

THE MUON COMPONENT OF EXTENSIVE AIR SHOWERS.

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ABSTRACT

The direct investigation of very high energy cosmic rays ($E > 10^{15}$ eV) by measurements at the top of the atmosphere is impracticable due to the low incident flux. Extensive air showers provide a practical but indirect method of detecting this radiation by measurements lower down in the atmosphere.

Studies of the muon component of showers in which the primary particle has undergone many interactions can reveal features of the primary particle composition. The general properties of very high energy nuclear interactions may also be deduced from measurements of the lateral density distribution and the energy spectrum of the air shower muon component.

This thesis describes the work carried out by the author as a member of the Imperial College Air Shower Group.

Chapter 1 reviews the main features of extensive air showers with particular emphasis being placed on the muon component. The design and operation of the lead shielded liquid scintillator muon detectors is described in Chapter 2. The third Chapter contains an account of an experiment carried out to determine the nature of the sea-level radiation responsible for the uninhibited detector response. The fourth Chapter describes the method of

analysis of the observed shower information which is carried out in order to find the basic parameters of each shower. Chapter 5 describes the experimental method and the data reduction of measurements made to determine the lateral distribution of muons of energy greater than 0.31GeV , at distances between 10m and 1000m from the core, and for showers of size $2 \cdot 10^7$ particles.

The variation of the muon lateral distribution with zenith angle for showers of size greater than about $5 \cdot 10^6$ particles, and the dependence of the relative response of the muon and Cherenkov detectors upon distance and zenith angle is described in Chapter 6.

Measurements on the dependence of the muon content of a shower upon shower size are reported in Chapter 7 for shower sizes ranging from $2 \cdot 10^5$ to 10^8 particles.

Finally conclusions are drawn about the usefulness of the experiment to detect changes in primary composition.

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CHAPTER ONE.

EXTENSIVE AIR SHOWERS (E.A.S.).

1.1. Introduction.

The primary cosmic radiation incident on the top of the atmosphere is known to consist mainly of atomic nuclei, the largest proportion of which are protons. At energies in excess of 10^{14} eV the interactions of primary cosmic rays with air nuclei result in a cascade of secondary particles which at first multiply in number, and then decline due to absorption in the atmosphere.

At the higher primary particle energies the maximum of the cascade occurs deeper in the atmosphere. The cascade is observable at ground level as the simultaneous arrival of a large number of particles spread over a wide area. This phenomenon is known as an extensive air shower (E.A.S.).

The direct detection of primary particles having an energy in excess of about 10^{14} eV by balloon or satellite is impractical due to the low flux of particles at these energies. This flux decreases rapidly with increasing primary energy. However, the magnification of the cross section of the primary particle by the atmospheric cascade makes possible the indirect detection of primaries of all energies in excess of 10^{14} eV, at an acceptable rate.

1.2. Historical Background.

In 1937 Schmeiser and Bothe (Bothe et al., 1937) suggested the occurrence of showers in free air as a result of measurements of the Rossi transition curve. A year later they showed that the particles in these showers could have separations of the order of 40cm. (Schmeiser and Bothe, 1938). Auger and Maze (1938) and Kohlhörster et al. (1938) using horizontally separated Geiger-Muller counters, detected coincidences between counters separated by several tens of metres.

The early work on electron-photon cascade theory (Carlson and Oppenheimer, 1937; Bhabha and Heitler, 1937; Landau and Rumer, 1938) appeared at about the same time as cloud chamber studies of E.A.S. by Janossy and Lovell (1938), in which the multiplication of particles by a lead plate showed that a large proportion of the particles were electrons. Auger (1938) showed the existence of a more penetrating radiation in E.A.S. capable of traversing more than 20cm of lead. This was confirmed by Daudin (1945) using a cloud chamber.

Primary particles of energy below 10^{13} eV were identified as nuclei by balloon flights carrying nuclear emulsions high into the atmosphere. (Freir et al., 1948). The nature of the primary particles of energy above about

10^{14} eV cannot be determined directly. However deductions can be made from measurements of the various components of the shower and on the properties of extensive air showers over a wide shower size range.

1.3. The Components of an Extensive Air Shower.

The particles which constitute an extensive air shower may be divided into three categories, namely:-

- (a) The Nuclear Active Component. This consists of strongly interacting particles such as nucleons, anti-nucleons, mesons and hyperons, [and high energy gamma rays.] The highest energy nuclear active particles are concentrated in the core region of the shower.
- (b) The Soft Component. This consists of electrons, positrons and photons. The soft component particles spread out to large distances from the core.
- (c) The Muon-neutrino Component. This consists of approximately equal numbers of positive and negative muons and approximately equal numbers of neutrinos and anti-neutrinos. The muons spread out to distances in excess of 1km from the core.

The nuclear cascade forms the backbone of a shower. The nucleon interaction mean free path is of the order of 100gm /cm^2 (Fukui et al., 1960A; Hasegawa et al., 1962),

so that the primary particle suffers approximately 10 collisions in the vertical atmosphere.

The main features of the interactions of energy between 10^{11} eV and 10^{16} eV have been summarised by McCusker (1965). They are:-

- (1) The interaction mean free path of protons in air is of the order of 90gm/cm^2 .
- (2) The elasticity of proton air nucleus collisions varies from about 0.3 to about 0.9 and on average, the protons retain about 55% of their pre-collision energy. The elasticity of pion-nucleus collisions is much smaller.
- (3) The production of pions arises mainly from the decay of a "fireball" (Cocconi, 1958; Ciolek et al., 1958), but a small contribution may arise from baryon isobars.
- (4) The "fireball" pions acquire a transverse momentum of about $400\text{MeV}/c$ whereas the nucleon acquires a somewhat higher transverse momentum. (Akashi et al., Japanese and Brazilian Emulsion Group, 1965).

In these high energy collisions, kaons and hyperons will also be produced. The charged pions (mean life $\simeq 2.6 \times 10^{-8}$ sec) can either decay into muons and neutrinos (or anti-neutrinos) or interact with air nuclei. (The nuclear interaction mean free path of charged pions is about 100gm/cm^2). The decay process is more favourable in

regions of low atmospheric density and for low pion energy. This is a result of the reduced thickness of matter encountered in a given time. Neutral pions have a very short mean lifetime ($\approx 10^{-16}$ sec) and decay into two gamma rays which, in turn, produce electron-positron pairs. The electrons and positrons produce gamma rays by the bremsstrahlung process. The alternate processes of pair production by gamma rays and of gamma ray production by electrons and positrons continue until the energy of the electrons and positrons has fallen below the critical energy in air (≈ 84 MeV).

Estimates of the total energy carried by the various components of an extensive air shower at maximum development have been made by Zatsepin et al., (1963). For a shower of size of the order of 3.5×10^5 particles at the observation level (3,860m a.s.l.), the total energies were found to be:-

$$\begin{aligned} \text{Electron Photon Component} &: (0.75 \pm 0.16) \times 10^{14} \text{ eV} \\ \text{Muon Component} &: \left(0.9 \pm 0.9 \right) \times 10^{14} \text{ eV} \\ \text{Nuclear Active Component} &: \left(0.3 \pm 0.3 \right) \times 10^{14} \text{ eV} \end{aligned}$$

From measurements of the Cherenkov light emitted by the charged particles above the observation level (Chudakov et al., 1960), an estimate of the energy lost by ionisation was made. This was found to be:-

$$\text{Energy lost by Ionisation} : (3.4 \pm 1) \times 10^{14} \text{ eV}$$

The energy carried by the neutrino flux at the observation level was estimated to be of the order of 0.65×10^{14} eV.

1.4 The High Energy Primary Cosmic Radiation.

The energy spectrum of cosmic rays having an energy E_0 in the range 10^{10} eV $< E_0 < 10^{14}$ eV is well established from geomagnetic data at the lower energies ($\approx 10^{10}$ eV) (McDonald, 1958; Webber, 1958); from emulsion measurements made at balloon altitudes (Lal, 1953; Kaplon et al., 1952), and from observations on the muon component of small extensive air showers underground. (Barrett et al., 1952). In this energy range the integral spectrum is well represented by a power law of the form:-

$$F(>E_0) = k(E_0/6.10^{14})^{-\gamma} \text{ h}^{-1} \text{ m}^{-2} \text{ ster}^{-1}$$

where E_0 is the particle energy in eV.

$$k = (1.9 \pm 0.4) \times 10^{-2}$$

$$\text{and } \gamma = (1.62 \pm 0.05) \quad (\text{cf. Zatsepin et al., 1963}).$$

At energies beyond 10^{14} eV the energy spectrum is deduced from measurements of the frequency of occurrence of a given total number of particles observed in extensive air showers. The calculated shower size primary energy relationship depends upon the nucleon-cascade model used. If a relationship of the form:-

$N = BE_0^{\lambda}$ is assumed, Olberts calculations (Olbert, 1957) for a primary proton of energy E_0 producing a shower at sea-level of size N give the following values for B and λ :-

$$B = 6.10^{-12} \text{ and } \lambda = 1.1 \text{ (Fermi model).}$$

$$B = 5.10^{-13} \text{ and } \lambda = 1.16 \text{ (Landau model).}$$

Ueda (1960) has computed the development of showers using models in which the inelasticity and interaction mean free paths are varied. The ratio of the primary energy E_0 to the shower size $N_{\max}$ at maximum development is found to vary in the range $(1.9 - 2.6).10^9 \text{ eV}$, the corresponding inelasticity values ranging from 0.3 to 0.7. Measurements of the energy carried by the secondary particles at the observation level can also be used to deduce the shower size primary energy relationships. (Greisen, 1960; Nikolskii, 1962).

Measurements of the size spectrum at both sea-level and at mountain altitudes have been carried out by many groups. At sea-level, most of the results show a marked steepening in the slope of the size spectrum at a size of $10^5 - 10^6$ particles, corresponding to a primary energy of approximately 10^{15} eV . (Allan et al., 1962; Zatsepin et al., 1963; Miura and Hasegawa, 1962; Fukui et al., 1960; Kulikov and Khristiansen, 1959). The change in the slope as observed at sea-level is also found to occur at mountain

altitudes. (Kameda et al., 1960; Clark et al., 1963; Bradt et al., 1965). Above a shower size of about 10^9 particles the data on the shower size spectrum is sparse. However the measurements made in this region (Linsley, 1963) suggest that the steepening of the slope in the size range $\approx 10^5$ - 10^6 particles is followed by a flattening at a size of about 10^9 particles. In addition to changes in the shower size spectrum at sizes of $\approx 10^5$ - 10^6 particles, other properties of extensive air showers are also found to change at about the same size. Chatterjee et al. (1963) and Danilova and Nikolskii (1963) have measured the variation of the total number of nuclear active particles as a function of shower size. The results of these two experiments have been compared by Danilova (1965). He finds that (for low energy nuclear-active particles) the experimental results do not contradict a gradual change of the exponent δ of the power law ($Nn.a.p. \sim N^\delta$) from a value of 0.36 for $N \approx 4 \times 10^4$ particles to unity for $N \approx 2 \cdot 10^6$ particles, with an increase in δ below $N \approx 10^4$ particles. It is suggested that, beginning with protons, the discontinuity in the dependence of $Nn.a.p.$ upon N may be a result of the transfer of a large portion of energy into the electron-photon component by a process other than ordinary pionisation (Nikolskii and Pomanski, 1960). The theories which have been proposed to

explain the variation of the dependence of the total number of nuclear active particles on shower size at a size of about $2 \cdot 10^5$ particles are as follows:-

- (a) A change occurs in the characteristics of the elementary act of the nuclear interaction at an energy of about $3 \cdot 10^{14}$ eV. (Nikolskii et al., 1956; Nikolskii and Pomanski, 1960).
- (b) A change occurs in the primary composition at a similar energy (Peters, 1961).
- (c) A change occurs in the role of the unstable particles with primary energy. (Narayan and Yodh, 1960).

Changes in the behaviour of the muon component have also been detected in showers of size $\approx 10^5 \pm 10^6$ particles. Miura and Hasegawa (1962) report a kink in the muon number spectrum in this shower size range. Since the muon component is fairly insensitive to the point of initiation of a shower, this kink is evidence for a corresponding change in the energy spectrum at an energy of the order of 10^{15} eV. A flattening in the slope of the number spectrum of high energy muons is found by Khristiansen et al. (1965). It is suggested that this may be correlated with the flattening of the shower size spectrum for showers of size greater than about 10^7 particles (Linsley, 1963).

In a shower size range of $10^5 - 10^7$ particles, the proportion of showers which exhibit multiple core structure

is found to increase (Bray et al., 1965; Wimm et al., 1965; McCusker, 1963). For showers of size between 10^5 and 10^6 particles the latter workers found approximately equal numbers of multiple cored and single cored showers. However, above a shower size of 10^6 particles, multiple cored showers are found to predominate. The maximum central density of single cored showers is found to be much greater, on average, than that of multiple cored showers. (i.e. for a given shower size). The overall conclusion is that single cored showers result from proton primaries, and multiple cored showers from heavier primaries. It is suggested that the lack of single cored events in showers of size greater than 10^6 is indicative of a change in the primary composition at an energy of about $3 \cdot 10^{15}$ eV; and that this is probably due to an upper limit to the particle rigidity in the acceleration of galactic primaries. The Kiel Group (Samorski et al., 1965) have used a large neon hodoscope (described by Bagge et al., 1965) of area 32m^2 to study the electro-magnetic core structure. In an analysis of two double core events they point out that, if one assumes that both cores are due to pions originating in the same nuclear interaction, then the minimum transverse momenta required would be approximately 10 times larger than those observed in nuclear emulsion measurements (Lattes et al., 1964). This is taken as evidence in support of the view that double cores are due to interactions of the nuclei

of a heavy primary particle.

Measurements on the absorption of shower particles in showers of size 10^4 - 10^6 particles (at an altitude of 3,860m.a.s.l.) reveal a change in the proportion of the energy carried by the nuclear-active and the electron-photon components as the shower size increases. This change occurs at a size of about 10^5 particles (Nikolskii and Pomanski, 1959).

In a shower size range 10^5 - 10^9 particles, deviations from a smooth variation with shower size are found to occur in the slope of the size spectrum, the muon size spectrum, the proportion of nuclear-active particles, and the proportion of showers having multiple cores. Various explanations of these discontinuities have been proposed.

One suggestion is that a change occurs in the nature of the nuclear interaction at an energy of the order of 10^{15} eV. Recent experimental evidence is however in favour of an explanation in terms of a kink in the energy spectrum of the primary cosmic radiation. This may be explained in terms of a model of diffusion of cosmic rays in the galaxy and metagalaxy in which the diffusion coefficient depends upon the energy per nucleon of the particles (Fomin and Khristiansen, 1964; Syrovatskii et al., 1964). An alternative (or complementary) explanation is that the coefficient of

diffusion of cosmic rays from their region of acceleration depends upon the energy per nucleon of the particles.

The very strong altitude dependence of the density spectrum observed by Swinson and Prescott (1965) and by McCaughan et al. (1965) suggest that there exists a cut-off energy per nucleon compatible with a break in the primary rigidity spectrum.

The proportion of heavy nuclei in the primary composition should increase as a consequence of a cut-off in the primary rigidity spectrum, corresponding to an energy per nucleon of about 10^{15} eV. The resultant cut-off of the galactic component at an energy of about 10^{17} eV should then reveal the lower intensity metagalactic component.

Fukui et al. (1960) suggested that a measurement of the proportion of muons in E.A.S. might reveal information on the nature of the primary particles. For this reason, an experiment was set up by the Imperial College Group to investigate the muon component of extensive air showers at sea-level in a shower size range 10^5 - 10^8 particles, corresponding to the region of primary energy of most interest.

1.5. The Muon Component.

Muons constitute the penetrating component of extensive air showers. They are the decay products of

charged π and K mesons which are produced in nuclear interactions in the atmosphere. Low energy muons of energy less than about 10^{11} eV are produced primarily by π^+ meson decay processes, with a contribution of uncertain magnitude from K-meson decay at the higher energies. Pion decay occurs when the energy of the particle has been degraded sufficiently for the probability of interaction to be small. The highest energy muons tend to come from the upper regions of the atmosphere where the density of matter is low and the probability of pion decay compares favourably with the interaction probability.

As a result of the preference of the low energy pions for decay rather than interaction, the lateral distribution and energy spectrum of low energy muons are more intimately connected with the low energy nuclear interactions ($E < 10^{12}$ eV) than with the nature of the collision of a primary particle with an air nucleus. Nuclear interactions at energies of the order of 30 GeV can be studied very precisely using particle accelerators. From the nuclear physics point of view therefore, there is little value in a measurement of the muon lateral distribution as a means of inferring the distribution of transverse momenta given to the parent pions in low energy interactions. A more useful quantity is the

total number of muons N_μ in a shower at the observation level, and the variation of this quantity as a function of shower size. A change in the primary composition should reveal itself in the E.A.S. muon-electron ratio, since showers produced by heavy primary nuclei should be relatively richer in muons.

Muon detectors take the form of charged particle detectors placed under thick absorbers or buried deep underground. Measurements of the energies of muons are made using large magnetic spectrographs placed beneath a thickness of absorber sufficient to eliminate the electron component.

The first measurements of the muon lateral distribution were made by Alekseev et al. (1948) by recording the number of coincidences observed by the detectors as a function of their separation. More recent measurements close to the core have been carried out by Abrosimov et al. (1958), Andronikashvili and Bibilashvili (1957), Vavilov et al. (1957), Khrenov (1962), Kazarov and Liudvigov (1959), and Matano (1962). At larger distances from the core ($> 100\text{m}$), investigations have been carried out by Antonov et al. (1957), Abrosimov et al. (1960), Porter et al. (1957), Linsley et al. (1962), Bennett and Greisen (1961), and Allan et al. (1965). The muon lateral

distribution is a function of the threshold energy of the detectors. In addition, measurements are carried out at various altitudes and over different ranges of shower size, complicating any direct comparison of results. It is generally agreed however, that the lateral distribution function is independent of shower size. No variation with altitude has been detected which is greater than the limits of experimental accuracy. Greisen (1960) has fitted a curve to the muon density measurements of the M.I.T. Group (Clark et al., 1958) and others, to produce an empirical relationship giving the density of muons ρ_μ of energy greater than 1 GeV in a shower of size N at a distance r metres from the axis:-

$$\rho_\mu = 18(N/10^6)^{0.75}(1+r/320)^{-2.5}r^{-0.75} \dots\dots\dots 1.1$$

valid for $20m \leq r \leq 500m$.

Nikolskii (1963) has also taken the data of many workers to give a generalised expression:-

$$\rho_\mu = ((0.22p(N/10^5)^{0.81})((2+r)^{-0.75}e^{-r/r'})) / (2.10^6(p-500)+E_\mu)^{0.25} \dots\dots\dots 1.2$$

validity:- $E_\mu \geq 200MeV$; $10^3 \leq N \leq 10^8$; $p \geq 500 \text{ gm/cm}^2$.

E_μ is the detector threshold energy in eV and p is the atmospheric depth at the observation level in gm/cm^2 .

r' depends upon E_μ and takes the values:-

$$r' = 300m \text{ for } E_\mu \geq 4.4 \times 10^8 \text{ eV.}$$

$$r' = 220m \text{ for } E \gtrsim 10^9 \text{ eV.}$$

$$r' = 100m \text{ for } E \gtrsim 5 \cdot 10^9 \text{ eV.}$$

Close to the axis the muon density decreases with distance approximately as r^{-1} steepening to a variation proportional to $r^{-2.2}$ at distances greater than 300m from the core.

The effects of Coulomb scattering and deflection in the earth's magnetic field on the muon lateral distribution have been shown to be small, (Vavilov et al., 1957; Antonov et al., 1957; Hillas, 1965) for vertically incident showers. The latter author shows however, that although the overall density distribution is not altered by the earth's field, the density distribution of positive and negative muons can be, for a shower which approaches from a direction perpendicular to the magnetic field lines.

The dominant factor responsible for the wide radial spread of the muon component is the transverse momentum p_T given to the parent pions in the nuclear interaction. The distribution of p_T found from emulsion and cloud chamber studies is well represented by:-

$$f(p_T) dp_T \sim p_T \exp(-2p_T/\bar{p}_T) dp_T$$

$$\text{where } \bar{p}_T \simeq 0.32 \text{ GeV/c.}$$

A number of theoretical calculations on the muon lateral distribution in air showers have been made (Hillas,

1965; Holyoak et al., 1965; Wdowczyk, 1965). The calculation of Wdowczyk was concerned with the muon distribution which arises from the photo-production of nuclear active particles in a shower initiated by a primary photon of energy 10^{15} eV.

The muon energy spectrum in E.A.S. has been measured over a wide range of shower size and distance from the core. Measurements of the integral energy spectrum of muons (using the absorption technique) have been made in an energy range of 0.3 GeV to 3.5 GeV by Dovzhenko et al. (1957) at an altitude of 3,860 m a.s.l. At sea level several investigations have been carried out. Sakvarelidze (1956), Andronikashvili and Kazarov (1960), Vernov et al. (1960) and Barrett et al. (1952) used the absorption technique whereas Rochester et al. (1965) and Bennett and Greisen (1961) used a magnetic spectrograph. The latter workers give an empirical relationship for the muon energy spectrum of the form:-

$$\rho(r, > E) \sim (51/(E+50))(3/(E+2))^{0.14} r^{0.375}$$

where E is the muon energy in units of one GeV and r is the distance (in metres) from the core. The equation is valid for $25\text{m} < r < 500\text{m}$.

De Beer et al. (1962) obtained a muon energy spectrum from observations of the scattering of muons in lead. Their results are inconsistent with those of the workers

listed above.

Spectrograph measurements of the charge ratio of muons in extensive air showers have been carried out by Rochester et al. (1965) and Bennett and Greisen (1961). Both sets of measurements are consistent with a value which does not differ significantly from unity, indicating the relative unimportance of K-meson production. (for which the plus-minus ratio is large.)

The dependence of the number of muons in a shower upon the shower size has been measured by many workers under various experimental conditions. Almost all of the measurements show that the fraction of muons in a shower decreases with increasing shower size. This was shown firstly by Cocconi et al. (1949). More recent measurements have been made in which the basic parameters of the shower (e.g. core location, zenith angle, shower size) have been accurately determined. (Linsley et al., 1962; Khrenov, 1962; Fukui et al., 1960; Clark et al., 1958; Vernov et al., 1965; Khristiansen et al., 1965; Chatterjee et al., 1965; Allan et al., 1965). Theoretical calculations on the muon number shower size relationship have been made by Hillas (1965) and Ueda and Ogita (1957). Nikolskii (1963) has summarised the data on the muon number shower size dependence and finds that the data available to him can be fitted, over a shower

size range of 10^4 to 10^9 particles by an expression:-

$$N_{\mu} \sim N^{0.81 \pm 0.03}$$

The existence of showers in which the muon content is low ($\leq 1/30$ of expected), consistent with the initiation of the shower by a gamma ray primary particle has been established by Firkowski et al. (1962), Firkowski et al. (1963), Hasegawa et al. (1962) and the B.A.S.J.E. Group, (Clark et al., 1962). More recent results presented by the B.A.S.J.E. Group (Toyoda et al., 1965) and by Gawin et al. (1965) and Matano et al. (1965) confirm the existence of mu-poor showers. In the primary energy range 10^{15} eV to 10^{16} eV the relative frequency of mu-poor showers is found to be approximately $2 \cdot 10^{-4}$ of the total. (i.e. at the same primary energy.) It is pointed out that this value is to be regarded as an upper limit for the frequency of the primary gamma rays, due to the possibility of extreme fluctuations in the first nuclear interaction producing a large number of neutral pions. Such a process has been shown to be possible by Narayan (1964). An anisotropy of the mu-poor showers may be expected if the most important mechanism for their production is the initiation by gamma rays which originate from the decay of π^0 mesons produced in the collisions of cosmic rays and hydrogen nuclei. This is because the distribution of cosmic rays and protons

in the universe is not necessarily uniform! On the other hand, no anisotropy would be observed if low- μ showers were produced as a result of extreme fluctuations in the interactions between the highly isotropic charged primary cosmic rays (energy less than about 10^{16} eV) with atmospheric nuclei. The B.A.S.J.E. Group found a statistically significant peak in the distribution of arrival direction in right ascension between 200° and 220° .

The main mechanism responsible for the production of primary gamma rays of energy above 10^{16} eV is neutral pion production arising from collisions of cosmic ray protons of energy $\geq 10^{17}$ eV with thermal photons in intergalactic space. (Hayakawa, 1962; Hayakawa and Yamamoto, 1963). The B.A.S.J.E. Group did not find any low μ showers in the energy range 10^{16} eV to 10^{17} eV. Making the assumption that the energy density of intergalactic thermal photons is ≈ 0.1 eV/cc (Gerasimova and Rozental, 1962), they came to the conclusion that, at an energy of $2 \cdot 10^{17}$ eV, the extra-galactic cosmic ray proton intensity is less than 6½% of the terrestrially observed cosmic ray proton intensity. Matano et al. (1965) give an upper limit of $5 \cdot 10^{-3}$ for the ratio of gamma ray intensity to proton intensity for showers of size greater than 10^7 particles at sea-level.

The recent discovery by Penzias and Wilson (1965) of

an isotropic radiation at a wavelength of 7.4 cm, later confirmed as thermal in character by the measurements of Roll and Wilkinson (1966) (at 3.2 cm wavelength) gives new meaning to the interpretation of the detection rate of mu-poor showers. The radiation is believed to be universal and to have a black body temperature of about 3.5°K . The energy density of black body radiation at this temperature is of the order of 1 eV/cc and the mean photon energy is about 10^{-3} eV so that the expected number density of these photons in the universe is appreciable. ($\approx 10^3/\text{cc}$).

Gould and Schröder (1966) have estimated the effect of high energy photon absorption as a result of electron positron pair production in photon-photon collisions. From the calculated absorption probability curve for high energy photons traversing a black body photon gas at 3.5°K , they predict that no mu-poor showers will be found as a result of primary photon interactions with atmospheric nuclei for primary energies of $10^{16}\text{eV} - 10^{17}\text{eV}$. This is in accord with the latest experimental observations on mu-poor showers.

On the other hand, showers which are rich in their muon content (mu-rich) are likely to have been produced by heavy primary particles. The first investigation of the anisotropy of mu-rich showers was reported by

Hasegawa et al. (1962A). Recent work on the anisotropy of mu-rich showers has been reported by Chatterjee et al. (1965) and Matano et al. (1965). The latter workers selected showers in which the muon content was more than 60% of the maximum of that in air showers of the same size. ($10^6 < N < 10^7$). They found a decrease in the distribution in right ascension between 15h and 21h. For showers of sizes between 10^5 and 10^6 particles the arrival directions of mu-rich showers are found to be isotropic (Matano et al. 1964). However Chatterjee et al. (1965) did not find any anisotropy of mu-rich air showers in the size range investigated. ($10^5 < N < 10^7$). The question of mu-rich showers is therefore uncertain at the present time.

1.6 The Muon Measurements at Haverah Park.

The shower detection equipment at Haverah Park (230m a.s.l.) is capable of detecting showers of sizes lying between about 10^5 - 10^8 particles. This corresponds to a range of primary energies in which the primary composition is thought to consist of an increasing proportion of heavy nuclei, up to a cut-off energy per nucleon of the order of 10^{15} ev. At energies above about 10^{17} ev the primary radiation is thought to consist of protons of metagalactic origin. Changes in the primary composition should be reflected in a

thorough investigation of the muon component over a wide shower size range.

The work described in this thesis concerns the investigation of the muon lateral distribution at distances lying between 10m and 1000m from the core, the variation of the lateral distribution with zenith angle, and the features of the muon content of showers as a function of shower size.

CHAPTER TWO.

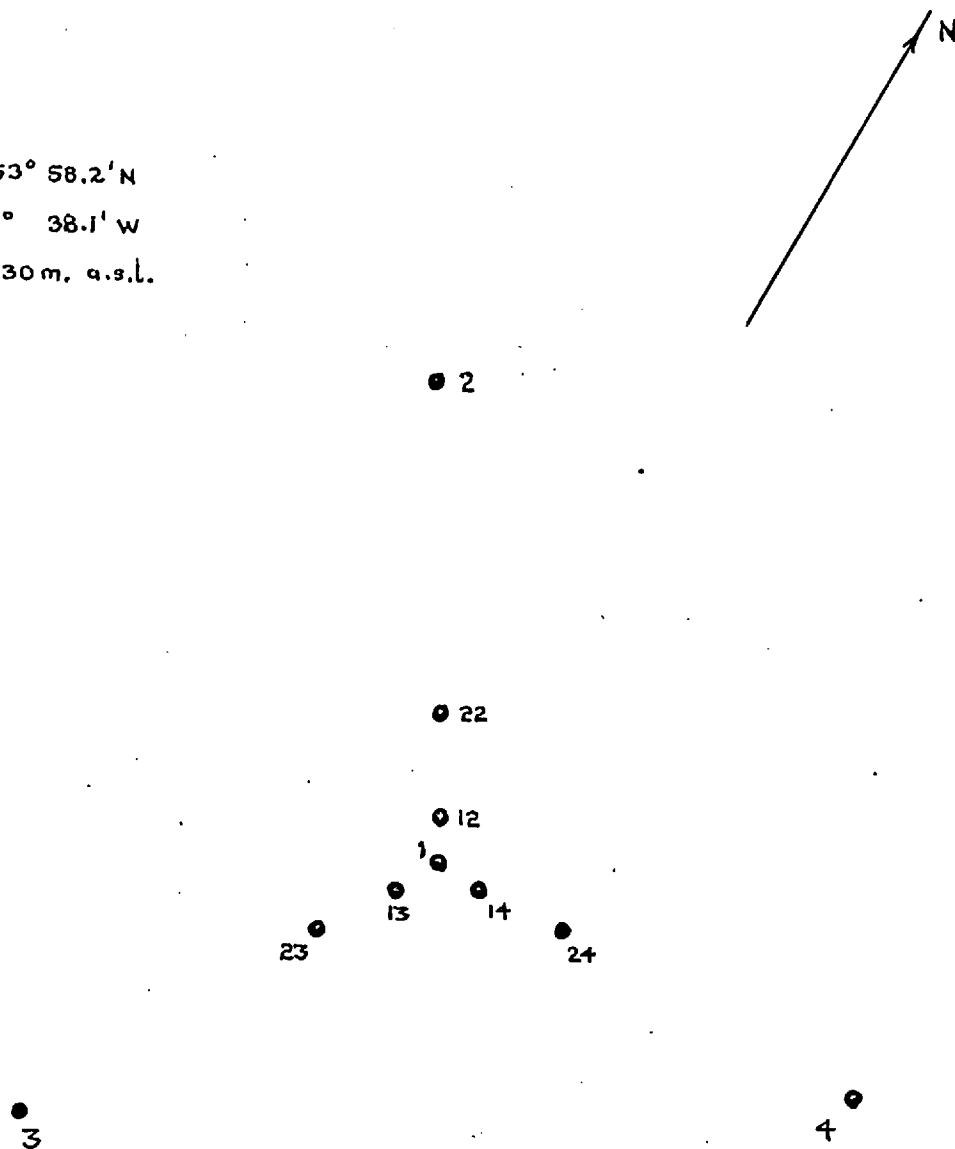
THE DETECTORS USED FOR MEASURING THE MUON COMPONENT OF E.A.S. AT HAVERAH PARK.

2.1. The Haverah Park E.A.S. Arrays.

The basic array configuration used at Haverah Park for the detection of extensive air showers is the same as that used by Allan et al. (1960) for the investigation of E.A.S. of sizes lying between $2 \cdot 10^5$ and $5 \cdot 10^6$ particles. At Haverah Park, three such arrays have been built in symmetrical formation as shown in figure 2.1.

Each array consists of three outside detecting stations situated at the corners of equilateral triangles and one common central station situated at the centroid of the triangle. The three arrays are referred to as the 50 metre, 150 metre and 500 metre arrays, these being the distances measured from the central station to the three outer detecting units. Each individual Cherenkov detector consists of a galvanised iron tank (1.83m x 1.22m x 1.22m deep) lined with diffusely reflecting white plastic (I.C.I. Darvic), filled with water to a depth of 1.1m and sealed with a 10 gauge aluminium lid, holding a photomultiplier head unit. A fraction of the Cherenkov light produced by relativistic charged particles passing through the water is converted by

LATITUDE : $53^{\circ} 58.2' N$
LONGITUDE : $1^{\circ} 38.1' W$
ALTITUDE : 230 m. a.s.l.



THE HAYERAH-PARK. E.A.S. ARRAY.

SCALE : METRES

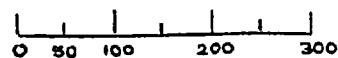


FIG 2.1

the 5" photomultiplier (E.M.I. type 9618YB) into an electrical pulse which, after passing through the head-unit has a rise time of less than 0.1 microseconds and a decay time of 8 microseconds. The pulses from each of the tanks at a particular station are added together, and those from the outer stations are passed down underground cable (Aerialite 363) to the central station where facilities exist for the photographic recording of pulses associated with E.A.S. on 35mm film.

The 500 metre array detecting stations each consist of 16 tanks, 15 of which are normally in use with one spare. The 50 metre and the 150 metre arrays have 2 tanks in each of the outer stations and 4 tanks in the central station. The 4 tanks at the central station are in fact four adjacent tanks of the 500 metre array.

Each array uses a four-beam oscilloscope (Allied Electronics) for the recording of shower pulses. The time bases of the oscilloscopes are triggered by the associated array whenever a certain density of particles is exceeded at two of the outer stations of an array and the central station. Each event is recorded on film for subsequent reading and analysis. Blanking pulses at a fixed distance apart (in time) are put on to each time base so that information about the direction of arrival of a shower can be obtained from

measurements of the relative positions of the pulses on the traces. The amplitudes of the pulses from each of the stations are converted to a nominal density of shower particles via a calibration procedure using the mean pulse height produced by single vertical muons passing through a detector.

2.2. The Muon Detectors.

The muon detectors used at Haverah Park during the past 20 months in conjunction with the arrays described in section 2.1. are liquid scintillation detectors shielded by 20cm of lead. The basic design work on this type of detector was done by Dr. J. C. Barton (Barton et al., 1962; Barton and Crispin, 1964). The scintillation liquid used is medicinal liquid paraffin (Boots) containing para-terphenyl and POPOP. The total sensitive area of the four detectors is 10.14 sq. m.

2.3. Basic Requirements for the Haverah Park Muon Detectors.

The type of detecting unit used for the Haverah Park muon experiment was decided after various considerations.

One very important factor was the minimum useful area. As the experiment was at first intended to measure muon densities out to distances of the order of 1000m where the

density of muons in a shower of size 10^8 particles is $\approx 0.1/\text{sq.m.}$ (Greisen, 1960), an area of 10 sq.m. would detect on average, one muon.

In order to absorb all of the electron-photon component of the E.A.S. the detector must be adequately shielded. To minimise the cost of expensive shielding material a shallow detector of large area seemed ideal. Another consideration was the minimisation of scattering from the lead shielding into the adjacent Cherenkov tanks.

The scintillator had to be chemically stable over a long period of time, at possibly high ambient temperatures, since the detectors could not readily be buried.

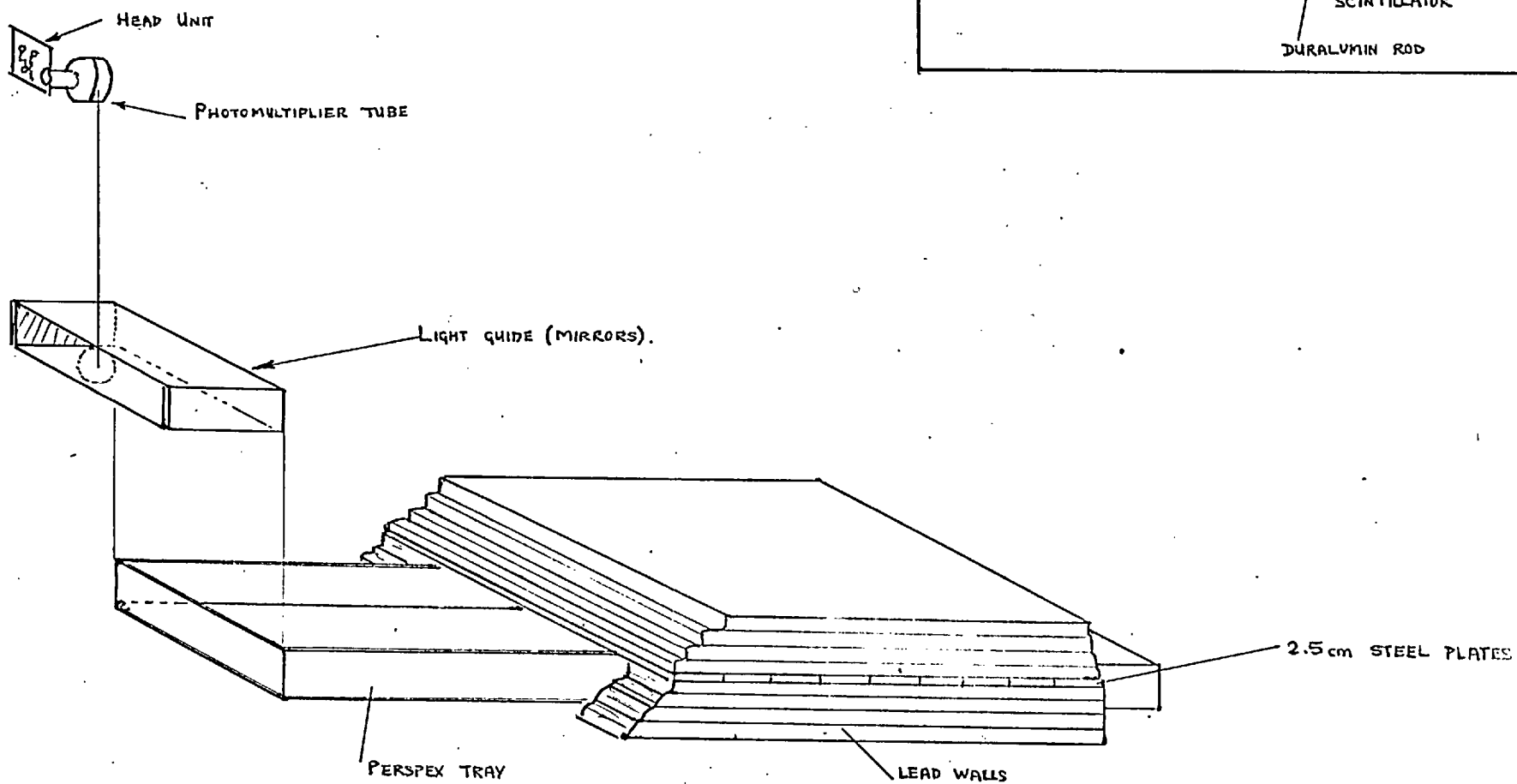
2.4. Design and Performance.

A sketch of the scintillator detector is shown in fig. 2.2. The scintillator is contained in a perspex tray (2.44m x 1.07m x 10cm deep) supported on six 1.25 cm. diameter duralumin rods providing an air gap for total internal reflection of the scintillation light at the perspex-air interface. Mirror light guides at both ends of the tray isolate the photomultiplier tubes so that particles cannot dissipate large amounts of energy by the scintillation process in the vicinity of the photo-cathodes.

The scintillator is Boots medicinal paraffin in which

FIG 2.2

THE LIQUID - SCINTILLATOR DETECTOR.



is dissolved 0.6 gm/litre of para-terphenyl and 6 milligrams/litre of POPOP. It was found necessary to heat the liquid paraffin to a temperature of $\approx 373^{\circ}\text{K}$ to facilitate the dissolving-process. This concentration of para-terphenyl was chosen to minimise the oversaturation and consequent deposition of crystals in the event of the scintillator temperature falling abnormally.

When a charged particle passes through an organic scintillator, energy is lost by ionisation, excitation and to a lesser extent, dissociation of the molecules along the particle path. The mechanism by which this energy is transferred to the molecules of the primary solute is not well known but collisions involving exciton transfer and resonance interaction are probably responsible.

The light output from organic scintillators is not proportional to the energy loss when the ionisation density along the particle track is large. This loss of uniformity is attributed to 'damage' of the solvent molecules when the dissipation of energy in a given distance is high. (Brooks, 1956). In the present experiment, however, the detectors are used exclusively for the detection of muons almost all of which will be near minimum ionisation.

2.5. Shielding.

Twelve thousand lead blocks of size 15.2cm x 5cm x 5cm were used to shield the detectors against the soft component particles. The complete absorption of the electron-photon component of showers was ensured using a 20cm thickness of lead shielding above and at the sides of the detector. (i.e. 39 radiation lengths of lead). This particular thickness of lead was decided from the results of a subsidiary experiment described fully by Blake (1966).

The lead above the detector was supported by steel plates 2.5 cm. thick forming a bridge between the lead walls. A 30 cm. overlap of lead was provided at each end of the detector. The whole detecting unit was therefore screened against all soft component particles except those having a high zenith angle, travelling in a narrow band of azimuth.

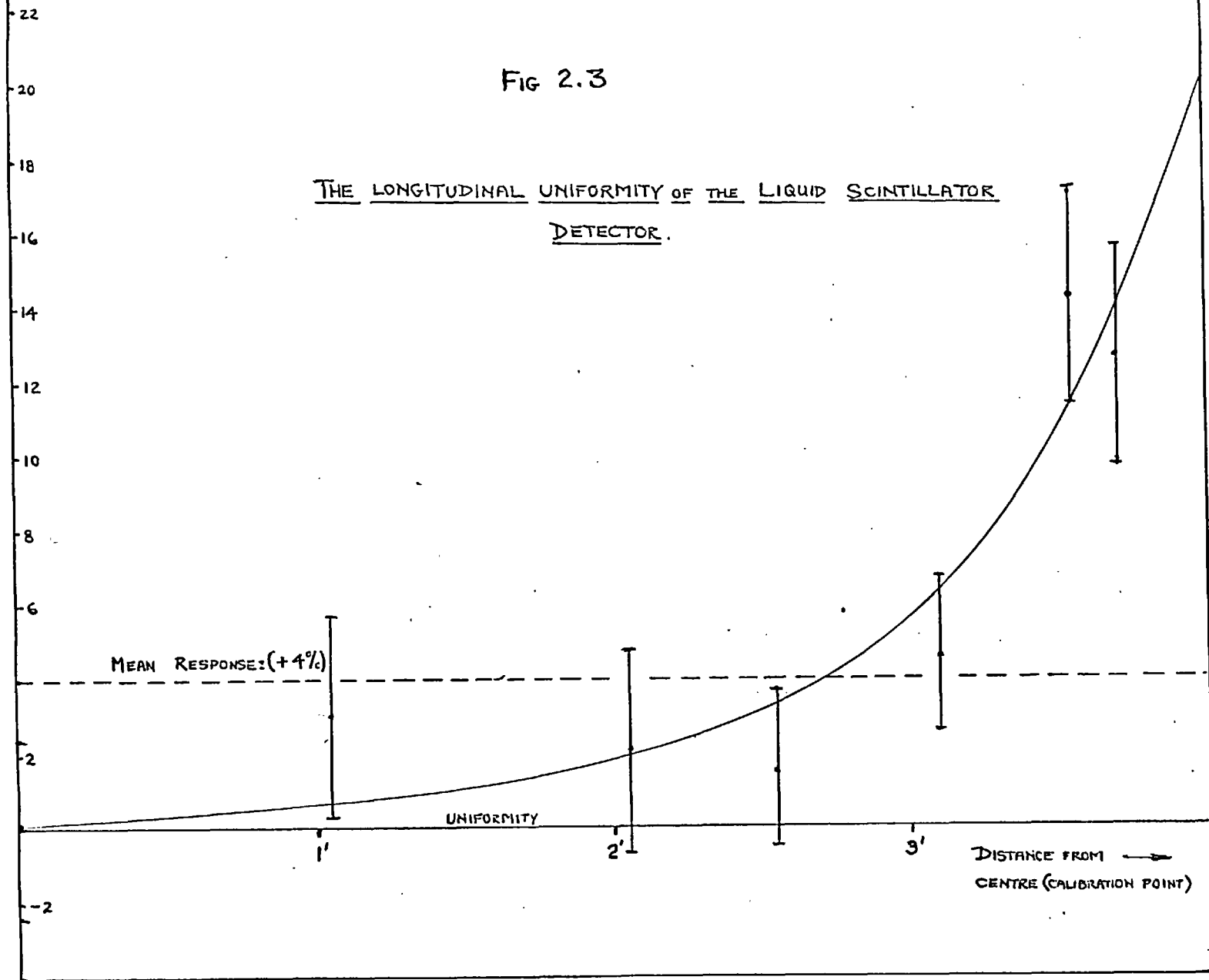
2.6. Response to Single Particles.

2.6.1. Uniformity.

The response of the detector to single vertical muons at various positions over the area of the detector is shown in fig. 2.3 as a percentage increase over the response at the centre. The mean response to vertically incident particles for the whole detector is approximately 4% above the value obtained by calibrating the detector at the central position.

FIG 2.3

THE LONGITUDINAL UNIFORMITY OF THE LIQUID SCINTILLATOR
DETECTOR.



The scintillation telescope was used to select muons travelling at an angle θ to the vertical and the response was found to vary as $\sec \theta$. Hence to a good approximation, neglecting edge effects, the density of muons in showers sampled by the detector is proportional to the pulse height, independent of zenith angle.

2.6.2. Calibration.

The number of millivolts output required per vertical muon was chosen, and a stable discriminator was then used to obtain a count-rate for each tube, corresponding to a level of two particles passing simultaneously through the detector. It was possible to equalise the rates to within 5% by fine adjustment of the photomultiplier E.H.T. supply voltage. The rate R , at which particles exceeding a number 'n' pass through the detector can be written as:

$$R(> n) = kn^{-\gamma}$$

where k is a constant and γ is the slope of the integral spectrum. In the $n = 2$ region γ is of the order of 3.

Now $\frac{dR}{R} = -\gamma \frac{dn}{n}$ so that an uncertainty of 5% in the integral count rate R would represent less than 2% uncertainty in the millivolts/particle output. An absolute measure of the output per particle was obtained by using a three-fold-coincidence narrow-angle plastic scintillator

telescope (selecting vertical muons passing through the centre of the detector), to gate detector pulses fed into a 400 channel pulse spectrometer, (Victoreen 400M). A mercury relay pulse generator producing simulated scintillator detector pulses was used to calibrate the spectrometer channels in terms of millivolts.

The single particle pulse height distribution obtained over a period of one week is shown in fig. 2.4 together with a fitted Poissonian distribution having a mean of 7. The broadness of the observed distribution results mainly from the relative inefficiency of the liquid scintillator. If an efficient plastic scintillator is used (Towers, 1966) a much greater light output is obtained for the same energy loss. This results in a larger number of photo-electrons emitted from the photo-cathode of the photomultiplier tube with the consequent narrowing of the distribution of the number of electrons produced in the multiplication by the first few dynodes. (At the first few dynodes the statistical variation in the number of emitted electrons has the most serious effect on the final output of the photomultiplier tube). The width of the single particle distribution will also be widened due to the contribution of a small percentage of knock-on electrons and imperfect experimental conditions, such as the finite

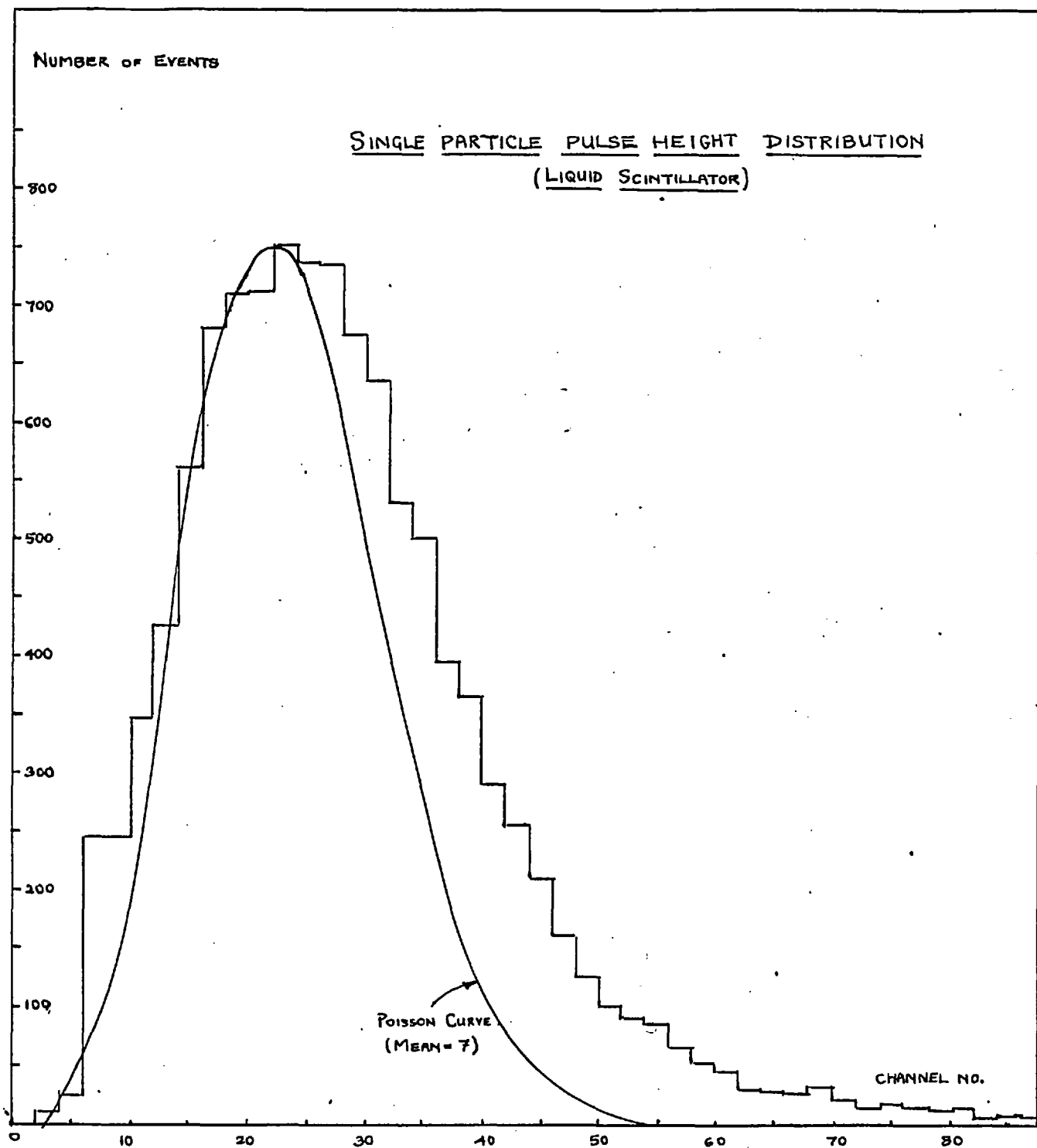


FIG 2.4

acceptance angle of the muon-telescope and the imbalance of the photomultiplier tubes.

2.6.3. Stability.

The power units supplying the E.H.T. voltage to the photomultiplier tubes, and the L.T. power supplies for the electronic circuitry were each run off a stabilised power supply (Servomex). The E.H.T. power supplies were also stabilised so that any changes in the gain of the system due to variations in supply voltages were reduced to negligible proportions. The temperature coefficient of the photomultiplier tubes used in the muon detectors (E.M.I. type 9530A) is $\simeq -1\%$ per $^{\circ}\text{C}$ so that large changes in environmental temperature could have a large influence on the gain stability of the photomultiplier tubes. The air temperature was kept constant to within a few degrees and the count rates were checked each day so that changes in gain over a long period were small. No perceptible deterioration of the scintillation liquid was detected over a period of two years' operation.

2.7. The Detector Cut-Off Energy.

At energies less than about 100 GeV the main source of energy loss for the muons in traversing the lead absorber will be due to collision processes. At energies beyond

100 GeV, bremsstrahlung and pair production become significant. (Cousins and Nash, 1962; Hayman et al., 1963). The mean energy of muons in E.A.S. is ≈ 7 GeV. (Bennett et al., 1962) and most of the muons have an energy lying in the range 1 GeV to 20 GeV so that collision losses predominate in the present experiment.

Sternheimer (1959) has calculated the range-energy relation for protons in lead taking into account the density effect correction. The range-energy relation for muons of energy between 200 MeV and 1 GeV has been obtained by using the conversion formula quoted by Sternheimer. The energy cut-off E_{μ}^c of the detectors to the muon component can be represented by an equation of the form:-

$$E_{\mu}^c = (13.5 R_{\text{cm Pb}} + 28.3) \text{ MeV.}$$

The lead shielding is 20 cm thick so that:-

$$E_{\mu}^c = (270.1 \sec \theta + 28.3) \text{ MeV.}$$

where θ is the zenith angle of the incident muons. For vertically incident muons, the threshold energy is 298 MeV.

2.8. Contribution to the Detector Response from Knock-on Electrons.

Measurements have been made at sea-level by many workers on the percentage of knock-on electrons which are produced by unassociated cosmic ray muons in a lead plate.

Hazen (1943) observed that on average, 7% of secondary electrons per muon emerged from a lead plate of 0.7cm thickness. The number of knock-on electrons emerging from a lead absorber as a function of the thickness of the absorber have been investigated by Stoker et al. (1961). Above a thickness of about 1cm the proportion of knock-on electrons was approximately constant at 6%. The number of knock-on electrons which emerged with an energy above 10MeV was about 1% of the incident muon flux. Therefore, the contribution to the muon detector response is small compared with uncertainties due to temperature fluctuations and to the detector calibration for an incident flux consisting of unassociated muons.

The unassociated muon energy spectrum at sea-level is very similar to the energy spectrum of muons in a cosmic ray air shower at a distance of about 500m from the shower axis. Close to the axis the energy spectrum of air shower muons flattens. However, the contribution of muon knock-on electrons to the detector response is relatively insensitive to the energy of the muon (Bhabha, 1938). Hence, the overall knock-on electron contribution to the detector response, although small, is not negligible. For this reason the mean value of the pulse heights produced by single vertical muons in the calibration measurements is taken,

rather than the mode. Therefore, in the summation over a large number of observations, the effect of muon knock-on electrons upon a particular evaluation of a mean density can be ignored.

CHAPTER THREE

THE PULSE HEIGHT SPECTRUM

PRODUCED BY SECONDARY COSMIC RAYS.

3.1. Introduction.

A close examination of the integral pulse height spectrum obtained by using two different types of cosmic ray detector, namely water Cherenkov detectors and liquid scintillator detectors, reveals two changes in the slope of the log-log plot. The first change occurs at a level corresponding to approximately four particles over a detector and the second at a level corresponding to about 40 particles over a detector. The experiments described below were carried out to investigate in particular the processes responsible for the region of the spectrum below the forty particles 'kink'. This region is of particular interest at Haverah Park in connection with the gain stability checks carried out on the detectors using the count rate in this region as an indication of the gain of the detectors.

3.2. Apparatus.

A description of the liquid paraffin scintillation detector is given earlier in Chapter 2. The first

experiment used two such units, one placed vertically above the other at a separation of about 30cm. A schematic diagram of the recording system used is shown in fig. 3.1.

The added output from the two photomultiplier tubes of the one detector was used to gate the added output of the other at discrimination levels corresponding to approximately 1.5, 2.5, 4.5, and 10 particles over the detecting area. The gated pulse heights were recorded by a 400 channel pulse spectrometer (Victoreen type 400M).

Both detector units were calibrated by means of a narrow angle scintillation telescope to select near vertical muons (zenith angle $< 10^\circ$) incident upon the detectors. The spectrometer was calibrated by using a mercury pulse generator giving simulated detector pulses of variable height. The experiment was carried out over a short period of time, thus minimising the possibility of a drift in the gain of the photomultiplier tubes, and wide changes in atmospheric pressure.

Initially the bottom detector was used to gate those pulses entering the spectrometer from the top detector. A gate pulse width of $1 \mu\text{sec}$ was produced from a stable discriminator and the gated pulses from the top detector were delayed by $\frac{1}{2} \mu\text{sec}$ with respect to the bottom. The whole process was repeated with the top and bottom detector

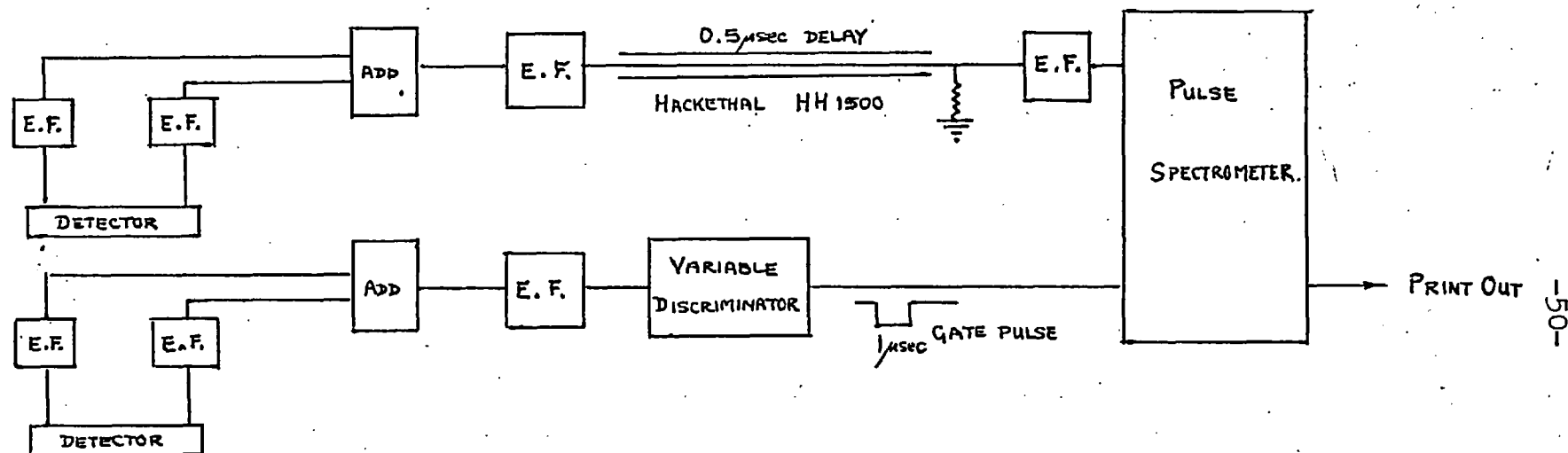


Fig 3.1 THE SANDWICH RECORDING SYSTEM.

channels reversed.

3.3. The Liquid Scintillator Sandwich Results.

Fig. 3.2. shows the integral pulse height spectra obtained from the liquid scintillators in both recording modes. The ungated spectra below 5mV mainly arise from noise at the photocathode of the photomultiplier. Between 5mV and 30mV the contribution is due to the distribution of detector response to single unassociated cosmic ray secondaries passing through the detector. It is evident from the absorption-in-lead experiment (described below) that these are mainly muons.

If we represent the rate of particles falling over the detecting area and exceeding a number n by:

$R(>n) = an^{-\gamma}$ then, three distinct values of γ may be assigned to different regions of the ungated spectrum of an unshielded detector.

Region (A)	$2p < n < 3$ particles	$\gamma = 3.3 \pm 0.1.$
Region (B)	$4p < n < 30$ particles	$\gamma = 3.0 \pm 0.1$
Region (C)	$n > 40$ particles	$\gamma = 1.5$

With regard to the gated spectra shown in fig. 3.2 the following points become evident upon close examination.

(a) A finite pulse height in one detector of the sandwich does not always coincide with a measurable pulse

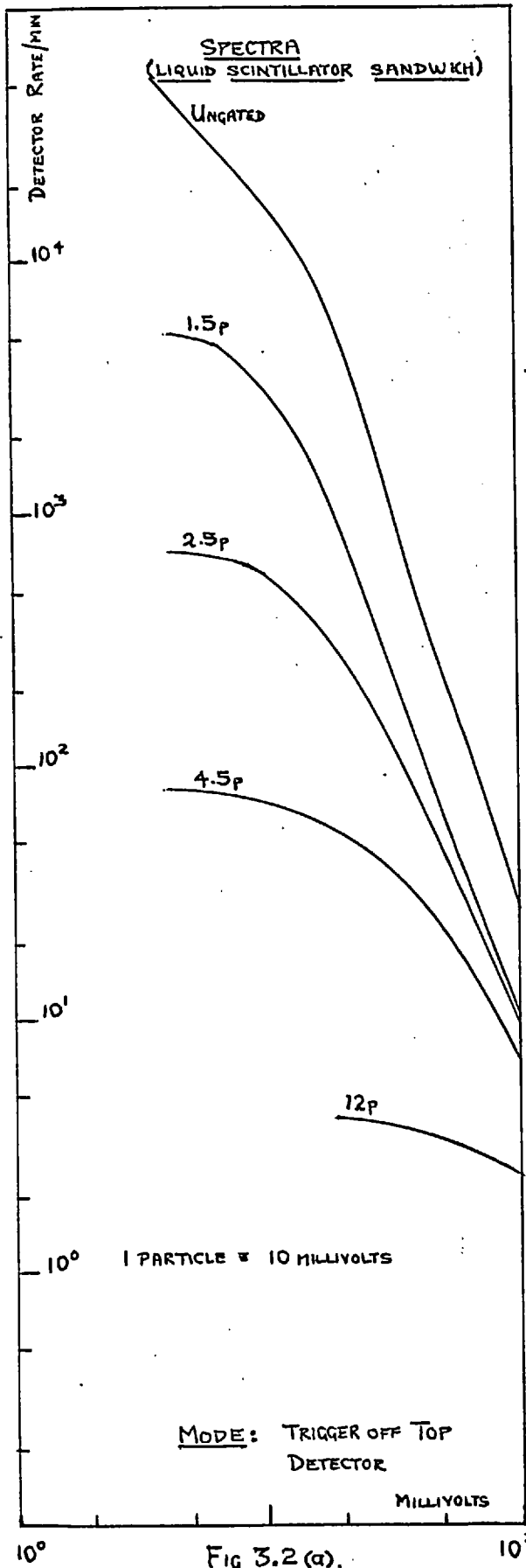


Fig 3.2 (a).

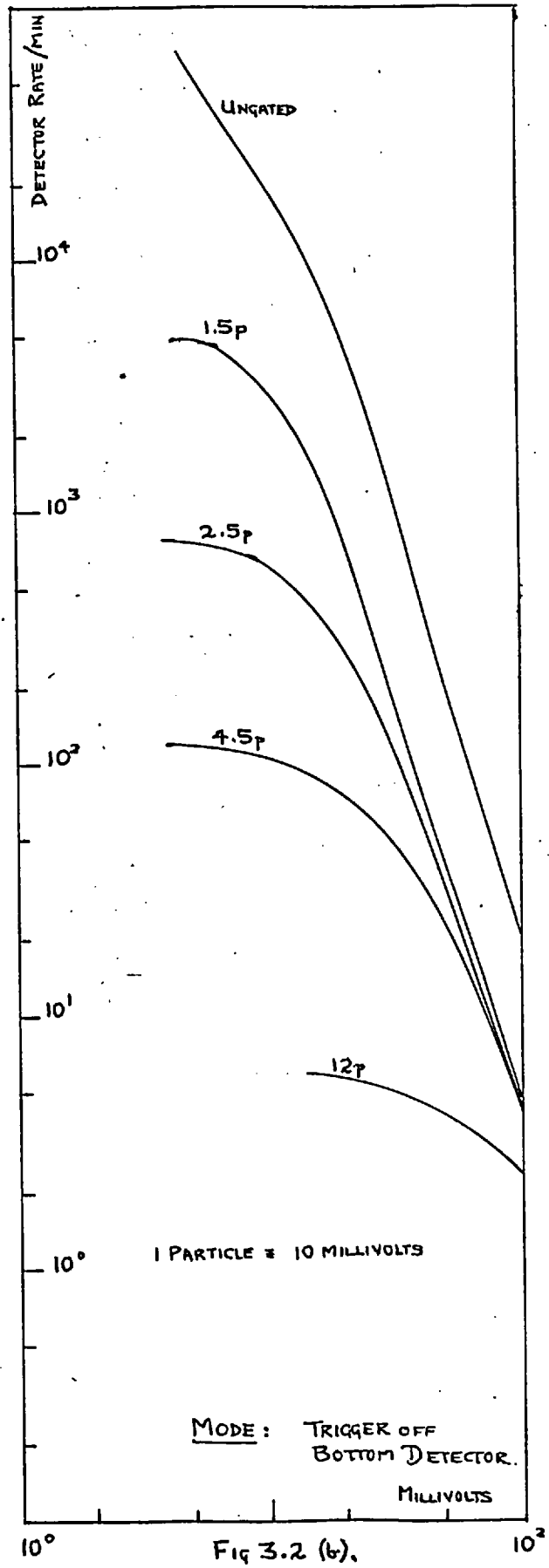


Fig 3.2 (b).

in the other. The number of cases in which the pulse height in the sampled detector is zero varies with the trigger level of the triggering detector as shown in fig. 3.3. The large number of 'zeros' can only be partially explained by the geometrical effect of particles which miss one of the detectors. The radiation responsible for the observed spectra must therefore contain a significant proportion of neutral particles.

(b) The number of events in which a certain pulse height, equivalent to n particles, is exceeded in the sampled detector as a percentage $P_{st}(n)$ of the number of events in which the same pulse height is exceeded in the triggering detector, is shown in table 3.1. for both recording modes. This shows that the radiation is becoming increasingly penetrating below the $1\frac{1}{2}$ particle level, consistent with a mainly muonic composition. The decrease of the percentage for both recording modes at the $2\frac{1}{2}$ and $4\frac{1}{2}$ particle levels indicates that at these levels interactions are taking place in the detector and that the remnants can be of such an energy as to be confined within the detector (of thickness 8gm/cm^2). At the ten particle level, the energy of the interaction is greater, with a resultant increase in the percentage $P_{st}(n)$.

The value of $P_{st}(10)$ is greater when the bottom

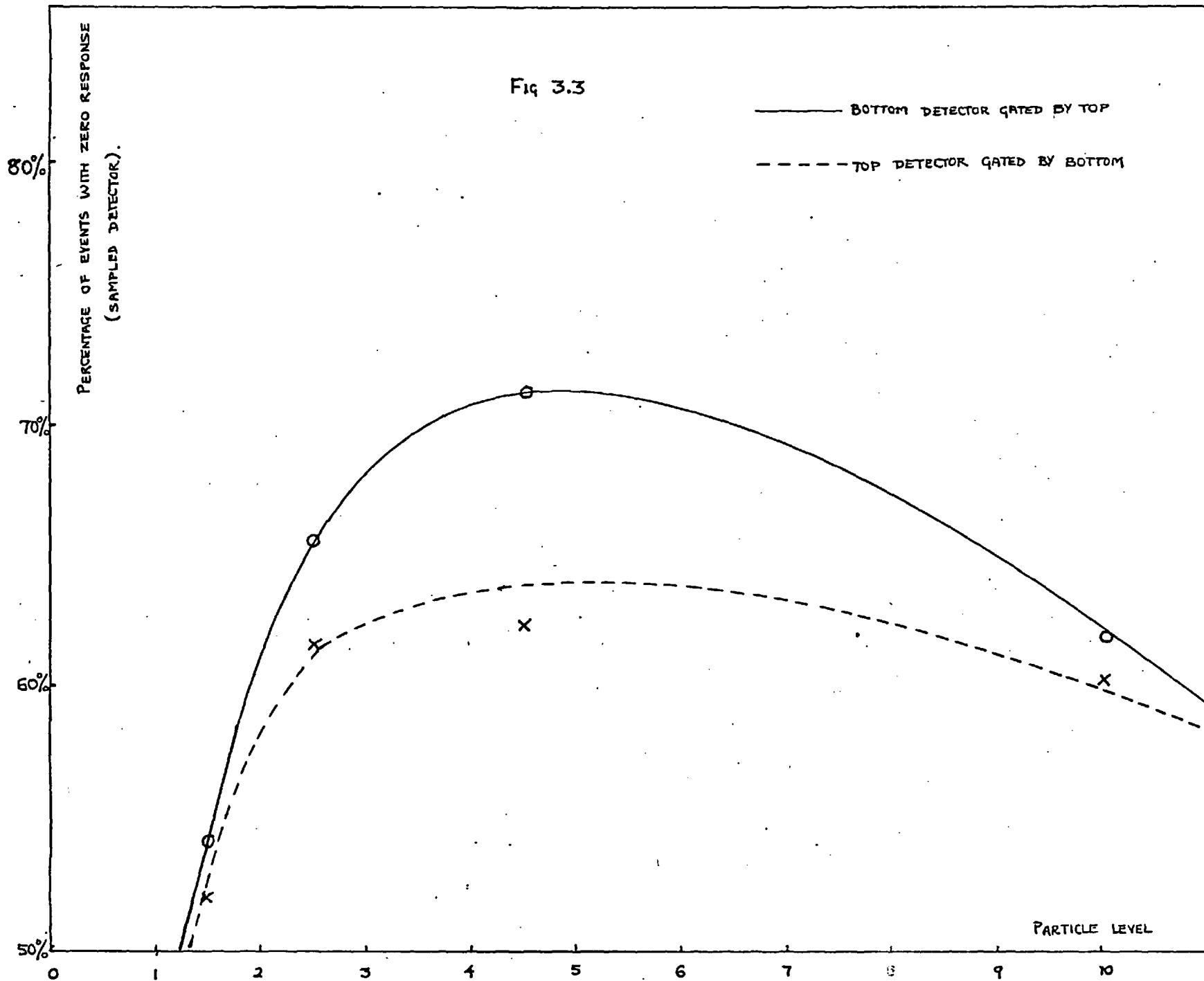


TABLE 3.1.

LEVEL	SAMPLED DETECTOR (TOP)	TRIGGERED DETECTOR (BOT)	P _{st} (%)
1 $\frac{1}{2}$	1.46 x 10 ³ /min	1.03 x 10 ⁴ /min	14.2
2 $\frac{1}{2}$	1.59 x 10 ²	1.98 x 10 ³	8.0
4 $\frac{1}{2}$	3.0 x 10 ¹	3.42 x 10 ²	8.8
10	1.86 x 10 ⁰	1.47 x 10 ¹	12.7

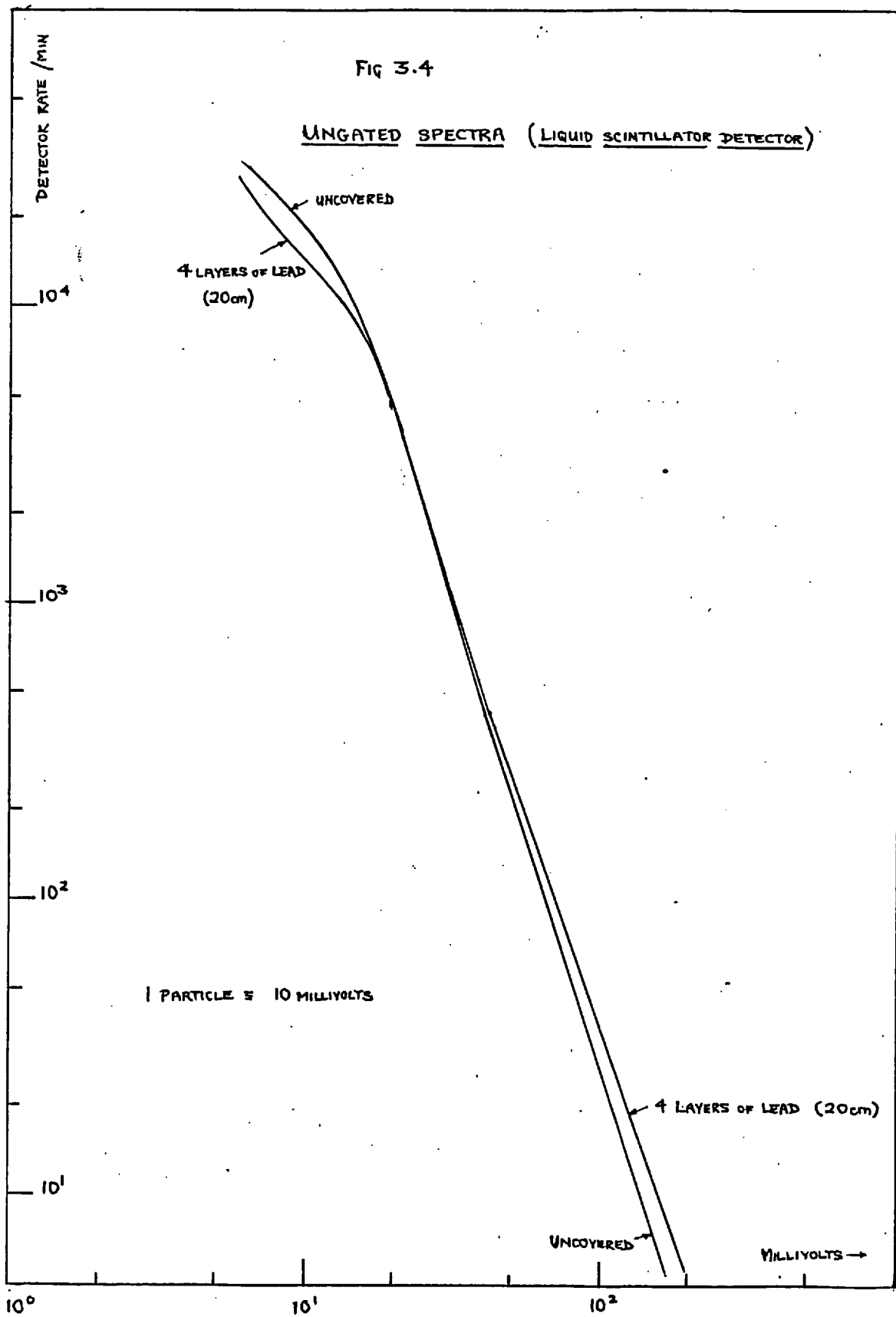
LEVEL	SAMPLED DETECTOR (BOT)	TRIGGERED DETECTOR (^{TOP} ~~BOT~~)	P _{st} (%)
1 $\frac{1}{2}$	1.53 x 10 ³ /min	1.11 x 10 ⁴ /min	13.8
2 $\frac{1}{2}$	1.69 x 10 ²	2.14 x 10 ³	7.9
4 $\frac{1}{2}$	2.50 x 10 ¹	2.93 x 10 ²	8.5
10	1.99 x 10 ⁰	1.09 x 10 ¹	18.3

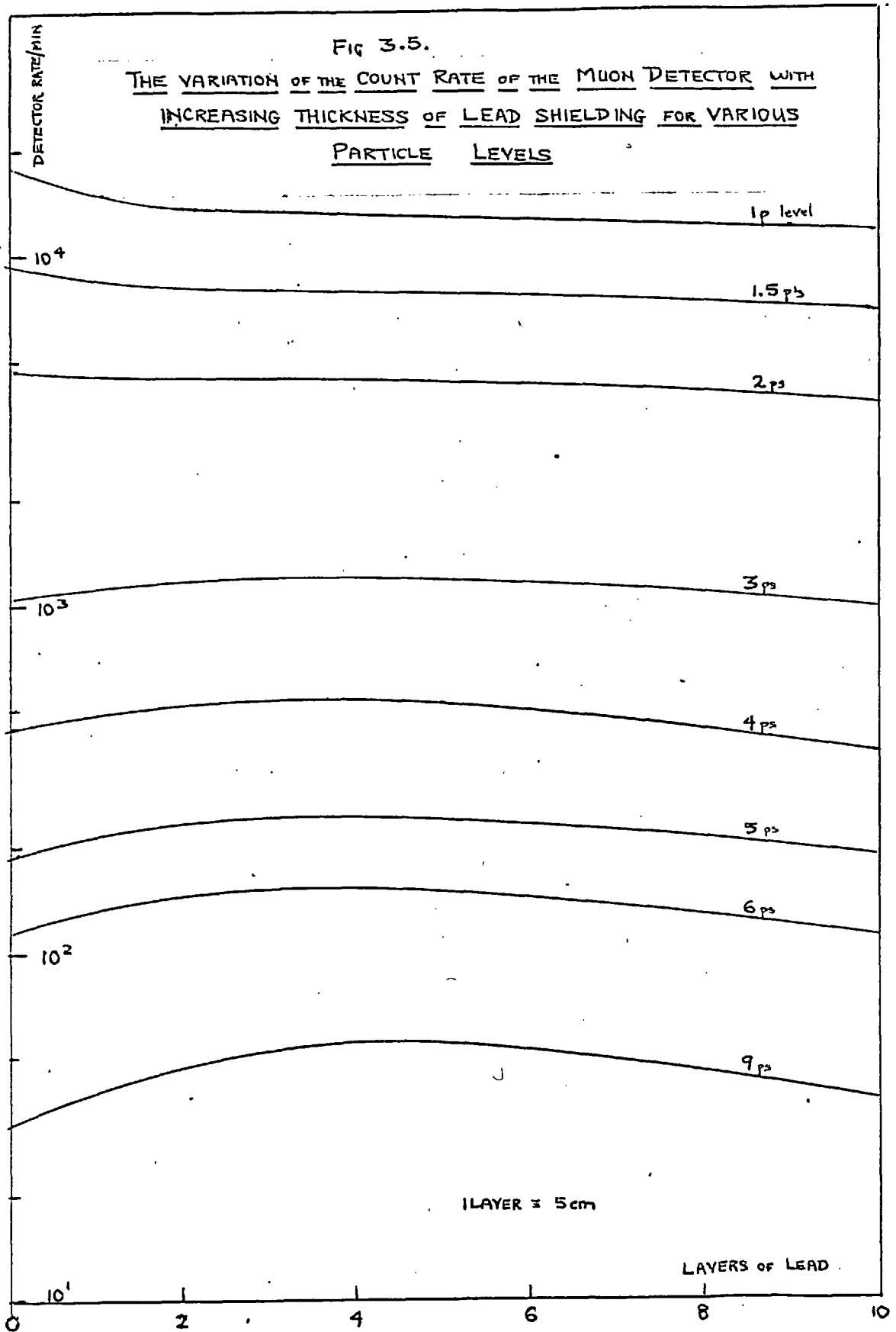
detector is sampled by triggering off the top detector than when triggering in the other mode. This is to be expected from a particle flux which is incident from above the detectors and which interacts in the detectors.

In order to elucidate the question of the composition of the radiation which is responsible for the pulse height spectrum in the region above the 4 particle 'kink', an absorption experiment was carried out by using a single detector to observe the ungated cosmic ray pulse height spectrum under varying thicknesses of lead.

3.4. The Absorption by Lead Experiment.

This experiment was performed on one detector only. The ungated pulse height spectrum was recorded under varying thicknesses of lead, up to a maximum thickness of 567gm/cm^2 (10 layers). A constant check was kept on the gain of the detector during the period of operation (one week), and also on the atmospheric pressure during running time. The integral spectra obtained with the detector uncovered, and under 20 cm of lead shielding, are shown in Fig. 3.4. Five measurements with the detector shielded by 10cm, 20cm, 30cm, 40cm and 50cm of lead were made. Fig. 3.5. shows the variation of the integral counting rate with increasing thicknesses of lead





absorber at various levels of particle numbers. The rapid absorption of the soft electron-photon component of the incident radiation at the 1 and $1\frac{1}{2}$ particle levels under the first layer of lead is followed by a slow absorption of the penetrating muon component.

At the three particle level a broad maximum in the count rate occurs, centred at a thickness of about 20 cm of lead (227gm/cm^2), followed by a slow absorption of the incident radiation. The appearance of a maximum counting rate (at levels greater than 3 particles over a detector) for a thickness of lead absorber of the order of 200gm/cm^2 strongly suggests a substantial nuclear-active particle contribution to the observed spectrum beyond the 3 particle level.

3.5. The Lateral Spread.

In order to investigate the lateral spread of the local showers responsible for the observed spectrum in the region between 4 and 40 particles, the following experiment was carried out.

The four separate outputs of the shielded scintillator detectors (arranged in the configuration shown in fig. 5.2(b)), were sampled individually whenever more than N particles fell over the four detecting units simultaneously.

The experiment was performed for values of N equal to 5, 10 and 20 particles. The output from the four detectors observed photographically, showed that in less than 1% of the recorded events, the final output originated from more than one detector for each value of N. The implication of this result is that the radiation, which is responsible for producing a response of greater than 5 particles in a shielded detector (up to 40 particles), must interact in the shielding of that detector or in the scintillation liquid. Unshielded Cherenkov detectors have a count rate proportional to detector area, for areas between 1m^2 and 30m^2 . This is to be expected if the incident radiation interacts in the detector or produces a narrow local shower of lateral spread less than 1m above the detector. (Allan et al., 1965A). Miyake et al. (1962) and Delvaille et al. (1960) measured the pulse height spectrum in the transition region, and then imposed the condition that at least one particle must be simultaneously recorded by a detector placed a few metres away. They found that the steep part of the spectrum disappeared, indicating a localised radiation responsible for the steep portion of the curve.

3.6. Interpretation.

3.6.1. The Single Particle Contribution.

This region corresponds to a response of up to 4 equivalent vertical muons, and is produced mainly by single muons passing through the detector. The integral intensity of muons in the vertical direction, above an energy of 307MeV, as measured by the Durham spectrograph group is $7.54 \times 10^{-3}/\text{cm}^2/\text{sec}/\text{ster}$ (Hayman and Wolfendale, 1962). If this figure were used, the rate expected over a flat scintillator of area 2.5 sq.m and with an energy cut-off of 300MeV would be somewhat greater than $2.26 \times 10^4/\text{min}$.

It was shown by Blake (1966) that, assuming an intensity dependence upon zenith angle θ given by

$$I(\theta) = I_0 \cos^2 \theta$$

the integral spectrum due to single muons should have an exponent of about -4. The observed slope of -3.3 is probably due to the statistical broadening of the response and the inaccuracy of the dependence upon zenith angle. That the statistical broadening of the response can produce pulse heights four times as large as that of a single muon with a comparable frequency, can be seen by plotting the integral pulse height spectrum for pulses produced by single vertical particles. From a single particle pulse height spectrum obtained over a period of one week's continuous

operation, it was found that the integral spectrum slope varied from -2.4 at the single particle level to -4.8 at the four particle level. That this region of the spectrum is produced almost entirely by muons, is further supported by the constancy of the slope of the spectrum when large thicknesses of lead absorber are placed over the detector.

On the other hand, as was pointed out in section 3.4, the sensitivity of the slope of the spectrum between the four and thirty particle levels to the thickness of lead absorber, indicates a very different type of radiation responsible for this region.

3.6.2. The Local Contribution.

The components of the secondary radiation which fall on an uncovered detector are in equilibrium, but when a large thickness of absorber is interposed, this equilibrium is destroyed, until the radiation has passed through an appreciable thickness of absorber. The increase in the count rate above the three particle level, as the thickness of lead is increased, is interpreted as being due to the increase in the number of interactions of the radiation incident on the detector, as a result of nuclear encounters in the lead. The nuclear-active particle interaction length in lead is of the order of 180gm/cm^2 . One would

therefore expect a maximum count rate of nucleon induced events under about 16cm of lead, as was observed.

3.7. The Nuclear-active Component.

When a neutron or proton with an energy of the order of a few GeV collides with a nucleus, a complicated chain of events follows, with the eventual production of a star. If the energy of the incident particle is large enough for multiple pion production, a star with an associated electron-photon cascade will be produced. The nuclear-active particles are released by the evaporation process, in which they gain sufficient energy to overcome the binding energy, and escape. The threshold energy for meson production is much lower when the target nucleus is complex. Meson production has been observed in carbon at a proton energy of 200MeV (Jones and White, 1950), whereas the threshold energy for the production of a π^+ , π^- and π^0 in an interaction between two protons is of the order of 900MeV.

In order to estimate an order of magnitude for the rate of nuclear interactions in the scintillator, it is convenient to know the secondary proton and neutron energy spectra at sea-level.

The proton energy spectrum for energies greater

than 60 MeV up to energies ≈ 150 GeV has been measured by various workers. The secondary neutron energy spectrum is not well known, however, and an estimation of the relative rates of secondary neutrons and protons at sea level was made, using a treatment given by Rossi (1952).

In this model, the cascade through the atmosphere is limited to one dimension, and the production spectrum of both neutrons and protons is based on the measurements of Camerini et al. (1950) using emulsions (which involved interactions with heavy nuclei).

The production spectrum of neutrons at a depth x' in the atmosphere for an energy E is well represented by the function:-

$$s_n(E, x') = \frac{A e^{-x'/L}}{(50 + E)^2}$$

A is a constant.
E is in MeV.
L is the attenuation mean free path.

Letting $j_n(E, h)$ represent the differential energy spectrum of neutrons at a depth h in the atmosphere then:-

$$j_n(E, h) = \int_0^h s_n(E, x') \exp((-h + x')/L_c) dx'$$

where L_c is the collision mean free path in the atmosphere. As an approximation the exponential term may be taken as unity, so that:-

$$j_n(E, h) = \frac{Ah \exp(-h/L)}{(50 + E)^2}$$

For protons of energy less than 500MeV, energy loss by ionisation becomes important and must be taken into account.

The proton intensity at a depth h in the atmosphere is then given by

$$j_p(E, h) = \frac{A \exp(-h/L)}{k(E)} \times \frac{(E_m - E)}{(50 + E)(50 + E_m)}$$

where E_m is defined by $R(E_m) = R(E) + h$

R being the residual range in air, and

$k(E)$ represents the loss of energy by ionisation for a proton of kinetic energy E .

The integral spectra of neutrons and protons at sea level ($h = 1000\text{gm/cm}^2$) are then given by:-

$$J_n(>E_0, h) = \int_{E_0}^{\infty} \frac{Ah \exp(-h/L)}{(50 + E)^2} dE \quad \dots\dots\dots 3.1$$

$$\text{and } J_p(>E_0, h) = \int_{E_0}^{\infty} \frac{A \exp(-h/L) (E_m - E)}{k(E) (50 + E) (50 + E_m)} dE \quad . \quad 3.2$$

These equations were integrated, in order to find the ratio of neutrons to protons of energy greater than E_0 at sea-level.

It was necessary to integrate equation 3.2 in small intervals of E taking average values of E_m and $k(E)$ for each interval. This process was carried out for each interval until the integrands converged sufficiently for an accurate summation to be made.

The ratios J_n/J_p as calculated are:-

E_0 (MeV) sea-level	Ratio (neutrons to protons).
50	9 : 1
100	5.9 : 1
150	4.7 : 1
300	3.1 : 1
1000	1.8 : 1
(cf. fig.3.6).	

Germain (1951) has shown that star production can occur in photographic emulsions for protons in the energy range 95MeV to 345MeV. The mean free path for star production decreased from 600 gm/cm² to $\simeq$ 300 gm/cm² in this energy range, and the average number of star prongs increased with energy. Powell and Occhialini (1947) have observed star production by low energy neutrons.

In order to estimate an order of magnitude for the rate of star production in the liquid scintillator

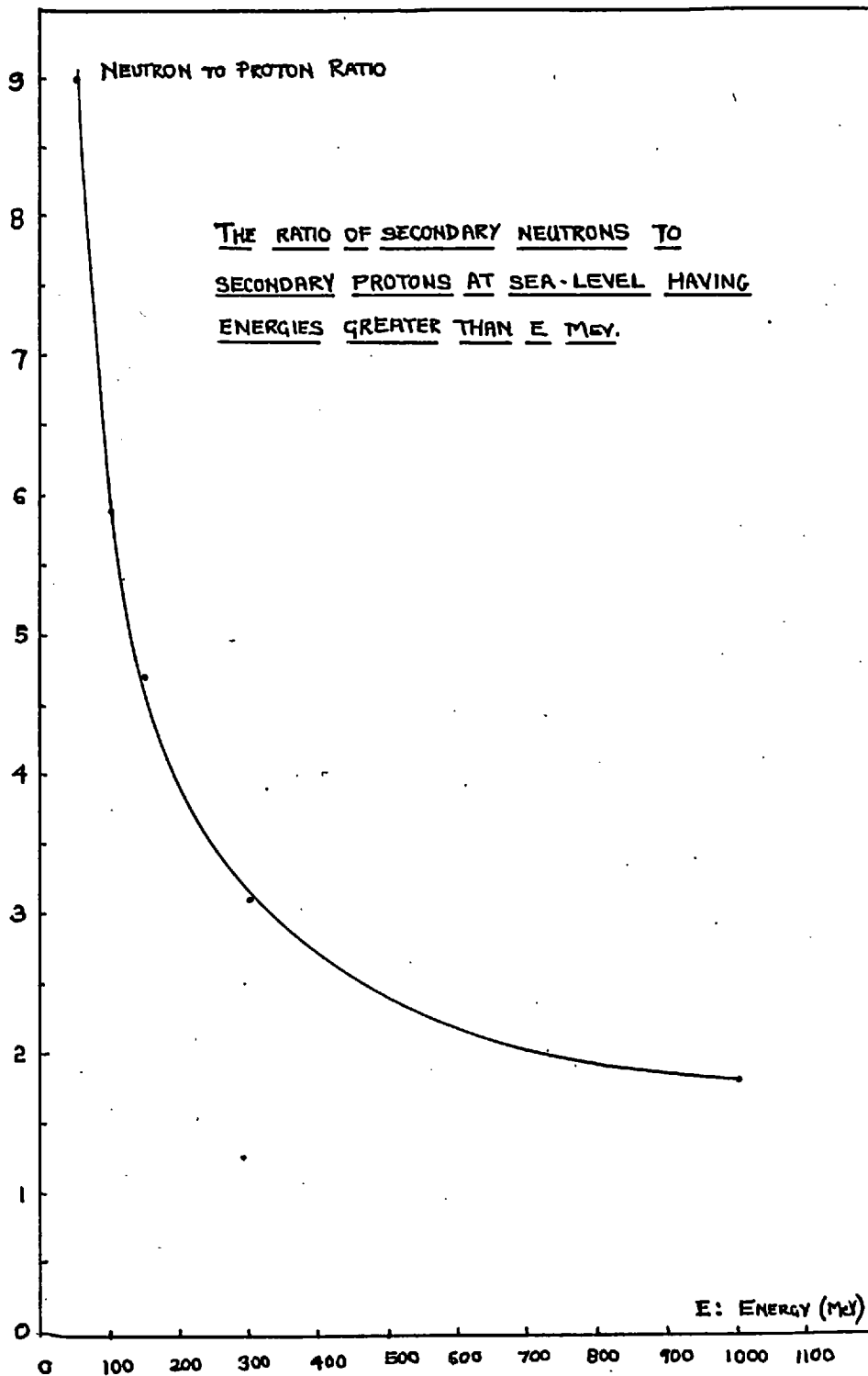


FIG 3.6

used in the present experiment, it is necessary to know the sea-level intensity of nuclear active particles. Hughes and Marsden (1966) used the differential proton energy spectra at sea-level determined by various workers (Meshikovski et al., 1958; Ogilvie, 1955; Mylroi and Wilson, 1951; Brooke and Wolfendale, 1964) and constructed a differential energy spectrum for protons of energy in excess of 30 MeV. They found that the omnidirectional intensity of protons of energy greater than 180 MeV is:-

$$(1.51 \pm 0.03) \times 10^{-4} / \text{cm}^2 / \text{sec}.$$

The calculation of the neutron to proton ratios at sea level, as a function of particle energy (cf. fig.3.6), shows that at an energy of 180 MeV the ratio of neutrons to protons is 4.3 : 1. The rate of neutrons and protons falling on a detector of area 2.5 sq m. is then of the order of $1.2 \times 10^3 / \text{min}.$

Assuming an interaction length for star production in the scintillation liquid $\simeq 80 \text{ gm/cm}^2$, for a thickness of 8 gm/cm^2 , the rate of star formation by nucleons of energy greater than 180 MeV will be $\simeq 1.2 \times 10^2 / \text{min}.$ The pion contribution will be small ($\simeq 1\%$), since the pion to proton ratio at sea-level falls rapidly with decreasing energy below 10 GeV. (The ratio at 1 GeV is about 10%) (Brooke et al., 1964A). This rate corresponds

to the rate of an unshielded scintillation detector at the six particle level. The energy dissipation in a scintillator of thickness 8gm/cm^2 required to produce a response of about 6 equivalent particles is of the order of 100MeV (for minimum ionising particles). The proportionality constant k between scintillator detector response R and energy dissipation E ($R = kE$), is unity for minimum ionising particles. The conclusion is, therefore, that if the proportionality k for low energy ($\simeq 100\text{MeV}$) protons and alpha-particles does not fall below about 0.5, then, the rate of nuclear-active particle-produced stars in the scintillator is sufficient to explain the uncovered scintillator response at the six particle level.

George and Jason (1949), working at 3,457 metres above sea-level investigated the decrease in star intensity as a result of shielding their recording apparatus by various thicknesses of lead.

They found the mean free path for star production in lead to be $310 \pm 20\text{gm/cm}^2$ ($\simeq 27\text{cm}$ of lead). The observation of a peak in the counting rate of the detector used in the present experiment, under about 240gm/cm^2 ($\simeq 21\text{cm}$) of lead, at particle levels greater than about four equivalent particles, also supports an explanation in terms of star production by nuclear-active particles in local interactions in the scintillator.

The mean interaction length in lead of the nuclear-active particle radiation is therefore of the order of 240gm/cm^2 .

3.8. Summary.

The conclusions which can be drawn from the results of the experiments described in the previous sections and the experiments of other workers concerning the cosmic ray pulse height spectrum produced by a liquid scintillator are, as follows:-

Below a level corresponding to about 3 particles over a detector, the detector response is almost entirely due to muons. Above the 'kink' in the slope of the spectrum, located at approximately the four particle level, the observed response is mainly due to star production in the scintillator liquid by nuclear-active particles. A similar conclusion was drawn by Green and Barcus (1959) to explain the response of liquid scintillators to the cosmic radiation at 1575 metres above sea-level. A direct comparison between nuclear emulsion observations and the latter experiment, confirmed the feasibility of an explanation in terms of star production.

CHAPTER FOUR.

THE ANALYSIS OF AIR SHOWER DATA.

4.1. Introduction.

The raw data consisted of a photographic record of each shower containing information on the time of occurrence of the event; the pulse heights from the four Cherenkov detecting units; and the relative times of arrival of the shower front at the four stations. The muon detector pulse heights were recorded on a separate film and correlated with the triggering array by neon indicator lights, and with the particular event by the message register number.

The message register number; the four pulse heights; and the time of occurrence of the pulses (measured from a standard blanking pulse on the oscilloscope trace) were read off the film. The ten pieces of information for each shower, (including the muon detector pulse height) were punched on to I.B.M. computer input cards for the 500m array, and on to 8-hole paper tape for the 50m array.

In addition to the data on individual showers, calibration information was punched, comprising (for each station) the standard pulse heights; the number of tanks in use; the mean response of the tanks to a vertical muon;

the message register numbers at the beginning and end of the run, and the calibration data for the muon detector.

The 500m array results were processed by the Imperial College I.B.M. 7090/1401 computer system and the 50m array results by the University of Leeds E.E.L.M. - KDF9 computer.

The rate of analysis averaged about one shower per second.

The various stages in the analysis are described below:

These are:-

- (a) Calculation of the zenith and azimuthal angles and radii of curvature.
- (b) The location of an approximate core position.
- (c) Calculation of the final core position and shower size, using a χ^2 fitting procedure.
- (d) Correction of the zenith and azimuthal angles.
- (e) Calculation of the arrival direction in galactic co-ordinates.

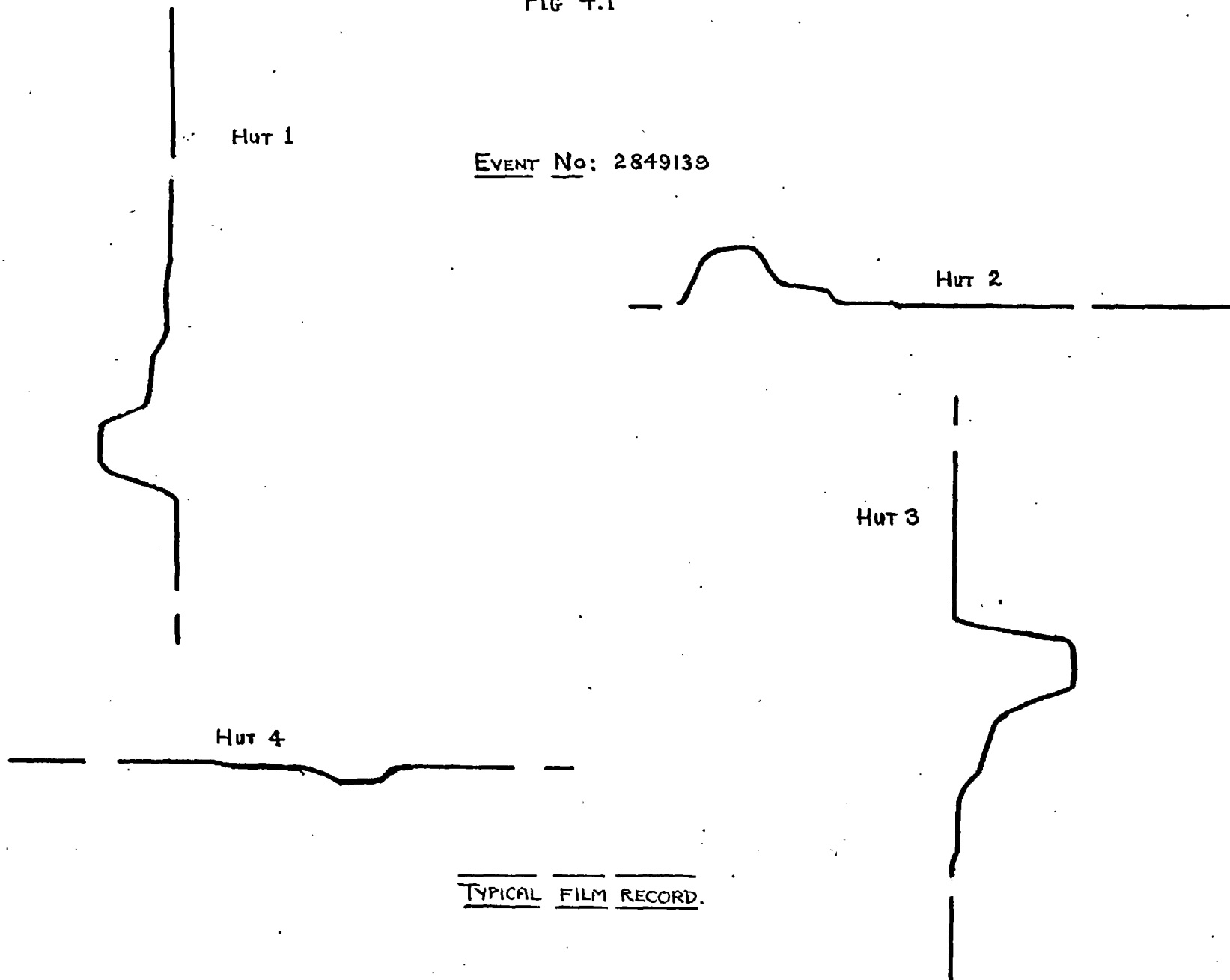
A flow-chart is shown in Appendix 3.

4.2. The Shower Arrival Directions and the Radius of Curvature.

Since timing information was not available for the 50m array, the arrival directions were not determined.

A typical photographic record from the 500m array is shown in fig. 4.1. The distance between the blanking markers corresponds to 6μ sec for the 500m array. The time of breakaway of the Cherenkov tank pulses with respect

Fig 4.1



to the blanking markers can then be determined in μ secs. A plane shower front incident on the array was assumed, and the relative times of arrival of the front at two of the outer stations with respect to the third were used to calculate the arrival directions in azimuth and zenith. If one of the times at an outer station was unknown (e.g. zero pulse height), the arrival directions were calculated by using arrival times at two of the outer huts with respect to the central station.

If t_a and t_b are the relative times of arrival in μ secs of the shower front at two of the outer huts with respect to either the central hut or the other outer hut, it can be shown that the zenith angle θ and the azimuthal angle ϕ measured east of north are given by:-

$$\sin \theta = \frac{2 \times c \times 3^{(n/2 - 1)}}{D} (t_a^2 - (-1)^n t_a \times t_b + t_b^2)^{\frac{1}{2}} \dots 4.1$$

and

$$\tan \phi = \frac{1}{3^{(n - \frac{1}{2})}} \times \frac{(t_a - t_b)}{(t_a + t_b)} + k \dots \dots \dots 4.2$$

where: D is the array spacing in metres.

c is the velocity of light in metres per microsecond.

n = 0 when the three outer stations are used and

n = 1 when the central station and two outer stations are used.

k is a constant depending upon which combination of huts is used, and upon the orientation of the array.

The perpendicular to the plane of the array is inclined at a zenith angle of 2° and at an azimuthal angle of 10° E of N. The zenith angle θ of the shower is corrected by the amount:

$$(-1)^m \tan^{-1} (|\cos (\phi - 10^\circ)| \times \sin 2^\circ)$$

where $m = 1$ if the azimuthal angle of the shower lies between 100° and 280° and $m = 0$ for an azimuthal angle lying between 280° and 100° .

The relative times of arrival of the shower front at the four stations were used to calculate the radius of curvature of the shower front from the formula:-

$$R = \frac{(3D^2 - c^2 \sum_i t_i^2)}{4cD \sum_i t_i} \dots\dots\dots 4.3$$

where D is the array spacing in metres.

c is the velocity of light in metres per μ sec.

t_i are the relative times of arrival in microseconds.

R is the curvature of the shower front in kilometres.

The derivation of the three expressions of the form of equations 4.1, 4.2 and 4.3 are given by Lilliecrap (1963).

4.3. Caluulation of an Approximate Core Position and the Best Fit Structure function.

The pulse height p , read in film reading units is converted to a nominal density d of particles in terms of the equivalent number of muons passing vertically through the detector, by the expression:-

$$d(i) = \frac{p(i) \times k'(i)}{\mu(i) \times S(i) \times t(i) \times 2.25} \dots\dots\dots 4.4$$

where $k'(i)$ is a constant depending upon the attenuation of the calibration pulse of known magnitude sent out from the central station before it reaches the Cherenkov adding unit.

$t(i)$ is the number of tanks in use at station i

$\mu(i)$ is the pulse height in mV produced by a vertical muon passing vertically through a tank.

$S(i)$ is the pulse height in film-reading units produced by a calibration pulse of magnitude $k(i)$ mV.

The area of each tank is 2.25 square metres.

The calculation of an approximate core position is as follows:-

A cartesian co-ordinate system in the configuration of fig.4.2 is used. If l and m are the direction cosines given by:-

$$l = \sin \theta \sin (\phi - 145.5^\circ)$$

CO-ORDINATE SYSTEM

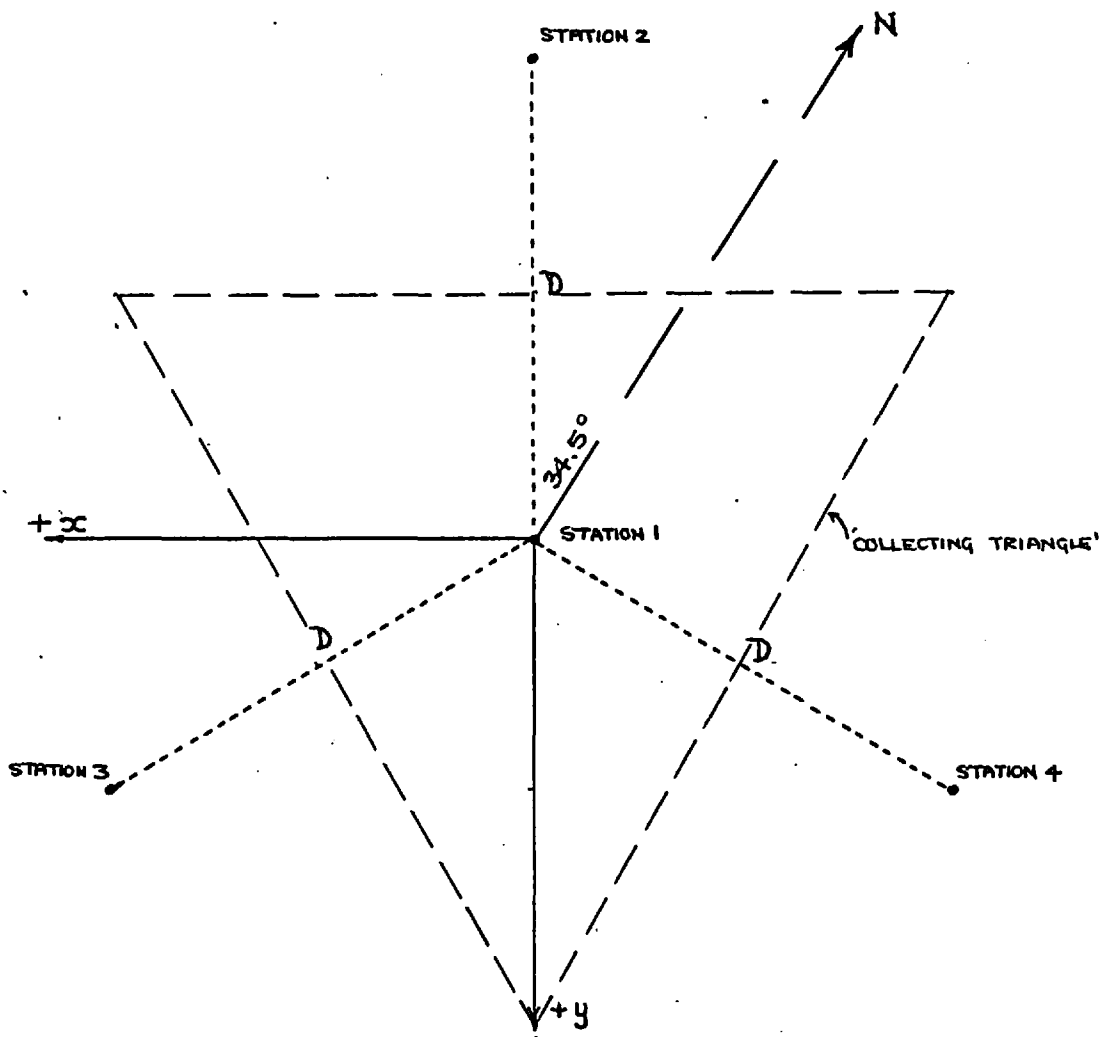


FIG 4.2

and $m = \sin \theta \cos (\phi - 145.5^\circ)$, then the perpendicular distances from the i^{th} detector to the core, having a location (X, Y) are given by:-

$$r(i) = ((X - x_s(i))^2 + (Y - y_s(i))^2 - (l(X - x_s(i)) + m(Y - y_s(i))))^2)^{\frac{1}{2}}$$

If this expression is expanded, substituting $A(i)$ for the coefficient of X , $B(i)$ for the coefficient of Y and $C(i)$ for the constant we have:

$$r(i)^2 = r(1)^2 + A(i)X + B(i)Y + C(i) \quad \dots\dots\dots 4.6$$

$$A(i) = 2(lm y_s(i) + l^2 x_s(i) - x_s(i)) \quad \dots\dots\dots 4.7$$

$$B(i) = 2(lm x_s(i) + m^2 y_s(i) - y_s(i)) \quad \dots\dots\dots 4.8$$

$$C(i) = (D^2 - (l x_s(i) + m y_s(i))^2) \quad \dots\dots\dots 4.9$$

The density is related to the nominal shower size N and perpendicular distance from the axis to the point of observation by an expression of the form:

$$d(i) = k * N * sf(i)$$

where $sf(i)$ is the structure function. Taking $sf(i)$ to be of the form $r(i)^{-n}$, then

$$d(i) = k * N * r(i)^{-n} \quad \dots\dots\dots 4.10$$

For the 500m array this expression gives a good representation of the variation of the density with distance from the core. For the 50m array it is simply used in the analysis as a convenient method of obtaining an initial but approximate core position. Equations 4.6 and 4.10 can be solved for X and Y in the following way:-

From 4.10, eliminating kN gives three equations of the form:-

$$\left[\frac{d(i)}{d(1)} \right]^{2/n} = \left[\frac{r(i)}{r(1)} \right]^2 = R(i) \dots\dots\dots 4.11$$

where i = 2,3,4.

Eliminating r(i) between equations 4.6 and 4.11 gives:-

$$r(1)^2(R(i) - 1) = A(i)*X + B(i)*Y + C(i) \dots\dots\dots 4.12$$

where i = 2,3,4.

Taking i=2 in equation 4.12 and dividing by equation 4.12 for i equal to 3 and 4 gives:-

$$\left[\frac{R(2) - 1}{R(i) - 1} \right] = \frac{A(2)*X + B(2)*Y + C(2)}{A(i)*X + B(i)*Y + C(i)} = BM(i) \dots\dots\dots 4.13$$

where i = 3,4.

Solving these two equations for X and Y gives:-

$$X = \frac{BS(4)*CS(3) - BS(3)*CS(4)}{AS(4)*BS(3) - AS(3)*BS(4)} \dots\dots\dots 4.14$$

$$Y = \frac{AS(4)*CS(3) - AS(3)*CS(4)}{AS(3)*BS(4) - AS(4)*BS(3)} \dots\dots\dots 4.15$$

where xS(i) = x(2) - BM(i)*x(i), for x = A,B,C.

The analogue method of finding the core position by the method of intersecting loci (Allan et al.,1960) requires a minimisation of the function:-

$$S = \sum_{i=1}^4 \left\| \frac{d(i)}{d(1)} - \frac{d(i)}{dc(i)} \right\| \dots\dots\dots 4.16$$

where $dc(i)$ is the calculated density at station i .
If we denote the starting value of the structure function exponent n by n_s then:-

$$\frac{d(i)}{dc(i)} = \left[\frac{r(i)}{r(1)} \right]^{n_s} \dots\dots\dots 4.17$$

Equation 4.16 then becomes:-

$$S = \sum_{i=2}^4 \left(\frac{d(i)}{dc(i)} - \left[\frac{r(i)}{r(1)} \right]^{n_s} \right) \dots\dots\dots 4.18$$

$r(i)$ for $i = 1, 2, 3, 4$ are obtained from the calculated values of X and Y for the starting value of n . (i.e. $n = n_s$).
The value of n_s is then varied and S recalculated for each new core position resulting from a variation in n_s .
The best fit structure function exponent is taken to be that value for which S is a minimum.

4.4 Calculation of the Final Core Position and Shower Size.

The calculation of an approximate core position is followed by a minimisation of the function:-
 $\chi^2 = \sum_{i=1}^4 w(i)(d(i) - dc(i))^2$ in order to obtain the core position and shower size giving a best fit to the observed data. This function is minimised by varying the position of the core in steps of r and θ' . (the polar co-ordinates of the core position in the plane of the array.). $w(i)$ are the weights of the observed densities given by:-

$$w(i) = 1/(\text{standard error})_i^2$$

For the 500m array the expression (given by Dennis, 1964) takes into account :-

(a) The Poissonian errors on the number of particles recorded, and on the number of photo-electrons produced at the photo-cathode.

(b) The errors in the muon calibrations and the uncertainty in the reading of the standard pulse heights and the individual detector pulse heights from the film.

The calculated densities ($dc(i)$) are related to the nominal shower size N and the structure function $sf(i)$ by the expression:-

$$dc(i) = k \cdot N^{n} \cdot sf(i).$$

The exponent of the structure function which is used in this analysis is the mean value for the particular zenith angle θ and is given by:-

$$\begin{aligned} n &= 11.75 - 8.7 \sec^{0.2}(\theta) && (\text{for } \theta \leq 51^{\circ}) \\ n &= -4.92 \sec^{0.25}(\theta) + 7.72 && (\text{for } 51^{\circ} < \theta < 59.3^{\circ}) \\ n &= 1.9 && (\text{for } \theta \geq 59.3^{\circ}) \end{aligned}$$

When the final core location has been determined, the zenith and azimuthal angles are corrected for the curvature of the shower front. This eliminates the systematic bias imposed on the calculation of θ and ϕ when a plane shower front is assumed. The expressions

for the corrected zenith and azimuthal angles (i.e. θ_c and ϕ_c) given by Dennis (1964) are:-

$$\phi_c = \tan^{-1} \left(\frac{R \sin \theta \times \sin \phi - X}{R \sin \theta \times \cos \phi - Y} \right) \dots\dots\dots 4.19$$

and $\theta_c = \tan^{-1} \left(\frac{R \sin \theta \times \sin \phi - X}{R \cos \theta \times \sin \phi_c} \right) \dots\dots\dots 4.20$

The right ascension and declination are determined from the expressions:-

R.A. = Local Sidereal Time + Hour Angle of the Shower
i.e. $R.A. = L.S.T. + \tan^{-1} \left(\frac{\sin \phi_c}{\cos \lambda \cdot \cot \theta_c - \sin \lambda \cdot \cos \phi_c} \right)$

and $Dec = \sin^{-1}(\sin \theta_c \cdot \cos \phi_c \cdot \cos \lambda + \cos \theta_c \cdot \sin \lambda)$
where λ is the latitude of the array.

Finally, the nominal shower size N is determined. The 'un-normalised' shower size kN is obtained by differentiating χ^2 partially with respect to kN at the final core position, whence:-

$$kN = \left(\sum_i w(i) \times d(i) \times sf(i) \right) / \left(\sum_i w(i) \times sf(i)^2 \right)$$

where $sf(i)$ is the structure function.

For the 500m array the normalisation constant k is given by:-

$k = \exp(-73.46 + 61.45n_s - 13.307n_s^2)$ for $n_s \leq 2.2$
and $k = \exp(4.46n_s - 12.45)$ for $n_s > 2.2$

For the 50m array the normalisation constant was

taken to be 1.11×10^{-3} (cf. sectn. 4.6).

The essential parameters of each analysed shower are punched onto I.B.M. output cards (500m array) or written on to magnetic tape (50m array) for ease of subsequent analysis of the muon component. The shower information was also printed out on to a line-printer.

4.5 The Fifty Metre Array and Five Hundred Metre Array Structure Functions.

The program used to calculate the core position and size of showers detected by the 50m array made use of a similar method of analysis to that of the 500m array program described above.

An initial core position was found by assuming a structure function of the form:-

$$sf_{50}(r_i) = r_i^{-n} \dots\dots\dots 4.21$$

The final core position was found using a structure function of the form:-

$$sf_{50}(r_i) = \frac{\exp(-r_i/80)}{r_i^\alpha} \dots\dots\dots 4.22$$

The analysis of about 1000 showers yeilded a mean value for α of 1.1. This value of α is the mean of the values of α found to give the best fit to the observed data. In the final analysis the mean value of α was

imposed upon the data for the χ^2 minimisation process. Such an imposition smooths out large fluctuations on the observed density which can have a serious effect on the analysis for an array of only four detectors.

The structure function used throughout the analysis of the 500m array showers (core to detector distances $> 100\text{m}$) was of the form of equation 4.21.

4.6 Comparison of the Results from the Two Arrays.

The 50m array records showers most efficiently, in a shower size range of 10^5 to 10^6 nominal particles, at distances $\simeq 10\text{m}$ to 100m from the core. For the 500m array the efficient shower size range is $5 \cdot 10^6$ to 10^8 particles, at distances greater than 100m from the core.

For the purposes of the present experiment an absolute shower size determination for each shower, although desirable, is not essential. Only relative shower sizes are important.

The deep water Cherenkov detectors used for the detection of showers at Haverah Park do not record particle density directly. Close to the core, cascading by the electron-photon component may produce a much greater nominal density than that which would be recorded by a thin scintillator. At larger distances $\gg 100\text{m}$ from the

core, the nominal density recorded by the Cherenkov detectors would be less than that recorded by conventional thin scintillators (Lillicrap et al., 1963).

A reasonably accurate normalisation of the 50m array shower sizes to the shower sizes determined by the larger array analysis was made by joining up the experimentally observed Cherenkov structure functions from the two arrays, for a shower size of $2 \cdot 10^7$ nominal particles. The overall lateral distribution function is shown in fig 4.3.

In the 50m array χ^2 fitting procedure, the four weights were taken to be unity due to the wide variation in film reading conditions. The 500m array showers having an exceptionally large value of χ^2 ($\chi^2 > 2$) were rejected in the subsequent analysis. The large values of χ^2 were usually associated with showers falling in areas outside the efficient collecting area of the array or in areas near to one of the detectors. In the 50m array analysis only those showers having a value of $\chi^2 > 20$ were rejected. (A larger percentage of high χ^2 values is expected for the 50m array where no account is taken of the arrival direction of the shower). The form of the structure function used in the final 50m array analysis was:-

$$sf(r_i) = k \times \frac{\exp(-r_i/80)}{r_i^{1.1}} \dots\dots\dots 4.23$$

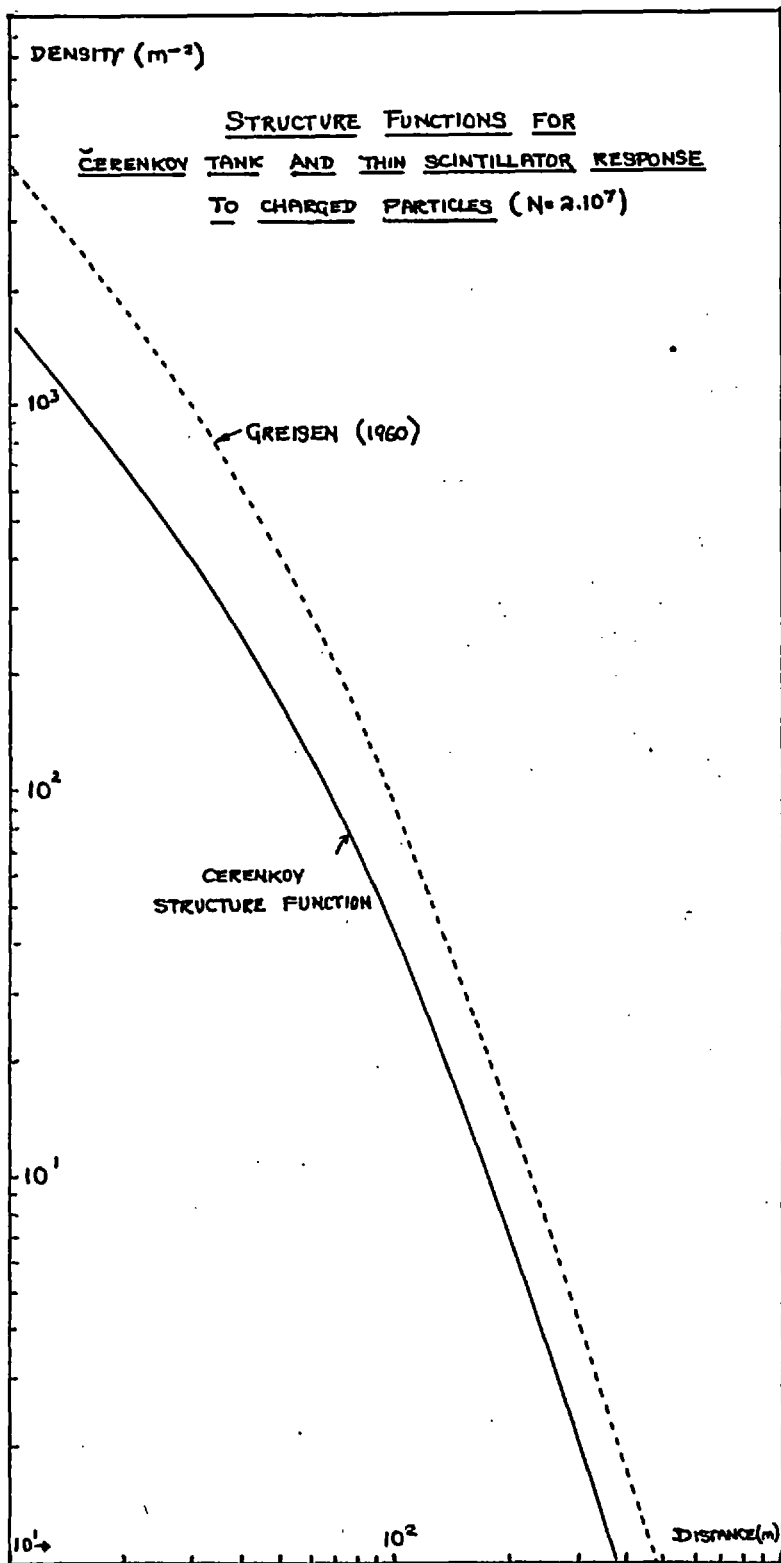


FIG 4.3

The zenith angle distribution of showers recorded by the 50m array is much narrower than that of showers recorded by the 500m array. The mean zenith angle is $\simeq 20^\circ$ for the 50m array compared with $\simeq 34^\circ$ for the 500m array. The structure function represented by equation 4.23 can then be regarded as applying to near vertical showers. The χ^2 rejection of badly fitted 50m array showers therefore eliminates most of the highly inclined showers recorded by the array.

4.7 Checks on the Computer Program.

The initial checks made on the functioning of the final extensive air shower analysis program confirmed that the computer-calculated arrival directions, shower size and core location coincided with the expected values. These checks were made by feeding in data calculated from the known relationships between density, core-detector distance and shower size.

Checks were built into the program to ensure that the air shower input data was compatible with a real shower. The expression for the radius of curvature (cf. equn 4.3) shows that for a plane shower front, the summation of the relative times of arrival at the three outer stations with respect to the central station

is zero. Showers having a value of $\sum t_i$ for which $\sum |t_i - 0.25| > 1 \mu\text{sec}$ were rejected. Such showers were usually associated with errors in the reading of the film records.

The triggering requirement of the array is that three of the Cherenkov detecting stations record a nominal density exceeding some fixed trigger level.

About 20% of all showers were rejected before analysis because the nominal densities at three or more of the stations did not exceed 10 equivalent particles (5 particles for the 50m array). A nominal density of 10 equivalent particles at a detecting station situated at a distance $> 100\text{m}$ from the core would, on average, represent a much greater number of actual particles. (e.g. 10 photons associated with 1 electron), so that the effect of fluctuations of showers just triggering the 500m array is small.

The next stage at which the analysis of a 500m array shower was curtailed, occurred when the structure function exponent n lay outside the limits 1 to 5. i.e. Exceptionally steep or flat showers were rejected. Large showers, falling at large distances from the array can have an apparently flat structure function whereas a shower with a core falling close to a detector may give, (for core-detector distances $\lesssim 10\text{m}$),

a large spurious pulse at that detector due to a nuclear interaction, the structure function therefore appearing abnormally steep. Approximately 3% of all showers were rejected by the 500m array program on this criterion. (This initial rejection criterion did not apply in the case of the 50m array for which the particle density distribution does not follow a simple power law at distances less than 100m from the core.

Having located the approximate core position, the χ^2 fit adjusted the core position to give the best weighted-fit to the data. Figure 4.4 shows a plot of the observed and calculated densities of showers having a zenith angle of less than 27° for the 500m array. The distribution of points confirms that no systematic biases occur in the computer analysis program.

Comparison with a large number of showers analysed by a similar 500m array program used independently by the Leeds University Group showed complete agreement between the two sets of results.

4.8 Errors on the Evaluated Quantities.

The errors in the calculated arrival directions result from experimental and film reading errors, the deviation of the radius of curvature from the mean,

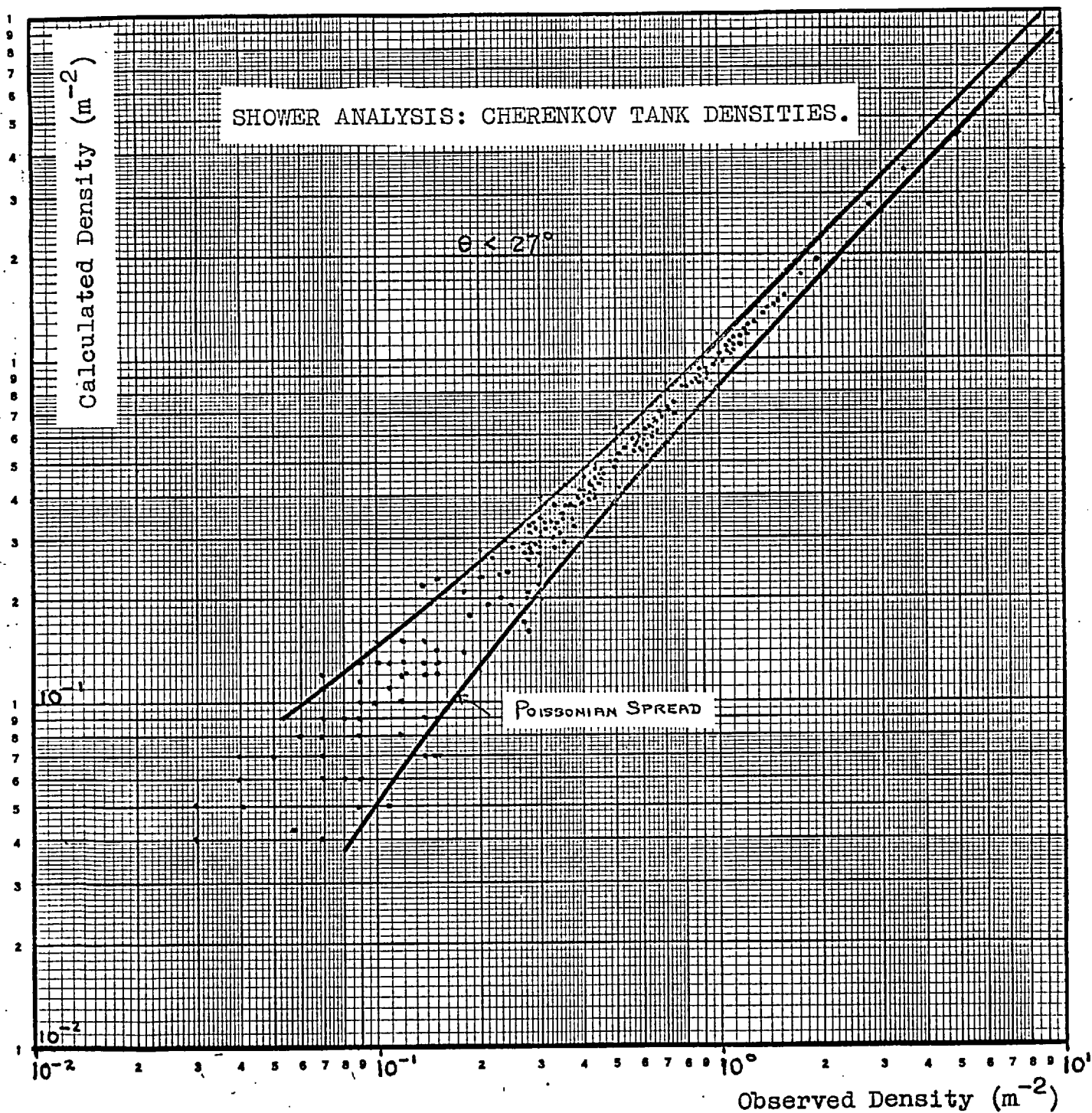


FIG. 4.4.

mis-location of the core position and fluctuations in the arrival time of the shower particles. For the experiment described in the following chapters, errors of the order of a few degrees in zenith angle (for angles up to about 70°) and produced by the above factors, are unimportant.

A typical error in the core location, as a result of film reading errors and fluctuations in the density at the detector, is about 50m for the 500m array and about 5m for the 50m array.

CHAPTER FIVE

THE LATERAL DISTRIBUTION OF MUONS IN SHOWERS OF
SIZE 10^5 TO 10^8 PARTICLES AT DISTANCES BETWEEN
10m AND 1000m FROM THE CORE.

5.1. Introduction.

The muon component of extensive air showers is characterised by its ability to traverse large thicknesses of absorber without interacting. In a shower of size 10^6 charged particles, the proportion of muons is approximately 7% of all charged particles at a distance of 100m from the axis, increasing to about 70% at 1000m from the axis. From a knowledge of the lateral density distribution of the muon component an estimate can be made of the total number of muons in a shower. The variation of this quantity with shower size is reported fully in Chapter 7.

NOTE: In the following chapters, references to shower size denoted by N refer to the nominal shower size derived from the computer analysis. This is related to the conventional shower size N_0 in Appendix 2.

Muons are the decay products of $\pi^{\pm}$ mesons (a far lesser proportion being the decay products of kaons) which are generated in the high energy interactions in the core region of the shower as it progresses through the atmosphere. Due to the degradation of the N - component energy and the greater probability of interaction of the pions in the denser, lower regions of the atmosphere, the high energy muons are mainly produced in the first few nuclear encounters.

The lateral spreading of the muon component is mainly due to the angular spread of the parent pions. Theoretical calculations (Hillas, 1965) predict only a small spreading effect due to Coulomb scattering. The magnetic field of the earth is also predicted to have a small effect on the overall distribution, but can distort the distribution of positive and negative particles by a significant amount, (approximately a threefold increase in the density at 100m east of the axis) if the shower approaches from geomagnetic north (in the northern hemisphere).

Due to the small cross-section for interaction of muons and the time dilation effect on the mean lifetime, the probability of reaching sea-level before decay is large. Hence, measurements on this component at sea-level reflect more directly the properties of the nuclear interaction and

the type of primary particle responsible. This aspect is discussed fully in Chapter 7.

The energy distribution of muons in extensive air showers is also a function of distance from the shower axis (Bennett et al., 1961), so that the lateral density distribution of muons will depend upon the minimum energy required to penetrate the detector shielding. In the present experiment this energy is $\simeq 300\text{MeV}$ for a vertically incident muon.

The M.I.T. Group (Clark et al., 1958) measured the muon density in showers of size greater than about $2 \cdot 10^5$ particles with detectors having a minimum penetration energy of 1GeV . Greisen (1960) has fitted these results to the curve

$$\rho(N, r) = 18(N/10^6)^{\frac{3}{4}} r^{-\frac{3}{4}} (1 + r/320)^{-2.5} \dots\dots 5.1$$

where ρ is the density of muons in a shower of size N , at a distance r from the shower axis, having energies in excess of 1GeV .

The lateral density distribution of muons of energy greater than 0.3 GeV in E.A.S. of size greater than 10^5 particles is presented below over a distance range of 10m to 1000m . The 500m array measurements are described firstly.

5.2. The Experimental Set-Up.

5.2.1. Background.

The Haverah Park E.A.S. detecting arrays have been described in Chapter 2.

To obtain a lateral density distribution over a large range of distances using this type of detecting array, it was necessary to perform the experiment at two different locations in the 500m array complex. The first location was at one of the outer stations (No 3, cf. fig. 2.1). This particular outer station was chosen for accessibility. The muon densities of approximately 5000 showers were recorded during the period of operation in station 3 before the detecting units were moved to the central location (station 1).

Fig. 5.1. shows the distributions of the perpendicular distances of the shower axes from stations 1 and 3 for all showers triggering the 500m array. These distributions illustrate the necessity of performing the experiment at the two locations. A lateral density distribution of muons at distances between 300m and 1000m from the core was obtained with the detectors located in station 3, and at distances between 200m and 600m in station 1. This gave sufficient overlap of the two distributions to produce a statistically good overall distribution in a reasonable period of time.

DISTRIBUTIONS OF MUON DETECTOR - CORE DISTANCES.

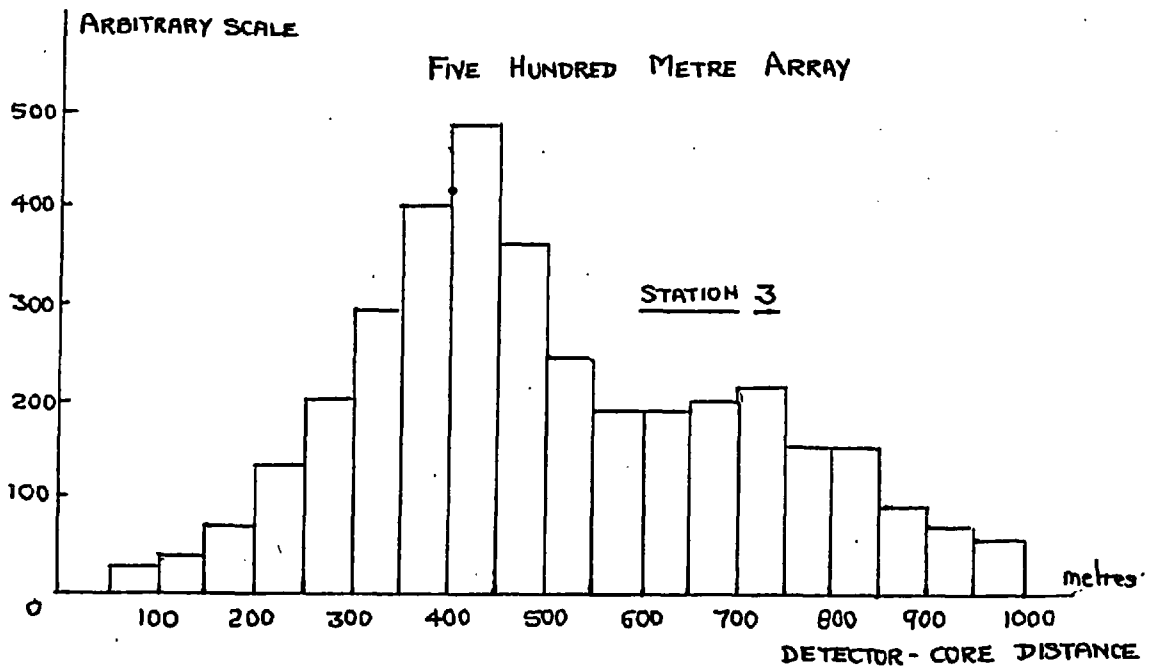
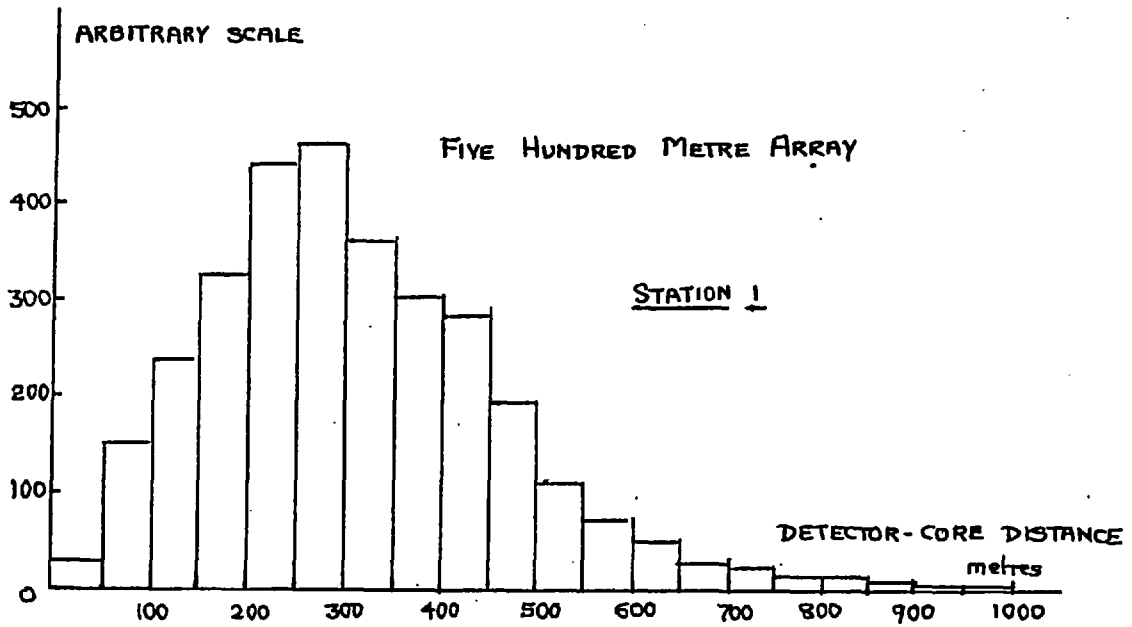


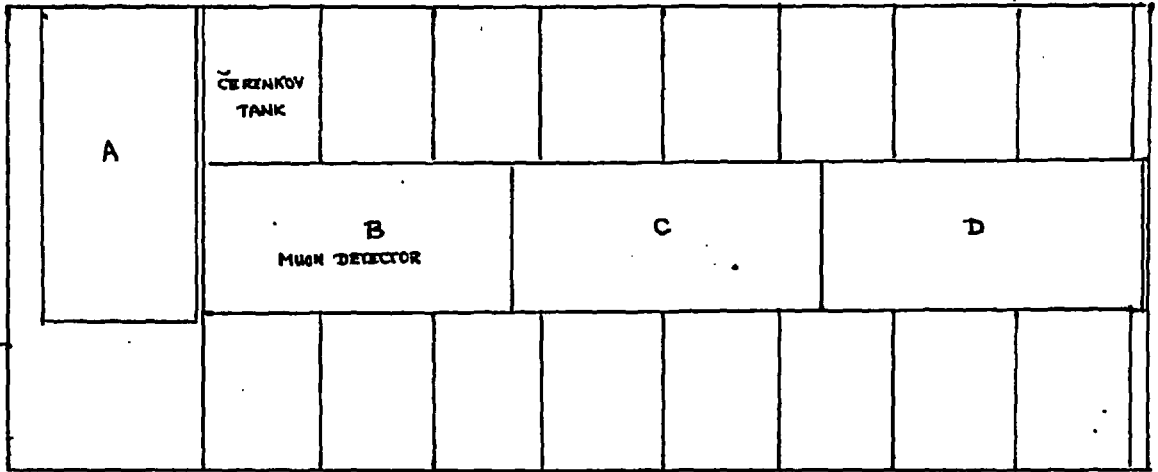
Fig 5.1

5.2.2. The Station Three Measurements.

At station 3 the four detecting units were arranged in the configuration shown in fig. 5.2.(a). The detectors were shielded against the soft component radiation by a 20cm (227gm/cm^2) thickness of lead covering the sides and the top of each detector. The ends of the detectors containing the photo-multiplier tubes and head-units were left uncovered for ease of maintenance. However, the overlap of lead at the ends of the detector ensured that only particles of zenith angle greater than 75 degrees, incident from a preferred azimuthal direction, would miss the shielding.

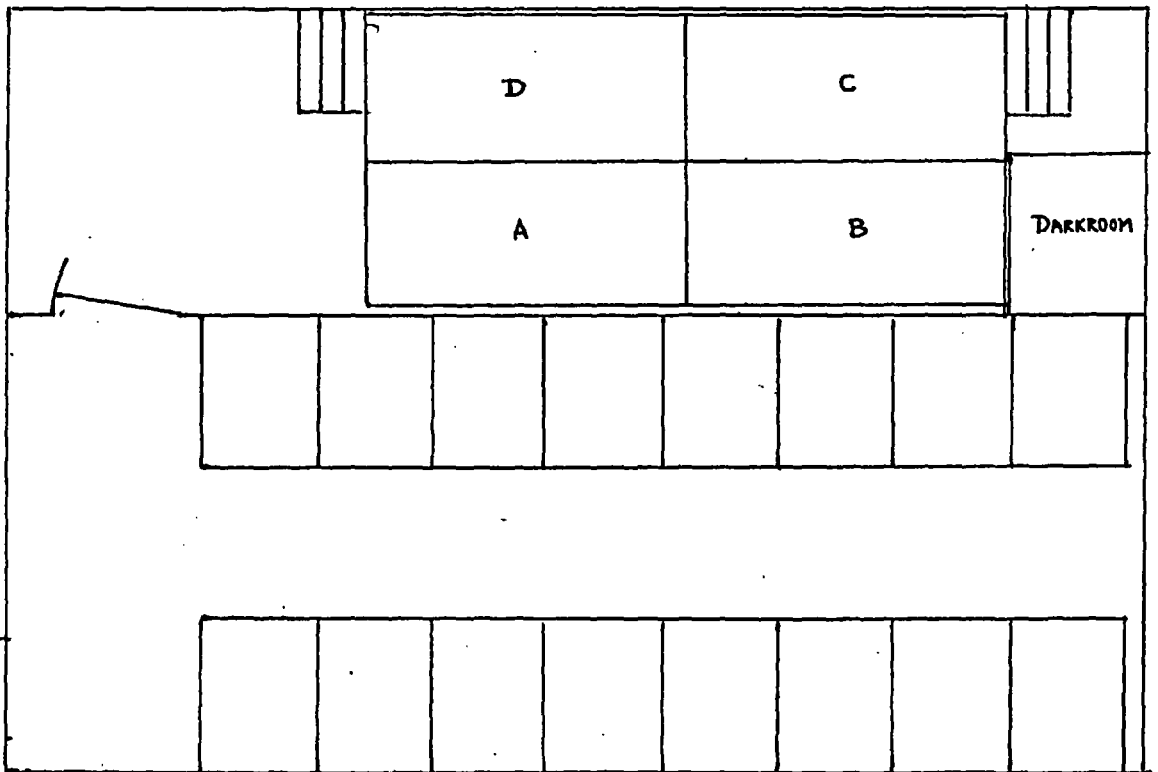
The outputs of the two photomultiplier tubes in each detector were firstly added together. The four added detector outputs were then added, clipped to a length of 1 μ sec using a shorted 100m length of U.R. 39 cable, delayed by 7 μ sec using Hackethal 1500 delay cable and finally fed down 550m of Aerialite 365 transmission cable to the central station. The pulses were displayed on a Tektronix 581A Oscilloscope.

Whenever the nominal density of shower particles at a station exceeds 10 particles $/\text{m}^2$, a trigger pulse is sent out from the Cherenkov tank adding unit to the main coincidence circuit located at the central station.



STATION 3

Fig 5.2 (a).



STATION 1

DETECTOR LAY-OUT

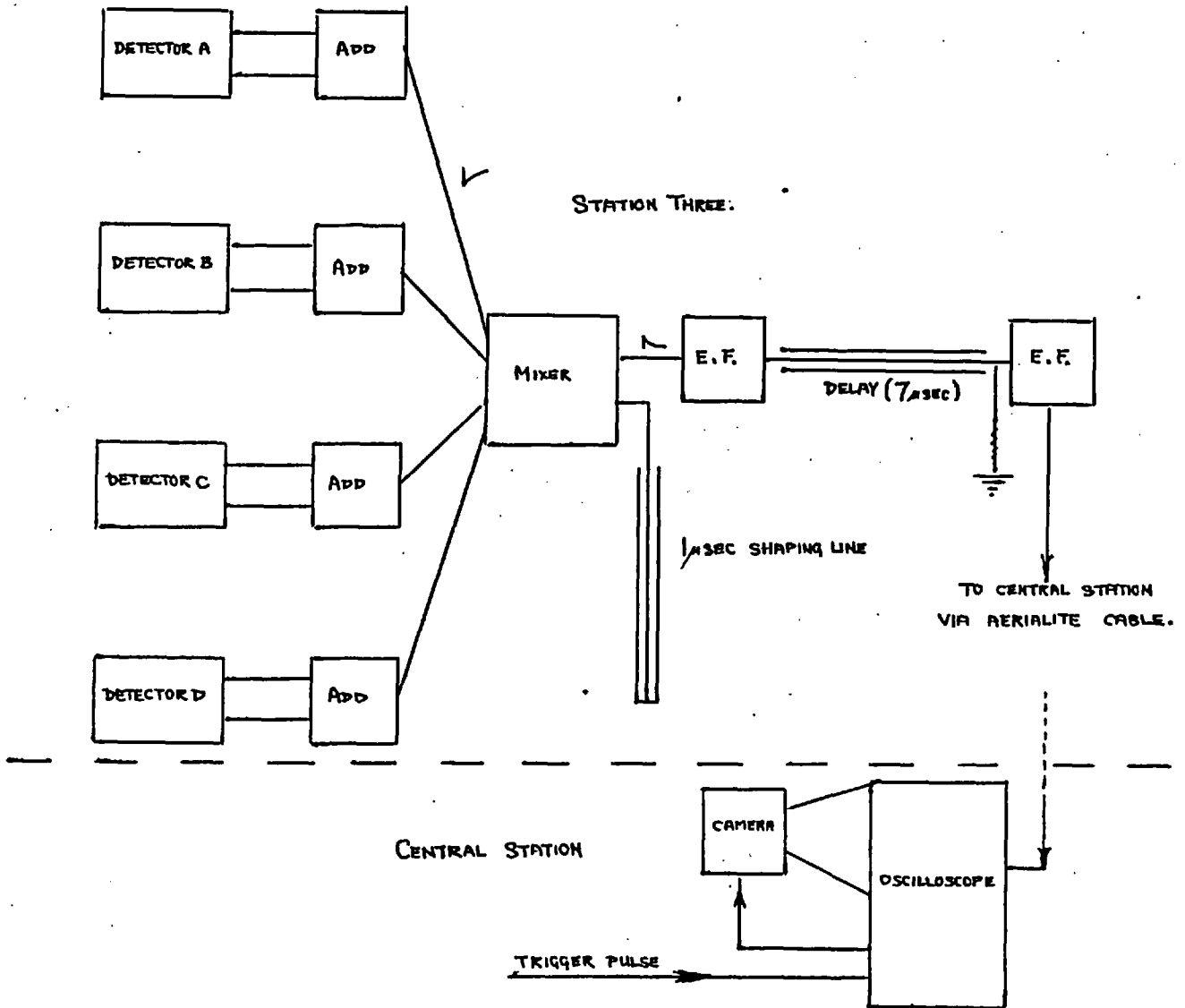
FIG 5.2 (b)

The trigger pulses from the outer stations have an inherent delay of 2.33μ secs after travelling down 600m of Aerialite cable. This is compensated for in the central station channel by a 2.33μ sec length of Hackethal delay cable (1500). When the coincidence circuit receives a trigger pulse from the central station and at least two of the outer stations, a master coincidence pulse is produced within 4μ sec. This is used directly to trigger the oscilloscope time-base. The muon pulse associated with that particular event is then recorded photographically using a Shackman electric recording camera.

The gate output pulse from the oscilloscope is used to drive the camera control unit which illuminates a message register for a fraction of a second and then advances the camera. The message register number is advanced every half minute. The time of a particular event can then be ascertained to within 30 seconds and the event correlated with the 500m array and other photographic records. A block diagram of the recording system used at station 3 is shown in fig. 5.3.

5.2.3. The Station One Measurements.

The detecting units were moved into the central station during September/October 1965. The arrangement of

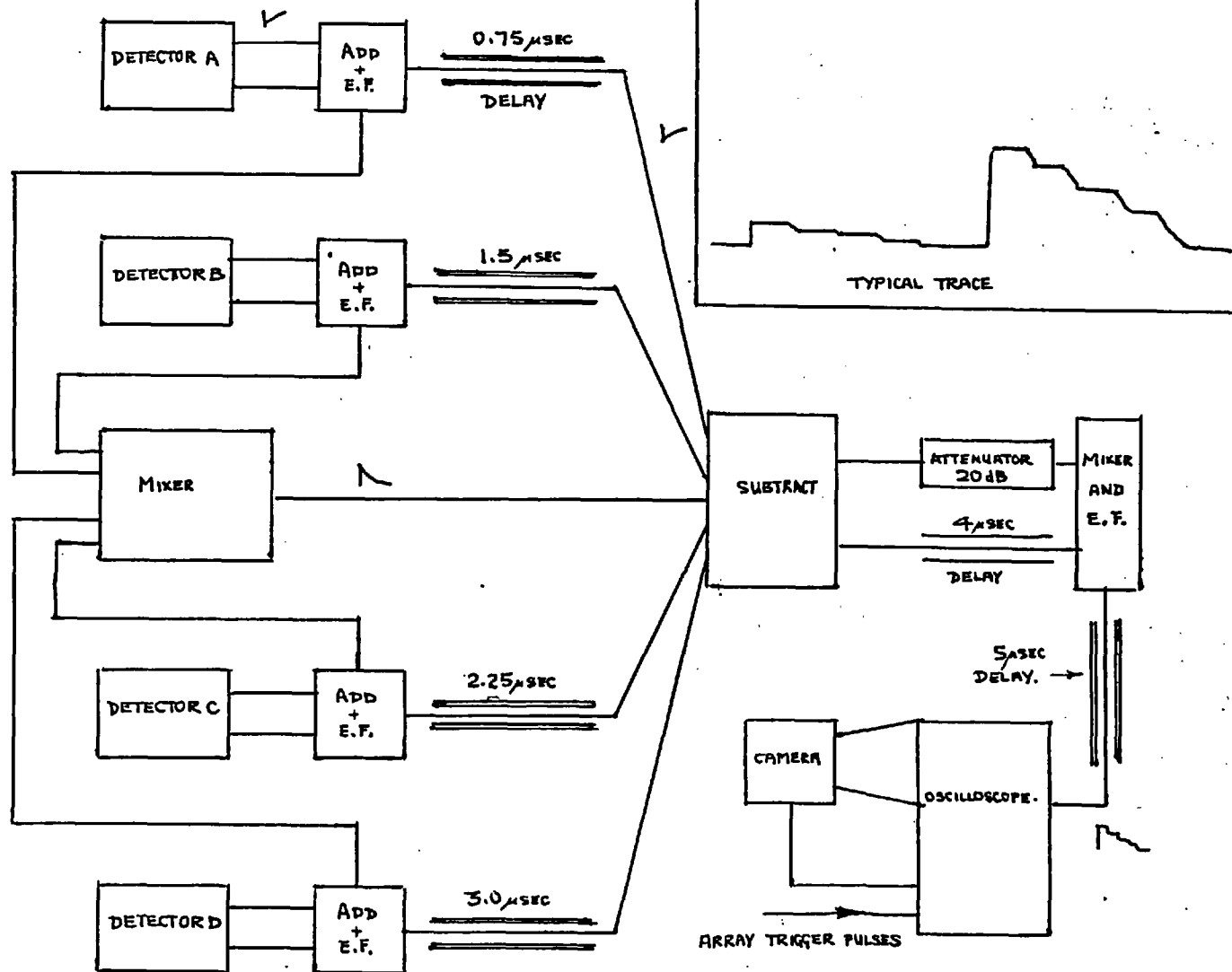


RECORDING SYSTEM (STATION THREE)

Fig 5.3

the detectors in their new location is shown in fig. 5.2(b). At the central station the muon detectors were used to sample the muon content of showers over a wider range of densities. To improve the dynamic range of the recording system the four-fold added pulse was preceded on the oscilloscope display by an attenuated pulse of 0.22 times the normal amplitude. The attenuated pulse was passed through 5 μ secs of Hackathal 1500 delay cable followed 4 μ secs later by the unattenuated pulse. The initial 5 μ secs allows for time delays of the main array trigger pulses from the outer stations and delays in the coincidence unit. A block diagram of the recording system used at station 1 is shown in fig. 5.4.

When recording showers close to the core using the 50m array, it was important to observe the detector responses individually. To avoid using large quantities of expensive delay cable a system was devised whereby the positive four-fold added pulse was clipped at intervals of 0.75 μ secs, 1.5 μ secs, 2.25 μ secs and 3 μ secs by the individual detector negative outputs. This gave a qualitative description of the distribution of particles over the detecting area.



RECORDING SYSTEM (STATION ONE)

FIG 5.4

5.3. Dynamic Range, Response, Calibration and Stability.

The dynamic range of the recording system was determined mainly by the linearity of the photomultiplier head-units at large input voltages. Each head amplifier was tested for linearity up to a matched input of 10 volts, provided by a mercury relay pulse generator. The output was fed into a 400 channel pulse spectrometer, which had been previously linearised and calibrated. The head amplifier was found to be linear from at least 5mV to over 2 volts. A working output voltage of about 30mV from each photomultiplier per vertical muon passing through the centre of the detector was chosen. This made it possible to record 40 particles in one detector (16 particles per square metre) before non-linearity set in. The electronics of the adding unit were tested and found to be linear up to 10 volts input. The maximum number of particles recordable linearly for an even distribution over the detectors was then approximately 160. The oscilloscope gain was set at a level giving readability of single particle pulse heights. With the oscilloscope gain set at 500mV/cm it was possible to obtain a photographic record of the number of particles passing through the detectors over the full linear region. Finally the optical system was checked for any serious aberrations.

The counting rates of each tube were adjusted to within 5% using a stable discriminator. An absolute calibration of the output voltage per incident vertical muon passing through the centre of the detector was obtained by using a three-fold coincidence, narrow angle, scintillation telescope. The count rates of the individual tubes and detectors were checked regularly and minor adjustments made when necessary.

5.4. The Five Hundred Metre Array.

5.4.1. Treatment of the Raw Data.

Each shower recorded during the period of operation of the muon detector was analysed by the Imperial College IBM 7090/1401 computer system. One punched card was obtained for each analysable shower containing information on all of the basic parameters of the shower. About 20% of all the input showers were rejected by the shower analysis program (cf. section 4.7.)

The average number of muons recorded in the station 1 and station 3 measurements were 20 and 2 respectively. With the single detecting unit used for the detection of muons in E.A.S. at Haverah Park, the parameters of a particular shower relating to this component cannot be determined sufficiently accurately for a detailed analysis of its

structure to be made. Instead, average values for the parameters obtained from the distributions of measurements of a large number of events are more meaningful. The quantities investigated and presented below are averages over a large number of showers.

The variation of the density of muons of energy greater than E in E.A.S. at sea-level, at distances greater than 200m from the core, was assumed to be of the form:-

$$\rho_{\mu}(>E) = k(\theta) N^{\beta(\theta)} r^{-n(\theta,N)} \dots\dots\dots 5.2$$

where θ is the zenith angle of the shower.

N is the shower size.

k and β are constants.

n is the structure function exponent

and r is the perpendicular distance from the shower axis.

The number of showers used in the final analysis was approximately 5000. The showers for which a full analysis was possible were broken down into five intervals of zenith angle, ten intervals of shower size and nine intervals of distance.

The first step towards the evaluation of the parameters k, β and n was to check that the mean muon density recorded at a particular distance from the core by the muon

detectors at stations one and three were equal. This was found to be the case and many of the results presented below are from the combined observations.

5.4.2. The Size Dependence of the Lateral Density

Distribution at Large Distances from the Core.

It was shown by Blake (1966) that the structure function exponent n is independent of shower size. By using a greater number of showers this result was investigated again, the density v distance curves for narrow intervals of shower size and zenith angle being fitted to a function of the form:-

$$\rho_{\mu} = k(\theta) r^{-n(\theta)} \quad \dots\dots\dots 5.3$$

by a χ^2 fitting procedure. No variation could be detected over a wide range of shower sizes ($10^6 - 6.10^7$). The mean value for n for a zenith angle interval of 0° to 27° was found to be 2.07 ± 0.16 . The errors on the value of n obtained do not, however, exclude a variation of n less than 0.16. The variation of n_{μ} with shower size was then ignored in the subsequent analysis and an empirical structure function taking the form of equation 5.4 was used.

$$\rho_{\mu} = k(\theta) N^{\beta(\theta)} r^{-n(\theta)} \quad \dots\dots\dots 5.4$$

The average size of all showers with zenith angles less than 27° recorded by the 500m array is 2.10^7 particles.

The normalisation of muon densities to a 'normalisation size' of 2.10^7 in this zenith angle interval results in an average muon density at a particular distance which is relatively insensitive to the value of $\beta(\theta)$. A function of the form of equation 5.3 was fitted to the density v distance curve by a χ^2 fitting procedure to obtain the best values of n and k. The densities at each distance were weighted proportionally to the inverse of the square of the associated standard error. For a mean zenith angle of 17° the best-fit exponent was found to be 2.19 ± 0.02 . By using this value of n for all showers with a zenith angle of less than 27° a least-squares fit was carried out on the density v distance curves for six different size intervals. From this analysis, six values of the average muon density (at an arbitrary distance from the shower axis = 300m) were plotted against the average shower sizes within the narrow shower size intervals chosen (cf. fig. 5.5). Again a χ^2 fitting procedure was used to fit a curve of the form

$$\rho_\mu = cN^\beta \dots\dots\dots 5.5$$

to this data.

From the combined station 1 and station 3 measurements the value of β was found to be:-

$$\beta = 0.73 \pm 0.09 \quad (\theta < 27^\circ)$$

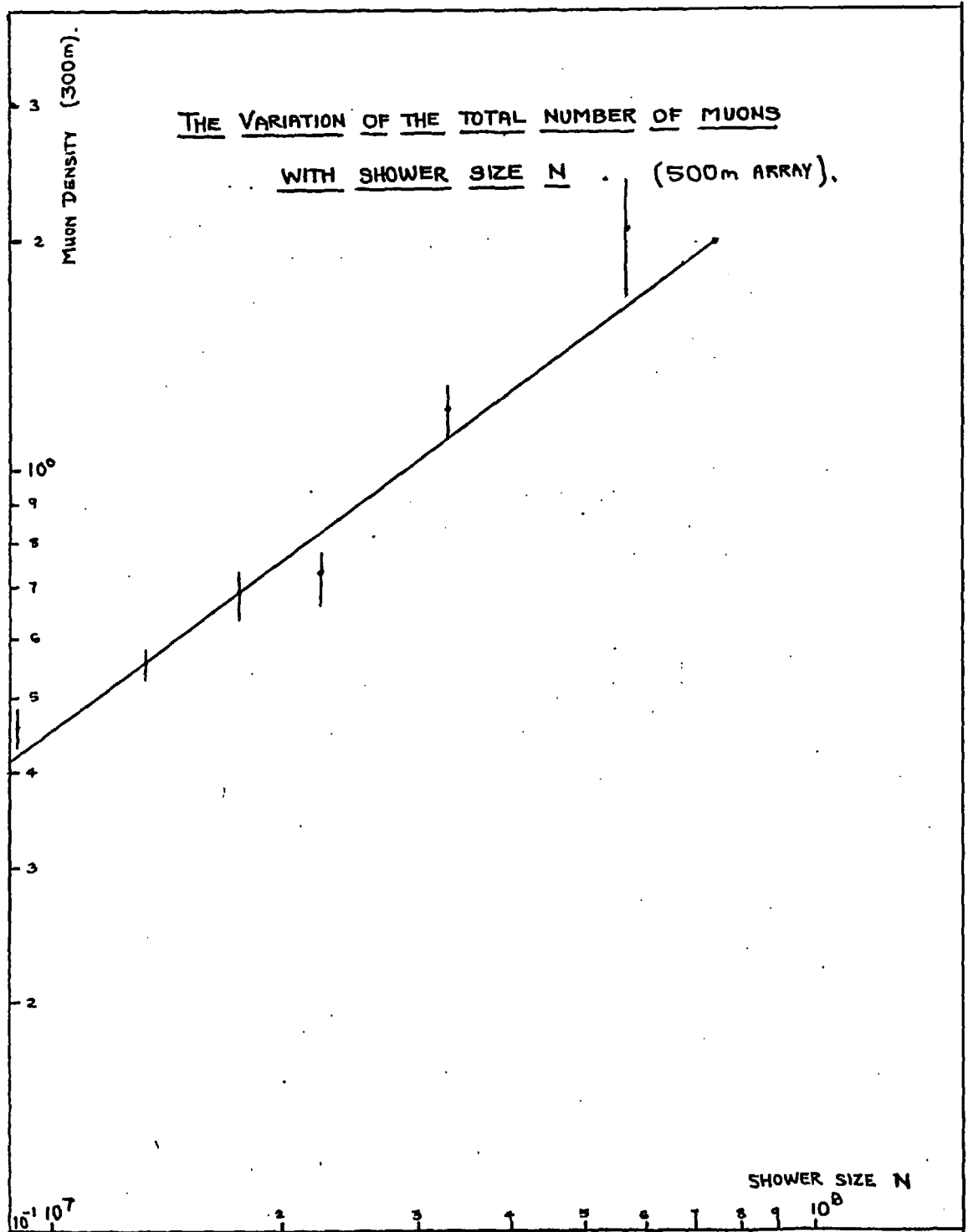


Fig. 5.5

5.4.3. The Mean Lateral Distribution of Muons.

It was established that any size dependence of the exponent of the lateral distribution is small, and a mean lateral distribution of muons at distances between 200m and 1000m from the core was then calculated. All recorded muon densities in showers with a zenith angle of less than 27° were divided into nine groups corresponding to nine intervals of distance. The mean muon density in each interval was then calculated for a nominal shower size of 2.10^7 particles, the normalisation being carried out using the relationship given by equation 5.5.

Using a χ^2 fitting procedure, a curve of the form

$$\rho_\mu = k(N/2.10^7)^{0.73} r^{-n} \dots\dots\dots 5.6$$

was fitted. The values of n and k obtained were:-

$$n = 2.19 \pm 0.02$$

and $k = 2.15 \times 10^5$

Each point was weighted by the square of the reciprocal of the associated standard error. A graph of the mean lateral distribution, for a mean zenith angle of 17° ($\theta < 27^\circ$) is shown in fig. 5.6 namely:-

$$\rho_\mu = 2.15 \times 10^5 (N/2.10^7)^{0.73} r^{-2.19} \dots\dots\dots 5.7$$

valid for $r > 250m$. N is the nominal shower size.

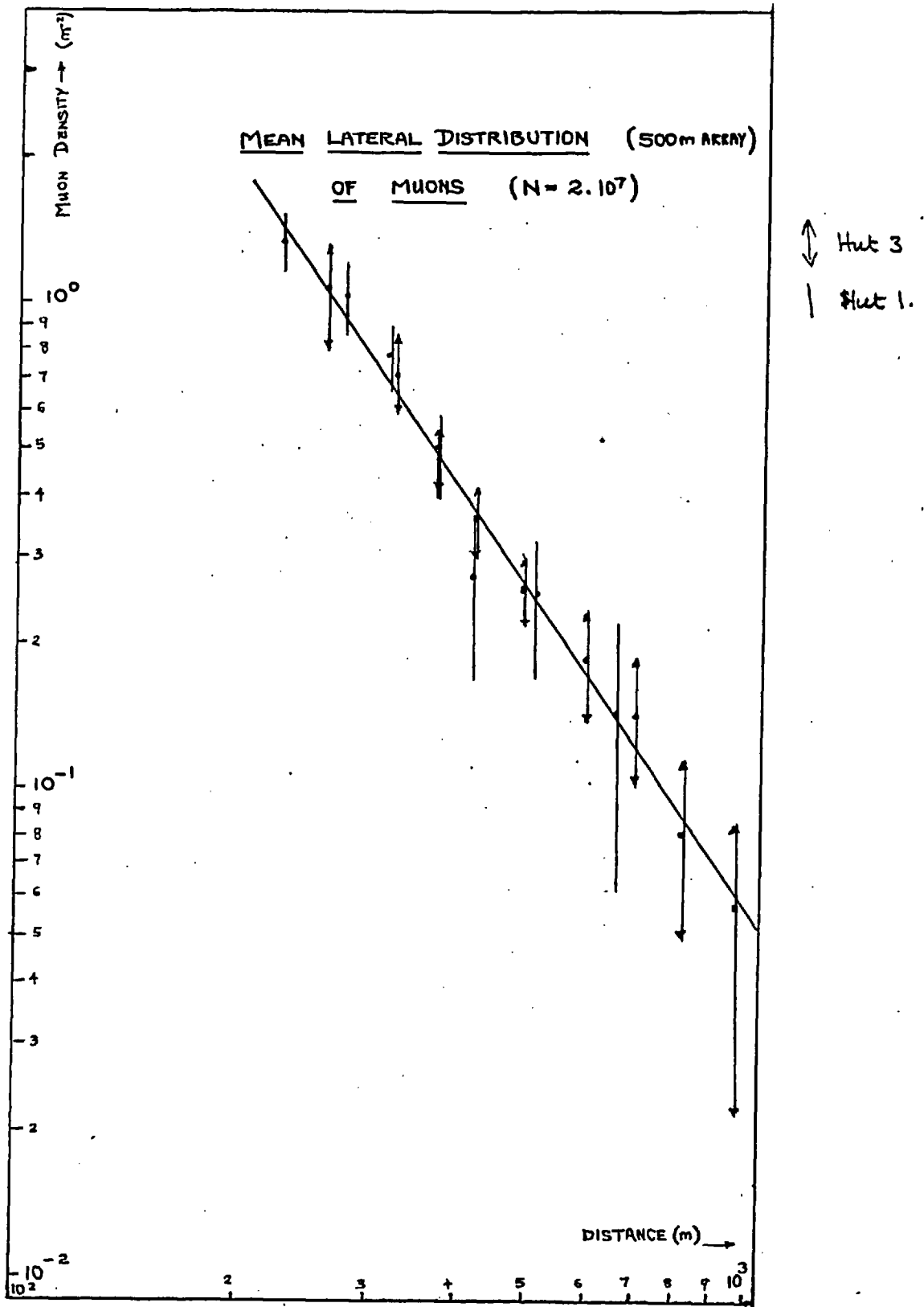


FIG 5.6

5.5. The Fifty Metre Array.

5.5.1. Preliminary Measurements of Muon Density.

A triangular array of four diffused-light water Cherenkov detectors, consisting of a central unit and three outer units placed at the corners of an equilateral triangle at distances of 50m from a central detector (cf. fig. 2.1), was used to detect showers of size between 10^5 and 10^6 particles. The rate of shower detection was approximately four hundred per day. From an analysis of the photographic records, the location of the core and the shower size were computed, assuming the showers to be incident vertically upon the array. The muon densities of approximately 2000 showers were measured during the period of operation of the muon detector. The distributions of the core to muon detector distances and the shower sizes of all analysable showers are shown in fig. 5.7.

5.5.2. Measurements of the Muon Density Close to the Core.

At the core to detector distances investigated by the 500m array, the probability of encountering a nucleon associated with the extensive air shower is remote. However, using the 50m array, showers falling within 10m of the detector were investigated, where the density of nuclear-active particles is not negligible. For example, in a shower of

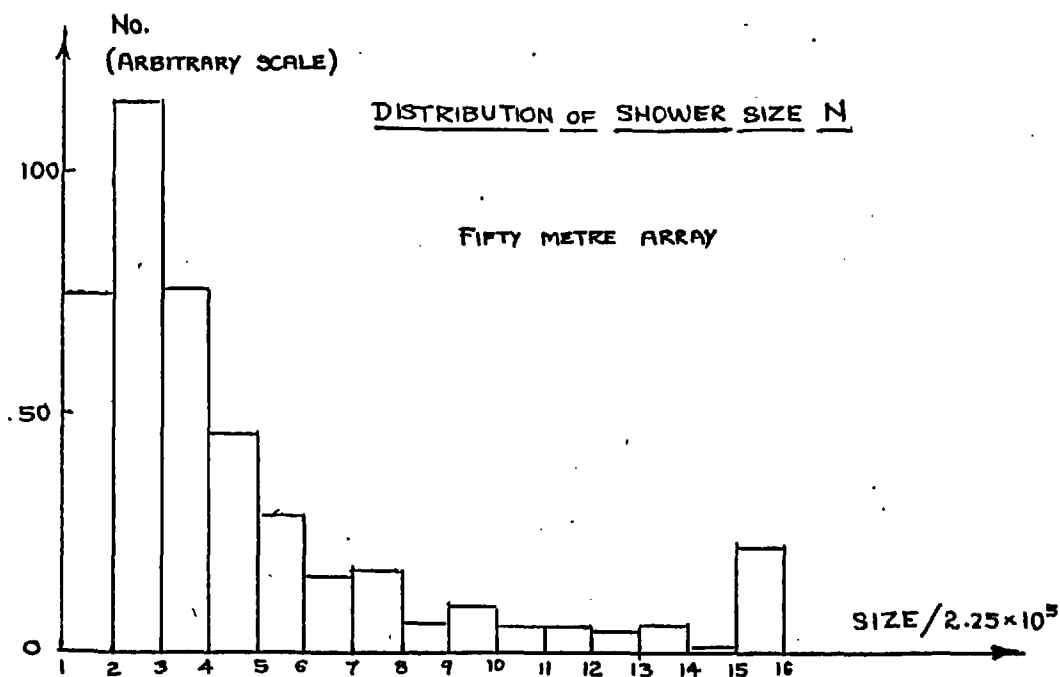
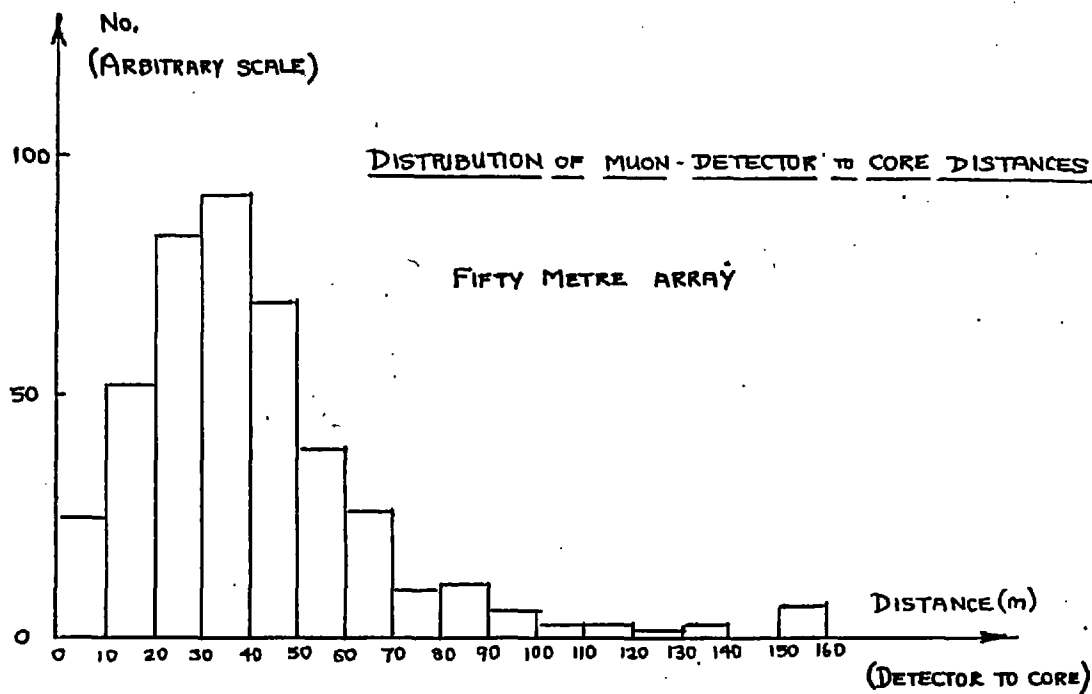


Fig 5.7

size $3 \cdot 10^5$ particles at 3860m altitude, at a distance of 10m from the core, the density of nuclear-active particles of energy greater than 1GeV is $\simeq 0.15$ per square metre, and the mean energy is 47GeV (Greisen, 1960). The rapid decline in the density of nuclear-active particles with increasing distance from the core, (described by $\rho_{\text{n.a.p.}} \sim e^{-r/r'}$ where $r' \sim 2\text{m}$ at a size of 10^5 particles) results in a low probability of recording a nuclear interaction over the sensitive area of the detector (10 sq.m) at distances greater than 10m.

The detecting unit consists of four detectors each of area 2.5 sq.m. The responses of each detector were recorded independently as described in section 5.2.3. This enables a qualitative description of the charged particle distribution over the detectors to be available. Those events, for which the pulse height distribution over the detectors was incompatible with a muon distribution, were rejected in the subsequent analysis. Apart from nuclear interactions in the lead, a large pulse in one detector can also be associated with the nuclear interaction of a high energy muon in the lead absorber.

A large pulse in one or more of the detectors may also be produced by an extreme fluctuation or even a muon beam (Vernov et al., 1962; Miyake et al., 1963).

The Russian group (Vernov et al., 1962) have observed bundles of muons consisting of three or more particles at distances of less than 10m from the core by using a Geiger-Muller counter hodoscope. The particles in the bundles were of energies greater than 10GeV, and lay within a projected distance of approximately 20cm.

The Japanese Group (Miyake et al., 1963) at Mount Norikuru (2,770m above sea-level), using a multiplate cloud chamber, recorded one event out of approximately 100 events in which five muons were observed to travel parallel to one another through 21 lead plates of 1cm thickness, without interaction. The axial distance was about 5m. The probability of a fluctuation of this magnitude is negligible. Although it is possible that such bundles are detected in the present experiment, they cannot be directly identified.

The frequency of occurrence of events in which the pulse height in one detector produced saturation, is shown in fig. 5.8 as a function of the distance from the shower axis for all analysable showers. The energy released in one detector in such an event exceeds 0.5GeV. These events are therefore associated with a nuclear interaction.

Figure 5.8. (overleaf) shows, as a function of core-detector distance, the relative frequency of analysable 50m array showers which produce a 'saturated' response in only one of the four muon detectors.

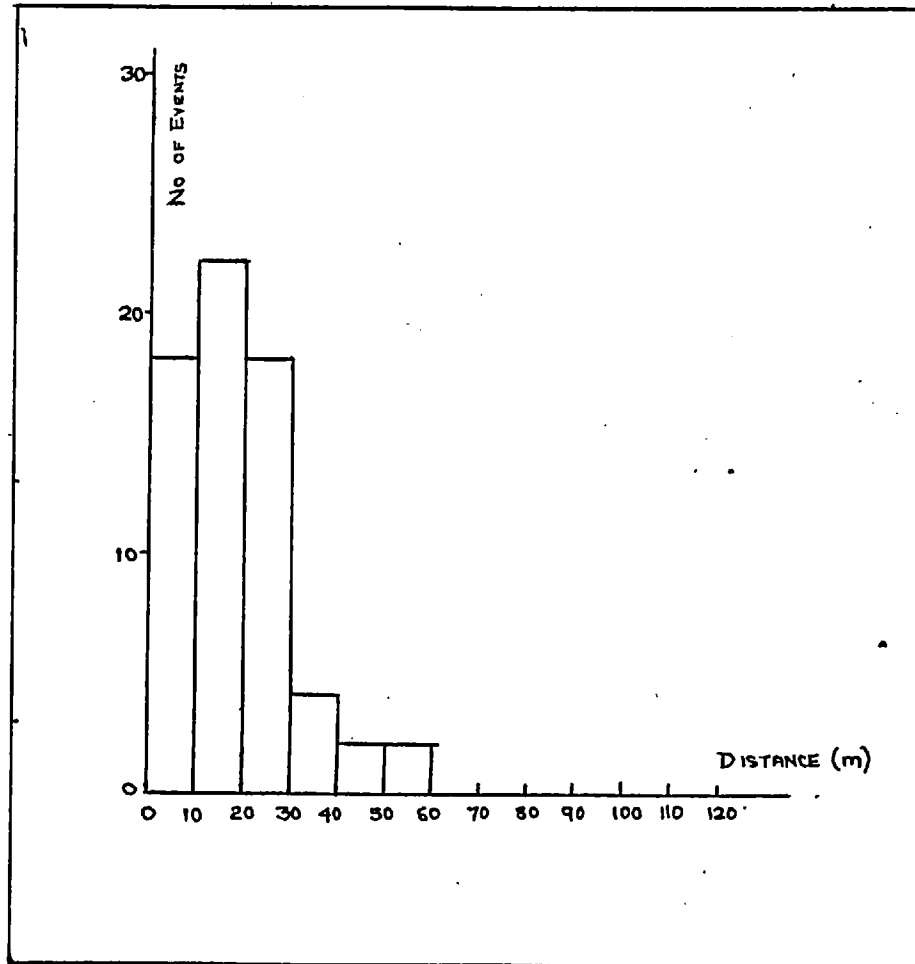


FIG 5.8

5.5.3. Treatment of the Raw Data.

The 50m array was used to select showers of sizes between 10^5 and $5 \cdot 10^6$ particles. The showers used in the analysis were assumed to come from a direction close to the vertical. (This assumption should not produce a serious effect on the analysis since the mean angle of arrival of these showers is about 20° from the vertical). These measurements were carried out during the same period as the recording of muon densities in showers triggering the 500m array.

The shower information recorded using the 50m array was punched on to paper tape and analysed using the University of Leeds E.E.L.M. - KDF9 computer.

The analysable showers were divided into five intervals of shower size and five intervals of distance. For each size interval a curve of the form:

$$P_\mu = (ke^{-r/320})/r^\alpha \dots\dots\dots 5.8$$

was fitted. The best-fit values of α did not indicate any change of the structure function with shower size greater than the limits of experimental error. Having established that the structure function was independent of shower size, the muon densities of all showers were normalised to an arbitrary normalisation size and a curve of the form of equation 5.8 was fitted.

The value of α giving the best fit was found to be:-

$$\alpha = 0.8 \pm 0.01$$

For each shower size interval this value for α was used, and the curve of equation 5.8 was fitted to the density v distance plots.

The density at 30m was then plotted as a function of the mean shower size, and a curve of the form of equation 5.5 was fitted by a χ^2 fitting procedure. This yielded a value for the exponent β of:-

$$\beta = 0.73 \pm 0.05$$

The dependence given by equation 5.5 with the above value for β was used to normalise the muon densities of all showers to a size of 2.10^7 particles. A least squares fitting procedure was used to fit a curve of the form of equation 5.8 with $\alpha = 0.8$ and the muon density at distances between 10m and 100m from the core was found to be given by the expression:

$$\rho = 2.36 \times 10^2 (N/2.10^7)^{0.73} e^{-r/320} / r^{0.8} \quad \text{..5.9}$$

Fig. 5.9 shows the composite lateral distribution obtained from the 50m and 500m arrays for a shower size of 2.10^7 nominal particles. The region between 100m and 200m from the core will be investigated using showers selected by the 150m array, when sufficient results have been accumulated.

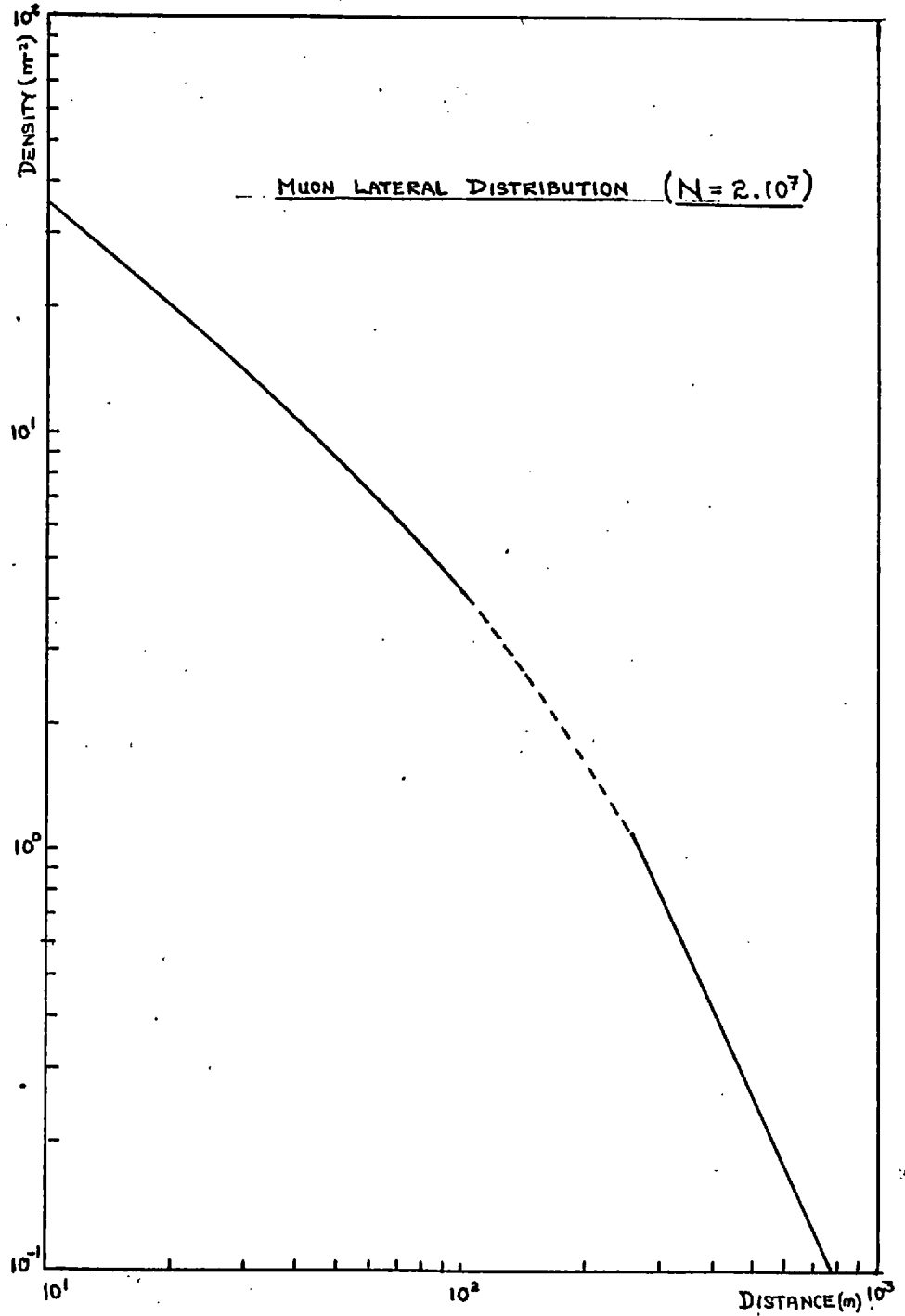


FIG 5.9

A more sophisticated form for the lateral distribution will then be fitted. Fig. 5.10 shows the lateral distributions given by Greisen (1960), Abrosimov et al. (1960) and the theoretically-predicted lateral distribution due to Hillas (1965).

5.6. Comparison with Other Workers.

The lateral distribution of muons at large distances ($> 100\text{m}$) from the core in showers of size greater than 10^6 particles has been measured by a number of workers (Clark et al., 1958; Abrosimov et al., 1960; Khrenov, 1962; Blake, 1966). The results of Clark et al. (1958) have been fitted by Greisen (1960) to the empirical relationship of equation 5.1.

The densities given by Greisen's formula for an energy threshold of 1GeV have been modified to a threshold of 0.31GeV by using the sea-level energy spectrum of air shower muons determined by Bennett and Greisen (1961). This curve is shown in fig. 5.10 for a shower size of $2 \cdot 10^7$ particles. The points in the same figure due to Abrosimov et al. (1960) were made by using counters under approximately 8cm of lead and 8cm of iron. The results were normalised to a shower size of $2 \cdot 10^7$ particles. A relation of the form of equation 5.5 with β equal to unity was used for the

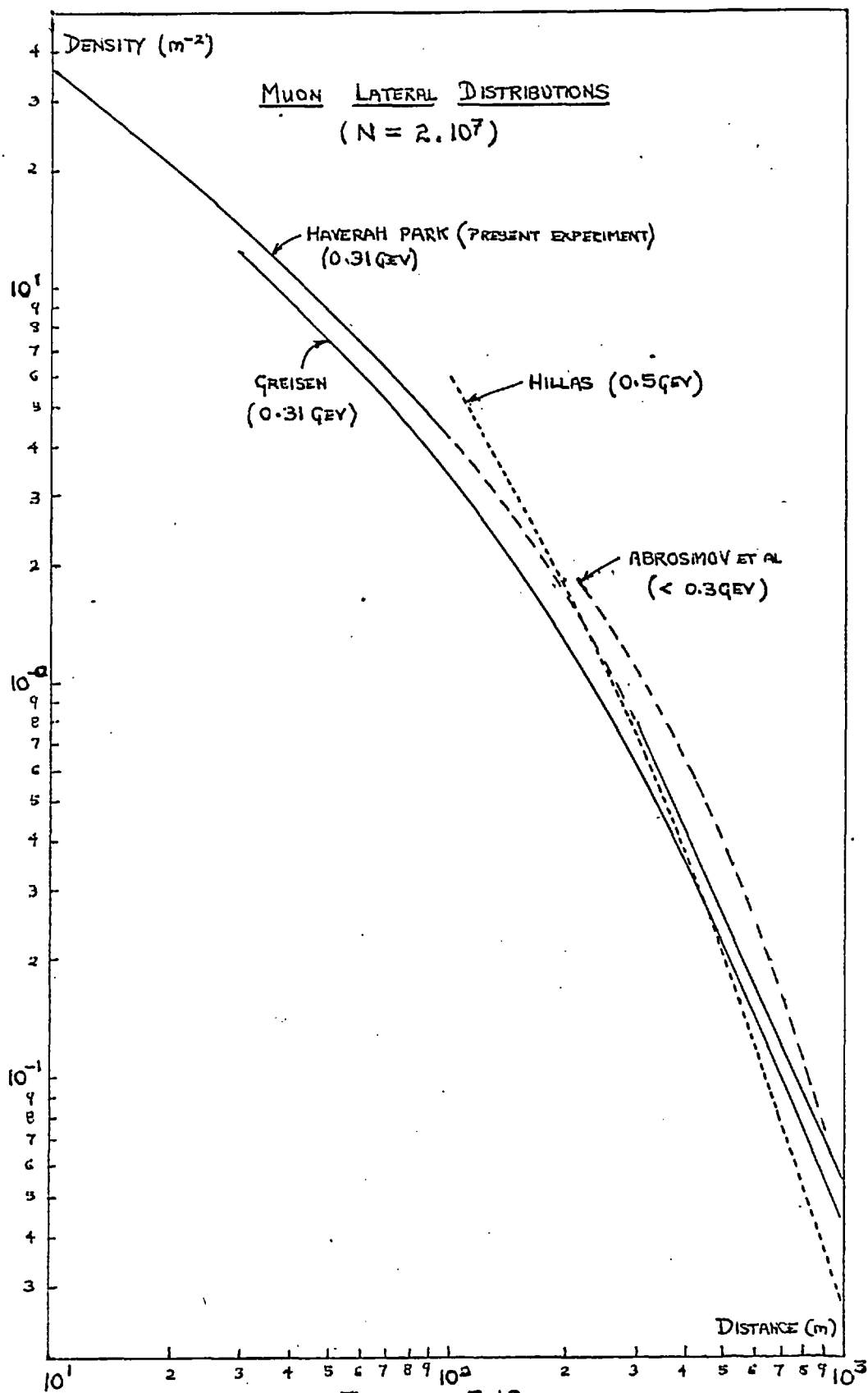


FIGURE 5.10

normalisation procedure. The shower sizes concerned were however spread about the normalisation size, so that the error generated by assuming a value of one for the exponent β is small.

For a nucleon initiated shower of size $2 \cdot 10^7$ particles, which is incident at a zenith of 10° , a theoretically derived lateral distribution of muons with energies above 0.5 GeV is also shown in fig. 5.10 (Hillas, 1965). The results obtained at Haverah Park are compatible with a dependence of the muon density upon distance described by a power law, for distances between 250m and 1000m from the core. Below 250m the lateral distribution flattens quite rapidly with decreasing distance.

In the present experiment the muon density measurements at distances between 10m and 1000m from the core in showers which approach from a direction close to the vertical ($\theta < 27^\circ$ for 500m array showers), are in good agreement with the values given by the empirical relationship due to Greisen (1960), being approximately 15% greater. They are less (by approximately 20%) than the values found by Blake (1966) at Haverah Park. However, the latter author used showers falling within 40° of the zenith. For an inclined shower, the proportion of muons will be greater than that in a vertical shower of the same size, since the showers

under observation are past maximum development. The mean zenith angle of all 500m array showers detected at Haverah Park within the range 0° to 40° is 25° , compared with 17° for the range 0° to 27° . When a particle attenuation length of $\simeq 300\text{gm/cm}^2$ (Suri, 1966) for a given primary energy is used, the shower size expected for a shower incident at 25° to the zenith will be 20% less than that of a shower incident at 17° to the zenith. (An additional atmospheric depth of 60gm/cm^2 is involved). Assuming that the loss of muons due to ionisation, decay and geometrical spreading is small ($\ll 20\%$) in the additional 60gm/cm^2 of atmosphere, the present results are in good agreement with those of Blake (1966).

The theoretical calculations due to Hillas (1965) for an energy threshold of 0.5GeV appear to diverge from the present measurements at large distances from the core. The mean muon energy becomes smaller, so that the detector threshold energy becomes an important factor. The computed energy spectrum of muons (Hillas, 1965) suggests that at distances greater than 500m from the core, the discrepancy between the theoretical and observed results is partly accounted for by the different threshold energies considered.

CHAPTER SIX.

THE ZENITH ANGLE DEPENDENCE OF THE MUON COMPONENT OF EXTENSIVE AIR SHOWERS OF SIZE GREATER THAN 10^6 NOMINAL PARTICLES.

6.1 Introduction.

Measurements of the characteristics of extensive air showers at various atmospheric depths in one experiment can be made, either by measurements at varying atmospheric pressures, or at various zenith angles. By the method of fast timing at four widely spaced detectors, the zenith angle can be measured to an accuracy of the order of a few degrees up to zenith angles of 70° .

6.2 The Zenith Angle Distribution.

The zenith angle distribution of all analysable showers recorded by the 500m array is shown in fig. 6.1. The mean zenith angle of showers detected by the array is approximately 34 degrees. This compares with a mean zenith angle of 23 degrees as found by Bennett et al. (1962) who used thin detectors. The difference in the mean value found by the two detectors is due to both physical and geometrical reasons, namely:-

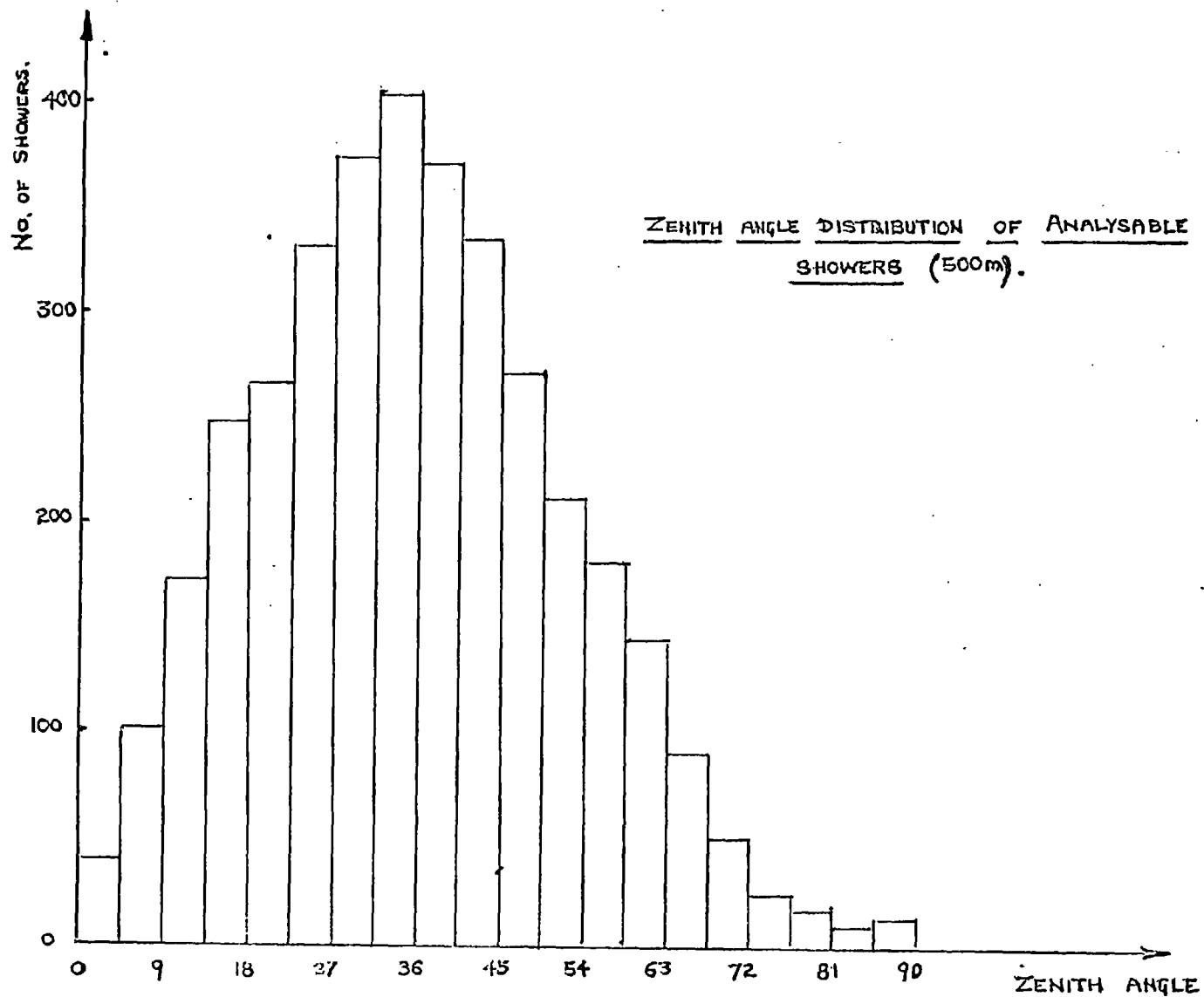


FIG 6.1

- (a) The different sensitivity of the Cherenkov tanks and thin scintillators to the electron-photon component.
- (b) The greater probability of the deep Cherenkov tanks intercepting an inclined particle.
- (c) In the plane of the shower front the projected separation of the detectors decreases with zenith angle.
- (d) The Cherenkov detector lateral structure function becomes flatter with increasing zenith angle.

6.3 The Variation of the Lateral Structure Function of the Cherenkov Tanks and Muon Detectors with Atmospheric Depth.

The structure functions of the lateral density distributions of both Cherenkov tanks and muon detectors have been investigated, and conclusions drawn about the structure of the shower as a function of atmospheric depth.

The muon densities of all analysable showers were sorted into intervals of zenith angle and distance of the detectors from the shower core. After establishing that any variation of the muon structure function exponent n_{μ} with

shower size is negligible (cf. chapter 5), the densities were normalised to a 'normalisation size' for each zenith angle interval. It was then possible to obtain n_μ for five different mean atmospheric depths. To obtain n_μ a curve of the form:

$$P_\mu = k_\mu N_\mu r^{-n_\mu} \dots\dots\dots 6.1$$

was fitted by a χ^2 fitting procedure to the P_μ vs r plots for distance intervals 200m to 600m from station one, and 400m to 1000m from station three. Fig. 6.2 shows the variation of the exponent n_μ with atmospheric depth $h_0 \sec \theta$. Also shown is the Cherenkov tank structure function exponent n_C as a function of atmospheric depth. A quantitative measure of the variation with distance from the core in a distance range 100m to 1000m is not yet possible, due to a lack of points of small statistical error at the larger distances. Beyond an atmospheric depth of approximately 1200 gm/cm^2 the exponent falls quite sharply with increasing depth and approaches the value of the exponent for the Cherenkov detectors at a depth of 2500 gm/cm^2 .

6.4. Observations on the Variation of the Muon Structure Function Exponent with Atmospheric Depth.

The variation of the muon structure function exponent n_μ with atmospheric depth will be influenced by two distinct

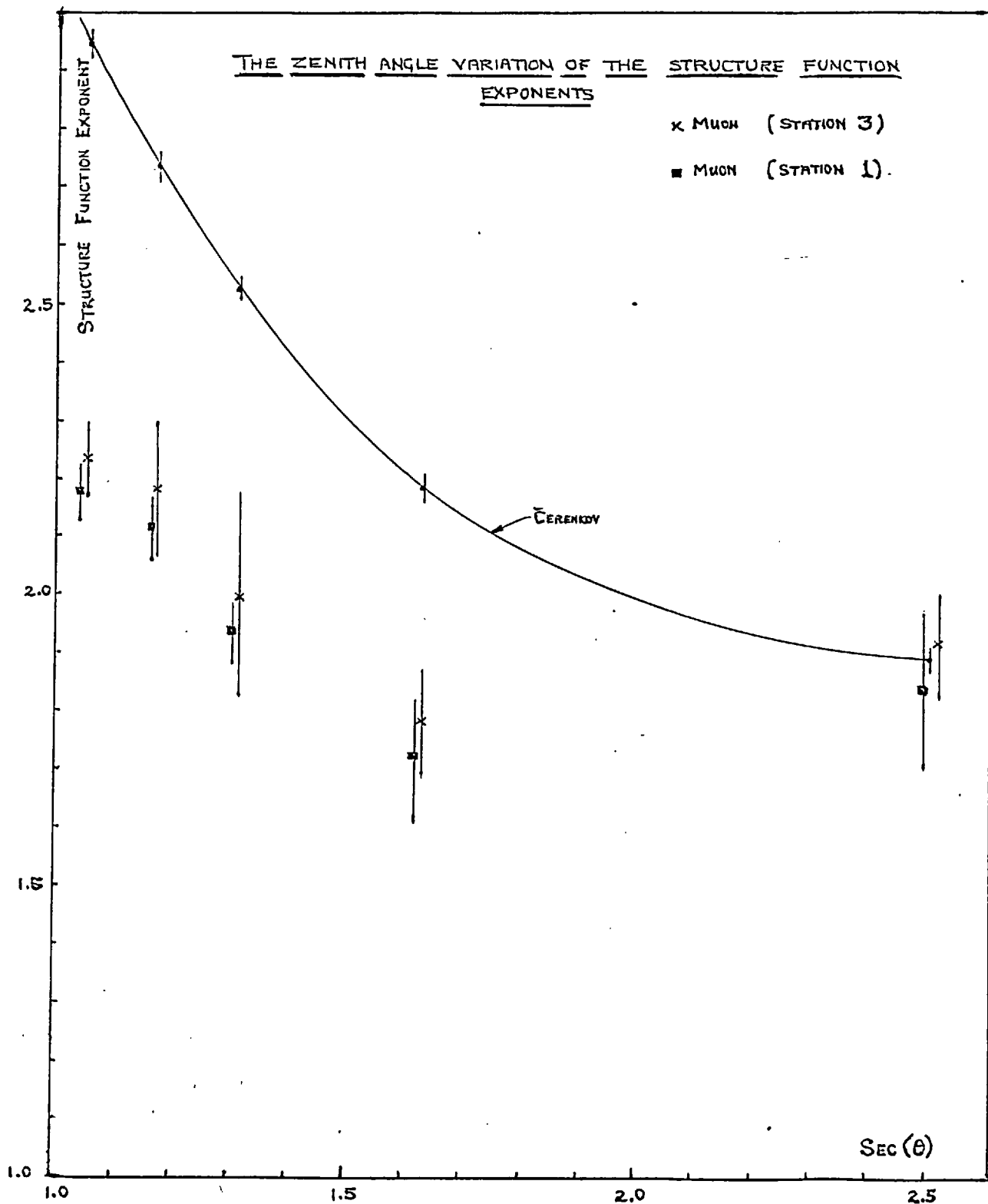


Fig 6.2

factors:-

- (a) Processes going on in the shower as the thickness of atmosphere traversed increases e.g. decay and ionisation loss.
- (b) At higher zenith angles, the parent pions will spend longer in regions of lower atmospheric density and the probability of decay into muons, rather than interaction with air nuclei, will increase.

The structure function exponents at both large ($\approx 600\text{m}$) and smaller ($\approx 300\text{m}$) distances from the core both vary in the same manner with atmospheric depth, indicating that the mean source of muons lies high enough in the atmosphere to have a widespread effect. Baxter et al. (1965) measured the pulse forms of the Cherenkov tanks at Haverah Park and interpreted their observation of a sharp shower front as a muon front. They deduced, therefore, that the mean source of muons is located at a height corresponding to the mean radius of curvature, as measured at Haverah Park. This is approximately 4km for vertical showers, increasing to 7.5km for showers inclined at an angle $\geq 60^\circ$ to the vertical (Suri, 1966). Bennett et al. (1962) found a value of 2.8km when using detectors of smaller area. Linsley and Scarsi (1962A) made measurements at 1800m altitude of the delay of single particles with respect to the shower front and obtained a value lying between

7km and 10km for the mean production height.

The percentage of low energy muons increases with distance from the core for vertical showers (Bennett et al., 1962), so that as the atmospheric depth traversed increases, the effects of decay and ionisation loss would be to increase the exponent of the structure function (if it were assumed that nearly all of the detected muons originated from a point source). The greatest rate of change of n_{μ} with atmospheric depth occurs between depths of 1100gm/cm^2 and 1600gm/cm^2 after which, the value remains fairly constant, to atmospheric depths in excess of $2,500\text{gm/cm}^2$.

The structure function of muons is known to flatten quite rapidly below about 300m distance from the core. A value of $n_{\mu} = 1.9$ ($\theta > 60^\circ$) may therefore be associated with the higher energy muons produced high in the atmosphere. The higher value of n_{μ} of 2.2 observed for zenith angles $< 27^\circ$ may be influenced by a component of lower mean energy, produced lower down in the atmosphere. The quite rapid flattening of the distribution between 1100gm/cm^2 and 1600gm/cm^2 suggests a cause, such as a loss of this lower energy contribution, due to decay and ionisation, and rapid dilution due to geometrical spreading. The Cherenkov tank structure function at atmospheric depths in the region 1100gm/cm^2 to 1600gm/cm^2 is dominated by the electron-photon

component, so that no quantitative information on the relative importance of loss due to ionisation or decay can be made from an investigation of the exponent n_0 with atmospheric depth.

A quantitative answer to this problem cannot be made until the distribution of muon energy in showers of size $10^6 - 10^8$ particles at large zenith angles has been measured by a magnetic spectrograph having a large acceptance solid angle.

6.5. The Relative Response of Muon Detectors and Cherenkov Detectors at Varying Atmospheric Depths and Core Distances.

The Cherenkov detector responds to both the muon and electron-photon components so that the nominal density P_C recorded by a Cherenkov detector may be written as:

$$P_C = P_\mu + B P_e \quad \dots\dots\dots 6.2$$

B is the average response, in terms of the response of a vertical equivalent muon, produced by one electron and the average number of photons accompanying one electron. B depends upon the energy of the electron-photon component and so varies with distance from the core for a deep Cherenkov tank (120gm/cm^2). B should also be dependent upon shower size.

At high zenith angles ($\theta > 60^\circ$), the electron-photon component should have been absorbed by the time the shower reaches sea-level for a shower of size between 10^6 and 10^8 particles. Thus the detectors will respond mainly to muons, and the Cherenkov tank response should equal that of the muon detector. The ratio of the total response of the Cherenkov detector to that of the muon detector is shown in fig. 6.3. as a function of zenith angle, for various distances from the core. This ratio approaches a value of unity for atmospheric depths in excess of 2000gm/cm^2 . Fig. 6.4. shows the variation of the ratio of the total muon detector response to the total Cherenkov detector response as a function of distance from the core for various intervals of zenith angle.

The average shower size recorded by the array is found to decrease with an increase in zenith angle. As shown in Chapter 5. the muon density varies with the nominal shower size as:

$$\rho \sim N^\alpha \quad \text{where } \alpha = 0.73 \quad \dots\dots\dots 6.3$$

The points shown on figs. 6.3. and 6.4. are taken for all showers over wide ranges of shower size. Therefore, as a consequence of the non-linear relationship between muon density and shower size at zenith angles $< 60^\circ$, no quantitative conclusions can be drawn. They do, however,

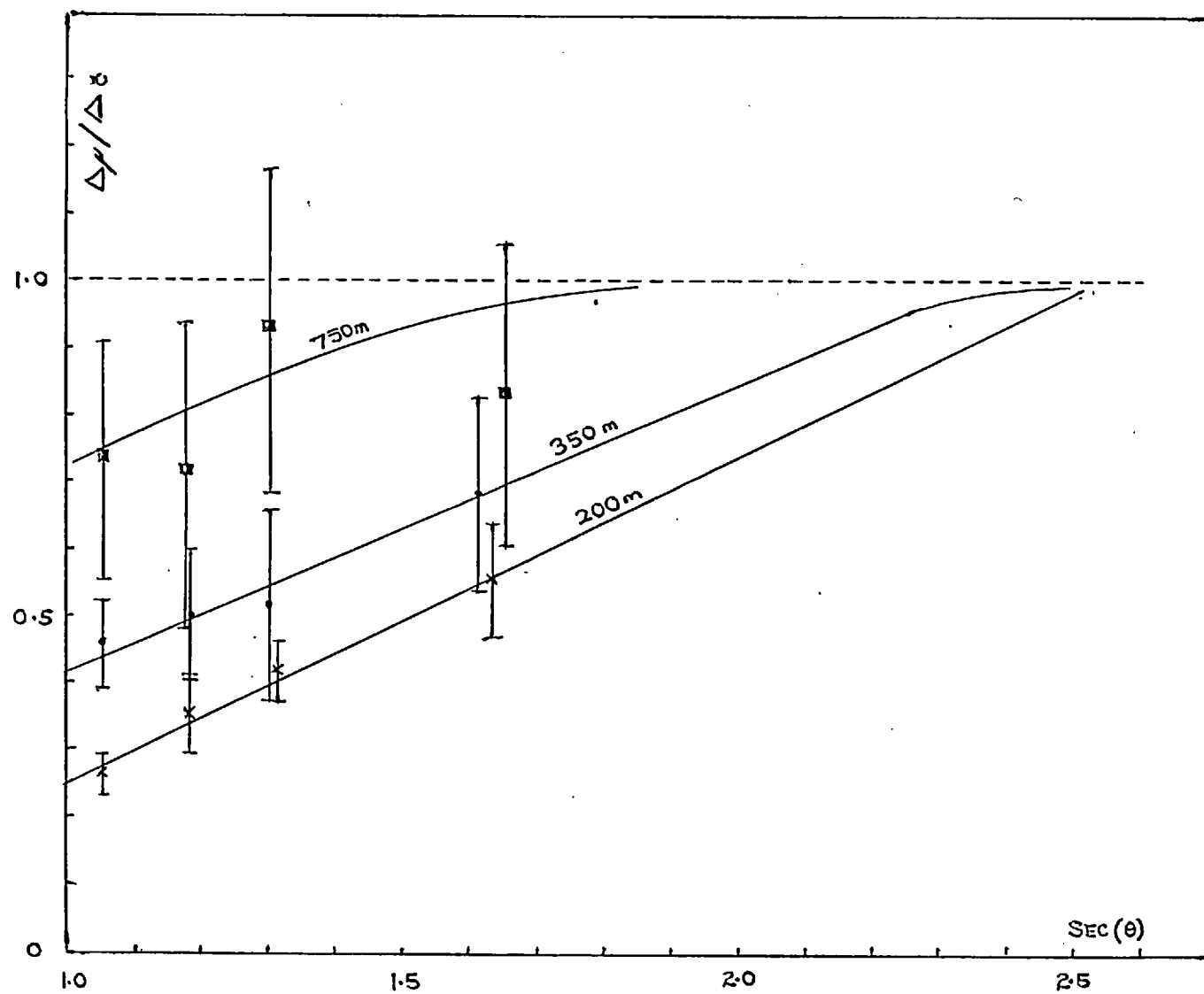


FIG 6.3

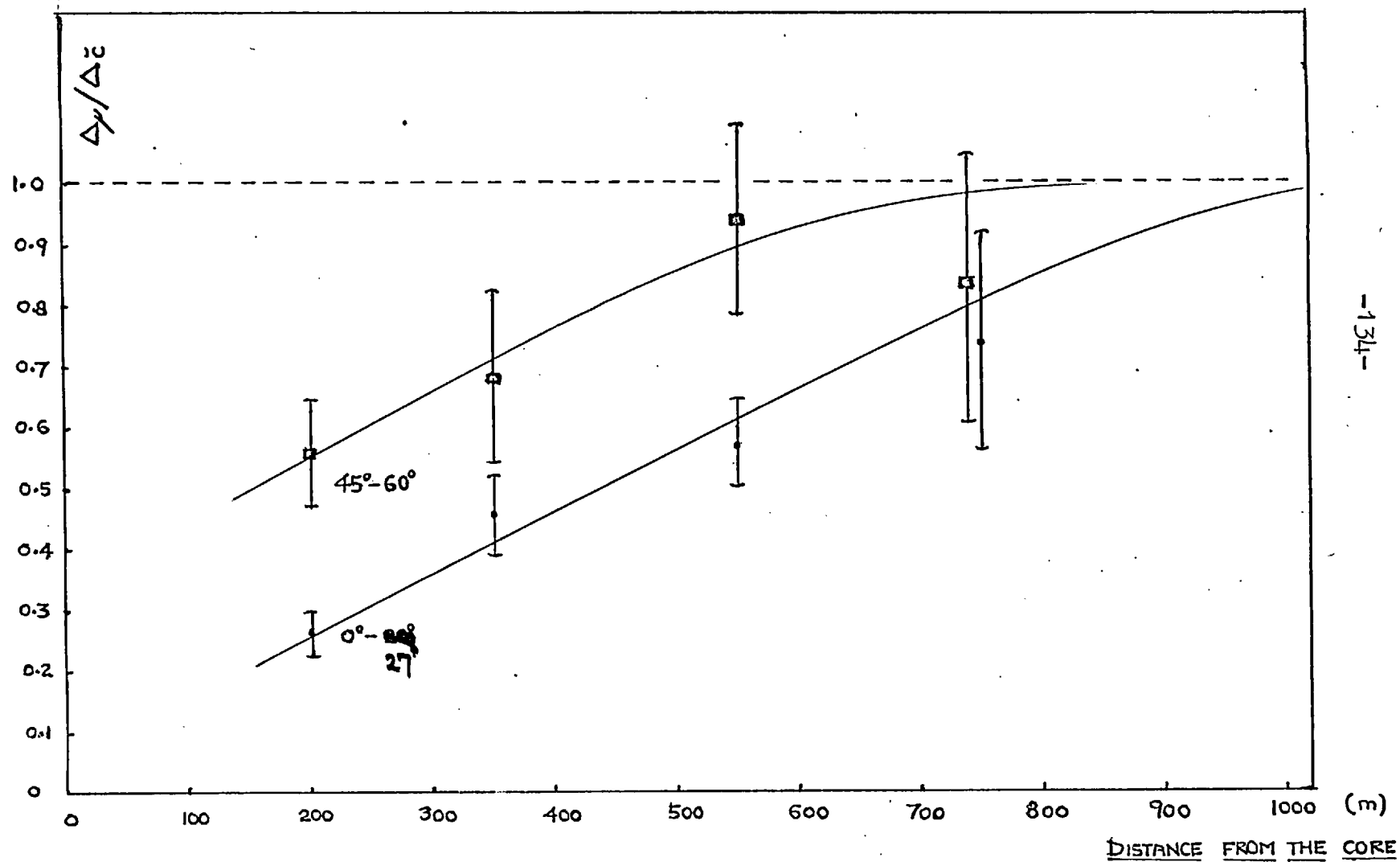


FIG 6.4

show that the ratio of the responses of the muon detector to Cherenkov detector, approaches unity at large distances ($\simeq 1000\text{m}$) from the core; and at large zenith angles ($> 60^\circ$) for all distances greater than 200m from the core.

A value of the order of unity is expected at large distances ($\simeq 1000\text{m}$) where the muon to electron ratio is approximately 60% in a shower of size 10^7 (Greisen, 1960). At large zenith angles a small reduction in the muon detector response of less than 10% compared with the Cherenkov tank response would be expected, due to the increasing energy threshold of the muon detectors with zenith angle. The Cherenkov detector response is expected to increase with respect to the muon detector at high zeniths, due to the greater probability of intercepting particles in low density showers, but this effect is probably off-set by the fact that the track length of particles passing through the Cherenkov detector does not increase linearly with $\sec \theta$ for increasing zenith angle. There will also be a small contribution to the Cherenkov tank response due to muon decay electrons and knock-on electrons. However, the ratio of the responses of the muon detector and Cherenkov detectors at high zenith angles is not inconsistent with unity at all distances greater than 200metres from the core.

6.6. Evaluation of the Cherenkov Detector Response to the Electron-photon Component.

As stated in section 6.5. the Cherenkov detector response may be written in the form:-

$$\rho_C = \rho_\mu + a\rho_e + b\rho_\gamma \dots\dots\dots 6.4$$

It has been shown by Allan et al. (1960) that, for vertical showers, the Cherenkov tank response to one electron indicates an energy dissipation of about 100MeV, but this is made up from the response to one electron and about ten photons, each dissipating about 10MeV. This is supported by the fact that the deviations in the observed Cherenkov detector density from the calculated density are much lower than would be expected on Poissonian statistics, assuming that the observed response to one electron resulted from one particle.

The calculations of Kamata and Nishimura (1958) also suggest that the electron component at large distances is produced from low energy photons travelling outwards from the core region.

Equation 6.4. may therefore be re-written as:

$$\rho_C = \rho_\mu + a\rho_e + (b\rho_e) \times n \dots\dots\dots 6.5$$

where a and b are functions of the energy of the electrons and photons respectively, and n is the number of photons (on average) accompanying one electron.

When $n \gg 1$ this expression reduces to the form of equation 6.2, namely:-

$$\rho_c = \rho_\mu + B \rho_e \dots\dots\dots 6.6$$

The relationships for the muon density ρ_μ and Cherenkov tank nominal density found in chapters 5 and 4 respectively, (for a shower size 2.10^7 nominal particles) were used in conjunction with the electron density in a shower of 2.10^7 particles, obtained from the expression given by Greisen (1960), to evaluate B for different distances from the shower core. An estimate of B close to the core (30m) was also made from the results of the 500m array analysis.

Fig. 6.5. shows the trend in the value of B with increasing distance from the core for a shower of size 2.10^7 nominal particles incident from the vertical direction ($\theta < 27^\circ$ for the 500m array points).

An interpretation of the variation of B with distance is made difficult by the fact that the mean energy (and consequently the absorption length) and also the number of photons accompanying one electron, both change with distance from the core. The results, however, do show that the final contribution of one electron to the Cherenkov tank response at distances lying between 100m and 500m from the core, does not change very rapidly with increasing

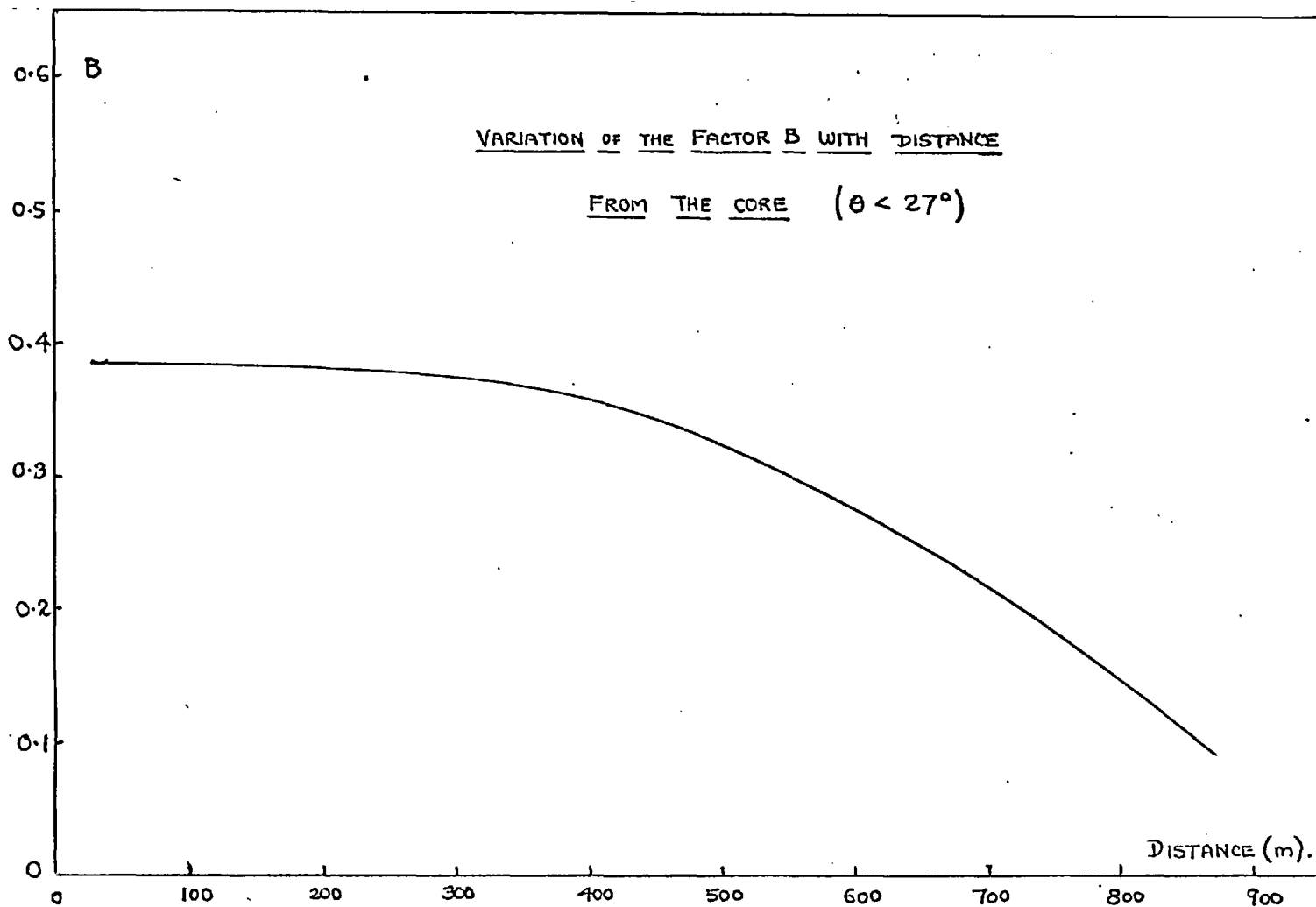


Fig 6.5

distance from the core. This is a consequence of the associated photon contribution. At distances greater than 500m from the core, the photon contribution falls.

A detailed investigation of the relative contributions of the electrons, photons and muons to the Cherenkov tank response at large distances from the shower core is being carried out by Mr. L. Towers at Haverah Park, the results of which, will be reported in a succeeding thesis.

Preliminary results of measurements made at distances between 400m and 1000m from the core (Dufresne et al., 1965) suggest that the number of photons accompanying one electron remains almost constant, but that the mean energy of the photons falls from about 10MeV at 400m to $\simeq$ 3MeV at 900m from the core.

6.7. The Variation of the Lateral Distribution and the Total Number of Muons with Zenith Angle.

A function of the form

$$\rho_r(\theta, N, r) = k(\theta) \left(\frac{N}{2 \cdot 10^7} \right) \beta(\theta) r^{-n(\theta)} \dots\dots\dots 6.7$$

has been fitted to the normalised density v distance distributions of muons for four intervals of zenith angle by a χ^2 fitting procedure. A shower size of $2 \cdot 10^7$ nominal particles was taken as a 'normalisation size' for all zenith angle intervals. $\beta(\theta)$, the exponent of the size

dependence for showers incident from a direction $> 60^\circ$ from the vertical, was assumed to be unity. Such an assumption is justified, if these showers are composed mainly of muons. If the value of β is taken as 0.75 for the zenith angle ranges $0^\circ/27^\circ$; $27^\circ/36^\circ$; $36^\circ/45^\circ$; $k(\theta)$ takes the values given in table 6.1. The table also shows the Cherenkov tank and muon structure function exponents, and the muon density at distances of 300m and 550m from the core for a shower size 2.10^7 particles.

The number of muons lying between 300m and 1000m from the core, and having energies greater than 1GeV for a shower size of 2.10^7 particles at Haverah Park, has been found to be 2.96×10^5 for zenith angles less than 27° . This compares with 2.73×10^5 particles calculated by integrating Greisen's expression (cf. equation 5.1) with respect to r , between the same limits of distance.

For showers of equal size at the observation level, the fact that, the density of muons at a particular distance from the core increases with increasing zenith angle, may be important, as is the case when measuring the variation of the muon content of a shower as a function of shower size over a wide range of zenith angle. In the case of Haverah Park, which has a wide zenith angle distribution, this is more important, since the zenith angle distribution of showers

TABLE 6.1.

ZENITH ANGLE RANGE.	0°-27°	27°-36°	36°-45°	60°
MEAN ZENITH ANGLE.	17°	32°	40°	67°
MUON STRUCTURE FUNCTION EXPONENT.	2.19	2.14	1.96	1.89
CHERENKOV STRUCTURE FUNCTION EXPONENT.	2.95	2.73	2.52	1.89
MUON PROPORTIONALITY CONSTANT $(k/10^5)$.	2.15	2.08	1.10	1.70
MUON-NUMBER SHOWER-SIZE RELATIONSHIP EXPONENT.	0.75	0.75	0.75	1.0
MUON DENSITY AT 300m FROM THE CORE (m^{-2}) ($N=2.10^7$).	0.81	1.04	1.54	3.54
MUON DENSITY AT 550m FROM THE CORE (m^{-2}) ($N=2.10^7$).	0.21	0.28	0.47	1.13
DETECTOR THRESHOLD ENERGY. (MeV).	310	350	380	720

of different sizes varies greatly (cf. Chapter 7, fig. 7.2). This result, and its implications with respect to the results given by Allen et al. (1965) for the variation of the total number of muons as a function of shower size are discussed fully in Chapter 7.

CHAPTER SEVEN.

THE DEPENDENCE OF THE MUON NUMBER UPON SHOWER SIZE.

7.1. Introduction.

It has been suggested by Fukui (1960), that measurements on the proportion of muons in E.A.S. might elucidate the question of the existence of heavy particles in the primary composition.

Recently this problem has been approached by the Moscow Group (Vernov et al., 1965) by measurements of the shower electron number N_e at a fixed muon number N_μ and vice-versa, (Vernov et al., 1965). They found that the $N_\mu - N_e$ ratios from both methods of selection are in very close agreement. This would be the expected behaviour, if the primary radiation at the energies concerned, consisted mainly of heavy nuclei, for which fluctuations in shower development are smaller. By comparison of the distributions of N_e at fixed N_μ with theoretical distributions, Khristiansen et al. (1965) suggest a weak change in primary composition with increasing primary energy.

A large proportion of the muons detected in an E.A.S. at sea-level are the decay products of charged pions produced in the first few nuclear interactions in the atmosphere. From a study of the high energy muons

($> 200\text{GeV}$) in E.A.S. at sea-level the Bombay Group (Chatterjee et al., 1965A) found a value for β (the exponent of muon-number shower-size relationship) of 0.47. This compares with a value for β of 0.75 for muons of energy greater than 1GeV (Greisen, 1960). Due to the degradation of the energy of the nuclear-active component of the shower, the high energy pions are produced preferentially higher up in the atmosphere and have a greater probability of interaction rather than decay, since they will encounter a greater thickness of matter during their lifetime. Hence, as the primary nucleon energy increases, the pions produced will become more energetic, the number of charged pions interacting will increase, and the proportion of muons in the shower will decrease compared with the electron-photon component originating from π^0 decay (i.e. assuming equal numbers of π^+ , π^- and π^0 mesons are produced at the energies concerned).

The following sections describe an experiment designed to investigate the variation of β over a wide range of shower sizes (10^5 to 10^8 particles).

In almost all of the measurements made on the number of muons N_μ in showers of various sizes, N_μ has been found to vary with the shower size N_0 as:-

$$N_\mu \sim N_0^\beta \quad \text{where } \beta < 1$$

If a shower produced by a primary particle of mass number A is treated as being equivalent to a superposition of A proton-like showers, each of size N_0/A , this relation becomes:

$$N_\mu(A) = \left(\frac{N_0}{A} \right)^\beta \times A$$

i.e. $N_\mu(A) \sim A^{1-\beta} N_0^\beta \dots\dots\dots 7.1.$

A measurement of the total number of muons over a shower size range in which the mean mass number of the primary particles changes, should reveal a change in the slope of the $N_\mu - N_0$ relationship.

Measurements on the muon content of showers initiated by primaries of energy $\simeq 10^{15}$ eV by Vavilov et al. (1957) suggest a smaller value of β than that observed by many workers in the region above 10^{15} eV which is associated with a mixed primary composition, (as distinct from one consisting mainly of protons.) Linsley (1962) suggested that particles with energies $\gg 10^{17}$ eV are homogeneous in their chemical composition, and are probably protons. By investigating the variation of β over a wide shower size range, an indication of a change in the chemical composition of the primary spectrum is sought.

7.2. The Variation of the Structure Function Exponent With Shower Size.

The results presented below were obtained by using both the 50m and 500m arrays to select showers in the size ranges 10^5 to 10^6 nominal particles and 10^6 to 10^8 nominal particles respectively.

No determination of shower directions was made for the 50m array showers. However, at the distances from the core concerned (10m to 80m), in showers of size 10^5 to 10^6 , the Cherenkov tank detector response is dominated by the electron-photon component, rather than by the muon component. This results in a zenith angle distribution of all showers which should be much narrower than that of the 500m array and the error involved in taking all showers into consideration regardless of zenith angle, should not be serious. In addition, showers which did not fit the mean Cherenkov structure function within certain limits, were rejected. These are showers which come from directions far from the vertical. The 500m array showers used in the size range 10^6 to 10^8 particles, have a zenith angle of less than 27° (mean angle 17°).

The first step was to determine the variation (if any) of the muon lateral structure function with shower size. The showers to be used were divided into narrow

intervals of shower size and muon-detector to core distance, and the lateral structure functions obtained for each size interval by a χ^2 fitting procedure. The experimental error on the points prevented any quantitative investigation of any structure function variation with shower size to be determined. The fits were not inconsistent with a lateral structure function which is independent of shower size over the whole shower size range. Small showers tend to be preferentially detected in regions close to the apexes of the collecting area of the array (cf. fig. 4.2), whereas large showers are more evenly distributed about the array. The fact that the structure function is independent of shower size indicates that no appreciable mis-location of the core position occurs at any size or for any particular region of the collecting area. On the assumption that the structure function is independent of shower size, the mean structure functions of muons for the 50m and 500m array showers (cf. Chapter 5) were imposed on the density v distance plots for each interval of shower size. By a least-squares fitting procedure, the best fitting curves were obtained for each size interval. For each array, the density at a 'normalisation distance' (30m for the 50m array and 300m for the 500m array) for each shower size interval, was plotted against the mean shower size within that interval.

A χ^2 fit on the two sets of points yielded slopes of:-

$$\beta (50m) = 0.73 \pm 0.05 \quad (10^5 < N < 10^6)$$

$$\text{and } \beta (500m) = 0.73 \pm 0.09 \quad (5 \cdot 10^6 < N < 10^8)$$

Fig. 7.1. shows the variation of the total number of muons in showers of size between 10^5 and 10^8 obtained by extrapolating the 50m array muon densities at 30m to the density expected at 300m by using the overall lateral distribution shown in fig. 5.9.

7.3. Discussion of results.

The value of β over the shower size range 10^5 to 10^8 particles is found to remain constant inside the limits of experimental accuracy. The value found from the 500m array results (for 10^6 to 10^8 particle showers), is 0.73 for zenith angles less than 27° as compared with a value of 0.56 found by Blake (1966) for showers of zenith angle less than 40° . The lower value of β may be expected if the zenith angle distribution of showers is a sensitive function of shower size.

Fig. 7.2. shows the distribution of zenith angles ($\theta < 40^\circ$) of all analysable 500m array showers for two different shower size ranges ($N < 7 \cdot 10^6$ and $N > 3 \cdot 10^7$ particles). At the lower shower sizes the mean zenith angle of approach is $\approx 35^\circ$ whereas at the higher shower sizes,

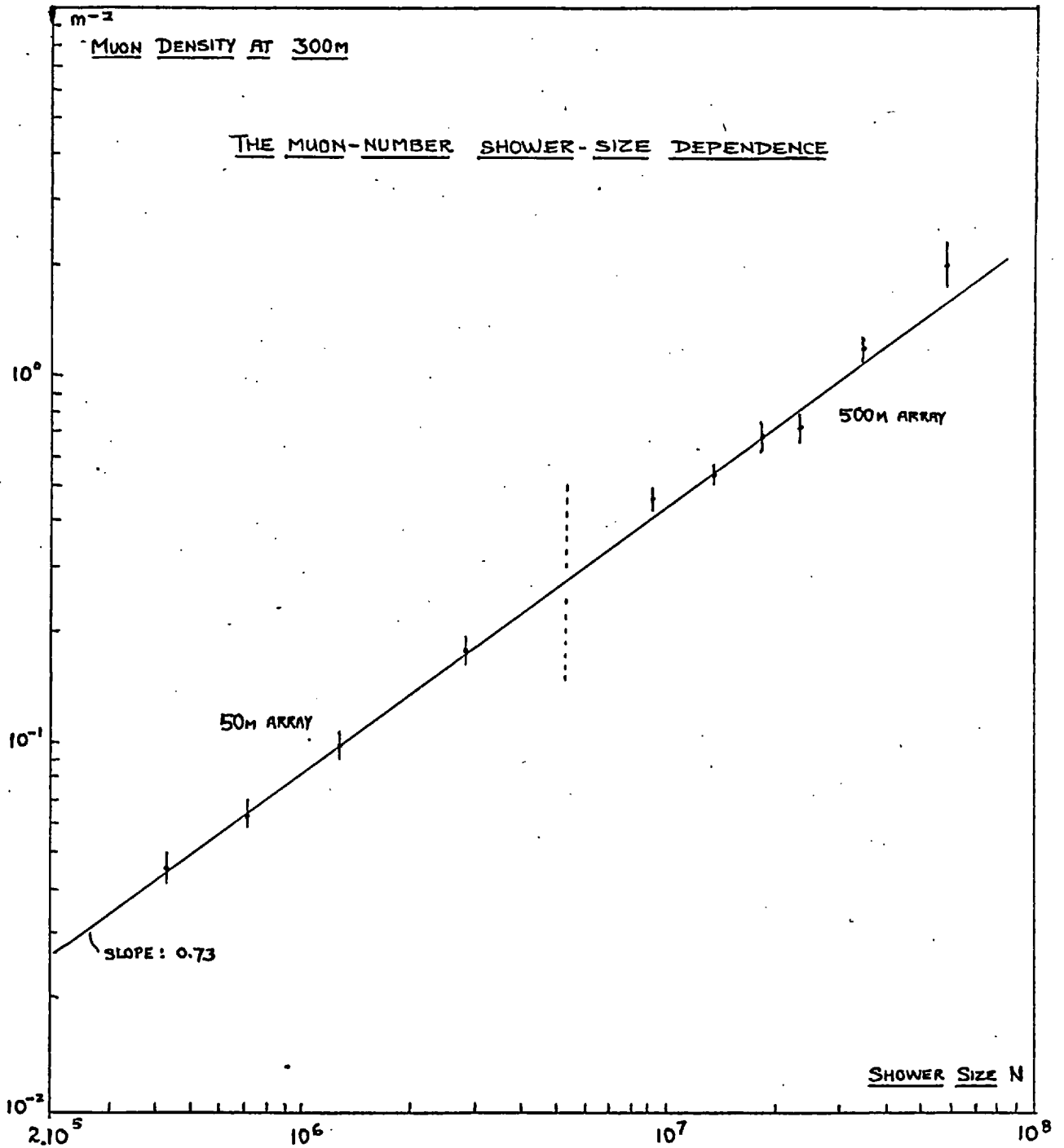


Fig 7.1

SHOWER SIZE DISTRIBUTIONS (FOR ZENITH
ANGLES OF LESS THAN 40°)

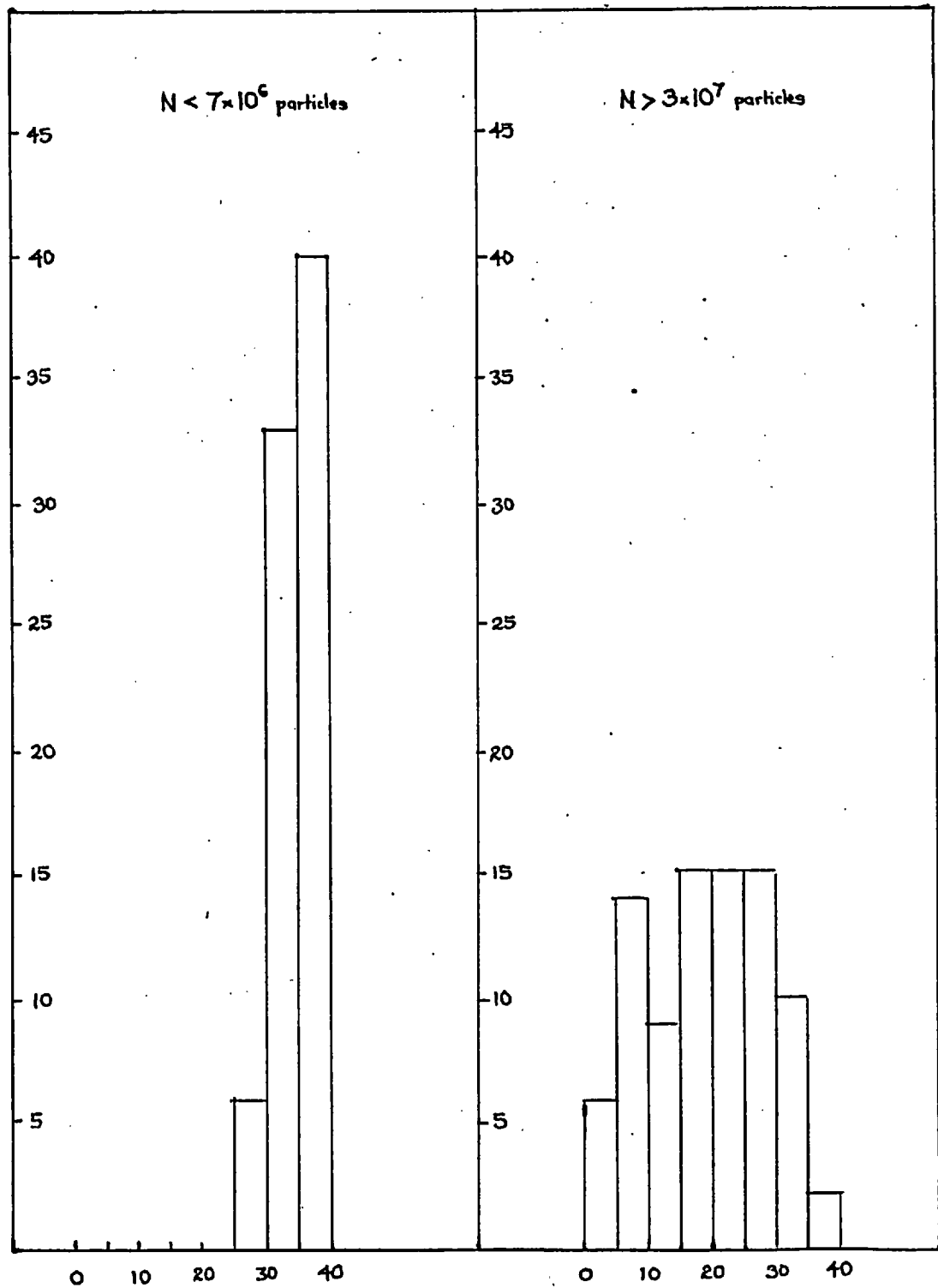


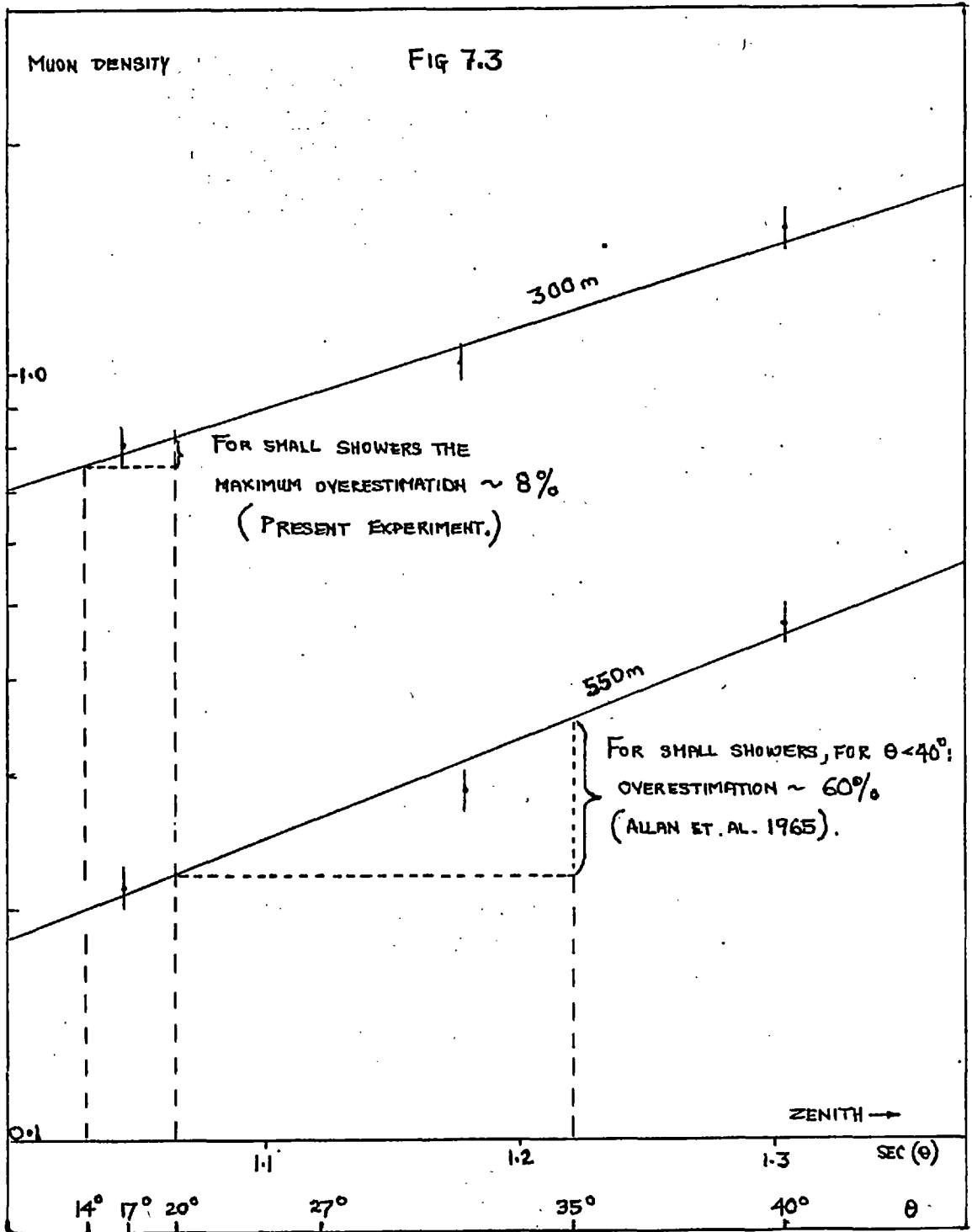
Fig 7.2

the mean zenith angle is $\simeq 20^\circ$. The results of section 6.7 on the density of muons at distances of 300m and 550m from the core in a shower of a fixed size at the observation level, as a function of zenith angle, will now be considered.

Fig. 7.3 shows the muon density at distances of 300m and 550m from the core in a shower of size 2.10^7 particles at the observation level, plotted as a function of zenith angle. (These figures are taken from Table 6.1). The shape of the curve should be independent of shower size. The points plotted for the zenith angle ranges 27° to 36° and $36^\circ/45^\circ$ were calculated by normalising muon densities to the 'normalisation size' using an expression of the form:-

$$\rho_{\text{norm}} = \rho_{\text{observed}} \times \left(\frac{2.10^7}{N_{\text{obs}}} \right)^{0.75}$$

It is likely that the exponent 0.75 increases with zenith angle in the range 0° to 45° . The slope of figure 7.3 should then be regarded as a lower limit since the muon densities for the 27° to 36° and 36° to 45° points will be underestimated. For the purposes of the present analysis, the number of muons in showers of different sizes is taken to be proportional to the muon density at a fixed 'normalisation distance' from the core.



Therefore, when considering particular shower size intervals at the observation level as is done in the computation of the $N_{\mu} \propto N$ relationship, any variation in the mean zenith angle with shower size, will produce serious biases in the number of muons found at different shower sizes. The number of muons will also be a function of the 'normalisation distance' chosen. By measuring the muon density at 550m from the core to find the muon content of a shower, in the particular case of $\theta < 40^\circ$, an over-estimation of approximately 60% in the muon number would occur for a shower of size $3 \cdot 10^6$ particles, as compared with the muon number of a shower of size 10^8 particles. This is a consequence of the different mean zenith angle of the two shower size categories. This is shown diagrammatically in fig. 7.3. As a result, the value of β the exponent of the $N_{\mu} \propto N$ relationship, is depressed by more than 0.1, as shown in figure 7.4. It is therefore essential, when investigating the variation of the number of muons in a shower as a function of shower size to take a narrow interval of zenith angle. In the present experiment a zenith angle range of 0° to 27° was used, which had a mean zenith angle of 17° . The mean zenith angle of showers in a particular shower size range used in the analysis, did not deviate from the overall mean of 17° by more than 3° .

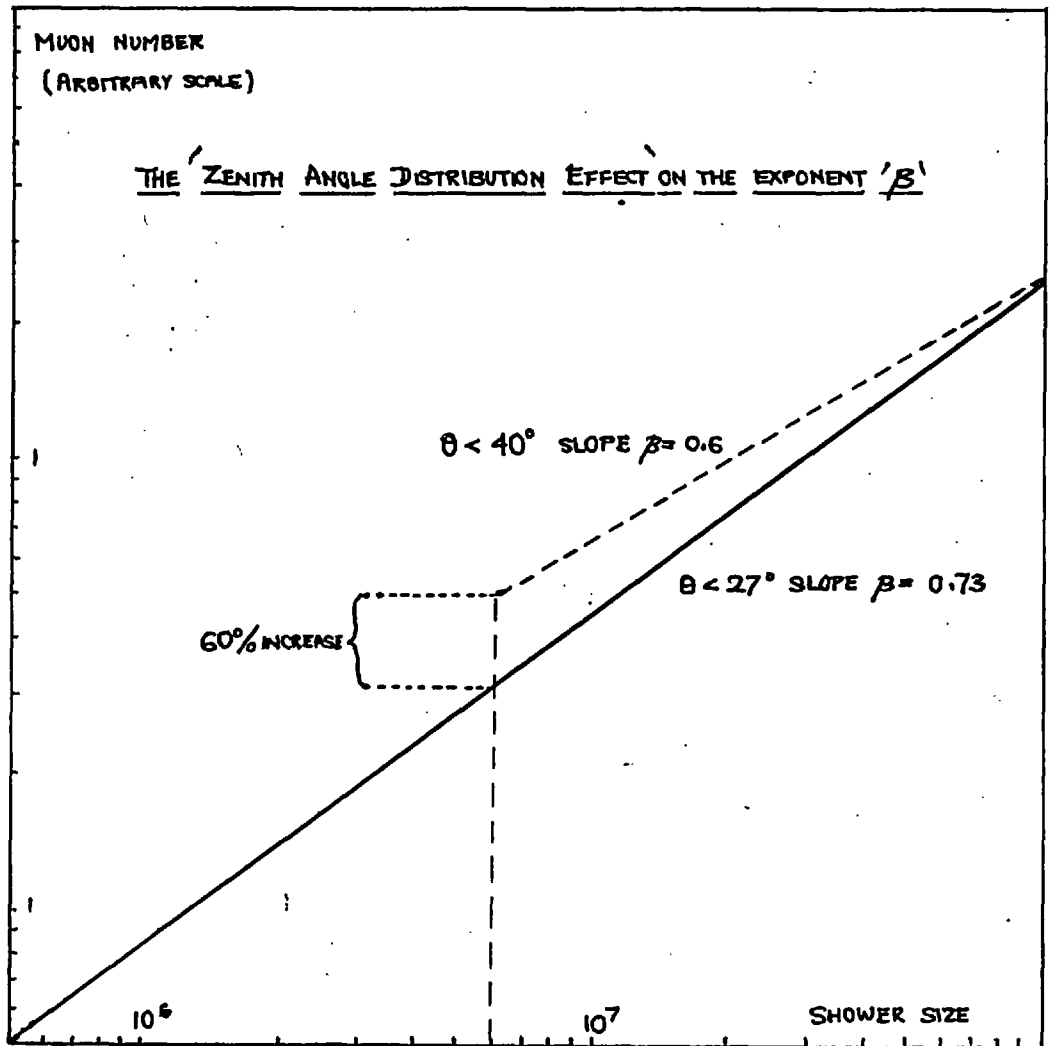


FIG 7.4

for any shower size interval (cf. fig. 7.3).

The muon number was taken to be proportional to the muon density at a 'normalisation distance' of 300m from the core. The zenith angle effect described above can then produce an error of less than 0.03 in the derived value of β .

The results presented by Allan et al. (1965) and Blake (1966) used showers falling within 40° of the zenith, to obtain a value for the exponent β of 0.5 (0.56 before a correction for the Cherenkov detector sensitivity). The 'normalisation distance' used to find the muon content of showers in different shower size ranges was 550m. As a result, the zenith angle effect described above produced a value of β which was seriously underestimated. Their explanation that the small value of β is indicative of a change in the primary composition from primaries consisting of heavy nuclei to primaries consisting of metagalactic protons (Allan et al., 1965) must therefore be discounted. As a final check on the zenith angle effect the author has recalculated the value of β for a zenith angle range of 0° to 40° and again finds a value lower by 0.1 than the value found when considering a range of 0° to 27° .

With regard to the 50m array result (appropriate to a shower size range 10^5 to 10^6 particles), due to the lack of reliable timing information the direction of arrival of

these showers has not been considered. However as pointed out previously, showers which gave a bad fit to the mean Cherenkov structure function used in the analysis were rejected, as the probability that these showers have a large zenith angle is high. Also the zenith angle distribution of the smaller showers detected by the 50m array is much narrower than that of the 500m array showers.

The investigation of the mean value of β over a wide shower size range does not show any change of value in the region where a transition in the primary composition from heavy primaries to protons of metagalactic origin is expected.

7.4. Comparison with Other Workers.

It is shown in Appendix 2 that the exponent $\beta = 0.73$ relating to the muon number nominal shower size dependence is reduced to a value of 0.72 when the relationship between muon number and conventional size N_0 is used.

Table 7.1. below gives a list of the measurements made by other workers of the exponent β relating the muon content N_μ of an E.A.S. to the total charged particle content N_0 .

$$\text{i.e. } N_\mu \sim N_0^\beta$$

TABLE 7.1.

AUTHOR	VALUE	ALTITUDE	ZENITH RANGE	THRESHOLD	ENERGY/SIZE RANGE
Cocconi et al. (1949)	0.87	3,260m	Not Measured		$5 \cdot 10^{14} - 8 \cdot 10^{16} \text{ eV}$
Vavilov et al. (1957)	0.6	3,860m	Not Measured	1 GeV	$10^{14} - 2 \cdot 10^{15} \text{ eV}$
Clark et al. (1958)	0.7	sea-level	$< 25^\circ$	1 GeV	$10^5 - 10^8 \text{ ptcls.}$
Hasegawa et al. (1962)	0.65	sea-level	$< 19^\circ$	1 GeV	$10^5 - 10^6 \text{ ptcls.}$
Khrenov (1962)	0.85	sea-level	Not Measured	5 and 10 GeV	$10^4 - 10^7 \text{ ptcls.}$
Linsley et al. (1962)	0.77	820gm/cm ²	$< 40^\circ$	10 cm Pb	$10^7 - 10^9 \text{ ptcls.}$
Matano et al. (1962)	0.8	sea-level	$< 40^\circ$	5 GeV	$10^5 - 10^8 \text{ ptcls.}$
Chatterjee et al. (1963)	0.44	800gm/cm ²	$< 40^\circ$	0.5 GeV	$10^5 - 10^7 \text{ ptcls.}$
	0.6	800gm/cm ²	$< 40^\circ$	1 GeV	$10^5 - 10^7 \text{ ptcls.}$
Vernov et al. (1965)	0.8	sea-level	Known	10 GeV	$10^5 - 10^6 \text{ ptcls.}$
Present Experiment	0.72	230m	$< 27^\circ$	0.31 GeV	$5 \cdot 10^6 - 10^8 \text{ ptcls.}$
	0.72	230m	Near Vertical	0.31 GeV	$10^5 - 10^6 \text{ ptcls.}$

The results show a wide spread in the values of β obtained. However, in very few of the experiments are the experimental conditions similar.

Results most directly comparable with the present experiment on measurements of the size dependence of the muon content of E.A.S. are those of Clark et al. (1958) who found a value of 0.7 and Hasegawa et al. (1962) who obtained a value of 0.65 for the upper edge of the $N_\mu - N$ plot, the mean value being somewhat higher. Most of the other determinations of β at sea-level used showers falling within a zenith angle range 0° to 40° . The basic assumptions in all of these experiments was that the muon structure function does not vary with shower size. An investigation of the variation of the muon lateral distribution at distances of the order of a few hundred metres from the core with zenith angle has revealed a marked change with increasing zenith angle (cf. section 6.7) for a shower of fixed size at the observation level. However, this effect is probably less serious when making measurements close to the core, where the mean muon energy is greater and perturbations such as geometrical spreading, decay and ionisation loss should not have such a pronounced effect.

The value of β found for the shower size range $10^5 - 10^6$ particles is in agreement with the values found by

other workers at sea-level. (Clark et al., 1958; Fukui et al., 1960; Hasegawa et al., 1962; Matano, 1962; Khrenov, 1962; Vernov et al., 1965).

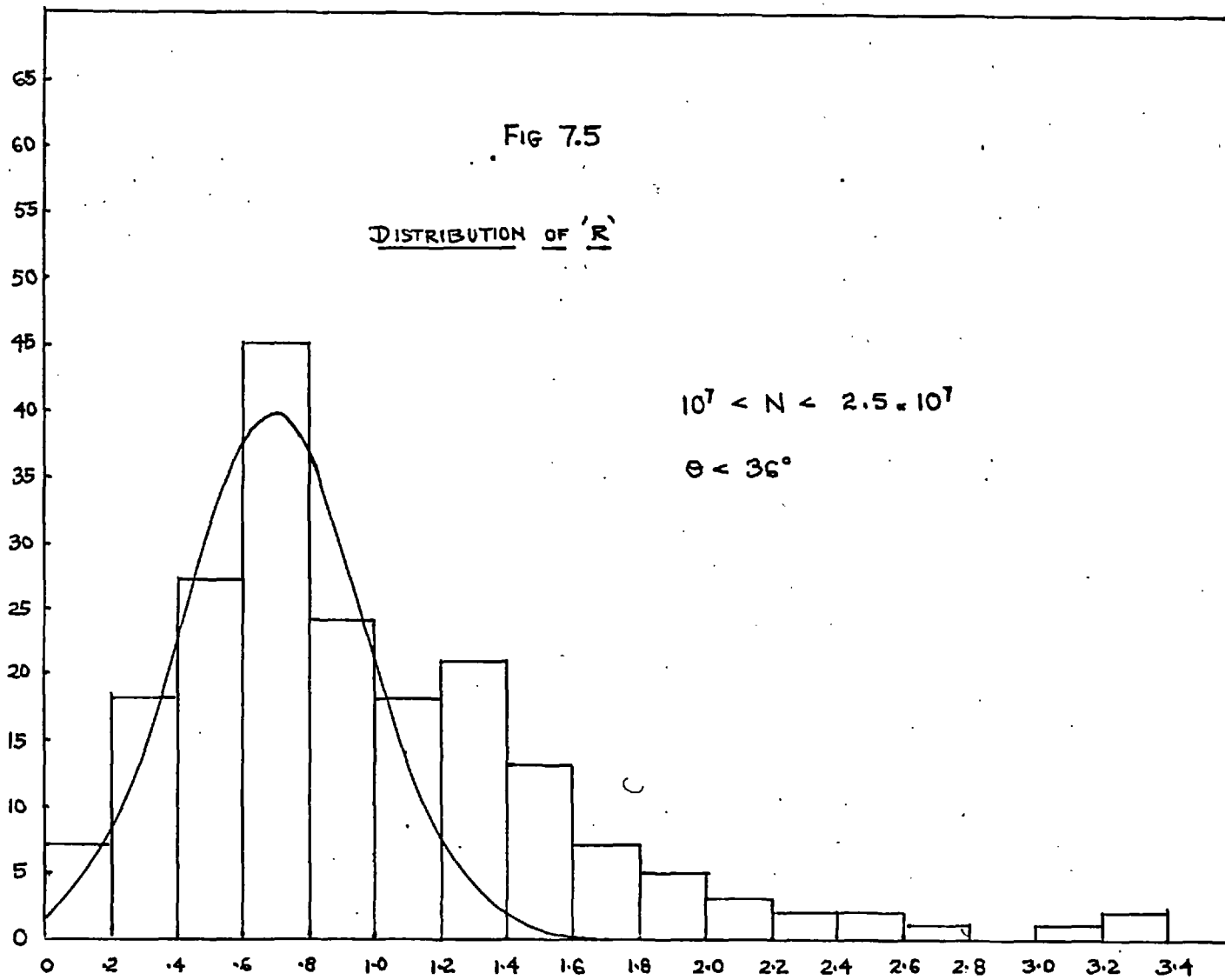
7.5. Speculation on the Primary Composition at Energies of the Order of 10^{17} eV.

The dependence of the total number of muons in a shower upon the shower size (cf. section 7.2) does not reveal any change in the exponent β with a change in shower size. As shown in section 7.1, the proportion of muons in a shower of a given size at the observation level is expected to be a function of the mass number of the primary particle. The distribution in different shower size ranges of the ratio R of the observed number of muons in a shower to the expected number should therefore reveal any marked change which takes place in the primary composition e.g. a change from a mixed, to a homogeneous radiation.

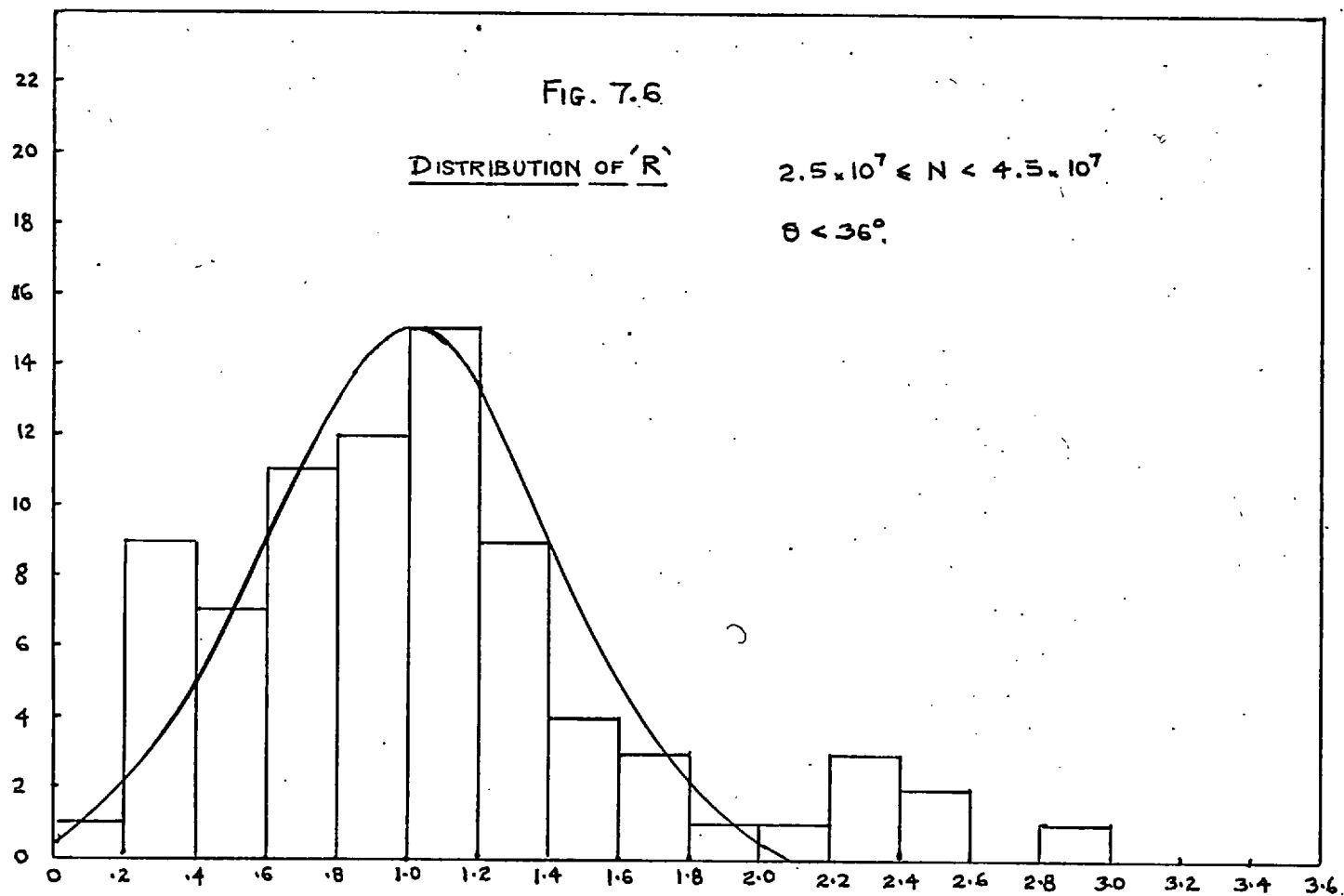
The error involved in the determination of the ratio of the observed to the expected muon density depends upon many factors, the main ones being considered in Appendix 1.

The distributions of the ratio R for showers lying in three shower size ranges having zenith angles of less than 36° are shown in figures 7.5, 7.6 and 7.7. (It was necessary to use a zenith angle interval of 0° to 36° to

No. of
events

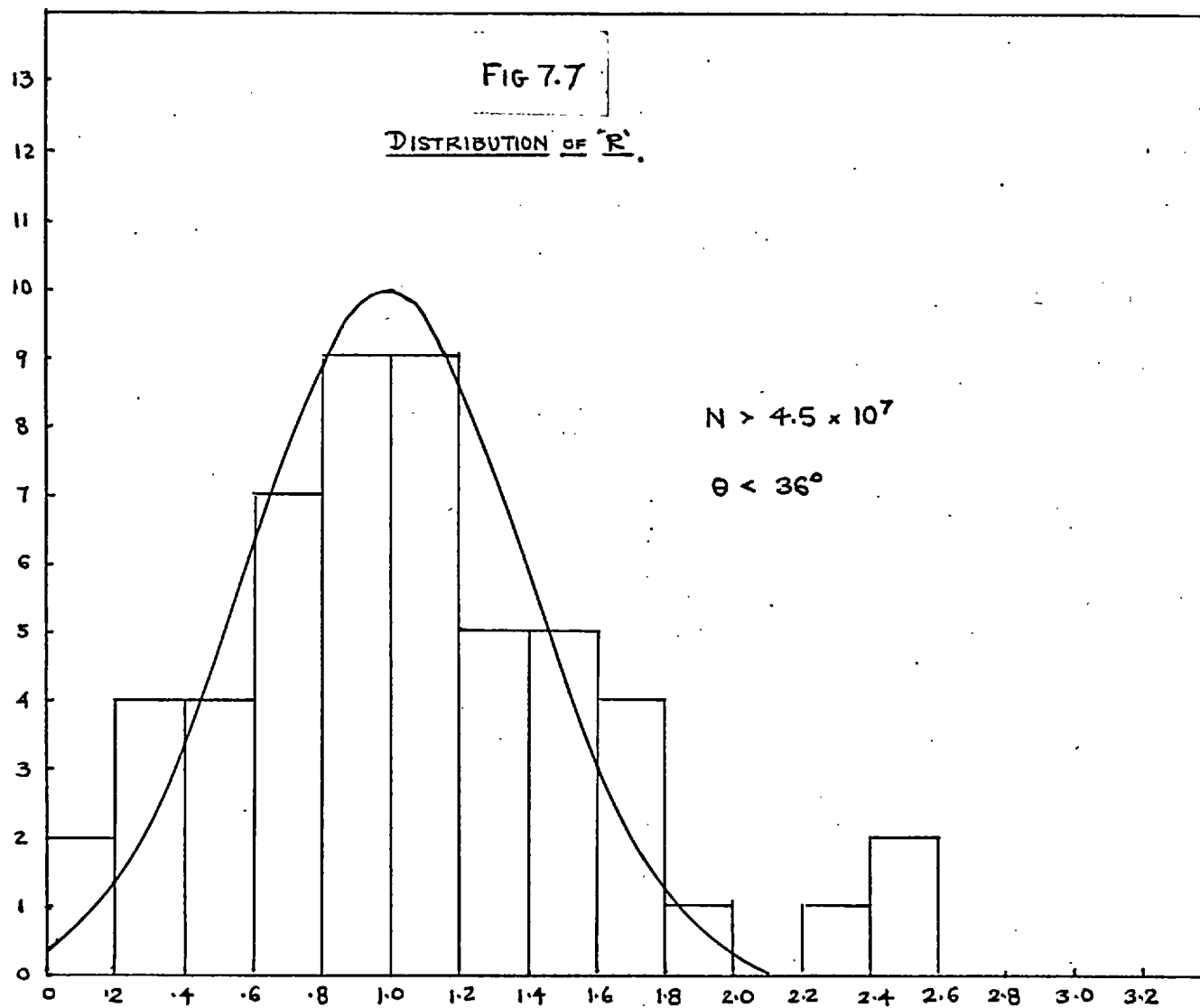


No. of
events



R

No. of
events



R

obtain a reasonable number of events for the R distributions). The calculated density takes into account the change in the muon lateral distribution with zenith angle (cf. Chapter 6). Superimposed on each distribution is a Gaussian distribution having standard deviation of 40% of the mean. (This value is taken as a close approximation to the expected experiment error (cf. Appendix 1). The main random errors arising in the value of R have been shown to result from uncertainties in the evaluation of the expected muon density due to errors in the determination of the shower size and the core location. This is not expected to differ by any significant amount between the shower size ranges considered.

The distribution of R is observed to change from a distribution which is incompatible with a Gaussian curve for showers in the size range $10^7 - 2.5 \cdot 10^7$ to a near Gaussian distribution for showers of size greater than 4.5×10^7 particles. These results suggest an explanation in terms of a trend towards homogeneity of the primary composition as the shower size increases.

The large experimental errors expected for the fifty metre array showers ($10^5 < N < 10^6$), and the increased effect of shower fluctuations, do not permit any interpretations of the R distribution to be made for these smaller showers.

The Gaussian error expected due to experimental uncertainties is of the order of 40%. It may be concluded therefore that the fluctuations in the shower size as detected by the Haverah Park array are less than the predicted fluctuations (De Beer et al., 1966). This may be explained however on the basis of the relatively greater sensitivity of the Cherenkov detectors to the muon component compared with the sensitivity of conventional thin detectors. (The average response of the Cherenkov detectors to the muon component is about 40% of the total Cherenkov detector response. This compares with about 10% for a conventional detecting array). The effect of fluctuations in the observed shower size will also be reduced for showers initiated by heavy primary particles.

The distribution of the ratio R for showers of size greater than 10^7 particles and less than 2.5×10^7 particles (cf. fig. 7.5) does not fit a Gaussian distribution with a spread of 40%. It is possible that this is due to a spread in the ratio resulting from showers initiated by primaries of widely different mass number.

Although these results are by no means conclusive, the indications are that a change occurs in the distribution of the ratio R as the shower size increases. The observations are consistent, either with the explanation that the showers of size greater than about 4.5×10^7 particles are initiated

by heavy galactic primaries, or that these showers are produced by metagalactic protons, (Linsley and Scarsi, 1962).

7.5.1. Conclusions.

The fluctuations in the shower size of showers initiated by primary particles of a given energy, are at a minimum at observation levels corresponding to the depth of maximum development of the shower. However, the showers detected by the 500m array at Haverah Park are past maximum development.

De Beer et al. (1966) have calculated the fluctuations expected at sea-level, in the shower size N and the muon number N_μ , for showers initiated by primary protons. The results show that the muon number distribution is much narrower than the shower size distribution. The latter distribution can spread over shower sizes differing by an order of magnitude. The distribution of the ratio of observed to expected muon densities for showers of size greater than 4.5×10^7 particles closely fits a Gaussian distribution with a standard deviation equal to 40% of the mean.

In the R distribution for showers of size greater than 4.5×10^7 particles (cf. fig. 7.7), the occurrence of 3 showers (constituting about 6% of the total) which lie outside three standard deviations (probability $\approx 0.3\%$),

favours the explanation that these showers are produced by very heavy primary particles, whereas the showers responsible for the distribution conforming to a Gaussian distribution are mainly initiated by protons. The distribution in R for the size range $10^7 - 2.5 \times 10^7$ particles (cf. fig. 7.5) suggests that these showers are initiated by primaries of widely differing mass number.

In conclusion, for showers of size between 10^7 and 10^8 particles, it appears that the primary particle composition changes from a composition of particles of widely differing mass number, to one of a more homogeneous nature. It is suggested that this homogeneous composition consists mainly of protons of metagalactic origin.

A similar analysis carried out on showers near maximum development (Linsley and Scarsi, 1962; Toyoda et al., 1965) predicted that the primary radiation at energies of about 10^{17} eV consists predominantly of protons.

7.6. General Conclusions and Future Work.

The 50m and 500m Cherenkov tank air shower detectors have been used to select showers falling as close as 10m and as distant as 1km from the muon detector. This has enabled a lateral distribution of muons, for near vertical showers, to be constructed with good statistical accuracy in the

distance ranges 10m to 100m and 250m to 1000m from the core. The 150m array will be used to select showers of size greater than 10^6 particles, falling at distances between 50m and 300m from the muon detector. A complete lateral density distribution for muons of energy greater than 300MeV will then be constructed for a distance range of 10m to 1000m from the core.

The dependence of the muon number upon the shower size for showers detected by the 150m array ($N > 10^6$) should confirm the value of the exponent β equal to 0.73, found in the present experiment, in regions of size above and below that detectable by the 150m array.

A measurement of the zenith angle dependence of the muon lateral density distribution at distances $\simeq 100$ m from the core should support the conclusion drawn in section 6.4 that the marked dependence found in the present experiment (at large distances) is a consequence of the rapid dilution by geometrical spreading of the low energy muons produced relatively low down in the atmosphere.

An analysis of the fluctuations in the divergence of the muon density from the expected density should confirm the indications of a large spread in the mass numbers of the primary particles responsible for the observed showers. More measurements on such fluctuations are required for the

largest showers detected by the 500m array to confirm (or deny) the suggested trend of an increasing homogeneity of the primary radiation responsible for these showers.

The directions of arrival will be available from the 150m and 500m array analyses for shower sizes greater than 10^6 particles. Over a fairly long period of efficient recording it will be possible to investigate the distribution of the arrival directions of mu-rich showers. (A mu-rich shower is defined (Hasegawa et al., 1962A) as a shower in which the muon number is more than 80% of the maximum number which can be contained in the shower, the maximum number being that indicated by the upper edge of the $N_\mu - N$ plot of fig. 7.8).

The present system which uses a single muon detecting unit for the measurement of muon densities at Haverah Park does not lend itself to the efficient measurement of low-mu showers. This is a consequence of the limitations on the core-location of efficient showers, imposed by the simple E.A.S. detector array pattern; and also by the lack of muon density recording apparatus at the outer stations. As a result, the average number of muons recorded in showers detected by the five hundred metre array is about 20, with the muon detector located at the central station. Half of the showers fall at distances greater than 300m from the

THE N_{μ} -N PLOT ($\theta < 27^\circ$)

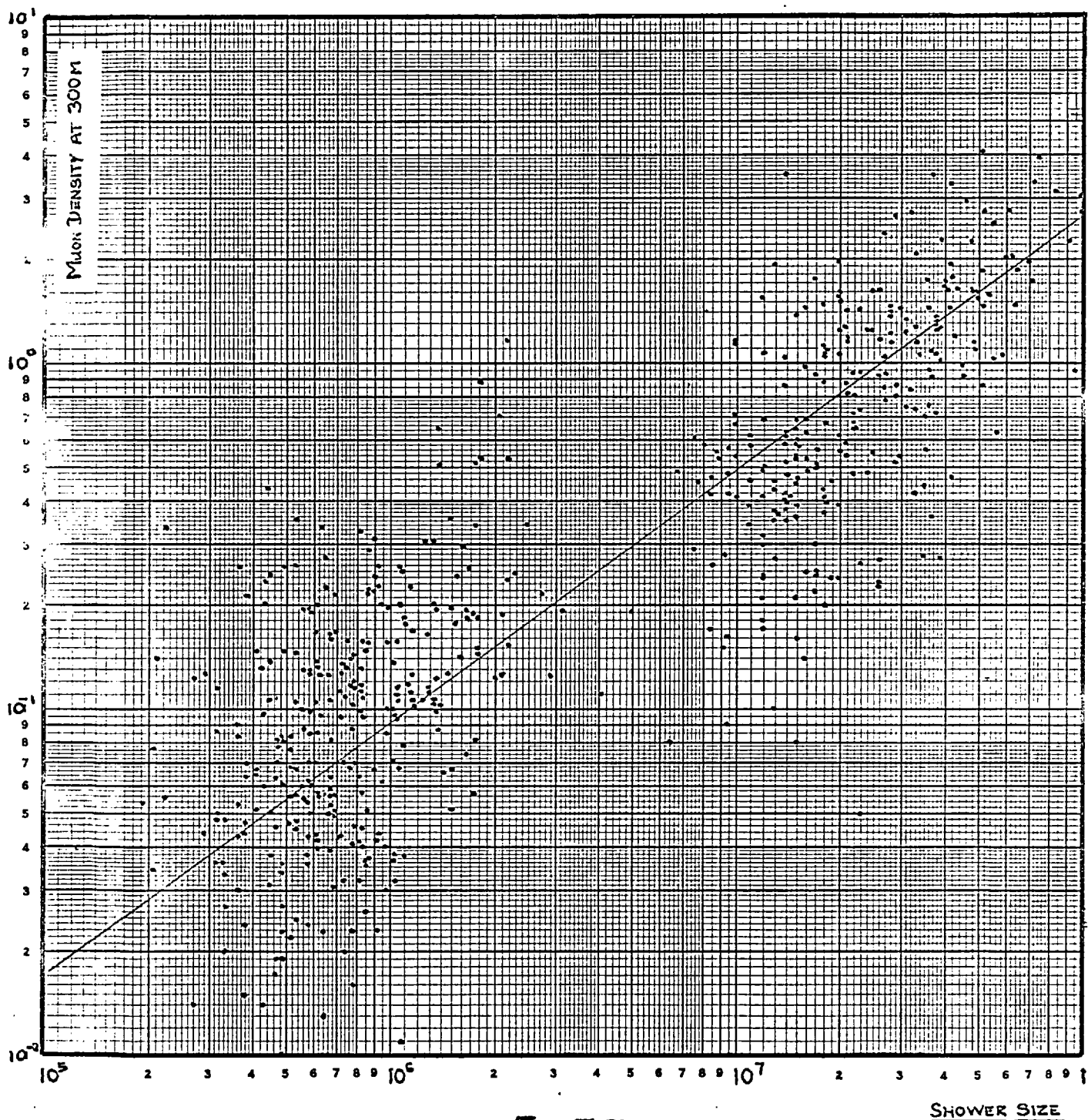


Fig 7.8

detector for which the average number of recorded muons is less than four. A low-mu shower is taken to be a shower in which the observed number of muons is less than one-thirtieth of the expected number. For the five hundred metre array it is necessary therefore, to measure the muon density of showers, the majority of which, fall within 50m of the core. This is not possible with the detector configuration used at the present time.

APPENDIX 1

THE ERRORS ARISING IN THE PRESENT EXPERIMENT IN THE RATIOS
OF THE OBSERVED TO THE EXPECTED MUON DENSITIES.

If it is assumed that the fluctuation in the muon lateral distribution is small, and that the lateral distribution is independent of shower size (Chapter 5), then the muon density at a particular distance is directly proportional to the total number of muons in the shower (N_μ).

In the present experiment, the error in the determination of a particular muon number N_μ depends upon many factors, the most important of which are considered below:-

(1) The spread in the response of the detector to a single vertical muon. This is fairly well represented by a Poissonian distribution of mean equal to 7. (Chapter 2.) The standard deviation associated with the measurement of a single muon will then be approximately 38% ($\sigma(1)$).

(2) The average number of muons passing through the detector. In showers triggered by the five hundred metre array the muon detector located at the central station records on average, about 20 muons. The average error on a muon measurement at the central station due to the spread in detector response is then given by $\sigma(1) / 20^{\frac{1}{2}}$.
i.e. About 8.5%

(3) Film-reading and calibration errors. The random error due to this source is of the order of 8%.

The total error of the measured muon density is then about 12%. The error in the observed to expected ratio of muons is also dependent upon the error in the evaluation of the expected number of muons. This error depends upon the accuracy of the shower size determination and the core location. A negligible error is introduced by the uncertainty in the zenith angle determination.

The error in the determination of the perpendicular distance $r(1)$ from the core to the muon detector is about 50 metres and the average distance of recorded showers from the central station is about 300m.. The percentage error in $r(1)$ is then about 17%. The error in the shower size determination is of the order of 20%.

$$\text{Now } \rho = k N^{\beta} r^{-n} \quad \text{where } n \simeq 2$$

$$\text{and } \beta = 0.73$$

The maximum percentage error in ρ is then given by:-

$$\frac{\Delta \rho}{\rho} = \left\{ \left(\beta \frac{\Delta N}{N} \right)^2 + \left(\frac{n \Delta r}{r} \right)^2 \right\}^{\frac{1}{2}} = 37\%$$

i.e. The total error of the expected muon density is $\simeq 37\%$.

The percentage error in the ratio R of the observed muon density $\rho(\text{obs})$ to the expected muon density $\rho(\text{exp})$

is then given by:-

$$\frac{\Delta R}{R} = \left\{ \left(\frac{\Delta \rho(\text{obs})}{\rho(\text{obs})} \right)^2 + \left(\frac{\Delta \rho(\text{exp})}{\rho(\text{exp})} \right)^2 \right\}^{\frac{1}{2}} \approx 39\%$$

i.e. The total experimental error is about 39% for the ratio R of the observed to expected muon densities.

The most serious source of error in the present experiment is the uncertainty in the core-location.

APPENDIX 2.

THE MUON-NUMBER, CONVENTIONAL SHOWER-SIZE DEPENDENCE.

Due to the relatively greater sensitivity of the Cherenkov detectors to the muon component compared with the sensitivity of conventional, thin detectors (e.g. thin scintillators), the nominal shower size N is not equal to the conventional shower size N_0 over the whole shower size range.

It was shown in Chapter 6 that the muon content of a shower of nominal size equal to 2.10^7 particles is almost equal to the muon content of a shower having a conventional size of 2.10^7 particles as determined from Greisen's (1960) empirical relationship.

If the number of muons N_μ is related to the conventional shower size N_0 by an expression:

$$N_\mu = aN_0^\alpha \dots\dots\dots A.2.1.$$

and to the nominal shower size N by an expression:

$$N_\mu = bN^\beta \dots\dots\dots A.2.2.$$

then, for $N = N_0 = 2.10^7$, equating equations A.2.2 and A.2.1 gives:

$$\frac{b}{a} = (2.10^7)^{(\alpha - \beta)} \dots\dots\dots A.2.3.$$

Therefore, for equations A.2.1 and A.2.2 to hold for all shower sizes we have:-

$$\left(\frac{N_0}{2 \cdot 10^7} \right)^\alpha = \left(\frac{N}{2 \cdot 10^7} \right)^\beta \dots\dots\dots \text{A.2.4.}$$

$$\text{Now } N = c(N_\mu + \bar{k}N_e) \dots\dots\dots \text{A.2.5.}$$

where c is a constant and $\bar{k} \simeq 0.4$ for both the 50m and the 500m arrays (c.f. Chapter 6).

$$\text{Also } N_0 = N_\mu + N_e \dots\dots\dots \text{A.2.6.}$$

Greisen (1960) gives an expression for the number of muons in a shower of size N_0 having an energy greater than 1GeV:-

$$N_\mu(>1\text{GeV}) = 9.5 \times 10^4 \left(\frac{N_0}{10^6} \right)^{0.75} \dots \text{A.2.7.}$$

Thus for a conventional shower size N_0 equal to $2 \cdot 10^7$ we have:-

$$N = 2 \cdot 10^7$$

$$\text{and } N_\mu = 9 \cdot 10^5$$

Equation A.2.5 therefore yields a value for c of 2.34.

Eliminating N and N_0 between equations A.2.4, A.2.5 and A.2.6 gives:-

$$\left(\frac{N_\mu + N_e}{2 \cdot 10^7} \right)^\alpha = \left(\frac{2.34(N_\mu + 0.4N_e)}{2 \cdot 10^7} \right)^\beta$$

whence:

$$\alpha = \frac{\beta \log \left(\frac{2.34(N_\mu + 0.4N_e)}{2 \cdot 10^7} \right)}{\log \left(\frac{N_\mu + N_e}{2 \cdot 10^7} \right)}$$

When $N_o = 10^6$; $N_\mu = 9.5 \times 10^4$; $N_e = 9.05 \times 10^5$;
we have:-

$$\underline{\alpha = 0.98\beta}$$

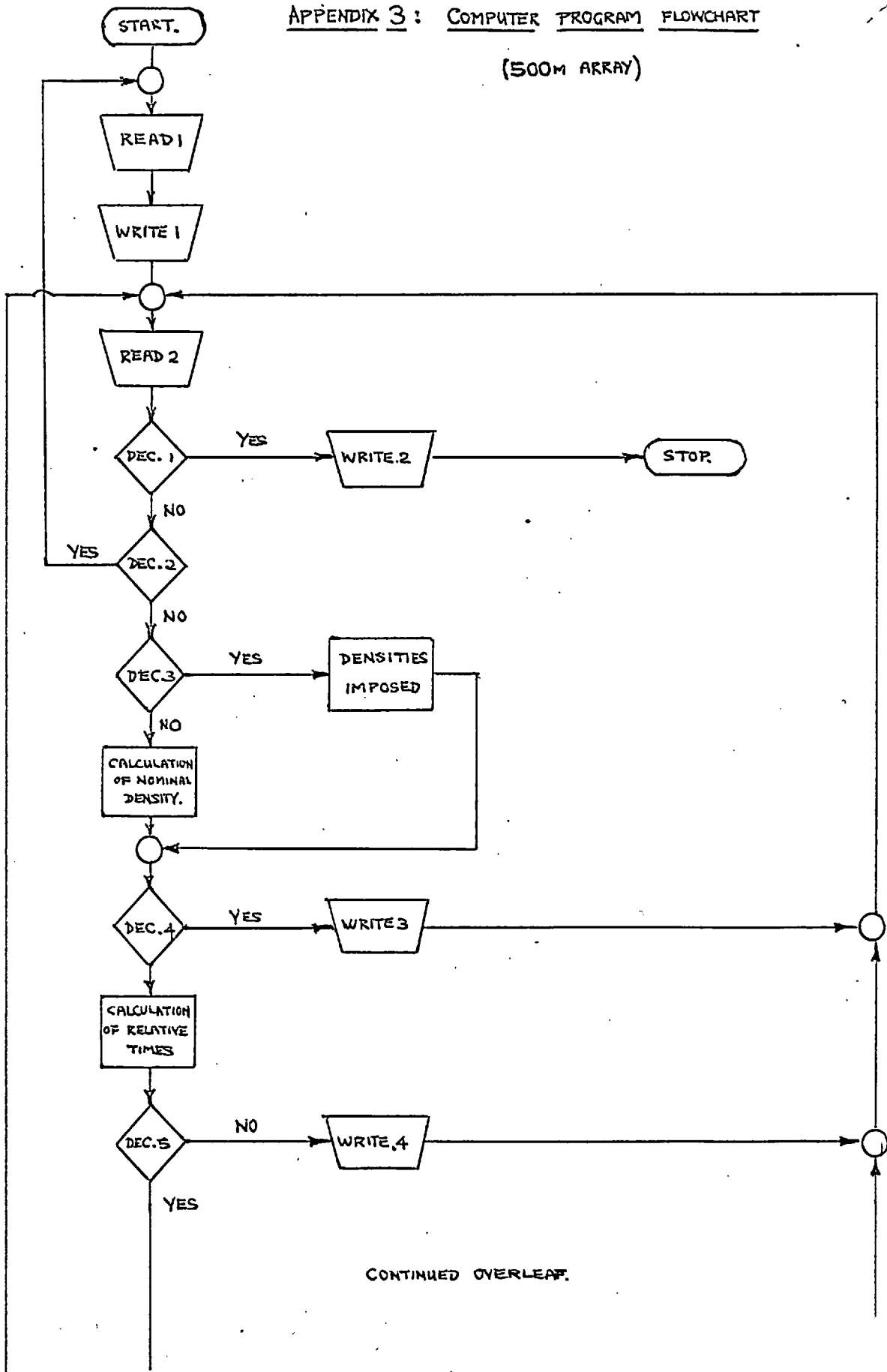
In the present experiment $\beta = 0.73$. For comparison
with other experiments this must be reduced by about 2%.

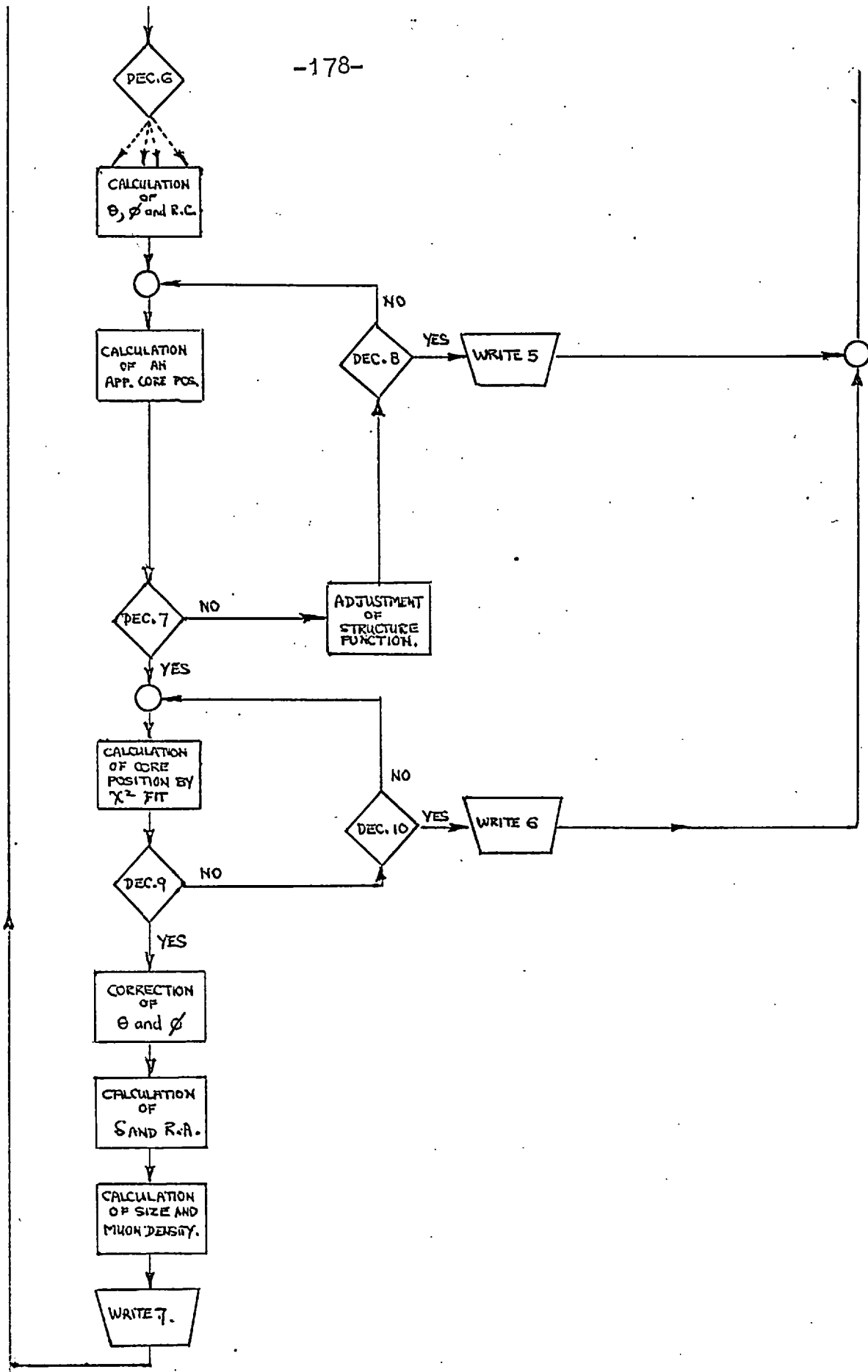
$$\text{i.e. } \underline{\alpha = 0.72}$$

This corrected exponent has been calculated for
muons of energy greater than 1GeV. The difference for
muons of energy greater than 300MeV is unlikely to be
significantly different.

APPENDIX 3: COMPUTER PROGRAM FLOWCHART

(500M ARRAY)





KEY TO THE FLOW-CHART.

READ 1:- Reads in the calibration data.

READ 2:- Reads in information on one shower.

WRITE 1:- Writes page heading after each calibration change.

WRITE 2:- Writes out a histogram of $r(1)$, $r(3)$, θ , ϕ , and χ^2 .

WRITE 3:- Indicates if data is not analysable.

WRITE 4:- Indicates timing information is incompatible with a shower.

WRITE 5:- Indicates failure of best-fit structure function fitting procedure.

WRITE 6:- Indicates failure of χ^2 fitting procedure.

WRITE 7:- Writes out analysed shower information.

DECISION 1:- Ends computation if TRUE.

DECISION 2:- Reads in new calibration data if TRUE.

DECISION 3:- Imposes a ceiling density if one pulse is off-screen.

DECISION 4:- Checks for zero pulse heights and too-small pulse heights.

DECISION 5:- Checks the validity of the timing information.

DECISION 6:- Routes the program for the calculation of θ and ϕ according to available timing information.

DECISION 7:- Checks if the best-fit structure function has been found.

DECISION 8:- Limits the number of loops in the best-fit structure function procedure.

DECISION 9:- Checks that a minimum χ^2 has been found.

DECISION 10:- Limits the number of loops in the χ^2 minimisation procedure.

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