

EXPERIMENTAL STUDIES OF ARRAY SYSTEMS USING SOFTWARE DEFINED RADIO TESTBED

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The logo features a dark blue background with a white wave-like shape at the bottom. The text "Imperial College London" is written in a dark blue, sans-serif font, with "Imperial College" on the first line and "London" on the second line.

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Abstract

This thesis is concerned with multiple-antenna wireless communication systems, initially, a forward-backward autoregressive process is presented to predict the downlink channel of a LTE-TDD multiuser MIMO system. With the proposed channel prediction algorithm, an integrated block diagonalisation (BD) precoding scheme is proposed for improving the sum system capacity in a time-variant Rayleigh fading channel environment based on a Jakes model. Furthermore, a hybrid testbed, constructed using National Instrument PXIe platform, was used to compare the performance with that of the conventional BD precoding method where channel prediction is not present.

Then various novel synchronisation and calibration schemes are presented and studied using another software defined radio (SDR) antenna array testbed which includes an array of USRP boards manufactured by Ettus Research and National Instrument. This testbed provides a flexible platform to perform a wide range experimental array processing studies with different geometries and a varying number of antennas. Furthermore, various localisation algorithms are evaluated within a controlled anechoic environment as well as outdoors. In particular, large aperture array (LAA) localisation techniques and multiple signal classification (MUSIC) techniques are studied for locating the position of a signal source operating in both in the far field and in the near-far field of the array. Finally, this thesis presents a novel array element pattern synthesis algorithm. Throughout the thesis, extensive experiments are presented to analyse the performance of the developed algorithms using an array SDR testbed made of off-the-shelf antennas.

Declaration of Originality

I hereby declare that this thesis is my own work. Where other sources of information have been used, they have been acknowledged.

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Notation

a, A	Scalar
$\underline{a}, \underline{A}$	Column Vector
$\mathbf{a}, \mathbb{A}$	Matrix
$(\cdot)^T$	Transpose
$(\cdot)^*$	Conjugate
$(\cdot)^H$	Hermitian transpose
$\widehat{(\cdot)}, \widetilde{(\cdot)}$	Nominal values and uncertainties
$\exp(\underline{A})$	Element by element exponential of vector $\underline{A}$
$\underline{A}^b$	Element by element power
$\mathbb{A}^\#$	Pseudo inverse of matrix $\mathbb{A}$
$\text{diag}(\underline{A})$	Diagonal matrix whose diagonal elements are elements of $\underline{A}$
$\underline{\text{diag}}(\mathbb{A})$	Vector containing the diagonal elements of matrix $\mathbb{A}$
$\text{diag}(\mathbb{A})$	Diagonal matrix whose diagonal elements are diagonal elements of $\mathbb{A}$
$\text{trace}(\mathbb{A})$	Sum of the diagonal elements of matrix $\mathbb{A}$
$\text{row}_k(\mathbb{A})$	k^{th} row of the matrix $\mathbb{A}$
$\text{col}_k(\mathbb{A})$	k^{th} column of the matrix $\mathbb{A}$
$ A $	Absolute value
$\ \underline{A}\ $	Euclidean norm of vector $\underline{A}$
$\ \mathbb{A}\ _F$	Frobenius norm of matrix $\mathbb{A}$
$\mathcal{E}\{\cdot\}$	Expectation operator
$\mathcal{L}(\mathbb{A})$	Linear subspace spanned by the columns of matrix $\mathbb{A}$
$\mathcal{L}(\underline{A})$	Linear subspace spanned by vector $\underline{A}$
$\approx$	Approximately equal

$\odot$	Hadamard product
$\oslash$	Hadamard division
$\circledast$	Convolution
$\forall$	For all
$\underline{0}_N$	Column vector of N zeros
$\underline{1}_N$	Column vector of N ones
$\mathbb{I}_N$	$N \times N$ Identity matrix
$\mathbb{O}_{N \times M}$	$N \times M$ matrix of zeros
$\mathcal{R}$	Set of real numbers
$\mathcal{C}$	Set complex numbers
pdf	Probability density function
$\arg(\cdot)$	Argument

Abbreviations

ADC	Analog-to-Digital Converter
AR	Autoregressive
AOA	Angle of Arrival
AWGN	Additive White Gaussian Noise
BD	Block Diagonalization
BER	Bit Error Rate
BS	Base Station
CDMA	Code Division Multiple Access
CP	Cyclic Prefix
CRB	Cramer-Rao Bound
CSI	Channel State Information
DDC	Digital Down-Converter
DF	Direction Finding
DFT	Discrete Fourier Transform
DOA	Direction of Arrival
DUC	Digital Up-Converter
EGC	Equal Gain Combining
DPC	Dirty Paper Coding
EM	Electromagnetic
EPR	Element Pattern Reconstruction
EPSVA	Element Pattern Synthesis using Virtual Arrays
EVA	Extended Vehicular A model

FDD	Frequency Division Duplexed
FIR	Finite Impulse Response
FFT	Fast Fourier Transform
FPGA	Field-programmable Gate Array
GPS	Global Positioning System
IBI	Inter Block Interference
IFFT	Inverse Fast Fourier Transform
ISI	Inter Symbol Interference
LAA	Large Aperture Array
LS	Least Squares
LTE	Long-Term Evolution
MAI	Multiuser Access Interference
MCM	Mutual Coupling Matrix
MMSE	Minimum Mean Square Error
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MIVO	Multiple-Input Vector-Output
MRC	Maximal Ratio Combining
MU	Multi-User
MUSIC	Multiple Signal Classification
OFDM	Orthogonal Frequency Division Multiplexing
PAPR	Peak-to-Average Power Ratio
PPS	Pulse Per Second
QPSK	Quadrature Phase Shift Keying
SIMO	Single-Input Multiple-Output
SISO	Single-Input Single-Output
SIVO	Single-Input Vector-Output

SNIR	Signal to Noise plus Interference Ratio
SNR	Signal to Noise Ratio
ST	Space Time
SVD	Singular Value Decomposition
Tx	Transmitter
TDL	Tapped Delay Line
TDD	Time Division Duplex
THP	Tomlinson-Harashima Precoding
TOA	Time of Arrival
UCA	Uniform Circular Array
UE	User Equipments
ULA	Uniform Linear Array
URA	Uniform Rectangular Array
Rx	Receiver
USRP	Universal Software Radio Peripheral
RMSE	Root Mean Square Error

Chapter 1

Introduction

In recent years, the ever increasing demand of high data rate transmission has led to a strong interest in multiple-input multiple-output (MIMO) systems. It has been shown in [1, 2, 3] that the channel capacity of a MIMO system with N_{TX} transmit antennas and N_{RX} receive antennas, in principle, grows linearly by the factor of $\min(N_{TX}, N_{RX})$ without increasing transmission power or bandwidth. In practice, the performance of MIMO systems is influenced by the received signal-to-noise-plus-interference ratio (SNIR) and correlation properties of the MIMO channel. To improve the system performance, signal processing techniques at the transmitter, known as precoding techniques, are often used. Several precoding techniques have been successfully adopted by the 3GPP Long Term Evolution (LTE) specifications [4]. The main difficulty of spatial multiplexed MIMO systems is the separation of the independent parallel data streams with limited channel state information (CSI) available at the transmitter side [5]. In [6], it is shown that the capacity of the MIMO channel can be achieved by using nonlinear precoding.

In frequency division duplexed (FDD) LTE systems, where uplink and downlink channels are in the separated frequencies, feedbacks from the receivers are used to obtain the CSI at the transmitter. However, the feedback information size grows exponentially as the number of antennas of the system increases. As in time division duplexed (TDD) LTE systems, where uplink and downlink channels share

the same transmission frequency, the CSI can be obtained using the reciprocal property of the channel. The problem with such a system is that the transmitter may have the outdated CSI due to the time varying property of the channel. In order to keep track of the channel change, in [7, 8, 9] Kalman-based algorithms are proposed for channel estimation and prediction of multi-user (MU) MIMO orthogonal frequency-division multiplexing (OFDM) systems. Furthermore, by using reference signals, Kalman filter can be trained to adapt to various channel fading characteristics with improved channel estimation and prediction performance [8].

Recent research has been shifted to massive MIMO systems to achieve higher spectral efficiency and higher link reliability [10, 11]. However, these technologies are not scalable from conventional MIMO systems to massive ones due to the large CSI overhead as the number of antennas increases. On the other hand, with the knowledge of the array geometry and the characteristics of the array elements, this information can be estimated in the form of so-called *array manifold* vector [12]. The array manifold vector (also known as source position vector) relies on the accurate information on the location of the sensors (array geometry), the carrier frequency and other array elements characteristics to provide reliable channel estimation. As the array geometry and the characteristics of the array elements are utilised, the cost of the pilot signal for channel estimation does not increase with the number of antenna elements.

The most important and widely used family of direction finding techniques is the family of subspace-type techniques which have super-resolution capabilities. Multiple Signal Identification and Classification (MUSIC) algorithm [13] is the most popular subspace technique and can be used to estimate various source parameters such as direction-of-arrival (DOA), time-of-arrival and transmission frequency without being impacted by the SNR on the array resolution. Other super-resolution techniques have also been proposed to estimate the DOAs of unknown signal sources such as the maximum likelihood localisation algorithm [14] and the ESPRIT algorithm [15]. Nonetheless, MUSIC and other signal-subspace

based estimation techniques [16, 17, 18, 19, 20] need precise knowledge of array response to provide good performance. However, in real systems there are various electrical and geometrical uncertainties (or errors). This thesis is concerned with various system uncertainties, and special attention is given to gain, phase, mutual coupling and the local oscillator (LO) uncertainties.

1.1 Precoding Techniques at Transmitter

In single user MIMO (SU-MIMO), precoding maps N_s data streams to N_{TX} transmitter antennas with appropriate weighting so that the throughput at the receiver output is maximized. Usually, the channel state information (CSI) at transmitter is essential for precoding.

Precoding algorithms are often sub-divided into linear and non-linear precoding strategies. Linear precoding (often called linear pre-equalisation) schemes such as zero-forcing (ZF) precoding [21, 22] and MMSE precoding [23] are well-known linear precoding methods that achieve reasonable performance with much lower complexity than non-linear precoding strategies. Transmitter pre-equalization usually outperforms the receiver side equalization due to the fact that the receiver side equalization suffers from noise enhancement [24]. The simulated bit-error-rate (BER) results in [24] also shows that MMSE pre-equalization outperforms receiver side MMSE equalization. Furthermore, a traditional receiver combining technique, the maximum ratio combining (MRC), has also been applied in [25] to transmitter, known as the maximum ratio transmission scheme (MRT), which has been shown to maximise the sum SNR.

If the channel state information is completely known by the transmitter side, singular value decomposition (SVD) precoding is used in [24] to achieve the MIMO channel capacity. It shows that the channel can be diagonalized through pre-processing and post-processing the two unitary matrices from SVD at the transmitter and receiver respectively. Then the MIMO channel can be re-visualized

as a set of virtual SISO channels each carries an independent data stream. With an appropriate power allocation algorithm, such as water filling, one data stream per SISO channel can be transmitted without creating any interference. However, SVD-based equalization cannot be used in the multi-user MIMO scenario. Instead, we use SVD to compare the usable channel capacity with multi-user MIMO precoding techniques.

A more common use of MIMO systems is when multiple users sharing the same radio resources as shown in [26]. It considers a multi-user MIMO system downlink channel (also called broadcast channel) with a single base station (BS) and K independent users. Suppose the BS is equipped with N_B antennas and each user is equipped with N_M antennas, the point-to-point system for downlink can be visualized as a $(N_B \times KN_M)$ MIMO system. This configuration results in an increasing in degrees of freedom as oppose to the single-user MIMO case.

The main consideration for broadcast channel in multiple-user MIMO (MU-MIMO) transmission is the interference cancellation at the BS. Four popular transmission precoding strategies are studied in [24]. These are: channel inversion, block diagonalization, dirty paper coding (DPC), and Tomlinson-Harashima precoding (THP).

Channel inversion precoding method presented in [27, 28] considers a MU-MIMO system with K users that each has one receiver antenna and the number of users equals to the number of antenna at BS, $K = N_B$. The system can be then considered to be the same as the SU-MIMO case. Channel inversion and regularized channel inversion are the same processes as ZF pre-equalization and MMSE pre-equalization for SU-MIMO. In this method, all signals other than the target signal are considered as interference and are cancelled from received signals. Since channel inversion processes cancel inter-antenna interference as well as inter-user interference in its own signal, noise enhancement becomes more severe for the target user hence resulting in decreasing in performance.

Block diagonalization (BD) [29, 30] method considers a more general case with

multiple users that each equips with multiple antennas. As oppose to channel inversion methods, only the interference from other users is cancelled in BD. The idea of BD is based on beam steering so that interference is nulled at the undesired users. Various signal detection processes, such as zero forcing (ZF) detection and MMSE detection, can be used to cancel the inter-spatial stream interference for each user. In BD, an effective channel matrix is constructed between BS and each user. The aim of BD method is to find a set of precoding matrices that are null-space of the effective channel matrix except for the target user. In other words, interference-free transmission can be realized if the effective channel matrix can be block-diagonalized. To design precoders, BD assumes total number of antennas of all users equal to the number of antennas of BS and also assumes all users have equal number of antennas on each mobile device. Then SVD can be performed on the downlink interference channel matrix for each user. We then take the right-singular vectors that correspond to vanishing (zeros) singular values of the channel matrix to produce the precoding matrix for the each user since these vectors span the null space of the interference channel matrix.

Dirty paper coding (DPC) [31, 32, 33] method proposes that virtual interference-free channels can be realized by subtracting the potential interferences before transmission provided the CSI is completely known by the transmitter. The idea of DPC in transmission is very similar to successive interference cancellation (SIC) in reception. The interferences due to the first up to $(k - 1)^{th}$ user can be cancelled successively from the k^{th} user by the mean of precoding. In DPC, LQ-decomposition is performed on the channel matrix of channel gains between BS and all users. The broadcast channel matrix can be then LQ-decomposed into a lower-triangular matrix $\mathbb{L}$ and an orthogonal matrix $\mathbb{Q}$. It can be seen that, by transmitting $\mathbb{Q}^H \underline{x}$, where $\underline{x}$ is the precoded user data, the effect of $\mathbb{Q}$ in the channel matrix can be eliminated leaving only the lower-triangular matrix L at receiver. The interference-free transmission can be realized if precoding weight matrix W can be found such that it eliminates all elements in $\mathbb{L}$ except for the diagonal ele-

ment, $\mathbb{L}\mathbb{W} = \text{diag}(\mathbb{L})$. The DPC precoding matrix is then a scaled inverse matrix of $\mathbb{L}$ such that $\mathbb{W} = \mathbb{L}^{-1}\text{diag}(\mathbb{L})$. Conjugate transposed orthogonal matrix $\mathbb{Q}^H$ should be used on top of the DPC precoding matrix $\mathbb{W}$ for transmission, or can be used at receiver for detection.

Tomlinson-Harashima (TH) precoding [34, 35, 36] is equivalent to the combination of DPC with symmetric modulo operation. The idea of TH precoding is to reduce the transmission power of DPC by bounding all user signals within the constellation map. The successive interference cancellation algorithm of DPC is performed on symmetric modulo version of previous signals in TH precoding. In comparison of DPC and TH-precoding, DPC outperforms THP in BER analysis. However, THP is transmitted with less power than DPC due to its modulo operation in the precoding process. [37] also shows how THP algorithms for MU-MISO OFDM systems can be implemented on an FPGA.

In [6], the TH precoding for uncoded (modulation scheme independent) MU-MIMO broadcasting systems is examined over linear pre-equalization schemes. It shows that non-linear precoding scheme-TH precoding, offers significant advantages over linear pre-equalisation schemes and outperforms popular the decision-feedback post-equalization scheme-Bell Laboratory Layered Space-Time (BLAST) systems [6], at the receiver, which suffers from noise enhancement. The study assumes perfect channel information is known by the transmitter, and assumes the MIMO channel is block fading over a reasonable blocklength. The capacity of THP is derived and is compared with the optimum precoding scheme for SU-MIMO, SVD. It shows that THP system capacity asymptotically converges with that of SVD at high SNRs. In other words, TH precoding is capable of using the underlying MIMO channel capacity that is described by SVD. Simulation results in [6] for different equalization strategies - THP, V-BLAST, SVD and Linear-precoding, using optimum rate and power loading as described in [38] and [39], also show that the substantial performance improvements are possible by optimally distributing rate and power over the independent spatial channels in precoding systems.

The main difficulty of spatial multiplexed MIMO systems is the separation of the independent parallel data streams with limited channel state information (CSI) available at the transmitter side [5]. Also, it is shown in [6], that the entire channel capacity of the MIMO channel can be achieved by using nonlinear precoding. In frequency division duplexed (FDD) LTE systems, where uplink and downlink channels are in the separated frequencies, feedbacks from the receivers are used to obtain the CSI at the transmitter. However, the feedback information size grows exponentially as the number of antennas of the system increases. As in LTE time division duplexed (TDD) systems, where uplink and downlink channels share the same transmission frequency, the CSI can be obtained using the reciprocal property of the channel. The problem with such a system is that the transmitter may have the outdated CSI due to the time varying property of the channel. Furthermore, how AR-based algorithms for channel estimation and prediction could be used in MIMO OFDM systems has been reported in [8] and [9]. On the other hands, various channel modelling approaches are studied in [40] showing that the least-squares (or forward-backward) AR-modelling has better performance than that of Yule-Walker algorithm as used in [8] and [9] in low AR orders.

1.2 Array Manifold and Uncertainties

Array manifold uncertainties refer to the gain, phase mutual coupling and geometrical errors in the antenna array system. Even when carefully picked identical antennas, cables, amplifiers and digitisers are deployed to form an array in precisely measured locations, the array may still have uncertainties due to manufacturing impairments of the components. Moreover, other factors such as aging, motion, temperature and installation error, will cause the gain and phase of each array element to fluctuate. On the other hand, if there are more than one LO distributing the clocks to all array elements, the array becomes incoherent. To achieve a coherent array system, these hardware dependent uncertainties must be estimated

and removed first. As aforementioned, super-resolution DOA finding algorithms are very sensitive to array uncertainties [41].

The unresolved modelling errors will severely degrade the capabilities of such systems. To demonstrate the significance of the modelling error on accuracy and sharpness of the MUSIC algorithm, Figure 1.1 shows the MUSIC spectrum when gain and phase perturbation is present on each element of an 8-element Uniform Circular Array (UCA) with half-wavelength inter-antenna spacing. The array operates in the presence of two far-field sources having azimuth angles of 65° and of 95° with respect to the array reference point. The gain and phase error is assumed to follow a uniform distribution with zero mean and variance equals to 0.01.

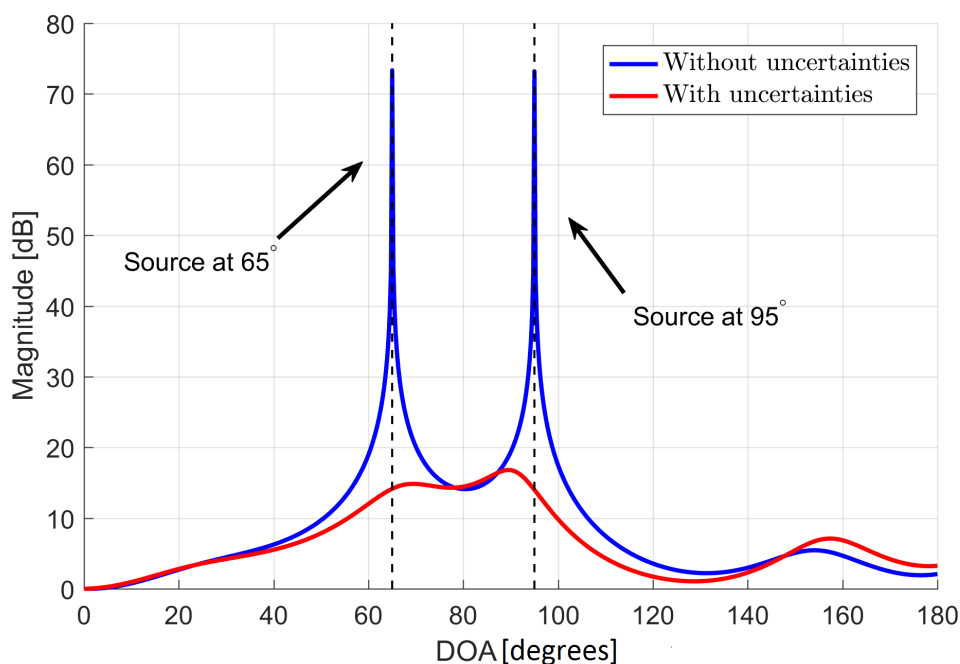


Figure 1.1: Example of the effect of antenna gain and phase errors on MUSIC.

It is clear in Figure 1.1 that in the presence of the array uncertainties, MUSIC fails to estimate the DOAs of the sources completely. Moreover, with more transmitting sources deployed in the presence of both gain, phase, mutual coupling and geometrical uncertainties as well as LO errors, the performance of MUSIC will only be more degraded.

Luckily, the modelling error of antenna array systems can be estimated and removed using calibration techniques [42, 43, 44, 45, 46]. One class of these calibration techniques make use of received signals that either correspond to the known or unknown transmitting sources so-called pilot sources to estimate the true array parameters. This thesis is focused on calibrating arrays using pilot sources.

1.3 Signal Modelling for Coherent Arrays

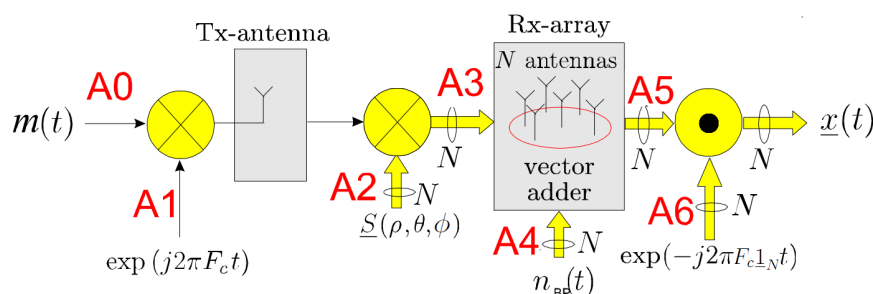


Figure 1.2: The single source line-of-sight (LOS) coherent array model

Consider an Rx-array of N coherent elements operating in the presence of a single narrowband source at location denoted by θ , ϕ and ρ , where θ is the azimuth angle in radius, ϕ is the elevation angle and ρ is the range of the source with respect to the array reference point as shown in Figure 1.2. In the case of a single line-of-sight (LOS) path, the received antenna array signal vector $\underline{x}(t) \in \mathcal{C}^{N \times 1}$ is modelled as:

$$\underline{x}(t) = \underline{S}(\rho, \theta, \phi) m(t) + \underline{n}(t), \quad (1.1)$$

where $m(t)$ is the signal from the source impinging on the array and $\underline{n}(t) \in \mathcal{C}^{N \times 1}$ is the baseband noise vector. Without loss of generality, the noise is assumed to be additive white Gaussian noise (AWGN) of zero mean and has its covariance matrix defined as $\sigma_n^2 \mathbb{I}_N$. It is worth noting that this thesis will focus on the LOS path model. In the presence of multipath, it is possible to remove non-LOS paths

by placing a pre-processor on the front-end of the system. However, resolving and separating these undesired non-LOS signal paths usually comes at a cost of increased signal bandwidth and computational complexity.

In Equation (1.1), the vector $\underline{S}(\rho, \theta, \phi) \in \mathcal{C}^{N \times 1}$ is the well-known *array manifold* vector or *array response* vector which can be represented as follows:

$$\underline{S}(\rho, \theta, \phi) = (\rho \cdot \underline{1}_N \odot \underline{\rho})^\alpha \odot \exp \left\{ -j \frac{2\pi}{\lambda} (\rho \cdot \underline{1}_N - \underline{\rho}) \right\}, \quad (1.2)$$

where $\odot$ and $\oslash$ denote Hadamard multiplication and Hadamard division operators respectively,

$$\underline{\rho} \triangleq \sqrt{\rho^2 \cdot \underline{1}_N + \text{diag}\{\mathbf{r}^T \mathbf{r}\} - \frac{\lambda}{\pi} \rho \mathbf{r}^T \underline{k}(\theta, \phi)} \in \mathcal{R}^{N \times 1}, \quad (1.3)$$

and α is the path loss exponent and $\mathbf{r}$ is the matrix with columns being the Cartesian coordinate of array elements. That is,

$$\mathbf{r} = [\underline{r}_1, \underline{r}_2, \dots, \underline{r}_N] = [\underline{r}_x, \underline{r}_y, \underline{r}_z]^T \in \mathcal{R}^{3 \times N}. \quad (1.4)$$

Furthermore, $\underline{k}(\theta, \phi) \in \mathcal{R}^{3 \times 1}$ denotes the wavenumber vector, defined as,

$$\underline{k}(\theta, \phi) \triangleq \frac{2\pi F_c}{c} \begin{bmatrix} \cos \theta \cos \phi \\ \sin \theta \cos \phi \\ \sin \phi \end{bmatrix}, \quad (1.5)$$

where c is the speed of light constant and F_c is the carrier frequency.

The appropriate propagation model to use can be dependant upon whether the source is located in the near-far or far field of the array. Consider a point source that undergoes a spherical wave propagation as described in Equation (1.2), the gain and phase of the signal received at each array element is related to the range of the source and the array, i.e. the $\underline{\rho}$ vector in Equation (1.3). However, as the source moves further away from the array, the difference in the propagation losses between the array elements becomes smaller. In the other words, as $\rho \rightarrow \infty$, the difference in the distances between the source and the array elements becomes

negligible and the wavefront can be approximated as planar. In this case, the array manifold vector in Equation (1.2) obeys a plane wave propagation model, that is,

$$\underline{S}(\theta, \phi) = \exp(-j\mathbf{r}^T \underline{k}(\theta, \phi)). \quad (1.6)$$

The source can be approximated to operate in the far field of the array and undergoes plane wave propagation if the following inequality is satisfied,

$$\rho \gg \frac{2D^2 F_c}{c} \quad (1.7)$$

where D is the array aperture defined as the largest distance between any two array elements.

1.4 Signal Modelling for Non-coherent Arrays

Consider the case, in which the array is configured such that each of the array elements have an independent LO. In this case, the uncertainties arise because each of the LOs are not tuned precisely to the same frequency due to the tolerances in the electronics, specially the clock reference sources.

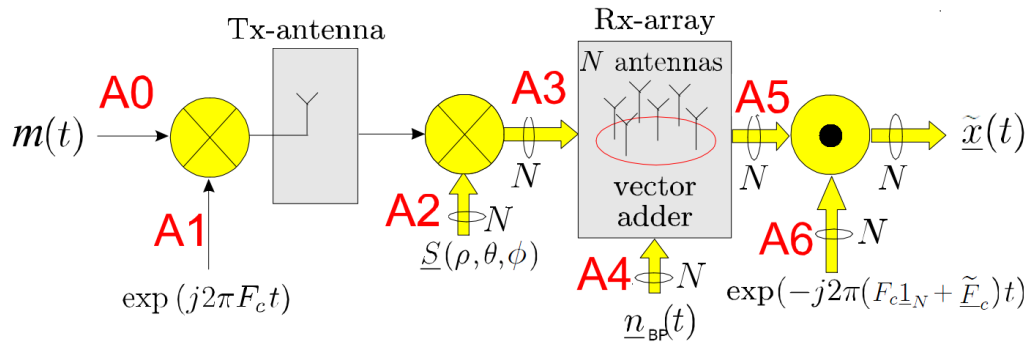


Figure 1.3: The single source line-of-sight (LOS) non-coherent array model

With reference to Figure 1.3, these frequency offsets are system uncertainties and can be modelled by

$$\underline{\tilde{F}}_c = [\tilde{F}_{c,1}, \tilde{F}_{c,2}, \dots, \tilde{F}_{c,N}]^T \in \mathcal{R}^{N \times 1} \quad (1.8)$$

where $\tilde{F}_{c,i}$ denotes the LO uncertainty associated with the i^{th} element and the received baseband antenna array signal vector $\tilde{\underline{x}}(t) \in \mathcal{C}^{N \times 1}$ can be modelled as:

$$\begin{aligned}\tilde{\underline{x}}(t) &= \underline{S} \odot \exp\left(-j2\pi\left(F_c \underline{1}_N + \tilde{\underline{F}}_c\right)t\right) \exp(j2\pi F_c t) m(t) \\ &\quad + \exp(-j2\pi\left(F_c \underline{1}_N - \tilde{\underline{F}}_c\right)t) \odot \underline{n}_{BP}(t) \\ &= \underline{S} \odot \exp(-j2\pi \tilde{\underline{F}}_c t) m(t) + \tilde{\underline{n}}(t),\end{aligned}\tag{1.9}$$

where $\underline{S}$ is given by Equations (1.2) or (1.6). The term $\underline{n}_{BP}(t) \in \mathcal{C}^{N \times 1}$ and $\tilde{\underline{n}}(t) \in \mathcal{C}^{N \times 1}$ are the bandpass noise vector and the baseband noise vector respectively. Without any loss of generality, the noise, $\tilde{\underline{n}}(t)$, is assumed AWGN and is redefined as $\underline{n}(t) \in \mathcal{C}^{N \times 1}$.

From the above discussion, it is clear that the estimation of $\tilde{\underline{F}}_c$ crucial in the case of non-coherent array. Once the LO frequency offsets have been estimated, they can be easily removed from Equation (1.9) via multiplication to produce the coherent signal model given in Equation (1.1), that is,

$$\begin{aligned}\underline{x}(t) &= \tilde{\underline{x}}(t) \odot \exp(j2\pi \tilde{\underline{F}}_c t) \\ &= \underline{S} m(t) + \underbrace{\tilde{\underline{n}}(t) \odot \exp(j2\pi \tilde{\underline{F}}_c t)}_{\text{redefined as } \underline{n}(t)} \\ &= \underline{S} m(t) + \underline{n}(t).\end{aligned}\tag{1.10}$$

It is important to note that the phase uncertainties introduced due to the LOs have the same effect on the array system as the phase uncertainties that arise due to uncertainties in the array sensors and electronics. Thus, they are modelled as part of the array manifold vector $\underline{S}$ for the purposes of this thesis.

1.5 Array Synchronisation

In traditional array systems, there is a single common local oscillator signal for all the RF front-end elements of the array. In [47], the author demonstrated a hardware synchronisation technique that a collection of individual software defined radio (SDR) units can be successfully synchronised using a common clock

reference to form a coherent array system. Figure 1.4 shows a coherent array system constructed from a collection of individual USRP[48] based SDR units.

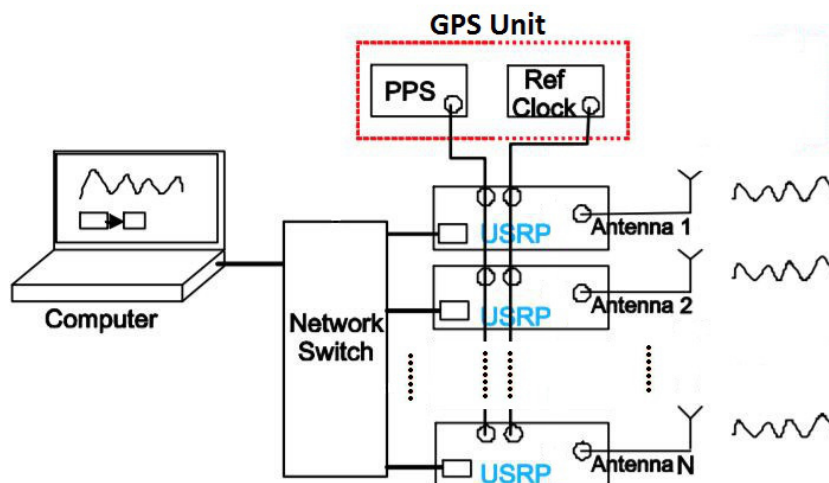


Figure 1.4: Array testbed setup comprising multiple USRP boards that are synchronised (hardware synchronisation) using a common clock reference and 1 PPS reference source.

Note that hardware synchronisation is not possible in the case of a large aperture array system where its elements are distributed in space having large distances between their elements. This implies each array element will have its own LO source operating independently from each other. In reality, these LOs may be locked to their own local reference clock sources which may have frequency uncertainties in the region of approximately 5kHz to 50kHz. Or these LOs may be locked using individual GPS units, which in turn locked to one or many satellites, as shown in Figure 1.5. However, these LOs will still contain small amounts of residual frequency offsets in the region of approximately 100Hz which have to be removed to obtain a fully coherent array system. In [45], the author proposed a pre-processor to estimate and to remove frequency offsets $\tilde{F}_c$ associated with the LO's. The proposed technique employs harmonic retrieval modelling in conjunction with a frequency based MUSIC algorithm for estimating the frequency offsets $\tilde{F}_c$. Once $\tilde{F}_c$ have been estimated, the array can be synchronised and Equation

(1.10) becomes valid, reflecting the received signal vector $\underline{x}(t)$ of a coherent array system of N antennas

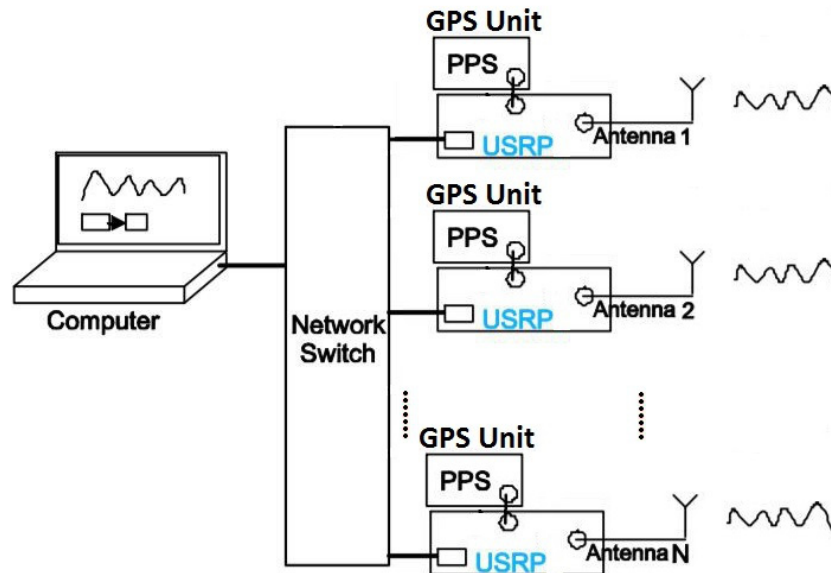


Figure 1.5: Array testbed setup comprising multiple USRP boards that are synchronized using GPS unit generated PPS clocks.

1.6 Array Calibration

A simple but tedious array calibration approach is to measure the array response of the field of interest at regular intervals in a controlled environment, such as an anechoic chamber. This is an offline process and requires interpolation to estimate the array response between quantisation points. An alternative approach to alleviate this problem is to treat array uncertainties as a parameter estimation problem. The author in [49] proposed an algorithm to estimate the array calibration matrix using a set of pilot sources in known locations. The array calibration matrix describes the combined effects of array electrical characteristics - mutual coupling and sensor gain and phase responses. Another category of array calibration is known as self-calibration or online calibration. As the name suggests, self-calibration does not require any additional sources to estimate the array prop-

erties. In [50, 51, 52], self-calibration algorithms for uniform linear array (ULA), uniform rectangular array (URA) and uniform circular array (UCA) are proposed respectively to estimate the array electrical properties. In [53], an algorithm to estimate array mutual coupling for arbitrary array is proposed which utilizes one addition source prior to array self-calibration. Also, in [54], a self-calibration algorithm for UCA has been proposed to simultaneously estimate the DOAs and coupling coefficients by using prior knowledge of the special structure of mutual coupling matrix (MCM).

A general pilot array calibration method is proposed in [55] for calibrating a planar array with respect to the unknown array location, phase and gain parameters. This is an offline calibration technique that requires three pilot sources from at least two different azimuth angles. An auto-calibration algorithm for an array is also proposed in [42] concerning with estimating geometrical, gain and phase uncertainties. As the name suggests, auto-calibration does not require external sources as in the case of pilot and self-calibration algorithms but requires the array elements to be capable of operating as transceivers. In [56], a pilot calibration technique that is capable of estimating location, gain, phase as well as mutual coupling parameters is proposed for a planar array. This is a pilot based offline method that requires three calibration sources. A maximum-likelihood calibration algorithm is proposed in [57] to estimate all types of array parameters using a large number of calibration sources.

In some practical array systems, the signal might get reflected, re-radiated or scattered by the other elements or obstacles within the radiative near field of the antennas. This includes antennas mounted on a complex environments such as buildings, airplanes, ships and ground vehicles. The interaction between the antennas and the obstacles can lead to severe coupling effect and hence alters the array characteristics such as antenna gain and phase patterns. In other words, the electromagnetic (EM) properties of a particular antenna in an array are influenced by the other elements in the proximity and influence the other elements at the

same time[58]. This influence usually depends on various factors, such as, antenna and obstacle types, spacing, antenna radiation patterns, bandwidth, coupling of the feed network, and so on. As a result, the element patterns are perturbed and will no longer be identical. Several studies have been made on the effect of coupling and different types of arrays and have attempted to predict the EM properties such as gain and phase pattern of antennas in complex environments[59, 60, 61, 62, 63, 64, 53].

Recently, a novel element pattern reconstruction (EPR) method was presented in [65] to calibrate the mutual coupling effect of a fixed shape array such as microstrip conformal antenna arrays. This method is based on the fact that the reconstructed element patterns can be transformed to the corresponding element patterns in their isolated states without the coupling effect. The modified EPR method was later proposed in [66] to cover wide-angle range by using multiple transformation matrices. This modified method, however, comes with a significant increase in complexity.

1.7 Localisation Techniques

The problem of localizing signal sources has attracted considerable attention over the last few decades. In many communication applications, the location of the source can be used to improve the performance and efficiency of the communication system[67]. Conventional localisation approaches, in general, use a number of sensing nodes at different known locations to estimate the spatial relationships between the source and the sensing nodes. These spatial information are used to compile the estimation of the source location by triangulation or trilateration.

Trilateration-based or range-based localisation techniques such as Received Signal Strength (RSS) and Time based approaches estimate the range between the source and each of the nodes. By fusing the results of several range estimates, the source location can be inferred by trilateration. RSS approaches such as the one

proposed in [68] measures the signal power received as each sensor when a signal of known power is transmitted from a source hence estimates the ranges between the source and the nodes using a path loss model. This approach is often cheap and does not require synchronization between the source and the nodes. Time based techniques such as the Time-of-Arrival (TOA) estimation approach in [69] requires that the source and nodes are accurately synchronized. In [70], the Time-Difference-of-Arrival (TDOA) localisation algorithm is proposed that only needs the nodes to be synchronized. However, the localisation accuracy of time based approaches is usually limited by the effective bandwidth of the signal, additive noise and fading effects[71].

In direction-based localisation approaches, two or more of direct finding (DF) array systems are usually employed to estimate the direction of a far-field source, e.g. using the MUSIC algorithm [72] or the Maximum Likelihood Estimation (MLE) algorithm [73]. Then a set of linear equations are used which utilize the estimated directions to locate the source under consideration.

Furthermore, hybrid approaches may also be employed to solve the localisation problem, e.g. TOA/TDOA [74] or single array DOA and TOA [75]. While the direction-based localisation techniques unable to localise a far field source with a single array, some array localisation techniques are able to locate near-far field source with a single array (e.g. the spherical-MUSIC[73]) which jointly estimates both DOA and range.

In [76], a localisation algorithm is proposed that uses a number of distributed sensors to form a large aperture array (LAA). Based on the spherical wave propagation model, the algorithm relies on rotating the array reference point to be at each of the array element and finally solving a set of linear equations to estimate the source location.

1.8 Thesis Scope and Organisation

The aim of this thesis is to analyse the performance of the existing and proposed novel array synchronisation and calibration methods with real data using an experimental array SDR testbed. Different types of SDR testbeds will be developed to suit different evaluations.

The rest of the thesis is organised as follows:

In Chapter 2, an AR-based channel estimation and prediction technique is proposed for the LTE-TDD MU-MIMO downlink system for a time-varying frequency-selective environment. By estimating the impulse response of OFDM channels, the proposed approach reduces the computational burden by only considering the most significant taps of the impulse response. Testbed results also prove that the proposed method is able to increase dramatically the system capacity under the described test conditions.

In Chapter 3, a low cost array testbed based on USRP boards manufactured by Ettus Research is presented. The testbed is highly reconfigurable and can be used in many applications without requiring major modifications or specialised equipment. However, such a system contains many array non-idealities including gain and phase uncertainties, mutual coupling effects and local oscillator offsets, which have to be addressed before performing further signal processing algorithms. These issues will be studied in this chapter and two array localisation algorithms (namely the LAA and the spherical-MUSIC algorithms) will be examined in detail. In addition, an outdoor field-test conducted in a native woodland at Silwood Park, Imperial College London is presented. As the array elements are distributed in space with large distances, the hardware array synchronisation technique becomes impractical. In this case, each array element will be locked to its own independent local oscillator. Tolerances in the independent local oscillators will introduce frequency offsets between array elements. This will prevent the system from operating as a coherent array. In this chapter, frequency offset estimation algorithm based on harmonic retrieval will be added as pre-processors. A pilot based

array calibration technique will also be presented. In addition, the performance of the LAA localisation algorithm will be studied for targets at located different locations.

Chapter 4, is concerned with the problem of element pattern synthesis, and a novel approach will be presented for an array with fixed geometry in the presence of a far-field source. It consists of constructing a virtual array model for each antenna element, which not only considers the coupling effect between antenna elements but also takes into account the influence of the scattering near-field obstacles of unknown locations. The influence of the isolated element pattern is also taken into account by the virtual array in order to optimise the reconstructed far field pattern covering a wide-angle range without added complexity. To validate the proposed technique, the SDR platform developed in Chapter 3 is used with an array system consisting of eight off-the-shelf omnidirectional antennas in a uniform circular geometry. Numerical examples using experimental data are presented showing that results obtained by the proposed method greatly outperform those obtained by conventional mutual coupling compensation techniques.

Finally, the thesis is concluded in Chapter 5 and several recommendations for future work are outlined.

Chapter 2

Channel Prediction for Multiuser Downlink LTE-TDD Systems

In this chapter, a multi-user (MU) MIMO OFDM system is proposed. The system is based on LTE-TDD which is composed of a block diagonalisation beamformer at the base station and spatial MMSE receivers at user equipment (UEs). Furthermore, a channel prediction scheme based on AR-modelling and linear interpolation is proposed. In this scheme, reference symbols, placed at the last OFDM symbol in the uplink frame of the UE, are used for channel estimation. AR based predictor performs recursive filtering on the channel measurements estimating CSI a few subframes in advance. Finally, with the proposed scheme, linear interpolation is performed to obtain downlink CSI for each OFDM symbol. A National Instrument (NI) PXIe based rapid prototyping system was also developed for testing the proposed algorithms.

2.1 Single-User MIMO System Model

Single-user spatial multiplexed MIMO systems can transmit data at a much higher rate than SISO systems and MIMO systems using antenna diversity techniques. Consider a $N_{RX} \times N_{TX}$ spatial multiplexed MIMO system with N_{RX} receive an-

tennas and N_{TX} transmit antennas. Let $\mathbb{H}$ denote the channel transfer matrix for one subcarrier in OFDM,

$$\mathbb{H} = \begin{bmatrix} h^{(1,1)}, & h^{(1,2)}, & \dots & h^{(1,N_{TX})} \\ h^{(2,1)}, & h^{(2,2)}, & \dots & h^{(2,N_{TX})} \\ \vdots & \vdots & \ddots & \vdots \\ h^{(N_{RX},1)}, & h^{(N_{RX},2)}, & \dots & h^{(N_{RX},N_{TX})} \end{bmatrix}, \quad (2.1)$$

where each entry h_{ij} represents the channel transfer function gain from the j th transmit antenna to the i th receive antenna, $i \in \{1, \dots, N_{RX}\}$ and $j \in \{1, \dots, N_{TX}\}$. Let the number of independent spatial streams be denoted by N_s , where $N_s = \min(N_{RX}, N_{TX})$. The transmission streams at each antenna can be represented as:

$$\underline{z} = \mathbb{F} \cdot \underline{x}, \quad (2.2)$$

where $\underline{x} = \begin{bmatrix} x_1, & x_2, & \dots & x_{N_s} \end{bmatrix}^T \in \mathcal{C}^{N_s \times 1}$ contains the transmit symbols for all streams, and $\mathbb{F}$ is a $N_{TX} \times N_{RX}$ complex precoding matrix that maps independent spatial streams to transmit antennas.

The received signal vector $\underline{r} = \begin{bmatrix} r_1, & r_2, & \dots & r_{N_{RX}} \end{bmatrix}^T \in \mathcal{C}^{N_{RX} \times 1}$ can be expressed as:

$$\begin{aligned} \underline{r} &= \mathbb{H} \underline{z} + \underline{n} \\ &= \mathbb{H} \mathbb{F} \underline{x} + \underline{n}, \end{aligned} \quad (2.3)$$

where $\underline{n}$ is the additive white Gaussian noise (AWGN) vector with power spectral density σ_n^2 . To detect the desired signals, the effect of the channel and the precoding matrix is cancelled by a weight matrix $\mathbb{W}$, such that

$$\begin{aligned} \underline{y} &= \mathbb{W} \underline{r} \\ &= \mathbb{W} (\mathbb{H} \underline{z} + \underline{n}) \\ &= \mathbb{W} \mathbb{H} \mathbb{F} \underline{x} + \mathbb{W} \underline{n}. \end{aligned} \quad (2.4)$$

2.1.1 Singular Value Decomposition (SVD)

The optimal SU-MIMO channel equalization method is based on SVD of the channel matrix $\mathbb{H}$. In the SVD method, channel equalization is split among transmitter and receiver; and it cannot be applied to multi-user MIMO. The channel matrix $\mathbb{H}$, can be decomposed in to its singular values as

$$\mathbb{H} = \mathbb{U}\mathbf{\Lambda}\mathbb{V}^H, \quad (2.5)$$

where $\mathbb{U}$ and $\mathbb{V}$ are matrices, and $\mathbf{\Lambda}$ is a real diagonal matrix. Applying $\mathbb{V}$ at the transmitter as the precoding matrix, the signals at the output of the channel can be written as:

$$\begin{aligned} \underline{r} &= \mathbb{H}\mathbb{V}\underline{x} + \underline{n} \\ &= \mathbb{U}\mathbf{\Lambda}\mathbb{V}^H\mathbb{V}\underline{x} + \underline{n} \\ &= \mathbb{U}\mathbf{\Lambda}\underline{x} + \underline{n}. \end{aligned} \quad (2.6)$$

At the receiver, if $\mathbb{U}^H$ was used as the receiving weight matrix, independent parallel channels can be visualised as,

$$\begin{aligned} \underline{y} &= \mathbb{U}^H \underline{r} \\ &= \mathbb{U}^H (\mathbb{U}\mathbf{\Lambda}\underline{x} + \underline{n}) \\ &= \mathbf{\Lambda}\underline{x} + \mathbb{U}^H \underline{n} \end{aligned} \quad (2.7)$$

where $\|\mathbb{U}^H \underline{n}\| = \|\underline{n}\|$. Since $\mathbb{U}$ and $\mathbb{V}$ are unitary matrices, neither the transmission power amplification nor the noise enhancement will occur.

2.2 Multi-User MIMO System Model

Now consider a simplified downlink M -user MIMO system (or broadcast system) with a single base station (BS). Suppose the BS is equipped with N_{TX} antennas and each user is equipped with N_{RX} antennas. In this paper, the point-to-point

system is visualized as a (MN_{RX}, N_{TX}) MIMO system with $MN_{RX} = N_{TX}$. With reference to Figure 2.1, the system is partitioned into three main blocks, the base station, the channel associated to the i th user, and the i th user's receiver. The main blocks of the system will be introduced and modelled in the following sections. For simplicity, IFFT and Cyclic Prefix (CP) appending blocks at the base station, and FFT and CP removal blocks at the i th user's receiver are removed in the system model.

2.2.1 Base Station Transmitter

Assume the number of independent transmission spatial streams (or spatial layers) per user N_S equals to N_{RX} , for simplicity. Let T_s denote the OFDM sampling period, and a subcarrier $k = 1, 2, \dots, N_C$, where N_C is total number of subcarriers, is assigned with frequency given by $F_k = k\Delta F$, where $\Delta F = 1/T_c$ is the subcarrier spacing. For the k^{th} subcarrier of the n th OFDM symbol of duration $T_{cs} = N_C T_s$, the precoded signals for the transmission can be expressed as

$$\underline{z}[k, n] = \mathbb{F}[k, n] \underline{x}[k, n]. \quad (2.8)$$

Dropping the subcarrier index, in our system model, the above expression is then simplified to

$$\underline{z}[n] = \mathbb{F}[n] \underline{x}[n] \quad (2.9)$$

where

$$\underline{x}[n] = \left[\underline{x}_1^T[n], \underline{x}_2^T[n], \dots, \underline{x}_M^T[n] \right]^T \in \mathcal{C}^{(MN_S) \times 1} \quad (2.10)$$

contains transmission symbols for all users.

$$\mathbb{F}[n] = \left[\mathbb{F}_1[n], \mathbb{F}_2[n], \dots, \mathbb{F}_M[n] \right] \in \mathcal{C}^{N_{TX} \times (MN_{RX})} \quad (2.11)$$

is the overall precoding matrix for all users and

$$\mathbb{F}_i[n] \in \mathcal{C}^{N_{TX} \times N_{RX}}, \forall_i \quad (2.12)$$

are complex precoding matrices for each user which will be discussed in details in Section III. Note that Equation (2.9) can be written (at point [D] in Figure 2.1) as

$$\underline{z}[n] = \sum_{i=1}^M \mathbb{F}_i[n] \underline{x}_i[n]. \quad (2.13)$$

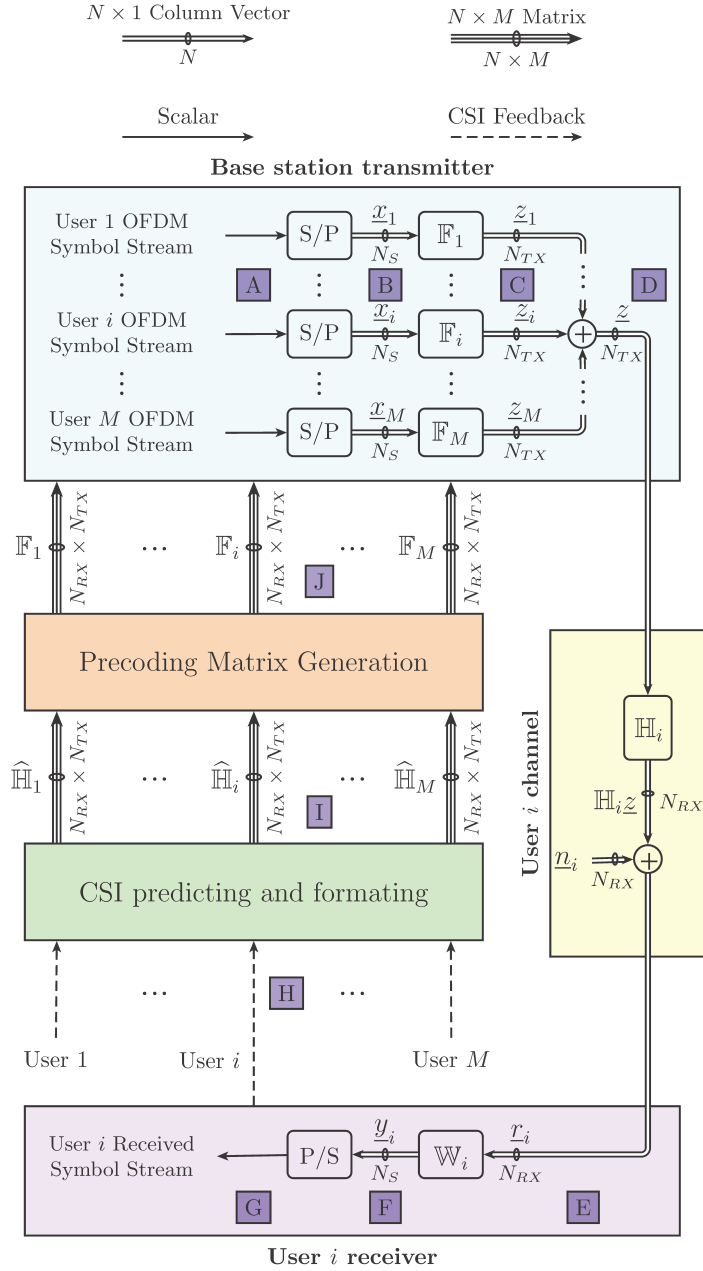


Figure 2.1: Simplified multiuser OFDM downlink system.

2.2.2 User i Downlink Channel

Assuming the cyclic prefix (CP) is long enough to cope with the maximum delay spread, the channel associated to the i th user is defined as:

$$\mathbb{H}_i[n] = \begin{bmatrix} h_i^{(1,1)}[n] & h_i^{(1,2)}[n] & \cdots & h_i^{(1,N_{TX})}[n] \\ h_i^{(2,1)}[n] & h_i^{(2,2)}[n] & \cdots & h_i^{(2,N_{TX})}[n] \\ \vdots & \vdots & \ddots & \vdots \\ h_i^{(N_{RX},1)}[n] & h_i^{(N_{RX},2)}[n] & \cdots & h_i^{(N_{RX},N_{TX})}[n] \end{bmatrix} \in \mathcal{C}^{N_{TX} \times N_{RX}} \quad (2.14)$$

where $h_i^{(m_{RX},m_{TX})}[n]$ denotes the complex channel transfer function gain from the BS transmitter antenna m_{TX} to the i th user equipment receiver antenna m_{RX} . Hence the i th user's received signal vector (at point **E** in Fig 2.1) can be written as follows:

$$\underline{r}_i[n] = \mathbb{H}_i[n] \underline{z}[n] + \underline{n} \quad (2.15)$$

where is the i th user's downlink channel transfer function matrix that is obtained using the reciprocal property of the uplink channel.

2.2.3 User- i Receiver

Referring to Equation (2.15), the received signal $\underline{r}_i$ for the i th user is recovered with a minimum-mean-square-error (MMSE) detector which compensates the amplitude and phase distortion and inter-antenna interference that introduced by the channel and the process of precoding. We assume complete channel state information (CSI) is available at the receiver, the normalized MMSE weight matrix for the i th user is given as:

$$\begin{aligned} \mathbb{W}_i[n] &= \left(\mathbb{F}_i^H[n] \mathbb{H}_i^H[n] \mathbb{H}_i[n] \mathbb{F}_i[n] + \sigma_n^2 \mathbb{I}_{N_{RX}} \right)^{-1} \\ &\quad \mathbb{F}_i^H[n] \mathbb{H}_i^H[n]. \end{aligned} \quad (2.16)$$

The output of the MMSE detector for the i th user can be then written as (at

point **F** in Figure 2.1)

$$\begin{aligned}
 \underline{y}_i[n] &= \mathbb{W}_i[n] \underline{r}_i[n] \\
 &= \mathbb{W}_i[n] (\mathbb{H}_i[n] \underline{z}[n] + \underline{n}) \\
 &= \mathbb{W}_i[n] \mathbb{H}_i[n] \underline{z}_i[n] + \mathbb{W}_i[n] \underline{n}.
 \end{aligned} \tag{2.17}$$

2.3 Zero Delay Feedback and Precoder Design

Zero-delay feedback can be achieved by means of channel prediction using well-known autoregressive modelling, which is a widely used prediction method that could be found in many applications including tracking of a radio channel in a time-varying environment [40]. Unlike deterministic curve fitting techniques, such as interpolation and extrapolation, AR modelling takes the advantage of the statistics of the data. Assuming the CSI controller at the base station (see Figure 2.1) has P perfect past CSI measurements, the current CSI can be estimated with certain amount of error. Let $h[n]$ denote the the complex channel $h_i^{(m_{RX}, m_{TX})}[n]$ for simplicity. It is shown in [8], that a state-space model can be constructed. Figure 2.2 shows a P order tapped delay line (TDL) channel predictor, that uses the past P channel measurements and multiplies them with the corresponding AR weights to predict the current state of the channel.

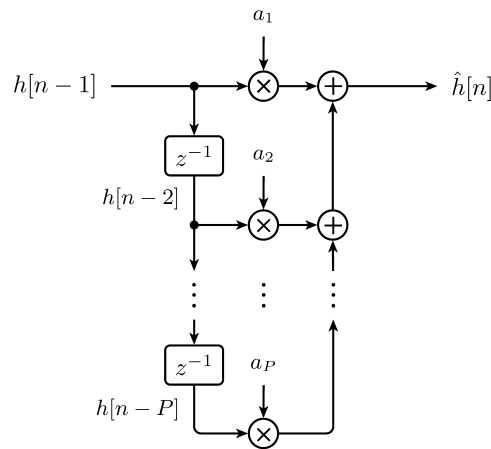


Figure 2.2: State-space diagram.

The process can be summarized by the following expression:

$$\begin{aligned}\widehat{h}[n] &= \sum_{u=1}^P a_u h[n-u] \\ &= \underline{a}^T \underline{h}[n]\end{aligned}\tag{2.18}$$

where $\widehat{h}[n]$ is the estimated channel transfer function. And

$$\underline{h}[n] = \begin{bmatrix} h[n-1], & h[n-2] & \cdots, & h[n-P] \end{bmatrix}^T\tag{2.19}$$

holds the past P samples of $h[n]$. The AR parameters

$$\underline{a} = [a_1, a_2, \dots, a_P]^T\tag{2.20}$$

can be estimated according to the environment. With increased estimation error, multiple-step prediction can be achieved by feeding back the current estimated channel state to the tap-delay line recursively.

2.3.1 Forward-Backward Estimation of AR Parameters

The well-known prediction model for a fixed order time series is usually referring to the forward-backward linear prediction model or the modified covariance method [77]. This method, based on the AR stochastic model, can provide stable estimates from short data sets and has the potential of improving the temporal resolution in the measurements, thus allowing the prediction of rapidly varying channel conditions. Let $h[1], h[2], \dots, h[L]$ denote L perfect channel measurements from a training sequence that are collected at point $\boxed{\text{H}}$ in Figure 2.1. To estimate the AR parameters, $\underline{a}$, in Equation (2.18), an overdetermined system model is firstly constructed:

$$\mathbb{A}\underline{a}=\underline{b}.\tag{2.21}$$

The least squares AR parameters $\underline{a}$ can be found by solving

$$\underline{a}=(\mathbb{A}^H\mathbb{A})^{-1}\mathbb{A}^H\underline{b}.\tag{2.22}$$

In the forward-backward method, the total squared sum of both the forward and the backward prediction errors are minimized. We can construct $\mathbb{A}$ and $\underline{b}$ as below:

$$\mathbb{A} = \begin{bmatrix} h[P], & h[P-1], & \cdots & h[1] \\ h[P-1], & h[P-2], & \cdots & h[2] \\ \vdots & \vdots & \ddots & \vdots \\ h[L-1], & h[L-2], & \cdots & h[L-P] \\ h^*[2], & h^*[3], & \cdots & h^*[P+1] \\ h^*[3], & h^*[4], & \cdots & h^*[P+2] \\ \vdots & \vdots & \ddots & \vdots \\ h^*[L-P+1], & h^*[L-P+2], & \cdots & h^*[L] \end{bmatrix}, \quad (2.23)$$

$$\underline{b} = \begin{bmatrix} h[P+1] \\ h[P+2] \\ \vdots \\ h[L] \\ h^*[1] \\ h^*[2] \\ \vdots \\ h^*[L-P] \end{bmatrix} \quad (2.24)$$

where $(\cdot)^*$ denotes complex conjugate.

2.3.2 Precoder

The main consideration for downlink broadcast channel in MU-MIMO transmission is the interference cancellation at the BS (the transmitter side). In this thesis, a BD precoder [29, 30] is considered. The idea of the BD precoder is based on beam steering so that interference is nulled at the undesired users. As oppose to other popular precoding schemes, such as DPC [31, 32, 33] and THP [34, 35, 36], BD cancels only the interference from other users with respect to the target user. Sig-

nal detection, such as MMSE detection, is required to mitigate the inter-antenna interference at each user.

In BD, an effective channel matrix is constructed between BS and each user. The aim of BD method is to find a set of precoding matrices that are null-space of the effective channel matrix except for the target user. In other words, interference-free transmission can be realized if the effective channel matrix can be block-diagonalized. To design precoders, BD assumes total number of antennas of all users equal to the number of antennas of BS and also assumes all users have equal number of antennas on each mobile device. We then take the right-singular vectors that correspond to vanishing (zeros) singular values of the channel matrix to produce the precoding matrix for each user since these vectors span the null space of the interference channel matrix.

Let $\mathbb{H}_i \triangleq \mathbb{H}_i[n]$ and $\mathbb{F}_i \triangleq \mathbb{F}_i[n]$, referring to Equations (2.8), and (2.15), it can be seen that, the interference-free transmission is possible if the channel can be block-diagonalized, that is,

$$\mathbb{H}_i \mathbb{F}_u = \mathbb{O}_{N_M \times N_M}, \forall u \neq i. \quad (2.25)$$

where $\mathbb{O}_{N_M \times N_M}$ denotes an $N_M \times N_M$ matrix of zeros.

To obtain the precoder weight matrix, let us first construct an interference channel matrix $\tilde{\mathbb{H}}_i \in \mathbb{C}^{(M-1)N_{RX} \times N_{TX}}$ that contains only the channel estimated gains from all other users except for the target i th user,

$$\tilde{\mathbb{H}}_i = \left[\hat{\mathbb{H}}_1^T, \hat{\mathbb{H}}_2^T, \dots, \hat{\mathbb{H}}_{i-1}^T, \hat{\mathbb{H}}_{i+1}^T \dots, \hat{\mathbb{H}}_M^T \right]^T \quad (2.26)$$

where $\hat{\mathbb{H}}_i$ is constructed from the estimated channel coefficients, $\hat{h}_i^{(m_{RX}, m_{TX})}$, from the previous section using Equation (2.14) that is (at point **I** in Fig 2.1)

$$\hat{\mathbb{H}}_i = \begin{bmatrix} \hat{h}_i^{(1,1)}, & \hat{h}_i^{(1,2)}, & \dots & \hat{h}_i^{(1,N_{TX})} \\ \hat{h}_i^{(2,1)}, & \hat{h}_i^{(2,2)}, & \dots & \hat{h}_i^{(2,N_{TX})} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{h}_i^{(N_{RX},1)}, & \hat{h}_i^{(N_{RX},2)}, & \dots & \hat{h}_i^{(N_{RX},N_{TX})} \end{bmatrix}. \quad (2.27)$$

Equation (2.25) can then be rewritten as:

$$\tilde{\mathbb{H}}_i \mathbb{F}_i = \mathbb{O}_{(M-1)N_{RX} \times N_{RX}}. \quad (2.28)$$

This implies that the precoding matrix $\mathbb{F}_i \in \mathcal{C}^{N_{TX} \times N_{RX}}$ must be designed to sit in the null space of $\tilde{\mathbb{H}}_i$ so that the interference-free transmission is warranted for the i th user. To obtain such a precoding matrix, we take the SVD of $\tilde{\mathbb{H}}_i$, that is

$$\begin{aligned} \tilde{\mathbb{H}}_i[n] &= \mathbb{U}_i \mathbf{\Lambda}_i \mathbb{V}_i^H \\ &= \mathbb{U}_i \left[\mathbf{\Lambda}_i^{non-zero} \mid \mathbf{\Lambda}_i^{zero} \right] \left[\mathbb{V}_i^{non-zero} \mid \mathbb{V}_i^{zero} \right]^H \end{aligned} \quad (2.29)$$

where

$$\mathbb{V}_i^{non-zero} \in \mathcal{C}^{N_{TX} \times (M-1)N_{RX}} \text{ and} \quad (2.30)$$

$$\mathbb{V}_i^{zero} \in \mathcal{C}^{N_{TX} \times N_{RX}} \quad (2.31)$$

are composed of right singular vectors that correspond to non-zero singular values

$$\mathbf{\Lambda}_i^{non-zero} \in \mathcal{R}^{(M-1)N_M \times (N_{TX} - N_{RX})} \quad (2.32)$$

and vanishing singular values

$$\mathbf{\Lambda}_i^{zero} = \mathbb{O}_{(M-1)N_{RX} \times N_{RX}} \quad (2.33)$$

respectively. Since the right-singular vectors corresponding to the vanishing singular values of $\tilde{\mathbb{H}}_i$ span the null space of $\tilde{\mathbb{H}}_i$, we have the following relationship

$$\tilde{\mathbb{H}}_i \mathbb{V}_i^{zero} = \mathbb{O}_{(M-1)N_{RX} \times N_{RX}}. \quad (2.34)$$

From the equation above, we can see that if the signal is transmitted with weight matrix $\mathbb{V}_i^{zero}$, all users other than the target user i receive no signal. Thus we can use the normalized $\tilde{\mathbb{V}}_i^{zero}$ as the precoding matrix for the i th user, that is (at point **J** in Fig 2.1)

$$\mathbb{F}_i = \frac{1}{\|\mathbb{V}_i^{zero}\|_F} \mathbb{V}_i^{zero}. \quad (2.35)$$

2.4 LTE Multipath Channel Model

When a signal propagates from a transmitter to a receiver, it usually travels multiple paths. The signal may follow the line-of-sight path between the transmitter and receiver, it may also bounce off the walls and other objects on its way to the receiver. This produces multiple copies of the same signal arrive at the receiver at different times and with different power levels, based upon the paths they have taken and various other factors.

A well-known technique to model a wireless channel is to model it as a *finite impulse response* (FIR) filter with its taps representing signal paths. A set of LTE channel models for the test and verification of radio transmission and reception has been defined in [4]. It specifies three channel delay profiles, Extended Pedestrian A model (EPA), Extended Vehicular A model (EVA), and Extended Typical Urban model (ETU). These three profiles represent a low, medium and high spread environment with their multipath delay profiles shown in Table 2.1, 2.2 and 2.3, respectively.

Table 2.1: Extended Pedestrian A (EPA) Delay Profile

Path #	Relative Delay (ns)	Relative Power (dB)
1	0	0.0
2	30	-1.0
3	70	-2.0
4	90	-3.0
5	110	-8.0
6	190	-17.2
7	410	-20.8

Table 2.2: Extended Vehicular A (EVA) Delay Profile

Path #	Relative Delay (ns)	Relative Power (dB)
1	0	0.0
2	30	-1.5
3	150	-1.4
4	310	-3.6
5	370	-0.6
6	710	-9.1
7	1090	-7.0
8	1730	-12.0
9	2510	-16.9

Table 2.3: Extended Typical Urban (ETU) Delay Profile

Path #	Relative Delay (ns)	Relative Power (dB)
1	0	-1.0
2	50	-1.0
3	120	-1.0
4	200	0.0
5	230	0.0
6	500	0.0
7	1600	-3.0
8	2300	-5.0
9	5000	-7.0

Furthermore, all the paths in the preceding tables are assumed to have a classical Doppler spectrum caused by motion of the transmitter, receiver, or intermediate objects in the channel. The coherence time and Doppler spread are inversely related:

$$\text{Coherence Time} \approx \frac{1}{\text{Doppler Spread}}. \quad (2.36)$$

Thus, if the motion is fast, the Doppler spread is large and the coherence time is small and vice versa. In addition to the multipath delay profile, a maximum Doppler spread is also specified for each propagation condition, as shown in Table 2.4.

Table 2.4: Typical Doppler Spreads and Coherence Times for LTE channels

Channel Model	Maximum Doppler Spread (Hz)	Coherence Time (ms)
EPA 5Hz	5	200
EVA 5Hz	5	200
EVA 70Hz	70	14.3
ETU 70Hz	70	14.3
ETU 300Hz	300	3.33

To apply a multipath delay profile to a signal, an FIR filter has to perform convolution operation on the signal uniformly sampled at 100MHz with very large number of taps. Thus real-time operation with a software implementation is almost impossible. However, since most of these taps have no power so the FIR filter can be efficiently implemented in hardware, such as an FPGA.

2.5 Rapid Prototyping Testbed

To test and verify the performance of the proposed algorithm under realistic wireless channel conditions, a rapid prototyping testbed has been configured to operate with an FPGA-based real-time channel emulator. The channel emulator has the ability to mimic the operations of wireless MIMO transceivers under real-world conditions by applying multipath profiles to the signals. The hardware of the rapid prototyping platform included an National Instrument (NI) PXIe-7966R FlexRIO FPGA module with an NI 5791 RF module installed on an NI PXIe 1062Q chassis as shown in Figure 2.3. The system was divided into two main parts:

- The host processing unit is based on a PXIe chassis that runs Labview on Windows OS. It handled high-complexity single and double precision floating-point operations and baseband signal processing such as message generation, symbol mapping/demapping, and modulation/demodulation for testing wireless communication systems. The source code and graphical user interface were developed in Labview with help from toolkits such as Modulation Toolkit and Digital Filter Design Toolkit.

- The channel processing unit operated in real-time on a field-programmable gate array (FPGA)-based channel emulator. It handled the time-critical computationally intensive signal processing such as sampling rate conversion, signal filtering, and convolution operations for applying multipath channel delay profiles.

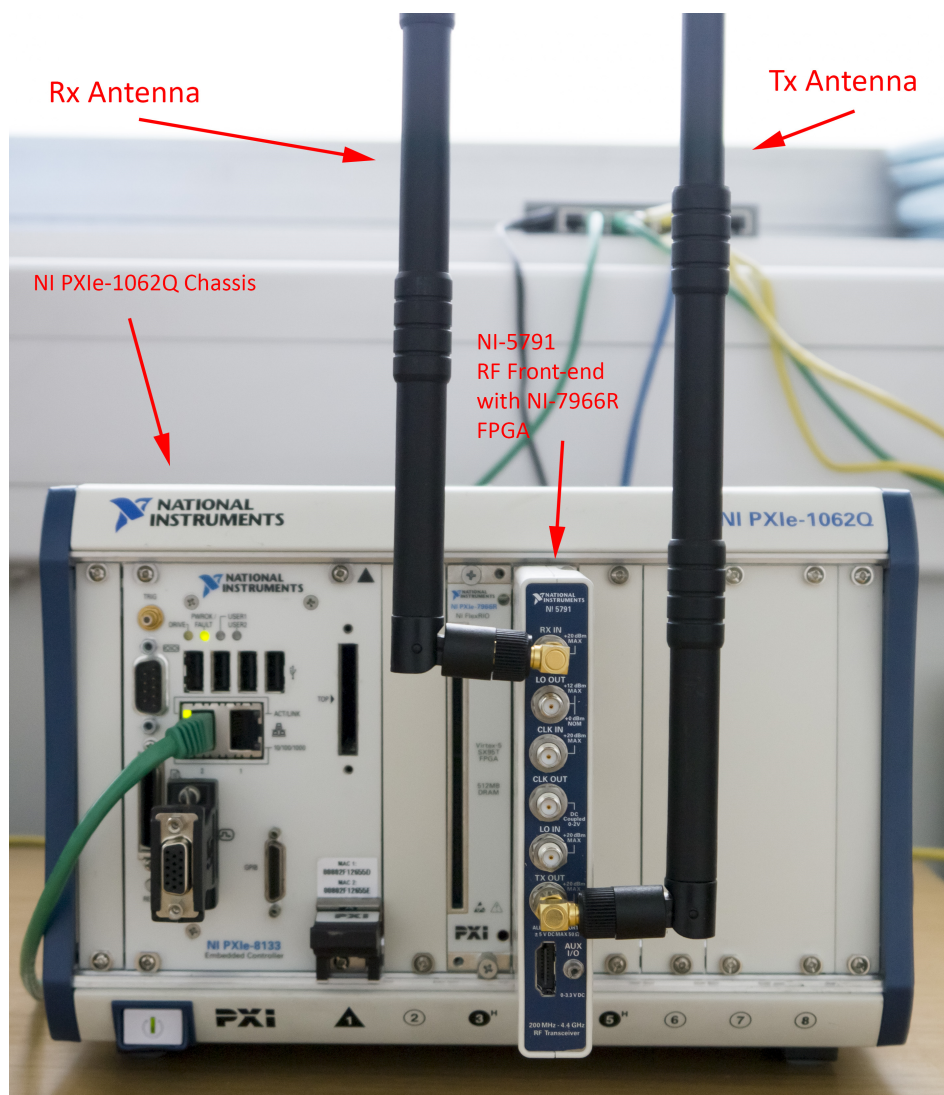


Figure 2.3: Rapid prototyping system based on NI PXIe-1062Q chassis and NI-5791 RF-frontend

2.5.1 Host Processing

The host processing unit of the rapid prototyping system handles high complexity floating-point operations. The processing includes most of the baseband operations such as signal generation, symbol mapping/demapping, modulating/demodulating, pre-equalization/post-equalization, and inverse fast Fourier transform (iFFT) with CP appending and fast Fourier transform (FFT) with CP removal. It also includes pilot signal insertion at the transmitter and estimation at the receiver. In addition, the host processing is also used to emulate channel fading signals and multipath channel coefficients for different power delay profiles standardized by various standardization organizations.

Figure 2.4 shows the host processing unit having a transceiver block which starts with a random message generator producing a stream of bits as the original transmission message. Depending on the modulation mode of choice, a symbol-mapping device then maps the bit stream to a complex symbol stream. For LTE systems, Quadrature Amplitude Modulation (QAM) will be used as the standard modulation and demodulation scheme. In spatial multiplexing LTE MIMO systems, the physical layer processing unit consists mainly of spatial multiplexing of the data and orthogonal frequency-division multiplexing (OFDM) with cyclic prefix appending. The spatial multiplexing will split the input symbol stream into multiple independent streams to increase the overall throughput of the system.

A pilot signal is inserted after the transmission signals are generated. This enables the channel estimation to be carried out at the receiver, irrespective of the physical layer processing for transmission. The chosen pilot signal bandwidth will cover the entire active bandwidth of the system. The pilot signal has a flat frequency power spectrum and a minimum peak-to-average power ratio (PAPR) to provide accurate channel estimation at the receiver for each subcarrier. The combined modulated and pilot signals are then passed to the channel processing unit to transmit either over a channel simulator or the FPGA based MIMO channel emulator.

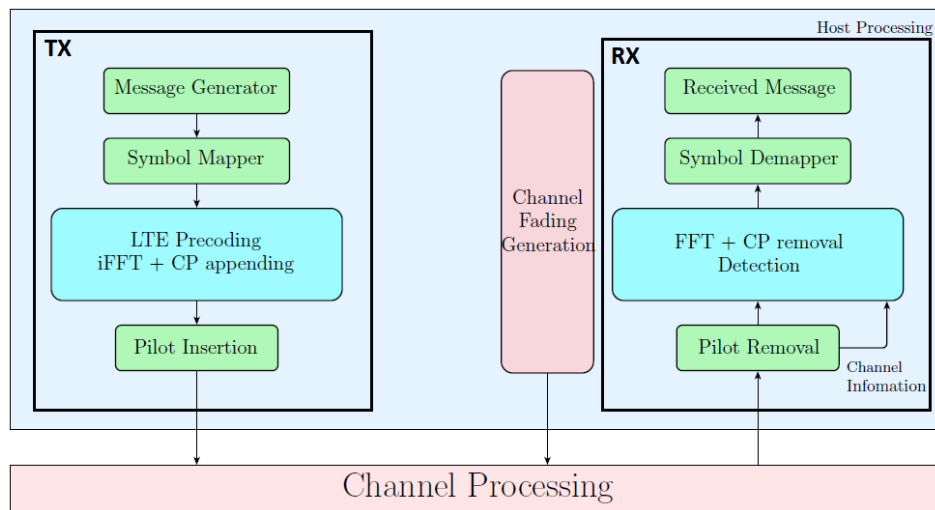


Figure 2.4: Rapid prototyping system: Host Processing

The combined signal will pass through the channel processing unit and the signals at the output of the channel processing unit will be passed to the host processing unit. The host processing unit at the receiver will use a channel estimator to extract the channel information from the appended pilot signal. The receiver processing unit then performs FFT and CP removal on the signals. It also recovers the transmitted message using the MMSE receiver as described in previous chapter with the extracted channel information. The message extraction at the host processing unit involves the recovery of transmitted symbols to produce the transmitted single bit stream.

The software channel simulator uses a channel fading generator operating as part of the host processing unit. The host processing unit generates multipath channel impulse responses in accordance with the power delay profiles at irregular sampling instances as specified by the standardization organizations. The multipath channel impulse response samples are generated at regular sampling intervals to be able to handle fading signals. The channel impulse responses are re-sampled to a higher sampling rate using a digital up-converter (DUC). These samples are then passed to the software channel simulator or the FPGA based real time channel emulator.

2.5.2 Channel Processing

Channel processing will be performed either using the off-line software channel simulator or a real-time FPGA channel emulator (Figure 2.5). The FPGA based channel emulator is equipped with RF front-ends so that real life channel effects are captured. Both software and real-time channels can be modelled in the same way as described in the following section. A N_{RX} -by- N_{TX} MIMO channel with N_{RX} receive antennas and N_{TX} transmit antennas can be re-modelled as $N_{RX}N_{TX}$ independent SISO channels processed separately and combined.

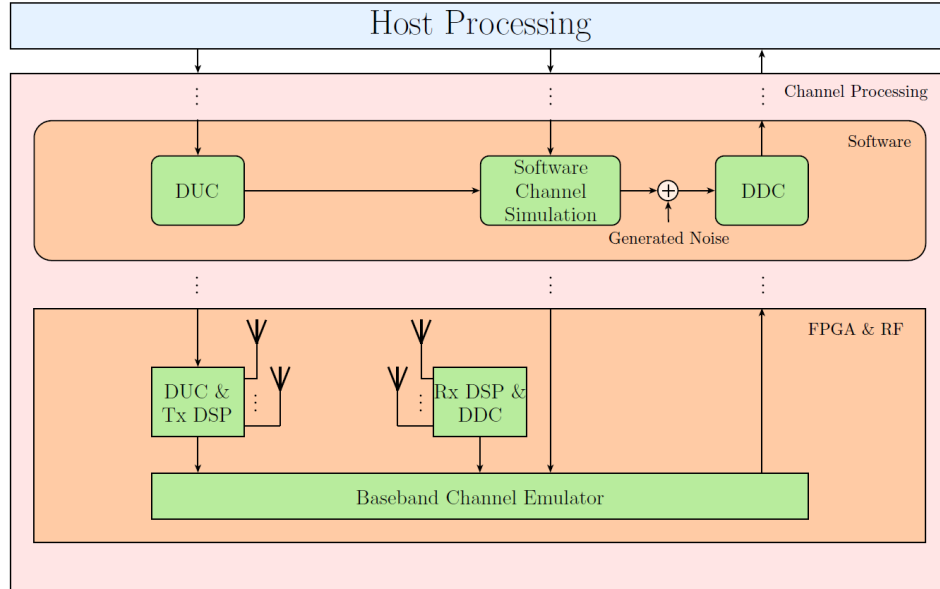


Figure 2.5: Rapid prototyping system: Channel Processing

Recalling that $\underline{z}[n] = \begin{bmatrix} z_1[n], & z_2[n], & \dots, & z_{N_{TX}}[n] \end{bmatrix}^T$ denotes the transmit symbol vector at transmission instance n , and are passed to the digital up-sampler (DUC), which has an interpolator and a pulse shaping filter (PSF). This allows channel coefficients at a different sampling rate to be inserted. Let $z'_j[n']$ denote the signal at the output of the DUC & PSF at instance n' of the original transmit symbol $z_j[n]$, where $i = 1, \dots, N_{RX}$ and $j = 1, \dots, N_{TX}$. Then the signal at the output of the channel filter is

$$r'_i[n] = \sum_{j=1}^{N_{TX}} \left(\beta_{i,j} \left(g_{i,j}[l] \otimes z'_j[n'] \right) \right) \quad (2.37)$$

where $\otimes$ denotes complex discrete convolution, $g_{i,j}(l)$ is the l^{th} complex channel fading coefficients and $\beta_{i,j}$ is the channel gain. A set of matched filter (MF) and digital down converter are used to convert $r'_i[n']$ back to its original sampling rate $r_i[n]$.

2.5.3 Channel Estimation Using Pilot

The least-square (LS) channel estimation using a pilot symbol is widely used for channel estimation due to its simplicity [78, 79]. Let the same system model in Equation (2.3) be used for pilot symbols based channel estimation. The LS estimates of the channel is simply given by

$$h_{i,j} = z_i^{-1} r_j \quad (2.38)$$

2.6 Numerical Results

In this section the total system capacity is presented, which illustrates the performance of a multiuser LTE-TDD system with AR based prediction. A MU-MIMO system with 8 transmit antennas at the BS and 4 UEs each equipped with 2 receive antennas is implemented. The major problem of the forward-backward estimation method is the selection of the AR model order. Insufficient order can produce very smoothed estimates, and on the other hand, too high orders can introduce spurious noise. A Monte Carlo simulation was used to find a range of suitable orders, from this range, the best model order was selected. Results are collected by averaging over 2048 simulation instances using Monte-Carlo method. For each simulation we consider an OFDM system with parameters complying LTE-TDD 5MHz transmission showing in Table 2.5.

Table 2.5: Simulation configurations

Modulation scheme	QPSK
Carrier frequency	2.4GHz
No. of subcarriers	300
Cyclic prefix	Long
Bandwidth	5MHz
Subcarrier spacing	15kHz
Sample rate	7.68MS/s
No. of users	2
Symbol Duration	83.3μs
Training length	100 OFDM symbols over 100ms
AR order	6
Prediction step size	1ms (every 12 OFDM symbols)

For each simulation, 12 OFDM symbols (1 pilot symbol and 11 payload symbols) are generated for each user. This is assumed that each user has one receiver antenna and perfect CSI is available at the transmitter for AR parameter estimation at 1ms step size. Multipath fading coefficients are generated based on Jake's model[80] using the EVA model with Doppler spread 5 and 70Hz and coherence time 200 and 14.3ms simulating the user travel speed of 2 and 32km/h, respectively. Typical CSI latencies, 1, 5, 10 and 20ms, at the base station of an LTE-TDD system are considered. The average signal-to-noise ratio (SNR) of all N_C subcarriers is given by

$$\text{SNR} = \frac{1}{N_C} \sum_{k=1}^{N_C} \frac{\beta^2[k] E_{TX}[k]}{2\sigma_n^2 N_{RX}}, \quad (2.39)$$

where

$$E_{TX}[k] = \mathcal{E} \left\{ \mathbf{z}^H[k, n] \mathbf{z}[k, n] \right\} \quad (2.40)$$

is the average total transmission energy. Note that, subcarrier index k and symbol index n are re-included. Furthermore $\beta^2[k]$ is the average channel power gain that is defined by

$$\beta^2[k] \triangleq \frac{1}{N_{TX}} \mathcal{E} \left\{ \|\mathbb{H}[k, n]\|_F^2 \right\}. \quad (2.41)$$

The ergodic system capacity is measured as the sum of all users' MMSE receiver outputs, that is

$$C = \frac{1}{N_C} \sum_{k=1}^{N_C} \mathcal{E} \left\{ \log_2 \left(1 + \frac{\underline{x}^H [k, n] \underline{x} [k, n]}{\underline{\epsilon}^H [k, n] \underline{\epsilon} [k, n]} \right) \right\} \text{ in bps/Hz,} \quad (2.42)$$

where

$$\underline{\epsilon} [k, n] = \underline{y} [k, n] - \underline{x} [k, n] \quad (2.43)$$

is the post-detection error and

$$\underline{y} [k, n] = \left[\underline{y}_1^T [k, n], \underline{y}_2^T [k, n], \dots, \underline{y}_M^T [k, n] \right]^T \quad (2.44)$$

contains recovered symbols for all the users.

As shown in Figure 2.6, for the EVA 5Hz channel, the proposed approach is able to recover the full system capacity for the entire SNR region that has been simulated. This shows the proposed method was able to predict the channel 10ms in advance with very little error. Compared to the case with no channel prediction, the system capacities dropped to 50% and 25% for 5ms and 10ms delays respectively at SNR = 30dB. As shown in Figure 2.7, the EVA 70Hz channel, which was a much more vigorous, fast varying channel, completely destroyed the MU-MIMO capacity when CSI delays are present. Without channel prediction, the system capacity dropped to 15% for 1ms delay and became unusable for 5ms at SNR = 30dB. The proposed method fully recovered the system capacity for 1ms CSI delay at entire SNR region and recovered over 60% of the capacity at SNR = 30dB for 5ms CSI delay. In Figure 2.8, further comparisons are made between the proposed method, linear extrapolation method, and 6th-order Wiener-Hopf FIR predictor with per symbol step. The performance of LTE downlink was significantly improved by using the proposed channel predictor with the BD precoder in a medium-high mobility environment.

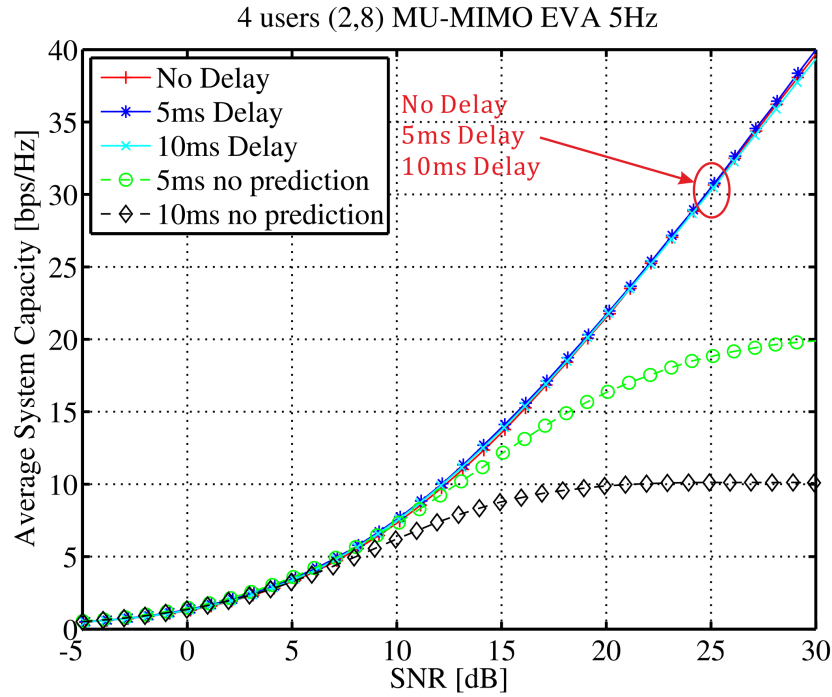


Figure 2.6: Ergodic system capacity [bps/Hz] against received SNR [dB] for various CSI delays with and without prediction on EVA 5Hz channel.

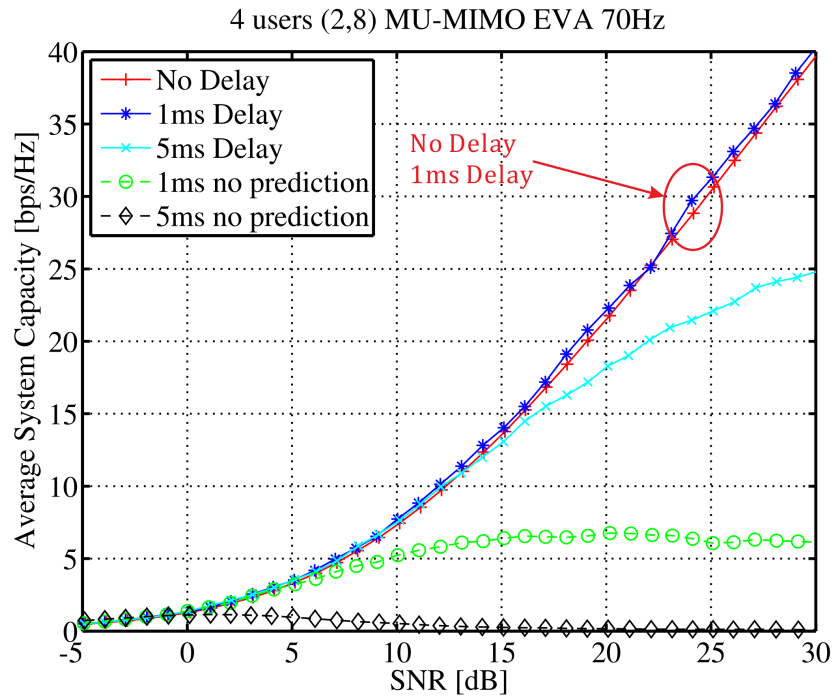


Figure 2.7: Ergodic system capacity [bps/Hz] against received SNR [dB] for various CSI delays with and without prediction on EVA 70Hz channel.

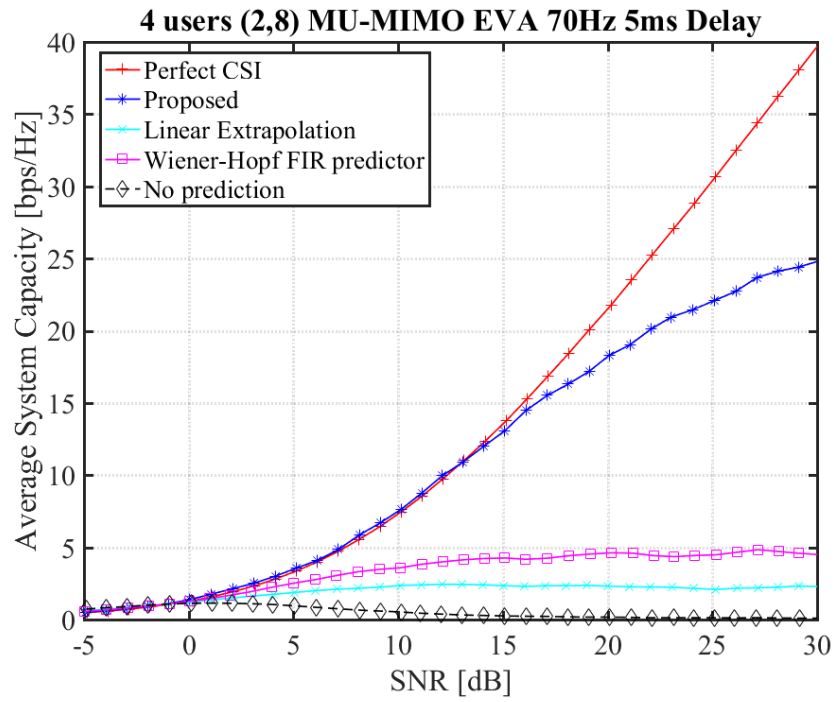


Figure 2.8: Ergodic system capacity [bps/Hz] against received SNR [dB] for 5ms CSI delays with various channel prediction methods on EVA 70Hz channel.

2.7 Summary

In this chapter, an AR-based channel estimation and prediction solution for the LTE-TDD multiuser downlink system is proposed for a time-varying frequency-selective environment. Testbed results also proved that the proposed method is able to increase the system capacity dramatically under the described test condition. The performance of the algorithm can be further improved by assimilating the antenna geometry [81] into the system model. For example, in [82], an asynchronous CDMA MIMO system has been modelled using the spatio-temporal array (STAR) manifold vector. Similarly, an LTE-TDD system can be modelled with the use of the multicarrier spatio-temporal array (MC-STAR) manifold vector [83], such that Equation (2.1) can be modelled as following

$$\mathbb{H}_i[n] \triangleq \mathbb{H}_i^{\text{MC-STAR}}(\underline{\theta}, \underline{\phi}, n, F_k, \mathbf{r}) \quad (2.45)$$

The new manifold vector is a function of the array geometry $\mathbf{r} \in \mathcal{R}^{3 \times N}$, and the directions of the signals $\underline{\theta} \in \mathcal{R}^{N_{TX} \times 1}$ and $\underline{\phi} \in \mathcal{R}^{N_{TX} \times 1}$ ($\underline{\theta}$ and $\underline{\phi}$ represent the azimuth and elevation angles respectively). Some of the parameters such as the array geometry $\mathbf{r}$, and the subcarrier frequency F_k in the manifold is known and fixed if the array is fully calibrated. The remaining channel parameters such as signal DOAs, $\underline{\theta}$ and $\underline{\phi}$, can be therefore estimated regardless of the array size N_{RX} . In addition, polarization information [84] and Doppler effect [85] can also be used to deal with correlated [86] sources thus to improve the detection and estimation capabilities.

Chapter 3

Experimental Study of Localisation Techniques Using an SDR Testbed

In this chapter, two array localization algorithms (namely the LAA and the spherical-MUSIC algorithms) are studied experimentally using multiple off-the-shelf universal software radio peripheral (USRP) hardware boards forming a circular antenna array system of nine elements operating in the presence of a source located in the near-far field of the array. In particular, the effects of gain and phase array uncertainties on these localization approaches are presented and their experimental accuracy is examined in terms of both range and direction RMSE.

3.1 Mathematical Model

Consider a fully synchronised array of N omnidirectional element with Cartesian coordinates described by the matrix $\mathbf{r} \in \mathcal{R}^{3 \times N}$ such that

$$\mathbf{r} = [r_1, r_2, \dots, r_N] = [\underline{r}_x, \underline{r}_y, \underline{r}_z]^T \quad (3.1)$$

where $\underline{r}_i$ with $i = 1, \dots, N$ denotes the location of the i^{th} antenna with respect to the array reference point. Assume the array operates in the presence of a near-

far field source emitting an unknown message $m(t)$ from an unknown position $\underline{r}_m \in \mathcal{R}^{3 \times 1}$ with Cartesian coordinates,

$$\underline{r}_m = \begin{bmatrix} \rho \cos(\theta) \cos(\phi), & \rho \sin(\theta) \cos(\phi), & \rho \sin(\phi) \end{bmatrix}^T \quad (3.2)$$

expressed as a function the azimuth θ , the elevation ϕ and the range $\rho = \|\underline{r}_m\|$. In this case, and with reference to the introductory material presented in Chapter 1, Equation (1.2) assumes that the array elements are isotropic and there are no mutual coupling effects. Based on Equation (1.1) the covariance matrix of $\underline{x}(t)$ can be expressed as

$$\begin{aligned} \mathbb{R}_{xx} &= \mathcal{E} \{ \underline{x}(t) \underline{x}^H(t) \} \in \mathcal{C}^{N \times N} \\ &= P_m \underline{S} \underline{S}^H + \sigma_n^2 \mathbb{I}_N \end{aligned} \quad (3.3)$$

where $P_m \triangleq \mathcal{E} \{ m^2(t) \}$ is the power of the received baseband message signal $m(t)$. In practice, Equation (3.3) is constructed by using a finite number of snapshots L as follows,

$$\mathbb{R}_{xx} = \frac{1}{L} \mathbb{X} \mathbb{X}^H \in \mathcal{C}^{N \times N} \quad (3.4)$$

with

$$\mathbb{X} = \begin{bmatrix} \underline{x}(t_1), & \underline{x}(t_2), & \dots, & \underline{x}(t_L) \end{bmatrix} \in \mathcal{C}^{N \times L}. \quad (3.5)$$

Performing an eigen-decomposition on $\mathbb{R}_{xx}$, the eigenvector $\underline{E}_s$ that corresponds to the principle eigenvalue belongs to one-dimensional subspace spanned by manifold vector $\underline{S}$. That is

$$\underline{E}_s \in \mathcal{L} \{ \underline{S} \} \quad (3.6)$$

which implies

$$\underline{E}_s = \lambda \cdot \underline{S} \quad (3.7)$$

where λ is a complex scaling constant.

In real system there are uncertainties and thus the true manifold vector is not given by Equation (1.2). Let Equation (1.2) be the "assumed" or the nominal manifold vector $\widehat{\underline{S}}$, the true manifold vector is given by

$$\underline{S} = \left(\mathbb{I}_N + \widetilde{\mathbb{C}} \right) \left((\underline{1}_N + \widetilde{\underline{g}}) \odot \widehat{\underline{S}} \right) \quad (3.8)$$

where $\tilde{\mathbb{C}}$ is the matrix with elements being the mutual coupling uncertainties whose diagonal elements are all equal to zero,

$$\tilde{\mathbb{C}} = \begin{bmatrix} 0, & \tilde{c}_{12}, & \tilde{c}_{13}, & \cdots, & \tilde{c}_{1N} \\ \tilde{c}_{21}, & 0, & \tilde{c}_{23}, & \cdots, & \tilde{c}_{2N} \\ \tilde{c}_{31}, & \tilde{c}_{32}, & 0, & \cdots, & \tilde{c}_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{c}_{N1}, & \tilde{c}_{N2}, & \tilde{c}_{N3}, & \cdots, & 0 \end{bmatrix} \in \mathcal{C}^{N \times N} \quad (3.9)$$

and

$$\underline{\tilde{g}} = \begin{bmatrix} \tilde{g}_1, & \tilde{g}_2, & \cdots & \tilde{g}_N \end{bmatrix}^T \in \mathcal{C}^{N \times 1} \quad (3.10)$$

with $\tilde{g}_i$ indicating the complex gain uncertainties of the i^{th} array element.

Note that Equation (3.8), in the absence of both mutual coupling and gain uncertainties, becomes $\underline{S} = \hat{\underline{S}}$. That is

$$\left. \begin{array}{l} \tilde{\mathbb{C}} = \mathbb{O}_{N \times M} \\ \underline{\tilde{g}} = \underline{0}_N \end{array} \right\} \implies \underline{S} = \hat{\underline{S}} \quad (3.11)$$

However, in a real uncalibrated array system Equation (3.8) can be re-written as follows

$$\underline{S} = \hat{\underline{S}} + \tilde{\underline{S}} \quad (3.12)$$

where

$$\begin{aligned} \tilde{\underline{S}} &= (\mathbb{I}_N + \tilde{\mathbb{C}}) (\underline{\tilde{g}} \odot \hat{\underline{S}}) + \tilde{\mathbb{C}} \hat{\underline{S}} \\ &= \underbrace{\left((\mathbb{I}_N + \tilde{\mathbb{C}}) \text{diag}(\underline{\tilde{g}}) + \tilde{\mathbb{C}} \right)}_{\triangleq \tilde{\mathbb{G}}} \hat{\underline{S}} \end{aligned} \quad (3.13)$$

and

$$\begin{aligned} \underline{S} &= (\mathbb{I}_N + \tilde{\mathbb{C}}) (\underline{1}_N \odot \hat{\underline{S}} + \underline{\tilde{g}} \odot \hat{\underline{S}}) \\ &= \hat{\underline{S}} + \underline{\tilde{g}} \odot \hat{\underline{S}} + \tilde{\mathbb{C}} \hat{\underline{S}} + \tilde{\mathbb{C}} (\underline{\tilde{g}} \odot \hat{\underline{S}}) \\ &= \hat{\underline{S}} + \underbrace{(\mathbb{I}_N + \tilde{\mathbb{C}}) (\underline{\tilde{g}} \odot \hat{\underline{S}}) + \tilde{\mathbb{C}} \hat{\underline{S}}}_{\triangleq \tilde{\underline{S}}} \end{aligned} \quad (3.14)$$

which implies

$$\underline{\tilde{S}} = \underline{\tilde{\mathbb{G}}} \underline{\hat{S}} \quad (3.15)$$

with

$$\underline{\tilde{\mathbb{G}}} = \left(\mathbb{I}_N + \underline{\tilde{\mathbb{C}}} \right) \text{diag}(\underline{\tilde{g}}) + \underline{\tilde{\mathbb{C}}}. \quad (3.16)$$

3.2 Robust Array Calibration Using Pilot Sources

Assuming $\underline{\tilde{\mathbb{G}}}$ is non-directional, i.e. it is independent of (θ, ϕ) parameters, and that there are K pilot signals then for the k^{th} pilot, Equation (3.7) can be written as follows

$$\begin{aligned} \underline{E}_k &= \lambda_k \cdot \underline{S}_k \\ &= \lambda_k \left(\underline{\hat{S}}_k + \underline{\tilde{S}}_k \right) \\ &= \lambda_k \left(\underline{\hat{S}}_k + \underline{\tilde{\mathbb{G}}} \underline{\hat{S}}_k \right) \\ &= \lambda_k \left(\mathbb{I}_N + \underline{\tilde{\mathbb{G}}} \right) \underline{\hat{S}}_k; \forall k = 1, 2, \dots, K. \end{aligned} \quad (3.17)$$

The set of equations described by Equation (3.17) can be written in a more compact form as

$$[\underline{E}_1, \underline{E}_2, \dots, \underline{E}_K] = \left(\mathbb{I}_N + \underline{\tilde{\mathbb{G}}} \right) \left[\underline{\hat{S}}_1, \underline{\hat{S}}_2, \dots, \underline{\hat{S}}_K \right] \underline{\Lambda} \quad (3.18)$$

where

$$\begin{aligned} \underline{\Lambda} &= \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_K \end{bmatrix} \\ &= \text{diag}(\underline{\lambda}). \end{aligned} \quad (3.19)$$

Let

$$\underline{\mathbb{E}} = [\underline{E}_1, \underline{E}_2, \dots, \underline{E}_K] \in \mathcal{C}^{N \times K} \text{ and} \quad (3.20)$$

$$\underline{\hat{\mathbb{S}}} = [\underline{\hat{S}}_1, \underline{\hat{S}}_2, \dots, \underline{\hat{S}}_K] \in \mathcal{C}^{N \times K} \quad (3.21)$$

Equation (3.18) can be written as

$$\begin{aligned}
\mathbb{E} &= \left(\mathbb{I}_N + \tilde{\mathbb{G}} \right) \hat{\mathbb{S}} \mathbf{\Lambda} \\
\Rightarrow \mathbb{E} - \hat{\mathbb{S}} \mathbf{\Lambda} &= \tilde{\mathbb{G}} \hat{\mathbb{S}} \mathbf{\Lambda} \\
\Rightarrow \left(\mathbb{E} - \hat{\mathbb{S}} \mathbf{\Lambda} \right) \mathbf{\Lambda}^{-1} &= \tilde{\mathbb{G}} \hat{\mathbb{S}} \\
\Rightarrow \mathbb{E} \mathbf{\Lambda}^{-1} - \hat{\mathbb{S}} &= \tilde{\mathbb{G}} \hat{\mathbb{S}}
\end{aligned} \tag{3.22}$$

The LS estimate of $\tilde{\mathbb{G}}$ is therefore given by

$$\tilde{\mathbb{G}} = \left(\mathbb{E} \mathbf{\Lambda}^{-1} - \hat{\mathbb{S}} \right) \hat{\mathbb{S}}^\# . \tag{3.23}$$

From Equation (3.17) we also have

$$\underline{E}_k^H \underline{E}_k = \|\lambda_k\|^2 \hat{\underline{S}}_k^H \left(\mathbb{I}_N + \tilde{\mathbb{G}} \right)^H \left(\mathbb{I}_N + \tilde{\mathbb{G}} \right) \hat{\underline{S}}_k = 1 \tag{3.24}$$

hence Equation (3.22) can be written as

$$\text{diag} \left(\mathbb{E}^H \mathbb{E} \right) = \text{diag} \left(\mathbf{\Lambda}^H \hat{\mathbb{S}}^H \left(\mathbb{I}_N + \tilde{\mathbb{G}} \right)^H \left(\mathbb{I}_N + \tilde{\mathbb{G}} \right) \hat{\mathbb{S}} \mathbf{\Lambda} \right) = \mathbb{I}_K . \tag{3.25}$$

By replacing $\tilde{\mathbb{G}}$ in Equation (3.25) with Equation (3.23), the only unknown is the diagonal matrix $\mathbf{\Lambda}$. The cost function with respect to $\mathbf{\Lambda}$ is given by

$$\begin{aligned}
\xi(\mathbf{\Lambda}) &= \text{Tr} \left\{ \left(\mathbb{I}_K - \hat{\mathbb{S}}^H \left(\hat{\mathbb{S}} \hat{\mathbb{S}}^H \right)^{-1} \hat{\mathbb{S}} \right)^H \mathbf{\Lambda}^H \mathbb{E}^H \mathbb{E} \mathbf{\Lambda} \left(\mathbb{I}_K - \hat{\mathbb{S}}^H \left(\hat{\mathbb{S}} \hat{\mathbb{S}}^H \right)^{-1} \hat{\mathbb{S}} \right) \right\} \\
&= \text{Tr} \left\{ \mathbf{\Lambda}^H \mathbb{E}^H \mathbb{E} \mathbf{\Lambda} \left(\mathbb{I}_K - \hat{\mathbb{S}}^H \left(\hat{\mathbb{S}} \hat{\mathbb{S}}^H \right)^{-1} \hat{\mathbb{S}} \right) \right\} .
\end{aligned} \tag{3.26}$$

It is important to point out that the parameters become not identifiable, i.e. the parameters are not uniquely defined, if there exists

$$\left(\mathbb{I}_N + \tilde{\mathbb{G}} \right) \hat{\mathbb{S}} \mathbf{\Lambda}^{-1} = \left(\mathbb{I}_N + \tilde{\mathbb{G}}' \right) \hat{\mathbb{S}} \left(\mathbf{\Lambda}' \right)^{-1} \tag{3.27}$$

with $\tilde{\mathbb{G}} \neq \tilde{\mathbb{G}}'$ and $\mathbf{\Lambda} \neq \mathbf{\Lambda}'$. In other words, if there exist $\tilde{\mathbb{G}}'$ and $\mathbf{\Lambda}'$ that satisfy Equation (3.27) then the parameters cannot be uniquely estimated. However, with a reasonable large number of well separated calibration sources, Equation (3.27) usually cannot be satisfied thus $\tilde{\mathbb{G}}$ and $\mathbf{\Lambda}$ are identifiable. Applying the

matrix identity, $\text{Tr} \{ \mathbf{C}^H \mathbf{A} \mathbf{C} \mathbf{B} \} = \underline{\text{diag}}(\mathbf{C})^H (\mathbf{A} \odot \mathbf{B}^T) \underline{\text{diag}}(\mathbf{C})$, Equation 3.26 can be rewritten as follows

$$\begin{aligned} \xi(\underline{\lambda}) &= \underline{\text{diag}}(\mathbf{\Lambda})^H \left(\mathbb{E}^H \mathbb{E} \odot \left(\mathbb{I}_K - \widehat{\mathbf{S}}^H \left(\widehat{\mathbf{S}} \widehat{\mathbf{S}}^H \right)^{-1} \widehat{\mathbf{S}} \right)^T \right) \underline{\text{diag}}(\mathbf{\Lambda}) \\ &= \underline{\lambda}^H \mathbf{Z} \underline{\lambda} \end{aligned} \quad (3.28)$$

where $\mathbf{Z} \triangleq \mathbb{E}^H \mathbb{E} \odot \left(\mathbb{I}_K - \widehat{\mathbf{S}}^H \left(\widehat{\mathbf{S}} \widehat{\mathbf{S}}^H \right)^{-1} \widehat{\mathbf{S}} \right)^T$. Given $\mathbf{Z}$ is a positive definite matrix, and under the constraint

$$\underline{\lambda}^H \underline{\lambda} = 1 \quad (3.29)$$

$\underline{\lambda}$ can be taken to be the eigenvector that corresponds to the minimum eigenvalue of $\mathbf{Z}$.

3.3 Array Gain and Phase Calibration using a Pilot Source

In the case of array elements are far apart enough that mutual coupling effect is negligible, this implies the MCM $\widetilde{\mathbf{C}} = \mathbf{O}_N$. Equation (3.16) can be simplified to

$$\begin{aligned} \widetilde{\mathbf{G}} &= (\mathbb{I}_N + \mathbf{O}_N) \text{diag}(\underline{\widetilde{g}}) + \mathbf{O}_N \\ &= \text{diag}(\underline{\widetilde{g}}) \end{aligned} \quad (3.30)$$

and consequently, Equation (3.12) can be simplified to

$$\begin{aligned} \underline{S} &= \widehat{S} + \widetilde{S} \\ &= \widehat{S} + \text{diag}(\underline{\widetilde{g}}) \widehat{S} \\ &= \text{diag}(\mathbf{1}_N + \underline{\widetilde{g}}) \widehat{S} \\ &= (\mathbf{1}_N + \underline{\widetilde{g}}) \odot \widehat{S}. \end{aligned} \quad (3.31)$$

Now consider a single pilot source is present at a known location with nominal manifold vector $\widehat{S}_k$, and its eigenvector $\underline{E}_k$ that corresponds to the principle eigenvalue, the complex gain of antennas can be easily estimated using the approach

presented in the previous section which is simplified to

$$\underline{\tilde{g}} = \frac{1}{K} \sum_{k=1}^K \left(\frac{1}{\lambda_k} \underline{E}_k \oslash \hat{\underline{S}}_k \right) - \underline{1}_N \quad (3.32)$$

with $\lambda_k = \underline{E}_k[1]$ assuming the first element of the array is the reference element.

3.4 Localization Algorithms

3.4.1 Source Localization using Large Aperture Arrays

In [76] a source localization approach is presented which uses a number of sensors (either surrounding or not surrounding the source) to form a large aperture array of known coordinates to perform localization by rotating the array reference point to be at each of the array elements. By subtracting $\mathbb{R}_{nn}$ from $\mathbb{R}_{xx}$, the signal covariance matrix can be constructed that now has theoretical structure

$$\mathbb{R}_{yy} = \mathbb{R}_{xx} - \mathbb{R}_{nn} \quad (3.33)$$

Now consider the i^{th} element of the array is the array reference point by denoting its manifold vector as $\underline{S}_i$. It can be proved that $\underline{S}_i$ can be directly related to $\underline{S}$ via the following expression,

$$\underline{S}_i = \frac{\underline{S}}{\underline{S}[i]} \quad (3.34)$$

where $\underline{S}[i]$ denotes the i^{th} element of vector $\underline{S}$. Combining Equation (3.34) and Equation (3.33), the covariance matrix with the reference point being at the i^{th} element is constructed by

$$\mathbb{R}_i = \frac{\mathbb{R}_{yy}}{\mathbb{R}_{yy}[i, i]} \quad \forall i = 1, 2, \dots N. \quad (3.35)$$

The corresponding signal eigenvalue λ_i at the i^{th} reference point can be estimated by taking the principle eigenvalue of the covariance matrix $\mathbb{R}_i$, that is

$$\lambda_i = \max(\text{eig}(\mathbb{R}_i)). \quad (3.36)$$

From Equations (1.2) and (3.35), it can be shown that the signal eigenvalue λ_i at the i^{th} reference point is related to its range to the source ρ_i as

$$\lambda_i = \rho_i^{2\alpha} \cdot \underline{1}_N^T \underline{\rho}^{-2\alpha}. \quad (3.37)$$

Hence, the ratio of ranges of the source with respect to the i^{th} array element, ρ_i , and the first array element ρ_1 is related to the ratio of their corresponding signal eigenvalues λ_i and λ_1 via the following expression,

$$\frac{\lambda_i}{\lambda_1} = \left(\frac{\rho_i}{\rho_1} \right)^{2\alpha}. \quad (3.38)$$

The vector $\underline{\mathcal{K}} \in \mathcal{R}^{(N-1) \times 1}$ that consists of all ratios of ranges to be used in the later ‘Metric-Fusion’ can be formed,

$$\underline{\mathcal{K}}^2 \triangleq \begin{bmatrix} \frac{\rho_2}{\rho_1} \\ \frac{\rho_3}{\rho_1} \\ \vdots \\ \frac{\rho_N}{\rho_1} \end{bmatrix}^2 = \begin{bmatrix} \frac{\lambda_2}{\lambda_1} \\ \frac{\lambda_3}{\lambda_1} \\ \vdots \\ \frac{\lambda_N}{\lambda_1} \end{bmatrix}^{\frac{1}{\alpha}}. \quad (3.39)$$

In the ‘Metric-Fusion’ phase the vector $\underline{\mathcal{K}}$ can be used to construct $N - 1$ disjoint loci. As a result, the common intersection point of these loci can be used to estimate the source location. It is important to note that in $\mathcal{R}^2$ space, a minimum of 3 loci and hence $N = 4$ are necessary to produce a unique estimate of the source location. Similarly, in $\mathcal{R}^3$ space, a minimum of 4 loci and $N = 5$ are required.

As proposed in [76], in the general $\mathcal{R}^3$ space, metric-fusion can be performed on the vector $\underline{\mathcal{K}}$ to estimate source location $\underline{r}_m$ and the range by taking the least square a set of linear equations:

$$\begin{bmatrix} \underline{r}_m \\ \rho_1^2 \end{bmatrix} = \mathbb{H}^H (\mathbb{H} \mathbb{H}^H)^{-1} \underline{b} \quad (3.40)$$

where

$$\mathbb{H} = [2 (\underline{1}_{N-1} \cdot \underline{r}_1^T - \underline{r}_0^T), (\underline{1}_{N-1} - \underline{\mathcal{K}}^2)] \in \mathcal{R}^{(N-1) \times 4} \text{ and} \quad (3.41)$$

$$\underline{b} = [\|\underline{r}_1\|^2 \underline{1}_{N-1} - \underline{r}_{0x}^2 - \underline{r}_{0y}^2 - \underline{r}_{0z}^2] \in \mathcal{R}^{(N-1) \times 1} \quad (3.42)$$

with

$$\mathbf{r}_0 = [r_2, r_3, \dots, r_N] = [r_{0x}, r_{0y}, r_{0z}]^T \in \mathcal{R}^{3 \times (N-1)}. \quad (3.43)$$

However, in the presence of array uncertainties, Equation (3.37) can be rewritten as follows,

$$\lambda_i = \rho_i^{2\alpha} \cdot \underline{1}_N^T \left(\left| \underline{\tilde{\Delta}}_i + \underline{1}_N \right|^2 \odot \underline{\rho}^{-2\alpha} \right) \quad (3.44)$$

where

$$\underline{\tilde{\Delta}}_i^2 = \frac{\left(\underline{\tilde{\Delta}} + \underline{1}_N \right)^2}{\left(\underline{\tilde{\Delta}}[i] + 1 \right)^2} - \underline{1}_N \in \mathcal{R}^{N \times 1}. \quad (3.45)$$

with $\underline{\tilde{\Delta}} = \underline{\tilde{G}} \underline{\hat{S}} \odot \underline{\hat{S}}$. In this case, Equation (3.38) becomes

$$\begin{aligned} \frac{\lambda_i}{\lambda_1} &= \frac{\rho_i^{2\alpha} \cdot \underline{1}_N^T \left(\underline{\tilde{\Delta}}_i + \underline{1}_N \right)^2}{\rho_1^{2\alpha} \cdot \underline{1}_N^T \left(\underline{\tilde{\Delta}}_1 + \underline{1}_N \right)^2} \\ &= \left(\frac{\rho_i}{\rho_1} \right)^{2\alpha} \cdot \left(\frac{\underline{1}_N^T \left(\underline{\tilde{\Delta}}_i + \underline{1}_N \right)^2}{\underline{1}_N^T \left(\underline{\tilde{\Delta}}_1 + \underline{1}_N \right)^2} \right). \end{aligned} \quad (3.46)$$

Let $\underline{\tilde{\Delta}}_i^2 \triangleq \frac{\underline{1}_N^T \left(\underline{\tilde{\Delta}}_i + \underline{1}_N \right)^2}{\underline{1}_N^T \left(\underline{\tilde{\Delta}}_1 + \underline{1}_N \right)^2} - 1$ for $i = 2, \dots, N$, the vector $\underline{\mathcal{K}} \in \mathcal{R}^{(N-1) \times 1}$ in Equation (3.39) can be expressed as

$$\begin{aligned} \underline{\mathcal{K}}^2 &= \left[\begin{array}{c} \frac{\lambda_2}{\lambda_1} \left(\underline{\tilde{\Delta}}_2^2 + 1 \right) \\ \frac{\lambda_3}{\lambda_1} \left(\underline{\tilde{\Delta}}_3^2 + 1 \right) \\ \vdots \\ \frac{\lambda_N}{\lambda_1} \left(\underline{\tilde{\Delta}}_N^2 + 1 \right) \end{array} \right]^{\frac{1}{\alpha}} \\ &= \underline{\hat{\mathcal{K}}}^2 + \underline{\tilde{\mathcal{K}}}^2 \end{aligned} \quad (3.47)$$

where the nominal ratio vector $\underline{\hat{\mathcal{K}}}^2$ and the uncertainty ratio vector $\underline{\tilde{\mathcal{K}}}^2$ are given as follows

$$\underline{\hat{\mathcal{K}}}^2 = \left[\begin{array}{c} \frac{\lambda_2}{\lambda_1} \\ \frac{\lambda_3}{\lambda_1} \\ \vdots \\ \frac{\lambda_N}{\lambda_1} \end{array} \right]^{\frac{1}{\alpha}} \quad (3.48)$$

and

$$\underline{\tilde{\mathcal{K}}}^2 = \underline{\hat{\mathcal{K}}}^2 \odot \left[\begin{array}{c} \tilde{\Delta}_2^2 \\ \tilde{\Delta}_3^2 \\ \vdots \\ \tilde{\Delta}_N^2 \end{array} \right]^{\frac{1}{\alpha}}. \quad (3.49)$$

The matrix $\mathbb{H}$ in Equation (3.41) can be expressed in terms of its nominal value $\hat{\mathbb{H}}$ that with $\underline{\hat{\mathcal{K}}}^2$ and the deviation from the true value $\tilde{\mathbb{H}}$ with $\underline{\tilde{\mathcal{K}}}^2$:

$$\mathbb{H} = \hat{\mathbb{H}} + \tilde{\mathbb{H}} \quad (3.50)$$

where

$$\hat{\mathbb{H}} = \left[2 \left(\mathbf{1}_{N-1} \cdot \mathbf{r}_1^T - \mathbf{r}_0^T \right), \left(\mathbf{1}_{N-1} - \underline{\hat{\mathcal{K}}}^2 \right) \right] \quad (3.51)$$

and

$$\tilde{\mathbb{H}} = \left[\mathbb{O}_{(N-1) \times 3}, -\underline{\tilde{\mathcal{K}}}^2 \right]. \quad (3.52)$$

Note that, by using more than minimum number of required array elements will result an overdetermined set of equations which allows the effects of finite observation interval, noise and array uncertainties to be reduced.

3.4.2 The spherical-MUSIC algorithm

Signal-subspace based superresolution techniques are considered to be as one of the most powerful approach to locate the source and they are based on the eigen-decomposition of the array data covariance matrix, $\mathbb{R}_{xx}$. Assume $M = 1$ and in $\mathcal{R}^2$ space (i.e. $\phi = 0$), the spherical-MUSIC algorithm based on the cost function ξ can be defined as

$$\xi_{MU}(\theta, \rho) = \underline{S}(\theta, \rho)^H \mathbb{E}_n \mathbb{E}_n^H \underline{S}(\theta, \rho) \quad (3.53)$$

where $\mathbb{E}_n \in \mathcal{C}^{N \times (N-M)}$ is the noise-space matrix, which consists of a set of $N - M$ eigenvectors corresponding to the $N - M$ smallest eigenvalues of the array data

covariance matrix $\mathbb{R}_{xx}$. The manifold vector $\underline{S}(\theta, \rho)$ is given by Equation (1.2). By searching Equation (3.53), the corresponding source location parameters such as the azimuth θ , and the range ρ can be estimated.

Now consider the array uncertainties in Equation (3.12), for simplicity, let $\hat{\underline{S}} \triangleq \hat{\underline{S}}(\theta, \rho)$ and $\tilde{\underline{S}} \triangleq \tilde{\underline{S}}(\theta, \rho)$, Equation (3.53) can be rewritten as:

$$\begin{aligned} \xi(\theta, \rho) &= \left(\hat{\underline{S}} + \tilde{\underline{S}} \right)^H \mathbb{E}_n \mathbb{E}_n^H \left(\hat{\underline{S}} + \tilde{\underline{S}} \right) \\ &= \hat{\xi}(\theta, \rho) + \tilde{\xi}(\theta, \rho) \end{aligned} \quad (3.54)$$

where $\hat{\xi}(\theta, \rho)$ is the nominal cost function,

$$\hat{\xi}(\theta, \rho) = \hat{\underline{S}}^H \mathbb{E}_n \mathbb{E}_n^H \hat{\underline{S}} \quad (3.55)$$

and $\tilde{\xi}(\theta, \rho)$ represents the error in the cost function as a result of array gain and phase uncertainties

$$\tilde{\xi}(\theta, \rho) = \hat{\underline{S}}^H \mathbb{E}_n \mathbb{E}_n^H \tilde{\underline{G}} \hat{\underline{S}} + \hat{\underline{S}}^H \tilde{\underline{G}}^H \mathbb{E}_n \mathbb{E}_n^H \hat{\underline{S}} + \hat{\underline{S}}^H \tilde{\underline{G}}^H \mathbb{E}_n \mathbb{E}_n^H \tilde{\underline{G}} \hat{\underline{S}}. \quad (3.56)$$

3.5 Indoor Performance Evaluation

3.5.1 Array Testbed Setup

In this section, two localization algorithms presented in the previous section were implemented and tested using a circular array of 9 elements (shown in Fig. 3.1). The array was constructed using nine USRP boards each equipped with a RFX2400 daughterboard which has an operational frequency range from 2300MHz to 2900MHz.

The array is constructed with the geometry shown in Table 4.1. The attributes of the antenna used in this paper are listed as follows:

- antenna elements are omnidirectional quarter wave monopoles in their x-y plane
- operational frequency range from 2400MHz to 2490MHz

- 0dBi nominal gain in the x-y plane
- 9-element array consists of a 8-element circular array and a centre element
- minimum element-to-element spacing is half wavelength at $F_c = 2.43\text{GHz}$ to mitigate coupling of the elements

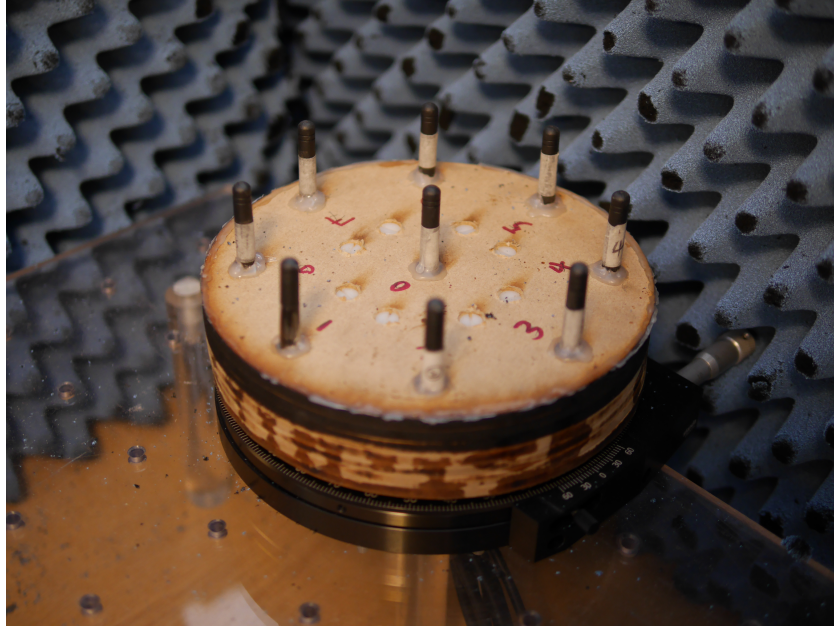


Figure 3.1: Nine-element planar array with geometry described in Table I.

Table 3.1: Cartesian coordinates of the array in centimetres

Element	x in cm	y in cm
Rx1	0	0
Rx2	-8.0652	0
Rx3	-5.7030	-5.7030
Rx4	0	-8.0652
Rx5	5.7030	-5.7030
Rx6	8.0652	0
Rx7	5.7030	5.7030
Rx8	0	8.0652
Rx9	-5.7030	5.7030

Each antenna array element is attached to a Universal Software Radio Peripheral (USRP) board as shown in Fig. 4.4. Each USRP board down-converts the

RF analog signal to its baseband which is then digitized to produce the complex (I and Q) digital data that can be used by the localization algorithms. The carrier frequency F_c to be used in the downconversion and the sampling rate F_s of the Analogue to Digital Converter (ADC) were selected to be $F_c = 2.43\text{GHz}$ and $F_s = 2\text{MS/s}$ ($0.5\mu\text{s}$ sampling period). The source consisted of a single USRP board transmitting only the carrier at a power of approximately 0dBm.



Figure 3.2: Multi-USRP hardware configuration for the 9-element array (8 out of 9 Rx USRP boards are shown here), USRP boards share a common clock that is generated by a GPS module.

3.5.2 Array Synchronisation and Calibration

All nine USRP boards must be fully synchronized to form a coherent antenna array system, since most both MUSIC and LAA algorithms assume that the same local oscillator and time reference were used between array elements. However, each board has its own local oscillator locked to its own internal clock source which will cause frequency and phase drift between array elements. Moreover, signals were sampled based on arbitrary internal clocks that are independent from board to board which will cause misalignment of samples. The problem can be

solved by providing a common 10MHz reference clock signal and a 1 PPS (Pulse Per Second) signal to each board. The 10MHz clock provides a reference for the local oscillator and the ADC converter on each USRP board. The 1 PPS signal provides a common reference frame that is used to reset the timestamp on each board. This allows multiple independent USRP boards to be tuned to the same carrier frequency and the same timing frame giving a fully synchronized system. For the purpose of the experiment, a GPS unit as shown in Figure 3.4 was used to generate both the 10MHz reference clock signal the 1 PPS (Pulse Per Second) signal feeding to all USRP units as shown in Fig. 4.3 and Fig. 4.4. Figure 3.5a and 3.5b show the system before and after synchronisation respectively.

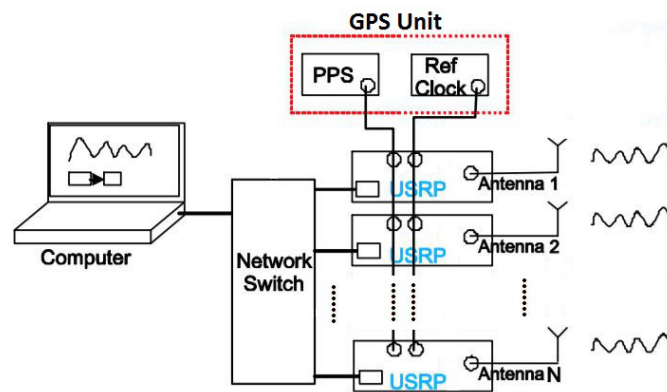
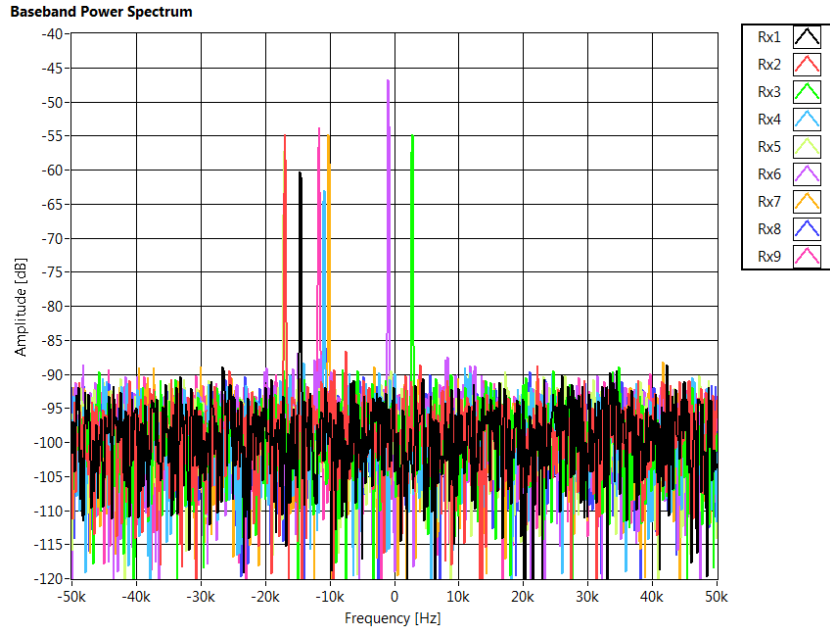


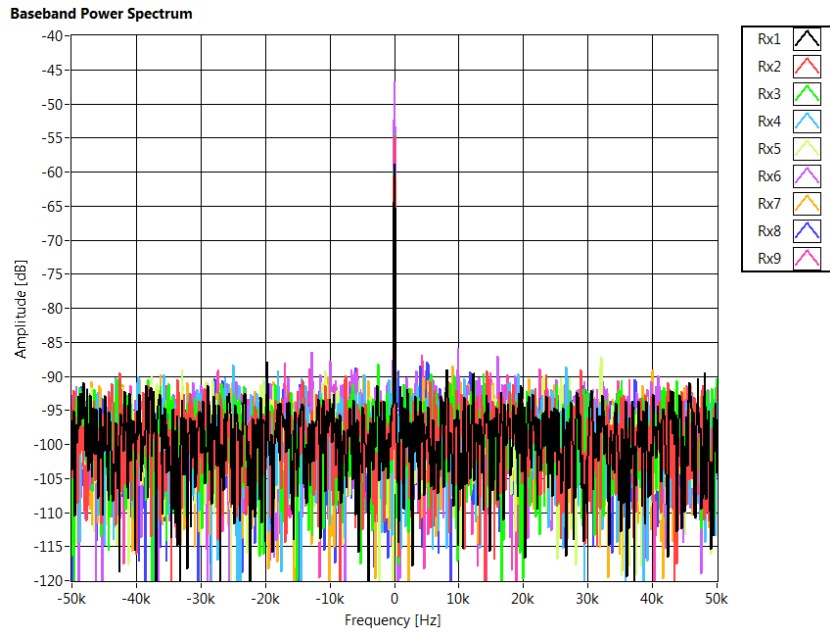
Figure 3.3: Array testbed setup comprising multiple USRP boards that are synchronized using a common clock reference and 1 PPS reference source.



Figure 3.4: GPS module used for array synchronisation.



(a) Before synchronisation



(b) After synchronisation

Figure 3.5: Array synchronisation using a common clock reference.

However, this alone is not enough to form a fully calibrated coherent array system. Due to antenna imperfections and the mutual coupling effect, further calibration is necessary. Consider a single Tx antenna (Figure 3.6) was used to generate multiple time disjoint measurements at known locations as illustrated in

Fig. 3.7 and Table 3.2 were used for array calibration using Equation 3.23. Following the calibration procedure, the matrix $\tilde{\mathbb{G}}$ can be estimated for this particular array setup.

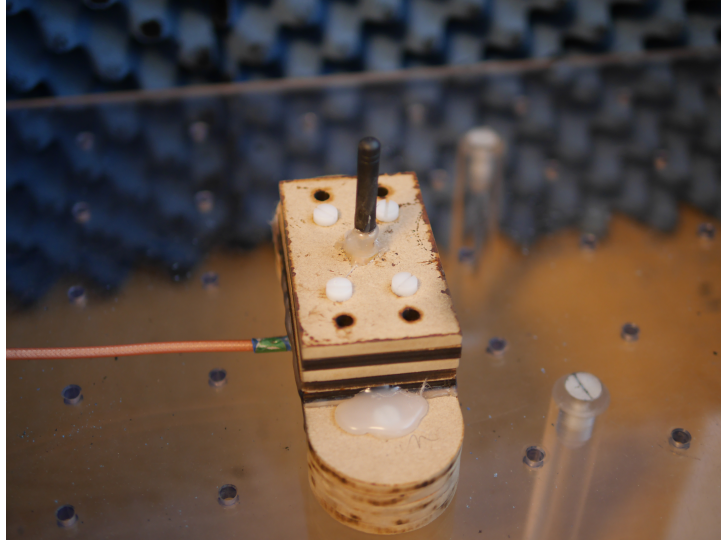


Figure 3.6: Tx Antenna.

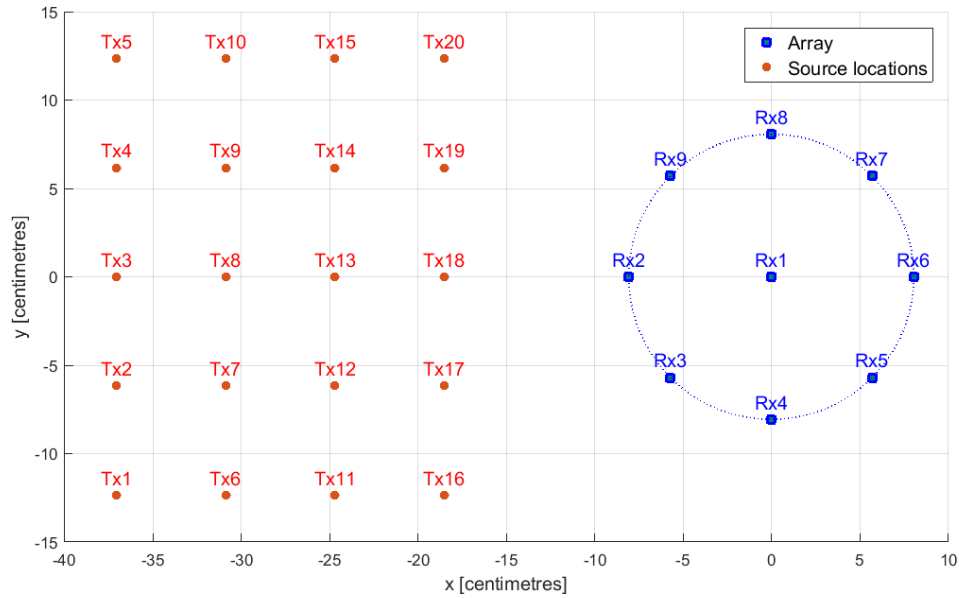


Figure 3.7: The location of the source with respect to the array in centimetres. The red square represents the source and the blue circles represent the array.

Table 3.2: Source locations and measured SNRs

Source	x (cm)	y (cm)	ρ (cm)	θ ($^\circ$)	SNR (dB)
Tx1	-37.0370	-12.3457	39.0405	198.4349	21.13
Tx2	-37.0370	-6.1728	37.5479	189.4623	19.80
Tx3	-37.0370	0	37.0370	180.0000	20.38
Tx4	-37.0370	6.1728	37.5479	170.5377	21.71
Tx5	-37.0370	12.3457	39.0405	161.5651	19.52
Tx6	-30.8642	-12.3457	33.2418	201.8014	24.09
Tx7	-30.8642	-6.1728	31.4754	191.3099	24.84
Tx8	-30.8642	0	30.8642	180.0000	23.72
Tx9	-30.8642	6.1728	31.4754	168.6901	24.30
Tx10	-30.8642	12.3457	33.2418	158.1986	22.27
Tx11	-24.6914	-12.3457	27.6058	206.5651	24.62
Tx12	-24.6914	-6.1728	25.4513	194.0362	26.14
Tx13	-24.6914	0	24.6914	180.0000	26.84
Tx14	-24.6914	6.1728	25.4513	165.9638	25.81
Tx15	-24.6914	12.3457	27.6058	153.4349	26.91
Tx16	-18.5185	-12.3457	22.2565	213.6901	28.42
Tx17	-18.5185	-6.1728	19.5202	198.4349	32.27
Tx18	-18.5185	0	18.5185	180.0000	32.95
Tx19	-18.5185	6.1728	19.5202	161.5651	32.45
Tx20	-18.5185	12.3457	22.2565	146.3099	31.53

3.5.3 Experimental Results

The array system was tested in an anechoic environment to eliminate multipath in which case free-space path loss exponent $\alpha = 2$ can be assumed. In the presence of multipath, the preprocessing algorithm in [87] can be employed to remove the contributions of all Non-Line-of-Sight (NLOS) paths. For each source position, 100 experiments were conducted.

In the case of $L = 1000$ snapshots, Table illustrate the DOA and the range estimation RMSEs of both localization algorithms. Results show that both LAA and spherical-MUSIC algorithms give very accurate DOA estimates for all source locations. As for the range estimates, the LAA localisation approach outperforms the spherical-MUSIC algorithm by a great margin.

Table 3.3: RMSE for each Tx position, $L = 1000$.

Source	LAA RMSE		spherical-MUSIC RMSE	
	$\tilde{\theta}$ ($^{\circ}$)	$\tilde{\rho}$ (%)	$\tilde{\theta}$ ($^{\circ}$)	$\tilde{\rho}$ (%)
Tx1	0.2073	2.3245	0.5715	14.8624
Tx2	0.1889	3.9381	0.5594	16.8154
Tx3	0.2569	25.1111	0.3990	10.0528
Tx4	0.1222	3.0262	0.6945	15.0913
Tx5	0.2286	3.2072	0.7627	17.2498
Tx6	0.3474	6.6004	0.5407	17.1883
Tx7	0.1150	1.5016	0.9280	14.1553
Tx8	0.2152	3.4036	1.3437	25.9418
Tx9	0.1112	1.2721	1.1633	22.6292
Tx10	0.0765	2.7017	0.7110	19.5538
Tx11	0.0604	1.8072	0.5468	19.5564
Tx12	0.0804	0.7136	0.8838	19.4576
Tx13	0.1145	1.5919	1.0106	30.5636
Tx14	0.3962	13.3488	1.0827	26.2597
Tx15	0.1018	0.8336	1.4522	19.9373
Tx16	0.7189	1.9073	1.8210	33.0294
Tx17	0.0520	0.1956	2.5905	33.7190
Tx18	0.0351	1.0340	1.5756	33.1101
Tx19	0.0273	0.7523	1.9731	40.7688
Tx20	0.0491	1.5573	1.6932	38.0550

Results of each trial and the RMSE of each of three selected source locations are given in Fig 3.8, 3.9 and 3.10. Once again it is clear that the performance of the LAA algorithm is much poorer for the far away sources than for the closer ones. On the other hand, the spherical-MUSIC algorithm yields more consistent estimates over the entire tested range. However, increasing the number of snapshots yields a significantly improved performance for the LAA algorithm whereas the spherical-MUSIC algorithm does not benefit from increased the number of snapshots as much.

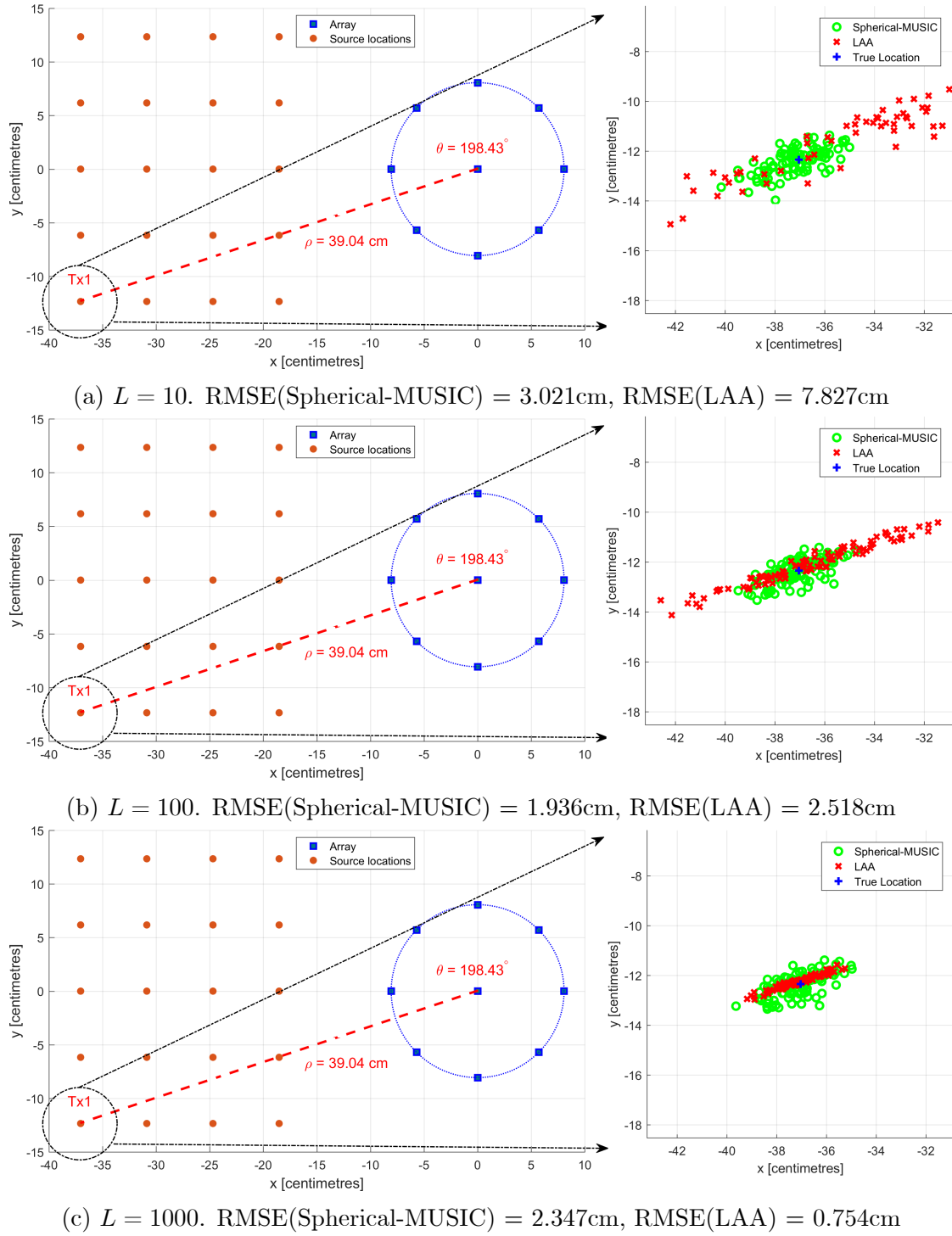


Figure 3.8: Localisation results over 100 experiments: green circles represent estimates using the spherical-MUSIC algorithm, red crosses represent estimates using the LAA algorithm, and blue plus represents the true source location.

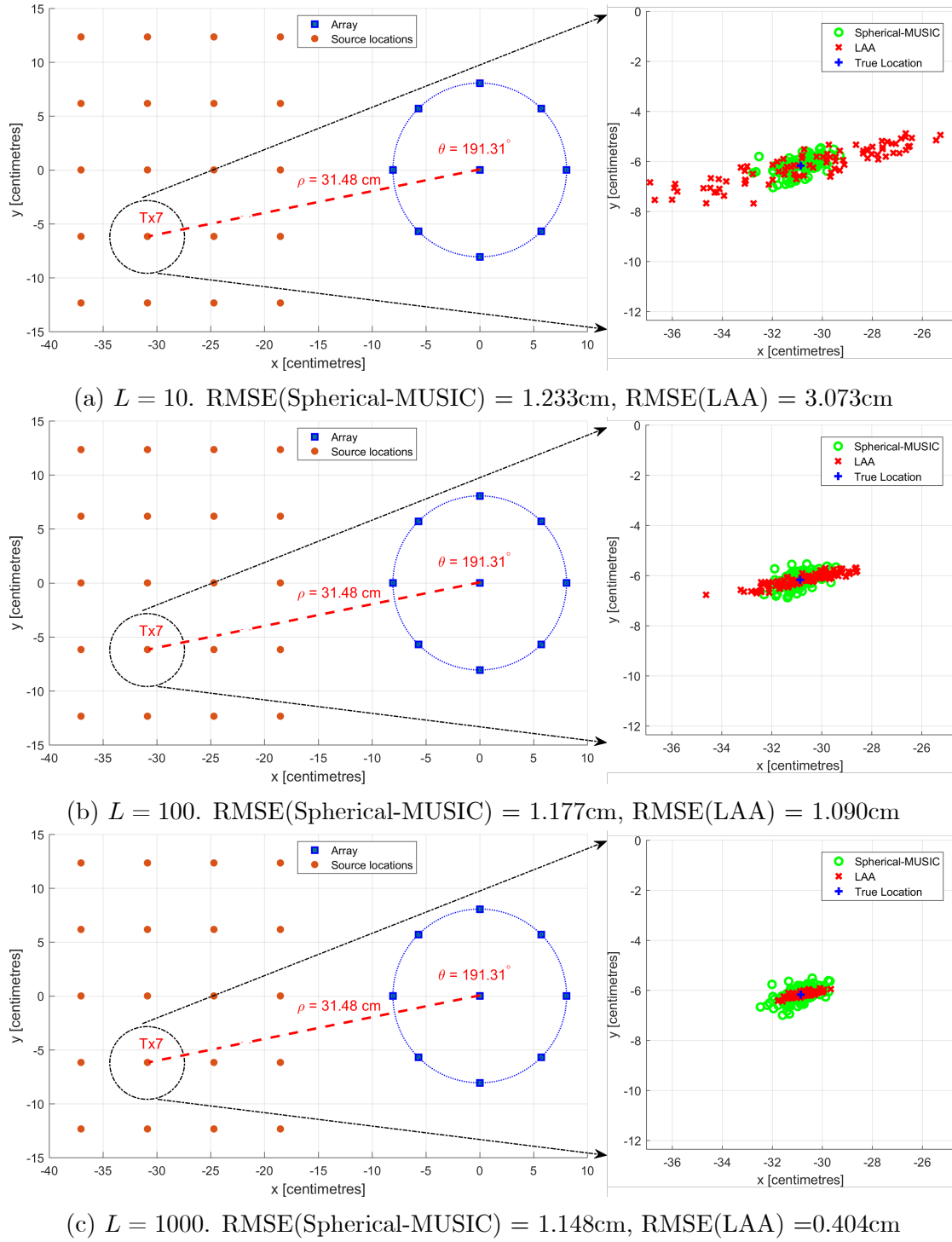


Figure 3.9: Localisation results over 100 experiments: green circles represent estimates using the spherical-MUSIC algorithm, red crosses represent estimates using the LAA algorithm, and blue plus represents the true source location.

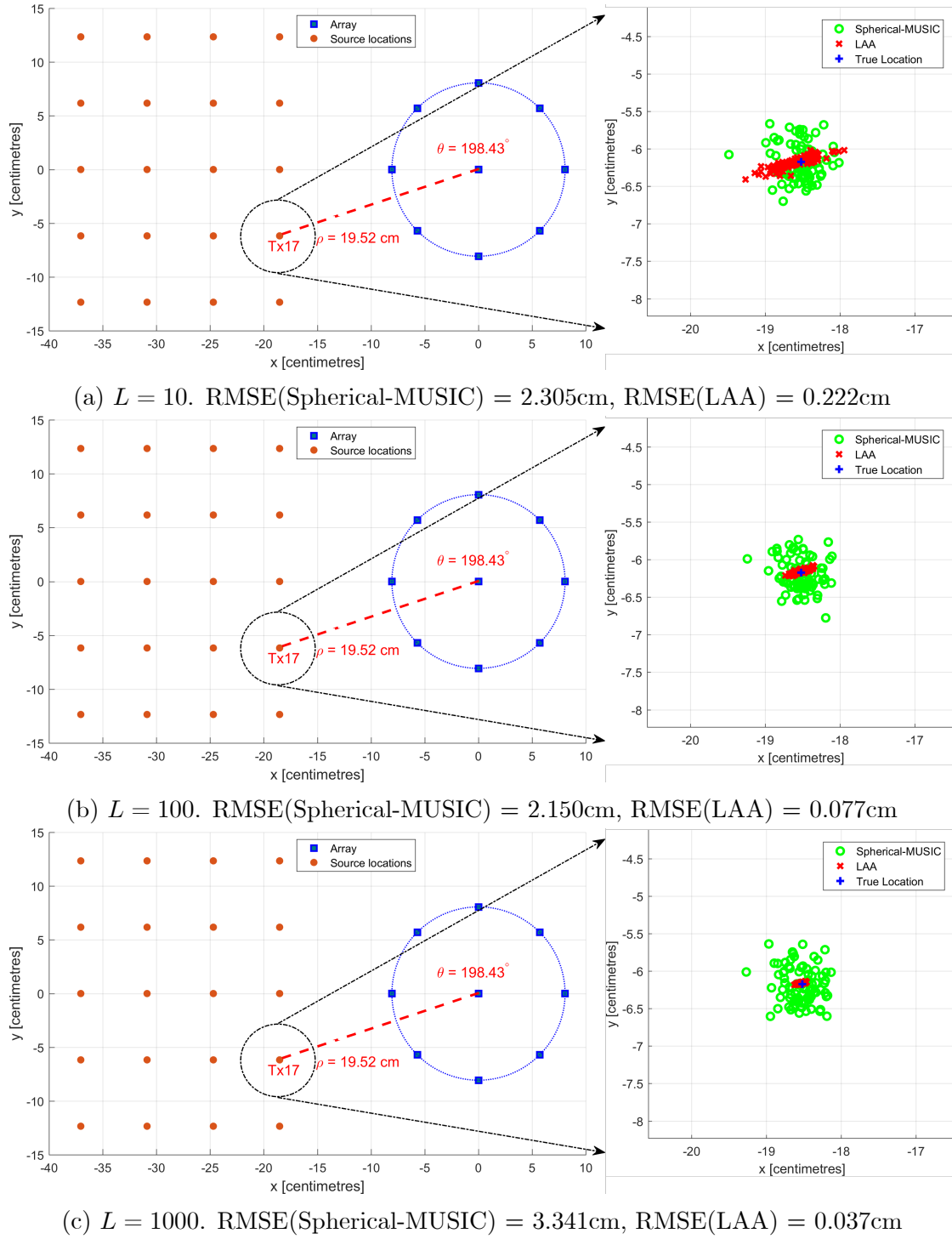


Figure 3.10: Localisation results over 100 experiments: green circles represent estimates using the spherical-MUSIC algorithm, red crosses represent estimates using the LAA algorithm, and blue plus represents the true source location.

3.6 Outdoor Experimental Study with Carrier Frequency Uncertainties

In this section, a field-test of the antenna array SDR testbed is presented and its results are studied. This field-test was conducted in collaboration with the Multidisciplinary Approach Life histories (MALO) group in a native woodland at Silwood Park, Imperial College London. As the array elements are widely distributed in space, each array element is locked to its own independent LO of slightly different frequencies than the assumed carrier frequency. This prevents the system from operating as a coherent array. Therefore, a carrier frequency offset estimation algorithm is employed as a pre-processor and used in conjunction with a pilot based array calibration technique. In addition, the performance of the LAA localisation algorithm is examined for localising targets at different positions.

3.6.1 Local Oscillator Frequency Offsets Estimation using Harmonic Retrieval

In the contents of array processing, it is generally assumed that all LOs behind each antenna run at the exactly same frequency. However, this is not always practical, large aperture array systems usually have their antennas far away from each other which makes it hard to have a common LO. Two synchronisation approaches are examined. One approach is to use the harmonic retrieval in conjunction with a frequency MUSIC (Multiple Signal Identification and Classification) algorithm. In [45], the author proposed a novel algorithm that estimates the frequency offsets in a widely distributed array system with independent LO. The approach utilises harmonic retrieval modelling based MUSIC algorithm to estimate the frequency offset of each array element from its source.

According to Equation (1.1), the received antenna array signal for the i^{th} an-

tenna element can be expressed as:

$$\tilde{x}_i(t) = S_i \exp(-j2\pi \tilde{F}_{c,i} t) m(t) + n_i(t); \forall_i \quad (3.57)$$

where S_i and $n_i(t)$ are the i^{th} element of the $(N \times 1)$ vectors $\underline{S}$ and $\underline{n}(t)$ respectively.

The received signal from i^{th} antenna element is sampled at T_s period and passed through a Tapped-Delay-Line (TDL) of length N_L as shown in Figure 3.11.

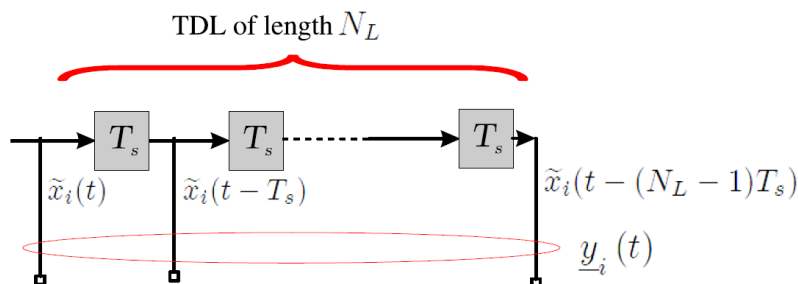


Figure 3.11: The i^{th} TDL of length N_L used for local oscillator frequency offset estimation.

The signal vector at the output of the TDL $\underline{y}_i(t) \in \mathcal{C}^{N_L \times 1}$ is modelled as:

$$\begin{aligned} \underline{y}_i(t) &= \begin{pmatrix} \tilde{x}_i(t) \\ \tilde{x}_i(t - T_s) \\ \tilde{x}_i(t - 2T_s) \\ \vdots \\ \tilde{x}_i(t - (N_L - 1)T_s) \end{pmatrix} \in \mathcal{C}^{N_L \times 1} \\ &= S_i \underline{A}_i m(t) + \underline{\tilde{n}}_i(t); \forall_i \end{aligned} \quad (3.58)$$

where

$$\underline{\tilde{n}}_i(t) = \begin{pmatrix} n_i(t) \\ n_i(t - T_s) \\ n_i(t - 2T_s) \\ \vdots \\ n_i(t - (N_L - 1)T_s) \end{pmatrix} \in \mathcal{C}^{N_L \times 1}; \forall_i \quad (3.59)$$

and the vector $\underline{A}_i \in \mathcal{C}^{N_L \times 1}$ is the so-called temporal manifold vector for the i^{th} antenna element defined as

$$\underline{A}_i \triangleq \underline{A}_i(\tilde{F}_{c,i}) = \exp \left(-j2\pi \tilde{F}_{c,i} \begin{pmatrix} 0 \\ 1 \\ \vdots \\ N_L - 1 \end{pmatrix} T_s \right); \forall_i \quad (3.60)$$

By constructing the covariance matrix $\mathbb{R}_{y_i y_i} = \mathcal{E} \left\{ \underline{y}_i(t) \underline{y}_i(t)^H \right\}$ of $\underline{y}_i(t)$, the frequency error $\tilde{F}_{c,i}$ can be estimated using the following MUSIC based expression, that is:

$$\tilde{F}_{c,i} = \arg \min_{\tilde{f}} \underline{A}_i^H(\tilde{f}) \mathbb{E}_{n,i} \mathbb{E}_{n,i}^H \underline{A}_i(\tilde{f}); \forall_i \quad (3.61)$$

where $\mathbb{E}_{n,i}$ is the noise subspace that can be obtained from the eigen-decomposition of $\mathbb{R}_{y_i y_i}$. Once the LO frequency of each element has been estimated, it can be easily removed by multiplication to obtain the baseband signal $\underline{x}(t)$ which corresponds to the received signal in the coherent array system, that is

$$\begin{aligned} \underline{x}(t) &= \tilde{\underline{x}}(t) \odot \exp(j2\pi \tilde{F}_c t) \\ &= \underline{S}m(t) + \underbrace{\tilde{\underline{n}}(t) \odot \exp(j2\pi \tilde{F}_c t)}_{\text{redefined as } \underline{n}(t)} \\ &= \underline{S}m(t) + \underline{n}(t). \end{aligned} \quad (3.62)$$

3.6.2 Outdoor Performance Evaluation

The outdoor field-test was conducted in collaboration with the Multidisciplinary Approach Life histOries (MALO) group in a native woodland at Silwood Park, Imperial College London. The goal was to develop a wireless tracking system that may lead to real-time monitoring of the populations of wood mice, which will allow scientists to understand and manage the biosphere. All natural features in the study site was provided by the MALO group at a 1m^2 resolution. The entire area measures 2.47 hectares and is populated with trees, logs, stumps and debris with their locations and dimensions measured and mapped as shown in Figure 3.12. Sensors and transmitters were carefully placed and their coordinates were measured at locations shown in Figure 3.13 using a high precision total station from Leica shown in Figure 3.14. The total station is capable of measuring angles and distances with an accuracy of about 2 arc-second and $1.0\text{mm} \pm 1.5\text{ppm}$ respectively. The coordinates were calculated from the measured angles and distances using trigonometry and triangulation. The sensors and transmitters were placed to avoid trees so that the environment is a line-of-sight environment. In addition, the study site can be considered as a multipath free environment since wood lumbers have very low reflection coefficients [88].

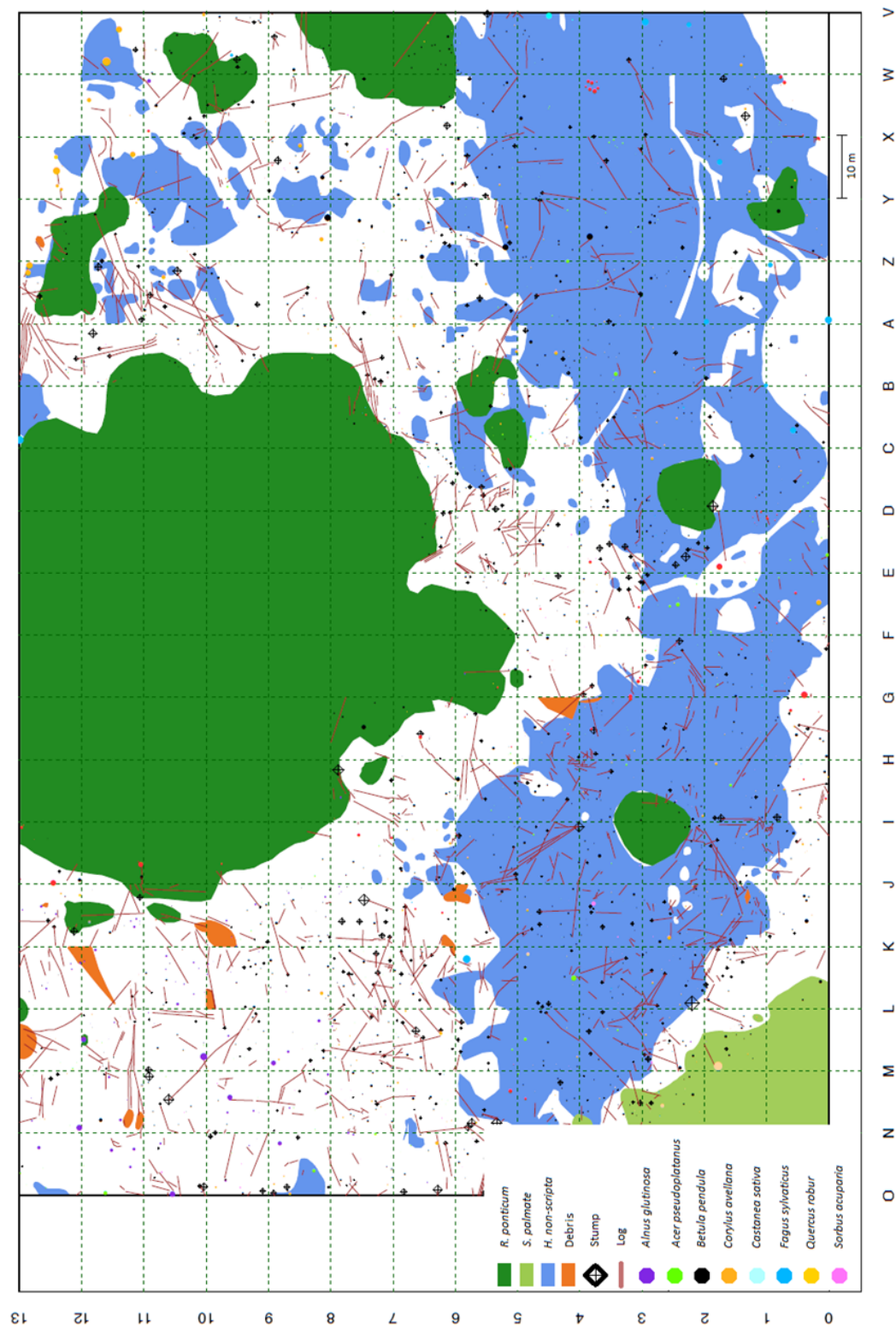


Figure 3.12: Map of the study site at Nash's Copse, Silwood Park, Imperial College London with a total area of 2.47 Hectare ($24700m^2$). Grids are $10 \times 10m$.

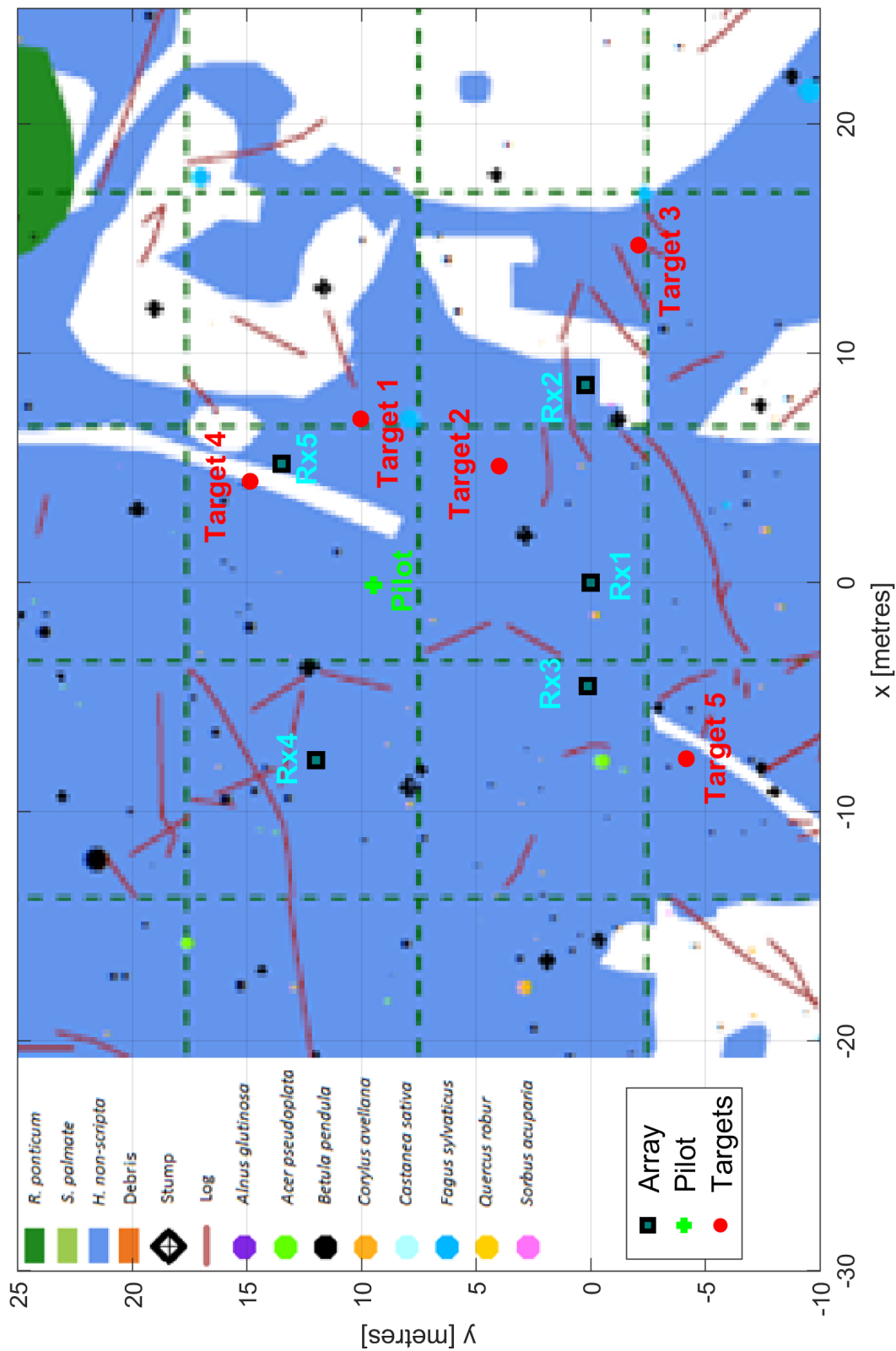


Figure 3.13: Zoomed in area of the field study site in more detail the array and the source locations.



Figure 3.14: Fieldwork in collaboration with the MALO group at the Silwood Park taking measurements for the study.

3.6.3 Array Testbed Setup

For this study, an array of $N = 5$ sensing antennas was constructed using five USRP boards with the same RFX2400 daughterboards that were used in the indoor study. Signal data was sampled at a rate of $F_s = \frac{1}{T_s} = 2\text{MS/s}$ ($0.5\mu\text{s}$ sampling period) and was transferred from and to the host computer via gigabit Ethernet. A single pilot source at a known location and five time disjoint targets whose locations are to be estimated were considered as shown in Figure 3.15 transmitting a 100kHz sine wave on a carrier at $F_c = 2.43\text{GHz}$ with a power of 10dBm. Omnidirectional monopole antennas with operational frequency range from 2400MHz to 2500MHz and 5dBi nominal gain in the x-y plane used. The LAA localisation algorithm was implemented using $L = 100$ snapshots.

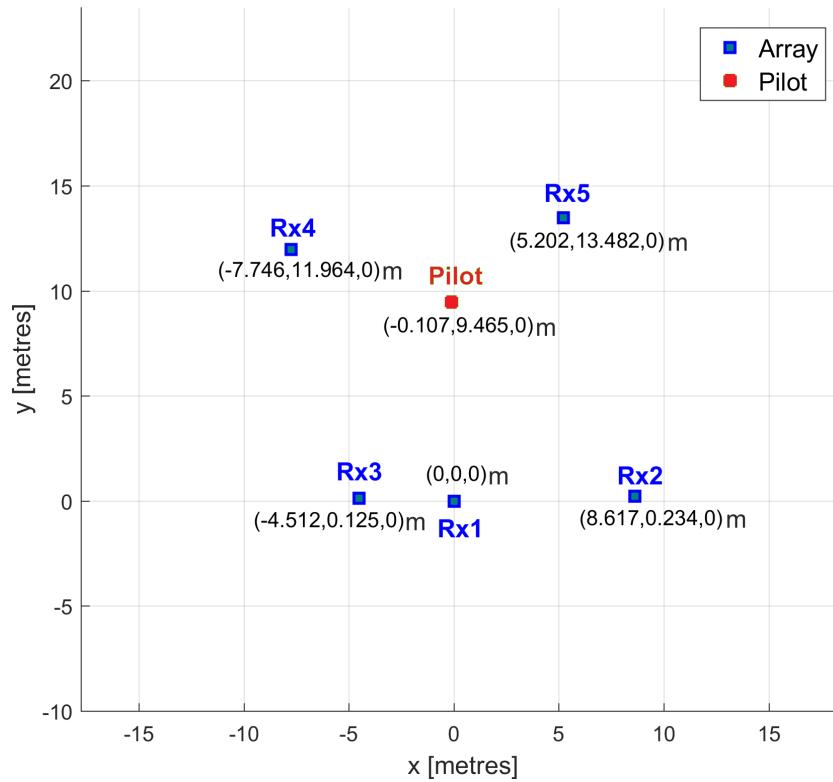


Figure 3.15: Fieldtest setup array geometry.

3.6.4 Array Synchronisation and Calibration

Due to the size of the field, using a common clock reference signal source and 1 PPS signal source was impractical. Instead, each board was connected to a 1 PPS signal of a GPS unit as shown in Figure 3.16. However, each USRP board contains its own local oscillator locked to its own internal 10MHz reference clock which will lead to frequency offset between array elements. One approach to synchronise the array is to use the harmonic retrieval frequency uncertainty estimation in [45]. Here, the pilot signal received from each of the array element was passed through a tapped delay line (TDL) of length $N_L = 20$. Figure 3.17 and Table 3.4 show the carrier frequency offset estimates using the harmonic retrieval algorithm. Figure 3.18a and 3.18b show the frequency offsets before and after the frequency synchronization respectively. However, frequency synchronization alone is not enough to fully calibrate an array. In addition, the complex gain uncertainties were estimated

using Equation (3.32) with a single pilot source.

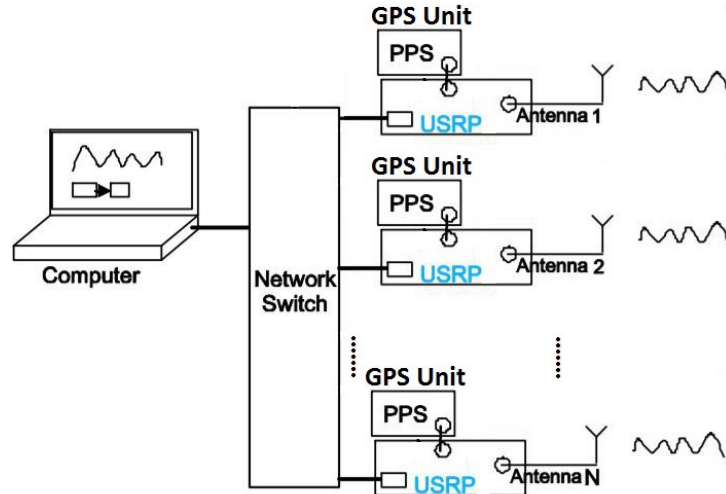


Figure 3.16: Array testbed setup comprising multiple USRP boards that are synchronized using GPS unit generated PPS clocks.

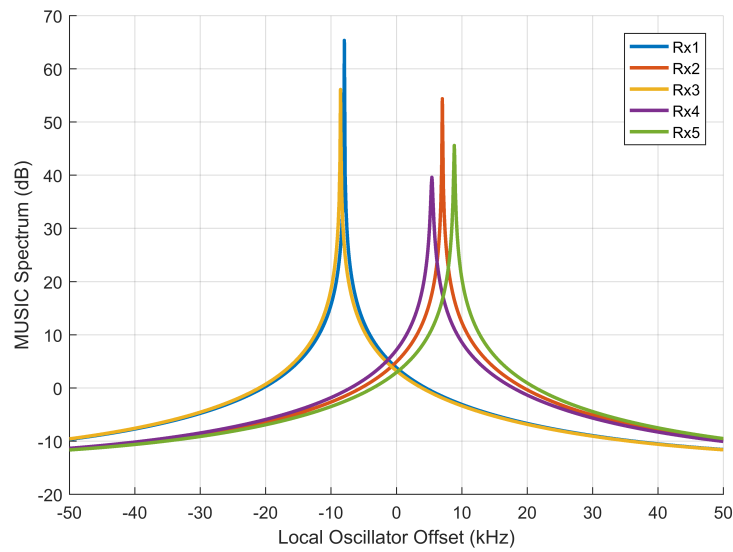
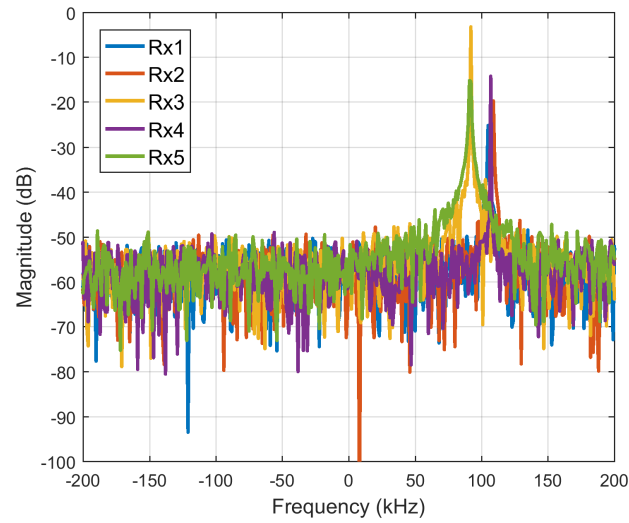


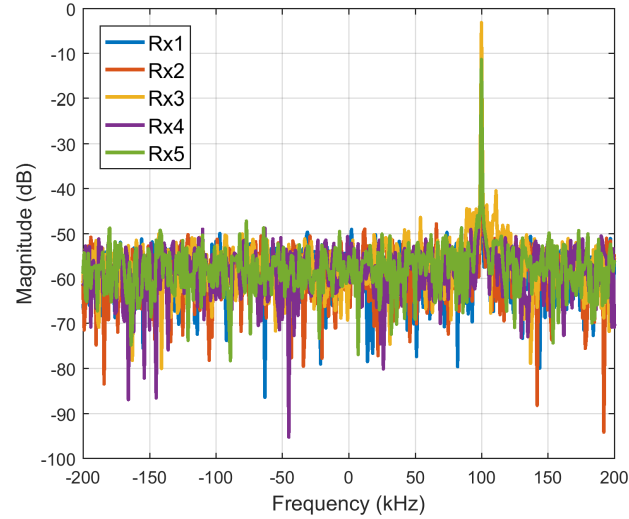
Figure 3.17: MUSIC spectra of the harmonic retrieval for the tested array with local oscillator offsets. $L = 100$ and $N_L = 20$.

Table 3.4: Table of local oscillator offset and measured SNR for each Rx element.

Element	$\tilde{F}_c$ in kHz	SNR in dB
Rx1	-7.953	8.24
Rx2	7.037	12.01
Rx3	-8.534	18.81
Rx4	5.435	15.95
Rx5	8.864	16.83



(a) Before frequency synchronization



(b) After frequency synchronization

Figure 3.18: Frequency synchronization of the array.

3.6.5 Experimental Results

The LAA localization approach used in the previous section is also used here to investigate the performance of the LAA in the described outdoor environment. The array calibration is performed for each of the 100 experiments using $L = 100$ snapshots and $F_s = 2\text{MS/s}$ for every target before applying the LAA localisation algorithm. However, even after the synchronisation and calibration, the spherical-MUSIC based algorithm still fails to produce any valuable location estimate for this array setup. This may be because the residual uncertainties are still too large for the very sensitive spherical-MUSIC algorithm to perform. The red dots in Figure 3.19, 3.20, 3.21, 3.22 and 3.23 illustrate the LAA estimates of each of the targets location over 100 realisations. RMSE results in metres calculated from these estimates are given in the figures as well as the actual target locations measured using the total station. Tables 3.5, 3.6, 3.7, 3.8 and 3.9 show the SNR at each sensing antenna for the corresponding target. It is clear that the estimates are better as the target moving closer to the array. RMSE results for "Target 1", "Target 2" and "Target 4" show the spatial resolution could come down to less than 0.2m.

Table 3.5: Target 1 SNR for each element in the array

Element	SNR in dB
Rx1	8.13
Rx2	11.95
Rx3	18.89
Rx4	15.97
Rx5	26.85

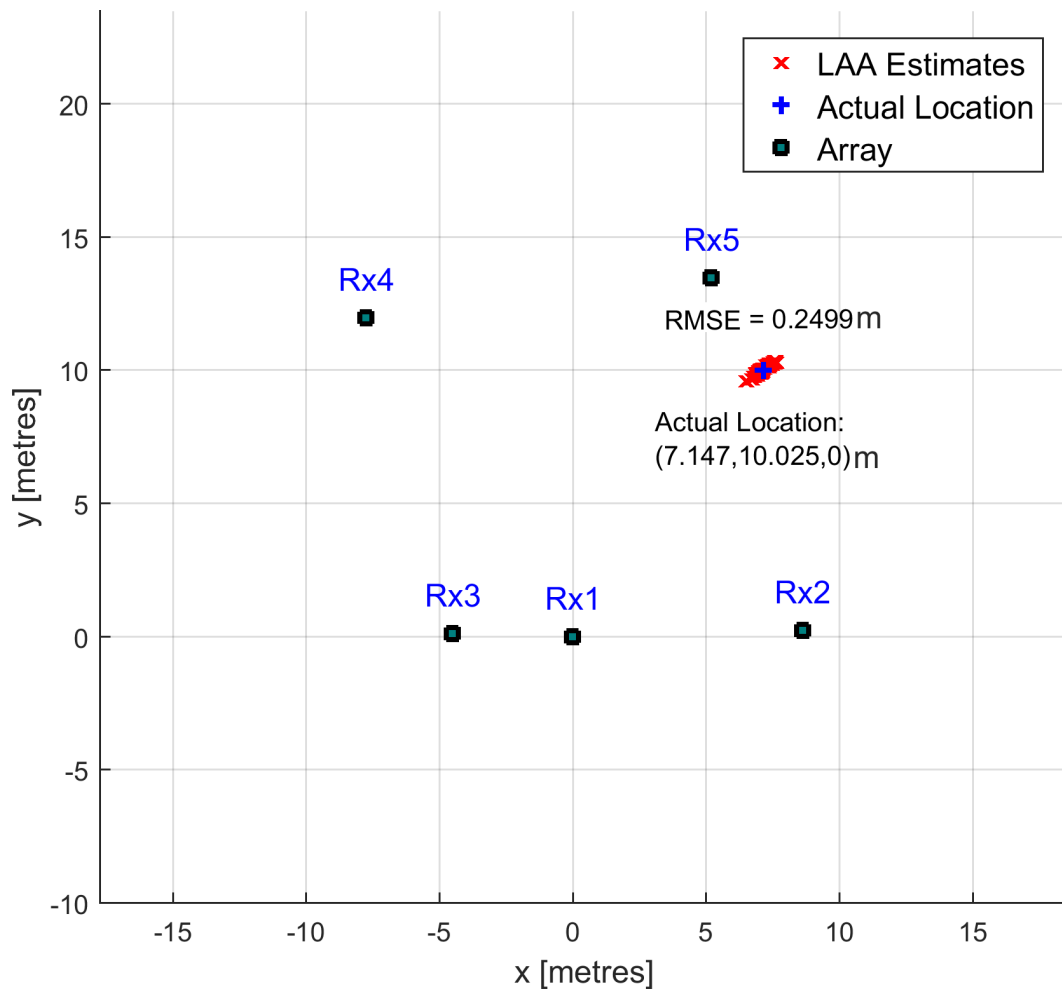


Figure 3.19: Target 1 Localisation results over 100 experiments: red crosses represent estimates using the LAA localisation algorithm, and blue plus represents the actual source location.

Table 3.6: Target 2 SNR for each element in the array

Element	SNR in dB
Rx1	19.93
Rx2	22.97
Rx3	13.38
Rx4	6.02
Rx5	17.43

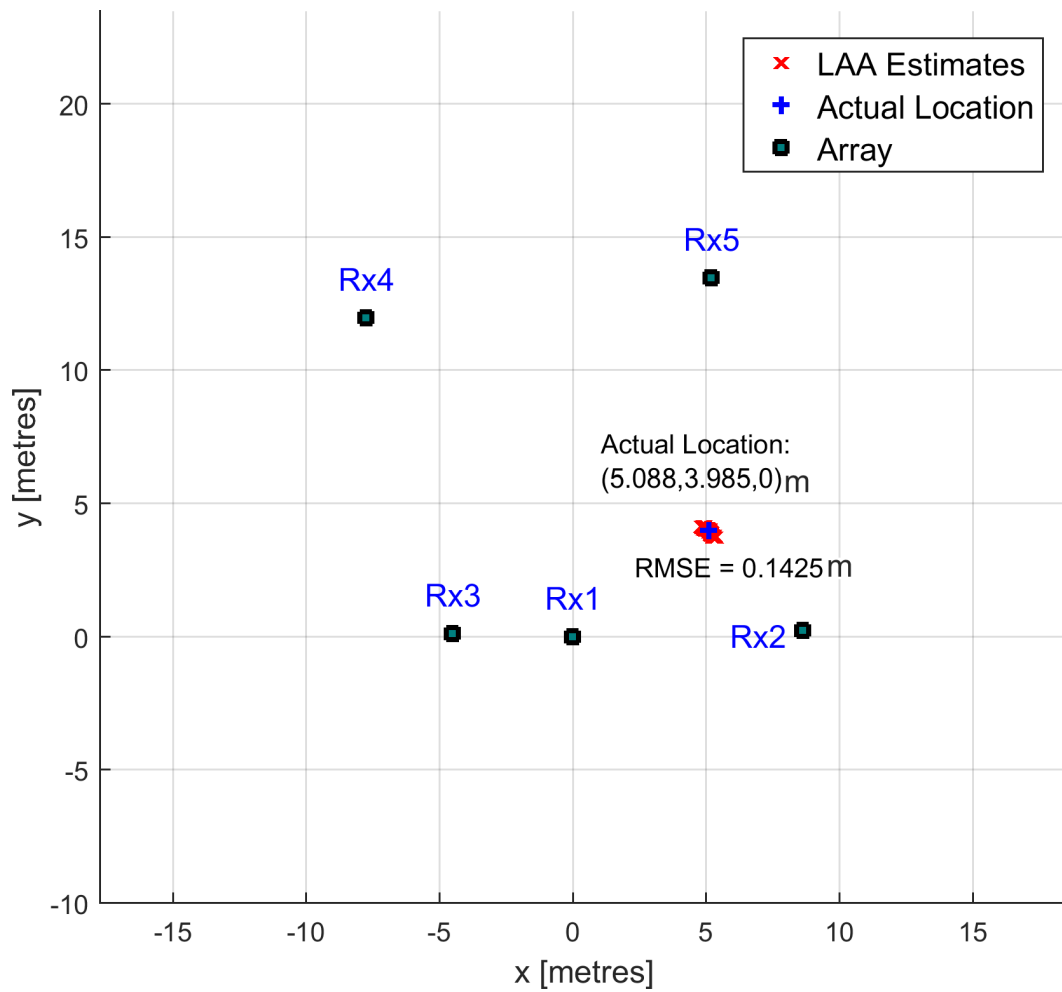


Figure 3.20: Target 2 Localisation results over 100 experiments: red crosses represent estimates using the LAA localisation algorithm, and blue plus represents the actual source location.

Table 3.7: Target 3 SNR for each element in the array

Element	SNR in dB
Rx1	12.95
Rx2	23.80
Rx3	10.91
Rx4	5.39
Rx5	12.36

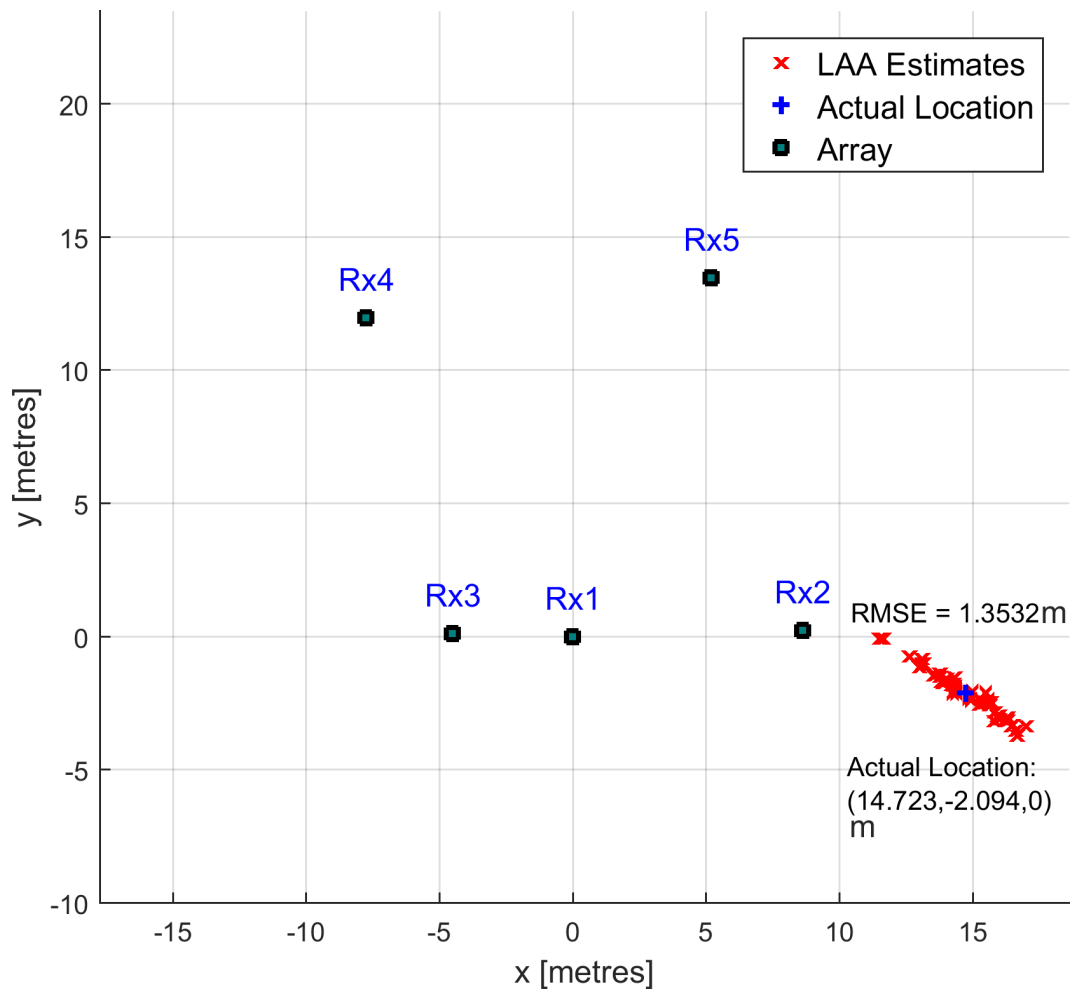


Figure 3.21: Target 3 Localisation results over 100 experiments: red crosses represent estimates using the LAA localisation algorithm, and blue plus represents the actual source location.

Table 3.8: Target 4 SNR for each element in the array

Element	SNR in dB
Rx1	10.47
Rx2	12.96
Rx3	10.26
Rx4	16.41
Rx5	33.10

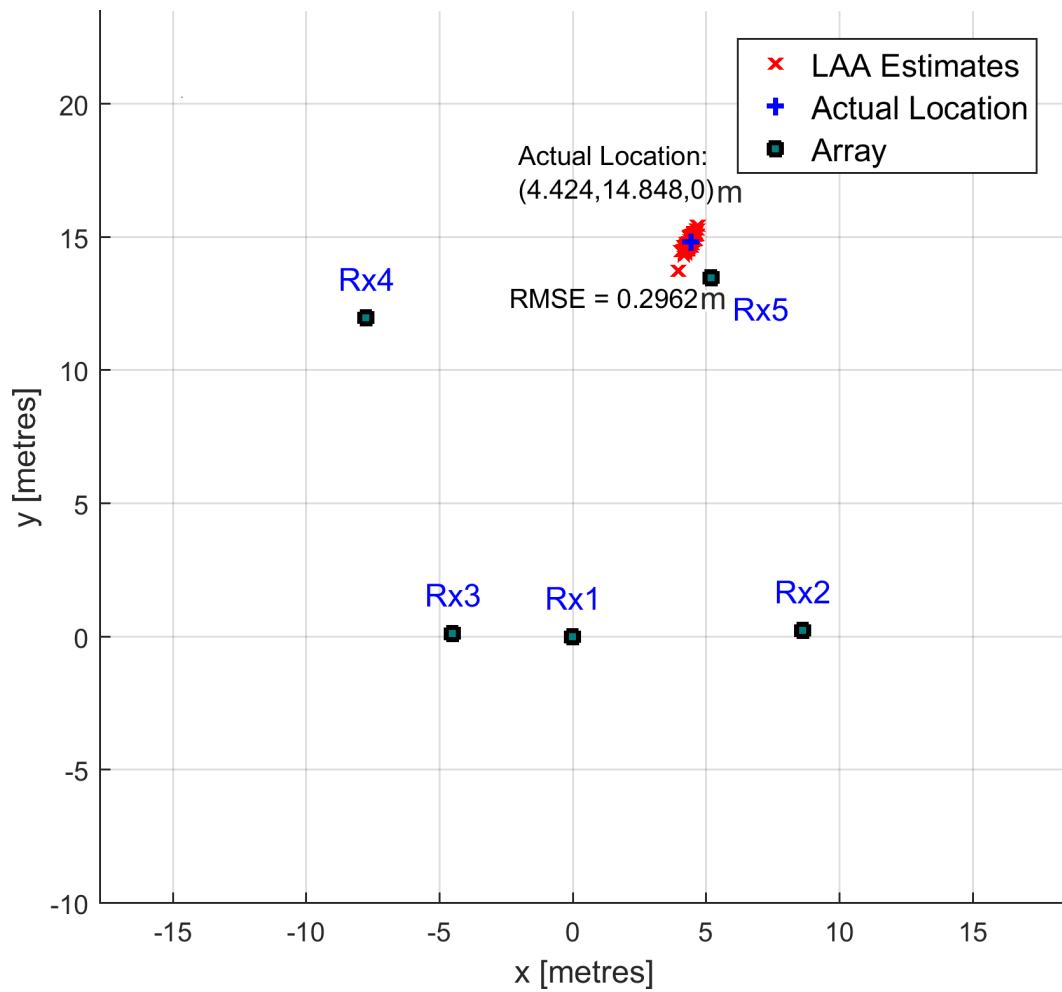


Figure 3.22: Target 4 Localisation results over 100 experiments: red crosses represent estimates using the LAA localisation algorithm, and blue plus represents the actual source location.

Table 3.9: Target 5 SNR for each element in the array

Element	SNR in dB
Rx1	20.94
Rx2	12.74
Rx3	18.03
Rx4	12.59
Rx5	7.60

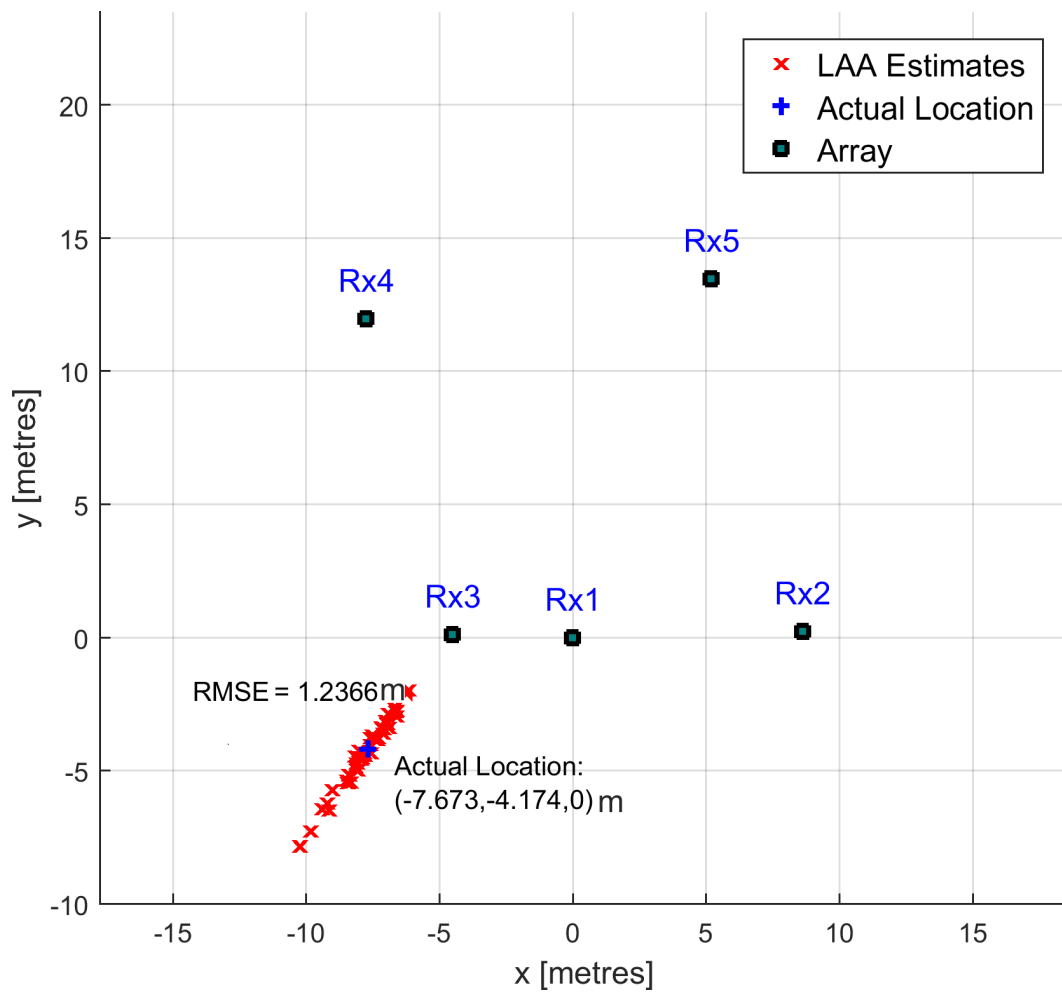


Figure 3.23: Target 5 Localisation results over 100 experiments: red crosses represent estimates using the LAA localisation algorithm, and blue plus represents the actual source location.

3.7 Summary

In this chapter, algorithms to estimate the location of a near-far field source using an array were presented and examined in the indoor and outdoor scenarios using SDR systems. In the indoor scenario, a synchronisation approach based on using a common clock reference was applied in association with the multiple pilots array calibration approach. The performance of two source localization algorithms namely the LAA algorithm and the spherical-MUSIC algorithm were examined experimentally using a circular antenna array of nine elements.

However, in the outdoor scenario, a common clock reference is not always possible. In the field study, a harmonic retrieval frequency estimation approach was presented for estimating the LO frequency offsets associated with each element of the array. In addition, a pilot source was used to further calibrate the array gain and phase uncertainties. Following this, the performance of the LAA localisation algorithm was examined for targets at different locations.

The results in this chapter show that a collection of individual USRP units can be successfully synchronised and calibrated to form different array systems. This provides a flexible platform to test different array processing algorithms in a wide variety of applications.

Chapter 4

Antenna Array Element Pattern Synthesis

In this chapter, a novel element pattern synthesis method is presented in this chapter for an array of fixed geometry. It consists of constructing a virtual array model for each antenna element, which not only considers the coupling effect between antenna elements but also takes into account the influence of the scattering near-field obstacles of unknown locations. The influence of the isolated element pattern is also taken into account by the virtual array in order to optimise the reconstructed far field pattern covering a wide-angle range without added complexity. To validate the proposed method, a narrowband normal-mode helical antenna array with eight elements arranging in a uniform circular structure is designed and built. Numerical examples using experimental data show that results obtained by applying the proposed method are greatly improved.

4.1 Mathematical Model

Now consider an coherent array of N elements that operates in the presence of a far-field narrow-band signal source $m(t)$, the signals at the output of the array is described by Equation (1.6) in Chapter 1.3. Furthermore, In Chapter 3, Equation

(3.8) assumes that the array elements are isotropic and mutual coupling effects are present. However, in some systems there are also uncertainties associated to the antenna patterns, which implies the complex gain uncertainties are directional, that is

$$\underline{\tilde{g}}(\theta, \phi) = \left[\tilde{g}_1(\theta, \phi), \tilde{g}_2(\theta, \phi), \dots, \tilde{g}_N(\theta, \phi) \right]^T \in \mathcal{C}^{N \times 1} \quad (4.1)$$

with $\tilde{g}_i(\theta, \phi)$ indicating the complex gain pattern uncertainties of the i^{th} array element. The nominal or isolated antenna pattern given by the antenna manufacturer is modelled by

$$\underline{\hat{g}}(\theta, \phi) = \left[\hat{g}_1(\theta, \phi), \hat{g}_2(\theta, \phi), \dots, \hat{g}_N(\theta, \phi) \right]^T \in \mathcal{C}^{N \times 1} \quad (4.2)$$

with $\hat{g}_i(\theta, \phi)$ indicating the complex nominal gain pattern of the i^{th} array element. For a set of identical antennas, the nominal pattern is assumed to be identical. That is

$$\hat{g}_1(\theta, \phi) = \hat{g}_2(\theta, \phi) = \dots = \hat{g}_N(\theta, \phi) \quad (4.3)$$

Assuming the array elements are isotropic, the nominal pattern is given by

$$\hat{g}_i(\theta, \phi) = 1; \forall \theta, \phi \quad (4.4)$$

for $i = 1, 2, \dots, N$. In this case, Equation (3.10) is a function of parameters (θ, ϕ) ,

$$\underline{S}(\theta, \phi) = \left(\mathbb{I}_N + \tilde{\mathbb{C}} \right) \left((\underline{1}_N + \underline{\tilde{g}}(\theta, \phi)) \odot \underline{\hat{S}}(\theta, \phi) \right). \quad (4.5)$$

Note that, in the absence of uncertainties,

$$\begin{aligned} \tilde{\mathbb{C}} &= \mathbb{O}_{N \times M} \\ \underline{\tilde{g}}(\theta, \phi) &= \underline{0}_N; \forall \theta, \phi \end{aligned} \quad (4.6)$$

Equation (3.8) becomes $\underline{S} = \underline{\hat{S}}$.

4.2 Element Gain and Phase Pattern Compensation Algorithms

4.2.1 Generalised Gain, Phase and Mutual Coupling Compensation

In [53], the author considered a far-field signal impinging on an array of isotropic elements. In the presence of mutual coupling, every element also receives signals which are re-radiated by other nearby elements. The amplitude and phase of these signals depend on the power of the reradiated signal and the propagation path which introduces an additional electrical phase from one element to another. Furthermore, according to the reciprocity theorem, the electrical characteristics of the elements when they act as receivers remain the same as when they act as transmitters. Assuming the antennas are isotropic of unity gain, Equation (4.5) is re-written as

$$\underline{S} = \left(\mathbb{I}_N + \tilde{\mathbb{C}} \right) \left((\underline{1}_N + \underline{\tilde{g}}) \odot \underline{\hat{S}} \right) \quad (4.7)$$

where

$$\underline{\tilde{g}} = \begin{bmatrix} \tilde{g}_1, & \tilde{g}_2, & \dots & \tilde{g}_N \end{bmatrix}^T \in \mathcal{C}^{N \times 1} \quad (4.8)$$

In general, $\tilde{\mathbb{C}}$ is a symmetric complex matrix ($\tilde{\mathbb{C}} = \tilde{\mathbb{C}}^T$) with the main diagonal elements all equal to zero as a consequence of the reciprocity theorem. Moreover, the magnitude of mutual coupling coefficients are inversely proportional to the distance between the antennas. This suggests that good ULAs will have a banded Toeplitz MCM structure with the mutual coupling effects between two elements that are far enough from each other approximated to be zero. Similarly, UCAs will exhibit a banded Toeplitz MCM structure and URA will have Toeplitz-block MCM. Now consider there are K pilot signals and for the k^{th} pilot, Equation (4.7)

can be written as follows

$$\begin{aligned}\underline{S}_k &= \left(\mathbb{I}_N + \tilde{\mathbb{C}} \right) \left((\underline{1}_N + \underline{\tilde{g}}) \odot \hat{\underline{S}}_k \right) \\ &= \left(\mathbb{I}_N + \tilde{\mathbb{C}} \right) \left(\mathbb{I}_N + \text{diag}(\underline{\tilde{g}}) \right) \hat{\underline{S}}_k\end{aligned}\quad (4.9)$$

The set of equations described by Equation (4.9) can also be written in a more compact form as

$$[\underline{S}_1, \underline{S}_2, \dots, \underline{S}_K] = \left(\mathbb{I}_N + \tilde{\mathbb{C}} \right) \left(\mathbb{I}_N + \text{diag}(\underline{\tilde{g}}) \right) [\hat{\underline{S}}_1, \hat{\underline{S}}_2, \dots, \hat{\underline{S}}_K] \quad (4.10)$$

Let

$$\begin{aligned}\hat{\mathbb{S}} &= [\hat{\underline{S}}_1, \hat{\underline{S}}_2, \dots, \hat{\underline{S}}_K] \in \mathcal{C}^{N \times K} \text{ and} \\ \mathbb{S} &= [\underline{S}_1, \underline{S}_2, \dots, \underline{S}_K] \in \mathcal{C}^{N \times K}\end{aligned}\quad (4.11)$$

Equation (4.10) can be written as

$$\tilde{\mathbb{C}} \left(\mathbb{I}_N + \text{diag}(\underline{\tilde{g}}) \right) \hat{\mathbb{S}} = \left(\mathbb{I}_N + \text{diag}(\underline{\tilde{g}}) \right)^{-1} \mathbb{S}. \quad (4.12)$$

Note the only unknowns are the complex symmetric MCM uncertainties $\tilde{\mathbb{C}}$ and non-directional complex gain uncertainties $\underline{\tilde{g}}$ which can be estimated as follows (in a step format).

1. Set the iteration counter, $p = 1$.
2. Set the maximum number of iterations, $p_{\max}$.
3. Set initial values for $\tilde{\mathbb{C}}^{[0]} = \mathbb{O}_N$
4. Estimate non-directional gain uncertainties $\underline{\tilde{g}}$ by fixing $\tilde{\mathbb{C}}$ in Equation (4.9), that is
$$\underline{\tilde{g}}^{[p]} = \frac{1}{K} \sum_{k=1}^K \left(\left(\mathbb{I}_N + \tilde{\mathbb{C}}^{[p-1]} \right)^{-1} \underline{S}_k \oslash \hat{\underline{S}}_k \right) - \underline{1}_N \quad (4.13)$$
5. Fixing $\underline{\tilde{g}}$, estimate $\tilde{\mathbb{C}}^{[p]}$ in Equation (4.12) using the algorithm for symmetric matrices proposed in [89].

6. Terminate the iterations if

$$\left\| \left(\mathbb{I}_N + \tilde{\mathbf{C}}^{[p]} \right) \left(\mathbb{I}_N + \text{diag} \left(\underline{\hat{g}}^{[p]} \right) \right) - \left(\mathbb{I}_N + \tilde{\mathbf{C}}^{[p-1]} \right) \left(\mathbb{I}_N + \text{diag} \left(\underline{\hat{g}}^{[p-1]} \right) \right) \right\|_F \leq \epsilon$$

where ϵ is a preset threshold or $p = p_{\max}$, else increment the iteration counter, $k = k + 1$, and return to Step 4.

4.2.2 Element Pattern Reconstruction

Element pattern reconstruction (EPR) method in [90] considers two types of element patterns. One is the element pattern in the array in the presence of other elements, the true complex antenna gain patterns $(\underline{\hat{g}}(\theta, \phi) + \underline{\tilde{g}}(\theta, \phi))$. The other is the element pattern in the isolated state, in other words, the element pattern that is measured with other elements removed from the array, $\underline{\hat{g}}(\theta, \phi)$. In the EPR method, it considers an antenna array composed of N identical antennas, and there are K antenna pattern measurements then for the k^{th} measurement, the following transformation relating to two kinds of element patterns can be written as

$$\underline{\hat{g}}(\theta_k, \phi_k) = \mathbb{M} (\underline{\hat{g}}(\theta_k, \phi_k) + \underline{\tilde{g}}(\theta_k, \phi_k)) \quad (4.14)$$

where $\mathbb{M} \in \mathcal{C}^{N \times N}$ is the element pattern reconstruction matrix. Substitute Equation (4.14) into Equation (3.8),

$$\begin{aligned} \underline{S}_k &= \left(\mathbb{I}_N + \tilde{\mathbf{C}} \right) \left((\underline{\hat{g}}(\theta_k, \phi_k) + \underline{\tilde{g}}(\theta_k, \phi_k)) \odot \underline{\hat{S}}_k \right) \\ &= \underbrace{\left(\left(\mathbb{I}_N + \tilde{\mathbf{C}} \right) \mathbb{M}^{-1} \right)}_{\triangleq \mathbb{G}_{EPR}} \left(\underline{\hat{g}}(\theta_k, \phi_k) \odot \underline{\hat{S}}_k \right) \end{aligned} \quad (4.15)$$

where

$$\mathbb{G}_{EPR} = \left(\mathbb{I}_N + \tilde{\mathbf{C}} \right) \mathbb{M}^{-1} \quad (4.16)$$

To estimate $\mathbb{G}_{EPR}$, a set of equations described by Equation (4.15) can be written in a compact form

$$[\underline{S}_1, \underline{S}_2, \dots, \underline{S}_K] = \mathbb{G}_{EPR} \left[\underline{\hat{g}}(\theta_K, \phi_K) \odot \underline{\hat{S}}_1, \underline{\hat{g}}(\theta_K, \phi_K) \odot \underline{\hat{S}}_2, \dots, \underline{\hat{g}}(\theta_K, \phi_K) \odot \underline{\hat{S}}_K \right] \quad (4.17)$$

The least-square (LS) estimate of $\mathbb{G}_{EPR}$ is therefore given by

$$\mathbb{G}_{EPR} = [\underline{S}_1, \underline{S}_2, \dots, \underline{S}_K] \left[\widehat{\underline{g}}(\theta_K, \phi_K) \odot \widehat{\underline{S}}_1, \widehat{\underline{g}}(\theta_K, \phi_K) \odot \widehat{\underline{S}}_2, \dots, \widehat{\underline{g}}(\theta_K, \phi_K) \odot \widehat{\underline{S}}_K \right]^\# \quad (4.18)$$

The matrix $\mathbb{G}_{EPR}$ is then used in Equation (4.15) to reconstruct the antenna patterns and to compensate for the mutual coupling effects of the array. The nominal/isolated element patterns $\widehat{\underline{g}}(\theta, \phi)$ are usually provided by the antenna manufacturer and can be assumed identical for the same antenna model. If the all antennas are isotropic, the nominal pattern is given by

$$\widehat{\underline{g}}(\theta_k, \phi_k) = \underline{1}_N; \forall k = 1, 2, \dots K. \quad (4.19)$$

4.2.3 Element Pattern Synthesis using Virtual Arrays

In this section, an element pattern synthesis method is proposed to estimate the patterns of antennas located in an array using the concept of virtual arrays. This method reconstructs the element pattern by modelling each element as a beam-former with fixed weights which takes into account the effect of both directional and non-directional complex gain uncertainties as well as mutual coupling effects in array system. Consequently, the relation between $\underline{S}$ and $\widehat{\underline{S}}$ of the i^{th} antenna can be modelled as

$$S_i(\theta, \phi) = \underline{w}_i^H \exp(-j \mathbf{r}_i^T \underline{k}(\theta, \phi)) \widehat{S}_i(\theta, \phi) \quad (4.20)$$

where $\underline{w}_i \in \mathcal{C}^{N_d \times 1}$ is the weighting vector and $\mathbf{r}_i \in \mathcal{R}^{3 \times N_d}$ represents the locations of the isotropic virtual array elements with respect to the i^{th} element. N_d denotes the number of virtual elements used to synthesise to the element pattern. To obtain a proper estimation of the element pattern, assume K measurements are taken and for the k^{th} measurement, Equation (4.20) can be written as follows

$$\begin{aligned} S_i(\theta_k, \phi_k) &= \underline{w}_i^H \exp(-j \mathbf{r}_i^T \underline{k}(\theta_k, \phi_k)) \widehat{S}_i(\theta_k, \phi_k) \\ \implies \underline{w}_i^H \exp(-j \mathbf{r}_i^T \underline{k}(\theta_k, \phi_k)) &= \frac{S_i(\theta_k, \phi_k)}{\widehat{S}_i(\theta_k, \phi_k)} \end{aligned} \quad (4.21)$$

and can also be written in a more compact form

$$\underline{w}_i^H \mathbb{A}(\mathbf{r}_i) = \underline{S}_i \oslash \widehat{\underline{S}}_i. \quad (4.22)$$

where

$$\mathbb{A}(\mathbf{r}_i) = \exp(-j\mathbf{r}_i^T [\underline{k}(\theta_1, \phi_1), \underline{k}(\theta_2, \phi_2), \dots, \underline{k}(\theta_K, \phi_K)]) \in \mathcal{C}^{N_d \times K}, \quad (4.23)$$

$$\underline{S}_i = [S_i(\theta_1, \phi_1), S_i(\theta_2, \phi_2), \dots, S_i(\theta_K, \phi_K)] \in \mathcal{C}^{1 \times K} \text{ and} \quad (4.24)$$

$$\widehat{\underline{S}}_i = [\widehat{S}_i(\theta_1, \phi_1), \widehat{S}_i(\theta_2, \phi_2), \dots, \widehat{S}_i(\theta_K, \phi_K)] \in \mathcal{C}^{1 \times K}. \quad (4.25)$$

It can be shown that $\underline{w}_i$ and $\mathbf{r}_i$ can be estimated by solving the following optimisation problem

$$\min_{\underline{w}_i, \mathbf{r}_i} \left\| \underline{w}_i^H \mathbb{A}(\mathbf{r}_i) - \underline{S}_i \oslash \widehat{\underline{S}}_i \right\|_F^2 \quad (4.26)$$

Under the assumption that $\mathbb{A}(\mathbf{r}_i)$ is a full rank matrix, the cost function in (4.21) is a separable non-linear least-squares optimisation problem [91] that is linear in the weight vector $\underline{w}_i$ but non-linear in the virtual array locations $\mathbf{r}_i$. Minimising $\underline{w}_i$ while keeping $\mathbf{r}_i$ fixed results in

$$\underline{w}_i = \mathbb{A}(\mathbf{r}_i) (\mathbb{A}^H(\mathbf{r}_i) \mathbb{A}(\mathbf{r}_i))^{-1} (\underline{S}_i \oslash \widehat{\underline{S}}_i)^H. \quad (4.27)$$

Substituting Equation (4.27) into Equation (4.26) leads the following optimisation problem for $\mathbf{r}_i$ that is independent of $\underline{w}_i$

$$\begin{aligned} \xi(\mathbf{r}_i) &= \left\| \underline{w}_i^H \mathbb{A}(\mathbf{r}_i) - \underline{S}_i \oslash \widehat{\underline{S}}_i \right\|_F^2 \\ &= \left\| (\underline{S}_i \oslash \widehat{\underline{S}}_i) (\mathbb{A}^H(\mathbf{r}_i) \mathbb{A}(\mathbf{r}_i))^{-1} \mathbb{A}^H(\mathbf{r}_i) \mathbb{A}(\mathbf{r}_i) - \underline{S}_i \oslash \widehat{\underline{S}}_i \right\|_F^2 \\ &= \left\| (\underline{S}_i \oslash \widehat{\underline{S}}_i) \left((\mathbb{A}^H(\mathbf{r}_i) \mathbb{A}(\mathbf{r}_i))^{-1} \mathbb{A}^H(\mathbf{r}_i) \mathbb{A}(\mathbf{r}_i) - \underline{1}_N^T \right) \right\|_F^2. \end{aligned} \quad (4.28)$$

The cost function (4.28) may have multiple local minima. However, a practical assumption for the initial locations is that the virtual array elements $\mathbf{r}_i^{[0]}$ is reasonably close to the physical antenna locations. Assuming $N_d \leq N$, $\mathbf{r}_i^{[0]}$ can be estimated to be the N_d closest near-field objects to the i^{th} array element.

4.3 Numerical Examples with Experimental Data

In this section, the proposed algorithm is applied to an array constructed with off-the-shelf antenna elements and multiple *Universal Software Radio Peripheral* (USRP) devices. To illustrate the performance of the proposed Element Pattern Synthesis using Virtual Arrays (EPSVA) algorithm, an eight-element uniform circular array with half wavelength inter-element spacing is considered as shown in Fig. 4.1.

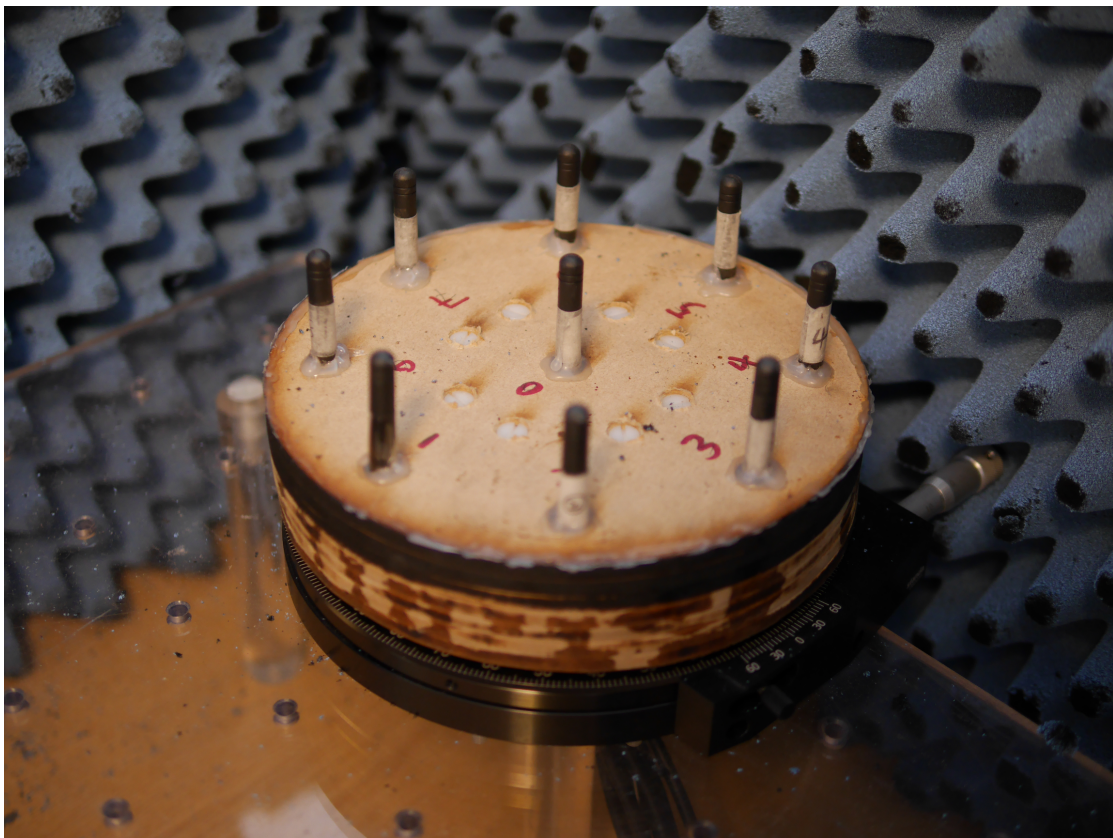


Figure 4.1: Eight-element uniform circular array with a central dummy element.

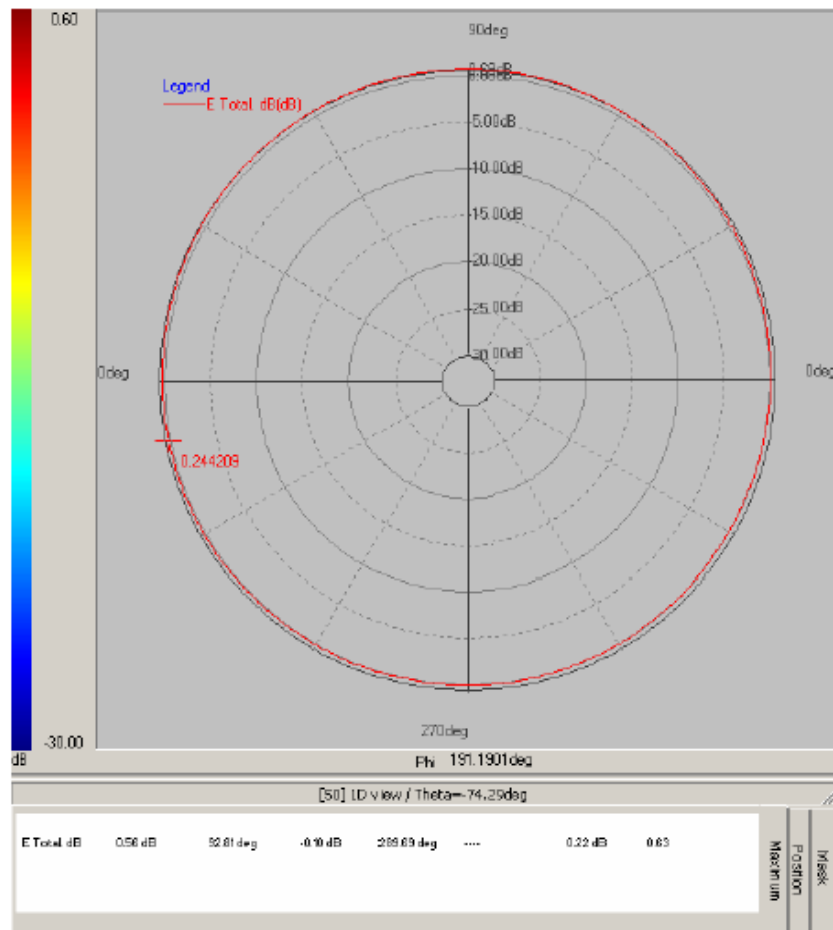
Table 4.1: Cartesian coordinates of the array in centimetres

Element	x (cm)	y (cm)
Rx1	8.0652	0
Rx2	5.7030	5.7030
Rx3	0	8.0652
Rx4	-5.7030	5.7030
Rx5	-8.0652	0
Rx6	-5.7030	-5.7030
Rx7	0	-8.0652
Rx8	5.7030	-5.7030

The array is constructed with the geometry shown in Table 4.1. The attributes of the antenna used in this paper are listed as follows:

- antennas are normal-mode helical antennas
- operational frequency range of 2400MHz to 2490MHz
- 0dBi nominal gain in the x-y plane
- 8-element uniform circular array with a centre dummy element (terminated/unused)
- inter-element spacing is half wavelength at $F_c = 2.43\text{GHz}$ to mitigate coupling of the elements

The isolated antenna gain pattern is shown in Fig. 4.2. It is safe to assume that antennas are isotropic, hence Equation (4.4) is valid. Each antenna array element is attached to a USRP board as shown in Fig. 4.4. Each USRP board down-converts the RF analog signal to its baseband which is then digitised to produce the complex (I and Q) digital data that can be used by the localisation algorithms. The carrier frequency F_c to be used in the downconversion and the sampling rate F_s of the ADC were selected to be $F_c = 2.43\text{GHz}$ and $F_s = 2\text{MS/s}$ ($0.5\mu\text{s}$ sampling period). All of USRP boards were connected to a common GPS clock source to synchronise their internal LOs, ADC clocks and timing clocks. The array is synchronised by using a common 10MHz reference clock signal and

March 1st 2010

Frequency (MHz): 2450.00

Pattern Field: H Plane

Average Gain: 0.22dB

Maximum Gain: 0.56 dB

Maximum Gain (degree): 92.81

Minimum Gain: -0.10 dB

Minimum Gain (degree): 289.69

Figure 4.2: Antenna gain pattern given by the manufacturer's datasheet

a 1 Pulse Per Second (PPS) signal to each board. The 10MHz clock provides a reference for the LO and ADC on each USRP board. The 1 PPS signal provides a common reference frame that is used to reset the timestamp on each board. This allows multiple independent USRP boards to be tuned to the same carrier frequency and the same timing frame giving a fully synchronised system. For the purpose of the experiment, a GPS unit was used to generate both the 10MHz reference clock signal the 1 PPS signal feeding to all USRP units as shown in Fig. 4.3 and Fig. 4.4.

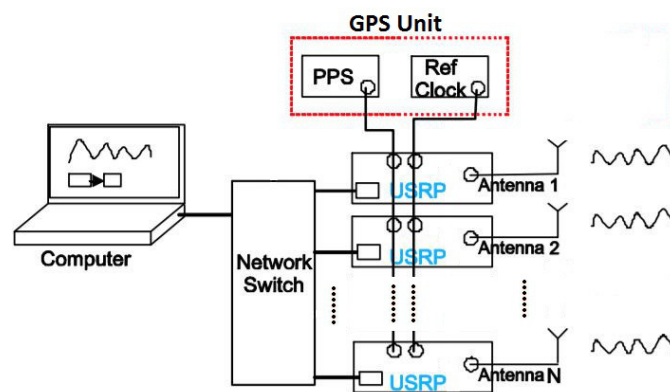


Figure 4.3: Array testbed setup comprising multiple USRP boards that are synchronized using a common clock reference and 1 PPS reference source.



Figure 4.4: Multi-USRP hardware configuration for the 8-element UCA, USRP boards share a common clock that is generated by a GPS module.

4.3.1 Element Patterns

In the first study, two element pattern reconstruction algorithms, EPR and EPSVA, have been applied to the described array system. The calibration data were measured with the array assembly mounted on a rotary stage (shown in Fig. 4.1) inside a room with a single pilot source in the far-field of the array on the x-y plane. 72 azimuthal directions ($K = 72$) are uniformly measured in the range of $[0^\circ, 360^\circ)$ with a 5° degree interval each containing 500 snapshots/samples with SNR of about 20dB to carry out the calibration. Each antenna is assumed to have isotropic gain and phase response in their isolated state according to their manufacturing datasheet.

Next, EPR and the proposed EPSVA with $N_d = 4$ and $N_d = 8$ were applied to reconstruct the array element gain and phase patterns. Tables 4.2 and 4.3 show the RMSEs of element patterns, gain and phase respectively, for the 8-element UCA. In Figures 4.5 to 4.12, the gain (left) and phase (right) patterns for the eight elements in the array are shown. It is clear the isolated patterns deviate quite a lot from the measured patterns in both the gain and the phase. Although the EPR algorithm attempts to reconstruct the patterns, but the residual errors are still far from ideal. On the other hand, the EPSVA reconstructed element patterns are almost identical to the measured patterns.

Table 4.2: Element gain patterns RMSE (volts) comparison

Element	Isolated	EPR	EPSVA, $N_d = 4$	EPSVA, $N_d = 8$
Rx1	0.4920	0.2647	0.0840	0.0277
Rx2	0.6505	0.3117	0.1060	0.0299
Rx3	0.4820	0.2566	0.0801	0.0370
Rx4	0.3466	0.2320	0.0916	0.0380
Rx5	0.6106	0.3102	0.1199	0.0424
Rx6	0.5327	0.2186	0.0930	0.0316
Rx7	0.5779	0.3189	0.1305	0.0425
Rx8	0.3557	0.2351	0.0952	0.0448

Table 4.3: Element phase patterns RMSE (degrees) comparison

Element	Isolated	EPR	EPSVA, $N_d = 4$	EPSVA, $N_d = 8$
Rx1	32.4581	20.1197	5.8585	3.7276
Rx2	45.2998	31.0282	8.4175	3.3244
Rx3	42.7143	23.8695	6.7256	4.5216
Rx4	42.6610	19.4479	7.7872	4.0663
Rx5	39.7539	23.1826	8.8026	4.4774
Rx6	44.1940	27.2158	9.8154	4.5666
Rx7	45.4795	26.1671	6.6630	4.5327
Rx8	29.6175	14.3383	6.2116	4.1745

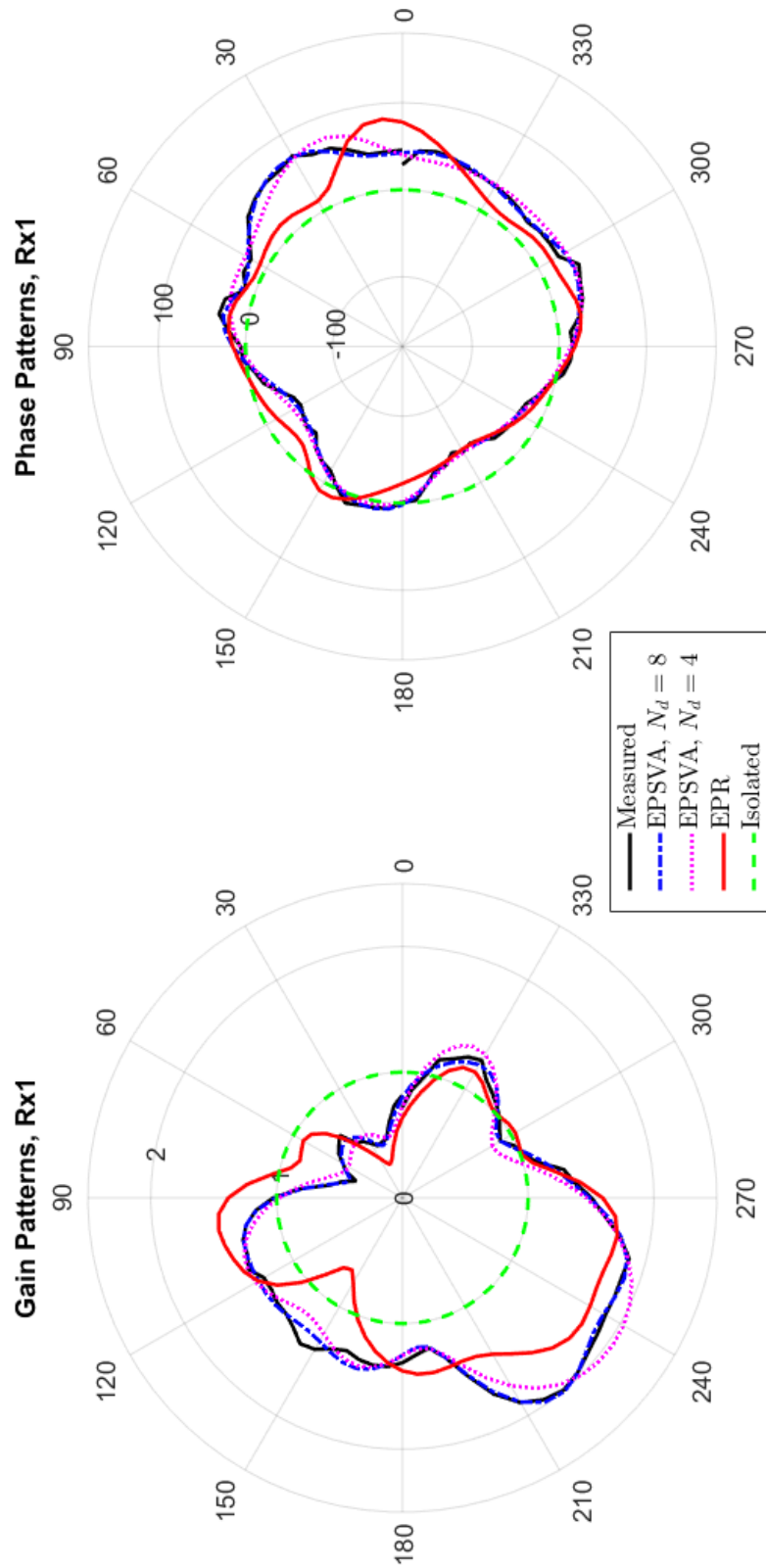


Figure 4.5: Rx1 Element, gain pattern (left) in volts and phase pattern (right) in degrees.

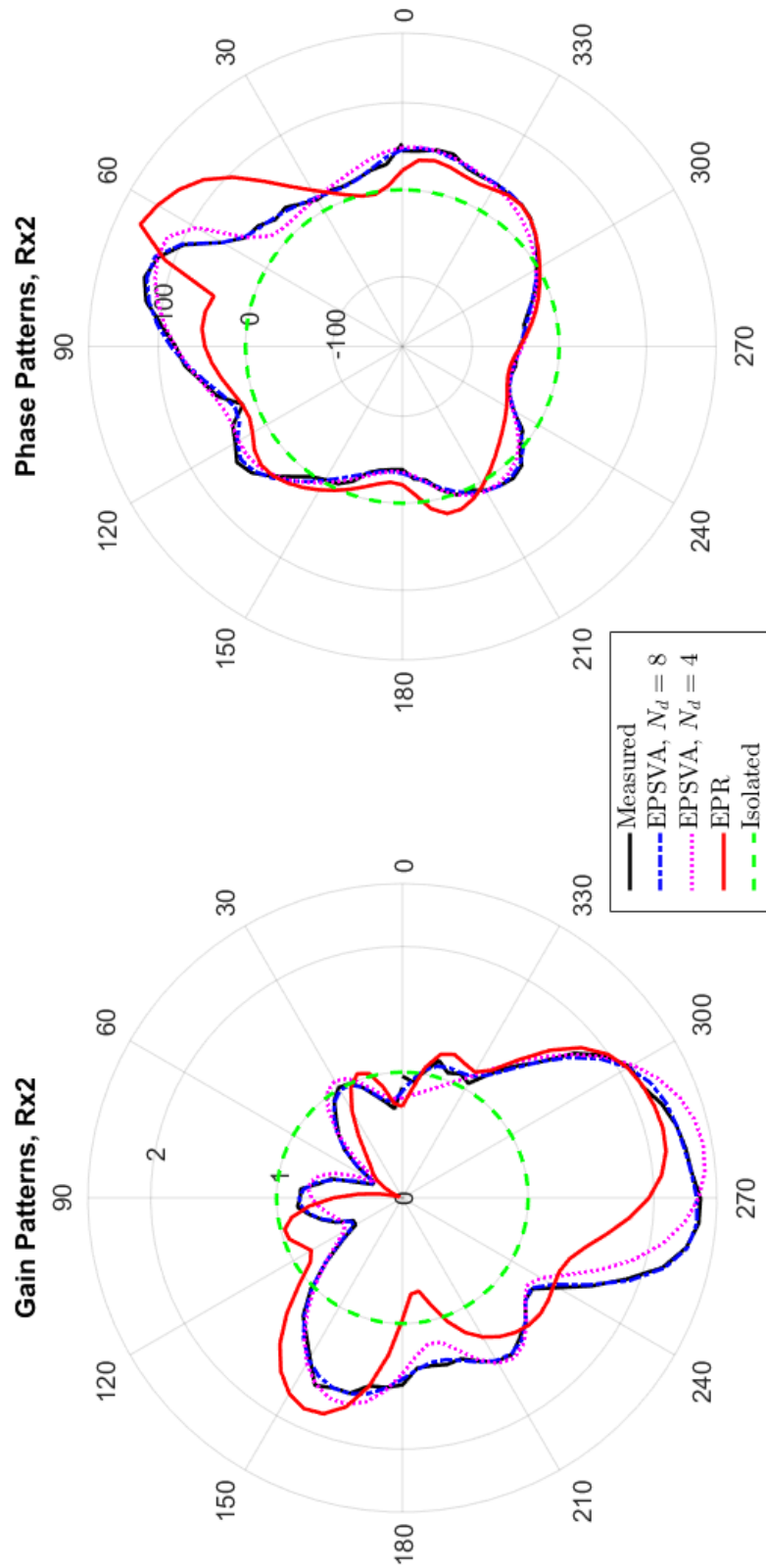


Figure 4.6: Rx2 Element, gain pattern (left) in volts and phase pattern (right) in degrees.

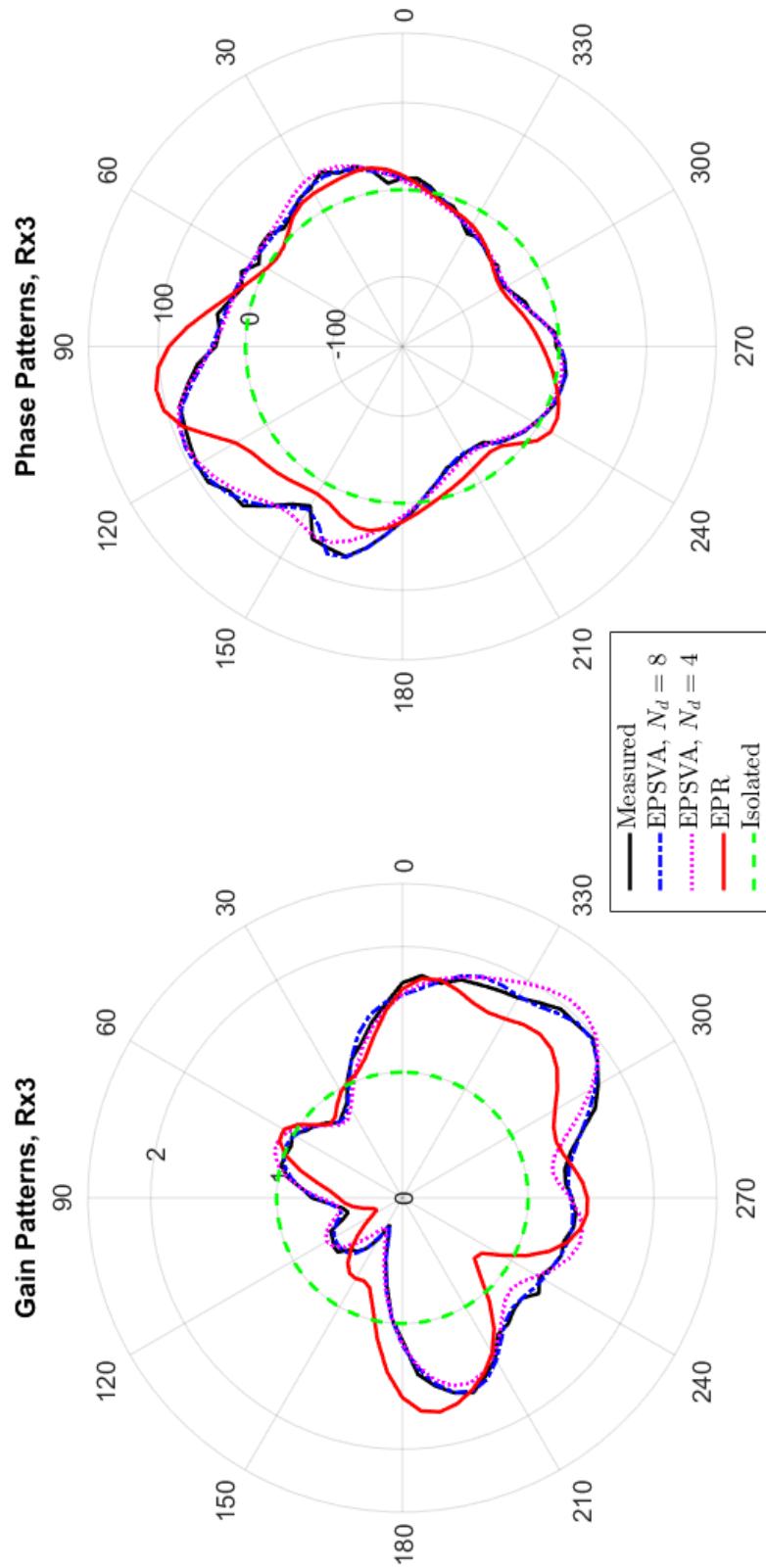


Figure 4.7: Rx3 Element, gain pattern (left) in volts and phase pattern (right) in degrees.

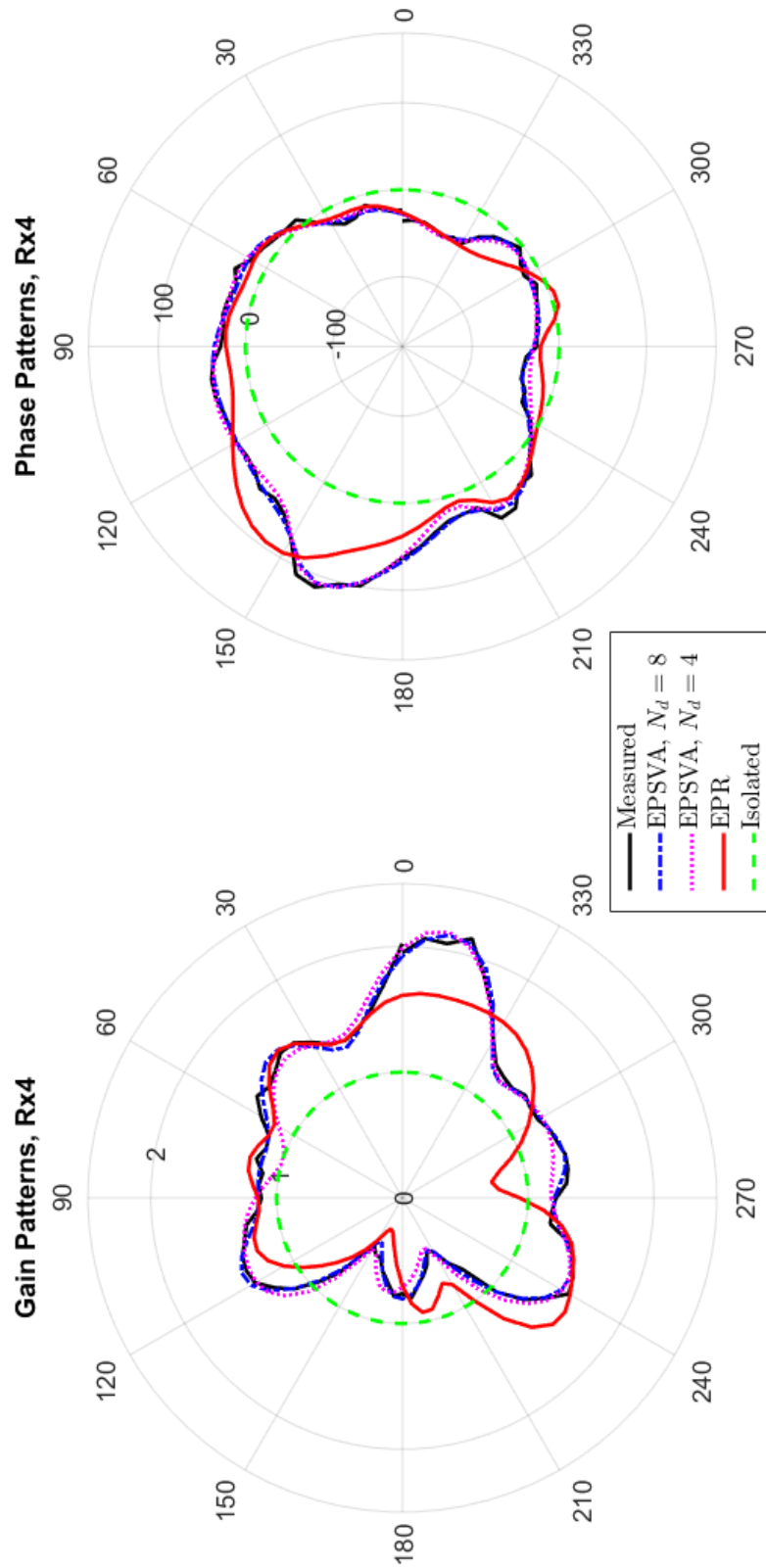


Figure 4.8: Rx4 Element, gain pattern (left) in volts and phase pattern (right) in degrees.

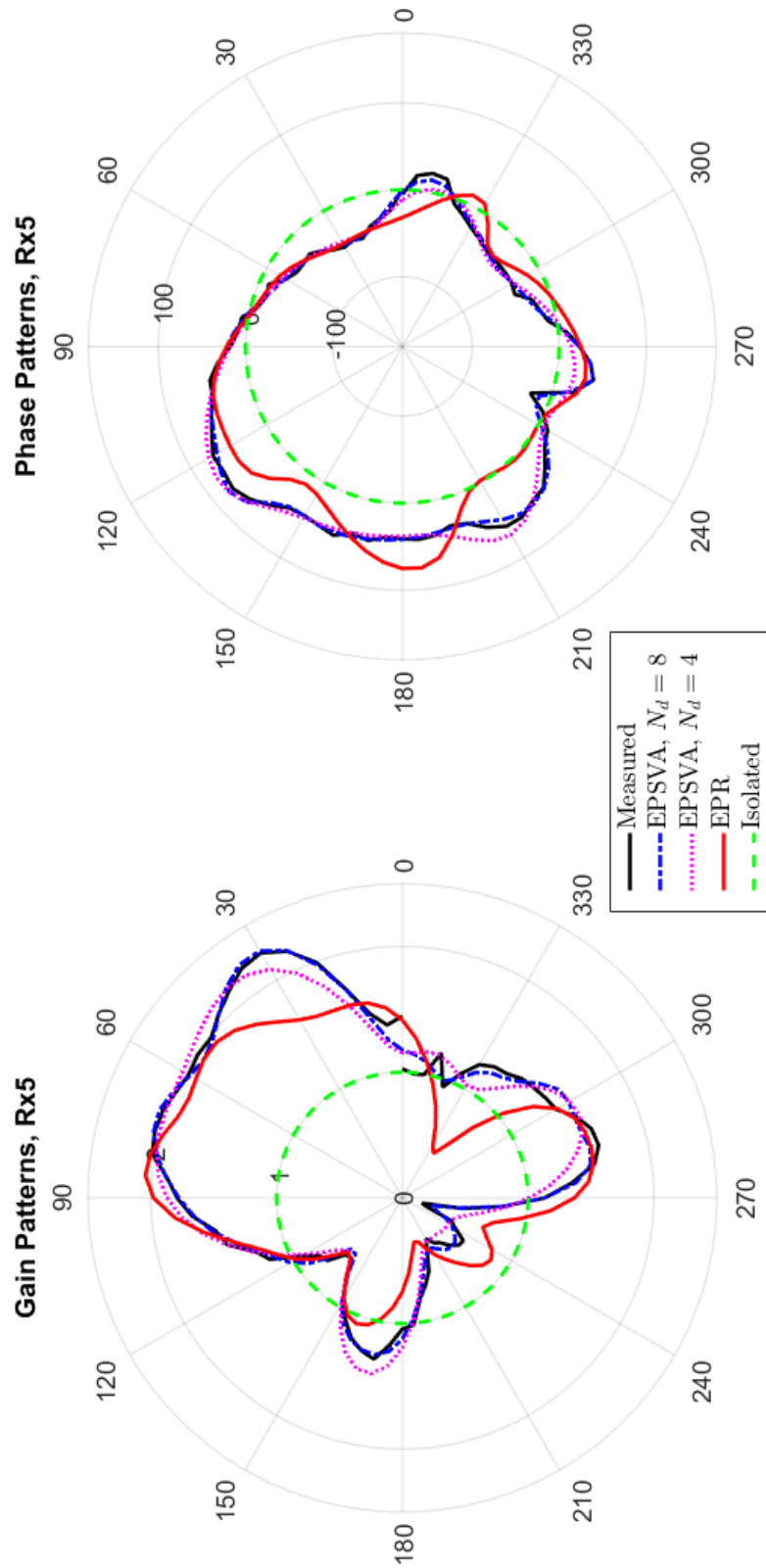


Figure 4.9: Rx5 Element, gain pattern (left) in volts and phase pattern (right) in degrees.

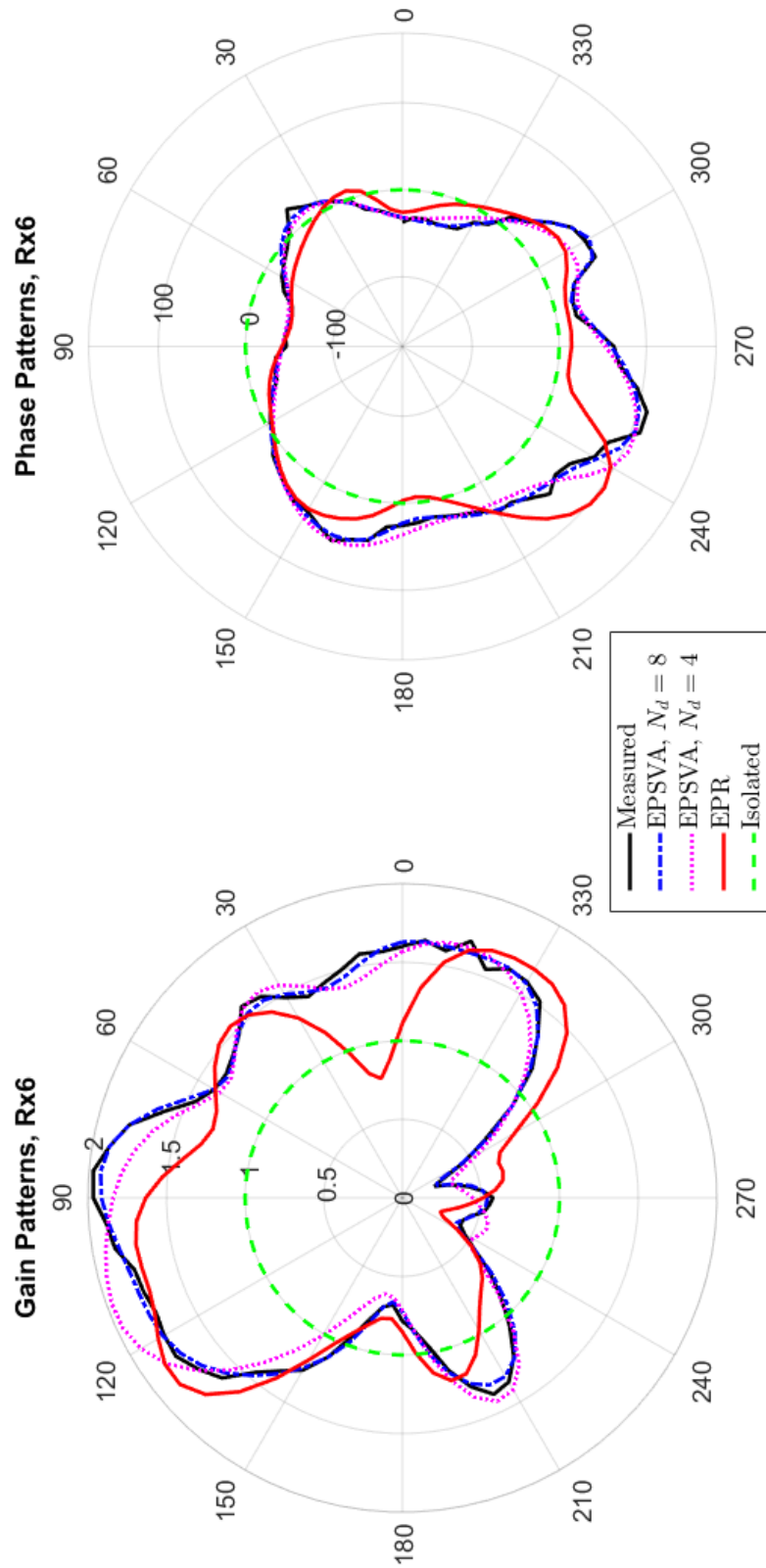


Figure 4.10: Rx6 Element, gain pattern (left) in volts and phase pattern (right) in degrees.

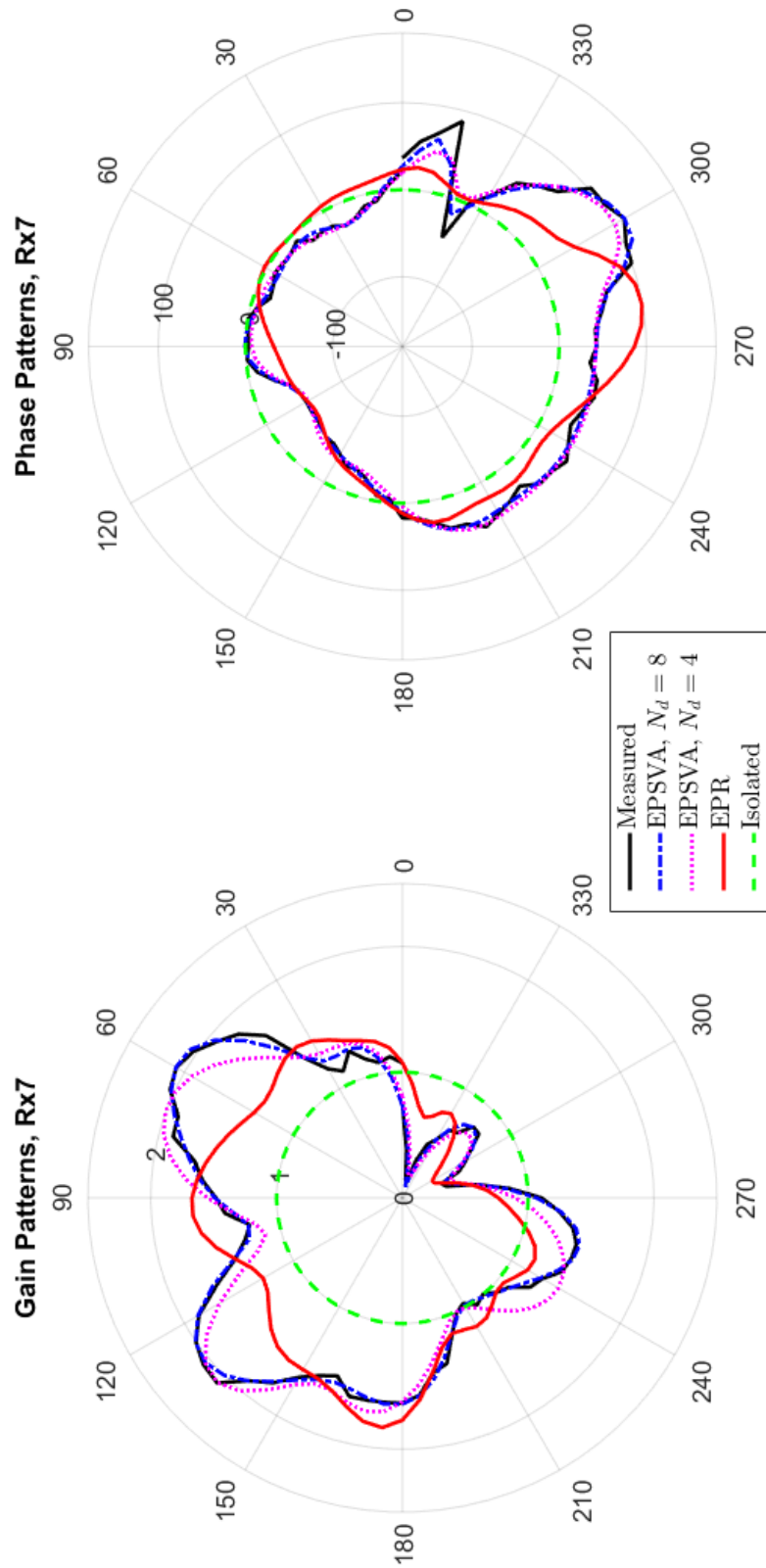


Figure 4.11: Rx7 Element, gain pattern (left) in volts and phase pattern (right) in degrees.

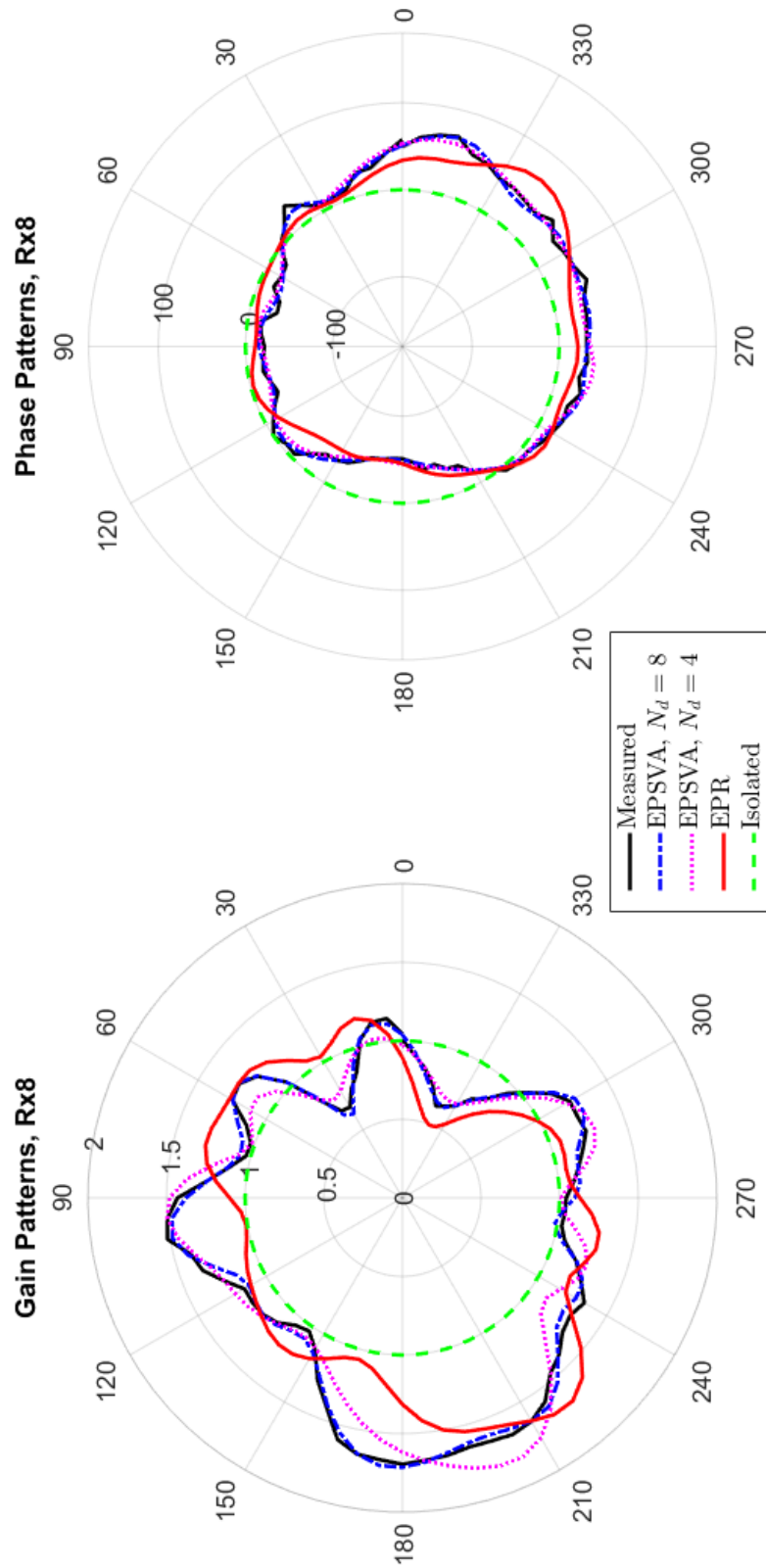


Figure 4.12: Rx8 Element, gain pattern (left) in volts and phase pattern (right) in degrees.

4.3.2 Direction-of-Arrival Estimates Analysis

In this study, the MUSIC performance of all three aforementioned methods, namely generalised mutual coupling compensation algorithm, EPR and EPSVA, will be demonstrated using the same array setups. In this experiment, three uncorrelated signal sources are located at 125° , 155° and 260° containing 500 snapshots/samples with SNR of about 20dB. Figure 4.13 shows the MUSIC spectrum using the isolated array element assumption failed to localise all three sources, as expected. Two MCM compensation methods performed similarly, they failed to resolve two closely placed sources. The EPR algorithm was able to give better DOA estimations but with very limited resolution. On the other hand, the EPSVA algorithm with $N_d = 4$ and $N_d = 8$ were both able to give better source separation and resolution.

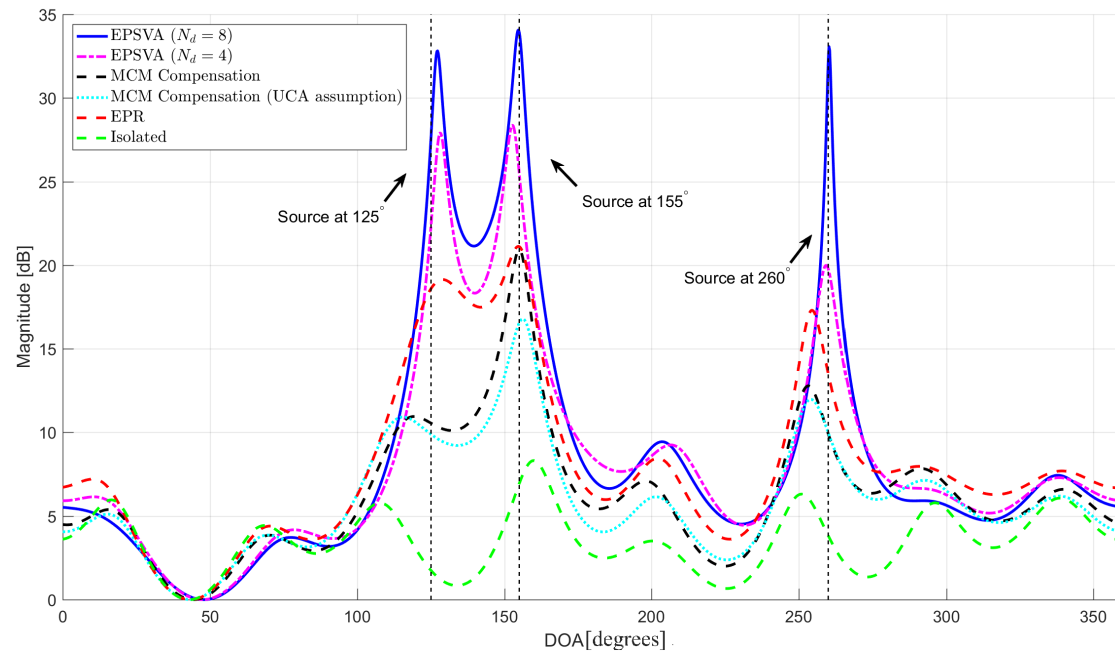


Figure 4.13: MUSIC spectrums of three uncorrelated signal sources located at 125° , 155° and 260° each with SNR of about 20dB. The true DOAs are indicated by vertical dashed lines.

Further tests were conducted concerning one signal source in the absence of multipath using MUSIC algorithm. The angle of arrival was scanned from 0° and

360° covering all operational angles of the array system with a 5° interval. At each angle, fifty independent measurements were taken each containing 500 snapshots with SNR of about 20dB. The direction of arrival was estimated by MUSIC using 10 and 100 snapshots from the total 500 snapshots to get some performance statistical measure under different number of snapshots.

In Figures 4.14 to 4.19, the mean value of the DOA estimation error over 50 measurements was shown in solid black and each individual measurement was shown in red star. Figure 4.14 shows that a significant error was introduced to DOA estimates when assuming antenna patterns were the same as the isolated antenna patterns, i.e. no mutual coupling between the elements or near-field objects. It is clear the error is quite large, over 10° in some directions. The error was reduced to somewhere between 2° to −7° with a slight negative bias as the EPR algorithm being applied as shown in Figure 4.15. Both MCM compensation algorithms (with and without UCA assumption) show similar DOA estimate errors as shown in Figures 4.16 and 4.17. On the other hand, Figures 4.18 and 4.19 clearly show a significant reduction in DOA estimate errors by introducing EPSVA. The error is quite small, within $\pm 2^\circ$, compared to the isolated, EPR and MCM compensation algorithms with no noticeable bias. Furthermore, the error standard deviation decreases as the number of assumed "dummy" elements N_d increases from $N_d = 4$ to $N_d = 8$. This agrees with the element pattern analysis in Figures 4.5 to 4.12. Moving from only 10 snapshots to 100 snapshots, the estimation error remains almost the same as shown in Figures 4.14 to 4.19. This is reasonable since the variations between independent measurements is the main source of error here.

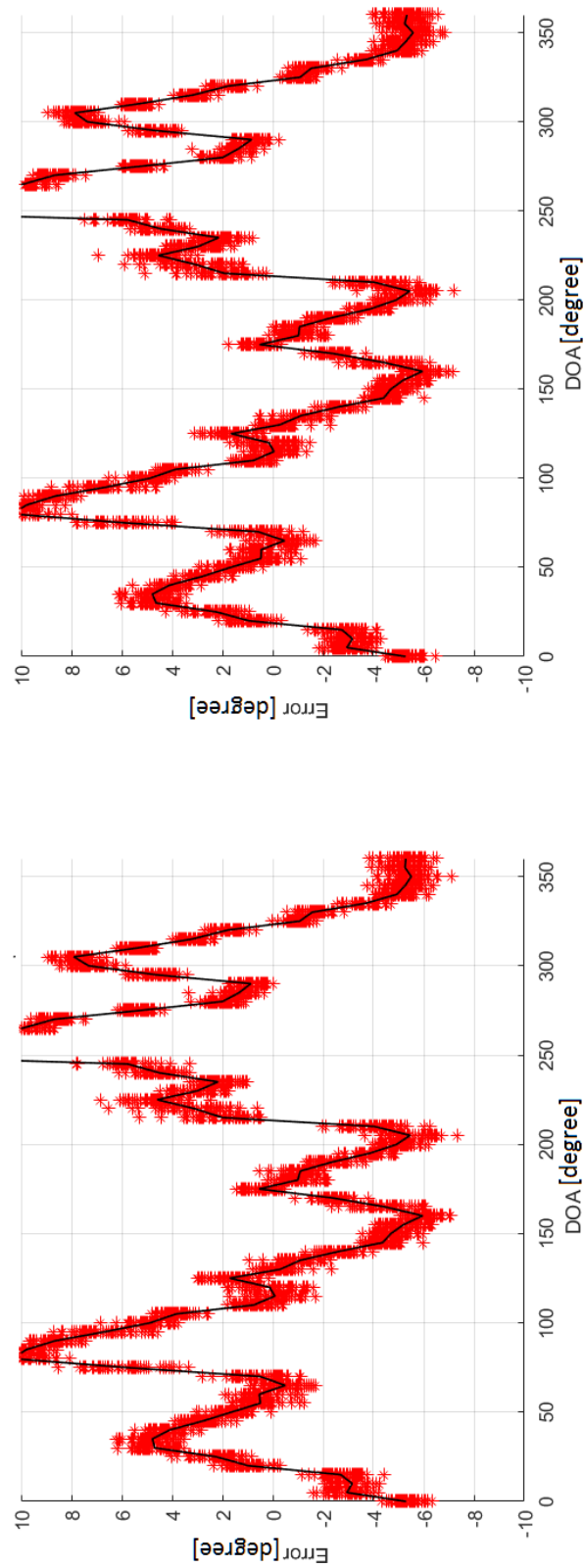


Figure 4.14: DOA estimation errors when isolated antenna pattern is used, $L = 10$ (left) and $L = 100$ (right)

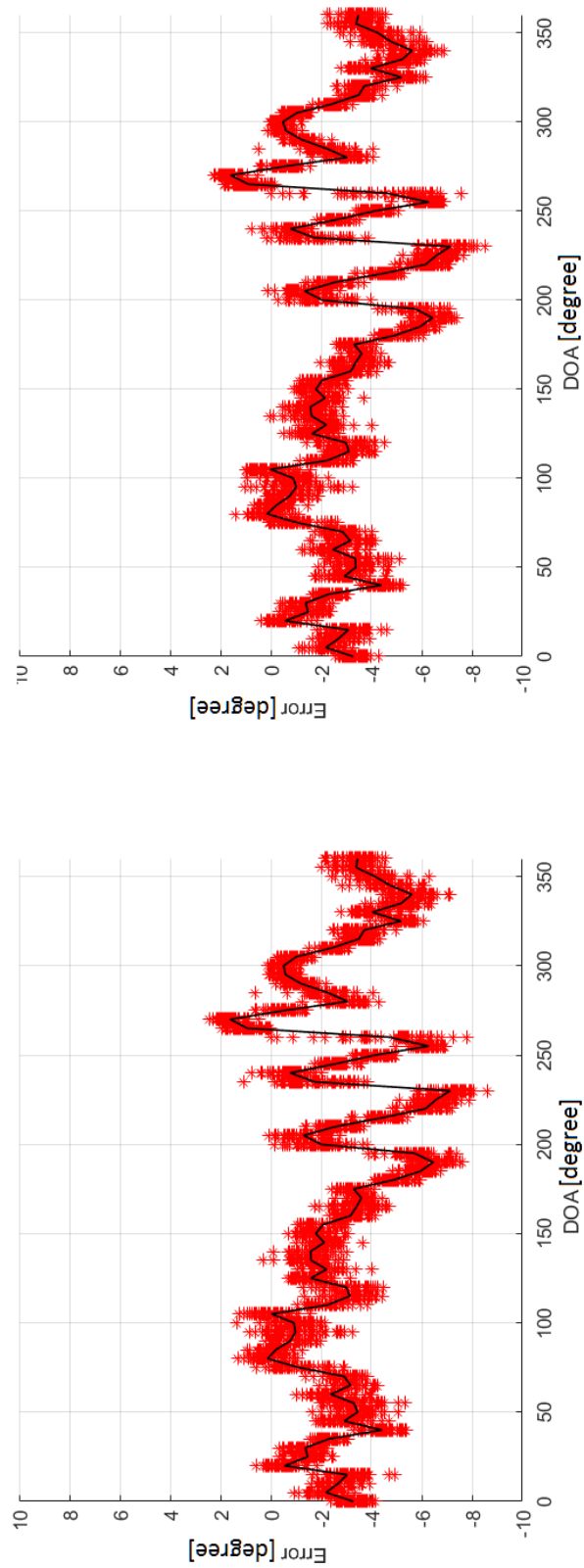


Figure 4.15: DOA estimation errors when EPR is used, $L = 10$ (left) and $L = 100$ (right)

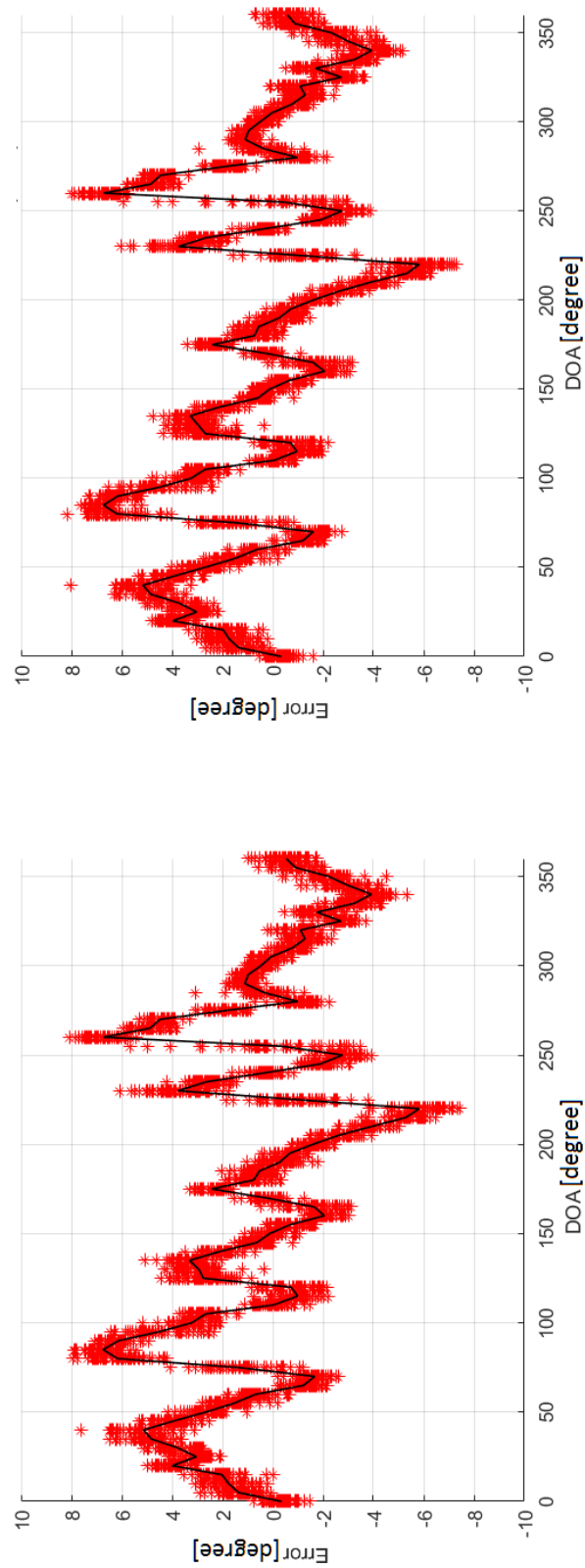


Figure 4.16: DOA estimation errors when MCM compensation is used, $L = 10$ (left) and $L = 100$ (right)

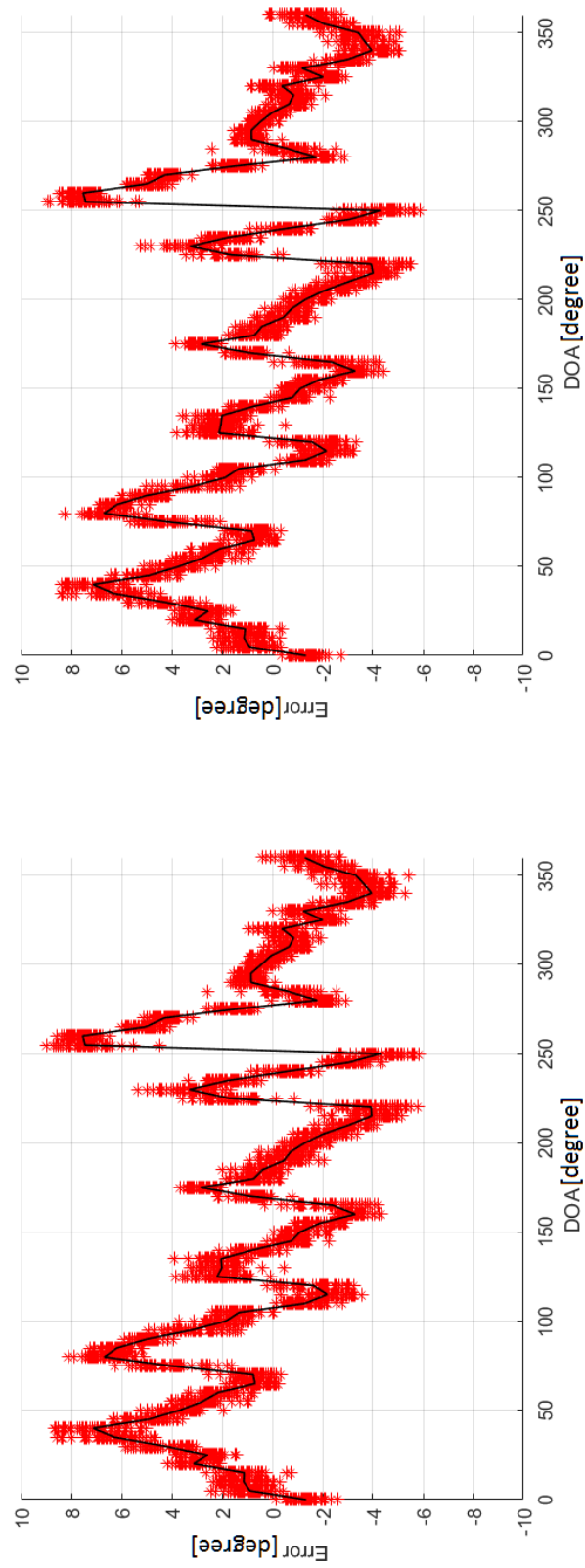


Figure 4.17: DOA estimation errors when MCM compensation with UCA assumption is used, $L = 10$ (left) and $L = 100$ (right)

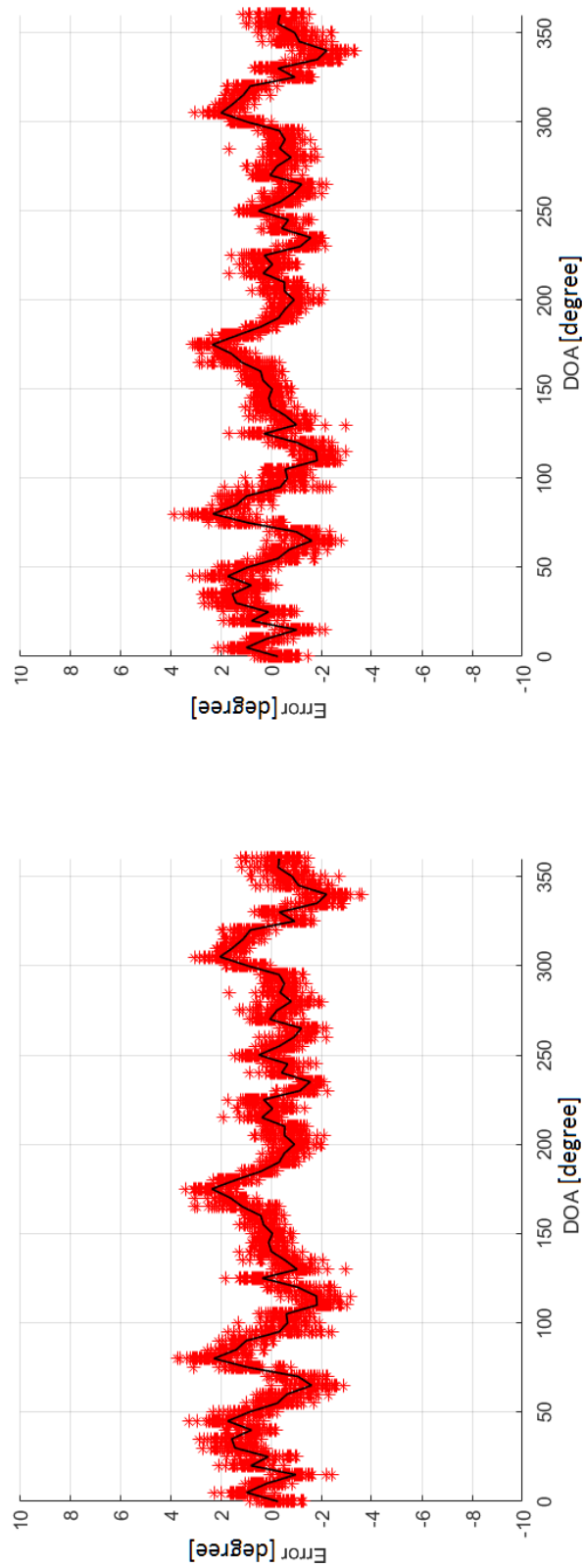


Figure 4.18: DOA estimation errors when EPSVA with $N_d = 4$ is used, $L = 10$ (left) and $L = 100$ (right)

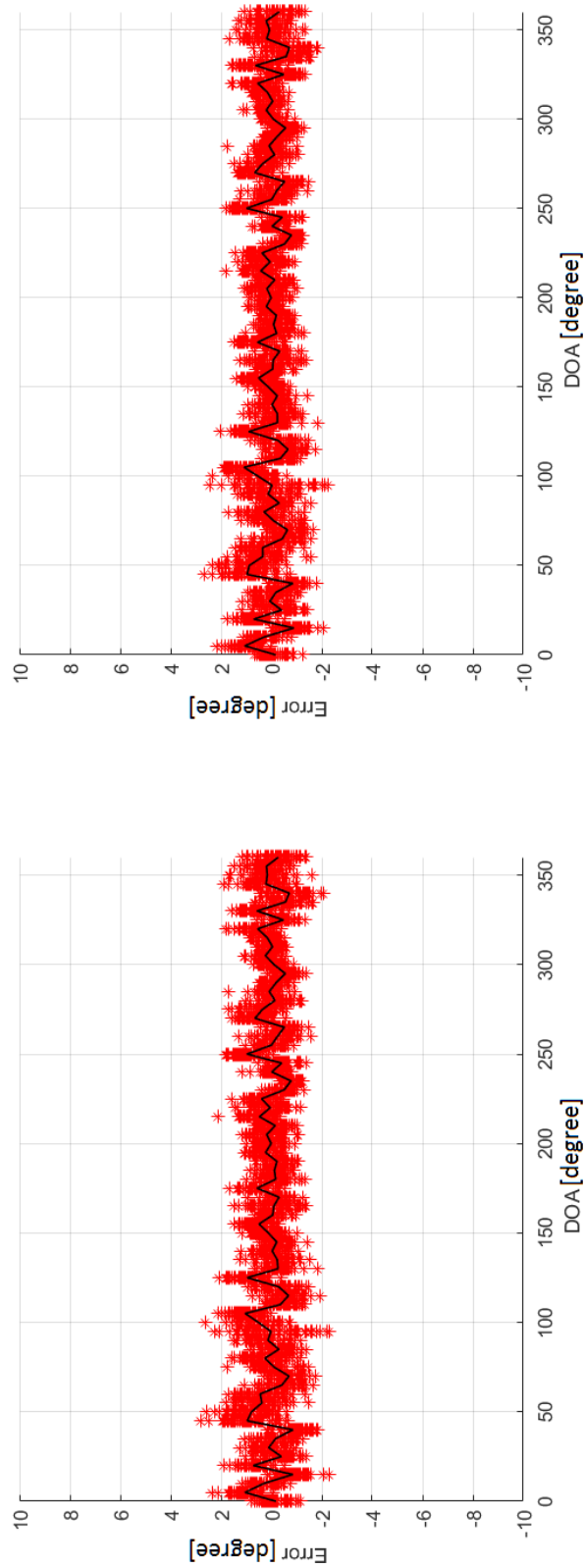


Figure 4.19: DOA estimation errors when EPSVA with $N_d = 8$ is used, $L = 10$ (left) and $L = 100$ (right)

4.3.3 Complexity Analysis

Table 4.4 and Table 4.5 show the complexity of the different algorithms. As for the given example, an 8-element UCA array, the generalised MCM compensation algorithm with UCA assumption exhibits a significant improvement over the isolated assumption with only a small amount of increase in the number of unknowns to estimate. The EPSVA with $N_d = 4$ has much better performance compared to the EPR algorithm with the same number of unknowns. The EPSVA with $N_d = 8$ clearly has the best performance overall but at an expense of much more unknowns to estimate.

Table 4.4: Complexity comparison in terms of the number of real unknowns to estimate

Isolated	$2N - 2$
MCM (UCA assumption)	$2 \lfloor \frac{N}{2} \rfloor + 2N - 2$
MCM	$N^2 + 3N - 2$
EPR	$2N^2 - 2$
EPSVA (1D array)	$3N_d N - 2$
EPSVA (2D array)	$4N_d N - 2$
EPSVA (3D array)	$5N_d N - 2$

Table 4.5: Complexity comparison in terms of the number of real unknowns to estimate under the setup described in the previous study

Isolated	14
MCM (UCA assumption)	22
MCM	86
EPR	126
EPSVA ($N_d = 4$)	126
EPSVA ($N_d = 8$)	254

4.4 Conclusions

In this chapter, a novel element pattern synthesis algorithm was proposed using a number of virtual arrays to reconstruct the distorted antenna patterns. It was evaluated and compared to the other antenna calibration methods with experimental data. The results showed that the EPSVA method is able to greatly decrease the deviation of the reconstructed antenna gain and phase patterns from the true ones compared to the EPR algorithm proposed in [90], without increasing the complexity. The studies have also shown that the gain and phase errors can be further reduced by introducing additional virtual array elements.

The direction of arrival analysis also shows that the EPSVA approach reduces the DOA estimate errors over the entire 360° operation range in both three-target and single-target estimates using MUSIC. This approach is also robust to array geometry errors as the virtual arrays have already taken the location of individual elements into account. On the other hand, a generalised mutual coupling compensation algorithm based on [53] was also proposed. It incorporates the estimation of MCM and gain and phase with minimal increase in the complexity.

Chapter 5

Conclusions and Further Work

This thesis has covered a variety of topics concerning MIMO channel estimation, SDR implementation, array calibration and measurements. The work has demonstrated that the proposed novel system designs make phased array signal processing practical in indoor and outdoor wireless and mobile communication applications. Important challenges that must be overcome if such a system is to be widely deployed have been addressed using off-the-shelf hardware.

In Chapter 2, a novel precoding scheme was presented based on AR-modelling and linear interpolation. The proposed integrated BD precoding and channel prediction method can improve the sum system capacity in a time-variant Rayleigh fading channel environment, which is based on a Jakes model. The performance was compared to the conventional precoding without channel prediction. An NI PXIe based flexible testbed was also presented consisting two main components:

- the host processing side for the flexibility of implementing different transmission schemes for various wireless standards. A computer with multi-core Central Processing Unit (CPU) will be used for high complexity floating-point operations in the host processing. It allows the easy implementation of different transceiver designs for testing various precoding schemes
- the channel processing side which offers either a software channel simulator

running on CPU for off-line simulations or a real-time FPGA based RF channel emulator. The channel processing is independent of the transmission or the modulation scheme that is in host processing, thus any transmission scheme can be implemented with the same channel processing module.

The testbed results indicate that there were significant system capacity improvements under certain conditions. Moreover, by estimating and predicting OFDM channels in the impulse response domain, the solution manages to reduce the computational burden by only considering the significant taps of the impulse response. The performance of the algorithm can be further improved by assimilating the antenna geometry into the system model.

In Chapter 3, a transition of focus was made from the conventional MIMO systems to the antenna array systems. An SDR antenna array testbed consisting of independent radio front-end units has been constructed using USRP boards by Ettus Research. This provides a flexible platform to perform wide range of array signal processing algorithms with configurable number of antennas and array shapes. Array synchronisation and calibration techniques were described for such a flexible system. The practical performance of a recently developed large aperture array localisation algorithm as well as the well-known spherical wave propagation based MUSIC algorithm were analysed in an indoor scenario. An array synchronisation technique using shared clock reference was presented. An generalised array calibration algorithm was also presented. It is clear from the extensive test, the synchronisation and calibration approaches allow both localisation algorithms to perform very well. Furthermore, a different SDR array setup and array synchronization and calibration approach was also presented in the outdoor field-test in Silwood Park woodland. In this field-test, the LAA localisation algorithm was studied and successfully estimated the source locations with high accuracy. The spherical-MUSIC localisation algorithm also examined but failed to produce any valid result as it is more sensitive to complex gain errors than the LAA algorithm.

In Chapter 4, a novel element pattern synthesis algorithm was proposed using a

number virtual arrays to reconstruct the distorted antenna patterns. The proposed algorithm was compared to the other array calibration and antenna pattern reconstruction techniques using the SDR antenna array testbed developed in Chapter 3. Results illustrated clearly that the proposed algorithm was able to reconstruct the antenna patterns with exceptionally high accuracy. It is also clear that, from the direction of arrival analysis, the proposed algorithm was able to reduce the DOA estimate errors over the entire 360° operation range in both three-target and single-target estimates using MUSIC.

5.1 List of Contributions

The following novel contributions have been presented in this thesis:

1. Propose a novel channel prediction technique for LTE-TDD MU-MIMO systems based AR-modelling and linear interpolation
2. Develop a hybrid rapid prototyping testbed that is capable of real-time channel emulation as well as high-level complexity floating-point operations
3. Develop a software defined radio (SDR) antenna array testbed of independent single channel USRP2 units synchronised using a common clock reference signal and 1 PPS signal (hardware synchronisation)
4. Propose a calibration approach for estimating the gain, phase and mutual coupling uncertainties for an array operating in the presence of a near-far field source using the SDR array testbed. Then investigate the impact of uncertainties on the performance of a large aperture array system using the developed SDR array testbed
5. Propose and analyse an element pattern synthesis algorithm for antennas in an array system for small aperture arrays

6. Generalise the mutual coupling modelling proposed in [53] to include gain and phase uncertainties. Then investigate the effectiveness of different mutual coupling compensation algorithms and array element pattern reconstruction algorithms on the performance of a small aperture array system using the developed SDR array testbed
7. Develop an SDR array testbed using individual USRP and GPS units synchronised using the harmonic retrieval technique (software synchronisation). Then investigate the performance of the large aperture array localisation algorithm in a field-test using the developed SDR array testbed

5.2 Suggestions for Further Work

The work presented in this thesis provides solutions to synchronisation and calibration problems of practical flexible antenna array testbed using SDR and paves the way for the future research in this area. This work could be extended with further research and some suggestions are:

- **Array Calibration in the Presence of Multipath**

In sophisticated environments such as indoor and urban scenarios multipath is very common. However, none of the array calibration techniques covered in this thesis has the capability to overcome the multipath effects in their current forms. As a result, these algorithms will not be as effective to be used in the calibration or re-calibration scenarios where multipath is severe. In [92], the author proposed a passive array pre-processing algorithm that suppresses all the multipath contributions in the received signals and leaves only the line-of-sight (LOS) one. The algorithm estimates the signal subspace of the LOS path using the temporal information extracted from the received signals. By employing such a pre-processor, the contributions of all non-line-of-sight (NLOS) paths can be removed hence the calibration algorithms proposed in the thesis can perform as normal. This will be desirable to

investigate the performance of the calibration algorithms with such a pre-processor using the developed SDR testbed in different multipath scenarios

- **Near-far Field Array Element Pattern Synthesis Algorithm with Antenna Location Uncertainties**

An array pattern synthesis algorithm was presented to allow better reconstruction of the antenna patterns in an array system. It considers a source operating at far-field of the array so the plane wave propagation is valid. If a source operates close to the array instead, a spherical wave propagation model must be used. Array geometrical uncertainties arisen due to the imperfect knowledge of the array element locations should also be considered.

- **Investigating the Performance of the Array Element Pattern Algorithm for Different Array Geometries**

Although the basic concepts presented in this thesis are valid for any array geometry, a uniform circular array has been employed in the investigation. More array types, such as the vertical uniform rectangular array with patch antennas shown in Figure 5.1 and the massive uniform rectangular array with chip antennas shown in Figure 5.2 should also be investigated.

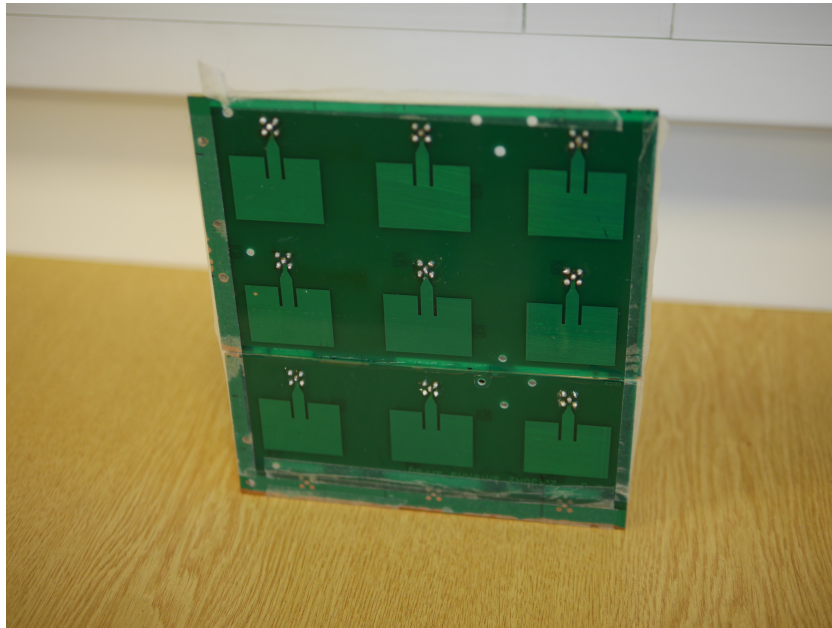


Figure 5.1: 9-element vertical uniform rectangular array with patched antennas

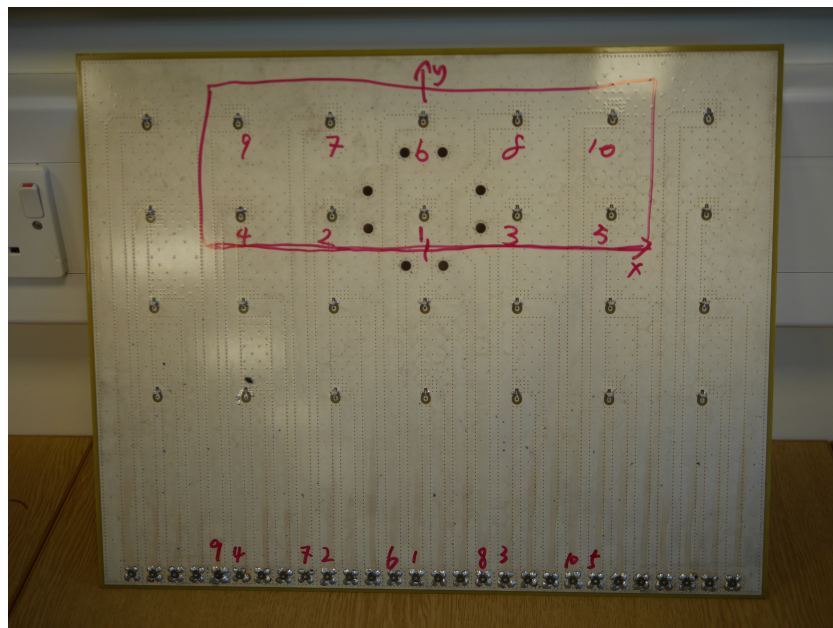


Figure 5.2: 28-element uniform rectangular array with chip antennas

Publications Related to this Thesis

The following papers were published during the PhD:

1. Q. Liu and A. Manikas, "Experimental Comparison of Localisation Techniques in the Presence of Array Uncertainties," European Conference on Antennas and Propagation (EUCAP), Paris, March 2017, pp. 2551-2555.
2. Q. Liu, "Channel prediction for multiuser downlink LTE-TDD systems," Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC), Hong Kong, December 2015, pp. 160-165.

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