

Optimal Design of Multi-echelon Supply Chain Networks

by

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A B S T R A C T

Much of the research in the area of supply chain management has been focused on individual aspects of the supply chain, such as purchasing, production scheduling, facility location, product pricing and transportation. The motivation for the work described in this thesis is the need for models that integrate many of these components within a single framework while taking account of some of the inherent uncertainty.

We consider geographically-distributed multi-echelon supply chain networks in which a set of manufacturing plants produce a relatively large number of products making use of shared production resources. The products may be delivered to customers either directly or via one or more intermediate warehousing stages. Demand for products may be both time-varying and uncertain, as described in terms of a number of possible scenarios.

The objective of the optimisation is to determine cost-optimal network structures that can deal with one or more demand scenarios, each of given probability. The network may comprise both existing manufacturing plants and warehouses, and new ones, the geographical locations of which are to be selected from a given set of candidates. The transportation links that are necessary for the network to operate properly are also to be determined by the optimisation. Operational decisions include the production rates of the various materials, the transportation flows, and the determination of how customers are to be served by the network, in view of the need for a single point of contact for each customer (“single-sourcing” constraints).

The optimisation seeks to minimise the combined annualised infrastructural and operational costs of the network. The former are associated with the establishment of manufacturing plants and warehouses, and the provision of appropriate levels of manufacturing resources at the plants. Operational costs comprise the expected values of the production, material handling and transportation costs over all the scenarios under consideration. Transportation costs may be subject to significant economies of scale.

The thesis considers several variations of the above general problem. The first problem assumes that the network comprises four echelons, namely those of manufacturing plants, major warehouses, minor distribution centres and customer zones, with material being allowed to flow only from one echelon to the next. Moreover, the manufacturing plants and customer zones are at fixed locations.

Steady-state and dynamic versions of this problem are considered, both being formulated as mixed integer linear programming (MILP) problems.

The thesis then proceeds to propose a more general model for supply chain networks that removes the restrictions mentioned above. The problem is formulated in terms of a network of generalised production-warehousing nodes that may or may not be established at a set of candidate locations. The optimisation calculation automatically determines the precise function (manufacturing or warehousing or both) of each node that is included within the network. No restriction on the flow of material is imposed. Thus, customers may be served directly from manufacturing plants, or indirectly via one or more warehousing stages; the number of the latter may differ from one customer to another. Also, nodes (e.g. manufacturing plants) may exchange intermediate materials with each other. Such generalised networks are found to provide more efficient solutions than their more restricted counterparts.

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“The more arguments you win, the fewer friends you’ll have.”

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Introduction

1.1. Definition of a supply chain

A supply chain is a network of facilities and distribution mechanisms that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and distribution of these products to customers (Ganeshan and Harrison, 1995). A similar definition has been given by Bhaskaran and Leung (1997) who describe the manufacturing supply chain as an integrative approach used to manage the inter-related flows of products and information among suppliers, manufacturers, distributors, retailers, and customers.

A typical supply chain (see figure 1.1) comprises suppliers, production sites, storage facilities and customers. It includes two basic processes tightly integrated with each other:

- the Production Planning and Inventory Control Process, which deals with manufacturing, storage and their interfaces;
- the Distribution and Logistics Process, that determines how products are retrieved and transported from the warehouse to retailers.

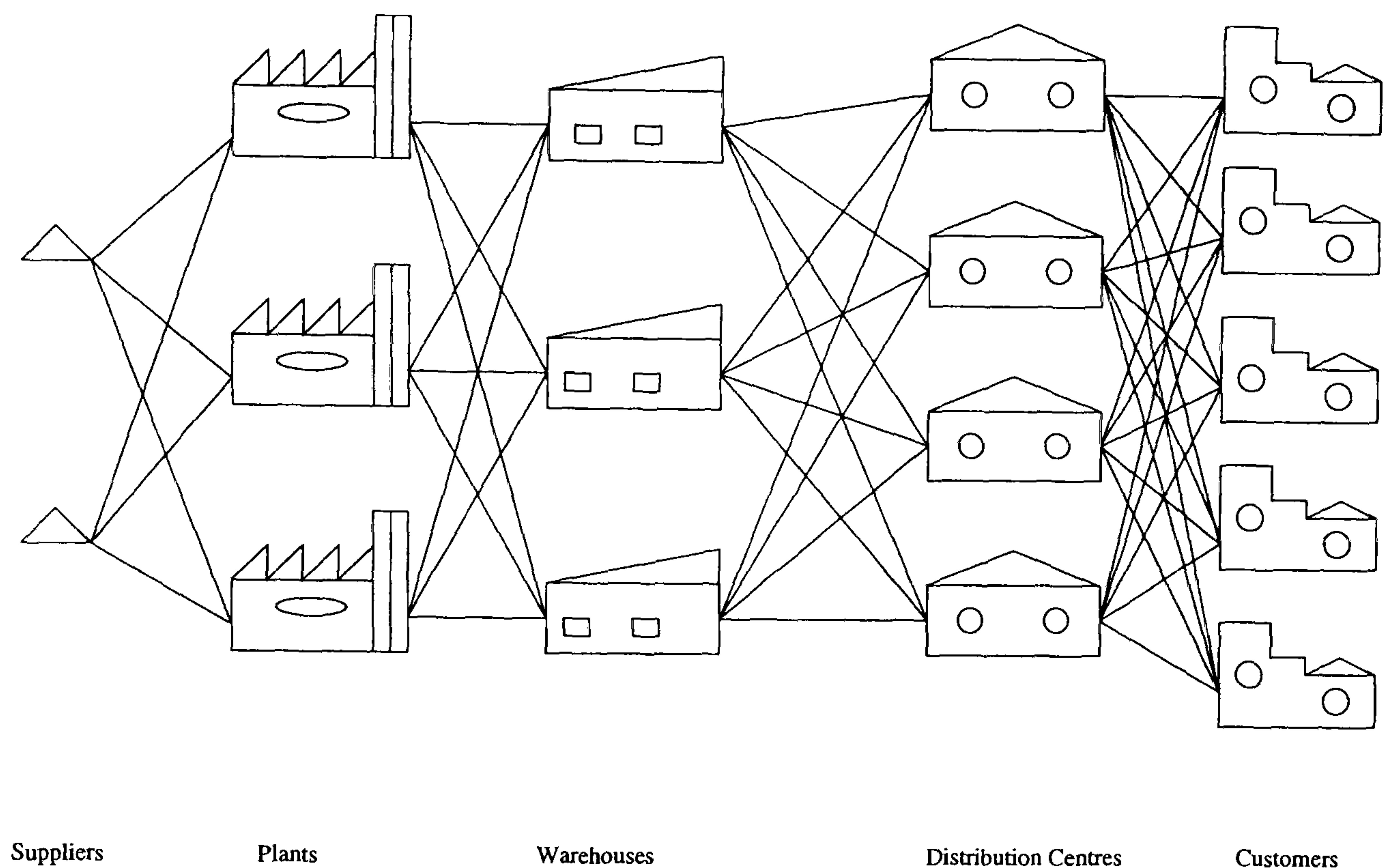


Figure 1.1: Typical supply chain network.

Suppliers are at the start of the supply chain providing raw material to the manufacturers. Each manufacturer may have more than one supplier.

The manufacturing sites of interest to this thesis are multipurpose production plants where a wide range of products can be produced. The production capacity of each site is determined by the detailed scheduling of each plant.

Before being distributed to the customers, the final products from production plants are often stored at one or more distinct stages in the supply chain. Figure 1.1 shows a typical situation involving storage at two stages, namely at major warehouses and at smaller distribution centres. Each warehouse may be supplied material from more than one manufacturing site. On the other hand, each distribution centre is usually supplied by only one warehouse. Both the storage and handling material capacity of warehouses and distribution centres are limited within certain bounds.

At the end of the supply chain, there are the customers. Usually, each customer is assigned to a single distribution centre which supplies all the required

material. Customers typically place their orders at distribution centres which pass this information to the upper levels until it gets to the suppliers. Thus, a key characteristic of the supply chain is the flow of material from suppliers towards the customers and the counter flow of information from customers to the suppliers (Beamon, 1998).

The places where inventory is maintained in the supply chain are called “echelons”. The complexity of a supply chain is directly related to the number of echelons that it incorporates.

1.2. Supply chain management

The operation of supply chains is a complex task due to the large-scale physical production and distribution network flows, the uncertainties associated with the external customer and supplier interfaces, and the nonlinear dynamics associated with internal information flows. It is, therefore, not surprising that the significance of many theories related to the corporate management of the supply chain is still questioned (Krautter, 1999).

In a highly competitive environment, a supply chain should be managed in the most efficient way, with the objectives of:

- *minimisation* of costs, delivery delays, inventories, investment;
- *maximisation* of profit, return on investment (ROI), customer service level, production.

The above tasks involve both strategic and operational decisions, with time horizons ranging from several years down to a few hours respectively (Ganeshan and Harrison, 1995):

1. *Location decisions* concern the number, size and physical location of production plants, warehouses and distribution centres.
2. *Production decisions* concern the products to be produced at each plant, and also the allocation of suppliers to plants, of plants to distribution centres, and of distribution centres to customers.

The detailed production scheduling at each plant must also be decided.

3. *Inventory decisions* are concerned with the management of the inventory levels.
4. *Transportation decisions* determine the transportation media to be used, and the size of each shipment of material.

As supply chains become increasingly global, additional aspects such as differences in tax regimes, duty drawback and avoidance, and fluctuations in exchange rates, also become important (Vidal and Goetschalckx, 1997).

1.3. Supply chain integration as a business process

1.3.1. The route to integration

The establishment of a supply chain seeks to integrate several operations that may have previously been independent cost centres within a certain organisation (or even different organisations), more-or-less in isolation and with little knowledge of each other's operating policies or even inventories. The operation of the supply chain directly affects five basic variables that are commonly used in measuring business performance, namely sales, cost of goods sold, expenses, inventory, and accounts receivable. Consequently, the establishment of an integrated supply

chain is a complex undertaking that may involve a substantial redesign of business processes.

The term *Business Process Reengineering* (BPR) has been defined by Hammer and Champy (1993) as “the fundamental rethinking and radical redesign of business processes to achieve dramatic improvements in critical, contemporary measures of performance, such as cost, quality, service and speed”. BPR comprises four basic stages, namely mobilisation of a team, diagnosis to identify weaknesses in existing design, redesign to create breakthroughs, and transition to roll out and institutionalise the results.

At a more detailed level, a four stage process for establishing an integrated supply chain has been described by Stevens (1989) and is illustrated in figure 1.2.

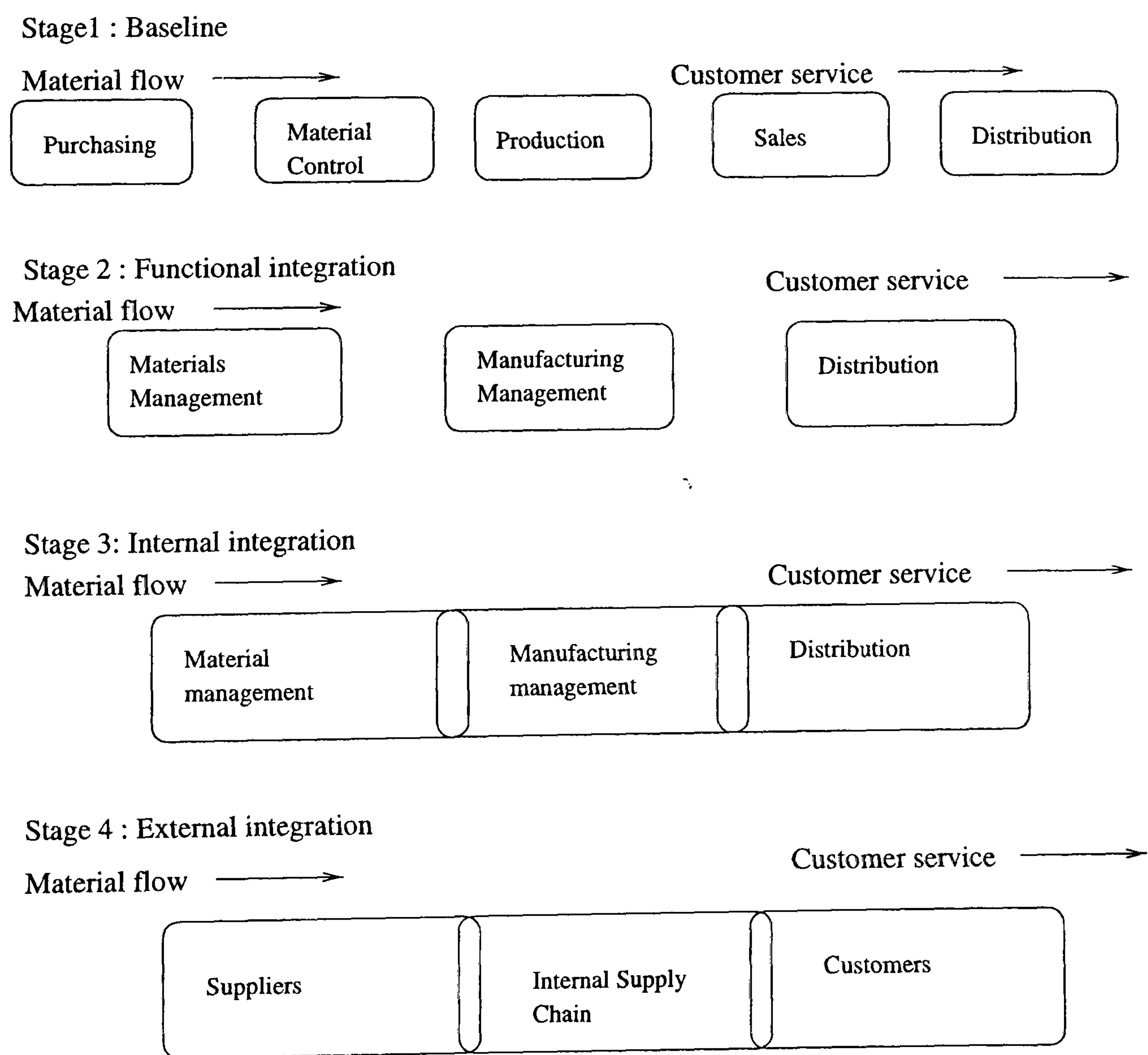


Figure 1.2: Stages in supply chain integration (Stevens, 1989)

Stage 1 is typical of a company with a fragmented supply chain. It is divided into departments, each operating independently from the rest. Activities cannot be synchronised, and staged inventories are held. Substantial organisational boundaries exist.

Stage 2 involves factory functional integration which focuses principally on the inward flow of goods. Emphasis is placed on cost reduction rather than performance improvement. Discrete business functions still exist. Companies apply planning to the materials and manufacturing management using MRP (material requirements planning) and MRP-II (manufacturing resource planning).

Stage 3. This stage recognises that there is little point on focusing only on material flow unless this flow is well managed on the way to the customer. Thus, a local integration of those aspects of the supply chain which are directly under the control of the company is undertaken. The company uses JIT (just-in-time) manufacturing techniques to support the execution of the material plan derived by the MRP-II system.

Stage 4. Full supply chain integration is achieved by extending the scope of management outside the company to embrace suppliers and customers. The company changes its focus from being product-orientated to customer-orientated, penetrating deep into the customer organisation to understand the products, culture, market and organisation.

1.3.2. Business aspects of supply chain management

The management of supply chains has been studied extensively in two well known papers. First, Lee and Billington (1992) discuss pitfalls and opportunities for managing supply chain inventory since, in their view, inventory levels are what the supply chain tries to control. The problems appearing in a supply chain environment are usually the outcome of the complex interactions among its con-

stituent parts since the management of a supply chain is very different from the management of a single site.

Lee and Billington (1992) consider three areas of potential difficulties: 1) pitfalls related to information definition and supply chain management, 2) operational related problems, and 3) strategic and design related problems. These problems include the absence of supply chain metrics, deficiencies in the definition of customer service, inefficient information systems, operating issues (like inventory stocking and coordination), and finally organisational issues related to the structure of the business, product design and development, and the nature of the supply chain.

In addressing the above pitfalls, a number of opportunities arise, especially in the areas of product design, integration and improvement of database systems, more robust decisions about planning and control, and finally better metrics to characterise supply chain performance.

The design problems and their complexity for a distribution network as part of the supply chain were identified by Geoffrion and Powers (1980) who focused on warehouse related issues. The latter involves four decisions: 1) warehouse mission, 2) warehouse design, 3) warehouse location, and 4) operational utilisation. They discuss a comprehensive distribution planning system which is composed of the “sources” (plants, copackers, vendors), existing and candidate “distribution centres (DCs)” (master distribution centres, regional distribution centres, branch warehouses), and “customers” (aggregations of individual customers).

Erenguc *et al.* (1999) examine integrated production and distribution planning in supply chains. They point out that the traditional way of determining the operating policy for companies has involved a choice between flexibility and quality on one hand, and product cost on the other. In order to obtain one of those, the other one had to be sacrificed, at least partially. However, today’s competitive business environment requires both high quality and low product cost. Customer

service levels should not be traded-off against other factors.

1.3.3. Supply chain metrics

Optimal supply chains seek to accomplish a number of targets (set by the business process) at minimum cost and maximum profit for the company. These targets may be product quality, deliveries at the right place, size and amount, etc. A key requirement for the optimal design and operation of a supply chain is the ability to measure its performance. A number of such measures were described by Beamon (1998) who categorised them as either qualitative or quantitative.

Qualitative performance measures include:

- customer satisfaction
- flexibility
- information and material flow integration
- effective risk management
- supplier performance.

Quantitative performance measures are categorised into:

1. Measures based on direct financial indicators such as: cost minimisation, sales maximisation, profit maximisation, inventory investment minimisation and return on investment maximisation.
2. Measures based on customer service level such as: fill rate maximisation, product lateness minimisation, customer response time minimisation, lead time minimisation and function duplication minimisation.

In a more recent work, Beamon (1999) presented a framework for the evaluation of the performance of supply chains. She points out that the process of

choosing appropriate measures is generally difficult due to the complex dynamics of the supply chain.

1.3.4. Supply chain partnerships

A *supply chain partnership* (or strategic alliance) is a relationship formed between two or more independent entities in supply channels to achieve specific objectives and benefits (Maloni and Benton, 1997). In particular, the partnership aims to provide the end-customer with higher quality in a better value package, while the partner companies reduce their production costs and inventories and increase the levels of shared information. This integration also results in a reduction of the supplier base, which leads to an abatement of adversarial attitudes, lower production switching costs and fewer shipping errors.

Maloni and Benton (1997) also point out that supply chain partnerships support a move away from the concept of discrete (“one-off”) transactions between organisations. Instead, they require the establishment of long-term relations, complex (two-way sharing) communication, extensive and sincere management support while the attitude must be open, trusting and cooperative. In addition, long-term plans and goals are established jointly while, benefits and risks are shared fairly between both parties (Gjerdrum *et al.*, 2001).

The critical elements of a supply partnership compared to traditional thinking are presented in table 1.1.

There are both benefits and risks associated with such partnerships. Suppliers gain from higher quality, and transaction costs may be reduced through economies of scale, decreased administrative and switching efforts, process integration, coordination of processes and quantity discounts. Additional benefits may be set-up time reduction, improved process-oriented layout and better product design. Furthermore, the partners can benefit from increased communication

Traditional supply relationships	Supply chain partnerships
Price emphasis for supplier selection	Multiple criteria for supplier selection
Short-term contracts for suppliers	Long-term alliances with suppliers
Bid evaluation	Intensive evaluation
Large supplier base	Few suppliers
Proprietary information	Shared information
Power driven problem solving	Mutual problem solving

Table 1.1: Traditional versus partnership supply strategies.

and goal congruence, leading to enhanced conflict resolution, less probability for opportunism and decreased risk from externalities. On the other hand, possible risks for the participants may include heavy reliance on a partner who does not meet expectations. Also, firms risk decreased competitiveness due to loss of partnership control, complacency and over-specialisation with an affirmed partner.

1.3.5. Product design and supply chain management

A new aspect of supply chain optimisation that has recently been receiving attention is the quest to improve the supply chain performance by optimising the design of the products themselves.

The design for supply chain management (DFSCM) theory of Lee and Sasser (1995) aims at designing products and processes so that the supply chain related costs and performance can be managed more effectively. The design of the product determines to a large extent the total cost of producing and delivering it, including both manufacturing and logistical elements.

The scope for restructuring or redesigning a supply chain at later stages is limited by the fact that a product has already been designed and introduced in the marketplace. Thus, the supply chain has to be designed in parallel with the product development. This concept was applied by the Hewlett-Packard company for a new product (Lee, 1996).

Lee and Sasser (1995) describe a model to evaluate logistics costs and service performance. The model is based on the assumptions of stationary demands and the same lead time for all markets. Product universality (i.e. having a unique product for all markets, and minimising differentiation) is achieved by the assumption of a key module for the product for any market. Then, an analysis is presented for the control of the inventory level for each key module. A safety factor (Lee and Billington, 1993) is introduced to ensure a specified service target.

Besides inventory, there are other factors that must be considered. First among these is the demand which varies through the life cycle of the product (Davis, 1993). The life cycle of each product is different and depends strongly on the product type. In general, however, life cycles can be separated into three distinct phases: the introductory, mature and end-of-life phases. The cost of holding inventory is the same for the first two phases. Another situation that often causes problems is stock-out which may lead to product delivery delays. Trans-shipment sometimes offers a solution to such problems, especially when the stock-out appears in the introductory phase. An issue that arises in this case is the cost of any rework (Lee and Tang, 1997) that might be necessary to customise the right product for each distinct market (e.g. of laptop computers having different voltage requirements in the U.S.A. and in Europe.)

1.4. Aims and structure of this thesis

Research in the area of production and distribution systems has been active for many years within the context of supply chain management. Much of this research has been focused on individual aspects of the production-distribution system, such as purchasing, production scheduling, facility location, product pricing and transportation. The absence of models that integrate all these components within a single framework provides the motivation for our work. Such a model would

allow a better understanding of the various interactions in these complex systems, ultimately permitting their optimal design; currently available techniques usually lead to merely feasible or, at best, near-optimal solutions. In view of the very large financial flows involved in supply chains, optimisation is important since even modest improvements can lead to major financial benefits.

The objective of this thesis is to develop a systematic approach to the planning of production at multiple manufacturing sites, as well as the distribution mechanisms between the sites, and also the design of such networks. We are particularly interested in sites that comprise multipurpose manufacturing facilities. Moreover, we want the optimal design of such networks to take account of changing circumstances, such as demand variations that cannot be predicted with certainty at the time when the design exercise is carried out.

Chapter 2 of this thesis presents a critical review of past work on supply chain modelling and design, and uses this to define the objectives and scope of this thesis.

In Chapter 3, the problem of optimal design of supply chains is formulated as a Mixed Integer Linear Programming (MILP) model. A scenario-based approach is used to take account of uncertainty in product demands. The latter includes some of the features that earlier models have failed to consider. A case study is presented in chapter 4 to illustrate the applicability of our model.

In contrast to the model described in chapter 3 which considers the steady-state (*i.e.* time-averaged) operation of the supply chain, chapter 5 describes a dynamic model that takes account of temporal demand variations. It also revisits the case study of chapter 4 in a time-varying demand context.

Chapters 3 and 5 are concerned with supply chain networks comprising a fixed number of echelons between suppliers and customers. Chapter 6 presents a general model for the design of supply chain networks that removes that restriction. The advantages of this approach compared with the fixed echelon structure approaches

are explained, and a case study demonstrating these points is presented.

Finally, chapter 7 concludes the thesis summarising the work that has been done and outlining possible directions for future work in the area of supply chain management and MILP solution methods.

Literature Review

In view of their importance in modern market economies, it is not surprising that supply chains have been on the research agenda of a variety of business and other academic disciplines for many years.

The tight integration of several, previously independent, businesses into a single supply chain is, in itself, a complex process having important business implications and involving many managerial and organisational issues. These have been discussed extensively in the business management literature which has been reviewed briefly in section 1.3.

On the more technical side, the design and management of supply chains involve a large number of interacting decisions (cf. section 1.2). This work is primarily concerned with models and tools for supporting these decisions, and the relevant literature is reviewed in section 2.1.

Section 2.2 provides a brief review of supply chain dynamics. Albeit of secondary interest to this work, this is an important aspect of the day-to-day operation of supply chains.

Finally, section 2.3 discusses the motivation and scope of this work in the light of the earlier work in this area.

2.1. Supply chain optimisation

Section 1.3 has reviewed some of the literature on the business management aspects of supply chain integration. In this section, we turn our attention to model-based methodologies that can provide *quantitative* support to the design and operation of supply chains. The major decisions that need to be made in this context have already been described in section 1.2 of this thesis.

2.1.1. Heuristic-based approaches

Williams (1981) presents seven heuristic algorithms for scheduling production and distribution operations in supply chain networks. The objective is to determine a minimum cost production and product distribution schedule, satisfying the product demands within a given distribution network. The seven models are compared with each other and with a dynamic programming model, using four different network structures namely series, assembly, arborescence and conjoined assembly-arborescence. Two simplifying assumptions are made for this study:

- Demand rate is constant.
- Processing is instantaneous and there are no delivery lags between facilities.

The risks arising from the use of heuristics in distribution planning were identified and discussed early on by Geoffrion and Roy (1979). They pointed out that heuristics (computerised or not) can lead to wrong decisions. Three examples were presented in the area of distribution planning considering questions such as the number, size, and location of plants and distribution facilities, stock of materials, and the policies regarding inventory and transportation.

All three examples demonstrate the failure of “common sense” methods to come up with the best possible solution. This is due to the failure to enumerate

all possible combinations, the use of local improvement procedures instead of global ones, and the failure to take into account the interactions in the system. Moreover, heuristic methods are particularly error-prone. On the other hand, it has to be recognised that heuristics may still be the only practical option: 1) when there are no realistic rigorous models, 2) in applications where repeated decisions must be made on a frequent basis and the problem is difficult to solve, and 3) in applications where sophisticated heuristics are available with tight error bounds.

2.1.2. Mathematical programming-based approaches

The alternative to heuristics is the use of mathematical models of supply chains. Such models can be classified into four categories (Beamon, 1998):

1. *Deterministic models*
2. *Stochastic models*
3. *Economic models*
4. *Simulation models.*

Workable and realistic models and algorithms began to emerge in mid-1970s with improvements in computational capabilities.

In their pioneering work, Geoffrion and Graves (1974) present a model to solve the problem of designing a distribution system with optimal location of the intermediate distribution facilities between plants and customers. In particular, they aim to determine which distribution centre (DC) sites to use, what size DC to have at each selected site, what customer zones to serve and the transportation flow for each commodity. The objective is to minimise the total distribution cost (transportation cost and investment cost), subject to a number of constraints such as supply constraints, demand constraints and specification constraints regarding

the nature of the problem. The problem is formulated as a mixed-integer linear program (MILP) which is solved using Benders decomposition. The model is applied to a case study for a supply chain comprising 17 commodity classes, 14 plants, 45 possible distribution centre sites and 121 customer demand zones.

Wesolowsky and Truscott (1975) present a mathematic formulation for the multiperiod location-allocation problem with relocation of facilities. They model a small distribution network comprising a set of facilities aiming to serve the demand at given points. The model incorporates two types of discrete decision, one concern the assignment of customers to facilities and the other the location of the nodes. They consider both steady-state and time-varying demands. In the latter case, a reallocation of the facilities is necessary during the time horizon of interest in order to operate at minimum cost but without sacrificing customer service. The solution procedure which is presented allows the static model to be applied to medium size cases. The dynamic formulation introduces two new discrete decisions to describe the relocation of the facilities. In order to find a feasible solution for the problem, it mainly relies on the solution of the static problem.

Williams (1983) develops a dynamic programming algorithm for simultaneously determining the production and distribution batch sizes at each node within a supply chain network. The average cost is minimised over an infinite horizon.

Another deterministic model presented by Ishii *et al.* (1988) aims to calculate the base stock levels and lead times associated with the lowest cost solution for an integrated supply chain in a finite time horizon.

Brown *et al.* (1987) present an optimisation-based decision algorithm for a support system used to manage complex problems involving facility selection, equipment location and utilisation, and manufacture and distribution of products. They focus on operational issues such as where each product should be produced, how much should be produced in each plant, and from which plant product should

be shipped to each customer. Some strategic issues are also taken into account such as location of the plants and the number, kind and location of facilities (plants). The resulting MILP model is solved using a decomposition strategy. The model has been applied to a real case study for the NABISCO company.

Breitman and Lucas (1987) present a modelling system named PLANETS (Production Location Analysis NETwork System) used to decide what products to produce, and when, where, and how to make these products. It also provides information on shipping allocations, capital spending schedules and resources usage. Unfortunately, no clear description of the mathematical basis of this systems is provided.

An MINLP formulation is presented by Cohen and Lee (1989) which seeks to maximise the total after-tax profit for the manufacturing facilities and distribution centres. Managerial (resource and production) as well as logical consistency (feasibility, availability, demands limits) constraints were applied. This work was extended by Cohen and Moon (1990) who developed a constrained optimisation model, called PILOT, to investigate the effects of various parameters on supply chain cost, and to determine which manufacturing facilities and distribution centres should be established. A supply chain consisting of raw materials suppliers, manufacturing facilities, distribution centres and retailers was used as an example. The PILOT model aims to minimise fixed and variable production and transportation costs, subject to supply, capacity, assignment, demand and raw material requirements constraints.

A two-phase approach was used by Newhart *et al.* (1993) to design an optimal supply chain. First, a combination of mathematical programming and heuristic models is used to minimise the number of product types held in inventory throughout the supply chain. In the second phase, a spreadsheet-based inventory model determines the minimum safety stock required to absorb demand and lead time fluctuations.

Chandra (1993) presents a model that plans deliveries to customers based upon inventories (both at warehouses and distribution centres) and vehicle routes. In a latter work, Chandra and Fisher (1994) consider the coordination of production and distribution planning.

Goetschalckx *et al.* (1994) report on a decision support tool for supply chains and industrial logistics systems. The tool is based on an integrated logistics model that tries to capture several components of the supply chain such as production, warehousing, transportation and customer service. The objective is to minimise the total costs occurring in the system including warehouse inventory and handling costs, production and material handling costs at plants, and transportation costs. Constraints concerning warehouse material handling capacity, production capacity and network flow are taken into account.

Pooley (1994) presents the results of an MILP formulation used by the Ault Foods company to restructure their supply chain. The model aims to minimise the total operating cost of a production and distribution network. Data are obtained from historical records; data collection is described as one of the most time consuming parts of the project. Binary variables characterise the existence of plants and warehouses, and the links between customers and warehouses. The model is used to answer questions like: where should the division locate plants and depots (DCs)? how should the production be allocated to different plants? how should the customers be served? Thirty different plant production strategies were investigated.

Arntzen *et al.* (1995) develop an MILP “global supply chain model” (GSCM) aiming to determine: 1) the number of echelons, 2) the number and location of distribution centres, 3) the assignment of customers to distribution centres, and 4) the assignment of products to plants. The objective of the model is to minimise a weighted combination of total cost and activity days. The total cost consists of production costs, inventory costs, transportation costs and fixed costs

of any kind. The constraints taken into account include: customer satisfaction, accounted production and inventory volumes, product recipes, limited production capacities, limited storage and shipping volumes. The model also includes duty drawbacks and duty reliefs. Potential applications of this model fall into three categories: 1) analysing the supply chain for new products, 2) analysing the supply bases for commodities, and 3) studying company-wide or division-wide supply chains. Some results of the application of the model in the Digital Equipment Corporation are reported.

Voudouris (1996) develops a mathematical model designed to improve efficiency and responsiveness in a supply chain, leading to enhanced overall flexibility. He identifies two types of manufacturing resources: activity resources (manpower, warehouse doors, packaging lines, forklift trucks, etc.) and inventory resources (volume of intermediate storage, warehouse area). The activity resources are related to time while the inventory resources are related to space. The objective function aims to represent the flexibility of the plant to absorb unexpected demands. The model can be used: 1) for plant simulation and resource analysis, 2) for statistical analysis of manufacturing resource availability, and 3) as a scheduling support tool.

Camm *et al.* (1997) develop an integer programming model for the Procter and Gamble corporation to determine the location of distribution centres and to assign those selected to customer zones. A variety of reasons (including the need to reduce transportation costs, the introduction of new products, new quality policies, the reduction in the life cycle of products, and relatively high number of plants) forced the company to restructure their supply chain. The models developed were a simple transportation model and an uncapacitated facility-location problem. They were formulated as an LP and an MILP model respectively.

An MILP facility location model is proposed by Canel and Khumawala (1997) who use it to optimise the global supply chain for a single product. The model

does not include transfer prices and transportation costs. The model is solved using a branch and bound algorithm. Some algorithmic issues relating to the solution approach are also discussed.

Pirkul and Jayarama (1996) study a tri-echelon multi-commodity system comprising production, transportation and distribution planning functions. A “single sourcing” constraint is imposed specifying that each customer is to be supplied from a single warehouse. Warehouses can receive products from several manufacturing plants. The objective is to minimise the combined costs of establishing and operating the plants and the warehouses, in addition to any transportation and distribution costs from plants to warehouses, and from warehouses to customers. Two key decisions are, first the choice of a desired number of warehouses out of a set of possible locations while assigning customers to them, and, second, the selection of plants out of a set of potential plants assigning plants to warehouses as well. Lagrangian relaxation is used to solve the problem which is formulated as an MILP. The solution obtained is not optimal but just feasible. A solution procedure is presented to address the problems involved by the relaxation. A number of case studies are presented.

In a later work, Pirkul and Jayarama (1998) present the PLANWAR model that seeks to locate a number of production plants and distribution centres so that the total operating cost for the distribution network is minimised. The network consists of a potential set of production plants and distribution centres, and a set of given customers. This model appears to have many similarities with the one presented by the same authors in 1996.

PLANWAR aims to determine the locations of a given number of plants and warehouses subject to capacity restrictions. It is assumed that customer locations and demands are known in advance, and so are the potential locations and capacities of warehouses and plants. The problem is solved via a heuristic solution procedure coupled with a Lagrangian relaxation scheme used to obtain lower

bounds. The solutions generated are feasible but not necessarily optimal. The authors argue that Benders decomposition and branch-and-bound methods are too time consuming for solving realistic problems.

Hindi and Pienkosz (1999) present a solution procedure for large-scale, single source, capacitated plant location problems (SSCPLP). They present an MILP formulation of the problem, with two types of decision variables relating to the existence of a plant and to the allocation of a customer to a plant, respectively. A major objective is to develop a solution procedure able to cope with large-scale problems. A number of other studies in the same area are presented, each using a variation of the Lagrangian relaxation combined with other methods.

A multi-period model for production and distribution systems is proposed by Dogan and Goetschalckx (1999) together with a decomposition algorithm applied in order to obtain solutions in reasonable computational times. Emphasis is placed on tactical decisions in the design of strategic supply chain networks determining tactical decisions. Demand is assumed to be subject to seasonal variations. A multi-period MILP formulation is proposed with the objective of minimising supply and manufacturing costs, warehousing and inventory costs, as well as any transportation costs. A case study illustrates the use of the model and the advantages of their decomposition method against other methods reported in the literature.

In a recent review, Schmidt and Wilhelm (2000) discuss the modelling of logistic networks in the context of strategic, tactical and operational decision support. They focus on a prototype formulation, stressing a number of issues arising in the current business environment. They point out many weaknesses of current modelling approaches, emphasising the lack of integration between different parts of the supply chain network. They also mention the failure to take account of flexible transportation costs and uncertainty in operations, and the fact that most current models study single product operations in a static mode. In general, they

argue that there is a lack of strategic supply chain models integrating all single components of the logistic networks. They also highlight the need for new solution methods which are currently limited to Benders decomposition and heuristic approaches.

Sabri and Beamon (2000) propose a mathematical model for supply chain design facilitating simultaneous strategic and operational decisions. The model incorporates production, delivery and demand uncertainty aspects and is divided into strategic and operational level sub-models. The strategic sub-model is deterministic and aims to optimise the supply chain configuration and the material flow. On the other hand, the operational sub-model includes stochastic aspects designed to accommodate the uncertainty, and also the cost analysis. The authors discuss a solution algorithm developed to integrate these two sub-models, and apply it to a case study.

Timpe and Kallrath (2000) present a detailed model for the optimal planning of large multi-site productions networks. They are primarily interested in the operational and utilisation aspects of the supply chain rather than its actual design. Different parts of the supply chain are considered, such as production and changeovers, material flow and transportation, and inventories. The problem is modelled as an MILP solved using commercial solvers employing the branch-and-bound method.

Nozick and Turnquist (2001) consider the interactions between inventory levels and the number of distribution centres established to serve the demand. They propose an MILP formulation to integrate the location problem of distribution centres with determination of the levels of inventory to be held at each centre.

2.1.3. Facility location

In addition to papers considering fairly general classes of problems in supply chain management (cf. sections 2.1.2 and 3.4.1), there is also a substantial literature on the more specific problem of facility location. Current *et al.* (1990) review the work in this area until 1990. They consider four types of objective: 1) cost minimisation, 2) demand satisfaction, 3) profit maximisation and 4) environmental concerns. Issues of interest are also the number and type of facilities being sited, their capacities, the nature of the problem (continuous or discrete), the nature of the parameters (stochastic or deterministic) and finally the nature of the model (static or dynamic).

The majority of the models reviewed by Current *et al.* (1990) concern uncapacitated multi-facility location, where the formulation of the problem is deterministic with a discrete solution space, and static planning horizon. Solutions methods for capacitated plant location problems are described in a review by Sridharan (1995). Both heuristic and exact solution procedures are reported. A number of different problems and solution approaches used for their solution are listed.

Revelle and Laporte (1996) consider the plant location problem using capacitated and uncapacitated mathematical models for both single and multiple product plants. A number of different objectives are considered including: 1) maximum return on investment, 2) minimum transportation and manufacturing costs and 3) on-time delivery for products.

In a more recent review, Owen and Daskin (1998) discuss developments in the area of location and strategic planning. Facility location decisions are considered as critical by a wide range of companies. Decision makers want to select sites that will not simply perform well according to the current system state, but will also continue to be profitable even under changes in the operational environment of the company. The authors divide the research in the area into three categories: 1) deterministic, static models, 2) dynamic models, and 3) stochastic models.

Deterministic, static models are further distinguished into:

- *Median problems* that measure effectiveness by determining the average distance between facilities.
- *Covering problems* that seek to maximise the amount of demand covered within an acceptable service distance.
- *Centre problems* that require coverage of all demands while minimising coverage distances.

Dynamic location models are divided into two main categories:

- Dynamic single facility location models.
- Dynamic multiple facility location models.

while stochastic location problems can be distinguished into:

- Probabilistic models.
- Scenario planning models.

An alternative to the use of MILP formulations for facility location, is the use of continuous functions to represent spatial distributions of cost and customer demand (Dasci and Verter, 2001). The authors argue that their model leads to more insight on the impact of different parameters on the actual location of the facilities. The underlying mathematical formulation can be characterised as a capacitated facility location problem.

2.2. Supply chain dynamics

Towill (1991) reviews the application of system dynamics models to supply chain problems. The main problem identified is the mismatch between the production rate and the actual sales rate to the consumers. In particular, the factory

production rate usually fluctuates much more widely than does the actual sales rate. The problem tends to be exacerbated as the supply chain becomes longer. For example, in a three level supply chain, the amplification at the factory can be six times as big as the swing in the marketplace. As a consequence, there are alternating periods of stock-outs and stock surpluses, the former resulting in business losses, and both to increased total cost. Increased complexity means higher possibility of counter-intuitive behaviour. Less complicated supply chains are preferable. The introduction of JIT (Just-In-Time) production approaches to improve supply chain dynamics provides an obvious strategy of using “competitive time” to advantage. However, as pointed out by Towill *et al.* (1992), this can prove an expensive and disastrous solution for a poorly designed supply chain due to its complex dynamics.

The above phenomenon was identified by the early work of Forrester (1961) who observed that: “*Very small variations in customer purchases lead to huge cycles in raw material demand.*” It was also codified in the ‘Law of Industrial Dynamics’ (Burbidge, 1984): “*If demand for products is transmitted along a series of inventories using stock control ordering, then the demand variation will increase with each transfer*”. A typical supply chain considered by Forrester is a three-stage link between suppliers/manufacturers, manufacturers/distributors and distributors/customers, as shown in figure 2.1. Each stage embraces the following constituents:

- perceived demand for orders
- a production or added value process
- information on current performance
- disturbances
- decision points

- transmission lags
- decision rules.

There is uncertainty associated with perceived demand, with the quality of information and with the value of transmission lags. Furthermore, it is not easy to comprehend where all the time is spent by materials flowing through a supply chain. In the example of figure 2.1, inventory is held at plants, warehouses, distributors and retailers. There is a delay in the information coming back, in the transportation and in the manufacturing times. These delays are responsible for fluctuations in the factory ordering policy and in the inventories held in the supply chain.

On the other hand, the information associated with the nature of these phenomena can be used to smooth out the dynamics of supply chains (Wikner *et al.*, 1991). Control theory can be employed to model the system and to study the effects of various parameters on the performance of such systems (Perea *et al.*, 2000).

2.3. Discussion

As discussed in section 1.3, effective supply chain integration is essential in the current business environment. Such integration often requires many organisational and managerial changes. The objective of the current project is to produce quantitative tools for supporting strategic business decisions towards supply chain integration.

In our opinion, models based on a formal mathematical programming offer the best scope for describing the multitude of interactions that exist in complex supply chains and for determining solutions that are either optimal or within a known margin from optimality. The models of this type that have been reviewed

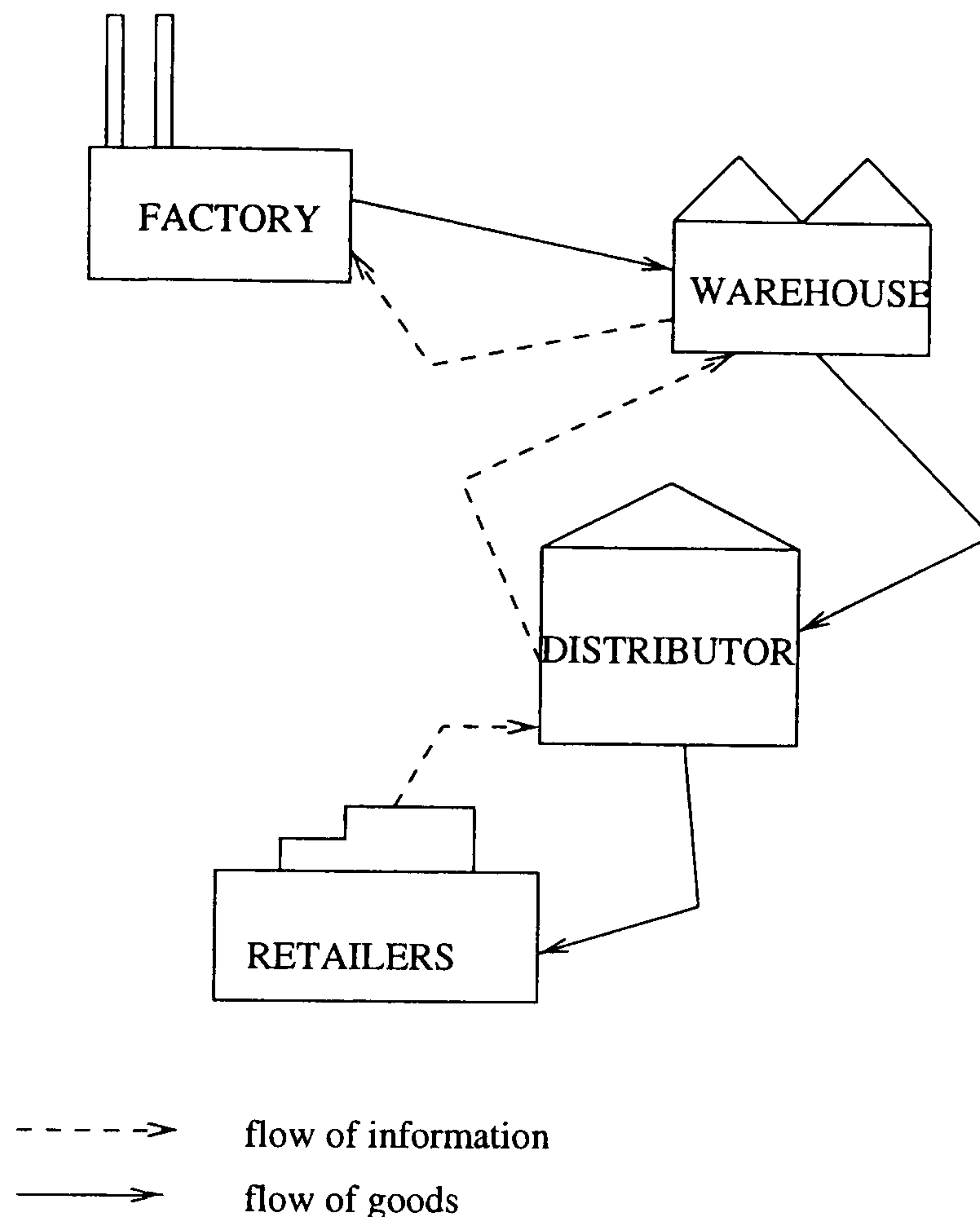


Figure 2.1: Forrester (1961) production-distribution system

in this chapter are summarised in table 2.1. As indicated by the information in the second column of the table, none of these models has considered the combined interactions between plants, warehouses and distribution centres. Moreover, the treatment of many important issues such as multipurpose production facilities, complex transportation cost structures, and product demands that are both time-varying and uncertain, either has not been undertaken at all or has never been combined in a single model. In fact, many of the model-based approaches cited in the table are relatively simple extensions of the uncapacitated facility location problem.

The main objectives of the work presented in this thesis are:

- to develop mathematical programming models that address the above issues;
- to test both the models and the solution algorithms on industrial problems of realistic complexity.

We should note here that further literature will be reviewed in later parts of this thesis whenever it is necessary to support the ideas presented.

Author(s)	Model Type	Decisions Considered						Objective Function		
					Operational		Strategic			
	Number of Echelons Considered	Heuristic	Deterministic	Stochastic	Production Scheduling	Production Assignment	Inventory Levels	W-DC Assignment/ Location	DC-C Assignment/ Location	
Arntzen <i>et al.</i> (1995)	Not Clear		*			*	*		*	cost
Brown <i>et al.</i> (1987)	No Echelons		*			*			*	cost
Camm <i>et al.</i> (1997)	No Echelons		*						*	cost
Chandra (1993)	Not Clear		*				*			cost
Chandra and Fisher (1994)	Not Clear		*				*			cost
Cohen and Lee (1988)	P,W			*			*			cost
Cohen and Lee (1989)	P,W		*		*	*	*		*	cost
Cohen and Moon (1990)	No Echelons		*		*	*				cost
Current <i>et al.</i> (1990)	No Echelons		*			*		*	*	cost
Dogan and Goetschalckx (1999)	DC		*				*			cost
Geoffrion and Graves (1974)	DC		*			*	*		*	cost
Goetschalckx <i>et al.</i> (1994)	W,DC		*			*		*	*	cost
Hindi and Pienkosz (1999)	No Echelons		*			*			*	cost
Lee and Billington (1993)	P			*			*			customer response
Lee and Sasser (1995)	No Echelons			*			*			customer response
Nozick and Turnquist (2001)	DC		*				*		*	cost
Pirkul and Jayarama (1996)	P,W		*			*			*	cost
Pirkul and Jayarama (1998)	W		*			*			*	cost
Pooley (1994)	Not Clear		*			*			*	cost
Pyke and Cohen (1993)	P			*			*			cost
Pyke and Cohen (1994)	P			*			*			cost
Sabri and Beamon (2000)	S,P,DC		*	*		*	*		*	cost/response/flex
Svoronos and Zipkin (1991)	P,W			*	*	*	*			cost/response
Timpe and Kallrath (2000)	P		*		*	*	*			cost
Voudouris (1996)		*				*	*			flexibility
Williams (1981)	No Echelons	*			*	*	*			cost
Williams (1983)	No Echelons	*	*		*	*	*			cost

Table 2.1: Supply chain models reviewed in this chapter.

Optimal Steady-State Design of Supply Chain Networks

3.1. Introduction

This chapter considers the design of multiproduct, multi-echelon production and distribution networks. As shown in figure 3.1, the networks of interest to this chapter comprise a number of existing multiproduct manufacturing sites at fixed locations, a number of warehouses and distribution centres of unknown location (to be selected from a set of potential candidates), and finally a number of customer zones at fixed locations.

In general, each product can be produced by several plants at different locations. The production capacity of each manufacturing site is modelled in terms of a set of linear constraints relating the mean production rates of the various products to the available resources. The material handling capacity of warehouses and distribution centres (if established) is subject to upper and lower bounds. Warehouses can be supplied from more than one manufacturing site and can supply more than one distribution centre. However, each distribution centre should normally be supplied by a single warehouse.

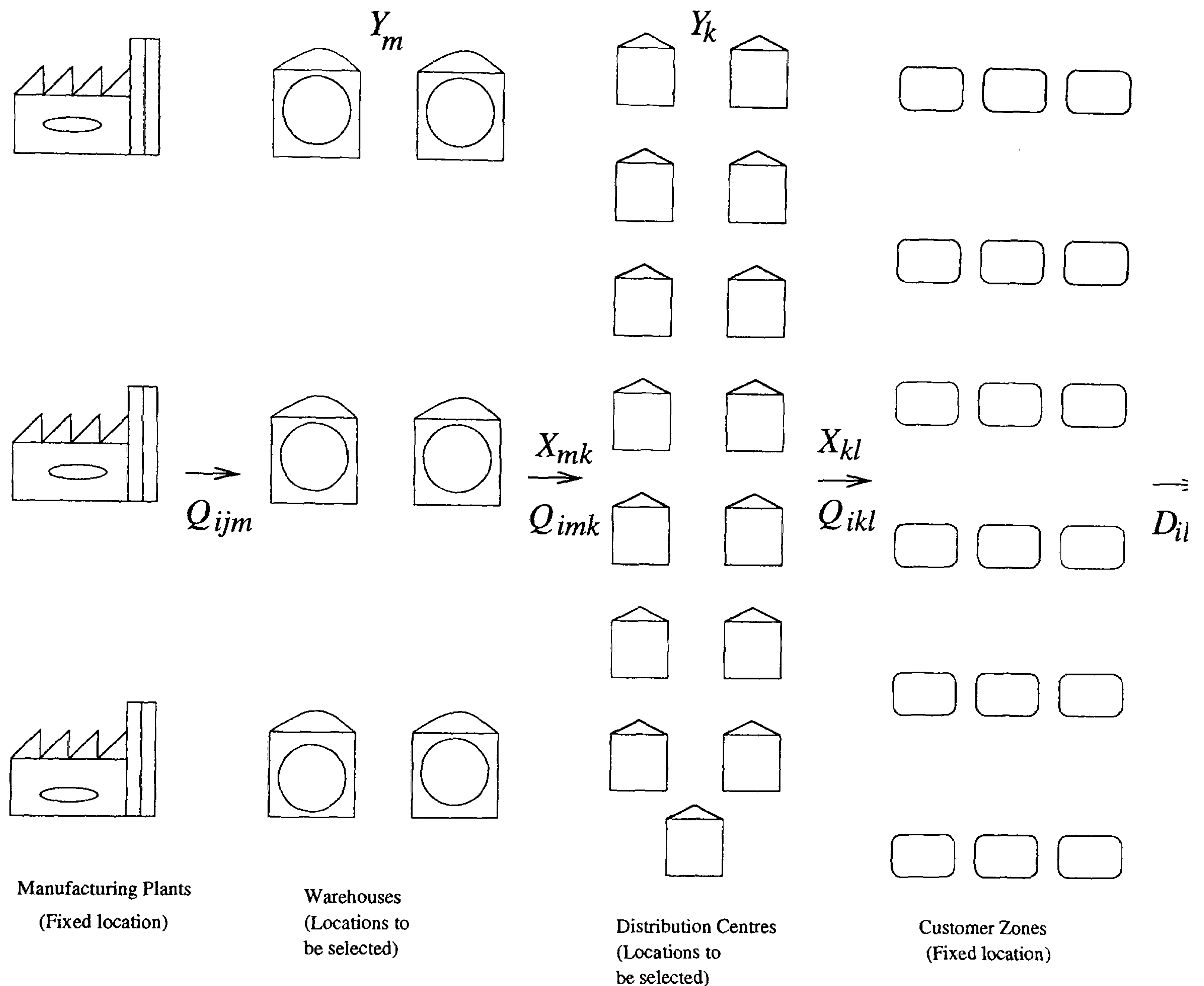


Figure 3.1: Overview of the supply chain networks of interest

Each customer zone places demands for one or more products. These demands may be assumed to be known a priori; alternatively, a number of demand scenarios, each with a given, non-zero probability may be considered. Single sourcing constraints, according to which each customer zone is served by a single distribution centre (to be determined by the optimisation), may be imposed.

The establishment of warehouses and distribution centres incurs a fixed infrastructure cost. Operational costs include those associated with production, handling of material at warehouses and distribution centres, and transportation. Transportation costs are assumed to be piecewise linear functions of the actual flow of the product from the source stage to the destination stage, and they may

include taxes and duties.

The decisions to be determined include the number, location and capacity of warehouses and distribution centres to be set up, the transportation links that need to be established in the network, and the flows and production rates of materials. The objective is the minimisation of the total annualised cost of the network, taking account of both infrastructure and operating costs.

This chapter considers a steady-state form of the above problem according to which demands are time-invariant (but possibly uncertain). Of course, this is usually a simplifying assumption since demands vary with time. For the purposes of the current chapter, suffice to say that all production and transportation flows determined by the optimisation are considered to be time-averaged quantities. A more thorough discussion of time variability in supply chain networks will be presented in chapter 5.

The rest of this chapter is structured as follows: section 3.2 introduces the necessary mathematical notation. Section 3.3 considers the deterministic case of known product demands, formulating the problem as an MILP. Section 3.4 extends the formulation to the case of uncertain demands described in terms of multiple scenarios. Finally, section 3.5 draws some conclusions from the work presented.

3.2. Notation

The notation to be used in this chapter is summarised below for ease of future reference:

3.2.1. Indices

e manufacturing resource (equipment, manpower, utilities, etc.)

f	transportation product families
i	products
j	plants
k	possible distribution centres
l	customer zones
m	possible warehouses
r	range of transportation flows with constant transportation cost
s	product demand scenarios (superscript used in section 3.4 only)

3.2.2. Sets

K^{ss}	set of distribution centres requiring single sourcing from warehouses
L^{ss}	set of customer zones requiring single sourcing from distribution centres

3.2.3. Parameters

$\overline{C}_{fjmr}$	transportation cost for products of family f from plant j to warehouse m over range r of transportation flow
$\overline{C}_{fkmr}$	transportation cost for products of family f from warehouse m to distribution centre k over range r of transportation flow
$\overline{C}_{fklr}$	transportation cost for products of family f from distribution centre k to customer zone l over range r of transportation flow
C_{im}^{WH}	unit handling cost for product i at warehouse m
C_{ik}^{DH}	unit handling cost for product i at distribution centre k
C_m^W	annualised fixed cost of establishing warehouse at location m
C_k^D	annualised fixed cost of establishing distribution centre at location k
C_{ij}^P	unit production cost for product i at plant j

$\bar{C}_r$	transportation cost which corresponds to interval r
$D_k^{\max}$	maximum distribution centre capacity
$D_k^{\min}$	minimum distribution centre capacity
$\mathcal{D}_{il}$	demand for product i in customer zone l
NI	number of products
NF	number of families
NR_{fjm}	number of transportation flow ranges, each with constant unit cost for transporting products of family f from plant j to warehouse m
NR_{fmk}	number of transportation flow ranges, each with constant unit cost for transporting products of family f from warehouse m to distribution centre k
NR_{fkl}	number of transportation flow ranges, each with constant unit cost for transporting products of family f from distribution centre k to customer l
NS	number of product demand scenarios
$P_{ij}^{\max}$	maximum production capacity of plant j for product i
$P_{ij}^{\min}$	minimum production capacity of plant j for product i
$Q_{mk}^{\min}$	minimum rate of flow of material that can practically and economically be transferred from warehouse m to distribution centre k
$Q_{kl}^{\min}$	minimum rate of flow of material that can practically and economically be transferred from distribution centre k to customer zone l
$Q_{ijm}^{\max}$	maximum rate of flow of product i that can be transferred from plant j to warehouse m
$Q_{imk}^{\max}$	maximum rate of flow of product i that can be transferred from warehouse m to distribution centre k
$Q_{ikl}^{\max}$	maximum rate of flow of product i that can be transferred from distribution centre k to customer zone l
$\bar{Q}_{fjmr}$	upper bound for range r of transportation flow of products of family f from plant j to warehouse m

$\overline{Q}_{f m k r}$	upper bound for range r of transportation flow of products of family f from warehouse m to distribution centre k
$\overline{Q}_{f k l r}$	upper bound for range r of transportation flow of products of family f from distribution centre k to customer l
$R_{j e}$	total rate of availability of resource e at plant j
$W_m^{\max}$	maximum warehouse capacity
$W_m^{\min}$	minimum warehouse capacity
$\alpha_{i m}$	coefficient relating capacity of warehouse m to flow of product i handled by the warehouse
$\beta_{i k}$	coefficient relating capacity of distribution centre k to flow of product i handled by the distribution centre
$\rho_{i j e}$	coefficient for rate of utilisation resource e in plant j to produce product i
ψ_s	probability of product demand scenario s occurring during the lifetime of the network

3.2.4. Continuous variables

$C_{f j m}$	total cost incurred per unit time in transporting products of family f from plant j to warehouse m
$C_{f k m}$	total cost incurred per unit time in transporting products of family f from warehouse m to distribution centre k
$C_{f k l}$	total cost incurred per unit time in transporting products of family f from distribution centre k to customer zone l
D_k	capacity of distribution centre k
$P_{i j}$	production rate of product i in plant j
$Q_{i j m}$	rate of flow of product i transferred from plant j to warehouse m
$Q_{i m k}$	rate of flow of product i transferred from warehouse m

	to distribution centre k
Q_{ikl}	rate of flow of product i transferred from distribution centre k to customer zone l
Q_{fjmr}	rate of flow of products of family f from plant j to warehouse m in r th transportation flow interval
$Q_{fmk r}$	rate of flow of products of family f from warehouse m to distribution centre k in r th transportation flow interval
Q_{fklr}	rate of flow of products of family f from distribution centre k to customer l in r th transportation flow interval
W_m	capacity of warehouse m

3.2.5. Binary variables

Y_m	1 if warehouse m is to be established, 0 otherwise
Y_k	1 if distribution centre k is to be established, 0 otherwise
X_{mk}	1 if material is to be transported from warehouse m to distribution centre k , 0 otherwise
X_{kl}	1 if material is to be transported from distribution centre k to customer zone l , 0 otherwise
Z_{fjmr}	1 if the rate of flow of material in family f from plant j to warehouse m is between $\bar{Q}_{fjm,r-1}$ and $\bar{Q}_{fjmr}$, 0 otherwise
$Z_{fmk r}$	1 if the rate of flow of material in family f from warehouse m to distribution centre k is between $\bar{Q}_{fmk,r-1}$ and $\bar{Q}_{fmk r}$, 0 otherwise
Z_{fklr}	1 if the rate of flow of material in family f distribution centre k to customer l is between $\bar{Q}_{fkl,r-1}$ and $\bar{Q}_{fklr}$, 0 otherwise

3.3. Mathematical formulation of deterministic problem

The mathematical model proposed for this problem is a Mixed Integer Linear Program (MILP) as described below.

3.3.1. Constraints

3.3.1.1. Network structure constraints

A link between a warehouse m and a distribution centre k can exist only if warehouse m also exists:

$$X_{mk} \leq Y_m, \quad \forall m, k \quad (3.1)$$

As has been mentioned in the introduction to this chapter, it is sometimes required that certain distribution centres be served by a single warehouse (single-sourcing). This can be enforced via the constraint:

$$\sum_m X_{mk} = Y_k, \quad \forall k \in K^{ss} \quad (3.2)$$

where K^{ss} is the set of distribution centres for which single sourcing is required.

In any case, if the distribution centre does not exist, then its links with warehouses cannot exist either. This leads to the constraint:

$$X_{mk} \leq Y_k, \quad \forall m, k \notin K^{ss} \quad (3.3)$$

We note that the above is written only for those distribution centres that are not single-sourced. For the rest of the distribution centres, constraint (3.2) already suffices.

The link between a distribution centre k and a customer zone l will exist only if the distribution centre also exists:

$$X_{kl} \leq Y_k, \quad \forall k, l \quad (3.4)$$

Some customer zones may be subject to a single sourcing constraint requiring that they be served by exactly one distribution centre:

$$\sum_k X_{kl} = 1, \quad \forall l \in L^{ss} \quad (3.5)$$

while L^{ss} is the set of customer zones for which single sourcing is required.

3.3.1.2. Logical constraints for transportation flows

Flow of material i from plant j to warehouse m can take place only if warehouse m exists:

$$Q_{ijm} \leq Q_{ijm}^{\max} Y_m, \quad \forall i, j, m \quad (3.6)$$

Flow of material i from warehouse m to distribution centre k can take place only if the corresponding connection exists:

$$Q_{imk} \leq Q_{imk}^{\max} X_{mk}, \quad \forall i, m, k \quad (3.7)$$

Flow of material i from distribution centre k to customer zone l can take place only if the corresponding connection exists:

$$Q_{ikl} \leq Q_{ikl}^{\max} X_{kl}, \quad \forall i, k, l \quad (3.8)$$

Appropriate values for the upper bounds $Q_{ijm}^{\max}$, $Q_{imk}^{\max}$, $Q_{ikl}^{\max}$ appearing on the right hand sides of constraints (3.6) - (3.8) can be obtained as described in section 3.3.3.

There is usually a minimum total flowrate of material (of whatever type) that is needed to justify the establishment of a transportation link between two locations in the network. This consideration leads to constraints of the form:

$$\sum_i Q_{imk} \geq Q_{mk}^{\min} X_{mk}, \quad \forall m, k \quad (3.9)$$

$$\sum_i Q_{ikl} \geq Q_{kl}^{\min} X_{kl}, \quad \forall k, l \quad (3.10)$$

for the links between a warehouse m and a distribution centre k , and between a distribution centre k and a customer zone l , respectively.

3.3.1.3. Material balances

The actual rate of production of product i by plant j must equal the total flow of this product from plant j to all warehouses m :

$$P_{ij} = \sum_m Q_{ijm}, \quad \forall i, j \quad (3.11)$$

Since, for the purposes of this chapter, we assume steady state operation, the total rate of flow of each product leaving a warehouse or a distribution centre must equal the total rate of flow entering this node of the supply chain network:

$$\sum_j Q_{ijm} = \sum_k Q_{imk}, \quad \forall i, m \quad (3.12)$$

$$\sum_m Q_{imk} = \sum_l Q_{ikl}, \quad \forall i, k \quad (3.13)$$

Finally, the total rate of flow of each product i received by each customer zone l from the distribution centres must be equal to the corresponding market demand:

$$\sum_k Q_{ikl} = \mathcal{D}_{il}, \quad \forall i, l \quad (3.14)$$

3.3.1.4. Production resources

An important issue in the operation of the distribution network is the ability of the manufacturing plants to cover the demands of the customers as expressed through the orders received from the warehouses.

The rate of production of each product at any plant cannot exceed certain limits. Thus, there is always a maximum production capacity for any one product; moreover, there is often a minimum production rate that must be maintained while the plant is operating:

$$P_{ij}^{\min} \leq P_{ij} \leq P_{ij}^{\max}, \quad \forall i, j \quad (3.15)$$

It is common in many manufacturing sites for some resources (equipment, utilities, manpower, etc.) to be used by several production lines and at different stages of the production of each product. This shared usage limits the availability of the resource that can be used for any one purpose as expressed by the following constraint:

$$\sum_i \rho_{ije} P_{ij} \leq R_{je}, \quad \forall j, e \quad (3.16)$$

The coefficient ρ_{ije} expresses the amount of resource e used by plant j to produce a unit amount of product i , while R_{je} represents the total rate of availability of resource e at plant j .

3.3.1.5. Capacity of warehouses and distribution centres

The capacity of a warehouse m generally has to lie between given lower and upper bounds, $W_m^{\min}$ and $W_m^{\max}$, provided, of course, that the warehouse is actually established (i.e. $Y_m = 1$):

$$W_m^{\min} Y_m \leq W_m \leq W_m^{\max} Y_m, \quad \forall m \quad (3.17)$$

Similar constraints apply to the capacities of the distribution centres:

$$D_k^{\min} Y_k \leq D_k \leq D_k^{\max} Y_k, \quad \forall k \quad (3.18)$$

The assumption of steady-state operation of the network does not permit the precise characterisation of the amounts of inventory held at warehouses and distribution centres (cf. section 5.3.1.5). To overcome this difficulty, we generally assume that the required capacities of the warehouses and the distribution centres are related linearly to the flows of materials that they handle. This is consistent with the commonly used inventory management policy of holding stock equivalent to a given time period's worth of demand. This is expressed via the constraints:

$$W_m \geq \sum_{i,k} \alpha_{im} Q_{imk}, \quad \forall m \quad (3.19)$$

$$D_k \geq \sum_{i,l} \beta_{ik} Q_{ikl}, \quad \forall k \quad (3.20)$$

where α_{im} and β_{ik} are given coefficients.

3.3.1.6. Non-negativity constraints

All continuous variables must be non-negative:

$$P_{ij} \geq 0 \quad \forall i, j \quad (3.21)$$

$$Q_{ijm} \geq 0, \quad \forall i, j, m \quad (3.22)$$

$$Q_{imk} \geq 0, \quad \forall i, m, k \quad (3.23)$$

$$Q_{ikl} \geq 0, \quad \forall i, k, l \quad (3.24)$$

3.3.2. Objective function

In general, a distribution network involves both capital and operating costs. The former are one-off costs associated with the establishment of the infrastructure of the network, and in particular its warehouses and distribution centres. On the other hand, operating costs are incurred on a daily basis and are associated with the cost of production of material at plants, the handling of material at warehouses and distribution centres, and the transportation of material through the network.

3.3.2.1. Fixed infrastructure costs

The infrastructure costs considered by our formulation are related to the establishment of a warehouse or a distribution centre at a candidate location. These costs are represented by the following objective function terms:

$$\sum_m C_m^W Y_m + \sum_k C_k^D Y_k$$

We assume that the production plants are already established. Therefore, we do not consider the capital cost associated with their design and construction. We also ignore any infrastructure cost associated with the customer zones.

3.3.2.2. Production cost

The production cost is given by the product of the production rate P_{ij} of product i in plant j , by the unit production cost C_{ij}^P . The corresponding term in the objective function is of the form:

$$\sum_{i,j} C_{ij}^P P_{ij}$$

3.3.2.3. Material handling costs at warehouses and distribution centres

Material handling costs can usually be approximated as linear functions of the total throughput. They can be expressed as follows:

$$\sum_{i,m} C_{im}^{WH} \left(\sum_j Q_{ijm} \right) + \sum_{i,k} C_{ik}^{DH} \left(\sum_m Q_{imk} \right)$$

3.3.2.4. Transportation costs

We start by considering generically the cost incurred in transporting a certain flow Q of a material in the arc between any two nodes in the supply chain network, as shown in figure 3.2. Usually the *unit* transportation cost is a non-increasing piecewise constant function of the rate of flow, reflecting economies of scale. Thus the total transportation cost is a piecewise linear function of flow of material, of the form shown in figure 3.3. The possible range of the transportation flow is divided into NR sub-ranges, each corresponding to a different (and progressively lower) unit transportation price C_r^T . The limits of interval $r \in [1, NR]$ are $\bar{Q}_{r-1}$ and $\bar{Q}_r$.

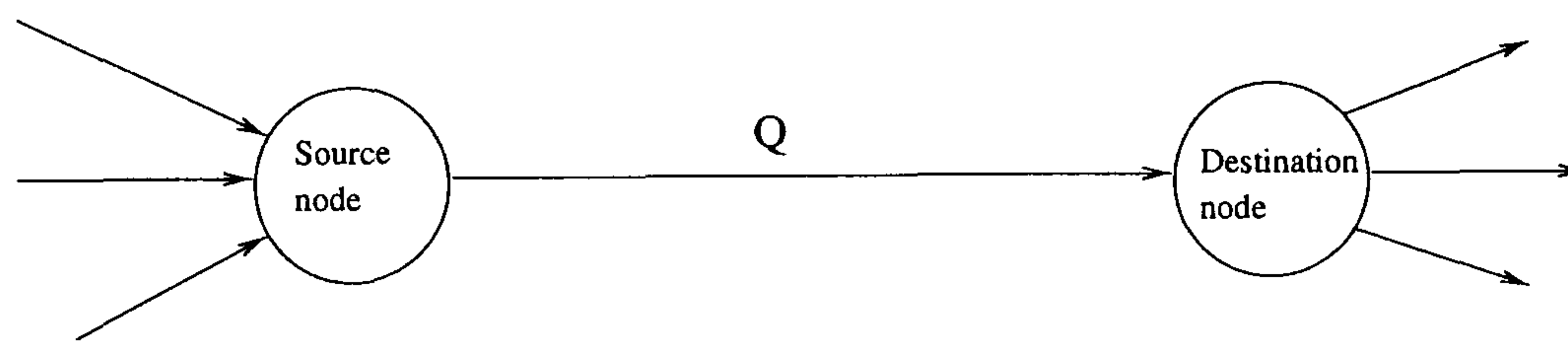


Figure 3.2: Transportation of material in an arc of the supply chain network.

We introduce a new set of binary variables Z_r :

$$Z_r = \begin{cases} 1, & \text{if } Q \in [\bar{Q}_{r-1}, \bar{Q}_r) \\ 0, & \text{otherwise} \end{cases}$$

The above definition can be effected via the linear constraints:

$$\bar{Q}_{r-1} Z_r \leq Q \leq \bar{Q}_r Z_r, \quad \forall r \quad (3.25)$$

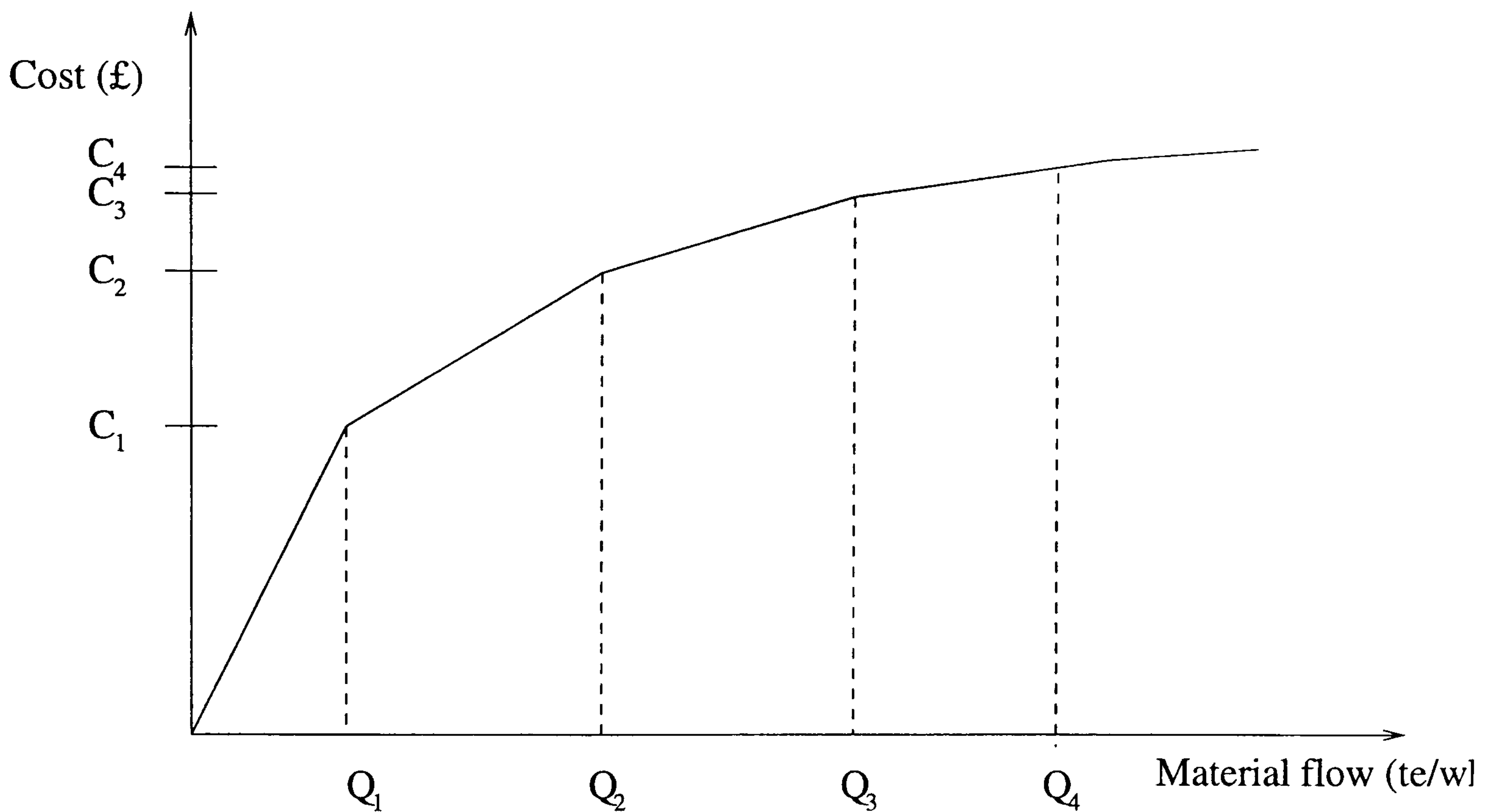


Figure 3.3: Transportation cost as piecewise linear function of material flow

$$\sum_{r=1}^{NR} Z_r = 1 \quad (3.26)$$

$$Q = \sum_{r=1}^{NR} Q_r \quad (3.27)$$

Constraint (3.26) ensures that only one of the variables Z_r (say, for $r = r^*$) takes a value of 1, all others being zero. Constraint (3.25) then forces Q_r to 0 for all $r \neq r^*$, while $Q_{r^*} \in [\bar{Q}_{r^*-1}, \bar{Q}_{r^*}]$. Finally, constraint (3.27) implies that $Q = Q_{r^*}$ and, therefore, $Q \in [\bar{Q}_{r^*-1}, \bar{Q}_{r^*}]$, as desired.

The total transportation cost is given by:

$$C = \sum_{r=1}^{NR} \left[\bar{C}_{r-1} Z_r + (Q_r - \bar{Q}_{r-1} Z_r) \frac{\bar{C}_r - \bar{C}_{r-1}}{\bar{Q}_r - \bar{Q}_{r-1}} \right]$$

We note that, because of constraints (3.25)-(3.27), only one of the terms in the above summation will be non-zero, effecting a linear interpolation between two points $(\bar{Q}_{r^*-1}, \bar{C}_{r^*-1})$ and $(\bar{Q}_{r^*}, \bar{C}_{r^*})$ on figure 3.3 (where r^* is such that $Z_{r^*} = 1$).

Having established how we can generically model piecewise linear transportation costs in a linear fashion, we can apply these manipulations to the flows of material specifically taking place in the supply chain network. Although we could assume that each product i under consideration has a different transportation cost, in reality many of the products in any given supply chain are likely to be very similar to each other. Therefore we introduce the concept of a product family as a subset of the products, all of which have the same unit transportation costs. We denote these families as I_f , $f = 1, \dots, NF$, where NF is the number of families. Each product belongs to exactly one family. Thus:

$$\bigcup_{f=1}^{NF} I_f = \{i \mid i = 1, \dots, NI\}$$

$$I_f \cap I_{f'} = \emptyset, \quad \forall f, f' \in \{1, \dots, NF\}, f \neq f'$$

where NI is the total number of products, $i = 1, \dots, NI$.

Since all products in a family have the same transportation cost, we can apply the above manipulations for the determination of transportation cost to their *combined* flow rate in each arc of the supply chain network. The total transportation

cost incurred in the network is given by:

$$\sum_{f,j,m} C_{fjm} + \sum_{f,m,k} C_{fmk} + \sum_{f,k,l} C_{fkl} \quad (3.28)$$

subject to the following constraints:

- For the transportation of material between plants and warehouses:

$$\sum_{r=1}^{NR_{fjm}} Z_{fjmr} = 1, \forall f, j, m \quad (3.29)$$

$$\bar{Q}_{fjm,r-1} Z_{fjmr} \leq Q_{fjmr} \leq \bar{Q}_{fjmr} Z_{fjmr}, \forall f, j, m, r = 1, \dots, NR_{fjm} \quad (3.30)$$

$$\sum_{i \in I_f} Q_{ijm} = \sum_{r=1}^{NR_{fjm}} Q_{fjmr}, \forall f, j, m \quad (3.31)$$

$$C_{fjm} = \sum_{r=1}^{NR_{fjm}} \left[\bar{C}_{fjm,r-1} Z_{fjmr} + (Q_{fjmr} - \bar{Q}_{fjm,r-1} Z_{fjmr}) \frac{\bar{C}_{fjmr} - \bar{C}_{fjm,r-1}}{\bar{Q}_{fjmr} - \bar{Q}_{fjm,r-1}} \right] \forall f, j, m \quad (3.32)$$

- For the transportation of material between warehouses and distribution centres:



$$\sum_{r=1}^{NR_{fmk}} Z_{fmr} = 1, \forall f, m, k \quad (3.33)$$

$$\bar{Q}_{fmk,r-1} Z_{fmr} \leq Q_{fmr} \leq \bar{Q}_{fmr} Z_{fmr}, \forall f, m, k, r = 1, \dots, NR_{fmk} \quad (3.34)$$

$$\sum_{i \in I_f} Q_{imk} = \sum_{r=1}^{NR_{fmk}} Q_{fmr}, \forall f, m, k \quad (3.35)$$

$$C_{fmk} = \sum_{r=1}^{NR_{fmk}} \left[\bar{C}_{fmr,r-1} Z_{fmr} + (Q_{fmr} - \bar{Q}_{fmr,r-1} Z_{fmr}) \frac{\bar{C}_{fmr} - \bar{C}_{fmr,r-1}}{\bar{Q}_{fmr} - \bar{Q}_{fmr,r-1}} \right] \forall f, m, k \quad (3.36)$$

- For the transportation of material between distribution centres and customer zones:

$$\sum_{r=1}^{NR_{fkl}} Z_{fklr} = 1, \forall f, k, l \quad (3.37)$$

$$\bar{Q}_{fkl,r-1} Z_{fklr} \leq Q_{fklr} \leq \bar{Q}_{fklr} Z_{fklr}, \forall f, k, l, r = 1, \dots, NR_{fkl} \quad (3.38)$$

$$\sum_{i \in I_f} Q_{ikl} = \sum_{r=1}^{NR_{fkl}} Q_{fklr}, \quad \forall f, k, l \quad (3.39)$$

$$C_{fkl} = \sum_{r=1}^{NR_{fkl}} \left[\bar{C}_{fkl,r-1} Z_{fklr} + (Q_{fklr} - \bar{Q}_{fkl,r-1} Z_{fklr}) \frac{\bar{C}_{fklr} - \bar{C}_{fkl,r-1}}{\bar{Q}_{fklr} - \bar{Q}_{fkl,r-1}} \right] \cdot \forall f, k, l \quad (3.40)$$

3.3.2.5. Overall objective function

By combining the cost terms derived in sections 3.3.2.1 - 3.3.2.4, we obtain the total cost of the supply chain network to be minimised by the optimisation:

$$\begin{aligned} & \min \sum_m C_m^W Y_m + \sum_k C_k^D Y_k + \\ & \sum_{i,j} C_{ij}^P P_{ij} + \sum_{i,m} C_{im}^{WH} \left(\sum_j Q_{ijm} \right) + \sum_{i,k} C_{ik}^{DH} \left(\sum_m Q_{imk} \right) + \\ & \sum_{f,j,m} C_{fjm} + \sum_{f,m,k} C_{fmk} + \sum_{f,k,l} C_{fkl} \end{aligned} \quad (3.41)$$

The above minimisation is subject to all the constraints presented in section 3.3.1 as well as constraints 3.29 - 3.40.

3.3.3. Calculation of upper bounds on network flows

Constraints (3.6) - (3.8) involve the upper bounds $Q_{ijm}^{\max}$, $Q_{imk}^{\max}$ and $Q_{ikl}^{\max}$. The tightness of the MILP formulation and, consequently, the efficiency of its solution will depend crucially on the quality of these bounds.

In order to obtain estimates for these bounds, consider the flow of a product i along the arc $A \rightarrow B$ connecting two nodes A and B in a network (see figure 3.4). We note that this flow, denoted by Q_{iAB} , cannot exceed either the total flow of product i entering node A or the total flow of product i leaving node B . Thus we obtain the expression:

$$Q_{iAB}^{\max} = \min \left(\sum_{C \in IN} Q_{iCA}^{\max}, \sum_{C \in OUT} Q_{iBC}^{\max} \right) \quad (3.42)$$

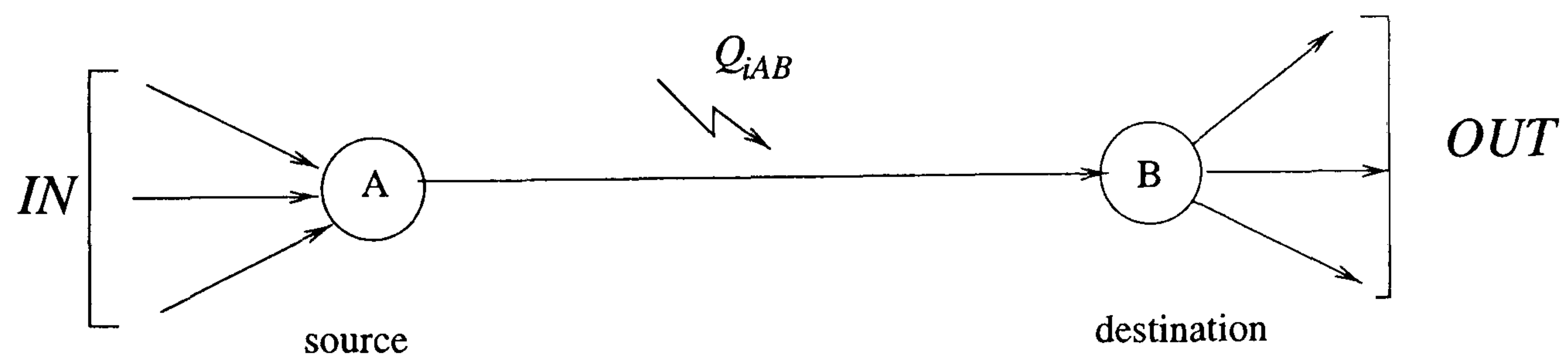


Figure 3.4: Obtaining upper bounds for network flows

Applying this expression (3.42) to the flows taking place in the supply chain network, we obtain:

$$Q_{ijm}^{\max} = \min \left(P_{ij}^{\max}, \sum_k Q_{imk}^{\max} \right) \quad \forall i, j, m \quad (3.43)$$

$$Q_{imk}^{\max} = \min \left(\sum_j Q_{ijm}^{\max}, \sum_l Q_{ikl}^{\max} \right) \quad \forall i, m, k \quad (3.44)$$

$$Q_{ikl}^{\max} = \min \left(\sum_m Q_{imk}^{\max}, D_{il} \right) \quad \forall i, k, l \quad (3.45)$$

Note that here we consider the production rate P_{ij} of product i at plant j as a flow notionally entering plant node j . Similarly, the demand rate D_{il} for product i at a customer zone l is notionally considered as a fixed flow leaving customer zone node l .

The above formulae need to be applied in an iterative manner. Initially, we can set $Q_{ijm}^{\max} = +\infty$, $Q_{imk}^{\max} = +\infty$ and $Q_{ikl}^{\max} = +\infty$. Then we repeatedly apply equations (3.43) - (3.45) until none of the above upper bounds changes.

Equation (3.42) takes account of all arcs potentially entering node A and all arcs potentially leaving node B . However, during the branch-and-bound search, some of these arcs may be deleted. For instance, if the search fixes a variable X_{mk} to zero, then the corresponding arc from plant j to warehouse m is deleted. Hence, $Q_{imk}^{\max}$ could be omitted from the summation on the right-hand side of (3.43). We could consequently tighten the various upper bounds by applying the above procedure at every node during the branch-and-bound solution.

We note that some arcs may be deleted explicitly by the branch-and-bound algorithm (i.e. by setting the corresponding X variables to zero) while others may be deleted by implication. Examples of the latter case include:

- Setting Y_m to zero deletes all arcs from any plant j to warehouse m , and from warehouse m to any distribution centre k (cf. constraint (3.1)).
- Setting Y_k to zero deletes all arcs from any warehouse m to distribution cen-

tre k , and from distribution centre k to any customer zone l (cf. constraints (3.2) and (3.4)).

- Setting X_{m^*k} to 1 for a particular warehouse m^* and distribution centre $k \in K^{ss}$ deletes all arcs from any other warehouse $m \neq m^*$ to distribution centre k (cf. constraint (3.2)).
- Setting X_{k^*l} to 1 for a particular distribution centre k^* and customer zone $l \in L^{ss}$ deletes all arcs from any other distribution centre $k \neq k^*$ to customer zone l (cf. constraint (3.4))

3.4. Supply chain network design under uncertainty in product demands

The formulation presented in section 3.3 assumes that the product demands, D_{it} , are known constants and aims to design a production and distribution network capable of handling them. We now proceed to consider the case where product demands are not known exactly but are subject to some uncertainty.

A good description of the uncertainties that occur throughout the entire supply chain network, affecting its performance and generally its operation, has been given by Davis (1993). He considers uncertainty arising from suppliers, manufacturing and customers. Suppliers can be characterised through their performance and their responsiveness can be predicted. Manufacturing problems can be addressed using reliability and maintenance analysis for the equipment. Finally, customer demands involve uncertainty which needs to be addressed via high quality forecasting methods.

At a more general level, Zimmermann (2000) identifies the sources of uncertainty as lack of information, complexity of information, conflicting evidence, ambiguity and measurement errors.

3.4.1. Handling uncertainty in supply chain optimisation

Most of the factors affecting the operation of the supply chain network can be classified as either short-term fluctuations or long-term trends. To a certain extent, short-term fluctuations are captured implicitly in steady-state models like the one presented in section 3.3 by averaging each flow in the network over a sufficiently long period of time. On the other hand, taking account of long-term variations necessitates a more direct approach.

Most research on addressing uncertainty can be distinguished into two primary

approaches, referred as the *probabilistic approach* and the *scenario planning approach*. As argued by Zimmermann (2000), the choice of the appropriate method is context-dependent, no single theory being sufficient to model all kinds of uncertainty.

Probabilistic models account for the uncertainty aspects of the supply chain by treating one or more parameters as random variables with known probability distributions (Owen and Daskin, 1998). This approach has been adopted by Cohen and Lee (1988), Svoronos and Zipkin (1991), Lee and Billington (1993), Pyke and Cohen (1993, 1994) and Lee *et al.* (1997).

On the other hand, scenario planning attempts to capture uncertainty by representing it in terms of a moderate number of discrete realisations of the stochastic quantities (Mulvey *et al.*, 1997). Each complete realisation of all uncertain parameters gives rise to a scenario (Owen and Daskin, 1998). The objective is to find robust solutions which perform well under *all* scenarios. In some applications, scenario planning replaces forecasting as a way of taking account of potential changes and trends in a business environment.

These are various common approaches to robust optimisation (Mulvey *et al.*, 1995) seeking, for example, to optimise the expected performance over all scenarios, or to optimise the worst-case scenario; or to minimise the expected or worst-case “regret” across all scenarios.

Mulvey (1996) uses scenario planning for formulating and solving operational problems, while Jenkins (1999) employed this approach to assess the environmental impact of possible disasters.

Mohamed (1999) uses a scenario approach to decide on the design of a production and distribution network that operates under varying exchange rates. A number of scenarios for different exchange rates aim to determine the production policy of the company, which operates in more than two countries.

Applequist *et al.* (2000) study the economic risks for the design and planning of a supply chain for uncertain product demands and inventory levels. The risk of investment according to the authors is directly associated with the production levels and the costs of stocking material in the system.

One important issue that arises in the context of the scenario planning approach is the increase in computational complexity as the number of scenarios increases (see e.g. Cheung and Powell, 1996). One approach towards addressing this concern is via the use of parallel computation (Nielsen and Zenios, 1996). Alternatively, specialised solution techniques have been considered (e.g. Ahmed and Sahinidis, 1998). Recently, MirHassani *et al.* (2000) have proposed a heuristic approach for handling very large numbers of scenarios.

3.4.2. Scenario generation

In our work, we adopt a scenario planning approach for handling the uncertainty in product demands. A question that needs to be addressed in this context concerns the generation of the scenarios to be considered. It is, of course, possible to assume that the demand for each product in each customer zone is an independent random parameter. However, more realistically, demands for similar products will tend to be correlated, and will ultimately be controlled by a small number of major factors such as economic growth, political stability, competitor actions, and so on. This view is consistent with that of Mobasheri *et al.* (1989) who describe scenarios as plausible possible states derived from the present state with consideration of potential major industry events.

The overall aim should be to construct a set of scenarios representative of both optimistic and pessimistic situations within a risk analysis strategy. An example of such an approach is the Towers Perin software tool described by Mulvey (1996). A 12-step procedure for generating appropriate scenarios and a discussion on the use of scenario planning techniques are presented by Vanston *et al.* (1977).

From the practical point of view, the main conclusion of the above discussion is that the total number of scenarios that have to be considered is typically much smaller than what might be expected given the (often large) numbers of products and customer zones. In any case, for the purposes of this work, we will assume that there is some systematic way of generating demand estimates $D_{il}^{[s]}$ for a number of scenarios $s = 1, \dots, NS$.

3.4.3. Mathematical formulation

The formulation of section 3.3 needs to be modified to take account of the multiple scenarios which are used to capture the uncertainty aspects. As we are still aiming to arrive at a *single* network design, the binary variables Y_m , Y_k , X_{mk} , X_{kl} and the capacity variables W_k and D_k remain unchanged. However, the operating variables relating to production and transportation flows will be different depending on which demand scenario materialises. Thus, we introduce a superscript $[s]$ on the corresponding variables which now become $P_{ij}^{[s]}$, $Q_{ijm}^{[s]}$, $Q_{imk}^{[s]}$ and $Q_{ikl}^{[s]}$. Any constraint that involves these variables must be enforced separately for each scenario.

In order to arrive at a meaningful objective function for the optimisation, we assume that the probability of scenario s occurring in practice is known and is denoted by ψ_s . These probabilities will generally satisfy:

$$\sum_{s=1}^{NS} \psi_s = 1 \quad (3.46)$$

Our aim is now to minimise the *expected value* of the cost of the network taken overall the scenarios. The modified formulation can be derived in a straightforward manner from that of section 3.2 and is summarised below:

$$\begin{aligned}
& \min \sum_m C_m^W Y_m + \sum_k C_k^D Y_k + \\
& \sum_{s=1}^{NS} \psi_s \left(\sum_{i,j} C_{ij}^P P_{ij}^{[s]} + \sum_{i,m} C_{im}^{WH} \left(\sum_j Q_{ijm}^{[s]} \right) + \sum_{i,k} C_{ik}^{DH} \left(\sum_m Q_{imk}^{[s]} \right) + \right. \\
& \left. \sum_{f,j,m} C_{fjm}^{[s]} + \sum_{f,m,k} C_{fmk}^{[s]} + \sum_{f,k,l} C_{fkl}^{[s]} \right) \tag{3.47}
\end{aligned}$$

subject to¹:

$$X_{mk} \leq Y_m, \quad \forall m, k \tag{3.1}$$

$$\sum_m X_{mk} = Y_k, \quad \forall k \in K^{ss} \tag{3.2}$$

$$X_{kl} \leq Y_k, \quad \forall k, l \tag{3.4}$$

¹Constraints that are exactly the same in this formulation as in the deterministic one retain the same number as in section 3.3. On the other hand, numbers of the form (3.x') denote a form of constraint (3.x) modified to take account of the existence of multiple scenarios.

$$\sum_k X_{kl} = 1, \quad \forall l \in L^{ss} \quad (3.5)$$

$$Q_{ijm}^{[s]} \leq Q_{ijm}^{[s],\max} Y_m, \quad \forall i, j, m, \quad s = 1, \dots, NS \quad (3.6')$$

$$Q_{imk}^{[s]} \leq Q_{imk}^{[s],\max} X_{mk}, \quad \forall i, m, k, \quad s = 1, \dots, NS \quad (3.7')$$

$$Q_{ikl}^{[s]} \leq Q_{ikl}^{[s],\max} X_{kl}, \quad \forall i, k, l, \quad s = 1, \dots, NS \quad (3.8')$$

$$\sum_i Q_{imk}^{[s]} \geq Q_{mk}^{[s],\min} X_{mk}, \quad \forall m, k, \quad s = 1, \dots, NS \quad (3.9')$$

$$\sum_i Q_{ikl}^{[s]} \geq Q_{kl}^{[s],\min} X_{kl}, \quad \forall k, l, \quad s = 1, \dots, NS \quad (3.10')$$

$$P_{ij}^{[s]} = \sum_m Q_{ijm}^{[s]}, \quad \forall i, j, \quad s = 1, \dots, NS \quad (3.11')$$

$$\sum_j Q_{ijm}^{[s]} = \sum_k Q_{imk}^{[s]}, \quad \forall i, m, \quad s = 1, \dots, NS \quad (3.12')$$

$$\sum_m Q_{imk}^{[s]} = \sum_l Q_{ikl}^{[s]}, \quad \forall i, k, \quad s = 1, \dots, NS \quad (3.13')$$

$$\sum_k Q_{ikl}^{[s]} = \mathcal{D}_{il}^{[s]}, \quad \forall i, l, \quad s = 1, \dots, NS \quad (3.14')$$

$$P_{ij}^{\min} \leq P_{ij}^{[s]} \leq P_{ij}^{\max}, \quad \forall i, j, \quad s = 1, \dots, NS \quad (3.15')$$

$$\sum_i \rho_{ije} P_{ij}^{[s]} \leq R_{je}, \quad \forall j, e, \quad s = 1, \dots, NS \quad (3.16')$$

$$W_m^{\min} Y_m \leq W_m \leq W_m^{\max} Y_m, \quad \forall m \quad (3.17)$$

$$D_k^{\min} Y_k \leq D_k \leq D_k^{\max} Y_k, \quad \forall k \quad (3.18)$$

$$W_m \geq \sum_{i,k} \alpha_{im} Q_{imk}^{[s]}, \quad \forall m, \quad s = 1, \dots, NS \quad (3.19')$$

$$D_k \geq \sum_{i,l,s} \beta_{ik} Q_{ikl}^{[s]}, \quad \forall k, \quad s = 1, \dots, NS \quad (3.20')$$

$$P_{ij}^{[s]} \geq 0 \quad \forall i, j, \quad s = 1, \dots, NS \quad (3.21')$$

$$Q_{ijm}^{[s]} \geq 0, \quad \forall i, j, m, \quad s = 1, \dots, NS \quad (3.22')$$

$$Q_{imk}^{[s]} \geq 0, \quad \forall i, m, k, \quad s = 1, \dots, NS \quad (3.23')$$

$$Q_{ikl}^{[s]} \geq 0, \quad \forall i, k, l, \quad s = 1, \dots, NS \quad (3.24')$$

$$\sum_{r=1}^{NR_{fjm}} Z_{fjmr}^{[s]} = 1, \quad \forall f, j, m, \quad s = 1, \dots, NS \quad (3.29')$$

$$\overline{Q}_{fjm,r-1}Z_{fjmr}^{[s]} \leq Q_{fjmr}^{[s]} \leq \overline{Q}_{fjmr}Z_{fjmr}^{[s]}, \quad \forall f, j, m, r = 1, \dots, NR_{fjm}, \quad s = 1, \dots, NS \quad (3.30')$$

$$\sum_{i \in I_f} Q_{ijm}^{[s]} = \sum_{r=1}^{NR_{fjm}} Q_{fjmr}^{[s]}, \quad \forall f, j, m, \quad s = 1, \dots, NS \quad (3.31')$$

$$C_{fjm}^{[s]} = \sum_{r=1}^{NR_{fjm}} \left[\overline{C}_{fjm,r-1}Z_{fjmr}^{[s]} + (Q_{fjmr}^{[s]} - \overline{Q}_{fjm,r-1}Z_{fjmr}^{[s]}) \frac{\overline{C}_{fjmr} - \overline{C}_{fjm,r-1}}{\overline{Q}_{fjmr} - \overline{Q}_{fjm,r-1}} \right], \quad \forall f, j, m, s \quad (3.32')$$

$$\sum_{r=1}^{NR_{fmk}} Z_{fmkr}^{[s]} = 1, \quad \forall f, m, k, \quad s = 1, \dots, NS \quad (3.33')$$

$$\overline{Q}_{fmk,r-1}Z_{fmkr}^{[s]} \leq Q_{fmkr}^{[s]} \leq \overline{Q}_{fmkr}Z_{fmkr}^{[s]}, \quad \forall f, m, k, r = 1, \dots, NR_{fmk}, \quad s = 1, \dots, NS \quad (3.34')$$

$$\sum_{i \in I_f} Q_{imk}^{[s]} = \sum_{r=1}^{NR_{fmk}} Q_{fmkr}^{[s]}, \quad \forall f, m, k, \quad s = 1, \dots, NS \quad (3.35')$$

$$C_{fmk}^{[s]} = \sum_{r=1}^{NR_{fmk}} \left[\overline{C}_{fmk,r-1}Z_{fmkr}^{[s]} + (Q_{fmkr}^{[s]} - \overline{Q}_{fmk,r-1}Z_{fmkr}^{[s]}) \frac{\overline{C}_{fmkr} - \overline{C}_{fmk,r-1}}{\overline{Q}_{fmkr} - \overline{Q}_{fmk,r-1}} \right], \quad \forall f, m, k, s \quad (3.36')$$

$$\sum_{r=1}^{NR_{fkl}} Z_{fklr}^{[s]} = 1, \forall f, k, l, \quad s = 1, \dots, NS \quad (3.37')$$

$$\bar{Q}_{fkl,r-1} Z_{fklr}^{[s]} \leq Q_{fklr}^{[s]} \leq \bar{Q}_{fklr} Z_{fklr}^{[s]}, \forall f, k, l, r = 1, \dots, NR_{fkl}, \quad s = 1, \dots, NS \quad (3.38')$$

$$\sum_{i \in I_f} Q_{ikl}^{[s]} = \sum_{r=1}^{NR_{fkl}} Q_{fklr}^{[s]}, \forall f, k, l, \quad s = 1, \dots, NS \quad (3.39')$$

$$C_{fkl}^{[s]} = \sum_{r=1}^{NR_{fkl}} \left[\bar{C}_{fkl,r-1} Z_{fklr}^{[s]} + (Q_{fklr}^{[s]} - \bar{Q}_{fkl,r-1} Z_{fklr}^{[s]}) \frac{\bar{C}_{fklr} - \bar{C}_{fkl,r-1}}{\bar{Q}_{fklr} - \bar{Q}_{fkl,r-1}} \right], \forall f, k, l, s \quad (3.40')$$

3.4.4. Scenario-dependent distribution network structure

The formulation presented above was based on the assumption that the structure of the distribution network (i.e. the transportation links between warehouses, distribution centres and customer zones) were independent of the scenario. In many cases, this is unnecessary since the costs associated with the establishment of a transportation link are relatively small. This is especially the case when transportation is outsourced to third parties. In such cases, we could allow the network of transportation links to be different from one scenario to another, effectively allowing for some reconfiguration of the supply chain network during operation.

The only change to the model formulation presented in section 3.4.3 is that variables X_{mk} and X_{kl} will now have a superscript $[s]$ to denote that they can change for each possible scenario of the operation of the network.

The constraints that need to change are:

$$X_{mk}^{[s]} \leq Y_m, \quad \forall m, k, s = 1, \dots, NS \quad (3.1')$$

$$\sum_m X_{mk}^{[s]} = Y_k, \quad \forall k, s = 1, \dots, NS \quad (3.2')$$

$$X_{kl}^{[s]} \leq Y_k, \quad \forall k, l, s = 1, \dots, NS \quad (3.4')$$

$$\sum_k X_{kl}^{[s]} \geq 1, \quad \forall l, s = 1, \dots, NS \quad (3.5')$$

$$Q_{imk}^{[s]} \leq Q_{imk}^{[s], \max} X_{mk}^{[s]}, \quad \forall i, m, k, \quad s = 1, \dots, NS \quad (3.7'')$$

$$Q_{ikl}^{[s]} \leq Q_{ikl}^{[s], \max} X_{kl}^{[s]}, \quad \forall i, k, l, \quad s = 1, \dots, NS \quad (3.8'')$$

$$\sum_i Q_{imk}^{[s]} \geq Q_{mk}^{[s],\min} X_{mk}^{[s]}, \quad \forall m, k, \quad s = 1, \dots, NS \quad (3.9'')$$

$$\sum_i Q_{ikl}^{[s]} \geq Q_{kl}^{[s],\min} X_{kl}^{[s]}, \quad \forall k, l, \quad s = 1, \dots, NS \quad (3.10'')$$

The objective function remains the same as before.

3.5. Concluding remarks

This chapter has presented a detailed mathematical formulation for the problem of designing supply chain networks comprising multiproduct production facilities with shared production resources, warehouses, distribution centres and customer zones. Demands for the various products at the latter may be specified either as given deterministic values or in terms of a number of likely scenarios. A flexible transportation cost model has also been introduced.

Compared to the models that have been presented in the literature to date (cf. chapter 2), our model integrates three distinct echelons of the supply chain within a single, mathematical programming-based formulation. Moreover, it takes account of the complexity introduced by the multiproduct nature of the production facilities, the economies of scale in transportation, and the uncertainty inherent in the product demands.

The next chapter presents a case study illustrating the applicability of this model to a realistic problem.

A Case Study in Steady-State Design of Supply Chain Networks

We consider three manufacturing plants producing 14 different types of products and located in three different European countries, namely the United Kingdom, Spain and Italy (see figure 4.1). Each plant produces several products using a number of shared production resources. However, no single plant produces the entire range of products.

Product demands are such that Europe can be divided into 18 customer zones located in 16 different countries. We consider the establishment of a sufficient network distribution centres (DCs) to cover the whole market. The distribution centres can be located anywhere in 15 countries, and are to be supplied by up to six warehouses, the location of which is also to be chosen among six candidate places.

Table 4.1 presents the one-to-one correspondence of symbols used for the locations to represent the countries on the map of Europe.

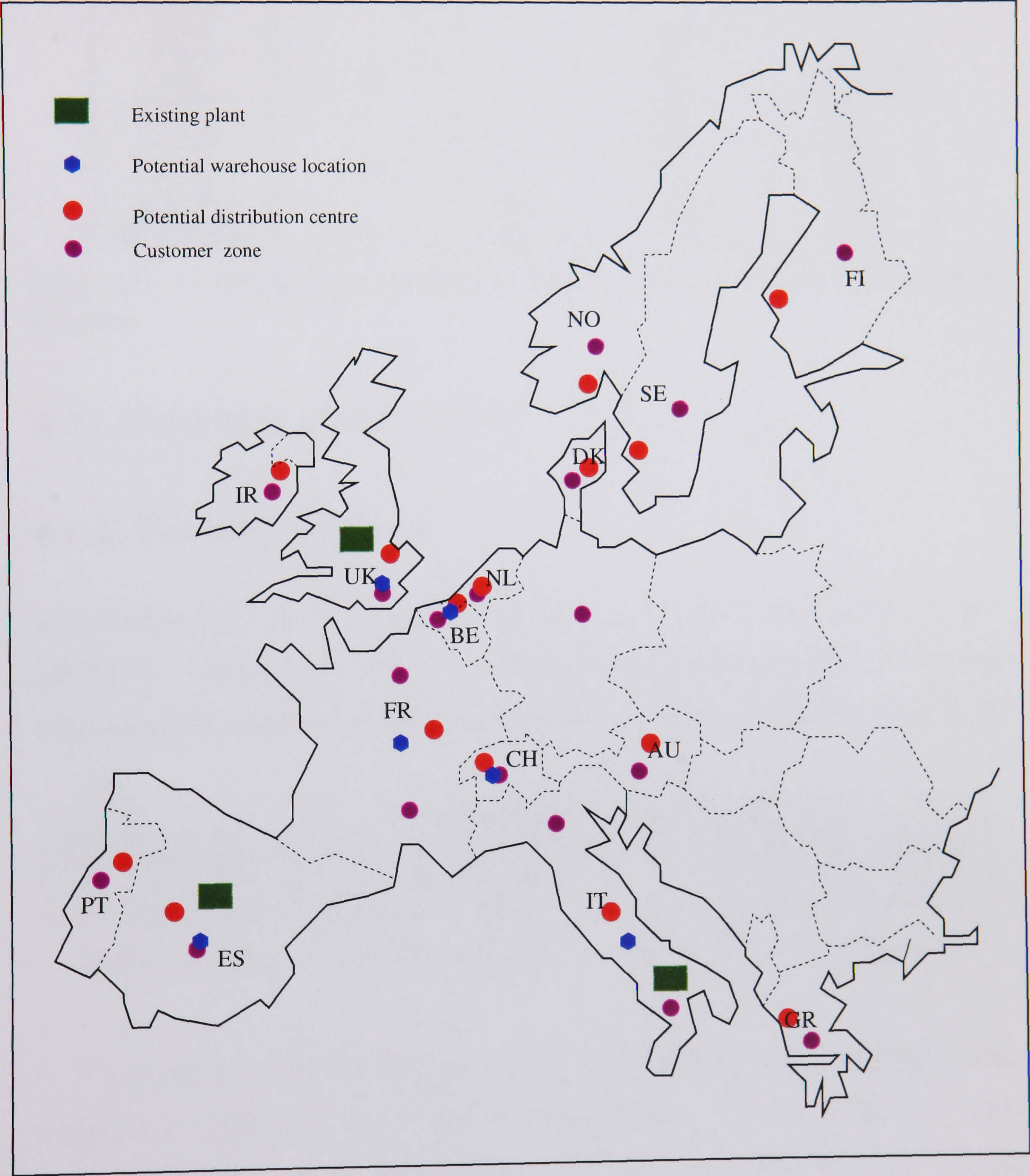


Figure 4.1: Europe-wide supply chain network design.

Location	Production Plant	Warehouse	Distribution Centre	Customer
Austria	PL3	W5	DC15	C15
Belgium			DC12	C12
Denmark			DC9	C9
Finland			DC10	C10
France		W4	DC4	C4(N), C16(S)
Germany				C18
Greece			DC8	C8
Ireland		W3	DC6	C6
Italy			DC3	C3(N), C17(S)
Netherlands			DC7	C7
Norway	PL2	W2	DC14	C14
Portugal			DC11	C11
Spain			DC2	C2
Sweden			DC5	C5
Switzerland	PL1	W6	DC13	C13
United Kingdom		W1	DC1	C1

Table 4.1: Correspondence of symbols used in the case studies to the actual locations.

4.1. Problem description

4.1.1. Production plants

The maximum production capacity (te/wk)¹ of each plant with respect to each product (i.e. the parameter $P_{ij}^{\max}$ of our formulation) is given in table 4.2. The corresponding minimum production rates are taken to be zero (i.e. $P_{ij}^{\min} = 0 \quad \forall i, j$).

Plant	Maximum production capacity (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
PL1	158	2268	1701	1512	0	812	642	482	320	504	0	661	441	221
PL2	0	1411	1058	1328	996	664	664	0	0	0	530	496	330	0
PL3	972	778	607	540	0	416	416	312	208	0	403	0	270	0

Table 4.2: Maximum production capacity of each plant for each product.

The production of each product at each plant makes use of shared equipment resources as indicated in table 4.3. The last column of this table lists the total rate of availability of each resource (in hours of useful operation per week). These are the parameters R_{je} in constraint (3.16) of our formulation.

The unit production costs for the products are listed in table 4.4.

¹te/wk = tonnes/week

4.1.2. Warehouses and distribution centres

The infrastructure costs for the establishment of the various warehouses and distribution centres under consideration are listed in table 4.5; these have already been amortised and are expressed in £/wk.

The warehouses and distribution centres are assumed to have maximum material handling capacities of 14000 te/week and 7000 te/week respectively. All minimum handling capacities are set to zero. The coefficients α_{im} and β_{ik} relating the capacity to the throughput of each material handled (cf. constraints (3.19) and (3.20)) are all taken to be unity. The unit handling costs (including labour or other operating costs) are listed in the last column of table 4.5; for the purposes of this case study, we assume that these costs are independent of the product being handled but may differ from one location to another.

4.1.3. Transportation costs

The transportation costs are generally assumed to depend on the geographical distances between the locations involved. For the purposes of transportation, the 14 products may be aggregated into the three families shown in table 4.6.

The baseline unit costs for transportation from plants to warehouses, warehouses to distribution centres, and distribution centers to customer zones are shown in tables 4.7, 4.8 and 4.9 respectively. These apply to transportation flows up to 40 te/wk for each family. Above this limit, it is possible to deploy alternative models of transportation that have lower average unit costs. The magnitudes of these costs (relative to the corresponding baseline values of tables 4.7-4.9) are shown in table 4.10.

Plant Resource	Shared equipment resource utilisation coefficient (hr/te)														Total resource availability (hr/wk)
	P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	P 9	P 10	P 11	P 12	P 13	P 14	
PL1.E1										0.2381					120
PL1.E2		0.0463	0.0617	0.0694			0.1634	0.2178	0.3268			0.2267	0.3401	0.6802	105
PL1.E3															105
PL1.E4															150
PL1.E5						0.1292									105
PL1.E6	0.6667														105
PL2.E1											0.1984	0.2118	0.3174		105
PL2.E2				0.0793	0.1054	0.1582	0.1582								105
PL2.E3		0.0740	0.1000												105
PL3.E1			0.1976	0.2222											120
PL3.E2						0.3968	0.3968	0.5291	0.7936						165
PL3.E3	0.1200	0.1543													120
PL3.E4											0.2976		0.4444		120

Table 4.3: Utilisation and availability data for shared manufacturing resources.

	Unit production costs (£/te)						
Plant	P1	P2	P3	P4	P5	P6	P7
PL1	61.27	61.27	61.27	61.27	0.0	61.27	256.90
PL2	59.45	59.45	59.45	59.45	59.45	59.45	268.50
PL3	61.44	61.44	61.44	61.44	0.0	61.44	270.80
	Unit production costs (£/te)						
Plant	P8	P9	P10	P11	P12	P13	P14
PL1	256.90	256.90	61.27	0.0	256.90	256.90	256.90
PL2	0.0	0.0	0.0	268.50	268.50	268.50	0.0
PL3	270.80	270.80	0.0	270.80	0.0	270.80	0.0

Table 4.4: Unit production costs.

Warehouse	Infrastructure cost (£/wk)	Handling cost (£/te)
W1	10000	4.25
W2	5000	4.55
W3	4000	4.98
W4	6000	4.93
W5	6500	4.06
W6	4000	5.28
Distribution centre	Infrastructure cost (£/wk)	Handling cost (£/te)
DC01	10000	4.25
DC02	5000	4.55
DC03	4000	4.98
DC04	6000	4.93
DC05	6500	4.85
DC06	4000	3.90
DC07	6000	4.06
DC08	4000	3.08
DC09	5000	6.00
DC10	3000	4.85
DC11	4500	4.12
DC12	7000	5.66
DC13	9000	5.28
DC14	5500	4.95
DC15	8500	4.83

Table 4.5: Fixed infrastructure and material handling costs for candidate warehouses and distribution centres.

Families	Products
F1	P1 - P6, P10
F2	P7 - P9
F3	P11 - P14

Table 4.6: Division of products into transportation families.

Transportation cost for family F1 products (£/te)						
	Warehouses					
Plant	W1	W2	W3	W4	W5	W6
PL1	1.24	58.56	62.30	26.16	17.44	36.13
PL2	60.82	1.68	70.96	43.93	70.96	55.76
PL3	76.16	79.21	1.52	54.83	68.54	41.12
Transportation cost for family F2 products (£/te)						
	Warehouses					
Plant	W1	W2	W3	W4	W5	W6
PL1	1.35	63.46	67.51	28.35	18.90	39.15
PL2	82.70	2.29	96.48	59.72	96.48	75.81
PL3	94.90	98.69	1.89	68.32	85.41	51.24
Transportation cost for family F3 products (£/te)						
	Warehouses					
Plant	W1	W2	W3	W4	W5	W6
PL1	1.46	68.88	73.28	30.77	20.51	42.50
PL2	79.69	2.21	92.97	57.55	92.97	73.05
PL3	92.82	96.53	1.85	66.83	83.54	50.12

Table 4.7: Baseline unit transportation costs from plants to warehouses.

Transportation cost for family F1 products (£/te)															
Distribution centres															
Warehouse	DC01	DC02	DC03	DC04	DC05	DC06	DC07	DC08	DC09	DC10	DC11	DC12	DC13	DC14	DC15
W1	0	74.40	76.13	25.96	69.21	29.41	17.30	117.66	44.99	110.74	76.13	12.11	39.79	64.02	60.56
W2	58.85	0	62.96	45.16	109.49	69.80	67.06	94.44	90.33	145.08	17.79	60.22	52.01	108.12	72.54
W3	72.83	76.14	0	49.66	94.35	99.32	62.90	43.04	79.45	129.12	94.35	62.90	33.10	104.29	28.14
W4	28.54	62.78	57.08	0	87.52	58.98	32.34	106.55	62.78	135.09	72.30	22.83	19.02	89.42	49.47
W5	16.51	73.58	57.06	25.52	48.05	37.54	0	93.10	25.52	84.09	78.08	7.50	30.03	45.05	42.04
W6	69.52	67.78	34.76	17.38	78.21	69.52	34.76	79.95	59.09	121.66	78.21	33.02	0	83.42	27.80
Transportation cost for family F2 products (£/te)															
Distribution centres															
Warehouse	DC01	DC02	DC03	DC04	DC05	DC06	DC07	DC08	DC09	DC10	DC11	DC12	DC13	DC14	DC15
W1	0	75.28	77.03	26.26	70.02	29.76	17.50	119.04	45.51	112.04	77.03	12.25	40.26	64.77	61.27
W2	60.87	0	65.12	46.71	113.25	72.20	69.36	97.68	93.43	150.06	18.40	62.29	53.79	111.84	75.03
W3	90.75	94.88	0	61.88	117.57	123.76	78.38	53.63	99.00	160.89	117.57	78.38	41.25	129.95	35.06
W4	28.88	63.54	57.77	0	88.58	59.69	32.73	107.83	63.54	136.72	73.17	23.10	19.25	90.50	50.06
W5	17.90	79.77	61.86	27.67	52.09	40.70	0	100.93	27.67	91.16	84.65	8.14	32.56	48.84	45.58
W6	86.62	84.46	43.31	21.65	97.45	86.62	43.31	99.62	73.63	151.59	97.45	41.14	0	103.95	34.65
Transportation cost for family F3 products (£/te)															
Distribution centres															
Warehouse	DC01	DC02	DC03	DC04	DC05	DC06	DC07	DC08	DC09	DC10	DC11	DC12	DC13	DC14	DC15
W1	0	69.15	70.78	24.14	64.32	27.33	16.08	109.35	41.81	102.92	70.76	11.25	36.98	59.50	56.28
W2	69.66	0	74.52	53.46	129.61	82.63	79.38	111.79	106.93	171.74	21.06	71.28	61.56	127.99	85.87
W3	92.01	96.19	0	62.73	119.19	125.47	79.46	54.37	100.37	163.11	119.19	79.46	41.82	131.74	35.55
W4	26.53	58.37	53.07	0	81.37	54.83	30.07	99.06	58.37	125.59	67.22	21.22	17.69	83.14	45.99
W5	20.49	91.29	70.80	31.67	59.62	46.58	0	115.51	31.67	104.33	96.88	9.31	37.26	55.89	52.16
W6	87.82	85.63	43.91	21.95	98.80	87.82	43.91	85.70	63.34	130.42	83.84	35.40	0	89.43	35.13

Table 4.8: Baseline unit transportation costs from warehouses to distribution centres.

Transportation cost for family F1 products (£/te) - Customer zones																		
DCs	CZ1	CZ2	CZ3	CZ4	CZ5	CZ6	CZ7	CZ8	CZ9	CZ10	CZ11	CZ12	CZ13	CZ14	CZ15	CZ16	CZ17	CZ18
DC1	0	75.61	54.51	12.30	70.34	29.89	17.58	119.57	45.72	112.54	77.37	12.30	40.44	65.06	61.54	45.72	100.23	38.68
DC2	73.55	0	78.68	73.55	136.84	87.23	83.81	118.02	112.89	181.31	22.23	75.26	64.99	135.12	90.65	34.21	71.84	94.07
DC3	73.28	76.61	21.65	49.96	94.93	99.93	63.28	43.30	79.94	129.90	94.93	63.28	33.31	104.92	28.31	51.63	21.65	58.29
DC4	26.58	58.47	53.16	14.17	81.51	54.93	30.12	99.23	58.47	125.81	67.33	21.26	17.72	83.28	46.07	54.93	17.72	40.75
DC5	77.16	154.33	109.96	84.88	0	90.67	59.80	136.97	27.00	48.23	160.12	69.45	86.81	21.22	77.16	106.10	131.18	50.15
DC6	27.08	84.65	79.57	38.93	79.57	0	42.32	143.90	57.56	118.51	77.87	38.93	67.72	67.72	88.03	67.72	126.97	64.33
DC7	19.97	88.99	67.19	25.42	58.11	0	0	112.60	30.87	101.70	94.44	9.08	36.32	54.48	50.85	45.40	94.44	21.79
DC8	118.02	121.49	64.22	109.34	123.23	147.53	107.61	0	116.29	149.27	142.32	109.34	79.84	137.12	60.74	86.78	27.77	93.72
DC9	42.04	106.72	61.44	48.51	22.63	54.98	27.49	108.34	0	64.68	109.96	35.57	54.98	24.25	53.36	67.91	85.70	24.25
DC10	107.82	178.57	116.24	117.92	42.11	117.92	94.34	144.88	67.38	0	181.94	104.45	117.92	50.54	104.45	134.77	144.88	84.23
DC11	75.72	22.37	87.77	67.12	142.85	79.17	89.49	141.13	117.03	185.87	0	79.17	79.17	137.68	104.98	60.23	115.31	103.26
DC12	11.91	74.91	44.26	13.62	61.29	39.15	8.51	107.26	37.45	105.56	78.31	0	30.64	61.29	49.37	39.15	88.53	25.53
DC13	69.11	67.38	13.82	29.37	77.75	69.11	34.55	79.47	58.74	120.94	77.75	32.82	0	82.93	27.64	19.00	58.74	34.55
DC14	63.10	134.74	88.69	71.63	18.76	68.22	51.16	134.74	25.58	51.16	136.44	61.40	81.86	0	80.16	95.51	127.92	51.16
DC15	56.52	85.60	17.76	50.06	64.60	83.98	45.22	56.52	53.29	100.13	98.52	46.83	25.84	75.90	0	41.99	45.22	30.68

Transportation cost for family F2 products (£/te) - Customer zones																		
DCs	CZ1	CZ2	CZ3	CZ4	CZ5	CZ6	CZ7	CZ8	CZ9	CZ10	CZ11	CZ12	CZ13	CZ14	CZ15	CZ16	CZ17	CZ18
DC1	0	73.12	52.71	11.90	68.02	28.90	17.00	115.63	44.21	108.83	74.82	11.90	39.11	62.91	59.51	44.21	96.92	37.41
DC2	73.20	0	78.31	73.20	136.20	86.82	83.42	117.47	112.36	180.46	22.13	74.91	64.69	134.49	90.23	34.05	71.50	93.63
DC3	81.65	85.36	24.12	55.67	105.78	111.34	70.52	48.25	89.07	144.75	105.78	70.52	37.11	116.91	31.54	57.52	24.12	64.95
DC4	24.76	54.48	49.53	13.20	75.95	51.18	28.06	92.46	54.48	117.22	62.74	19.81	16.51	77.60	42.92	51.18	16.51	37.97
DC5	77.52	155.04	110.47	85.27	0	91.09	60.08	137.60	27.13	48.45	160.86	69.77	87.21	21.31	77.52	106.59	131.79	50.39
DC6	32.65	102.06	95.93	46.94	95.93	0	51.03	173.50	69.40	142.88	93.89	46.94	81.64	81.64	106.14	81.64	153.09	77.56
DC7	19.54	87.05	65.73	24.87	56.85	44.41	0	110.15	30.20	99.49	92.38	8.88	35.53	53.30	49.74	44.41	92.38	21.32
DC8	127.18	130.92	69.20	117.83	132.79	158.98	115.96	0	125.31	160.85	153.37	117.83	86.03	147.76	65.46	93.52	29.92	101.00
DC9	48.45	123.01	70.82	55.91	26.09	63.36	31.68	124.87	0	74.55	126.73	41.00	63.36	27.95	61.50	78.27	98.78	27.95
DC10	111.91	185.36	120.66	122.40	43.71	122.40	97.92	150.38	69.94	0	188.85	108.41	122.40	52.46	108.41	139.89	150.38	87.43
DC11	81.65	24.12	94.64	72.37	154.03	85.36	96.50	152.17	126.19	200.42	0	85.36	85.36	148.46	113.20	64.95	124.33	111.34
DC12	12.44	78.21	46.21	14.22	63.99	40.88	8.88	111.98	39.10	110.21	81.76	0	31.99	63.99	51.55	40.88	92.43	26.66
DC13	58.96	57.48	11.79	25.05	66.33	58.96	29.48	67.80	50.11	103.18	66.33	28.00	0	70.75	23.58	16.21	50.11	29.48
DC14	60.11	128.35	84.48	68.23	17.87	64.98	48.74	128.35	24.37	48.74	129.97	58.48	77.98	0	76.36	90.98	121.85	48.74
DC15	60.97	92.32	19.16	54.00	69.68	90.58	48.77	60.97	57.48	108.00	106.26	50.51	27.87	81.87	0	45.29	48.77	33.09

Transportation cost for family F3 products (£/te) - Customer zones																		
DCs	CZ1	CZ2	CZ3	CZ4	CZ5	CZ6	CZ7	CZ8	CZ9	CZ10	CZ11	CZ12	CZ13	CZ14	CZ15	CZ16	CZ17	CZ18
DC1	0	72.82	52.50	11.85	67.74	28.79	16.93	115.17	44.03	108.39	74.52	11.85	38.95	62.66	59.27	44.03	96.54	37.26
DC2	69.04	0	73.86	69.04	128.46	81.89	78.68	110.80	105.98	170.21	20.87	70.65	61.02	126.85	85.10	32.11	67.44	88.31
DC3	74.54	77.92	22.02	50.82	96.56	101.64	64.37	44.04	81.31	132.13	96.56	64.37	33.88	106.72	28.79	52.51	22.02	59.29
DC4	28.74	63.22	57.48	15.32	88.13	59.39	32.57	107.29	63.22	136.03	72.80	22.99	19.16	90.05	49.81	59.39	19.16	44.06
DC5	65.66	131.33	93.57	72.23	0	77.15	50.89	116.56	22.98	41.04	136.26	59.10	73.87	18.05	65.66	90.29	111.63	42.68
DC6	31.07	97.12	91.29	44.67	91.29	0	48.56	165.10	66.04	135.96	89.35	44.67	77.69	77.69	101.00	77.69	145.68	73.81
DC7	20.58	91.67	69.22	26.19	59.87	46.77	0	116.00	31.80	104.77	97.29	9.35	37.42	56.13	52.38	46.77	97.29	22.45
DC8	121.40	124.97	66.05	112.48	126.76	151.75	110.69	0	119.62	153.54	146.40	112.48	82.12	141.04	62.48	89.27	28.56	96.41
DC9	55.75	141.52	81.48	64.32	30.02	72.90	36.45	143.66	0	85.77	145.81	47.17	72.90	32.16	70.76	90.06	113.64	32.16
DC10	106.08	175.70	114.37	116.03	41.44	116.03	92.82	142.55	66.30	0	179.02	102.77	116.03	49.72	102.77	132.60	142.55	82.88
DC11	74.81	22.10	86.72	66.31	141.13	78.21	88.42	139.43	115.62	183.64	0	78.21	78.21	136.03	103.72	59.51	113.92	102.02
DC12	13.58	85.41	50.47	15.53	69.88	44.64	9.70	122.30	42.70	120.36	89.29	0	34.94	69.88	56.29	44.64	100.94	29.11
DC13	72.36	70.55	14.47	30.75	81.40	72.36	36.18	83.21	61.50	126.63	81.40	34.37	0	86.83	28.94	19.90	61.50	36.18
DC14	72.66	155.15	102.12	82.48	21.60	78.56	58.92	155.15	29.46	58.92	157.12	70.70	94.27	0	92.30	109.98	147.30	58.92
DC15	62.62	94.83	19.68	55.47	71.57	93.04	50.10	62.62	59.05	110.94	109.15	51.89	28.63	84.10	0	46.52	50.10	33.99

Table 4.9: Baseline unit transportation costs from distribution centres to customer zones.

Amount transported (te/wk)	Unit transportation cost relative to baseline (cf. tables 4.7-4.9)
0-40	1.00
40-100	0.95
100-1000	0.89
1000-5000	0.80

Table 4.10: Economies of scale in transportation costs.

4.2. Case Study 1: Deterministic product demands

We start by considering a deterministic problem aiming to satisfy the demands shown in table 4.11.

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	18	0	0	106	0	252	0	0	43	0	0	0	70	34
CZ02	0	99	55	203	76	0	30	0	0	0	20	0	0	0
CZ03	0	155	50	266	0	66	17	0	0	0	0	0	15	0
CZ04	15	150	126	0	0	0	5	27	0	0	0	25	0	0
CZ05	0	0	92	0	0	0	0	0	21	0	0	0	50	0
CZ06	0	0	50	0	0	68	0	0	20	0	0	0	10	10
CZ07	0	114	0	140	0	0	0	0	34	0	0	0	68	0
CZ08	0	50	0	45	40	23	0	0	5	0	0	0	0	0
CZ09	0	50	0	0	0	0	52	0	7	0	0	0	0	0
CZ10	0	0	0	17	0	0	0	0	5	0	0	0	16	0
CZ11	0	50	0	31	20	0	0	0	0	0	20	0	15	20
CZ12	10	31	0	0	0	100	13	0	0	38	0	0	0	0
CZ13	0	21	0	100	0	0	15	0	0	0	0	0	50	0
CZ14	0	0	0	50	0	0	0	7	0	0	0	0	0	0
CZ15	0	0	0	150	0	0	10	0	0	0	15	0	0	10
CZ16	15	0	68	0	0	0	5	20	0	0	0	20	0	0
CZ17	0	103	0	110	0	44	12	0	0	0	0	0	13	0
CZ18	0	0	0	0	0	100	0	0	0	266	0	0	0	0

Table 4.11: Product demands by customer zone.

The optimal solution of the model of section 3.3 leads to the network structure shown in figure 4.2. The number above each arc denotes the corresponding total material flow (in te/wk). The optimal objective function corresponds to a minimum cost of £677,458 per week. A breakdown of this cost is shown in table 4.12. Tables 4.13 - 4.16 present more details of the costs and the flows for each stage in the supply chain.

Cost element	Amount (£/wk)	%
Production	436,892	64.49
Warehouse infrastructure	19,000	0.03
Distribution centre infrastructure	19,000	0.03
Material handling costs	40,060	0.06
Material transportation	142,506	35.39
Total	677,458	100.00

Table 4.12: Costs breakdown for optimal solution.

It can be seen that the main contributor to costs (64.5%) is production, with transportation being another major contributor (35.4%). The cost of establishing and running warehouses and distribution centres is relatively small in this case. In fact, it could be argued that, for this particular problem, the distribution centres may be unnecessary as they are placed at the same locations as the corresponding warehouses.

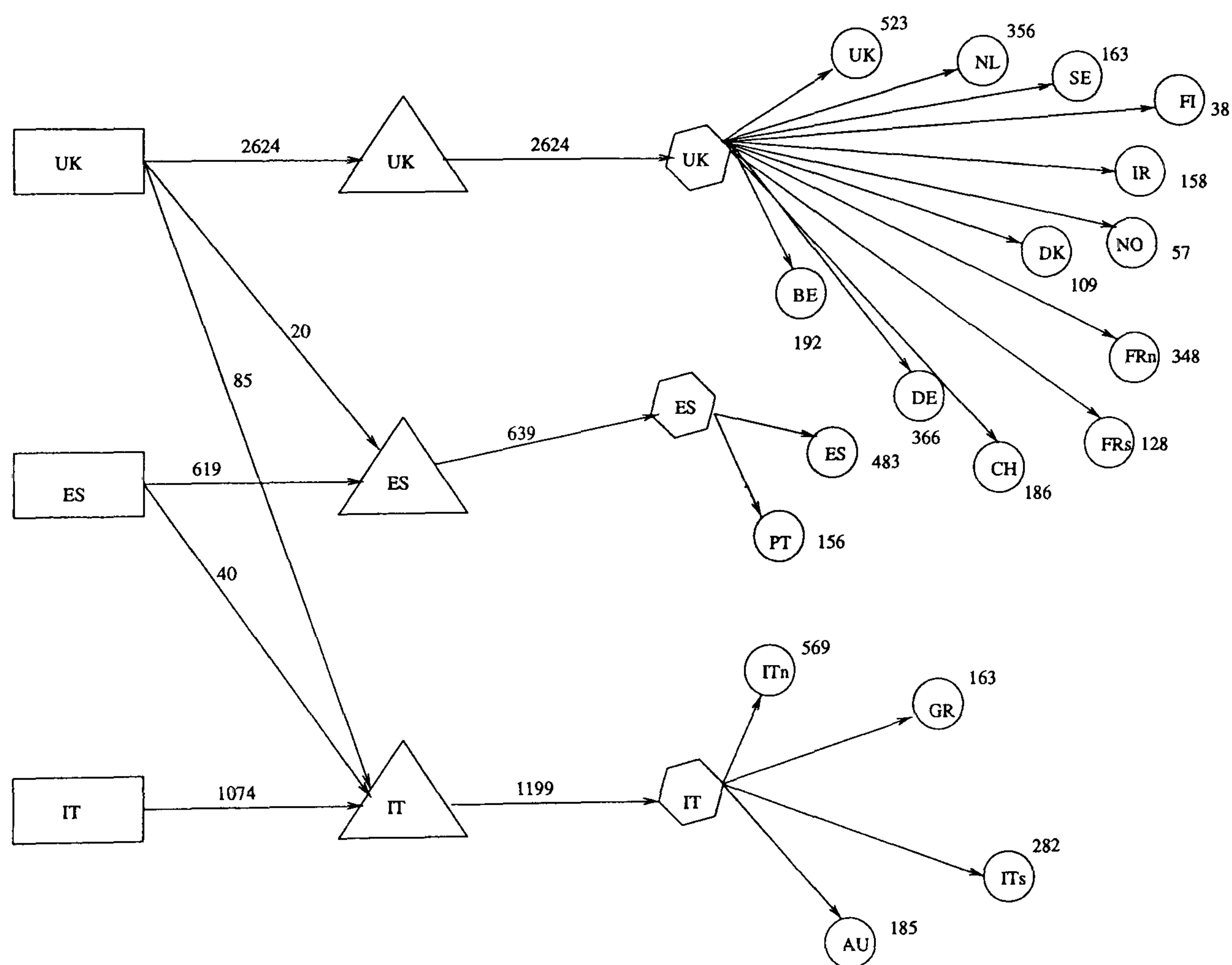


Figure 4.2: Optimal network for deterministic product demands.

The market is served by three warehouses and three distribution centres lo-

cated close to the production sites. This is primarily a result of the demand patterns considered since the three countries which host the manufacturing plants are also the biggest customers.

Plant	Production (te/wk)	Manufacturing cost (£/wk)
UK	2729	295516
ES	659	57201
IT	1074	84175
Total	4462	436892

Table 4.13: Costs associated with manufacturing plants.

Warehouse	Infrastructure cost (£/wk)	Throughput (te/wk)	Handling cost (£/wk)
UK	10000	2624	11152
ES	5000	639	2907
IT	4000	1199	5971
Total	19000	4462	20030

Table 4.14: Costs associated with warehouses.

Distribution Centre	Infrastructure cost (£/wk)	Throughput (te/wk)	Handling cost (£/wk)
UK	10000	2624	11152
ES	5000	639	2907
IT	4000	1199	5971
Total	19000	4462	20030

Table 4.15: Costs associated with distribution centres.

The corresponding computational statistics for this problem are summarised in table 4.17. The MILP problems were solved using the CPLEX v6.5 (ILOG, 1998) code embedded within *ooMILP* (Tsiakis *et al.*, 1999). As can be seen from the results of table 4.17, computational demands are relatively modest in this case despite the relatively high number of integer variables. This can be attributed primarily to the low integrality solution gap of this formulation.

Transportation costs									
Plant	Warehouse			Distribution centre			Customer zone		
Location	Location	Flow (te/wk)	Cost [†] (£/wk)	Location	Flow (te/wk)	Cost [†] (£/wk)	Location	Flow (te/wk)	Cost [†] (£/wk)
UK	UK	2634	3936	UK	2634	0.0	UK	523	1167
							FRn	198	5484
							SE	163	16152
							IR	158	6524
							NL	356	8136
							DK	109	7108
							FI	38	6267
							NO	7	5408
							BE	192	3086
							CH	186	10360
							FRs	128	8314
							DE	366	17451
ES	ES	619	1380	ES	639	0.0	ES	483	1034
							PT	156	4802
IT	IT	1074	2046	IT	1199	0.0	ITn	569	15235
							GR	163	9351
							AU	185	7161
							ITs	282	7872
Total		4462	21464		4462	0.0		4462	141002

† This is the cost incurred for material to be transported to this destination.

Table 4.16: Transportation costs throughout the system

Number of Constraints	17949
Number of Integer Variables	4917
Number of Continuous Variables	11025
Optimal Objective Value	677458
Fully Relaxed LP Objective Function	677458
Integrality Solution Gap	0.00%
Optimality Margin	0.1 %
Number of Branch & Bound Nodes	2634
CPU time (SUN-Ultra60)	1082s
Solver	CPLEX6.5

Table 4.17: Computational statistics for deterministic design problem.

4.3. Case Study 2: Uncertain product demands

We now consider a case with three possible product demand scenarios. The first one is the same as that used for the deterministic case considered in section 4.2 (cf. table 4.11). The demands for the second and third scenarios are given in tables 4.18 and 4.19 respectively. We assume that all three scenarios are equally probable, i.e. $\psi_1 = \psi_2 = \psi_3 = 0.333$. Our aim is to design a network that can handle all three scenarios while minimising the expected cost.

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	18	0	0	506	0	452	0	0	43	0	0	0	120	34
CZ02	0	499	155	203	76	0	30	0	0	0	20	0	0	0
CZ03	0	155	0	166	0	66	17	0	0	0	0	0	15	0
CZ04	15	0	126	0	0	0	5	27	0	0	0	25	0	0
CZ05	0	0	92	0	0	0	0	0	21	0	0	0	50	0
CZ06	0	0	0	0	0	68	0	0	20	0	0	0	10	10
CZ07	0	14	0	40	0	0	0	0	34	0	0	0	68	0
CZ08	0	0	0	45	0	23	0	0	5	0	0	0	0	0
CZ09	0	0	0	0	0	0	52	0	7	0	0	0	0	0
CZ10	0	0	0	17	0	0	0	0	5	0	0	0	16	0
CZ11	0	0	0	31	0	0	0	0	0	0	0	0	15	0
CZ12	0	31	0	0	0	0	13	0	0	38	0	0	0	0
CZ13	0	21	0	0	0	0	15	0	0	0	0	0	0	0
CZ14	0	0	0	0	0	0	0	7	0	0	0	0	0	0
CZ15	0	0	0	0	0	0	10	0	0	0	15	0	0	0
CZ16	15	0	68	0	0	0	5	20	0	0	0	20	0	0
CZ17	0	103	0	110	0	44	12	0	0	0	0	0	13	0
CZ18	0	0	0	0	0	0	0	0	0	266	0	0	0	0

Table 4.18: Product demands by customer zone for scenario 2.

In order to gain some understanding of the implications of the three demand patterns, we first consider each scenario in isolation by formulating and solving the deterministic design problem corresponding to it as described in section 3.3. The optimal network structures for the three scenarios are shown in figures 4.2, 4.3 and 4.4 respectively. The three structures are similar in terms of the locations of warehouses and distribution centres. However, they differ in the allocation of products to plants which depends on the demand patterns imposed. More specifically, in the first scenario, the Italian part of the distribution network needs contributions from the UK and Spanish plants to cover the demands placed by its customers, while in the other two scenarios, it is capable of supplying the

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	38	0	0	206	0	252	0	0	43	30	0	0	120	34
CZ02	0	199	155	103	76	0	30	0	0	0	20	0	0	0
CZ03	0	155	0	80	0	66	17	0	0	0	0	0	15	0
CZ04	15	0	126	0	0	0	5	27	0	0	0	25	0	0
CZ05	0	0	92	100	0	0	0	0	21	0	45	0	50	0
CZ06	0	0	0	0	36	68	15	0	20	30	0	0	10	10
CZ07	0	14	0	80	0	0	0	40	34	0	0	0	68	0
CZ08	0	0	0	45	0	23	0	0	5	0	40	0	0	0
CZ09	0	150	0	0	0	0	52	20	7	0	0	0	0	0
CZ10	0	0	0	17	0	0	0	0	5	0	0	0	16	0
CZ11	0	150	0	31	0	0	0	0	0	0	30	0	15	30
CZ12	0	31	0	0	30	0	13	0	0	38	0	0	0	0
CZ13	0	21	0	86	0	50	15	0	0	0	0	0	0	0
CZ14	0	0	0	120	0	0	0	7	20	0	0	0	0	0
CZ15	0	0	0	0	0	0	10	0	0	0	15	0	0	0
CZ16	15	0	68	0	0	0	5	20	0	0	0	20	0	55
CZ17	0	103	0	110	0	44	12	0	0	0	0	0	13	0
CZ18	0	0	0	0	17	0	0	0	0	266	0	0	50	0

Table 4.19: Product demands by customer zone for scenario 3.

required amounts out of its own production. The set of customers assigned to each distribution centre is also different in each scenario, which demonstrates the effects of transportation costs on the network structure.

We now consider all three scenarios simultaneously, making use of the scenario planning formulation of section 3.4. The optimal network structure obtained is shown in figure 4.5. The three numbers above each arc denote the total flowrate (in te/wk) of material over this link.

It is interesting to compare the expected costs of the above network to the costs that would be incurred if one were to fix the structure² to one of those shown in figures 4.2, 4.3 and 4.4 respectively, and to optimise the operational decisions only. As can be seen from table 4.20, the fixed structure of figure 4.2 has the same expected cost as the optimal solution for the three scenarios case. This is due to the fact that the corresponding network configurations are the same (cf. figures 4.2 and 4.5). On the other hand, the networks of figures 4.3 and 4.4 result in more expensive operations than the optimal solution of figure 4.5.

Tables 4.21 - 4.24 present the costs and the flows for each stage in the dis-

²In terms of both warehouse and distribution centre location, and network connectivity.

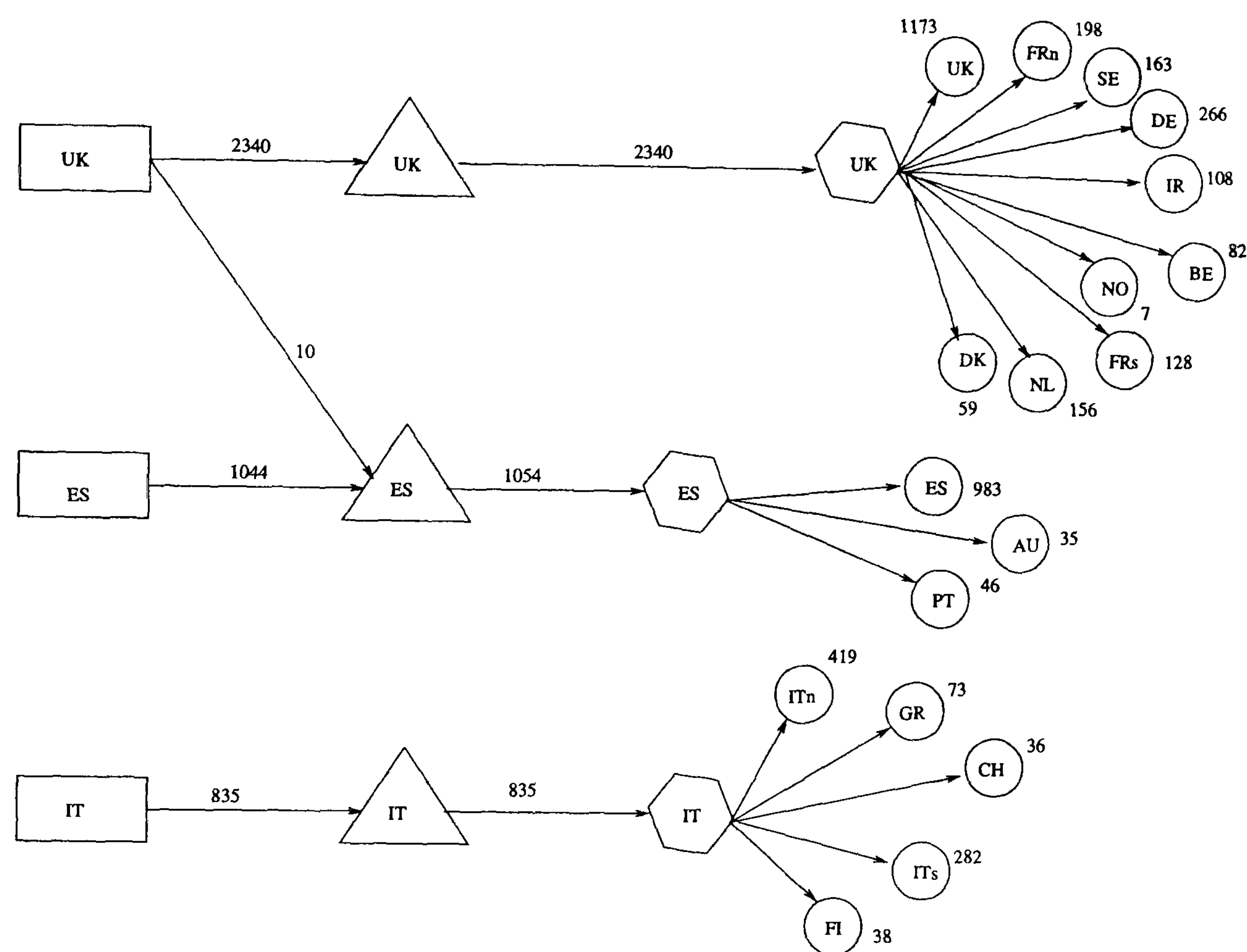


Figure 4.3: Optimal network structure for scenario 2.

tribution chain for each scenario in the optimal solution of the scenario planning formulation.

Network structure	Minimum expected cost over 3 demand scenarios (£/wk)	Difference from optimal solution (£/wk)
Figure 4.2	677,285	0
Figure 4.3	684,388	7,103
Figure 4.4	679,171	1,886
Figure 4.5	677285	0

Table 4.20: Comparison of expected performance of fixed network structures with performance of optimal network structure of figure 4.5.

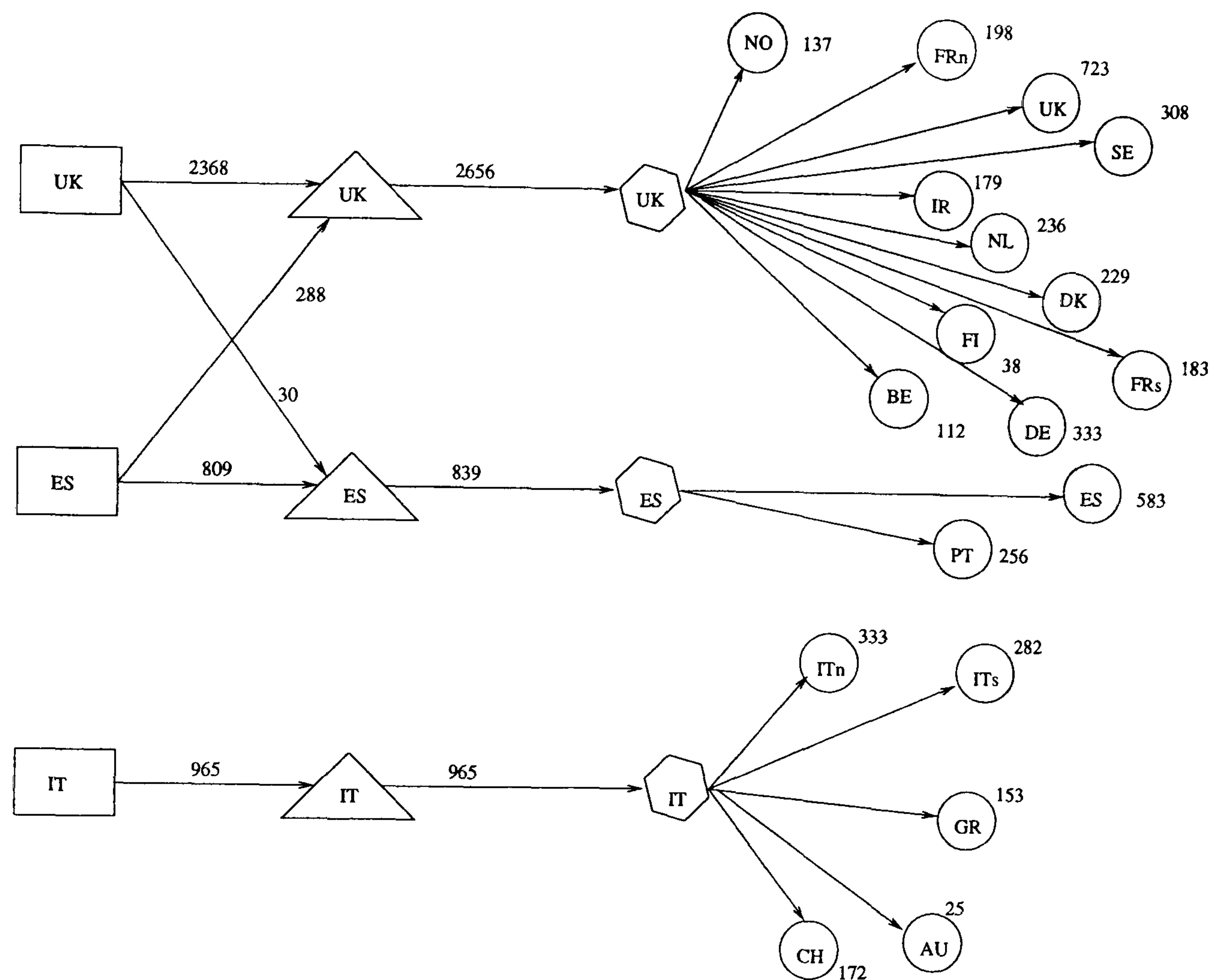


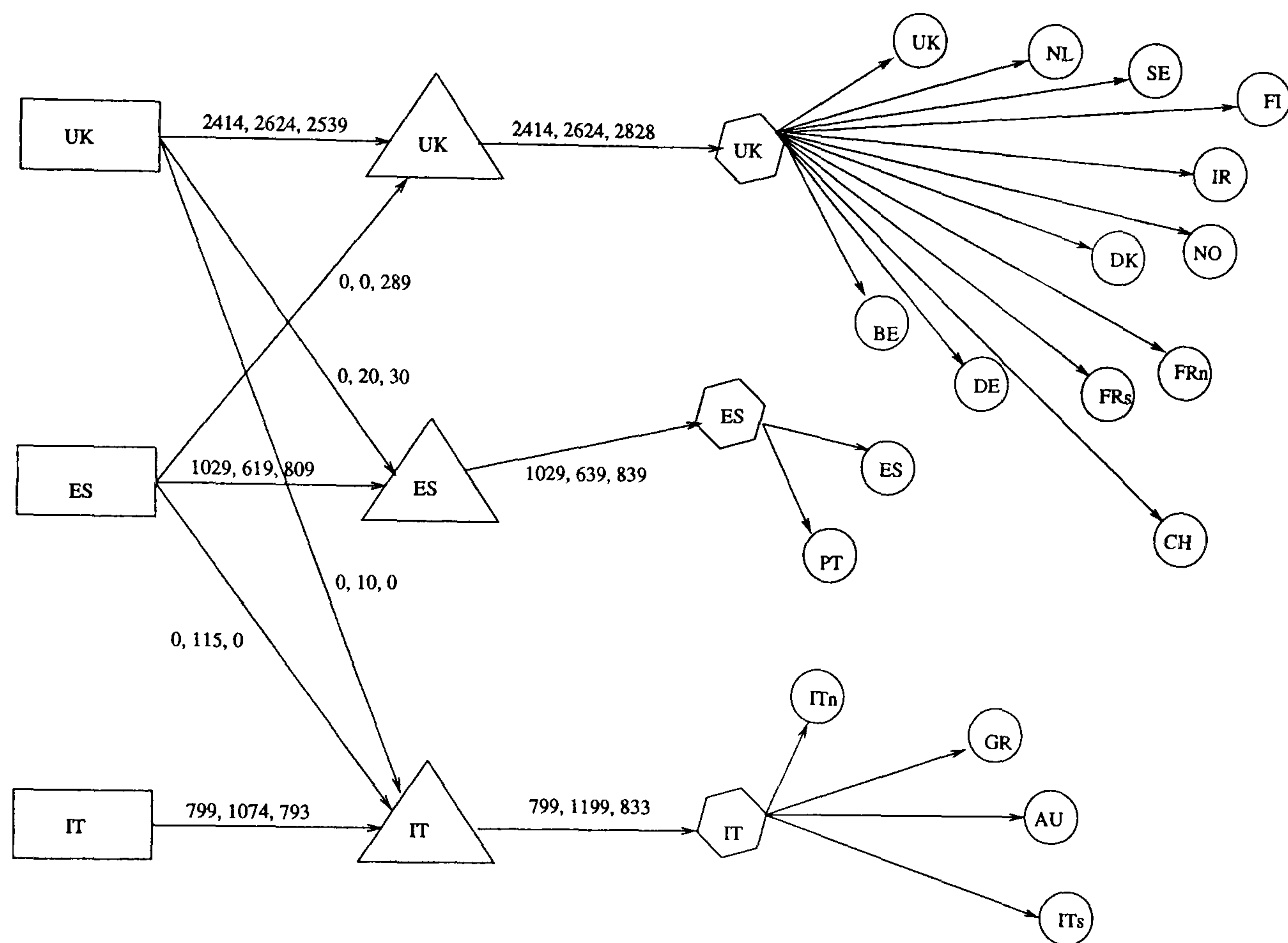
Figure 4.4: Optimal network structure for scenario 3.

Plant	Production (te/wk)			Manufacturing cost (£)		
	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
UK	2654	2378	2569	290896	270566	285749
ES	734	1029	1099	61684	74762	128186
IT	1074	835	763	84175	67304	83685
Total	4462	4242	4460	436755	412632	497620
Expected Value	4383			448555		

Table 4.21: Costs associated with manufacturing plants for scenario-based design problem.

Warehouse	Infrastructure cost (£/wk)	Throughput (te/wk)			Handling cost (£/wk)		
		Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
UK	10000	2624	2414	2828	11152	10983	12019
ES	5000	639	1029	829	2907	4681	3817
IT	4000	1199	799	793	5971	3955	4073
Total	19000	4462	4242	4460	20030	19619	18680
Expected Value	19000	4383			19423		

Table 4.22: Costs associated with warehouses for scenario-based design problem.



Transportation costs																						
Plant		Warehouse						Distribution centre						Customer zone								
Location	Location	Flow(te/wk)			Cost† (£/wk)			Location	Flow(te/wk)			Cost† (£/wk)			Location	Flow(te/wk)			Cost† (£/wk)			
		Sc 1	Sc 2	Sc 3	Sc 1	Sc 2	Sc 3		Sc 1	Sc 2	Sc 3	Sc 1	Sc 2	Sc 3		Sc 1	Sc 2	Sc 3	Sc 1	Sc 2	Sc 3	
UK	UK	2624	2378	2539	3936	3657	3816	UK	2624	2378	2828	0.0	0.0	0.0	UK FRn SE IR NL DK FI NO BE CH FRs DE	523	1173	723	1167	2503	1575	
ES	ES	20		30	2066		3099	ES							ES PT	483	983	583	1034	2041	1235	
	IT	10			1099												156	46	256	4802	1501	7451
	UK			289			28238															
	IT	115			11223																	
IT	IT	1074	809	793	2046	1554	1600	IT	1199	809	823	0.0	0.0	0.0	GR	163	73	153	9351	4561	1007	
															ITn AU ITs	569 185 282	419 35 282	333 35 282	15235 7161 7872	11423 1118 7872	923 111 7872	
Total		4462	4242	4460	21464	7386	38521		4462	4242	4460	0.0	0.0	0.0		4462	4242	4460	141002	92176	15116756	
Expected Value		4383			22434				4383			0.0				4383			128108			

† This is the cost incurred for material to be transported to this destination.

Table 4.24: Transportation costs throughout the system for scenario-based design problem.

Some computational results for this case are shown in table 4.25. A comparison with the corresponding statistics for the deterministic design problem (cf. table 4.17) indicates that both the integrality solution gap of the formulation and the performance of the branch-and-bound algorithm remains reasonable (e.g. with respect to the number of nodes examined). However, the overall computational cost grows significantly due to the increase in the number of constraints and variables, and the consequent increase of the cost of solving the LP at each node of the branch-and-bound tree.

Number of Constraints	54228
Number of Integer Variables	13989
Number of Continuous Variables	34176
Optimal Objective Value	677285
Fully Relaxed LP Objective Function	670287
Integrality Solution Gap	1.0 %
Optimality Margin	1.0 %
Number of Branch & Bound Nodes	2419
CPU time (SUN-Ultra60)	8924s
Solver	CPLEX6.5

Table 4.25: Computational statistics for scenario-based design problem.

4.4. Concluding remarks

This chapter has studied several variations of a case study concerned with the establishment of Europe-wide supply chain network. The results obtained provide a good indication of the value of having a model that takes into account the complex interactions that exist in such networks. In particular, it has been shown how changes in the numerical values of various problem parameters and/or the consideration of demand uncertainty may lead to significant changes in the structure of the optimal supply chain network.

A characteristic of the solution obtained is the establishment of distribution centres very close to major warehouse locations. As has already been mentioned, this would probably be unnecessary in practice. However, it is an inevitable

consequence of the fixed number of echelons in the formulations of chapter 3. We will return to remove this restriction in chapter 6.

The computational cost associated with the problems considered here has been found to be modest. However, it has to be remembered that real-life problems may be much larger, involving hundreds of products. Moreover, the satisfactory modelling of demand uncertainty may require the consideration of many more scenarios than the three used in this case study of section 4.3.

Optimal Design of Supply Chain Networks under Uncertain Transient Demand Variations

5.1. Introduction

The operation of a supply chain network does not normally occur at a steady state. Usually, product demands vary with time as a result of fluctuations in consumption patterns and finite product life cycles. For this reason, companies are forced to predict demand variation and to be prepared to face possible increases or decreases. A *dynamic* model for supply chain operation and design, in which demands can be both uncertain *and* time-varying, is presented in this chapter.

Steady-state models are a justified approximation of the operation of supply chains when we are not interested in long-term variability. This is because short-term fluctuations are captured implicitly, to a certain extent, by averaging the material flows in the network over a sufficiently long period of time. On the other hand, modelling of long-term variations, such as systematic variation of

demand with time (e.g. seasonality or growing/declining markets) necessitates a time-dependent approach.

Due to the long planning horizon intrinsic in these kinds of problems, a high level of uncertainty is often encountered. The handling of this uncertainty raises many challenges, from the choice of an appropriate modelling method, to the solution method selected to attack the problem (Wets, 1996). In the case of supply chain networks, as in any production and distribution system, we are mainly interested in the uncertainty in product demands, prices and costs, and availability of resources. Dynamic mathematical formulations approach uncertainty in two ways (Ierapetritou *et al.*, 1996):

- scenario or multi-period approaches, often including discretization applied to a continuous uncertain parameter space,
- probabilistic approaches, using two-stage stochastic programming.

The decisions taken in the presence of uncertainty can be classified into “wait-and-see” and “here-and-now”. “Here-and-now” decisions have to be implemented *before* the uncertainty is resolved as new information becomes available. A good example of such decisions are those concerning the structure of the supply chain networks. For example, the location and size of each warehouse have to be decided before the actual demands pertaining to some future time period become known. On the other hand, “wait-and-see” decisions do not actually have to be implemented before additional information becomes available at some future time. For example, the rates of production and transportation of various products in a supply chain network in some future time period can, to a large extent, be revised in view of the most up-to-date information on product demands.

Of course “wait-and-see” and “here-and-now” decisions interact strongly with each other. Consequently, “wait-and-see” decisions also need to be taken into account at the initial stage of designing the supply chain network – even if they

do not have to be implemented right away, and can, indeed, be revised at a later date.

The scenario approach to modelling of uncertainty (cf. section 3.4) is particularly well suited to problems involving a combination of “wait-and-see” and “here-and-now” decisions. The nature of the scenarios that need to be postulated depends on the existence of “wait-and-see” decisions. If the problem under consideration involves no such decisions, then each scenario is a distinct profile of product demand(s) over time, of the type shown in figure 5.1. In this case, all decisions are of the “here-and-now” type and are determined only on the basis of information available at the initial stage.

On the other hand, in problems involving a combination of “wait-and-see” and “here-and-now” decisions, the postulated scenarios are typically of the form shown in figure 5.2. Here the information pertaining to product demands in a given period becomes available at the end of the preceding period – and this results in each scenario branch breaking into multiple branches at these points (cf. figure 5.2). In many cases, there is little or no uncertainty regarding the very first period; this results in a single scenario branch over this period, as shown in figure 5.2.

Despite their significant differences, from the point of view of mathematical formulation, the two cases illustrated in figures 5.1 and 5.2 are quite similar. In essence, the optimisation has to determine one set of values for all operational decisions over each distinct branch shown in these figures.

In the supply chain literature, uncertainty in time-varying demands has received attention in the context of the planning and control of inventories in multi-echelon networks. Some of the mathematical programming models tend to require substantial amounts of data and to make many restrictive assumptions, which renders them impractical to some extent (Stenger, 1996). On the other hand, heuristic models fail to consider the interactions between echelons and frequently

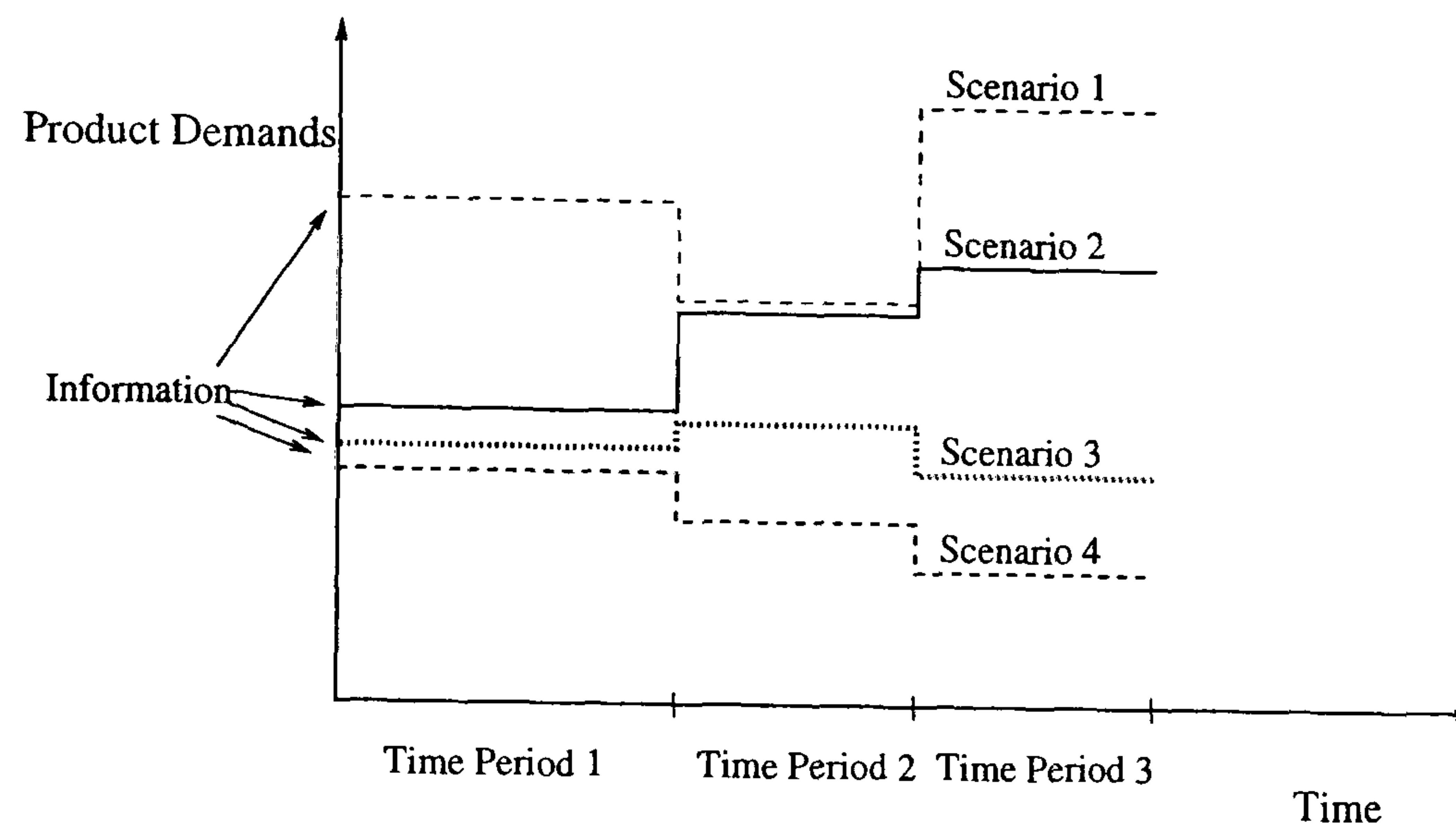


Figure 5.1: Product demand scenarios for problems involving only “here-and-now” decisions.

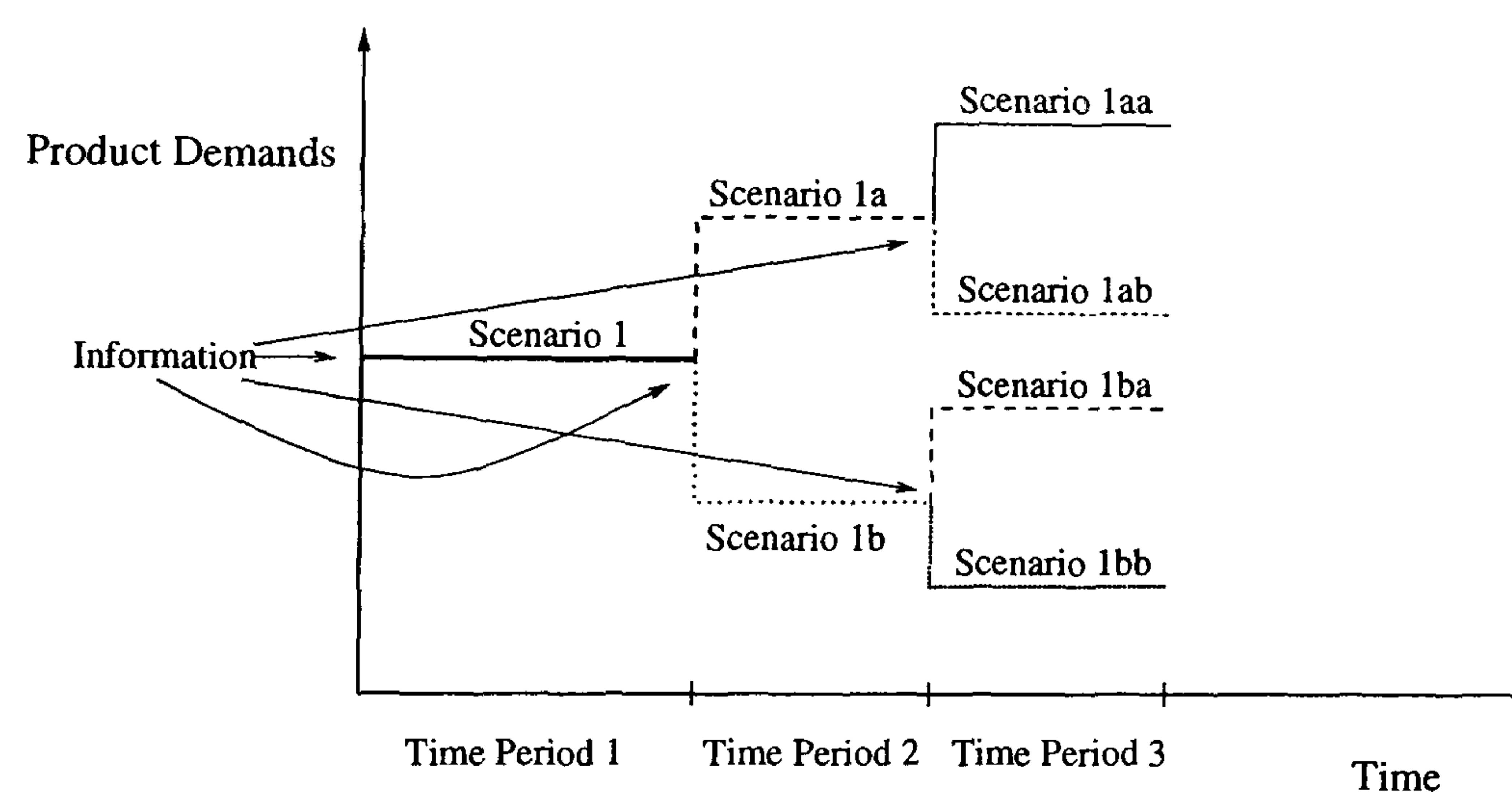


Figure 5.2: Scenarios for problems involving both “here-and-now” and “wait-and-see” decisions.

result in excessive inventories. Classical control theory has also been used to model dynamically the inventories in the supply chain (Perea *et al.*, 2000).

The design of an efficient logistics system requires the identification of locations for distribution centres as well as the determination of appropriate inventory levels to maintain high customer service levels (Nozick and Turnquist, 2001). Usually, there is an interaction between the actual inventory levels and the number of distribution centres established to serve the demand.

This chapter considers a dynamic form of the problem described in chapter

3, allowing demands to be both time-varying and uncertain. Inventory can be carried over from one time period to the next. As pointed out by Shah (1998), failure to consider such inventory carry-over can compromise the optimality of the solution obtained by forcing each product to be produced in all periods in which a non-zero demand (however small) exists for it. Moreover, the network capacity must be designed for a peak demand period, which is particularly constraining for problems with seasonal demand variations.

Section 5.2 introduces the notation used. Section 5.3 presents an MILP mathematical formulation for this problem. Section 5.4 revisits the case study of chapter 4 in a time-varying demand context.

5.2. Notation

The notation to be used in this chapter is consistent with that used for the steady-state model of chapter 3. This section summarises only those symbols that are different to those listed in section 3.2.

5.2.1. Indices

t time period

5.2.2. Parameters

C_{ijt}^I unit inventory handling cost for product i at plant j
over time period t

C_{imt}^I unit inventory handling cost for product i at warehouse m
over time period t

C_{ikt}^I unit inventory handling cost for product i at distribution centre k

	over time period t
$I_{ijt}^{[s],\min}$	minimum inventory of product i held in plant j at the end of time period t under scenario s
$I_{imt}^{[s],\min}$	minimum inventory of product i held in warehouse m at the end of time period t under scenario s
$I_{ikt}^{[s],\min}$	minimum inventory of product i held in distribution centre k at the end of time period t under scenario s
n^{DC}	minimum inventory held at distribution centres expressed in terms of number of days equivalent of material handled
n^P	minimum inventory held at production plants expressed in terms of number of days equivalent of material handled
n^W	minimum inventory held at warehouses expressed in terms of number of days equivalent of material handled
γ_{im}	coefficient relating capacity of warehouse m to inventory of product i held
γ_{ik}	coefficient relating capacity of distribution centre k to inventory of product i held
ΔT_t	duration of time period t

5.2.3. Continuous variables

$C_{fjmt}^{[s]}$	transportation cost of products of family f from plant j to warehouse m during time period t under scenario s
$C_{fkm}^{[s]}$	transportation cost of products of family f from warehouse m to distribution centre k during time period t under scenario s
$C_{fkl}^{[s]}$	transportation cost of products of family f from distribution centre k to customer zone l during time period t under scenario s
$I_{ijt}^{[s]}$	inventory level of product i being held at plant j

	at the end of time period t under scenario s
$I_{imt}^{[s]}$	inventory level of product i being held at warehouse m
	at the end of time period t under scenario s
$I_{ikt}^{[s]}$	inventory level of product i being held at
	distribution centre k at the end of time period t under scenario s
$P_{ijt}^{[s]}$	production rate of product i in plant j
	during time period t under scenario s
$Q_{ijmt}^{[s]}$	rate of flow of product i transferred from plant j
	to warehouse m during time period t under scenario s
$Q_{imkt}^{[s]}$	rate of flow of product i transferred from warehouse m
	to distribution centre k during time period t under scenario s
$Q_{iklt}^{[s]}$	rate of flow of product i transferred from distribution centre k
	to customer zone l during time period t under scenario s

5.2.4. Binary variables

$Z_{fjmt}^{[s]}$	1 if the rate of flow of products of family f from plant j
	to warehouse m is between $\overline{Q}_{fjm,r-1}^{[s]}$ and $\overline{Q}_{fjmr}^{[s]}$
	during time period t under scenario s , 0 otherwise
$Z_{fmkt}^{[s]}$	1 if the rate of flow of products of family f from warehouse m
	to distribution centre k is between $\overline{Q}_{fmk,r-1}^{[s]}$ and $\overline{Q}_{fmk}^{[s]}$
	during time period t under scenario s , 0 otherwise
$Z_{fklrt}^{[s]}$	1 if the rate of flow of products of family f distribution centre k
	to customer l is between $\overline{Q}_{fkl,r-1}^{[s]}$ and $\overline{Q}_{fklr}^{[s]}$
	during time period t under scenario s , 0 otherwise

5.3. Mathematical formulation

As in chapter 3, we consider a four stage network of the type shown in figure 5.3. The locations of the plants and customers are assumed to be fixed and known. A number of warehouses and distributions centres must be selected from a set of possible locations.

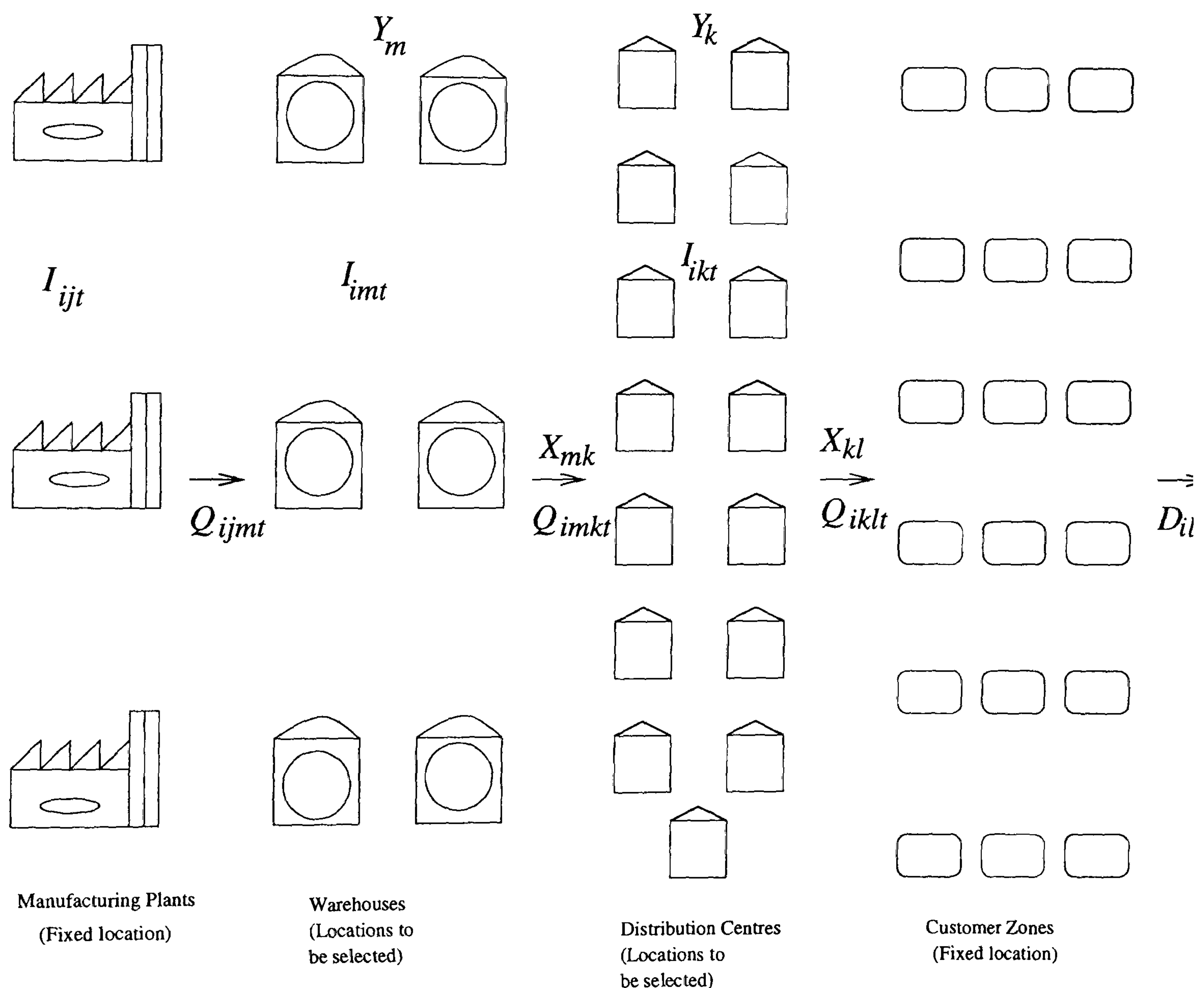


Figure 5.3: Overview of the supply chain networks of interest.

Product demands are assumed to vary as piecewise constant functions of time defined over a number of time periods of given (but not necessarily equal) duration. The uncertainty in these demands is taken into account by postulating a number of scenarios $s = 1, \dots, NS$, each with a potentially different set of piecewise constant

demand functions. Our objective is to design a network that can handle any one of these scenarios, should it materialize at some point during the lifetime of the network, so as to minimise the combined operating and capital cost of the network.

In order to arrive at a meaningful objective function for the optimisation, we assume that the probability of scenario s occurring in practice is known and is denoted by ψ_s . These probabilities will generally satisfy:

$$\sum_{s=1}^{NS} \psi_s = 1 \quad (5.1)$$

Our aim is to minimise the *expected value* of the cost of the network taken over all the scenarios and during the operation of the network.

The mathematical model proposed for this problem is an MILP as described below.

5.3.1. Constraints

5.3.1.1. Network structure

The network structure constraints are identical to those in the steady-state formulation (cf. section 3.3.1.1). They are repeated here for ease of reference.

$$X_{mk} \leq Y_m, \quad \forall m, k \quad (5.2)$$

$$\sum_m X_{mk} = Y_k, \quad \forall k \in K^{ss} \quad (5.3)$$

$$X_{mk} \leq Y_k, \quad \forall m, k \notin K^{ss} \quad (5.4)$$

$$X_{kl} \leq Y_k, \quad \forall k, l \quad (5.5)$$

$$\sum_k X_{kl} = 1, \quad \forall l \in L^{ss} \quad (5.6)$$

5.3.1.2. Logical constraints for transportation flows and inventories

These are similar to those of section 3.3.1.2, with the difference that they have to be enforced over both each scenario, s and each time period t . Thus, we have:

$$Q_{ijmt}^{[s]} \leq Q_{ijm}^{[s],\max} Y_m, \quad \forall i, j, m, t, s = 1, \dots, NS \quad (5.7)$$

$$Q_{imkt}^{[s]} \leq Q_{imk}^{[s],\max} X_{mk}, \quad \forall i, m, k, t, s = 1, \dots, NS \quad (5.8)$$

$$Q_{iklt}^{[s]} \leq Q_{ikl}^{[s],\max} X_{kl}, \quad \forall i, k, l, t, s = 1, \dots, NS \quad (5.9)$$

$$\sum_i Q_{imkt}^{[s]} \geq Q_{mk}^{\min} X_{mk}, \quad \forall m, k, t, s = 1, \dots, NS \quad (5.10)$$

$$\sum_i Q_{iklt}^{[s]} \geq Q_{kl}^{\min} X_{kl}, \quad \forall k, l, t, s = 1, \dots, NS \quad (5.11)$$

5.3.1.3. Material balances

If no product inventories were held at the plant locations, the actual rate of production of product i by plant j would equal the total flow of this product from plant j to all warehouses m . However, if inventory of product i is allowed to be held in plant j at time t , then the material balance on the plant over period t becomes:

$$I_{ijt}^{[s]} = I_{ij,t-1}^{[s]} + \left(P_{ijt}^{[s]} - \sum_m Q_{ijmt}^{[s]} \right) \Delta T_t, \quad \forall i, j, t, s = 1, \dots, NS \quad (5.12)$$

Constraint (5.12) states that the available inventory $I_{ijt}^{[s]}$ of product i held in plant j at the end of period t is equal to the inventory held at the end of period $t - 1$ plus any product accumulated in the plant due to the production during the period, minus any product transported from the plant to warehouses during the same period. Since both production and transportation are expressed as flows of material over time (e.g. te/wk), we calculate the total amount of material during period $[t - 1, t]$ by multiplying these rates by the duration ΔT_t of time period t . Typically, the durations ΔT_t range from a few weeks to several months depending on the actual planning procedures of the corporation.

Similarly, we can formulate the following constraints for the warehouses and

distribution centres:

$$I_{imt}^{[s]} = I_{im,t-1}^{[s]} + \left(\sum_j Q_{ijmt}^{[s]} - \sum_k Q_{imkt}^{[s]} \right) \Delta T_t, \quad \forall i, m, t, s = 1, \dots, NS \quad (5.13)$$

$$I_{ikt}^{[s]} = I_{ik,t-1}^{[s]} + \left(\sum_m Q_{imkt}^{[s]} - \sum_l Q_{iklt}^{[s]} \right) \Delta T_t, \quad \forall i, k, t, s = 1, \dots, NS \quad (5.14)$$

Customer zones do not normally hold significant amounts of inventory. Consequently, the total flow of each product i received by each customer zone l from the distribution centres is assumed to be equal to the corresponding market demand:

$$\sum_k Q_{iklt}^{[s]} = \mathcal{D}_{ilt}^{[s]}, \quad \forall i, l, t, s = 1, \dots, NS \quad (5.15)$$

The initial inventories, $I_{ij0}^{[s]}$, $I_{im0}^{[s]}$ and $I_{ik0}^{[s]}$ are assumed to be given as part of the specification of each scenario s .

5.3.1.4. Production resources

Again, these constraints are similar to those of section 3.3.1.4 but they are enforced for each and every scenario s and time period t :

$$P_{ijt}^{[s],\min} \leq P_{ijt}^{[s]} \leq P_{ijt}^{[s],\max}, \quad \forall i, j, t, s = 1, \dots, NS \quad (5.16)$$

$$\sum_i \rho_{ije} P_{ijt}^{[s]} \leq R_{je}, \quad \forall j, e, t, s = 1, \dots, NS \quad (5.17)$$

5.3.1.5. Capacity of warehouses and distribution centres

The capacity of a warehouse m generally has to lie between given lower and upper bounds, $W_m^{\min}$ and $W_m^{\max}$, provided, of course, that the warehouse is actually established (i.e. $Y_m = 1$):

$$W_m^{\min} Y_m \leq W_m \leq W_m^{\max} Y_m, \quad \forall m \quad (5.18)$$

Similar constraints apply to the capacities of the distribution centres:

$$D_k^{\min} Y_k \leq D_k \leq D_k^{\max} Y_k, \quad \forall k \quad (5.19)$$

In the steady-state formulation (cf. section 3.3.1.5), we assumed that the capacities of the warehouses and the distribution centres are somehow related to the flows of materials that they are to handle. The dynamic formulation of this chapter allows an arguably more precise characterisation of these capacities in terms of the actual inventory being held. More specifically, the capacity of a warehouse or a distribution centre cannot be less than the combined inventory to be held there at any time period under any scenario. This leads to constraints of the form:

$$W_m \geq \sum_i \gamma_{im} I_{imt}^{[s]}, \quad \forall m, t, s = 1, \dots, NS \quad (5.20)$$

$$D_k \geq \sum_i \gamma_{ik} I_{ikt}^{[s]}, \quad \forall k, t, s = 1, \dots, NS \quad (5.21)$$

where γ_{im} and γ_{ik} are given coefficients expressing the amount of warehousing capacity required to hold a unit amount of a particular product i at a warehouse m or a distribution centre k respectively.

5.3.1.6. Safety stock constraints

Maintaining a safety stock (also known as “buffer inventory”) is often desirable, providing a means of overcoming unforeseen production disturbances or unexpected product demands. In general, the higher the level of inventory, the better the customer service, with fewer stockouts. On the other hand, excess inventory causes higher operating costs. Consequently, safety stock is usually only as much as is necessary to keep the network functioning for a short period of time (e.g. from a few days to a week) in case of disruption at one or more of its nodes.

The need for safety stock can be expressed by the following constraints:

$$I_{ijt}^{[s]} \geq I_{ijt}^{[s],\min}, \quad \forall i, j, t, s = 1, \dots, NS \quad (5.22)$$

$$I_{imt}^{[s]} \geq I_{imt}^{[s],\min} Y_m, \quad \forall i, m, t, s = 1, \dots, NS \quad (5.23)$$

$$I_{ikt}^{[s]} \geq I_{ikt}^{[s],\min} Y_k, \quad \forall i, k, t, s = 1, \dots, NS \quad (5.24)$$

The above constraints ensure that inventory is kept at warehouses or distribution centres only if these are established. We also note that the minimum inventory handling requirements may vary from scenario to scenario and from one time period to another. In fact, they are often expressed as functions (e.g. constant multiples) of the corresponding material flows delivered by each plant, warehouse or distribution centre node to all other nodes that are served by it (cf. section 5.4.4).

Since we assume constant rates of production, transportation and demand over each time period, inventories vary linearly with time during a period. Consequently, it suffices to enforce constraints (5.20) - (5.24) at the time period boundaries for them to hold at all times during the planning time horizon.

5.3.1.7. Additional constraints

All continuous variables must be non-negative:

$$P_{ijt}^{[s]} \geq 0 \quad \forall i, j, t, s = 1, \dots, NS \quad (5.25)$$

$$Q_{ijmt}^{[s]} \geq 0, \quad \forall i, j, m, t, s = 1, \dots, NS \quad (5.26)$$

$$Q_{imkt}^{[s]} \geq 0, \quad \forall i, m, k, t, s = 1, \dots, NS \quad (5.27)$$

$$Q_{iklt}^{[s]} \geq 0, \quad \forall i, k, l, t, s = 1, \dots, NS \quad (5.28)$$

$$I_{ijt}^{[s]} \geq 0, \quad \forall i, j, t, s = 1, \dots, NS \quad (5.29)$$

$$I_{imt}^{[s]} \geq 0, \quad \forall i, m, t, s = 1, \dots, NS \quad (5.30)$$

$$I_{ikt}^{[s]} \geq 0, \quad \forall i, k, t, s = 1, \dots, NS \quad (5.31)$$

5.3.2. Objective function

The objective of the optimisation is to minimise the overall cost of the supply chain network over a planning time horizon. This includes both (annualised) capital costs and operating costs, and lead to an objective function of the form:

$$\min \left(\sum_m C_m^W Y_m + \sum_k C_k^D Y_k \right) \sum_t \Delta T_t +$$

$$\begin{aligned}
 \sum_{s=1}^{NS} \psi_s \left(\sum_t \Delta T_t \left(\sum_{i,j} C_{ij}^P P_{ijt}^{[s]} + \sum_{i,m} C_{im}^{WH} \left(\sum_j Q_{ijmt}^{[s]} \right) + \sum_{i,k} C_{ik}^{DH} \left(\sum_m Q_{imkt}^{[s]} \right) \right. \right. \\
 \left. \left. + \sum_{f,j,m} C_{fjmt}^{[s]} + \sum_{f,m,k} C_{fmk}^{[s]} + \sum_{f,k,l} C_{fkl}^{[s]} + \right. \right. \\
 \left. \left. \sum_{i,j,t} C_{ijt}^I \frac{I_{ijt}^{[s]} + I_{ij,t-1}^{[s]}}{2} + \sum_{i,m,t} C_{imt}^I \frac{I_{imt}^{[s]} + I_{im,t-1}^{[s]}}{2} + \sum_{i,k,t} C_{ikt}^I \frac{I_{ikt}^{[s]} + I_{ik,t-1}^{[s]}}{2} \right) \right) \quad (5.32)
 \end{aligned}$$

It will be noted that this is generally similar to the objective function of the steady-state problem (cf. equation 3.47), with the following differences:

- The objective function reflects actual total cost incurred over the planning horizon rather than rate of expenditure (i.e. it is measured in $\mathcal{L}$ instead of $\mathcal{L}/wk$).
- The (annualised) capital costs are multiplied by the total length of the time horizon under consideration $\sum_t \Delta T_t$.
- The rate of operating expenditure varies from one time period to the next. Consequently, the operating cost in each period t is multiplied by the corresponding duration ΔT_t .
- In addition to the production, material handling and transportation costs, the operating cost also includes those associated with holding inventory at plants, warehouses and distribution centres (cf. last term of (5.32)). In general, the cost incurred over a given time period t is proportional to the average amount of inventory held over this period. As already noted in section 5.3.1.6, inventories vary linearly over each time period. Consequently, the average inventory is given by the arithmetic mean of the starting and finishing inventories for this period.

5.3.3. Scenario-dependent distribution network structure

As in the steady-state (cf. section 3.4.4), it may be possible in some cases to vary the connectivity of the supply chain network both over time and depending on the scenario instead of treating these decisions as “here-and-now” variables to be fixed once and for all at the design stage.

The required changes in the mathematical formulation are minimal. More specifically, the binary variables X_{mk} and X_{kl} are replaced by variables $X_{mkt}^{[s]}$ and $X_{klt}^{[s]}$ respectively, and constraints (5.2) - (5.6) and (5.8) - (5.11) are modified to:

$$X_{mkt}^{[s]} \leq Y_m, \quad \forall m, k, t, s = 1, \dots, NS \quad (5.2')$$

$$\sum_m X_{mkt}^{[s]} = Y_k, \quad \forall k \in K^{ss}, t, s = 1, \dots, NS \quad (5.3')$$

$$X_{mkt}^{[s]} \leq Y_k, \quad \forall m, k \notin K^{ss}, t, s = 1, \dots, NS \quad (5.4')$$

$$X_{klt}^{[s]} \leq Y_k, \quad \forall k, l, t, s = 1, \dots, NS \quad (5.5')$$

$$\sum_k X_{klt}^{[s]} = 1, \quad \forall l \in L^{ss}, t, s = 1, \dots, NS \quad (5.6')$$

$$Q_{imkt}^{[s]} \leq Q_{imk}^{[s],\max} X_{mkt}^{[s]}, \quad \forall i, m, k, t, s = 1, \dots, NS \quad (5.8')$$

$$Q_{iklt}^{[s]} \leq Q_{ikl}^{[s],\max} X_{klt}^{[s]}, \quad \forall i, k, l, t, s = 1, \dots, NS \quad (5.9')$$

$$\sum_i Q_{imkt}^{[s]} \geq Q_{mk}^{\min} X_{mkt}^{[s]}, \quad \forall m, k, t, s = 1, \dots, NS \quad (5.10')$$

$$\sum_i Q_{iklt}^{[s]} \geq Q_{kl}^{\min} X_{klt}^{[s]}, \quad \forall k, l, t, s = 1, \dots, NS \quad (5.11')$$

This option gives the network an additional degree of flexibility. Only the locations of warehouses and distribution centres are constant through the operation of the system, while the links between nodes may be re-allocated in every time period and scenario in order to satisfy customer demand in the most cost-efficient manner.

5.4. Case study revisited

In this section, we revisit the case study of chapter 4, this time taking account of long-term demand variations. The problem data concerning production sites, warehouses and distribution centres, as well as transportation costs have already been presented in section 4.1. In this section, only additional data are presented.

5.4.1. Inventory costs

The costs of holding inventory at different parts of the supply chain network are assumed to be independent of the product being held, but vary from one location to another, as shown in table 5.1.

Production site	Inventory holding cost (£/te)
PL1	8.25
PL2	8.55
PL3	8.98
Warehouse	Inventory holding cost (£/te)
W1	8.25
W2	8.55
W3	8.98
W4	8.93
W5	8.06
W6	10.28
Distribution centre	Inventory holding cost (£/te)
DC01	8.25
DC02	8.55
DC03	8.98
DC04	8.93
DC05	8.85
DC06	6.90
DC07	8.06
DC08	6.08
DC09	12.00
DC10	8.85
DC11	8.12
DC12	10.66
DC13	10.28
DC14	8.95
DC15	8.83

Table 5.1: Inventory holding costs.

5.4.2. Product demands

We consider a planning horizon comprising three four-week time periods (i.e. $\Delta T_1 = \Delta T_2 = \Delta T_3 = 4$ weeks). Product demands over the first period are assumed to be known with certainty, and are shown in table 5.2. However, there are two distinctly different predictions for demands over the second period, shown in tables 5.3 and 5.4 respectively. Moreover, each of these demand predictions for the second period leads to two distinct demand predictions for the third and fourth period (see tables 5.5 - 5.8).

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	18	0	0	506	0	452	0	0	43	0	0	0	120	34
CZ02	0	499	155	203	76	0	30	0	0	0	20	0	0	0
CZ03	0	155	0	166	0	66	17	0	0	0	0	0	15	0
CZ04	15	0	126	0	0	0	5	27	0	0	0	25	0	0
CZ05	0	0	92	0	0	0	0	0	21	0	0	0	50	0
CZ06	0	0	0	0	0	68	0	0	20	0	0	0	10	10
CZ07	0	14	0	40	0	0	0	0	34	0	0	0	68	0
CZ08	0	0	0	45	0	23	0	0	5	0	0	0	0	0
CZ09	0	0	0	0	0	0	52	0	7	0	0	0	0	0
CZ10	0	0	0	17	0	0	0	0	5	0	0	0	16	0
CZ11	0	0	0	31	0	0	0	0	0	0	0	0	15	0
CZ12	0	31	0	0	0	0	13	0	0	38	0	0	0	0
CZ13	0	21	0	0	0	0	15	0	0	0	0	0	0	0
CZ14	0	0	0	0	0	0	0	7	0	0	0	0	0	0
CZ15	0	0	0	0	0	0	10	0	0	0	15	0	0	0
CZ16	15	0	68	0	0	0	5	20	0	0	0	20	0	0
CZ17	0	103	0	110	0	44	12	0	0	0	0	0	13	0
CZ18	0	0	0	0	0	0	0	0	0	266	0	0	0	0

Table 5.2: Product demands by customer zone over first period (all scenarios).

Overall we need to consider four distinct scenarios (i.e. $s = 1, \dots, 4$) organised in a tree structure of the type shown in figure 5.4. The formulation of section 5.3 is directly applicable to this situation, provided:

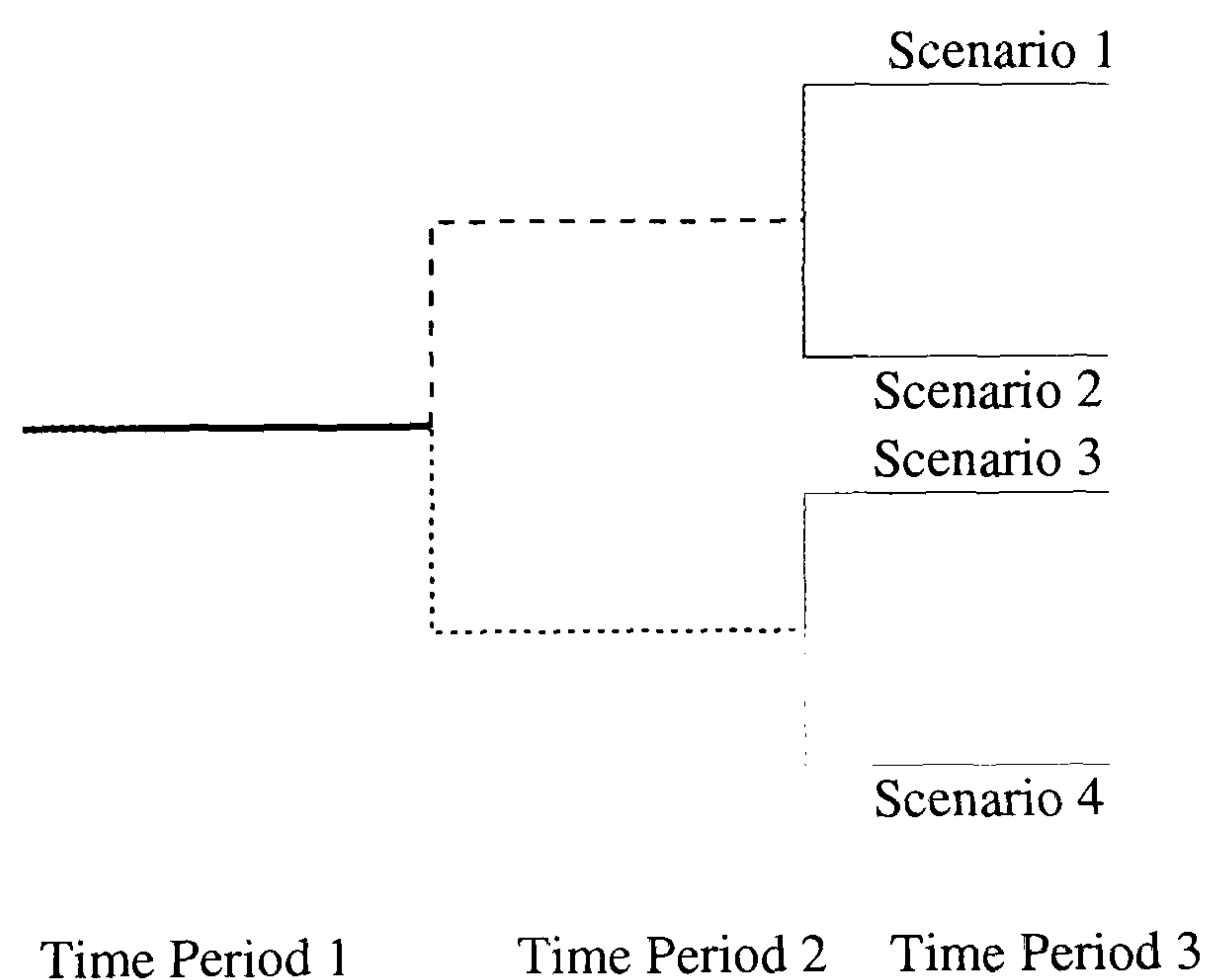


Figure 5.4: Time-varying scenario tree.

- the operational variables of all four scenarios are treated as identical in the first time period, e.g.

$$P_{ij1}^{[1]} = P_{ij1}^{[2]} = P_{ij1}^{[3]} = P_{ij1}^{[4]}, \forall i, j$$

- the operational variables of scenarios 1 and 2 are treated as identical in the second time period, e.g.

$$P_{ij2}^{[1]} = P_{ij2}^{[2]}, \forall i, j$$

- the operational variables of scenarios 3 and 4 are treated as identical in the second time period, e.g.

$$P_{ij2}^{[3]} = P_{ij2}^{[4]}, \forall i, j$$

In practice, the above equalities are used to eliminate *a priori* a large proportion of the problem variables.

All four scenarios are assumed to be equally probable (i.e. $\psi_s = 0.25, s = 1, \dots, 4$).

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	14	0	0	547	0	412	0	0	46	0	0	0	94	30
CZ02	0	612	144	248	83	0	33	0	0	0	24	0	0	0
CZ03	0	117	0	202	0	70	21	0	0	0	0	0	12	0
CZ04	17	0	157	0	0	0	6	25	0	0	0	27	0	0
CZ05	0	0	97	0	0	0	0	0	19	0	0	0	56	0
CZ06	0	0	0	0	0	65	0	0	16	0	0	0	10	11
CZ07	0	14	0	40	0	0	0	0	42	0	0	0	63	0
CZ08	0	0	0	51	0	25	0	0	5	0	0	0	0	0
CZ09	0	0	0	0	0	0	49	0	8	0	0	0	0	0
CZ10	0	0	0	17	0	0	0	0	5	0	0	0	13	0
CZ11	0	0	0	33	0	0	0	0	0	0	0	0	11	0
CZ12	0	26	0	0	0	0	14	0	0	38	0	0	0	0
CZ13	0	21	0	0	0	0	11	0	0	0	0	0	0	0
CZ14	0	0	0	0	0	0	0	8	0	0	0	0	0	0
CZ15	0	0	0	0	0	0	11	0	0	0	14	0	0	0
CZ16	11	0	51	0	0	0	5	24	0	0	0	20	0	0
CZ17	0	124	0	135	0	36	14	0	0	0	0	0	20	0
CZ18	0	0	0	0	0	0	0	0	0	241	0	0	0	0

Table 5.3: Product demands by customer zone over second period (scenarios 1 and 2).

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	18	0	0	436	0	568	0	0	54	0	0	0	149	31
CZ02	0	633	154	195	62	0	25	0	0	0	26	0	0	0
CZ03	0	172	0	189	0	58	16	0	0	0	0	0	15	0
CZ04	14	0	159	0	0	0	6	33	0	0	0	29	0	0
CZ05	0	0	74	0	0	0	0	0	17	0	0	0	57	0
CZ06	0	0	0	0	0	57	0	0	18	0	0	0	11	12
CZ07	0	16	0	41	0	0	0	0	44	0	0	0	67	0
CZ08	0	0	0	46	0	19	0	0	6	0	0	0	0	0
CZ09	0	0	0	0	0	0	54	0	6	0	0	0	0	0
CZ10	0	0	0	22	0	0	0	0	4	0	0	0	20	0
CZ11	0	0	0	28	0	0	0	0	0	0	0	0	14	0
CZ12	0	29	0	0	0	0	12	0	0	45	0	0	0	0
CZ13	0	20	0	0	0	0	15	0	0	0	0	0	0	0
CZ14	0	0	0	0	0	0	0	8	0	0	0	0	0	0
CZ15	0	0	0	0	0	0	10	0	0	0	19	0	0	0
CZ16	13	0	62	0	0	0	4	19	0	0	0	21	0	0
CZ17	0	97	0	122	0	51	13	0	0	0	0	0	16	0
CZ18	0	0	0	0	0	0	0	0	0	327	0	0	0	0

Table 5.4: Product demands by customer zone over second period (scenarios 3 and 4).

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	19	0	0	528	0	559	0	0	50	0	0	0	106	40
CZ02	0	657	164	279	82	0	42	0	0	0	22	0	0	0
CZ03	0	113	0	205	0	95	28	0	0	0	0	0	12	0
CZ04	21	0	196	0	0	0	6	30	0	0	0	33	0	0
CZ05	0	0	112	0	0	0	0	0	23	0	0	0	65	0
CZ06	0	0	0	0	0	77	0	0	14	0	0	0	15	16
CZ07	0	16	0	45	0	0	0	0	46	0	0	0	79	0
CZ08	0	0	0	70	0	24	0	0	6	0	0	0	0	0
CZ09	0	0	0	0	0	0	50	0	7	0	0	0	0	0
CZ10	0	0	0	17	0	0	0	0	6	0	0	0	12	0
CZ11	0	0	0	44	0	0	0	0	0	0	0	0	13	0
CZ12	0	30	0	0	0	0	17	0	0	51	0	0	0	0
CZ13	0	27	0	0	0	0	13	0	0	0	0	0	0	0
CZ14	0	0	0	0	0	0	0	11	0	0	0	0	0	0
CZ15	0	0	0	0	0	0	13	0	0	0	14	0	0	0
CZ16	14	0	56	0	0	0	5	29	0	0	0	27	0	0
CZ17	0	160	0	172	0	35	20	0	0	0	0	0	12	0
CZ18	0	0	0	0	0	0	0	0	0	331	0	0	0	0

Table 5.5: Product demands by customer zone over third period (scenario 1).

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	18	0	0	761	0	551	0	0	55	0	0	0	125	37
CZ02	0	706	209	282	104	0	38	0	0	0	34	0	0	0
CZ03	0	152	0	233	0	88	24	0	0	0	0	0	15	0
CZ04	19	0	164	0	0	0	7	30	0	0	0	35	0	0
CZ05	0	0	115	0	0	0	0	0	27	0	0	0	80	0
CZ06	0	0	0	0	0	81	0	0	16	0	0	0	11	15
CZ07	0	15	0	42	0	0	0	0	44	0	0	0	82	0
CZ08	0	0	0	65	0	30	0	0	6	0	0	0	0	0
CZ09	0	0	0	0	0	0	67	0	8	0	0	0	0	0
CZ10	0	0	0	21	0	0	0	0	7	0	0	0	15	0
CZ11	0	0	0	37	0	0	0	0	0	0	0	0	14	0
CZ12	0	36	0	0	0	0	14	0	0	39	0	0	0	0
CZ13	0	22	0	0	0	0	16	0	0	0	0	0	0	0
CZ14	0	0	0	0	0	0	0	10	0	0	0	0	0	0
CZ15	0	0	0	0	0	0	16	0	0	0	18	0	0	0
CZ16	12	0	72	0	0	0	6	33	0	0	0	25	0	0
CZ17	0	169	0	194	0	35	15	0	0	0	0	0	13	0
CZ18	0	0	0	0	0	0	0	0	0	329	0	0	0	0

Table 5.6: Product demands by customer zone over third period (scenario 2).

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	24	0	0	420	0	772	0	0	58	0	0	0	169	40
CZ02	0	680	175	220	62	0	32	0	0	0	24	0	0	0
CZ03	0	165	0	191	0	78	22	0	0	0	0	0	16	0
CZ04	17	0	198	0	0	0	6	41	0	0	0	35	0	0
CZ05	0	0	85	0	0	0	0	0	21	0	0	0	65	0
CZ06	0	0	0	0	0	67	0	0	16	0	0	0	15	16
CZ07	0	18	0	46	0	0	0	0	48	0	0	0	84	0
CZ08	0	0	0	63	0	18	0	0	8	0	0	0	0	0
CZ09	0	0	0	0	0	0	55	0	6	0	0	0	0	0
CZ10	0	0	0	21	0	0	0	0	6	0	0	0	19	0
CZ11	0	0	0	37	0	0	0	0	0	0	0	0	16	0
CZ12	0	34	0	0	0	0	15	0	0	60	0	0	0	0
CZ13	0	25	0	0	0	0	18	0	0	0	0	0	0	0
CZ14	0	0	0	0	0	0	0	11	0	0	0	0	0	0
CZ15	0	0	0	0	0	0	12	0	0	0	19	0	0	0
CZ16	16	0	68	0	0	0	4	23	0	0	0	27	0	0
CZ17	0	124	0	155	0	50	18	0	0	0	0	0	16	0
CZ18	0	0	0	0	0	0	0	0	0	448	0	0	0	0

Table 5.7: Product demands by customer zone over third period (scenario 3).

Customer zone	Product demands (te/wk)													
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14
CZ01	24	0	0	606	0	760	0	0	64	0	0	0	198	37
CZ02	0	730	223	222	78	0	29	0	0	0	37	0	0	0
CZ03	0	222	0	218	0	72	19	0	0	0	0	0	19	0
CZ04	15	0	165	0	0	0	7	41	0	0	0	37	0	0
CZ05	0	0	88	0	0	0	0	0	24	0	0	0	80	0
CZ06	0	0	0	0	0	71	0	0	18	0	0	0	11	16
CZ07	0	17	0	43	0	0	0	0	45	0	0	0	87	0
CZ08	0	0	0	59	0	23	0	0	7	0	0	0	0	0
CZ09	0	0	0	0	0	0	73	0	7	0	0	0	0	0
CZ10	0	0	0	27	0	0	0	0	6	0	0	0	24	0
CZ11	0	0	0	30	0	0	0	0	0	0	0	0	18	0
CZ12	0	41	0	0	0	0	12	0	0	46	0	0	0	0
CZ13	0	21	0	0	0	0	21	0	0	0	0	0	0	0
CZ14	0	0	0	0	0	0	0	9	0	0	0	0	0	0
CZ15	0	0	0	0	0	0	14	0	0	0	24	0	0	0
CZ16	14	0	87	0	0	0	5	26	0	0	0	25	0	0
CZ17	0	131	0	174	0	50	14	0	0	0	0	0	18	0
CZ18	0	0	0	0	0	0	0	0	0	446	0	0	0	0

Table 5.8: Product demands by customer zone over third period (scenario 4).

5.4.3. Safety stock requirements

We examine two distinct cases with different levels of safety stock held in the system. As explained in section 5.3.1.6, the amount of safety stock to be held at each plant, warehouse or distribution centre node is usually expressed in terms of a given number of days' equivalent of material flow delivered by the node to all nodes supplied by it. Here we assume that this number of days is the same for all nodes of the same type (e.g. plants, warehouses or distribution centres). Thus, the safety stock is given by:

$$I_{ijt}^{[s],\min} = \frac{n^P}{7} \sum_m Q_{ijmt}^{[s]}, \quad \forall i, j, t, s = 1, \dots, NS \quad (5.33)$$

$$I_{imt}^{[s],\min} = \frac{n^W}{7} \sum_k Q_{imkt}^{[s]}, \quad \forall i, m, t, s = 1, \dots, NS \quad (5.34)$$

$$I_{ikt}^{[s],\min} = \frac{n^{DC}}{7} \sum_l Q_{iklt}^{[s]}, \quad \forall i, k, t, s = 1, \dots, NS \quad (5.35)$$

where n^P , n^W , n^{DC} are the numbers of days equivalent for plants, warehouses and distribution centres respectively, and the division by 7 reflects the fact that all our material flows are expressed as tonnes per week. The right hand sides of (5.33) - (5.35) are used to replace the minimum inventory quantities appearing in the right hand sides of constraints (5.22) - (5.24) respectively.

The two cases considered differ in the values of the parameters n^P , n^W , and n^{DC} as summarised in table 5.9.

Initial inventories for each product for the production plants are assumed to be

Safety stock held (in no. of days equivalent)			
Case	n^P	n^W	n^{DC}
C1	1	1	1
C2	7	4	2

Table 5.9: Minimum inventory requirements.

known and at a certain level. This is equal to the maximum production capacity of the plants for a week. Warehouses and distribution centres are assumed to hold zero initial inventory.

5.4.4. Results

The optimal network structure for the first case is shown in figure 5.5. Each arc denotes the total material flow. Green arcs denote the connections and transportation of material between the production plants and warehouses. Blue arcs denote the flow of material from warehouses to distribution centres, while red arcs denote the paths of material flowing from distribution centres to customer zones.

The market is served by three warehouses and three distribution centers located close to the production sites, and one distribution centre established in Ireland to serve the local market. This structure is primarily a result of the demand patterns considered since the three countries which host the manufacturing plants are also the biggest customers, and the demand is mainly concentrated in the northern part of Europe. However, since none of the plants can produce the whole range of products, and plant capacity is restricted due to production constraints, all plants provide products to all warehouses.

The optimal network for the second case is shown in figure 5.6. The northern part of Europe places the highest demands so the transportation costs are lower if they are supplied by the UK distribution centre. Also, the distribution centre of Ireland is supplied by the UK warehouse, rather than of Italy's. The South France customer zone is assigned from the distribution centre of Italy to the distribution

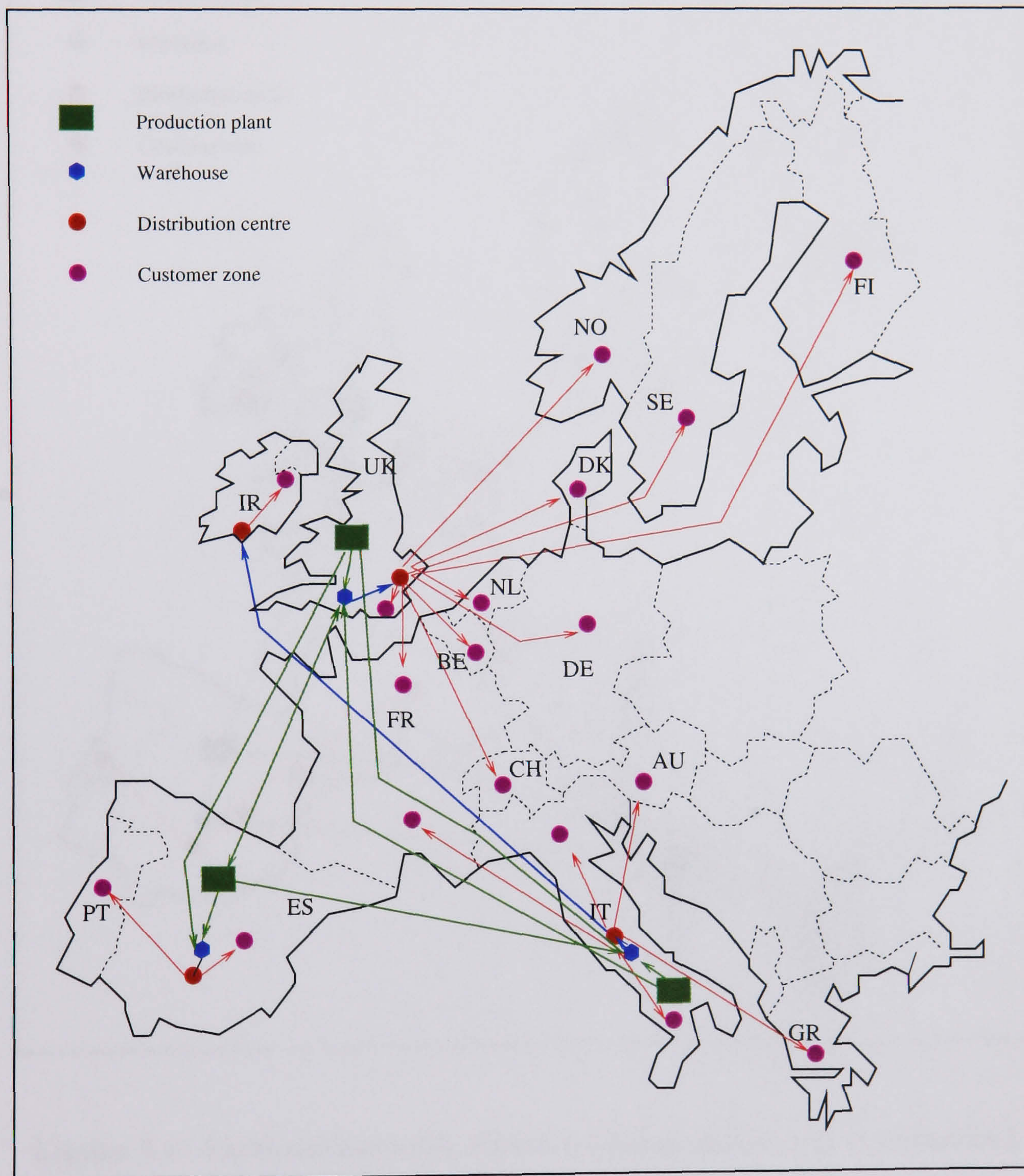


Figure 5.5: Optimal network structure (low minimum inventories).

centre of Spain, while the Swedish market is served by the Italian distribution centre.

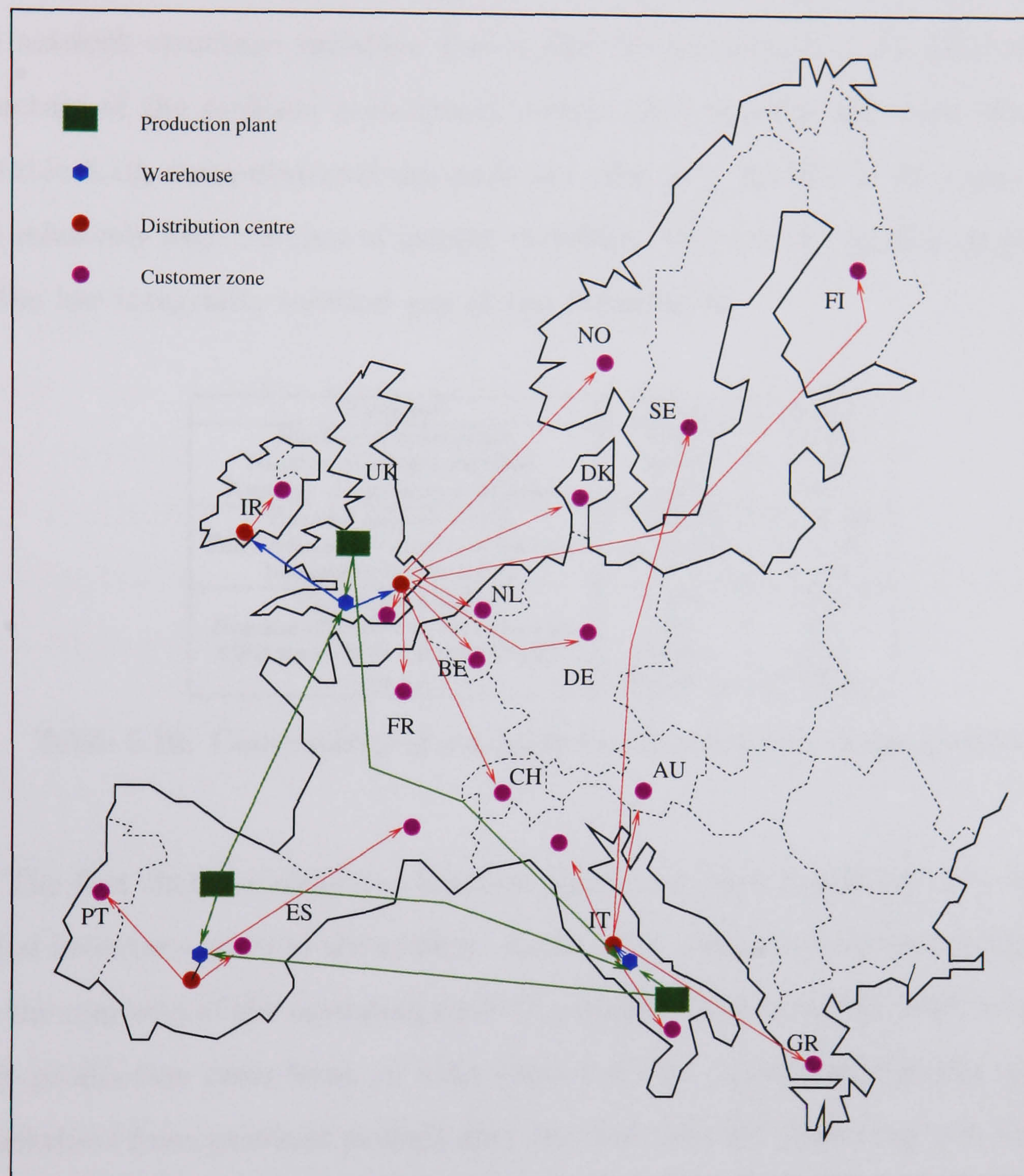


Figure 5.6: Optimal network structure (high minimum inventories).

We can see that for the same demand patterns but for different minimum levels of inventory required by the system we obtain different network structures. In particular, it has been shown how changes in the numerical values of the inventory levels in the system and various problem parameters under demand uncertainty may lead to changes in the structure of the optimal supply chain network.

The corresponding computational statistics for both cases are summarized in table 5.10. The MILP problems were solved using the CPLEX (ILOG, 1998) code embedded within *ooMILP* (Tsiakis *et al.*, 1999). A higher priority is given to the network structure variables due to their importance and the block-diagonal structure of the problem occurrences matrix. As can be seen from the results of table 5.10, computational demands are relatively modest in this case despite the relatively high number of integer variables. This can be attributed primarily to the low integrality solution gap of this formulation.

Statistics	Case 1	Case 2
Number of constraints	113947	113947
Number of integer variables	24195	24195
Number of continuous variables	74473	74473
Optimal objective value	7,189,264	7,404,628
Fully relaxed LP objective function	6,829,706	7,272,937
Integrality Solution Gap	5.0 %	5.0 %
Optimality margin	5.00%	1.78%
Number of branch & bound nodes	3778	301
CPU time (SUN UltraSPARC60)	111219s	8956s
Solver	CPLEX6.5	CPLEX6.5

Table 5.10: Computational statistics for deterministic design problem.

The fact that the objective function values are close is attributed to the high initial inventory levels of the system. As we have seen before the production costs are the majority of the operating costs of a supply chain network. High inventories keep production costs lower in both cases since for some scenarios the inventory maintained from previous periods may be used without producing new material.

It is interesting to compare the expected costs of the networks that would be incurred if one was to fix the structure of the network to either cases. If we fix the structure obtained from the first case as is shown in figure 5.5, and we solve the problem with the inventory levels of the second case we obtain that the network operates, at a cost of £8,099,605 which is higher than £7,404,628 that we obtained solving the problem originally. If we now fix the structure to the solution of the second case shown in figure 5.6, and solve the problem with the inventory levels corresponding to the first case, we get an expected cost of £7,083,430 instead of

£7,189,265 which is lower but actually falls within the 5% optimality gap of the solution of the original case.

5.5. Concluding remarks

In addition to the formulations presented in Chapter 3, this chapter has presented a detailed mathematical formulation for the problem of designing supply chain networks comprising multiproduct production facilities with shared production resources, warehouses, distribution centres and customer zones and operating under demand uncertainty, while inventory may be kept at different stages in the network. Uncertainty is captured in terms of a number of likely scenarios possible to materialise during the life time of the network. A flexible transportation cost model has been used to capture economies of scale.

This chapter has studied a case study concerned with the establishment of Europe-wide supply chain. The results obtained provide a good indication of the value of having a model that takes into account the complex interactions that exist in such networks and the effect of inventory levels to the design and operation.

The computational cost associated with the problems considered here has been found to be high due to the large size of the problems. The special structure of the multi-period problem occurring in scenario-based optimisation, gives rise to the development of special decomposition techniques. Real life applications may include hundreds of products resulting in very large problems making them intractable using currently existing solution techniques (Sherali and Driscoll (2000) and Hoffman (2000)).

Optimal Design of Generalised Supply Chain Networks

The supply chain models presented in chapters 3 and 5 assume a given fundamental structure of the network in terms of the echelons involved, namely those corresponding to suppliers, manufacturing plants, warehouses, distribution centres and customers. Moreover, material is allowed to flow only from each echelon to the one immediately following it. For example, it is not possible for a warehouse (or a production plant) to supply a customer directly, by-passing the distribution centre echelon.

In fact, in many cases, the distinction between “warehouse” and “distribution centre” is somewhat arbitrary, and insisting upon having two separate stages of material storage throughout the supply chain network may lead to uneconomical decisions. On the other hand, in some cases, especially when extended geographical distances are concerned, it may be advantageous to establish three or more stages of warehousing between production plants and customers. This would allow the exploitation of economies of scale in transportation as a higher proportion of the material flow shares common routes over longer distances.

This chapter presents a general framework that integrates the different components of a supply chain without any *a priori* assumption on the fundamental structure of the network. The optimal design of such networks is formulated as an MILP. The fundamental concepts of this approach are presented in section 6.1. Section 6.2 summarises the notation used while section 6.3 presents the mathematical formulation. A case study based on the model is presented in section 6.4.

6.1. Fundamental concepts

In the proposed approach, general supply chains are represented as networks of potential suppliers, customers and generalised production/warehousing (PW) nodes (see figure 6.1). In principle, the latter can:

- be located at any one of a set of candidate locations;
- produce one or more products using one or more shared resources;
- hold inventories of the above products as well as of any other material (e.g. raw material or intermediate) in the network;
- receive any material from any potential supplier and/or any other PW node;
- deliver any material to any customer and/or any other PW node.

In practice, it may be desirable to impose certain restrictions on a node-by-node basis (e.g. a PW node may already exist at a given location; connectivity between certain locations may be impossible or prohibitively expensive; and so on). However, the important thing to note is that PW nodes represent a generalisation of the plants, warehouses and distribution centres that were treated as three distinct types of entity in earlier supply chain models. The main difference is that the function of each PW node is not specified *a priori* but is left to be

determined by the optimisation within any constraints that the designer may wish to impose.

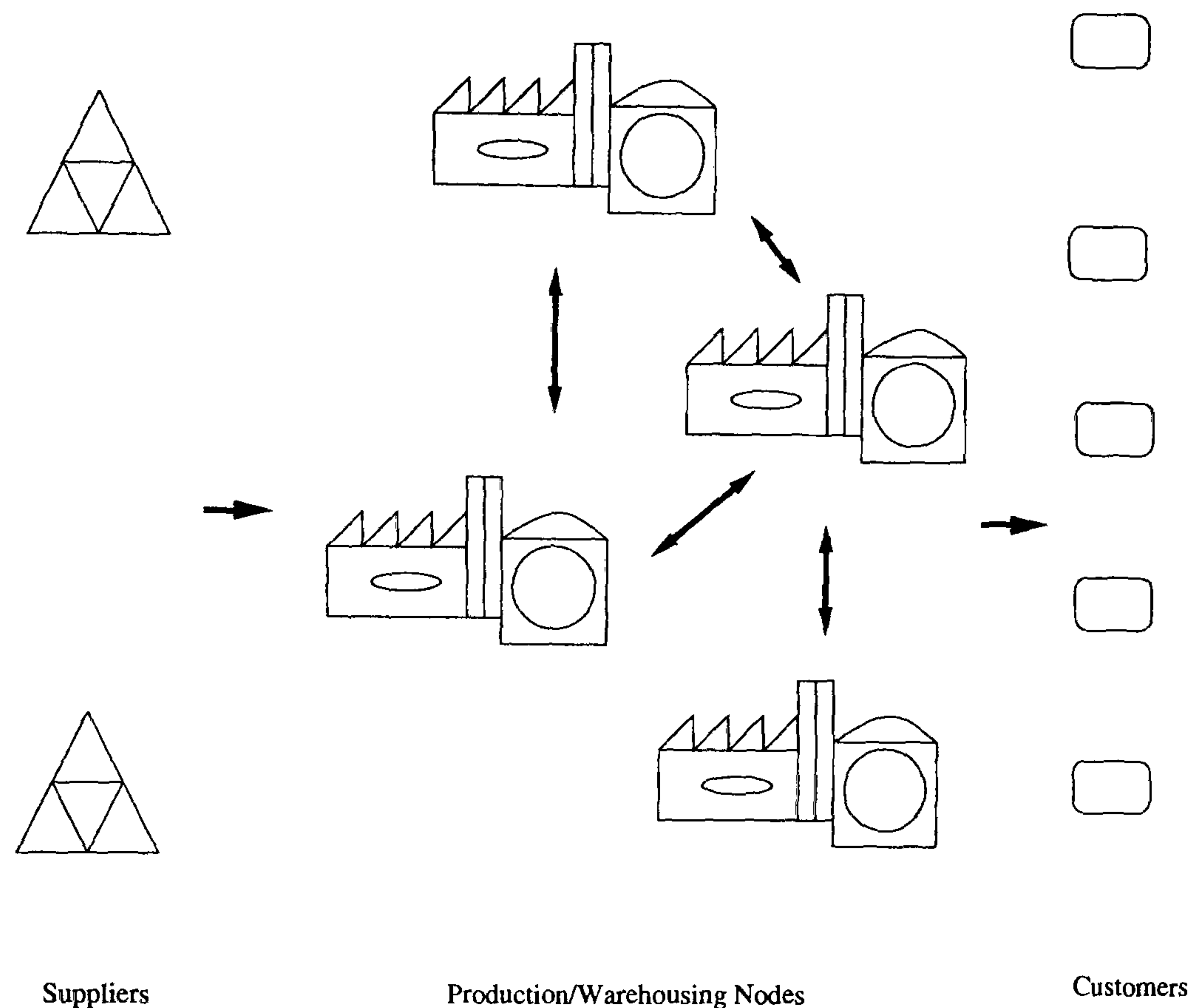


Figure 6.1: Supply chain network representation

We denote the set of all nodes in the network as $\mathcal{N}$; this includes not only the PW nodes mentioned above, but also the suppliers and customer zones. Arcs represent the connections throughout the network and the possible paths of flow of material.

The supplier nodes are denoted as $\mathcal{S} \subset \mathcal{N}$. Each supplier may provide a number of raw materials to the production plants or assembly points, generally in limited amounts that are normally determined by contracts. Customer zones form a subset $\mathcal{C}$ of nodes that place demands on the end-products of the network. Overall, we have:

$$\mathcal{N} = \mathcal{S} \cup \mathcal{PW} \cup \mathcal{C} \quad (6.1)$$

where $\mathcal{PW}$ is the set of (potential) production/warehousing nodes, and the three sets $\mathcal{PW}$, $\mathcal{S}$ and $\mathcal{C}$ are generally disjoint.

6.2. Notation

The notation to be used in this chapter is summarised below for ease of future reference:

6.2.1. Indices

e	manufacturing resource (equipment, manpower, utilities, etc.)
f	transportation product families
i	materials (including raw materials, intermediates and end-products)
k	tasks
n	nodes
r	range of transportation flows with the same unit transportation cost
s	product demand scenarios

6.2.2. Sets

$\mathcal{C}$	set of customer nodes
$\mathcal{C}^{ss}$	subset of customer nodes requiring single sourcing
K_n	set of production tasks that can be performed at node n
$\mathcal{N}$	set of nodes of the network to be designed
$\mathcal{PW}$	set of nodes with production and warehousing ability
$\mathcal{S}$	set of material supplier nodes

6.2.3. Parameters

$\overline{C}_{fnn'r}$	transportation cost for products of family f from node n to node n' over range r of transportation flow
$\overline{C}_r$	transportation cost which corresponds to interval r
C_{in}^P	unit production cost for product i at plant at PW node n
C_{in}^S	unit purchase price of material i from supplier $n \in \mathcal{S}$
C_n^S	annualised fixed cost of establishing relationship with supplier $n \in \mathcal{S}$
C_n^W	annualised fixed cost of establishing a warehousing capability at PW node n
C_{in}^{WH}	unit handling cost for product i at warehouse existing at node n
D_{in}	demand for product i in customer zone $n \in \mathcal{C}$
$E_{en}^{\max}$	maximum rate of provision of resource e at node n
$E_{en}^{\min}$	minimum rate of provision of resource e at node n
NI	number of products
NF	number of families
$NR_{fnn'}$	number of transportation flow ranges, each with constant unit cost for transporting products of family f from node n to node n'
NS	number of product demand scenarios
$Q_{inn'}^{\max}$	maximum rate of flow of material i that can be transferred from node n to node n'
$Q_{nn'}^{\max}$	maximum combined rate of flow of material that can be transferred from node n to node n'
$Q_{inn'}^{\min}$	minimum rate of flow of material i that can be practically and economically transferred from node n to node n'
$Q_{nn'}^{\min}$	minimum combined rate of flow of material that can be practically and economically transferred from node n to node n'
$\overline{Q}_{fnn'r}$	upper bound for range r of transportation flow of product family f from node n to node n'
$S_{in}^{\max}$	maximum rate of provision of material i by supplier $n \in \mathcal{S}$

$S_{in}^{\min}$	minimum rate of provision of material i by supplier $n \in \mathcal{S}$
$W_n^{\max}$	maximum warehouse capacity (if any) that can be established at PW node n
$W_n^{\min}$	minimum warehouse capacity (if any) that can be established at PW node n
α_{in}	coefficient relating capacity of warehouse at node n to flow of product i handled
γ_{en}^P	cost associated with the establishment of a unit of production resource e at PW node n
γ_n^W	cost associated with the establishment of a unit amount of warehousing capacity at PW node n
δ_{en}^P	unit cost of consumption of resource e at node n
λ_{ek}	amount of resource e required to perform unit amount of task k
ν_{ki}	amount of material i produced by unit amount of task k
ψ_s	probability of product demand scenario s occurring during the lifetime of the network

6.2.4. Continuous variables

$C_{inn}^{[s]}$	actual transportation cost of product i from node n to node n' under scenario s
E_{en}	total rate of provision of resource e at node n
$Q_{inn'}^{[s]}$	rate of flow of product i transferred from node n to node n' under scenario s
$Q_{fnn'r}^{[s]}$	rate of flow of material of family f from node n to node n' in r th transportation flow interval under scenario s
$S_{in}^{[s]}$	rate of provision of material i by supplier $n \in \mathcal{S}$ under scenario s
W_n	capacity of warehouse established at PW node n
$\xi_{kn}^{[s]}$	rate of operation of task k at PW node n under scenario s

6.2.5. Binary variables

Y_n	1 if PW node n is to be established, 0 otherwise
Y_n^P	1 if a production capability is to be established at PW node n , 0 otherwise
Y_n^S	1 if supplier $n \in \mathcal{S}$ is selected to provide material to the network, 0 otherwise
Y_n^W	1 if a warehousing capability is to be established at PW node n , 0 otherwise
$X_{nn'}$	1 if material flow is to be allowed from node n to node n' , 0 otherwise
$Z_{fnn'r}$	1 if the rate of flow of material of family f from node n to node n' is between $\overline{Q}_{fnn',r-1}$ and $\overline{Q}_{fnn'r}$, 0 otherwise

6.3. Optimal steady-state design formulation

The model presented in this chapter is a steady-state one, analogous to that of chapter 3. A dynamic model analogous to that of chapter 5 can also be formulated in a straightforward manner. The design formulation aims to minimise the expected capital and operating costs of the network, determining the optimal structure of the network in terms of:

- the new PW nodes to be established, selected from a given superset of PW nodes, each at a given location;
- whether a PW node is a production plant or a warehousing facility, or a combination of the two;
- the necessary connections between suppliers, PW nodes and customers;
- the production and/or inventory holding capabilities of each PW node.

The mathematical model proposed for this problem is an MILP as described below.

6.3.1. Network structure constraints

Assuming a PW node n is established, this may be a production plant or a warehouse or both:

$$Y_n \geq Y_n^P, \quad \forall n \in \mathcal{PW} \quad (6.2)$$

$$Y_n \geq Y_n^W, \quad \forall n \in \mathcal{PW} \quad (6.3)$$

If node $n \in \mathcal{PW}$ is established, then it must receive material from at least one other node n' . The latter may be a supplier or another PW node. Thus a production plant may receive material both from suppliers and from other plants (e.g. for further processing); also a warehouse may receive material from another warehouse in addition to any material received from the production plants:

$$Y_n \leq \sum_{n' \in \mathcal{N} \setminus \{n, \mathcal{C}\}} X_{n'n}, \quad \forall n \in \mathcal{PW} \quad (6.4)$$

Similarly, each established node $n \in \mathcal{PW}$ must send material to at least one other node n' which may be either another PW node or a customer zone. Thus, customer zones may, in principle, receive material directly from production plants as well as from warehouses:

$$Y_n \leq \sum_{n' \in \mathcal{N} \setminus \{n, \mathcal{S}\}} X_{nn'}, \quad \forall n \in \mathcal{PW} \quad (6.5)$$

In order for a connection between two nodes to exist, both nodes must be established:

$$X_{nn'} \leq Y_n, \quad \forall n \in \mathcal{PW}, n' \in \mathcal{N} \setminus \{n\} \quad (6.6)$$

$$X_{n'n} \leq Y_n, \quad \forall n \in \mathcal{PW}, n' \in \mathcal{N} \setminus \{n\} \quad (6.7)$$

If a single sourcing restriction needs to be enforced for a certain customer zone n , requiring that it will be served by exactly one warehouse or production plant, the following additional constraint is introduced:

$$\sum_{n' \in \mathcal{PW}} X_{n'n} = 1, \quad \forall n \in \mathcal{C}^{ss} \quad (6.8)$$

6.3.2. Transportation flow constraints

A flow of material i from node n to node n' can take place only if a connection of the corresponding directionality is established. In such a case, the flow must be maintained between given lower and upper bounds:

$$Q_{inn'}^{\min} X_{nn'} \leq Q_{inn'}^{[s]} \leq Q_{inn'}^{\max} X_{nn'}, \quad \forall i, n, n' \in \mathcal{N}, n \neq n' \quad (6.9)$$

In addition to the bounds on the flows of individual products, it will typically be necessary or desirable to impose bounds on the total flow of material from one node to another:

$$Q_{nn'}^{\min} X_{nn'} \leq \sum_i Q_{inn'}^{[s]} \leq Q_{nn'}^{\max} X_{nn'} \quad \forall n, n' \in \mathcal{N}, n \neq n' \quad (6.10)$$

6.3.3. Production constraints

A production capability established at a node $n \in \mathcal{PW}$ will generally involve a set of production tasks K_n . Each task $k \in K_n$ will make use of production resources (e.g. equipment, utilities, manpower etc.) to transform certain materials into other materials.

The materials involved will generally be those that can be transported between nodes in the supply chain network as well as other intermediates that exist only within the manufacturing site itself. The overall balance on a material i at node $n \in \mathcal{PW}$ is of the form:

$$\sum_{n' \in S \cup \mathcal{PW} \setminus \{n\}} Q_{in'n}^{[s]} - \sum_{n' \in C \cup \mathcal{PW} \setminus \{n\}} Q_{inn'}^{[s]} + \sum_{k \in K_n} \nu_{ik} \xi_{kn}^{[s]} = 0, \quad \forall i, n \in \mathcal{PW}, s \quad (6.11)$$

The first two terms in the above constraint represent the flowrates of material i into and out of node n respectively. The first term includes inflows from both suppliers and other PW nodes. The second term includes outflows to both customer zones and other PW nodes. For intermediate materials i that exist only within the node, both of these terms will be zero¹.

The last term in constraint (6.11) represents the rate of generation of material i at node n . Here, $\xi_{kn}^{[s]}$ is the rate at which task $k \in K_n$ is to operate at node n under scenario s . The parameter ν_{ik} represents the amount of material i that is

¹In practice, the transportation variables $Q_{in'n}^{[s]}$ and $Q_{inn'}^{[s]}$, corresponding to such materials would be completely omitted from the formulation.

produced per unit amount of task k performed².

The various tasks taking place at a node $n \in \mathcal{PW}$ will generally share production resources. This shared resource usage leads to constraints of the form:

$$\sum_{k \in K_n} \lambda_{ek} \xi_{kn}^{[s]} \leq E_{en}, \quad \forall n \in \mathcal{PW}, e, s \quad (6.12)$$

Coefficient λ_{ek} is the amount of resource e required by task k per unit extent of that task, while E_{en} is the total rate of provision of resource e that is to be made available at node n . This will generally have to lie between given lower and upper bounds, provided that a production capability is actually established at node n (i.e. $Y_n^P = 1$):

$$E_{en}^{\min} Y_n^P \leq E_{en} \leq E_{en}^{\max} Y_n^P, \quad \forall e, n \in \mathcal{PW}, s \quad (6.13)$$

6.3.4. Material supply constraints

The total amount of material i provided by a supplier node $n \in \mathcal{S}$ is given by:

$$\sum_{n' \in \mathcal{PW}} Q_{in'n}^{[s]} = S_{in}^{[s]}, \quad \forall i, n \in \mathcal{S}, s \quad (6.14)$$

Usually, this amount has to be within certain contracted limits, i.e.:

²A negative ν_{ik} indicates that material i is actually consumed by task k .

$$S_{in}^{\min} \leq S_{in}^{[s]} \leq S_{in}^{\max}, \forall i, n \in \mathcal{S}, s \quad (6.15)$$

In some cases, it may be desired to select the appropriate suppliers as part of the overall optimisation of the supply network design. This can be achieved by modifying the above constraint to:

$$S_{in}^{\min} Y_n^S \leq S_{in}^{[s]} \leq S_{in}^{\max} Y_n^S, \forall i, n \in \mathcal{S}, s \quad (6.15')$$

where the binary variable Y_n^S has a value of 1 if the supplier node $n \in \mathcal{S}$ is included in the network, and 0 otherwise.

6.3.5. Product demand constraints

The total rate of each product i arriving at a customer node n from all PW nodes n' must equal the demand for that product:

$$\sum_{n' \in \mathcal{PW}} Q_{in'n}^{[s]} = D_{in}^{[s]}, \forall i, n \in \mathcal{C}, s \quad (6.16)$$

6.3.6. Warehousing capacity constraints

The warehousing capacity to be provided at a PW node $n \in \mathcal{PW}$ generally has to lie between given lower and upper bounds, $W_n^{\min}$ and $W_n^{\max}$, provided, of course, that it is decided to establish a warehousing capability at this node (i.e. $Y_n^W = 1$):

$$W_n^{\min} Y_n^W \leq W_n \leq W_n^{\max} Y_n^W, \quad \forall n \in \mathcal{PW} \quad (6.17)$$

We generally assume that the capacities of the warehouses are linearly related to the flows of materials that they handle (cf. section 3.3.1.5). This is expressed via the constraints:

$$W_n \geq \sum_{i,n' \in \mathcal{S}} \alpha_{in} Q_{in'n}^{[s]} + \sum_{i,n' \in \mathcal{PW} \setminus \{n\}} \alpha_{in} (Q_{in'n}^{[s]} + Q_{inn'}^{[s]}) + \sum_{i,n' \in \mathcal{C}} \alpha_{in} Q_{inn'}^{[s]}, \quad \forall n \in \mathcal{PW}, s \quad (6.18)$$

where α_{in} are given coefficients. We note that the above constraint will lead to the establishment of some warehousing capacity at each and every node that is included in the network since all such nodes handle a non-zero amount of material.

6.3.7. Additional constraints

All continuous variables representing flows of material or task processing rates must be non-negative:

$$Q_{inn'}^{[s]} \geq 0, \quad \forall i, n, n' \in \mathcal{N}, n \neq n', s \quad (6.19)$$

$$\xi_{kn}^{[s]} \geq 0, \quad \forall n \in \mathcal{PW}, k \in K_n, s \quad (6.20)$$

$$S_{in}^{[s]} \geq 0, \quad \forall i, n \in \mathcal{S}, s \quad (6.21)$$

6.3.8. Objective function

In general, a distribution network involves both capital and operating costs. The former are one-off costs associated with the establishment of the infrastructure of the network. On the other hand, operating costs are incurred on a daily basis and are associated with the cost of production of material at plants, the handling of material at warehouses, and the transportation of material throughout the network.

6.3.8.1. Fixed infrastructure costs

The infrastructure costs considered by our formulation are related to the establishment of a warehouse or a production plant at a particular node $n \in \mathcal{PW}$.

Generally speaking, the cost associated with establishing a production capability at node $n \in \mathcal{PW}$ will comprise a fixed element C_n^P as well as elements that are proportional to the amounts of the various production resources e to be installed at this node; we denote the corresponding unit cost coefficients by γ_{en}^P .

Similarly, there will be both a fixed cost C_n^W and a variable (unit) cost γ_n^W associated with establishing a warehousing capability at node n .

It is assumed that all the above cost coefficients are annualised. Thus the overall contribution of node $n \in \mathcal{PW}$ to the infrastructure cost is given by:

$$C_n^P Y_n^P + \sum_e \gamma_{en}^P E_{en} + C_n^W Y_n^W + \gamma_n^W W_n$$

6.3.8.2. Material handling costs

Material handling costs are assumed to be linear functions of the total throughput.

The cost incurred at a node $n \in \mathcal{PW}$ is given by:

$$\sum_i C_{in}^{\text{MH}} \left(\sum_{n' \in \mathcal{S}} Q_{in'n}^{[s]} + \sum_{n' \in \mathcal{PW}} (Q_{inn'}^{[s]} + Q_{in'n}^{[s]}) + \sum_{n' \in \mathcal{C}} Q_{inn'}^{[s]} \right)$$

where C_{in}^{MH} is the cost of handling a unit amount material i at node n .

6.3.8.3. Production cost

The cost of production at a node n is normally related to both the provision and the utilisation of the various resources. The capital cost associated with the provision of resources has already been accounted for in section 6.3.8.1. For many resources, such as processing equipment, this is by far the dominant factor. On the other hand, for “consumable” resources, such as steam and electricity, there is also a significant operating cost that is proportional to the actual utilisation of each resource which is given by $\sum_{k \in K_n} \lambda_{ek} \xi_{kn}^{[s]}$ (cf. equation 6.12). Thus the total operating cost associated with resource consumption at node n is:

$$\sum_e \delta_{en}^P \sum_{k \in K_n} \lambda_{ek} \xi_{kn}^{[s]}$$

where δ_{en}^P is the unit cost of resource e at node n .

6.3.8.4. Transportation costs

The modelling of transportation costs occurring in the network between nodes n and n' is performed in the way described in section 3.3.2.4. This gives rise to the

following constraints:

$$\sum_{r=1}^{NR_{fnn'}} Z_{fnn'r}^{[s]} = 1, \forall f, n, n' \in \mathcal{N} \quad s = 1, \dots, NS \quad (6.22)$$

$$\bar{Q}_{fnn',r-1} Z_{fnn'r}^{[s]} \leq Q_{fnn'r}^{[s]} \leq \bar{Q}_{fnn'r} Z_{fnn'r}^{[s]}, \forall f, n, n' \in \mathcal{N}, r = 1, \dots, NR_{fnn'}, \quad s = 1, \dots, NS \quad (6.23)$$

$$\sum_{i \in I_f} Q_{inn't}^{[s]} = \sum_{r=1}^{NR_{fnn'}} Q_{fnn'r}^{[s]}, \forall f, n, n' \in \mathcal{N}, \quad s = 1, \dots, NS \quad (6.24)$$

$$C_{fnn'}^{[s]} = \sum_{r=1}^{NR_{fnn'}} \left[\bar{C}_{fnn',r-1} Z_{fnn'r}^{[s]} + (Q_{fnn'r}^{[s]} - \bar{Q}_{fnn',r-1} Z_{fnn'r}^{[s]}) \frac{\bar{C}_{fnn'r} - \bar{C}_{fnn',r-1}}{\bar{Q}_{fnn'r} - \bar{Q}_{fnn',r-1}} \right], \forall f, n, n', s \quad (6.25)$$

6.3.8.5. Cost of raw materials

An important element of the cost of operating most supply chains is given by:

- a fixed cost C_n^S of establishing a relationship with a supplier organisation $n \in \mathcal{S}$;
- a variable cost (i.e. unit price) C_{in}^S of purchasing a unit amount of raw material i from supplier $n \in \mathcal{S}$.

Thus, the overall cost of supply from a supplier $n \in \mathcal{S}$ under scenario s is given by (cf. section 6.3.4):

$$C_n^S Y_n^S + \sum_i C_{in}^S S_{in}^{[s]} \quad (6.26)$$

Of course in many cases, discounts may be available for bulk purchases from the same supplier. These can be modelled in a similar fashion to that used for modelling economies of scale in transportation (cf. section 6.3.8.4).

6.3.8.6. Overall objective function

The objective is to minimise the total cost of the supply chain given by:

$$\begin{aligned} \min \sum_{n \in \mathcal{PW}} \left(C_n^P Y_n^P + \sum_e \gamma_{en}^P E_{en} + C_n^W Y_n^W + \gamma_n^W W_n \right) + \sum_{n \in \mathcal{S}} C_n^S Y_n^S + \\ \sum_{s=1}^{NS} \psi_s \left(\sum_{n' \in \mathcal{S}} Q_{in'n}^{[s]} + \sum_{n \in \mathcal{PW}} \sum_i C_{in}^{\text{MH}} \left(\sum_{n' \in \mathcal{PW} \setminus \{n\}} (Q_{inn'}^{[s]} + Q_{in'n}^{[s]}) + \sum_{n' \in \mathcal{C}} Q_{inn'}^{[s]} \right) + \right. \\ \left. \sum_e \delta_{en}^P \sum_{k \in K_n} \lambda_{ek} \xi_{kn}^{[s]} + \sum_i \left(\sum_{n' \in \mathcal{PW} \setminus \{n\}} (C_{inn'}^{[s]} + C_{in'n}^{[s]}) + \sum_{n' \in \mathcal{C}} C_{inn'}^{[s]} \right) + \sum_{s \in \mathcal{S}} \sum_i C_{in}^S S_{in}^{[s]} \right) \end{aligned} \quad (6.27)$$

6.4. A case study

The formulation presented above is applied to a case study similar to the one presented in chapter 4. We consider 18 candidate PW nodes as shown in figure 6.2.

In the interests of comparability with the earlier case study, we allow production to take place only at the first three of these nodes³, with fixed levels of provision of production resources. Thus, we fix $Y_n^P = 0$ at all other nodes $n = 4, \dots, 18$. Moreover, we adopt a simple manufacturing model for the first three nodes according to which each product i is produced by a unique task k with a coefficient $\nu_{ik} = 1$. The raw materials consumed by these tasks are ignored, which obviates the need to model the supplier nodes in the network.

On the other hand and in contrast with the case studies of chapter 4, no distinction is made here between warehouses and distribution centres. Warehousing capacity is allowed to be established at any node – including the three that also have production capabilities. Furthermore, material is allowed to flow from any node to any other.

6.4.1. Warehouses

The infrastructure costs for the establishment of warehousing capability at each node under consideration are listed in table 6.1; these have already been amortised and are expressed in £/wk.

The warehouses are assumed to have a maximum material handling capacities of 6000 te/wk. All minimum handling capacities are set to zero. The coefficients α_{in} relating the capacity to the throughput of each material handled (cf. constraint (6.18)) are all taken to be unity⁴. The unit material handling costs C_{in}^{WH} (including labour or other operating costs) are also listed in table 6.1; for the purposes of this case study, we assume that they are the same for all products but may differ from one location to another.

³These correspond to the locations of the manufacturing plants in the case studies of chapter 4.

⁴This is equivalent to requiring that each node must be capable of holding two weeks' inventory of each material it handles; note that constraint (6.18) accounts for material twice, i.e. both when it enters and when it leaves the node.

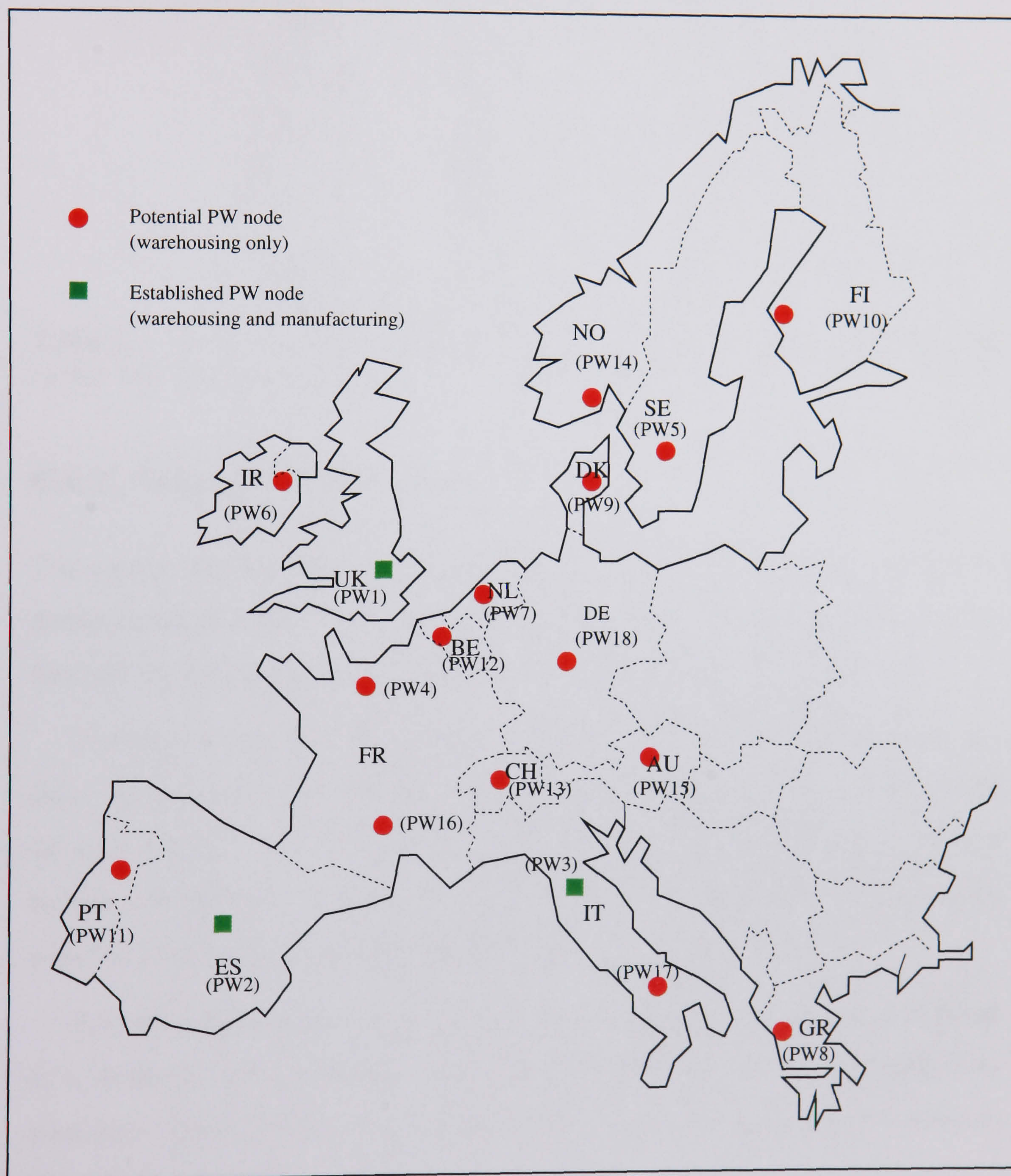


Figure 6.2: Europe-wide supply chain network design.

PW node	Infrastructure cost (£/wk)	Handling cost (£/te)
PW01	10000	4.25
PW02	5000	4.55
PW03	4000	4.98
PW04	6000	4.93
PW05	6500	4.85
PW06	4000	3.90
PW07	6000	4.06
PW08	4000	3.08
PW09	5000	6.00
PW10	3000	4.85
PW11	4500	4.12
PW12	7000	5.66
PW13	9000	5.28
PW14	5500	4.95
PW15	8500	4.83
PW16	9800	4.55
PW17	7700	4.93
PW18	16000	6.00

Table 6.1: Fixed infrastructure and material handling costs for candidate warehouses and distribution centres.

6.4.2. Transportation costs

The transportation costs are generally assumed to depend on the geographical distances between the locations involved. For the purposes of transportation, the 14 products may be aggregated into the three families shown in table 4.6.

The baseline unit costs for transportation between each pair of PW nodes are shown in tables 6.2 - 6.4. These apply to transportation flows up to 500 te/wk for each family. Above this limit, it is possible to deploy alternative models of transportation that have lower average unit costs. The magnitudes of these costs (relative to the corresponding baseline values) are shown in table 6.5.

For the purposes of computing the costs incurred in transporting material to a particular customer zone, the location of the latter (and, consequently, the associated transportation costs) is treated as being approximately the same as that of the nearest PW node.

Product family F1: Unit transportation cost (£/te)																		
Destination																		
Origin	PW1	PW2	PW3	PW4	PW5	PW6	PW7	PW8	PW9	PW10	PW11	PW12	PW13	PW14	PW15	PW16	PW17	PW18
PW1	0	75.61	54.51	12.30	70.34	29.89	17.58	119.57	45.72	112.54	77.37	12.30	40.44	65.06	61.54	45.72	100.23	38.68
PW2	73.55	0	78.68	73.55	136.84	87.23	83.81	118.02	112.89	181.31	22.23	75.26	64.99	135.12	90.65	34.21	71.84	94.07
PW3	73.28	76.61	21.65	49.96	94.93	99.93	63.28	43.30	79.94	129.90	94.93	63.28	33.31	104.92	28.31	51.63	21.65	58.29
PW4	26.58	58.47	53.16	14.17	81.51	54.93	30.12	99.23	58.47	125.81	67.33	21.26	17.72	83.28	46.07	54.93	17.72	40.75
PW5	77.16	154.33	109.96	84.88	0	90.67	59.80	136.97	27.00	48.23	160.12	69.45	86.81	21.22	77.16	106.10	131.18	50.15
PW6	27.08	84.65	79.57	38.93	79.57	0	42.32	143.90	57.56	118.51	77.87	38.93	67.72	67.72	88.03	67.72	126.97	64.33
PW7	19.97	88.99	67.19	25.42	58.11	45.40	0	112.60	30.87	101.70	94.44	9.08	36.32	54.48	50.85	45.40	94.44	21.79
PW8	118.02	121.49	64.22	109.34	123.23	147.53	107.61	0	116.29	149.27	142.32	109.34	79.84	137.12	60.74	86.78	27.77	93.72
PW9	42.04	106.72	61.44	48.51	22.63	54.98	27.49	108.34	0	64.68	109.96	35.57	54.98	24.25	53.36	67.91	85.70	24.25
PW10	107.82	178.57	116.24	117.92	42.11	117.92	94.34	144.88	67.38	0	181.94	104.45	117.92	50.54	104.45	134.77	144.88	84.23
PW11	75.72	22.37	87.77	67.12	142.85	79.17	89.49	141.13	117.03	185.87	0	79.17	79.17	137.68	104.98	60.23	115.31	103.26
PW12	11.91	74.91	44.26	13.62	61.29	39.15	8.51	107.26	37.45	105.56	78.31	0	30.64	61.29	49.37	39.15	88.53	25.53
PW13	69.11	67.38	13.82	29.37	77.75	69.11	34.55	79.47	58.74	120.94	77.75	32.82	0	82.93	27.64	19.00	58.74	34.55
PW14	63.10	134.74	88.69	71.63	18.76	68.22	51.16	134.74	25.58	51.16	136.44	61.40	81.86	0	80.16	95.51	127.92	51.16
PW15	56.52	85.60	17.76	50.06	64.60	83.98	45.22	56.52	53.29	100.13	98.52	46.83	25.84	75.90	0	41.99	45.22	30.68
PW16	45.72	34.21	51.63	54.39	106.10	67.72	45.40	86.78	67.91	134.77	60.23	39.15	39.00	79.90	41.99	0	55.32	70.68
PW17	100.23	71.84	21.65	17.72	131.18	126.97	94.44	27.77	85.70	144.88	115.31	88.53	58.74	127.92	88.99	41.99	0	60.18
PW18	38.68	94.07	58.29	40.75	50.15	64.33	21.79	93.72	24.25	84.25	103.26	25.55	34.55	51.16	34.55	41.99	45.22	0

Table 6.2: Baseline unit transportation costs between PW nodes (family 1).

Product family F2: Unit transportation cost (£/te)																		
Destination																		
Origin	PW1	PW2	PW3	PW4	PW5	PW6	PW7	PW8	PW9	PW10	PW11	PW12	PW13	PW14	PW15	PW16	PW17	PW18
PW1	0	73.12	52.71	11.90	68.02	28.90	17.00	115.63	44.21	108.83	74.82	11.90	39.11	62.91	59.51	44.21	96.92	37.41
PW2	73.20	0	78.31	73.20	136.20	86.82	83.42	117.47	112.36	180.46	22.13	74.91	64.69	134.49	90.23	34.05	71.50	93.63
PW3	81.65	85.36	24.12	55.67	105.78	111.34	70.52	48.25	89.07	144.75	105.78	70.52	37.11	116.91	31.54	57.52	24.12	64.95
PW4	24.76	54.48	49.53	13.20	75.95	51.18	28.06	92.46	54.48	117.22	62.74	19.81	16.51	77.60	42.92	51.18	16.51	37.97
PW5	77.52	155.04	110.47	85.27	0	91.09	60.08	137.60	27.13	48.45	160.86	69.77	87.21	21.31	77.52	106.59	131.79	50.39
PW6	32.65	102.06	95.93	46.94	95.93	0	51.03	173.50	69.40	142.88	93.89	46.94	81.64	81.64	106.14	81.64	153.09	77.56
PW7	19.54	87.05	65.73	24.87	56.85	44.41	0	110.15	30.20	99.49	92.38	8.88	35.53	53.30	49.74	44.41	92.38	21.32
PW8	127.18	130.92	69.20	117.83	132.79	158.98	115.96	0	125.31	160.85	153.37	117.83	86.03	147.76	65.46	93.52	29.92	101.00
PW9	48.45	123.01	70.82	55.91	26.09	63.36	31.68	124.87	0	74.55	126.73	41.00	63.36	27.95	61.50	78.27	98.78	27.95
PW10	111.91	185.36	120.66	122.40	43.71	122.40	97.92	150.38	69.94	0	188.85	108.41	122.40	52.46	108.41	139.89	150.38	87.43
PW11	81.65	24.12	94.64	72.37	154.03	85.36	96.50	152.17	126.19	200.42	0	85.36	85.36	148.46	113.20	64.95	124.33	111.34
PW12	12.44	78.21	46.21	14.22	63.99	40.88	8.88	111.98	39.10	110.21	81.76	0	31.99	63.99	51.55	40.88	92.43	26.66
PW13	58.96	57.48	11.79	25.05	66.33	58.96	29.48	67.80	50.11	103.18	66.33	28.00	0	70.75	23.58	16.21	50.11	29.48
PW14	60.11	128.35	84.48	68.23	17.87	64.98	48.74	128.35	24.37	48.74	129.97	58.48	77.98	0	76.36	90.98	121.85	48.74
PW15	60.97	92.32	19.16	54.00	69.68	90.58	48.77	60.97	57.48	108.00	106.26	50.51	27.87	81.87	0	45.29	48.77	33.09
PW16	45.72	34.21	51.63	54.39	106.10	67.72	45.40	86.78	67.91	134.77	60.23	39.15	39.00	79.90	41.99	0	55.32	70.68
PW17	100.23	71.84	21.65	17.72	131.18	126.97	94.44	27.77	85.70	144.88	115.31	88.53	58.74	127.92	88.99	41.99	0	60.18
PW18	38.68	94.07	58.29	40.75	50.15	64.33	21.79	93.72	24.25	84.25	103.26	25.55	34.55	51.16	34.55	41.99	45.22	0

Table 6.3: Baseline unit transportation costs between PW nodes (family 2).

Product family F3: Unit transportation cost (£/te)																		
		Destination																
Origin	PW1	PW2	PW3	PW4	PW5	PW6	PW7	PW8	PW9	PW10	PW11	PW12	PW13	PW14	PW15	PW16	PW17	PW18
PW1	0	72.82	52.50	11.85	67.74	28.79	16.93	115.17	44.03	108.39	74.52	11.85	38.95	62.66	59.27	44.03	96.54	37.26
PW2	69.04	0	73.86	69.04	128.46	81.89	78.68	110.80	105.98	170.21	20.87	70.65	61.02	126.85	85.10	32.11	67.44	88.31
PW3	74.54	77.92	22.02	50.82	96.56	101.64	64.37	44.04	81.31	132.13	96.56	64.37	33.88	106.72	28.79	52.51	22.02	59.29
PW4	28.74	63.22	57.48	15.32	88.13	59.39	32.57	107.29	63.22	136.03	72.80	22.99	19.16	90.05	49.81	59.39	19.16	44.06
PW5	65.66	131.33	93.57	72.23	0	77.15	50.89	116.56	22.98	41.04	136.26	59.10	73.87	18.05	65.66	90.29	111.63	42.68
PW6	31.07	97.12	91.29	44.67	91.29	0	48.56	165.10	66.04	135.96	89.35	44.67	77.69	77.69	101.00	77.69	145.68	73.81
PW7	20.58	91.67	69.22	26.19	59.87	46.77	0	116.00	31.80	104.77	97.29	9.35	37.42	56.13	52.38	46.77	97.29	22.45
PW8	121.40	124.97	66.05	112.48	126.76	151.75	110.69	0	119.62	153.54	146.40	112.48	82.12	141.04	62.48	89.27	28.56	96.41
PW9	55.75	141.52	81.48	64.32	30.02	72.90	36.45	143.66	0	85.77	145.81	47.17	72.90	32.16	70.76	90.06	113.64	32.16
PW10	106.08	175.70	114.37	116.03	41.44	116.03	92.82	142.55	66.30	0	179.02	102.77	116.03	49.72	102.77	132.60	142.55	82.88
PW11	74.81	22.10	86.72	66.31	141.13	78.21	88.42	139.43	115.62	183.64	0	78.21	78.21	136.03	103.72	59.51	113.92	102.02
PW12	13.58	85.41	50.47	15.53	69.88	44.64	9.70	122.30	42.70	120.36	89.29	0	34.94	69.88	56.29	44.64	100.94	29.11
PW13	72.36	70.55	14.47	30.75	81.40	72.36	36.18	83.21	61.50	126.63	81.40	34.37	0	86.83	28.94	19.90	61.50	36.18
PW14	72.66	155.15	102.12	82.48	21.60	78.56	58.92	155.15	29.46	58.92	157.12	70.70	94.27	0	92.30	109.98	147.30	58.92
PW15	62.62	94.83	19.68	55.47	71.57	93.04	50.10	62.62	59.05	110.94	109.15	51.89	28.63	84.10	0	46.52	50.10	33.99
PW16	45.72	34.21	51.63	54.39	106.10	67.72	45.40	86.78	67.91	134.77	60.23	39.15	39.00	79.90	41.99	0	55.32	70.68
PW17	100.23	71.84	21.65	17.72	131.18	126.97	94.44	27.77	85.70	144.88	115.31	88.53	58.74	127.92	88.99	41.99	0	60.18
PW18	38.68	94.07	58.29	40.75	50.15	64.33	21.79	93.72	24.25	84.25	103.26	25.55	34.55	51.16	34.55	41.99	45.22	0

Table 6.4: Baseline unit transportation costs between PW nodes (family 3).

Amount transported (te/wk)	Relative transportation cost
0-500	1.00
500-1000	0.95
1000-5000	0.90

Table 6.5: Economies of scale in transportation costs.

6.4.3. Optimal network design

We consider the three product demand scenarios given in tables 4.11-4.19, assuming they are all equally probable, i.e. $\psi_1 = \psi_2 = \psi_3 = 0.33$. Our aim is to design a network that can handle all of them while minimising the total expected cost.

The mathematical formulation of section 6.3 was implemented in *ooMILP* (Tsiakis *et al.*, 1999) and solved to optimality using the CPLEX v6.5 (ILOG, 1998) solver.

The optimal solution obtained is shown in figure 6.3. As can be seen, some warehousing capacity is established next to each of the three production plants. In fact, the majority of the customers are served directly from these warehouses. As none of the plants is capable of producing the entire range of products (cf. table 4.2), some material transportation between these locations is inevitable.

We can also see the establishment of a regional warehouse in the Netherlands. This is created primarily to exploit economies of scale in transportation as it is cheaper to transfer material in bulk to a regional warehouse, and then to distribute it from there to more than one customer in its vicinity. Thus, material is transferred in bulk amounts to the Dutch warehouse, from where it is subsequently distributed to the markets of the Netherlands, Germany and Sweden.

Another reason for this structure is the interaction between, on one hand, the single source constraint for the customer zones and, on the other, the inability of any single plant to supply all the material necessary to cover the corresponding demand. Again, geography and transportation costs are important factors. In this

case, it is cheaper to establish a local warehouse receiving material from both the UK and Spanish plants, rather than to transport the material from one of these two plants to the other, and from there to the German and Swedish customer zones.

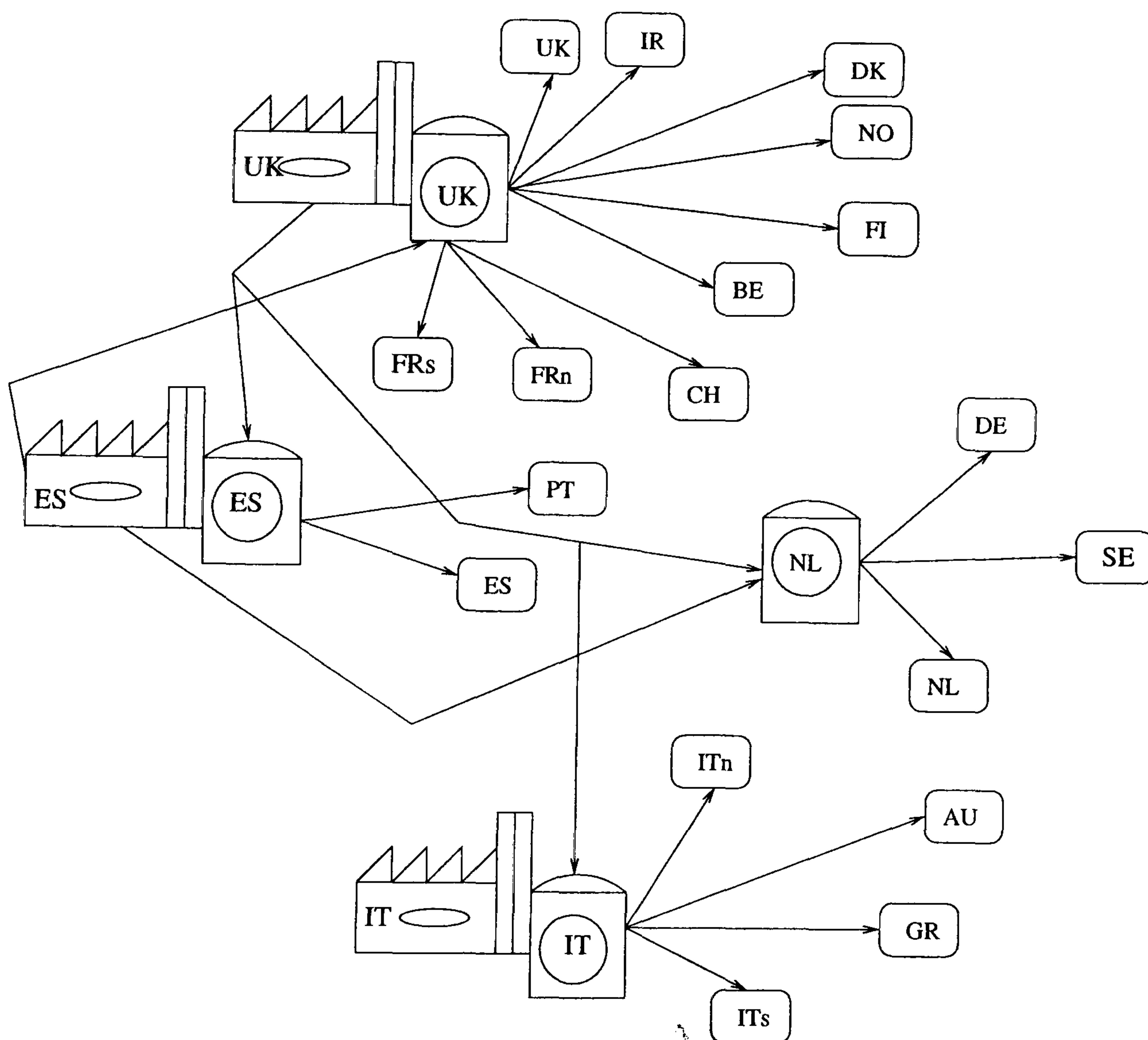


Figure 6.3: Optimal supply chain network.

Table 6.6 compares the various elements of cost in the solution obtained with the formulation of chapter 3 (cf. section 4.3) with the corresponding costs in the solution presented here. It can be seen that the additional flexibility of the formulation in this chapter results in a more than 10% reduction in cost. This is due to two main factors. First, the ability to serve customers directly from plants allows production to be redistributed to take advantage of lower costs. Secondly, much of the infrastructural and, more importantly, operational costs that are

incurred by forcing the network adhere to a given echelon structure are now avoided; instead, independent warehouses are established only when absolutely necessary.

Cost element	Fixed echelon structure (chapter 3)		Generalised network (chapter 6)	
	amount (£/wk)	%	amount (£/wk)	%
Production	448,555	66.22	440,157	73.15
Infrastructure	38,000	0.06	24,000	0.04
Material handling costs	38,826	0.06	20,228	0.03
Material transportation	150,542	33.66	115,457	19.20
Total	677,285	100.00	600,251	100.00

Table 6.6: Comparison of optimal solutions from different mathematical formulations.

6.4.4. Optimal network design under increased demands

In order to demonstrate the flexibility of the proposed formulation, we repeat the optimal network design exercise using three demand scenarios in which all demands are increased by 20% over those shown in tables 4.11-4.19.

The problem is solved to optimality and the results are shown in figure 6.4. Comparing this with the optimal solution of figure 6.3, we note that there is still a single regional warehouse but this is now located in Ireland rather than the Netherlands. Also, there are changes to the plants that supply some of the customer zones. For example, the Austrian market has been shifted from the Italian to the UK plant.

The above changes in the optimal network structure are primarily due to the single source constraints for the customers and their interactions with the production constraints. For example, the demands of the Irish market can no longer be satisfied by a single plant; this, together with the single sourcing constraint makes the establishment of a regional warehouse in Ireland inevitable.

Computational statistics relating to the solution of the problems considered in sections 6.4.3 and 6.4.4 are presented in table 6.7. Despite the size of the

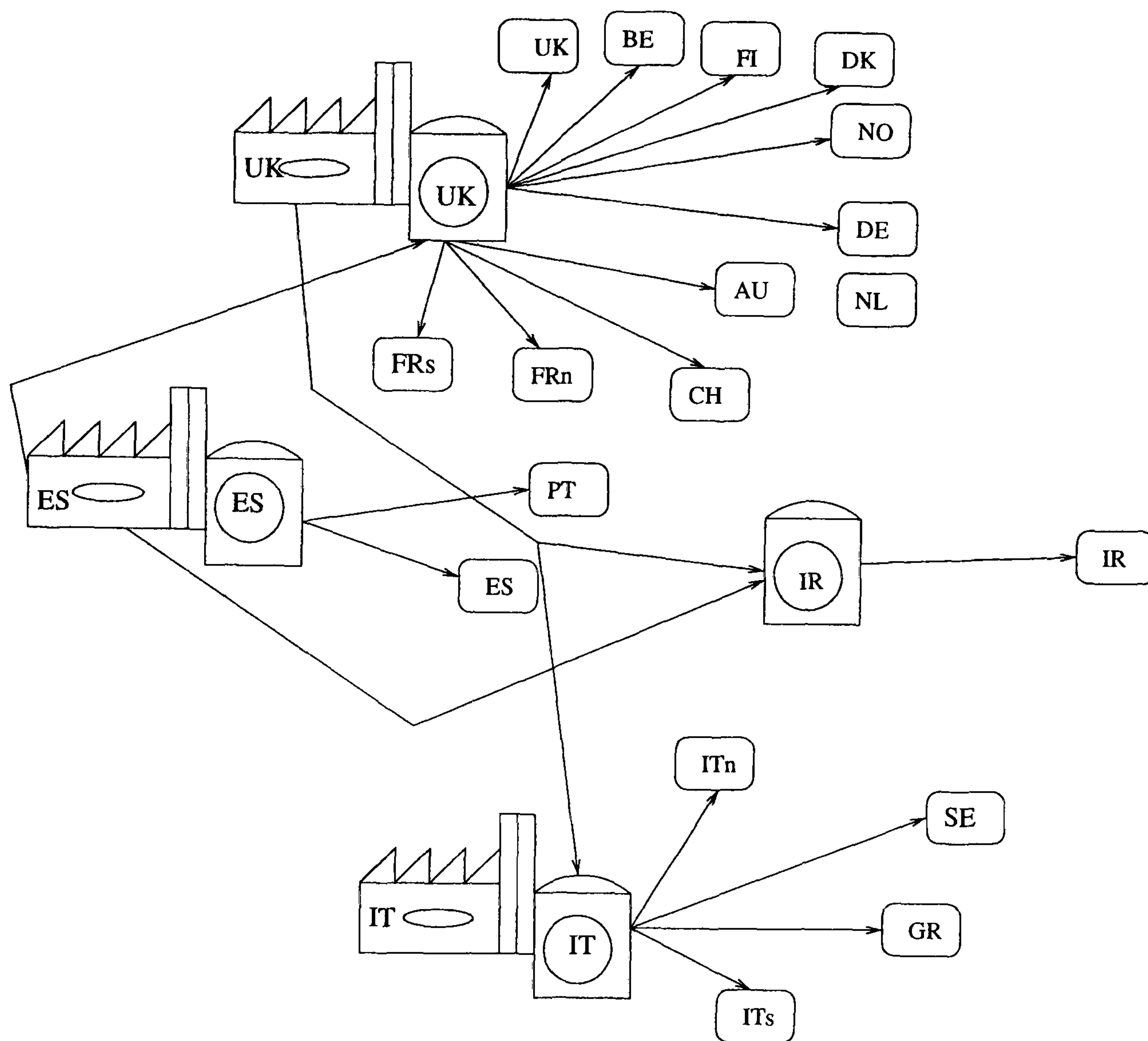


Figure 6.4: Optimal supply chain network for increased demands.

problems, the computational costs are relatively modest. This may be attributed to the small integrality solution gap of the formulation.

6.5. Concluding remarks

This chapter has described an MILP mathematical model for the optimisation of the design and operation of generalised supply chain networks that do not involve any *a priori* assumptions on the number of echelons. As in previous chapters, uncertainty in product demands is captured using demand scenarios.

The generalised nature of the formulation presented leads to leaner networks where warehousing is established only when necessary. The number of echelons

Statistics	Section 6.4.3	Section 6.4.4
Number of Constraints	127116	127116
Number of Integer Variables	18198	18198
Number of Continuous Variables	77562	77562
Optimal Objective Value	600197	722474
Fully Relaxed LP Objective Function	600251	722474
Integrality Solution Gap	0.01%	0.00%
Optimality margin	1.00%	1.00%
Number of Branch & Bound Nodes	59	41
CPU time (SUN-Ultra60)	604s	643s
Solver	CPLEX6.5	CPLEX6.5

Table 6.7: Computational statistics for the network design problem.

between manufacturing plants and customer zones may vary within the same network. In fact, where appropriate, customers may be supplied directly by the plants removing any other costs associated with the establishment of, and material handling at warehouses or distribution centres. On the other hand, one or more intermediate warehousing eschelons are established automatically where this leads to economies of scale in transportation and/or is necessary in order to satisfy single sourcing constraints.

Conclusions

This chapter provides a summary of the work described in this thesis, and provides some ideas for further research in the area.

7.1. Contributions of this thesis

The design of supply chain networks is a difficult task due to the intrinsic complexity of the major sub-systems of these networks and the many interactions among these, as well as other factors such as the considerable uncertainty in product demands. In the past, this complexity has forced much research to focus on individual sub-processes within supply chain networks, such as production scheduling and planning, inventory management and transportation planning. Recently, however, attention has increasingly been placed on the performance, design and analysis of the supply chain as a whole due to the large scale economic benefits that may be realised by such an integrated approach.

The aim of this work has been to develop quantitative tools that can support strategic decisions in supply chain network design while addressing the complexity of the problem as outlined above. Our models consider flexible production facilities

in which a number of products are produced making use of shared resources; they also take account of flexible transportation modes and uncertainty in product demands.

Our initial work focused on structured supply chain networks with given numbers of echelons in both material and information flow. A steady-state model using time-average material flows was developed (chapter 3) for the design of complex, multiple product, production and distribution systems with several echelons. The model took account of the uncertainty arising from long-term variations in product demands using a scenario-based approach. The model was then extended to consider some dynamic aspects of the supply chain (chapter 5) taking account of product demands that are both time-varying and uncertain.

The applicability of the two models mentioned above was illustrated through two case studies for the design and operation of a Europe-wide production and distribution network (chapters 4 and 5).

Based on the experience gained from these case studies, a more general model was proposed in chapter 6. The problem is formulated in terms of a network of generalised production-warehousing nodes that may or may not be established at a set of candidate locations. The optimisation calculation automatically determines the precise function (manufacturing or warehousing or both) of each node that is included within the network. No restriction on the flow of material is imposed. Thus, customers may be served directly from manufacturing plants, or indirectly via one or more warehousing stages; the number of the latter may differ from one customer to another. Also, nodes (e.g. manufacturing plants) may exchange intermediate materials with each other.

The advantages of the generalised model over its more restricted counterparts were explored through a case study. It was found that generalised networks tend to be more compact as they avoid the establishment of unnecessary intermediate warehousing stages. On the other hand, they can also better exploit economies of

scale in transportation by establishing as many intermediate warehousing nodes as necessary for serving a particular customer or group of customers.

Supply chain management is an enormously wide field, with many organisational, human and technical aspects. Inevitably, this thesis has dealt with only a small part of this entire spectrum of constraints, objectives and decisions. Nevertheless, we believe that the work presented has resulted in tools that are capable of quantifying the interactions among many facets of the supply problem network to the extent necessary for supporting strategic decision making.

7.2. Recommendations for further work

The research presented in this thesis has identified a number of issues that need to be further investigated. These can be classified in two main categories relating, respectively, to the modelling of the supply chain and to the solution methods. We consider each of these separately below.

7.2.1. Supply chain modelling

The models presented in this thesis take a more detailed view of manufacturing operations than many of those used in earlier work, in that they recognise explicitly the existence of multiple products competing for scarce resources. The model of chapter 6 goes even further to include not just the products but also the raw materials and intermediates, as well as the various processing steps (cf. section 6.4.3). This provides a more accurate representation of production capability, especially in cases in which several products share common intermediates. It also allows the modelling of interactions between manufacturing plants (e.g. one plant supplying an intermediate to another) and between plants and suppliers (cf. section 6.4.4).

Although the above represents some progress in the representation of manufacturing, there are still questions relating to the consistency of the models used here with the more detailed models that are used for planning and scheduling. This is particularly important in the case of existing manufacturing facilities, for which the planning and scheduling models may already exist. An interesting question in this context is whether it is possible to derive automatically the coarser models used for decisions at the level of supply chain networks from the finer models used for plant planning and scheduling. This would avoid both the possibility of errors and inconsistencies, and the effort involved in constructing and maintaining two separate models. A possible solution that may be worth investigating is the use of formal aggregation techniques being applied to the detailed scheduling models to derive coarse, yet accurate, aggregate models (Wilkinson, 1996).

As has been demonstrated by the various case studies presented in this thesis, the optimal structure of supply networks is a strong function of the assumed product demands imposed. The use of multiple demand scenarios represents an attempt to reduce this sensitivity by taking account of a range of possible demands. Nevertheless, the importance of the availability of good demand estimates cannot be overemphasised. Thus, the techniques presented in this thesis need to be tightly coupled with accurate demand forecasting tools. Conversely, once an optimal network has been designed, it would be highly desirable to have techniques that can quantify its “operating envelope”, i.e. the space of demands that can be met by this network, and the dependence of operating costs on such demands.

Finally, this thesis has treated the supply chain network as effectively belonging to a single organisation with a centralised planning capability. However, recent industrial trends point towards an ever increasing degree of outsourcing – not only in traditional areas such as warehousing and transportation, but also in manufacturing. So-called Third-Party Logistic (3GL) and Fourth-Party Logis-

tic¹ (4GL) networks already operate within many enterprises, especially in the semi-conductor and computer hardware sectors. It is, therefore, important for mathematical formulations such as those presented in this thesis to be extended to recognise explicitly the existence of multiple organisations within the supply chain, each with potentially different objectives and constraints, and with limited sharing of information among these organisations.

7.2.2. Solution Algorithms

This thesis has demonstrated that the use of MILP formulations allows the detailed modelling of many of the important features of supply chain networks. However, as can be seen from the case studies presented, the resulting mathematical problems are quite large. This is particularly the case for problems involving multiple product demand scenarios and/or multiple time periods.

Fortunately, our supply chain models can be decomposed in a number of possible ways. The sub-models resulting from such decompositions may correspond to individual manufacturing locations (spatial decomposition), time periods (temporal decomposition) or product demand scenarios (scenario decomposition), with the coupling between different sub-models being confined to relatively few “complicating” variables.

In view of the above, the use of decomposition techniques that exploit the special structure of our models appears both feasible and highly desirable. A complication that arises in this context is the existence of discrete operational decisions (e.g. with respect to transportation) within each of the sub-models. This reduces the applicability of decomposition techniques that rely on the availability of gradient information from the sub-problems. Some decomposition techniques for solving MILP problems with integer decisions within the sub-problems have

¹ “Fourth-Party Logistics” is a trademark of Andersen Consulting LLP.

been proposed by Dimitriadis (2000). It is recommended that the use of these techniques in the context of supply chain models be investigated.

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