A Spin Entanglement Witness for Quantum Gravity

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Understanding gravity in the framework of quantum mechanics is one of the great challenges in modern physics. However, the lack of empirical evidence has lead to a debate on whether gravity is a quantum entity. Despite varied proposed probes for quantum gravity, it is fair to say that there are no feasible ideas yet to test its quantum coherent behaviour directly in a laboratory experiment. Here, we introduce an idea for such a test based on the principle that two objects cannot be entangled without a quantum mediator. We show that despite the weakness of gravity, the phase evolution induced by the gravitational interaction of two micron size test masses in adjacent matter-wave interferometers can detectably entangle them even when they are placed far apart enough to keep Casimir-Polder forces at bay. We provide a prescription for witnessing this entanglement, which certifies gravity as a quantum coherent mediator, through simple spin correlation measurements.

Quantizing gravity is one of the most intensively pursued areas of physics [1, 2]. However, the lack of empirical evidence for quantum aspects of gravity has lead to a debate on whether gravity is a quantum entity. This debate includes a significant community who subscribe to the breakdown of quantum mechanics itself at scales macroscopic enough to produce prominent gravitational effects [3–7], so that gravity need not be a quantized field in the usual sense. Indeed it is quite possible to treat gravity as a classical agent at the cost of including additional stochastic noise [8–11]. Moreover, oft-cited necessities for quantum gravity (e.g. the Big Bang singularity) can be averted by modifying the Einstein action such that gravity becomes weaker at short distances and small time scales [12]. Thus it is crucial to test whether fundamentally gravity is a "quantum" entity. Proposed tests of this question have traditionally focussed on specific models, phenomenology and cosmological observations (e.g. [2, 13–16]) but are yet to provide conclusive evidence. **More recently, the idea of laboratory probes (proposed originally by Feynman [17]) that emphasize the interaction of a probe-mass with the gravitational field created by another mass [18–23], has started to take hold. However, this approach does not yet clarify how the possible quantum coherent nature of gravity can be unambiguously certified in an experiment. In this Letter, we present the scheme for an experimental that not only would certify the potential quantum coherent behaviour of gravity, but would also offer a much more prominent witness of quantum gravity than existing laboratory based proposals.**

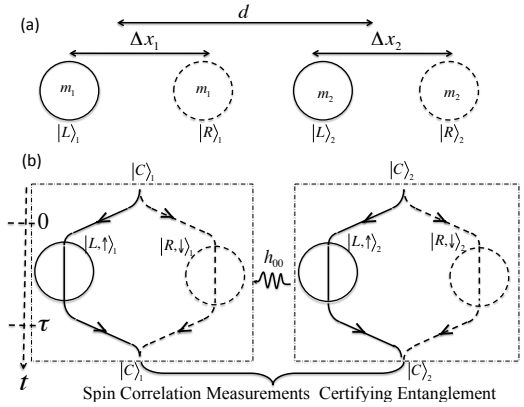


FIG. 1. **Adjacent interferometers to test the quantum nature of gravity:** (a) Two test masses held adjacently in superposition of spatially localized states $|L\rangle$ and $|R\rangle$. (b) Adjacent Stern-Gerlach interferometers in which initial motional states $|C\rangle_j$ of masses are split in a spin dependent manner to prepare states $|L,\uparrow\rangle_j + |R,\downarrow\rangle_j$ $(j = 1, 2)$. Evolution under mutual gravitational interaction for a time τ entangles the test masses by imparting appropriate phases to the components of the superposition. This entanglement can only result from the exchange of quantum mediators – if all interactions aside gravity are absent, then this must be the gravitational field (labelled h_{00} where $h_{\mu\nu}$ are weak perturbations on the flat space-time metric $\eta_{\mu\nu}$). This entanglement between test masses evidencing quantized gravity can be verified by completing each interferometer and measuring spin correlations.

We show that the growth of entanglement between two mesoscopic test masses in adjacent matter-wave interferometers (Fig.1(b)) can be used to certify the "quantum" character of the mediator (gravitons) of the gravitational interaction – in the same spirit as a Bell-inequality cer-

tifies the "non-local" character of quantum mechanics. We make two striking observations that make the test for quantum gravity accessible with feasible advances in interferometry: (i) For mesoscopic test masses $\sim 10^{-14}$ kg (with which intereference experiments might soon be possible [24]) separated by ~ 100 μm, the quantum mechanical phase $\frac{E\tau}{\hbar}$ induced by their gravitational interaction (with E being their gravitational interaction energy, and $\tau \sim 1$s their interaction time) is significant enough to generate an observable entanglement between the masses; (ii) If we use test masses with embedded spins and a Stern-Gerlach scheme [25, 26] to implement our interferometry, then, at the end of the interferometry, the gravitational interaction of the test masses actually entangles their spins which are readily measured in complementary bases (necessary in order to witness entanglement). Additionally, although our approach is independent of the specifics of any quantum theory of gravity (in the same spirit as using entanglement to study the nature of unknown processes[27, 28]), we show, in the supplementary material, that off-diagonal terms between coherent states (a signature of the quantum superposition principle) of the Newtonian gravitational field is necessary for the development of the entanglement between the test masses.

Our proposal relies on two simple assumptions: (a) the gravitational interaction between two masses is mediated by a gravitational field (in other words, it is *not* a direct interaction-at-a-distance), (b) the validity of a central principle of quantum information theory: entanglement between two systems *cannot* be created by Local Operatons and Classical Communication (LOCC) [29]. **It can readily be proved that, in the absence of close time-like loops [30] (i.e. under the assumption of validity of the chronology protection conjecture [31]) and as long as the notion of classicality itself is not extended significantly [32], LOCC keeps any initially unentangled state separable**. Translating to our setting of two test masses in adjacent interferometers, any external fields (including the gravitational fields from other masses around them) can only make local operations (LO) on their states, while a classical gravitational field propagating between the test masses can only give a classical communication (CC) channel between them. These LOCC processes cannot entangle the states of the masses. Thus it immediately follows that if the mutual gravitational interaction entangles the state of two masses, then the mediating gravitational field is necessarily quantum mechanical in nature.

Entanglement due to gravitational interaction:- We first consider a schematic version that clarifies how the states of two neutral test masses 1 and 2 (masses m_1 and m_2), each held steadily in a superposition of two spatially separated states $|L\rangle$ and $|R\rangle$ as shown in Fig.1(a) for a time τ, get entangled. Imagine the centres of $|L\rangle$ and $|R\rangle$ to be separated by a distance Δx, while each of the states $|L\rangle$ and $|R\rangle$ are localized Gaussian wavepackets with widths $<< \Delta x$ so that we can assume $\langle L|R\rangle = 0$. There is a separation d between the centres of the superpositions as shown in Fig.1(a) so that even for the closest approach of the masses $(d - \Delta x)$, the short-range Casimir-Polder force is negligible. Distinct components of the superposition have distinct gravitational interaction energies as the masses are separated by different distances and thereby will have different rates of phase evolution. Under these circumstances, the time evolution of the joint state of the two masses is purely due to their mutual gravitational interaction, and given by

$$|\Psi(t=0)\rangle_{12} = \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1)\frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2) \tag{1}$$

$$\rightarrow |\Psi(t=\tau)\rangle_{12} = \frac{e^{i\phi}}{\sqrt{2}}\{|L\rangle_1\frac{1}{\sqrt{2}}(|L\rangle_2 + e^{i\Delta\phi_{LR}}|R\rangle_2) + |R\rangle_1\frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|L\rangle_2 + |R\rangle_2)\} \tag{2}$$

where $\Delta\phi_{RL} = \phi_{RL} - \phi, \Delta\phi_{LR} = \phi_{LR} - \phi$, and

$$\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)},\ \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d+\Delta x)},\ \phi \sim \frac{Gm_1m_2\tau}{\hbar d}.$$

One can now think of each mass as an effective "orbital qubit" with its two states being the spatial states $|L\rangle$ and $|R\rangle$, which we can call orbital states. As long as $\frac{1}{\sqrt{2}}(|L\rangle_2 + e^{i\Delta\phi_{LR}}|R\rangle_2)$ and $\frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|L\rangle_2 + |R\rangle_2)$ are not the *same* state (which is very generic, happens for any $\Delta\phi_{LR} + \Delta\phi_{RL} \neq 2n\pi$, with integral n), it is clear that the state $|\Psi(t=\tau)\rangle_{12}$ cannot be factorized and is thereby an entangled state of the two orbital qubits. Witnessing this entanglement then suffices to prove that a quantum field must have mediated the gravitational interaction between them.

It makes sense to start with particles of the largest possible masses, namely $m_1 \sim m_2 \sim 10^{-14}$ kg for which there have already been realistic proposals for creating superpositions of spatially separated states such as $|L\rangle$ and $|R\rangle$ [24]. Note that we are constrained to design an experiment in which only the gravitational interaction is active. This means that the allowed distance of closest approach is $d - \Delta x \approx 200\mu$m, which is the distance at which the Casimir-Polder interaction [33] $\sim \frac{1}{(4\pi\epsilon_0)^2}\frac{23\hbar cR^6}{4\pi(d-\Delta x)^7}(\frac{\epsilon-1}{\epsilon+2})^2 \sim 0.1$ of the gravitational potential, where, to take an explicit material, we have assumed $R \sim 1\mu$m radius diamond microspheres with dielectric constant $\epsilon \sim 5.7$. Note that we can get

$$\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)} \sim O(1) \tag{3}$$

if the duration for which we can hold the superposition without decoherence is $\tau \sim 2$s. Such a significant phase accumulation will leads to a significant entanglement between the masses as the entanglement increases monotonically over $\Delta\phi_{LR}+\Delta\phi_{RL}$ evolving from 0 to π and reaches

maxiaml value for π. In practice, it is very difficult to witness directly the entanglement between the dichotomized spatial orbital degrees of freedom as generated above as, for that, one will need to measure the spatial degrees of freedom in more than one spatial bases (which involves constructing ideal two port beam-splitters for massive objects). We next show how we naturally solve this problem by resorting to Stern-Gerlach (SG) interferometry which has recently been achieved with neutral atoms [26], and proposed for freely propagating nano-crystals with embedded spins [25].

Gravitational Entanglement Witnessing in SG Interferometry:- The SG interferometry [cf. Fig. 1(b)] includes the following three steps:

Step 1: A spin dependent spatial splitting of the centre of mass (COM) state of a test mass m_j in an inhomogeneous magnetic field depicted by the evolution:

$$|C\rangle_j \frac{1}{\sqrt{2}}(|\uparrow\rangle_j + |\downarrow\rangle_j) \rightarrow \frac{1}{\sqrt{2}}(|L,\uparrow\rangle_j + |R,\downarrow\rangle_j), \qquad (4)$$

where $|C\rangle$ is the initial localized state of m_j at the centre of the axis of the SG apparatus and $|L\rangle$ and $|R\rangle$ are separated states localized on its opposite sides along the axis (these are qualitatively the same ones as shown in Fig.1).

Step 2: "Holding" the coherent superposition created above (Eq.(4) for a time τ (Consider the magnetic field of the SG effectively switched off for a duration τ).

Step 3: The third and final step brings back the superposition through the unitary transformations

$$|L,\uparrow\rangle_j \rightarrow |C,\uparrow\rangle_j,\ |R,\downarrow\rangle_j \rightarrow |C,\downarrow\rangle_j, \qquad (5)$$

which is, essentially, a refocussing SG apparatus with magnetic field homogeneity oriented oppositely to the apparatus in step 1 (although, in practice, it is best to keep the same magnetic field inhomogeneity and simply flip the spin so as to reverse the SG effect of step 1).

Let us now assume that two such SG interferometers with neutral test masses m_1 and m_2 operate in close proximity (but masses do not come so close as to have a significant Casimir-Polder interaction) as depicted in Fig.1(b). Moreover, we assume temporarily that the evolution time in steps 1 and 3 (when the spin-dependent splitting and recombination takes place) is much smaller than the time needed for the accumulation of a non-negligible gravitational phase. Then during the step 2 of the SG interferometry, due to the mutual gravitational interaction, the joint state of the two test masses evolves exactly as in Eq.(1)-Eq.(2) with the orbital qubit states $|L\rangle_j$ and $|R\rangle_j$ replaced by "spin-orbital" qubit states $|L,\uparrow\rangle_j$ and $|R,\downarrow\rangle_j$. When we follow-up the evolution of Eq.(2) of spin-orbital qubits with the step 3 of Eq.(5), then we obtain the state at the end of the SG interferometry to be

$$|\Psi(t=t_{\rm End})\rangle_{12} = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_1 \frac{1}{\sqrt{2}}(|\uparrow\rangle_2 + e^{i\Delta\phi_{LR}}|\downarrow\rangle_2) + |\downarrow\rangle_1 \frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|\uparrow\rangle_2 + |\downarrow\rangle_2)\}|C\rangle_1|C\rangle_2,$$

where the unimportant overall phase factor outside the state has been omitted. The above is manifestly an entangled state of the spins of the two test masses. It can be verified by measuring the spin correlations in two complementary bases in order to estimate the entanglement witness $\mathcal{W} = |\langle\sigma_x^{(1)} \otimes \sigma_z^{(2)}\rangle - \langle\sigma_y^{(1)} \otimes \sigma_z^{(2)}\rangle|$. If $\mathcal{W}$ is found to exceed unity then the state is proven to be entangled, and, thereby, the mediator, the gravitational field, a quantum entity.

An Explicit Scheme:- We now outline an explicit interferometer. Each SG interferometer has to be fed in by neutral masses with an embedded electronic spin, a very low internal crystal temperature and operate under very low ambient pressure (the latter two conditions are required for suppressing decoherence over relevant time scales as described in the supplementary material). We assume a scenario where they are released simultaneously from two adjacent traps separated by $d \sim 450$ μm, and fall vertically through their respective interferometers[25, 34]. Micro-diamonds with an embedded **Nitrogen Vacancy (NV)** centre spin is one candidate for the test masses – they can be trapped in diamagnetic traps [35] and cryogenically cooled. Alternatively objects such as Yb microcrystals with a single doped atomic two-level system in optical traps can be cooled in their internal temperature by laser refirgeration. Any charges should be neutralized immediately following their release from their traps by demonstrated means [36]. The core aim is to drop two objects simultaneously – one through each interferometer – so that their states can become entangled through their mutual gravitational interaction while they traverse their respective interferometers. To this end, we adopt, in each interferometer, a modified version of the SG interferometry scheme of Ref.[25] for splitting into two parts and then recombining the wavepacket of each mass in the horizontal direction while they fall vertically through the interferometer. In step 1 of the SG interferometer described schematically by Eq.(4), the test masses are subjected to an inhomogeneous magnetic field gradient in the horizontal direction for a time $\tau_{\rm acc}$ with a spin-flip (by a short microwave $\pi/2$ pulse) exactly midway at time $\tau_{\rm acc}/2$. Thus the initial state of each mass (say, a Gaussian wavepacket just below their respective trap location) is subjected to a spin dependent acceleration and deccelleration in sequence, to reach at time $\tau_{\rm acc}$ a superposition of spatially separated states $|L,\uparrow\rangle_j$ and $|R,\downarrow\rangle_j$ centred at $x_{j,L}$ and $x_{j,R}$ respectively with a spatial separation of

$$\Delta x = |x_{j,L} - x_{j,R}| \sim \frac{1}{2}\frac{g\mu_B \partial_x B}{m_j}\tau_{\rm acc}^2, \qquad (6)$$

where μ_B is the Bohr magneton, $g \sim 2$ the electronic g-factor and $\partial_x B$ the field gradient in the horizontal (x) direction. For a micro-object of mass $m \sim 10^{-14}$ kg, a magnetic field gradient of $\sim 10^6$ T m^{-1} [25] and a time $\tau_{\rm acc} \sim 500$ ms, $\Delta x \sim 250\mu$m. At this stage, step 2 is carried out: A microwave pulse is used to swap the electronic state to the nuclear spin state, so that the masses are not subjected to spin dependent forces any more, and evolve by falling in parallel next to each other for a time τ. If we allow only a time of $\tau \sim 2.5$ s for this step, then the masses continue to fall parallel to each other to a very good approximation: their movement towards each other due to their gravitational acceleration towards each other $Gm/(d-\Delta x)^2 \sim 10^{-16}$ms^{-2} is truly negligible. Under these circumstances, given the different steady separations $|x_{1,\xi} - x_{2,\xi'}|$ (where $\xi, \xi^{'} \in \{L, R\}$) the phases $\Delta\phi_{LR} \sim -0.2$ and $\Delta\phi_{RL} \sim 0.7$ accumulated due to the gravitational interaction over the time $\tau \sim 2.5$s. This phase accumulation alone gives $\mathcal{W} \sim 1.16$ implying a gravitationally mediated spin entanglement **(the strength of the direct spin-spin dipolar interaction is $\sim 10^{-8}$Hz, so that it hardly entangles the spins in the time-scale of the experiment)**. In practice, the witness will give a larger value as phase accumulation and the adjoining entanglement growth happens also during steps 1 and 3 of the SG interferometry. A discussion of how to overcome the challenges of large superpositions necessarily accompanying our scheme, as well the efficacy of the scheme when the scale of superpositions is smaller, is discussed in the supplementary material.

Decoherence:- We require both the orbital and spin degree of freedom of the mases to remain coherent for the whole duration of the experiment. As we map to nuclear spins for step 2 of the interferometry with their very long coherence times, we only require electronic spins coherent for 1s (in steps 1 and 3), which should be possible for micro-diamond below 77 K [37] with dynamical decoupling pulses on its spin bath [38]. To estimate collisional and thermal decoherence times [39–41] of the orbital degree of freedom we consider the pressure $P = 10^{-15}$ Pa and the temperature 0.15 K: the collisional decoherence time for a superposition size of $\Delta x \sim 250\,\mu$m is the same order of magnitude as the total microsphere's fall time $\tau + 2\tau_{\rm acc} \sim 3.5$ s, while the thermal decoherence mechanism, due to scattering, emission and absorption of environmental photons, is negligible. Note that speculated spontaneous collapse mechanisms [4–6], if true, will typically lead to a strong loss of coherence on the time-scale of the experiment and inhibit the gravitationally mediated entanglement. A pivotally important stage preceding the entangling experiment is to take the interferometers far apart from each other to characterize the relative phases between the two paths in each SG interferometer as affected by nearby surfaces, other masses etc. While these are LO and thereby cannot give spurious entanglement between the test masses, the spin operators used in the Witness $\mathcal{W}$ will have to readjusted in accordance to these local phases. Note that although the internal cooling is necessary, the centre of mass motion of the test masses, if originally released from ~ 1 MHz traps, are allowed to have a temperature as high as 100 K as that causes only a factor of $\sim 10^{-2}$ change in the gravitational phase, while the change due to spreading of the wavepacket during the experiment is truly negligible.

Summary:- While gravity is one of the fundamental forces, its weakness has made it difficult to test theories on its nature. In particular, in order to treat gravity in the context of quantum mechanics, it is important to answer the question, is gravity a quantum entity? Lack of a scheme to test this question has been a long-standing issue. In this paper, based on the principle that classical mediators *cannot* entangle [29], we introduce an idea to solve this problem: to observe the entanglement of two test masses to ascertain whether the gravitational field is a quantum entity. As regards to "which" quantum aspect, the discussion in the supplementary material indicates that it should be that the gravitational field obeys the principle of quantum superposition. Instead of using the gravity of one test mass to change the position of another[18–20, 42, 43], which is a tiny effect to measure for a test mass as small as those for which large quantum superpositions are feasible, we consider a change of the phase affected by the gravitational interaction, which we find to be much larger. The test described here is several orders of magnitude stronger than other predictions in the low-energy long-distance sector of quantum gravity such as post-Newtonian corrections [44, 45] and decoherence induced by the gravitational field background [46–48]. Moreover, its prominence stems from a very simple and aesthetically beautiful fact: a Planck's constant in the denominator fighting with the Gravitational constant in the numerator of a relevant phase factor. The prescriptions we have provided for overcoming the challenges will set out a roadmap towards quantum gravity experiments and could have other beneficial spin-offs on the way, such as the measurement of the Newtonian potential for microspheres, given that so far it has only been measured for much larger masses (this will only need one interferometer and a proximal mass) [42]. Thus the idea and scheme presented in this paper arguably opens up the shortest route known to date for establishing/falsifying the quantum nature of gravity through a laboratory experiment.

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