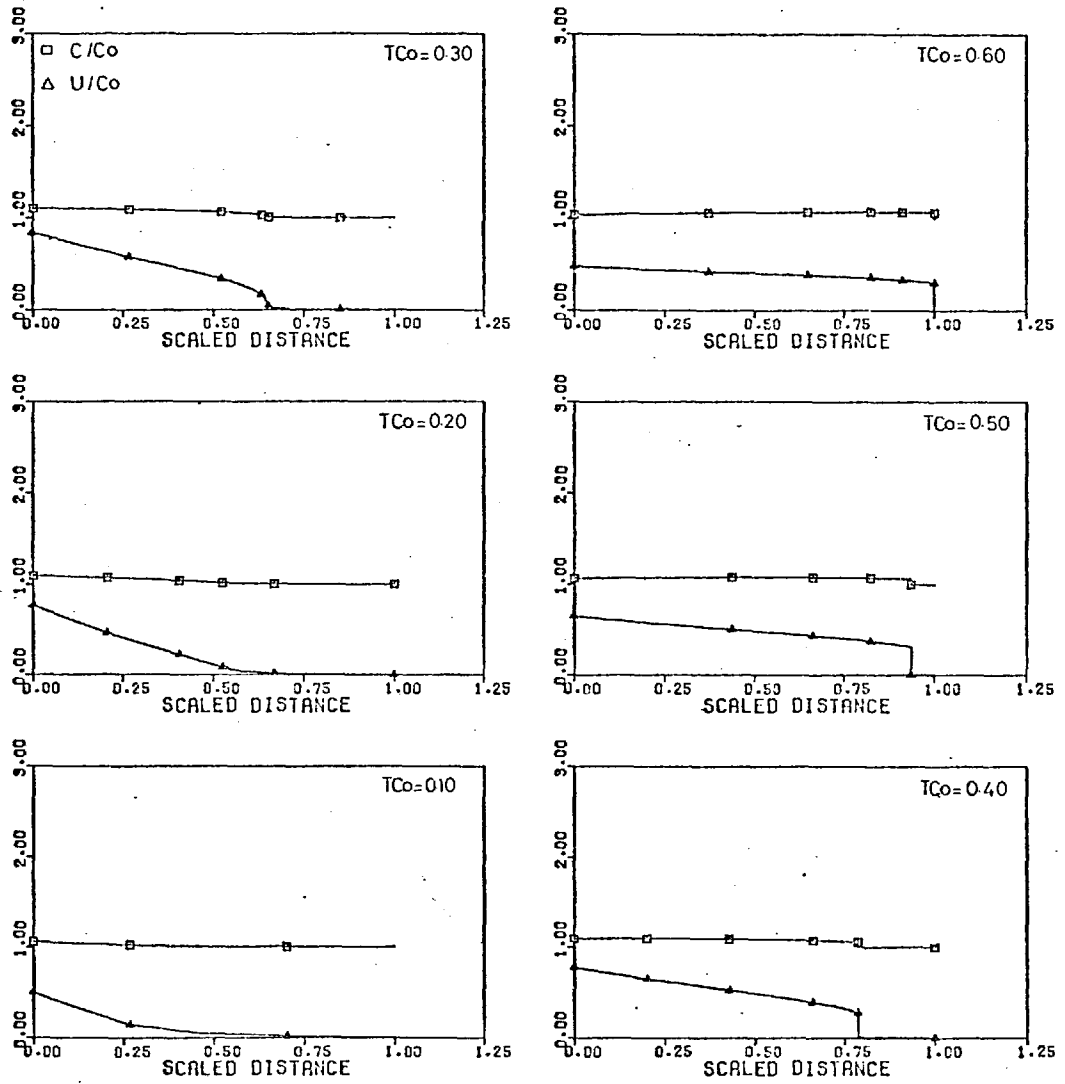
a 2 in. diameter vent; the positive characteristic directions are included. The trajectory of the piston is of the form previously described. As would be expected for a spherical one-dimensional flow the shockwave forms inside the acoustic wave, gradually accelerates as the waves from behind catch up and coalesce, and emerges into the undisturbed region. The period of piston acceleration gives rise to a compression wave which influences the shock development upto the point where the tail of the wave makes an intersection. The tail has its origin where the piston trajectory supports a point of inflection, i.e. where the acceleration is zero. The subsequent deceleration period causes a rarefaction wave to develop, the head of which is co-linear with the tail of the compression wave. The effect of the rarefaction wave on the shock development is one of attenuation; this of course is amplified by the spherical co-ordinates of the system and is present throughout the whole of the period investigated. However at small times the forces present are at their greatest and overcome the attenuation effects to give a nett accelerating motion to the gas flow. The sonic and gas velocity profiles between the piston path and the outermost disturbance are shown in Figure 5.8 for a number of TC_0 values. The formation of the shock wave inside the acoustic line and its advancement to the front of the disturbed region are clearly illustrated. The strength of the shock wave attained in the explosion depends on the magnitude and duration of the accelerating phase which is a function of both the rate of release of the energy and the period of influence of the contact surface on the surrounding gas.

Figures 5.9 to 5.11 are a series of plots illustrating the changes brought about in the shock formation and its subsequent development by different piston accelerations. The one pertaining to the air blast shows a much later shock formation time than those of the helium blasts. Also the distance between the piston surface and the shock wave is substantially

FIGURE 5-8

Run number 70A/2

Sonic and Gas Velocity Profiles



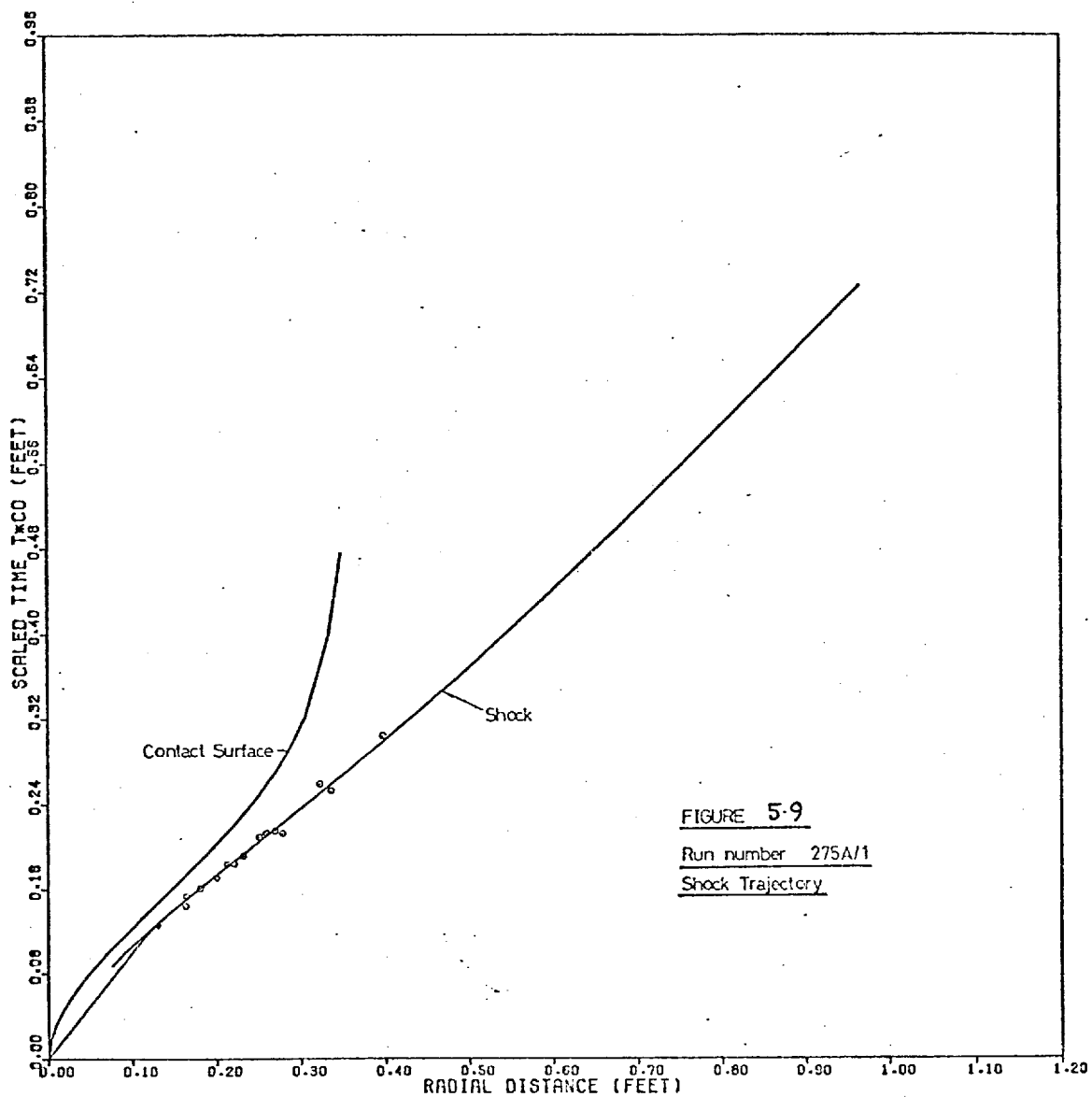
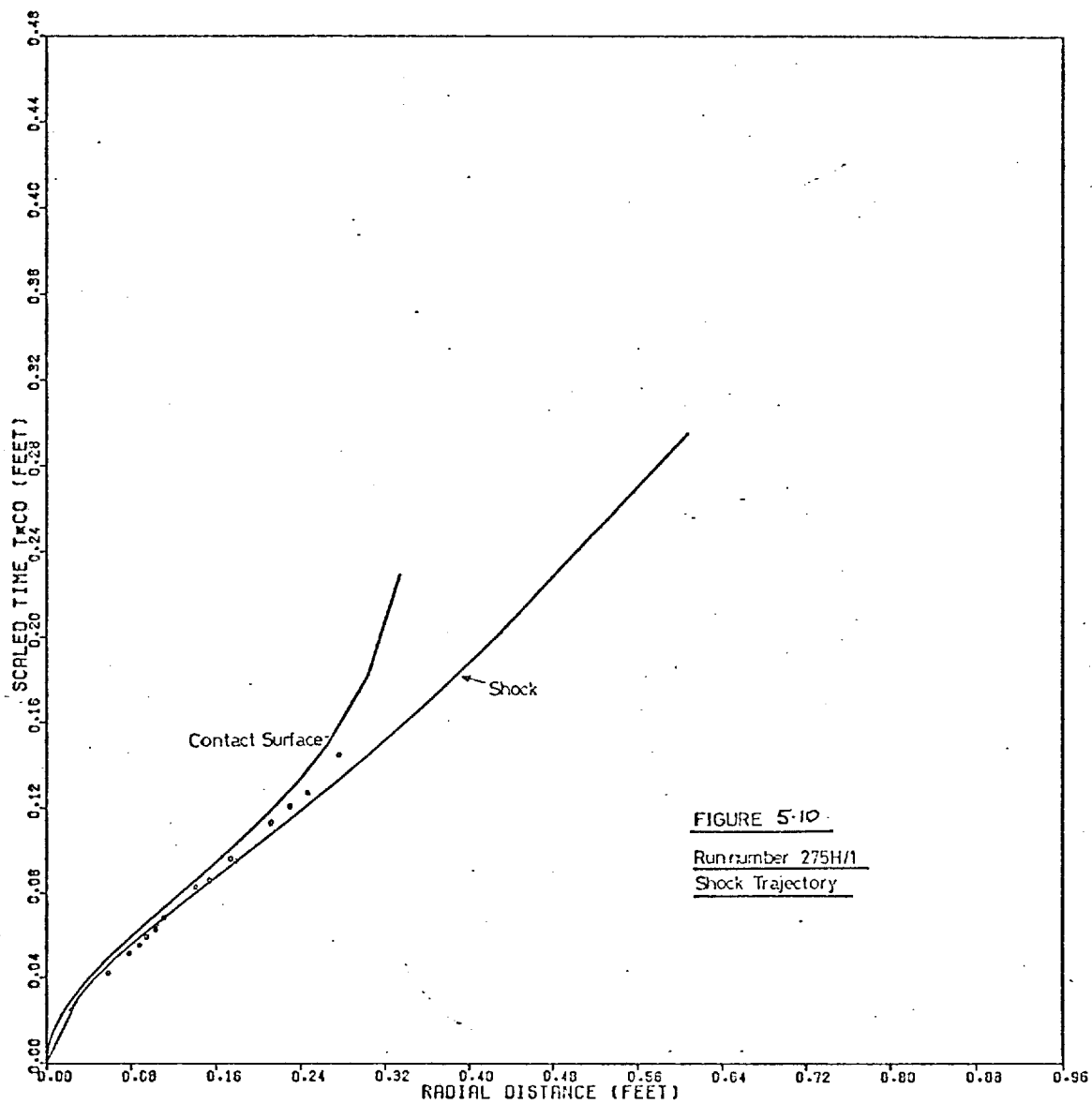
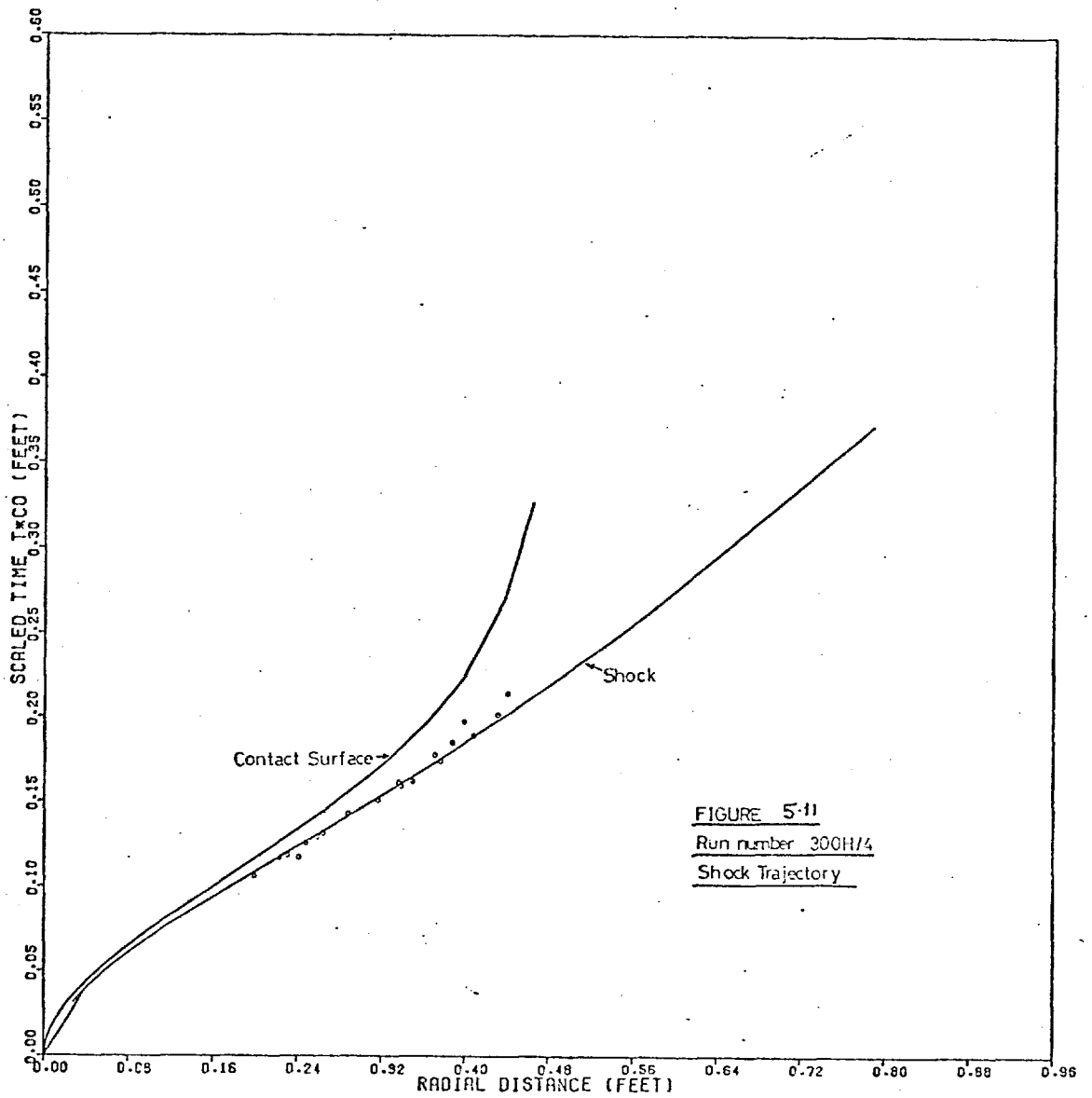


FIGURE 5-9

Run number 275A/1

Shock Trajectory





greater for the air blast. This observation is endorsed by the measurements taken off the photographs of this particular distance and is represented by the points on each graph.

The sonic and gas velocity profiles for the different venting conditions are compared in Figures 5.12 to 5.14. The state of each shock development at different intervals of time is clearly shown, the later values on the graph being whilst the shock is attenuating. The computer representation of the flow suggests that the method of shock formation can be described by a series of compression waves coalescing over a period of time to form a single large amplitude wave with a gradually steepening front which eventually becomes discontinuous. The position of the shock formation is at the point most favourable for a discontinuity to exist. At a scaled distance of 1.0, the furthest distance from the piston surface at which a shock wave could be formed, the attenuation effects are at a maximum and consequently the compression waves will be the weakest. Towards the piston surface, however, the spherically symmetric geometry of the system will mean a minimum of attenuation, but here there has not been sufficient time for a discontinuity to develop in the flow. The most favourable point in the flow for shock formation is therefore at the compromise between the effects of attenuation and the duration of the flow development. Table 5.2 gives the shock formation co-ordinates for the piston motion derived from each set of experimental conditions.

The duration of the shock wave acceleration and the maximum shock velocity reached depends on the magnitude and period of influence of the contact surface. This in turn is related to the time dependance of the energy released at the vessel orifice and will be discussed in more detail at a later stage. Table 5.3 lists the co-ordinates at which the maximum shock velocity is attained together with its magnitude at this position.

FIGURE 5-12

Run number 275A/1

Sonic and Gas Velocity Profiles

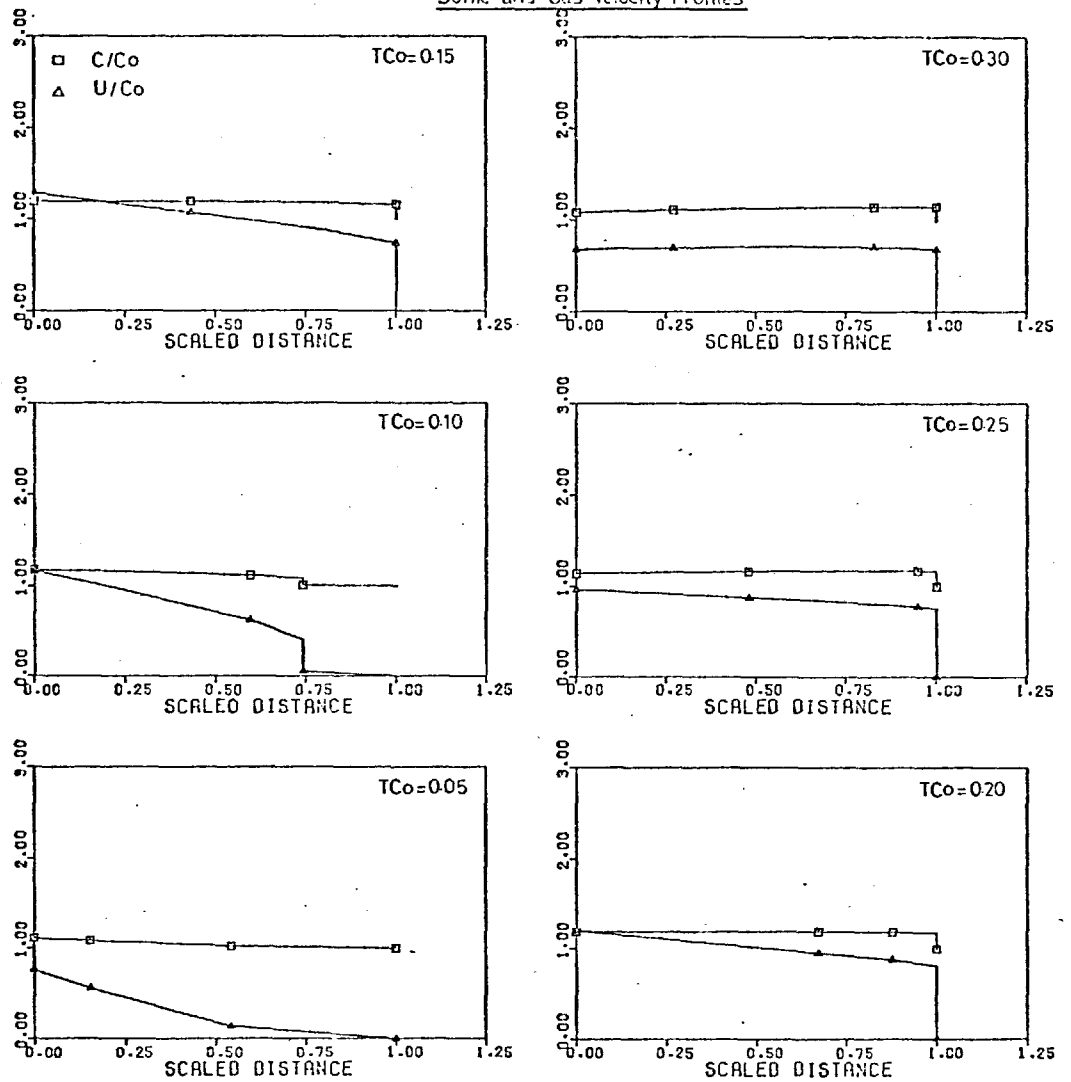


FIGURE 5-13

Run number 275H/1

Sonic and Gas Velocity Profiles

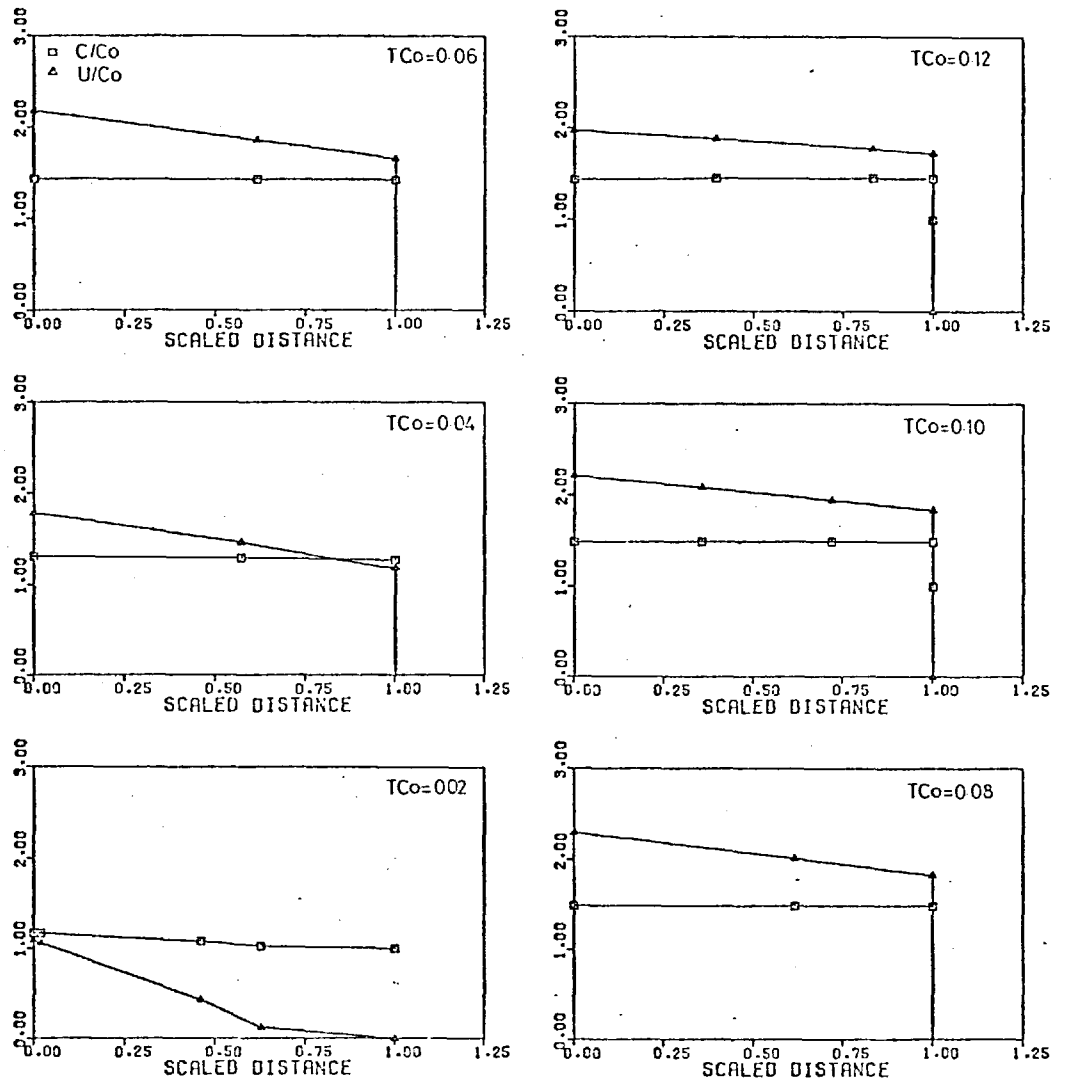


FIGURE 5-14

Run number 300H/4

Sonic and Gas Velocity Profiles

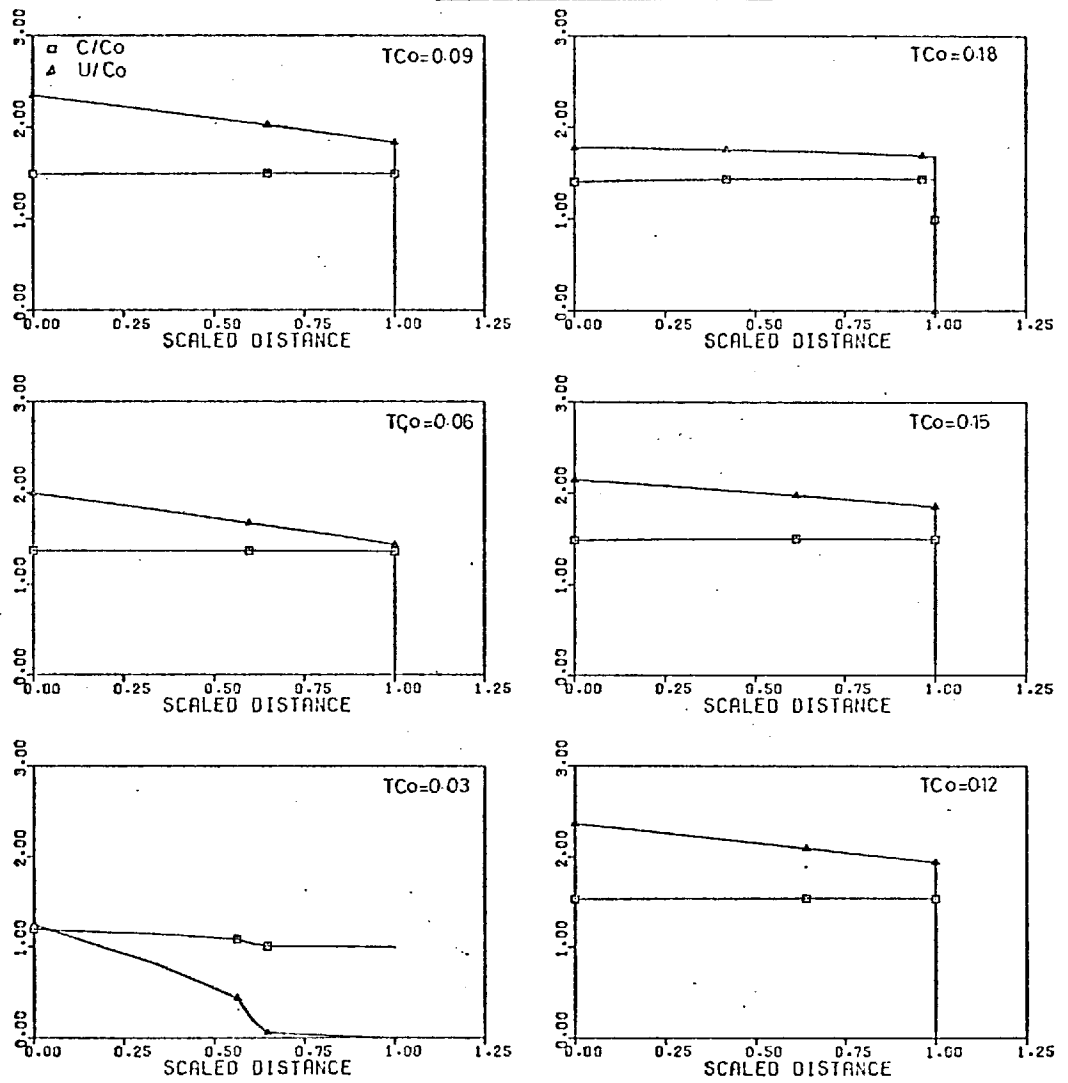


TABLE 5.2

SHOCK FORMATION CO-ORDINATES FOR AN ACCELERATING PISTON

Run Number	X_s (ft)	$T_s C_o$ (ft)
70A/2	0.266	0.309
100A/2	0.245	0.286
110A/2	0.190	0.221
145A/2	0.182	0.215
95A/3	0.181	0.214
200A/2	0.146	0.172
130A/3	0.101	0.118
100A/4	0.127	0.150
130A/4	0.085	0.100
200A/4	0.077	0.089
260A/3	0.071	0.082
275A/2	0.086	0.100
275A/1	0.088	0.096
250A/4	0.061	0.069
140H/4	0.020	0.022
250H/4	0.022	0.025
275H/1	0.022	0.025
200H/2	0.029	0.033
300H/4	0.028	0.031

5.3 Shock Wave Arrival Times

The arrival time of the shock wave is measured by a series of piezo-electric gauges, the sensitive elements of which respond to the compressive action of the wave front. The location of these gauges must be as near to the vessel axis as possible so as to eliminate the need to correct each result for any non-spherically symmetric shape that the shock wave might adopt. This way of positioning the detector gauges could possibly lead to shock wave distortion being introduced by the first gauge, causing subsequent gauges to give an incorrect result. However, provided the gauge is small

TABLE 5.3
CO-ORDINATES AND MAGNITUDE OF THE POINT OF MAXIMUM SHOCK VELOCITY

Run Number	x (ft)	TC _o (ft)	M = $\frac{V}{C_o}$
70A/2	0.504	0.513	1.205
100A/2	0.478	0.481	1.239
110A/2	0.398	0.391	1.280
145A/2	0.397	0.390	1.294
95A/3	0.432	0.415	1.305
200A/2	0.378	0.346	1.416
130A/3	0.254	0.234	1.413
100A/4	0.305	0.287	1.378
130A/4	0.297	0.237	1.704
200A/4	0.262	0.210	1.675
260A/3	0.192	0.164	1.655
275A/2	0.250	0.212	1.615
275A/1	0.209	0.179	1.611
250A/4	0.231	0.171	1.864
140H/4	0.204	0.094	3.146
250H/4	0.222	0.107	2.916
275H/1	0.171	0.092	2.604
200H/2	0.190	0.112	2.338
300H/4	0.234	0.122	2.696

the disturbance it sets up in the flow will be dissipated before the shock reaches the next detector gauge location. Plate 1 illustrates the shock wave(S) approaching one of the detector gauges which is located two feet away from the vessel. There is an initial delay of 1.661 msec. to the first frame and then a further 20 μ sec. to each subsequent frame. The shock wave is seen to be of a spherical nature, its surface showing no irregularities which would lead to further errors in the arrival time measurements. The curvature of the shock wave at this distance is minimal over the small angle that the gauge subtends making the contact between it

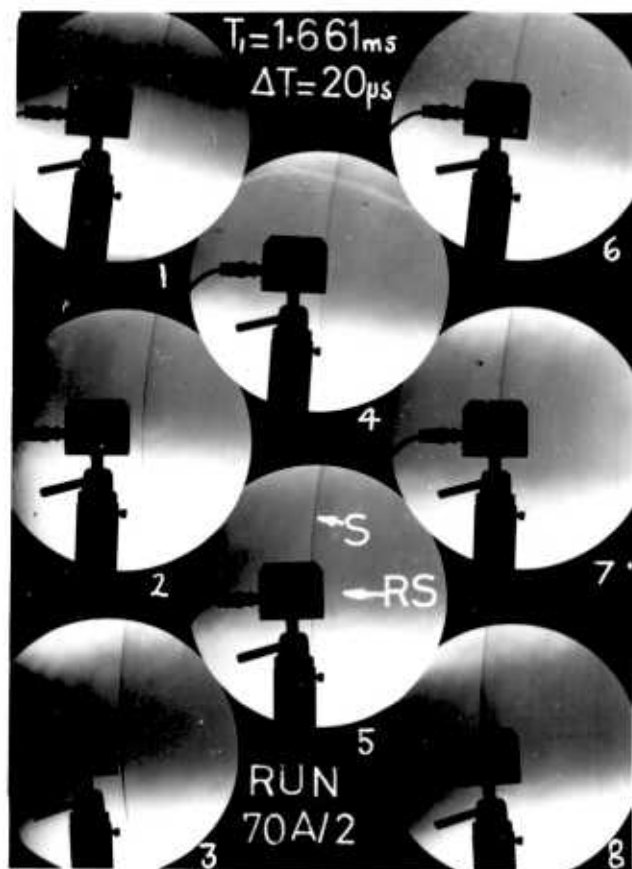


PLATE ONE



frame 5

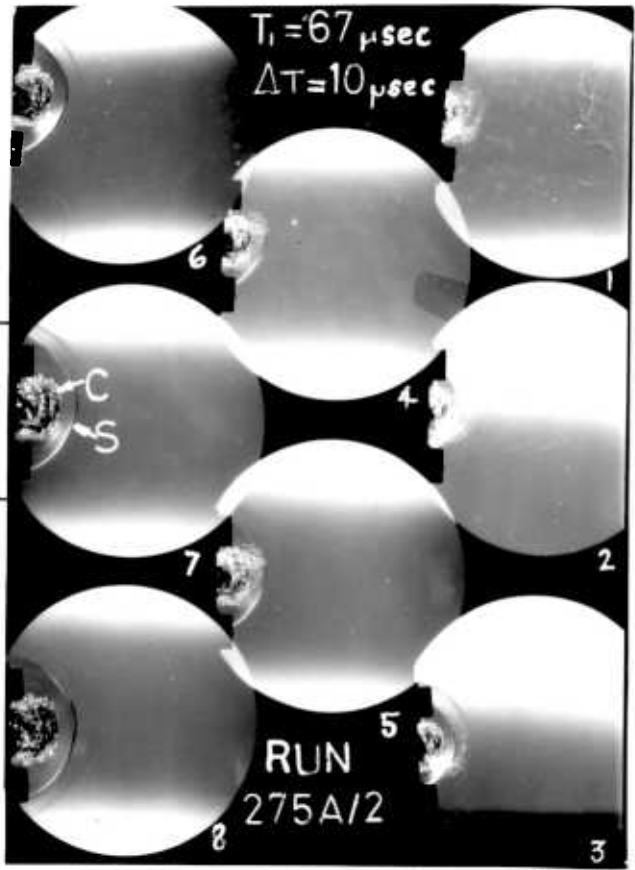
and the sensitive element virtually planar. The wave (RS) is reflected from the front face of the gauge and travels against the main flow towards the vessel, gradually decreasing in strength. The propagated wave makes an angle of approximately 90° with the side of gauge and any reflection is therefore difficult to observe on the photograph. Beyond the gauge the complete spherical form of the shock wave will be re-established and the next gauge which is located at a minimum of six inches further down stream will be met with an undistorted spherical wave front.

The average arrival time of the shock waves at each gauge location are given in Table 4.2 (Chapter 4) along with their standard deviations. The shock wave strength and hence its arrival time at a certain position obviously depends on the experimental conditions under which the shock was generated. The deviations of these times from the mean will therefore depend upon a number of experimental discrepancies. Firstly the bursting pressure of the disc will vary about the nominal value due to imperfections in the material used, and further the orifice diameter will be slightly less than the nominal value due to fragments of the disc folding around the plate and causing obstructions to the flow. These particular discrepancies will have a small and perhaps negligible effect on the amount of energy transferred to the shock wave. A more significant effect on the shock wave trajectory is brought about by the way in which the bursting disc opens. Ideally the time between the initial rupture of the material and the disc opening out to form an unrestricted orifice should be negligible. In practice the mode by which the disc opens is unpredictable and can considerably effect the flow development. The following series of photographic plates illustrates the way in which the bursting of the disc can lead to widely differing flow patterns.

Each plate shows a series of exposures of a blast wave produced by a gas being vented to the atmosphere from a cylindrical vessel through a circular orifice. The vessel is located to the left hand side of each frame as shown schematically against frame 7 of Plate 2a, the front flange being clearly visible. In each case the experimental conditions of bursting pressure, venting area and the type of gas contained in the vessel are indicated by the run number at the bottom of the plate. The exposures are numbered in sequence and their absolute times are determined from T_1 , the delay between the opening of the bursting disc and the first exposure together with ΔT , the time interval between successive exposures. Where there is considerable detail in a frame which is discussed further, an enlargement is made and positioned below the main sequence with the frame number indicated along side. Most of the frames unfortunately show a faint double image caused by reflections in adjacent prisms of the camera head. Steps taken to eliminate this fault were not successful.

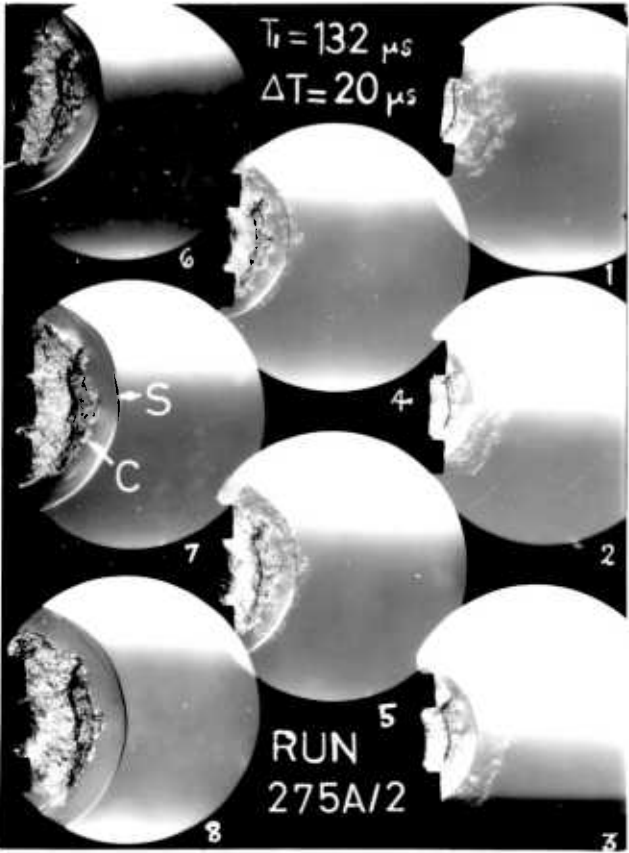
Plates 2a and 2b are both of 275 psig. air blasts over a 2 in. diameter orifice. They are taken from two different runs under the same experimental conditions and illustrate the extent of reproducibility of the blast wave. Plate 2a covers the early development of the flow pattern from 67 μ sec. to 137 μ sec. In frame 1 the bursting disc is only open a small amount and the gas that was originally contained inside the vessel is seen to be emerging into the atmosphere. A weak shock wave, S, forms after about 87 μ sec. and is strengthened by the action of the contact surface, C. Initially the shock profile is ellipsoidal in form due to the energy injection point not being spherically symmetric. The mode by which the bursting disc opens appears to be that of the petal formation described earlier, there being evidence of the orifice area increasing with time. The way in which the contact surface is emerging from the vessel in Plate 2b suggests that a

pressure
vessel



(a)

PLATE TWO



(b)

large fragment of the disc has been ripped away in the bursting process leaving the maximum venting aperture from the start. Under these conditions the axially symmetric flow that exists in the orifice is maintained into the surroundings for a greater length of time and thus the contact surface and shock wave initially present a flatter profile. A comparison of the two series of frames for these experimental conditions is best made by using the shock wave positions. Frame 8 on Plate 2a has approximately the same shock position as Frame 4 on Plate 2b, but the exposure times are 137 μ sec. and 192 μ sec. respectively indicating that the non-ideal mode of bursting of the latter leads to a slower shock development.

An interesting feature is shown on Plate 3 where the bursting disc has failed to open fully. A four inch diameter orifice should have been formed through which helium gas is vented to the atmosphere. However, after the disc had been distorted a pin-hole appeared at the apex of the dome and part of the gas was vented over this small area in the form of a jet. Due to the extremely rapid movement of helium this quantity of gas is capable of forming a weak shock wave directly down stream, and numerous compression waves, each with a different origin, intersect the sides of the jet. Eventually the bursting disc will rupture completely and the sudden increase in the rate of energy release will cause a second shock wave to develop which will catch up and coalesce with the primary wave. Due to the poor opening performance of the disc the arrival time measurement for this particular run will be considerably greater than the average for these conditions. Unfortunately it is not possible to present shock arrival times for photographically recorded runs as it was found that interference from the spark head caused the readings from the chronometer to be mostly unreliable.

Plate 4 again shows helium being vented over an orifice with a nominal 4 in. diameter, but this time with the bursting pressure at 140 psig. The

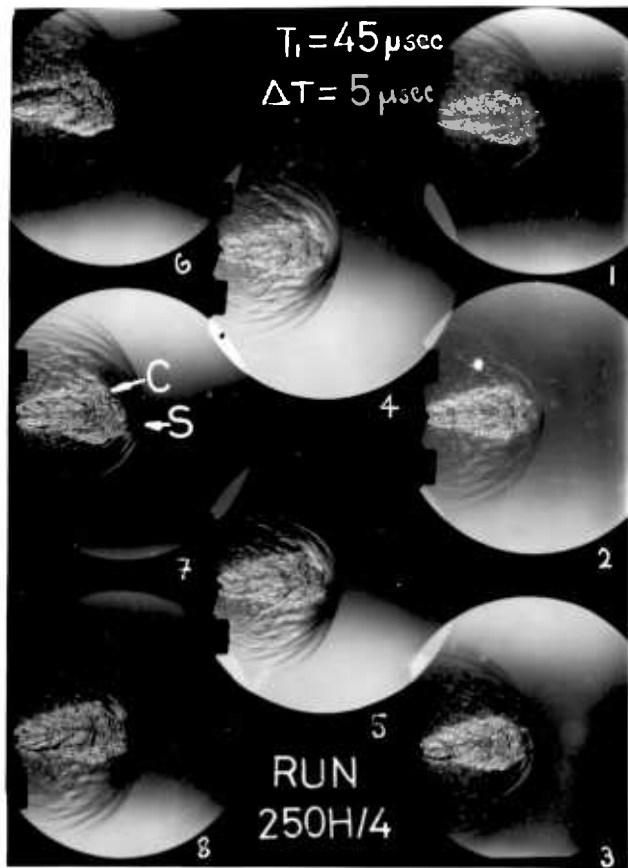
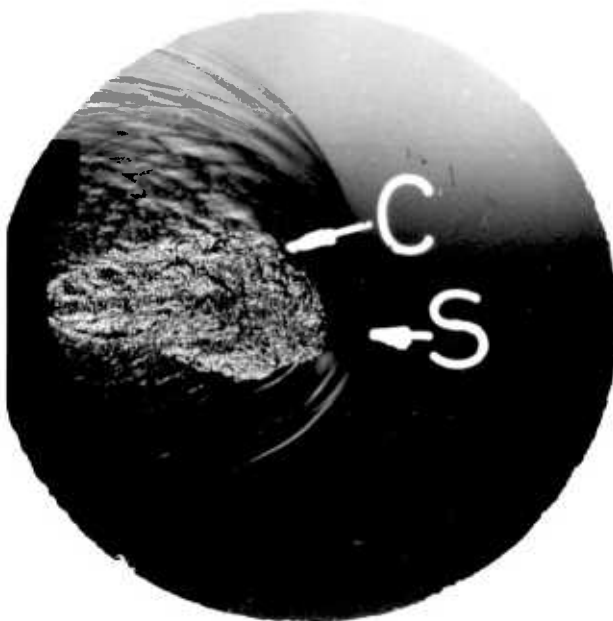


PLATE THREE



frame 7

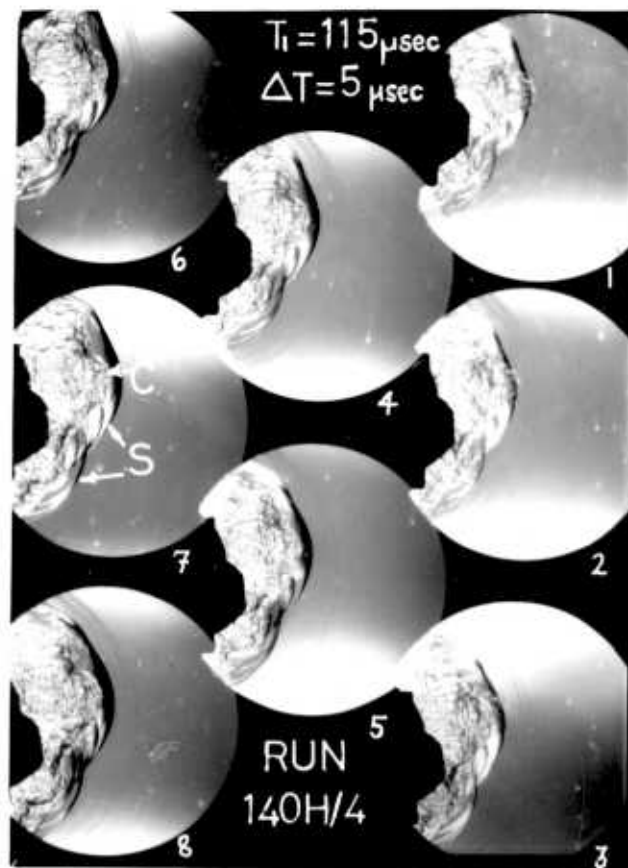
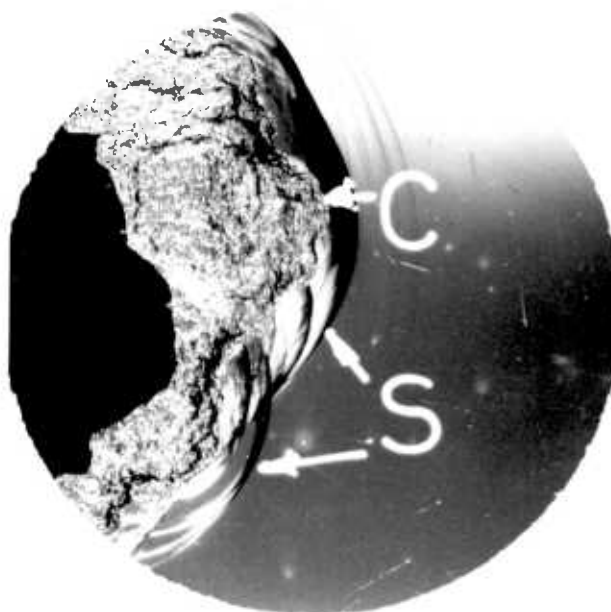


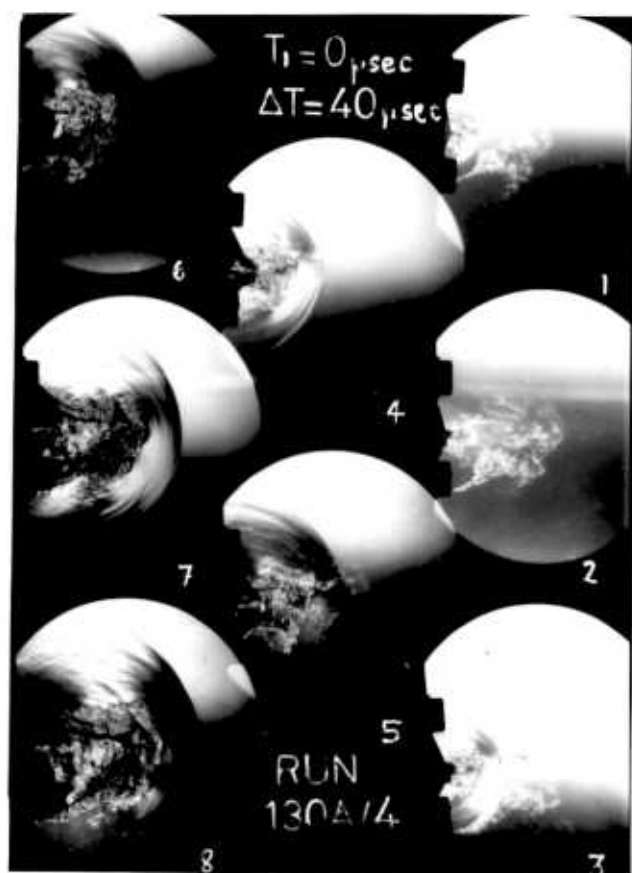
PLATE FOUR



frame 7

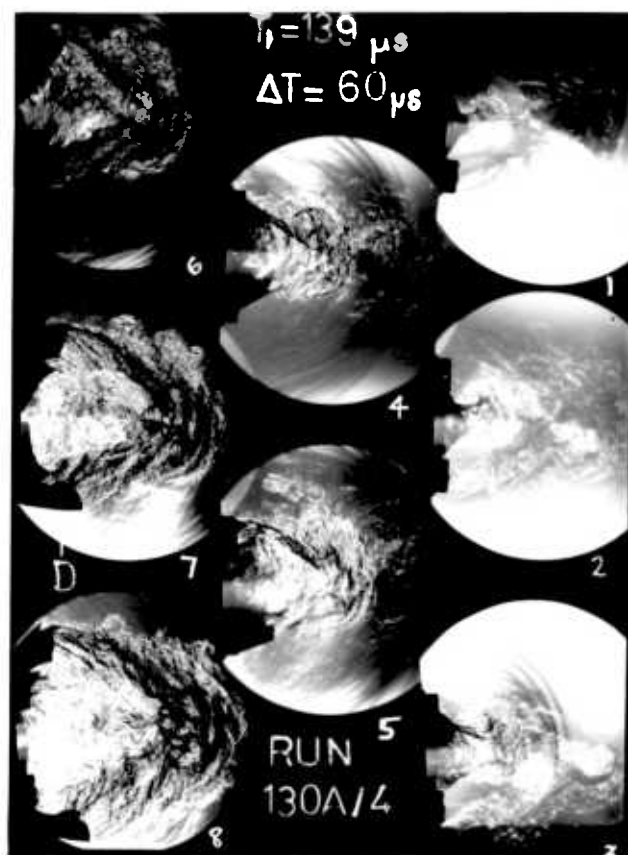
disc rupture is far from ideal and the appearance of a large jagged segment illustrates how uneven the aperture is likely to be. The resulting flow is thus far from symmetrical and consists of a number of individual shock fronts being driven independently by different areas of the contact surface. The unevenness in the flow results from the source of energy injection not being distributed symmetrically around the vessel axis, the irregularities in the disc rupture causing random jets of gas to be vented at angles which depend upon the obstruction caused by the disc fragments. Once the orifice is completely clear the gas flow becomes symmetric and the initial irregularities are dissipated. Eventually the main shock front will take on a continuous and spherical form. Frame 7 of this plate provides experimental evidence of the conclusions drawn from the computer simulation of one-dimensional spherical flow, that is the shock wave is formed ahead of the contact surface and behind the initial acoustic disturbance. In this enlarged frame a series of compression waves are clearly visible ahead of the main shock wave. Their smooth profiles suggest that they originate from the place of the initial disc rupture and have not been effected by the subsequent irregular gas motion. Another feature that is also apparent from the photographs after studying both air and helium venting systems is the relative position of the shock wave with respect to the contact surface. As indicated in the discussion of the computer results the contact surface and shock wave are much closer when the blast is driven by a helium venting system for which the average speed of the contact surface is approximately twice that for air under the same experimental conditions.

Plates 5a and 5b show two different sequences of an air blast over a four inch diameter orifice with a nominal bursting pressure of 130 psig. The disc opening is, in both cases, very nearly symmetrical about the vessel axis and consequently the shock wave has a smooth yet elipsoidal profile.



(a)

PLATE FIVE



(b)

In frames 1, 2 and 3 of Plate 5a the disc has begun to tear at its centre and in Frame 3 these tears are propagating along the radius leaving petals which begin to fold back onto the wall of the orifice due to the force exerted by the gas issuing from the vessel. The remaining frames illustrate this process and show the area available for flow rapidly increasing. Plate 5b extends the time covered for the observation of venting under these conditions to 559 μ sec. and in Frame 8 the petals have almost been folded back to their limit. Due to the high degree of turbulence in the flow it is usual at this stage for fragments of the disc to be severed and carried down stream as small projectiles. For disc ruptures such as these that approach the ideal case, the greater reproducibility of the flow pattern after the initial opening can be shown by a chronological comparison of the frames from each sequence.

The highly turbulent conditions present during the venting period are due to the very high Reynolds number of the flow ($\sim 10^6$) and also to the jagged edges of the torn bursting disc. The effect the bursting disc petals have on the flow whilst they are being folded back can be seen in Plate 6. A late developing, 100psig., air blast over a four inch diameter orifice is recorded. The bursting disc does not begin to form petals until approximately 500 μ sec. after its initial opening whereupon they fold back rapidly. As soon as the gas passes over the edge of the disc the increased resistance met by compressing the gas ahead sets up a stationary shock wave in the flow. The enlargement of Frame 5 shows in detail the series of eddies that are formed at the contact surface. These continue in the flow until their energy has been dissipated as is almost the case in Frame 8.

An examination of each disc after rupture suggested that the thinner discs with the smaller bursting diameter conformed most readily to the ideal case. These showed that the bursting was accomplished by an initial rupture in the

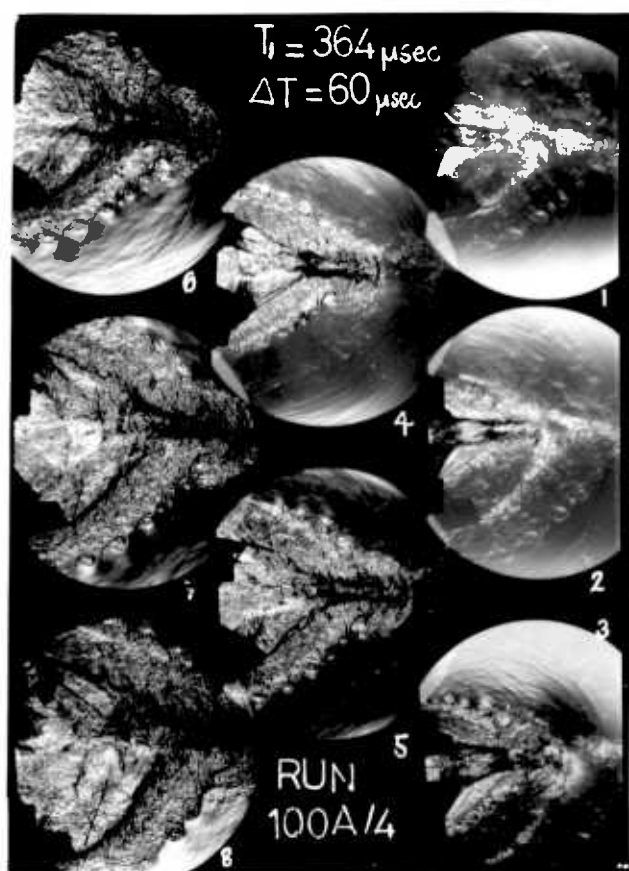
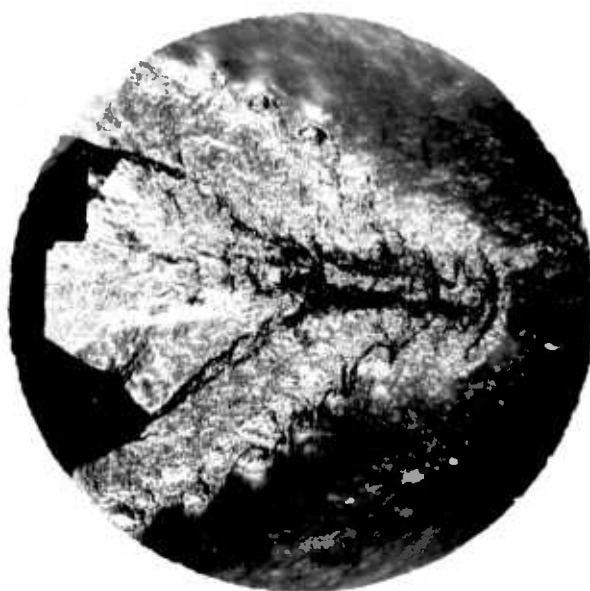


PLATE SIX



frame 5

centre and a propagation of tears to the outside forming a number of petals (normally six for the material used in this work) which folded back against the sides of the orifice plate to leave an aperture. With the larger venting areas and thicker bursting discs the behaviour was somewhat erratic, the centre of the disc being ripped out as a single fragment on most occasions. These tendencies are reflected by the standard deviations of the arrival times, those for the lower energy blasts being small and those for the higher energy blasts considerably larger. Particularly notable are the large values for the cases where two layers of foil were used in the bursting disc. By adding further complications to the mode of bursting a greater deviation from the ideal case would be expected.

5.4 Choking Times and Steady State Jet Formation

The time taken for the vessel orifice to become choked plays an important part in the flow development. During this period unsteady subsonic flow exists in the throat as the pressure falls from its value when the disc ruptured to the one corresponding to sonic flow. The process is one of gas expansion and acceleration whereby energy is imparted to the surrounding medium across the contact surface. When the orifice becomes choked a steady state gas jet will begin to form and further venting will not have any effect on the shock wave development. The orifice choking time is defined as that period over which unsteady-state conditions prevail. It is determined by measuring the pressure transition time at the vessel rear during venting and subtracting from this the theoretical transition time for a centred rarefaction wave leaving the vessel front under identical conditions. The time for the pressure in the orifice to become constant depends on two factors, firstly the rate at which the gas accelerates and secondly the rate at which the bursting disc opens. If the bursting disc opening rate were the limiting factor the aperture made available for flow would become

choked from the start and would stay so throughout the remainder of the opening time. Alternatively the bursting disc opening time may be short compared with the time for the gas to accelerate to sonic velocity and thus it would be the gas properties alone that determine the duration of unsteady state conditions in the vessel orifice. No experimental measurements are available for the bursting disc opening times in this work but a semi-empirical relationship derived by Curzon and Phillips (1971) from a simple model of the disc opening process is used to give a good approximation to these times. Table 5.4 lists the centred rarefaction wave reflection time off the vessel rear, the duration of the unsteady state conditions in the vessel orifice and the bursting disc opening time for each of the experimental conditions. In all cases the time for the pressure to become constant in the orifice is substantially greater than the bursting disc opening time and therefore the latter should not change the strength of the blast wave eventually developed. It is worth noting that the duration of the unsteady state period in the vessel orifice for helium is considerably less than that for air due to the greater sound speed; the disc opening times, however, are the same in each case. Plate 7 illustrates the formation of a steady state jet for the case of an air blast over a two inch diameter orifice with an initial pressure of 145 psig. Frame 1, at 232 μ sec., is taken whilst unsteady state conditions prevail in the vessel orifice. At 150 μ sec. later the orifice is choked (Frame 2) and the steady state jet is beginning to develop. At more advanced times the characteristics of a steady state jet begin to appear, the enlarged Frame 7 showing the Mach Bridge about four inches down stream from the orifice. Fragments of bursting disc (D) can be seen in Frames 5, 6, 7 and 8, the last three frames also showing the main shock wave which is actually passing through the incident beam of the schlieren system and is some 12 inches ahead of its position in these images.

TABLE 5.4

DURATION OF UNSTEADY STATE PROCESSES WITHIN A
VENTING PRESSURE VESSEL

Run Number	Centred Rarefaction Wave Reflection time (μ sec)	Duration of Unsteady State conditions in the vessel orifice (μ sec)	Bursting Disc opening time (μ sec)
70A/2	85	388	172
100A/2	88	329	160
110A/2	90	315	168
145A/2	86	315	169
95A/3	221	519	255
200A/2	91	337	161
130A/3	211	558	244
100A/4	455	741	322
130A/4	428	624	346
200A/4	413	671	322
260A/3	208	416	244
275A/2	79	353	173
275A/1	20	122	86
250A/4	429	697	328
140H/4	161	480	344
250H/4	170	561	328
275H/1	8	79	86
200H/2	33	303	161
300H/4	165	591	332

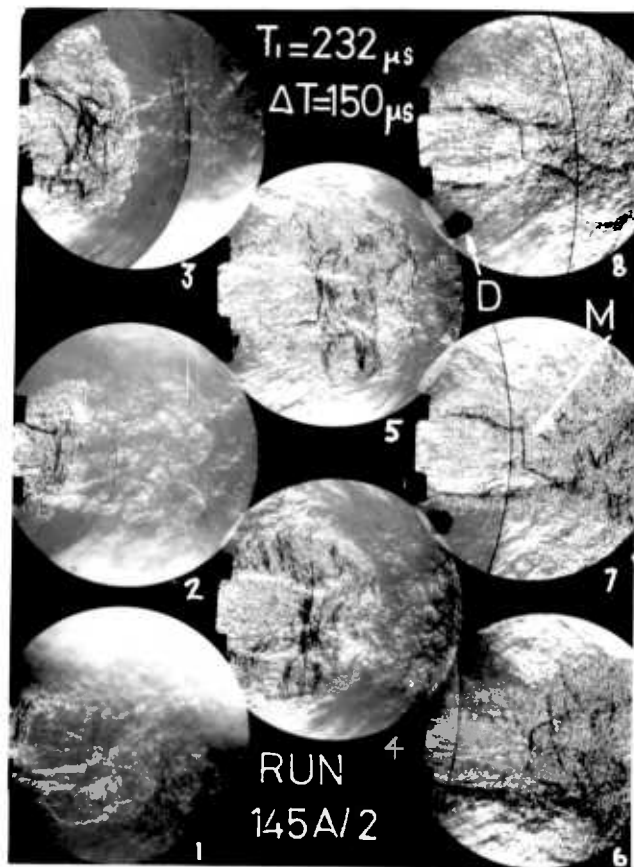
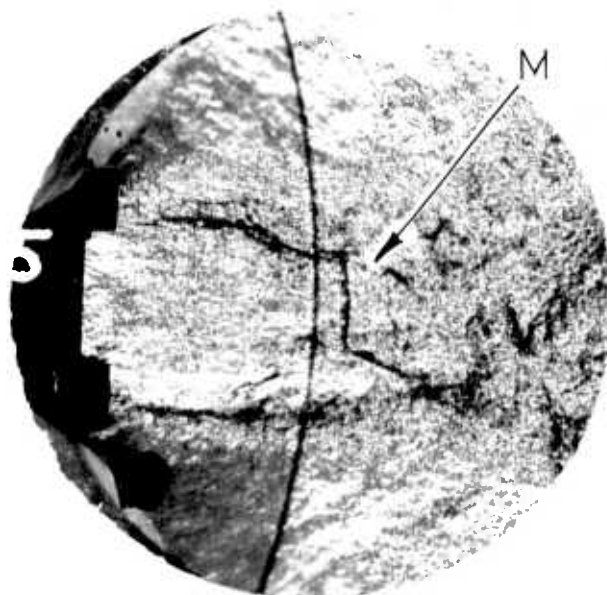


PLATE SEVEN



frame 7

5.5 The Blast Wave Energy and Scaling Length

The amount of energy vented from the vessel up to the time of steady state conditions developing in the orifice is calculated as outlined in Chapter 2. This value of E_0 is used to calculate the scaling length (ϵ_2) of the blast wave used in the expression given on page 23, Chapter 1. A comparison is made with the scaling length (ϵ_1) derived by Munday (1971) given by equation 1.18, page 24. Table 5.5 presents these results for each set of experimental conditions used in this work. A comparison of the two independently derived scaling lengths gives a reasonable agreement for the two inch diameter orifice size and smaller, but with the larger areas there is an increased discrepancy. This can be put down to the assumptions made in the derivation of equation 1.18, these being (i) the gas velocity inside the vessel is zero and (ii) the pressure remains constant during the initial stages of venting, both being better approximations for the smaller orifice areas. As there is no way of estimating which scaling length is the more effective, the choice must be swung in favour of that derived by Munday solely on the success of applying a similarly derived scaling length in the field of hydraulic analogy. The scaling of the shock wave arrival times will therefore be discussed using this scaling length and comments will be made on the outcome of using the one based on the amount of energy vented up to the time of steady state conditions in the vessel orifice.

5.6 The Discrepancy Between the Point Source Solution and the Scaled Experimental Results

The shock wave arrival times at the locations laid down in Table 4.2 have been scaled using the scaling length (ϵ_1) derived by Munday (Table 5.5) and are presented in Table 5.6. Also the shock wave trajectories predicted by program PISTON have been similarly scaled, but these results being too numerous have not been tabulated. The pressure transition times at the

TABLE 5.5

BLAST WAVE ENERGIES AND SCALING LENGTHS

Run Number	Energy (ft lbf)	Scaling Length (ft)	
		ϵ_2	ϵ_1
70A/2	66	0.3154	0.3188
100A/2	79	0.3347	0.3709
110A/2	84	0.3418	0.3868
145A/2	103	0.3653	0.4377
95A/3	261	0.4976	0.5442
200A/2	156	0.4190	0.5075
130A/3	356	0.5520	0.6250
100A/4	652	0.6755	0.7419
130A/4	662	0.6787	0.8333
200A/4	1037	0.7884	1.0150
260A/3	500	0.6184	0.8611
275A/2	194	0.4509	0.5895
275A/1	19	0.2064	0.2948
250A/4	1368	0.8646	1.1271
140H/4	890	0.7493	1.3640
250H/4	N/D	N/D	1.7843
275H/1	21	0.2135	0.4666
200H/2	224	0.4731	0.8034
300H/4	2264	1.0227	1.9455

front of the vessel provide an indication of the blast wave energy injection period^{and} in order to make comparisons between different strength blasts it is necessary to scale these figures also. The method of scaling the transition times is that used for shock wave arrival times, i.e.

$$\tau_r = \frac{\tau_r c_o}{\epsilon_1}$$

TABLE 5.6

SCALED SHOCK WAVE ARRIVAL TIMES AND PRESSURE TRANSITION TIMES

Run Number	Scaled Distance (λ)	Scaled Time (τ)	Scaled Pressure Transition Time (τ_r)
70A/2	7.94	7.86	1.37
	9.50	9.40	
	11.07	10.96	
	12.64	12.48	
100A/2	4.12	4.04	1.00
	5.47	5.38	
	6.82	6.71	
	8.17	8.02	
	9.52	9.35	
	10.86	10.65	
110A/2	3.95	3.88	0.916
	5.25	5.14	
	6.54	6.43	
	7.83	7.68	
145A/2	3.49	3.34	0.810
	4.64	4.43	
	5.78	5.55	
	6.92	6.66	
	8.06	7.77	
	9.21	8.89	
95A/3	2.81	2.78	1.07
	3.73	3.67	
	4.65	4.58	
	5.57	5.48	
200A/2	3.01	2.81	0.749
	4.00	3.77	
	4.98	4.70	
130A/3	2.45	2.38	1.00
	3.25	3.20	
	4.05	3.90	
100A/4	2.06	2.12	1.12
	2.74	2.79	
	3.41	3.46	
	4.08	4.07	
130A/4	1.84	1.66	0.844
	2.44	2.23	
	3.04	2.79	
	3.64	3.38	

contd/...

TABLE 5.6 contd.

200A/4	1.51	1.33	0.742
	2.00	1.78	
	2.49	2.23	
	2.98	2.71	
260A/3	1.77	1.53	0.544
	2.36	2.07	
	2.94	2.59	
	3.52	3.15	
275A/2	2.59	2.24	0.674
	3.44	3.04	
	4.29	3.84	
	5.14	4.67	
275A/1	5.19	4.67	0.465
	6.88	6.32	
	8.58	7.96	
	10.28	9.64	
250A/4	1.80	1.64	0.697
	2.24	2.03	
	2.68	2.45	
	3.13	2.88	
140H/4	1.48	1.07	0.395
	1.85	1.37	
	2.22	1.70	
	2.59	2.03	
250H/4	1.14	0.80	0.354
	1.42	1.02	
	1.70	1.26	
	1.98	1.51	
275H/1	4.35	3.48	0.190
	5.42	4.47	
	6.49	5.48	
	7.56	6.53	
200H/2	2.53	1.99	0.425
	3.15	2.55	
	3.77	3.13	
	4.39	3.73	
300H/4	1.04	0.72	0.343
	1.30	0.91	
	1.56	1.13	
	1.81	1.36	

The results for each case are also given in Table 5.6. A comparison of these experimental shock wave characteristics is made with the point source solution derived by Kinney (1962) and not that derived by Brode (1955). The reason for this choice lies in the differing definitions of the energy term, E_0 , in the expression for the scaling length ϵ . Brode incorporates terms for the gas internal energy, the preshock internal energy of the gas enclosed by the wave and the kinetic energy of the wave, whereas Kinney makes use of a simple internal energy scaling term in a similar way to Munday.

In Figures 5.15 a to d the shock strength, as predicted by the numerical solution of the relevant equations, is plotted as a function of scaled distance for each set of experimental conditions alongside Kinney's point source solution. To make use of a logarithmic scale the shock strength is expressed by $M - 1$, where M is the shock Mach number.

All the plots of the shock strength against distance show an initial period of acceleration followed by one of attenuation. As the distance approaches infinity the experimental results and the point source solution, on a linear set of axes, will approach asymptotically because in each case the front of the blast wave decays to a sound wave. The initial development of the shockwave depends on the rate of energy release and therefore as would be expected the experimental conditions that possess a high initial pressure and a large venting area show a much earlier shock development. The extremes of these conditions being illustrated by the curves for Runs 70A/2 and 300H/4, the former achieving its maximum velocity after the point source solution and the latter somewhat before.

There are several conditions that can be used to show the effectiveness of the chosen scaling factors for a set of particular blast waves. Their relative magnitudes can be assessed by an examination of the degree by

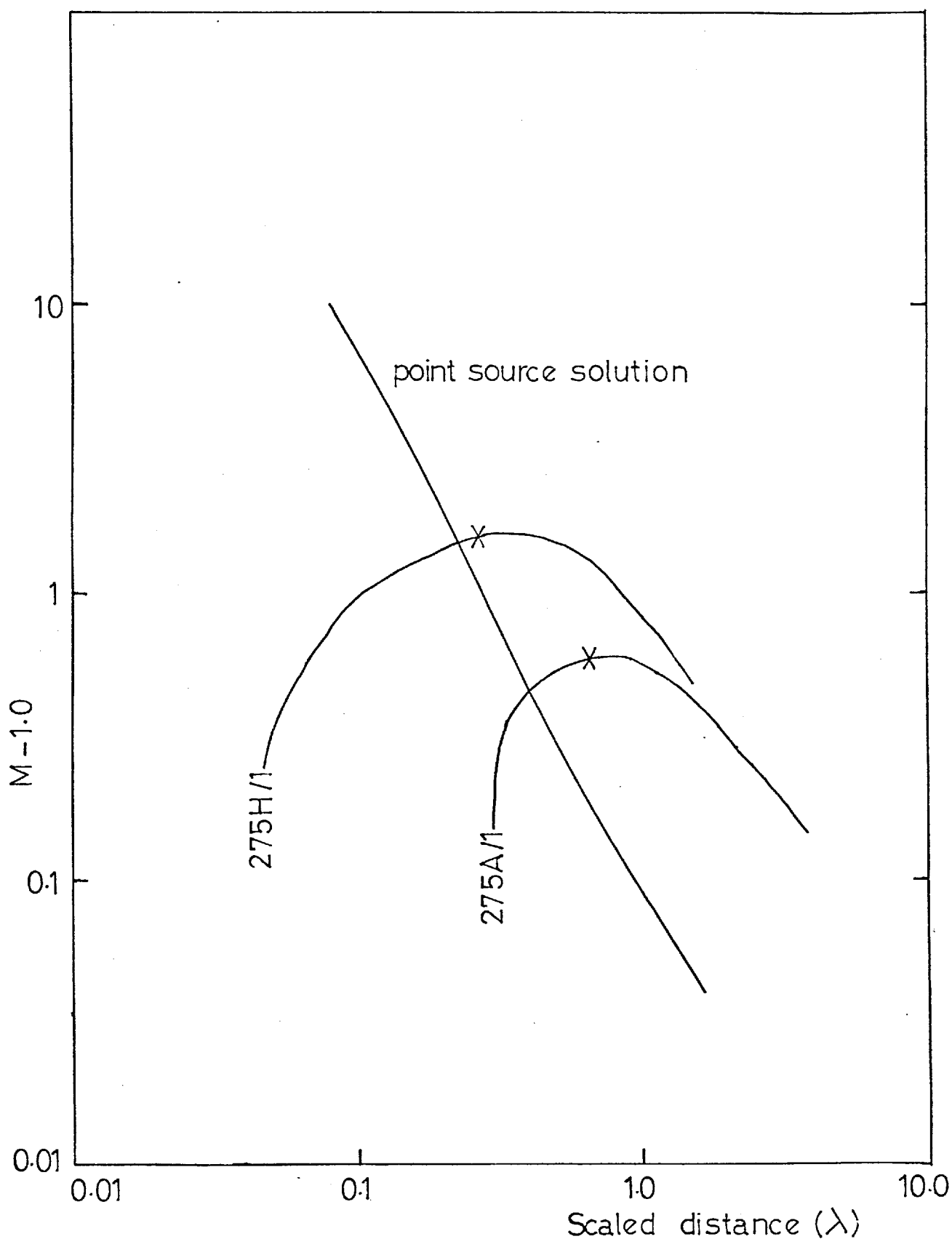


Figure 5.15a Blast wave strengths for a
one inch diameter vent

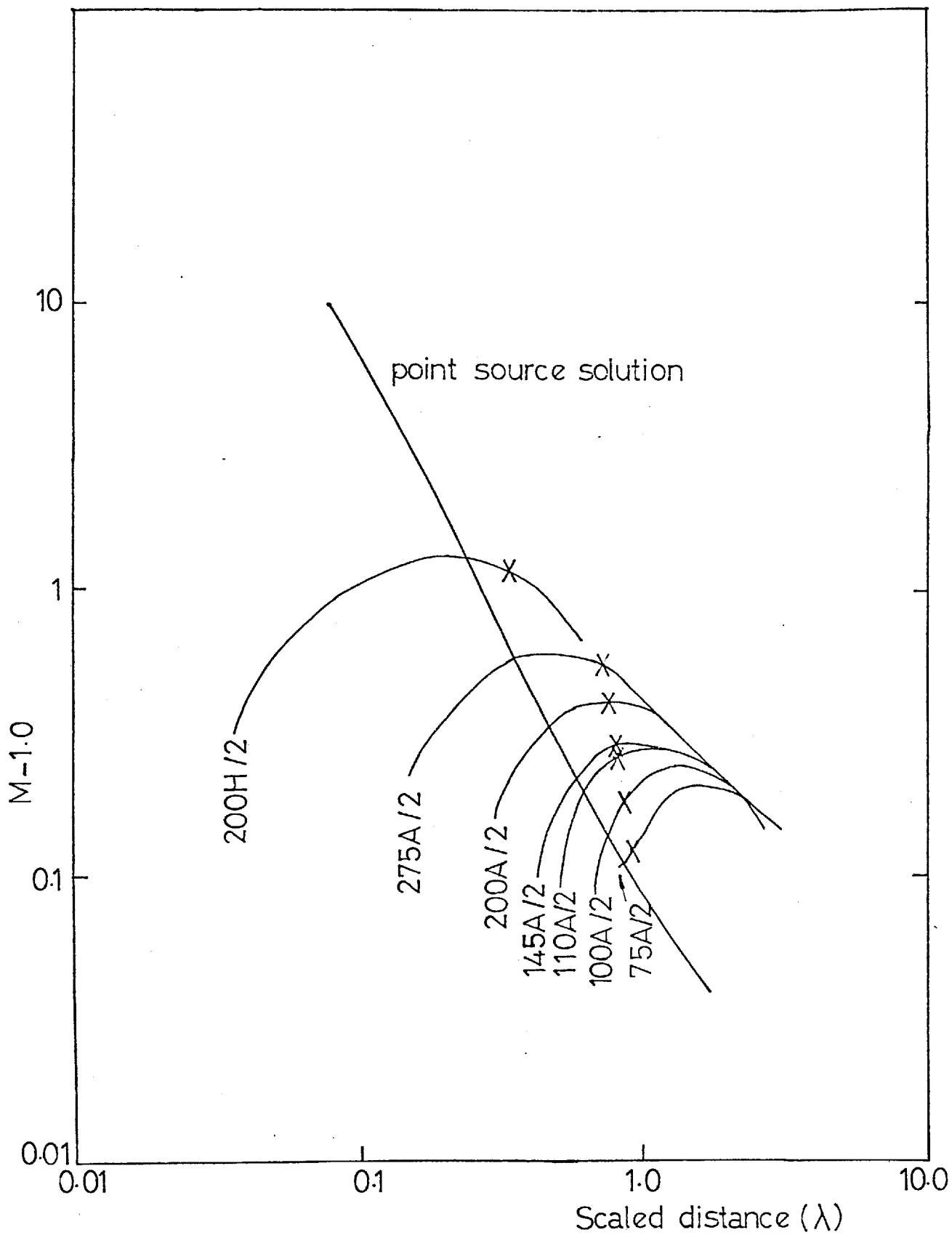


Figure 5.15b Blast wave strengths for a
two inch diameter vent

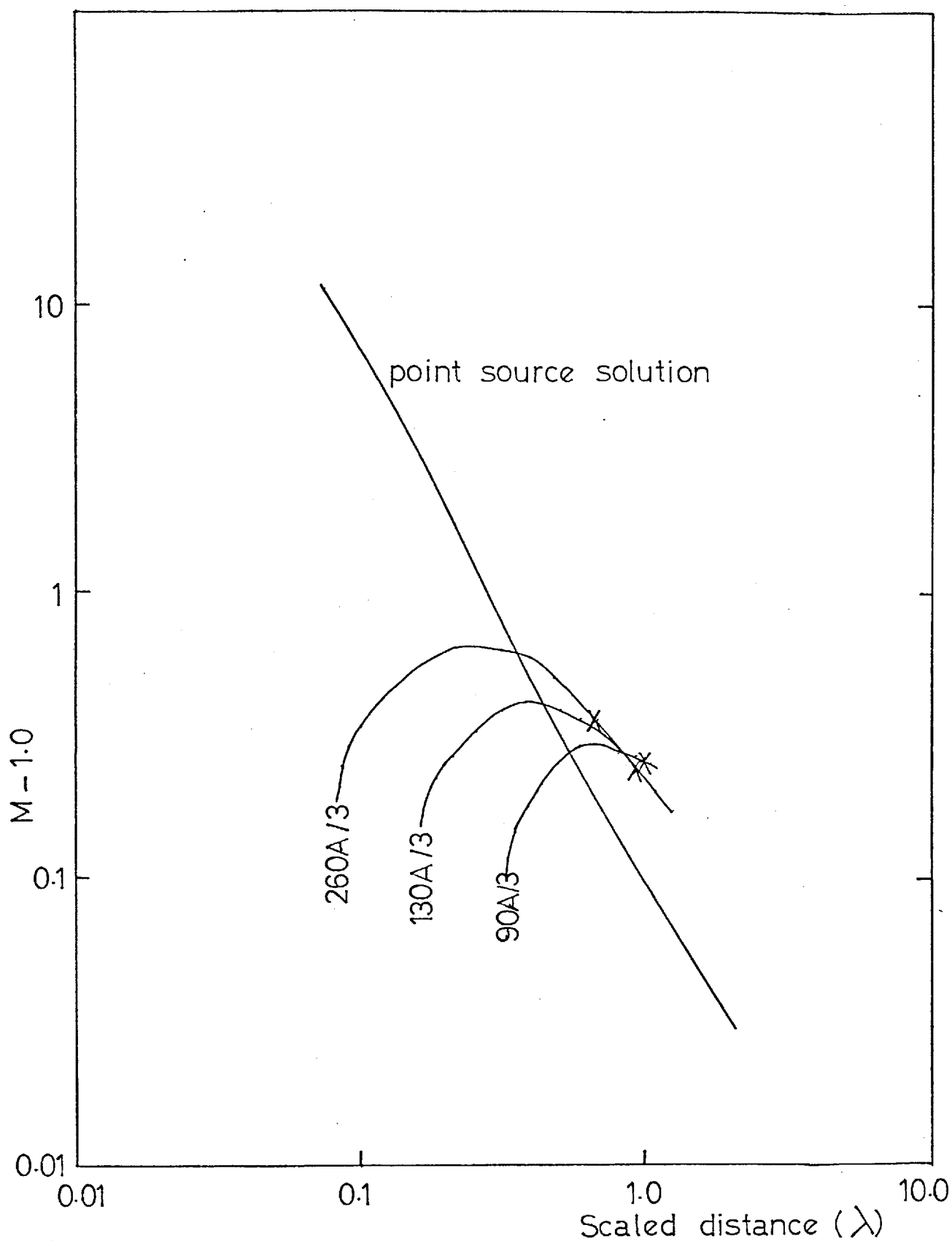


Figure 5.15c. Blast wave strengths for a three inch diameter vent

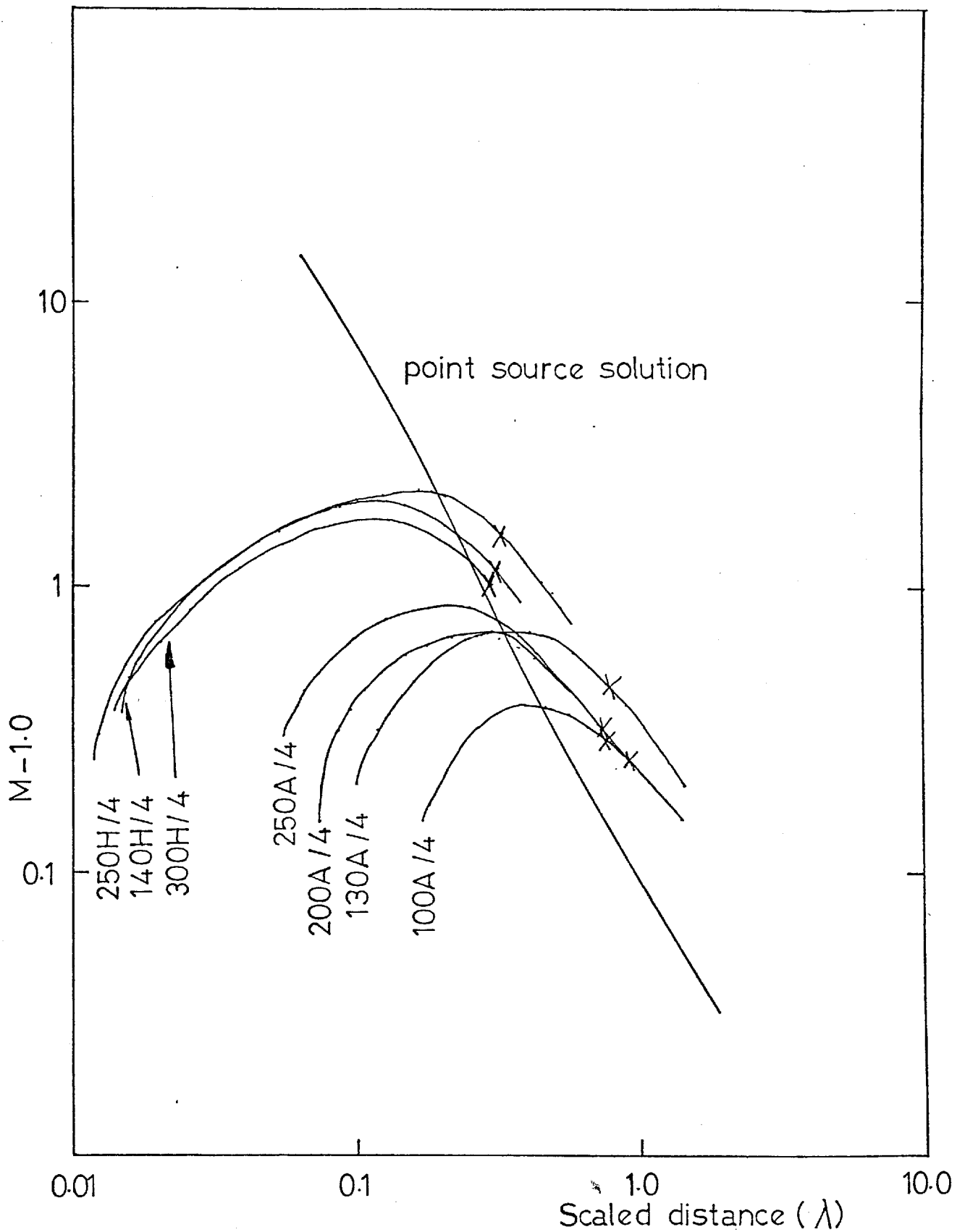


Figure 5.15d

Blast wave strengths for a
four inch diameter vent

which the shock trajectories merge into one another and become colinear after the early development stages have passed. This effect is shown to a greater extent in the curves for the 1, 2 and 3 in. diameter vents and to a lesser extent for the 4 in. diameter vent. In all cases the line into which the shock trajectories eventually merge is displaced to the right of the point source solution which suggests that the constant in Munday's scaling length has to be increased.

The absolute magnitudes of the scaling lengths can thus be assessed using the following reasoning with reference to Figure 5.16 a and b below.

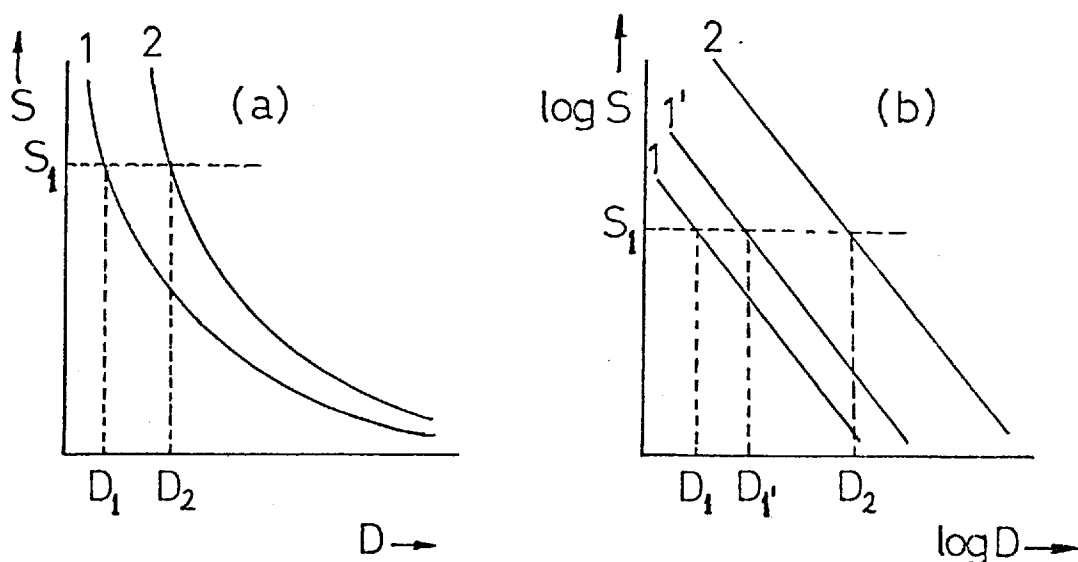


Figure 5.16 Plot of shock wave magnitude against distance

The shock wave magnitude (S) (which can be represented by $M - 1$ as in this work) is plotted against distance D , using both linear and logarithmic scales. Curve (1) is for a point source blast with unit energy release whereas curve (2) is a typical experimental blast wave. The plots cover a region in which each blast wave has a similar form and scaling lengths can be applied meaningfully. At a value of shock strength S_1 the positions of

the two blast waves are D_1 and D_2 respectively. To superimpose the two curves the experimental distances must be scaled by a length ϵ which is defined by the following relationship:

$$D_1 = D_2 / \epsilon \quad (5.2)$$

and is true for any value of S in this region. On the logarithmic plot at S_1 equation 5.2 takes the form:

$$\log D_1 = \log D_2 - \log \epsilon \quad (5.3)$$

that is the horizontal distance between the curves is equal to $\log (\epsilon)$. As this is constant for all values of S the two curves in Figure 5.16b are parallel. Since the scaling length for a real blast wave is derived from nominal experimental conditions it is subject to error. Such a scaling length ϵ_1 gives a curve $(1')$, which is displaced to the right of the point source curve (1) . The horizontal distance between curves (1) and $(1')$ is equal to $\log (C)$, where C is a correction factor for ϵ_1 such that:

$$\epsilon = C \epsilon_1 \quad (5.4)$$

The value of this correction factor at each orifice diameter is given in Table 5.7 below.

TABLE 5.7
CORRECTION FACTORS FOR SCALING LENGTH ϵ_1

Orifice Diameter (in)	1	2	3	4
A_1/A_2	36	9	4	2.25
C	3.1	3.0	1.8	1.6

In general the scaling lengths ϵ_2 of Table 5.5 are less than the corresponding values for ϵ_1 which means that a larger correction factor has to be applied.

The relative effectiveness of each scaling length can be assessed in a quantitative manner using the observations of Brode (1955) on an exploding sphere. His scaled results were shown to agree with the point source solution to within 10% when the shock wave had engulfed ten times the mass of gas released from the ruptured sphere. The total available energy was assumed to contribute to the blast strength, whereas for a partially failed vessel this is not so. For reasons stated previously it is reasonable to assume that the energy associated with the type of blast of interest here is released up to the time of steady state conditions in the orifice. Ideally on reaching these conditions a final disturbance will travel from the orifice to the shock front with a velocity corresponding to the local velocity of sound relative to the gas motion, i.e. $u + c$. It is only after this wave impinges upon the shock front that the steady state conditions present in the vessel orifice begin to influence the shock wave trajectory. The time at which this occurs is difficult to determine as a solution of the gas flow between the orifice and the contact surface is involved. However, a reasonable approximation to this time can be made by assuming the disturbance transit time of the region between the orifice and the contact surface is negligible compared with that of the region between the contact surface and the shock front. The results of program PISTON can then be used to trace a disturbance path or characteristic direction from the contact surface to the shock front.

Table 5.8 summarises the results obtained for testing the relative effectiveness of each scaling length. Column two indicates whether the plot of shock strength, as a function of scaled distance, becomes colinear

TABLE 5.8

SCALED SHOCK WAVE LOCATIONS USED TO EXAMINE THE RELATIVE
EFFECTIVENESS OF THE CHOSEN SCALING LENGTHS

Run Number	Approach of Curves to within 10%	Location of Shock for 10% approach	Location of Shock for 10 x mass vented	Location of Shock at Intersection of last disturbance
275A/1	Yes	2.0	0.672	0.71
275H/1	Yes	0.7	0.270	0.45
70A/2	Yes	2.5	0.947	2.98
100A/2	Yes	2.0	0.874	1.92
110A/2	Yes	2.0	0.854	1.81
145A/2	Yes	2.0	0.808	-
200A/2	Yes	1.0	0.798	-
275A/2	Yes	0.8	0.740	2.05
200H/2	Yes	0.35	0.347	-
90A/3	No	-	0.884	-
130A/3	Yes	0.7	0.854	-
260A/3	Yes	0.4	0.694	-
100A/4	Yes	0.8	0.882	-
130A/4	No	-	0.787	-
200A/4	Yes	0.5	0.750	-
250A/4	Yes	0.35	0.740	-
140H/4	No	-	0.324	-
250H/4	No	-	0.333	-
300H/4	No	-	0.310	-

(to within 10%) with similar curves for other experimental conditions whilst Column three gives the position at which this occurs. Column four indicates the scaled position at which the shock wave has engulfed ten times the mass of gas vented from the vessel up to obtaining steady state conditions in the orifice. These locations are also shown on each curve of Figures 5.15 a, b, c, and d. Finally the remaining Column gives, where

possible, an estimate of the scaled position at which the last disturbance to leave the vessel orifice during the initial unsteady period would intersect the shock wave. Unfortunately it was not possible to determine these locations for every set of conditions as those with the larger energy release needed a finer characteristic network to accommodate the more rapid motions and a sacrifice had to be made in the duration over which the complete flow pattern was solved. A comparison of columns 3 and 4 of Table 5.8 and referring to Figures 5.15 a, b, c, and d indicates the relative effectiveness of the "energy vented" scaling length ϵ_2 . It appears that the energy associated with the 1 in. diameter vent blast waves is greater than the amount considered in evaluating ϵ_2 . The lower pressure blasts for the 2 in. diameter vent give similar results whereas at higher pressures there is good agreement with Brode's hypothesis which is maintained for the case of the 3 in. diameter vent. The group of curves for the 4 in. diameter vent, even though they do not show the same tendency to become co-linear at later stages, all have the point of interest similarly placed. It is located to the right of where the curve begins to straighten which indicates that for such conditions the blast wave has become fully developed before the shock has engulfed the amount of gas in question. The limited number of results in the last column of Table 5.8 are sufficient to confirm that the choking time of the vessel throat is an important factor in the development of the blast wave.

To summarise the merits of each scaling length in Table 5.5 it is clear that both ϵ_1 and ϵ_2 are on the low side and the amount of energy that is associated with the blast wave is higher than predicted in each case. It therefore appears that neither of these methods as they stand are suitable for predicting the absolute value of the scaling length for the experimental conditions present in this work. The relative effective-

ness of each factor is represented in its ability to translate changes in experimental conditions to degrees of variation in the energy associated with the blast wave. It appears that E_1 is the more effective in dealing with differing pressures and gases for the small venting areas. For the more intense blast waves the energy associated with the mass of gas vented up to steady state conditions is a better approximation for the energy term used in determining the scaling length. In general E_1 can be used for scaling blast waves with values of $\frac{A_1}{A_2} > 4$ and an extension to higher pressures and other types of gas can be made with reasonable confidence. For values of $\frac{A_1}{A_2} < 4$ the throat choking time must be determined to calculate the energy associated with the blast wave. At present this has to be done by empirical means as insufficient knowledge is available to derive an analytical expression. Factors which influence this time are, (i) the type of gas being vented, (ii) the orifice area, (iii) the material and thickness of the bursting disc. With so many variables extrapolation of the results for the experimental ground covered here must only be done with utmost caution.

A more rigorous assessment of these results can only be made by determining the approach to the point source solution over a greater distance by further experimentation. (In this work the computational results cannot be extended further due to lack of additional boundary conditions and the shock arrival time gauges are not suitable for measurement of shock strength). However, looking at the results at hand in a different light leads to a further method for measuring the deviation of a real blast wave from a point source blast. This is achieved by making a comparison of the scaled shock wave trajectories for each different blastwave with that of the point source solution as illustrated in Figure 5.17 for a typical case. The point source blast wave has a higher velocity throughout its history and therefore will arrive at a given location at a time increment ΔT ahead of the real

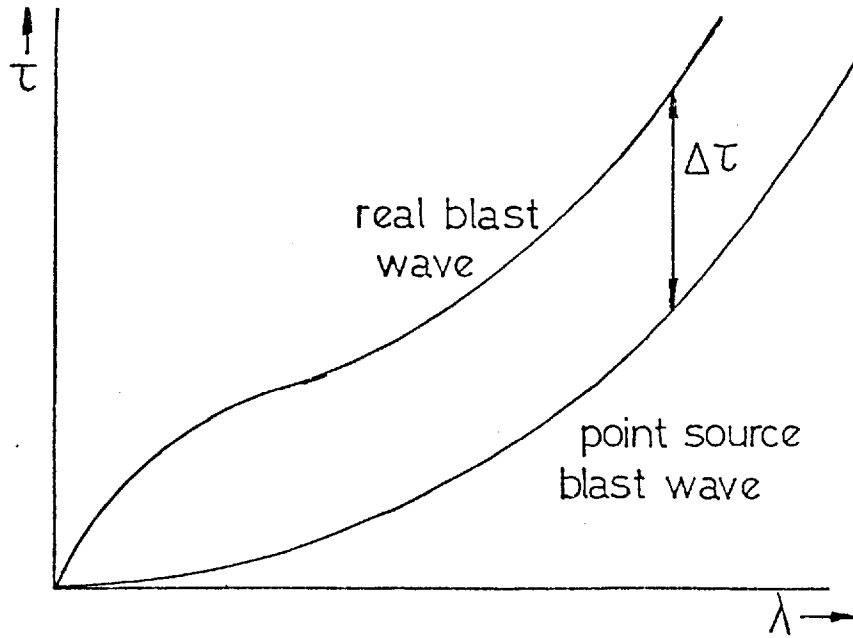


Figure 5.17 Comparison of a real and a point
source blast wave trajectories

blast wave. At large values of λ both types of blast waves will have dissipated much of their energy and their fronts will be travelling at almost sonic velocity. The trajectories of each wave will then be parallel and the value of $\Delta \tau$ at any location downstream will be constant. This reasoning can be used with the results obtained from the shock arrival time gauges as their positioning in all cases was such that the passing shock wave velocity was approximately equal to that of sound. Obviously the faster the blast wave develops the less the shock wave trajectory will differ from that of the point source and therefore these measurements will give a good indication of the relative time over which the blast wave is energised.

Calculations are made by taking each value of the scaled gauge location and the corresponding scaled arrival time from Table 5.6. The arrival time for the point source solution at this same gauge location is determined and subtracted from that of the real blast wave. These are then averaged for all the gauge locations of one particular set of experimental conditions and the

results are presented in Table 5.9 below.

TABLE 5.9
TIME INCREMENT BETWEEN THE ARRIVAL OF A PEAK AND
POINT SOURCE BLAST WAVE AT LARGE DISTANCES

Run Number	$\Delta \tau$
70A/2	0.63
100A/2	0.56
110A/2	0.58
145A/2	0.46
95A/3	0.59
200A/2	0.42
130A/3	0.54
100A/4	0.66
130A/4	0.39
200A/4	0.36
260A/3	0.30
275A/2	0.24
275A/1	0.12
250A/4	0.39
140H/4	0.07
250H/4	0.13
275H/1	0.27
200H/2	0.03
300H/4	0.13

A comparison of these time increments with the scaled pressure transition times (Table 5.6) shows that there is a linear relationship between the two. In figure 5.18 the two variables in question are plotted to confirm their relationship. The experimental conditions that approximate most readily to an instantaneous energy injection into the blast wave, i.e. those with small pressure transition times in the vessel orifice, show the least deviation from the point source blast wave whilst those with the longer pressure transition times show the greatest deviation. By referring back to section

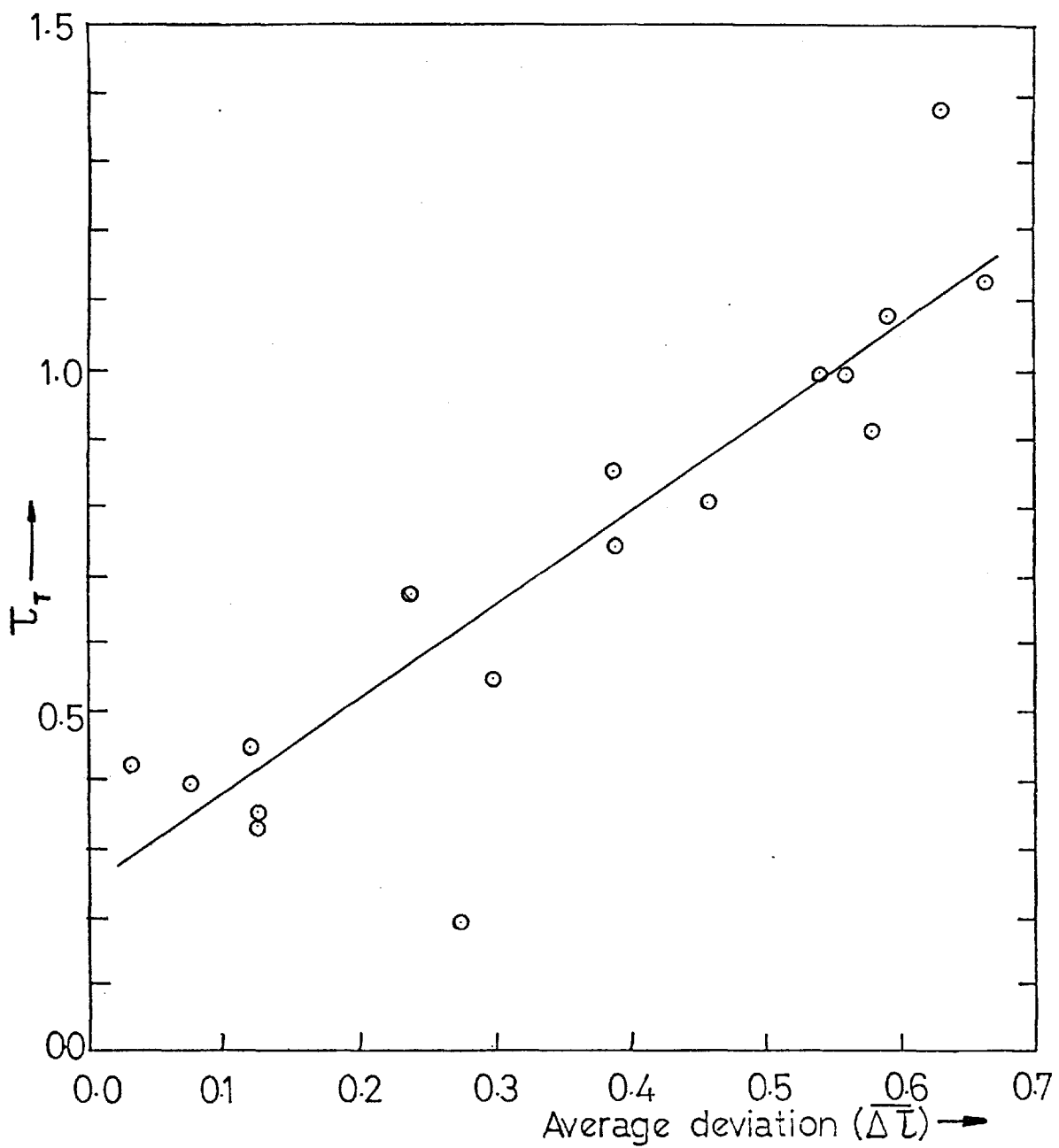


Figure 5.18 Plot of pressure transition time (scaled)
against shock trajectory deviation
from the point source solution

3.6, which deals with the experimental errors, it is possible to show that the two parameters τ_r and $\overline{\Delta\tau}$ are subject to errors of approximately 2 and 10% respectively which indicates that most of the scatter in the experimental points is due to errors in $\overline{\Delta\tau}$.

Further evidence of the blast wave being developed from only part of the total available energy is provided in some recent work by Garrison (1973). In a comparison of the method of T.N.T. scaling (the weight of T.N.T. with energy equivalent to the total vessel contents) with a hydrodynamic model for predicting the blast strength from a ruptured pressure vessel the following conclusions were drawn:

The former method gave unrealistically high overpressures near the vessel and values too low at distances far removed. The hydrodynamic solution which does not include the total available energy in determining the blast strength provided a better correlation with the available data across both regions.

5.7 The Apparent Vessel Discharge Co-efficient

The discharge co-efficient for steady flow through an orifice is a measure of the degree of flow reduction from the theoretically calculated value. It can be expressed in a variety of ways including (i) the ratio of the effective area to the actual area of the orifice (C_D^A) and (ii) the ratio of the actual mass flow rate through the orifice to the theoretical value (C_D^G). There is little information available on the values of discharge co-efficients for quasi-steady flow through a sharp edged orifice. The data that are available for steady flow [Coulson and Richardson (1964)] show the discharge co-efficient (C_D^G) for a sharp edged orifice to be dependant both on the Reynold's number and the area ratio A_1/A_2 , but at high values of the former it is essentially constant at 0.62. Grace and Lapple

(1951) give values of C_D^G for an orifice rounded on the upstream face varying between 0.7 and 0.88 depending upon the degree of curvature and the size of the orifice.

An attempt has been made to deduce values for both these forms of discharge co-efficient under the experimental conditions considered here. Firstly for C_D^A a method is used which determines the value of the Mach Number for the gas approaching the vessel orifice; this is a sole function of the ratio P_3/P_0 and is given by:

$$M_1 = \frac{2}{\gamma - 1} \left\{ \frac{1 - \left\{ P_3/P_0 \right\}^{\frac{\gamma-1}{2\gamma}}}{1 + \left\{ P_3/P_0 \right\}^{\frac{\gamma-1}{2\gamma}}} \right\} \quad (5.5)$$

P_3/P_0 is found from the experimental records of pressure variation in the vessel and the apparent area ratio $A_1/A_2 = \mu$ is then calculated from the expression:

$$\mu^2 = \frac{1}{M_1^2} \left\{ \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M_1^2 \right\}^{\frac{\gamma+1}{\gamma-1}} \quad (5.6)$$

The discharge co-efficient C_D^A is determined by taking the ratio of μ and the actual value of A_1/A_2 . To use the nominal orifice area for the latter ratio would incur an error caused by the fragments of the bursting disc which tend to reduce the size of this aperture. An estimate of this area reduction based on the bursting disc nominal thickness has been made and is incorporated into Table 5.10. A list of the apparent area ratios and also the calculated values of the discharge co-efficient C_D^A is made along with the Reynold's Number for the flow through the orifice. The latter is defined by:

$$Re = \frac{\rho_2 d_2 u_2}{\mu_1} \quad (5.7)$$

TABLE 5.10

THE DISCHARGE CO-EFFICIENTS C_D^A FOR THE QUASI-STEADY
FLOW THROUGH A SHARP EDGED ORIFICE

Run Number	Nominal Orifice Area Ratio	Actual Orifice Area Ratio	Apparent Orifice Area Ratio (μ)	C_D^A	Reynolds Number $\times 10^6$
275A/1	36	37.20	42.00	0.885	5.45
70A/2	9	9.07	8.98	1.010	2.95
100A/2	9	9.09	9.16	0.993	4.00
110A/2	9	9.12	9.28	0.984	4.35
145A/2	9	9.14	9.97	0.918	5.55
200A/2	9	9.18	9.79	0.938	7.49
275A/2	9	9.31	10.82	0.860	10.1
95A/3	4	4.04	3.92	1.031	5.08
130A/3	4	4.05	4.22	0.959	6.72
260A/3	4	4.11	4.51	0.912	12.7
100A/4	2.25	2.27	2.19	1.037	6.15
130A/4	2.25	2.28	2.28	1.000	7.74
200A/4	2.25	2.29	2.46	0.931	11.5
250A/4	2.25	2.31	2.43	0.952	14.2
275H/1	36	37.20	42.91	0.867	1.86
200H/2	9	9.18	10.19	0.900	2.52
140H/4	2.25	2.29	2.31	0.992	2.65
250H/4	2.25	2.31	2.37	0.975	4.52
300H/4	2.25	2.32	2.44	0.952	5.39

Olson (1974) has investigated the effects of both the Reynold's number and the Mach Number on the discharge co-efficient C_D^A . Theoretical treatment of the flow through a standard ASME nozzle did not agree fully with his experimental work, however, results for an orifice which was designed to minimise two-dimensional effects did show good agreement. By calculation of the laminar boundary layer displacement thickness the value of C_D^A was determined and shown to increase with both Reynold's Number and Mach Number.

The results of this work produce an opposite effect, that is the discharge co-efficient decreases with Reynold's Number, the Mach Number in each case being unity. This effect is illustrated in Figure 5.19 where the data of columns 5 and 6 of Table 5.10 are plotted. An estimated line is drawn for each different orifice diameter to make the trend of the plotted data more apparent. It seems unlikely at the high Reynold's Number of the flow investigated here that any laminar boundary layer will be formed. As the Reynold's Number increases two-dimensional effects will cause smaller vena contracta cross-sectional areas and hence smaller discharge co-efficients.

The second discharge co-efficient considered is defined by G/G' , i.e. the ratio of the actual to the theoretical mass flow rates through the orifice. During the steady state period in the vessel an application of the principle of mass conservation shows the flow rate in the main vessel to be equal to that in the orifice and therefore the above ratio can be expressed in the following form:

$$C_D^G = \frac{G}{G'} = \frac{A_1 u_1 \rho_1}{(A_1 u_1 \rho_1)'} = \frac{M_1 c_1 \rho_1}{(M_1 c_1 \rho_1)'} = \frac{M_1}{M_1'} \left(\frac{P_1/P_o}{P_1'/P_o} \right)^{\frac{\gamma+1}{2\gamma}} \quad (5.8)$$

where $\frac{P_1}{P_o}$ can be found from the expression

$$\frac{P_1}{P_o} = \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{\frac{2\gamma}{1-\gamma}} \quad (5.9)$$

A knowledge of both the experimental and theoretical values of M_1 will therefore be sufficient to determine C_D^G . Equation 5.5 with values of P_3/P_o from the experimental work determines the former value of M_1 and a reference to the work of Giffen (1940) mentioned in Chapter 2 will determine the latter value. The figures obtained for C_D^G for each set of experimental

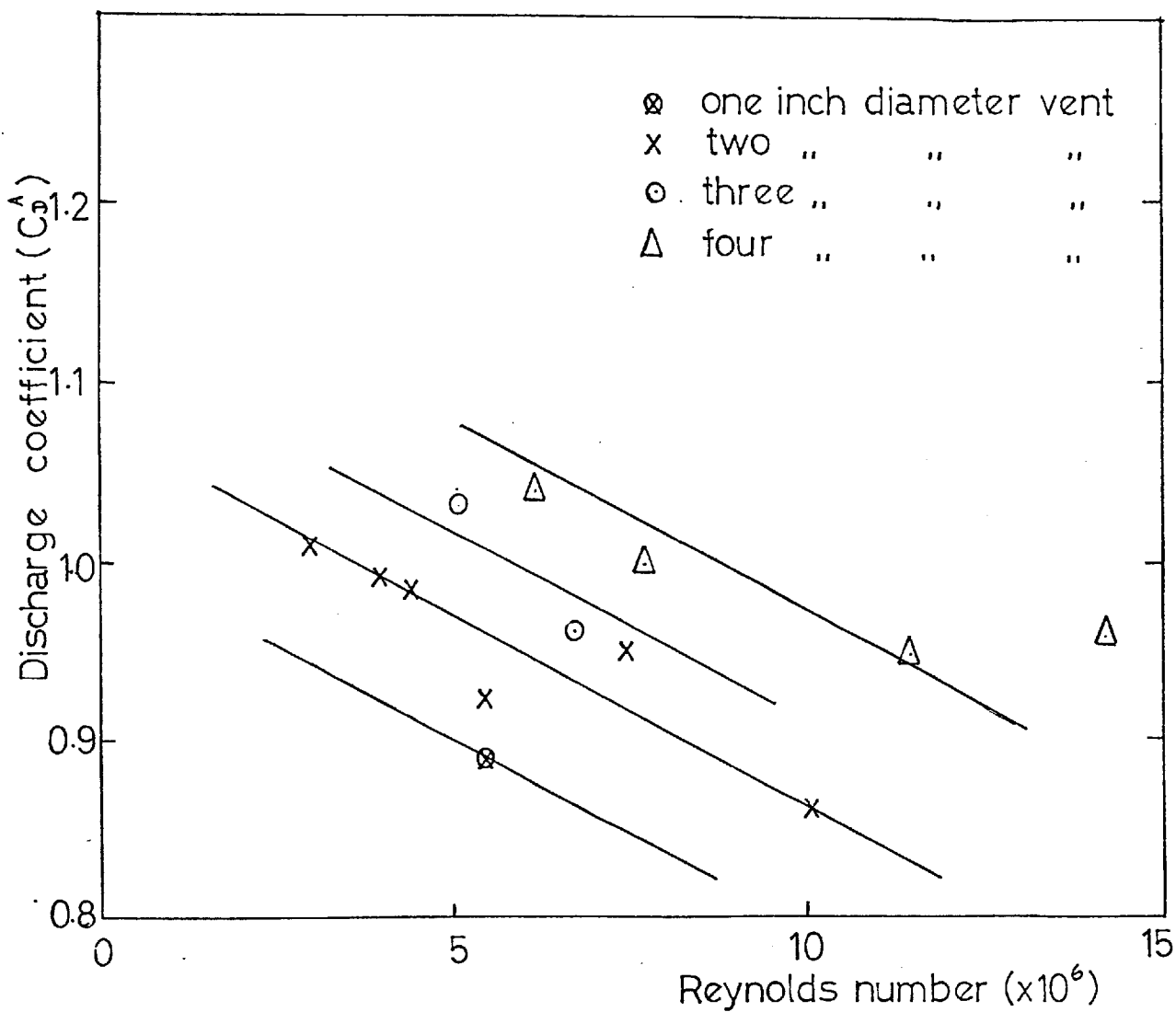


Figure 5.19 Plot of discharge coefficient C_d^A against Reynolds number for quasi-steady sonic flow

conditions are given in Table 5.11 below. These results are very close to the ones for C_D^A and show similar trends, thus confirming that the values of the discharge co-efficient given in the literature for a sharp edged orifice cannot normally be used for this type of flow.

TABLE 5.11

DISCHARGE CO-EFFICIENTS BASED ON MASS FLOW RATES (C_D^G)

Run Number	C_D^G
275A/1	0.863
70A/2	1.007
100A/2	0.988
110A/2	0.977
145A/2	0.914
200A/2	0.929
275A/2	0.847
95A/3	1.017
130A/3	0.955
260A/3	0.901
100A/4	1.016
130A/4	0.988
200A/4	0.931
250A/4	0.941
275H/1	0.842
200H/2	0.890
140H/4	0.938
250H/4	0.960
300H/4	0.979

5.8 An Assessment of the Shock Wave Distortion From a Spherical Form

The spherical shape of the shock wave assumed in this work was based on the source of energy injection approximating to that of a point. However, in the early stages of the blast formation the geometry of the gas release

mechanism is far from spherically symmetric and it is only at much later stages of the blast development that this approximation can be considered. A measurement of the shock wave shape for an eccentric source of energy injection has been made by James (1965) and Mawbey (1965) who both showed the initial form to be ellipsoidal, changing to spherical at later stages. In an attempt to assess the deviation from spherical symmetry of the blast waves produced under the conditions of this work, shock wave arrival times have been measured at different angles to the vessel axis, but maintaining the same distance from the centre of the bursting disc. The arrival times are expressed as the ratio of that on the vessel axis (T_B) to that at the angle under consideration (T_A). An average is taken for each location and the results plotted in Figure 5.20 for a series of different angles and at three positions in front of the vessel. The ratio T_B/T_A for the 300 psig. air blast over a 4 in. diameter orifice increases with both angle and reduced distance from the vessel. This indicates that the shock wave has the greatest deviation from a spherical form at the early stage of development and as it moves away from the vessel, the energy becoming redistributed, it approaches a spherical shape. A 200 psig. blast over a 2 in. diameter orifice reaches a spherical shape much sooner; the results showing that arrival times at locations up to 20° to the axis can be almost identical to those on the axis. It is also evident from Figure 5.20 that the arrival times measured during the main experimentation would not differ from those on the vessel axis beyond the accuracies of the measuring equipment because at no time did the gauges subtend an angle of more than five degrees to the vessel axis.

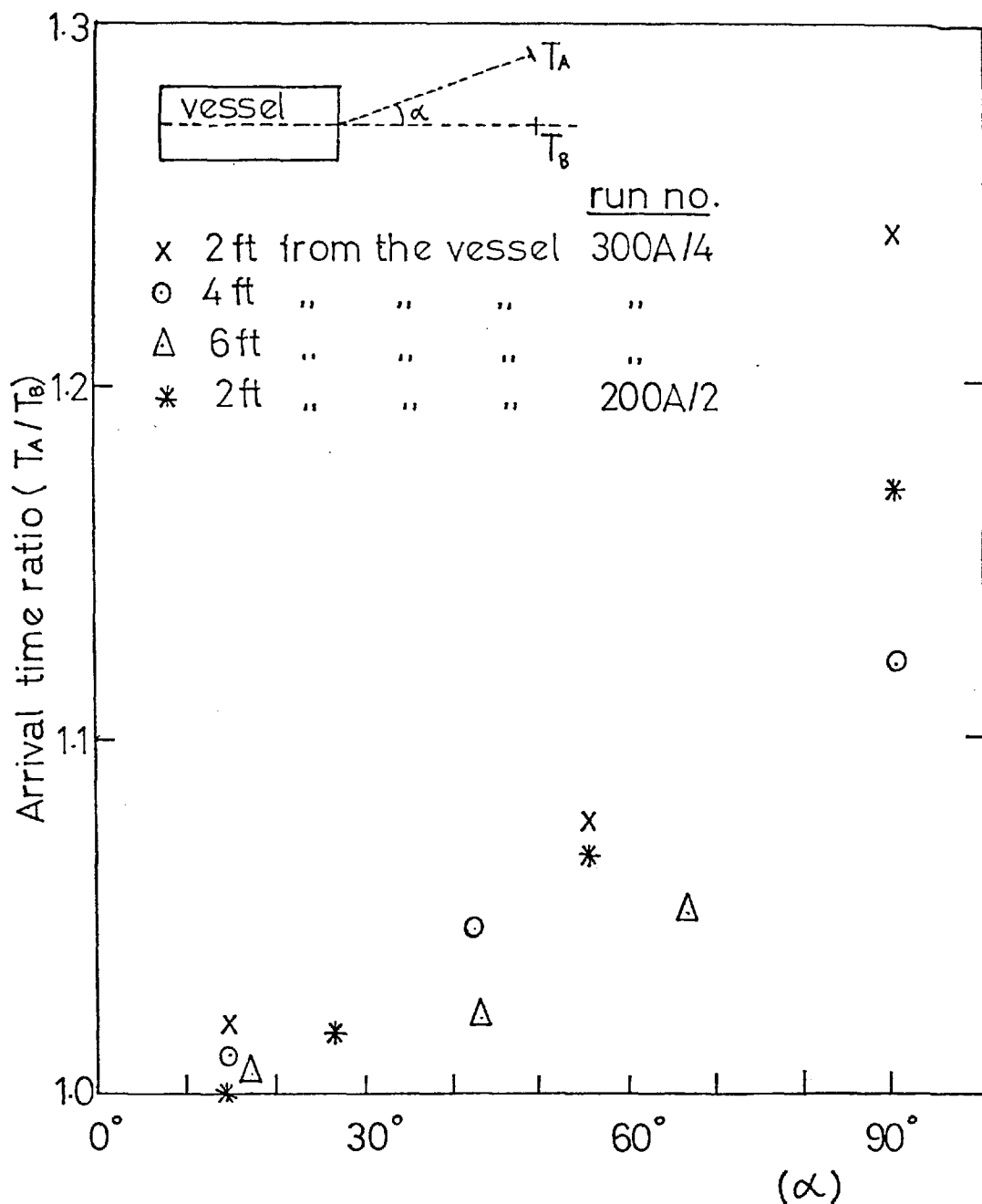


Figure 5.20 Plot of the shock wave arrival time as a function of the angle made with the vessel axis

5.9 Summary and Conclusions

To summarise this work the important points from each section of the results discussion are presented below along with the conclusions.

(i) The computer simulation of the various types of compressible flow associated with an expanding piston gives a clear picture of the development of a discontinuity from a compression wave. The results indicate that the shock wave is formed on the most advanced disturbance set up by a one-dimensional plane piston moving with a uniform acceleration. For this same piston moving with a non-uniform acceleration and for a spherically symmetric piston moving with a uniform acceleration the position of the shock wave formation is, in most cases, between the piston surface and the most advanced disturbance. However, for the latter co-ordinate system, as the initial piston radius increases the shock formation point returns to the outer most disturbance and then approaches asymptotically the value for the plane piston motion. The distance the shock formation point falls short of the outermost disturbance also depends upon the magnitude of the piston acceleration. With the coefficient of the 2nd order term in the equation representing the piston trajectory at values of 0.5, 1.0 and 5.0 the ratios of the shock formation distance co-ordinate to that of the outermost disturbance are 0.83, 0.86 and 1.0 respectively.

(ii) The flow associated with the discharge of a high pressure gas from a cylindrical vessel through an orifice can be determined by computer simulation. A representation of the contact surface trajectory by a fourth order polynomial shows a shock wave to develop at a position that depends on the initial gas pressure, the type of gas and the orifice diameter. For a weak blast (Run No. 70A/2) the shock wave forms at a distance of 0.266 ft. downstream from the orifice whereas the corresponding figure for a strong blast (Run No. 300H/4) is 0.028 ft. The predicted shock trajectory is in good agreement with experimentally determined values and the distance reduction between the contact surface and the shock wave is confirmed for

helium venting systems. The flow pattern developed in this way for a high pressure gas released from one end of a cylindrical vessel is very similar to that recorded by photographic techniques for the same experimental conditions.

(iii) The shock wave arrival times are dependent on the mode of rupture of the bursting disc. This causes 'lost time' in the early stages of shock formation which can never be recovered. At a fixed distance away from the vessel for blast waves produced under identical conditions the standard deviation of the arrival times is a minimum for those conditions under which the opening of the bursting disc approaches the ideal case, that is for thin, small diameter discs. A series of photographic plates demonstrates the number of ways in which a bursting disc can rupture and the types of flow that are subsequently developed. From this evidence it appears that the flow irregularities caused by the bursting disc opening are eventually smoothed out and the subsequent motion is controlled only by the gas expansion process.

(iv) The choking time of the vessel orifice is considerably greater than the bursting disc opening time. This indicates that the energy release rate from the vessel is controlled mainly by the thermodynamic properties of the gas being vented and to a small extent by the mechanical properties of the bursting disc material.

(v) The two scaling lengths ϵ_1 and ϵ_2 derived independently, the first assuming steady state conditions inside the vessel during discharge to derive an analytical expression and the second determined empirically from a knowledge of the energy release time dependence, show a good agreement for the low intensity blast waves. The assumptions used in the derivation of ϵ_1 do not possess the same degree of validity for the larger blasts and therefore a discrepancy appears between the two values.

(vi) In an assessment of both the absolute and relative merits of ϵ_1 and ϵ_2 for scaling the shock wave strength for comparison with the point source blast wave the following points emerged. Of the two scaling lengths ϵ_1 gives a marginally better prediction of the true blast strength but as they stand neither method is effective in obtaining a reliable estimate. The relative effectiveness of ϵ_1 in predicting the change of blast strength brought about by varying bursting pressure, venting area and the type of gas released is good for small venting areas ($A_1/A_2 > 4$) and can be extended to higher pressure blasts with reasonable confidence. For larger venting areas, ($A_1/A_2 < 4$) ϵ_2 is more effective in representing the change in blast strength from different initial conditions, however, extrapolation to blast waves outside the range of this work must only be performed with caution. From these results it appears that it would not be wise to predict blast wave strengths produced by a system with a dis-similar geometrical form. To do this it would be necessary to perform further small scale experimentation to ascertain the effectiveness of the scaling technique under the new conditions. To use either method without analysing the situation before hand would probably mean the strength of the blast wave predicted could be grossly underestimated which would have serious consequences when designing for blast containment. There is, however, a strong case for the relaxation of the constraints placed on the safety margins that are normally allowed from the use of conventional blast scaling techniques.

Further results have shown that there is a linear relationship between the scaled value of the throat choking time and the difference between the arrival time at a fixed location of a real and a point source blast wave. This suggests that the energy associated with the blast wave must be released from the vessel over this period.

(vii) Discharge co-efficients have been calculated for the sonic flow of a gas through an orifice and a strong dependence on both Reynolds number and

orifice diameter is apparent for the conditions of this work. It is therefore not advisable to make use of discharge coefficients from the standard tables for this type of flow as in many cases the value approaches unity.

5.10 Recommendations for Further Work

There are numerous aspects of this work that can be explored further and in so doing add to the knowledge already gained regarding shock wave development and its ultimate strength when driven by an accelerating contact surface.

The rate of transfer of energy across such a surface is an important factor to determine in assessing the blast wave strength. A useful exercise would be to execute program PISTON until a known amount of energy has been transferred across the contact surface. By then bringing the piston to rest instantaneously the resulting shock wave could be analysed. A similar investigation was made by Kiwan (1970), however, in his work the piston motion was uniform and the shock formation was assumed to be instantaneous.

The accuracy of program PISTON is in doubt over extended computation times, it is recommended that before using it for further work the effect of mesh size on the results should be carefully assessed.

An interesting application of computational techniques to shock wave formation is given by Satofuka (1970). By using the Fluid-in-cell (FLIC) method proposed by Gentry et al (1966) and a model for a diaphragm opening process the shock formation was investigated in cylindrical and two-dimensional shock tubes. The attraction of this method is that it solves for the flow between the orifice and the contact surface and also the effect of the diaphragm opening time on the flow development can be obtained. An extension of this method to the venting of a gas from a cylindrical vessel to atmosphere should therefore show some interesting features.

The measurement of the shock wave overpressure to supplement the arrival time measurements at fixed locations downstream from a venting vessel is recommended. This will provide a more accurate assessment of the blast strength at these later stages and provide further data for extending the plots in Figures 5.15 a, b, c and d.

For a short vessel the assumptions made about throat choking times cannot hold as the rarefaction wave will be reflected from the vessel rear and be returned within this period. It is therefore likely that the blast strength will show a dependence on the vessel length below some critical value which is open for investigation.

Most arrangements for venting contents of a vessel include a piping system to carry the vessel contents to a safe area. The back pressure caused by restricting the flow from the vessel will reduce the effectiveness of the relief area in venting an explosion. Further useful experimentation could therefore be performed on venting into pipe systems of different bore and also those that include bends of various angles.

Finally the extension of the experimental techniques used in this work to the other industrial hazard outlined in Chapter One, that is vapour cloud explosions, would be advantageous. Program PISTON can easily be adapted by providing a new set of boundary conditions in place of those at the piston surface that are compatible with flame transition. The photographic equipment can be used directly for obtaining records of the disturbances set up during the early stages of flame propagation. To perform this work in the laboratory would require the careful design of an explosion chamber with observation windows so as to give adequate protection for both equipment and personnel.

NOMENCLATURE

The symbols and meanings listed below are in their order of appearance in each Chapter. From Chapter Two onwards only new symbols are listed, the meanings of the old symbols being taken as those in the previous chapter.

CHAPTER ONE

u	particle velocity
c	sonic velocity
γ	ratio of gas specific heats
M	shock wave Mach number
T	gas temperature
ρ	gas density
P	gas pressure
S	gas specific entropy
C_v	specific heat of gas at constant volume
x, r	distance
t	time
a_2, a_3	constant coefficients
E_0	explosion energy
η	similarity variable
λ	scaled distance
τ	scaled time
ϵ_1	scaling length
A_2	orifice area

Sub and superscripts

$-$	ambient conditions
1	refers to immediately behind the shock wave
0	refers to immediately ahead of the shock wave or to undisturbed gas
p	refers to a piston motion
s	refers to the point of shock wave formation

CHAPTER TWO

η	parameter to indicate the co-ordinate system
λ	a constant multiplying factor
$\lambda_{1,2,3}$	identifying symbol for each characteristic direction
r	New dependent variable
s	New dependent variable
C_p	specific heat of gas at constant pressure
A_1	vessel cross-sectional area
z	vessel length
E	energy
M_o	mass flow rate
Δx	step length for numerical integration
P_c	critical pressure

Subscripts

0, 3	refers to conditions in the vessel when the gas is stationary
1, 4	refers to conditions in the vessel after the passage of the rarefaction wave
2	refers to conditions in the vessel orifice

CHAPTER THREE

t	diaphragm thickness
a	diaphragm radius
R	strain gauge radius
ϵ_{min}	minimum diaphragm design strain
S_{max}	maximum diaphragm design stress
S_1	maximum working stress of diaphragm material
f	frequency
f_{min}	minimum diaphragm design frequency

CHAPTER FOUR

t	time
a_0, a_1, a_2	constant coefficients
T	time

CHAPTER FIVE

X	distance
C_0	sound speed in undisturbed gas
T_s	shock formation time
$\epsilon, \epsilon_1, \epsilon_2$	scaling lengths
τ_T	scaled pressure transition time
T_T	pressure transition time
S	shock wave strength
D	distance
C	scaling length correction factor
C_D^G	discharge coefficient based on mass flow rates
C_D^A	discharge coefficient based on effective orifice area
Re	Reynolds number
μ	area ratio (A_1/A_2)
d_2	orifice diameter
μ_1	gas viscosity in the orifice
G	mass flow rate

APPENDIX A

W	shock wave velocity
v	particle velocity relative to the shock wave
V	gas specific volume

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APPENDIX A

The Classical Shock Wave Theory

The classical shock wave theory regards a shock as a pure discontinuity and disregards the properties of real gases that tend to smooth out all such discontinuities; these being viscosity, heat conduction and mass diffusion effects. The general laws of conservation of mass, momentum and energy can be applied to give relationships between the gas properties in front and behind the shock wave.

It is convenient to utilise a co-ordinate system which has the shock plane itself as the reference datum. Denoting gas properties by subscript 'o' in the preshock region and by subscript '1' in the shocked region and with reference to Figure A1 we have:

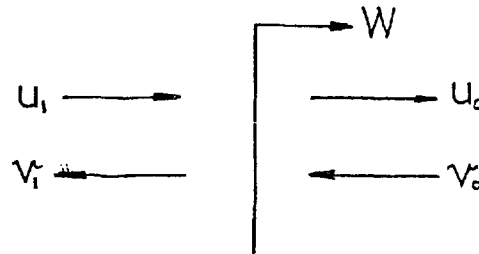


Figure A1 Coordinate system for a shock wave

$$v_o = W - u_o$$

$$v_1 = W - u_1$$

and from the laws of conservation:

$$\text{Mass} \quad \frac{v_1}{V_1} = \frac{v_o}{V_o} \quad (A.1)$$

$$\text{Momentum} \quad P_1 + \frac{v_1^2}{V_1} = P_o + \frac{v_o^2}{V_o} \quad (A.2)$$

$$\text{Energy} \quad \frac{1}{2}v_1^2 + \frac{\gamma}{\gamma-1} P_1 V_1 = \frac{1}{2}v_o^2 + \frac{\gamma}{\gamma-1} P_o V_o \quad (A.3)$$

From Equations (A.1) and (A.2) the relative gas velocities become

$$v_o^2 = v_o^2 \frac{P_1 - P_o}{V_o - V_1}$$

$$v_1^2 = v_1^2 \frac{P_1 - P_o}{V_o - V_1}$$

and by substituting in Equation (A.3) the following relationship is obtained:

$$\frac{P_1}{P_o} = \frac{(\gamma + 1)V_o - (\gamma - 1)V_1}{(\gamma + 1)V_1 - (\gamma - 1)V_o} \quad (A.4)$$

Equation (A.4) is known as the shock adiabat or the Hugoniot relation. The curve is shown in Figure A 2 passing through the point (P_o, V_o) and giving the loci of all other shocked conditions that can be reached from the initial state. It can be shown that for a given shock velocity there is a unique value for the shocked conditions.

From Equations (A.1) and (A.2) the mass flux across the shock is given by:

$$M^2 = \left\{ \frac{v_o}{V_o} \right\}^2 = \frac{P_1 - P_o}{V_o - V_1} = \text{const.} \quad (A.5)$$

Therefore, starting from (P_o, V_o) on the Hugoniot curve, a line can be drawn with slope $-\left\{ \frac{v_o}{V_o} \right\}^2$ and the values of P_1 and V_1 are determined at the

intersection with the Hugoniot curve.

Points on the curve to the right and below (P_o, V_o) cannot represent real solutions for the shock transition.

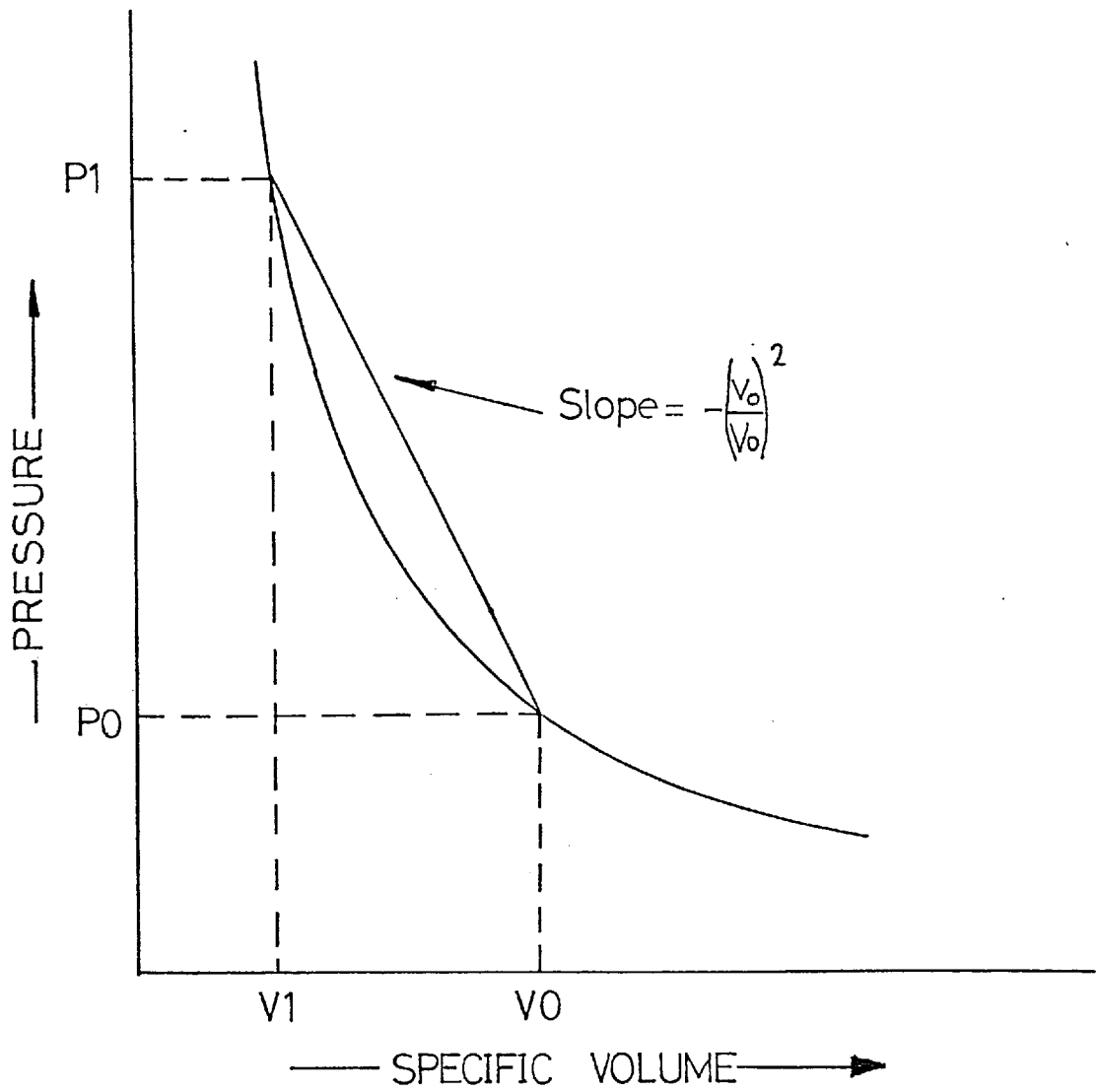


FIGURE A2 HUGONIOT CURVE

Derivation of the Rankine-Hugoniot Relations for Shock Transition

The Rankine-Hugoniot relations express the change in properties of a gas after it has passed through a shock wave. They are normally given as a function of shock strength: in this case the Mach number is used and defined below:

$$M = \frac{W - u_o}{c_o} = \frac{v_o}{c_o}$$

Putting Equation (A.3) in the form

$$\frac{P_1 v_1}{v_1} = \frac{\gamma - 1}{\gamma} \frac{k}{v_1} - \frac{\gamma - 1}{2\gamma} v_1 \quad (\text{A.6a})$$

$$\text{and } \frac{P_o v_o}{v_o} = \frac{\gamma - 1}{\gamma} \frac{k}{v_o} - \frac{\gamma - 1}{2\gamma} v_o \quad (\text{A.6b})$$

where k = constant, rearranging Equation A.2 and combining with Equations (A.6a) and (A.6b)

$$v_1 v_o = 2 \frac{\gamma - 1}{\gamma + 1} k \quad (\text{A.7})$$

Replacing the expression for k and using the definition of the speed of sound we have:

$$v_1 = \frac{2}{\gamma + 1} \frac{c_o^2}{v_o} + \frac{\gamma - 1}{\gamma + 1} v_o \quad (\text{A.8})$$

from which

$$\frac{v_1 - v_o}{c_o} = \frac{2}{\gamma + 1} \frac{c_o}{v_o} - \frac{2}{\gamma + 1} \frac{v_o}{c_o} \quad (\text{A.9})$$

and

$$\frac{u_1 - u_o}{c_o} = \frac{2}{\gamma + 1} \left(M - \frac{1}{M} \right) \quad (\text{A.10})$$

By rearranging Equations (A.4) and (A.5) and substituting for the speed of sound:

$$\frac{2\gamma v_o^2}{c_o^2} = (\gamma + 1) \frac{P_1}{P_o} + (\gamma - 1) \quad (A.11)$$

$$\therefore \frac{P_1}{P_o} = 1 + \frac{2\gamma}{\gamma + 1} (M^2 - 1) \quad (A.12)$$

A rearrangement of Equation (A.4) and substitution for P_1/P_o from Equation (A.12)

$$\frac{v_o}{v_1} = \frac{\rho_1}{\rho_o} = \left[1 - \frac{2}{\gamma + 1} \left\{ 1 - \frac{1}{M^2} \right\} \right]^{-1} \quad (A.13)$$

The ratio of the sound velocities is obtained from the equation of state which, after substituting for P_1/P_o and ρ_1/ρ_o and rearranging yields

$$\frac{c_1^2}{c_o^2} = \frac{T_1}{T_o} = 1 + 2 \frac{(\gamma - 1)}{(\gamma + 1)^2} \left[\gamma M^2 - \frac{1}{M^2} - (\gamma - 1) \right] \quad (A.14)$$

Similarly the equation of state can be expressed in the following form:

$$\frac{P_1}{P_o} = \left\{ \frac{\rho_1}{\rho_o} \right\}^\gamma \exp \left\{ \frac{S_1 - S_o}{C_v} \right\} \quad (A.15)$$

and substitution for P_1/P_o and ρ_o/ρ_1 gives

$$\frac{S_1 - S_o}{C_v} = \ln \left[1 + \frac{2\gamma}{\gamma + 1} (M^2 - 1) \right] + \gamma \ln \left[1 - \frac{2}{\gamma + 1} \left\{ 1 - \frac{1}{M^2} \right\} \right] \quad (A.16)$$

APPENDIX B

A listing of program PISTON can be found on the following pages, immediately after which is a table of the principle names and their meanings. Names which have not been explained are usually intermediates in the calculation of a defined variable, others are integers that make up the program logic. A series of flow charts, one for each routine, follows which describe the logic system in more detail.

```

PROGRAM PISTON (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE62)
DIMENSION XX(100),TT(100)
COMMON      X(50,50),T(50,50),U(50,50),C(50,50),S(50,50),XS(50),TS(5
10),          JEND(50),US1(50),US0(50),CS1(50),CS0(50),SS1(50
2),SS0(50)
COMMON/ONE/GAMMA,C0,X0,ETA,I,J,VELSH,IP,WORKT
WRITE(6,597)
597 FORMAT(/1X,*.....OUTPUT FROM DECK NUMBER 1.....*)
IP=6
WORKT=0.0
X0=1.
C0=1126.0
TSTOP=20.0
TSTOP=TSTOP*C0/(X0*10.0**3)
GAMMA=1.4
ETA=2.0
ETA=0.0
PIST=1.0
PIST=PIST/12.0
XSTEP=0.1
JSE=49
JSTOP=JSE
DO 400 I=1,JSE
JEND(I)=JSE
400 CONTINUE
DO 20 J=1,JSE
JS=J
XSTART=PIST+FLOAT(J-1)*XSTEP
DO 21 I=1,J
IS=I-1
IF(I-1) 25,25,26
25 X(I,J)=(XSTART+PIST)/2.0
T(I,J)=X(I,J)-PIST
U(I,J)=0.0
C(I,J)=1.0
S(I,J)=1.0
GO TO 27
26 IF(I-J) 23,22,22
22 CALL PATH
GO TO 24
23 CALL PT3
IF(T(I,J)-T(I-1,J)) 28,28,29
29 CONTINUE
24 CONTINUE
27 CONTINUE
WRITE(6,31)I,J,X(I,J),T(I,J),U(I,J),C(I,J),S(I,J)
31 FORMAT(1X,2I5,5(4X,F12.6))
21 CONTINUE
20 CONTINUE
WRITE(6,33)
33 FORMAT(1X,*NO SHOCK FORMATION IN THE REGION OF INTEGRATION*)
GO TO 32
28 I=IS+1
J=JS
XT1=(X(I,J)-X(I,J-1))/(T(I,J)-T(I,J-1))
XT2=(X(I-1,J)-X(I-1,J-1))/(T(I-1,J)-T(I-1,J-1))
TS(JS)=(X(I-1,J-1)-X(I,J-1)+XT1*T(I,J-1)-XT2*T(I-1,J-1))/(XT1-XT2)
XS(JS)= X(I-1,J-1)+XT2*(TS(JS)-T(I-1,J-1))

```

```

WRITE(6,30)IS,JS,XS(JS),TS(JS)
30 FORMAT(1X,2(5X,I3),5X,*THE SHOCK BEGINS TO FORM AT X=*,F6.3,*AND A
1TIME OF*,F6.3)
32 CONTINUE
J=JS
JSS=JS
IU=IS
JEND(IS)=JS
JEND(IS+1)=JS
IO=JSS-IS
IOST=IS+1
VELSH=U(IOST,JSS-1)+C(IOST,JSS-1)
XSTART=PIST+FLOAT(JSS-1)*XSTEP+XS(JSS)-X(IOST,JSS-1)
34 CONTINUE
DO 131 I=1,IU
IF (I-1) 35,35,35
35 X(I,J)=(XSTART+PIST)/2.0
T(I,J)=X(I,J)-PIST
U(I,J)=0.0
C(I,J)=1.0
S(I,J)=1.0
GO TO 37
36 CALL PT3
37 CONTINUE
131 CONTINUE
XERR=XS(JSS)-X(IU,JSS)
IF (ABS(XERR)-0.00001) 38,33,39
39 XSTART=XSTART+XERR
GO TO 34
38 CONTINUE
DO 601 I=1,IU
WRITE(6,120) I,J,X(I,J),T(I,J),U(I,J),C(I,J),S(I,J)
120 FORMAT(1X,2I3,5X,5F10.5)
601 CONTINUE
US0(JSS)=U(IU,JSS)
CS0(JSS)=C(IU,JSS)
SS0(JSS)=S(IU,JSS)
42 CALL SHOCK
U(IOST,JSS)=US1(JSS)
C(IOST,JSS)=CS1(JSS)
S(IOST,JSS)=SS1(JSS)
X(IOST,JSS)=XS(JSS)
T(IOST,JSS)=TS(JSS)
C**** CHECK THE SHOCK VELOCITY HAS BEEN CHOSEN CORRECTLY
R1=(U(IOST,JSS-1)/2.0)+C(IOST,JSS-1)/(GAMMA-1.0)
S1=(U(IOST,JSS-1)/2.0)-C(IOST,JSS-1)/(GAMMA-1.0)
RSH1=(U(IOST,JSS)/2.0)+C(IOST,JSS)/(GAMMA-1.0)
SSH1=(U(IOST,JSS)/2.0)-C(IOST,JSS)/(GAMMA-1.0)
SR1=(R1-S1+RSH1-SSH1)/2.0
SR2=(R1**2-S1**2+RSH1**2-SSH1**2)/2.0
X1=(X(IOST,JSS)+X(IOST,JSS-1))/2.0
R3=R1+SR1*(S(IOST,JSS)-S(IOST,JSS-1))/4.0-((GAMMA-1.0)/4.0)*SR2*(E
1T4/X1)*(T(IOST,JSS)-T(IOST,J-1))
RERR=R3-RSH1
IF (ABS(RERR)-0.00001) 40,40,41
41 VELSH=VELSH+RERR
GO TO 42
40 CONTINUE

```

```

WRITE(6,117) VELSH,J,RERR
117 FORMAT(1X,*A SHOCK VELOCITY OF *,F10.5,*ON J=*,I3,*GIVES AN RERR
10F*,F10.5)
WRITE(6,700) J,VELSH
700 FORMAT(1X,I5,4X,F8.4,*      US      CS      SS*)
WRITE(6,701)US0(J),CS0(J),SS0(J)
701 FORMAT(1X,*      0*,3(4X,F8.5))
WRITE(6,702)US1(J),CS1(J),SS1(J)
702 FORMAT(1X,*      1*,3(4X,F8.5))
C*** PROCEED FROM THE SHOCK TO THE PISTON PATH
IOST1=IOST+1
DO 43 I=IOST1,JSS
IS=I-1
JS=J
IF (I-J) 44,45,45
45 CALL PATH
GO TO 45
44 CALL PT3
IF (T(I,J)-T(I-1,J)) 47,48,48
48 CONTINUE
46 CONTINUE
WRITE(6,121) I,J,X(I,J),T(I,J),U(I,J),C(I,J),S(I,J)
121 FORMAT(1X,2I3,5X,5F10.5)
43 CONTINUE
GO TO 99
47 WRITE(6,49) I,J
49 FORMAT(1X,*SORRY, CHOSEN INCORRECT SHOCK STARTING POINT*,2I4)
GO TO 29
99 CONTINUE
IU=IU-1
IOST=IOST+1
JS1=JS+1
DO 50 J=JS1,JSE
C*** CALCULATE INTERSECTION OF SHOCK WITH CHARACTERISTIC BEHIND
UC=U(IOST,J-1)+C(IOST,J-1)
TINT1=(X(IOST,J-1)-XS(J-1)+VELSH*TS(J-1)-UC*T(IOST,J-1))/(VELSH-UC
1)
XINT1=X(IOST,J-1)+UC*(TINT1-T(IOST,J-1))
IF(IU-1) 52,64,64
C*** CALCULATE INTERSECTION OF SHOCK WITH CHARACTERISTIC IN FRONT
64 UC=U(IU,J-1)+C(IU,J-1)
TINT0=(X(IU,J-1)-XS(J-1)+VELSH*TS(J-1)-UC*T(IU,J-1))/(VELSH-UC)
XINT0=X(IU,J-1)+UC*(TINT0-T(IU,J-1))
IF (XINT0-XINT1) 51,51,52
51 IWER=0
XS(J)=XINT0
TS(J)=TINT0
GO TO 53
52 IWER=1
XS(J)=XINT1
TS(J)=TINT1
53 CONTINUE
DXS=XS(J)-XS(J-1)
IF (DXS-0.5) 980,981,981
981 JSE=J-1
IF (JSTOP.EQ.JSE+1) JSTOP=JSE
GO TO 100
980 CONTINUE

```

```

      IF(JSTOP-JSE) 997,996,996
996 IF(TSTOP-T(J-1,J-1)) 999,999,998
999 JSTOP=J-1
997 ISTOP=JSTOP
      GO TO 995
998 ISTOP=J
999 CONTINUE
      WRITE(6,107) J, XS(J),TS(J),IWER
107 FORMAT(/1X,I3,*XS=*,F10.5,*TS=*,F10.5,*IWER=*,I3)
      IF (IJ-1) 130,131,131
131 CONTINUE
      XSTART=PIST+FLOAT(J-1)*XSTEP+XS(J)-XS(J-1)
54 CONTINUE
      DO 55 I=1,IU
      IF(I-1) 55,56,57
56 X(I,J)=(XSTART+PIST)/2.0
      T(I,J)=X(I,J)-PIST
      U(I,J)=0.0
      C(I,J)=1.0
      S(I,J)=1.0
      GO TO 58
57 CALL PT3
58 CONTINUE
55 CONTINUE
      IF (IWER) 59,59,60
60 I=IU+1
      UC=U(I-1,J)-C(I-1,J)
      VUC=VELSH-UC
      TX=(X(I-1,J)-XS(J-1)+TS(J-1)*VELSH-(T(I-1,J)*UC))/VUC
      XX=X(I-1,J)+UC*(TX-T(I-1,J))
      XERR=XS(J)-XX
      IF(ABS(XERR)-0.00001) 65,65,66
66 XSTART=XSTART+XERR
      GO TO 54
65 CONTINUE
      DO 602 I=1,IU
      WRITE(6,122) I,J,X(I,J),T(I,J),U(I,J),C(I,J),S(I,J)
122 FORMAT(1X,2I3,5X,5F10.5)
602 CONTINUE
      I=IU+1
      XWAIT=X(I,J-1)
      TWAIT=T(I,J-1)
      UWAIT=U(I,J-1)
      CWAIT=C(I,J-1)
      SWAIT=S(I,J-1)
C***** CALCULATE THE CONDITIONS AT THE SHOCK BOUNDARY BY PROJECTING AN EXT
C*****+IVE CHARACTERISTIC BACK TO J-1
      TBACK=TS(J)-T(I-1,J)+T(I-1,J-1)
63 UC=(UWAIT+U(I-1,J-1)-CAWAIT-C(I-1,J-1))/2.0
      XBACK=X(I-1,J-1)+UC*(TBACK-T(I-1,J-1))
      CBACK=C(I-1,J-1)+((CAWAIT-C(I-1,J-1))*(TBACK-T(I-1,J-1)))/(TWAIT-T(I-1,J-1))
      UBACK=U(I-1,J-1)+((UWAIT-U(I-1,J-1))*(TBACK-T(I-1,J-1)))/(TWAIT-T(I-1,J-1))
      SBACK=S(I-1,J-1)+((SWAIT-S(I-1,J-1))*(TBACK-T(I-1,J-1)))/(TWAIT-T(I-1,J-1))
      X(I,J-1)=XBACK
      T(I,J-1)=TBACK

```

```

U(I, J-1)=UBACK
C(I, J-1)=CBACK
S(I, J-1)=SBACK
CALL PT3
XERR=XS(J)-X(I, J)
IF (ABS(XERR)-0.00001) 61,61,62
62 TBACK=TBACK-XERR
GO TO 63
61 CONTINUE
X(I, J-1)=XWAIT
T(I, J-1)=TWAIT
U(I, J-1)=UWAIT
C(I, J-1)=CWAIT
S(I, J-1)=SWAIT
US0(J)=U(I, J)
CS0(J)=C(I, J)
SS0(J)=S(I, J)
GO TO 182
180 CONTINUE
US0(J)=0.0
CS0(J)=1.0
SS0(J)=1.0
182 CONTINUE
JEND(I0ST)=J
VELSH=U(I0ST, J-1)+C(I0ST, J-1)
69 CALL SHOCK
U(I0ST, J)=US1(J)
C(I0ST, J)=CS1(J)
S(I0ST, J)=SS1(J)
X(I0ST, J)=XS(J)
T(I0ST, J)=TS(J)
R1=(U(I0ST, J-1)/2.0)+C(I0ST, J-1)/(GAMMA-1.0)
S1=(U(I0ST, J-1)/2.0)-C(I0ST, J-1)/(GAMMA-1.0)
RSH1=(U(I0ST, J)/2.0)+C(I0ST, J)/(GAMMA-1.0)
SSH1=(U(I0ST, J)/2.0)-C(I0ST, J)/(GAMMA-1.0)
SR1=(R1-S1+RSH1-SSH1)/2.0
SR2=(R1**2-S1**2+RSH1**2-SSH1**2)/2.0
X1=(X(I0ST, J)+X(I0ST, J-1))/2.0
R3=R1+SR1*(S(I0ST, J)-S(I0ST, J-1))/4.0-((GAMMA-1.0)/4.0)*SR2*(ETA/X
11)*(T(I0ST, J)-T(I0ST, J-1))
RERR=R3-RSH1
IF (ABS(RERR)-0.00001) 67,67,68
63 VELSH=VELSH+RERR
IF (VELSH.LT.1.0) VELSH=1.0
GO TO 69
67 CONTINUE
WRITE(6,110) VELSH, J, RERR
110 FORMAT(1X, 'A SHOCK VELOCITY OF ', F10.5, ' ON J=', I3, ' GIVES AN RERR
10F', F10.5)
WRITE(6,800) J, VELSH
800 FORMAT(1X, I5, 4X, F6.4, ' US CS SS*)
WRITE(6,801) US0(J), CS0(J), SS0(J)
801 FORMAT(1X, ' 0*, 3(4X, F8.5))
WRITE(6,802) US1(J), CS1(J), SS1(J)
802 FORMAT(1X, ' 1*, 3(4X, F8.5))
I0ST=I0ST+1
DO 70 I=I0ST, ISTOP
IF (I-J) 71,72,72

```

```
72 CALL PATH
GO TO 73
71 CALL PT3
IF (T(I,J)-T(I-1,J)) 74,74,75
75 CONTINUE
73 CONTINUE
WRITE(6,123) I,J,X(I,J),T(I,J),U(I,J),C(I,J),S(I,J)
23 FORMAT(1X,2I3,5X,5F10.5)
70 CONTINUE
GO TO 77
74 WRITE(6,76)
75 FORMAT(1X,*SECOND SHOCK FORMATION*)
77 CONTINUE
GO TO 50
59 XERR=XS(J)-X(IU,J)
IF (ABS(XERR)-0.00001) 80,80,81
61 XSTART=XSTART+XERR
GO TO 54
60 CONTINUE
DO 603 I=1,IU
WRITE(6,111) I,J,X(I,J),T(I,J),U(I,J),C(I,J),S(I,J)
11 FORMAT(1X,2I3,5X,5F10.5)
03 CONTINUE
JEND(IU)=J
US0(J)=U(IU,J)
CS0(J)=C(IU,J)
SS0(J)=S(IU,J)
84 CALL SHOCK
I=IOST-1
X(I,J)=XS(J)
T(I,J)=TS(J)
U(I,J)=US1(J)
C(I,J)=CS1(J)
S(I,J)=SS1(J)
** CALCULATE THE INTERSECTION OF THE CHARACTERISTIC BEHIND THE SHOCK WITH J-1
UC1=U(I,J)+C(I,J)
UC2=(US1(J-1)+U(IOST,J-1))/2.0-(CS1(J-1)+C(IOST,J-1))/2.0
UC3=UC1-UC2
TX=(XS(J-1)-X(I,J)-UC2*TS(J-1)+UC1*T(I,J))/UC3
XX=XS(J-1)+UC2*(TX-TS(J-1))
UX=US1(J-1)+((U(IOST,J-1)-US1(J-1))*(TX-TS(J-1)))/(T(IOST,J-1)-TS(
1J-1))
CX=CS1(J-1)+((C(IOST,J-1)-CS1(J-1))*(TX-TS(J-1)))/(T(IOST,J-1)-TS(
1J-1))
SX=SS1(J-1)+((S(IOST,J-1)-SS1(J-1))*(TX-TS(J-1)))/(T(IOST,J-1)-TS(
1J-1))
R1=UX/2.0+CX/(GAMMA-1.0)
S1=UX/2.0-CS/(GAMMA-1.0)
RSH1=US1(J)/2.0+CS1(J)/(GAMMA-1.0)
SSH1=US1(J)/2.0-CS1(J)/(GAMMA-1.0)
SR1=(R1-S1+RSH1-SSH1)/2.0
SR2=(R1**2-S1**2+RSH1**2-SSH1**2)/2.0
X1=(XX+XS(J))/2.0
R3=R1+SR1*(SS1(J)-SX)/4.0-((GAMMA-1.0)/4.0)+SR2*(ETA/X1)*(TS(J)-TX
1)
RERR=R3-RSH1
IF (ABS(RERR)-0.00001) 82,82,83
83 VELSH=VELSH+RERR
```

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      IF (VELSH.LT.1.0) VELSH=1.0
      GO TO 84
82  CONTINUE
      WRITE(6,112) VELSH,J,RERR
112  FORMAT(1X,*A SHOCK VELOCITY OF *,F10.5,*ON J=*,I3,*GIVES AN RERR
      1OF*,F10.5)
      WRITE(6,900) J,VELSH
900  FORMAT(/1X,I5,4X,F6.4,*      US      CS      SS*)
      WRITE(6,901)US0(J),CS0(J),SS0(J)
901  FORMAT(1X,*      0*,3(4X,F8.5))
      WRITE(6,902)US1(J),CS1(J),SS1(J)
902  FORMAT(1X,*      1*,3(4X,F8.5))
      DO 85 I=I0ST,ISTOP
      IF (I-J) 85,87,87
87  CALL PATH
      GO TO 88
86  CALL PT3
      IF (T(I,J)-T(I-1,J)) 89,89,90
90  CONTINUE
88  CONTINUE
      WRITE(6,124) I,J,X(I,J),T(I,J),U(I,J),C(I,J),S(I,J)
124  FORMAT(1X,2I3,5X,5F10.5)
85  CONTINUE
      IU=IU-1
      GO TO 92
89  WRITE(6,91)
91  FORMAT(1X,*SECOND SHOCK FORMATION*)
92  CONTINUE
50  CONTINUE
100 CONTINUE
C*****  PLOT THE CHARACTERISTIC NETWORK
      CALL START(2)
      XX(1)=0.0
      XX(2)=XS(JSE)
      TT(1)=0.0
      TT(2)=T(ISTOP,JSE)
      CALL SCALE (XX,12.0,2,1)
      CALL SCALE (TT,12.0,2,1)
      FIRSTX=XX(3)
      DELTX=0.25
      DELTX=XX(4)
      FIRSTT=TT(3)
      DELTT=0.25
      DELTT=TT(4)
      CALL AXIS(0.0,0.0,22HRADIAL DISTANCE (FEET),-22,12.0,0.0,FIRSTX,DE
1LTX)
      CALL AXIS(0.0,0.0,23HSCALED TIME T*CO (FEET),23,12.0,90.0,FIRSTT,D
1ELTT)
      CALL PLOT (0.0,12.0,3)
      CALL PLOT (12.0,12.0,2)
      CALL PLOT (12.0,0.0,2)
C*****  PLOT THE PISTON PATH
      JP=JSTOP
      DO 101 J=1,JP
      XX(J)=X(J,J)-PIST
      TT(J)=T(J,J)
101  CONTINUE
      XX(JP+1)=FIRSTX

```

```
XX(JP+2)=DELTX
TT(JP+1)=FIRSTT
TT(JP+2)=DELTT
CALL LINE (XX,TT,JP,1,0,1)
C*****+ PLOT THE +IVE CHARACTERISTICS
JS1=1
JS1=JSTOP
DO 102 I=1,JS1
IF(JEND(I).GT.JSE) JEND(I)=JSE
JS2=JEND(I)
DO 103 J=I,JS2
JP=J-I+1
XX(JP)=X(I,J)-PIST
TT(JP)=T(I,J)
103 CONTINUE
XX(JP+1)=FIRSTX
XX(JP+2)=DELTX
TT(JP+1)=FIRSTT
TT(JP+2)=DELTT
CALL LINE (XX,TT,JP,1,0,1)
102 CONTINUE
C*****+ PLOT THE SHOCK PROFILE
DO 106 J=JS,JSE
JP=J+1-JS
XX(JP)=XS(J)-PIST
TT(JP)=TS(J)
106 CONTINUE
XX(JP+1)=FIRSTX
XX(JP+2)=DELTX
TT(JP+1)=FIRSTT
TT(JP+2)=DELTT
CALL LINE(XX,TT,JP,1,0,1)
CALL ENPLOT
JS=JS+1
STOP
END
```

COMPILER SPACE

```

SUBROUTINE PTS
  DIMENSION XX(100), TT(100)
  COMMON      X(50,50), T(50,50), U(50,50), C(50,50), S(50,50), XS(50), TS(5
10), JEND(50), US1(50), US0(50), CS1(50), CS0(50), SS1(50
2), SS0(50)
  COMMON /ONE/ GAMMA, CO, XC, ETA, I, J, VELSH, IP, WORKT
  IT=-1
  ITT=-1
  ITT=1
  R1=(U(I, J-1)/2.0)+C(I, J-1)/(GAMMA-1.0)
  S1=(U(I, J-1)/2.0)-C(I, J-1)/(GAMMA-1.0)
  R2=(U(I-1, J)/2.0)+C(I-1, J)/(GAMMA-1.0)
  S2=(U(I-1, J)/2.0)-C(I-1, J)/(GAMMA-1.0)
  RS1=(GAMMA+1.0)*R1/2.0-(GAMMA-3.0)*S1/2.0
  RS2=(3.0-GAMMA)*R2/2.0+(GAMMA+1.0)*S2/2.0
10  T(I, J)=(X(I-1, J)-X(I, J-1)+RS1*T(I, J-1)-RS2*(T(I-1, J)))/(RS1-RS2)
  X(I, J)=X(I, J-1)+RS1*(T(I, J)-T(I, J-1))
  IF(IT) 11,12,12
11  CONTINUE
  R4=(R1+R2)/2.0
  S4=(S1+S2)/2.0
  RS4=R4+S4
  XT2=(X(I, J-1)-X(I-1, J))/(T(I, J-1)-T(I-1, J))
12  T4=(X(I, J)-X(I-1, J)-T(I, J)*RS4+T(I-1, J)*XT2)/(XT2-RS4)
  X4=X(I, J)-RS4*(T(I, J)-T4)
  SS4=S(I-1, J)+(S(I, J-1)-S(I-1, J))*(X4-X(I-1, J))/(X(I, J-1)-X(I-1, J))
  S(I, J)=SS4
  IF(IT) 13,14,14
13  CONTINUE
  SR1=R1-S1
  SR2=R1**2-S1**2
  X1=(X(I, J-1)+X(I, J))/2.0
14  R3=R1+SR1*(S(I, J)-S(I, J-1))/4.0-((GAMMA-1.0)/4.0)*SR2*(ETA/X1)*(T(
11, J)-T(I, J-1))
  IF (IT) 15,16,16
15  CONTINUE
  SR3=S2-R2
  SR4=S2**2-R2**2
  X2=(X(I-1, J)+X(I, J))/2.0
16  S3=S2+SR3*(S(I, J)-S(I-1, J))/4.0-((GAMMA-1.0)/4.0)*SR4*(ETA/X2)*(T(
11, J)-T(I-1, J))
  U(I, J)=R3+S3
  C(I, J)=(R3-S3)*(GAMMA-1.0)/2.0
  IF(IT) 17,18,18
17  CONTINUE
  IF(ITT) 18,21,21
21  CONTINUE
  RS11=(GAMMA+1.0)*R1/2.0-(GAMMA-3.0)*S1/2.0
  RS12=(GAMMA+1.0)*R3/2.0-(GAMMA-3.0)*S3/2.0
  RS1=(RS11+RS12)/2.0
  RS21=(3.0-GAMMA)*R2/2.0+(GAMMA+1.0)*S2/2.0
  RS22=(3.0-GAMMA)*R3/2.0+(GAMMA+1.0)*S3/2.0
  RS2=(RS21+RS22)/2.0
  RS4=(R4+S4+R3+S3)/2.0
  SR1=(R1-S1+R3-S3)/2.0
  SR2=(R1**2-S1**2+R3**2-S3**2)/2.0
  SR3=(S2-R2+S3-R3)/2.0
  SR4=(S2**2-R2**2+S3**2-R3**2)/2.0

```

```
X1=(X(I,J-1)+X(I,J))/2.0  
X2=(X(I-1,J)+X(I,J))/2.0  
IT=1  
GO TO 10  
10 CONTINUE  
RETURN  
END
```

COMPILER SPACE

```

SUBROUTINE PATH
DIMENSION XX(100), TT(100)
COMMON      X(50,50), T(50,50), U(50,50), C(50,50), S(50,50), XS(50), TS(5
10),                      JEND(50), US1(50), US3(50), CS1(50), CS3(50), SS1(50
2), SS3(50)
COMMON/ONE/GAMMA, C0, X0, ETA, I, J, VELSH, IP, WORKT
A0=0.0
A1=1.0
A2=0.0
A4=1.0
A4=A4/12.0
IF (IP) 1,1,2
1 WRITE(6,3) A0,A1,A2
3 FORMAT(/1X,*A0=*,E11.5,*A1=*,E11.5,*A2=*,E11.5)
IRET=0
2 CONTINUE
IP=1
T(I,J)=T(I-1,J)
11 TREAL=T(I,J)*X0*10.0**3/C0
XREAL=(TREAL**2)*(A0+A1*TREAL+A2*TREAL**2)+A4
UREAL=2.0*A0*TREAL+3.0*A1*TREAL**2+4.0*A2*TREAL**3
X(I,J)=XREAL/X0
U(I,J)=(UREAL/C0)*10.0**3
TNEW=((X(I-1,J)-X(I,J))+T(I,J)*U(I,J)-(U(I-1,J)-C(I-1,J))*T(I-1,J)
1)/ (U(I,J)-(U(I-1,J)-C(I-1,J)))
TOLD=T(I,J)
T(I,J)=TNEW
TDIFF=ABS(TNEW-TOLD)
IF (TDIFF-0.00001) 10,10,11
10 CONTINUE
S2=(U(I-1,J)/2.0)-C(I-1,J)/(GAMMA-1.0)
R2=(U(I-1,J)/2.0)+C(I-1,J)/(GAMMA-1.0)
SR4=S2**2-R2**2
X2=(X(I-1,J)+X(I,J))/2.0
S3=S2-((GAMMA-1.0)/4.0)*SR4*(ETA/X2)*(T(I,J)-T(I-1,J))
C(I,J)=((U(I,J)/2.0)-S3)*(GAMMA-1.0)
S(I,J)=S(I-1,J)
IF (U(J,J)-U(J-1,J-1)) 22,22,13
22 IRET=1
13 IF (IRET) 15,15,15
16 IF (U(J,J)-U(J-1,J-1)) 17,18,18
18 U(J,J)=U(J-1,J-1)
C(J,J)=C(J-1,J-1)
S(J,J)=S(J-1,J-1)
15 CONTINUE
17 CONTINUE
P0=14.7
PP1=C(I-1,J-1)**((2.0*GAMMA)/(GAMMA-1.0))
PP2=C(I,J)**((2.0*GAMMA)/(GAMMA-1.0))
WORK=(PP1+PP2)*3.14159*(X(I,J)**3-X(I-1,J-1)**3)/3.0
WORK=WORK+P0*144.0
WORKT=WORKT+WORK
WR=PP2*P0*144.0*2.0*3.14159*X(I,J)**2*U(I,J)*C0
WRITE(6,12) I,WORK,WORKT,WR
12 FORMAT(1X/,I5,5X,*INCREMENT AND TOTAL WORK*,2F15.6,*FT LBF*,
1*RATE OF WORK*,F15.6,*FT LBF/SEC*)
RETURN
END

```

```
SUBROUTINE SHOCK
DIMENSION XX(100),TT(100)
COMMON      X(50,50),T(50,50),U(50,50),C(50,50),S(50,50),XS(50),TS(5
10),
JEND(50),US1(50),US3(50),CS1(50),CS3(50),SS1(50
2),SS3(50)
COMMON/ONE/GAMMA,30,X0,ETA,I,J,VELSH,IP,WORKT
SHMACH=(VELSH-US0(J))/CS0(J)
SHMAC2=SHMACH**(-2)
US1(J)=US0(J)+2.0*SHMACH*(1.0-SHMAC2)/(GAMMA+1.0)
PS0=CS0(J)**((2.0*GAMMA)/(GAMMA-1.0))
PS1=PS0*(1.0+2.0*GAMMA*(SHMACH**2-1.0)/(GAMMA+1.0))
RHOS0=PS0** (1.0/GAMMA)
RHOS1=RHOS0/(1.0-2.0*(1.0-SHMAC2)/(GAMMA+1.0))
ENT1=(1.0/GAMMA)*ALOG(1.0+2.0*GAMMA*(SHMACH**2-1.0)/(GAMMA+1.0))
ENT2=ALOG(SHMAC2*(1.0+(GAMMA-1.0)*(SHMACH**2-1.0)/(GAMMA+1.0)))
SS1(J)=SS0(J)+ENT1+ENT2
GM1=GAMMA*SHMACH**2-SHMAC2-(GAMMA-1.0)
GH1=GM1*2.0*(GAMMA-1.0)/((GAMMA+1.0)**2)
CS1(J)=CS0(J)*SORT(1.0+GM1)
RETURN
END
```

COMPILER SPACE

Principle Names used by Program PISTON

(in order of appearance)

MAIN ROUTINE

I, J	refers to the point of intersection of the I^{th} positive and the J^{th} negative characteristic	
XX	distance	)
TT	time	)
UU	particle velocity	)
CC	sonic velocity	)
PP	gas pressure	)
used in the graph plotting routine		
X	distance at a point on the characteristic network	
T	time at a point on the characteristic network	
U	particle velocity at a point on the characteristic network	
C	Sonic velocity at a point on the characteristic network	
S	entropy at a point on the characteristic network	
XS	distance co-ordinate of the shock wave	
TS	time co-ordinate of the shock wave	
US1	particle velocity behind the shock wave	
US0	particle velocity in front of the shock wave	
CS1	sonic velocity behind the shock wave	
CS0	sonic velocity in front of the shock wave	
SS1	entropy behind the shock wave	
SS0	entropy in front of the shock wave	
GAMMA	ratio of the gas specific heats	
CO	sound speed in air at 20°C	
XO	scaling length (set to unity in this work)	
VELSH	shock wave velocity	
WORKT	total work done by the piston (Ft. lbs F)	
TSTOP	time at which the computation is terminated	

PIST	starting distance co-ordinate of the piston
JSE	number of negative characteristics that are computed
XSTEP	integration step length
IU	number of positive characteristics ahead of the shock wave
IOST	number of positive characteristics behind the shock wave
J END(I)	negative characteristic at which the positive characteristic (I) terminates on the shock path

SUBROUTINE PT3

R1, R2, R3, R4,	dependent variables at points 1,2,3 and 4
S1, S2, S3, S4	dependent variables at points 1,2,3 and 4
SSL4	entropy at point 4

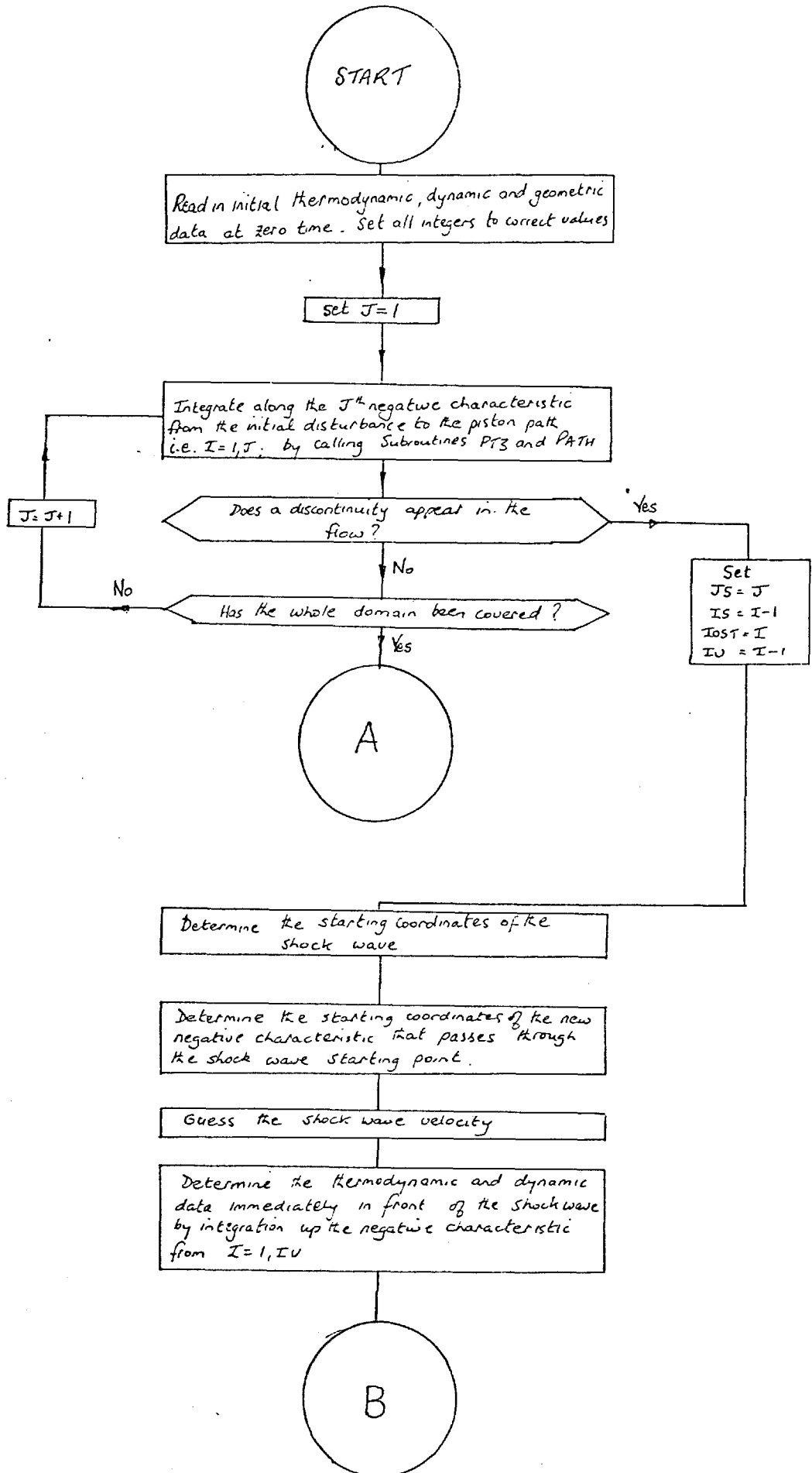
SUBROUTINE PATH

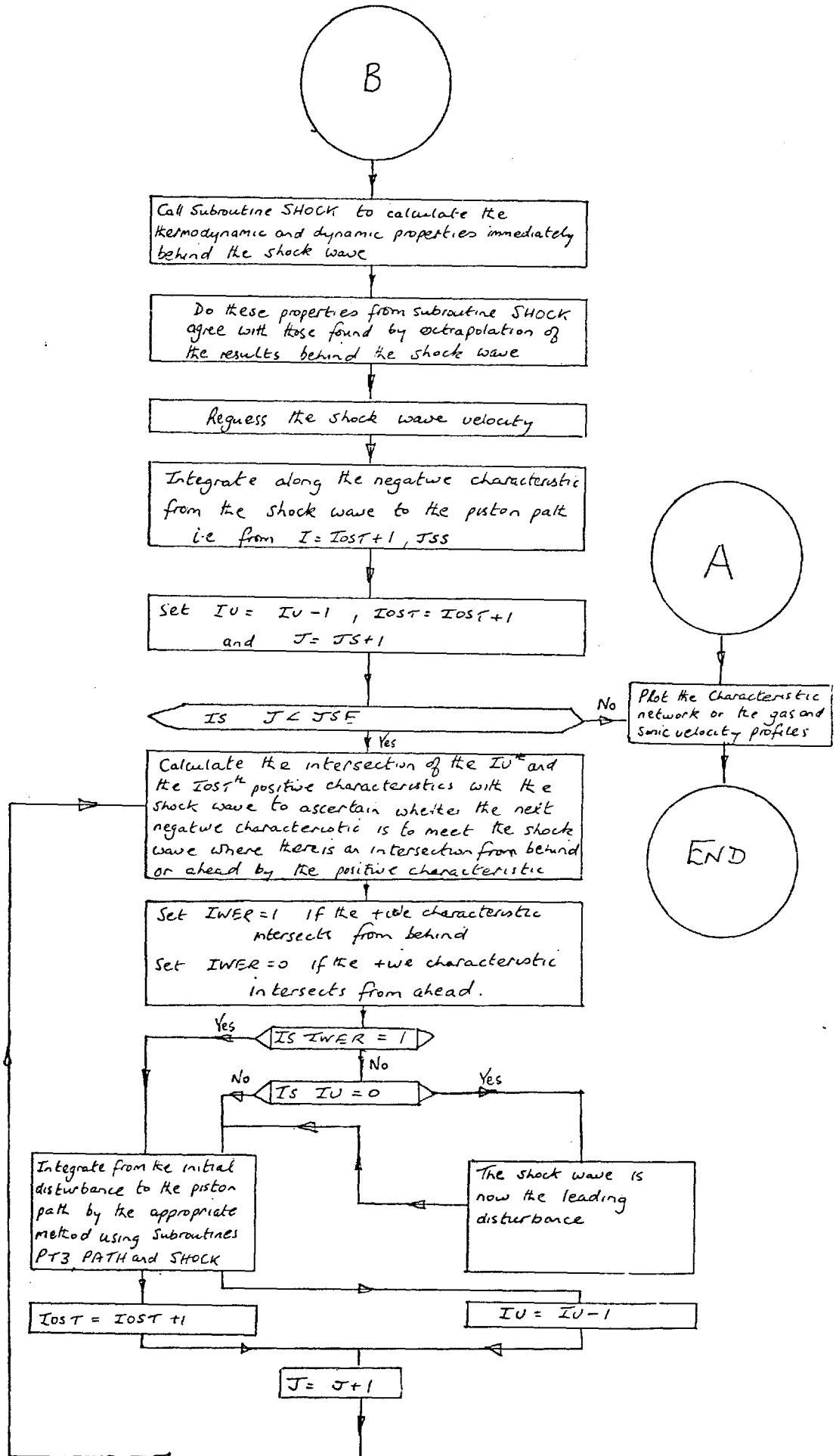
AO, A1, A2, A3	coefficients of the polynomial representing the piston path
WORK	work increment done by the piston during a single integration step

SUBROUTINE SHOCK

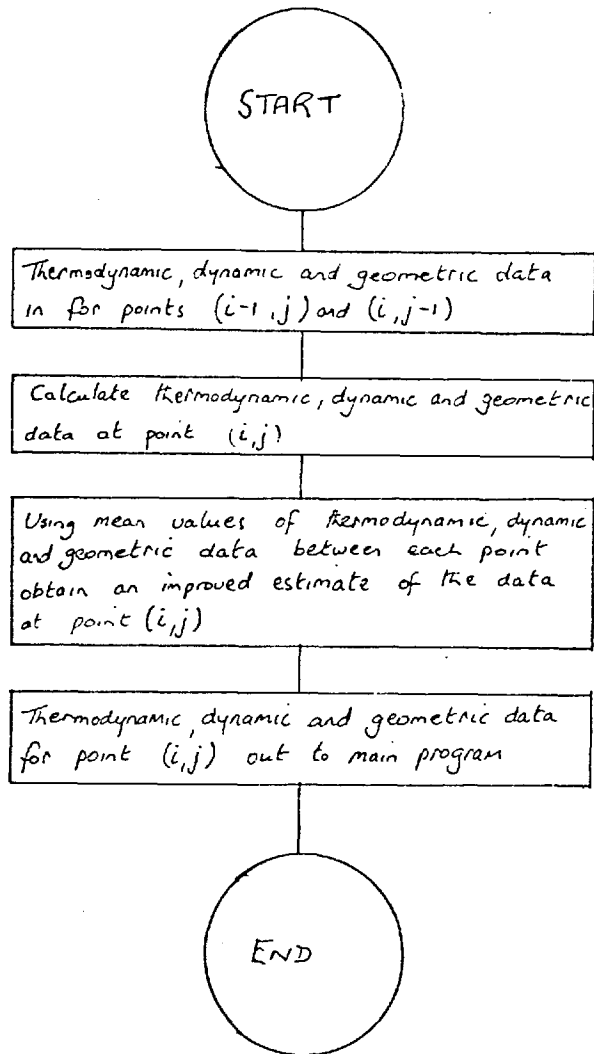
SHMACH	Shock wave Mach number
--------	------------------------

FLOWCHART FOR PROGRAM PISTON

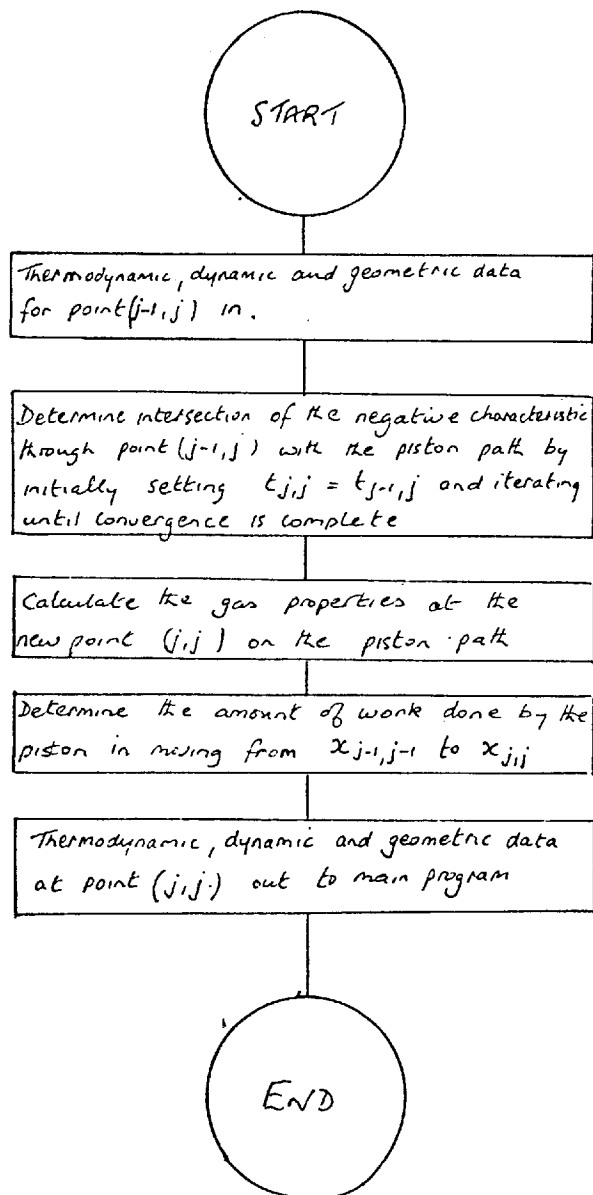




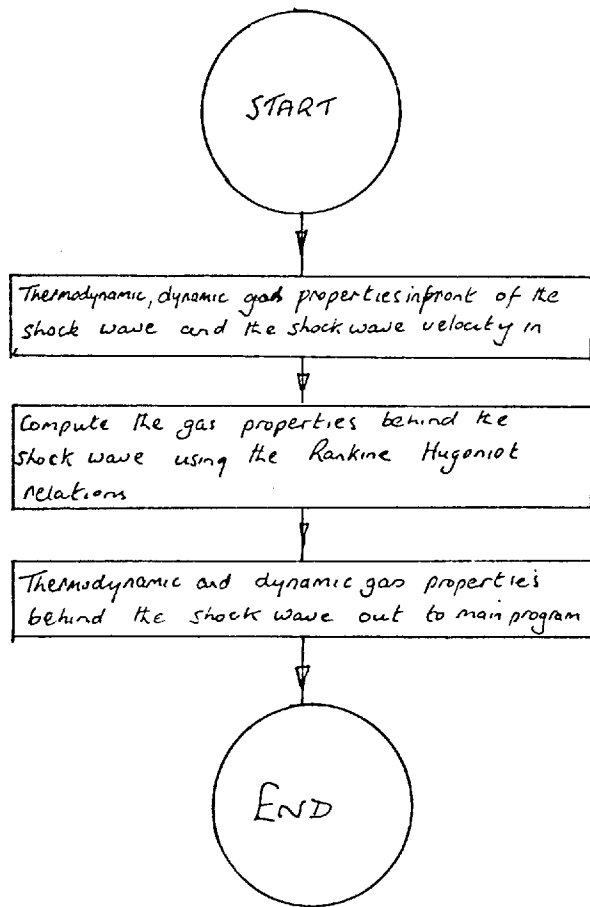
FLOWCHART FOR SUBROUTINE PT3



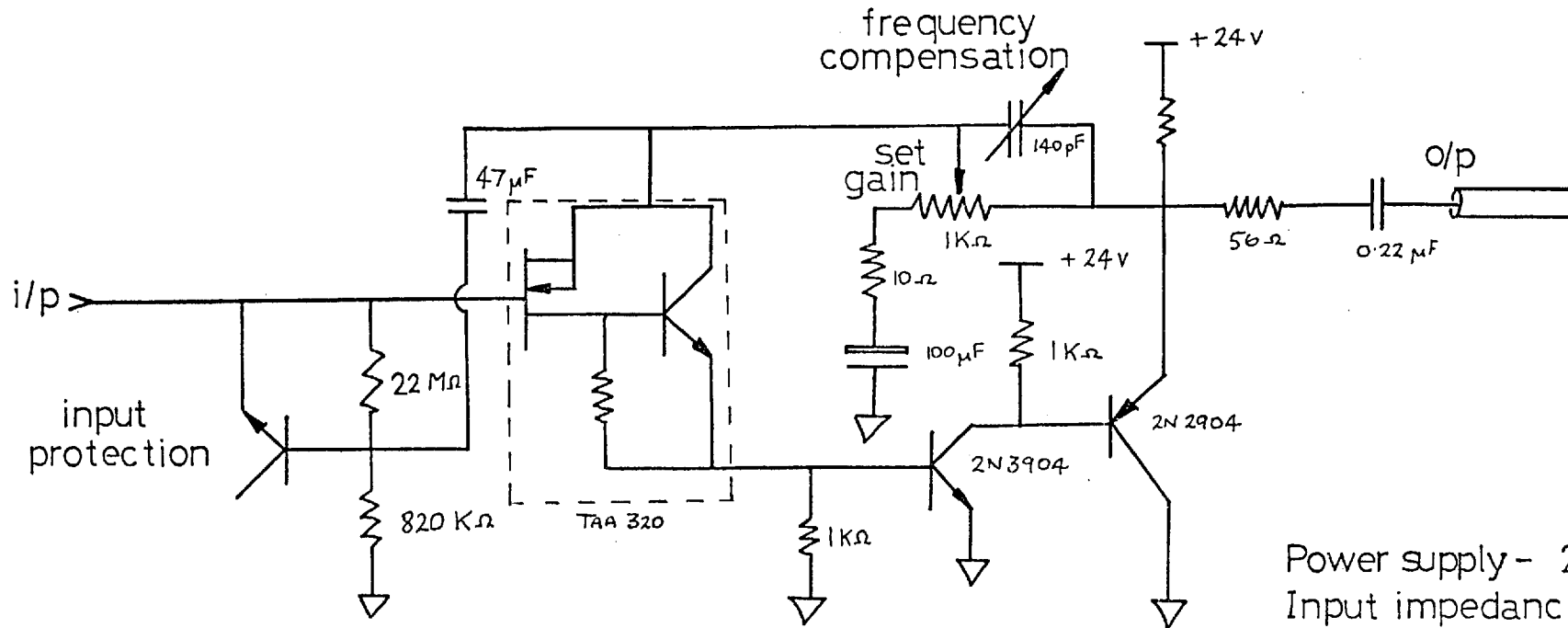
FLOWCHART FOR SUBROUTINE PATH



FLOWCHART FOR SUBROUTINE SHOCK



SHOCK DETECTOR GAUGE AMPLIFIER



Power supply - 24volts
 Input impedance $\approx 100 \text{ M}\Omega$
 Output impedance $\approx 50 \Omega$
 Voltage gain = 1 - 100
 Risetime $\neq 100 \text{ nS}$

STRAIN GAUGE PRESSURE TRANSDUCER AMPLIFIER

