

FRACTURE MECHANICS ANALYSIS
OF FATIGUE CRACK GROWTH
IN VISCOELASTIC SOLIDS

by:

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A thesis submitted for the award of the degree of
Doctor of Philosophy in the Faculty of Engineering
of the University of London.

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University of London. NOVEMBER 1972

ABSTRACT

The programme of studies presented in this thesis was aimed at the development of procedures for the characterisation of the fatigue failure process in thermoplastic materials through the application of the fracture mechanics principles. The total energy balance concept of failure analysis has been applied to the formulation of a theoretical method for the prediction of the cyclic crack growth initiation life and the determination of the fatigue endurance limit in viscoelastic solids. The fatigue crack propagation process has been extensively studied by performing a comprehensive programme of empirical studies on four characteristically different thermoplastics (polymethylmethacrylate, polycarbonate, nylon 6.6 and polyacetal) and the effects on the rate of crack growth of such parameters as the amplitude and the mean level of the stress intensity factor, the loading frequency, the load cycle waveform, the molecular orientation in anisotropic materials and the environmental temperature have been investigated. Subsequently a model for the determination of the cyclic rate of crack growth was proposed. The specific form of this model enables its representation in terms of such concepts as the crack opening displacement and the crack tip plastic zone size. The correspondence of the predictions of this model to some fatigue data on metals, with subsequent simplification of the data selection process at the design stage, has also been discussed.

The problem of failure under multiaxial loading conditions has been investigated through the analysis of the failure process in specimens containing cracks of arbitrary orientation and subjected to uniaxial and biaxial loading conditions. Variations of the fracture toughness parameter and hence the initiation life in fatigue as well as the changes in fatigue strength as a function of the extent of load

biaxiality have been discussed. From the consideration of the results of the finite element stress analysis of the uncracked specimens and the physical path of fracture obtained from the empirical studies, a procedure for the determination, at the design stage, of the path of crack extension in a component subjected to complex loading conditions has been proposed.

ACKNOWLEDGEMENTS

The author wishes to acknowledge with appreciation the support and the co-operation of numerous members of the Mechanical Engineering Department from which he has considerably benefited throughout the period of engagement on the present project. In particular he wishes to extend his gratitude to Drs. L.E. Culver and J.C. Radon whose advice and encouragement he has found extremely valuable.

The kind support of Dr. C.E. Turner in securing the College appointment for the author and also his contribution to the discussions related to the study of the process of failure under multiaxial loading conditions are gratefully acknowledged.

Prof. M.L. Williams, Dean of the Faculty of Engineering, University of Utah, is specially thanked for providing the stimulus and encouragement for the analytical study of the fatigue failure process in viscoelastic solids.

The co-operation of the members of the technical staff in the laboratories, the departmental workshops and the drawing offices in overcoming the problems associated with the preparation of the experiments is gratefully acknowledged.

Special thanks are due to Mr. N. Afshar and Mr. S. Smart for their assistance in the preparation of the figures. Co-operation of Mr. J.D. Page and Mr. S. Worley who performed part of the tests on the modulus measurements and the fracture toughness studies as their final year projects and the Stress Analysis Group of Rolls Royce Ltd. in providing some photoelastic data on stress distribution in the biaxially loaded specimens are acknowledged.

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CHAPTER ONE

INTRODUCTION

This introductory chapter is aimed at providing a brief survey of some of the most important contributions to the field of fatigue failure analysis, with the basic purpose of outlining the advantages and/or shortcomings of the various empirical and analytical approaches to the characterisation of failure under cyclic loading conditions. The effects of some of the important factors influencing the fatigue life of engineering materials are also indicated.

1.1 The Fatigue Failure Problem:

The subject of fatigue failure analysis of metals and polymers is concerned, fundamentally, with the apparent fact that these materials will fail, often in the absence of excessive elongation or gross plastic flow, upon the repeated application of a stress which the material would withstand if applied in a static manner.

The interest in the ability to predict the specific behaviour, under fatigue loading conditions, of various materials stems from the fact that a very large number of engineering components (such as rotating parts, pressure vessels and vibrating parts) are subjected to cyclic loading conditions in their service. Very often during the manufacturing processes defects are produced resulting in the presence of undesirable stress concentrations which can act as sources of subcritical crack nucleation. Such cracks will subsequently grow to a critical size and catastrophic failure of the component will ensue.

The fatigue life of a component can, in general, be divided into two stages: (a), the period during which a crack-like flaw is initiated and (b), the period

embracing the propagation of a macrocrack leading to unstable failure. These two distinct periods have been designated the stage I and stage II growths (Forsyth, 1961). In metals stage I takes place through a slipping process on the most favourable crystallographic plane (i.e. the plane closely aligned with the maximum shear stress direction) resulting in the generation of "intrusions" and "extrusions". When the microcrack reaches a point where the ratio of the local shear stress to tensile stress is sufficiently low, it turns into stage II growth which will be on a plane perpendicular to the maximum tensile stress at this point. During the latter stage ripple markings (fatigue striations) are produced on the fracture surface.

There are numerous examples of experimental data relating to the comparative lengths of stages I and II. The question as to at what size or at which point in time a microcrack becomes a macrocrack can not, of course, be divorced from consideration of the microstructural integrity of the material, the overall geometry of the component and the conditions of loading as well as the characteristics of the environment (Forsyth, 1961). A crack should be considered as a macrocrack if it is large enough to permit the application of the notions of a homogeneous continuum.

Clearly, in low cycle (and/or high stress) fatigue, the initiation period is not expected to be large and inversely with low stress cycling, stage I may assume the major part of the total fatigue life.

In addition, if the component is rather bulky with no distinct stress raisers, the nucleation stage will be very long compared to the propagation period. On the other hand in structures with severe stress concentrations, and particularly in thin plates and shells, formation of a macrocrack takes place relatively early in the fatigue life and hence in terms of the number of load cycles the propagation phase constitutes the major part of the total life.

It is of interest to consider the proposals put forward in relation to the mechanism of crack propagation, associated with stage II. Laird and Smith (1962) and McEvily et al. (1963) have suggested a crack blunting and sharpening process, i.e. that the crack grows during the tensile portion of the load cycle until the radius at the root is increased (blunting) so much that the stress concentration is sufficiently relieved so as to arrest any further growth. Electron fractography of failure surfaces has shown that there is a very good correlation between the number of load cycles and the number of fatigue striations.

The complexity of the mechanism involved in fatigue failure, is mainly due to the cyclic nature of the loading process. Many factors which exert little, if any, influence on the static strength properties of a material, may be the cause of marked changes in their fatigue characteristics. A general, even brief, consideration of the whole spectrum of the factors influencing the fatigue failure process, will be outside the reasonable scope of any single treatise on the subject.

However, it is possible to list a range of elements which are of obvious major influence: (a) The metallurgical processes used in metal production; (b) Characteristics of the manufacturing process (e.g. casting, rolling, etc.); (c) Characteristics of the external loading conditions; (d) The detailed design characteristics of the component; (e) Additional post-production treatments of the component (e.g. surface hardening and polishing) and (f) the conditions of service environment.

The present study of fatigue crack propagation in thermoplastic materials, will be mainly concerned with part (c) in the above list with further exploratory studies associated with parts (b) and (f).

1.1.1 A historical survey of various approaches to the analysis and characterisation of the fatigue failure process:

The fatigue failure phenomenon has been recognised by technologists for well over a century. Indeed, the first attempt to study the failure of a machine component subjected to repeated loading was made by Albert in 1829, who studied the fatigue strength of welded mine hoist chains. However, the first original systematic testing method is due to Wöhler who, in 1852, published the results of his studies of the failure of railroad axles. Wöhler's representation of his results in the form of graphs of applied stress against fatigue life (so called S-N curves) have since been extensively used. Today, in situations where the crack initiation period occupies by far the largest part of the total fatigue life (conditions of low stress, large components), the S-N type data are used by some designers.

Based upon his test data, Wöhler proposed the two fundamental laws of fatigue:

(1) That iron and steel may fracture under a stress not merely less than the static rupture stress, but also less than the elastic limit, if the application of the stress is repeated a sufficient number of times; and (2) No matter how many times the stress cycle is repeated, rupture will not take place if the range between the maximum and the minimum levels of stress is less than a certain limiting value. These two laws have formed the basis of the major part of the conventional methods of fatigue failure analysis, i.e. in the determination of endurance limit and fatigue strength of materials.

S-N type data, while being of some qualitative use in aiding material selection, do not indicate the effect of discontinuities or stress concentrations on the overall fracture resistance. Endurance limits obtained from S-N curves show the

maximum nominal stress to avoid crack initiation, whereas in structural components in service this nominal stress is readily exceeded at discontinuities, or at points of stress concentration. Consequently, using this approach the predicted time to failure of a given component can be severely overestimated.

The fundamental ideas relating to what is known as the cumulative damage process were proposed by Orowan in 1959, who considered the existence of weak spots within the bulk of a material as sources of stress concentration. It was postulated that the ability of a material to flow plastically enhances its rupture strength. Thus the failure of a metal after repeated application of a stress cycle, with peak stress level less than the critical stress for the specific material, was attributed to the gradual loss in its ability to flow as a result of work hardening. The cumulative damage concept may become so complicated in the case of some materials that its accurate application would be impractical (Christensen, 1954).

1.1.2 A brief review of the quantitative theories of fatigue failure:

The quantitative theories of fatigue crack propagation in metals may be divided into the following categories:

(a) The metal physics approach which embodies the advantages of consideration of structural lattice defects in relation to microcrack growth, Machlin (1949) and Valluri (1961). Growth of the crack is attributed to the movement of dislocation, at the edge of the flow, generated by the application of a shear stress cycle. Such an approach to failure analysis, while undoubtedly important in aiding the discussions of the mechanics of crack growth, has been of limited practical use.

(b) The empirical approach suggested by Manson (1953) and Coffin (1954), which consisted of a method of separation of the plastic strain, ϵ_p , from the total strain, ϵ_T , in a load cycle and then relating the plastic component to the

total cyclic life, N_f . A linear relationship (of slope 0.5) was shown to exist in a logarithmic plot of ϵ_p against N_f . A considerable quantity of experimental data has been analysed using this approach.

However, difficulties arise in attempting to use the plastic strain range as a damage criterion for low ductility metals in the case of which the plastic strains are too small to be measured or calculated accurately and hence can not be easily correlated with parameter, N_f (Mehring and Felgar, 1960).

(c) The increasing interest in the failure life prediction of components subjected to loading conditions in which the initiation period occupies a relatively small fraction of the total fatigue life, has led to the direction of the research efforts towards the measurement of the actual cyclic rate of crack propagation in various materials. This aspect of fatigue failure has, in recent years, been the focus of a most concentrated research effort, utilising both the conventional stress analysis methods and, lately, the relatively more powerful linear elastic fracture mechanics (LEFM) techniques.

At the initial stage of development of such approaches, the stress distribution around fatigue cracks were analysed using the continuum mechanics concepts (McLintock, 1963), hence taking into consideration the influence of the component configuration and the crack geometry. Subsequently a number of crack growth equations based on empirical data were proposed. These equations may be expressed in the general form:

$$\frac{da}{dN} = \phi(\sigma, a, C) \quad (1.1)$$

where a is the crack length; N is the number of cycles; σ is the applied stress level and C is a parameter whose magnitude depends on a range of factors such as loading frequency, material properties and testing conditions.

More specifically, the first crack growth equation of this kind was proposed by Head (1953), which was of the form:

$$\frac{da}{dN} = a \chi \quad (1.2)$$

χ being the parameter dependent upon the stress amplitude.

This equation was later revised by Frost and Dugdale (1958) and expressed in the following form:

$$\frac{da}{dN} = C \sigma^3 a \quad (1.3)$$

The above equations and those of similar form proposed by others may be expressed in the general form of

$$\frac{da}{dN} = C \sigma^{n_1} a^{n_2} \quad (1.4)$$

This general model for crack propagation will be referred to again in Section (2.3) where the more efficient LEFM based models will be discussed.

(d) At a more fundamental level, the concepts of general process rate theories have been used by Yokobori (1969) in the development of a failure criterion based upon the dissociation mechanisms at the atomic level. Although in an approach of this nature, the fundamental aspects of failure are dealt with (as in the ultimate analysis fracture of metallic and polymeric solids occurs as a consequence of the rupture of bonds which normally bind the atoms together), the practical applications of such a model are at present very limited. The main reason being the lack of quantitative data related to the magnitudes of the parameters involved in the analysis at the atomic level. However, some useful information with regards to the relative role of such parameters may be obtained from the outcome of such analyses (Yokobori, 1971).

(e) The energy based approach: This is the most fundamental approach to failure analysis at the continuum level. Through an examination, at the crack tip, of all the energy components, general criticality criteria are derived. There have been two distinct parts to this approach in the past: (1) the global energy consideration of Feltner and Morrow (1961) and (2) consideration of the local dissipation processes by Liu (1963).

The total thermodynamic energy balance approach has been successfully used (Griffith 1921, 1924) for the development of the fundamental principles of LEFM. More recently it has been used by Cherepanov (1968) to develop a rigorous analysis of failure of solids in general terms. This approach has been utilised in the present study, in the derivation of a theoretical model for prediction of the cyclic initiation life, N_i , in viscoelastic solids (Chapter 4).

1.2 Fatigue Failure of Thermoplastics:

Polymeric materials (in the form of plastics, fibres and rubbers) are now frequently used by engineering designers, wherever they can suitably replace the conventional materials. The comparatively favourable strength to weight ratio, excellent damping properties, good wear resistance and the facility of economic manufacture of intricately shaped components, are some of the advantages of polymeric materials, highly attractive in the design of components such as light duty gears, bellows, bearings, bushes, dampers, etc.

Thus, in recent years with the continuous development of industrial plastics of improved mechanical properties, a tremendous amount of interest has been created in the knowledge of the behaviour of these materials under monotonic and cyclic loading conditions.

The static fracture has received by far the largest attention, especially in the case of glassy thermoplastics, where concepts of linear elastic fracture mechanics (LEFM) have been successfully utilised in the analysis of craze (local yielding) growth and the measurement of such parameters as the critical stress and flaw size for fracture under various loading conditions (see for example, Rosen, 1964).

Attempts to analyse the fatigue failure process, however, have been relatively scarce, mainly due to the inherent complexity of the fatigue failure mechanisms as well as the complexity of the polymer molecular structure and the subsequent lack of sufficient understanding of the effects of structural changes on physical behaviour. The initial studies have, inevitably, been carried out on elastomers (Andrews, 1961; Thomas, 1967).

In the case of thermoplastics the fatigue failure studies were initially based upon the conventional techniques of empirical analysis used for metals; i.e. the determination of the total fatigue life of specimens of such materials from plate-bending, rotational bending and reversed shear (torsion) tests. Results obtained from such tests were normally presented in the form of the S-N (Stress-Number of cycles) curves as was the case for metals. However, the total fatigue life studies have, in practice, often proved to be of limited importance in the analysis of service failure problems - as indicated in the preceding section. The structural inhomogeneity of polymeric solids (due to irregularities in molecular chain entanglements plus the fact that during the processing stage, there exists a strong possibility of inclusion of external particles and gas bubbles) inevitably results in the presence of a number of stress concentration points within the bulk of the material. Thus, the cyclic crack growth initiation life in these solids are often relatively short (McEvily and Boetner, 1963); and hence the importance of the information on the effects of various parameters influencing the cyclic rate of crack propagation becomes

apparent.

It is pertinent to remark here, that due to the wide variation in the viscoelastic and mechanical properties of polymeric solids, attempts to generalise the results obtained from tests on one material must be cautiously treated. The quasi-brittle glassy thermoplastics, behave in many ways similar to metals. For example, the fatigue behaviour of PMMA and its fracture surface appearance were studied by Feltner (1967) from rotating bending tests. His results demonstrated that the failure occurred in a cycle dependent manner with surface striations similar to those in metals. McEvily et al. (1963) studied the high stress, short life fatigue crack propagation in polycarbonate, PC and polyethylene, PE, reaching generally similar conclusions. However, as it will become clear later, important differences in material behaviour due to basic structural properties may also be observed (Von Jacoby and Cramer, 1969).

1.2.1 Some general aspects of fatigue failure process in polymeric materials:

Empirical studies of fatigue failure in polymeric materials have revealed that the failure of these materials under repeated loading conditions, takes place as a consequence of the occurrence of one of the following three phenomena:

- 1) Creep (strain accumulation) due to cyclic loading.
- 2) Propagation of flaws up to a critical length.
- 3) 'Softening' of the material due to an excessive rise in temperature within its bulk (cyclic thermal softening).

Fatigue failures in the above three modes have been observed in cyclic axial loading, rotating bending, plate bending and three point bending tests. In mode 1 (equivalent to cumulative damage process in metals) and mode 3 failures, the direct

manifestations of the time-temperature dependence of the mechanical properties of polymers are observable.

The strain accumulation will occur if the periodic time of the load cycle is short enough not to allow complete strain recovery in each cycle; and the high temperature rise which occurs under high frequencies is attributed to the frictional movement between molecular chains within the polymer bulk.

The second mode of fatigue failure in these materials is the main focal point of the empirical analysis presented in Chapter 5. It is influenced by a very wide range of parameters such as the maximum and minimum levels of the load cycle, the load frequency, the cyclic waveform, crack sharpness (crack tip radius) and the environmental conditions.

Mode 1 will be used in the present study (Chapter 4) as the criterion for initiation of crack growth in a flawed viscoelastic body subjected to cyclic loading.

1.2.2 A discussion of mode 3 fatigue failure:

Considerable amount of experimental evidence exists which points to the fact that all polymeric materials under certain conditions of strain (or stress) and loading frequency, will exhibit a tendency to "self-heat", with the temperature in the stressed zone rising gradually to a threshold level where it increases very rapidly, thus causing melting of the material and its subsequent failure (Koo et al., 1967; Opp et al., 1969 and Constable et al., 1970).

This phenomenon occurs as the direct consequence of high damping capacity (which is the result of frictional movement between the molecular chains) and the low thermal conductivity of polymeric solids. In metals, the high thermal conductivity coupled with relatively low cyclic hysteresis loss (damping) enables the cyclically loaded material to maintain its initial mechanical characteristics. As

the physical properties are functions of temperature themselves, so in polymers the energy dissipated per cycle does not itself stay constant for a particular set of loading conditions. These changes can not be neglected (Constable et al., 1970) and although thermal failure models have been proposed (Opp et al., 1969; Koo et al., 1967), their accurate application is by no means well established. Attempts at quantitative formulation of the temperature rise, inevitably have met with practical difficulties such as the accurate measurement of temperature (One method for the measurement of small temperature rises has been devised by Higuchi and Imai, 1970.) and also the uncertainties arising from the fact that while linear viscoelastic damping is well defined and indeed is a basic tool of polymer characterisation, the complex and non-linear nature of damping under fatigue loading conditions has not yet been analysed.

1.2.3 Factors influencing the fatigue life:

In the previous section, a passing reference was made to some of the most important parameters which have a marked influence on fatigue life of polymeric components. It is thought desirable to reconsider the specific effect of some of these parameters, as the necessary background for the discussion of the experimental results presented in Chapter 5.

Two excellent reviews of the contributions to the study of various aspects of fatigue failure in thermoplastics have been prepared (Dillon, 1950; Andrews, 1969a); the consideration here will be confined to some of the most illuminating and closely relevant observations:

Findley (1941) who studied the behaviour of cellulose acetate from test results in the form of S-N curves, showed that the endurance limit of this material increased when each of the following steps was taken:

The minimum stress was held at zero level; Specimen surfaces were machined; Loading frequency was reduced; Specimens were cooled (say by air blast) and the specimen cross-section was changed from rectangular to circular. (In the case of rubbers, the fatigue life increases if the minimum stress is not allowed to fall to zero; Andrews, 1969b.

In polymers increasing the mean stress always tends to decrease the fatigue life (Mukherjee and Burns, 1971). The effect of loading frequency has been studied both from tests in which excessive temperature rise was experienced (Opp et al., 1969) and also from tests in which non-thermal failure was observed (von Jacoby and Cramer, 1968; Hertzberg et al., 1970; Mukherjee and Burns, 1971). The general conclusion being that for the majority of the materials, the increase in frequency leads to a decrease in the total fatigue life (with the exception of PC for which variations from this trend of behaviour have been observed in some cases, von Jacoby and Cramer (1968). Williams (1967) analysis of the growth of a spherical flaw under pulsating loading conditions also resulted in a growth function related to frequency in accordance with the above mentioned empirical evidence.

1.2.4 Analysis of the crack propagation process:

Theoretical approaches to the analysis of the fatigue crack propagation process in polymeric solids, can be fundamentally divided into two categories:

- (1) The molecular rupture analysis
- and (2) the continuum mechanics approach to failure. Contributions to each of these two fields will be separately considered:

Treating the polymer failure under cyclic loading as basically an "irreversible accumulative" process, Zhurkov and Tomashovskii (1959) postulated a principle of impairment superposition. Their analysis was fundamentally based upon Eyring's

(1936) reaction rate theory and assumed that there exists a direct relationship between rupture under constant load (static) and that under cyclic load. The time to failure, t_f , was obtained from:

$$\int_0^{t_f} \frac{dt}{\tau [\sigma(t)]} = 1 \quad (1.5)$$

where $\tau [\sigma(t)] = \tau_0 \exp [(U_0 - \gamma_0 \sigma(t)) / \kappa T]$ and it was suggested that coefficients U_0 , τ_0 and γ_0 can be determined from static tests; κ and T being the Boltzmann's Constant and the absolute temperature. The stress function $\sigma(t)$ can take any form. Subsequent to this analysis there have been a number of other contributions, notably in the Soviet Union, which have confirmed the above hypothesis and compared it to test data, (Leksovskii and Regel, 1965).

A similar approach to failure analysis was adopted by Pervorsek and Lyons (1964) and Kargin and Slovinskii (1967) who studied the characteristics of the chemical structure and the effects of various parameters on the failure processes in bulk polymers. However, as previously indicated, treatments at the atomic level suffer from the paucity of comprehensive quantitative information with regards to the structural parameters involved.

From a direct engineering application point of view, approaches based on continuum mechanics concepts have proved to be by far the most useful; Lake and Lindley (1964) on rubbers; Borduas et al. (1968) on PMMA; Mukherjee et al. (1969) on PMMA and PC; Andrews and Walker (1971) on polyethylene.

Opp et al. (1967) studied the fatigue failure of a set of homogeneous and heterogeneous polymers (cast PMMA, rigid PVC, acetal homopolymer and nylon 6.6). The possibility of direct application to polymeric solids, of some of the

fatigue failure theories developed for metals was examined. The empirical models based on plastic strain measurement (Coffin, 1954) were found to be not applicable to viscoelastic materials due to high sensitivity of these materials to temperature and strain rate variation.

A total hysteresis energy model (Feltner and Morrow, 1959) was also considered but rejected on the basis of its theoretical deficiency in application to polymers as well as its incapability to deal with such important factors as relaxation and creep response, load rate, temperature and geometrical dimensions.

Studies of Gent et al. (1964), Lake and Lindley (1964) and Thomas (1967) on fatigue of rubbers and that by Andrews and Walker (1971) on polyethylene have all been discussed in terms of the "tear energy" concept. This particular approach to failure of polymers was initially derived by Rivlin and Thomas (1953) and was based upon the brittle fracture criterion of Griffith (1921). Fatigue studies showed that the incremental growth of a crack, with each stress cycle, obeyed a relationship of the form:

$$\frac{d(a)}{dN} = \phi (\mathcal{T}) \quad (1.6)$$

$\mathcal{T}$ being the parameter representing the energy available for crack propagation or alternatively describing the stress distribution at the crack tip. More specifically, the following form of the above equation was obtained:

$$\frac{da}{dN} = B \mathcal{T}^n \quad (1.7)$$

B and n being functions of a range of parameters which influence the fatigue life.

On a logarithmic scale, $\frac{da}{dN}$ is linearly related to $\mathcal{T}$, with n as the slope of the line. It has been shown that for some materials, depending on the specific

testing range, different values of B and n may be obtained (Andrews and Walker, 1971) - an observation comparable to the distribution of the crack propagation data reported for some metals, Barsom (1971a).

For an edge crack, in a semi-infinite sheet of a highly elastic material, Rivlin and Thomas (1953) defined:

$$\mathcal{T} = 2k a W_0 \quad (1.8)$$

where W_0 is the term representing the stored strain energy density ($= \frac{\sigma^2}{2E}$); a is the crack length and k is a numerical factor which varies between π and 1 (Andrews, 1969b).

Waters (1966) also observed that PMMA exhibits the phenomenon of small scale crack growth, when subjected to cyclic loading and with the value of $\mathcal{T}$ chosen such that it fell well below the level of the critical tear energy, $\mathcal{T}_c$, for the material.

In line with the development of the LEFM techniques and their application to fatigue failure of metals, some attempts have been made to test the applicability of LEFM concepts to the fatigue failure analysis of polymeric materials (Borduas et al., 1968; Hertzberg et al., 1970). These approaches will be further discussed in the following chapter. The basic impediment to the direct application of the LEFM concepts, is the strong non-linearity of some polymeric materials. In such cases the energy based approach will be preferred. However, for a wide range of polymers, approximate linearity can be assumed within specified limits of load duration and level. Thus prior to the direct application of the LEFM concepts, these limits must be identified for a particular material. The fundamental value of the LEFM approach, in the case of polymeric solids - as also for metals - lies in the fact that this method of failure behaviour characterisation considers a material

simply as an elastic continuum and hence requires no knowledge of the specific nature of the microscopic structure.

CHAPTER TWO

FRACTURE MECHANICS CONCEPTS AND FATIGUE FAILURE ANALYSIS

In this chapter a brief review of the pertinent aspects of linear elastic fracture mechanics (LEFM) principles is presented and the uses of LEFM concepts in empirical and theoretical analysis of the fatigue failure problem in metals and polymers are surveyed.

2.1 Some Pertinent Aspects of LEFM Concepts:

2.1.1 Introduction:

Most engineering structures contain flaws or cracks that are created either during the manufacturing process (e.g. inclusion of external particles and voids), or at the fabrication stage (e.g. weld defects) or during service (e.g. corrosion pits). Unstable fracture occurs when these flaws develop to a critical size that is a function of both the applied (or residual) stress acting on the structure and the toughness of the material. The presence of these flaws results in reducing the potential theoretical strength of solids to a relatively much lower level (e.g. The theoretical strength inherent in a perfect structural lattice in a metal will typically be of the order of 2500 tons/in^2 !).

Recognition of the inherent weaknesses in practical engineering materials (i.e. the existence of non-continuity and non-homogeneity) has led to a total re-thinking in the approach to the design of engineering components, specially with regards to the use of high strength-low ductility materials.

Basically there are two methods of evaluation of failure strength: (1) The ductile-brittle transition temperature approach, which is an empirical method of

determination of the temperature ranges over which material ductility is lost; and (2) the analytical approach which is based on the evaluation of the stress field parameters in the vicinity of the crack. Within the context of the latter approach, accurate analyses of fundamental parameters such as the relationship between the flaw size, the external loading, and the characteristic mechanical properties of a material, etc. are required. LEFM concepts have been developed to simply provide the tools with which such correlations between the influential parameters in failure analysis can be obtained. Comprehensive reviews of the method and its fundamental aspects are now numerous (Weiss and Yukawa, 1965; Paris and Sih, 1965; Tetelman, 1970). Here we shall only refer to those aspects which will be relevant to the analysis and discussion of the outcome of the present programme of studies..

2.1.2 A note on the plane-strain, plane-stress and ductile, brittle fractures:

Assuming that the conditions of fracture can be divided into plane-strain and plane-stress, the following comments may be made:

In plane-stress, where the components of stress $\sigma_{zz} = \sigma_{yz} = \sigma_{xz} = 0$, the material when pulled in one direction, is free to deform in the transverse direction. In so doing it sets up resistance to further fracture. This is a 'tough' mode of failure. On the other hand, plane-strain is a state of stress that is characteristic of thick or bulky parts for which the stress adjacent to a flaw is triaxial tension. The triaxial stress condition is a result of either the bulk restraint or the material brittleness. Plane-strain leads to rapid fracture as a flaw in triaxial state of stress can have little local yielding. Material yielding in plane-strain is usually doing so in its weakest manner and will fracture below the stress level required for failure in plane-stress.

There are two main situations in which brittle fracture can occur in engineering materials: 1) By placing a normally tough, medium strength material into a "brittle" environment. For instance, one can introduce flaws into the structure during the material solidification stage, in metal forming operations, during heat treatment and in welding. One may also use the material in very low temperature environments, where it is known that tough materials become brittle. The rate of loading could also cause the material to react in a brittle manner rather than yielding gradually. If the component is loaded in two orthogonal directions at once, under certain stress conditions, its ability to yield in transverse direction will be limited and hence brittle behaviour may be observed (see Chapter 7). And (2) the case of high strength, low ductility materials which are known to be highly brittle in fracture mode. It is, thus, clear that a primary goal of LEFM methods of analysis will be to determine which particular condition of fracture prevails. If failure occurs in plane-stress, the usual yield strength criteria are often sufficient to render a safe design. However, for the more usual plane-strain case, fracture mechanics tests must be performed to determine the exact nature of parameters associated with fracture.

2.1.3 Thermodynamics of fracture; the energy balance concept; fracture toughness and crack tip plasticity:

Studies of fracture behaviour in terms of phenomenological, structural and atomic or molecular aspects involved in the failure process, require some means of intercorrelation so as to develop a unified theory of mechanical response in fracture. On the continuum level the fracture behaviour is described in terms of the relations between stress and strain and their time derivative (e.g. Griffith 1921, Cherepanov, 1968). On the atomic or molecular levels, the behaviour of the discrete particles

is described by their relative positions in space, their velocities and the force of interaction between them (Barenblatt, 1962, treatment of cohesive fracture). The concept of energy balance has the same meaning at all levels of structural subdivision. Therefore the problem of not being able to provide an equation of state for real materials can be by-passed and the mechanical response determined by a unified theory of fracture based upon a thermodynamic energy approach.

Considering a solid body containing a crack, Griffith (1921, 1924) was able to demonstrate that work must be done to supply the energy necessary to create new surfaces and hence extend a crack. The well known Griffith equation was derived by considering an ellipsoidal crack (initially analysed by Inglis, 1913) in an elastic medium. The criterion for crack extension being that the energy required for separating the surfaces was just less than the amount of elastic energy released as a consequence of crack propagation. Griffith's (1921) calculation of the required stress to initiate fracture resulted in the following relationship:

$$\sigma = \left(\frac{\gamma E}{a} \right)^{\frac{1}{2}} \quad (2.1)$$

where σ is the required stress (fracture strength), E is the Young's modulus of elasticity, γ is the surface energy and a is the crack length. Inevitably the most useful application of this equation depends simply on the accuracy with which the above parameters can be evaluated. In Griffith's case the solution was obtained for a perfectly elastic solid. Such an ideally brittle material is rarely found in practice and for engineering materials, specifically, such perfect elasticity may not be assumed.

In quasi-brittle and ductile materials, due to very high concentration of stresses at the crack tip, plastic yielding will take place. The nature of the fracture process that follows is dependent on the extent and form of the plastic deforma-

tion. In developing a quantitative treatment of the effects of plasticity in fracture, it is necessary to consider the work done by deforming a localised region as the crack progresses. Irwin (1948, 1957) and Orowan (1949) independently analysed the way in which the work of plastic deformation can be incorporated in the crack propagation equation. This is achieved simply by replacing the true surface energy term, γ , in equation (2.1) by another term γ^* , where

$$\gamma^* = \gamma_p + \gamma \quad (2.2)$$

γ_p is the irreversible work absorbed in local plastic deformation per unit area of crack extension. γ^* may be 1000 times higher than γ (Spretnak, 1961) except, perhaps, in the presence of a strongly corrosive atmosphere which reduces ductility; γ_p varies widely for different materials. Cherepanov (1967) suggested that the ratio γ^*/γ in many cases is of the order of E/σ_{yp} for a specific material (σ_{yp} being the yield stress in tension). The magnitude of γ^* is dependent on parameters such as temperature and crack speed; for example, for metals it can increase rapidly with a small increase in temperature. However, in the case of PMMA material, γ^* increases with the lowering of temperature (Pratt, 1968). In both metals and polymers the work of plastic deformation increases with crack speed, an effect evident from the analytical solution of Goodier and Field (1963) and the empirical data of Berry (1964). It is clear that, the magnitude of γ^* , in real materials, is a measure of the work required for the extension of cracks. This characteristic parameter of a material (which is defined under specific conditions of loading) is called the material fracture toughness. Thus the practical and physical significance of 'fracture toughness' is that it is essentially a measure of the ease and speed with which a crack propagates through the damaged material under selected stress and environmental conditions. From the above discussion: in metals the

plastic deformation and hence the fracture toughness value may be suppressed under the conditions of high crack velocity and low temperature. An increase in the degree of brittleness in fracture under these conditions can be observed. The fracture toughness of high strength materials also decreases asymptotically with increasing thickness as the fracture mode changes from fully plane-stress to fully plane-strain.

Irwin and Kies (1952, 1954) analyses of the balance between the input elastic energy and the irreversible work of deformation discussed above, resulted in the development of a criterion for fracture which involves the following two quantities: (a) The strain energy release rate, denoted by G , which represents the elastic stored energy released per unit crack extension; And (b) A term denoted by G_c (the limiting value of G) which is a measure of the fracture toughness of the material as discussed above. $G_c = 2\gamma^*$ as two surfaces are created in crack extension.

The strain energy release rate is a useful parameter in the phenomenological study of the crack stability; however, it does not lend itself to aid the mechanistic study of fracture as in such a study detailed analysis of the structure at the crack tip will be required. Parameter G can be determined experimentally as well as analytically. A comprehensive treatment of the experimental aspects of fracture is provided by Brown and Srawley (1966). If the direction of crack extension is collinear with the plane of the crack, the strain energy release rate can be calculated using the analytical method of Irwin (1957); see also Chapter 5.

In considering the input and the absorbed energies, one is faced with the problem of variations in the plastic deformation work with increasing crack length, as this will decide the degree of stability of a crack. In the presence of plastic flow,

as the crack starts to propagate, the stiffness of the component and the nature of the loading mechanism determine the continuation of its growth. Propagation of the crack reduces the component stiffness. Under "dead load" conditions (i.e. when work is being done at the boundary as the crack extends) the displacement will increase automatically to compensate for loss of stiffness and catastrophic failure will ensue. However, under "fixed grip" conditions, the loss of stiffness may unload the specimen and hence arrest the crack. In other words the work of plastic deformation will become so large that without an increase in the boundary loading the crack propagation can not continue. Note that a large specimen under fixed grip conditions may contain sufficient strain energy to cause complete fracture when the point of instability has been reached.

The causes of increase in the plastic deformation energy demand vary with materials. In highly ductile materials, it is the result of increase in the size of the plastic zone at the crack tip. Since the stress increase ahead of the crack is limited by yield stress, the increased disturbance to the original field is balanced by a larger zone within which stress has reached yield load. The energy of deformation increases as it is proportional to the deformed area. Glucklich (1971) proposed that in quasi-brittle materials, increase in plastic energy demand is due to the development of microcracks in the highly stressed zone ahead of the crack. The energy transformed to the surfaces of these microcracks is lost as the majority of these cracks do not join to form the main crack. In glassy polymers, the material adjacent to the fracture plane, thus, develops the so-called "crazes", dissipating an appreciable amount of energy in the process. The density of these microcracks in the body of the material increases with increasing crack length and hence the plastic deformation work per unit crack extension rises in magnitude as the crack grows in length. This phenomenon has been observed in PMMA (Berry, 1964).

The smaller the work of plastic deformation, the less stable will the crack growth become. In static fracture (i.e. under monotonic loading conditions) and in fatigue failure after the initial crack has grown to an appropriate limiting length, two singular events occur: Namely the start of slow (stable) crack growth and the onset of rapid crack propagation. The value of G_c , referred to above, is normally taken as that which corresponds to the latter stage. The extent of slow crack growth will be less the more brittle is the material behaviour and in the limit of extreme brittleness the start of slow crack propagation and the onset of rapid fracture will tend to coincide; Cherepanov (1968) and Wnuk (1971). Hence in practice, the more brittle a material, the more accurate is the determination of the fracture toughness parameter. Obviously the study of factors which may influence the extent of slow growth is of special significance. Some of the most important parameters are: material properties, section size, load history, temperature and corrosive environment.

2.1.4 The concept of stress intensity factor:

Linear elastic fracture mechanics techniques make use of the elastic stress analysis of a cracked part to define the conditions under which an existing crack will extend. The basic assumptions are the same as those in theories of strength of materials: the material is a homogeneous isotropic continuum; strains are small; stress is proportional to strain and distortions are neglected. In addition, non-linear effects such as those due to yielding, internal stresses and local irregularities in the crack surface are assumed not to affect the general characteristics of the stress field (Paris and Sih, 1965). Thus, a necessary requirement in the applicability of the method is the confinement of the plastically deformed zone to a region at the crack tip, small in comparison to the characteristic dimensions of the structure,

such as its thickness.

Irwin (1957, 1958) noted that basically there are three kinematically admissible modes of crack extension (Figure 1); and that all other types of crack opening may be considered as a combination of these three modes, which result from three types of loading: in-plane tensile; in-plane shear and anti-plane shear. Subsequently the elastic stress field associated with each mode of opening was analysed. The stresses and strains were shown to have a singularity of the form $(r)^{-\frac{1}{2}}$ where r is the distance from the crack tip (Williams, 1957, Irwin, 1958). It was shown that whatever the geometry or self-equilibrating loading conditions, the environment in the immediate vicinity of a crack tip can be described by a single parameter, which was designated the stress intensity factor, K ; subscripts I, II and III are assigned corresponding to three types of opening. In an isotropic material, brittle fracture usually occurs in mode I. Crack growth in the form of shear lip is a combination of modes I and III, but is usually treated as mode I in calculations. In two-dimensional cases and shell problems only modes I and II can exist. Mode II displacements exist only at internal or very deep external cracks. Knowledge of K_I , K_{II} and K_{III} at the crack tip, will completely specify the stress environment at the tip. For the stresses:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} \begin{bmatrix} 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \\ 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \\ \sin \frac{\theta}{2} \cos \frac{3\theta}{2} \end{bmatrix} + \frac{K_{II}}{(2\pi r)^{\frac{1}{2}}} \sin \frac{\theta}{2} \begin{bmatrix} -2 - \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \\ \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \\ \cot \frac{\theta}{2} - \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \end{bmatrix} \quad (2.3)$$

and

$$\begin{bmatrix} \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \frac{K_{III}}{(2\pi r)^{\frac{1}{2}}} \begin{bmatrix} \sin \frac{\theta}{2} \\ \cos \frac{\theta}{2} \end{bmatrix}$$

or more briefly for mode I:

$$\sigma_{ij} = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \phi_{ij}(\theta) + \text{other terms bounded at the crack tip.} \quad (2.4)$$

Strains and displacements are obtained in a similar manner. The set of functions $\phi_{ij}(\theta)$ are the same for all symmetrically loaded crack problems. For an infinite plate, containing a crack of length $2a$ loaded at the boundary (Figure 2):

$$\sigma_{xx} = \frac{K}{\sqrt{2\pi r}} \quad (2.5)$$

where

$$K = \sigma_{\infty} \sqrt{2\pi a} \quad (2.6)$$

The nature of the above relationships point to the fact that linear factors K_I , K_{II} and K_{III} have no bias towards any particular field parameter and although the current usage describes them as stress intensity factors, they could equally well be described as strain intensity factor, displacement intensity factor, etc.

2.1.5 Methods of determination of the stress intensity factor parameter:

Various techniques have been developed for the purpose of calculation of the stress intensity factor in flawed bodies. The choice rests, in effect, on the selection of an appropriate method for analysing the stress field in a plane elasticity problem. For practical cases of reasonable simplicity, complex stress functions have been derived (Muskhelishvili, 1953). However, for relatively more complicated problems reasonable accuracy can be obtained only by implementation of

numerical techniques. A summary of the important and useful techniques available, will be presented here:

(A) The boundary collocation technique:

This method has been extensively used in derivation of the ASTM standard formulae for determination of stress intensity factors (Brown and Srawley, 1966). It is easily applicable to problems involving simple crack-body configurations (in the slightly complicated cases, elaborate computational analysis may be involved; no general proof of convergence for the method is available at present).

Basically the technique consists of the selection of stress functions, usually in the form of polynomials, which satisfy boundary conditions on the crack face and also have the desired angular behaviour in the vicinity of the crack tip. The free parameters in the stress function are then collocated so that the remaining boundary conditions are approximately satisfied.

(B) Muskhelishvili stress functions:

Analyses of elastic stress fields in solid bodies containing cracks - used initially by Westergaard (1939) in the form of complex stress functions - have been formulated in a very comprehensive form by Muskhelishvili (1953). Two complex stress functions are defined for any particular flawed body/crack configuration and boundary conditions. Component stresses and strains are given in terms of these complex functions; for instance in an infinite plate containing a crack of length $2a$ and loaded at infinity in a direction perpendicular to crack plane inclination:

$$\begin{aligned}\sigma_{yy} + \sigma_{xx} &= 2 [\phi'(z) + \overline{\phi'(\bar{z})}] \\ \sigma_{yy} - \sigma_{xx} + 2i\sigma_{xy} &= 2 [\bar{z} \phi''(z) + \psi'(z)]\end{aligned}\tag{2.7}$$

where $z = x + iy$ and $\phi(z)$ and $\psi(z)$ are the complex stress functions, defined for a particular configuration.

For the crack tip at point $z = z_i$, the magnitudes of the stress intensity factor can be found from:

$$K_I - i K_{II} = 2(2\pi)^{\frac{1}{2}} \lim_{z \rightarrow z_i} (z - z_i)^{\frac{1}{2}} \phi'(z) \quad (2.8)$$

for the example of centre-notched plate:

$$\phi'(z) = \frac{\sigma}{2} \left[\frac{z}{(z^2 - a^2)^{\frac{1}{2}}} - 1 \right] \quad (2.9)$$

where σ is the applied stress.

$$\begin{aligned} \text{Thus } K_I - i K_{II} &= \sigma (2\pi)^{\frac{1}{2}} \lim_{z \rightarrow a} \left[\frac{z(z-a)^{\frac{1}{2}}}{(z-a)^{\frac{1}{2}} (z+a)^{\frac{1}{2}}} - (z-a)^{\frac{1}{2}} \right] \\ &= \sigma \sqrt{\pi a} \end{aligned} \quad (2.10)$$

$$\text{Hence } K_I = \sigma \sqrt{\pi a} \quad (2.11)$$

$$\text{and } K_{II} = 0.$$

(C) The approximate method

This method, proposed by Williams and Isherwood (1968), provides solutions with approximations of about 20% (improved results have been obtained over certain ranges of crack length). It consists of substitution of a weighted average value for parameter σ in equation (2.11).

$$\text{Thus } K_I = \sigma_m \sqrt{\pi a} \quad (2.12)$$

Example methods of calculation of σ_m' are provided in the above reference.

(D) The finite elements method:

This method, by virtue of its ability to deal with very complicated geometries and loading conditions, is by far the most versatile technique for the accurate calculation of the stress intensity factor parameter. A comprehensive review of the contribution to the development of this method and its applications may be found in Hayes (1970) and Miyamoto et al. (1971).

2.1.6 Temperature and load rate dependency of fracture toughness:

In Section (2.1.3), in the discussion of the fracture toughness parameter, the important effect of the loading rate was referred to. In the case of a large number of quasi-brittle and ductile materials, the K_{IC} parameter (termed the plane-strain fracture toughness value) is significantly affected by variation in the load rate, as indicated by the differences in the results of the slow-bend and the Charpy tests. The discussion previously presented in relation to the increase in the degree of brittleness and the subsequent decrease in the level of the required energy for crack extension under various conditions, should be reconsidered. Thus in practice a drop in the value of K_{IC} in the presence of an increasing brittle behaviour may be anticipated for a large number of materials.

Temperature variations also influence the value of K_{IC} . If a lowering of temperature enhances the brittle behaviour of a material, a lowering of K_{IC} value may be expected. A model for the prediction of the temperature dependence of K_{IC} in low alloy steels exists (Tetelman, 1970), in which it is postulated that the K_{IC} variation occurs as a result of variations with temperature, in yield stress, σ_{yp} , of the material. Thus if the value of K_{IC} is known at one temperature and

σ_{yp} is known as a function of temperature variation, the value of K_{IC} at various levels of temperature can be calculated.

The temperature and load rate (frequency) dependence of the K_{IC} parameter is obviously of considerable importance in fatigue failure studies; hence, in Chapter 5, such effects with special reference to polymers will be reconsidered.

2.1.7 The concept of crack opening displacement (COD):

When a specimen containing a crack is loaded, the crack tip opens without extension of the crack. This movement is called the Crack Opening Displacement (COD) and is associated with the development of the crack tip plastic zone (Wells 1965a, 1968); i.e. the material contained within a local zone of appropriate dimension at the crack tip is plastically strained to accommodate the crack tip displacement. In a non strain hardening material the stresses in the plastic zone are of the order of the yield stress, σ_{yp} . Consequently the work of plastic deformation (or G) at the crack tip is approximately equal to the product of the local stress acting on the volume element adjacent to the tip and the displacement through which it acts (McLintock and Irwin, 1965). It has been found that, (Wells, 1965a):

$$G = \frac{3\pi^2}{8} \sigma_{yp} (\text{COD}) \quad (2.13)$$

The critical value of COD, associated with a certain load rate, is denoted δ_C and may be used as a tool in assessing the fracture resistance of materials (Nichols et al., 1969). From the consideration of a simple model of elastic-plastic state of stress in the vicinity of the crack tip, Wells (1965b) related the COD to the crack tip extension force. Subsequently Williams and Swedlow (1967), using numerical analysis methods, demonstrated that COD in the presence of general

yielding is related to the fracture toughness of the material. Kobayashi et al.

(1969) analysis showed that, excluding a segment of length of approximately $a/10$ from the crack tip, any other position along the crack surface may be suitably used for COD measurements as an index of fracture toughness of ductile materials.

For low strength, high ductility materials, the COD concept has the advantage that it still has meaning at and beyond the general yield point (Burdekin, 1969). For brittle materials, fracture takes place at δ_C without any prior crack growth; and in the transition range from brittle to ductile behaviour, unstable cleavage fracture may be preceded by a small amount of ductile crack growth (Nichols et al., 1969). The possible use of COD concept in fatigue failure studies will be discussed in Chapter 6.

2.1.8 The contour integral, J:

The contour integral technique is used basically to calculate the change in the energy of a body as a consequence of crack extension. It was developed by Rice and Rosengern (1968) through consideration of the behaviour of linear and non linear elastic bodies. The change in energy is expressed in terms of a line integral which has the property of being path independent, with a magnitude designated J . This integral is a function of stress, displacement gradient and strain energy density. Hence these parameters must all be defined along the path of integration; thus along a path Γ , (Rice, 1968 a)

$$J = \int_{\Gamma} (W(\epsilon_{mn}) dy - T_i \left(\frac{\partial u_i}{\partial x_i} \right) dS) \quad (2.14)$$

where the strain energy density,

$$W(\epsilon_{mn}) = \int_0^{\epsilon_{mn}} \sigma_{ij} d\epsilon_{ij} \quad (2.15)$$

ϵ_{mn} is the infinitesimal strain tensor

T_i is the traction force

u_i is the displacement

and dS is an element of arc length along Γ .

Parameter J may be regarded as the single parameter associated with the stress environment near the crack tip from which all parameters of current interest in fracture mechanics may be deduced. Note that the prerequisite of linear behaviour of material, a fundamental necessity in calculating the stress intensity factor, does not apply in the case of J .

By considering the stress field singularity terms associated with the stress environment near the crack tip in a linear elastic body with small scale yielding, and evaluating the integral, the value of J is found to be (Rice, 1968b):

$$J = \left(\frac{1-\nu^2}{E}\right)(K_I^2 + K_{II}^2) + \left(\frac{1+\nu}{E}\right) K_{III}^2 \quad (2.16)$$

and for the special case of mode I crack opening,

$$J_I = \left(\frac{1-\nu^2}{E}\right) K_I^2 = G_I \quad (2.17)$$

which, again may be taken as a general proof of the equivalence between stress intensity factor and the crack extension force.

The J integral parameter was evaluated employing a total (deformation) theory of plasticity. Two dimensional deformation fields (plane strain, generalised plane stress and anti-plane strain) have been considered. Experimental evidence for the usefulness of J parameter as a crack growth initiation criterion is gradually being collected (Begley and Landes, 1971). There is, as yet, no substantial proof for its applicability to propagating cracks, which will naturally be immediately relevant to the fatigue problem. In the discussion of the experimental fatigue data

(Chapter 6), we will return to equation (2.17) to explore the possibility of application of the J factor. An inherent advantage of parameter J is that, like other energy based criteria, it may be used in problems dealing with large scale plasticity (Rice, 1968a) as obviously in the presence of gross plastic strain, the linear elastic stress intensity analysis will prove deficient, even though its empirical use in such situations has met with some success (McEvily and Johnston, 1966).

2.2 Application of the LEFM Concepts to the Characterisation of Failure in Polymeric Solids:

Failure in polymeric solids may occur either by excessive permanent deformation or by quasi-brittle fracture. While cross-linked polymers fail only by fracture, linear polymers may fail both by flow and fracture depending upon the rate of load application and the temperature.

At the continuum level, the distinctive characteristics of polymeric solids may be identified as follows:

- (a) Strong temperature and time dependence of elastic constants.
- (b) Relatively low elastic constants.
- (c) Non-Hookean elasticity.
- (d) Viscoelasticity (which gives rise to pronounced mechanical losses during the cyclic deformation in the elastic range).

Variation in each of the above characteristics may have a significant effect on the fatigue failure process in polymers.

In addition, there are distinct variations in the degree of non-linearity in mechanical behaviour of polymeric solids and although the fundamental stress analysis of these materials is based upon linear viscoelasticity theories, prior to the application of the principles of LEFM, the limits of strain and loading time within

which approximate linearity may be assumed, ought to be identified. In the case of the thermoplastic materials used in the present study, such limits were obtained from the consideration of the room temperature (21°C) isochronous and isometric data available; see Table (1).

There is a considerable amount of empirical data available as evidence of the possibility of application of LEFM concepts in the analysis and characterisation of the fracture and fatigue failure processes in polymers. Glassy polymers such as PMMA, PC and Polystyrene (PS) as well as semicrystalline thermoplastics such as rigid Polyvinylchloride (PVC), Nylon and Polyacetal (PA), together with most thermosetting polymers and other highly crystalline materials can be studied in terms of the LEFM concepts; (Boogart and Turner, 1963; Hertzberg et al., 1968; Andrews, 1968 and 1969a; Mukherjee et al., 1969; Key et al., 1968).

However, elastomers and soft crystalline polymers, such as Polyethylene (PE), and anisotropic polymers, such as fibres and films, deviate strongly from linear behaviour in elastic range and thus prohibit the application of any theory which is based upon classical elasticity concepts and infinitesimal strain assumptions.

Hence it may be stated that at least for the case of materials enumerated above, all aspects of the fundamental concepts of the LEFM techniques (Section 2.1) can, within the limits discussed, be equally applied to the polymeric materials; with realization of the fact that some parameters, such as the fracture energy term γ^* and the plane-strain fracture toughness, K_{IC} , may respond in a manner different from the case of metals when important factors such as strain rate, temperature and structural properties are modified.

Quantitative studies of the above phenomena have been relatively scarce. Increase in the fracture toughness of PMMA with increasing crack speed and with a rise in the molecular weight of the material was demonstrated by Berry (1964).

Radon (1971) reported a set of data to relate the variation of the K_{IC} parameter in PVC as a function of temperature and load rate. Key et al. (1968) measured the fracture toughness of PMMA and PS under various load rates and temperatures.

From a general consideration of the data referred to above, the following comments on some aspects of polymer fracture may be made:

The fracture surface energy (and hence the fracture toughness value) in polymers increases as the following steps are taken:

Reduction of temperature; increase in crack velocity; increase in molecular weight of the material; increase in the angle between the molecular orientation direction and the crack inclination (maximum when angle = $\pi/2$). Well documented expositions of various aspects of fracture of polymers may be found in Rosen (1964), Lindsey (1967) and Andrews (1968).

The crack tip plastic deformation associated with fracture has also been studied in some materials. For example, Brinson (1970) showed the possibility of application of the Dugdale (1960) model of plastic flow, to the fracture process in PC material. The crack tip deformation has been more extensively studied for PMMA, often using optical interference techniques (Theocaris, 1970). The general problem of craze growth - i.e. local yielding not necessarily at the tip of a crack - has also been studied using the LEFM concepts. An aspect of craze formation process, particularly useful in the discussion of local yielding during fatigue crack propagation process, is the time and strain dependency of craze size. From the studies of Kambour (1968) and LeGrand (1969) the following conclusions have emerged:

A critical stress (or strain) and time limit controls the craze formation process; i.e. with a smaller strain a longer loading time will be necessary to develop a craze of a specific size. The importance of this phenomenon in relation to frequency dependence of fatigue failure will be discussed in Chapter 6.

There have also been a number of attempts to develop theoretical models for fracture of viscoelastic solids. Evidence presented by Berry (1961) on PMMA and PC indicates the existence of a submicroscopic flaw which satisfies the Griffith criterion for fracture. Thus, based upon an energy balance concept, the theoretical formulation (Williams, 1962 and 1965; Knauss and Dietman, 1970; Wnuk and Knauss, 1970; Mueller, 1971) provide solutions for the onset of fast fracture in polymeric solids under various loading conditions. These solutions are inevitably in terms of the mechanical properties of the material. Hence, except in a few cases, the paucity of accurate and comprehensive data on structural properties and physical behaviour of polymers has severely limited the extent to which comparisons of the analytical solutions with the test data could be carried out. Thus such theoretical solutions, irrespective of their apparent success, should be regarded as attempts to explore certain concepts rather than as definitive statements.

2.3 Application of the LEFM Principles to Fatigue Failure Analysis:

In Section 1.1, in the discussion of the various techniques of fatigue failure analysis, the specific advantages of the use of LEFM concepts were referred to. In the conventional fatigue design theories, the energy required to initiate crack propagation was not considered and hence the ability to prevent the growth of a crack was severely limited. Fracture mechanics concepts treat this energy as well as taking into account the effects of such parameters on the component geometry.

The continuum models of failure all aim at the provision of a quantitative tool to aid the designer. Hence while it must be ascertained that such models have a sound physical basis, they must also be in a relatively simple form and contain only the field parameters of the system which would be easily available. Considera-

tion of the vast variety in the composition and the microstructural characteristics of actual materials, the diversity in geometry, loading conditions and the environment, clearly indicates that search for a general and unified model of failure applicable to all materials will, at present, probably be a futile task. Hence the great quantity of empirical data on the fatigue crack growth rate in metals under various loading conditions. However, in the case of polymeric materials, such data are scarce and, as yet, no comprehensive study of the cyclic crack propagation phenomena in these materials has been carried out. As one of the main purposes of the present study is to develop an empirical model for the prediction of the cyclic crack growth rates in polymers, models presently available for metals and their possible applicability to polymers will be surveyed in the following section.

2.3.1 Empirical analysis of the cyclic crack propagation process:

The output from the vast quantity of experimental studies of the fatigue crack propagation process, has been in the form of a specific relationship between the crack length, a , and the number of cycles, N . In all crack growth models, it has been assumed that da/dN (hereafter abbreviated to $\dot{a}_N$), which is the cyclic increase in crack length, is a continuous function of such variables as the external load, geometrical dimensions and material properties.

As mentioned in Section 1.1, the first cyclic crack growth equation of this type was proposed by Head (1953) in the form of a rather complex relationship between stress levels and the crack length. The model contained the idea of a plastic element at the crack tip surrounded by an elastic continuum. The complexity of determination of the size of the plastic particle was the main disadvantage of Head's equation.

At a later stage, a growth model was proposed by Frost and Dugdale (1958) which indicated a direct correlation between $\dot{a}_N$, crack length and the applied stress to the power three. A similar model was proposed by Liu (1961) relating $\dot{a}_N$ to a in the following form:

$$\dot{a}_N = F a \quad (2.18)$$

where F was a function of the alternating and the mean stress levels; i.e.

$$F = F(\sigma_a, \sigma_m) \quad (2.19)$$

Equation (2.18) can thus be put in the more general form of:

$$\dot{a}_N = \phi(\sigma_a, \sigma_m, a) \quad (2.20)$$

However, the initial introduction of the LEFM concepts to the analysis of the cyclic crack propagation process is attributed to Paris (1963) who proposed that since the elastic stress field and the plastic deformation in the vicinity of the crack tip can be characterised by the stress intensity factor (K_I , for in-plane extension mode), the rate of crack propagation should be some function of K_I . Subsequently the analysis of zero-tension test data on an aluminium alloy resulted in an expression of the following form:

$$\dot{a}_N = C (\Delta K)^m \quad (2.21)$$

where $\Delta K = K_{\max} - K_{\min}$; C is a numerical parameter dependent upon material properties and testing conditions and m is a numerical index. Both C and m have to be determined by empirical methods.

In a review of the crack propagation models available at the time, Paris and Erdogan (1963) - and in the discussion of this review by McEvily - it was pointed out that the cyclic rate of fatigue crack propagation is proportional to a term σ_a^{2n} i.e.:

$$\dot{a}_N = \phi (\sigma^{2n} a^n) \quad (2.22)$$

Similar formulations have been independently arrived at by McLintock (1962, 1965) and Schijve(1961). Thus equation (2.21) is a special case of equation (2.22). McEvily in the discussion of Paris and Erdogan (1963) also pointed out that equation (2.21) did not apply very accurately to the test data obtained on some copper alloys. He argued that the growth rate would be proportional to the energy stored in the plastic zone; and since the quantity of this energy is related to K_I^2 , then

$$\dot{a}_N \approx b K_I^2 \quad (2.23)$$

where b is a measure of plastic zone size. Thus depending on the exact nature of b (e.g. with small scale yielding $b \approx K_I^2$), $\dot{a}_N$ may be proportional to K^2 raised to a power not necessarily equal to 2 as suggested by Paris (1963). Thus, the main objection to the form of equation (2.21) is that the effect of mean level of stress intensity factor, K_m , is not catered for and as will be shown later, this parameter may have a significant influence on the rate of crack growth.

The crack propagation models discussed above, do not include, in an explicit form, the effect of frequency variation on $\dot{a}_N$ parameter.

(A) The effect of mean load on $\dot{a}_N$:

The criterion for crack growth rate prediction has been considerably developed since the initial formulation of equation (2.21) by Paris (1963). A wide range of experimental studies of the effect of the mean stress on the rate of crack propagation in a number of materials have demonstrated that in a lot of cases, simple correlation with the range of variation of stress (or K) will not yield an accurate prediction of $\dot{a}_N$. (Broek and Schijve, 1963; Frost, 1966; Roberts and Erdogan, 1967; Taira

and Abe, 1969 and Frost and Greenan, 1970).

Roberts and Erdogan (1967) proposed that the rate of fatigue crack propagation is probably more fundamentally related to the size of the plastic zone ahead and in the plane of the fatigue crack. Their analysis lead to the conclusion that the level of $K_{\max}$ reached in a loading cycle must be given a more prominent part in the formulation of a crack propagation law, instead of the use of the term ΔK as in equation (2.21). The effect of variation in K_m was also recognised. Thus a modified crack propagation equation was proposed:

$$\dot{a}_N = C K_{\max}^p \Delta K^q \quad (2.24)$$

From fatigue testing in tension and bending of Al-alloys (2024-T3) and (7075-T6). They found that $p = q = 2$.

The effect on $\dot{a}_N$ of variations in mean stress was investigated by Broek and Schijve (1963) using a parameter $R = \frac{\sigma_{\min}}{\sigma_{\max}}$; Forman et al. (1967) proposed a crack propagation equation which included the term R explicitly and also incorporated the conditions of fast fracture as $K_{\max}$ approached K_{IC} in a particular loading programme; thus:

$$\dot{a}_N = \frac{C(\Delta K)^m}{(1 - R) K_C - \Delta K} \quad (2.25)$$

C and m have to be determined empirically, for a material under specific loading conditions.

Both equations (2.24) and (2.25) have been examined by a number of investigators (e.g. Hudson, 1969 with data on high strength aluminium alloys and Crooker and Lange, 1968 with data on high strength steels), and reasonable success was achieved in fitting the empirical data to these models of crack growth; except for equation (2.24) when correlated with the test data at the high growth rates (above

5×10^{-3} in/cycle) in (7075-T6) aluminium alloy (Hudson 1969).

The influence of the load ratio R , was also examined by Pearson (1972), on six aluminium alloys of low and high fracture toughness and it was confirmed that the magnitude of $\dot{a}_N$ increases as the level of R is raised for alloys with relatively lower fracture toughness while R has a relatively smaller influence on $\dot{a}_N$ in high fracture toughness alloys.

Equation (2.25) was modified by Roberts and Kibler (1971a) such that a term called the effective stress intensity factor, K_e , was introduced. Denoting the stress intensity factors in tension and bending as K_t and K_b , K_e was defined as:

$$K_e = K_t \text{ for plane extension}$$

$$K_e = K_b/2 \text{ for bending}$$

$$K_e = K_t + (K_b/2) \text{ for combined loading and extension.}$$

And thus the modified equation took the form:

$$\dot{a}_N = \frac{C(1+D)(\Delta K_e)^2}{K_C - (1+D)\Delta K_e} \quad (2.26)$$

where C and D are constants to be determined empirically.

One difficulty which may arise in the application of equation (2.25 or 2.26) for rate sensitive materials at a particular loading frequency, is the availability of data on parameter K_C (or K_{IC}) as a function of load rate; as K_{IC} may undergo substantial changes when loading rate is varied. In some metals, when the data over a range of frequencies are being analysed, an average value for K_{IC} parameter must be used (Bucci, 1970).

The numerical parameters C and m in the above crack propagation laws have to be accurately determined for each material under specific conditions. There have been a number of attempts to analyse the nature of dependency of these para-

meters on material properties (e.g. Yokobori, 1969, 1971). McLintock (1966) showed that C is a function of the ultimate strength, the Young's modulus and the fracture strain. However, as yet there is no generally acceptable analytical method of determination of this parameter.

As far as the power law index m is concerned, a considerable amount of test data on various metals point to a value of $m \approx 4$. However, it is also recognised that the value of m may change considerably as a function of variations in loading conditions such as frequency (for rate sensitive materials), environmental conditions and also the range of applied stress. Barsom (1971a) discussed some fatigue data on steel in which a logarithmic plot of $\dot{a}_N$ against ΔK , yielded three distinct regions of varying slope, m . The middle range was the region for which most of the usual fatigue data are obtained. It was also proposed that the rise in m , under specified loading conditions, was due to the combined contributions of a ductile tear and a cyclic damage type of growth. It was further indicated that the onset of the transition in most metals is associated with the level of COD reaching a value of approximately 1.68×10^{-3} inch. The transition of growth rates has also been observed by Clark (1968) and Pearson (1972). The value of index m will also change if there is a significant variation in the degree of ductility of a material; for example, if the material exhibits a high degree of brittleness under higher load rates (or loading frequencies), m will have a higher value, Miller (1968).

(B) Crack tip plastic zone size and the COD as criteria for crack propagation:

Crack propagation models based on the stress intensity factor variation do not, in general, throw much light on the nature of the mechanism responsible for the propagation of fatigue cracks. There have been a few attempts to relate the rate of crack propagation to the crack tip plastic deformation process - as a consequence of which the fatigue crack propagates - (Paris, 1963; McEvily et al., 1963; von

Jacoby and Cramer, 1968). Rice's (1967) model of plastic yielding and Johnson and Paris's (1968) analysis of the deformation zone, as it passes through a cycle of loading and unloading, resulted in the following relationship for the determination of the extent of deformation:

$$w_{dd} = \frac{(\Delta K)^2}{2\pi(2\sigma_{yp})^2} \quad (2.27)$$

This relationship was subsequently presented as a justification for the use of the crack growth model, equation (2.21). However, the extent of accurate information on the nature of the cyclic crack tip deformation is at present very limited and hence it has not been possible to use, in an explicit and direct form, the crack tip plastic zone size as a criterion for prediction of crack growth rate.

Closely related to the plasticity aspects of crack tip stress field, is the concept of crack opening displacement (COD). It is now possible to measure this parameter fairly accurately and hence a number of attempts have been made to develop a criterion for cyclic crack growth rate prediction based upon the COD parameter, denoted by δ ; McLintock and Pelloux (1968); Hall and Shah (1971) and Dover (1972). The crack growth equation used was of the form:

$$\dot{a}_N = C \delta^m \quad (2.28)$$

Use of the plastic zone and COD in empirical models of fatigue crack propagation will be further discussed in Chapter 6.

2.3.2 Some comments on the crack growth initiation process :

The concepts of LEFM have in general been applied to the characterisation of the process of crack propagation and thus comparatively little effort has been devoted to the analysis of the crack growth initiation process. It is thought desirable to con-

sider at this stage, some of the parameters which influence the cyclic initiation life, N_i , as an aid to the discussion, in Chapters 4 and 6, of the proposed theoretical method for prediction of N_i .

Nakozawa et al. (1969) studies on carbon steel (S35C) and Jack and Price (1970) analysis, showed a decrease in the value of N_i as the level of the applied mean stress was raised. Frost and Dugdale (1958) performed similar studies on non-propagating cracks in mild steel; and Bilby and Heald (1968) used a plastic relaxation model to develop an expression relating N_i term to the level of stress intensity factor.

Nemec (1970) approach in developing the notion that the first stage of failure process occurs in the thin surface layer of the body, resulted in a relationship of the form:

$$N_i \propto \frac{\sigma_{yp}}{E\sqrt{q}} \phi(\sigma_{ef}) \quad (2.29)$$

where q is a measure of average grain size and σ_{ef} is given by the local effective cyclic stressing in the notch root region. The higher the value of σ_{ef} , the lower the value of N_i will be.

$\phi(\sigma_{ef})$ in the above equation will include the effects of such elements as the notch root radius and the quality of surface.

Jack & Price (1970) results on mild steel showed that, at a particular level of applied stress, N_i was independent of crack tip radius, ρ , up to a value of $\rho \approx 0.01$ in (≈ 0.25 mm). This limiting value of ρ was found to be independent of crack length (or notch depth), a , and the stress level. Data from tests with notch root radius greater than the above value, indicated an increase in N_i as ρ increased, the approximate relationship being $N_i \propto \rho^2$. A plot of N_i against ΔK values used in the above report for specimens of various width and

thickness values, resulted in a linear distribution with a slope approximately equal to $-1/4$, indicating that for the range studied:

$$N_i \propto \frac{1}{(\Delta K)^4} \quad (2.31)$$

Unfortunately there is no other information of similar nature available at present and hence further examination of a relationship such as equation (2.31) is not possible. One difficulty in obtaining data on initiation of crack growth has been the problem associated with the accurate measurement of the crack length and the onset of growth.

It is pertinent at this point to make a brief comment on the possibility of the presence of loading situations in which N_i acquires an infinite value, i.e. a 'threshold' level for the applied stress intensity factor. In corrosion fatigue tests such a threshold does exist, denoted by K_{ISCC} (Barsom 1971b, Bucci 1970). The case of fatigue in air is discussed by Johnson and Paris (1968) and it is suggested that for some materials no threshold value can be found. However, Tetelman and McEvily (1967) argued that it is more appropriate to use equation (2.21) in a modified form, such as:

$$\dot{a}_N = C (K_{\max} - K_o)^m \quad (2.32)$$

where K_o is the 'threshold' stress intensity below which no growth will occur.

Adams (1970) observed a threshold value in tests of high strength steels. Similar conclusions have been drawn by Weibull (1963) and Klesnil and Lukas (1972).

2.4 Application of Empirical Laws to Fatigue Crack Propagation in Polymeric Materials

Development of empirical models for cyclic failure of polymeric materials has progressed along two conceptually similar, although mathematically different, paths. One approach which has initially, and mainly, been used in the study of elastomers is based on an energy concept; the other has been the direct application of empirical models discussed in the previous section, using the concept of stress intensity factor.

The development of a modified form of Griffith's (1921) fracture energy criterion as the tearing energy concept in rubbers (Rivlin and Thomas 1953) in which the strain energy density within the body and the length of cracks are used as the main parameters influencing the failure mechanism, have been successfully extended to the case of cyclic failure in elastomers (Gent et al., 1964; Lake and Lindley, 1964). Empirical data has shown that the cyclic rate of crack growth could be related to the tearing energy, $\mathcal{T}$, through a relationship of the form:

$$\dot{a}_N = B \mathcal{T}^n \quad (2.33)$$

where $\mathcal{T} = \phi(\text{strain energy density and cut length})$, B is the crack growth constant; and n was found to be equal to 4 in tests on SBR but had a value of 2 for natural rubbers. Investigations also showed that B was an inverse function of the loading frequency; $\dot{a}_N$ decreased with increasing frequency.

Concepts such as the stress intensity factor, which is based on the assumption of linear elastic behaviour, can not be applied to elastomers, or indeed to any other polymeric material which behaves in a highly non-linear manner. Andrews and Walker's (1971) study of fatigue crack growth in polyethylene is thus based upon a similar criterion as equation (2.33). It is interesting that in the logarithmic plots

of $\dot{a}_N$ against $\mathcal{T}$, from data on polyethylene, regions of varying slope were observed (similar to the case of metals discussed previously).

Andrews and Walker (1971) attribute the transition region in PE data to a change from quasi-brittle to ductile fracture. The exact position of the transition region was said to be a function of stress and temperature. Waters (1966) also used equation (2.33) to study the process of cyclic crack propagation in a brittle thermoplastic, PMMA. However, experimental results of Watts and Burns (1967) and Borduas et al. (1968) showed that the LEFM concepts could successfully be applied to the study of crack growth process in PMMA. These two reports and also that of Mukherjee et al. (1969) which deals with PMMA and PC were devoted to the study of the applicability to these materials of the crack propagation law, equation (2.21).

Tests were carried out under constant cyclic stress intensity factor conditions, in zero-tension loading. Both Borduas et al. (1968) and Mukherjee et al. (1969) tested specimens of different thicknesses and found no deviation from the behaviour predicted by equation (2.21). Mukherjee found the value of index m in this equation, to be approximately equal to 5.1 for PC and 3.6 → 4.6 for PMMA. These tests were performed at a loading frequency of 0.5 Hz. von Jacoby and Cramer (1968) in a study of fatigue failure of polycarbonate also showed that a growth model in the form of equation (2.21) could be used in characterisation of the fatigue failure process.

Studies of similar nature (i.e. the attempt to relate the macroscopic cyclic crack growth rate to the stress intensity factor) was carried out on Nylon 6.6 by Hertzberg et al. (1970) whose work also included investigation of the behaviour of PMMA, PC and low density polyethylene.

From the foregoing discussion it is evident that fatigue failure studies of polymeric materials, using the LEFM principles, have been carried out in a limited and basically exploratory manner. However, it has been shown that at least for some polymeric solids, the LEFM concepts can be applied as useful tools in the analysis and characterisation of the fatigue failure process. The experimental research programme, to be discussed in Chapters 5 and 6, has been aimed at the study of the effects of parameters such as the range and the mean level of the stress intensity factor (denoted by ΔK and K_m) as well as frequency and temperature, so as to enable the development of a model for the prediction of the cyclic rate of crack propagation in thermoplastic materials.

CHAPTER THREE

STRUCTURAL PROPERTIES OF POLYMERS AND THEIR INFLUENCE ON THE FATIGUE FAILURE PROCESS

In this chapter some aspects of theories of linear viscoelasticity relevant to cyclic loading process are considered. The importance of identification of the degree of non-linearity in mechanical properties of polymers and the effects of relaxation and creep processes are discussed. The physical nature of the cyclic hysteresis energy and its dependence on loading frequency and temperature are described. The role of molecular parameters, such as the degree of crystallinity and orientation, in determining the mechanical behaviour of polymers is indicated.

3.1 The Extent of Linearity in Mechanical Behaviour of Viscoelastic Materials :

In an earlier discussion (Chapter 2), it was indicated that the approximate limits of linearity in mechanical behaviour of viscoelastic material must be identified prior to the application of methods of stress analysis based upon linear elasticity theories.

Linear viscoelasticity implies that, at any instant, the magnitude of a time dependent response is directly proportional to the magnitude of the applied time dependent input. Linear viscoelasticity is a typical property of the polymeric materials which have constant structural formation within the time span of a loading operation; and the non-linear behaviour, observed at higher stress and deformation levels, is associated with structural changes in polymers.

For linear materials, it is sufficient to specify the mechanical response of the material by defining a parameter such as the creep compliance function and the

linear relationship between the stress and strain permits the prediction of the behaviour of the material under any type of external loading pattern through the application of the Boltzman superposition principle (Ferry, 1970). Such a superposition technique cannot be utilised in characterisation of non-linear behaviour (Sharma and Sen, 1965).

In the present study the limits of loading time and stress level below which approximate linear behaviour may be assumed, were identified from the consideration of isochronous and isometric data at room temperature (Ogorkiewicz, 1971); see Table (1).

3.2 Relaxation and Creep Behaviour – Determination of the Glassy and the Rubbery Moduli

The constitutive equations for viscoelastic solids are formulated in terms of functions representing the cumulative influence of time. The stress relaxation and creep phenomena in these materials are described by time dependent moduli relating the instantaneous value of strain to the stress level. Ferry (1970) and Williams (1964) provide excellent discussions of the time dependent mechanisms and their specific role in influencing the mechanical behaviour of polymeric solids.

As an accurate realisation of the phenomenological aspects of the behaviour of polymers under cyclic loading conditions requires a knowledge of such parameters as the relaxation modulus and the frequency dependent loss modulus, etc., the specific characteristics of such elements will be briefly discussed.

The "mechanical" modulus of polymers is defined as the complex sum of a storage component, E' , and a loss component, E'' . E' being controlled mainly by the motion of the molecular chain backbone and E'' by the movement of the chain

side groups. Hence the complex modulus is defined as

$$E^* = E' + iE'' \quad (3.1)$$

There have been numerous attempts to investigate the variation in the magnitudes of E' and E'' as a function of changes in time and temperature, Ferry (1970) and Williams et al. (1955). In studies of the mechanical behaviour it is essential that this type of information is available.

The relaxation spectra for these materials have been described through analytical formulations which include the glassy (or short time) value, E_g , and the rubbery (or long time) value, E_e . The glassy modulus is, thus, associated with loading situations in which the effects of viscosity are largely suppressed.

The specific form of the functions characterising the relaxation and creep spectra depends on the assumed structural model for the viscoelastic material. For example, for a three element model (spring and dashpot in parallel, in series with another spring) also called the single relaxation time model, the time dependent modulus is defined as:

$$E(t) = E_g - \frac{E_g - E_e}{(1 + \frac{t}{\tau})^n} \quad (3.2)$$

and the creep compliance function is:

$$D(t) = D_e + \frac{D_e - D_g}{(1 + \frac{t}{\tau})^n} \quad (3.3)$$

where τ is the relaxation time and n is obtained from the slope of $D(t)$ (or $E(t)$) against time graph.

Schapery (1964) has proposed a more general formulation of the above equations, which thus is independent of the specific choice of the basic model used for material characterisation:

$$E_{(t)} = E_e + \sum E_i e^{-\frac{t}{\tau_i}} \quad (3.4)$$

and

$$D_{(t)} = D_e - \sum D_i e^{-\frac{t}{\tau_i}} \quad (3.5)$$

For the case of a simple, single relaxation time, model

$$D_i = D_e - D_g \quad (3.6)$$

and thus

$$D_{(t)} = D_e - \frac{D_e - D_g}{e^{t/\tau}} \quad (3.7)$$

These simplified equations will be used in the theoretical analysis of the failure process, discussed in Chapter 4.

For most thermoplastic materials, room temperature values of E_g and E_e parameters have not yet been determined. The dearth of such data is mainly due to the inherent difficulty of obtaining empirical data at the two extremes of the experimental time scale. E_g values have to be determined from measurements at very short times (much lower than 0.1 second) so as to prohibit the experience of the effect of viscous deformation of the material; and accurate measurement of the E_e values may require years as the appropriate length of the experimentation period.

A solution to this practical problem seems to be the use of time-temperature superposition principle. However, the use of such methods should be combined with a check on the possibility of performing a time-temperature shift, Williams et al. (1955), for a specific material. In contrast to elastomers, it has not been possible to carry out this type of analysis on some thermoplastic materials due to presence of non-linearity in mechanical behaviour, Ferry (1970). However, it is clear that through such studies, it will be possible to obtain good approximations to

the values of E_g and E_e parameters; see also Chapter 5 in which a method for the measurement of E_g parameter is fully described. Briefly, in order to obtain the value of E_g ($\approx \frac{1}{D_g}$) a test programme was devised to measure this modulus by performing a range of deformation tests at a temperature of -197°C . Through this approach it was hoped that the viscous elements would be "frozen" and hence a value for the modulus associated with negligible viscosity (or short times) would be obtained.

3.3 Cyclic Viscous Energy and Its Dependence on Frequency and Temperature:

Under cyclic loading conditions, a part of the total input energy is dissipated in viscous deformation. The magnitude of this 'lost' energy, W_v , is dependent upon the specific value of E'' , the loss component of the complex modulus, which itself is a function of disturbance frequency and temperature of the environment. Calculation of W_v in terms of loading parameters is important as: (a) it influences the magnitude of the energy available for crack propagation, and (b) it is a measure of the internal temperature rise due to viscous deformation. Williams (1964) has provided a solution for determination of W_v in terms of the ratio E''/E' (usually denoted by $\tan \delta$, called the loss tangent). The following analysis is provided so as to demonstrate the effect of the mean stress, σ_m :

Let the stress cycle have the form:

$$\sigma = \sigma_m + \sigma_a \sin \omega t \quad (3.8)$$

where σ_m and σ_a are the mean level and the amplitude of the applied stress cycle and ω is the circular frequency.

For a unit volume the viscous energy W_v , absorbed per cycle of load application, will be

$$W_v = \int_0^{T^*} \sigma'' \dot{e}'' dt \quad (3.9)$$

where T^* is the periodic time and $\dot{e}$ is the strain rate; σ'' and $\dot{e}''$ are the quadrature components of σ and $\dot{e}$.

$$T^* = \frac{2\pi}{\omega} = \frac{1}{f}$$

and

$$\sigma'' = E'' e' \quad (3.10)$$

also

$$\dot{e}'' = \frac{\omega \sigma'}{E'} = \omega e' \quad (3.11)$$

Thus:

$$W_v = \frac{\omega E''}{E'^2 + E''^2} \int_0^{\frac{2\pi}{\omega}} \sigma^2 dt \quad (3.12)$$

or

$$W_v = \frac{\omega E''}{E'^2 + E''^2} \int_0^{\frac{2\pi}{\omega}} [\sigma_m^2 + \sigma_a^2 \sin^2 \omega t] dt$$

and, from equation (3.1)

$$W_v = \frac{\pi E''}{E'^2 + E''^2} [2\sigma_m^2 + \sigma_a^2] \text{ per cycle} \quad (3.13)$$

(A) The effect of frequency:

Equation (3.12) shows the interdependence of W_v and E'' terms. However, in order to realise the effect of a change in frequency on the magnitude of W_v , it is necessary to study the nature of variation in E'' as a function of frequency. E'' has a low value at the two extremes of the frequency range, a behaviour for which the following explanation in terms of the molecular kinetics concepts may be offered.

When the frequency is low, the retarded elasticity component of deformation can fully develop within the time scale of stressing and the static loading conditions will be approximated; hence E' and E'' parameters will both have small values.

However, when the frequency is increased to an intermediate level (say $\omega \approx \frac{1}{\tau} = \omega_m$,

where τ is the relaxation time and ω_m is the value of frequency at which maximum relaxation occurs) the polymer chains do not have time to uncoil to their equilibrium state during the period of stress application. This means that the amplitude of deformation will be decreased. Thus if failure is to occur at a certain critical level of strain accumulation, then at higher frequencies, the number of load cycles required for the development of a critical strain will be larger than that for low frequencies (Williams, 1967).

Evidently, E'' peaks can be established for a material at various temperatures. Depending on which side of the maximum point (corresponding to the so-called resonance frequency level) one chooses a test range, the effect of frequency variation on the term W_v , and hence on the level of available energy for crack extension, will be different.

Assuming that the tests are carried out to the left of the E'' peak, for example, increasing frequency will result in a gradual increase in E'' and hence a higher value of W_v will prevail.

To summarise, the energy loss occurs if the circumstances are such that all input elastic strain energy can not be recovered. At low frequencies because of the possibility for molecular chains to rearrange faster than the stress variation, the strain energy is almost completely recovered (Lord and Wetton, 1961); and at very high frequencies, due to the short loading time the viscous deformation (molecular rearrangement) can not take place in an extensive manner, and the material behaviour will approach the elastic conditions. In both cases the energy loss (and E'') will have a small value.

Finally, in real materials more than one peak may be observed in $E'' - f$ distributions. The discussion in the following section will deal with the specific

nature of these maximum loss situations.

(B) The effect of temperature:

As a direct consequence of the temperature dependence of mechanical behaviour, it is essential that the influence of temperature variations on E'' and ω_m parameters be established prior to the application of a relationship such as equation (3.13).

It has been shown that an increase in temperature causes a displacement of the loss modulus maxima towards a higher frequency (Ferry, 1970), confirming the relaxation origin of these peaks. Thus the effect of temperature may be simply exemplified by its influence on relaxation time, τ .

From the transition state theory, for small strains:

$$\tau = \frac{1}{\omega_m} = C \exp(U_0/RT) \quad (3.13)$$

where U_0 is the activation energy

R is the gas constant

T is the absolute temperature

and C is equal to: $\frac{\text{Plank's Constant}}{2(\text{Boltzman's Constant} \times \text{Absolute temperature})}$

Parameter C may be treated as a constant over a certain range of temperature.

Thus, τ will be large at low temperatures and it will have a small value at high temperatures. The arguments presented in the preceding section in terms of frequency dependence of E'' and E' can be rephrased for the description of the effect of temperature. With the same reasoning, E'' will have a small value under the temperature conditions in which the molecular chain mobility is highly restricted.

The existence of local mobility in molecular chains has been confirmed from the empirical studies of the dynamic properties of polymers. It has been shown that

several maxima are obtained in the temperature-loss modulus distributions. These maxima, caused by the absorption of energy during the rotation of small kinetic units, are called the α , β , γ , δ etc. peaks in a descending order of temperature level. The α transition occurs at very high temperatures and it is associated with crystallite melting. The β transition occurs around the glass transition temperature region; hence the cyclic loading studies performed at room temperature (such as the one discussed in Chapter 5) are associated with γ and δ peaks. It is thus essential to ascertain the frequency levels at which the peaks occur in order to be able to determine the position of a specific range of test frequency and hence predict the expected trend of variation of E'' with frequency.

(C) The influence of molecular parameters:

Experimental studies of the parameters associated with the chemical structure of polymers have shown that the mechanical behaviour of these materials may be significantly influenced by the variations in parameters such as the molecular orientation, the degree of crystallinity, the molecular weight, the monomer friction coefficient, etc. Williams and Kelly (1970) have provided a useful summary of the data relating the effects of some of the parameters associated with the molecular structure, on the magnitudes of properties such as the loss and the storage moduli.

It is anticipated that a higher degree of molecular chains cross-linking and material crystallinity will result in a rise in the value of the complex modulus. The empirical data also indicate that the storage and loss moduli will be independent of the variations in the molecular weight if it exceeds a certain level for a specific material, (e.g. approximately 10^4 for polystyrene).

The anisotropy introduced as a result of molecular chains orientation will have obvious effects on the mechanical properties of a polymer. In an extruded sheet,

for example, there will be a large degree of orientation along the direction of extrusion and the material will exhibit a higher strength in that direction, Hsiao (1964). This effect has been studied under fatigue failure conditions and will be discussed in Chapters 5 and 6.

As a subsidiary part of the present studies, an attempt was made to develop a molecular rupture criterion for the prediction of failure of polymeric solids, so as to enable the qualitative analysis of the relative effects of the various parameters associated with the molecular bond dissociation process. This analysis is presented in Appendix (A).

CHAPTER FOUR

DEVELOPMENT OF AN ANALYTICAL SOLUTION FOR THE DETERMINATION OF THE CYCLIC INITIATION PERIOD IN VISCOELASTIC SOLIDS

4.1 Introduction

Closed form solutions for the problem of failure in polymeric materials under cyclic loading conditions are very rare. Indeed, only the contributions of Williams (1967) and Yokobori (1969) can be referred to in this context. Yokobori (1969) analysis was based on chemical structural properties of the material and utilised the dissociation rate theory (see also Appendix A); and Williams (1967) analysed the special case of the cyclic growth of a spherical flaw under repeated hydrostatic tension loading. The application of the former approach has been limited due to the complexity of the solution and also the scarcity of the required data on basic material properties. The latter approach was based upon the energy balance at the continuum level, and enables, at least, the direct calculation, for the special case treated, of the cumulative flaw growth without the semi empirical fitting of experimental data.

Andrews and Walker (1971) also, have proposed a solution based upon the magnitudes of the transformed energy components in cyclic loading, so as to substantiate the use of the tearing energy concept in the analysis of the fatigue failure process in polyethylene. However, this particular solution does not include the effects of such parameters as the relaxation modulus or the creep compliance of the material. For polymers, a fundamental analysis of the cyclic damage process must be explicitly in terms of the time dependent properties of the material. Hence, in the method of analysis presented here the effects of such properties will be considered.

The procedure is based upon the total energy balance approach to failure, proposed by Cherepanov (1967, 1968) dealing initially with the conditions of monotonic fracture.

Cherepanov's approach to the analysis of fracture of solids is by far one of the most rigorous techniques available at present. It treats the failure phenomenon as a consequence of the achievement of limiting conditions arrived at through the disturbance of the balance between the internal energies of the system and the external energies imposed upon it. The only assumption, in relation to material characteristics, is continuity of the medium. Consequently it is possible to treat elastic-plastic or viscoelastic materials through the same general approach. Derivation by Cherepanov (1968) of the limiting conditions for fracture of viscoelastic solids will be briefly described for the sake of completeness. This analysis will then be combined with the contributions of Williams (1965, 1971) to develop a solution for the determination of the cyclic crack growth initiation life in viscoelastic solids.

4.2 A Summary of Cherepanov's Approach to Failure Analysis:

From consideration of the general case of a crack in a semi-infinite plate subjected to uniform loading at infinity, and assuming that in the near field of the crack tip some physical law relating the parameters of the medium and their fundamental time and spatial characteristics exists, Cherepanov (1968) formulated the conditions of total energy balance in a small region at the crack tip:

$$\dot{A} + \dot{Q} = \dot{E}_k + \dot{W} + \dot{\Pi} \quad (4.1)$$

where

$\dot{A}$ is the work of body surface tractions on the boundary of the local zone

Q is the contribution of the input thermal energy

E_k is the kinetic energy term

W is the internal energy

and Π is the energy for surface extension

(Superfix 'dot' signifies differentiation with respect to time).

Subsequently, the conditions of criticality were derived for a fixed crack in an elastic medium which, neglecting the contributions of the kinetic energy term (for a stable crack) and the body forces, takes the form:

$$R \int_{-\pi}^{+\pi} (B_* \cos \theta - A_*) d\theta = 2\gamma^* \quad (4.2)$$

where

$$B_* = \int \sigma_x d\epsilon_x + \sigma_y d\epsilon_y + 2\tau_{xy} d\epsilon_{xy} \quad (4.3)$$

and

$$A_* = (\sigma_x \cos \theta + \tau_{xx} \sin \theta) \frac{\partial u}{\partial x} + (\sigma_y \sin \theta + \tau_{xy} \cos \theta) \frac{\partial v}{\partial x} \quad (4.4)$$

γ^* is the surface energy term and R is the radius of the small zone under consideration.

The equation (4.2) above was then suitably modified so as to be applicable to the specific case of solid bodies with time dependent mechanical properties. In this type of materials, the stress and strain values will be related through cumulative time operators.

Thus by substitution of the time dependent parameters and also converting the stress and crack length terms into the stress intensity parameter, K , the following equation, representing the criticality conditions in viscoelastic solids was obtained:

$$\pi \int K \frac{1 - \nu(t)}{E(t)} \dot{K} dt + \frac{\pi}{2} \frac{K}{E(t)} [3 + \nu(t)] K = 2\gamma^* \quad (4.5)$$

For which a limit check, as $t \rightarrow 0$, yields:

$$E \gamma^* = \pi K^2 \quad (4.6)$$

i.e. Irwin's limiting conditions for brittle cracks in plane stress.

The stress intensity factor in the above equation can be related to an external loading system of arbitrary form (step, ramp, sinusoidal, etc.). In the present case a sinusoidal stress function will be used.

4.3 Determination of the Cyclic Initiation Life, N_i :

4.3.1 The crack growth initiation process:

In the presence of a flaw in a material, the application of a stress cycle of appropriate level will result in an extension of the flaw. In many cases, however, if the flaw is small enough and the maximum applied stress is below a certain level, the component may endure an excessively large number of applied load cycles prior to the commencement of the macroscopic crack propagation process (N_i may have an infinite value in some cases - the concept of fatigue endurance).

If, however, the stress intensity factor (combining the initial flaw size and the value of the applied load) is higher than a specified level, after the application of a finite number of load cycles, N_i , the crack propagation process will be initiated.

A possible criterion for the endurance of N_i cycles prior to the initiation of growth may be expressed as follows:

Viscoelastic deformation is not in phase and lags behind the applied stress.

For a linear material the strain will follow the stress but with a phase angle. Due to the concentration of stress in the region of a discontinuity in the material, plastic flow will take place which may induce non-linearities in the mechanical response of

the local material. The plastic flow will cause a permanent separation of the crack faces, and the magnitude of this separation will increase gradually according to a damage accumulation law which, as will be shown, may be formulated in terms of the stress intensity factor parameter.

The extent of the permanent deformation continues to rise until the achievement of a critical level, when the deformed material at the tip of the crack can no longer accommodate any further increases in flow and hence rupture results.

Williams (1967) showed that the rate of damage accumulation is a function of the applied loading frequency. A higher level of frequency results in a shorter initiation life.

4.3.2 A theoretical method for the determination of N_i :

The conditions of limiting stability for a crack in a viscoelastic solid, as derived by Cherepanov (1968) in the form of equation (4.5), can be extended to the case of a material subjected to an external disturbance of cyclic form, in order to determine the value of the N_i parameter in terms of the stress intensity factor and the material properties.

The following assumptions are made in order to simplify the analysis:

(a) The creep function is chosen to be of the form (equation 3.6):

$$D_{\text{crp}}(t) = D_e - \frac{D_e - D_g}{e^{xt}}$$

where $x = \frac{1}{\tau}$

D_e and D_g are the rubbery and the glassy values of the creep compliance function and τ is the material relaxation time.

(b) The case of an incompressible material with Poisson's ratio $\nu = \frac{1}{2}$ is considered.

(c) The value of fracture energy term, γ^* , is considered to be a constant.

Let the time dependent stress intensity factor be defined as:

$$K(t) = \sigma(t) \sqrt{\frac{a(t)}{2}} \quad (4.7)$$

For a crack length of a_0 under plane strain conditions, and using equations (3.7), (4.7) and the general equation (4.5), Williams (1965, 1971) derived the following relationship:

$$\sigma(t) \sqrt{\frac{a_0}{2}} \left[D_g \sigma(t) \sqrt{\frac{a_0}{2}} + \sqrt{\frac{a_0}{2}} \int_0^t \sigma(\xi) \frac{\partial D_{crp}(t-\xi)}{\partial (t-\xi)} d\xi \right] = \frac{4}{3\pi} \gamma^* \quad (4.8)$$

ξ being a time variable.

If the initiation of growth occurs at $t = t_f$:

$$D_g \sigma^2(t_f) + \sigma(t_f) \int_0^{t_f} \sigma(\xi) \frac{\partial D_{crp}(t_f - \xi)}{\partial (t_f - \xi)} d\xi = \frac{8}{3\pi} \frac{\gamma^*}{a_0} \quad (4.9)$$

In this equation, the stress function $\sigma(t)$ can take any form.

With the application of an external disturbance of the form:

$$\sigma(t) = \sigma_0 + \sigma_1 \sin \omega t \quad (4.10)$$

where σ_0 is the mean level and σ_1 is the amplitude of the stress cycle, equation (4.9) becomes:

$$\begin{aligned}
& D_g \left[\sigma_0^2 + \sigma_1^2 \sin^2 \omega t_f + 2\sigma_0\sigma_1 \sin \omega t_f \right] + \\
& \quad (I) \left[\sigma_0 + \sigma_1 \sin \omega t_f \right] \left[\sigma_0 \int_0^{t_f} x(D_e - D_g) e^{-x(t_f - \xi)} d\xi \right] + \\
& \quad (II) \int_0^{t_f} \sigma_1 \sin(\omega \xi) x (D_e - D_g) e^{-x(t_f - \xi)} d\xi = \frac{8}{3\pi} \frac{\gamma^*}{a_o} \quad (4.11)
\end{aligned}$$

In the above equation

$$(I) \equiv \sigma_0 x (D_e - D_g) \left[\frac{1}{x} e^{-x(t_f - \xi)} \right]_0^{t_f} = \sigma_0 (D_e - D_g) (1 - e^{-xt_f})$$

and

$$\begin{aligned}
(II) & \equiv \int_0^{t_f} \sigma_1 \sin(\omega \xi) x (D_e - D_g) e^{-x(t_f - \xi)} d\xi = \\
& \frac{\sigma_1 x (D_e - D_g)}{x^2 + \omega^2} \left[x \sin \omega t_f - \omega \cos \omega t_f - \omega e^{-xt_f} \right]
\end{aligned}$$

Thus (4.11) becomes:

$$\begin{aligned}
& D_g \left[\sigma_0^2 + \sigma_1^2 \sin^2 \omega t_f + 2\sigma_0\sigma_1 \sin \omega t_f \right] + \left[\sigma_0 + \sigma_1 \sin \omega t_f \right] \\
& \left\{ \sigma_0 (D_e - D_g) (1 - e^{-xt_f}) + \frac{\sigma_1 x (D_e - D_g)}{x^2 + \omega^2} \left[x \sin \omega t_f - \right. \right. \\
& \left. \left. \omega \cos \omega t_f - \omega e^{-xt_f} \right] \right\} = \frac{8}{3\pi} \frac{\gamma^*}{a_o} \quad (4.12)
\end{aligned}$$

A limit check for $t \rightarrow 0$ yields:

$$\sigma_0^2 = \frac{8}{3\pi} \frac{\gamma^* E_g}{a_o} \quad (4.13)$$

i.e., Irwin's critical stability conditions for fracture in plane-strain, which is given

as $K^2 = \frac{E \gamma^*}{(1-\nu^2)}$, with the use of assumption (b) above and equation (4.7).

Assuming that the time t_f corresponds to the completion of N_i load cycles,

i.e.,

$$\omega t_f = 2 \pi N_i \quad (4.14)$$

Equation (4.12) becomes:

$$D_g \sigma_0^2 + \sigma_0 \left[\sigma_0 (D_e - D_g) (1 - e^{-\frac{x}{\omega} (2\pi N_i)}) - \frac{\sigma_1 x (D_e - D_g)}{x^2 + \omega^2} (\omega + e^{-\frac{x}{\omega} (2\pi N_i)}) \right] = \frac{8}{3\pi} \frac{\gamma^*}{a_o} \quad (4.15)$$

Thus:

$$D_g \sigma_0^2 + \sigma_0^2 (D_e - D_g) (1 - e^{-\frac{x}{\omega} (2\pi N_i)}) - \frac{\sigma_0 \sigma_1}{x^2 + \omega^2} x (D_e - D_g) \left[\omega + e^{-\frac{x}{\omega} (2\pi N_i)} \right] = \frac{8}{3\pi} \frac{\gamma^*}{a_o} \quad (4.16)$$

This equation represents the general conditions of limiting stability corresponding to a situation in which N_i cycles have been applied.

For the special conditions of minimum stress $= 0$ (i.e. $\sigma_0 = \sigma_1$) and also assuming that the glassy value of the creep compliance is much smaller than the rubbery value (i.e. $D_g \ll D_e$), equation (4.16) becomes:

$$\sigma_0^2 D_e \left[1 - e^{-\frac{x}{\omega} (2\pi N_i)} \right] - \frac{\sigma_0^2 x}{x^2 + \omega^2} D_e \left[\omega + e^{-\frac{x}{\omega} (2\pi N_i)} \right] = \frac{8}{3\pi} \frac{\gamma^*}{a_o} \quad (4.17)$$

The above equation can be solved in terms of N_i :

$$N_i = \frac{\omega}{2\pi x} \ln (\Delta) \quad (4.18)$$

where

$$\Delta = \frac{3\pi\sigma_0^2 a_o D_e (x^2 + \omega^2 + x)}{3\pi\sigma_0^2 a_o D_e (x^2 + \omega^2 - \omega x) - 8\gamma^* (x^2 + \omega^2)}$$

From equation (4.18) an "endurance limit" may be determined ($N_i = \infty$):

$N_i \rightarrow \infty$ as $\Delta \rightarrow \infty$; for which

$$3\pi\sigma_0^2 a_o D_e (x^2 + \omega^2 - \omega x) - 8\gamma^* (x^2 + \omega^2) = 0$$

i.e.

$$\sigma_0^2 = \frac{8\gamma^*}{3\pi a_o D_e} \left[\frac{x^2 + \omega^2}{x^2 + \omega^2 - \omega x} \right] \quad (4.19)$$

And in terms of the stress intensity factor, using equation (4.7)

$$K_o = \frac{4\gamma^*}{3\pi D_e} \left[\frac{x^2 + \omega^2}{x^2 + \omega^2 - \omega x} \right] \quad (4.20)$$

Hence for a certain material with specified values of D_e , γ^* and x , subjected to a loading frequency of ω , the cyclic crack extension will be avoided if the level of the stress intensity factor is chosen to be less than that of K_o .

The above results will be compared with some experimental data in Chapter 6.

4.4 Analysis of the crack propagation process:

An attempt was made to extend the above analysis to the study of the process of cyclic crack propagation. However, the outcome of this second stage in the theoretical characterisation of the fatigue failure process in viscoelastic solids indicated that in the absence of some modifications to the assumed (simplified) energy conservation relationships, equation (4.2), an accurate formulation of the behaviour of cyclically propagating cracks will be impossible.

It is thought that the inclusion of the kinetic energy term and also the consideration of thermo-mechanical coupling processes will be steps in the right direction. See also Chapter 6.

CHAPTER FIVE

THE EMPIRICAL ANALYSIS

Experimental studies under discussion in this section, were devised with the aim of investigating the specific roles of a number of parameters which may have a marked influence upon the fatigue crack propagation process. These parameters were chosen through consideration of the fatigue failure process in the light of the discussions presented in Chapters 1 to 4. The analysis has been carried out in terms of the linear elastic fracture mechanics concepts.

5.1 Introduction to the Test Programme:

The discussion of fatigue failure process in polymers, presented in previous sections, was aimed mainly at the exposition of some of the parameters which significantly influence the fatigue life (both the initiation and propagation stages) in these materials. The experimental programme in the present study has been designed so as to provide data which will identify the exact role of some of the most important of such parameters.

The influence of the amplitude of variation and the mean level of stress intensity factor (ΔK and K_m) and the loading frequency on four thermoplastic materials with different structural characteristics were investigated. The effect of variations in the environmental temperature, with its implicit influence on such parameters as the E'' peak frequency and the value of K_{IC} for each material were examined.

The effects on the cyclic crack growth rate of variations in the load cycle waveform and molecular orientation in the material have also been explored.

The programme of empirical studies was divided into the following test series:

- Series 1 Fracture energy (and K_{IC}) measurements using constant compliance (contoured triangular) specimens.
- Series 2 Measurement of the temperature dependency of the instantaneous modulus and determination of an approximate value for the glassy modulus, E_g .
- Series 3 Fatigue crack propagation in PMMA; study of the effects of ΔK , K_m and frequency.
- Series 4 Fatigue crack propagation in PC; study of the effects of ΔK , K_m , frequency and the cyclic waveform.
- Series 5 Fatigue crack propagation in Nylon 6.6; study of the effects of ΔK , K_m , frequency and molecular orientation.
- Series 6 Fatigue crack propagation in PA copolymer; study of the effects of ΔK and K_m .
- Series 7 Study of the effect of environmental temperature variation on fatigue crack growth rate in PMMA and Nylon 6.6.
- Series 8 Determination of crack growth path under complex loading conditions. Fatigue crack growth studies in PMMA under biaxial loading conditions; effect of crack orientation on cyclic initiation life.

In Chapters 5 and 6 test series 1 to 7 will be discussed. Series 8 will be described in Chapter 7.

5.2 Fracture Energy (and K_{IC}) Measurement Using Constant Compliance

Specimens

(Test Series 1):

The aim of this series was two-fold: 1) To explore the possibility of use of a triangular crack-line-loaded specimen which, under constant applied loading conditions, would have the property of maintaining a constant level of the stress intensity factor irrespective of the crack length. Such specimens have been used in fracture studies of metals (Srawley and Gross 1967; Mostovoy et al. 1967 and Radon et al., 1971). Their use in fatigue failure studies would significantly simplify the process of control of the applied load limits in order to maintain specific stress intensity factor levels as the crack propagates. And 2) to attempt to measure the fracture energy and hence the value of the critical stress intensity factor for the PMMA material, under certain cross-head speed conditions. The knowledge of the K_{IC} parameter associated with certain strain rates will be useful in determination of the maximum level of the stress intensity factor to be applied in fatigue tests.

5.2.1 Design and selection of the specimens:

If a specimen is so designed that its compliance, C , changes linearly with crack length, a , then $\frac{dC}{da}$ will have a constant value. Irwin and Kies (1954) relationship between the energy release rate in crack extension

$$G_{IC} = \frac{P^2}{2 B_c} \frac{dC}{da} \quad (5.1)$$

where P is the applied load and B_c is the crack width; and

$$K_{IC}^2 = \frac{E}{1-\nu^2} G_{IC} \quad (\text{plane-strain}) \quad (5.2)$$

can then be used to calculate the value of G_{IC} , the critical level of the energy

release rate corresponding to unstable fracture. From the above equations it is clear that for a constant value of $\frac{dC}{da}$, the relationship between the G_{IC} or K_{IC} and the load P will be independent of the crack length.

Constant $\frac{dC}{da}$ specimens designed by Srawley and Gross (1967) and Mostovoy et al. (1967), Figure (3), were both tested in the present study. As shown in the figure, the specimens had surface V-grooves to control the crack growth direction. Square slots (0.006 inch wide and 0.33 inch deep) were added to the V-grooves at a later stage so as to improve the direction control. The cracks were initiated from a dovetail shaped stress raiser, Figure (3).

Preliminary tests with Mostovoy et al. (1967) specimens proved to be not very successful as the initial crack tended to deviate from the centre line (out of the groove) and hence premature fracture of one specimen arm resulted. It was decided that Srawley and Gross (1967) specimen with straight edges and its improved form with contoured edge (Radon et al., 1971) will be used in the present test series.

5.2.2 Experimental method and results:

The test procedure consisted mainly of the following steps:

A load-deflection (P v δ) plot at a certain crosshead speed was obtained from a test on an Instron testing machine. (Crosshead speeds of 0.05 and 0.1 inch/minute were used in these tests.) During the test the crack length was marked at regular intervals on the output chart. The compliance values $C = \frac{\delta}{P}$ (in/lbs) were then calculated at various points and the results were plotted in the form of a compliance against crack length graph. For a well-designed specimen this graph will be approximately linear and its slope will yield the parameter $\frac{dC}{da}$ in equation (5.1) above. In Table (2) a set of data obtained from one test are presented.

The above procedure was repeated in a number of tests at the two crosshead speeds of 0.05 and 0.1 in/min. The K_{IC} values calculated from these tests ranged from 1050 to 1200 psi $\sqrt{\text{in}}$; see, for example, the data in Table (2). This range of values for K_{IC} were in agreement with the data previously obtained (Borduas et al. (1968)). In the following sections, the value of $K_{IC} \approx 1100 \text{ psi}\sqrt{\text{in}}$ will be used as the value from whose fractions the maximum limits of the applied load cycles will be selected..

5.3 Measurement of Short-time (Glassy) Modulus, E_g : (Test Series 2):

5.3.1 Introduction:

The value of the modulus of elasticity parameter, E , used in the conversion of strain energy release rate to the stress intensity factor (see equation 5.2) is that pertaining to a linear elastic material. In polymeric solids this modulus is of a time dependent nature and hence in an application of the above equation to the dynamic loading conditions, the instantaneous (or short time) modulus of the material should be used. It is assumed that the value of this modulus will approximate the glassy value of $E(t)$ which is defined as E_g in the relaxation spectra. The value of E_g for most thermoplastics is not available in the literature as the normal test procedures adopted in relaxation studies would require measurements of load and strain at exceedingly short times and hence are often impractical. A range of room temperature relaxation data for PMMA, Nylon 6.6 and PA are given in (Ogorkiewicz, 1970). The shortest time interval used in obtaining these results was of the order of 1/100 of an hour and in general such data seemed to be far from reaching a plateau region in such spectra, expected to identify the approximate E_g levels, Ferry (1970). In

the present test series, it was proposed to make relaxation measurement tests in gradually decreasing environmental temperature conditions until the achievement of cryogenic temperature (i.e. from 21°C to -197°C). At the lowest temperature, the viscous behaviour of these materials is assumed to be largely restricted and hence the modulus measurements obtained under such conditions will approximate the value of E_g .

5.3.2 Specimen preparation and the testing apparatus:

All specimens used in this series of tests were in the form of bars of rectangular cross-section. The specimens had the dimensions of standard Charpy specimens ($10 \times 10 \times 55 \text{ mm}$), Figure (4a). They were cut from cast plates of PMMA and extruded sheets of PC, Nylon 6.6 and PA, in a direction perpendicular to that of extrusion.

A temperature controlled bath containing a support frame (Figure 4b) was mounted on an Instron testing machine and load-deflection readings were obtained using an output x-y plotter.

5.3.3 Methods of experimentation:

Compliances and moduli are usually determined by applying forces to a body and measuring the resulting deformation. In the present studies the specimens were placed lengthwise on the supports and the striker was lowered until it just made contact with the specimen. Tests were carried out at a constant crosshead speed of 0.2 in/min and measurements were taken after approximately 1 minute. The temperature range used was from $+20^{\circ}\text{C}$ to -197°C .

To achieve the conditions of cryogenic temperature, the bath was filled with liquid nitrogen. Other modulus measurements, at higher temperatures were carried

out in baths filled with various mixtures of liquid nitrogen and petroleum ether. Test readings were recorded after the specimen had been allowed to reach a steady temperature. Most tests were repeated 3-4 times so as to ascertain the repeatability of the results.

5.3.4 Discussion of results:

Data relating the changes in modulus to the value of environmental temperature for PC, PMMA, N6.6 and PA are given in Figures (5 and 6). In these figures similar data obtained for polyvinyl chloride (PVC) are also included so that a general comparison of the behaviour of these different materials may be made. The figures show the general trends of behaviour in the modulus/temperature relationships and one may assume that an approximate plateau region has been achieved in the data on PVC, PMMA and PC. However for PA and more prominently for N6.6 the plateau region is less pronounced. Indeed, the modulus of N6.6 seems to be still rising at a temperature of -197°C . The specific values of the moduli at -197°C and at 20°C are given in Table (3).

The room temperature data in this table were compared with relaxation data given in (Ogorkiewicz, 1970) and a reasonably good degree of correspondence was observed. Also, for N6.6 the value of modulus obtained at -197°C was compared to the data provided in the Cryogenic Materials Data Handbook (1968). The results of tests on round bars of 0.25 inch diameter were considered and excellent correlations were obtained, Figure (6).

5.4 Fatigue Crack Propagation Studies:

(Test Series 3-6):

5.4.1 Materials selection:

With the aim of testing thermoplastic materials of widely different structural properties, the following four materials were selected:

Polymethylmethacrylate (PMMA), on which most of the initial work on crack propagation studies has been carried out, was selected as the representative of quasi-brittle, glassy type thermoplastics with the additional aim of developing it as a model material to be used in studies of phenomenological behaviour of propagating cracks (see Chapter 7). PMMA (ICI Perspex) was supplied in the form of cast sheets of dimensions 4' x 6' of various thicknesses.

Polycarbonate of Bisphenol A (PC), is also an amorphous thermoplastic; however it was selected because of its relatively more ductile behaviour in medium rate fracture tests at room temperature. PC (Makrolon Bayer) was supplied in the form of extruded sheets of dimensions 12" x 48" x $\frac{1}{4}$ " inch thick.

Nylon 6.6 (N6.6) and polyacetal copolymer (PA) were chosen as representatives of semi-crystalline thermoplastics. These materials also exhibit enhanced ductility in fracture at low load rate at room temperature. N6.6 (ICI Maranyl) and PA (ICI Kematal) were supplied in the form of extruded sheets of dimensions 12" x 48" x $\frac{1}{4}$ " in thick.

The above materials differ from one another in their basic polymerisation processes. PMMA and PC are obtained through the addition polymerisation process; and N6.6 and PA are obtained through the condensation polymerisation process. The former group are of much higher molecular weights than the latter.

Uses of N6.6 (and PA as its replacement) in engineering applications are much more proliferate than those of PMMA and PC. This is due to the more favourable

mechanical and wear properties of N6.6 and PA under cyclic loading conditions. However it is essential that comprehensive data on such different materials be collected so that: (a) the effects of structural properties on failure strength of materials are identified; and (b) the analytical solutions to the failure problem, upon which no a priori limiting assumptions on structural characteristics of the material are imposed, can be fully examined.

5.4.2 Specimen design and preparation:

In the design of specimen configurations, general principles laid down for fracture and fatigue studies by the American Society for Testing and Materials were taken into consideration. These principles have been proposed in various ASTM publications; see for example ASTM STP 410. The following are some of the important fundamental points to be observed:

- (1) Point of load application must be remote from the region of crack extension.
- (2) The specimen dimensions must be sufficiently large so as to provide the necessary elastic constraint around the plastic zone developed at the crack tip prior and during the stable crack propagation.
- (3) The adequacy of the sharpness of the starting crack. This point is especially important in the light of the discussions in Section 2.3.2 on the effect of crack tip radius on initiation life in fatigue.

With the additional requirement of being able to obtain results over a reasonably long crack growth range (at least 2-3 inches) so that the accuracy of the determination of the rate of growth is enhanced, the following specimen dimensions were selected:

PMMA	Rectangular plate	$13\frac{1}{2} \times 15 \times \frac{1}{4}$ inch thick
PC	" "	$7 \times 12 \times \frac{1}{4}$ " "
N6.6	" "	$7 \times 12 \times \frac{1}{4}$ " "
PA	" "	$7 \times 12 \times \frac{1}{4}$ " "

PA, PC and N6.6 specimens were cut out of the extruded sheets with the 12 inch side perpendicular to the direction of extrusion.

A $\frac{1}{4}$ inch diameter hole was drilled at the centre of all specimens and an initial crack of length of 1 inch was produced in a direction perpendicular to that of the external loading; see Figure (7). At each end of the centre crack, a sharp notch was created by means of a razor blade which was placed at the root of the cut and forced into the specimen body. The length of this flaw was approximately 0.020 inch. Care was taken that the flaw was produced ahead of the razor blade tip.

Two rows of $\frac{1}{4}$ inch diameter, equally spaced, holes were drilled on the two sides of each specimen so as to enable the uniform application of the external load.

5.4.3 Methods of experimentation:

Fatigue crack growth studies in this series of tests were in general carried out under constant stress intensity factor conditions, at room temperature (21°C) and in air. Magnitudes of cyclic loads were gradually reduced as the crack grew in length so as to maintain the applied K_I levels at constant values. The increase in length on the two sides of the centre crack was monitored using a cathetometer.

The stress intensity factors were calculated initially using the following equation known as the tangent formula (Paris and Sih, 1965):

$$K_1 = \sigma (\pi a)^{\frac{1}{2}} \left[\frac{w}{\pi a} \tan \frac{\pi a}{w} \right]^{\frac{1}{2}} \quad (5.3)$$

At a later stage in experimental programme, the following improved equation derived by Brown and Srawley (1966) was used:

$$K_1 = \frac{PY}{wh} \sqrt{a} \quad (5.4)$$

where

$$Y = 1.77 + 0.277 Z - 0.510 Z^2 + 2.70 Z^3$$

and

$$Z = \frac{2a}{w}$$

In the above equations σ is the applied stress, P is the applied load, w and h are the specimen width and thickness and a is the half crack length.

Tables of K_1 , a and P appropriate to specific specimen geometries were produced using a computer programme. The load levels were chosen such that the maximum limit of the applied stress intensity factor did not exceed the level of 80% of the estimated value of K_{IC} for the material under test. In general the fatigue tests were carried out with K_{max} levels varying between 0.7 and 0.2 of the K_{IC} value. Corresponding maximum applied stress levels hence fell far below levels which would be associated with gross deformation. Similar arguments have been used in the preparation of data for tests on metals (Frost et al., 1971).

The degree of accuracy in the application of the above equation for parameter K_1 falls rapidly as the crack length exceeds 0.5 of specimen width, w . Hence, most tests were terminated when $2a$ reached a value of about $0.4w$.

Tests in which a grossly uneven crack front prevailed and also tests in which the crack on one side propagated much faster than the other - hence perpetuating the non-uniformity in the stress distribution - were abandoned. These defects were

usually present as a consequence of non-uniformity of the initial razor notch.

Results of tests were checked in a random manner. Deviations of up to 6 per cent were observed.

5.4.4 Description of the experimental apparatus:

Fatigue crack propagation tests were in the main carried out using the Dowty electrohydraulic fatigue testing machine. Only a section of 0.1 Hz frequency studies on PMMA were performed on a specially converted Denison T42C2 testing machine. Any possible discrepancy in results arising from the use of two machines was checked by performing similar tests on both; such discrepancies were found to be negligible. The Denison T42C2 testing machine was of limited capacity in terms of the loading frequency (maximum less than 1 Hz).

However, the Dowty electrohydraulic machine, which is comprehensively described by Knight (1965), was capable of cycling up to 100 Hz at loads ranging up to 12000 lbf in the available model. Various major components of the machine are shown in the circuit diagram, Figure (8). Briefly, a sine wave input (or a wave-form of any other kind), produced by a signal generator, passes through an amplifier and subsequently operates the actuator. The signal from a load cell is monitored by means of a graduated oscilloscope on the control console. There were facilities on the machine to vary the load whilst a test was in progress.

The specimens were bolted to a pair of steel holding plates and then placed between the load cell and the actuator.

5.4.5 Fatigue crack growth studies in polymethylmethacrylate: (Test Series 3):

Initial experimental studies on PMMA were performed in continuation of the studies of Watts and Burns (1967), Borduas et al. (1968) and Arad (1970) which were

basically aimed at the confirmation of the possible application of Paris's (1963) model of crack growth based upon $\Delta K = K_{\max} - K_{\min}$ parameter. Borduas et al. (1968) and Arad (1970) referred to the influence on the cyclic crack growth rate, $\dot{a}_N$, of the mean level of the stress and hence the stress intensity factor, K_m . Thus the K_m parameter was selected to be exhaustively studied.

In order to establish the possible limits of the external load cycle in these tests, preliminary monotonic fracture tests were performed to obtain a value for the critical stress intensity factor for this material (Section 5.2). K_{IC} values thus obtained ranged between 1100 and 1200 $\text{psi}\sqrt{\text{in}}$, in close agreement with the data of Borduas et al. (1968). The highest level of $K_{\max}$ used in the present series of tests was chosen at 900 $\text{psi}\sqrt{\text{in}}$ (see Section 5.4.3). Conditions of plane strain were assumed in all tests, as the plastic zone size, r_p , calculated from a relationship based upon equation (2.6) for the special case of yield under plane-strain conditions, with $K = 900 \text{ psi}\sqrt{\text{in}}$ and an approximate value of σ_{yp} chosen at 11000 psi was found to be equal to 0.35×10^{-3} inch. This value of r_p was considered to be sufficiently small when compared to the specimen thickness of 0.25 inch.

(A) The effect on $\dot{a}_N$ of the maximum and the minimum levels of the stress intensity factor:

Through a careful selection of the values of $K_{\max}$ and $K_{\min}$ for a wide range of tests, it was possible to establish crack growth rates over a large spectrum of values of K_m and ΔK parameters.

A typical set of results, obtained at a loading frequency of 5 Hz, showing the relationship between crack length and load cycles is given in Figure (9). A similar linear relationship was obtained under all testing conditions; and this is in

agreement with the previous work of Watts and Burns (1967), Borduas et al. (1968) and Mukherjee et al. (1969) concerning tests in which K_{min} was always maintained at zero.

From consideration of the various test results, it became immediately apparent that tests conducted with very similar ΔK ranges were associated with widely different crack growth rates. For instance, tests 1 and 4, having ΔK values equal to 440 and 450 $\text{psi}\sqrt{\text{in}}$, gave crack growth rates (obtained from the slopes of graphs such as those in Figure 9) of 13.8×10^{-6} and 3.14×10^{-6} in/cycle respectively. Furthermore, a comparison of tests 2 and 3 in this figure shows that the test in which ΔK was least (Test 2, $\Delta K = 300 \text{ psi}\sqrt{\text{in}}$ compared with Test 3, $\Delta K = 360 \text{ psi}\sqrt{\text{in}}$) produced the higher crack growth rate (9.05×10^{-6} in/cycle compared with 6.1×10^{-6} in/cycle).

A study of all the results (Table 4) indicated that there is no single controlling parameter in these tests at constant speed of cycling. Two further major test series were therefore undertaken to clarify the effects of the range of stress intensity factor, ΔK , and the mean value, K_m . In the first series, Figure 10, ΔK was maintained constant and K_m was varied by factors of 3 and 5. Each test produced a linear curve and the crack growth rate increased non-linearly with K_m ; (the results indicated a fifteen-fold increase in crack growth rate for a five-fold increase in K_m). Similar results were obtained for other constant, but different, ΔK tests in which K_m was similarly varied, Figure (11).

In the second series of tests, K_m was maintained constant throughout but ΔK was varied in each test. The results (Figure 12) for $K_m = 400 \text{ psi}\sqrt{\text{in}}$ and ΔK variable between 200 and 700 $\text{psi}\sqrt{\text{in}}$ were similar to the previous series in that crack growth rate increased rapidly although non-linearly but this time with an increase in ΔK . Similar results were obtained from a further series of tests with

$K_m = 750, 500, 375$ and $275 \text{ psi}\sqrt{\text{in}}$ and at different ΔK ranges. These results are included in Table (4) from which it is clear that both ΔK and K_m are of considerable importance in influencing fatigue crack growth rate in PMMA. The combined effects of K_m and ΔK with reference to the values of $K_{\max}$ have been presented in Figure (13). The limiting curve to the left is the locus of the results of tests with $K_{\min} = 0.$, i.e. $K_{\max} = \Delta K$.

(B) The effect of variation in the loading frequency:

The discussion presented in Chapter 2 in relation to the cyclic crack growth process in rate sensitive materials showed that significant frequency effects may be experienced by such materials. A very limited amount of data is at present available in the literature concerning the manner in which a change in frequency affects the cyclic rate of fatigue crack propagation, $\dot{a}_N$, in thermoplastics. Published data indicate a simple dependence of viscoelastic energy absorbed - and hence the energy available for crack propagation - on the cyclic frequency; Riddell et al. (1966), Opp et al. (1969) and Hertzberg et al. (1970).

Present studies were commenced by testing the effect of K_m parameter at different loading frequencies. Results of tests performed at a frequency of 0.1 Hz clearly indicated that the mean level of the stress intensity factor has considerable influence on the value of $\dot{a}_N$ at the frequency of 0.1 Hz as well as 5 Hz. More extensive investigation of the influence of cyclic frequency was then undertaken.

The results of tests with $K_{\max} = 750 \text{ psi}\sqrt{\text{in}}$ and $K_{\min} = 250 \text{ psi}\sqrt{\text{in}}$ (i.e. $\Delta K = 500$ and $K_m = 500 \text{ psi}\sqrt{\text{in}}$) at loading frequencies of 20, 5 and 0.1 Hz are given in Figure (14). Evidently, decreasing the frequency from 20 Hz to 5 Hz and then to 0.1 Hz (under the same $K_{\max}$ and $K_{\min}$ conditions) resulted in an increase in the magnitude of the cyclic growth rate, $\dot{a}_N$ (see also Table 5). However, when

the results were arranged in terms of the crack growth rate with respect to time, $\dot{a}_t$, the order of the slopes changes, Figure (15). Here, the slopes for 20, 5 and 0.1 Hz are 1.0, 0.7 and 0.41×10^{-4} in/s respectively. Thus, as expected, faster load cycling of PMMA leads to a shorter total fatigue life, if all other parameters are maintained constant.

The relationship between $\dot{a}_N$ and the frequency is shown in Figure (16), where the results of tests at a range of values of ΔK and K_m are presented, it is evident from these results that the rate of variation of $\dot{a}_N$ with changing frequency is a function of the stress intensity factor, viz:

$$\frac{d(\dot{a}_N)}{df} = \phi(K_m, \Delta K) \quad (5.5)$$

At lower frequencies, the effect on $\dot{a}_N$ of variations in K_m is more pronounced. This may be attributed to longer hold times under low frequency loading conditions. In Figure (17) the logarithmic plots of $\dot{a}_N$ and frequency for a range of values of K_m at constant ΔK are presented so as to demonstrate the apparent relatively higher effect of K_m at lower frequencies.

5.4.6 Fatigue crack growth studies in polycarbonate:

(Test Series 4):

(A) Determination of the applied load and the frequency levels:

Various attempts have been made to study the process of fatigue failure in PC. von Jacoby and Cramer (1968) described the fatigue mechanism in this material as a process of superposition of crazing, orientation and physical cross-linking and subsequently proposed a model similar to the models of fatigue crack growth put forward for metals in terms of a relationship between the applied stress amplitude and the rate of crack growth (as discussed in Chapter 2). Also Hertzberg et al. (1970), from

the results of some exploratory tests suggested the applicability of equation (2.21) to fatigue crack growth process in PC. Thus it was decided that a test programme, similar to the case of PMMA, should be carried out to investigate the crack growth process in this material. However, some preliminary studies were necessary to determine the range of frequencies to be tested at room temperature. The temperature rise due to load cycling at various levels has been investigated by Higuchi and Ishii (1968) and Higuchi and Imai (1970). In tension cycling with the mean stress range extending to above 5000 psi and alternating stress values above 4000 psi at a frequency of 1 Hz, the maximum temperature rise obtained was about 0.4°C above room temperature. (The rise in temperature was related to viscoelastic strain and was also associated with an increase in the stiffness.)

Heijboer (1968) studied the variations of material damping (measured by the loss tangent $\tan \delta = \frac{E''}{E'}$) with changes in temperature and showed that the damping maximum of PC at room temperature occurs at frequencies above 10^4 Hz. At a frequency of 20 Hz (the highest used in the present series of tests) a loss tangent peak was observed at a temperature of about -75°C .

Fracture toughness measurements for the polycarbonate material have been carried out at various strain rates (Key et al., 1968). Although the exact nature of the behaviour of this material in failure under various loading conditions is not fully understood, the available data indicate a value for K_{IC} at moderate loading rates (corresponding to cross-head speeds of 0.02 in/min) of about $3200 \text{ psi}\sqrt{\text{in}}$. This approximate value was confirmed from some exploratory tests on Charpy type specimens manufactured out of the PC material and in the fatigue tests, Figure (18). The maximum limit of K_{max} parameter in this series of tests was chosen to be equal to $2400 \text{ psi}\sqrt{\text{in}}$.

(B) The combined effect of K_{max} and K_{min} on $\dot{a}_N$:

As in the case of PMMA, the linear relationship between total crack length, $2a$, and the total number of cycles, N , in tests with constant K limits was found to be a linear one (Figure 19). At a frequency of 0.1 Hz, the data obtained from these tests (Table 6) show the marked dependence of $\dot{a}_N$ on K_m as well as ΔK . Similar to the case of PMMA, by appropriate choice of the K_{max} and the K_{min} it was possible to obtain higher crack propagation rates at smaller ΔK values. In Table (6) data on the combined effect of K_m and ΔK parameters on the cyclic crack growth rate in this material are presented. It is thus evident that at a loading frequency of 0.1 Hz, the trend of variations in $\dot{a}_N$ as a function of the applied load levels is similar in both PC and PMMA materials.

(C) The effect of cyclic waveform:

In order to investigate the effect of input waveform on the crack growth pattern a number of tests were performed with a triangular input waveform. The results are included in Figure (20) for comparison with the sinusoidal waveform case. It is evident that $\dot{a}_N$ values obtained in triangular waveform tests are generally smaller than those from sinusoidal waveform tests; although the magnitudes of variations in the results from the two test series were very small.

(D) The effect of loading frequency:

Similar to the case of PMMA, a series of tests were performed on PC, at frequencies of 5 and 20 Hz and the results were compared to the data from tests with frequency of 0.1 Hz described in the previous section. An interesting point arose from consideration of data obtained at a frequency of 5 Hz: that the direct dependence of $\dot{a}_N$ on both ΔK and K_m , observed previously, did not exist for the

whole set of data. Closer examination indicated a point of transition in the relative effects of ΔK and K_m . It was found that beyond a certain level of $\dot{K}$ (obtained by dividing the ΔK in a test by half the periodic time) the effect of K_m became very small and the influence of ΔK became the predominant factor. This transition value of $\dot{K}$, $\dot{K}^*$, was found to be in the region of $4000 \text{ psi}\sqrt{\text{in}} \text{ s}^{-1}$.

Figure (21) shows the results of tests at a frequency of 5Hz where $\dot{K}$ is below $4000 \text{ psi}\sqrt{\text{in}} \text{ s}^{-1}$. It is noted that the maximum $\dot{K}$ achieved in the 0.1 Hz tests was $140 \text{ psi}\sqrt{\text{in}} \text{ s}^{-1}$.

Wherever the value of $\dot{K}$ exceeded the $\dot{K}^*$ value, it was sufficient to use only the changes in ΔK , to predict the value of $\dot{a}_N$. From Figure (22) it is evident that when $\dot{K}$ is smaller than $4000 \text{ psi}\sqrt{\text{in}} \text{ s}^{-1}$, the influence of K_m on $\dot{a}_N$ is significant, as exemplified by the results of tests at $\Delta K = 300 \text{ psi}\sqrt{\text{in}}$. However for $\dot{K}$ larger than $\dot{K}^*$, as noted from the results of tests at $\Delta K = 500 \text{ psi}\sqrt{\text{in}}$, the effect of variation in K_m is very small.

The stress intensity factor rate effects were confirmed when tests were carried out at a frequency of 20 Hz. The minimum K value applied in this series of tests was $8000 \text{ psi}\sqrt{\text{in}} \text{ s}^{-1}$. Thus if the observations from Figure (22) were correct, it would be expected that the K_m effect would be of secondary importance when compared to the influence of ΔK . Figure (23), a logarithmic plot of $\dot{a}_N$ against ΔK , confirms the previous observations.

The above results (Tables 6, 7 and 8) indicated that the fatigue failure characteristics of PC can not be simply determined from a consideration of the applied stress intensity factor levels without the appreciation of specific loading frequency effects which seem to be unique to this material. When the problem is approached using the $\dot{K}$ parameter, the frequency effect takes a different form, according to the testing region in which one is interested. For all the tests with $\dot{K}$ greater than

$\dot{K}^*$, the influence of frequency variation on $\dot{a}_N$ was as shown in Figure (24). These results indicate a small drop in the value of $\dot{a}_N$ as the frequency is reduced, Figure (25). In this figure it is shown that for $\dot{K}$ values below the level of $\dot{K}^*$, the behaviour is similar to that of PMMA; Figure (16). However, above $\dot{K}^*$ level, the resistance to cyclic crack growth tends to decrease as frequency is raised. This indicated a behaviour in opposition to the observations on PMMA. These results were considered as confirmation of the existence of some unique characteristics in cyclic failure properties of PC material, as indicated by von Jacoby and Cramer (1968) and Hertzberg et al. (1970); in the latter work it was shown that there was a large increase in the value of $\dot{a}_N$ as the loading frequency was raised from 0.33 to 10 Hz. The characteristic behaviour of the PC material at load frequency levels tested (0.1 to 20 Hz) may possibly be attributed to the particular damping properties of this material. It is possible that at still higher frequency levels the trend may change again.

Data relating the relaxation properties of PC and its behaviour in fracture may prove to be useful in the development of an acceptable explanation for the occurrence of failure phenomena described above. Such data, available for PMMA (Johnson and Radon 1972), help to clarify the nature of the processes relating the fracture strength of the material - in the stable growth region - to the load rate parameter. A peculiarity in the relaxation data on PC has been the observation of an intermediate relaxation peak, situated between the α and β peaks (Illers and Breuer, 1963). This small peak has been observed both in mechanical and dielectric tests and at present there is no explanation for its existence. It is experienced in oriented polymers (drawn or extruded) and not in annealed ones (McCrum et al., 1967); (In the present study extruded sheets have been used).

5.4.7 Fatigue crack growth studies in Nylon 6.6: (Test Series 5):

This particular test series was aimed at the investigation of the relative effects of ΔK and K_m parameter on the fatigue failure process in a semicrystalline thermoplastic. Nylon 6.6 material is widely used in engineering design processes (design of gears, bushes, bearings, etc.) and hence it was felt that the availability of fatigue data on this material will be very useful.

Fracture toughness tests on this material indicated that Nylon 6.6 behaves in a relatively ductile manner in failure under low loading rates at room temperature. At a crosshead speed of about 0.002 in/min a wide scatter in K_{IC} measurements have been observed; Figure (26). However at higher crosshead speeds above 2 in/min a quasi-brittle type of failure was experienced and consistent values for K_{IC} parameter could be obtained. Under cyclic loading conditions, even at the lowest frequency used (0.1 Hz) the rate of load application falls into the quasi-brittle behaviour region.

It was also thought desirable to test the effect of anisotropy of the material in relation to the fatigue crack growth resistance. Advantage was taken of the molecular orientation characteristics in an extruded material by performing crack propagation tests in two perpendicular directions under similar loading conditions.

(A) The effect of ΔK and K_m on $\dot{a}_N$:

A test series similar to those of PMMA and PC, but with K_{max} ranging from 1000 to 2000 $\text{psi}\sqrt{\text{in}}$ and K_{min} from 0 to 1300 $\text{psi}\sqrt{\text{in}}$ was carried out. The results obtained at a loading frequency of 5 Hz have been presented in Table (10). Again, under constant K_{max} and K_{min} conditions, a straight line relationship between the crack length and the total number of cycles was obtained; Figure (27).

As in PMMA and PC, the significant effect on $\dot{a}_N$ of K_m as well as ΔK parameters, was observed. For example in a test with $K_{\max} = 1800$ and $K_{\min} = 1300$ psi $\sqrt{\text{in}}$ ($K_m = 1550$ and $\Delta K = 500$ psi $\sqrt{\text{in}}$) a crack growth rate of 0.160 in/cycle was obtained; whereas in a test with $K_{\max} = 1000$ and $K_{\min} = 300$ psi $\sqrt{\text{in}}$ (i.e. $K_m = 650$ and $\Delta K = 700$ psi $\sqrt{\text{in}}$) the value of $\dot{a}_N$ was 0.010 in/cycle. Comparison of the data in Table (10) confirms the conclusion from previous test series that highly erroneous predictions of $\dot{a}_N$ value will result if the important influence of K_m parameter is discarded.

(B) The effect of loading frequency:

Cyclic crack propagation rate in Nylon 6.6 is influenced by changes in loading frequency in a manner similar to that in PMMA. Experimental studies at three frequency levels of 0.1, 5 and 20 Hz indicated that the value of $\dot{a}_N$ gradually decreases as the loading frequency is raised. Testing conditions under which these results were obtained have been tabulated in Tables (9, 10, and 11). The trend in the variation of $\dot{a}_N$ with frequency has been demonstrated in Figure (28).

Material properties data in Table (12) show that the relaxation peaks (maxima of E'' parameter) at room temperature (21°C) occur at very high frequencies (above 10^4 Hz). This may be taken as an indication that within the limits of the frequency range used in the present study, the value of E'' will be gradually increasing with increasing frequency. From the discussions in Chapter 3, it is expected that the level of the viscous energy absorbed per cycle will rise and hence the magnitude of the energy available for crack extension per cycle will be lowered. A decrease in the value of $\dot{a}_N$ may then be expected.

Data on the fracture toughness values related to crack speed are not yet available for N6.6; hence the discussion of this important aspect of the failure phenomena

has to be left out. Additional arguments in relation to the effect of loading frequency will be presented in the following Chapter.

(C) The effect of molecular orientation:

In extruded sheets of polymeric materials, the effect of molecular orientation in the direction of extrusion, manifests itself in a higher fracture strength in that direction. The same effects are observed in the case of rolled and extruded metals.

Some exploratory work was carried out to test the effect of anisotropy on fatigue crack growth rate in the Nylon 6.6 material: specimens were cut from the supplied extruded sheets such that the centre notch was perpendicular to the extrusion direction. Fatigue tests were carried out at a loading frequency of 5 Hz. The results have been included in Figure (27) so that a direct comparison with the previously discussed data could be made. From such a comparison it became evident that cyclic growth rates for cracks extending in a direction perpendicular to that of extrusion, are about 15 per cent lower than the values corresponding to the cracks extending in parallel to the direction of extrusion.

5.4.8 Fatigue crack growth in studies in polyacetal copolymer:
(Test Series 6)

The main purposes of this particular study were the following:

(1) To study the fatigue behaviour of a copolymer which has frequently been used as a substitute for Nylons in engineering applications; and to provide data on the combined effects of ΔK and K_m levels on the $\dot{a}_N$ parameter.

(2) To study the possibility of using constant stress cycle tests in place of constant stress intensity factor tests, as a more convenient procedure for gathering data on fatigue behaviour of a material.

Preparation of specimens was according to the procedure described in Section 5.4.2.

(A) Cyclic crack propagation under the conditions of constant applied stress levels:

A test was carried out with the maximum applied load of 2160 lbs and the minimum of zero. The cyclic rate of growth increased with increasing crack length. Figure (29) shows the relationship between crack length and the total number of cycles. As expected an exponential pattern of behaviour (accelerating crack) was observed.

It was indicated above that one of the purposes of the present section of the empirical studies was to investigate the possibility of derivation of crack growth rates related to stress intensity factor data. In the above figure, the slope of $2a$ against N graph at each value of a yields the rate of variation, $\dot{a}_N$. The transient maximum and minimum value of K_I corresponding to a particular crack length can be found using the instantaneous crack length and applied stress values. In Table (13) the magnitude of stress intensity parameter and the corresponding value of $\dot{a}_N$, obtained at each value of a , are presented. Figure (30) shows these results plotted on the basis of the parameter $K_{\max}$ ($= \Delta K$). Evidently a linear relationship has been obtained.

In order to investigate the degree of accuracy of the above data, a series of tests were performed in each of which the appropriate value of the stress intensity factor was maintained constant. Results of these tests have been superimposed on the previous set of data in Figure (30).

The limited extension of the data does not allow a full analysis of the results. However a small percentage of deviation in the data obtained from the constant stress

test is clearly evident. Nevertheless, it is reasonable to assume that whenever a good approximation to the actual value of $\dot{a}_N$, is sufficient to serve a certain purpose, the above method will yield much faster results, in that only one or two tests are required to be carried out and the subsequent analysis will also be relatively simple.

5.5 The Effect of the Environmental Temperature on the Cyclic Rate of Crack Propagation: (Test Series 7):

The studies of the effect of the environmental temperature on the fracture strength of polymeric solids (e.g. Key et al., 1968, on PC and PMMA; Benbow and Roseler, 1957, on PMMA; Johnson and Radon, 1972, on PMMA), have indicated that the variations in temperature may have a significant influence on the value of the fracture toughness parameter, K_{IC} , for these materials. The rate of crack propagation under cyclic loading conditions, is influenced by the variations in the value of K_{IC} parameter (Miller, 1968) and hence it was thought desirable to obtain some data indicating the extent of the effect of environmental temperature on the value of the $\dot{a}_N$ parameter.

Ideally, the test programme should consist of the study of the combined effects of the loading frequency and the temperature, so as to be able to relate the effect of variations in frequency, on the $\dot{a}_N$ parameter, to the changes in the material properties such as the loss modulus distribution and the relaxation peaks which would experience a shift to higher frequencies as the temperature level is raised. However, this would be a major study in its own right and thus within the context of the present programme of studies, the test series (7) was aimed at the measurement of the extent of the effect of the environmental temperature on the cyclic rate of crack

propagation.

5.5.1 Materials and the specimens:

The materials selected for testing were PMMA and Nylon 6.6, for both of which room temperature crack propagation data at three frequency levels were available; (Test series 3 and 5).

The specimens used were of the following dimensions: $13\frac{1}{2} \times 12 \times \frac{1}{4}$ inch thick plates for PMMA and $7 \times 12 \times \frac{1}{4}$ inch thick plates for Nylon 6.6. All specimens contained an initial centre notch of length about 1 inch.

5.5.2 Equipment and the test procedure:

All tests were carried out at a frequency of 5 Hz using a specially designed environmental chamber which could be added to the Dowty electrohydraulic testing machine (see Figure 31a). The general arrangement of the test chamber is shown in Figures (32 and 33).

The main unit, A, was manufactured out of $1\frac{1}{2}$ inch thick wood and was covered by an external metallic shield. The subsidiary equipment consisted of a heater-refrigerator unit (Gebrüder Haake, Model KT32) which provided a circulating liquid at temperatures over the range of -30° to 100°C . This liquid was subsequently passed through the two copper coils in the unit B. An electric fan, also placed in this unit, circulated the air past these coils hence providing the environment of appropriate temperature inside the chamber. The unit B could be arranged in any desired position in relation to the chamber itself. In the present tests it was placed in parallel and adjacent to the walls of the main unit so as to reduce the length of the external pipings and hence lower the level of heat losses from them.

The dimensions of the test chamber ($30 \times 22 \times 18$ inches) were so chosen as to allow the use of the 12×15 inches plate specimens. However, the large surface

area resulted in considerable heat losses. In order to reduce the volume of air to be controlled at fixed temperatures, slabs of foamed polystyrene material were placed in the chamber so as to fill the space between the walls and the specimen. The external surface of the whole unit was covered with layers of "glass-wool" material so as to limit the extent of heat losses. These measures did result in an extension of the available temperature range. Tests proved that temperature levels from (+3°C) to (+70°C) could be easily maintained over long periods of testing which are normally required in fatigue failure studies.

In the present test series, the crack propagation process at two temperature levels of (+3°C) and (+38°C) were studied and the results were compared to the data obtained at room temperature (+21°C). All tests were performed at a loading frequency of 5 Hz.

The following two points should also be noted: (a) Enough space was available inside the chamber so as to allow at least one other specimen to be placed inside it while a test was being performed. This led to a reduction in the length of time (about two hours) normally allowed in a test, for the specimen to reach a steady state condition in relation to its environment; and (b) the liquids used in the temperature control unit may be selected from the following on the basis of suitability for tests over a certain temperature range:

From -30 to +50°C	Methyl or Ethyl alcohol
From +1 to 95°C	Distilled water
Up to 100°C	Pure Ethylene Glycol, Kerosene, Glycerine or low viscosity mineral oil.

In the present tests methyl alcohol was used.

5.5.3 Discussion of results:

For each of the two materials selected, a number of tests were performed at the two temperature levels of 3°C and 38°C . The results have been presented in Table (14) in which the room temperature (21°C) data have also been included to facilitate the comparison. In the case of the PMMA material, lowering of the environmental temperature to 3°C resulted in a considerable increase in the cyclic rate of crack propagation. In all cases the values of the $\dot{a}_N$ parameter were more than 50% higher than those obtained at room temperature. Data obtained at a temperature of 38°C also indicated a lowering of the resistance of the material to cyclic crack extension. These results, although slightly lower than the data obtained at 3°C , indicated a rate of crack propagation much higher than that at room temperature. The above results have been presented in Figure (34).

In the case of the Nylon 6.6 material also, under the same load levels, variations in the environmental temperature conditions (from 21°C to 3°C and 38°C) resulted in significant changes in the cyclic rate of crack propagation. The trend of this variation, however, was opposite to that observed in PMMA. As indicated by the result of tests presented in Table (14) and Figure (35), the fatigue resistance of this material increased at both temperature levels of 3°C and 38°C in comparison to that at 21°C . The crack growth rates obtained at 3°C , particularly, were of much lower magnitudes than those at room temperature; Figure (35).

The above results, although covering a limited range of tests, were taken as the evidence for the important fact that relatively small variations in the environmental temperature levels can cause drastic changes in the fatigue crack propagation rate in the materials tested.

A plausible explanation for the considerable variation in the value of the $\dot{a}_N$ parameter at different temperatures may be based upon the analysis of the effect of temperature on the specific fracture energy (or the K_{IC} parameter) of these materials.

Recent developments in the studies related to the fracture phenomena in polymers in connection with the molecular kinetic processes, have demonstrated the correspondence of a rise in the measured fracture toughness value of a material, to a rise in the extent of relaxation processes taking place in deformation. It has been shown that the temperature and the load rate (frequency) levels associated with the β relaxation process, correspond to a peak in the distribution of the fracture toughness data (Johnson and Radon, 1972). Hence, if under certain loading conditions, variations in one parameter (e.g. the temperature, as in the present study) leads to a change in the value of the K_{IC} parameter, it is expected that the rate of crack extension under cyclic loading conditions will also be affected. From Figure (6) in Johnson and Radon (1972), in which data on PMMA have been presented, it is evident that at a frequency of 5 Hz, a loss peak (dielectric) will occur at a temperature of about 20°C. A reduction in the value of the K_{IC} parameter, as a consequence of the temperature level being raised to 38°C or lowered to 3°C, will thus be anticipated. (An exploratory measurement of the K_{IC} parameter value for PMMA at 3°C yielded the value of 850 psi $\sqrt{\text{in}}$, i.e. about 25% lower than the 1200 psi $\sqrt{\text{in}}$ value obtained at room temperature and at the same load rate.) The reduction in the value of the K_{IC} parameter will lead to a higher rate of crack propagation under cyclic loading condition, as indicated by the results of the present test series.

Similar data on Nylon 6.6 material are not, at present, available. It is proposed that phenomena similar to those described above, were responsible for the

variations in the fatigue strength of this material. It is also possible that at a temperature of 38°C the extent of non-linearity in mechanical response and also the level of ductility of this material are enhanced to such a degree that fundamental modifications to the material characterisation procedures are required.

The exposition of the important effect of the environmental temperature variation on the value of the $\dot{a}_N$ parameter, again points to the fact that the room temperature data discussed in previous sections, should be used only in conjunction with an exact identification of the specific environmental conditions. The present test ranges will obviously have to be extended in future studies, in order to establish the exact form of the effect of temperature on the $\dot{a}_N$ parameter.

CHAPTER SIX

DISCUSSION

In the present section an attempt will be made to discuss the general trends of material behaviour apparent in all tests.

A model for prediction of fatigue crack propagation rate based upon the empirical data and results of the analytical solution will be proposed.

6.1 Analysis of the Crack Initiation and the Crack Propagation Processes:

Theoretical analysis of the fatigue failure process in viscoelastic solids was carried out through the application of a thermodynamic energy balance approach. Cyclic initiation period and the rate of crack propagation were analysed independently. The process of initiation was discussed in terms of an accumulation of cyclic strain in the region of maximum stress concentration, up to a critical limit where the strain at the crack tip (which can be discussed in terms of separation of the two crack faces) is sufficiently high so as to induce the initiation of the macroscopic crack propagation process.

From the discussion of the 'slow' growth zone in the fracture of quasi-brittle materials (Chapter 2), it is evident that in the case of even the most brittle thermoplastics, such zones are expected to exist - except for the purely idealistic case in which the testing parameters such as the load rate or temperature are so chosen as to create the conditions of the perfectly brittle response of the material. In such a case, initiation of failure will result in catastrophic fracture. However, for the case of the stable growth mechanism, which is an inherent characteristic of the fatigue process, the strain gradient in the stress field region extending from the crack

tip into the uncracked body, is such that a small extension, δa , reduces the maximum strain level to a value less than the critical limit required for crack extension. This drop is, however, compensated by the application of the next load cycle which raises it again to a critical level and thus the cyclic propagation process proceeds. This seems to be the explanation for the "per cycle growth" pattern observed in these tests. Although there have been some suggestions of a multi-cycle dependent propagation (Elinck et al., 1971), observations from the present tests indicated a growth pattern per cycle of load application. This was clearly evident from fracture surface studies, Section 6.6. Fatigue literature related to metals, contains a considerable amount of evidence in favour of a crack propagation pattern based on crack extension per each load cycle (e.g. ASTM STP 415).

6.1.1 Results of the theoretical analysis of the crack growth initiation process:

The cyclic initiation period, N_i , as determined from equation (4.18) was compared to the experimental data obtained from tests on PMMA. The important limiting factor in such tests of correspondence between empirical and analytical results related to the tested materials, has been the scarcity of data on γ^* , the specific fracture energy values for these materials. For PMMA the value of this parameter has been measured by a number of authors. Results of Pratt (1968) have been used here.

As indicated in Chapter 2 the specific fracture energy, γ^* , is itself a function of temperature and load rate, Bennet et al. (1970); hence in order to be able to substitute the correct value of γ^* in the analytical solution, data on variation of γ^* with load rate and temperature must be available. Such data for the commonly used thermoplastics are at present very scarce.

For the majority of tests in the present studies, the very sharp through-thickness razor notches resulted in the commencement of crack propagation after the application of the first load cycle. However, in a few cases a small number of cycles pertaining to the initiation period could be detected. Such data on the crack growth initiation life in PMMA compared reasonably well with the theoretical values (Figure 36). The apparent overestimation of N_i from the analysis was thought to be due to the introduction of approximation in the values of Poisson's ratio (assumed $\nu = \frac{1}{2}$), creep compliance and specific fracture energy terms. The specific form of equation (4.18) enables the determination of an endurance limit (similar to the conventional stress-number of cycles (S-N) curves in fatigue) below which an infinite crack growth initiation life is predicted. (An endurance limit for PMMA has been observed in plate-bending type tests, Riddell et al., (1966).)

Attempts to extend the analysis to the conditions of macroscopic crack propagation were not successful. An exponential dependency on the K^2 term was obtained for the cyclic crack propagation rate. However in the final analysis, the specific form of the crack growth function indicated a negative dependency of $\dot{a}_N$ on the stress intensity factor term. Cherepanov (1968) and Wnuk (1971) in the general analyses of the crack propagation processes chose to leave out the case of fatigue failure in viscoelastic materials.

More significantly, in Cherepanov (1968) solution of cyclic crack propagation problem in elastoplastic materials, the final function contains a negative sign, indicating a reduction in the cyclic rate of crack growth as the crack length is increased, while boundary loading is maintained at the same level. The significance of this particular result was not discussed.

It is suggested that the inclusion of such terms as the

kinetic energy and the thermo-mechanical coupling (ignored in the present analysis) may help the derivation of a correct solution.

6.2 An Empirical Model for Fatigue Crack Propagation:

Review of contributions to the field of LEFM and its application to the analysis of fatigue crack propagation with the various methods of approach based upon parameters such as the stress intensity factor, K , Paris (1963), the crack opening displacement, COD, Dover (1972), and crack tip plastic zone size, r_p , Erdogan (1968), etc. (see also Chapter 2), indicates the need to develop a novel model which is capable of replacing those available at present in such a way that it throws a unifying light upon them, as they each have been demonstrated to be specifically applicable under certain conditions. Such a development would drastically simplify the task of the designers.

Crack propagation laws in use at present, do not, in general, aid in the exposition of the specific nature of the mechanisms which may be responsible for propagation of a crack. On the continuum level, it seems reasonable to assume that the rate of crack growth is controlled by the pattern of transformations, during each load cycle, of various energy components.

However, even an intuitive consideration of the characteristics of the parameters which, as indicated above, have been shown to have controlling influence under certain conditions (such as COD and r_p), lends itself to a postulation of the cyclic crack growth rate dependency on parameter K^2 . In a zero-tension load cycle, $\dot{a}_N$ will be related to $K_{\max}^2$ and when $K_{\min} \neq 0$, $\dot{a}_N$ is expected to be proportional to $K_{\max}^2 - K_{\min}^2$.

From the above consideration, the following crack growth equation is proposed:

$$\dot{a}_N = \beta (\lambda)^n \quad (6.1)$$

$$\text{where} \quad \lambda = K_{\max}^2 - K_{\min}^2 \quad (= 2\Delta K K_m) \quad (6.2)$$

β and n are the numerical factor and the exponent and values of both vary as functions of the load frequency, environment and other testing conditions.

Evidently, in all cases where $K_{\min} = 0$ (i.e. $K_{\max} = \Delta K$) the above equation may be written as:

$$\dot{a}_N = \beta (K_{\max}^2)^n \quad (6.3)$$

$$\text{and} \quad \dot{a}_N = \beta (\Delta K)^{2n} \quad (6.4)$$

This equation is of the same form as that proposed by Paris (1963); equation (2.21). Similarly, in the cases where K_m is held at a constant level, variations in $\dot{a}_N$ may simply be described in terms of changes in ΔK .

The crack growth data for the four materials under test, which were previously discussed in terms of the influence of ΔK and K_m parameters, were replotted in terms of the parameter λ . Figures (37 to 39) for PMMA, (40, 41) for PC, (42 to 44) for Nylon 6.6 and (45) for PA show a linear relationship, on a logarithmic scale, between $\dot{a}_N$ and λ parameters; thus confirming the accuracy of the specific formulation of equation (6.1). It is evident that, the results of all the above tests may be discussed in terms of the λ parameter and irrespective of the specific values of the ΔK and K_m terms. For a certain part of data on PC material, where no significant influence of K_m on $\dot{a}_N$ was observed, analysis of data in terms of the ΔK parameter was found to be sufficiently accurate (Section 5.4.6).

In the following section, an attempt will be made to demonstrate the facility with which the above model, equation (6.1), can be converted into the form of a model based upon the cyclic strain energy release rate, the crack opening displace-

ment and the crack tip plasticity. Correspondence of empirical data to the above model for crack propagation will be demonstrated in Section 6.3.

6.2.1 The energy approach:

The failure of solids, particularly polymers, must be treated in the light of the general phenomena governing the nature of temperature-time dependence of strength. Such phenomena can be characterised in terms of the energy concepts. The 'total thermodynamic energy' criteria applied to the failure in monotonic fracture and cyclic fatigue of solids (Griffith, 1921; Williams, 1965, 1967; Cherepanov, 1967, 1968) have proved to be reasonably adequate. The approach to fatigue in elastomers based on the tear energy concept has already been mentioned; (Gent et al., 1964). Also, the empirical models of cyclic failure which are based on the total hysteresis energy, plastic strain energy, etc., developed for metals and described in Chapter 1, point to the possibility of the development of a unified approach to fatigue failure based upon the cyclic energy balance and irrespective of the nature of the particular continuum. Cherepanov (1967, 1968) analytical treatment (see Chapter 4) may be considered as the theoretical justification for such an approach.

The stress intensity factor, K_I , is related to the strain energy release rate per unit crack extension in the form of the following equation:

$$G = \frac{K_I^2}{E} (1 - \nu^2) \quad (6.5)$$

for plane-strain conditions.

It is evident that equation (6.1) can be readily converted to an energy formulation by substituting for K , in terms of G_1 :

$$\dot{a}_N = \beta * (G_{\max} - G_{\min})^n \quad (6.6)$$

where
$$\beta^* = \beta \left(\frac{E}{1 - \nu^2} \right)^n \quad (6.7)$$

The term G in such cases will be calculated from an equation such as the one used for rubber (Gent et al., 1964) and for polyethylene (Andrews and Walker, 1971).

The direct use of the energy term, eliminates the linearity limitation which is a prerequisite to the application of the stress intensity factor concept. Only when the material is linear or the displacements are very small, the two approaches will correspond to each other.

The discussion of the energy term G , leads to the addition of a note on the possibility of inclusion of the term J , the path independent contour integral (Chapter 2). Within the bounds of the present programme of research there has been no direct experimental study of the J parameter and its application to the propagating crack situations; and as far as it could be ascertained data of this nature have not been reported elsewhere. However, it is clear that, the form of the proposed crack growth model (equation 6.1) is such that it is capable of conversion to include the term J . This model, when duly examined by empirical methods, may be applicable to the case of linear and non-linear elastic behaviour (Note the special relevance to the case of polymers.) and also to the fatigue failures involving large plasticity.

The energy based model can thus be written as:

$$\dot{a}_N = M A^n \quad (6.8)$$

where
$$A = G_{\max} - G_{\min} \quad (= J_{\max} - J_{\min}) \quad (6.9)$$

M and n being the numerical factor and the exponent whose values are dependent upon material properties and loading conditions.

Crack growth data presented as a function of A will be very similar to those produced as a function of the parameter λ . Except that the inclusion of the modulus parameter in this equation facilitates the prediction of the variations in the fatigue failure characteristics of a material subjected to various loading conditions from the knowledge of the manner in which the modulus of elasticity of the material is influenced by such conditions.

6.2.2 Consideration of the COD and the crack tip plasticity:

(A) Implementation of the COD concept:

Fatigue failure is a strain dependent process; thus it is reasonable to assume that the rate of crack growth would be a function of the actual separation of the two crack surfaces during each load cycle. The concept of crack opening displacement (COD), Wells (1968), is implicitly related to the plastic zone size and to the energy criterion, thus

$$\text{COD} \equiv \delta = \frac{G_I}{\sigma_{yp}} = \frac{\xi K_I^2}{E \sigma_{yp}} \quad (6.10)$$

Clearly, wherever convenient, equation (6.1) can be expressed in terms of the COD parameter through the replacement of K_I^2 by $\frac{E \delta}{\xi} \sigma_{yp}$; thus:

$$\dot{a}_N = \beta \left(\frac{E \sigma_{yp}}{\xi} \right)^n (\delta_{\max} - \delta_{\min})^n \quad (6.11)$$

Dover (1972) demonstrated the possibility of relating $\dot{a}_N$ to the specific levels of a COD cycle.

(B) Crack tip plasticity:

In a similar manner to the above, relating the crack tip strain to the crack tip plastic deformation, as a consequence of which fatigue crack growth occurs

(McLintock, 1962; Valluri, 1961; Tomkins, 1968; Buck et al., 1972) the crack growth rate can be predicted in relation to the crack tip plastic zone formation.

The size of the plastic zone is dependent upon the upper bound of the stress intensity factor cycle, i.e. $K_{\max}$; and the length of the region within the plastic zone starting from the crack tip which undergoes reversed deformation as unloading takes place can be reasonably assumed to be dependent on the magnitude of the reduction in $K_{\max}$ (see also Paris, 1963). The small cyclic increase in the crack length is related to the length of this region (Rice, 1967) and hence to the difference in the plastic zone size, r_p , corresponding to $K_{\max}$ and $K_{\min}$; and r_p is related to the stress intensity factor, as in:

$$r_p = \frac{1}{4\sqrt{2}\pi} \left(\frac{K_I}{\sigma_{yp}} \right)^2 \quad (\text{plane-strain}) \quad (6.12)$$

$r_{p_{\max}}$ and $r_{p_{\min}}$ will correspond to $K_{\max}^2$ and $K_{\min}^2$ respectively and hence the cyclic rate of crack growth may be taken as related to the difference of these squared terms:

$$\dot{a}_N = \beta (4\sqrt{2}\pi \sigma_{yp}^2)^n (r_{p_{\max}} - r_{p_{\min}})^n \quad (6.13)$$

In rate dependent materials, the yield stress σ_{yp} will be affected by changes in frequency (or load rate) and thus the value of r_p will be correspondingly influenced. The above equation has the advantage of being capable of application to fatigue under various multiaxial stress conditions, if an estimate of the plastic zone size under such conditions can be made (see Chapter 7).

(C) A note on the use of the plastic zone size and the plastic strain in the fatigue failure models:

The following remarks may be made in relation to the importance of the plastic zone size and plastic strain concepts in the analysis of the fatigue data:

1) In the presence of excessive boundary loading conditions (where σ_{∞} approaches σ_{yp}), it will be very difficult to justify the use of the stress intensity factor parameter in characterising the failure process. Under such conditions the plastic strains and the plastic zone size will yield a rigorous measure of the mechanical phenomena taking place.

2) In certain configurations with the theoretically same stress intensity factor, it is possible to have different crack growth rates (Erdogan, 1968) - plate bending compared to plate extension is one example of such circumstances. The plastic zone size will provide an accurate correlation in such cases. Other examples include comparison of flat with shear, or plane-strain with plane-stress, modes of crack propagation in relatively thin plates.

3) The plastic yielding phenomena conveniently combine the mechanical variables at the continuum level and the microstructural properties of the material. Hence in the development of an integrated crack growth model such properties of the material may be represented by the characteristics of the plastic deformation process.

4) Under complex loading conditions, a knowledge of the variations in the extent of plastic yielding due to the presence of multiaxial loading conditions may prove very useful in determining the upward or downward change in the fatigue resistance of a material; see Chapter 7.

6.3 Survey of the Empirical Data and Comparison with Various Crack Growth Models:

6.3.1 The effect of the amplitude and the mean level of the stress intensity factor:

Experimental data produced from the present test series indicated that a crack growth model formulated such that a simple relationship between $\dot{a}_N$ and ΔK is

satisfied, will, in many instances, yield erroneous predictions. Thus although the initial studies of the fatigue crack propagation process were based upon the effect of ΔK parameter (Borduas et al., 1968; Mukherjee et al., 1969; Hertzberg et al., 1970), recently some evidence of the significant effect of K_m has been obtained (Mukherjee and Burns, 1971; Hertzberg et al., 1972). Results of the present testing programme demonstrated that by an appropriate choice of K_{max} and K_{min} levels it is possible to obtain widely different $\dot{a}_N$ values under constant ΔK conditions (Tables 4, 6, 10, 13). From the data presented in these tables, it is evident that if the level of K_m is appropriately modified, one may obtain a larger crack growth rate value for a smaller ΔK load cycle.

The significant effect of K_m on the rate of crack propagation was observed in tests on all four materials under investigation, Figures (37, 40, 42 and 45). Similar behaviours were observed at all three frequency levels of 0.1, 5 and 20 Hz. The specific influence on $\dot{a}_N$ of the frequency variation will be described in the following section. In the case of PC the combined effect of ΔK and K_m was found to be dependent on the specific level of the applied cyclic load rate, Figures (22, 23).

The special characteristics of the behaviour of PC material within the limits of the test range in the present study was described in Section (5.4.6).

The experimental results of the study of ΔK and K_m parameters may now be analysed in relation to the proposed crack propagation model, equation (6.1).

The value of λ parameter in each test was calculated from equation (6.2). These values, as presented in Tables (4, 6, 9, 13), indicate that irrespective of the values of K_{max} and K_{min} , the crack growth rate will stay the same if the appropriate level of the λ parameter is maintained. Graphs obtained from the plots of $\dot{a}_N$ against λ demonstrate the applicability of equation (6.1) to fatigue crack propagation process in the four materials tested; Figures (37, 40, 42 and 45). A per-

fectly linear relationship (on a log-log plot) would not be expected, however, within the limits of experimental scatter - unavoidable due to the anomalies involved in fatigue tests - there is a unique relationship between parameter λ and $\dot{a}_N$. The logarithmic plots of the results, obtained at various frequency levels, show good linear distributions and allow the values of parameters n and β in equation (6.1) to be calculated (Table 15).

To illustrate the effectiveness of the parameter λ , tests were conducted in which ΔK and K_m were allowed to vary arbitrarily but λ was maintained at a constant value. The results, in the form of logarithmic plots of $\dot{a}_N$ against λ , are shown in Figure (37) for PMMA. Thus application of the λ model leads to the following simplification in the analysis of the effect on $\dot{a}_N$ of the stress intensity factor values: that irrespective of the specific values of K_{max} and K_{min} (and hence ΔK and K_m) the cyclic rate of crack growth will stay constant if parameter λ is maintained at a certain level. Assuming, of course, that all other testing parameters have remained the same. See, for example, Figures (37 to 39) in which $\dot{a}_N$ against λ data for PMMA at three loading frequencies have been presented.

6.3.2 Correspondence of the empirical data to other crack growth models:

The limitations inherent in the crack growth models available in the literature on the fatigue problem became immediately apparent when an attempt was made to analyse the data obtained from the present study. The application of Paris's (1963) model, equation (2.21), was found to be limited to the conditions of constant K_m . More specifically, as discussed before, by an appropriate choice of K_{max} and K_{min} levels, it was shown that it is possible to obtain an increased $\dot{a}_N$ value corresponding to a reduced ΔK level. Hence, any prediction of $\dot{a}_N$ based solely upon ΔK parameter, and irrespective of the value of K_m , will yield erroneous results.

Amongst the other crack growth models, the one proposed by Roberts and Erdogan (1967) and the equation recently used by Mukherjee and Burns (1971) were considered. These two equations are very similar to the model proposed here (equation 6.1), specially that by Mukherjee and Burns (1971) which contains both ΔK and K_m parameters as:

$$\frac{d(2a)}{dN} = C_1 f^{\alpha_1} \Delta K^{\alpha_2} K_m^{\alpha_3} \quad (6.14)$$

where f is the loading frequency and C_1, α_1, α_2 and α_3 are numerical constants.

This equation has been based on a general consideration of the fatigue data on PMMA. The exponents α_2 and α_3 were found to be 2.13 ± 0.18 and 2.39 ± 0.18 . Bearing in mind the nature of the experimental work, it may be stated that α_2 and α_3 have approximately equal values, i.e. the exponents of ΔK and K_m terms in the above equation are equal. This would then support the form of the proposed model (equation 6.1). Equation (6.14) will produce a reasonably good fit to the empirical data obtained from the present study. This model, however, does not possess the specific advantages arising from the special form of equation (6.1) based on parameter λ , as described in earlier parts of the present discussion.

In the analysis of the fatigue data on metals, a frequently used crack growth equation is that proposed by Forman et al. (1967); (see equation 2.25). When the data obtained from fatigue tests on viscoelastic materials were analysed using this model, it was found that its predictions did not accurately correspond to experimental results. The specific conditions set for this comparison have been given in Table (16). Contents of this table show that, under certain K_{max} and K_{min} levels where approximately equal rates of crack propagation were obtained in the tests, the predictions of the Forman et al. (1967) model are widely different; whereas the

values of λ parameter are nearly equal. Equation (2.25) clearly can not be used in relation to the present test data. In Section 6.4 it will be shown that the use of the model in place of Forman's equation (which includes a constant value for $R = K_{\min}/K_{\max}$ in each case) in application to fatigue data on metals, will result in considerable simplification of the task of the designers.

6.3.3 Discussion of the effect of frequency:

From the discussions in Chapter 3, it is expected that the mechanical response of a viscoelastic material in terms of the viscous dissipation energy, the extent of yielding, etc., will be highly rate dependent. In fatigue cycling tests such effects have a pronounced influence on the rate of crack propagation. This is indeed demonstrated by the results of tests carried out at three frequency levels of 0.1, 5 and 20 Hz on four materials under consideration.

Except for a section of the results on PC material (discussed in Section 5.4.6), the general trend of behaviour of PMMA, PC, N6.6 and PA may be described as follows.

If all parameters associated with the conditions of loading, including stress intensity factor levels, are maintained constant, an increase in frequency will lead to a decrease in the cyclic rate of crack propagation, $\dot{a}_N$. However, if crack growth rate is determined as a function of the time elapsed, faster cycling leads to a shorter life time. Due to the very complex nature of the mechanisms involved in the extension of cyclic damage, no simple explanation can be offered as the phenomenological substantiation of the observed frequency effect. However, an attempt will be made to present explanations which seem to be reasonably plausible.

The trend of change in $\dot{a}_N$ values as a consequence of frequency variation may be attributed to the following sources:

(A) The effects of parameters associated with the external loading conditions:

During the high frequency loading process (sharper sinusoidal waveform), the material is subjected to the high loads for a shorter period of time in each cycle. The rate of growth of fatigue cracks is proportional to the extent of plastically deformed zone at the crack tip (Frost, 1966; Rice, 1967), and as indicated by equation (6.13). In the case of the glassy polymers, the permanently yielded zone may conveniently be considered as the crazed material through which the crack propagates. There is evidence of time and strain dependency of craze growth in PMMA (Higuchi, 1965), and in PC (Burnier and Kambour, 1968); namely that if the same strain is applied for a shorter period of time, the size of the crazed zone which consequently develops will be smaller. Observations of the fracture surface produced under various load frequencies also confirm the hypothesis of smaller plastic deformation at higher frequencies. The study of the fracture surfaces produced in monotonic fracture tests at various load rates, has indicated that with increasing load rate, ductile behaviour of the material becomes more restricted and consequently a more smooth fracture surface results.

In the discussion of variations with load rate of the energy required for creation of a unit fracture surface one must also refer to the data relating the fracture toughness value of the material to the speed of crack propagation. Such data on PMMA, for instance, show a rise in K_{IC} (indicating a higher fracture strength) as the crack speed increases up to a level of about $1700 \text{ psi}\sqrt{\text{in}}$ and then starts to decrease; see Figure (2) in Johnson and Radon (1972). The first part of this graph (i.e. up to crack speed of about 3 in/s) is considered as corresponding to the stable crack growth situation, with which conditions of crack growth under cyclic loading conditions may be associated; i.e. one would expect an increasing resistance to

crack extension (higher K_{IC}) as the load rate is raised. As yet crack speed/fracture toughness data are available only for PMMA (Williams et al., 1968; Johnson and Radon, 1972).

(B) The influence of the intrinsic viscoelastic properties of the material :

From the consideration of the analytical treatment of the viscoelastic fracture process by Williams (1965), as referred to in Chapter 4, it is evident that the role of the viscoelastic absorbed energy may be detrimental to the availability of propagation energy required for crack extension. The magnitude of this loss energy, designated W_v , as calculated in Chapter 3, is a function of E'' parameter (the loss component of the complex modulus, E^*). The loss modulus itself is a function of the loading frequency. At low frequencies, where large recoveries may occur in the delayed elastic response of the material, and at high frequencies when there is not sufficient time for extensive stress relaxation to take place, E'' has a small value. (A full description of this aspect of dynamic response has been provided in Chapter 3.) In the intermediate range, E'' has a peak at a level of frequency termed the resonance frequency. Thus depending on which side of the E'' peak a frequency range is selected, the loss modulus will increase or decrease with increasing frequency levels. The specific trend of variations in E'' will appropriately influence the W_v term above and hence the magnitude of the energy available for crack propagation will be affected. Room temperature relaxation data covering a wide frequency range are as yet scarcely available for most thermoplastics. Comprehensive data of this nature are required so as to enable the establishment of the relative positions of the relaxation peaks (α , β , γ etc.) at fixed temperatures for each polymer and hence aid the accurate estimation of the W_v -frequency relationship.

A final point may be made in relation to the association of the impact strength of a material with viscoelastic energy absorption and hence its resistance to fatigue crack growth. High impact toughness in a material will be associated with conditions under which extensive relaxation (in the form of chain rotation) occurs in response to dynamic loading situations. Hence a rise in impact strength will result in a lower cyclic crack growth rate for a given stress intensity factor level.

* * * * *

The discussion in Sections (A) and (B) above point to the possible effect of two sources of influence which have rate dependent characteristics. It is, however, important to at least recognise, from the limited amount of information available at present, that in most situations where conditions of stable crack propagation (in the absence of high material temperature rise) prevail, a considerably larger proportion of the mechanical work done per cycle is stored rather than converted into heat (Tanchert, 1970). This means that the arguments put forward in part 1 of the above discussion in terms of the load rate dependency of fracture toughness etc. should perhaps be treated as the relatively more important factors affecting the rate of crack propagation.

6.3.4 The effect of the cyclic waveform:

The foregoing discussion in relation to the effect on $\dot{a}_N$ of loading frequency variation may be reconsidered in the context of the input waveform effects. Such effects are expected to be more predominant in the slow cycling tests as in high frequency tests a sinusoidal waveform test, for example, will closely approximate to the one with a triangular waveform.

In the present programme of studies the effect of change from a sinusoidal to a triangular input waveform on the cyclic crack growth rate in PC has been studied

(Figure 40). The $\dot{a}_N$ values obtained with a triangular input waveform were generally smaller than those obtained from a sinusoidal waveform test. Similar results obtained from tests on metals have been discussed in Bradshaw et al. (1970). The trend of behaviour in both thermoplastics and metals seem to be similar. In general as the waveform changes from triangular to sinusoidal, then to trapezoidal and then to square forms, the amount of damage due to each load cycle tends to increase. One explanation for such a behaviour may be put in terms of the effect of the increase in hold time at maximum load, as a triangular waveform is gradually changed to a square one.

6.3.5 Variations in parameters β and n as a function of test conditions:

Values of the parameters β and n in the crack growth model, equation (6.1), are functions of the material properties such as the loading frequency, the environmental temperature (as indicated in Section 5.5) and the chemical properties of the containing medium. Yokobori (1969, 1971) analyses attempt to identify the specific role of some of the above parameters in determining the crack propagation rate. Discussions presented in Section 6.2 also demonstrates the manner in which a parameter such as β is related to σ_{yp} and E . Variations in the material properties as a function of the loading conditions, affect the value of β , resulting in a parallel shift - upwards or downwards - of the linear graph of the logarithmic plot of $\dot{a}_N$ against λ .

The slope of such a plot yields the value of parameter n . The present test results indicate significant variations in the value of n as a function of test frequency and temperature. Mukherjee et al. (1969) from tests on PMMA at a frequency of 0.65 Hz, found the approximate value of m (index in equation 2.21) to be equal to 5, which will corresponds to a value of $n = 2.5$ in the λ -model.

(See Section 6.2.) From Figures (37 to 39) the values of n were found to be 2.3, 2.2 and 3.5 at frequencies of 20, 5, and 0.1 Hz. No specific trend in variation of slope n with frequency can be identified and any general conclusions are avoided, since data are limited to only three frequency levels. However, from the results of Mukherjee et al. (1969) and the present test programme, it is proposed that for medium and high frequencies an average value of $n = 2.5$ will yield good approximations to the value of $\dot{a}_N$.

In the case of polycarbonate, due to the unique behaviour of this material it was not possible to compare values of index n in λ -based equation (6.1) as the relative effect of K_m was found to be insignificant, above a certain level of parameter $\dot{K}$. However, one can refer to the results of tests at three frequency levels under constant K_m conditions. Figure (46) shows plots of some data of this nature. The slopes of the graphs change from 3.35 to 3.5 to 4.0 as the frequency (loading rate) is raised from 0.1 to 5 and then to 20 Hz.

For Nylon 6.6 materials, plots of $\dot{a}_N$ against λ data in Figures (42 to 44) show slopes of 3.1, 3.3 and 1.8 for frequency levels of 0.1, 5 and 20 Hz respectively. From comparison of the above values of the parameter $n^\dagger$ it is evident that due to the dearth of sufficient data, no special significance can, at present, be attached to the relationship between the slopes of the graph and changes in loading frequency. It can only be stated that the maximum value of parameter n in equation 6.1, does not exceed 3.5 for the materials tested at three frequency levels. The large differences in the value of n in some cases, may be attributed to the possibility of inclusion of data from tests in which limits of approximate linearity

[†] Note: Only for the PC material the value 4.0 corresponds to parameter m as it is obtained from an $\dot{a}_N \propto \Delta K$ plot.

have inadvertently been exceeded. In such case linear elastic fracture mechanics analysis will fail to produce very accurate data. LEFM concepts will be unable to account for the specific nature of the mechanism (involving inelasticity) of response of the material to cyclic loading even though such concepts as the stress intensity factor may successfully characterise the crack growth process. (A similar point may be raised as an addition to the discussions in relation to the unique behaviour of the PC material.)

In Andrews and Walker (1971) studies of fatigue crack growth in polyethylene (PE), an analysis based on energy density parameters was employed (similar to the model used by Gent et al. (1964) in the failure studies in rubber). The energy level approach is applicable to the non-linear behaviour of PE. Three distinct regions were obtained from plots of crack growth data: Region I, of slope of 3.5, region II a transition zone of not a distinct slope and region III, which contained by far the majority of the experimental results, of slope 2 to 2.5 depending on the specific PE material tested.

Variations in the slope of the logarithmic plots of $\dot{a}_N$ against stress intensity factor dependent terms have been observed also in metals. For example, Miller (1968) found an increase in the slope of such graphs as the effective K_{IC} value for the material was lowered as a result of variations in the test conditions. Barsom (1971a) tests on steels observed an increase in the slope of the graph pertaining to a particular value of ΔK (or K_{max}) parameter. It was proposed that after a certain level of crack opening displacement ($\delta = 1.6 \times 10^{-3}$ inch) had been surpassed, a ductile tear process accompanied the true fatigue damage process and hence higher values for $\dot{a}_N$ were obtained. Hence it is accepted that for some materials, over a very wide test range, graphs of singular slope may not be obtained.

A final point ought to be made in relation to the comparison of slopes from various sets of data: when an attempt is made to relate data on the same material obtained from various test programmes performed by different workers, care must be taken to check for any differences in the basic material properties due to the methods of production and specimen preparation. For example whether the materials possess similar temperature and time histories in their processing stage; whether the manufacturing techniques have differed from one case to another (such as cast or extruded material) and also whether there have been any subsequent (post-processing) treatments such as annealing or drawing. Insufficient care in this respect will yield erroneous conclusions in attempts to correlate various sets of data.

6.3.6 The heat affected zone at the crack tip:

Heat generation at the crack tip and the surrounding material under certain cyclic loading conditions, has a considerable effect upon the rate of crack propagation. The magnitude of the temperature rise, which occurs as a consequence of viscous energy dissipation in polymeric solids, will be a function of the specific material properties in terms of the characteristics of the chemical structure (degree of crystallinity, molecular friction coefficient, etc.), the strain levels and the loading frequency. Hence, under similar loading conditions, the heat generation due to viscous deformation and the frictional movement between the molecular chains is expected to be lower for amorphous thermoplastics than semi-crystalline ones.

Clearly the region in which the highest degree of deformation and reorientation of molecular chains occur will be the plastic yield zone at the crack tip. Hence it may be expected that in thermoplastic materials significant rises in temperature may be associated with certain cyclic loading conditions in which the applied load and frequency levels are such that large crack tip plasticity, at high rates of deformation,

is involved.

Higuchi and Imai (1970) measured the rise in temperature in PC material when subjected to cyclic loading conditions and subsequently proposed a relationship between the temperature rise and the load and frequency levels. The general applicability of this relationship to other materials has not yet been demonstrated and hence it will not be discussed here; however, the results of the above analysis identified the limits of the applied load and frequency levels below which the rise in temperature, for PC, will be insignificant.

At the highest level of frequency used in the present programme (20 Hz), the rises in temperature in all four materials were found to be negligible. This general observation could be confirmed from studies of the fatigue crack surfaces, which clearly showed a step-wise propagation process. In the presence of large internal temperature, thermal softening will occur and the material will lose its basic integrity (Constable et al., 1970).

However, in an exploratory test on PVC ($7 \times 12 \times \frac{1}{2}$ inch thick specimen) with load levels of 2100 - 0 lbf and at a frequency of 20 Hz the following results were obtained:

a) The crack propagated in cyclic steps and a uniform rate of growth was obtained; and

b) At a crack length of about 2.75 inches, when the applied stress in the ligament was equal to about 1000 psi, a very large temperature rise at the crack tip was experienced and material softening occurred. The heat affected zone subsequently extended to a length of about 1.5 inches in front of the crack, Figure (31b). Crack extension at this stage was no longer along the crack plane. The process of fatigue failure under conditions is not measurable in terms of the crack propagation relationship previously discussed.

6.4 Application of the Model to the Fatigue Crack Growth Process in Metals:

In an attempt to test the possibility of application of equation (6.1) to the fatigue failure process in metals, data were extracted from the available literature on the studies of the relationship between the cyclic crack growth rate, $\dot{a}_N$, the range of stress intensity factor, ΔK , and ratio $R (= K_{\min}/K_{\max})$. Data on metals are often presented in terms of $\dot{a}_N$ and ΔK at a fixed value of R . These results have been plotted on the basis of the parameter λ in Figures (47 to 50). λ was calculated in each case from the values of ΔK and R :

$$\text{as} \quad K_{\max} = \frac{\Delta K}{1 - R} \quad (6.15)$$

$$\text{and} \quad K_{\min} = \frac{R \Delta K}{1 - R} \quad (6.16)$$

Then

$$\lambda = \frac{(\Delta K)^2 (1 - R^2)}{(1 - R)^2} \quad (6.17)$$

The data on two Aluminium Alloys (2024-T3 and 7075-T6), obtained at a frequency range of 0.5 to 13.7 Hz, were taken from Hudson (1969); those on (9Ni - 4Co - 0.25C) steel, obtained at a frequency of 0.1 Hz, from Crooker and Lange (1968) and those on cold rolled mild steel from Frost et al. (1971).

The encouraging outcome of this exercise demonstrated that at least for the case of the above metals, the crack growth rate can be determined from a single set of data irrespective of values of ΔK and R . From the designer's point of view successful application of the model will be considered as an important step in simplifying the $\dot{a}_N$ prediction procedure and reduce the complexity of the empirical data as exist in the present form, with the index and constant in the power law model (equation 2.21) having a different value at various R levels.

In some metals the effect of K_m has been shown to be relatively small, Frost et al. (1971) on some aluminium alloys and steels for example; hence it is required that further checks on the behaviour of metals are carried out before the λ relationship (equation 6.1) is adopted as a criterion for the prediction of fatigue crack growth rate in these materials.

The use of the parameter λ may open the possibility of treating the combined creep and fatigue data. Extensive study of this particular aspect of failure will depend upon the availability of the LEFM based creep and fatigue data obtained under similar environmental conditions.

Finally, correspondence of the λ model to fatigue crack propagation in metals and polymers suggests that, at the macroscopic level, the controlling mechanisms of crack growth process may be similarly characterised by the stress intensity factor concept.

6.5 The State of Stress and the Shape of the Crack Front:

The rate of growth of fatigue cracks in a material, will be influenced by the variations in the fracture toughness value of the material. If component geometry and loading arrangements are such that plane-stress or plane-strain conditions prevail, different crack propagation rates will be observed. In plane-strain conditions, where the critical stress intensity factor will be lower than plane-stress values, application of similar loading conditions will yield a higher rate of growth. Dependence of $\dot{a}_N$ on material fracture toughness value has been discussed by Miller (1968) in terms of the variations in the power law index. The value of index n in equation (2.21) was greater for a material with lower K_{IC} .

In the present study of behaviour of polymers, $\dot{a}_N$ data under various loading frequencies can be taken as evidence of similar behaviour. As the fracture toughness in the stable range increases with the load rate (or crack speed), under the same applied load levels, a smaller crack growth rate is obtained.

Transition from the plane-stress to the plane-strain state of stress (in the regions of the specimen surface and interior) and subsequent anticipated larger growth at the centre of the specimen is evident from the parabolic shape of the crack front obtained in all tests in this series. Similar crack front shapes were obtained by Mukherjee et al. (1969).

6.6 Fracture Surface Studies:

The study of the specific features of the crack surfaces in failed specimens can make a significant contribution to the recognition of the particular pattern of crack propagation under the static or the cyclic loading conditions.

From amongst the materials tested in the present study, only in the case of the PMMA was it possible to observe and identify regular features representing various patterns of crack propagation under different loading conditions. These observations (obtained at a magnification of 24:1) will be briefly described in the following section.

6.6.1 Comments on the specific features of the fatigue failure surfaces in PMMA:

The appearance of the failure surface of a fatigued specimen of PMMA has been previously described (e.g. Mukherjee et al., 1969). For centre-notched specimens it consists of the following three distinct regions: (a) the initial saw-cut with sharp stress raisers on each side; (b) the fatigued zone (containing the striations) which has a silver-coloured appearance and (c) the unstable fracture zone which usually

contains 'fan' marks, i.e. lines originating from the crack tip and fanning out towards the specimen boundaries.

In the case of PMMA, the presence of distinct fatigue striations were observed at a magnification of 24:1, Figure(51a). The two regions shown in this micrograph correspond to two different levels of the applied frequency (0.06 Hz and 0.1 Hz). Fatigue striations of equal spacing in each region correspond to an approximately constant rate of crack propagation per cycle of load application. The effect of frequency variation on the value of the $\dot{a}_N$ parameter is also evident from the decrease in the spacing between the lines as the frequency level is raised.

The striations observed in the fractography of the fatigued region should be distinguished from another set of lines which are also observed on the failure surface. These lines are of similar form but the spacing between them increases as the free boundary of the specimen is approached, Figure (51b). They correspond to the fast (unstable) fracture process which occurs as the fatigue crack reaches a critical length, and are associated with the interaction of the stress waves travelling towards the free boundary and those reflected from it; see also Wolock and Newman (1964).

The surface features of the fatigued zone in Figure (51a) correspond to brittle fracture striations as discussed by Frotyth (1961). For relatively more ductile type of fatigue crack growth (i.e. in the presence of extensive plasticity) the fatigued surface in metals consists mainly of cyclic striations. However, for the case of brittle crack propagation an additional feature in the form of narrow bands (ridges) approximately parallel to the crack growth direction may be observed in fractography. These bands indicate the presence of cleavage mechanisms. The presence of such bands is expected to be more prominent in fatigue failures under the environmental conditions which enhance the brittle behaviour of a material.

For PMMA, the study of the fatigue failure surfaces obtained from tests at a temperature of 3°C confirmed the suggestion that the failure at the lower end of the temperature range occurred under the conditions of enhanced brittle behaviour of the material. The cyclic crack propagation pattern was again observed (Figure 51c); however the narrow bands associated with the presence of cleavage mechanisms were found to occupy the surface to an extent greater than that on the surfaces obtained from room temperature tests.

The effect of high environmental temperature was also observed on the surface of the fatigue zone in a specimen tested at a temperature of 38°C (Figure 51d). Micrographs c and d were obtained from specimens subjected to the same stress intensity factor levels and the same loading frequency ($K_{\max} - K_{\min} = 600 - 200 \text{ psi}\sqrt{\text{in}}$; $f = 5 \text{ Hz}$). Thus, a direct comparison of these two micrographs will yield some information on the relative states of material ductility under the two test conditions. In Figure(51d) large regions, apparently featureless at the magnification of 24:1, can be observed. The existence of these features indicates the possible presence of such processes as the material 'healing'. The failure process still occurs in a cyclic manner as indicated by the presence of small striations.

CHAPTER SEVEN

GROWTH OF CRACKS UNDER COMPLEX LOADING CONDITIONS

The main purpose of the discussion in this chapter is to present a method for the determination of the path of crack extension in fatigue and fracture under arbitrary loading conditions. The effects of initial crack orientation direction on the fracture toughness value of the material and on the cyclic initiation life and rate of crack propagation in fatigue are also indicated.

7.1 Introduction:

The study of the growth of cracks or flaws arbitrarily oriented in relation to the boundary loading direction is a natural extension to the considerable amount of research work directed towards the analysis of the fracture processes under uniaxial loading conditions. In its earliest stages, the problem of failure of components subjected to biaxial loading conditions was examined empirically from tests such as fracture and fatigue of internally pressurised cylinders or rotating discs. Valluri (1965) and Dong (1970) in studies of the biaxial failure of metals and polymers developed failure envelope concepts in which failure planes described by the principal stress components were used as loci of points of limiting stability for fracture under multi-axial loading conditions. A similar criterion based upon the equivalent-stresses, used initially by Griffith (1924), has been employed by Joshi and Shewchuck (1970). The exact form of this criterion will be described later. Tuba and Wilson (1970) proposed the use of a criticality plane described in terms of the three stress intensity factor components K_I , K_{II} and K_{III} , so as to enable the application of the LEFM parameters to the extension of cracks subjected to combined mode crack opening con-

ditions. Direct application of the LEFM concepts to these loading conditions will be discussed in a separate section. It suffices to say that at present no general closed form solutions for the combined mode failure problem are available. However, from the practical application viewpoint, as will become clear later, some success has been achieved in the analysis of the effect of the crack orientation direction on parameters such as the fracture toughness of a material.

A factor of considerable importance in the process of fracture under arbitrary loading conditions, is the determination of the path of crack extension. Irwin (1957), in the discussion of the stress field parameters which significantly influence the fracture process, proposed that cracks will grow in a direction controlled by the orientation of the maximum stress direction at the crack tip. At a later stage Forsyth (1961) proposed the hypothesis of division of the fatigue failure process in metals to stages I and II (see section 1.1), where in stage I, flaw growth occurs parallel to the crystallographic plane of maximum shear and in stage II, the macroscopic crack propagates in a direction perpendicular to that of the local maximum stress. The hypothesis that cracks will extend in a radial direction, from the tip, normal to the direction of maximum tangential stress was confirmed by Erdogan and Sih (1963) and also by Cotterell (1965 and 1966) who from the consideration of the second term in the power series solution of the stress field in a cracked body, concluded that a crack will propagate along the principal stress plane passing through its tip.

The aim of the present study was to investigate the specific role of such parameters as the component geometry, the initial crack orientation and applied load levels, in the determination of the path of crack extension under complex loading conditions. Some exploratory work on the effect of crack inclination on the critical

load at fracture, the crack growth initiation life and the rate of cyclic crack propagation in fatigue will also be discussed. The experimental work was performed on a quasi-brittle thermoplastic (PMMA). However, the phenomenological aspects of crack behaviour in fracture analysis can be equally applied to the failure of metals.

7.2 Analysis of the Stress Field in the Vicinity of an Inclined Crack:

In addition to the analyses based upon the equivalent stress term, used initially by Griffith (1924), various other attempts have been made to study the characteristics of the crack tip stress field. The effect of crack inclination angle, β , on the shear stress distribution in the neighbourhood of the crack was analysed by Schijve (1964); similar studies were carried out by Hasseem and Affimiwala (1972) who investigated the stress distribution pattern using photoelasticity techniques. Studies of variations in strain concentration with increased biaxiality were performed by Wilson and White (1971) who used both the finite elements analysis and photoelastic measurements for the analysis of stress fields in cross-shaped specimens.

In terms of the LEFM concepts, in the presence of both the opening mode and the sliding mode crack face movements, the strength of stress singularity at the crack tip is controlled by the stress intensity factors K_I and K_{II} (Paris and Sih, 1965, Tuba and Wilson, 1970). Hence in the analysis of the fracture strength of a material subjected to biaxial loading conditions, a relationship combining K_I and K_{II} must be employed. Attempts have been made to develop such a relationship by Erdogan and Sih (1963) and Tuba and Wilson (1970) for isotropic materials and by Wu (1968) for anisotropic materials.

The instability criterion proposed by Erdogan and Sih (1963) was of the general form:

$$a_{11} K_{IC}^2 + 2a_{12} K_{IC} K_{IIc} + a_{22} K_{IIc}^2 = 1 \quad (7.1)$$

The three coefficients a_{11} , a_{12} and a_{22} must be determined experimentally. (Note that when the crack propagates in its own plane, $a_{11} = a_{22}$ and $a_{12} = 0$. Thus only one coefficient, a_{11} , must be determined by empirical methods.)

Use of such a relationship would require the knowledge of K_I and K_{II} parameters determined from the component geometry and loading conditions. However, analytical difficulties have prevented the extensive development of such solutions (Brezhinskii, 1965) and only with the aid of such techniques as the boundary collocation method or the finite elements analysis has it been possible to obtain reasonably accurate solutions for K_I and K_{II} parameters (Wilson, 1969; Iida and Kobayashi, 1969).

7.2.1 Plastic deformation at the crack tip:

The influence of biaxial loading on the extent of plastic deformation at the crack tip has not yet been formulated in terms of a general criterion. On the one hand some experimental data - to be discussed in Section 7.8 - indicate an increase in the degree of brittleness of the material when it is subjected to biaxial loading conditions, hence implying that the extent of crack tip plastic deformation is reduced when the load parallel to the crack plane is increased (Joshi and Shewchuck, 1970). On the other hand there is a considerable amount of evidence that in many other cases the introduction of external load biaxiality enhances the fracture toughness of the material (Pook, 1970; Kibler and Roberts, 1970). Also, Ang and Williams (1961) showed that if von Mises criterion for yield is adopted, under the conditions of constant normal stress, the estimated plastic zone size increases as the stress parallel to the crack is increased.

Due to the complexity of the problems associated with the presence of a degree of non-linearity introduced by crack tip plasticity and the important influence of parameters such as the state of stress (plane-stress or plane-strain) and Poisson's ratio (Swedlow, 1965) a specific criterion uniquely describing the effect of biaxiality in all materials is as yet unavailable.

The crack tip plastic zone sizes, related to the three modes of crack opening are given as follows (Pook, 1970):

$$r_p = \frac{1}{2\pi\sigma_{yp}^2} \left[K_I^2 + 3K_{II}^2 \right] \quad \text{plane-stress} \quad (7.2)$$

and

$$r_p = \frac{1}{2\pi\sigma_{yp}^2} \left[K_I^2(1-2\nu)^2 + 3(K_{II}^2 + K_{III}^2) \right] \quad \text{plane-strain} \quad (7.3)$$

In plane problems, in the presence of modes I and II, the plastic zone is no longer symmetrical about the crack plane, Figure (52). The direction of initial crack growth will be approximately the direction in which the extent of yielding is minimum (Kobayashi et al., 1969). This will be the direction in which the shear component of the local stress has the least possible value (zero) i.e. in the direction of a principal plane. The discussion of the crack growth direction is deferred to Section 7.6.3.

7.2.2 Determination of the K_I and K_{II} parameters:

A general closed form solution for the combined mode problems is as yet unavailable. However, for the simple case of an inclined crack in an infinite plane subjected to uniaxial loading, K_I and K_{II} parameters may be calculated from (Sih et al., 1962):

$$K_I = \sigma \sqrt{\pi a} \sin^2 \beta \quad (7.4)$$

and

$$K_{II} = \sigma \sqrt{\pi a} \sin \beta \cos \beta \quad (7.5)$$

where β is the angle between the crack plane and the loading direction and σ is the magnitude of the applied boundary stress.

Various combinations of numerical and empirical techniques have been used to calculate K_I and K_{II} parameters, by varying degrees of approximation.

For the general case of a crack loaded in such a manner so that both the opening and sliding modes are present, the elastic state of stress in the vicinity of its tip may be described in terms of the local polar coordinate system as (Paris and Sih, 1965):

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] - \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left[2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right] \quad (7.6)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (7.7)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} + \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] \quad (7.8)$$

The maximum in-plane shear stress, τ^0 , can be obtained from:

$$\tau^{02} = \left[\frac{\sigma_y - \sigma_x}{2} \right]^2 + \tau_{xy}^2 \quad (7.9)$$

Using equations (7.5, 7.6 and 7.7):

$$\tau^0 = \frac{1}{2\sqrt{2\pi r}} \left[(K_I \sin \theta + 2K_{II} \cos \theta)^2 + (K_{II} \sin \theta)^2 \right]^{\frac{1}{2}} \quad (7.10)$$

Differentiation of τ^0 with respect to θ will yield the angular direction, θ_m , along which the maximum shear stress occurs (Erdogan and Sih, 1963; Smith and Smith, 1970):

$$\left(\frac{K_{II}}{K_I}\right)^2 - \frac{4}{3} \left(\frac{K_{II}}{K_I}\right) \cot(2\theta_m) - \frac{1}{3} = 0 \quad (7.11)$$

This relationship thus yields a function for θ_m in terms of K_{II}/K_I ratio.

It is possible, through the implementation of the photoelastic studies, to determine another relationship between K_I and K_{II} using equation (7.10). From isochromatic fringe patterns, for each of which the value of τ^0 is known, values of r , θ and τ^0 are substituted in equation (7.10). Angle θ_m will again correspond to the direction in which τ^0 has reached a maximum (Smith and Smith, 1970). K_I and K_{II} are hence determined from the two relationships thus obtained.

7.2.3 Use of the finite elements method in the calculation of the stress intensity factors:

If the states of stress or displacements in the neighbourhood of the crack tip can be determined with reasonable accuracy, then the stress intensity factors K_I and K_{II} may be calculated from the stress field equations presented in the previous section. Facilities offered by a technique, such as the finite elements method, thus immediately become apparent. As a rule of thumb, the magnitudes of the stress components in a region $r < \frac{a}{20}$ have to be calculated with the maximum possible accuracy. Although Kobayashi et al. (1969) proposed that this restriction may be relaxed significantly along the crack boundary, i.e. for $\theta = \pi$.

Iida and Kobayashi (1969) used the finite elements technique to calculate the K_I and K_{II} components of the combined mode stress intensity factor. These calculations were carried out over a range of crack lengths corresponding to the results of

fatigue crack growth tests. Plots of K_{II} and K_I as a function of crack extension indicated an immediate drop in K_{II} component and a rise in K_I . In other words, the crack turned to grow under the conditions of maximum K_I .

7.3 Empirical Studies of the Crack Growth Path:

Experimental studies carried out in the present work may conveniently be divided into the following two sections:

- 1) Uniaxially loaded specimens containing inclined centre notches (Test series UA and AG).
- 2) Biaxially loaded specimens containing arbitrarily oriented side and centre notches (Test series RR, ROR and RTR).

In the following sections, the general trends of behaviour, observed from these tests will be discussed.

7.3.1 Equipment:

Uniaxial loading tests (Series UA and AG) in both fatigue and fracture studies were performed on an Avery hydraulic testing machine with a maximum load capacity of 25000 lbs.

Biaxial loading tests (Series RR, ROR and RTR) were carried out using the Dowty electrohydraulic fatigue testing machine (Fully described in Section 5.4.4.) . A loading frame was designed and added to this machine in order to enable the application of loads in two perpendicular directions; see figure (53). The frame, which was placed between the two main columns of the Dowty machine, supports side loads applied through a steel wire - pulley arrangement shown in the figures. Maximum side load used in this series was 100 lbs on each side.

7.3.2 Material selection and specimen design:

(A) The material:

The material selected for use in these tests was polymethylmethacrylate (PMMA) supplied in the form of 0.25 inch thick and 1 inch thick cast sheets. The main reasons for this choice were the quasi-brittle behaviour of this thermoplastic material and its transparency. The latter characteristic enables very precise monitoring of the crack position and its progress through the body of the material.

In addition, availability of the data from uniaxial loading tests in this material, enables a direct comparison with the results of arbitrarily loading test in order to analyse the specific effects of multiaxial loading conditions.

The PMMA material is being employed as a convenient model material to study the phenomenological aspects of fracture and fatigue processes and hence the results may equally well be applied to the failure processes in metals.

(B) The specimens:

General arrangements of the specimens are presented in Figure (54).

Specimens used in the uniaxial loading tests were of dimensions $5\frac{1}{2} \times 11 \times \frac{1}{4}$ inch in UA series and $5\frac{1}{2} \times 11 \times 1$ inch thick in AG series. A centre hole of $\frac{1}{4}$ inch diameter was drilled in these specimens and using a hack-saw an initial crack of length of 1 inch was produced at angles of 15° , 30° , 45° and 60° to vertical direction. Sharp stress raisers were created at the ends of the centre notch using razor blades.

In AG series of tests, surface grooves were produced at various angles, through a vertical milling process. Subsequently centre cracks colinear with the grooves were produced as described above.

In biaxial loading, specimens of dimensions $9 \times 18 \times \frac{1}{4}$ inch were used, Figure (54c,d). Dovetail forms were cut on the sides of these specimens and suitably designed blade-form components (Figure 55) which fitted the dovetails closely, were used to apply the side loads. This particular design of the specimen and the loading components were chosen so as to represent the practical engineering problem of the failure in turbine rotors and blades.

At the initial stage four blades on each side were used; however this arrangement was found to be unsuitable as it resulted in extremely long initiation lives. In the present test series only two blades on each side were employed.

Specimens used in ROR tests, contained a circular hole of diameter of $2\frac{1}{2}$ inches, at the centre. Whereas in RR tests initial cracks of length of about $\frac{1}{8}$ inch were produced at appropriately chosen angles at the root of the dovetail formations, in ROR tests initial notches were created at points around the periphery of the centre hole at various selected angles.

In RTR tests, specimens of the type used in RR tests were employed. However, in this series of tests the specimens were loaded in a transverse direction (see Figure 53b) so as to study the failure of the dovetails as representative of the failure in blade-root regions around the periphery of the turbine rotors.

7.3.3 Methods of experimentation:

(A) Uniaxial loading tests:

Monotonic fracture and fatigue tests under uniaxial loading conditions were carried out with initial centre cracks inclined at angles of 15, 30, 45 and 60 degrees to the loading direction. The cyclic tests were carried out at a frequency of 0.25 Hz. The growth of crack on two fronts was monitored by a cathetometer. In fatigue tests the effect of initial crack angle on N_i , the cyclic crack growth initia-

tion period, was evaluated. The specific level of the critical stress intensity factor corresponding to final unstable fracture was also measured.

(B) Biaxial loading tests:

Tests under static and cyclic biaxial loading conditions were performed with initial cracks at various angles selected on the basis of the results of the finite elements analysis of the stress fields in the specimen. In some cases the initial notches were so produced as to fall on a major principal plane and in others they were of arbitrary orientation.

In RR tests the curvature of path of crack growth in the fractured specimens was examined using a Hilger optical projector. Magnification level of X25 was found to be sufficient.

(C) Determination of the applied load levels:

Stress ratios in RR, RTR and ROR tests were calculated on the basis of stress levels acting across the narrowest section of the dovetails. For example: if the applied blade load is equal to 30 lbs on each side, then each blade will carry 15 lbs and hence the load acting on the narrowest section of the dovetail formation ($\delta = 10$ mm, Figure 55) will be equal to 30 lbs. This load will give rise to an average stress of:

$$\frac{15 \times 25.4}{10 \times 0.25} = 152.4 \text{ psi .}$$

This stress was designated the blade (or radial) stress σ_B and the levels of the hoop stress, σ_H , were so chosen as to give various values to the ratio σ_B/σ_H .

7.4 The Method of Analysis of the Stress Fields in the Test Specimens:

Stress distribution in RR, RTR and ROR specimens subjected to various boundary loading conditions were analysed using the finite elements stress analysis technique. Elastic solutions were found for all the specimens in uncracked state.

The computer program used for this analysis was based on the program developed by Hayes (1970). The program treats a displacement based method of discrete element analysis with the fundamental assumption that the displacements within each triangular element are linear functions of the coordinates. A full description of the basic program may be found in Hayes (1970).

In its present form, additional subroutines have been introduced into the program so as to make possible the calculation of the principal stress values and the angle of inclination of principal planes at the nodes of the finite elements mesh. These results were obtained in the form of punched cards and could subsequently be plotted using Calcomp and Kingmatic plotting facilities to obtain the principal stress distributions or isoclinic contours. (Isoclinics are the loci of nodal points at which the principal plane inclinations relative to the horizontal direction, are equal.)

In Figures (56 to 58) the finite elements meshes used for RR and ROR specimens are shown. Advantage was taken of the specimen symmetry and only a quarter of each plate was analysed.

7.5 Distribution of the Stress Fields in the Specimens:

Results of the finite elements analysis of stress distribution in RR and ROR specimens were plotted using the Calcomp and Kingmatic computer plotting facilities. As the main interest lay in the study of possible crack growth directions, the plotting programs were so arranged that the outputs were in the form of isoclinic contours.

The choice of isoclinic contours rather than the principal stress plane contours, was based on the fact that some photoelastic data of similar nature were available and hence a direct check on the accuracy of the results of the finite elements analysis could be made, see Figure (59).

From the distribution of the isoclinics, plots of principal plane directions could easily be obtained. One way of achieving this would be to use the specific angular value by which a contour is designated. This will be the value of the angle at which all principal planes situated on the particular contour are inclined to the horizontal. By drawing an appropriately inclined short line at each point on the curves, contours of principal plane directions will be constructed.

Another method would be to take advantage of the fact that the isoclinics cross the free boundary of the body at a point positioned such that the tangent to the boundary curvature at that point is parallel to the direction of one of the principal planes. This particular characteristic arises from the fact that at the free boundary the major principal stress acts in a direction tangential to the surface.

The isoclinic patterns obtained in RR specimens corresponding to RR1 loading conditions, were compared to the results from photoelastic analysis and a very good degree of correspondence was observed. In Figures (60 and 61) a sample plot of isoclinic contours for biaxial loading conditions under two extreme cases of $L_x = 0$ and $L_y = 0$ are presented, (L_x and L_y being the load in x and y directions, respectively).

Principal plane contours obtained from these plots have been sketched onto these graphs. The minimum principal stress trajectories cross the periphery of the circular centre hole in a radial direction, as maximum principal stresses act tangentially in relation to the circle.

7.6 Discussion of Results:

7.6.1 The effect of crack inclination on fracture toughness:

A series of tests were carried out, using the UA specimen (rectangular plates with inclined centre cracks), to study the variations with angle β of the apparent fracture toughness, K_{AC} , of the material. The term apparent is employed as the crack length parameter used in the calculation of the critical stress intensity factor values did not represent a measurement along the whole length of the crack. It represented the length of the horizontal section, ℓ , of the crack plus the projected length of the inclined section at the centre, Figure (54a), thus:

$$2a^* = 2a \sin \beta + 2\ell \quad (7.12)$$

In Tables 17 and 18 a summary of the results obtained from this series of tests is presented.

It is apparent that irrespective of the initial angle of crack inclination, the crack continues to propagate in a cyclic manner until the achievement of a critical length. This limiting value of crack length, $2a$, combined with the maximum applied stress level, yields values of parameter K_{AC} , the apparent fracture toughness of the material, Table 18. Relatively small variations in the level of the K_{AC} parameter were obtained from the results of various tests. Under uniaxial loading conditions ($\beta = 90^\circ$), the fracture toughness value, K_{IC} , was found to be about $1100 \text{ psi}\sqrt{\text{in}}$; data in Table 18 show that the K_{AC} levels reached at the end of fatigue loading processes examined cover a range from 745 to $796 \text{ psi}\sqrt{\text{in}}$. The relatively smaller values of K_{AC} may be attributed to the fact that the crack length used in the calculations, $2a^*$, is smaller in magnitude than the physical length of the crack. The case, in fact, corresponds to the conditions of the presence of a zig-zag crack.

From this set of data it is also evident that the effective total crack length, $2a^*$, is nearly equal for all values of β , when applied load levels are maintained at a constant level: 1.30, 1.37 and 1.37 inches under load limits of 560-224 lbf and 1.70 and 1.80 inches under load limits of 448-112 lbf. This indicates that the extent of crack growth in the direction perpendicular to that of loading, prior to the achievement of unstable fracture conditions, increases with decreasing angle β .

The angle of inclination has a very significant influence on the value of the effective stress intensity factor at the tip of the initial crack in a static fracture process. This influence plus the effect of angle β on the cyclic initiation life and crack propagation rate will be discussed in the following sections.

7.6.2 The cyclic initiation life and the subsequent rate of crack propagation:

The cyclic initiation life, N_i , discussed in Chapter 4 is primarily influenced by the magnitude of the stress intensity factor parameter which characterises the stress field in the vicinity of the crack tip. Hence it is expected that in such cases where the initial crack direction is inclined to the loading direction, the value of N_i will be influenced by the magnitude of angle β . As discussed in a previous section (7.2.2), the value of K_I component of the combined stress intensity factor increases with increasing β . Thus it is anticipated that a shorter initiation life will result from an increase in the value of β . Results of experimental studies, Table 17, show that such a prediction based upon the above analysis, is indeed accurate. In tests UA2, UA3 and UA4 where the angle β had values of 60° , 30° and 45° respectively, the initiation life, N_i , was found to be 6500, 25700 and 13400 cycles. These results indicate a ratio of 1:2.0 : 3.5 in N_i values as angle β is varied from 60° to 45° to 30° .

In the second set of tests (UA5 and UA6) performed under the conditions of a lower level of maximum applied load, relatively larger values for N_i were obtained; however the ratio of N_i values for the cases of $\beta = 60^\circ$ and 30° is approximately the same as above, i.e. 1:4.

In this series of tests, the initiation period constituted by far the largest proportion of the total fatigue life (N_i values in tests with $\beta = 90^\circ$ and under similar loading conditions were of the order of a few hundred cycles). In Table 17 total number of cycles to failure are also included. The figures indicate an increase in total life with decreasing angle β . This increase was mainly due to the differences in the initiation period, although the rates of crack propagation in all cases were also different. Wherever the value of N_i is smaller, the rate of propagation in the subsequent stage of the fatigue process is greater. This behaviour would be expected as both processes are, in the main, influenced by the K_{IC} term.

In propagation, the large value of β means a larger effective initial length ($\sin \beta \rightarrow 1$ as $\beta \rightarrow \pi/2$) and under constant applied stress level, this would correspond to a higher level of stress intensity factor. Hence a greater value for crack propagation rate is expected.

7.6.3 Determination of the crack growth path in fatigue and fracture under complex loading conditions:

From the finite elements stress analysis of RR specimens the position and the pattern of distribution of principal stress planes around the periphery of the dovetail roots at the sides of the specimen, were determined. The first set of tests in RR series were carried out with initial cracks produced along a maximum principal plane contour. In Table 19 a record of various test parameters (initial crack orientation angle and biaxial load levels) are provided.

(A) The RR tests:

Test RR1 was carried out in order to obtain a measure of the fracture load with an initial crack inclined at a small angle to the horizontal direction, $\theta = 10^\circ$ in Figure(55a). It was found that with blade loads of 50 lbs on each side (applied through two blades) the load L_y could be raised up to 2000 lbs prior to fracture. Test RR2 was performed under $\sigma_y : \sigma_x$ stress ratio of 0.8 : 1 with initial crack inclination of $\theta = 25^\circ$. After 4×10^6 cycles no growth was observed. This behaviour may again be discussed in terms of the relationship between the cyclic initiation life, the crack angle and applied stress levels. Evidently in this particular case, much higher values of L_y would be needed in order to initiate the crack propagation process. Subsequent tests, RR3, RR4 and RR5, were performed under stress ratios of 4.3:1, 3.5:1 and 4.3:1, respectively. The initial angle of crack inclination varied from $\theta = 10^\circ$ to $\theta = 15^\circ$. Lower values of θ were not chosen as the test would then approach the case of the horizontal crack problem and a larger value of θ was avoided as it would result in the immediate fracture of the dovetail. The latter phenomenon, however, has been separately investigated using the RTR specimens (Part C).

The specific values of angle θ used in these tests were selected from the photo-elastic and the finite elements studies (Table 20). These values correspond to the angular position of the peak stress concentration point appropriate to each level of the applied stress ratios.

Crack growth paths obtained from the above tests demonstrated that if the initial crack lies along a minimum principal stress trajectory, it continues to grow along that contour. The correspondence of experimental observation to the results of the finite elements analysis is demonstrated in Figure (62). The dotted lines in this

figure are the isoclinic contours and the full lines are the appropriate minimum principal stress trajectories. The path of crack growth is indicated by the direction of the arrows.

In practice, however, the crack in its original stage may have an arbitrary orientation and may not necessarily lie on a plane passing through the theoretical peak stress concentration point. Test RR7 was devised to study this particular aspect. With an applied stress ratio of 4.3:1 the maximum stress concentration occurs at $\theta = 10^\circ$; however the initial crack was positioned at $\theta = 30^\circ$. The result showed that the crack continues to propagate along the principal plane passing through its tip. This plane obviously becomes the specific principal stress plane, perpendicular to which the maximum principal stress of highest magnitude acts. This outcome confirms the proposals of Cotterell (1966) in relation to the expected path for crack propagation, as discussed earlier.

(B) The ROR tests:

Due to the specific geometrical configuration of the RR specimens, the contours of maximum principal stress plane do not possess a pronounced curvature and hence use of optical magnifiers is often necessary in order to establish the exact contour of the fracture planes. This geometry was thus modified into the form of ROR specimens where the presence of the centre hole of $2\frac{1}{2}$ inches diameter ensures the existence of principal plane contours of large curvature. The stress field in this specimen subjected to biaxial loading conditions has been analysed, Figures (60 and 62). The principal stress contours are again indicated by full lines in these figures. Due to the circular shape of the boundary, the minimum and the maximum principal stresses at each point on the periphery will be in radial and tangential directions respectively.

Data for ROR tests have been presented in Table 19. In tests ROR1, two initial notches were produced at $\beta = 45^\circ$ position: one along the 45° radius and the other at 30° to the horizontal direction, see Figure (63a). Under cyclic loading conditions at a frequency of 5 Hz and with $L_y = 840 \pm 0$ lbs and static loading of $L_x = 15$ lbs on each side, fatigue crack growth was observed only on the 30° crack side, indicating again a much lower initiation life for the more favourably positioned crack. In ROR2 test with both cracks oriented in a radial direction and positioned at $\beta = 60^\circ$, fatigue crack growth was observed on both sides.

A monotonic fracture test was also carried out with radial cracks at $\beta = 45^\circ$ position (Test ROR3). The path of crack extension under unstable fracture conditions was found to be closely coincident with the predicted contour, Figures (63b and 64b).

In test ROR4, with the minimum level of the applied load cycle $\neq 0$, the two cracks were produced one along $\beta = 15^\circ$ radius and the other at $\beta = 45^\circ$ position but in a horizontal direction, Figure(63c). The crack growth path obtained in this specimen (where one of the initial cracks was not oriented along a principal plane) is indicated in this figure. It was found that the horizontal crack turned to grow along the minimum principal stress trajectory passing through its tip. The exact contour of this plane had been predicted from elastic stress analysis of the uncracked specimen.

(C) The RTR tests:

A brief series of tests were performed to study the pattern of behaviour of cracks leading to the fracture of dovetails supporting the blades. Clearly, this type of failure occurs when the $L_x : L_y$ load ratio is such that the highest stress is applied across the neck of the dovetail. Distribution of the minimum principal stress trajec-

tories in the region of dovetail root is shown in Figure (59). In RRT tests the RR specimens were held in a transverse direction (Figure 53b) and various initial cracks were used to study the fracture path. Figure (61) shows that, again, the path of crack propagation was coincident with the principal plane contour.

(D) Comments on the crack growth path determination:

The above results clearly indicate that it is possible to predict the crack growth direction in a component subjected to arbitrary loading conditions. Stress analysis of the uncracked body yields information on the distribution of the minimum principal stress trajectories and previous discussions of experimental data showed that cracks continue to grow in a direction closely approximating those predicted from analysis of the stress field. Naturally, growth of the crack disturbs the stress distribution pattern in the specimen. This will cause a change in the magnitudes of the specific values assigned to each principal plane, i.e. the plane on which the crack is propagating acquires the highest level. The principal plane is a plane of zero shear and the extension of the crack along it does not disturb the pattern of local symmetry, in relation to this plane, of the stress field. As indicated by Cotterell (1965, 1966) the crack propagation will occur along this plane of symmetry, i.e. along the major principal plane.

For the case of an initially mis-oriented crack, one must consider the shape of the crack tip curvature itself. Principal plane contours will cross this curve in radial directions. Schematic representation of the configuration of principal plane contours in a rectangular plate, containing a circular or elliptical centre hole are given in Figure (65). Depending upon the ratio of σ_x / σ_y the stress concentration point will move around the periphery of the centre hole. For example, for the case of the circular arc in Figure (65b), if $\sigma_x = 0$, the maximum stress concentra-

tion points will occur at regions 1 and 3. If $\sigma_y = 0$, these points will move to 2 and 4. And for any other values of ratio σ_x/σ_y , as σ_x is raised, the maximum stress concentration points move gradually along 1-4, 1-2 and 3-2, 3-4 paths. Thus, variation of the applied load loads does not cause any changes in the configuration of these planes and only the specific stress magnitudes are influenced.

Figure (65c) shows the direction of principal planes intersecting the large radius arcs at the two ends of the ellipse. If $\sigma_y > \sigma_x$, the maximum stress concentration point will be in regions 1 and 3, as confirmed by the photoelastic stress analysis data of Kobayashi et al. (1969) and Hasseem and Affimiwala (1972).

In a rectangular plate the direction of principal planes will be parallel to the plate boundaries, Figure (65a). If in practice it were possible to produce cracks with zero tip radii, then the crack, irrespective of its orientation, would turn to grow in a direction parallel to the boundary carrying the greater load; however, the finite radius at the crack tip and subsequent curvature of the principal planes in the vicinity of the tip, signify that a small curvature in crack path, at the initial stage of propagation, should be expected. A short distance away from the crack tip, the contours become straight again.

This behaviour is, indeed, evident from the fracture contours obtained in UA series of tests.

The principal plane direction (with shear stress equal to zero) can be theoretically determined in a stress field for which a closed form solution exists. For instance, for the case of uniaxially loaded rectangular plate containing a slanted centre crack, Erdogan and Sih (1963) equated the shear stress equation to zero to find the maximum principal plane direction:

$$\sin \theta_0 + (3 \cos \theta_0 - 1) \cot \beta = 0 \quad (7.13)$$

It is thus possible to produce a range of data relating the angle of orientation to the angle of extension over the range of $0 \leq \beta \leq \pi/2$; Erdogan and Sih (1963), Williams and Ewing (1970).

Theoretical predictions have compared favourably with experimental data to a large extent (Erdogan and Sih, 1963). However, in the case of a vertical crack, i.e. $\beta = 0$, the above equation predicts an extension angle of $\theta_0 = 70.5$ degrees, assuming the case of a crack in pure shear conditions. However, the crack is not in pure shear (from equation 7.4, as $\beta \rightarrow 0$, $K_{II} \rightarrow 0$) and as shown by Williams and Ewing (1970) a crack extension angle of about 90 degrees is obtained in practice, confirming the hypothesis of growth along a principal plane direction. In the latter report it was restated that the discrepancy with the analytical solution in part of the test data of Erdogan and Sih (1963) was due to the omission of the effect of the stress component parallel to the crack surface (Cotterell, 1966). The corrected formulation showed a better correspondence to the test data over the whole range of $0 < \beta \leq \pi/2$.

(E) Crack growth path in cross-shaped specimens:

As an additional confirmation of the "path selection" aspect of the phenomenological behaviour of cracks, the experimental studies of Kibler and Roberts (1970) were analysed: these authors studied the fatigue and fracture processes in cross-shaped specimens of aluminium alloys (6061-T4) and (6061-T6). The specimen contained centre cracks parallel to one arm and were subjected to biaxial loadings with various L_x/L_y ratios. The symmetrical geometry of this type of specimen allowed the designation of a finite elements mesh confined to only a quarter of the specimen, Figure (66). Plots of isoclinic contours obtained from the stress analysis are presented in Figure (67). The actual crack growth directions obtained by Kibler and

Roberts (1970) have been superimposed onto these figures. These directions coincide very closely with the predicted principal plane contours. Clearly the results of these tests could be predicted from the stress analysis of uncracked plates.

As previously mentioned, in all specimens whose geometry consists of a single rectangle or as in the present case a combination of rectangular forms, irrespective of the initial crack orientation the crack will continue to grow - or will immediately turn to extend - in a direction parallel to the side which is subjected to the higher load. This will be true for a large section of the crack path. However, when the crack has grown a considerable length in relation to specimen size, extensive bending may occur and consequently the crack will turn away from its prescribed path due to this major change in loading arrangement. See, for example, the final section of crack path in tests of Kibler and Roberts (1970), Figure (67).

(F) The minimum system energy concept:

As indicated by Erdogan and Sih (1963), prediction of the direction of crack propagation may be based upon a total system energy concept. Namely that the crack will propagate in a direction along which the elastic strain energy release per unit crack extension is maximum. This statement is based upon the accepted hypothesis that all systems will tend to move towards equilibrium conditions associated with the minimum total energy. Indeed, as indicated by Anderson et al. (1971) the finite elements technique may be used to determine the path of crack extension by simply calculating the total strain energy of the system, with a small crack extension assumed over a range of directions and subsequently selecting the direction which is associated with the minimum possible energy level. The process may be repeated in steps, to cover the whole width of a component. The required computer analysis will be reasonably complicated as with each small extension of the crack,

element mesh pattern will have to be suitably modified at least in the neighbourhood of the crack tip.

(G) Crack growth path under reversed loading conditions:

Roberts and Kibler(1971b) found that under the conditions of application of a fully reversed load cycle, the direction of crack growth on a macroscopic scale, was in the plane of the initial crack. Consideration of crack growth at a relatively lower scale, proved the growth mechanism to be the same as that described in previous sections, i.e. small extension in the direction perpendicular to that of the maximum principal stress. However due to the reversing nature of the load cycle, in each full cycle two principal stress directions, distributed symmetrically about the crack plane, are obtained and thus the crack tends to propagate in a saw-tooth pattern, Cotterell (1966). Clearly this is an important aspect of the whole process of crack path determination and will have to be more extensively studied.

7.7 Further Studies of the Crack Growth Direction: the Results of Tests with Inclined-Grooved Specimens:

This series of tests (AG Series) were devised for the purpose of further investigation of crack growth path of inclined cracks. The results of UA series of tests indicated that irrespective of the values of σ_x and σ_y parameters, it was impossible for an inclined crack to extend in its own plane. For the AG series of tests specimens of dimensions $5\frac{1}{2} \times 12 \times 1$ thick with deep narrow surface grooves (1/8 inch wide, 0.1-0.45 inches deep) along the crack orientation plane were used, similar to the specimens in Figure (54). All cyclic loading tests were performed at a frequency of 0.25 Hz.

The surface grooved specimens have been widely employed in fracture tests, for the purpose of control of crack growth direction (Radon et al., 1971). The use of surface grooves was first proposed by Hill (1953) and later used by Ellington (1958). Deep grooving results in a thin section of the material separating two relatively more bulky parts and thus, under load, ensures a good approximation to a plastically deformed region surrounded by two elastic zones.

7.7.1 Discussion of results:

A general feature of the outcome of this series of tests was that irrespective of the groove orientation angle, the crack did not extend along the plane of the groove. However, the specific characteristics of crack propagation process could be influenced by the depth of the surface grooves.

Up to a groove depth of approximately 0.3 inches (i.e. sixty per cent reduction in specimen thickness) the initial crack turned immediately to form along a minimum principal stress trajectory until it reached the unstable fracture point. These results confirmed the outcome of UA series of tests discussed previously, Figure (64c).

Using a specimen of larger dimensions ($13\frac{1}{2} \times 15 \times 1$ inch thick), the groove depth was further increased to about 0.45 inch (i.e. ninety per cent reduction of specimen thickness) the following pattern of growth was obtained.

The crack initially started to grow out of the plane of the groove as before; however, after a growth of about 0.15 inch on each side, the relative strength of the specimen in the two directions of extended crack and groove itself, became very much smaller in the groove due to its very narrow section. Consequently a finite rupture took place along the groove. The occurrence of this phenomenon transformed the situation to those pertaining at the initial stage except now a larger crack

along the groove existed. The process was repeated six times until the final unstable fracture point. Thus under these conditions the separation of the two parts of the specimen occurred along the drastically reduced section of the material. The actual cyclic crack extension never occurred along the groove and it was always in the preferred principal plane direction. The extent of growth into the bulk material became smaller from one stage to the other. This was due to the gradual weakening of the thinned section as the effective length of the crack that it contained became larger from one stage to another.

The importance of the outcome of this series of tests lies in the following observation: that although the direction of crack extension is monotonic fracture and under cyclic loading conditions will be determined from the orientation of the minimum principal stress trajectories, for certain component configurations where conditions of AG specimens in terms of thickness variations are approximated, actual material separation may occur along a highly weakened geometrical plane (in this case due to the presence of deep grooves) and away from the expected crack growth direction.

7.8 The Effect of Load Biaxiality on the Fracture Strength and the Fatigue Crack Growth Rate:

Linear elastic fracture mechanics theories state that, in a biaxial stress field, the stress parallel to the fracture direction does not contribute to crack growth (Paris and Sih, 1965; Sih, 1967). This statement can only apply in the limit to the case of a perfectly brittle material, where plasticity phenomena are not considered to be relevant. However, as the discussion of the stress field analysis in Section 7.2 indicated, in real materials the process of plastic yielding at the crack tip is considerably influenced by the presence of biaxial loading conditions. The exact

nature of such an influence is not unique and indeed as indicated by Swedlow (1965) the stress applied parallel to the crack plane may either strengthen or weaken a cracked plate depending upon the state of stress (plane-stress or plane-strain) and the value of Poisson's ratio for the material. Clearly the presence of hydrostatic stress conditions will limit the extent of plasticity and lead to an enhanced brittle behaviour.

Wilson et al. (1968) and Pook (1970) proposed that the extent of plastic yielding at the crack tip is increased when the transverse load is added. If the crack tip plasticity is taken as a measure of the energy required for crack extension, then it is expected that the apparent fracture toughness value of the material (calculated irrespective of the crack angle) will be larger under biaxial loading conditions.

The apparent increase in fracture strength under biaxial loading was observed in fracture and fatigue tests performed by Kibler and Roberts (1970) using cross-shaped specimens of cold rolled (6061-T4) and (6061-T6) aluminium alloys, under plane-stress conditions. Data from these tests indicate a 25 per cent rise in fracture toughness for $\sigma_x / \sigma_y = 1$. Fatigue data showed a gradual decrease in the magnitude of the exponent m and coefficient C in the power law model for fatigue crack growth, hence indicating a drop in the cyclic rate of crack growth as load biaxiality was introduced.

On the other hand the experimental results of Joshi and Shewchuck (1970) show an increase in the rate of crack propagation under cyclic loading conditions, when circular and elliptical specimens of (2024-T35) aluminium alloy, 0.25 inch thick, were tested under fully reversed bending conditions. The highest rate of crack growth was obtained for $\sigma_x = \sigma_y$ indicating enhanced brittle behaviour under biaxial loading. These authors analyse their results in terms of an equivalent stress

intensity factor term calculated on the basis of an equivalent stress, σ_e , which in maximum strain energy theory of failure, has the following form:

$$\sigma_e^2 = \sigma_1^2 + \sigma_2^2 - \nu\sigma_1\sigma_2 \quad (7.14)$$

where σ_1 and σ_2 are the principal stresses and ν is the Poisson's ratio.

Fatigue crack growth studies of Iida and Kobayashi (1969) on aluminum alloy (7075-T6) tested in the form of rectangular plates of 0.032 inch thickness, with central inclined cracks subjected to uniaxial loading, also showed an increase in the cyclic rate of crack growth of the order of 15 per cent when K_{II} component was present even at relatively low ratios of K_{II}/K_I (i.e. for large values of β). Hence, with respect to the influence of biaxiality, the trend of these results is in general agreement with the data of Joshi and Shewchuck (1970).

Based on the foregoing discussions the following general comments may be made:

If the addition of the load component parallel to the crack plane enhances the brittle behaviour of the material, the fracture toughness level, in monotonic fracture, will be reduced. Under cyclic loading conditions, the cyclic initiation life, N_i , will be reduced and the subsequent rate of crack propagation, $\dot{a}_N$, will be raised. Conversely, if larger plasticity is experienced as a consequence of introduction of load biaxiality (increase in ductility associated with a lowering of yield stress, σ_{yp}), the fracture toughness level will be increased and subsequently, under fatigue loading conditions, N_i will be raised and the $\dot{a}_N$ will be appropriately lowered.

In the present tests, as previously indicated, an increase in the apparent fracture toughness of the PMMA material was observed as the angle β was lowered. For instance the value of the apparent fracture toughness term obtained from a test with a rectangular specimen of dimensions of $9 \times 18 \times \frac{1}{4}$ inch thick, with a side notch of

0.2 inch long and with $\beta = 60^\circ$, was found to be approximately equal to 1300 psi $\sqrt{\text{in}}$. This value is about 15-20 per cent higher than the K_{IC} values of about 1100 psi $\sqrt{\text{in}}$ obtained with $\beta = 90^\circ$ (see Section 5.1).

The cyclic initiation life, N_i , was also extended as a result of a decrease in the value of the angle β . A smaller cyclic rate of growth was observed as β was lowered while maintaining the applied load levels at constant values (see Table 17).

For PMMA, the value of K_{IIC} has been found to be approximately equal to that of K_{IC} ($K_{IIC} = 0.9 K_{IC}$, Erdogan and Sih, 1963). A knowledge of K_{IC} , and the resolved value of K_I (from equation 7.4) however, are insufficient to predict the criticality conditions for the initiation of crack growth. The assumption that the crack will start to propagate when the resolved value of K_I reaches the level of K_{IC} may prove to be dangerously optimistic (Pook, 1970), particularly in situations where the resolved value of K_{II} is equal or greater than the resolved level of K_I .

7.8.1 A note on the applicability of the empirical crack growth models to the conditions of biaxial loading:

The test series UA, the results of which have been discussed in Sections 7.8 and 7.10, were of exploratory nature and consequently the limited extent of the result does not allow the possibility of application of the λ -model (equation 6.1) to the prediction of cyclic crack growth rate, under biaxial loading conditions. The present study will benefit from a future extension in such a direction.

Roberts and Kibler (1971a) indicated that the crack propagation models available at present - mainly based upon data from uniaxial loading tests involving cracks perpendicular to load direction - may be used for the purpose of prediction of crack growth rate under biaxial loading conditions after some of the parameters used in

such models are suitably modified. For example the term K_C in the Forman et al. (1967) model should be replaced by the apparent fracture toughness of the material which is appropriately influenced by biaxiality; or the power law exponent in the model proposed by Erdogan and Roberts (1965) should be interpreted as being a function of the apparent fracture toughness, as indicated by Miller (1968). Roberts and Kibler (1971b) have also proposed a possible model for crack propagation which includes both the terms K_{IC} and K_{IIIC} . As yet substantial data to verify any such model have not been presented.

CHAPTER EIGHT

DESIGN AGAINST FATIGUE FAILURE IN THERMOPLASTICS

Conclusions drawn from the discussions presented in the foregoing sections dealing with a range of phenomena associated with the process of crack growth initiation and cyclic propagation, yield sufficient information to enable the development of a failure prediction technique to be employed in the design of polymeric components subjected to cyclic loading conditions. From the discussions in Chapter 4 on the initiation process, it is evident that at the design stage an optimum combination of the material properties and the loading conditions may be found, which would correspond to the ideal conditions of infinite cyclic initiation life. This will be carried out through the selection of an appropriate material and its subjection to specified load levels and frequencies. Failing this, or in a situation where a relatively large initial crack exists, a procedure based upon equation $\dot{a}_N = \beta \lambda^n$ can be adopted to predict the rate of propagation of cracks and determine the cyclic life of a flawed component.

8.1 Cyclic life computation:

At the design calculations stage, the applied stress limits, the loading frequency and an estimated or calculated initial flaw size are usually available. Under constant applied stress conditions, an increase in the length of the crack results in a rise in the magnitude of the applied stress intensity factor; hence, there will be an accelerating rate of crack propagation leading to the final unstable fracture.

Taking the two limits of the applied stress as $\sigma_{\max}$ and $\sigma_{\min}$, a ratio $R = \sigma_{\min} / \sigma_{\max}$ can be defined. From equation (2.11), for any arbitrary crack length, this ratio will also be equal to $R = K_{\min} / K_{\max}$; then

$$\lambda = K_{\max}^2 - K_{\min}^2 = (1 - R^2) K_{\max}^2 \quad (8.1)$$

and hence equation (6.1) can be rewritten as:

$$\dot{a}_N = \beta \lambda^n = \beta (1 - R^2)^n (K_{\max}^2)^n \quad (8.2)$$

Since in most practical cases $\sigma_{\max}$ and $\sigma_{\min}$ are constants associated with the characteristics of the external disturbance, the growth of the crack will not change the value of R , even though the values of the $K_{\min}$ and $K_{\max}$ parameters increase gradually. Thus, in equation (8.2), $(1 - R^2)^n$ is a constant and hence a plot of $\dot{a}_N$ against λ as in Figure (37) can be changed to a plot of $\dot{a}_N$ against $K_{\max}^2$, with β replaced by $\beta(1 - R^2)^n$.

The following procedure can be used to estimate the cyclic fatigue life:

From the results of the type shown in Figure (37), a small cyclic period is chosen to suit a particular loading situation, during which it is assumed that the rate of change of $\dot{a}_N$ is very small. Then, using the value of the maximum design stress, the initial flaw size and the appropriate stress intensity factor relations, the value of $K_{\max}$ is computed. From the relationship between $\dot{a}_N$ and $K_{\max}^2$ (equation 8.2), the value of $\dot{a}_N$ corresponding to the calculated value of $K_{\max}$ is obtained. This magnitude of $\dot{a}_N$ is then used to calculate the increase in crack length, δa , after the first cyclic period has elapsed. δa is added to a , and then a new value for $K_{\max}$ is calculated. The process is repeated until the value of $K_{\max}$ becomes equal to the appropriate level of K_{IC} , the critical value of K_I corresponding to unstable fracture for the specific material. The achievement of the condition $K_{\max} = K_{IC}$ is treated as the criterion for final catastrophic fracture of the component. These calculations are carried out using a suitable computer program in which, for each case, the parameters β and n from the graphs of $\dot{a}_N$

against λ and also the values of $\sigma_{\max}$ and $\sigma_{\min}$ are specified.

It is pertinent to reiterate briefly a point previously raised in connection with the rate dependency of the plane-strain fracture toughness parameter, K_{IC} , which is used in the above procedure: whenever the data covering a range of cyclic loading frequencies are to be analysed, special attention must be paid to the selection of a correct value for the K_{IC} . If the variations in K_{IC} over the range of strain rates (corresponding to the frequency range under consideration) are reasonably small, an average value for K_{IC} should be used in the above procedure. However, if such variations are considerable, at each frequency level an appropriate value for K_{IC} must be used; hence, the importance of the fracture toughness v load rate data for the rate-sensitive materials becomes evident.

Typical results obtained from the above described computation procedure have been presented in Figures (68 and 69) for PMMA, Figures (70 and 71) for PC and Figures (72 and 73) for Nylon 6.6. On the basis of such results (i.e. total number of cycles to failure related to the initial flaw size), at the design stage, materials can be compared in terms of their resistance to fatigue crack growth. Clearly these results could also be expressed in terms of the $\frac{K_I \text{ (applied)}}{K_{IC}}$ ratio related to N_f , the cyclic fatigue life. The $K_I \text{ (applied)}$ term will be calculated in the present case from equation (2.11) which represents the general case of a cracked plate loaded at infinity. σ will be given the value of $\sigma_{\max}$ in each case (Figure 68) and a will be the appropriate initial flaw size.

In Figure (68) the value of $\sigma_{\max}$ is fixed and calculations are carried out for the various $\sigma_{\min}$ levels. These calculations correspond to constant frequency conditions as the values of β and n parameters change from one frequency to another. Figure (69) shows the effect of variations in the value of $\sigma_{\max}$ on the relationship between the initial flaw size and the total number of cycles to failure.

Thus, when the whole range of data covering various frequencies and load levels have been prepared, and having the initial flaw size (which can be inspected or assumed to exist) and the minimum number of load cycles that a component must be capable of enduring prior to the point of catastrophic failure, an appropriate decision can be made on the choice of the most suitable material, the frequency level and the limits of the applied stress cycle as required.

8.2 Limitations of the Applicability of the Above Procedure:

The procedure outlined in the preceding section is based totally on the LEFM parameters; hence it is obvious that necessary precautions must be taken so as not to exceed the limits set upon the range of accuracy of the relationship based upon the LEFM concepts, specifically with regards to parameters associated with component geometry. For example, acceptable limits of component thickness and plastic zone size relationship are defined by:

$$B_s \geq 0.25 \left(\frac{K_{IC}}{\sigma_{yp}} \right)^2 \quad (8.3)$$

so as to provide for the applicability of the assumption of small scale yielding.

The relationship between the crack length, a , and the component width, w , is also important in that as the ratio a/w increases above a certain limit (often 0.4-0.5), the accuracy of the equations used for the calculation of the stress intensity factor itself is reduced. Such limits will depend on the choice of a particular stress intensity factor equation for which one can normally find graphs describing the percentage error in relation to a/w ratio (Paris and Sih, 1965).

In the computation procedure described in Section 8.1, the general infinite plate solution for the stress intensity factor term equation (2.11) has been used. This

equation must be replaced by the appropriate relationship applicable to a specific case.

CONCLUSIONS

The concepts of linear elastic fracture mechanics can be successfully used to characterise the fatigue failure process in thermoplastic materials whose mechanical behaviour under small scale yielding conditions can be assumed to be approximately linear. The results of a comprehensive programme of empirical studies indicated that variations in such parameters as the amplitude and the mean level of the stress intensity factor (ΔK and K_m respectively), the loading frequency, the cyclic waveform, the molecular orientation in anisotropic materials and the environmental temperature conditions may have a significant effect on the cyclic rate of crack propagation, $\dot{a}_N$, in polymeric solids. An increase in the levels of ΔK and K_m leads to a rise in the value of $\dot{a}_N$. A rise in the level of the applied load frequency resulted in a decrease in the cyclic rate of crack propagation; however, in terms of the total fatigue life, the time to failure was shorter at higher frequencies. Variation of the cyclic waveform from triangular to sinusoidal led to a small increase in the level of $\dot{a}_N$ and tests on extruded sheets showed that the greater material strength in the direction of extrusion, manifests itself in the form of a reduced cyclic rate of crack extension along a direction perpendicular to that of extrusion. The effect of the environmental temperature was found to be very significant. A relatively small change in the level of the temperature was found to have a very considerable influence on the value of the $\dot{a}_N$ parameter.

Consideration of the data gathered from the empirical studies indicated the possibility of use of the following model for the prediction of the cyclic rate of crack propagation:

$$\dot{a}_N = \beta \lambda^n$$

$$\text{where } \lambda = K_{\max}^2 - K_{\min}^2 = 2\Delta K K_m$$

The form of this model implies that the value of $\dot{a}_N$ parameter can be predicted from the consideration of the value of λ and irrespective of the specific values of ΔK and K_m . It was also shown that this equation can be used in the prediction of the fatigue crack propagation rate in some metals; consequently, irrespective of the specific values of $K_{\max}$ and $K_{\min}$, all the data on these materials could be represented by a single graph.

The specific form of the above model also enables it to be related to the crack opening displacement (COD) and the crack tip plasticity (r_p) concepts through a simple conversion of the K^2 term into the appropriate COD or r_p functions.

The total energy balance concept of failure analysis was used in the derivation of a theoretical model for the determination of the cyclic initiation period which may precede the crack propagation process. The analytical solution also indicated the possibility of the determination of the fatigue endurance limit under specified loading conditions.

Based upon the outcome of studies of both the initiation and the propagation processes, a design procedure for the prediction of fatigue failure life in thermo-plastic components was devised.

From the consideration of the process of failure under multiaxial loading conditions the following conclusions were drawn: the crack will grow along the minimum principal stress trajectory passing through its tip. This phenomenon was demonstrated by performing uniaxial and biaxial loading tests on specimens containing cracks of arbitrary orientation. The stress fields in the uncracked specimen (in the form of contours of maximum principal stress planes) were determined using the finite elements method. The cyclic initiation life was found to increase as the magnitude of the angle between the crack inclination and the load direction was reduced.

The presented results will be of direct interest to materials scientists who are interested in the comparative study of the behaviour of elastic and viscoelastic solids; to engineering designers using polymeric materials as a convenient substitute for the conventional materials - and also to the designers of such components as the turbine rotors and blades, gear wheels, etc. in which the specific direction of crack propagation prior to the final catastrophic failure may have a considerable influence on the consequences of the failure process.

Finally, in dealing with a discipline such as viscoelasticity, which although being rapidly established, is nonetheless as yet so young in its fundamental development, it is perhaps inevitable that due to the paucity of comprehensive data on material properties a few rather speculative remarks enter into the general discussion of the behaviour of polymeric solids. However, it is hoped that a much more than proper balance has been maintained between such aspects and the firmly established parts of the data.

PROPOSALS FOR FUTURE WORK

The discussions presented in this thesis in relation to the applicability of the linear elastic fracture mechanics concepts to the characterisation of the fatigue failure process in polymeric materials, have clearly indicated the need for further studies of a large number of parameters which may have a significant influence on the fatigue strength of polymeric solids.

In addition to the need for the data on the basic material properties, the following studies may be considered as the immediate extension of the present programme of research:

The analytical studies of failure should be further extended to a formulation of the rate of crack propagation. Also, an attempt should be made to appropriately modify the present analysis for the prediction of the initiation life so as to explore its applicability to the case of failure under multiaxial loading conditions.

The empirical model proposed should be further studied so as to identify the exact form of the dependence on the material properties of parameters β and n . Attempts should also be made to test the applicabilities of this model to other polymeric and metallic materials. The possibility of the use of the above model for the treatment of the combined creep and fatigue failures should also be investigated.

The empirical studies discussed in this thesis concerned the tensile fatigue failure process. An obvious extension of the work will be the study of the applicability of the empirical model to the case of failure under a tension-compression loading pattern. The investigation of the effects of the temperature and the chemical composition of the environment on the fatigue crack growth rate in polymers is a field of research in which very little work has yet been carried out. Provision of data related to the effects of the above parameters plus the study of the influence of

such elements on the material processing conditions, with its subsequent effects on the mechanical properties of the material, will prove to be extremely useful in the development of design procedures.

The combined effects of the environmental temperature and the loading frequency should be further studied with the aim of provision of data on the variation of the fracture toughness parameter as a function of the load rate (frequency) and temperature and also on the extent of the shifts in the distribution of the loss modulus data with temperature and frequency and its subsequent effect on the rate of crack growth in a material.

The failure under multiaxial loading conditions should be extended in the following directions:

The variation in fracture toughness of a specific material as a function of the changes in the angle of crack inclination should be studied. Hence the effects of load biaxiality on the cyclic initiation life and the rate of crack propagation in the material may be investigated. The possibility of the employment of the proposed model based upon parameter λ , for predicting the rate of crack growth under biaxial loading conditions should be studied. Further extension of the tests with the grooved specimens to study the effects of 'weakening' in geometry is also desirable.

A computer program should be developed for the purpose of optimisation of the role of various parameters involved in determining the nature of the stress field distribution in a component of arbitrary shape. The geometrical factors (for example the design of the turbine blade root) and the loading conditions should be combined in order to achieve the preferred paths of crack extension in engineering components.

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TABLE 1

PERCENTAGE STRAIN DATA ON THE FOUR MATERIALS STUDIED

(Test of approximate linear behaviour)

(a) PMMA (PERSPEX) (65% RH; 20°C)

Stress (psi)	Loading time (hrs)				
	0.01	1	100	1000	10000
500	0.12	0.12	0.13	0.15	0.20
1000	0.23	0.24	0.29	0.35	0.44
2000	0.44	0.52	0.66	0.78	0.97
3000	0.70	0.84	1.10	1.30	-
4000	0.95	1.22	1.70	2.08	-

(b) PC (MAKROLON) (20°C)

Stress (psi)	Loading time (hrs)	
	100	1000
500	0.14	0.16
1000	0.29	0.30
2000	0.56	0.56
3000	0.84	0.87
4000	1.08	1.24

(c) PA (KEMATAL)

Stress (psi)	Loading time (hrs)				
	0.01	1	100	1000	10000
500	0.12	0.17	0.24	0.31	0.37
1000	0.25	0.33	0.50	0.63	0.77
2000	0.52	0.68	1.05	1.33	1.63
3000	0.82	1.08	1.73	2.20	2.69
4000	1.17	1.55	2.61	-	-

(d) N6.6 (MARANYL)

Stress (psi)	Loading time (hrs)				
	0.01	1	100	1000	10000
500	0.11	0.11	0.15	0.23	0.44
1000	0.22	0.23	0.31	0.46	0.88
2000	0.45	0.47	0.61	0.95	1.80
4000	0.92	0.97	1.30	2.4	-

TABLE 2

MEASUREMENT OF THE G_{IC} AND THE K_{IC} PARAMETERS FOR PMMA

Crack length (a) (in)	Load (P) (lb)	Time (Min)	Deflection (δ) (in)	Compliance (δ/P) (in/lb)	G_{IC} (lbf/in)	K_{IC} psi $\sqrt{in}$
3.22	6.5	4.0	0.200	0.0307	1.88	940
3.62	6.8	4.50	0.225	0.0330	2.05	980
4.02	7.1	4.90	0.245	0.0345	2.25	1020
4.40	7.5	6.00	0.300	0.0400	2.35	1040
4.80	7.9	7.25	0.365	0.0460	2.73	1120
5.20	8.3	7.40	0.370	0.0445	3.07	1190
5.60	8.5	9.00	0.450	0.0530	3.15	1210
6.00	8.7	9.90	0.495	0.0570	3.25	1235
6.36	8.9	10.55	0.527	0.0590	3.53	1255

TABLE 3

MODULUS MEASUREMENTS AT THE MAXIMUM AND
THE MINIMUM LIMITS OF THE TEMPERATURE RANGE

Material	Temperature ($^{\circ}\text{C}$)	
	20	-197
N 6.6	180,000	936,000
PC	310,000	622,000
PVC	364,000	800,000
PA	371,000	1,510,000
PMMA	419,000	942,000

TABLE 4

CYCLIC CRACK GROWTH RATE DATA FOR PMMA AT A FREQUENCY OF 5 Hz

Test No.	$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)	ΔK (psi $\sqrt{\text{in}}$)	K_m (psi $\sqrt{\text{in}}$)	λ lbf ² in ⁻³	$\dot{a}_N$ (in/cycle)
1	800	700	100	750	150,000	1.42×10^{-6}
2	850	650	200	750	300,000	5.65 "
3	500	300	200	400	160,000	1.02 "
4	250	0	250	125	625,000	0.39 "
5	500	250	250	375	187,500	2.15 "
6	750	500	250	625	312,500	6.10 "
7	900	600	300	750	450,000	12.50 "
8	750	450	300	600	360,000	8.50 "
9	660	350	310	505	313,600	6.10 "
10	610	250	360	430	309,600	6.00 "
11	600	200	400	400	320,000	7.10 "
12	770	330	440	550	502,900	13.80 "
13	500	50	450	275	247,500	3.14 "
14	770	300	470	535	502,000	13.80 "
15	750	250	500	500	500,000	13.60 "
16	500	0	500	250	250,000	3.20 "
17	560	0	560	280	312,500	6.10 "
18	675	75	600	375	450,000	12.40 "
19	750	50	700	400	560,000	19.70 "
20	750	0	750	375	562,500	19.60 "

TABLE 5

CYCLIC CRACK GROWTH RATE DATA FOR PMMA AT FREQUENCIES OF 0.1 AND 20 Hz

Test	K_{max}	K_{min}	K_m	ΔK	λ	$\dot{a}_N$ (in/cycles)	
No.	($\text{psi}\sqrt{\text{in}}$)	($\text{psi}\sqrt{\text{in}}$)	($\text{psi}\sqrt{\text{in}}$)	($\text{psi}\sqrt{\text{in}}$)	($\text{lb}f^2/\text{in}^3$)	0.1 (Hz)	20 (hz)
1	600	550	575	50	57,500	0.5×10^{-6}	0.025×10^{-6}
2	500	300	400	200	160,000	1.7 "	0.05 "
3	500	200	350	300	210,000	5.0 "	0.70 "
4	550	250	400	300	240,000	20.0 "	2.10 "
5	500	50	275	450	247,500	40.0 "	1.25 "
6	650	350	500	300	300,000	122.0 "	2.50 "
7	750	500	625	250	312,500	210.0 "	2.80 "
8	750	450	600	300	360,000	260.0 "	3.0 "
9	700	200	450	500	450,000	380.0 "	4.50 "
10	750	250	500	500	500,000	440.0 "	4.80 "
11	750	50	400	700	560,000	480.0 "	5.30 "
12	800	100	450	700	630,000	560.0 "	91.0 "
13	850	50	450	800	720,000	600.0 "	130.0 "

TABLE 6

CRACK GROWTH RATE DATA FOR PC AT A FREQUENCY OF 0.1 Hz

Test No.	$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)	ΔK (psi $\sqrt{\text{in}}$)	K_m (psi $\sqrt{\text{in}}$)	λ (lbf ² in ⁻³)	$\dot{a}_N$ (in/cycle)
1	250	50	200	150	60,000	0.09×10^{-6}
2	500	300	200	400	160,000	0.69 "
3	500	250	250	375	187,500	1.57 "
4	550	250	300	400	240,000	1.57 "
5	500	50	450	275	247,500	1.96 "
6	650	350	300	500	300,000	1.72 "
7	750	500	250	625	312,500	1.92 "
8	750	450	300	600	360,000	3.90 "
9	650	150	500	400	400,000	5.10 "
10	700	200	500	450	450,000	10.5 "
11	700	0	700	350	490,000	12.0 "
12	750	250	500	500	500,000	15.03 "
13	750	50	700	400	560,000	27.5 "
14	800	100	700	450	630,000	31.3 "

TABLE 7

CRACK GROWTH RATE DATA FOR PC AT A FREQUENCY OF 5 Hz

Test No.	$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)	ΔK (psi $\sqrt{\text{in}}$)	K_m (psi $\sqrt{\text{in}}$)	λ (lbf ² in ⁻³)	$\dot{a}_N$ (in/cycle)
1	250	50	200	150	60,000	.03 $\times 10^{-6}$
2	350	150	200	250	100,000	.063 "
3	500	300	200	400	160,000	.120 "
4	450	200	250	325	162,500	.140 "
5	500	250	250	375	187,500	.194 "
6	550	250	300	400	240,000	.197 "
7	650	350	300	500	300,000	.268 "
8	750	450	300	600	360,000	.430 "
9	775	475	300	625	375,000	.800 "
10	1700	1400	300	1550	930,000	1.470 "
11	1600	1200	400	1400	1,120,000	1.970 "
12	1500	1100	400	1300	1,040,000	1.380 "
13	1000	600	400	800	640,000	1.100 "
14	600	200	400	400	320,000	1.400 "
15	500	50	450	275	247,500	2.500 "
16	1100	650	450	875	787,500	2.200 "
17	1700	1250	450	1475	1,327,500	2.950 "
18	700	250	450	475	427,500	2.990 "
19	650	150	500	400	400,000	2.500 "
20	700	200	500	450	450,000	2.500 "
21	750	250	500	500	500,000	2.400 "
22	1600	1100	500	1350	1,350,000	2.900 "
23	1000	450	550	725	797,500	2.900 "
24	1100	550	550	825	907,500	3.600 "
25	1600	1050	550	1325	1,457,500	5.500 "
26	800	200	600	500	600,000	4.600 "
27	1100	500	600	800	960,000	5.00 "
28	1500	900	600	1200	1,440,000	3.940 "
29	750	150	600	900	540,000	4.700 "
30	900	250	650	575	747,500	6.100 "
31	1050	400	650	725	942,500	6.800 "
32	750	50	700	400	560,000	10.600 "

TABLE 8

CRACK GROWTH RATE DATA FOR PC AT A FREQUENCY OF 20 Hz

Test No.	$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)	ΔK (psi $\sqrt{\text{in}}$)	K_m (psi $\sqrt{\text{in}}$)	λ (lbf ² in ⁻³)	$\dot{a}_N$ (in/cycle)
1	250	50	200	150	60,000	0.04×10^{-6}
2	500	300	200	400	160,000	0.10 "
3	500	250	250	375	187,500	0.20 "
4	550	250	300	400	240,000	0.396 "
5	650	350	300	500	300,000	0.150 "
6	750	450	300	600	360,000	0.276 "
7	1700	1400	300	1550	930,000	0.290 "
8	1200	850	350	1025	717,500	0.390 "
9	1500	1100	400	1300	1,040,000	1.38 "
10	600	200	400	400	320,000	1.25 "
11	500	50	450	275	247,500	2.10 "
12	1100	650	450	875	787,500	1.28 "
13	1700	1250	450	1475	1,327,500	1.57 "
14	700	250	450	475	427,500	2.30 "
15	650	150	500	400	400,000	3.50 "
16	700	200	500	450	450,000	4.40 "
17	750	250	500	500	500,000	5.0 "
18	1450	950	500	1200	1,200,000	4.1 "
19	1000	450	550	725	797,500	2.8 "
20	1100	550	550	825	907,500	2.6 "
21	800	200	600	500	600,000	5.6 "
22	700	100	600	450	480,000	5.4 "
23	1100	500	600	800	960,000	7.8 "
24	1600	1000	600	1300	1,560,000	6.8 "
25	900	250	650	575	747,500	9.5 "
26	1250	600	650	925	1,202,500	5.9 "
27	700	0	700	350	490,000	9.8 "
28	1800	1100	700	1450	2,030,000	15.0 "
29	1800	1000	800	1400	2,240,000	20.6 "

TABLE 9
CYCLIC CRACK GROWTH RATE DATA FOR NYLON 6.6 AT A FREQUENCY
OF 0.1 Hz

Test No.	K_{max} (psi $\sqrt{in}$)	K_{min} (psi $\sqrt{in}$)	ΔK (psi $\sqrt{in}$)	K_m (psi $\sqrt{in}$)	λ lbf ² in ⁻³	$\dot{a}_N$ (in/cycle)
1	1000	100	900	550	990,000	0.66×10^{-6}
2	1150	450	700	800	1,120,000	0.92 "
3	1500	1000	500	1250	1,250,000	1.00 "
4	1800	1300	500	1550	1,550,000	1.70 "
5	1420	600	820	1010	1,646,400	2.30 "
6	1400	470	930	935	1,739,100	3.50 "
7	1400	0	1400	700	1,960,000	3.23 "
8	1500	500	1000	1000	2,000,000	4.05 "
9	1500	0	1500	750	2,250,000	4.88 "
10	1600	400	1200	1000	2,400,000	6.20 "
11	1600	200	1400	900	2,520,000	6.20 "
12	1600	100	1500	1350	2,550,000	9.20 "
13	1800	800	1000	1300	2,600,000	12.4 "
14	1700	400	1300	1050	2,730,000	15.1 "
15	1700	300	1400	1000	2,800,000	16.0 "
16	1700	200	1500	950	2,850,000	20.1 "
17	1700	100	1600	900	2,880,000	17.8 "
18	1800	400	1400	1100	3,080,000	30.0 "
19	1800	380	1420	2180	3,095,600	30.0 "
20	1800	300	1500	1050	3,150,000	30.2 "
21	1825	0	1825	912.5	3,330,625	31.6 "

TABLE 10
CYCLIC CRACK GROWTH RATE DATA FOR NYLON 6.6 AT A FREQUENCY
OF 5 Hz

Test No.	$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)	ΔK (psi $\sqrt{\text{in}}$)	K_m (psi $\sqrt{\text{in}}$)	λ lbf ² in ⁻³	a_N (in/cycle)
1	1000	0	1000	500	1,000,000	.014 $\times 10^{-6}$
2	1150	500	650	825	1,190,000	.04 "
3	1400	800	600	1100	1,320,000	.05 "
4	1400	750	650	1075	1,397,000	.08 "
5	1750	1250	500	1500	1,500,000	.12 "
6	1800	1300	500	1550	1,550,000	.17 "
7	1850	1300	550	1575	1,732,500	.21 "
8	1800	1200	600	1500	1,800,000	.35 "
9	1800	1200	600	1500	1,800,000	.38 "
10	1850	1200	650	1525	1,982,500	.60 "
11	1500	500	1000	1000	2,000,000	.66 "
12	1450	200	1250	825	2,062,500	1.0 "
13	1550	400	1150	975	2,252,500	0.82 "
14	1575	400	1175	986	2,320,625	1.32 "
15	1600	200	1400	900	2,520,000	1.78 "
16	1725	400	1325	1063	2,715,625	2.09 "
17	1700	300	1400	1000	2,800,000	2.40 "
18	1750	350	1400	1050	2,940,000	2.63 "
19	1800	200	1600	1000	3,200,000	3.80 "
20	2150	0	2150	1075	4,620,000	4.18 "
21	2150	0	2150	1075	4,620,000	5.5 "

TABLE 11.

CYCLIC CRACK GROWTH RATE DATA FOR NYLON 6.6 AT A FREQUENCY
OF 20 Hz

Test No.	$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)	ΔK (psi $\sqrt{\text{in}}$)	K_m (psi $\sqrt{\text{in}}$)	λ lbf ² in ⁻³	$\dot{a}_N$ (in/cycle)
1	1500	1200	300	1350	810,000	$.028 \times 10^{-6}$
2	1700	1300	400	1500	1,200,000	.042 "
3	1000	500	500	750	750,000	.025 "
4	1500	1000	500	1250	1,250,000	.045 "
5	1600	1100	500	1350	1,350,000	.046 "
6	1700	1200	500	1450	1,450,000	.050 "
7	1800	1300	500	1550	1,550,000	.064 "
8	1500	1000	500	1000	1,250,000	.042 "
9	1610	1100	510	1355	1,380,000	.056 "
10	1600	1050	550	1325	1,457,500	.051 "
11	1200	600	600	900	1,080,000	.038 "
12	1700	1100	600	1400	1,680,000	.090 "
13	1800	1200	600	1500	1,800,000	.110 "
14	850	150	700	500	700,000	.024 "
15	1000	300	700	650	910,000	.031 "
16	1000	300	700	650	910,000	.034 "
17	1400	700	700	1050	1,470,000	.048 "
18	1400	600	800	1000	1,600,000	.075 "
19	1400	500	900	950	1,710,000	.095 "
20	1010	100	910	555	1,010,100	.035 "
21	1500	500	1000	1000	2,000,000	1.14 "

TABLE 12

DATA ON THE RELAXATION PROPERTIES OF THE MATERIALS USED

(Sources: McCrum et al., 1967 and Ferry, 1970)

Material	Glass Transition Temperature (T _g .°C)	α	β	γ
PMMA	105	120°C (1 Hz)	40°C (1 Hz) 21°C (0.727 Hz) -20°C (0.8 × 10 ⁻² Hz) -50°C (2.99 × 10 ⁻⁴ Hz)	25°C (3 × 10 ⁵ Hz) -100°C (1 × 10 ³ Hz) -173°C (1 Hz)
PC	150	160°C (1 kHz)	-40°C (20 kHz)	-
Nylon 6.6	70	60°C (4 Hz) 65°C (5 Hz) 90°C (70 Hz)	-70°C (1 Hz)	-

TABLE 13

CRACK GROWTH RATE DATA FOR PA AT A FREQUENCY OF 5 Hz

Test No.	$K_{\max} = \Delta K$ psi $\sqrt{\text{in}}$	λ lbf ² in ⁻³	$\dot{a}_N$ (in/cycle)	
			Constant Applied Stress	Constant Stress Intensity Factor
1	1700	2,890,000	7.5×10^{-8}	-
2	1800	3,240,000	9.1 "	8.0×10^{-8}
3	1950	3,802,500	14.4 "	-
4	2100	4,410,000	18.8 "	16.7 "
5	2300	5,290,000	21.1 "	19.9 "
6	2450	6,002,500	25.1 "	-
7	2500	6,250,000	-	28.0 "
8	2650	7,022,500	35.4 "	-

TABLE 14

CRACK GROWTH RATE DATA FOR PMMA AND NYLON 6.6 AT VARIOUS

TEMPERATURES

(Test frequency = 5 Hz)

PMMA

$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)	λ lbf ² in ⁻³	$\dot{a}_N$ (in/cycle)		
			3°C	21°C	38°C
250	50	60,000	0.55×10^{-6}	0.39×10^{-6}	0.48×10^{-6}
500	250	187,500	3.6 "	2.15 "	2.9 "
500	50	247,500	5.6 "	3.14 "	4.6 "
600	200	320,000	12.9 "	7.10 "	10.5 "
675	75	450,000	19.4 "	12.4 "	18.7 "

NYLON 6.6

$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)	λ lbf ² in ⁻³	$\dot{a}_N$ (in/cycle)		
			3°C	21°C	38°C
1500	1000	1,250,000	0.01×10^{-6}	0.05×10^{-6}	0.03×10^{-6}
1400	700	1,470,000	0.03 "	0.18 "	0.11 "
1500	500	2,000,000	0.15 "	0.70 "	0.45 "
1600	400	2,400,000	0.40 "	1.30 "	0.80 "
1700	300	2,800,000	0.80 "	2.10 "	1.40 "

TABLE 15

TABULATION OF PARAMETERS n AND β FOR VARIOUS TEST CONDITIONS(n and β from equation (6.1); m and c from equation (2.21))

Material	Frequency (Hz)	n	β
PMMA	0.1	3.5	2.8×10^{-24}
PMMA	5	2.2	1.2×10^{-17}
PMMA	20	2.3	4.8×10^{-19}
PC	0.1	3.35	1×10^{-24}
PC	5	1.8	4×10^{-17}
PC	20	m = 4.2	c = 1.3×10^{-17}
Nylon 6.6	0.1	3.1	1.1×10^{-25}
Nylon 6.6	5	3.3	1.0×10^{-27}
Nylon 6.6	20	1.8	7.0×10^{-19}
PA	5	1.8	1.4×10^{-19}

TABLE 16
TEST OF APPLICABILITY OF FORMAN ET AL. (1967) MODEL TO DATA
ON POLYMERS

Material : PMMA

Arbitrary values of $m = 3$ and $c = 1$ are used

Loading frequency = constant = 0.1 Hz

$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)	λ lbf ² in ⁻³	$\dot{a}_N$ (Test data) (in/cycle)	Crack Growth Rate predicted by Forman et al. (1967)
750	500	312500	210×10^{-6}	35511.1
560	50	311100	213×10^{-6}	19788.0
610	250	309600	107×10^{-6}	15250.0
660	350	313100	210×10^{-6}	8990.0

Note: The approximately constant value of λ corresponds to $\dot{a}_N$ values which are nearly equal, whereas the Forman et al. (1967) model predicts widely different values for $\dot{a}_N$.

TABLE 17

RESULTS OF UA SERIES OF TESTS (PART I), FREQUENCY = 0.35 Hz

Test No.	Specimen Dimensions	β (degrees)	Applied Load Limits (tons)	Cyclic Initiation Period (N_i)	Total Cycles to Failure
UA2	$5\frac{1}{2} \times 11 \times \frac{1}{4}$	60	0.32 - 0.1	6500	8474
UA4	"	45	" "	13400	16000
UA3	"	30	" "	25700	29976
UA5	"	60	0.29 - 0.05	11450	13900
UA6	"	30	" "	45300	59350
UA7	$13\frac{1}{2} \times 15 \times \frac{1}{4}$	30	0.58 - 0.1	78630	153691
UA8	"	45	0.58 - 0.15	49700	90434

TABLE 18

RESULTS OF UA SERIES OF TESTS (PART 2)

Test No.	B	Total Cyclic Growth Prior to Final Failure (in)	Effective Total Crack Length $2a^*$ (in)	$\frac{2a^*}{\text{Plate Width}}$ (%)	Maximum Stress at Point of Fracture (psi)	Apparent Fracture Toughness based on a^* (psi $\sqrt{\text{in}}$)
UA2	60	0.5	1.30	23.6	533.3	761
UA4	45	0.6	1.37	24.9	"	780
UA3	30	0.75	1.37	24.9	"	780
UA5	60	0.70	1.70	31.7	472.0	771
UA6	30	1.10	1.80	32.7	"	795
UA8	45	1.53	2.33	17.3	388.0	745

TABLE 19

DATA ON THE RR, ROR AND RTR SERIES OF TESTS

Test	Angle (θ) (to the horizontal) (degrees)	Stress Ratio	Static Load per blade (lbs)
RR1	10	Static test	25
RR2	10	0.8 : 1	25
RR3	10	4.3 : 1	12.5
RR4	15	3.5 : 1	12.5
RR5	10	4.3 : 1	25
RR6	14	3.5 : 1	12.5
RR7	30	4.3 : 1	7.5

Test	Angle (θ)	Cyclic Load (lbs)	Static Load per blade (lbs)
RTR1	14	480 - 0	10
RTR2	25	480 - 0	7.5
RTR3	20	300 - 120	7.5
RTR4	30	120 - 60	7.5

Test	Crack 1 Radial Position (degrees)	β	Crack 2 Radial Position (degrees)	β	Cyclic Load (lbs)	Static Load per blade (lbs)
ROR1	45	45	45	60	840 - 0	7.5
ROR2	30	60	30	60	720 - 0	7.5
ROR3	45	45	45	45	Static	7.5
ROR4	75	15	45	90	1800-600	7.5

TABLE 20
DATA ON STRESS DISTRIBUTION IN THE DOVETAIL REGION OF THE BIAXIALLY
LOADED SPECIMENS

(Obtained from photoelastic studies)

Test No.	Radial Nominal Stress	Hoop Nominal Stress	Ratio	Maximum Radial Stress		Maximum Hoop Stress	
				Value	Position	Value	Position
1	2.0	1.71	0.855	12.0	115°	3.37	22°
2	1.0	4.30	4.3	5.17	119°	6.81	9°
3	0	4.38	∞	-	-	6.88	0°

Position θ°	Stress for Test No.		
	1	2	3
0	2.90	6.35	6.80
10	3.00	6.80	6.70
20	3.30	6.30	6.20
30	3.30	5.25	5.65
40	3.10	3.85	3.90
50	2.80	2.50	1.55
60	2.75	1.40	0.65
70	3.05	1.50	-0.55
80	3.65	2.50	-1.05
90	6.30	4.40	-1.20
100	8.10	4.95	-1.05
110	9.70	5.15	-0.70
120	9.70	5.55	-0.55
130	8.85	4.60	-0.55

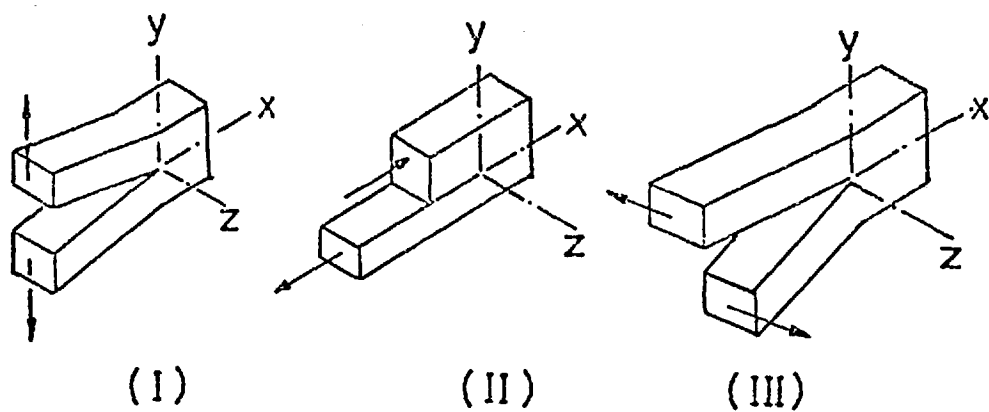


FIG (1) THE BASIC MODES OF CRACK OPENING

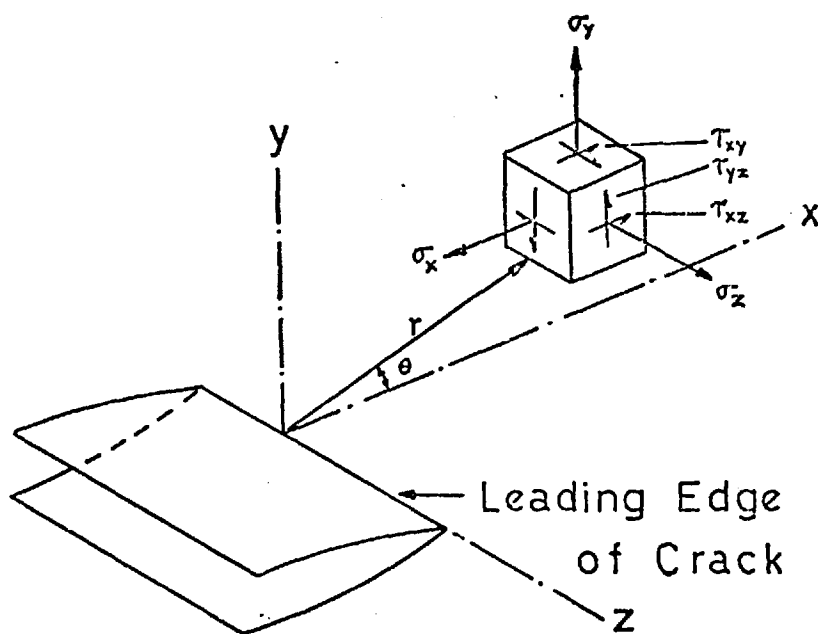
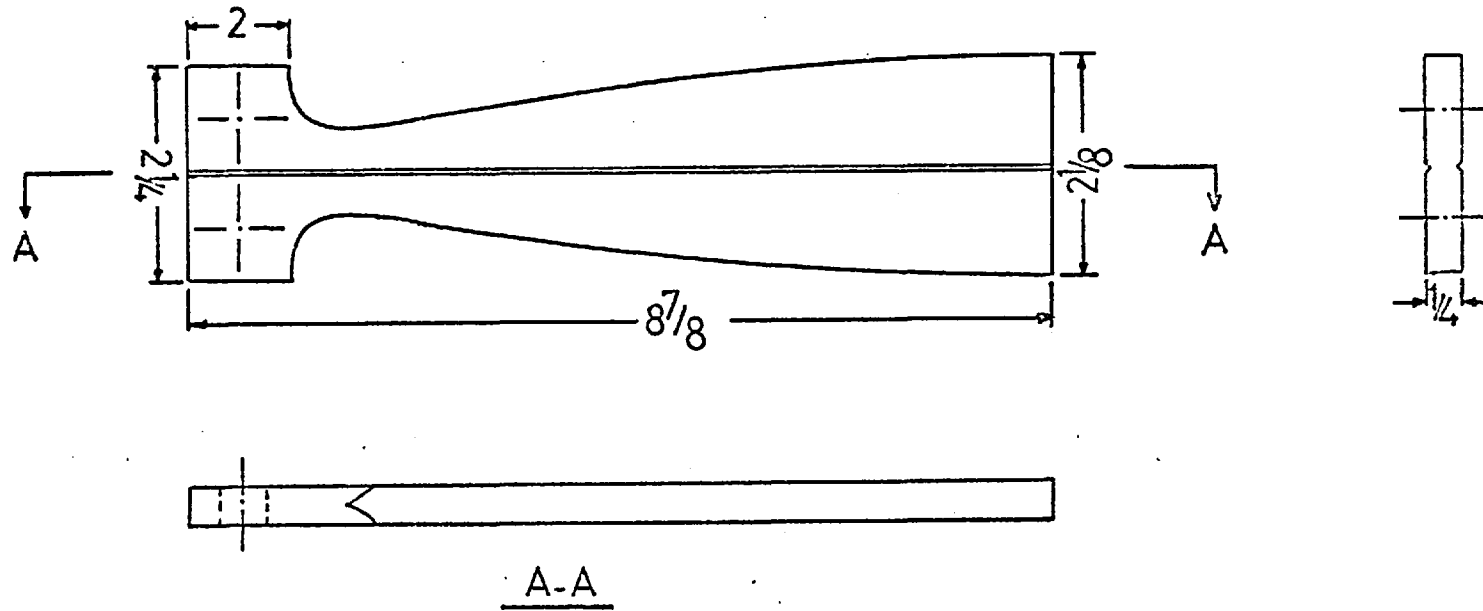


FIG (2) STRESS COMPONENTS IN THE CRACK TIP STRESS FIELD

(a)



(b)

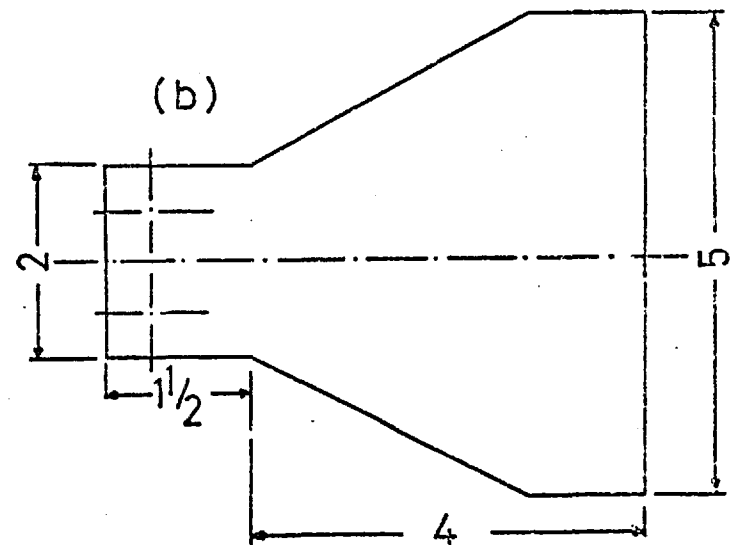
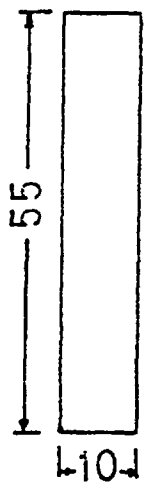
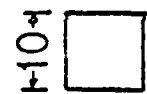
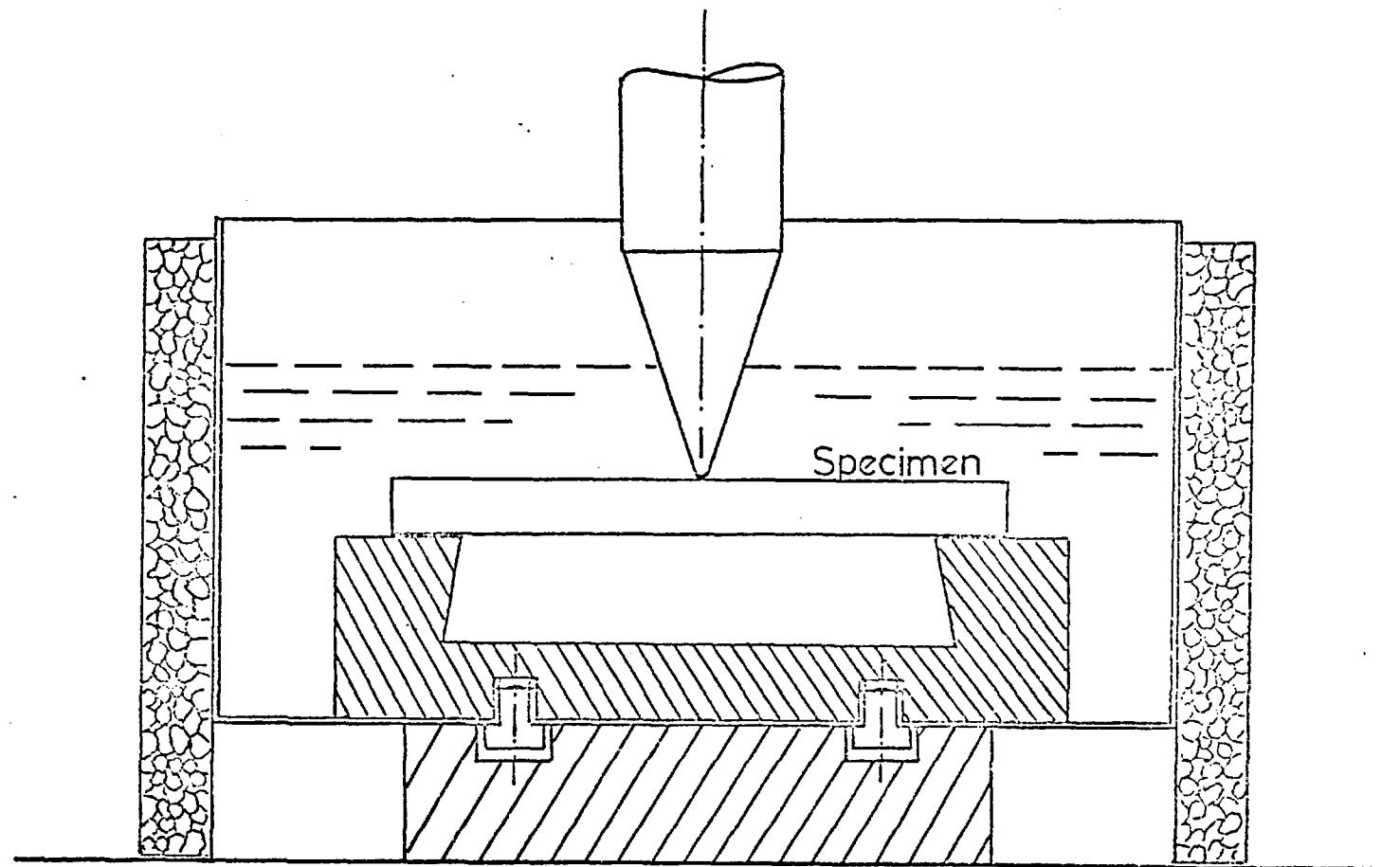


FIG (3)

- (a) THE SPECIMEN USED IN TEST SERIES (I)
- (b) THE SPECIMEN USED BY MOSTOVOY ET AL. (1967)



(a)



(b)

FIG (4)

(a) THE SPECIMEN USED FOR E_g MEASUREMENTS

(b) TEMPERATURE CONTROLLED BATH (TEST SERIES 2)

(Dimension in millimetres)

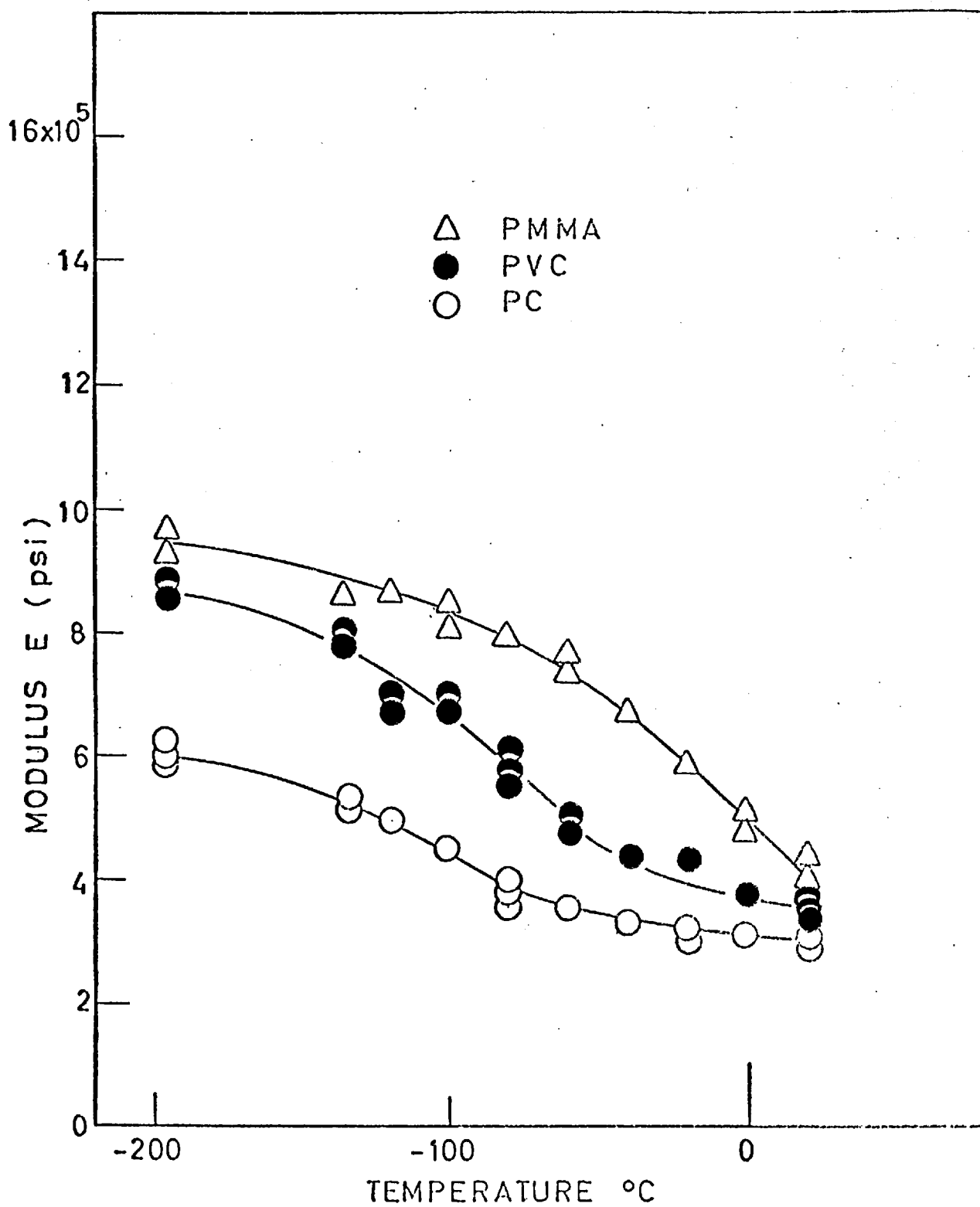


FIG (5) VARIATION OF THE TIME DEPENDENT MODULUS OF PMMA, PVC AND PC AS A FUNCTION OF TEMPERATURE

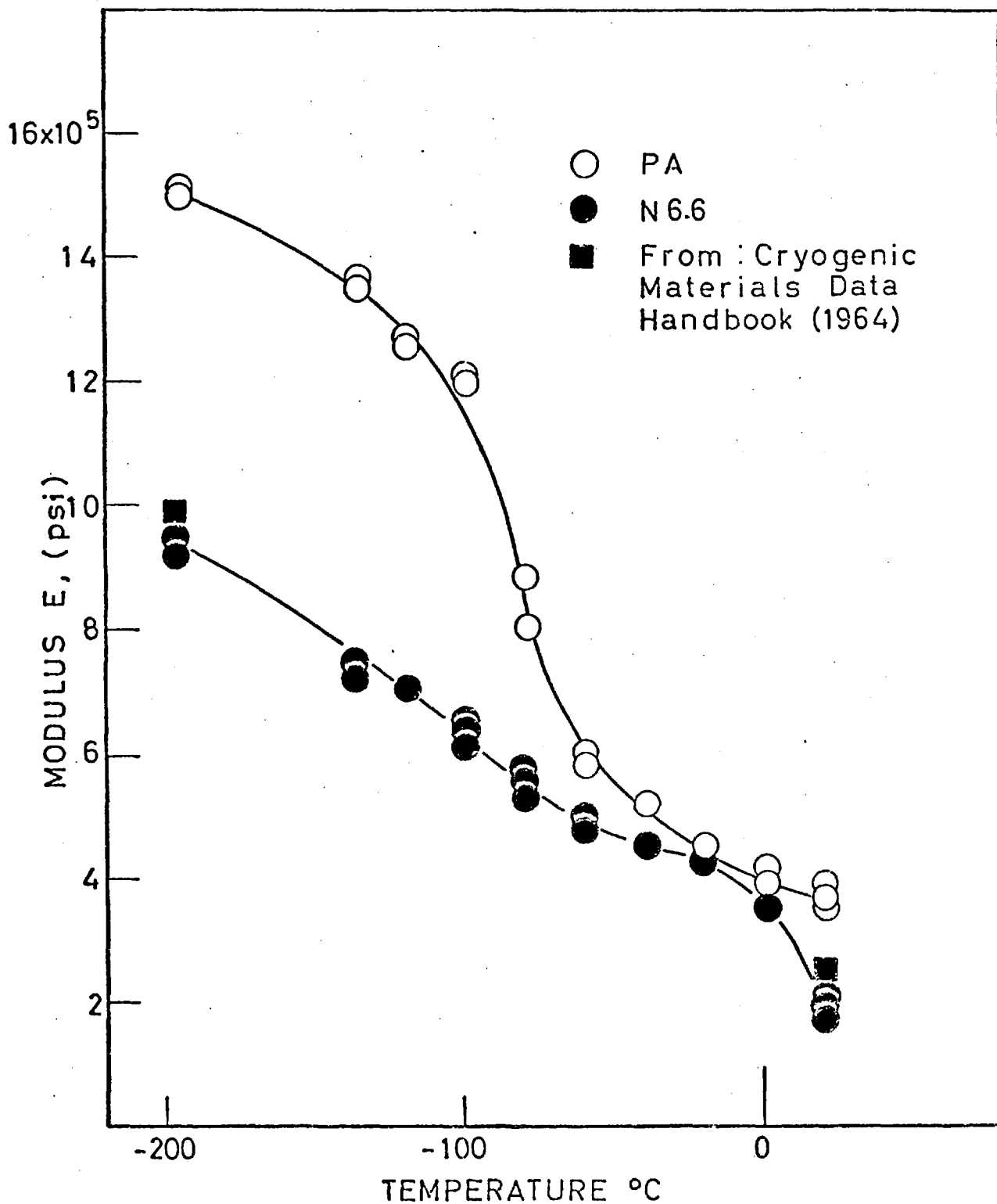
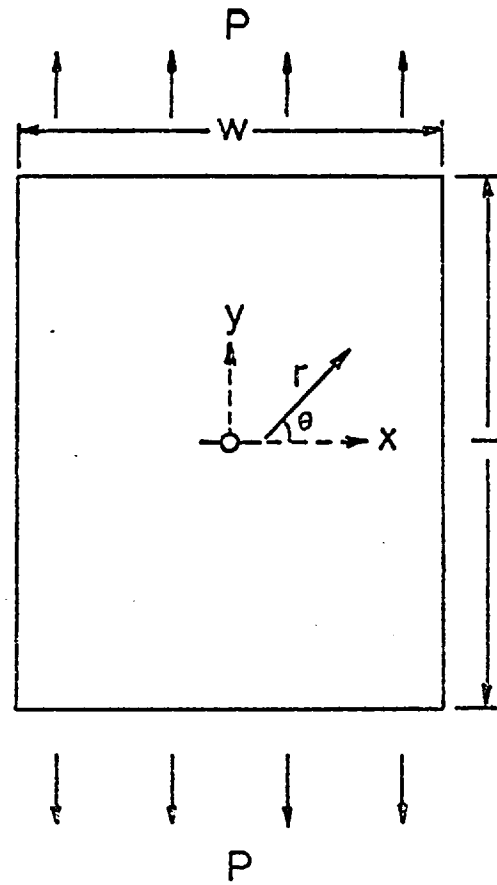
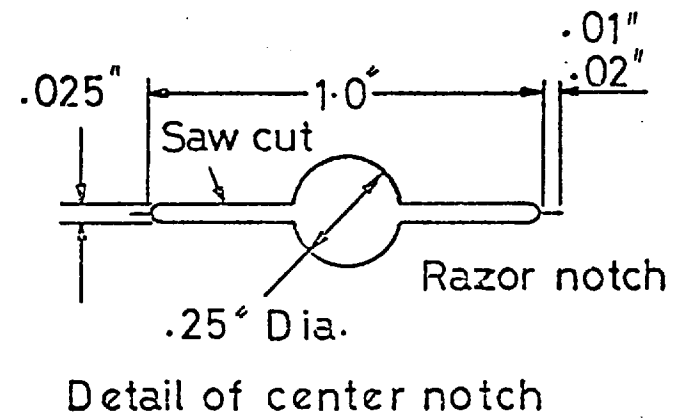


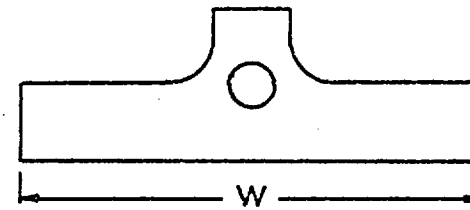
FIG (6) VARIATION OF THE TIME DEPENDENT MODULUS OF PA AND NYLON 6.6 AS A FUNCTION OF TEMPERATURE



Specimen



Detail of center notch



Specimen holder

FIG (7) DETAILS OF THE SPECIMEN AND THE CENTRE NOTCH (TEST SERIES 3-6)

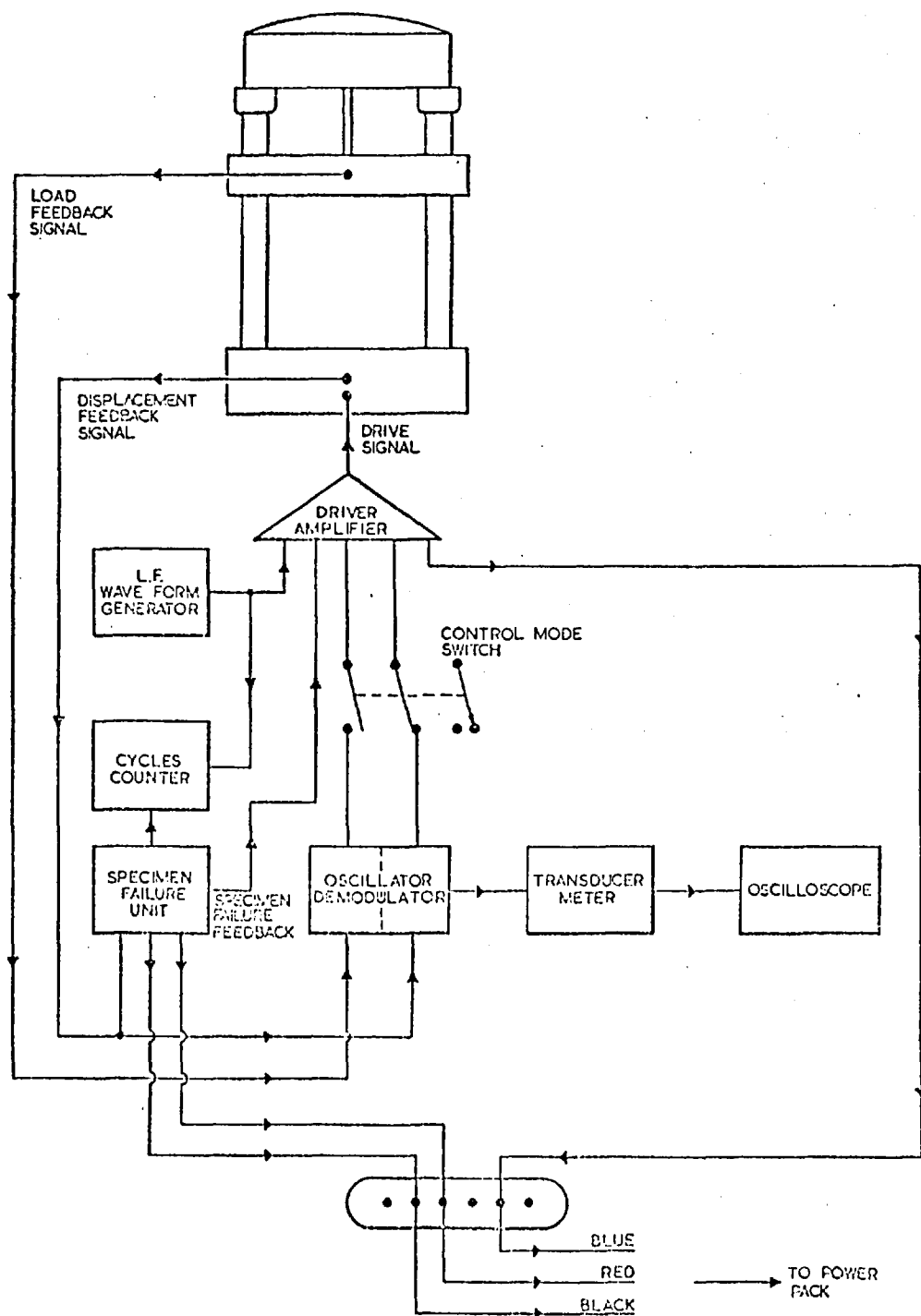


FIG (8) GENERAL ARRANGEMENT OF THE DRIVE AND THE CONTROL COMPONENTS OF THE DOWTY TESTING MACHINE

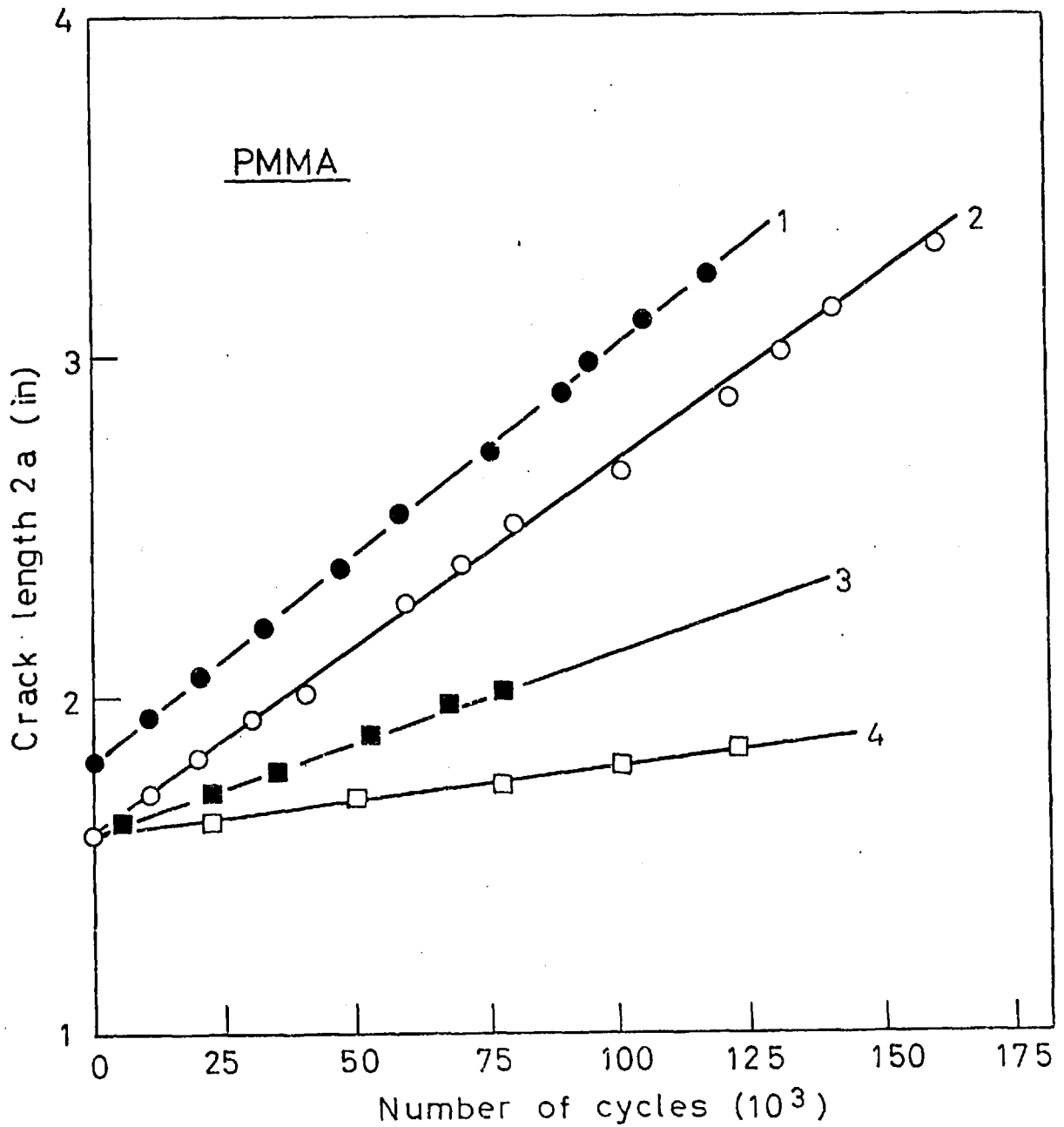
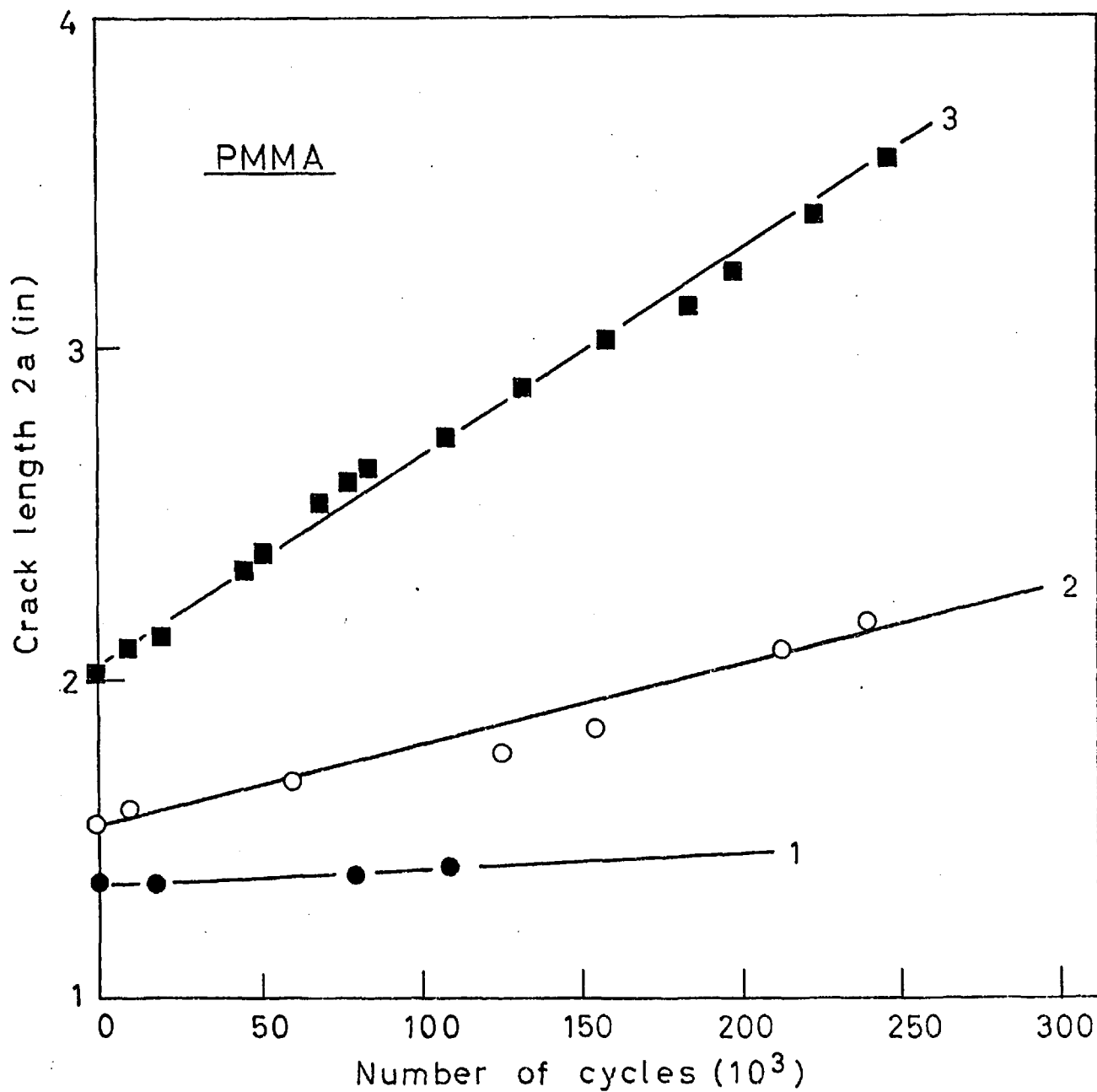


FIG (9) CRACK GROWTH PATTERN AT DIFFERENT VALUES OF ΔK AND K_m IN PMMA



Curve	1	2	3
K_m (psi $\sqrt{\text{in}}$)	125	375	625

FIG (10) VARIATION WITH K_m OF CRACK GROWTH RATE IN PMMA;
 $\Delta K = \text{Constant} = 250 \text{ psi}\sqrt{\text{in}}$

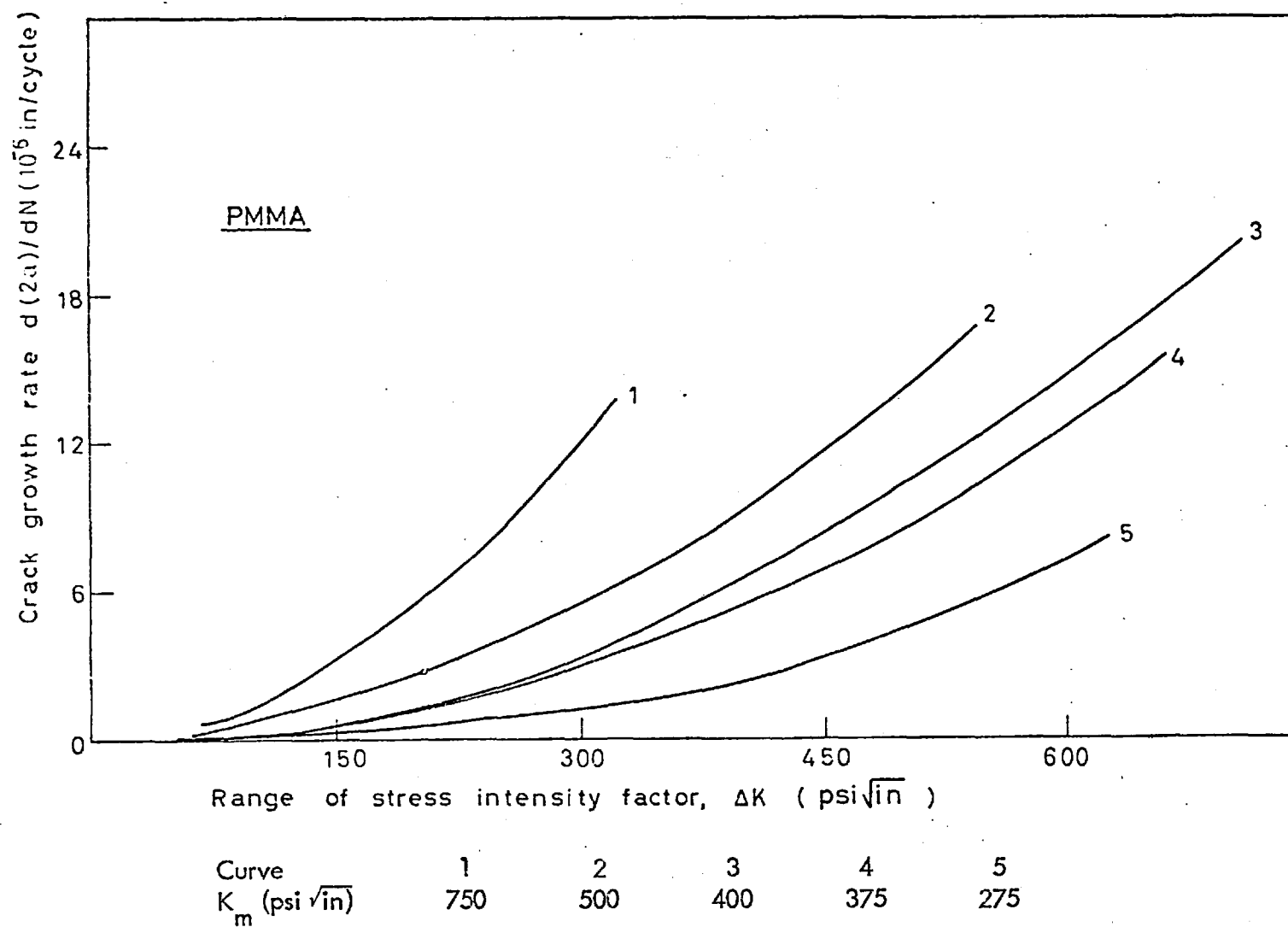


FIG (11) VARIATION WITH ΔK OF CRACK GROWTH RATE IN PMMA AT DIFFERENT VALUES OF K_m

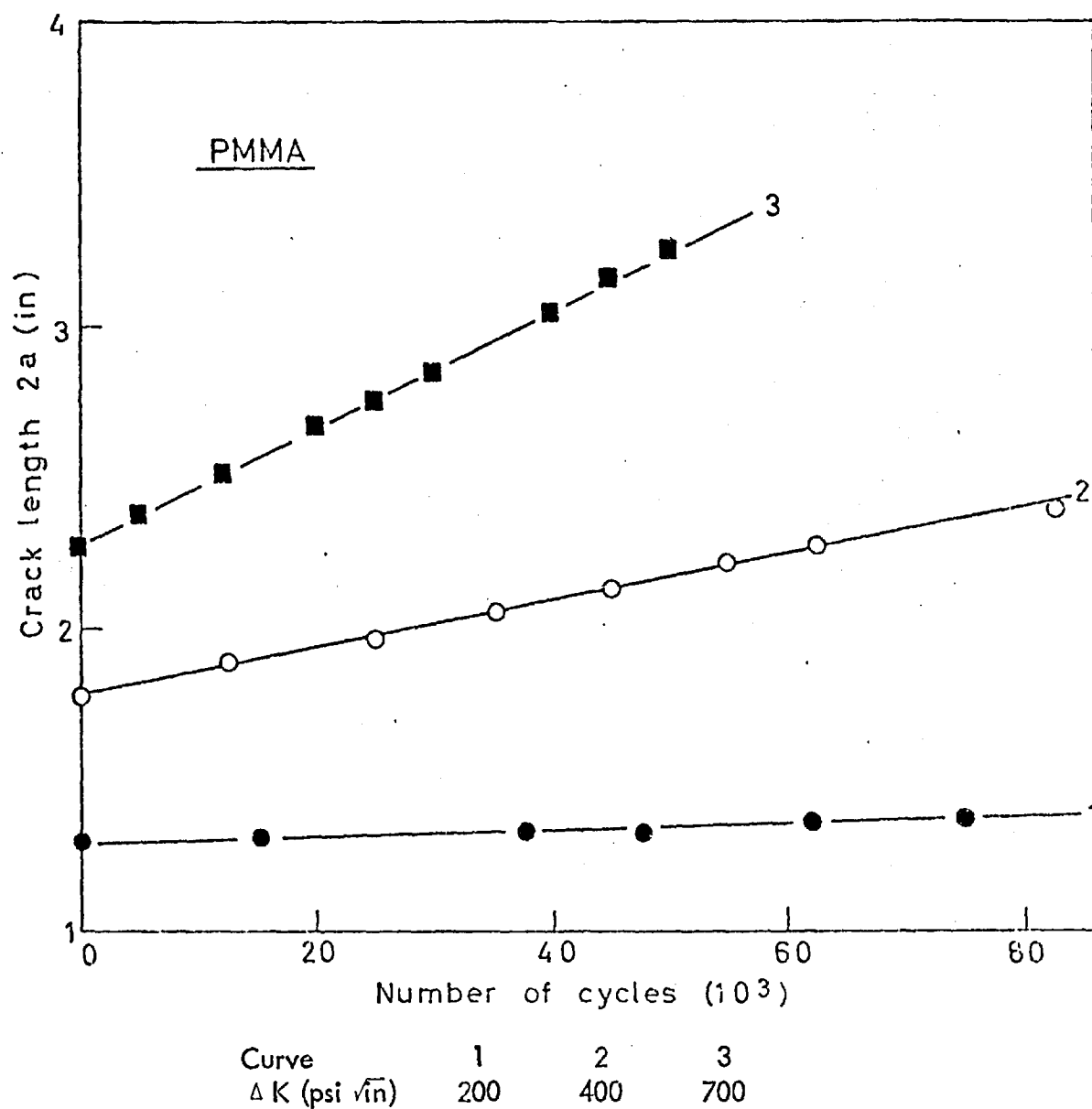


FIG (12) VARIATION WITH ΔK OF CRACK GROWTH RATE IN PMMA,
 $K_m = \text{CONSTANT} = 400 \text{ psi}\sqrt{\text{in}}$

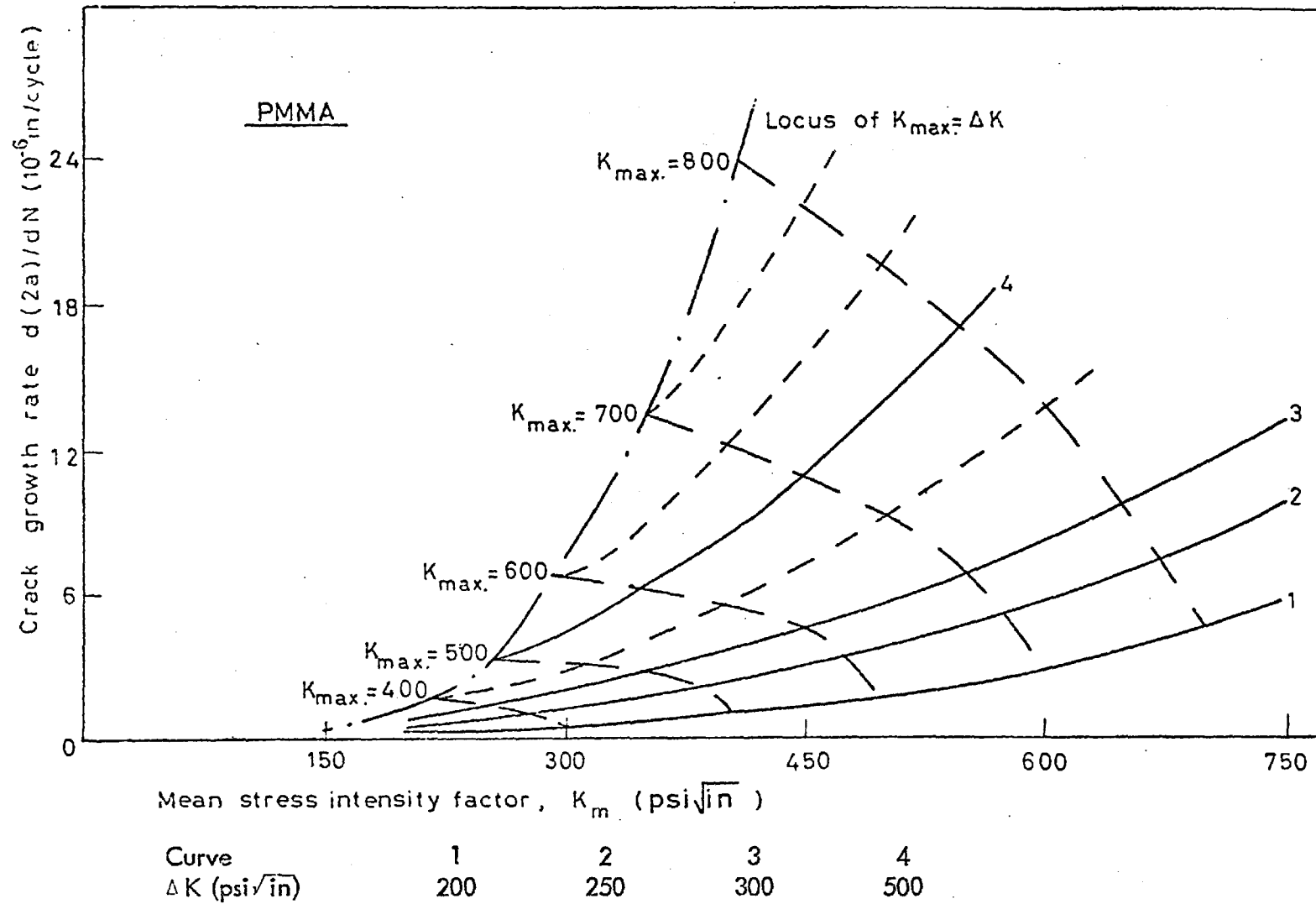


FIG (13) VARIATION WITH K_m OF CRACK GROWTH RATE IN PMMA AT DIFFERENT VALUES OF ΔK

Curve	Frequency (Hz)	$\dot{a}_N$ (in/cycle)
1	20	4.8×10^{-6}
2	5	13.8×10^{-6}
3	0.1	440×10^{-6}

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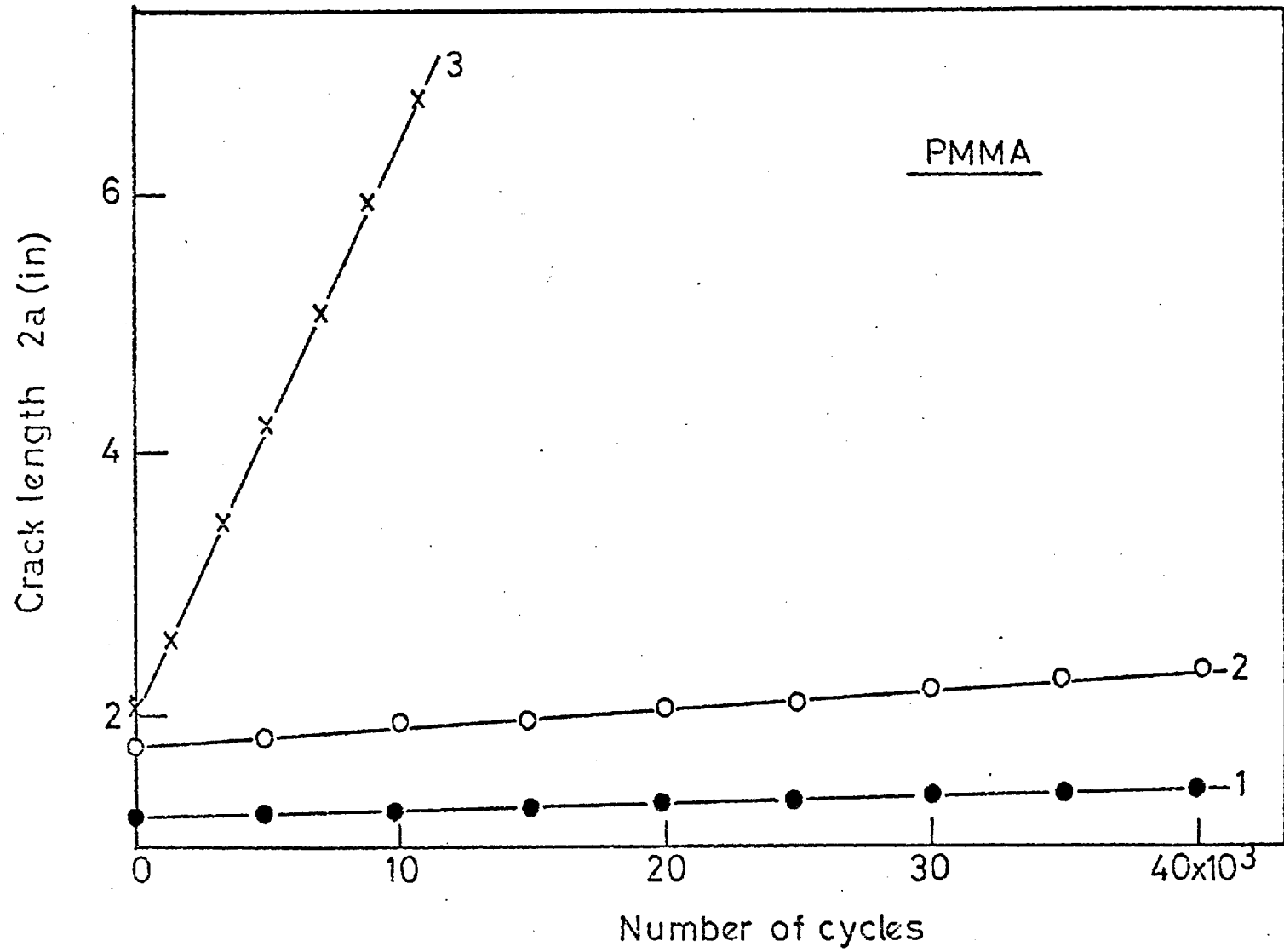


FIG (14) VARIATION OF $\dot{a}_N$ WITH FREQUENCY IN TESTS ON PMMA UNDER $K_{\max} = 750$, $K_{\min} = 250$ psi $\sqrt{\text{in}}$ CONDITIONS

Curve	Frequency (Hz)	$\dot{a}_t$ (in/s)
1	20	1.0×10^{-4}
2	5	0.7×10^{-4}
3	0.1	0.42×10^{-4}

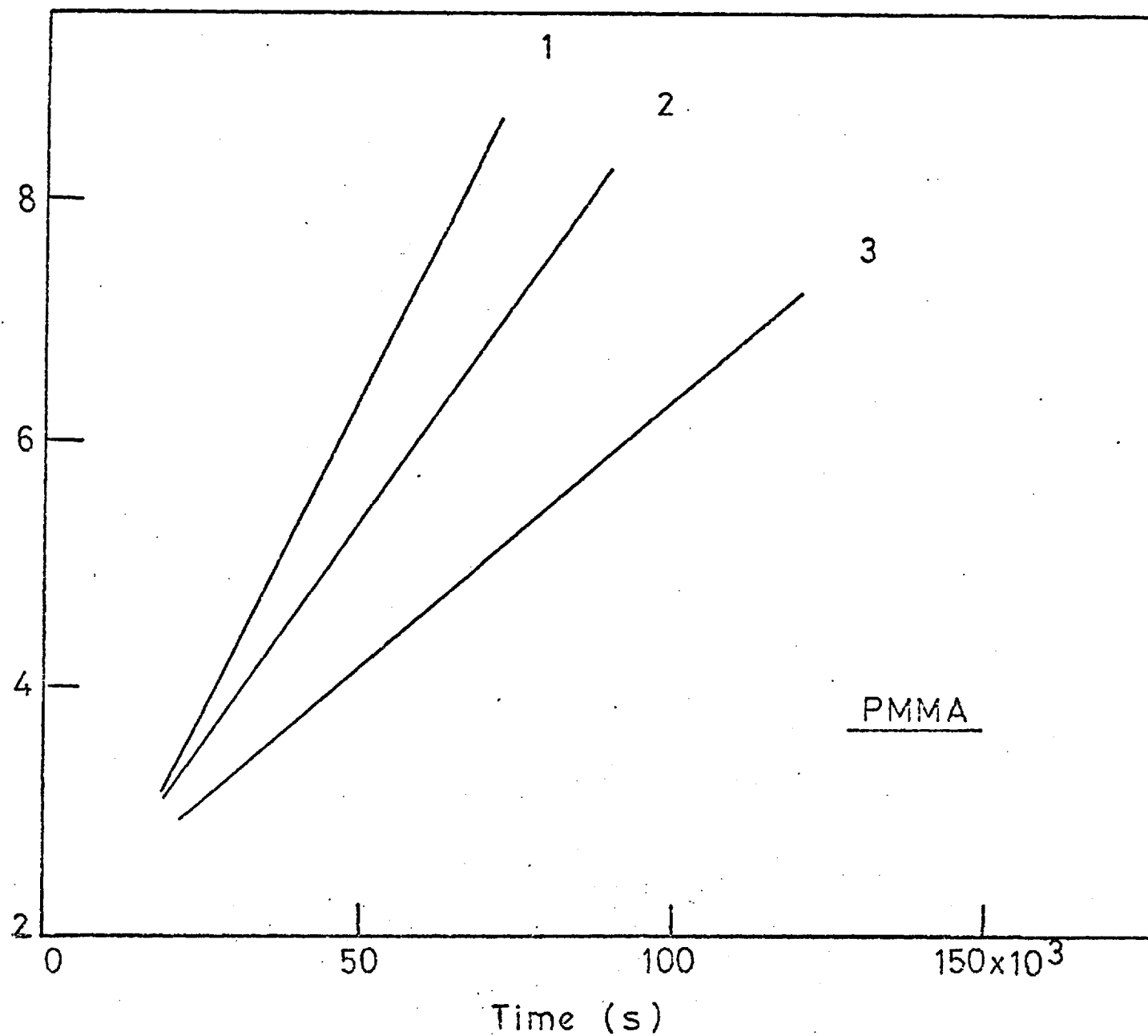
Crack length $2a$ (in)

FIG (15) VARIATION OF CRACK LENGTH WITH TIME AT DIFFERENT FREQUENCIES

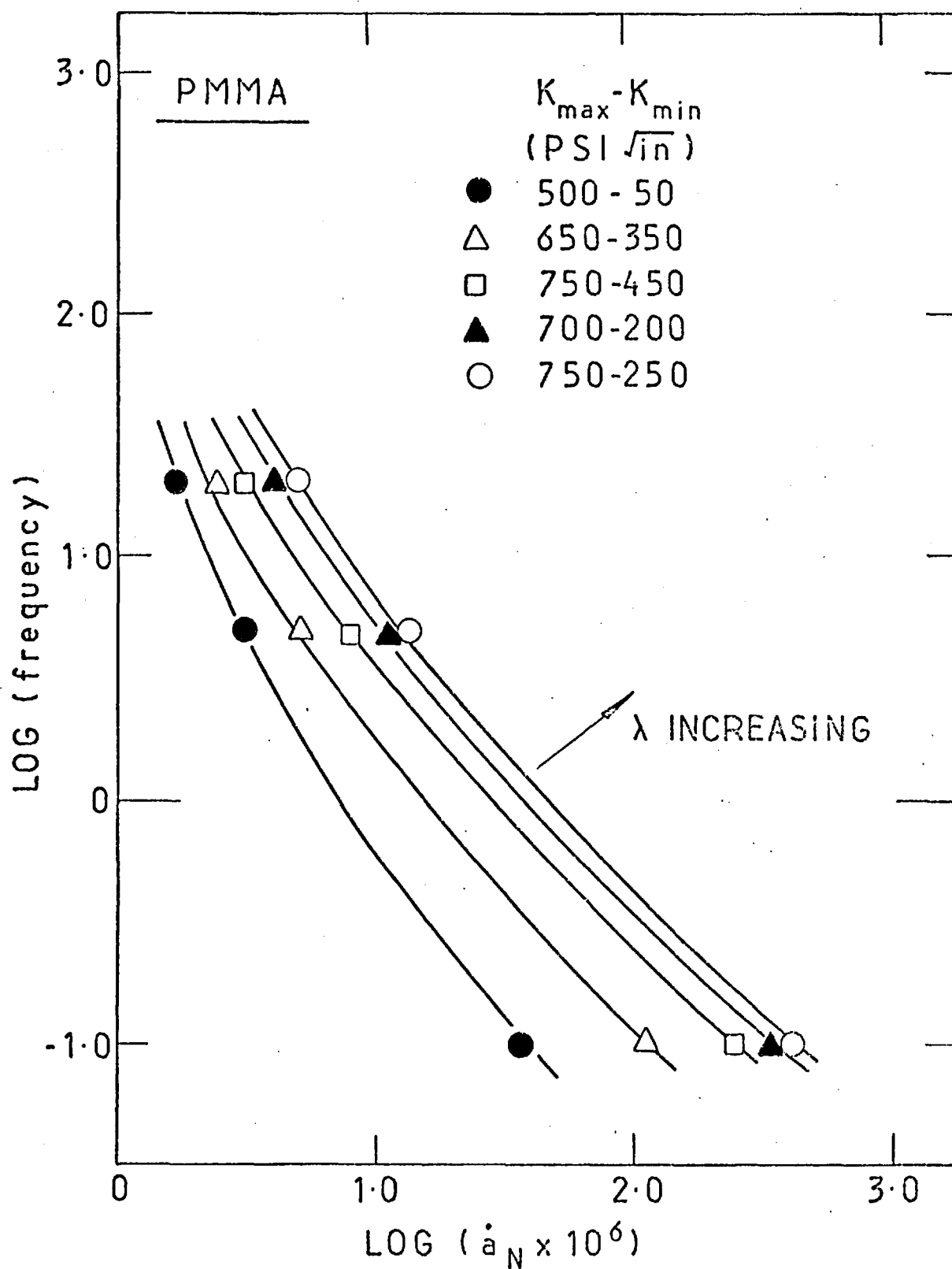


FIG (16) VARIATION WITH FREQUENCY OF $\dot{a}_N$ IN PMMA FOR A RANGE OF VALUES OF $K_{\max}$ AND $K_{\min}$

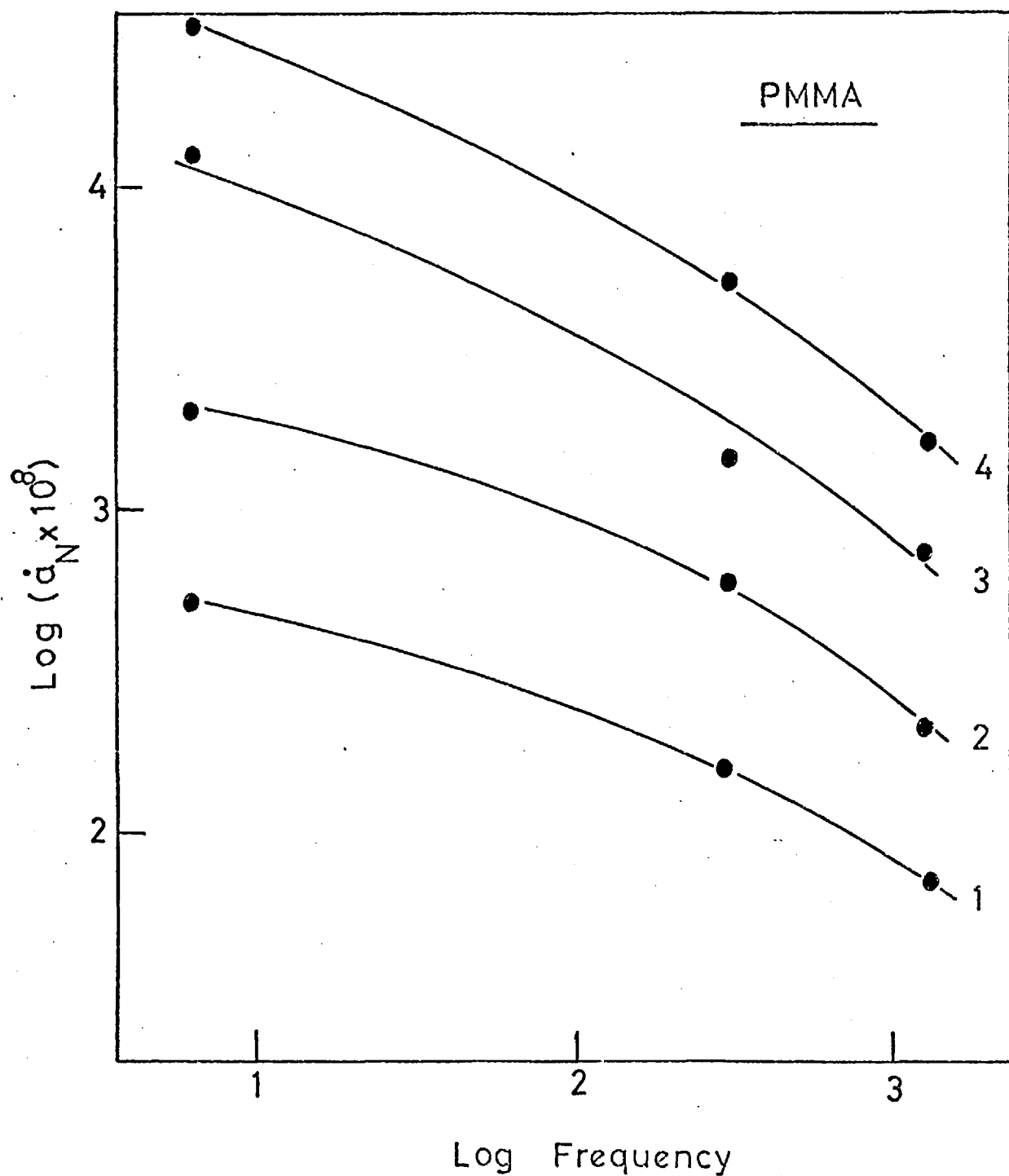
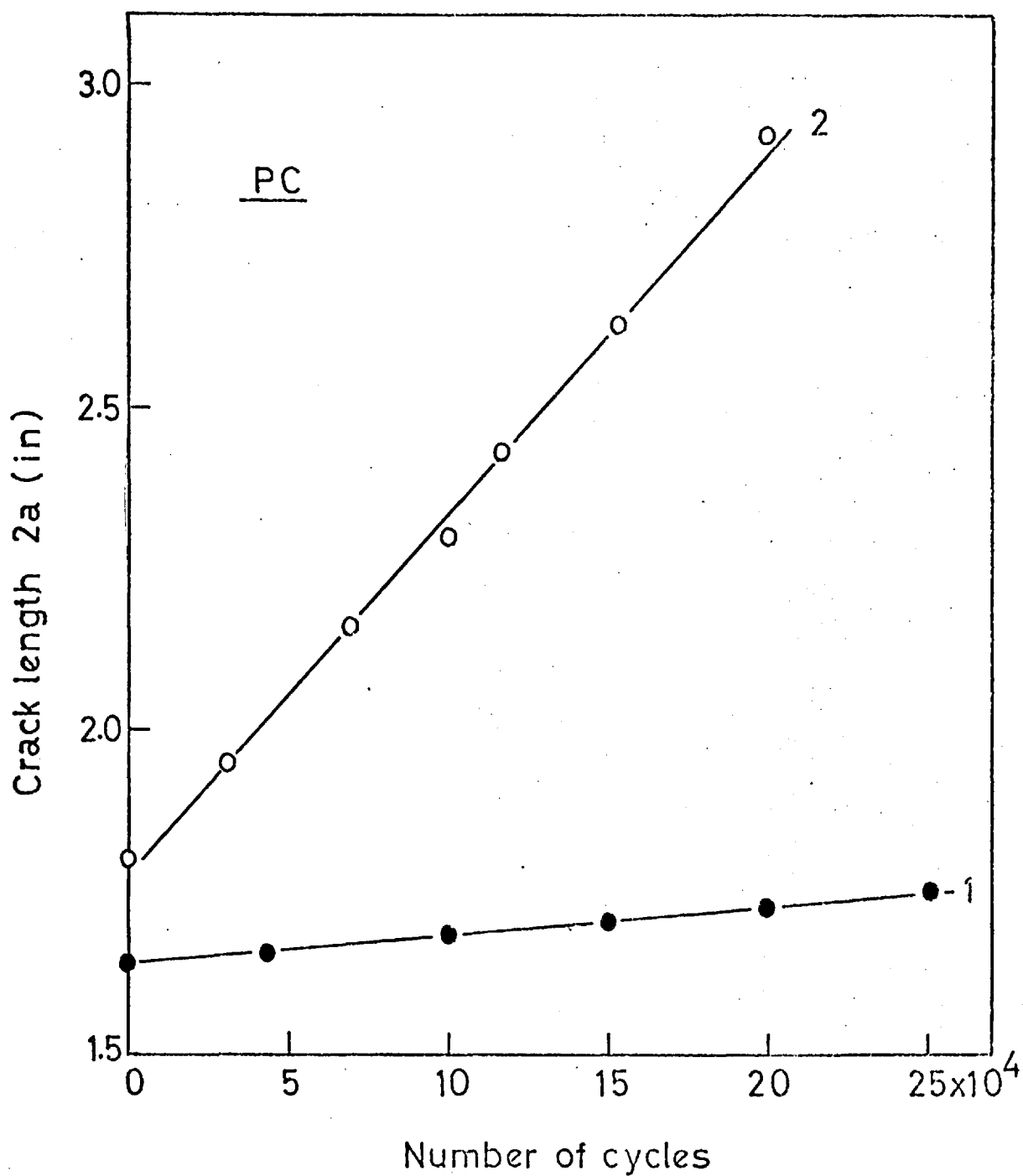


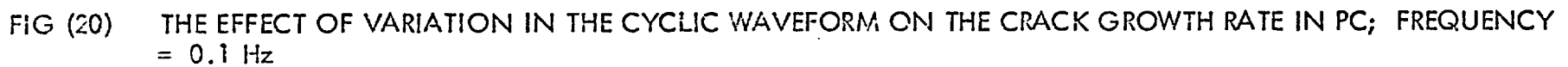
FIG (17) THE INFLUENCE OF K_m ON $\dot{a}_N$ IN PMMA AT DIFFERENT FREQUENCIES

FIG (18) VARIATION OF THE FRACTURE TOUGHNESS OF POLYCARBONATE AS A FUNCTION OF THE CROSSHEAD SPEED



Curve	1	2
$K_{\max}$ (psi $\sqrt{\text{in}}$)	750	650
$K_{\min}$ (psi $\sqrt{\text{in}}$)	450	150

FIG (19) CRACK GROWTH PATTERN AT DIFFERENT VALUES OF K_m AND ΔK IN PC, FREQUENCY = 5 Hz



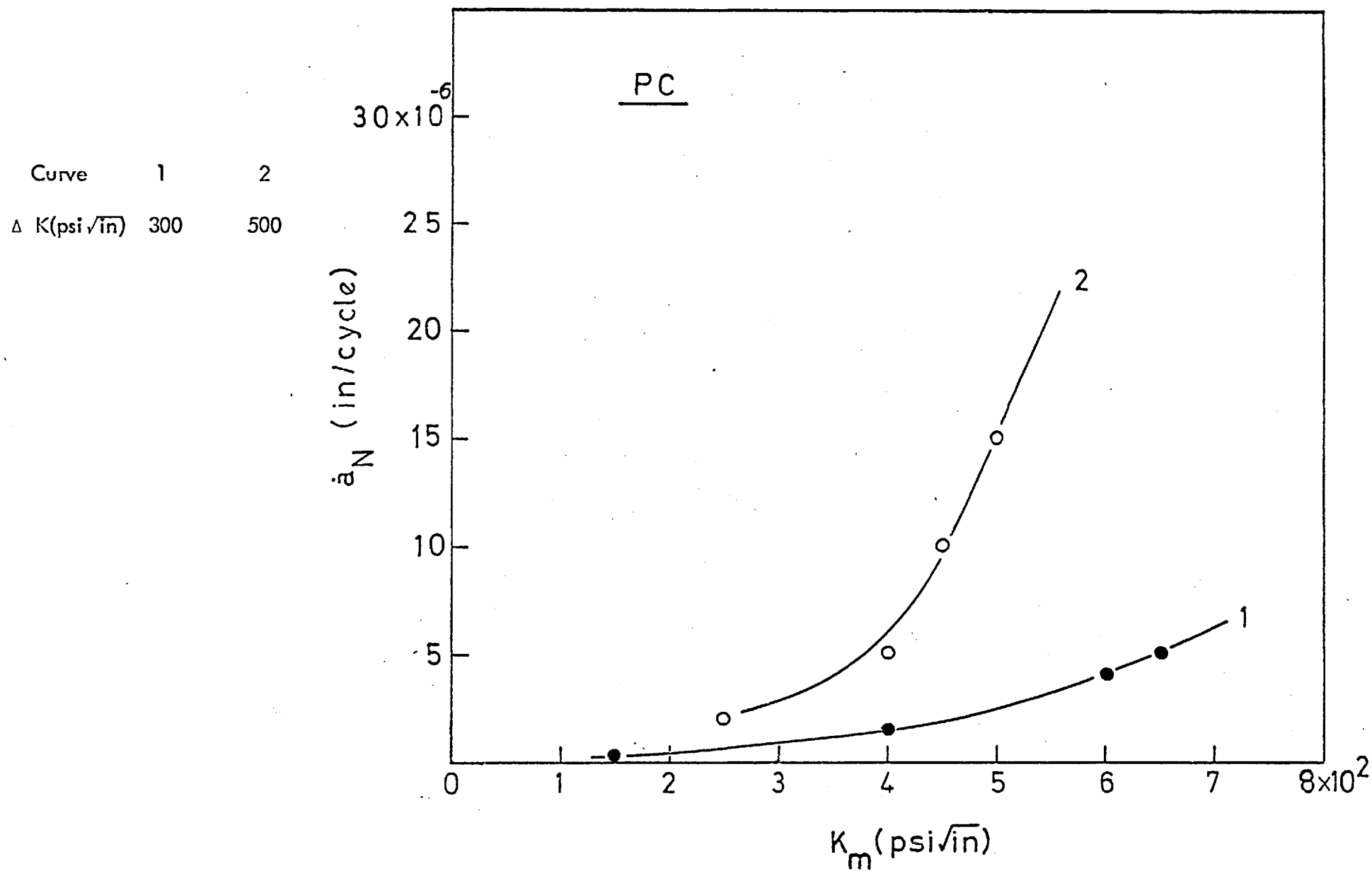


FIG (21) THE EFFECT OF K_m ON THE CRACK GROWTH RATE IN PC; FREQUENCY = 0.1 Hz

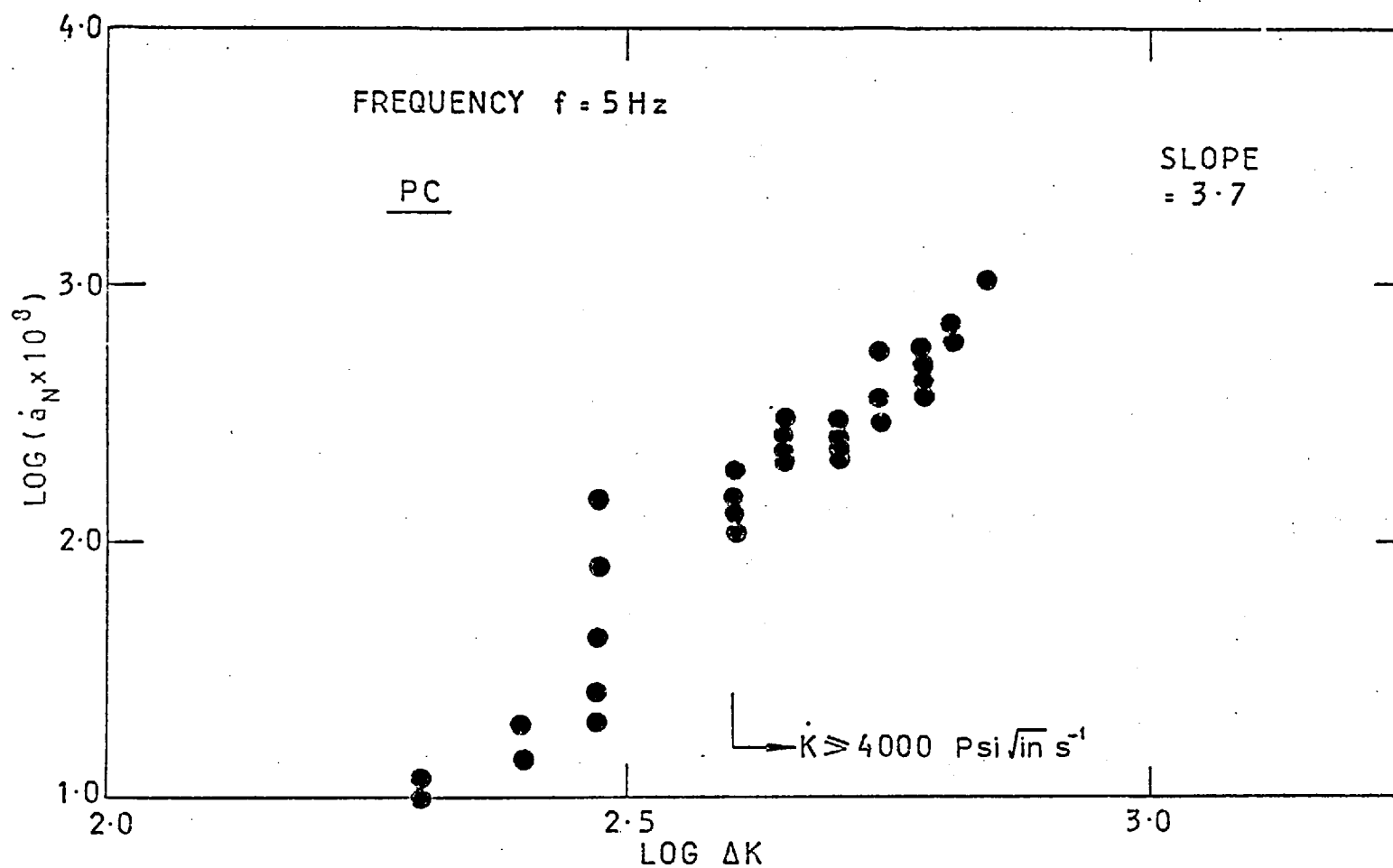


FIG (22) RESULTS OF TESTS AT A FREQUENCY OF 5 Hz ON PC, SHOWING THE COMPARATIVELY SMALL EFFECTS OF K_m , WHEN K EXCEEDS $400 \text{ psi}\sqrt{\text{in}} \text{ s}^{-1}$

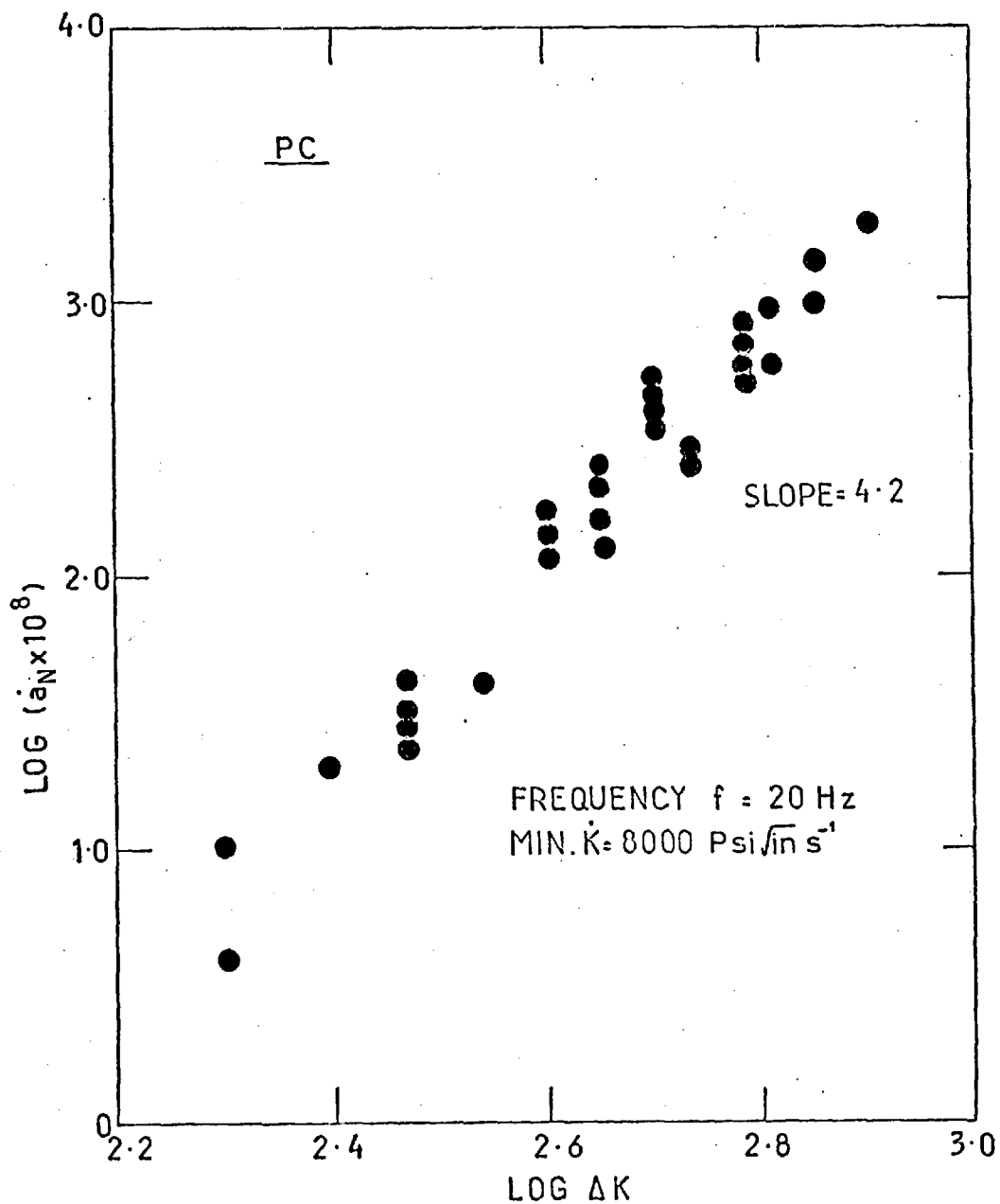


FIG (23) RESULTS OF TESTS ON PC. AT A FREQUENCY OF 20 Hz. MINIMUM $\dot{K} = 8000$ psi $\sqrt{\text{in}} \text{ s}^{-1}$

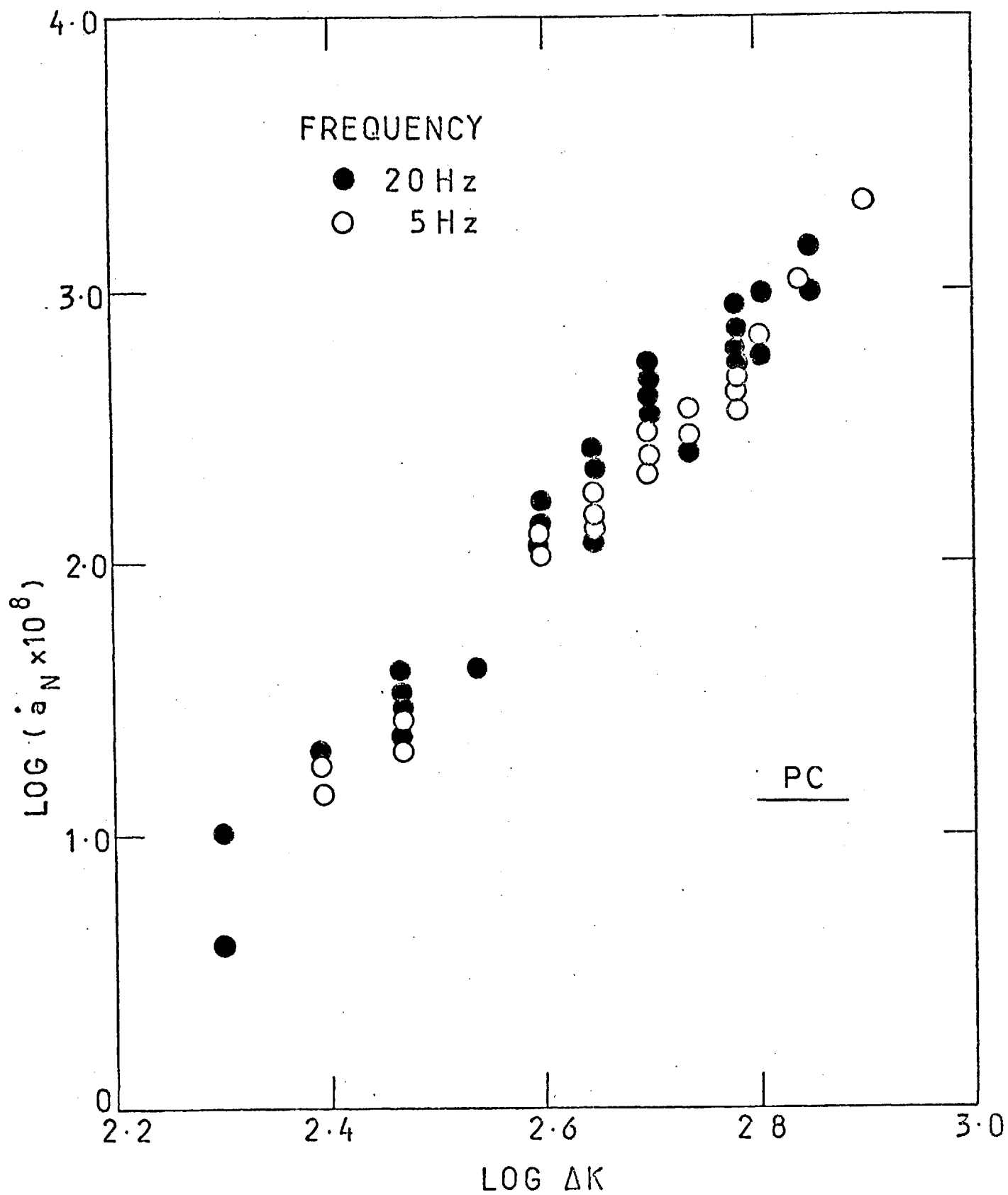
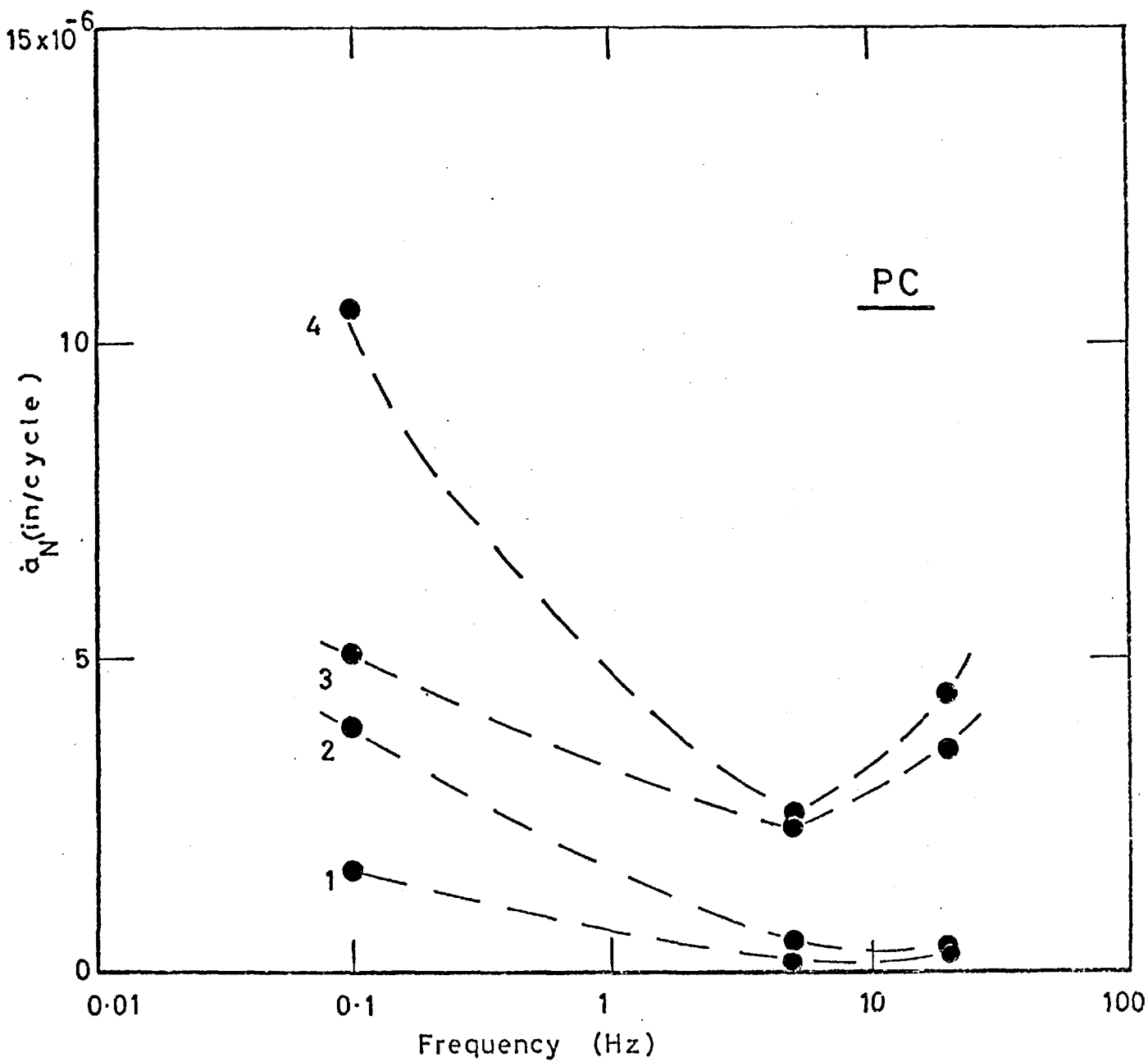


FIG (24) THE EFFECT OF CHANGE IN FREQUENCY ON $\dot{a}_N$ IN PC



Curve	$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)
1	550	250
2	750	450
3	650	150
4	700	200

FIG (25) VARIATIONS IN THE INFLUENCE OF THE FREQUENCY CHANGES ABOVE AND BELOW THE $\dot{K}^*$ LEVEL

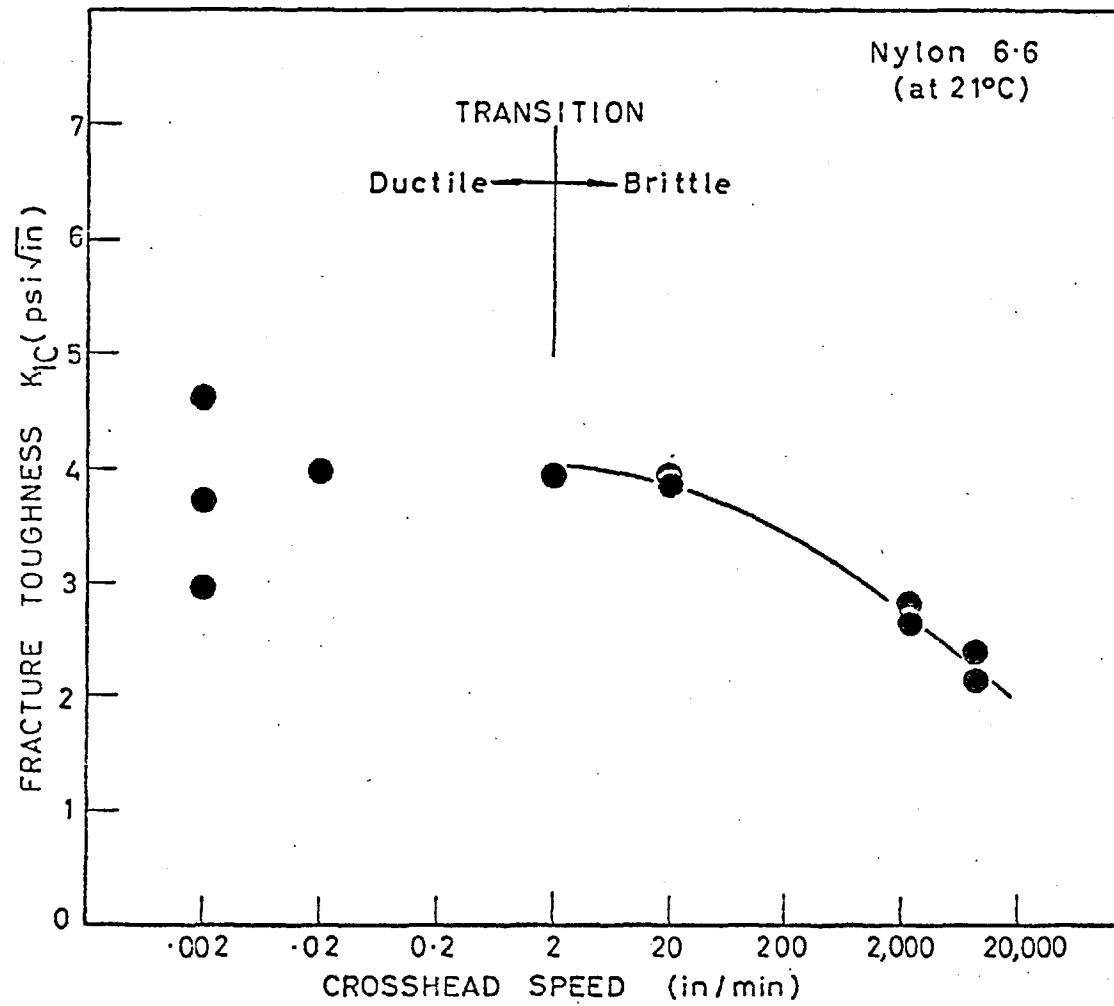


FIG (26) VARIATION OF THE FRACTURE TOUGHNESS OF NYLON 6.6
AS A FUNCTION OF THE CROSSHEAD SPEED

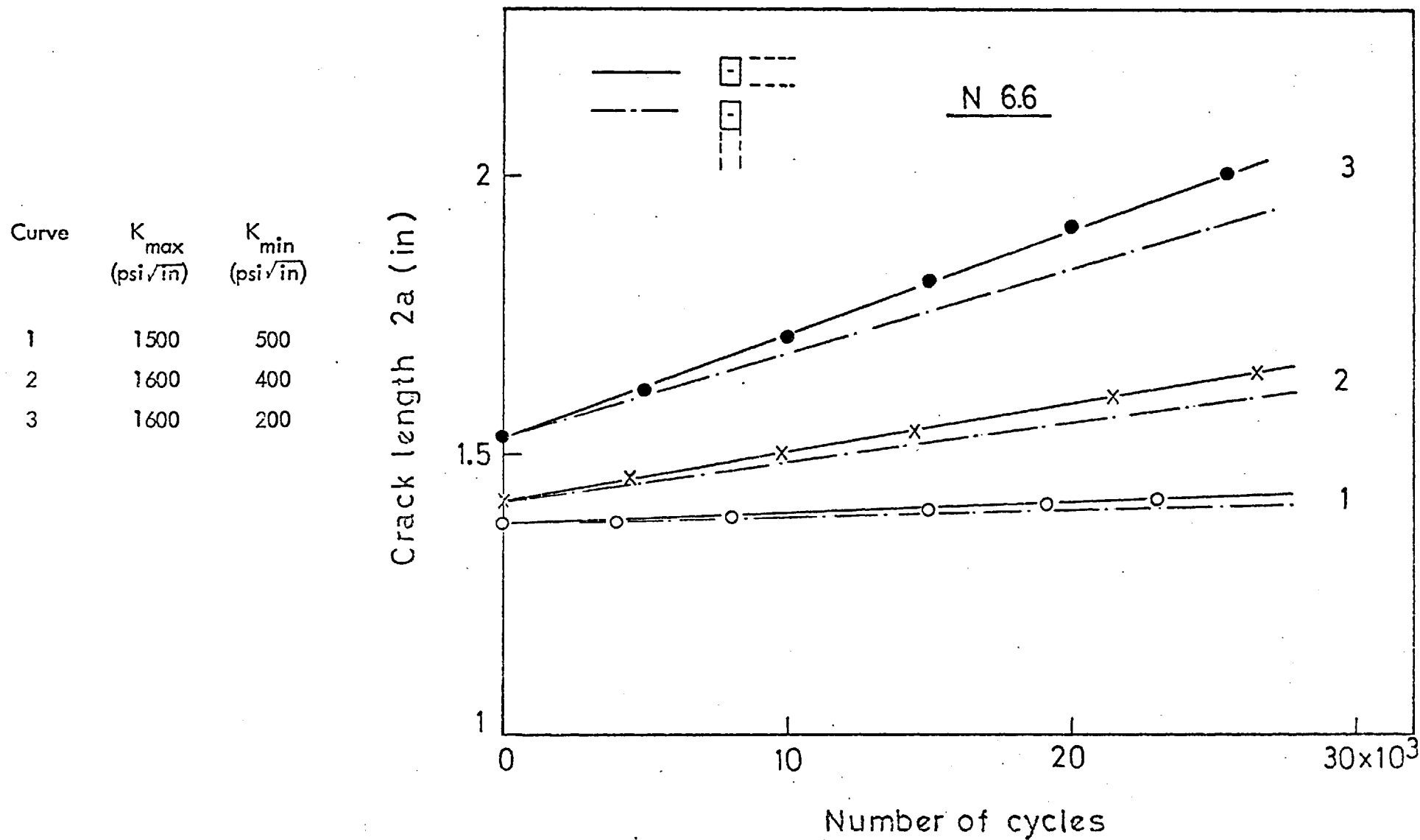


FIG (27) TEST OF THE EFFECT OF VARIATION IN THE DIRECTIONAL STRENGTH IN EXTRUDED SHEETS OF NYLON 6.6,
FREQUENCY = 5 Hz

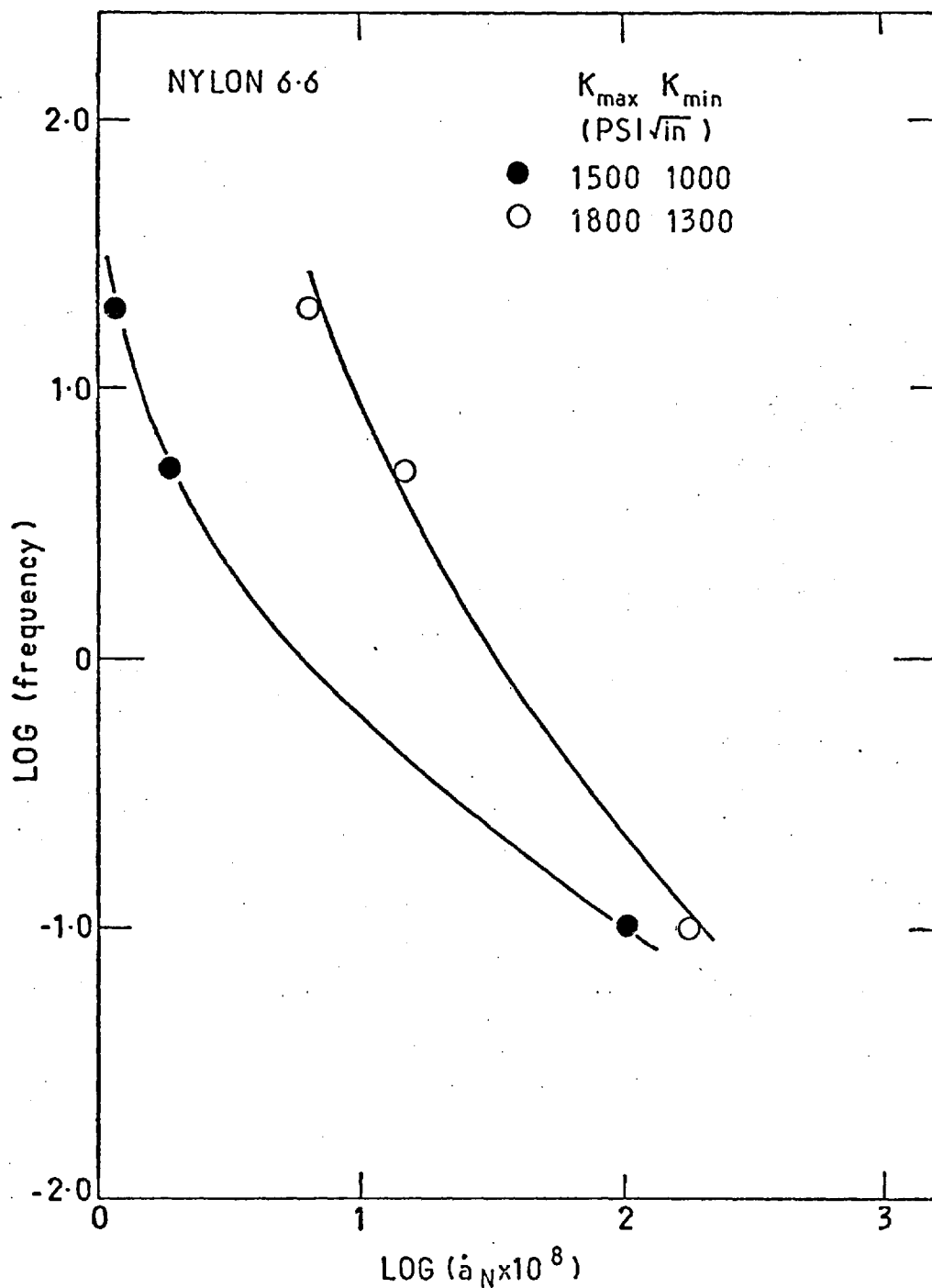


FIG (28) VARIATION OF THE CYCLIC CRACK GROWTH RATE IN NYLON 6.6 WITH CHANGES IN LOADING FREQUENCY

Stress levels 1300 - 0 psi

Fracture at $K_{IC} \approx 2700 \text{ psi}\sqrt{\text{in}}$

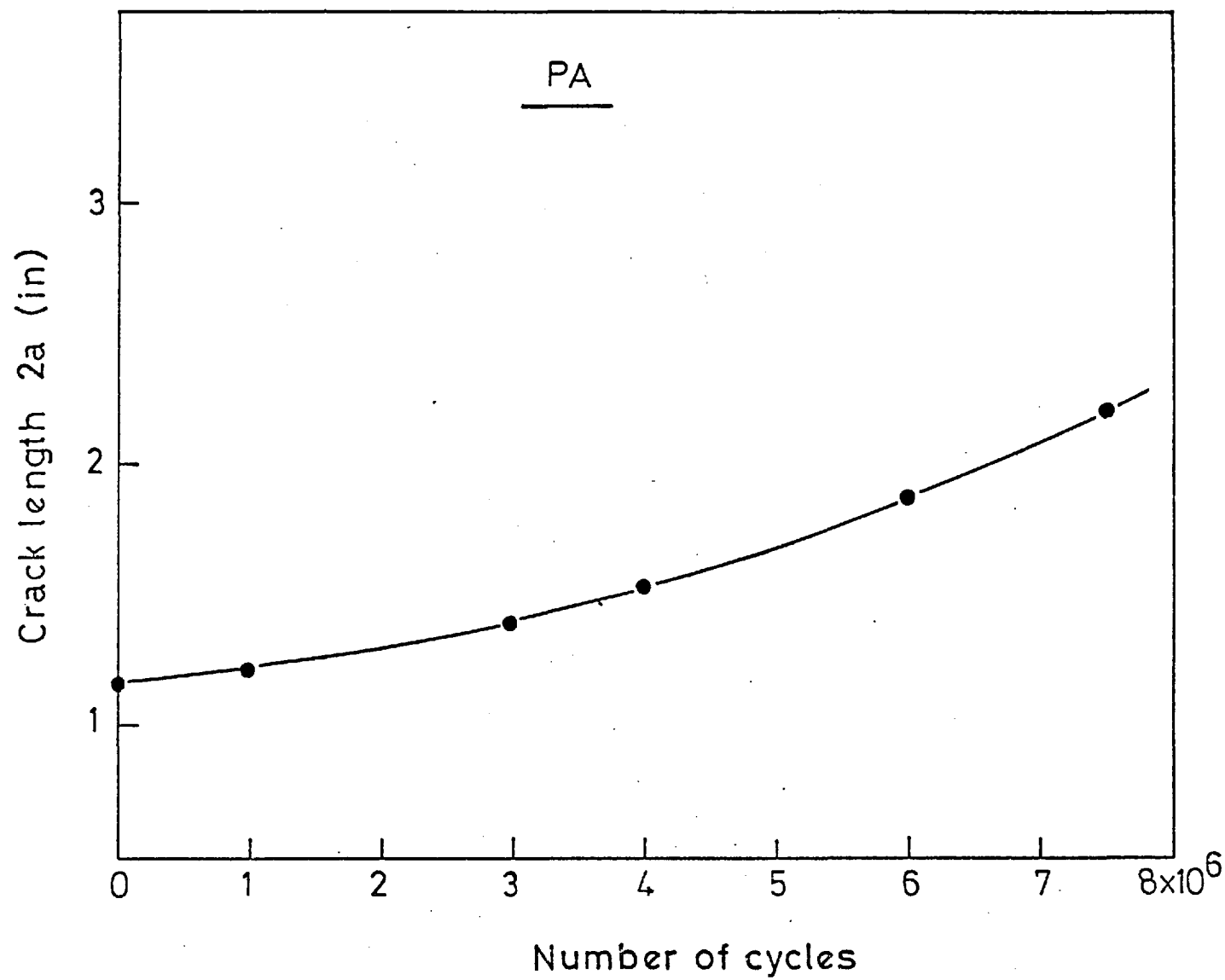


FIG (29) CYCLIC CRACK PROPAGATION IN PA AT A FREQUENCY OF 5 Hz and UNDER THE CONSTANT APPLIED STRESS CONDITIONS

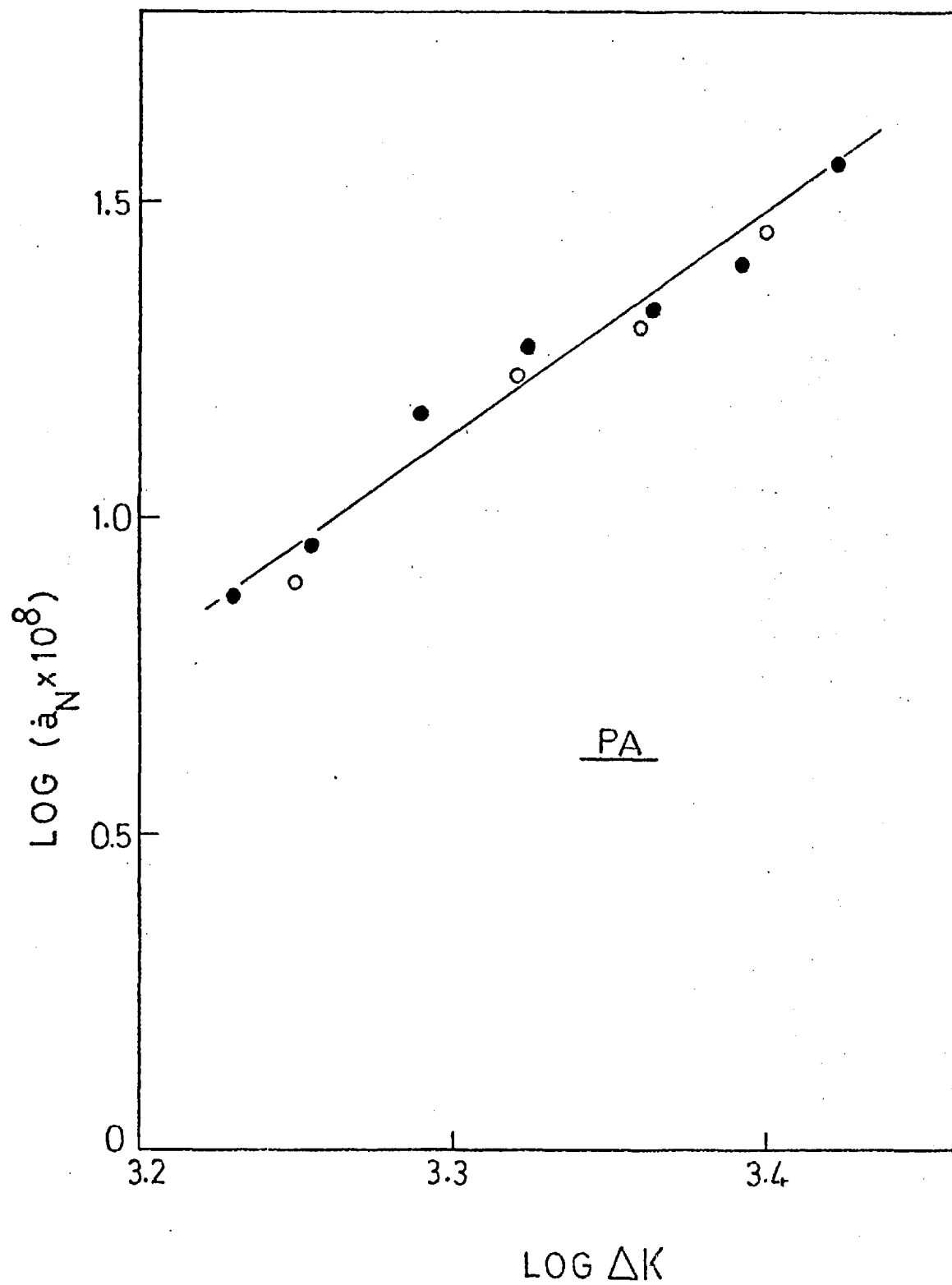
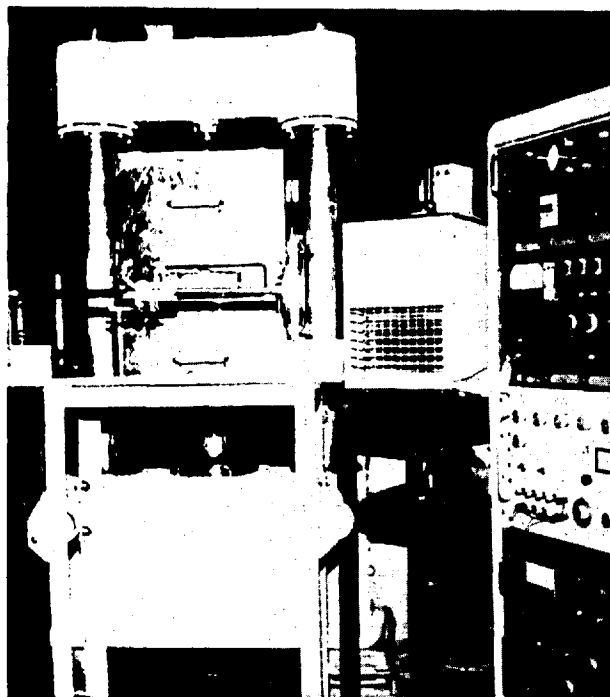
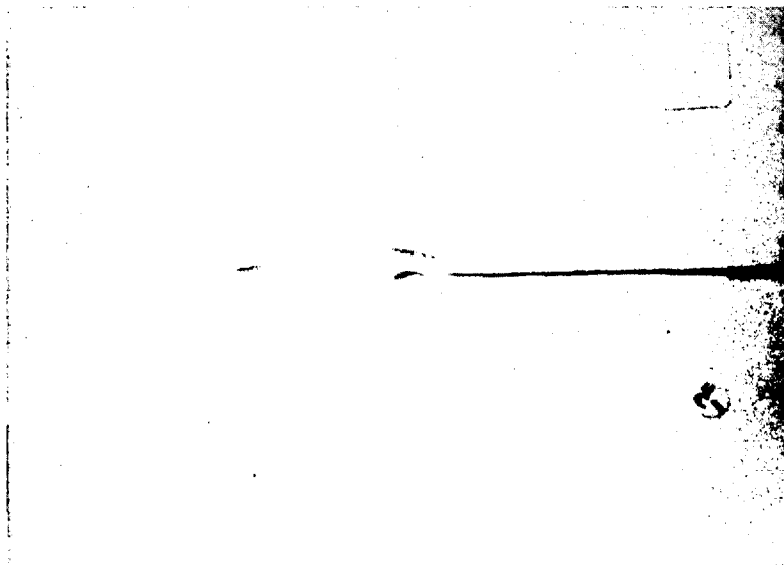


FIG (30) CYCLIC CRACK GROWTH DATA ON PA; FREQUENCY = 5Hz

- (o) Constant ΔK conditions
- (●) Constant applied load conditions



(a)



(b)

FIG (31) (a) ARRANGEMENT OF THE EQUIPMENT FOR THE ENVIRONMENTAL TEMPERATURE CONTROLLED TESTS
(b) THE HEAT AFFECTED ZONE AT THE CRACK TIP; WHITE PVC SPECIMEN

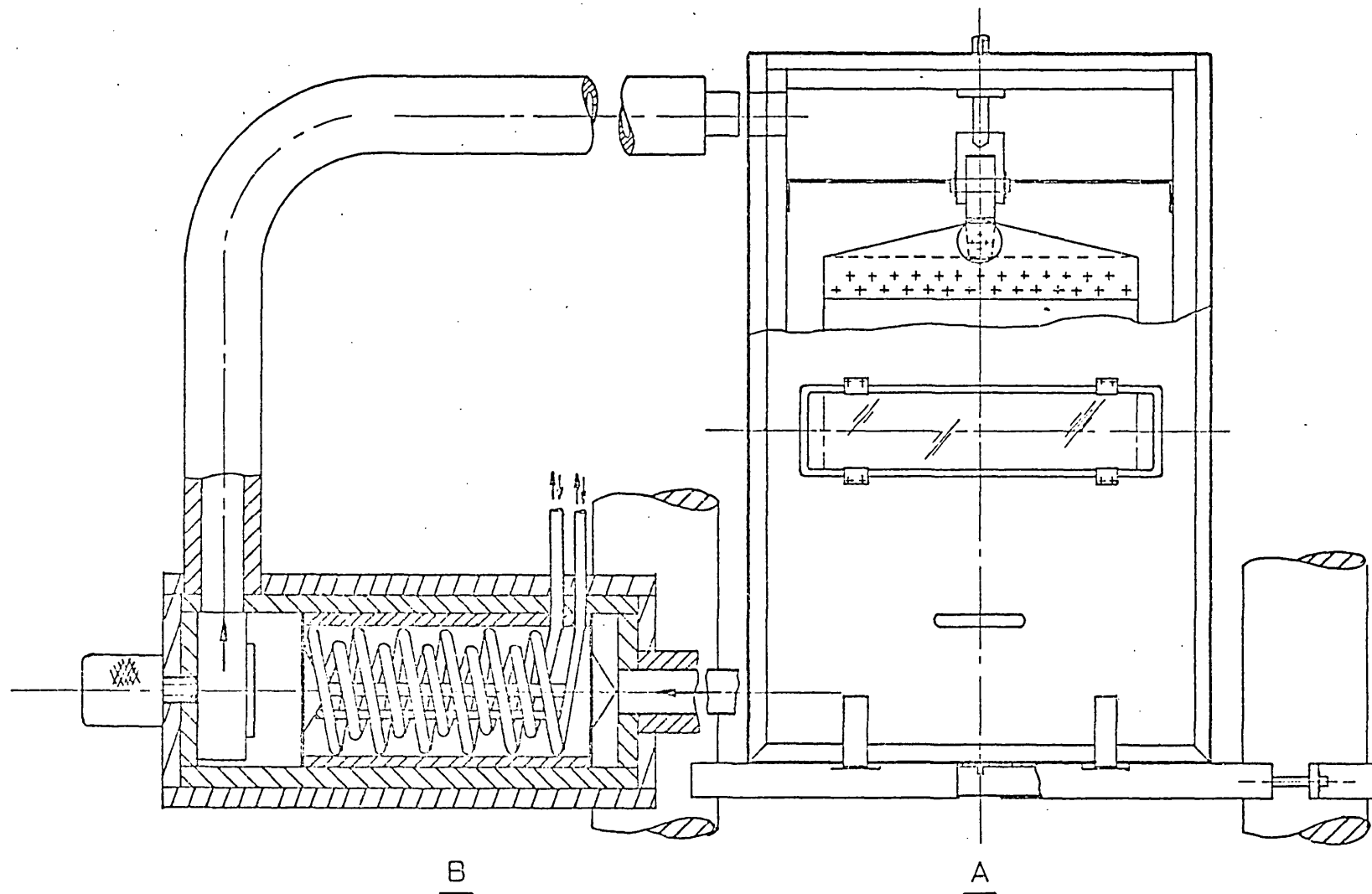


FIG (32) THE TEMPERATURE CONTROLLED ENVIRONMENTAL CHAMBER
(A) SPECIMEN CONTAINER
(B) COIL AND THE CIRCULATOR FAN

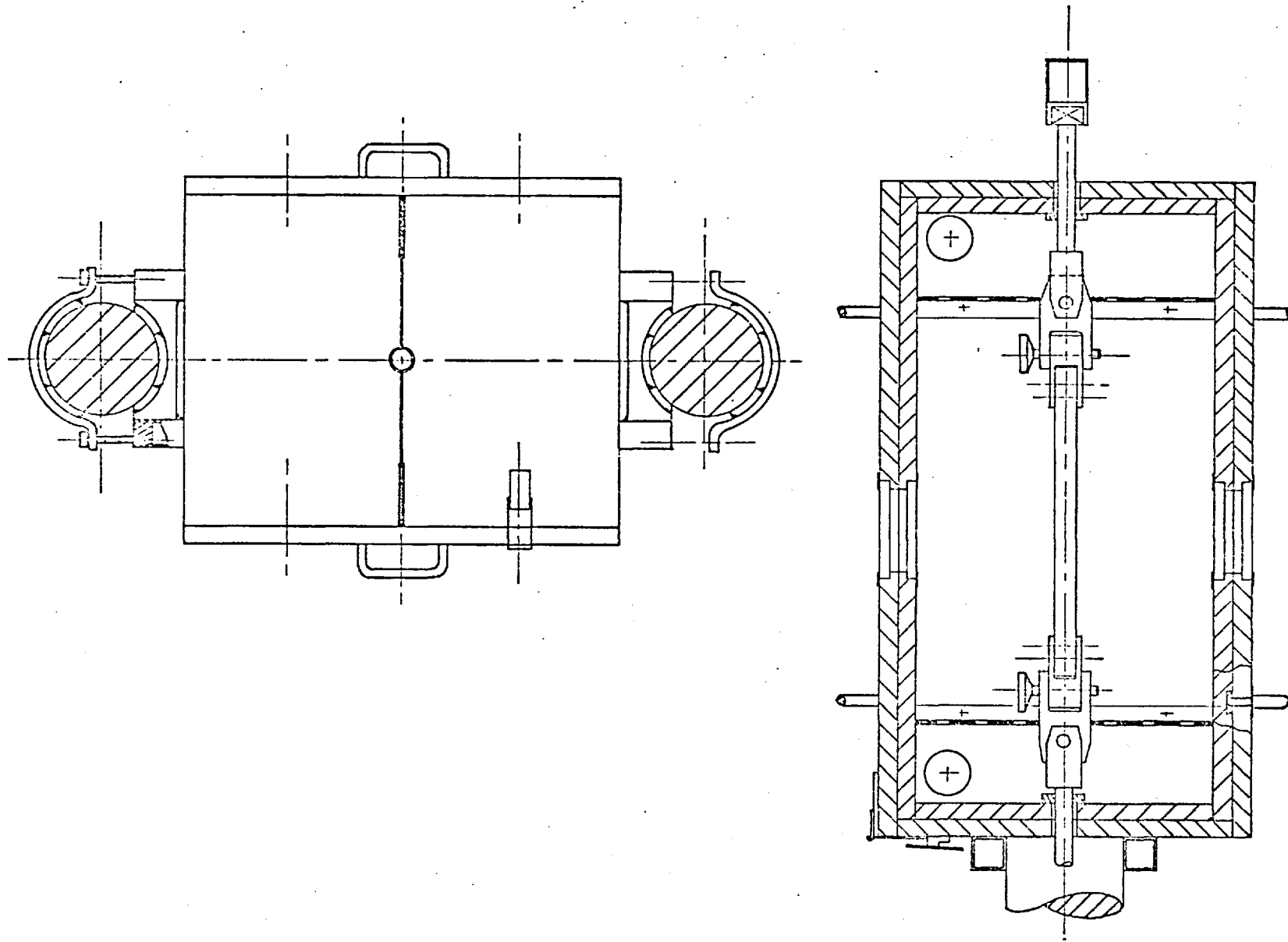
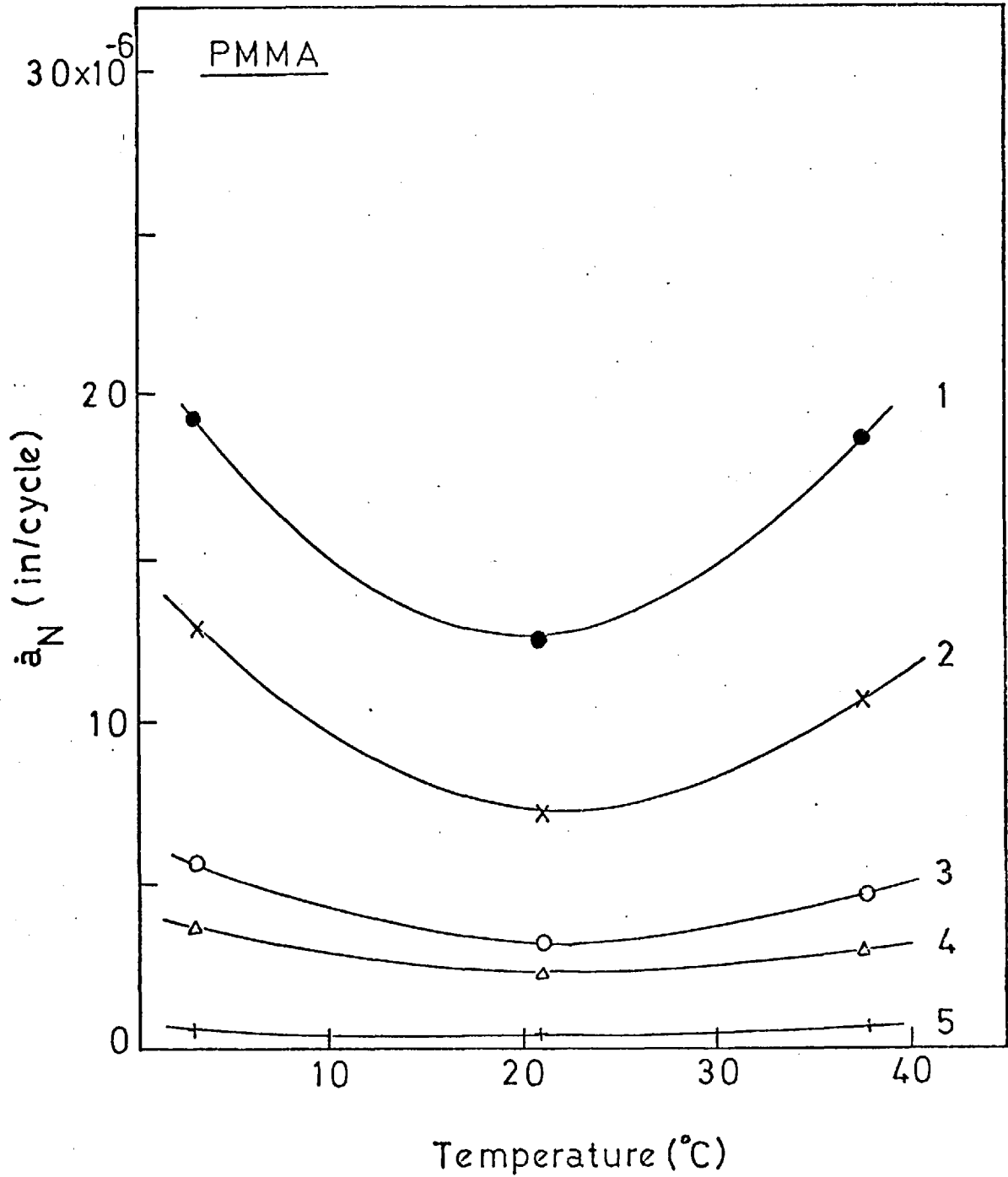
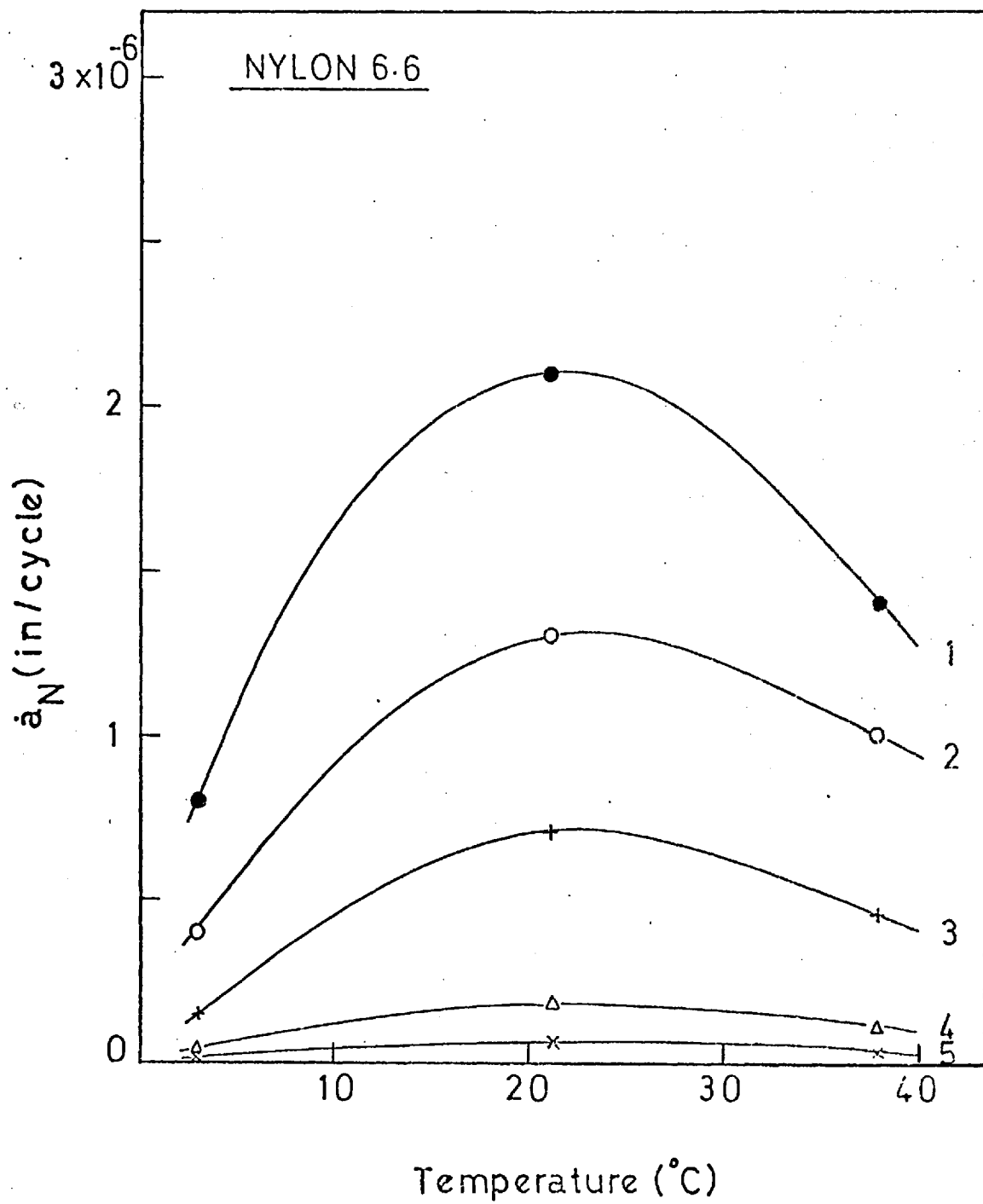


FIG (33) DETAILS OF THE CONSTRUCTION OF THE TEMPERATURE CONTROLLED CHAMBER



Curve	$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)
1	675	75
2	600	200
3	500	50
4	500	250
5	250	50

FIG (34) THE EFFECT OF VARIATION IN THE ENVIRONMENTAL TEMPERATURE ON THE VALUE OF $\dot{a}_N$ IN PMMA; FREQUENCY = 5 Hz.



Curve	$K_{\max}$ (psi $\sqrt{\text{in}}$)	$K_{\min}$ (psi $\sqrt{\text{in}}$)
1	1700	300
2	1600	400
3	1500	500
4	1400	700
5	1500	1000

FIG (35) THE EFFECT OF VARIATION IN THE ENVIRONMENTAL TEMPERATURE ON THE VALUE OF $\dot{a}_N$ IN NYLON 6.6; FREQUENCY = 5 Hz

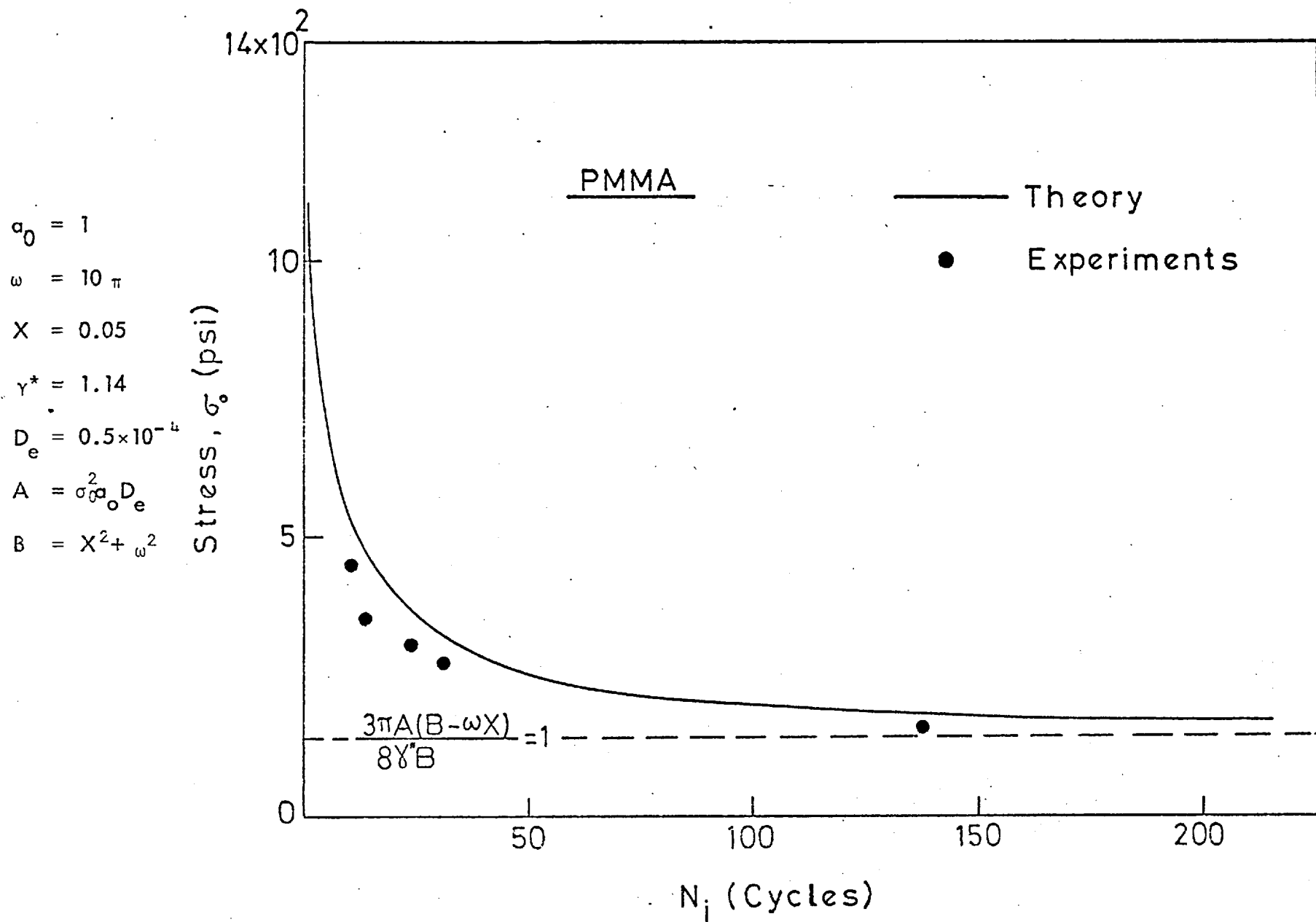


FIG (36) VARIATION OF THE CYCLIC INITIATION LIFE AS A FUNCTION OF THE APPLIED STRESS IN PMMA

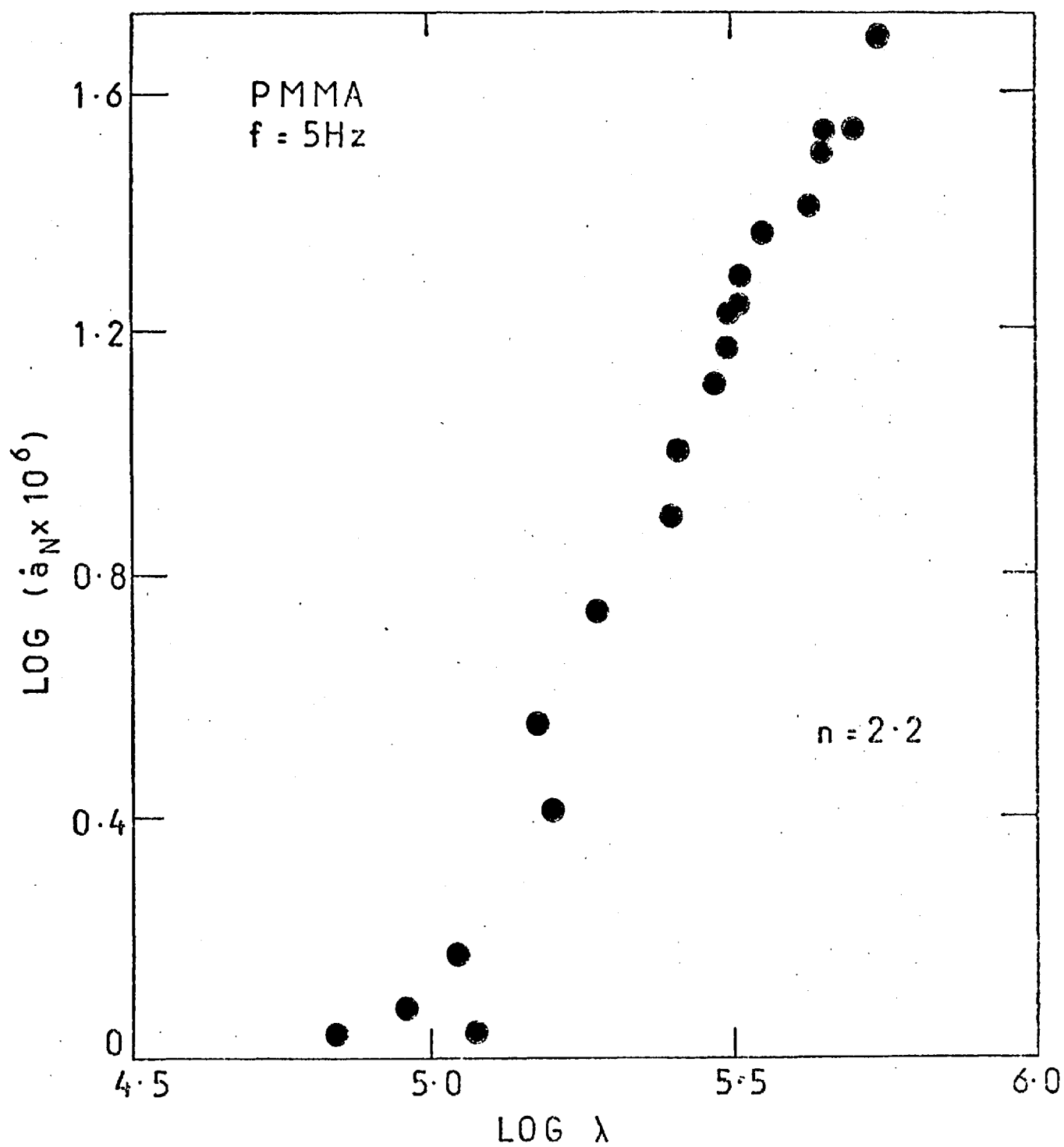


FIG (37) VARIATION OF CRACK GROWTH RATE WITH λ IN PMMA

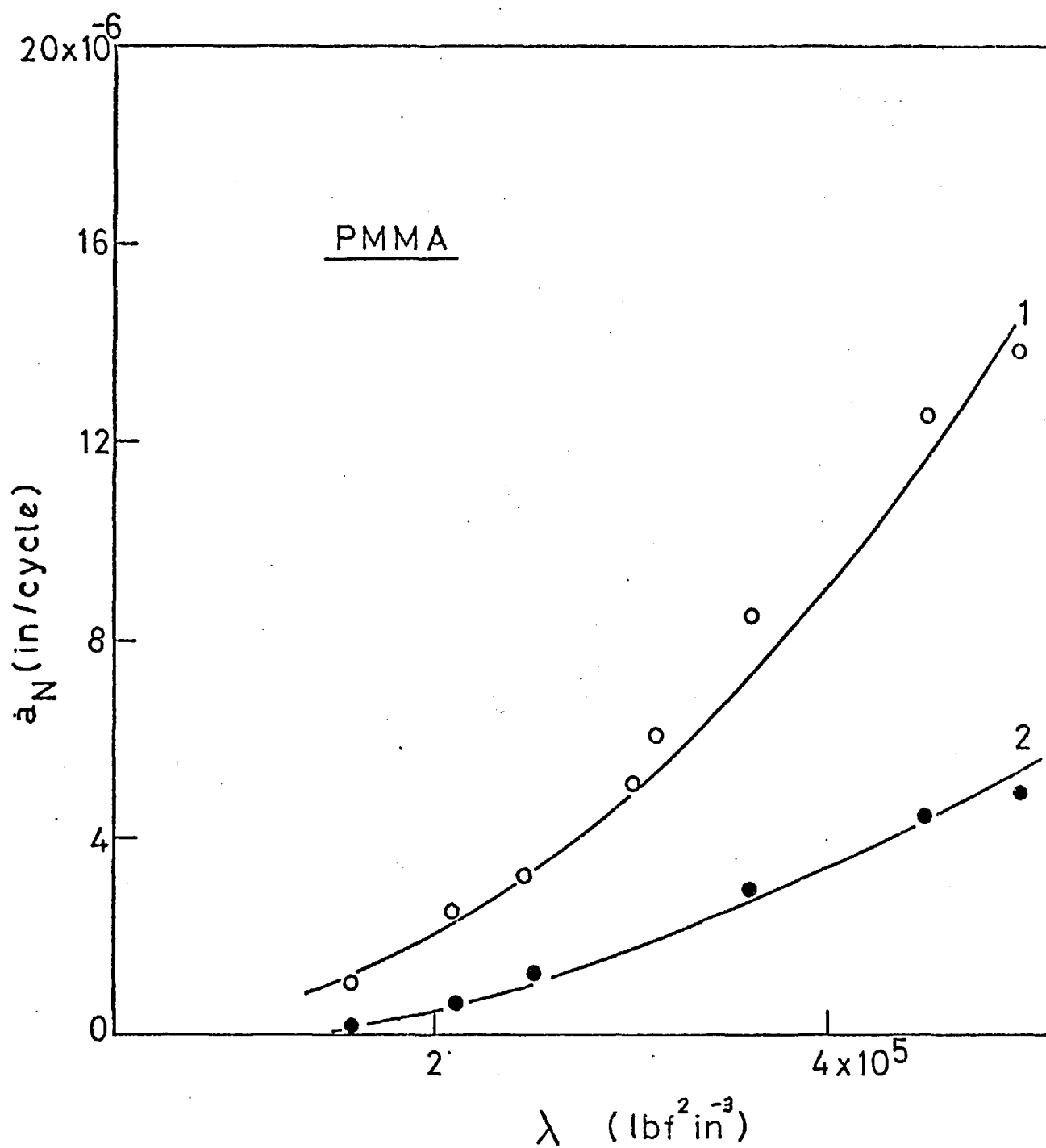


FIG (38) THE RELATIONSHIP BETWEEN $\dot{a}_N$ AND λ AT FREQUENCIES OF 5 AND 20 Hz in PMMA

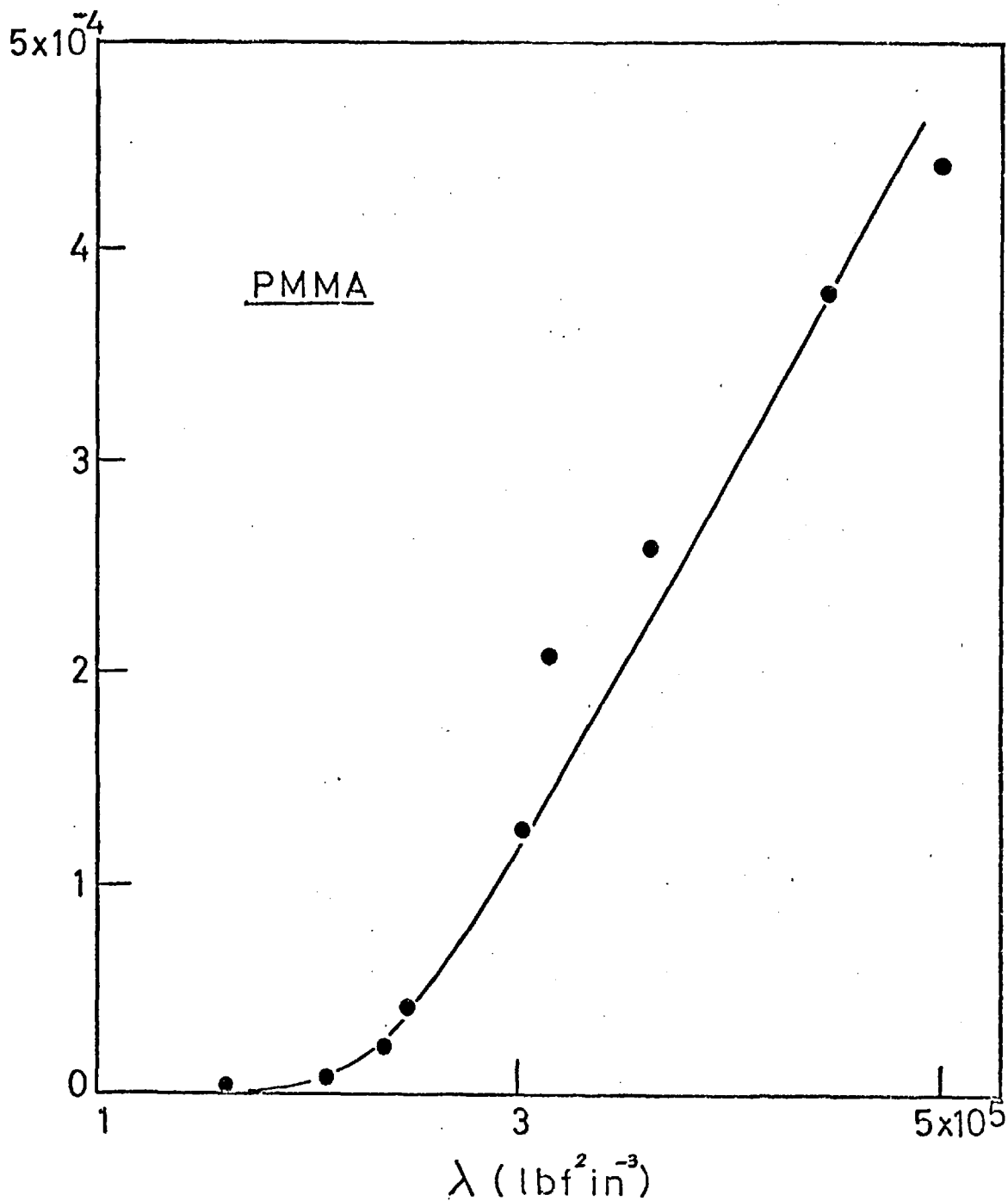


FIG (39) THE RELATIONSHIP BETWEEN $\dot{a}_N$ AND λ AT A FREQUENCY OF 0.1 Hz IN PMMA

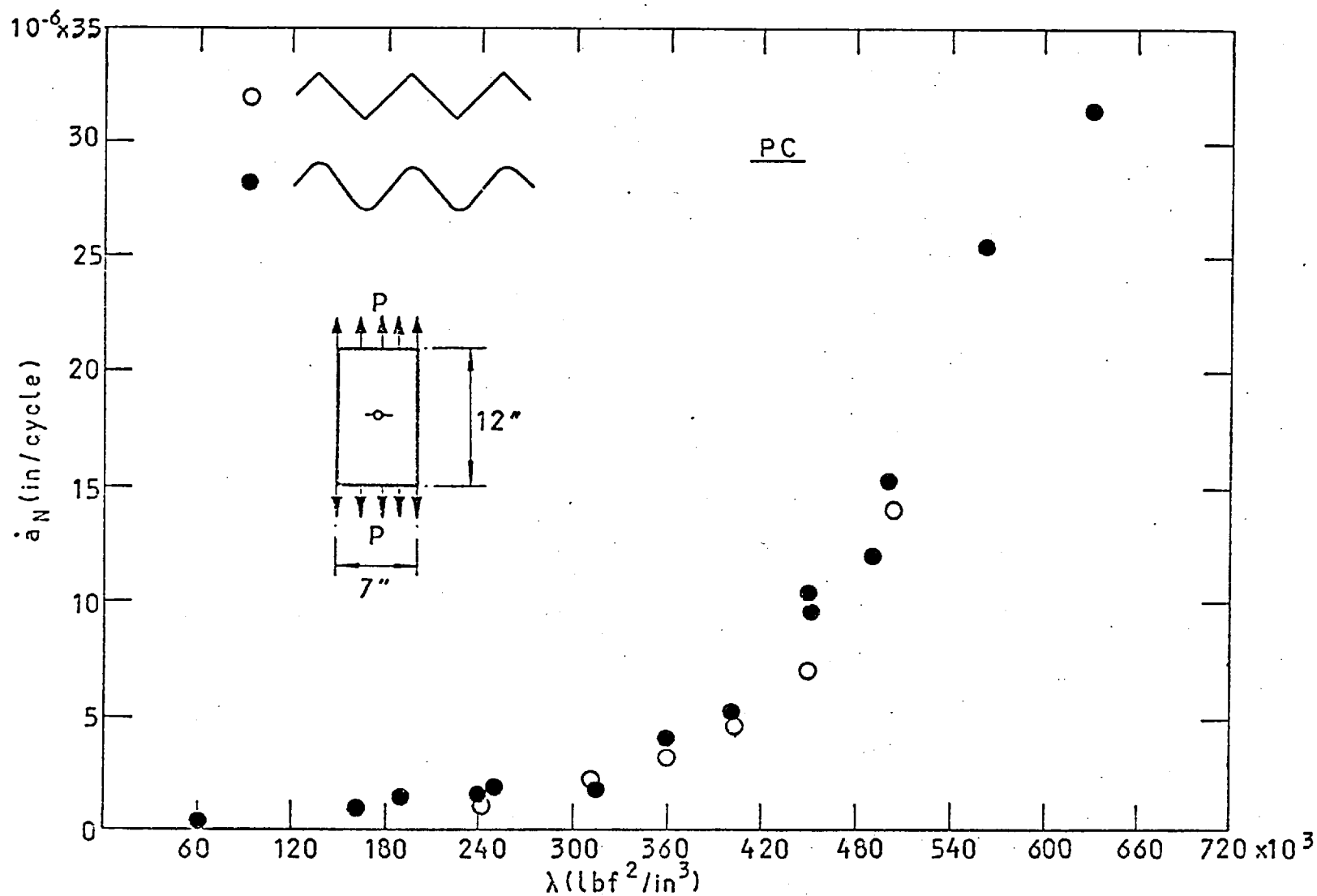


FIG (40) RESULTS OF TEST ON PC AT A FREQUENCY OF 0.1 Hz, FOR SINUSOIDAL AND TRIANGULAR INPUT WAVEFORMS

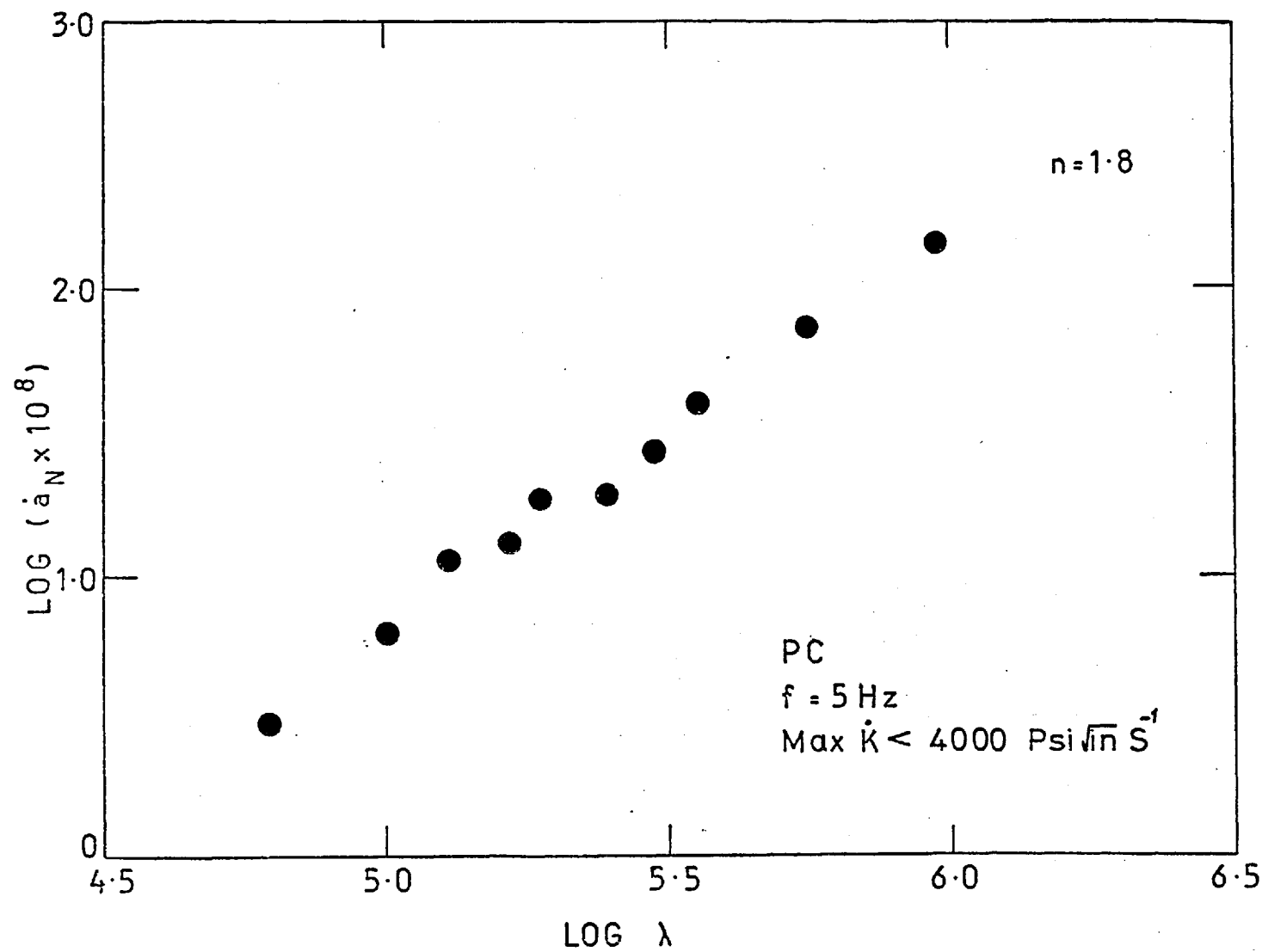


FIG (41) RESULTS OF TESTS ON PC AT A FREQUENCY OF 5 Hz WITH MAXIMUM $\dot{K} < 4000 \text{ psi} \sqrt{\text{in}} \text{ s}^{-1}$

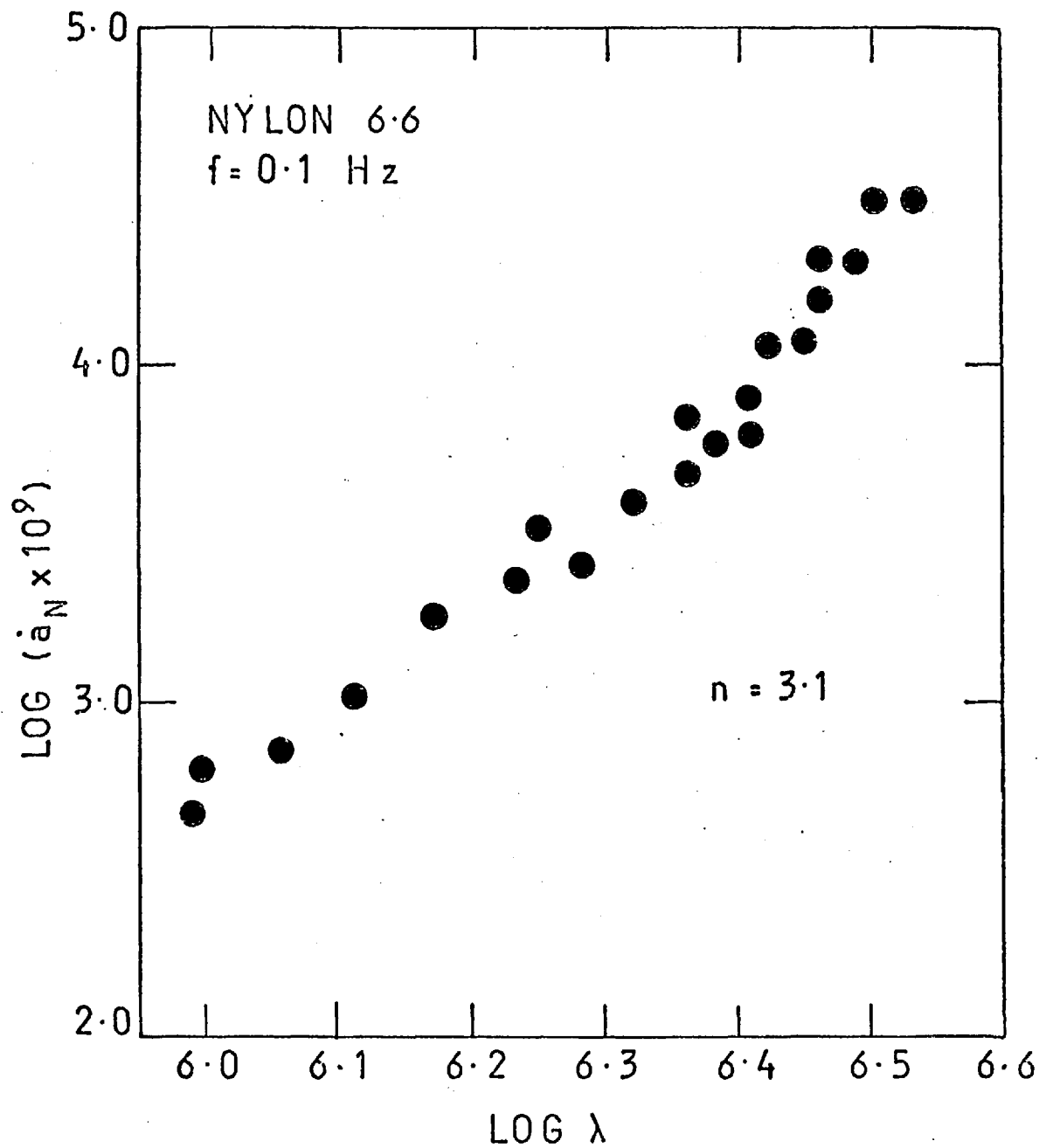


FIG (42) RATE OF CRACK PROPAGATION AS A FUNCTION OF λ FOR NYLON 6.6 AT A FREQUENCY OF 0.1 Hz

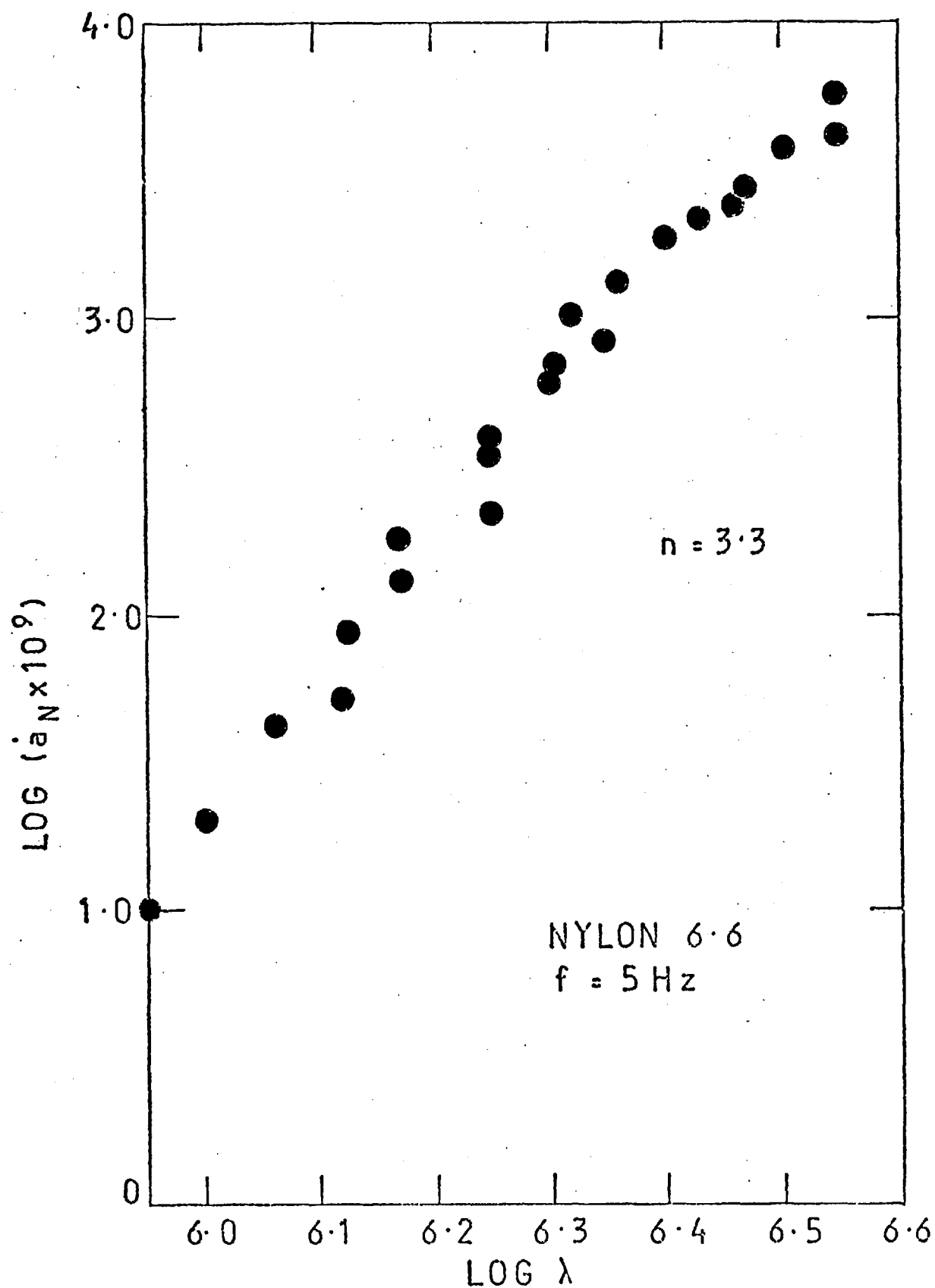


FIG (43) RATE OF CRACK PROPAGATION AS A FUNCTION OF λ FOR NYLON 6.6 AT A FREQUENCY OF 5 Hz

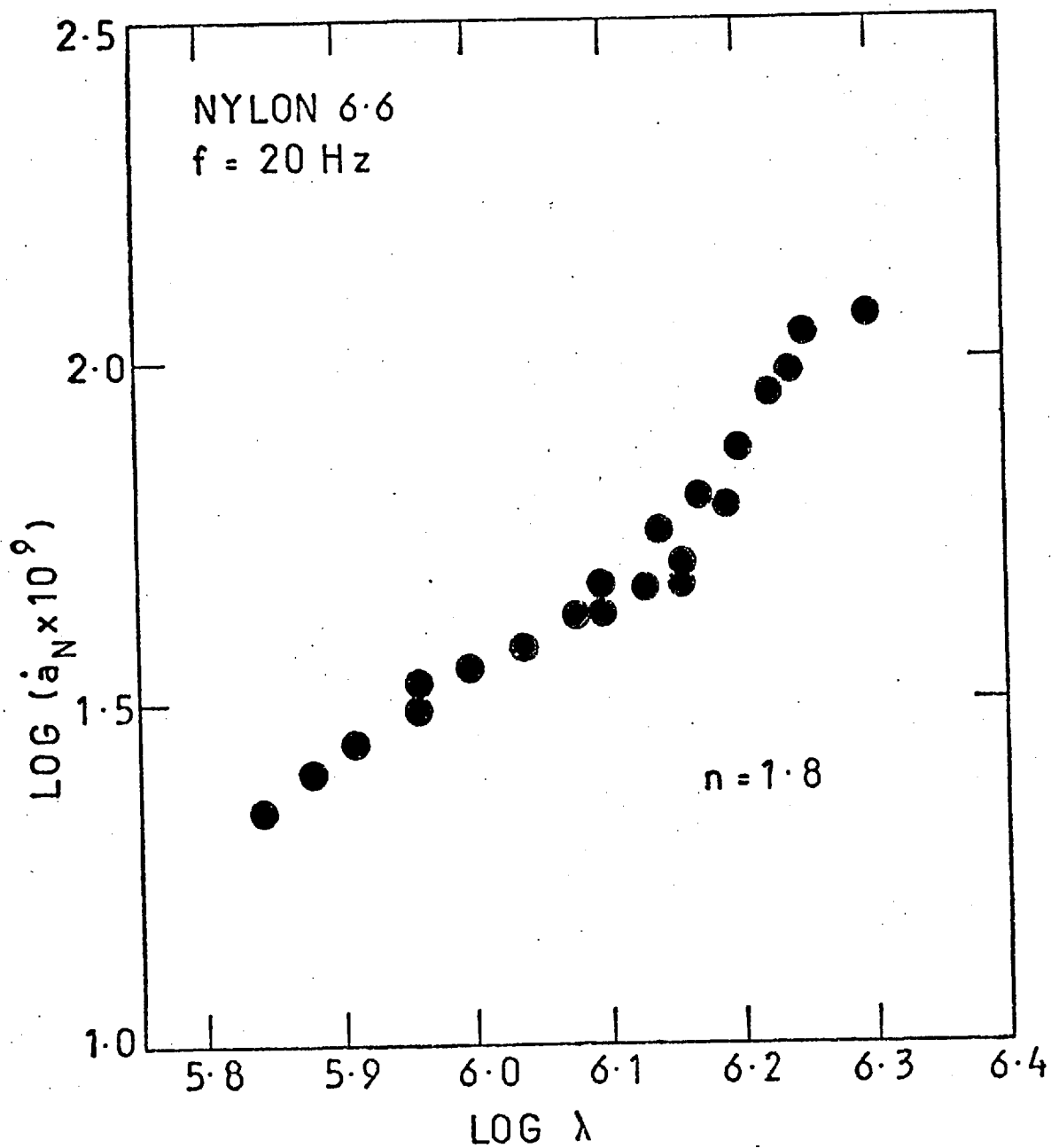


FIG (44) RATE OF CRACK PROPAGATION AS A FUNCTION OF λ FOR NYLON 6.6 AT A FREQUENCY OF 20 Hz

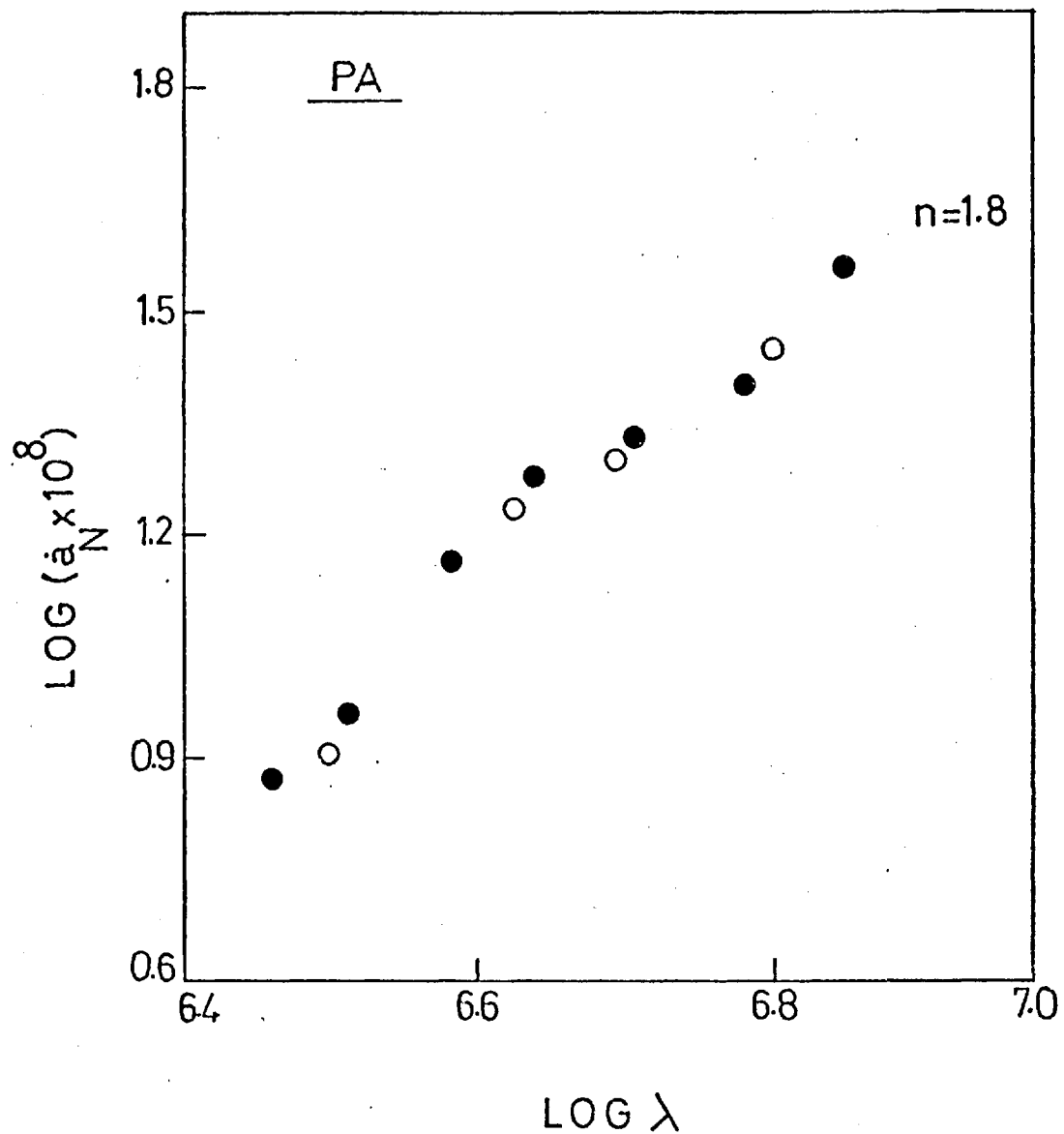


FIG (45) RATE OF CRACK PROPAGATION AS A FUNCTION OF λ FOR POLYACETAL AT A FREQUENCY OF 5 Hz

- Constant applied load tests
- Constant stress intensity factor tests

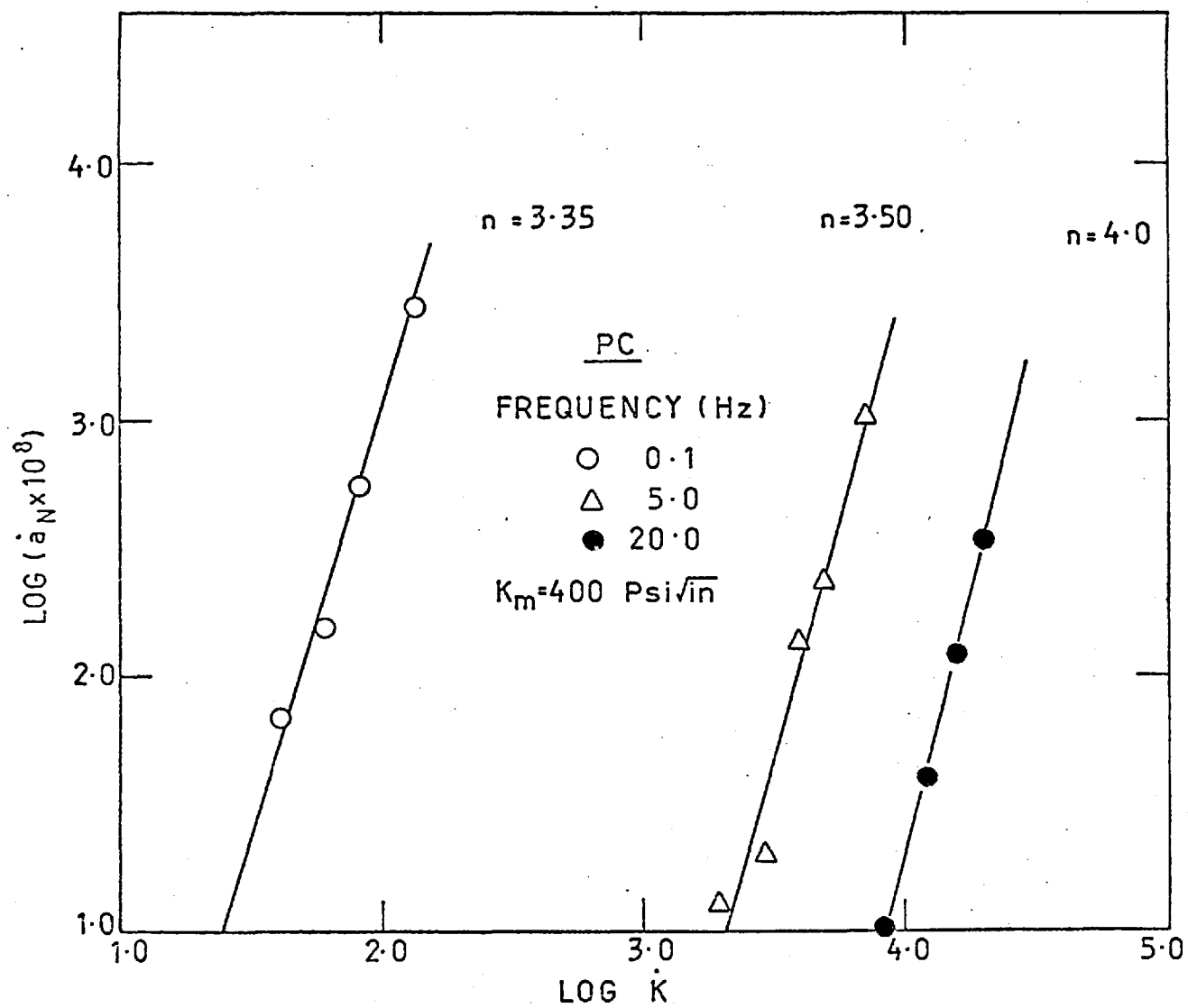


FIG. (46) THE EFFECT OF CHANGE IN FREQUENCY ON THE VALUE OF INDEX n IN PC

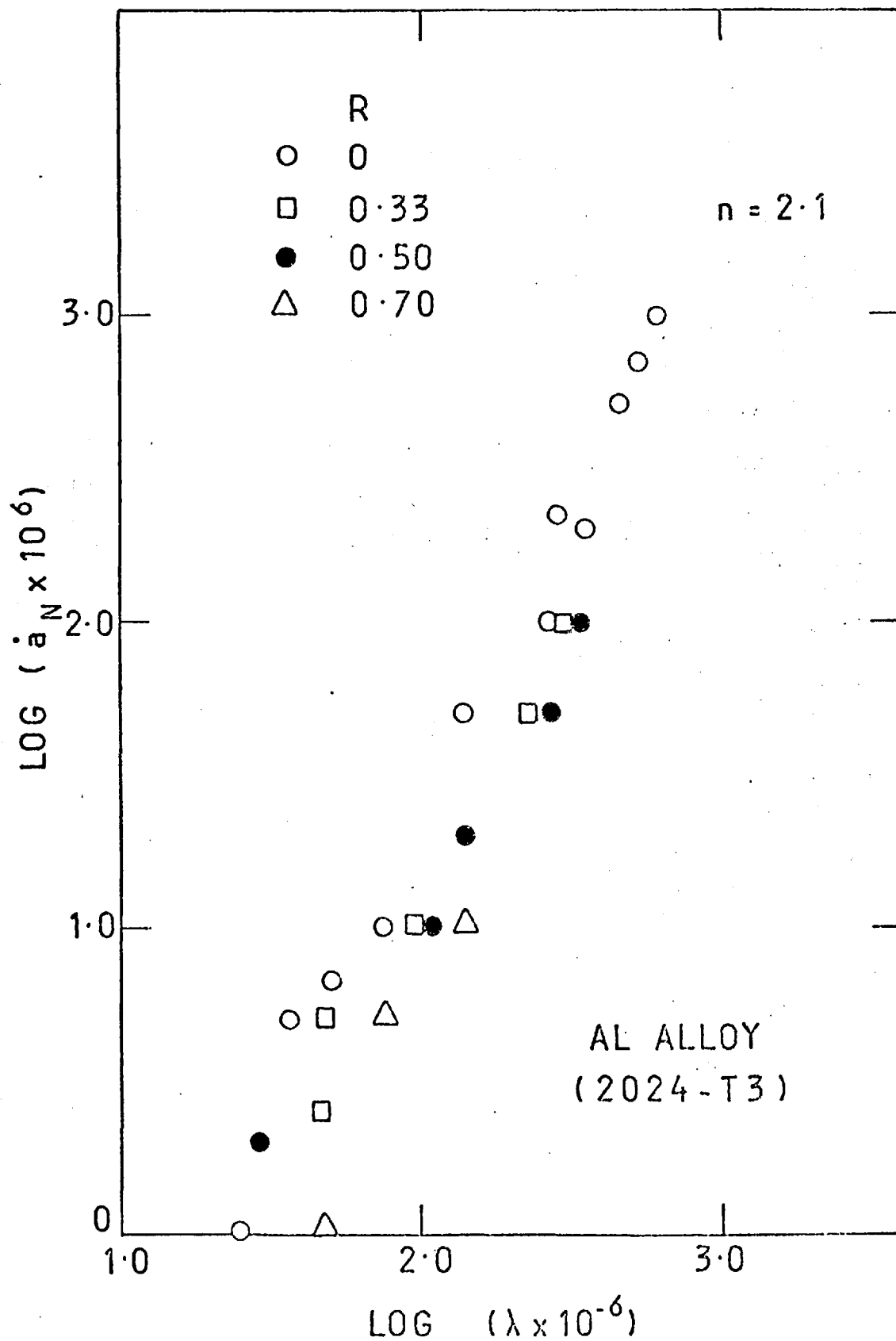


FIG. (47) CYCLIC CRACK GROWTH DATA FOR ALUMINIUM ALLOY (2024-T3); HUDSON (1969)

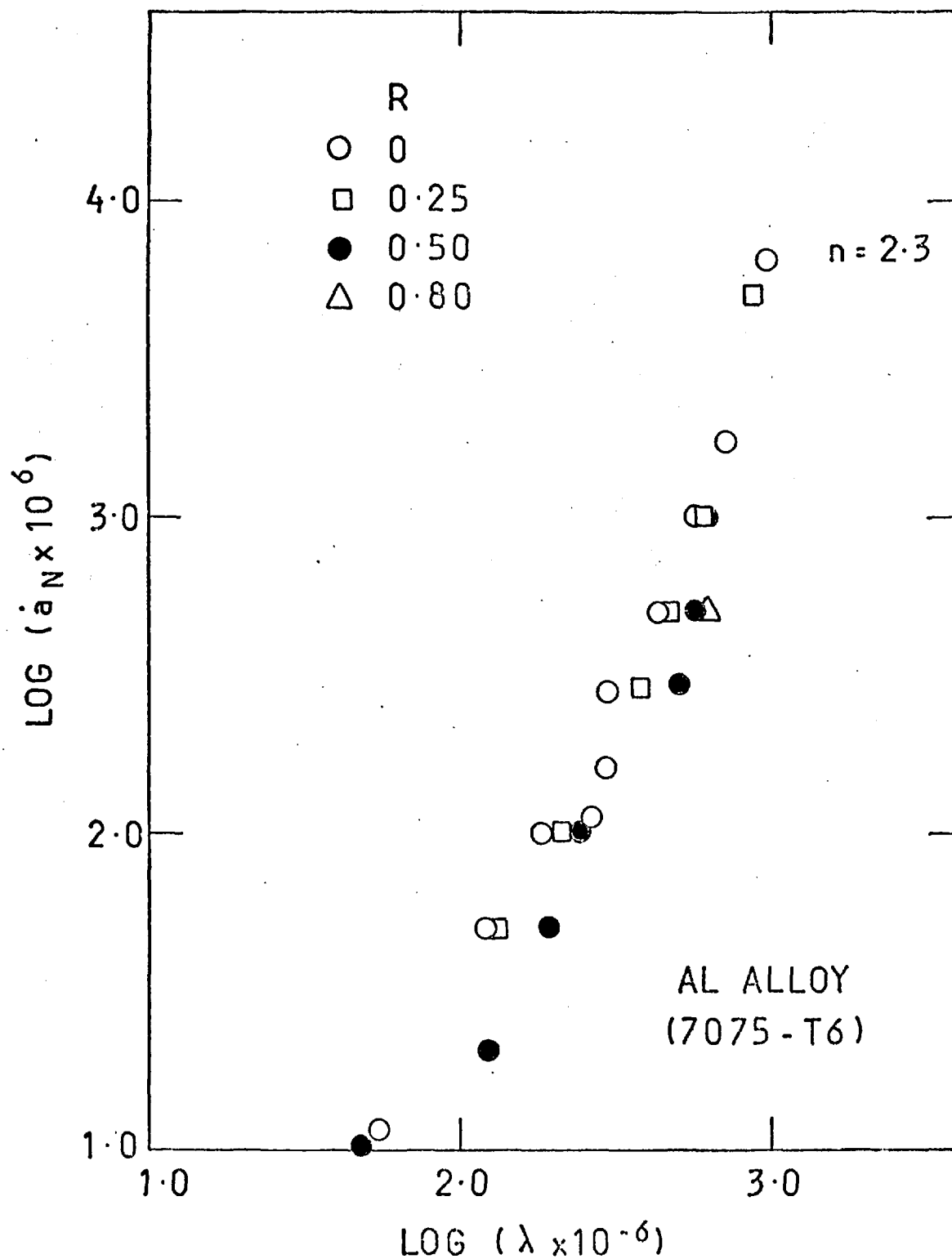


FIG (48) CYCLIC CRACK GROWTH DATA FOR ALUMINIUM ALLOY (7075-T6); HUDSON (1969)

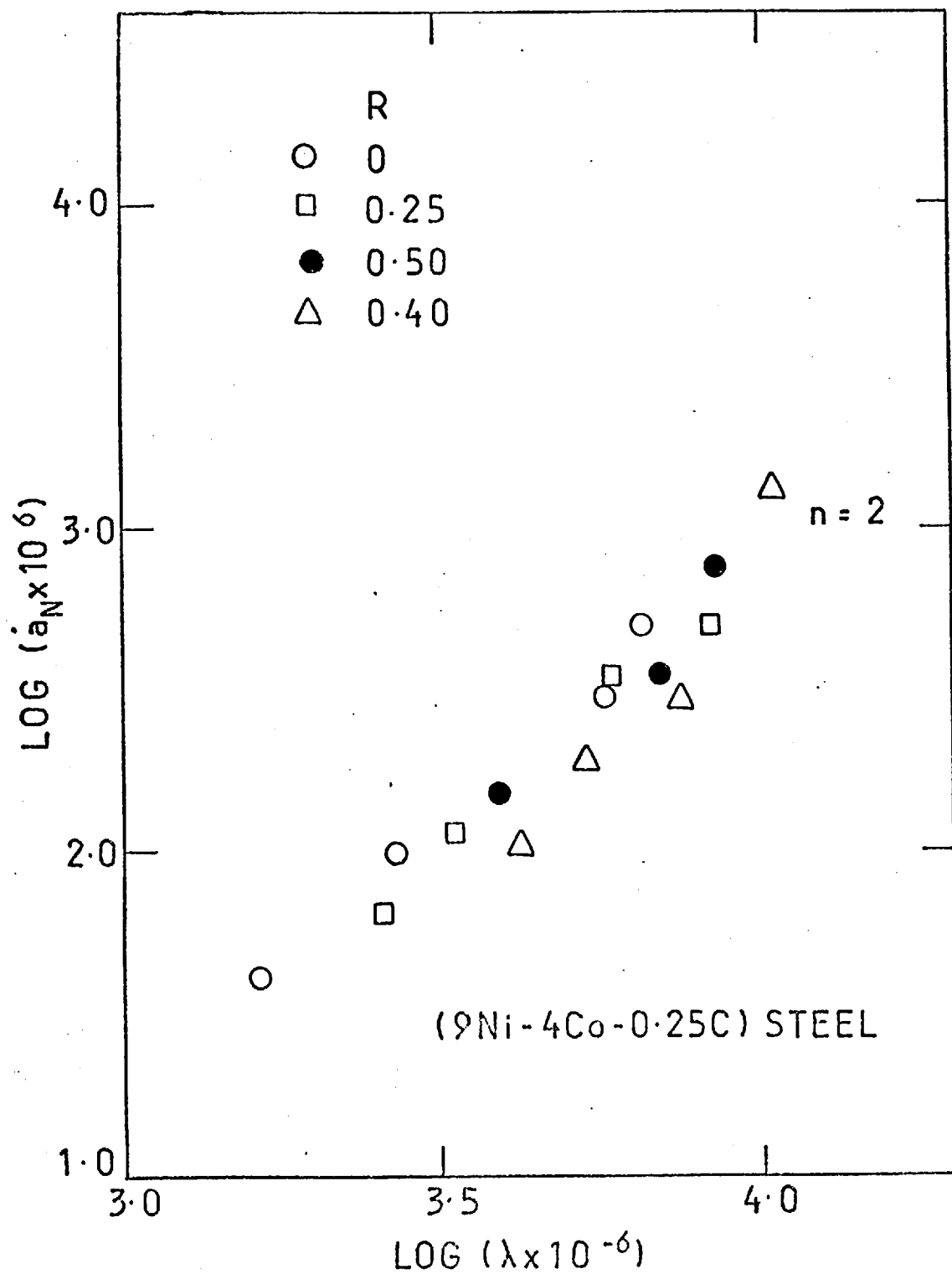


FIG (49) CYCLIC CRACK GROWTH RATE DATA FOR (9Ni-4Co-0.25C) STEEL; CROOKER AND LANGE (1968)

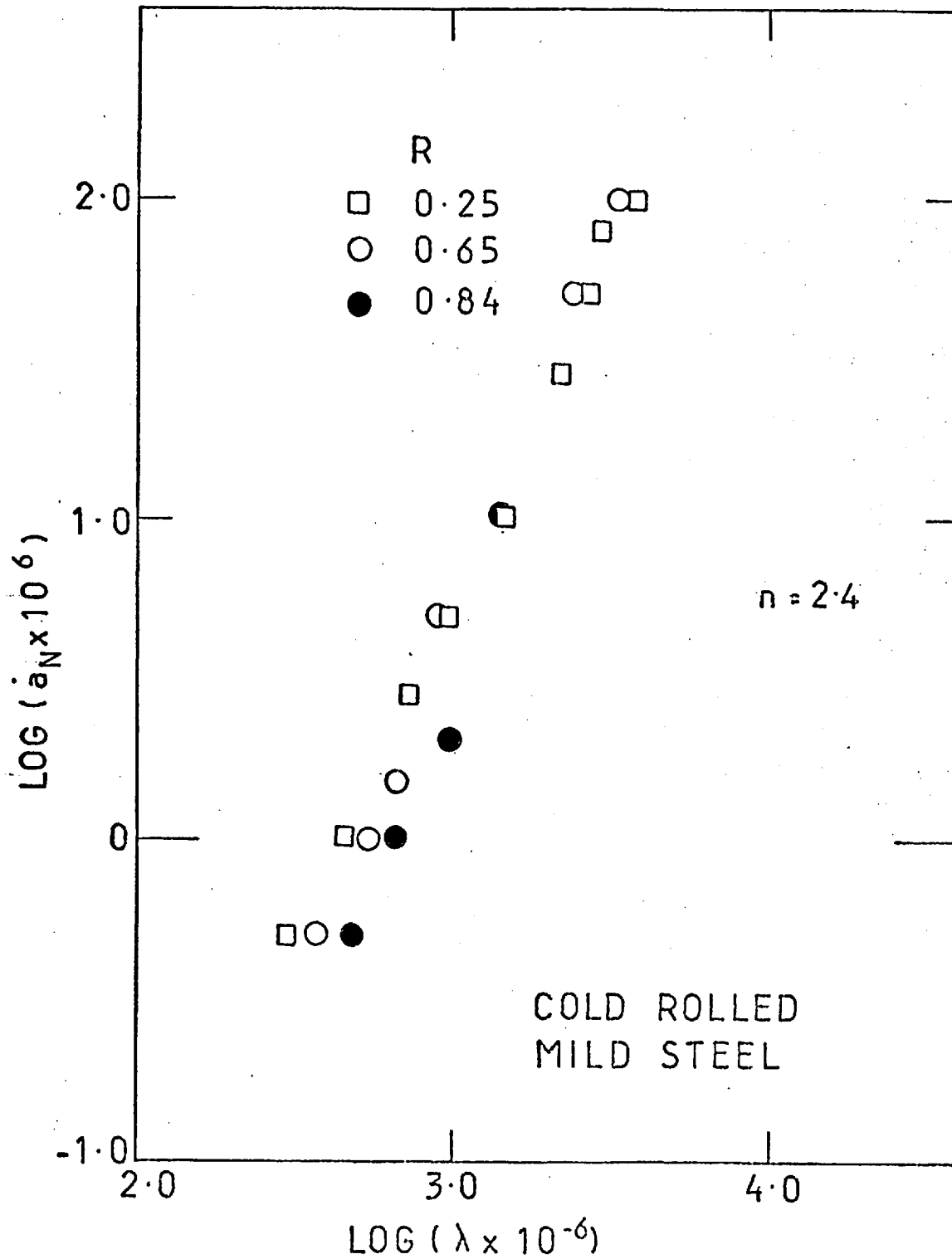
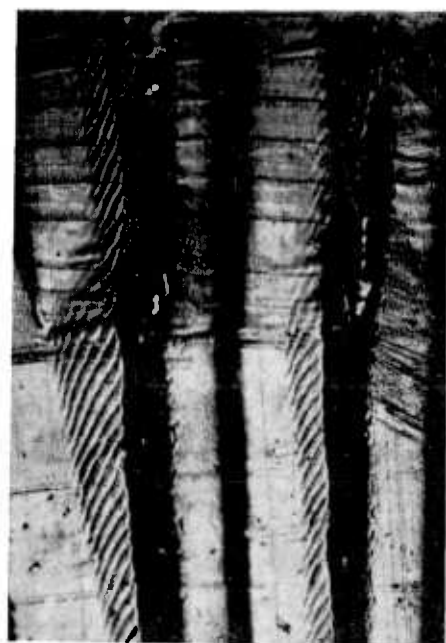
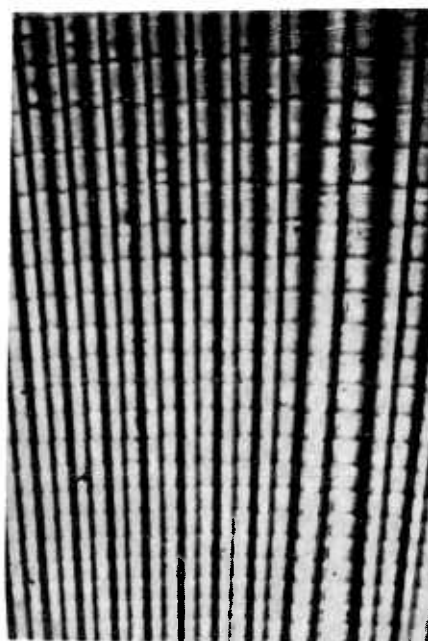


FIG (50) CYCLIC CRACK GROWTH RATE DATA FOR COLD ROLLED MILD STEEL; FORST ET AL. (1971)



(a)
 $K_{\max} = 800$
 $K_{\min} = 100 \text{ psi}\sqrt{\text{in}}$



(b)



(c)
 $K_{\max} = 600 \text{ psi}\sqrt{\text{in}}$
 $K_{\min} = 200$



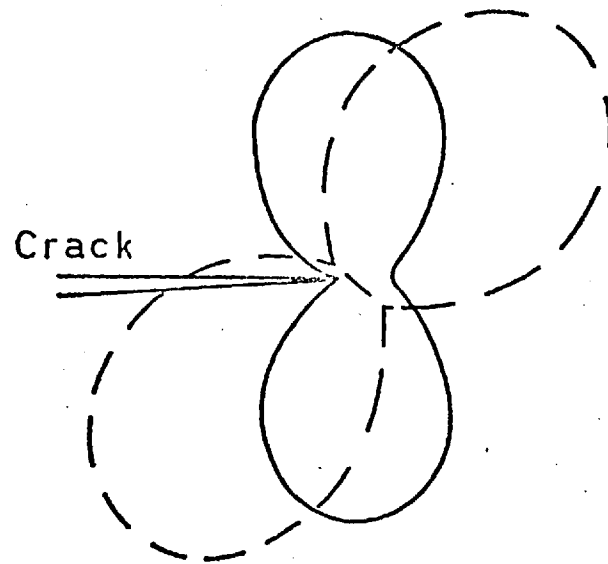
(d)
 $K_{\max} = 600 \text{ psi}\sqrt{\text{in}}$
 $K_{\min} = 200$

CRACK GROWTH DIRECTION


FIG (51) FRACTOGRAPHY OF THE FAILURE SURFACES IN PMMA

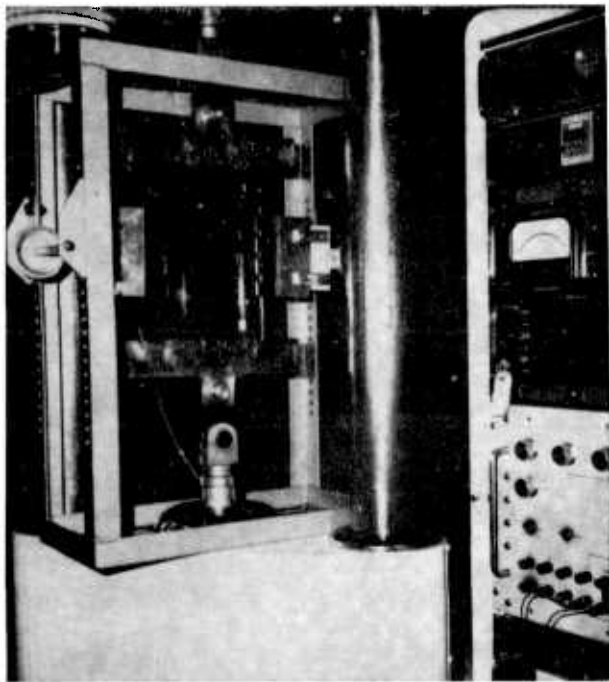
(a) FATIGUE STRIATIONS; FREQUENCY = 0.1 Hz (TOP)
 AND 0.06 Hz (BOTTOM). (b) SLOW GROWTH REGION
 (c) TEMPERATURE = 3°C (d) TEMPERATURE = 38°C

(MAGNIFICATION 24:1)

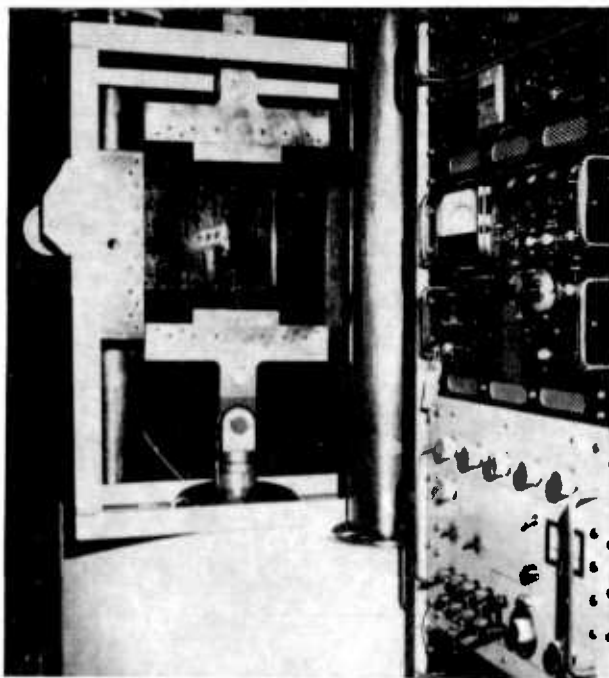


— $\beta = 90^\circ$
- - - $\beta = 60^\circ$

FIG (52) PLASTIC ZONE SHAPE FOR SYMMETRIC AND ASYMMETRIC LOADING



(a)



(b)

FIG (53) THE ARRANGEMENT OF EQUIPMENT FOR THE BIAXIAL LOADING TESTS

- (a) RR AND ROR TESTS
- (b) RTR TESTS

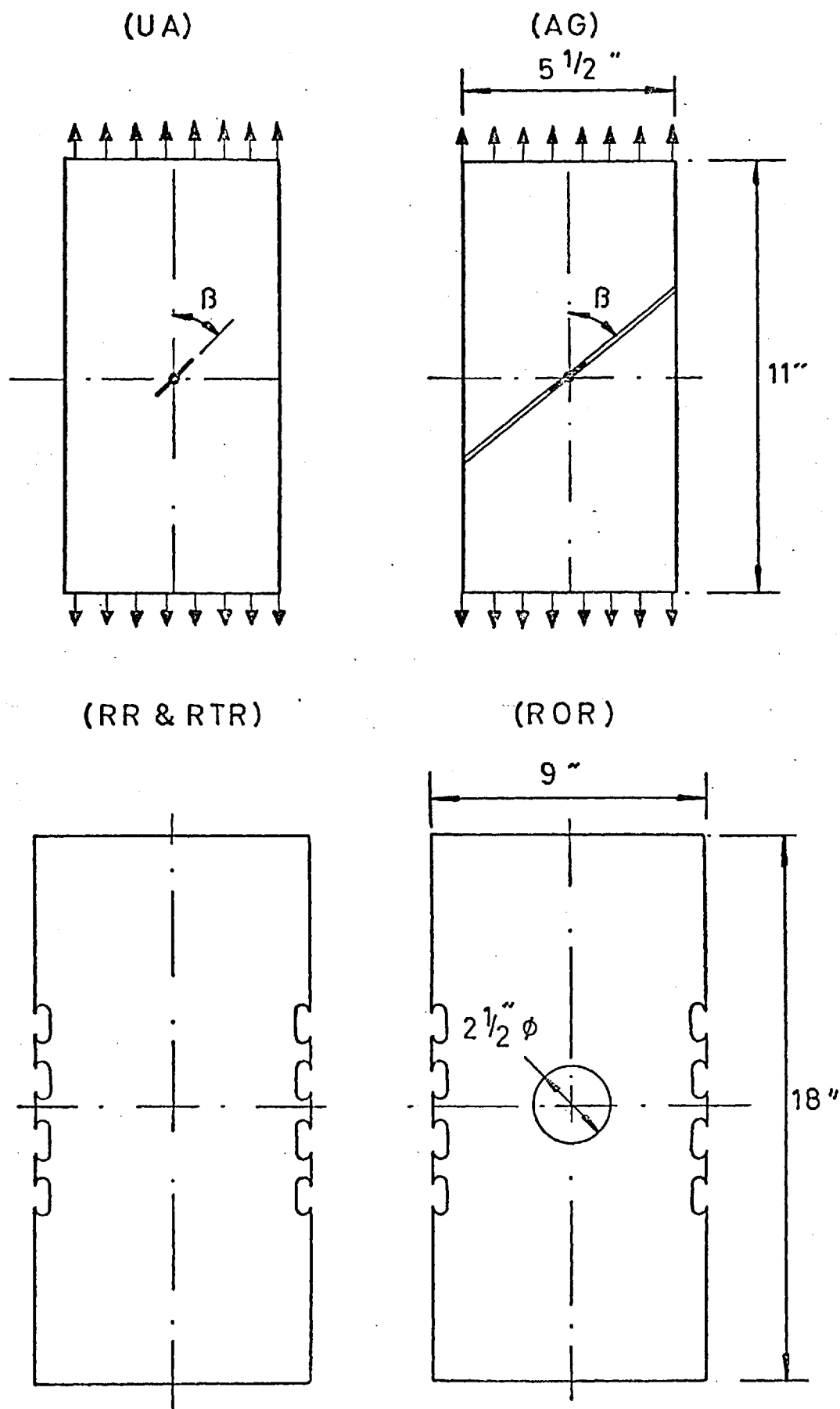
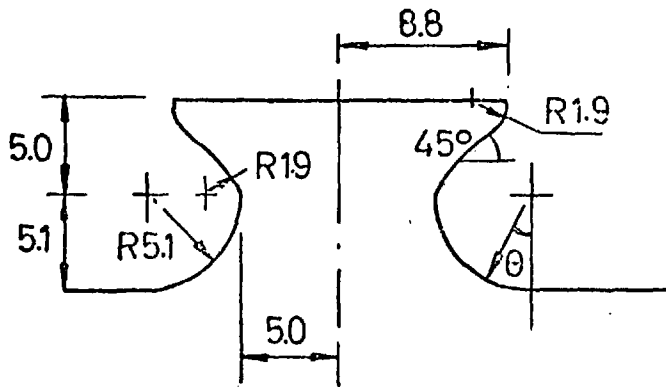


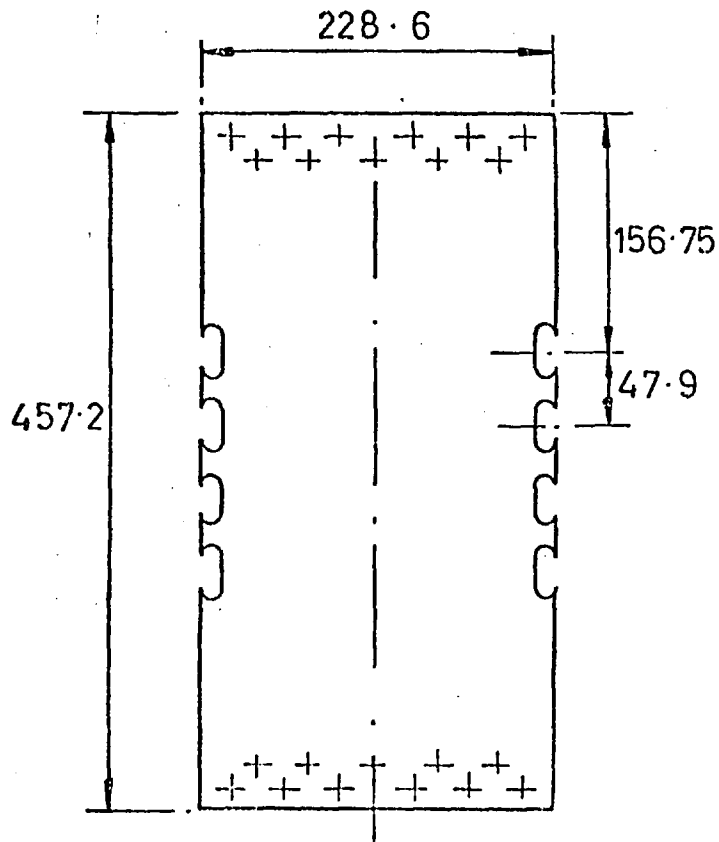
FIG (54) THE SPECIMENS USED IN THE TEST SERIES UA, AG, RR, RTR AND ROR

RR Specimen details



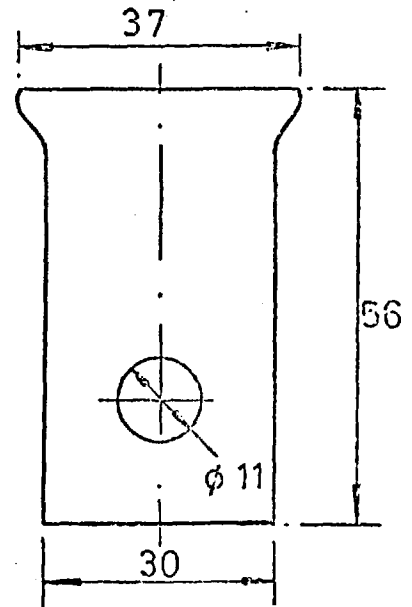
Dimensions
in mm

(a)



(b)

Blade details



(c)

FIG (55) DETAILS OF THE DESIGN OF THE DOVETAIL ROOTS AND THE BLADES

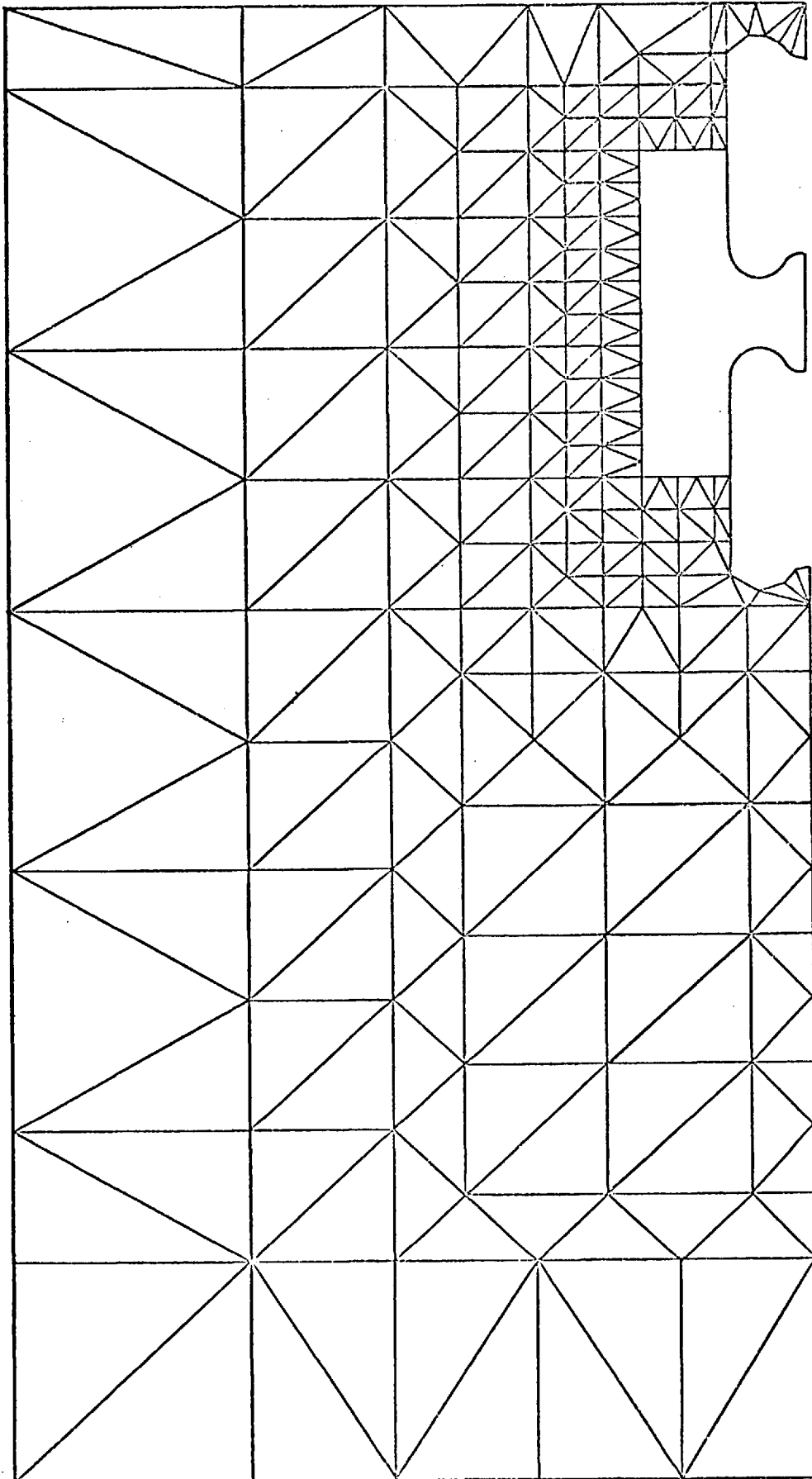


FIG (56) THE FINITE ELEMENTS MESH USED FOR THE ANALYSIS OF THE RR SPECIMEN

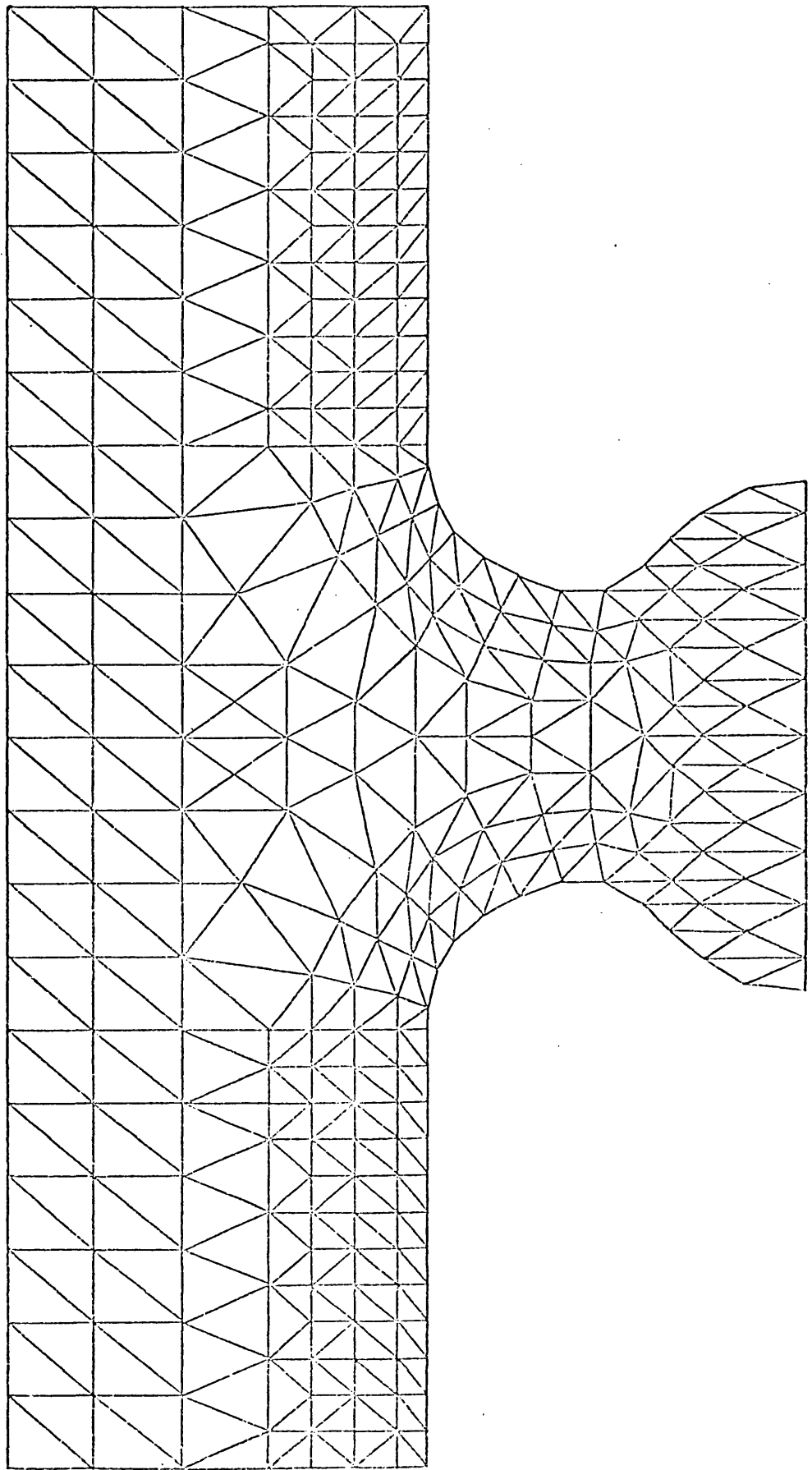


FIG (57) DETAILS OF THE MESH USED IN THE DOVETAIL REGION

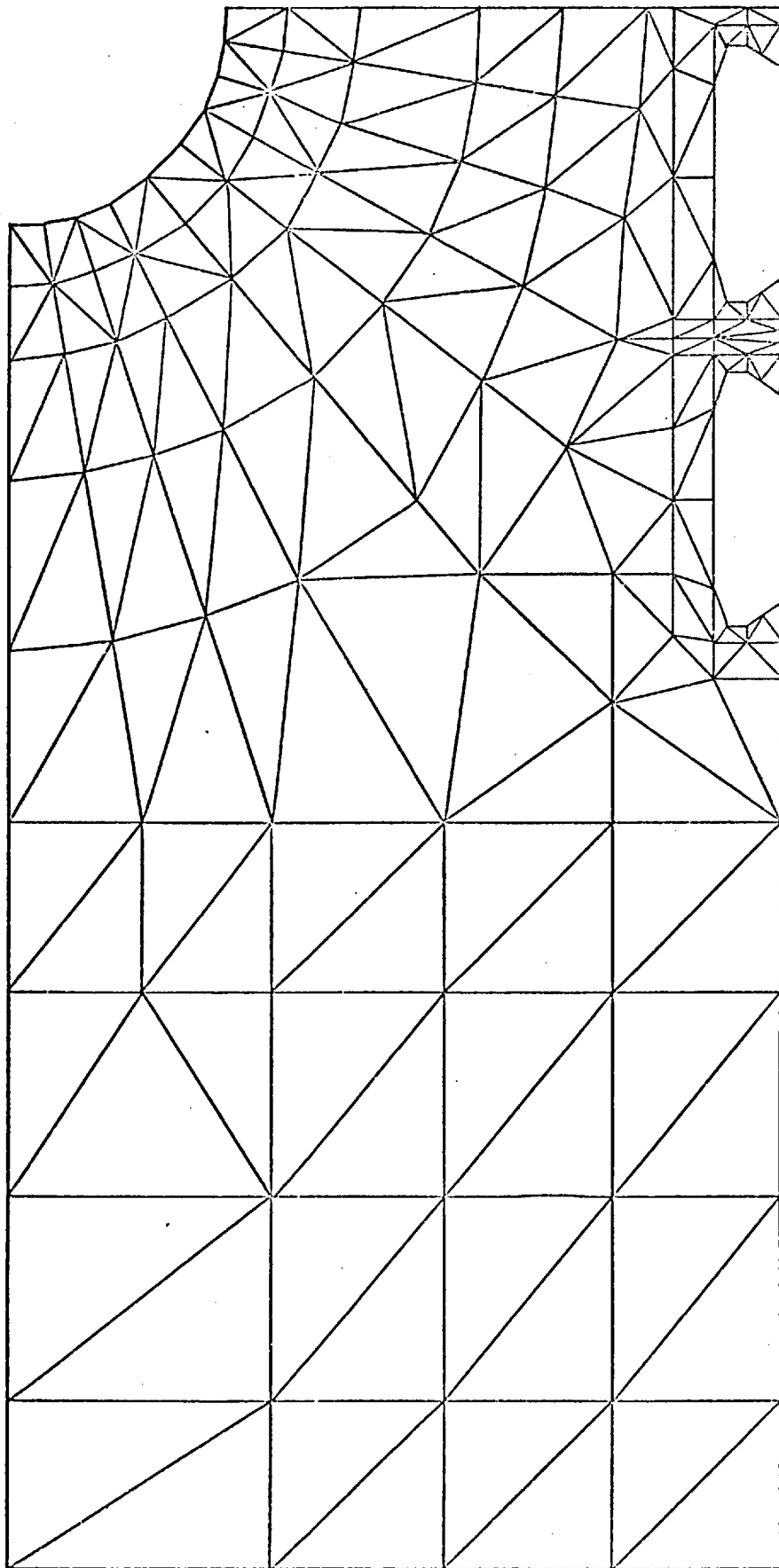


FIG (58) THE FINITE ELEMENTS MESH USED FOR THE ANALYSIS OF THE ROR SPECIMEN

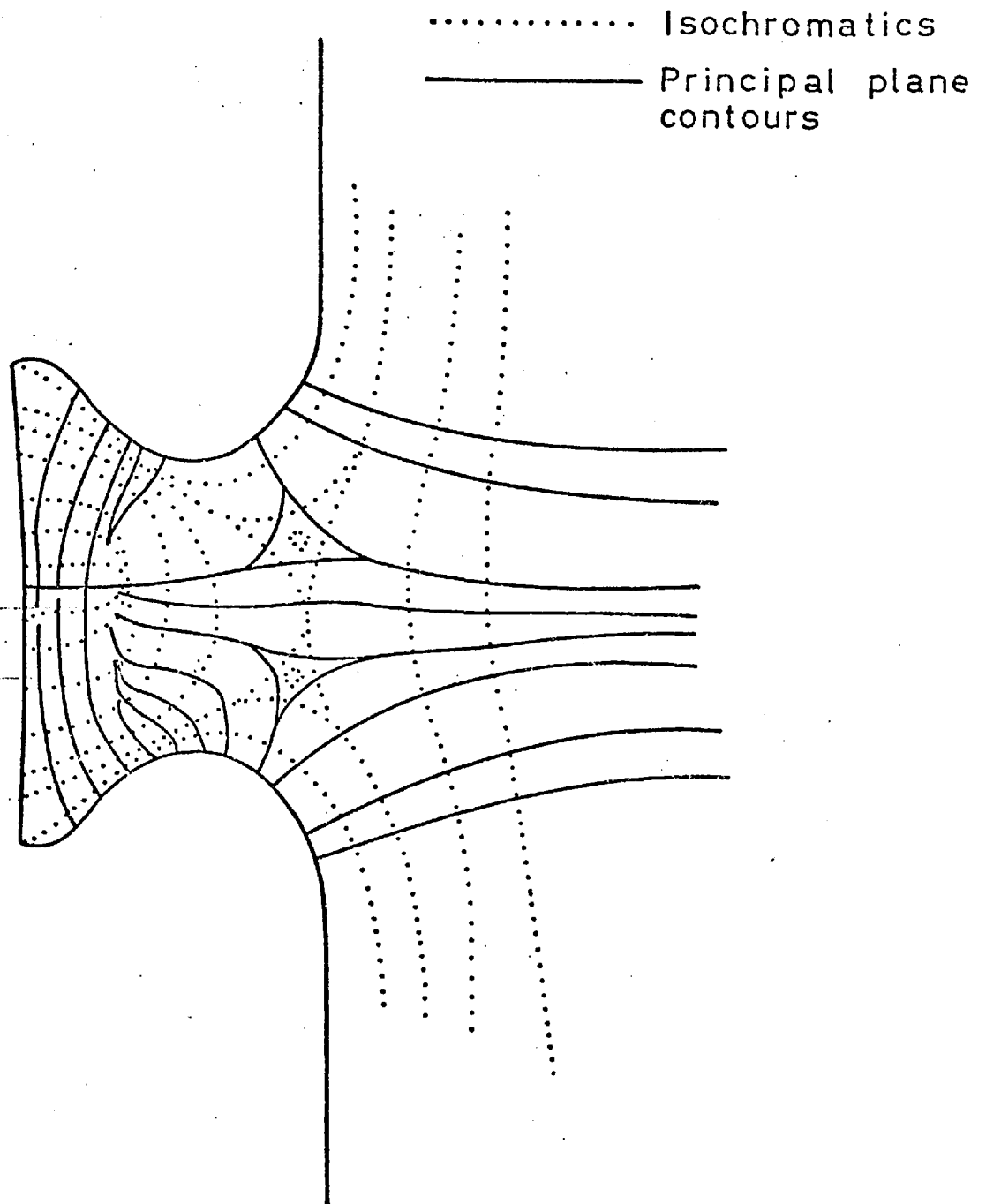


FIG (59) DISTRIBUTION OF THE MINIMUM PRINCIPAL STRESS TRAJECTORIES IN THE REGION OF THE DOVETAIL (FROM PHOTOELASTIC STUDIES)

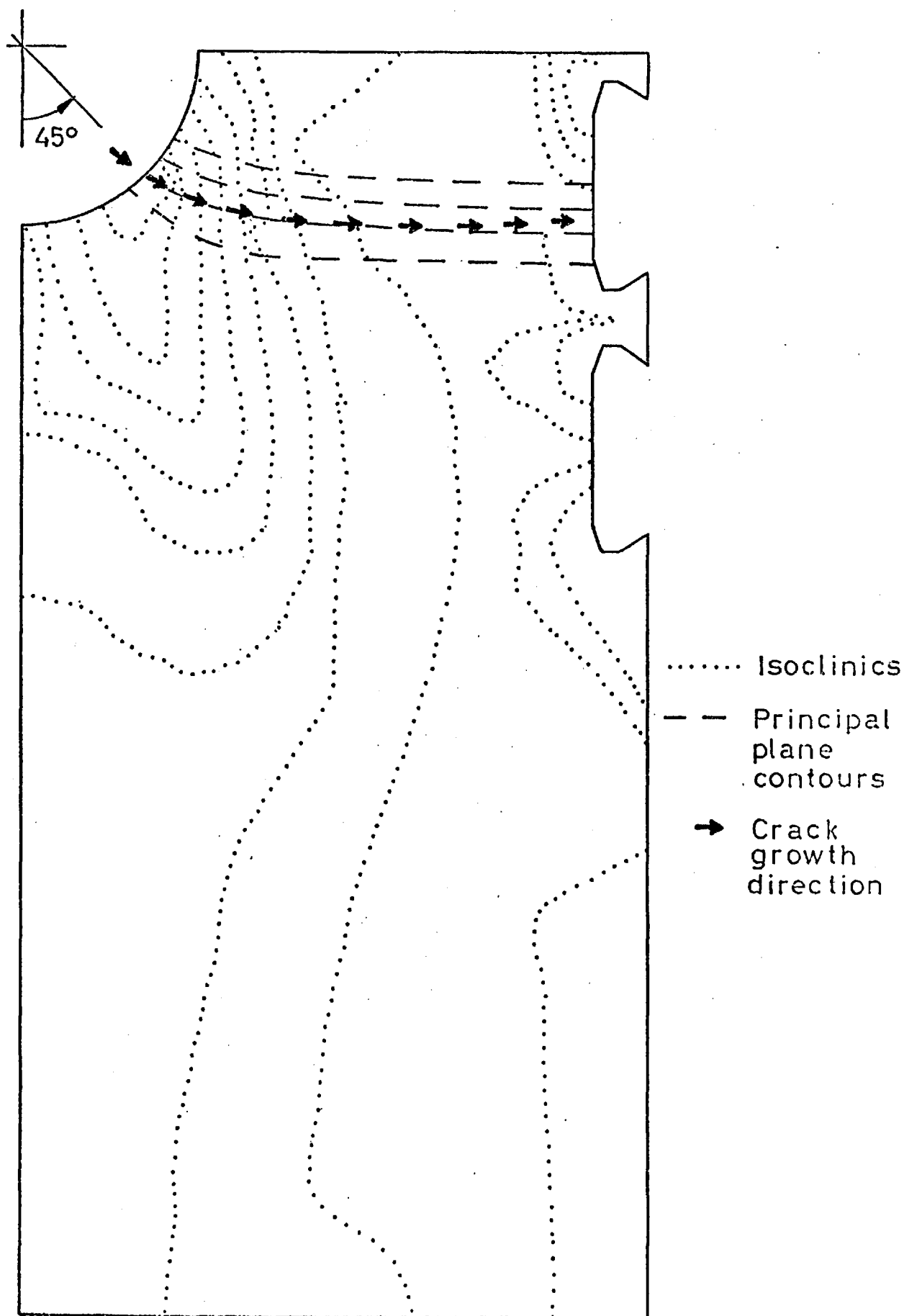


FIG (60) CONTOURS OF ISOCLINICS, PRINCIPAL PLANES AND THE CRACK GROWTH DIRECTION; $L_x = 0$

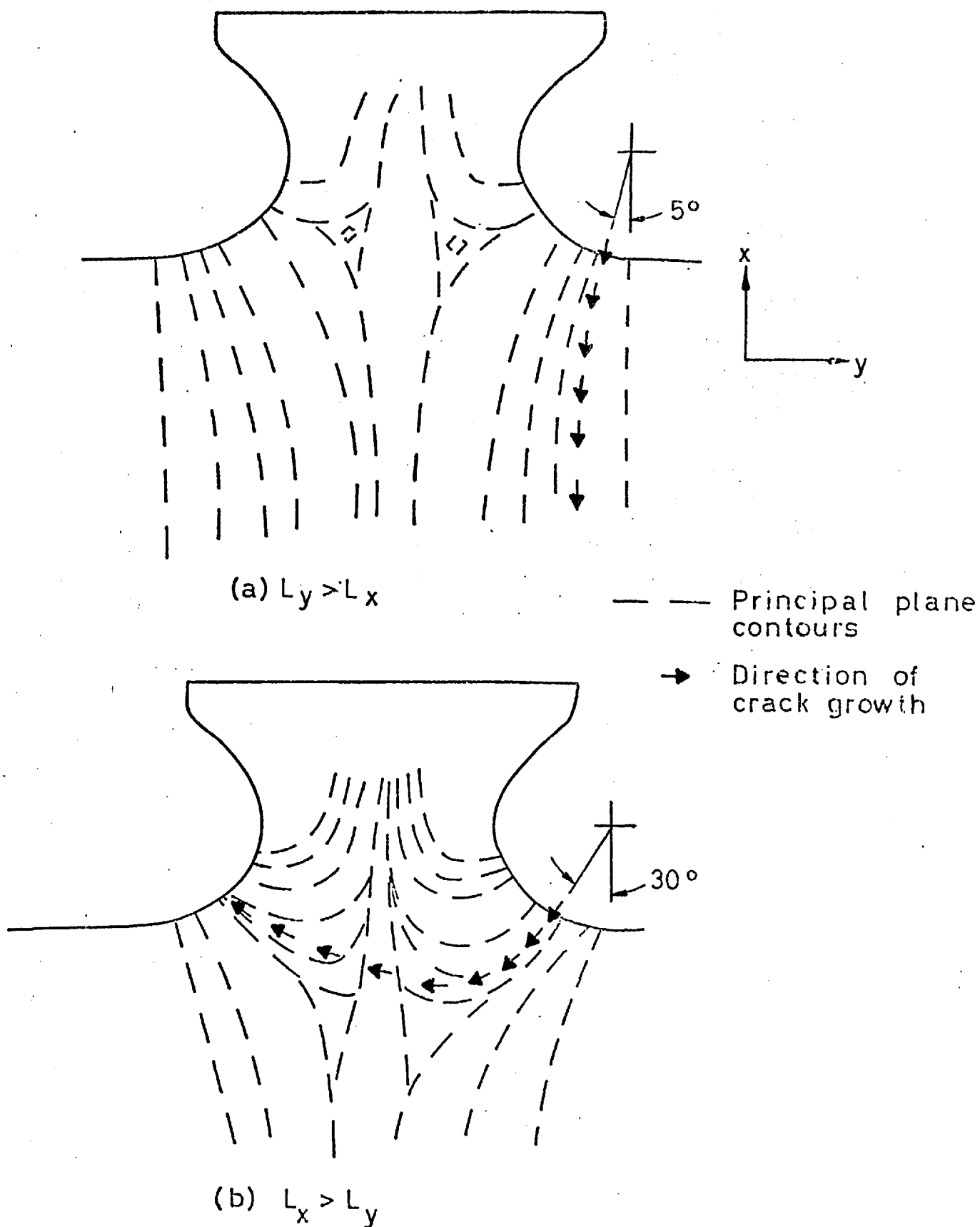


FIG (61) CONTOURS OF ISOCLINICS, PRINCIPAL PLANES AND THE CRACK GROWTH DIRECTIONS; $L_y = 0$ FOR THE CASE (b).

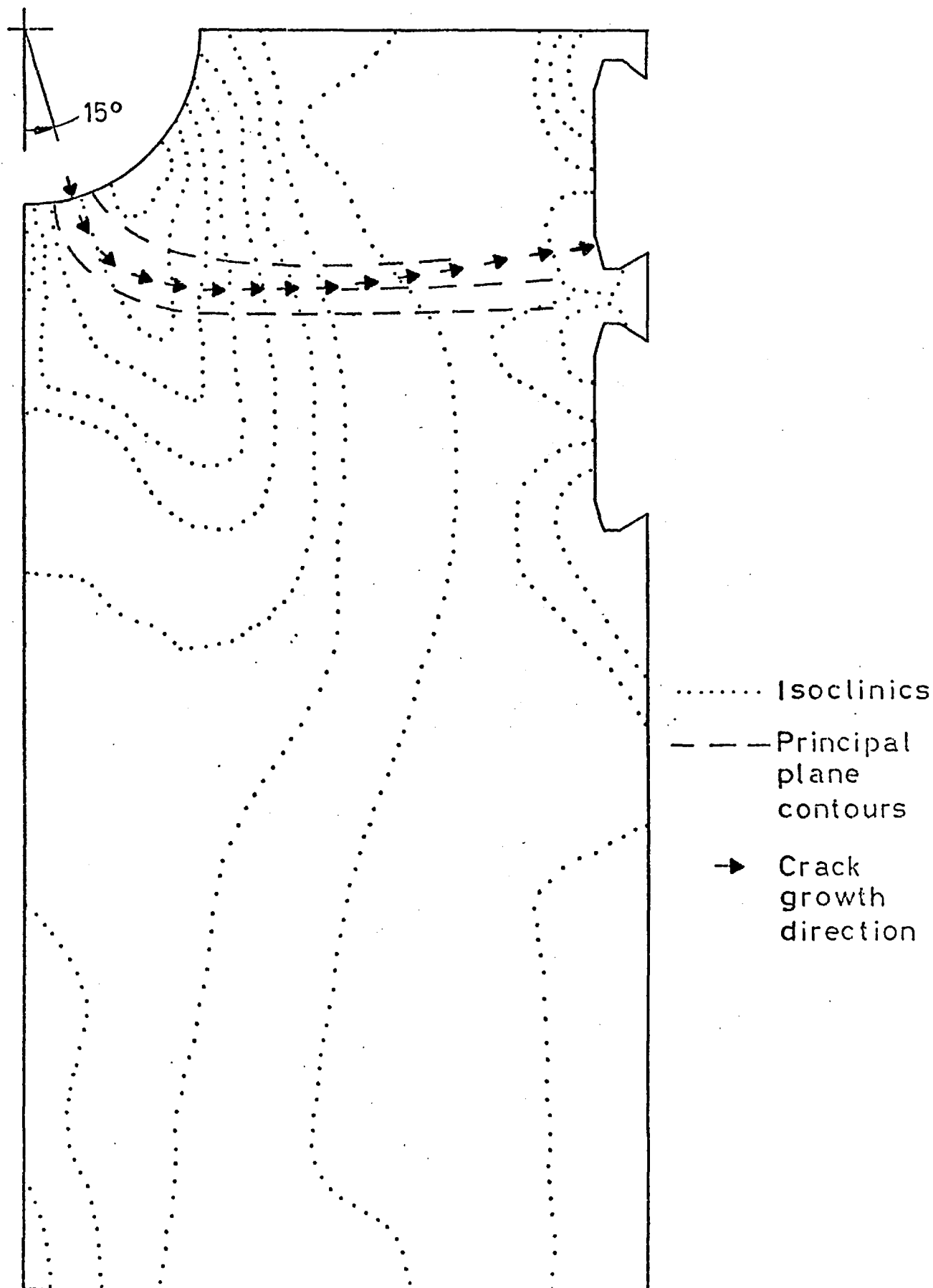
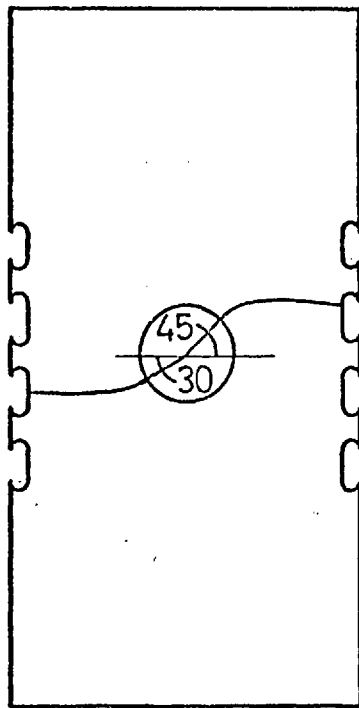
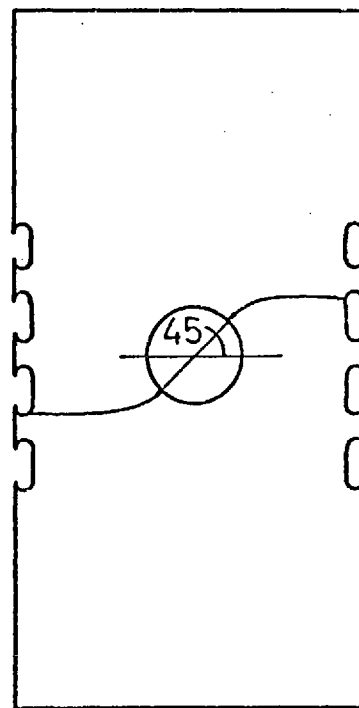


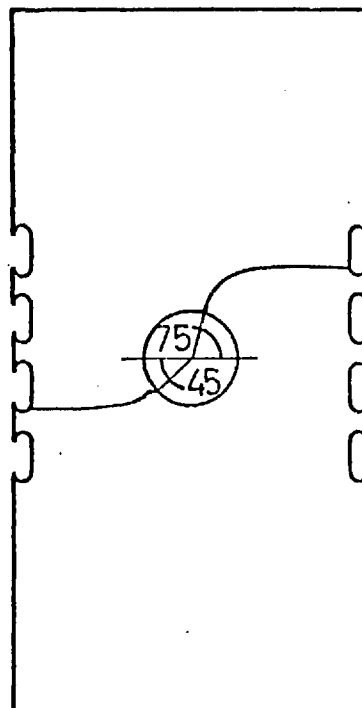
FIG (62) CORRESPONDENCE OF THE EXPERIMENTAL CRACK GROWTH DIRECTION TO THE PREDICTIONS OF THE FINITE ELEMENTS STRESS ANALYSIS (See also Fig 60)



(a)



(b)

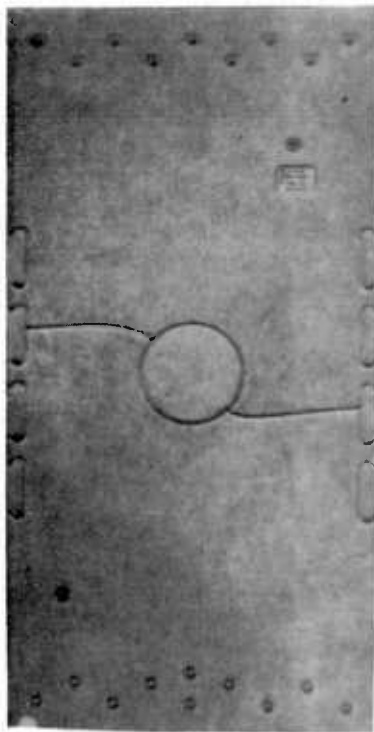


(c)

FIG (63) PATH OF CRACK EXTENSION IN BIAXIAL LOADING OF ROR SPECIMENS CONTAINING INITIAL CRACKS AROUND THE PERIPHERY OF THE CENTRE HOLE



(a)



(b)



(c)

FIG (64) PATH OF EXTENSION OF INCLINED CRACKS:

- (a) UA SERIES
- (b) ROR SERIES
- (c) AG SERIES

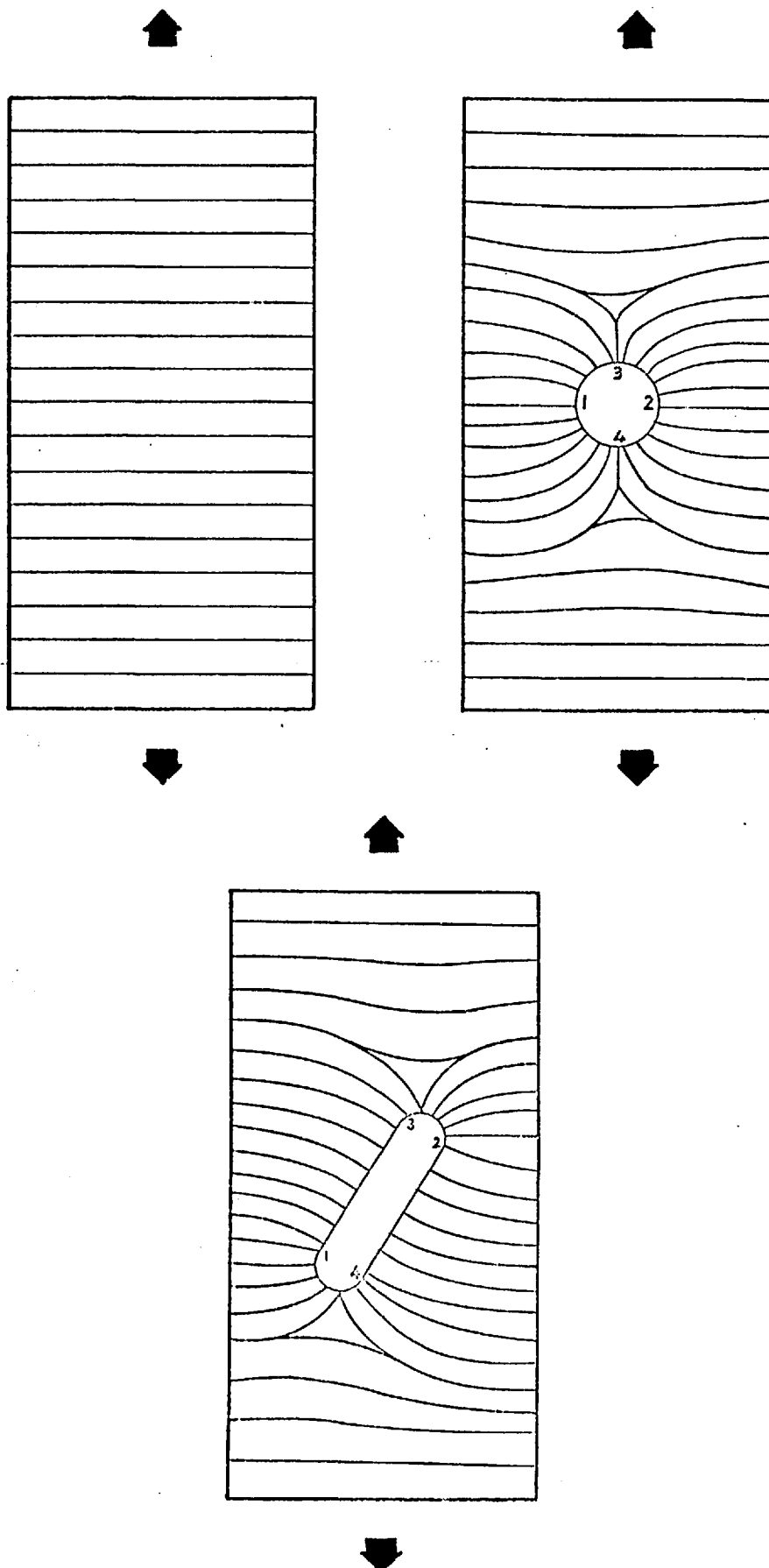


FIG (65) SCHEMATIC REPRESENTATION OF THE CONFIGURATION OF THE MAXIMUM PRINCIPAL PLANE CONTOURS

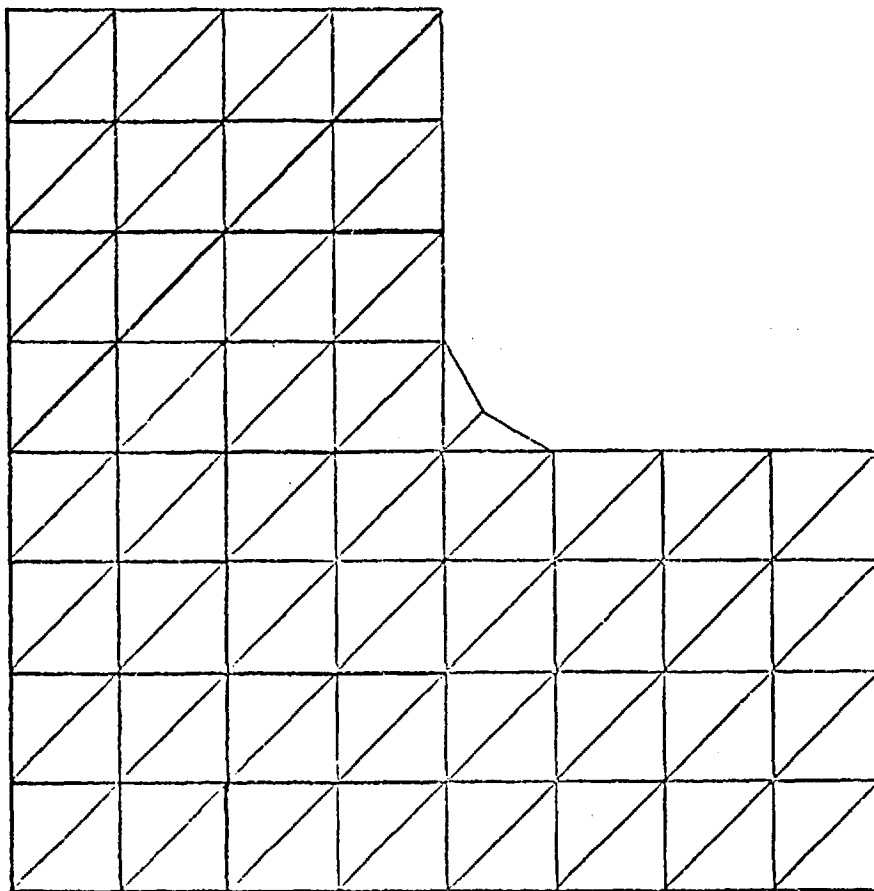
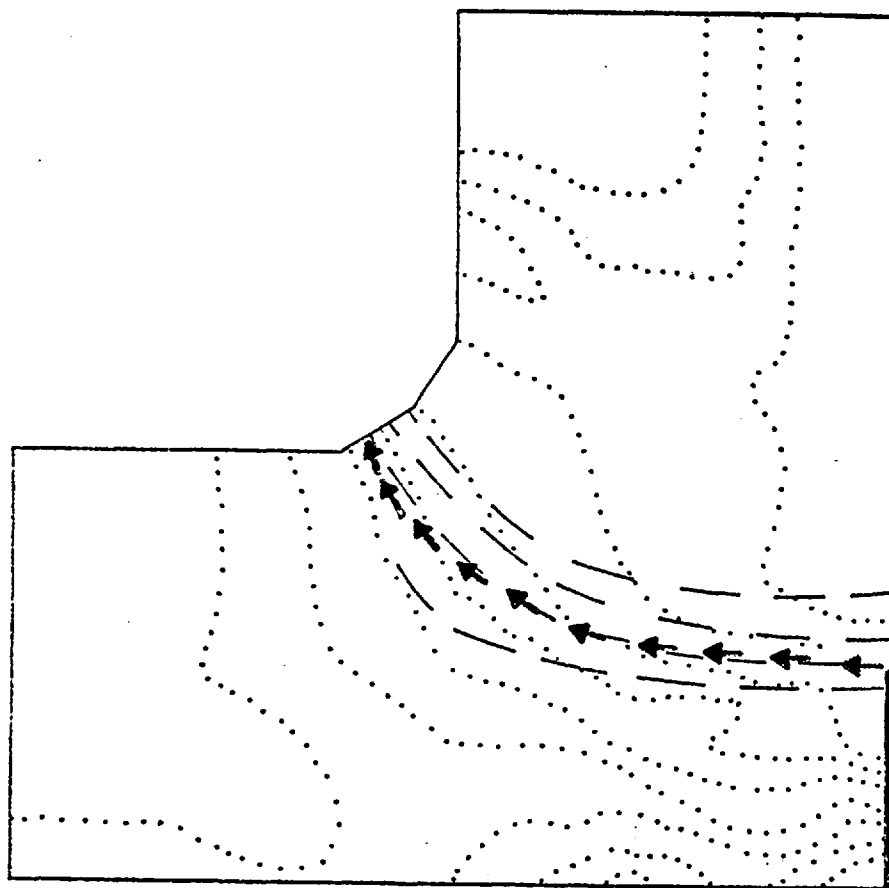
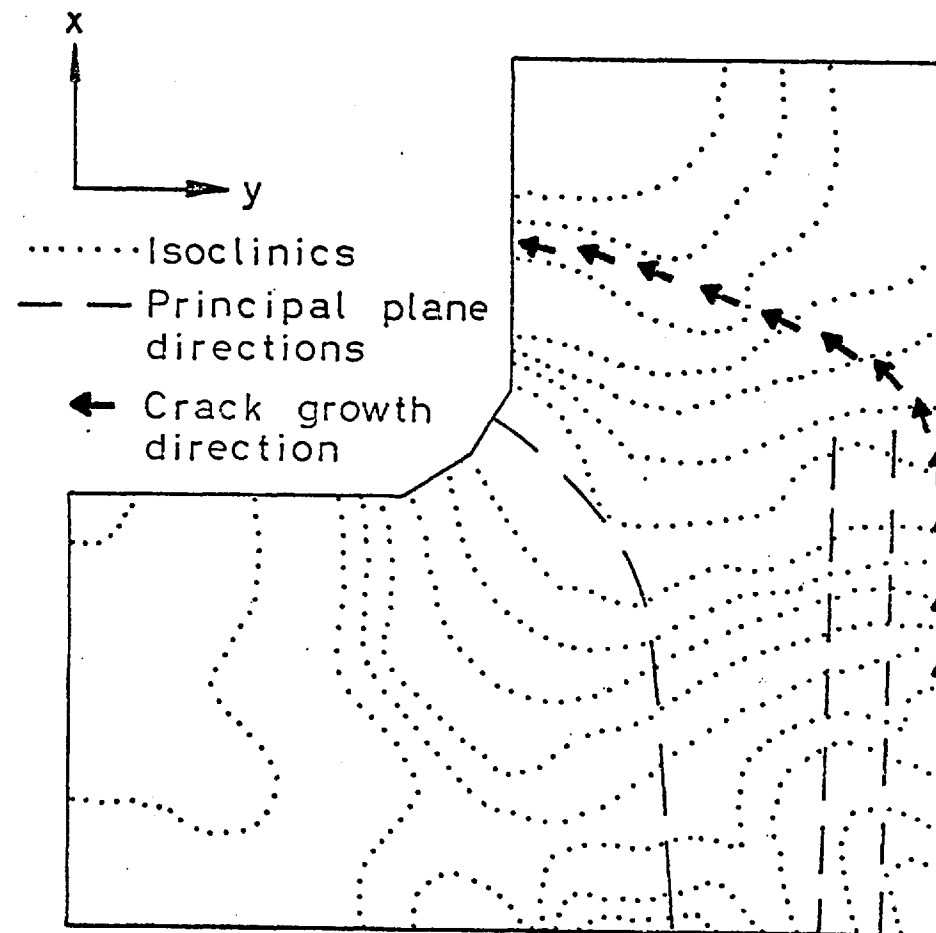


FIG (66) THE FINITE ELEMENTS MESH USED FOR THE ANALYSIS OF THE KISLER AND ROBERTS (1970) SPECIMEN



(a) $L_x > L_y$



(b) $L_y > L_x$

FIG (67) MAXIMUM PRINCIPAL PLANE CONTOURS AND THE DIRECTION OF CRACK GROWTH IN CROSS-SHAPED SPECIMENS

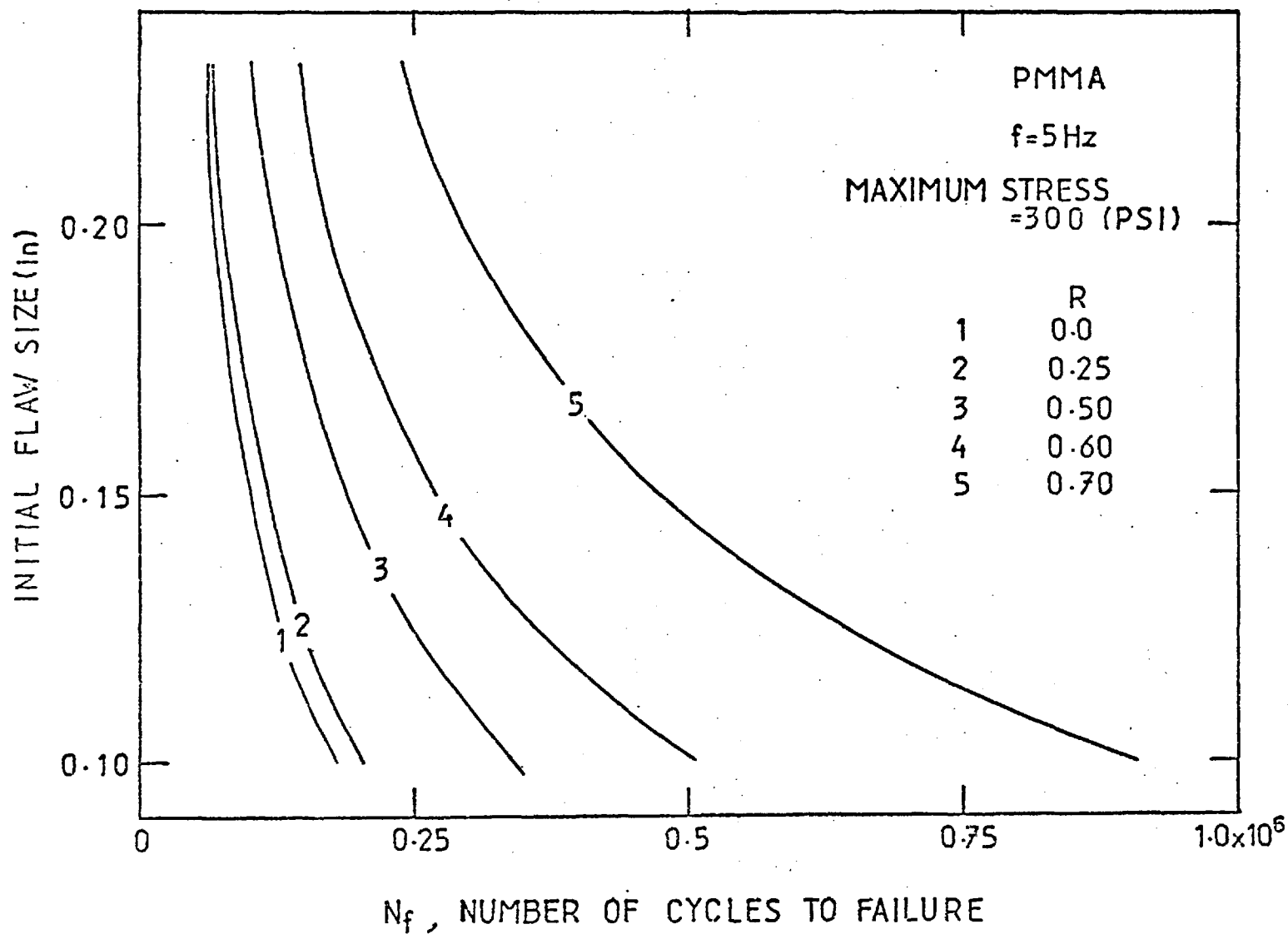


FIG (68) ESTIMATION OF THE FATIGUE LIFE AS A FUNCTION OF THE INITIAL FLAW SIZE; PMMA AT A FREQUENCY OF 5 Hz

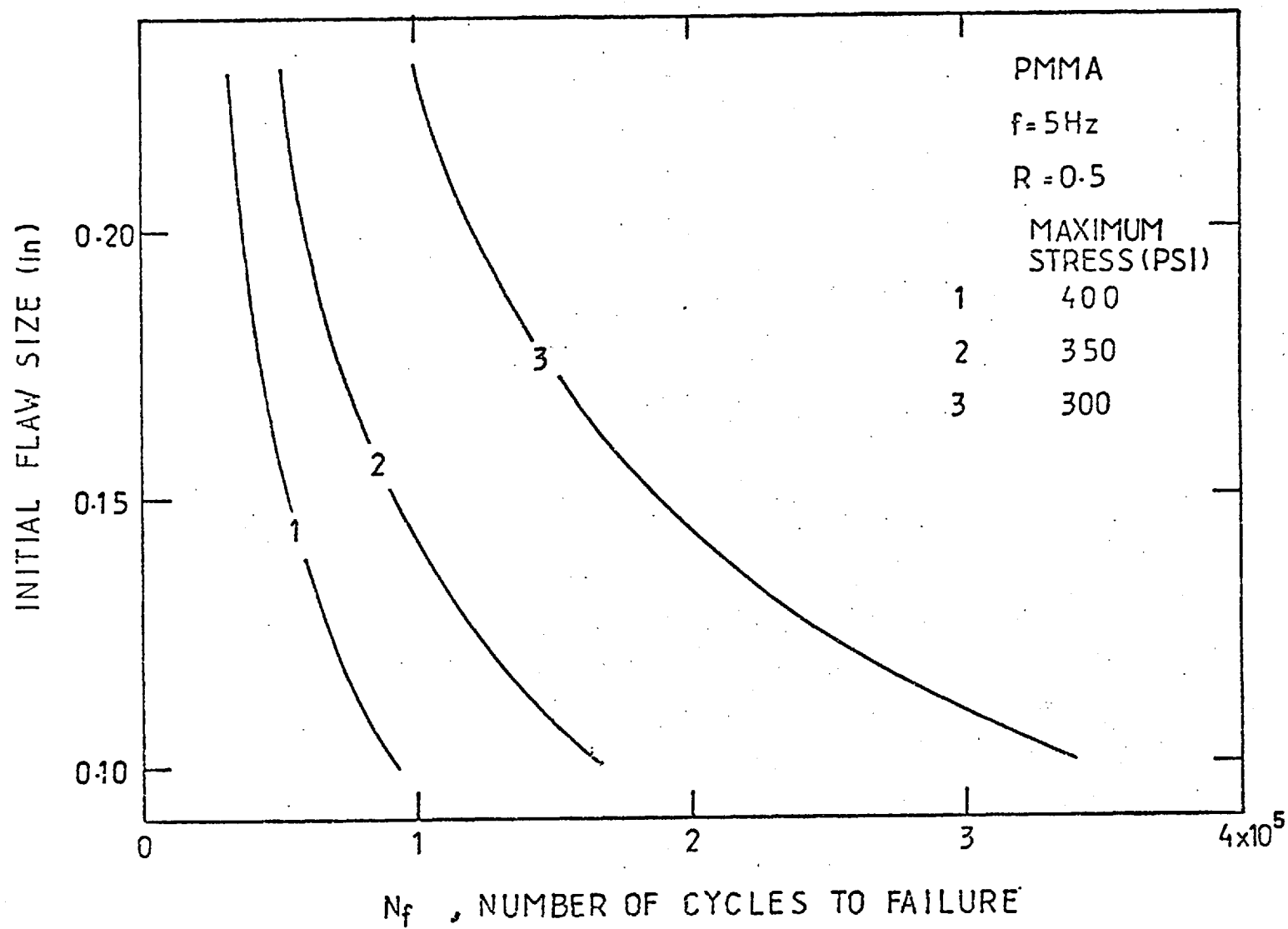


FIG (69) THE EFFECT OF CHANGES IN THE LEVEL OF THE MAXIMUM APPLIED STRESS ON THE FATIGUE LIFE OF PMMA

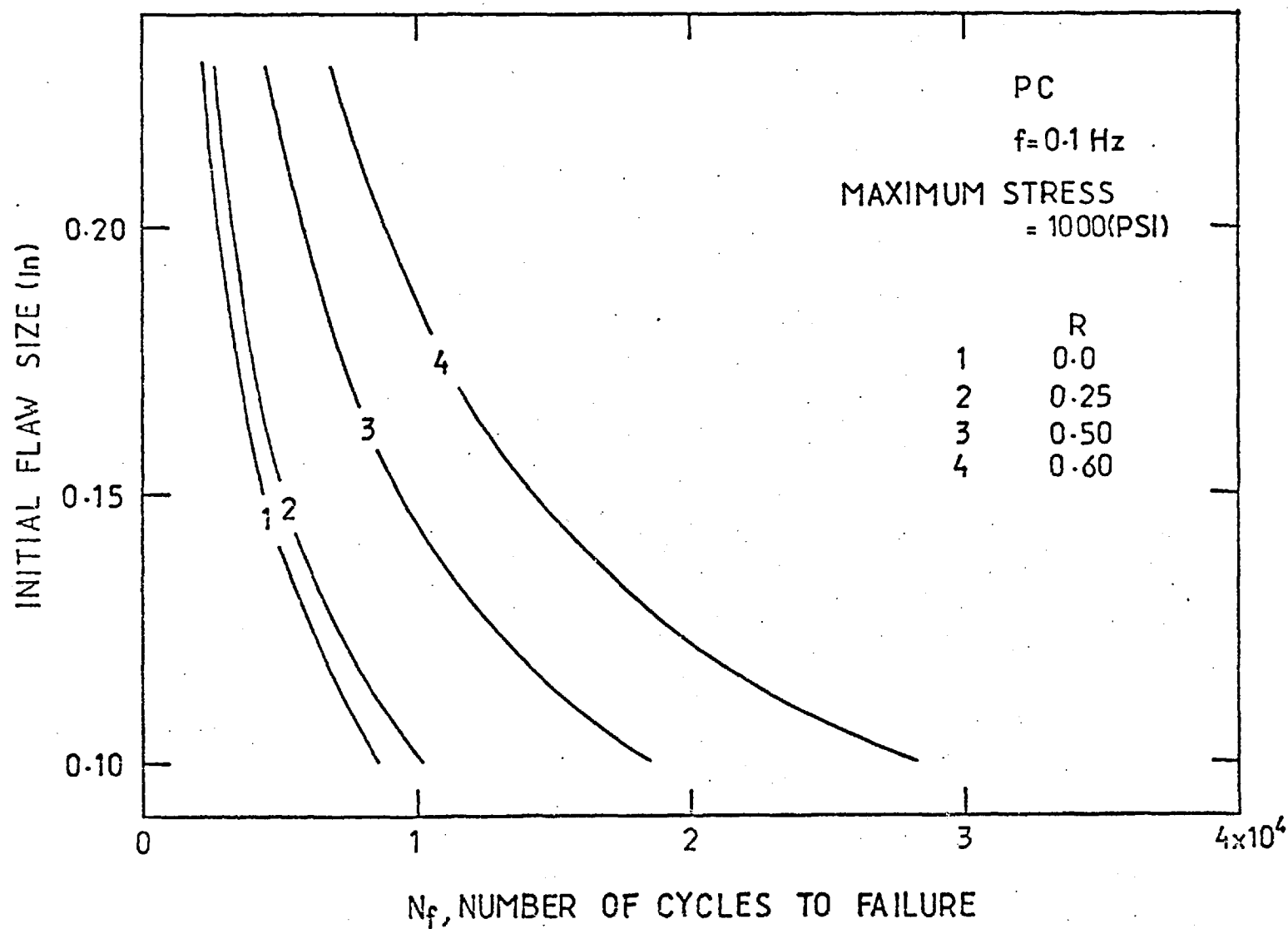


FIG (70) ESTIMATION OF THE FATIGUE LIFE AS A FUNCTION OF INITIAL FLAW SIZE IN PC

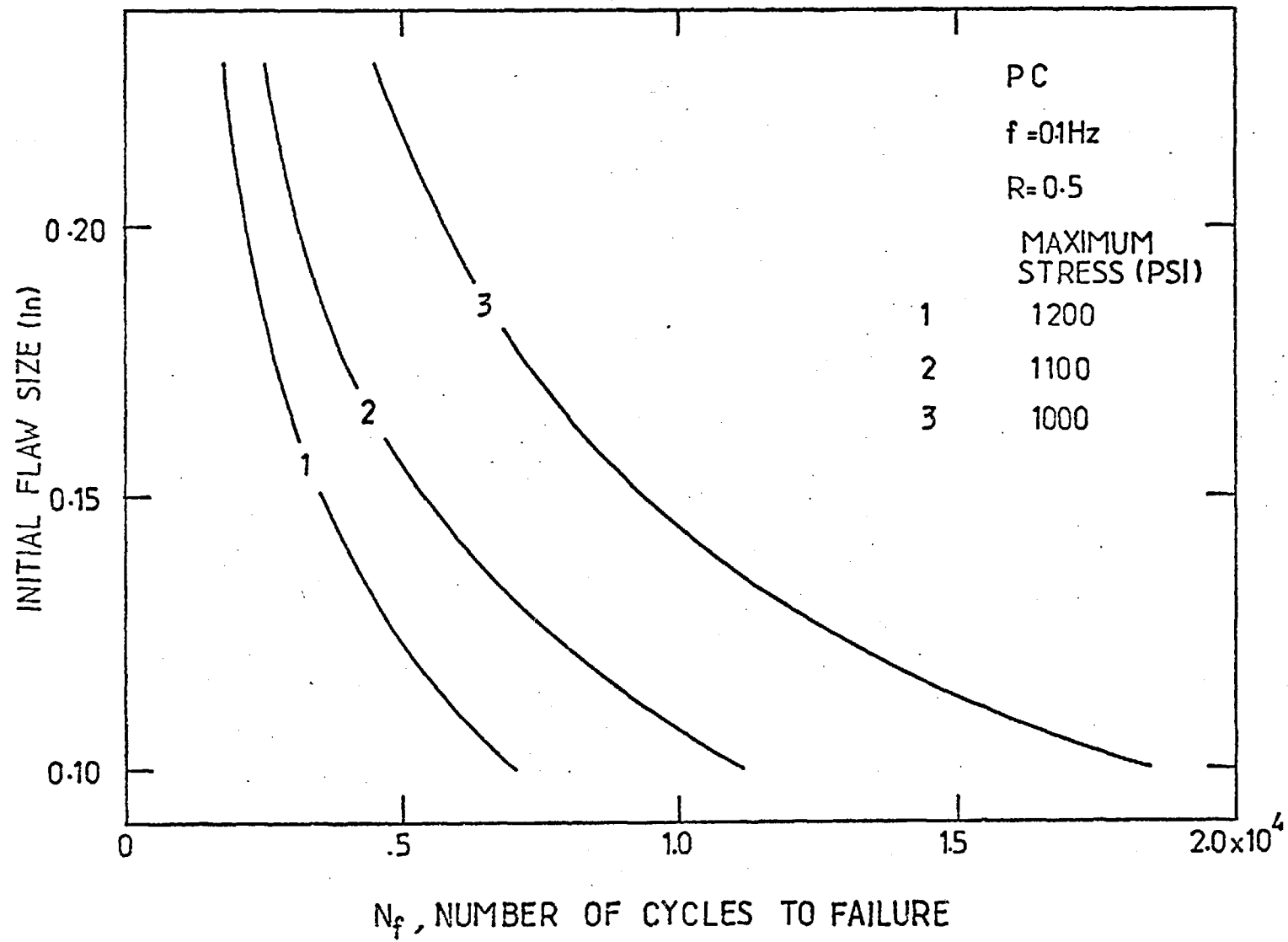


FIG (71) THE EFFECT OF CHANGES IN THE LEVEL OF THE MAXIMUM APPLIED STRESS ON THE FATIGUE LIFE OF PC

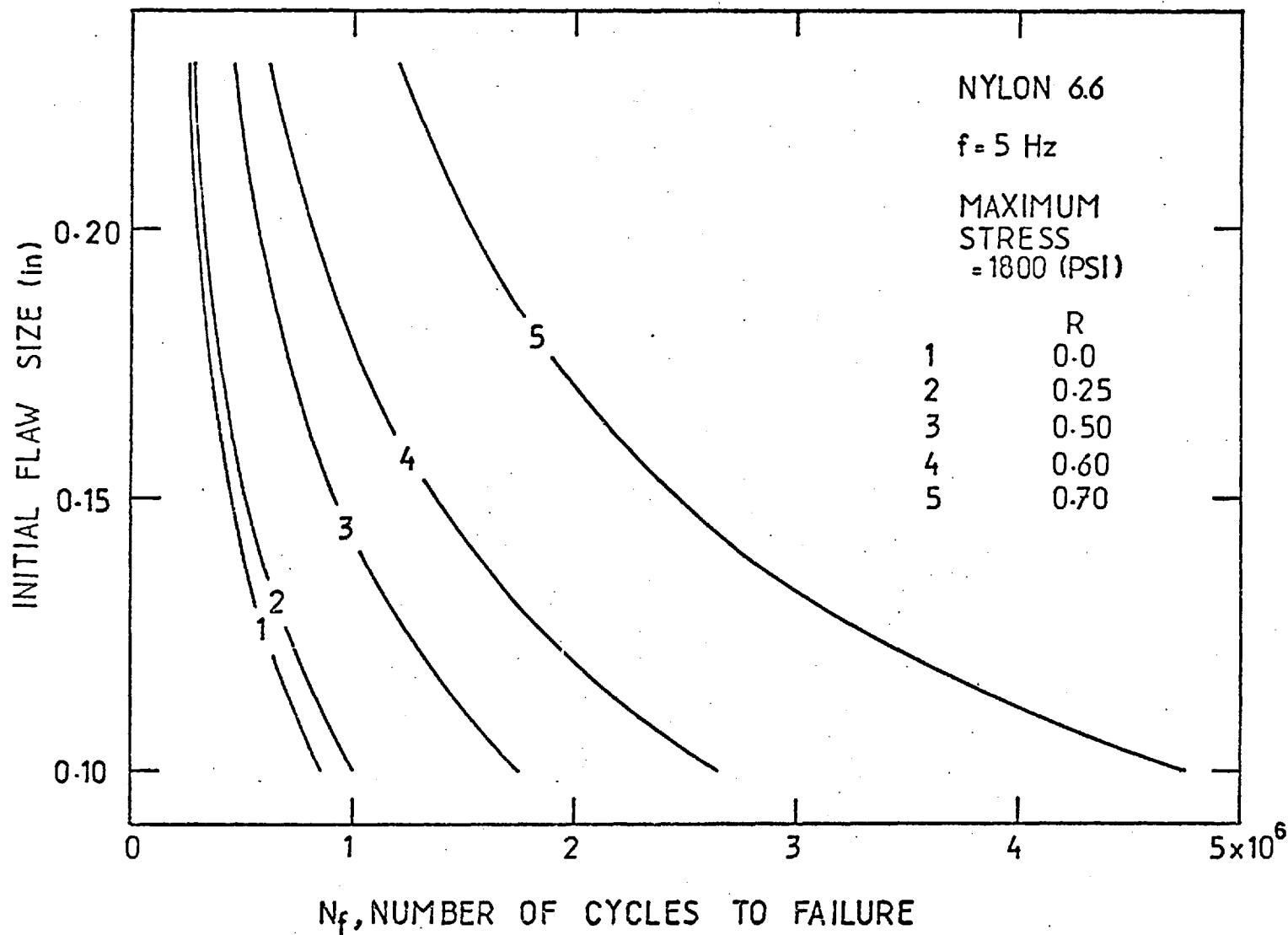


FIG (72) ESTIMATION OF THE FATIGUE LIFE AS A FUNCTION OF THE INITIAL FLAW SIZE IN NYLON 6.6

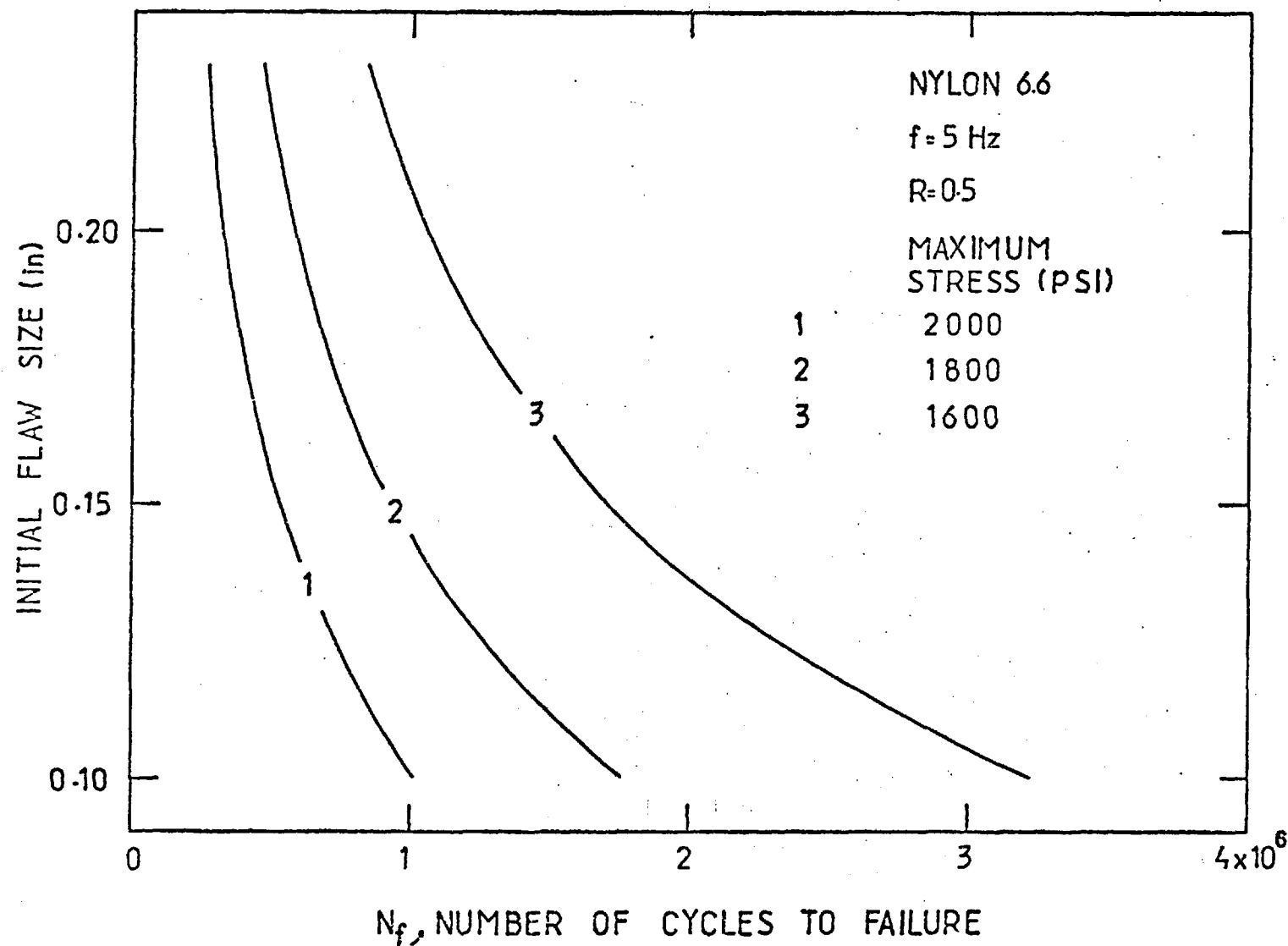


FIG (73) THE EFFECT OF CHANGES IN THE LEVEL OF THE MAXIMUM APPLIED STRESS ON THE FATIGUE LIFE OF NYLON 6.6

APPENDIX A

CONSIDERATION OF FAILURE AT THE MOLECULAR LEVEL

In Chapter 4 the interrelations between the chemical structure and the mechanical properties of viscoelastic materials were referred to. Recognition of the importance of the influence of molecular kinetics on the physical behaviour at the macroscopic level has led to the development of various methods of failure analysis in viscoelastic solids, formulated in terms of the fundamental concepts of dissociation processes at the atomic level; e.g. the recent contributions of Kawabata et al. (1970), Kaush (1971) and Landel and Fedors (1971). The most important achievement in this field has, perhaps, been the development of theories related to physics of rubber elasticity. As the analytical approaches based upon the atomic bond rupture mechanisms contain the largest possible number of basic material properties, they enjoy the highest degree of adaptability and when fully developed, they will provide the most accurate solution to the failure problem. Useful reviews of the reaction rate models of failure have been provided by Martin (1969) and Henderson et al. (1970).

The fundamental subject matter to be covered prior to the achievement of a reasonable level of realisation of the extent of the influence of molecular mechanisms on the failure process, is extremely broad. Hence, it is pertinent to reiterate that the underlying philosophy of the analysis presented here has been to demonstrate the possibility of derivation of a quantitative relationship between the parameters associated with the chemical structure and those related to the macroscopic mechanical behaviour. It is hoped that the exposition of the specific role and the relative importance, with regards to the failure process, of such parameters as the molecular chain length, the dissociation energy and other parameters of the theories related to

chain scission and factors such as the temperature and the nature of the external disturbance, even at its simplest form, will be useful in the prediction of possible variations in the mode and the extent of failure as a consequence of changes in the material properties and the environmental conditions. The analysis will be aimed at the derivation of a relationship for the determination of the time to failure under the conditions of application of an arbitrary load function.

In the following analysis, the dissociation rate theory will be used to characterise processes such as the molecular bond strain and rupture for a certain volume of the material containing an arbitrary number of bonds. These results will be related to the physical parameters used in the continuum approaches to the analysis of the strength of the material.

(a) Changes in the macroscopic dimensions:

Consider a stress region of dimensions $d_0 \times 1 \times 1$ (Fig. A.1) in a linear viscoelastic material.

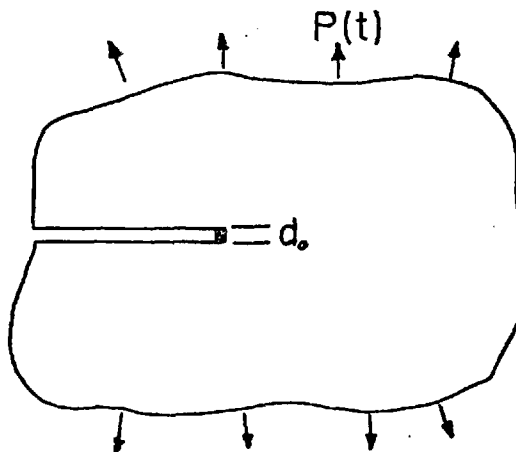


Fig. (A.1) The stressed region under consideration.

Upon the application of load $P(t)$, sustained for a period t , dimension d_o acquires a new value $d(t)$. $d(t)$ increases gradually up to a certain critical level, d_f , at which point material failure occurs, thus:

$$d(t) = Y(t) d_o \quad (1)$$

and

$$d(t) \Big|_{t=t_f} \equiv d_f = Y_f d_o$$

$Y(t)$ is a parameter which characterises the material behaviour. This can be written as:

$$d(t) = \left[1 + D_{\text{crp}}(t) P(t) \right] d_o \quad (2)$$

where the creep compliance function

$$D_{\text{crp}}(t) = D_g + \frac{D_e - D_g}{e^{t/\tau}} \quad (3)$$

D_e and D_g are the rubbery and the glassy levels of $D_{\text{crp}}(t)$; τ is the relaxation time; (see Chapter 3) for the case of $D_g \approx 0$

$$d(t) = d_o \left[1 + \frac{D_e}{e^{t_f/\tau}} P(t) \right]$$

and the total extension at failure:

$$d(t) \Big|_{t=t_f} \equiv d_f = d_o \left[1 + \frac{D_e P(t)}{e^{t_f/\tau}} \right] \quad (4)$$

and

$$Y_f = \left[1 + \frac{P(t)}{E_e e^{t_f/\tau}} \right]$$

(b) Variations in dimensions at the molecular level:

The dimensional changes defined by equation (4) can be related to the variations in the physical dimensions at the molecular level.

The molecular chain model of Rouse (termed the bead-spring model) was selected for consideration. If such a chain is subdivided into Z sub-molecules each with n links ($N = Zn$, where N is the total number of links), then the end-to-end distance, R_o , at time $t = 0$, of a freely orienting chain (i.e. a chain of sufficient length so that completely free segmental orientation can take place) will be given in terms of a root-mean-square function:

$$R_o = N \ell_o^2 \quad (5)$$

where R_o is the end-to-end distance vector
and ℓ_o is the initial bond length; Figure (A.2).

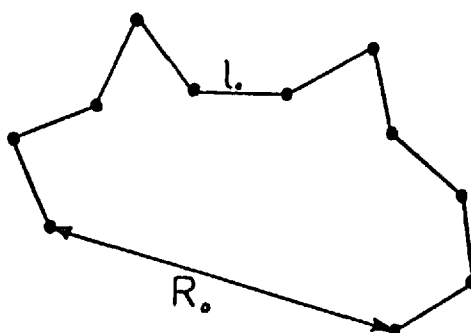


Fig. A.2 Representation of the end-to-end distance and the bond length of a molecular chain.

In the coiled position, R_o will be much smaller than the contour length of the chain. Let us assume that for a configuration such as that in Figure (A.2) one may write:

$$R_o = N \ell_o \cos \theta D \quad (6)$$

for all values of $0 \leq D \leq 1$.

$D = 1$ corresponds to the fully stretched conditions and $D = 0$ corresponds to the conditions in which the two chain ends are close together.

Upon the application of the arbitrary load $P(t)$, R_o will increase up to the fully stretched point and further (with the introduction of pairing bond strain) until a value R_f has been reached which corresponds to the conditions of chain scission at a strained bond length of ℓ_f :

$$R_f = B_f R_o \quad (7)$$

and as $D = 1$ under these conditions, then from equation (6):

$$R_f = N \ell_f \cos \theta \quad (8)$$

Parameter B_f which is a measure of the extension of the end-to-end distance, can be related to the individual bond strain $X(t)$:

As

$$X(t) = \frac{\ell(t) - \ell_o}{\ell_o}$$

and

$$X_f = \frac{\ell_f - \ell_o}{\ell_o} \quad (9)$$

and also

$$B_f = \frac{R_f}{R_o} = \frac{N \ell_f \cos \theta}{R_o} = \frac{N \cos \theta \ell_o}{R_o} \left[1 + \frac{\ell_f - \ell_o}{\ell_o} \right]$$

hence

$$B_f = \frac{N \ell_o \cos \theta}{R_o} (1 + X_f) \quad (10)$$

From equations (6) and (10):

$$D B_f = 1 + X_f \quad (11)$$

(c) A criterion for pairing bond dissociation and chain rupture:

The failure process is essentially a non-equilibrium process which involves the molecular bond rupture and viscous flow. The analysis of the chain failure mechanisms described here, is based upon the chemical rate process theory of Arrhenius as developed by Eyring (1936). The theory assumes that a process which occurs at a constant rate is the result of the movement of a unit of flow (a molecule or a larger unit) past a potential energy barrier which opposes this motion, Figure (A.3). A considerable amount of input energy is required to enable this 'activation' process to take place:

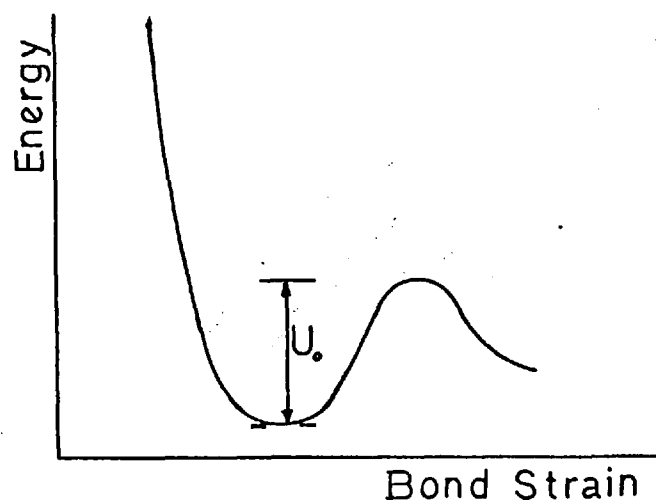


Fig. A.3 The relationship between bond strain and potential energy.

In the absence of an external input energy, the rate of transition into the activated state is given by:

$$R^* \approx \frac{kT}{h} \exp \left[\frac{-U_o}{R_1 T} \right] \quad (12)$$

where

k is the Boltzmann's constant

h is the Plank's constant

R_1 is the Gas constant

T is the absolute temperature.

If the equilibrium state is disturbed by the introduction of an external force, parameter U_o above can be replaced by $[U^* = U_o - \phi(\sigma)]$, where $\phi(\sigma)$ is a function of the applied stress, Zhurkov (1965).

For an isothermal proces $\frac{kT}{h} = J$ is a constant, the magnitude of which is of the order of bond vibration frequency.

The quantity $R dt$ will thus be the rate at which the activation process resulting in bond rupture takes place during the time dt . Thus:

$$R^* dt = J \exp \left[\frac{-U^*}{R_1 T} \right] dt \quad (13)$$

The chain extending bond in most polymers is the carbon to carbon single bond. Both the interatomic distance between the carbons and the valence angle (i.e. the angle between successive carbon-carbon bonds) are independent of the chain length. In the present analysis, dealing with the failure in linear polymers, it is assumed that the bond rupture will be preceded by full chain extension (i.e. extension into zig-zag form with valence angle, θ). The pairing bonds will subsequently be strained until one of them fails. The assumption of orientation before failure is based on the experimental observation of Novikov (1968).

Real polymers possess molecular chains of various lengths. If the number of bonds (links in the chain backbone) in a particular chain is N , then a chain length (N) against the number of chains (Q_N) distribution of the following form may be assumed. (Flory, 1969):

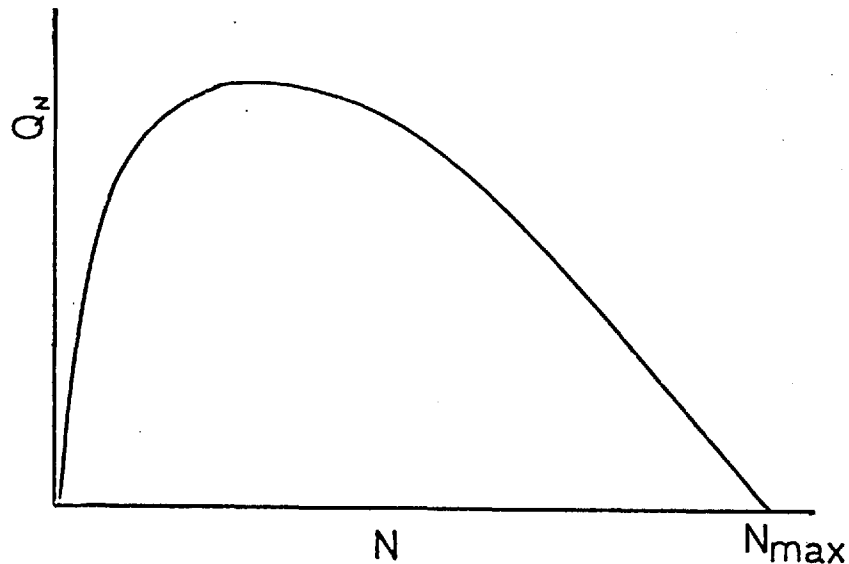


Fig. A.4 Chain length - number of chains distribution.

Thus Q_N is the number of chains with N links and N_{max} is the number of bonds in the longest possible chain. The rate of chain failure is dependent on chain length. The shorter chains are stretched out in a shorter period of time and thus will fail before the longer chains.

The total number of chains within the volume under consideration is denoted by Q_{T_0} . It is assumed that the failure of the stressed region will occur when a critical number of chains Q_{T_f} have been ruptured (Higuchi, 1965). There will, thus, be a certain value of N , denoted by N_f , such that at the instant when chains with N_f links have failed and only Q_{T_f} number of chains have remained, material failure occurs.

Parameter Q_{T_f} may be calculated as follows:

U^* in equation (13) can be written as:

$$U^*(X) = U(X_2) - U(X_1) \quad (14)$$

i.e. the difference in energies corresponding to two bond strain levels.

From Bueche (1962):

$$U^*(X) = |U_o| (\alpha - \beta X) \quad (15)$$

and at the instant of rupture, substituting for X_f from equation (11)

$$U^*(X_f) = U_o \left[\alpha - \beta (D B_f - 1) \right] \quad (16)$$

Thus the rate of bond failure, for a chain of N bonds, when subjected to an energy transformation of magnitude U^* for a period dt , is obtained from equation (13):

$$R^* dt = N J \exp \left[\frac{-U^*}{R_T} \right] dt \quad (17)$$

or

$$R^* dt = N J \exp \left[\frac{U_o}{R_T} \right] \left[\beta (D B_f - 1) - \alpha \right] dt \quad (18)$$

The relative rate of chain scission amongst the chains of various lengths must also be determined.

At the instant $t = 0$, there exist Q_{N_o} chains each with N links. At the constant $t > 0$ the rate of decrease in the number of these chains is given by (Tobolsky and Eyring, 1943):

$$-d Q_{N_t} = Q_{N_t} J N \exp (\Delta) dt \quad (19)$$

where Q_{N_t} is the number of chains with N links at instant t , and

$$\Delta = \frac{U_o}{R_T} \left[\beta (D B_f - 1) - \alpha \right] \quad (20)$$

Note that the equation (19) is formulated on the basis of the assumption that chains rupture without reforming. The following comments may justify such an assumption (Bueche, 1962):

The number of chains at any instant is directly proportional to the applied stress at that instant. The broken chains are not always 'dead'. They may react with each other or with another unbroken chain. Since the stretched chain will retract very rapidly after scission, it is reasonable to assume that chain reformation will always occur in the relaxed state. As a result these newly formed chains will not contribute to the relief of stress in the yet unbroken portions of the chains.

Assuming that at time $t = 0$, Q_{N_0} is a certain fraction f of the total number of chains, Q_{T_0} , i.e.

$$Q_{N_0} = f Q_{T_0} \quad (21)$$

and also from Figure (A.4) †

$$Q_{T_0} \approx \int_0^{N_{\max}} Q_{N_0} dN \quad (22)$$

At an instant $t > 0$, all chains with number of links smaller than N_t have ruptured. The total number of the remaining chains, Figure (A.5):

$$Q_{T_t} = \int_{N_t}^{N_{\max}} Q_{N_t} dN \quad (23)$$

† Approximation is introduced due to the unrealistic assumption of zero as the lower limit of N .

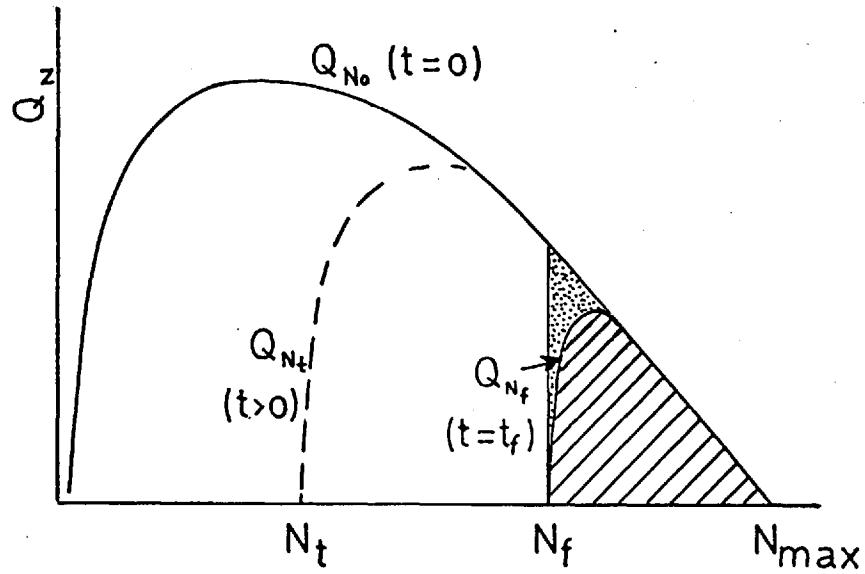


Fig. A.5 Number of chains with N links at various instances during the loading period.

At the point of total failure of the stressed zone:

$$Q_{T_f} = \int_{N_f}^{N_{\max}} Q_{N_f} dN \quad (24)$$

From Figure (A.5), an approximate value of Q_{T_f} is found:

$$Q_{T_f} \approx \int_{N_f}^{N_{\max}} Q_{N_0} dN = Q_{N_0} (N_{\max} - N_f) \quad (25)$$

From equation (19) and (21) parameter Q_{N_t} is obtained in terms of Q_{N_0} :

$$\int_{Q_{N_0}}^{Q_{N_t}} \frac{dQ_{N_t}}{Q_{N_t}} = \ln Q_{N_t} \bigg|_{Q_{N_0}}^{Q_{N_t}} = -J \int_0^t N_t \exp(\Delta) dt \quad (26)$$

hence

$$Q_{N_t} = Q_{N_o} \exp \left[-J \int_0^t N_t \exp (\Delta) dt \right] \quad (27)$$

Thus

$$Q_{T_t} \Big|_{t=t_f} = Q_{t_f} = Q_{T_o} \int_{N_f}^{N_{\max}} f \exp \left[-J N_f \int_0^{t_f} \exp (\Delta) \right] dN \quad (28)$$

(d) The relationship between the zone extension $Y(t)$ and the chain extension $B(t)$:

This relationship is found through the assumption that the chains are not allowed to fail at the stressed zone boundaries. This is a reasonable assumption as the strongest pull takes place in the middle of the chain. The assumption implies that all the chains situated within the zone, are positioned as shown in Figure (A.6):

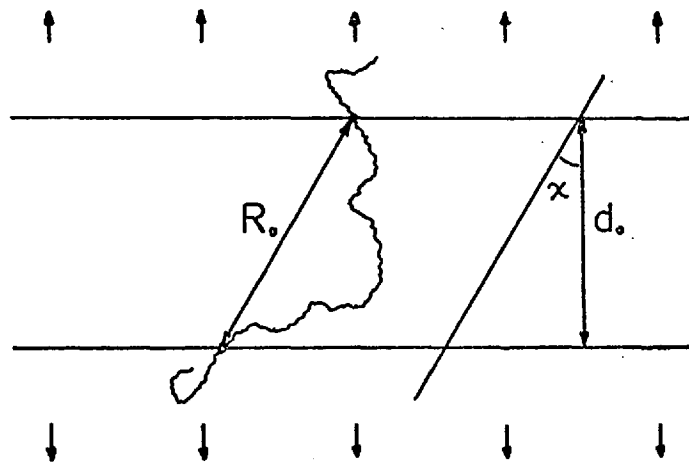


Figure A.6 Position of a chain in the stressed zone.

For the sake of generality it is assumed that the end-to-end vector R_o makes an angle χ with the loading direction. From Figure (A.6):

$$d_o = R_o \cos \chi \quad (29)$$

The time dependent chain extension

$$B(t) = \frac{R(t)}{R_o} = Y(t) \cos \chi$$

and

$$B_f = Y_f \cos \chi = Y_f \frac{d_o}{R_o} \quad (30)$$

Equation (20) becomes:

$$\Delta = \frac{U_o}{R_f T} \left[\beta \left(Y_f \frac{d_o}{R_o} - 1 \right) - \alpha \right] \quad (31)$$

with $D = 1$ at failure.

Thus using equation (21) and (25) equation (28) becomes:

$$Q_{T_f} = Q_{T_o} \left(\frac{Q_{N_o}}{Q_{T_o}} \right)^{\frac{(N_{\max} - N_f)}{\exp(J N_f)}} \exp \left[\int_0^{t_f} \exp(\Delta) dt \right] \quad (32)$$

Therefore

$$J N_f = \int_0^{t_f} \exp(\Delta) dt \quad (33)$$

or

$$\int_0^{t_f} \exp \left[\frac{U_o}{R_f T} \right] \left[\beta \left(\frac{d_f}{R_o} - 1 \right) - \alpha \right] dt = J N_f \quad (34)$$

In this equation parameters U_o , R_o , T , R_f , β , α and J are known. d_f can be substituted in terms of d_o , $P(t)$ and t_f . The parameter N_f for a particular polymer will be determined from empirical studies such as creep failure under isothermal conditions; d_f and t_f can be measured and hence N_f can be calculated. Subsequently graphs of N_f v T distribution will be prepared.

Discussion:

The formulation of a function for the determination of time to failure under arbitrary loading conditions (equation 34), can, no doubt, be further refined. As indicated in the introduction, the above analysis should be treated simply as an enquiry into the possible effects of some of the parameters associated with the chemical structure. Such analytical studies should be coupled with extensive experimentation on materials with well defined structural properties so as to verify the relevance and the accuracy of the basic assumptions made throughout the theoretical treatment. For example, there may be doubts about the specific form of the distribution of the chain length against the number of chains, Figure (A.4); specially whether the same type of distribution can be assumed for the products of both the addition and the condensation polymerisation processes. It is, however, possible to determine the chain length distribution for a certain polymer by the electron paramagnetic resonance technique (De Vris et al., 1971).

There may also be some differences of opinion with regards to the choice of the criterion of equal load distribution for all chains of the same orientation. This is a less realistic choice than that in the model of Hsiao (1959). However the mathematical complexity of this model has prevented its further development.

Further complexities in the mathematical formulation will arise when the function $P(t)$ is substituted for in terms of a sinusoidal load function. In the past some attempts have been made to incorporate the effect of periodic stress on the kinetic behaviour of the molecular deformation unit (Coleman, 1956). Ree et al. (1951) and Lyons (1958) also introduced into the reaction rate theory a concept relating the frequency dependence of the mechanical properties. It was proposed that under the periodic stress conditions the activation energy may vary as a function of frequency.

With each periodic application of stress, the probability of bond breakage is increased above the level prevailing in the simple constant-load case.

It is suggested that the incorporation of the macroscopic strain $d(t)$ in terms of a sinusoidal function for $P(t)$ in equation (34), will be capable of incorporating the cyclic damage accumulation process (Chapter 4). Inevitably, the relationship will acquire a mathematically more complex formulation; see also Yokobori (1969). However, it will not be too difficult to perform such analyses with the aid of computers (De Vries et al., 1971).

Finally, the importance of the molecular failure analysis with regards to the exposition of the specific role of parameters associated with the chemical structure lies in the fact that subsequently more accurate predictions of the properties of new materials can be made. The results will also yield guidance to the attempts to synthesise materials with optimum properties.

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APPENDIX B
NOMENCLATURE

Symbol	Meaning
A	Work of surface tractions
a	Half crack length
a_0	The value of a at $t = 0$
$\dot{a}_N$	Rate of change of a per load cycle
$\dot{a}_t$	Rate of change of a per unit time
B	Constant in the crack growth equation
B_s	Component thickness
B_c	Crack width
b	Constant in the crack growth equation
C	Constant in the crack growth equation
c	Compliance
D	Constant in the crack growth equation
$D_{crp}(t), D(t)$	Time dependent creep compliance
D_e	Rubbery level of the creep compliance
D_g	Glassy level of the creep compliance
E	Young's Modulus
$E_{rel}(t), E(t)$	Time dependent Modulus
E_e	Rubbery value of the time dependent modulus
E_g	Glassy value of the time dependent modulus
E^*	The complex modulus of a viscoelastic material
E'	Real (storage) component of the complex modulus

Symbol	Meaning
E''	Imaginary (loss) component of the complex modulus
E_K	Kinetic energy term in the energy balance equation
e	Strain
f	Loading frequency
G	Strain energy release rate
G_c	Critical level of G
G_I	Value of G for mode I crack opening
G_{max}	Maximum level of G
G_{min}	Minimum level of G
h	Specimen height
J	Path independent contour integral
J_{max}	Maximum level of J
J_{min}	Minimum level of J
K	Stress intensity factor
K_I	The value of K under mode I crack opening
K_{II}	The value of K under mode II crack opening
K_{III}	The value of K under mode III crack opening
K_{IC}	The critical value of K under plane strain conditions - termed the fracture toughness of the material
K_{AC}	"Apparent" level of fracture toughness
K_b	Value of K for pure bending conditions
K_C	The critical value of K
K_e	The "effective" level of K
K_{max}	Maximum level of K_I in a load cycle

Symbol	Meaning
K_{min}	Minimum level of K_I in a load cycle
K_m	Mean level of K_I in a load cycle
K_o	Threshold level of K
$K(t)$	Time dependent stress intensity factor
$\dot{K}$	Rate of change of K with respect to time
k	Constant in the crack growth equation
M	Constant in the crack growth equation
m	Numerical index in the crack growth equation
N	Number of load cycles
N_i	Cyclic initiation life
n	Numerical index in the crack growth equation
P	Applied load level
p	Numerical index in the crack growth equation
Q	Input thermal energy term in the energy balance equation
q	Numerical index in the crack growth equation
R	Ratio of K_{min}/K_{max}
R_I	Universal Gas Constant
r	Radial distance measured from the crack tip
r_p	Plastic zone size at the crack tip
$r_{p_{max}}$	Maximum level of r_p
$r_{p_{min}}$	Minimum level of r_p
T	Absolute temperature
T_i	Traction forces
t	Time

Symbol	Meaning
t_f	Time to failure
U_o	Activation energy
u_i	Displacements
W	Internal energy term in the energy balance equation
W_o	Strain energy density
W_v	Viscous energy lost per load cycle
w	Specimen width
w_{dd}	Extent of plastic deformation at the crack tip
$\alpha_1, \alpha_2, \alpha_3$	Numerical indices in the crack growth equation
β	Constant in the crack propagation model
β	Angle of inclination of a crack with respect to the loading direction
γ	Specific surface energy
γ^*	Specific fracture energy
γ_p	Irreversible work of deformation
γ_o	Coefficient in the time to failure equation
δ	Extent of crack opening displacement
ϵ_{mn}	Strain tensor
ζ	$\left. \begin{array}{l} 1 - \nu^2 \\ 1 \end{array} \right\}$ plane strain plane stress
κ	Boltzmann's Constant
λ	Term representing the stress intensity factor parameter in the crack growth equation
ν	Poisson's ratio
ξ	A time function

Symbol	Meaning
Π	Surface energy term in the energy balance equation
σ	Stress level
σ', σ''	Real and imaginary components of complex stress function
σ_{ij}	Stress tensor
σ_o', σ_m	Mean level of stress
σ_l', σ_a	Amplitude of stress cycle
σ_m'	Weighted average value of stress
σ_{yp}	Yield stress in tension
τ	A time function
τ_{ij}', τ_{xy}	Components of shear stress
τ^0	Maximum in plane shear stress level
$\mathcal{T}$	Parameter representing the level of available energy in crack propagation equation
χ	Parameter in the crack propagation equation - a function of the applied stress level
ω	Circular frequency

LIST OF PUBLICATIONS

"Fatigue Crack Propagation in Polymethylmethacrylate - The Effect of Mean Value of Stress Intensity Factor; "

Journal of Mechanical Engineering Science, Vol. 13, P 75, (1971)

"Growth of Fatigue Cracks in Polycarbonate; "

Polymer Engineering and Science, Vol. 12, P 193, (1972)

"Fatigue Crack Propagation in PMMA - The Effect of Frequency; "

Journal of Mechanical Engineering Science, Vol. 14, P 328, (1972)

"Design Against Fatigue Failure in Thermoplastics; "

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Published in:

Journal of Engineering Fracture Mechanics, Vol. 4, P 511, (1972)

"Rate Dependency of Failure Processes in Polycarbonate and Nylon 66; "

To be published in:

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"Fatigue Failure in Thermoplastics; "

To be presented at:

3rd International Conference on Fracture, Munich, (1973)

(Paper 88)

"Growth of Fatigue Cracks in Metals and Polymers;"

To be presented at:

Canadian Applied Mechanics Conference (CANCAM '73), 1973

(Paper 196)

FATIGUE CRACK PROPAGATION IN POLYMETHYLMETHACRYLATE; THE EFFECT OF THE MEAN VALUE OF STRESS INTENSITY FACTOR

By S. Arad*, J. C. Radon† and L. E. Culver‡

Polymeric components often contain structural defects which give rise to regions of internal stress concentration. As the engineering application of these materials expands, the need to understand their behaviour under various loading conditions becomes more necessary.

This paper reports the results of part of a programme of research undertaken to study fatigue crack propagation phenomena in thermoplastics.

A fracture mechanics approach is used and the effects of the mean stress intensity factor, K_m , and the range of stress intensity factor, ΔK , on crack propagation phenomena in polymethylmethacrylate are studied.

Based on the experimental data available, a relationship of the following form, between the cyclic crack growth rate $d(2a)/dN$ and the tensile loading levels, has been proposed

$$d(2a)/dN = \beta(K_{\max}^2 - K_{\min}^2)^n$$

where $K_{\max}$ and $K_{\min}$ are the maximum and minimum values of the stress intensity factor in each loading cycle. In tests at room temperature (21°C), in air, at a loading frequency of 5 Hz, n was found to be equal to 2.5; $\log \beta$ equalled -12.37.

INTRODUCTION

POLYMERS are finding an increasing use in engineering applications and if they are to be developed to their fullest extent, a complete understanding of their behaviour under all types of loading is necessary. Previous work (1)-(3) has shown that linear-elastic fracture mechanics concepts may be successfully used to describe the propagation of through-thickness and part-through-thickness cracks under fatigue conditions in polymethylmethacrylate. It has also been suggested that for through-thickness cracks, the crack growth rate could be described as

$$\frac{d(2a)}{dN} = C(\Delta K)^m \quad (1)$$

where $2a$ is the crack length, N is the number of cycles, ΔK is the range of stress intensity factor and C and m

are constants for the material, mean load and frequency used.

This law, originally proposed by Paris (4), is typical of those, based on empirical data, suggested for crack propagation in metals. The fact that crack growth in polymethylmethacrylate was found to follow the same law suggested that it might be a very useful material for model studies of the growth of fatigue cracks in metallic components in which it is often difficult to monitor such growth. However, the law represented by equation (1) does not give a clear indication of the effects of the mean value of the stress intensity factor; the previous work had been conducted using a minimum value of the stress intensity factor K , equal to zero for all tests. Thus, as ΔK was increased, there was always a proportionate increase in the mean value, K_m , whose effect therefore became masked.

For metals, it had previously been suggested (4) that K_m was of minor importance when compared to ΔK but more recently attention has been drawn to the influence of K_m on the rate of fatigue crack propagation. In particular, in tests on round bars of carbon steel (S35C) evidence has shown that the mean stress (proportional to K_m) cannot be discarded as having only secondary influence on crack growth (5). Similar conclusions have

The MS. of this paper was received at the Institution on 27th July 1970 and accepted for publication on 24th November 1970. 33

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been drawn for aluminium alloys of high and low fracture toughness (6). Forman, Kearney and Engle (7), following tests on aluminium alloys, have proposed a fatigue crack propagation model which relates the cyclic growth of the crack to the range of stress intensity factor and the ratio of its minimum and maximum levels.

The purpose of the work assessed in this paper was to study further those variables which control the growth of fatigue cracks in glassy plastics and, in particular, to assess the influence of the mean value of the stress intensity factor, K_m , in relation to the range, ΔK , used. Repeated tension tests have been carried out on sheet specimens of polymethylmethacrylate, $\frac{1}{4}$ in thick, each containing a central through-thickness crack. A concurrent study of the influence of frequency of loading is being continued.

Notation

a	Half crack length.
C	Numerical constant.
f	Frequency.
K	Stress intensity of crack tip stress field.
K_I, K_{Ic}	Crack tip stress intensity factor in mode I fracture. Subscript c refers to critical value.
K_{max}	Maximum
K_{min}	Minimum
K_m	Mean
} levels of the stress intensity factor K .	
m, n	Numerical exponents.
N	Number of loading cycles.
r_p	Radius of plastic zone at the crack tip.
W	Width of specimen.
β	Numerical constant.
ΔK	The range of variation of stress intensity factor.
λ	Parameter dependent on K_m and ΔK .
σ_y	Tensile yield stress of the material.

FRACTURE MECHANICS CONCEPTS

The differences between experimental and theoretical values of tensile strengths have been attributed by Griffith (8) to the presence of inherent flaws which act as stress raisers. Fracture mechanics has thus evolved as a study of flawed components represented by notched specimens. Early work was couched in terms of the energy in the system but Irwin (9) gave stress-function solutions relating the stress field in the vicinity of a crack tip to the applied stress.

For an infinite plate containing a central crack (Fig. 1) the elastic stress (p_{ij}) close to the crack tip may be represented by

$$p_{ij} = \frac{K}{(2\pi r)^{1/2}} f(\theta) \quad (2)$$

where K is the stress intensity factor and r and θ are the polar co-ordinates with origins at the crack tip. Since the stress field is essentially described in terms of K , any fracture criterion must be related to K or to the applied stress p , but experience shows that p , whether based on

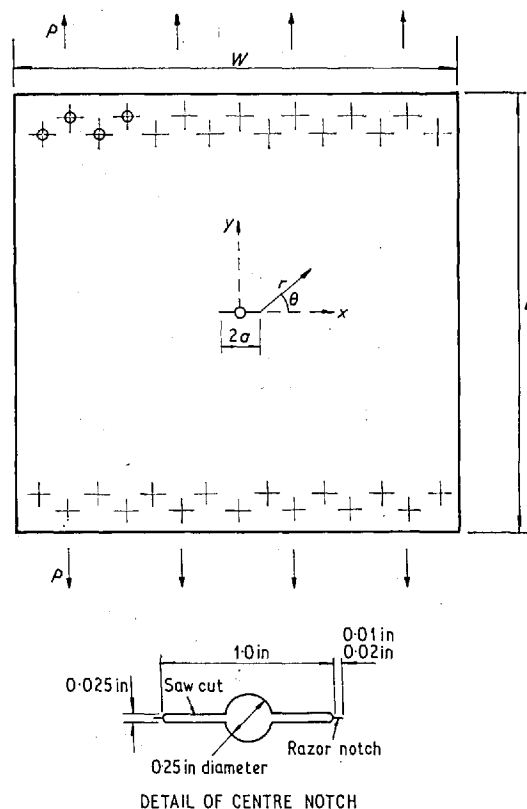


Fig. 1. Specimen geometry

gross or net area, is not constant at fracture. It is therefore postulated that K has a constant, critical, value K_c at fracture.

The value of K needs to be determined for the particular specimen geometry being used. For a centre notched plate of width W (Fig. 1) its value is closely approximated by

$$K = p(\pi a)^{1/2} \left(\frac{W}{\pi a} \tan \frac{\pi a}{W} \right)^{1/2} \quad (3)$$

if the total crack length $2a$ is less than half the plate width (10). This value of K , often designated as K_I , applies to the opening mode of fracture. It refers to the case where the applied load is in the y direction and the crack surface lies in the x - z plane. Only this mode is considered in this paper.

It is a basic requirement that the radius of the plastic zone, r_p , at the tip of the crack should be small compared to the crack length and specimen thickness (11). Its value may be calculated, for plane strain or plane stress conditions, respectively, from

$$r_p = \frac{K_I^2}{6\pi\sigma_y^2} \quad \text{or} \quad r_p = \frac{K_I^2}{2\pi\sigma_y^2} \quad (4)$$

where σ_y is the tensile yield stress of the material under the appropriate testing conditions.

TEST PROGRAMME

The polymethylmethacrylate specimens, $13\frac{1}{2} \times 15 \times \frac{1}{4}$ in thick, were cut from cast sheets and the central notch was produced by drilling a central $\frac{1}{4}$ in diameter hole from which the slot was saw-cut to the required length (Fig. 1). A sharp crack was then formed by forcing a sharp razor blade into the root of the saw cut. In the early stages of the fatigue test, uniformity of the initial notch front has been found to be very important in determining the accuracy of data on the rate of crack growth (2) but the two ends of the central crack will normally start to propagate in a symmetrical manner if sufficient care is taken in producing the initial notch. Those specimens which did not produce symmetric growth were discarded. All specimens were allowed to normalize at a temperature of $21 \pm 1^\circ\text{C}$ and a relative humidity of 50 per cent.

The specimens were assembled in turn in a Dowty electro-hydraulic fatigue testing machine. This machine is capable of cycling from 2 c/h to 6000 c/min at loads ranging from 600 lbf to 12 000 lbf and is fully described in (12). Briefly, a sine wave input, produced by a signal generator, passes through an amplifier and subsequently operates the actuator. The signal from a load cell is monitored by means of a graduated oscilloscope on the control console. There are facilities on the machine to vary the load or strain whilst a test is in progress.

The specimens were bolted to a pair of steel holding plates placed between the load cell and the actuator and were then cycled in tension at a constant frequency of 5 Hz between predetermined limits of stress intensity factor, $K_{\max}$ and $K_{\min}$. These values, being dependent upon crack length (equation (3)) were maintained constant by suitably adjusting the load limits as the crack propagated. The crack growth was monitored by using a cathetometer until the total crack length reached 0.4 times the plate width, when the specimen was pulled directly to fracture.

By carefully selecting the values of $K_{\max}$ and $K_{\min}$ for each test, it was possible to establish crack growth rates over a wide range of values of mean stress intensity factor K_m and the range of stress intensity factor ΔK independently. The highest value of $K_{\max}$ used in the programme was $900 \text{ lbf/in}^{3/2}$. It was found, from preliminary tests, that at even higher values of $K_{\max}$ slow crack growth interfered with the fatigue process and further investigation of this aspect is currently under consideration.

Values of the stress intensity factor at the start of unstable crack growth K_{Ic} were also obtained from simple tension tests on specimens of the same geometry. They varied between 1100 and $1200 \text{ lbf/in}^{3/2}$, thus agreeing with previous established values (2). The plastic zone size calculated for plane strain conditions from equation (4) for K_{Ic} equal to $1200 \text{ lbf/in}^{3/2}$ and using $\sigma_y = 11\,000 \text{ lbf/in}^2$ was $0.640 \times 10^{-3} \text{ in}$. For the highest value of $K_{\max}$ used in these tests, i.e. $900 \text{ lbf/in}^{3/2}$, r_p amounted only to $0.355 \times 10^{-3} \text{ in}$; this value was considered negligible when compared with the specimen thickness of 0.25 in.

RESULTS AND DISCUSSION

A typical set of curves showing the dependence of crack growth on the number of load cycles for four different specimens is given in Fig. 2. A linear relationship exists for all the test conditions used which is in agreement with the previous work (1)–(3) using tests in which $K_{\min}$ was always zero. What is immediately significant, however, is that tests conducted with very similar ΔK ranges (the upper and lower curves of Fig. 2 having $\Delta K = 440 \text{ lbf/in}^{3/2}$ and $450 \text{ lbf/in}^{3/2}$ respectively) have very different crack growth rates (c.g.r.), namely 13.8×10^{-6} and $3.14 \times 10^{-6} \text{ in/cycle}$ as given by the slopes of the graphs. Furthermore, a comparison of the central two curves in Fig. 2 shows that the test in which ΔK was least ($300 \text{ lbf/in}^{3/2}$ compared with $360 \text{ lbf/in}^{3/2}$) produced the higher crack growth rate ($9.05 \times 10^{-6} \text{ in/cycle}$ compared with $6.1 \times 10^{-6} \text{ in/cycle}$). Similar results were obtained in tests conducted at a frequency of 0.1 Hz (see Table 1).

A study of the results indicated that there is no single controlling parameter in these tests at constant speed of cycling. Two further major test series were therefore undertaken to clarify the effects of the range of stress intensity factor ΔK and the mean value, K_m . In the first series (Fig. 3) ΔK was maintained constant and K_m was varied by factors of 3 and 5. Each test produced a linear curve and the crack growth rate increased non-linearly with K_m . The results indicated a fifteen-fold increase in crack growth rate for a five-fold increase in K_m . Similar results were obtained for other constant, but different, ΔK tests in which K_m was similarly varied (Fig. 5).

In the second major series, K_m was maintained constant throughout but ΔK was varied from specimen to

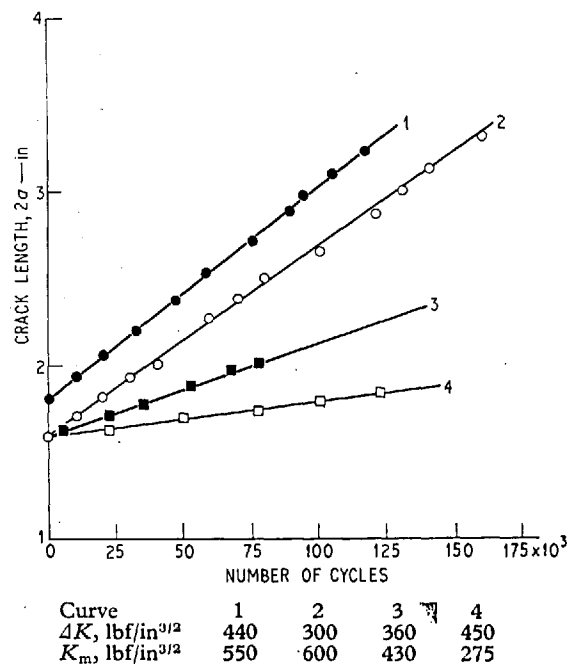


Fig. 2. Crack growth pattern at different values of ΔK and K_m

Table 1

K_{max} , lbf/in ^{3/2}	K_{min} , lbf/in ^{3/2}	ΔK , lbf/in ^{3/2}	K_m , lbf/in ^{3/2}	λ , lbf ² /in ³	Frequency, Hz	c.g.r., 10 ⁻⁶ in/ cycle	Ref. to Figs
770	330	440	550	502 900	5	13.80	2 (1)
750	450	300	600	360 000	5	8.50	(2)
610	250	360	430	312 500	5	6.10	(3)
500	50	450	275	247 500	5	3.14	(4)
750	450	300	600	360 000	0.1	260.0	
610	250	360	430	309 600	0.1	207.0	
250	0	250	125	62 500	5	0.39	3 (1)
500	250	250	375	187 500	5	2.15	(2)
750	500	250	625	312 500	5	6.10	(3)
750	250	500	500	500 000	5	13.60	
500	0	500	250	250 000	5	3.20	
500	300	200	400	160 000	5	1.02	4 (1)
600	200	400	400	320 000	5	7.10	(2)
750	50	700	400	560 000	5	19.70	(3)
900	600	300	750	450 000	5	12.50	
850	650	200	750	300 000	5	5.65	
800	700	100	750	150 000	5	1.42	
500	250	250	375	187 500	5	2.15	
675	75	600	375	450 000	5	12.40	
750	0	750	375	562 500	5	19.60	
560	0	560	280	312 500	5	6.10	
610	250	360	430	309 600	5	6.00	
660	350	310	505	313 600	5	6.10	
750	500	250	625	312 500	5	6.10	
708	0	708	354	500 000	5	13.60	
770	300	470	535	502 000	5	13.80	
750	250	500	500	500 000	5	13.60	
675	75	600	375	450 000	5	12.40	
900	600	300	750	450 000	5	12.50	
750	500	250	625	312 000	0.1	210.0	9 (1)
660	350	310	505	313 100	0.1	213.0	(2)
610	250	360	430	309 600	0.1	207.0	(3)
560	50	510	305	311 100	0.1	210.0	(4)

specimen. The results (Fig. 4) for $K_m = 400$ lbf/in^{3/2} and ΔK variable between 200 and 700 lbf/in^{3/2} were similar to the previous series in that crack growth rate increased rapidly although non-linearly but this time with an increase in ΔK . Similar results were obtained from a further series of tests with $K_m = 750, 500, 375$ and 275 lbf/in^{3/2} and at different ΔK ranges. These results are included in Table 1 and it is clear that both ΔK and K_m are of considerable importance in influencing fatigue crack growth rates in polymethylmethacrylate.

From the results it is possible to construct families of curves. Fig. 5 represents the effects of variations in ΔK on crack growth rates when K_m is maintained constant, and Fig. 6 represents the effects of variations in K_m when ΔK is maintained constant.

In addition to these two parameters, however, it seemed possible that the maximum stress intensity factor reached in any cycle, K_{max} , should have an influence on the rate of fatigue crack growth. A number of additional tests

were therefore performed under constant K_{max} conditions whilst allowing ΔK , and hence K_m , to vary from specimen to specimen. The results have been superimposed on Fig. 6 to form a grid representing the influence of three major variables on the crack growth rate for a given frequency of loading.

Consideration of all the available results led to the possibility of forming another law, more general than the law of equation (1), governing the crack growth rates under the range of variables used. Any such law would need to be similar in general form to equation (1) since that has previously been shown to be satisfactory for the test conditions then used and a possible form for the specified test conditions is

$$\frac{d(2a)}{dN} = \beta(\lambda)^n \quad \dots \quad (5)$$

where β and n are constant and λ is a parameter to be established.

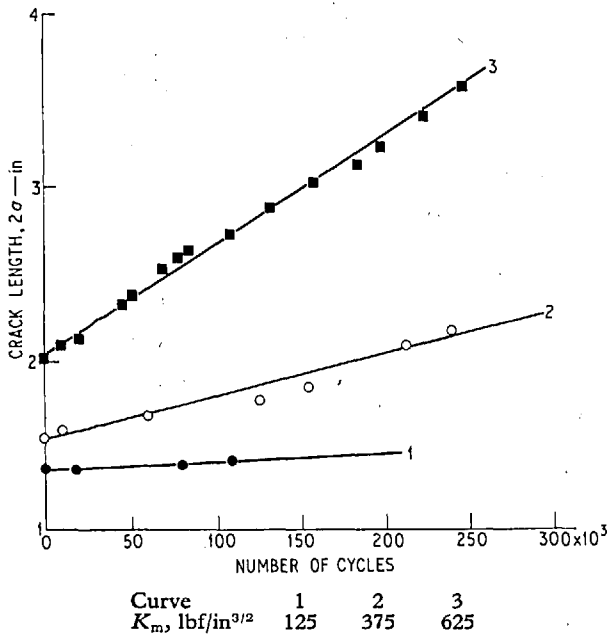


Fig. 3. Variation of crack growth rate with K_m ,
 $\Delta K = \text{constant} = 250 \text{ lbf/in}^{3/2}$

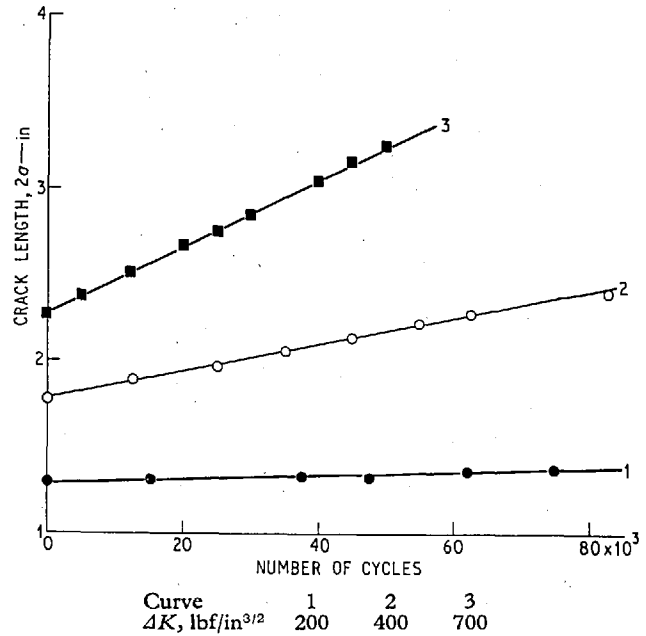


Fig. 4. Variation of crack growth rate with ΔK ,
 $K_m = \text{constant} = 400 \text{ lbf/in}^{3/2}$

The present work suggests that λ is a direct function of both $\Delta K = (K_{\max} - K_{\min})$ and $K_m = \frac{1}{2}(K_{\max} + K_{\min})$ and a reasonable assumption is to take

$$\lambda = f(K_{\max}^2 - K_{\min}^2) \quad (6)$$

which has the virtue of incorporating all the three variables considered.

The value of λ for each of the tests now reported has been calculated and is included in all the results. It is clear that even in tests conducted with different con-

trolling parameters, the crack growth rate is similar between tests, provided the factor λ is also similar, despite the anomalies discussed earlier. A linear relationship would not be expected but within the limits of experimental scatter, there is a unique relationship between the parameter and the crack growth rate. A logarithmic plot of the results (Fig. 7) as suggested by the form of the law now proposed

$$\frac{d(2a)}{dN} = \beta(K_{\max}^2 - K_{\min}^2)^n \quad (7)$$

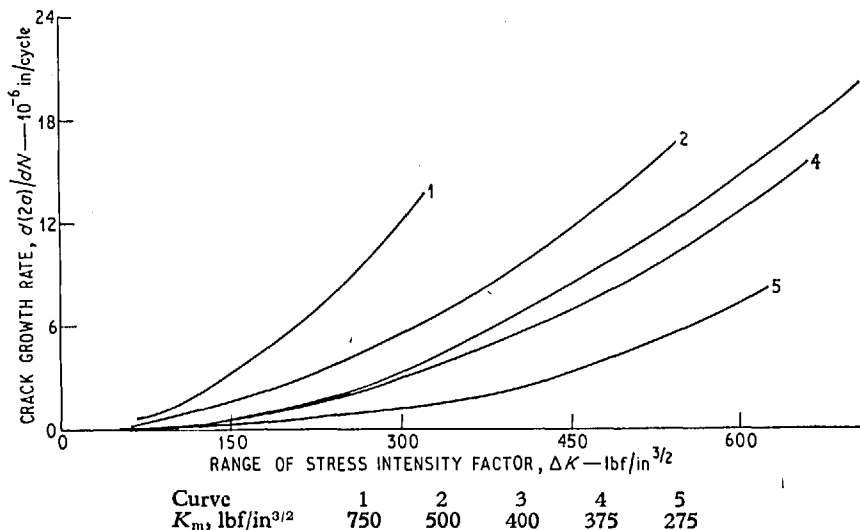


Fig. 5. Variation of crack growth with ΔK at different values of K_m

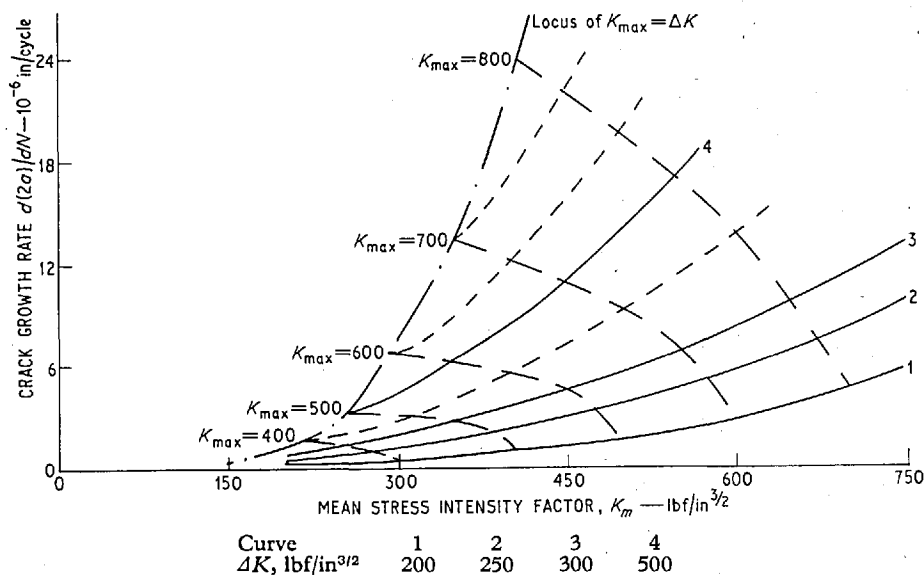


Fig. 6. Variation of crack growth rate with K_m at different values of ΔK

shows a good linear distribution and allows the constants n and $\log_{10} \beta$ to be calculated as 2.5 and -12.37 respectively.

Finally, to illustrate the effectiveness of the parameter λ , further tests were conducted in which ΔK and K_m were allowed to vary but λ was maintained constant throughout. The results are shown in Fig. 8 which confirms λ as the parameter controlling crack growth rate. Similar results were obtained from a few check tests run under similar conditions but at a frequency of 0.1 Hz (Fig. 9).

CONCLUSIONS

The propagation of fatigue cracks in centre-notched sheets of polymethylmethacrylate under cyclic tension conditions in air have been shown, in terms of fracture mechanics concepts, to be dependent upon the range of stress intensity factor ΔK and its mean value, K_m ; but apparent inconsistencies can occur unless care is taken in interpreting the results.

These inconsistencies can be avoided by describing crack growth rate in terms of a parameter $\lambda = (K_{\max}^2 - K_{\min}^2)$. It has been shown that, over the range of values

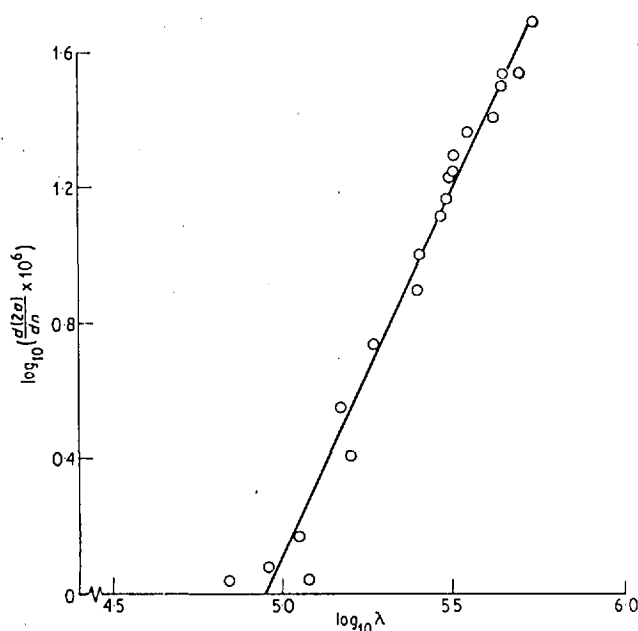


Fig. 7. Variation of crack growth rate with λ

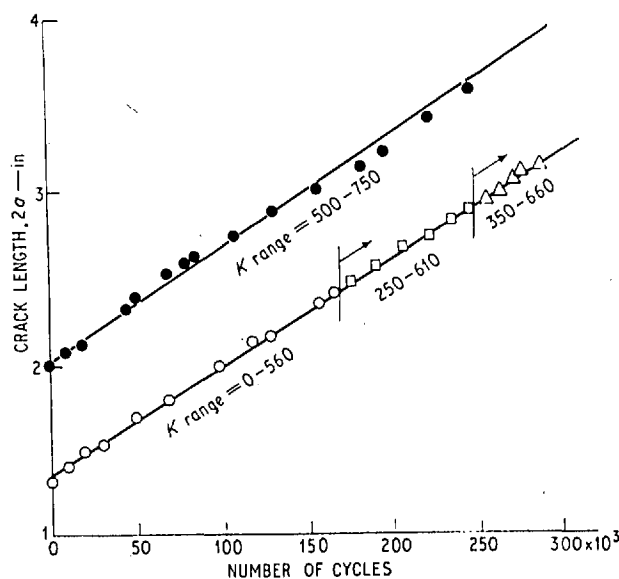


Fig. 8. Dependence of crack growth on λ at various values of K_m and ΔK

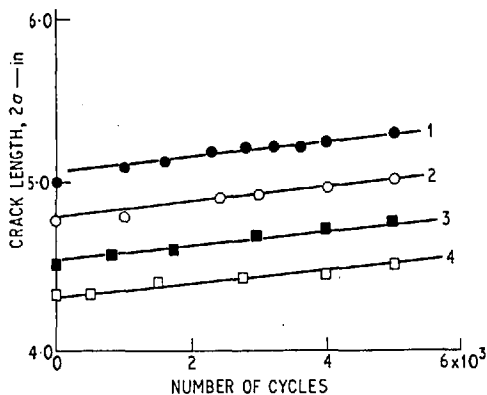


Fig. 9. Dependence of crack growth on λ at a lower frequency (see Table 1)

in the variables ΔK and K_m tested, the crack growth rate $d(2a)/dN$ is constant if λ is constant.

ACKNOWLEDGEMENTS

The investigation, which is continuing, was supported by the Science Research Council and the polymethylmethacrylate was kindly donated by the Plastics Division, Imperial Chemical Industries Ltd.

APPENDIX

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Growth of Fatigue Cracks in Polycarbonate

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The rapid increase in the rate of application of thermoplastics in engineering design problems and the interest in the structural use of these materials have resulted in the requirement of comprehensive information about the behaviour of thermoplastics when subjected to cyclic loading conditions. In addition to the "total fatigue life" data already available for many materials, attempts have been made to analyse the crack initiation and steady crack growth processes and determine the effects of parameters such as mean load, frequency and crack geometry on the rate of crack propagation. The results of an investigation of these aspects of fatigue crack growth in a brittle thermoplastic, polymethylmethacrylate (PMMA), have already been reported. In this paper, the results of a test program devised to study the behaviour, at room temperature and in air, of a polycarbonate, (PC), under similar loading conditions, are presented. Fracture Mechanics concepts have been used to analyse the results. It was found that a relationship of the form $\dot{a}_N = \beta \lambda^n$ already shown to predict the cyclic fatigue crack propagation rate in PMMA, is also applicable to polycarbonate. However, when the effects of frequency and loading rate were studied, it was found that after the magnitude of parameter $\dot{K}$ ($= \Delta K / \text{half the periodic time}$) exceeded $4000 \text{ lbf in.}^{-3/2} \text{ s}^{-1}$, the influence of the mean level of stress intensity factor, K_m , became negligible in comparison to the effect of ΔK .

INTRODUCTION

The fatigue failure process in polymeric solids has, in recent years, attracted considerable attention as there has been a continuous increase in the rate of application of these materials in the design of engineering components subjected to cyclic loading, as well as an increased interest in their structural application in both unreinforced and reinforced forms. A considerable amount of data has been gathered from "total-fatigue-life" studies using plane bending or rotating bending types of tests. A model for the fatigue failure process in polymers related to the temperature rise due to repeated loading, which is a consequence of the relatively high damping capacity and low thermal conductivity of these materials, has been proposed (1). However, there is no reason to assume that the failure due to cyclic loading will be merely temperature dependent. In fact, in many cases, within reasonably wide ranges of variation in the mean load, the alternating load levels and the

frequency, the temperature rise has been shown to be very small (2).

In a study of the behaviour of polymeric materials under fatigue conditions, the influence of a range of parameters needs to be investigated. The most important of these are: range of stress, mean stress, frequency of loading, waveform of the cyclic loading process, crack sharpness, damping characteristics of the material, and its thermal conductivity, as well as the conditions of the environment such as temperature, humidity, and presence of chemically harmful agents.

In the presence of a crack, an energy balance equation for a polymeric body under load, relating the input energy to the elastically stored energy, the surface energy, and the viscoelastically dissipated energy can be formulated (3). The rate of heat build up within the material is directly dependent on the rate of viscoelastic energy loss, W , the magnitude of which is related to the loading characteristics (S_m

and S_a), the frequency (f) and E'' in the following form (4):

$$\dot{W} = \frac{\pi f \epsilon''}{E'} [2 S_m^2 + S_a^2] \quad (1)$$

This equation shows that the manner in which the E'' characteristics of the material as well as the frequency of loading vary, directly influences the $\dot{W}$ term and hence affects the magnitude of the energy available for crack propagation. In dynamic loading studies, basic data relating the parameters in the energy balance equation to strain rate and temperature variations are required. Variations of loss tangent with frequency in constant temperature tests and with temperature in constant frequency tests and variations of the surface energy term in relation to the ductile-brittle transition phenomena are only a few examples of the type of information which will be helpful when an attempt is made to interpret fatigue crack growth data under various loading conditions and to produce a general and comprehensive model for crack growth.

The physical properties of polymeric solids depend upon molecular motions which can occur in these materials in the solid state. Heijboer's work (5) to study the variations in shear modulus and damping (measured by the loss tangent $\tan \delta$) with changes in temperature, showed that the damping maximum of PC, at room temperature occurs at frequencies above 10^4 Hz. At a frequency of 20 Hz (the highest used in tests described in this paper) the loss peak occurred at about -75°C .

The importance of the availability of fundamental data, such as the characteristics of the relaxation spectra of the material under various conditions in the analysis of the fatigue failure process is recognised but complete data for this material have not yet been collected. Some information related to the above mentioned parameters is available from non-deformation (e.g., dielectric) tests (6), as is data from dynamic mechanical behaviour tests conducted at one frequency but at various temperatures (7, 8). However, the effects of frequency variations at room temperature have not been fully studied.

The static fracture of PC has been studied by workers using both the conventional stress analysis techniques and the linear elastic fracture mechanics principles. Linear Elastic Fracture Mechanics (LEFM), in providing a unique design concept that yields a quantitative relationship between applied stress, component geometry, flaw size and material properties, has been successfully applied to the analysis of the fracture and fatigue failure processes in many metals. The special parameter used to characterise the crack tip environment, in relation to the applied load and crack-loading geometric relationship is the stress intensity factor, denoted by K , which in the present work has been calculated as in the Appendix. The application of LEFM principles to

fracture and fatigue failure in a number of thermoplastics has been demonstrated (9-11).

The fracture toughness characteristic of the material and its variations with strain rate and temperature were required in order to estimate the ratio of the maximum applied stress intensity factor K , in fatigue cycling, to the critical value of K under the particular conditions of the test. Experimental data reported in (10), partially cover these requirements. Changes in the fracture toughness of glassy thermoplastics (PMMA, PS, PC) were measured under various temperature and strain rate conditions, using specimens of different thicknesses. The fracture toughness parameters, K_{Ic} , of PMMA and PC, at room temperature, tended to reduce as the strain rate was increased.

The influence of the variation in the value of K_{Ic} on the results of fatigue tests on polycarbonate, at high frequencies (20 Hz) and at lower frequencies when the load amplitude was large, will be shown later. In the case of PMMA, reported previously (4, 12) this variation did not significantly change the outcome of tests, within the range of frequencies used (0.1 to 20 Hz).

Various attempts have been made to study the process of fatigue failure in PC. Jacoby and Cramer (13), described the fatigue mechanism in this material as a process of superposition of crazing, orientation and physical cross-linking and subsequently proposed a model similar to the models of fatigue crack growth put forward for metals in terms of a relationship between the applied stress amplitude and the rate of growth of the crack.

Temperature rises due to load cycling at various levels have been investigated by Higuchi, et al (2, 14). In tension cycling, with the mean stress range extending to above 5000 lbf in.⁻² and alternating stress values above 4000 lbf in.⁻² at a frequency of 1 Hz, the maximum temperature rise obtained was about 0.4°C above room temperature. The rise in temperature was related to viscoelastic strain and was also associated with an increase in the stiffness (14). Measurements reported by the above workers show an initial rise in temperature and then an approximately constant level of temperature until the final fracture point, when there is a sharp rise in the temperature level. Hertzberg, et al (9), have reported on their studies of the applicability to the fatigue failure process in PC, of a power law relationship relating the cyclic rate of growth to the change in the range of stress intensity factor. From the results of tests with $K_{max} = \Delta K$, it was concluded that the following relationship originally used for metals, could be applied to PC:

$$d(2a)/dN = C(\Delta K)^m \quad (2)$$

In previous papers (4, 12) the present authors have discussed the influence of the mean level of the stress intensity factor K , as well as its amplitude, on the rate of growth of fatigue cracks in PMMA. The effect of variations in the loading frequency was also

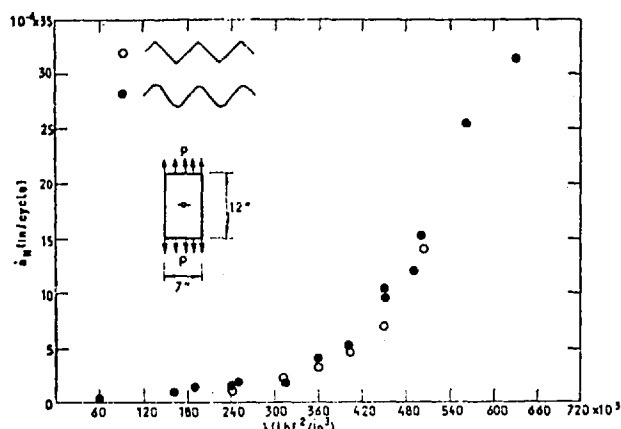


Fig. 1. Results of tests at a frequency of 0.1 Hz, for sinusoidal and triangular input waveforms.

evaluated. Subsequently a model for crack growth rate of the following form was proposed:

$$\dot{a}_N = \beta \lambda^n \quad (3)$$

and n , the numerical exponent, in the case of PMMA was shown to be approximately equal to 2.5 and to vary very slightly with change of frequency.

The aim of the present study has been to investigate the fatigue failure process in another glassy thermoplastic, polycarbonate (PC), which in static fracture behaves differently to PMMA, and to demonstrate whether the more general crack growth model (Eq 3) is applicable to PC.

MATERIALS AND EXPERIMENTAL PROCEDURES

The material chosen was polycarbonate of Bisphenol A (Makrolon, Bayer) in the form of extruded sheets. Plate specimens of dimensions $7 \times 12 \times \frac{1}{4}$ -in. thick, (Fig. 1), were used in all tests. A $\frac{1}{4}$ -in. diam hole was drilled in the centre of the plate and a flaw of 1-in. length was produced parallel to the direction of extrusion, using a miniature hacksaw. The initial sharp notches at the ends of the saw cut were produced using a razor blade.

The testing machines employed in these tests consisted of a Denison T42C2 Hydraulic machine and a Dowty Electrohydraulic system. The characteristics of these machines have been described elsewhere (4, 12). Briefly, the output from a signal generator is amplified and then used to drive an actuator which is connected to the test specimen and a load cell. A sinusoidal waveform has been used for the loading process.

In a similar manner to that described in (12), a series of cyclic tension tests in which the two limits of the stress intensity factor, $K_{\max}$ and $K_{\min}$ were kept constant were performed to study the relationship between the total crack length and the number of cycles. Also, by appropriate choice of K limits, the effects on the crack growth rate of K_m , the mean level, and ΔK , the range of variation of K were investigated.

The stress intensity factor limits were maintained constant throughout each test by adjusting the applied load levels according to the current crack length. Crack length measurements were carried out at regular intervals using a cathetometer. A series of random check tests were performed, and the deviations in the growth rates were found not to exceed 8%.

DISCUSSION OF RESULTS

As in the case of PMMA, the relationship between the total crack length, $2a$, and the total number of cycles, N , in tests with constant K limits, was found to be a linear one. Cyclic rates of crack growth ($\dot{a}_N$), were calculated from the slopes of these graphs. At a frequency of 0.1 Hz, the data obtained from the experiments* shows a marked dependence of $\dot{a}_N$ on K_m as well as on ΔK . The crack growth rate does not vary with a simple dependence on ΔK , and in fact, by appropriate choice of $K_{\max}$ and $K_{\min}$, it is possible to obtain higher crack propagation rates at smaller ΔK values. Test results indicate that with equal ΔK values one may obtain completely different growth rates depending on the choice of K_m . Using this set of results it was shown that at a frequency of 0.1 Hz, the fatigue failure of PC may be predicted from a model similar to that used for PMMA, in Eq 3. This is evident from Fig. 1 where the cyclic rates of growth have been plotted against the parameter λ , ($= K_{\max}^2 - K_{\min}^2$). On a log-log scale, as will be shown later, this graph is a straight line, the slope of which gives the value of the exponent n in Eq 3.

To investigate the effect of the input waveform on the growth pattern, an effect which would be more predominant in the slow cycling tests, a number of tests were performed with a triangular input waveform. The results are included in Fig. 1, so that a comparison can be made with the sinusoidal input waveform. It is evident that the variations in results from the two series of tests are generally very small.

Applicability of Eq 3 for the prediction of crack growth rates at various frequencies was shown (4) for the case of PMMA. To study the behavior of PC, a similar series of tests was performed at frequencies of 5 and 20 Hz. An interesting point arose from consideration of the data obtained at a frequency of 5 Hz: that the direct dependence of $\dot{a}_N$ on λ , observed previously, does not exist for the whole set of data. Closer examination indicated a point of transition in the relative effects of ΔK and K_m . It was found that beyond a certain level of $\dot{K}$ (obtained by dividing ΔK in a test by half the periodic time) the effect of K_m became very small and the influence of ΔK became the predominant factor. The transition value of $\dot{K}$, $\dot{K}^*$, was found to be in the region of $4000 \text{ lbf in.}^{-3/2} \text{ S}^{-1}$. Figure 2 shows the results of all tests at 5 Hz where $\dot{K}$ is below $4000 \text{ lbf in.}^{-3/2} \text{ S}^{-1}$, and there the applicability of parameter λ is clearly demonstrated in a log-log plot. It is noted that the maximum

* The numerical data presented in the graphs will be supplied to readers on request.

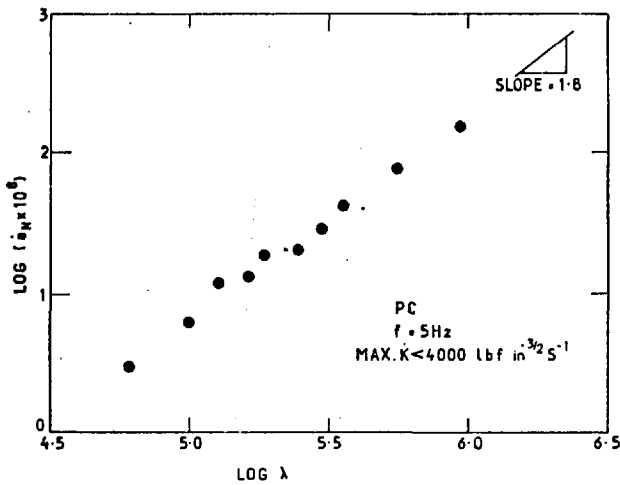


Fig. 2. Results of tests at a frequency of 5 Hz with maximum $\dot{K} = 4000 \text{ lbf in.}^{-3/2} \text{ S}^{-1}$.

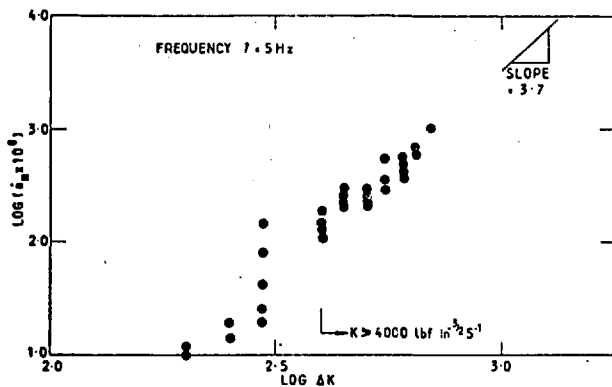


Fig. 3. Results of tests at a frequency of 5 Hz showing the comparatively small effects of K_m , when $\dot{K}$ exceeds $4000 \text{ lbf in.}^{-3/2} \text{ S}^{-1}$.

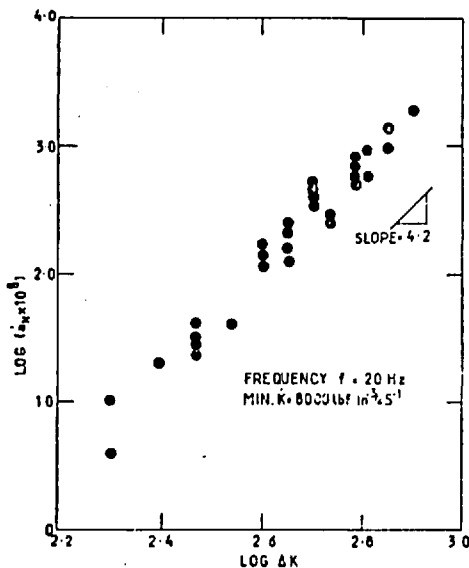


Fig. 4. Results of tests at a frequency of 20 Hz. Minimum $\dot{K} = 8000 \text{ lbf in.}^{-3/2} \text{ S}^{-1}$.

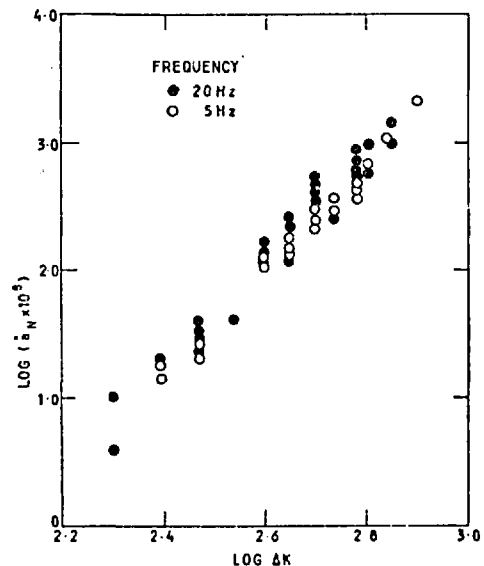


Fig. 5. The effect of change in frequency on $\dot{a}_N$: 20 Hz $\bullet$; 5 Hz $\circ$.

$\dot{K}$ achieved in the 0.1 Hz tests was $140 \text{ lbf in.}^{-3/2} \text{ S}^{-1}$. Where the value of $\dot{K}$ exceeded the $\dot{K}^*$ value, it was sufficient to use only the changes in ΔK , to predict the value of $\dot{a}_N$. From Fig. 3 it is evident that when $\dot{K}$ is smaller than $4000 \text{ lbf in.}^{-3/2} \text{ S}^{-1}$ the influence of K_m on $\dot{a}_N$ is significant, as exemplified by the results of tests at $\Delta K = 300 \text{ lbf in.}^{-3/2}$. However, for $\dot{K}$ larger than $\dot{K}^*$, as can be noted from the results of tests at $\Delta K = 500 \text{ lbf in.}^{-3/2}$, the effect of variation in K_m is very small.

The stress intensity factor rate effects were confirmed when tests were carried out at a frequency of 20 Hz. The minimum K value applied in this series of tests was $8000 \text{ lbf in.}^{-3/2} \text{ S}^{-1}$. Thus if the observations from Fig. 3 were correct, it would be expected that the K_m effects would be of secondary importance when compared to the influence of ΔK . Figure 4, a logarithmic plot of $\dot{a}_N$ against ΔK , confirms the previous observations.

From the above results it was concluded that the fatigue failure characteristics of polycarbonate cannot be simply determined from a consideration of the applied K levels and an appreciation of the loading frequency effects, as was the case in PMMA (4). In the case of PC, the empirical data point to a much higher rate-dependency effect. When the problem is approached using the $\dot{K}$ parameter, the frequency effect takes a different form, according to the testing region in which one is interested. For all the tests with $\dot{K}$ greater than $\dot{K}^*$, the influence of frequency variation on $\dot{a}_N$ is as shown in Fig. 5. These results indicate a small drop in the value of $\dot{a}_N$ as the frequency is reduced, Fig. 6. Similar results were reported in (9, 13). The increase in material resistance to the cyclic rate of growth at lower frequencies was attributed by Hertzberg, et al (9), to the particular

damping characteristics of this material. It must be noted that the effect of frequency variation in the above case is opposite to that in polymethylmethacrylate. However, when the results from tests with $\dot{K}$ smaller than $\dot{K}^*$ (where K_m influenced $\dot{a}_N$ significantly) are considered, the effect of frequency variation was similar to that in PMMA. In Fig. 6 these trends in the frequency effect are shown. With the ΔK values chosen in these tests, the $\dot{K}^*$ values occurred at a frequency of 5 Hz and thus this value of f corresponds to the transition frequency in Fig. 6. In order to analyse the effect of frequency on the value of the exponent n in Eq 3, the results of a series of tests at a constant value of K_m were plotted in Fig. 7. The value of n increases from 3.35 to 4.0 as the loading rates are increased. It has already been mentioned that the K_{Ic} value for polycarbonate drops significantly at high loading rates (10), and thus it is expected that under similar applied K_{max} conditions, at higher loading rates, a higher crack growth rate will be achieved as the ratio of K_{max} to the current fracture toughness of the material will be comparatively higher. Observations of a similar nature on the effect of fracture toughness values on the fatigue crack growth in metals were reported in (15).

The results of tests on PMMA, PC (and on Nylon 6.6 on which some preliminary tests have been performed) indicate that the parameter K^2 ought to be given the dominating part in any mathematical formulation of the fatigue crack growth process. It is significant to note that the crack opening displacement (16), the plastic zone size at the crack tip, given by

$$r_p = \frac{1}{4\sqrt{2\pi}} \cdot \left(\frac{K_{Ic}}{S_{yp}} \right)^2 \quad (4)$$

and also the term denoting the elastic energy release rate (17), are proportional to K^2 . Considering the plastic deformation process at the crack tip, as a consequence of which the fatigue crack propagates, in a loading cycle with minimum load = 0 the load is completely removed on the return half and thus the whole of the plastically deformed zone undergoes a reversed loading; however, when only a proportion of the maximum load is removed (minimum load $\neq 0$) then: considering that there will be r_{pmax} and r_{pmin} values corresponding to K_{max} and K_{min} , at the end of the return half cycle only a part of the r_{pmax} region, from the crack tip, will have undergone the reversed loading. It is hypothesised that the growth will be proportional to the actual dimensions of the zone of double deformation. When $K_{min} = 0$, then $r_{pmin} = 0$, and this is the case for the applicability of Eq 2 since $(\Delta K)^m$, for $\Delta K = K_{max}$, can be treated as $(K_{max}^2)^\xi$ where $\xi = m/2$.

The plastic zone size consideration is also useful when loading rate effects are being investigated. PC has a decreasing fracture toughness at high loading rates. The fracture surfaces of this material at very high rates are increasingly more smooth. It is as-

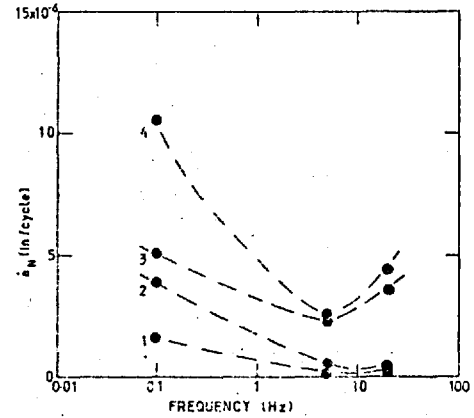


Fig. 6. Variation in the influence of frequency changes above and below the $\dot{K}^*$ level.

Curve	$K_{max} - K_{min} (lb\ f\ in^{-3/2})$	$\Delta K (lb\ f\ in^{-3/2})$
1	550 - 250	300
2	750 - 450	300
3	650 - 150	500
4	700 - 200	500

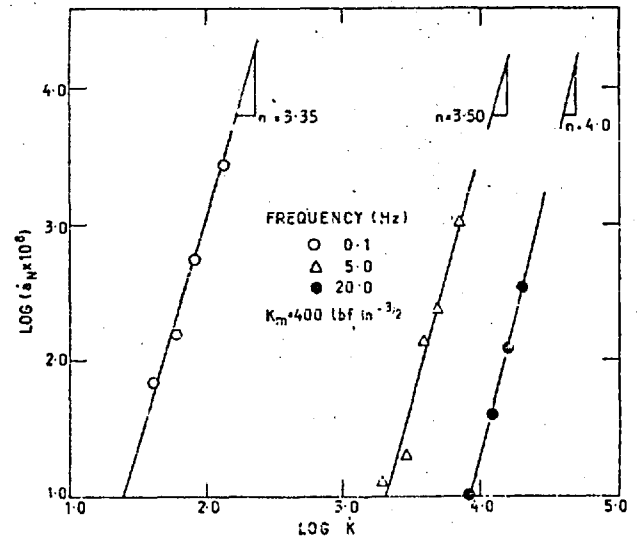


Fig. 7. The effect of change in frequency on the value of index n . Frequency (Hz): 0.1 $\circ$; 5.0 Δ ; 20.0 $\bullet$; $K_m = 400\ lb\ f\ in^{-3/2}$.

sumed that at high loading rates, there is not sufficient time for the plastic zones to form in the manner described above and a different mechanism dominates the fatigue crack growth progress.

A minimum time and strain limit does exist in the crazing process of polymers (18). Data obtained by Higuchi, et al (14), show that for any given strain, at higher loading rates, a longer time is required for initiation of crazing. Thus the enlargement of the crack by crazing at the tip is strain rate dependent. Under the same strain, smaller crazes are produced at higher rates. This phenomenon can easily be related to the observed influence of the loading rate on $\dot{a}_N$. Load rate dependency is an important aspect of

crazing process in polymers, not yet fully documented, which will be immediately useful in the analysis of fatigue crack growth phenomena in these materials.

CONCLUSIONS

The growth of cracks in polycarbonate specimens due to cyclic loading in air and at room temperature have been studied.

It was found that the fatigue failure process can be described in terms of the application of LEFM concepts in the form of an equation:

$$\dot{a}_N = \beta \lambda^n$$

which includes the effect of mean stress intensity factor, as well as the range of its variation.

However, when the effects of loading frequency were investigated, it was found that a transition point in the value of the $\dot{K}$ term existed above and below which the mean stress and frequency effects were different. Above the value of $\dot{K} = 4000 \text{ lbf in.}^{-3/2} \text{ S}^{-1}$ the K_m effect was small and an increase in frequency led to larger cyclic crack growth rates. However, below this value of $\dot{K}$, K_m had a significant effect on $\dot{a}_N$ under the same applied $K_{\max}$ and $K_{\min}$.

Data obtained from tests at a frequency of 0.1 Hz with sinusoidal and triangular waveforms of the same amplitude varied only by a small amount.

NOMENCLATURE

$2a$	=	total crack length
$\dot{a}_N$	=	cyclic rate of crack growth = $d(2a)/dN$
C	=	a constant
E'	=	storage component of complex modulus
E''	=	loss component of complex modulus
f	=	frequency
h	=	specimen thickness
K_I	=	stress intensity factor in mode one opening of the crack
K_{IC}	=	critical value of K_I
$K_{\max}$	=	maximum
K_m	=	mean
$K_{\min}$	=	minimum
} levels of the stress intensity factor K		
ΔK	=	$(K_{\max} - K_{\min})$
$\dot{K}$	=	$\Delta K / \frac{1}{2} T$
m, n	=	numerical exponents
N	=	number of loading cycles
P	=	applied load
r_p	=	radius of plastic zone at the crack tip
S_a	=	amplitude of the applied stress
S_m	=	mean value of the applied stress
S_{yp}	=	yield stress
T	=	periodic time
w	=	specimen width

$\dot{W}$	=	the rate of viscoelastic energy loss
β	=	numerical constant
δ	=	$\tan^{-1} \left(\frac{E''}{E'} \right)$
ξ	=	numerical exponent
λ	=	$(K_{\max}^2 - K_{\min}^2)$

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APPENDIX

The stress intensity factor, K , corresponding to the "opening mode" for a plate specimen and allowing for finite width effects, were calculated by Brown and Srawley (19) using the following equation:

$$K_I = \frac{PY}{wh} \cdot \sqrt{a}$$

where,

$$Y = 1.77 + 0.227 Z - 0.510 Z^2 + 2.7 Z^3$$

and

$$Z = \frac{2a}{w}$$

FATIGUE CRACK PROPAGATION IN POLYMETHYLMETHACRYLATE; THE EFFECT OF LOADING FREQUENCY

S. Arad* J. C. Radon† L. E. Culver‡

The influence of loading frequency on the cyclic rate of crack propagation $\dot{a}_N$, in a glassy thermoplastic (polymethylmethacrylate, p.m.m.a.) in air, at a temperature of $21^\circ \pm 1$ degC and relative humidity of approximately 50 per cent, has been investigated and linear-elastic fracture mechanics concepts have been used to analyse the results. Tests have been conducted on sheet specimens, fatigue loaded in tension, under conditions of constant maximum and minimum values of stress intensity factor, K , and at three frequency levels, namely 0.1, 5 and 20 Hz. It was found that, under these conditions, an increase in frequency led to a decrease in $\dot{a}_N$. Also, the results showed that when K_{max} achieved levels equal to or greater than 750 lbf/in^{3/2}, the contribution of 'dynamic creep' type damage to the crack growth process became more significant. The lower the frequency, the greater was the effect of K_m , the mean level of K , on $\dot{a}_N$. It was also found that for the material under investigation the crack propagation power law index, n in equation $\dot{a}_N = \beta(\lambda)^n$, where $\lambda = K_{max}^2 - K_{min}^2$, had an average value approximately equal to 2.25 and varied only by a small amount with frequency changes.

1 INTRODUCTION

FATIGUE FAILURE in polymeric materials may occur as the result of the following loading processes.

(1) A process which produces negligible internal heat build-up in the material and results in component failure due to pure fatigue growth which may, in some cases, be accompanied by some degree of creep damage.

(2) A process which causes a considerable temperature rise inside the material (due mainly to the relatively high damping capacity and low heat conductivity of polymeric substances) and as a result 'thermal failure' takes place.

The present paper is concerned with the former failure mechanism.

The empirical data available on the effect of variation of loading frequency on the rate of fatigue crack growth in some metals, point to the very small influence of changes in frequency within reasonably wide testing ranges (1)§. However, in the case of all rate-sensitive materials, such as polymers, changes in frequency of loading may have a considerable effect on the fatigue behaviour of the material.

Very little data is available concerning the manner in which a change in frequency affects the cyclic rate of fatigue crack propagation, $\dot{a}_N$, in thermoplastics. Some results have been published which indicate a simple dependence of viscoelastic energy absorption on frequency (2) (3). However, when stress cycles of the following form are applied

$$S = S_m + S_a \sin \omega t \quad . \quad . \quad . \quad (1)$$

then, for a unit volume, the viscous energy W_v absorbed per cycle of load application will be

The MS. of this paper was received at the Institution on 13th February 1972 and accepted for publication on 3rd March 1972. 43
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§ References are given in the Appendix.

$$W_v = \int_0^{t^*} S'' e'' dt \quad . \quad . \quad . \quad (2)$$

where $t^* = 2\pi/\omega = \frac{1}{f}$

S'' and e'' are quadrature components of S and e

$$S'' = E' e'; \quad e'' = \frac{\omega \sigma'}{E'} = \omega e'$$

Hence

$$\begin{aligned} W_v &= \frac{1}{\mu} \int_0^{2\pi/\omega} S^2 dt, \quad \text{where } \mu = \frac{E'^2 + E''^2}{\omega E''} \\ &= \frac{1}{\mu} \int_0^{2\pi/\omega} (S_m + S_a \sin \omega t)^2 dt \\ &= \frac{1}{\mu} \left[S_m^2 \left(\frac{2\pi}{\omega} \right) + \frac{S_a^2}{2} \left(\frac{2\pi}{\omega} \right) \right] \end{aligned}$$

i.e.,

$$W_v = \frac{\pi}{\mu \omega} (2S_m^2 + S_a^2) \quad . \quad . \quad . \quad (3)$$

E'' , the loss modulus = $\phi(f, T)$, we have

$$W_v = \frac{\pi E''}{E'^2 + E''^2} (2S_m^2 + S_a^2) \quad \text{per cycle} \quad . \quad (4)$$

The loss modulus E'' increases as a function of frequency up to a peak value and then decreases, the position of the peak being a function of temperature (4). The frequency level at which E'' reaches a peak is termed the resonance frequency. The increase or the decrease in the level of the cyclic absorbed energy as a function of a rise in frequency will depend on whether the test frequency range is selected to be below or above the resonance frequency at test temperature.

1.1 Notation

a Half crack length.
 $\dot{a}_N$ Cyclic rate of crack growth.
 $\dot{a}_t$ Crack propagation rate with respect to time
(= $\dot{a}_N f$).

E'	'Storage' component of the complex modulus.
E''	'Loss' component of the complex modulus.
$\dot{\epsilon}$	Strain rate.
f	Frequency.
h	Specimen thickness.
K	Stress intensity factor (s.i.f.) of the elastic field in the vicinity of the crack tip.
K_1	S.i.f. referring to the opening mode of crack extension.
K_{1C}	Critical value of K_1 .
K_m	The mean value of K_1 .
$K_{\max}$	The maximum value of K_1 .
$K_{\min}$	The minimum value of K_1 .
m, n	Power law indices.
P	Applied load.
r_p	Radius of the crack tip plastic zone.
S	Applied stress level.
S_a	The amplitude of the stress cycle.
S_m	The mean stress applied.
$S_{\max}$	The maximum value reached in the stress cycle.
S_{yp}	Yield stress of the material.
T	Temperature.
t	Time.
t^*	Periodic time.
W_v	Viscous energy absorbed.
w	Width of specimen.
β	Power law parameter dependent upon loading frequency, material properties and environmental conditions.
ΔK	$= K_{\max} - K_{\min}$.
λ	A parameter $(= K_{\max}^2 - K_{\min}^2)$.
ω	Circular frequency $= 2\pi f$.

2 EQUIPMENT AND TEST PROCEDURE

The material chosen for the tests was p.m.m.a. as it has previously been shown to be a convenient material for crack propagation studies using linear-elastic fracture mechanics concepts (5) (6). The specimens were in the form of rectangular plates ($13\frac{1}{2}$ in $\times$ 15 in $\times$ $\frac{1}{4}$ in thick) containing a central slit 1 in long. Sharp notches at the ends of the central slit were produced using a razor blade. These specimens and the notching procedure have been fully described elsewhere (5) (7).

Cyclic tension tests conducted at frequencies of 5 and 20 Hz were performed in a 'Dowty' electrohydraulic fatigue testing machine (7), whilst some of those at 0.1 Hz were in the 'Dowty' machine and others in a suitably adapted 'Denison T42C2 Hydraulic Universal Testing Machine'. The possibility of differences in the results arising purely from the use of two different testing machines was checked and found to be negligible. Also, the repeatability of the results was examined and the discrepancies found never exceeded 6 per cent.

In each test the maximum and minimum levels of crack tip stress intensity factor, $K_{\max}$ and $K_{\min}$, were maintained at specific values by adjusting the applied load limits as the crack propagated and the growth of the crack was monitored using a cathetometer.

3 APPLICATION OF LINEAR FRACTURE MECHANICS CONCEPTS

Values of the stress intensity factor, K , corresponding to the 'opening mode' for a plate specimen and allowing for

finite width effects, were calculated using the following equation from (8):

$$K_1 = SY\sqrt{a} \quad (5)$$

where

$$S = \frac{P}{wh} \quad (6)$$

$$Y = 1.77 + 0.227Z - 0.510Z^2 + 2.7Z^3 \quad (7)$$

and

$$Z = \frac{2a}{w}$$

At the crack tip, the size of the plastic zone was calculated from

$$r_p = \frac{1}{4\sqrt{2\pi}} \left[\frac{K_1}{S_{yp}} \right]^2 \quad (8)$$

At the highest K_1 level applied (850 lbf/in^{3/2}) and taking $S_{yp} = 10\,000$ lbf/in², the value of r_p is equal to 4×10^{-4} in. This value of r_p is very small in comparison with the plate thickness and enables plane-strain conditions to be assumed throughout the tests.

Values of $K_{\max}$ in these series of tests were chosen as fractions of some critical level of stress intensity factor K_{1C} (here assumed to be equal to 1200 lbf/in^{3/2} (5)), and never exceeded the level of $0.8K_{1C}$. However, in rate sensitive materials, K_{1C} itself is dependent upon the rate of loading and hence on frequency, and thus when equation (9) in Section 4 is being applied to a range of frequencies, an average value for K_{1C} is used.

4 CYCLIC RATE OF CRACK GROWTH

As a result of an earlier series of tests to investigate the effect of mean stress on the rate of fatigue crack growth (7), it was found that the following expression relates the cyclic rate of crack propagation and the stressing conditions

$$\dot{a}_N = \beta(\lambda)^n \quad (9)$$

where $\lambda = (K_{\max}^2 - K_{\min}^2)$, n is a numerical parameter and β is a function of loading frequency, material properties and environmental conditions.

5 DISCUSSION OF RESULTS

At a particular value of the parameter λ (here specified by certain ΔK and K_m levels), and at a constant frequency, the relationship between the total crack length, $2a$, and the total number of cycles of the applied load was found to be a linear one. Fig. 1 shows the results of tests with $K_{\max} = 750$ lbf/in^{3/2} and $K_{\min} = 250$ lbf/in^{3/2} (i.e., $\Delta K = 500$ lbf/in^{3/2}, $K_m = 500$ lbf/in^{3/2}) at frequencies of 20, 5 and 0.1 Hz. This linear behaviour has previously been established for glassy thermoplastics tested at constant frequency (6) (9), and also for metals (1). In the present tests, decreasing the frequency from 20 Hz to 5 Hz and then to 0.1 Hz (under the same $K_{\max}$ and $K_{\min}$ conditions) resulted in an increase in the magnitude of the cyclic growth rate, $\dot{a}_N$ (see Fig. 1 and Table 1). However, when the results are arranged in terms of the crack growth rate with respect to time, $\dot{a}_t$, the order of the slopes changes (Fig. 2). Here, the slopes for 20, 5 and 0.1 Hz are 1.0, 0.7 and 0.42×10^{-4} in/s, respectively. Thus, as expected, faster load cycling of p.m.m.a. leads to a shorter total time life, if all other parameters are maintained constant.

The relationship between $\dot{a}_N$ and frequency is shown in Fig. 3, where the results for a range of values of λ

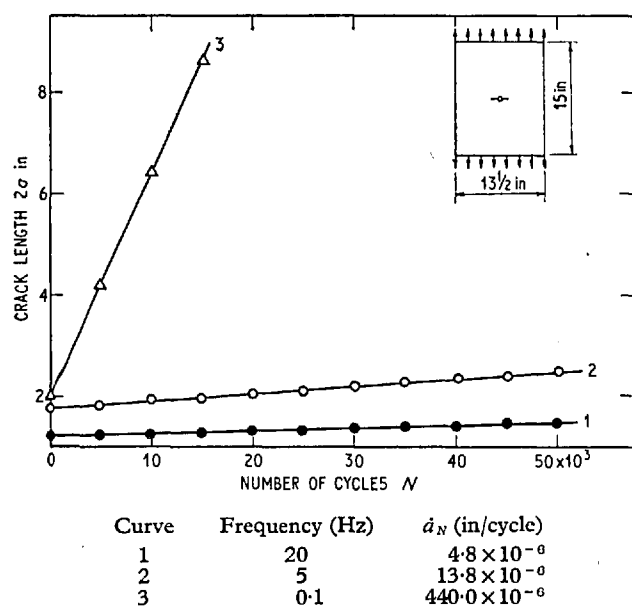


Fig. 1. Variation of $\dot{a}_N$ with frequency in tests under $K_{\max} = 750$, $K_{\min} = 250$ lbf/in^{3/2} conditions

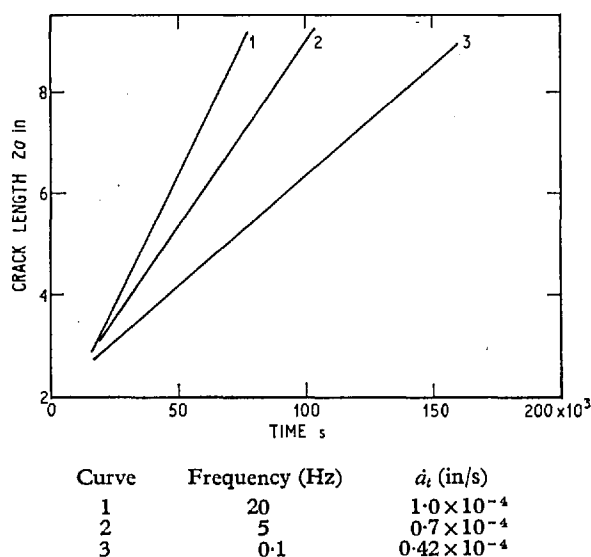


Fig. 2. Variation of crack length with time at different frequencies

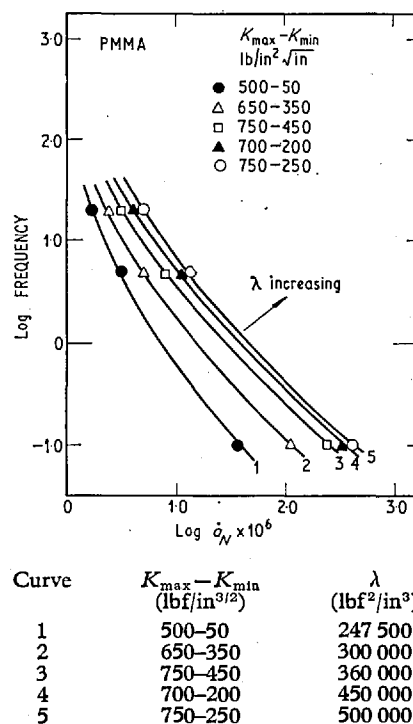


Fig. 3. Variation of $\dot{a}_N$ with frequency for a range of values for λ

(corresponding to various combinations of ΔK and K_m) are presented. The reduction in the cyclic rate of crack growth can be attributed to (1) the possibility of increase in visco-elastic energy absorbed per cycle as the frequency is increased and (2) the increase in the effective yield strength of the material with increasing strain rate which reduces the size of the plastic zone at the crack tip, approximately calculated from equation (8). This is similar to the strain rate sensitivity of the craze growth pattern in p.m.m.a. reported in (10). It must also be noted that, in faster cycling tests, the material is under high stress intensity factor levels for a shorter period per cycle and this might also tend to reduce the size of the plastically damaged zone at the crack tip, in comparison to the case of slow cycling.

Fig. 3 shows that the rate of variation of $\dot{a}_N$ with changing frequency is a function of the stress intensity factor limits, namely,

$$d(\dot{a}_N)/df = \phi(K_m, \Delta K) \dots (10)$$

Table 1

No.	$K_{\max}$	$K_{\min}$	K_m	ΔK	$\lambda = K_{\max}^2 - K_{\min}^2$ (lbf ² /in ³)	$\dot{a}_N$ (in/cycle) at frequency (Hz)		
						0.1	5	20
1	600	550	575	50	57 500	0.5×10^{-6}	0.46×10^{-6}	0.025×10^{-6}
2	500	300	400	200	160 000	1.7×10^{-6}	1.02×10^{-6}	0.05×10^{-6}
3	500	200	350	300	210 000	5.0×10^{-6}	2.50×10^{-6}	0.70×10^{-6}
4	550	250	400	300	240 000	20.0×10^{-6}	—	2.10×10^{-6}
5	500	50	275	450	247 500	40.0×10^{-6}	3.14×10^{-6}	1.25×10^{-6}
6	650	350	500	300	300 000	122.0×10^{-6}	5.10×10^{-6}	2.50×10^{-6}
7	750	500	625	250	312 500	210.0×10^{-6}	6.10×10^{-6}	2.80×10^{-6}
8	750	450	600	300	360 000	260.0×10^{-6}	8.5×10^{-6}	3.0×10^{-6}
9	700	200	450	500	450 000	380.0×10^{-6}	12.5×10^{-6}	4.5×10^{-6}
10	750	250	500	500	500 000	440.0×10^{-6}	13.8×10^{-6}	4.8×10^{-6}
11	750	50	400	700	560 000	480.0×10^{-6}	19.7×10^{-6}	5.3×10^{-6}
12	800	100	450	700	630 000	560.0×10^{-6}	100.0×10^{-6}	91.0×10^{-6}
13	850	50	450	800	720 000	600.0×10^{-6}	180.0×10^{-6}	130.0×10^{-6}

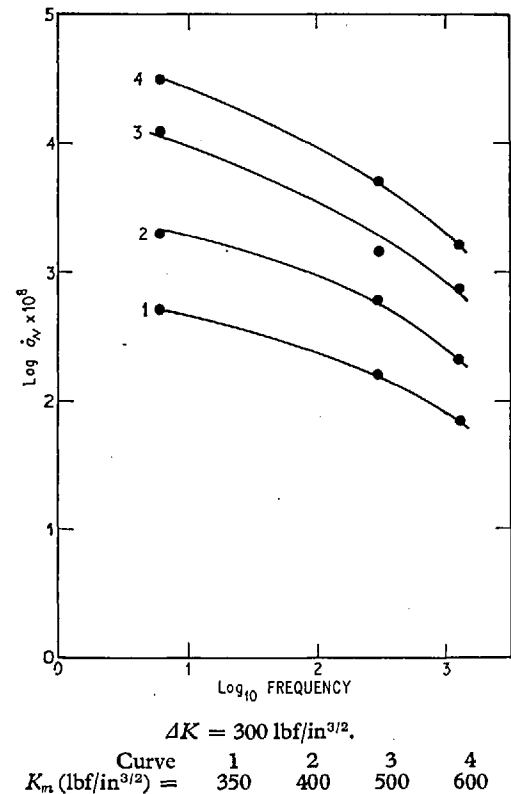


Fig. 4. Influence of K_m on $\dot{a}_N$ at different frequencies

The effect of variation in λ on $\dot{a}_N$ which increases as λ increases, is more pronounced the slower the cycling conditions. It is suggested that in tests at 0.1 Hz, crack growth is relatively more influenced by 'dynamic' creep damage.

In Fig. 4, logarithmic plots of $\dot{a}_N$ and frequency for a range of values of K_m at constant ΔK are presented to demonstrate the important effect of K_m on the rate of change of $\dot{a}_N$ with respect to frequency. The graphs show that K_m has a relatively higher effect at lower frequencies.

In addition, as is evident from Figs 5 and 6, the parameter λ conveniently relates the loading limits and cyclic

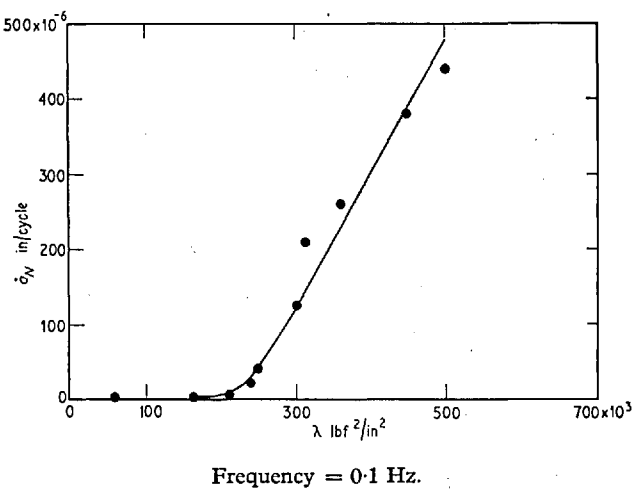


Fig. 6. Relationship between $\dot{a}_N$ and λ at a frequency of 0.1 Hz

crack propagation rate $\dot{a}_N$ for each frequency. This simply confirms the suggestions made in (7) in relation to the validity of equation (9) in predicting the crack growth rate, under various cyclic load limits and testing frequencies, in this material. However, the results in Table 1, for tests in which very high K_{max} levels were applied, show that when K_{max} is equal to or greater than 750 lbf/in^{3/2}, values of $\dot{a}_N$ obtained are much greater than that which would be expected from the relative magnitude of λ . This is especially evident from the results of the last three series of tests in Table 1. Such an observation is in accordance with the conclusion in (5) of the results of a series of cyclic fatigue and static creep tests, where, for values of K_{max} greater than 750 lbf/in^{3/2}, 'cyclic' creep had a pronounced effect on the crack growth rate. Results of a concurrent programme of research reported in (11), have also pointed to the importance of the effect of variations in K_m and frequency on the cyclic rate of crack growth in p.m.m.a. Also, the results of tests on low toughness Al alloys and a medium strength tough steel reported in (12), showed the

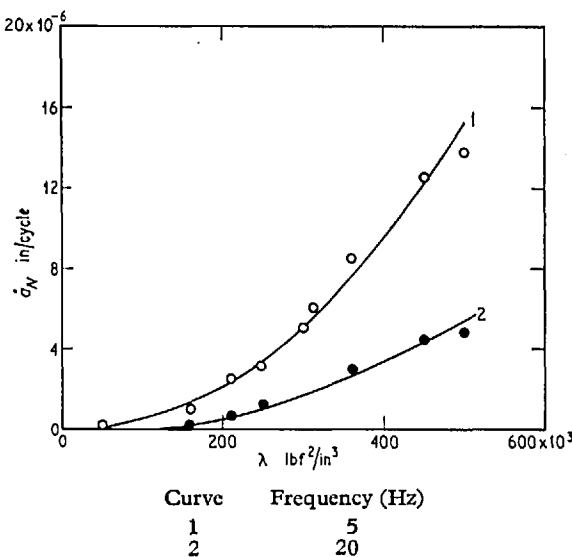


Fig. 5. Relationship between $\dot{a}_N$ and λ at frequencies of 5 and 20 Hz

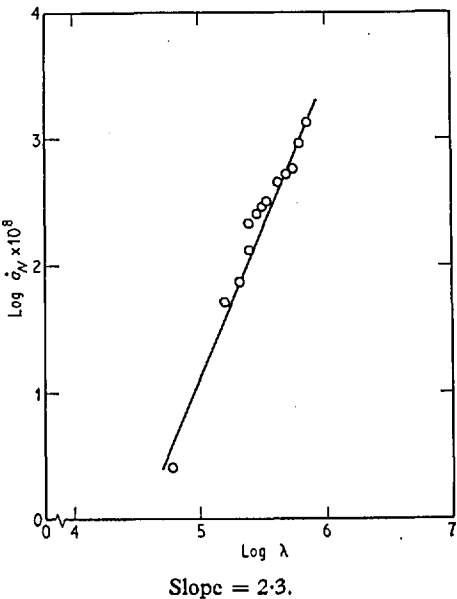


Fig. 7. Variation of $\dot{a}_N$ with λ at a frequency of 20 Hz

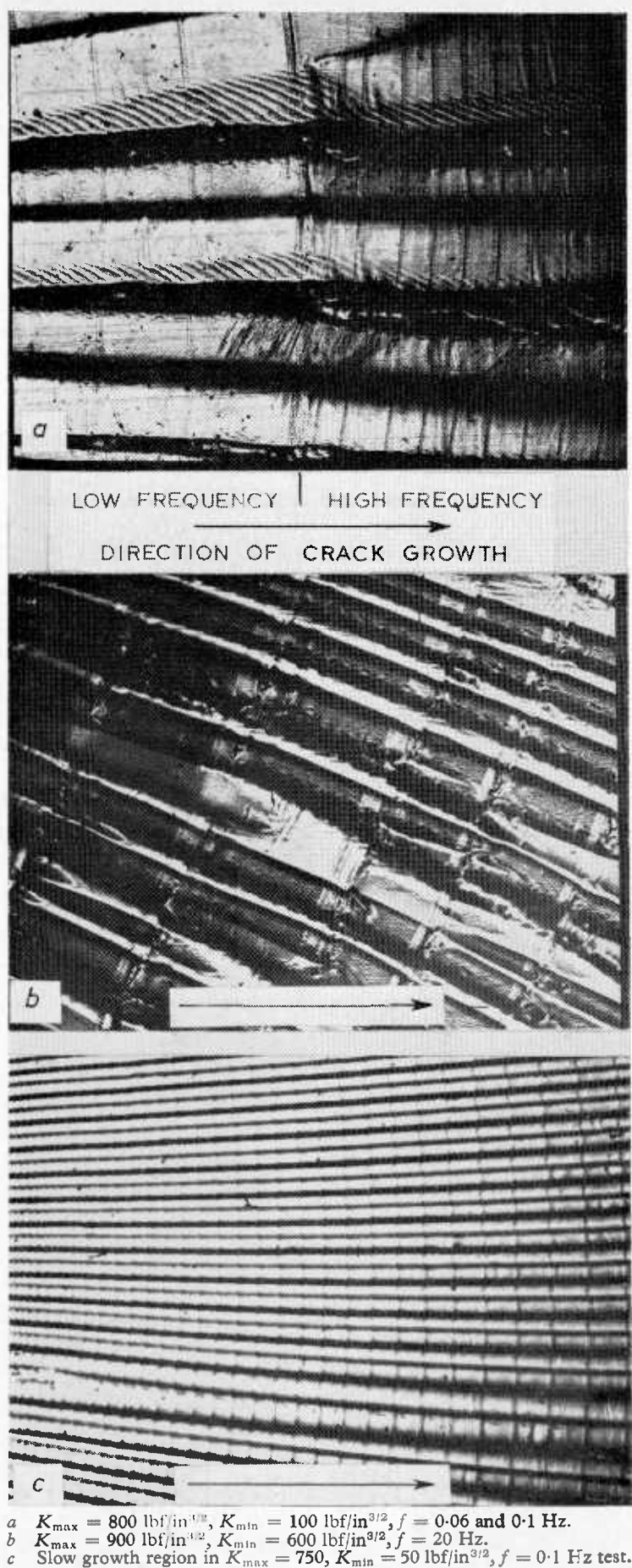


Fig. 8. Micrographs of fatigued and fractured surfaces

marked influence of the 'slow growth' type propagation on $\dot{a}_N$.

Some attention is now given to the variations in the value of parameter n in equation (9) with respect to changes in frequency. For metals, based on an equation of the form

$$\frac{d(2a)}{dN} = C(\Delta K)^m \quad (11)$$

it was originally suggested (1) that a value of $m = 4$ is applicable in many cases. Various modifications have been made to this relation (13) (14). For unreinforced glassy thermoplastics, values reported for m have been approximately equal to 5 (6). Logarithmic plots of the results in Figs 5 and 6 yielded straight lines, such as those shown in Fig. 7 for example, the slopes of which gave the values for index n in equation (9). The values were 2.3, 2.2 and 3.5 at 20, 5 and 0.1 Hz, respectively. The results at 0.1 Hz show a relatively higher degree of non-uniformity and scatter and in general tend towards a line of higher slope than the other two sets of results. This difference in crack growth behaviour at very low frequencies has already been referred to in terms of K_m effects. Thus, considering the value of m reported in (6) for tests at frequencies of about 0.65 Hz and results of present tests at 5 and 20 Hz with n (from equation (9)) equal to 2.2 and 2.3, it may be stated that the value of the parameter m in the power law equation, which is an index to the stress intensity factor term, is approximately equal to 4.5 for this material and is almost independent of the loading frequency for the pure cyclic fatigue damage processes, but it acquires a higher value when cyclic creep damage contributes significantly to the crack growth process. Variation of this parameter with other characteristics of the loading process have been dealt with for the case of metals (15). In thermoplastics, it may be expected that n will depend on testing conditions, such as very high frequencies which, as already mentioned, may result in internal heat build up in the material and cause failures due to thermal softening. Such a failure process ought to be treated totally independently from the type of fatigue damage discussed here. The value of n might also vary with significant variations in environmental temperature, as under such conditions other material properties such as yield and ultimate strength, relaxation modulus and fracture toughness (16) are expected to vary.

Examination of the fracture surfaces of the specimens used in the 0.1 Hz tests revealed the presence of definite fatigue striations as well as markings of the kind normally observed in the slow growth region (17) in static fracture tests. This lends support to the experimental observations on the rate of growth. Fig. 8a shows two regions in which the spacing of the striations is different. The right-hand side corresponds to a frequency of 0.1 Hz with $K_{\max} = 800 \text{ lbf/in}^{3/2}$ and $K_{\min} = 100 \text{ lbf/in}^{3/2}$, and the left-hand side corresponds to a region of yet slower cycling, 0.06 Hz, under the same K limits, which resulted in cyclic crack growth of about three times that at 0.1 Hz. Fig. 8b shows the fracture surface of a specimen which was cycled at 20 Hz with $K_{\max}$ and $K_{\min}$ equal to 900 and 600 $\text{lbf/in}^{3/2}$ respectively. In this case, at the magnification depicted in Fig. 8a, striations cannot be observed on the crack surface. Due to the very large value of $K_{\max}$ and the high rate of cycling, the growth pattern was found to be similar to the case of 'slow growth' before final fracture, i.e., a fast, but

steady, and clearly visible extension of crack under repeated loading conditions. The line markings on the micrograph are 'stop marks', obtained by stopping the cycling process every 50 s (or 1000 cycles), specially produced to examine the manner in which the crack grew under these conditions. The 'stop marks' are approximately equally spaced, showing that the crack was growing at a constant rate. In the region of slow growth, prior to the ultimate unstable fracture, the increase in the spacing of the markings shown in Fig. 8c indicates an accelerating 'slow growth'.

6 CONCLUSIONS

The influence of variations in frequency of loading on fatigue crack propagation in p.m.m.a. at room temperature in air has been examined. Under cyclic loading conditions, if the maximum and the minimum levels of the stress intensity factor are maintained constant, the relationship between the total crack length and the total number of cycles of applied load is a linear one. In constant $K_{\max}$ and $K_{\min}$ tests, any increase in frequency results in a decrease in the cyclic crack growth rate, $\dot{a}_N$, the effects of frequency variation becoming more marked at higher stress intensity factor levels.

At all frequencies, under conditions of $K_{\max} \geq 750 \text{ lbf/in}^{3/2}$ the contribution of 'dynamic creep' to the damaging process becomes more significant and also at very low frequencies, as tested here, cyclic creep influences the fatigue failure process, even at relatively lower $K_{\max}$ levels.

The crack propagation power law index, n , for p.m.m.a., is independent of frequency variations under pure fatigue failure conditions.

7 ACKNOWLEDGEMENTS

This investigation was carried out with the support of the Science Research Council and the p.m.m.a. was kindly donated by the Plastics Division of Imperial Chemical Industries Ltd.

APPENDIX

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DESIGN AGAINST FATIGUE FAILURE IN THERMOPLASTICS†

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Abstract—Empirical studies of fatigue failure in polymeric solids have led to the development of a cyclic crack propagation model of the following form:

$$\frac{d(2a)}{dN} = \dot{a}_N = \beta \lambda^n$$

where

$$\lambda = (K_{\max}^2 - K_{\min}^2)$$

Applicability of this model to the analysis of fatigue failure in polymethylmethacrylate and polycarbonate has previously been demonstrated. In this paper data on the behaviour of Nylon 6.6 under cyclic loading conditions are presented and on the basis of the above fatigue failure model a procedure for the computation of cyclic fatigue life is described. The effect on the rate of crack propagation of variations in loading frequency is also indicated.

Nomenclature

a	Half crack length
a_N	Cyclic rate of crack growth
f	Loading frequency
h	Specimen thickness
K_I	Stress intensity factor in mode I crack opening
$K_{\max}$ $K_{\min}$ K_m	} The maximum, the minimum and the mean levels of stress intensity factor
K_{IC}	
ΔK	
$\dot{K}$	$= \Delta K / \text{half periodic time}$
m, n	Indices of the power law equations
N	Number of load cycles
N_f	Number of load cycles at failure (cyclic life)
P	Applied load
R	$= K_{\min} / K_{\max}$
S	Applied stress
$S_{\max}$ $S_{\min}$	} The maximum and the minimum levels of the applied stress cycle
S_{yp}	
W	Specimen width
β	A parameter in the fatigue crack growth model
λ	$= K_{\max}^2 - K_{\min}^2 = 2\Delta K \times K_m$

†Presented at the Symposium on Fracture and Fatigue at the School of Engineering and Applied Science, George Washington University, Washington, D.C. May 3-5, 1972.

1. Introduction

The increasing rate of application of thermoplastics in the design of engineering components which are subjected to repeated loadings, such as gears and bearings, has resulted in a demand for information on the behaviour of these materials in terms of their resistance to fatigue crack growth. However guide lines or exact information in the form of mathematical models or empirical data (which can be gathered from investigations of the effects of various parameters on fatigue life such as the amplitude and the mean level of the applied load, the frequency and the characteristics of the load cycle) exist only in a very limited quantity. Until recently, the types of data available in this field consisted largely of those obtained through rotating bending and plate bending types of tests, i.e. total fatigue life data which did not lead to the further understanding of the influence of the basic factors affecting the rate of crack growth. Such information could not readily be used in cases where initial flaws and notches existed, as the exact influence of the parameters which affect the mechanics of crack propagation could not be included in the fatigue life estimation.

During the last few years, following the development and application of Linear Elastic Fracture Mechanics (LEFM) concepts to fatigue crack propagation in metals, some data have been published relating the cyclic crack growth rate in polymers to the variation of a stress field parameter known as the stress intensity factor, K_I , which for the case of an infinite plate containing a crack of length a , is obtained from:

$$K_I = S\sqrt{\pi a} \quad (1)$$

The results pointed to a fatigue strength characteristic in many ways similar to that of metals. The rate of crack growth could be predicted from an equation of the form [1]

$$\frac{d(2a)}{dN} = C(\Delta K)^m \quad (2)$$

when such parameters as the frequency and the environment were maintained constant. The above model did not originally include the effect of variations in the mean level of the stress intensity factor, K_m . However, with further appreciation of the importance of K_m in crack growth rate determination in fatigue failure processes in metals, various modified forms of equation (2) have been proposed [2-4].

In the following paper the results of a comprehensive test programme intended to study the effects of frequency and the mean level and the amplitude of the stress intensity factor on the growth of fatigue cracks in three characteristically different thermoplastics (Polymethylmethacrylate, (PMMA), Polycarbonate, (PC) and Nylon 6.6) are reported. Previous papers by the present authors have discussed crack growth data for (PMMA) and (PC) in terms of fracture mechanics parameters [5-7]. The current paper is concerned with the application of LEFM to Nylon 6.6 and discusses a crack propagation model of the form

$$\dot{a}_N = \beta \lambda^n \quad (3)$$

The parameter $\lambda (= K_{\max}^2 - K_{\min}^2 = 2\Delta K K_m)$ includes the influence of the mean level of the stress intensity factor, K_m . In addition, a computational method for the estimation of the cyclic fatigue life, which takes into account the cumulative effect of the rise in

stress intensity factor due to crack growth is described. The outcome of investigations into the effects of frequency and the ratio of the minimum to maximum levels of stress intensity factor are also presented through initial flaw size/cyclic fatigue life data.

2. Empirical analysis

The specimens used were rectangular plates of dimensions $7 \times 12 \times \frac{1}{4}$ in. thick for all tests except those on PMMA where the dimensions were $15 \times 13\frac{1}{2} \times \frac{1}{4}$ in. thick. All specimens contained an initial central notch of length 1 in. Tests were carried out on a Dowty Electrohydraulic fatigue testing machine, the characteristics of which have been previously described[5], and tension cycling with a sinusoidal input waveform at frequencies of 0.1, 5 and 20 Hz were used. The stress intensity factors, calculated from[12]:

$$K_1 = PY\sqrt{a/Wh} \quad (4)$$

where

$$Y = 1.77 + 0.227Z - 0.510Z^2 + 2.7Z^3 \quad \text{and} \quad Z = \frac{2a}{W},$$

were maintained constant throughout each test by appropriately adjusting the levels of applied load, as the crack increased in length.

As previously reported for PMMA and PC[5-7], the results obtained indicated that changes in K_m and in the loading frequency strongly influenced the cyclic rate of crack growth. It was shown that for these materials, a crack propagation equation which does not include the K_m term will not yield an accurate prediction of crack growth rate. Figures (1 and 2) demonstrate the effectiveness of equation (3), in which

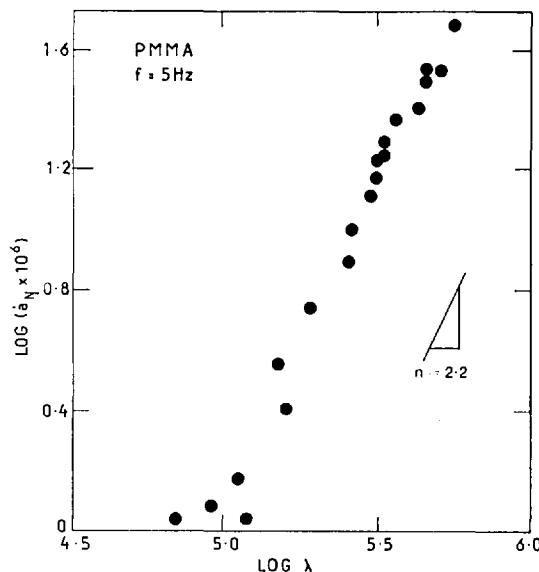


Fig. 1. Rate of crack propagation as a function of λ for PMMA at frequency of 5 Hz.

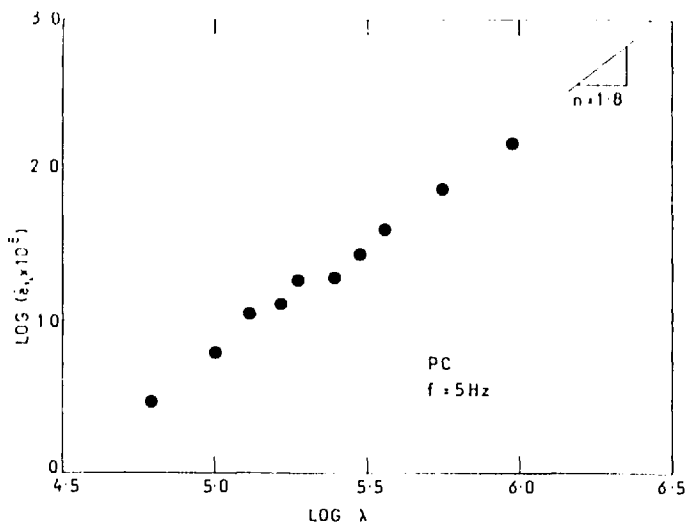


Fig. 2. Rate of crack propagation as a function of λ for PC at frequency of 5 Hz.

the parameter $\lambda = (K_{\max} + K_{\min})(K_{\max} - K_{\min})$ is included. These results were obtained from a range of tests in which arbitrary $\Delta k (= K_{\max} - K_{\min})$ and K_m levels were chosen and it was shown that with an appropriate value of K_m , a smaller ΔK may result in a higher crack growth rate than a larger ΔK with a different K_m . This effect has now been demonstrated for the case of PMMA [5], PC [7] and for Nylon 6.6 (Figs. 3a-c). The influence of the cyclic frequency has also been investigated and it has been found that unlike most metals, where within a reasonably wide range, the effect of changes in frequency on the crack growth rate was very small, in the case of polymeric materials there is a marked change in the cyclic growth rate as the loading frequency is altered, Figs. (4a and b). $\dot{a}_N$ decreases as the frequency is increased; however, the rate of crack growth with respect to time increases as the frequency is increased [6].

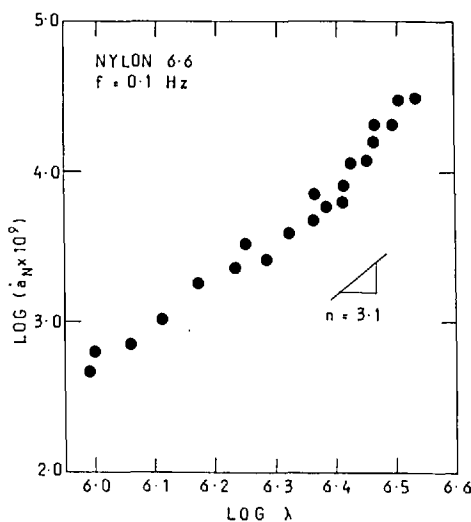


Fig. 3(a).

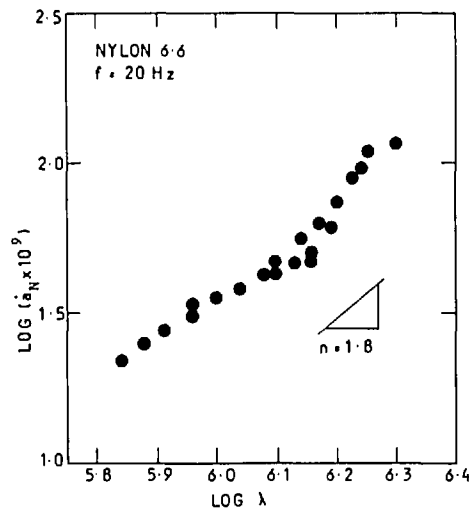


Fig. 3(b).

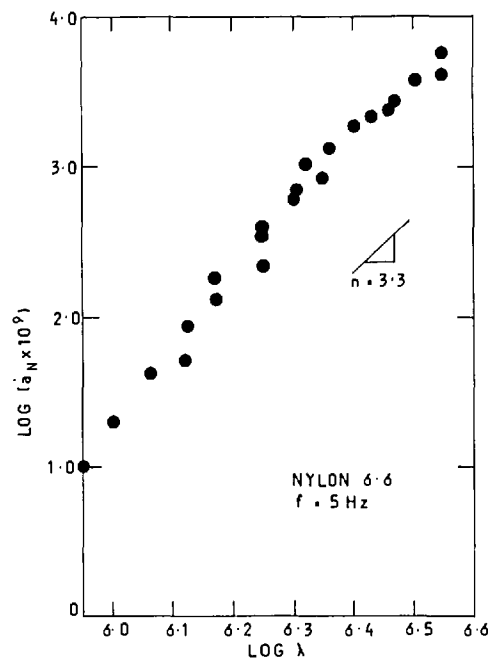


Fig. 3(c).

Fig. 3. Rate of crack propagation as a function of λ for Nylon 6.6 at frequencies of (a) 0.1 Hz, (b) 5 Hz and (c) 20 Hz.

3. A note on the proposed model

In the case of polymeric materials, as mentioned above, the effect of K_m and frequency on the rate of crack growth have been shown to be quite significant and hence only criteria in which the role of K_m , as well as ΔK , is fundamentally embedded, will

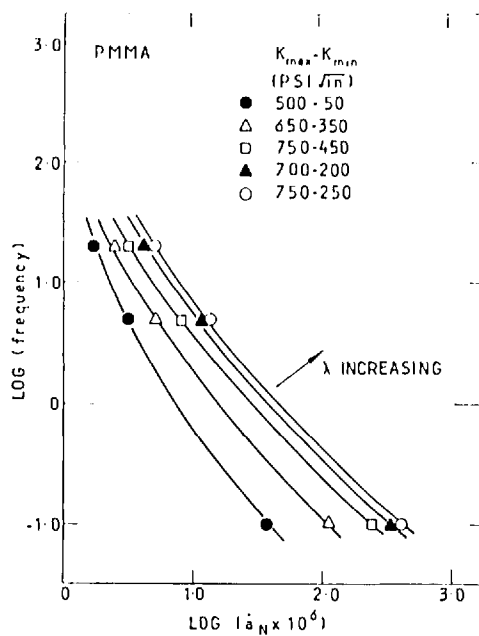


Fig. 4(a).

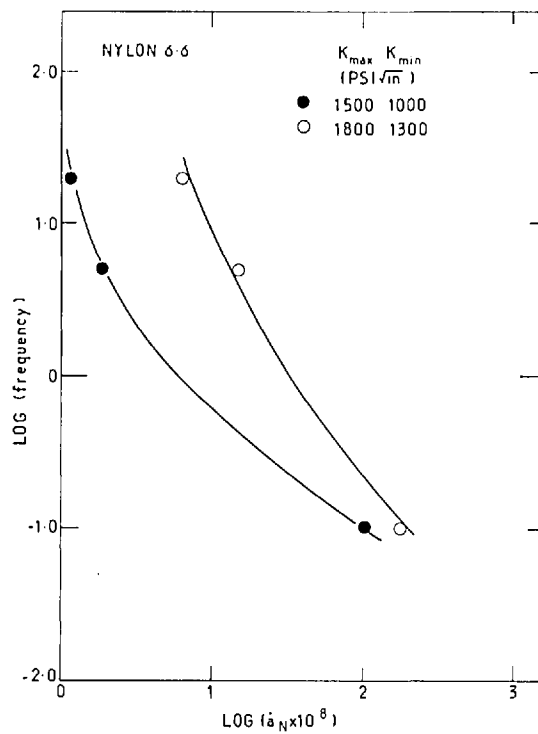


Fig. 4(b).

Fig. 4. Variation of cyclic crack growth rate with changes in loading frequency; (a) PMMA, (b) Nylon 6.6.

find accurate and acceptable applications. The crack propagation model proposed here can be taken as one such criterion and although it is based on empirical analysis, it is in a novel form which seems to offer some clarification of the mechanics of crack growth. Implicit in the proposition of the proportionality of the rate of crack propagation to the K^2 term is the exposition of the dependency of cyclic damage to the strain energy release rate G [8], the crack opening displacement [9] and the crack tip plastic zone size [10]. Thus the model appears to project a unifying pattern on most of the previously proposed fatigue laws which have been obtained both analytically and experimentally and have treated the fatigue failure phenomena in terms of local yielding, total energy transformation, applied stress parameters, etc. Further explanation of the characteristics of equation (3) and its applicability to the cyclic crack growth rate prediction in some aluminium alloys and steels have been given in [11].

4. Cyclic life computation

During the process of design calculations for the fatigue life of an engineering component, the design stress limits, the loading frequency and an estimated or calculated initial flaw size are available. As the crack increases in length, an increase in the magnitude of the stress intensity factor is implied and hence there is an accelerating rate of crack propagation leading to eventual fracture.

Taking the two limits of the applied stress as S_{max} and S_{min} , a ratio $R = S_{min}/S_{max}$ can be defined. For any arbitrary crack length this ratio will also be equal to $R = K_{min}/K_{max}$,

Then

$$\lambda = K_{max}^2 - K_{min}^2 = (1 - R^2)K_{max}^2$$

and hence equation (3) can be written as

$$\dot{a}_N = \beta \lambda^n = \beta (1 - R^2)^n (K_{max}^2)^n \quad (5)$$

Since in each practical case S_{max} and S_{min} are constants, the growth of crack will not change the value of R , even though the values of K_{min} and K_{max} gradually increase. Thus, in equation (5), $\beta(1 - R^2)^n$ is a constant and hence a plot of $\dot{a}_N$ against λ as in Fig. (1) can be changed to a plot of $\dot{a}_N$ against K_{max}^2 , with β replaced by $\beta(1 - R^2)^n$.

The following procedure can be used to estimate the cyclic fatigue life: From results of the type shown in Fig. (1), a small 'cyclic period' is chosen to suit a particular case, during which it is assumed that the rate of change of $\dot{a}_N$ is very small. Then using the maximum design stress and the initial flaw size and the appropriate stress intensity factor relations, the value of K_{max} is computed. From the relationship between $\dot{a}_N$ and K_{max}^2 (equation 5), corresponding to the calculated value of K_{max} , a value for $\dot{a}_N$ is obtained. This magnitude of $\dot{a}_N$ is then used to calculate the increase in crack length, δa , after the first cyclic period has elapsed. δa is added to a , and then a new value for K_{max} is calculated. The process is repeated until K_{max} becomes equal to the appropriate K_{IC} , the critical value of K_I corresponding to unstable fracture for the specific material, which is used as a criterion for final catastrophic fracture of the component. These calculations are, of course, carried out using a suitable computer programme in which, for each case, the parameters β and n from Figs. (1-3) and S_{max} and S_{min} are specified. One point of practical importance is the choice of an appropriate value of K_{IC} . For

many materials this is a highly rate dependent parameter and thus in order to enable the designer to make the right decision on its value, when dealing with fatigue problems at, say, various loading frequencies, data on variations of K_{IC} with loading rate must be available. In the case of some polymeric materials such measurements have been carried out but the available data are far from complete in many cases. Data reported in [13] for PMMA and PC point to a drop in the value of K_{IC} for these materials as the

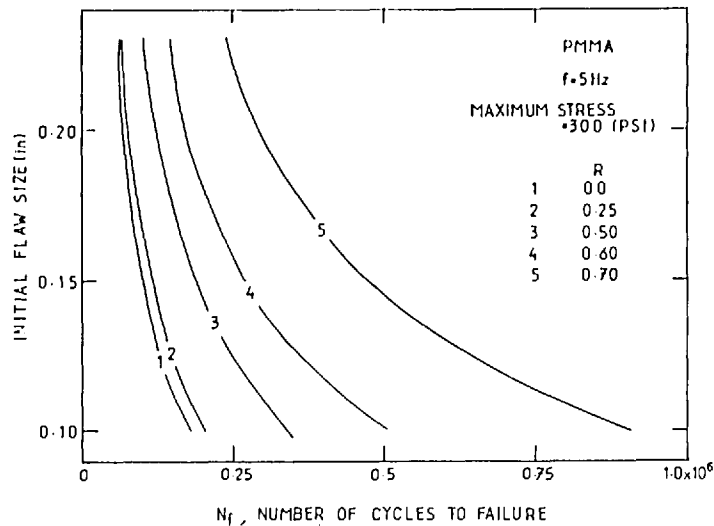


Fig. 5. Estimation of fatigue life as a function of the initial flaw size; PMMA at frequency of 5 Hz.

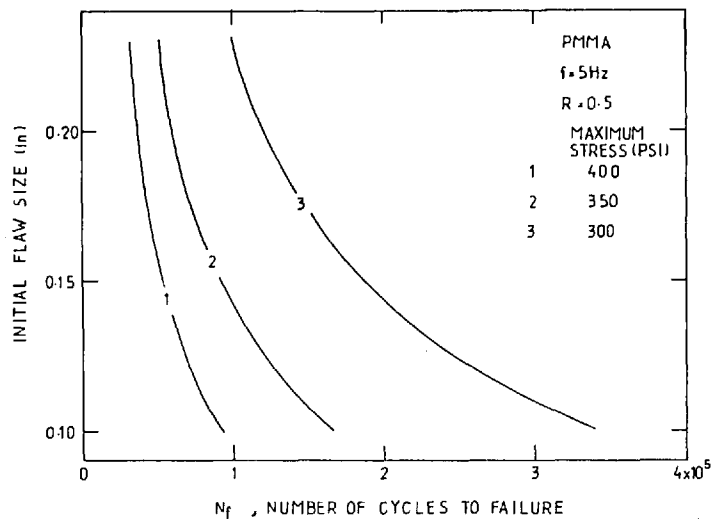


Fig. 6. Effect of changes in the maximum level of the applied stress cycle on the fatigue life of PMMA at frequency of 5 Hz.

load rate (cross head speed) is raised. As LEFM concepts are being implemented here it will be more useful to establish the relationship between $\dot{K}$ and K_{IC} , where

$$\dot{K} = (K_{\max} - K_{\min})/\text{half periodic time}$$

an increase in $\dot{K}$ corresponds to a decrease in K_{IC} within the range tested.

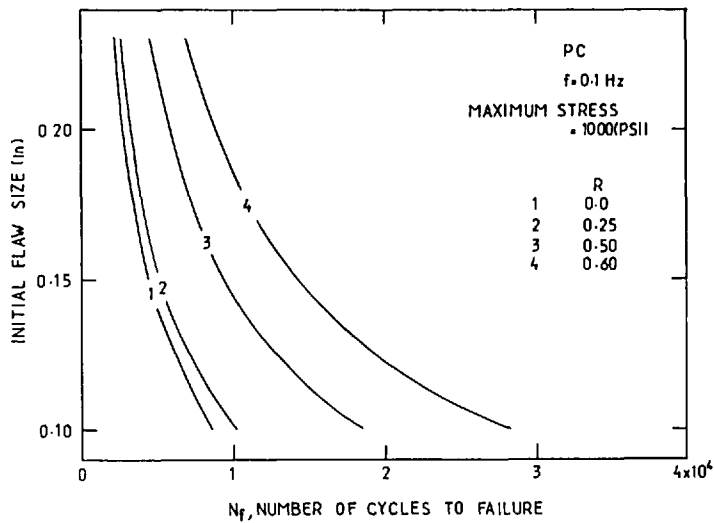


Fig. 7. Estimation of fatigue life as a function of initial flaw size; PC at frequency of 0.1 Hz.

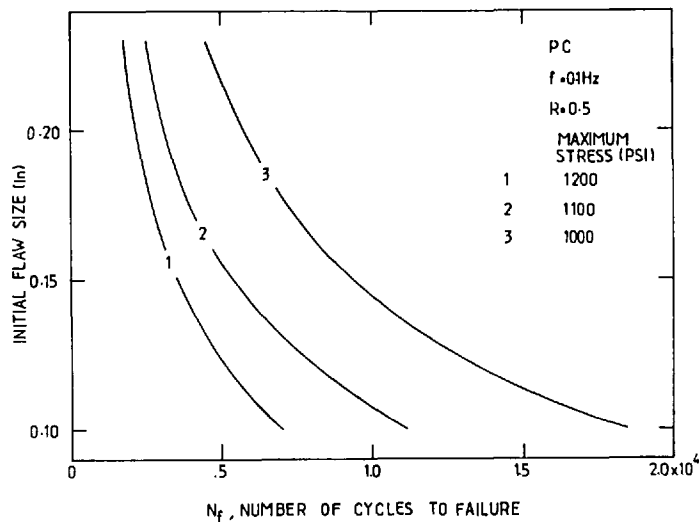


Fig. 8. Effect of changes in the maximum level of the applied stress cycle on the fatigue life of PC at frequency of 0.1 Hz.

On the basis of results such as those in Figs. (5-11) (i.e. total number of cycles to failure related to the initial flaw size), at the design stage, materials can be compared in terms of their resistance to fatigue crack growth: Data of the type presented in Figs. (5 and 6) for PMMA, in Figs. (7 and 8) for PC, and Figs. (9-11) for Nylon 6.6, will be obtained from the computational procedure described above. In Fig. (5), the value of S_{max} is fixed and calculations are carried out for various S_{min} levels. These calculations

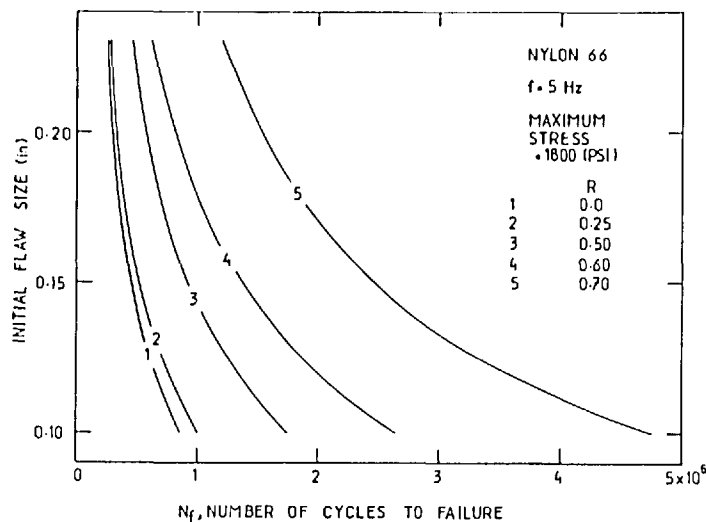


Fig. 9. Estimation of fatigue life as a function of the initial flaw size for Nylon 6.6 at frequency of 5 Hz.

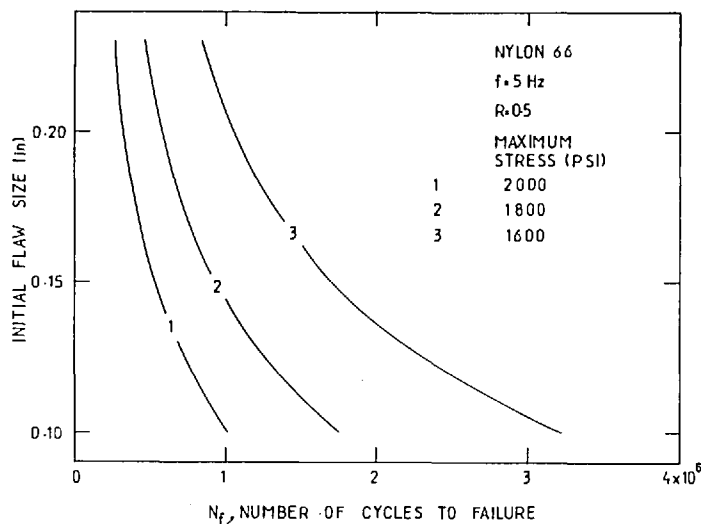


Fig. 10. Effect of changes in the maximum level of the applied stress cycle on the fatigue life of Nylon 6.6 at a frequency of 5 Hz.

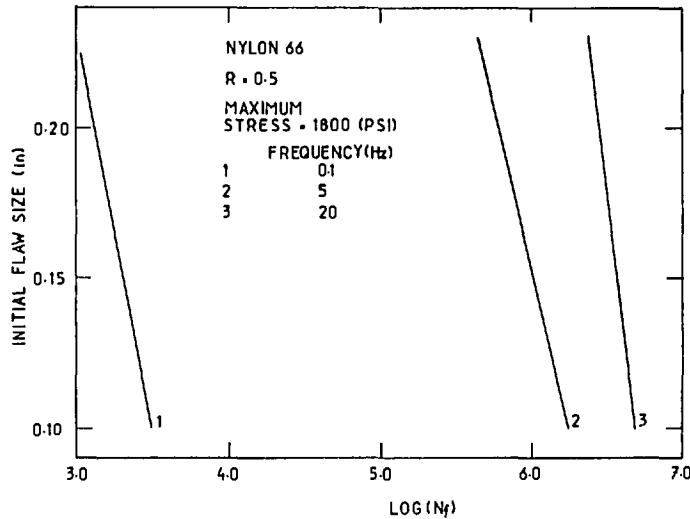


Fig. 11. The influence of variations in the loading frequency on fatigue life of Nylon 6.6.

are all for one frequency, as the values of β and n change from one frequency to another. Fig. (6) shows the effect of variations in the value of $S_{\max}$ on the relationship between the initial flaw size and the total number of cycles to failure.

Thus, when the whole range of data covering various frequencies and load levels have been prepared, and having the initial flaw size (that can be inspected or assumed to exist) and the minimum number of load cycles that a component must be capable of undergoing prior to catastrophic failure, an appropriate decision can be made on the choice of the most suitable material, the frequency level and the limits of the applied stress cycle as required.

An important point which must not be overlooked is that this type of data can only be used when the requirements for the applicability of LEFM concepts, in the case of a particular component, are satisfied. One such criterion being that the following relationship is satisfied:

$$h \geq 0.25 \left(\frac{K_{Ic}}{S_{yp}} \right)^2 \quad (6)$$

where h is the thickness of the material and S_{yp} is the appropriate yield stress.

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(Received 4 March 1972)

STRAIN RATE DEPENDENCY OF FAILURE PROCESSES IN

POLYCARBONATE AND NYLON

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September, 1972.

SUMMARY

A study has been made of two types of failure, namely monotonic fractures using Charpy type specimens and fatigue crack propagation using rectangular plates containing an initial central notch. The work was conducted on an amorphous polymer (polycarbonate) and a semi-crystalline polymer (Nylon N 6.6). Monotonic tests were performed in an Instron testing machine between 0.002 and 20 in/min, and in a Charpy testing machine, between 2000 and 11800 in/min. The cyclic tests (under constant K conditions) were carried out at frequencies that ranged from 0.1 to 20 Hz.

A model for the relationship between the cyclic rate of crack growth and appropriate LEFM parameters, previously described, has now been converted into cyclic strain energy transformations. In computing the strain energy the value of the time dependent modulus of the material was used and under cyclic loading conditions the glassy (short time) value was employed. The authors have proposed that the modulus measurements obtained at very low temperatures, where the viscous response of the material is highly restricted, will approximate the glassy value, E_g , found by conducting relaxation measurement tests at different temperatures down to -197°C .

Within the range of tests conducted, the fracture toughness values of both PC and N 6.6 apparently decrease with increase in loading rate. Fatigue crack growth in both materials is influenced by loading frequency and cyclic waveform. These variations may be related to the magnitude of the viscous energy loss and hence to the available energy for crack extension per cycle.

NOTATIONS

A	$G_{\max} - G_{\min}$
$\dot{a}_n$	Cyclic rate of crack propagation da/dN
$\dot{a}_t$	Crack speed da/dt
$E(t)$	Time dependent modulus of polymers
E_g	Glass modulus
E^*	Complex modulus
E'	Real part of E^* (Storage modulus)
E''	Imaginary part of E^* (Loss modulus)
G	Strain energy release rate
$G_{\max}$	Maximum and minimum levels of G
$G_{\min}$	
K_1	Stress intensity factor in mode 1 crack opening
$K_{\max}$	Maximum and minimum levels of K_1
$K_{\min}$	
K_{1C}	Critical value of K_1 - corresponds to unstable fracture conditions
$K_1 \text{ INIT}$	Stress intensity factor calculated at the point of initiation of slow (stable) growth
S_a	Amplitude and mean level of applied stress cycle
S_m	
M	Numerical constant in crack growth equation
n	Exponent in crack growth equation
$\dot{W}_v$	Viscous energy absorbed per cycle of loading
β	Numerical parameter in crack growth equation
λ	$K_{\max}^2 - K_{\min}^2$

1. INTRODUCTION

An analysis of the failure strengths of polymeric materials may be conveniently expressed in terms of the time and temperature dependencies of their mechanical properties. Irrespective of the nature of the external load, the response of viscoelastic solids will be conditioned by the time and temperature histories of the loading process imposed upon them.

This behaviour is indicative of the complexity of the nature of such problems as the determination of the combined effects of, say the relaxation moduli, the mechanical hysteresis energies and the characteristics of the boundary loadings combined with a formulation of the time and temperature effects on each of these parameters. However, in recent years, as the use of polymeric solids has increased the failure of such materials under static and cyclic loading conditions has been the subject of numerous investigations.

Monotonic fracture and fatigue studies have been carried out employing the concepts of linear-elastic-fracture-mechanics (LEFM), [1], [2]. Successful applications of LEFM concepts to a number of thermoplastics have been demonstrated, although such applications are limited to a range of load time and strain in each material, where approximate linear elastic behaviour may be assumed [3]. Consequently, failures containing large amounts of plastic deformation are not discussed.

Linear elastic fracture mechanics has provided the designer with quantitative relationships between various parameters influencing the failure of bodies containing flaws.

A parameter which represents the combined influence of applied stress, body geometry and the current crack length is introduced into the field of failure analysis. This parameter, termed the stress intensity factor, K , has been tabulated for a wide range of stress fields in flawed bodies [4,]. Achievement of a critical level of the stress intensity factor corresponds to catastrophic failure conditions.

In the present paper, two types of failure studies will be discussed in detail: 1) monotonic fracture studies using standard Charpy-type specimens and 2) fatigue crack propagation studies using large rectangular plates containing an initial centre notch. The stress intensity factors for these two geometries have been given previously [6, 7].

It was decided to study the behaviour of an amorphous polymer (polycarbonate, PC) and a semicrystalline polymer (Nylon, N 6.6) under the above two loading conditions. Monotonic fractures at various strain rates and cyclic failure patterns under different loading frequencies were investigated. A model for the relationship between the cyclic rate of crack growth and appropriate parameters associated with LEFM principles, which has previously been described [8, 9] is now presented in terms of cyclic strain energy transformations.

For the computation of strain energy, the value of the time dependent modulus of the material, $E(t)$, was used, but under cyclic loading conditions (and also under the conditions of fast dynamic fracture), it was assumed that the use of the glassy (short time) value E_g , would be sufficiently accurate.

A convenient procedure for the determination of the E_g modulus for thermoplastic materials is described and the

results of such measurements on a number of thermoplastics (including PC and N 6.6) are presented.

2. MATERIALS, SPECIMEN DESIGN AND TEST PROCEDURES

Materials used were polycarbonate of Bisphenol A (Makrolon, Bayer) and Nylon 6.6 (ICI Maranyl) which had been produced in the form of extruded sheets. Methods of preparation of the three-point-bend specimens (dimensions 10 x 10 x 55 mm) used in the static fracture tests and the centre-notched plates (dimensions 7 x 12 x $\frac{1}{4}$ in) used in the fatigue studies have been fully described in [6] and [8] respectively. Descriptions of the associated equipment used may also be found in the above papers.

The dimensions of the rectangular-plate fatigue specimens were selected such that crack growth over a length of about 3 in could be monitored. These tests, in tension, were carried out under the conditions of constant maximum levels of the stress intensity factor, the conditions being achieved by a gradual reduction in the level of applied load as the crack grew in length.

In the static fracture tests a 10,000 lb capacity Instron tensile testing machine was used for the low load rates and a 120 Kg m capacity instrumented Charpy impact machine for the high load rates.

In the modulus measurement studies, unnotched bars of similar form to those used in the Charpy tests were employed and the tests were performed at crosshead speeds of 0.2 in/min at cryogenic temperature (-197°C , obtained in a liquid nitrogen bath). Other modulus measurements, at higher temperatures, in the range between $+20^{\circ}\text{C}$ to -197°C were carried

out in baths filled with various mixtures of liquid nitrogen and petroleum ether. Test readings were recorded after the specimen had reached a steady temperature.

3. STATIC FRACTURE TESTS

The critical value of the stress intensity factor at the point of instability (catastrophic fracture) is termed the fracture toughness of the material, and in plane strain conditions it is designated by K_{1C} . This parameter is a measure of the energy required for crack extension in the material and it is to be expected that, in rate dependent materials (such as polymers), K_{1C} will change as a function of variations in the applied loading rate, environment and temperature. It is anticipated that at lower temperatures and higher loading rates, the material will behave in an increasingly less ductile manner.

In order to analyse the behaviour of PC and N 6.6 under such conditions, a range of tests was carried out to study the variation of fracture toughness value with change in loading rate, other conditions being constant.

3.1 Load Rate Effects

The influence of loading rate on K_{1C} was studied in three-point bend tests at room temperature (21°C) in air, over a range of crosshead speeds between 0.002 and 11800 in/min. The tests were performed in accordance with the ASTM standard [7] and the fracture toughness values were calculated using the same procedure as described in [6], i.e. by calculating the strain energy release rate and then converting it to the stress intensity factor term. The

results of these tests have been presented in Figures 1 and 2 which clearly indicate the variations in K_{1C} as the loading rate is increased. The variations at relatively high crosshead speeds, however, do not seem to be as large as those suggested in [10] for PC. The discrepancy may lie in basic differences in the materials tested and in testing techniques including the nature of the initial notch in the specimens.

Slow bend fractures in Nylon 6.6 were in the form of very ductile failures; under these conditions the LEFM concepts are not applicable and the quoted K_{1C} values may be described only as "apparent" toughnesses; the three tests carried out at this rate exhibited a large deviation in results, Figure (2). However, as the loading rate was increased, a gradually decreasing ductility was observed and the measurements of K_{1C} became more consistent.

Fracture surface studies have also confirmed such a gradual transition in behaviour pattern; see for example [11] for studies on PC. Quasi-brittle materials usually exhibit a stable crack growth period (termed the slow growth) preceeding the final unstable fracture. The length of this slow growth zone can be correlated with the material ductility and if $K_{1 \text{ INIT}}$ is calculated at the point of initiation of slow growth rather than at the catastrophic failure point, a value different from and lower than K_{1C} will be obtained; in the presence of an increasing slow growth region a decreasing value for $K_{1 \text{ INIT}}$ will result. Similar behaviour may be observed with increasing test temperatures. Thus, as the degree of ductility is reduced through variations in testing conditions, smaller slow growth regions will be

obtained and the fracture toughness values calculated at the beginning and at the end of this region will become closer to each other.

4. CYCLIC CRACK PROPAGATION

A comprehensive programme of experimentation, results of which have in part been reported previously, [8, 12, 13], has shown that a crack propagation model of the following form may successfully be used in predicting the cyclic rate of crack growth:

$$\dot{a}_n = \beta \lambda^n \quad (3)$$

$$\text{where } \lambda = K_{\max}^2 - K_{\min}^2$$

β and n are numerical parameters - to be determined empirically - dependent upon frequency of loading, material properties and other testing conditions.

The form of this crack growth model, enables the crack propagation rate to be expressed in terms of G , the strain energy release rate, in a load cycle [3]. The K^2 term is related to G by the following [14].

$$G = \frac{K^2}{E} \zeta \quad (4)$$

$$\text{where } \zeta = \begin{cases} 1 - \nu^2 & \text{plane strain} \\ 1 & \text{plane stress} \end{cases}$$

Equation (3) can thus be converted into the following form

$$\dot{a}_N = \beta \left(\frac{E}{\zeta}\right)^n [G_{\max} - G_{\min}]^n \quad (5)$$

or briefly:

$$\dot{a}_N = M A^n \quad (6)$$

where $A = G_{\max} - G_{\min}$

In [3] a review of instances of application of an energy based model for crack propagation as well as its specific advantages have been presented.

4.1 Effect of Variation in Frequency on Cyclic Damage

Evidence indicating the strong frequency dependence of cyclic crack growth rate in polymeric materials has been produced [8, 9, 13]. It has been found that in general, under constant stress level conditions, an increase in frequency leads to a decrease in the cyclic rate of growth, Fig. 3. However, in terms of time to failure, a rise in frequency will result in a shorter total life [13].

In the case of N 6.6 data from tests at three frequency levels namely 0.1, 5 and 20 Hz, has clearly indicated such a trend [9]. However, in the case of PC, Fig. 4, a deviation from this pattern of behaviour is observed above a certain frequency level [8]. Other studies [2, 11] have shown that it is possible to obtain an increase in the cyclic rate of crack propagation with increasing frequency. The following discussion may explain, at least in part, the above mentioned frequency effects:

a) The influence attributed to the external loading:

During the fast cycling process (sharper sinusoidal wave form) the material is subjected to high loads for a shorter period of time per cycle. The crack growth rate is proportional to the size of the plastically deformed zone at the crack tip, as indicated by equation (3) [3, 8]. The plastic

zone may be considered as the crazed material at the crack tip through which the crack propagates. There is evidence of the time and strain dependency of craze growth in PMMA [15] and PC [16]; namely that if the same strain is applied for a shorter period of time, the size of the crazed zone developed will be smaller. Observation of the fracture surfaces produced under various load rates also confirms such a hypothesis: as the loading rate is increased the fracture surfaces become more smooth, indicating a more brittle type of behaviour. In the discussion of variations, with load rate, of the energy required to create a unit fracture surface however, one must also refer to the data relating the fracture toughness value to the crack speed, a_t , in the material. Such data on PMMA, for example, shows a rise in K_{1C} indicating a higher fracture toughness as a_t increases up to a certain level corresponding to crack speeds of a few inches per second. Subsequently K_{1C} remains constant but the situation becomes more complicated as a_t continues to rise to very high values (1000 ft/s) and K_{1C} then begins to rise again [17]. Clearly if the crack growth rate in fatigue falls within the first section of this behaviour pattern, one would expect an increasing resistance to crack growth as the loading rate (or frequency) is increased.

Data relating crack speed and fracture toughness are not, unfortunately available yet for most materials although some have recently been produced on PC [18] and hence detailed discussion of such results at present, has to be limited. The fracture toughness measurements discussed in section 3, clearly indicate the gradual change in the specific

fracture energy of PC and N 6.6; however such data must be extended to a wider range of loading rates before a general discussion covering behaviour of stable and accelerating cracks is possible.

b) The influence of intrinsic viscoelastic properties of the material:

From consideration of analytical solutions of viscoelastic fracture processes [19], it is evident that the role of the viscoelastic absorbed energy is important in the availability of propagation energy required for crack extension. This loss energy, $\dot{W}_V$, is a function of E'' (the loss component of the complex modulus of a polymeric material, $E^* = E' + iE''$). $\dot{W}_V$, is related to E'' and E' in the following form [8]:

$$\dot{W}_V = \frac{\pi E''}{E''^2 + E'^2} [2S_m + S_a]^2 \quad \text{per cycle}$$

S_m and S_a being the mean level and amplitude of the applied stress cycle.

The loss modulus E'' is itself a function of frequency. At low frequencies, where large recoveries may be made in delayed elasticity, as well as at high frequencies when there is not sufficient time for molecular motions required for stress relaxation to take place, E'' has a small value. In the intermediate range, E'' has a peak at a level of frequency termed the resonance frequency. Clearly, depending on which side of the E'' peak a frequency range is chosen, the loss modulus may increase or decrease with increasing frequency levels. The specific trend of variations in E'' will thus influence $\dot{W}_V$ and consequently the magnitude of the energy

available for crack propagation will be affected. Quantitative substantiation of the above is not possible at present as comprehensive relaxation data for PC and Nylon 6.6 at room temperature are not yet available.

5. MEASUREMENT OF GLASSY MODULUS E_g

The value of E in equation (4) is that pertaining to a linear elastic material; hence in an application of this equation to polymers, the value of E must correspond to the instantaneous modulus of the material which will be an appropriate choice for all dynamic loading conditions. It is assumed that the value of this modulus will approximate the short time (glassy value) of $E(t)$ which is defined as E_g in relaxation spectra. The value of E_g for most thermoplastics is not available in the literature as the normal procedures of relaxation studies would require measurements of load and strain at exceedingly short times and hence are often impractical. In the present programme of studies, it was proposed to make relaxation measurement tests in gradually decreasing environmental temperatures until the achievement of cryogenic temperature (i.e. from $+21^{\circ}\text{C}$ to -197°C). Data relating the value of the modulus to temperature, measured after approximately 1 minute, at a constant crosshead speed of 0.2 in/min, for PC and N 6.6 are given in Figures (5 and 6). In these figures similar data obtained on Polyvinyl Chloride (PVC), polyacetal (PA) and polymethylmethacrylate (PMMA) are also included so that a general comparison of the behaviour of these different materials may be made. The figures show the general trends of behaviour in the modulus/temperature relationships and one may assume

that an approximate plateau region has been achieved in the data on PVC, PMMA and PC. However for PA and more prominently for N 6.6 the plateau region is less pronounced, indeed, the modulus of N 6.6 seems to be still rising at a temperature of -197°C . The specific values of the moduli at -197°C and at 20°C are:

Material	Modulus (psi)	
	20°C	-197°C
PVC	364,000	800,000
PA	371,000	1,510,000
PC	310,000	622,000
N 6.6	180,000	936,000
PMMA	419,000	942,000

The room temperature data in the above table were compared with relaxation data given in [20] and a reasonably good degree of correspondence was observed. Also, for N 6.6, the value of the modulus obtained at -196°C was compared to data provided in [21] and excellent correlation was obtained, Figure (6).

6. DISCUSSION

Application of energy based approaches to failure prediction and analysis in engineering materials, have been immensely fruitful. Studies of ductile-brittle fractures of metals, originally initiated by the work of Griffith [22], fracture in elastic and viscoelastic polymers [19, 23], tear and fatigue of rubbers [24] and fatigue of polyethylene

[25] have indicated the possibility of development of a unified approach to the analysis of failure processes in general. Equation (3), which has been based on empirical data from fatigue crack growth studies, is in the most suitable form to be converted into an energy based criterion, equation (6). Possibility of the application of this growth model to fatigue failure in metals has been explored with some degree of success in [3]. It is within the context of such an energy based criterion for failure, that one may be able to incorporate with facility, the effects of various parameters associated with loading conditions. For instance, the influence of loading rate (or frequency) and temperature can be easily predicted, in the light of the empirical data and the discussions presented in previous sections, from the analysis of the effects of such external parameters on various energy components involved in a failure process, as formulated in [19].

The specific role of strain rate in influencing the amount of energy available for crack extension may be discussed in terms of the magnitude of the hysteresis energy loss. In the monotonic fracture tests, this phenomenon is observed in the form of a reduction in total energy required for crack extension as the loading rate is increased and thus the material is increasingly more restricted in exercising its viscous response. In cyclic loading, the effect of an increase in frequency manifests itself in terms of a reduction in the cyclic rate of crack growth which may, again, be partly explained in terms of the variation in viscous energy components.

From the above discussion the following conclusions

may be drawn with regard to the effect of loading rate on the behaviour of PC and N 6.6 in failure:

The fracture toughness values of both PC and N 6.6 decrease with an increase in loading rate, at least within the presented range of experimentation. Fatigue crack growth rate in both materials is influenced by variations in loading frequency and cyclic waveform as these parameters have a considerable effect on the magnitude of the available energy for crack extension. The rate of growth can be directly related to this energy, equation (6).

The trend in the variation of the cyclic rate of growth, $\dot{a}_N$, with frequency was found to be consistent throughout the range of tests on PMMA reported in [13]; i.e. a gradual decrease in $\dot{a}_N$ as frequency is raised. In the case of PC, however, this trend is not continuous throughout the whole range of test frequencies: $\dot{a}_N$ initially decreases with an increase in frequency, but subsequently increases again as the frequency is raised further (Figure 3). It is suggested that this variation in behaviour is possibly related to a change in frequency/ E'' and crack speed/ K_{1C} dependencies.

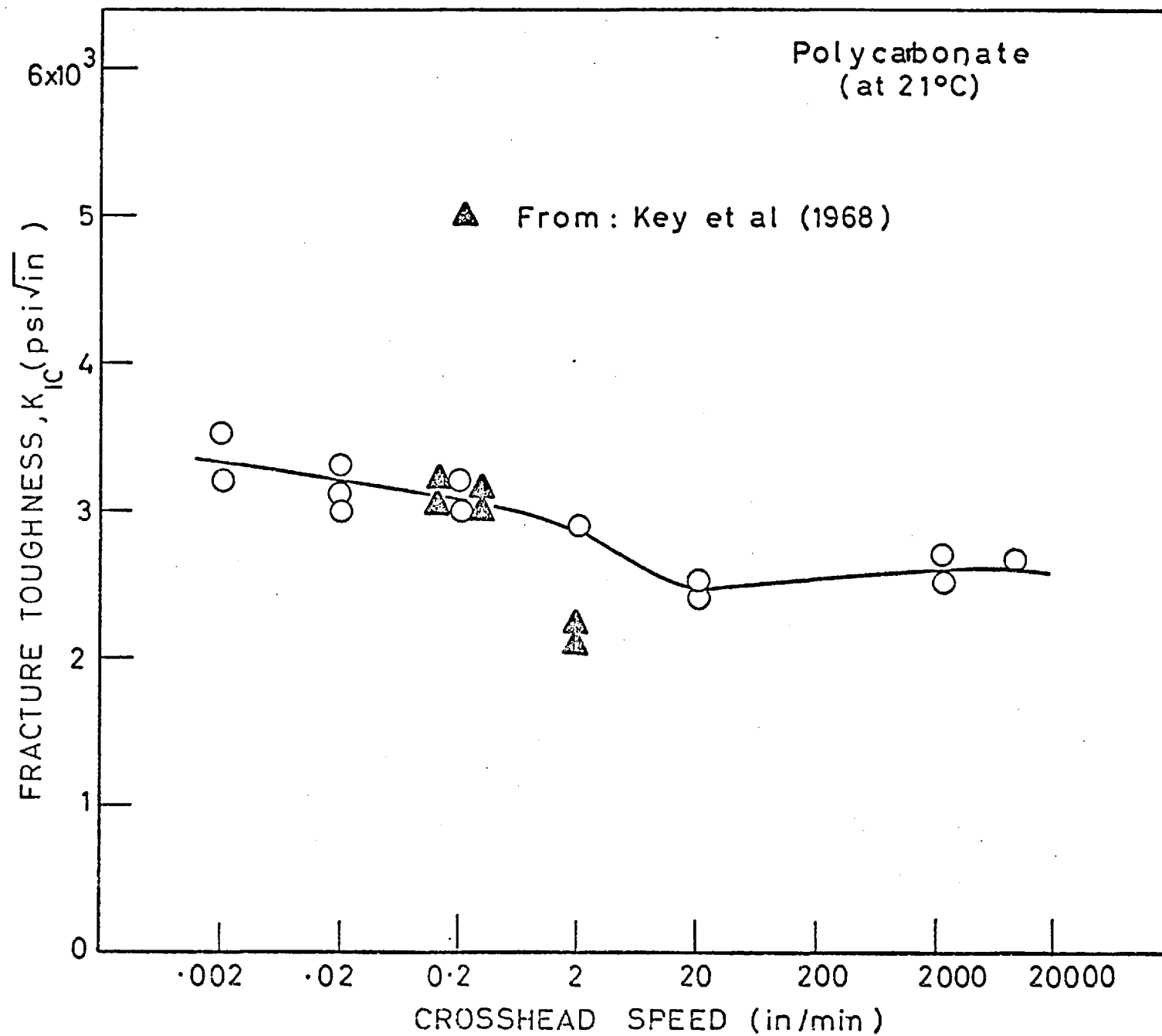
LEGENDS

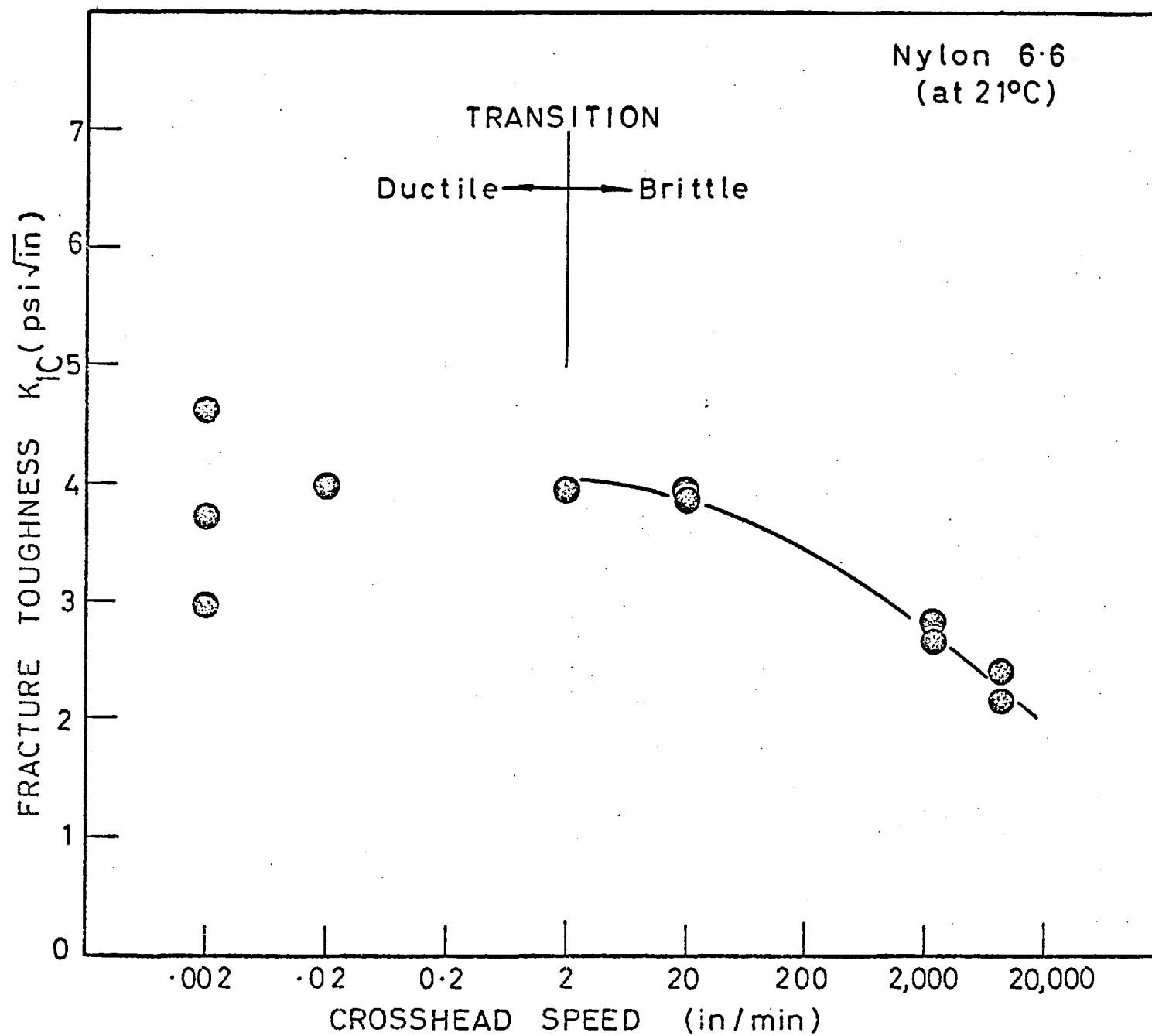
- Fig. 1 Variation of fracture toughness of polycarbonate with crosshead speed at 21°C.
- Fig. 2 Variation in fracture toughness of Nylon 6.6 with crosshead speed at 21°C.
- Fig. 3 Effect of frequency variation on cyclic crack growth rate in polycarbonate at 21°C.
- Fig. 4 Dependence of cyclic crack propagation rate in N 6.6 on parameter λ at 21°C.
- Fig. 5 Variation in time dependent modulus of PMMA, PVC and PC with temperature.
- Fig. 6 Variation in time dependent modulus of PA and N 6.6 with temperature.

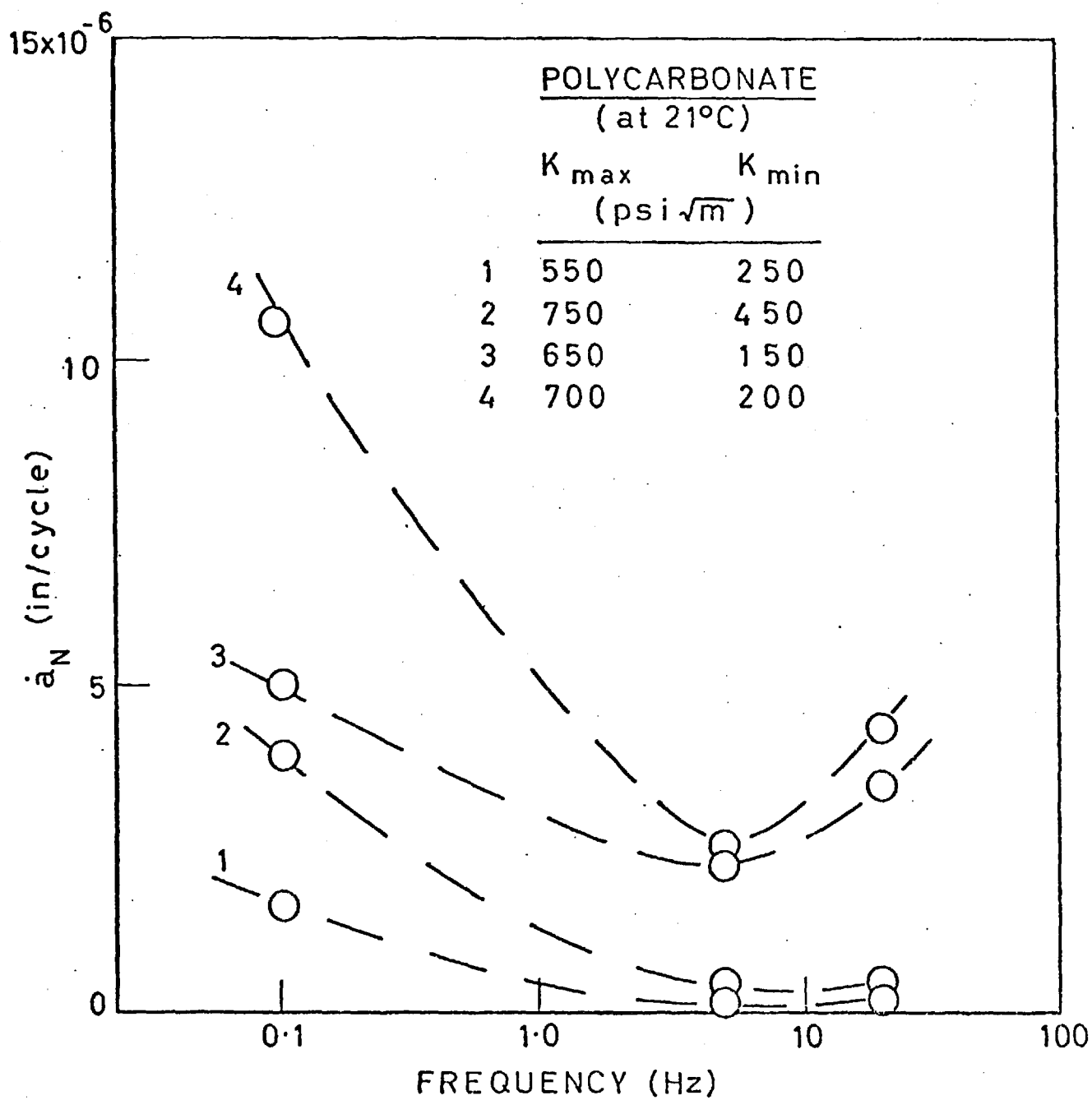
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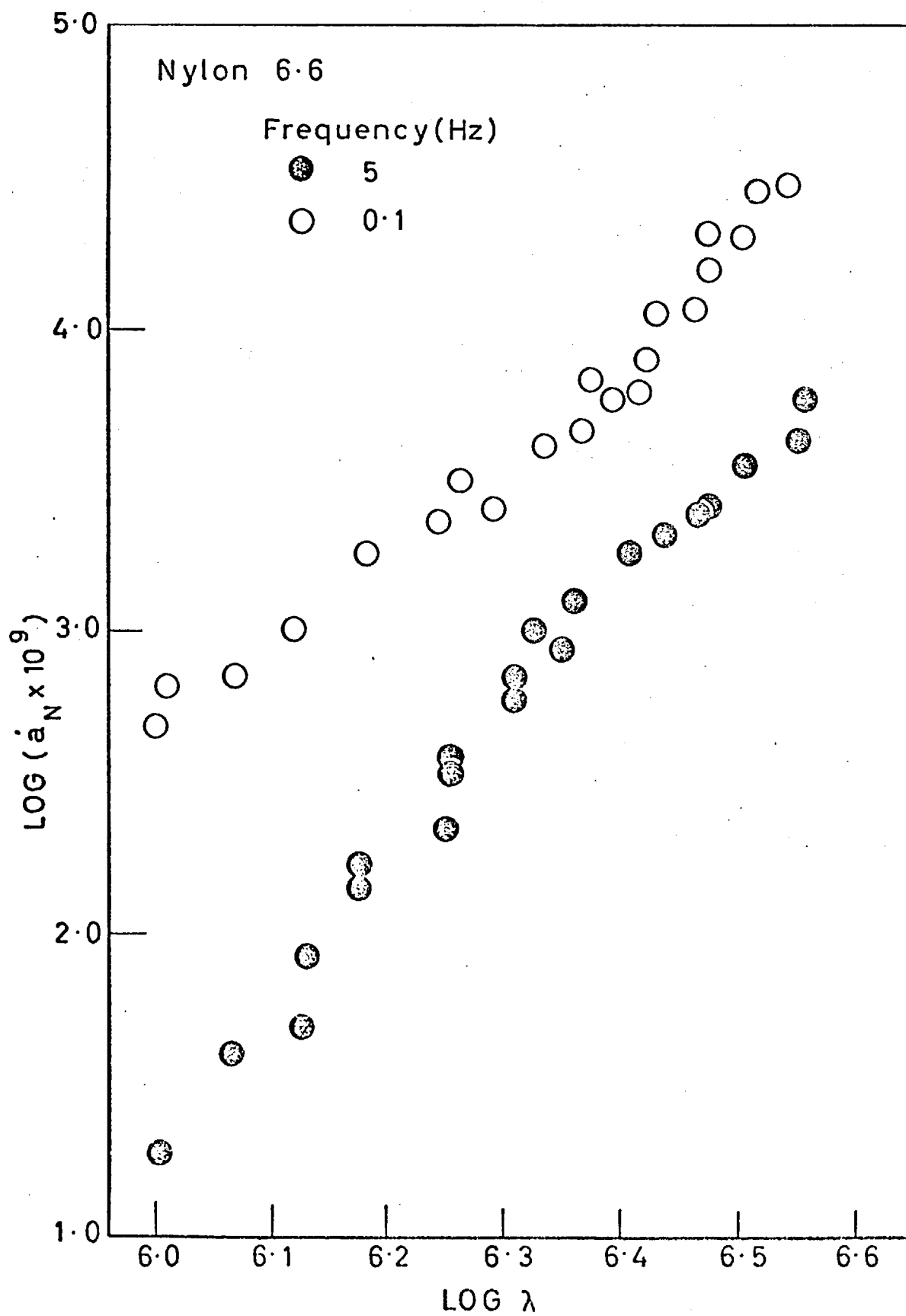
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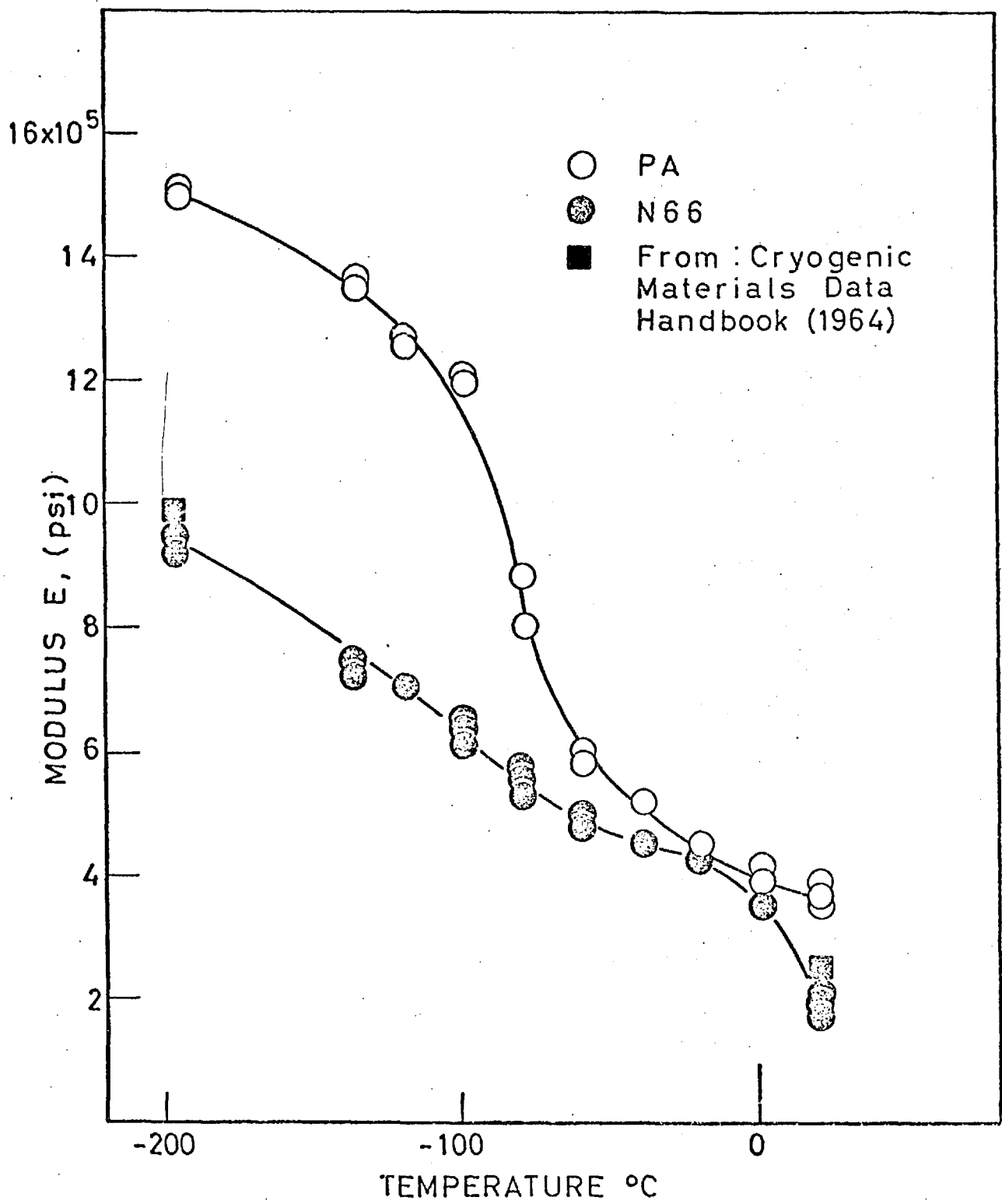
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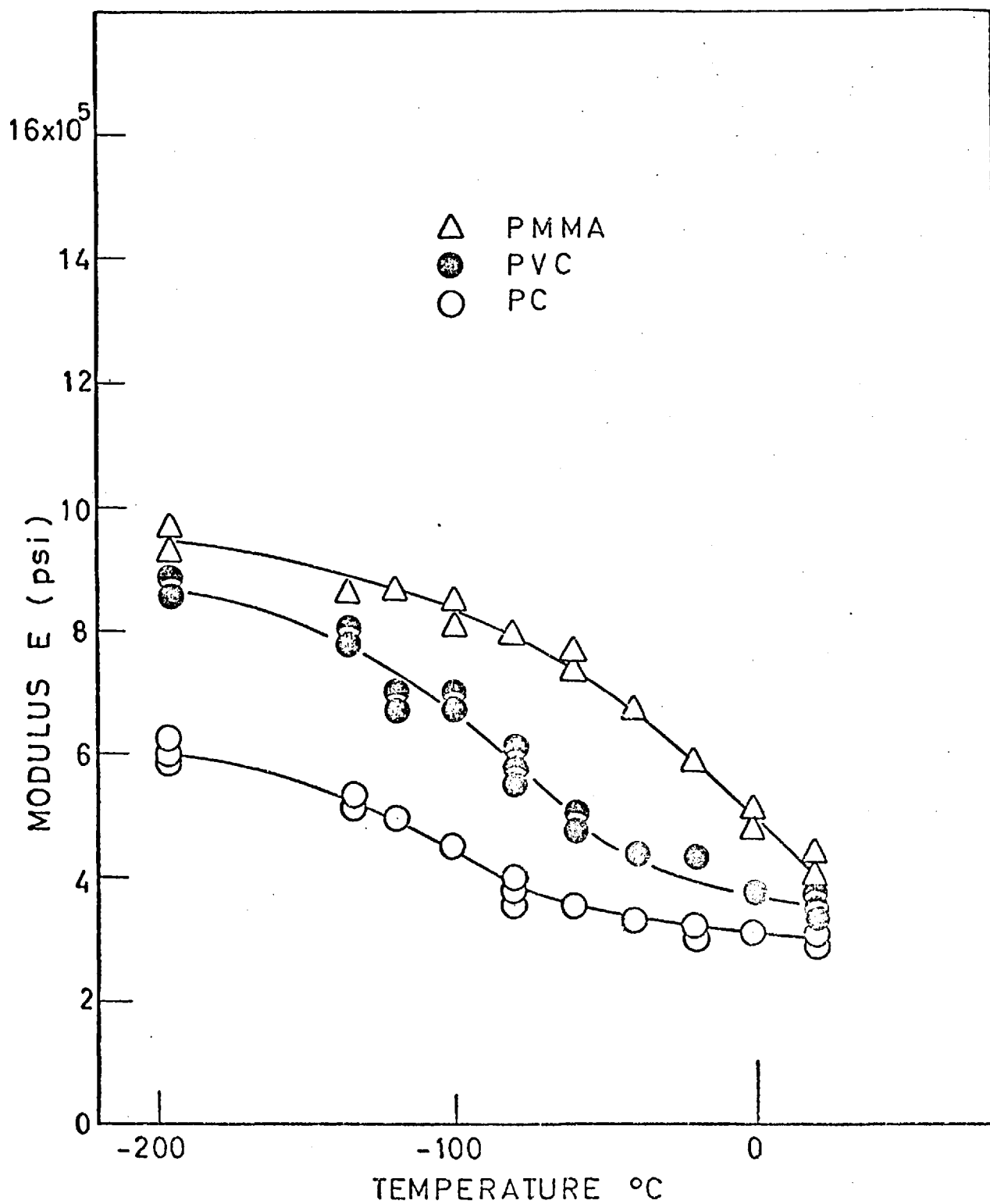












GROWTH OF FATIGUE CRACKS IN THERMOPLASTICS

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1. Introduction Empirical studies of fatigue failure in polymeric materials have revealed that the failure of these materials under cyclic loading conditions takes place as a consequence of the occurrence of one of the following three phenomena:

- 1) Creep (strain accumulation) due to cyclic loading
- 2) Propagation of flaws up to a critical length
- 3) "Softening" of the material due to an excessive rise in temperature within its bulk. (Cyclic thermal softening).

In the present paper fatigue failure in the absence of high material temperature will be discussed; the failure process is divided into initiation and propagation.

2. Determination of Crack Growth Initiation Life Cherepanov (1968) analysis of failure in solids which is founded upon thermodynamic energy balance concepts, resulted in the following criticality condition for viscoelastic solids:

$$\pi \int K \frac{1-\gamma(t)}{E(t)} \dot{K} dt + \frac{\pi}{2} \frac{K}{E(t)} [3+\gamma(t)] K = 2\gamma_* \quad (1)$$

where K is the stress intensity factor, γ_* is the specific fracture energy and $E(t)$ and $\gamma(t)$ are time dependent modulus and Poisson's ratio. As discussed by Williams (1971), for an incompressible material ($\gamma=\frac{1}{2}$) with a simple 3-element material model the creep compliance is given by

$$D_{crp}(t) = D_e - \frac{D - D_e}{e^{xt}} \quad (2)$$

where $x = \frac{1}{r}$, D_g is the glassy value and D_e the rubbery value of the compliance.

let
$$K(t) = \sigma(t) \left[\frac{a(t)}{2} \right]^{\frac{1}{2}} \quad (3)$$

with an initial crack length of a_0 and an external disturbance of the form

$$\sigma(t) = \sigma_0 + \sigma_1 \sin \omega t \quad (4)$$

from the above equation, the time to failure $t_f (= \frac{2\pi N_i}{\omega})$ is calculated (Arad 1972) where N_i = the number of cycles for initiation:

$$N_i = \frac{\omega}{2\pi x} \ln \Delta$$

where
$$\Delta = \frac{3\pi\sigma_0^2 a_0 D_e (x^2 + \omega^2 + x)}{3\pi\sigma_0^2 a_0 D_e (x^2 + \omega^2 - \omega x) - 8 \gamma_* (x^2 + \omega^2)} \quad (5)$$

with $N_i = \infty$, the level of maximum applied stress intensity factor corresponding to fatigue strength is found to be:

$$K_* = \frac{4\gamma_*}{3\pi D_e} \left(\frac{x^2 + \omega^2}{x^2 + \omega^2 - \omega x} \right) \quad (6)$$

Fig.1 shows a plot of N_i against σ_0 . Some experimental data obtained from tests on Polymethylmethacrylate were superimposed on the theoretical curve and reasonably good correspondence was observed.

3. Macroscopic Crack Propagation Under Cyclic Loading Conditions

3.1. The effect of the amplitude and the mean level of stress

intensity factor: A comprehensive programme of testing on the three materials (PMMA, PC, N 6.6) revealed very significant effects on the cyclic rate of crack propagation, $\dot{a}_N$, of the mean level and amplitude of the stress intensity factor (K_m and ΔK). It was shown that by an appropriate choice of the maximum and minimum limits of the load cycle, it was possible to obtain higher crack growth rates at

smaller ΔK values. Based upon experimental data, and from a general analysis of the crack propagation process (in terms of the relationships between $\dot{a}_N$, the COD_A^{and} crack tip plasticity) the following model for crack propagation was proposed:

$$\dot{a}_N = \beta \lambda^n \quad (7)$$

where $\lambda = K_{\max}^2 - K_{\min}^2 (= 2\Delta K K_m)$ and β and n are constants.

It was thus demonstrated that irrespective of values of $K_{\max}$ and $K_{\min}$ the rate of crack propagation may be described by parameter λ . This model has been shown to be applicable to fatigue failure process in some metals (Arad et al 1973). Such data on metals, which are normally presented in the form of graphs of $\dot{a}_N$ related to ΔK at various values of $R = K_{\min}/K_{\max}$, were shown to fall on a single line when presented on the basis of λ model.

3.2. The effect of loading frequency: The effect of frequency on the value of $\dot{a}_N$ was investigated at 0.1, 5 and 20Hz. It was found that, contrary to the case in most metals, in general in polymers the loading frequency had a significant effect on $\dot{a}_N$. The cyclic rate of crack growth decreased as frequency was raised. The effect of frequency may be attributed to two sources:

1) The effect of the external parameters: Here the change in fracture toughness level of the material with increase in crack speed (or rate of load application) may be considered. It has been shown that for example for PMMA, K_{1c} rises with crack speed in the stable growth region. This is expected to result in a smaller rate of growth per cycle under high frequency fatigue conditions.

In association with this effect, the influence of load rate on the extent of crack tip plasticity is important. The existence of strain and time limits for development of crazes of a certain size in some

glassy polymers have been demonstrated. A stress applied for a shorter time will result in a crazed zone smaller than one which will result from application of the same stress for a longer time. This aspect of craze formation in polymers (not yet fully documented) will be immensely useful in the analysis of the fatigue failure process.

2) The effect of parameters associated with molecular structure:

Analysis of relaxation data for thermoplastic materials yields information on the specific frequency levels which result in achievement of maxima in loss modulus (E'') values at a certain temperature. Clearly, depending upon the position of the test frequency range in relation to such peaks, the value of E'' will increase or decrease with increasing frequency. The extent of viscous energy absorbed per cycle $\dot{W}_v$ is related to the loss modulus parameter:

$$\dot{W}_v = \frac{\pi E''}{E'^2 + E''^2} (2\sigma_m^2 + \sigma_a^2) \quad (8)$$

with a constant total input energy, if $\dot{W}_v$ increases with increasing frequency, the amount of energy available for crack propagation per cycle will be reduced. Hence a reduction of $\dot{a}_N$ is expected.

3.3. The effect of cyclic waveform: This effect was studied from tests with sinusoidal and triangular waveforms. It was found that the value of $\dot{a}_N$ under sinusoidal loading conditions were slightly higher than those under triangular ones. It is expected that, under the same loading conditions, as the waveform is varied from triangular via sinusoidal and trapezoidal to a square wave, the extent of damage per cycle is increased. Such an effect may be attributed to an increase in 'hold time' under higher loads, as the waveform is changed in the above manner.

3.4. The effect of orientation: Tests were carried out on Nylon 6.6 specimens cut from extruded sheets, in two perpendicular directions

(one in the direction of extrusion). The effect of molecular orientation in the direction of extrusion was observed in the form of a 10-15% smaller rate of growth for fatigue cracks propagating in a direction perpendicular to the direction of extrusion.

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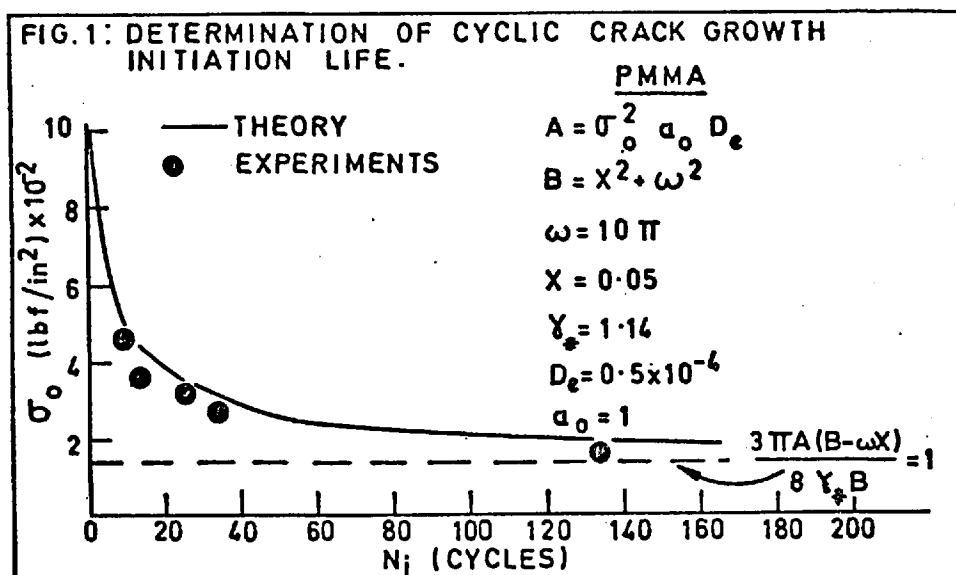
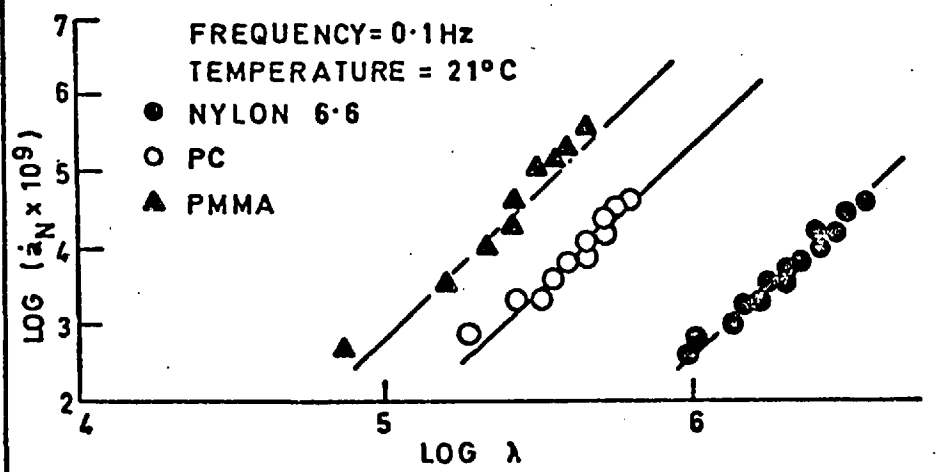


FIG.2: FATIGUE CRACK GROWTH DATA RELATED TO PARAMETER λ .



GROWTH OF FATIGUE CRACKS IN METALS AND POLYMERS

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SUMMARY

Empirical data on the propagation of tensile fatigue cracks in metals and thermoplastics have been examined. It was found that a cyclic crack propagation relationship, based on the stress intensity factor concept, exists which can be successfully utilised for both types of materials. The possible modified forms of such a relationship in terms of strain energy release rate, the crack tip yielding and the crack opening displacement concepts are also indicated.

1. INTRODUCTION

The exact nature of the mechanisms involved in the fatigue failure of engineering materials is by no means well understood. Even though a great amount of research effort has been focused on the study of various aspects of the phenomena associated with component failure under repeated loading conditions, a general criterion for the analysis of such failures has not yet been established.

Various empirical as well as analytical approaches have been adopted ranging from cyclic plastic strain measurement [1,2] to the implementation of dislocation mechanics techniques [3,4], and the development of an analytical failure model based upon the reaction rate theory [5]. However, many of the conventional approaches suffer from the lack of generality and the difficulty of application.

With the development of linear elastic fracture mechanics concepts (hereafter referred to as LEFM) a considerable number of basic problems associated with the assessment of service life of engineering components in relation to the available sample test data have been solved. LEFM, in providing a unique design concept that yields a quantitative relationship between the applied stress, component geometry, flaw size and material properties, has been successfully applied to the analysis of fracture and fatigue failure processes in many metals and also in some polymeric solids.

The aim of this paper is to examine the empirical evidence on the pattern of crack propagation in some metals and polymers in the light of LEFM concepts and to demonstrate that a particular criterion, related to a parameter known as the stress

intensity factor, K , can be successfully applied to both types of materials. In addition, the possibility of the development of a general failure criterion based on the strain energy release rate, G , the crack tip plastic zone size, or the crack opening displacement (COD) is pointed out.

2. The Fracture Mechanics Approach

The concept of applying the stress intensity factor, K , [6] for the determination of fatigue crack growth rate was originally used by Paris [7]. The following relationship was demonstrated to describe the changes in the cyclic rate of crack growth in relation to the changes in the range of variation of K (denoted by ΔK):

$$\frac{d(2a)}{dN} = C (\Delta K)^m \quad (1)$$

where m is a numerical constant and C is a numerical factor dependent on loading conditions, the mean level of K , (K_m), the material properties and the characteristics of the environment. Both C and m have to be determined experimentally. The implication of the above relationship was that K_m should be treated as of secondary importance compared to ΔK . This line of approach has since been considerably developed. The influence of K_m on the crack growth rate has been further appreciated and various modified forms for equation (1) including the parameter K_m or a ratio $R = K_{\min}/K_{\max}$ have been proposed [8,9,10].

One particular model which has been examined by a number of investigators is that due to Forman et al [9] and is of

the following form:

$$\frac{d(2a)}{dN} = C \frac{\Delta K^m}{(1-R) K_{1C} - \Delta K} \quad (2)$$

The term K_{1C} in this equation refers to a critical level of stress intensity factor, corresponding to unstable fracture. K_{1C} has been shown to be a rate dependent parameter for many materials and hence, when fatigue data for rate sensitive materials at a range of frequencies are being analysed, an average value for K_{1C} must be used.

Equation (2) has found applications in various metals [11]. However, when the data obtained from fatigue tests on viscoelastic materials were analysed using the same equation, no success was achieved. [See Table 1 in which serious discrepancies are observed in the Forman predicted values although the experimentally observed values of $\dot{a}_N$ are very similar. The Forman model values should also have been approximately equal at all values of $K_{\max}$ and $K_{\min}$ for any particular values of C and m].

3. Consideration of Viscoelastic Materials

Conventional methods of fatigue failure analysis and life determination (e.g. plane-bending and rotating-bending tests) have been used in the past to provide design data for the prediction of failure of polymers under cyclic loading conditions. Only recently, following the application of LEFM principles to the fracture analysis of some viscoelastic materials, efforts have been made to use these concepts in relation to the fatigue failure process in this type of material. Clearly appropriate

checks must be made on the extent of non-linearity of a particular viscoelastic material, before the LEFM concepts are applied. These were carried out in the case of the materials discussed here (polymethylmethacrylate, PMMA; polycarbonate, PC; and nylon 6.6, N 6.6) by examining the characteristic isochronous and isometric data and the limits of time and load within which approximate linear behaviour can be assumed were determined.

Associated with studies of the fatigue failure process in polymeric materials, is an examination of the influence of the viscous energy loss, W_v , during each loading cycle. W_v is a function of the loss modulus (E'' , the imaginary component of the complex modulus) which in turn is a function of frequency. Data on fatigue crack growth rate in various viscoelastic materials shows a strong frequency dependence [12, 13, 14]. Hence, in order to determine the exact role of W_v in the cyclic energy balance equation, the characteristic dependence of E'' on frequency must be known. Additionally, due to the poor thermal conductivity of viscoelastic materials, there is the possibility of a very high temperature rise in the material as a consequence of cyclic loading. This would lead to thermal failure [15]. Data referred to in this paper were obtained from tests in which frequency levels were chosen such that the rise in temperature was insignificant and thermal softening thus avoided.

Based on the outcome of an extensive test programme covering a wide range of maximum and minimum levels of the stress intensity factor (K_{max} and K_{min}) at various loading frequencies and using the three materials referred to above, the following relationship has been proposed as the equation

governing the fatigue crack growth rate [12, 13]:

$$\frac{d(2a)}{dN} = \beta \lambda^n \quad (3)$$

where $\lambda = (K_{\max}^2 - K_{\min}^2) = 2\Delta K K_m$, n is a numerical constant and β is a numerical factor dependent on frequency, material characteristics and loading conditions. Curves illustrating the relationship are included as Figs. 1 - 3.

It is interesting to compare equation (3) with the crack growth relationship recently proposed in [16], which has the following form:

$$\frac{d(2a)}{dN} = C_1 f^{\alpha_1} \Delta K^{\alpha_2} K_m^{\alpha_3} \quad (4)$$

and which is based on a general consideration of fatigue data on PMMA. The exponents of α_2 and α_3 were found to be 2.13 ± 0.18 and 2.39 ± 0.18 . Bearing in mind the nature of experimental work it may be stated that α_2 and α_3 are approximately equal; i.e. the exponents of the K_m and ΔK terms in the model are equal which then supports the form of the proposed equation (3). However there are advantages arising from the implementation of equation (3) in the form given.

4. Application of Equation (3) to Metals

In an attempt to test the possibility of applying equation (3) to the fatigue failure process in metals, data were extracted from the available literature on the relationship between $d(2a)/d(N)$, ΔK and the ratio R . The data on two Al-alloys (2024-T3, Fig. 4, and 7075-T6, Fig. 5, (obtained at a frequency

range of 0.5 to 13.7 Hz) were taken from [11]. Those on (9Ni-4Co-0.25C) steel, Fig. 6, (obtained at a frequency of 0.1 Hz) from [17] and those on cold rolled mild steel, Fig. 7, from [18]. These results have been plotted on the basis of parameter λ in Figs. 4 - 7.

The encouraging outcome of this exercise demonstrated that at least for the case of the above metals, the crack growth rate can be determined from a single set of data irrespective of the values of ΔK and R . Obviously similar checks on other metals must be made before the relationship is adopted as a general criterion for the prediction of fatigue crack growth rate. It may be noted that equation (1) is a special case of equation (3) since it was based on tests in which $K_{\min} = 0$, i.e. $K_{\max} = \Delta K$; hence it may be written as:

$$\frac{d(2a)}{dN} = C K_{\max}^2)^n (\beta \lambda^n) \quad (5)$$

when $n = \frac{m}{2}$.

5. The Energy Approach

The failure of solids, particularly polymers, must be treated in the light of the general phenomena governing the temperature-time dependence of strength. The total thermodynamic energy criteria applied to failure in monotonic fracture and cyclic fatigue of solids [19, 20, 21, 22, 23] have proved to be reasonably adequate. The fatigue problem has also been treated using a bulk plastic hysteresis absorption criterion [24]; through the development of a cyclic growth

relationship based on the crack driving force [25] and by the adoption of an approach based on the local dissipative processes near the tip of the crack [26]. Significantly the analysis in [26] resulted in a relationship of the form:

$$\frac{d(2a)}{dN} = \xi K^2 \quad (6)$$

pointing to the dependence of $d(2a)/dN$ on the term K^2 . However this analysis cannot be completely adopted as tests on its correspondence to empirical results have not been entirely successful.

The critical stress intensity factor, K_{1C} , is related to the strain energy release rate per unit crack extension G_{1C} , in the following form[27]:

$$G_{1C} = \frac{\xi K_{1C}^2}{E} \quad (7)$$

$$\text{where } \xi = \begin{cases} (1 - \nu^2) & \text{plane strain} \\ 1 & \text{plane stress} \end{cases}$$

and ν is the Poisson's ratio.

It is evident that equation (3) can be readily converted to an energy formulation by substituting for K_1 in terms of G_1 from the above relationship:

$$\frac{d(2a)}{dN} = \beta^* (G_{\max} - G_{\min})^n \quad (8)$$

$$\text{where } \beta^* = \beta \left(\frac{E}{\xi}\right)^n \quad (9)$$

It has already been shown that a total energy criterion can be applied to the analysis of the fatigue failure process

in viscoelastic materials [21]. In elastomers the cyclic rate of crack propagation has been determined [28, 29] from an energy function of the following form:

$$\frac{d(2a)}{dN} = \kappa (2k W a)^\alpha \quad (10)$$

where W is the term representing the strain energy density. Also the fatigue failure mechanism in polyethylene has been treated through implementation of equation (10), [30].

Thus there exists a considerable amount of data (largely empirical) pointing to the possibility of the development of a unified approach to fatigue failure based on cyclic energy balance and irrespective of the nature of the particular continuum. Cherepanov's [22, 23] analytical treatment can be considered the theoretical justification for such an approach. Whereas one of the limitations of the LEFM principles would be the required linear behaviour of the material, the energy based approach is not unduly held back by this factor. However, when the material under consideration is linear, the two approaches will correspond to each other.

A more generalised form of equation (8) can be obtained by replacing the energy term G by another parameter recently introduced [31, 32], namely the path independent contour integral, J . The applicability of the J integral to monotonic fracture is gradually being recognised [33]. At the present stage of its development, the line integral approach can be applied to two dimensional plane strain or generalised plane stress problems. In the case of stable fatigue crack propagation where only a small zone of plastic deformation is created at the crack tip during each cycle, the particular

case of

$$J_1 = G_1 = \frac{\zeta K_1^2}{E} \quad (11)$$

is considered. Equation (8) can thus be converted into the following form:

$$\frac{d(2a)}{dN} = \dot{a}_N = M A^n \quad (12)$$

$$\text{where } A = G_{\max} - G_{\min} \quad (= J_{\max} - J_{\min}). \quad (13)$$

$G_{\max}$ and $G_{\min}$ have been calculated from the corresponding maximum and minimum levels of the stress intensity factor achieved in each cycle.

Figs. 8 and 9 illustrate the $\dot{a}_N$ vs A relationship for some metals. Some of the data were obtained from plane strain and some from plane stress conditions and this has been appropriately taken into account when calculating G .

For the case of the two Al-alloys, Fig. 8, where the values of the modulus of elasticity E and also the values of $\dot{a}_N$ at the same levels were comparable, a single curve may be drawn with a slope of approximately 2.1. Such a single curve could not be drawn for the data on the two steels, Fig. 9. In this case the slope n increases from approximately 2 for the high strength steel to approximately 2.3 for the cold rolled mild steel. The fracture toughness K_{1C} of the latter material is sensitive to the variations in the loading rate however and the change in slope may thus be a consequence of changes in loading frequency [34]. Empirical data quoted for a number of metals indicate that the index n varies

between 2 and 2.4; hence an average value of 2.2 will give a reasonably good approximation to the actual crack growth rate.

For comparative purposes, in Figs. 10 and 11, part of the data on the three thermoplastic materials are similarly given in terms of the $\dot{a}_N$ vs A relationship. The G parameters for these materials were calculated using the glassy value E_g , of the time dependent modulus $E(t)$, in each case (Table 2). The magnitudes of the glassy modulus parameter for various materials were obtained from a study of the pattern of behaviour of the relaxation modulus data over a temperature range of (+21°C) to (-197°C), [35]. Obviously, due to the very high rate sensitivity of these materials, values of M (and in many cases also the index n) are influenced by changes in loading frequency. The values of n are indicated in each of the figures and values of M are brought together in Table 3.

6. The Crack Opening Displacement and the Plastic Zone Size

Fatigue failure is a strain dependent process, thus it is reasonable to assume that the rate of crack growth would be a function of the actual separation of the two crack surfaces during each cycle. The concept of crack opening displacement (COD), [36], for a specific strain rate and stress state is related to the energy criterion:

$$\delta = \text{COD} = \frac{G_1}{\sigma_{yp}} = \frac{\zeta K_1^2}{E \sigma_{yp}} \quad (14)$$

Clearly, wherever convenient, equation (3) can be expressed

in terms of COD through a replacement of K_1^2 by $\frac{E \delta}{\zeta} \sigma_{yp}$:

$$\dot{a}_N = \beta \left(\frac{E \sigma_{yp}}{\zeta} \right)^n (\delta_{max} - \delta_{min})^n \quad (15)$$

In a similar manner, relating the crack tip strain to the crack tip plastic deformation, as a consequence of which fatigue crack growth occurs [37, 4, 38], the crack growth rate can be predicted in relation to the crack tip plastic zone formation. The size of the plastic zone is dependent upon the upper bound of the stress intensity factor cycle, i.e. K_{max} ; and the length of the region within the plastic zone starting from the crack tip which undergoes reversed deformation as unloading takes place can reasonably be assumed to be dependent on the magnitude of the reduction in K_{max} . The small cyclic increase in the crack length is related to the length of this region and hence to the difference in the plastic zone size (r_p) corresponding to K_{max} and K_{min} . Also r_p is related to the stress intensity factor [6] by

$$r_p = \frac{1}{4/2 \pi} \left(\frac{K_1}{\sigma_{yp}} \right)^2 \quad \text{plane strain.} \quad (16)$$

$r_{p_{max}}$ will correspond to K_{max}^2 and $r_{p_{min}}$ may be thought of in terms of K_{min}^2 and hence the cyclic rate of crack growth will be related to the difference of these squared terms:

$$\dot{a}_N = \beta (4/2 \pi \sigma_{yp}^2)^n (r_{p_{max}} - r_{p_{min}})^n \quad (17)$$

Again, in rate dependent materials, the yield stress, σ_{yp} , will be affected by changes in frequency (or load rate)

and thus the value of r_p will be correspondingly influenced.

7. Conclusions

Presentation of tensile fatigue data on metals and polymers in the above form points to the possibility of further development of an energy based criterion, such as equation (12), for the prediction of fatigue crack growth rates in both types of materials, under various K_{max} , K_{min} and frequency levels. An obvious advantage of the energy formulation, which introduces the material characteristics into the crack growth equation will be its applicability to a very wide spectrum of media. In addition it is possible that the failure cases in which reasonably large plastic deformations are involved - thus prohibiting the direct application of the stress intensity factor concept - could also be treated.

The correspondence between the strain energy, the crack opening displacement and the plastic zone size approaches has been pointed out in terms of the modified forms of the equation:

$$\dot{a}_N = M A^n$$

The proposed model appears to project a unifying pattern on most of the previously adopted fatigue laws which have been obtained both empirically and analytically.

For metals, the value of index n varies between 2 and 2.4 for the range of data considered; thus it could be stated that an average $n = 2.2$ would give a good approximation to

the value of $\dot{a}_N$. The task of experimentalists would then consist mainly of tabulating the values of M for various materials (see Table 3), the knowledge of which will be sufficient to enable a designer to predict by a good approximation, the crack growth rate at a particular value of A .

In the case of polymeric materials, both n and M will be frequency dependent and thus need to be specified as an average for a given frequency range.

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NOTATION

a	Half crack length
A	$G_{\max} - G_{\min}$
C, C_1	Constants in fatigue crack growth equation
E	Young's modulus of elasticity
$E(t)$	Time dependent modulus (viscoelastic materials)
E''	Loss component of the complex modulus (viscoelastic materials)
E_g	Glassy modulus (viscoelastic materials)
f	Loading frequency
G_1	Strain energy release rate per unit crack extension corresponding to mode 1 crack opening
G_{1c}	Critical level of G_1
$G_{\max}, G_{\min}$	Maximum and minimum values of G_1
J_1	The path independent contour integral
$J_{\max}, J_{\min}$	Maximum and minimum values of J_1
k	Numerical parameter in crack growth equation
K_1	Stress intensity factor for mode 1 opening
K_{1c}	Critical value of K_1
$K_{\max}, K_{\min}, K_m$	Maximum, minimum and mean values of K_1
ΔK	$K_{\max} - K_{\min}$
M	Numerical factor in crack growth equation
m, n	Numerical exponents in crack growth equation
N	Number of cycles
R	$K_{\min}/K_{\max}$
r_p	Plastic zone size at the crack tip

W	Strain energy density
W_v	Viscoelastic energy absorbed in each load cycle
$\alpha_{1,2,3}$	Exponents in crack growth equation
β, β^*	Numerical factor in crack growth equation
δ	Crack opening displacement
λ	$K_{\max}^2 - K_{\min}^2 = 2\Delta K K_m$
ν	Poisson's ratio
ξ, κ	Constants in crack growth equations
σ_{yp}	Yield stress

LEGENDS

- Fig. 1 Cyclic crack growth rate data for polymethylmethacrylate at a loading frequency of 5 Hz.
- Fig. 2 Cyclic crack growth rate data for polycarbonate at a loading frequency of 5 Hz.
- Fig. 3 Cyclic crack growth rate data for nylon 6.6 at a loading frequency of 0.1 Hz.
- Fig. 4 Cyclic crack growth rate data for aluminium alloy (2024-T3); Hudson (1969).
- Fig. 5 Cyclic crack growth rate data for aluminium alloy (7075-T6); Hudson (1969).
- Fig. 6 Cyclic crack growth rate data for (9Ni-4Co-0.25C) steel; Crooker and Lange (1968).
- Fig. 7 Cyclic crack growth rate data for cold rolled mild steel; Frost et al (1971).
- Fig. 8 Data on two aluminium alloys plotted on the basis of parameter A.
- Fig. 9 Data on cold rolled mild steel and (9Ni-4Co-0.25) steel plotted on the basis of parameter A.
- Fig. 10 Fatigue crack growth rate data for polymethylmethacrylate and polycarbonate plotted as a function of parameter A.
- Fig. 11 Cyclic crack growth rate data for nylon 6.6 at two frequencies plotted as a function of parameter A.

Table (1)

Test of Forman et al (1967) model for data on polymers. Arbitrary values of $m = 3$ and $C = 1$ are used. Loading frequency is constant in all tests (0.1 Hz).

$K_{\max}$	$K_{\min}$	$\dot{a}_N$ (in/cycle) (Experimentally determined)	Predictions from Forman et al model
lbf/in ^{3/2}			
750	500	210×10^{-6}	35511.1
560	50	210×10^{-6}	19788.0
610	250	207×10^{-6}	15250.0
660	350	213×10^{-6}	8990.0

Table (2)

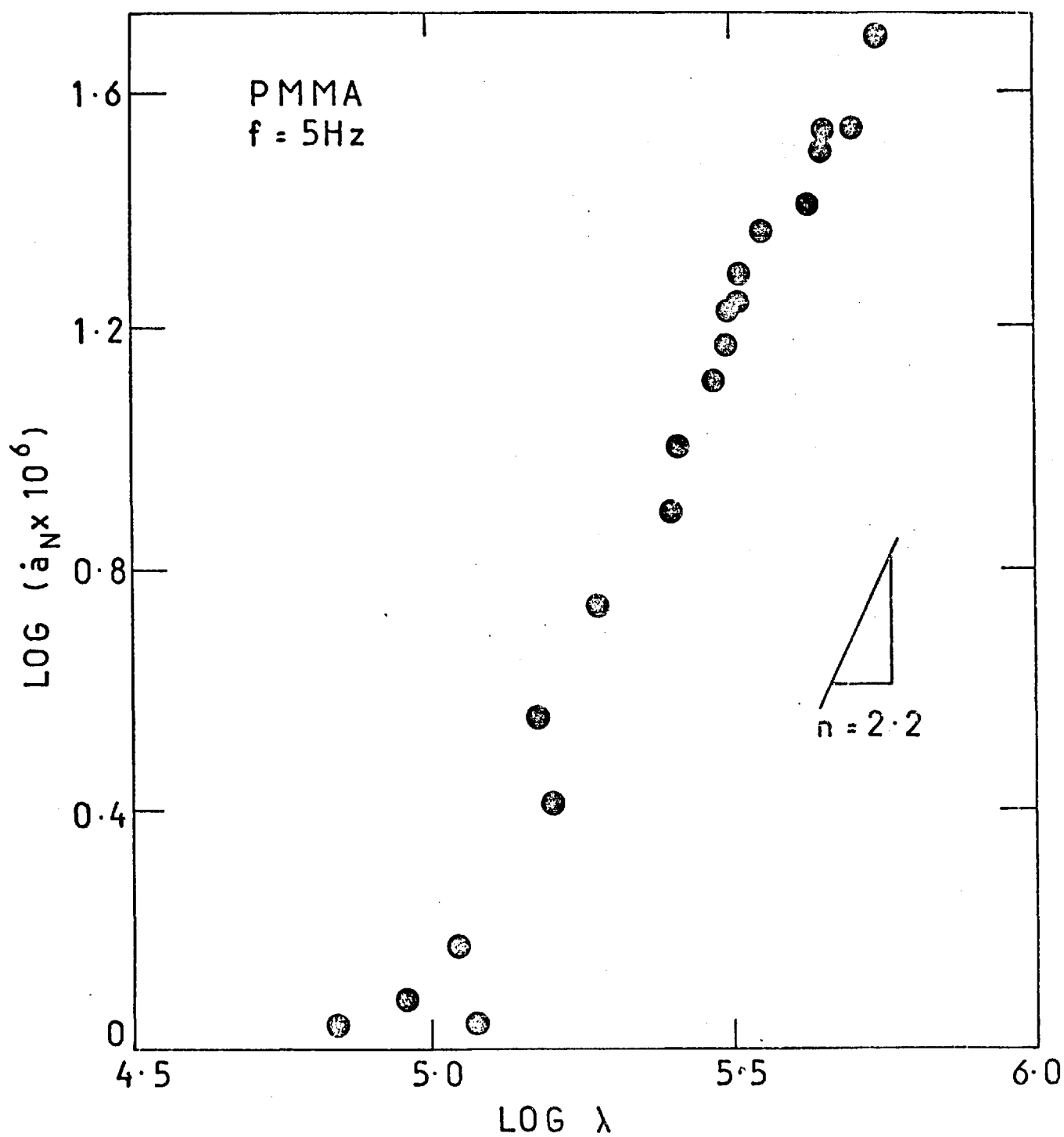
Approximate Values of the Glassy Moduli

Material	PMMA	N 6.6	PC
E_g (PSI)	950,000	950,000	600,000

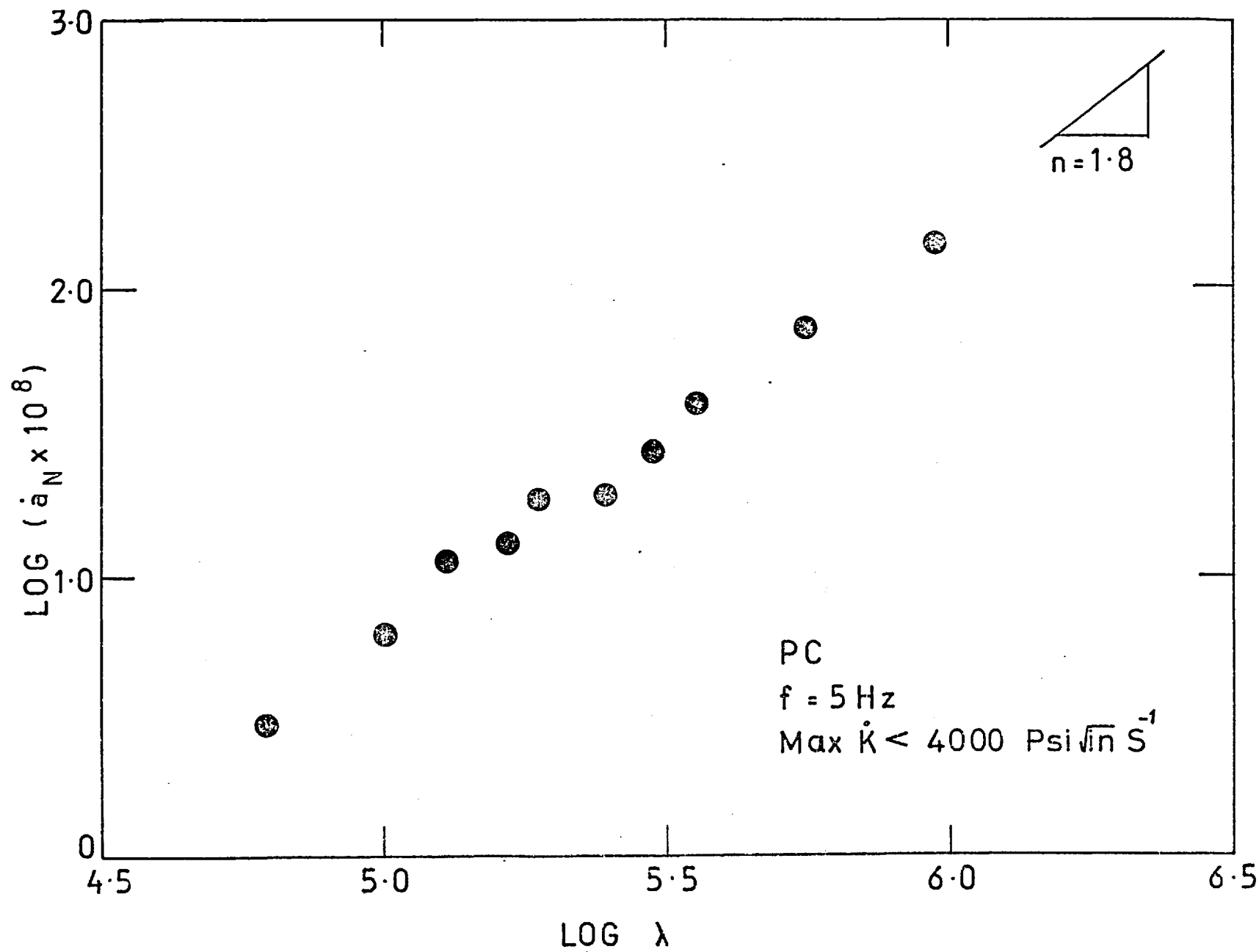
Table (3)

Tabulation of Values of Parameter M in Equation (12)
for Various Materials

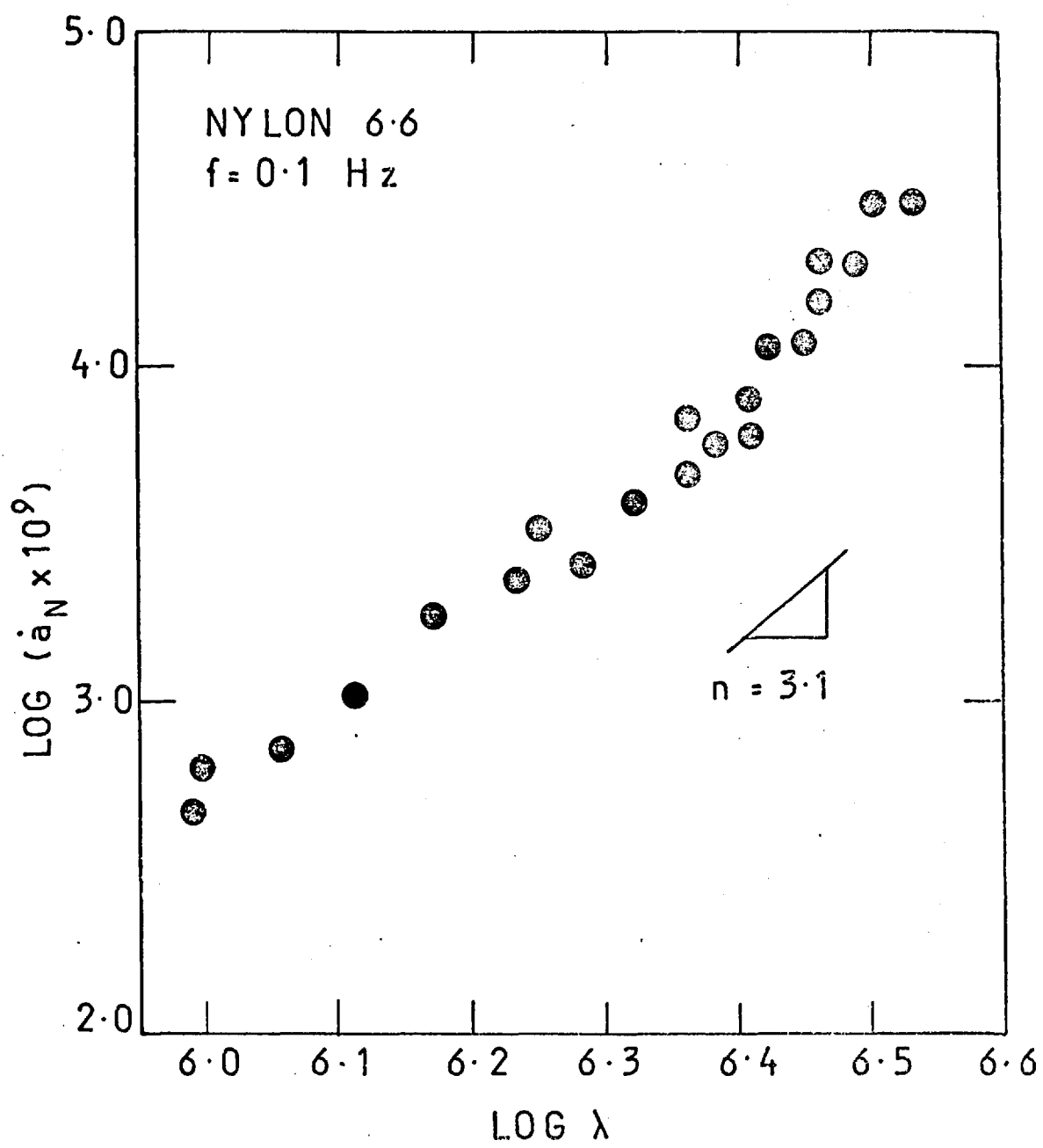
MATERIAL		M
Al Alloy (7075 - T6)		1.0×10^{-7}
Al Alloy (2024 - T3)		1.0×10^{-7}
Cold Rolled Mild Steel		3.4×10^{-7}
(9Ni - 4Co - 0.25C) Steel		1.6×10^{-7}
	Frequency (Hz)	
	5	2.0×10^{-6}
	5	1.0×10^{-4}
	0.1	1.1×10^{-6}
	5	0.6×10^{-7}



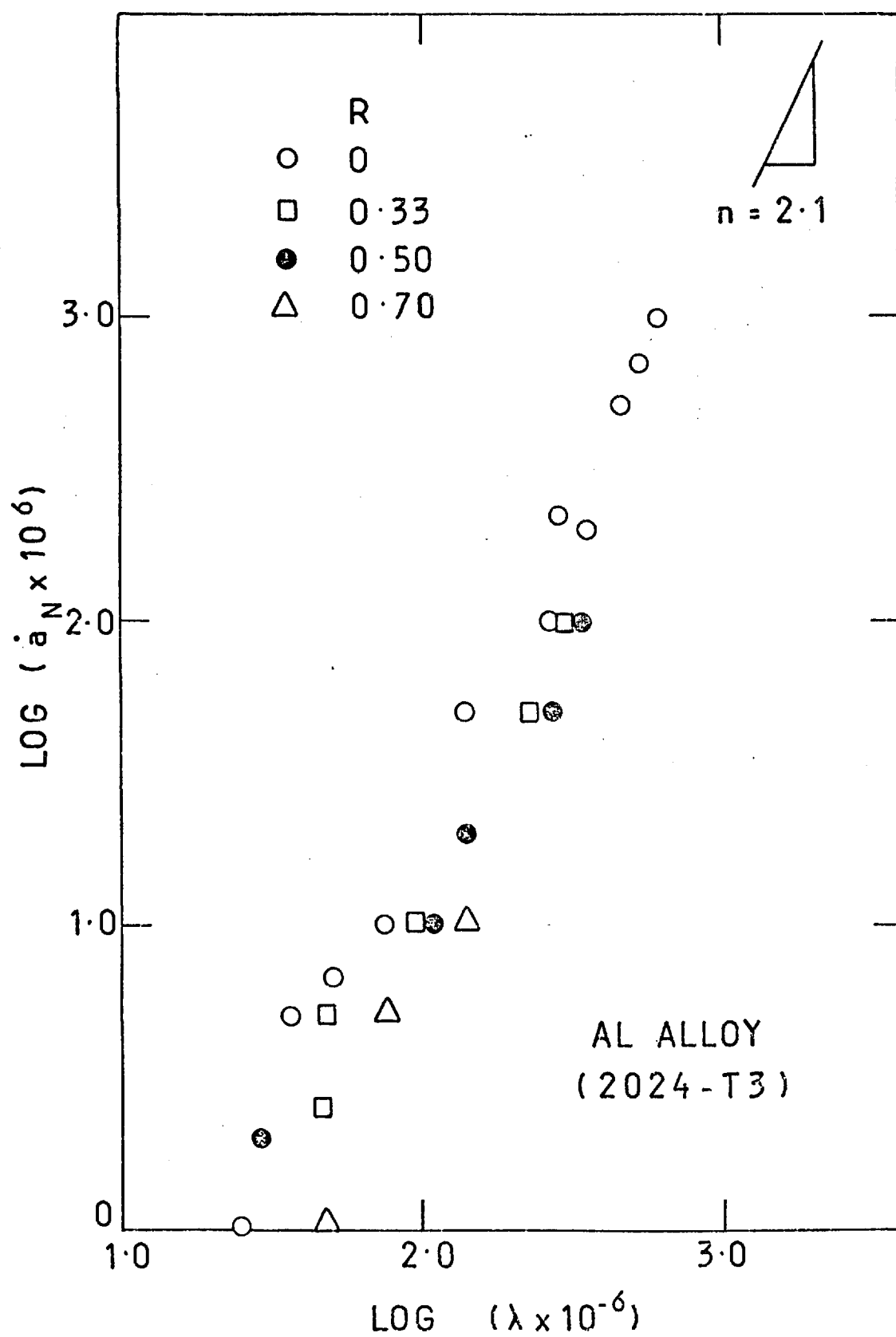
(1)

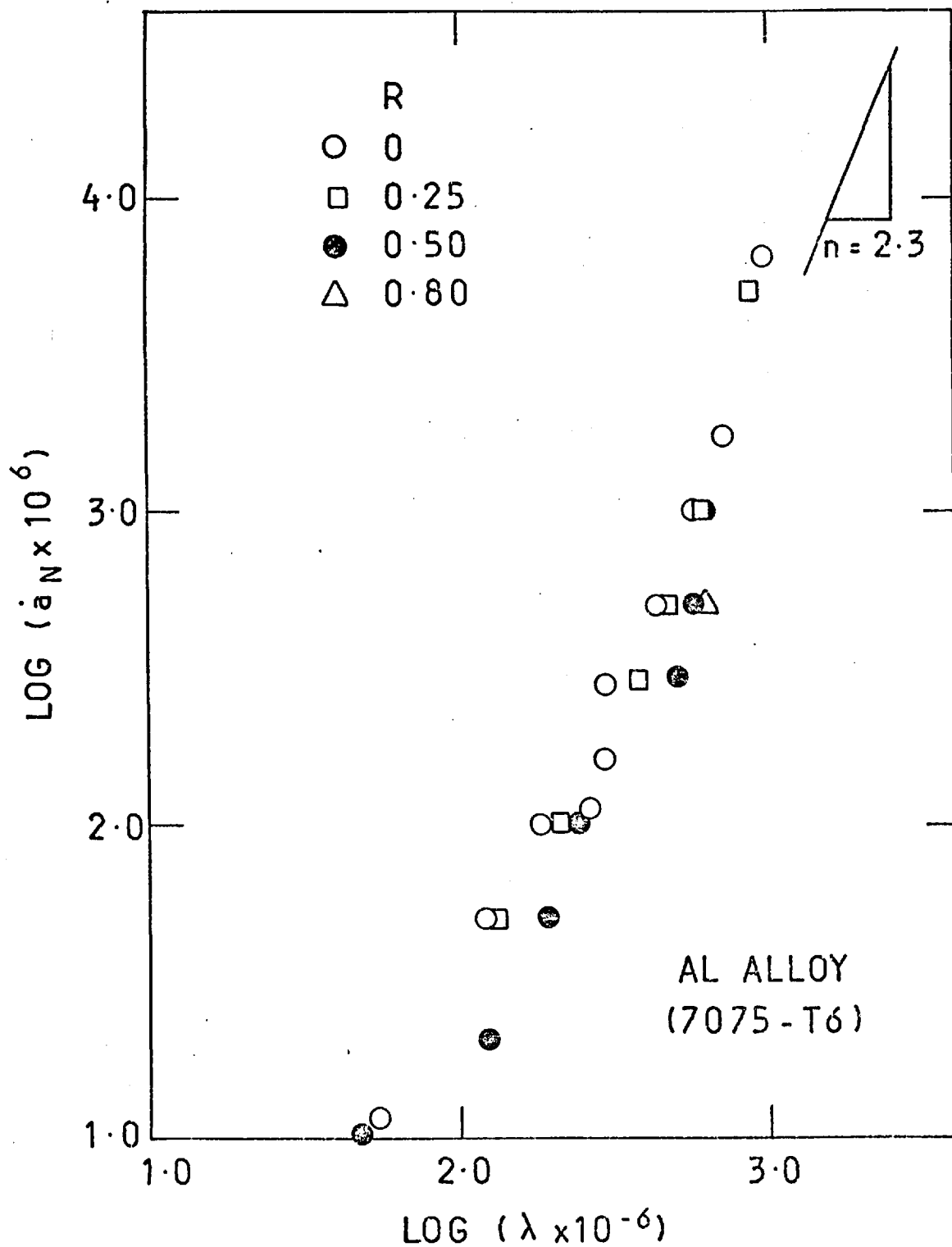


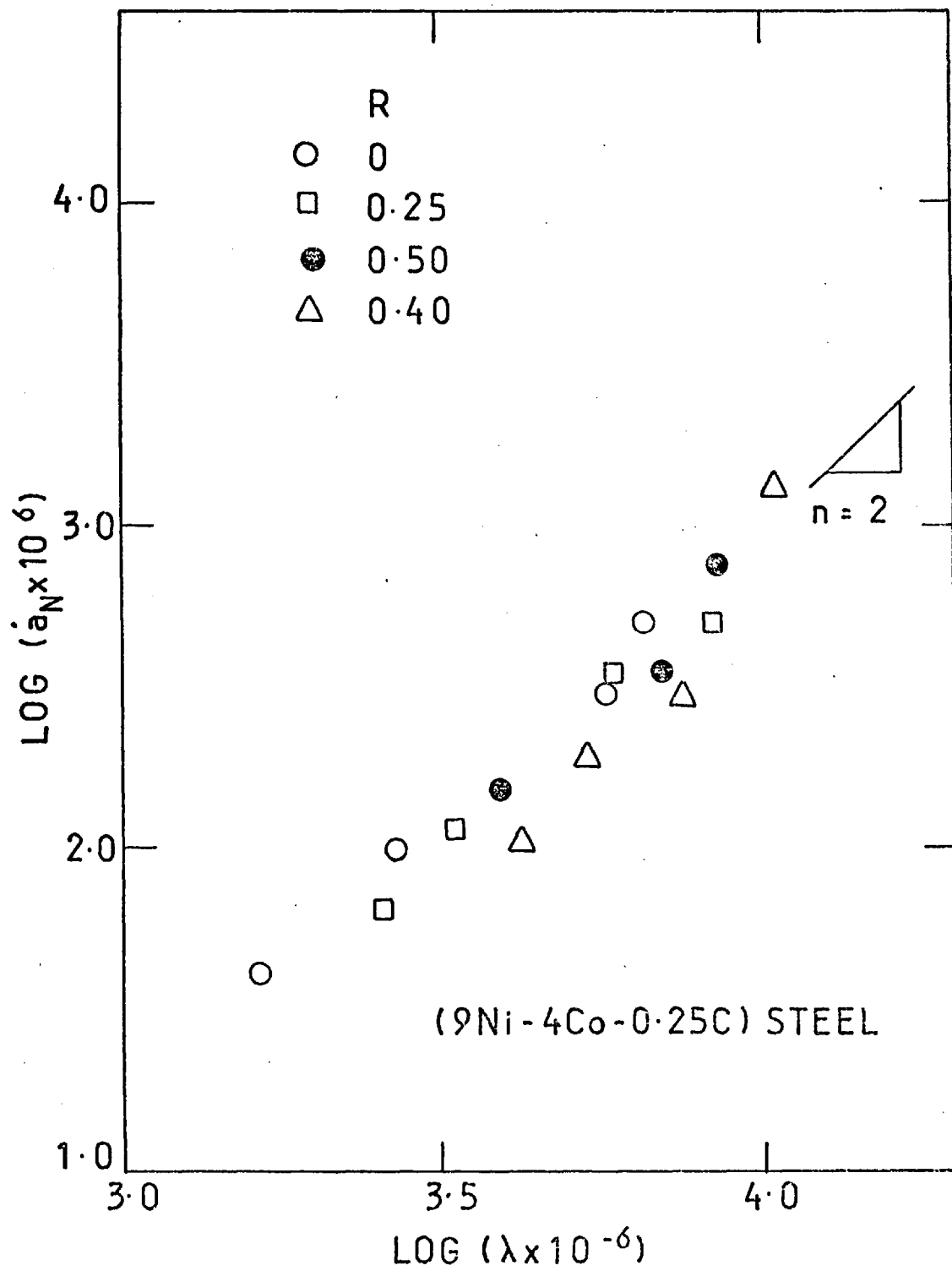
(2)



(3)







(b)

