Powder Technology 396 (2022) 689–695

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Short Communication

Elongation, flatness and compactness indices to characterise particle form

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ARTICLE INFO

Article history:
Received 15 September 2021
Received in revised form 9 November 2021
Accepted 10 November 2021
Available online 16 November 2021

Keywords:
Particle form
Morphology characterisation
Particle shape classification
Non-spherical particles

ABSTRACT

A century after the first attempts of Wentworth to characterise the shape of cobbles, our understanding of particle morphology is still expanding. A plethora of shape indices has been proposed in the literature to characterise the morphology of individual particles. This study aims to shed light on the merits and limitations of the indices currently used to characterise particle elongation, flatness and compactness, adopting a unified classification framework. Second, new indices are proposed to address the identified shortcomings. Third, a new particle classification system derived from the proposed indices is illustrated. It is shown the new system overcomes the misclassification of a range of particles that are incorrectly classified as bladed in the Zingg system.

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1. Introduction

Particle morphology has been shown to bear significant influence on the mechanical, rheological and hydraulic behaviour of granular materials (*e.g.* [8,6,17,12,19]). Note that the terms *shape* and *morphology* are used interchangeably in this communication, not relating to any particular scale. Particle morphology is typically characterised in terms of three aspects, namely form (first order approximation of shape), roundness (second order) and roughness (third order) as in Barrett [4]. Other aspects of particle shape have been discussed in the literature, such as sphericity and irregularity [5]. While sphericity [26] is considered by many as an index of form (*e.g.* [9]), others consider it to be affected by both form and roundness [5].

The aforementioned aspects of particle morphology affect the physical behaviour of particles in bulk differently. For instance, particle form has been demonstrated to affect the critical state shear strength [11,1], packing [14], fabric [20], flowability [23] and jamming [18] of particulate materials, roundness has been associated with the shear strength [27] and stress concentrations within particulate assemblies [15], while roughness has been shown to control the small strain stiffness [21] normal contact behaviour and bulk deformation [16] of particles with rough surfaces. It is evident that no single shape aspect controls the link between particle morphology and mechanical behaviour. This study deals with form, as multiple formulae proposed in the literature often lead in contradicting results; thus a direct comparison of their values is of interest.

The characterisation of the form of three-dimensional particles is mostly performed via the calculation of two independent indices: flatness and elongation (*e.g.* as in Zingg [28]). Flatness is meant to express how flat (or flaky or platy) a particle is, while elongation is meant to express how elongated (or rod-like) it is. Some approaches consider a third index as well, compactness (see [5]). However, compactness is calculated from flatness and elongation so the latter two indices are enough to fully characterise the overall form of a particle.

All traditional formulae for flatness and elongation make use of the three main dimensions of a particle, here called $a > b > c$ as in Zingg [28]. In the case of regular particle geometries (*e.g.* cuboids or ellipsoids), the main particle dimensions are unambiguously defined, whereas for irregular particles, the definition of these dimensions can be ambiguous. These dimensions provide a simplified description of the form of a particle, and can be calculated using a variety of methods. For instance, Fonseca [10] assumes a,b,c as the dimensions of an oriented bounding box, calculated using Principal Component Analysis, while Potticary et al. [22] assume the axes of an equivalent ellipsoid as the main particle dimensions.

This study first describes the classification system proposed by Zingg [28] and then compares the indices proposed by Kong and Fonseca [13], Bagi and Orosz [3] and Potticary et al. [22] in a Zingg plot. This enables a direct comparison of the indices for particles of the same aspect ratios. By clarifying the advantages and limitations of each set of indices, the case is laid out for the introduction of a new set of indices. The advantages of the new indices are illustrated. Then, a new particle

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https://doi.org/10.1016/j.powtec.2021.11.027
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classification system derived from the new indices which overcomes the shortcomings of the Zingg system is described.

2. Methods

2.1. Interpretations of flatness and elongation

Zingg [28] proposed one of the most popular classification systems for form, considering two aspect ratios to define the form of a particle: c/b being particle flatness and b/a being elongation (as shown in Eq. (1)).

$$\begin{aligned} el &= \frac{b}{a} \\ fl &= \frac{c}{b} \end{aligned} \tag{1}$$

Plotting the Zingg parameters c/b and b/a in a chart provides an intuitive visualisation of the form of a particle, in what is widely referred to as a *Zingg plot*. Zingg [28] categorised particle morphology in four classes: oblate if $c/b < 2/3$ and $b/a > 2/3$; compact if $c/b > 2/3$ and $b/a > 2/3$; blade-like if $c/b < 2/3$ and $b/a < 2/3$ or prolate if $c/b > 2/3$ and $b/a < 2/3$, as shown in Fig. 1a. Although Zingg [28] did not use the term flatness nor elongation, b/a and c/b have been referred to as elongation and flatness respectively in the subsequent literature (*e.g.* [5]).

Kong and Fonseca [13] proposed new formulae for flatness and elongation given in Eq. (2). These indices can be thought of as the complementary to unity of the ratios in Eq. (1).

$$\begin{aligned} el &= 1-\frac{b}{a} \\ fl &= 1-\frac{c}{b} \end{aligned} \tag{2}$$

The advantage of the Kong and Fonseca [13] formulae is that a flat particle takes a high value of flatness, an elongated particle takes a high value of elongation, a blade-like particle (both flat and elongated) takes a high value of both flatness and elongation, while a compact particle takes a low value for both indices. Also these indices are easy to understand and visualise (see the Zingg plots of Fig. 2). However, a limitation of the indices is that they do not express flatness nor elongation as percentages of an overall form. For example, a particle with $fl = 0.8$, so $c/b = 0.2$, is not compact, but it could be either flat, if $el < 1/3$, so $b/a > 2/3$, or bladed (flat and elongated) otherwise (see Fig. 1b). So, according to Kong and Fonseca's [13] definition of flatness and elongation, particles featured by the same value of fl may feature different degrees of flatness, depending on their elongation values.

Bagi and Orosz [3] proposed the use of a surface orientation tensor to characterise the form of a particle as a solution to the problem of defining the main dimensions of irregular particles. The tensor is used to calculate elongation, flatness and compactness of the tessellated surface of a particle. In case of cuboidal particles, calculation of the surface orientation tensor leads to the following formulae for elongation, flatness and compactness:

$$\begin{aligned} el &= \frac{c}{b}-\frac{c}{a} \\ fl &= 1-\frac{c}{b} \\ co &= \frac{c}{a} \end{aligned} \tag{3}$$

Contour maps of these indices produced on a Zingg plot are shown in Fig. 3. From Fig. 3a emerges that the *el* index takes low values for particles on the lower left of the Zingg plot (*i.e.* for $c/b<0.2$ and $b/a<0.2$). These are particles that are in fact flat and elongated. However, most geomaterials do not exhibit such high elongation and flatness values. Nevertheless a merit of these indices is that they add up to unity ($el + fl + co = 1$), so they can be interpreted as percentile representations of an overall particle form, *i.e.* a real three-dimensional particle will always have some degree of elongation, flatness and compactness even if it is predominantly elongated, flat or compact. Moreover, for values at the extremes of the spectrum, one of the indices is enough to determine the class of the particle form, *e.g.* a flatness of 0.8 guarantees that a particle is flat, regardless of its values of compactness or elongation. However Bagi and Orosz [3] did not attempt to employ these indices to improve on the current particle classification systems.

On the other hand Potticary et al. [22] considered an equivalent scalene ellipsoid to define the main dimensions of irregular particles, arriving at a different definition for *el*, *fl* and *co*:

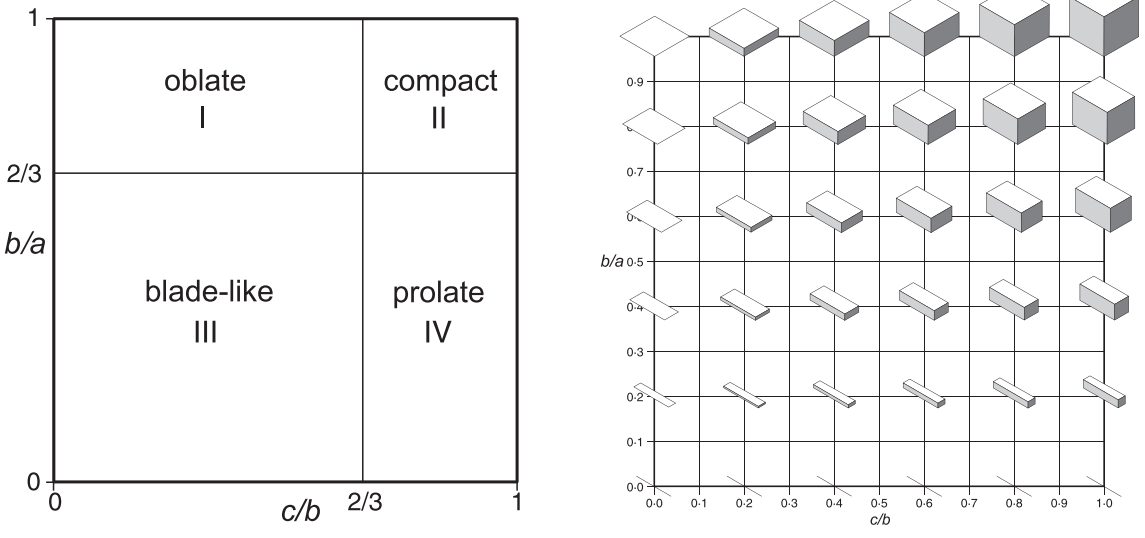

Fig. 1. (a) The shape classification system of Zingg [28]; (b) cuboids with varying aspect ratios on a Zingg plot (modified from Blott and Pye [5] with permission from Wiley & Sons).

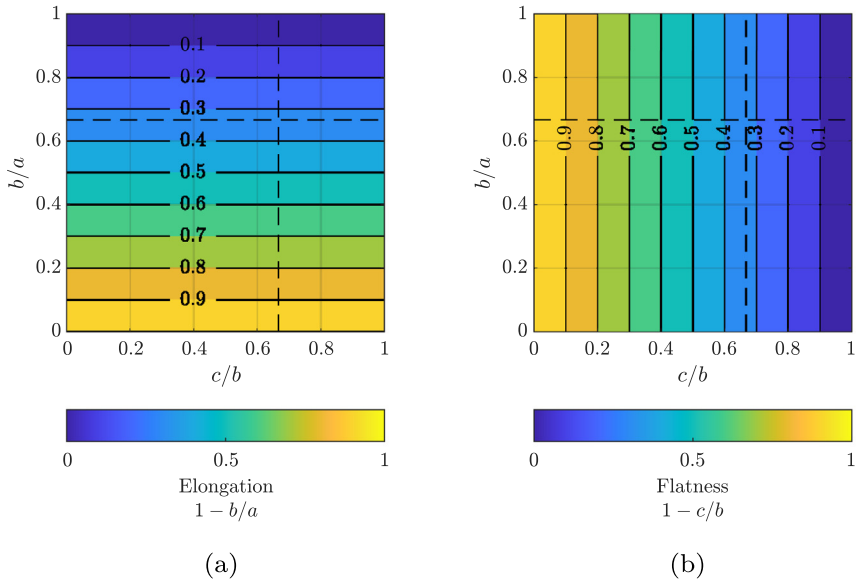

Fig. 2. Indices of (a) elongation and (b) flatness proposed by Kong and Fonseca [13].

$$\begin{aligned} el &= \frac{a-b}{a+b+c} \\ fl &= \frac{2(b-c)}{a+b+c} \\ co &= \frac{3\cdot c}{a+b+c} \end{aligned} \tag{4}$$

These indices can be seen as percentages of an overall form, while their distribution on a Zingg plot seems in agreement with the shape of the cuboids of varying aspect ratios of Fig. 1b. With regard to very bladed particles (*i.e.* for $c/b < 0.2$ and $b/a < 0.2$), the indices show an opposite trend of the indices of Bagi and Orosz [3], featuring high elongation and low flatness values for particles that are both flat and elongated. Contour maps of these indices produced on a Zingg plot are visible in Fig. 4.

2.2. New indices for elongation, flatness, compactness

A new set of indices to characterise particle elongation, flatness and compactness is here proposed. The indices (Eq. (5)) were designed to add up to unity and take 'reasonable' values for the whole range of particle aspect ratios, *i.e.* for c/b and b/a ranging within (0,1].

$$\begin{aligned} el &= \frac{a\cdot c}{a\cdot c+b^2} - \frac{c}{a+c} \\ fl &= \frac{b^2}{a\cdot c+b^2} - \frac{c}{a+c} \\ co &= \frac{2\cdot c}{a+c} \end{aligned} \tag{5}$$

Mapping the indices on a Zingg plot (see Fig. 5), makes it apparent that the new flatness and elongation indices are symmetric along the

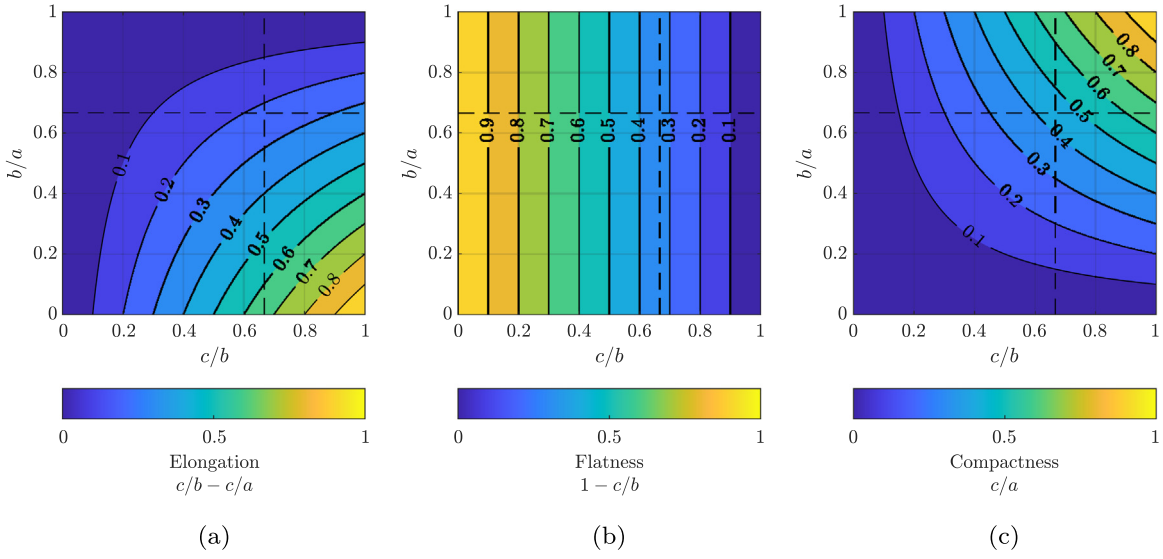

Fig. 3. Indices of (a) elongation, (b) flatness and (c) compactness proposed by Bagi and Orosz [3].

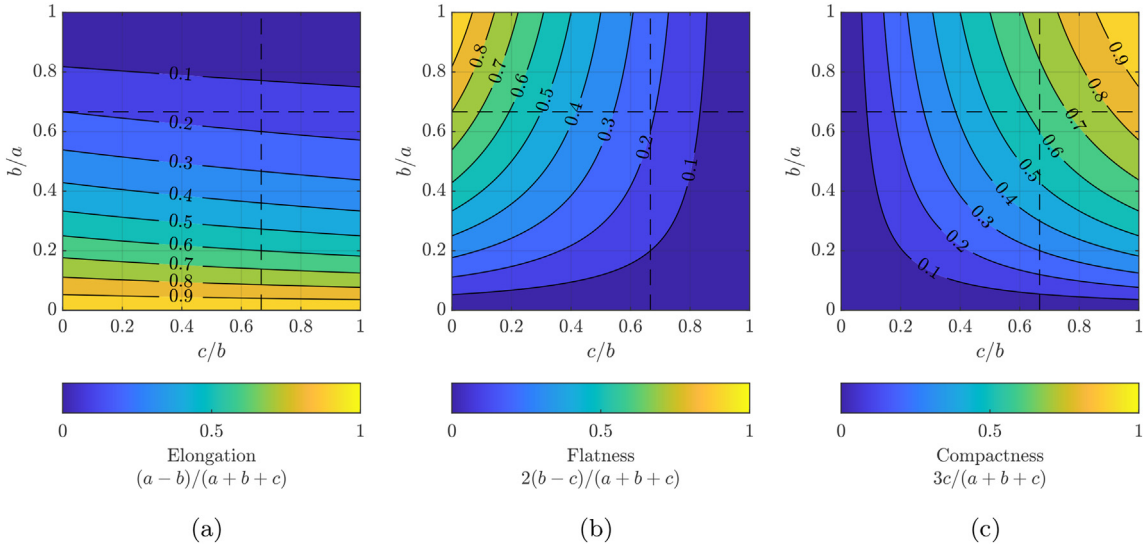

Fig. 4. Indices of (a) elongation, (b) flatness and (c) compactness proposed by Potticary et al. [22].

bisect line of the map, $c/b = b/a$, making their values easier to interpret. The proposed *fl* and *el* indices show a general agreement with the ones proposed by Bagi and Orosz [3] and Potticary et al. [22] in the regions I, II and IV of the Zingg plot (see Fig. 1a), featuring flat, compact and elongated particles, respectively. Instead, this is not the case for the flatness and elongation values of particles falling in the bladed region of the Zingg classification system, *i.e.* $c/b < 2/3$ and $b/a < 2/3$ (lower left region bounded by dashed lines). For example, for particles featured by small c/b values the *el* index of Potticary et al. [22] takes high values although they are predominantly flat. But their flatness is captured by the new proposed indices. For instance let us consider a very flat particle featured by $c/b = 0.01$ and $b/a = 0.2$ (see the red circle in Fig. 6c), meaning that its short axis c is 100 times smaller than the intermediate axis b, while the intermediate axis is only 5 times smaller than the long axis a. According to Potticary et al. [22] $fl = 0.329$ and $el = 0.666$, whereas using the proposed new indices $fl = 0.950$ and $el = 0.046$. Moreover, for particles with $b/a = 0.2$ (see the green band in Fig. 6c) their morphology becomes more elongated for increasing c/b values. This trend is well captured by the proposed *fl* and *el* indices whereas *el* of Potticary et al. [22] takes nearly a constant value irrespective of the c/b values. Finally with regard to the proposed compactness index, it exhibits similar values to the ones taken by the indices of Bagi and Orosz [3] and Potticary et al. [22].

2.3. New classification system for particle form

As early as 1958, Sneed and Folk highlighted that the Zingg classification system underestimates the range of elongated and flat particles, while overestimates the range of bladed particles. The latter occupies the majority of the Zingg plot [24] with a range of both flat and elongated particles misclassified as bladed [25].

To address the current shortcoming of the Zingg classification system, a sought new classification system needs 1) to keep unchanged region II of the compact particles while 2) enlarging the regions of both flat and elongated particles to the expense of the region of the bladed ones. The first requirement, i.e. keeping region II unchanged, implies that the region needs to be delimited by the same boundaries as those of the Zingg classification system, i.e. $b/a = 2/3$ and $c/b = 2/3$. These

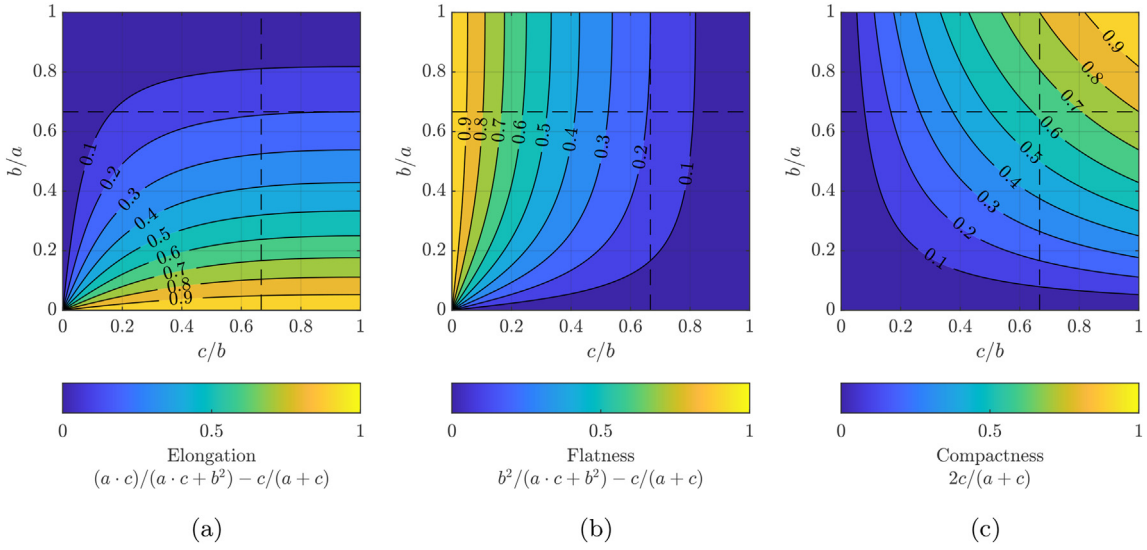

Fig. 5. Indices of (a) elongation, (b) flatness and (c) compactness proposed in this study.

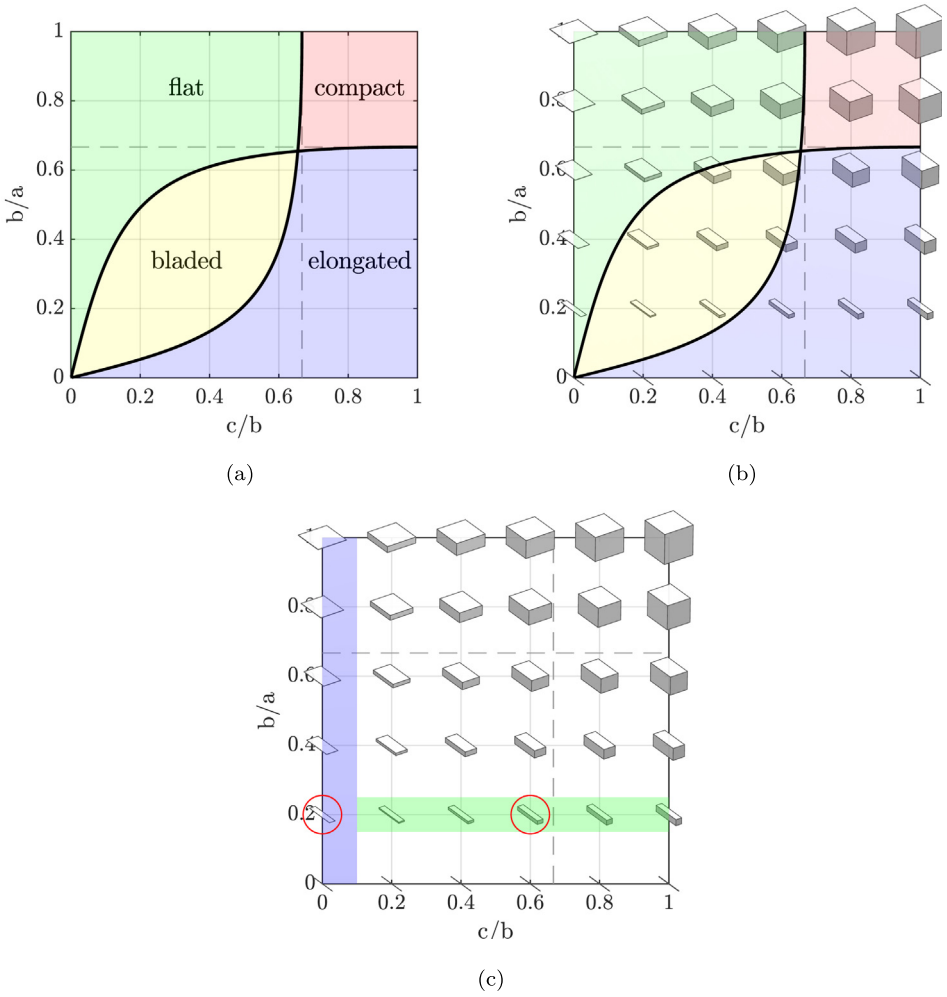

Fig. 6. (a) The proposed classification system with the solid lines denoting the boundaries among bladed, elongated, compact and flat particles. The dashed lines denote the Zingg classification system; (b) cuboids with varying aspect ratios plotted over our and Zingg's classification system; (c) particles in the coloured bands were employed for the comparison exercise between form indices.

values plugged into our proposed new indices result into $fl = 0.2$ and $el = 0.2$. In Fig. 6a the curves corresponding to $fl = 0.2$ and $el = 0.2$ are plotted as solid lines on a Zingg plot together with the modified regions I, II, III and IV of the new proposed classification system, where the boundaries between regions are now provided by $fl = 0.2$ and $el = 0.2$ instead of $b/a = 2/3$ and $c/b = 2/3$ (drawn as dashed lines in Fig. 6a). Visually it is immediately evident that in the new system, regions I and IV corresponding to flat and elongated particles respectively are larger while the size of region III of the bladed particles is smaller, fulfilling the second aforementioned requirement. In Fig. 6b cuboids of various dimensions are superimposed on the plot of Fig. 6a so that some specific example cases of particles can be analysed in order to assess the accuracy of the new proposed classification system. For example, let us consider the cuboid featured by $c/b = 0.6$ and $b/a = 0.2$, marked by a red circle in Fig. 6c. This is clearly an elongated rather than a bladed particle as per the Zingg classification system. Second, let us consider the cuboid featured by $c/b = 0.01$ and $b/a = 0.2$. This is a predominantly flat particle rather than flat and elongated (*i.e.* bladed) as per the Zingg classification system. In summary, in the proposed new classification system and unlike the Zingg one, particles featured by $b/a < 2/3$ and small c/b values are correctly classified as flat and particles featured by $c/b < 2/3$ and small b/a values as elongated.

The proposed indices and classification system are well defined for particles of all possible aspect ratios ($c/b > 0$ and $b/a > 0$). Focusing on the lower-left region of a Zingg plot, which corresponds to particles with $c/b < 0.1$ and $b/a < 0.1$, three particle shapes will be considered: (i) a flat particle with $c/b = 0.01$ and $b/a = 0.1$, (ii) a blade-like particle with $c/b = 0.1$ and $b/a = 0.1$ and (iii) an elongated particle with $c/b = 0.1$ and $b/a = 0.01$, aiming to demonstrate the robustness of calculations for three extreme particle shapes. Cuboids of these aspect ratios are shown in Fig. 7, where the morphological difference among these particles becomes apparent. Moreover, it should be noted that most natural materials, such as sands or silts, do not feature such extreme particle shapes, as these tend to break in nature, and evolve into smaller, more regular fragments [7].

3. Concluding remarks

In this study, the most popular sets of indices from the literature employed to characterise and classify particle form were critically reviewed with the aid of a Zingg plot, where all possible combinations of aspect ratios are represented. This way of visualising the values of form indices for particles of all possible aspect ratios in a single diagram, proved to be an insightful tool in exploring and better understanding what each of the analysed shape indices represents.

New indices for elongation, flatness and compactness were proposed in the attempt to overcome the limitations identified for the indices currently in use. The newly proposed elongation and flatness indices give rise to Zingg plots which are symmetric along the bisect line of the plot, making their values easier to interpret. Also, they take values

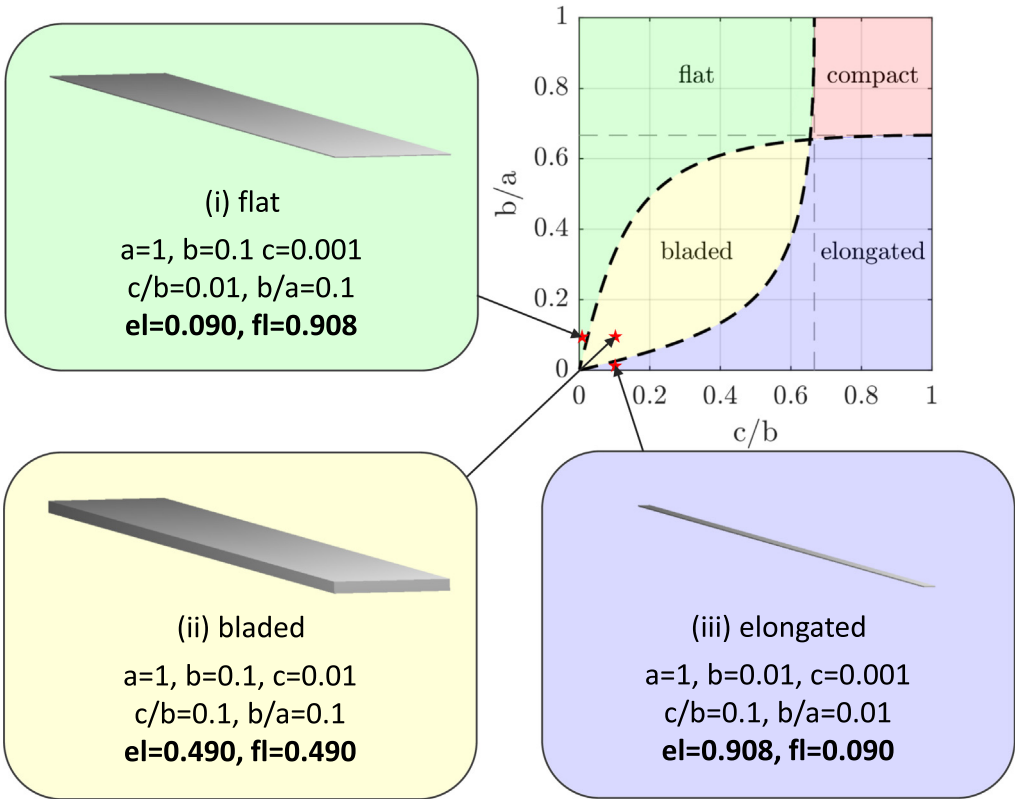

Fig. 7. Classification of three cuboids with extreme aspect ratios using the proposed formulation.

complementary to unity so they can be interpreted as percentages of an overall form. Examples of cuboids are analysed to illustrate the appropriateness of the indices to describe particle form.

Finally, a new particle classification system derived from the proposed indices which overcomes the misclassification of a range of particles that are incorrectly classified as bladed in the Zingg system, is illustrated.

Data availability

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request. The indices proposed in this study have been implemented in the open-source code `SHAPE: SHape Analyser for Particle Engineering` [2], where they can be used to characterise three-dimensional particles.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

We acknowledge Newcastle University for the doctoral scholarship of the first author. This work was partially supported by EPSRC grant No. EP/V053655/1 RAILSANDING - Modelling Particle Behaviour in the Rail-Wheel Interface.

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