

IMPERIAL COLLEGE LONDON

SHEAR-HORIZONTAL GUIDED WAVE TOMOGRAPHY

by

Andreas Armin Ernst Zimmermann

A thesis submitted to Imperial College London for the degree of
Doctor of Engineering

Department of Mechanical Engineering
Imperial College London
London SW7 2AZ

September 2021

Declaration of Originality

The content of this thesis is my own research work, which was completed with the supervision of Doctor Peter Huthwaite and Professor Michael Lowe. Where I have made use of the work of others, I have made this clear and provided appropriate references.

Andreas Armin Ernst Zimmermann

22/09/21

This page intentionally left blank

Copyright Declaration

The copyright of this thesis rests with the author. Unless otherwise indicated, its contents are licensed under a Creative Commons Attribution-Non Commercial 4.0 International Licence (CC BY-NC). Under this licence, you may copy and redistribute the material in any medium or format. You may also create and distribute modified versions of the work. This is on the condition that: you credit the author and do not use it, or any derivative works, for a commercial purpose. When reusing or sharing this work, ensure you make the licence terms clear to others by naming the licence and linking to the licence text. Where a work has been adapted, you should indicate that the work has been changed and describe those changes. Please seek permission from the copyright holder for uses of this work that are not included in this licence or permitted under UK Copyright Law.

This page intentionally left blank

Abstract

The petrochemical industry's pipe-network is increasingly suffering from corrosion-induced shutdowns and thus a quantitative measurement approach is vital for making adequate service-life predictions and to guarantee safety. At pipe-supports, access is limited and traditional point-inspection methods such as ultrasonic thickness (UT) testing are not feasible. Guided wave tomography has been proposed as a solution to this problem. In this technique ultrasonic guided waves are excited from an array around the defect and received signals capture the interaction of the defect with these waves. From these signals, velocity maps are reconstructed and the dispersive nature of the waves utilised to invert to thickness maps. Recently, it was found, that although reconstruction algorithms such as the hybrid algorithm for robust breast ultrasound tomography (HARBUT) allow for very high-resolution reconstructions, in practice, the mismatch of traditional guided wave scattering of Lamb waves used in guided wave tomography with the acoustic scattering models on which algorithms are based lead to poor resolution.

In this thesis, another type of guided wave mode is investigated instead, shear-horizontal (SH) guided waves. Traditionally, these have been harder to excite but recent advances in electromagnetic acoustic transducer (EMAT) excitation made them a practical alternative. Since the fundamental SH0 mode is non-dispersive, the aim of this thesis is to assess whether the first higher-order SH mode, SH1, is applicable to guided wave tomography. SH1 exists at a higher frequency than those used in guided wave tomography with fundamental Lamb wave modes, leading to a smaller wavelength and it offers no out-of-plane displacement, which should avoid leakage into any pipe-loading. The thesis presents a thorough numerical investigation of the SH1 mode, examining its scattering behaviour and reconstruction of various defect types using three different SH excitation sources; (i) pure SH1 point source (ii) SH surface point source and (iii) directional SH surface source.

These numerical results were validated experimentally, confirming the robustness of SH1 based guided wave tomography. In addition, the thesis investigates a technique to separate out individual modes via the application of the nonuniform fast Fourier transform (NUFFT) algorithm to non-uniformly sampled guided wave data.

The benefits of SH1 for guided wave tomography are shown throughout the thesis and have been demonstrated by accurate and reliable thickness mapping of the defects. The SH1 wave was able to achieve a 2.4 times better resolution than was previously achieved using Lamb waves; yielding the best numerical and experimental guided wave tomography reconstructions to date. The application of the NUFFT algorithm showed how filtering in the frequency-wavenumber domain could be used to separate modal components even without the application of time-based windowing approaches. This technique can also be used to robustly determine bulk shear velocity even from directional data. Finally, this thesis demonstrated proof of concept for SH1 guided wave tomography, suggesting that this technique should be further investigated for employment in the accurate sizing and quantification of corrosion in pipelines.

Acknowledgements

I can count myself extremely lucky that Dr Peter Huthwaite was my principal academic supervisor during these past five years. He has given me the most fascinating project to work on and I really cannot imagine a better supervising experience than mine. He has created several wonderful tools and allowed me to go play with them and I had a tremendous amount of fun. Thank you for being so incredibly patient and for taking so much of your time to listen, to explain things to me, to build me up when I was struggling to overcome obstacles, and to point me towards footholds so I could eventually manage to climb over them. I will miss our chats at Eastside and the Weir and look forward to many more conversations whenever I'm back in London for a visit.

I also owe a lot of thanks to my second academic supervisor, Prof Mike Lowe, creator of Disperse and the current 'skipper' of the Department of Mechanical Engineering. From our very first lunch at SCR I could tell what a warm and kind person you are. You made me feel welcome from the beginning, encouraged me and I will look back fondly on our conversations in France, Italy, and the USA.

A massive thank you to my industrial supervisor, Dr Brian Pavlakovic. Your brilliant developments at Guided Ultrasonics Ltd. were the foundation to the success of my experiments. Thank you for your software support and guidance during this very crucial part of my thesis and for taking time out of your busy schedule for our meetings.

The whole team at Guided Ultrasonics Ltd. has been brilliant to have as support and has been fundamental for my success, thank you all. I must thank Dr. David Alleyne especially for being so fantastically welcoming and keen to bring together academia and business, making us PhD and EngD students feel part of GUL. Also, many thanks to Mr Nick Kemp and Mr Rob Tyson for their tremendous support during the experiments.

ACKNOWLEDGEMENTS

Prof Peter Cawley is owed many thanks for enduring my inquisitive nature during group outings ever so graciously and for establishing the fantastic NDE group at Imperial.

I owe thanks to Dr Fred Cegla for his vital critique during my first year, his encouragement and the many great group dinners.

Many thanks to Dr Christina Fraij for introducing me to her professor in Munich at WCNDT 2016. Her supervisor, Prof Robert Smith introduced me to the RCNDE EngD scheme and encouraged me to apply. Thank you both so very much! Prof Keith Newton was also one of my first contacts at Imperial and a fantastic director of the RCNDE and the Quantitative NDE CDT.

Thank you Prof. Nagy for all the fun conversations at Eastside.

Another hugely important person to the NDE group is our fantastic administrator, Ms Nina Hancock. Thank you for helping me sort out countless issues and for teaching me how to write emails that are much more concise and to the point than this acknowledgements section.

The NDE group in CAGB 563 has been a fantastic source of fun, knowledge, luncheons, dinners, sports and support. Thank you all for making the past five years spectacular: Dr Chien Chua thank you so much for your help and for adopting me into your family, Dr Stewart Haslinger thank you for making me a Spurs fan and all the fantastic football nights and your encouragement throughout this project. Dr Ming Huang thank you for your kindness and support. Many thanks are also owed to Dr Sebastian Eckel, Filip Szlaszynski, Jaime Parra Raad, Dr Yuan Liu, Dr Sebastian Heinlein, Dr Fan Shi, Dr Glenn Jones, Dr Anton van Pamel, Dr Xiaotang Gu, Dr Stefano Mariani, Dr Bálint Herdovics, Dr Pouyan Khalili, Dr Paweł Waszczuk, Dr Jack Egerton, Dr Jinrui Huang, Dr Joe Corcoran, Dr Bo Lan, Dr Kostas Zarogoulidis, Dr Michael Leung, Dr Peng Zuo, Dr Rollo Jarvis, Dr Richard Howard, Dr Richard Phillips, Gabor Gubicza, Antonio De Sanctis, Filip Szlaszynski, , Yasel I Quintero Lares, Aaron Chung, Edmund Jones, George West, Panpan Xu, Yiannis Simillides, Yifeng Zhang, David Shepherd, Euan Rodgers, Georgios Sarris, Jenny Jobling, Jerome Combaniere, Melody Png, Nick Mroszczak, Richard Eager, Wei Yi Yeoh, Toko Hamaguchi and all the other former and current members that I had the pleasure of meeting!

Another group of friends that I owe gratitude to is the members and alumni of the Imperial College Yacht Club: Alex, Bob, Panos, Charlotte, Tomasz, thanks for escaping the city together every once in a while.

Many many thanks to all my friends who supported me, especially Gregorio and Charlotte for organising all the awesome socials and for letting me stay with you when I first arrived in London. Moreover, a massive thank you to Dr Thomas Küstner for all the awesome moments we got to share in London, your very frequent reality checks, and support on all levels of this EngD project and life!

This endeavour would not have been possible without the love, support and encouragement by my family. Thank you so much to my parents, to my brother Thomas and everyone who visited me. I am so happy that I got to share so many precious memories of London with you.

Finally, the biggest thank you of all to Sina-Bianca, for your strength, determination, and serving as a role model for accomplishing big things. Your love, laughter, and encouragement have been the source of strength that I needed to go on whenever things wouldn't converge, crash or just not work.

Danke!

This page intentionally left blank

Contents

1	Introduction	3
1.1	Motivation	4
1.2	Aim	6
1.3	Thesis outline	6
2	Theoretical Background	9
2.1	Introduction	9
2.2	Bulk Waves	9
2.3	Guided waves	12
2.3.1	Shear-horizontal guided waves	14
2.4	Guided wave tomography	17
2.4.1	Forward scattering theory	18
2.4.2	Reconstructing thickness from velocity maps	21
2.4.3	Resolution in guided wave tomography	23
2.5	Finite element simulations	26
2.5.1	SH guided wave excitation approach for FE	28
2.6	Summary	30
3	Numerical investigation of SH guided wave tomography	33
3.1	Introduction	33
3.2	Plate model	34
3.3	Scattering of SH guided waves from a point-like defect	36

3.3.1	Model	37
3.3.2	Data processing	38
3.3.3	Results	38
3.3.4	Summary	42
3.4	SH1 guided wave tomography from pure SH1 point source data	42
3.4.1	Model	42
3.4.2	Data processing	44
3.4.3	Results	47
3.4.4	Summary	50
3.5	SH1 guided wave tomography from SH point source data	51
3.5.1	Model	51
3.5.2	Data processing	52
3.5.3	Results	54
3.5.4	Summary	55
3.6	SH1 guided wave tomography from directional SH source data	55
3.6.1	Model	56
3.6.2	Results	57
3.6.3	Summary	60
3.7	Performance Analysis	61
3.7.1	Model	61
3.7.2	Results	61
3.7.3	Summary	63
3.8	SH0 guided wave tomography	64
3.8.1	Model	64
3.8.2	Data processing	65
3.8.3	Results	65
3.8.4	Summary	66
3.9	Summary	66

4	Experimental verification of SH1 guided wave tomography	67
4.1	Introduction	67
4.2	Experiment setup	67
4.3	Data processing	70
4.4	Results	70
4.4.1	Defect-Free plate	70
4.4.2	Axisymmetric defect	75
4.4.3	Realistic defect - 20% wall loss	77
4.4.4	Realistic defect - 50% wall loss	80
4.5	Summary	82
5	Modal decomposition of non-uniformly sampled SH guided wave data	85
5.1	Introduction	85
5.2	Frequency-wavenumber diagrams from signals acquired at equispaced receivers	87
5.3	Frequency-wavenumber diagrams from signals acquired at nonequispaced receivers	93
5.4	SH0 removal from FE acquired complex defect point source SH signals . .	98
5.5	Investigation of directional source SH signals with NUFFT-based frequency wavenumber diagrams	103
5.6	Determining bulk shear velocity from NUFFT-based f - k diagrams	105
5.7	Summary	108
6	Conclusions	111
6.1	Thesis review	111
6.2	Main findings	112
6.3	Future work	113
	References	115
	List of Publications	123

This page intentionally left blank

List of Figures

1.1	Spotlight on a corrosion patch at pipe support. In order to assess service-life, quantitative measurement of the remnant wall thickness at this point is vital. Source: Guided Ultrasonics Ltd., January 2022	4
2.1	Schematic of an isotropic plate of thickness T with internal reflecting waves and indication of shear horizontal displacement.	13
2.2	a) Phase velocity dispersion curves for Lamb and shear-horizontal guided waves in a steel plate. SH guided wave mode shapes for b) SH0 & c) SH1 for a single frequency. The grey arrows in b) and c) indicate the direction of propagation.	16
2.3	Schematic of the full view guided wave tomography layout employed in this thesis: A circular array of radius R is placed on a plate. The transducers are used for excitation as well as reception. The arrow pointing towards the defect indicates an excitation event at a transmitting transducer Tx. A defect is placed in the centre of the plate. The incident waves are interacting with the defect and the scattered waves (arrows pointing away from the defect) are picked up by the receiving transducers.	17

- 2.4 Schematic of mapping scattered information from the spatial domain to the spatial frequency domain called k -space: (a) shows a circular transducer array setup and the excitation vector $\hat{\mathbf{s}}_0$ in red and the receiving direction $\hat{\mathbf{s}}$. Different receiving directions are indicated with the different colours shown. (b) presents the spatial frequency domain of the object, known as k -space, into which the measured scattered components can be mapped, according to eq. (2.44). The component corresponding to $\hat{\mathbf{s}}_0$ incident and $\hat{\mathbf{s}}$ in reception in (a) is mapped to (1, -1) as illustrated. By changing the receiver direction $\hat{\mathbf{s}}$ around 360 degrees, a circle is mapped out, illustrated by the colours shown in both plots. Changing the source direction then allows this circle to be iteratively moved around, with all the measurements ultimately populating the disk shown. 21
- 2.5 Mapping velocity to thickness: A velocity map was reconstructed from captured SH1 signals (a). By fixing frequency to 350 kHz, the velocity becomes thickness dependent and the thickness values are looked up in the dispersion curve to calculate a thickness map (c). The green cross is a point of nominal thickness and the red cross depicts a location with 50% wall loss. 23
- 2.6 k -space representation and reconstruction of images after applying an inverse 2D-FFT: Inverting the full k -space image K (a) leads to the original image (e). Removal of low spatial frequency components at the centre (b) leads to a reconstruction of the finer details (f), encoded in the high spatial frequency components. Inversely, removal of the higher spatial frequency components (c-d) leads to a drop in resolution but maintains the low spatial frequency information of the imaged object (g-h). The more high spatial frequency components are removed, the ‘blurrier’ the image and greater the loss of resolution. 25

2.7	a) Omnidirectional shear excitation on all nodes of four elements. The source can be positioned arbitrarily by weighting the displacements of the four elements and superposing the signals together. (b) shows how this shear source can be applied to the top and bottom surfaces; excitation by twisting in opposite directions top to bottom will excite the asymmetric SH modes and pure SH1, assuming excitations well below the SH3 cut-off frequency.	28
2.8	Shear-displacement u_y profiles for the first four SH guided wave modes: (a) SH0, (b) SH1, (c) SH2 and (d) SH3. The two symmetric modes are plotted in red and the asymmetric modes in blue.	29
2.9	Through-thickness nodal displacement schematic for pure SH0 (a) and SH1 (b) point source excitation in a 10 mm thick plate: Here the transducer is positioned at $[x, y] = [200, 0]$ mm. The 9 node columns are colour coded by thickness from 0 mm = darkblue to 10 mm = yellow for all 21 model layers. The red vectors indicate the displacement in counter-clockwise direction and blue arrows point in the clockwise direction. For SH0 (a) this leads to a uniform displacement profile through-thickness while for SH1 (b) this leads to anti-symmetric displacement profile through-thickness. Note that the centre of the transducer position is located perfectly on the grid and therefore the nodal displacement in the centre is zero.	30
3.1	Plate modelling approach: 2D thickness maps were used to squash node layers, ‘imprinting’ the defects into the structured mesh. Here, a cut-out of the total model is shown, highlighting the thickness profile through the deepest point of the defect and the squashed elements.	34
3.2	The seven defect models used in the numerical study: Hann-shaped axisymmetric <i>Hann</i> , Tukey-shaped axisymmetric <i>Tukey</i> and laser scanned defects of corrosion under pipe support (CUPS): <i>Alpha-Echo</i>	35
3.3	Schematic defining the scattering angle θ for a point scatterer.	37

3.4	Comparison of acoustic and guided wave scattered fields for a 3 mm wide and 0.5 mm deep point-like defect in a mild steel plate: (a) A0 (green) at 50 kHz, (b) SH0 (red) at 200 kHz and (c) SH1 (blue) also at 200 kHz. The peak of the acoustic amplitude is indicated by the dashed grey circle. . . .	39
3.5	Point source transducer signal processing: a) Magnitude of the pure SH1 wavefield at $t = 0.13$ ms after excitation at Tx(1). b) Magnitude of the Hilbert envelope of the received SH1 time traces for the excitation at Tx(1) and reception at all 120 transducers. c) 5-cycle Hann shaped excitation signal and the received pure SH1 signal.	45
3.6	HARBUT reconstruction from point source SH transducer FE simulations. (a) - (g) show true thickness maps, (h) - (n) display the reconstructed thickness maps and (o) - (u) show line profile through the deepest points for the true (black) and reconstructed depths (red).	48
3.7	HARBUT reconstruction from point source SH1 transducer FE simulations at a centre frequency of 350 kHz. (a) and (d) depict the two investigated defect models, <i>Hann</i> & <i>Echo</i> . The HARBUT reconstructions from FE data are shown in (b) and (e). As can be seen from the thickness profiles in (c) and (f) both defect shape and thickness were captured with extreme accuracy.	50
3.8	Example of full SH point source excitation of a 5-cycle Hann shaped pulse at 250 kHz: a) Magnitude of displacement wavefield at $t = 0.11$ ms after excitation of SH0 in addition to SH1 when modal decoupling has occurred. b) Magnitude of the Hilbert envelope of the received time traces and the respective arrival times for of SH0 (red), SH1 (green) and the calculated SH1 window boundary times: t_1 (light grey) and t_2 (light grey dashed). c) Full centre time trace (black) and Tukey windowed SH1 component (red). .	52

3.9	HARBUT reconstruction from point source SH transducer data of FE simulations at a centre frequency of 350 kHz. (a) depicts the investigated defect model, <i>Echo</i> . The HARBUT reconstruction from SH point source FE data is shown in (b). As can be seen from the thickness profiles in (c), both defect shape and thickness were captured accurately.	55
3.10	Directional transducer (width $w = 5$ mm, length $l = 20$ mm) modelling approach.	56
3.11	Directional transducer signal processing: a) Magnitude of displacement wavefield at $t = 0.11$ ms after excitation of SH0 in addition to SH1 due to directional excitation, measured on the top surface of the plate. Modal decoupling has occurred. b) Magnitude of the Hilbert envelope of the received time traces and the respective arrival times for of SH0 (red), SH1 (green) and the calculated SH1 window boundary times: t_1 (light grey) and t_2 (light grey dashed). c) Full centre time trace (black) and Tukey window based on t_1 and t_2 (black dashed) and windowed SH1 component (red). . .	56
3.12	HARBUT reconstruction from directional SH transducer FE simulations. (a) - (g) show true thickness maps, (h) - (n) display the reconstructed thickness maps and (o) - (u) show line profile through the deepest points for the true (black) and reconstructed depths (grey).	58
3.13	HARBUT reconstruction of a 50% maximum wall loss defect from directional SH transducer FE data. (a) shows the input model, (b) is the HARBUT reconstruction after 26 iterations and (c) shows the thickness profile of the reconstruction (red) compared to the input model (black). . .	60
3.14	Comparison of HARBUT reconstructions using Lamb wave modes A0, S0 and shear-horizontal modes SH0 and SH1 for Tukey-type defects with widths ranging from 1T to 10T, where T is the nominal plate thickness. . .	62
3.15	Global error plot of the Tukey defect diameter sweep for A0 (dashed), S0 (dot-dashed) and SH1 (solid).	63

3.16	HARBUT reconstruction of pure SH0 guided wave tomography data. (a) shows the previously used <i>Echo</i> input model, (b) shows the Laplacian of the k-Space filtered input model and (c) shows the HARBUT reconstruction.	65
4.1	The thickness map templates which were machined into mild steel plates and used in experiments.	68
4.2	Experiment setup: (A) custom plywood rig with 120 array positions, (B) & (C) QSR1 [®] transducers attached to wooden sleds, (D) Laptop computer for data acquisition, (E) QSR1 [®] base unit, (F) mild-steel plate with machined defects (shown here: <i>Echo</i>).	69
4.3	Experiment data acquired from excitation at a centre frequency of 250 kHz: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the border times (dashed blue) and the theoretical SH1 arrival time (blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), the magnitude of the Fourier transform at the frequency is plotted in a polar diagram.	72
4.4	Experiment data acquired from excitation at a centre frequency of 325 kHz: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the border times (dashed blue) and the theoretical SH1 arrival time (blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), the magnitude of the Fourier transform at the frequency is plotted in a polar diagram.	74

4.5	Experiment data acquired from excitation at a centre frequency of 250 kHz for the axisymmetric defect: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the border times (dashed blue) and the theoretical SH1 arrival time (blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), the magnitude of the Fourier transform at the frequency is plotted in a polar diagram.	76
4.6	HARBUT reconstruction of the axisymmetric machined defect from Experiment data at 250 kHz: (a) shows the true thickness map, (b) displays the reconstructed thickness map and (c) shows the line profiles through the deepest point for the true (black) and reconstructed depths (gray).	77
4.7	Experiment data acquired from excitation at a centre frequency of 250 kHz for the realistic defect at 20% maximum wall loss: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the first arrival times T1 (dashed light blue), the theoretical SH1 arrival times T2 (dashed dotted light blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), this data is used to populate a complex amplitude matrix.	78
4.8	HARBUT reconstruction of the 20% realistic machined defect from experimental data at 250 kHz: (a) shows the true thickness map, (b) displays the reconstructed thickness map and (c) shows the line profiles through the deepest point for the true (black) and reconstructed depths (gray).	79

4.9	Experiment data acquired from excitation at a centre frequency of 325 kHz for the realistic defect at 50% maximum wall loss: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the first arrival times T1 (dashed light blue), the theoretical SH1 arrival times T2 (dashed dotted light blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), this data is used to populate a complex amplitude matrix.	81
4.10	HARBUT reconstruction of the 50% realistic machined defect from experimental data at 335 kHz: (a) shows the true thickness map, (b) displays the reconstructed thickness map and (c) shows the thickness profiles through the deepest point for the machined (black) and reconstructed depths (red).	83
5.1	Model setup and steps from time-distance to frequency-distance and frequency-wavenumber diagrams: The first line shows the model setup of uniformly spaced receivers on a plate. Next, the Hilbert envelopes of the received time-traces at every receiver position are shown as well as the magnitude of the FFT of these signals and the frequency wavenumber diagrams for SH0 (first column), SH1 (second column), SH2 (third column) and all SH modes (fourth column). Notice how the three modes are distinctly separated only in the f - k diagram.	88
5.2	From frequency-wavenumber back to time-distance: Applying inverse Fourier transforms to invert (a) the f - k diagram to (b) the frequency domain and finally to (c) the time domain. The last plot (d) shows the perfect agreement of the inverted time-domain signal with the original signal from fig. 5.1(e).	89

5.3	Application and inversion of two SH0 windows in the frequency wavenumber diagram. (a) shows SH0WinA. (b) is a window based on the bandwidth of the excitation pulse. (c) shows the windows applied to the centre frequency f-k components for SH0 (grey), SH with window <i>SH0WinA</i> (red dashed-dotted) and window <i>SH0WinB</i> (blue dashed-dotted). (d) and (e) are the Fourier spectra after applying an iFFT to the windowed f-k diagrams. (f) shows the magnitude at centre frequency. (g) and (h) are the Hilbert enveloped time-traces from applying an iFFT to the Fourier spectra. (i) shows the final time-traces for the true SH0 signal and the two windowed SH0 components. (j) compares the phase differences of the two filtered cases with the true SH0 phase angles at the centre frequency. . . .	90
5.4	Comparison of the performance of SH0 removal by the two window types. (a) shows Fourier spectrum ground truth when subtracting SH0. (b) shows the subtraction of the SH0 components filtered by applying <i>SH0WinA</i> and (c) <i>SH0WinB</i> to the f-k diagram. (d) shows the SH1 Fourier spectrum obtained after subtracting the true SH0 and SH2 components and (e) and (f) show the Fourier spectrum when subtracting SH2 and the f-k windowed SH0 components. (g) compares the phase difference at centre frequency for the three cases and shows the excellent performance of the f-k filtered SH0 component subtraction.	92
5.5	Model setup and steps from time-distance to frequency-distance and frequency-wavenumber diagrams: (a) shows the model setup of a circular array on a plate. In the next line, the signals are shown for (b) SH0, (c) SH1, (d) SH2 and (e) full SH. Below, the Fourier spectra are plotted for (f) SH0, (g) SH1, (h) SH2 and (i) full SH. The bottom line shows the f-k diagrams for the four cases. In (m), the expected frequency wavenumber dispersion curves are plotted for SH0 (red), SH1 (blue) and SH2 (green).	94

5.6	Investigation of the NUFFT performance: (a) shows the f - k diagram of the full SH spectrum. (b) shows the spectrum after applying the inverse NUFFT to the f - k diagram in (a). (c) shows a comparison of the magnitude at the centre frequency for the true SH spectrum (grey) and for the recovered magnitude from the NUFFT f - k diagram (black). (d) Compares the phase angle at the centre frequency for SH (grey) and SH from NUFFT (black).	96
5.7	Comparison of the performance of SH0 removal after applying the two f - k SH0 windows to the NUFFT-based f - k diagram. (a) shows the true SH0 Fourier spectrum, (b) shows the SH0 spectrum reconstructed after applying the window $SH0WinA$ to f - k_{NUFFT} and (c) is the reconstructed Fourier spectrum from $SH0WinB$ applied to f - k_{NUFFT} . In (d), the phase difference over distance is compared for true SH0 (grey), $SH0WinA$ (red dashed-dotted) and $SH0WinB$ (blue dashed-dotted). In (e),(f),(g) the SH0, $SH0WinA$ and $SH0WinB$ components are subtracted from the full SH Fourier spectrum. SH2 components are removed as well which would be achieved through signal windowing. In (h), the phase difference is compared for SH-SH0-SH2 (grey), SH- $SH0WinA$ -SH2 (red dashed-dotted) and SH- $SH0WinB$ -SH2 (blue dashed-dotted).	97
5.8	Model setup and steps from time-distance to frequency-distance and frequency-wavenumber diagrams: (a) shows the model setup of a circular array on a plate. Next, the Hilbert envelopes of the received time-traces at every receiver position are shown (b-e), as well as magnitude of the Fourier spectra of the signal components (f-i) and the NUFFT-based frequency wavenumber diagrams (j-m).	99

5.9	Comparison of the performance of SH0 removal after applying the two f - k SH0 windows to the NUFFT-based f - k diagram from complex defect FE point source data. (a) shows the true SH0 spectrum, (b) shows the SH0 spectrum reconstructed after applying the window <i>SH0WinA</i> to f - k_{NUFFT} and (c) is the reconstructed spectrum from <i>SH0WinB</i> applied to f - k_{NUFFT} . In (d), the phase difference over distance is compared shown for true SH0 (grey), <i>SH0WinA</i> (red dashed-dotted) and <i>SH0WinB</i> (blue dashed-dotted). In (e),(f),(g) the SH0, <i>SH0WinA</i> and <i>SH0WinB</i> components are subtracted from the full SH spectrum. SH2 components are removed as well which would be achieved through signal windowing. In (h), the phase difference over distance is compared and shown for SH-SH0-SH2 (grey), SH- <i>SH0WinA</i> -SH2 (red dashed-dotted) and SH- <i>SH0WinB</i> -SH2 (blue dashed-dotted).	101
5.10	HARBUT results after SH0 removal. (a) shows the HARBUT result from section 3.5.3, (b) shows the result after applying the NUFFT-based f - k -window type A and (c) the result after applying the NUFFT-based f - k -window type B. (d) compares the true (grey) thickness profile with the reconstructed ones.	102
5.11	f - k diagrams from FE simulation generated directional transducer signals: (a) received signals from a directional transducer. (b) shows the windowed SH0 components of (a). (c) shows the f - k diagram calculated from the data in (a) and in (d), the f - k diagram calculated from the windowed SH0 components is shown. Overlaid on (c) and (d) are the f - k dispersion curves of SH0, SH1 and SH2.	104
5.12	Results from 250 kHz experimental datasets: From time-traces to f - k diagram linear regression based shear velocity: (a) Received experimental time-traces, (b) windowed SH0 components and (c) their Fourier transform. (d) SH0 f - k diagram and (e) the maxima (black crosses) at several frequencies used to fit the SH0 dispersion curve. (f) shows the velocities found for all 120 excitations.	106

5.13 Results from 325 kHz experimental datasets: From time-traces to f - k diagram linear regression based shear velocity: (a) Received experimental time-traces, (b) windowed SH0 components and (c) their Fourier transform. (d) SH0 f - k diagram and (e) the maxima (black crosses) at several frequencies used to fit the SH0 dispersion curve. (f) shows the velocities found for all 120 excitations.	107
--	-----

List of Tables

3.1	Error and performance values for point source finite-element results: The global error e , the error at maximum defect depth e_m , the number of iterations per case and the defect area.	47
3.2	Error and performance values for directional source finite-element results: The global error e , the error at maximum defect depth e_m , the number of iterations per case and the defect area.	59

This page intentionally left blank

Acronyms

BRT bent ray tomography. 22

CPU central processing unit. 26

CUDA compute unified device architecture. 26

CUPS corrosion under pipe support. 3

DT diffraction tomography. 22

EMAT electromagnetic acoustic transducer. v

FE finite element method. 9

FFT fast Fourier transform. 24

GPU graphics processing unit. 26

HARBUT hybrid algorithm for robust breast ultrasound tomography. v

HARBUTISM HARBUT with an improved scattering model. 5

NUFFT nonuniform fast Fourier transform. vi, 7

QSR quantitative short range. 68

SH shear-horizontal guided wave mode. v, 6

UT ultrasonic testing. v

This page intentionally left blank

Chapter 1

Introduction

Corrosion is a major problem in pipework [1] and leads to yearly costs of £28 billion in the UK or £4 trillion globally and over 20% of all major gas and oil incidents within the EU since 1984 are associated with it¹. Moreover, a lot of assets in the North Sea have exceeded their design life and most pipe failures are caused by corrosion. Therefore, accurate estimation of the remnant wall thickness of pipelines and storage vessels in the presence of corrosion damage is highly important for making service-life prediction and for avoiding environmental disaster in the petrochemical industry.

Knowledge of remnant wall thickness is especially required at locations with limited access such as pipe-supports, where corrosion under pipe support (CUPS), which is also known as touch point corrosion or contact point corrosion, is one of the common causes of pipe failure (see fig. 1.1). Guided wave tomography has been proposed as a solution to this problem by reconstructing thickness maps from guided wave signals that were transmitted through the region of interest utilising the dispersive nature of certain guided wave modes.

It is of great importance to industry to reliably size defects down to dimensions of $3T \times 3T$, where T is the nominal wall thickness [2]. For a nominal wall thickness of $T = 10$ mm, this would correspond to 30 mm. It is noted that this measure does not make any assessment of defect shape, so the actual in-plane resolution would need to exceed $3T$ in order to capture the necessary complexity present in true corrosion defects to establish maximum depth. In terms of depth resolution, estimates are required to be within 10% of

¹Source: S. Burch, *HOIS joint industry project managed by ESR technologies*, Accessed September 2021.



Figure 1.1: Spotlight on a corrosion patch at pipe support. In order to assess service-life, quantitative measurement of the remnant wall thickness at this point is vital. Source: Guided Ultrasonics Ltd., January 2022

wall thickness, i.e. $0.1 T$; while this itself has been achieved through various approaches, the necessary in-plane resolution to achieve this accuracy in practice places a significant requirement on the capabilities of the technology, and one which has not been reliably met up to this point.

1.1 Motivation

The typical approach in guided wave tomography is to utilise the measured signals to reconstruct a velocity map. Each velocity value can then be directly mapped to a corresponding thickness value by the guided wave dispersion curves, ultimately producing the required thickness map. This approach has been the basis of many approaches in the past, mainly utilising Lamb waves [3–8]. There have been experimental investigations that have shown 100% sensitivity to damage detection and even in challenging scenarios such as pipe bends [9]. Also, guided wave tomography has been investigated for structural health monitoring purposes [10–12]. Another approach that is utilised for velocity inversion is full waveform inversion (FWI), where an iteratively updated acoustic finite difference model is matched to measurements and has been investigated using Lamb

guided wave modes [13, 14]. However, in all of these investigations it was found that the remnant wall thickness cannot be captured accurately when relying on Lamb waves.

The underlying assumption in most of the previous investigations is that guided waves scatter from varying thickness in the same manner as acoustic waves in a 2D medium would scatter from the corresponding change in the velocity field. This is a reasonable assumption provided the thickness variation is gradual, such that the dispersion curves, derived assuming constant thickness, can capture the behaviour. As thickness varies faster, Huthwaite [15] showed that the approach cannot capture the guided wave scattering behaviour, which manifests itself as a resolution limit of around two wavelengths for the fundamental asymmetric Lamb mode A0 at 50 kHz, which corresponds to $8T$ in-plane resolution.

There are two options to improve the resolution further. The first option is to develop a model that better captures the guided wave behaviour. The HARBUTISM [16] approach develops a more sophisticated model which aims to capture the guided wave scattering behaviour more accurately than the dispersion curve and hence enable higher resolution. Francis Rose et al. [17–19] have developed Mindlin plate theory along similar lines for both composite delamination and plate corrosion damage. However, both approaches are typically more complex and less robust, since they are sensitive to discrepancies (e.g. small perturbations in local properties such as wall loss) in the modelled behaviour of guided wave scattering [20, 21].

The other option is to select a guided wave mode and frequency such that the wavelength becomes shorter, which typically would improve resolution. There are practical issues around this, such as the reduction in signal mode purity due to the existence of other guided wave modes. For example, in the case of Lamb waves [22], the next higher order modes, the asymmetric A1 and symmetric S1, will co-exist with the two fundamental Lamb wave modes A0 and S0, and for the typical propagation distances in guided wave tomography, will overlap heavily in the time-domain making any single-mode analysis extremely complicated. In addition, whenever the guided waves interact with a defect, mode-conversion may occur at sharp boundaries, which will further add to the complexity of the signal. Another issue is that Lamb waves display out-of-plane displacement. This is an issue whenever the plate or pipe boundary is in contact with a liquid, e.g. water in

a pipeline. The energy of the waves will inevitably leak into the liquid which adds to the difficulty by attenuating the signal. Moreover, if this energy can couple into the liquid, it can provide a mechanism for additional wave-paths through the space between the source and receiver, adding unwanted coherent noise in the data (see [12]). All of these aspects will pollute the signal and typically increase attenuation (see [23] for a study of various operating frequencies and modes).

1.2 Aim

In this thesis, shear-horizontal (SH) guided waves will be investigated for use in guided-wave tomography. With the fundamental mode SH0 being non-dispersive and thus unsuitable for thickness inversion from dispersion curves, the first higher-order SH mode, SH1, will be the main focus. This mode operates at higher frequency than the fundamental modes, and the shear form of the wave avoids much of the attenuation associated with liquid loading, which is problematic for most usable Lamb modes. This is due to the SH guided wave's lack of vertical, out-of-plane displacement. There will be some shear coupling into any liquid loading but compared to the amount of vertical coupling by Lamb waves it will be much lower. Also, despite being a higher order mode, it is relatively straightforward to extract pure measurements since below SH2 cut-off only SH1 and SH0 exist and SH0 may be separated more easily because it has a very different wave speed and crucially, is non-dispersive, thus keeping its waveform.

Therefore, this thesis aims to explore the possibility of using the SH1 mode to enable guided wave tomography to achieve its potential of imaging defects of $3T \times 3T$ reliably and robustly.

1.3 Thesis outline

Chapter 2 will provide background on guided wave tomography and the algorithm that was chosen for thickness inversion. Chapter 3 will present the numerical model that was used for pure SH1 excitation and demonstrate the suitability by reconstructing various defects from pure SH1 point source data generated from 3D finite-element simulations.

As pure SH1 excitation is practically hard to achieve without access to both surfaces of the pipe, reconstructions from surface-only point source excitations were investigated. Next, a more realistic, directional excitation of SH guided waves will be explained and the same defects as before reconstructed from numerical data. Furthermore, results from a numerical study comparing the performance of the fundamental Lamb wave modes A0 and S0 against SH1 guided wave tomography of axisymmetric defects of varying width will be presented. The final section in the numerical investigation will present an interesting reconstruction from pure SH0 data. In chapter 4, the method is tested and validated experimentally by reconstructing thickness maps from defects that were machined into mild-steel plates. Given the issue of mode-conversion and modal-overlap at higher frequencies, chapter 5 applies the nonuniform fast Fourier transform (NUFFT) based strategies for modal decomposition and velocity determination. The final chapter 6 will discuss the results, the main findings of this project and outline potential future work.

This page intentionally left blank

Chapter 2

Theoretical Background

2.1 Introduction

In this chapter, a brief introduction of the theory of ultrasonic waves in solids is given, which come in the form of bulk waves (section 2.2) and guided waves (section 2.3). Given the focus on shear-horizontal (SH) guided waves within this thesis they will be introduced in a more detailed fashion in section 2.3.1 and followed by an introduction to guided wave tomography (section 2.4). Finally, in section 2.5, an overview of the Finite Element (FE) method will be provided and the SH guided wave excitation model used in this thesis will be explained (section 2.5.1). Parts of this chapter have been adapted and extracted from publication [P1].

2.2 Bulk Waves

Bulk waves are governed by Euler's time dependent equation of motion, which is defined as

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \boldsymbol{\sigma} , \quad (2.1)$$

where ρ is the density of the material, $\mathbf{u}$ is the particle displacement vector and ∇ is the vector differential operator ($\nabla = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the unit vectors along the x , y and z axes, respectively) acting on the stress tensor $\boldsymbol{\sigma}$. In an isotropic

2. Theoretical Background

material, the stress tensor can be related to the material strain ϵ using Hooke's law:

$$\boldsymbol{\sigma} = \mathbf{C} \cdot \boldsymbol{\epsilon}, \quad (2.2)$$

where $\mathbf{C}$ is the stiffness tensor. For the case of a linear elastic, isotropic and homogeneous material, the theory of elasticity states that one may reduce the components of the stiffness tensor to the two Lamé constants, λ and μ . Furthermore, it is possible to express the stress tensor in terms of displacements, by transforming Hooke's law into

$$\boldsymbol{\sigma} = \lambda \mathbf{I} \nabla \cdot \mathbf{u} + \mu (\nabla \cdot \mathbf{u} + \mathbf{u} \nabla^T), \quad (2.3)$$

where $\mathbf{I}$ is the identity matrix. The Lamé constants are also related to Poisson's ratio ν

$$\nu = \frac{\lambda}{2(\lambda + \mu)} \quad (2.4)$$

and to the modulus of elasticity E which is given by

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}. \quad (2.5)$$

Substituting eq. (2.3) into eq. (2.1) yields Navier's differential equation of motion for an isotropic elastic material:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla \nabla \cdot \mathbf{u}, \quad (2.6)$$

where ∇^2 is the scalar differential operator $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})$. Expanding this vector equation into its spatial components (x, y, z) leads to a set of scalar linear equations. This linearity guarantees that the superposition of several valid solutions will itself be a solution. However, these equations cannot be integrated directly and therefore a proper, application-dependent solution must be presumed. By applying the Helmholtz decomposition, the particle displacement vector $\mathbf{u}$ may be written as

$$\mathbf{u} = \nabla \phi + \nabla \times \boldsymbol{\psi}, \quad (2.7)$$

where ϕ is a compressional scalar potential and ψ is the shear vector potential. Clearly, $\nabla\phi$ is the irrotational component, and $\nabla \times \psi$ the rotational component, where $\nabla\phi = 0$, also known as the gauge condition [24]. Substituting eq. (2.7) into eq. (2.6) gives

$$\nabla \left[(\lambda + 2\mu) \nabla^2 \phi - \rho \frac{\partial^2 \phi}{\partial t^2} \right] + \nabla \times \left[\mu \nabla^2 \psi - \rho \frac{\partial^2 \psi}{\partial t^2} \right] = 0, \quad (2.8)$$

which is satisfied as long as both terms in the square brackets are equal to zero. This leads directly to the Helmholtz differential equations: the first one governs longitudinal waves

$$\frac{\partial^2 \phi}{\partial t^2} = c_L^2 \nabla^2 \phi, \quad (2.9)$$

and the second one shear waves

$$\frac{\partial^2 \psi}{\partial t^2} = c_S^2 \nabla^2 \psi, \quad (2.10)$$

where c_L is the phase velocity of longitudinal waves, defined as

$$c_L = \left(\frac{\lambda + 2\mu}{\rho} \right)^{\frac{1}{2}} \quad (2.11)$$

and c_S is the phase velocity of shear waves, defined as

$$c_S = \left(\frac{\mu}{\rho} \right)^{\frac{1}{2}}. \quad (2.12)$$

Due to eq. (2.9) and eq. (2.10) being independent from another, shear and longitudinal waves propagate independently in an unbounded isotropic medium, without interacting. There exist general solutions for the propagation of longitudinal- and shear waves and for the case of propagation in the z -direction they are defined as

$$\phi = \phi_0 e^{i(k_L z - \omega t)} \quad (2.13)$$

and

$$\psi = \psi_0 e^{i(k_S z - \omega t)}, \quad (2.14)$$

where ϕ_0 and ψ_0 are arbitrary amplitudes, $i = \sqrt{-1}$ is the imaginary number, z is the spatial coordinate of the wave and $k_{L,S}$ are the longitudinal (L) and shear wavenumbers (S), defined as

$$k_{L,S} = \frac{\omega}{c_{L,S}}, \quad (2.15)$$

with $\omega = 2\pi f$ being the angular frequency and f being the ordinary frequency.

2.3 Guided waves

Bulk waves have been derived for infinite structures and are solely dependent on the material properties. However, they do interact with boundaries, where refraction, reflection and mode conversion can occur. It is the interaction of bulk waves with boundaries that leads to the occurrence of guided waves. The Rayleigh surface wave [25] is one example, which propagates along a boundary. For the case of an infinite plate (see fig. 2.1), two other types of guided waves may be excited, namely the Lamb waves [22] and the shear-horizontal guided waves [24,26,27]. Guided waves share the wave propagation equations with bulk waves, however, guided waves depend not only on the material properties but also on the material boundaries. Thus, any solution is required to satisfy the wave propagation equations and the physical boundary conditions.

Let us analyse the case of a guided wave in an unbounded stress free and isotropic plate of infinite extent. Two popular solutions are the method of potentials and the partial wave technique. While the latter is able to describe the occurrence of guided waves through the superposition of the waves reflected at the boundaries, and Snell's law governing the reflection of waves, the method of potentials may be used to solve eq. (2.9) and eq. (2.10) with ϕ and ψ as unknown variables.

Lamb waves are a solution for the case of a plane wave propagating along the z -axis, if the

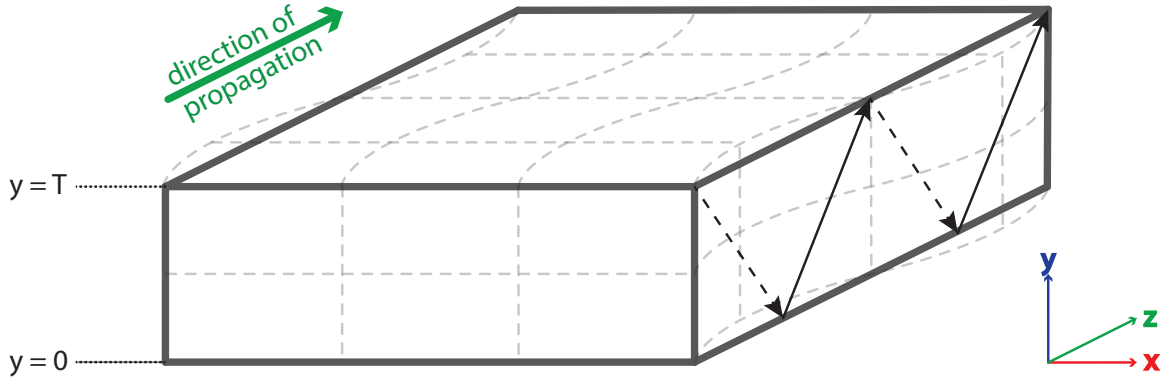


Figure 2.1: Schematic of an isotropic plate of thickness T with internal reflecting waves and indication of shear horizontal displacement.

assumption is made, that particle displacement in the x -axis is zero (i.e. $u_x = 0$). From this, it follows that $\frac{\partial}{\partial x} = 0$ and that only x -axis rotations are allowed ($\psi_z = \psi_y = 0$). This leads to a simplified set of Helmholtz equations,

$$\frac{\partial^2 \phi}{\partial t^2} = c_L^2 \left(\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right), \quad (2.16)$$

and

$$\frac{\partial^2 \psi}{\partial t^2} = c_S^2 \left(\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right), \quad (2.17)$$

to which the solutions are the Lamb waves.

For the opposite case, where no out of plane displacement is permitted, i.e. any displacement along the y and z -axis is assumed to be zero (i.e. $u_y = u_z = 0$) and neglecting any compressional scalar potential ($\phi = 0$), eq. (2.9) cancels out and eq. (2.10) reduces to

$$\frac{\partial^2 \psi}{\partial t^2} = c_S^2 \left(\frac{\partial^2 \psi}{\partial x^2} \right), \quad (2.18)$$

for which the shear-horizontal (SH) guided waves are the solutions. The displacement of these SH guided waves is purely perpendicular to the direction of propagation. Given that SH guided waves are of particular importance in this thesis, we refer the reader to [26] for further discussion of Lamb waves. It has to be noted however, that exact Lamb wave solutions have to be derived numerically and approximations may be derived analytically

2. Theoretical Background

through Mindlin plate theory (see [17, 19]). Analytical expressions for SH guided waves will be presented in the following subsection.

2.3.1 Shear-horizontal guided waves

The SH guided wave solution to eq. (2.18) is given by the following equation of wave propagation:

$$\psi = \psi_0(y) e^{i(k_z z - \omega t)}, \quad (2.19)$$

where ψ_0 is the displacement amplitude and k_z is the wavenumber in the propagation direction. There is a similarity to eq. (2.14) but in this case, the displacement amplitude is thickness dependent as it is a function of the spatial term y , thus describing a potential of thickness-dependent sinusoids:

$$\psi_0(y) = A \sin(k_y y) + B \cos(k_y y), \quad (2.20)$$

where A and B are arbitrary constants and k_y is the through-thickness wavenumber. For traction free boundary conditions applying $y = T$ and $y = 0$ to eq. (2.19) yields

$$A \cos(k_y T/2) - B \sin(k_y T/2) = 0, \quad (2.21)$$

$$A \cos(k_y T/2) + B \sin(k_y T/2) = 0, \quad (2.22)$$

resulting in the frequency equation

$$\cos(k_y T/2) \sin(k_y T/2) = 0, \quad (2.23)$$

which is only satisfied if

$$k_y = \frac{\pi n}{T} \quad (n = 0, 1, 2, 3, \dots) \quad (2.24)$$

at integer values n , showing that SH guided waves may propagate only under certain conditions. Using the relationship between the bulk shear wavenumber k_S and k_y ,

$$k_y^2 = k_S^2 + k_z^2, \quad (2.25)$$

the substitution of eq. (2.15) for shear (S) and eq. (2.24) leads to the SH guided wave dispersion relation

$$k_z^2 = \left(\frac{\pi n}{T}\right)^2 - \left(\frac{\omega}{c_S}\right)^2. \quad (2.26)$$

From this, the frequency f and thickness T dependent phase velocity of SH guided waves is found by utilising $k_z = \omega/c_p$ [26]:

$$c_p(f T) = \pm 2 c_S \left(\frac{f T}{\sqrt{4(f T_0)^2 - n^2 c_S^2}} \right), \quad (2.27)$$

as well as the group velocity of SH guided waves, which describes the velocity of the wave pulse envelope:

$$c_g = c_S \sqrt{1 - \frac{(n/2)^2}{(f T_0/c_S)^2}}. \quad (2.28)$$

From both equations it is immediately observable that for the fundamental SH guided wave mode SH0, $n = 0$ and both group and phase velocities are equal to the phase velocity of the bulk shear wave, c_S . In fig. 2.2a, the phase velocity dispersion curves for SH guided waves and the two fundamental Lamb wave modes are plotted. Moreover, it is shown that the higher order SH guided wave modes do not exist below certain cut-off frequency-thickness products, which are defined by

$$f T = \frac{n c_S}{2}, \quad (2.29)$$

stating for a particular integer value of n , solutions have to be greater than a corresponding $f T$ value. This property is solely dependent on the material parameters which set the bulk shear wave velocity.

2. Theoretical Background

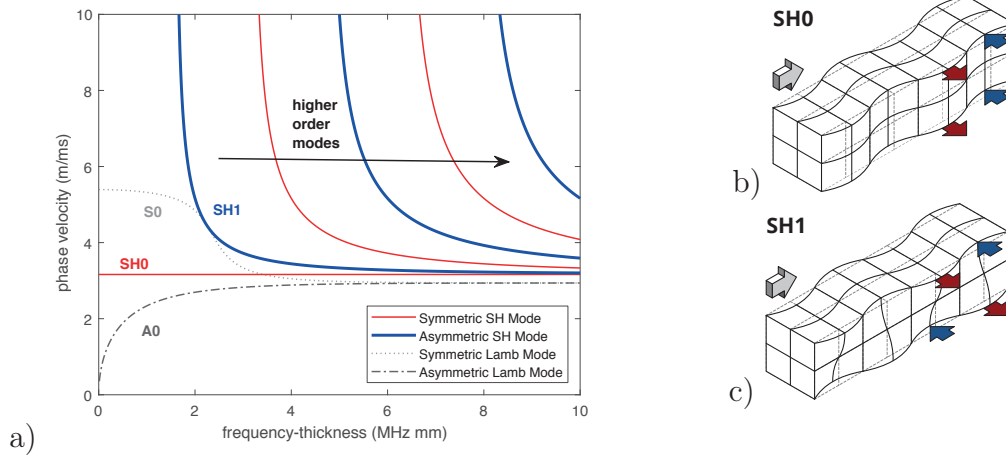


Figure 2.2: a) Phase velocity dispersion curves for Lamb and shear-horizontal guided waves in a steel plate. SH guided wave mode shapes for b) SH0 & c) SH1 for a single frequency. The grey arrows in b) and c) indicate the direction of propagation.

The mode shapes of the fundamental SH mode (SH0) and of the first higher order mode (SH1) are shown in fig. 2.2b,c. SH guided waves do not have any out-of-plane displacement, which makes them less susceptible to attenuation due to liquid loading, since, neglecting viscosity, no part of the shear motion can be transmitted into the liquid. The dispersion relationship between thickness and phase velocity is frequently used to determine remnant wall thickness from wave velocity measurements.

The fundamental SH0 mode is non-dispersive and therefore unsuitable for this type of thickness inversion method. The first anti-symmetric mode, SH1, does not exist below a cut-off frequency-thickness product of $f \cdot T \approx 1.6$ MHz mm but in contrast is highly dispersive up to a frequency-thickness product of 5 MHz mm, making it a promising candidate for thickness inversion guided wave tomography. The working point on the dispersion curve depends on the required depth penetration. For the case of SH1, the working point would move towards higher frequency-thickness products when greater depth penetration is required. Given the steepness of the slope at lower frequency-thickness values between cut-off and 3 MHzmm, SH1 will be most sensitive for shallower defects and thickness inversion will be more challenging when moving to greater frequency-thickness values for greater depth penetration. Here, the accuracy of the velocity measurements will determine how accurately greater depths can be measured.

The following section will briefly explain acoustic wave scattering before explaining guided

wave tomography in the section thereafter.

2.4 Guided wave tomography

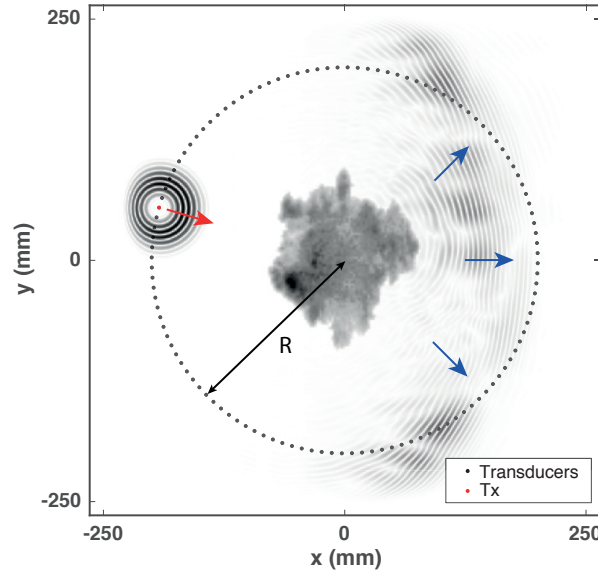


Figure 2.3: Schematic of the full view guided wave tomography layout employed in this thesis: A circular array of radius R is placed on a plate. The transducers are used for excitation as well as reception. The arrow pointing towards the defect indicates an excitation event at a transmitting transducer Tx. A defect is placed in the centre of the plate. The incident waves are interacting with the defect and the scattered waves (arrows pointing away from the defect) are picked up by the receiving transducers.

The goal of guided wave tomography is to reconstruct a thickness map of a region of interest (ROI) using guided waves which are transmitted through the ROI and received at various receiving positions by an array placed around it (see fig. 2.3). This reconstruction is possible since dispersive guided waves modes are dependent on the product of frequency and thickness and by fixing frequency, their group or phase velocity become thickness dependent. Typically, the aim is to generate a velocity map from the measurements and utilising the dispersion relationship to convert that velocity map to a thickness map. As mentioned previously, the SH1 mode seems to be a particularly suitable candidate, as it offers shorter wavelength than the Lamb waves typically considered, meaning it has potential for improved resolution. Using even higher frequencies would lead to a multitude of modes being present, which adds complexity to the signal processing, due to

2. Theoretical Background

mode conversion and overlapping wave modes in both time and space. Higher order Lamb modes are also not considered because generating a pure mode is practically challenging. The next section will explain the basic acoustic scattering model which is used for velocity inversion.

2.4.1 Forward scattering theory

Commonly applied reconstruction algorithms use the change in phase to infer velocity which may then be used to infer thickness; assuming an applicable dispersion relationship is known. It is worth noting that these algorithms typically assume an acoustic scattering model. In this section, the model used in this thesis will be explained briefly by investigating the scattering in the frequency domain. The Helmholtz equation describes the wave equation in the frequency domain

$$[\nabla^2 + k(\mathbf{x})^2]\phi = 0, \quad (2.30)$$

where k is the wavenumber at position $\mathbf{x}$ and ϕ is the scalar field potential. In the frequency domain, the response for a background wavenumber k_0 can be defined by an object function

$$O(\mathbf{x}) = k_0^2 \left[\left(\frac{c_0}{c(\mathbf{x})} \right)^2 - 1 \right], \quad (2.31)$$

which, when substituted into the Helmholtz equation (2.30) yields

$$[\nabla^2 + k_0^2]\phi = -O\phi. \quad (2.32)$$

We can find an equation for the scattered field $\phi_s = \phi - \phi_0$ using the free space Helmholtz equation

$$[\nabla^2 + k_0^2]\phi_0 = 0, \quad (2.33)$$

and subtracting it from eq. (2.32), which gives the Helmholtz equation in free space acting on the scattered field ϕ_S that produces the strength response $-O\phi$:

$$[\nabla^2 + k_0^2]\phi_S = -O\phi. \quad (2.34)$$

Using the Green's function solution, we can write eq. (2.34) as

$$[\nabla^2 + k_0^2]G(\mathbf{y}, \mathbf{x}) = \delta(\mathbf{x} - \mathbf{y}), \quad (2.35)$$

where $\mathbf{x}$ and $\mathbf{y}$ are the position vectors of the source and receiver locations. The analytical solution for the two-dimensional case is given by

$$G(\mathbf{y}, \mathbf{x}) = -\frac{i}{4}H_0^{(1)}(k_0r), \quad (2.36)$$

where

$$r = |\mathbf{x} - \mathbf{y}| \quad (2.37)$$

and $H_0^{(1)}$ is the Hankel function of the first kind.

We can now multiply the right-hand-side (RHS) of eq. (2.34), $-O\phi$, with eq. (2.35) and find

$$-O\phi \cdot [\nabla^2 + k_0^2]G(\mathbf{y}, \mathbf{x}) = -O\phi \cdot \delta(\mathbf{x} - \mathbf{y}). \quad (2.38)$$

Integrating both over $d\mathbf{x}$ gives

$$\int -O\phi \cdot [\nabla^2 + k_0^2]G(\mathbf{y}, \mathbf{x})d\mathbf{x} = \int -O\phi \cdot \delta(\mathbf{x} - \mathbf{y})d\mathbf{x}. \quad (2.39)$$

The operator on the left-hand-side (LHS), $[\nabla^2 + k_0^2]$, can be moved out of the integral and the RHS simplifies to $-O\phi$:

$$[\nabla^2 + k_0^2] \int -O(\mathbf{x})\phi(\mathbf{x})G(\mathbf{y}, \mathbf{x})d\mathbf{x} = -O\phi, \quad (2.40)$$

2. Theoretical Background

which, compared to eq. (2.34) leads to the Lippmann-Schwinger equation:

$$\phi_S = - \int O(\mathbf{x}) \phi(\mathbf{x}) G(\mathbf{y}, \mathbf{x}) d\mathbf{x}. \quad (2.41)$$

In order to solve the Lippmann-Schwinger equation an approximation is needed and the most commonly used one is the Born approximation [28]. The main assumption of the Born approximation is that the total field is approximately equal to the incident field $\phi \approx \phi_0$, which when substituted into eq. (2.41) leads to

$$\phi_S = \int -O(\mathbf{x}) \phi_0(\mathbf{x}) G(\mathbf{y}, \mathbf{x}) d\mathbf{x}. \quad (2.42)$$

This may be solved by assuming a far-field array which models the incident field and the Green's function as plane waves [29] so that

$$\phi_S = \int -O(\mathbf{x}) e^{ik(\hat{\mathbf{s}} - \hat{\mathbf{s}}_0) \cdot \mathbf{x}} d\mathbf{x}, \quad (2.43)$$

where $\hat{\mathbf{s}}$ represents the scattered wave direction to the receiver and $\hat{\mathbf{s}}_0$ is the incident wave direction from the source (see fig. 2.4a). This is commonly recognised to be a two-dimensional Fourier transform

$$\phi_S(\hat{\mathbf{s}}, \hat{\mathbf{s}}_0) \approx -\hat{O}[k(\hat{\mathbf{s}} - \hat{\mathbf{s}}_0)]. \quad (2.44)$$

By exciting and receiving from all possible angles from 0 to 2π it is possible to fill the Ewald limiting disc [28] of radius $2k_0$ (see fig. 2.4b). This can be achieved by plotting the measured components into a uniform grid in ' k -space', the spatial frequency domain. For an array with 120 positions, all populatable k -space positions are depicted as grey dots in fig. 2.4(b) within the Ewald limiting circle with radius $2k_0$ (black circle). For a fixed source position (red circled dot in fig. 2.4(a)) each measurement is mapped to a particular point in k -space. The multi-coloured arrows indicate that through-transmission information (from light blue, green to yellow) is mapped to the low-spatial frequency region in k -space, whereas reflection information (blue, red to orange) is stored towards the high-spatial frequency region close to the $2k_0$ limit. By performing all transmit-receive

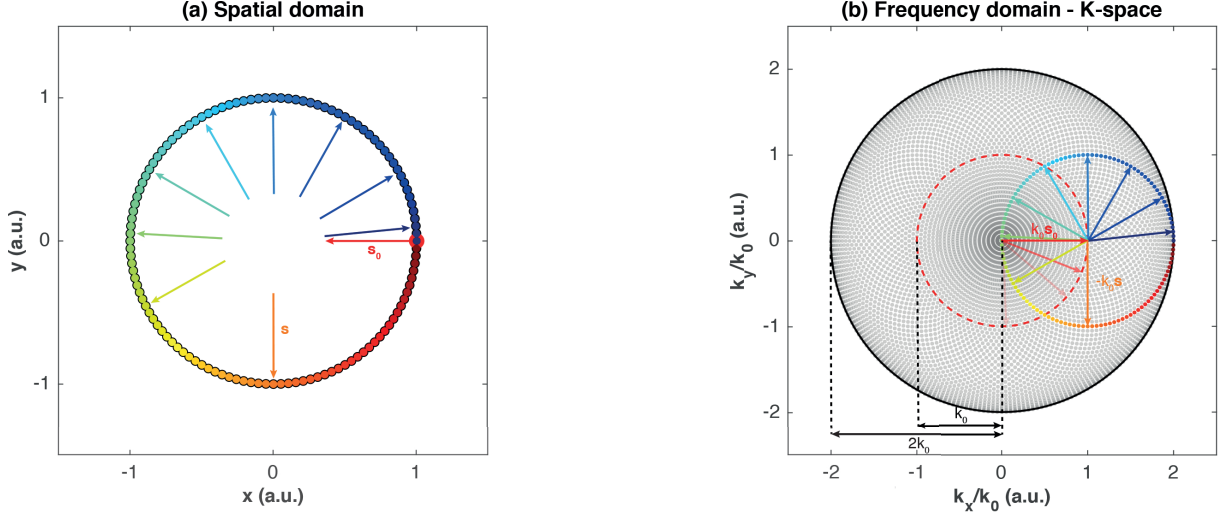


Figure 2.4: Schematic of mapping scattered information from the spatial domain to the spatial frequency domain called k -space: (a) shows a circular transducer array setup and the excitation vector $\hat{s}_0$ in red and the receiving direction $\hat{s}$. Different receiving directions are indicated with the different colours shown. (b) presents the spatial frequency domain of the object, known as k -space, into which the measured scattered components can be mapped, according to eq. (2.44). The component corresponding to $\hat{s}_0$ incident and $\hat{s}$ in reception in (a) is mapped to (1, -1) as illustrated. By changing the receiver direction $\hat{s}$ around 360 degrees, a circle is mapped out, illustrated by the colours shown in both plots. Changing the source direction then allows this circle to be iteratively moved around, with all the measurements ultimately populating the disk shown.

combinations, all possible positions in k -space are filled. After application of an inverse 2D Fourier transform, the scattered object may be reconstructed. The next section will explain how this principle is employed in guided wave tomography.

2.4.2 Reconstructing thickness from velocity maps

In order to generate the velocity map which can then be converted to a thickness map, a velocity inversion algorithm is needed and typically, this assumes an acoustic wave behaviour. One of the simpler algorithms is to model the waves as rays and to use their arrival times for estimating the velocity. While this can allow a defect to be located, the resolution of this approach is very poor as it neglects diffraction which occurs when the scattering object is small compared with the wavelength. Many approaches in tomography have utilised ray theory, including Hinders [30,31], Volker [5], Simonetti [32] and Belanger [33].

Another possibility is the use of the diffraction tomography algorithm. This is, however,

2. Theoretical Background

limited by its underlying use of the Born approximation which limits the contrast and size of the object which can be reconstructed [15, 29]. An alternative approximation that may be used with DT is the Rytov approximation [29], which has been considered for guided wave tomography in the past [34]. Here, the phase of the field has to be unwrapped which is challenging particularly for complex scatterers and if any noise is present in the signals. The HARBUT algorithm, which relies on a modification of the Born approximation, has been shown to have good performance for guided wave tomography velocity inversion [15]. In essence, HARBUT is a modification of diffraction tomography (DT). DT produces a reconstruction by recognising that the wave scattering process, under the Born approximation and due to the sinusoidal nature of waves at a single frequency, forms a Fourier transform of the scatterer. Hence, an inverse Fourier transform may be used to obtain an image of the scatterer. HARBUT modifies DT to incorporate an inhomogeneous background from bent ray tomography (BRT), accounting for the wave aberrations with an Eikonal solver, ultimately increasing the contrast and size of objects which can be imaged.

Thus, by combining the complementary strengths of bent ray and diffraction tomography, the algorithm is capable of achieving high resolution results at very low computational intensity. HARBUT has been shown to be robust for a huge range of different defects across different modes and with different frequencies [35].

The highly-dispersive nature of the SH1 mode would traditionally limit the propagation distance of the wave. However, since HARBUT operates in the frequency domain rather than the time domain where dispersion would be an issue, we bypass this challenge. The maximum propagation distance depends on energy lost from the wave; dispersion does not remove energy but simply redistributes it in time and therefore, frequency domain methods are insensitive to this. It is for these reasons that HARBUT has been chosen as the algorithm for velocity reconstruction in this investigation.

Once a velocity reconstruction has been achieved (see fig. 2.5(a)), the velocity values are looked up in a dispersion curve (see fig. 2.5(b)) and maybe be converted to thickness values (see fig. 2.5(c)).

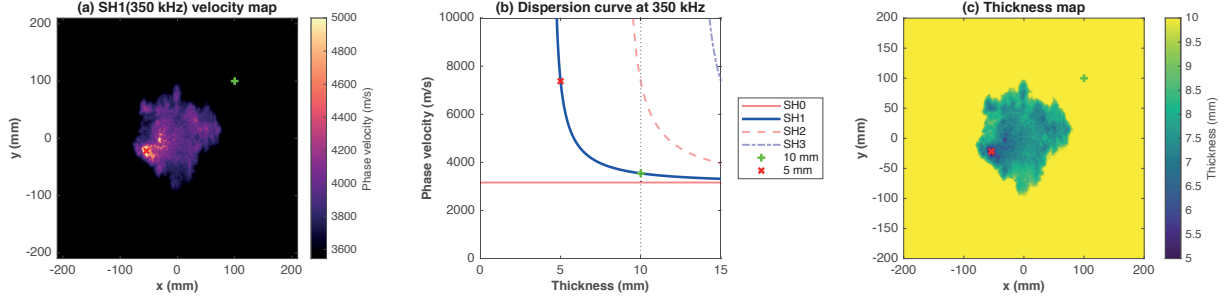


Figure 2.5: Mapping velocity to thickness: A velocity map was reconstructed from captured SH1 signals (a). By fixing frequency to 350 kHz, the velocity becomes thickness dependent and the thickness values are looked up in the dispersion curve to calculate a thickness map (c). The green cross is a point of nominal thickness and the red cross depicts a location with 50% wall loss.

2.4.3 Resolution in guided wave tomography

The resolution of an imaging modality determines how well features may be resolved. The finer the details that are desired to be imaged are, the higher the requirement in terms of resolution. The achievable resolution of a guided wave tomography system broadly depends on four parameters: the type of the transducer array, the number of transducers, the chosen centre frequency f_c (i.e. wavelength $\lambda = c/f$) and finally, the applicability of the reconstructing algorithm to the elastic wave model. In order to capture the full scattering information from a defect, a circular, ‘full-view’ configuration is desired over for example a pair of parallel arrays, which only offers ‘limited view’ of a region of interest because the ROI is not illuminated from certain directions.

Another factor is the number of transducers, which is dictated by the sampling criterion that is dependent on the wavelength, which is determined by the chosen centre frequency. One fundamental aspect to the imaging problem that has been underestimated in the past is the applicability of the reconstruction algorithm to guided waves [16]. The typical acoustic scattering model used has been the Born approximation which holds for small perturbations in velocity. The approximation assumes that the scattered field is the superposition of infinitesimally small elementary scatterers, which in this case, scatter acoustically or rather in an omnidirectional fashion. Any density variations will have dipolar scattering. It should be noted that while the Born approximation is used to analyse and describe the scattering here, the conclusions are general to guided wave scattering and not just applicable to reconstructions achieved via the Born approximation.

2. Theoretical Background

Recent work by Huthwaite [15] has revealed that for Lamb waves, the scattered amplitude from a point-like defect does not exhibit acoustic scattering behaviour but rather shows a highly directional scattering profile. This was shown through a comparative study between simulated data from 2D acoustic and 3D elastic simulations.

As has been described previously, the resolution limit of diffraction tomography and HARBUT is $\lambda/2$ [29,36], which may be achieved if the field is fully sampled and the scattering behaviour is fully acoustic. A frequently used tool to determine the spatial resolution of an image is to determine the wavenumber components that were captured in the frequency domain. For diffraction tomography, the maximum wavenumber components that can be captured are located within the Ewald limiting circle in ' k -space'. k -space is the 2D Fourier transform of an image as illustrated in fig. 2.6 and by choosing an investigation centre frequency, a background wavenumber k_0 can be determined. Using this background wavenumber, the axes of the k -space image can be normalised to multiples of this wavenumber.

It is useful to know that low spatial frequency (or wavenumber) components are located at the centre of k -space, whereas high frequency components are located at the outer regions. The consequences of this can be readily demonstrated by applying two types of windows to the k -space image. First, we apply a circular binary window w_o of radius k_0 to the k -space image, setting any wavenumber components lower than k_0 to zero:

$$w(\mathbf{k})_o = \begin{cases} 0 & \mathbf{k} \leq k_0, \\ 1 & \mathbf{k} > k_0, \end{cases} \quad (2.45)$$

where $\mathbf{k}$ is the wavenumber position in k -space. Applying the inverse 2D FFT to this windowed k -space image yields an image that contains only the details of the image. It is directly obvious that higher spatial frequency components correspond to the finer details within the image. Applying the inverse of w_o to the k -space image shown in fig. 2.6(c), where

$$w(\mathbf{k})_o = \begin{cases} 1 & \mathbf{k} \leq k_0, \\ 0 & \mathbf{k} > k_0, \end{cases} \quad (2.46)$$

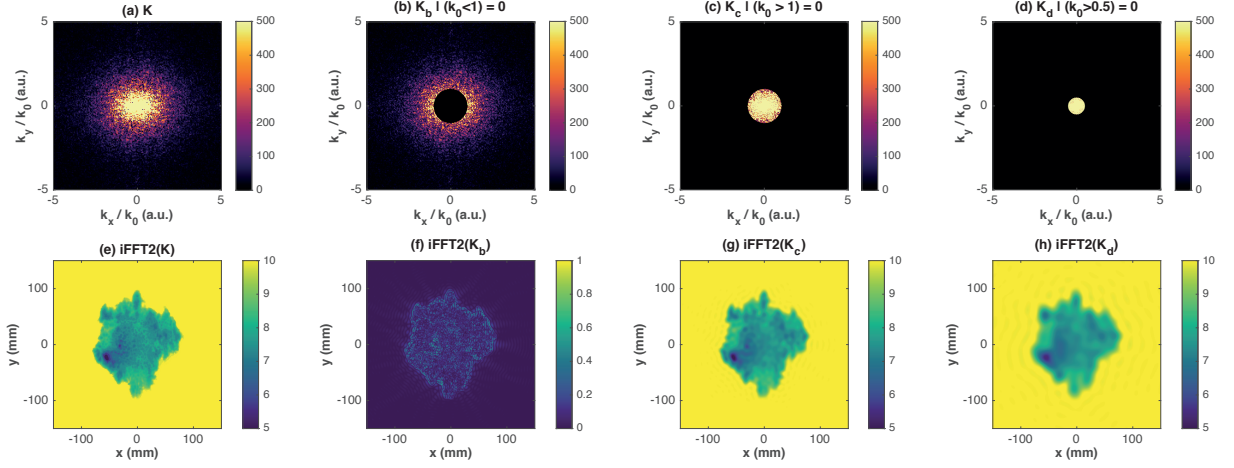


Figure 2.6: k -space representation and reconstruction of images after applying an inverse 2D-FFT: Inverting the full k -space image K (a) leads to the original image (e). Removal of low spatial frequency components at the centre (b) leads to a reconstruction of the finer details (f), encoded in the high spatial frequency components. Inversely, removal of the higher spatial frequency components (c-d) leads to a drop in resolution but maintains the low spatial frequency information of the imaged object (g-h). The more high spatial frequency components are removed, the ‘blurrier’ the image and greater the loss of resolution.

followed by application of the inverse 2D FFT leads to a reasonably accurate reconstruction of the original image but a noticeably blurrier result as can be seen in fig. 2.6(g). From the figure, it is shown that the finer details are missing. Reducing the window radius to $0.5k_0$ as shown in fig. 2.6(d) leads to greater loss of resolution and to a much blurrier result shown in fig. 2.6(h). Any high spatial frequency components in regions $> 2k_0$ are not captured using image reconstruction techniques based on the Born approximation.

As stated previously, the wavelength and array parameters should have been sufficient to achieve a resolution of $\lambda/2$ but due to the mismatch in scattering behaviour between Lamb and acoustic waves insufficient scattering information is captured, leading to a loss of wavenumber components in k -space and ultimately a significant loss in resolution. Expanding the resolution criteria to the upper limit of wavenumber components, we can determine which wavenumber components need to be captured in order to achieve sufficient resolution to capture the depth accurately, while retaining good in-plane resolution.

In the following section, the fundamentals of the FE method used for the numerical investigation is discussed and the modelling approach for pure SH1 excitation is explained.

2.5 Finite element simulations

In the last decades, numerical simulation tools have been developed that allow for highly realistic simulation data. One of the most important types of modelling tools is the FE method, where large models are segmented into small, finite elements and thus finding approximate solutions to partial differential equations with boundary value problems is possible for even complex geometries. In this thesis, FE modelling was employed to investigate guided wave propagation in isotropic steel plates. For this, the steel plate was discretised into millions of finite elements. Next, the partial differential equation with boundary values is solved at every node of these elements which provides approximate solutions at a specific timestep. The calculations are ideally solved explicitly. This means that the next timestep value is based solely on known values from the current and previous timestep without an interdependence in the following ones.

The FE package used in this thesis was Pogo, which utilises graphics processing units (GPUs) to solve the explicit time-domain FE model which significantly speeds up the simulation, compared with conventional FE software that runs on central processing units (CPUs). The main advantage is that Pogo is an Nvidia compute unified device architecture CUDA enabled solver, solving explicit time domain problems on one or multiple GPUs allowing for a dramatic speed in solving time compared to CPU-based commercial FE packages.

More specifically, Pogo solves the dynamic equilibrium equation in its finite form, which is given by

$$[M]\ddot{\mathbf{u}} + [C]\dot{\mathbf{u}} + [K]\mathbf{u} = [F_{ext}], \quad (2.47)$$

where $\ddot{\mathbf{u}}$ is the acceleration vector, $\dot{\mathbf{u}}$ the velocity vector and $\mathbf{u}$ the previously introduced displacement vector. $[M]$ is the diagonal mass matrix describing material density, $[C]$ is the viscous damping matrix (which is zero for the case of an elastic material), $[K]$ is the static stiffness matrix that encodes Young's modulus and Poisson's ratio and $[F_{ext}]$ are the external forces that are applied to the system.

Using the finite difference scheme, the acceleration, velocity and displacement vectors in

the dynamic equilibrium equation may be defined as

$$\ddot{\mathbf{u}} = \frac{\mathbf{u}_{t+1} - 2\mathbf{u}_t + \mathbf{u}_{t-1}}{\delta t^2}, \quad (2.48)$$

$$\dot{\mathbf{u}} = \frac{\mathbf{u}_{t+1} - \mathbf{u}_{t-1}}{2\delta t}, \quad (2.49)$$

$$\mathbf{u} = \mathbf{u}_t, \quad (2.50)$$

using the displacement vectors of the previous timestep $\mathbf{u}_{t-1}$, of the current timestep $\mathbf{u}_t$ and the next timestep $\mathbf{u}_{t+1}$.

Careful selection of the time increment δt and the element size Δx is required for accurate numerical results. Drozd [37] recommended the following relation

$$\delta t \approx \frac{x_{min}}{c_0 \sqrt{G}}, \quad (2.51)$$

where x_{min} is the smallest nodal distance in the mesh, c_0 the velocity of the fastest propagating wave and G the dimensional order of the model. Essentially, δt needs to be smaller than the time it takes for the fastest propagating wave to transverse between the smallest nodal distance x_{min} . It is thus equally important to select a suitable element size Δx as numerical stability also heavily depends on this. For this, Drozd suggested a mesh density of 30 elements per wavelength for a triangular mesh

$$\Delta x \approx \frac{\lambda}{30}, \quad (2.52)$$

which should lead to sufficiently accurate quantitative results that nevertheless need to be verified in case of greater defect complexity. For the case of a tetrahedral mesh, Carandente [38] used 22 elements per wavelength for greater defect complexity. Thus, in this thesis the numerical simulations were performed with at least 20 elements per wavelength. Another important factor in FE modelling of guided waves is the excitation approach. The next subsection will explain the technique that was employed to generate pure SH guided wave excitation data.

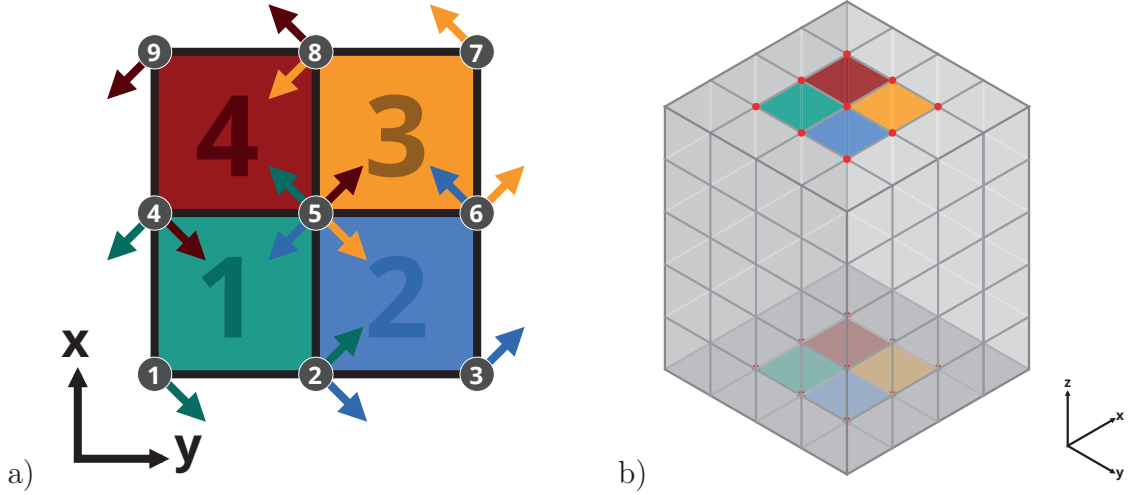


Figure 2.7: a) Omnidirectional shear excitation on all nodes of four elements. The source can be positioned arbitrarily by weighting the displacements of the four elements and superposing the signals together. (b) shows how this shear source can be applied to the top and bottom surfaces; excitation by twisting in opposite directions top to bottom will excite the asymmetric SH modes and pure SH1, assuming excitations well below the SH3 cut-off frequency.

2.5.1 SH guided wave excitation approach for FE

The finite element package Pogo was employed to capture the full complexity of guided wave scattering as it provides full three-dimensional elastodynamic simulation capabilities [39]. In order to excite and receive pure SH guided wave signals from an array of point sources, an omnidirectional excitation model had to be developed. The array of transducers was modelled by exciting forces and measuring the displacement of nodes around the transducer positions. This can be seen in fig. 2.7 by considering a single colour. Note the depicted situation only occurs if the desired point source location happens to lie perfectly at the centre of an element. In practice, one solution is to accept a slight shift in the transducer position and move it to be centred on an element, however, in this thesis this excitation is applied to each of the four adjacent elements, shown as different colours in fig. 2.7.

Next, the weighting of each element is determined by a linear interpolation approach, such that the centre of mass of the source lies at the desired location. The case depicted in fig. 2.7(a) shows the weighting that would be applied for the ideal case of the transducer position being exactly on the centre node (number 5), although it should be noted that the central forces will balance and hence cancel each other out to become zero. The

principle of reciprocity means that the same weightings that are used during excitation are also used during reception, summing the measured displacement values on each degree of freedom with the same weighting as the excitation term.

During lower frequency investigation, SH1 was excited by an opposing set of twisting sources on the top and bottom surfaces of the plate; the twisting avoids longitudinal excitation because it has zero divergence, and the opposing sources prevent the fully symmetric SH0 and SH2 from being generated. This was sufficient, as long as the frequency content of the excitation pulse was below the cut-off frequencies of higher order asymmetric SH modes (such as SH3 and SH5).

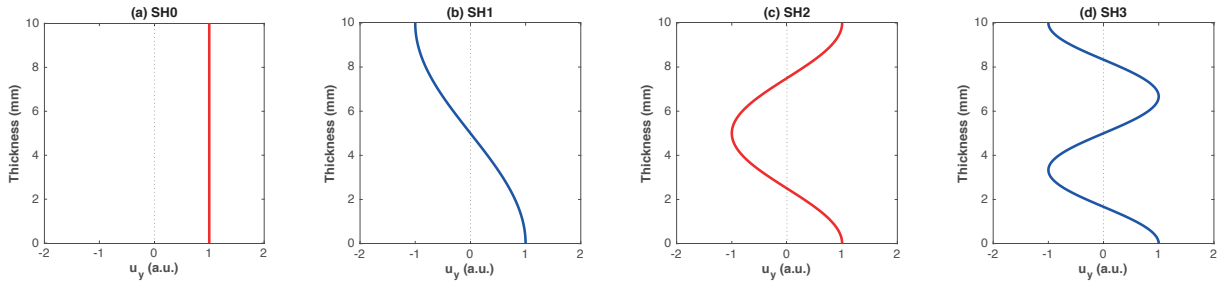


Figure 2.8: Shear-displacement u_y profiles for the first four SH guided wave modes: (a) SH0, (b) SH1, (c) SH2 and (d) SH3. The two symmetric modes are plotted in red and the asymmetric modes in blue.

When investigating deeper defects, however, the excitation had to be performed at higher frequencies, where the bandwidth of the used pulses might contain frequency components that are greater than e.g. SH2 and SH3 cut-off frequencies. This could lead to higher order symmetric or asymmetric modes being picked up by the previous excitation scheme, where excitation and reception only occurred at the top and bottom surfaces. In order to compensate for this effect, the through-thickness displacement curves were calculated for every mode within the frequency range used in this thesis (SH0 - SH3) and are shown in fig. 2.8 .

The displacement was then coupled with the weighting of all excitation nodes on every thickness layer of the model. For example, for SH0, the displacement through-thickness is a straight line as it is the fundamental symmetric SH mode. This was modelled by selecting 9 nodes for every thickness layer of the model and applying the weighting described above multiplied with the displacement curve. Hence, pure modal excitation and reception were achieved by multiplying the mode dependent displacement shape with the element

2. Theoretical Background

weighting for every thickness layer. The scheme is shown for SH0 and SH1 in fig. 2.9 in a 10 mm thick plate which is close to the thickness of a 12 inch schedule 40 pipe. Thus, true pure SH mode excitation and reception were achieved at higher frequencies.

In practice, pure omnidirectional point source SH excitation is very challenging to achieve. The same holds for pure through-thickness modal excitation as this requires access to both surfaces of the pipe. In the field, the access will usually be limited to the surface. In the following chapter this is acknowledged by increasing realism from pure and through-thickness excitation over surface point source excitation to directional surface excitation. Nevertheless, the here presented pure excitation model is vital for careful modal analysis.

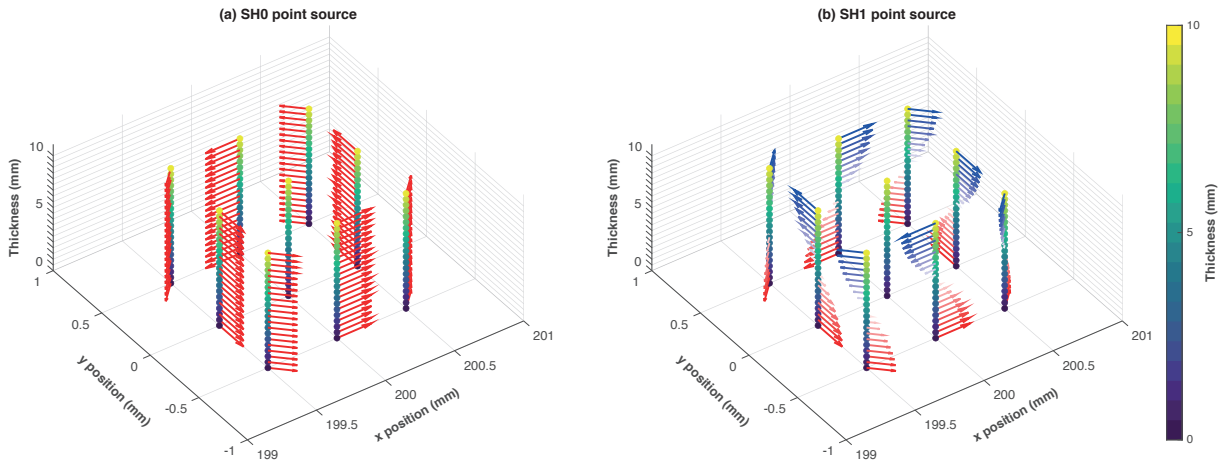


Figure 2.9: Through-thickness nodal displacement schematic for pure SH0 (a) and SH1 (b) point source excitation in a 10 mm thick plate: Here the transducer is positioned at $[x, y] = [200, 0]$ mm. The 9 node columns are colour coded by thickness from 0 mm = darkblue to 10 mm = yellow for all 21 model layers. The red vectors indicate the displacement in counter-clockwise direction and blue arrows point in the clockwise direction. For SH0 (a) this leads to a uniform displacement profile through-thickness while for SH1 (b) this leads to anti-symmetric displacement profile through-thickness. Note that the centre of the transducer position is located perfectly on the grid and therefore the nodal displacement in the centre is zero.

2.6 Summary

In this chapter, the fundamentals of bulk waves, guided waves and more specifically SH guided waves were presented. Guided wave tomography was explained and the various stages demonstrated. Next, the FE method was discussed and the special approach to exciting pure modes of SH guided waves was explained. This was the fundamental problem necessary to solve to reach a thorough understanding of the scattering phenomena which

affect the quality of reconstructions in guided wave tomography. The next chapter will see the application of these fundamentals in numerical simulations.

This page intentionally left blank

Chapter 3

Numerical investigation of SH guided wave tomography

3.1 Introduction

In this chapter, SH guided wave tomography was investigated numerically. As mentioned in the introduction of this thesis, Lamb waves exhibit a markedly different point scattering pattern compared with acoustic waves, which leads to an underperformance of the HARBUT reconstruction algorithm. It was thus necessary to first identify the point scattering pattern of SH guided waves in order to determine whether any increase in frequency would lead to an improvement in resolution. The results of this investigation are presented in section 3.3. This is followed by results of SH1 point source guided wave tomography for several types of defects. Naturally, the idealised pure mode excitation approach is currently not practical for field applications and therefore a directional SH source was modelled. The guided wave tomography results using this type of transducer are presented in section 3.6. This is followed by a performance analysis, comparing SH1 with the two fundamental Lamb wave modes, A0 and S0. Next, HARBUT is used for reconstruction from SH0 signals yielding an interesting result. Finally, the chapter is summarised in section 3.9. Parts of this chapter have been adapted and extracted from publication [P1].

3.2 Plate model

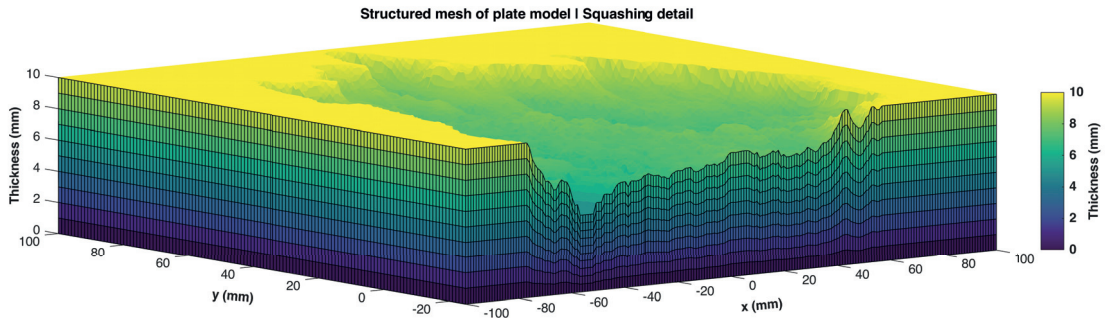


Figure 3.1: Plate modelling approach: 2D thickness maps were used to squash node layers, ‘imprinting’ the defects into the structured mesh. Here, a cut-out of the total model is shown, highlighting the thickness profile through the deepest point of the defect and the squashed elements.

As curvature can be neglected for the pipe-radii of interest in industry [40], the investigation was simplified by considering corrosion patches in 10 mm thick isotropic mild-steel plates ($E = 210$ GPa, $\nu = 0.3125$, $\rho = 8000$ kg/m³), which had to be modelled numerically. The thickness value of 10 mm has been chosen throughout this thesis as it allows for better comparison with previous work (see for example [13, 34, 36]), allows scalability of the results and is quite close to the thickness of a commonly used pipe type, the 12-inch schedule 40 pipe. It is noted that during manufacturing, pipe or plate thickness may vary slightly and variations of 10% are not uncommon for oil industry applications. Any such thickness change will cause a corresponding velocity change. However, it should be possible to spot in the signals when assessing bulk velocity from calibration signals. Another factor that might affect wave velocity is material anisotropy due to uneven rolling of the pipes during manufacturing. In this thesis, such effects are neglected and isotropic material properties as well as nominal thickness are assumed.

The plates were modelled in the form of structured meshes with 8-element hexahedral elements following the approach used in [8] and using Pogo’s mesh generation routines for MATLAB (see fig. 3.1). For frequencies up to 300 kHz an element size of $\Delta x = 0.5$ mm was chosen which for a 0.5 m wide quadratic plate led to 21 million nodes ($1001 \times 1001 \times 21$). For higher frequencies, an element size of $\Delta x = 0.25$ was selected, increasing the model size to 160 million nodes ($2001 \times 2001 \times 41$). In order to avoid signal from boundary

reflections, 60 mm of stiffness reduction method (SRM) boundaries [41] were applied along the edges of the plate model. For the defect region, axisymmetric defects were calculated and laser scanned defects imported and used as templates for element-wise ‘squashing’ in the out-of-plane z -direction in order to model the thickness changes inherent within the defects. A set of seven thickness maps of two categories were created (see fig. 3.2);

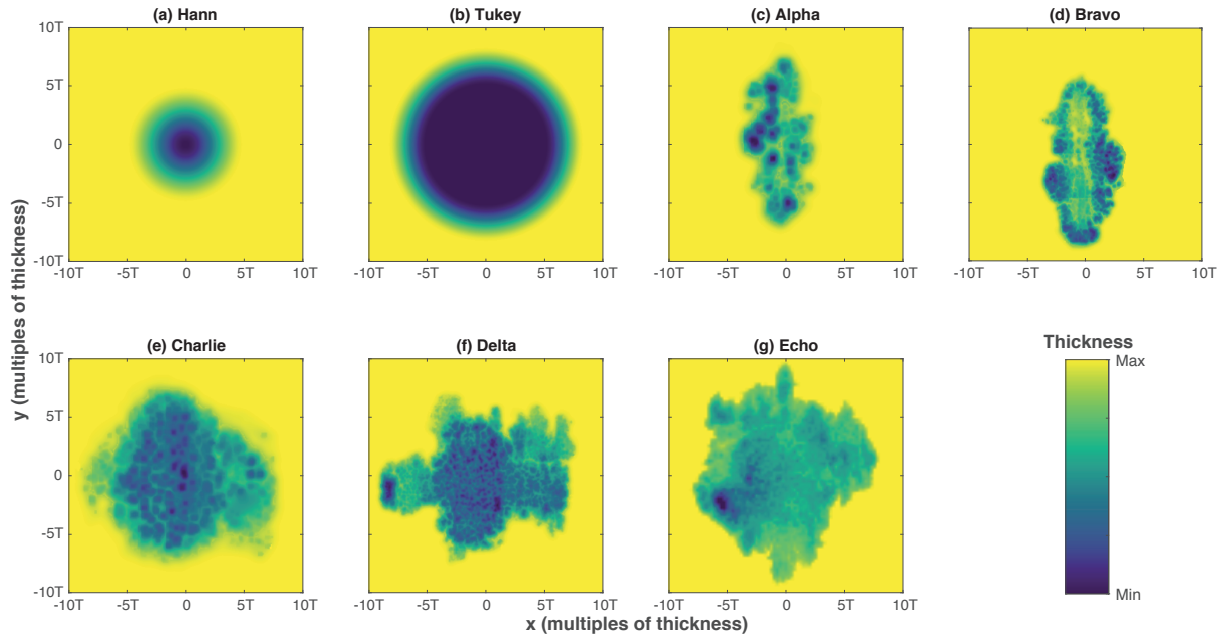


Figure 3.2: The seven defect models used in the numerical study: Hann-shaped axisymmetric *Hann*, Tukey-shaped axisymmetric *Tukey* and laser scanned defects of corrosion under pipe support (CUPS): *Alpha-Echo*.

two synthetic axisymmetric defects and five rough defects based on real laser scanned corrosion patches provided by industry. This range of defects is shown in fig. 3.2 for the case of 20% maximum wall loss. Whenever greater wall loss was required, the thickness maps were scaled linearly. The axisymmetric defects were based on the Tukey [42] and Hann [43] window functions. These were chosen as they are axisymmetric, which reduces simulation duration, as a single excitation event may be used to create a full dataset. Furthermore, the Tukey window was chosen for a cylindrical flat-bottomed defect type as it is slightly more realistic to assume shallow slopes in contrast to sharp boundaries. Furthermore, this helped by reducing mode-conversion, adding situational control.

For the Hann-shaped defect (*Hann*), the thickness function $T(\mathbf{x})$ was defined as

$$T(\mathbf{x}) = \begin{cases} T_0 - \frac{d}{2} \left[1 + \cos \left(\frac{2\pi|\mathbf{x}|}{w} \right) \right] & |\mathbf{x}| < \frac{w}{2} \\ T_0 & |\mathbf{x}| \geq \frac{w}{2}, \end{cases} \quad (3.1)$$

where d is the depth of the defect and w the width. These values were set to $d = 2$ mm and $w = 100$ mm for *Hann*. In order to model an axisymmetric flat-bottomed defect, a Tukey window function was used as a template and adapted to

$$T(\mathbf{x}) = \begin{cases} T_0 - \frac{d}{2} \left[1 + \cos \left(\pi \frac{(|\mathbf{x}| - w/2)}{W/2 - w/2} \right) \right] & \frac{w}{2} < |\mathbf{x}| < \frac{W}{2} \\ T_0 - d & |\mathbf{x}| \leq \frac{w}{2} \\ T_0 & |\mathbf{x}| > \frac{W}{2}, \end{cases} \quad (3.2)$$

where w is the width of the bottom hole and W the width of the defect at the surface. The window becomes rectangular when $W = w$ and a Hann window when $w = 0$. In the case of defect *Tukey*, the width was set to $w = 100$ mm and an outer width of $W = 200$ mm. For the five realistic defects, the spatial resolution of the models was limited to 2.5 mm, the maximum diameter was scaled to be ≤ 200 mm and the depths were linearly scaled such that the maximum depth in each case was set to 2 mm. These defects can be grouped into two classes, pitting-corrosion (*Alpha* and *Bravo*) and larger corrosion patches (*Charlie*, *Delta*, *Echo*).

The Pogo model was run on eight NVIDIA Quadro RTX 8000 GPUs which required 29 minutes to run for the case of 120 excitation events.

3.3 Scattering of SH guided waves from a point-like defect

Recent work by Huthwaite [15] identified a mismatch in scattering behaviour between acoustic and elastic waves. It is this mismatch that leads to the under-performance of HARBUT, which assumes acoustic wave scattering. This was demonstrated for the case of a realistic, laser-scanned defect model where HARBUT performed as expected with data

from acoustic frequency domain finite difference (FDFD) simulation data and achieved good resolution. When reconstructing from elastic simulation data, however, the resolution could not be matched although all parameters such as wavelength, material properties, and number of transducers were constant. This became especially apparent after analysing the spatial frequency components of the reconstructed images, revealing that only wavenumber components within a circle of radius $0.63 k_0$ were acquired, instead of the to be expected $1.4 k_0$ radius for full-view through transmission data. In order to assess the origin of this mismatch, Huthwaite performed an analysis of the scattering amplitude from a point-like defect, which was modelled in form of a shallow and small (diameter $\lambda/4$) flat-bottomed defect of depth 5% of the remnant wall thickness. The specific sizes are unimportant as long as the scatterer may be considered elemental compared to the wavelength.

The purpose of this study is to determine the scattering pattern of SH guided waves in order to assess whether their use could offer a better match to the acoustic case and thus lead to a resolution improvement.

3.3.1 Model

The radius of the array was set to $R = 200$ mm and an excitation frequency of 200 kHz for the SH modes was chosen. At this frequency, SH0 has a phase velocity of 3162 m/s for all frequencies and a wavelength of 15.8 mm. The SH1 mode, at the same frequency, has a phase velocity of 5164 m/s and a wavelength of 25.8 mm. This is already well below the 40 mm of A0 at 50 kHz. As the wavelength of SH0 is much smaller than that of

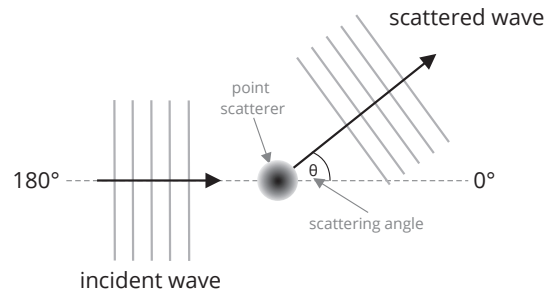


Figure 3.3: Schematic defining the scattering angle θ for a point scatterer.

SH1, the width of the point scatterer was chosen to be a quarter of SH0's wavelength at 200 kHz, leading to a defect width of $\frac{\lambda_{SH0}}{4} = \frac{15.8 \text{ mm}}{4} \approx 4 \text{ mm}$. The depth was chosen to be 5 % of the nominal wall thickness leading to a depth of 0.5 mm, which is in line with the defect setup used by Huthwaite [15]. The circular array sampling criterion was set to $N = 128$. Although SH0 is non-dispersive and therefore unsuitable for thickness inversion from velocity, its scattering behaviour might still be of interest for reconstructing the change in effective density for cases where modal purity is impossible to achieve.

3.3.2 Data processing

In order to extract the scattering amplitude, the received time-traces after an excitation at a single source are recorded and a Fourier transform is applied. Taking the absolute of the values of each receiving position per centre frequency leads to the complex amplitude of the total field. In order to retain the scattered amplitude, an identical defect-free model was run and the same data processing steps were applied. This allowed for the subtraction of the incident field complex amplitude $A_{inc}(\theta)$ from the total field complex amplitude $A_{tot}(\theta)$ which yields the scattered field's complex amplitude:

$$A_{scat}(\theta) = A_{tot}(\theta) - A_{inc}(\theta). \quad (3.3)$$

The scattering pattern is the absolute of the complex amplitude and was plotted as a polar plot, showing the scattered amplitude versus scattering angle.

3.3.3 Results

The resulting scattering amplitudes for the acoustic case, A0 at 50 kHz, SH0 and SH1 at 200 kHz are plotted in fig. 3.4. A0 exhibits the expected directional dipolar pattern, with a larger reflected component and a smaller through-transmission lobe. Notably, this is limited to only small angles, and matches the acoustic case only up to a scattering angle of $\approx 37^\circ$. The fundamental SH guided wave mode, SH0 also scatters in a directional manner, with the biggest amplitude appearing in the reflected amplitude. Interestingly, the through-transmission components are almost reduced to zero except for a very small

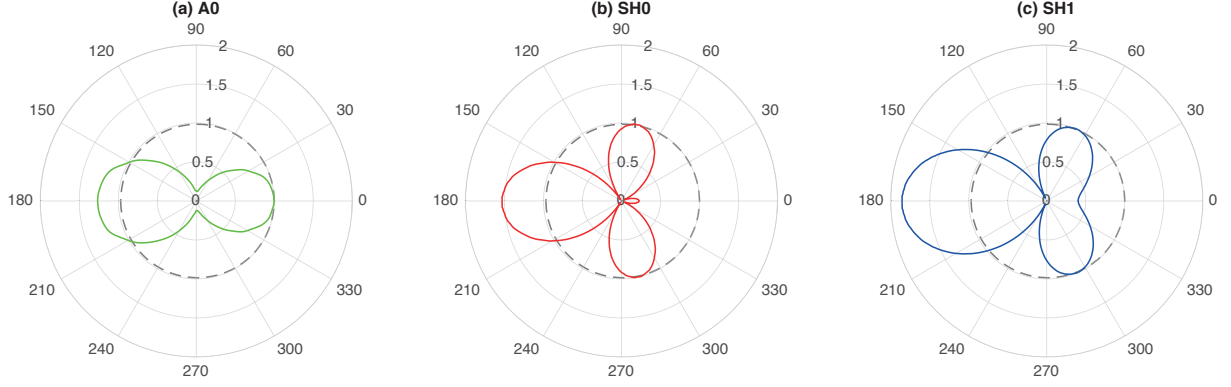


Figure 3.4: Comparison of acoustic and guided wave scattered fields for a 3 mm wide and 0.5 mm deep point-like defect in a mild steel plate: (a) A0 (green) at 50 kHz, (b) SH0 (red) at 200 kHz and (c) SH1 (blue) also at 200 kHz. The peak of the acoustic amplitude is indicated by the dashed grey circle.

amount of amplitude in direct through-transmission and two larger side-lobes between 60° and 90° . Here, the through transmission parts will correspond to the slowly varying features and should match the acoustic model perfectly in the limit as the scattering angle disappears to zero. For SH0 the lack of dispersion means that there should be no velocity variation and therefore no scattering in through-transmission.

The first higher-order SH mode, SH1, on the other hand shows a vastly higher amount of through-transmitted amplitude with the two side lobes being less pronounced than was the case for SH0. However, the wavelength is also significantly larger. Nevertheless, the through-transmission components match the acoustic case much better and most crucially, seem to retain more higher spatial frequency components than for the case of A0. The next step was to assess the matching area more quantitatively in order to be able to infer a resolution prediction from the scattering angle.

3.3.3.1 Resolution prediction

In guided wave tomography, the amount of captured scattering information dictates the achievable resolution. By relying on through-transmission data and neglecting any reflected information, the scattering angle θ is already limited to a range between 0° to 90° and 270° to 360° . As mentioned in section 2.4.3, the scattered amplitudes may be directly mapped to K-space, which is the two-dimensional Fourier transform of the spatial domain. For a given excitation position $\hat{s}_0$ and measurement direction $\hat{s}$ the component

3. Numerical investigation of SH guided wave tomography

of the scattered field maps to a point at $k_0(\hat{\mathbf{s}}_0 - \hat{\mathbf{s}})$. Given that a circular, full-view array was employed, any directional bias in the maximum wavenumber can be neglected and only the magnitude of the vector may be considered, which dictates the maximum wavenumber that may be obtained and in turn sets the resolution limit. Hence, we can set

$$k_0 |\hat{\mathbf{s}}_0 - \hat{\mathbf{s}}| = k_0 \sqrt{(\hat{\mathbf{s}}_0 - \hat{\mathbf{s}}) \cdot (\hat{\mathbf{s}}_0 - \hat{\mathbf{s}})} \quad (3.4)$$

where $\cdot$ is the standard dot product. It is now possible to transform eq. (3.4) to find

$$k_0 |\hat{\mathbf{s}}_0 - \hat{\mathbf{s}}| = k_0 \sqrt{\hat{\mathbf{s}}_0 \cdot \hat{\mathbf{s}}_0 - 2\hat{\mathbf{s}}\hat{\mathbf{s}}_0 + \hat{\mathbf{s}} \cdot \hat{\mathbf{s}}} \quad (3.5)$$

$$= k_0 \sqrt{2 - 2\cos(\theta)} \quad (3.6)$$

$$= 2k_0 \sqrt{\frac{1 - \cos(\theta)}{2}} \quad (3.7)$$

$$= 2k_0 \sin\left(\frac{\theta}{2}\right) \quad (3.8)$$

where θ is the scattering angle between $\hat{\mathbf{s}}_0$ and $\hat{\mathbf{s}}$. Using the 3 dB envelope criterion, is it possible to determine a maximum scattering angle at each frequency up to which we can expect a reasonable match with the acoustic scattering amplitude. For scattering in the perpendicular direction,

$$\theta = 0 + \frac{\pi}{2}, \quad (3.9)$$

which leads to a maximum wavenumber of

$$2k_0 \cdot \sin(\pi/4) \approx 2k_0 \cdot 0.71 \quad (3.10)$$

$$= 1.42k_0. \quad (3.11)$$

For A0 at 50 kHz, a maximum matching scattering angle of 0.64 radians [15] leads to a maximum wavenumber of

$$2k_0 \sin(0.64/2) = 0.63k_0, \quad (3.12)$$

in turn corresponding to a wavelength of

$$\lambda = \frac{2\pi}{k} \tag{3.13}$$

$$\Rightarrow \lambda_{A0}(50 \text{ kHz}) = \frac{2\pi}{0.63 k_0} \tag{3.14}$$

$$\approx \frac{2\pi}{0.63 \cdot 166.8 \text{ m}^{-1}} \tag{3.15}$$

$$= 59.8 \text{ mm} \tag{3.16}$$

or in terms of resolution θ :

$$\theta_{A0}(50 \text{ kHz}) \approx 1.5 \lambda_{A0}(50 \text{ kHz}). \tag{3.17}$$

This is clearly far below the theoretically achievable resolution of $\lambda/2$ and leads to a significant loss in resolution which explained the poor results of the reconstructions from elastic data in [15].

For SH0, a determination of the through transmission resolution is not of much relevance, as within the through-transmission envelope only between $\pi/3$ and $\pi/2$ components are captured and therefore a huge loss of low spatial frequency components can be expected. This would lead to a significant drop in contrast and given that the majority of the signal is reflected, only the contours of scattering regions should be detectable. As SH0 cannot be used for thickness inversion, this result is noteworthy but not problematic, except that in the case of coexisting SH0 and SH1 signals, higher wavenumber components might overlap with SH1 components, which would require proper modal decomposition of the signals.

It can be shown that SH1 offers the best match of the guided wave modes with the acoustic model. The full range of through-transmission scattering angles may be captured, which leads to a maximum wavenumber of $1.42 k_0$. For the SH1 wavelength at the given frequency, this leads to a maximum resolution θ of

$$\theta_{SH1}(200 \text{ kHz}) = 0.7 \lambda_{SH1}(200 \text{ kHz}) \approx 18 \text{ mm}. \tag{3.18}$$

This would be a significant improvement over A0 at a centre frequency of $f = 50$ kHz and more importantly significantly exceeds the target of $3T$ in plane resolution set by industry.

3.3.4 Summary

This section has investigated the point scattering behaviour of SH0 and SH1 and compared it to that of the fundamental asymmetric Lamb mode, A0. Although the SH guided wave modes both exhibit directional scattering behaviour, it was shown for the dispersive SH1 mode, how the greater range of scattering angle coverage should lead to an improvement in resolution. In the next section these results will be tested by running three-dimensional elastic FE simulations and applying HARBUT to the data.

3.4 SH1 guided wave tomography from pure SH1 point source data

Having shown the theoretical improvement in resolution based on the more favourable scattering pattern of SH1, in this section, several defect types were modelled and investigated. For the first part of this section, defects with a maximum thickness loss of 20% are considered which can be broadly categorised into artificial axisymmetric type defects and more realistic defects which have been sourced from laser scans of real corrosion patches. Next, the most interesting defects were scaled for a maximum thickness loss of 50% which required excitation at a higher frequency. First, a brief description of the models for these plates with defects will be given and it will be explained how they are introduced into the plate model. This is followed by the data processing that was involved until finally, the results are presented.

3.4.1 Model

For excitation, a 5-cycle Hann-shaped pulse was used to create the experimental guided wave data which was excited at 120 transducer positions with a circular array of radius

$R = 200$ mm. For the initial investigation of 20% wall loss, a centre frequency of $f = 250$ kHz was chosen. At this frequency, SH1 has a wavelength of $\lambda = 16.3$ mm and with the cut-off frequency thickness product being at about 1.6 MHz mm, a maximum wall thickness-loss of

$$d_{max}(250 \text{ kHz}) = 10 \text{ mm} - \frac{1.6 \text{ MHz mm}}{250 \text{ kHz}} = 10 \text{ mm} - 6.4 \text{ mm} = 3.6 \text{ mm} \quad (3.19)$$

was expected to be suitable for imaging. However, for these initial simulations the maximum wall loss was set to 20%, which, at 2 mm wall loss, is well within the limit posed by the cut-off frequency thickness product. In order to reach greater depth, a higher frequency is required as has been discussed previously. In order to penetrate 50% wall loss, a centre frequency of 350 kHz was selected, offering a wavelength of $\lambda = 10.1$ mm. For the selected material parameters, this should allow for a depth penetration of up to

$$d_{max}(350 \text{ kHz}) = 10 \text{ mm} - \frac{1.6 \text{ MHz mm}}{350 \text{ kHz}} = 5.3 \text{ mm}. \quad (3.20)$$

The choice of 120 transducers was made using the sampling criterion for a circular array by Simonetti et al. [44]:

$$N \approx \frac{4\pi r_0}{\lambda}, \quad (3.21)$$

where r_0 is the radius of a circle which contains the region of interest with a resolution of half the wavelength. If r_0 is to be equal to the array radius R , the number of required transducers for $\lambda(200 \text{ kHz}) = 16.3$ mm would have to be $N(200 \text{ kHz}) \approx 155$. As the defects considered in this thesis will not exceed a diameter of 300 mm, this region is set to 75% of the array radius R or $r_0 = 150$ mm. This leads to $N \approx 116$ but has been increased to $N = 120$ to be able to move in 3° steps during the experimental stage of the project. Also, for a wavelength of $\lambda(350 \text{ kHz}) = 10.1$ mm the required number of transducers to achieve half the wavelength sampling rate would be $N(350 \text{ kHz}) \approx 155$.

The maximum resolution using HARBUT is set by the Born approximation and is limited to half the wavelength. The wavelength and thus resolution in turn is limited by whatever frequency is chosen. Any complexity of a defect and its finer features will not be captured

if their size is smaller than half the wavelength, even if an infinite number of transducers is used. For the case of a full-view circular array configuration, the features that are bigger than half the wavelength will be captured as long as the defect is within the circle of radius r_0 , where the sampling criterion holds and a resolution of half the wavelength is possible. Therefore, the number of sensors required has to be adjusted so that the full extent of the defect remains within the circle at the centre of the array in order to capture its complexity to the extent that is possible at the set frequency.

3.4.2 Data processing

Owing to the modal purity of the data (see fig. 3.5), no windowing of the signals was necessary and, for every transmit-receive combination, an FFT of the signals was performed in order to obtain the required frequency domain data for HARBUT. This is because any source will produce an arbitrary amplitude change and phase shift (both in simulations and experimentally) calibration of the measurement data is required. It would in theory be possible to measure or calculate these, but in practice it is simpler to scale the measurements based on a known value. Typically, one chooses a measurement whose ray path did not interact with the defect or from the incident field, if available. Since an array of measurements are taken there are multiple transmit receive combinations that may be suitable for calibration purposes. If convergence fails, other such combinations might have to be considered or another section of the pipe might have to be used for preliminary measurements. For the actual calibration, a matrix is calculated by assuming an incident field generated by unit point sources [8], given by the 2D-Green's function

$$G(r) = -\frac{i}{4}H_0^{(1)}(k_0r) \quad (3.22)$$

where r is the distance from the source to the receiver, $H_0^{(1)}$ the Hankel function of the first kind and k_0 the background wavenumber. For every SH1 simulation dataset, a calibration value was generated from a transducer pair that did not interact with the defect.

Within this thesis, the HARBUT algorithm was used starting from a homogeneous background; it was found that with suitable regularisation, convergence was still achieved. The regularisation steps taken were to a) apply a Gaussian filter (with the standard deviation

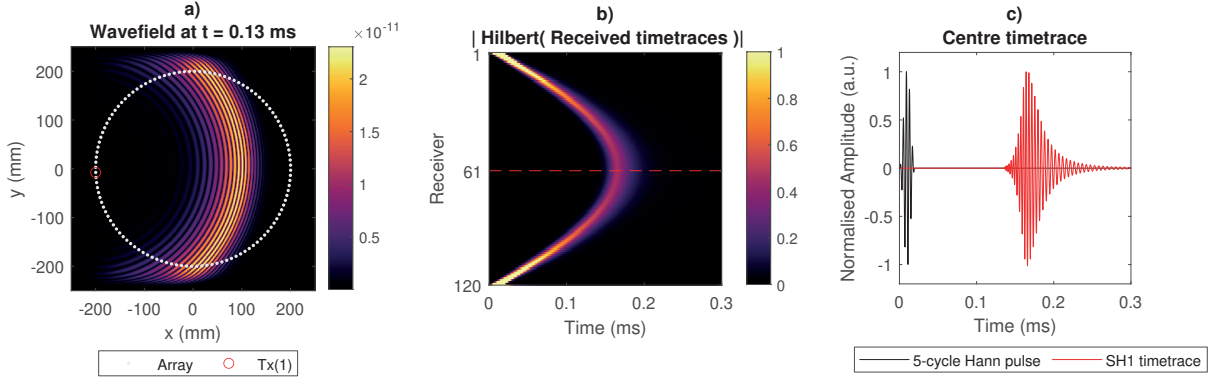


Figure 3.5: Point source transducer signal processing: a) Magnitude of the pure SH1 wavefield at $t = 0.13$ ms after excitation at Tx(1). b) Magnitude of the Hilbert envelope of the received SH1 time traces for the excitation at Tx(1) and reception at all 120 transducers. c) 5-cycle Hann shaped excitation signal and the received pure SH1 signal.

of the Gaussian distribution set to $\sigma = 3$) to the velocity map of the previous step, which served as background for the next one and b) to use an additional positivity constraint, that any value lower than the background velocity would imply an increased thickness which is non-physical and therefore any such values were corrected to the background velocity.

Since only the through-transmission data is used for reconstruction ($\theta < 90^\circ$; which corresponds to the low spatial frequency components [29] discussed in section 2.4.1), we set the limiting spatial frequency components to values corresponding to

$$k_{\text{Lim}} = \left[1.1 \cdot 2 \cdot \sin\left(\frac{\theta}{2}\right) \right] \cdot k_0 \quad (3.23)$$

$$\underset{\theta=90^\circ}{\approx} [1.1 \ 1.4] \cdot k_0, \quad (3.24)$$

where θ is the scattering angle. Applying a step filter, which removes all frequency content above a specific value, will result in ringing artefacts appearing in the image. Therefore we use a sine-shaped taper in the filter, from 1 to 0. The two values given correspond to the start and end of the filter. This upper limit was used for every defect type and the absence of any large ring artefacts suggests that the sampling was sufficient and effects from the incident field low [45].

The convergence criterion Q was based on the average thickness change over the sum of the region of interest relative to the nominal thickness, following the approach in [8],

3. Numerical investigation of SH guided wave tomography

where the parameter Q is defined by

$$Q^{(n)} = \frac{\int |T^{(n)}(\mathbf{x}) - T^{(n-1)}(\mathbf{x})| S(\mathbf{x}) d\mathbf{x}}{T_0 \int S(\mathbf{x}) d\mathbf{x}}. \quad (3.25)$$

In this equation, T_0 is the nominal thickness, $T^{(n)}(\mathbf{x})$ is the thickness at location $\mathbf{x}$ for iteration n and S is a map indicating the extent of the damage area. S is a binary map which was created by setting any reconstructed depth greater than 5% of the background thickness to 1 and everything else to zero:

$$S(\mathbf{x}) = \begin{cases} 1 & \frac{T_0 - T^{(n)}(\mathbf{x})}{T_0} \geq 0.05 \\ 0 & \frac{T_0 - T^{(n)}(\mathbf{x})}{T_0} < 0.05. \end{cases} \quad (3.26)$$

We set the convergence value to $Q \leq 4 \cdot 10^{-3}$ as this led to fewer than 10 iterations required for convergence in most cases. This value was found through trial and error and will have to be revisited for other configurations.

Two error metrics were calculated for all defects by comparing them to the values of the true thickness maps. The global error e is based on the quality of shape assessment used in [15], which is based on a root mean squared (RMS) error:

$$e = \sqrt{\frac{\int [T(\mathbf{x}) - T^*(\mathbf{x})]^2 w(\mathbf{x}) d\mathbf{x}}{T_0^2 \int w(\mathbf{x}) d\mathbf{x}}}, \quad (3.27)$$

where $T(\mathbf{x})$ is the true thickness map, $T^*(\mathbf{x})$ is the reconstructed thickness map and w is a windowing function that biases the error metric towards depth changes towards the centre of the array of radius R :

$$w(\mathbf{x}) = \begin{cases} \frac{1}{2}[1 + \cos(\frac{\pi \mathbf{x}}{R})] & \mathbf{x} < R \\ 0 & \mathbf{x} \geq R. \end{cases} \quad (3.28)$$

Essentially, this biases the global error towards the region where the defect is most prominent and neglects the outer regions closer to the array positions, where no defect is to be expected. It has to be noted that this is an error metric specific to this full view circular array configuration.

The relative error at maximum defect depth was calculated by:

$$e_m = \frac{|T_m - T_m^*|}{T_0}, \quad (3.29)$$

where T_m is the true thickness at maximum depth, T_m^* is the ‘measured’ thickness at maximum depth and T_0 the nominal thickness of the plate.

3.4.3 Results

3.4.3.1 20% wall loss

The reconstruction results for the pure SH1 point source data excited at a centre frequency of 250 kHz are shown in fig. 3.6 and all results are listed in table 3.1. Qualitatively, the geometry and features of the defects were captured extremely well. The two axisymmetric defects stood out for being very accurate reconstructions. At the location of greatest depth, their respective errors were extremely low at $e_m = 0.2\%$ for the Hann-shaped and $e_m = 0.1\%$ relative background thickness for the Tukey-shaped defect. Their global error percentage was also below 1%, which suggests that the shape of the defects was also captured very well.

Table 3.1: Error and performance values for point source finite-element results: The global error e , the error at maximum defect depth e_m , the number of iterations per case and the defect area.

Metric:	e (%)	e_m (%)	Iterations	Defect area (mm²)
<i>Hann</i>	0.2	0.2	5	7.8
<i>Tukey</i>	0.2	0.1	17	21.8
<i>Alpha</i>	0.8	1.7	5	17.3
<i>Bravo</i>	1.0	1.1	5	11.8
<i>Charlie</i>	0.7	2.1	10	34.8
<i>Delta</i>	0.9	1.3	8	22.6
<i>Echo</i>	0.7	0.6	7	18.2

The five realistic defects also showed very good agreement with the true values. They ranged from 1.5% for *Alpha* to 2.1% for *Delta*, which is extremely low. At the maximum depth, the error values were again extremely low, with *Echo* showing the greatest accuracy with only $e_m = 0.6\%$ deviation from the background thickness. Defect *Charlie* showed the

3. Numerical investigation of SH guided wave tomography

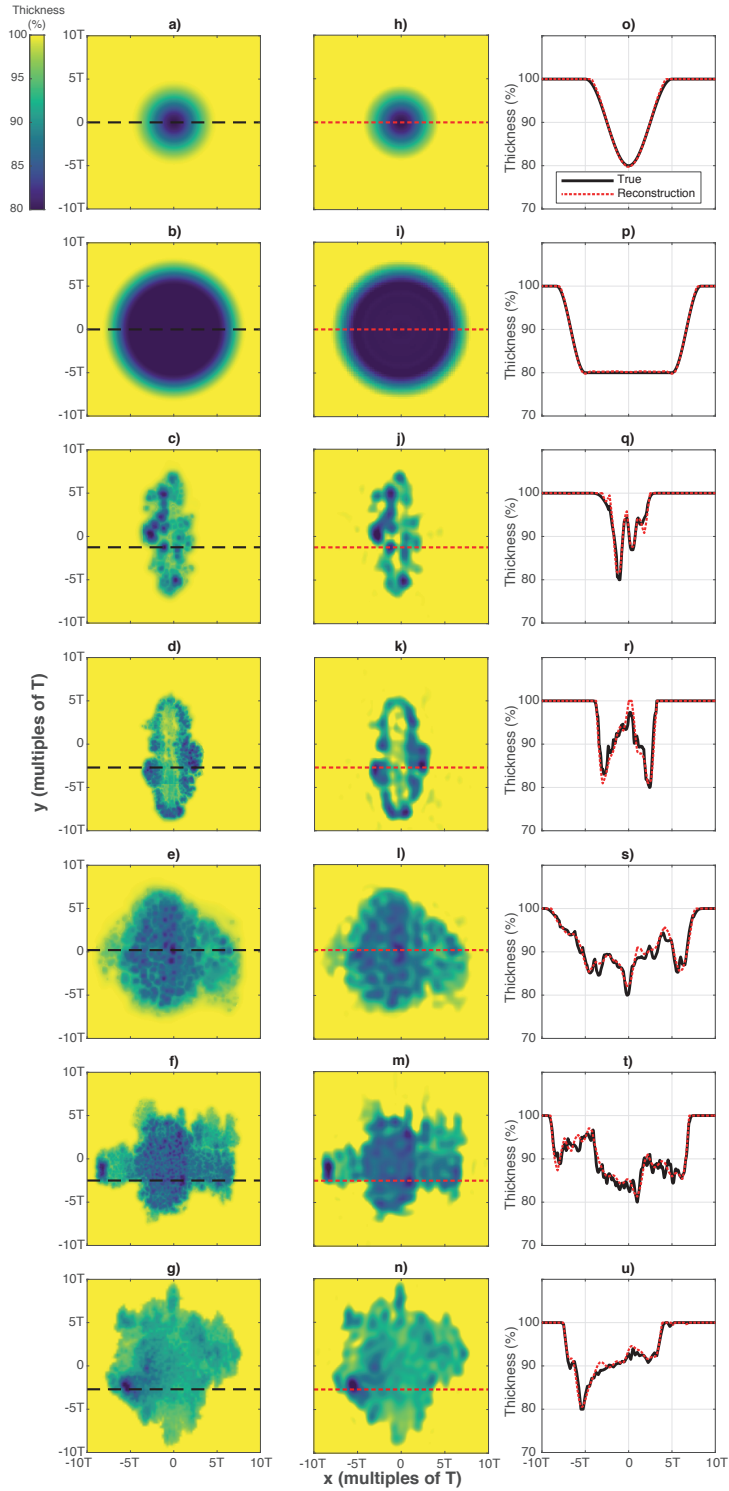


Figure 3.6: HARBUT reconstruction from point source SH transducer FE simulations. (a) - (g) show true thickness maps, (h) - (n) display the reconstructed thickness maps and (o) - (u) show line profile through the deepest points for the true (black) and reconstructed depths (red).

greatest error at 2.1 % which corresponds to a deviation of 0.21 mm, which is importantly within 10 % desired by industry.

Another interesting result is the number of iterations that were required for convergence. The two axisymmetric defects show the lowest and highest number of iterations at 5 (*Hann*) and 17 (*Tukey*). Similar numbers of iterations were observed for the more realistic defects where smaller defects (*Alpha & Bravo*) required fewer iterations than the larger ones. This is likely to correspond to the phase shift of the guided wave passing beneath the defect. Essentially, the increase in phase shift pushes the scattering behaviour further away from the Born approximation, which assumes weak scattering, and therefore more iterations are required to account for this shift. Given that *Tukey* and *Charlie* are among the largest defects, thus increasing phase shift, their higher iteration counts corroborate this assumption as convergence was still achieved although iteration counts were notably higher.

3.4.3.2 50% wall loss

The initial results for 20% wall loss at an excitation frequency of 250 kHz were excellent and provided good reason to investigate defects of greater depth. Therefore, the axisymmetric *Hann* and the realistic *Echo* defect were selected for the 50% wall loss investigation. As before, the defects were linearly scaled to reach 5 mm depth at their points of greatest wall loss. In order to achieve proper depth penetration, a sufficiently high frequency had to be chosen. The choice of 350 kHz was made in light of SH1's maximum depth penetration of 4.8 mm at this frequency. The results are shown in fig. 3.7. The *Hann* shape is captured equally well as for 20% wall loss as is evident from fig. 3.7(b) and (c). Both the global error at $e = 0.2\%$ and the error at maximum $e_m = 0.4\%$ are extremely small. The *Echo* defect is again captured near-perfectly with even finer features such as the smaller pit to the top right of the largest pit being visible in fig. 3.7(e). As is shown in the thickness profile plot in fig. 3.7(f), the geometry of the defect is captured very well. Most importantly, the error at maximum depth is extremely small, at 0.4 % and the global error is 1.6 % which is mainly due to the loss of high resolution components and minimal artefacts around the defect. It is noted that each iteration typically takes around 1 second on an Intel i7 desktop computer; meaning that the algorithm run time is not a

3. Numerical investigation of SH guided wave tomography

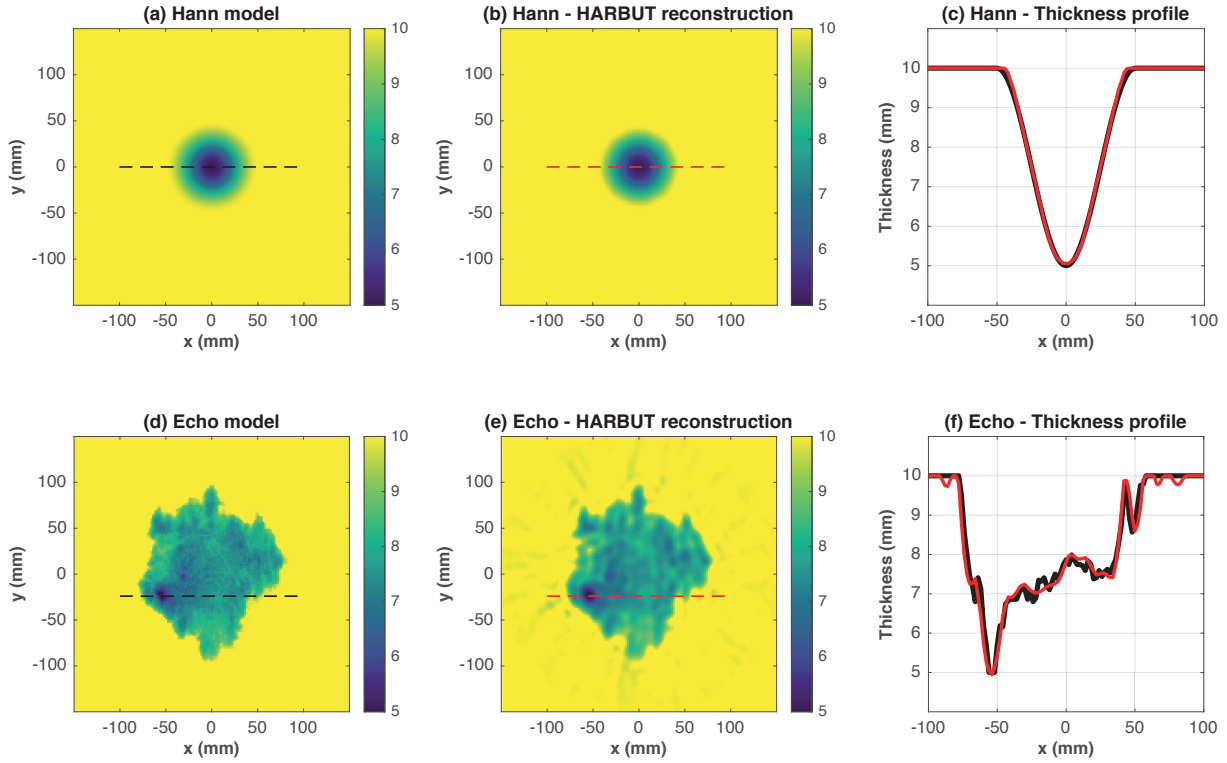


Figure 3.7: HARBUT reconstruction from point source SH1 transducer FE simulations at a centre frequency of 350 kHz. (a) and (d) depict the two investigated defect models, *Hann* & *Echo*. The HARBUT reconstructions from FE data are shown in (b) and (e). As can be seen from the thickness profiles in (c) and (f) both defect shape and thickness were captured with extreme accuracy.

consideration in the majority of cases. This is an important factor for in-field operation as this will allow for rapid data analysis in contrast to full-waveform inversion approaches that require more powerful computational resources and processing time.

3.4.4 Summary

This section showed that for the case of pure SH1 signals, the expected resolution was met and that it exceeds that of A0 guided wave tomography. This is made possible by the superior scattering match of SH1 with the acoustic case, leading to the capture of up to $0.7 \cdot \lambda_{\text{SH1}}$ which at 350 kHz leads to a resolution up to 8.5 times smaller than what was previously possible with A0 at 50 kHz ($0.7 \lambda_{\text{SH1}}(350 \text{ kHz}) = 0.7 \cdot 10.1 \text{ mm} \approx 7 \text{ mm} \Rightarrow \frac{59.6 \text{ mm}}{7 \text{ mm}} \approx 8.5$). The next task will be to increase the modelling realism by exciting a surface-only point source. This will lead to a loss of modal control, which in turn will lead to the excitation of other SH guided wave modes. For lower frequencies, this will be

the non-dispersive SH0 and the first higher order mode SH1, but when the penetration of greater depth is required, higher frequencies and thus higher order SH guided wave modes will have to be dealt with. The following section will lay out the strategies that were employed to deal with this and present the results from surface-only SH point source excitation simulations.

3.5 SH1 guided wave tomography from SH point source data

Having demonstrated the increase in resolution from pure SH1 data, the next step was to investigate how well SH1 data can be extracted from more realistic, full SH mode component signals. This is necessary, as it will be extremely challenging to excite a pure SH1 mode in practice, especially, when there is only access to one surface of a plate or pipe. Therefore, careful analysis of the multi-mode situation is vital.

At frequencies lower than the SH2 cut-off frequency, the main obstacle is the excitation of SH0. In order to penetrate greater depths, however, frequencies will have to be chosen where higher order modes such as SH2 and even SH3 may appear. Fortunately, these will tend to travel at much lower group velocity and be sufficiently separate from the SH1 components at the chosen array radius of this investigation. However, whenever SH0 and SH1 scatter at deeper thickness points, mode conversion will occur, leading to the superposition of SH0, SH1 and mode converted components. In order to deal with these difficulties, the previously employed strategies had to be adjusted and will be explained in the next subsection.

3.5.1 Model

Compared to the previous section's defect model, all parameters are identical except that the point source excitation is only applied to the 9 surface nodes per transducer. The most immediate effect of the surface SH excitation at $f = 250$ kHz compared with the idealised omnidirectional source discussed earlier is the additional excitation and detection of the

3. Numerical investigation of SH guided wave tomography

fundamental SH mode, SH0, which appears within the measured time traces (see fig. 3.8). The next higher order mode, SH2 was not picked up as the excited frequencies were well below SH2 cutoff. The primary signal processing problem to solve was the isolation of SH1 from the SH0 + SH1 time traces. For the case of excitation at $f = 350$ kHz, two problems arise. First, SH1 is less dispersive and travels at more similar velocity to SH0 leading to an overlap of the tail of SH0 and the front of SH1 components. Second, the next higher order mode SH2 is excited but as it is extremely dispersive and much slower at close to its cut-off frequency, the previously employed time-gating approaches should be applicable.

3.5.2 Data processing

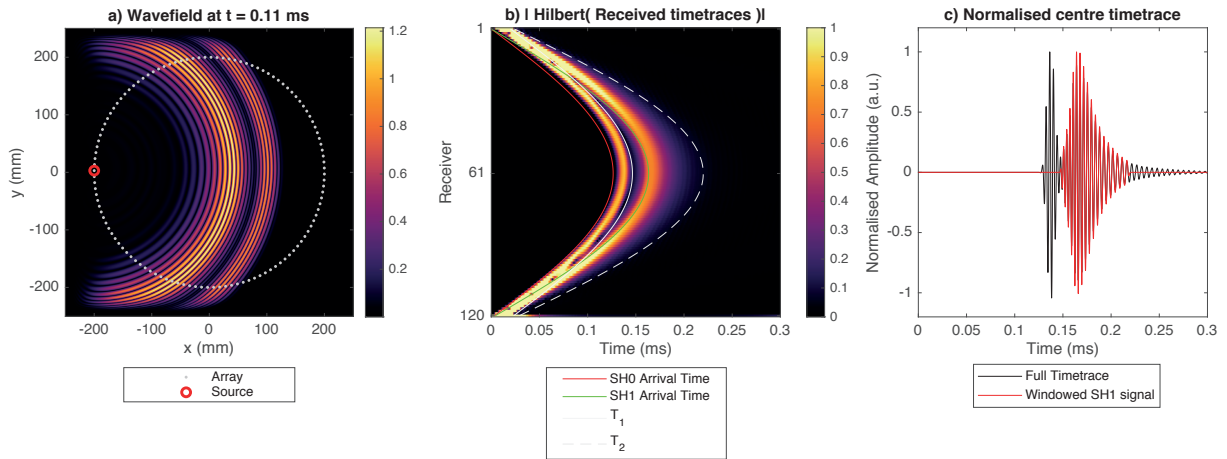


Figure 3.8: Example of full SH point source excitation of a 5-cycle Hann shaped pulse at 250 kHz: a) Magnitude of displacement wavefield at $t = 0.11$ ms after excitation of SH0 in addition to SH1 when modal decoupling has occurred. b) Magnitude of the Hilbert envelope of the received time traces and the respective arrival times for of SH0 (red), SH1 (green) and the calculated SH1 window boundary times: t_1 (light grey) and t_2 (light grey dashed). c) Full centre time trace (black) and Tukey windowed SH1 component (red).

Indeed the removal of unwanted modes was achieved by applying a simple adaptive windowing technique that relied on knowing the distance between transducers as well as the group velocities (see fig. 3.8(b) & (c)). The latter were taken from the dispersion curves calculated using Disperse [46,47], which made the calculation of theoretical arrival times straightforward. For the case of exciting at 250 kHz these matched very well for the non-dispersive SH0 mode but overestimated the arrival time of SH1, which is primarily caused

by the wavepacket dispersing and hence part of it arriving earlier. A separation point between the two modes was taken to be the mean of the two arrival times. The length of the SH1 signal component was calculated by multiplying the calculated SH1 arrival time by 1.5 which was found to capture the extent of the useful SH1 data sufficiently, accounting for the dispersion. These two signal SH1 window boundary times are defined as

$$t_1(250 \text{ kHz}) = \frac{l_n}{c_{g,\text{SH0}}} + \frac{1}{2} \cdot \left(\frac{l_n}{c_{g,\text{SH1}}} - \frac{l_n}{c_{g,\text{SH0}}} \right)$$

$$t_2(250 \text{ kHz}) = 1.5 \cdot \frac{l_n}{c_{g,\text{SH1}}},$$

where $c_{g,\text{SH1/SH0}}$ are the group velocities of the fundamental mode SH0 and the first higher order mode SH1 and l_n the distances between every transducer pair. Using these two arrival times, the signals were windowed using a Tukey window (see fig. 3.8c) defined by the following equation:

$$h(t) = \begin{cases} 0 & 0 < t < t_1 \\ \frac{1}{2} \left[1 + \cos \left(\frac{\pi(t-t_1)}{\alpha(t_2-t_1)} \right) \right] & t_1 \leq t < t_1 + \alpha(t_2 - t_1) \\ 1 & t_1 + \alpha(t_2 - t_1) \leq t < t_2 - \alpha(t_2 - t_1) \\ \frac{1}{2} \left[1 + \cos \left(\frac{\pi(t-t_2)}{\alpha(t_2-t_1)} \right) \right] & t_2 - \alpha(t_2 - t_1) \leq t < t_2 \\ 0 & t_2 \leq t \end{cases} \quad (3.30)$$

where the length of the window was defined by t which is a linear-spaced vector with length $t_2 - t_1$. Setting $\alpha = 0.1$ led to an almost rectangular Tukey window with a steep slope, therefore not altering the bulk of the signal as would have been the case with e.g. a Hann window. For the higher excitation frequency of 350 kHz, the SH1 window boundary times were chosen differently. This was due to SH1 travelling at higher velocity and being less dispersive. Thus the SH1 wave-packet's front and the tail of SH0 components were overlapping even at maximum travelling distance. This could be readily solved by increasing the transducer array radius which would allow for proper signal separation. However, there could be situations where such an increase would not be feasible and the task was set to investigate whether a proper window could be found that allowed

3. Numerical investigation of SH guided wave tomography

for sufficiently pure signals. Fortunately, because of the lower dispersion of the SH1 components, the theoretical SH1 arrival time was more representative of all frequencies in the wavepacket than was the case for 250 kHz and could be chosen as the first window boundary time. The closing window boundary time had to be chosen to avoid capturing any SH2 components as well as sufficiently capturing those SH1 components that had been significantly slowed down by the greater defect depth. Thus, the initial excitation pulse duration was multiplied by 1.15, leading to the following window

$$t_1(350 \text{ kHz}) = \frac{l_n}{c_{g,\text{SH1}}}$$
$$t_2(350 \text{ kHz}) = 1.15 \cdot \left(\frac{l_n}{c_{g,\text{SH1}}} + \Delta T_{\text{SH1}} \right),$$

where ΔT_{SH1} is the pulse duration of the 5-cycle Hann-shaped excitation pulse.

Further processing was carried out as described in section 3.4.2. The only difference being that instead of using through-thickness excitation, only 9 nodes at the plate surface are selected for weighted excitation. As can be seen in the wavefield in fig. 3.8(a), multiple modal components are excited when using the directional transducers. The Hilbert envelopes of the received signals shown in fig. 3.8(b) highlight how the chosen transducer model only receives SH0 and SH1 modes, which was desired. Moreover, due to the difference in group velocity and the non-dispersive nature of SH0, the two modes separate out well at the through-transmission region between receivers #30 and #90 (see fig. 3.8(b)). This allowed for a straightforward separation method using a Tukey window. This type of window is a tapered cosine function, allowing for relatively sharp boundaries while retaining most of the desired signal components (see fig. 3.8(c)).

3.5.3 Results

The result for 50 % wall loss is shown in fig. 3.9. Owing to the proper windowing of the signals, the SH1 components at centre frequency could be isolated. This made the accurate HARBUT reconstruction shown in fig. 3.9(b) possible, which converged after 37 iterations. Apart from some minor artefacts around the defect, the overall shape was captured very well and the two largest pits can be identified easily. The thickness profile

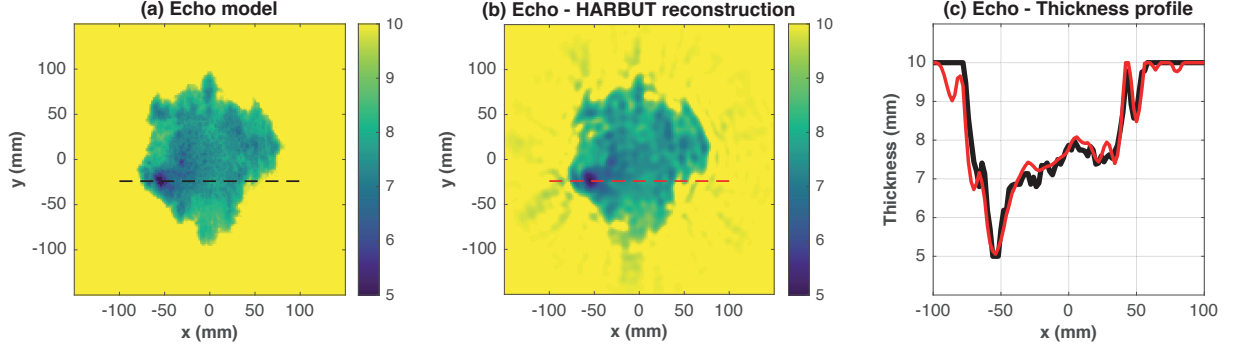


Figure 3.9: HARBUT reconstruction from point source SH transducer data of FE simulations at a centre frequency of 350 kHz. (a) depicts the investigated defect model, *Echo*. The HARBUT reconstruction from SH point source FE data is shown in (b). As can be seen from the thickness profiles in (c), both defect shape and thickness were captured accurately.

through the point of maximum depth (red) plotted in fig. 3.9(c) against the true thickness profile (black) showed that not only the depth was accurately recovered but also that the overall geometry was followed nicely. The error at maximum wall loss was thus very low at $e_m = 0.5\%$ and the global error slightly larger at $e = 1.3\%$.

3.5.4 Summary

The result shown in this section showed how the employed time-based mode-windowing was successfully employed to enable extremely good HARBUT reconstructions even at up to 50% wall loss from full SH mode component signals. It was already mentioned that pure SH1 excitation is extremely challenging and omnidirectional point source excitation is only slightly simpler. More realistic excitation uses directional SH transducers and the next section will present the results from investigation into this type of approach.

3.6 SH1 guided wave tomography from directional SH source data

In a practical physical setup, pure omnidirectional SH1 transducers are difficult to implement without simultaneous access to both sides of the plate or the inside of a pipeline, which is unfeasible. So far, highly accurate reconstructions were produced from pure SH1 point source data as well as from full SH point sources where SH1 components were win-

3. Numerical investigation of SH guided wave tomography

dowed. Both of these setups are idealised. In an experimental setup, directional SH transducers are more commonly used. Next, the modelling approach of a directional SH transducer is explained and the reconstruction results from this type of data presented.

3.6.1 Model

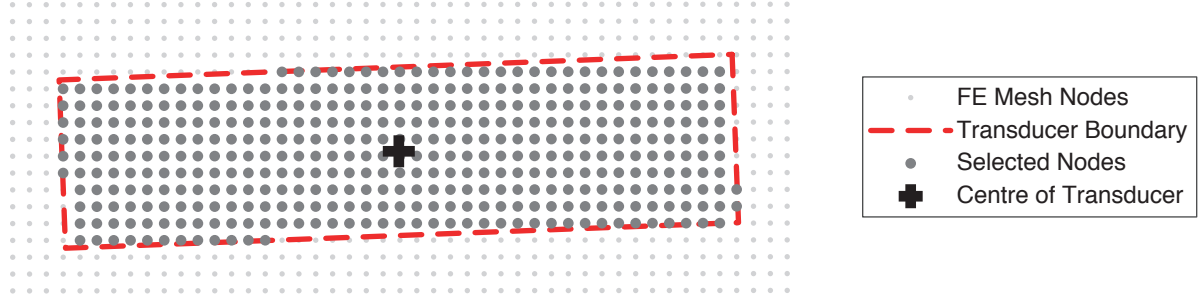


Figure 3.10: Directional transducer (width $w = 5$ mm, length $l = 20$ mm) modelling approach.

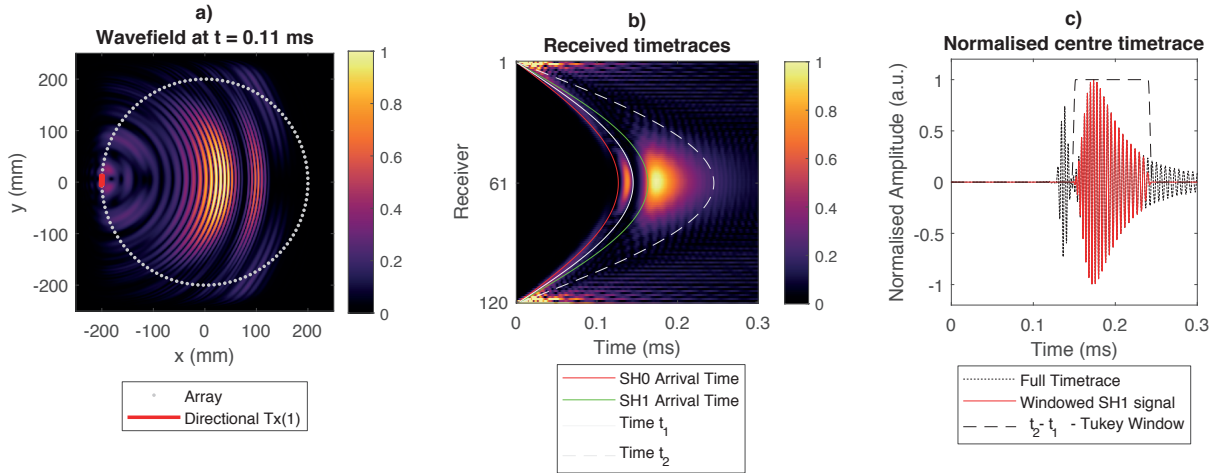


Figure 3.11: Directional transducer signal processing: a) Magnitude of displacement wavefield at $t = 0.11$ ms after excitation of SH0 in addition to SH1 due to directional excitation, measured on the top surface of the plate. Modal decoupling has occurred. b) Magnitude of the Hilbert envelope of the received time traces and the respective arrival times for of SH0 (red), SH1 (green) and the calculated SH1 window boundary times: t_1 (light grey) and t_2 (light grey dashed). c) Full centre time trace (black) and Tukey window based on t_1 and t_2 (black dashed) and windowed SH1 component (red).

The excitation of SH waves can be achieved with both EMATs and shear piezoelectric transducers, and in general, the function of these are to excite a force parallel to the surface in a particular region. Following this, the SH transducer was modelled as a

simple rectangular shape with a width of 5 mm and a length of 20 mm (see fig. 3.10). These parameters were found to be a good compromise between a wide beam spread to ensure the whole of the defect lies inside the transducer wavefield, and directionality to ensure that sufficient energy is transmitted into the medium. The transducer model is made in this way to approximately represent the behaviour and dimensions of the true transducer to be used in experiments. In theory, it would be possible to model the resulting forces arising from a full electromagnetic model of the EMAT, but this is seen as unnecessary given the intention is to demonstrate the general performance with a more realistic, directional transducer rather than focusing on the specific example.

A rectangular mask was calculated for every array transducer and the nodes within the rectangular area were selected. In order to generate a homogeneous shearing traction on the surface, all nodes within one transducer area received the same forcing, weighted between the two in-plane degrees of freedom in order to excite the force parallel to the longer dimension of the transducer. The forcing applied in the x and y directions for each transducer is given as

$$\begin{aligned} x_n &= \sin(\theta_n) \\ y_n &= -\cos(\theta_n), \end{aligned}$$

where n is the number of the transducer and θ the angular coordinate of the transducer. Due to the use of a structured mesh, transducers that align perfectly with the mesh (e.g. 0° , 90° , 180° , etc.) held more nodes than those at an angle which resulted in uneven signal amplitudes. In order to reduce this effect, a bias angle of 2.14° was added to the angular coordinate θ , which helped to even out the number of nodes within all transducers.

3.6.2 Results

3.6.2.1 20% wall loss

The reconstructions from windowed SH1 data using directional transducers are shown in fig. 3.12 and the results are listed in table 3.2. The same reconstruction parameters were used as in the reconstructions from point source data ($k_{\text{Lim}} = [1.1, 1.4]$, $\sigma = 3$,

3. Numerical investigation of SH guided wave tomography

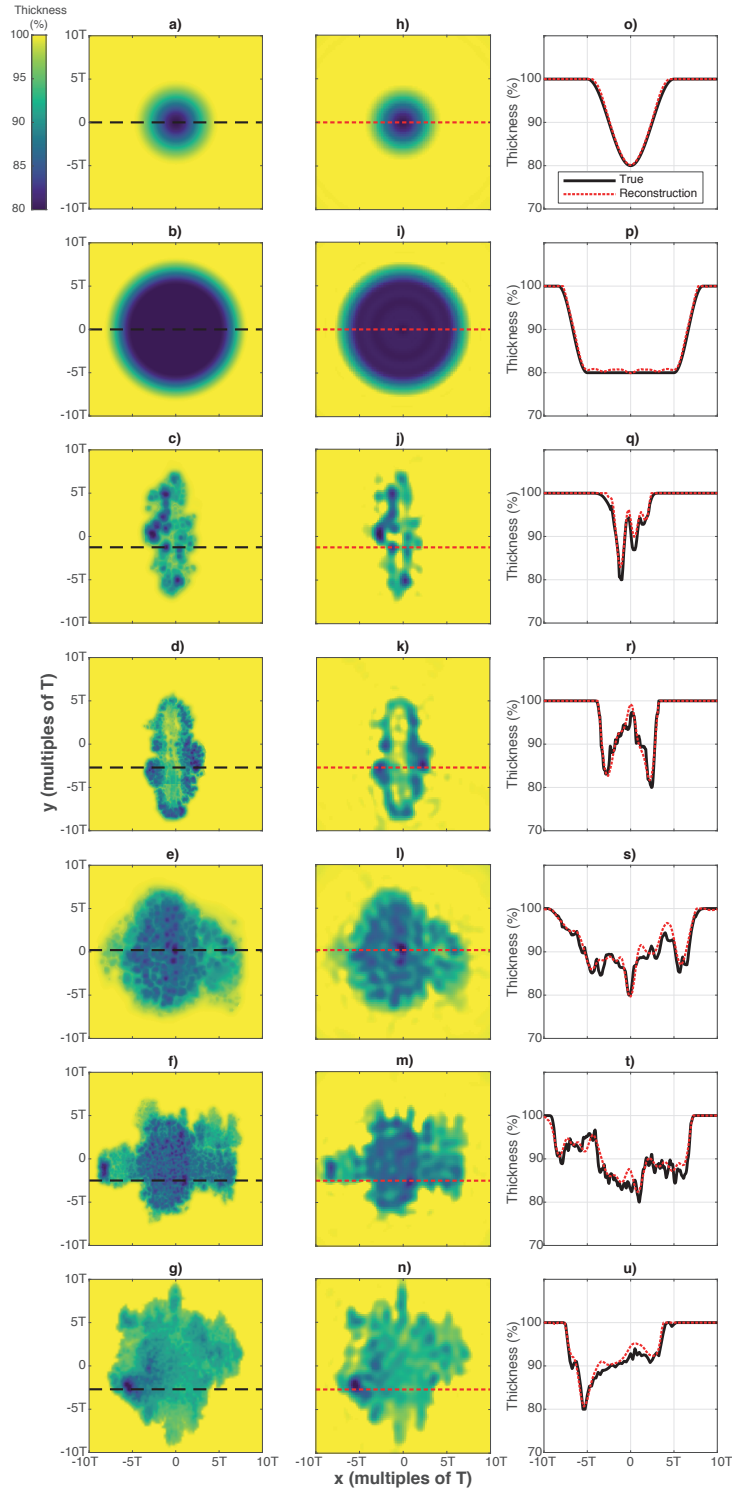


Figure 3.12: HARBUT reconstruction from directional SH transducer FE simulations. (a) - (g) show true thickness maps, (h) - (n) display the reconstructed thickness maps and (o) - (u) show line profile through the deepest points for the true (black) and reconstructed depths (grey).

$Q_{\text{Lim}} = 0.004$). The defect shape and details are reconstructed extremely accurately, with the axisymmetric defects achieving the best values and lowest error values across all metrics. The geometry of the defects is reconstructed very well and the overall error values are well below 2% for both the global error and the error at minimum thickness. The number of iterations for convergence increased as well which can be explained by the added ring artefacts due to the use of the directional surface transducer model (see for example fig. 3.12(j,k)). It seems that in these cases the chosen reconstruction limits might be slightly too high, leading to Gibbs ringing artefacts [29]. Nevertheless, they are consistent with the number of iterations required for the ideal point source dataset.

Table 3.2: Error and performance values for directional source finite-element results: The global error e , the error at maximum defect depth e_m , the number of iterations per case and the defect area.

Metric:	e (%)	e_m (%)	Iterations	Defect area (mm²)
<i>Hann</i>	0.3	0.2	5	7.8
<i>Tukey</i>	0.5	0.1	19	21.8
<i>Alpha</i>	1.0	3.0	4	17.3
<i>Bravo</i>	1.0	2.5	5	11.8
<i>Charlie</i>	0.9	0.1	9	34.8
<i>Delta</i>	1.2	2.7	7	22.6
<i>Echo</i>	1.2	0.6	5	18.2

3.6.2.2 50% wall loss

As stated in the previous sections when using SH1 point source and SH point source data, greater depth penetration required an increase of operating frequency. As this leads to a smaller wavelength, the number of transducers required to sample the full Ewald limiting circle increases. However, it was shown how, even for a sub-sampled array, data from pure SH1 and full SH point source data produced highly accurate reconstructions and in order to allow for better comparison the array parameters were kept constant, with a circular array of $N = 120$ transducers.

The *Echo* case was investigated and the reconstruction and thickness profile can be seen in fig. 3.13. Reconstruction proved challenging as the larger level of phase change in addition to the effect of directional transducers led to non-convergence of HARBUT. This was remedied by adding another level of regularisation, setting all values to the background

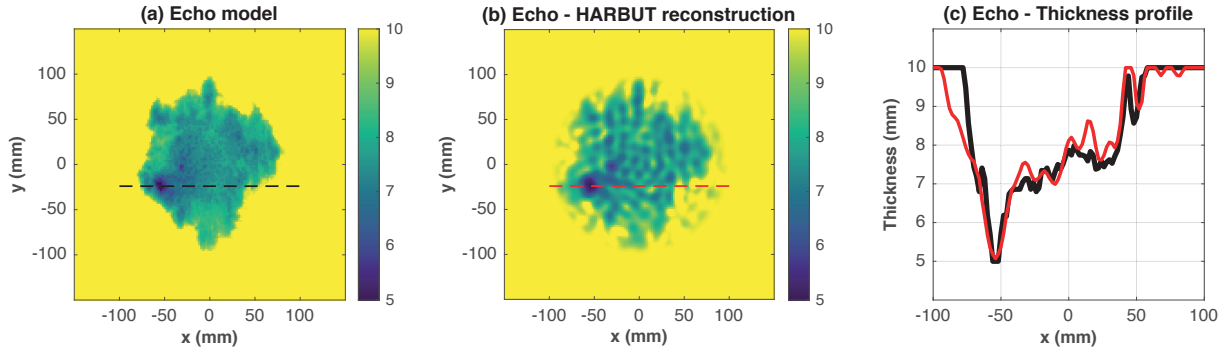


Figure 3.13: HARBUT reconstruction of a 50% maximum wall loss defect from directional SH transducer FE data. (a) shows the input model, (b) is the HARBUT reconstruction after 26 iterations and (c) shows the thickness profile of the reconstruction (red) compared to the input model (black).

velocity which were outside of a circle at the origin with a radius of $r_0 = 100$ mm. Adding this constraint led to convergence after 26 iterations. The shape of the overall defect was captured as well as the two deeper pits. The error at maximum depth was very low at $e_m = 0.7\%$. The global error on the other hand was slightly higher at $e = 1.8\%$ which is of course biased by the regularisation that removed smaller artefacts in the non-defective regions of the model. Given that industry is mostly interested in knowing the minimum remnant wall thickness, larger errors at higher remnant thickness locations are tolerable as long as the minimum remnant wall thickness measured is highly accurate as is the case here. Thus, the result showed that even under adverse conditions, defects of greater depth should be detected with high accuracy.

3.6.3 Summary

As has been stated previously, any directionality of the scattering pattern leads to a loss of captured scattering angles and therefore to a loss in resolution. Nevertheless, sufficiently sampled arrays yield good results and confirm the improvement in resolution seen in the previous cases. Given the high fidelity of the FE simulations, similar results can be expected from experimental data which will be investigated in the next chapter. The next section, however, will present results of a performance analysis between A0, S0 and SH1 reconstructions of axisymmetric defects of various widths and fixed depth.

3.7 Performance Analysis

Having demonstrated the concept with complex defects from numerical data, the performance of SH1 compared with the two main modes considered previously, A0 and S0, will be investigated. For these purposes, the focus will be set on simple axisymmetric defects of the same type as the Tukey defect used previously.

3.7.1 Model

During these simulations, A0 was excited at 50 kHz on the 10 mm thick plate, S0 was excited at 175 kHz and SH1 was excited at 250 kHz. These operation points are the ones commonly identified for guided wave tomography, recognising practical constraints [23]. At these operating points, the wavelength was $3.8T$ for A0, $2.8T$ for S0 and $1.6T$ for SH1. In all cases, point sources were used and simulations performed using FE, as discussed in section 3.2. The data was post-processed in the same way and images produced via the HARBUT algorithm. The filter was set to $[1.1, 1.4]$ for all reconstructions.

3.7.2 Results

Firstly, the performance of SH1 compared with the A0 and S0 modes is evaluated. Tukey-type defects of width $10T$ down to $1T$ and $0.1T$ depth are used for this. Figure 3.14 shows the line plots of the true (grey) thickness profiles and of the reconstructions of A0, S0 and SH1. From these, it is clear that the A0 mode performs the worst through all the widths, indicating that it has the lowest resolution. SH1 appears to have the best resolution, particularly capturing the shape for width $1T$ and maintaining a good accuracy as the width increases. There is a clear ‘overshoot’ visible for both S0 and SH1 at $1.5 - 2T$ for SH1 and $2.5 - 3T$ for S0. This is very likely to be associated with the change in effective density of the plate causing additional scattering to the guided wave, which is not accounted for in this thesis, and is most significant for small defects of the scales considered here. As explained in [35], the effective density is assumed to be proportional to the mass per unit area and therefore proportional to thickness if there are no material density changes, and this only plays a role whenever there are sharp boundaries. This is

3. Numerical investigation of SH guided wave tomography

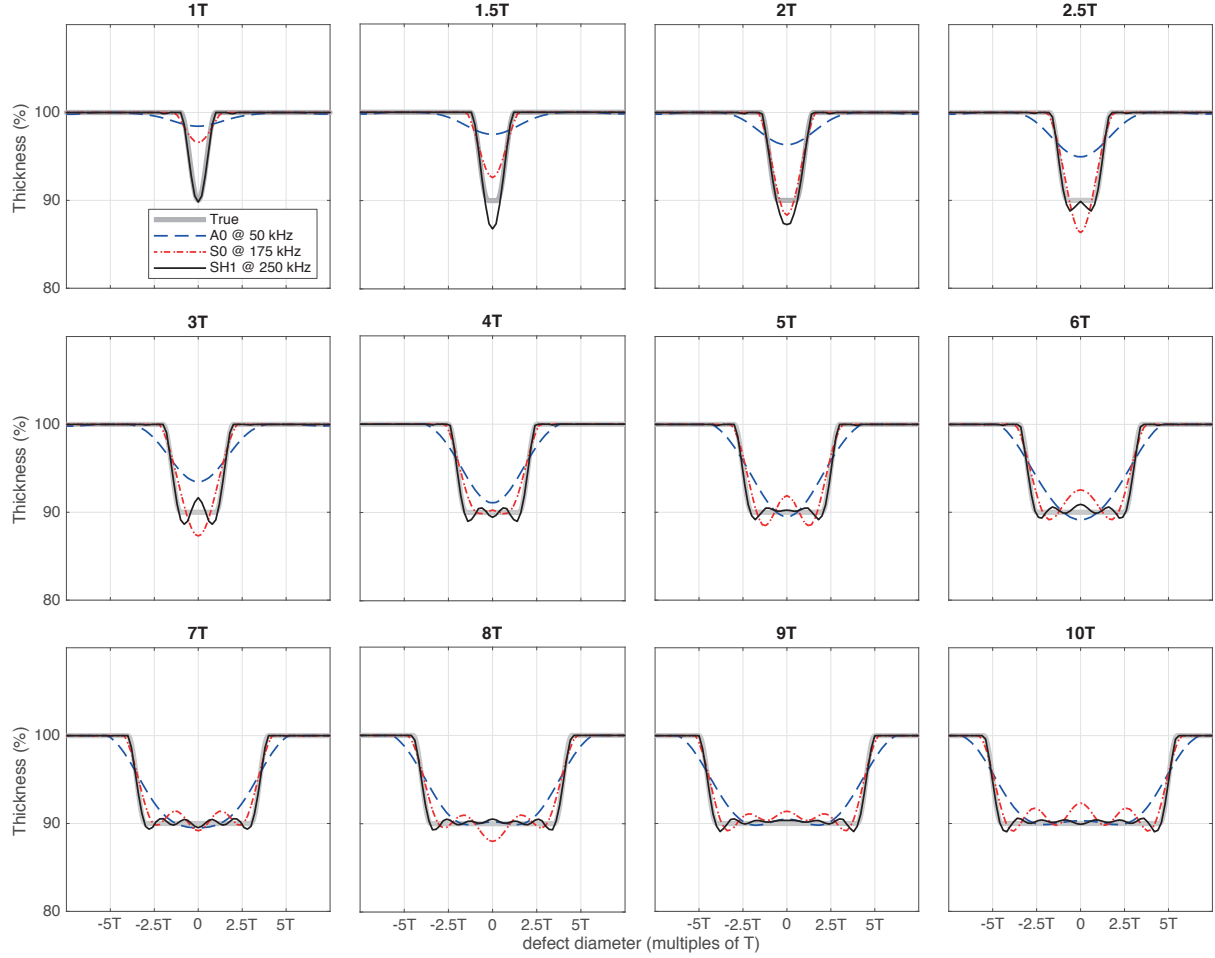


Figure 3.14: Comparison of HARBUT reconstructions using Lamb wave modes A0, S0 and shear-horizontal modes SH0 and SH1 for Tukey-type defects with widths ranging from 1T to 10T, where T is the nominal plate thickness.

demonstrated for both A0 and S0 modes, and it is reasonable to expect that other modes, including SH1, exhibit this too.

For the smallest defects, the S0 reconstruction is resolution-limited so it underestimates the wall loss, an effect noted in [48]. As the defect becomes larger, resolution is less of an issue so the wall loss estimate increases, but at one point the mode overestimates the value because of the density effect discussed above. As the defect width increases further, the density effect becomes less significant compared with the velocity contribution, and the overshoot reduces. This same pattern is visible with SH1, although the transitions occur at smaller defect widths. The overshoot of SH1 is small at widths of $2.5T$ and above, indicating that SH1 is reliable for reconstructing all defects in this range and hence demonstrating that the $3T$ requirement can be met. It is noted that the performance

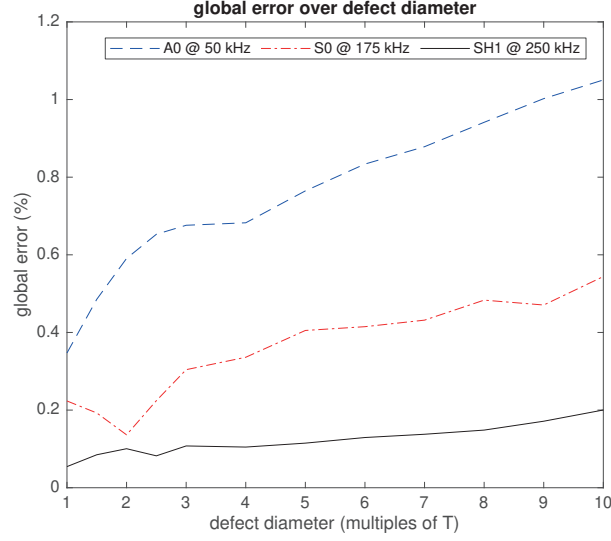


Figure 3.15: Global error plot of the Tukey defect diameter sweep for A0 (dashed), S0 (dot-dashed) and SH1 (solid).

below $2.5T$ is still good. In contrast, S0 requires defects of width $4T$ and above to minimise the overshoot.

Moreover, errors at fringes are much smaller in the case of SH1 than for the other modes, suggesting that SH1 will lead to superior defect sizing performance as well.

While these images give a good qualitative overview, they are not quantitative. Figure 3.15 compares the global error (see eq. (3.27)) across all defects for the three modes; this error indicates the overall behaviour of the modes and avoids sensitivity to overshoots. As can be seen, SH1 at 250 kHz outperforms A0 at 50 kHz and S0 at 175 kHz for all investigated defect diameters.

3.7.3 Summary

In this section the two fundamental Lamb wave modes A0 and S0 were compared with the SH1 mode at the typical excitation frequencies when performing guided wave tomography. The results show how SH1 outperforms the two Lamb wave modes at the individual frequencies and across all investigated defect diameters.

3.8 SH0 guided wave tomography

The SH0 mode is non-dispersive and thus not suitable for thickness inversion but its analogous pipe mode T(0,1) is the fundamental tool for long range guided wave testing [49]. Although it is to be expected that the interaction of SH0 with a defect will be relatively weak in through-transmission, it is nevertheless of interest to identify what kind of reconstructions are possible, as the shape of the defect and possibly its finer features might be encoded in through-transmission signals. This is caused by the small wavelength even at 150 kHz which is $\lambda_{\text{SH0}}(150 \text{ kHz}) = 21 \text{ mm}$. Unfortunately, the non-dispersive nature of SH0 does not allow for thickness to be extracted from a velocity map. Therefore, the thickness-dependent scattering behaviour can be explained by the change in relative density that can be observed at any boundary changes such as the defect edges and whenever the thickness gradient is high [35]. The resulting map could provide valuable information about the extent and shape of the defect and identify pits and other finer details that might be hidden in the reconstructions from SH1 with larger wavelengths. If these pits were not captured in the SH1 reconstructions, they could hint at deeper and smaller pits and add to inspection evaluation.

3.8.1 Model

The through-thickness SH point source excitation allowed for pure SH0 signals to be received by fixing the weighted degrees of freedom of all nine nodes in every layer of the FE model. Thus, even at excitation frequencies that are above the cut-off frequency of higher order SH modes, pure SH0 signals may be received. The modelling of the plate is identical to the SH1 *Echo* case with a maximum wall loss of 50% (see fig. 3.16(a)). Here, an excitation frequency of 350 kHz was chosen which leads to a wavelength of 9 mm for the SH0 mode. Matching previous models, the number of transducers is kept at $N = 120$, which leads to a gap of 10.5 mm between the transducers.

3.8.2 Data processing

Data processing was performed matching previous sections. Owing to the modal purity of the received SH0 signals, no windowing scheme was required. During the final reconstruction steps, the final thickness-inversion steps were neglected, as velocity remains constant over thickness and only one iteration was performed.

3.8.3 Results

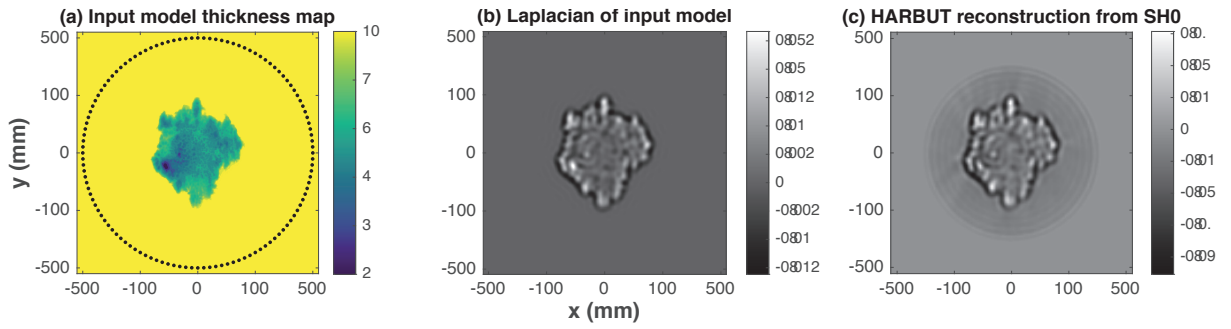


Figure 3.16: HARBUT reconstruction of pure SH0 guided wave tomography data. (a) shows the previously used *Echo* input model, (b) shows the Laplacian of the k-Space filtered input model and (c) shows the HARBUT reconstruction.

The HARBUT reconstruction from SH0 point source data is shown in fig. 3.16(c). The defect shape and dimensions are captured very well, as well as details such as the location of deepest pit in the lower left corner as well as the minor pit to the top right of the largest pit. The SH0 transmission data appears to capture the scattering at areas with a sudden thickness change, which corresponds to sudden changes in effective density.

As discussed, the SH0 phase velocity is constant and thus HARBUT reconstructs the density term (as a velocity) which forms a Laplacian, i.e. the second derivative. When Fourier transforming this operator, it introduces a weighting proportional to k^2 - effectively removing the constant component and minimising lower wavenumber values, and amplifying the higher wavenumbers. This is demonstrated in fig. 3.16(b), where the Laplacian was calculated for the thickness map. This is a qualitative result but the clear matching with the HARBUT velocity reconstruction indicates that it is an effective density effect that was captured in this reconstruction.

3.8.4 Summary

The theoretical predictions were confirmed for the case of SH0 tomography. The SH0 reconstructions offer valuable information about defect complexity and appear similar to the high frequency component filtered reconstruction seen in fig. 2.6 in section 2.4.3. The high resolution reconstructions of the complex defects showed that SH0 scattering captures sudden thickness changes at the boundary of the defect as well as at major pits. While this does offer certain insights, it also leads to the generation of higher order modes through mode conversion. Unfortunately, these will interfere with the desired SH1 scattering information. Nevertheless, proper windowing and modal separation should allow for the compensation of these effects. The next section will conclude this chapter, summarising the main results.

3.9 Summary

This chapter is an example for the high fidelity of state of the art FE simulation capability. The good agreement with the resolution prediction of the scattering analysis of SH1 and the latter achieved results highlight the potential of SH1 based guided wave tomography. The robustness of this approach was demonstrated by increasing simulation realism gradually from the ideal SH1 point source over full SH modal point sources up to the case closest to real world application: directional SH excitation. In all of these scenarios, SH1 outperformed previous results. This was confirmed by the performance analysis, where the established modes A0 and S0 were compared with SH1. Finally, pure SH0 data was investigated and it was shown that although it does not offer an inversion mechanism using the dispersion curves, effective density changes at defect boundaries and sudden thickness changes such as pits are captured in the SH0 through-transmission signals which may be used to reconstruct the dimensions of the defect and highlight the location of deeper corrosion pits. The following chapter will present the results from an experimental verification study of SH1 guided wave tomography.

Chapter 4

Experimental verification of SH1 guided wave tomography

4.1 Introduction

In this chapter, the findings from the numerical study were experimentally validated. In the first section, the set up of the experiment is described, which involves steel plates with machined defects and commercially available SH transducers. This is followed by a brief description of the signal processing steps of the experimentally collected data. Next, the results from the various experiments are discussed. Finally, a summary of the chapter will be provided.

4.2 Experiment setup

For the validation experiment, several mild steel plates ($1000\text{ mm} \times 1000\text{ mm} \times 10\text{ mm}$) were acquired and synthetic defects were machined into them. Two types of defects were manufactured with a maximum thickness loss of 20 %: an axisymmetric defect (*Fox-trot*) and the complex corrosion patch *Echo* considered for the numerical simulations (see fig. 4.1). For this, 2D thickness maps were generated in MATLAB and imported into plate models in SolidWorks[®] which were used as templates during metal milling. The spatial resolution of the models sent for machining was limited to 2.5 mm, which is the

4. Experimental verification of SH1 guided wave tomography

tip size of the metal milling tool. In order to assess proper machining, epoxy moulds of the defects were taken and a tyre tread depth gauge was used to confirm thickness at the deepest points as the whole machined plates could not be placed in a laser scanner and no laser camera was available at the time of the experiments.

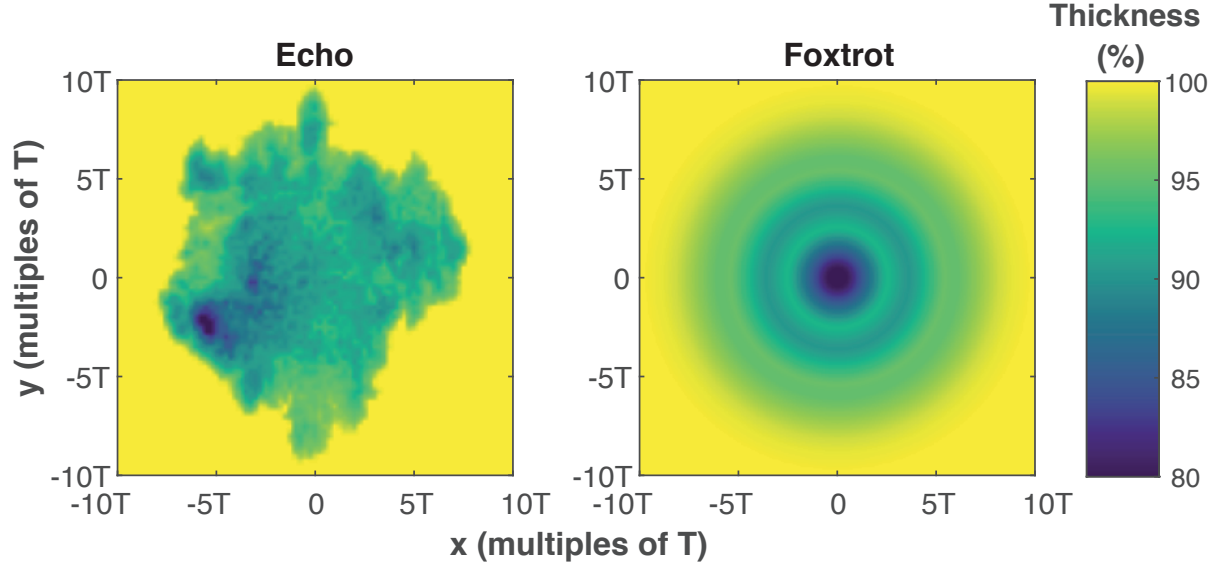


Figure 4.1: The thickness map templates which were machined into mild steel plates and used in experiments.

In order to guarantee repeatable accurate positioning of the transducers, a plywood rig was designed (see fig. 4.2). The rig consisted of a plywood frame that was attached securely to the steel plates via datum holes in the four corners of the steel plates. A ring with 120 positioning holes and identifier labels was fastened to the frame and served as a rail for two wooden sleds which were attached to the two transducers. The sleds fit the curvature of the inner and outer diameter of the ring and two steel rods were used to fix the positions of the sleds on the ring and therefore the attached transducers.

For signal excitation and reception, a modified QSR1[®] system by Guided Ultrasonics Ltd. was employed [50]. The use of SH1 relies upon directional EMAT transduction, with a racetrack coil applied to the specimen in conjunction with alternating permanent magnets [51]. This is a key distinction from the A0 mode, which was readily excited with point transducers [6, 52]. The time-traces were captured with an in-house acquisition software and saved individually for every excite and receive component.

For the axisymmetric defect, data acquisition from one transmission and 51 receiving

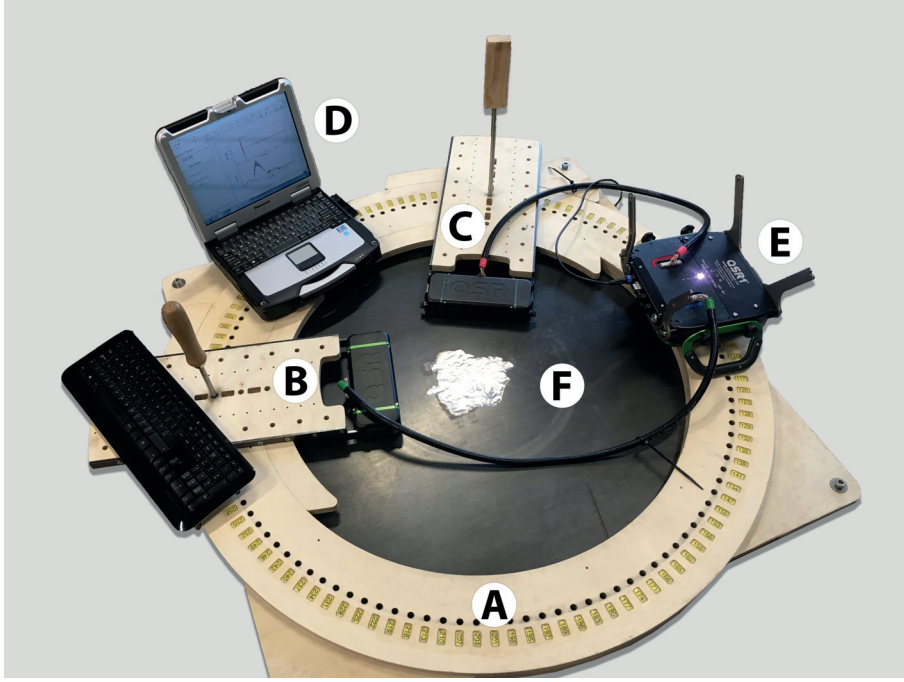


Figure 4.2: Experiment setup: (A) custom plywood rig with 120 array positions, (B) & (C) QSR1[®] transducers attached to wooden sleds, (D) Laptop computer for data acquisition, (E) QSR1[®] base unit, (F) mild-steel plate with machined defects (shown here: *Echo*).

points was sufficient, since a full dataset can be produced by replicating those measurements and exploiting the symmetry of the defect. This was not possible for the rough defect and therefore 120 transmission and 51 receiving positions led to a total of 6120 single measurements. In all measurements a 5-cycle Hann-shaped pulse was excited and 16 averages per acquisition were taken to minimise incoherent noise. This number of averages was chosen as this was the point from which no further increase in SNR was observed. Data was acquired using two electromagnetic acoustic transducers (EMAT) of the QSR1[®] system and recorded on a laptop.

Throughout this thesis, a full view configuration is used, with sources and receivers placed 360 degrees around the scatterer which may not be practical in many scenarios. The intention is to demonstrate the capability of the method. However, it should be expected that the limitations of other setups are aligned with the common issues seen with limited view velocity reconstructions, namely artefacts and resolution loss [53,54].

Parts of this chapter have been adapted and extracted from publication [P1].

4.3 Data processing

One of the most notable differences between experiment and simulation data is the presence of noise. Owing to signal averaging, the time traces were already low in incoherent noise. Thus, the experiment data was processed in an analogous manner to the directional FE data, except for the application of a frequency filter to eliminate most of the high-frequency noise. For this, a Tukey window was placed in the frequency spectrum of every time trace, filtering out any signals below 150 kHz and above 400 kHz, which helped with determining the correct SH0 arrival time as this discarded most of the incoherent noise and led to smoother Hilbert envelopes of the signals (see for example fig. 4.3).

Since the exact values for the elastic modulus and density of the experiment plates were unknown, the dispersion curve had to be calculated using the analytical equation for SH phase velocity (eq. (2.27)). For the case of $n = 0$, the shear velocity equals the phase velocity of SH0 so extracting this information is all that is needed to enable the calculation of the dispersion curves. The arrival times of SH0 could be determined by identifying the arrival of the maximum of the Hilbert envelope of SH0 for every transducer arriving at its opposite receiver (e.g. 0° and 180° , 3° and 183° etc.), then used to estimate the velocity since the distance between the transducers was known. From this velocity, theoretical arrival times were calculated which were used to calculate SH1 windows using the scheme developed during numerical investigation. The windowed SH1 components were then Fourier transformed and information at the frequency for every captured send-receive combination used to fill a complex amplitude input matrix for HARBUT. The next section will present the results gained from experiments.

4.4 Results

4.4.1 Defect-Free plate

For initial testing purposes, a defect-free mild steel plate was purchased and a set of time traces acquired. The magnitude of the Hilbert enveloped raw time traces can be seen in fig. 4.3(a) for the case of excitation with a 5-cycle Hann shaped pulse with a centre

frequency of 250 kHz at position 000° and reception from positions 090° to 270° .¹ From this and the plot of the centre-time trace in fig. 4.3(b), the low amount of noise can be seen as well as the separation of the SH0 and SH1 components of the initial pulse. Moreover, the most prominent effect of a real plate is visible, as at $t \approx 0.31\text{ms}$, the two back-wall reflections of the non-dispersive SH0 mode are received at the same time.

For example, if excitation occurs at coordinate $[x, y] = [-0.2, 0]$, the pulse will propagate to the left (towards the edge of the plate) and the right (towards the centre of the array), where it will be reflected at the boundaries of the plate. The left-travelling pulse will be reflected by the left boundary of the plate, reflected and return to the excitation position after propagating for 0.6 m of distance and continue to propagate through the defect to arrive at the receiver at the same time as the reflection of the first transmitted pulse, that was first received at $t \approx 0.12\text{ms}$. The first arriving pulse, after travelling through the defect, will continue to propagate and be reflected from the other back-wall of the plate and, since the array is symmetric, arrive at the same time.

Selecting the maxima of the Hilbert envelopes to be the arrival times yields an arrival time of $t_{\text{SH0}} = 0.122\text{ ms}$ for the initial SH0 pulse and $t_{\text{SH0_BW}} = 0.309\text{ ms}$ for the arrival of the backwall reflections. This signal is a useful tool to determine the bulk shear velocity of the plate which is required to calculate the dispersion curves. Given that the distance from the receiver to the back-wall is 0.3 m, the total distance travelled will be 0.6 m and this can be used to calculate the bulk shear velocity c_{Sh} as

$$c_{\text{Sh},B} = \frac{0.6\text{ m}}{t_{\text{SH0_BW}} - t_{\text{SH0}}} = 3212\text{ m/s}, \quad (4.1)$$

where $t_{\text{SH0_BW}}$ is the arrival time of the maximum of the Hilbert envelope of the back-wall reflected SH0 signal and t_{SH0} the arrival time of the maximum of the Hilbert envelope of the first arrival of SH0. This can now be used to confirm the velocity taken from the first arrival of SH0 and dividing this from the array diameter

$$c_{\text{Sh},A} = \frac{0.4\text{ m}}{0.122\text{ ms}} \approx 3280\text{ m/s}. \quad (4.2)$$

¹The horizontal bar at $t = 0\text{ ms}$ in fig. 4.3(a) is the excitation signal that was picked up by the receiver due to crosstalk.

4. Experimental verification of SH1 guided wave tomography

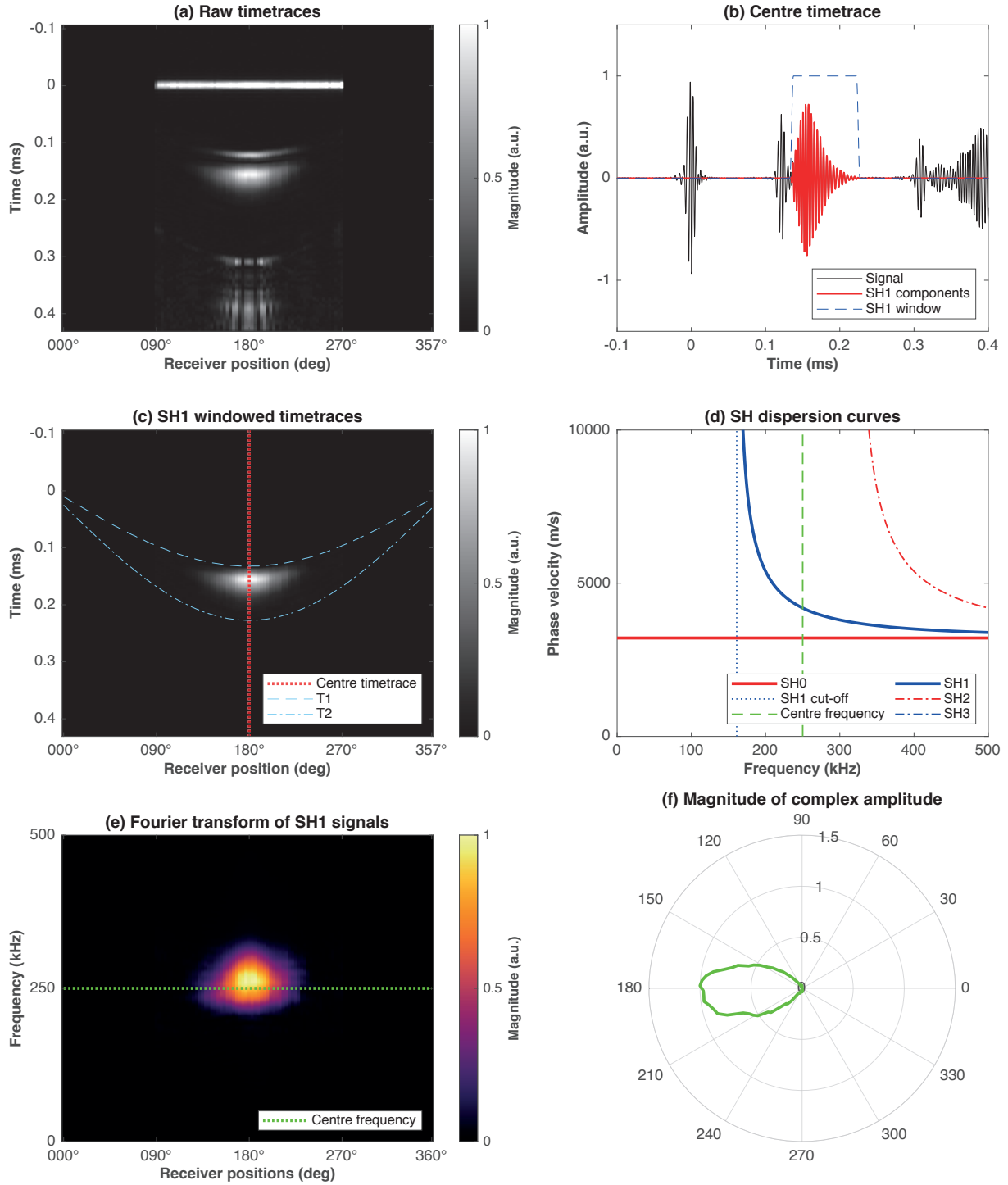


Figure 4.3: Experiment data acquired from excitation at a centre frequency of 250 kHz: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the border times (dashed blue) and the theoretical SH1 arrival time (blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), the magnitude of the Fourier transform at the frequency is plotted in a polar diagram.

The slight discrepancy of the velocities can be due to multiple reasons. Firstly, it is unclear how the measurement system set $t = 0$. Thus, it was deemed more accurate to use the time difference between the first arrival and the back-wall reflection for determining the bulk shear velocity required for the calculation of the dispersion curves shown in fig. 4.3(d). As in the previous chapter, the theoretical arrival times were calculated and used to window the SH1 components (shown in fig. 4.3(c)) which were Fourier transformed (fig. 4.3(e)) in order to find the complex amplitude at the frequency (green line). The directional nature of the transducer is emphasised in fig. 4.3(f) by plotting the magnitude of the complex amplitude at the frequency in a polar plot. In order to penetrate greater depths, a higher frequency was required and, based on the dispersion curves calculated using the bulk shear velocity, a centre frequency of 325 kHz should be sufficient and the results from this are presented in fig. 4.4. In hindsight, a more conservative selection would have been 350 kHz. As discussed previously, the bandwidth of the pulse will very likely overlap with the cut-off frequency of SH2 and indeed, as can be seen in fig. 4.4(b) in the range $t = 0.2 - 0.3$ ms, SH2 components were received. Again, the SH0 back-wall reflections were picked up and using the two techniques, the bulk shear velocity was acquired:

$$c_{\text{Sh},A} \approx 3255 \text{ m/s}, \quad (4.3)$$

$$c_{\text{Sh},B} \approx 3233 \text{ m/s}. \quad (4.4)$$

Here, the discrepancy is much smaller and their values lie between the ones found during the 250 kHz excitation. While $c_{\text{Sh},B}$ was used for the calculation of the dispersion curves, $c_{\text{Sh},A}$ was more useful for calculation of the SH1 windows. The pulse-duration can be estimated by

$$\Delta t_{\text{pulse}} \approx n_{\text{cyc}} \cdot \frac{1}{f_{\text{centre}}}, \quad (4.5)$$

where n_{cyc} is the number of pulse cycles and f_{centre} is the centre frequency of the pulse. By subtracting half of the pulse duration from the selected arrival time, the start of the SH1 window was found and for the end of the window, a multiplication factor of 1.15 for a centre frequency of 250 kHz and 1.25 for 325 kHz was applied. These were determined by observing the data and estimating suitable values. As can be seen in fig. 4.3(b)

4. Experimental verification of SH1 guided wave tomography

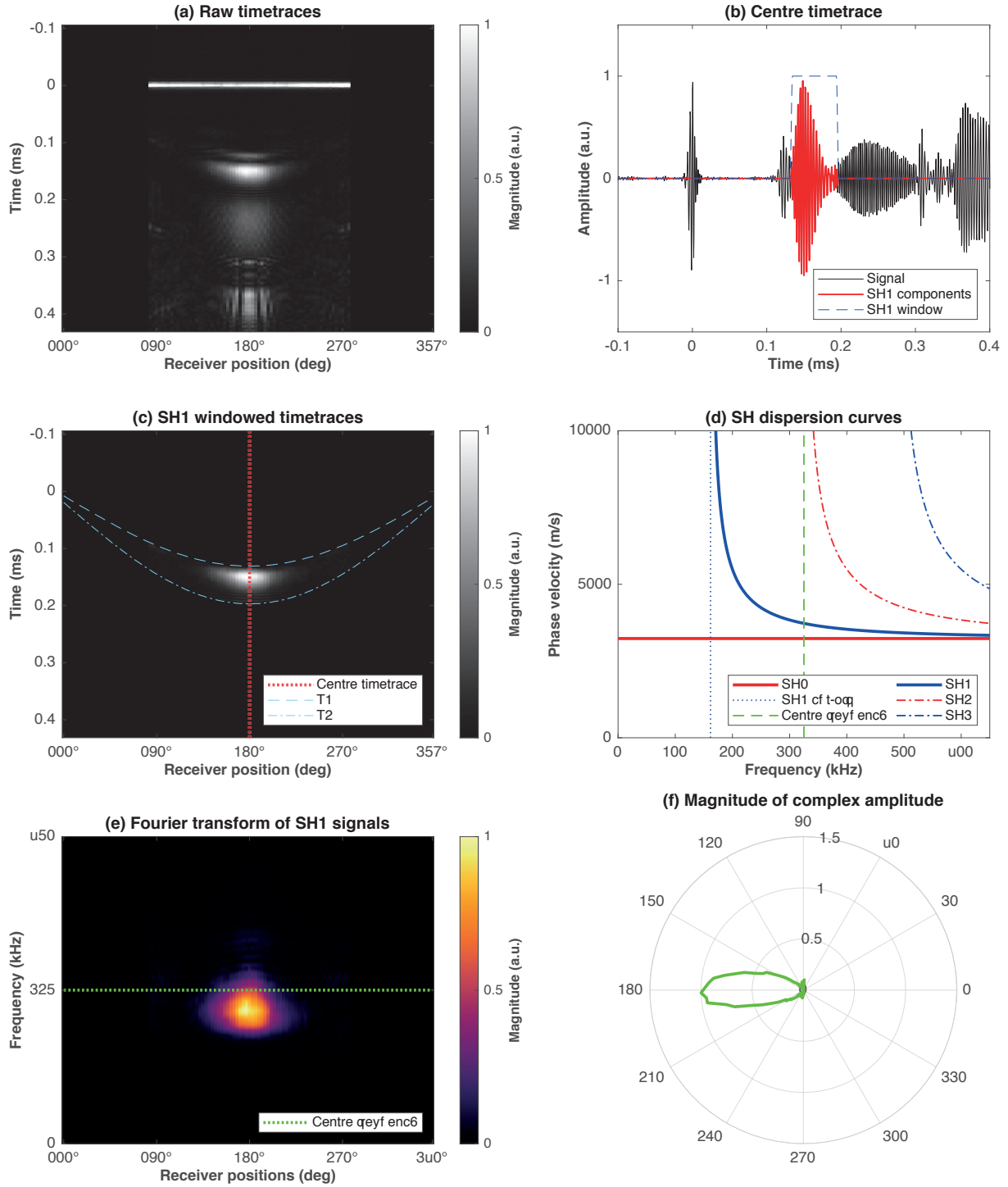


Figure 4.4: Experiment data acquired from excitation at a centre frequency of 325 kHz: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the border times (dashed blue) and the theoretical SH1 arrival time (blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), the magnitude of the Fourier transform at the frequency is plotted in a polar diagram.

and fig. 4.4(b), the window envelope times were well suited to discard SH0 and SH2 signals components as well as any later arriving back-wall reflected signal components. As has been observed in the previous chapters, at higher frequencies, the SH0 tail and SH1 first arriving components tended to overlap. Thus, a certain amount of the SH1 pulse was inevitably discarded. It can also be observed that the modal conversion could play a more prominent role at higher frequencies due to the overlapping arrivals of the SH1 waves. Next, results from data acquired from plates with machined defects will be discussed, starting with the case of an axisymmetric defect.

4.4.2 Axisymmetric defect

Having established the experimental data processing steps on the defect-free plate, a mild steel plate with an axisymmetric defect was investigated next. The magnitude of the Hilbert enveloped raw time-traces can be seen in fig. 4.5(a). The first arrival of the SH0 pulse's peak maximum was found at $t = 0.122$ ms leading to a velocity of $c_{Sh_A} = 3270$ m/s. From this, the SH1 window was calculated and are shown as light blue dashed lines in fig. 4.5(c). The backwall reflection at about $t = 0.3$ ms was used to calculate the bulk shear velocity $c_{Sh_B} = 3192$ m/s from which the dispersion curves were determined (see fig. 4.5(d)). The HARBUT input data matrix was created from the Fourier transformation of the signals, shown in fig. 4.5(e); the complex amplitudes were extracted from this at the centre frequency. These complex amplitudes were used to populate an input matrix for HARBUT and have been plotted in fig. 4.5(f). Owing to the symmetry of the defect, a synthetic full set of data could be created by shifting and repeating the time-traces. As can be seen, there is an asymmetry in the magnitude of the complex amplitude, which could be a material-dependent effect. As mentioned, this complex matrix was input into the iterative HARBUT algorithm with k_{Lim} set to $[1.3, 1.4]$ and $\sigma = 3$ and the reconstruction converged after 4 iterations.

The experimental results are summarised in fig. 4.6. The template sent for machining is plotted as reference in fig. 4.6(a) and the reconstruction from experimental data based on fig. 4.5(f) is plotted in fig. 4.6(b). Most notably, the maximum depth was captured very well with the value being within 0.14% of the true thickness. From fig. 4.6(c)

4. Experimental verification of SH1 guided wave tomography

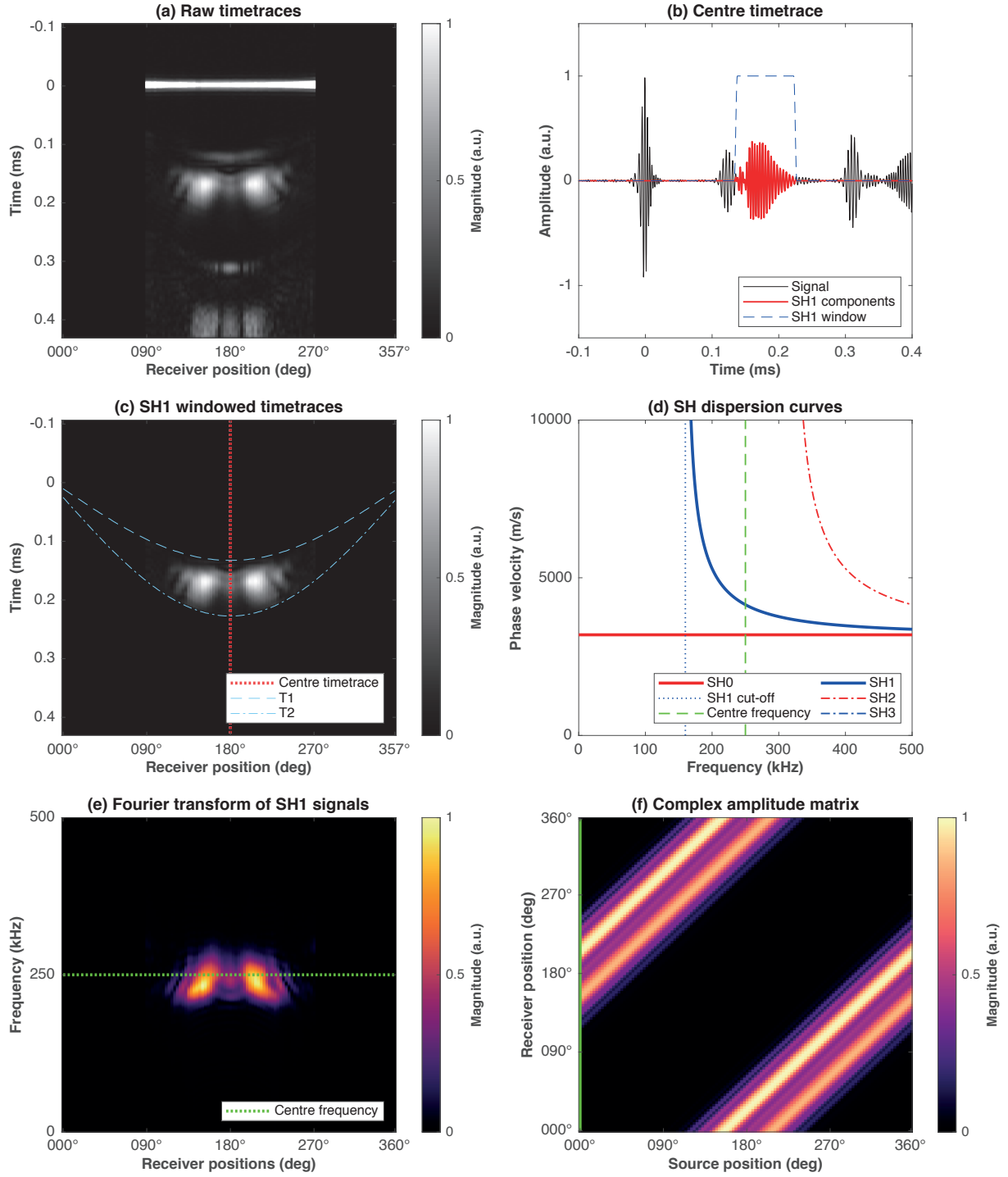


Figure 4.5: Experiment data acquired from excitation at a centre frequency of 250 kHz for the axisymmetric defect: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the border times (dashed blue) and the theoretical SH1 arrival time (blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), the magnitude of the Fourier transform at the frequency is plotted in a polar diagram.

it can be seen that the depth profile changed at the correct points as in the model. Quantitatively, however, the depth values are incorrect in the region between $\pm 2.5T$ and $\pm 9T$. At the centre point of greatest depth however, the thickness change was captured near perfectly. This could be due to potential discrepancies between the model that was sent for machining and the real defect. Furthermore, as shown by Belanger [6] and Huthwaite [15], reconstructions from a single set of received data that has been formed into a full array from axisymmetric data is prone to exaggerate any errors present within that data set. In contrast, utilising a full data set will average out more of the incoherent errors. Nevertheless, the global error value of 1.7% is again very low. The next step of

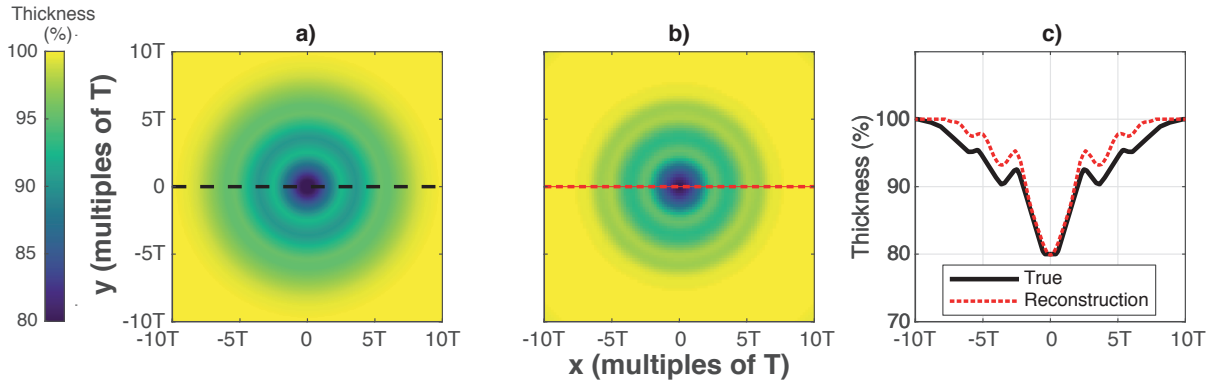


Figure 4.6: HARBUT reconstruction of the axisymmetric machined defect from Experiment data at 250 kHz: (a) shows the true thickness map, (b) displays the reconstructed thickness map and (c) shows the line profiles through the deepest point for the true (black) and reconstructed depths (gray).

this investigation was to analyse data from a defect based on the *Echo* template used during the numerical investigation.

4.4.3 Realistic defect - 20% wall loss

The next experiment was conducted on a mild steel plate which saw defect *Echo* from the previous chapter machined into it. The resolution of the model was 2.5 mm/pixel due to the limitations of the machining tool. This time, measurements had to be performed at 120 excitation positions. For each excitation position, signals were received at 51 positions, leading to a total number of 6120 measurements. The time-traces for the case of exciting at position 000° are shown in fig. 4.7(a). Similar to before, the first arrival of SH0 can be observed at $t \approx 0.12$ ms, leading to a measured bulk shear velocity of

4. Experimental verification of SH1 guided wave tomography

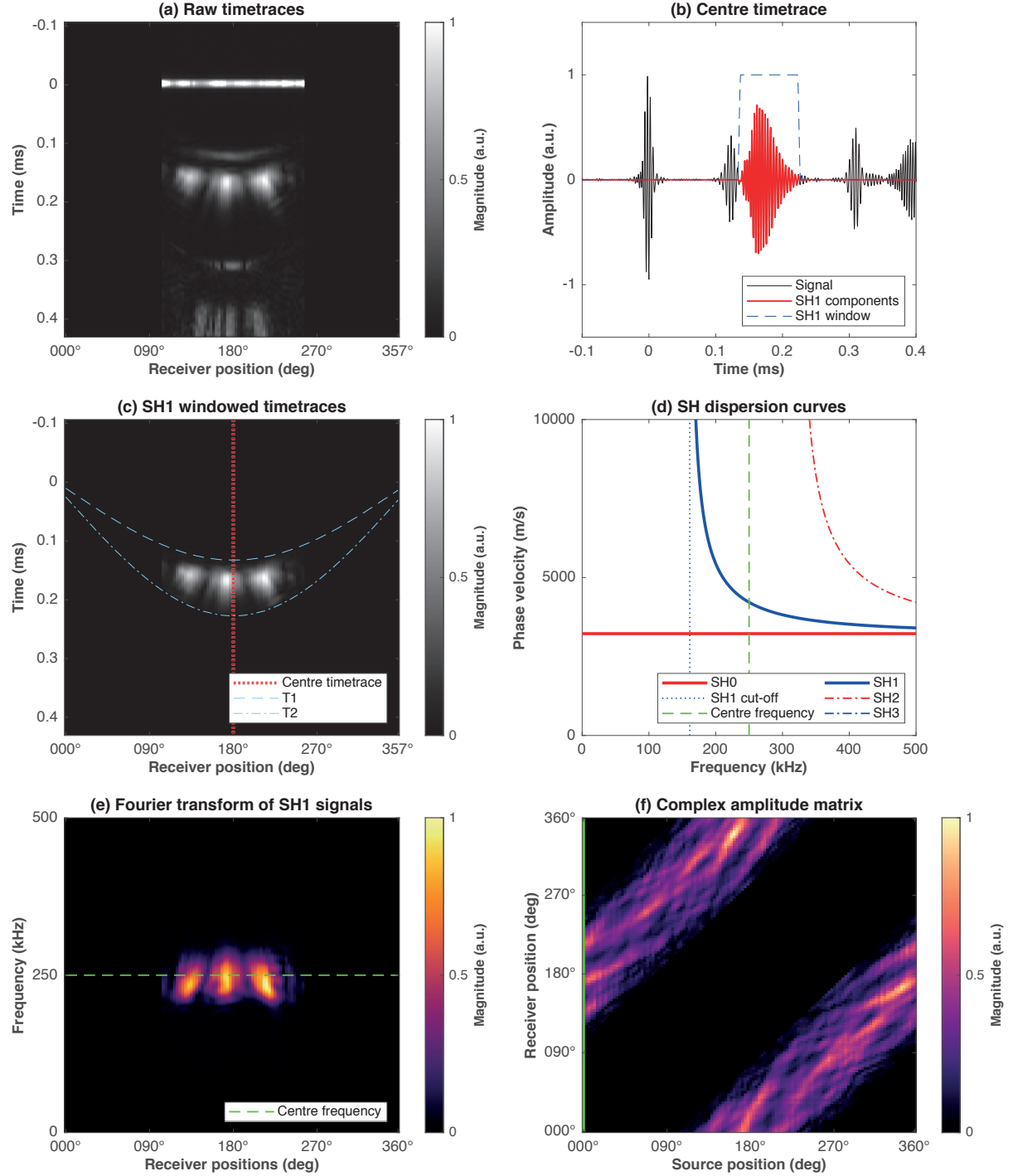


Figure 4.7: Experiment data acquired from excitation at a centre frequency of 250 kHz for the realistic defect at 20% maximum wall loss: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the first arrival times T1 (dashed light blue), the theoretical SH1 arrival times T2 (dashed dotted light blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), this data is used to populate a complex amplitude matrix.

$c_{Sh_A} \approx 3272 \text{ m/s}$, which was used for the windowing of the signals shown in fig. 4.7(d). The bulk shear velocity for the dispersion curves shown in fig. 4.7(d) was determined from the four excitation and reception position pairs $000^\circ - 180^\circ$, $090^\circ - 270^\circ$, $180^\circ - 000^\circ$ and $270^\circ - 090^\circ$. Here, the distance travelled between the first arriving SH0 pulses and the SH0 back-wall reflections can be assumed to be 0.6 m. Thus, the bulk shear velocity was calculated as the mean of the four velocities:

$$c_{Sh_B} \approx 3222 \text{ m/s}. \quad (4.6)$$

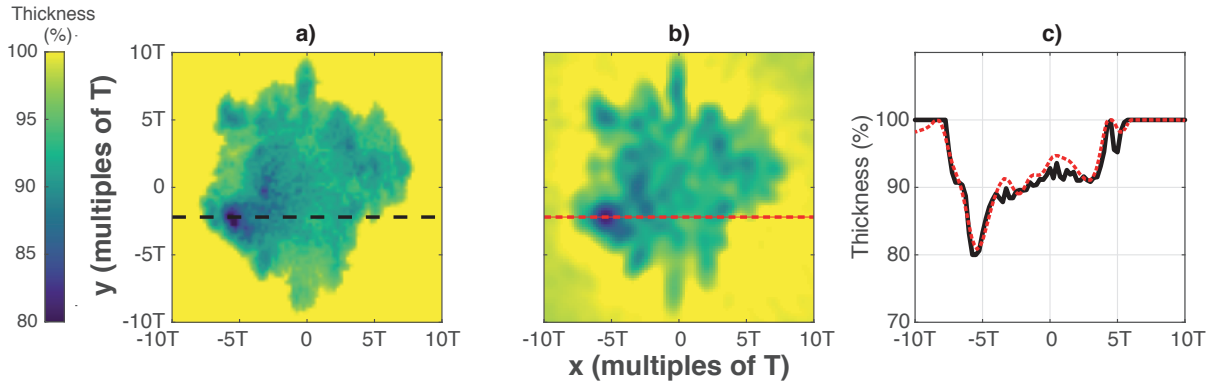


Figure 4.8: HARBUT reconstruction of the 20% realistic machined defect from experimental data at 250 kHz: (a) shows the true thickness map, (b) displays the reconstructed thickness map and (c) shows the line profiles through the deepest point for the true (black) and reconstructed depths (gray).

The complex amplitude input matrix was populated from all 120 excitation events (see fig. 4.7(f)) after SH1 windowing was applied and the data was Fourier transformed as shown in fig. 4.7(e).

Next, iterative HARBUT was employed and after 7 iterations, convergence was achieved. The global error was very low, at 1.1% and the error at the global minimum was 0.71%. The reconstruction is shown in fig. 4.8(b) and as can be seen, the defect was reconstructed extremely well because both of the profiles are nearly identical in fig. 4.8(c). Some artefacts can be seen in fig. 4.8(b), between 12 and 3 as well as between 6 and 9 o'clock, in the outer regions of the field of view. This seemed to indicate some variance in bulk shear velocity which is a common feature in steel from rolling manufacturing. Overall, the

experiment results for the 20% wall loss case are in line with the numerical predictions, which, importantly, is a demonstration of the highest resolution experimentally achieved in guided wave tomography to date. The next step was to determine whether the predictions could also be met for a defect with a greater maximum depth of 50% maximum wall loss. The results of this are presented in the next section.

4.4.4 Realistic defect - 50% wall loss

For the realistic machined defect with a maximum depth of 5 mm, the defect template *Echo* was linearly scaled to match that depth at the global minimum and the spatial resolution was set to 2.5 mm. The data was acquired from a set of 120 excitation positions with 41 receiving positions each. Thus, a total of 4920 measurements were acquired. For the excitation at 000° , the raw time traces are shown in fig. 4.9(a) and for the case of receiving at position 180° , the time-trace is plotted in fig. 4.9(b). The SH0 components were relatively unaffected and arrived first, with the back-wall reflections arriving later, at $t \approx 0.31$ ms. The centre frequency was chosen to be 325 kHz and from fig. 4.9(c), it can be observed that the greater depth led to most of the SH1 components being reflected, which was expected. As before, the velocity measured from the first arrivals was used for windowing of the signals and the velocities measured from the back-wall echo differences at 000° , 090° , 180° and 270° were used for calculation of the bulk shear velocity. The velocities were $c_{Sh_A} = 3257$ m/s and $c_{Sh_B} = 3192$ m/s. From this the dispersion curves were calculated which are plotted in fig. 4.9(d) and the cut-off frequency was found to be $f_{\text{cut-off}} \approx 159.912$ kHz which led to a maximum penetration depth of ≈ 4.9 mm at the centre frequency of 325 kHz. An example of the windowed time traces can be seen in fig. 4.9(b&c) from which the Fourier transform was calculated (fig. 4.9(e)). Taking the complex amplitude at the frequency, the complex amplitude matrix was populated for every transmit-receive combination which is shown in fig. 4.9(f).

From the FE simulations of directional excitation, convergence of iterative HARBUT was achieved after 58 iterations, which indicates a much higher level of phase shift due to the greater depth of the defect. However, for convergence, the field of view had to be reduced. This was achieved by setting all values at positions with radii from the centre

4. Experimental verification of SH1 guided wave tomography

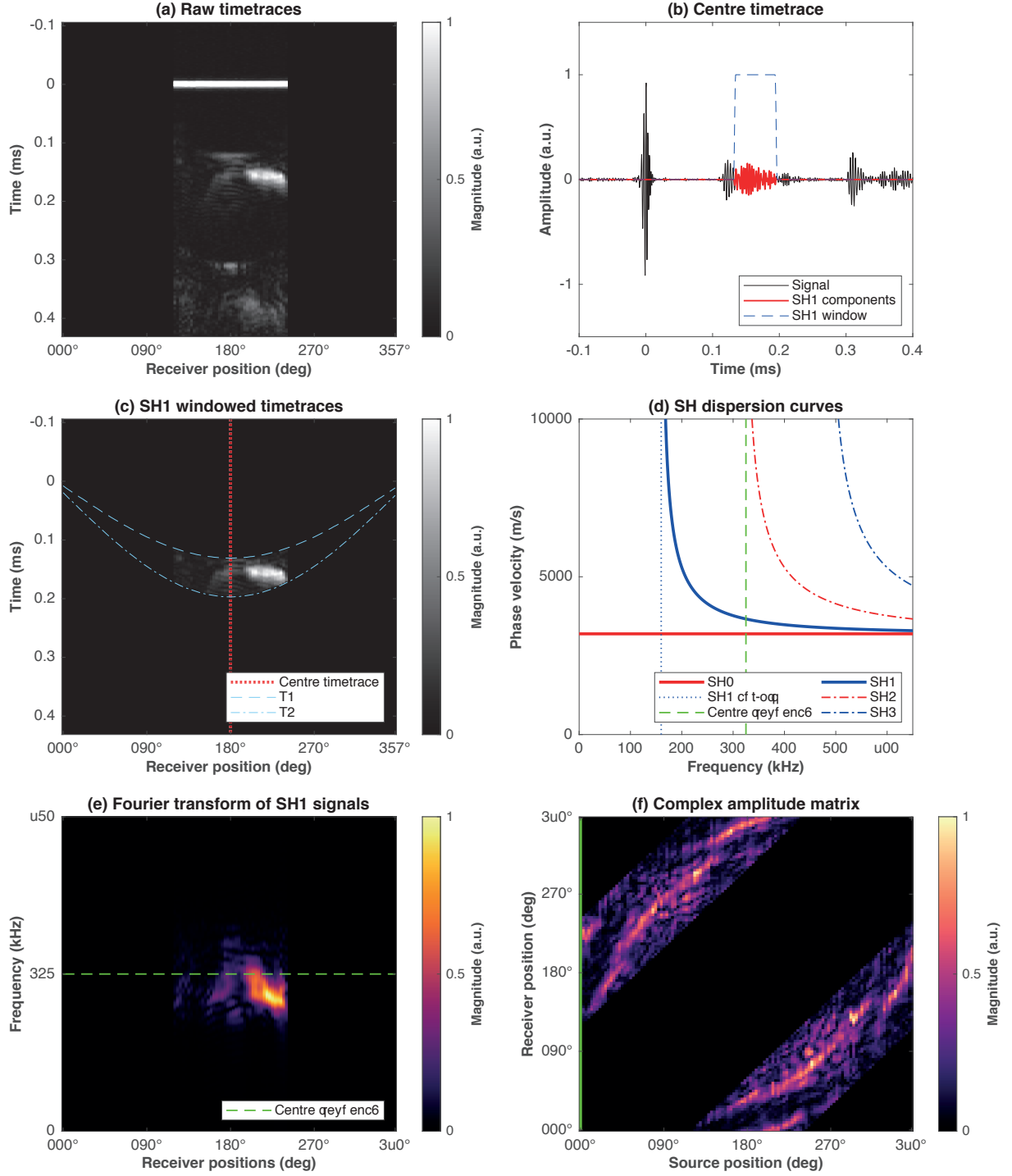


Figure 4.9: Experiment data acquired from excitation at a centre frequency of 325 kHz for the realistic defect at 50% maximum wall loss: (a) the magnitude of the Hilbert enveloped time traces for all receiver positions, (b) the centre time trace from receiver position 180°, (c) the windowed SH1 components with the first arrival times T1 (dashed light blue), the theoretical SH1 arrival times T2 (dashed dotted light blue) and (d) the calculated SH dispersion curves. In (e) the magnitude of the Fourier transform of the windowed signal is shown and in (f), this data is used to populate a complex amplitude matrix.

larger than $r_0 = 0.45 \cdot R$ to the background velocity. The reason this was required was the undersampling of the total field of view, using only $N = 120$ transducer positions, as well as because of the lower number of received time-traces which led to fewer data points from signals that have traversed nominal plate areas. However, given the sampling criterion by Simonetti et al. [44], this allowed for a higher sampling of k-Space during reconstruction, making it possible to set $k_{\text{Lim}} = [1.3, 1.4]$.² Still, the global error was larger than in previous results, at 3.7% as well as the error at the global minimum, at 5%. This corresponds to 0.5 mm and is thus still quite low.

Results are shown in fig. 4.10(b) and (c). The reconstructions are not as pristine as those of the 20 % remnant wall thickness defect, however, this was predicted by FE results and can be attributed to the use of directional transducers, the lower number of receiving positions and the challenge of finding a suitable calibration value and finally, the chosen centre frequency being slightly too low. Nevertheless, this 50% wall loss experiment matches the FE results extremely well and successfully concluded the experiments.

4.5 Summary

In this chapter, the experimental proof of concept of SH1 guided wave tomography was presented. This was possible owing to the design of the experiment test rig, which allowed for the repeated accurate positioning of the transducers. The experimental data processing very much benefited from the groundwork performed during the numerical testing. The results were in line with numerical predictions. The shallow defects were reconstructed very well. Somewhat surprisingly, the complex defect overall geometry was reconstructed more faithfully than the axisymmetric defect's geometry. This was most likely due to a mismatch between the machined defect and the model. The result for the 50% defect shows that the loss of resolution is well understood and that a transducer more similar to an SH point source is required to better capture the scattering information. Moreover, the field of view being too narrow is another cause of this reduction in resolution. Nevertheless, the reconstruction was much better than was previously achieved for Lamb wave based guided wave tomography and shows how robust the use of SH1 guided waves is. Also, modal

²See section 3.4.1 for a discussion of the sampling criterion.

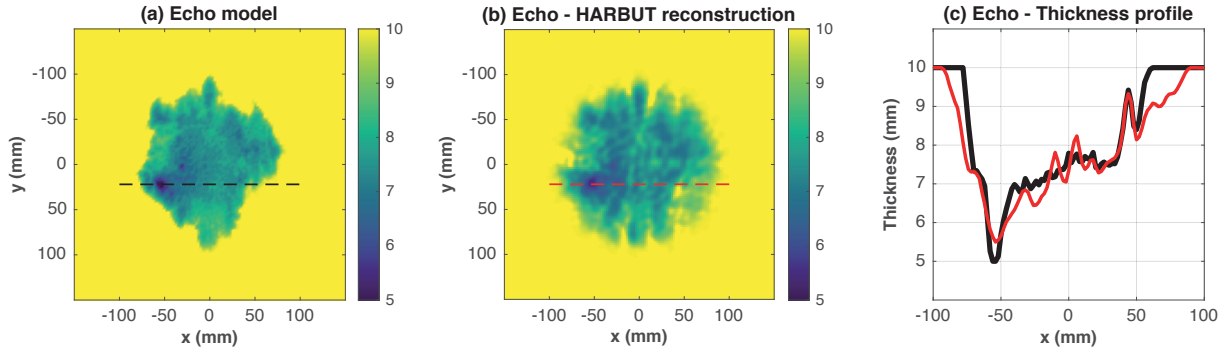


Figure 4.10: HARBUT reconstruction of the 50% realistic machined defect from experimental data at 335 kHz: (a) shows the true thickness map, (b) displays the reconstructed thickness map and (c) shows the thickness profiles through the deepest point for the machined (black) and reconstructed depths (red).

conversion signals will have played a role in reducing the quality of the reconstructions and the next chapter will present strategies that were investigated to remedy the impact of these factors as well as present results of techniques applied to the data acquired in previous chapters.

This page intentionally left blank

Chapter 5

Modal decomposition of non-uniformly sampled SH guided wave data

5.1 Introduction

The key to a high performance guided wave system has always been to reduce the number of modes present in the captured signals. Similarly, this is also desired in guided wave tomography to achieve the best corrosion mapping outcome using the velocity-thickness method implemented in this work. The presence of unwanted guided wave modes leads to coherent noise; especially when they exist at the same frequencies as the SH1 signals. Unwanted guided wave modes can also be generated when the SH1 waves interact with the sharp boundaries of deep defects. The higher the frequency, the more SH modes may be excited which may be combined together upon arrival at the receiver locations. Simple 1-dimensional Fourier transformation without windowing will lead to a Fourier spectrum showing interference patterns of the multiple modes. This is especially the case when multiple modes exist at the excited frequencies. One way of separating the modes from this type of data is to apply the 2-D Fourier transform in order to generate data in the frequency-wavenumber domain, which is often referred to as the frequency-wavenumber (f - k) diagram [55–58]. The f - k diagram allows to distinguish signal components or modes according to their velocity by frequency and wavenumber, revealing dispersion relationships.”

In this chapter, the 2-D FFT technique will be applied to SH guided waves. It was applied to Lamb guided waves for the measurement of amplitudes and velocities by Alleyne [2] for the first time and later used by Hayashi [59] to extract single Lamb wave modes. However, this technique allows for the separation of multiple modes when receiving signals at equidistant receiving positions only. Crucially, it requires uniformly sampled data in the time-domain as well as in the spatial-domain. In the setup considered in this thesis, the measurements are taken around a circular array. These are equidistant from each other. However, as a wave is radiated from an individual source, the distances between that source and the remaining transducers in the array will not be equally spaced and thus, the spatial Fourier transform fails to properly separate the wavenumber components. Instead, the non-uniform Fast Fourier Transform (NUFFT) algorithm [60, 61] will be applied to this problem for the first time, showing how it allows for the separation of the SH guided wave modes for signals received from a circular array. It should be noted that this could also be applicable to other guided wave modes but given the focus of this thesis on SH guided waves, the investigation was limited to them.

First, the original 2D-FFT method will be explained for a set of signals received by uniformly spaced receivers and the f - k diagram will be introduced. The strategy for extracting the non-dispersive SH0 components from the full SH f - k diagrams will be explained and evaluated. Next, the application of the NUFFT algorithm for the creation of f - k diagrams from circular array signals will be shown for the defect free case. This is followed by application to FE data of complex defects. Given the limited amount of information captured by the directional excitation, the technique suffers from a loss of resolution which makes modal separation near impossible. Nevertheless, a useful result was the development of an approach for determining phase velocity from NUFFT-based f - k diagrams, which will be explained and applied to data from the experiment. Finally, the results will be discussed and recommendations for the application of this technique will be provided.

5.2 Frequency-wavenumber diagrams from signals acquired at equispaced receivers

In this subsection, we will demonstrate the original 2D-FFT technique on analytically calculated SH signals for equispaced receivers placed in a line on a mild-steel plate. In order to penetrate greater depths in mild steel plates, higher excitation frequencies have to be selected and thus an excitation frequency of 350 kHz was chosen throughout this chapter. At this frequency, three SH guided wave modes can exist: the fundamental SH mode, SH0, the first higher order asymmetric SH mode, SH1 and above its cut-off frequency of 330 kHz, the SH2 mode is received as well (see fig. 5.1(b-d)). The signals were received at 120 receiving positions at a distance step of $\delta x \approx 5$ mm over a maximum distance of 600 mm. As shown by the time-traces in fig. 5.1(e), the signals are separable after having travelled about 400 mm by applying proper time-windowing techniques, however, the combined Fourier spectrum in fig. 5.1(i) shows the aforementioned interference pattern. This pattern emerges after addition of the Fourier spectra of SH0, SH1 and SH2 shown in fig. 5.1(f-h). Since the complex amplitude used to populate the input matrices used in the HARBUT algorithm stems from this Fourier spectrum, the decomposition of signals is of vital importance. By performing another Fourier transform along the distance axis, frequency-wavenumber diagrams (fig. 5.1(j-l)) are obtained and as can be seen in fig. 5.1(m), all three modes can be separated from one another. However, at these higher frequencies, the SH0 and SH1 components are placed ever closer to one another making very narrow windowing of the SH0 components a priority. Owing to the linear nature of the Fourier transform, the inverse Fourier transform will return the original signal (after division by the number of frequency components) as can be seen in fig. 5.2. Given that SH2 at 350 kHz excitation frequency will travel much slower and should be easily removable via time-gating, the main obstacle is the SH0 components overlapping with SH1 components leading to corrupted frequency domain information. Thus, the focus here is on the removal of the SH0 components.

Two window types were investigated: *SH0WinA* is shown in fig. 5.3(a) and is created by subtraction of SH1 and SH2 components in the frequency domain and the creation of a binary map from the thus created pure SH0 f - k diagram by setting values below the full-

5. Modal decomposition of non-uniformly sampled SH guided wave data

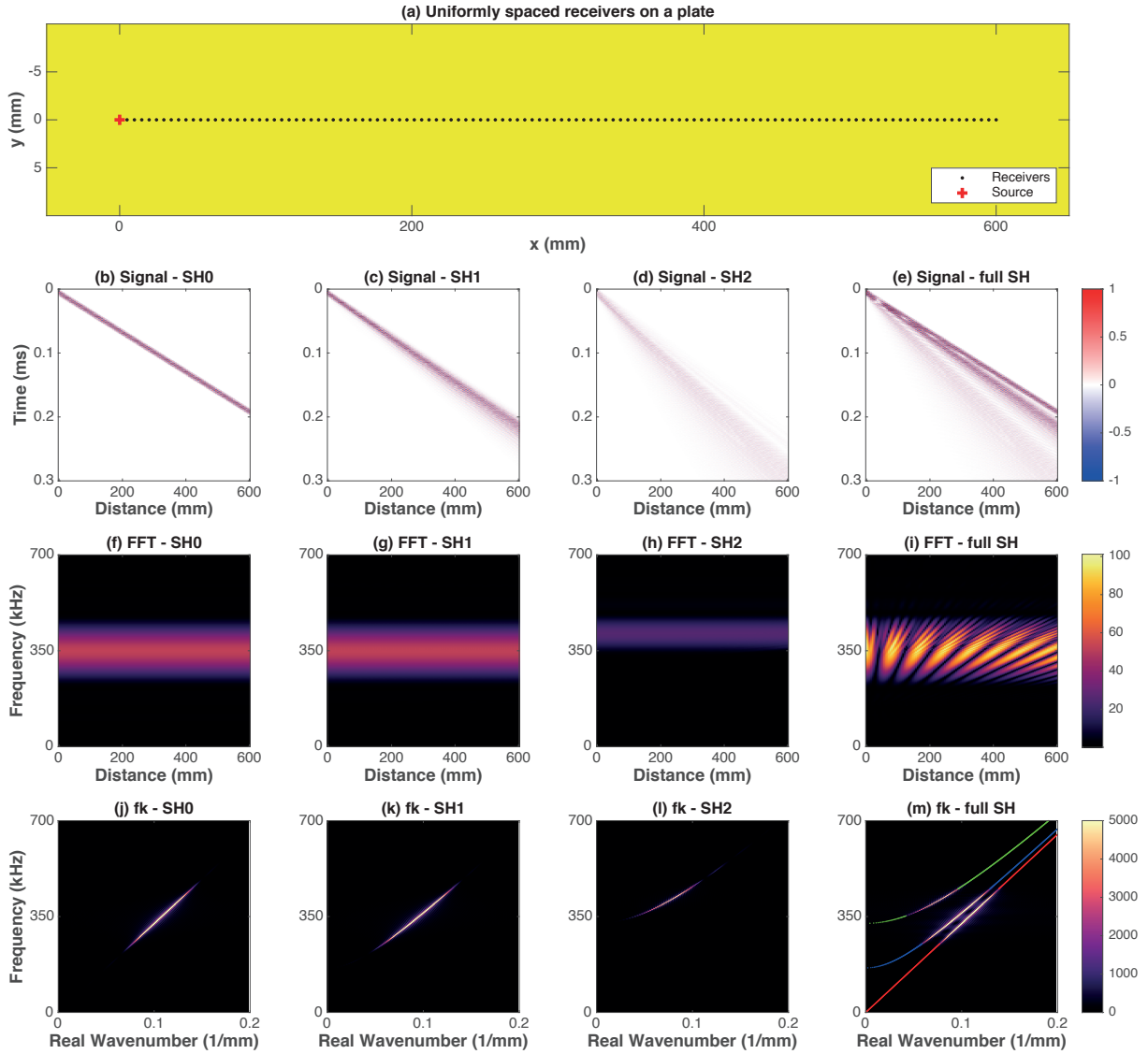


Figure 5.1: Model setup and steps from time-distance to frequency-distance and frequency-wavenumber diagrams: The first line shows the model setup of uniformly spaced receivers on a plate. Next, the Hilbert envelopes of the received time-traces at every receiver position are shown as well as the magnitude of the FFT of these signals and the frequency wavenumber diagrams for SH0 (first column), SH1 (second column), SH2 (third column) and all SH modes (fourth column). Notice how the three modes are distinctly separated only in the f - k diagram.

width-at-half-maximum value to zero and all values higher than that to one. The other window type, *SH0WinB* is a parallelogram that follows the linear SH0 dispersion curve and uses the excitation pulse frequency bandwidth (see y-axis in fig. 5.2(b)) as height and the width of the main lobe at the centre frequency as base (see blue dashed-dotted line in fig. 5.3(c)).

Next, these windows are applied to the full SH f - k diagram and an inverse Fourier trans-

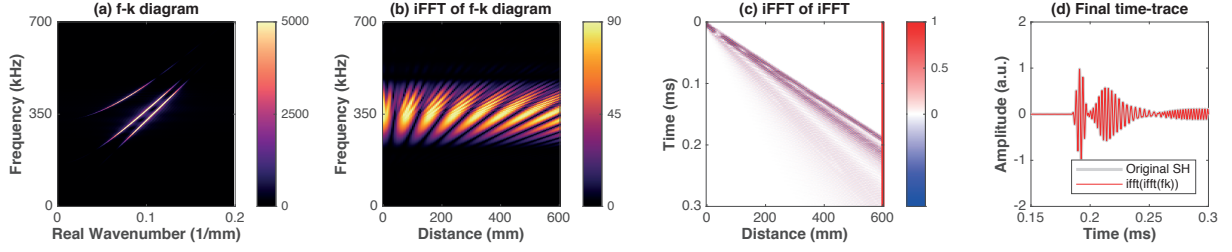


Figure 5.2: From frequency-wavenumber back to time-distance: Applying inverse Fourier transforms to invert (a) the f - k diagram to (b) the frequency domain and finally to (c) the time domain. The last plot (d) shows the perfect agreement of the inverted time-domain signal with the original signal from fig. 5.1(e).

form is performed, which leads to the frequency distance spectra shown in fig. 5.3(d) and (e). In a perfect scenario this would isolate only SH0 components and lead to the pure SH0 spectrum seen in fig. 5.1(f).

For *SH0WinA*, the smaller ellipsoidal window clearly cuts off the lower and higher frequency components and the smooth distribution in fig. 5.3(d) points to an elimination of other modal components that would be visible in the form of interference patterns. The same may be said for *SH0WinB* in fig. 5.3(e), however, due to the larger size of the parallelogram-type window, more frequency bands are captured.

For both window types the magnitude is lower at the beginning and end of the spectrum and increased at the centre. This is shown in fig. 5.3(f), where the centre frequency components are compared to the magnitude of the pure SH0 Fourier spectrum at the centre frequency (grey). As the whole process is linear, this magnitude shift was inherited in the time-distance domain signals as their amplitude was higher at the centre (see fig. 5.3(g-h)). However, as can be seen in fig. 5.3(i), the cycles of the reconstructed signals matched well.

Most crucially, the phase information of SH0 at the centre frequency was captured very well by both windows as is evident from the very low phase difference values plotted in fig. 5.3(j). The phase difference was calculated by dividing the recovered complex amplitude at centre frequency by the true complex amplitude of SH0 and calculating the phase angle. The phase difference of SH0 with SH0 is of course zero (grey). Although the phase differences of both *SH0WinA* (red dashed-dotted line) and *SH0WinB* (blue dashed-dotted line) were very low, the *SH0WinA* phase difference remained relatively constant whereas the phase difference of *SH0WinB* was worse at the beginning and the end but

5. Modal decomposition of non-uniformly sampled SH guided wave data

close to zero around 300 mm distance.

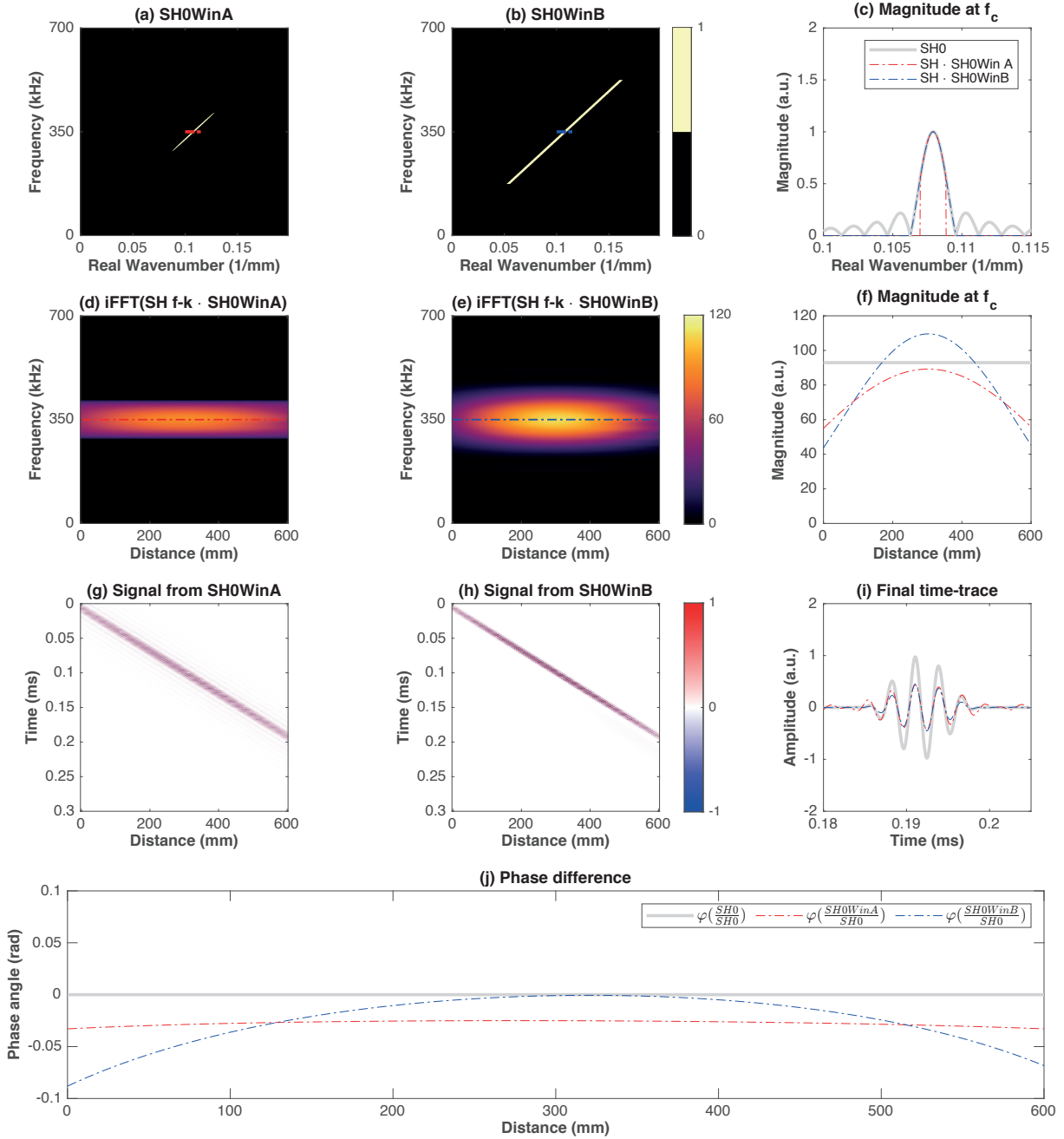


Figure 5.3: Application and inversion of two SH0 windows in the frequency wavenumber diagram. (a) shows SH0WinA. (b) is a window based on the bandwidth of the excitation pulse. (c) shows the windows applied to the centre frequency f - k components for SH0 (grey), SH with window *SH0WinA* (red dashed-dotted) and window *SH0WinB* (blue dashed-dotted). (d) and (e) are the Fourier spectra after applying an iFFT to the windowed f - k diagrams. (f) shows the magnitude at centre frequency. (g) and (h) are the Hilbert enveloped time-traces from applying an iFFT to the Fourier spectra. (i) shows the final time-traces for the true SH0 signal and the two windowed SH0 components. (j) compares the phase differences of the two filtered cases with the true SH0 phase angles at the centre frequency.

The ultimate goal of this procedure will be to remove SH0 components in the Fourier spectra (frequency-distance domain) which is used to generate the k-matrices that are the fundamental input for HARBUT. Thus, the SH Fourier spectra after subtracting the recovered SH0 components were investigated (see fig. 5.4).

Here, the amplitudes of the filtered SH0 components of *SH0WinA* and *SH0WinB* were normalised by dividing by the maximum magnitude per receiver channel (or rather, distance) and multiplying with the magnitude at the centre frequency of the SH0 spectrum.¹

The first sub-figure, fig. 5.4(a), shows the SH Fourier spectrum after subtraction of the true SH0 components, where the underlying smooth SH1 spectrum is visible and where, at frequencies above SH2 cut-off frequency, the interference pattern of SH2 with SH1 is visible. Given the narrower Fourier spectrum of the recovered *SH0WinA* filtered SH0 components, the SH0 removal is only observable in the region between 300 – 400 kHz (see fig. 5.4(b)).

For *SH0WinB*, a larger bandwidth region was available for SH0 removal and thus, the qualitative resemblance with the subtraction spectrum for real SH0 was the highest. This becomes more clear when subtracting the SH2 components as well. During experiments, this could be achieved by time-gating the slower moving SH2 components. Essentially, this leaves the pure SH1 components (fig. 5.4(d)) and again, *SH0WinB* appears to be a better window when more frequencies than just the centre frequency are required.

Most crucially, however, both windows performed very well at the centre frequency where, based on comparison of the phase difference at the centre frequency, they matched the phase information of SH1 very well (see fig. 5.4(g)). As before, the phase difference was constant for the values from the *SH0WinA* data while phase difference from *SH0WinB* data matched better at the centre than at the beginning and end. This seems to be an effect from the applied windows and could be beneficial for circular array data, when the defect interaction will be located at the centre of the Fourier spectrum.

Thus, a viable method for modal decomposition was demonstrated that could be a powerful tool in combination with other modal suppression methods such as the time-windowing methods described in earlier chapters. It was shown that for a uniformly spaced receiver

¹This is an optimisation problem that will have to be addressed in the later investigations, where pure SH0 components may be unavailable.

5. Modal decomposition of non-uniformly sampled SH guided wave data

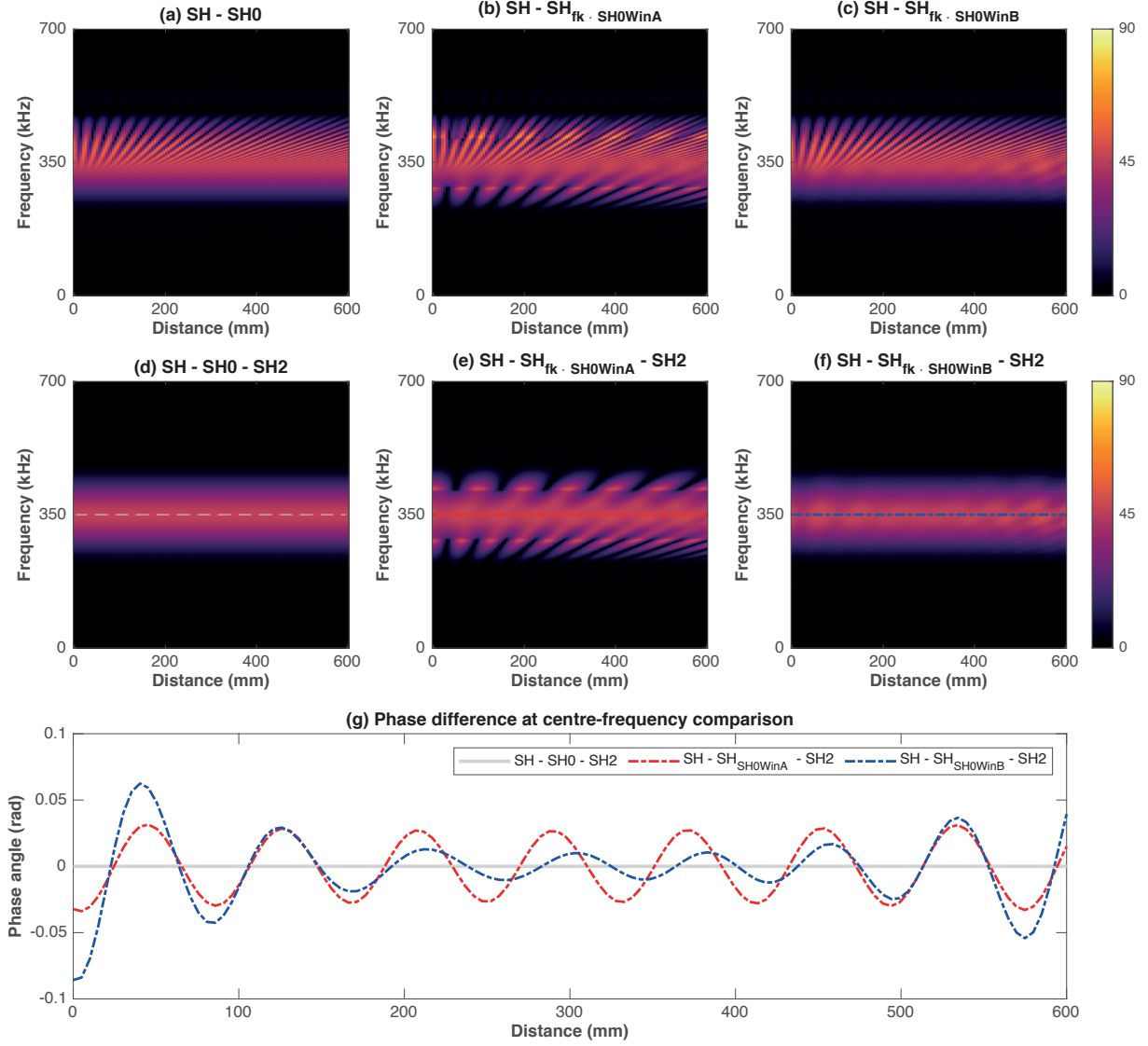


Figure 5.4: Comparison of the performance of SH0 removal by the two window types. (a) shows Fourier spectrum ground truth when subtracting SH0. (b) shows the subtraction of the SH0 components filtered by applying *SH0WinA* and (c) *SH0WinB* to the f - k diagram. (d) shows the SH1 Fourier spectrum obtained after subtracting the true SH0 and SH2 components and (e) and (f) show the Fourier spectrum when subtracting SH2 and the f - k windowed SH0 components. (g) compares the phase difference at centre frequency for the three cases and shows the excellent performance of the f - k filtered SH0 component subtraction.

array, f - k diagrams could be calculated and how, through proper windowing techniques, individual modes could be separated. Next, the circular array setup will be investigated, where the non-uniform placement of receivers will lead to the break-down of the spatial Fourier transform, requiring the application of the NUFFT algorithm.

5.3 Frequency-wavenumber diagrams from signals acquired at nonequispaced receivers

The chief guided wave tomography array utilised in this thesis is a type of ‘full-view’ array, the circular array. For this, N transducers were positioned at equidistant positions on a circle around the region of interest. In this subsection, a 10 mm mild steel plate is investigated upon which a circular array with radius $R = 200$ mm and $N = 120$ transducers was placed. SH signals were calculated analytically in the form of 5-cycle Hann-shaped pulses with a centre frequency of 350 kHz, leading to the appearance of the fundamental mode SH0 and the first and second higher order modes SH1 (cut-off-frequency at approximately 160 kHz) and SH2 (cut-off-frequency at approximately 330 kHz).

This setup is shown in fig. 5.5(a) and the resulting signals for excitation at a single source (red cross) and receiving at all receivers are shown in fig. 5.5(b-e) for SH0, SH1, SH2 and full SH. In fig. 5.5(f-i) the corresponding Fourier spectra are plotted. The plots in fig. 5.5(j-m) are the f - k diagrams of the above Fourier spectra. For the linear array in the previous section, another application of the Fourier transform along the distance axis would lead to a frequency-wavenumber diagram, revealing the dispersion relationship for the various modes. As can be seen in fig. 5.5(j-m), this step fails for this configuration and although there are differences in the f - k diagrams, there is no distinct revelation of SH mode components and they do not correspond to the expected dispersion curves plotted in fig. 5.5(m).

This was due to the requirement of the Fourier transform for uniformly sampled data. For the case of the circular array, the time-traces are sampled uniformly at a constant time-step, but crucially, the signals are not captured at equispaced positions from the source. This becomes clear when plotting the Euclidean distance of the receivers from the source on a line; receivers close to the source show a greater distance-step than the ones around the receiver opposite from the source, where the distance-steps are much smaller. In summary, these non-uniformly sampled spatial steps are causing the break-down of the spatial Fourier transform.

One solution to overcome this problem is the application of the non-uniform fast Fourier transform (NUFFT), which has been developed extensively by several groups [60,61] over

5. Modal decomposition of non-uniformly sampled SH guided wave data

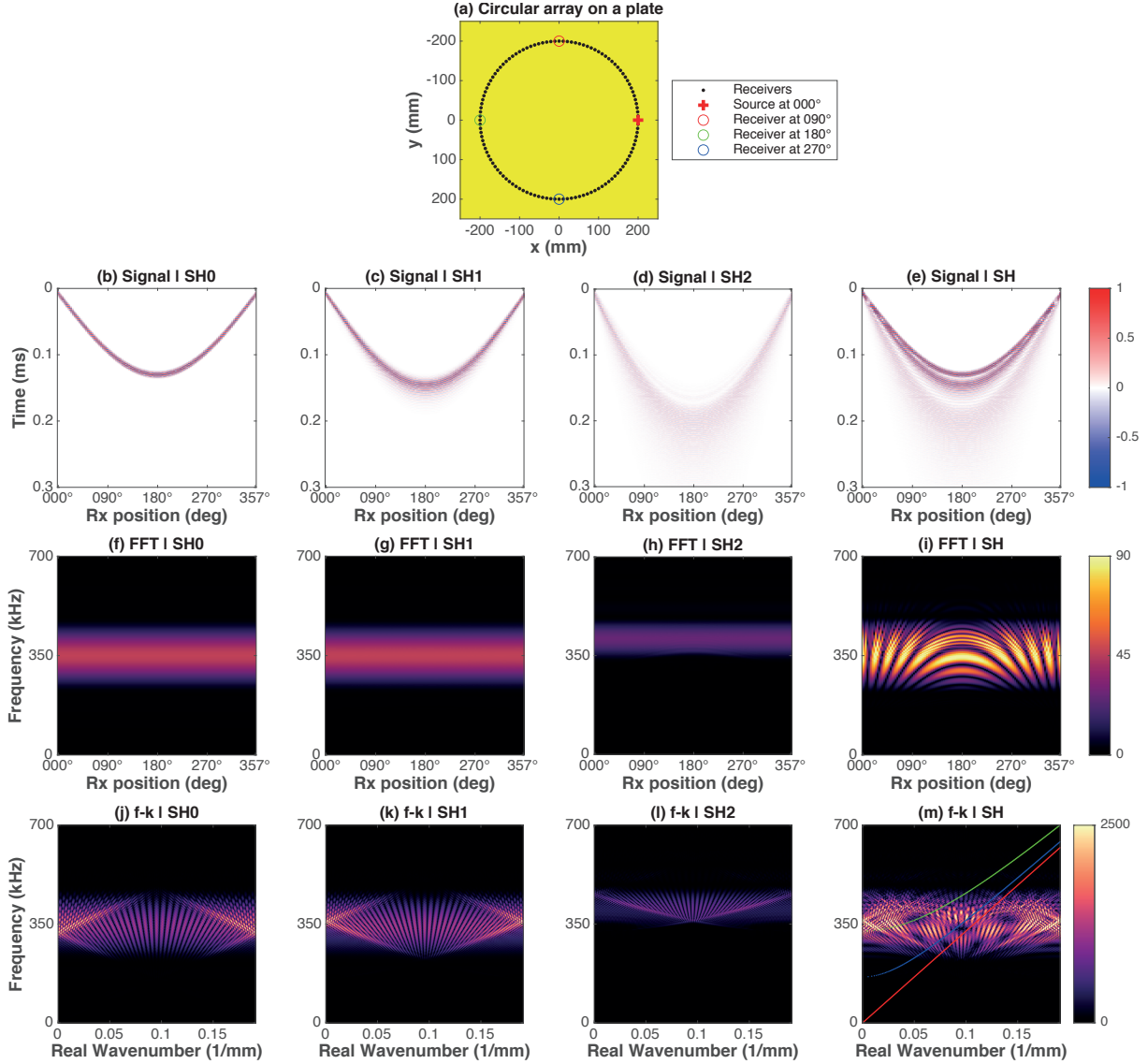


Figure 5.5: Model setup and steps from time-distance to frequency-distance and frequency-wavenumber diagrams: (a) shows the model setup of a circular array on a plate. In the next line, the signals are shown for (b) SH0, (c) SH1, (d) SH2 and (e) full SH. Below, the Fourier spectra are plotted for (f) SH0, (g) SH1, (h) SH2 and (i) full SH. The bottom line shows the f - k diagrams for the four cases. In (m), the expected frequency wavenumber dispersion curves are plotted for SH0 (red), SH1 (blue) and SH2 (green).

the past decades and has recently (MATLAB R2020a) been introduced as a function in MATLAB. The nonuniform discrete Fourier transform of a spectrum X is defined as

$$Y(k) = \sum_{j=1}^n \mathbf{X}(j) e^{-2\pi i t(j) f(k)}, \quad (5.1)$$

where $\mathbf{X}$ is a vector of length n , t are sample points, f are frequency query points and

where $k = 1, 2, \dots, m$. In order to calculate the f - k diagram with the NUFFT function, the Euclidean distance for every source-receiver combination is needed, as well as a wavenumber vector with a sampling wavenumber step-size of

$$k_{\text{snu}} = \frac{2\pi}{k_0}, \quad (5.2)$$

where k_0 is the wavenumber at the centre frequency. For n query points, the wavenumber-step is calculated from

$$\Delta k_{\text{nu}} = \frac{k_{\text{snu}}}{n}, \quad (5.3)$$

and used to calculate a wavenumber vector $\mathbf{k}_{\text{nu}}$ of n evenly spaced points between 0 and $\Delta k_{\text{nu}} \cdot n$. By providing the NUFFT function with this wavenumber vector and the non-uniform distance vector $\mathbf{d}$, the frequency-wavenumber diagram of the circular array based SH Fourier spectrum can be calculated.² This is shown in fig. 5.6(a), where the frequency-wavenumber dispersion relationship of the three modes SH0 (red), SH1 (blue) and SH2 (green) was properly captured as indicated by the dispersion plots.

Compared to the f - k diagram for the equispaced receiver case shown in fig. 5.2(a), there are significantly more artefacts in the lower (left) and higher (right) real wavenumber components. Nevertheless, a distinction of the three modal component areas is possible and they agree very well with the analytically calculated dispersion curves. Given the non-uniformity of the distance vector $\mathbf{d}$, the inverse Fast Fourier Transform (iFFT) is unsuitable as well. Thus, the inverse was found by applying the NUFFT algorithm to $Y(k)'$ and by switching the query with the sample points.

This led to the result shown in fig. 5.6(b). Qualitatively, the shape of the Fourier spectrum resembles the spectrum in fig. 5.5(i), however, the regions close to the source and the ones at the centre of the spectrum are showing exaggerated magnitudes. This is especially apparent in fig. 5.6(c), where the magnitude at the centre frequency is plotted for the original SH Fourier spectrum (grey) and the one reconstructed using the NUFFT algorithm

²This of course requires accurate knowledge of the position of the transducers. From simple reconstruction trials it was shown that position errors should not exceed ± 0.5 mm in order to avoid artefacts in the reconstructions which could lead to a tight tolerance for industry.

5. Modal decomposition of non-uniformly sampled SH guided wave data

(black). Up to distances of ≈ 300 mm, the minima and maxima are captured well but especially in the regions around the receiver opposite from the source, the magnitude is almost four-times that of the original SH magnitude.

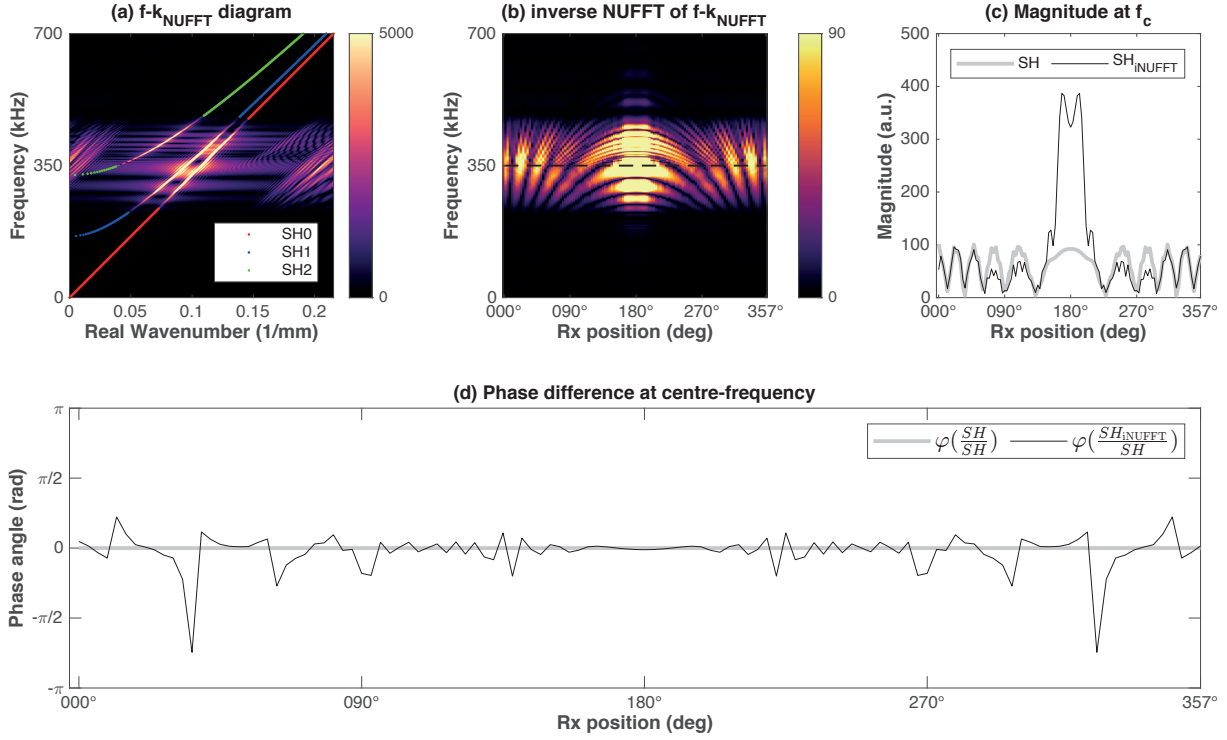


Figure 5.6: Investigation of the NUFFT performance: (a) shows the f - k diagram of the full SH spectrum. (b) shows the spectrum after applying the inverse NUFFT to the f - k diagram in (a). (c) shows a comparison of the magnitude at the centre frequency for the true SH spectrum (grey) and for the recovered magnitude from the NUFFT f - k diagram (black). (d) Compares the phase angle at the centre frequency for SH (grey) and SH from NUFFT (black).

This is most likely due to interpolation errors of the NUFFT algorithm which are introduced by the spatial sampling being very small in these regions, as the distance steps between these receivers are significantly smaller than the ones less than 200 mm away from the source.

The most crucial information is the phase as this is used by HARBUT to reconstruct phase velocity. Thus, the NUFFT-based SH phase angle at the centre frequency is compared to the true SH Fourier spectrum's phase angle at the centre frequency and displayed in fig. 5.6(d). Interestingly, the phase matches well in the regions where the magnitude is exaggerated and shows worse agreement at positions closer to the source. It therefore seemed promising to apply the inverse NUFFT calculation to an SH0-windowed f - k diagram. The two SH0 f - k windows, $SH0WinA$ and $SH0WinB$ were multiplied with the

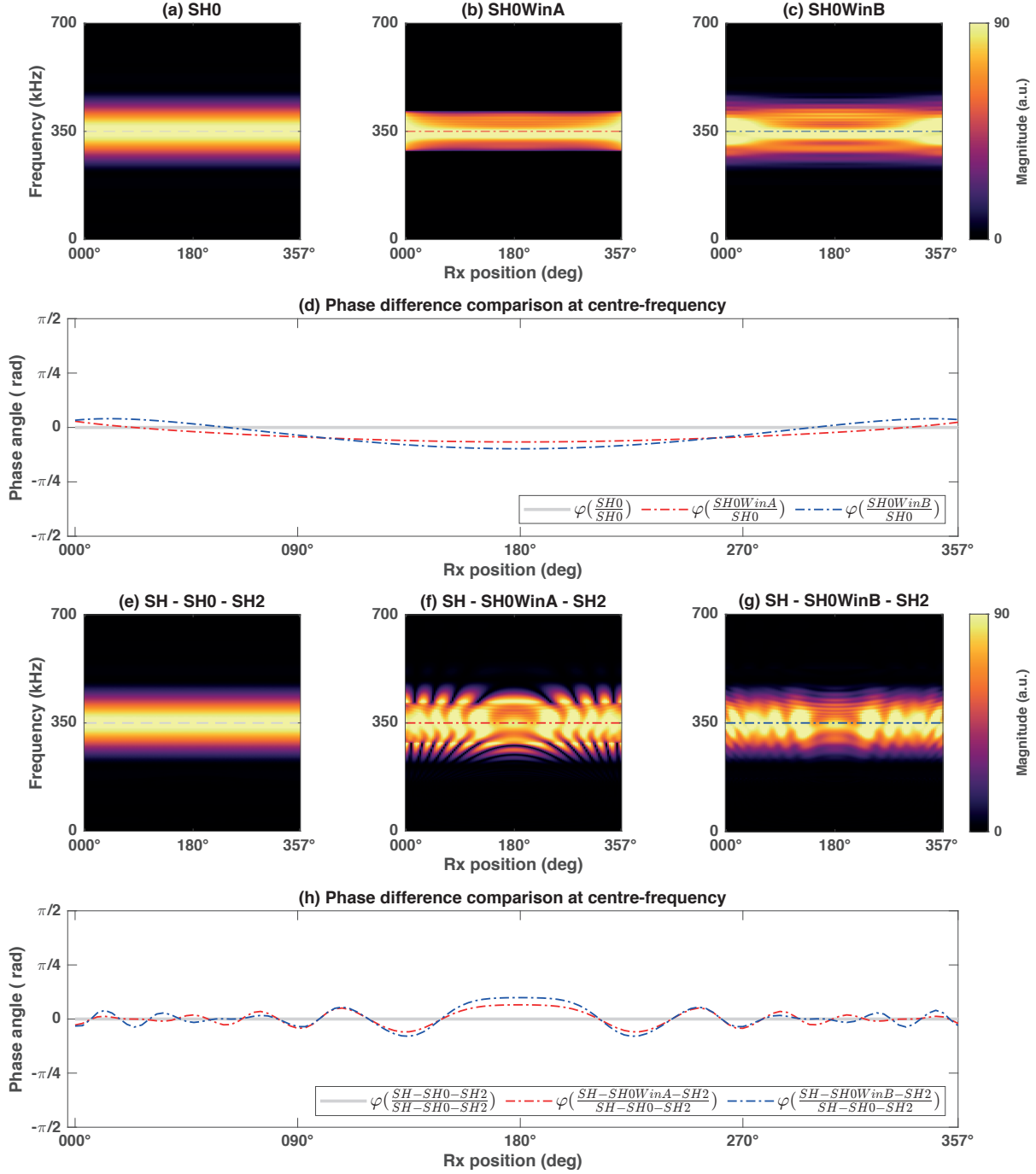


Figure 5.7: Comparison of the performance of SH0 removal after applying the two f - k SH0 windows to the NUFFT-based f - k diagram. (a) shows the true SH0 Fourier spectrum, (b) shows the SH0 spectrum reconstructed after applying the window $SH0WinA$ to f - k_{NUFFT} and (c) is the reconstructed Fourier spectrum from $SH0WinB$ applied to f - k_{NUFFT} . In (d), the phase difference over distance is compared for true SH0 (grey), $SH0WinA$ (red dashed-dotted) and $SH0WinB$ (blue dashed-dotted). In (e),(f),(g) the SH0, $SH0WinA$ and $SH0WinB$ components are subtracted from the full SH Fourier spectrum. SH2 components are removed as well which would be achieved through signal windowing. In (h), the phase difference is compared for SH-SH0-SH2 (grey), SH- $SH0WinA$ -SH2 (red dashed-dotted) and SH- $SH0WinB$ -SH2 (blue dashed-dotted).

f - k _{NUFFT} diagrams and the inverse NUFFT strategy used to calculate the Fourier spectra shown in fig. 5.7(b)&(c). Figure 5.7(d) provides a comparison of the phase angle of SH0 at the centre frequency taken from the true SH0 frequency spectrum (see fig. 5.7(a)) with the ones recovered from the two windows and yield very similar results and matching the true SH0 values well in most areas. Finally, the *SH0WinA* and *SH0WinB* spectra were subtracted from the full SH spectra along with the true SH2 and compared with the subtraction of true SH0 components (see fig. 5.7(e-g)). As can be seen in fig. 5.7(h), the resulting phase angle matches the one found by subtracting true SH0 and SH2 very well. In summary, the f - k diagram for a circular, non-uniformly spaced array was successfully calculated, using a NUFFT algorithm implemented in MATLAB. Moreover, an inverse NUFFT was found to yield satisfactory results, allowing for the application of previously found SH0 windows to the f - k diagram. Usage of these SH0-windowed Fourier spectra for SH0 removal from full SH spectra led to promising results as can be seen from the well matching phase angles at the centre frequency. The next subsection will see the application of this strategy to FE signals with a complex defect.

5.4 SH0 removal from FE acquired complex defect point source SH signals

Having demonstrated the application of the NUFFT algorithm for f - k diagram calculation from circular array data and the subsequent implementation of the SH0- f - k -filtering scheme, this section will showcase the results from application to signals acquired after the SH guided waves have travelled through a complex defect. This is a significant increase in complexity as no mode conversion occurred in the two models of the previous sections. In this scenario, the interaction of the SH guided waves with a rough defect with a maximum of 50% wall loss will lead to the inevitable creation of mode-converted signal components. However, the main focus will be on the removal of the first arriving SH0 components, as for the distances and excitation frequencies considered in this thesis, SH0 and SH1 will overlap partly in the time-domain. This in turn causes problems when applying the time-windows.

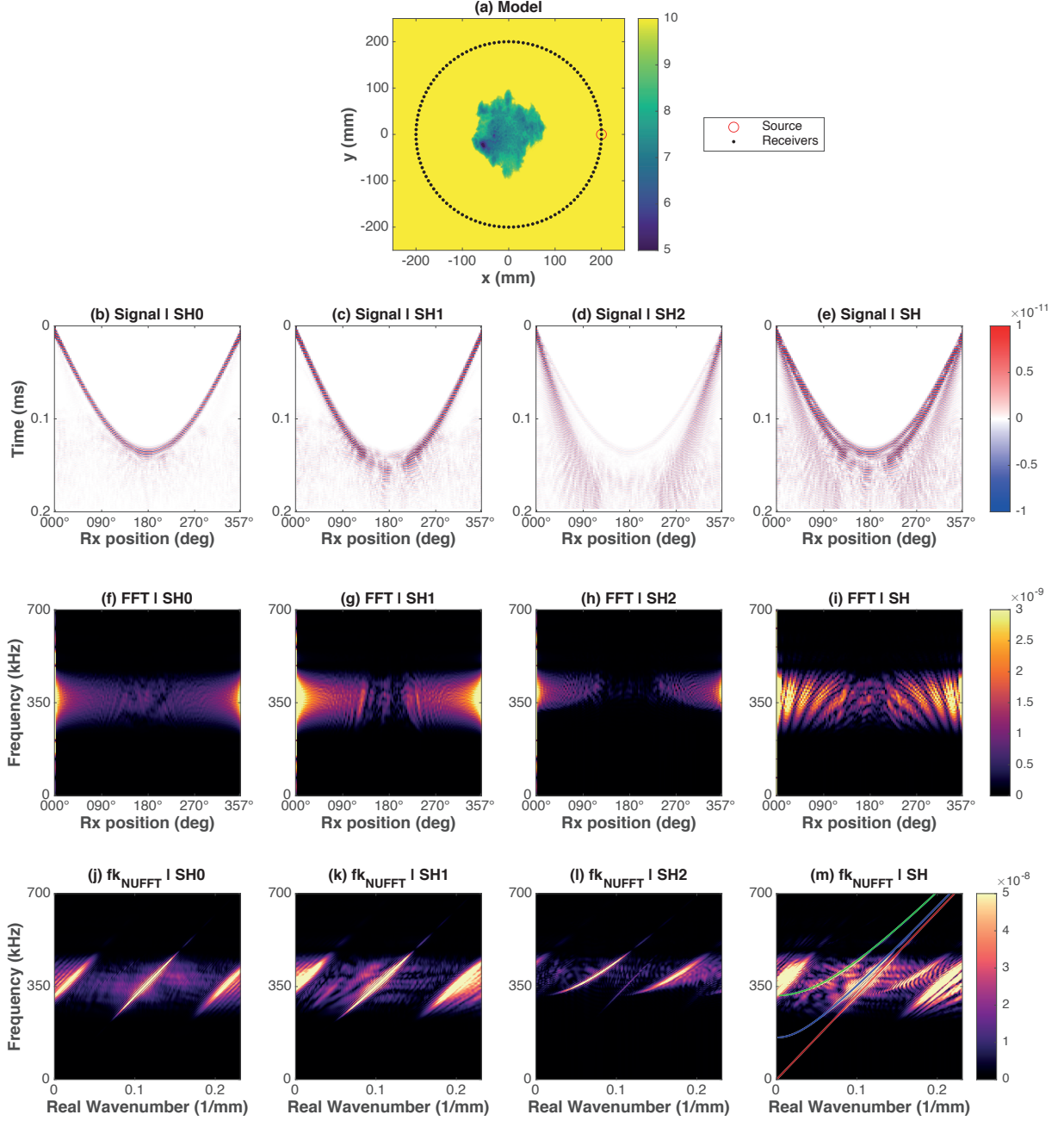


Figure 5.8: Model setup and steps from time-distance to frequency-distance and frequency-wavenumber diagrams: (a) shows the model setup of a circular array on a plate. Next, the Hilbert envelopes of the received time-traces at every receiver position are shown (b-e), as well as magnitude of the Fourier spectra of the signal components (f-i) and the NUFFT-based frequency wavenumber diagrams (j-m).

Figure 5.8(a) shows the model setup which is equivalent to the one used in the previous section, apart from the presence of a rough corrosion patch with a maximum wall loss of 50 %. The magnitude of the Hilbert envelopes of the received signals are shown in fig. 5.8(b-e). In fig. 5.8(b), some mode-converted SH0 components are visible at around

$t = 0.15$ ms. The strong interaction of SH1 with the defect is clearly visible in fig. 5.8(c). SH2 also interacts heavily with the defect as can be observed in fig. 5.8(d). Most of SH2's frequency components are unable to propagate through the defect due to the remaining thickness being below the cutoff frequency-thickness value for that mode. Another favourable effect is the relatively slow propagation of SH2 which allows for almost complete SH2 removal by time-windowing in future steps. The full SH time-trace received from the SH surface-only point receiver shows again how SH2 should be relatively straightforward to separate, whereas SH0 will be more challenging to remove as for the distance used in this model, the modes are much closer to one another in the time-domain (see fig. 5.8(e)). The level of interaction of SH0 with the defect is more subtle in the SH0 Fourier spectrum in fig. 5.8(f), which is in contrast to the significant interaction of SH1 in fig. 5.8(g) and SH2 in fig. 5.8(h). Next, the NUFFT function was applied to calculate the f - k diagrams and the results are shown in fig. 5.8(j-m). The NUFFT-based f - k diagrams are capturing the main modal components, however, there are artefacts visible at small and higher wavenumbers in all four f - k diagrams. Given that the f - k SH0 windowing approach neglects these regions, they are of no concern.

Having applied the previously discussed SH0 f - k window approaches to this data the results can be compared to pure mode data to evaluate the performance. In fig. 5.9(a-c) the pure SH0 Fourier spectrum is compared to the Fourier spectra reconstructed from f - k diagrams after windowing with *SH0WinA* (fig. 5.9(b)) and *SH0WinB* (fig. 5.9(c)) and applying the inverse NUFFT approach. From a qualitative point of view these are quite different and the defect interaction seems implied by the drop in magnitude at the opposing receiver positions at centre. As can be seen in fig. 5.9(d), the phase differences at the centre frequency of the recovered complex amplitude are larger than in the defect free case shown in the previous section but still remains below $\pi/2$, which is encouraging.

The next step saw comparison of the windowing approaches with a perfect subtraction of SH0 and SH2 from the full SH data. The ideal case in fig. 5.9(e) was achieved by subtracting the true Fourier spectra of SH0 and SH2 from the full SH which left the sought after SH1 components. In reality, this would be hard to achieve as there would inevitably remain overlapping SH0 components. SH2 components are easier to remove as they travel at much lower group velocity and for this configuration can be removed in

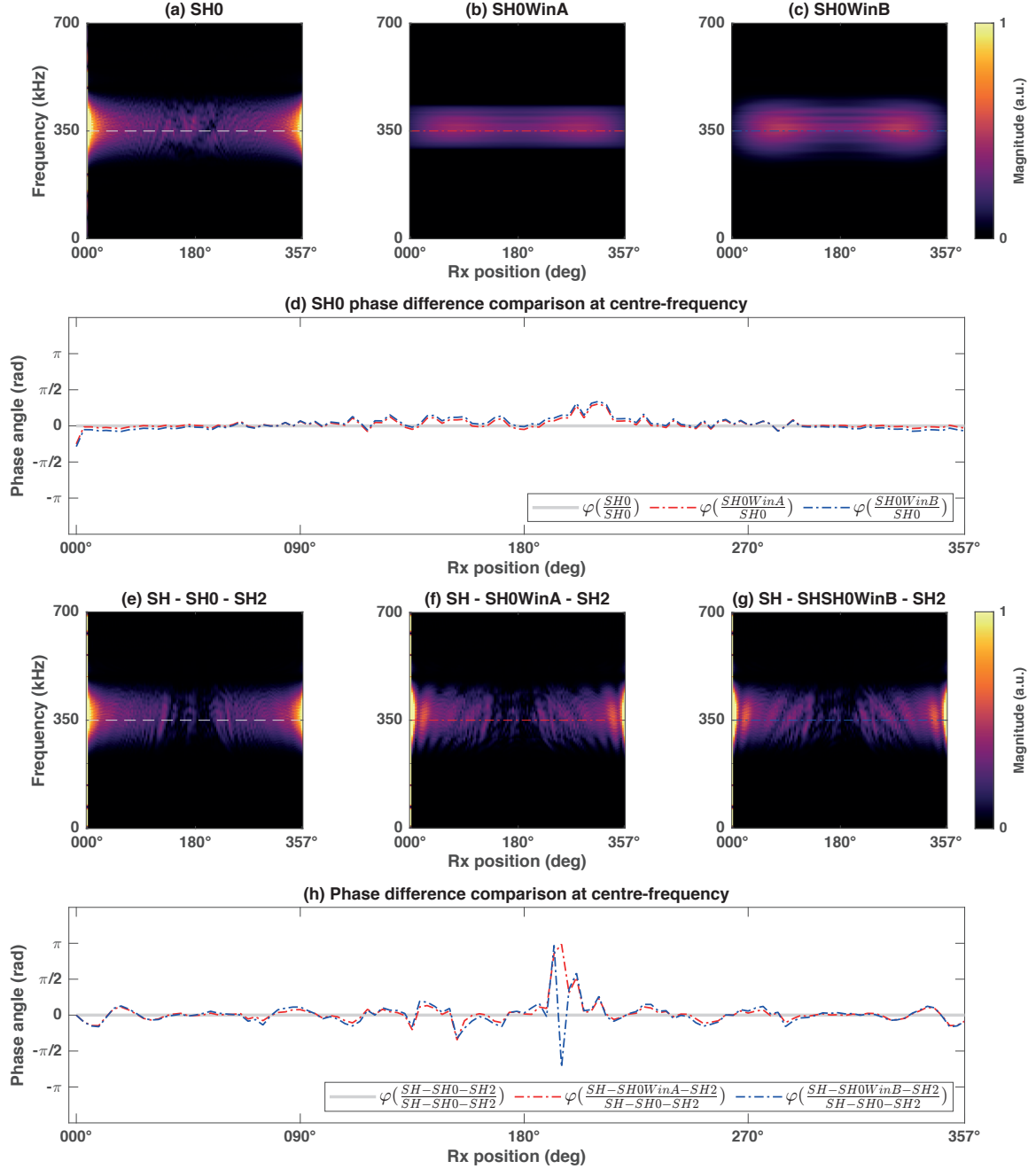


Figure 5.9: Comparison of the performance of SH0 removal after applying the two f - k SH0 windows to the NUFFT-based f - k diagram from complex defect FE point source data. (a) shows the true SH0 spectrum, (b) shows the SH0 spectrum reconstructed after applying the window $SH0WinA$ to f - k_{NUFFT} and (c) is the reconstructed spectrum from $SH0WinB$ applied to f - k_{NUFFT} . In (d), the phase difference over distance is compared shown for true SH0 (grey), $SH0WinA$ (red dashed-dotted) and $SH0WinB$ (blue dashed-dotted). In (e), (f), (g) the SH0, $SH0WinA$ and $SH0WinB$ components are subtracted from the full SH spectrum. SH2 components are removed as well which would be achieved through signal windowing. In (h), the phase difference over distance is compared and shown for SH-SH0-SH2 (grey), SH- $SH0WinA$ -SH2 (red dashed-dotted) and SH- $SH0WinB$ -SH2 (blue dashed-dotted).

the time-domain. For comparison-sake, however, the SH2 Fourier spectra were subtracted along with the f - k windowed SH0 components from the full SH f - k diagrams. The results of these subtractions are shown in fig. 5.9(f) for *SH0WinA* and in fig. 5.9(g) for *SH0WinB*. Compared with the original full SH Fourier spectrum in fig. 5.8(i), the SH2 interference pattern at frequencies above 330 kHz are removed and the interference pattern from the interaction of SH0 and SH1 is reduced. This seems to be more the case for *SH0WinB* data than for *SH0WinA* data. However, the most important aspect is the recovery of the phase at the centre frequency. This is again analysed through calculation of the phase difference which is shown in fig. 5.9(h). For the first time in this analysis, the phase difference approached values in the order of π for components received between receiver positions 180° and 225° . These receivers are in direct line with the deepest corrosion pit of the defect shown in fig. 5.8(a) and therefore the region where the greatest amount of phase difference is to be expected. In other regions, the phase difference values remain below $\pi/2$. Next, this windowing and subtraction was repeated for all 120 excitation events and then used to populate an input matrix for use in HARBUT.

The final assessment of the separation routine performance is to compare the resulting reconstructions. The HARBUT reconstruction results are shown in fig. 5.10. The time-domain window approach led to a slightly better result than the NUFFT-based f - k SH0 windowed reconstructions; it converged after 36 iterations compared to 65 (*SH0WinA*) and 59 (*SH0WinB*), had a lower error at maximum depth with 0.2% compared to 0.7% (*SH0WinA*) and 0.6% (*SH0WinB*) as well as a lower global error at 2.3% compared to 3.6% (*SH0WinA*) and 3% (*SH0WinB*). Also, in order to achieve convergence, the two f - k -

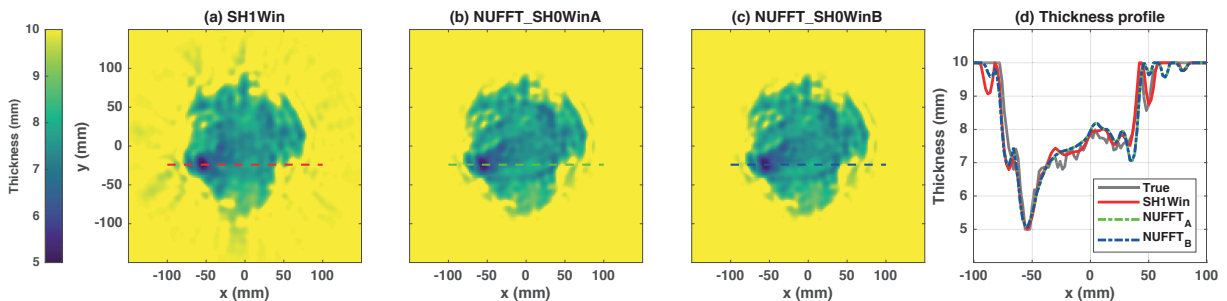


Figure 5.10: HARBUT results after SH0 removal. (a) shows the HARBUT result from section 3.5.3, (b) shows the result after applying the NUFFT-based f - k -window type A and (c) the result after applying the NUFFT-based f - k -window type B. (d) compares the true (grey) thickness profile with the reconstructed ones.

filtered datasets had to be regularised during reconstruction by setting all values at radii larger than $0.55 \cdot R$ to the background velocity. This is noticeable in fig. 5.10(b&c) by the lack of artefacts compared to the outer regions of fig. 5.10(a). Nevertheless, this is a very good result as this demonstrates the possibility of accurate thickness reconstruction from full SH mode data after application of SH0 removal through NUFFT based f - k filtering.

5.5 Investigation of directional source SH signals with NUFFT-based frequency wavenumber diagrams

In the previous subsections, the NUFFT-based f - k diagram windowing of SH0 has been applied to point source FE data. It was shown that using the NUFFT algorithm, this allowed for reasonable decomposition of the various modal components. For experiments, a directional SH guided wave transducer was used which led to acceptable results in the 20% case but less optimal results in the 50% wall loss case. This was mainly due to the limited view that was created by limiting the received time-traces per source in order to reduce measurement time. However, another factor seemed to have been the inaccurate measurement of the fundamental shear velocity of the mild steel plate used. It is therefore of interest to see whether an f - k diagram may be calculated from directional data which could be used to calculate the dispersion relationships.

Thus, an FE model was run that had the same setup as the previous one, albeit without a defect and using the rectangular surface excitation approach introduced in a previous chapter. The centre frequency was set to $f_c = 350$ kHz, the transducer width was set to $Tx_W = 5$ mm and the transducer length was set to $Tx_L = 20$ mm.

The resulting time-traces for excitation at source Tx(1) and receiving at all receivers Rx are shown in fig. 5.11. Due to the directionality of the transducer, the desired SH1 components are only received at receivers that are opposite to the source and at least 350 mm away. Compared to the point source data, a lot of spatial frequency components are therefore missing and a poor separation in the frequency-wavenumber domain may be expected. In fact, the interfering signals received at the other receivers lead to a frequency wavenumber diagram without a clear dispersion pattern visible as can be seen

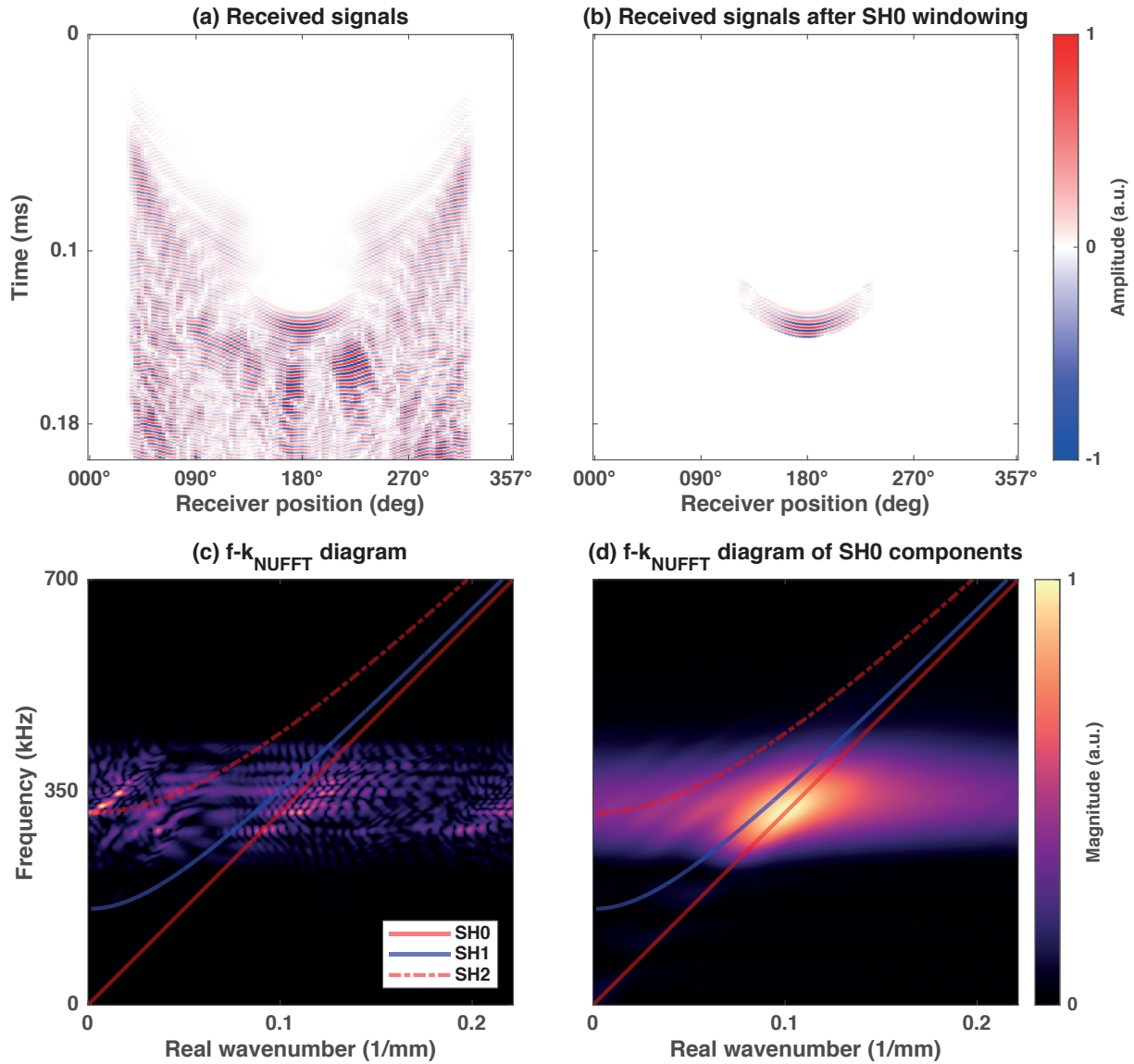


Figure 5.11: f - k diagrams from FE simulation generated directional transducer signals: (a) received signals from a directional transducer. (b) shows the windowed SH0 components of (a). (c) shows the f - k diagram calculated from the data in (a) and in (d), the f - k diagram calculated from the windowed SH0 components is shown. Overlaid on (c) and (d) are the f - k dispersion curves of SH0, SH1 and SH2.

in fig. 5.11(c). To check whether this was due to the algorithm under-performing, the SH0 components in fig. 5.11(a) were windowed following the approach from previous chapters. The f - k diagram calculated from these windowed components is shown in fig. 5.11(d) with the expected SH f - k dispersion curves overlaid. The lack of interference patterns indicates that it was indeed possible to window the SH0 components, however, the blurry ‘blob’ indicates a lack of wavenumber-resolution as the algorithm fails to build a clearly defined region. Also, it can be seen how the captured SH0 components leak into the SH1 ‘region’

which undermines the argument of modal separation through time-based windowing or f - k diagram based filtering.

Although this result makes it impossible to perform any meaningful f - k diagram based filtering of SH0, it might yet prove useful. Qualitatively, it seems as if the maxima of the blurry SH0 region are close to the SH0 f - k dispersion curve. We know that this plot contains accurate data as it stems from FE data. Furthermore, since SH0 is non-dispersive we know that it must be zero at origin which can be useful for performing linear regression in order to determine the SH0 f - k dispersion curve for experimental data. This in turn should theoretically enable the calculation of SH0's phase velocity, which as previously stated is equal to the bulk shear velocity. This velocity is fundamentally important for calculating the dispersion curves used for inversion. The next section will present the investigation of this approach.

5.6 Determining bulk shear velocity from NUFFT-based f - k diagrams

As has been shown in the previous subsection, the directionality of the data leads to poor spatial frequency (wavenumber) resolution which does not allow for proper modal separation as was the case for point source-excited SH data. Furthermore, the uncertainty of the wavenumber components also leads to indistinguishable full mode f - k diagrams and without signal processing, any dispersion curve fitting seems arbitrary.

In previous chapters, it was shown how a very simple windowing approach was used to determine the shear velocity which is equal to the phase- and group-velocity of SH0, as SH0 is non-dispersive. This was achieved by utilising the prior knowledge of the transducer positions and thus the distances between source and receiver pairs. Given the good results of the reconstructions, it seems as if this way of determining the velocity is sound. For greater depths, however, where the velocity gradient in the dispersion curves is reduced, the better this velocity estimate, the better the reconstruction.

In this section, the NUFFT algorithm is applied to experimental data in order to generate f - k SH0 diagrams, which allow for the determination of the shear velocity through linear

5. Modal decomposition of non-uniformly sampled SH guided wave data

regression. For this, the available limited dataset of captured time traces for every source excitation was used to determine the group velocity of SH0, in order to calculate an arrival time based SH0 window. The width of the window was found using the pulse duration of the 5-cycle Hann shaped pulse.

Next, the Fourier transform was performed on the windowed signals and the NUFFT algorithm applied in order to generate a frequency-wavenumber diagram. Consequently, the maxima of the relevant frequency components were determined and a fit was found through linear regression. The slope of the fit may then be used to determine the shear velocity at the centre frequency. This strategy was applied to the two experimental data sets discussed in the previous chapters for 20% and 50% wall loss.

Figure 5.12 shows the individual steps and results for the 20% wall loss case with an excitation frequency of 250 kHz. In fig. 5.12(a), the raw data is shown along with the theoretical arrival times for SH0 (red line) and SH1 (blue line). The theoretical SH0 arrival times $t_{arr,SH0}$ are used to calculate an SH0-window, using the 5-cycle Hann shaped

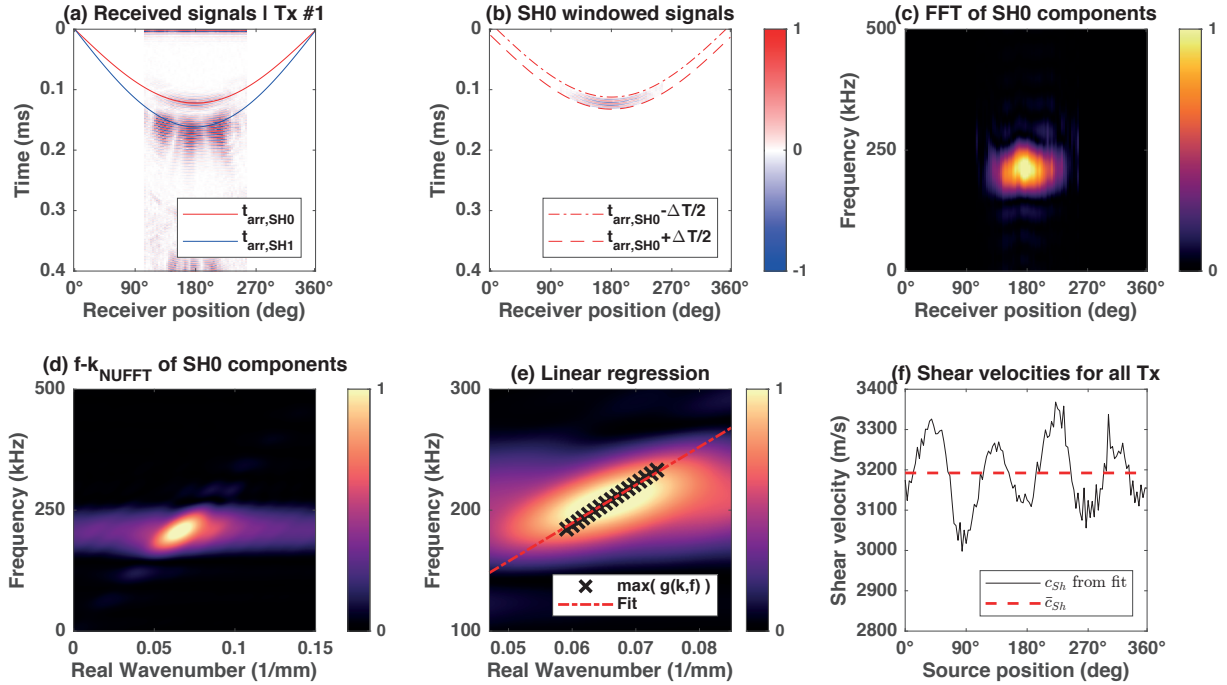


Figure 5.12: Results from 250 kHz experimental datasets: From time-traces to f - k diagram linear regression based shear velocity: (a) Received experimental time-traces, (b) windowed SH0 components and (c) their Fourier transform. (d) SH0 f - k diagram and (e) the maxima (black crosses) at several frequencies used to fit the SH0 dispersion curve. (f) shows the velocities found for all 120 excitations.

5. Modal decomposition of non-uniformly sampled SH guided wave data

pulse duration (fig. 5.12(b)) and are then Fourier transformed using the FFT algorithm (fig. 5.12(c)). Next, the NUFFT algorithm is applied and the frequency-wavenumber diagram calculated (fig. 5.12(d)) and the maxima per frequency and wavenumber at several frequencies around the centre frequency are determined (black crosses in fig. 5.12(e)). Using linear regression, a line is fitted against the found values and the slope values are used to calculate the SH0 phase velocity, which is equal to the SH0 group velocity and the material's bulk shear velocity. This is repeated for every received dataset at all 120 transmission positions and which yielded the results plotted in fig. 5.12(f). As can be seen, the bulk shear velocity is anisotropic and encouragingly, the values calculated from opposite source positions are in good agreement. Here, a mean shear velocity of $c_{Sh} = 3188 \text{ m/s}$ was found (red dashed line in fig. 5.12(f)), which is slightly lower than what was found through peak detection. From this velocity, a dispersion curve plot may be calculated and the velocity can be used during HARBUT reconstruction.

As was shown in previous sections, the 50% wall loss data seems to be more complicated. The same steps as before were applied to the available limited dataset. In fig. 5.13(a),

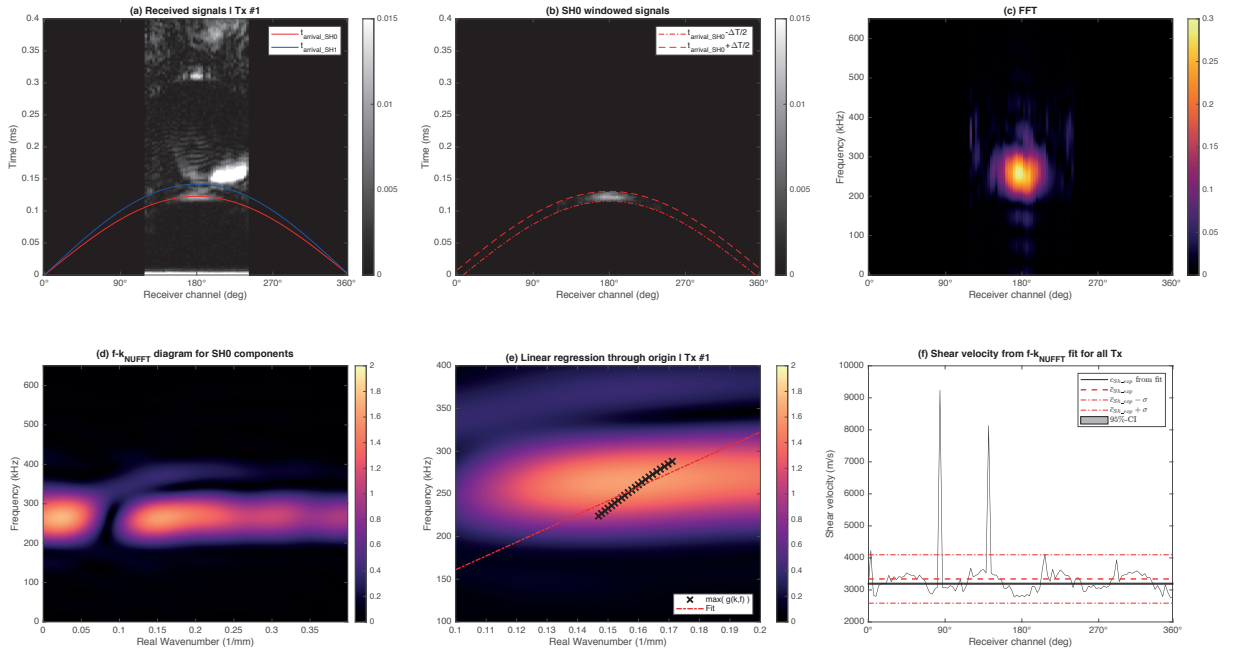


Figure 5.13: Results from 325 kHz experimental datasets: From time-traces to f - k diagram linear regression based shear velocity: (a) Received experimental time-traces, (b) windowed SH0 components and (c) their Fourier transform. (d) SH0 f - k diagram and (e) the maxima (black crosses) at several frequencies used to fit the SH0 dispersion curve. (f) shows the velocities found for all 120 excitations.

the raw time-traces for excitation at Tx #1 (position 000°) are shown along with the two theoretical arrival times $t_{arr,SH0}$ in red and $t_{arr,SH1}$ in blue. These were calculated using the peak of the Hilbert envelope of the SH0 pulse and prior knowledge of the diameter of the array. The SH0 window was calculated using the pulse-duration and the windowed data was Fourier transformed (see fig. 5.13(c)). The next steps were performed as before, creating a NUFFT-based frequency-wavenumber diagram (fig. 5.13(d)) and executing the linear regression fit to find the slope of the SH0 values (see fig. 5.13(e)). In fig. 5.13(f) the recovered velocity is plotted, which also shows anisotropic variation. There are about four outliers which were most likely due to mode-converted signal components overlapping the SH0 pulses and by excluding these from calculating the mean shear velocity was found to be $\bar{c}_{Sh} = 3234$ m/s. In contrast to the 250 kHz data, this value is much closer to the mean velocity found from peak detection.

5.7 Summary

This chapter dealt with the application of the NUFFT algorithm to non-uniformly sampled array data in order to achieve modal separation of the signals. For this, various scenarios were investigated, starting with uniformly sampled data, which allowed for the application of the FFT algorithm to find the frequency wavenumber diagram. It was shown how only the NUFFT algorithm allowed for proper modal separation of the circular array based received signals in the frequency wavenumber domain. For point source based excitation, it was shown how two types of SH0 frequency wavenumber windows allowed for acceptable SH0 removal in the frequency domain. Moreover, the HARBUT reconstructions using this type of filtered input data were shown to yield extremely similar results to the time-domain windowing based approach, confirming ability of the NUFFT algorithm to capture accurate phase information.

Moreover, the constraints of the method were demonstrated as it was shown how directional source data provides too few wavenumber data points to allow for modal separation in the frequency wavenumber domain. Nevertheless, the method was successfully used in an algorithm to determine the bulk shear velocity, which was applied to experimental data that was excited and received using directional transducers. Thus, the found velocity was

shown to provide slightly better reconstructions and proved sensitive enough to reveal the slight inhomogeneous nature of the rolled plate material.

This page intentionally left blank

Chapter 6

Conclusions

6.1 Thesis review

The guided wave tomography method has been adapted for the use of the first order shear-horizontal guided wave mode, SH1 after previous work showed that Lamb wave based guided wave tomography approaches do not deliver the required resolution for algorithms that assume acoustic wave behaviour. The analytical and numerical findings showed that this approach offers an improved resolution, which was also experimentally validated.

In chapter 2 of the thesis, the theoretical background of guided waves, SH guided waves, guided wave tomography and 3D FE simulations was outlined.

Chapter 3 discussed the numerical investigation of SH guided wave scattering as well as SH-based guided wave tomography for pure SH1, multi-mode SH point sources as well as full-mode directional surface sources.

Chapter 4 presented the methods used in experiments and the results for three defect types, which includes an axi-symmetric defect, a rough laser-scanned defect with 20% wall loss at maximum depth and the same defect linearly scaled to up to 50% wall loss at maximum depth.

Having demonstrated the proof of concept of the method as well as improving resolution 2.4-fold in previous chapters, chapter 5 investigated a NUFFT-based frequency-wavenumber diagram for use in modal decomposition, ultimately trying to remove un-

wanted modal data, as well as for use in determining the true bulk shear wave velocity.

6.2 Main findings

One of the main limitations for A0 Lamb wave based guided wave tomography is its directional scattering behaviour failing to match the assumed acoustic omnidirectional pattern. This is limiting the scattering angles covered in through transmission, which leads to a significant reduction of resolution. This is the main reason why the extensive work on guided wave tomography done in the past [5, 9, 31, 34] has not been able to improve resolution beyond 1.5λ . Using Full-Waveform-Inversion (FWI), where an iteratively updated acoustic finite difference model is matched to measurements [16], is limited in very much the same way when relying on Lamb guided wave modes [13, 14].

In this thesis, the scattering pattern from SH1 showed a much greater match with the acoustic case, suggesting a higher suitability of SH1 for use with reconstruction algorithms that assume acoustic scattering, such as HARBUT. For the case of excitation at 200 kHz for instance, the matching scattering angles on through transmission suggested a resolution of up to $0.7\lambda_{SH1}$ which is significantly better than $1.5\lambda_{A0}$ for A0. The other important factor is that the wavelength of SH1 at the chosen excitation frequencies in this thesis is also much lower than the wavelength of A0 at the typical excitation frequency of 50 kHz. The more compatible SH1 scattering behaviour in combination with the smaller wavelength of SH1 are what enabled the highly accurate reconstructions in this thesis.

The ability to implement high-fidelity three-dimensional FE models to simulate the propagation of pure SH1 excitation in realistic mild-steel plates, and solving a full model with 120 excitations, has enabled very accurate reconstructions of both simple axisymmetric and real corrosion-like defect models. Crucially, the reconstruction error at maximum defect depth was below 5% and the global errors below 2% for all cases, confirming that pure SH1 through-transmission data comes very close to the theoretical resolution limit which may be achieved with acoustic scattering. This also confirms that the scattering analysis gives a good estimate of the achievable resolution.

The results from the experimental study validated the simulation based results and

provided a proof of concept. Here, accurate results were achieved for 20% maximum wall loss defects and reasonable results were achieved for a 50% wall loss defect. It was found that this was mainly due to the directionality of the transducer type as well as to the limited number of scattering angle coverage during reception.

Finally, the application of the NUFFT algorithm for modal decomposition through frequency wavenumber filtering was investigated. With a rudimentary SH0 window in the f - k domain, the SH0 components could be successfully filtered out without any time-domain windowing of the signals.

While the technique fails to be applicable to directional data due to the limited amount of receiver channels, the technique proved to be useful for determining the bulk shear velocity of the plates, revealing an anisotropic bulk shear velocity of the samples.

Overall, while directional transducers lead to acceptable results, it was found that point-source like transducers offer much higher stability, higher quality reconstructions and ultimately better resolution.

6.3 Future work

As mentioned above, this thesis served as a proof of concept for SH1 guided wave tomography. Given the advances made in this thesis there still remain some limitation that need to be addressed prior to industrial application of SH1 guided wave tomography.

For instance, the impact of limited view array configurations needs to be investigated thoroughly as this will inevitably lead to a loss in resolution. There have been some developments to account for them in the recent past [62,63] and a first step would be to apply these techniques to SH1 guided wave tomography data. Again, FE simulations will allow for an accelerated investigation.

Another limitation that needs to be addressed is EMAT size. In order to reduce acquisition time, it is imperative to receive at multiple locations simultaneously or shortly after another. Using the SH specific EMATs by GUL enabled the excellent results shown during experiments, however, their size will not allow for the employment of multiple receivers next to each other and therefore more development is required to reduce transducers.

6. Conclusions

Potentially this might require use of piezoelectric transducer arrays in order to increase the number of transducers.

Given the wealth of corrosion types and the readily available computational power these days, one possible future project would be to do a comprehensive study of various defect types, depths, frequencies and array set ups.

Another aspect worth considering is to harness the broadband signal information by investigating the combination of HARBUT reconstructions from several frequencies.

This study focused exclusively on isotropic mild steel plates but other materials such as aluminium or anisotropic materials such as CFRP or GFRP plates would be interesting to investigate. In the latter, delamination detection is an important problem in the aerospace industry.

Also, modelling of SH1 guided wave tomography on pipes will be of interest to take the technique further and to deal with any required adjustments due to the helical pathways. In some cases, pipe-supports are welded to the pipe which might be a particular challenging test of the performance of the technique.

Moreover, modelling a loaded pipe might become possible in the future which may be used to confirm the assumed lesser attenuative effect from a liquid loading on SH1 guided waves due to their nonexistent vertical displacement.

Concerning the NUFFT-based modal decomposition approaches, experimental verification from point source-like SH guided wave transducers would be of interest as well as experimental verification of the NUFFT-based f-k diagram velocity determination technique and comparison with other velocity determination methods.

The combination of Lamb-wave and SH guided wave based approaches might be helpful for the case of defects with greater depth when phase distortion increases even further. Here, the Lamb-wave based lower-resolution reconstructions might be useful when serving as background for the SH guided wave tomography reconstructions.

References

- [1] P. Cawley, F. Cegla, and M. Stone, “Corrosion Monitoring Strategies—Choice Between Area and Point Measurements,” *Journal of Nondestructive Evaluation*, vol. 32, pp. 156–163, 6 2013.
- [2] D. N. Alleyne and P. Cawley, “The excitation of Lamb waves in pipes using dry-coupled piezoelectric transducers,” *Journal of Nondestructive Evaluation*, vol. 15, pp. 11–20, 3 1996.
- [3] W. Wright, D. Hutchins, D. Jansen, and D. Schindel, “Air-coupled Lamb wave tomography,” *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, vol. 44, pp. 53–59, 1 1997.
- [4] K. R. Leonard, E. V. Malyarenko, and M. K. Hinders, “Ultrasonic Lamb wave tomography,” *Inverse Problems*, vol. 18, pp. 1795–1808, 12 2002.
- [5] A. Volker, A. Mast, J. Bloom, D. O. Thompson, and D. E. Chimenti, “Experimental Results of Guided Wave Travel Time Tomography,” in *Review of Progress in Quantitative Nondestructive Evaluation Vol. 29* (D. E. A. I. o. P. Thompson, D. O.; Chimenti, ed.), pp. 2052–2059, 2010.
- [6] P. Belanger, P. Cawley, and F. Simonetti, “Guided wave diffraction tomography within the Born approximation,” *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, vol. 57, pp. 1405–1418, 6 2010.
- [7] P. B. Nagy, F. Simonetti, and G. Instanes, “Corrosion and erosion monitoring in plates and pipes using constant group velocity Lamb wave inspection,” *Ultrasonics*, vol. 54, pp. 1832–1841, 9 2014.

- [8] P. Huthwaite and F. Simonetti, “High-resolution guided wave tomography,” *Wave Motion*, vol. 50, pp. 979–993, 7 2013.
- [9] A. J. Brath, F. Simonetti, P. B. Nagy, and G. Instones, “Guided Wave Tomography of Pipe Bends,” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 64, pp. 847–858, 5 2017.
- [10] T. Druet, B. Chapuis, and E. Moulin, “Feasibility of Passive Tomography of Extended Defects Using Ambient Elastic Noise Correlation,” *Physics Procedia*, vol. 70, pp. 640–643, 2015.
- [11] T. Druet, J.-L. Tastet, B. Chapuis, and E. Moulin, “Autocalibration method for guided wave tomography with undersampled data,” *Wave Motion*, vol. 89, pp. 265–283, 6 2019.
- [12] F. Simonetti and M. Y. Alqaradawi, “Guided ultrasonic wave tomography of a pipe bend exposed to environmental conditions: A long-term monitoring experiment,” *NDT & E International*, vol. 105, pp. 1–10, 7 2019.
- [13] J. Rao, M. Ratassepp, and Z. Fan, “Limited-view ultrasonic guided wave tomography using an adaptive regularization method,” *Journal of Applied Physics*, vol. 120, p. 194902, 11 2016.
- [14] J. Rao, M. Ratassepp, and Z. Fan, “Investigation of the reconstruction accuracy of guided wave tomography using full waveform inversion,” *Journal of Sound and Vibration*, vol. 400, pp. 317–328, 7 2017.
- [15] P. Huthwaite, “Evaluation of inversion approaches for guided wave thickness mapping,” *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 470, pp. 20140063–20140063, 4 2014.
- [16] P. Huthwaite, “Guided wave tomography with an improved scattering model,” *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science*, vol. 472, p. 20160643, 11 2016.

- [17] L. R. F. Rose and C. H. Wang, “Mindlin plate theory for damage detection: Source solutions,” *The Journal of the Acoustical Society of America*, vol. 116, pp. 154–171, 7 2004.
- [18] A. Rohde, M. Veidt, L. Rose, and J. Homer, “A computer simulation study of imaging flexural inhomogeneities using plate-wave diffraction tomography,” *Ultrasonics*, vol. 48, pp. 6–15, 3 2008.
- [19] L. R. Francis Rose, T. H. Wang, and C. H. Wang, “Mindlin plate theory for damage detection: Imaging of flexural inhomogeneities,” *The Journal of the Acoustical Society of America*, vol. 127, no. 754, 2010.
- [20] L. F. Rose, E. Chan, and C. H. Wang, “A comparison and extensions of algorithms for quantitative imaging of laminar damage in plates. I. Point spread functions and near field imaging,” *Wave Motion*, vol. 58, pp. 222–243, 11 2015.
- [21] E. Chan, L. F. Rose, and C. H. Wang, “A comparison and extensions of algorithms for quantitative imaging of laminar damage in plates. II. Non-monopole scattering and noise tolerance,” *Wave Motion*, vol. 66, pp. 220–237, 11 2016.
- [22] F. R. S. Lamb, “On waves in an elastic plate,” *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, vol. 93, pp. 114–128, 3 1917.
- [23] P. Huthwaite, R. Ribichini, P. Cawley, and M. J. S. Lowe, “Mode selection for corrosion detection in pipes and vessels via guided wave tomography,” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 60, pp. 1165–1177, 6 2013.
- [24] J. Miklowitz, *The theory of elastic waves and waveguides*. North-Holland series in applied mathematics and mechanics, Amsterdam ;: North Holland Pub. Co, 1976.
- [25] L. Rayleigh, “On the Free Vibrations of an Infinite Plate of Homogeneous Isotropic Elastic Matter,” *Proceedings of the London Mathematical Society*, vol. s1-20, pp. 225–237, 11 1888.
- [26] J. L. Rose, *Ultrasonic Guided Waves in Solid Media*. New York: Cambridge University Press, 2014.

- [27] J. Achenbach, *Wave Propagation in Elastic Solids*. Elsevier, 1975.
- [28] M. Born and E. Wolf, *Principles of Optics*. Cambridge: Cambridge University Press, 1999.
- [29] A. C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging*. Society for Industrial and Applied Mathematics, 1 2001.
- [30] K. R. Leonard and M. K. Hinders, “Multi-mode Lamb wave tomography with arrival time sorting,” *The Journal of the Acoustical Society of America*, vol. 117, pp. 2028–2038, 4 2005.
- [31] M. K. Hinders and J. C. P. McKeon, “Lamb Wave Tomography for Corrosion Mapping,” in *The Second Joint NASA/FAA/DoD Conference on Aging Aircraft*, pp. 732–740, 1999.
- [32] C. L. Willey, F. Simonetti, P. B. Nagy, and G. Instanes, “Guided wave tomography of pipes with high-order helical modes,” *NDT and E International*, vol. 65, pp. 8–21, 7 2014.
- [33] P. Belanger and P. Cawley, “Feasibility of low frequency straight-ray guided wave tomography,” *NDT and E International*, vol. 42, pp. 113–119, 3 2009.
- [34] P. Belanger, *Feasibility of Thickness Mapping using Ultrasonic Guided Waves*. PhD thesis, 2009.
- [35] P. Huthwaite, “Improving accuracy through density correction in guided wave tomography,” *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science*, vol. 472, p. 20150832, 2 2016.
- [36] P. Huthwaite and F. Simonetti, “High-resolution imaging without iteration: a fast and robust method for breast ultrasound tomography,” *The Journal of the Acoustical Society of America*, vol. 130, pp. 1721–1734, 9 2011.
- [37] M. Drozd, E. Skelton, R. V. Craster, and M. J. S. Lowe, “Modeling Bulk and Guided Waves in Unbounded Elastic Media Using Absorbing Layers in Commercial Finite Element Packages,” *AIP Conference Proceedings*, vol. 894, p. 87, 3 2007.

- [38] R. Carandente, J. Ma, and P. Cawley, “The scattering of the fundamental torsional mode from axi-symmetric defects with varying depth profile in pipes,” *The Journal of the Acoustical Society of America*, vol. 127, pp. 3440–3448, 6 2010.
- [39] P. Huthwaite, “Accelerated finite element elastodynamic simulations using the GPU,” *Journal of Computational Physics*, vol. 257, pp. 687–707, 1 2014.
- [40] A. Velichko and P. D. Wilcox, “Excitation and scattering of guided waves: Relationships between solutions for plates and pipes,” *The Journal of the Acoustical Society of America*, vol. 125, pp. 3623–3631, 6 2009.
- [41] J. Pettit, A. Walker, P. Cawley, and M. Lowe, “A Stiffness Reduction Method for efficient absorption of waves at boundaries for use in commercial Finite Element codes,” *Ultrasonics*, vol. 54, pp. 1868–1879, 9 2014.
- [42] C. Bingham, M. Godfrey, and J. Tukey, “Modern techniques of power spectrum estimation,” *IEEE Transactions on Audio and Electroacoustics*, vol. 15, pp. 56–66, 6 1967.
- [43] A. V. Oppenheim and R. W. Schaffer, *Discrete Time Signal Processing*. 2nd ed., 1998.
- [44] F. Simonetti, L. Huang, and N. Duric, “On the spatial sampling of wave fields with circular ring apertures,” *Journal of Applied Physics*, vol. 101, p. 083103, 4 2007.
- [45] P. Huthwaite, “Eliminating incident subtraction in diffraction tomography,” *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science*, vol. 472, p. 20160707, 11 2016.
- [46] B. Pavlakovic, M. Lowe, D. Alleyne, and P. Cawley, “Disperse: A General Purpose Program for Creating Dispersion Curves,” in *Review of Progress in Quantitative Nondestructive Evaluation*, pp. 185–192, Boston, MA: Springer US, 1997.
- [47] M. Lowe, “Matrix techniques for modeling ultrasonic waves in multilayered media,” *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, vol. 42, pp. 525–542, 7 1995.

REFERENCES

- [48] P. Huthwaite, M. Lowe, and P. Cawley, “Guided wave tomography performance analysis,” in *42nd Annual Review of Progress in Quantitative Nondestructive Evaluation*, vol. 1706, p. 020021, 2016.
- [49] C. A. Chua and P. Cawley, “Crack growth monitoring using fundamental shear horizontal guided waves,” *Structural Health Monitoring*, vol. 19, pp. 1311–1322, 9 2020.
- [50] J. Fong and T. Pialucha, “Quantitative Short Range Guided Wave System,” in *15th Asia Pacific Conference for Non-Destructive Testing*, 2017.
- [51] M. Clough, M. Fleming, and S. Dixon, “Circumferential guided wave EMAT system for pipeline screening using shear horizontal ultrasound,” *NDT & E International*, vol. 86, pp. 20–27, 3 2017.
- [52] M. Seher, P. Huthwaite, M. J. S. Lowe, and P. B. Nagy, “Model-Based Design of Low Frequency Lamb Wave EMATs for Mode Selectivity,” *Journal of Nondestructive Evaluation*, vol. 34, p. 22, 9 2015.
- [53] M. E. Davison, “The Ill-Conditioned Nature of the Limited Angle Tomography Problem,” *SIAM Journal on Applied Mathematics*, vol. 43, pp. 428–448, 4 1983.
- [54] J. Friel, “Sparse regularization in limited angle tomography,” *Applied and Computational Harmonic Analysis*, vol. 34, pp. 117–141, 1 2013.
- [55] R. W. Pratt, “The Interpretation of Space-Time Spectral Quantities,” *Journal of the Atmospheric Sciences*, vol. 33, pp. 1060–1066, 6 1976.
- [56] Y. Hayashi, “Space-Time Spectral Analysis of Rotary Vector Series,” *Journal of the Atmospheric Sciences*, vol. 36, pp. 757–766, 5 1979.
- [57] H. v. Storch and F. W. Zwiers, *Statistical Analysis in Climate Research*. Cambridge University Press, 2 1984.
- [58] N. A. C. Cressie and C. K. Wikle, *Statistics for spatio-temporal data*. Wiley series in probability and statistics, Hoboken, New Jersey: Wiley, 2011.

- [59] T. Hayashi and K. Kawashima, “Single Mode Extraction from Multiple Modes of Lamb Wave and Its Application to Defect Detection,” *JSME International Journal Series A*, vol. 46, no. 4, pp. 620–626, 2003.
- [60] S. F. Potter, N. A. Gumerov, and R. Duraiswami, “Fast interpolation of bandlimited functions,” *ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing - Proceedings*, pp. 4516–4520, 6 2017.
- [61] A. Dutt and V. Rokhlin, “Fast Fourier Transforms for Nonequispaced Data,” *SIAM Journal on Scientific Computing*, vol. 14, pp. 1368–1393, 11 1993.
- [62] P. Huthwaite, A. A. Zwiebel, and F. Simonetti, “A new regularization technique for limited-view sound-speed imaging,” *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, vol. 60, pp. 603–613, 3 2013.
- [63] F. Shi and P. Huthwaite, “Ultrasonic wave-speed diffraction tomography with under-sampled data using virtual transducers,” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 65, pp. 1226–1238, 7 2018.

This page intentionally left blank

List of Publications

Written publications

- [P1] A.A.E. Zimmermann, P. Huthwaite and B. Pavlakovic. “High-resolution thickness maps of corrosion using SH1 guided wave tomography” in *Proc. R. Soc. A, Vol. 477: 20200380* <http://doi.org/10.1098/rspa.2020.0380>

Authors’ contributions: All authors contributed to the writing of [P1] and A.A.E.Z. wrote the bulk of the paper. B.P. provided access to and support for experimental equipment and data capturing. P.H. provided substantial contributions to conception and design of the model and interpretation of the results. A.A.E.Z. performed simulations, experiments, method development and implementation as well as data analysis. All authors approved the final version and agreed to be accountable for all aspects of the work.

Separate oral presentations

- [P2] A.A.E. Zimmermann, P. Huthwaite and B. Pavlakovic. “Investigation of shear-horizontal guided wave tomography”, at *18th Anglo-French Physical Acoustics Conference (AFPAC)*, Fréjus, France. 2019.
- [P3] A.A.E. Zimmermann, P. Huthwaite and B. Pavlakovic. “SH1 guided wave tomography” at *46th Review of progress in quantitative nondestructive evaluation*, Portland, OR, USA. 2019.
- [P4] A.A.E. Zimmermann, P. Huthwaite and B. Pavlakovic. “SH1 for high resolution and accurate guided wave tomography mapping of corrosion at pipe supports”, at *NDT 2019 - 58th Annual Conference*, Telford, UK. 2019.

REFERENCES

- [P5] A.A.E. Zimmermann, P. Huthwaite and B. Pavlakovic. “High-resolution corrosion thickness mapping using SH1 guided wave tomography”, at *NDT 2020 - 59th Annual Conference*, Telford, UK. 2020.