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Design of cold-formed steel SHS and RHS beam-columns considering the influence of steel grade

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Abstract:

The structural behaviour and design of cold-formed steel square and rectangular hollow section (SHS and RHS) beam-columns, made of both normal and high strength steels, are investigated in the present study through a comprehensive numerical modelling programme. Refined finite element (FE) models were first established and calibrated against a collection of existing cold-formed steel member test results from the literature. Following this, an extensive parametric study was undertaken to expand the cold-formed steel beam-column data pool, covering a wider range of steel grades, cross-section geometries, member slendernesses and loading combinations. Results from the parametric study were then used to examine the accuracy of the current codified beam-column design methods, provided in the European Standards EN 1993-1-1 (2005) and EN 1993-1-12 (2007) as well as the American Specification AISC 360-16 (2016). The comparisons revealed that the codified design rules provide varying levels of accuracy in predicting the ultimate resistances of cold-formed steel beam-columns depending on the steel grade. An improved design approach, compatible with current codified design rules in EN 1993-1-1 (2005), is proposed that features (1) recently developed column buckling curves that reflect the increasing normalised

column buckling resistance with increasing steel yield strength; (2) bending moment resistances calculated using the Continuous Strength Method (CSM) and (3) new interaction curves for cold-formed steel beam-columns, which are anchored to these more accurate end points. The newly proposed design approach is shown to yield more accurate and consistent resistance predictions over current design provisions for cold-formed steel SHS and RHS beam-columns made of different steel grades, and its reliability has also been statistically verified in accordance with EN 1990.

Keywords: Beam-columns; Continuous strength method; Cold-formed steel; Finite element modelling; High strength steel; Structural steel design; Square and rectangular hollow sections.

1. Introduction

Cold-formed steel square and rectangular hollow section (SHS and RHS) members are becoming increasingly popular in structural applications, mainly due to their excellent structural properties, ease of prefabrication and inherent aesthetic advantages. Advances in metallurgical science and fabrication techniques, such as thermomechanical rolling and quenching and tempering, enable the economic production of cold-formed steel tubular members with yield strengths up to 1100 MPa. It is known that, due to the effects of work-hardening experienced during sheet and section forming, cold-formed steels differ significantly from hot-rolled steels in their stress-strain behaviour, with the former being characterised by a rounded response with no sharply defined yield point [1]. The varying amount of cold-work experienced at different positions around cold-formed steel cross-section profiles also results in non-uniformity of material properties, with strength enhancements in the corner regions being more pronounced than those in the flat regions [2]. Hot-rolled and cold-

formed steel members are, however, often treated the same at the cross-section level in existing design provisions [3-5] on the basis of an elastic, perfectly plastic material model, which is generally suitable for hot-rolled steels but fails to reflect the rounded nature of cold-formed steels. At the member level, existing design provisions [3-5] employ a single column buckling curve for cold-formed SHS and RHS of different steel grades (e.g. buckling curve c in Eurocode 3 [3,4]), and the influence of yield strength on the buckling reduction factor is ignored [6]. The current European standards EN 1993-1-1 [3] and EN 1993-1-12 [4] are applicable to cold-formed steel structural members with steel grades up to S700 (i.e. a nominal 0.2% proof stress of 700 N/mm²), while the current American specification AISC 360 [5] only allows the design of cold-formed steel structural members up to a nominal 0.2% proof stress of 485 N/mm². An investigation into the feasibility of using the current codified design rules for cold-formed high strength steel (HSS) structural members is thus deemed necessary.

With the increasing use of cold-formed steel SHS and RHS members in construction, extensive research has been carried out in recent decades to investigate their structural behaviour under isolated compression [6-10] and bending [11-15]. However, to date, investigations into the structural performance and design of cold-formed steel SHS and RHS members subjected to compression plus bending have been rather limited [8,16]. This prompted a systematic numerical study undertaken by the authors, aimed to devise more accurate and efficient design rules for cold-formed steel SHS and RHS beam-columns of varying steel grades up to S1100. The work presented herein is based upon, and complementary to, the study into the stability design of cold-formed steel SHS and RHS columns conducted by Meng and Gardner [6], where new design provisions were proposed that reflect the influence of yield strength on flexural buckling resistance. In the

present study, finite element (FE) models that are capable of replicating the structural response of cold-formed steel SHS and RHS beam-columns are established and validated against available experimental results collected from the literature [8,16]; this is followed by extensive parametric studies to produce further FE data covering a broad range of steel grades, member slendernesses, cross-section geometries and loading combinations. The codified beam-column design rules set out in the European standards EN 1993-1-1 [3] and EN 1993-1-12 [4], as well as the American specification AISC 360 [5], are subsequently assessed by comparing the resistance predictions with the combined set of experimental and FE data. Finally, an improved design approach, based on the design provisions of Meng and Gardner [6] for SHS and RHS columns and the Continuous Strength Method (CSM) [11,12] for SHS and RHS in bending, is proposed for cold-formed steel SHS and RHS beam-column members with steel grades up to S1100. The proposed method is shown to yield significantly improved accuracy and consistency in the resistance predictions compared with the existing codified design approaches, and its reliability is confirmed by carrying out statistical analyses according to EN 1990 [17].

2. Numerical modelling programme

A comprehensive numerical modelling programme was carried out to generate extensive structural performance data on cold-formed steel SHS and RHS beam-columns; these numerically obtained data were used to assess the accuracy of the codified design provisions set out in [3-5], as well as to underpin the development of the proposed new design method, as described in the following sections. The finite element (FE) models were created using the ABAQUS [18] FE package; the essential modelling assumptions and validation of the FE models are described in this section.

2.1. FE modelling approach

The general FE modelling approach adopted herein was similar to that successfully employed in several previous studies on cold-formed steel structural elements [11,19-21]; the key modelling assumptions are described in this section. The four-noded finite-strain shell element S4R with reduced integration was used for all simulations in the present study. A characteristic (approximately square) element size of $\sim(B+H)/40$, where B and H are the outer breadth and height respectively of the modelled cold-formed steel SHS or RHS, was applied within the flat portions of the members, while using a finer mesh of six elements was used to discretise each corner region; the longitudinal mesh size in the corner regions matched that employed in the flat portions. To simulate the pin-ended boundary conditions typically adopted in eccentric compression tests (i.e. beam-column tests) [8,16], two reference points (RF1 and RF2, as shown in Fig. 1) were defined at the centre of the top and the bottom knife edges, the degrees of freedom (DOFs) of which were kinematically coupled to the DOF of the nodes at the corresponding end section. Suitable boundary conditions (i.e. pinned in-plane about the axis of bending and fixed out-of-plane) were then enforced by restraining the appropriate DOF of the two defined reference points. A typical SHS beam-column model is shown in Fig. 1, where U_1 , U_2 and U_3 represent the translations along the X, Y and Z axes, respectively, and UR_1 , UR_2 and UR_3 represent the rotations about the X, Y and Z axes, respectively.

Both global and local geometric imperfections were explicitly considered in the FE models. The global geometric imperfections were defined in the form of a half-sine wave along the longitudinal direction of the modelled members; for the local imperfections, the lowest elastic local buckling mode shape with an odd number of buckling half-waves [11,23,24], derived from a prior linear

bifurcation analysis (LBA), was taken as the representative shape. Note that the LBA was performed on cold-formed steel members with a modified (thinner) thickness of 1 mm, thus enabling the first eigenmode to be a local buckling-dominant mode, noting that, for plate buckling, the buckling shape would not be expected to alter with thickness. A sensitivity analysis was carried out to identify the influence of different imperfection amplitudes on the ultimate resistances of the cold-formed steel SHS and RHS beam-columns. Three global imperfection amplitudes (namely the measured value from the respective experiment δ_0 , $L_e/1000$ and $L_e/1500$, where L_e is the effective length of the member) and five local imperfection amplitudes (namely the measured value from the respective experiment w_0 if available, and four fractions of the clear cross-section height h , i.e. $h/100$, $h/200$, $h/300$ and $h/400$) were considered, resulting in a total of nine imperfection amplitude combinations, as shown in [Table 1](#). Explicit modelling of residual stresses was deemed unnecessary for the simulated cold-formed tubular members for the reasons explained and verified in previous studies [\[6,8,11,19,26\]](#).

2.2. Validation of FE models

The fidelity of the adopted FE modelling approach, as summarised above, was affirmed by comparing the numerical predictions with the key results of 26 experiments on cold-formed steel SHS and RHS beam-columns from the literature [\[8,16\]](#). The measured stress-strain curves from the coupon tests reported in [\[8,16\]](#) were employed for the validation of the corresponding beam-column test specimens. Note that the measured engineering stress-strain curves were transformed into true stress-logarithmic plastic strain curves before inputting into the shell FE models. The corner strength enhancements of the cold-formed SHS and RHS members, resulting from the plastic deformations during the manufacturing process, were also considered by assigning the

measured corner material properties (featuring an increased yield strength but reduced ductility) to the corner regions plus adjacent flat plate portions extending beyond the corners by a distance of $2t$, where t is the thickness of the modelled cross-section, in view of the recommendations of [22].

Table 1 summarises the comparisons between the ultimate resistances obtained from the physical experiments $N_{u,\text{test}}$ and the FE models $N_{u,\text{FE}}$ considering the different imperfection amplitude combinations, together with the mean values and coefficients of variation (COV) of the ratios of $N_{u,\text{FE}}/N_{u,\text{test}}$. It can be seen in Table 1 that the ultimate resistances of the cold-formed steel SHS and RHS columns were sensitive to the global imperfection amplitude, while the sensitivity tends to be greatly reduced for beam-columns, especially those subjected to high bending moments (i.e. members with large initial loading eccentricity e_0); this is because the lateral deflections induced by bending swamp the initial imperfection. The ultimate resistances were also shown to be relatively insensitive to variation of the local imperfection amplitude, especially for members with non-slender cross-sections (i.e. the experiments conducted by Ma et al. [14]) which are the focus of the present study. Overall, the numerical resistance predictions with the global and local imperfection amplitudes taken as $L_e/1000$ and $h/200$, respectively, are in close agreement with those obtained from the experiments, with a mean value of $N_{u,\text{FE}}/N_{u,\text{test}}$ of 1.00 and a corresponding COV of 0.034. Note that the global imperfection amplitude of $L_e/1000$ is the same as that assumed in the development of the European column buckling curves [3,4] and the adopted local imperfection amplitude of $h/200$ is consistent with that recommended in EN 1993-1-5 [27]; these assumptions have also been successfully implemented in various previous numerical studies [6,10,11,24,28]. The experimental load-mid-height lateral deflection curves were also accurately

reproduced by their FE counterparts with the aforementioned imperfection amplitude combinations, examples of which are shown in [Fig. 2](#). It can therefore be concluded that the FE models described in this section are capable of providing accurate and reliable structural performance data on cold-formed steel SHS and RHS beam-columns.

The successfully validated FE models were used in parametric studies to create additional numerical data on cold-formed steel SHS and RHS columns and beam-columns with varying cross-sectional slendernesses and steel grades. Details on the column parametric studies are presented in Sections 3.1.4, while the beam-column parametric studies are described in Section 4.4.

3. Design of cold-formed steel SHS and RHS under isolated loading conditions

The accuracy in determining the ultimate resistances of cold-formed steel SHS and RHS beam-columns depends greatly on the accuracy to which the column buckling resistances and cross-sectional bending resistances serving as the end points of the design interaction curves for beam-columns, can be predicted. In this section, the codified design methods [\[3-5\]](#) for cold-formed steel SHS and RHS under isolated loading conditions (i.e. compression or bending), as well as recently proposed design methods [\[6,11,12\]](#), which are shown to provide more accurate and reliable ultimate resistance predictions, are introduced.

3.1. Design of cold-formed steel SHS and RHS columns

3.1.1. EN 1993-1-1 [3] and EN 1993-1-12 [4] (EC3)

In Eurocode 3 (EC3) [3,4], Eq. (1) is employed for the calculation of the flexural buckling resistance of cold-formed steel SHS and RHS columns with non-slender cross-sections (i.e. Class 1-3 cross-sections):

$$N_{b,EC3,Rd} = \frac{\chi_{EC3} N_{c,EC3}}{\gamma_{M1}} = \frac{\chi_{EC3} A f_y}{\gamma_{M1}}, \text{ for non-slender SHS and RHS} \quad (1)$$

where $N_{b,EC3,Rd}$ is the EC3 design column buckling resistance, $N_{c,EC3}$ is the EC3 cross-section resistance in compression which is equal to the product of the cross-sectional area A and the yield strength f_y for non-slender cross-sections, χ_{EC3} is the EC3 column buckling reduction factor and γ_{M1} is the partial safety factor for the resistance of members, taken equal to unity. Note that, throughout the present study, f_y is taken as the weighted (by area) average yield strength, allowing for the influence of the corner strength enhancements in the studied cold-formed SHS and RHS, as given in Eq. (2):

$$f_y = \frac{f_{y,f} A_f + f_{y,c} A_c}{A} \quad (2)$$

where $f_{y,f}$ and $f_{y,c}$ correspond to the yield strengths of the flat and corner portions, respectively, and A_f and A_c are the areas of the flat and corner (including the corner extensions of $2t$ as described in Section 2) portions, respectively. Originating from the Ayrton-Perry formula [29], the EC3 column buckling reduction factor χ_{EC3} is calculated from Eqs. (3) to (5):

$$\chi_{EC3} = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, \text{ but } \chi_{EC3} \leq 1.0 \quad (3)$$

$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} \quad (4)$$

$$\Phi = 0.5 \left(1 + \eta_{EC3} + \bar{\lambda}^2 \right) = 0.5 \left[1 + \alpha (\bar{\lambda} - 0.2) + \bar{\lambda}^2 \right] \quad (5)$$

where $\bar{\lambda}$ is the non-dimensional relative member slenderness, N_{cr} is the Euler buckling load for the relevant in-plane flexural buckling mode and η_{EC3} is the EC3 generalised initial imperfection factor equal to $\alpha(\bar{\lambda} - 0.2)$, in which α is an imperfection factor that accounts for the combined influence of the initial geometric imperfections and residual stresses; for cold-formed steel SHS and RHS columns, α is taken as 0.49, corresponding to buckling curve c in EN 1993-1-1 [3].

3.1.2. AISC 360-16 [5] (AISC)

The formula for the calculation of the design flexural buckling resistance $N_{b,AISC,Rd}$ of cold-formed steel columns with non-slender SHS and RHS, as specified in the American Specification AISC 360-16 [5], is given by Eq. (6):

$$N_{b,AISC,Rd} = \phi_c \chi_{AISC} Af_y, \text{ for non-slender SHS and RHS} \quad (6)$$

where χ_{AISC} is the AISC column buckling reduction factor given by Eq. (7), and ϕ_c is the resistance factor (i.e. equivalent to the inverse of the partial safety factor γ_{M1} in Eq. (1)) with a recommended value of 0.9 in [5].

$$\chi_{\text{AISC}} = \begin{cases} 0.658 \bar{\lambda}^2 & \text{for } \bar{\lambda} \leq 1.5 \\ \frac{0.877}{\bar{\lambda}^2} & \text{for } \bar{\lambda} > 1.5 \end{cases} \quad (7)$$

3.1.3. Modified EC3 approach proposed by Meng and Gardner [6]

A modified EC3 approach was developed in [6] for the design of cold-formed steel SHS and RHS columns with steel grades up to S900, adopting the format of Eq. (1) used in the current EC3 [3,4] but with a modified column buckling reduction factor $\chi_{\text{mod-EC3}}$, as given by Eqs. (8) and (9):

$$\chi_{\text{mod-EC3}} = \frac{1}{\Phi_{\text{mod-EC3}} + \sqrt{\Phi_{\text{mod-EC3}}^2 - \bar{\lambda}^2}}, \text{ but } \chi_{\text{mod-EC3}} \leq 1.0 \quad (8)$$

$$\Phi_{\text{mod-EC3}} = 0.5 \left(1 + \eta_{\text{mod-EC3}} + \bar{\lambda}^2 \right) = 0.5 \left[1 + 0.56\varepsilon (\bar{\lambda} - 0.1) + \bar{\lambda}^2 \right] \quad (9)$$

The modified generalised initial imperfection factor $\eta_{\text{mod-EC3}}$ in Eq. (9) includes a modified imperfection factor $\alpha_{\text{mod-EC3}}$ of 0.56ε (where ε is the material factor equal to $\sqrt{235/f_y}$), which is linked to the yield strength of the material, and a shortened plateau length (i.e. from 0.2 in Eq. (5) to 0.1 in Eq. (9)).

3.1.4. Assessment of different design methods

The accuracy and reliability of the codified column buckling design methods [3-5], as well as the modified design approach of Meng and Gardner [6], have been assessed in [6] for cold-formed steel SHS and RHS columns with steel grades up to S900 based on a combined dataset of tests and FE results. It was revealed that the proposed modified EC3 approach is able to provide more consistent and reliable buckling resistance predictions for cold-formed steel SHS and RHS columns compared to the current codified design methods [3-5]. This is primarily due to the fact that the trend of the reducing relative influence of imperfections (both the global imperfections and residual stresses) on the column buckling resistance with increasing steel grades, which is not well reflected in existing design provisions, is appropriately captured by the modified imperfection factor $\alpha_{\text{mod-EC3}}$. In this section, the applicability of the modified approach [6] to the design of cold-formed S1100 steel SHS and RHS columns is appraised based on the freshly generated numerical data, as described below.

The validated FE models, as described in Section 2, are employed in this section to perform a parametric study to acquire additional data on cold-formed steel SHS and RHS columns with varying cross-section geometries and relative slendernesses. Columns in S1100 steel are the focus of the parametric study, but the lower steel grades of S355, S460, S700 and S900 are also considered, providing further verification of the accuracy and consistency of the modified EC3 design approach. In the parametric study, the full stress-strain curves for the S355 and S460 steel grades were generated using the predictive model proposed by Gardner and Yun [1] while, considering that only a limited number of experimental stress-strain curves on cold-formed HSS were available to underpin the developed material model in [1], the measured stress-strain curves

for S700, S900 and S1100 steel reported in [26] were used instead. The adopted full stress-strain curves for the five investigated steel grades are displayed in Fig. 3, while the corresponding key material properties, including the Young's modulus E , the yield strength f_y , the ultimate strength f_u and the ultimate strain ε_u , are given in Table 2, where the suffixes "f" and "c" denote the flat and corner portions of the cross-sections, respectively. Two non-slender cross-sections – $150 \times 100 \times 7$ and $200 \times 100 \times 10$ ($H \times B \times t$, in mm) were analysed in the parametric study. Note that throughout the numerical simulation programme, the inner corner radius r_i of each investigated cross-section was set equal to the corresponding cross-section thickness t . For each cross-section and for each axis of buckling, the column lengths were varied from 400 mm to 12000 mm, resulting in a spectrum of relative slenderness values $\bar{\lambda}$ ranging from 0.2 to 2.6. Overall, a total of 1130 cold-formed steel SHS and RHS columns was simulated in the parametric study.

The column buckling curves from the codified design provisions [3-5] and the modified EC3 approach [6] are plotted in Fig. 4, along with the numerically generated data (i.e. the numerical column buckling resistance $N_{b,FE}$ normalised by the yield load Af_y) on cold-formed S1100 steel SHS and RHS columns. It can be seen that, relative to the codified column buckling curves, the modified EC3 column buckling curve generally provides more accurate resistance predictions for the S1100 columns, though are slightly conservative for columns with $\bar{\lambda} \leq 0.8$. The improved accuracy can also be seen in Figs. 5(a)-5(c), where the numerically derived buckling resistances $N_{b,FE}$ for the S1100 columns are normalised by the unfactored resistance predictions from the EC3, AISC 360 and modified EC3 design approaches ($N_{b,EC3}$, $N_{b,AISC}$ and $N_{b,mod-EC3}$), respectively, and plotted against the relative slenderness $\bar{\lambda}$. Quantitative comparisons of the different design approaches for columns in grade S1100 steel are also given in Table 3, showing that the mean

value of the ratio of $N_{b,FE}/N_{b,mod-EC3}$ is closer to unity than that resulting from the current EC3 (i.e. $N_{b,FE}/N_{b,EC3}$) and the COV value for the modified EC3 is maintained similar to that for the current EC3. Therefore, it can be concluded that the modified EC3 approach [6] provides improved accuracy relative to the current codified design approaches, and its applicability can be extended to cold-formed S1100 steel SHS and RHS columns. The numerically derived results for columns with the lower steel grades (i.e. S355, S460, S700 and S900), combined with the existing tests results and numerical data obtained in [6], are also plotted in Fig. 5 and statistically assessed in Table 3, confirming the findings in [6] that the modified EC3 approach yields improved accuracy and consistency over the existing design provisions for the design of cold-formed steel SHS and RHS columns, owing to its rational consideration of the influence of the yield strength on the column buckling reduction factor. The modified EC3 approach [6] is therefore used in place of the current EC3 approach to determine the compression end points in the new beam-column design proposals.

3.2. Design of cold-formed steel SHS and RHS in bending

In both EC3 [3,4] and AISC 360 [5], the nominal (unfactored) bending resistances for Class 1 and 2 (compact) box-shaped sections are limited to the plastic bending moment capacity $M_{pl} = W_{pl}f_y$, where W_{pl} is the plastic section modulus about the axis of bending. With regards to Class 3 (semi-compact) box-shaped sections, the current version of EC3 [3,4] prescribes the use of the elastic bending moment capacity $M_{el} = W_{el}f_y$, where W_{el} is the elastic section modulus about the axis of bending, ignoring the influence of the partial spread of plasticity; this latter effect is considered in AISC 360 [5] and the upcoming revision to EC3 [30], resulting in bending resistances for Class 3 cross-sections lying between M_{el} and M_{pl} . The existing codified methods for calculating cross-

sectional resistances in bending are based on the assumption that the material obeys a linear elastic, perfectly plastic stress-strain law, which is at odds with the rounded material response of cold-formed steel SHS and RHS that exhibits gradual yielding merging into strain hardening [1]. The codified methods thus ignore the beneficial influence of strain hardening, which has been shown to result in under-predictions of the bending resistances for cold-formed steel SHS and RHS, especially those within the Class 1 domain [11-14].

To address the above-mentioned shortcomings and, more widely, to facilitate a more rational system-level advanced design approach [31-33], the deformation-based Continuous Strength Method (CSM) has been developed. Provisions for the design of cold-formed steel SHS and RHS in bending were established in [11], while extension to HSS tubular sections with steel grades up to S1100 was presented in [12]. The CSM design cross-sectional resistance for cold-formed steel non-slender SHS and RHS in bending $M_{\text{csm,Rd}}$ is given by Eq. (10):

$$M_{\text{csm,Rd}} = \frac{W_{\text{pl}} f_y}{\gamma_{\text{M0}}} \left[1 + \frac{E_{\text{sh}}}{E} \frac{W_{\text{el}}}{W_{\text{pl}}} \left(\frac{\varepsilon_{\text{csm}}}{\varepsilon_y} - 1 \right) - \left(1 - \frac{W_{\text{el}}}{W_{\text{pl}}} \right) \right] / \left(\frac{\varepsilon_{\text{csm}}}{\varepsilon_y} \right)^2, \text{ for } \bar{\lambda}_p \leq 0.68 \quad (10)$$

in which γ_{M0} is the partial safety factor for cross-section resistances with a recommended value of unity, E_{sh} is the strain hardening slope associated with the adopted CSM bi-linear material model [11], and ε_{csm} is the CSM strain limit, which represents the maximum strain that a cross-section can endure prior to failure. Note that the existing slenderness limit of $\bar{\lambda}_p = 0.68$ has been shown in several existing studies [12–14] to be suitable for the design of cold-formed high strength steel

SHS and RHS in bending. The CSM strain ε_{csm} can be determined from the CSM base curve [34,35], as expressed by Eq. (11):

$$\frac{\varepsilon_{\text{csm}}}{\varepsilon_y} = \frac{0.25}{\bar{\lambda}_p^{3.6}}, \text{ but } \frac{\varepsilon_{\text{csm}}}{\varepsilon_y} \leq \min\left(15, \frac{0.4\varepsilon_u}{\varepsilon_y}\right) \quad (11)$$

in which $\bar{\lambda}_p$ is the cross-section slenderness defined by Eq. (12).

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{\text{cr,cs}}}} \quad (12)$$

In Eq. (12), $\sigma_{\text{cr,cs}}$ is the elastic local buckling stress of the full cross-section, which can be numerically determined e.g. through the finite strip software CUFSM [36], or alternatively through simplified analytical expressions e.g. those proposed by Gardner et al. [37]. Note that Lan et al. [12] proposed a modified base curve for determining ε_{csm} for HSS SHS and RHS subjected to bending; however, the deviation from the original base curve of Eq. (11) is small for non-slender cross-sections with $\bar{\lambda}_p \leq 0.68$. The CSM has been consistently shown to provide more accurate and consistent bending resistance predictions than traditional design methods, as demonstrated for the case of cold-formed steel non-slender SHS and RHS in [11,12], and is hence adopted in the new beam-column design proposal for the determination of the bending end points.

4. Design of cold-formed steel SHS and RHS beam-columns

The in-plane stability design of beam-columns under combined axial load plus uniaxial bending is investigated in this section, with a focus on cold-formed SHS and RHS members with Class 1 and 2 cross-sections. The design provisions set out in EC3 [3,4] and AISC 360 [5] are firstly briefly introduced, followed by the development of a new proposal for the design of cold-formed steel SHS and RHS beam-columns, including due allowance for the influence of steel grade. The new design proposal features the utilisation of more accurate end points (i.e. the column buckling resistances and cross-sectional bending resistances) calculated by the recently developed methods as discussed above, and newly derived interaction factors for the axial-moment interaction curve; the concept of the design proposals is anticipated to be applicable to beam-column members with other forms of cross-sections or with a Class 3 or Class 4 cross-sections, though further investigation into this topic is deemed necessary.

4.1. EN 1993-1-1 [3] and EN 1993-1-12 [4] (EC3)

The EN 1993-1-1 [3] and EN 1993-1-12 [4] formulae for the design of cold-formed steel SHS or RHS beam-columns with Class 1 or Class 2 cross-sections are given by Eqs. (13) and (14) for the cases of axial compression plus major and minor axis bending, respectively.

$$\frac{N_{Ed}}{\chi_{EC3,y} A f_y / \gamma_{M1}} + k_{yy} \frac{M_{Ed,y}}{W_{pl,y} f_y / \gamma_{M1}} \leq 1 \quad (13)$$

$$\frac{N_{Ed}}{\chi_{EC3,z} A f_y / \gamma_{M1}} + k_{zz} \frac{M_{Ed,z}}{W_{pl,z} f_y / \gamma_{M1}} \leq 1 \quad (14)$$

In Eqs. (12) and (13), N_{Ed} is the design axial force, M_{Ed} is the design first-order bending moment and k_{yy} and k_{zz} are the interaction factors with respect to the major and minor axis, respectively, as determined from Eqs. (15) and (16), where C_m is the equivalent uniform moment factor which is taken equal to unity for the studied loading case of uniform bending. The EC3 column buckling reduction factor χ_{EC3} is determined using the traditional Ayrton-Perry type formula [29], as presented in Subsection 3.1.1, where a single buckling curve (i.e. curve c in EC3 [3]) is adopted for the design of cold-formed steel SHS and RHS columns with steel grades up to S700. Note that the subscripts “y” and “z” used in the symbols of M_{Ed} , W_{el} , W_{pl} , χ_{EC3} , χ_{AISC} , $\chi_{mod-EC3}$, C_m and $\bar{\lambda}$ denote bending/buckling about the major and minor axes of the cross-section, respectively.

$$k_{yy} = C_{my} \left[1 + (\bar{\lambda}_y - 0.2) \frac{N_{Ed}}{\chi_{EC3,y} A f_y / \gamma_{M1}} \right] \leq C_{my} \left(1 + 0.8 \frac{N_{Ed}}{\chi_{EC3,y} A f_y / \gamma_{M1}} \right) \quad (15)$$

$$k_{zz} = C_{mz} \left[1 + (\bar{\lambda}_z - 0.2) \frac{N_{Ed}}{\chi_{EC3,z} A f_y / \gamma_{M1}} \right] \leq C_{mz} \left(1 + 0.8 \frac{N_{Ed}}{\chi_{EC3,z} A f_y / \gamma_{M1}} \right) \quad (16)$$

4.2. AISC 360-16 [5] (AISC)

AISC 360 [5] adopts a bilinear interaction curve for the design of compact tubular beam-column members, as given by Eqs. (17) and (18) for the cases of axial compression plus major and minor axis bending, respectively.

$$\begin{cases} \frac{N_{Ed}}{\phi_c \chi_{AISC,y} A f_y} + \frac{8 \alpha_{amp} M_{Ed,y}}{9 \phi_b W_{pl,y} f_y} \leq 1, \text{ for } \frac{N_{Ed}}{\phi_c \chi_{AISC,y} A f_y} \geq 0.2 \\ \frac{1}{2} \frac{N_{Ed}}{\phi_c \chi_{AISC,y} A f_y} + \frac{\alpha_{amp} M_{Ed,y}}{\phi_b W_{pl,y} f_y} \leq 1, \text{ for } \frac{N_{Ed}}{\phi_c \chi_{AISC,y} A f_y} < 0.2 \end{cases} \quad (17)$$

$$\begin{cases} \frac{N_{Ed}}{\phi_c \chi_{AISC,z} A f_y} + \frac{8 \alpha_{amp} M_{Ed,z}}{9 \phi_b W_{pl,z} f_y} \leq 1, \text{ for } \frac{N_{Ed}}{\phi_c \chi_{AISC,z} A f_y} \geq 0.2 \\ \frac{1}{2} \frac{N_{Ed}}{\phi_c \chi_{AISC,z} A f_y} + \frac{\alpha_{amp} M_{Ed,z}}{\phi_b W_{pl,z} f_y} \leq 1, \text{ for } \frac{N_{Ed}}{\phi_c \chi_{AISC,z} A f_y} < 0.2 \end{cases} \quad (18)$$

Note that the notation used in Eqs. (17) and (18) has been adjusted in accordance with the design formulae provided in EC3 (i.e. Eqs. (13) and (14)) [3,4], facilitating the direct comparison of the two codified design approaches. In Eqs. (17) and (18), χ_{AISC} is the AISC column buckling reduction factor as introduced in Subsection 3.1.2, α_{amp} is the amplification factor that accounts for second-order effects, as determined from:

$$\alpha_{amp} = \frac{1}{1 - \frac{N_{Ed}}{N_{cr}}} \quad (19)$$

and ϕ_b and ϕ_c are resistance factors (i.e. equivalent to the inverse of the partial safety factor in EC3 [3,4]) for compression and flexure, respectively, both of which are taken equal to 0.9 [5].

4.3. New design proposal

The proposed design formulae for cold-formed steel SHS and RHS beam-columns with Class 1 or Class 2 cross-sections follow the general format used in EC3 [3,4], but with modified end points,

as expressed by Eqs. (20) and (21) for the cases of axial compression plus major and minor axis bending, respectively.

$$\frac{N_{Ed}}{\chi_{\text{mod-EC3,y}} A f_y / \gamma_{M1}} + k_{yy,\text{mod}} \frac{M_{Ed,y}}{W_{\text{csm,y}} f_y / \gamma_{M1}} \leq 1 \quad (20)$$

$$\frac{N_{Ed}}{\chi_{\text{mod-EC3,z}} A f_y / \gamma_{M1}} + k_{zz,\text{mod}} \frac{M_{Ed,z}}{W_{\text{csm,z}} f_y / \gamma_{M1}} \leq 1 \quad (21)$$

In Eqs. (20) and (21), $M_{\text{csm,y}}$ and $M_{\text{csm,z}}$ are the unfactored CSM cross-sectional bending resistances about the major and minor axes, respectively, as determined from Eq. (10), and $k_{yy,\text{mod}}$ and $k_{zz,\text{mod}}$ are the newly derived interaction factors for the major and minor axes, respectively. The design proposal features the use of the modified EC3 proposals [6] and the CSM [11] for the determination of more accurate column buckling and cross-sectional bending resistances, respectively, which act as the end anchor points of the axial load-moment interaction curve, and the development of modified interaction factors (i.e. $k_{yy,\text{mod}}$ and $k_{zz,\text{mod}}$) linked to the yield strength of the material, thus capturing its influence on the shape of the interaction curve. To be compatible with the current EC3 approach [3,4], the equations for the modified interaction factors adopt a similar bilinear form, as given by Eqs. (22) and (23) for $k_{yy,\text{mod}}$ and $k_{zz,\text{mod}}$, respectively:

$$k_{yy,\text{mod}} = C_{my} \left[1 + D_{1,y} \left(\bar{\lambda}_y - D_{2,y} \right) n_{\text{mod},y} \right] \leq C_{my} \min \left(1 + D_{1,y} n_{\text{mod},y}, D_{3,y} \right) \quad (22)$$

$$k_{zz,\text{mod}} = C_{mz} \left[1 + D_{1,z} \left(\bar{\lambda}_z - D_{2,z} \right) n_{\text{mod},z} \right] \leq C_{mz} \min \left(1 + D_{1,z} n_{\text{mod},z}, D_{3,z} \right) \quad (23)$$

where $n_{\text{mod},y}$ and $n_{\text{mod},z}$ are the axial compressive load levels considering buckling about the major axis and minor axis, respectively, as given by Eqs. (24) and (25):

$$n_{\text{mod},y} = \frac{N_{\text{Ed}}}{\chi_{\text{mod-EC3},y} A f_y / \gamma_{\text{M1}}} \quad (24)$$

$$n_{\text{mod},z} = \frac{N_{\text{Ed}}}{\chi_{\text{mod-EC3},z} A f_y / \gamma_{\text{M1}}} \quad (25)$$

and $D_{1,y}$, $D_{1,z}$, $D_{2,y}$, $D_{2,z}$, $D_{3,y}$ and $D_{3,z}$ are yield strength-dependent auxiliary coefficients which are derived by means of calibration against the obtained FE data, as described in the following paragraph.

The two cross-section sizes (RHS $150 \times 100 \times 7$ and RHS $200 \times 100 \times 10$) and five steel grades (S355, S460, S700, S900 and S1100) employed in the column parametric study, as described in Subsection 3.1.4, were also selected in the present numerical study of cold-formed steel beam-columns. For each axis of buckling, the considered relative slenderness $\bar{\lambda}$ of the beam-columns (determined from Eq. (4)) varied from 0.2 to 2.5, and the considered axial compressive load levels (i.e. $n_{\text{mod},y}$ and $n_{\text{mod},z}$) ranged from 0.2 to 0.8, which was achieved by varying the initial loading eccentricities e_0 from 5 mm to 2000 mm. For each relative member slenderness $\bar{\lambda}$ and axial compressive load level (i.e. $n_{\text{mod},y}$ or $n_{\text{mod},z}$), the required modified interaction factors $k_{yy,\text{FE}}$ and $k_{zz,\text{FE}}$ for each FE model can be determined by rearranging the interaction formulae given by Eqs. (20) and (21), respectively, resulting in the following expressions:

$$k_{yy,FE} = \left(1 - \frac{N_{Ed}}{\chi_{mod-EC3,y} A f_y / \gamma_{M1}} \right) / \left(\frac{M_{Ed,y}}{M_{csm,y} / \gamma_{M1}} \right) \quad (26)$$

$$k_{zz,FE} = \left(1 - \frac{N_{Ed}}{\chi_{mod-EC3,z} A f_y / \gamma_{M1}} \right) / \left(\frac{M_{Ed,z}}{M_{csm,z} / \gamma_{M1}} \right) \quad (27)$$

where N_{Ed} is taken as the ultimate load $N_{u,FE}$ obtained from the FE model and M_{Ed} is the ultimate first order bending moment equal to $N_{u,FE} e_0$. The proposed formulae for $k_{yy,mod}$ and $k_{zz,mod}$, as given by Eqs. (22) and (23), were then fitted to the upper (i.e. safe-sided) bound of the FE data points as shown in Figs. 6-10, where the required modified interaction factors $k_{yy,FE}$ and $k_{zz,FE}$ derived from the FE results are plotted against the relative member slenderness $\bar{\lambda}$ for four different axial compressive load levels (i.e. 0.2, 0.4, 0.6 and 0.8), together with the corresponding proposed curves for the modified interaction factors $k_{yy,mod}$ and $k_{zz,mod}$. The determined auxiliary coefficients are given in Eqs. (28)-(30), and are a function of the material factor $\varepsilon = \sqrt{235/f_y}$.

$$D_{1,y} = D_{1,z} = 2.6\varepsilon \quad (28)$$

$$D_{2,y} = D_{2,z} = 0.9(1 - \varepsilon) \quad (29)$$

$$D_{3,y} = D_{3,z} = 1.6 + \varepsilon \quad (30)$$

As indicated in Figs. 6-10, the proposed curves for $k_{yy,mod}$ and $k_{zz,mod}$ follow the general trend of the back-calculated results from the FE study (i.e. $k_{yy,FE}$ and $k_{zz,FE}$) and provide a suitable upper bound. Note that the predicted interaction factors are less accurate for beam-columns of high slenderness $\bar{\lambda}$ under high axial load levels, but the interaction factors have only a marginal influence on the accuracy of the resistance predictions in these instances since the axial load term in Eqs. (20) and (21) is dominant [38].

4.4. Assessment of different design methods

The accuracy of the existing codified methods [3-5], as well as the new proposed method described in Subsection 4.3, for the in-plane stability design of cold-formed steel SHS and RHS beam-columns is assessed against a large number of numerical data generated through an extensive parametric study. In this parametric study, three outer cross-section dimensions with different cross-section aspect ratios – 100×100 , 150×100 and 200×100 ($H \times B \times t$, in mm) were considered, with their thicknesses varied within the Class 1 and 2 ranges according to the slenderness limits in EC3 [3,4]. This led to a range of cross-section slenderness values $\bar{\lambda}_p$ (in compression) between 0.20 and 0.62 being investigated. The member lengths of the beam-columns were varied to generate a spectrum of relative global slenderness $\bar{\lambda}$ values from 0.2 to 2.5 for each buckling axis of the modelled SHS and RHS beam-columns. Finally, while various initial loading eccentricities e_0 were considered, resulting in a wide range of axial compressive load levels from 0.1 to 0.9. Overall, a total of 15,000 cold-formed steel SHS and RHS beam-columns was simulated in the parametric study, with 3000 numerical results for each of the five investigated steel grades (i.e. S355, S460, S700, S900 and S1100).

The ultimate resistances $N_{u,FE}$ obtained from the FE models are normalised by the predicted resistances using the different design methods and plotted against the radial angle θ in Figs. 11-15 for beam-columns in different steel grades. The radial angle θ is defined by Eq. (31):

$$\theta = \tan^{-1} \left(\frac{N_{Ed}/N_{b,R}}{M_{Ed}/M_{c,R}} \right) = \tan^{-1} \left(\frac{M_{c,R}}{e_0 N_{b,R}} \right) \quad (31)$$

where $N_{b,R}$ and $M_{c,R}$ are the unfactored column buckling and cross-sectional bending resistances, respectively, determined using the considered design method, as summarised in Section 3. The radial angle θ is an indicator of the ratio of axial compression to bending moment applied to a beam-column specimen, with $\theta = 0^\circ$ and 90° representing members subjected to pure bending and pure compression, respectively, and θ between 0° and 90° corresponding to a combination of axial compression and bending; the axial compression becomes more dominant with increasing values of θ . In Figs. 11-15, $N_{u,EC3}$, $N_{u,AISC}$ and $N_{u,prop}$ denote the ultimate resistances of beam-columns determined from the EC3 [3,4], AISC 360 [5] and proposed design rules, respectively. Note that all partial safety factors were set equal to unity to facilitate a direct comparison among the different design methods.

As shown in Figs. 11-15, overall, the current EC3 design approach leads to conservative resistance predictions for cold-formed steel SHS and RHS beam-columns, but there are a large number of results lying on the unsafe side for beam-column specimens with S355 and S460 steel grades. The AISC 360 design approach yields somewhat scattered ultimate resistance predictions for beam-columns of all investigated steel grades, with the predictions generally lying on the unsafe side for the S355 and S460 steel beam-columns. The proposed approach results in significantly improved

accuracy and reduced scatter in the ultimate resistance predictions, as also revealed by the quantitative results presented in Table 4. Overall, as shown in Figs. 11-15 and Table 4, the proposed design approach provides more accurate and generally safe-sided ultimate resistance predictions for cold-formed steel SHS and RHS beam-columns with different steel grades, with the COV values for the proposed approach being generally one half those of existing design methods.

5. Reliability analysis

Statistical analyses were conducted in compliance with EN 1990 [17] to quantitatively assess the reliability of the proposed design approach. The variability parameters (i.e. the ratio between mean and nominal values X_m / X_n , where X represents the basic variable, and the corresponding COV) of the basic material and geometric properties, were taken in accordance with Annex E of prEN 1993-1-1 [30] and are summarised in Table 5. The COV of the cross-sectional area A , as denoted by V_A , was determined based on the variability parameters of the basic cross-sectional dimensions H , B and t following the procedure described in [39]. As a result, an average value of V_A equal to 0.026 was adopted for the studied SHS and RHS.

The beam-column design function can be expressed in the form of Eq. (32), where C_0 is a constant, and C_1 , C_2 and C_3 are the participation coefficients associated with the variables f_y , A and E respectively, which can be determined following the approach detailed in [39]. The combined COV of the material and geometric basic parameters V_{rt} can be then determined using Eq. (33), where V_{f_y} and V_E are the COV of the yield strength f_y and Young's modulus E respectively, as given in Table 5.

550

$$N_{b,R} = C_0 f_y^{C_1} A^{C_2} E^{C_3} \quad (32)$$

552

$$V_r = \sqrt{(C_1 V_{fy})^2 + (C_2 V_A)^2 + (C_3 V_E)^2} \quad (33)$$

554

555 According to the recommendation of [28], the correction factor b was calculated using Eq. (34)
 556 instead of the codified least-squares method to avoid biasing the resulting values towards the data
 557 with higher ultimate loads. In Eq. (34), n is the number of data points, and $r_{e,i}$ and $r_{t,i}$ are the
 558 numerical resistance and theoretical (i.e. predicted) resistance respectively. Other key statistical
 559 parameters, including the design fractile factor $k_{d,n}$, the COV of the FE resistances relative to the
 560 predictions from the resistance model V_δ and the combined COV incorporating the variability of
 561 the resistance model and the basic variables V_r , were calculated in accordance with EN 1990 [17].

562

$$b = \frac{1}{n} \sum_{i=1}^n \frac{r_{e,i}}{r_{t,i}} \quad (34)$$

564

565 The required partial safety factors, denoted γ_{M1}^* , were calculated for the different steel grades from
 566 S355 to S1100. For each considered steel grade, the dataset was divided further into subsets based
 567 on the relative member slenderness ($\bar{\lambda} \leq 0.5$ and $\bar{\lambda} > 0.5$) and compression-bending combinations
 568 ($\theta \leq 45^\circ$ and $\theta > 45^\circ$). The values of b and V_δ were determined for each subset, while the overall
 569 γ_{M1}^* was calculated by averaging the derived γ_{M1}^* values for all considered design cases. The key
 570 reliability analysis results are summarised in Table 6. Overall, the required partial safety factors
 571 γ_{M1}^* for the proposed design method are either below or marginally above the target value of 1.0

for all steel grades up to S1100. Therefore, the current EC3 partial safety factor $\gamma_{M1} = 1.0$ is deemed to be suitable for use with the proposed design method for cold-formed SHS and RHS beam-columns.

6. Conclusions

A comprehensive numerical investigation of cold-formed steel SHS and RHS beam-columns with non-slender cross-sections has been performed in the present study. Finite element models were first established and validated against a set of existing test results collected from the literature, and subsequently employed in a series of parametric studies to generate additional beam-column data covering a broad range of steel grades, cross-sectional sizes, relative slendernesses and combinations of compression and bending moment. On the basis of 15,000 numerically derived data, the accuracy of current design methods given in the EC3 [3,4] and AISC 360 [5], were assessed. It was shown that both the current codified design methods lead to somewhat inaccurate and scattered resistance predictions for cold-formed steel SHS and RHS beam-columns, with a large number of resistance predictions for the S355 and S460 steel beam-columns lying on the unsafe side. A new design approach, featuring the use of the recently proposed modified EC3 method [6] and the Continuous Strength Method (CSM) [11] for the calculation of more accurate column buckling and cross-sectional bending resistances, respectively, and the development of new yield strength-dependent interaction factors that are compatible with the adopted compression and bending end points, has been proposed. The new design approach was shown to provide more accurate, consistent and generally safe-sided ultimate resistance predictions for cold-formed steel SHS and RHS beam-columns with steel grades up to S1100. The reliability of the new design approach was evaluated in accordance with EN 1990 [17], revealing that a member buckling partial

safety factor γ_{M1} of unity, as adopted in EC3 [3,4], is also applicable to the new design proposals. It is considered that the form of the new design proposals is also applicable to beam-column members with slender cross-sections and with different cross-section shapes, which will be the focus of future work.

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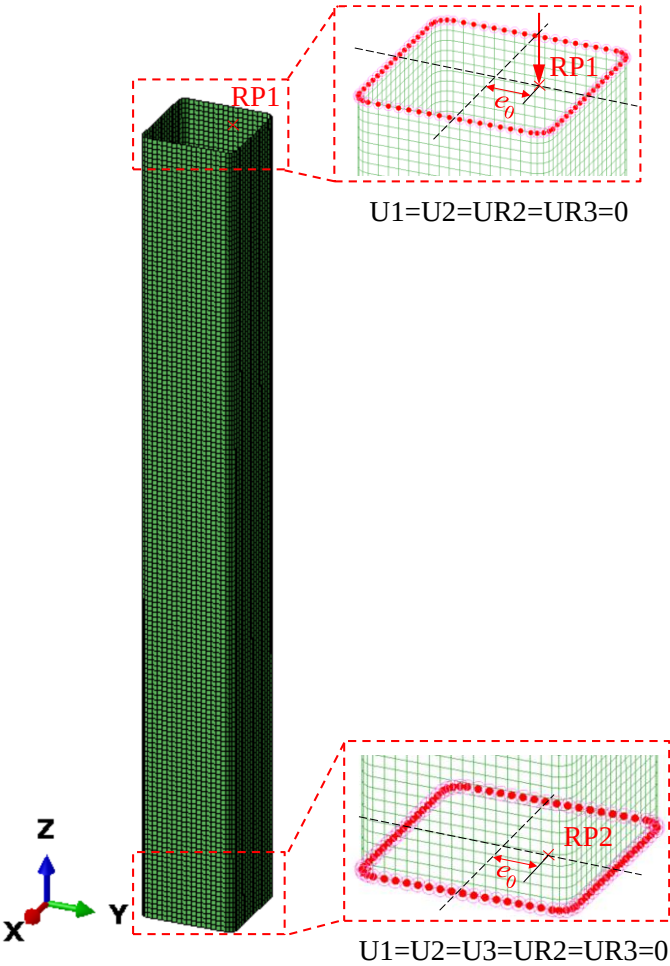
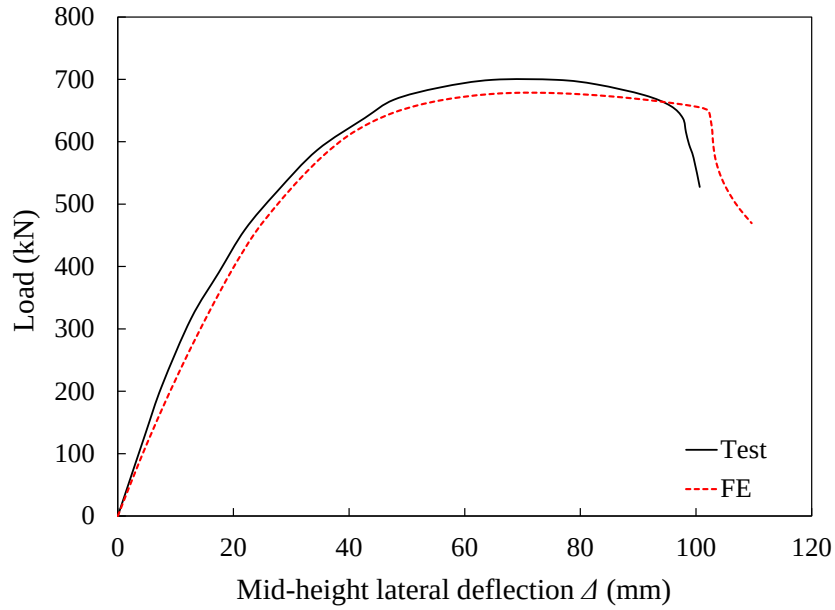
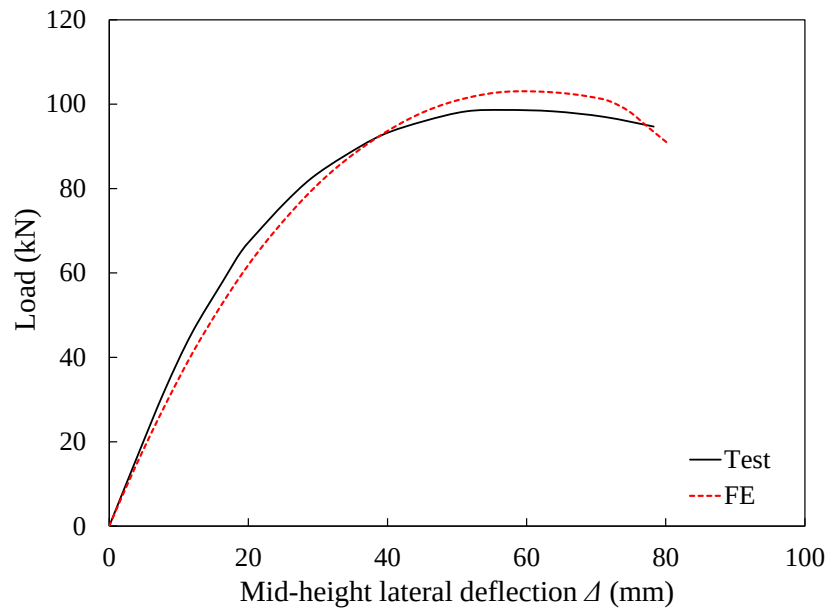


Fig. 1. Typical FE model of an SHS beam-column



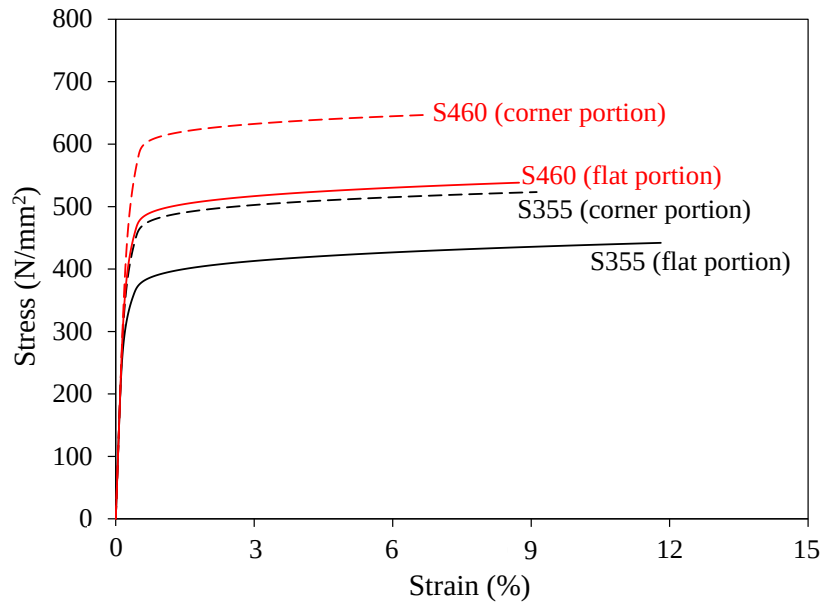
(a) RHS CF 220 × 120 × 6 T4 in S355 steel [16]



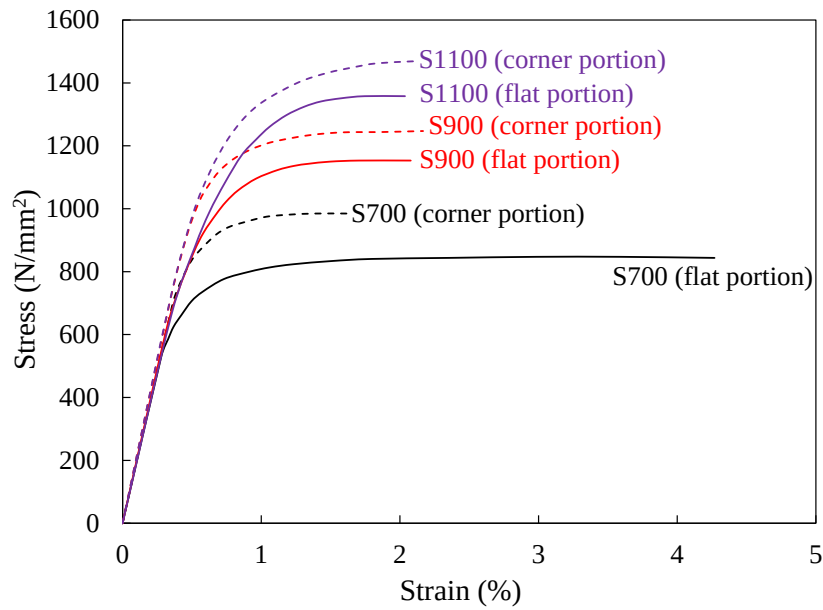
(b) H100 × 50 × 4-BC-e80 in S700 steel [8]

Fig. 2. Comparisons between experimental and numerical load-mid-height lateral deflection curves for typical cold-formed steel RHS beam-columns.

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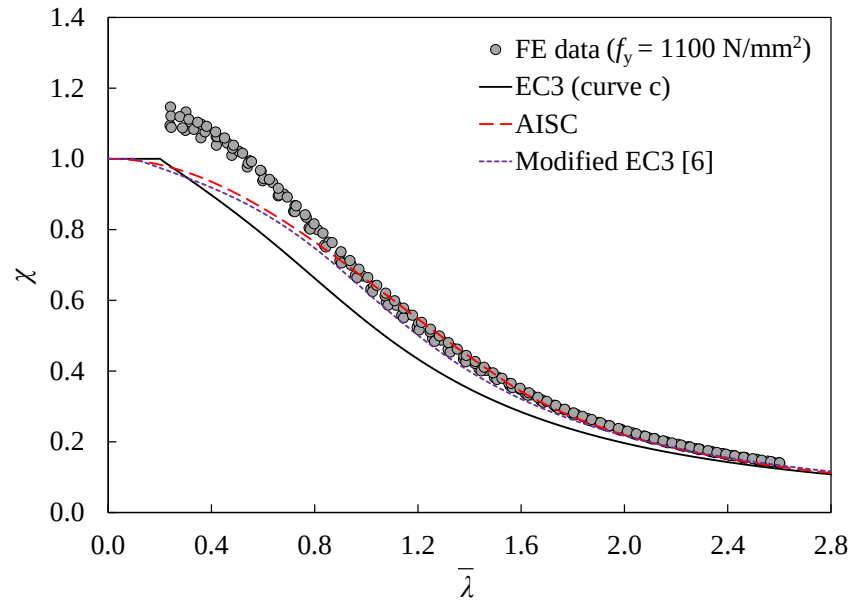
(a) S355 and S460 steel grades [1]



(b) S700, S900 and S1100 steel grades [26]

Fig. 3. Full stress-strain curves of cold-formed NSS and HSS employed in the parametric study.

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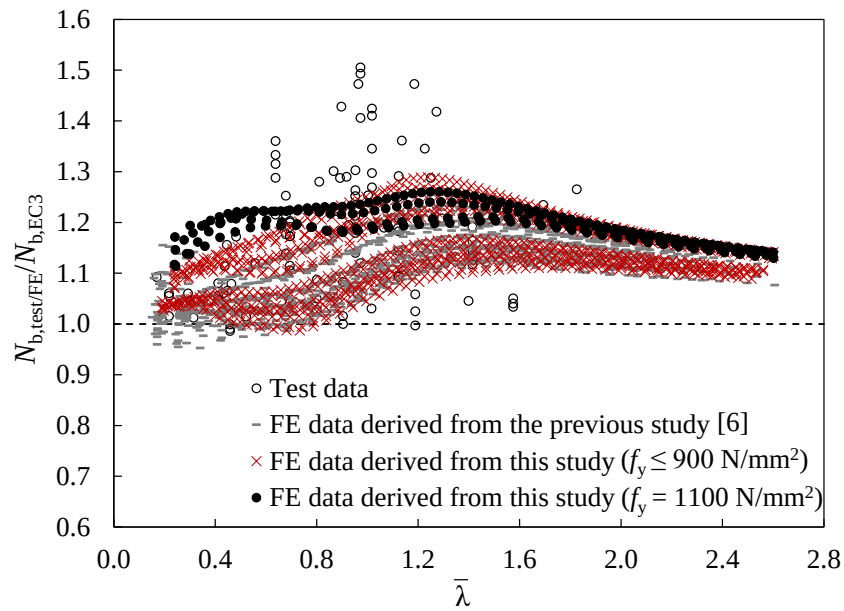


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709 **Fig. 4.** Comparisons of different column buckling curves with numerically derived data for S1100 steel
 710 columns.

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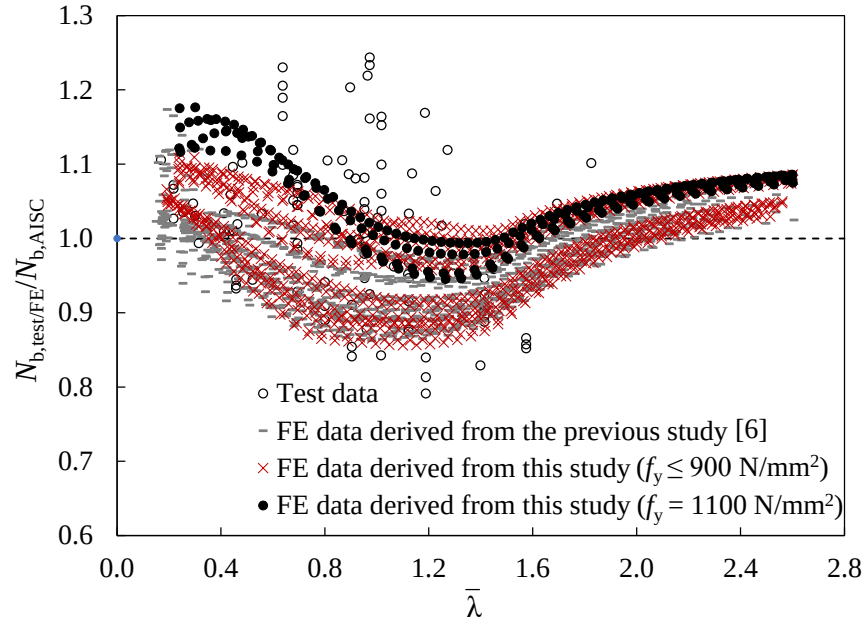


(a) EC3 [3,4]

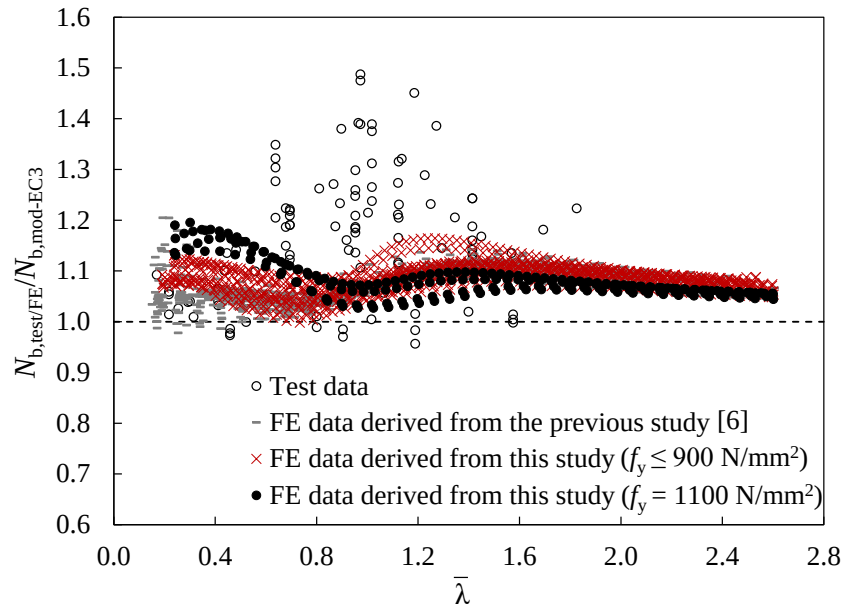
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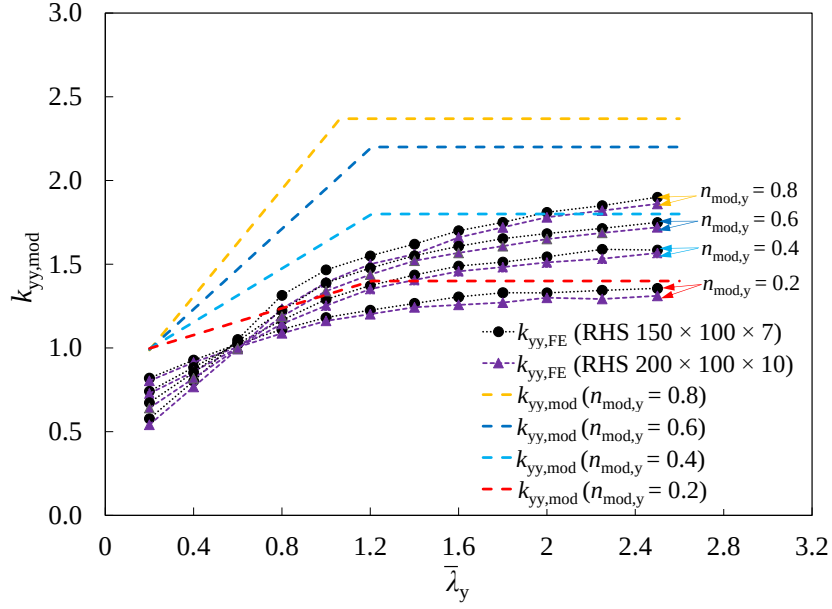


(b) AISC [5]

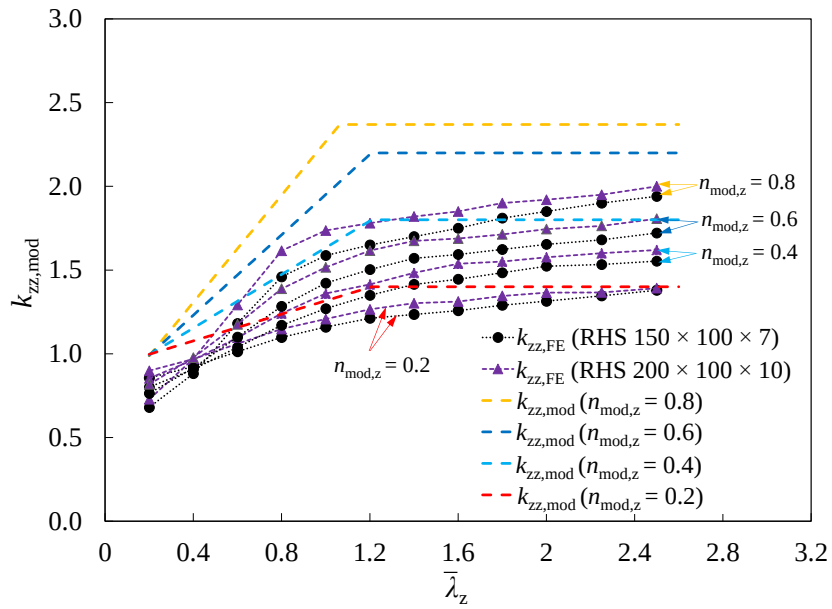


(c) Modified EC3 [6]

Fig. 5. Comparisons of test and FE column buckling resistances with predicted resistances obtained from different design methods.

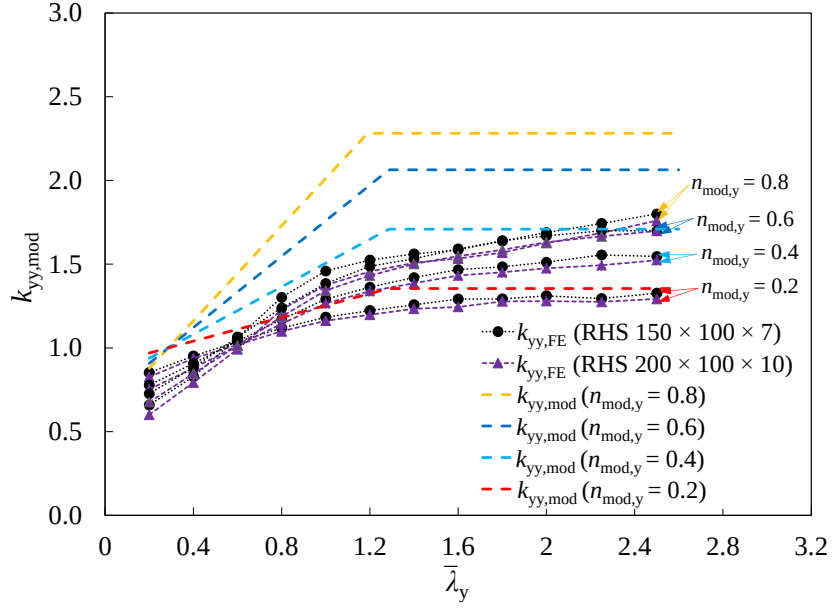


(a) $k_{yy,mod}$

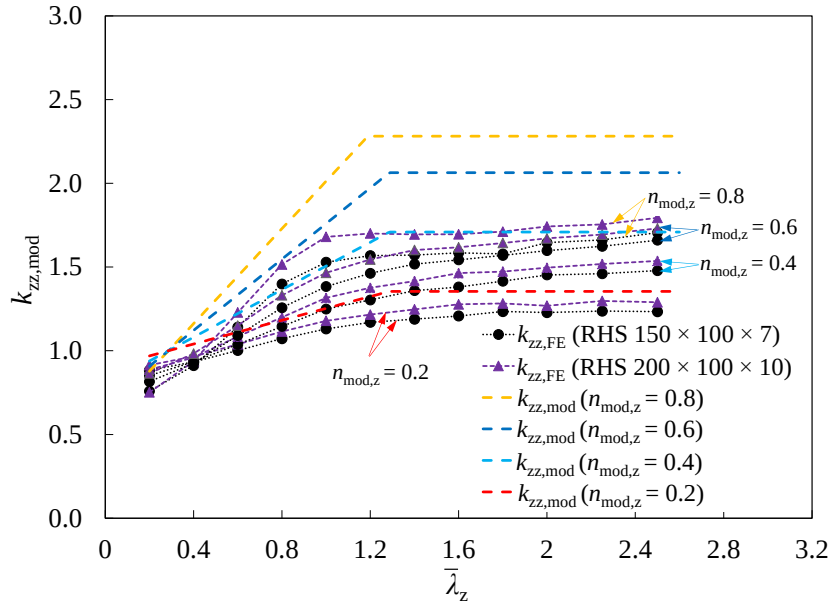


(b) $k_{zz,mod}$

Fig. 6. Calibration of the modified interaction factors $k_{yy,mod}$ and $k_{zz,mod}$ for cold-formed S355 steel beam columns.

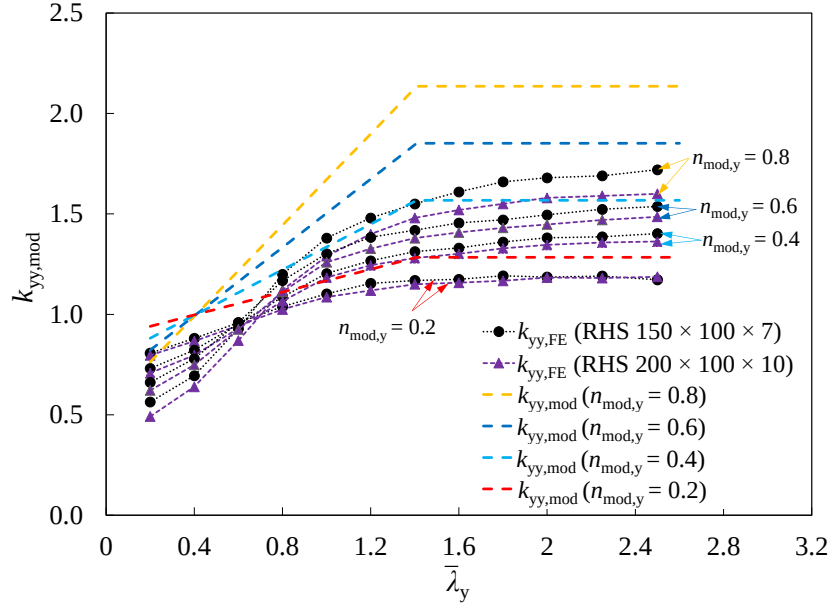


(a) $k_{yy,mod}$

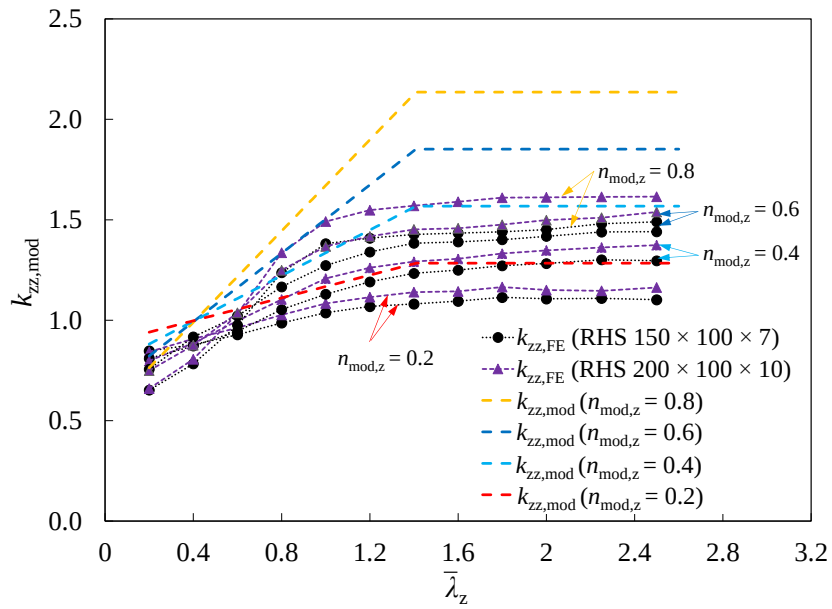


(b) $k_{zz,mod}$

Fig. 7. Calibration of the modified interaction factors $k_{yy,mod}$ and $k_{zz,mod}$ for cold-formed S460 steel beam columns.

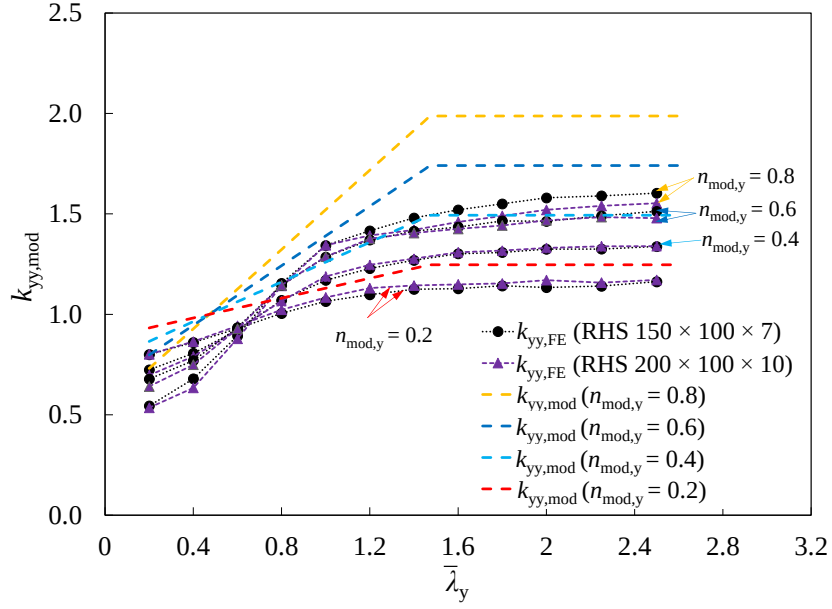


(a) $k_{yy,mod}$

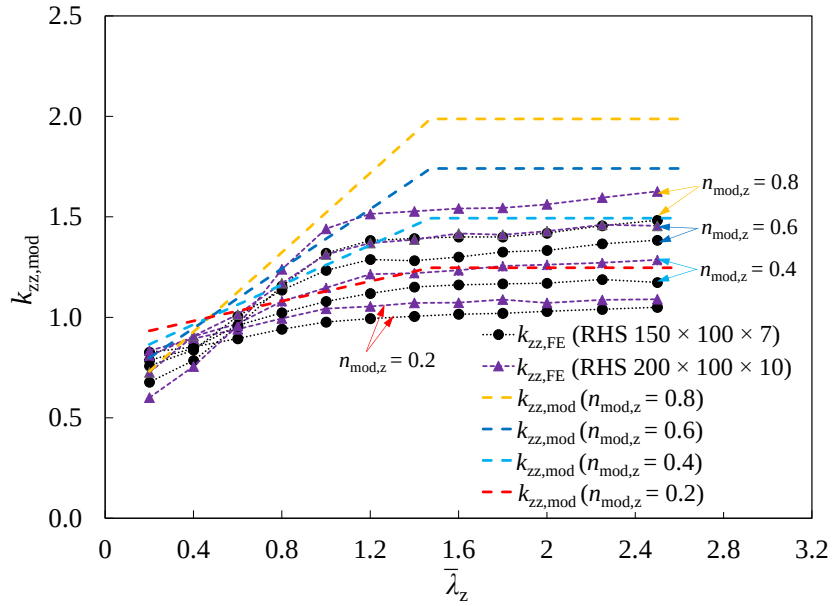


(b) $k_{zz,mod}$

Fig. 8. Calibration of the modified interaction factors $k_{yy,mod}$ and $k_{zz,mod}$ for cold-formed S700 steel beam columns.



(a) $k_{yy,mod}$



(b) $k_{zz,mod}$

Fig. 9. Calibration of the modified interaction factors $k_{yy,mod}$ and $k_{zz,mod}$ for cold-formed S900 steel beam columns.

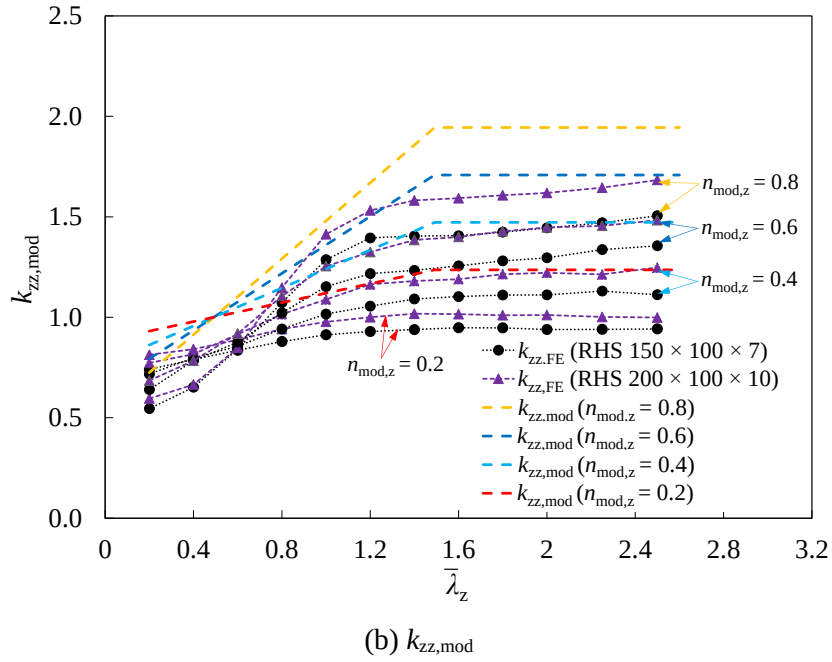
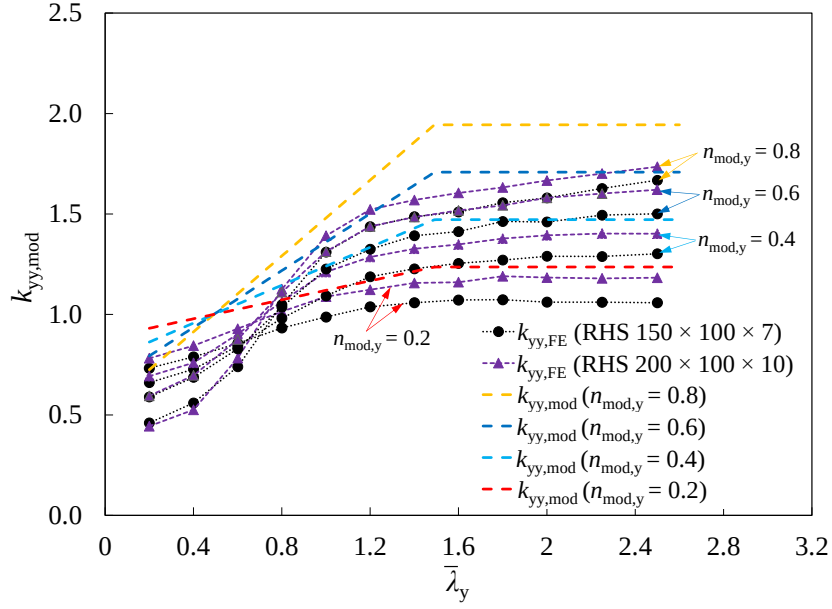
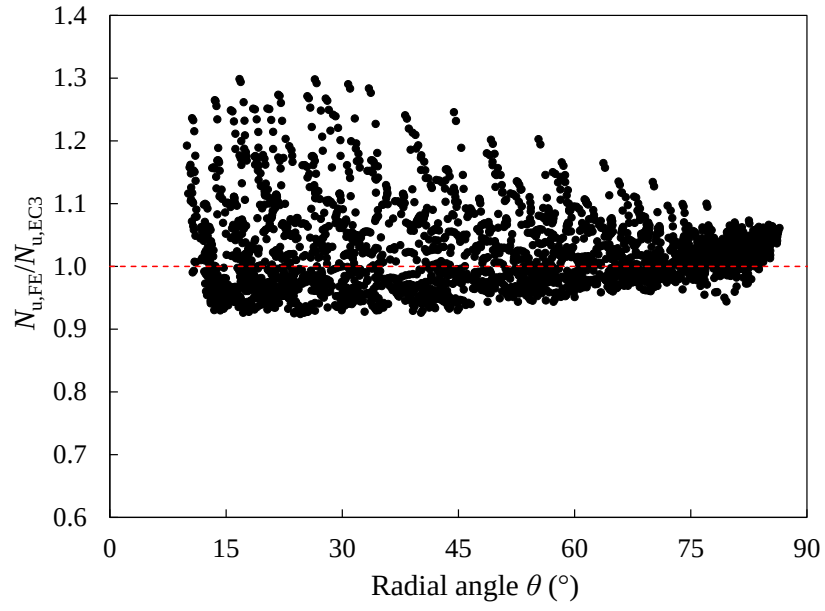
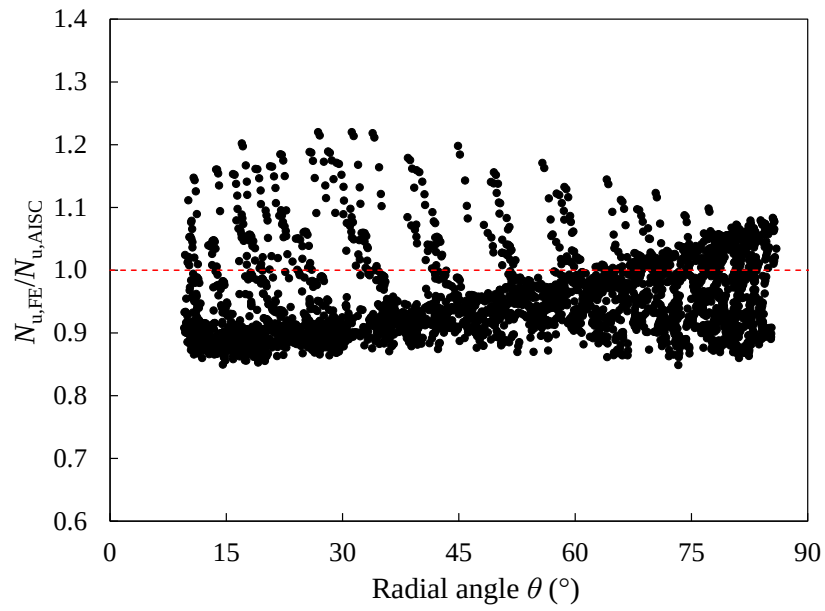


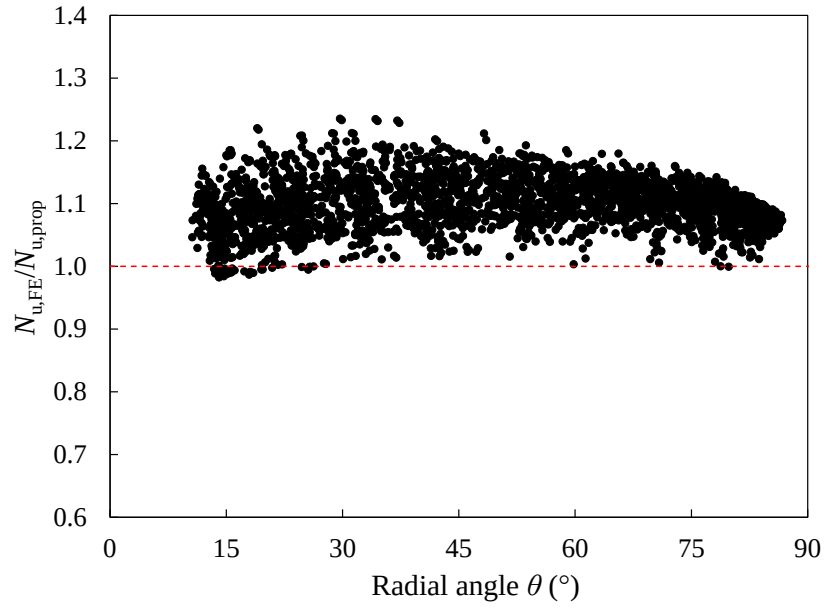
Fig. 10. Calibration of the modified interaction factors $k_{yy,mod}$ and $k_{zz,mod}$ for cold-formed S1100 steel beam columns.



(a) EC3 [3,4] – S355

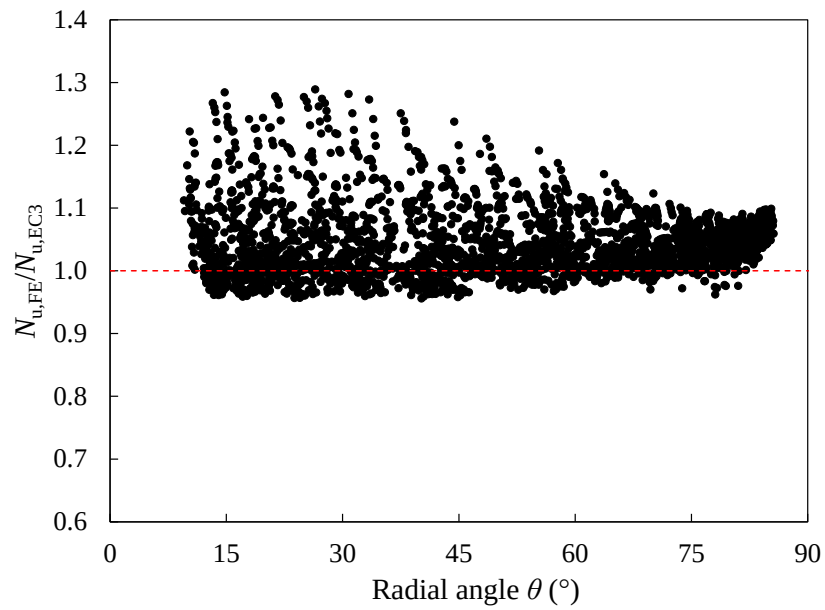


(b) AISC [5] – S355



(c) Proposed method – S355

Fig. 11. Comparison of FE results with unfactored resistance predictions calculated from different design methods for cold-formed S355 steel SHS and RHS beam-columns.



(a) EC3 [3,4] – S460

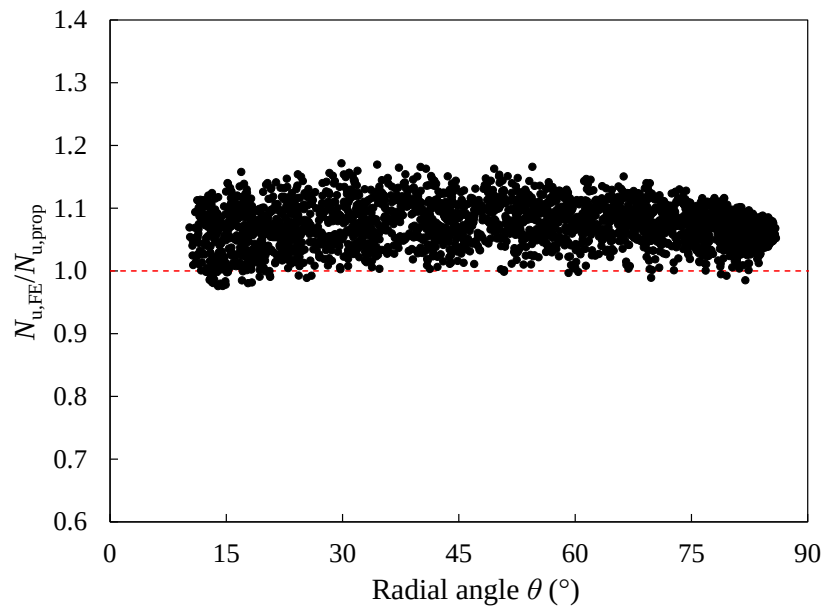
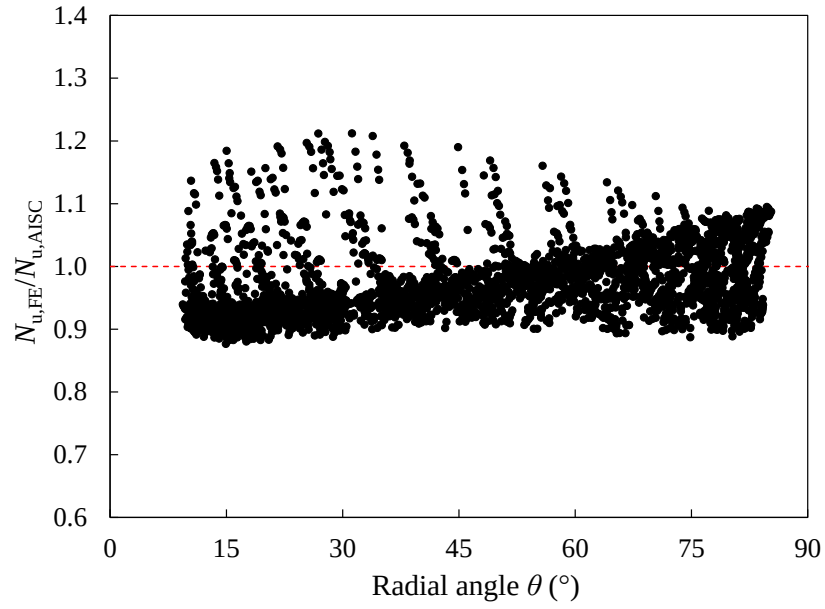
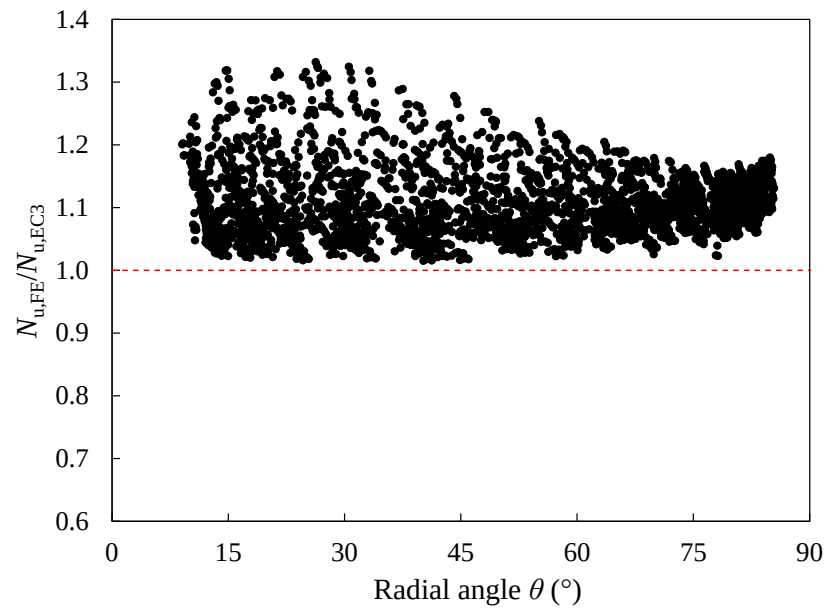
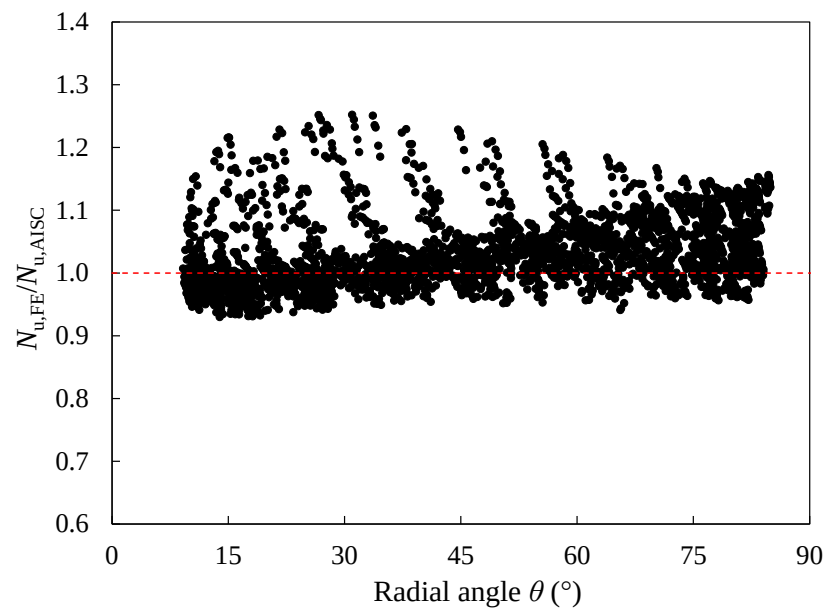


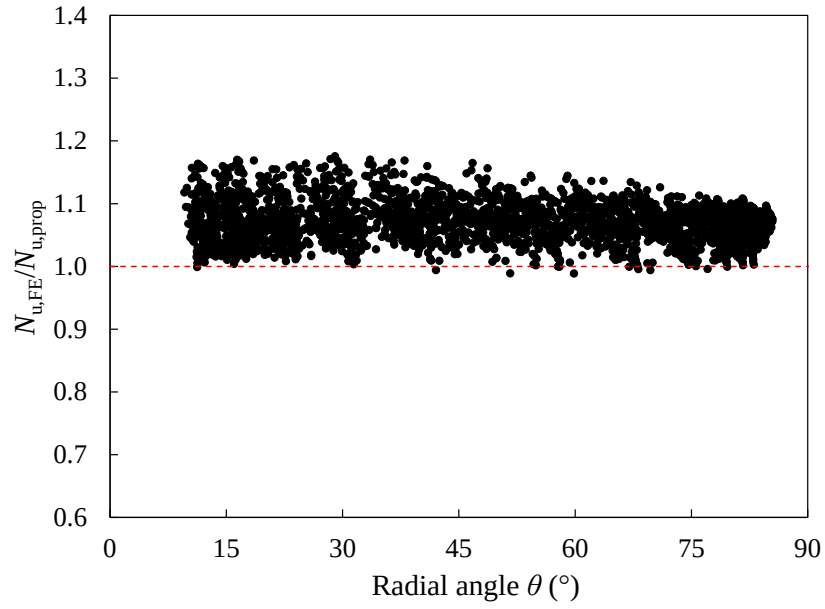
Fig. 12. Comparison of FE results with unfactored resistance predictions calculated from different design methods for cold-formed S460 steel SHS and RHS beam-columns.



(a) EC3 [3,4] – S700

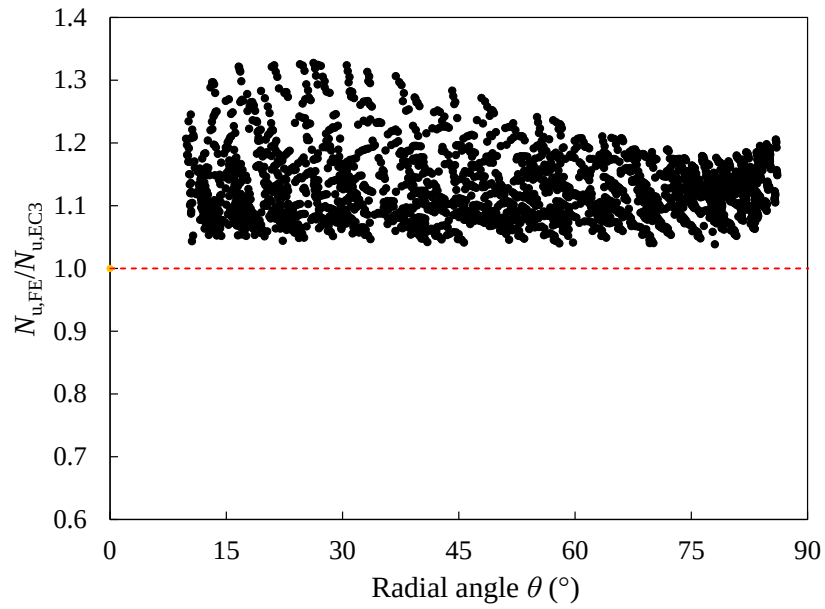


(b) AISC [5] – S700

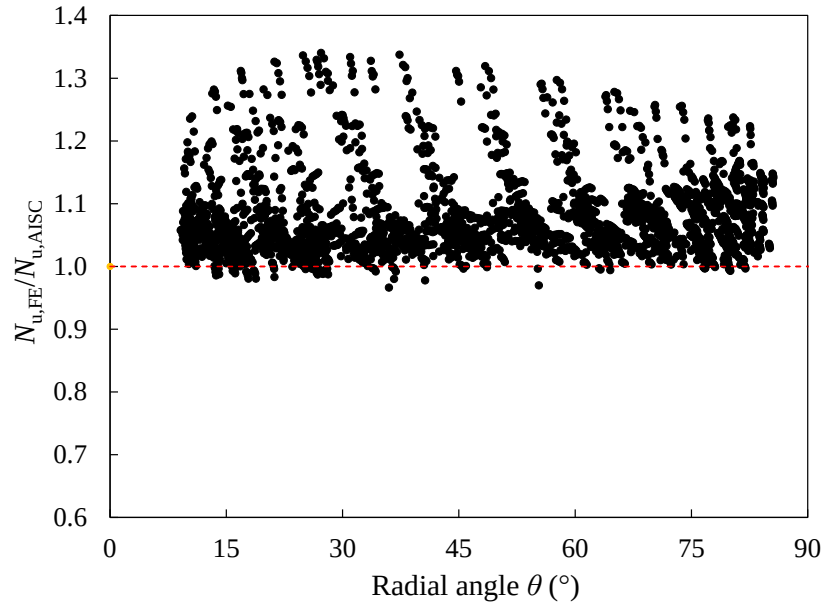


(c) Proposed method – S700

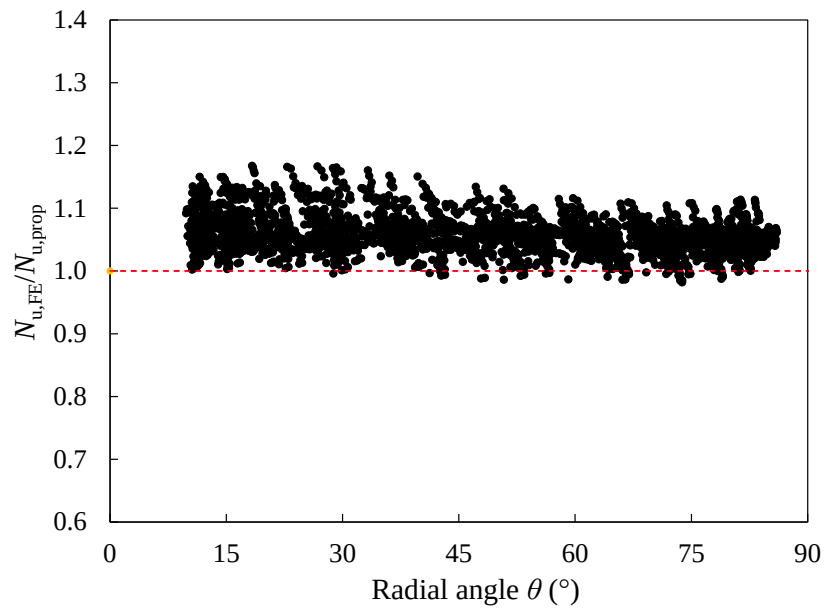
Fig. 13. Comparison of FE results with unfactored resistance predictions calculated from different design methods for cold-formed S700 steel SHS and RHS beam-columns.



(a) EC3 [3,4] – S900

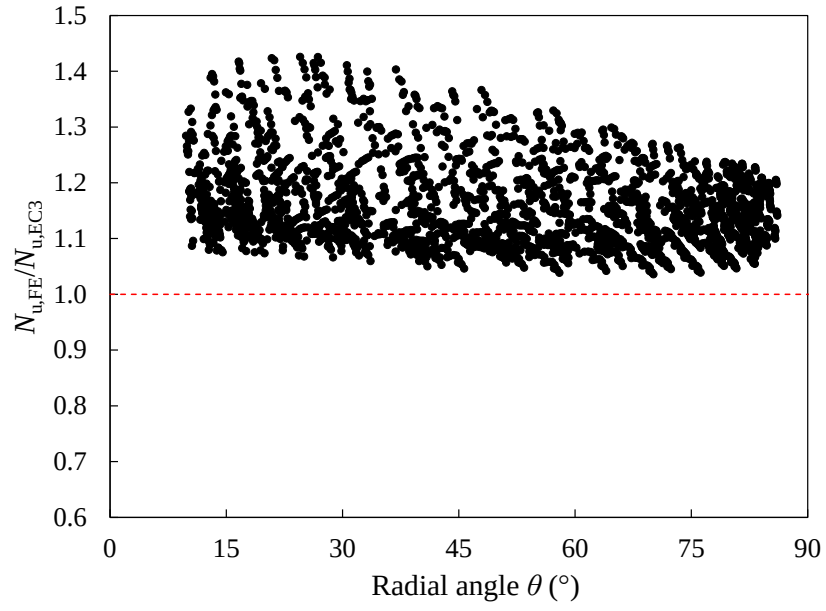


(b) AISC [5] – S900

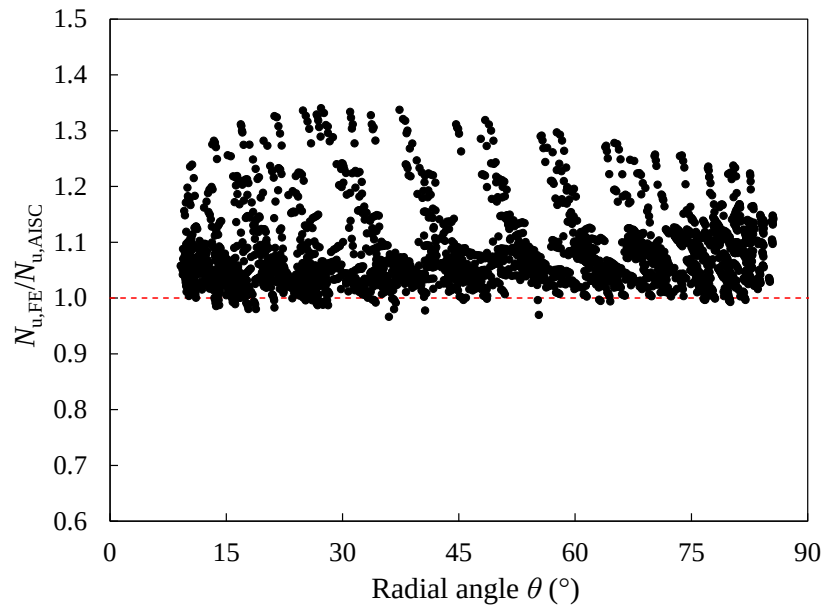


(c) Proposed method – S900

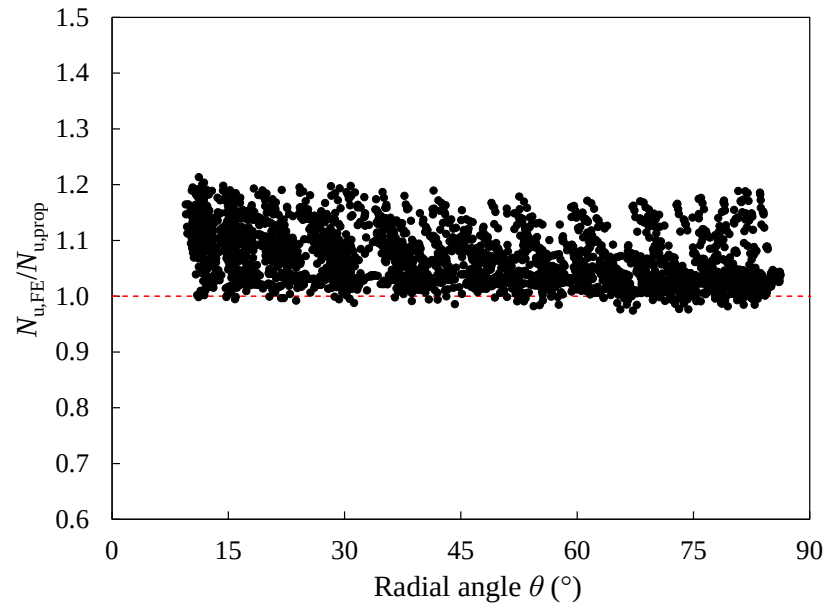
Fig. 14. Comparison of FE results with unfactored resistance predictions calculated from different design methods for cold-formed S900 steel SHS and RHS beam-columns.



(a) EC3 [3,4] – S1100



(b) AISC [5] – S1100



(c) Proposed method – S1100

Fig. 15. Comparison of FE results with unfactored resistance predictions calculated from different design methods for cold-formed S1100 steel SHS and RHS beam-columns.

Table 1: Comparison of ultimate resistances obtained from experiments and FE models with varying imperfection magnitudes.

Reference	Steel Grade	Specimen	$w_0 + \delta_0$	$N_{u,FE}/N_{u,test}$							
				$h/100+$ $L_e/1000$	$h/200+$ $L_e/1000$	$h/300+$ $L_e/1000$	$h/400+$ $L_e/1000$	$h/100+$ $L_e/1500$	$h/200+$ $L_e/1500$	$h/300+$ $L_e/1500$	$h/400+$ $L_e/1500$
Hayeck et al. [16]	S355	RHS CF $200 \times 100 \times 4$ T1	1.03	0.94	0.97	0.99	1.01	0.95	0.99	1.00	1.01
		RHS CF $220 \times 120 \times 6$ T4	0.98	0.94	0.95	0.96	0.97	0.94	0.97	0.97	0.97
		H80 $\times$ 80 $\times$ 4-BC-e0	1.13	1.02	1.02	1.02	1.03	1.05	1.06	1.06	1.06
		H80 $\times$ 80 $\times$ 4-BC-e5	1.02	0.97	0.98	0.98	0.98	0.99	0.99	1.00	1.00
		H80 $\times$ 80 $\times$ 4-BC-e10	1.00	0.96	0.97	0.97	0.98	0.97	0.98	0.98	0.98
		H80 $\times$ 80 $\times$ 4-BC-e20	1.04	1.01	1.02	1.02	1.02	1.02	1.03	1.03	1.03
		H80 $\times$ 80 $\times$ 4-BC-e20#	1.04	1.01	1.02	1.02	1.02	1.02	1.03	1.03	1.03
		H80 $\times$ 80 $\times$ 4-BC-e40	1.05	1.02	1.03	1.03	1.03	1.03	1.04	1.04	1.04
		H80 $\times$ 80 $\times$ 4-BC-e80	1.05	1.03	1.04	1.04	1.05	1.04	1.05	1.05	1.05
		H80 $\times$ 80 $\times$ 4-BC-e150	1.02	1.00	1.01	1.01	1.02	1.01	1.02	1.02	1.02
Ma et al. [8]	S700	H100 $\times$ 50 $\times$ 4-BC-e0	1.01	0.90	0.92	0.92	0.92	0.92	0.94	0.94	0.94
		H100 $\times$ 50 $\times$ 4-BC-e5	1.02	0.97	0.99	0.99	1.00	0.98	1.00	1.00	1.01
		H100 $\times$ 50 $\times$ 4-BC-e15	1.03	0.99	1.00	1.01	1.02	1.00	1.02	1.02	1.03
		H100 $\times$ 50 $\times$ 4-BC-e30	1.05	1.00	1.02	1.03	1.04	1.01	1.03	1.04	1.04
		H100 $\times$ 50 $\times$ 4-BC-e50	1.04	0.98	1.01	1.01	1.02	0.99	1.01	1.02	1.02
		H100 $\times$ 50 $\times$ 4-BC-e50#	1.05	0.99	1.02	1.02	1.03	1.00	1.02	1.03	1.03
		H100 $\times$ 50 $\times$ 4-BC-e80	1.05	1.00	1.03	1.04	1.04	1.00	1.03	1.04	1.04
		H100 $\times$ 50 $\times$ 4-BC-e130	1.03	0.97	1.00	1.01	1.01	0.97	1.00	1.01	1.02
		H50 $\times$ 100 $\times$ 4-BC-e0	0.99	0.92	0.93	0.93	0.94	0.94	0.96	0.96	0.96
		H50 $\times$ 100 $\times$ 4-BC-e3	1.07	1.00	1.00	1.01	1.02	1.02	1.03	1.03	1.03
		H50 $\times$ 100 $\times$ 4-BC-e10	1.07	1.02	1.02	1.03	1.04	1.03	1.04	1.05	1.05
		H50 $\times$ 100 $\times$ 4-BC-e10#	1.07	1.02	1.03	1.03	1.03	1.03	1.04	1.04	1.04
		H50 $\times$ 100 $\times$ 4-BC-e20	1.07	1.03	1.04	1.05	1.05	1.04	1.05	1.05	1.05
		H50 $\times$ 100 $\times$ 4-BC-e40	1.08	1.05	1.06	1.06	1.06	1.06	1.07	1.07	1.07
		H50 $\times$ 100 $\times$ 4-BC-e80	1.03	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.03
		H50 $\times$ 100 $\times$ 4-BC-e150	1.02	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01
		Mean	1.04	0.99	1.00	1.01	1.01	1.00	1.02	1.02	1.02
		COV	0.030	0.037	0.034	0.033	0.033	0.035	0.031	0.031	0.030

Table 2: Key material properties employed in parametric study.

Steel grade	E_f (E_c) N/mm ²	$f_{y,f}$ ($f_{y,c}$) N/mm ²	$f_{u,f}$ ($f_{u,c}$) N/mm ²	$\varepsilon_{u,f}$ ($\varepsilon_{u,c}$) %
S355	210,000 (210,000)	355 (444)	442 (523)	11.8 (9.1)
S460	210,000 (210,000)	460 (575)	538 (647)	8.7 (6.7)
S700	212,000 (212,000)	719 (897)	840 (983)	4.3 (1.6)
S900	208,000 (209,000)	982 (1138)	1149 (1245)	2.1 (2.2)
S1100	205,000 (206,000)	1073 (1245)	1356 (1470)	2.0 (2.1)

Table 3: Comparisons of test and FE column buckling resistances with predicted resistances obtained from different design methods.

Data source	Evaluation parameter	EC3 [3,4] ($N_{b,test}/N_{b,EC3}$)	AISC [5] ($N_{b,test}/N_{b,AISC}$)	Modified EC3 [6] ($N_{b,test}/N_{b,mod-EC3}$)
226 FE data on columns made of S1100 steel from this study	Mean	1.19	1.05	1.08
	COV	0.031	0.052	0.035
108 tests + 1150 FE data from [6] + 1130 FE data from this study	Mean	1.13	0.99	1.08
	COV	0.062	0.065	0.040

Table 4: Comparisons of FE results with unfactored resistance predictions calculated from different design methods for cold-formed steel SHS and RHS beam-columns.

Steel grade	No.	$N_{u,FE}/N_{u,EC3}$		$N_{u,FE}/N_{u,AISC}$		$N_{u,FE}/N_{u,prop}$	
		Mean	COV	Mean	COV	Mean	COV
S355	3000	1.02	0.060	0.95	0.069	1.10	0.036
S460	3000	1.05	0.052	0.98	0.061	1.07	0.034
S700	3000	1.11	0.050	1.03	0.057	1.07	0.029
S900	3000	1.13	0.047	1.08	0.059	1.06	0.028
S1100	3000	1.16	0.063	1.08	0.059	1.06	0.045
All	15000	1.09	0.078	1.02	0.080	1.07	0.040

839 **Table 5:** Adopted variability parameters of material and geometric properties in accordance with prEN
840 1993-1-1 [30].

Parameter	X_m / X_n	COV
f_y (S355, S420)	1.20	0.050
f_y (S460)	1.15	0.045
f_y (above S460)	1.10	0.035
E	1.00	0.030
H	1.00	0.009
B	1.00	0.009
t	0.99	0.025

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843 **Table 6:** Reliability analysis results for the proposed design method.

Steel grade	No. of data	$k_{d,n}$	b	V_δ	V_r	γ_{M1}^*
S355	3000	3.093	1.100	0.035	0.069	0.978
S460	3000	3.093	1.074	0.034	0.061	1.055
S700	3000	3.093	1.079	0.029	0.059	1.044
S900	3000	3.093	1.056	0.028	0.057	1.059
S1100	3000	3.093	1.057	0.049	0.064	1.086

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