

Exact Inference Techniques for the Analysis of Bayesian Attack Graphs (Supplementary Material)

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1 CONDITIONAL PROBABILITY TABLES

In Table 1 we show an example of how to compute the conditional probability tables for the AND and OR cases for a node X_4 which has three parent nodes X_1, X_2, X_3 , i.e. $p(X_4|X_1, X_2, X_3)$, with the associated vulnerabilities v_1, v_2, v_3 that have a probability of successful exploitation of those vulnerabilities of p_{v_1}, p_{v_2} , and p_{v_3} respectively. To simplify the notation in the results of the table we use $q_{v_i} = 1 - p_{v_i}$.

TABLE 1
Conditional probability table $p(X_4|X_1, X_2, X_3)$ for the AND and OR cases.

$\mathbf{Pa}_4$			AND		OR	
X_1	X_2	X_3	$\Pr(X_4 = T)$	$\Pr(X_4 = F)$	$\Pr(X_4 = T)$	$\Pr(X_4 = F)$
0	0	0	0	1	0	1
0	0	1	0	1	p_{v_3}	q_{v_3}
0	1	0	0	1	p_{v_2}	q_{v_2}
0	1	1	0	1	$1 - q_{v_2}q_{v_3}$	$q_{v_2}q_{v_3}$
1	0	0	0	1	p_{v_1}	q_{v_1}
1	0	1	0	1	$1 - q_{v_1}q_{v_3}$	$q_{v_1}q_{v_3}$
1	1	0	0	1	$1 - q_{v_1}q_{v_2}$	$q_{v_1}q_{v_2}$
1	1	1	$p_{v_1}p_{v_2}p_{v_3}$	$1 - p_{v_1}p_{v_2}p_{v_3}$	$1 - q_{v_1}q_{v_2}q_{v_3}$	$q_{v_1}q_{v_2}q_{v_3}$

2 EXAMPLE WITH BELIEF PROPAGATION

To illustrate the message passing process of BP, in Table 2 we show the corresponding messages that need to be computed for the factor graph in Figure 5 in the paper. In this case, we start computing the messages from variables A_1, A_2, A_3 traversing the tree until reaching node G . Then, we traverse back the tree from G to compute the rest of the messages.

3 EXAMPLE WITH JUNCTION TREE

In Table 3 we show the list with all the messages that are needed to calculate the marginal probabilities for the clique tree shown in Figure 6.(b) in the paper. We start computing the messages from the leaf node that contains variables A and B .

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TABLE 2
BP message passing for the factor graph in Figure 5 in the paper.

From nodes to factors	From factors to nodes
$\mu_{A_1, f_1} = 1$	$\mu_{f_1, D} = \sum_{A_1} f_1(A_1, D) \mu_{A_1, f_1} = p(D)$
$\mu_{A_2, f_2} = 1$	$\mu_{f_2, B} = \sum_{A_2} f_2(A_2, B) \mu_{A_2, f_2} = p(B)$
$\mu_{A_3, f_3} = 1$	$\mu_{f_3, C} = \sum_{A_3} \sum_B f_3(A_3, B, C) \mu_{A_3, f_3} \mu_{B, f_3} = p(C)$
$\mu_{B, f_3} = \mu_{f_2, B}$	
$\mu_{C, f_4} = \mu_{f_3, C}$	$\mu_{f_4, E} = \sum_C f_4(C, E) \mu_{C, f_4} = p(E)$
$\mu_{E, f_5} = \mu_{f_4, E}$	$\mu_{f_5, F} = \sum_E \sum_D f_5(D, E, F) \mu_{E, f_5} \mu_{D, f_5} = p(F)$
$\mu_{D, f_5} = \mu_{f_1, D}$	
$\mu_{F, f_6} = \mu_{f_5, F}$	$\mu_{f_6, G} = \sum_F f_6(F, G) \mu_{F, f_6} = p(G)$
$\mu_{G, f_6} = 1$	$\mu_{f_6, F} = \sum_G f_6(F, G) \mu_{G, f_6} = 1$
$\mu_{F, f_5} = \mu_{f_6, F}$	$\mu_{f_5, E} = \sum_D \sum_F f_5(D, E, F) \mu_{D, f_5} \mu_{F, f_5} = 1$
$\mu_{D, f_5} = \mu_{f_1, D}$	
$\mu_{F, f_5} = \mu_{f_6, F}$	$\mu_{f_5, D} = \sum_E \sum_F f_5(D, E, F) \mu_{E, f_5} \mu_{F, f_5} = 1$
$\mu_{E, f_5} = \mu_{f_4, E}$	
$\mu_{D, f_1} = \mu_{f_5, D}$	$\mu_{f_1, A_1} = \sum_D f_1(A_1, D) \mu_{D, f_1} = p(A_1)$
$\mu_{E, f_4} = \mu_{f_5, E}$	$\mu_{f_4, C} = \sum_E f_4(C, E) \mu_{E, f_4} = 1$
$\mu_{C, f_3} = \mu_{f_4, C}$	$\mu_{f_3, A_3} = \sum_B \sum_C f_3(A_3, B, C) \mu_{B, f_3} \mu_{C, f_3} = p(A_3)$
$\mu_{B, f_3} = \mu_{f_2, B}$	
$\mu_{C, f_3} = \mu_{f_4, C}$	$\mu_{f_3, B} = \sum_{A_3} \sum_C f_3(A_3, B, C) \mu_{A_3, f_3} \mu_{C, f_3} = 1$
$\mu_{A_3, f_3} = 1$	
$\mu_{B, f_2} = \mu_{f_3, B}$	$\mu_{f_2, A_2} = \sum_B f_2(A_2, B) \mu_{B, f_2} = p(A_2)$

TABLE 3
Message passing for the JT in Figure 6.(b) in the paper.

From nodes to factors	From factors to nodes
$\mu_{AB, f_1} = 1$	$\mu_{f_1, CD} = \sum_A \sum_B f_1(A, B, C, D) \mu_{AB, f_1}$
$\mu_{CD, f_2} = \mu_{f_1, CD}$	$\mu_{f_2, DE} = \sum_C f_2(C, D, E) \mu_{CD, f_2}$
$\mu_{DE, f_3} = \mu_{f_2, DE}$	$\mu_{f_3, F} = \sum_D \sum_E f_3(D, E, F) \mu_{DE, f_3}$
$\mu_{F, f_4} = \mu_{f_3, F}$	$\mu_{f_4, G} = \sum_F f_4(F, G) \mu_{F, f_4}$
$\mu_{G, f_4} = 1$	$\mu_{f_4, F} = \sum_G f_4(F, G) \mu_{G, f_4} = \sum_G p(G F) = 1$
$\mu_{F, f_3} = \mu_{f_4, F} = 1$	$\mu_{f_3, DE} = \sum_F f_3(D, E, F) \mu_{F, f_3} = \sum_F p(F D, E) = 1$
$\mu_{DE, f_2} = \mu_{f_3, DE} = 1$	$\mu_{f_2, CD} = \sum_E f_2(C, D, E) \mu_{DE, f_2} = \sum_E p(E C) = 1$
$\mu_{CD, f_1} = \mu_{f_2, CD} = 1$	$\mu_{f_1, AB} = \sum_C \sum_D f_1(A, B, C, D) \mu_{CD, f_1} = p(A) p(B A)$