

THE HYDRAULIC CONVEYING OF SOLIDS
IN A ONE-INCH DIAMETER PIPE.

A Thesis submitted
for the degree
of
Doctor of Philosophy
in the
Faculty of Engineering
in the
University of London
by
Clifton Arnold Shook, rmc., B.Sc.(Alberta)

Department of Chemical Engineering,
Imperial College of Science & Technology,
London, S.W.7.

September, 1960.

ACKNOWLEDGMENTS.

The author wishes to thank Professor D.M. Newitt, D.Sc., F.R.S., and Dr. (now Professor) J.F. Richardson, D.Sc., for the advice and supervision given during the course of the investigation.

In addition he wishes to thank Mr. F.L.W. Binnie who constructed the camera and the staff of the laboratory workshop under Mr. A.C.V. Sweeting for assistance in the fabrication of equipment.

Further thanks are due to his wife for typing the manuscript of the thesis.

Finally, the author is gratefully indebted to the National Research Council of Canada and the Management Committee of the Athlone Fellowships Scheme for providing funds for his support.

ABSTRACT.

The Hydraulic Conveying of Solids

In a One-Inch Diameter Pipe

Previous investigations of the hydraulic conveying of solids in pipelines have usually been confined to variables measured externally and a satisfactory dimensionless correlation of friction losses has been produced for materials with specific gravities less than 4.60. Very few measurements, however, have been made on the microscopic scale.

In this investigation, experiments were conducted in closed-circuit plant with a one-inch pipeline. An attempt was made to measure the distribution of discharge solids concentration, the water velocity profile and the mean slip velocity as functions of mean solids concentration and hydraulic gradient. Five materials were chosen for investigation, covering a range of particle diameters from 0.0046 to 0.061 inch, of specific gravity from 1.17 to 4.60 and of settling velocity from 0.0648 to 0.695 ft./sec..

The behaviour of these materials illustrated that **they** could be classified according to the mechanism by which they were supported in the fluid. Three mechanisms were examined theoretically and all three types of behaviour were exhibited.

Small particles with settling velocities lower than the turbulent lifting component appeared to be suspended by fluid turbulence. A unique method for correlation of concentration profiles is presented, permitting the prediction of concentration profiles for these materials under conditions considerably removed from those encountered in this investigation.

Particles likely to be influenced by spin effects were shown to exhibit anomalous concentration profiles.

Coarse particles, supported by a dispersive stress resulting from interparticle contacts, were found to exhibit flow with a moving bed at high concentrations. Empirical correlations are shown for the effect of solids concentration and friction velocity upon mean slip velocity, the water velocity profile and the distribution of discharge solids concentration. In addition, it was possible to estimate the transport concentration distribution, solids velocity profile and local slip velocity for the case of flow with a bed.

CONTENTS.

	Page No.
Chapter I: Introduction	1
Chapter II: Literature Review	
II-1 Introduction	4
II-2 Hydraulic Conveying in Pipelines - Macroscopic Considerations	5
II-3 Hydraulic Conveying in Pipelines - Microscopic Considerations	11
II-4 Conveying in Open Channels	15
II-5 Vertical Hydraulic Conveying	18
Chapter III: Analytical Considerations	
III-1 Introduction	21
III-2 The Mean Solids Velocity	22
III-3 Concentration Profile Measurements	23
III-4 The Concentration Profile - Suspended Particles	26
III-5 Effects Of Particle Spin	30
III-6 Bagnold's Internal Stress Considerations	32
Chapter IV: Topics of Investigation	
IV-1 Introduction	36
IV-2 Concentration Measurements	37
IV-3 Mean Velocities	38

Chapter IV: (Continued)

IV-4	Water Velocity Profiles	38
IV-5	Particle Properties	39

Chapter V: Experimental Apparatus

V-1	General Layout of the Plant	40
V-2	The Pipeline	40
V-3	The Pump	42
V-4	Pressure Measurement	43
V-5	The Pitot Tube	44
V-6	The Flow Divider	47
V-7	Time Measurement - Discharge	50
	Velocities	
V-8	Salt Injection Circuit	51
V-9	Electrodes	51
V-10	Time Measurement - Water	52
	Velocities	

Chapter VI: Experimental Techniques

VI-1	Introduction	54
VI-2	Starting the Plant	54
VI-3	Elimination of Temperature Rise	55
VI-4	Pressure Gradient, Mean Velocity	56
	and Concentration Measurements	
VI-5	Shutting Down the Plant	58
VI-6	Water Velocity Profile	59
	Measurements	

Chapter VI: (Continued)

VI-7	Porosity and Density Determinations	62
VI-8	Settling Velocity Determinations	62
VI-9	Size Distribution Analyses	63

Chapter VII: Experimental Results

VII-1	Introduction	64
VII-2	Coarse Sand	64
VII-3	Gravel	70
VII-4	Perspex	72
VII-5	Fine Sand	75
VII-6	Zircon	77
VII-7	Particle Property Determinations	80

Chapter VIII: Correlations and Derived Data

VIII-1	Introduction	81
VIII-2	Concentration Profiles - Coarse Particles	82
VIII-3	Concentration Profiles - Fine Particles	87
VIII-4	Water Velocity Profiles (Coarse Sand and Gravel)	90
VIII-5	Mean Slip Velocities	93
VIII-6	Derived Data	94

Chapter IX: Conclusions

IX-1	Introduction	99
------	--------------	----

Chapter IX: (Continued)

IX-2	Coarse Particles	101
IX-3	Fine Particles	103
IX-4	Large Particles of Low Density	105
IX-5	Further Work	107
IX-6	Summary	109

SYMBOLS.

a	depth of channel
b	width of channel
C	local discharge concentration of solids
C_y	local discharge concentration of solids at distance y from pipe bottom
C^*	predicted local discharge concentration of solids at distance y from pipe bottom
$\bar{C}$	mean discharge concentration of solids
C_d	drag coefficient
D	pipe diameter
d	particle diameter
e	base of Napierian logarithms
f	Fanning friction factor
g	gravitational acceleration constant, ft/sec ²
h	pitot tube pressure - ft. water
i	subscript for impact pressure; hydraulic gradient in ft. water per ft. pipe
K	von Karman Universal Constant
l	Prandtl mixing length
l'	length term appropriate for momentum transfer process, analogous to mean free path of the Kinetic Theory of Gases
m	subscript referring to discharged mixture or suspension
N	speed of rotation in revolutions per second
P	dispersive stress
$\frac{1}{1-C}$	
$\frac{C}{1-C} e$	

q	local transport concentration of solids
q'	assumed local transport concentration of solids
$\bar{q}$	mean transport concentration of solids
r	pipe radius
S	specific gravity of solids; subscript for suction pressure; subscript for solids
T	tangential stress resisting motion of a bed of solids
V'_w	local water velocity
V'_m	maximum water velocity within the pipe
V_w	assumed mean water velocity along a chord
$\bar{V}$	mean velocity
v	fraction of total flow
v_B	fraction of total flow discharged below a given plane
W	hindered settling velocity of particles
W_o	settling velocity of an individual particle
w	subscript referring to water
y	distance from pipe bottom
y'_m	distance of point of maximum water velocity from pipe bottom
y_o	distance from pipe bottom to plane of zero shear
z	distance from pipe boundary
α_c	coefficient of internal friction for flow within a bed
α	coefficient of internal friction for a moving bed
β	ratio ϵ_s / ϵ_m
ϵ_m	coefficient of momentum transfer for fluid (eddy kinematic viscosity)

ρ	density
τ_0	shear stress at walls
τ_y	shear stress at plane y
η	kinematic viscosity of fluid

CHAPTER IIntroduction

The hydraulic conveying of solids has been employed industrially for nearly fifty years and examples of its use may be found in every industrialised country.

It has long been realized that the introduction of solid particles into a stream of fluid requires extra energy to be imparted to the fluid; this is revealed in the form of an increased pressure drop. A considerable amount of experimental effort has been devoted to establishing a relationship between this excess pressure drop and the external variables of the system: delivered solids concentration, the discharge velocity of the mixture of particles and fluid, pipe diameter and the properties of the particles. This has been done satisfactorily for the conveying of sands and gravels and the dimensionless correlation produced has been found applicable to particles with specific gravities less than 4.60. The various regimes of flow exhibited in hydraulic conveying have been considered to be governed by the diameter of the particles. This may be due to the fact that most of the experiments have been conducted over a rather narrow range of experimental conditions; only the particle diameter has been varied widely. Indeed, this criticism gains force when it is considered that experiments conducted in large

pipelines were carried out at low hydraulic gradients so that the friction velocity corresponded with that applying in a smaller pipe at higher hydraulic gradients.

Theoretical investigation of the problem from the external or macroscopic point of view has been relatively unsuccessful and it has been impossible to explain fully any of the equations put forward. Since the inertia of the solid particles is greater than that of the fluid, it is apparent that some difference between the velocities of the fluid and the solids will exist. If this is the case, the solids concentration in the discharged mixture will be less than the transport concentration inside the pipe. Studies of the sedimentation of particles have shown that the slip velocity between fluid and solids governs the momentum transfer process. It is therefore apparent that the slip velocity will be related to the excess pressure drop in those circumstances where hydraulic conveying is essentially a momentum transfer process. It has been found that small particles appear to remain suspended in the fluid and it is probable that a relationship between excess pressure gradient and slip velocity can be fairly readily obtained. It must be pointed out, however, that measurement of these slip velocities is extremely difficult for small suspended particles. Coarse particles, of greater inertia, have been treated theoretically and experimentally for two-dimensional flow in an open channel.

and it has been shown (46) that the conveying of these materials is not simply a momentum exchange process.

Because of their greater inertia, interparticle contacts are of importance and the problem is essentially one of the flow of granular solids. Motion is caused by a stress on the particles which is derived from the forward momentum of the fluid.

In the present investigation, which has been focussed upon the microscopic scale, significant differences between the regimes of flow are shown to exist. It has been possible to classify these forms of behaviour according to the mechanism by which the particles are supported in the fluid.

It will be shown that the five materials studied in this investigation exhibited three distinct forms of behaviour. These are considered analytically in Chapter III, Experimental results obtained at various pressure drops and mean solids concentrations are presented in Chapter VII. Although some of the techniques met with limited success or failure, it was still possible to make significant deductions about the behaviour of the solids on the microscopic scale.

CHAPTER II.

Literature Review.

1. Introduction.

The hydraulic conveying of solids in a pipe, like pneumatic conveying and open-channel transport, is a special case of the general problem of momentum transfer from a fluid to immersed solid particles. Because the density of air is so much less than that of water and because so little is known about the momentum exchange processes on the microscopic scale, it has not been possible to reconcile many macroscopic observations made for hydraulic and pneumatic conveying.

As an industrial operation, it is not surprising that hydraulic conveying first emerged in placer mining operations. Nardi (1) states that in the late 1850's, water under pressure was used to raise gold-bearing gravel distances of 30 to 55 feet for discharge to sluices. Horizontal hydraulic conveying was first employed industrially in London in 1914 when a pipeline 1750 feet in length and 8 inches in diameter was used to transport coal from barges moored in the Thames. Since this time the technique has found wide application in Canada, France, Russia and the United States for the transport of coal, sand, ores, mine and mill tailings and washery sludges.

Because special equipment was often not necessary, scientific investigation of the problem has lagged behind

industrial practice and today the engineer frequently finds himself knowing little about the fundamentals of a particular process.

It has been realized, however, that it is the diameter of the particles which is of primary importance in determining behaviour and each of the design formulae which have been produced covers only a limited size range. Suspensions containing fine particles less than 30 μ in diameter exhibit an increased resistance to laminar shear and the particles must be considered intimately enough associated with the suspending fluid for an internal structure to be established. Coarser particles exhibit differing regimes of behaviour as the conditions of flow are varied, but the criterion governing transition from one regime to another is usually still considered to be the diameter of the particles. It is these coarser particles with which the present series of experiments is concerned.

2. Hydraulic Conveying in Pipelines - Macroscopic Considerations.

External measurements made on the hydraulic conveying of solids have been concerned with:

- (i) the pressure gradient along the pipe and the difference between this quantity and that required to transport clear water at the same velocity;

- (ii) the critical velocity at which a bed of particles is formed.

The excess pressure gradient provides an indication of the amount of extra energy which must be imparted to the fluid to transport the solids while the critical velocity permits optimum operating conditions to be selected. It will be shown that a coherent picture of the relationship between pressure gradient and system variables has been established, although this has not been satisfactorily related to the internal mechanism of the process.

In a series of experiments on one-inch brass and iron pipes, Blatch (2) measured the discharge concentration and velocity and the excess head loss for the conveying of (-20 + 40) mesh and (-60 + 100) mesh sands. At any given velocity the excess head loss was proportional to discharge concentration. At very low velocities the excess head loss was independent of velocity until the critical velocity was passed; thereafter it decreased with velocity. At very high velocities the total head loss, expressed in terms of the mixture, was the same as for clear water flowing at the same velocity. The breadth of the transition region separating this last phenomenon from the critical velocity was observed to increase with the size range of the particles.

Hazen and Hardy (3) performed a similar series of experiments on sand particles less than 0.5 mm. in diameter in 3 and 4 inch pipes. The excess head loss was again found

to be a direct function of concentration and the general observations of flow regime made by Blatch were substantiated. Gilbert (4) concluded that particle saltation was the dominant mode of transport and reported an increase in the capacity of the pipeline when particles of two different size ranges were conveyed. This he attributed to a change in the roughness of the surface of the bed. Durepaire (5) stated that for total suspension the hydraulic gradient was the same as for water flowing at the same velocity. For the case of flow with a deposit the head losses were said to be a function of concentration and particle size. Howard (6) conducted a systematic investigation of the effect of mean concentration ($\bar{C}$) upon excess hydraulic gradient (i_s) and suggested the relationship:

$$i = i_w + i_s = i_w + A\bar{C} \quad (1)$$

where A was a function of pipe diameter and the sand characteristics. In addition, a qualitative description of the water velocity profile was given in which the maximum velocity was found at a point above the centre of the pipe. In the discussion of the paper Gadfelter and Howard produced a plot showing the variation of solids concentration over the pipe. The vertical diameter of the pipe was seen to be an axis of symmetry and some variation over a horizontal chord was evident.

Wilson (7) derived an equation for excess pressure

gradient by assuming that the work required to maintain suspension could be added to that required to produce the same water velocity:

$$i = i_w + i_s = \frac{f \bar{V}^2}{2 g D} + \frac{K \bar{C} W_c}{\bar{V}} \quad (2)$$

In the discussion of Wilson's paper, Einstein objected to the addition of energy terms in view of the possible influence of particles upon the state of turbulence.

Turtle (8) measured the discharge velocity and mean concentration for a considerable range of particle sizes and densities, and, with Worster's assistance, succeeded in producing a correlation based upon theoretical and empirical conclusions from open channel flow. This expression was observed to be least suitable for large particles.

It was not until Durand's comprehensive series of experiments (9, 10) that a macroscopic equation adequate for the design of industrial pipelines was produced. His experiments covered a wide range of the major system variables except particle density and may be presented in the form:

$$\frac{i - i_w}{C i_w} = 96 \frac{g D}{\bar{V}^2} \frac{(s - 1)^{1.5}}{C_d} \quad (3)$$

where the factor $(s - 1)$ was introduced by Worster (11) from Turtle's results to take account of particle density. This

equation has been found to apply to a wide range of concentrations, velocities and particle diameters. It has not been found applicable to particles of specific gravity greater than 4.60.

Abbott (12, 13) continued Turtle's series of experiments with uniform size sands and produced an alternative to Durand's equation. He adopted Wilson's approach, assumed that the solids velocity was a constant fraction of the mixture velocity, and obtained an expression which differed from Equation (II - 3) chiefly in that the pipe diameter was raised to the power 1 instead of 1.5. His assumption about the behaviour of the solids velocity received some support from pneumatic conveying observations. Chin Yung Wen and Simons (14) reported that the mean solids velocity was a constant fraction of the mean discharge velocity, while Mehta et al. (15) observed the same effect for the conveying of 36μ glass spheres. Larger (97μ) spheres, however, were shown to reach a maximum velocity.

By considering the effect of suspensions of small particles in reducing the settling velocity of larger particles, he showed how his observed increase of pipeline capacity could be explained.

The qualitative terms 'small' and 'large' will be given quantitative limits in the next section.

Craven (16) attempted a form of dimensional analysis with rather limited success but did succeed in establishing

a criterion for the formation of a bed:

$$\frac{\bar{V}}{\sqrt{d \frac{\gamma_o}{\rho_w}}} < 3.2 \quad (4)$$

where $\bar{V}$ is the mean discharge velocity.

Smith (17) suggested an equivalent diameter, d_e , for use in equation (II - 3) in the conveying of mixtures:

$$d_e = \frac{\sum W_o}{\sum \frac{W}{d}} \quad (5)$$

Spells (18) derived expressions for the critical and standard velocities from dimensional analysis and evaluated the constants from existing data. The standard velocity is defined as that at which the pressure gradient is the same as for clear water.

Sinclair (19) studied the transport of heavy particles, including steel and copper, through pipes of 1" diameter or less. His investigations were confined to the conditions where a bed was found. He showed that many of Durand's and Abbott's predictions were not substantiated in a system where the inertia of the fluid was much less than that of the particles. Unfortunately, the microscopic measurements, which might have explained the difference, were not attempted.

Additional data and reviews of hydraulic conveying may

also be found in the literature (20, 21, 22).

3. Hydraulic Conveying in Pipelines - Microscopic Considerations.

Early workers in the field of hydraulic conveying augmented their external measurements with visual observations of the flow pattern. Because the introduction of measuring instruments into the flow is fairly simple with open channels and because their usual rectangular shape permits the assumption of two-dimensional flow, most workers who have attempted microscopic measurements have done so on such a system. More will be said about these measurements in a later section.

By shearing suspensions in an annulus, Bagnold (23) established the existence of two distinct regimes of shear which he defined as inertial and viscous. In each of these he showed that the dispersive stress, to which the applied stress was proportional, was a function of concentration, particle diameter and solids velocity gradient. In the viscous shear region the fluid viscosity was observed to be of importance. He managed to show that the disappearance of particle saltation which has been observed to occur between $0.20 < d < 2.0$ mm. corresponds with a change of the shearing process from the inertial to the viscous regime. He reinforced his conclusion that saltation is a phenomenon associated with the inertial shearing process by using Jeffrey's (24) description of the development of drag upon a particle.

Durand and Worster have classified particles for the purposes of hydraulic conveying according to their diameters and the regimes of flow they exhibit as follows:-

1. Particles forming Homogeneous Mixtures

$$d \leq 30 \mu$$

2. Intermediates

$$30 \mu \leq d \leq 50 \mu$$

3. Particles forming Hetrogeneous Mixtures

(a) Suspended Particles

$$50 \mu < d < 0.20 \text{ mm.}$$

(b) Transition Category

$$0.20 \text{ mm.} < d < 2.0 \text{ mm.}$$

(c) Saltation Region

$$d > 2.0 \text{ mm.}$$

This classification has been drawn up on the basis of measurements with sands and gravels. For particles of different density the criteria will be different, because a particle travels in suspension if its settling velocity is less than the turbulent velocity fluctuation. Further, since the turbulent velocity fluctuation will vary with the Reynolds number and friction velocity, and hence with the diameter of the pipe, the applicability of these same criteria to pipes of widely different diameter must be doubtful.

Because the inertia of the small particles is low their velocities in the pipe tend to be high and visual observations are impossible. The behaviour of larger particles travelling in saltation was described in detail by Durand (25). Smith (17) and Novak (26) noticed that these particles tended to collect into isolated clouds and disperse as the fluid velocity was increased. No explanation for the phenomenon has been suggested.

Durand (27) has published the results of his determinations of local concentration and fluid velocity and has quoted isolated values of the local slip velocity. Unfortunately his descriptions of the experimental techniques were vague and even Hallbronn's report (28) on the concentration measuring instrument leaves much to be desired.

Worster (29, 30) determined transport concentration by isolating a section of the pipe. Like Bagnold, he recognised the importance of the shear stress on the upper surface of a bed of particles and he attempted to deduce its value. He then defined a coefficient of sliding friction between solids and pipe using the shear stress and the immersed weight of the particles. In addition he gave qualitatively the fluid velocity profile within a pipe, investigated the economics of hydraulic conveying, and showed how the present experimental knowledge influences the design of industrial equipment.

It is unfortunate that more of the work on hydraulic

conveying has not been conducted in a closed rectangular channel, for which, as will be shown in the next chapter, the applied shear stress may be more easily estimated in terms of the distance from the bottom of the conduit. However, Ismail (31) measured the concentration and velocity profiles for a fine sand in such a conduit. He reported a fluid velocity gradient increased by the presence of the solids and a concentration profile of the form:

$$\log \frac{C}{C_a} = \frac{W_o}{E_s} (y - a) \quad (6)$$

The behaviour of particles smaller than the microscale of turbulence has been described theoretically by Friedlander (32) but a great deal of experimental work must be done before his predictions can be tested and applied to hydraulic conveying. In their work on sedimentation, Dell and Whelan (33) suggested that a highly angular particle may have enough fluid permanently associated with it to form a pocket of intermediate density. It is quite possible that a similar phenomenon may be observed to apply to hydraulic conveying. Under such conditions both the quantities d and $(s - 1)$ could vary with particle shape.

In conclusion, the following major facts may be said to have emerged from the literature survey on horizontal hydraulic conveying in pipelines.

1. A simple, dimensionally correct, empirical correlation of friction losses against system variables has been produced for the sand - water system.
2. Some doubt exists as to the applicability of this equation where the specific gravity of the material is greater than 4.60, or where the size of the particles is not uniform.
3. Only a few internal measurements have been made and even these are subject to some doubt.
4. An explanation has been produced for the dispersion of particles too large to be supported by turbulence, but this has not been related to conveying in pipelines.

The key to an explanation of phenomena observed in hydraulic conveying will undoubtedly be found in microscopic measurements of transport concentration, fluid velocity and solids velocity. At the present time it is these measurements which deserve most attention.

4. Conveying in an Open Channel.

The hydraulic transport of solids in an open channel has received experimental attention primarily as a consequence of its importance in river flow and irrigation projects. The first contribution to the general problem of the

transport of solids in fluids was the definition of the terms bed load and suspended load. These were considered to be the portions of the total solids flow supported by the dispersive stresses of the bed and turbulent fluid shear stresses respectively.

For the case when the particles are supported by fluid turbulence, Lane and Kalinske (34) attempted, with limited success, to derive an expression for the transported concentration of solids by considering the turbulent lifting component which they assumed to be proportional to the friction velocity. Vanoni (35, 36) measured the water velocity distribution for the transport of fine sand at low ($\leq 1.5\%$) concentrations, and like Ismail interpreted the increased velocity gradient near the bottom of the pipe in terms of a 'damped' coefficient of momentum transfer ϵ_m . Einstein (37) suggested that for suspension the diameter of the particle must be considerably less than the mixing length in the fluid and, since near the walls of the channel the mixing length approaches zero, the mechanism by which particles were maintained against their own weight was, in that region, not turbulent suspension but 'hydrodynamic lift'. This hypothesis has not received support from subsequent workers in the field. Richardson (38, 39) sampled the discharge at the mouth of the river Tyne and obtained verification of his theoretical predictions. Like most attempts at explanation of the concentration profile, this was based

upon the approach due to Schmidt (40), Rouse (41), and Prandtl (42). Hunt (43) noticed that a small difference existed between the velocities of the fluid and the particles and produced a modification of the Schmidt-Rouse equation referred to in Chapter III, without assuming that the eddy viscosity with solids present was the same as for clear water. Morris (44) showed that if the local solids velocity is a constant fraction of the fluid velocity, the equation is unaffected by slip.

It was Einstein who first introduced the concept of bed-load, but the definition given earlier in this section was put forward by Bagnold (45, 46) who verified his analytical predictions by conveying very light (S.G. - 1.004) particles in an open flume. It was shown that, even with particles so near the density of water, only slightly more than one half were supported by fluid turbulence. In addition, he established that where the local solids concentration was high the turbulence of the fluid was damped considerably. Further, he showed that under such conditions the dispersion of the particles was produced by a dispersive stress resulting from interparticle contacts. For this mechanism he obtained good agreement between measurements made on:

- (a) Open channels - sand conveying
- (b) Open channels - light particle conveying

- (c) Air blown sands
- (d) Suspensions sheared in an annulus.

It is most unfortunate that the present state of knowledge does not permit such a treatment for conveying in pipelines.

5. Vertical Hydraulic Conveying.

The conveying of particles against their own weight is essentially a much simpler process than horizontal hydraulic conveying because the distribution of solids is symmetrical about the axis of the pipe. In addition, the particle weight is balanced by a colinear drag force. Nevertheless, the problem has received rather less attention than horizontal conveying, probably as a result of its more limited industrial use.

Durand (47) and Worster (48) conducted experiments on the vertical conveying of solids and concluded that the head loss due to friction, expressed in terms of the mixture, was the same as for clear water flowing at the same velocity. Gliddon (49) showed by experiments on 1 and 2 inch pipes that Durand's empirical equation, for excess pressure gradient for flow in a horizontal pipe in the absence of a bed, applied to vertical conveying. Such a conclusion is hardly surprising because, as the distribution of particles over the cross-section becomes more uniform, the essential difference

between the flow patterns diminishes.

Durand presented measurements of solids concentration and water velocity gradients within the pipe. The water velocity did not appear to be altered much from that for clear water and the solids distribution was uniform over the pipe cross-section. Gliddon obtained evidence to the contrary in both cases, showing that the particles tended to concentrate along the pipe axis and that a corresponding reduction of the local fluid velocity was produced in this region.

Probably the most ambitious attempt at a description of the flow process has been made by Maude and Whitmore (50) who derived, from considerations of drag and the velocity fluctuations of turbulent flow, expressions for mixing length, concentration profile and friction factor for fine ($20 - 40 \mu$) particles in small (2 and 5 mm.) tubes. Their prediction of a region of high local concentration near the walls of the conduit, together with Gliddon's observation of the reverse for the case of larger particles appears to bear out Bagnold's hypothesis that for the latter the influence of inertia and wall collisions is more important than that of the turbulent velocity fluctuations. Particles larger than 30μ in diameter, but fine enough to settle in the Stokes' Law region, were studied by Bhattacharya and Roy (51) who used a modification of the Vogt and White (52) method of correlating pneumatic conveying data to derive

an equation for excess pressure drop from considerations of dimensional analysis.

In conclusion it may be seen that although theoretical treatment is least difficult for vertical conveying, the problem has not received a great deal of experimental attention.

Significant theoretical and experimental progress has however been made for the case of open channel conveying. Horizontal conveying in a pipeline represents the most difficult problem theoretically, but the industrial importance of the process has resulted in considerable investigation. Unfortunately, very little is known about the flow mechanism on the microscopic scale.

CHAPTER III.Analytical Considerations.1. Introduction.

It has been shown in the Literature Review that equations based on macroscopic considerations of hydraulic conveying have been derived either from dimensional analyses or using rather questionable hypotheses, and it is doubtful whether further contributions to the science will be made from this approach. At the present time, the difficulties associated with the alternative microscopic analysis are intractable because so little is known about the variables concerned. What is possible, however, is an illustration of how relatively simple experimental techniques permit some of the relevant variables, notably mean solids velocity and solids concentration profile, to be determined. Existing experimental data for the lift force experienced by a sphere rotating in a stream of fluid permit an assessment of the relative importance of particle spin for particles of different size and density. A critical examination of the Schmidt-Rouse equation for the solids concentration profile in a circular pipe may also be made.

In addition, Bagnold's relationships based on the mechanisms of the system have been found to apply with such success to open-channel conveying that their application to flow in circular pipes is worthy of consideration.

Consideration will now be given to the factors involved in determining the way in which the particles are distributed over the cross-section, the mechanism by which they are dispersed or suspended, and the relative velocity between the liquid and the solids. Subsequently, possible methods of measuring some of these quantities and the development of suitable techniques will be discussed.

2. The Mean Solids Velocity.

The interaction between the liquid and the solids at any position within the pipe will be a function of the relative velocity. It is therefore important to consider how this quantity can be determined in terms of variables which can easily be measured.

Mass balances on fluid and particles give:

$$(1-\bar{C}) \bar{V}_m = (1-\bar{q}) \bar{V}_w \quad (1)$$

$$\bar{C} \bar{V}_m = \bar{q} \bar{V}_s \quad (2)$$

Eliminating $\bar{q}$ which cannot be measured nearly as readily as $\bar{C}$, and solving for $\bar{V}_s$

$$\bar{V}_s = \frac{\bar{C} \bar{V}_m \bar{V}_w}{\bar{V}_w - (1-\bar{C}) \bar{V}_m} \quad (3)$$

and

$$\bar{q} = \frac{\bar{V}_w - (1-\bar{C}) \bar{V}_m}{\bar{V}_w} \quad (4)$$

The mean slip velocity defined as $\bar{V}_w - \bar{V}_s$ is given by

$$\bar{V}_w - \bar{V}_s = \bar{V}_w \left[1 - \frac{\bar{C} \bar{V}_m}{\bar{V}_w - (1-\bar{C}) \bar{V}_m} \right] \quad (5)$$

$$= \frac{\bar{V}_w (\bar{V}_w - \bar{V}_s)}{\bar{V}_w - (1-\bar{C}) \bar{V}_m} \quad (6)$$

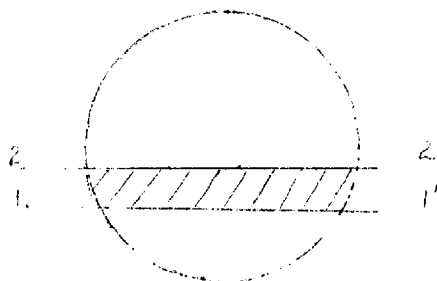
Since $\bar{C}$ and $\bar{V}_m$ are given by discharge measurements, a means of evaluating $\bar{V}_w$ permits $\bar{V}_s$, $\bar{q}$ and $\bar{V}_w - \bar{V}_s$ to be determined. It must be pointed out that the mean slip velocity defined as $\bar{V}_w - \bar{V}_s$ is not the mean value $\overline{V_w - V_s}$ of the local slip over the cross-section. In fact, as Durand has pointed out, even if the local slip is negligible a mean slip $\bar{V}_w - \bar{V}_s$ will exist if the particles concentrate in a region where V_w is less than $\bar{V}_w$.

The selection of a suitable method of measuring the water velocity will be discussed in Chapter IV.

3. Concentration Profile Measurements.

The concentration of particles will generally decrease from the bottom to the top of the pipe cross-section. In

order to obtain a measurement of the concentration profile it is necessary to devise a method of sampling which will provide data which can be used for this purpose. Discharge concentrations are more readily measured than transport concentrations, and attention will be confined to measurements of the former.



Consider the circular cross-section of the pipe cut by the horizontal chords 1 - 1' and 2 - 2'. The concentration of particles in the shaded area is given by either of the two expressions:

$$C_{1-2} = \frac{C_{B2} v_{B2} - C_{B1} v_{B1}}{v_{B2} - v_{B1}} \quad (7)$$

$$C_{1-2} = \frac{C_{A1} v_{A1} - C_{A2} v_{A2}}{v_{A1} - v_{A2}} \quad (8)$$

where

C_{1-2} Concentration in the area bounded by planes 1-1' and 2-2'

C_{B2} Concentration in the area below plane 2-2'

C_{B1} Concentration in the area below plane 1-1'

C_{A2}	Concentration in the area above plane 2-2'
C_{A1}	Concentration in the area above plane 1-1'
v_{B1}	Fraction of total flow discharged below 1-1'
v_{A1}	Fraction of total flow discharged above 1-1'
v_{B2}	Fraction of total flow discharged below 2-2'
v_{A2}	Fraction of total flow discharged above 2-2'

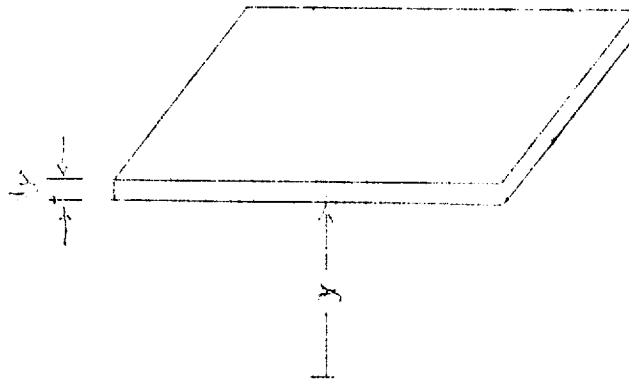
If the solids concentration is assumed constant along a series of horizontal chords as observed by Howard for small particles, there must be a point on the vertical axis of the pipe where the local concentration is the same as that given by Equations III (7) or III (8). The location of this point is governed by the position of the shaded area in a circular pipe. It would only be at the centroid of the area concerned if the concentration and discharge velocity profiles did not change appreciably with distance above the bottom of the pipe; under these circumstances it would not matter much where the point was located. At the top of the pipe the velocity gradient would tend to put the point lower than the centroid, but in this region the concentration profile does not change very much. At the bottom of the pipe the normal rapid rate of change of concentration with height would tend to lower the point while the velocity gradient reduced the effect somewhat. Provided the interval were small enough, one would be justified in assuming that the appropriate point lay at the centre of the area concerned.

By determining the quantities involved in equations III-(7) and III-(8) at various positions of the divider, it was possible to determine the concentration profile.

4. The Concentration Profile - Suspended Particles.

In order to explain the shape of the concentration profile, consideration must be given to the mechanisms by which particles are supported. Two cases will be considered, first, when the particles are suspended by turbulent velocity fluctuations, and secondly when their immersed weight is borne by stresses associated with a flowing bed.

For the case where suspension results from fluid turbulence, consider a horizontal element of pipe volume of unit cross-sectional area, thickness dy , mean particle concentration q at a distance y from the bottom of the pipe



Net rate of particle sedimentation from the element:

$$= 1 \times 1 \times W \times q \times dy \quad \frac{ft^4}{sec.} \quad (9)$$

Net rate of lift into the element

$$= 1 \times 1 \times \ell' \times v' \times dq \quad \frac{ft^4}{sec.} \quad (10)$$

Consequently, in the steady state

$$\ell' v' dq + W q dy = 0 \quad (11)$$

This equation can be applied only if the quantities ℓ' and v' can be estimated.

The Schmidt-Rouse analysis continues at this point by replacing the product $\ell' v'$ with an eddy diffusivity

$$\ell' v' = \epsilon_s = \beta \epsilon_m \quad (12)$$

ϵ_m has been defined (53) by the expression

$$\epsilon_m = \frac{\tau_y}{\rho_w} \times \frac{1}{\frac{du}{dy}} \quad (13)$$

At this point, Prandtl assumed that τ_y in the expression

$$\frac{du}{dy} = \frac{1}{\ell} \sqrt{\frac{\tau_y}{\rho_w}} \quad (14)$$

could be replaced by the boundary value τ_o . On the other hand, τ_y in (13) is determined from τ_o by the expression

$$\tau_y = \tau_o \frac{(y_o - y)}{y_o} \quad (15)$$

The inconsistency is obvious and of course unjustifiable.

By substitution, however,

$$l'v' = \beta l \sqrt{\frac{\gamma_o}{\rho_w}} \left(\frac{y_o - y}{y_o} \right) \quad (16)$$

Assuming, after Prandtl,

$$l = K y \quad (17)$$

Thus

$$l'v' = \beta K \sqrt{\frac{\gamma_o}{\rho_w}} \quad (18)$$

(11) then becomes

$$\frac{dq}{q} = \frac{-W y_o}{\beta K \sqrt{\frac{\gamma_o}{\rho_w}}} \frac{dy}{y (y - y_o)} \quad (19)$$

When the integrated form of (19) is applied to the case of hydraulic conveying in an open channel, the distance y_o is the depth of the conduit. Durand and Abbott in the interpretation of Durand's data substituted the pipe diameter for y_o . This is of course impossible because equation (III - 15) has been used in the derivation. In addition (17) is open to question in its application to regions outside the laminar sublayer. Furthermore, it is doubtful

whether the product $\mathcal{L}'v'$ should be replaced by an expression found adequate for the mechanism of momentum transfer between successive layers of fluid.

It is therefore desirable to deduce a relationship which is not dependent upon the mixing length theory. Abbott considered the effect of solids concentration upon the settling velocity to be given by the equation

$$W = W_o (1 - q)^{2.4} \quad (20)$$

Expressing the power to the nearest integral number to permit integration leaves the approximation

$$W \approx W_o (1 - q)^2 \quad (21)$$

From Laufer's data (54) in which the RMS. values of the velocity fluctuations for the flow of air in a circular pipe were expressed in terms of the friction velocity, it was found that the vertical component of the velocity fluctuations was roughly constant at a value of unity outside the laminar sublayer. Certainly the replacement of v' by $\sqrt{\frac{\tau_o}{\rho_w}}$ is less open to question than the assumptions made in the Schmidt-Rouse analysis.

There remains the problem of the length term $\mathcal{L}'$. Certainly the diameter of the particle must exert some influence upon the mechanism of momentum transfer vis-a-vis

the scale of the turbulence. In the absence of a closely deduced analysis one must be content with the observation that if the distance from the bottom of the pipe is expressed in terms of the particle diameter, equation (11) becomes:

$$\frac{dq}{q(1-q)^2} = \frac{-W_o}{d \sqrt{\frac{\tau_o}{\rho_w}}} dy \quad (22)$$

Integrating, and substituting the boundary condition that when $y = 0$

$$q = q_o \quad (23)$$

$$\frac{q_y}{1-q_y} e^{\frac{1}{1-q_y}} = \frac{q_o}{1-q_o} e^{\frac{1}{1-q_o}} \times e^{-\frac{W_o y}{d \sqrt{\frac{\tau_o}{\rho_w}}}} \quad (24)$$

It will be shown in a later section that although (24) does not apply, the dimensionless quantity $\frac{W_o y}{d \sqrt{\frac{\tau_o}{\rho_w}}}$ forms a

useful dependent variable for the presentation of concentration profile measurements.

5. Effects of Particle Spin.

In the preceding section, the maintenance of particles

in suspension by the turbulent eddies in the fluid has been considered. An additional mechanism may be superimposed when the particle is spinning.

For the case of a sphere rotating in a stream of fluid, Goldstein (55) cites results obtained at particle Reynolds numbers between 60,000 and 100,000 in which the coefficient of lift was found to be small and negative until the peripheral speed exceeded one-half the fluid velocity. Much the same effect was observed for the case of a rotating cylinder. Although the particles conveyed in this investigation were certainly not spheres and the Reynolds numbers were undoubtedly considerably lower than 60,000, the possibility of such a criterion applying is worth consideration.

If, for any lift to result

$$\text{Peripheral Velocity} > 0.5 (V_w - V_s) \quad (25)$$

must apply, then the necessary speed of rotation N in revolutions per second is given by the figures quoted in Table (III - 1)

TABLE III - 1

MATERIAL	$N_{r.p.s.}$
Perspex	26.1 $(V_w - V_s)$
Gravel	31.3 $(V_w - V_s)$
Coarse Sand	70.0 $(V_w - V_s)$
Fine Sand	267.3 $(V_w - V_s)$
Zircon	415.0 $(V_w - V_s)$

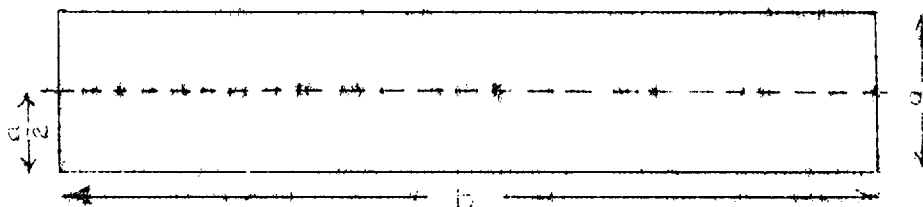
It may be seen that it is the large, light particles of perspex, for which $(V_w - V_s)$ is probably low, which are likely to be most influenced by spin. In addition, when the coefficient of lift is finite and positive these particles would be most easily supported because their immersed weight is small.

6. Bagnold's Internal Stress Considerations.

When the particles are not suspended in the fluid, but are present in such concentrations that a dilated bed is formed, the mechanism by which they are supported is considerably different. Under these circumstances the eddy velocity of the fluid may be considerably less than the settling velocity of the particles. This problem has been considered by Bagnold.

(a) The Applied Stress.

When a particle in a bed is subjected to the action of a fluid stream, there will be an applied stress which results from the external impelling forces. For the broad flat, enclosed horizontal duct shown below, the hydraulic gradient, i , expressed in terms of the conveying fluid, can be related to the stress at the walls.



$$- a b \rho_w g i = (2a + 2b) \tau_o \quad (26)$$

$$\tau_o = \frac{-a}{2a + \frac{2a}{b}} \rho_w i g \quad (27)$$

$$\approx \frac{-a}{2} \rho_w i g \quad (28)$$

At any distance z from one of the longer boundaries

$$\tau_z \approx - \rho_w \left(\frac{a}{2} - z \right) i \quad (29)$$

It is obvious from the form of (29) that for a circular pipe it would not be permissible to assume that τ_z was constant along any horizontal plane.

(b) The Resisting Stresses.

In a series of experiments in which suspensions of particles were sheared in the absence of gravitational effects, Bagnold established that the dispersive stress P was proportional to the tangential stress T :

$$T = P \tan \alpha \quad (30)$$

Since the resistance to shear within a bed of particles results from the presence of the solids and since in the steady state the stress P must be in equilibrium with the

Meander Features in Connection with Ripple Formation.

It was observed that if the channel was set at a particular slope, for certain ratio of the transported solids to water discharge, a well marked diagonal pattern or skew shoals would form up supperimposed over the normal ripple pattern. Plates

I I-I

immersed weight of the particles bearing upon a plane within the bed, equation (III - 30) can be used to give the resisting stress along that plane.

It has been observed that the regions of a conduit in which solids are present may be divided into two: one region generally called the bed, in which the solids concentration does not change much with depth; and the second in which the rate of change of q with y is rapid.

If the boundary between these two regions is located at y_t , at any distance y ($y < y_t$) from the bottom of the pipe, the load of particles per unit length of Conduit, where q_- is the solids concentration in the bed, is given by the expression:

$$(\rho_s - \rho_w) b g \left[\int_{y_t}^{\infty} q \, dy + q_- (y_t - y) \right] \quad (31)$$

Thus the resisting stress equals,

$$(\rho_s - \rho_w) b g \left[\int_{y_t}^{\infty} q \, dy + q_- (y_t - y) \right] \tan \alpha_- \quad (32)$$

This relationship for the resisting stress will only apply within the bed when the turbulent action of the fluid is damped considerably. Above the bed a term to account for the resistance of the fluid to shear must be added.

It is unfortunate that the limitations of the present state of knowledge on the microscopic scale do not permit theoretical justification of the observation that

$$i_s = i - i_w = 0.8 C (s - 1) \quad (33)$$

for the case of flow with a bed.

From Zaki's (56) measurements of the effect of particle concentration on the drag coefficient Bagnold was able to derive expressions for the solids transport rate, velocity profile and the local slip velocity, and it is most unfortunate that this cannot be done for the case of conveying in a circular pipe.

CHAPTER IV.Topics of Investigation.1. Introduction.

Analyses of hydraulic conveying in a pipeline are usually based upon the fairly easily obtained external variables: pressure gradient, pipe diameter, particle properties, mixture discharge velocity, and mean concentration. Macroscopic equations involving these variables are adequate for design purposes, permitting the various regimes of flow behaviour to be described.

In an examination of hydraulic conveying from the microscopic point of view the additional variables of interest are: the solids distribution within the pipe and the velocities of the fluid and the conveyed particles. The experiments were confined to one pipeline, one conveying fluid and uniform size natural particles, and thus the effects of a number of other important factors, including pipe diameter, fluid properties, particle size distribution and particle shape, were not investigated.

Measurements of the solids concentration and fluid velocity profiles in the pipe permitted the observations of other investigators working with open channels to be tested. The solids velocity was determined by the method outlined in Section III - 2, and the results were used to

test the assumptions made about the microscopic behaviour of the particles in the Schmidt-Rouse and Wilson-Abbott analyses. In addition it was possible to make an estimate of the variation of the local solids velocity in those regions where the concentration approached its limiting maximum value.

2. Concentration Measurements.

No systematic attempt has previously been made to investigate the manner in which the distribution of particles varies over the cross-section for a range of experimental conditions, and in view of the importance of such internal measurements, a method was developed for measuring the concentration distribution. The observations were confined to measuring the vertical distribution of discharge concentration because the difficulties of determining transport concentration directly were almost insuperable.

Silica particles with diameters spanning the transition region of 0.2 to 2.0 mm. quoted by Durand were chosen so that it could be seen whether a systematic change in the form of the concentration profile existed. The importance of particle diameter as a criterion of whether or not solids travelled in suspension was then to be checked by conducting experiments with a fine zircon sand. Because of its high density, the settling velocity of the

zircon particles corresponded to that of a silica particle within the transition region. Furthermore, in previous work (57) it had been shown that large, light particles exhibited anomalous behaviour and the experiments were repeated with 0.073 inch perspex to verify the observations and to see if some explanation could be produced.

3. Mean Velocities.

The solids velocity and the slip velocity are factors of great importance in the process of hydraulic conveying because they have a direct bearing on the flow pattern and solids distribution and because they determine the relationship between the transport and discharge concentration. Equation III - 1 shows that the mean solids velocity $\bar{V}_s$ may be obtained from a knowledge of the mean water velocity $\bar{V}_w$. For this purpose the salt injection technique described in detail by Allen and Taylor (58) and Taylor (59) was adopted. Values of $\bar{V}_w$ were found to be great assistance in assessing observations of local water velocity. The method consisted essentially of measuring the time required for a pulse of salt solution to travel between two points in the pipe.

4. Water Velocity Profiles.

Although some investigators have already attempted measurements of the water velocity profile within a pipe,

it was considered desirable to attempt to relate the velocity profile to the distribution of solids in the pipeline. It was realized that the technique was crude and rather inflexible and insuperable experimental difficulties might fairly easily arise. It was hoped that at least some of the phenomena observed in an open channel might be shown to exist for the case of flow through a pipe.

5. Particle Properties.

Standard techniques were used for the determination of particle size and density and the porosity of a packed bed of solids. By this means the results could be related to those obtained by other workers.

In the following chapters the apparatus is described and details are given of the experimental techniques developed for measuring the quantities referred to above.

CHAPTER V.Experimental Apparatus.1. General Layout of the Plant.

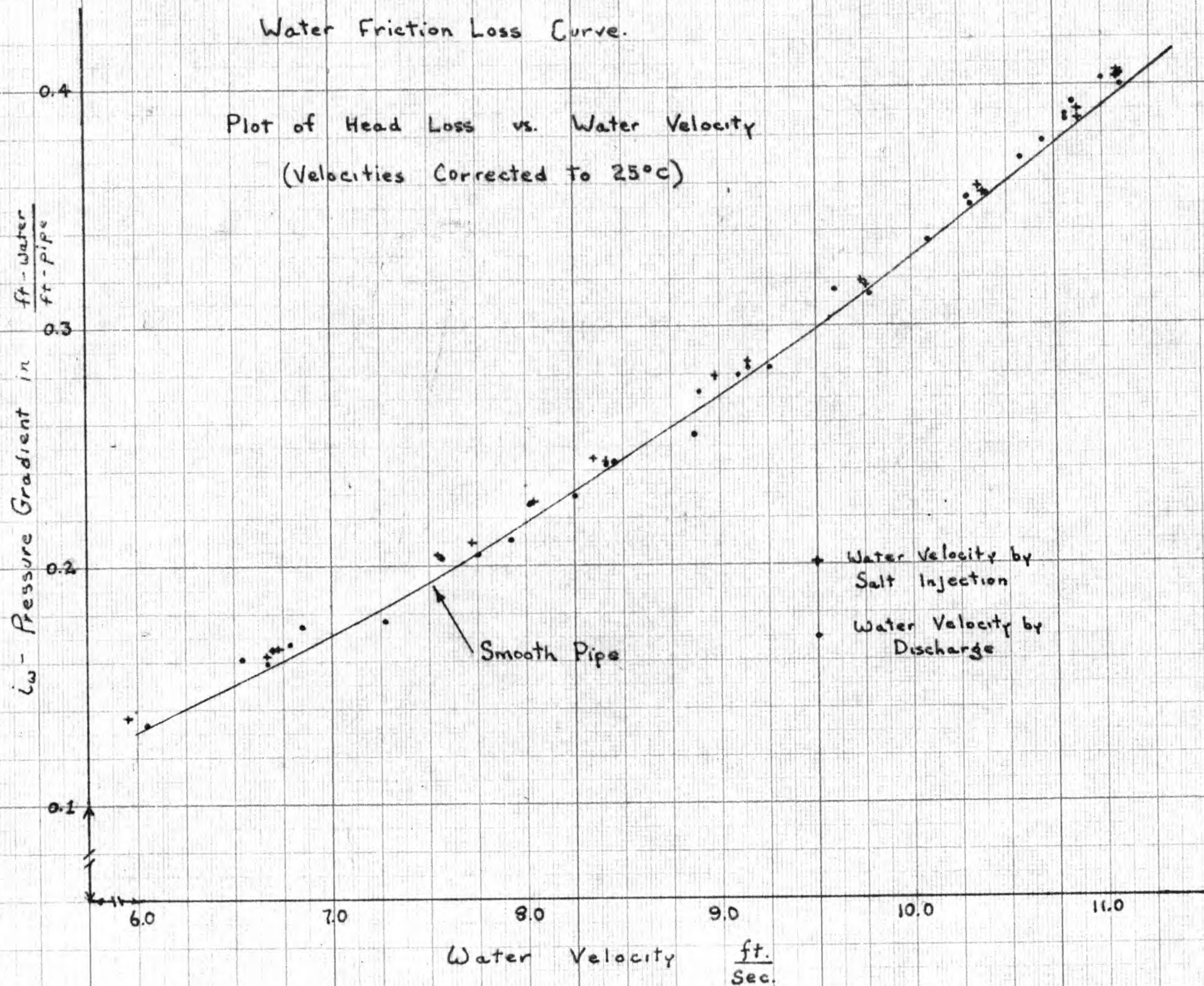
The apparatus was essentially that designed and operated initially by Turtle and subsequently modified by Abbott who improved the method of sampling. For the present work a number of modifications were made so as to permit measurement of several important variables. These modifications included the addition of a system for injecting a small pulse of salt solution and the detection of its passage at two points in the pipe; a device for splitting the flow and collecting a sample of each part; and a pitot tube for determining the water velocity profile.

Basically, the system consists of a horizontal loop one one-inch pipe, fed by a two-inch line from a centrifugal pump, discharging into a tank. A second section of two-inch pipe connects the tank to the suction side of the pump. Provision is made for varying the velocity of the fluid in the one-inch pipe by diverting a portion of the discharge from the pump directly into the tank.

2. The Pipeline.

A detailed description of the basic plant and the methods of construction may be found in Turtle's thesis, and only an abbreviated version will be given here. Two

FIG. V-1



straight limbs of 1.25 inch O.D. 10 S.W.G. brass pipe separated by a semicircular bend six feet in diameter comprise the test sections. Both sections were constructed of 8'6" lengths of pipe linked by Ermetco pressure couplings. The first limb, 42 feet 1 inch in length, contains the electrodes for detecting the passage of the salt pulse while the second or returning limb, 43 feet $\frac{1}{2}$ inch in length, contains in succession the pressure tappings, the pitot tube, the sight glass, and at its discharge, the flow divider.

The diameter of the pipe enters into the relationship between pressure gradient and fluid velocity to the 4.8 power when the velocity is computed from the discharge, and it is therefore obvious that the pipe diameter must be known accurately at all stages of the investigation. Before any experiments were conducted with the plant, one section of the pipe in the limb over which pressure measurements were made was removed and its diameter determined by filling it with water. Values of the water velocity obtained from the discharge rate were then compared with the absolute values given by the salt injection technique. Agreement was good and the diameter of the pipe was subsequently computed from the total discharge and the water velocity determined by salt injection. During the experiments the diameter of the pipe was found to increase from 1.010 to 1.017 inches, representing the

SCHEMATIC REPRESENTATION OF FLOW CIRCUIT
ISOMETRIC VIEW

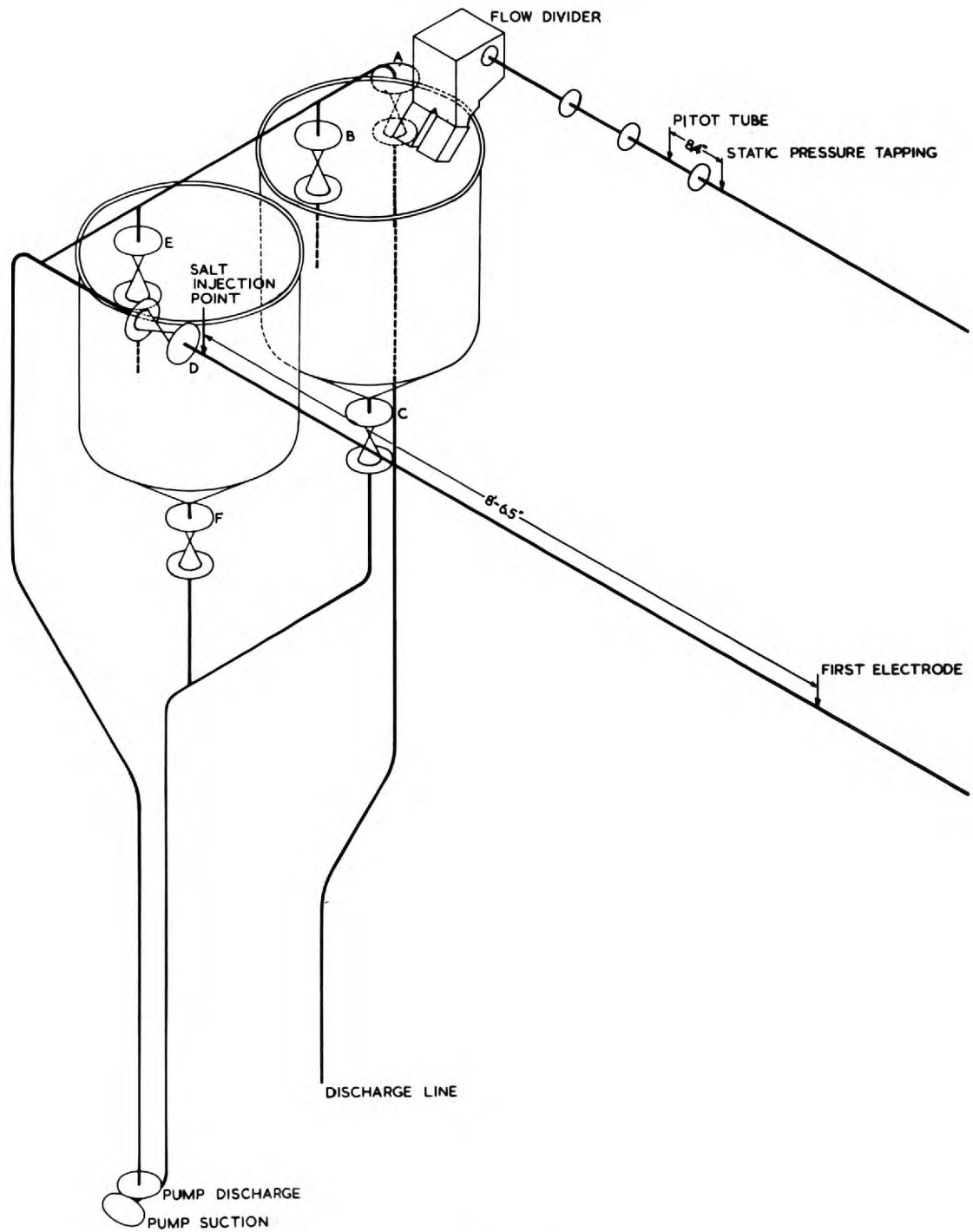


FIG. V-2

removal by abrasion of about 25 cubic inches of brass. Rather more than half of this was removed in a period of about fifty hours pumping at modest (less than 12%) discharge concentrations. At one time or another every bend and tee in the two-inch section of the plant, as well as every valve diaphragm, had to be replaced as a direct consequence of abrasion. In the particular case of the valve on the discharge bypass shown as B in fig. (V - 2), the disturbance caused to the flow by the valve resulted in the wall of the pipe immediately downstream being worn completely through.

3. The Pump.

The centrifugal pump has a number of advantages over other types, including simplicity, high running speed and steady delivery. It may be employed for the hydraulic conveying of solids provided the case and impeller are especially designed for the purpose. A $10\frac{1}{2}$ inch Vacseal Grit Pump, running at 1170 r.p.m., driven by a triple V-belt pulley system connected to a 10 - horsepower three-phase A.C. induction motor, proved adequate for the range of variables studied. Vacseal Grit Pumps are lined with rubber throughout to minimise abrasive wear, and vanes on the back of the impeller maintain a region of low pressure in the vicinity of the bearings to prevent fouling by solid particles. Unfortunately, this reduced pressure made the

provision of an air-tight seal in the gland very difficult. It was found necessary to keep the gland packing rings tightly compressed around the shaft, and at the resultant high running temperatures the packing tended to lose its lubricant very rapidly. By completely repacking the stuffing box at intervals of about fifty hours running time it was found possible to minimise the leakage of air into the casing. It is suggested that wherever possible further experiments be conducted with the gland secured with a mechanical seal, or flooded with a supply of the conveying fluid.

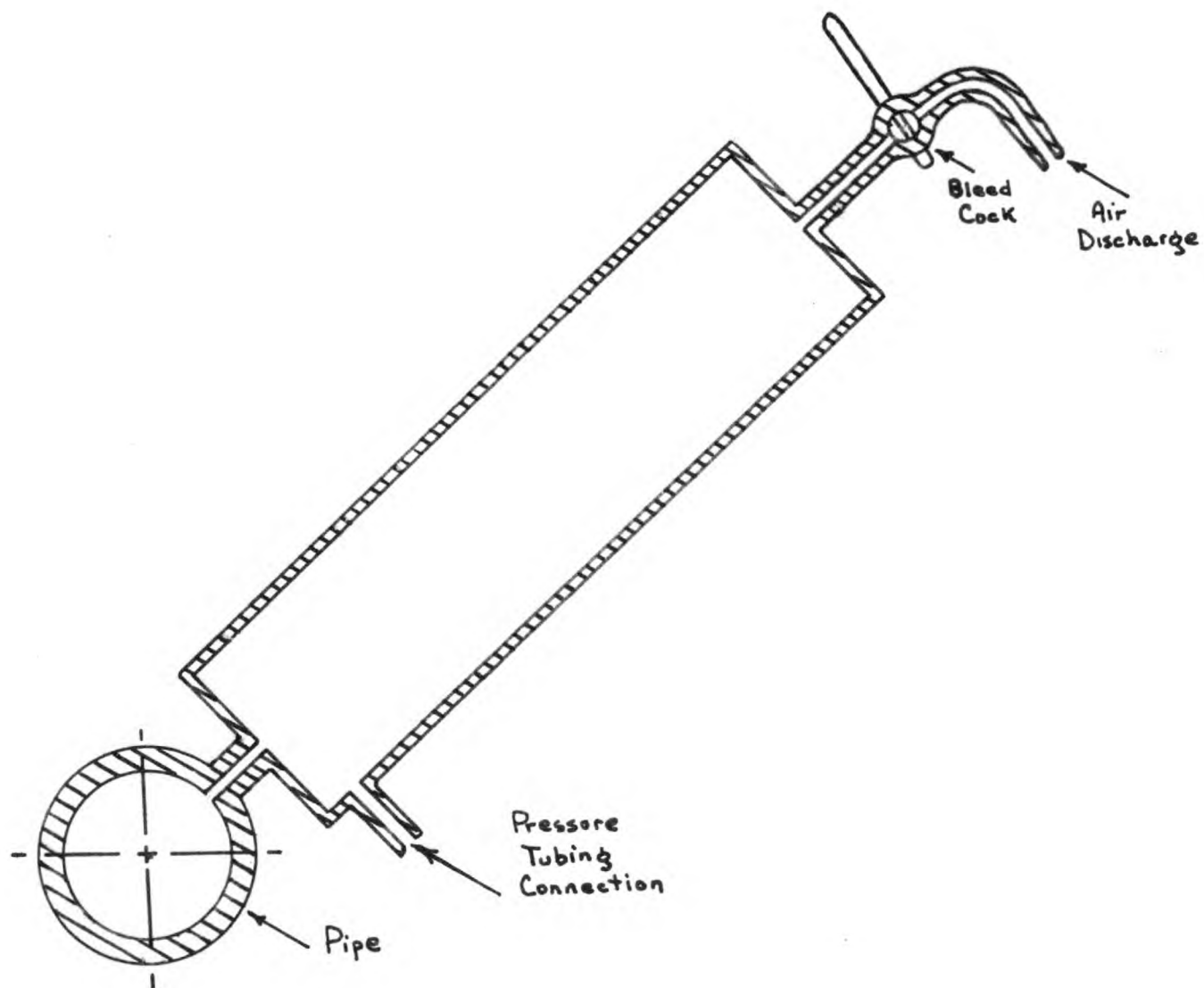
After about one-third of the experimental work had been done, it was found impossible to eliminate the air leakage into the plant. The motor and pump were replaced by the unit from the vertical conveying plant, described in detail by Gliddon. Roughly another third of the work was completed with this pump, and when the bearing of the original had been replaced by the manufacturers, it was returned to the circuit.

4. Pressure Measurement.

The static pressure of the fluid is measured at two points 4 feet $3\frac{1}{2}$ inches and 43 feet $\frac{1}{2}$ inch respectively from the discharge. Each pressure tapping, shown in fig. (V - 3) consists of a piezometer of Turtle's design, $4\frac{1}{2}$ inches long and $1\frac{1}{4}$ inch in diameter, terminating in a

FIG. V-3

SECTION THROUGH PIPE SHOWING
PIEZOMETER AND CONNECTIONS.



1/16 inch radial hole drilled through the pipe wall. Small bleed cocks permit the purging of air from the piezometers and lengths of $\frac{1}{4}$ inch soft copper tubing connect each pressure tapping to one limb of a mercury manometer inclined at 18.4 degrees to the horizontal. In no way does this arrangement differ from that described in detail by Abbott (12).

5. The Pitot Tube.

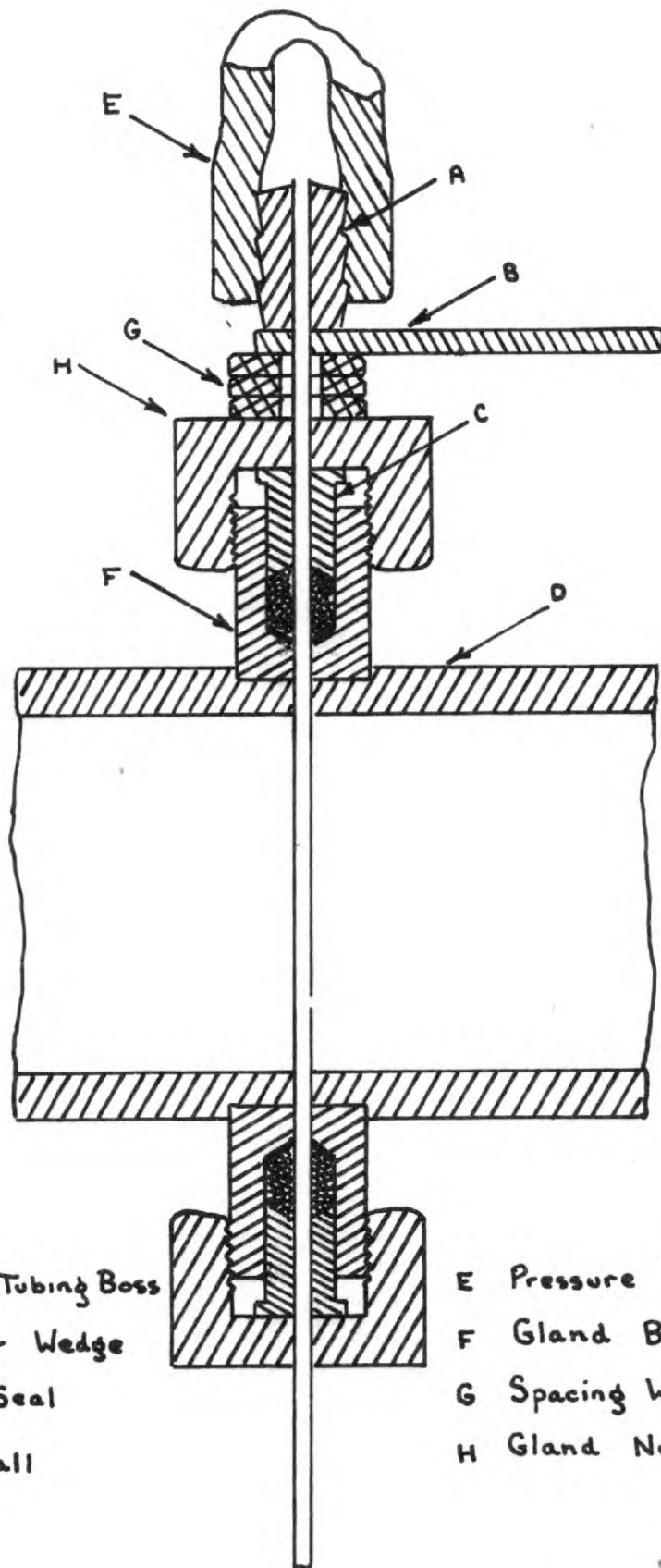
The simplest device for measuring the water velocity at a point within a pipe is certainly the pitot tube.

Having regard to the difficulty of producing a normal L-shaped pitot tube of sufficient rigidity to withstand the drag of the conveyed mixture, it was decided to employ a transverse tube spanning the pipe across a vertical diameter. The first tube was constructed by drilling a hole 0.013 inch in diameter through one wall of a piece of 22 gauge stainless steel hypodermic tubing. A tee was then constructed by soldering a $\frac{1}{4}$ inch length of the same tube over this hole. To the top of the tube was soldered a fitting for rubber pressure tubing and a long, narrow

wedge of strip brass was attached to indicate the position of the teepiece. The tube was mounted in the pipe through a pair of diametrically opposite glands, and it was here that the chief disadvantage of the assembly arose. It had been found impracticable to produce a removable screwed

FIG. V-4

SECTION
THROUGH PIPE
SHOWING
PITOT TUBE
INSTALLATION



A Pressure Tubing Boss
B Indicator Wedge
C Gland Seal
D Pipe Wall

E Pressure Tubing
F Gland Boss
G Spacing Washers
H Gland Nut

gland for a $1\frac{1}{4}$ " O.D. pipe with a wall $\frac{1}{8}$ inch thick and it was therefore necessary to solder the gland bosses in place. Replacement of the pitot tube was a tedious operation involving unsoldering this boss. Later pitot tubes were produced without a teepeece over the aperture and the velocity profile observed was identical. Similarly, it was found that tubes could be constructed from 0.050 inch diameter hypodermic tubing without giving significantly different results. Fig. (V - 5) shows a comparison of the water velocity profiles obtained with the first pitot tube and with one of 0.050 inch diameter without a teepeece.

The measurement of the distance of the pitot tube orifice from the pipe wall is usually made by means of a micrometric traversing system. It was found possible to provide an inexpensive alternative which proved satisfactory throughout the investigations. The short section of pipe containing the pitot tube was removed and the datum distance of the orifice from the bottom of the pipe measured with a cathetometer, when the tube was in its lowest possible position.

Filletted bright steel turned washers could then be inserted between the gland nut and the boss on the pitot tube. The thickness of each washer was $0.056" \pm 0.001"$, and the position of the pitot tube was thereby known at various points over the cross-section. Since a symmetrical velocity profile was obtained with the first pitot tube, further

FIG. V-5

Velocity Profiles As Measured By Impact Pressure
(Water Only)

V/V_m
 $\log_{10}$

-0.05

-0.10

-0.15

-1.00

-0.50

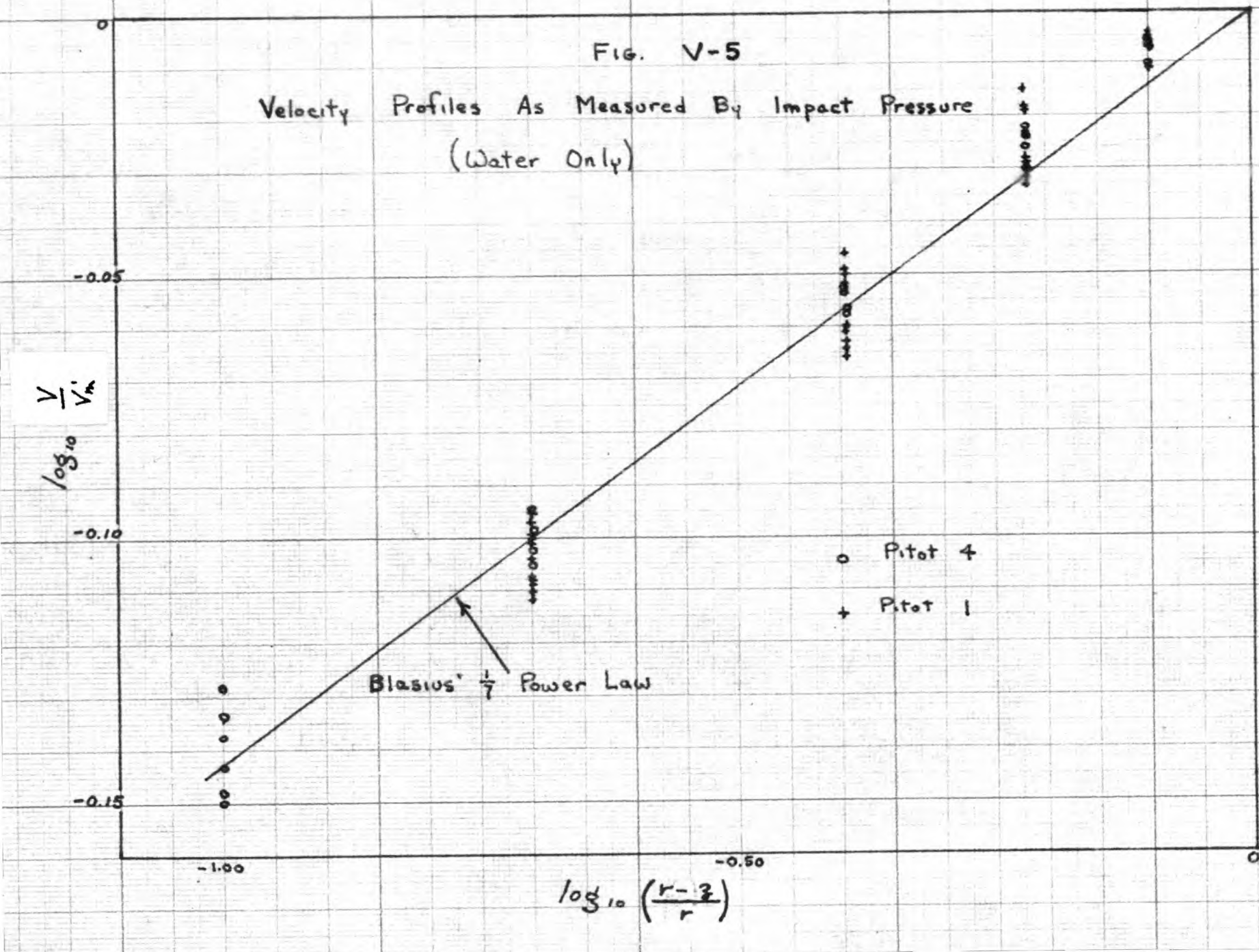
0

Blasius' $\frac{1}{7}$ Power Law

o Pitot 4

+ Pitot 1

$\log_{10} \left(\frac{r-2}{r} \right)$

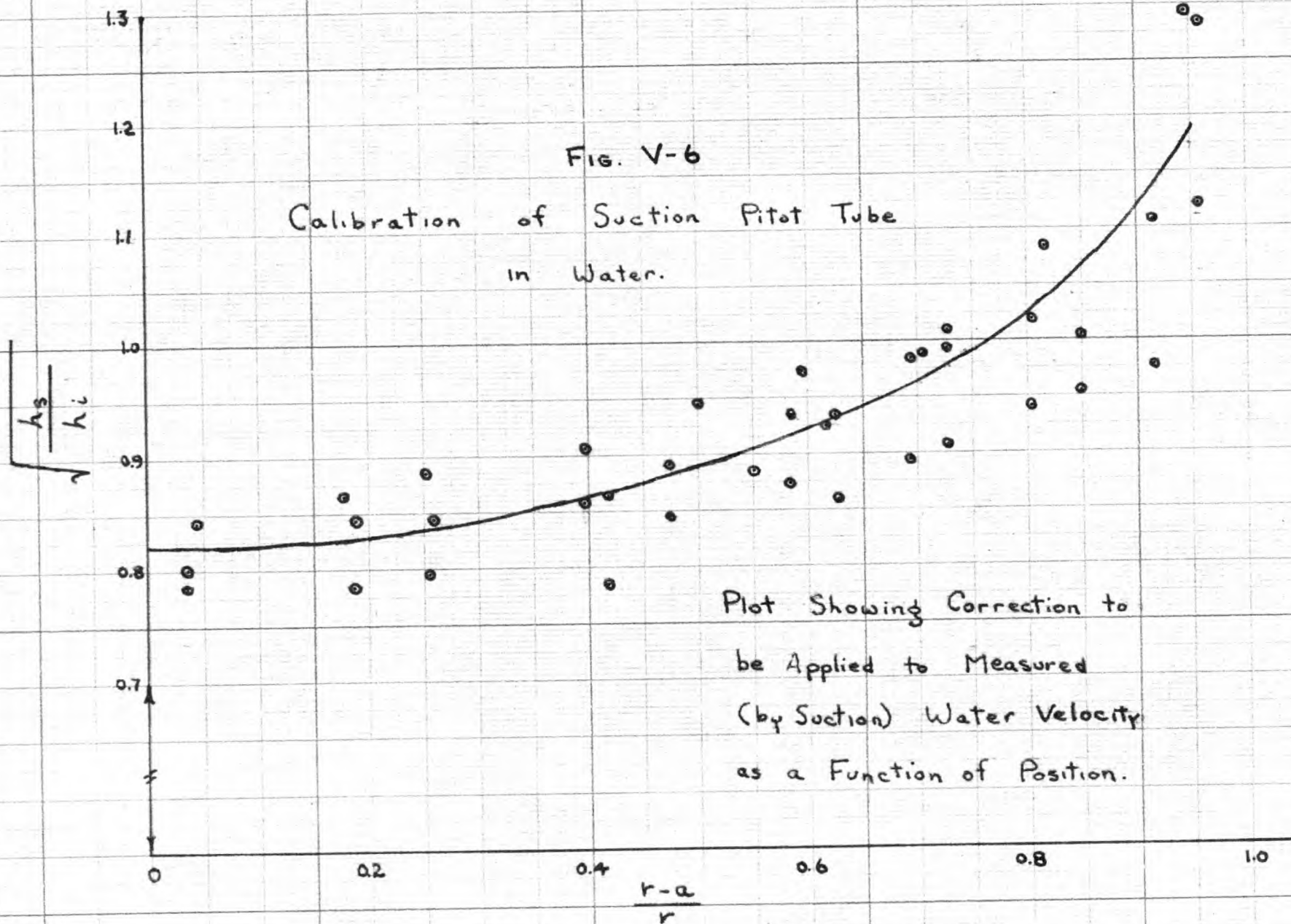


instruments were calibrated on the assumption that the axis of symmetry of the water velocity profile coincided with the axis of the pipe.

The pitot tube was connected to a bromoform (S.G. = 2.890) U - tube manometer by a length of 4 mm. I.D. rubber pressure tubing interrupted at intervals with glass tube which permitted the presence of air bubbles to be observed. Bubbles could usually be eliminated by sucking fluid through the needle of a hypodermic syringe inserted obliquely through the wall of the pressure tubing. The static pressure limb of the manometer was connected to the piezometer which was 8.4 inches upstream of the pitot tube; it was accordingly necessary to correct the static pressure gradient in the pipe.

Although it was desirable that the orifice of the pitot tube be as small as possible to prevent the entry of solid particles, the local Reynolds number of the orifice was kept greater than 38 to avoid the anomalous results often obtained with very small pitot tubes.

With the pitot tube facing upstream in the normal manner, it was found that near the bottom of the pipe negative readings of the bromoform manometer were obtained at successively higher levels as solids were added to the system. This phenomenon was attributed to the formation of a stationary layer of particles in front of the pitot tube. It was found that under these conditions the suction



ISOMETRIC VIEW
OF
FLOW DIVIDER

- A REPLACEABLE KNIFE EDGE
- B KNIFE EDGE DOGS
- C SPRING LOADED GUIDES
- D PARTITION
- E DIVIDER BOX
- F DISCHARGE - UPPER PORTION
- G CHUTE - LOWER PORTION
- H HINGE
- J TRAVERSING SCREW

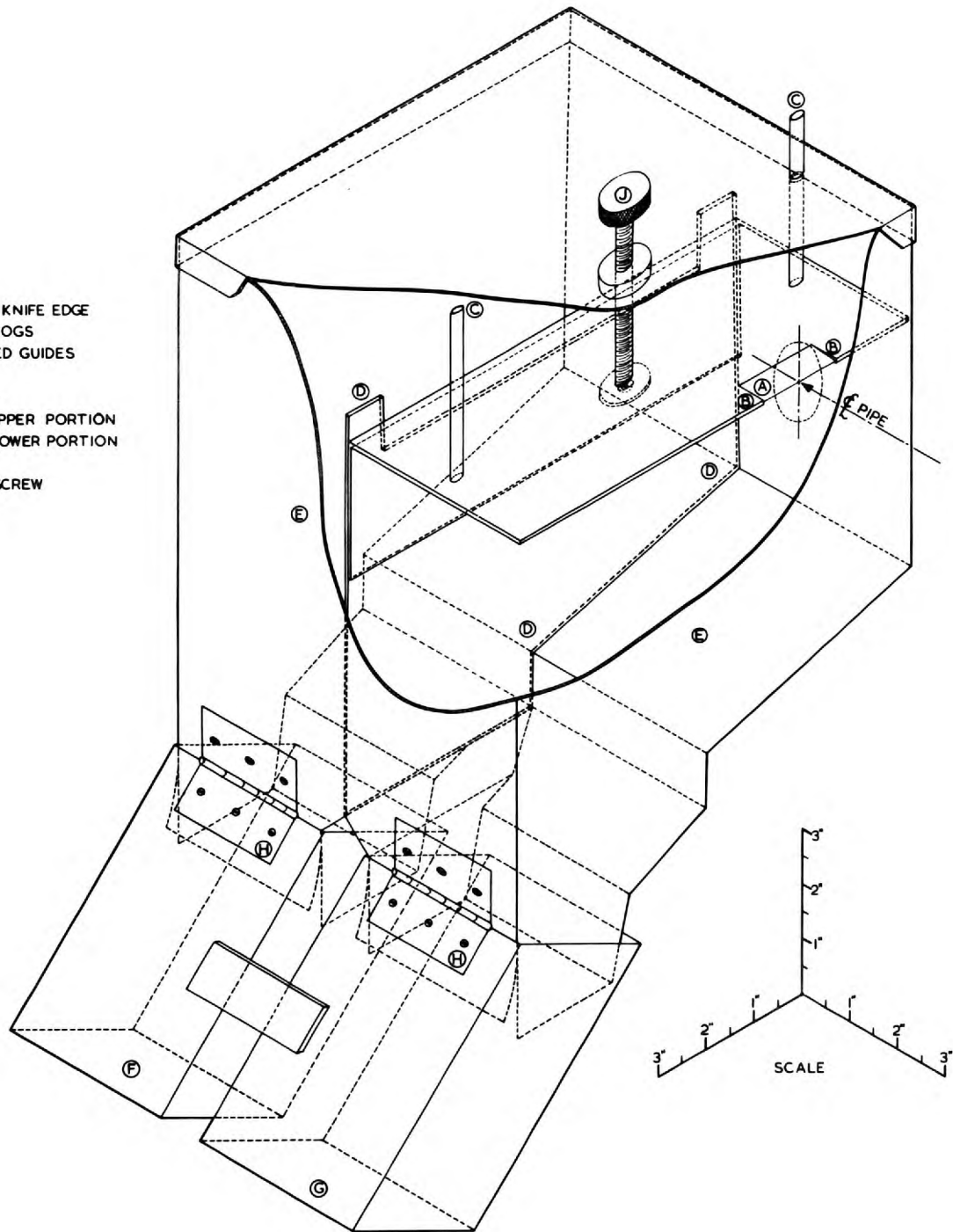


FIG. V-7

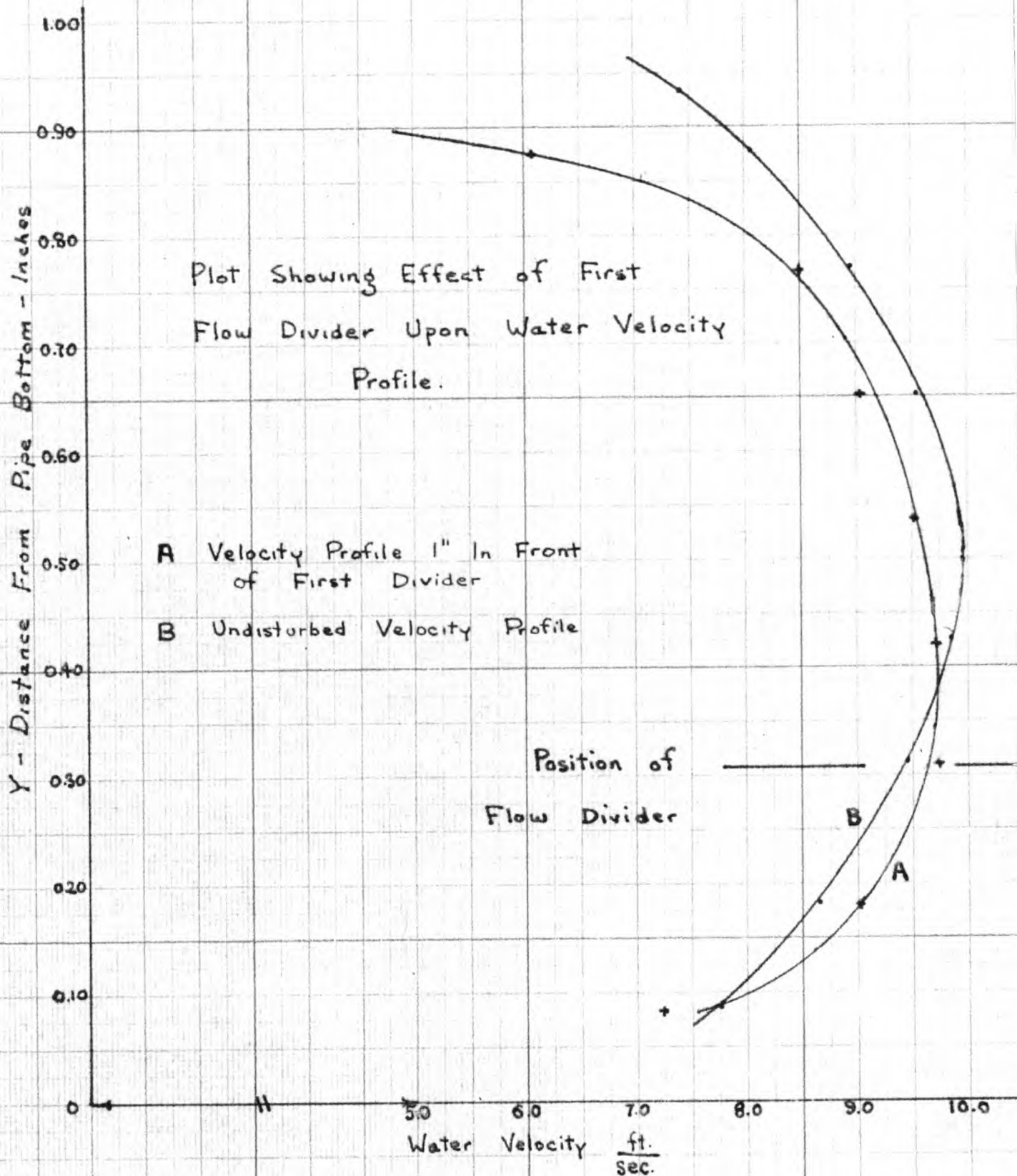
pressure produced with the tube orifice facing downstream seemed to give a reasonable estimate of the water velocity.

An examination of the manner in which the pressure behind the transverse tube without a tee-piece varied with fluid velocity was then conducted. It was found that at any point the velocity as computed from the suction pressure was a constant fraction of that obtained with the tube facing upstream. The ratio of suction pressure velocity to impact pressure velocity expressed as the ratio $\sqrt{\frac{h_s}{h_i}}$ was found to vary with distance from the pipe wall according to the calibration curve shown in fig. (V - 6).

6. The Flow Divider.

The device illustrated isometrically in fig. (V - 7) was used for dividing the flow obtained from the upper and lower portions of the pipe. It represented a development of the initial flow divider in which a knife edge situated in the pipe separated the flow into two streams which discharged through separate sections of one-inch pipe into the tank. Because the resistances of the pipes downstream from this divider were nearly the same, the flow through the areas above and below the knife edge tended to be equalised and false values were obtained for the discharge rates. The points marked "first divider" on fig. (V - 9) illustrate the effect, and the water velocity profile shown in fig. (V - 8) was obtained one inch in front of the first divider.

Fig. V-8



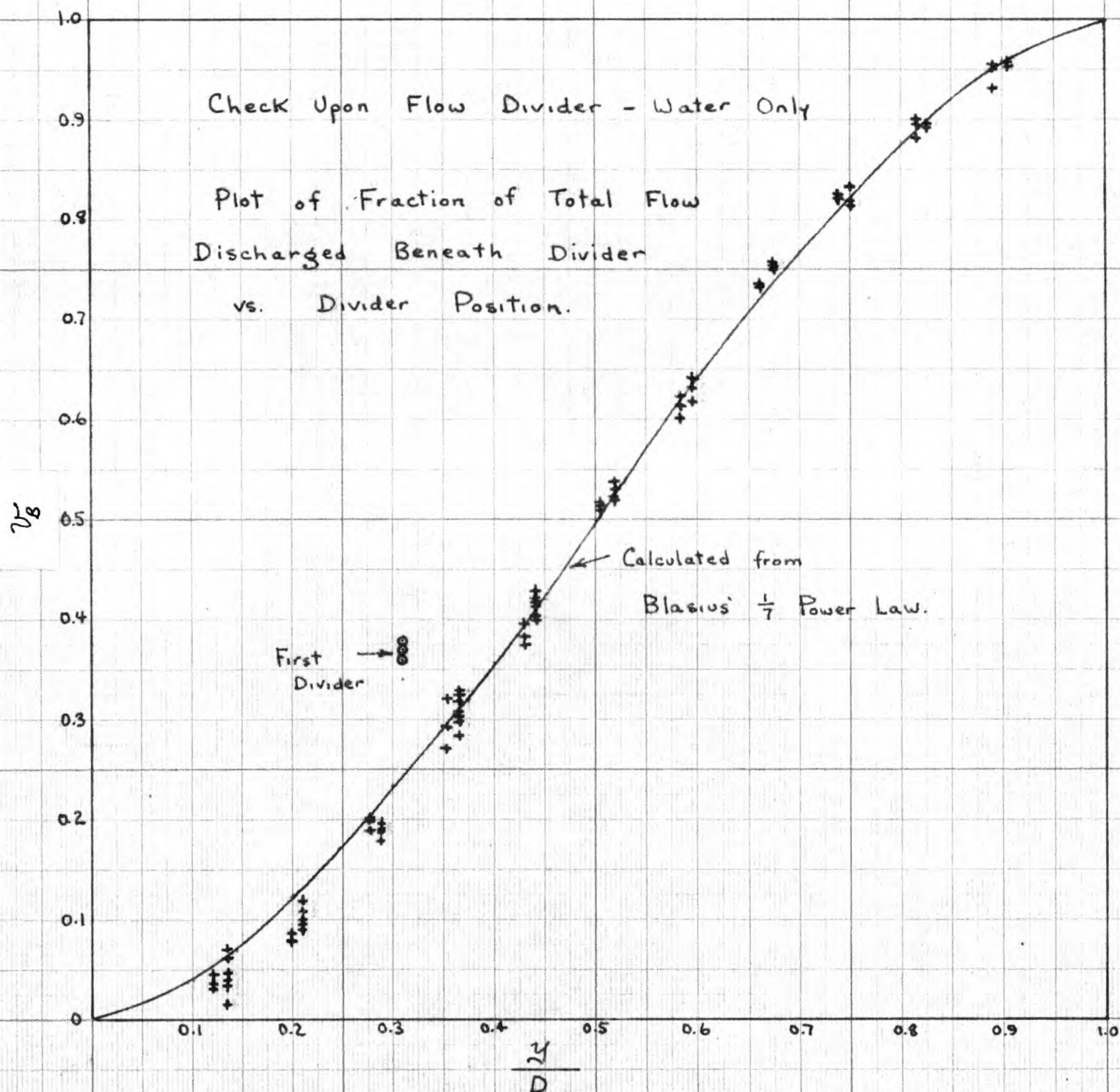
In the modified divider, the fluid discharged from the end of the pipe impinges upon the replaceable knife edge fashioned from a fragment of hacksaw blade. Particles and fluid travelling beneath the divider at the instant of separation are deflected by the vertical plane behind the knife edge and discharged through the front aperture. Similarly, the flow above the divider is allowed to strike the rear wall of the box and discharge through the second aperture. The box itself, the deflecting plate containing the knife edge and the diverting chutes were all fabricated from 16 gauge sheet steel.

It was realized that it might be possible for the deflecting wall to influence the discharge from the end of the pipe, and a series of experiments was conducted to try to ascertain whether or not the $3\frac{1}{2}$ inch gap allowed in this instrument was sufficient. Coloured fluid was injected continuously into a stream of water deflected as in the divider, and it was decided that at the range of velocities which would be encountered in this investigation, the influence of the deflecting wall could safely be ignored.

Movement of the dividing plane was accomplished by means of the traversing screw (pitch $1/26$ inch) passing through a retaining boss on the top of the box, and spring-loaded guides ensured that the plane was level.

When the diverting chutes were in the position shown in fig. (V - 7) the mixture discharged through them and

FIG. V-9



into the collecting vessels. Since they are hinged at H they could be removed simultaneously at the end of the sampling interval.

A check upon the effectiveness of the flow divider was performed by comparing the amount of fluid discharged beneath the divider without solids in the system with that predicted from the Universal Velocity distribution. Some discrepancy is evident in the lower part of the pipe because fragments of gland packing, causing fouling of the knife edge, tended to travel in this region. It was observed that when the knife edge was removed after having been in the centre or upper part of the pipe, it was relatively clean, whereas on removal from the lowest regions it showed the presence of a considerable amount of fibrous material. It was also observed that the presence of a small quantity of coarse sand caused the knife edge to remain relatively clean.

The datum level of the divider in its lowest position was obtained from the assumption that the discharge from the pipe was symmetrical about the horizontal plane passing through its axis. The fact that fig. (V - 9) shows reasonable agreement in the region $0.20" < Y < 0.90"$ meant that the location of the divider could be determined by making reference to the graph. Although the method was obviously less satisfactory than one giving a direct indication of divider position, it is felt that the error involved was probably never greater than 0.015 inch.

Fig. V-10

SIDE VIEW
OF
FLOW DIVIDER

- A Insulated Wires
- B Diverting Chutes Handle
- C Ebonite Cylinder (Rod)
- D Spring
- E Bronze Sphere
- F Brass Inset
- G Diverting Chutes
- H Hinges
- J Fibreboard Plate

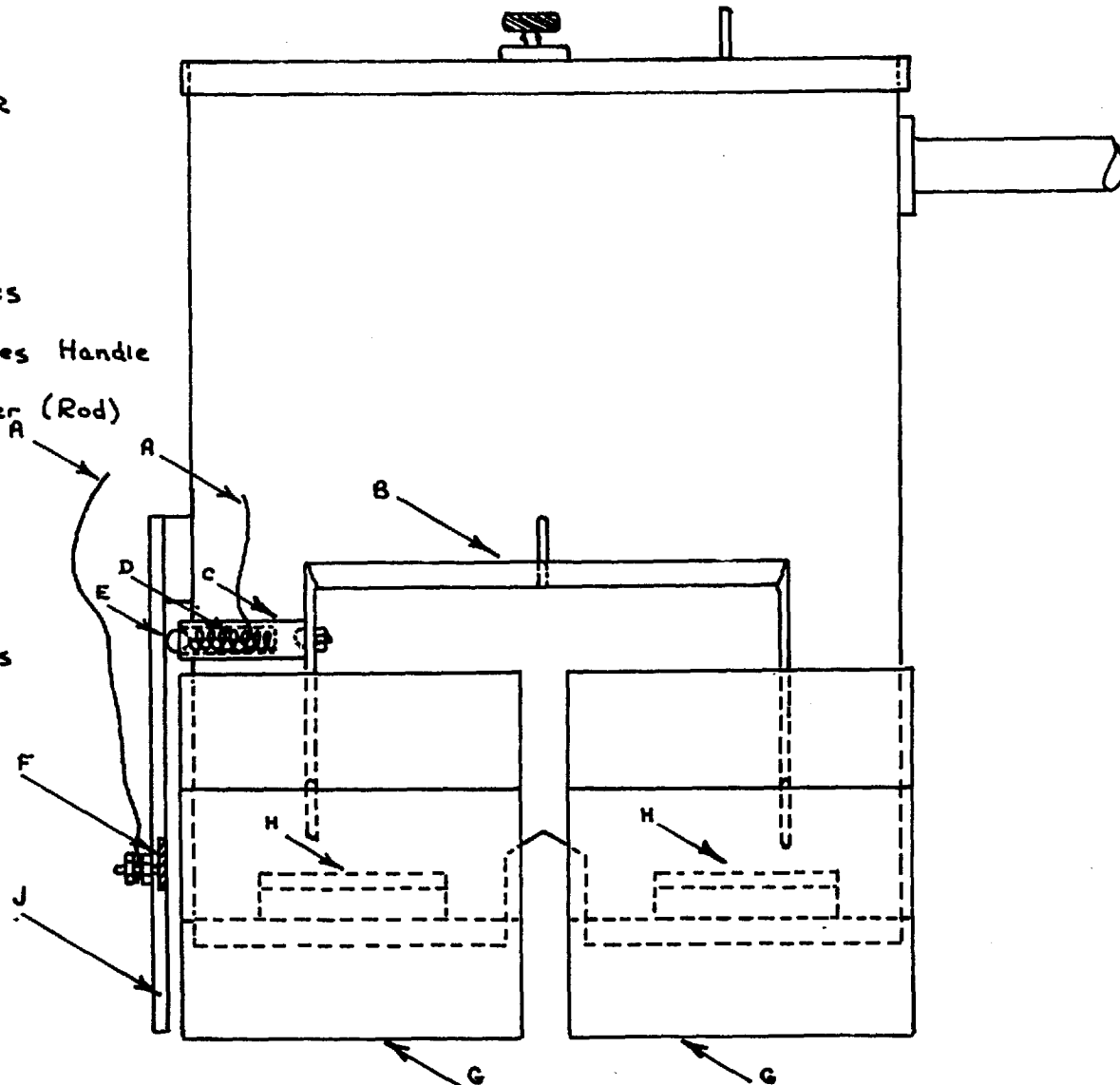
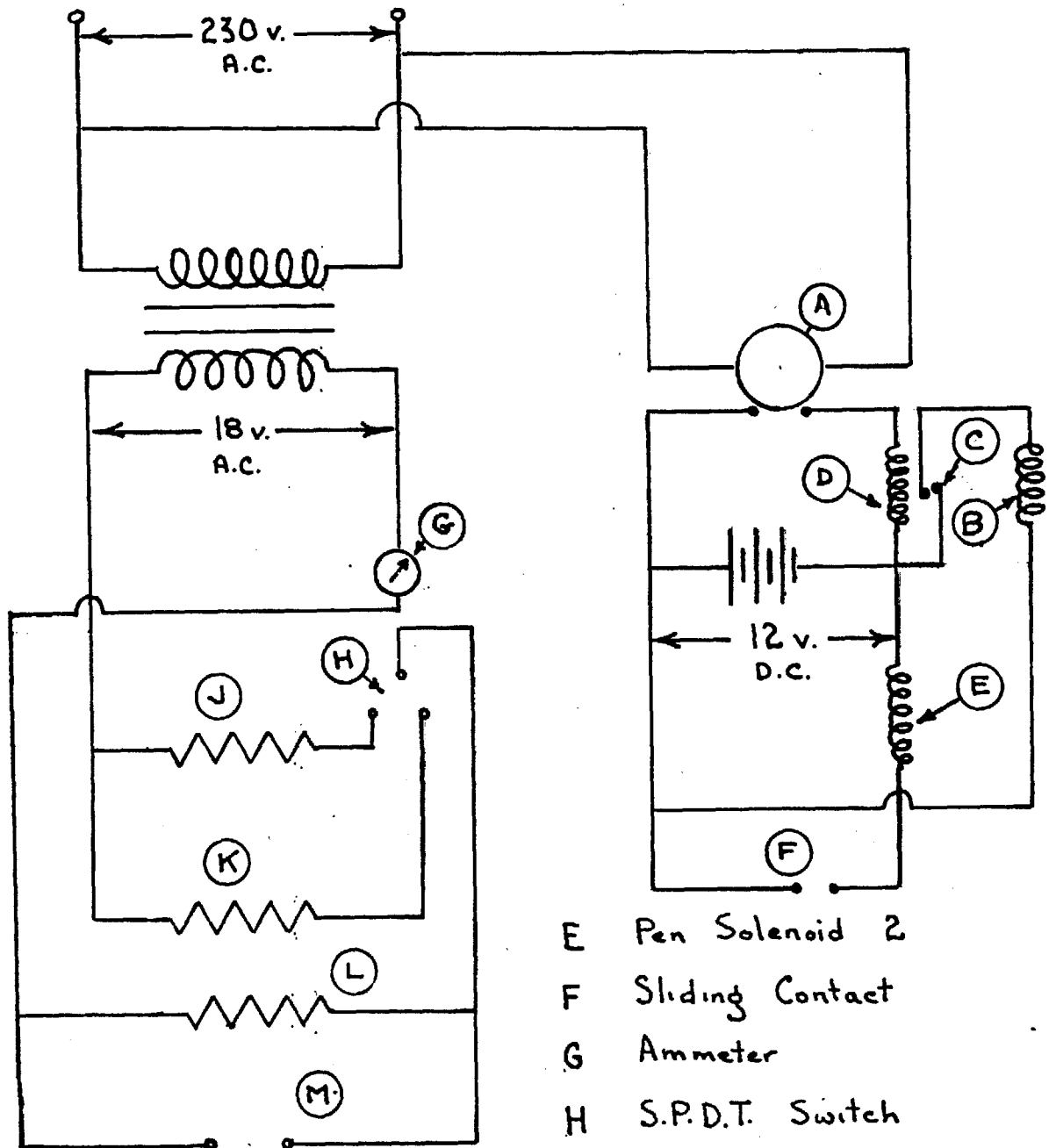


FIG. V-11



- A Synchronous Motor
- B Pen Solenoid 1
- C Relay
- D Relay Solenoid

- E Pen Solenoid 2
F Sliding Contact
G Ammeter
H S.P.D.T. Switch
J Electrode 1
K Electrode 2
L 10 Ohm Resistor
M C.R.O.

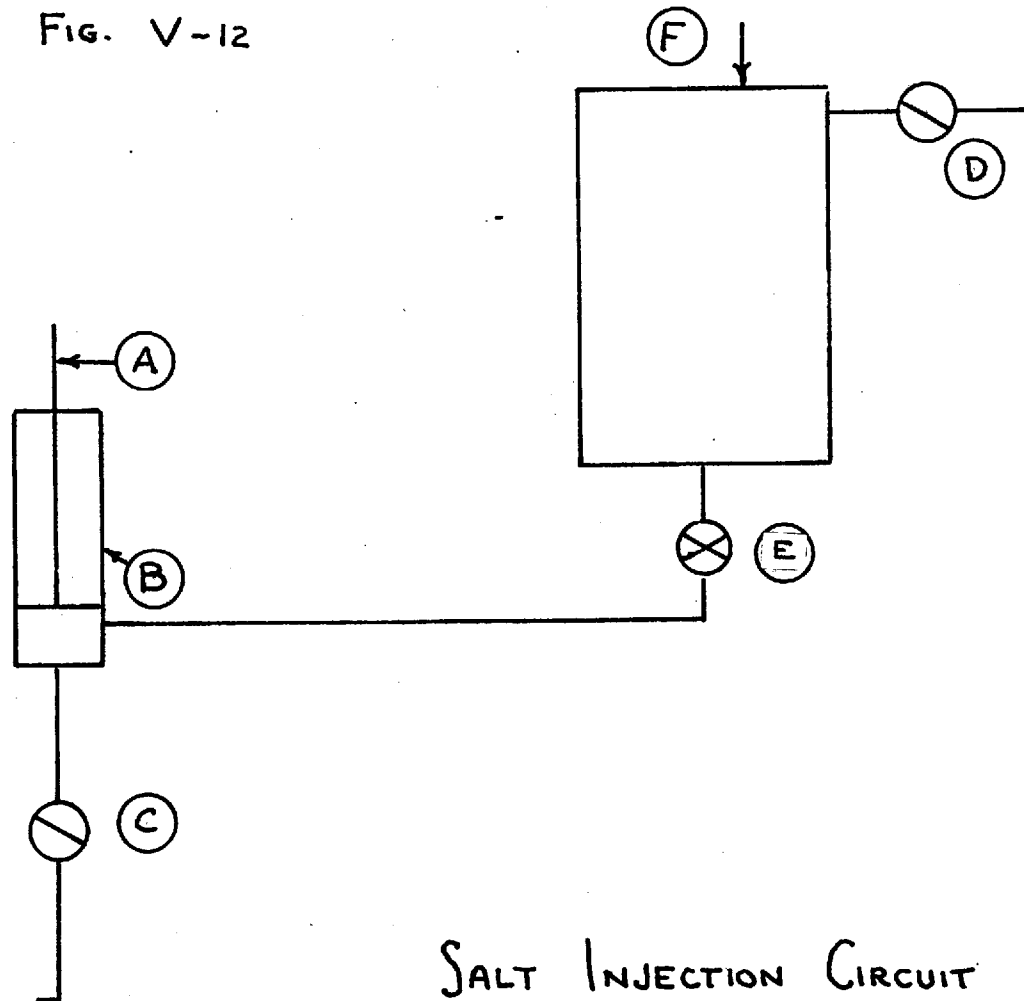
7. Time Measurement - Discharge Velocities.

Abbott improved Turtle's technique for measuring the length of the discharge sampling interval by employing a Cambridge recording chronograph in place of the stopwatch. This instrument was used throughout these investigations without modification, and a detailed description of its operation may be obtained from Abbott's thesis.

A spring-loaded bronze sphere contained in an ebonite tube screwed to the handle (shown in fig. (V -10)) of the diverting chutes of the flow divider made contact with a narrow brass strip inset as it slid along the face of a fibreboard plate fastened to the back of the flow divider. Movement of the sphere was possible only when the chutes were raised or lowered for the collection of a sample of the discharged mixture. At the instant of contact the impressed potential difference of the storage battery shown in fig. (V - 11) caused a current to flow in relay solenoid 2, deflecting the pen of the chronograph and indicating either the start or the finish of the sampling period. The synchronous motor provided an indication of the speed of the strip chart by driving a fibre contact wheel at one revolution per second and producing deflections of the other pen through circuit 1 at one second intervals.

FIG. V-12

- A Spring-Loaded Piston
- B Injection Cylinder
- C Plug Cock - Salt Line
- D - Air Line
- E Inlet Valve
- F Salt Filling Point



SALT INJECTION CIRCUIT
(SCHEMATIC)

8. Salt Injection Circuit.

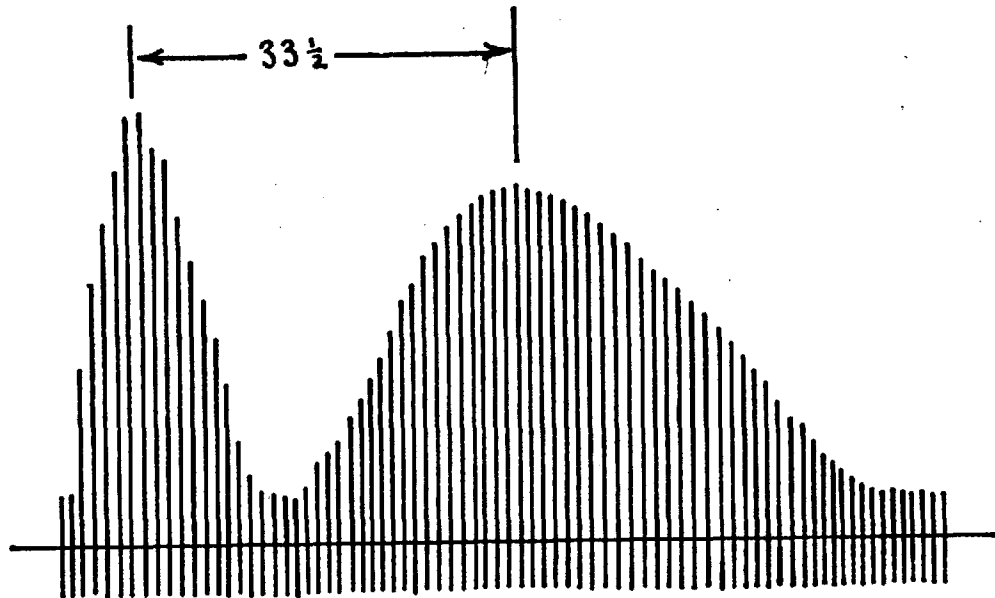
The injection circuit shown schematically in fig. (V - 12) consists of a spring-loaded brass piston $1\frac{1}{4}$ inch in diameter operating in a brass cylinder 2 inches O.D. x $5\frac{3}{8}$ inches high. Salt solution from the reservoir is maintained at 100 p.s.i.g. by compressed air and was forced into the cylinder by opening valve E. When about 7 cc. had been admitted, the spring restraining movement of the piston became compressed to balance the pressure of the fluid. Valve E was then closed and when the plug cock C was opened, the contents of the cylinder were discharged rapidly into the pipe at the 2" - 1" reducer. The system worked well, and the only experimental difficulty arose from wear in plug cock C.

9. Electrodes.

The passage of the salt pulse was detected by means of two pairs of electrodes inserted through the hexagonal portions of the Ermetco pressure couplings situated respectively 8 feet $6\frac{1}{2}$ inches and 42 feet 1 inch downstream of the injection point. A gland was screwed into a $\frac{3}{8}$ inch tapped hole in each coupling and a 0.266 inch perspex rod passed into the pipe. Each perspex rod was drilled lengthwise with two holes through which passed a pair of spring steel wires projecting about $\frac{1}{8}$ inch beyond the tip of the rod.

FIG. V-13

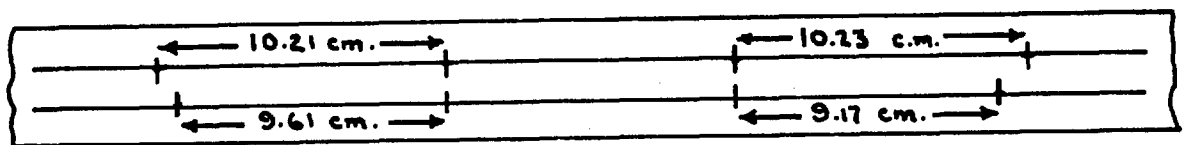
Determination of Salt Injection Interval



$$\text{Interval} = 33\frac{1}{2} \times 0.02 = 0.67 \text{ sec.}$$

FIG. V-14

Determination of Volume Sampling Interval



$$\begin{aligned} \text{Interval} &= 1.000 + \frac{9.61}{10.21} + \frac{9.17}{10.23} \text{ sec.} \\ &= 2.836 \text{ sec.} \end{aligned}$$

Amyl acetate cement was used to form a watertight seal between wires and rod. When the single-pole double-throw switch was in position, the impressed 18.6 V. A.C. potential difference between the wires caused a current to flow through the fluid, permitting the detection of the passage of the salt pulse by the variation in current as it passed the electrodes. The electrodes functioned satisfactorily at all times, the only maintenance found necessary being the removal of fibrous matter from the electrode tips when water had been pumped alone in the circuit for some time.

10. Time Measurement - Water Velocities.

Several attempts were made to modify the recording chronograph to measure the time interval between the passage of the salt pulse at the two electrodes. No matter how much the inertia of the relay systems was reduced, it was not possible to measure the water velocity with a systematic error of less than 5%.

It was therefore decided to employ an oscilloscope and a model 1049 mark III Cossor double-beam CRC was connected so as to permit a visual observation of the longitudinal concentration profile of the salt pulse to be made as it passed each electrode. The advantage of this particular instrument was that it had a sweep speed scale which extended to 1.5 seconds, making it possible for salt pulses of the length produced by simple injection devices to be detected

at distances considerably removed from each other. From the diagram V - 11, it may be seen that the current flowing between the elements of the electrode was used to give a potential difference across the 10-ohm resistor and that this was applied to the Y - plates of the CRO.

Since it was found desirable to have a permanent record of the measurement of the time interval, a photographic technique was evolved. A Kodak 122 folding camera was modified internally so that it would take a size 120 roll film and a special f.3.5 lens system was fitted to permit focusing upon objects at distances of from 8 to 12 inches from the aperture. A hood constructed from a length of 5 inch O.D. brass pipe was fitted over the face of the CRO. and the eyepiece of the camera was clamped into the opening in the hood facing the screen. This technique worked admirably and at least 500 photographs were taken without wastage.

Fig. (V - 13) illustrates the form of the photographic records of salt concentration at the two pairs of electrodes. The number of complete sweeps of the oscilloscope timebase separating the two photographed pulse forms, was of course unknown. Since the mean water velocity did not differ much from the mixture velocity, it was relatively simple to determine the number of integral sweeps to which the increment given by the relative positions of the pulses must be added.

CHAPTER VI.Experimental Techniques.1. Introduction.

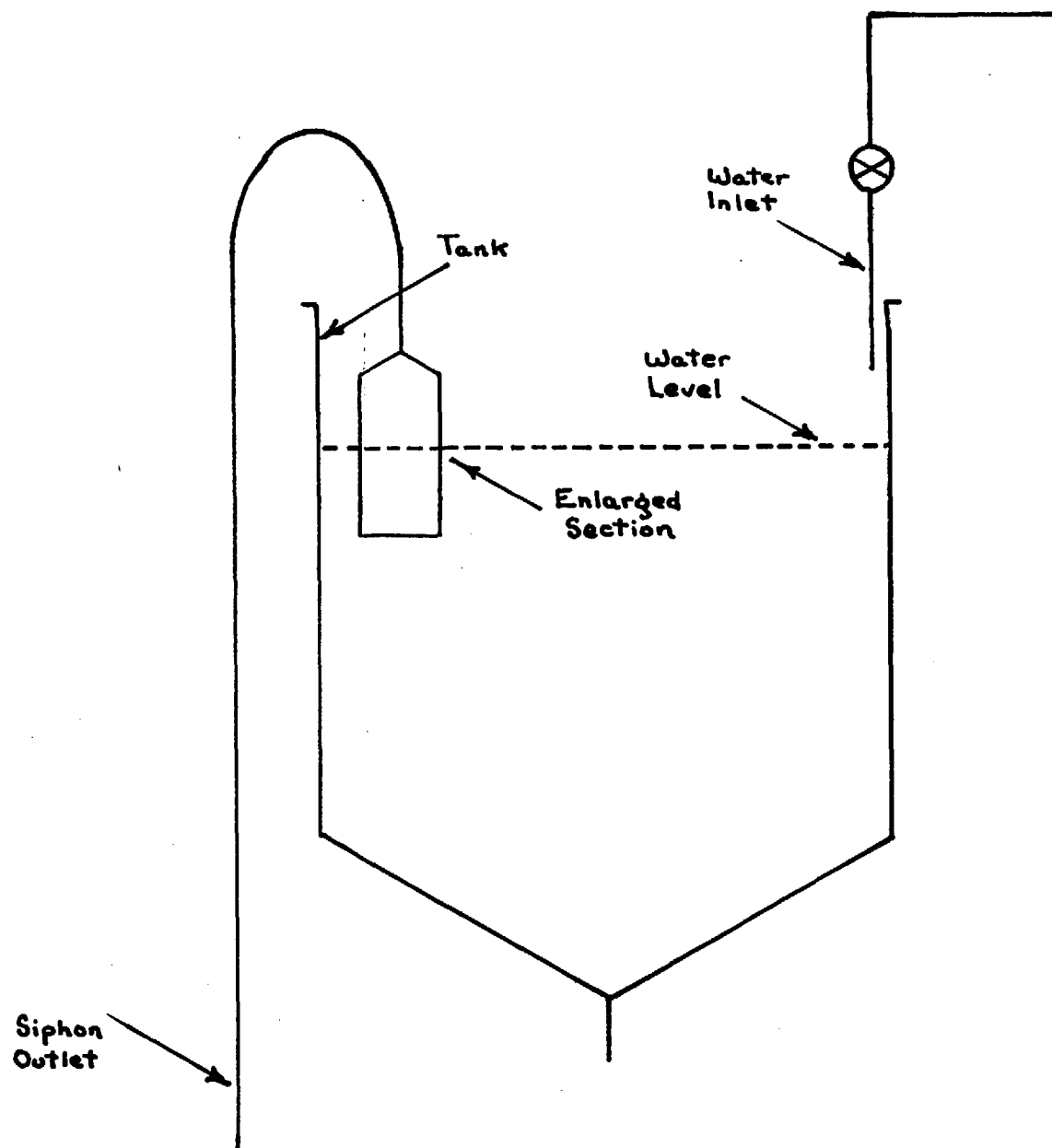
Inasmuch as the basic layout of the flow circuit has remained intact since Turtle's time, many of the experimental techniques described in detail in his and Abbott's theses were adopted for the determination of particle size distribution, bed porosity, particle density and settling velocity.

In order to maintain a steady temperature in the conveying liquid, a siphon was used to remove continuously a portion of the conveying fluid which was replaced by an equal amount of cold water. The modifications to the circuit described in the previous section made it necessary that new techniques be devised for starting and shutting down the plant and for measuring the concentration distribution, the discharge velocity, the mean water velocity and the water velocity profile.

2. Starting the Plant.

The 60-gallon copper tank into which the return limb of the flow circuit discharged was filled with water to a level about 12-14 inches above the conical shoulder. With every valve in the system tightly closed except B the pump was

FIG. VI-1
SCHEMATIC REPRESENTATION
OF SIPHON SYSTEM



started. Valve C in the pump suction line was then opened and water circulated through the bypass. When D was opened a portion of the pump discharge circulated through the pipeline. The piezometers were purged of air, the manometer opened to the test section, the siphon from the tank (shown schematically in fig. (VI - 1) started, and the stream of cold water to replace the siphoned amount turned on. Up to this point no solids had been added to the tank.

3. Elimination of Temperature Rise.

Early in the investigation it was discovered that the temperature rise associated with pumping was too large for a mean temperature to be attributed to a series of measurements. Although Abbott and Turtle completely replaced the conveying fluid intermittently, it was considered simpler to replace a portion of the conveying fluid continuously with water at a lower temperature. A siphon was constructed from a length of 7 mm. I.D. rubber tubing terminating in an enlarged section 5 inches in diameter, which permitted fluid to be removed at the rate of 1600 cc./minute without carrying away any of the particles. An additional advantage of the siphon was that the fine particles produced by abrasion were removed. It was found that although the water often appeared extremely turbid its density, as determined by a hydrometer bulb, was indistinguishable from that of pure water at the same temperature.

4. Pressure Gradient, Mean Velocity and Concentration Measurements.

Once the plant had been started in the manner described in Section (VI - 2), a quantity of solids was added slowly to the tank and allowed to circulate until freely dispersed through the system. By adjusting valve B on the bypass line, the pressure gradient could be fixed at one of the three standard values chosen for the investigation. These were usually 0.160, 0.280 and 0.400 ft./ft., but in the case of the fine sand the highest pressure gradient used was 0.372 ft./ft. The reason for this reduction was that the pump speed was reduced from 1170 r.p.m. to 950 r.p.m. during the series of experiments in order to eliminate the air leakage into the system. At this lower pump speed the capacity of the system at high concentrations of solids was lowered, and it was necessary to reduce the value of the standard pressure gradient. Adjustment of the pressure gradient was of course most difficult in the case of large particles and accounted for much of the scatter of the observed values of mixture velocity for perspex and gravel.

For the determination of discharge velocity and the concentration profile, two sampling vessels were placed on the wooden platform in the tank, the chronograph started and the diverting chutes allowed to fall into the position shown in fig. (V - 7). When the desired quantity of the discharge mixture had been collected, the chutes were lifted rapidly

and pinned in position and the chronograph stopped. A check upon the manometer reading was made, and if it had not altered to any significant degree, the vessels were removed and the volumes of mixture and solids determined by pouring into an 1300 ml. graduated cylinder. The strip chart was removed and marked and the temperature of the mixture was measured. The sample was then returned to the tank and, after the pressure gradient had been adjusted, the operation was repeated. When a series of measurements was completed at one position of the divider, it was raised, usually $1/13''$, by turning the traversing screw.

After about one half of the measurements had been made with a given quantity of solids in the system, measurements of water velocity were made by the salt injection method. The reservoir was filled with 5% NaCl solution and the top screwed in tightly. Plug cock D in the air line was then opened and a trial injection in the manner described in Section (V - 8) was conducted with the aperture on the oscilloscope hood opened to permit observation. After the aperture in the hood had been closed, the camera was loaded with film and a series of measurements of mean water velocity made at each pressure gradient. Ammeter G in the circuit provided a visual indication of the passage of the salt pulse at each electrode and permitted manipulation of the single-pole double-throw switch. Although the CRO was left turned on for as long as five days at a time, the brightness scale

was turned down at all times except when an observation or a photograph was being taken. The length of the timebase sweep varied from day to day, and it was calibrated by timing fifty sweeps twice with a stopwatch. If the periods as measured by the watch did not agree within 0.5%, the process was repeated until a reliable value of the sweep speed was obtained.

The second half of the experimental work was then completed and after the particles had been removed and the manometers isolated, plug cock D was closed and the pressure of the air in the reservoir used to force the remaining salt solution into the pipeline. The plant could then be shut down.

5. Shutting Down the Plant.

In order to avert the risk of blockage in the pump, it was considered desirable to remove the solids from the system before stopping the pump. By allowing the diverting chutes to fall into the position shown in fig. (V - 7) it was possible to replace the sampling vessels by a series of buckets and remove very nearly all the particles from the system. The residual solids were composed primarily of the fine products of abrasion, and as such were best removed from the system entirely with the conveying water and rejected.

After the separation of the solids, the manometer lines were isolated and valve D closed. When the flow of siphon makeup water had been stopped, valve A could be opened while B was simultaneously closed. The circuit emptied rapidly into the discharge tank, and when it could be heard that the pump suction was filled with air the motor was turned off and valve C closed.

6. Water Velocity Profile Measurements.

After a few attempts had been made, it was decided that it was not practicable to measure the water velocity by means of a pitot tube at the same time as the other quantities determined as in Section (VI - 4). Consequently a parallel series of experiments over the same concentration range was performed.

Solids were added to the system and the pressure gradient fixed as before. The pitot tube orifice was arranged at a given point on the vertical axis of the pipe which was determined by the number of steel washers separating the pressure tubing boss and the gland nut. Great care was taken to ensure that the pipeline was level at all times during the experiments. After the reading of the bromoform manometer had become steady, usually after three to five minutes, the readings of both manometers were taken and the pitot tube moved to another position. Samples of the discharged mixture were taken, usually three times so that the

solids concentration could be calculated. With the pitot orifice facing downstream, the manometer reading was occasionally affected by the collection of small air bubbles in the low pressure region behind the tube. These could either be removed by a hypodermic syringe, or eliminated by tightening the gland packing of the pump axle.

Although a calibration curve, fig. (V - 1) was obtained for the effect of position on the reading of suction pressure of the pitot tube, the velocity gradient as well as the local velocity were probably important. The failure of the calibration coefficient to apply more nearly when solids were present was probably a consequence of the fact that the velocity gradient was different and could not be allowed for.

Although the siphon system described in Section (VI - 3) maintained the fluid at a temperature which varied only by ± 2 degrees Centigrade or less during a six-hour period of operation, the value at which the temperature stabilized depended upon the temperature of the water in the head tank. In view of the fact that some correction of the observed water velocity was necessary to take account of small fluctuations of temperature, it was decided to avoid the tedious business of varying the siphoned amount to take account of changes in water supply temperature.

Accordingly a correction was applied which admittedly is unjustifiable theoretically, but which was probably not far wrong in practice.

For the case of water flowing alone in a pipe, the hydraulic gradient is given by the Fanning equation

$$i = \frac{f \bar{V}^2}{2gD} \quad (1)$$

where $\bar{V}$ denotes the mean velocity of the fluid. The mean velocity observed under conditions i and f may be corrected to standard conditions i_o and f_o by the relationship:

$$\frac{\bar{V}}{\bar{V}_o} = \sqrt{\frac{i f_o}{i_o f}} \quad (2)$$

Since the shape of the velocity profile does not change much as long as the flow remains turbulent, at any distance z from the pipe walls

$$\frac{V_z}{\bar{V}} = \frac{V_{zo}}{\bar{V}_o} \quad (3)$$

Hence a local water velocity measurement may be corrected by combining (2) and (3)

$$\frac{V_z}{V_{zo}} = \sqrt{\frac{i f_o}{i_o f}} \quad (4)$$

The appropriate friction factors are evaluated for the mean water velocities $\bar{V}$ and $\bar{V}_0$ and the fluid viscosities at $\bar{T}$ and T_0 . Now if the roughness of the pipe is known, the friction factor for a particular pipe will be a function of the quantity $\frac{i}{\eta^2}$ where η is the kinematic viscosity of the fluid. It was assumed that the relationship between f and $\frac{i}{\eta^2}$ was the same whether or not solids were present. Since the correction was never applied to a temperature difference greater than 5 degrees Centigrade the error involved was small. The effect of i was included in the correction to take account of the random fluctuations in pressure gradient experienced during a series of experiments.

7. Porosity and Density Determinations.

A 250 ml. graduated vessel was partially filled with water and a carefully weighed quantity of solids added. The cylinder was vibrated until the level of the particles had reached its lowest limit. Recordings were then made of the initial and final volumes of the fluid and the weight and apparent volume of the solids - from which both porosity and density could be determined. At least three determinations were made for each material.

8. Settling Velocity Determinations.

An 1800 ml. graduated cylinder, $2\frac{1}{2}$ " I.D. was marked at points 18 inches apart and filled to within $\frac{1}{2}$ " of the

top with water. Particles were placed under the surface of the water, released, and the time required to travel between the two points measured with a stopwatch. Particles which tended to move towards the sides of the vessel were ignored and the temperature of the water was taken at intervals during the experiment. At least fifty values were taken of the time required for the particle to settle through the fixed distance and the settling velocity was then calculated.

9. Size Distribution Analyses.

A carefully weighed sample of the material was placed on a Greening B.S. Test Sieve and vibrated on a G.E.C. Sherwen Electric Vibrator for forty minutes. At the end of this time the fraction which had passed through the screen was removed and the sample vibrated for another five minute period. If no further particles had passed through the sieve at the end of the second period, the process was repeated with a smaller sieve. The successive fractions were removed and weighed, permitting a size analysis curve to be prepared.

CHAPTER VII.Experimental Results1. Introduction.

In this chapter the data obtained from the measurements described in Chapter IV will be presented. Derived data and correlations will be shown in Chapter VIII.

It was found that although the technique for measuring the solids concentration profile functioned well for all materials, considerable difficulty was experienced in measuring the water velocity profile and in determining the difference between the mixture and water velocities for the more slowly settling materials. It could be concluded, therefore, that the solids velocity, for zircon, fine sand and perspex was very nearly the same as the fluid velocity.

One section of this chapter is devoted to each of the five materials studied and separate subsections deal with the results of the determinations of solids concentration, velocity profiles and mean velocities.

2. Coarse Sand, ($d = 0.0275$ inches.)(a) Concentration Profiles.

The results presented in Graphs VII - 1, 2 and 3, and Tables A1, 2 and 3, illustrate that the cross-section may be divided into the two regions based on the distribution of

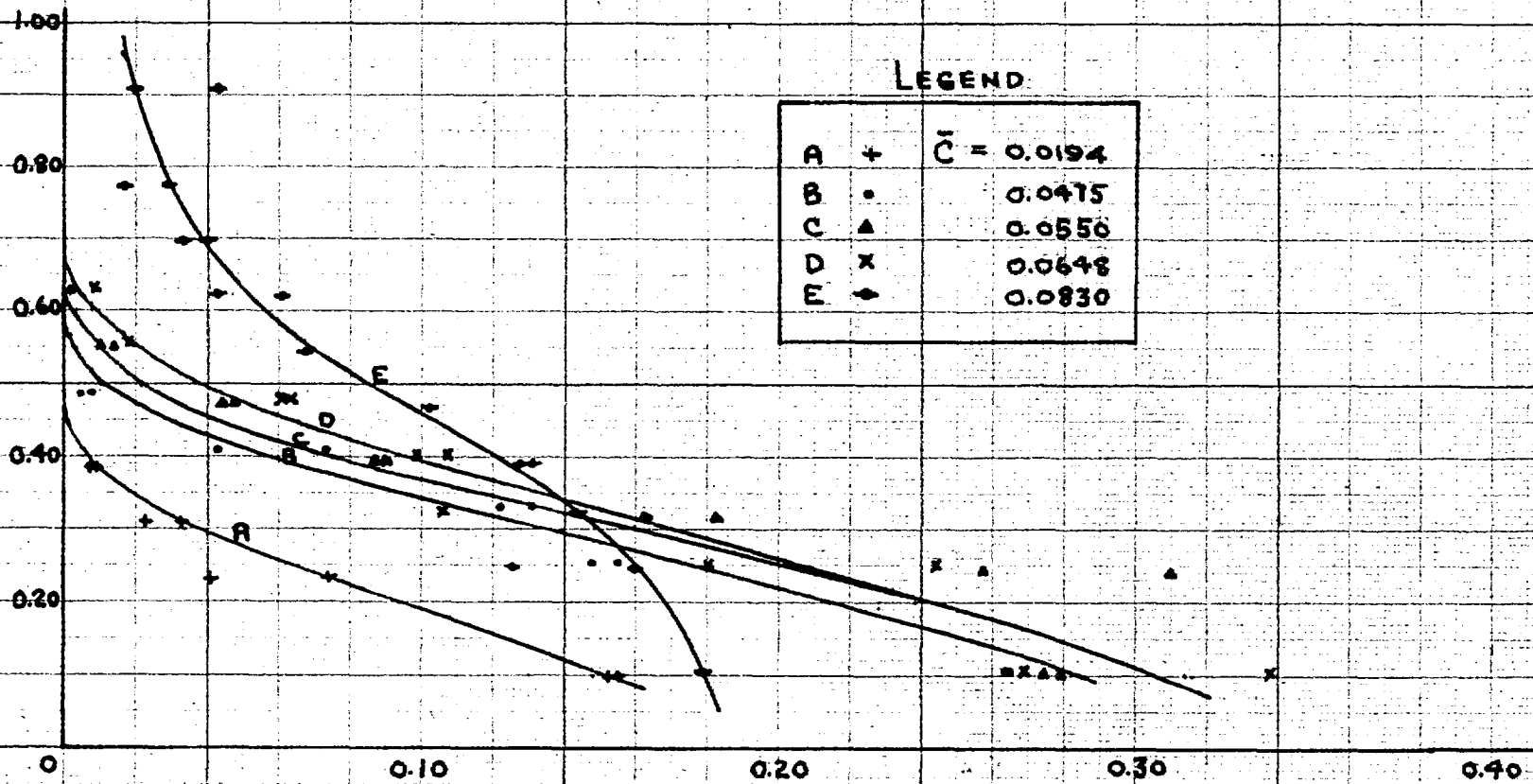
Fig. VII-1

Discharge Concentration Profiles

Coarse Sand

$\bar{C} = 0.160$ ft./ft.

y Distance From Pipe Bottom - Ins.



C_y Discharge Solids Concentration

FIG. VII-2

Discharge Concentration Profiles

Coarse Sand

$i = 0.286 \text{ ft./ft.}$

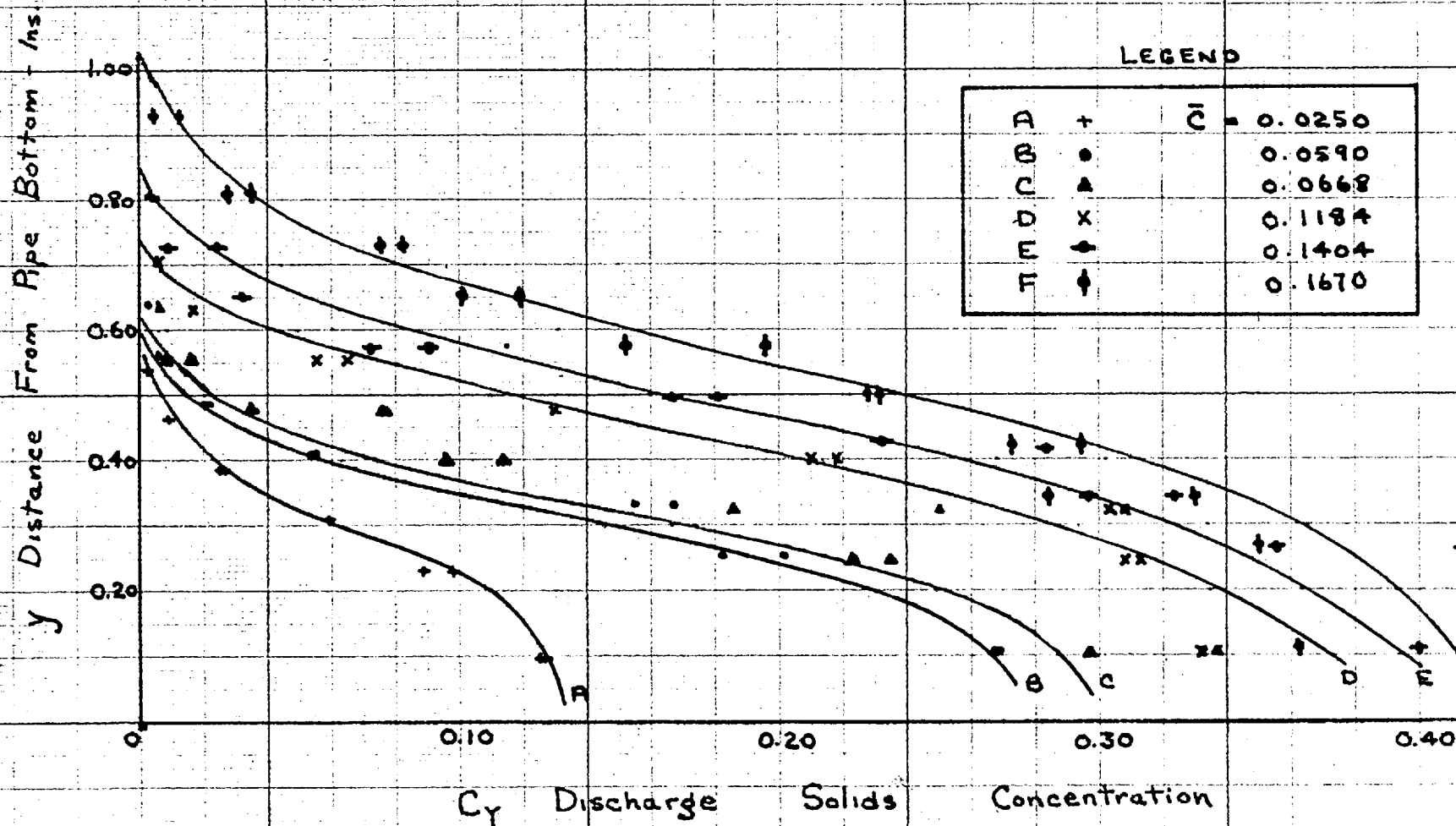


FIG. VII-3

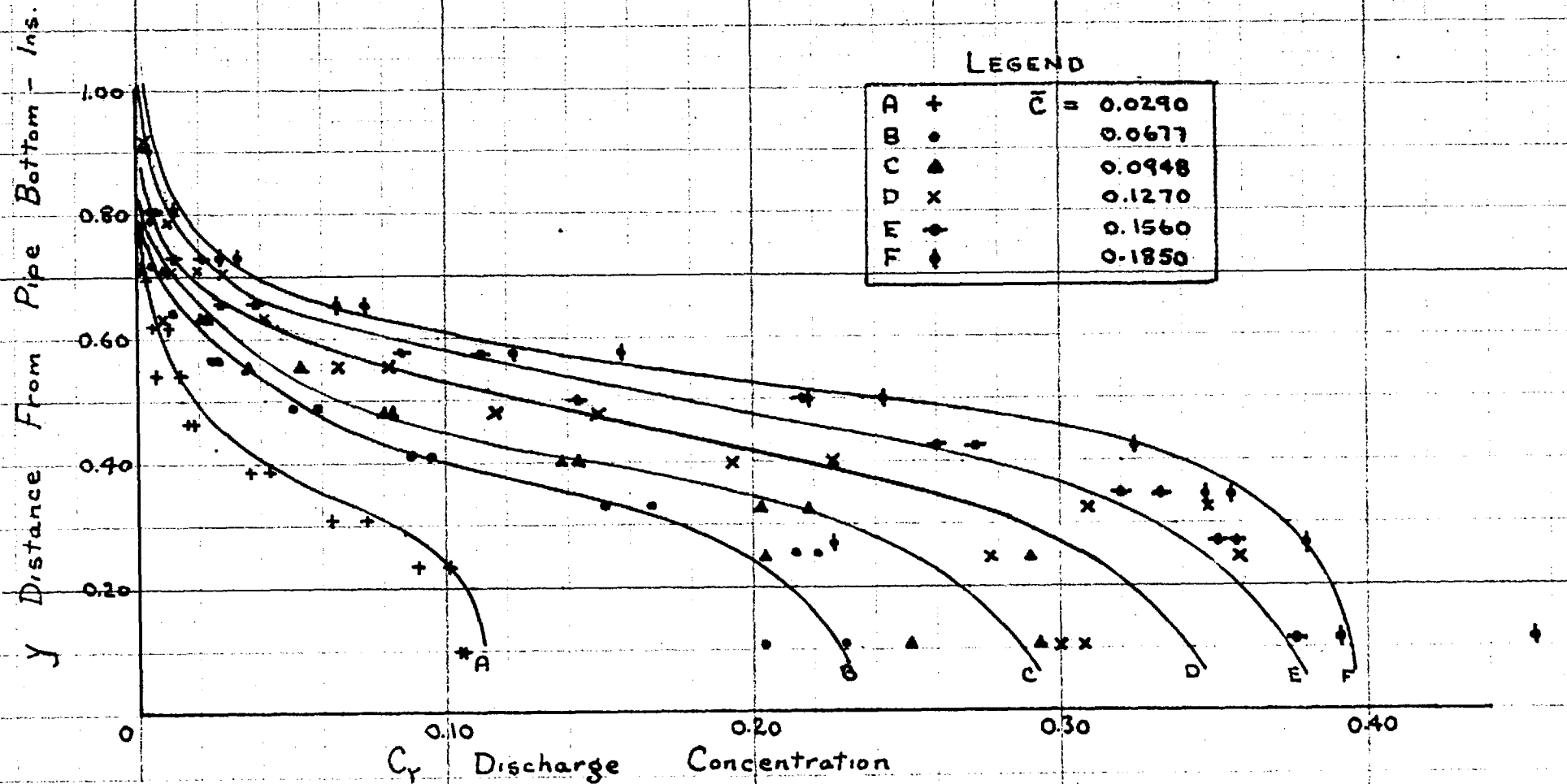
DISCHARGE CONCENTRATION PROFILES

COARSE SAND

$i = 0.400$ ft./ft.

LEGEND

A	+	$\bar{C} = 0.0290$
B	•	0.0677
C	▲	0.0948
D	x	0.1270
E	◆	0.1560
F	†	0.1850



solids, as described in Chapter III. Sand particles of this size are large enough for the mechanism of particle suspension to be that described by Bagnold and the general form of the concentration profiles appears to agree with his predictions, particularly when the solids concentration was high and/or the pressure gradient low.

It is evident that the addition of solids to the system causes particles to travel in successively higher regions of the pipe. Consequently, in these regions the concentration at a given point increases rapidly with $\bar{C}$ once a critical value of mean concentration is passed. In addition, it is apparent that the second derivative of point concentration with respect to mean concentration is negative in the lowest areas of the pipe. Although it is obvious that loading of the pipe with solids to the point where the bed ceases to move forward must produce a situation whereby the experimental technique fails, the condition is interesting and merits special consideration. Fig. VII - 1 illustrates that between $\bar{C} = 0.0550$ and $\bar{C} = 0.0648$ the discharge concentration below $y = 0.20$ passes through a maximum. The concentration profile E obtained for the case where $\bar{C} = 0.083$ illustrates that the decrease of local concentration with $\bar{C}$ in this region is rapid once the point of maximum C_y has been passed. Under these conditions the bed was observed to travel forward in jerks with a velocity of about 0.5 ft./second. If there were no slip between fluid and particles in the region

of high solids concentration, this tendency to produce a maximum in the relationship between C_y and $\bar{C}$ would not be observed; the local concentration would merely increase with the addition of solids until it reached its limiting value and the bed stopped. It can be concluded therefore that a positive indication of the existence of local slip within a bed has been obtained.

In addition, at values of discharge concentration between 0.0648 and 0.083, as the bed slows its forward movement a dramatic increase in the solids concentration in the upper half of the pipe is apparent. If it were generally observed that the presence of particles at a considerable distance from the bed accompanied its stoppage, the following hypothesis could be justified. Bagnold (46) has pointed out that the condition necessary for a bed of particles to move is that

$$\frac{\partial}{\partial y} (\text{Applied Stress}) \geq \frac{\partial}{\partial y} (\text{Resisting Stress}) \quad (1)$$

where the applied and the resisting stresses are those discussed in Section III - 6. If the applied stress on the particles is confined to those regions where particles are actually present, the observation that solids added to a conveying system tend to be transported at higher regions of the pipe would suggest that an increase in $\bar{C}$ reduced the applied stress gradient. Once solids were actually present at the top of the pipe, however, any further reduction of

the gradient of the applied stress could only be produced by a reduction of the applied stress on the particles. The simplest way in which this could be produced would be for the solid particles to diminish the forward momentum of the fluid. When the bed finally ceased its forward movement, solids added to the system could only be transported when a fraction of their number had so raised the level of the bed that Equation VII (1) was satisfied. Under these conditions, the system would become less and less efficient as a hydraulic conveying device. This fact was noticed macroscopically by Abbott and Turtle.

From this hypothesis it would appear that the effect of adding solids to the system would be to reduce the applied stress gradient in two ways:

- (1) By increasing the vertical distance over which the stress on the solids is applied,
- (2) By diminishing the forward momentum of the fluid and so reducing the applied stress itself.

When the applied stress is known as a function of solids concentration, pressure gradient, solids properties and position within the pipe, it should be possible to attack the problem of hydraulic conveying in pipelines theoretically from the microscopic point of view. The special difficulties introduced by the fact that the pipe has a circular cross-section may prove intractable. It is probable that an analysis based upon the two-dimensional system of a closed

rectangular channel would prove applicable to circular pipes, but empirical modification may be necessary.

(b) Water Velocity Profiles.

Figures VII - 4 to 6 and AVII - 1 to 13 presenting the data shown in Tables A16 to A18 for each of the standard hydraulic gradients, illustrate that the presence of solid particles in the pipe tended to reduce water velocity and that this reduction was greatest in those regions near to or where the solids concentration was high. It is not surprising that no direct relationship between local concentration and reduction in fluid velocity is apparent since the existence of a region of high solids concentration would be expected to reduce the water velocity in neighbouring areas even if they were free from particles.

It is apparent that the agreement between values of water velocity as measured by pitot impact pressure and suction pressure was fair only in the upper half of the pipe - the blanks left in the tabulated data refer to cases where the negative pressure phenomenon mentioned in Section V - 5 was observed. From the graphs it would appear that the calibration coefficient for the suction pressure velocities where solids are present does not produce the accuracy usually associated with the use of a pitot tube. Since the deviation was greatest near the centre of the pipe and appeared to increase with pressure gradient, the hypothesis advanced in

FIG. VII-4

SMOOTHED WATER VELOCITY PROFILES

AT VARIOUS DISCHARGE CONCENTRATIONS

COARSE SAND

$i = 0.160$ ft./ft.

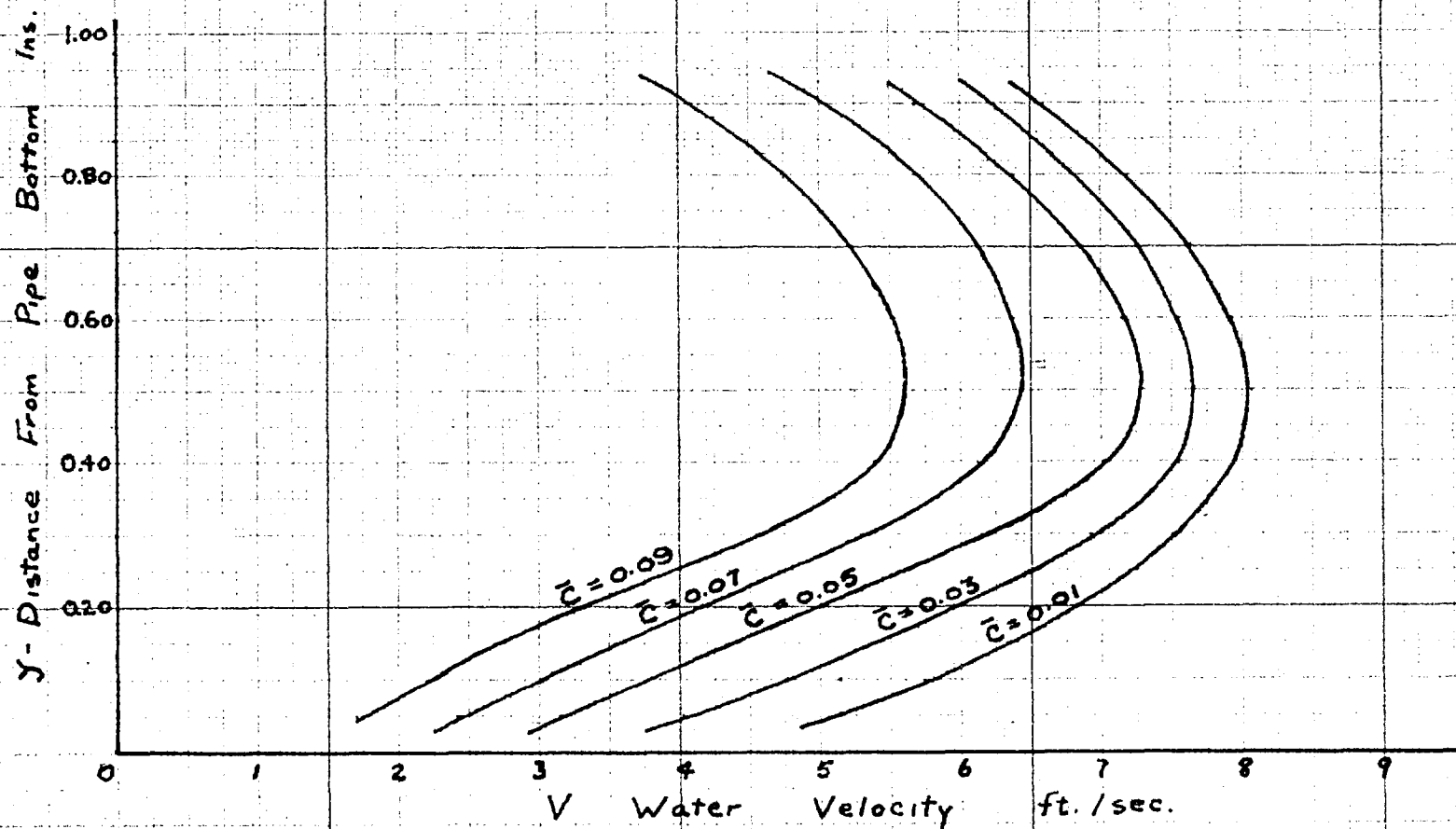


FIG. VII-5

SMOOTHED WATER VELOCITY PROFILES
AT VARIOUS DISCHARGE CONCENTRATIONS

COARSE SAND

$i = 0.280$ ft./ft.

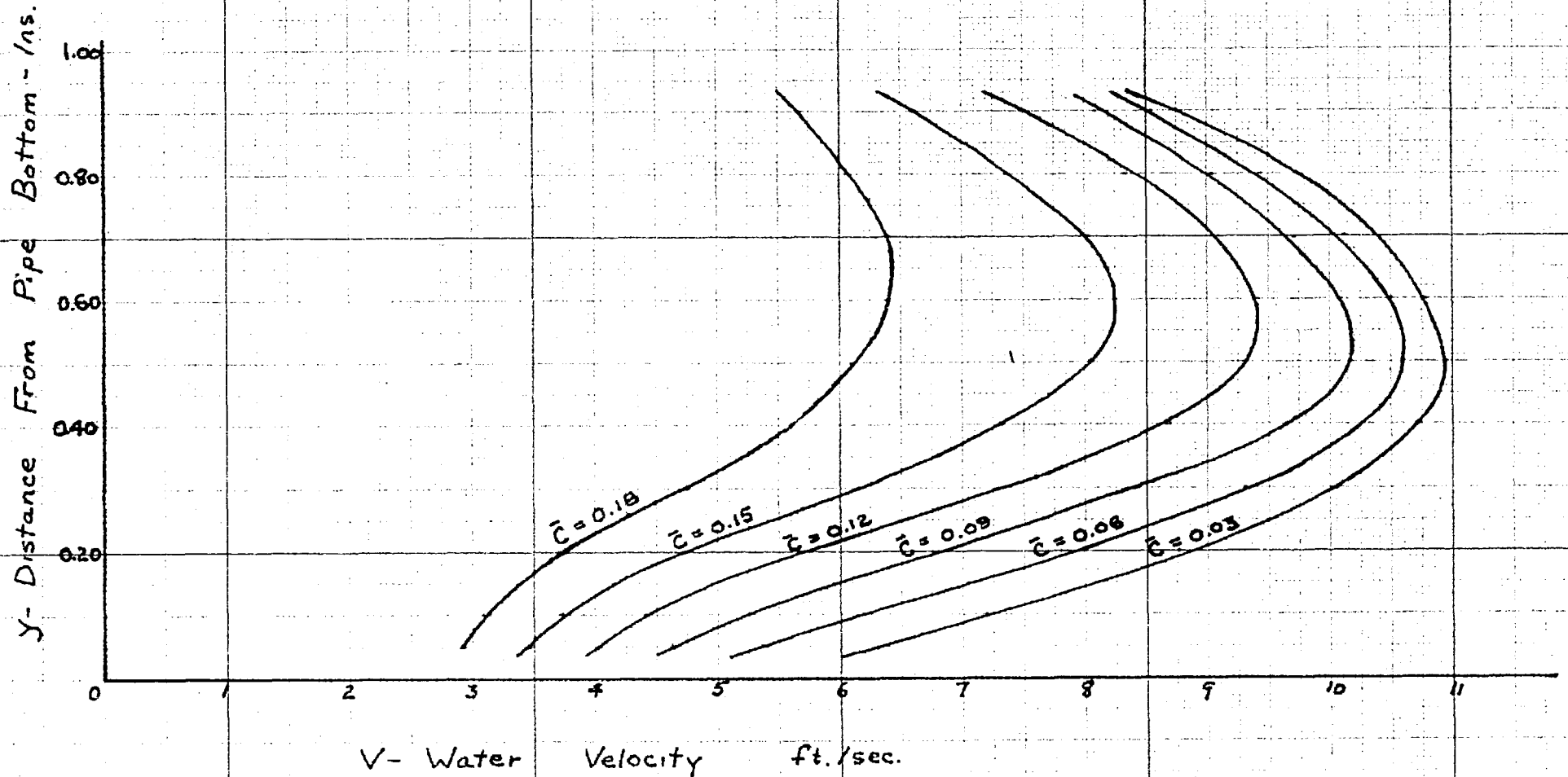
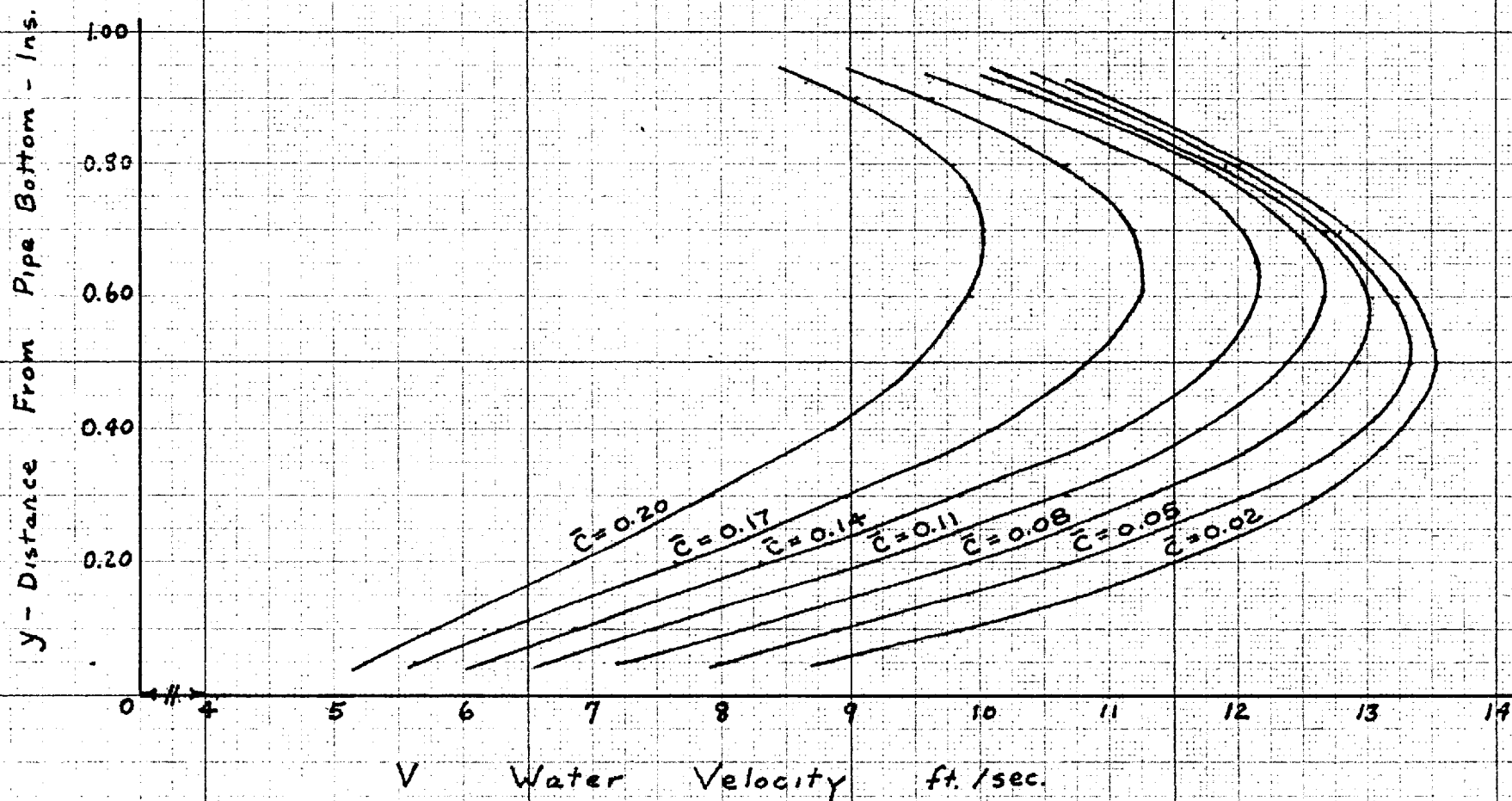


FIG. VII-6

SMOOTHED WATER VELOCITY PROFILES
AT VARIOUS DISCHARGE CONCENTRATIONS
COARSE SAND $i = 0.400$ ft./ft.



Section VI - 6 would appear to be substantiated. Reference to the smoothed plots, Figures VII - 4, 5 and 6, shows that the velocity gradient was altered most near the centre of the pipe and at high pressure drops. Bagnold (46) suggested that the water velocity profile would pass through a point of inflection in the lowest regions of the conduit when the local solids concentration was high. Some indication of this effect was obtained at the lower pressure gradients. It would appear that the point of maximum water velocity was raised by the addition of solids and by increasing the pressure gradient along the pipe. The reduction of the maximum velocity appeared to be greatest when the solids concentration was high and the pressure gradient low. Although the discharge concentrations for the case where $i = 0.160 \frac{\text{ft}}{\text{ft}}$ were fairly low, it will be shown in a later section that the transport concentrations were high and at the extreme conditions of $\bar{C} = 0.09$ the turbulent action of the fluid was probably damped to a considerable extent.

(c) Mean Velocity Measurements.

Figure VII - 7 presents the mean values of mixture discharge velocity and water velocity obtained as functions of the mean discharge solids concentration for the three standard pressure drops. The mixture velocity was found to decrease with $\bar{C}$ in the same way observed by Abbott and Turtle while a difference between $\bar{V}_m$ and $\bar{V}_w$ was indicated under all

FIG. VII-7

Effect of Solids
Concentration Upon
Mixture and Water
Velocities.

Coarse Sand

+ $\bar{V}_w$
• $\bar{V}_m$

$\bar{C}$ Discharge
Concentration of Solids.

0

1

2

3

4

5

6

7

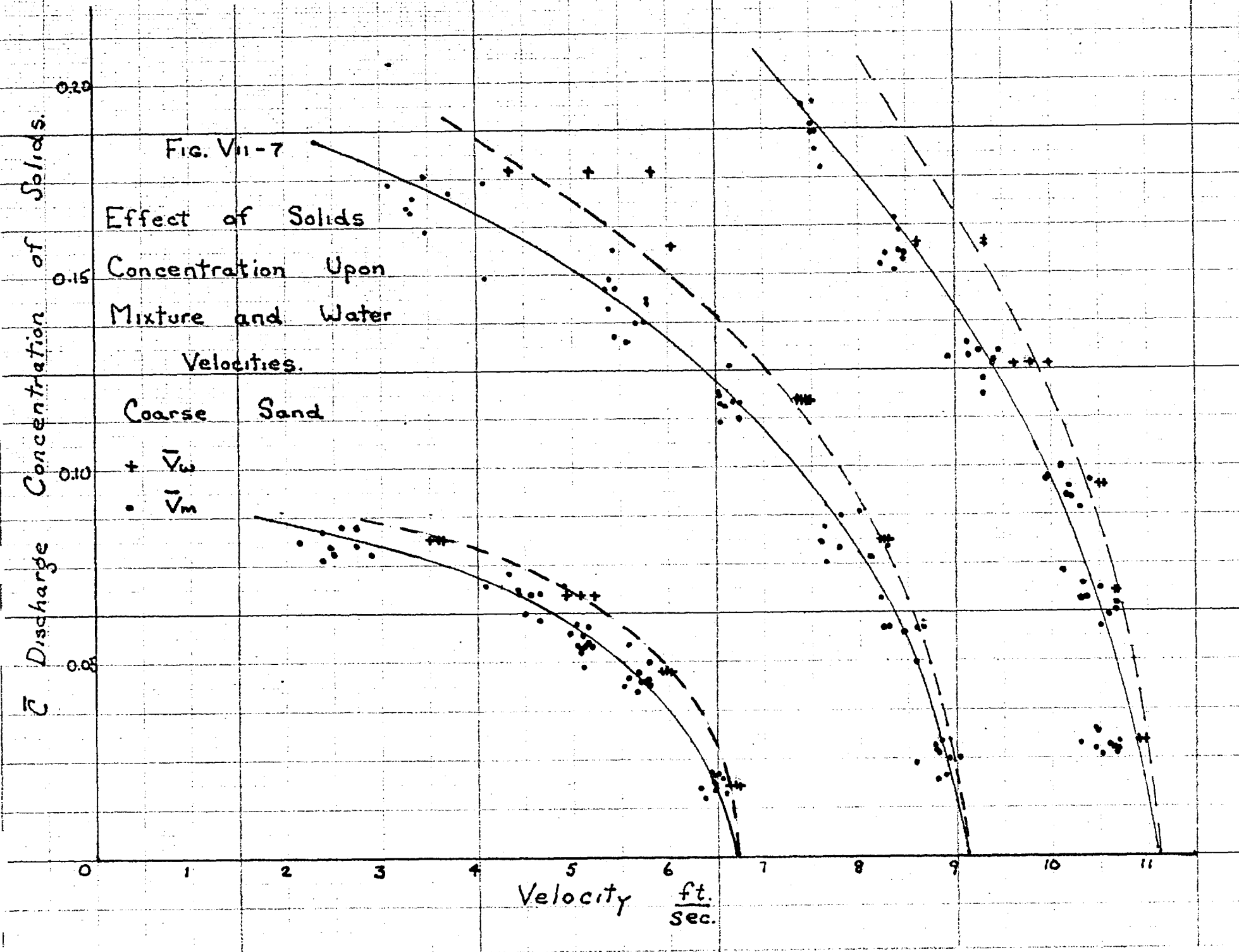
8

9

10

11

Velocity $\frac{\text{ft.}}{\text{sec.}}$



conditions. It was therefore possible for the mean slip $\bar{V}_w - \bar{V}_s$ to be calculated. The results will be shown, together with an empirical correlation, in Chapter VIII.

3. Gravel, (d = 0.061 inches).

(a) Concentration Profiles.

The data presented in Tables A4 to 6, and Figs. VII - 8, 9 and 10, show that the general form of the concentration profiles obtained for gravel of this size was much the same as for coarse sand (d = 0.0275"). At the two higher pressure gradients there was some indication that the larger particles appeared to be rather more evenly distributed over the cross-section. In the lowest regions of the pipe this could be a consequence of higher local slip reducing the discharge concentration. Above $y = 0.50$ " the phenomenon would be consistent with the existence of saltation, although the diameter of the particles was rather less than the minimum (2.0 mm.) quoted by Durand for saltation. Nevertheless, it is large enough for the shearing process as described by Bagnold to be predominantly inertial, and it is quite likely that the particles did travel by saltation. It may be seen that the behaviour of C_y at a given point as a function of $\bar{C}$ is much the same as that observed for coarse sand in both the upper and the lower regions of the pipe.

FIG. VII-8

DISCHARGE CONCENTRATION PROFILES
GRAVEL $i = 0.160 \text{ ft./ft.}$

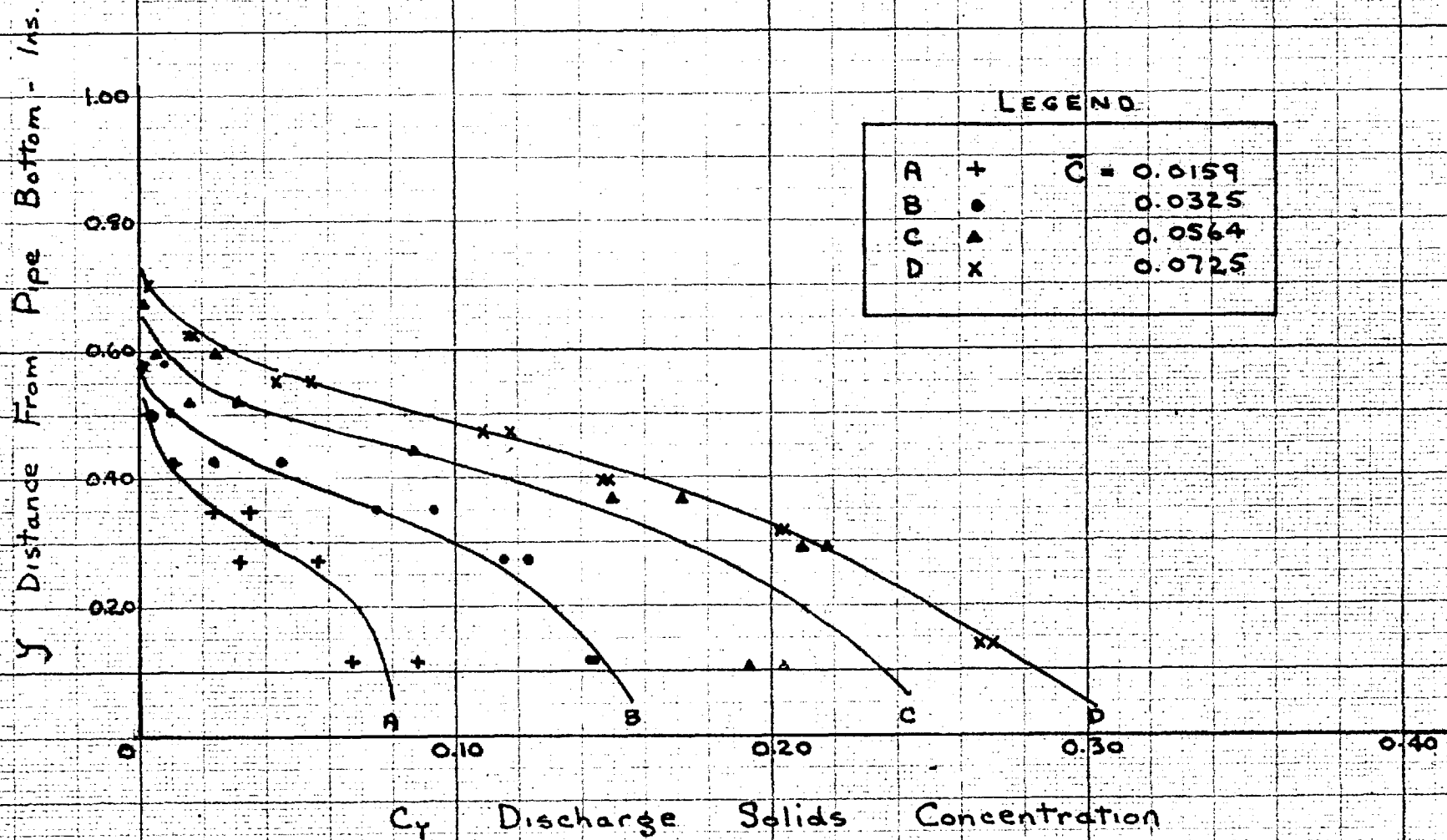


FIG. VII-9

DISCHARGE CONCENTRATION PROFILES

GRAVEL

$\bar{L} = 0.280 \text{ ft./ft.}$

LEGEND

A	+	$\bar{C} = 0.0205$
B	•	0.0377
C	▲	0.0677
D	x	0.0911
E	◆	0.1155

y Distance From Pipe Bottom Ins

1.00
0.80
0.60
0.40
0.20
0

C_y Discharge Solids Concentration

0.10

0.20

0.30

0.40

A

B

C

D

E

FIG. VII-10

DISCHARGE CONCENTRATION PROFILES

GRAVEL

$i = 0.400$ ft./ft.

LEGEND

A	+	$\bar{C} = 0.0203$
B	•	0.0394
C	▲	0.0655
D	x	0.0881
E	⊕	0.1093

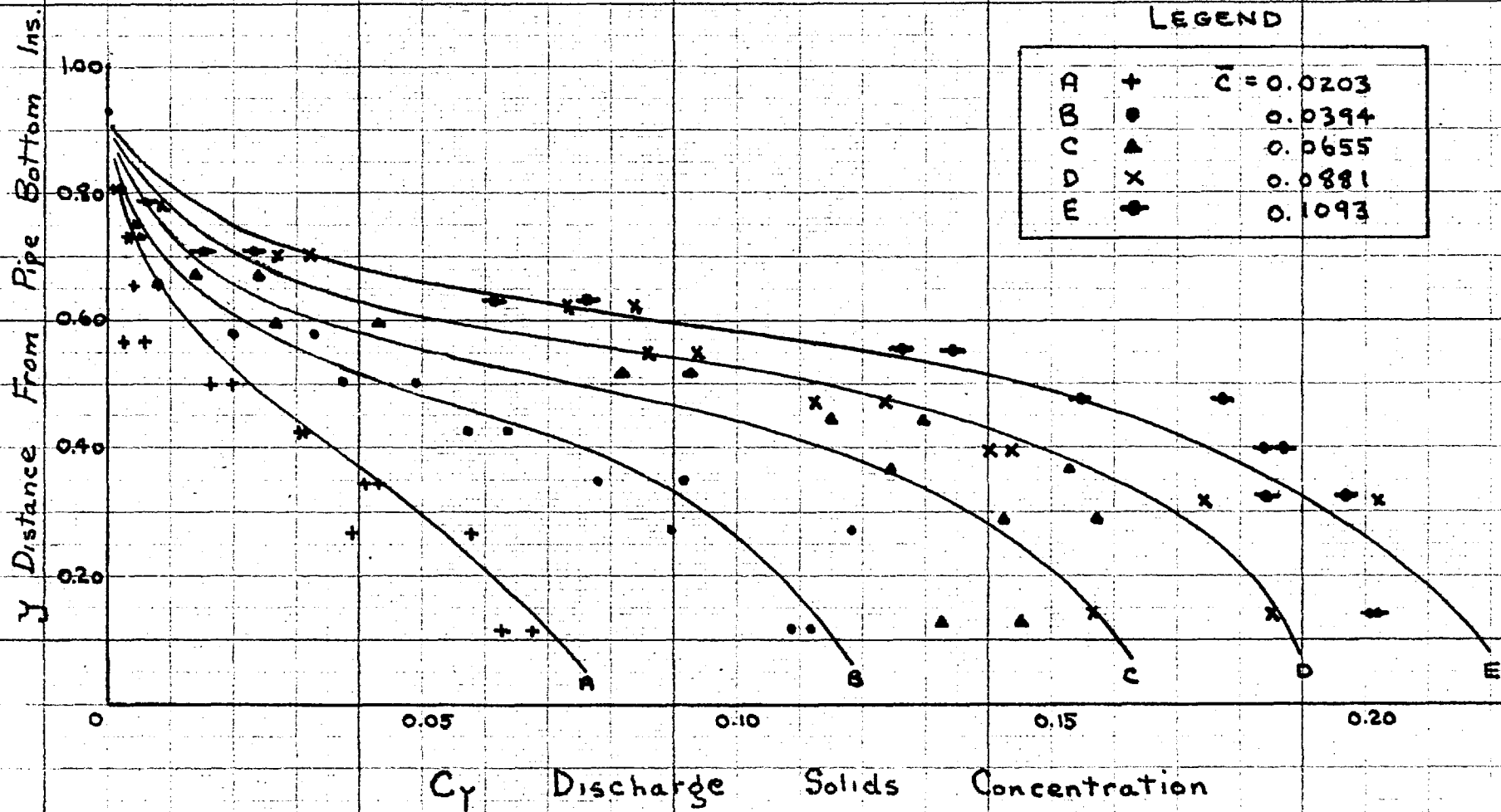


FIG. VII-11

SMOOTHED WATER VELOCITY PROFILES AT VARIOUS
DISCHARGE CONCENTRATIONS

GRAVEL

$i = 0.280$ ft./ft.

y - Distance From Pipe Bottom - hs .

1.00

0.80

0.60

0.40

0.20

0

1

2

3

4

5

6

7

8

9

10

11

V

Water

Velocity

ft./sec.

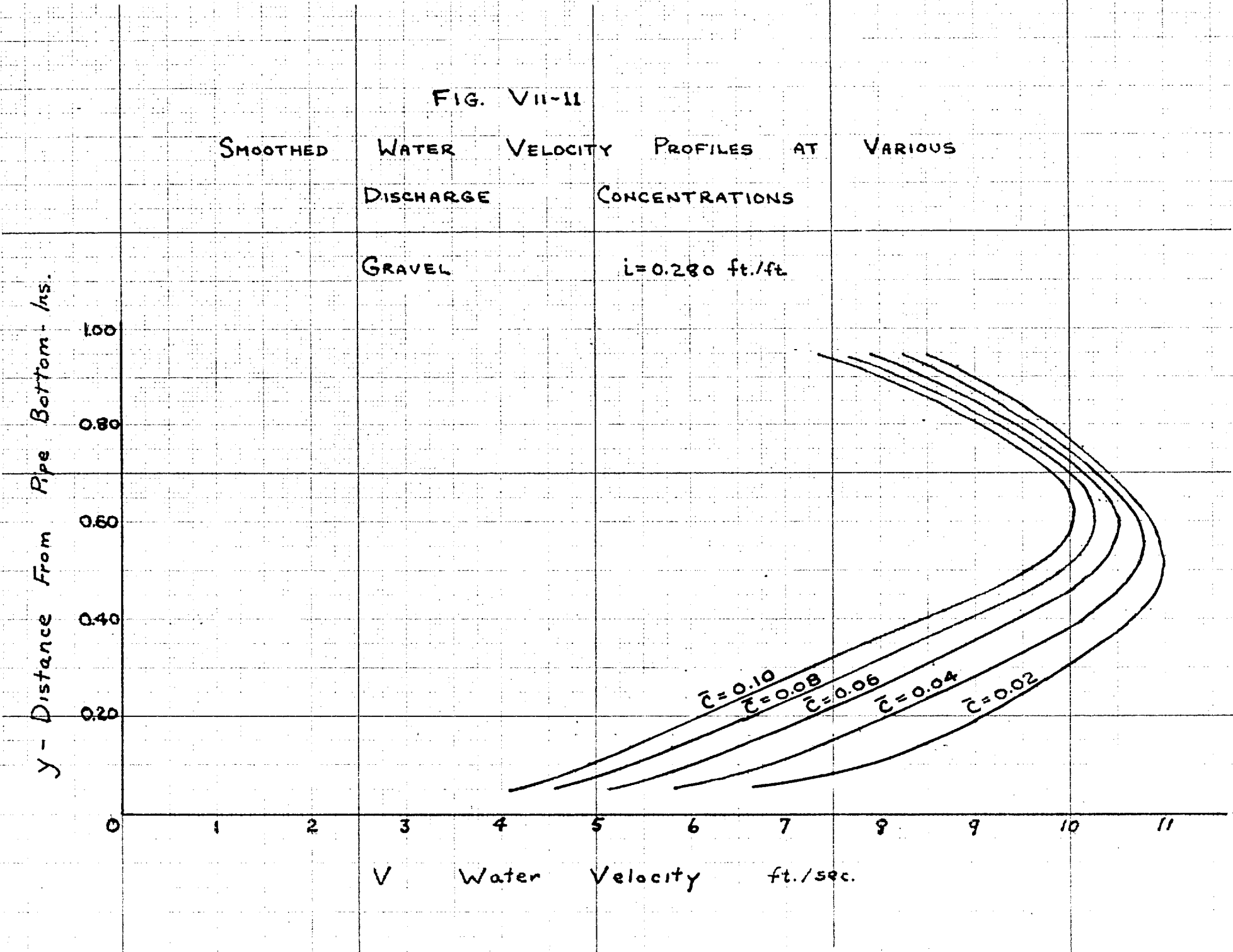
$\bar{c} = 0.10$

$\bar{c} = 0.08$

$\bar{c} = 0.06$

$\bar{c} = 0.04$

$\bar{c} = 0.02$



(b) Water Velocity Profiles.

Figures AVII - 14 to 26 and VII - 11 presenting the data of Table A19, illustrate that the presence of solid particles modifies the fluid velocity profile in much the same way as for the coarse sand. The abrasion of the pitot tube produced by these particles was severe and experiments were restricted to suction pressure measurements at a pressure gradient of $0.280 \frac{\text{ft}}{\text{ft}}$. It is particularly unfortunate that the pitot tube did not last long enough for impact pressure readings to be taken in the upper part of the pipe, for the values of water velocity obtained in this region were most unsatisfactory. Nevertheless, a consistent indication of the reduction of local water velocity by increasing $\bar{C}$ was obtained and the smoothed velocity profiles shown in Fig. VII - 11 are probably not much in error. The effect of mean concentration upon the maximum water velocity was much the same as that observed for coarse sand. No indication of the predicted point of inflection in the water velocity profiles was observed, possibly because the mean concentration achieved was not high enough.

(c) Mean Velocities.

Figure VII - 12 illustrates that although the scatter of the experimental data was large, because of the difficulty experienced in maintaining a constant pressure gradient

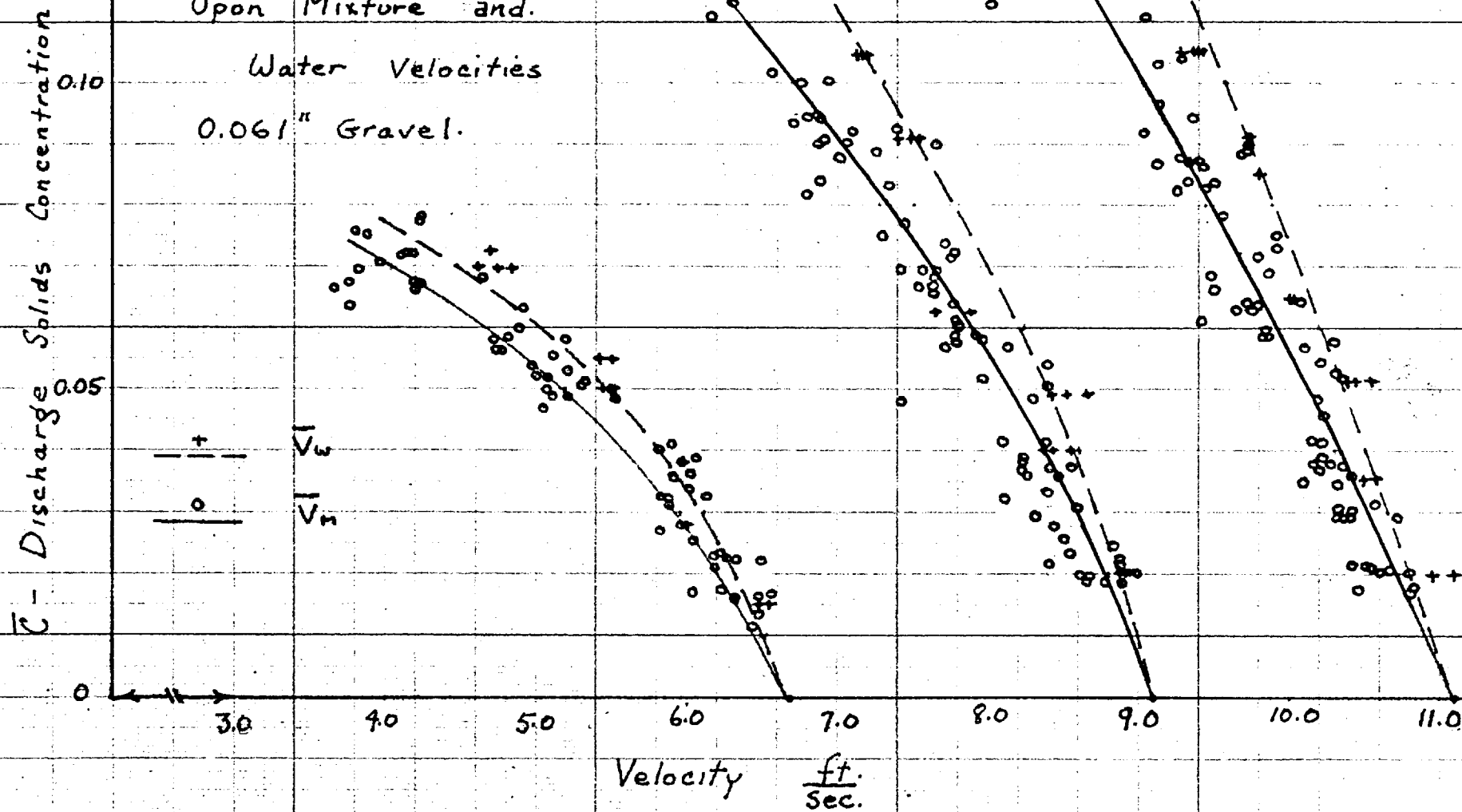
Fig. VII-12

Effect of Solids Concentration

Upon Mixture and

Water Velocities

0.061" Gravel.



during a sampling interval, a consistent indication of a difference between $\bar{V}_m$ and $\bar{V}_w$ was obtained. It was therefore possible to compute the mean solids velocity and the mean slip $\bar{V}_w - \bar{V}_s$ by the method described in Section III - 2. As for coarse sand, these results will be shown, together with an empirical correlation, in Chapter VIII.

4. Perspex, (d = 0.073 inches).

(a) Concentration Profiles.

Reference to Figs. VII - 13, 14 and 15, presenting the data contained in Tables A7, A8 and A9, illustrates that concentration profiles of a significantly different form were obtained with perspex. All the profiles exhibit a maximum point concentration at a distance from the bottom of the pipe; this distance increases with both pressure gradient and the addition of solids. The phenomenon was first observed by Garrett (57) using the first flow divider described in Chapter V, and was attributed to the presence of a large local slip velocity in the lowest regions of the pipe. If this were in fact the case, the transport concentration would not necessarily show a reversal. Unfortunately the techniques for measuring the water velocity in this investigation accomplished little more than to indicate that the water velocity profile was not affected much by the presence of the solids.

FIG. VII-13

DISCHARGE CONCENTRATION PROFILES

PERSPEX

$l = 0.160$ ft./ft.

LEGEND

A	+	C = 0.0291
B	•	0.0399
C	▲	0.0626
D	*	0.0800

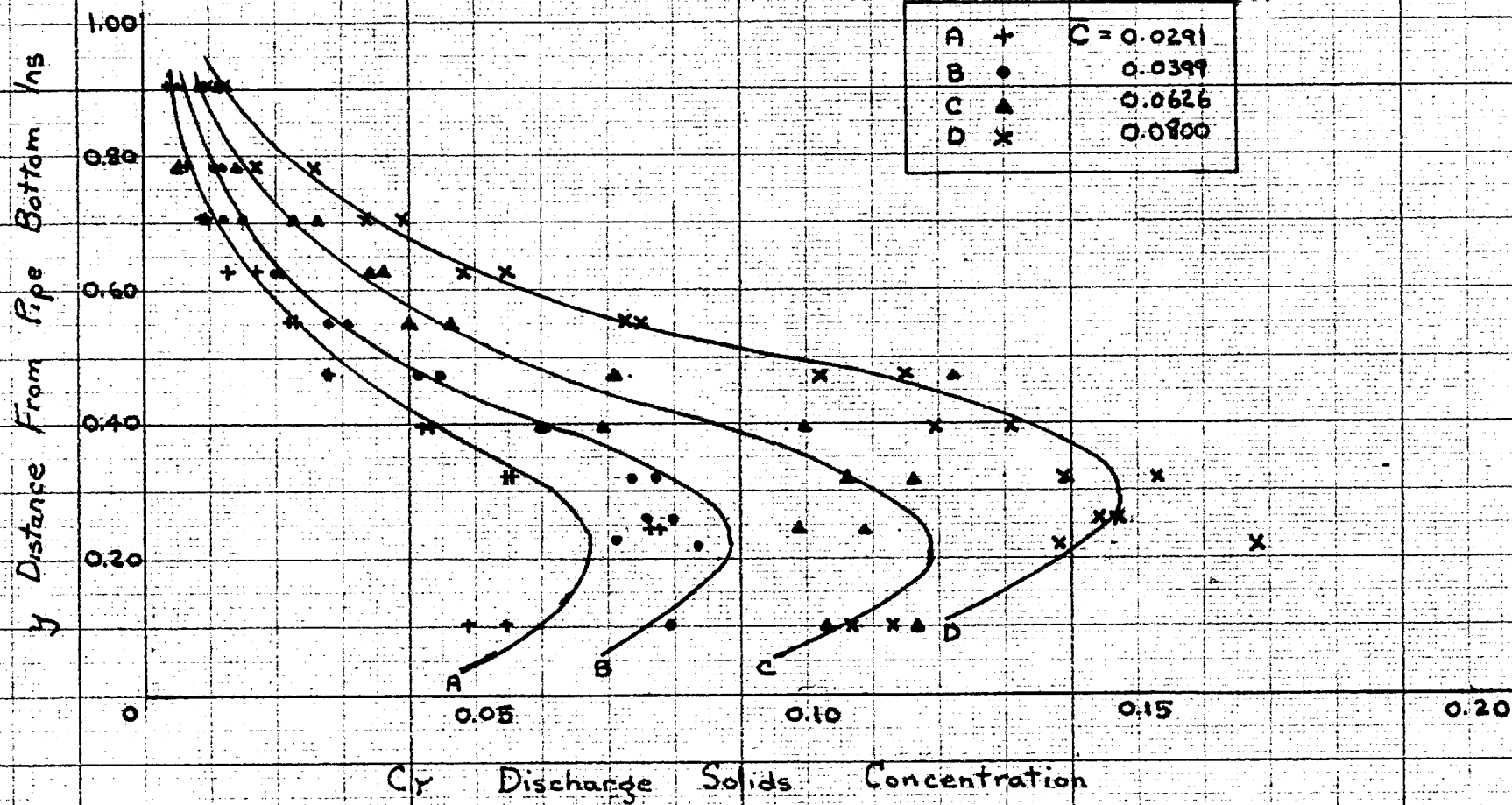


FIG. VII-14

DISCHARGE CONCENTRATION PROFILES

PERSPEX

$L = 0.280 \text{ ft./ft.}$

LEGEND

A	+	C	=	0.0282
B	•			0.0432
C	▲			0.0659
D	x			0.0875

y Distance From Pipe Bottom Ins

100
0.80
0.60
0.40
0.20
0

0.05

0.10

0.15

0.20

C_y Discharge Solids Concentration

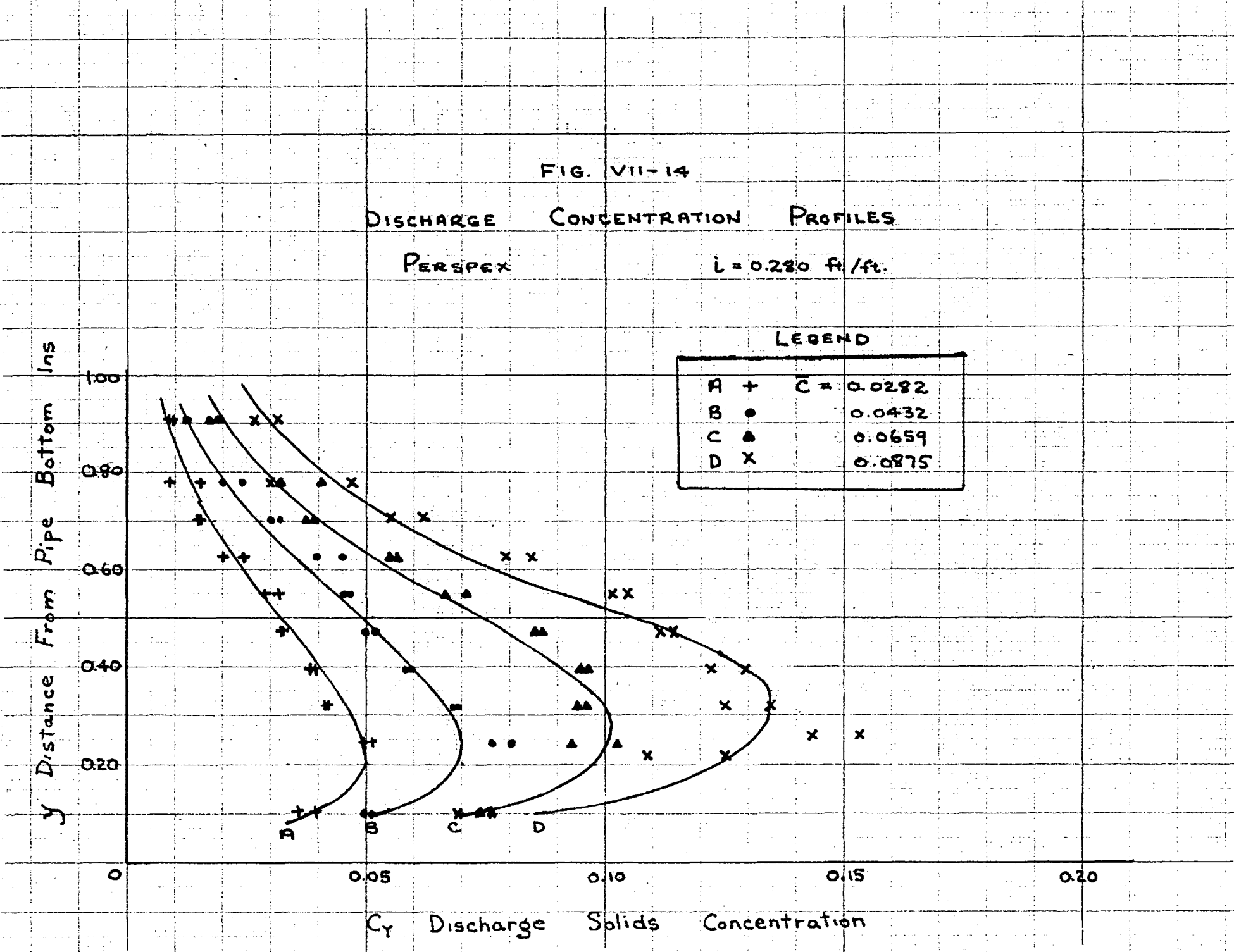


FIG. VII-15

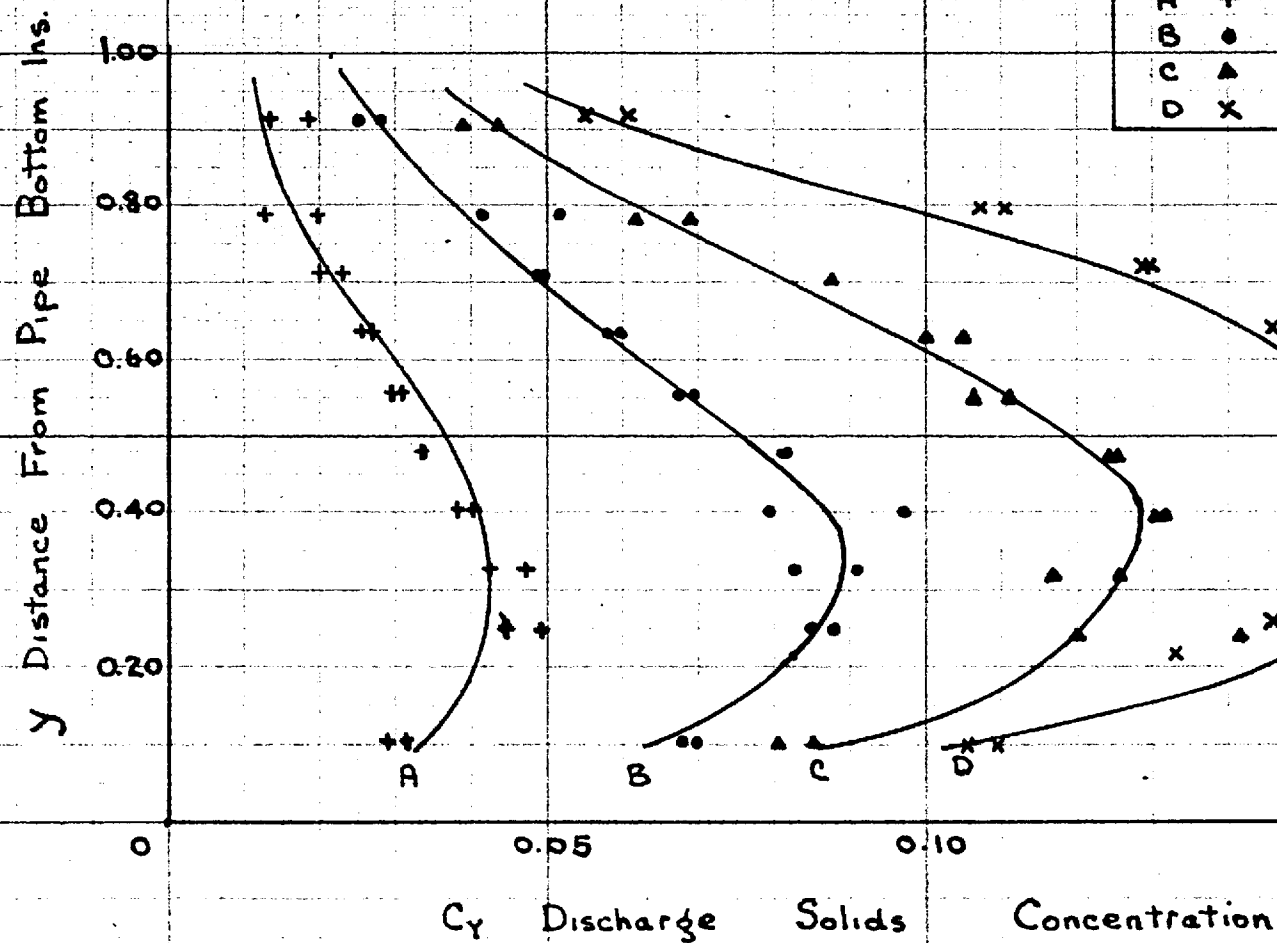
DISCHARGE CONCENTRATION PROFILES

PERSPEX

$i = 0.400$ ft./ft.

LEGEND

A	+	$\bar{C} = 0.0295$
B	•	0.0645
C	▲	0.0983
D	x	0.1330



It has been shown in Section III - 5 that particles of this relatively large diameter and low specific gravity (1.17) would be most likely to be affected by the lift force resulting from spin. Since the rotation of the particles must be caused in the first instance by the shape of the fluid velocity profile, one would expect that the effects of spin would be considerably different above and below the plane of maximum water velocity. Further, any decrease in local slip velocity from the bottom to the centre of the pipe would tend to improve the transporting qualities in the latter region at the expense of the former by decreasing the rotational speed necessary for lift, or if the lift were actually experienced, increase it considerably. It may be seen therefore that the particular qualities of this material make it quite likely that the transport concentration does not increase rapidly in the lower half of the pipe. Further, the dangers inherent in the quotation of particle diameter as a criterion of flow behaviour are made apparent by experiments under these extreme conditions. A blind interpretation of Durand's criteria quoted in Chapter II might lead to the supposition that this material travelled by saltation. Bagnold, however, showed that saltation was probably a manifestation of the effects of particle inertia vis-a-vis diameter upon the development of drag upon a particle.

The effect of particle density upon the discharge concentration profiles for the coarser materials shows that a

device for the continuous separation of particles of different density could be designed on the principle of the knife-edge of the flow divider used in these investigations. Such a device could find application in the separation of rock from coal.

(b) Water Velocity Profiles.

Attempts were made to determine the water velocity profiles accompanying transport of these particles by both the impact and suction pressure methods. It was discovered that whereas local values of water velocity by suction pressure were too large, being greater than the values for pure water at all points in the pipe, those obtained from impact pressure appeared to be slightly lower than the discharged mixture velocity. Since the effect of solids concentration upon the mixture velocity was small, it could only be concluded that the presence of the solids modified the water velocity profile only slightly.

(c) Mean Velocities.

Figure VII - 16 illustrates that the scatter of experimental data observed with this material was large, for the reasons mentioned previously. The mean velocities were observed to decrease only slightly with $\bar{C}$ and no difference between $\bar{V}_w$ and $\bar{V}_m$ could be determined to the degree of accuracy necessary to justify the calculation of $\bar{V}_w - \bar{V}_s$.

Fig. VII-16

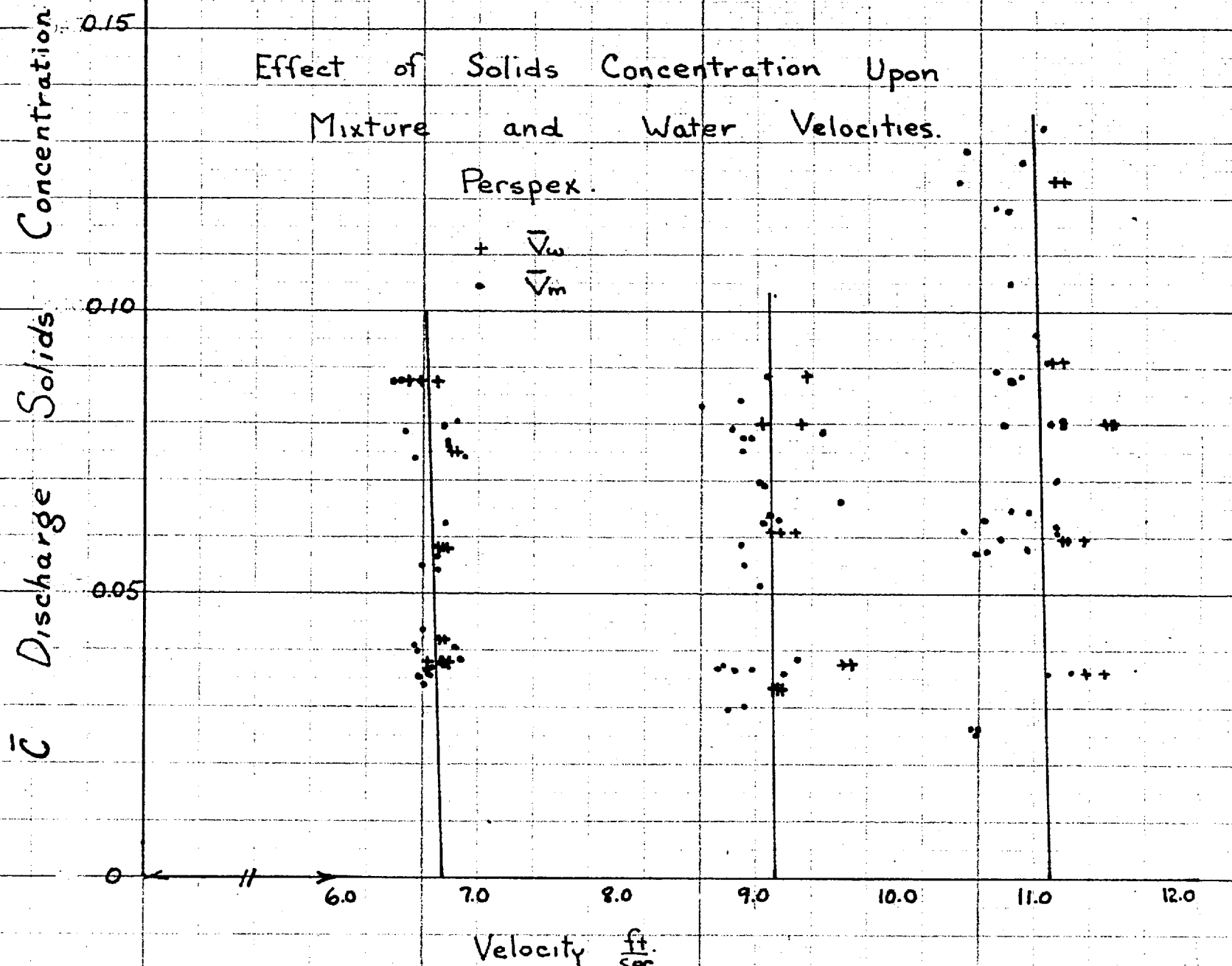
Concentration
Solids
Discharge
 $\bar{C}$

Effect of Solids Concentration Upon
Mixture and Water Velocities.

Perspex.

+ $\bar{V}_w$
• $\bar{V}_m$

Velocity $\frac{\text{ft.}}{\text{sec.}}$



The observation made in the preceding subsection, that the water velocity profile was relatively unaffected by the addition of solids, would appear to be justified. It can only be concluded that the local slip velocity $V_w - V_s$ was small, and that consequently the mechanism of spin could easily have contributed to the lift of the particles.

5. Fine Sand, ($d = 0.0073$ inches).

(a) Concentration Profiles.

Tables A10 to 12 and Figs. VII - 17, 18 and 19 illustrate that although the diameter of this material was roughly one and a half times that of the zircon sand dealt with in the next subsection, it was much more evenly distributed over the cross-section. The effect of increasing pressure gradient was seen to be the same as that observed with the other materials - to reduce the rate of change of C_y with y . Generally speaking C_y at any point appeared to increase fairly uniformly with $\bar{C}$ until, between $\bar{C} = 0.17$ and 0.20 at the higher pressure gradients, the rate of change of C_y with $\bar{C}$ decreased in the lowest regions and increased in the highest. This is much the same phenomenon, qualitatively, as that observed for the coarser silica particles. However, since the solids concentration was considerably lower than that associated with the formation of a bed, it was presumed that the effect was attributable to the

FIG. VII-17

DISCHARGE CONCENTRATION PROFILES

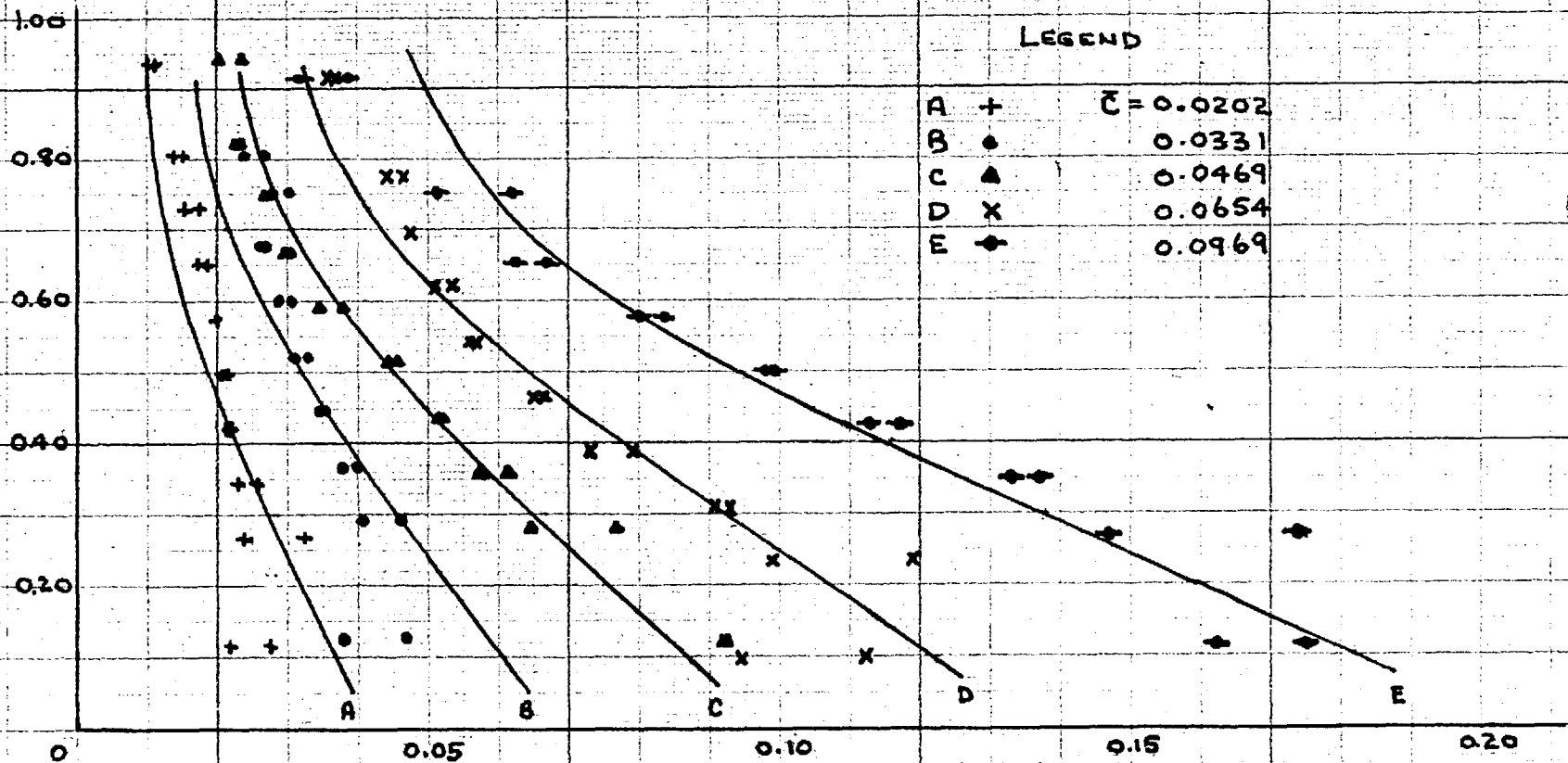
FINE SAND

$i = 0.160$ ft./ft.

y Distance From Pipe Bottom Ins

LEGEND

A	+	$\bar{C} = 0.0202$
B	•	0.0331
C	▲	0.0469
D	x	0.0654
E	◆	0.0969



C_r Discharge Solids Concentration

FIG. VII-18

DISCHARGE SOLIDS CONCENTRATION

PROFILES

FINE SAND

$l = 0.280$ ft./ft.

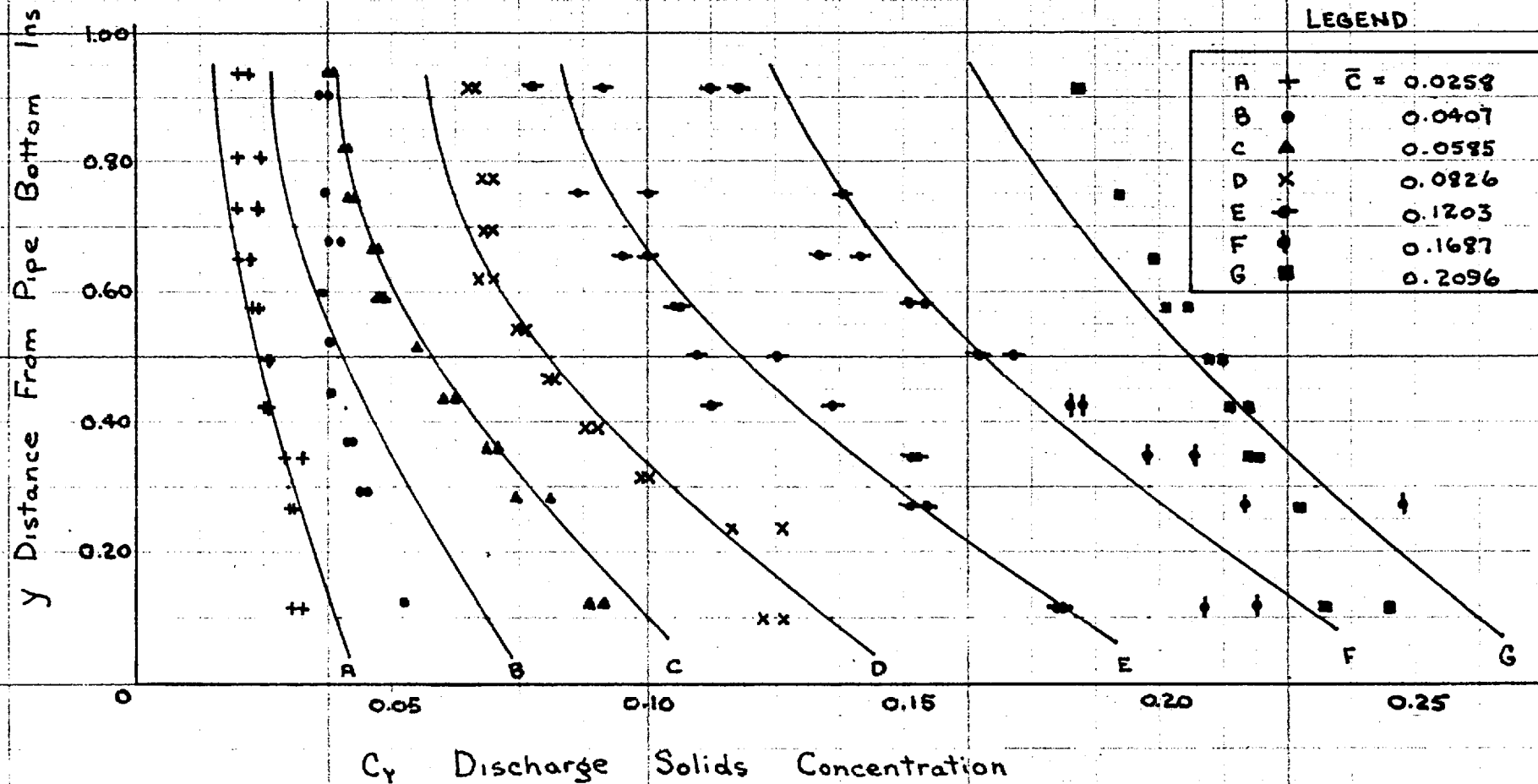


FIG. VII-19

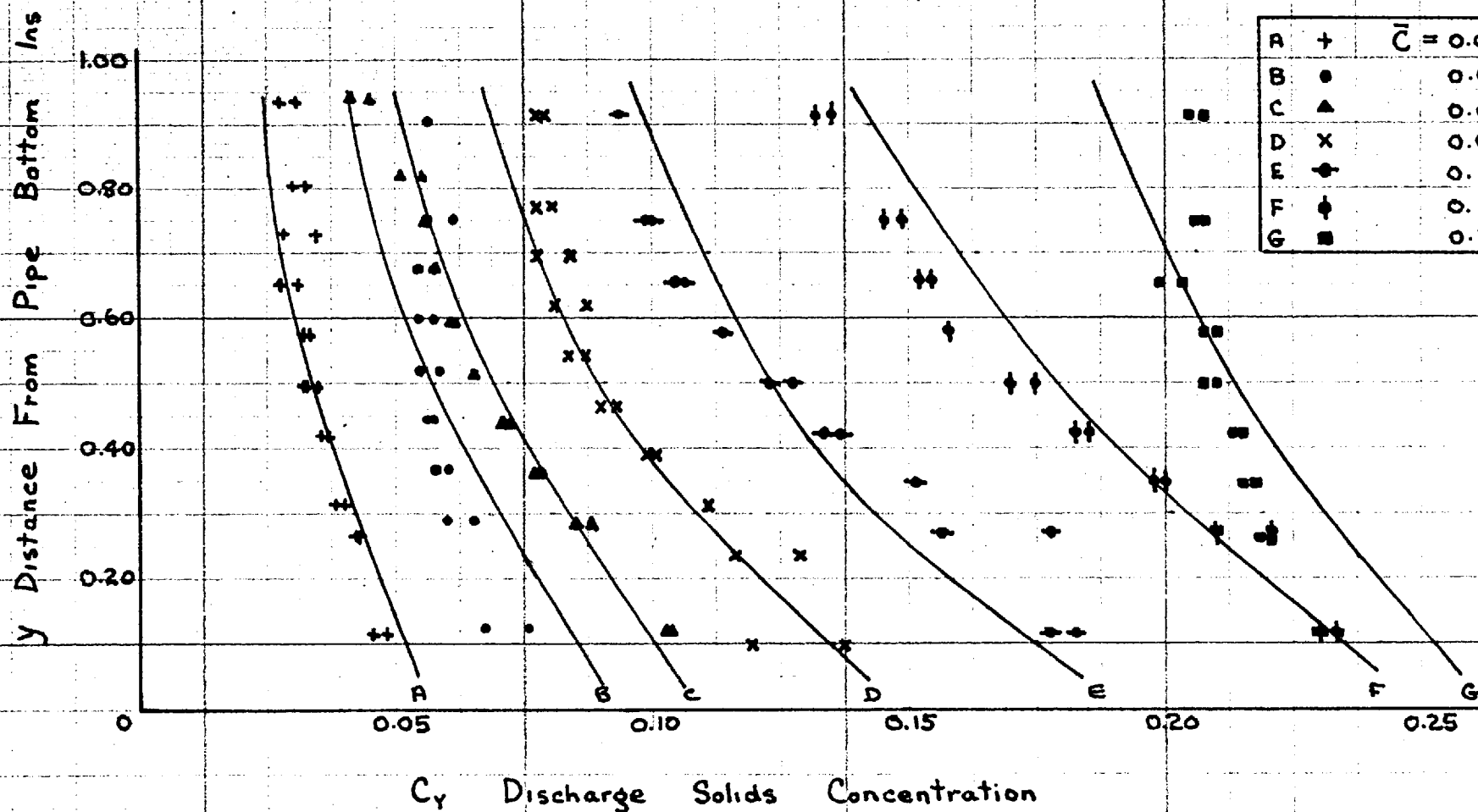
DISCHARGE CONCENTRATION PROFILES

FINE SAND

$l = 0.372 \text{ ft./ft.}$

LEGEND

A	+	$\bar{C} = 0.0358$
B	•	0.0586
C	▲	0.0686
D	x	0.0940
E	⊕	0.1264
F	⊙	0.1729
G	■	0.2133



modification of the settling velocity in the concentrated suspension.

Scatter of the order of $\pm 10\%$ was evident in the values of discharge concentration - undoubtedly a consequence of errors made in the positioning of the knife edge in the flow divider.

(b) Water Velocity Profiles.

The mean diameter of these particles (0.0073") was roughly half the diameter of the hole in the wall of the pitot tube and accordingly, measurements of impact pressure were impossible. Suction pressure readings were taken over the whole range of mean concentration but in view of the doubt as to whether the density of water or of the suspension should be used in the calculation of fluid velocities, these results have not been reported. If the density of the mixture as given by the concentration profiles were used to calculate the fluid velocity, a velocity profile whose maximum was about the same as the mean velocity determined by the salt injection method was obtained. If the density of pure water is used, a symmetrical water velocity profile is obtained and the measured mean value appears to justify this method of calculation. Nevertheless, solid particles often blocked the pitot tube and if these were present in the tube at all times, they would have reduced the suction pressure reading of the manometer. Accordingly, the values of water

Fig. VII-20

Effect of Solids
Concentration Upon
Mixture and Water
Velocities.

Fine Sand.

Concentration

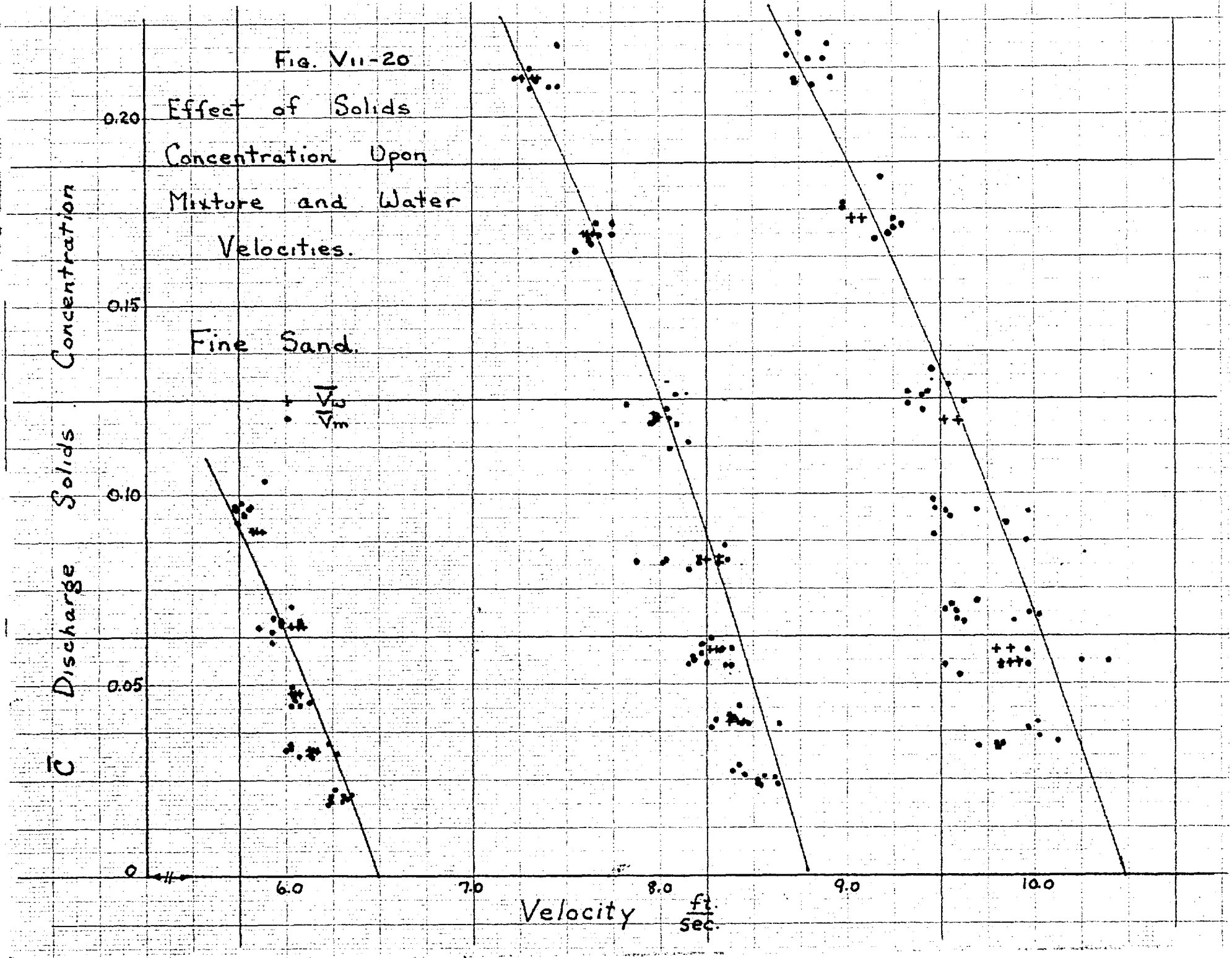
Solids

Discharge

$\bar{C}$

$\frac{V}{V_m}$

Velocity $\frac{\text{ft.}}{\text{sec.}}$



velocity obtained must be regarded with suspicion.

(c) Mean Velocities.

Figure VII - 20 illustrates that the mean velocities of both the mixture and the water decrease slowly with the addition of solids. Since no difference between $\bar{V}_w$ and $\bar{V}_m$ was apparent, it can only be concluded that within the limits of accuracy of the method no slip could be detected. Since the particles were fairly evenly distributed over the cross-section, the conclusion is not surprising. The absence of an appreciable slip velocity may be taken as an indication that the particles probably travelled in suspension. Some deviation of the order of 2 - 3% from the solid line is obvious at low concentrations because the mixture temperature under these conditions was low.

6. Zircon, (d = 0.0046 inches).

(a) Concentration Profiles.

The data contained in Tables A13 to 15 and presented in Figs. VII - 21, 22 and 23, illustrate that the solids concentration profiles for this material were as would be expected if the turbulent velocity fluctuations were responsible for suspending the particles. The settling velocity of this material was considerably less than the friction velocity, even at the lowest pressure drop. It would appear

FIG. VII-21

DISCHARGE

CONCENTRATION

PROFILES

ZIRCON

$i = 0.160$ ft/ft.

y Distance From Pipe Bottom Ins

LEGEND

A	+	$\bar{C} = 0.0071$
B	•	0.0152
C	▲	0.0222
D	x	0.0342

1.00
0.80
0.60
0.40
0.20
0

C_y

Discharge

Solids

Concentration

0.05

0.10

0.15

0.20

A

B

C

D

FIG. VII-22

DISCHARGE CONCENTRATION PROFILES

ZIRCON

$l = 0.280$ ft./ft.

LEGEND

A	+	$\bar{C} = 0.0063$
B	●	0.0179
C	▲	0.0251
D	x	0.0380
E	⊙	0.0571

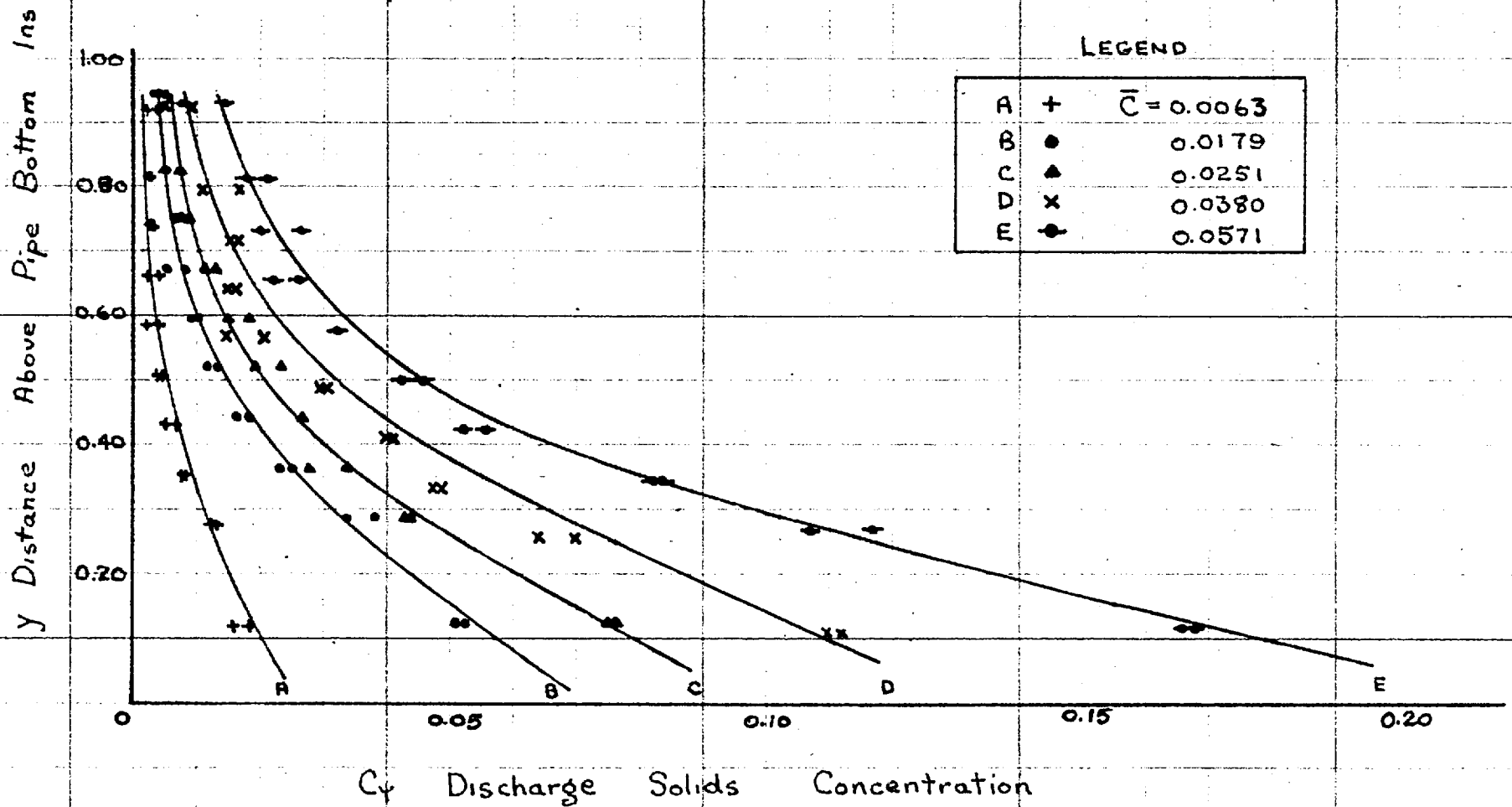
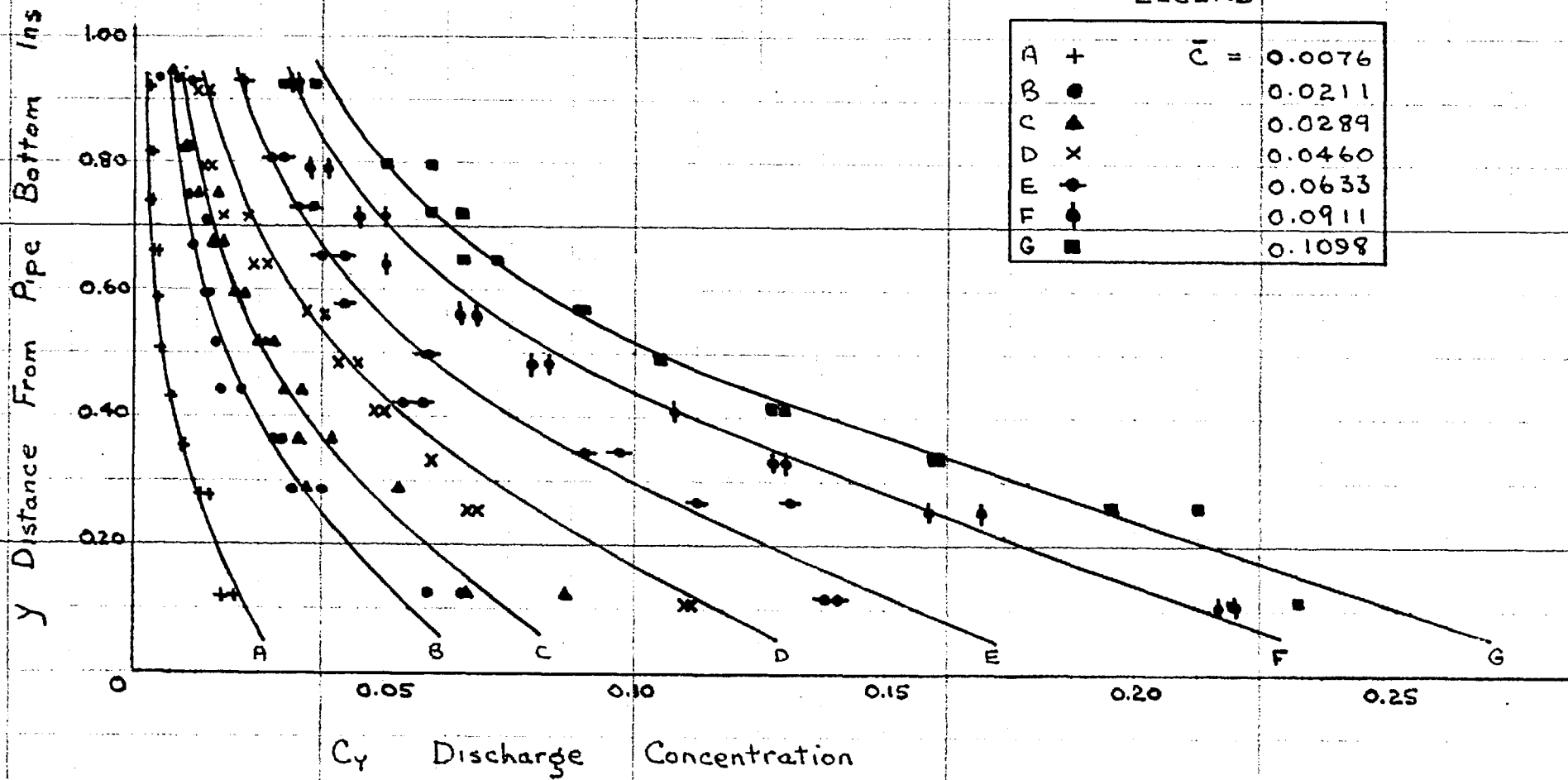


FIG. VII-23

DISCHARGE CONCENTRATION PROFILES

ZIRCON

$L = 0.400$ ft./ft.



that the effect of solids concentration upon fluid turbulence in the lowest regions of the pipe was not great enough to make the turbulent lifting velocity less than the settling velocity of the particles. The solids concentration at any point was observed to increase with $\bar{C}$ in a linear manner in the range studied. It was particularly unfortunate that the mean concentration could not be increased further without overloading the motor. The condition where a bed was just beginning to form would have been particularly interesting to study, since the transition from the fully suspended case to bed flow must inevitably occur.

The rapid rate of change of C_y with y observed for this material tended to exaggerate the difference, as seen through the sight glass, between the upper and lower regions of the pipe. The dangers inherent in macroscopic observations such as those made by Sinclair are obvious, for in this case the casual observer might be led to suppose that a bed existed even when the local concentration was not greater than 10%.

(b) Water Velocity Profiles.

Attempts to measure the fluid velocity by means of the pitot tube met with even less success than for the fine sand. The tube was constantly being plugged by particles and it was impossible to obtain consistent results. As with fine sand there was the suggestion that the particles were not

FIG. VII-24

Effect of Solids Concentration
Upon Mixture and Water
Velocities.

0.0046" ZIRCON

$\frac{V_m}{V_w}$

$\bar{C}$ - Discharge Solids Concentration

0.10

0.05

0

60

70

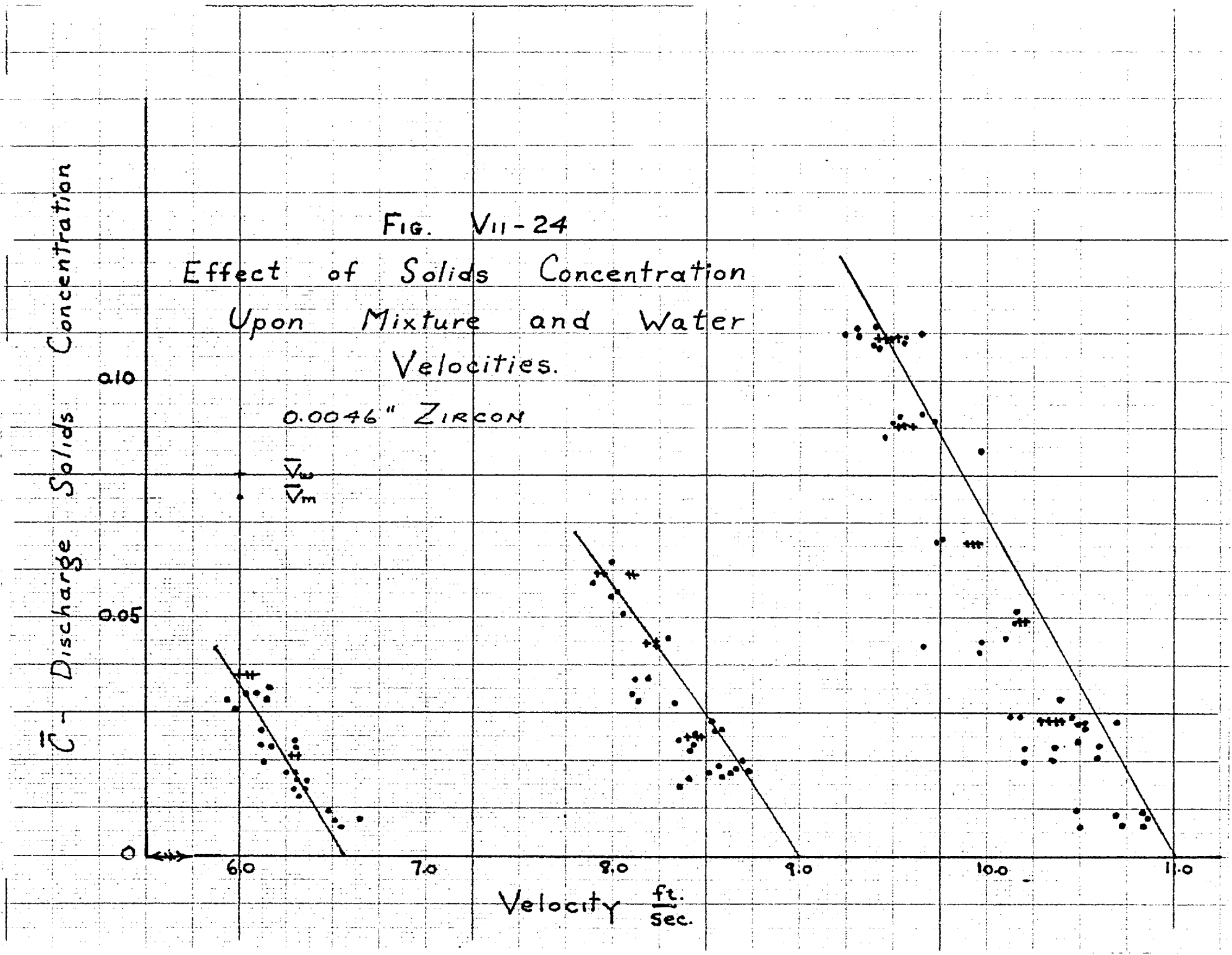
80

90

100

110

Velocity $\frac{\text{ft.}}{\text{Sec.}}$



associated closely enough with the fluid to permit the mean density of the mixture to be used in calculations of fluid velocity. It is quite possible, however, that this was merely the result of partial blockage of the pitot tube.

(c) Mean Velocities.

Figure VII - 24 illustrates the measured values of mixture and mean water velocities obtained at various discharge concentrations. It is apparent that a slight difference between $\bar{V}_m$ and $\bar{V}_w$ might exist at the lowest pressure gradients, but this is presumed to be the effect of the concentration profile mentioned in Section III - 2. It can be taken as reasonably well established that the local slip is small and that the particles are quite probably supported in suspension.

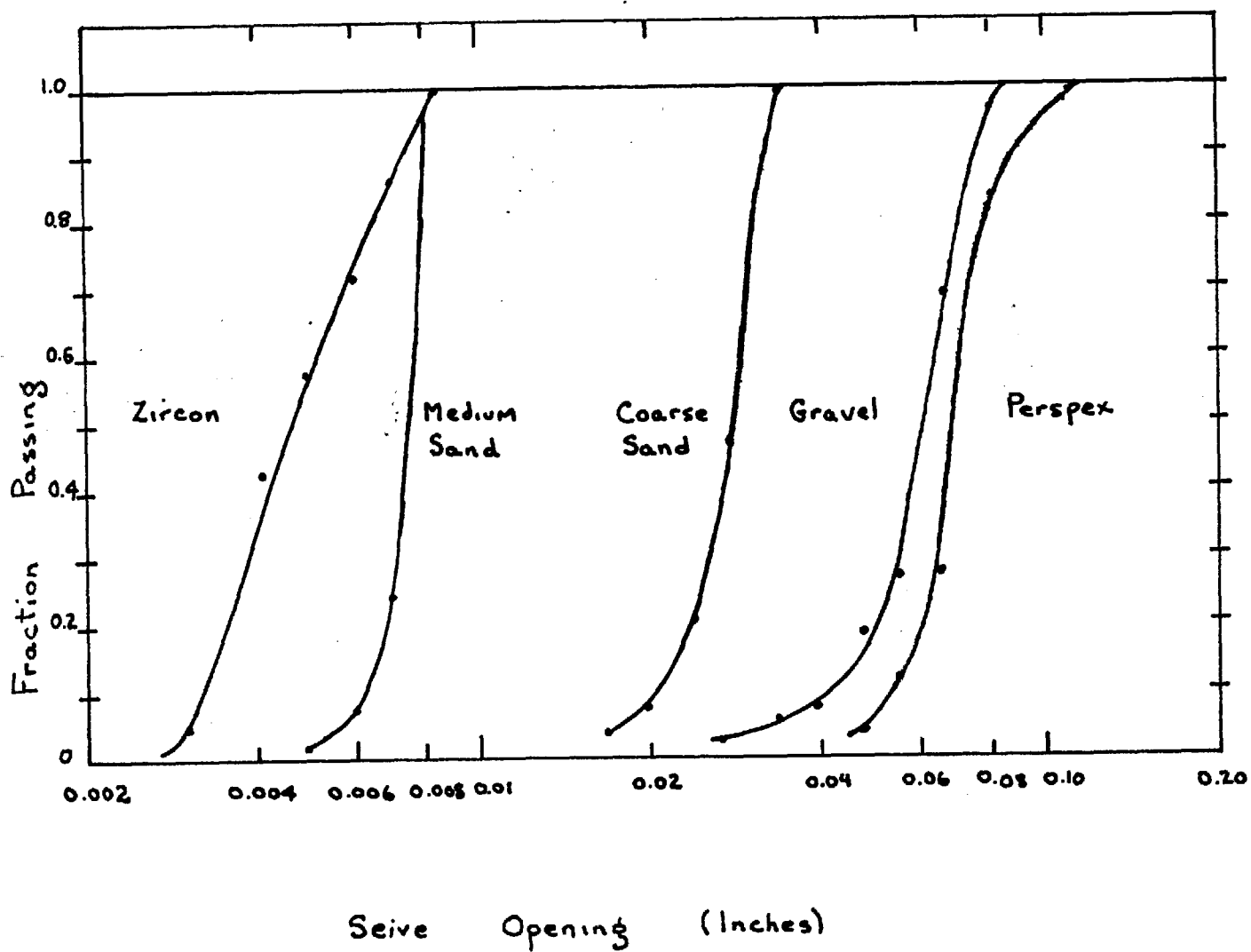
Some systematic deviation, of the order of 2 - 3% in the form of the $\bar{C} - \bar{V}$ curve, is apparent at the highest pressure drop. These velocities have not been corrected for temperature fluctuations and it is considered that the effect arose from two causes:

- (1) The water supply for the experiments at lower concentrations was much colder than that used at higher concentrations.
- (2) At the highest pressure drop the pump and motor appeared to act more efficiently. Consequently the temperature of the conveyed fluid was

TABLE VII-1 SUMMARY OF PARTICLE PROPERTY DETERMINATIONS.

MATERIAL	PERSPEX	GRAVEL	COARSE SAND	FINE SAND	ZIRCON					
POROSITY	0.345	0.391	0.383	0.410	0.328					
SPECIFIC GRAVITY	1.17	2.67	2.66	2.65	4.60					
SETTLING VELOCITY ft./sec.	0.173	0.695	0.313	0.0648	0.0886					
MEAN DIAMETER Inches.	0.073	0.061	0.0275	0.0072	0.0046					
PARTICLE SIZE ANALYSES	MESH No.	% PASSING	MESH No.	% PASSING	MESH No.	% PASSING	MESH No.	% PASSING		
	6	99.5	8	96.8	18	99.8	72	99.9	72	99.8
	8	83.3	10	69.1	22	46.9	85	24.3	85	86.5
	10	27.5	12	27.0	25	20.6	100	7.1	100	74.0
	12	1.2	14	18.5	30	7.5	120	1.6	120	57.5
	14	0.4	16	7.5	36	3.6			150	42.6
			18	5.9					200	4.7
		22	2.5							

FIG. VII-25



PARTICLE SIZE ANALYSES

observed to fall as the hydraulic gradient approached $0.400 \frac{\text{ft}}{\text{ft}}$. An increase in the transported concentration of solids was observed to decrease the magnitude of this effect, probably as a result of decreasing pump efficiency.

The most important conclusion which can be drawn from these plots is that the difference between the mean velocities of water and mixture is very small. This, of course, is unaltered by the effects just mentioned.

7. Particle Property Determinations.

Table VII - 1 presents the results of the determinations of porosity, specific gravity, settling velocity and size distribution. The densities compare favourably with those quoted in a standard handbook (60) of 1.18 for perspex 2.65 for silica and 4.60 for zircon. The size range of zircon, shown on Fig. VII - 25, was considerably greater than that observed for the other materials and as a consequence the bed porosity was lower.

CHAPTER VIII.Correlations and Derived Data.1. Introduction.

Inasmuch as previous investigations of hydraulic conveying have generally been confined to external measurements and little has been discovered about the internal mechanisms, many of the methods of correlation of the data presented in Chapter VII must be empirical. In this chapter the concentration profiles for the coarse particles will be shown as functions of friction velocity, mean concentration and particle properties. Results obtained with zircon and fine sand will be used to test the analysis shown in Section III - 4 applying to conditions where the particles are fully suspended. Velocity profiles obtained with gravel and coarse sand will be correlated with respect to friction velocity and mean concentration. In addition, an empirical correlation of the mean slip velocities for these two materials will be given.

Finally, it was possible by making certain assumptions to estimate the important local variables: transport concentration, solids velocity and slip velocity, for those cases where the concentration in the lowest region of the pipe approached its limiting maximum value.

2. Concentration Profiles - Coarse Particles.

It was found that the slope of the discharge concentration profiles in the upper half of the pipe was given by the expression:

$$C_y \frac{dy}{dC_y} = -\frac{0.601}{W_o} \sqrt{\frac{d}{C_d} \frac{\tau_o}{\rho_w}} \quad (1)$$

where C_d is the drag coefficient for the particle settling with a velocity W_o . On integration this gives $\log C_y$ as a function of y . Results were therefore plotted as $\log C_y$ vs. y .

Expressions for the effect of mean concentration and friction velocity upon the $\log C_y$ intercept gave the following equation for the linear portion of the $y - \log C_y$ curve:

$$\begin{aligned} 1.38 \sqrt{\frac{\tau_o d}{\rho_w W_o}} \log C_y = & -2.295 \sqrt{W_o C_d} y + 0.849 \sqrt{\frac{d}{W_o}} \log \frac{\bar{C}}{\sqrt{\frac{\tau_o}{\rho_w} (1-\bar{C})^2}} \\ & + 1.078 \sqrt{W_o C_d} - 676 d^2 \frac{\tau_o}{\rho_w} \sqrt{\frac{d}{W_o}} + 1.52 \sqrt{\frac{\tau_o}{\rho_w} \frac{d}{W_o}} \\ & - \frac{1.21 \times 10^{-6} C_d \sqrt{\frac{\tau_o d}{\rho_w W_o}} \left(1 - \frac{0.361 \bar{C}}{\frac{\tau_o}{\rho_w}} \right)}{4 \sqrt{\frac{\tau_o}{\rho_w}} d^4} \end{aligned} \quad (2)$$

or for each of the three materials individually:-

Coarse Sand:

$$\log C_y = \frac{-2.30}{\sqrt{\frac{\tau_o}{\rho_w}}} y + \frac{0.614}{\sqrt{\frac{\tau_o}{\rho_w}}} \log \frac{\bar{c}}{\sqrt{\frac{\tau_o}{\rho_w}} (1-\bar{c})^2} + \frac{1.08}{\sqrt{\frac{\tau_o}{\rho_w}}} - 0.451 \sqrt{\frac{\tau_o}{\rho_w}} + 1.221 - 0.820 \sqrt[4]{\frac{\tau_o}{\rho_w}} \left(1 - \frac{0.361 \bar{c}}{\sqrt{\frac{\tau_o}{\rho_w}}} \right) \quad (3)$$

Gravel:

$$\log C_y = \frac{-2.45}{\sqrt{\frac{\tau_o}{\rho_w}}} y + \frac{0.614}{\sqrt{\frac{\tau_o}{\rho_w}}} \log \frac{\bar{c}}{\sqrt{\frac{\tau_o}{\rho_w}} (1-\bar{c})^2} + \frac{1.14}{\sqrt{\frac{\tau_o}{\rho_w}}} - 2.18 \sqrt{\frac{\tau_o}{\rho_w}} + 1.221 - 0.170 \sqrt[4]{\frac{\tau_o}{\rho_w}} \left(1 - \frac{0.361 \bar{c}}{\sqrt{\frac{\tau_o}{\rho_w}}} \right) \quad (4)$$

Perspex:

$$\log C_y = - \frac{0.715}{\sqrt{\frac{\tau_o}{\rho_w}}} y + \frac{0.614}{\sqrt{\frac{\tau_o}{\rho_w}}} \log \frac{\bar{c}}{\sqrt{\frac{\tau_o}{\rho_w}} (1-\bar{c})^2} + \frac{0.335}{\sqrt{\frac{\tau_o}{\rho_w}}} - 2.60 \sqrt{\frac{\tau_o}{\rho_w}} + 1.10 - 0.0138 \sqrt[4]{\frac{\tau_o}{\rho_w}} \left(1 - \frac{0.361 \bar{c}}{\sqrt{\frac{\tau_o}{\rho_w}}} \right) \quad (5)$$

These equations are explicit in terms of C_y and y , but suffer from several obvious limitations. First, in common

with all empirical equations expressed in logarithmic form, a small inadequacy in the terms on the right of the equation may lead to a considerable error in the estimated value of C_y . This is particularly true at low pressure gradients when C_y changes very rapidly with y and $\bar{C}$. Secondly, the equations are elaborate and clumsy. Thirdly, they are not dimensionless. Although it would be possible to construct dimensionless terms from the parameters of Equation (2) and the independent variables of the system, pipe diameter and fluid kinematic viscosity, this has not been done because it is felt that this might tend to obscure the purely empirical basis of derivation.

Implicit in this inadequacy is the further disadvantage that from a mechanistic point of view, Equation VIII - 2 and its companion applying to the lower region of the pipe, are valueless. This is patently a consequence of the fact that there are more variables involved in the relationships between fluid properties and solids concentration than those shown in Equation VIII - 2. More will be said about this later, but it will be quite obvious to the reader that an attempt to correlate data obtained for those cases where saltation, spin effects and bed shearing consequences apply to different degrees must be based upon internal and not external considerations. None of the quantities in Equation VIII - 2 except C_y was obtained from internal measurements.

In the lower half of the pipe the deviation of the observed concentration profiles from Equation VIII - 2 was systematic and can be given by the following equations, where C^* is the value of C_y predicted from Equations VIII - 2,3,4 and 5.

Generally,

$$\begin{aligned}
 1.38 \sqrt{\frac{\tau_o}{\rho_w} \frac{d}{W_o}} \log \frac{C^*}{C} = & -y + \frac{0.655 \bar{C}}{(1-\bar{C})^2 \sqrt{\frac{\tau_o}{\rho_w}}} + 0.150 \left(\frac{C^*}{C} \right)^{-\frac{3.50}{1.58 \sqrt{\frac{W_o \bar{C}}{d}}}} \\
 & + 242 (s-1)^2 d^{5/2} \sqrt{\frac{\tau_o}{\rho_w}} - \frac{0.32 (s-1)}{\rho_w} \frac{W_o \tau_o}{C_d} \\
 & - \frac{0.583 d \sqrt{\frac{\tau_o}{\rho_w} \frac{d}{W_o}}}{(s-1) \left(\frac{\tau_o}{\rho_w} \right)^2} + 0.304 \sqrt{\frac{\tau_o}{\rho_w} \frac{d}{W_o}} \quad (6)
 \end{aligned}$$

More particularly, for each material:

Coarse Sand:

$$\begin{aligned}
 \log \frac{C^*}{C} = & - \frac{2.43 y}{\sqrt{\frac{\tau_o}{\rho_w}}} + \frac{1.595 \bar{C}}{\frac{\tau_o}{\rho_w} (1-\bar{C})^2} + 0.356 \left(\frac{C^*}{C} \right)^{-\frac{0.659}{\sqrt{\bar{C}}}} \\
 & - 1.326 \sqrt{\frac{\tau_o}{\rho_w}} - \frac{0.00704}{\left(\frac{\tau_o}{\rho_w} \right)^2} + 2.5693 \quad (7)
 \end{aligned}$$

Gravel:

$$\log \frac{C^*}{C} = - \frac{2.43}{\sqrt{\frac{\tau_o}{\rho_w}}} y + \frac{1.595 \bar{C}}{\frac{\tau_o}{\rho_w} (1-\bar{C})^2} + \frac{0.356}{\sqrt{\frac{\tau_o}{\rho_w}}} \left(\frac{C^*}{C} \right)^{\frac{-0.659}{\sqrt{\bar{C}}}} - 3.965 \sqrt{\frac{\tau_o}{\rho_w}} \\ - \frac{0.0156}{\left(\frac{\tau_o}{\rho_w} \right)^2} + 3.6410 \quad (8)$$

Perspex:

$$\log \frac{C^*}{C} = - \frac{1.115}{\sqrt{\frac{\tau_o}{\rho_w}}} y + \frac{0.731 \bar{C}}{\frac{\tau_o}{\rho_w} (1-\bar{C})^2} - \frac{0.1672}{\sqrt{\frac{\tau_o}{\rho_w}}} \frac{C^*}{C}^{\frac{-1.438}{\sqrt{\bar{C}}}} - 0.0608 \sqrt{\frac{\tau_o}{\rho_w}} \\ - \frac{0.1678}{\left(\frac{\tau_o}{\rho_w} \right)^2} + 0.8235 \quad (9)$$

All the disadvantages of Equation VIII - 2 apply to those given above, with the added limitation that Equation VIII - 6 is not explicit in terms of $\frac{C^*}{C}$.

From Figures VIII - 1 to 9, it is evident that Equation VIII - 6 fails badly at high concentrations of the slower settling materials when the pressure gradient is low. The utility of these equations is confined to the condition where

$$\frac{\bar{C}}{w_o \sqrt{\frac{\tau_o}{\rho_w}}} < 1.10 \quad (10)$$

FIG. VIII-1

PLOTS SHOWING COMPARISON OF OBSERVED
CONCENTRATION PROFILES WITH THOSE
PREDICTED BY EMPIRICAL EQUATIONS.
COARSE SAND $i = 0.160$ ft./ft.

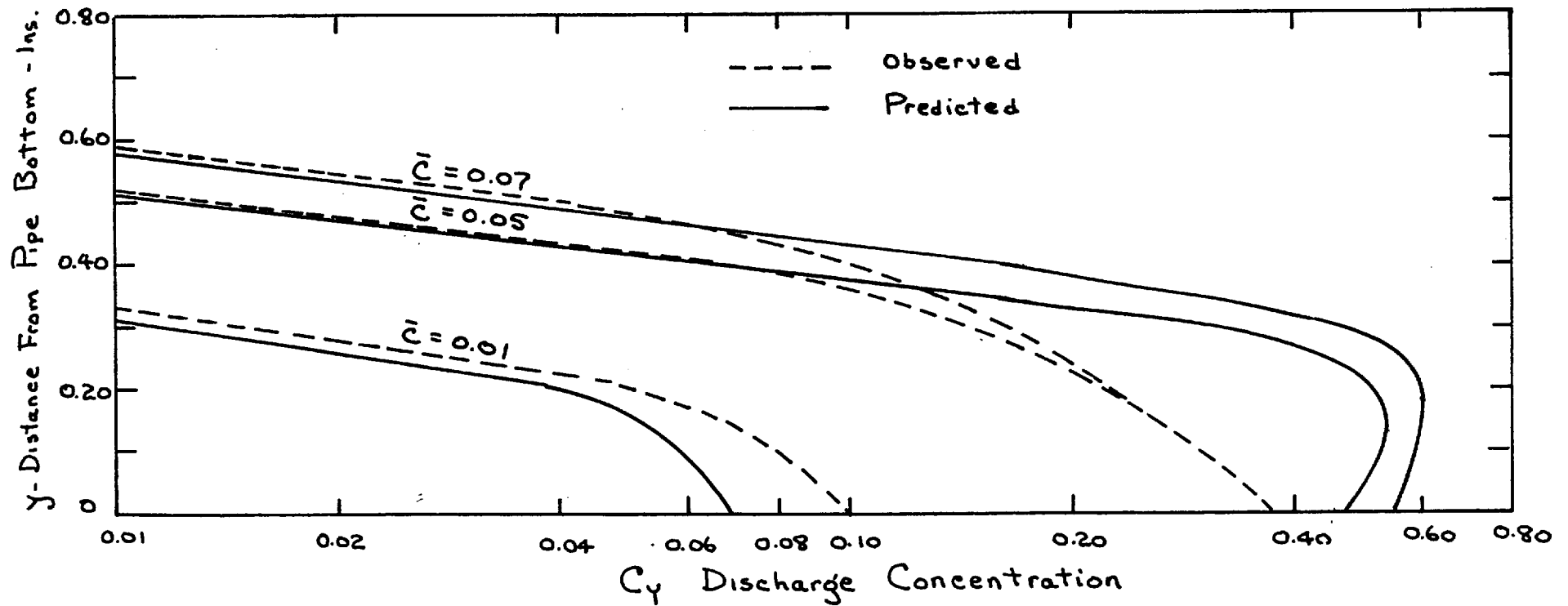


FIG. VIII-2

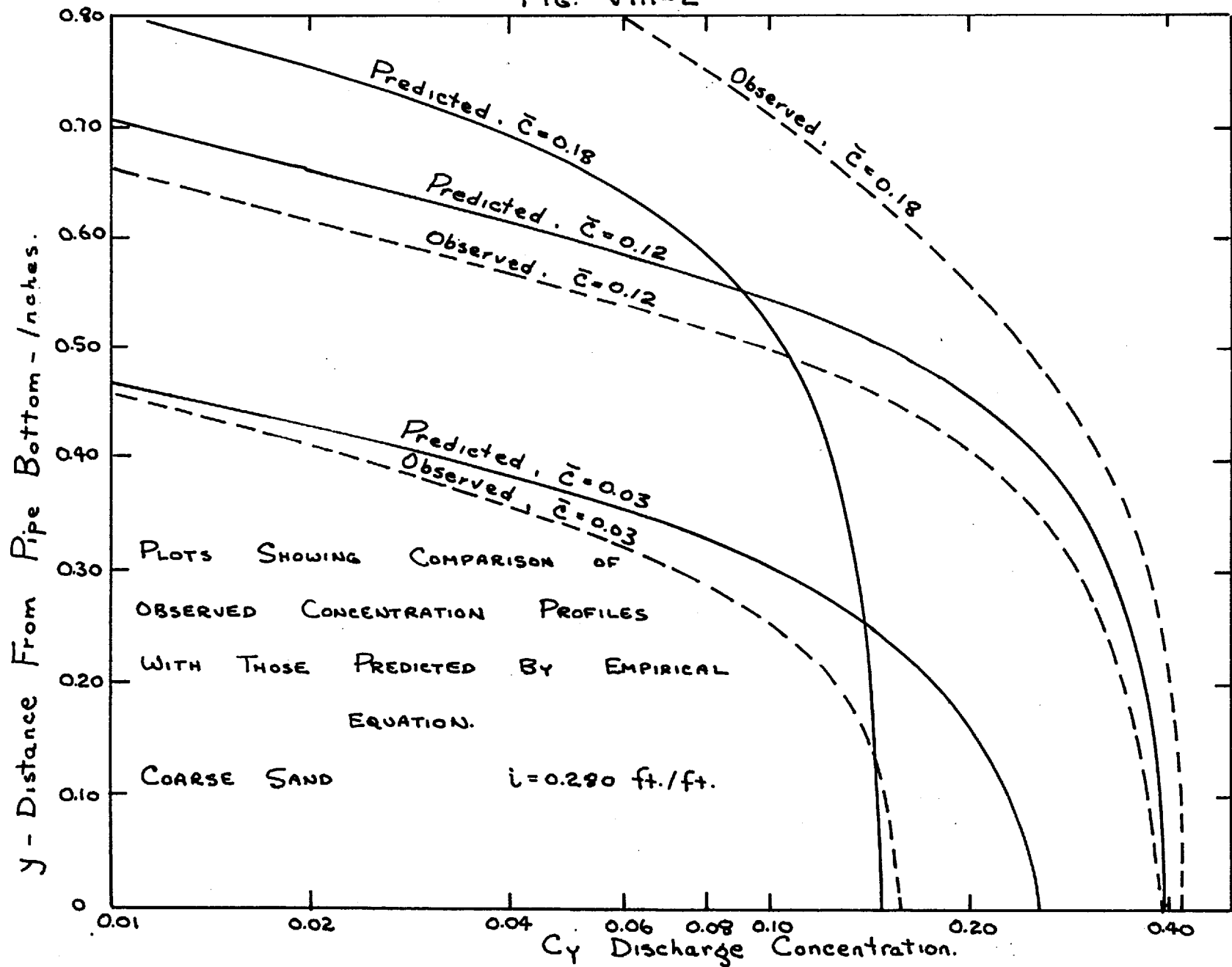


FIG. VIII-3

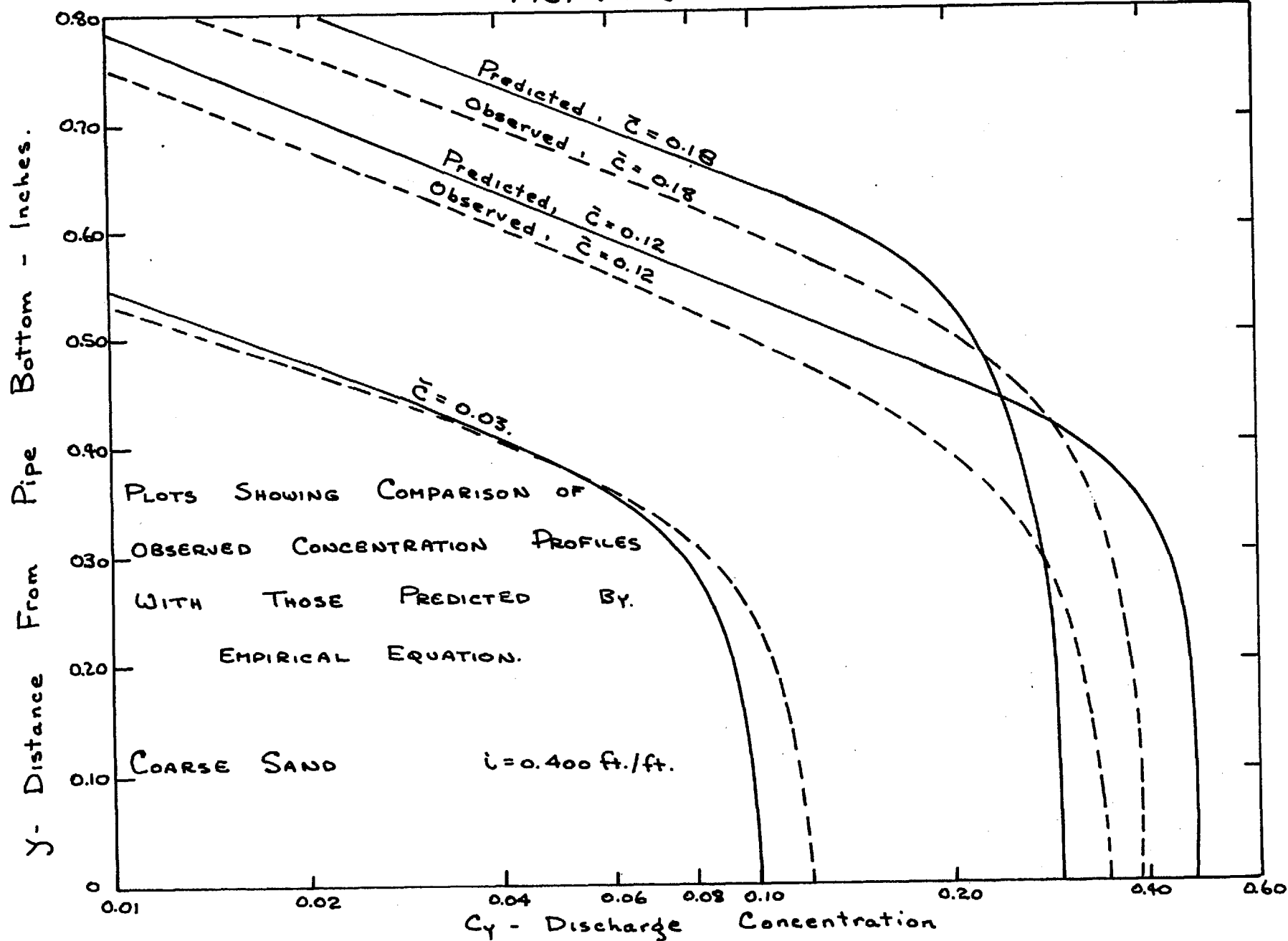


FIG. VIII-4

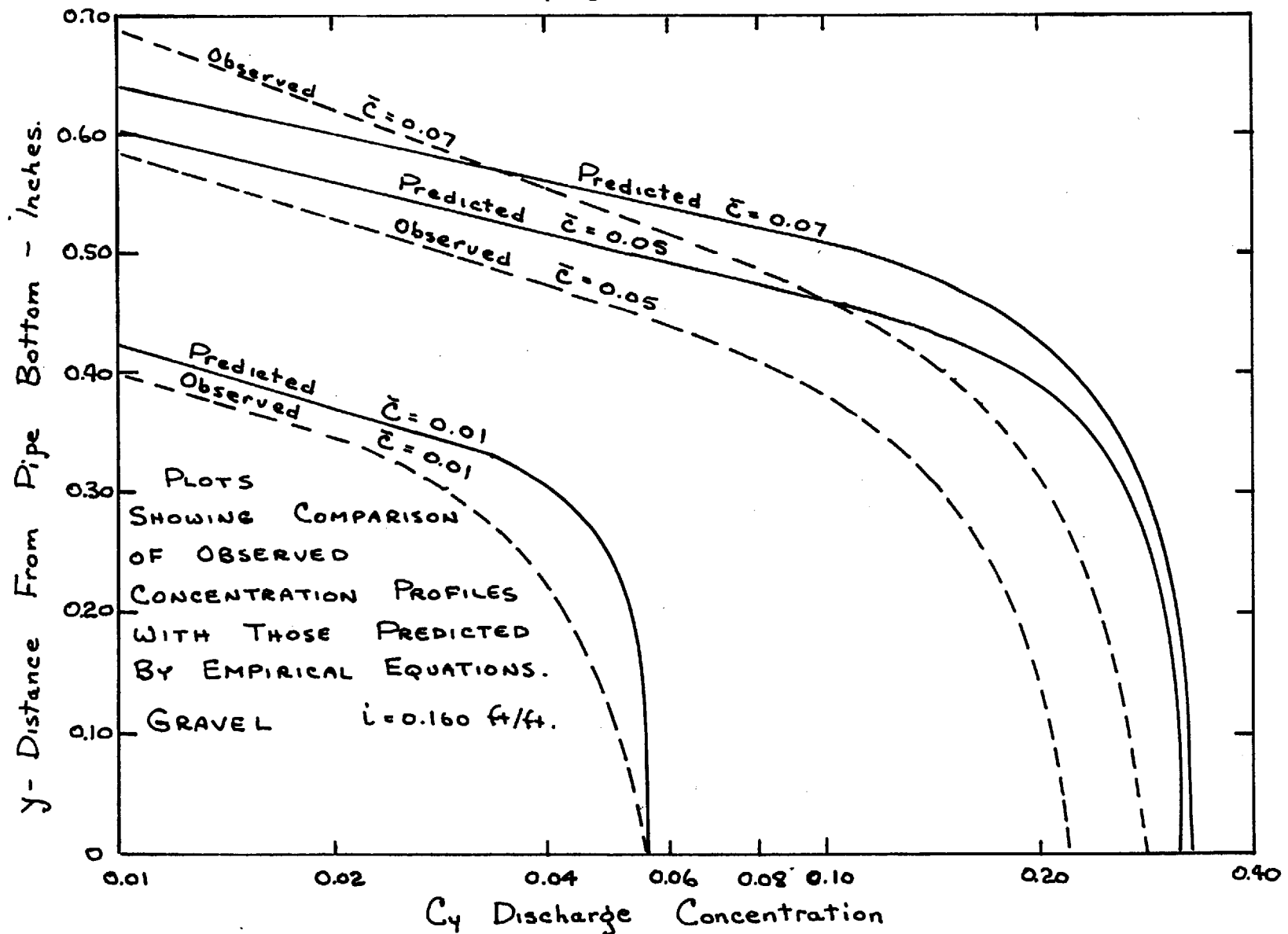


Fig. VIII-5

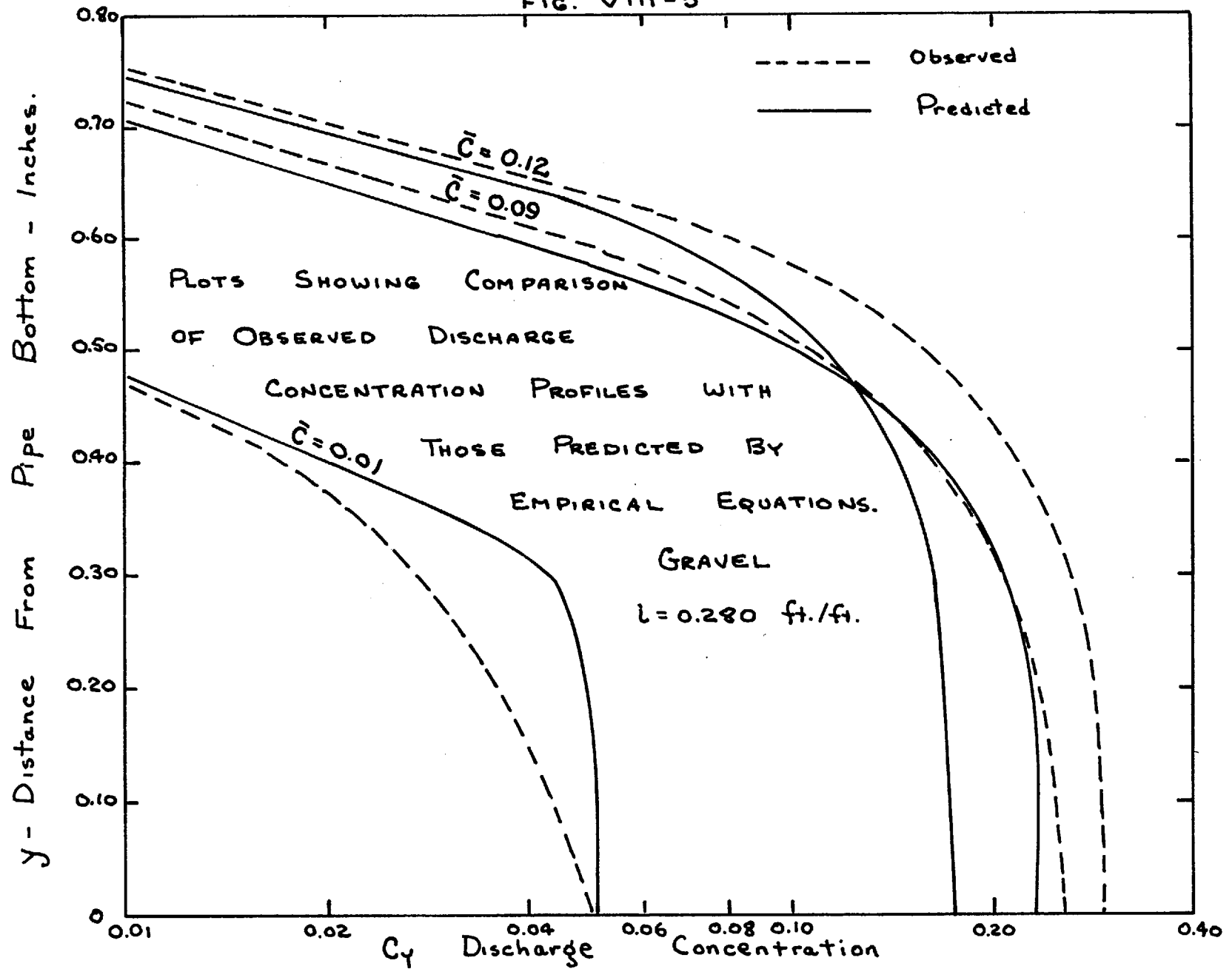


FIG. VIII-6

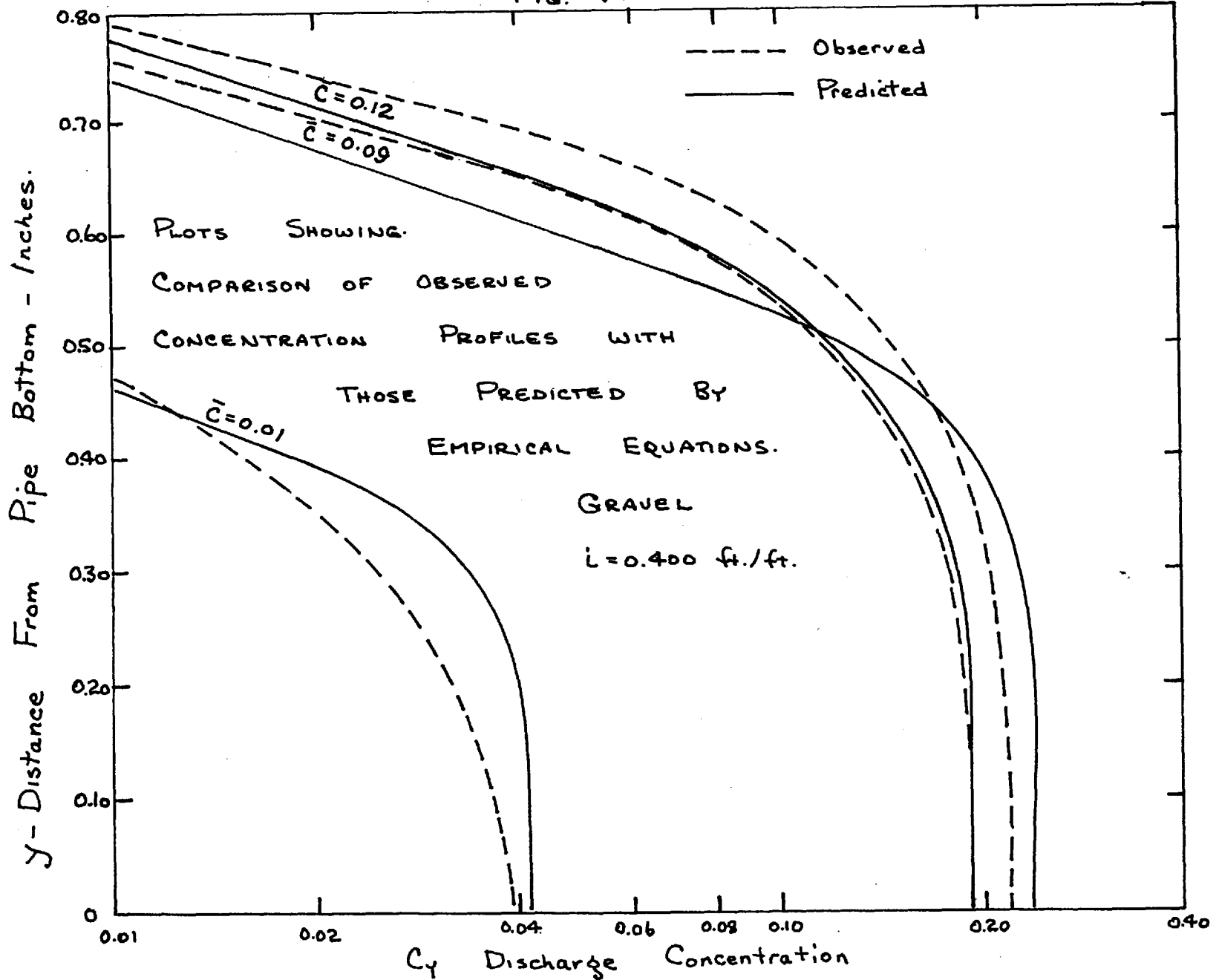


FIG. VIII-7

PLOT SHOWING COMPARISON OF OBSERVED
CONCENTRATION PROFILES WITH THOSE
PREDICTED BY EMPIRICAL EQUATION
PERSPEX $i = 0.160$ ft./ft.

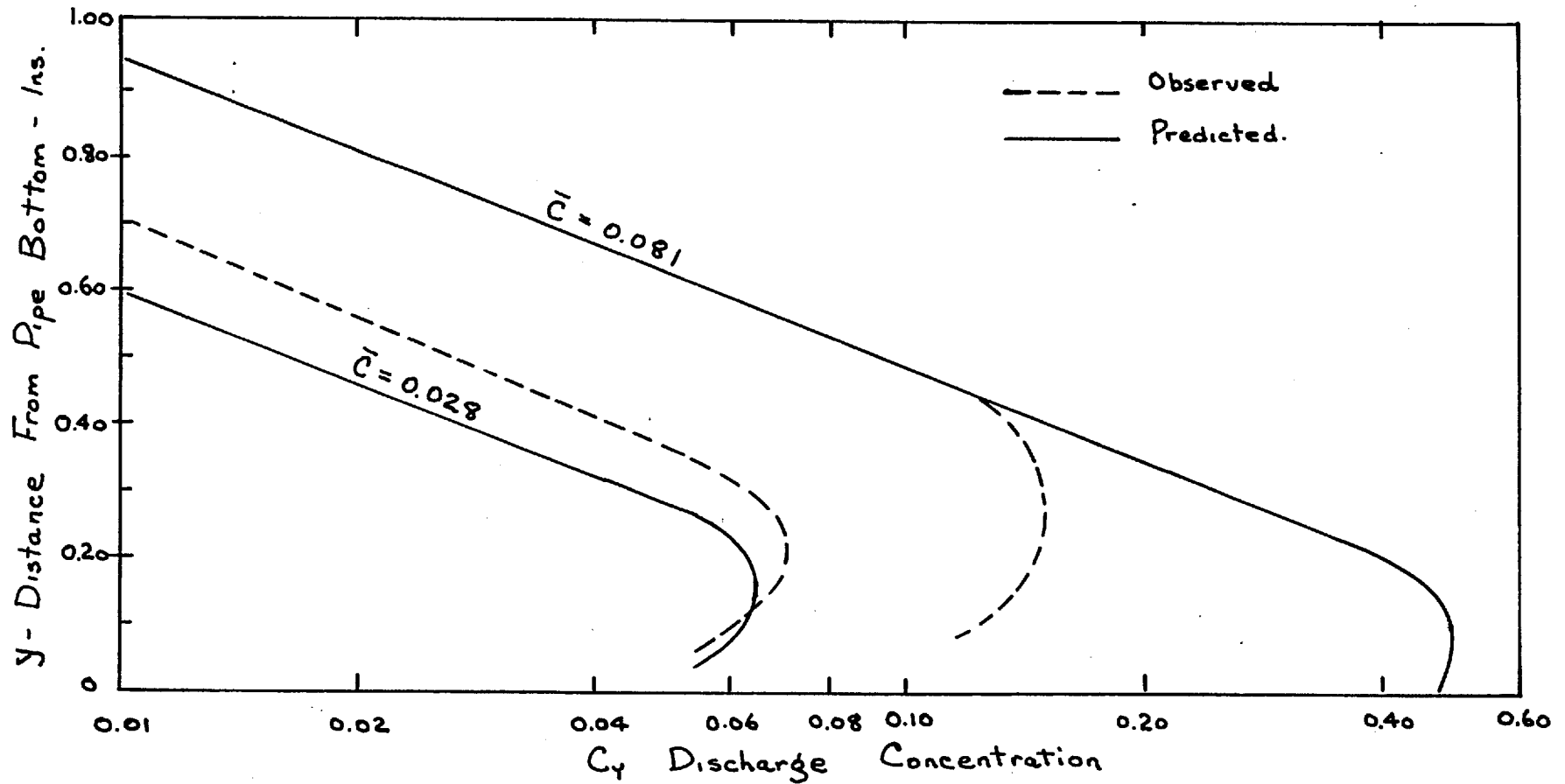


FIG. VIII-8

PLOT SHOWING COMPARISON OF OBSERVED
CONCENTRATION PROFILES WITH THOSE
PREDICTED BY EMPIRICAL EQUATION.

PERSPEX

$l = 0.280 \text{ ft./ft.}$

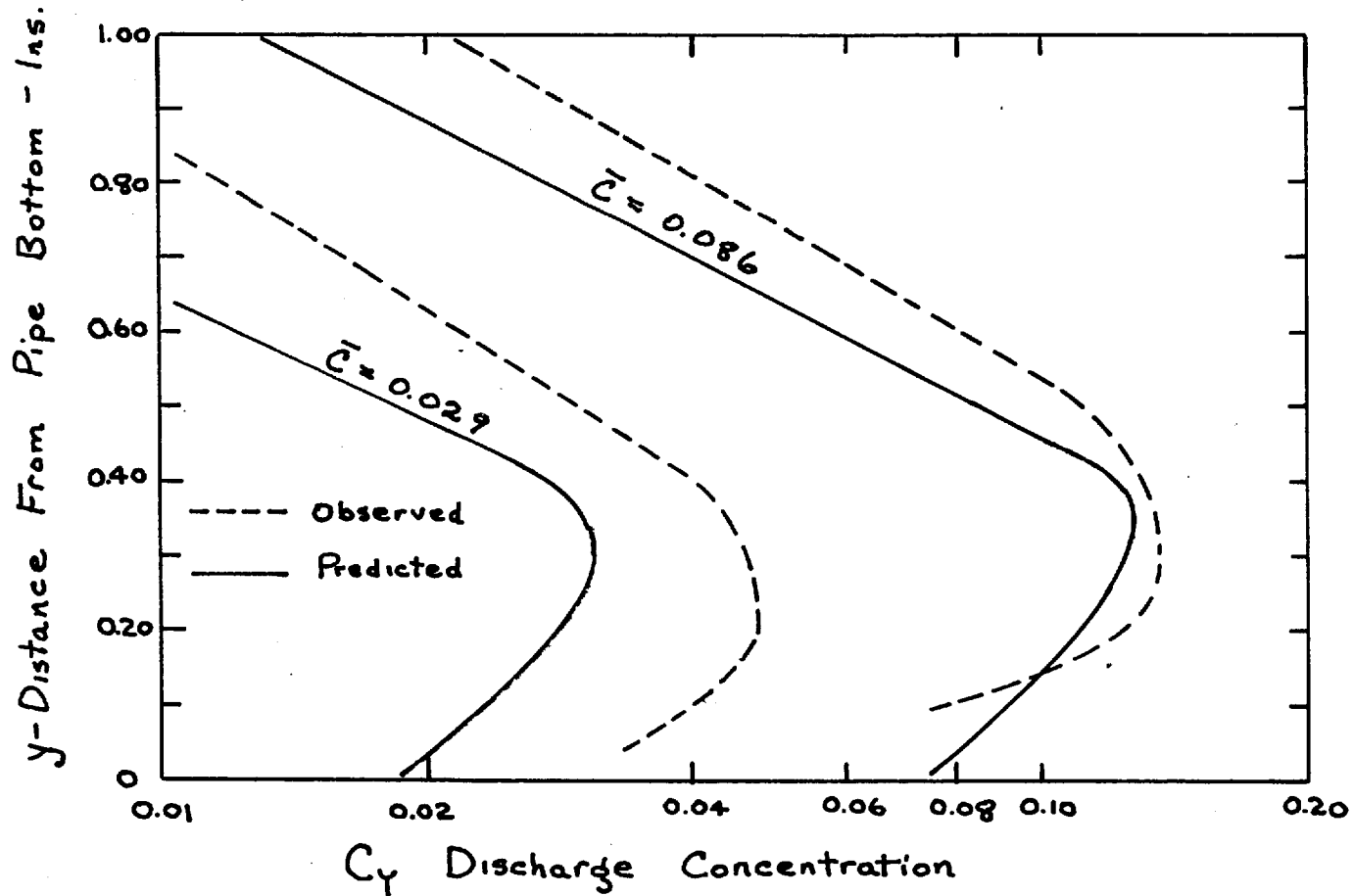
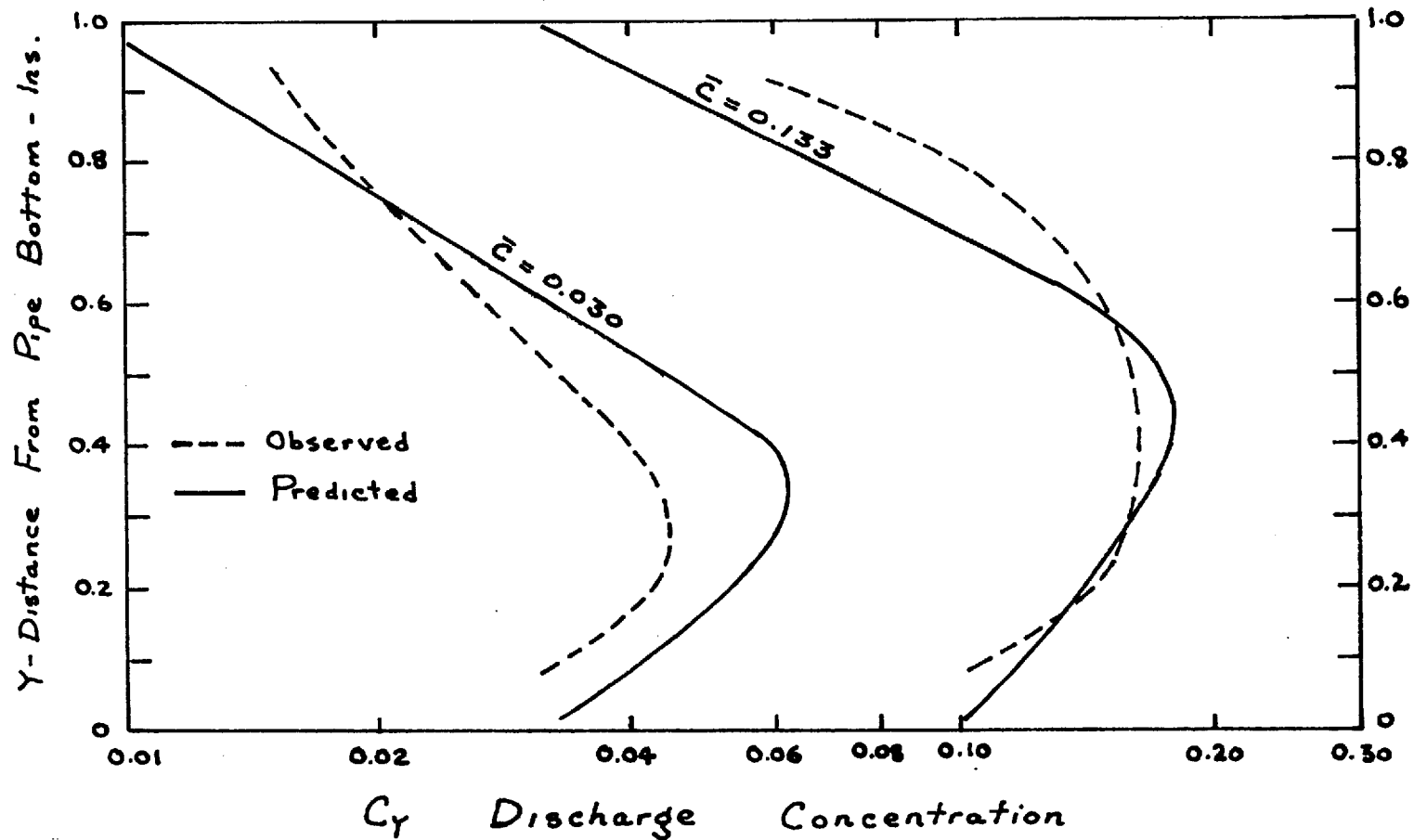


FIG. VIII-9

PLOT SHOWING COMPARISON OF OBSERVED
CONCENTRATION PROFILES WITH THOSE
PREDICTED BY EMPIRICAL EQUATION.

PERSPEX

$\dot{L} = 0.400 \text{ ft./ft.}$



The inadequacy of Equation VIII - 6 under conditions where the criterion expressed in Equation VIII - 10 is unsatisfied, may be attributed to a change of the flow regime in the lower half of the pipe at extreme conditions where variables not mentioned in Equations VIII - 1 to 9 are undoubtedly of some importance. These include particle shape and size distribution, coefficients of friction between particles and between particles and the pipe, the shape of the pipe and the rates of change of the velocities and concentrations associated with transport. It will be seen that these equations contain expressions which are used to give a numerical estimate of the variation of quantities not included in the empirical analyses. As such, Equations VIII - 2 to 9 cannot be considered a significant contribution to the science of hydraulic conveying.

3. Concentration Profiles - Fine Particles.

The data presented in Chapter VII for the measurements of local concentration are shown in the form of plots of

$$\frac{c_y}{1-c_y} e^{\frac{1}{1-c_y}} \div \frac{c_o}{1-c_o} e^{\frac{1}{1-c_o}} \text{ vs. } \frac{y W_o}{d \sqrt{\frac{\tau_o}{\rho_w}}} \text{ in Fig. VIII - 10.}$$

It is apparent that the data fit a mean curve fairly well but the linear relationship suggested by Equation III - 24 is not substantiated. Nevertheless, the quantity

FIG. VIII-10

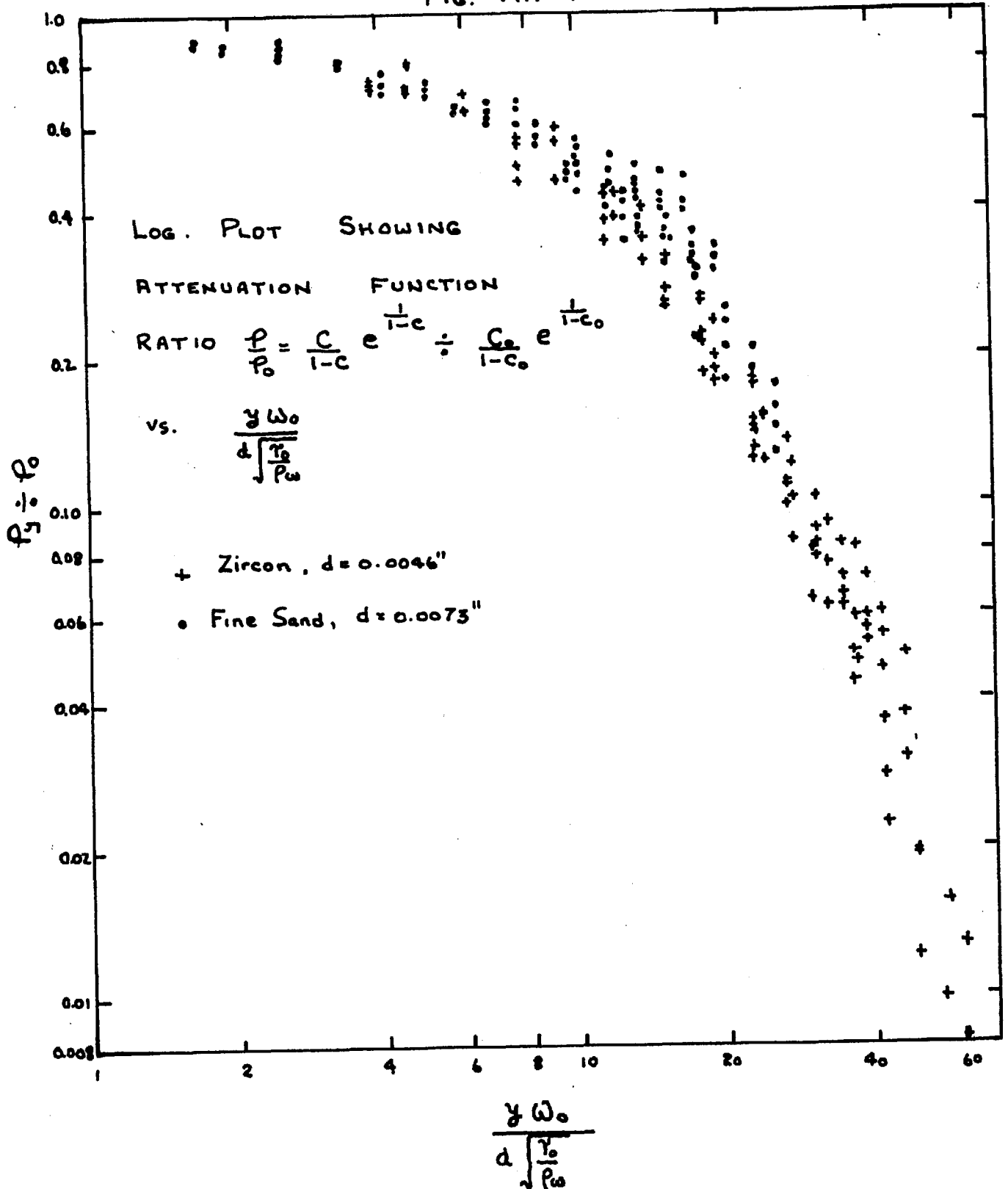
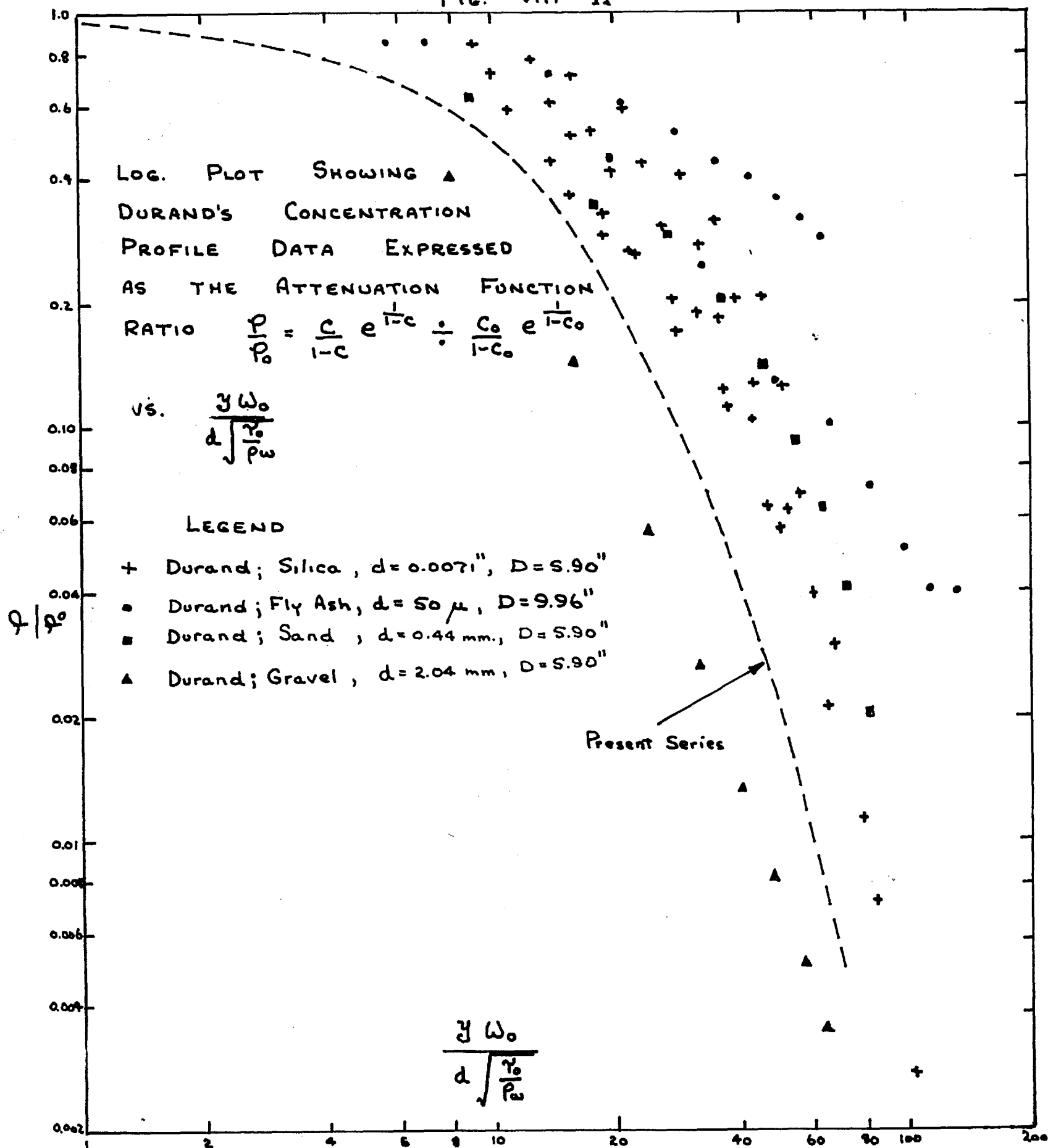


Fig. VIII-11



$\frac{y W_o}{d \sqrt{\frac{\tau_o}{\rho_w}}}$ seems to form a useful dependent variable for the presentation of data.

Figure VIII - 11 and Tables A20 and 21 show values of transport concentration obtained by Durand expressed in a similar manner. The pipe diameters and flow rates represent a considerable departure from the conditions of these experiments and the quantities $\frac{\tau_o}{\rho_w}$ and W_o had to be calculated from incomplete information. Since presentation in this form demands a knowledge of the concentration at the bottom of the pipe, this quantity had to be obtained by extrapolation. It is known that for the case of the 50 fly ash this extrapolated value was too low - which could easily account for the deviation observed with this material. For the 0.18 mm. and 0.44 mm. sands in the 5.90" pipe, an error in the calculated value of $\sqrt{\frac{\tau_o}{\rho_w}}$ was probably responsible for the deviation because, as the mean concentration increased, the concentration profiles approached those obtained in the present series. This would agree with Durand's observation that his equation for excess pressure became more accurate as the mean concentration was increased. The same general behaviour of $\frac{P}{P_o}$ as a function of $\frac{y W_o}{d \sqrt{\frac{\tau_o}{\rho_w}}}$ is apparent in all cases, and in view of the estimations which were made, the agreement can be regarded as encouraging. Table VIII - 1 shows that the quantity $p = \frac{C}{1-C} e^{\frac{1}{1-C}}$ is particularly sensitive to changes in C when

TABLE VIII - 1

C	$\frac{C}{1-C}$ e $\frac{1}{1-C}$
0.550	11.26
0.450	5.04
0.350	2.51
0.250	1.262
0.150	0.571
0.050	0.151
0.025	0.071
0.010	0.028
0.006	0.017
0.002	0.006

C is greater than 0.20. Most of Durand's data were determined under conditions where C_0 was estimated to be greater than this. Consequently the difference between Durand's data and those presented in Fig. VIII - 11 is exaggerated. As long as the discharge concentration data obey the same relationship between $\frac{p}{p_0}$ and $\frac{y W_0}{d \sqrt{\frac{\gamma_0}{\rho_w}}}$ as transport concentration data, the local slip velocity must be negligible. If this is the case the particles may be assumed to have been suspended by turbulent velocity fluctuations. There is, of course, no proof that the particles were in fact suspended: Bagnold has shown that a dispersive pressure can exist for the shear of fine particles. Two facts cast doubt upon any assumption of dispersion by such a stress under these conditions. First, the existence of this stress resulting from interparticle contacts demands the existence of a high concentration in the lowest regions of the pipe and, since no major deviation was observed between low and high concentration data, the possibility must be discounted. Secondly, the data observed for the gravel, 2.04 mm. in diameter, appear to agree fairly well with the other observations. Particles of this diameter would certainly be supported by an inertial dispersive stress, and it would be quite surprising if concentration profiles expressed in this manner agreed with each other when the regime of shear was different.

The data obtained with the gravel were particularly valuable because the particle diameter was greater than Durand's own limit of 2.00 mm. marking the initiation of the saltation regime. The calculated settling velocity was 0.804 ft/sec., while the estimated friction velocity was 0.738 ft/sec. Since the turbulent lifting component was found to be roughly equal to the friction velocity, the agreement of the concentration profile with those obtained for suspended particles would tend to reinforce the conclusion that for low concentrations at least, the criterion governing the transport in suspension is that the settling velocity of the particles must be less than the friction velocity.

4. Water Velocity Profiles. (Coarse Sand and Gravel).

It was observed that each of the water velocity profiles could be divided into two regions separated by the point at which the maximum velocity occurred:-

(a) Lower Section.

For this region, in which nearly all the solids were transported, the velocity profiles deviated from those for pure water by an amount directly proportional to mean concentration and the distance from the point of maximum velocity. In addition, this deviation was found to be inversely proportional to the friction velocity. Empirically, the relationship shown in Fig. VIII - 12 was obtained. It

TABLE VIII - 2
WATER VELOCITY PROFILES AT 25°C.
(SMOOTHED DATA)
COARSE SAND

$i = 0.160 \text{ ft/ft.}$

$\bar{C} =$	0.01	0.03	0.05	0.07
y ins.	Water velocities in ft/sec.			
0.05	5.13	4.02	3.15	2.41
0.10	5.79	4.76	3.76	3.00
0.20	6.82	5.94	5.01	4.16
0.30	7.52	6.92	6.20	5.35
0.40	7.94	7.53	7.01	6.13
0.50	8.05	7.65	7.29	6.45
0.60	7.92	7.56	7.15	6.38
0.70	7.64	7.25	6.86	6.13
0.80	7.18	6.85	6.38	5.67
0.90	6.55	6.24	5.69	4.99

TABLE VIII - 2

(Continued)

WATER VELOCITY PROFILES AT 25°C.

(SMOOTHED DATA)

COARSE SAND

$i = 0.280 \text{ ft/ft}$

$\bar{C} =$	0.03	0.06	0.09	0.12	0.15	0.18
y ins.	Water velocities in ft/sec.					
0.05	6.30	5.38	4.63	4.02	3.46	2.92
0.10	7.20	6.18	5.26	4.43	3.72	3.09
0.20	8.77	7.81	6.78	5.67	4.70	3.78
0.30	10.04	9.31	8.41	7.30	6.15	4.76
0.40	10.72	10.32	9.65	8.68	7.32	5.62
0.50	10.95	10.60	10.16	9.30	8.04	6.09
0.60	10.77	10.51	10.09	9.41	8.29	6.39
0.70	10.42	10.12	9.72	9.07	8.03	6.40
0.80	9.74	9.45	9.03	8.35	7.36	6.06
0.90	8.75	8.54	8.09	7.48	6.61	5.56

TABLE VIII - 2

(Continued)

WATER VELOCITY PROFILES AT 25°C.

(SMOOTHED DATA)

COARSE SAND

$i = 0.400$ ft/ft.

$\bar{C} =$	0.02	0.05	0.08	0.11	0.14	0.17	0.20
Y ins.	Water velocities in ft/sec.						
0.05	8.74	7.87	7.18	6.66	6.17	5.62	5.25
0.10	9.85	8.88	8.14	7.49	6.85	6.27	5.77
0.20	11.46	10.65	9.92	9.16	8.20	7.65	6.85
0.30	12.57	12.02	11.32	10.67	9.85	8.96	7.90
0.40	13.26	12.95	12.34	11.68	11.08	10.10	8.86
0.50	13.54	13.35	12.91	12.36	11.79	10.81	9.52
0.60	13.37	13.23	13.05	12.50	12.04	11.19	9.92
0.70	12.87	12.78	12.68	12.50	12.04	11.19	10.02
0.80	12.08	11.99	11.90	11.80	11.32	10.67	9.81
0.90	11.02	10.87	10.68	10.53	10.06	9.64	9.04

TABLE VIII - 3
WATER VELOCITY PROFILES AT 25°C.
(SMOOTHED DATA)
GRAVEL.

$i = 0.280 \text{ ft/ft.}$

$\bar{C}$	0.02	0.04	0.06	0.08	0.10
Y ins.	Water velocities in ft/sec.				
0.05	6.60	5.80	5.14	4.54	4.16
0.10	7.87	6.82	6.00	5.37	4.93
0.20	9.07	8.08	7.28	6.66	6.10
0.30	9.94	9.18	8.41	7.81	7.26
0.40	10.65	10.13	9.44	8.93	8.46
0.50	10.97	10.68	10.26	9.90	9.54
0.60	10.84	10.72	10.51	10.25	10.02
0.70	10.40	10.27	10.13	9.99	9.80
0.80	9.79	9.61	9.43	9.24	9.04
0.90	8.98	8.74	8.46	8.21	8.01

FIG. VIII-12

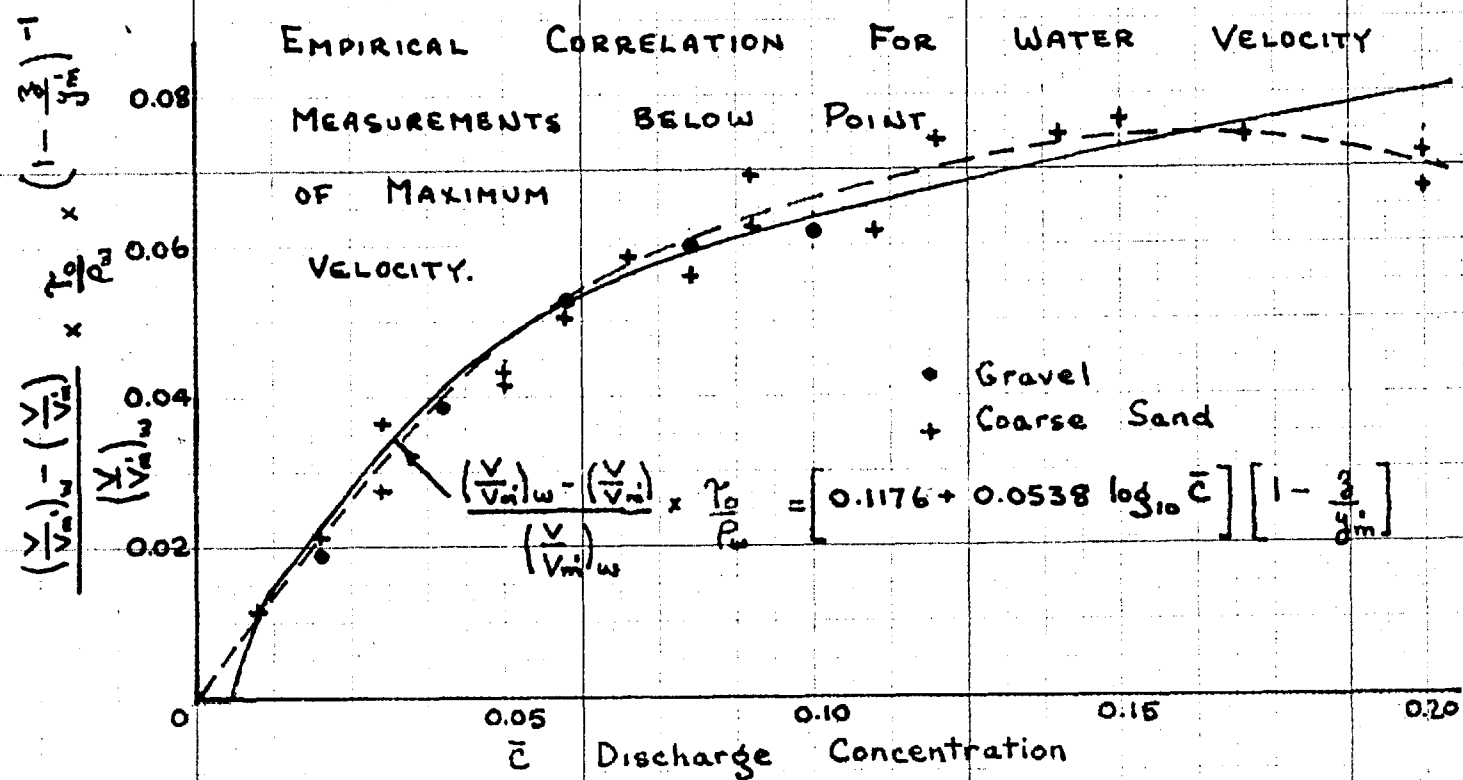
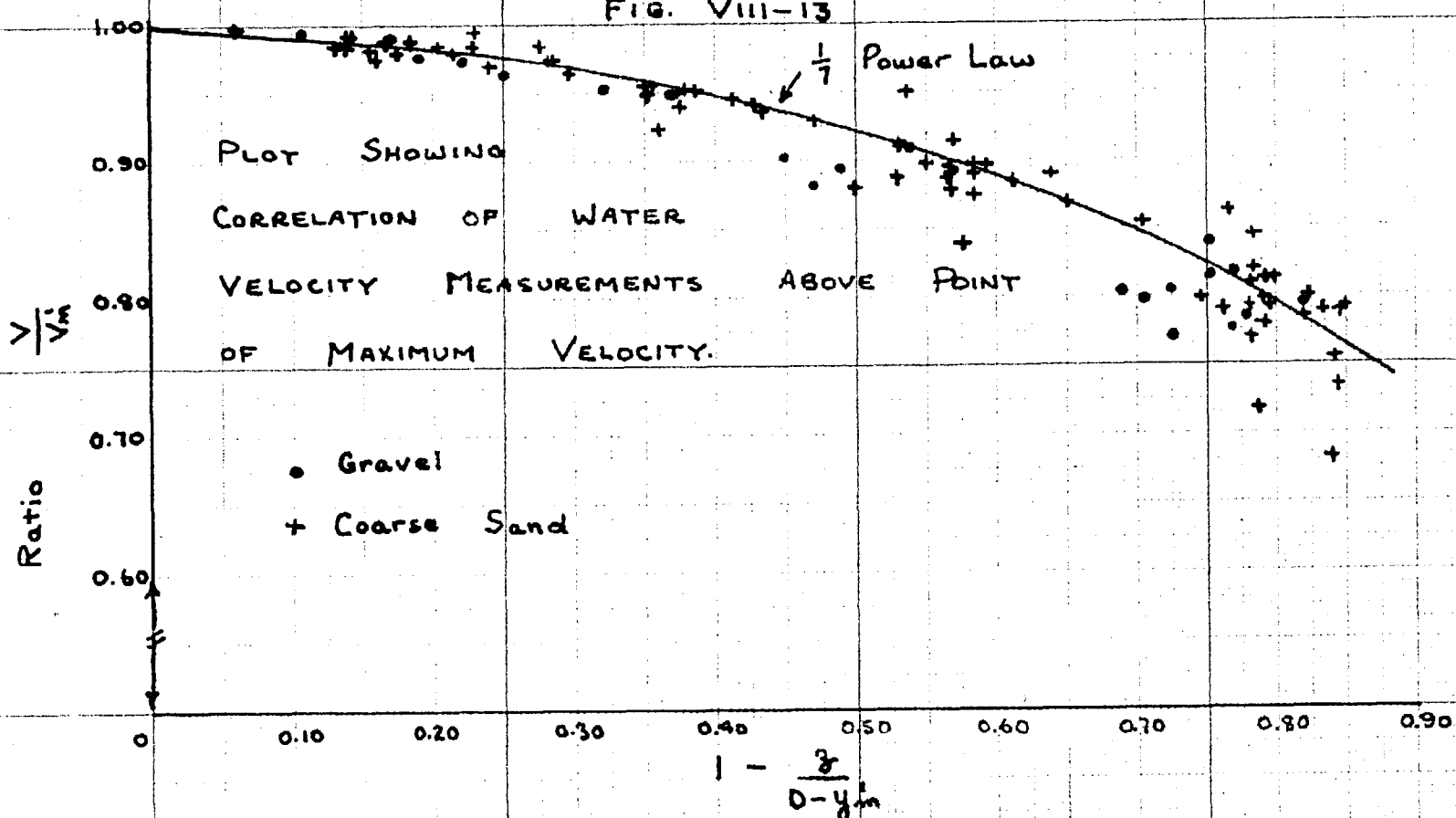


FIG. VIII-13



was found that for $\bar{C} = 0.01$ the data could be represented by the relationship

$$\frac{\left(\frac{V}{V'_m}\right)_w - \frac{V}{V'_m}}{\left(\frac{V}{V'_m}\right)_w} \times \frac{\gamma_o}{\rho_w} = (0.1176 + 0.0538 \log \bar{C}) \left[1 - \frac{z}{y'_m}\right] \quad (11)$$

where

$(V'_m)_w$ = maximum fluid velocity (solids absent)

V'_m = maximum fluid velocity (solids present)

z = distance from pipe boundary (bottom)

y'_m = distance of point of maximum fluid velocity from pipe bottom

V = fluid velocity measured at z

The dashed curve representing the experimental points, and the solid line representing Equation VIII - 11 may be seen to apply equally well to coarse sand and gravel.

Durand (27) reported the results of some measurements made of the water velocity profile for sand in the form:

$$\frac{V_m - V}{V_m} = \phi \left(\frac{y}{r}\right) \quad (12)$$

and indicated that ϕ was the same as for water flowing in the absence of solids. Unfortunately he failed to describe his technique and in view of the dangers, described in Chapter V, associated with measurements of water velocity made with the pitot tube orifice facing upstream, his results must be regarded with some suspicion.

FIG. VIII-14

PLOT SHOWING REDUCTION
OF MAXIMUM WATER VELOCITY
AS A FUNCTION OF $\frac{\bar{C}}{\sqrt{\frac{\gamma_0}{\rho_w}}}$

$10 \left\| \frac{1}{\bar{C}} \right\|$

$$\Delta V = 0.172 e^{+ \frac{1.63 \bar{C}}{\sqrt{\gamma_0 / \rho_w}}} \text{ ft./sec.}$$

+ Coarse Sand
• Gravel

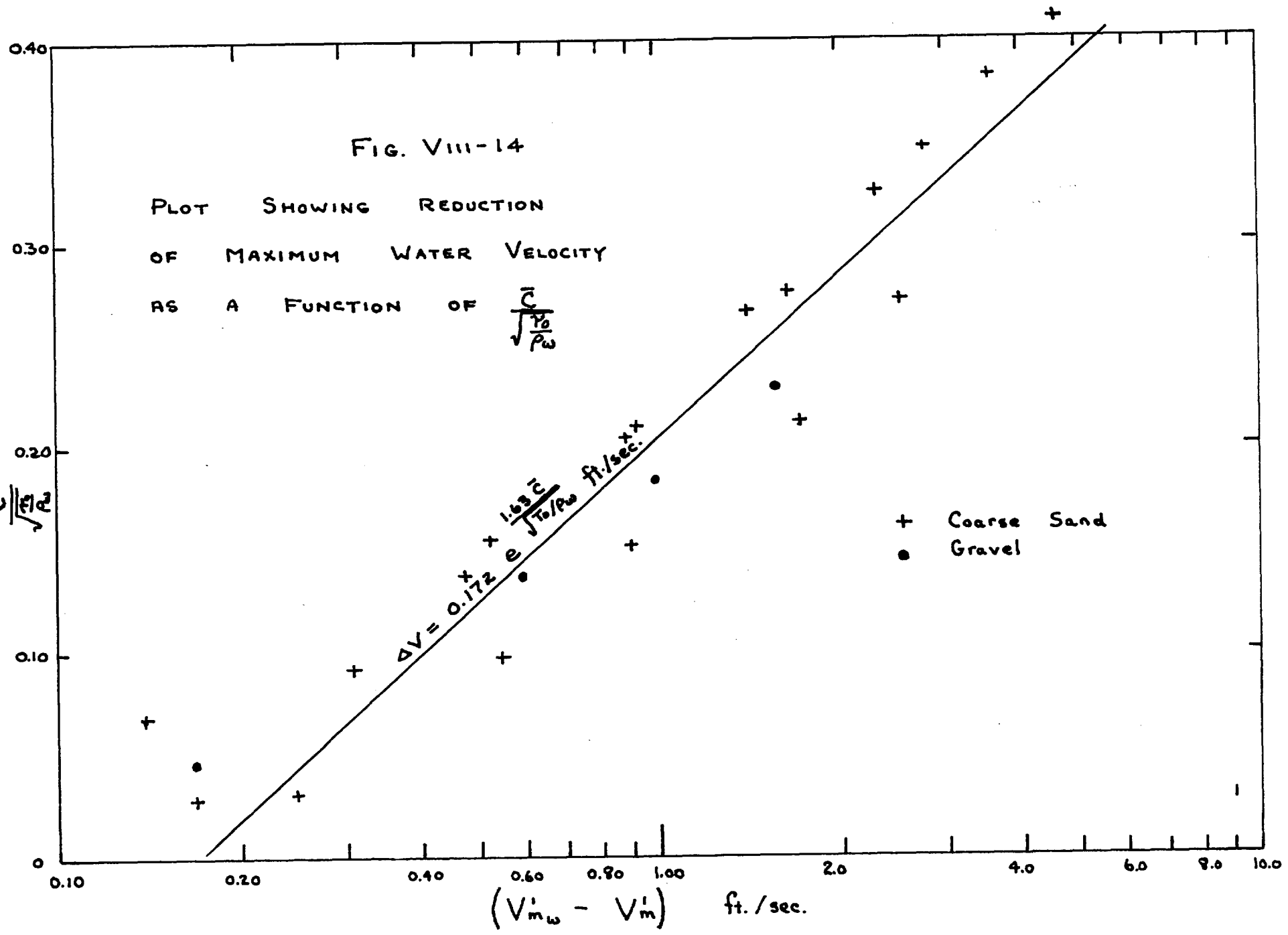
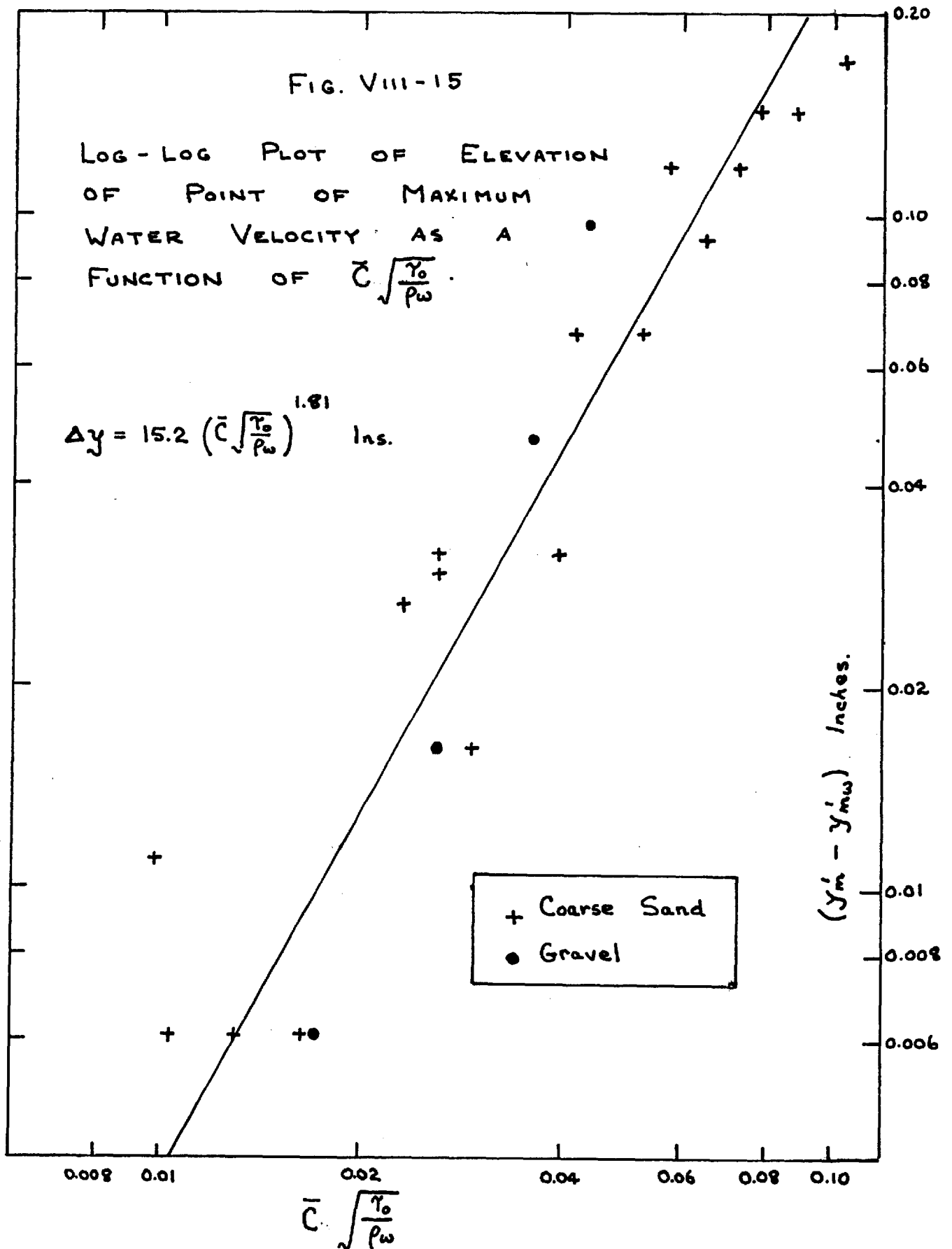


FIG. VIII-15

LOG-LOG PLOT OF ELEVATION
OF POINT OF MAXIMUM
WATER VELOCITY AS A
FUNCTION OF $\bar{C} \sqrt{\frac{\gamma_o}{\rho_w}}$

$$\Delta y = 15.2 \left(\bar{C} \sqrt{\frac{\gamma_o}{\rho_w}} \right)^{1.81} \text{ Ins.}$$



(b) Upper Section.

Figure VIII - 13 illustrates that, within the admittedly wide limits of experimental error, the data obtained above the point of maximum velocity may be represented by a plot of $\frac{V}{V'_m}$ vs. $1 - \frac{z}{D - y'_m}$, where z = distance from pipe (upper) boundary. In the absence of solids this plot reduces to the Blasius $\frac{1}{7}$ Power Law.

(c) Effect of Concentration upon Maximum Water Velocity.

The maximum water velocity was observed to decrease with mean discharge concentration and friction velocity according to the relationship:

$$V'_{m_w} - V'_m = 0.172 \cdot \frac{1.63 \bar{C}}{\sqrt{\frac{\tau_o}{\rho_w}}} \frac{\text{ft.}}{\text{sec.}} \quad (13)$$

as shown in Fig. VIII - 14.

(d) Effect of Concentration upon Point of Maximum Water Velocity.

From Fig. VIII - 15 it is evident that the maximum velocity was found to rise with both the addition of solids and the friction velocity. The data may be represented by the empirical equation.

$$y'_m - y'_{m_w} = 1.52 \left(\bar{C} \frac{\tau_o}{\rho_w} \right)^{1.31} \text{ ins.} \quad (14)$$

5. Mean Slip Velocities.

Tables VIII - 4 and 5 summarise the results of the attempts at estimation of the difference between the mean velocity of the fluid and that of the particles according to the method described in Section III - 2. In this method the mean discharge velocity and solids concentration are used with the mean water velocity by salt injection to compute, through the mass balances on fluid and solids, the transport concentration and mean solids velocity. It is obvious that for the case of the coarse sand the mean slip $\bar{V}_w - \bar{V}_s$ increases with solids concentration and decreases with pressure gradient. This second effect is presumed to be a consequence of the fact that an increased pressure gradient tends to distribute the solids more evenly over the cross-section. Considerable scatter of the derived data is evident for the case of the gravel - undoubtedly a consequence of the pronounced scatter of the experimental data.

Figure VIII - 16 illustrates that the slip velocity for the gravel may remain constant at about 3.30 ft/sec., but in the absence of conclusive evidence it is felt that the expression

$$\bar{V}_w - \bar{V}_s = 3.70 - 0.0397 \frac{(1-\bar{q})}{W_o} \bar{V}_w \text{ ft/sec.} \quad (15)$$

represents the data adequately for both gravel and coarse sand.

TABLE VIII - 4

SUMMARY OF MEAN VELOCITY DETERMINATIONS.COARSE SAND.

i (ft/ft)	$\bar{C}$	$\bar{V}_m$ (ft/sec)	$\bar{V}_w$ (ft/sec)	$\bar{V}_s$ (ft/sec)	$\bar{V}_w - \bar{V}_s$ (ft/sec)
0.160	0.01	6.61	6.67	3.54	3.07
	0.03	6.23	6.42	3.16	3.07
	0.05	5.52	5.89	2.62	3.27
	0.07	4.35	4.85	1.60	3.25
	0.08	2.82	3.87	0.64	3.23
0.280	0.03	8.80	8.93	6.27	2.66
	0.06	8.41	8.65	5.85	2.80
	0.09	7.64	8.05	5.03	3.02
	0.12	6.62	7.28	4.00	3.28
	0.15	5.12	6.02	2.72	3.30
	0.18	2.78	4.36	1.06	3.30
0.400	0.02	10.95	10.97	8.79	2.18
	0.05	10.70	10.85	8.53	2.32
	0.08	10.35	10.60	8.15	2.45
	0.11	9.78	10.16	7.53	2.63
	0.14	9.05	9.59	6.76	2.83
	0.17	8.17	8.88	5.81	3.07
	0.20	7.17	8.18	4.90	3.28

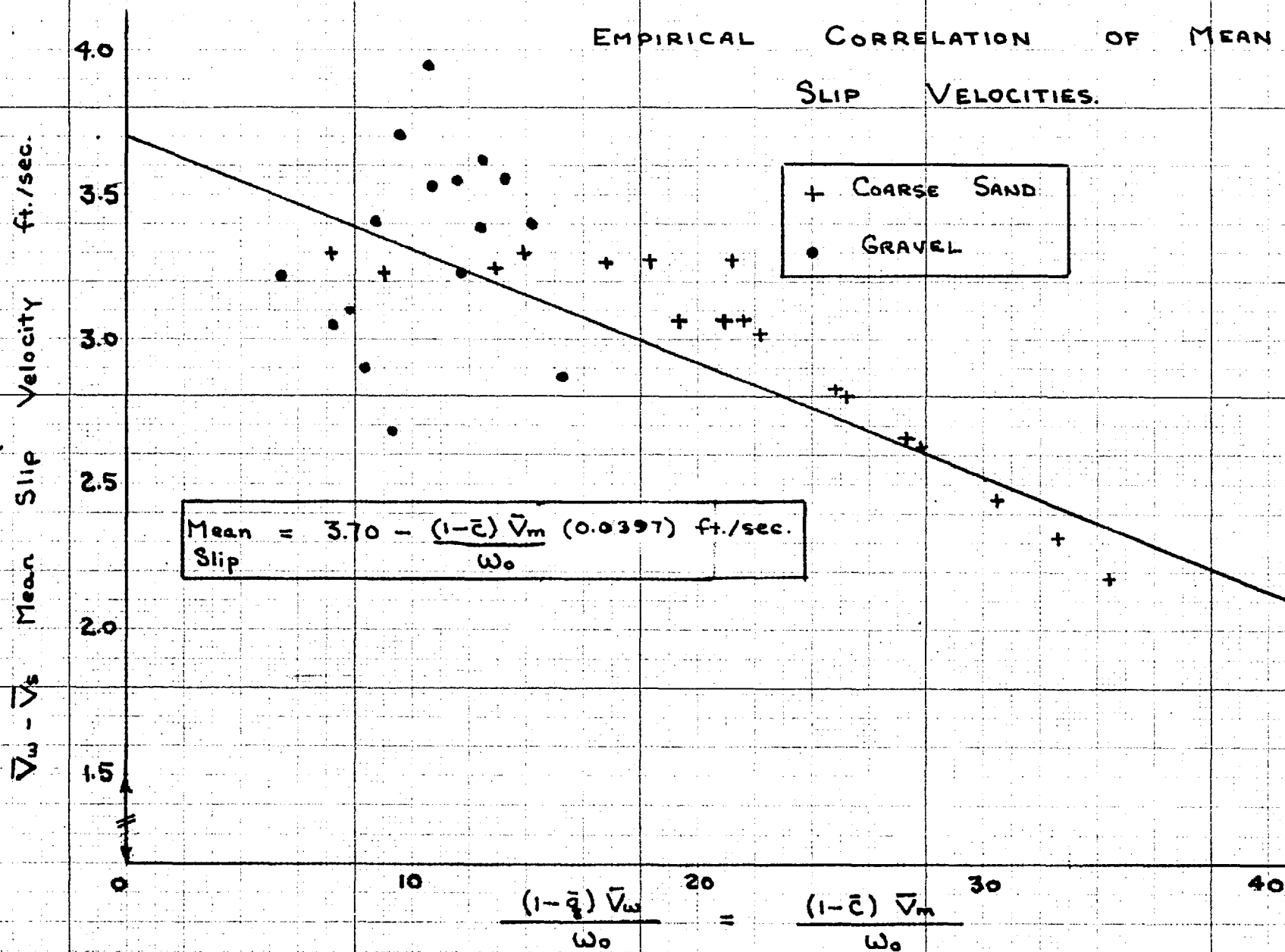
TABLE VIII - 5

SUMMARY OF MEAN VELOCITY DETERMINATIONS.GRAVEL.

$\bar{d}$ (ft/ft)	$\bar{C}$	$\bar{V}_m$ (ft/sec)	$\bar{V}_w$ (ft/sec)	$\bar{V}_s$ (ft/sec)	$\bar{V}_w - \bar{V}_s$ (ft/sec)
0.160	0.01	6.49	6.53	3.85	2.68
	0.03	5.96	6.12	3.22	2.90
	0.05	5.19	5.51	2.46	3.05
	0.07	4.04	4.67	1.45	3.22
0.280	0.02	8.80	8.91	5.41	3.39
	0.04	8.37	8.61	5.06	3.55
	0.06	7.82	8.25	4.30	3.95
	0.08	7.27	7.80	4.04	3.71
	0.10	6.71	7.31	3.90	3.41
	0.12	6.17	6.80	3.70	3.10
0.400	0.02	10.73	10.81	7.94	2.87
	0.04	10.30	10.50	7.10	3.40
	0.06	9.87	10.19	6.63	3.56
	0.08	9.45	9.88	6.25	3.62
	0.10	9.03	9.58	6.35	3.23
	0.12	8.61	9.25	5.72	3.53

FIG. VIII-16

EMPIRICAL CORRELATION OF MEAN
SLIP VELOCITIES.



It must be pointed out that these measurements strike at the heart of the equations derived by Wilson and Abbott by refuting the assumption that the mean slip is a constant fraction of the discharge velocity.

6. Derived Data.

(a) Introduction.

The mass balances given by Equations III - 1 and III - 2 may be applied at any point in the pipe provided a steady-state relationship between the variables is assumed. At high concentrations the velocity profile along an horizontal chord should be flat enough to permit the maximum velocity, as measured in these experiments, to be used to estimate the mean value. In addition, the measurements from which the discharge concentration profiles were obtained, permitted the estimation of the mixture discharge velocity profile. Since the discharge concentration C was known, the solids velocity V_s and the transport concentration q could be estimated in the lower regions of the pipe. It was discovered that the predictions of transport concentration were usually higher than the maximum possible concentration of about 0.54 in the lowest regions of the pipe.

This difference was attributed to the fact that the mean water velocity V'_w along a chord would be less than the

FIG. VIII-17

PLOT SHOWING DISCHARGE AND ASSUMED TRANSPORT CONCENTRATIONS

COARSE SAND $l = 0.160$ ft./ft. $\bar{C} = 0.05$

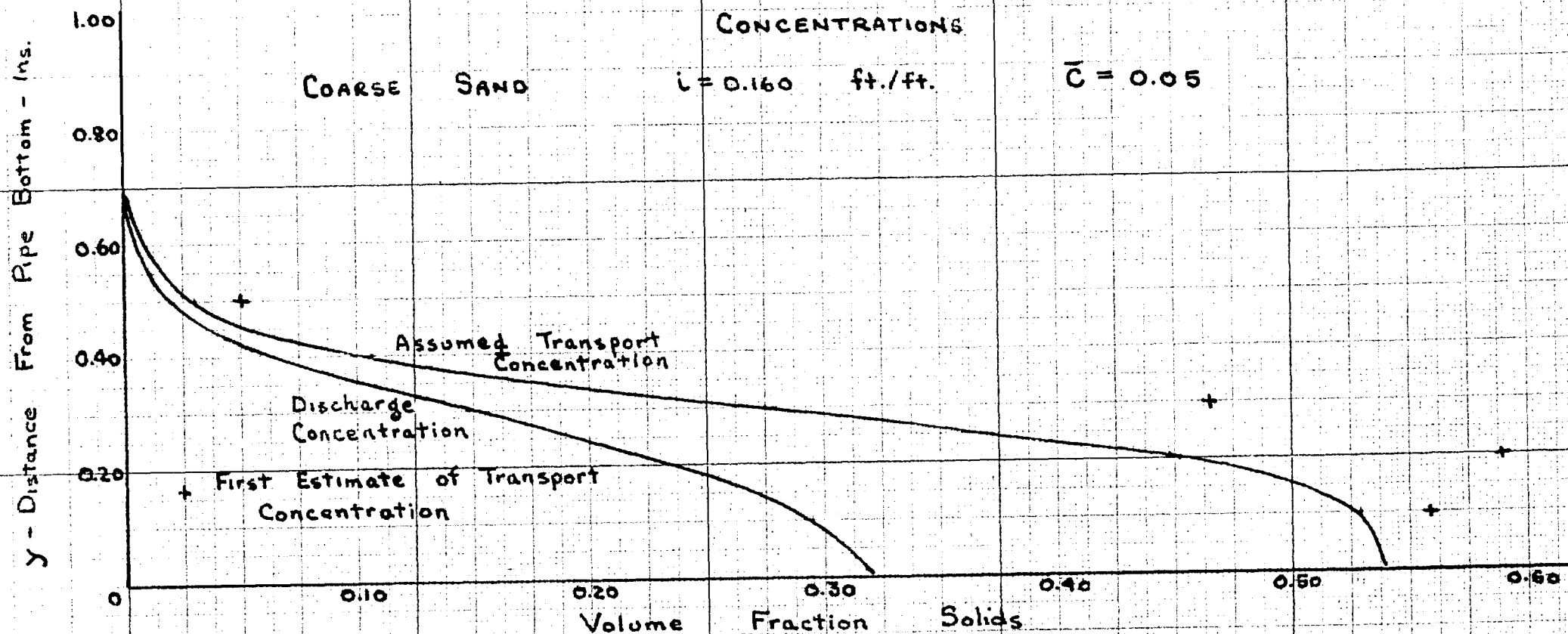


FIG. VIII-18

PLOT SHOWING RESULTS OF LOCAL VELOCITY ESTIMATIONS.

COARSE SAND $\bar{C} = 0.05$

$l = 0.160$ ft./ft.

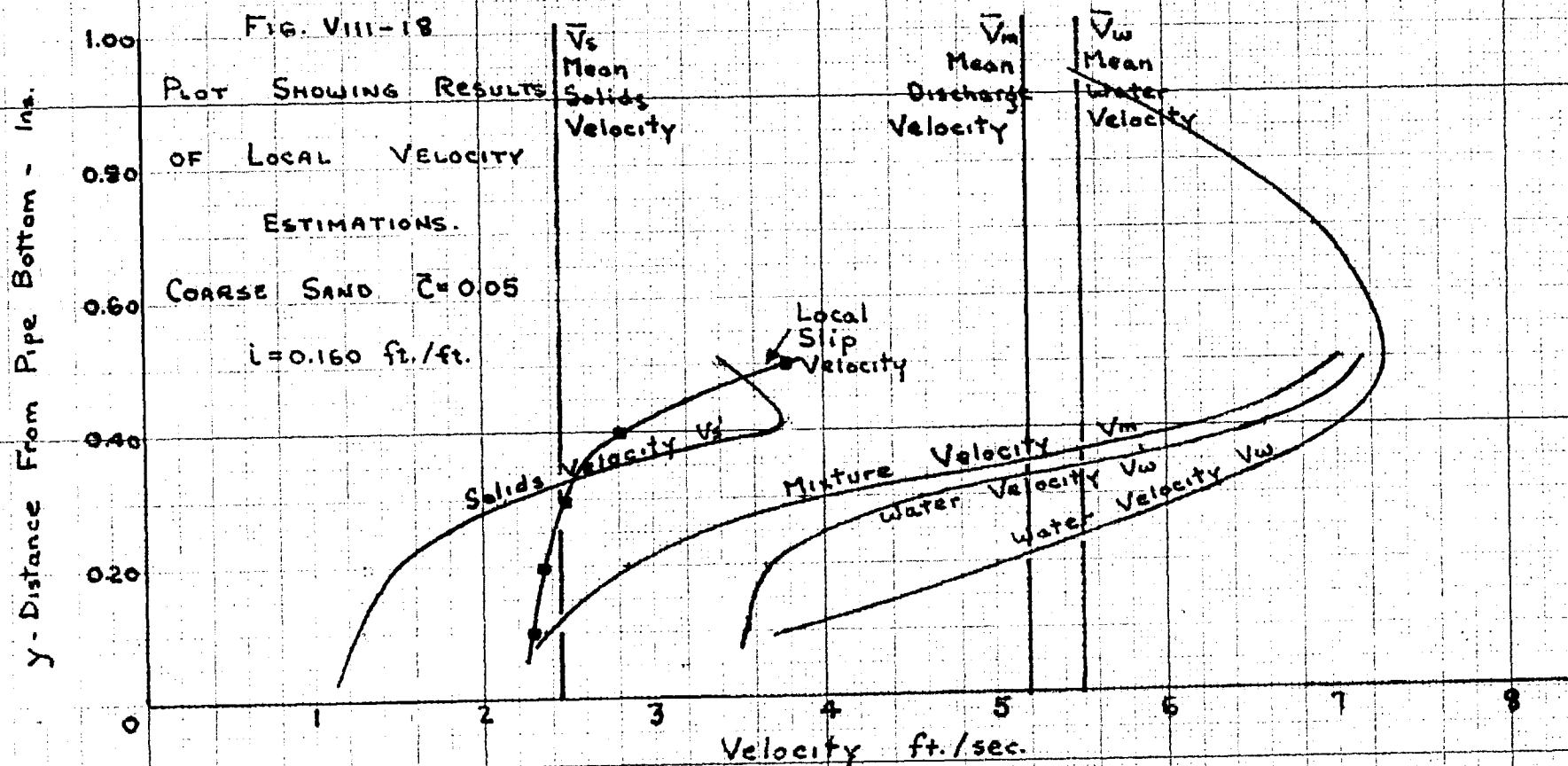


FIG. VIII-19

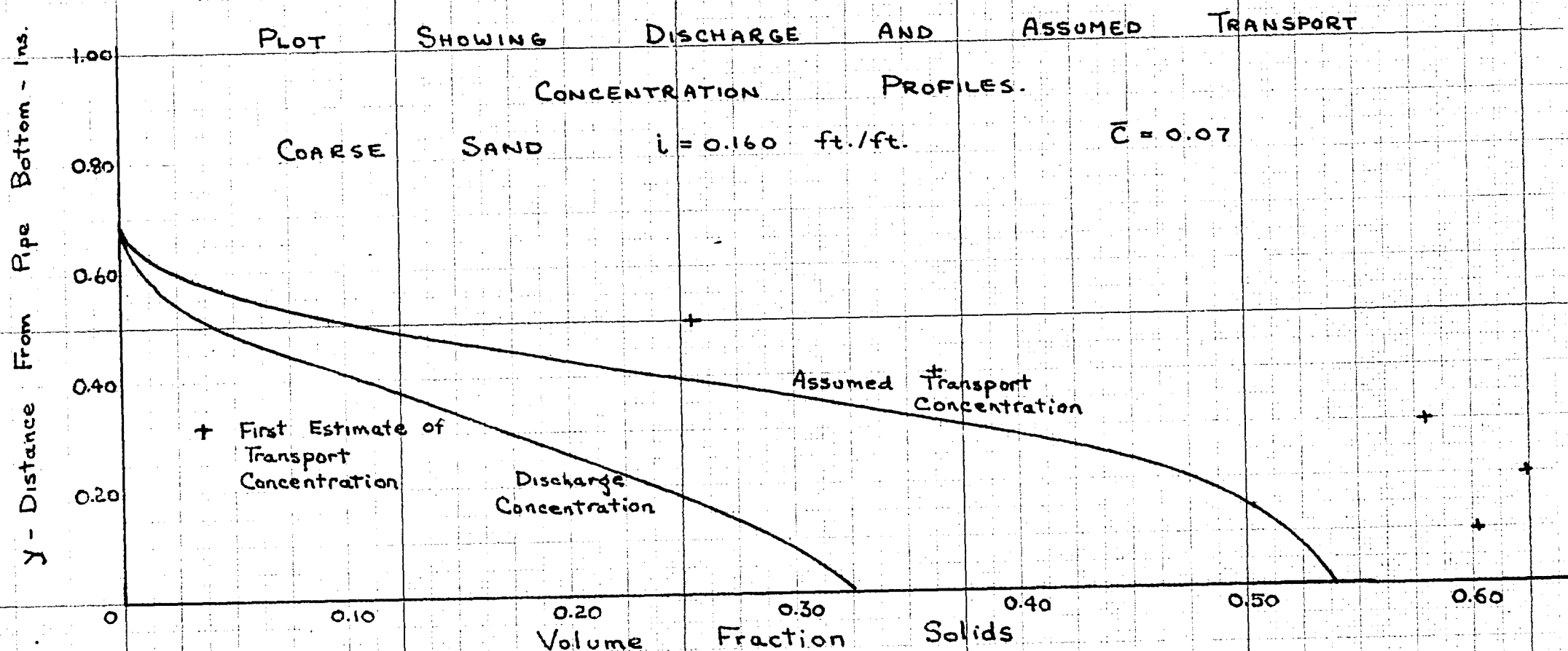


FIG. VIII-20

PLOTS SHOWING RESULTS OF LOCAL VELOCITY ESTIMATIONS.

COARSE SAND $\bar{C} = 0.07$
 $i = 0.160$ ft./ft.

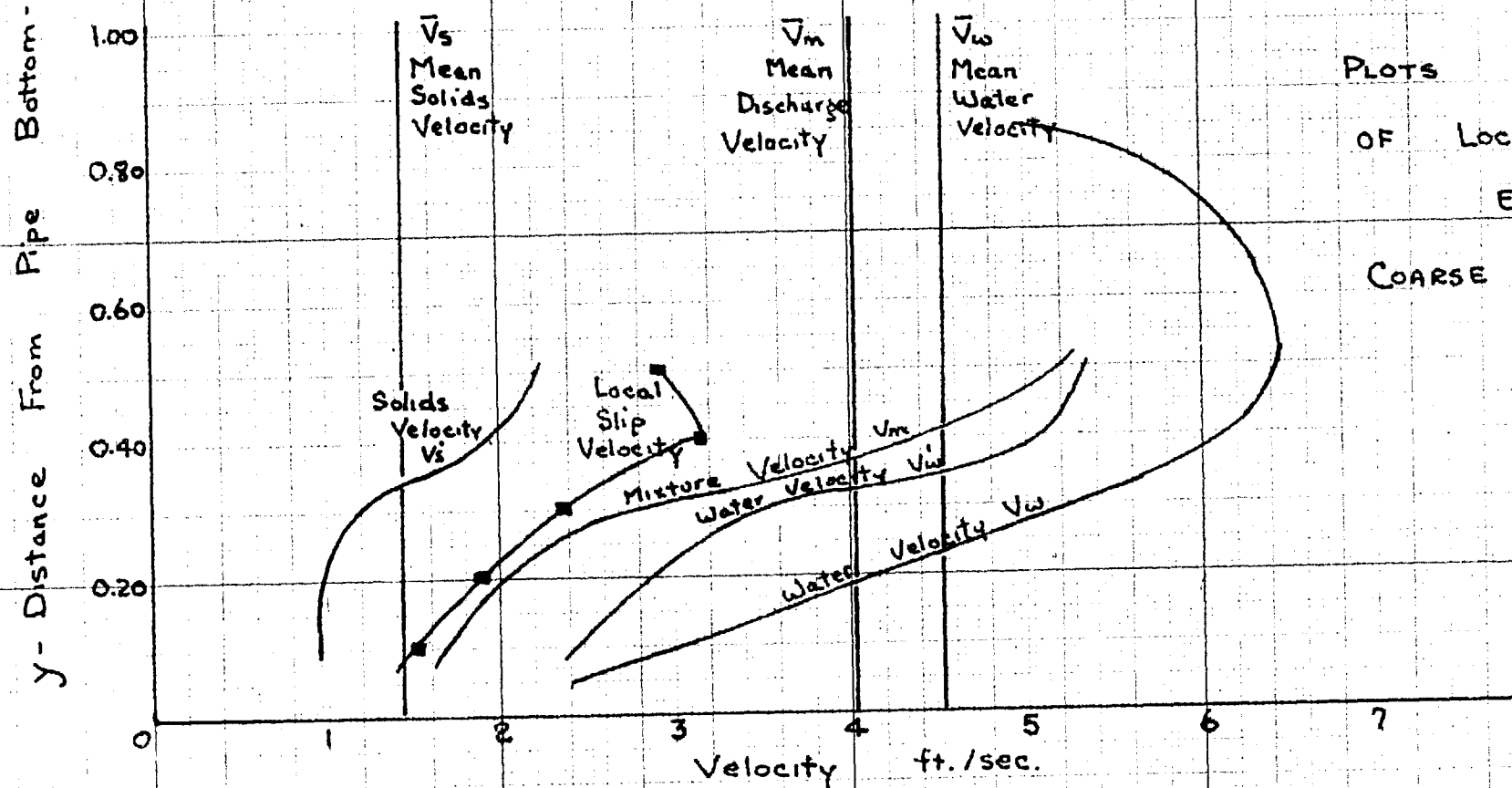


TABLE VIII - 6
DERIVED QUANTITIES
COARSE SAND.

i = 0.160 ft/ft.

$\bar{C}$	y ins.	C	V_m ft/sec.	V_w' ft/sec.	V_s' ft/sec.	q	q'
0.05	0.1	0.295	2.36	3.77	1.32	0.559	0.529
	0.2	0.223	2.65	5.01	1.31	0.589	0.452
	0.3	0.148	3.89	6.20	2.09	0.465	0.275
	0.4	0.064	6.28	7.02	3.79	0.163	0.106
	0.5	0.014	7.04	7.30	3.40	0.050	0.031
0.07	0.1	0.297	1.69	3.00	0.96	0.603	0.521
	0.2	0.223	2.01	4.15	0.97	0.624	0.461
	0.3	0.162	2.68	5.36	1.19	0.581	0.365
	0.4	0.106	4.36	6.13	1.96	0.362	0.236
	0.5	0.041	5.01	6.45	2.21	0.255	0.093
<u>i = 0.280 ft/ft.</u>							
0.15	0.1	0.409	2.28	3.72	1.74	0.668	0.536
	0.2	0.379	2.65	4.72	1.90	0.652	0.528
	0.3	0.339	3.49	6.14	2.34	0.624	0.506
	0.4	0.287	5.19	7.72	3.52	0.521	0.422
	0.5	0.201	6.73	8.04	5.18	0.331	0.261
	0.6	0.117	7.35	8.29	3.94	0.218	0.130
0.18	0.1	0.430	1.64	3.09	1.33	0.698	0.536
	0.2	0.405	2.07	3.78	1.59	0.674	0.527
	0.3	0.374	2.58	4.75	1.91	0.660	0.505
	0.4	0.323	3.80	5.62	2.80	0.542	0.438
	0.5	0.258	5.32	6.09	4.28	0.341	0.321
	0.6	0.180	6.36	6.38	5.56	0.182	0.206

value V_w measured by pitot suction pressure. Indeed, if the first estimate of transport concentration had not been too high, this would have indicated an appreciable error in one or more of the experimental measurements.

The inadequacy of the water velocity measurements was eliminated by the following technique. Since the transport concentration must vanish at the point on the concentration profile where $C = 0$ (unless the particles are stationary) the first estimates of transport concentration could be used as guides in drawing an assumed transport concentration profile between the limits $q = 0$ and $q = 0.54$. New values of water velocity V'_w , solids velocity V'_s and local slip velocity could then be calculated from the mass balances using the mixture velocity profile and the discharge and assumed transport concentration profiles. These data are shown in Tables VIII - 6 and 7 where q denotes the first and q' the second estimates of transport concentration and V'_w the derived mean water velocity.

(b) Coarse Sand.

Figures VIII - 17 to 28 and Table VIII - 6 show the results of those estimations for the cases where the bed concentration was considered high enough to reach its maximum value. It will be seen that near the bottom of the pipe the solids velocity seems to change very slowly with y . This observation is compatible with the existence of a bed,

FIG. VIII-21

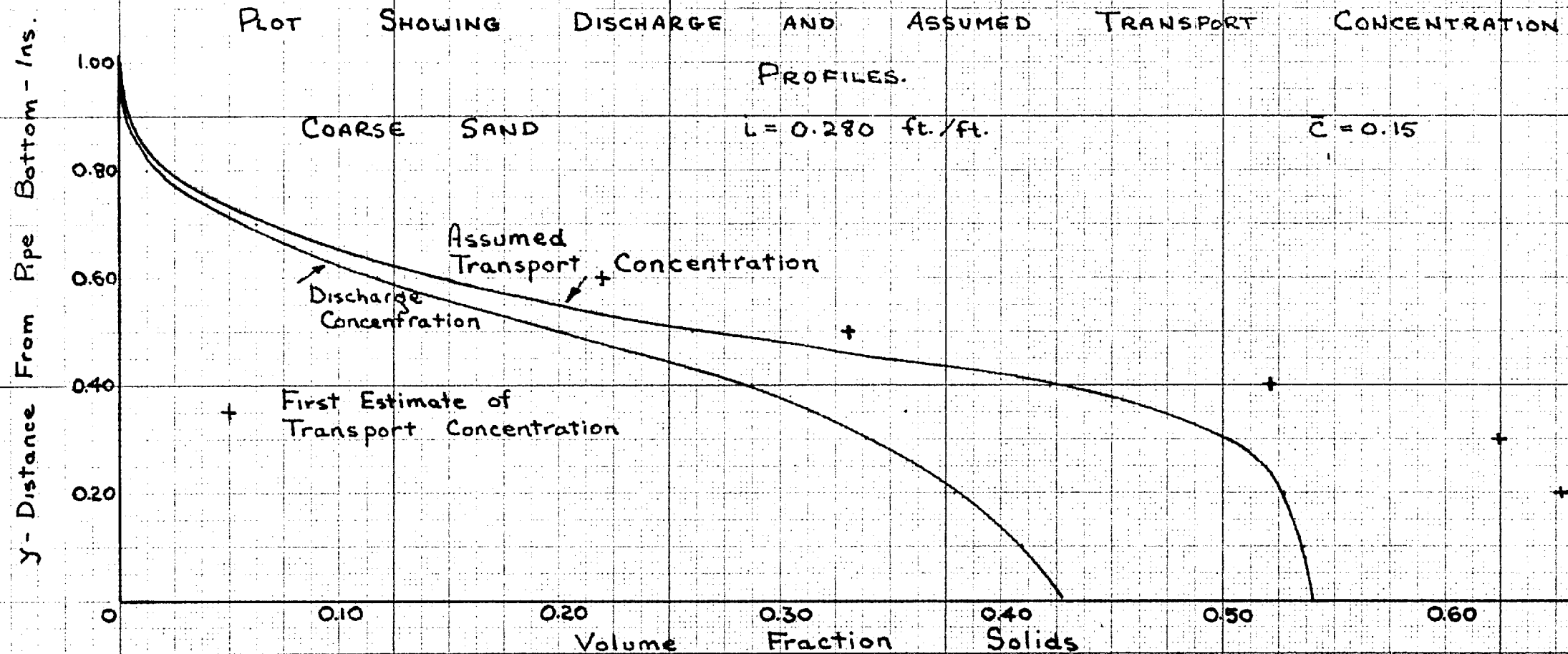


FIG. VIII-22

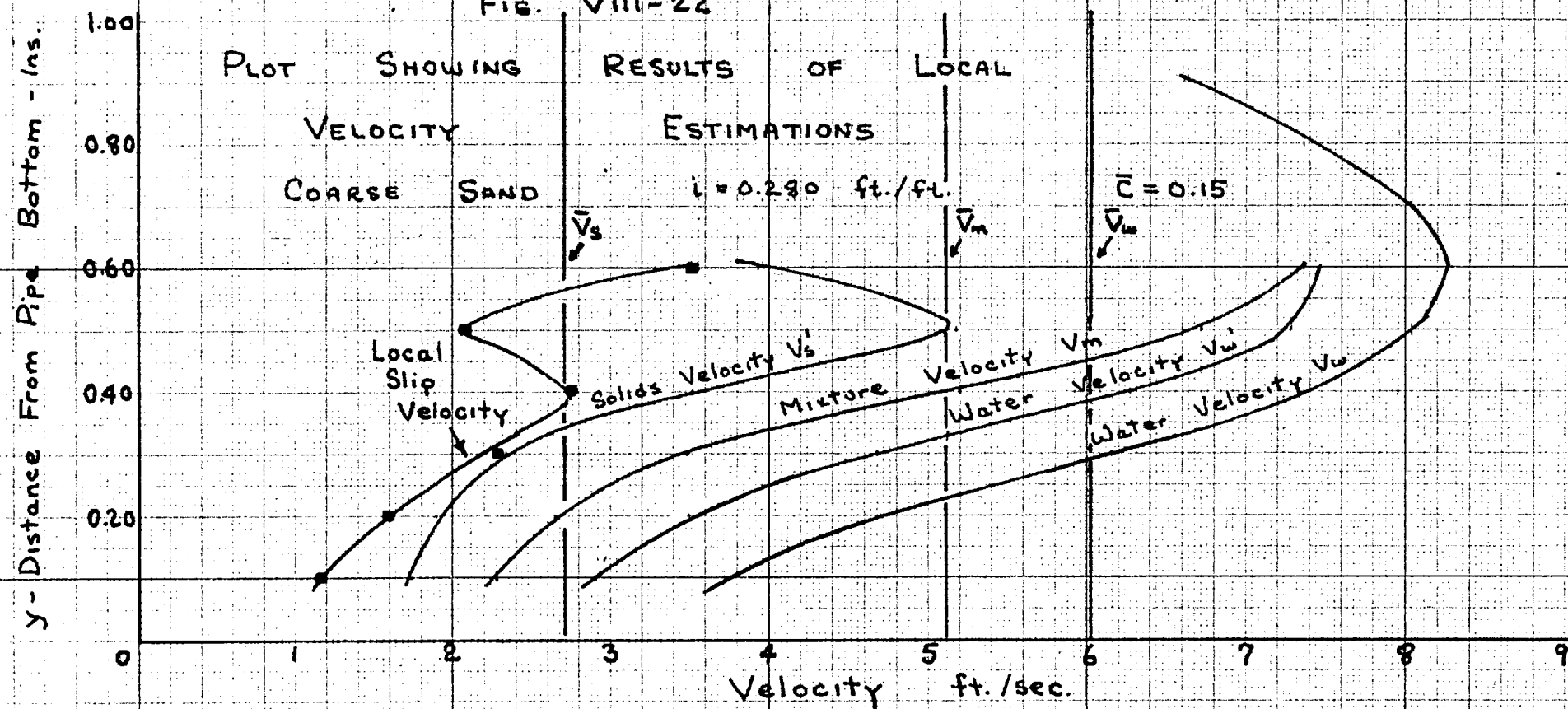


FIG. VIII-23

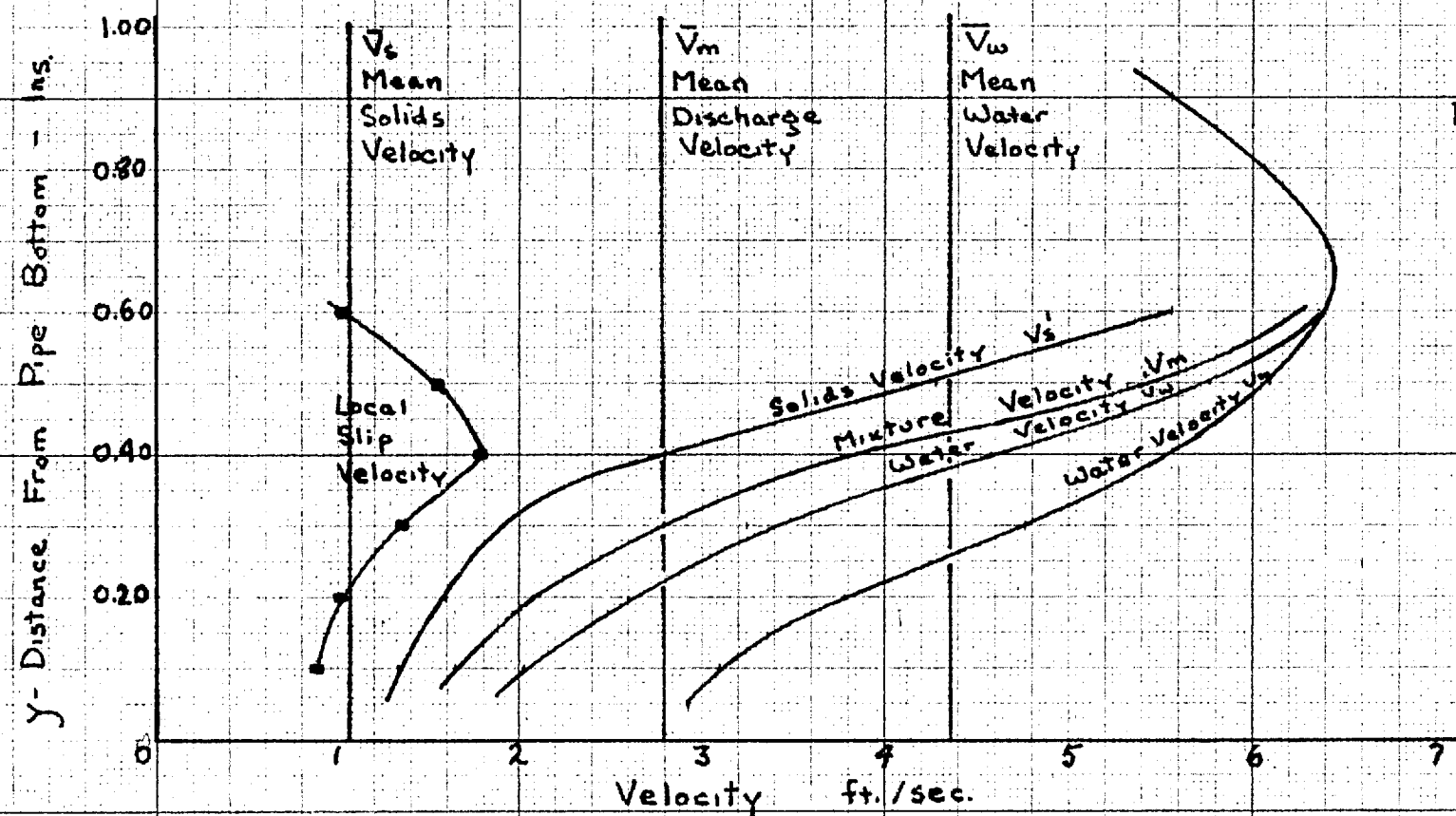
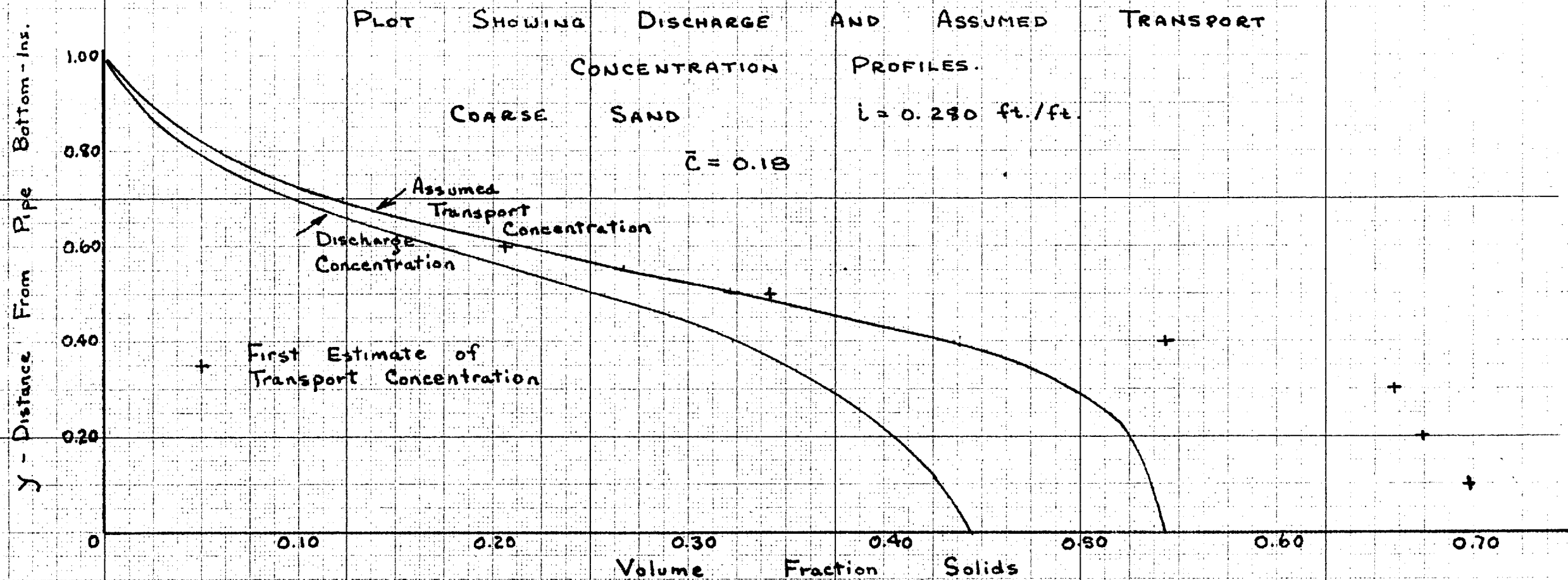


FIG. VIII-24

PLOT SHOWING RESULTS OF LOCAL VELOCITY ESTIMATIONS.

COARSE SAND $l = 0.290$ ft./ft. $\bar{C} = 0.18$

agrees with Bagnold's predictions and is probably correct because the high concentrations minimise the error resulting from inaccuracies in V'_m , C and q . In addition, in the condition bordering upon the formation of a stationary bed ($i = 0.160 \frac{ft}{ft}$, $\bar{C} = 0.07$) in Fig. VIII - 19 and 20 the rate of change of V_s with y is only very slight. As Bagnold predicted, this region is separated from one in which V_s increases more rapidly by a point of inflection in the concentration profile. At higher levels, where C falls below about 0.10 it is not considered that the values of V_w , V_s and especially $V_w - V_s$ are nearly as accurate. It may be seen that although the local slip velocity does not change much over the cross-section, there is some indication that it tends to increase with distance from the bottom of the pipe. The opposite variation is necessary to explain the observed form of the concentration profiles for perspex in the absence of the effects of spin. The peculiar S-shape of the curves for local slip velocity may be attributed to the aforementioned errors in C , q and V_m when the absolute values of C and q are low.

It would appear that the high concentrations of particles, for which these estimations can be made, exerted a considerable influence upon the turbulence of the fluid because the settling velocity of this material, 0.313 ft/sec. was less than the friction velocity at the lowest hydraulic

TABLE VIII - 6

(Continued)

DERIVED QUANTITIESCOARSE SAND.

$$i = 0.400 \text{ ft/ft.}$$

$\bar{C}$	y ins.	C	V_m ft/sec.	V_w' ft/sec.	V_s' ft/sec.	q	q'
0.17	0.1	0.385	4.67	6.27	3.36	0.542	0.535
	0.2	0.375	5.22	7.63	3.75	0.572	0.522
	0.3	0.357	6.75	8.96	5.10	0.516	0.473
	0.4	0.304	8.44	10.08	7.08	0.416	0.362
	0.5	0.195	9.37	10.74	7.91	0.297	0.231
	0.6	0.079	9.72	11.12	7.05	0.186	0.108
0.20	0.1	0.401	4.27	5.76	3.26	0.557	0.537
	0.2	0.398	4.93	6.85	3.74	0.566	0.525
	0.3	0.390	6.35	7.89	4.98	0.509	0.494
	0.4	0.362	7.77	8.80	6.39	0.436	0.440
	0.5	0.309	8.62	9.38	7.30	0.365	0.355
	0.6	0.133	8.98	9.77	6.28	0.295	0.190

FIG. VIII-25

PLOT SHOWING DISCHARGE AND ASSUMED.

TRANSPORT CONCENTRATION PROFILES.

COARSE SAND

$i = 0.400 \text{ ft./ft.}$

q'_s Assumed Transport Concentration $\bar{C} = 0.17$

Discharge Concentration

+ First Estimate of Transport Concentration

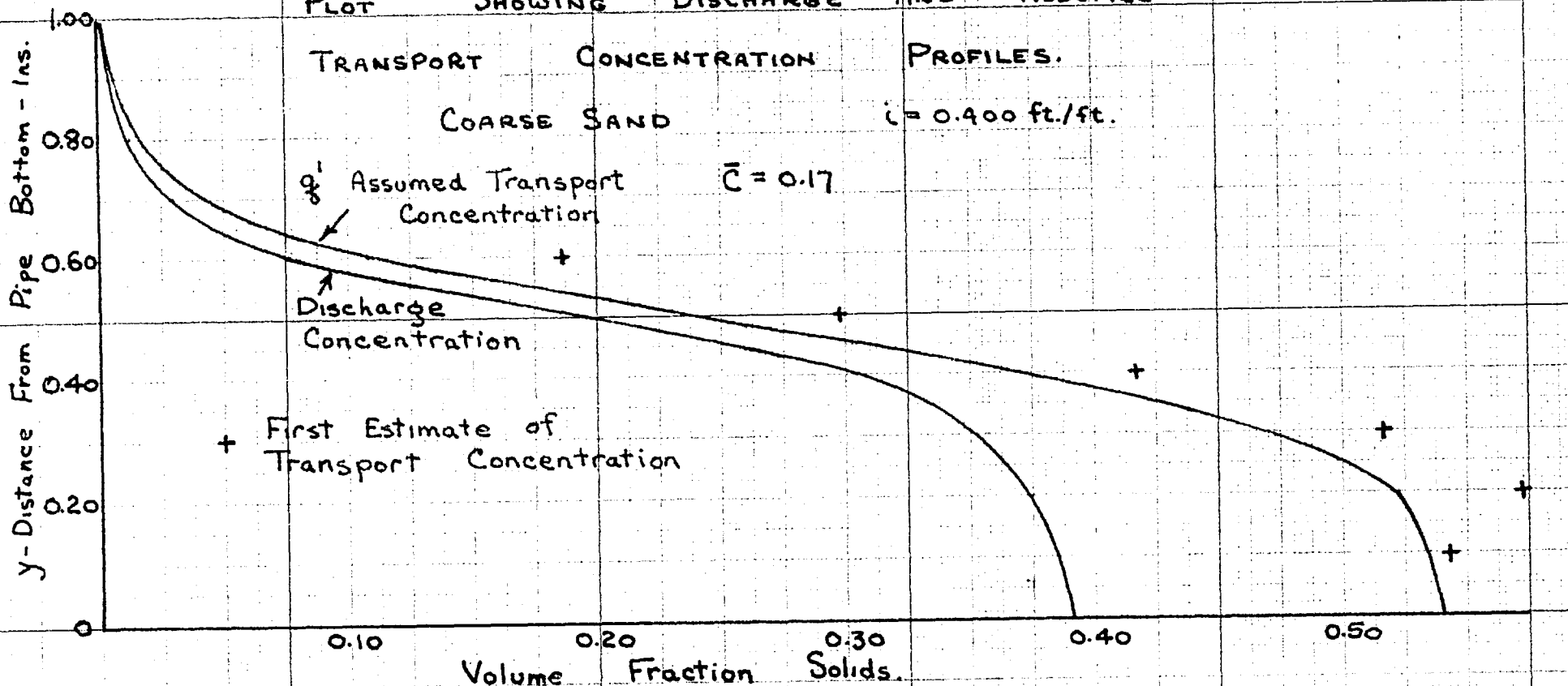


FIG. VIII-26

PLOT SHOWING RESULTS OF LOCAL VELOCITY ESTIMATIONS.

COARSE SAND

$i = 0.400 \text{ ft./ft.}$

$\bar{C} = 0.17$

Local Slip Velocity

$\bar{V}_s$
Mean Solids Velocity

$\bar{V}_m$
Mean Discharge Velocity

$\bar{V}_w$
Mean Water Velocity

Solids Velocity V'_s

Mixture Discharge Velocity V'_m

Water Velocity V'_w

Water Velocity V'_w

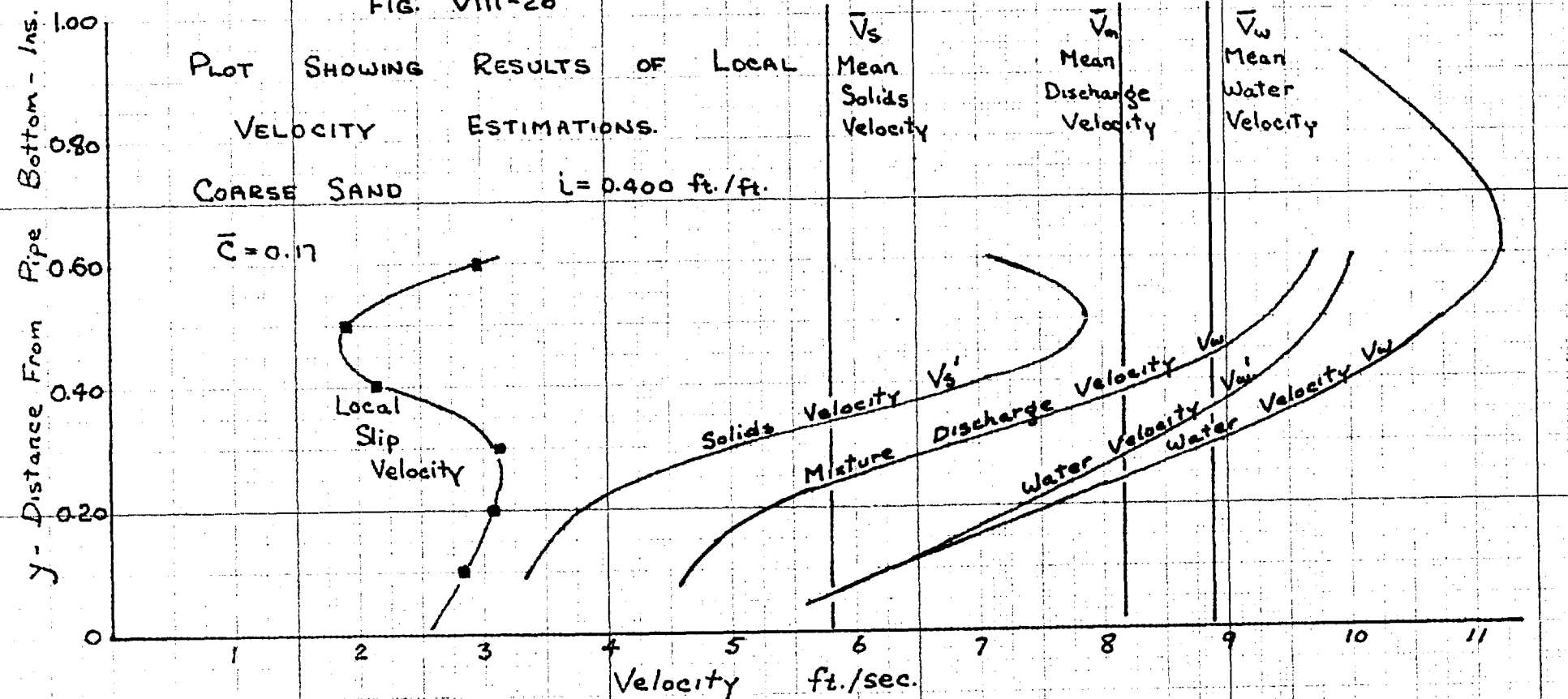


FIG. VIII-27

PLOT SHOWING DISCHARGE AND ASSUMED TRANSPORT CONCENTRATION PROFILES

COARSE SAND $i = 0.400$ ft./ft.

$\bar{C} = 0.20$

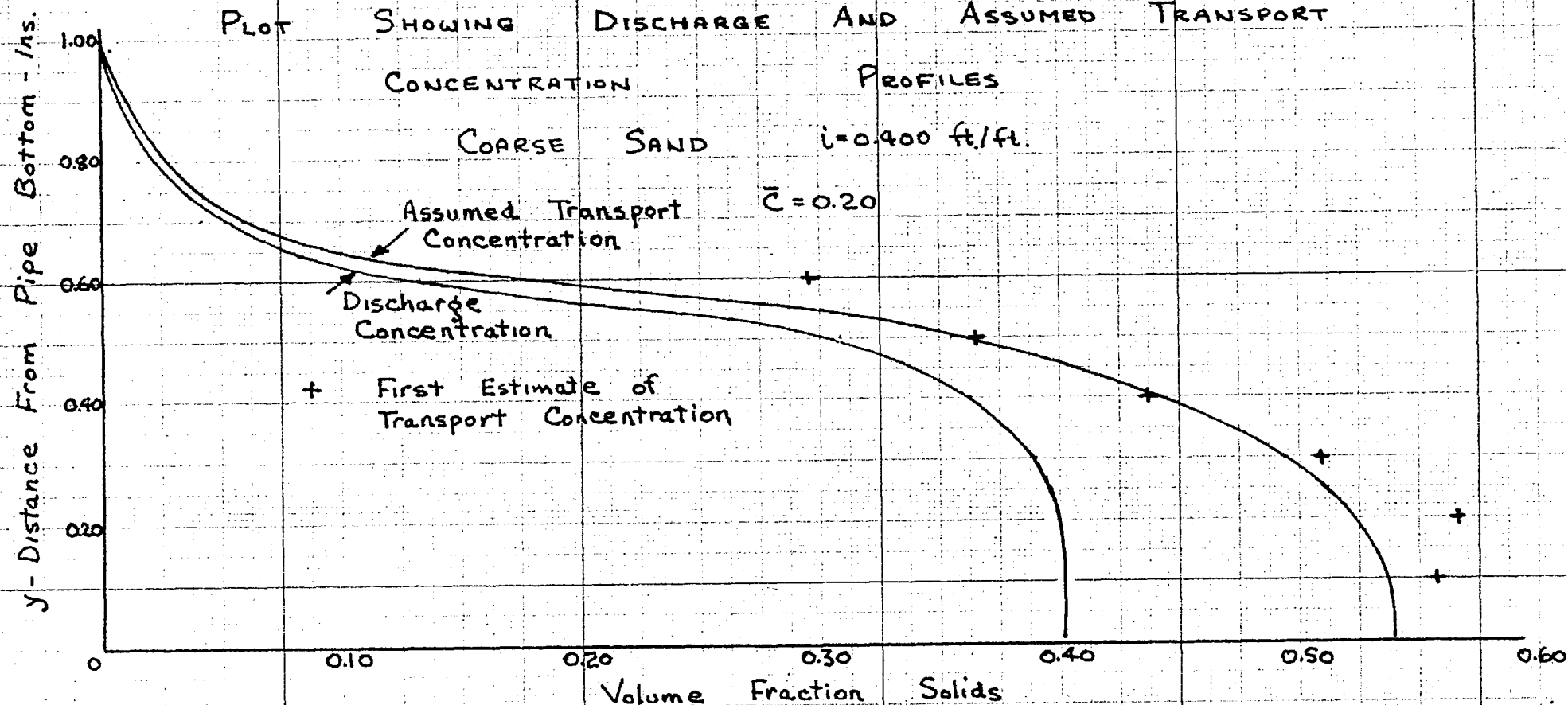


FIG. VIII-28

PLOT SHOWING RESULTS OF LOCAL VELOCITY ESTIMATIONS

COARSE SAND $i = 0.400$ ft./ft.

$\bar{C} = 0.20$

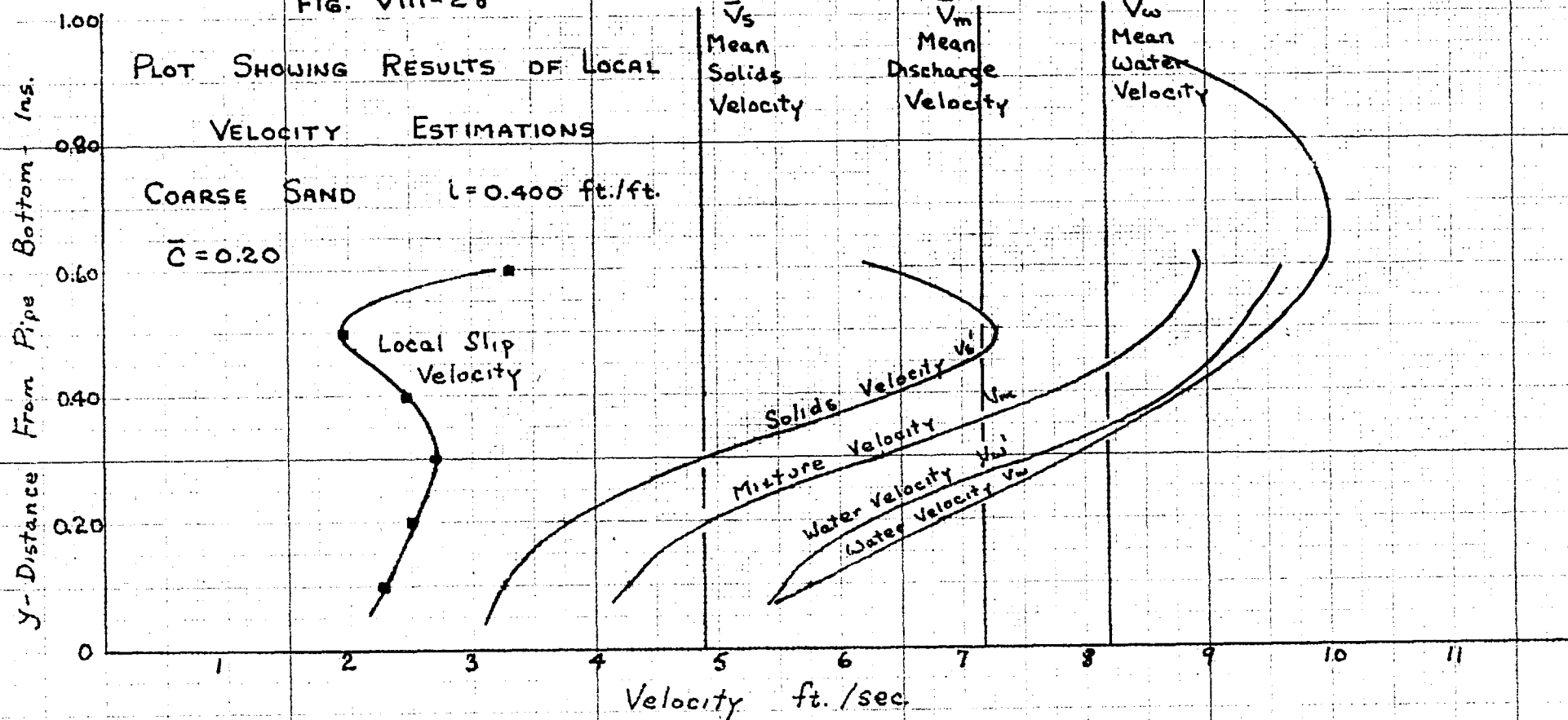


TABLE VIII - 7
DERIVED QUANTITIES
GRAVEL.

$i = 0.280 \text{ ft/ft.}$

$\bar{C}$	y ins.	C	V_m ft/sec.	V_w' ft/sec.	V_s' ft/sec.	q	q'
0.10	0.1	0.279	2.66	4.96	1.48	0.613	0.500
	0.2	0.265	3.31	6.03	1.97	0.596	0.446
	0.3	0.244	5.00	7.12	3.37	0.469	0.362
	0.4	0.210	6.97	8.17	5.15	0.326	0.284
	0.5	0.144	8.03	9.17	6.02	0.250	0.192

FIG. VIII-29

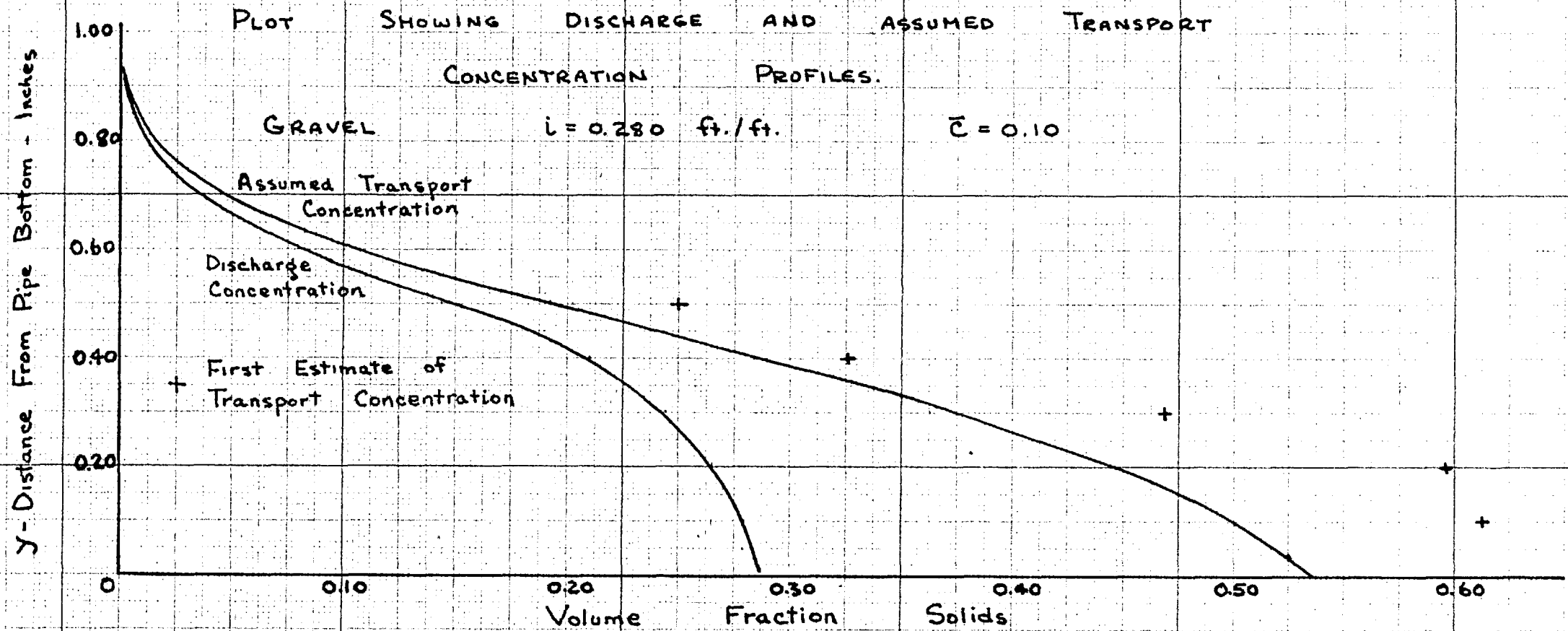
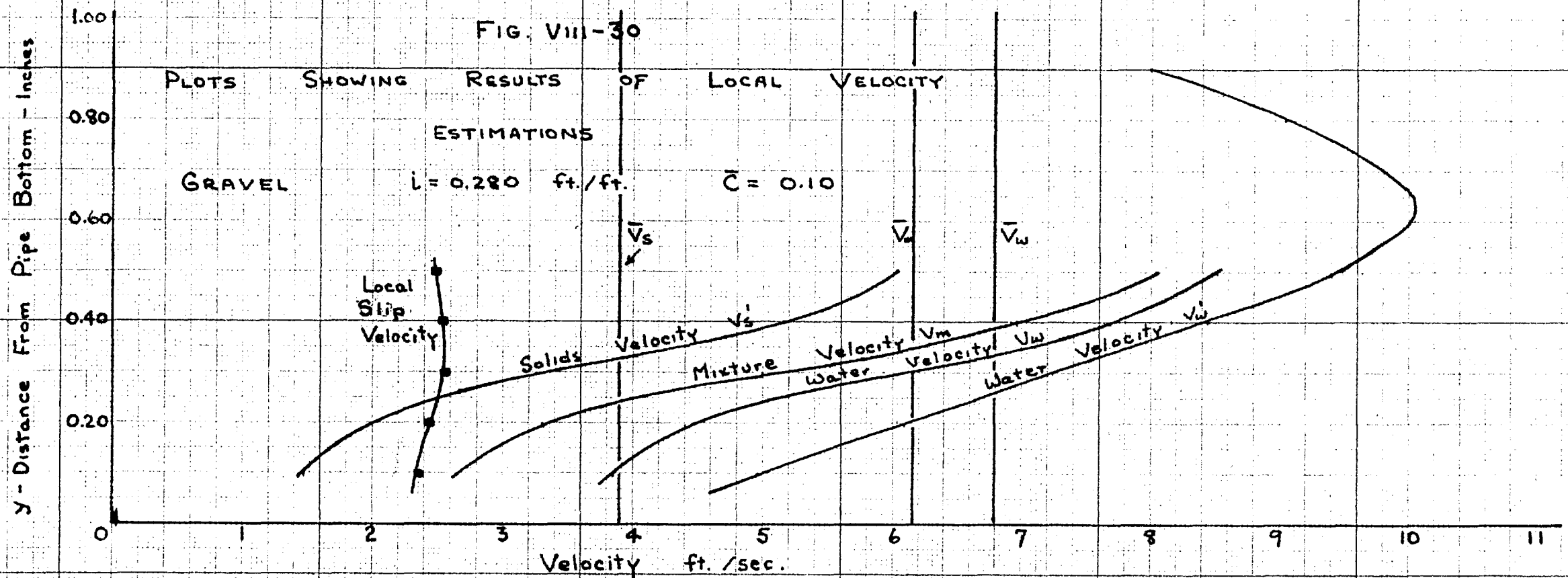


FIG. VIII-30



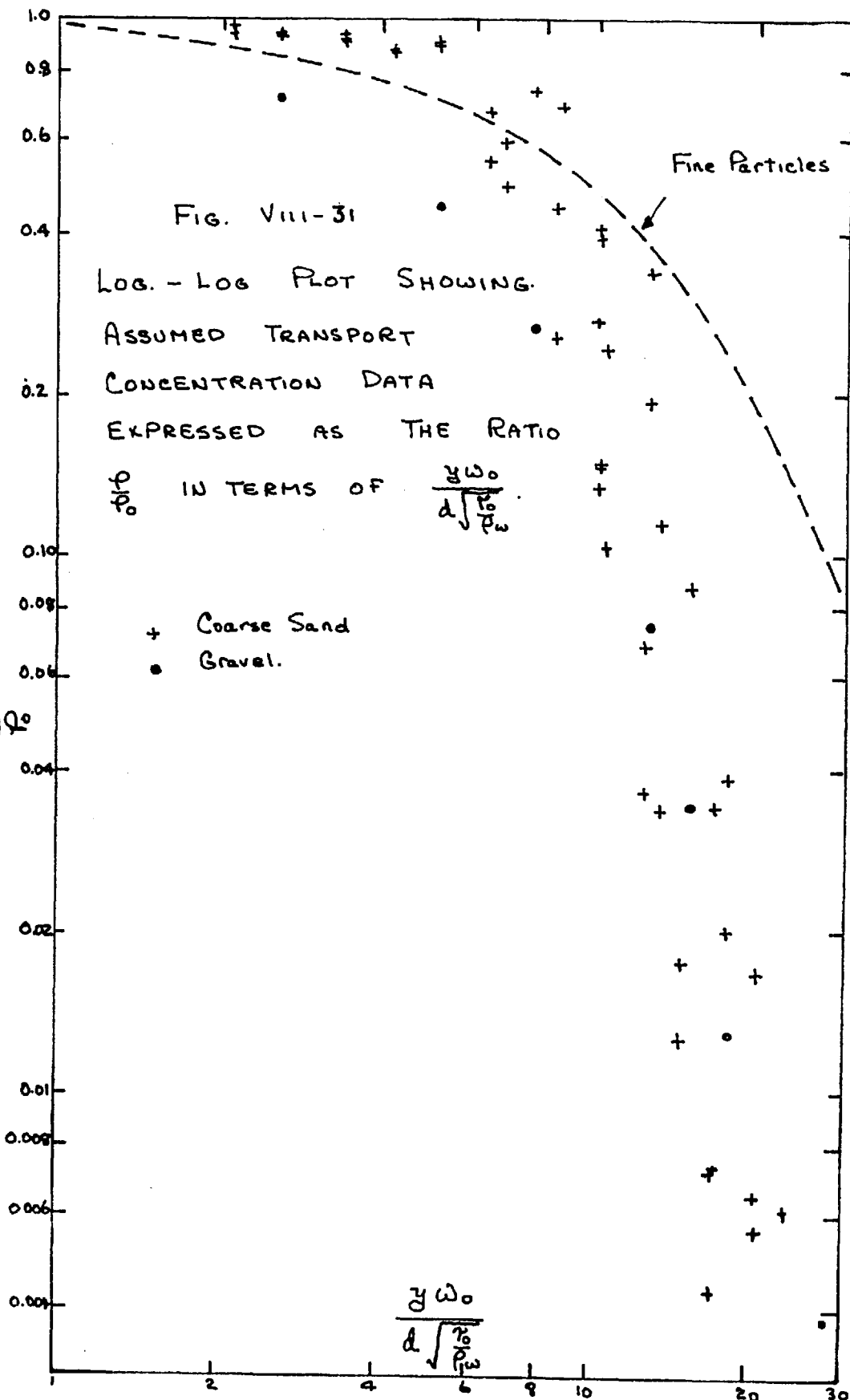
gradient. If a considerable local slip exists at lower mean concentrations where the settling velocity was still less than the turbulent velocity fluctuation, the mechanism of suspension must be rather different from the case in which the local slip is small.

(c) Gravel.

Similar results were obtained for gravel as for coarse sand. Figures VIII - 29 and 30 and Table VIII - 7 illustrate that the local slip velocity does not change much over the cross-section although if the solids concentration does not approach its fluid limit, the local slip velocity may increase from the bottom to the top of the pipe.

The assumed transport concentration profiles are expressed in the form of plots of $\frac{p}{p_c}$ vs. $\frac{y}{d} \frac{W_0}{\sqrt{\frac{\gamma_c}{\rho_c}}}$ in Fig. VIII - 31. The deviation from the dashed curve obtained for the fine particles is severe and much more significant than the deviation observed for Durand's data because W_0 and $\frac{\gamma_c}{\rho_c}$ could be determined accurately for the data obtained in this investigation. Three possible explanations for the behaviour of the data may be given:

- (1) Although the friction velocity is greater than the settling velocity at all pressure gradients for the coarse sand, the high concentrations of solids may so damp the turbulence that the



particles cannot be held in suspension. Under these conditions the mechanism of suspension would be that described by Bagnold.

- (2) The transport concentration may not reach its limit or approach it so closely as has been assumed.
- (3) Secondary flow of the nature predicted by Bagnold, produced by the modification of the fluid velocity profile by the solids, could produce a considerable lateral variation of the solids concentration in the region above the bed. Under these conditions the average concentration along a horizontal chord might not obey the $\frac{p}{p_o}$ vs.

$$\frac{y W_o}{d \sqrt{\frac{\gamma_o}{\rho_w}}} \text{ relationship.}$$

Explanations (1) and (3) seem most likely and it is not considered that the plot would apply to other experimental conditions.

CHAPTER IXConclusions1. Introduction

In the review of the literature it was established that the major contributions which have previously been made to our knowledge of hydraulic conveying in pipelines have been concerned with externally measured variables such as pressure gradient and discharge concentration. The flow circuit of the plant used in the present investigation has been employed previously (8,12) in attempts to produce correlations for the friction loss associated with flow and for the investigation of the macroscopic effects of flow with mixtures of particles of two different sizes.

This investigation represents an attempt at measurement or estimation of some of the internal variables which affect the process of momentum exchange between fluid and particles on the microscopic scale. Five materials were chosen, varying in size from 0.0046 to 0.061 inch in diameter, in specific gravity from 1.17 to 4.60, and in settling velocity from 0.0648 to 0.695 ft./sec.. For each of these materials attempts were made to determine the distribution of the discharged solids over the cross-section, the velocity profile of the fluid, and the difference between the mean velocities of the fluid and the solids at

each of three hydraulic gradients: 0.160, 0.280, and either 0.372 or 0.400 ft.(water)/ft.(pipe). The experimental techniques involved were simple and met with varying degrees of success. Nevertheless, it was possible to classify the materials according to the mechanism by which they were supported in the fluid. This classification may be given as follows:

(i) Coarse Particles (Gravel and Coarse Sand)

During the conveying of these materials at high concentrations, a moving bed develops in the lower part of the pipe. Under these conditions the particles are probably supported by dispersive stresses resulting from interparticle contacts. At lower concentrations the particles appear to travel in saltation when their settling velocities are greater than the lifting component of turbulence.

(ii) Fine Particles (Fine Sand and Zircon)

The settling velocities of these materials were low enough for them to have been supported by the turbulent velocity fluctuations.

(iii) Large Particles of Low Density (Perspex)

It was quite likely that particle spin was an important factor in the supporting of these particles against their own weight.

Experimental observations and conclusions will be presented separately for each group of materials.

2. Coarse Particles

(i) Concentration Profiles

At high mean concentrations of solids the rate of change of local concentration with height was observed to be slow near the bottom of the pipe, suggesting the existence of a bed. Above this region the solids concentration decreased very rapidly with distance from the bottom of the pipe.

(ii) Velocity Profiles

The presence of solids in the system was observed to reduce the maximum fluid velocity and raise the point at which it occurred. The fluid velocity was reduced, in the lower section of the pipe, by an amount proportional both to the distance from the point of maximum velocity and to the mean solids concentration. Above the point of maximum velocity, the local solids concentration was low and the velocity profile corresponded with the Universal Velocity Distribution. At high concentrations and low pressure gradients, some evidence of a point of inflection on the water velocity velocity profile was obtained.

(iii) Mean Slip Velocities

It was observed that the difference between the the mean velocity of the fluid and that of the solids was directly proportional to solids concentration and

varied inversely with the pressure drop. In addition, the mean slip velocity was observed to be greater for gravel than for coarse sand,

(iv) Derived Data

By making certain assumptions, it was possible to estimate the local slip velocity for those cases where the solids concentration in the lowest regions of the pipe approached its limiting maximum value. There was little evidence of a substantial variation of slip velocity over the cross-section. The local solids velocity was observed to increase slowly with distance from the bottom of the pipe until the region of high solids concentration was passed. Thereafter, its rate of change was rapid.

An attempt at empirical correlation of discharge concentration profiles met with failure because the important local variables such as transport concentration, solids velocity gradient and slip velocity were not known. It is quite likely that the data will be of use when the analysis made by Bagnold (46) for the two-dimensional case can be applied to a circular pipe. Considerably more experimental work will have to be done before this is possible. The fact that both the concentration and fluid velocity profiles indicated flow with a bed at high

mean solids concentrations for coarse sand, indicates that the turbulence of the fluid was considerably damped by the existence of high concentrations of solids. At lower mean concentrations, the settling velocity of the particles may be less than the turbulent velocity fluctuation, and the solids may in fact be suspended. Under these conditions, however, the measurements of mean slip velocity suggest that the slip velocity may be appreciable (greater than 1 ft./sec.) and the mechanism of suspension may differ from that obtaining where the local slip velocity is negligible.

It has been shown that the mean slip velocity decreases as the mean velocity of the mixture is increased. This is in contradiction with the assumptions of Wilson and Abbott, who took the mean slip velocity as a constant fraction of the mixture velocity. Their expression is the only one to date which is based on sound theoretical grounds. It would appear that a successful explanation of external phenomena must be sought on the microscopic scale.

3. Fine Particles

(i) Concentration Profiles

Although the relationship suggested by analytical considerations, equation III-24, was not substantiated the dimensionless quantity $\frac{y W_o}{d \sqrt{\frac{\rho_o}{\rho_w}}}$ which was produced in the analysis, formed a useful depen-

dent variable for the presentation of experimental data. Results obtained by Durand under considerably different conditions gave encouraging agreement, especially in view of the estimations which had to be made where precise figures were not supplied.

(ii) Water Velocity Profiles

Attempts at measuring the distribution of water velocity over a vertical axis through the pipe were unsuccessful because the particles were smaller than the pitot tube orifice.

(iii) Mean Slip Velocities

Since no difference was apparent between the mean velocity of the fluid and the discharged mixture velocity, it was concluded that the mean slip velocity was very small.

The settling velocities of these materials were considerably less than the turbulent velocity fluctuations of the fluid in the absence of the solids at all three hydraulic gradients. Since no change of regime was apparent as the mean solids concentration was increased, it was concluded that, unlike coarse sand, these materials did not damp the turbulence to such an extent that suspension could not exist. From Durand's data obtained with larger particles in pipes of greater diameter, it would appear that the criterion governing flow regime is not necessarily the diameter of the particle. It seems more likely that

particles remain in suspension only so long as their settling velocities are less than the turbulent velocity fluctuation. It is considered that the major contribution of this investigation has been made for the case of particles supported by fluid turbulence.

4. Large Particles of Low Specific Gravity

(i) Concentration Profiles

Anomalous concentration profiles were obtained during the series of experiments on perspex. The concentration was observed to exhibit a maximum in the lower part of the pipe. The distance of this maximum from the bottom of the pipe increased with mean concentration and with pressure gradient.

(ii) Water Velocity Profiles

As with the fine particles, attempts at measuring the fluid velocity distribution met with failure, largely because the inertia of the particles affected the reading of the pitot tube.

(iii) Mean Slip Velocities

It was concluded that the mean slip velocity was very small for this material at the range of hydraulic gradients studied.

The explanation of the anomalous discharge concentration profiles advanced by a previous worker (57) : that the local slip velocity is very high in the lower part of the

pipe, must be regarded as unproven because the mean slip velocity was small. It was shown analytically that these particles are most susceptible to the effects of their own rotation and it is quite likely that a lift force resulting from spin was an important factor in overcoming gravitational effects. Since it is impossible to deduce the magnitude of the lift force in terms of the variables of the system until the actual speed of rotation of the particles is known, this explanation of the anomalous behaviour must remain hypothetical.

It may be seen that the present state of knowledge varies considerably for the three groups of particles. If the the relationship between the concentration profiles and the quantity $\frac{y W_c}{d \sqrt{\frac{\gamma_0}{\rho_w}}}$ is verified by subsequent investigation, it should be possible to predict the shape of the concentration profile for suspended particles under conditions considerably different from those encountered in this investigation. The behaviour of coarse particles has been analysed for the two-dimensional case by Bagnold, but until the variation of the applied stress at a point on the axis of the pipe is established, this microscopic treatment will not be possible for conveying in pipes. Finally, a great deal of experimental work remains to be done before the shape of the discharge concentration

profiles obtained for perspex can be explained quantitatively.

In order to obtain a full understanding of the influence of the various factors which come into play during hydraulic conveying, a considerable amount of investigation is required. The main lines of work suggested are given below for each of the three groups of particles referred to in this work.

5. Further Work

(i) Coarse Particles

In order to permit an examination of hydraulic conveying in pipelines from the microscopic point of view in a manner analogous to that of Bagnold (46), it is suggested that efforts be concentrated upon measuring:

- (a) the distribution of the applied stress resulting from the dissipation of the energy of the fluid,
- (b) the solids concentration profile and the distribution of solids velocity over the cross-section.

(ii) Fine Particles

These particles, settling more slowly than the turbulent velocity fluctuations deserve experimental attention for the determination of:

- (a) the local slip velocity between particles and fluid,
- (b) the fluid velocity distribution,

- (c) the influence of solids concentration upon fluid turbulence.

The energy required for solids transport, manifested in the form of an increased pressure drop, may eventually be predicted from particle drag considerations and it is towards this end that research should be directed. Since the slip velocities for these particles will be very small, a technique providing greater accuracy than that obtained in the present investigation must be employed. The manner in which a bed is formed in the conveying of small particles will be of special interest and may be studied concurrently with the influence of solids concentration upon fluid turbulence.

(iii) Large Particles of Low Specific Gravity

Attention should be directed towards the determination of the speeds of rotation and the lift forces experienced by these particles.

Wherever possible, it is suggested that these experiments be conducted in pipes of greater diameter than one inch, to permit larger particles to be used, to check the influence of pipe diameter upon the flow regime, and to permit devices to be inserted into the pipe with less disturbance to the flow than would be encountered in a one-inch pipe.

It may be seen therefore, that the lines of conduct for further experiments have been fairly clearly establish-

ed. The chief difficulties which will be encountered will undoubtedly be those associated with the evaluation of point values of internal variables.

6. Summary

In conclusion, the positive achievements of this investigation may be summarised as follows:

- (a) a method has been developed for correlation of measurements of the distribution of solids concentration in terms of the quantity $\frac{y}{d} \frac{W_0}{\sqrt{\frac{\tau_0}{\rho_w}}}$ for the case of suspension by fluid turbulence;
- (b) velocity and concentration profile measurements obtained with coarse sand and gravel verified the existence of a moving bed at high mean solids concentrations;
- (c) these measurements agreed qualitatively with measurements and predictions made for the two-dimensional case by Bagnold;
- (d) mean slip velocity measurements showed that the mean slip is not a constant fraction of the mixture velocity, as suggested by Wilson and Abbott, but increases with solids concentration and decreases as the velocity is increased;
- (e) the previous classification of flow regime, given by Durand, on the basis of particle diameter, has been modified to include more of the variables of

the system including hydraulic gradient, solids density and pipe diameter;

- (f) derived data were produced which showed the variation of solids velocity, slip velocity and transport concentration for flow with a bed.

REFERENCES.

1. Nardi, J., J. Assoc. Prof. Engrs. of British Columbia, 10, 9, 28-36, (1959)
2. Blatch, N.S., Trans. A.S.C.E. 57, 400-408, (1906)
3. Hazen, A. and Hardy, E.D., Trans. A.S.C.E., 57, 307-454, (1906)
4. Gilbert, G.K., U.S. Geol. Survey Paper No. 86 (1914)
5. Durepaire, R.P., Ann. des Ponts et Chaussees, 1 er partie, 109, 7, 165-254, (1939)
6. Howard, G.W., Trans. A.S.C.E., 104, 1334-1376, (1939)
7. Wilson, W.E., Trans. A.S.C.E., 107, 1576-86, (1942)
8. Turtle, R.B., Ph.D. Thesis, University of London (1952)
9. Durand, R., Proc. Minn. Intl. Hyd. Conf. IARH (1953)
10. Durand, R., La Houille Blanche, 6, 609-19 (1951)
11. Worster, R. and Denny, D.F., Proc. 3rd Annual Conf. B.H.R.A. Part II, (Sept. 1954)
12. Abbott, M., Ph.D. Thesis, University of London, (1955)
13. Newitt, D.M., Richardson, J.F., Turtle R.B., Abbott, M., Proc. Inst. Chem. Engrs., 33, 2, 93-110, (1955)
14. Chin Yung Wen and Simons, H.P., A.I. Ch.E. Journal, 5, 2, 263-266 (1959)
15. Mehta, N.C., Smith, J.M., and Comings, E.W., Ind. Eng. Chem., 49, 6, 986-92, (1957)
16. Craven, J.P., Proc. 5th Hyd. Conf. Iowa State Univ., 67-76 (June 1952)
17. Smith, R.A., Trans. Inst. Chem. Engrs., 33, 2, 85-92, (1955)
18. Spells, K.E., Trans. Inst. Chem. Engrs., 33, 2, 79-84, (1955)

19. Sinclair, C.G., Ph.D. Thesis, Sydney University (1959)
20. Anon., Colliery Eng., 33, 423-8 (1956)
21. Bonnington, S.I., B.H.R.A. Publication SP 624 (1959)
22. Yufin, A.P., Ugol., 31, 4, 15-18, (1956)
23. Bagnold, R.A., Phil. Trans. Roy. Soc. 'A', 249, 235-298, (1957)
24. Jeffreys, H., Proc. Camb. Phil. Soc. 25, 592-600 (1929)
25. Durand, R., and Condolios, P., Mem. et Travaux de la Societe Hydratic. de France, 8, 2, 217-233, (1953)
26. Novak, J., Strojirenstvi, 8, 5, 350-61, (1958)
27. Durand, R., La Houille Blanche, 6, 384-93, (May - June 1951)
28. Hallbronn, G., Mem. et Travaux de la Societe Hydrotec. de France, 6, 2, 128-9, (1951)
29. Worster, R.C., B.H.R.A. Publication 477 (Jan.1954)
30. Worster, R.C., B.H.R.A. Publication RR 482 (Mar.1954)
31. Ismail, H.M., Proc. A.S.C.E., 77, 56, 1-26, (1951)
32. Friedlander, S.K., A.I.Ch.E., Journal, 3, 3, 381-85, (1957)
33. Dell, C.C., and Whelan F., J. Inst. Fuel, 28, 462, (1955)
34. Lane, E.W., and Kalinske, A.A., Trans. Am. Geophys. Union, 632-41, (1939)
35. Vanoni, V., Trans. A.S.C.E., III, 67-133, (1946)
36. Vanoni, V., Proc. 5th Hyd. Conf. Iowa State Univ., 137-158, (June 1952)
37. Einstein, H.A., U.S. Bureau of Agriculture Tech. Bulletin 1026 (1950)

38. Richardson, E.G., Proc. Roy. Soc. A. 162, 911, 583-597, (1937)
39. Richardson, E.G., Phil. Mag. 17, 769-83 (1934)
40. Schmidt, W., Der Massenaustach in Frier Luft, H. Grand, Hamburg (1925)
41. Rouse, H., Fluid Mechanics for Hydraulic Engineers, McGraw-Hill, New York (1938)
42. Prandtl, L., The Essentials of Fluid Dynamics, Blackie & Sons, London (1952)
43. Hunt, J.N., Proc. Roy. Soc. 'A', 224, 1158, 322-336, (1954)
44. Morris, A.W., Con. J. Tech., 33, 470, (1955)
45. Bagnold, R.A., Proc. Inst. Civ. Engrs. III, 4, 1, 174-205 (1955)
46. Bagnold, R.A., Proc. Roy. Soc. 'A' 225, 1160, 49-63 (1954)
47. Durand, R., Mem. et Travaux de la Societe Hydrotec. de France, 8, 1, 18-23, (1953)
48. Worster, R.C., and Denny, D.F., Proc. Inst. Mech. Engrs., 169, 32, 564-586, (1955)
49. Gliddon, B., Ph.D. Thesis, University of London (1959)
50. Maude, A.D., and Whitmore, R.L., Trans. Inst. Chem. Engrs., 36, 4, 296-304, (1958)
51. Bhattacharya, A., and Roy, A.N., Ind. Eng. Chem., 47, 2, 268-274 (1955)
52. Vogt, E.G., and White, R.R., Ind. Eng. Chem. 40, 9, 1731-8, (1948)
53. Goldstein, S., Modern Developments In Fluid Dynamics, Vol.II, 153, Oxford University (1938)
54. Laufer, J., N.A.C.A. Technical Note TN 2954, June 1953

55. Goldstein, S., Modern Developments In Fluid Dynamics, Vol.II, 221, Oxford University (1938)
56. Richardson, J.F. and Zaki, W.N., Trans. Inst. Chem. Engrs., 32, 1, 35-52, (1954)
57. Garrett, K., Private Communication (1957)
58. Allen, C.M., and Taylor, E.A., Trans. A.S.M.E., 45, 285-341, (1923)
59. Taylor, G.I., Proc. Roy. Soc. 'A', 223, 1155, 446-68, (1954)
60. Prockter, C.E., Kempe's Engineers Year Book. Morgan Bros. London (1958)

TABLE A1 CONCENTRATION PROFILE DATA FOR COARSE SAND; $l = 0.160$ ft./ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0194	0.097	0.1552	0.1533	0.1552	0.0475	0.109	0.2636	0.2636	0.2670
	0.232	0.0740	0.0400	0.0721		0.256	0.1552	0.1480	0.1628
	0.309	0.0231	0.0324	0.0272		0.333	0.1311	0.1220	0.1090
	0.386	0.0064	0.0065	0.0056		0.410	0.0737	0.0435	0.0571
						0.487	0.0074	0.0093	0.0065
						0.564		0.0014	0.0005
						0.641		0.0008	
						0.718		0.0005	
0.0550	0.102	0.2790	0.2742	0.3180	0.0668	0.104	0.2692	0.3381	0.3040
	0.242	0.3100	0.2570	0.2060		0.247	0.1808	0.2440	0.2032
	0.319	0.1826	0.1622	0.1471		0.324	0.1055	0.1055	0.1484
	0.396	0.0901	0.0872	0.0868		0.401	0.1068	0.0986	0.0951
	0.473	0.0445	0.0465	0.0415		0.478	0.0631	0.0605	0.0471
	0.550	0.0139	0.0105	0.0139		0.555		0.0179	0.0148
	0.627		0.0024	0.0037		0.632		0.0087	0.0035
	0.704		0.0012			0.709		0.0043	
	0.781		0.0009			0.786		0.0009	
	0.858		0.0004						

TABLE A1 CONTINUED
 CONCENTRATION PROFILE DATA
 COARSE SAND $\lambda = 0.160$ ft/ft.

$\bar{C}$	$Y_{ins.}$	C_b	C_T	C_y Smoothed
0.083	0.104	0.179	0.179	0.1795
	0.247	0.160	0.125	0.1595
	0.324	0.143	0.145	0.1445
	0.391	0.131	0.127	0.1276
	0.468	0.102	0.102	0.1000
	0.545	0.064	0.086	0.0675
	0.622	0.043	0.061	0.0468
	0.699	0.033	0.040	0.0360
	0.776	0.029	0.017	0.0285
0.907	0.043	0.020	0.0199	

TABLE A2 CONCENTRATION PROFILE DATA FOR COARSE SAND; $l = 0.280$ ft./ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_T Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_T Smoothed
0.0250	0.097	0.1249	0.1258	0.1251	0.0590	0.109	0.2671	0.2680	0.2620
	0.232	0.0980	0.0988	0.0944		0.256	0.2017	0.1802	0.1911
	0.309	0.0589	0.0590	0.0540		0.333	0.1549	0.1677	0.1210
	0.386	0.0250	0.0260	0.0262		0.410	0.0548	0.0539	0.0549
	0.463		0.0093	0.0090		0.487	0.0201	0.0216	0.0205
	0.540		0.0021	0.0022		0.564	0.0059	0.0059	0.0072
	0.617		0.0009	0.0006		0.641		0.0024	0.0024
	0.694		0.0005			0.718		0.0006	
	0.771		0.0002			0.794		0.0006	
0.0668	0.104	0.3363	0.2965	0.3150	0.1184	0.104	0.332	0.3363	0.3350
	0.247	0.2227	0.2348	0.2457		0.247	0.308	0.3122	0.3095
	0.324	0.1855	0.2502	0.1774		0.324	0.308	0.3032	0.2740
	0.401	0.1139	0.0965	0.1096		0.401	0.218	0.2101	0.2161
	0.478	0.0763	0.0348	0.0471		0.478	0.130	0.1290	0.1350
	0.555	0.0073	0.0162	0.0130		0.555	0.0548	0.0649	0.0555
	0.632		0.0057	0.0044		0.632		0.0163	0.0171
						0.709		0.0060	0.0063
						0.786		0.0022	0.0027

TABLE A2 CONCENTRATION PROFILE DATA FOR COARSE SAND; $i = 0.280$ ft/ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.1404	0.114	0.3999	0.4235	0.4156	0.167	0.115	0.3617	0.5355	0.3860
	0.267	0.3560	0.4135	0.3547		0.269	0.3503	0.2430	0.3472
	0.344	0.3237	0.2967	0.3150		0.346	0.3300	0.2843	0.3190
	0.421	0.2840	0.2320	0.2587		0.423	0.2725	0.2944	0.2800
	0.498	0.1811	0.1672	0.1732		0.500	0.2310	0.2275	0.2238
	0.575	0.0707	0.0952	0.0869		0.577	0.1965	0.1516	0.1609
	0.652	0.0316	0.0314	0.0305		0.654	0.1008	0.1192	0.1071
	0.729	0.0240	0.0086	0.0080		0.731	0.0816	0.0756	0.0703
	0.806		0.0041	0.0028		0.808	0.0276	0.0344	0.0407
	0.922		0.0004			0.929	0.0121	0.0045	0.0070

TABLE A3 CONCENTRATION PROFILE DATA FOR COARSE SAND ; $l = 0.400$ ft./ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0290	0.097	0.1060	0.1050	0.1050	0.0677	0.109	0.2300	0.2035	0.2357
	0.232	0.0913	0.1012	0.0966		0.256	0.2208	0.2142	0.1997
	0.309	0.0744	0.0634	0.0707		0.333	0.1516	0.1673	0.1615
	0.386	0.0430	0.0363	0.0373		0.410	0.0956	0.0898	0.0900
	0.463	0.0191	0.0159	0.0174		0.487	0.0501	0.0590	0.0478
	0.540	0.0059	0.0142	0.0100		0.564	0.0264	0.0244	0.0242
	0.617	0.0106	0.0049	0.0056		0.641		0.0108	0.0109
	0.694	0.0023	0.0029	0.0031		0.717		0.0047	0.0043
	0.771		0.0018	0.0016		0.794		0.0041	0.0012
	0.848		0.0012	0.0008					
	0.951		0.0005						
0.0948	0.104	0.2513	0.2935	0.2760	0.1270	0.104	0.3000	0.3078	0.3676
	0.247	0.2902	0.2040	0.2459		0.247	0.3584	0.2763	0.3022
	0.324	0.2027	0.2190	0.2120		0.324	0.3483	0.3087	0.2635
	0.401	0.1382	0.1429	0.1391		0.401	0.1931	0.2262	0.2035
	0.478	0.0798	0.0825	0.0805		0.478	0.1164	0.1498	0.1382
	0.555	0.0532	0.0355	0.0474		0.555	0.0821	0.0656	0.0702
	0.632	0.0202	0.0218	0.0240		0.632	0.0086	0.0419	0.0379
	0.709		0.0011	0.0083		0.709	0.0287	0.0196	0.0218
	0.786		0.0006	0.0013		0.786		0.0098	0.0111
						0.918		0.0016	0.0011

TABLE A3 CONCENTRATION PROFILE DATA FOR COARSE SAND; $i = 0.400$ ft./ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_y Smoothed
0.156	0.114	0.3770	0.3770	0.3762	0.185	0.115	0.3910	0.4645	0.4150
	0.269	0.3567	0.3512	0.3495		0.269	0.3798	0.2262	0.3920
	0.346	0.3327	0.3198	0.3224		0.346	0.3475	0.3560	0.3716
	0.423	0.2738	0.2603	0.2743		0.423	0.3236	0.3236	0.3270
	0.500	0.1432	0.2161	0.1875		0.500	0.2421	0.2180	0.2560
	0.576	0.1119	0.0855	0.0915		0.577	0.1220	0.1571	0.1391
	0.653	0.0388	0.0274	0.0598		0.654	0.0739	0.0647	0.0661
	0.730	0.0208	0.0120	0.0176		0.731	0.0324	0.0268	0.0294
	0.807	0.0037	0.0058	0.0075		0.808		0.0116	0.0133
	0.922		0.0020	0.0020		0.933		0.0032	0.0025

TABLE A4 CONCENTRATION PROFILE DATA FOR GRAVEL; $i = 0.160$ ft/ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0159	0.115	0.0673	0.0877	0.0798	0.0325	0.117	0.1422	0.1436	0.1428
	0.269	0.0565	0.0318	0.0450		0.272	0.1150	0.1230	0.1160
	0.346	0.0348	0.0236	0.0215		0.349	0.0750	0.0933	0.0846
	0.423	0.0112	0.0104	0.0103		0.426	0.0448	0.0236	0.0248
	0.500	0.0025	0.0033	0.0037		0.503	0.0027	0.0097	0.0088
	0.577	0.0041	0.0013	0.0012		0.580	0.0078	0.0015	0.0025
	0.654		0.0002	0.0003		0.657		0.0004	0.0004
0.0564	0.126	0.1930	0.2040	0.2410	0.0714	0.140	0.2695	0.2660	0.2675
	0.290	0.2178	0.2109	0.1670		0.318	0.2043	0.2022	0.2040
	0.367	0.1495	0.1720	0.1495		0.395	0.1488	0.1465	0.1620
	0.444	0.0872	0.0872	0.0625		0.472	0.1087	0.1137	0.1055
	0.521	0.0155	0.0315	0.0243		0.549	0.0543	0.0429	0.0545
	0.598	0.0239	0.0052	0.0170		0.626	0.0164	0.0157	0.0198
	0.675		0.0014	0.0025		0.703	0.0029	0.0030	0.0038
						0.780		0.0005	0.0010

TABLE A5 CONCENTRATION PROFILE DATA FOR GRAVEL ; $i = 0.280$ ft./ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0205	0.115	0.0805	0.0818	0.0813	0.0377	0.117	0.1301	0.1267	0.1289
	0.269	0.0555	0.0533	0.0545		0.272	0.0999	0.0838	0.0974
	0.346	0.0386	0.0386	0.0373		0.349	0.0804	0.0513	0.0750
	0.423	0.0242	0.0240	0.0210		0.426	0.0574	0.0750	0.0445
	0.500	0.0133	0.0099	0.0117		0.503	0.0364	0.0212	0.0219
	0.577	0.0008	0.0059	0.0063		0.580	0.0100	0.0086	0.0106
	0.654	0.0029	0.0027	0.0028		0.657	0.0002	0.0037	0.0037
	0.731	0.0016	0.0008	0.0006		0.734		0.0015	0.0010
0.0677	0.126	0.1821	0.1712	0.2157	0.0911	0.140	0.2330	0.2412	0.2320
	0.290	0.1921	0.2140	0.1865		0.318	0.2060	0.2050	0.2075
	0.367	0.1510	0.1570	0.1505		0.395	0.1802	0.1912	0.1813
	0.444	0.1161	0.1168	0.1002		0.472	0.1430	0.1702	0.1492
	0.521	0.0544	0.0639	0.0505		0.549	0.1042	0.0656	0.0700
	0.598	0.0257	0.0240	0.0262		0.626	0.0300	0.0310	0.0313
	0.675	0.0248	0.0088	0.0118		0.703	0.0273	0.0108	0.0120
	0.752		0.0030	0.0075		0.780		0.0015	0.0025

TABLE A5 CONTINUED
 CONCENTRATION PROFILE DATA
 GRAVEL ; $i = 0.280$ ft/ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.1155	0.142	0.2660	0.2635	0.2785
	0.323	0.2424	0.2635	0.2440
	0.400	0.1982	0.2050	0.2141
	0.477	0.1581	0.1772	0.1660
	0.554	0.1083	0.1145	0.1102
	0.631	0.0702	0.0410	0.0518
	0.708	0.0199	0.0126	0.0175
	0.785		0.0027	0.0030

TABLE A6 CONCENTRATION PROFILE DATA FOR GRAVEL; $i = 0.400$ ft./ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0203	0.115	0.0624	0.0676	0.0656	0.0394	0.117	0.1084	0.1119	0.1160
	0.269	0.0578	0.0388	0.0522		0.272	0.0898	0.1181	0.0966
	0.346	0.0410	0.0430	0.0424		0.349	0.0915	0.0780	0.0831
	0.423	0.0314	0.0308	0.0301		0.426	0.0572	0.0635	0.0642
	0.500	0.0198	0.0161	0.0162		0.503	0.0489	0.0374	0.0420
	0.577	0.0026	0.0058	0.0089		0.580	0.0326	0.0197	0.0213
	0.654	0.0079	0.0034	0.0043		0.657		0.0079	0.0103
	0.731	0.0036	0.0020	0.0016		0.734		0.0034	0.0043
	0.808		0.0009			0.811		0.0019	0.0014
						0.930		0.0006	
0.0655	0.126	0.1322	0.1448	0.1572	0.0881	0.140	0.1564	0.1852	0.1872
	0.290	0.1422	0.1572	0.1422		0.318	0.2022	0.1741	0.1698
	0.367	0.1526	0.1244	0.1299		0.395	0.1402	0.1436	0.1539
	0.444	0.1297	0.1150	0.1119		0.472	0.1121	0.1235	0.1251
	0.521	0.0820	0.0928	0.0824		0.549	0.0935	0.0858	0.0904
	0.598	0.0262	0.0429	0.0349		0.626	0.0732	0.0838	0.0571
	0.675	0.0239	0.0140	0.0140		0.703	0.0269	0.0318	0.0280
	0.752		0.0022	0.0041		0.780		0.0083	0.0098

TABLE A6 CONTINUED
 CONCENTRATION PROFILE DATA
 GRAVEL; $l = 0.400$ ft./ft.

$\bar{C}$	$Y_{ins.}$	C_B	C_T	$C_{y \text{ Smoothed}}$
0.1093	0.142	0.2017	0.2007	0.2030
	0.323	0.1968	0.1837	0.1902
	0.400	0.1870	0.1837	0.1795
	0.477	0.1547	0.1773	0.1600
	0.554	0.1258	0.1344	0.1240
	0.631	0.0760	0.0612	0.0538
	0.708	0.0230	0.0153	0.0273
	0.785		0.0061	0.0074

TABLE A7 CONCENTRATION PROFILE DATA FOR PERSPEX; $l = 0.160$ ft/ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0291	0.103	0.0545	0.0489	0.0595	0.0399	0.102	0.0796	0.0796	0.0760
	0.244	0.0778	0.0763	0.0725		0.219	0.0835	0.0710	0.0803
	0.321	0.0556	0.0547	0.0587		0.258	0.0798	0.0752	0.0781
	0.398	0.0434	0.0420	0.0413		0.320	0.0734	0.0773	0.0713
	0.475	0.0277	0.0280	0.0297		0.397	0.0606	0.0597	0.0590
	0.552	0.0230	0.0221	0.0218		0.474	0.0448	0.0410	0.0410
	0.629	0.0169	0.0128	0.0133		0.551	0.0279	0.0308	0.0286
	0.706	0.0093	0.0090	0.0099		0.628	0.0196	0.0206	0.0202
	0.783	0.0064	0.0055	0.0065		0.705	0.0148	0.0120	0.0143
	0.906	0.0038	0.0036	0.0040		0.782	0.0116	0.0109	0.0101
0.0626	0.100	0.1030	0.1166	0.1082	0.080	0.102	0.1128	0.1066	0.1220
	0.241	0.1089	0.0986	0.1142		0.219	0.1382	0.1640	0.1464
	0.318	0.1161	0.1062	0.1092		0.258	0.1442	0.1468	0.1494
	0.395	0.0996	0.0692	0.0970		0.320	0.1390	0.1528	0.1447
	0.472	0.0710	0.1220	0.0732		0.397	0.1306	0.1195	0.1185
	0.549	0.0561	0.0498	0.0468		0.474	0.1150	0.1020	0.1070
	0.626	0.0358	0.0341	0.0320		0.551	0.0755	0.0726	0.0736
	0.703	0.0264	0.0227	0.0220		0.628	0.0482	0.0546	0.0515
	0.780	0.0049	0.0139	0.0154		0.705	0.0336	0.0392	0.0356
	0.904	0.0113	0.0084	0.0084		0.782	0.0168	0.0260	0.0241
						0.906	0.0124	0.0099	0.0108

TABLE A8 CONCENTRATION PROFILE DATA FOR PERSPEX; $L = 0.280$ ft/ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0282	0.103	0.0395	0.0361	0.0400	0.0432	0.101	0.0495	0.0510	0.0513
	0.244	0.0494	0.0514	0.0477		0.241	0.0765	0.0804	0.0741
	0.321	0.0420	0.0415	0.0449		0.318	0.0682	0.0692	0.0699
	0.398	0.0392	0.0383	0.0403		0.395	0.0584	0.0575	0.0619
	0.425	0.0327	0.0324	0.0329		0.472	0.0518	0.0497	0.0529
	0.548	0.0318	0.0287	0.0257		0.549	0.0466	0.0454	0.0440
	0.625	0.0245	0.0200	0.0197		0.626	0.0450	0.0395	0.0364
	0.702	0.0152	0.0149	0.0156		0.703	0.0320	0.0302	0.0280
	0.779	0.0088	0.0152	0.0125		0.780	0.0242	0.0199	0.0213
	0.906	0.0096	0.0085	0.0087		0.909	0.0121	0.0082	0.0120
0.0659	0.101	0.0738	0.0762	0.0729	0.0875	0.101	0.0682	0.0762	0.0803
	0.241	0.0928	0.1025	0.0960		0.219	0.1089	0.1254	0.1278
	0.318	0.0960	0.0942	0.0975		0.258	0.1533	0.1430	0.1332
	0.395	0.0965	0.0948	0.0940		0.318	0.1346	0.1251	0.1340
	0.472	0.0867	0.0851	0.0846		0.395	0.1220	0.1290	0.1260
	0.549	0.0665	0.0710	0.0729		0.472	0.1142	0.1115	0.1043
	0.626	0.0565	0.0558	0.0573		0.549	0.1015	0.1047	0.0985
	0.703	0.0374	0.0390	0.0420		0.626	0.0795	0.0848	0.0770
	0.780	0.0406	0.0320	0.0309		0.703	0.0620	0.0551	0.0556
	0.909	0.0184	0.0161	0.0185		0.780	0.0299	0.0470	0.0426
						0.909	0.0316	0.0269	0.0299

TABLE A9 CONCENTRATION PROFILE DATA FOR PERSPEX; $l = 0.400$ ft./ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0295	0.105	0.0292	0.0315	0.0348	0.0645	0.104	0.0698	0.0680	0.0688
	0.249	0.0493	0.0445	0.0450		0.248	0.0880	0.0853	0.0848
	0.326	0.0472	0.0422	0.0438		0.325	0.0909	0.0812	0.0878
	0.403	0.0381	0.0401	0.0398		0.402	0.0792	0.0972	0.0866
	0.480	0.0333	0.0339	0.0346		0.479	0.0816	0.0812	0.0802
	0.557	0.0295	0.0307	0.0294		0.556	0.0695	0.0674	0.0686
	0.634	0.0269	0.0257	0.0253		0.633	0.0598	0.0581	0.0566
	0.711	0.0201	0.0230	0.0218		0.710	0.0495	0.0489	0.0465
	0.787	0.0127	0.0198	0.0191		0.787	0.0515	0.0412	0.0382
	0.913	0.0136	0.0183	0.0154		0.912	0.0280	0.0252	0.0277
0.0983	0.102	0.0801	0.0850	0.0850	0.133	0.100	0.1095	0.1054	0.1081
	0.242	0.1200	0.1414	0.1212		0.219	0.1330	0.1544	0.1486
	0.319	0.1254	0.1168	0.1279		0.258	0.1459	0.1600	0.1548
	0.396	0.1314	0.1302	0.1296		0.335	0.1544	0.1733	0.1621
	0.473	0.1254	0.1244	0.1255		0.412	0.1643	0.1530	0.1640
	0.550	0.1110	0.1063	0.1138		0.489	0.1702	0.1724	0.1601
	0.627	0.1000	0.1050	0.0993		0.566	0.1629	0.1485	0.1540
	0.704	0.0875	0.0876	0.0820		0.643	0.1458	0.1485	0.1430
	0.781	0.0690	0.0616	0.0649		0.720	0.1300	0.1287	0.1252
	0.905	0.0389	0.0433	0.0416		0.797	0.1073	0.1107	0.1018
						0.918	0.0608	0.0553	0.0561

TABLE A10 CONCENTRATION PROFILE DATA FOR FINE SAND; $i = 0.160$ ft./ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0202	0.114	0.0275	0.0218	0.0290	0.0331	0.126	0.0468	0.0380	0.0465
	0.266	0.0239	0.0324	0.0256		0.291	0.0405	0.0458	0.0395
	0.343	0.0230	0.0258	0.0238		0.368	0.0378	0.0396	0.0368
	0.420	0.0220	0.0232	0.0220		0.445	0.0344	0.0348	0.0340
	0.497	0.0207	0.0212	0.0204		0.522	0.0308	0.0328	0.0317
	0.574	0.0199	0.0200	0.0194		0.599	0.0284	0.0305	0.0296
	0.651	0.0187	0.0174	0.0172		0.676	0.0257	0.0268	0.0280
	0.728	0.0175	0.0156	0.0157		0.753	0.0279	0.0303	0.0267
	0.805	0.0153	0.0137	0.0144		0.905	0.0268	0.0241	0.0242
	0.936	0.0103	0.0111	0.0124					
0.0469	0.122	0.0919	0.0920	0.0897	0.0654	0.098	0.1125	0.0947	0.1240
	0.282	0.0645	0.0768	0.0680		0.234	0.0990	0.1194	0.0987
	0.359	0.0612	0.0675	0.0595		0.311	0.0905	0.0929	0.0863
	0.436	0.0519	0.0517	0.0500		0.388	0.0792	0.0728	0.0739
	0.513	0.0442	0.0456	0.0420		0.465	0.0665	0.0654	0.0654
	0.590	0.0346	0.0378	0.0357		0.542	0.0562	0.0568	0.0577
	0.667	0.0302	0.0297	0.0302		0.619	0.0511	0.0535	0.0520
	0.744	0.0266	0.0261	0.0264		0.696	0.0477	0.0477	0.0470
	0.821	0.0224	0.0229	0.0232		0.773	0.0466	0.0441	0.0428
	0.939	0.0234	0.0202	0.0203		0.915	0.0357	0.0363	0.0365

TABLE A10 CONTINUED
CONCENTRATION PROFILE DATA

FINE SAND		$l = 0.160 \text{ ft./ft.}$		
$\bar{C}$	$Y_{ins.}$	C_B	C_T	C_y Smoothed
0.0969	0.116	0.1750	0.1622	0.1790
	0.270	0.1468	0.1738	0.1455
	0.347	0.1330	0.1360	0.1292
	0.424	0.1128	0.1173	0.1126
	0.501	0.0994	0.0976	0.0960
	0.578	0.0836	0.0801	0.0811
	0.655	0.0625	0.0672	0.0664
	0.750	0.0620	0.0513	0.0525
	0.914	0.0320	0.0388	0.0365

TABLE AII CONCENTRATION PROFILE DATA FOR FINE SAND ; $i=0.280$ ft./ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_r Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_r Smoothed
0.0258	0.114	0.0326	0.0306	0.0339	0.0407	0.126	0.0527	0.0527	0.0516
	0.266	0.0300	0.0303	0.0302		0.291	0.0455	0.0438	0.0450
	0.343	0.0285	0.0330	0.0294		0.368	0.0422	0.0415	0.0426
	0.420	0.0272	0.0254	0.0268		0.445	0.0382	0.0040	0.0407
	0.497	0.0258	0.0260	0.0252		0.522	0.0381	0.0394	0.0391
	0.574	0.0230	0.0240	0.0240		0.599	0.0368	0.0361	0.0377
	0.651	0.0224	0.0207	0.0229		0.676	0.0375	0.0401	0.0364
	0.728	0.0203	0.0240	0.0220		0.753	0.0369	0.0369	0.0352
	0.805	0.0243	0.0201	0.0212		0.905	0.0386	0.0362	0.0332
	0.936	0.0220	0.0181	0.0200					
0.0585	0.122	0.0914	0.0885	0.0923	0.0826	0.098	0.1266	0.1224	0.1315
	0.282	0.0743	0.0810	0.0757		0.234	0.1116	0.1227	0.1100
	0.359	0.0710	0.0692	0.0678		0.311	0.1000	0.0985	0.0990
	0.436	0.0627	0.0600	0.0607		0.388	0.0903	0.0880	0.0900
	0.513	0.0551	0.0545	0.0538		0.465	0.0819	0.0804	0.0815
	0.590	0.0469	0.0485	0.0485		0.542	0.0760	0.0748	0.0757
	0.667	0.0471	0.0464	0.0450		0.619	0.0671	0.0699	0.0707
	0.744	0.0411	0.0431	0.0422		0.696	0.0654	0.0681	0.0673
	0.821	0.0404	0.0413	0.0403		0.773	0.0698	0.0668	0.0646
	0.939	0.0385	0.0376	0.0381		0.915	0.0665	0.0655	0.0605

TABLE AII CONCENTRATION PROFILE DATA FOR FINE SAND ; $l=0.280$ ft./ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.1203	0.116	0.1812	0.1797	0.1798	0.1687	0.117	0.2440	0.2083	0.2459
	0.270	0.1509	0.1546	0.1515		0.272	0.2163	0.2474	0.2138
	0.347	0.1530	0.1519	0.1379		0.349	0.1973	0.2066	0.1969
	0.424	0.1126	0.1361	0.1253		0.426	0.1845	0.1828	0.1803
	0.501	0.1092	0.1252	0.1153		0.503	0.1642	0.1714	0.1660
	0.578	0.1052	0.1062	0.1065		0.580	0.1540	0.1512	0.1519
	0.655	0.0996	0.0949	0.0990		0.657	0.1367	0.1412	0.1409
	0.749	0.1001	0.0873	0.0921		0.753	0.1391	0.1299	0.1298
	0.914	0.0914	0.0772	0.0840		0.915	0.1125	0.1177	0.1167
0.2096	0.114	0.2445	0.2323	0.2450					
	0.267	0.2275	0.2275	0.2289					
	0.344	0.2197	0.2179	0.2010					
	0.421	0.2170	0.2135	0.2144					
	0.498	0.2096	0.2125	0.2085					
	0.575	0.2020	0.2059	0.2025					
	0.652	0.1990	0.1990	0.1976					
	0.748	0.1922	0.1922	0.1913					
	0.912	0.1846	0.1827	0.1832					

TABLE A12 CONCENTRATION PROFILE DATA FOR FINE SAND; $l=0.372$ ft./ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_r Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_r Smoothed
0.0358	0.114	0.0483	0.0456	0.0479	0.0586	0.126	0.0758	0.0680	0.0707
	0.266	0.0430	0.0420	0.0424		0.291	0.0649	0.0595	0.0626
	0.313	0.0400	0.0382	0.0404		0.368	0.0603	0.0586	0.0600
	0.420	0.0370	0.0363	0.0369		0.445	0.0562	0.0572	0.0580
	0.497	0.0330	0.0350	0.0349		0.522	0.0545	0.0582	0.0566
	0.574	0.0318	0.0333	0.0335		0.599	0.0539	0.0570	0.0555
	0.651	0.0304	0.0334	0.0325		0.676	0.0534	0.0571	0.0548
	0.728	0.0307	0.0346	0.0316		0.753	0.0611	0.0563	0.0545
	0.805	0.0328	0.0306	0.0310		0.905	0.0560	0.0562	0.0538
	0.936	0.0308	0.0295	0.0308					
0.0646	0.122	0.1038	0.1022	0.1048	0.0940	0.098	0.1368	0.1193	0.1355
	0.282	0.0849	0.0883	0.0845		0.234	0.1160	0.1290	0.1176
	0.359	0.0765	0.0779	0.0759		0.311	0.1108	0.1108	0.1075
	0.436	0.0716	0.0704	0.0705		0.388	0.1008	0.0990	0.1000
	0.513	0.0648	0.0648	0.0653		0.465	0.0900	0.0933	0.0926
	0.590	0.0615	0.0607	0.0612		0.542	0.0835	0.0885	0.0871
	0.667	0.0577	0.0577	0.0575		0.619	0.0887	0.0858	0.0839
	0.744	0.0557	0.0534	0.0543		0.696	0.0773	0.0841	0.0815
	0.821	0.0555	0.0506	0.0518		0.770	0.0773	0.0806	0.0798
	0.939	0.0412	0.0450	0.0489		0.913	0.0773	0.0782	0.0775

TABLE A12 CONCENTRATION PROFILE DATA FOR FINE SAND; $i=0.312$ ft./ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.1264	0.116	0.1828	0.1775	0.1884	0.1729	0.117	0.2334	0.2293	0.2370
	0.270	0.1564	0.1772	0.1626		0.272	0.2100	0.2209	0.2082
	0.347	0.1512	0.1512	0.1492		0.349	0.2000	0.1970	0.1952
	0.424	0.1368	0.1335	0.1370		0.426	0.1823	0.1852	0.1820
	0.501	0.1272	0.1224	0.1240		0.503	0.1746	0.1697	0.1701
	0.578	0.1122	0.1112	0.1135		0.580	0.1581	0.1581	0.1611
	0.655	0.1040	0.1064	0.1060		0.657	0.1539	0.1521	0.1548
	0.749	0.0989	0.0997	0.0995		0.753	0.1490	0.1452	0.1466
	0.914	0.0933	0.0933	0.0935		0.915	0.1319	0.1359	0.1372
0.2133	0.114	0.2340	0.2312	0.2328					
	0.267	0.2210	0.2188	0.2219					
	0.344	0.2178	0.2157	0.2175					
	0.421	0.2133	0.2157	0.2134					
	0.498	0.2078	0.2098	0.2098					
	0.575	0.2100	0.2080	0.2069					
	0.652	0.2031	0.1990	0.2039					
	0.748	0.2057	0.2078	0.2010					
	0.912	0.2086	0.2055	0.1969					

TABLE A13 CONCENTRATION PROFILE DATA FOR ZIRCON ; $i=0.160$ ft./ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0071	0.119	0.0237	0.0243	0.0261	0.0152	0.124	0.0610	0.0577	0.0602
	0.276	0.0155	0.0177	0.0138		0.287	0.0316	0.0324	0.0288
	0.353	0.0086	0.0080	0.0082		0.364	0.0169	0.0173	0.0196
	0.430	0.0069	0.0048	0.0049		0.441	0.0120	0.0125	0.0125
	0.507	0.0032	0.0033	0.0032		0.518	0.0049	0.0088	0.0085
	0.584	0.0024	0.0024	0.0023		0.595	0.0055	0.0056	0.0058
	0.661	0.0014	0.0012	0.0017		0.672	0.0056	0.0041	0.0039
	0.738	0.0021	0.0017	0.0012		0.749	0.0020	0.0032	0.0030
	0.815	0.0007	0.0009	0.0010		0.826		0.0026	0.0022
	0.920	0.0015	0.0015	0.0008		0.945		0.0020	0.0019
0.0222	0.124	0.1008	0.0695	0.0804	0.0342	0.108	0.1270	0.1400	0.1356
	0.287	0.0335	0.0637	0.0353		0.254	0.0536	0.0738	0.0721
	0.364	0.0260	0.0293	0.0235		0.331	0.0526	0.0413	0.0523
	0.441	0.0209	0.0171	0.0166		0.408	0.0380	0.0369	0.0349
	0.518	0.0122	0.0187	0.0117		0.485	0.0256	0.0219	0.0210
	0.595	0.0082	0.0091	0.0077		0.562	0.0109	0.0106	0.0122
	0.672	0.0053	0.0071	0.0054		0.639	0.0075	0.0058	0.0077
	0.749		0.0047	0.0040		0.716	0.0038	0.0041	0.0049
	0.826		0.0036	0.0029		0.793		0.0031	0.0039
	0.945		0.0020	0.0020		0.923		0.0027	0.0030

TABLE A14 CONCENTRATION PROFILE DATA FOR ZIRCON; $l = 0.280$ ft./ft.									
$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0063	0.119	0.0182	0.0157	0.0187	0.0179	0.124	0.0523	0.0510	0.0544
	0.276	0.0123	0.0127	0.0115		0.297	0.0336	0.0383	0.0328
	0.353	0.0081	0.0074	0.0082		0.364	0.0252	0.0232	0.0240
	0.430	0.0050	0.0065	0.0059		0.441	0.0182	0.0163	0.0174
	0.507	0.0035	0.0043	0.0041		0.518	0.0115	0.0131	0.0128
	0.584	0.0018	0.0039	0.0032		0.595	0.0088	0.0102	0.0099
	0.661	0.0021	0.0038	0.0028		0.672	0.0052	0.0082	0.0078
	0.738	0.0030	0.0025	0.0024		0.749	0.0082	0.0064	0.0061
	0.815	0.0026	0.0023	0.0022		0.826	0.0068	0.0052	0.0048
	0.920	0.0035	0.0018	0.0020		0.945	0.0030	0.0036	0.0035
0.0251	0.124	0.0760	0.0743	0.0696	0.0380	0.108	0.1125	0.1098	0.1160
	0.287	0.0429	0.0435	0.0416		0.254	0.0638	0.0699	0.0730
	0.364	0.0276	0.0336	0.0320		0.331	0.0486	0.0471	0.0547
	0.441	0.0264	0.0264	0.0251		0.408	0.0410	0.0407	0.0408
	0.518	0.0233	0.0198	0.0200		0.485	0.0304	0.0296	0.0310
	0.595	0.0183	0.0148	0.0152		0.562	0.0144	0.0205	0.0235
	0.672	0.0130	0.0113	0.0112		0.639	0.0160	0.0148	0.0192
	0.749	0.0075	0.0090	0.0082		0.716	0.0152	0.0163	0.0158
	0.826	0.0043	0.0075	0.0062		0.793	0.0167	0.0106	0.0131
	0.945		0.0053	0.0045		0.923	0.0046	0.0091	0.0098

TABLE A14 CONTINUED
 CONCENTRATION PROFILE DATA
 ZIRCON $i = 0.280 \text{ ft./ft.}$

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0571	0.115	0.1680	0.1658	0.1750
	0.269	0.1069	0.1166	0.1113
	0.346	0.0822	0.0834	0.0800
	0.423	0.0555	0.0520	0.0560
	0.500	0.0457	0.0423	0.0421
	0.577	0.0320	0.0320	0.0331
	0.654	0.0217	0.0263	0.0262
	0.731	0.0263	0.0200	0.0216
	0.808	0.0212	0.0177	0.0182
	0.930	0.0074	0.0143	0.0139

TABLE A15 CONCENTRATION PROFILE DATA FOR ZIRCON ; $l = 0.400$ ft./ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0076	0.119	0.0200	0.0178	0.0213	0.0211	0.124	0.0586	0.0642	0.0640
	0.276	0.0135	0.0154	0.0128		0.287	0.0374	0.0314	0.0382
	0.353	0.0093	0.0100	0.0100		0.364	0.0294	0.0283	0.0292
	0.430	0.0071	0.0071	0.0075		0.441	0.0212	0.0148	0.0214
	0.507	0.0058	0.0056	0.0058		0.518	0.0166	0.0261	0.0170
	0.584	0.0041	0.0045	0.0048		0.595	0.0157	0.0139	0.0140
	0.661	0.0049	0.0038	0.0041		0.672	0.0119	0.0119	0.0120
	0.738	0.0033	0.0036	0.0037		0.749	0.0144	0.0106	0.0105
	0.815	0.0033	0.0037	0.0034		0.826	0.0106	0.0091	0.0091
	0.920	0.0028	0.0035	0.0033		0.945	0.0046	0.0080	0.0080
0.0289	0.124	0.0861	0.0661	0.0720	0.0460	0.108	0.1108	0.1100	0.1051
	0.287	0.0344	0.0529	0.0430		0.254	0.0666	0.0690	0.0718
	0.364	0.0327	0.0393	0.0346		0.331	0.0583	0.0583	0.0597
	0.441	0.0304	0.0338	0.0295		0.408	0.0483	0.0501	0.0505
	0.518	0.0278	0.0246	0.0232		0.485	0.0405	0.0446	0.0427
	0.595	0.0217	0.0199	0.0190		0.562	0.0382	0.0340	0.0350
	0.672	0.0156	0.0176	0.0154		0.639	0.0262	0.0239	0.0286
	0.749	0.0168	0.0130	0.0124		0.716	0.0225	0.0175	0.0230
	0.826	0.0113	0.0107	0.0101		0.793	0.0133	0.0156	0.0182
	0.945	0.0078	0.0075	0.0075		0.923	0.0129	0.0156	0.0132

TABLE A15 CONCENTRATION PROFILE DATA FOR ZIRCON ; $l = 0.400$ ft./ft.

$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed	$\bar{C}$	$Y_{INS.}$	C_B	C_T	C_Y Smoothed
0.0633	0.115	0.1416	0.1380	0.1484	0.0911	0.108	0.2204	0.2170	0.2198
	0.269	0.1126	0.1310	0.1068		0.254	0.1586	0.1694	0.1560
	0.346	0.0968	0.0898	0.0885		0.331	0.1303	0.1275	0.1290
	0.423	0.0575	0.0532	0.0720		0.408	0.1066	0.1066	0.1080
	0.500	0.0588	0.0582	0.0584		0.485	0.0829	0.0793	0.0740
	0.577	0.0418	0.0418	0.0479		0.562	0.0647	0.0684	0.0662
	0.654	0.0418	0.0374	0.0390		0.639	0.0501	0.0501	0.0537
	0.731	0.0367	0.0329	0.0318		0.716	0.0501	0.0447	0.0442
	0.808	0.0298	0.0272	0.0260		0.793	0.0355	0.0383	0.0375
	0.930	0.0114	0.0215	0.0209		0.923	0.0310	0.0328	0.0318
0.1098	0.111	0.2196	0.2325	0.2495					
	0.260	0.1954	0.2130	0.1933					
	0.337	0.1603	0.1613	0.1619					
	0.414	0.1295	0.1272	0.1345					
	0.491	0.1052	0.1050	0.1100					
	0.568	0.0900	0.0889	0.0904					
	0.645	0.0658	0.0725	0.0755					
	0.722	0.0658	0.0593	0.0622					
	0.799	0.0593	0.0503	0.0521					
	0.926	0.0296	0.0362	0.0391					

COARSE SAND DATA
TABLE A16 WATER VELOCITY PROFILE MEASUREMENTS; $i = 0.160 \text{ ft./ft.}$

$Y_{\text{INS.}}$	Suction Pressure Velocities at Various Discharge Concentrations.				Impact Pressure Velocities at Various Discharge Concentrations.				
	0.0235	0.0434	0.0553	0.0896	0.0160	0.0299	0.0424	0.0565	0.0794
0.027	3.90 $\frac{\text{ft.}}{\text{sec.}}$	3.16 $\frac{\text{ft.}}{\text{sec.}}$	2.69 $\frac{\text{ft.}}{\text{sec.}}$	1.49 $\frac{\text{ft.}}{\text{sec.}}$	3.03 $\frac{\text{ft.}}{\text{sec.}}$	—	—	—	—
0.093	4.68	3.97	3.07	2.44	4.06	2.09 $\frac{\text{ft.}}{\text{sec.}}$	1.45 $\frac{\text{ft.}}{\text{sec.}}$	—	—
0.139	5.31	4.73	3.46	2.41	4.92	3.26	2.86	0.54 $\frac{\text{ft.}}{\text{sec.}}$	—
0.195	6.14	5.39	4.49	2.95	5.45	4.28	4.08	2.78	—
0.307	7.68	6.92	6.02	4.63	6.82	6.46	6.03	4.80	2.64 $\frac{\text{ft.}}{\text{sec.}}$
0.419	7.95	7.66	7.11	5.75	7.61	7.32	7.08	6.16	4.41
0.531	7.83	7.26	6.85	{ 5.91 5.65	7.91	7.57	7.56	6.78	4.61
0.643	7.48	6.95	6.88	{ 5.10 4.80	7.85	7.60	7.25	6.98	5.24
0.755	7.35	6.67	6.17	5.45	{ 7.30 7.29	6.98	6.93	6.66	5.24
0.812	6.99	6.52	6.00	4.68	{ 6.96 6.78	6.88	6.77	6.35	5.19
0.869	6.80	5.95	5.65	4.38	{ 6.04 6.55	6.55	6.68	5.81	4.90
0.925	6.46	5.65	5.33	4.20	{ 6.18 6.08	6.28	6.24	5.15	4.59

COARSE SAND DATA
TABLE A176 WATER VELOCITY PROFILE
MEASUREMENTS $i = 0.280$ ft./ft.

Y INS.	Suction Pressure Velocities at Various Discharge Concentrations						
	0.0264	0.0455	0.0698	0.0976	0.1266	0.1518	0.1770
0.027	6.81 $\frac{\text{ft.}}{\text{sec.}}$	5.15 $\frac{\text{ft.}}{\text{sec.}}$	4.48 $\frac{\text{ft.}}{\text{sec.}}$	4.29 $\frac{\text{ft.}}{\text{sec.}}$	3.71 $\frac{\text{ft.}}{\text{sec.}}$	3.24 $\frac{\text{ft.}}{\text{sec.}}$	2.90 $\frac{\text{ft.}}{\text{sec.}}$
0.083	7.11	6.55	5.48	4.77	4.41	3.76	2.96
0.139	7.90	7.28	6.35	5.43	4.75	4.06	3.10
0.195	8.96	7.94	7.96	6.42	5.71	4.86	3.44
0.307	10.81	9.92	8.83	7.98	7.45	6.29	4.50
0.419	11.70	11.29	10.66	10.02	8.74	7.35	5.58
0.531	11.52	11.22	11.01	10.63	9.70	8.33	6.40
0.643	11.12	10.85	10.57	9.84	9.23	8.17	6.54
0.755	10.71	10.42	9.83	9.02	8.29	7.30	6.08
0.812	10.26	10.44	9.41	8.45	7.73	6.88	5.73
0.869	9.85	9.44	8.91	7.90	7.16	6.41	5.30
0.925	9.32	$\begin{cases} 8.05 \\ 8.63 \end{cases}$	8.30	7.43	6.50	5.97	4.89

COARSE SAND DATA
TABLE A176 WATER VELOCITY PROFILE
MEASUREMENTS. $L = 0.280$ ft./ft.

Y INS.	Impact Pressure Velocities at Various Discharge Concentrations.					
	0.0197	0.0410	0.0624	0.1032	0.1320	0.1575
0.027	5.25 $\frac{\text{ft.}}{\text{sec.}}$	3.25 $\frac{\text{ft.}}{\text{sec.}}$	1.98 $\frac{\text{ft.}}{\text{sec.}}$	—	—	—
0.083	6.43	4.26	2.63	—	—	—
0.139	7.32	5.42	1.96	2.03 $\frac{\text{ft.}}{\text{sec.}}$	—	—
0.195	8.03	6.37	4.86	3.79	—	—
0.307	9.45	8.27	7.18	6.64	3.41 $\frac{\text{ft.}}{\text{sec.}}$	1.14 $\frac{\text{ft.}}{\text{sec.}}$
0.419	10.45	9.75	8.96	9.35	6.37	3.57
0.531	10.84	10.42	10.05	10.05 9.94	8.34	5.36
0.643	10.51	10.61	10.33	9.82 9.62	9.14	7.23
0.755	10.16	10.23	10.02	9.30 9.57	9.46	7.68
0.812	9.90	9.88	9.20	9.10 9.23	8.55	7.20
0.869	8.97	9.29	8.68 8.50	8.79 8.56	8.15	7.11
0.925	8.30	8.65	8.12	8.09	7.71	6.45

COARSE SAND DATA
TABLE A18a WATER VELOCITY PROFILES

	Suction Pressure Velocities at Various Concentrations						
Y _{INS.}	i = 0.400 ft./ft.						
	0.0299	0.0536	0.0799	0.1147	0.1376	0.1656	0.1832
0.027	7.89 $\frac{\text{ft.}}{\text{sec.}}$	6.76 $\frac{\text{ft.}}{\text{sec.}}$	6.62 $\frac{\text{ft.}}{\text{sec.}}$	6.96 $\frac{\text{ft.}}{\text{sec.}}$	5.88 $\frac{\text{ft.}}{\text{sec.}}$	5.35 $\frac{\text{ft.}}{\text{sec.}}$	4.93 $\frac{\text{ft.}}{\text{sec.}}$
0.083	8.80	8.06	8.06	8.31	7.03	6.21	5.51
0.139	9.98	9.20	8.70	8.54	7.87	6.87	6.37
0.195	10.67	10.48	9.75	9.20	8.80	7.49	7.15
0.307	13.13	12.38	11.29	10.80	9.95	9.08	8.66
0.419	14.64	13.91	13.29	13.13	11.82	10.77	10.09
0.531	14.82	14.49	14.21	13.93	13.28	12.21	11.08
0.643	14.37 14.00	14.05	13.72	13.50	12.68	12.47	11.68
0.755	13.81	13.20	13.04	12.74	11.29	11.11	10.30
0.812	13.28	12.61	12.26	12.10	10.73	10.71	9.90
0.869	12.68	11.45	11.59	11.09	9.83	9.89	9.28
0.925	11.58	11.04	10.86	10.13	8.99	9.08	8.68

COARSE SAND DATA
TABLE A186 WATER VELOCITY PROFILES

Y INS.	Impact Pressure Velocities at Various Concentrations i = 0.400 ft./ft.					
	0.0263	0.0481	0.0785	0.1195	0.1442	0.1735
0.027	6.48 $\frac{\text{ft.}}{\text{sec.}}$	4.93 $\frac{\text{ft.}}{\text{sec.}}$	2.89 $\frac{\text{ft.}}{\text{sec.}}$	1.93 $\frac{\text{ft.}}{\text{sec.}}$	—	—
0.083	7.94	6.40	4.22	3.10	—	—
0.139	9.15	7.11	5.26	$\frac{12.44}{11.97}$	—	—
0.195	9.79	8.06	6.15	4.42	—	—
0.307	11.28	10.29	8.29	5.05	4.54 $\frac{\text{ft.}}{\text{sec.}}$	—
0.419	12.68	11.70	10.45	8.28	7.11	5.71 $\frac{\text{ft.}}{\text{sec.}}$
0.531	13.30	12.70	12.39	11.08	10.72	8.45
0.643	13.22	12.91	12.35	12.22	12.06	$\frac{10.91}{10.62}$
0.755	12.61	12.32	12.22	12.04	11.77	11.12
0.812	12.14	11.96	11.82	11.53	11.40	10.83
0.869	11.53	11.34	10.70	11.06	10.92	10.54
0.925	10.82	10.68	10.42	9.73	9.52	9.75

GRAVEL DATA
TABLE A19 WATER VELOCITY PROFILES

Y INS.	Section Pressure Velocities at Various Concentrations $i = 0.280 \text{ ft./ft.}$							
	0.0081	0.0389	0.0389	0.0613	0.0639	0.0909	0.0948	
0.026	7.30 $\frac{\text{ft.}}{\text{Sec.}}$	5.70 $\frac{\text{ft.}}{\text{Sec.}}$	5.82 $\frac{\text{ft.}}{\text{Sec.}}$	5.04 $\frac{\text{ft.}}{\text{Sec.}}$	5.00 $\frac{\text{ft.}}{\text{Sec.}}$	4.40 $\frac{\text{ft.}}{\text{Sec.}}$	4.23 $\frac{\text{ft.}}{\text{Sec.}}$	
0.082	8.32	6.58	6.80	5.05	5.63	5.03	4.89	
0.138	8.96	7.35	7.29	6.27	6.06	5.64	5.36	
0.194	10.14	8.02	8.10	7.15	6.81	6.08	6.18	
0.306	11.80	9.52	9.53	8.57	8.23	7.62	7.35	
0.418	12.72	11.30		10.12	10.23	9.10	8.66	
0.530	12.53	11.67		11.38	11.50	10.50	{ 10.22 10.27	
0.642	12.17	11.09	11.40	11.09	11.32	10.97	10.72	
0.754	11.14	9.54	10.61	9.87	10.45		9.57	
0.810	10.41	10.31	9.71	8.98	9.77		{ 8.63 8.44	
0.866	10.79	10.19	10.33	10.02	10.08		{ 9.21 9.21	
0.922	10.21	9.77		8.70	9.32		{ 8.60 8.38	
0.978	8.95	8.56	8.10	7.95	8.14	7.79	{ 7.08 7.29	

TABLE A20 CONCENTRATION PROFILE CORRELATION DATA - FINE SAND

$\bar{L} = 0.400 \text{ ft./ft.}$

y Inches	$\bar{C} = 0.04$			$\bar{C} = 0.08$			$\bar{C} = 0.12$			$\bar{C} = 0.16$			$\bar{C} = 0.20$			$\frac{y \omega_0}{d \sqrt{P_0}}$
	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	
0	0.0665	0.206	1.000	0.1297	0.478	1.000	0.1809	0.780	1.000	0.2395	1.14	1.000	0.2620	1.38	1.000	0
0.10	0.0602	0.184	0.893	0.1184	0.422	0.885	0.1715	0.700	0.898	0.2180	1.00	0.877	0.2460	1.23	0.892	1.71
0.20	0.0540	0.164	0.796	0.1082	0.374	0.783	0.1582	0.620	0.795	0.2015	0.892	0.781	0.2305	1.11	0.805	3.41
0.30	0.0488	0.146	0.709	0.0966	0.326	0.682	0.1447	0.562	0.720	0.1866	0.794	0.696	0.2170	0.995	0.720	5.12
0.40	0.0438	0.131	0.636	0.0875	0.289	0.605	0.1314	0.485	0.621	0.1720	0.702	0.615	0.2098	0.920	0.666	6.82
0.50	0.0394	0.117	0.568	0.0800	0.259	0.542	0.1206	0.433	0.545	0.1590	0.627	0.550	0.1940	0.836	0.606	8.53
0.60	0.0362	0.106	0.515	0.0738	0.234	0.490	0.1111	0.389	0.499	0.1475	0.568	0.498	0.1844	0.770	0.558	10.23
0.70	0.0338	0.100	0.485	0.0687	0.215	0.450	0.1040	0.357	0.455	0.1395	0.522	0.458	0.1762	0.722	0.522	11.93
0.80	0.0322	0.093	0.451	0.0654	0.203	0.425	0.0989	0.336	0.431	0.1343	0.500	0.438	0.1697	0.686	0.497	13.64
0.90	0.0309	0.0892	0.433	0.0632	0.194	0.406	0.0956	0.322	0.413	0.1307	0.482	0.423	0.1656	0.663	0.480	15.35
1.00	0.0300	0.0862	0.418	0.0618	0.190	0.398	0.0934	0.313	0.401	0.1289	0.475	0.416	0.1631	0.650	0.470	17.06

$\bar{L} = 0.280 \text{ ft./ft.}$

y Inches	$\bar{C} = 0.04$			$\bar{C} = 0.08$			$\bar{C} = 0.12$			$\bar{C} = 0.16$			$\bar{C} = 0.20$			$\frac{y \omega_0}{d \sqrt{P_0}}$
	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	
0	0.0737	0.234	1.000	0.1429	0.543	1.000	0.2007	0.887	1.000	0.2444	1.220	1.000	0.2740	1.502	1.000	0
0.10	0.0654	0.202	0.863	0.1284	0.470	0.863	0.1815	0.760	0.857	0.2262	1.060	0.869	0.2550	1.310	0.872	1.96
0.20	0.0575	0.175	0.748	0.1144	0.405	0.745	0.1633	0.650	0.733	0.2062	0.918	0.752	0.2385	1.170	0.778	3.92
0.30	0.0503	0.152	0.650	0.1007	0.344	0.634	0.1470	0.565	0.637	0.1869	0.795	0.651	0.2233	1.040	0.692	5.88
0.40	0.0442	0.132	0.564	0.0889	0.295	0.543	0.1318	0.488	0.550	0.1712	0.700	0.573	0.2087	0.930	0.619	7.84
0.50	0.0390	0.1165	0.498	0.0785	0.252	0.464	0.1192	0.425	0.480	0.1576	0.620	0.508	0.1957	0.842	0.560	9.80
0.60	0.0348	0.103	0.440	0.0698	0.219	0.404	0.1081	0.384	0.433	0.1458	0.565	0.463	0.1844	0.772	0.514	11.76
0.70	0.0314	0.0902	0.385	0.0630	0.194	0.357	0.0988	0.328	0.370	0.1360	0.507	0.415	0.1748	0.718	0.477	13.72
0.80	0.0280	0.0833	0.356	0.0584	0.179	0.330	0.0920	0.307	0.346	0.1278	0.470	0.385	0.1665	0.670	0.445	15.68
0.90	0.0273	0.0784	0.335	0.0553	0.169	0.311	0.0865	0.284	0.320	0.1215	0.438	0.359	0.1590	0.630	0.411	17.64
1.00	0.0262	0.0748	0.319	0.0540	0.164	0.302	0.0822	0.269	0.303	0.1166	0.414	0.339	0.1528	0.596	0.396	19.60

TABLE A20 CONCENTRATION PROFILE CORRELATION DATA - FINE SAND
 $l = 0.160$ ft./ft.

y Inches.	$\bar{c} = 0.025$			$\bar{c} = 0.05$			$\bar{c} = 0.075$			$\bar{c} = 0.10$			$\frac{y \omega_0}{d \sqrt{\rho_0}}$
	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	
0	0.0522	0.157	1.000	0.1048	0.358	1.000	0.1586	0.625	1.000	0.2110	0.950	1.000	0
0.10	0.0462	0.138	0.879	0.0926	0.311	0.870	0.1396	0.526	0.842	0.1875	0.795	0.837	2.58
0.20	0.0402	0.120	0.765	0.0803	0.261	0.730	0.1209	0.435	0.696	0.1643	0.658	0.692	5.16
0.30	0.0344	0.102	0.650	0.0684	0.215	0.601	0.1036	0.355	0.568	0.1396	0.525	0.553	7.74
0.40	0.0293	0.084	0.535	0.0582	0.178	0.498	0.0987	0.294	0.470	0.1167	0.416	0.438	10.32
0.50	0.0244	0.0693	0.441	0.0497	0.149	0.417	0.0752	0.240	0.384	0.0971	0.327	0.344	12.90
0.60	0.0204	0.0574	0.365	0.0421	0.125	0.349	0.0635	0.196	0.314	0.0805	0.262	0.276	15.48
0.70	0.0172	0.0479	0.305	0.0352	0.105	0.293	0.0527	0.160	0.256	0.0668	0.208	0.219	18.06
0.80	0.0144	0.0397	0.253	0.0292	0.084	0.235	0.0441	0.132	0.211	0.0558	0.170	0.179	20.64
0.90	0.0122	0.0333	0.212	0.0242	0.0686	0.192	0.0371	0.111	0.178	0.0470	0.141	0.149	23.22
1.00	0.0100	0.0275	0.175	0.0200	0.0584	0.158	0.0312	0.0903	0.145	0.0410	0.122	0.129	25.80

TABLE A21 CONCENTRATION PROFILE CORRELATION DATA - ZIRCON
 $l = 0.400$ ft./ft.

y Inches.	$\bar{c} = 0.03$			$\bar{c} = 0.06$			$\bar{c} = 0.09$			$\bar{c} = 0.12$			$\frac{y \omega_0}{d \sqrt{\rho_0}}$
	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	
0	0.0975	0.331	1.000	0.2015	0.887	1.000	0.260	1.360	1.000	0.328	2.15	1.000	0
0.10	0.0761	0.243	0.735	0.1590	0.630	0.710	0.215	0.980	0.720	0.278	1.54	0.715	3.89
0.20	0.0603	0.185	0.559	0.1197	0.430	0.485	0.175	0.618	0.454	0.229	1.09	0.587	7.78
0.30	0.0472	0.142	0.429	0.0940	0.334	0.377	0.137	0.514	0.378	0.183	0.764	0.355	11.68
0.40	0.0357	0.106	0.320	0.0712	0.223	0.252	0.107	0.371	0.272	0.145	0.554	0.258	15.58
0.50	0.0272	0.078	0.236	0.0548	0.167	0.188	0.083	0.272	0.200	0.111	0.386	0.179	19.47
0.60	0.0210	0.0595	0.180	0.0426	0.126	0.142	0.063	0.195	0.143	0.085	0.280	0.130	23.4
0.70	0.0162	0.0449	0.136	0.0332	0.0963	0.109	0.050	0.151	0.111	0.068	0.213	0.099	27.2
0.80	0.0126	0.0345	0.104	0.0257	0.0732	0.0827	0.040	0.120	0.0883	0.055	0.168	0.078	31.1
0.90	0.0100	0.0275	0.083	0.0207	0.0583	0.0657	0.033	0.0957	0.0704	0.045	0.134	0.0623	35.0
1.00	0.0086	0.0238	0.072	0.0178	0.0496	0.0560	0.0282	0.0810	0.0595	0.038	0.111	0.053	38.9

TABLE A21 CONCENTRATION PROFILE CORRELATION DATA - ZIRCON

$L = 0.280 \text{ ft./ft.}$

y Inches.	$\bar{C} = 0.02$			$\bar{C} = 0.04$			$\bar{C} = 0.06$			$\frac{y \omega_0}{d \sqrt{P_0}}$
	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	
0	0.0761	0.245	1.000	0.1538	0.601	1.000	0.2319	1.130	1.000	0
0.10	0.0632	0.195	0.796	0.1260	0.460	0.765	0.1887	0.801	0.709	4.65
0.20	0.0485	0.145	0.592	0.0974	0.331	0.550	0.1459	0.535	0.458	9.30
0.30	0.0338	0.100	0.408	0.0675	0.211	0.350	0.1010	0.355	0.374	13.95
0.40	0.0225	0.0637	0.260	0.0450	0.134	0.223	0.0674	0.210	0.186	18.6
0.50	0.0156	0.0432	0.1765	0.0312	0.090	0.1495	0.0467	0.140	0.124	23.3
0.60	0.0109	0.0297	0.1211	0.0218	0.0618	0.1025	0.0327	0.095	0.084	27.9
0.70	0.0082	0.0226	0.0922	0.0164	0.0456	0.0759	0.0246	0.0698	0.0618	32.6
0.80	0.0065	0.0201	0.0821	0.0130	0.0356	0.0592	0.0194	0.0545	0.0483	37.2
0.90	0.0049	0.0148	0.0604	0.0098	0.0269	0.0447	0.0148	0.0409	0.0362	41.9
1.00	0.0041	0.0122	0.0498	0.0082	0.0226	0.0375	0.0124	0.0345	0.0306	46.5

$L = 0.160 \text{ ft./ft.}$

y Inches.	$\bar{C} = 0.02$			$\bar{C} = 0.04$			$\frac{y \omega_0}{d \sqrt{P_0}}$
	C_y	P_y	$\frac{P}{P_0}$	C_y	P_y	$\frac{P}{P_0}$	
0	0.1087	0.379	1.000	0.2167	0.998	1.000	0
0.10	0.0808	0.262	0.691	0.1600	0.635	0.637	6.12
0.20	0.0555	0.165	0.435	0.1103	0.385	0.386	12.24
0.30	0.0343	0.101	0.266	0.0678	0.212	0.2125	18.36
0.40	0.0204	0.0573	0.151	0.0406	0.122	0.1222	24.48
0.50	0.0115	0.0306	0.0808	0.0229	0.0646	0.0647	30.60
0.60	0.0069	0.0191	0.0504	0.0121	0.0435	0.0436	36.72
0.70	0.0039	0.0107	0.0282	0.0072	0.0225	0.0226	42.84
0.80	0.0027	0.0075	0.0198	0.00438	0.0121	0.0121	48.96
0.90	0.00212	0.0059	0.0156	0.00357	0.0099	0.0099	55.08
1.00	0.00175	0.0049	0.0129	0.00295	0.0082	0.0082	61.20

TABLE A 22 DURAND'S CONCENTRATION PROFILE DATA; $D=5.90"$, $d=0.18$ mm.

$\bar{V}_m$ ft./sec.	y Inches	q_y	P_y	$\frac{P}{P_0}$	$\frac{y\omega_0}{d\sqrt{\frac{P_0}{P_w}}}$	$\bar{V}_m$ ft./sec.	y Inches.	q_y	P_y	$\frac{P}{P_0}$	$\frac{y\omega_0}{d\sqrt{\frac{P_0}{P_w}}}$
7.67	0	0.42	4.11	1.000	0	10.0	0	0.320	2.05	1.000	0
	0.79	0.24	1.19	0.289	18.6		0.844	0.179	0.740	0.361	15.5
	1.19	0.17	0.690	0.168	28.0		1.18	0.143	0.545	0.266	21.7
	1.576	0.125	0.455	0.111	37.0		1.48	0.117	0.416	0.203	27.2
	1.97	0.081	0.262	0.064	46.2		1.97	0.083	0.272	0.123	36.2
	2.74	0.030	0.0864	0.021	64.5		2.68	0.0392	0.117	0.057	49.4
	3.54	0.009	0.0300	0.0073	83.3		3.24	0.0215	0.0605	0.0295	59.5
	4.34	0.004	0.0110	0.0027	102.0		4.22	0.0083	0.0229	0.0112	77.7
$\bar{V}_m$ ft./sec.	y Inches.	q_y	P_y	$\frac{P}{P_0}$	$\frac{y\omega_0}{d\sqrt{\frac{P_0}{P_w}}}$	$\bar{V}_m$ ft./sec.	y Inches.	q_y	P_y	$\frac{P}{P_0}$	$\frac{y\omega_0}{d\sqrt{\frac{P_0}{P_w}}}$
14.4	0	0.198	0.864	1.000	0	11.8	0	0.257	1.44	1.000	0
	0.844	0.136	0.506	0.586	11.1		0.844	0.160	0.635	0.440	14.0
	1.18	0.121	0.434	0.503	15.6		1.18	0.128	0.470	0.326	18.7
	1.48	0.104	0.356	0.412	19.6		1.48	0.1075	0.373	0.259	22.4
	1.97	0.083	0.272	0.303	26.0		1.97	0.083	0.272	0.189	31.2
	2.68	0.052	0.157	0.182	35.4		2.68	0.0475	0.143	0.1035	42.4
	3.24	0.0368	0.110	0.127	42.8		3.24	0.0313	0.0903	0.0627	51.2
	4.22	0.0208	0.0588	0.068	55.8		4.22	0.0154	0.0430	0.0298	66.7

TABLE A22 DURAND'S CONCENTRATION PROFILE DATA ; $D=5.90''$; $d=0.18$ mm.

$\bar{V}_m$ ft./sec.	y Inches.	q_y	P_y	$\frac{P}{P_0}$	$\frac{y W_0}{d \sqrt{\frac{P_0}{\rho_w}}}$	$\bar{V}_m$ ft./sec.	y Inches.	q_y	P_y	$\frac{P}{P_0}$	$\frac{y W_0}{d \sqrt{\frac{P_0}{\rho_w}}}$
16.16	0	0.159	0.630	1.000	0	18.15	0	0.125	0.455	1.000	0
	0.844	0.1254	0.450	0.715	10.0		0.844	0.1092	0.380	0.835	9.0
	1.18	0.1092	0.380	0.603	14.0		1.18	0.1020	0.349	0.767	12.6
	1.48	0.0981	0.333	0.519	17.6		1.48	0.0944	0.317	0.696	15.8
	1.97	0.083	0.272	0.432	23.4		1.97	0.0818	0.265	0.582	21.0
	2.68	0.0566	0.173	0.274	31.8		2.68	0.0600	0.184	0.405	28.6
	3.24	0.0430	0.129	0.205	38.5		3.24	0.0475	0.143	0.314	34.5
	4.22	0.0272	0.0782	0.124	50.2		4.22	0.0321	0.0928	0.204	45.0

TABLE A23 DURAND'S CONCENTRATION PROFILE DATA ; $D=9.96''$; $d=50 \mu$.

$\bar{V}_m$ ft./sec.	y Inches.	q_y	P_y	$\frac{P}{P_0}$	$\frac{y W_0}{d \sqrt{\frac{P_0}{\rho_w}}}$	$\bar{V}_m$ ft./sec.	y Inches.	q_y	P_y	$\frac{P}{P_0}$	$\frac{y W_0}{d \sqrt{\frac{P_0}{\rho_w}}}$
1.48	0	0.0295	0.0850	1.000	0	0.58	0	0.0250	0.0712	1.000	0
	0.79	0.0263	0.0726	0.855	5.51		1.18	0.0116	0.0317	0.445	19.3
	0.99	0.0253	0.0720	0.847	6.90		1.97	0.0063	0.0174	0.244	32.1
	1.97	0.0216	0.0609	0.716	13.8		2.96	0.00332	0.0092	0.129	48.2
	2.96	0.0184	0.0519	0.610	20.7		3.94	0.00243	0.0072	0.101	64.3
	3.94	0.0158	0.0439	0.516	27.6		4.92	0.00184	0.0051	0.0715	80.3
	4.92	0.0137	0.0375	0.441	34.5		5.90	0.00131	0.0036	0.0505	96.3
	5.90	0.0126	0.0345	0.406	41.4		6.89	0.00105	0.0029	0.0407	111.5
	6.89	0.0116	0.0305	0.359	48.3		7.88	0.00105	0.0029	0.0407	128.0
	7.88	0.0100	0.0275	0.324	55.2						
	8.86	0.00895	0.0246	0.290	62.0						

TABLE A24 DURAND'S CONCENTRATION PROFILE DATA, SAND

$d = 0.44 \text{ mm.}$, $D = 5.90 \text{ in.}$, $\bar{V} = 14 \text{ ft./sec.}$, $w_0 = 0.165 \frac{\text{ft.}}{\text{sec.}}$

y/D	C	P	P/P_0	$yw_0/d \sqrt{\frac{P_0}{P}}$
0	0.220	1.02	1.000	0
0.1	0.162	0.645	0.630	9.05
0.2	0.103	0.352	0.345	18.10
0.3	0.090	0.299	0.293	27.15
0.4	0.067	0.208	0.204	36.20
0.5	0.048	0.144	0.141	45.25
0.6	0.0325	0.094	0.0921	54.30
0.7	0.0230	0.065	0.0637	63.35
0.8	0.015	0.0415	0.0407	72.40
0.9	0.0075	0.0208	0.0204	81.45

TABLE A25 DURAND'S CONCENTRATION PROFILE DATA

$d = 2.04 \text{ mm.}$, $D = 5.90 \text{ ft./sec.}$, $\bar{V} = 14 \text{ ft./sec.}$

y/D	C	P	P/P_0	$yw_0/d \sqrt{\frac{P_0}{P}}$
0	0.430	4.40	1.000	0
0.1	0.299	1.77	0.402	8
0.2	0.162	0.644	0.146	16
0.3	0.077	0.247	0.056	24
0.4	0.0385	0.115	0.0262	32
0.5	0.021	0.0594	0.0135	40
0.6	0.013	0.0358	0.0082	48
0.7	0.008	0.0222	0.0051	56

FIG. AVII-1

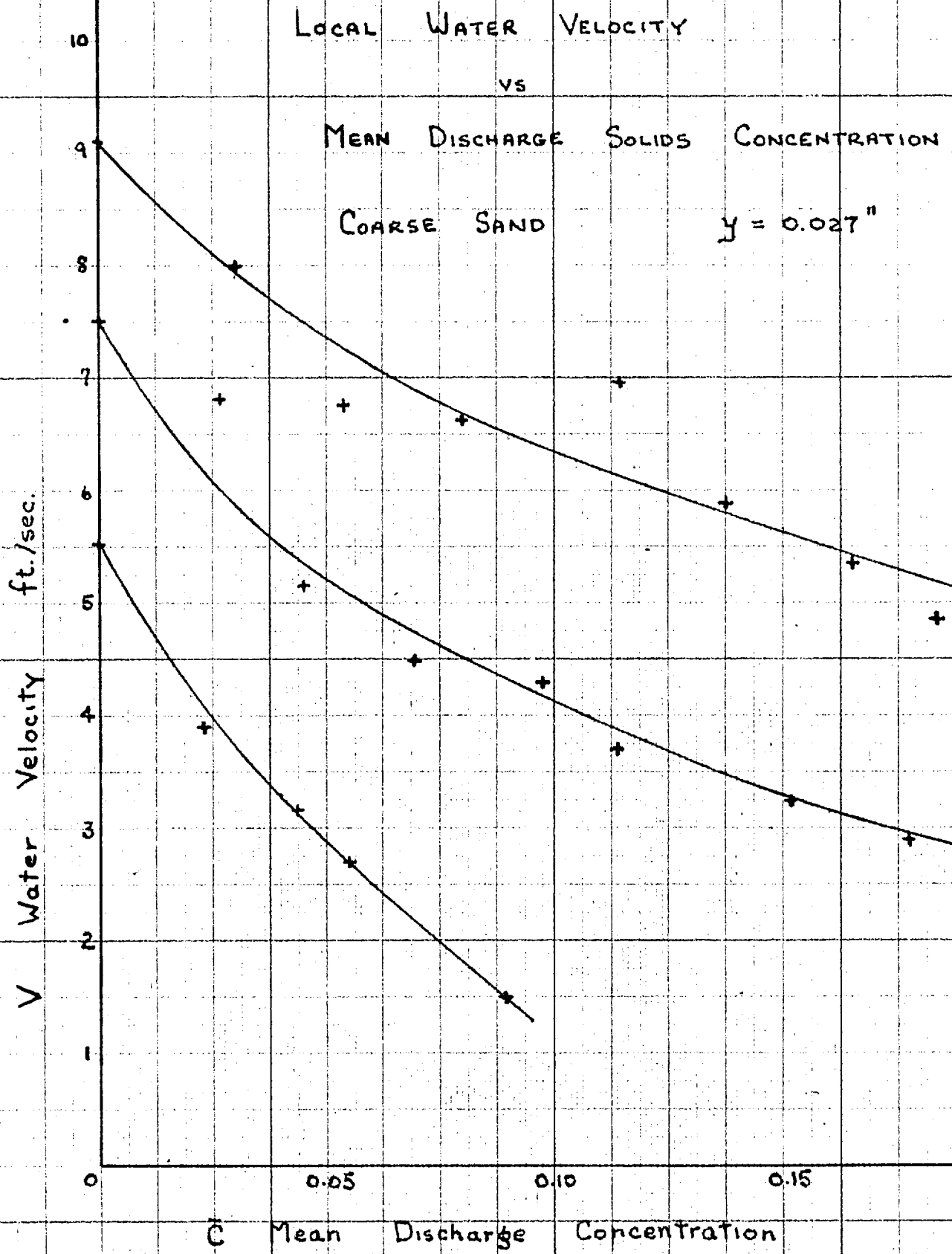


FIG. AVII-2

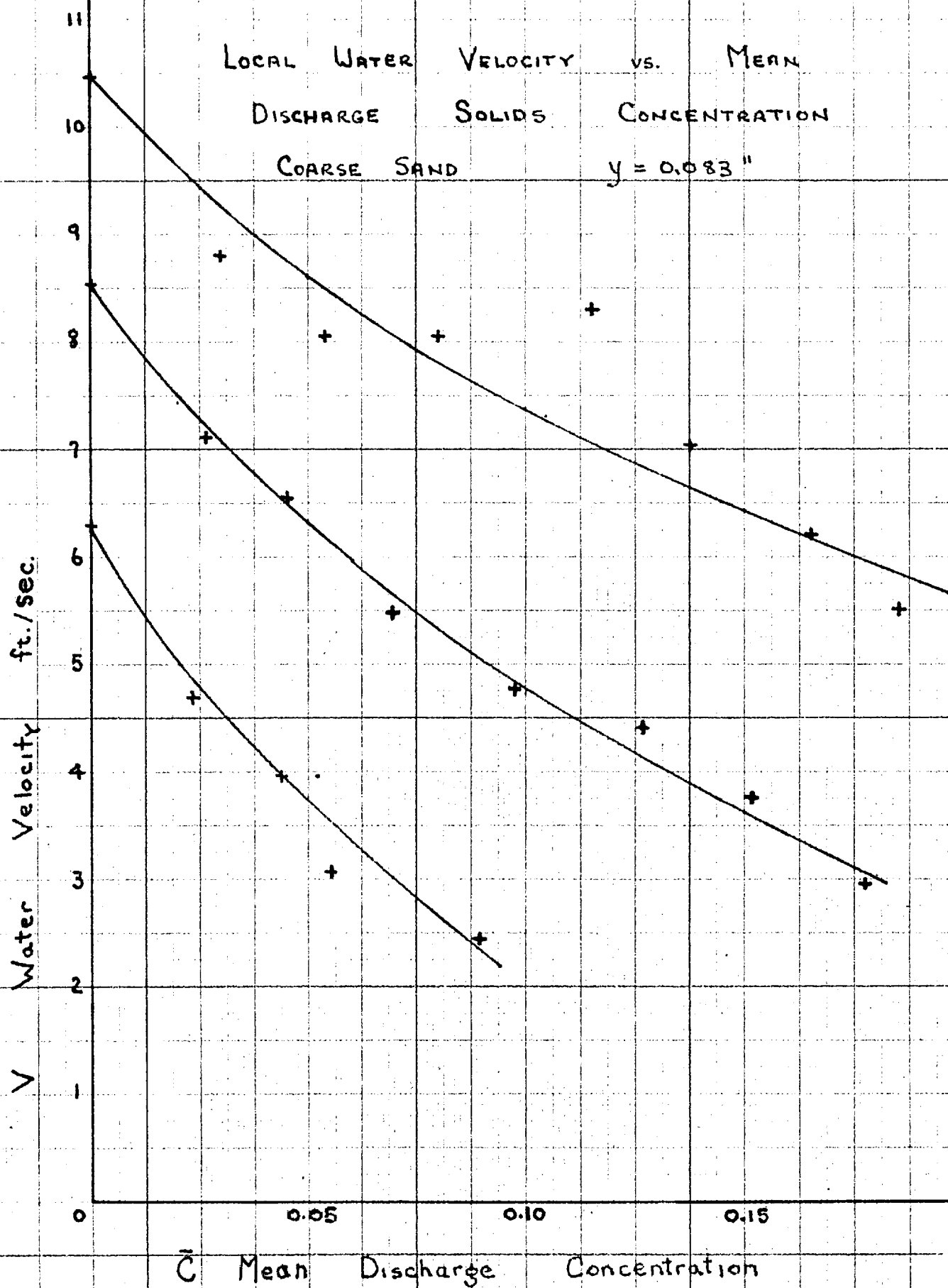


FIG. AVII-3

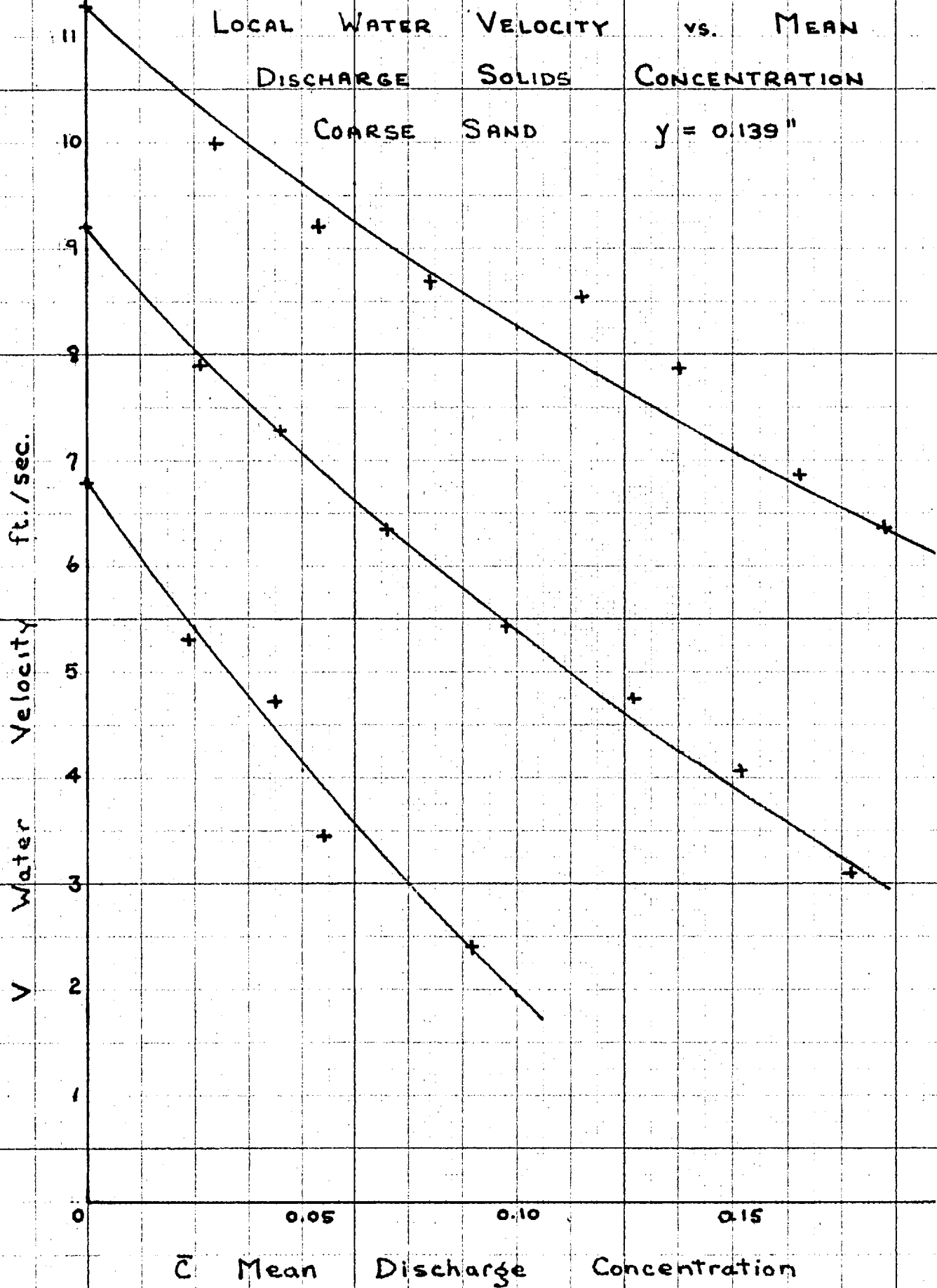


FIG. AVII-4

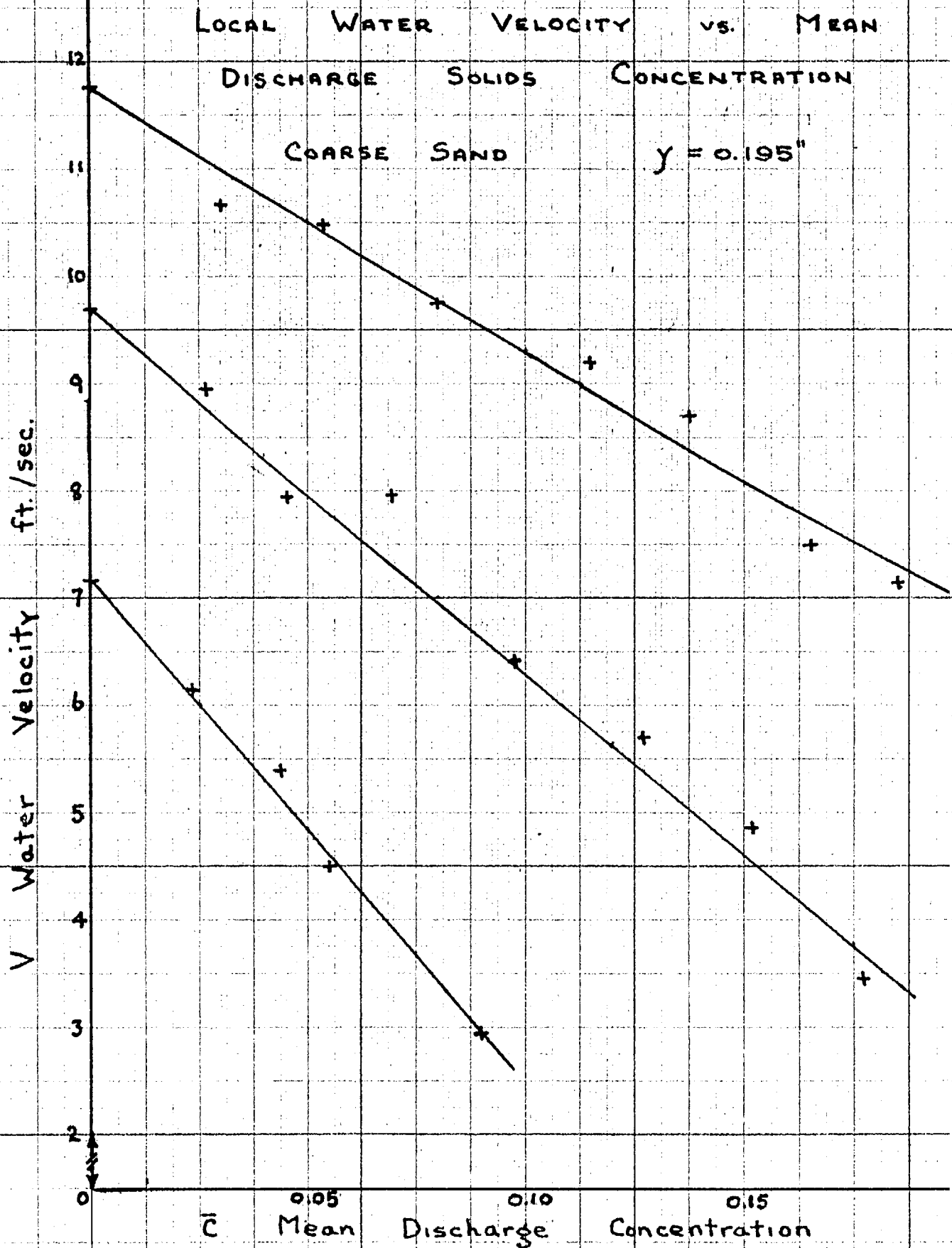


FIG. AVIII-6

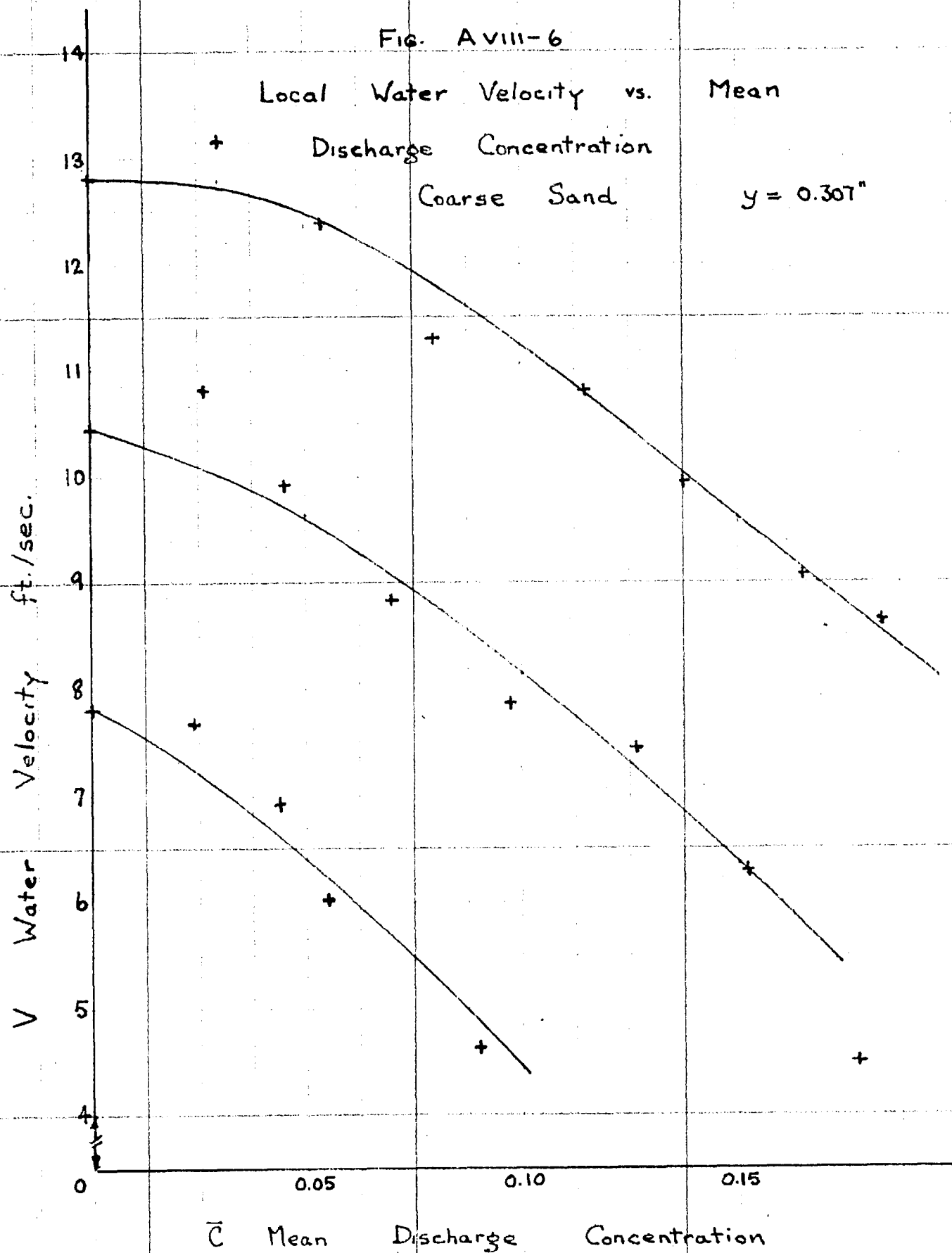


FIG. AVIII-7

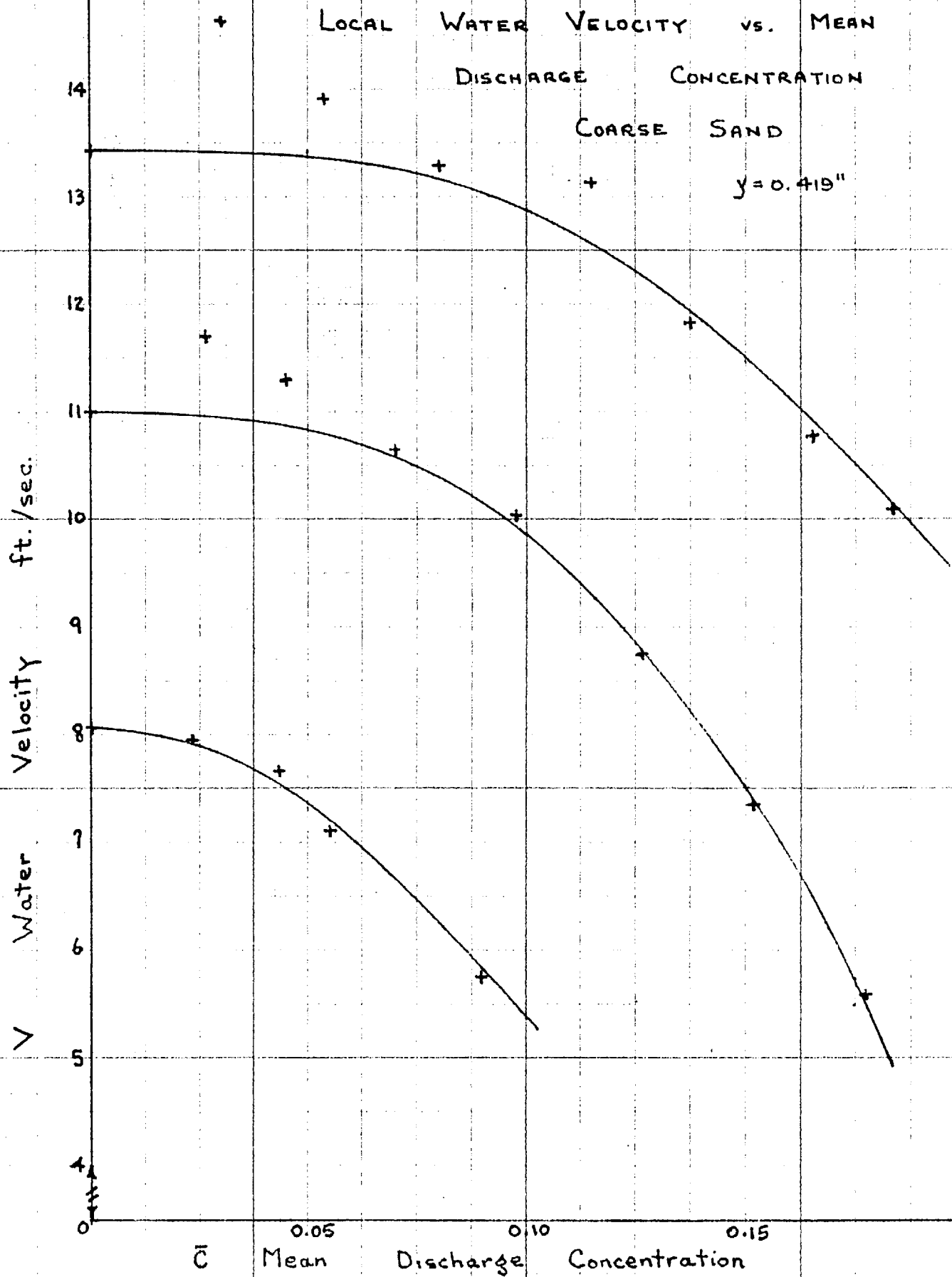


FIG. A VIII-8

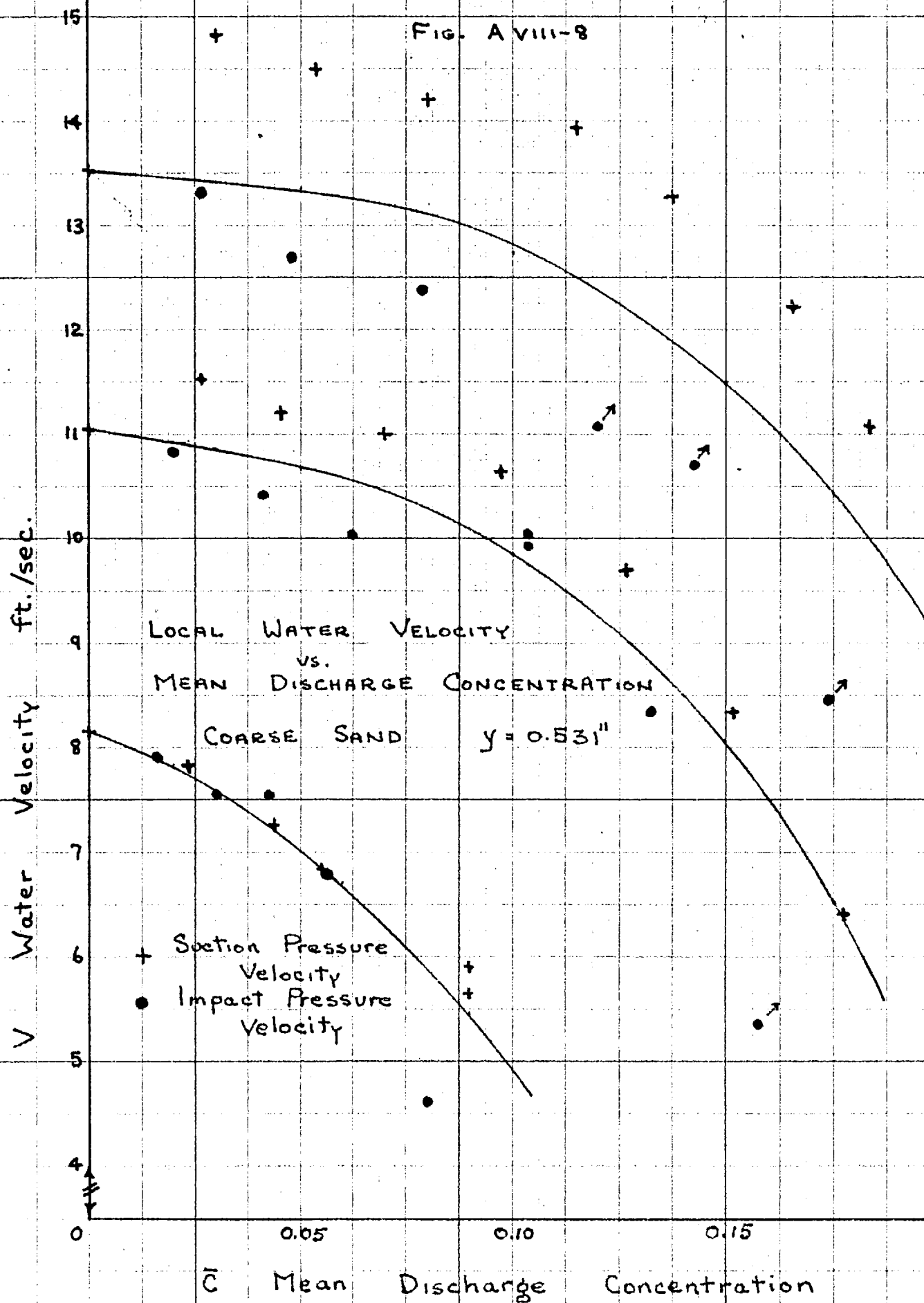


FIG. AVIII-9

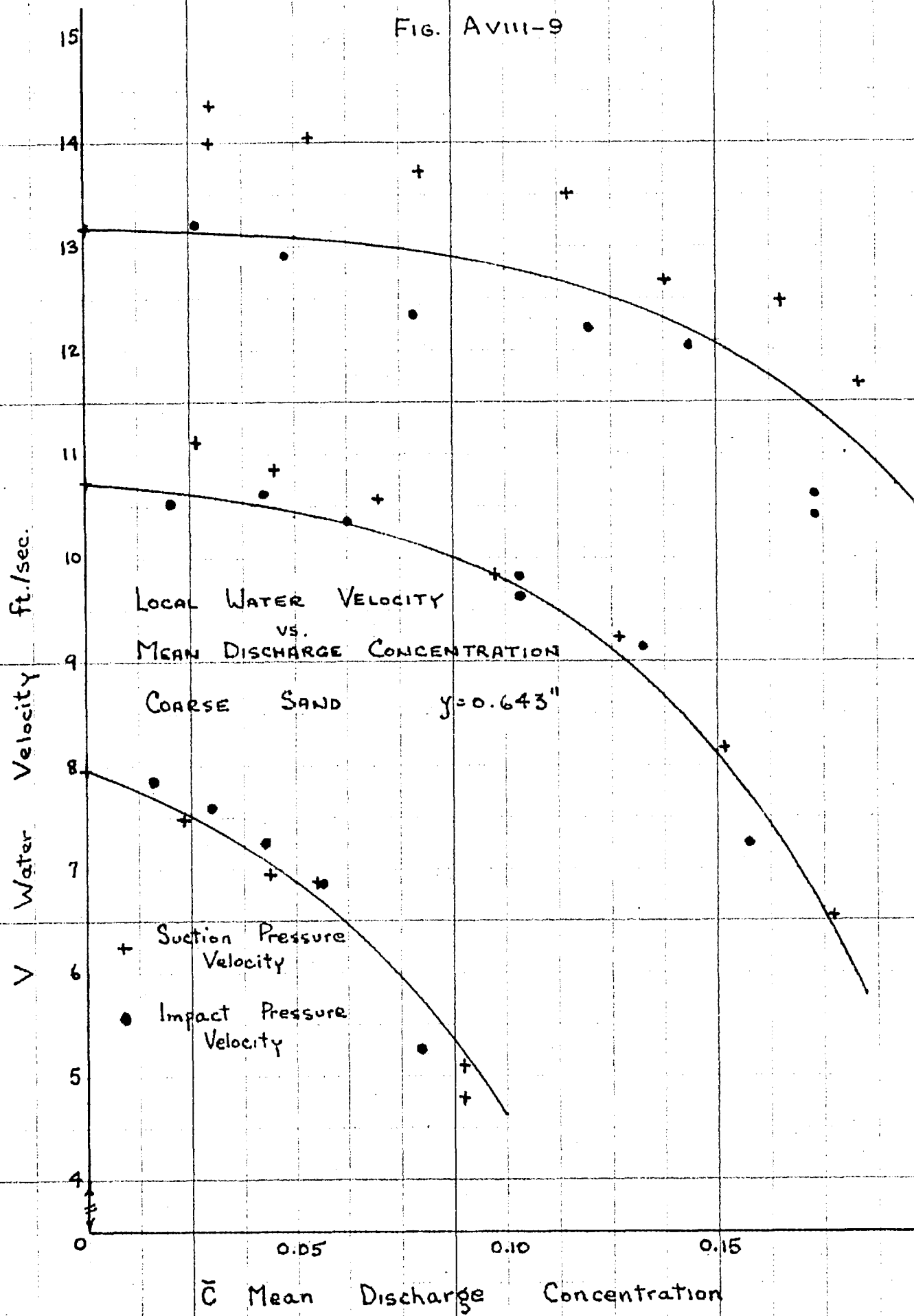


FIG. A VIII-10

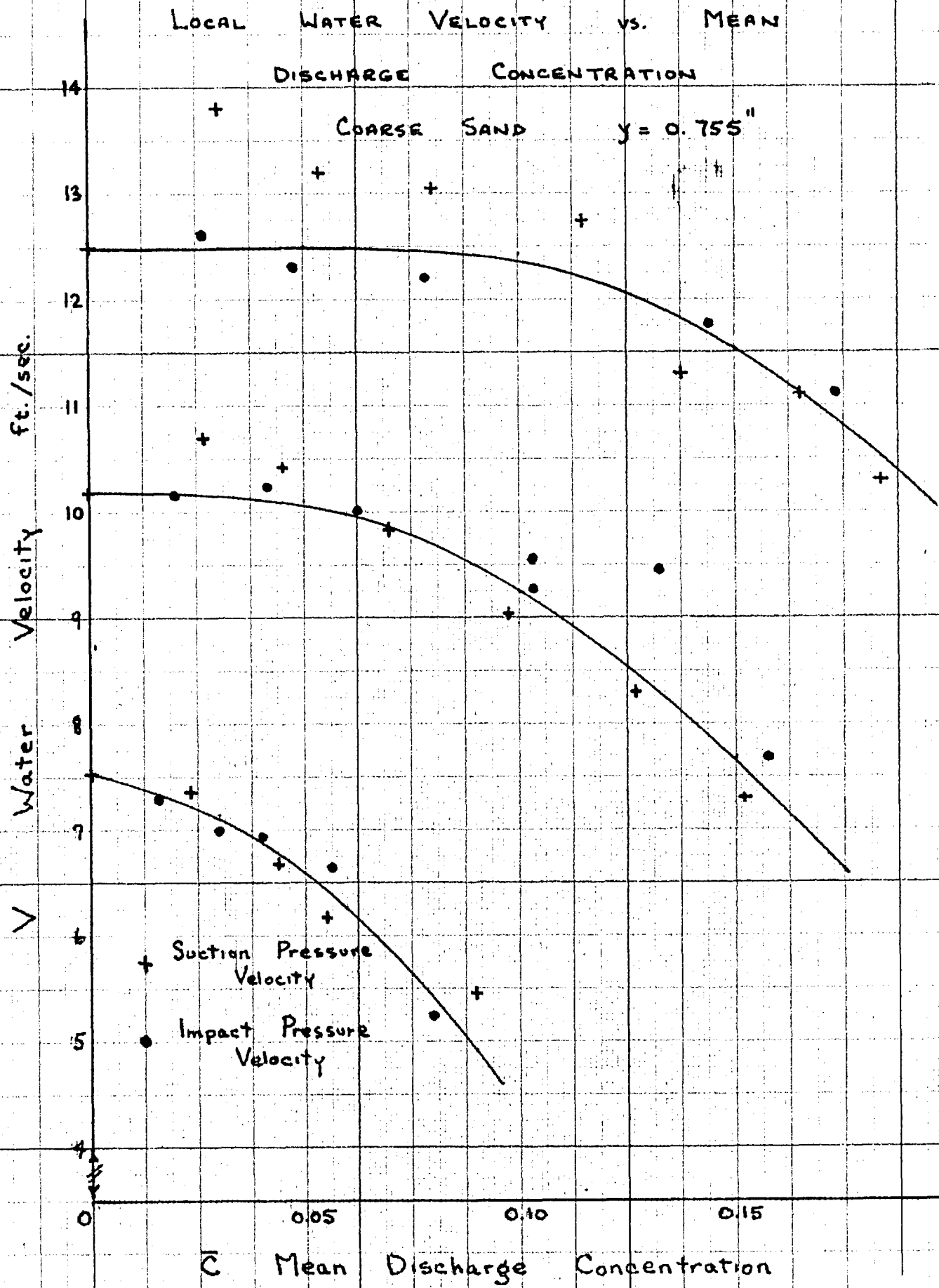


FIG. AVIII-11

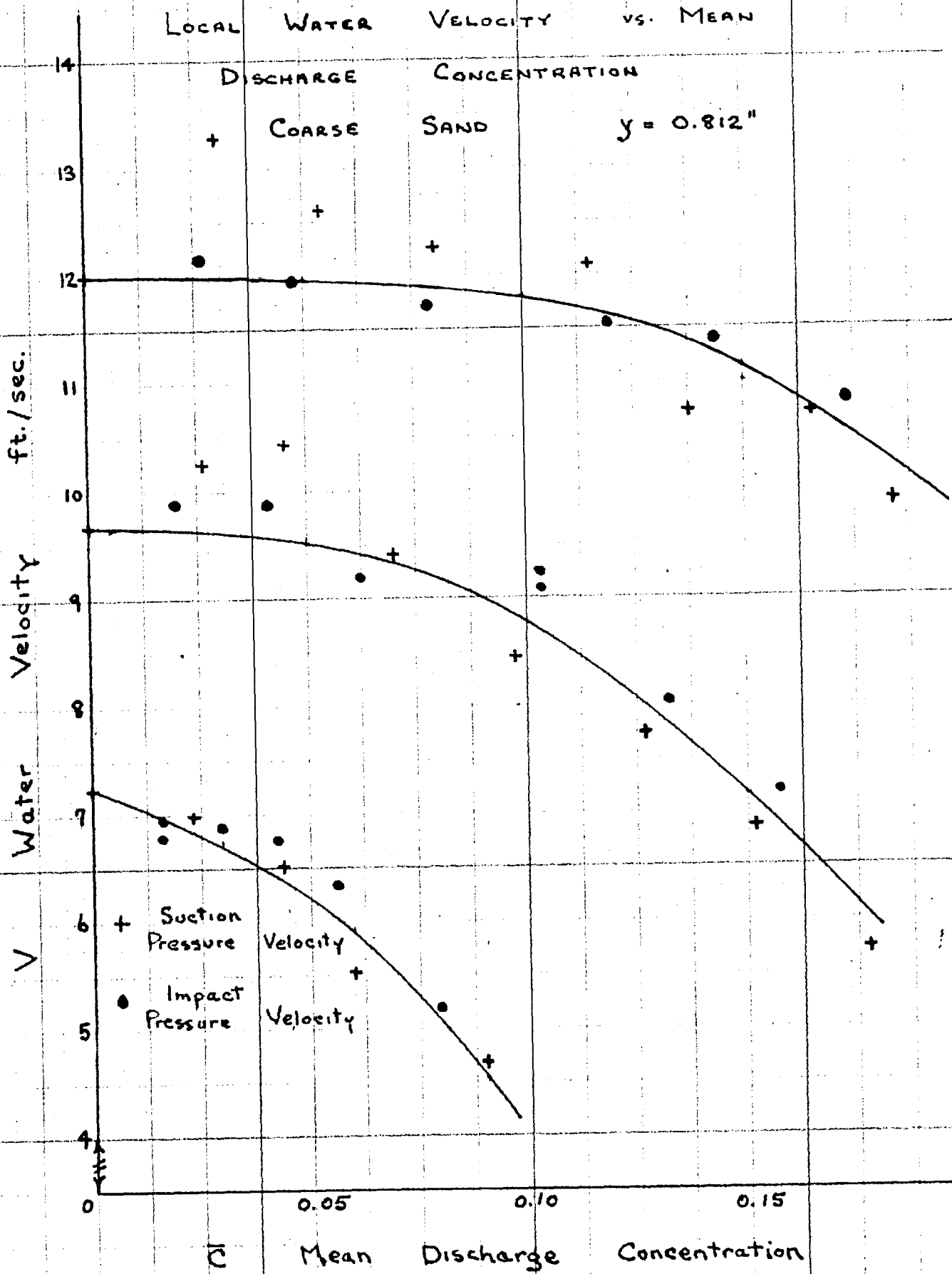


FIG. A VII-12

LOCAL WATER VELOCITY VS. MEAN DISCHARGE CONCENTRATION

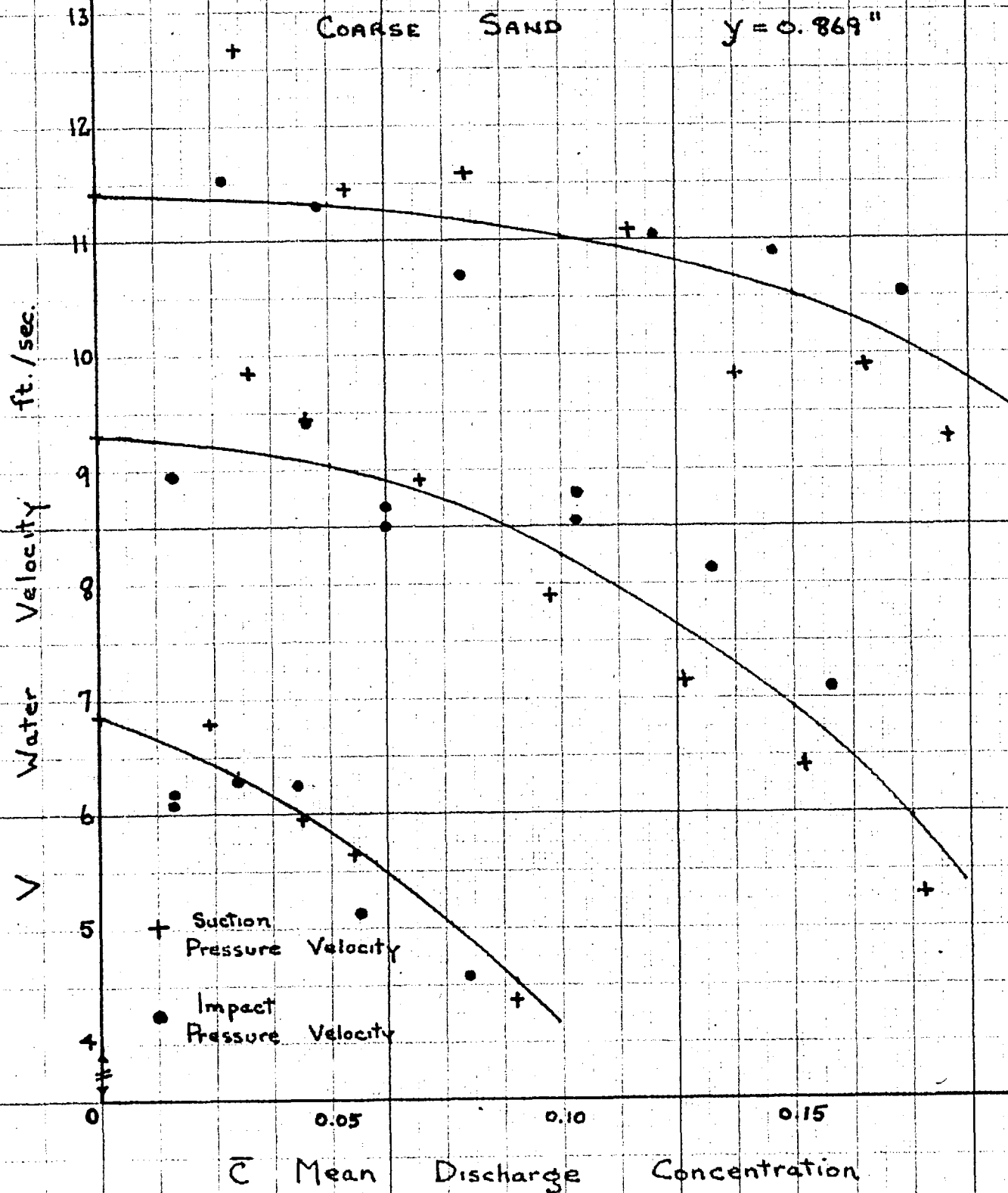


FIG. A VII-13

LOCAL WATER VELOCITY vs. MEAN
DISCHARGE CONCENTRATION

COARSE SAND

$$y = 0.925^x$$

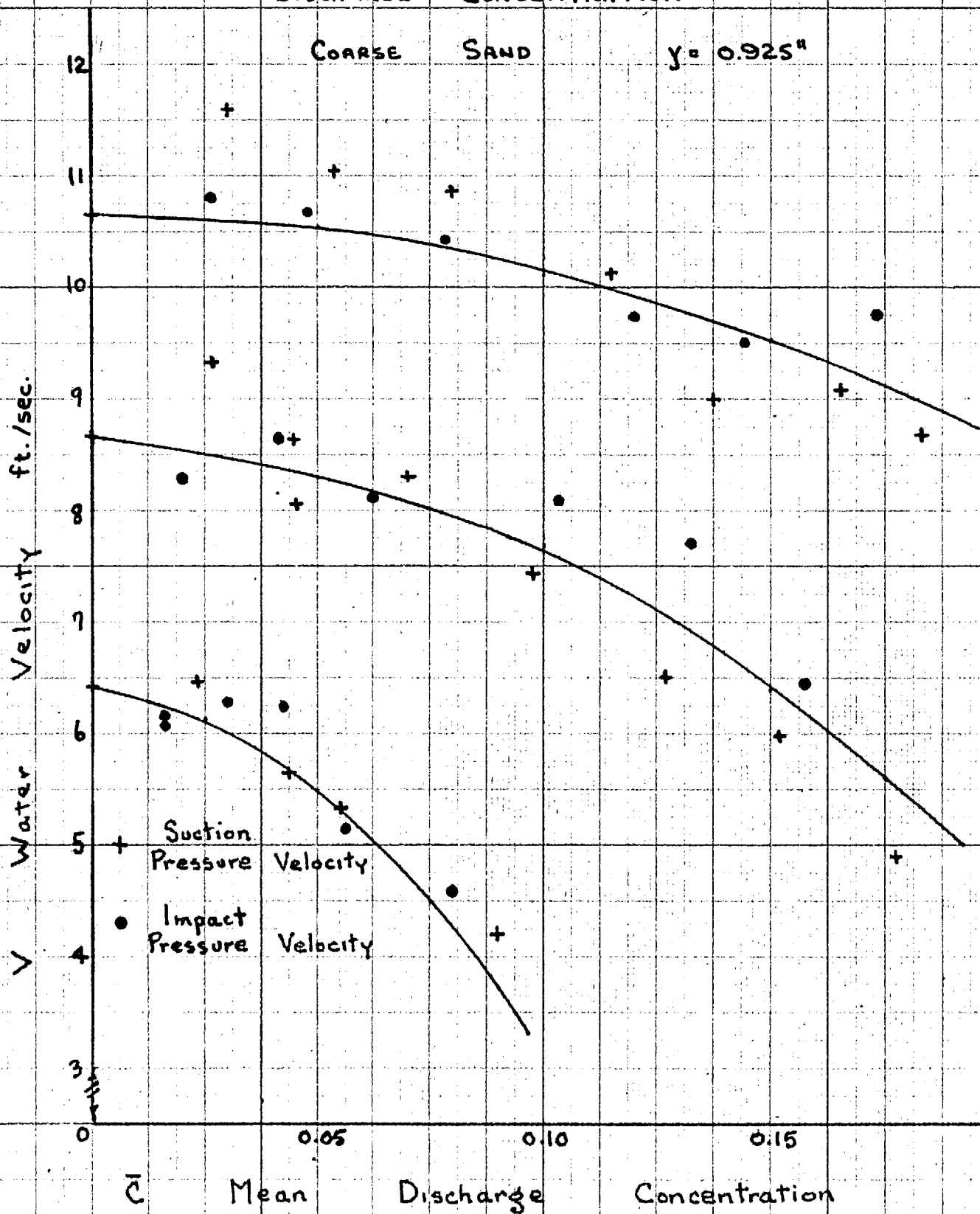


FIG. AVII-15

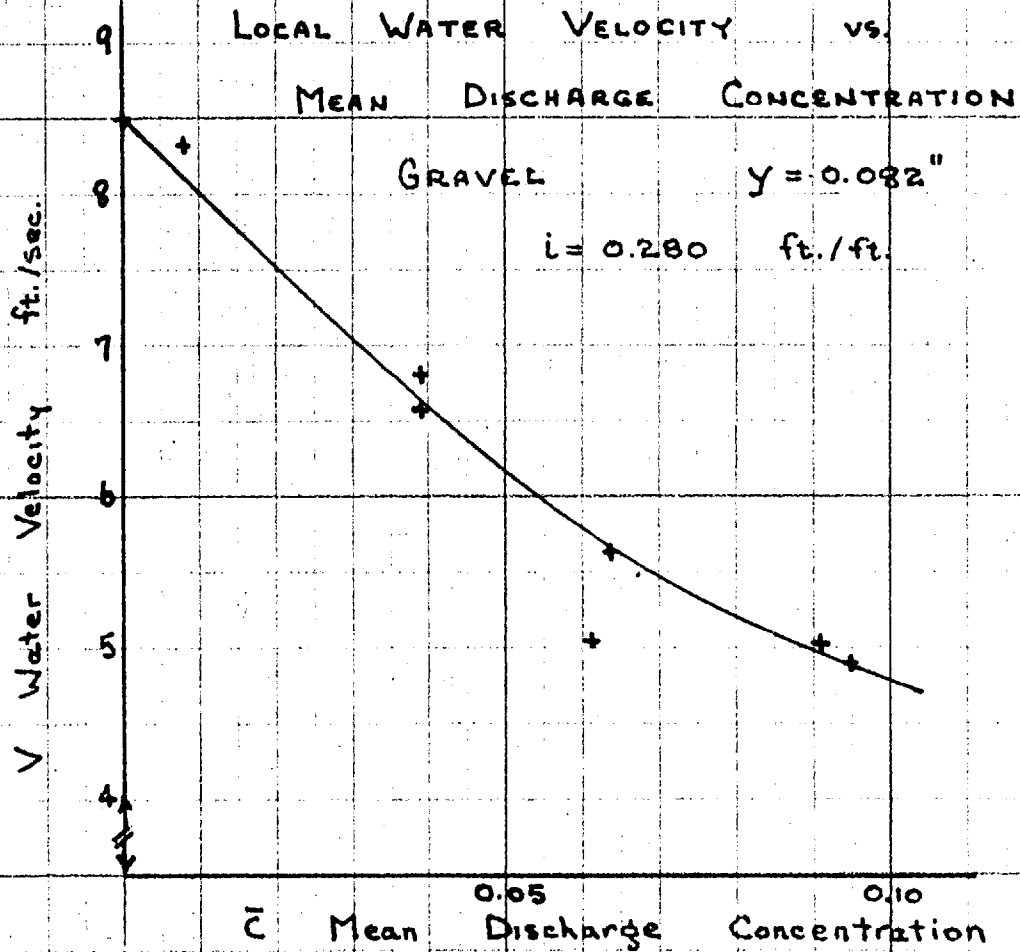


FIG. AVII-14

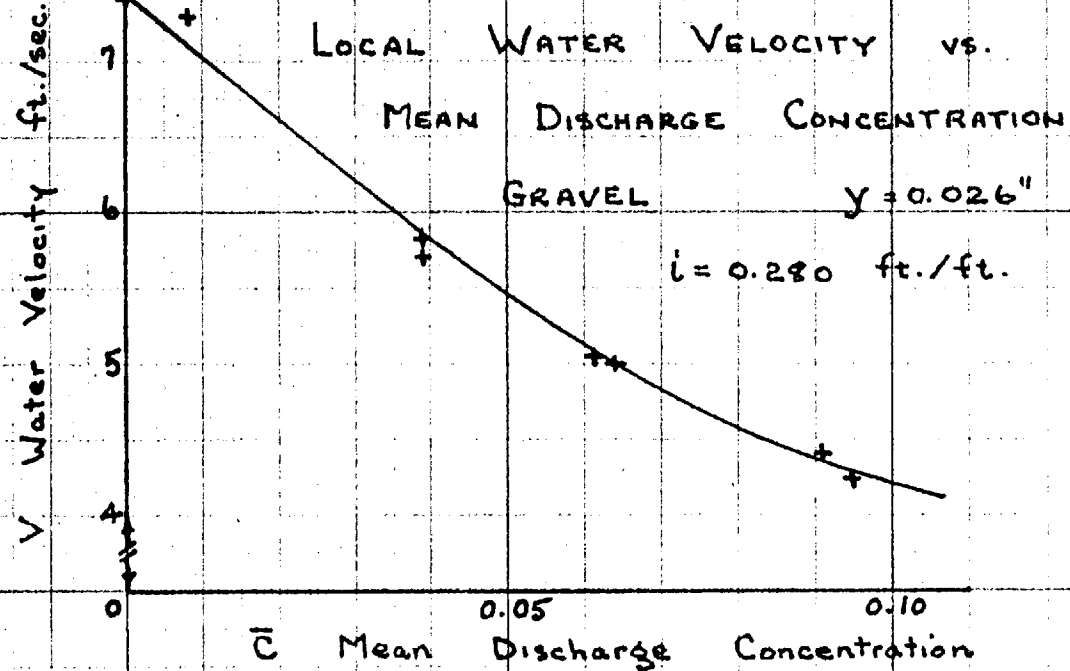


FIG. A VII-17

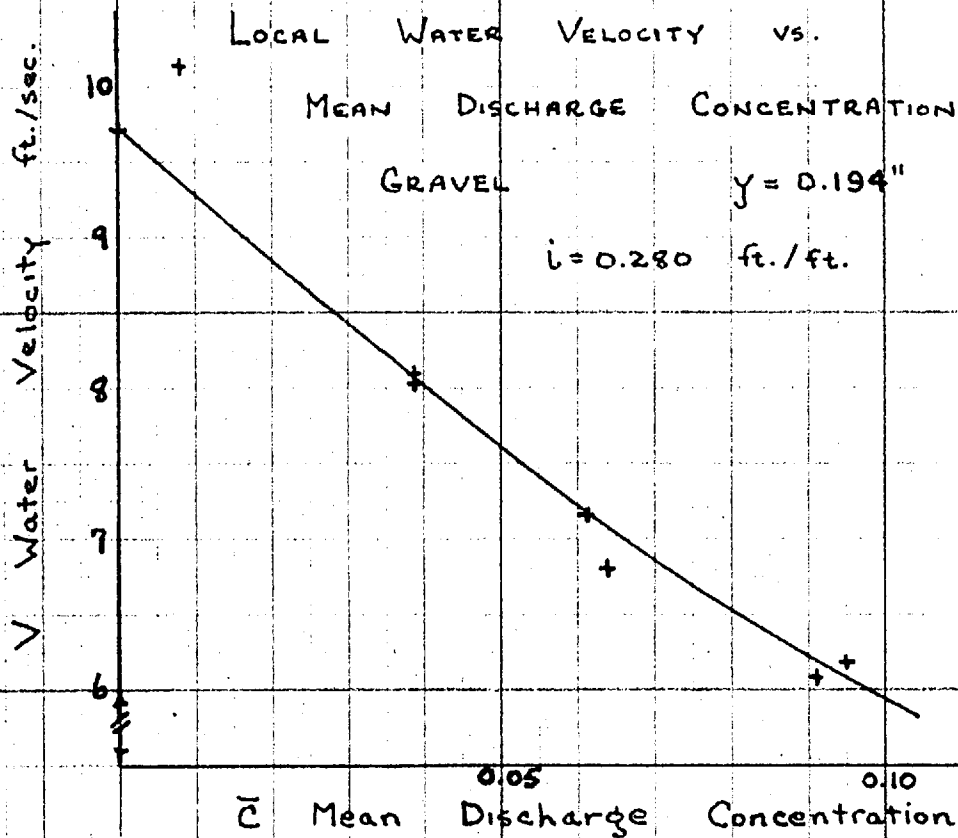
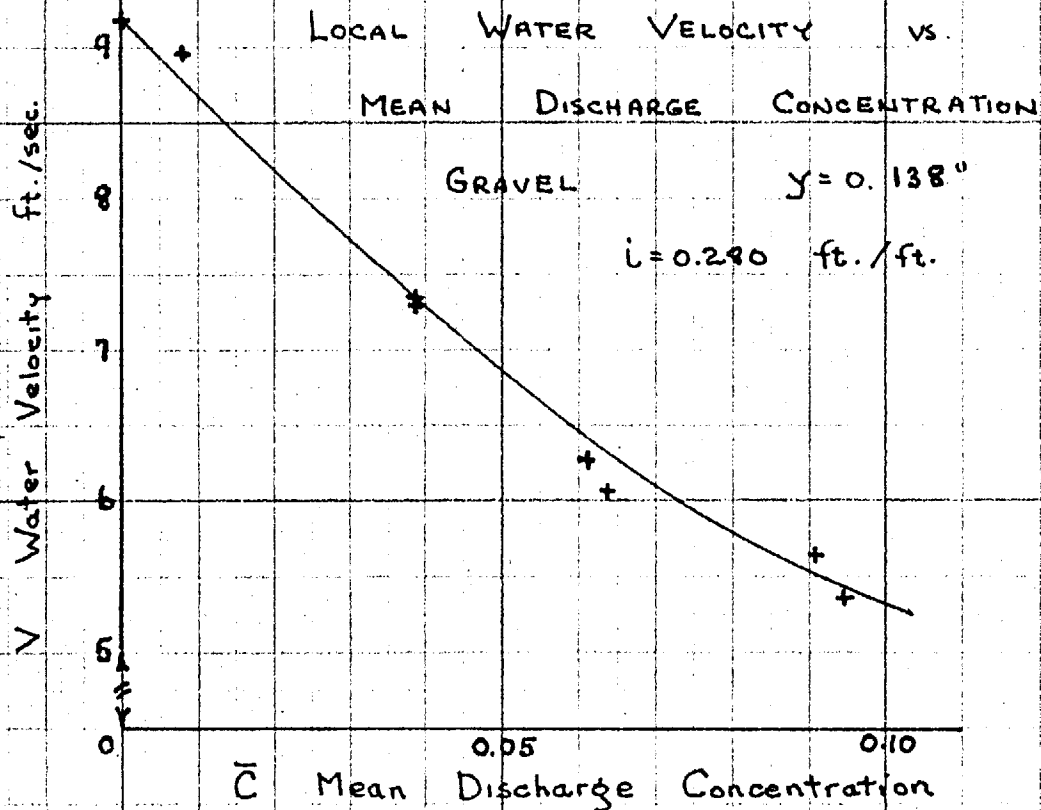


FIG. A VII-16



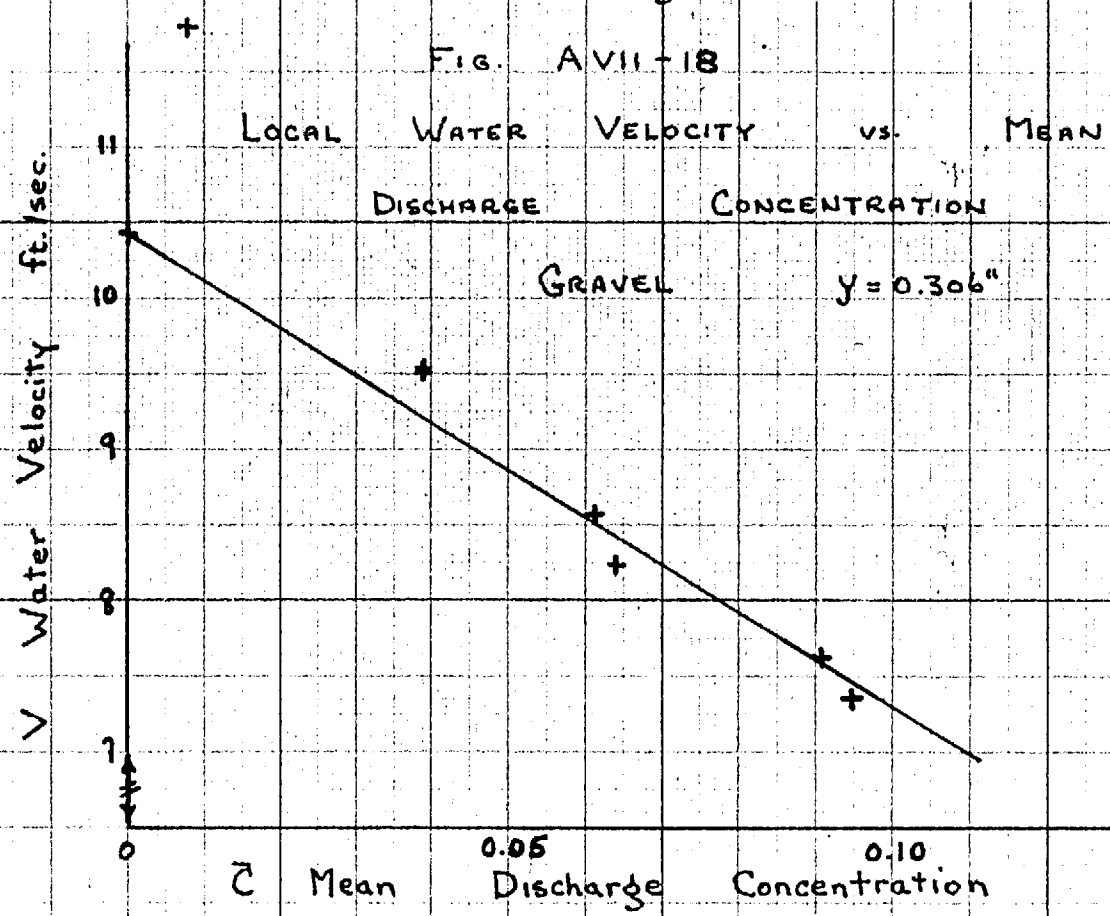
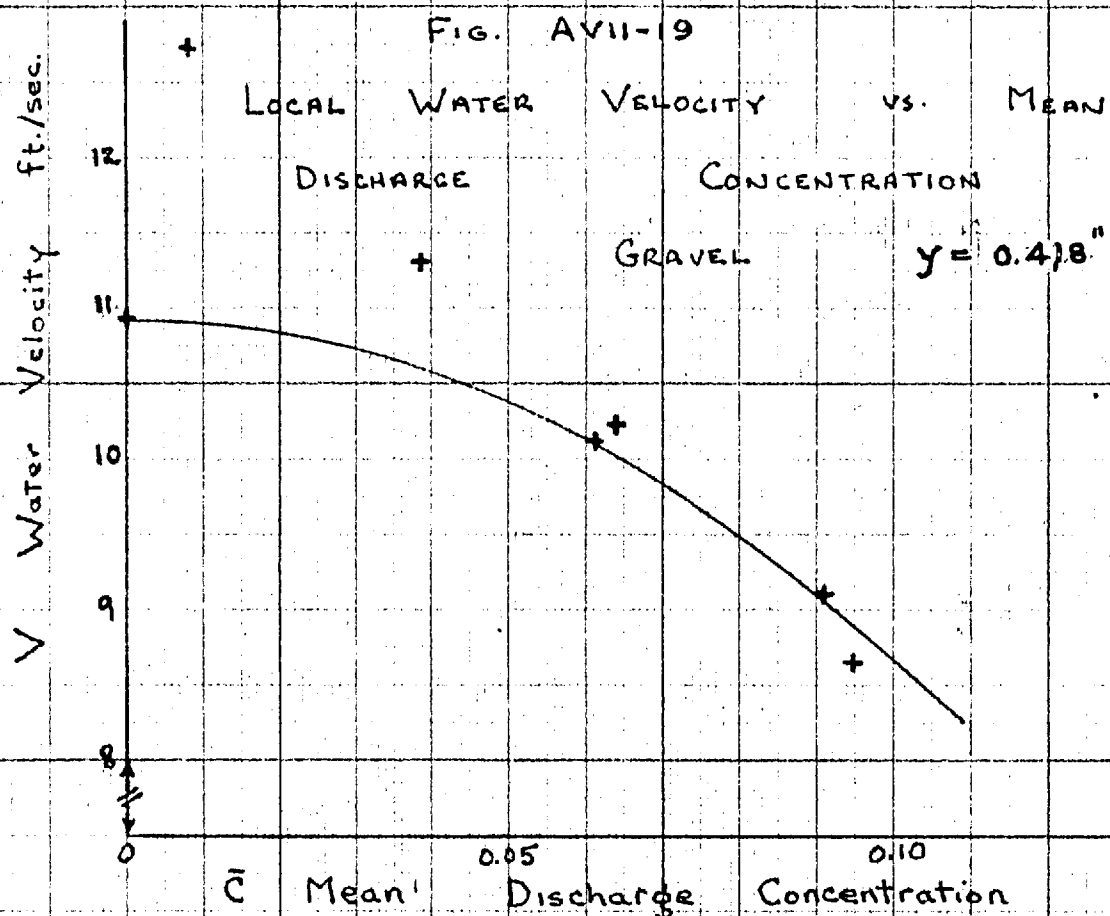


FIG. AVII-21

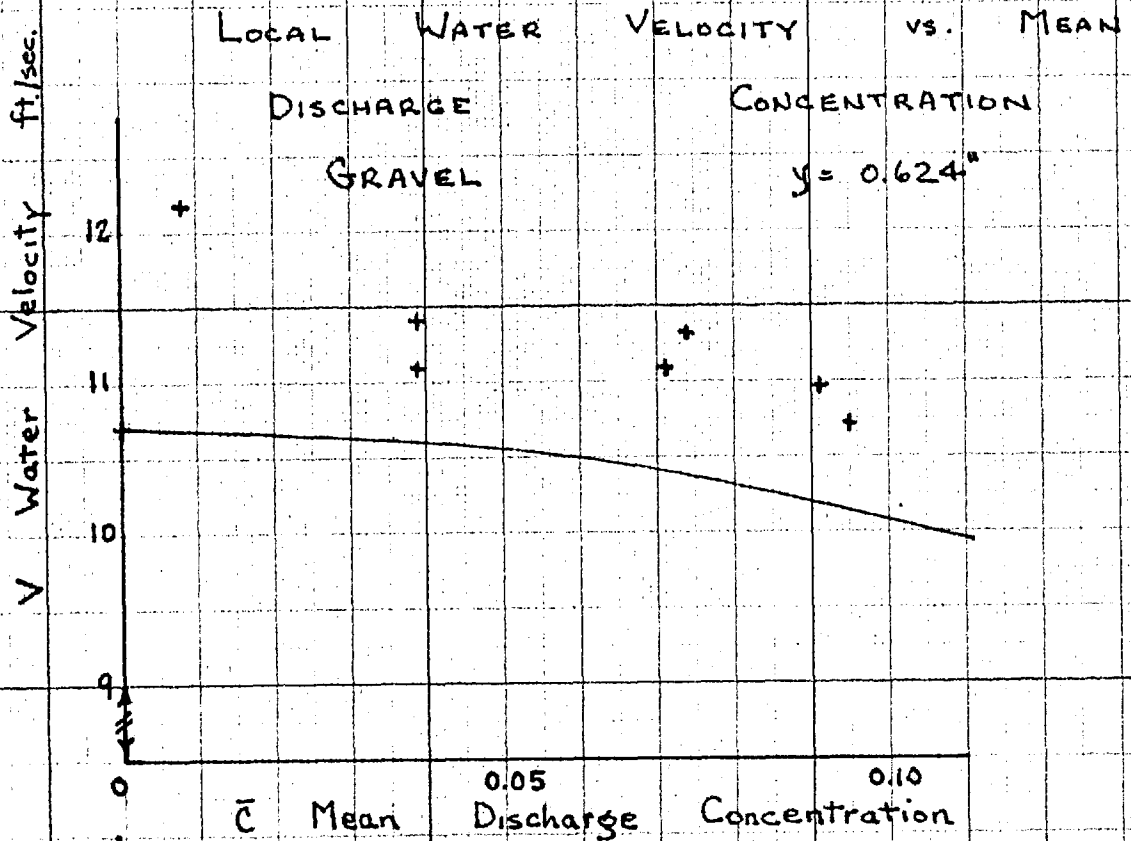


FIG. AVII-20

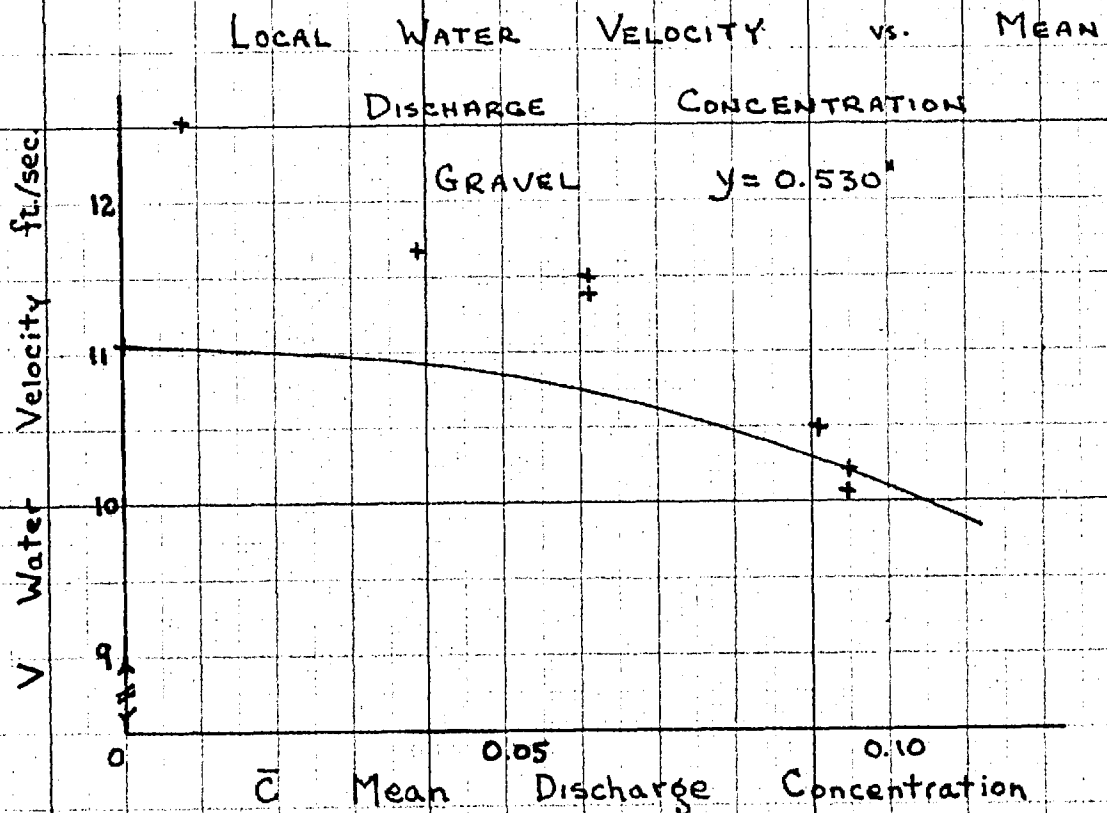


FIG. AVII-23

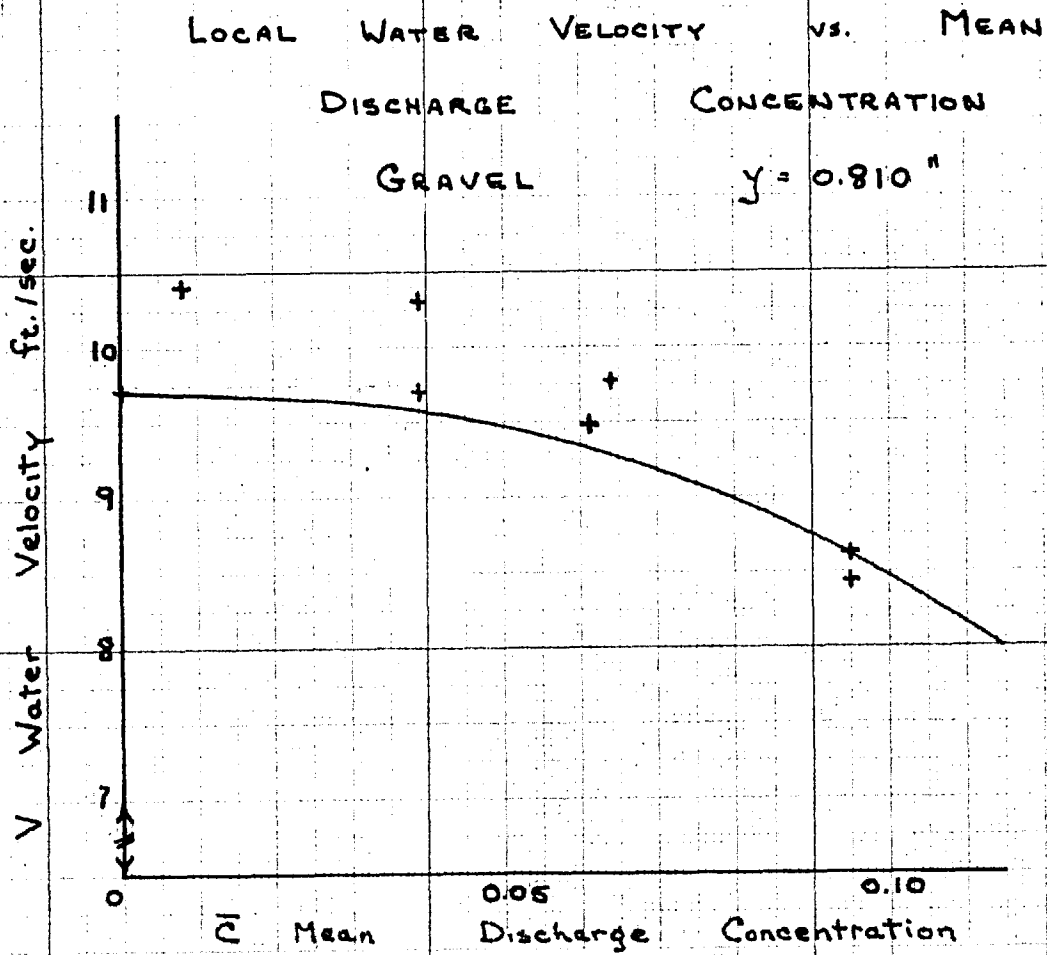


FIG. AVII-22

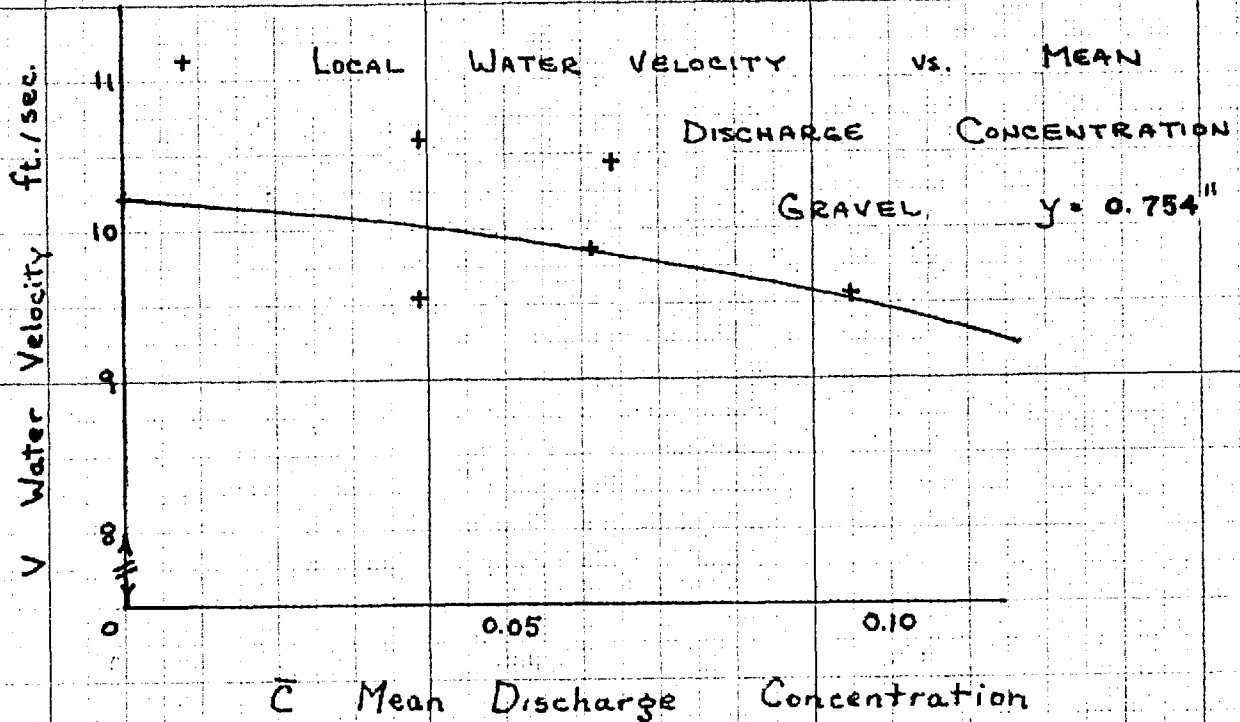


FIG. AVII-26

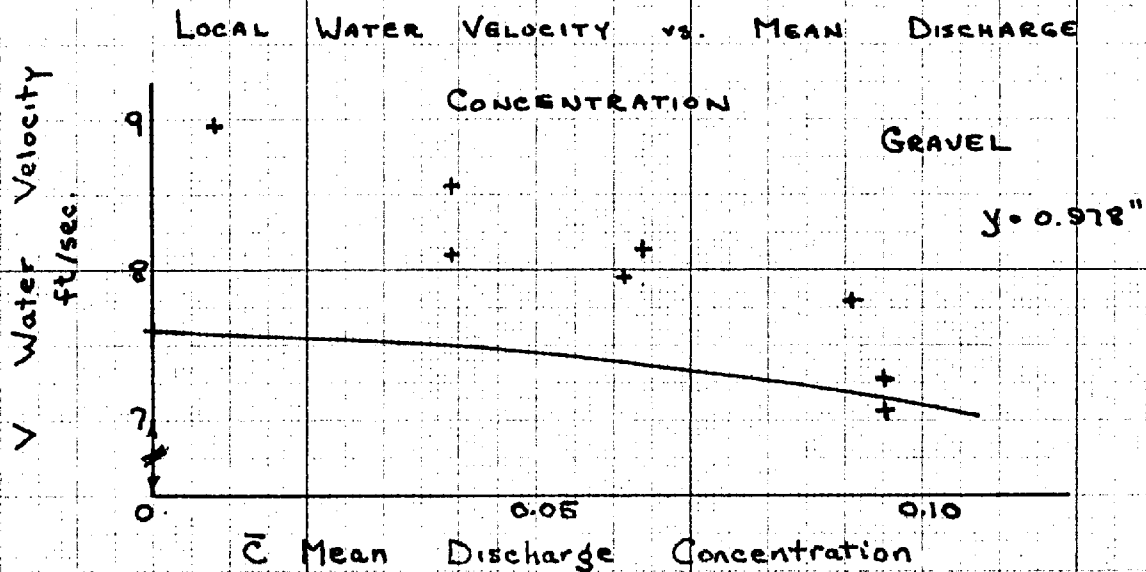


FIG. AVII-25

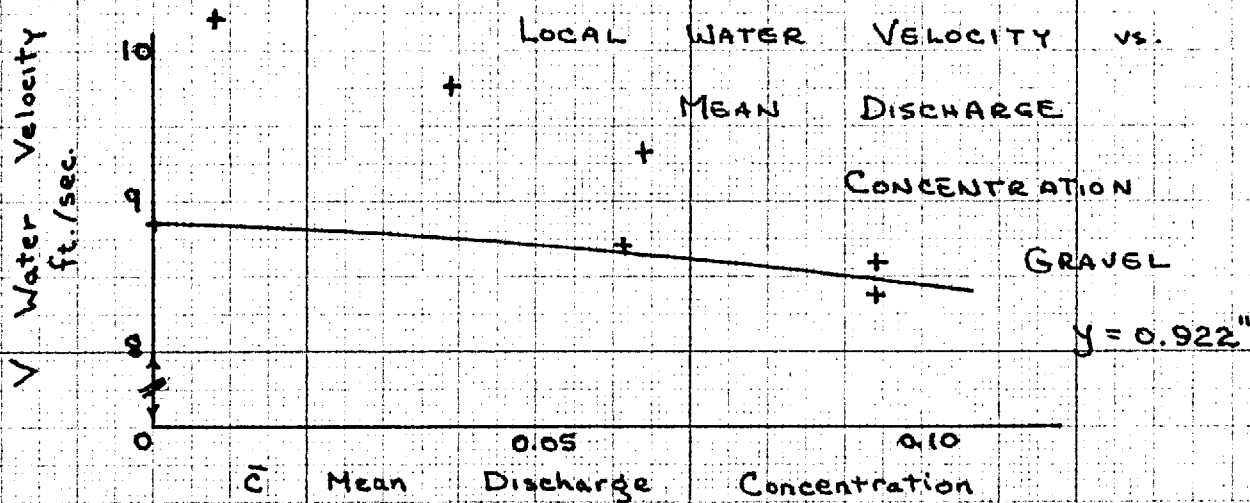


FIG. AVII-24

