Imperial College of Science and Technology

University of London

School of Management

Market Models and Exposure Management in Foreign Exchange

Robert D. Benson

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Abstract

This thesis is concerned with the management of exposures faced by companies which result from the fluctuations in exchange rates. In particular, we examine the implications of inaccuracies in the specification of the underlying process of exchange rate dynamics, for the way in which these exposures are managed. The first part of the thesis presents an empirical study of daily and weekly spot and forward exchange rates including some new tests for nonlinear dependence. We find that the highly kurtotic distribution of exchange rate changes is due to instability in the process which is caused by a changing variance. This changing variance in turn may be due to a form of multiplicative nonlinear dependence.

Two theories which have been applied to exposure management, Portfolio Theory and Option Theory, are then described and models based on them constructed. These models are examined for their sensitivity to the assumed market description. Some new ideas on the application of the portfolio model are described at this stage which arise because of both the inaccuracy in the underlying assumptions behind the model and the constraints which apply in practice. A more flexible approach than that used in investment portfolios is described based on a more detailed analysis of the risks within the portfolio.

An analysis on the efficient use of some new derivative options in the management of more complex exposures is then presented. We describe four such instruments and derive new valuation formulas where needed. Each of these options is found to have a payoff which more efficiently hedges a particular complex exposure than that available from conventional instruments. However we find that options are only appropriate when forward prices are biased, or when the firm has flexibility in operating decisions.

Finally we present an integrated approach to exposure management based on the appropriate use of available instruments combined with an understanding of the assumptions underlying the models. The strategy provides a logical sequence of steps, from analysis of the actual exposures to the implementation of hedging transactions, which is designed to produce a more efficient portfolio of hedging instruments. Analysing exposures within this framework provides a more structured approach to the selection of hedging products and thus a link be' veen strategic decisions related to exposures and market efficiency, and tactical decisions on the application of hedging instruments in practice.

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"The sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretations, describes observed phenomena. The justification of such a mathematical construct is solely and precisely that it is expected to work."

John von Neumann

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Chapter 1

The Foreign Exchange Market

1.1 Introduction

This opening chapter provides a background to the following research by briefly describing the history of the foreign exchange market up to the present day, its current structure and the subsequent need for exposure management techniques. It is not included simply as an overview of the problem. An appreciation of the development of the market is necessary, not only in order to understand the motivation behind the research, but as an important qualification for the theoretical and empirical analysis which follows. In any research which is concerned fundamentally with the way in which the complexities of the real world can be described in simpler terms, the changing nature of the problem itself must be continually borne in mind. The foreign exchange market is perhaps a classic example of how external influences and technological advances have brought about major structural changes in a market whose basic function and purpose has remained the same. Thus the market has from time to time been either highly regulated or completely unregulated with exchange rates themselves being either fixed, pegged or allowed to vary freely. The speed and frequency of these changes, especially in the recent past, has made it all the more important to be aware of the problem of trying to describe a structure which is not static, but continually evolving at a rapid pace.

As in previous work, the research that follows effectively examines the market in isolation with the current structure of the market assumed to remain essentially unchanged by external events. It is the *internal* behaviour of the market, in particular, the fluctuations in the rates of exchange, which is of concern rather than the effects of major changes in the economic or political environment on the basic structure of the market even though these will inevitably influence the movement of the rates themselves. Obviously this simplification is unrealistic, especially given the history of the market to date, but the problem of describing the role of the market in the context of the global macroeconomic and political environment is beyond the scope of this work. Major structural changes will no doubt continue to occur. They are however, in most cases, outside the control of market participants. Thus in terms of providing a description of the market for use in day to day management of foreign exchange exposures, which *is* the concern of this research, a narrow view of the problem probably represents the most appropriate approach. The purpose of this introduction is simply to serve as a reminder of how outside events have played a significant part in the development of the market so far and as such have influenced the behaviour of exchange rates themselves.

The rest of the chapter is divided into three parts. The first part describes how the market in foreign currencies developed from early coin exchanges in the Middle East through to the growth in international trade during the late nineteenth and twentieth centuries. The workings of the market today, its size, structure and participants, and the influence of advances in information technology are explained in section two. Finally the problems which are faced by those who need to buy and sell foreign currency, both corporations and financial institutions, and therefore the need for exposure management techniques are dealt with in section three.

1.2 A Short History of the Foreign Exchange Market up to 1973

The origins of foreign exchange transactions lie in the movement away from simple bartering and exchange of goods in trade, to the use of recognised coinage. For a long time, foreign and local coins were only exchanged on the basis of their metallic content and by weight. As such the market was more of a bullion market than foreign exchange market. Only when certain coins became accepted without weighing did a system of exchange rates become established.

Early foreign exchange markets developed in the Middle East and Ancient Greece although records relating to their activities are patchy. It seems likely that gatherings took place around ports and other commercial centres where money changers would exchange coins from different regions. One of the earliest records of this trade is found in a text by Josephus at the time of the Temple in Jerusalem. In this he refers to Jewish money dealers being involved in transfering funds between Palestine and Babylonia. Since interest rates were extremely high in Babylonia at this time and the Jews were prevented from charging interest in their own country, some form of interest arbitrage may have been practiced. Another form of arbitrage which occured at this time involved the ratio between gold and silver. For many years the ratio in use in the Persian Empire undervalued silver causing large quantities to flow out of the empire. It is unlikely that professional arbitragers operated at this time however, but foreign merchants probably exploited these discrepancies in chosing their means of payment.

The earliest record of professional bankers involved in foreign exchange appears in Athens from around 400 BC. The Athenian silver coin had become highly dependable and so was widely accepted throughout Ancient Greece. These early bankers, called "trapezitai" (meaning table, after the tables on which they laid out their scales and weights), were also involved in loan transactions and other commercial activities. In a speech by Demosthenes (383- 322 BC) before the Athens law courts, mention is made of an "exchange" at which business in loans and foreign exchange was systematically transacted. At around the same time "argentarii", the Roman equivalent of the trapezitai, were engaged in exchanging foreign coins, which flooded into Rome during the centuries of Roman expansion, for domestic coins at official rates. The argentarii charged a commision for these transactions. Roman coins, the gold Aureus and later the silver Dinarius, soon became highly acceptable, even as far away as China.

At this stage in the development of the market the main problem was that of debasement. In Rome, Greece and Ancient Egypt, periods of coin debasement occured which often resulted in highly inflationary consequences. During these extreme economic conditions the exchange of foreign coinage frequently reverted to a system of weighing, which effectively reduced the market to a bullion market. The period of decline after the end of the Pax Romana and throughout the dark ages, saw a drastic reduction in foreign trade and a virtual elimination of the foreign exchange market altogether.

When the market re-emerged in the early middle ages, the main centres of coin exchange had shifted from the mediterranean exchanges at Byzantium and Alexandria, to the new commercial centres in France, Italy and Spain. The term "banker" itself, originates from the benches used by the money changers in Genoa and the other Italian cities. The gradual recovery in international trade led to the development of merchant banking houses, mainly in Italy, which maintained branches and correspondents in foreign centres. New methods of foreign exchange transactions using notarial contracts, bills, letters of credit and the equivalent of modern mail transfers, were also developed. Indeed, from the thirteenth century onwards, the importance of the so-called "manual" or coin exchanges declined significantly as exchange by bills increasingly became the main method of serving the needs of trade.

The first known foreign exchange contract dates from June 8th 1156 in which two brothers, having received 115 Genoese pounds, promise to reimburse 460 bezants one month after their arrival in Constantinople. The terms of the contract also included provisions in the case of default under which the creditor could claim 500 bezants if payment was delayed and then 250 Genoese pounds if that payment was defaulted.

This period in the history of the market is the first instance of the importance of external regulations on the development and conduct of the market itself. At this time the influence of the Church was very great and the imposition of laws on the payment and receipt of interest was widespread. These strict anti-usary laws in fact provided a significant boost to the growth of the foreign exchange market. Since interest on domestic currency could not be paid, the use of foreign currency denominated bills enabled the inclusion of interest to be made in the agreed rate of exchange applied. Thus the laws could be effectively circumvented through the use of these foreign bills, which were then no longer simply issued to back up trade in foreign goods.

The market in these foreign bills originally developed in quarterly fairs, held in foreign countries to avoid problems with the church authorities, at which bankers would deal in bills maturing at that time and for the following fair. A system of exchange rate fixing also occured at the fairs. At the start of the fair a representative body of merchant bankers would gather to fix the rates through mutual agreement, whilst the brokers who dealt in the bills waited outside. The rates were not official however and dealers could and did depart from them. The purpose of the system in fact was almost certainly to satisfy the church that the rates were in accordance with the anti-usary laws. Another development at this time was the use of bills denominated in fictitious units which were settled by means of book transfers with only the balance being settled in existing currency. Bills drawn on the Champagne fair in France for example, were sometimes issued in the fictitious unit, *ecu de marc*. One of the reasons for this was the continiung problem of depreciation in the value of coinage through changes in the metallic content. These debasements and devaluations led to frequent speculation on the foreign exchange markets and so affected the bill rates too. The use of fictitious units however, led to a more stable rate.

As the market in bills grew, permanent centres also became established in the main commercial centres. These were mostly in the Italian cities of Venice, Genoa, Florence, Milan and Rome. In the rest of Europe, the markets were also run mainly by Italians, in such places as Paris and Avignon in France, Seville and Valladolid in Spain, and Bruge and later Antwerp in Belgium. London lagged behind, with English bankers not taking an active part until the Sixteenth Century.

As regards official activity in the markets, periodic imposition of exchange controls occured along with fixing of official rates. In the main however the market in coins was more highly regulated than the bill market since the latter served an important function in many official transactions. No open dealing in the foreign exchange markets, to influence the rates themselves, took place until the Sixteenth Century.

The following two centuries saw a continuing development of the markets. The anti-usary laws were gradually relaxed and eventually abandoned although the system of including interest in the rates continued for some time. The value of transactions increased, due firstly to the fact that bills became transferable and secondly as a result of the growth in the size and importance of official business. The monopolistic position of the Italian bankers slowly weakened as did the importance of the fairs which ceased to play any part after the Seventeenth Century. Also during this period early forms of forward exchange facilities became noticable. To begin with these consisted of a form of betting on the future level of the exchange rate which was common in the Netherlands and Spain. In 1702 a forward agreement was signed between Sterling and Guilders in connection with the financing of Marlborough's army on the continent. Sight bills were to be delivered during seven months for sums not exceeding £50,000 a month, at a rate of 10 Guilders 17 Stuyvers per £1 and also £20,000 immediately at 11 Guilders. There was therefore a premium of 3 Stuyvers on the bills.

Problems with the quality of coinage persisted however, with natural deterioration compounded by the practice of clipping and counterfeiting. These influences combined with the effects of changes in the gold-silver ratio added to the volatility and uncertainty of exchange rates throughout this time.

The role played by governements and other official agencies during this period became more and more significant as the importance of the markets in economic affairs widened. To a large extent this was due to the advancements which were being made in the area of economic, and particularly foreign exchange, theory. The effect of exchange rates on prices, trade balance theory, supply and demand theory, interest rate theory and purchasing power parity theory were all being developed at this time. The implications of this new knowledge led to a more active part being taken by governments in the markets through increasingly sophisticated intervention. Apart from the periodic introduction of bans on the import and export of foreign coins and the imposition of licences and duties and other restrictions on the flow of money, taxes on foreign exchange transactions were also brought in and revaluations and devaluations carried out to manipulate the balance of trade. Official intervention in the markets themselves also took place although of a more severe form than those carried out today. An early example of this took place in 1552 and 1553, when the cloth fleet of the Merchant Adventurers was detained in English ports by Gresham until the exporters agreed to sell to the government an amount of Flemish currency at rates less favourable than the current market rates. This drastic action was taken in order to pay for the governments large external debt and also to support Sterling on the Antwerp exchange. A more advanced form of policy, also advocated by Gresham, was to raise the level of Sterling by creating artificial monetary tightness at home. Although this advice was probably not acted on, the idea itself appears well ahead of its time.

Towards the end of this period and into the Nineteenth Century an attitude of *laisser faire* towards the foreign exchange markets set in. This was mainly due to the increased stability of the economic environment which lasted up to the French Revolution.

The period between 1789 and the start of the First World War saw the development in many respects of the modern foreign exchange market of today. From the introduction of better communication systems, first by mail transfer and later by telegraph, and the subsequent move towards continuous trading throughout business hours, to dealing in paper currencies, the use of inconvertible currencies and the general increase in volume, this century produced some of the most important characteristics of the current market.

The gradual move towards a gold standard was one of the main factors in this improvement. Another was the development of large markets in paper money, both spot and forward, after the French Revolution. The convertibility of these notes varied from country to country and from time to time due to civil wars, revolutions and similar upheavals. The market in foreign notes however was still overshadowed by the bill market right up to the end of the Nineteenth Century when the relative importance of mail and later telegraphic transfers, increased considerably. There were also developments in the differentials between classes of bills traded, so that a seperate market in bank bills and commercial or trade bills developed. During this time the practice, still used, of two day settlement was introduced. Previously "post days" (Tuesday and Friday) were used whereby the buyer was required to settle on the next post day after purchase.

The location of the main centres also changed during this period with the decline of Amsterdam, which had taken over from the Italian centres earlier on, and the rise of London and Paris after the battle of Waterloo, even though Sterling was inconvertible from 1815 to 1821. At the same time the market in New York developed, although because of the great distance, its growth proceeded at a slower pace. By the end of the century however, the use of cable transfers, first recorded in 1879, had improved the links between distant countries considerably. The importance of this link to the United States is emphasised in the continued use of the term "cable" as the market jargon for the exchange rate between Sterling and US Dollars. At the same time, the relative economic stability led to an improvement in banking relations between different centres. The development of mail transfers and increasing confidence led to the habit of keeping continual balances with banks in other financial centres and the practice of allowing overdraft facilities to foreign banks in their own currency without insisting on receiving bills of exchange.

The improvement in communication systems also led to changes in the method of dealing, with an increasingly large proportion of business being transacted away from the formal markets. The beginnings of the modern market, consisting of banks dealing through brokers in their own centre or directly with overseas banks, can be seen in the latter years of the century with the increasing use of the telephone. Although the formal meeting places continued to be used side by side with the new system, with rates fixed at the start of each day, their importance became less and less. The Royal Exchange in London for example only met on the two post days and the rates fixed there were often quickly departed from.

Perhaps the most important change during this time however was the development of regular and active forward markets. Prior to this, exchange risk was covered using long dated bills. The problem with this however was that an initial cash outlay or bank loan was required whereas in the forward market only a small deposit was occasionally needed. Vienna was probably the first modern market to deal systematically in forward exchange. These dealings were more akin to futures markets today with fixed delivery dates usually at month end. The market was actively used for interest arbitrage, with the exchange risk covered, between Vienna and Berlin, due to the instability of the Austrian Gulden.

As for the movement of the rates themselves, the period prior to 1914 was probably the most stable, at least in Western Europe and the United States, since the time of Ancient Rome. Exchange rates fluctuated during times of war, especially in France after the revolution and during the Napoleonic wars, and during the Civil war in America, but with the increasing adoption of the gold standard the overall stability of exchange rates improved. The position of Britain as the largest international trading centre, led to Sterling becoming effectively the world's reserve currency and the Sterling-Dollar rate became the most important rate. It was not overly volatile however and indeed during the thirty five years prior to 1914 the rate kept within gold points, conforming to the rules of the classical gold standard.

In terms of new ideas on the theory of foreign exchange, the Nineteenth Century provided few significant breakthroughs. Purchasing power parity theory was rediscovered and elaborated upon and a rudimentary theory of forward exchange developed, but much time was also spent on the so-called bullionist controversy, a long running debate on the reasons for the high price of gold. The most important contribution was undoubtedly provided by Goshen in his classic text, *The Theory of Foreign Exchange*, in which he brought together and clarified much of the existing theory. The relationship between interest rates and exchange rates, and between interest differentials and specie points were clearly stated. In addition by pointing out that even an anticipation of a change in interest rates would effect the exchange rate, he introduced ideas of psychology into exchange rate behaviour.

Due to the relative stability of the exchanges direct government involvement in the markets also declined during this period. This was in keeping with the general policy of *laisser faire* towards the business and economic environment. What actions were taken tended to be more subtle than before with the use of interest rates and the control of the volume of currency and credit being the most common, and indeed effective, weapons. Direct official intervention on the exchanges was rare, with one of the few occurances being during the American Civil War, when the US Treasury sold Sterling bills at lower rates than those in the market in order to force down the premium on gold. This was largely ineffective however, with the Treasury being forced to raise its selling rate three days later.

The development of the market from 1914 up to 1973 was in many respects dominated by the two World Wars and the subsequent turmoil in the political and economic environment which resulted from them. Despite these upheavels though, the market maintained a continuing growth in volume, both spot and forward, and an increasing sophistication in both communication systems and in the type of transactions performed. Although free exchange dealing was suspended during both World Wars and also for a time afterwards, the markets were able to recover and indeed continue to grow on each occasion. The most significant development was undoubtedly the emergence, after the Second World War, of a coordinated international monetary system. The realisation of the importance of exchange rates in influencing the recovery from the economic devastation of the war brought the market a higher profile in international political affairs than ever before.

This realisation came from the experiences gained after the first war. At the outbreak of the war the belief that it would not last long lead to an appreciation in Sterling which was being quoted at around $7 to the pound. However as the fighting dragged on confidence fell and the British government was forced into a devaluation to $4.51 in September 1915. Subsequently, after official intervention, the rate was stabalised at $4.765 where it was fixed for the rest of the war. A similar devaluation occured in the case of the Franc which was also pegged at a lower level. The real problems developed however once the war was over. Although both currencies were unpegged immediately, in a hope that pre-war parities could be restored, the economic problems were underestimated and a more severe loss of confidence resulted. This led to more drastic devaluations, which together with the collapse of the German mark and the other defeated countries currencies, created a high level of uncertainty and volatility in the foreign exchanges.

These problems persisted through to the early 1920's when a relative calm gradually returned to the markets. This lasted long enough to allow Sterling to return to the gold standard at its pre-war parity in 1925. Speculative attacks on the Franc in particular continued however, although the stability of most other currencies improved. The German mark was finally stabalised at 4,200,000,000,000 reichsmarks compared to 4.20 marks in 1914. Between 1927 and 1931 exchange rate volatility continued to fall with the return to the gold standard appearing to provide a stabalising influence. The Wall Street Crash of 1929 however put inevitable strains on the system and led to an eventual Sterling crisis in 1931. This resulted in a suspension of the gold standard by Britain in September of that year which was followed by most other major countries, except France and the United States, immediately. The Sterling Dollar rate continued to fluctuate however, between $3 and $4 up to 1933 when Roosevelt was finally forced by speculative pressures and the banking crisis, to follow Britain and suspend the gold standard as well. There then followed a depreciation race with the exchange rate rising to $5.25 in November 1933 and eventually fluctuating around $5 for the next few years.

The 1930's then came to be characterised by beggar-thy-neighbour policies adopted by individual countries, which attempted to use trade restrictions, subsidies and competitive depreciation of exchange rates to relieve domestic unemployment problems brought on by the depression. The market then, although essentially unregulated, became highly influenced by government actions. As a consequence rates were volatile and a large amount of speculation took place which added to the problems faced by international trading companies. It was largely as a reaction to these events that the development of the post World War II international monetary system proceeded as it did.

At the onset of the Second World War, Sterling was devalued immediately from $4.48 to $4.04 where it remained fixed throughout the war. Most business in foreign exchange was suspended with only the smaller neutral exchanges conducting any trade. Discussions on the establishment of a new monetary system after the war were taking place however, notably between John Maynard Keynes in Britain, who acted as a government advisor, and the American Treasury official Harry Dexter White. The result of their work was the so-called Bretton Woods agreement of 1944 which established both the International Monetary Fund and the World Bank, and a system of par values for individual currencies. Each member of the fund was commited to maintaining its currency within 1% of its par value by direct intervention in the market. Thus began the first intance of sustained and deliberate government intervention in the markets in order to stabalise the fluctuation of exchange rates and suspend the forces of supply and demand.

This system remained in place right up to 1973, and in its early years was fairly succesful in providing a stable environment for the recovery of the European economies which had been devastated by the war. Although periodic devaluations did occur, notably to the Franc in 1948 and Sterling in 1949 which went from $4.02 to $2.80, for most of the late 1940's exchange rates stayed within their agreed parities. The growth of the European economies during this period, with the aid of Marshall Plan support from the United States, was rapid and continued into the 1950's. When Marshall Plan loans stopped in 1952, increased military spending by the US continued to bolster European growth, which resulted in a 30% rise in GNP for the countries of the OEEC (Organisation for European Economic Cooperation) from 1953 to 1959. Although Sterling suffered a series of crises in 1951, 1956 after Suez, and 1957, the only official devaluation was that of the Franc which was devalued twice in the period 1957-58, by a total of 29%. It is interesting to note that during each of these Sterling crises, although the spot exchange rate stayed within its support points, speculation in the forward market caused considerable depreciation in those rates.

By the end of the 1950's the dollar had become the most important international currency, not only due to its use in intervention, but also through its growing use as a reserve currency. From 1949 to 1959 U.S. liabilities to foreign monetary authorities grew to over $10bn, equal to that of Britain. The increased importance of the dollar however, did not result in a move away from London as the main centre for foreign exchange business. London's continued domination though owed more to its geographical position than to the strength of the economy or the importance of Sterling itself. The growth of the market in terms of increasing volume, along with that of the world economy in general, also created problems for those engaged in foreign exchange dealing. The relative stability of the pegged exchange rate system provided fewer speculative profit opportunities, and even the steady profits from market making were reduced as spreads between buying and selling prices narrowed. In addition, arbitrage between different centres became more difficult with improved communications. Against this, the growth in the forward market and the development of integral markets in foreign currency deposits, created new profit opportunities. As mentioned above, these markets provided openings for trading interest rate differentials, which in turn caused the large speculative outflows experienced by Sterling and the Franc during this period. Indeed this problem was one of the failings of the Bretton Woods agreement which eventually led to its downfall in 1973.

The collapse of Bretton Woods was preceeded by a decade of increasing tensions within the international monetary system. Although the first half of the

1960's were relatively settled, with speculation being mainly confined to the forward rates so that spot rates remained within their agreed 1% bands, by 1964 renewed strains began to be felt. The first sign of this came during the summer of that year when a deterioration in the UK balance of payments brought sterling under pressure on the exchanges. The Bank of England raised interest rates and used official reserves to defend the parity level but by November the crisis had intensified. Up to $1bn were used in official intervention and the base rate was raised by 2%. Finally an arrangement of $3bn in swap credits through the IMF for support of the pound, convinced the markets that devaluation would not be allowed and the crisis subsided.

The respite was short-lived however and by the summer of the following year, a series of poor trade figures had once again begun to put pressure on the pound. Although a package of measures to restrain demand was announced, by early August selling pressure on the exchanges had reached a critical point and the Bank of England had to use $225m to support the rate in the first week. This intervention was followed by a classic "bear squeeze" in early September when coordinated sterling purchases by the Federal Reserve, a number of European banks, and the Bank of Japan, put a halt to sterling's fall and an end temporarily, to fears of devaluation.

The travail of sterling continued in 1966 however. The Six Day War and the closing of the suez canal, led to fears of a withdrawl of sterling funds by the Arabs and throughout the summer the Bank of England was once again forced to intervene heavily in the markets. By the autumn the situation had become critical and the decision to devalue was finally taken. On Saturday November 18th, after heavy sterling selling on the foreign exchanges the previous day, the announcement was made of a devaluation from $2.80 to $2.40.

Pressures within the exchange rate system did not ease however. The threat of further devaluations by other currencies and possibly the dollar, and a movement of funds into gold putting pressure on the fixed price of $35, persisted. Although sterling was able to maintain its new parity fairly effectively, the affair had demontrated some of the major weaknesses of the Bretton Woods system. The complacent view that exchange rates of the major countries could be regarded as fixed was severely undermined and was to be shattered over the following two years.

The next major crisis occured in France as a result of the student uprisings and general strike of May 1968. Throughout that summer the Franc was under continuous pressure and speculation about a devaluation and possible upward revaluation of the German mark, lead to heavy trading on the foreign exchanges. Between April and November the Bank of France used up nearly $3bn of the country's reserves leaving them down at less than $4bn. President De Gaulle however was adamant that devaluation should not be allowed, prefering to achieve a similar result by waiting for a German revaluation. Conflicting statements by the German government only increased uncertainty with funds flowing into and out of marks throughout 1969. Finally in the August of that year the French announced a devaluation of 11.1%, as much to put an end to the speculation as for economic reasons. This was followed in October by a revaluation of the mark by 9.3%. The method of German revaluation, whereby the currency was allowed to float temporarily until a new parity was fixed, was the first instance of free-floating since the Second World War.

These changes in parities and the speculative pressures which preceeded them, increased debate on the possibility of a re-introduction of a flexible exchange rate system. Indeed some form of change was felt to be inevitable, either by widening the existing fluctuation margins around parity or by the use of a so-called "crawling peg" system of frequent small adjustments. In the summer of 1970 the German mark was again allowed to float temporarily due to speculative pressures, and by the end of July had risen by 5%. The slow deterioration in the system continued into 1971, so that by

August President Nixon was forced by the huge outflow of dollars from the US, to suspend the gold convertibility of the dollar.

The next four months were characterised by confusion and turmoil in the financial markets. All the major currencies, except the franc, began to float. Crisis meetings were held amongst the Group of Ten and there was much debate over the abandoning of the fixed rate system or the devaluation of the dollar in terms of gold. Eventually the so-called Smithsonian Agreement was reached in December which amounted to a devaluation of the dollar of about 10% along with a widening of the fluctuation margins, although the inconvertibility of the dollar remained.

The agreement however failed to address the real problems of the existing system, that is it's inability to allow for adjustments in rates without causing massive speculation and subsequent political and economic crises. Perhaps more fundamentally however, the emerging strength of the European and Japanese economies made the Bretton Woods system, which placed a greater role on the United States, obsolete. A fairer, more symmetric, system was actually required. In the event, the Smithsonian Agreement lasted for just fifteen months. In February 1973, a further devaluation of the dollar brought an end to the pegged exchange rate system and ushered in the period of floating rates which remains in place today.

1.3 The Modern Foreign Exchange Market

Since the demise of the Bretton Woods agreement the structure of the foreign exchange market has developed rapidly. The importance of the market in global economic affairs has increased as has the volume of business and number of active participants. The level of technology has also advanced dramatically, bringing in many ways a significant influence to the movement of the rates themselves. Finally the volatility of exchange rates has fluctuated increasingly providing both problems and opportunities to all those who are involved in the market.

The market today is almost entirly free-floating with the exception of some minor currencies which are pegged, usually, to the US Dollar. Currencies are traded on a twenty four hour basis during weekdays with the largest centres being in London, Tokyo and New York. Trading also takes place in a limited way on Saturday mornings in Tokyo although this market is also effectively closed during lunchtime on weekdays. The most recent survey by the Bank for International Settlements conducted in April 1989, estimated that $640bn of foreign exchange transactions took place daily, double the amount three years earlier. This massive turnover is mainly handled by a relatively small number of large international commercial banks, together with a larger pool of second tier banks trading on their customers' behalf or for speculation. Liquidity however varies, both in currencies and at different times of the day, and large buy or sell orders can still at times affect the market on an intraday basis. The London market is the main centre (daily turnover $187bn), followed by New York ($129bn) and Tokyo ($115bn). More recently Tokyo may have overtaken New York as the growth in Japanese trade has continued.

The mechanics of the market has developed rapidly in recent years led by the introduction of new technology. Trading takes place usually in one of three forms; through inter-dealer brokers connected by direct telephone lines and voice boxes, between banks directly over the telephone, or using a direct dealing system, typically Reuters. Customer orders are mainly taken over the telephone and passed on to the trader by dedicated sales people. The flow of information is maintained by on screen news agencies, eg R›uters, Telerate, Knight Ridder, and by brokers continually calling out prices and the size of transaction. More recently touchscreen dealing terminals and electronic writing pads have been introduced to enable dealers to quote and input trades in an even faster and efficient way. As yet however the role of the junior trader or position keeper has yet to be replaced.

The market is fast flowing with bid/offer spreads being relatively tight especially in the major rates such as US Dollar versus Deutschmark or Yen. For sizes above $1-2m the normal spread for Dollar/Deutschmark would be 5 points or 0.05 pfennigs, for Dollar/Yen 0.05 Yen and for Dollar/Sterling 0.05 cents. Typical size for interbank trades is $5m to $10m with some of the larger banks having up to $20m relationships with each other. This means that they will agree to quote on a single deal up to $20m thus enabling large orders to be unwound with fewer calls. When large orders ie above $100m, are received the bank will often ask whether this size represents the full amount of the business. If the customer is also dealing with another bank then this will make it more difficult to unwind the position and so the spread will be widened. If alternatively the order is for the full amount then a tighter price can be made since the bank will have more time to close out its risk in the market before prices begin to move against it.

It is almost certain that the speed with which news can be passed around the market, for example when economic figures are released, has contributed to the continued growth in the volume of trades. It may also be true that this fast flow of information has contributed to exchange rate volatility, on an intraday basis, with at times wild gyrations occuring just after news is flashed onto the screens. Although the market is not entirely transparent, it is probably the most open and visible over the counter market.

The continuing importance of exchange rates to governments' economic policy has maintained the role of the central banks as important market participants. Although the range of exchange rate fluctuations has undoubtedly declined in recent years, direct intervention by central banks to support or cap particular currencies still takes place. The advent of the European Monetary System (EMS) which establishes bands for the fluctuation of most European currencies, has forced the European central banks in particular to monitor the movements of the markets closely on a day to day basis. In general however, attention is focussed mainly on the US Dollar, the Deutschmark and the Japanese Yen with the dollar still being the largest traded currency. Cross rates such as Deutschmark/Yen and Deutschmark/Swiss Franc however have also become increasingly active spurred on to some extent by the relative stability of the dollar. More recently the opening up of the Eastern European economies has begun to offer new opportunities to trade.

The forward market has also continued to develop, alongside the Eurocurrency deposits market. Forward deals can now be conducted out to five years or more, although credit considerations are begining to have an impact on these trades. The relationship between the foreign exchange market and the parallel cash and futures markets have continued to grow, with many of the traditional space and time arbitrages being removed. In particular at the longer end of the market, the trading of foreign exchange has become closely integrated with the market for interest rate swaps. A new market in synthetic forward exchange agreements (SAFEs) has also recently developed, analogous to the market in forward interest rate agreements (FRAs).

One of the biggest developments however is that of the currency options market. This market has grow dramatically in the past five years with estimates of around $25bn a day in turnover on the over the counter market. Foreign exchange options are also traded on listed exchanges in Philadelphia and Chicago with volume of $6bn to $7bn a day. Originally offered as hedging instruments to corporate customers options have developed into an active interbank instrument with specialised brokers providing liquidity on a 24 hour basis. Traders quote all the major currencies with maturities out to five years in some cases and often deal in terms of a volatility price alone. This is due to the widespread acceptance of the Black and Scholes model for option valuation, which has undoubtedly provided a boost to the growth of the market. Complex option strategies can be traded frequently in sizes of up to $1bn in a single deal and the more recent growth in specialised or so-called exotic option products has widened yet further the opportunites available to both hedgers and speculators.

1.4 The Need for Exposure Management

The growth in global trade and the expansion of multinational companies in the last twenty years has given rise to a whole new set of problems for managers of trading companies. It is no longer the case that manufacturing at a low cost and selling at a higher price can guarantee profitability. Entire annual profits have been wiped out and embarassing losses sustained, not by bad product design or lack of cost control but through adverse movements in exchange rates. One only has to look through the annual reports of the companies listed on the stock market to read of the impact of exchange rates on the costs of raw materials or the price of the finished product. It is also interesting to note how often exchange rate movements are reported to have been an adverse influence on profits but rarely a beneficial influence.

Management nowadays must look not only at the direct effects of exchange rates on their imports or exports but also the secondary effects on their competitors who may be domestic or, ever more frequently, based in a foreign country. It is certainly true that the fluctuations of the Dollar or the Deutschmark need to be carefully analysed by UK treasury managers in order to understand the impact they have on the company's bottom line. For the commercial banks operating in the foreign exchange markets too, growth in the volume of transactions has also given rise to a closer examination of the risks involved in running large positions in a wide range of currencies.

Before describing the different types of foreign exchange exposure we should address an important point which is often raised, that is, why should a firm hedge its foreign exchange exposure at all? The first issue here, as discussed in Makin (1978) and Ware and Winter (1988), is whether the value of the firm should reflect the risks involved in performing its business and as such shareholders expect this value to fluctuate as demand for its products varies and exchange rates move to provide opportunities in different markets. Therefore, if we assume that perfect capital markets exist and transaction costs are zero, shareholders could if they wished, hedge the foreign exchange risk in their investments themselves for the same cost as the firm. In this case companies should allow these exposures to remain open. In practice since hedging costs to individual shareholders are greater than for the firm and since the cost of monitoring this exposure is also higher, it is more practical for the firm to hedge these risks on behalf of the shareholder. It is also questionable whether an adequate response to shareholders of losses due to exchange rate movements would be that they should have been aware of these exposures and could have hedged them themselves.

A second issue arises when we consider the exposure problem in the context of competition. In a market in which all participants face the same foreign exchange exposures but all adjust prices to reflect movements in rates, there is little incentive for a firm to hedge itself independantly. This is because as long as the firms are able to effectively pass on the exposure to their customers, the risks to their profitability are eliminated. A firm which decides to hedge will find itself gaining in periods when its competitors are forced to raise prices, but losing when its competitors are able to lower prices. In this situation then the firm has little incentive to hedge from a profitability point of view, since it actually increases the volatility of profits, and may additionally be reluctant to do so for marketing reasons. This is because it may be felt to be more acceptable to adjust prices along with the competition, rather than to appear to be at odds with the "market price" especially in situations where prices are falling. In this case stability in prices, which results from being hedged, may act against the firm. The important question is whether and to what extent, customers put a value on price stability. In a market where customer loyalty is low and price is the most important determinant of sales, hedging by an individual firm may act against its longer term interests. An example of such a market if that for basic foods, where the large supermarkets, who purchase much of their stock overseas, are reluctant to hedge for the reasons described above. As long as they all pass on exchange rate and indeed commodity price risk to their customers none has an incentive to hedge it themself. In many other markets however hedging has become the norm and so the management of exchange rate risk is required for competitive reasons to reduce its impact on output prices. Since these markets are in general more common the analysis of exposure management is important.

Traditionally, exchange rate exposure has been classified into three types; transactional exposure, translational exposure and economic exposure. Transactional exposure arises through the payment or receipt of foreign currency in the course of, for example, paying for foreign materials or receiving payment for exported goods. The risks here are that the expected or budgetted cost or income in domestic currency terms is adversely effected by the rate of exchange at which the foreign currency is bought or sold. A secondary problem concerns the timing of the actual cashflow and its exact size, which are sometimes unknown at the time the exposure is taken on. The literature on transactional exposure demonstrates that, where the forward rate is assumed to be unbiased and the size and timing of the foreign currency cashflow is known, this risk can be completely hedged at no cost in the forward market. Thus a firm expecting to receive a known amount X of foreign currency at some time in the future can sell this to receive FX domestic currency at that date where F is the current forward rate (in domestic currency units per one unit of foreign currency). A further assumption here is that the risk in the real value of this domestic currency is small, ie inflation effects are negligible, compared to the risks in the nominal value due to changes in the nominal exchange rate. This result is important since it suggests that currency options would not be used to hedge a transactional exposure since this risk can be hedged completely with no loss of expected return, by hedging in the forward market. Of course, where the decision maker believes that forward prices are biased then the use of options may be appropriate.

Translational exposure arises from the conversion of foreign assets or liabilities into domestic currency for the purpose of financial reporting. As the growth in cross border acquisitions has increased and the number of multinational companies has risen, this problem has become more and more acute. The actual exposure here does not relate to a cashflow, but to an adverse revaluation of foreign holdings which could impact on the balance sheet and so effect shareholder confidence. Various techniques for mitigating this exposure have been used such as balancing foreign assets and liabilities through raising debt in foreign currency or more recently denominating the entire balance sheet in terms of another currency, for example, the Ecu. Often average exchange rates are used as conversion rates since these are felt to better reflect the ongoing exposure to the foreign currency and as averages these rates are less volatile than the underlying exchange rate. New complex options are now available for hedging these average rates.

Finally, economic exposure has been defined as the uncertainty of future profits arising from changes in the domestic currency value of traded inputs and outputs. Another way of describing this would be the risk of a loss in competitive advantage due to changes in exchange rates either because of a relative increase in input costs of output prices. This type of exposure has only recently come to the attention of companies as they have begun to suffer from losses due to changes in the exchange rates of competitor companies operating in different countries. Traditionally company treasurers were concerned mainly with the foreign exchange exposure arising from specific transactions.

As previous research has shown (see Ware and Winter (1988)), this type of exposure is much more complicated to hedge especially where inputs and outputs are sourced from several countries and competitors exist both domestically and outside the country. Thus even a local company with domestic inputs and only selling at home may have foreign exchange exposure if a competitor is either based abroad or is sourcing its materials from a foreign country. In these situations straightforward forward agreements cannot provide the required protection and even conventional options can only give a degree of protection under certain circumstances. More complex options whose payoff is a function of several exchange rates are needed, some of which are now available in the market. Interestingly however, Ware and Winter have shown that in order to hedge this exposure a company would effectively *write* these more complex options rather than buying them since it is giving up the opportunites available in its original unhedged position. This contradicts the more widespread view which puts forward the purchase of options as a means of protecting against transactional and economic exposure or as a hedge against contingent cashflows. In fact it can be shown that hedging of the classic "tender to contract" situation is not achieved by the use of conventional options either since the exercise decision is based only on the spot rate, not on the success of the tender. Using this approach economic exposure can in some sense be seen as an opportunity rather than a risk, which can be exploited or sold on to the market depending upon the situation. In the chapter on option theory, these more complex options are considered in more detail.

1.5 Summary

This chapter has provided an overview of the problem of exposure management in foreign exchange by describing the development of the foreign exchange market, its current structure and the type of risk which both international and domestic companies face from fluctuations in exchange rates. The choice of technique for managing these exposures is the critical problem for treasury managers. The actual application of a specific exposure management model however is a function of the biases that the model contains due to the assumptions about the market on which it is based. A critical examination of the market and the alternative models of price behaviour is therefore required in order to assess the inaccuracy of these assumptions. By studying the actual dynamics of exchange rate changes the potential weaknesses of the theoretical models become apparent and a more informed application of the exposure management tools can be made.

The next chapter deals with this issue by presenting the results of two empirical studies of exchange rates, both spot and forward, over the last decade and compares the results with those of earlier work. Alternative models for the behavior of market prices are described and some conclusions drawn on the process underlying exchange rate movements. These results are later used in the analysis of alternative hedging strategies in terms of the assumptions underlying the models or the pricing of particular instruments and the effects these assumptions may have on hedging performance.

Chapter 2

Empirical Examination of Exchange Rate Movements

2.1 Introduction

This chapter is concerned with the behaviour of exchange rates and the models which have been proposed for describing their movements. We begin with a brief survey of the economic models which have been put forward and some of the results of tests which have been carried out on their performance. We then describe in more detail some of the tests which have been done on the specific problem of exchange rate distributions before going on to present the results of two studies which we have carried out into the statistical distributions of both spot and forward rates using data from the 1980's. Finally we present some conclusions on the performance of the different models of exchange rate behaviour and the distribution of exchange rate movements.

2.2 Survey of Foreign Exchange Market Models

Attempts to study the structure of the foreign exchange market and the behaviour of exchange rates have produced a wide body of literature on both the theory and reality of exchange rate movements, in particular since the start of generalised floating post 1973. On the theoretical side the earliest models of exchange rate determination were based on the monetary approach of Mundell (1968) and Fleming (1962) which analysed the foreign exchange market in terms of the supply and demand for currencies[1]. This flow model was subsequently extended by Johnson and Frenkel (1978) to incorporate the stock of, as well as demand for, the two currencies. These early models however essentially assumed that prices would adjust continuously so that Purchasing Power Parity (PPP) would not be violated. Experience quickly suggested that these so-called

1 A detailed survey of the work done on the macroeconomic analysis of exchange rates can be found in Macdonald and Taylor (1989)

Flexi-Price monetary models were inappropriate since violations of PPP over the short term were often observed. It was apparent that prices did not adjust as quickly as exchange rates moved.

A development of the monetary model was proposed by Dornbusch (1976) which allowed for short term overshooting in the exchange rate. These Sticky-Price monetary models appeared to describe the behaviour of exchange rates better. A further development of these models was the Portfolio Balance Model (PBM) (see for example Branson (1977)). In this model the assumption of perfect substitutibility of foreign and domestic non-monetary assets, which lay behind the earlier models, was relaxed. In the PBM wealth is divided into domestic bonds, foreign bonds and money and the model could be used to explain overshooting with or without sticky prices. A subsequent extension of the PBM by Branson (1983) incorporated expectations, so that the expectation of future changes in current account imbalances would impact immediately on the exchange rate as the market looked ahead.

Tests of these alternative economic models, all of which essentially attempt to relate exchange rate movements to fundamental economic variables, have proved so far to be largely unsuccessful, especially on data post 1978. Difficulties in estimating the models have resulted in autocorrelation and multicolinearity problems, and results have often produced coefficients with opposite signs to the theoretical priors. Several tests have focussed on the differences between the monetary model and the PBM in terms of their assumptions regarding the attitude of speculators towards risk. The monetary approach assumes speculators are risk neutral and there is perfect substitutibility between foreign and domestic assets and perfect capital mobility. This in turn implies that uncovered interest parity (UIP) must hold. The PBM on the other hand assumes that speculators are risk averse and the exchange rate is determined by money and bond market equilibrium. This implies that there is a risk premium ie there is a difference between the forward rate and the expected future spot rate. Several studies have been made to discover the existence of a risk premium in order to support the PBM.

The overall conclusion from the empirical studies to date is that the performance of these models both in and out of sample is extremely poor. In an important study Meese and Rogoff (1983) compared several versions of the monetary model and the PBM to a naive random walk for forecasting future spot rates. Their results showed that none of the economic models outperformed the random walk. This rather devastating result brings forth the question of whether the exchange rate is an asset price and therefore behaves analogously to other asset prices such as stocks. The idea here, put forward by Frenkel and Mussa (1980) in the "asset market theory" of the exchange rate, is that the rate can be viewed as a highly sensitive asset price which is immediately affected by the arrival of new information or news. If this is the case then the Efficient Market Hypothesis (EMH) suggests that instead of regressing the exchange rate on actual asset stocks one should regress unanticipated exchange rate movements on unanticipated movements or news in asset markets. Several empirical studies have been carried out to test whether the monetary models perform better when analysed in a news framework, but as yet only mixed results have been produced.

The question remains then of whether the foreign exchange market in fact behaves consistently with the EMH. Fama (1970) defines an efficient market as one which "fully reflects" all relevant information instantly. Thus abnormal profits are not possible. The

EMH is a joint hypothesis of risk neutrality and rationality[2]. It should be noted however that randomness in prices is neither a necessary nor sufficient condition for an efficient market (Levich (1979)). For example, a market can exhibit serial correlation in prices and returns due to a changing equilibrium, but still be efficient.

The main conclusioı .rom the large body of empirical work on the EMH is that the joint hypothesis must be rejected. The existence of a risk premium which could explain this when risk aversion is present, is not conclusively proven either and what evidence there is suggests that even if it is present it is extremely small. More recently a new explanation for this rejection has focussed on the rational expectation rather than the risk neutrality aspect of the hypothesis. The existence of rational bubbles (and the so called Peso Problem) have been put forward to explain the apparent irrational behaviour of speculators. Examples of these situations are the extreme movements prior to anticipated revaluations (such as in the EMS). The Peso Problem itself refers to the persistent non zero forward premium on the Mexican Peso versus US Dollar exchange rate prior to the devaluation of the Peso in 1976, even though the exchange rate was fixed. Some success has been achieved in proving that bubbles exist in exchange rate data.

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2 Risk neutrality implies that;

$$S_t^e = f_{t-1}$$

where S_t^e is the expected spot rate at time t and f_{t-1} is the forward rate at time t-1. Rationality implies that;

$$S_t = S_t^e + u_t$$

where S_t is the spot rate at time t, $S_t^e = E(S_t \mid I_{t-1})$ ie the expected spot rate give the information set at time t-1 and u_t is white noise and $E(u_t \mid I_{t-1}) = 0$. The EMH can then be written as;

$$S_t = f_{t-1} + u_t$$

Running in parallel to the empirical work on testing the economic models of exchange rate determination, a series of tests have been carried out by several researchers into the distribution of exchange rate changes. The concern here is that in studies of the EMH for example, distribution-dependant measures of dispersion such as the variance of exchange rate changes, are typically used. If the actual distribution of price changes is non-normal then an alternative measure of dispersion may be appropriate. Furthermore the economic models are often tested using regression analysis. Thus a non-normal distribution for exchange rate changes may lead to non-normal regression disturbances so requiring alternative techniques and hypothesis tests. Finally, and in relation to our particular research, knowledge of the actual distribution of exchange rates is important for Mean Variance or Portfolio models where the assumption of normality is often made and for option pricing models based on the standard Black and Scholes (1973) or Garman and Kohlhagen (1983) model.

Some of this previous work on testing the statistical distribution of exchange rates is surveyed in more detail in the next section which is followed by two studies which we have carried out into the distribution of both spot and forward rates using data from the 1980's.

2.3 Previous Work on the Distribution of Exchange Rates

Early work on studying the statistical distribution of exchange rates was carried out by Giddy and Dufey (1975). Using daily data on the CAD, FRF, and GBP rates versus the USD in the 1920's and 1970's up to 1974 they found evidence for non-normality and intertemporal changes in variances. They also reported that the distributions had longer tails than those of the normal and some degree of leptokurtosis.

Confirmation of leptokurtosis was provided by Burt, Kaen and Booth (1977) in their analysis of the daily spot rates of GBP, DEM and CAD versus the USD from 1973 to 1975. Measuring exchange rate movements as the discrete percentage change in the rate, they performed both parametric and non-parametric time series tests from which they found that serial independence could be accepted for the GBP and DEM but not for the CAD. This was explained as either the result of central bank intervention or the institutional nature of the CAD market. They erroneously concluded however that this implied that the GBP and DEM markets were efficient (see Levich (1979)).

The first major work on exchange rate distributions however was done by Westerfield (1977) using weekly spot and forward rate data from both fixed and floating periods for five currencies versus the USD (CAD,GBP,DEM,CHF and NLG). In common with the research done on stock and bond markets, the measure of exchange rate change was the natural logarithm of the ratio between successive observations. This measure overcomes the problem of non-stationarity in exchange rate levels and also eliminates the units problem. This approach has been adopted by all future studies except that of Calderon-Rossell and Ben-Horim (1982) who use percentage changes in the same way as Burt et al. For small changes the two measures should be virtually the same.

Westerfield's results showed that most of the distributions were approximately symmetric but showed a significant degree of excess kurtosis. An alternative distribution, the stable Paretian, which could account for these fat tails, was proposed. This distribution is defined by the log of its characteristic function:

$$\log_e \Phi_x(t) = \log_e\left[\int_{-\infty}^{\infty} e^{itx} dF(x)\right] = i\delta t - \gamma|t|^{\alpha} \qquad 2.1$$

where x is the random variable, t is any real number and i is $\sqrt{-1}$. Symmetric stable distributions have three parameters: δ a location parameter, γ a dispersion parameter, and α the characteristic exponent. The characteristic exponent measures the height of the tails of the density function and varies from $0 < \alpha < 2$. When α is 2 the distribution is normal and when α is 0 the distribution is Cauchy.

The Fama-Roll (1968,1971) technique was used to estimate the parameters of the stable Paretian distribution for each exchange rate in periods of fixed and floating. She found that the characteristic exponent was less than 2 for every series, in both regimes, and observed that it was relatively stable when estimated using non-overlapping sums of weekly observations. Her conclusion was that this alternative distribution provided a better fit than the normal and that the underlying distribution was probably stable rather than unstable such as the Student.

A second examination of the same data set was carried out by Rogalski and Vinso (1978). Following on from similar work by Blattberg and Gonedes (1974) in examining stock prices, they tested for an alternative explanation of the fat-tails by analysing the performance of the Student distribution against that of the stable Paretian. The Student distribution is defined by the density function:

$$f(x \mid \delta, c, d) = \frac{\sqrt{c}\, d^{\left(\frac{1}{2}\right)d}}{B\left(\frac{1}{2}, \frac{1}{2}d\right)} [d + c(x-\delta)^2]^{-\frac{1}{2}(d+1)} \tag{2.2}$$

where x is a random number, B(*,*) is the beta function, δ is the location parameter, c is the scale parameter and d > 0 is the number of degrees of freedom. The first two moments exist as long as d > 2. The beta function is defined as;

$$B(m,n) = \int_0^1 t^{m-1}(1-t)^{n-1} dt \tag{2.3}$$

Using log likelihood functions they found that the Student provided a better fit during the floating period. They also confirmed that exchange rate changes were serially uncorrelated implying independence in the weekly returns ie a random walk.

Another examination of the stable Paretian was carried out by Islam (1982). Using daily data from 1973 to 1981 for the DEM versus USD spot rate he found that the characteristic exponent was again less than 2 but converged towards 2 as the interval between successive observations was increased from one day to a quarter. Contrary to Westerfield these results suggested that the underlying distribution was unstable.

Further evidence for instability was provided by Calderon-Rossell and Ben-Horim (1982). They examined daily spot rates for fourteen major currencies from 1974 to 1977. Their analysis showed that different currencies tended to display different distributional characteristics but could be grouped together into roughly similar types. Some evidence for skewness was reported and instability in the distributional parameters was also found. Using non-parametric tests they found that the rejection of iid was due to changing means in nine currencies and changing skewness in another four. In examining the stable Paretian distribution they found that nine of the currencies had characteristic exponents of around 1.5 with a relatively high degree of confidence. However they concluded that no unique distribution represented the behaviour of all the currencies they studied although nine of them could be characterised by a symmetric distribution with non-stationary mean and most likely following a symmetric stable Paretian model with a characteristic exponent close to 1.5.

MacFarland, Pettit and Sung (1982) examined the possibility that day of the week effects may be present in the market which might give rise to the leptokurtosis. Using chi-square goodness-of-fit tests on daily spot and forward rate data on seven currencies from 1975 to 1979, they found that the stable Paretian was rejected, although the estimated characteristic exponent was less than 2. Splitting their data into days of the week they then estimated the stable Paretian parameters separately and found that they were all below 2 but varied form one day of the week to the next. They concluded that the underlying distribution may be made up of a mixture of stable Paretians with different parameters. In a comment on this paper by So (1987) however it was suggested that the skewness of the distributions may have effected their results, which were based on the Fama-Roll technique which assumes a symmetric distribution. An asymmetric stable Paretian distribution was put forward as a better approximation.

In a further study on the nature of the observed leptokurtosis, Friedman and Vandersteel (1982) examined the stability of the characteristic exponent. Using daily data on nine currencies from 1974 to 1979 they again recorded the existence of moderate skewness and significant excess kurtosis. By randomising their data before taking sums they then tested for the alternatives of a mixture of distributions and a normal distribution with time varying parameters. Their results suggested that time varying parameters were a more likely explanation. They also tested for autocorrelation, concluding that no significant serial dependence existed, thus supporting previous evidence for a random walk.

The first study of intra-day exchange rates was carried out by Wasserfallen and Zimmermann (1985). Using every price quoted by a Swiss dealer (Union Bank of Switzerland) during a selected nine days from 1978 to 1980 for the CHF versus USD, they were able to analyse minute by minute changes. By examining the effect of lengthening the time interval over which changes were calculated they found that the normal distribution was rapidly approach in some cases although some significant skewness was also reported. Differences between days were found and they concluded that a single stable distribution was unlikely to be a correct description of the underlying distribution. A possible explanation, that of time varying parameters as previously suggested by Friedman and Vandersteel and others, was put forward. Time series analysis revealed that intra-day changes were adequately described by a random walk, although over some short time intervals some significant, unstable, autocorrelations were found.

Boothe and Glassman (1987) attempted to fit three alternative non-normal, as well as the normal, distributions to daily exchange rate changes for the GBP, CAD, DEM and JPY against the USD, using data from 1973 to 1984. Using the likelihood ratio approach and chi-square goodness-of-fit tests they found that lengthening the time interval from daily to quarterly improved the normal approximation (in agreement with Islam). For the daily changes however a Student or mixture of normals provided the best fit. They also found some evidence for time varying parameters and suggested that this may have been an important influence on the results.

Further evidence of instability in the distributional parameters was found by Tucker and Scott (1987). Their data consisted of daily spot rates for GBP, CAD, DEM, FRF, JPY and CHF versus the USD from 1980 to 1984. The problem of excess kurtosis was examined by using a model which related the variance to the level of the exchange rate (the constant elasticity of variance (CEV) model). They found that for most of the currencies a significant positive association between return variance and value existed when maximum likelihood tests were used. These coefficients however were unstable over time so that they concluded that both stable lognormal and CEV models were inadequate descriptions of the underlying distribution and proposed instead a process by which the variance would change randomly through time.

Hseih (1988a) confirmed the rejection of the stable distribution hypothesis in favour of one in which the parameters are time dependant in tests of daily GBP, CAD, DEM, JPY and CHF spot rates from 1974 to 1983. In testing directly for iid by subdividing the series into different periods, he found that the hypothesis was rejected irrespective of where the data was split. He then tested for whether the rejection was due to serial dependence or non identical distributions. In autocorrelation tests however little serial correlation was found. This once again confirmed the random walk. Testing for day of the week effects, as MacFarland et al had done, also showed little significant results as did tests for changing means. Changing variances however, and changing means and variances together did provide some significant results in explaining the rejection of iid however he concluded that this was not sufficient to fully explain the observed lepto-kurtosis.

The latest research on exchange rate changes follows on from these results. Hseih (1988b) used new tests developed by Brock, Dechert and Scheinkman (1987) to test for non-linear dependence. These results suggested that non-linear dependence was present in the same data set and that it was multiplicative rather than additive. This means that the non-linearity enters through the variance rather than the mean of the process and so it is essentially the general form of conditional heteroskedasticity as in Engle (1982). Tests of a generalised autoregressive conditional heteroskedasticity (GARCH) model were found to explain a large part of the non-linearity in all five currencies.

In summary, the empirical studies to date have developed from early work which recognised the departure from normality, in particular because of a high degree of excess kurtosis. This was followed by a series of studies proposing alternative stable distributions, ie Stable Paretian and mixtures of normals, or unstable distributions with time varying parameters. This later line of work appears to have been most successful so the emphasis of current research has been on analysing the form of this instability. Randomness in the variance of the underlying process has been examined in greater detail and new tests for non-linear, perhaps even chaotic behaviour, have begun to be used.

In the following two sections some empirical work is presented which attempts to add to this previous work by examining both spot and forward data over more recent periods. In the second set of tests in particular, some of the recent issues regarding the source of the instability in exchange rate distributions are examined along the lines of the work by Hseih (1988b).

2.4 Tests of Weekly Spot and Forward Exchange Rates 1981 to 1986

2.4.1 Data and Methodology

This section presents the results of a series of tests on weekly spot and forward rates for eight currencies from December 1981 to December 1986. The currencies used were GBP, DEM, CHF, FRF, JPY, NLG, CAD and ITL all against the USD. The data consisted of weekly closing spot rates (offer side) and eurodeposit rates for each currency and the USD for periods of 1, 3, 6 and 12 months (see Figures 1 to 17). From these rates the forward rates and forward points were calculated using the arbitrage equation (eqn 2.4) below. Overall there were 40 time series (8 currencies and 5 maturities for each) each of 263 observations giving a total of 10,520 observations.

From each of the time series for spot rates, a series of weekly returns was calculated as the change in rate over the week divided by the final rate (as in Calderon-Rossell and Ben-Horim). For the forward rates a new approach to measuring the return in the forward market was used. First a series of outright rates was calculated using;

$$F(t) = S\left[\frac{1 + r_f\left(\frac{t}{B}\right)}{1 + r_d\left(\frac{t}{360}\right)}\right] \qquad 2.4$$

where F(t) is the forward rate for maturity t days, S is the spot rate (in terms of foreign currency per USD), r_f is the foreign currency eurodeposit rate (where 10 % is 0.10), r_d is the USD eurodeposit rate and B is the interest rate day basis for the foreign currency (360 for all currencies except GBP which is 365). This calculation is consistent with the approach adopted by market makers in deriving forward prices for maturities under one year.

From the outright forward rate F(t) the forward discount or premium (also known as the forward swap points), SP(t), was calculated from,

$$SP(t) = F(t) - S \tag{2.5}$$

From the series of forward points a series of returns was then calculated by taking the difference between the opening and closing points and dividing by the spot rate. The rationale behind the use of this measure was, firstly, to enable the distribution of forward returns to be examined as essentially the change in the interest rate differential. If one looks at the outright rate (as in Westerfield) the results would be expected to be virtually the same as for the spot rate since the variability of the spot rate is many time larger in absolute terms than that of the forward points. Thus changes in the forward outright rate are almost identical to those of the spot rate and hence the distributional characteristics are the same. Here we have tried to isolate the effect of changes in the interest differential between the two currencies by looking at the forward points alone. Secondly, the actual calculation is in fact analogous to a common transaction in the forward market known as a forward swap. This transaction involves the simultaneous purchase and sale of one currency for another for delivery on different dates in the future. A notional swap would be,

At time t(0)	Buy +1 USD	Sell -S(0) Currency	SPOT
	Sell -1 USD	Buy +F(0) Currency	FWD

This transaction is then unwound one period later by a reverse transaction ie,

At time t(1)	Sell -1 USD	Buy +S(1) Currency	SPOT
	Buy +1 USD	Sell -F(1) Currency	FWD

The return on these transactions is then given by[3],

$$USD\,ForwardReturn = \left[\frac{S(1)-S(0)+F(0)-F(1)}{S(1)}\right] = \left[\frac{SP(0)-SP(1)}{S(1)}\right] \qquad 2.6$$

In the analysis of the Portfolio model in Chapter 3 the significance of knowledge about the distribution of swap returns is demonstrated.

2.4.2 Results

To begin with, a visual examination of the spot rates shows that the period being studied was characterised by an initial steady rise in the value of the USD up to about March 1985 followed by a steep decline over the remaining period. In particular Figures 1,2,3,4,6 and 8 display very similar patterns. This might be expected since the DEM, FRF, NLG, and ITL were all members of the exchange rate mechanism of the EMS during this period. Thus the cross rates, for example the DEM/FRF rate, were confined within maximum fluctuation limits and so all of these currencies tend to move in approximately the same way relative to currencies outside the system ie the USD. The CHF and GBP also follow similar patterns, probably due to the close ties between these economies and the other European countries. Over the entire period the USD finished weaker than it began against all these currencies and so some skewness might be expected in the distribution of exchange rate returns.

The two non-European currencies, the JPY and CAD follow different patterns. The JPY (Fig 5) traded within a fairly narrow range for most of the period up to about August 1985 after which it strengthened significantly against the USD with the rate falling by nearly 50%. Over this period its behaviour appears more like that of the European

3 Strictly the forward dates should coincide and the nominal amounts of USD should have the same net present value. Some tests using more complex formulas however gave essentially the same results (since differences are being taken this is not surprising) and so the simpler formula shown above was used.

currencies. The CAD (Fig 7) on the other hand displays a steady weakness against the USD throughout the entire period so that one might expect to find some positive skewness in the distribution of returns. The range of movement however appears to be less than that of the other currencies which may be due to the close links between the Canadian and US economies.

The eurodeposit rates, shown in Figures 9 to 17, show a much less consistent pattern of behaviour across currencies. In general all of the currencies ended the period with lower rates than they began with the sharpest falls tending to be in the period before about March 1983 (except for the JPY (Fig 13)). After this the remaining period was fairly stable expect for the GBP which remained volatile, and some rather dramatic spikes in both the FRF and ITL rates (Figs 12 and 16) which we discuss later. The JPY rates also show a sharp upward and then downward move after August 1985 which coincides with the strong fall in the JPY/USD exchange rate. These rate cuts can be seen as attempts to control the rise in the JPY on the foreign exchanges. In terms of our analysis, the main observation is that the differentials between rates tended to narrow over the entire period as interest rates converged, particularly within the EMS currencies, although this convergence was fairly unsteady.

The first test examined whether each series of returns showed any significant serial dependence. Spearman rank autocorrelation coefficients were calculated using a one week lag. The results are shown in Table 2.1 (results using the product moment measure are qualitatively the same) from which it can be seen that in general the data showed little evidence for significant autocorrelation. None of the values for the spot rates were significant at a 5% level and only one of the thirty two forward rates showed any significant difference from zero at this confidence level. Roughly half (21 out of 40) of the autocorrelations are negative.

From these results it would appear that both the spot and forward markets follow a random walk (in agreement with previous research) at least for weekly intervals.

The next two tests examined the higher moments of the distributions; skewness and kurtosis. For a symmetrical distribution the measure of skewness should be zero and for a distribution of the same peakedness as the normal distribution the measure of excess kurtosis should also be zero. Table 2.2 shows the skewness results, in which it can be seen that 33 of the 40 sample distributions are significantly skewed at the 5% level including all the spot rates. Note that the most skewed rate is the CAD as we anticipated. These results are in agreement with some of the later work by Wasserfallen and Zimmermann who also found significant skewness which had sometimes been overlooked in earlier work. This would suggest that tests for symmetric distributions, particularly those based on the Fama-Roll technique, should not be used on this data.

The FRF 6 and 12 month forward distributions showed by far the most dramatic skewness. Examination of the data revealed a large increase in these rates to levels of 30% and more in February and March 1983 (see Figure 12). These events in fact coincided with a realignment of the European Monetary System (EMS). This realignment had been widely anticipated in the markets and an expectation of a devaluation of the FRF had led to massive speculation. The FRF was being borrowed to be sold short on the foreign exchanges thus forcing up the eurodeposit rates. Once the devaluation had occurred the rates fell back to previous levels of about 15%. A similar pattern emerged in ITL rates at the same time although the increase was not as extreme, and an opposite effect of a drop in DEM rates occurred as large speculative flows went into Germany in anticipation of an upward revaluation.

A further realignment of the EMS in January 1986 produced similar effects although to a lesser degree since the authorities now had some experience in handling these situations.

Of those which showed significant skewness, no consistent pattern emerged except that in the spot rates the decline in the value of the USD was confirmed by negative skewness in all the spot rates except the CAD, against which the USD strengthened. Overall 20 distributions were positively skewed and 13 negatively. These results confirm some of the earlier work which also found significant skewness in spot rates, and points to a similar problem in the forward market too.

The results of the tests for kurtosis are shown in Table 2.3. For all the series significant excess kurtosis at the 5% level was found. Once again the effect of the EMS realignment can be seen in the FRF and ITL results which show the largest kurtosis, especially in the forward points. The forward points distributions seem in general to be more kurtotic than the spot rates, except for the JPY and to a lesser extent the CAD. Perhaps the nature of interest rate movements which tend to be characterised by periods of relative stability interspersed with jumps when official rates are moved, gives an intuitive explanation for these results.

The results confirm the findings of all the previous work on spot rates, that exchange rate returns, even over a weekly interval, are significantly leptokurtotic. The new results for the forward market also provide some evidence for excess kurtosis in this market too.

A final test on this data set was carried out in order to examine the stability of the parameters of the spot rate distributions alone. If we assume, as in many financial models, that the data is from a single normal distribution then the only explanation for the observed kurtosis would be time varying parameters. A simple test for changes in the variance is to compare the variance of a particular period calculated directly from the data, with an estimate calculated from the same data but for a different interval which, has been scaled. In this test we calculate the actual weekly standard deviation and then estimate it from the actual monthly, quarterly, semi-annual and annual standard deviations. This is done by multiplying the standard deviation by one over the square root of the number of weeks in the interval ie;

$$\tilde{\sigma}_w = \sigma_T \sqrt{\frac{7}{T}} \qquad 2.7$$

where $\tilde{\sigma}_w$ is an estimate of the true sample weekly standard deviation σ_w, σ_T is the sample standard deviation for a T day interval. If the distribution is normal with constant variance then the actual and estimated values for the weekly volatility should be the same[4].

The results of this test are shown in Table 2.4. As can be seen, for all the currencies except the CAD, the estimates are higher than the actual standard deviation and generally increase the longer the measuring interval. This would suggest that there is some time dependence in the variance of the spot rate process or perhaps the distribution is made up of a mixture of distributions. This hypothesis however was rejected in favour of time varying parameters by Friedman and Vandersteel.

This particular result has some important implications for currency option traders. In this market estimates of historical longer term volatility are typically made by calculating daily volatility and annualising by multiplying by the square root of the number of (working) days in a year. The results here would suggest that this calculation would systematically underestimate actual historic annual volatility by as much as 7%

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4 The spot rate is assumed to follow an ito process of the form;

$$dS = \mu S\,dt + \sigma S\,dz$$

where dz is a Weiner process ie $dz = \varepsilon\sqrt{dt}$ and ε is a random sample from a standardised normal distribution.

in the case of the NLG one year volatility estimated form one week volatility. Thus long term options may be undervalued if the pricing is based on these estimates of long term volatility.

2.4.3 Conclusion

The conclusion from this first set of tests is that both spot and forward markets follow approximately random walks, for weekly intervals, but that their distributions tend to be both asymmetric and highly kurtotic, especially for the forward returns. In addition some evidence for instability in the variance of the spot rate distributions was found. In order to examine in more detail the nature of this instability a second series of tests on daily spot rate data were carried out. These tests are described in the next section.

2.5 Tests of Daily Spot Exchange Rates 1983 to 1988

2.5.1 Data and Methodology

In this section the results of tests on the stability of the distribution of eight spot exchange rates are presented. The data consisted of six years of daily spot rates from 4th January 1983 to 23rd December 1988 taken from the Financial Times (mid rates). These rates actually come from the Reuters monitor (page FXFX) at 5 pm London time and so originate from a variety of Banks who supply rates to this system. The currencies used are the GBP, DEM, CHF, FRF, JPY, NLG, CAD and ITL all against the USD and expressed as currency units per USD. Each series consisted of 1514 observations giving a total of 12,112 pieces of data altogether. The raw data can be seen in Figures 18 to 25.

For each exchange rate a series of daily returns was calculated as the natural logarithm of the ratio of successive observations ie

$$R_t = \mathrm{Ln}\left[\frac{S_t}{S_{t-1}}\right] * 100 \qquad 2.8$$

where R_t is the return for time t, and S_t, S_{t-1} are the spot rates for time t and t-1. This approach is consistent with nearly all the previous work on exchange rate distributions in particular Hseih (1988a and 1988b). All the following tests were carried out on these series of daily returns.

Initially summary statistics are calculated for each currency and the presence of leptokurtosis is confirmed. Tests on the time series of each rate are then carried out by examining the linear and non-linear dependence of the daily returns. These provide some evidence to support the view that significant non-linear dependence may be present. Finally tests are done to examine the nature of the observed non-linear dependence and some conclusions presented.

2.5.2 Results

As in the first study, inspection of the spot rate data series' show that for all of the currencies the period under examination was characterised by an initial strengthening of the USD up to about the beginning of 1985 followed by long decline over the remaining time. For the CAD this decline however began at the start of 1986 but was rather steeper (in relative terms). As before the European currencies, including those outside the EMS, display very similar patterns whereas the JPY and CAD are slightly different. For the JPY (Fig 22) however, although it remains fairly stable initially, as the decline in the USD continues its movements tend to more closely follow the European currencies. The CAD (Fig 24) on the other hand remains fairly unique in its movements with, for example, a fairly sharp fall in the rate beginning in late 1986 which was not present in the other data.

The first series of tests calculated summary statistics for each currency followed by a chi-square goodness-of-fit test for the normal distribution. The results, shown in Table 2.5, show that for all the currencies the mean return was approximately zero although all have a slight negative drift due to the overall depreciation in the USD. All the distributions are unimodal and all but the CAD and ITL are significantly skewed at the 95% level. All have a high degree of kurtosis relative to the normal as with previous studies.

The results of the chi-square goodness-of-fit test confirm the departure from normality for all the distributions and this can be seen graphically in the histograms in Figures 26 to 33. These show the actual and expected distributions under the assumption of normality and clearly display the higher than expected number of small returns and the larger number of very high returns in the tails of the distributions characteristic of leptokurtosis.

Next, two tests for serial dependence were carried out. Firstly autocorrelation coefficients for lags up to 20 on the actual returns were calculated and these are shown in Table 2.6. None of these are greater than 0.1 which suggests that little serial dependence exists. The standard errors shown in brackets are calculated using the heteroskedasticity consistent estimator suggested by Diebold (1986)[5]. These show that only 4 of the 160 autocorrelation coefficients are significant at the 1% level. The adjusted Box-Pierce

5 Diebold suggests that $\sqrt{1/n}$ may underestimate the standard error in the presence of heteroskedasticity. The following alternative is given for the qth sample autocorrelation coefficient,

$$S(q) = \sqrt{[1/n]\left[\frac{1+V(q)}{\sigma^4}\right]}$$

where V(q) is the qth sample autocovariance of the squared data and σ is the sample standard deviation of the data. Also the adjusted Box-Pierce statistic is given by $\sum_{q=1}^{k}\left[\frac{\rho(q)}{S(q)}\right]^2$, which is asymptotically a chi-square distribution with k degrees of freedom.

statistics for 20 lags, reject independence at 1% for the GBP and DEM, at 5% for the FRF, JPY and ITL, and accept independence for the CHF, NLG and CAD. These results therefore provide mixed evidence for independence.

A second, non-parametric, test was then carried out based on the Rank von Neumann ratio[6]. This test has been shown to have power against various distributional assumptions when testing for first order autocorrelations (see Bartels (1982)). The results of these tests are shown in Table 2.7 which indicate that independence can not be rejected for any of the currencies.

The tests for linear dependence seem to generally confirm earlier work suggesting that serial dependence if present is certainly only in a weak form and in certain currencies.

We then tested for non-linear dependence by performing the same two tests as before but on the squared returns (as in Hseih (1988b)). The results are shown in Tables 2.8 and 2.9. From the first test it can be seen that many of the autocorrelation coefficients are greater than 0.1 with the largest being 0.4285 for the CAD at lag 1. Using conventional standard errors a large number of the coefficients are significant at 1% particularly for the GBP and DEM, and to a lesser extent the NLG. The Box-Pierce statistics for 20 lags are highly significant for all the currencies. It is also interesting to note that most of the significant coefficients occur in the first few lags, up to about lag 10 (except for the GBP which has significant autocorrelation up to lag 20). This would seem to indicate that the

6 The Rank von Neumann ratio is given by,

$$v = \frac{\sum_{t=1}^{n} (R_t - R_{t-1})^2}{\frac{n(n^2-1)}{12}}$$

where $R_1 \ldots R_n$ are the ranks associated with the series of returns. For large n, v is approximately distributed as N(2,4/n).

nature of the non-linear dependence is short-lived, that is, only due to prices in the recent past. Thus for forecasting purposes only recent price movements would be needed to be analysed in order to capture information about future changes.

The Rank von Neumann test on the squared returns confirms the previous evidence for non-linear dependence. Independence is rejected for all the currencies, except the GBP at 5% and all but the CHF are also rejected at 1%. The lack of rejection for the GBP is surprising given the strong evidence for non linear dependence in the previous test. Perhaps the distribution dependant nature of the previous test provides some explanation for these apparently contradictory results implying that the process for the GBP is more complex.

Overall, the results so far have suggested that linear dependence if present is only in a weak form, but non-linear dependence appears to be significant but probably only in the first few lags. We now go on to test for the type of non-linear dependence that we have found.

In Hseih (1988b) a new test has been suggested which discriminates between two types of non-linear dependence, additive and multiplicative. Additive dependence can be expressed as,

$$r_t = u_t + f(S_t, \ldots, S_{t-k}, r_t, \ldots, r_{t-k}) \qquad 2.9$$

where u_t is an independent identically distributed (iid) random variable with zero mean, and f() is an arbitrary function of the previous spot rates $S_t, \ldots, S_{t-k}$ and previous returns $r_t, \ldots, r_{t-k}$ for some k. As we have seen the value of k in practice might be around 10 for our data. In this type of non-linear dependence the non-linearity enters through the mean of the process.

Multiplicative dependence on the other hand is defined by,

$$r_t = u_t f(S_t, \ldots, S_{t-k}, r_t, \ldots, r_{t-k}) \qquad 2.10$$

and implies that the non-linearity enters through the variance of the process.

The basis of the test is that although both types of non-linearity imply autocorrelation in the squared returns, the expected value of u_t conditional on the previous spot rate returns, in the case of multiplicative returns is zero, whereas for additive dependence it is not. By assuming that f() is at least twice continuously differentiable it can be approximated by a Taylor series expansion around zero. Then multiplicative dependence implies that u_t is not correlated with the higher order terms such as $S_{t-i}S_{t-j}$, $S_{t-i}r_{t-j}$, and $r_{t-i}r_{t-j}$. The test is then carried out by examining whether the value $\rho_{rrr}(i,j) = E[r_t r_{t-i} r_{t-j}]/\sigma_r^3$ is equal to zero for all i,j > 0, for multiplicative dependence, or $\rho_{rrr}(i,j) = 0$ for some i,j > 0 for additive dependence. In practice $\rho_{rrr}(i,j)$ is estimated by,

$$Z_{rrr} = \frac{\left(\frac{1}{T}\right)\sum r_t r_{t-i} r_{t-j}}{\left[\left(\frac{1}{T}\right)\sum r_t^2\right]^{1.5}} \qquad 2.11$$

and $\sqrt{T}Z_{rrr}(i,j)$ is normally distributed with zero mean and variance $w(i,j)/\sigma_t^6$ which can be estimated by $\frac{\left(\frac{1}{T}\right)\sum r_t^2 r_{t-i}^2 r_{t-j}^2}{\left[\left(\frac{1}{T}\right)\sum r_t^2\right]^3}$.

The results of this test are shown in Table 2.10 which gives the value of Z_{rrr} for different i and j up to lag 5. Only two of these values are significantly different from zero at 1% (two tailed test), the $Z_{rrr}(2,1)$ for the JPY and the $Z_{rrr}(5,2)$ for the GBP. This evidence would tend to suggest that the dependence is multiplicative rather than additive so the nonlinearity is due to changing variance rather than changing mean which confirms the results of Hseih (1988b) on earlier data. As we mentioned, this might point to some form of GARCH process for the exchange rates. Newer work on alternative asset prices (Sheinkman and LeBaron (1988)) has suggested that the dependence in some asset prices may be a form of low dimensional deterministic chaos, which would not be inconsistent with our results.

2.5.3 Conclusions

An examination of the daily returns for eight major exchange rates against the USD has been carried out for the period 1983 to 1988. The results have shown that the distributions of returns are significantly leptokurtotic and non-normal for all currencies and unstable over time. Although there is mixed evidence for linear dependence, non-linear dependence, particularly in the first few lags has been found to be significant. The form of this dependence has been found to be multiplicative suggesting that the instability arises from changing variance rather than changing mean, thus confirming recent work. The implications of these results and some overall conclusions on the nature of the market are presented in the final section.

2.6 Final Conclusions and Some Implications of the Empirical Work

This chapter began by describing briefly some of the models of exchange rate determination which have been proposed in the literature. Early monetary and portfolio balance models attempted to relate changes in the level of exchange rates to changes, or anticipated changes, in fundamental variables. Problems with estimating these models however, and their lack of forecasting power when compared to a simple random walk, has focussed attention onto the modelling of exchange rate behaviour as an asset price. Unfortunately, evidence for an Efficient Market has yet to be discovered although more sophisticated explanations of the rejection of the EMH have suggested that apparently irrational behaviour (rational bubbles), may be the problem. If one observes how prices actually change, at least over the short term, the lack of support for a fundamentalist view is not surprising. Most operators in the market have very short term horizons and so tend to base their trading decisions, not on fundamental economic data, but on whether they feel the market is rising or falling in the next day or so. These movements are more a reflection of the positions being held by the majority of market players, ie is the market overbought or oversold, rather than due to fundamentals. A more realistic model of the market should include both short term and long term influences in order to describe the behaviour of prices more accurately.

If the exchange rate does approximately follow a random walk as most evidence suggests, the distribution of exchange rate changes is of concern to both theoreticians and empiricists alike. Both rely on assumptions about this distribution in the theories they construct and the tests they perform on these models. The wide body of literature on the form of this distribution has however yet to provide any firm answers. In fact the more work that is done the more complex the problem appears to be.

Having rejected any form of stable distribution in favour of a process whose parameters are time varying, the consensus seems to be that the observed leptokurtosis in the distribution of exchange rate returns is probably due to some form of changing variance which may be the result of non-linear dependence or some GARCH model or may even be a form of low dimensional chaos. The fact that there may be an element of deterministic behaviour, combined perhaps with a random walk, does not make the prospects for forecasting any easier. It is almost certain that the system is far too complex and the environment too changeable for any type of forecasting other than for very short intervals, to be successful. It may be that techniques used by so called Chartists or Technical Analysts, who provide short term forecasts using support and resistance levels based on market psychology, might provide some additional insight. Some research has suggested that these techniques are not inconsistent with a more complex non-linear deterministic model.

Any conclusive explanation of exchange rate behaviour then is still some way away. Many hopeful avenues for research remain but it seems certain that those who operate in the market will have to use some form of simplifying assumption about the movement of prices in order to make any structured analysis of their exposure. Given the lack of forecasting ability, the Gaussian random walk will probably be at the heart of most of these approaches and it is therefore important to have an understanding of the inadequacies of this assumption and the particular aspects of it which deviate the most from reality. It has been the aim of this chapter to highlight these inadequacies and provide some further evidence on the underlying process.

The main implication from these results concerns the use of variance as a measure of risk. The mean-variance portfolio model or Capital Asset Pricing Model, as well as the commonly used Black and Scholes (1973) approach to option pricing, both rely on the variance of the underlying price to be measurable and independent of both the level of the price and its past behaviour, ie constant. If the variance is in fact unstable and possibly dependant upon previous price changes, then more complex models would be needed. In the following chapters we demonstrate the use of both a Portfolio and Option approach to managing exchange rate risks and present an integrated theory on exchange rate management. The models we construct however are based fundamentally on the constant variance Gaussian random walk even though we have shown it to be unrealistic. We shall attempt to point out the potential problems with this assumption in the models we construct but hope that the work presented in this chapter will provide a reminder of the actual underlying process, as it is understood so far. In fact the theory we present for managing foreign exchange exposures incorporates the inadequacies of the distributional assumption within it. By being aware of the deficiencies we show how it is possible to benefit from this knowledge by looking for opportunities which may arise, for example in mispricing, or in the specification of the models being used. A unified approach must look not only at all the instruments and techniques available for managing any exposure profile but the assumptions underlying them. In this way both problems and opportunities are highlighted which have implications for the final strategy proposed.

The next chapter describes a particular approach to exchange rate exposure management based on a mean-variance portfolio model. Some tests of the model are carried out using the database of actual exchange rates used in the first empirical study presented in this chapter. Some implications of the distributional assumption are demonstrated and conclusions on the strategies suggested by the model are presented.

Table 2.1 Spearman First Order Rank Autocorrelation Coefficients for Weekly Spot and Forward Returns December 1981 to December 1986

Currency	Maturity				
	Spot	1 Mth	3 Mth	6 Mth	12 Mth
GBP	-0.0456	-0.0012	-0.0808	*-0.1421	0.0395
DEM	0.0700	-0.0195	-0.0584	-0.0326	0.0450
CHF	0.0646	0.0505	-0.1131	-0.0396	0.0451
FRF	0.0502	0.0010	-0.1056	0.1040	0.1176
JPY	0.1011	0.0659	-0.0396	0.0221	0.0754
NLG	0.0878	-0.0327	-0.1072	-0.0608	0.0156
CAD	-0.0703	-0.0243	-0.0964	-0.1208	-0.0575
ITL	0.0797	0.0419	-0.0220	-0.0837	0.1200

* marks a value outside two standard deviations

Table 2.2 Skewness Statistics for the Distributions of Weekly Spot and Forward Returns December 1981 to December 1986

Currency	Maturity				
	Spot	1 Mth	3 Mth	6 Mth	12 Mth
GBP	*-0.6170	*0.9530	*0.3180	*-0.2220	*-0.4500
DEM	*-0.1420	*0.5620	*0.3880	*0.4190	*0.3040
CHF	*-0.3810	*-0.1480	*0.4300	*0.2460	0.0990
FRF	*-0.2310	*0.1480	*0.3650	*-4.1940	*-2.7330
JPY	*-0.2720	*0.7370	*0.7590	*0.5950	*0.3850
NLG	*-0.4300	0.0010	*0.1910	-0.0300	*0.2010
CAD	*1.1520	*1.1050	*0.4650	*0.1250	0.0490
ITL	*-0.3020	-0.1050	-0.0990	-0.0820	*-0.1610

* marks a value outside two standard deviations

Table 2.3 Excess Kurtosis in the Distributions of Weekly Spot and Forward Returns December 1981 to December 1986

Currency		Maturity			
	Spot	1 Mth	3 Mth	6 Mth	12 Mth
GBP	2.23	4.42	3.67	3.98	3.26
DEM	1.33	2.97	2.74	5.22	3.43
CHF	1.17	1.57	3.03	4.69	2.32
FRF	2.96	10.50	20.50	44.30	25.40
JPY	6.62	3.71	5.09	5.01	3.39
NLG	1.00	3.92	2.20	3.30	2.18
CAD	5.05	6.40	3.86	3.61	2.86
ITL	1.07	16.40	4.25	4.15	4.70

ALL values outside two standard deviations

Table 2.4 Actual and Estimated Standard Deviations of Weekly Spot Rate Returns December 1981 to December 1986

Currency	Actual	Estimate Intervals			
	1 Wk	1 Mth	3 Mth	6 Mth	12 Mth
GBP	0.0167	0.0167	0.0170	0.0193	0.0210
DEM	0.0186	0.0189	0.0178	0.0208	0.0254
CHF	0.0197	0.0204	0.0195	0.0225	0.0261
FRF	0.0188	0.0190	0.0185	0.0218	0.0258
JPY	0.0175	0.0200	0.0191	0.0211	0.0239
NLG	0.0164	0.0171	0.0173	0.0212	0.0269
CAD	0.0055	0.0055	0.0048	0.0044	0.0039
ITL	0.0156	0.0162	0.0163	0.0194	0.0241

Table 2.5 Summary Statistics for Daily Spot Returns 4th January 1983 to 23rd December 1988

	GBP	DEM	CHF	FRF	JPY	NLG	CAD	ITL
mean	-0.0068	-0.0188	-0.0186	-0.0065	-0.0401	-0.0174	-0.0018	-0.0028
Median	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0179
Std. Dev.	0.7169	0.7013	0.7841	0.7054	0.6001	0.7067	0.2969	0.6805
Skewness	-0.4387	-0.2497	-0.4213	-0.1420	-0.2844	-0.2746	0.0082	-0.1121
	[0.0630]	[0.0630]	[0.0630]	[0.0630]	[0.0630]	[0.0630]	[0.0630]	[0.0630]
Kurtosis	6.4452	5.2608	8.4645	6.5818	6.7334	5.3376	17.2066	7.0288
	[0.1259]	[0.1259]	[0.1259]	[0.1259]	[0.1259]	[0.1259]	[0.1259]	[0.1259]
Maximum	2.9365	3.0013	5.2747	4.6652	3.3263	2.7780	2.6030	4.7659
Minimum	-4.6674	-4.0361	-6.2321	-3.9071	-3.5404	-4.1050	-2.5726	-3.9242
Studentised range	10.607	10.304	14.675	12.153	11.422	9.739	17.430	12.771
Chi-square	168.9	164.73	155.48	201.93	270.22	172.97	1393.83	202.12
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Standard errors are in brackets for skewness and kurtosis given by $\sqrt{6/NOBS}$ and $\sqrt{24/NOBS}$ respectively. Significance level for chi-square test also in brackets.
Note : Number of intervals, k, for chi-square test is 70 using $k = 3.7653(n-1)^{2/5}$ from Mandansky (1988) page 47.

Table 2.6 Autocorrelation Coefficients for Daily Returns

Lag	GBP	DEM	CHF	FRF	JPY	NLG
1	0.0898a	0.0439	-0.0062	0.0295	0.0518	0.0221
	[0.0280]	[0.0268]	[0.0256]	[0.0265]	[0.0275]	[0.0263]
2	0.0059	-0.0079	0.0104	0.0053	0.0408	0.0013
	[0.0259]	[0.0255]	[0.0259]	[0.0258]	[0.0271]	[0.0257]
3	-0.0009	0.0655	0.0521	0.0455	0.0653	0.0413
	[0.0257]	[0.0274]	[0.0268]	[0.0269]	[0.0279]	[0.0268]
4	-0.0635a	-0.0471	-0.0379	-0.0417	-0.0143	-0.0287
	[0.0240]	[0.0244]	[0.0249]	[0.0246]	[0.0252]	[0.0250]
5	0.0248	0.0315	0.0037	0.0123	0.0033	0.0388
	[0.0263]	[0.0265]	[0.0258]	[0.0260]	[0.0258]	[0.0267]
6	0.0365	0.0256	0.0478	0.0243	0.0380	0.0180
	[0.0266]	[0.0264]	[0.0267]	[0.0263]	[0.0270]	[0.0262]
7	-0.0110	0.0093	-0.0023	0.0307	0.0231	0.0226
	[0.0254]	[0.0259]	[0.0257]	[0.0265]	[0.0265]	[0.0263]
8	0.0531	0.0577	0.0681	0.0555	0.0508	0.0569
	[0.0271]	[0.0272]	[0.0271]	[0.0271]	[0.0275]	[0.0271]
9	0.0174	0.0262	-0.0224	0.0075	0.0347	0.0265
	[0.0262]	[0.0264]	[0.0252]	[0.0259]	[0.0269]	[0.0264]
10	-0.0389	-0.0324	-0.0360	-0.0534	-0.0249	-0.0394
	[0.0247]	[0.0248]	[0.0249]	[0.0243]	[0.0248]	[0.0247]
11	0.0003	-0.0122	-0.0108	-0.0299	0.0424	-0.0168
	[0.0257]	[0.0254]	[0.0255]	[0.0249]	[0.0272]	[0.0253]
12	-0.0541	-0.0382	-0.0350	-0.0210	-0.0154	-0.0289
	[0.0243]	[0.0247]	[0.0250]	[0.0252]	[0.0252]	[0.0250]
13	0.0231	0.0089	0.0205	0.0076	-0.0101	0.0133
	[0.0263]	[0.0259]	[0.0261]	[0.0259]	[0.0253]	[0.0260]
14	0.0337	0.0575	0.0335	0.0392	0.0153	0.0524
	[0.0266]	[0.0272]	[0.0264]	[0.0267]	[0.0262]	[0.0270]
15	0.0243	-0.0130	0.0083	0.0090	0.0166	0.0034
	[0.0263]	[0.0254]	[0.0259]	[0.0259]	[0.0263]	[0.0258]
16	-0.0057	-0.0137	0.0062	-0.0408	-0.0183	-0.0145
	[0.0253]	[0.0253]	[0.0258]	[0.0246]	[0.0250]	[0.0253]
17	-0.0101	-0.0140	-0.0562	-0.0328	-0.0508	-0.0421
	[0.0254]	[0.0246]	[0.0245]	[0.0248]	[0.0238]	[0.0246]
18	-0.0461	-0.0524	-0.0012	-0.0310	0.0028	-0.0443
	[0.0245]	[0.0243]	[0.0257]	[0.0249]	[0.0258]	[0.0245]
19	0.0298	0.0324	0.0217	0.0404	0.0340	0.0337
	[0.0265]	[0.0265]	[0.0262]	[0.0267]	[0.0269]	[0.0266]
20	0.0524	0.0318	0.0001	0.0256	-0.0268	0.0039
	[0.0270]	[0.0265]	[0.0257]	[0.0264]	[0.0247]	[0.0258]
Adjusted	44.09	40.02	29.20	32.30	31.99	29.67
Box-Pierce	[0.0019]	[0.0050]	[0.0869]	[0.0419]	[0.0448]	[0.0790]

Adjusted standard errors and significance levels for Box-Pierce statistics in brackets
a denotes significance at 1%

Table 2.6 Autocorrelation Coefficients for Daily Returns (Continued)

Lag	CAD	ITL
1	-0.0568a	0.0067
	[0.0153]	[0.0259]
2	-0.0004	-0.0107
	[0.0257]	[0.0254]
3	-0.0082	0.0255
	[0.0245]	[0.0264]
4	-0.0153	-0.0403
	[0.0234]	[0.0246]
5	-0.0087	0.0333
	[0.0244]	[0.0266]
6	-0.0037	0.0200
	[0.0252]	[0.0263]
7	0.0386	0.0096
	[0.0308]	[0.0260]
8	-0.0145	0.0716a
	[0.0235]	[0.0276]
9	0.0245	-0.0009
	[0.0291]	[0.0257]
10	-0.0263	-0.0410
	[0.0215]	[0.0245]
11	-0.0371	-0.0269
	[0.0196]	[0.0250]
12	-0.0228	-0.0094
	[0.0221]	[0.0254]
13	0.0094	-0.0090
	[0.0270]	[0.0255]
14	0.0151	0.0582
	[0.0278]	[0.0273]
15	-0.0213	0.0020
	[0.0244]	[0.0258]
16	-0.0209	-0.0337
	[0.0225]	[0.0248]
17	-0.0110	-0.0205
	[0.0244]	[0.0251]
18	-0.0087	-0.0697
	[0.0298]	[0.0237]
19	0.0305	0.0385
	[0.0279]	[0.0268]
20	0.0160	0.0251
	[0.0257]	[0.0264]
Adjusted	26.94	35.79
Box-Pierce	[0.1482]	[0.0178]

Adjusted standard errors and significance levels for Box-Pierce statistics in brackets
a denotes significance at 1%

Table 2.7 Rank von Neumann Ratios for Daily Returns

	GBP	DEM	CHF	FRF	JPY	NLG	CAD	ITL
Ratio	1.9003	1.9209	1.9818	1.9410	1.9894	1.9363	2.0091	1.9459
	[0.0514]	[0.0514]	[0.0514]	[0.0514]	[0.0514]	[0.0514]	[0.0514]	[0.0514]

Standard errors in brackets.
Note: Expected value is 2

Table 2.8 Autocorrelation Coefficients for Squared Daily Returns

Lag	GBP	DEM	CHF	FRF	JPY	NLG
1	0.0873a	0.0590	0.2707a	0.0384	0.1184a	0.0859a
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
2	0.0721a	0.0693a	0.0410	0.0856a	0.0727a	0.0392
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
3	0.0471	0.0880a	0.0641	0.0793a	0.1480a	0.0976a
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
4	0.1702a	0.1000a	0.0662	0.0718a	0.0453	0.1046a
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
5	0.0748a	0.0722a	0.0179	0.1559a	0.0592	0.0478
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
6	0.1132a	0.1317a	0.0770a	0.0849a	0.0278	0.0913a
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
7	0.0617	0.0597	0.1026a	0.0418	0.0299	0.0467
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
8	0.0940a	0.1307a	0.1464a	0.1009a	0.0939a	0.1189a
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
9	0.0781a	0.0401	0.0562	0.0132	0.0250	0.0340
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
10	0.0887a	0.0839a	0.0286	0.0463	0.0419	0.0819a
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
11	0.0887a	0.0886a	0.0425	0.0450	0.0731a	0.0697a
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
12	0.0714a	0.0641	0.0179	0.0283	0.0209	0.0318
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
13	0.0541	0.0424	0.0160	0.0159	-0.0071	0.0412
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
14	0.0704a	0.0946a	0.0558	0.0482	0.0587	0.0827a
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
15	0.0982a	0.0565	0.0514	0.0268	0.0644	0.0429
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
16	0.1006a	0.0041	-0.0005	-0.0035	-0.0078	0.0009
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
17	0.0483	0.0298	0.0334	0.0124	0.0107	0.0237
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
18	0.0809a	0.0653	0.0748a	0.0366	0.0164	0.0737a
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
19	0.0669a	0.0269	0.0390	0.0235	0.0179	0.0372
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
20	0.1232a	0.0387	0.0134	0.0322	0.0188	0.0447
	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]	[0.0257]
Adjusted	241.3	169.41	216.07	114.43	112.78	136.81
Box-Pierce	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Adjusted standard errors and significance levels for Box-Pierce statistics in brackets

a denotes significance at 1%

Table 2.8 Autocorrelation Coefficients for Squared Daily Returns (Continued)

Lag	CAD	ITL
1	0.4285a	0.1713a
	[0.0257]	[0.0257]
2	0.0666a	0.0739a
	[0.0257]	[0.0257]
3	0.0556	0.0892a
	[0.0257]	[0.0257]
4	0.0147	0.0709a
	[0.0257]	[0.0257]
5	0.0117	0.0410
	[0.0257]	[0.0257]
6	-0.0048	0.0601
	[0.0257]	[0.0257]
7	-0.0002	0.0355
	[0.0257]	[0.0257]
8	-0.0095	0.0695a
	[0.0257]	[0.0257]
9	-0.0084	0.0147
	[0.0257]	[0.0257]
10	0.0153	0.0256
	[0.0257]	[0.0257]
11	-0.0039	0.0429
	[0.0257]	[0.0257]
12	0.0005	-0.0083
	[0.0257]	[0.0257]
13	0.0000	-0.0071
	[0.0257]	[0.0257]
14	0.0019	0.0458
	[0.0257]	[0.0257]
15	0.0241	0.0364
	[0.0257]	[0.0257]
16	0.0211	-0.0040
	[0.0257]	[0.0257]
17	0.0200	0.0021
	[0.0257]	[0.0257]
18	0.0034	0.0380
	[0.0257]	[0.0257]
19	0.0007	0.0102
	[0.0257]	[0.0257]
20	-0.0097	0.0228
	[0.0257]	[0.0257]
Adjusted	73.86	94.27
Box-Pierce	[0.0000]	[0.0000]

Adjusted standard errors and significance levels for Box-Pierce statistics in brackets
a denotes significance at 1%

Table 2.9 Rank von Neumann Ratios for Squared Daily Returns

	GBP	DEM	CHF	FRF	JPY	NLG	CAD	ITL
Ratio	1.9123	1.8211	1.8926	1.7681	1.7680	1.8226	1.6447	1.8062
	[0.0514]	[0.0514]	[0.0514]	[0.0514]	[0.0514]	[0.0514]	[0.0514]	[0.0514]

Standard errors in brackets.
Note: Expected value is 2

Table 2.10 Third Order Moments of Daily Returns

Lag i	Lag j	GBP	DEM	CHF	FRF	JPY
1	1	-0.0083	-0.1264	-0.3420	-0.0490	-0.1507
		[-0.0983]	[-1.8192]	[-1.3448]	[-0.7061]	[-1.3856]
2	1	-0.0393	-0.0320	0.0268	0.0063	-0.1372a
		[-0.9136]	[-1.0040]	[0.4329]	[0.1808]	[-2.7937]
2	2	-0.1705	-0.1544	-0.2346	-0.0831	-0.2424a
		[-2.0704]	[-2.0567]	[-2.3507]	[-0.9845]	[-2.6418]
3	1	-0.0395	-0.0328	-0.0663	-0.0674	-0.1168
		[-0.9502]	[-0.9637]	[-1.5714]	[-1.9952]	[-1.8772]
3	2	-0.0370	-0.0164	-0.0537	-0.0122	-0.0842
		[-0.8142]	[-0.4032]	[-1.1629]	[-0.2942]	[-1.7469]
3	3	-0.0096	-0.0516	-0.0826	-0.0597	-0.2593
		[-0.1300]	[-0.6721]	[-0.7439]	[-0.7945]	[1.8587]
4	1	-0.0627	-0.0105	-0.1023	-0.0072	-0.0699
		[-1.4532]	[-0.2909]	[-1.1721]	[-0.2097]	[-1.4738]
4	2	0.0342	0.0254	0.0461	0.0148	0.0427
		[0.9683]	[0.7309]	[1.3204]	[0.3604]	[0.8574]
4	3	0.0244	0.0048	0.0947	-0.0169	-0.0487
		[0.5013]	[0.1332]	[1.1470]	[-0.4436]	[-1.1395]
4	4	-0.0121	0.0178	-0.0602	0.0301	-0.1363
		[-0.0909]	[0.2347]	[-0.6240]	[0.3990]	[-1.6490]
5	1	0.0506	0.0642	0.0428	0.0709	-0.0317
		[1.1964]	[1.7673]	[1.1532]	[1.9323]	[-0.5572]
5	2	0.0963a	0.0157	0.0443	0.0263	-0.0294
		[2.6836]	[0.4309]	[1.2504]	[0.6130]	[-0.6148]
5	3	-0.0278	-0.0261	-0.0585	-0.1039	-0.0424
		[-0.7401]	[-0.7162]	[-1.7531]	[-1.8079]	[-0.9524]
5	4	-0.0190	0.0123	-0.0262	0.0118	-0.0435
		[-0.3071]	[0.3373]	[-0.7394]	[0.3009]	[-1.0902]
5	5	-0.1218	-0.1582	-0.0732	-0.0185	-0.1274
		[-1.5525]	[-2.2995]	[-1.1748]	[-0.1575]	[-1.4816]

a denotes values significant at 1% (2 sided test)

Table 2.10 Third Order Moments of Daily Returns (Continued)

Lag i	Lag j	NLG	CAD	ITL
1	1	-0.0720	-0.4756	-0.2781
		[-0.9073]	[-0.9275]	[-1.9862]
2	1	0.0100	0.0585	0.0755
		[0.2961]	[0.6770]	[1.3917]
2	2	-0.1329	0.0106	-0.0843
		[-2.1425]	[0.0621]	[-1.0181]
3	1	-0.0347	-0.0482	-0.0403
		[-1.0932]	[-0.4672]	[-0.8984]
3	2	-0.0064	-0.2221	-0.0537
		[-0.1751]	[-1.5461]	[-1.1847]
3	3	-0.1261	-0.0936	-0.1085
		[-1.4617]	[-0.5506]	[-1.3282]
4	1	0.0160	-0.0298	0.0228
		[0.4190]	[-0.6184]	[0.4291]
4	2	0.0197	-0.0495	-0.0061
		[0.5826]	[-0.8141]	[-0.1555]
4	3	0.0141	0.0080	-0.0155
		[0.3234]	[0.1073]	[-0.3544]
4	4	-0.0256	-0.0658	0.0087
		[-0.3357]	[-0.7209]	[0.1075]
5	1	0.0520	-0.0520	0.0768
		[1.4493]	[-0.9331]	[2.1677]
5	2	0.0220	-0.0304	0.0039
		[0.6200]	[-0.5868]	[0.1072]
5	3	-0.0763	-0.0015	-0.0352
		[-2.1056]	[-0.0396]	[-0.9655]
5	4	0.0273	-0.0164	0.0318
		[0.7384]	[-0.2990]	[0.6973]
5	5	-0.1104	0.1505	-0.1595
		[-1.8140]	[1.7566]	[-1.8598]

a denotes values significant at 1% (2 sided test)

Figure 1 : Closing Weekly Spot Rate for GBP/USD December 1981 to December 1986

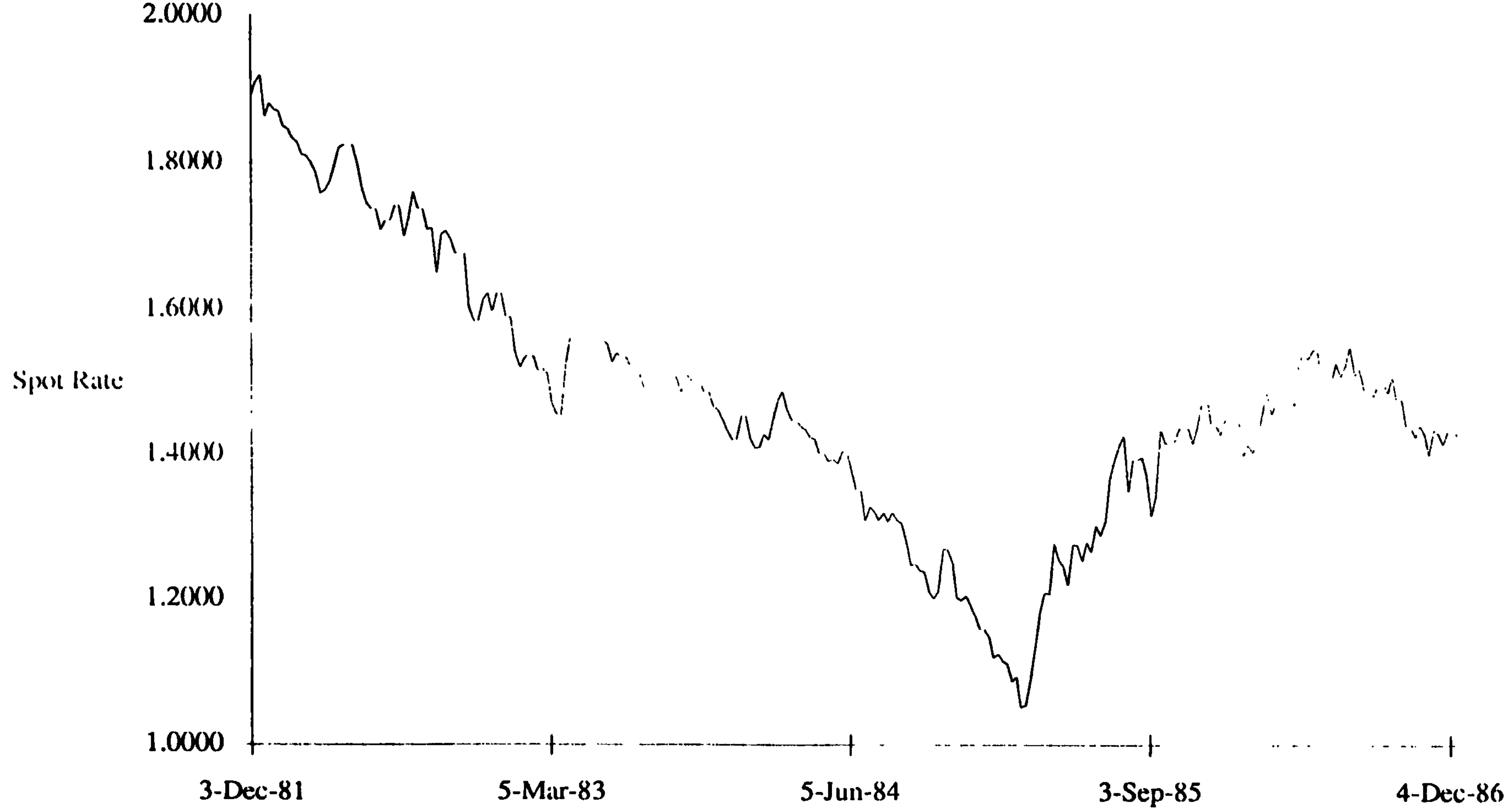


Figure 2 : Closing Weekly Spot Rate for DEM/USD December 1981 to December 1986

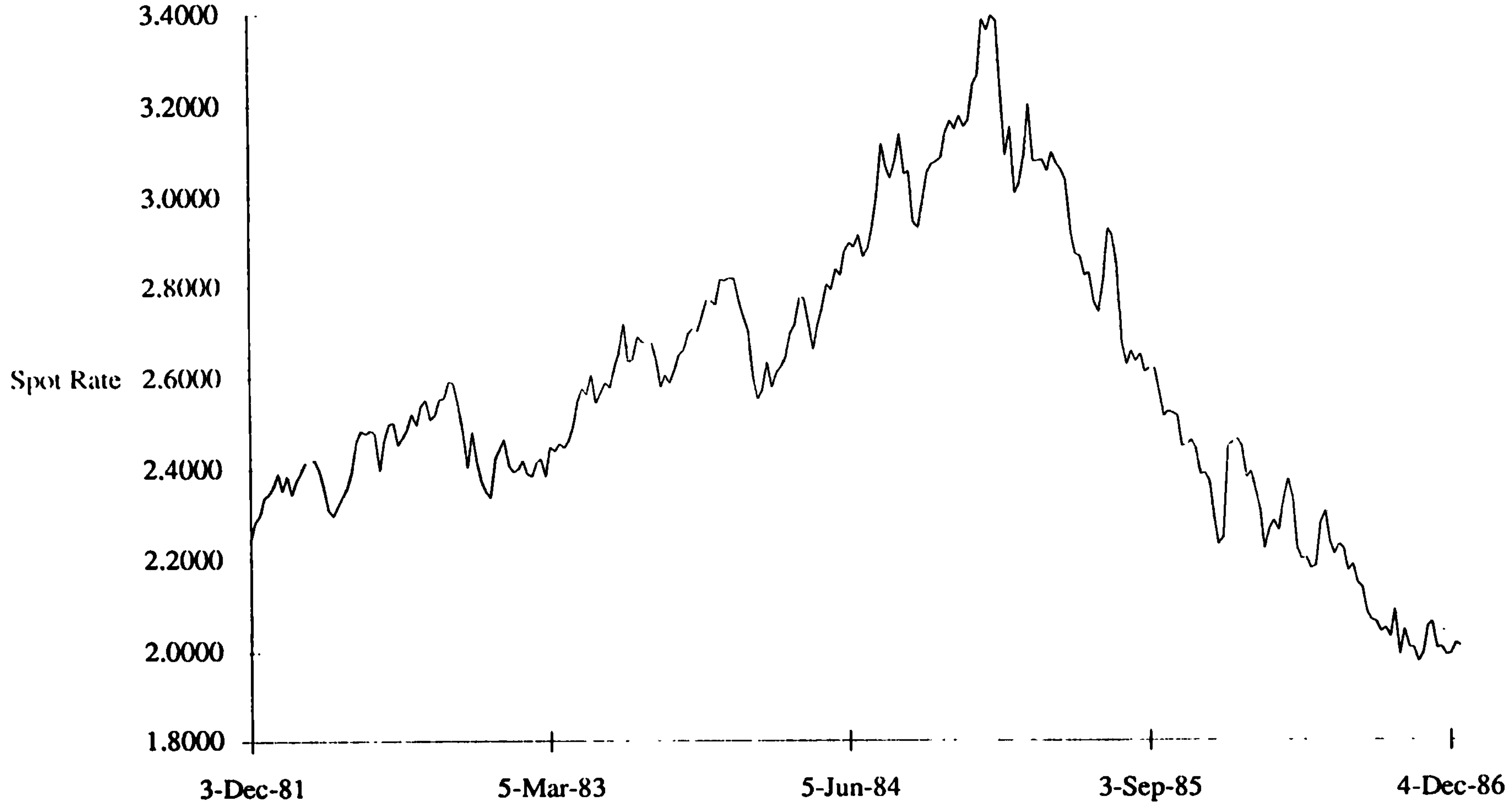


Figure 3 : Closing Weekly Spot Rate for CHF/USD December 1981 to December 1986

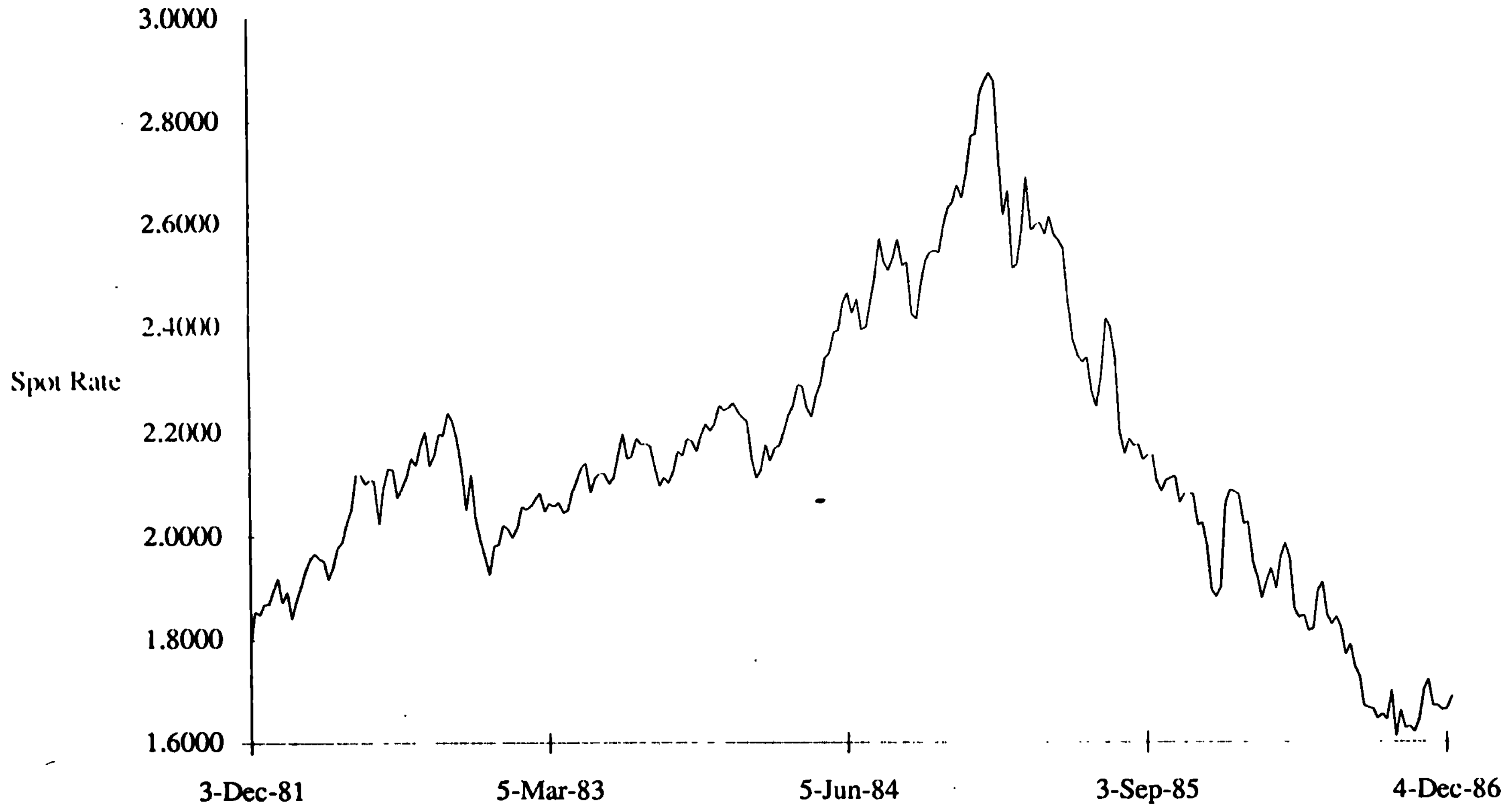


Figure 4 : Closing Weekly Spot Rate for FRF/USD December 1981 to December 1986

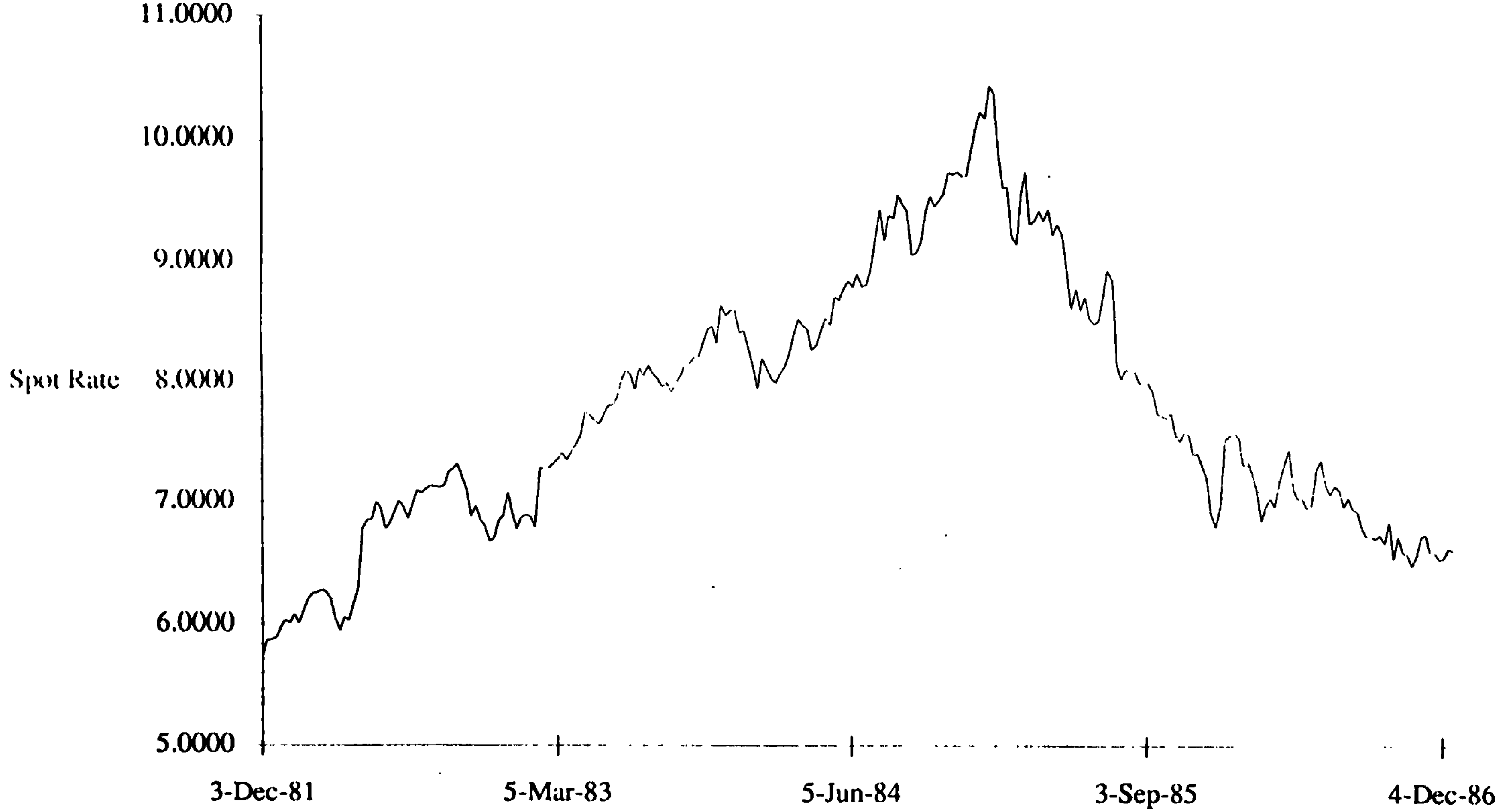


Figure 5 : Closing Weekly Spot Rate for JPY/USD December 1981 to December 1986

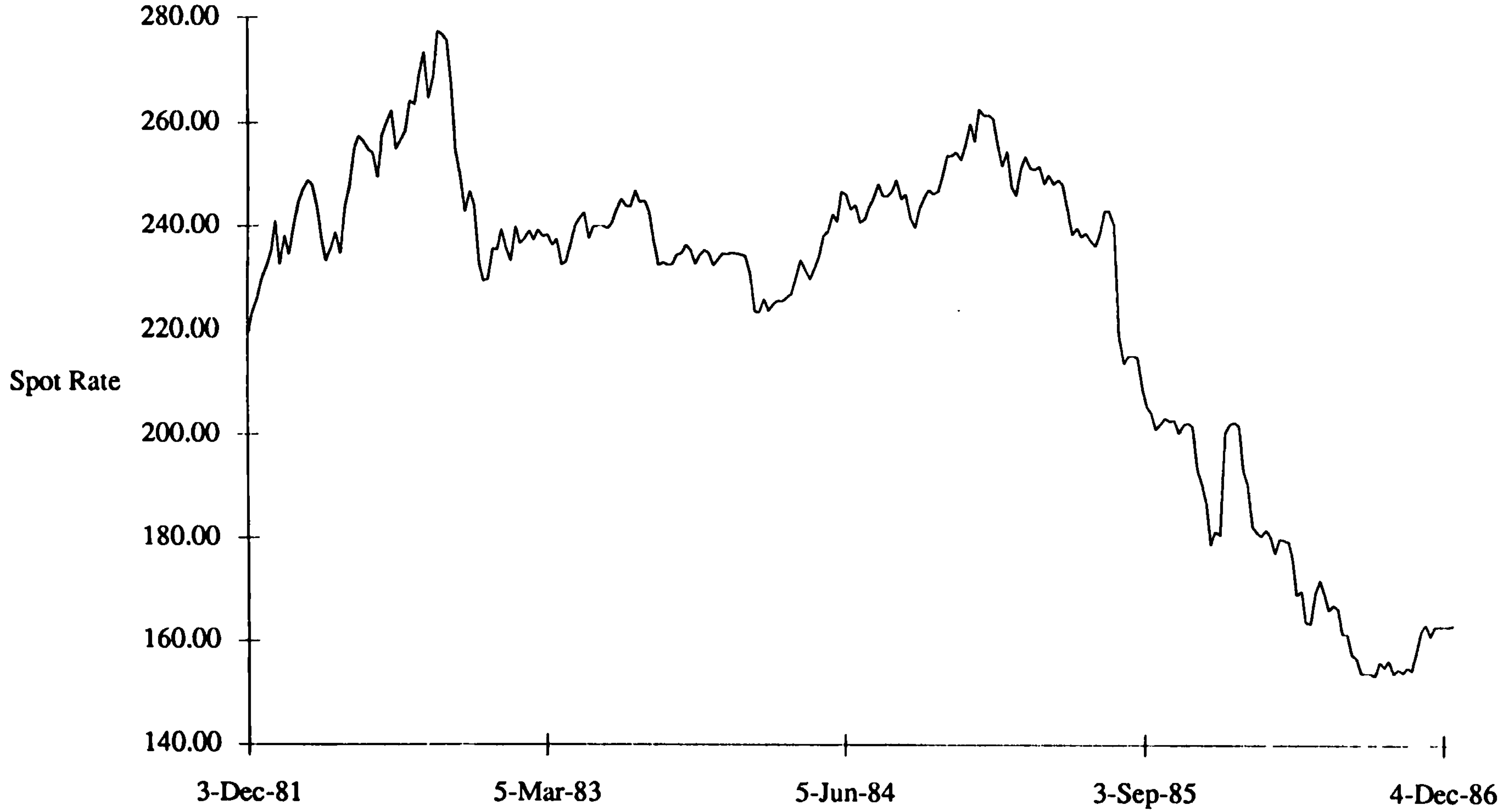


Figure 6 : Closing Weekly Spot Rate for NLG/USD December 1981 to December 1986

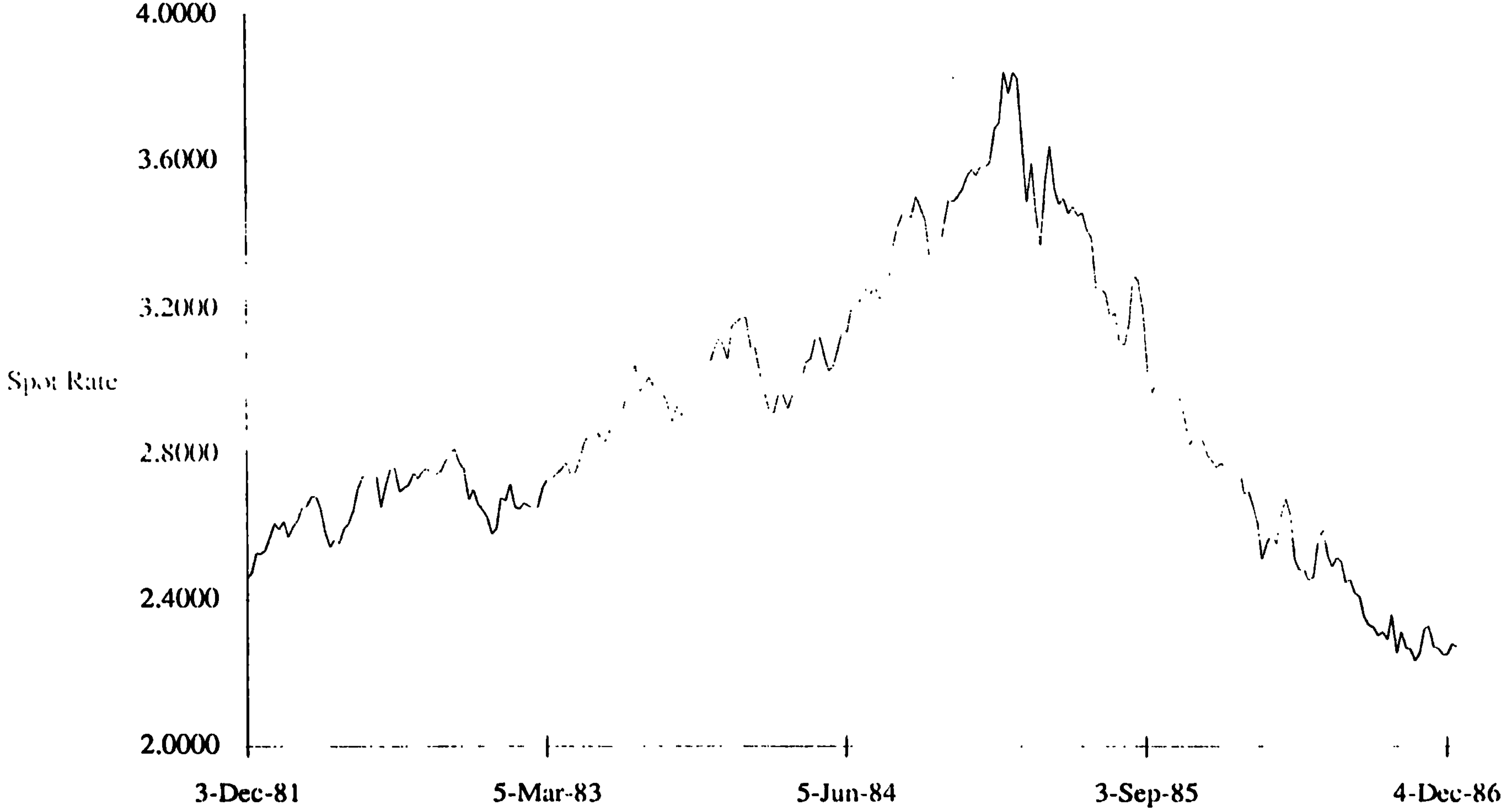


Figure 7 :Closing Weekly Spot Rate for CAD/USD December 1981 to December 1986

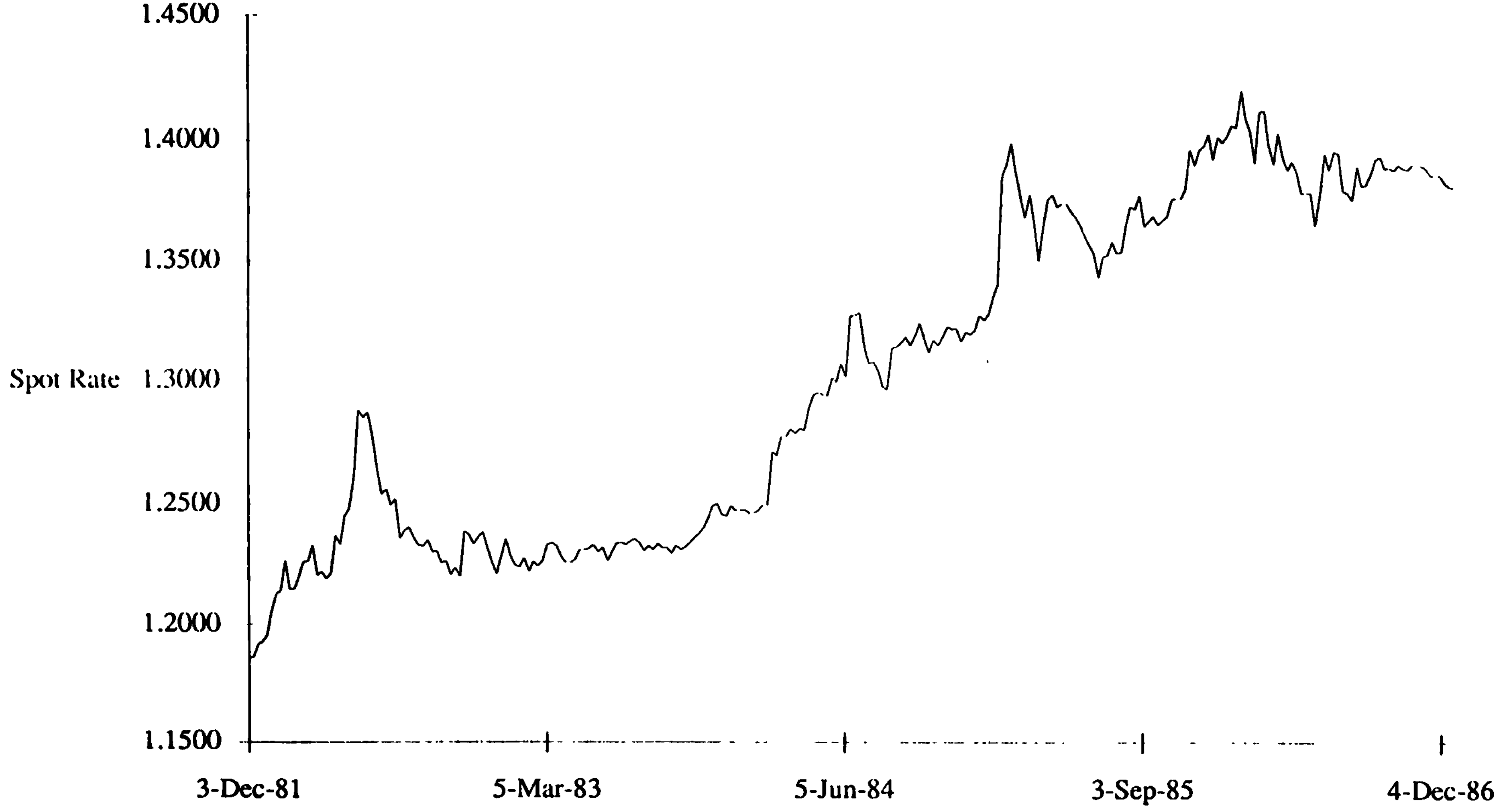


Figure 8 : Closing Weekly Spot Rate for ITL/USD December 1981 to December 1986

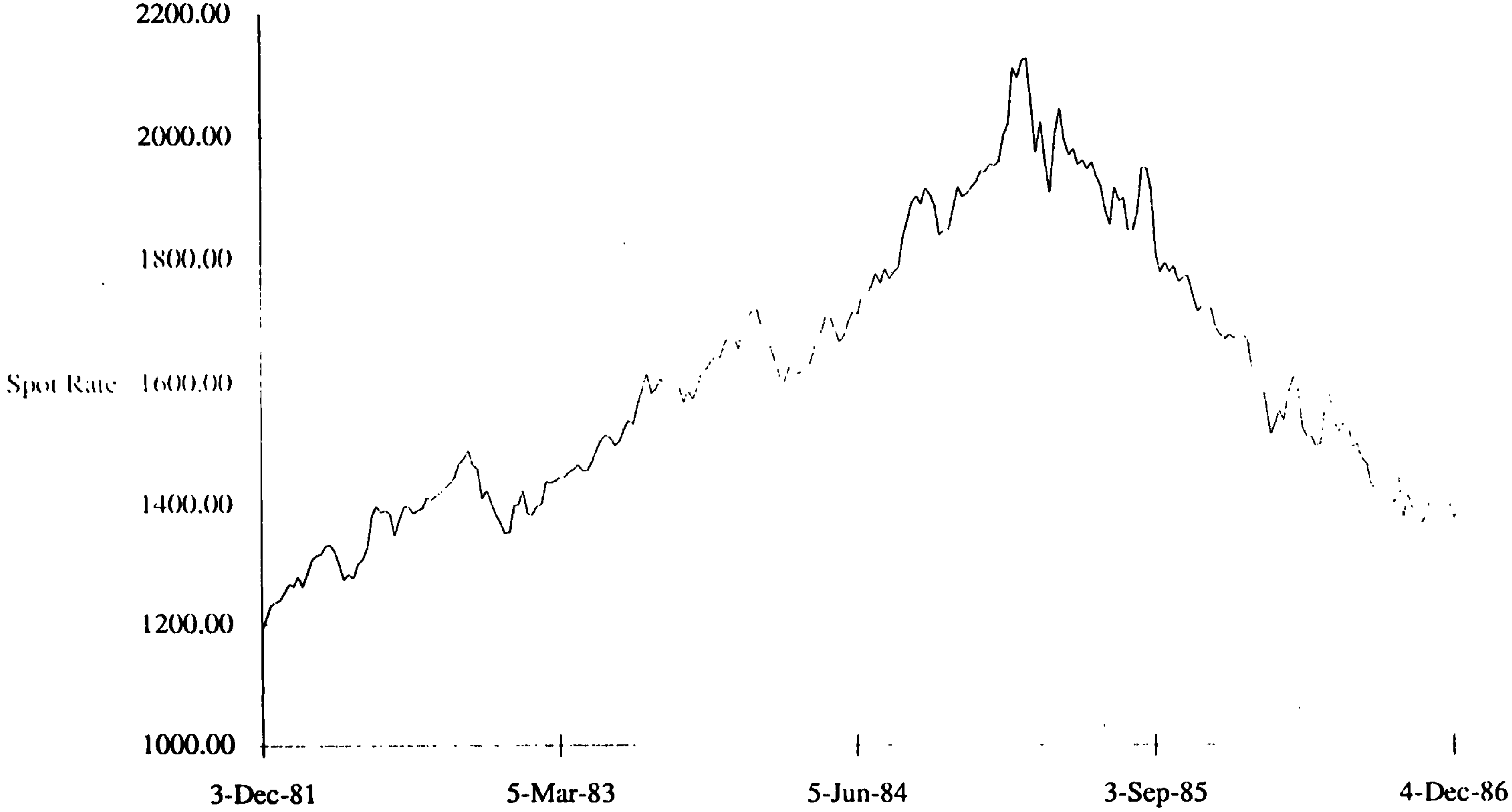


Figure 9 : Closing Weekly Eurodeposit Rates for GBP December 1981 to December 1986

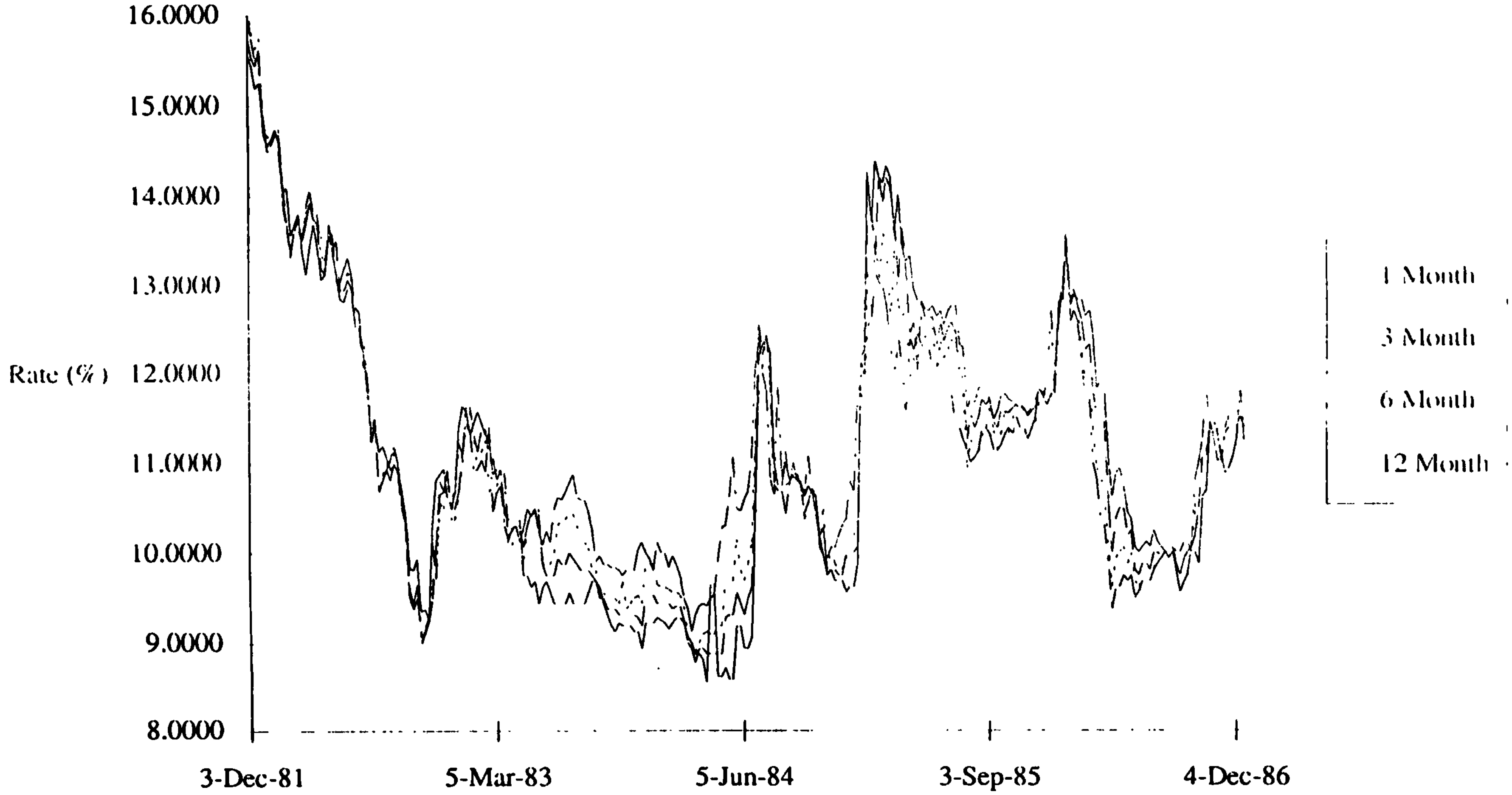


Figure 10 : Closing Weekly Eurodeposit Rates for DEM December 1981 to December 1986

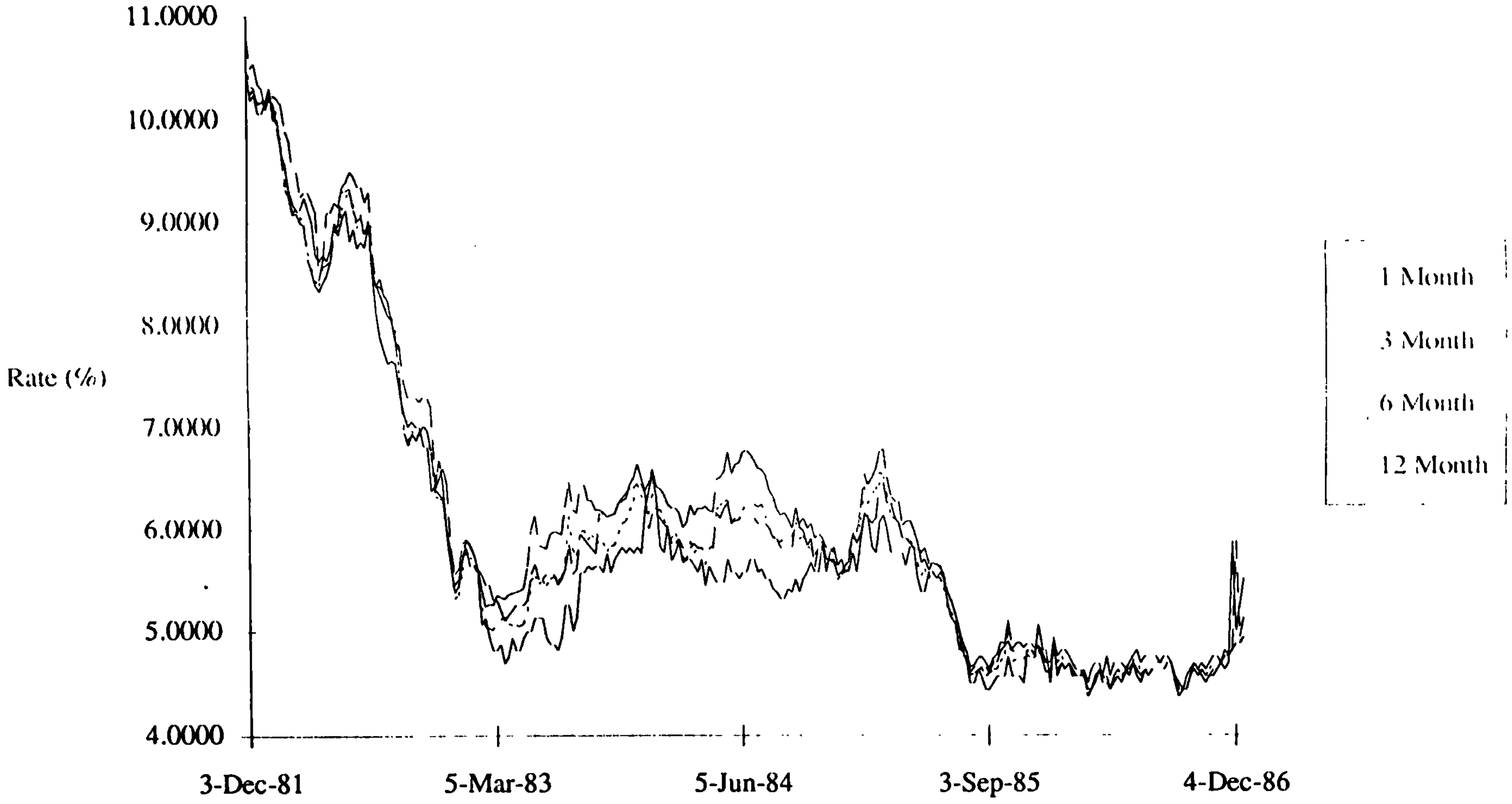


Figure 11 : Closing Weekly Eurodeposit Rates for CHF December 1981 to December 1986

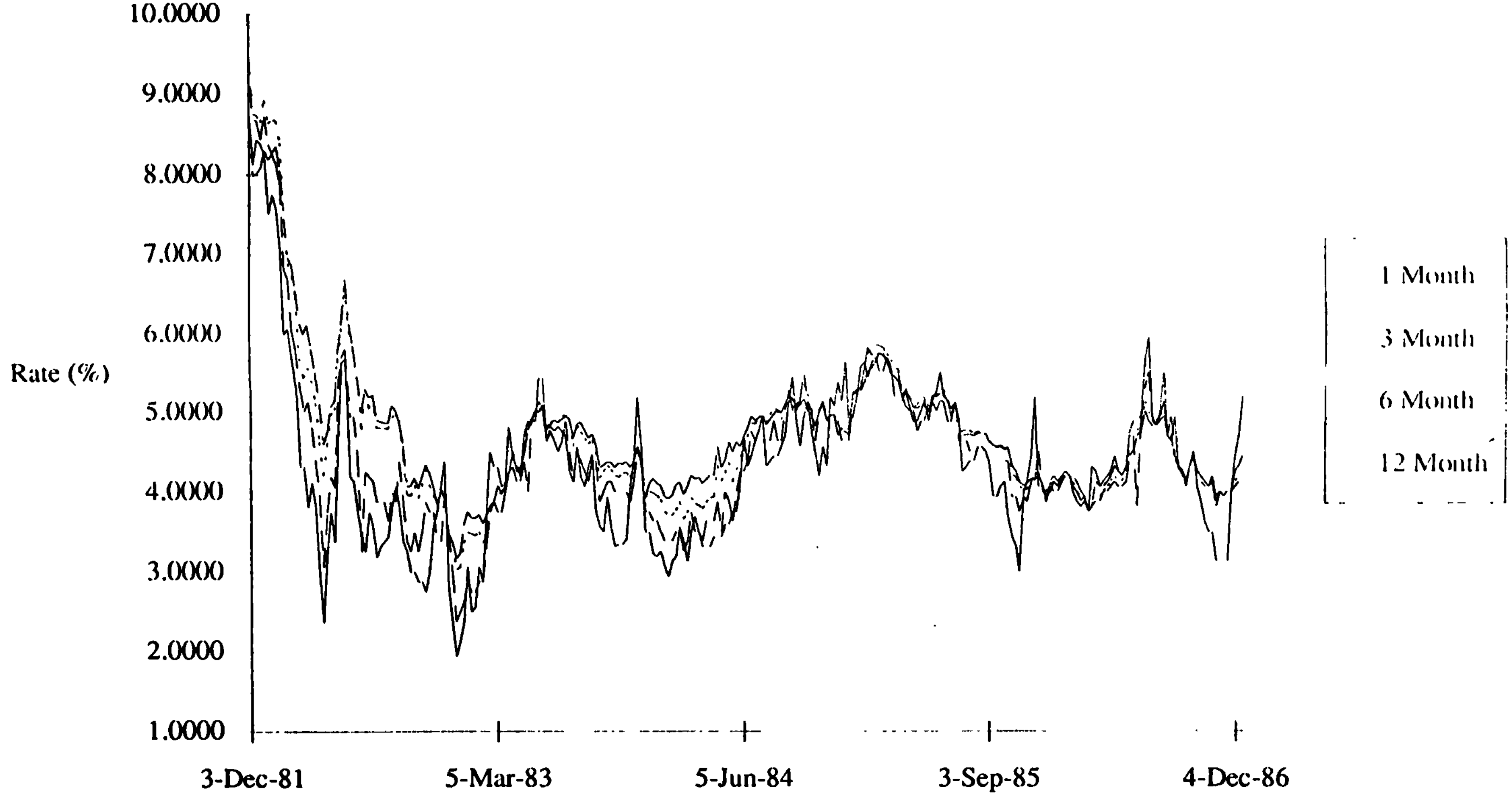


Figure 12 : Closing Weekly Eurodeposit Rates for FRF December 1981 to December 1986

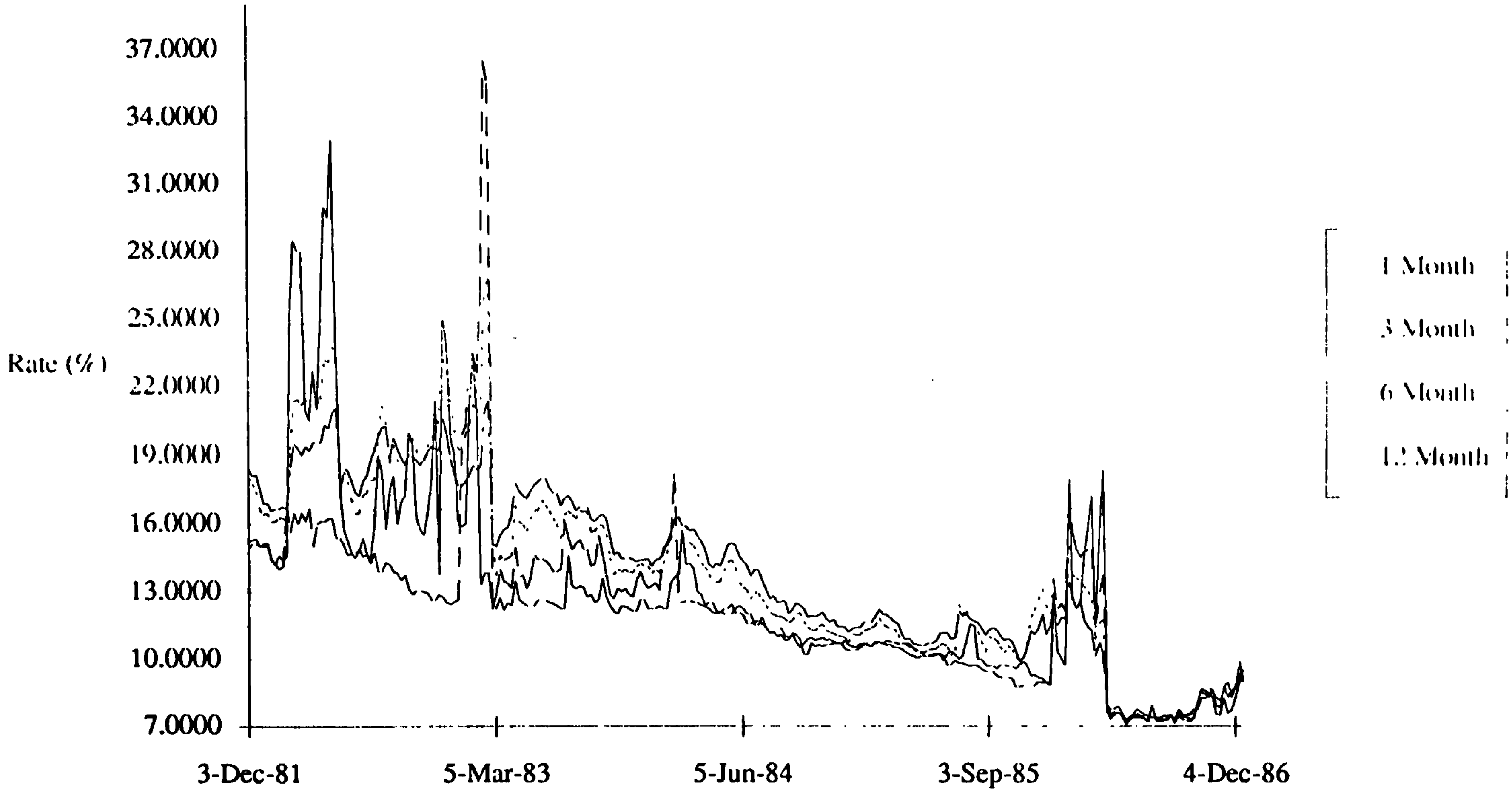


Figure 13 : Closing Weekly Eurodeposit Rates for JPY December 1981 to December 1986

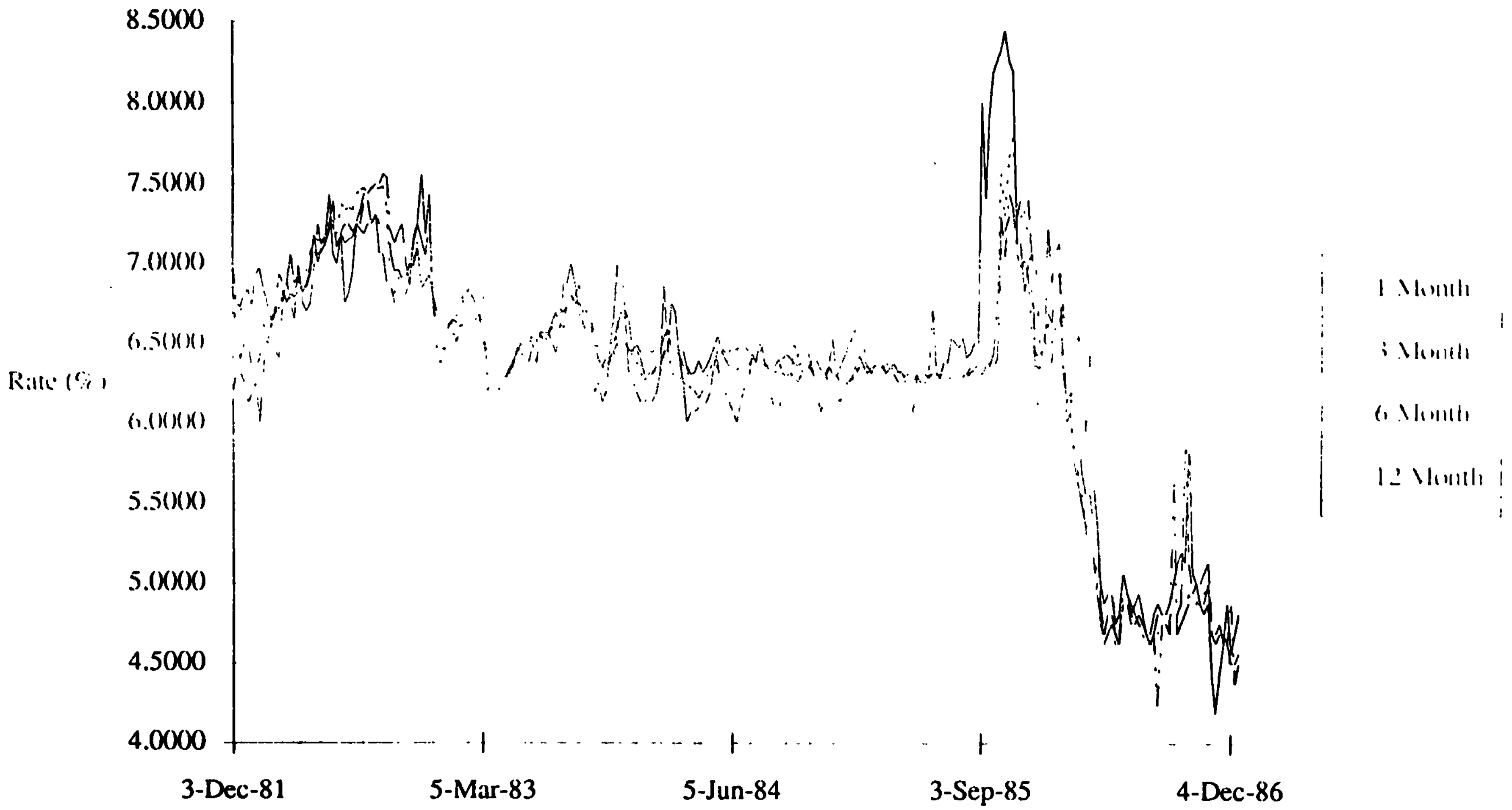


Figure 14 : Closing Weekly Eurodeposit Rates for NLG December 1981 to December 1986

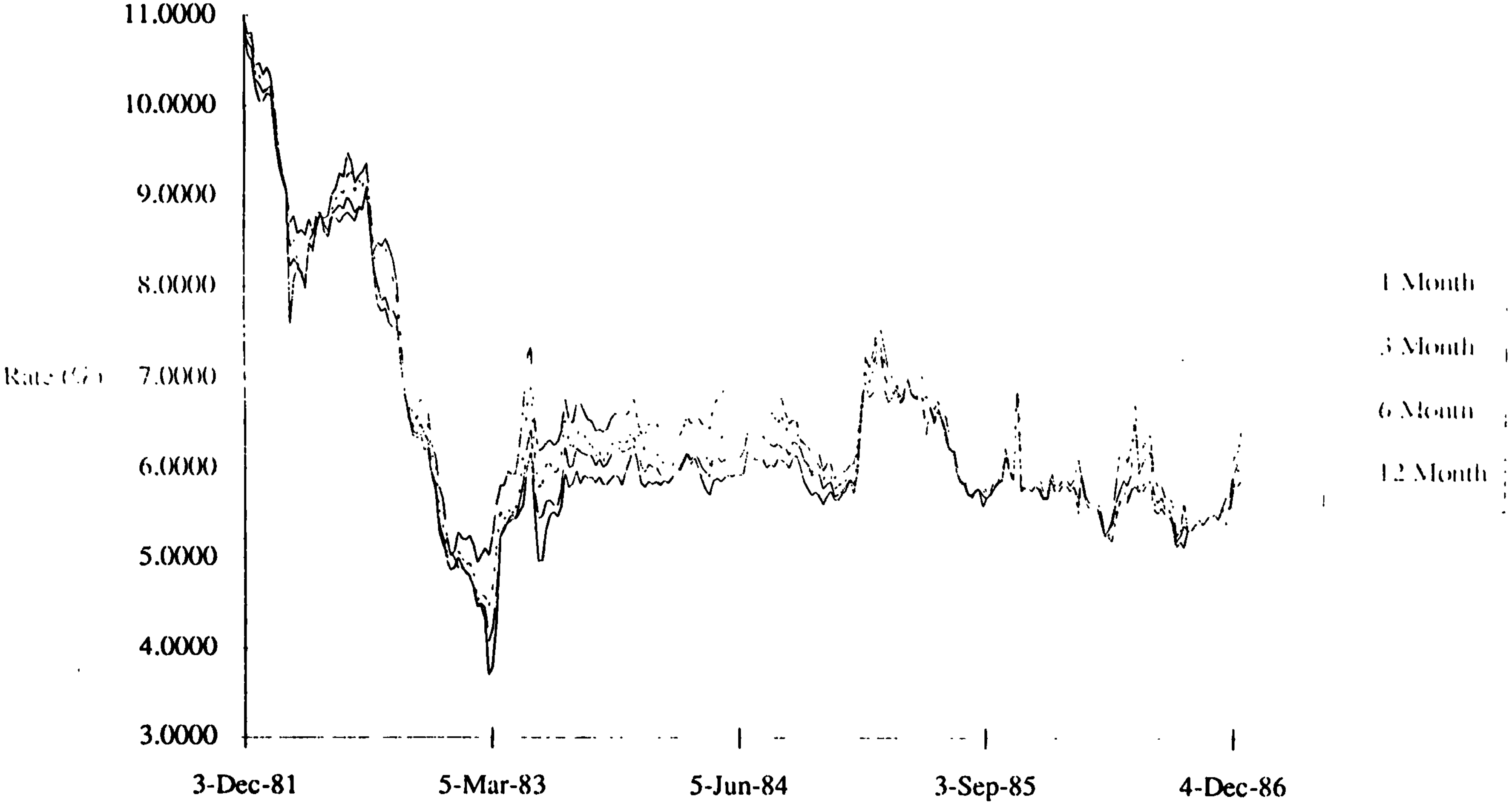


Figure 15 : Closing Weekly Eurodeposit Rates for CAD December 1981 to December 1986

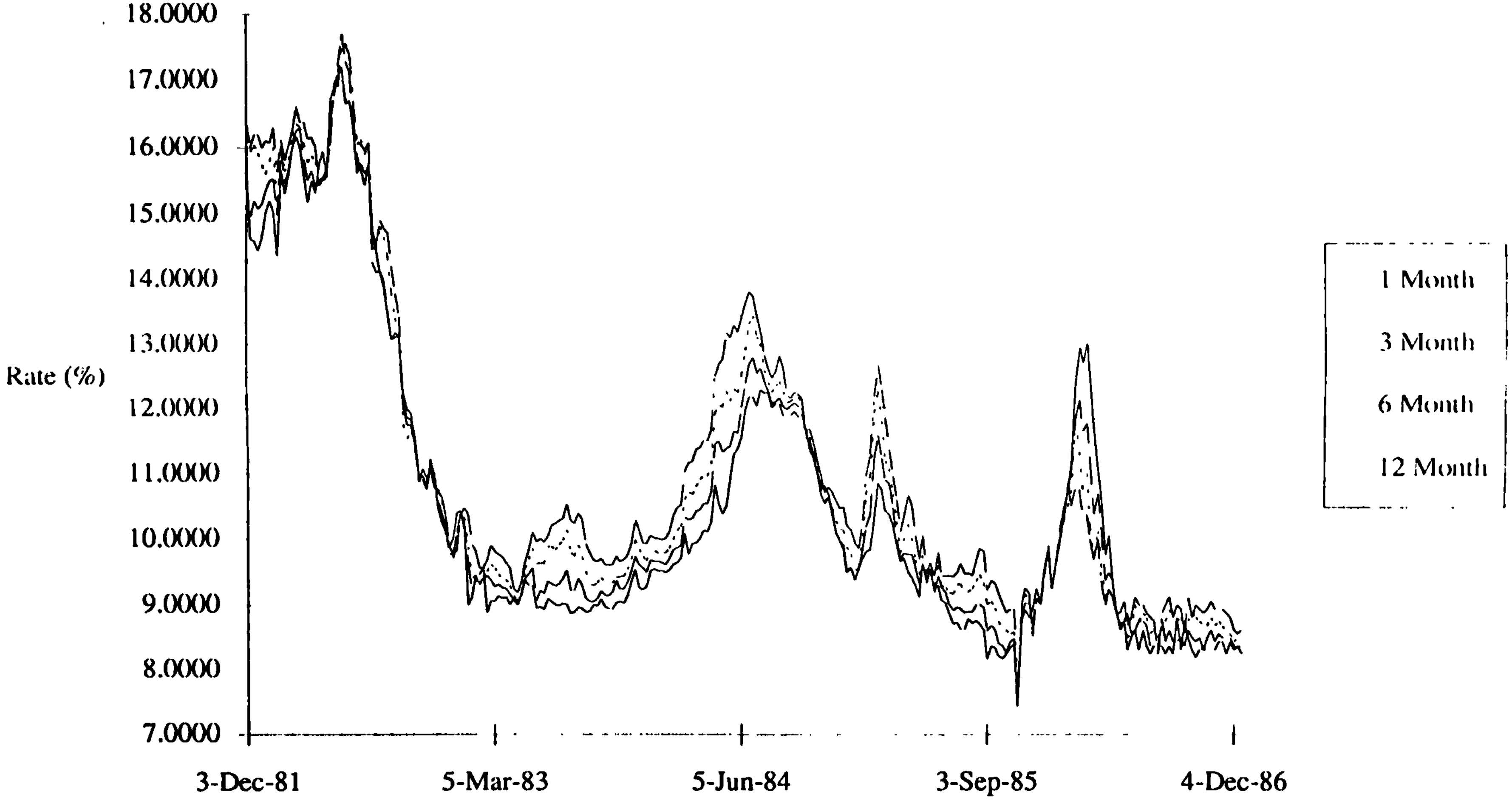


Figure 16 : Closing Weekly Eurodeposit Rates for ITL December 1981 to December 1986

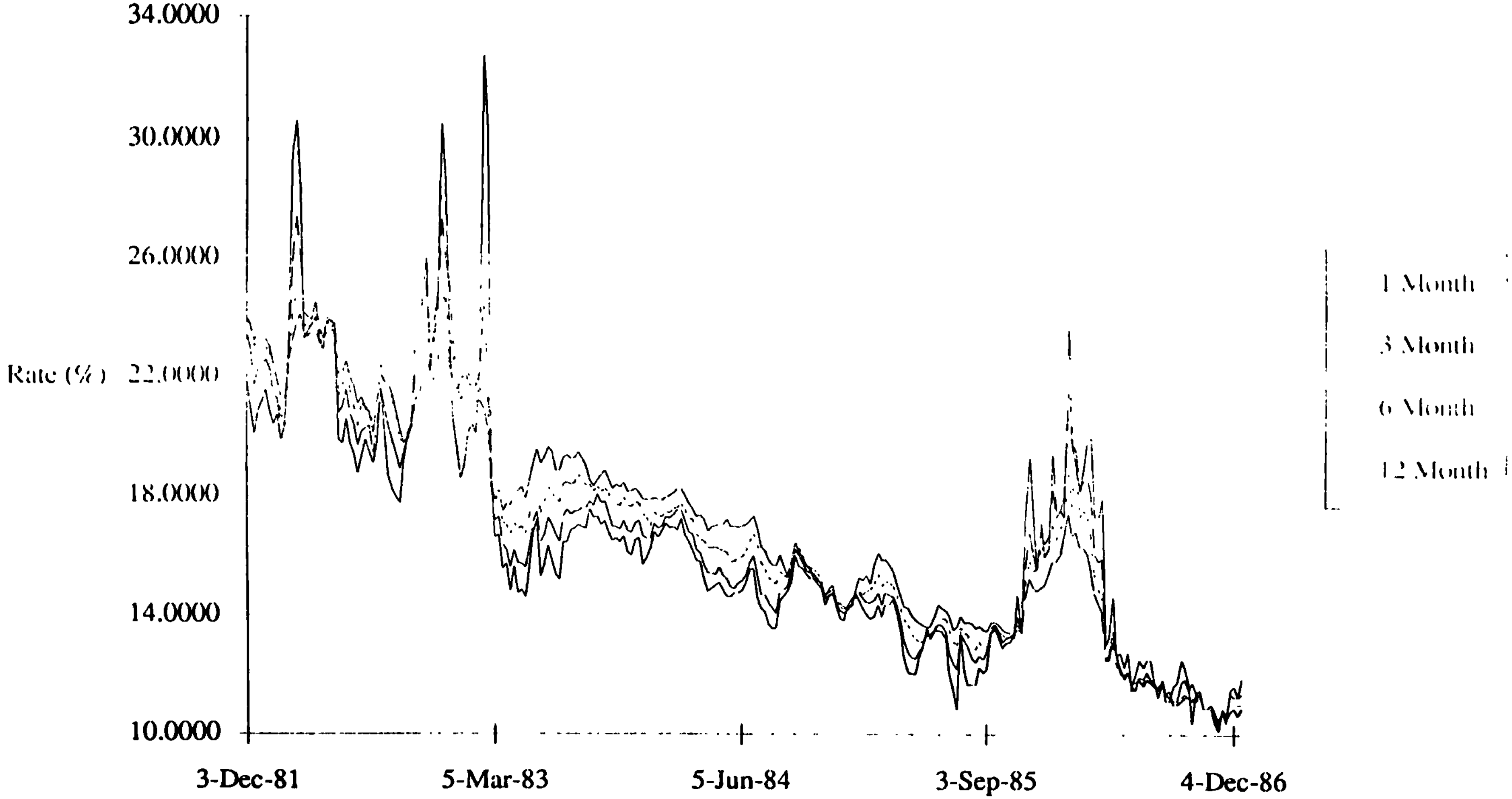


Figure 17 : Closing Weekly Eurodeposit Rates for USD December 1981 to December 1986

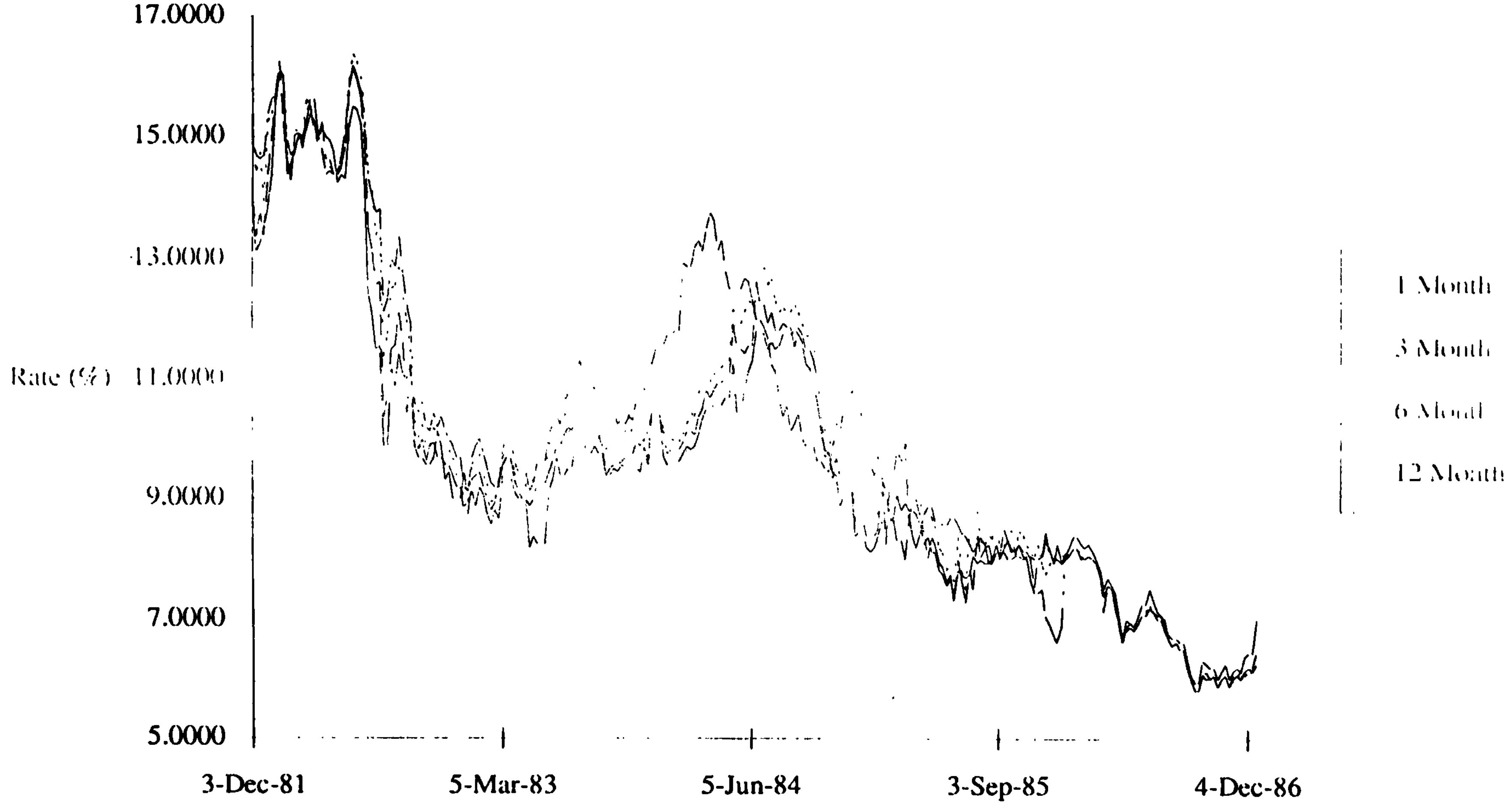


Figure 18 : Closing Daily Spot Rate for GBP/USD January 1983 to December 1988

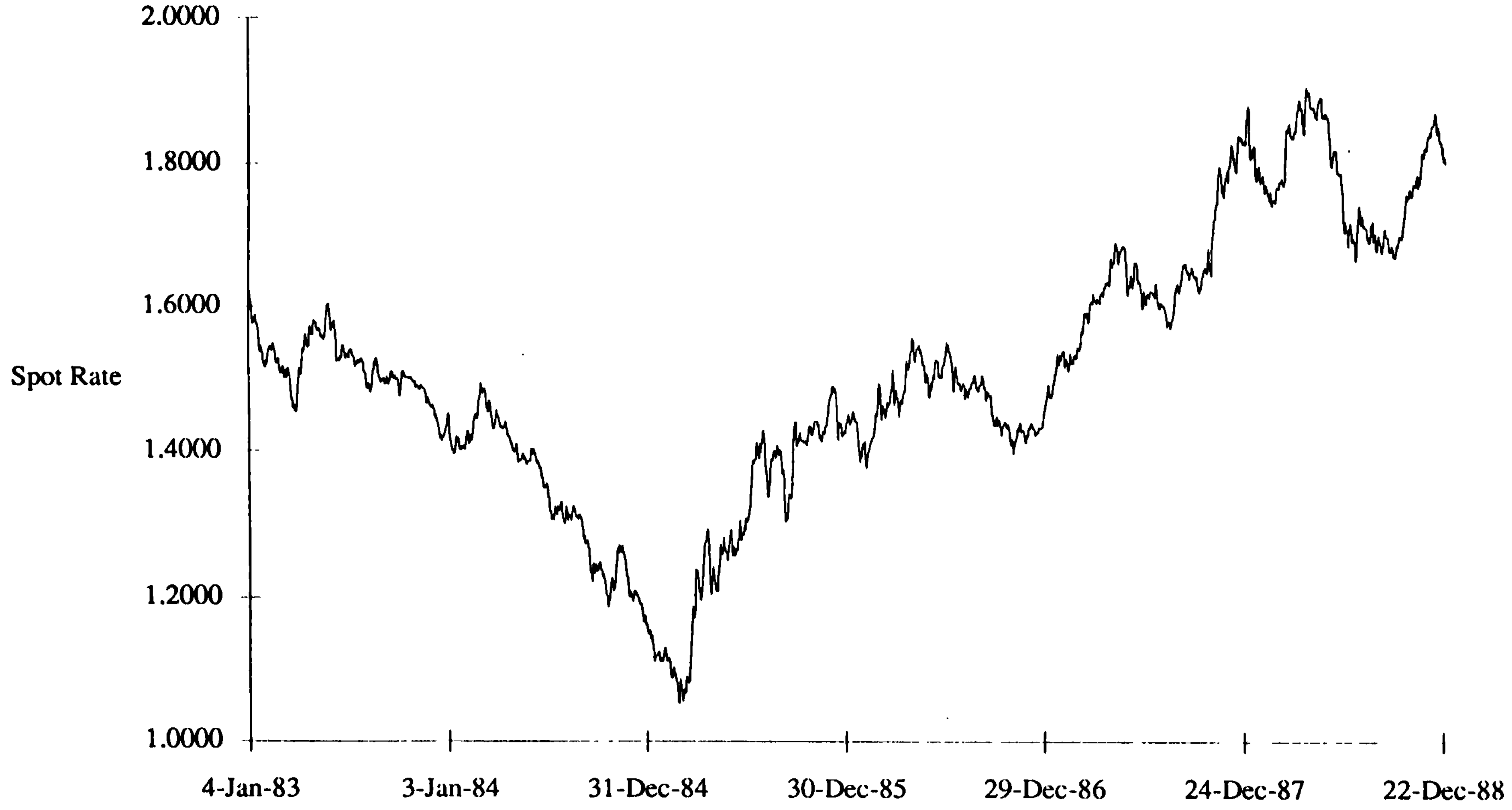


Figure 19 : Closing Daily Spot Rate for DEM/USD January 1983 to December 1988

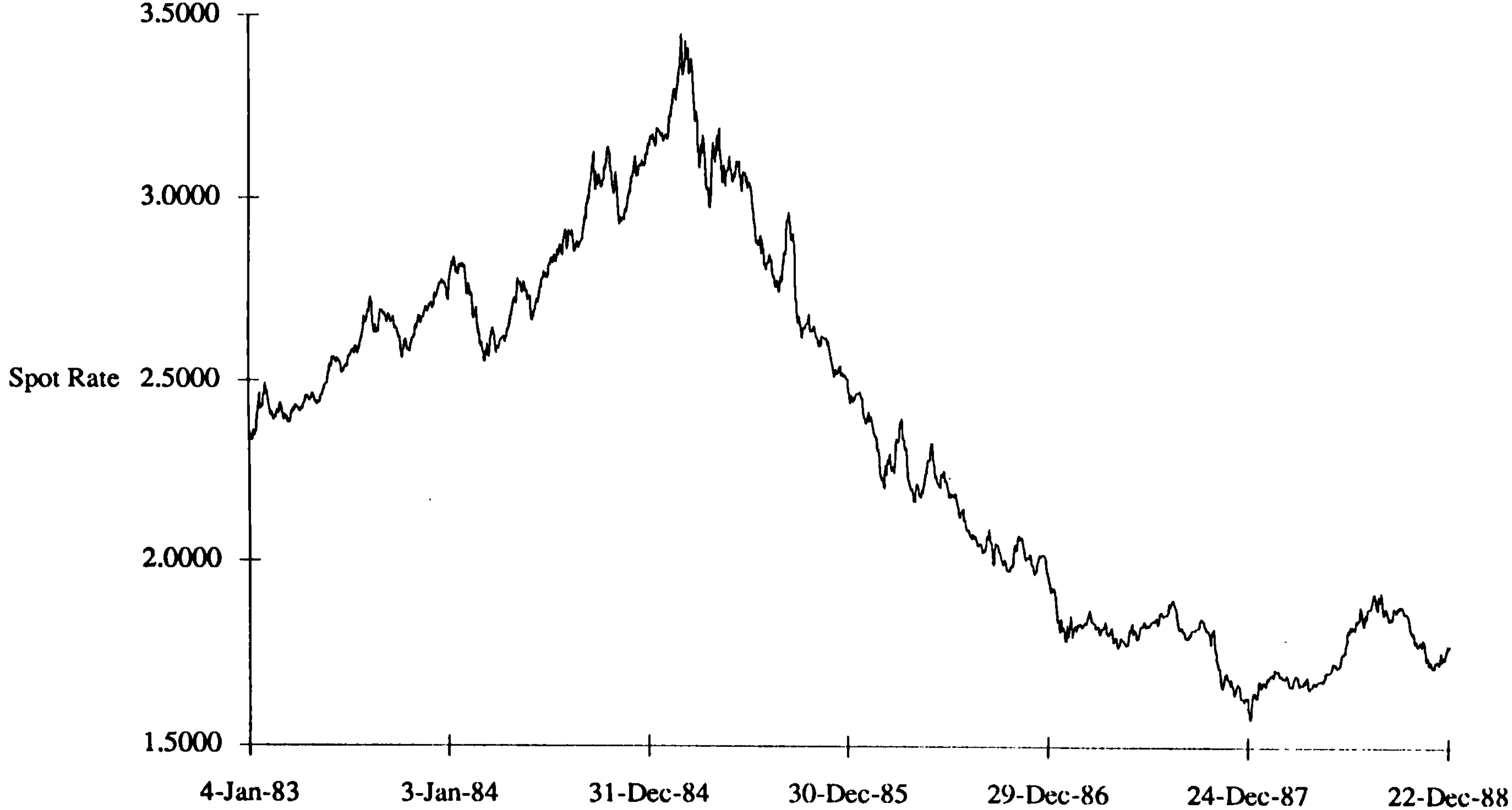


Figure 20 : Closing Daily Spot Rate for CHF/USD January 1983 to December 1988

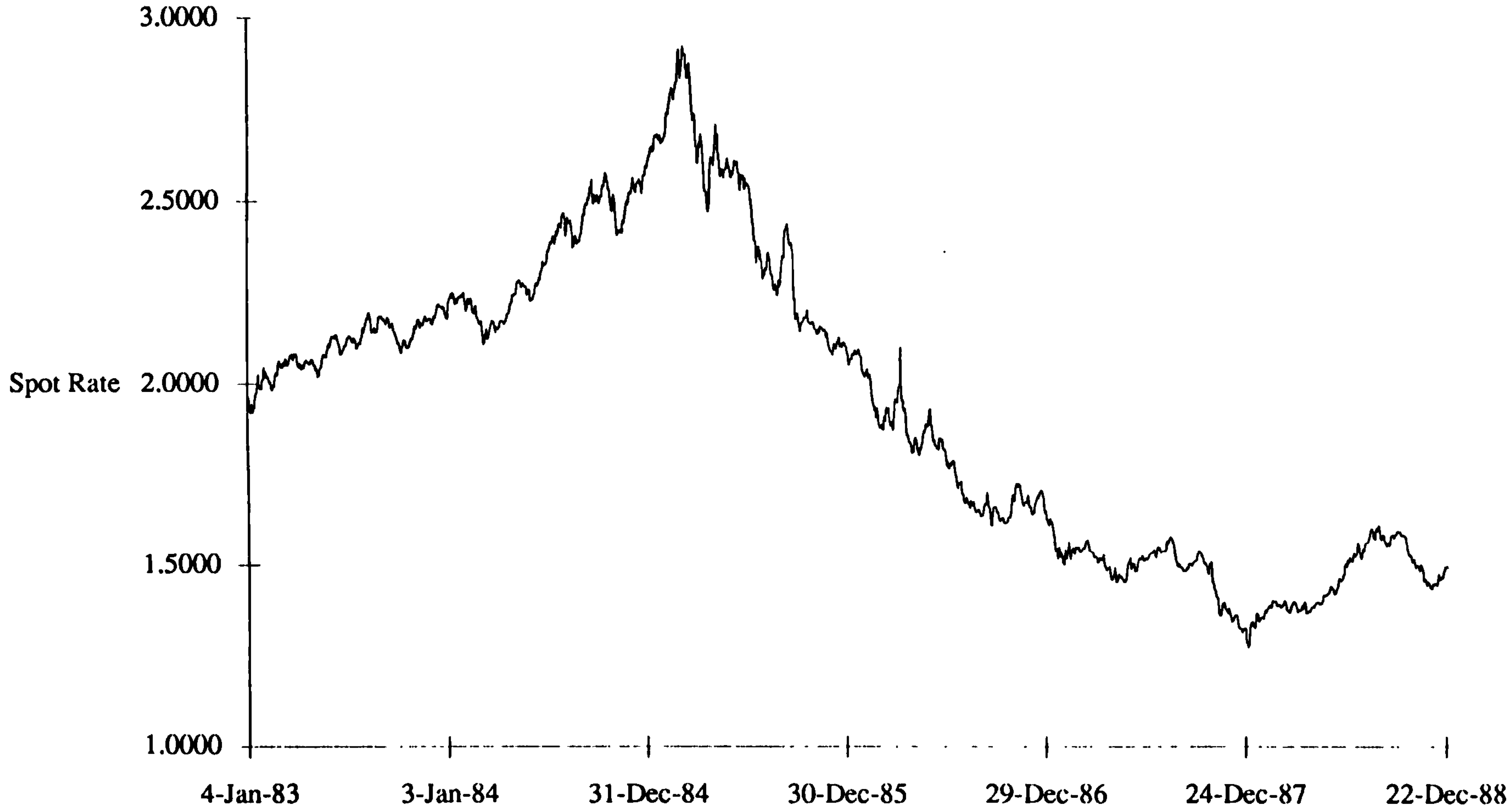


Figure 21 : Closing Daily Spot Rate for FRF/USD January 1983 to December 1988

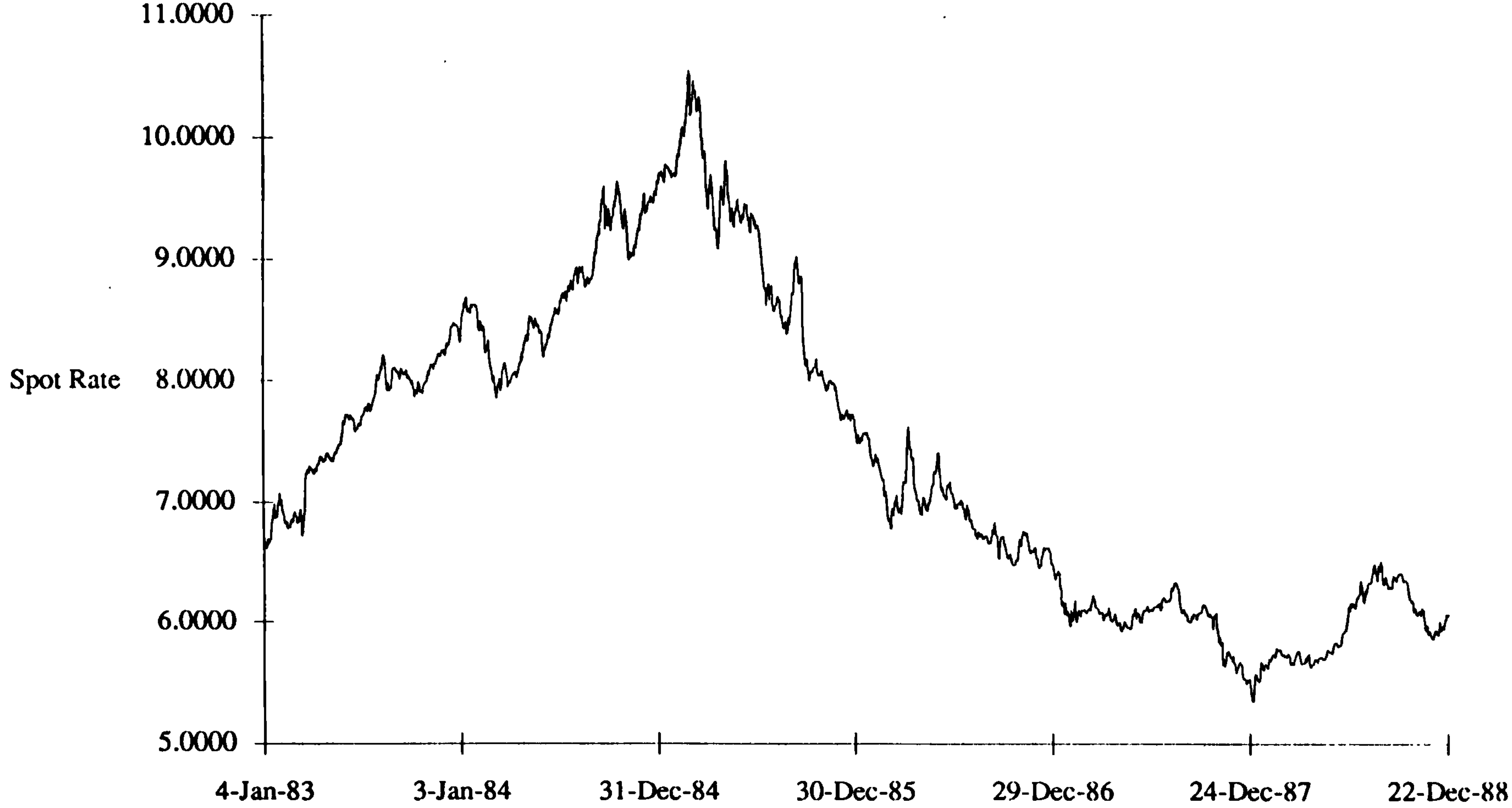


Figure 22 : Closing Daily Spot Rate for JPY/USD January 1983 to December 1988

Figure 23 : Closing Daily Spot Rate for NLG/USD January 1983 to December 1988

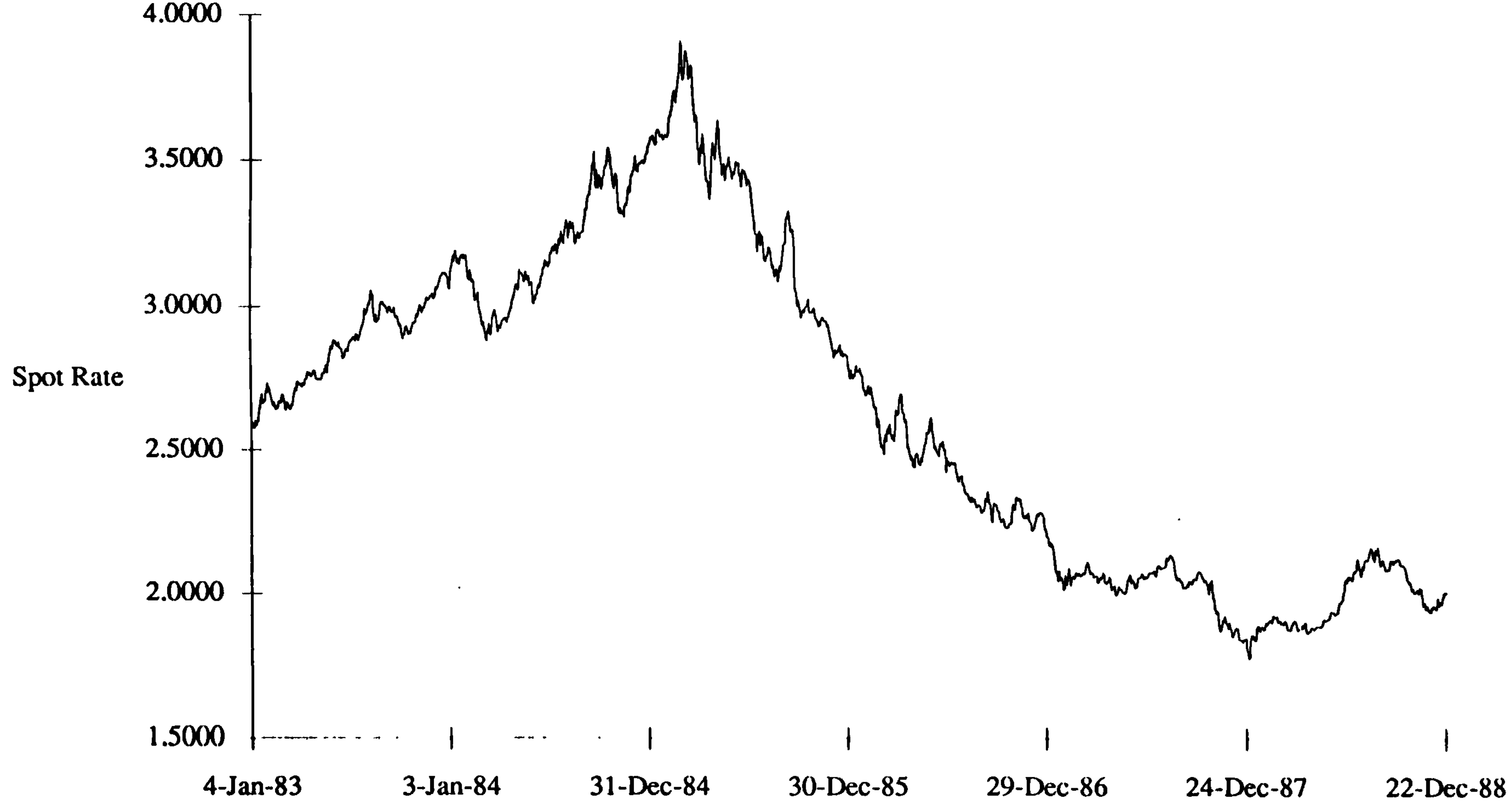


Figure 24 : Closing Daily Spot Rate for CAD/USD January 1983 to December 1988

Figure 25 : Closing Daily Spot Rate for ITL/USD January 1983 to December 1988

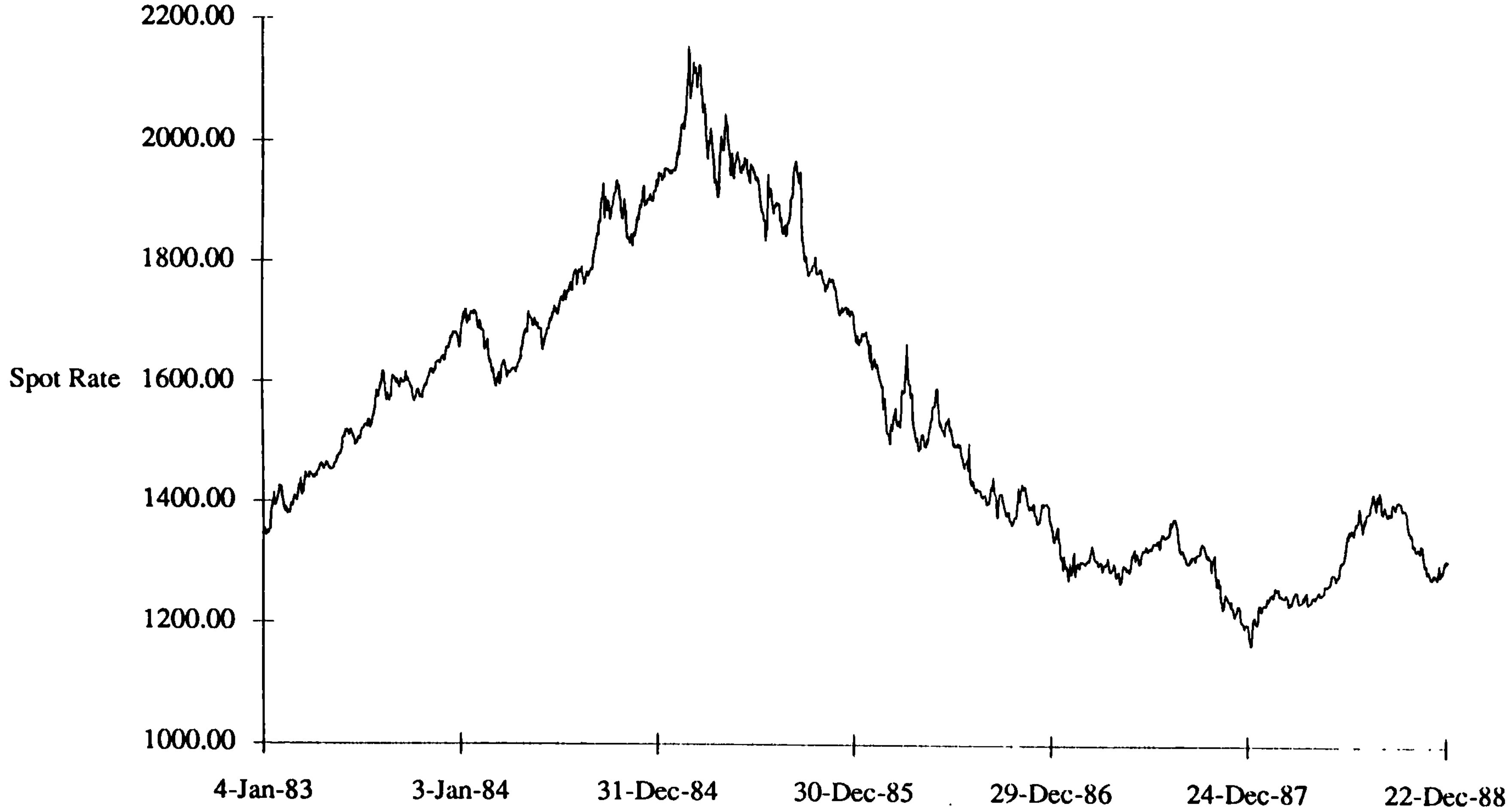


Figure 26 : Histogram of Log Price Changes for GBP/USD with Normal Approximation

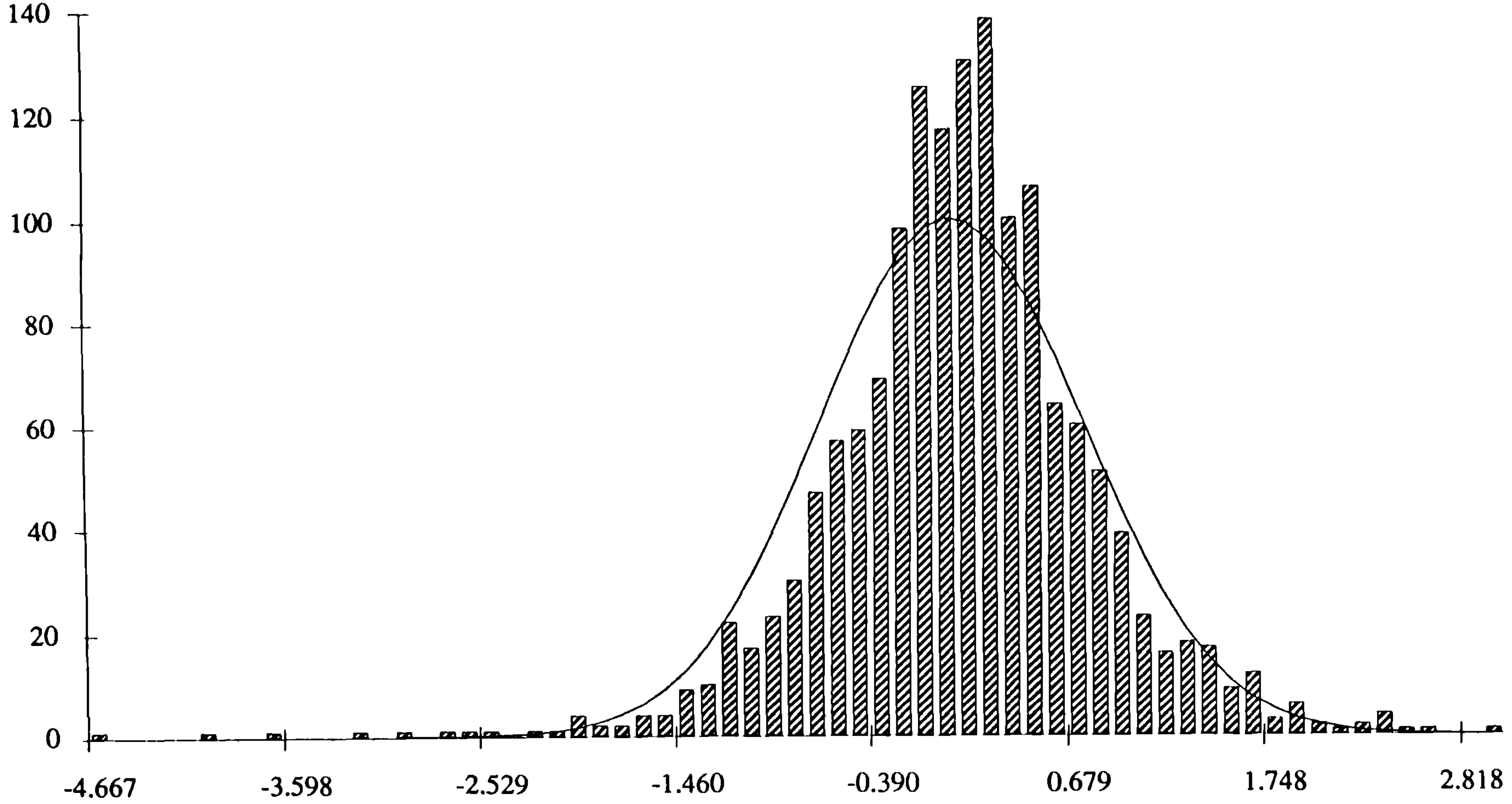


Figure 27 : Histogram of Log Price Changes for DEM/USD with Normal Approximation

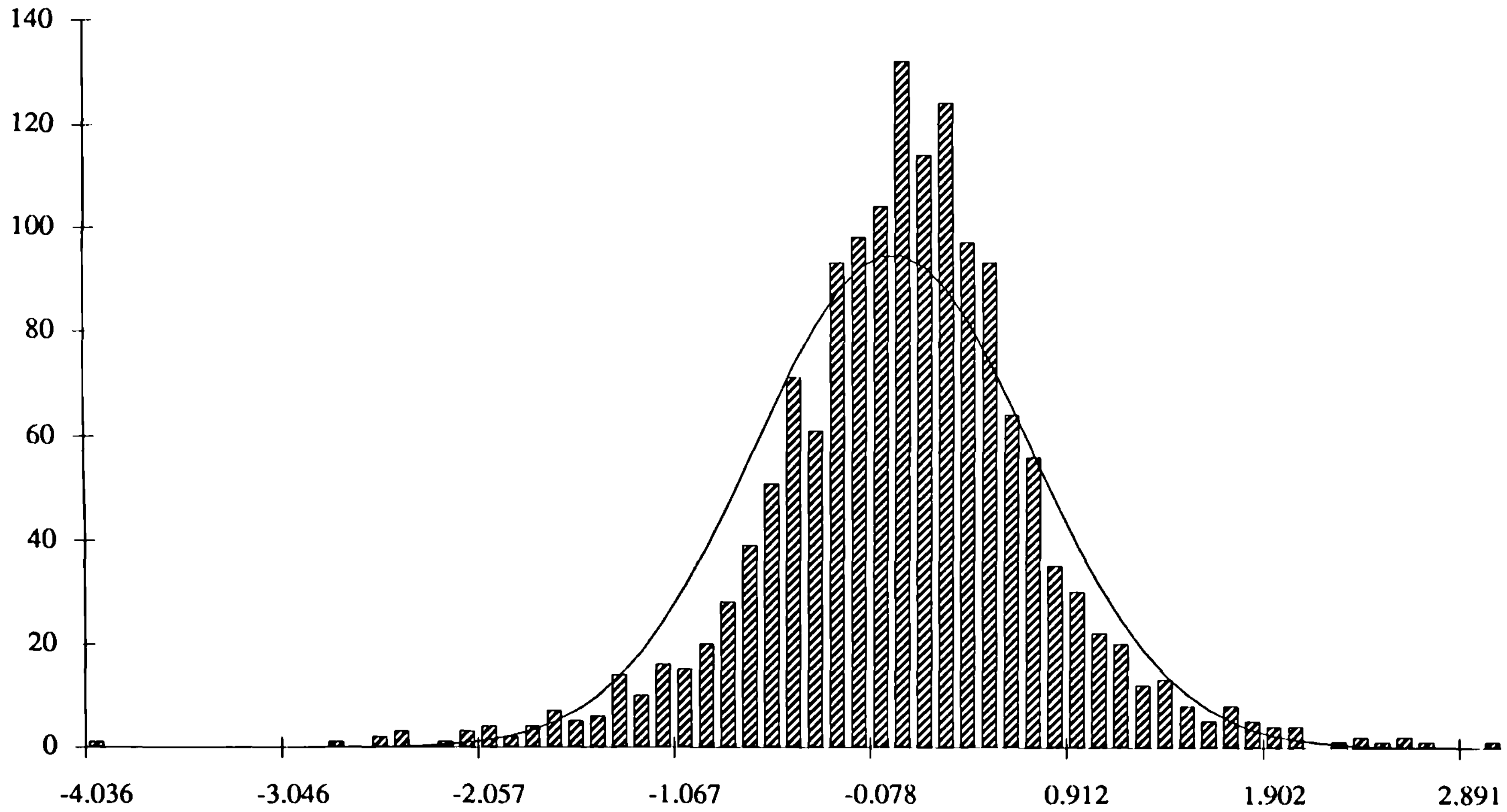


Figure 28 : Histogram of Log Price Changes for CHF/USD with Normal Approximation

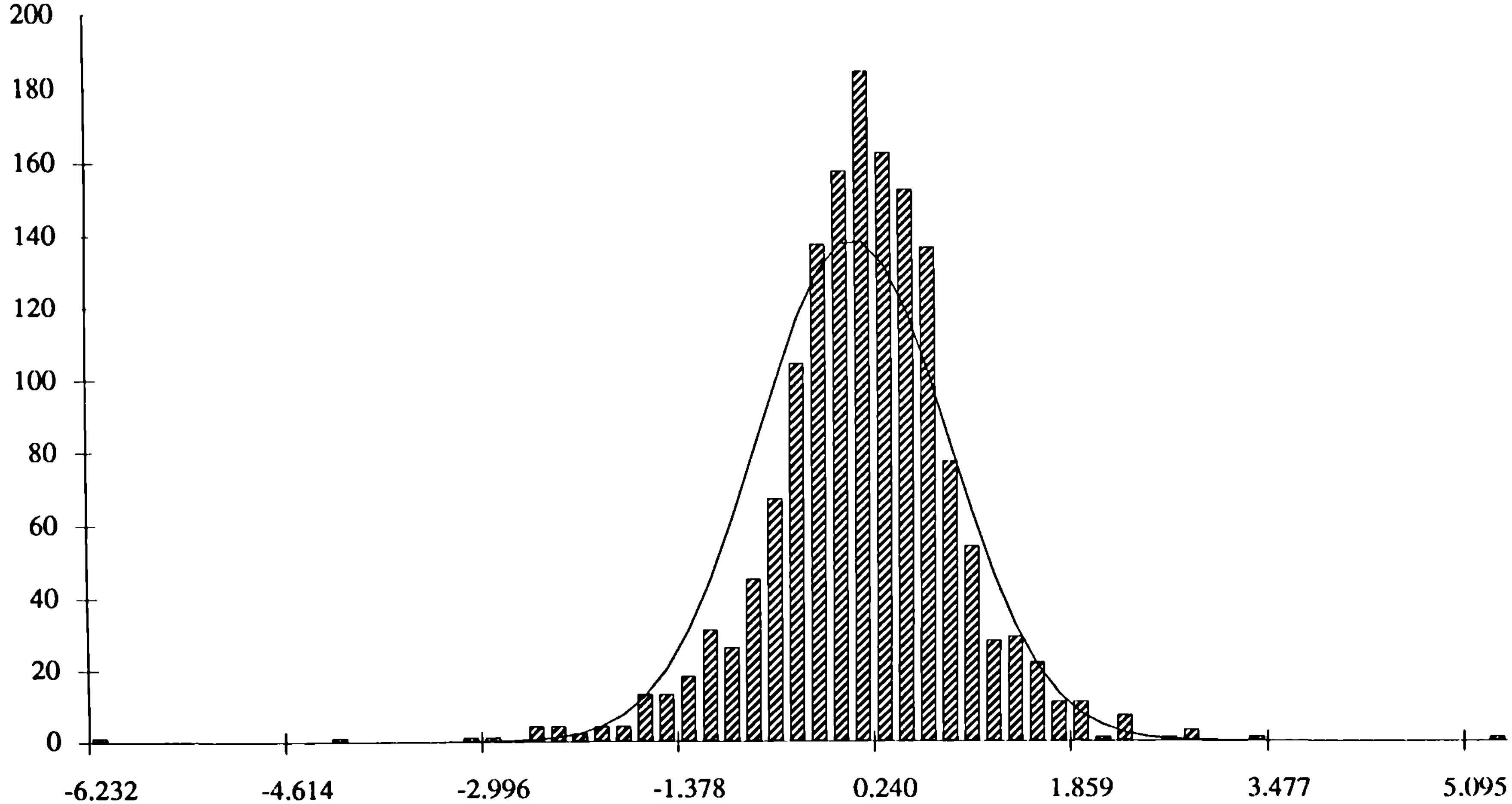


Figure 29 : Histogram of Log Price Changes for FRF/USD with Normal Approximation

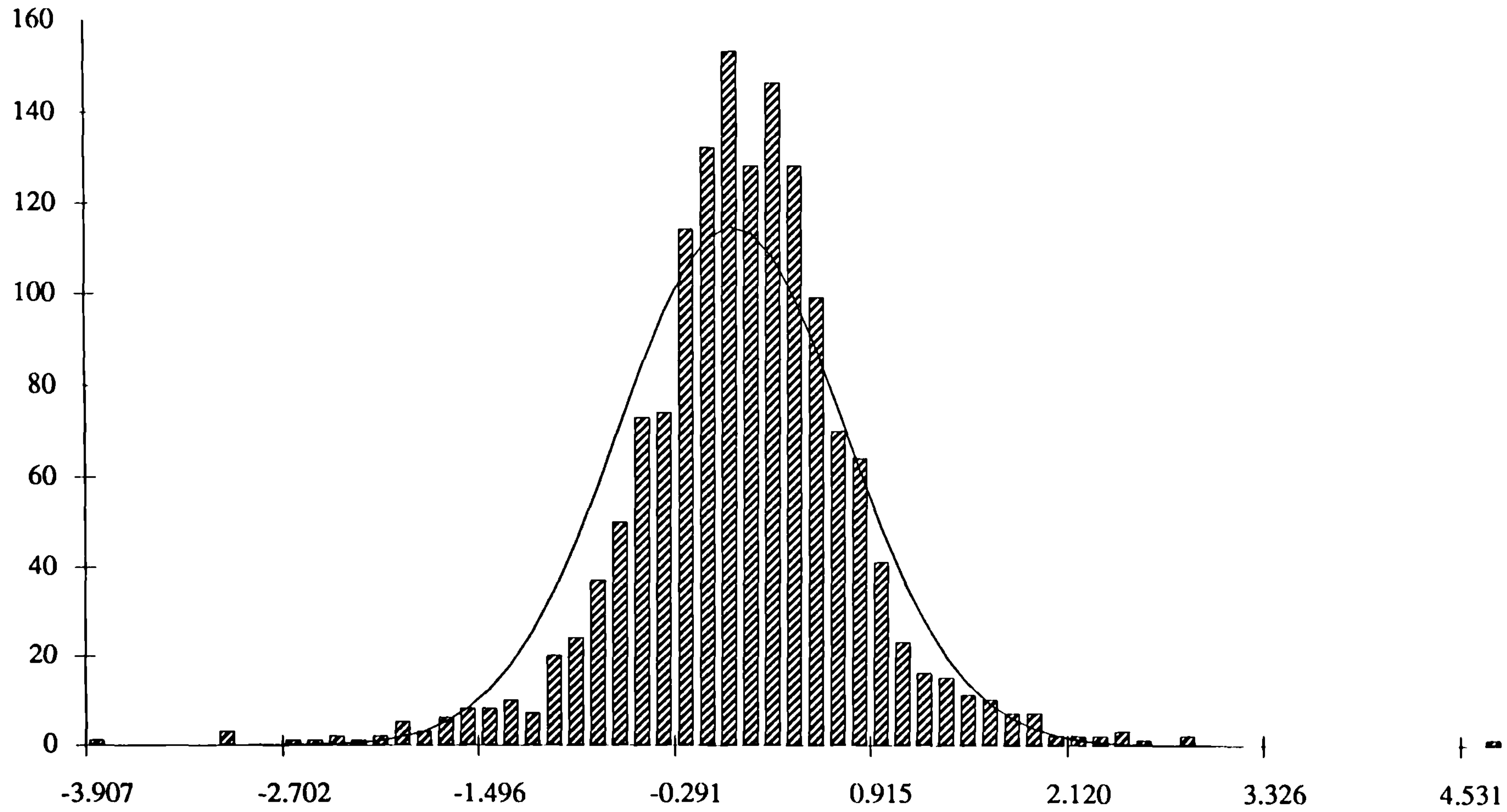


Figure 30 : Histogram of Log Price Changes for JPY/USD with Normal Approximation

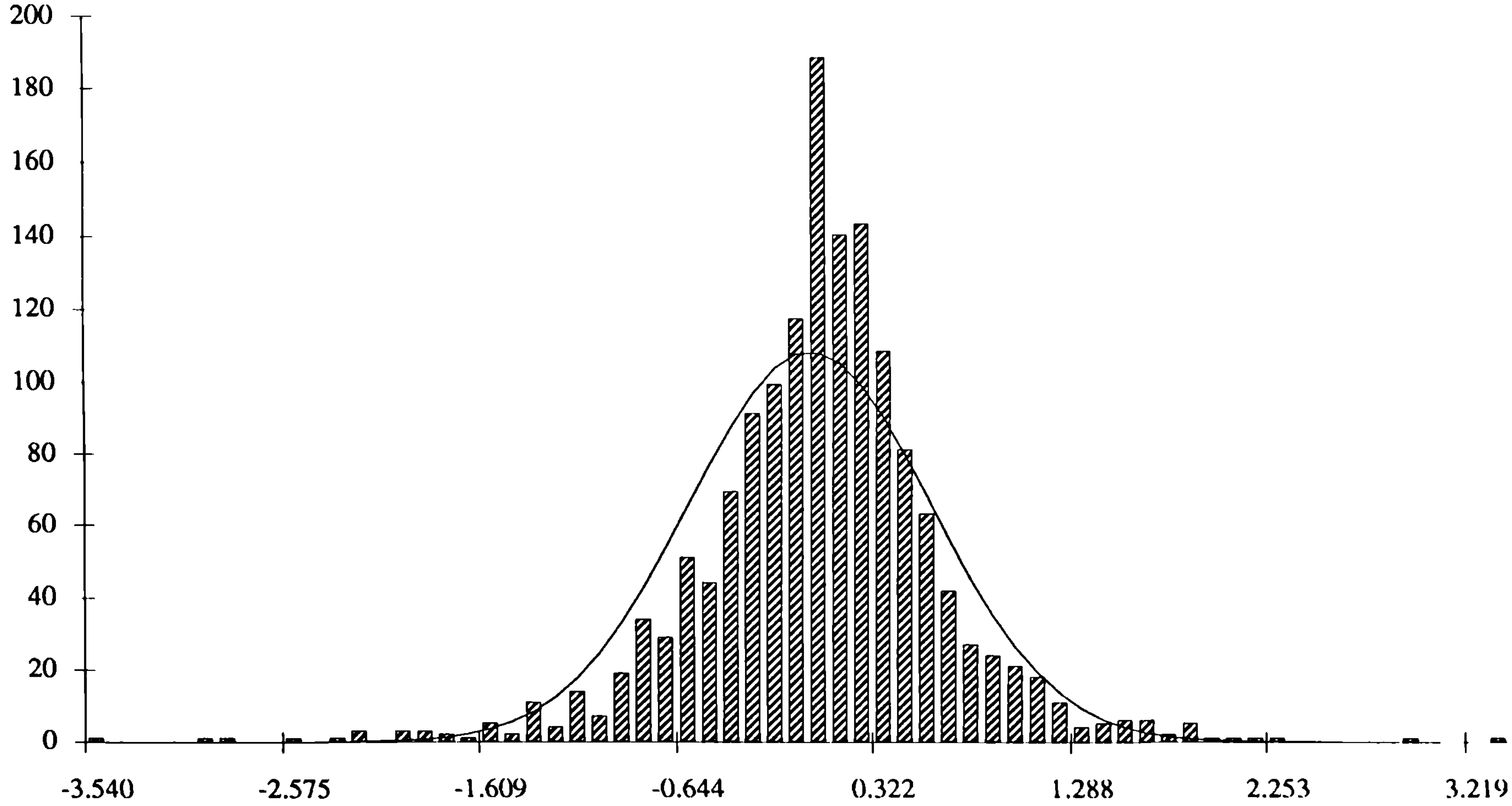


Figure 31 : Histogram of Log Price Changes for NLG/USD with Normal Approximation

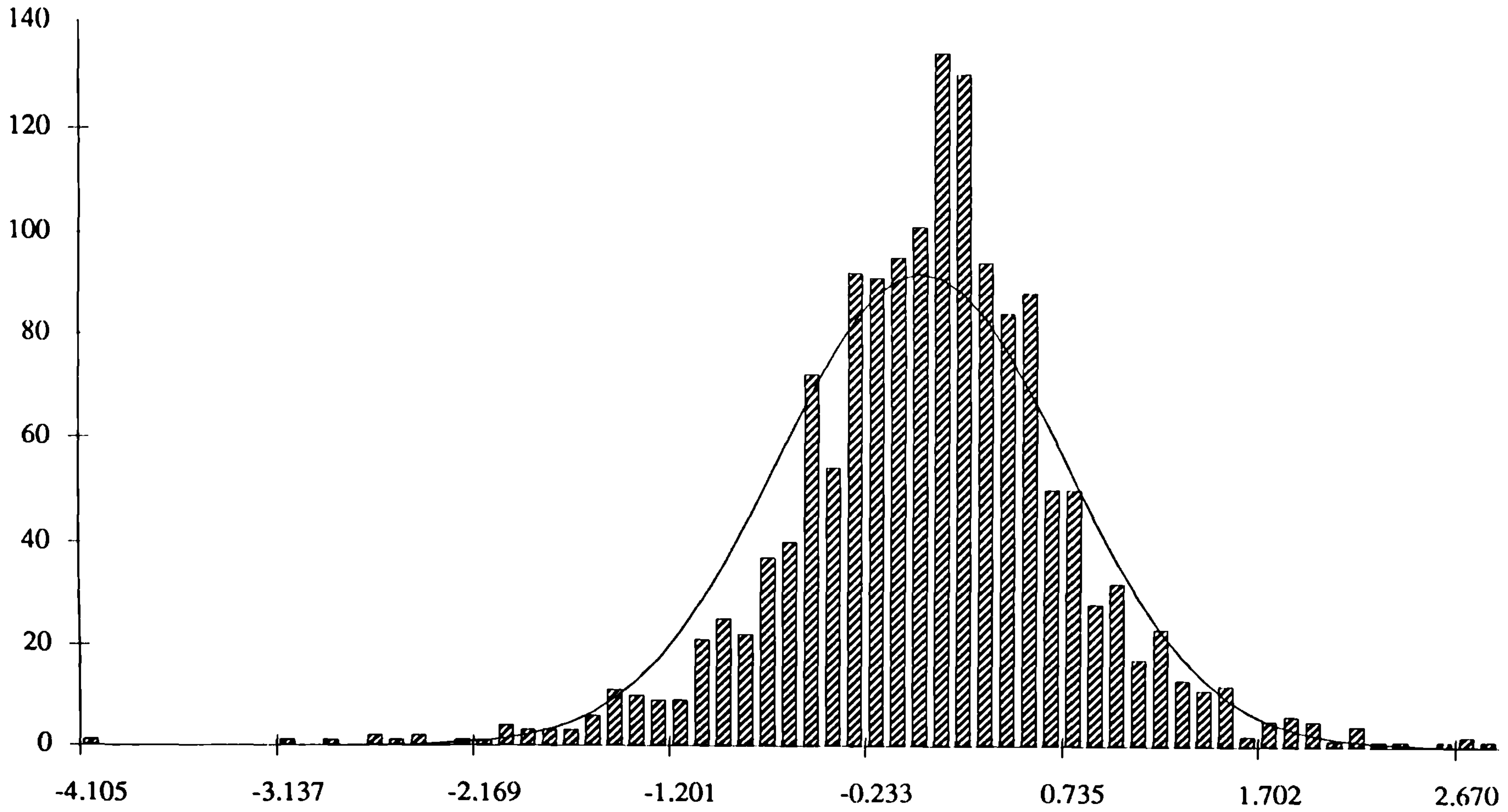


Figure 32 : Histogram of Log Price Changes for CAD/USD with Normal Approximation

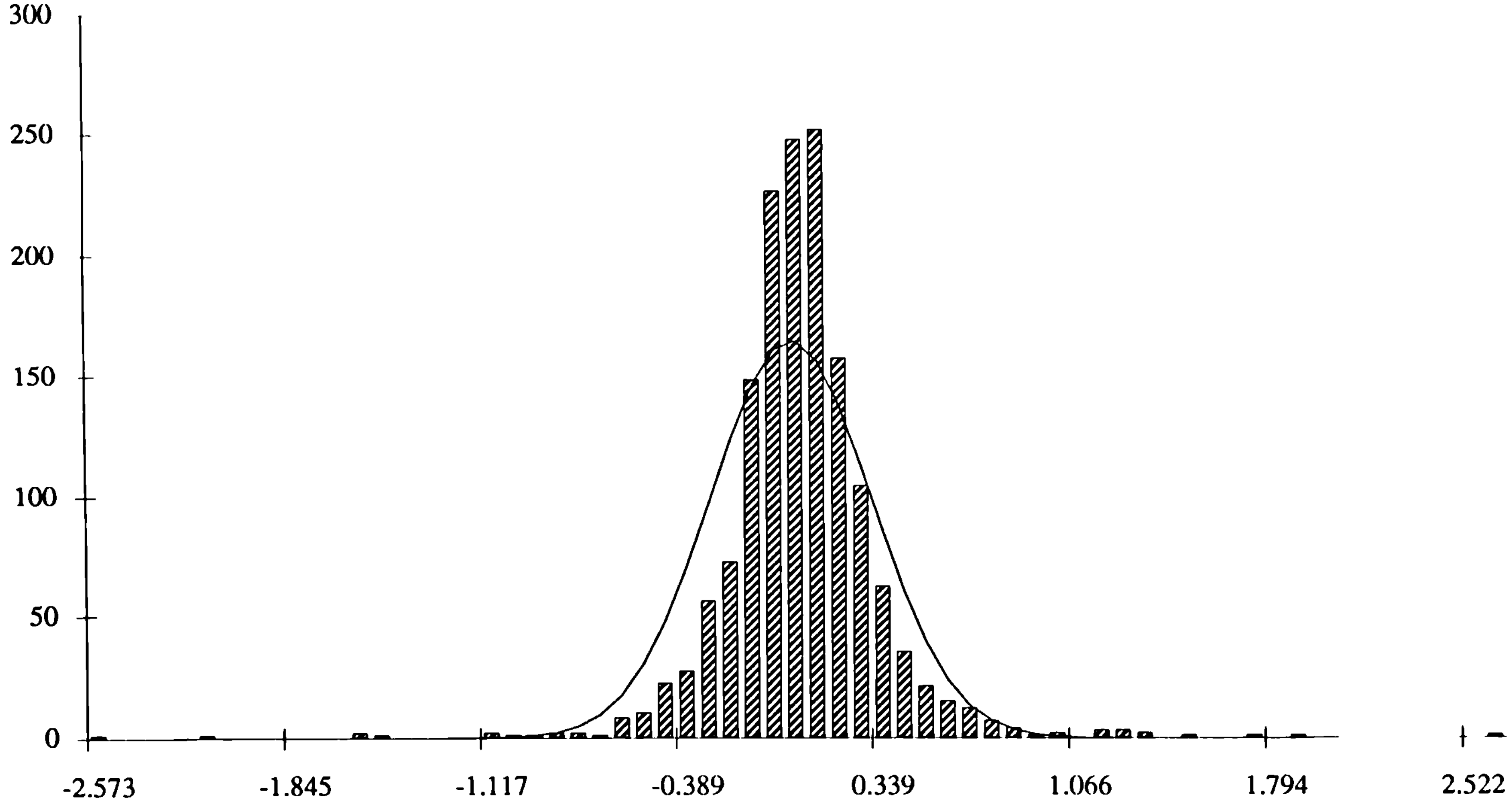
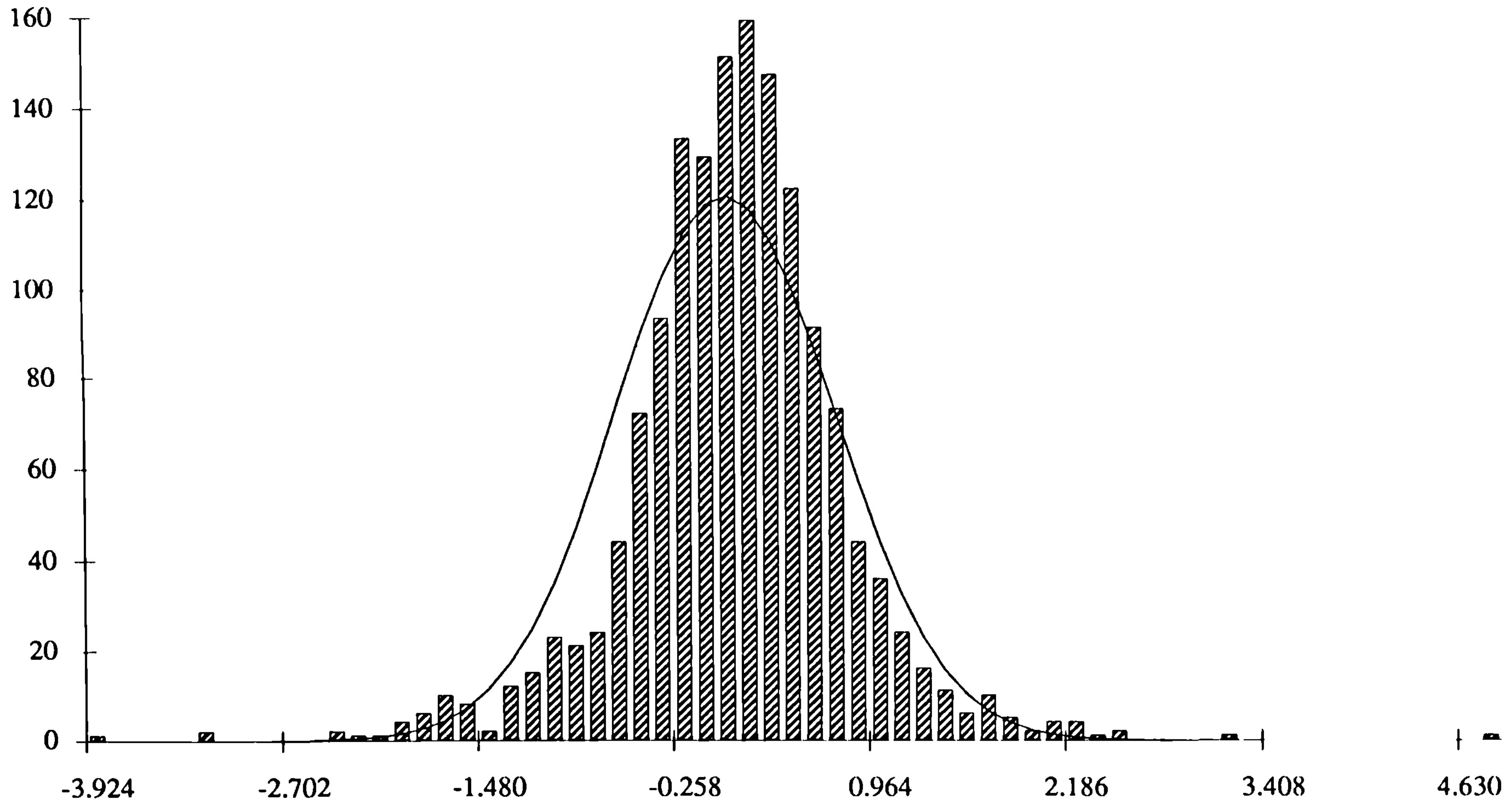


Figure 33 : Histogram of Log Price Changes for ITL/USD with Normal Approximation



Chapter 3

A Portfolio Approach to Foreign Exchange Exposure Management

3.1 Introduction

The expansion in multinational trade over the last decade, combined with the continuing volatility in exchange rates, has made the management of diversified multi-currency portfolios an increasingly common problem for a wide range of corporations. Although a large body of research has been carried out into the management of investment portfolios, containing both domestic and foreign holdings, the measurement and reduction of risk for managers of straightforward portfolios of foreign currency cashflows has received relatively little attention. More recently the mean-variance approach to portfolio analysis has begun to be applied to this problem and in this chapter we aim to describe this technique when applied to exposure management and pursue further some of the current work.

It should be noted that the type of exposure we are concerned with here is transactional exposure. Economic exposure, whilst by its nature involving several currencies, is not an appropriate candidate for the portfolio approach since the risks are contingent. An alternative strategy which we discuss later based on complex options, is the preferred approach to this risk. Similarly, translational exposure, being both notional and often a function of average exchange rates, requires a different technique.

We begin with a brief description of the mean-variance approach to portfolio analysis and its application to currency portfolios, followed by a survey of the previous work in this area. A model is then constructed to demonstrate an application of this approach to a portfolio of both spot and forward currency positions typical of those held by large multinational corporations and commercial banks. Some empirical tests are carried out on the model and results presented. The implications of these results are then discussed followed by general conclusions on the use of this approach in a unified exposure management strategy.

3.2 Some Notes on Mean-Variance Portfolio Analysis[1]

The basic mean-variance portfolio problem begins by assuming that investors' preferences can be represented by a utility function defined over the mean and variance of a portfolio's return. In general it is further assumed that asset prices follow a Gaussian normal distribution with constant mean and variance. In this case the expected return and variance of a portfolio is given by;

$$\overline{Z} = \mathbf{w}'\overline{\mathbf{z}} + w_0 R = \sum_{i=0}^{N} w_i \overline{z}_i \qquad 3.1a$$

$$\sigma^2 = \mathbf{w}'\Sigma\mathbf{w} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \qquad 3.1b$$

where $\mathbf{w}$ is the vector of weightings of the assets in the portfolio ($\sum_{i=0}^{N} w_i = 1$ and in general w_i can be less than zero implying a short sale), $\overline{\mathbf{z}}$ is the vector of asset returns, R is the riskless return, Σ the covariance matrix between the returns on the risky assets and N the number of assets.

The assumption is made that investors prefer higher means and lower variances so that the optimal portfolios are those with the highest level of return for a given level of variance or the lowest variance for a given level of return. In fact if short sales are allowed then the first condition is sufficient. These portfolios are called mean-variance efficient.

1 A more thorough explanation of the mean-variance portfolio approach can be found in Ingersoll (1987)

We shall restrict our exposition to the case where there is no investment in the riskless asset. This is because for the moment we will regard domestic currency cashflows as riskless and so outside the portfolio being hedged. In fact we show later that there are several reasons why the domestic currency may not be riskless, ie cashflows in domestic currency can have a foreign exchange exposure, but the implications of this problem are analysed later.

The portfolio with the lowest variance for a particular level of return μ is called the minimum variance portfolio and is found by solving,

$$\text{Min}\,\frac{1}{2}\mathbf{w}'\Sigma\mathbf{w} \qquad 3.2a$$

subject to the conditions,

$$\mathbf{1}'\mathbf{w} = 1, \bar{\mathbf{z}}'\mathbf{w} = \mu \qquad 3.2b$$

The solution set is given by,

$$\mathbf{w}^* = \lambda\Sigma^{-1}\mathbf{1} + \gamma\Sigma^{-1}\bar{\mathbf{z}} \qquad 3.3$$

where,

$$\lambda = \frac{C - \mu B}{D} \qquad \gamma = \frac{\mu A - B}{D}$$

$$A = \mathbf{1}'\Sigma^{-1}\mathbf{1} > 0, \qquad B = \mathbf{1}'\Sigma^{-1}\bar{\mathbf{z}} \qquad 3.4$$

$$C = \bar{\mathbf{z}}'\Sigma^{-1}\bar{\mathbf{z}} > 0, \qquad D = AC - B^2 > 0$$

The equation for the minimum variance set is given by,

$$\sigma^2 = \frac{A\mu^2 - 2B\mu + C}{D} \tag{3.5}$$

which is a parabola or a hyperbola in mean-standard deviation space. The global minimum variance portfolio is at the apex of the parabola and has weightings,

$$\mathbf{w}_{Min} = \frac{\Sigma^{-1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}} \tag{3.6}$$

Note that this means that the composition of the global minimum variance portfolio is only a function of the covariance matrix and not of the vector of asset returns. Also in general the weightings can be positive or negative implying short selling. This is appropriate for currencies since there is effectively no restriction on short sales. In addition short positions arise naturally through foreign currency payments.

When applying this model in practice several issues regarding the estimation of the vector of returns and the covariance matrix must be addressed. Typically these parameters are estimated from historic exchange rate movements over a particular period prior to the calculation date for example, the previous month or year, since they are both easy to calculate and free from any subjective judgement. The first problem here is choice of historic period and interval ie daily, weekly etc. In general the interval chosen should be equal to the interval over which changes to portfolio weightings can be made. Thus if the composition of the portfolio changes daily then daily is the appropriate measurement interval. The length of period should reflect the speed with which changes in exchanges rates are to impact on the composition of the portfolio. Thus a covariance matrix calculated over a yearly period will be more stable but take longer to respond to changes in the structure of exchange rate changes than one calculated over a monthly interval. It is important that these decisions are consistent with the use to which the analysis is going to be put. The implications of these choices are discussed in the analysis of the model presented in section 3.4.

The second problem is that of estimation error. This issue has been looked at by Jorion (1985,1986) and Frost and Savarino (1986) who carried out some simulations using US stock prices. The problem is that the impact of parameter uncertainty on optimal portfolio selection can be serious and they define estimation risk as the utility loss due to a portfolio choice based on sample estimates rather than true parameters. The key point here is that although the sample mean is optimal when estimated individually, in a portfolio context minimising estimation risk requires an alternative measure of expected return which turns out to be closer to the global mean of all the assets. Their results suggest that the use of Bayesian estimation procedures as opposed to classical sample means produce selection of superior portfolios in terms of mean variance efficiency.

An alternative approach for estimating the means which avoids this problem is to use the forward rate to calculate expected spot returns since this is consistent with the EMH. Thus if the forward is unbiased then the expected change in the spot rate will be towards the forward rate. The evidence from the empirical work however suggests that this may not be the best estimate and given the success of naive random walk models in forecasting the future spot rate a better estimate might be the historic or previous change in the rate as described above.

Another approach for estimating the variance which has not been used in research to date is to use implied volatilities taken from currency option prices. Due to the widespread acceptance of the Black and Scholes model for option pricing, these volatilities are widely available through several brokers who now operate in the OTC market. Since volatilities are quoted for any maturity up to one year and even beyond for some currencies, it should be possible to find the appropriate maturity for the period being used in the portfolio model. Since the Black and Scholes model makes the same assumption about a Gaussian normal distribution for spot rate returns, these volatilities are consistent with the variances used in standard portfolio models. As yet no empirical work to test whether implied volatility is a better predictor of future volatility than historic volatility has been done. The fact that these implied volatilities should be market estimates of future volatility however might suggest that they could provide a useful alternative to the historic measures. Of course only implied volatilities for at-the-money options should be used and the problem of liquidity or distortions in supply and demand in the options market, may lead to problems when using these estimates.

Option prices can also be used to find implied correlations. If prices for cross-currency options are available then their implied volatilities can be calculated (for some widely quoted cross-rates such as DEM/JPY implied volatilities are available from brokers). These can then be used together with the implied volatility on the USD based options to find an implied correlation. For example, if we define σ_1 as the implied volatility for USD/DEM, σ_2 as that for USD/JPY and σ_3 as that for DEM/JPY, then the implied correlation between USD/DEM and USD/JPY ρ_{12} can be calculated from,

$$\sigma_3^2 = \sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{12} \qquad 3.7$$

As with implied volatilities, the performance of implied correlations relative to historic correlations has yet to be tested empirically.

A final issue regards the assumption of normality. In the previous chapter the inadequacy of this assumption as a description of exchange rate returns was demonstrated especially with regard to the observed leptokurtosis. It is therefore important to look at the implications this has when applying a portfolio model based on the normal assumption. An interesting result in this context has been presented by Frankfurter and Lamoureux (1987) who looked at the use of the more general stable Paretian distribution as opposed to the normal distribution in the construction of optimal portfolios using data on US stocks. Since the stable Paretian contains the normal distribution as a member of the family (when the characteristic exponent $\alpha = 2$) one would expect that portfolios constructed using the more general distribution would perform better. Their results however suggest the opposite, that is the more simple assumption outperforms the more general model. They attribute this result to the law of parsimony in estimation and point to the fact that this explanation has also been used to explain the outperformance of the Sharpe (1963) simplification of the full variance covariance portfolio model when there is limited sample data. Their results suggest that the use of the normal approximation in a portfolio context, may not be too serious since in reality estimation problems are more important than a more accurate choice of underlying distribution. The use of the normal distribution can still lead to problems however, especially when used for risk analysis, and we demonstrate some of these problems in the model we construct.

The previous work on applying a portfolio approach to currencies has been mainly concerned with minimising risk for a given return μ by solving equation 3.2 or by approaching the global minimum portfolio given by w_{Min}. In the approach we propose however a different application of the mean-variance procedure is suggested. In our analysis we focus on σ^2 in equation 3.1b as a measure of risk and the ways in which it can be used to manage the exposure of a particular portfolio given that we know the normal approximation is inadequate.

In the next section we survey the previous work on the application of the mean-variance approach to foreign currency exposures before going on to present our model in section 3.4.

3.3 Survey of Previous Research

The literature on the use of Portfolio Theory in the management of foreign exchange exposure can be divided into two groups. The first, and by far the larger, is concerned with the effects of exchange rate movements on international investment portfolios. Early work by Levy and Sarnat (1978) looked at the use of foreign currencies as alternative investments pointing out that attractive returns could be made in the post 1973 floating regime. They found that a US investor would have incorporated foreign currencies as additional assets in the construction of an efficient portfolio when analysing data from the floating period. Later work by Levy (1981) extended this analysis by incorporating short positions and examined the effect of different investment horizons. He also demonstrated that the composition of efficient portfolios varied from different home currency perspectives. This is because as alternative home currencies are chosen the basic series' of exchange rate returns also change. Further work by Solnick and Noetzlin (1982), Adler and Simon (1986), Lee(1987) and Perold and Schulman (1988) continued to examine the effects of exchange rates on investment portfolios and the ways in which these effects could be measured or managed.

The second area of work, and that which concerns this research, has looked at the use of the mean-variance approach as a tool for managing foreign exchange exposure from the point of view of the corporate treasurer. Here the problem is to minimise the effects of exchange rate fluctuations on the home currency value of foreign currency payments or receipts. The major work in this area was carried out by Makin (1978). In this paper a portfolio approach was proposed as an alternative to the traditional hedge-no-hedge approach to managing foreign exchange exposure. The later strategy was based on the use of forecasts of exchange rates as the basis of decisions on whether or not to hedge individual cashflows on a selective basis. One of the arguments behind this approach was that the forward rate did not represent a good estimate of the future spot rate, especially in the new floating regime, and so some other forecast should be used in order to base hedging decisions on. Indeed the performance of the forward rate as an optimal or unbiased predictor of the spot rate has been found by empirical studies to be relatively poor (see amongst others Hansen and Hodrick (1980) and Bilson (1981)) so there was some justification for this approach. In fact if the forward rate is unbiased then assuming risk aversion implies that the forward market will always be used for hedging. This is because the forward will always reflect the expected spot rate change and so a risk averse decision maker would always chose to use the forward market since he could hedge the cashflow with no loss of expected return. In fact Makin does not use the difference between the current spot and forward rates as the expected change in the spot rate, but instead uses the current change in the spot rate. Implicitly therefore he is assuming that the forward rate is not unbiased. The portfolio approach though can in general be used in situations where the forward price is biased or unbiased since it provides a framework for analysing the risk under either assumption. When forward prices are biased however a more efficient hedge would be probably be to use options as we see later.

The main benefit of using a portfolio approach over a selective approach is that it allows the information in the covariance matrix of exchange rate changes to be used as additional information, aside from forecasts of expected changes. Makin uses the example of an expected DEM inflow to illustrate the problem. Suppose this cashlow is hedged independently, then presumably a forecast of the future spot rate is used to decide whether to sell the DEM in the forward market or wait and sell them spot. If an outflow of CHF is also expected then using the selective approach a similar judgement must again be made. Using the portfolio approach however the covariance between the expected returns on the CHF and DEM exchange rates is also considered. If the correlation is expected to be high then the preferred decision would be to wait and sell the DEM for CHF in the spot market when the cashflows arose. Makin therefore suggests that correlation hedging is an alternative to use of the forward market.

The results of simulations were presented using data from 1971 to 1975 with different adjustment speeds to the optimal portfolio. One year of quarterly data was used for the calculation of the covariance matrix and the expected changes were taken as the historic mean although some tests were also done using the change in the forward rate. A relatively high level of risk aversion was assumed and the results showed that the portfolio approach offered a viable alternative to the hedge-no-hedge approach and could produce results comparable with those achieved on the "market" portfolio ie stocks and bonds even using naive forecasts taken from historic data.

Several points are raised however by the Makin paper. First the assumption of a stable covariance matrix and a normal distribution, as he himself stresses, is problematic especially in the light of recent empirical evidence. The work on estimation risk has pointed out the problems with using sample means when constructing optimal portfolios and the use of one year of quarterly data also means that very few actual pieces of data were used in calculating the necessary parameters. Thus one would expect a fairly high degree of instability in the covariance matrix in particular. This means that the efficient boundary may move considerably each period so generating fairly radical revisions in the optimal portfolio. Secondly, the inclusion of the home currency as a riskless asset can be questioned on two grounds. Eaker (1981) observed that this assumption ignores the impact of exchange rates on the purchasing power of imported goods. Thus a portfolio model might allocate too high a proportion of currency in the home currency. He suggests the use of an index for measuring the purchasing power of a unit of home currency which would be used to adjust the returns of all currencies. The essence of this argument is that *real* rather than nominal returns should be used as a starting point for the portfolio approach. Makin, in a comment on the paper however suggests that it is not the responsibility of the firm to manage real returns since these are in fact specific to individual investors, depending on their actual expenditure patterns, and so unknown to the firm management. Thus the firm should concern itself with managing nominal exposure and leave the shareholders to manage their real return.

A second problem with this assumption is that it ignores the competitive exposure problem. A completely domestic producer for example, who sells all his product on the home market will still have foreign exchange exposure if one of his competitors is foreign or sources their inputs from abroad. This is because any exchange rate movements which benefit his competitors will have an indirect impact on his own domestic cashflows, ie through loss of sales. This impact therefore can be interpreted as an exposure to foreign exchange even though no actual foreign currency cashflow is present. This issue is increasingly important as the volume of international trade rises. In a multicurrency context the problem of competitive exposure is however too complicated to incorporate into a portfolio model since these economic exposures are in fact contingent and so a form of option must be used to protect against the impact of these movements in exchange rates. This is because the exposure is not to *all* the currencies of ones competitors but only to the one which gives the most benefit to the competition. The particular options which can be used to hedge this are described in the next chapter. From the point of view of using portfolio models within a unified exposure management strategy then, inclusion of the base currency is undesirable since it could create exposures to movements in competitors currencies which would need to be hedged separately.

The most important point raised by the Makin paper is that one can analyse foreign currency cashflows as if they are investments and as such apply the well established techniques of optimal portfolio selection to the management of these exposures. Thus the treasury manager may decide to "invest" in one currency or another irrespective of his actual position, simply to reach the optimal portfolio. In some sense the idea is to use the natural positions created by importing and exporting, to construct the optimal portfolio just as if one were doing so from the point of view of an investment manager. Obviously by possessing already some natural positions the transaction costs of setting up the portfolio are expected to be are less (although it is possible that the optimal portfolio is completely different to the original portfolio). This approach is obviously an improvement on the simple hedge-no-hedge strategy, not only because it incorporates information about the correlations between currencies, but because it provides a framework in which the treasury manager can analyse his entire position. This is important since it allows a degree of objectivity and quantitative analysis to be applied to the management of these exposures which had not be used before.

It is our belief however that the application of the investment approach to the portfolio problems faced by corporate treasurers is unlikely to be appropriate in practice. The main problem is that for most treasury managers the objective is not to maximise the return on a portfolio of foreign currency positions, as if they were running a kind of investment fund, but it is to minimise the fluctuations in domestic currency terms of their natural foreign currency cashflows. In reality the size and timing of these cashflows is a considerable problem in itself so that as Aliber (1978) has pointed out, the act of hedging at all ie using forwards, may be more risky than doing nothing. The manager therefore will be reluctant to treat even his natural positions as investments and will certainly be wary of adding new positions in alternative currencies even if his portfolio model tells him that these would give him a more optimal position. The hedging decisions faced by a treasury manager are not the same as those faced by the investment manager, even though both may be managing portfolios of foreign currencies.

This is not to say that the treasury manager is any more risk averse than the portfolio manager as Makin implies by using a very conservative measure of risk aversion. The difference is that for the investment manager, the maximisation of return for a given level of risk, is the primary concern. For a treasury manager, the control of irregular cashflows, their variability in size, and the uncertainty of cancellation or new flows arising is a significant problem which must affect his approach to the use of a portfolio strategy which requires him to adjust to an optimal portfolio whose composition reflects only the mean-covariance parameters of the exchange rates and his level of risk aversion but not his actual underlying cashflows. The strategy which we suggest therefore is not the application of the investment approach in entirety. Instead we propose to use the ideas behind the portfolio approach as a way of simply measuring the risk in a portfolio of currencies at any point in time. The idea is to use the structure or framework which the portfolio method provides, as a basis for analysing a given portfolio so that a relative increase or decrease in the diversification of the portfolio can be measured. In this way an objective measure of the risk in the portfolio can be made and changes monitored. The management of the portfolio then is a two stage process. First the calculation of some measure of risk or diversification, which we call the Portfolio Risk Capital[2], which is monitored over a period of time. Secondly, by analysing the portfolio in terms of generic spot and swap positions, trades can be done to reduce this risk within individual currencies or across the portfolio as a whole. The aim is to adjust the actual portfolio, which is itself not static, at the margin, by adding new positions and monitoring their effect on the level of risk. In this sense it is a dynamic strategy, but it is not as revolutionary as the investment approach. The objective is not to construct and maintain an optimal portfolio, but simply to keep the risk in the current portfolio under a given level. Since the portfolio is changing temporally in an irregular way, as cashflows change in timing

2 The use of the word "capital" stems from the original application of this technique as an alternative procedure for calculating the position risk element of the Bank of England minimum capital requirements for commercial banks.

and size, this less ambitious approach should be more practical to use. In addition it does not rely on estimates of exchange rate returns, but only the covariance matrix which as Jorion (1986) has suggested, provide less of an estimation problem in portfolio analysis.

The next section describes the model which we have constructed in more detail. Results of simulations carried out on the same data set used for the first empirical study in the previous chapter are then presented followed by a discussion of their implications and conclusions.

3.4 A Portfolio Model of Spot and Forward Currency Positions

3.4.1 The Model

In order to apply a portfolio approach to currencies a choice of home or domestic currency must be made so that all positions and returns are calculated in this currency. This choice is important because the exposure on the same portfolio will be different from different home currency perspectives. For example, suppose the home currency is USD and the risk on the portfolio is found to be one million USD. If the USD/GBP exchange rate is 2 USD per GBP the risk in GBP is not 500,000 GBP. This is because as one changes the home currency, all the exchange rate returns must be recalculated producing a different covariance matrix and so a different portfolio exposure. In the tests we perform on the model the home currency is USD since the exchange rates we have for testing the model are all in terms of currency per USD.

Another problem arises with respect to the inclusion of forward as well as spot positions. Since forward contracts are available for any date up to five years in the future, and sometimes further, a rigourous approach would separate each of these dates and treat positions which fall on them as separate assets. However this would create a very large number of possible assets and so greatly increase the time required for calculation. In order to avoid this problem we group together forward positions in each currency into bands. Thus all positions under 2 days are called spot, positions between 2 days and a week are grouped in the one week band, positions between 7 days and 30 days are grouped into the one month band, and so on. Two issues arise with the use of this technique. Firstly, the width of the bands effects the accuracy of the model and so the closer together the near and far ends of the bands the better. There is obviously a trade off between a large number of bands and calculation time. Secondly, a problem can arise as positions move from one band to another as they approach maturity. Where these positions are large, they may cause fluctuations in the level of portfolio risk even when no new deals are added and the covariance matrix is unchanged. With a large enough diversified portfolio these effects should not be too great, but one must be aware of them. In fact with regard to the issue of appropriate bands it should be noted that in reality only a limited number of forward rates are actually traded. Thus the one week, one month, two month, etc rates on any day, will be actively quoted but the rate for dates in between will simply be interpolated. Thus there should be a high degree of correlation between the returns on positions within each band[3].

In setting up the portfolio model it is necessary to define the assets which make up the portfolio. In the case of currency portfolios which are made up not just of spot positions but forward positions too, the traditional approach (as in Makin) has been to regard all the cashflows as outright positions. Thus a cashflow in three months is regarded as an outright position that is, as a kind of "three month asset". The return on the three month outright forward rate would then be used to calculate the variance and covariance of this position with all other positions. There are two potential problems with this

3 An exception to simple interpolation is when the "traded" dates are either side of a year end. In this case care must be taken in allowing for the effect of the "turn" that is the abnormal overnight rates which occur over the year end.

approach however. Firstly, the change in value of the three month outright rate can be interpreted as a made up of a change in the spot rate and a change in the forward points (ie the interest rate differential). Most of the absolute change is due to changes in the spot rate since this is more volatile in absolute terms than the forward points. The correlation between the outright forward and the spot rate is consequently very high and so the effect of movements in the forward points is to some extent hidden by the spot rate return. It is therefore difficult to distinguish between the risk on this cashflow arising from spot movements and that arising from changes in the forward points. It was for this reason that our empirical work on this data base examined spot rate and forward points returns rather than forward outrights. Some way of separating these two components of the exposure is similarly required in a portfolio model.

The second problem is that by regarding all the positions as outrights, hedging the exposures is naturally performed through outright forwards too. This however is contrary to market practice since in reality forward positions are hedged by forward swaps (as described in chapter two). Indeed this is the way in which banks manage their positions, that is by having all the spot positions in one portfolio and the forward swaps in another. When asked to price a forward outright therefore two separate prices, a spot price and a forward points price, are combined and once the deal is done the two parts of the exposure are hedged separately. It is desirable therefore that in setting up the assets in our portfolio we do so with these factors in mind.

In order to overcome these problems we set up our assets as spot positions and swap positions separately. Obviously a swap is made up of two cashflows, usually on the spot date and some other date in the future such as the one month date for a one month swap. So for example, a DEM inflow arriving in three months would be split up into a spot inflow of DEM and a three month swap with an outflow of DEM spot and an inflow in three months time. The net effect is obviously the same but by decomposing the outright cashflow into a spot and a swap position we can more easily examine the components of the risk. Thus we may decide to hedge the spot position but not the swap position depending on the overall composition of the portfolio.

One implication of this approach is that it explicitly incorporates the correlation between the spot rate and the interest rate differential. Thus the risk on an outright position calculated in this way may give a higher or lower Portfolio Risk Capital amount relative to the more traditional approach depending on the correlation between the spot rate and forward points. If the monetary model of exchange rates holds then obviously this correlation should be high since changes in the exchange rate should respond to changes in the relative interest rates of the two countries through supply and demand.

For a portfolio of currencies, held both spot and forward, the variance of the portfolio is calculated from the individual variances and covariances between the returns for each spot rate and swap points across currencies and maturity bands, together with the proportion of the total portfolio held in that "asset", using equation 3.1b. We then define the Portfolio Risk Capital (PRC) as,

$$PRC = K\sigma T \qquad 3.8$$

where K is a constant reflecting the level of confidence, σ is the portfolio volatility (standard deviation from eqn 3.1b) and T is the domestic currency value of the portfolio.

The Portfolio Risk Capital is simply the maximum expected positive or negative return on the portfolio, over the next period, to a level of confidence given by the choice of K. Thus if K is chosen as 1.96, since the portfolio return is assumed to be normally distributed, there would be assumed to be a 5% probability of the future portfolio return being outside plus or minus the PRC amount. In other words, since the distribution is assumed symmetric, if we set aside an amount of money equal to the PRC then there is just a 2.5% probability that this will not cover an adverse change in value of the portfolio over the next period.

The PRC therefore gives a measure of the exposure of the current portfolio and can be monitored to see whether the portfolio is becoming more or less exposed both as new positions are added and as the relationships amongst exchange rates, in the form of the covariance matrix, change. For a portfolio of constant size, the PRC is only a function of the covariance matrix so no forecasts or estimates of expected exchange rate returns are needed. Estimates of the variances and correlations between the currencies do however need to be calculated, either using historic data or in the case of variances and correlations, from option prices, or taken from subjective forecasts. In the tests which we describe later simple estimates are calculated from historic data, updated weekly. In practice a weighted average of estimates from different historic periods could be used as an alternative or some other form of exponential weighting.

Two issues arise in the use of the PRC measure. Firstly, its accuracy in absolute terms is highly dependant on the accuracy of the value of K in reflecting the probability of returns outside certain limits. Since we know that the distributions for individual currency returns are highly kurtotic this parameter should not be viewed with much confidence. As we shall see from the simulation results, even when we look at a portfolio, the leptokurtosis gives rise to large deviations between the expected size of the tails of the distribution and their actual size.

Secondly, the PRC amount has value as a relative measure of risk. Thus even if its absolute value is not an accurate reflection of the size of probable future returns, it can still be used as the basis for testing the sensitivity of the portfolio, in particular to the addition of new positions. By looking at the effect a proposed new deal would have on the PRC amount an idea of its impact on the overall portfolio risk can be made. This is important when assessing whether on not to hedge a new cashflow. By examining the effect that adding this cashflow has on the portfolio it is easy to decide whether or not a hedge is required.

The next section describes simulation tests carried out on the model and the results obtained from them.

3.4.2 Simulations and Results

The model was tested by simulating over the same data set as that used for the first set of empirical tests described in chapter two. This consisted of weekly spot and forward rates for the eight currencies from December 1981 to December 1986 (see Figs 1 to 17). As we have shown, all the currencies, both spot and forward, exhibit significant kurtosis and so the use of the PRC measure as a guide to the exposure on the portfolio, will be potentially inaccurate as an absolute number. This is because the confidence level attached to the PRC amount is a function of the parameter K which in the presence of leptokurtosis will not accurately reflect the probability of returns outside the assumed limits. In order to examine this problem the weekly returns for all the spot rates were calculated and plotted together on a frequency histogram (see Figure 34). The standard deviation was calculated and the percentage of actual returns outside +/- 1, 1.96 and 2.58 standard deviations was recorded. These results are shown overleaf;

Weekly spot rate returns for all currencies;

Standard deviation = 0.0166

% of observations outside +/- 1 std = 25.29 (530)

% of observations outside +/- 1.96 std = 8.11 (170)

% of observations outside +/- 2.58 std = 5.44 (114)

Number of observations in brackets.

The effect of the leptokurtosis can be easily seen as the percentage of observations outside 1 standard deviation is less than that of the assumed normal (ie 33%) but the extreme tails are much fatter. Thus the expected 95% confidence level, at 1.96 standard deviations, in fact only covers about 92% of the observations and the 99% level, at 2.58 standard deviations, covers only about 95% of the observations.

These results demonstrate that care must be taken in using the absolute PRC amount as a guide to the actual exposure with a particular probability. If the distribution were stable then of course alternative values for K could be used which more accurately reflected the actual confidence limits. Thus we could use 2.58 for the 95% level instead of 1.96 for example. The evidence presented in the previous chapter however suggests that the distributions for the individual spot rates are probably unstable, with changing variances. The degree of kurtosis therefore is likely to change through time. If the absolute value for the PRC amount is to be used then a conservative estimate of the confidence limits should be made and regularly checked against actual returns.

The second test simulated the calculation of the PRC amount for a particular portfolio over the actual data. The aim was to examine the effects of a changing covariance matrix on the PRC amount when the actual portfolio weightings remained constant. As figures 35 to 42 show the annualised standard deviations of the spot rates, calculated over a rolling thirteen week interval, vary significantly for all the spot rates during the entire period. The European currencies, with the exception of the GBP, all follow very similar patterns (see Figures 36,37,38,40 and 42). The JPY (Fig. 39) and the CAD (Fig. 41) are both much less volatile in absolute terms but again changes in the level of volatility through time can be large. In several of these graphs one can see the effect of a large movement in the rate over one week pushing up the volatility and keeping it high as long as that return stays in the calculation sample. A corresponding drop in the volatility as the return falls out of the sample can then be seen. This highlights one of the problems of using historic measures of variance suggesting that implied volatilities or weightings, as we discuss below, may be more appropriate.

In Figures 43 to 45 three sample correlations are shown, calculated using the same technique as for the volatilities. Once again the instability is evident, particularly for the GBP/USD versus DEM/USD and the GBP/USD versus JPY/USD correlations. The DEM/USD versus JPY/USD correlation shown in Figure 45 is more stable, around 0.8 for most of the period, but significant temporary drops can be observed, down to zero at one point. This implies that the cross rate DEM/JPY was fairly stable, more recently this rate has become a lot more volatile and so this correlation would be expected to have dropped. The question is therefore, to what extent does this instability in the covariance matrix effect the PRC amount on a typical diversified portfolio.

In order to examine this we constructed a portfolio which contained just spot positions, equally weighted, in all eight currencies. The PRC amount was computed from a covariance matrix calculated from the first thirteen weekly returns (ie three months) and then this "window" was rolled forward one week at a time. This historic period was chosen to provide a reasonable number of data points but also to be fairly reactive to market movements. One of the practical problems with using a covariance matrix calculated from historic returns is that if a relatively long period is used any extreme movements stay in the data long after they have actually occurred and it also takes longer to react to changes in volatility or correlations. One solution to this problem is to use shorter historic periods but this reduces the number of data points being used and can lead to instability. An alternative is to use a reasonably long period but weight the more recent changes higher, perhaps using exponential weighting. If the model is to be used for risk management rather for optimisation a longer period is preferable since this will give a more stable matrix.

The results of this simulation are shown graphically in Figure 46 where a value for K of 1.96 was used. As can be seen the PRC amount on this static portfolio varies between about 0.1% and 0.45% of the portfolio value over this period. A conservative treasury manager therefore might set aside 0.5% of the portfolio value as a reserve against a large loss on any one week. The PRC however is volatile, reflecting the volatility of the individual variances and correlations, so it is important that it is monitored closely, especially when in reality the portfolio size and the proportions of currencies held are also changing. A useful approach would be to examine the effect of the recalculated covariance matrix on the PRC before adding any new or changed positions. In this way the "market related" risk can be separated from the "position related" risk. The total change in the PRC can then be explained as made up of movements in the market and changes in the structure of the portfolio. This provides additional insight into the causes of any increase or decrease in the level of risk.

So far we have looked only at portfolios of spot positions. We now examine the exposure of swap positions and in particular the effect of different near and far maturity dates on the risk of these positions. The separation between spot and forward (ie interest rate) risk is possible because we have analysed the portfolio cashflows as separate spot and swap positions. Of course the entire portfolio is still managed with respect to the overall PRC amount so that all the benefits of diversification can be gained. The split between spot and swap positions is only done so that a closer analysis of possible hedge deals can be carried out. This is important because our approach uses the PRC amount as a measure of risk but provides no rules to reduce this risk as the optimal portfolio approach does. This brings out a crucial distinction between the two approaches. In our approach, the reduction in risk is carried out in a less formalised way. Rather than continually adjusting the portfolio in order to maintain the optimal position in an mechanical way, we propose that analysis of the risk is carried out in terms of the elements which make it up. In this way alternative hedging strategies are suggested which can then be tested for their overall effect on the portfolio position.

Management of this forward risk can be approached in either of two ways. Firstly, the risk in the swap positions can be analysed in a similar way to that of yield curve risk in for example, bond portfolios, using the techniques of duration and convexity. In a simple duration hedge the portfolio would be split into maturity bands by currency and the net duration weighted position of each currency "ladder" would be calculated using;

$$Net \quad Duration \quad Weighted \quad Position = \sum_{i=1}^{N} P_i T_i \qquad 3.9$$

where P_i is the domestic currency value of the position (positive or negative) in the ith maturity band, T_i is the maturity of this position (ie for six months T = 0.5) and N is the number of bands. This value can then be used to calculate the size of a hedge deal in any band which would give a zero overall duration weighted position. Thus formally we solve;

$$P_j T_j = -\sum_{i=1}^{N} P_i T_i \qquad 3.10$$

where P_j is the size of the hedge position at maturity T_j. If the yield curves of both currencies were flat and moved in parallel, then this hedge deal would completely immunise that currency from movements in interest rates.

An alternative technique would be to again examine each currency ladder in isolation. Each ladder however would be regarded as a portfolio and the PRC amount calculated. The value of a swap with a specific maturity can then be calculated which would minimise the PRC for that currency. Thus effectively, we solve for the minimum variance portfolio for that currency, given that all the weightings except one (that of the maturity of the proposed hedge deal) are fixed. This is a restricted application of the optimal portfolio approach, where only one currency is analysed, only one weighting is allowed to change and the solution is independent of the degree of risk aversion. In other words we are moving the portfolio a small way towards the global minimum variance portfolio for that currency by adjusting just one asset weighting.

Both these approaches ignore the covariances between different currencies but recognise the practical considerations involved in hedging. The portfolio approach however is at least consistent with the use of covariances in the calculation of the PRC on the entire portfolio. In addition the duration approach, and even a modified duration approach, rely on some fairly unrealistic assumptions about yield curve movements. As a way of condensing the portfolio into a single equivalent position however the duration technique may prove valuable as an additional risk measure.

If we are to applying the portfolio approach it is useful to examine the relationship between the risk on different swap positions. This is because the problem of changing variances again becomes an issue when using this technique. An understanding of the way in which this instability impacts on swaps of different maturity is needed so that the implications of alternative hedge deals can be appreciated. For example, the choice of which maturity to hedge in may not simply be a function of liquidity. The resultant position must be analysed in terms of its risk to changing variances so the effects of instability on different swaps must be carefully examined.

We carried out several simulations for a single currency, the GBP, by calculating the PRC amount for individual swaps over the same data period using the same approach as for the spot portfolio described above. In all, ten different swap maturities, including forward/forward swaps, were simulated and the results are shown in Figures 47 to 56 and Table 3.1. The first observation is that, as we might have expected, the amount of PRC increases both as the gap between the near and far ends of the swap increases and the further out the far end is. In Table 3.1 the gap is ranked by size along with the average PRC amount so that the high degree of correlation can be easily seen. A second observation is that a kind of portfolio effect is evident in the relative risks on different positions. For example, the risk on a $1m forward/forward 6m v 12m swap is $6,581 on average, which is less than the sum of the risks on spot v 6m and spot v 12m positions even though the net position could be the same in both cases (depending on whether the positions had opposite signs). In other words, if each swap was analysed separately, the total risk would be greater than that calculated after netting off positions which offset.

A further observation is that the average size of the PRC tends to increase almost linearly with increasing far date maturity. Thus all the 12m far dates have about $6,500 of risk, the 6m far dates about $3,000, the 3m about $1,500 and the 1m about $500. This suggests that a duration weighting approach may not produce results too dissimilar from a portfolio approach. This is because if we suppose we just had a single swap to hedge, say a $5m spot v 12m, then if we were to use a spot v 6m swap as a hedge under the duration approach the size of this swap would be $10m. From our simulation results it appears that a spot v 6m swap is about half as risky, that is it takes half the PRC of a spot v 12m swap. If we assumed that the correlations between the two swaps was high, in fact it would be perfect under the duration assumptions, then the minimum variance solution would give a swap amount in the 6m band of about double the 12 swap. Thus the two approaches discussed above might in fact give similar solutions when there is only one swap to be hedged. Of course in a more diversified ladder the differences would probably be larger as all the covariances effected the final solution.

By comparing the shape of the graphs in Figures 47 to 56 another interesting result can be seen. It appears that the way in which the risk changes temporally is very similar for swaps with the same far maturity date. Thus for example all the 12 month far date swaps (Figures 50,53,55 and 56) follow the same pattern of change in the PRC over the period. Similar behaviour can be seen in the 6 month far date swaps (Figures 49,52 and 54). Therefore although it is both the size of the gap and far date maturity which determine the level of risk it is the far date alone which appears to dominate in determining the way in which the risk changes through time. This has important implications for the type of swap used in hedging when we recognise that the covariance matrix is unstable. Since swaps with the same far date appear to behave in a similar way under changing variances, it may be better to choose to hedge at the far end of the maturity ladder rather than the near end even though the transaction costs will probably be higher. Thus if we had, for example, a 6m v 12m position it would be better to hedge this with a spot v 12m swap rather than a spot v 6m swap since the former will respond in a similar way to changing variances as the original position. This hedge would also be preferable since it would leave a spot v 6m position which appears to carry less PRC on average than a spot v 12m position which would be left if a spot v 6m hedge had been used.

3.4.3 Discussion and Conclusions

In this section we have proposed a new approach to managing a portfolio of foreign currency cashflows which is based on using the portfolio variance to calculate the

Position Risk Capital (PRC) amount measure of exposure. The approach differs from the traditional application of mean-variance portfolio analysis to hedging currency portfolios in two key respects. Firstly, the calculation of an optimal portfolio of currency positions is not required. Rather the practical problems of changing cashflows, reluctance to wholesale changes in positions or overtrading, together with problems such as mandates and credit lines not previously mentioned, are recognised. These practical considerations suggest a less ambitious approach which is still based on an objective measure of risk but uses a more flexible technique for reducing this risk which requires only marginal changes to the underlying portfolio. The approach eliminates the need for the treasury manager to specify his level of risk aversion explicitly. Instead his aim is assumed to be simply to manage the exposure using some objective criteria for the level of risk. He is then allowed some flexibility in the choice of hedge deal but has a procedure for assessing which deals may be appropriate and analysing the effect these deals will actually have on his overall position.

The second difference is in the way in which the structure of the portfolio is defined. We propose an approach in which future cashflows are decomposed into spot and swap positions rather than being regarded as outright positions. This has the benefit of enabling the spot risk and the interest rate risk to be analysed and potentially hedged separately. Thus the treasury manager can use the model to directly examine the effect of deals he might perform in the market to hedge his portfolio. Since the approach we have put forward does not explicitly generate any hedge deals, as the optimal portfolio approach would, it is important that the treasury manager can examine his positions in greater detail. The splitting of outright cashflows into spot and swap elements is also consistent with the way in which the market operates in practice.

The approach then is two staged. First the PRC amount is calculated to provide an absolute or more likely a relative measure of the riskiness of a given portfolio. It can then be used to monitor changes in exposure as cashflows change in size and timing or as the basis for "what if" scenario testing when examining the effect of proposed hedge deals. The analysis of the relationship between different forward swap positions which we have presented suggests possible approaches to hedging or reducing the PRC amount in response to new cashflows, or a changing exchange rate environment. This second stage is in a sense subjective with a range of hedge deals possible , however the maintenance of the PRC within a given limit provides a restriction on this range which is objective.

One of the central rationales behind the model is that any use of mean-variance portfolio analysis is subject to the problem of the normal assumption and perhaps more importantly of instability in the covariance matrix of asset returns. The empirical work presented in the previous chapter suggests strongly that these issues must be of some concern in the context of currency portfolios both for spot and swap positions. Since the optimal portfolio approach is reliant to a large extent on the validity of these assumptions it must be suspect when we know that severe departures from a stable normal distribution are found. In our approach the covariance matrix is also central to the procedure. However it is only used to calculate a relative measure of risk. No specific portfolio is aimed at but a flexible approach based on staying within certain risk levels is suggested. Certainly, the question of whether the variance is a valid measure of variability is important (the second moment does not exist if the distribution is a Student with $d < 2$) but the reliance of the optimal portfolio approach on accurate estimates of both means and the covariance matrix suggests that it is more exposed to the validity of these underlying assumptions. The approach which we propose has been developed with the inadequacies of the underlying assumptions in mind. It is for this reason that no specific target for the PRC is suggested. The model in fact simply provides a framework in which exposures can be analysed with as little reliance on assumptions about the underlying price process as possible. No direct risk reduction strategies are put forward but techniques for analysing the components of the risk and for calculating possible hedging deals are suggested and the assumptions underlying them clearly stated. When examining alternative swap deals for example we have looked at the effects of changing variance on different maturity positions. It is up to the treasury manager to evaluate possible strategies within this framework and find his preferred mix of hedging deals which give the desired level of risk and reliance on distributional assumptions.

3.5 Final Conclusions

In this chapter we have examined the use of mean-variance portfolio analysis in the management of foreign exchange exposure. More specifically we have looked at multicurrency portfolios of both spot and forward cashflows from the point of view of the treasury manager wishing to minimise the fluctuation of the domestic currency value of these cashflows. Thus we have focussed particularly on the hedging of transactional exposure. Although a large body of work has been carried out on the use of the mean-variance approach in managing international investment portfolios relatively little has been done from the treasury managers viewpoint. We have nevertheless reviewed the major work that has been done in this area.

We have then proposed an alternative approach to managing this exposure. The model is based on a similar objective measure of risk, calculated from the portfolio variance, as that used in previous work. However, a more flexible approach to managing the exposure is adopted. This flexibility is backed up by a more detailed separation of assets within the portfolio which allows the elements of the underlying risk to be more easily analysed. Simulations have been done to test the model over actual data in order to highlight some of the implications of this approach when the assumptions, in particular the stable normal distribution, are inaccurate. The rationale behind the model is to understand the inadequacies of the assumptions underlying the portfolio approach and the practical problems involved in implementing a full optimal portfolio strategy. In this way a more realistic approach to the management of these risks can be developed.

Some important issues remain to be addressed. Firstly the problem of illiquid currencies is a practical consideration for many treasury managers. The portfolio approach provides an ideal framework for managing these exposures. Using the ideas developed by Sharpe, we can use a reduced form of the full variance-covariance approach in order to efficiently incorporate these positions into the model. The essence of this approach is to group together assets, in our case currencies, which have similar behaviour. Thus in the case of stocks we may have a group for banks, another for chemicals, etc. For currencies, we may look at the EMS currencies together, those that are pegged to the dollar, etc. Since most of the illiquid currencies can be grouped with a more liquid currency, it is possible to reduce the actual number of currencies to be analysed to perhaps only five or six. This also reduces the computation time allowing more detailed analysis of possible hedge deals.

Secondly, we have not included conventional loans and deposits or other financing transactions such as interest rate swaps which the corporation may have entered into. If these are denominated in a foreign currency then it is of course possible to incorporate these cashflows into the portfolio model. In most corporations the financing decisions however are usually taken by a different area and before adding them to the natural cashflow positions the tax treatment of these transactions must be carefully examined. The whole issue of the tax treatment of foreign exchange gains and losses is in fact important but it is outside the scope of this research. It is of course possible to use dummy cashflows so that the foreign exchange exposure of these transactions is incorporated into the overall position.

Finally the model which we propose fits into the general approach which we put forward to managing all three types of foreign exchange exposure. That is, it is based on acknowledging both the problems of unreliable assumptions about market behaviour and the practicalities involved in implementing any hedging strategy. This provides the basis for a structured but at the same time flexible strategy which still allows decisions to be made using an analytical approach to the examination of the exposure. The idea is to be aware of the inadequacies of the assumptions underlying these techniques however so that a more informed decision can be made. In the next chapter we will describe the use of currency options in hedging more complex exposures. The calculation of a fair value for these options also relies on assumptions, many of them the same as the portfolio approach. We shall examine some of the implications that these assumptions might have and look at opportunities which they may present.

Table 3.1 Average PRC ($) for Typical $1m Swap Positions in GBP/USD March 1982 to December 1986

Near Date	Far Date	Gap	Rank	Average	Rank
Spot	1m	1m	1	658	1
Spot	3m	3m	3=	1,940	3
Spot	6m	6m	6=	3,904	6
Spot	12m	12m	10	6,717	10
1m	3m	2m	2	1,531	2
1m	6m	5m	5	3,572	5
1m	12m	11m	9	6,478	8
3m	6m	3m	3=	2,770	4
3m	12m	9m	8	6,314	7
6m	12m	6m	6=	6,581	9

Figure 34 : Frequency Distribution of all Weekly Spot Rate Returns December 1981 to December 1986

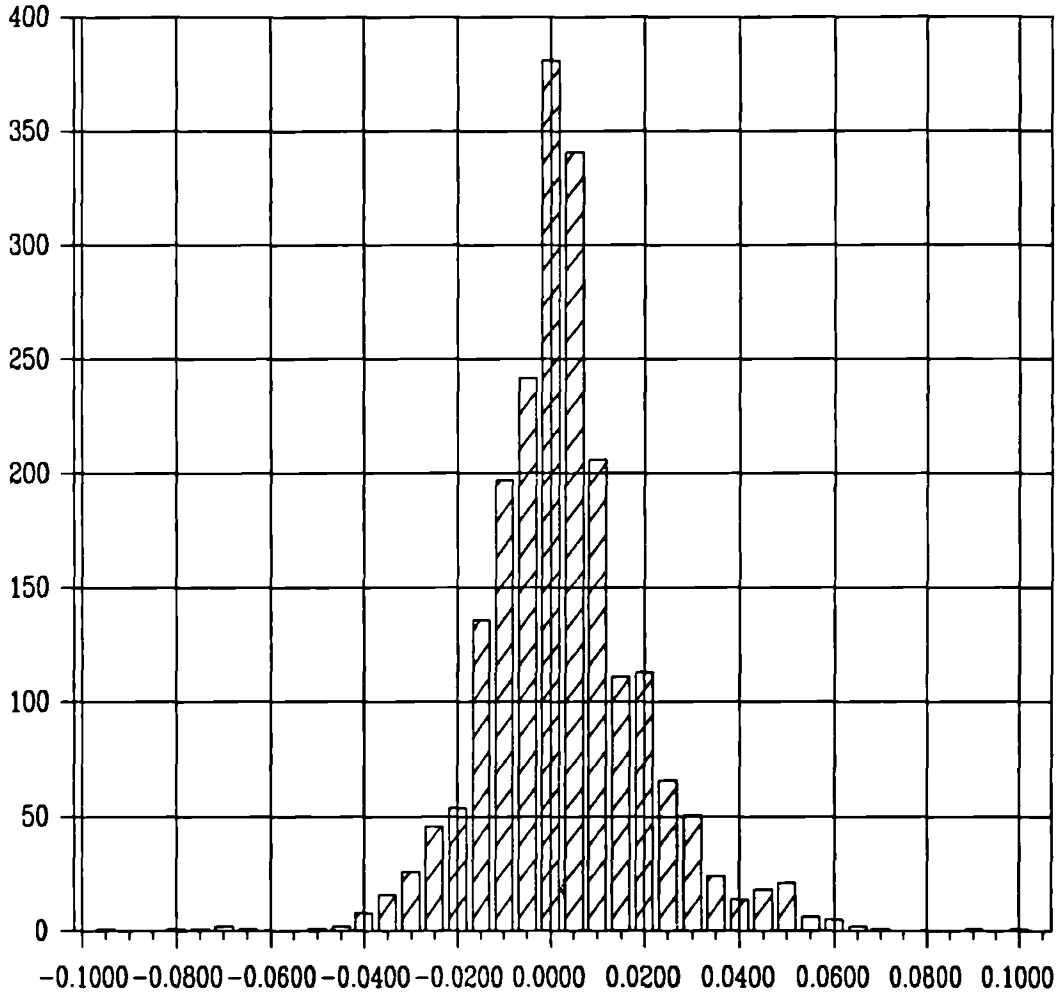


Figure 35 : Historic Volatility of GBP/USD Spot Rate December 1981 to December 1986

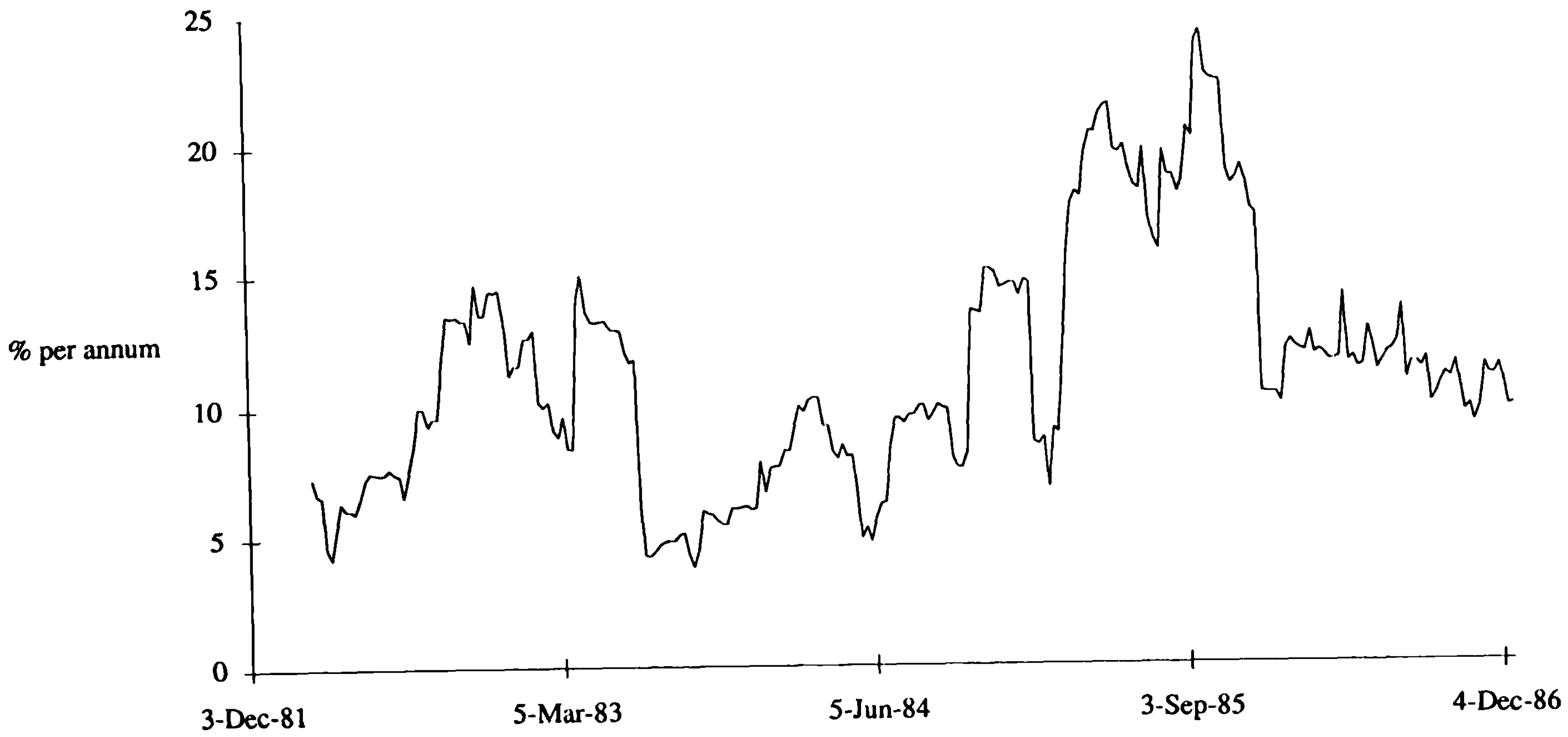


Figure 36 : Historic Volatility of DEM/USD Spot Rate December 1981 to December 1986

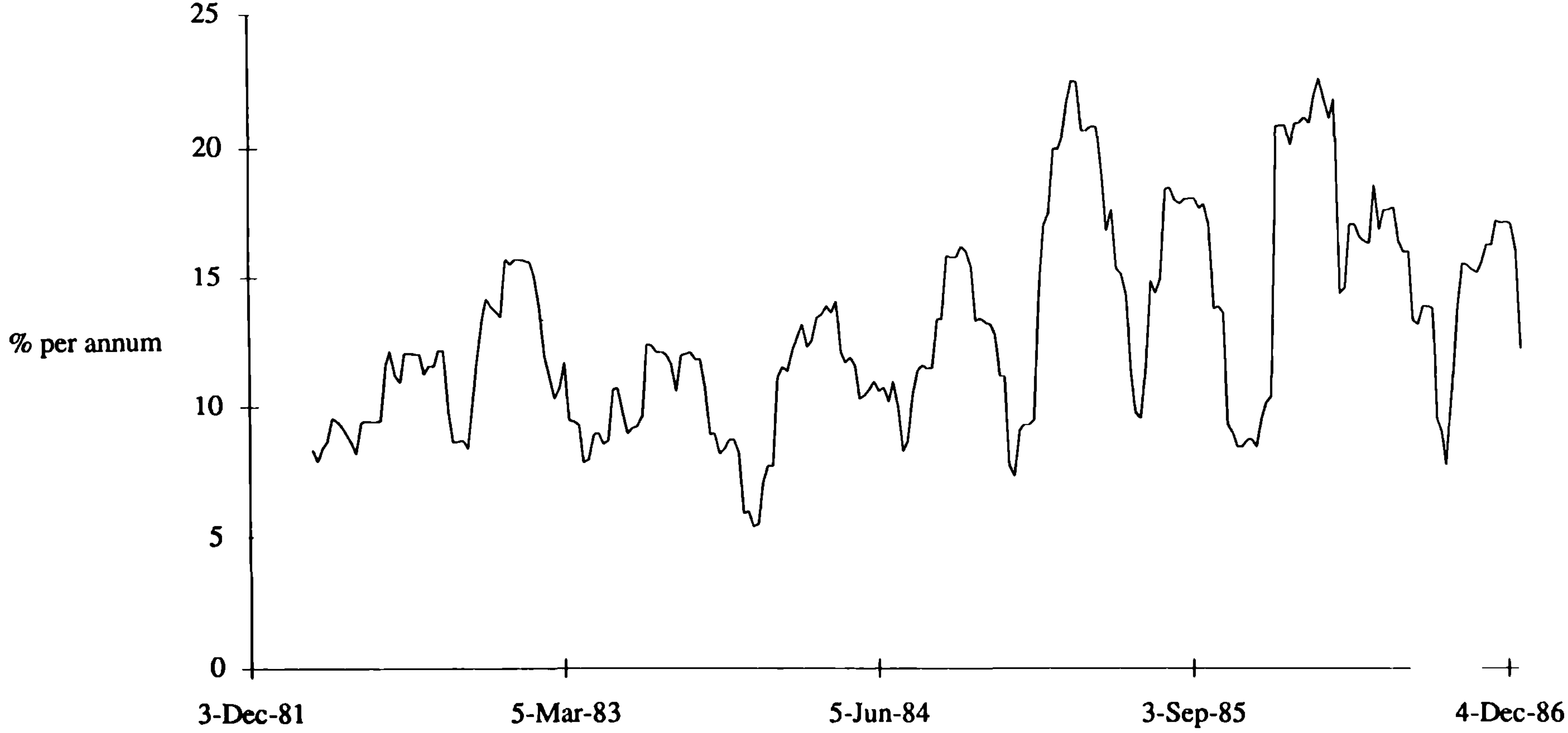


Figure 37 : Historic Volatility of CHF/USD Spot Rate December 1981 to December 1986

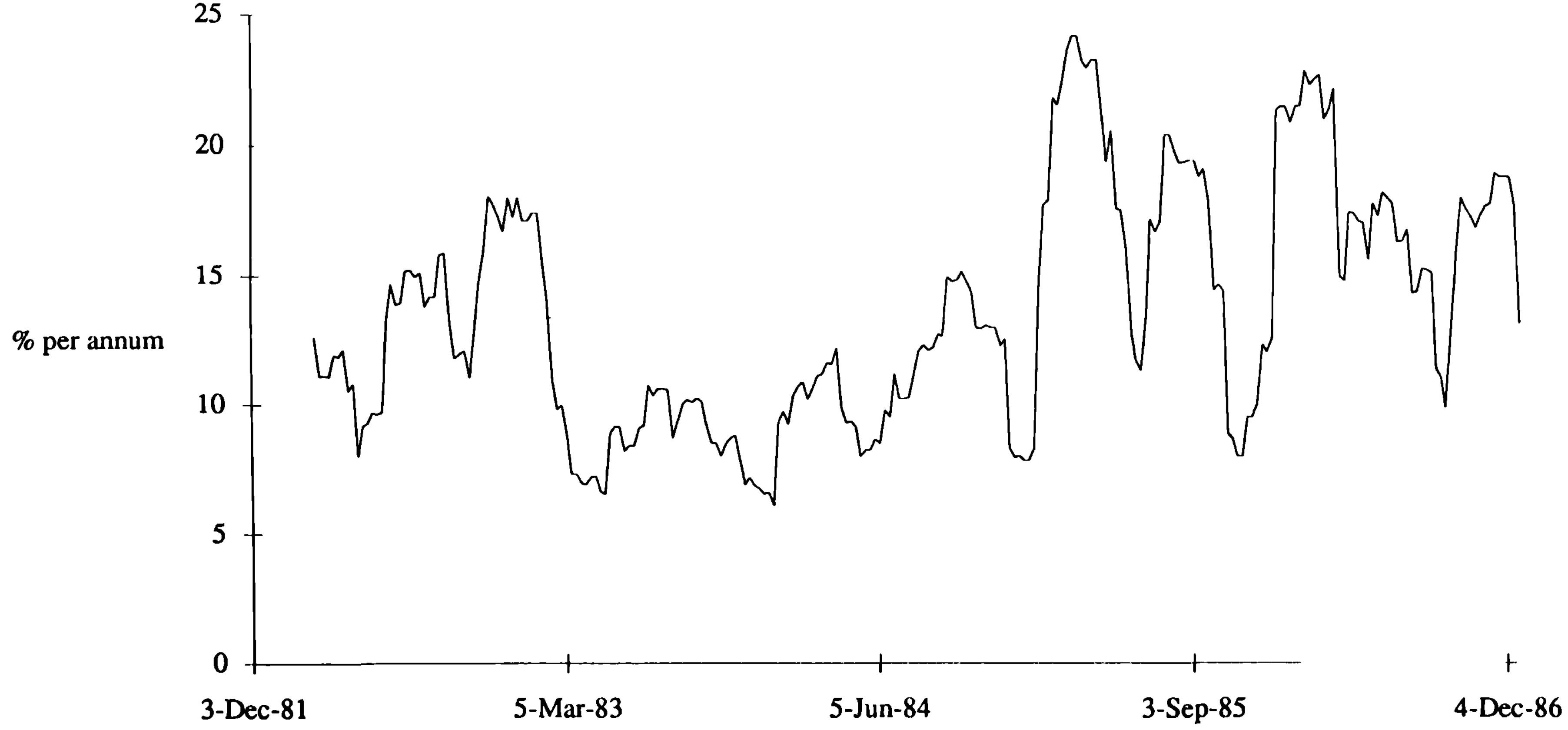


Figure 38 : Historic Volatility of FRF/USD Spot Rate December 1981 to December 1986

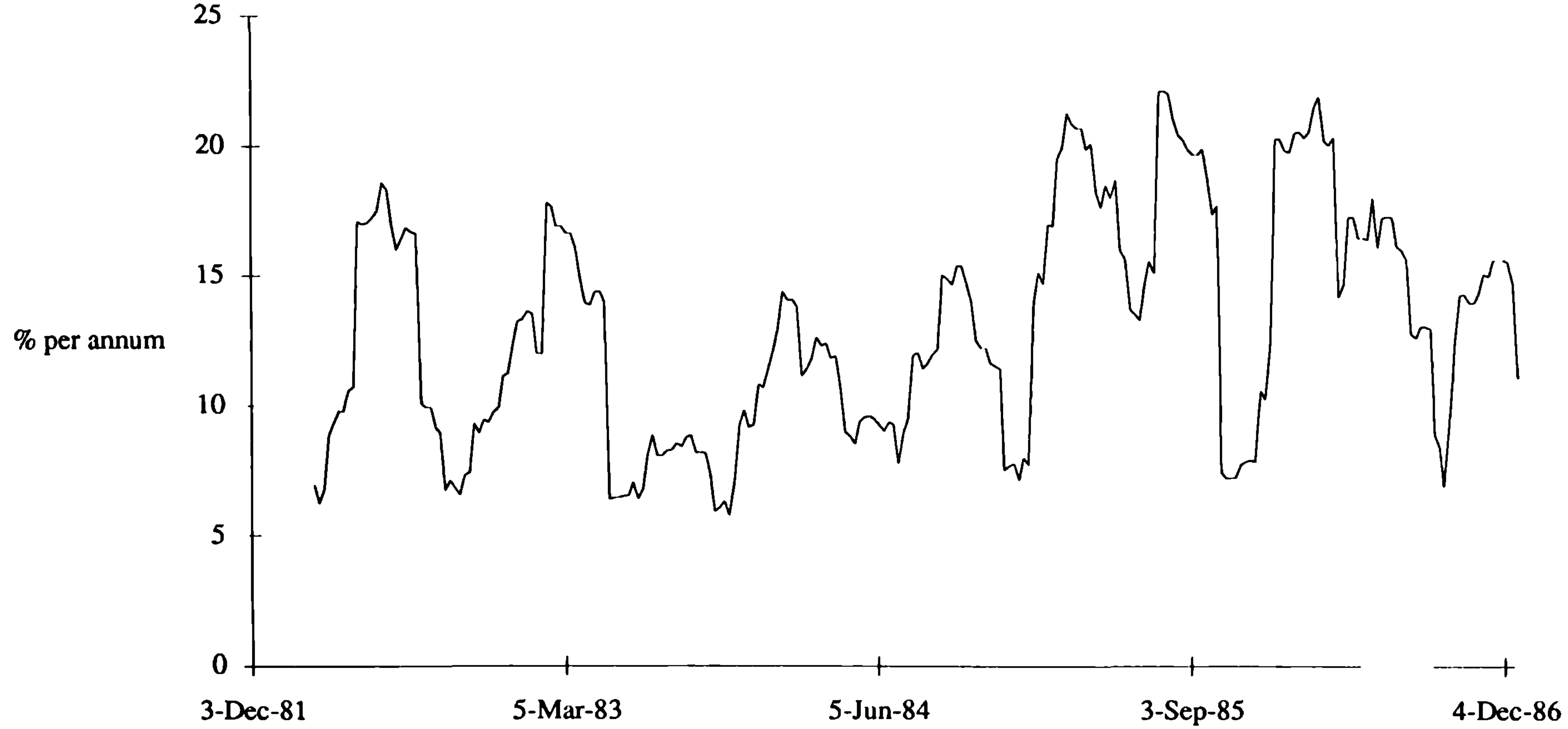


Figure 39 : Historic Volatility of JPY/USD Spot Rate December 1981 to December 1986

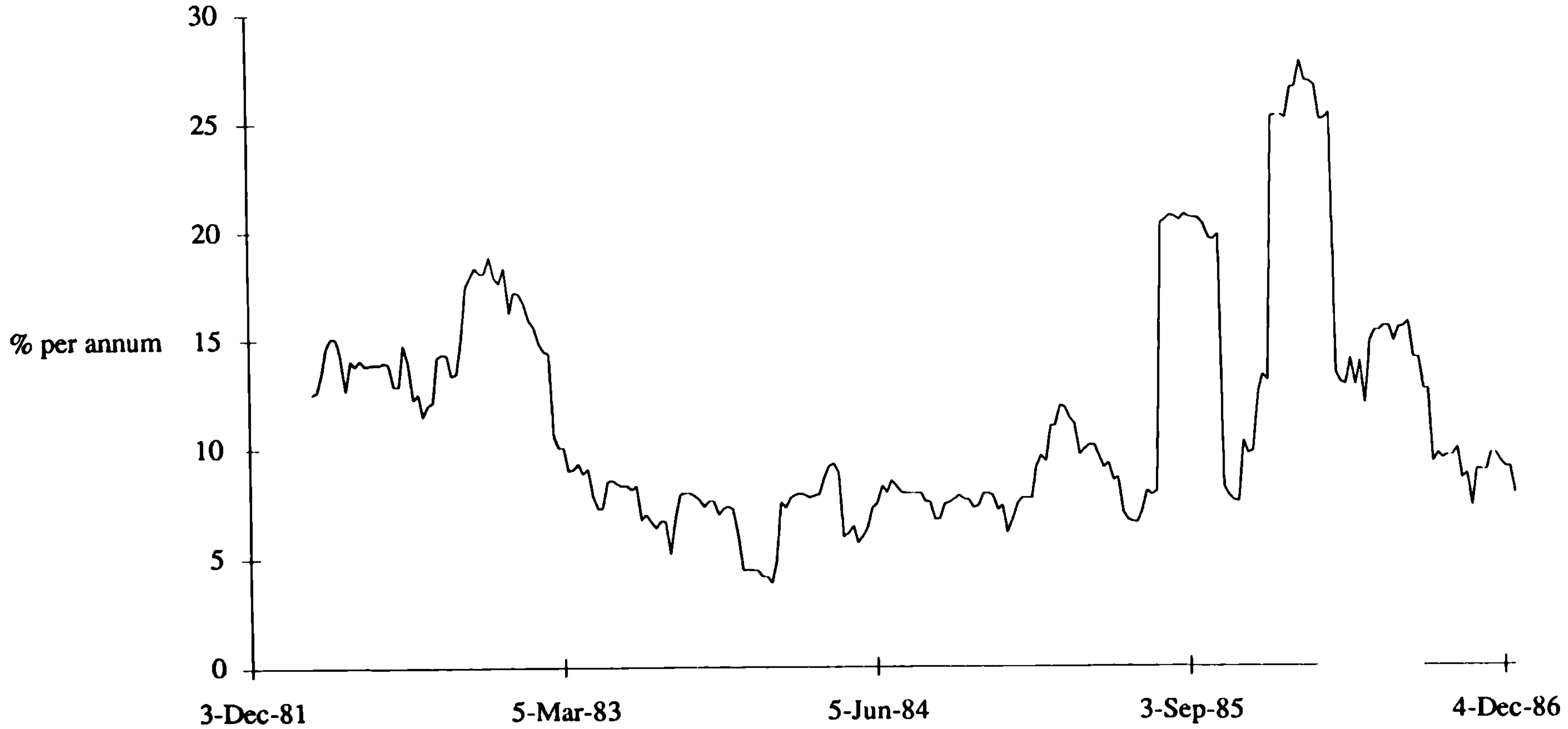


Figure 40 : Historic Volatility of NLG/USD Spot Rate December 1981 to December 1986

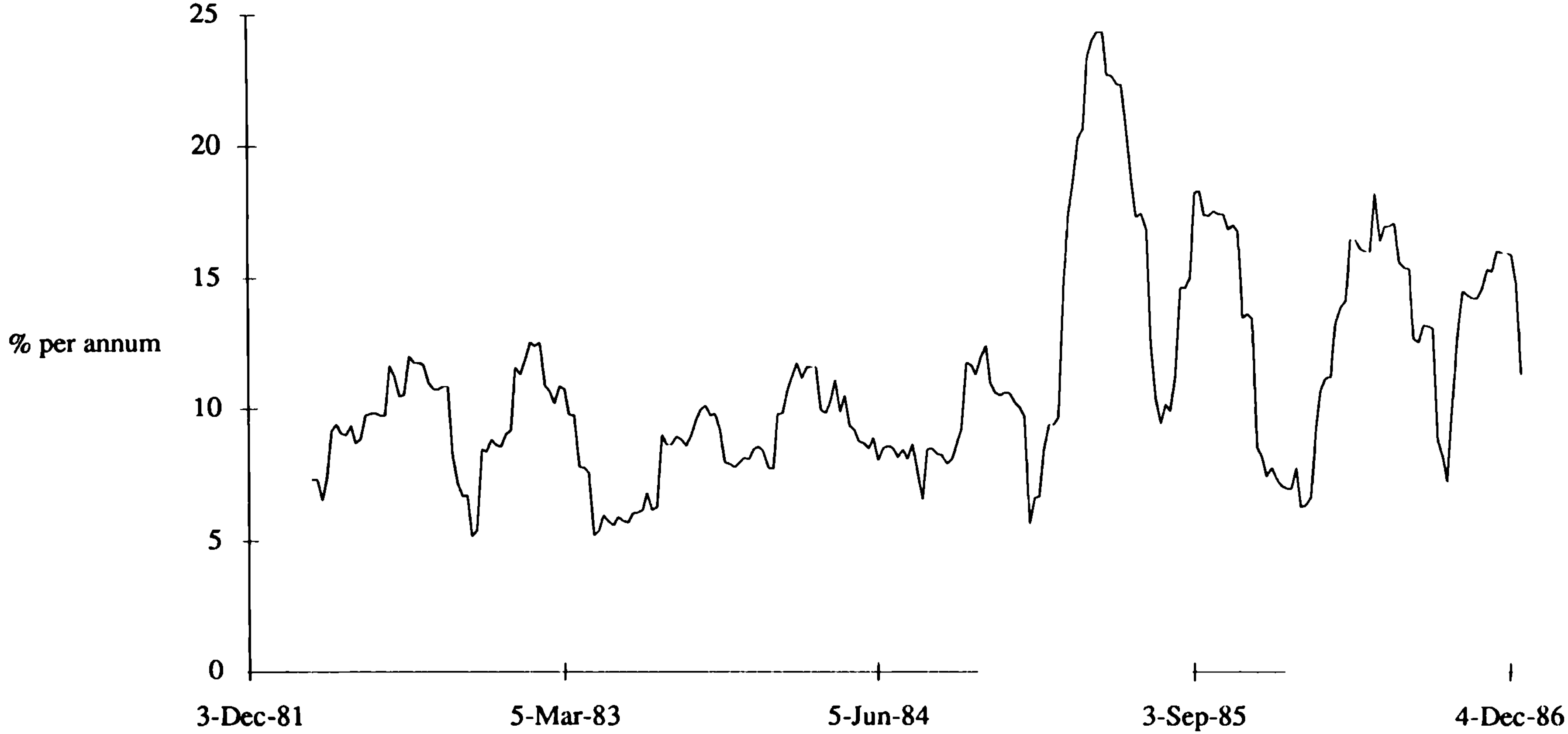


Figure 41 : Historic Volatility of CAD/USD Spot Rate December 1981 to December 1986

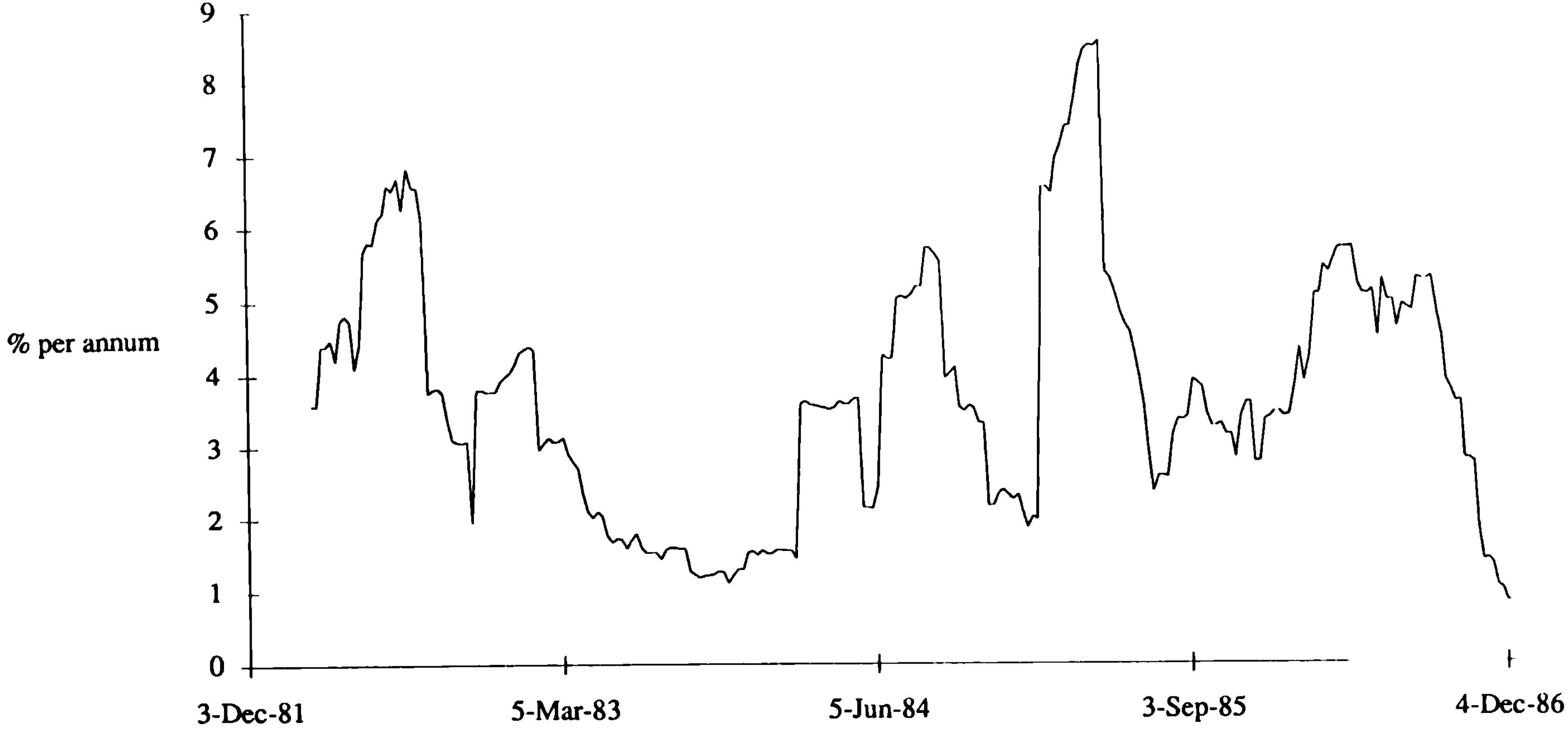


Figure 42 : Historic Volatility of ITL/USD Spot Rate December 1981 to December 1986

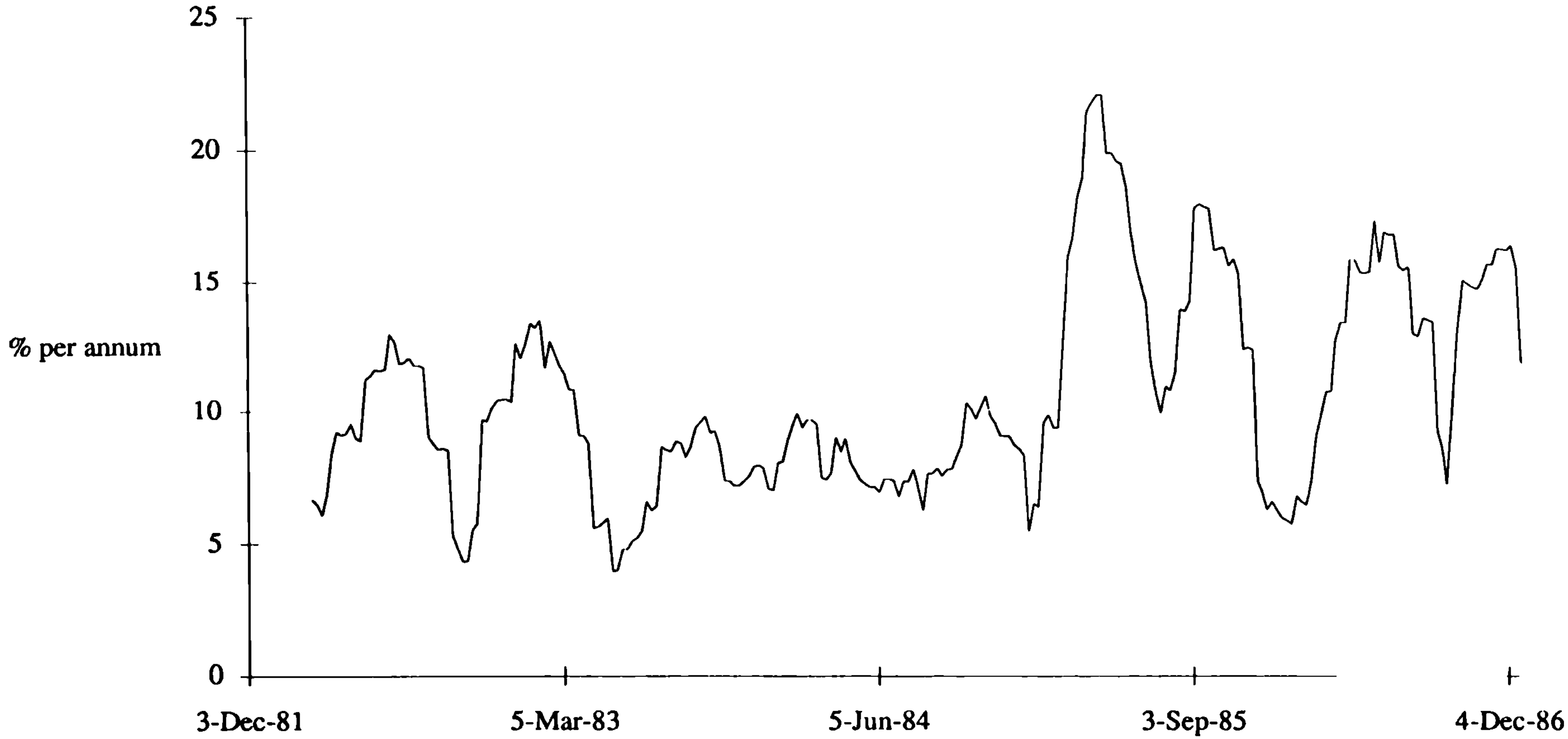


Figure 43 : Historic Correlation between GBP/USD and DEM/USD Spot Rates December 1981 to December 1986

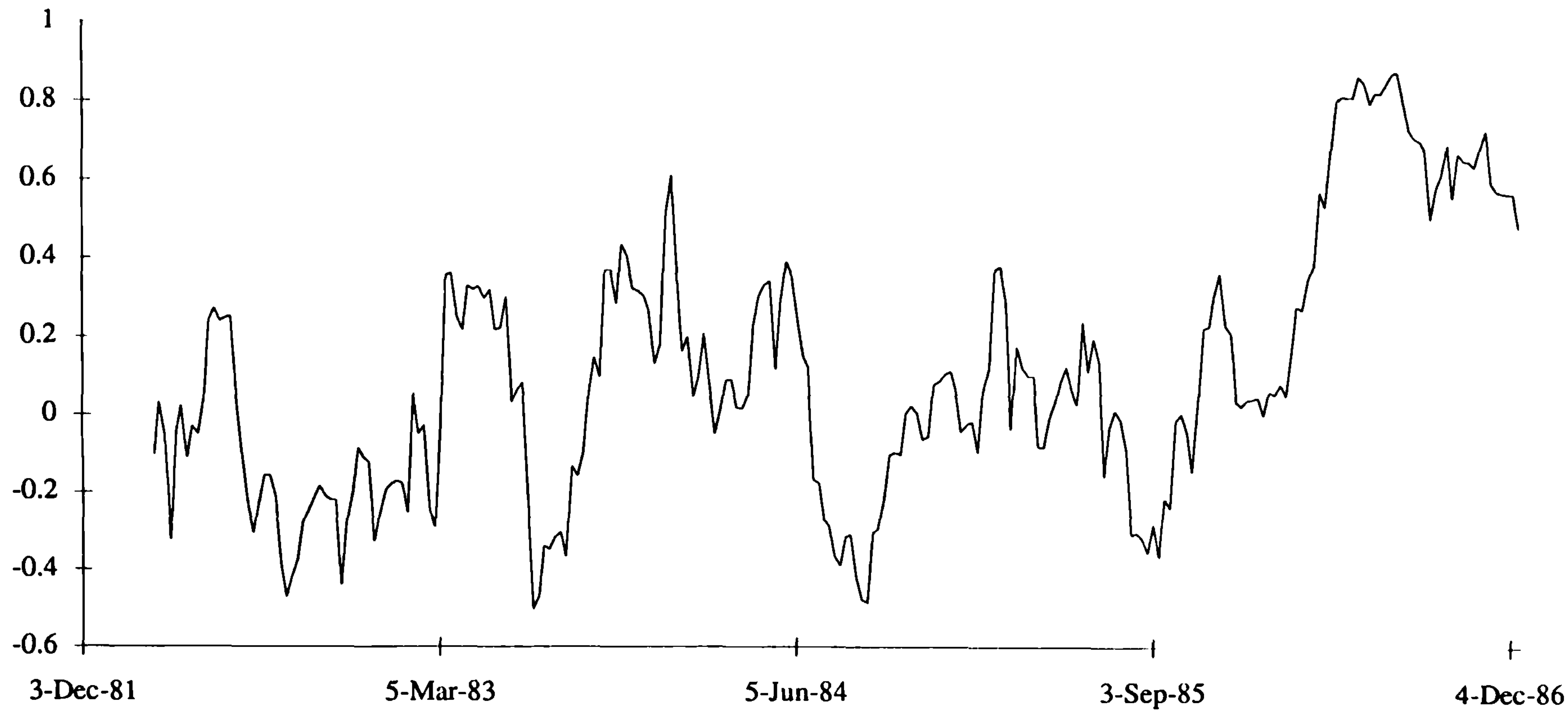


Figure 44 : Historic Correlation between GBP/USD and JPY/USD Spot Rates December 1981 to December 1986

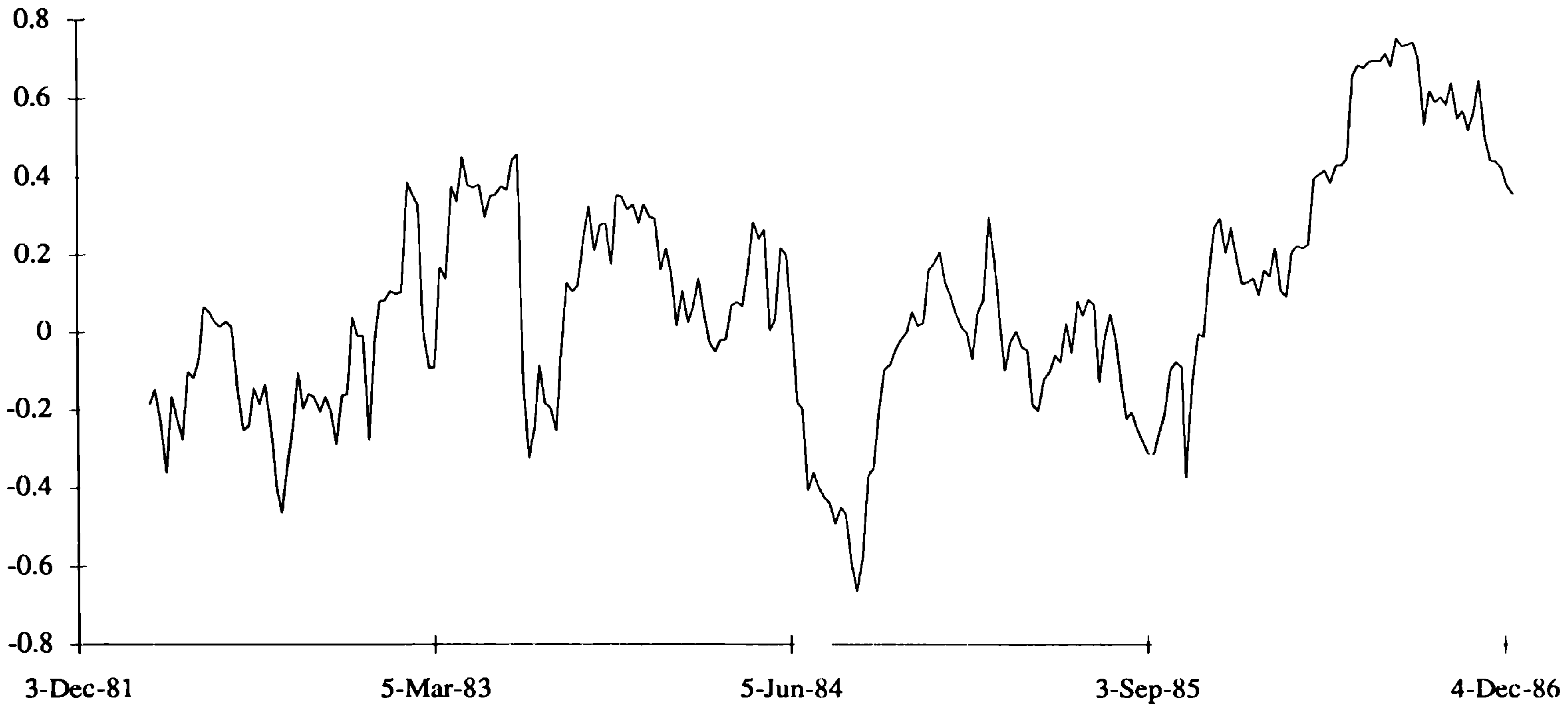


Figure 45 : Historic Correlation between DEM/USD and JPY/USD Spot Rates December 1981 to December 1986

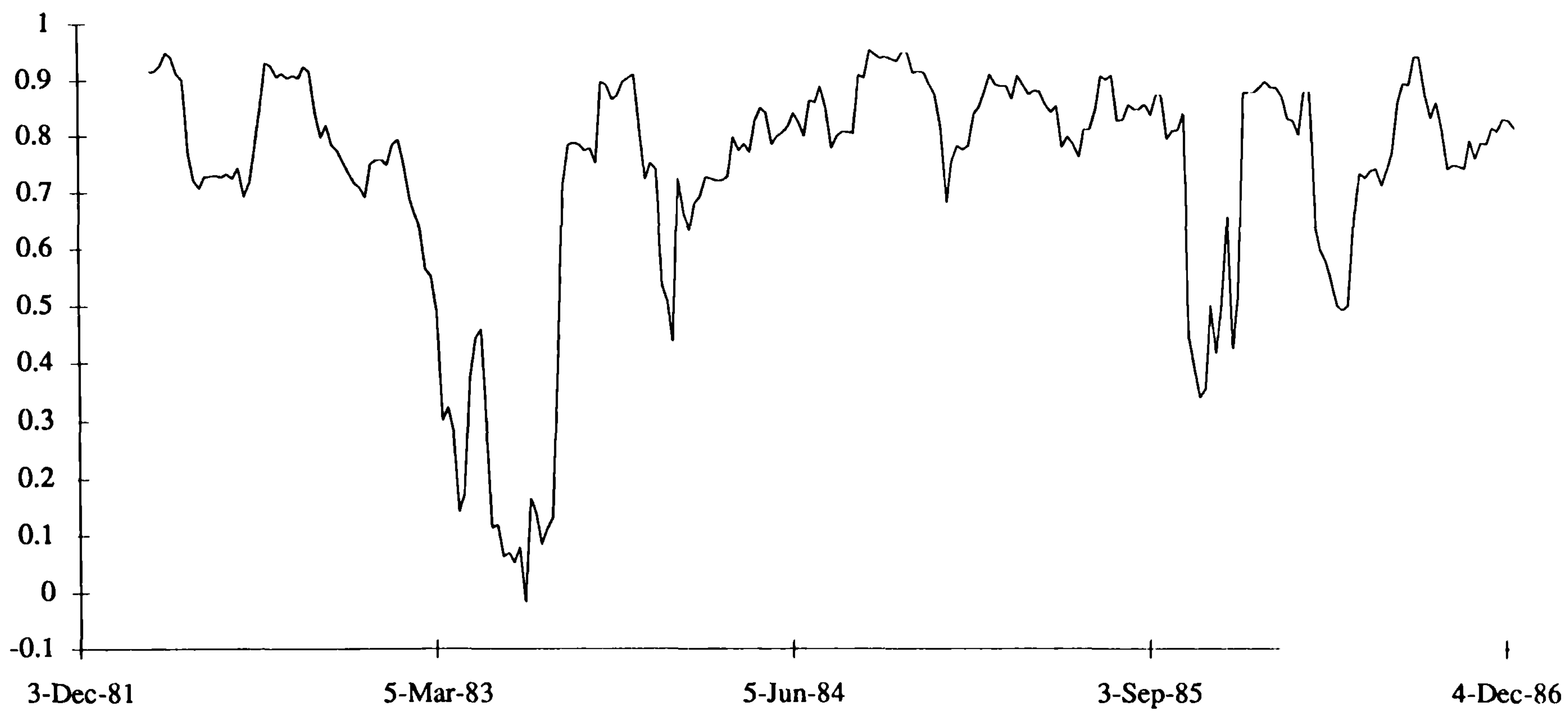
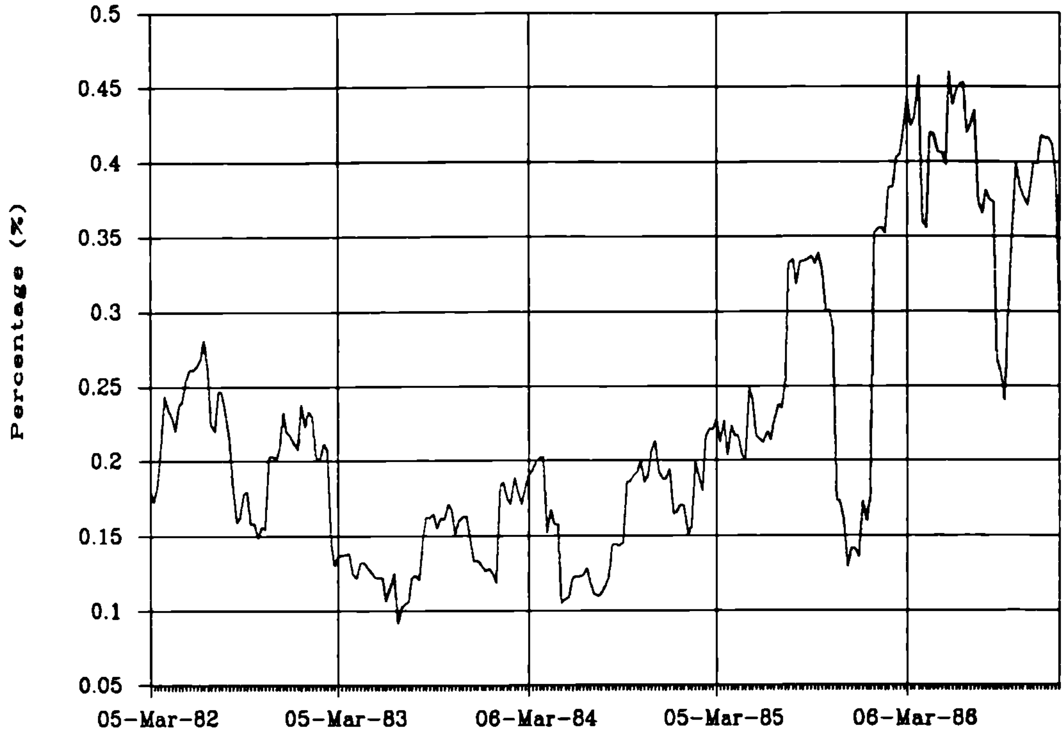


Figure 46 : PRC at 95% Confidence on a Portfolio of Eight Currencies held in Equal Proportions, as a % of Portfolio Value



Figures 47 - 56 : Position Risk Capital on Typical Swap Positions in GBP/USD
March 1982 to December 1986

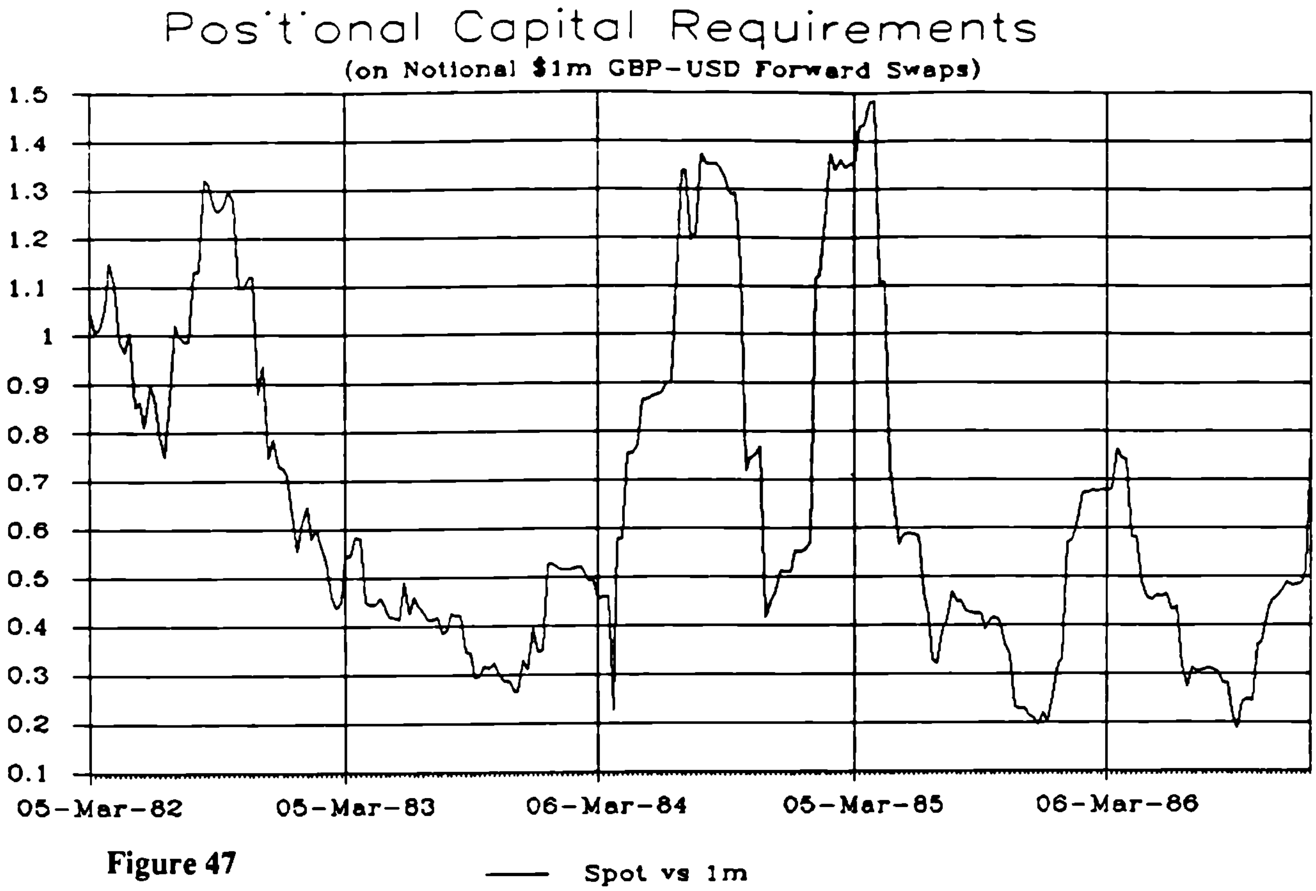


Figure 47

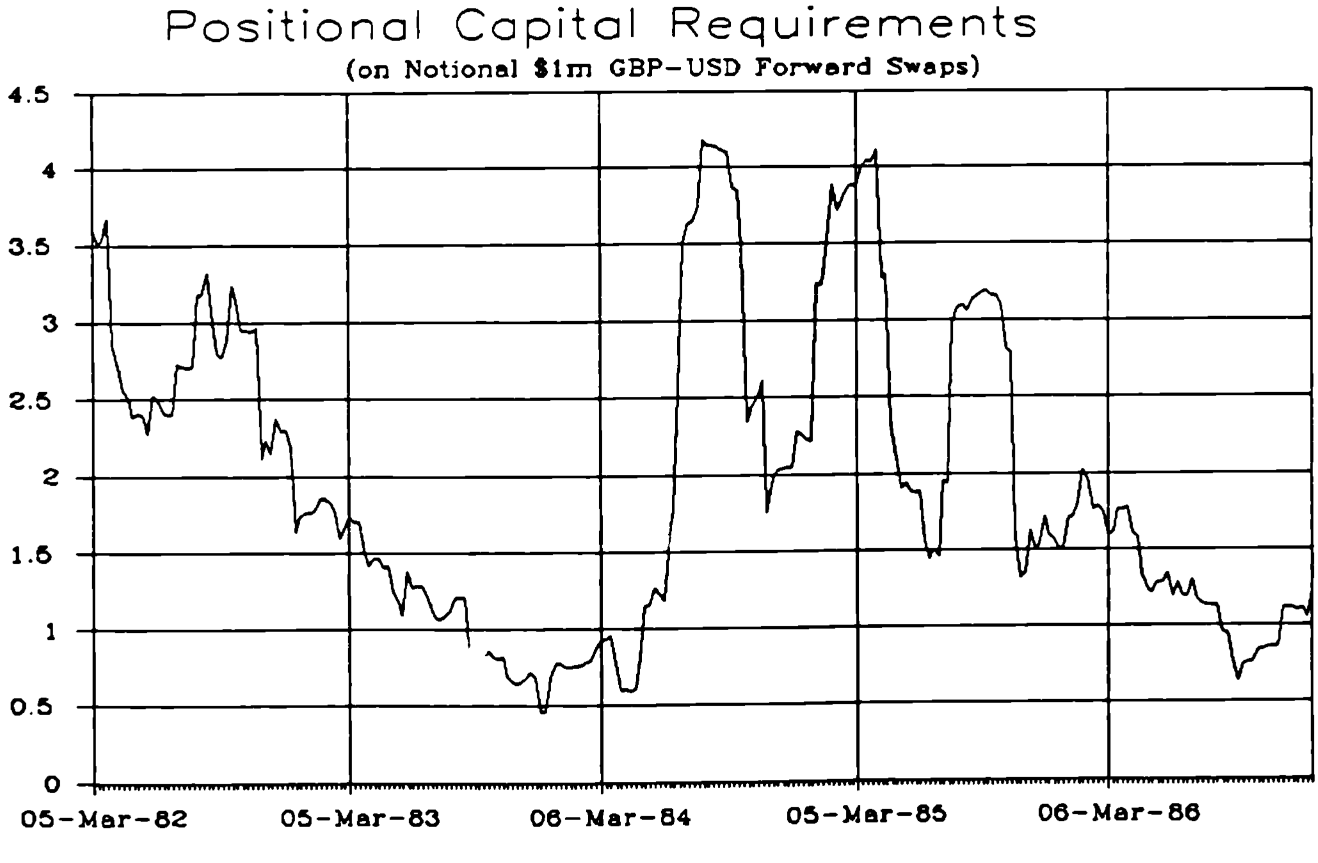


Figure 48

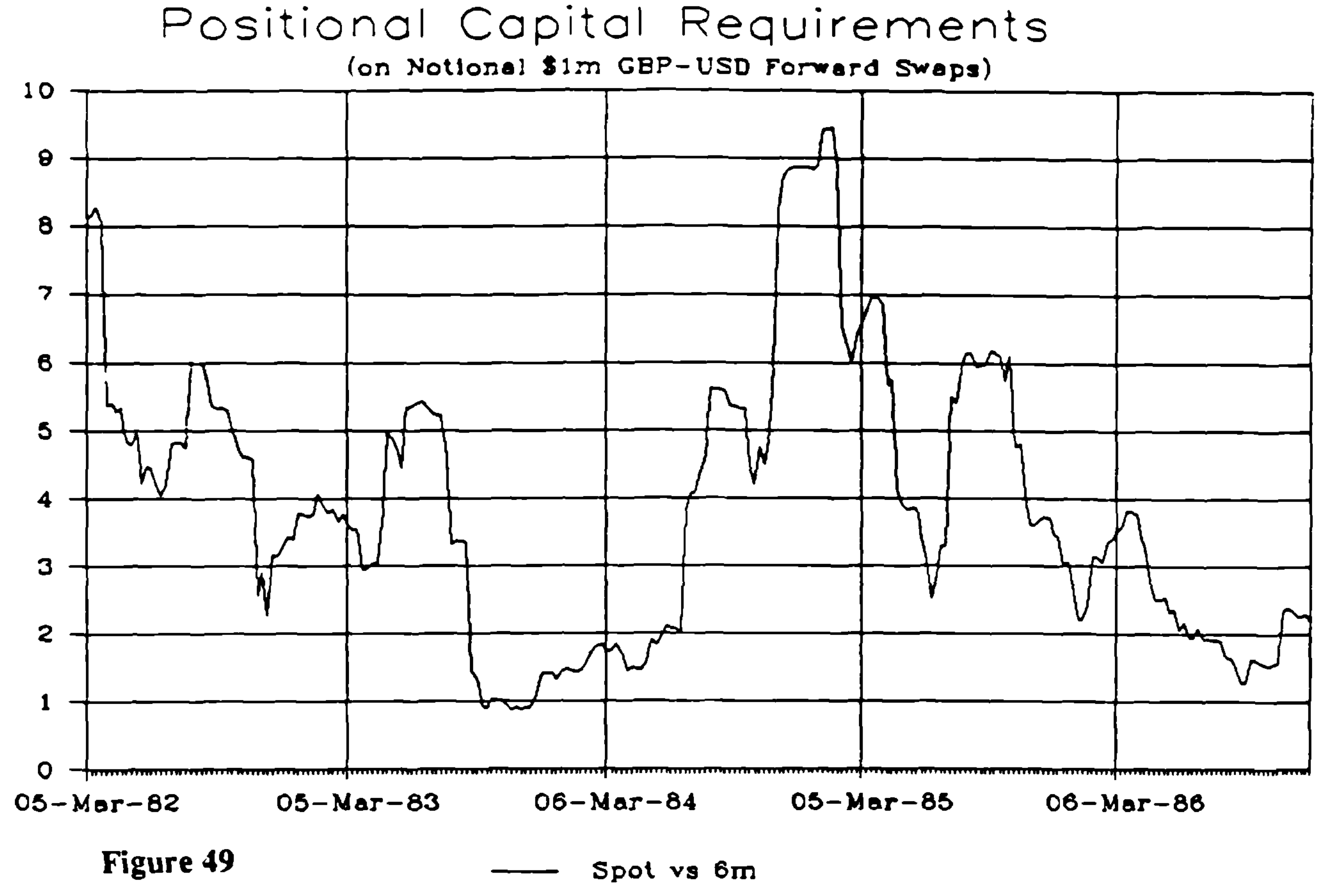


Figure 49

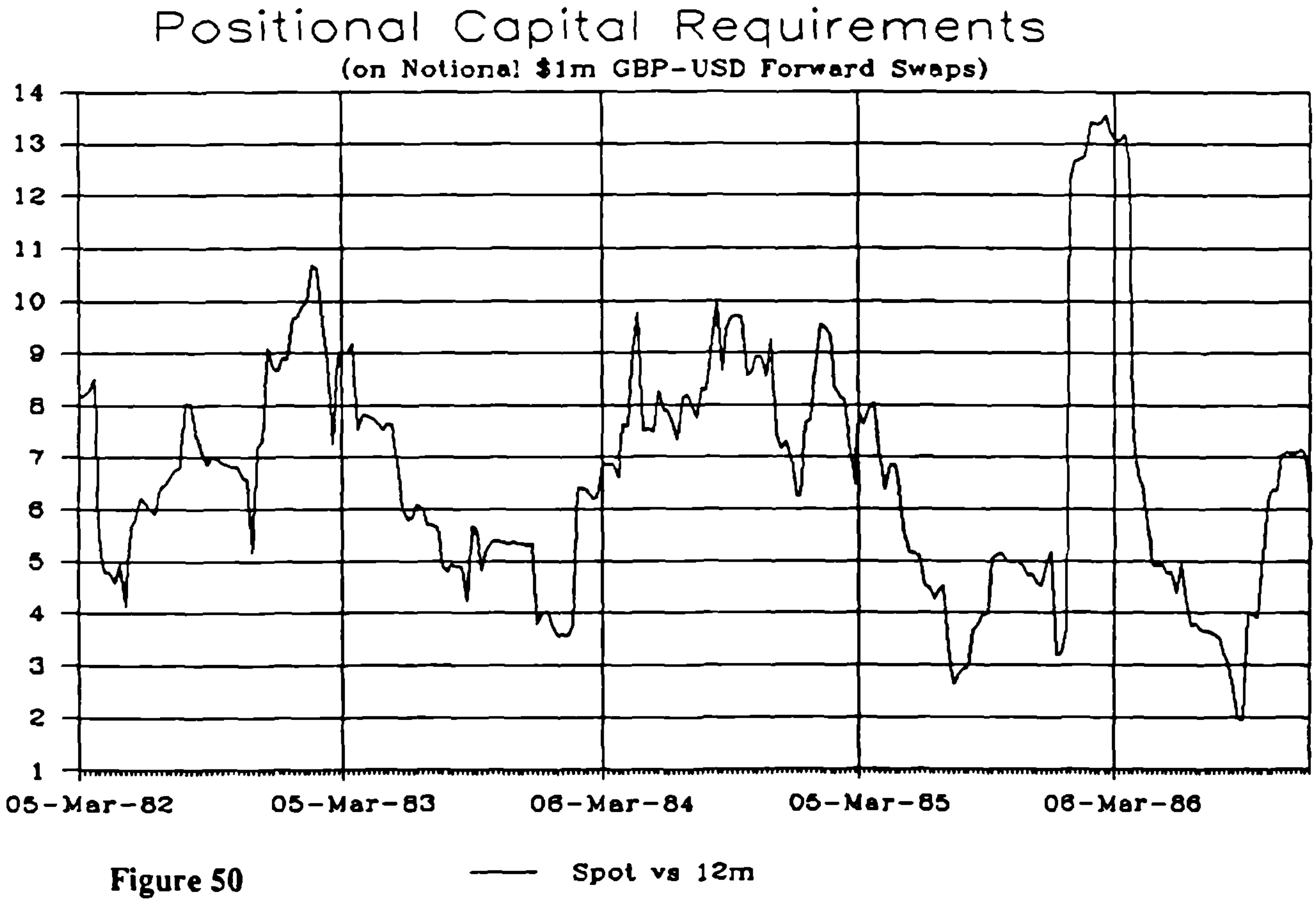


Figure 50

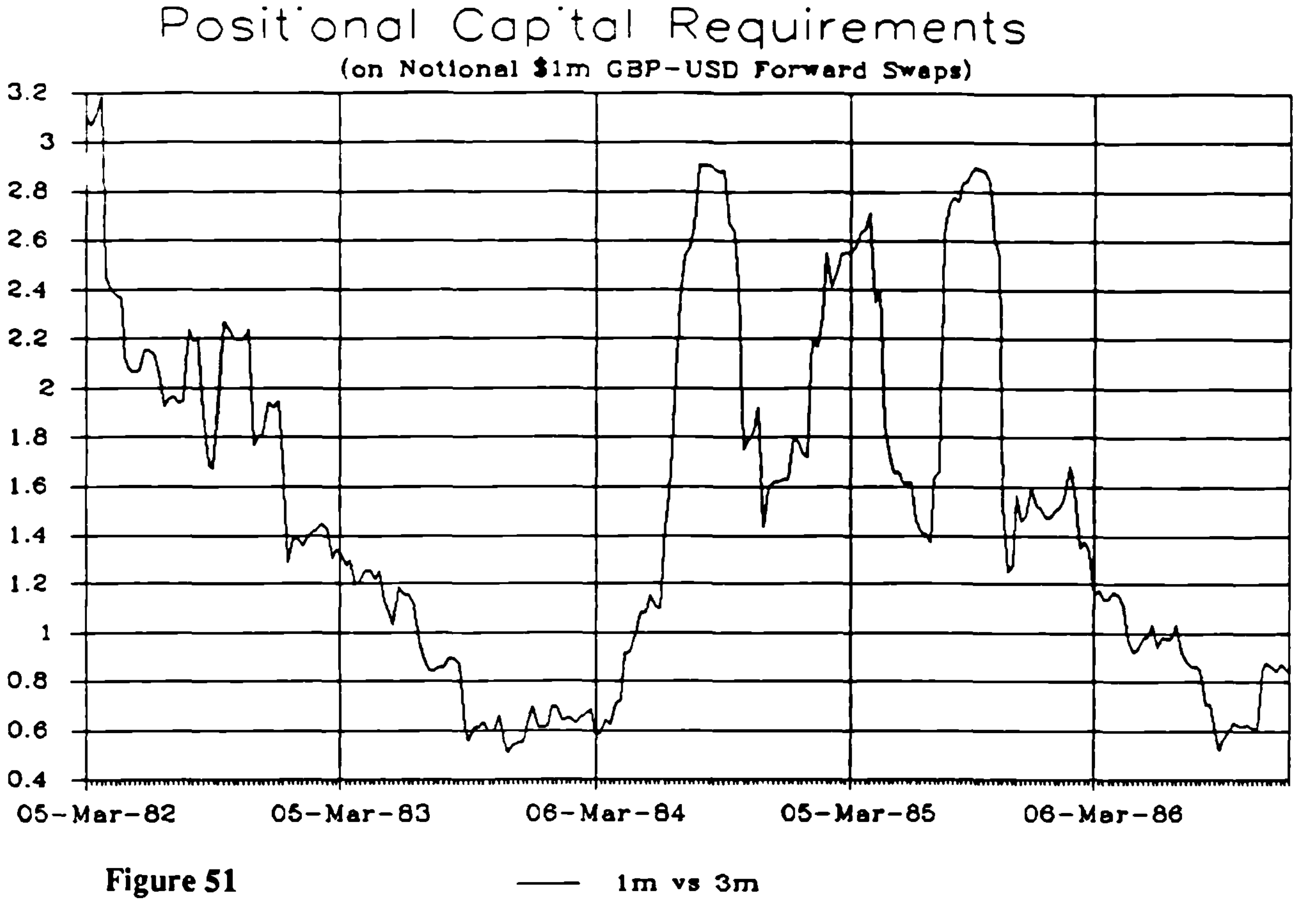


Figure 51

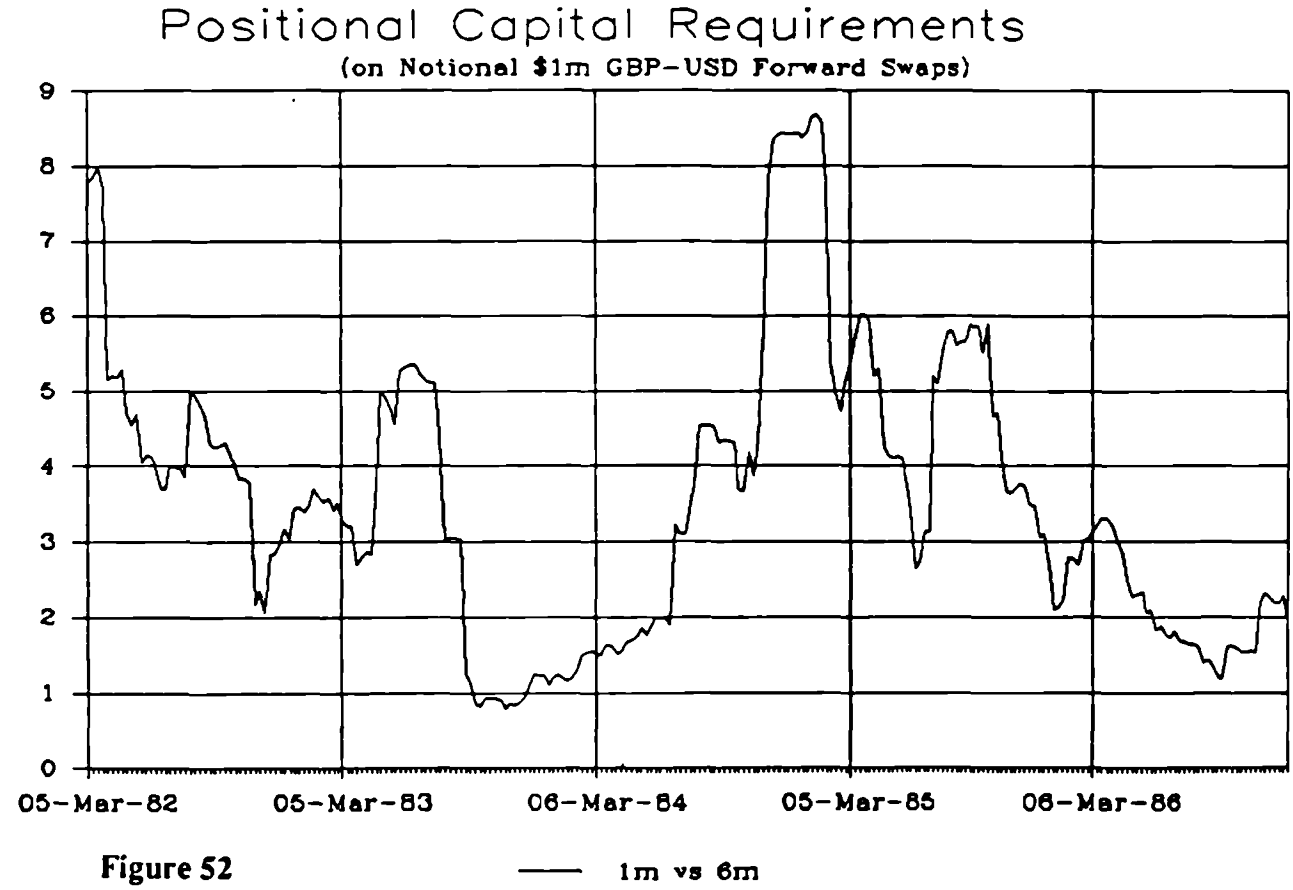


Figure 52

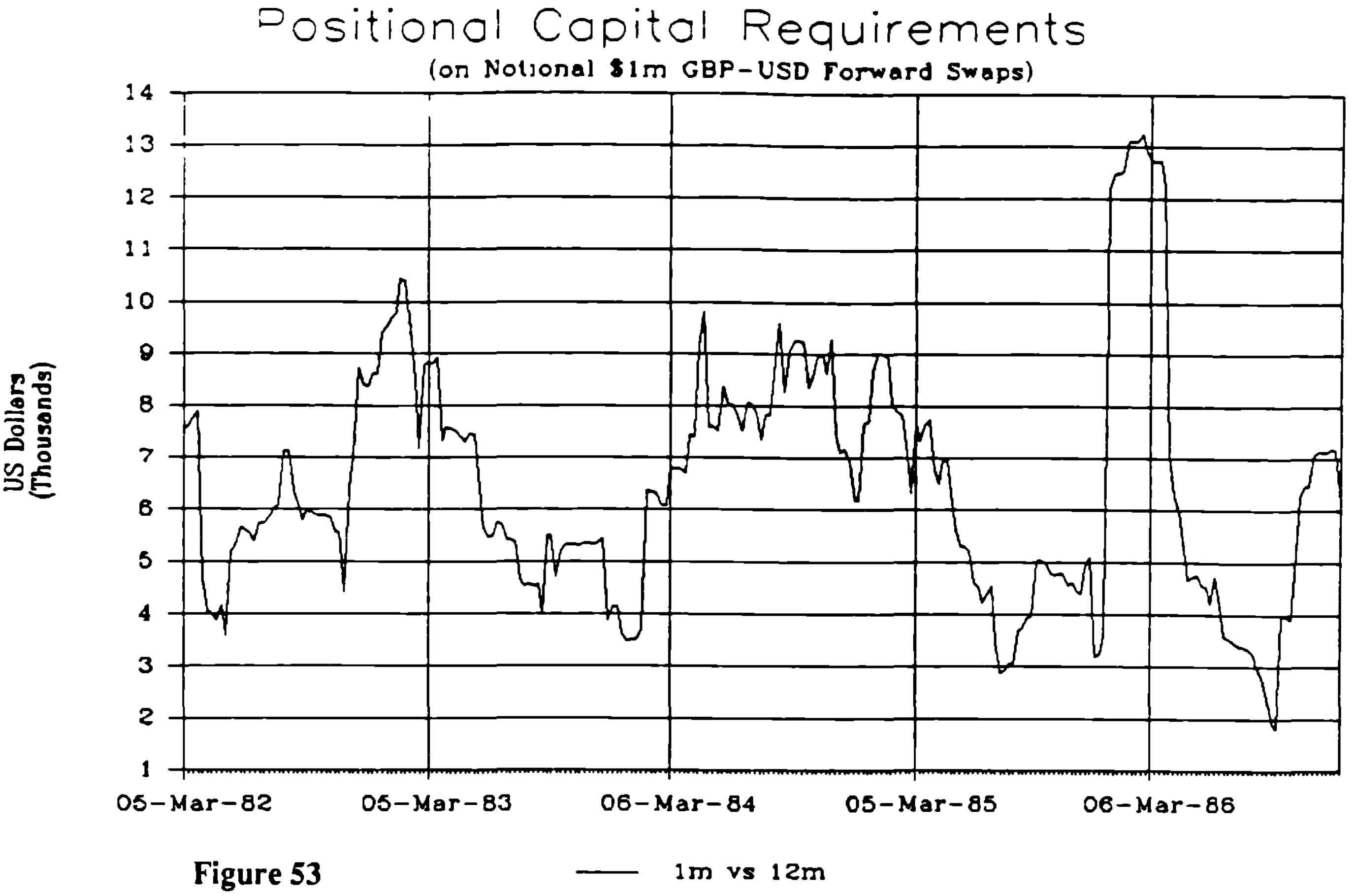


Figure 53

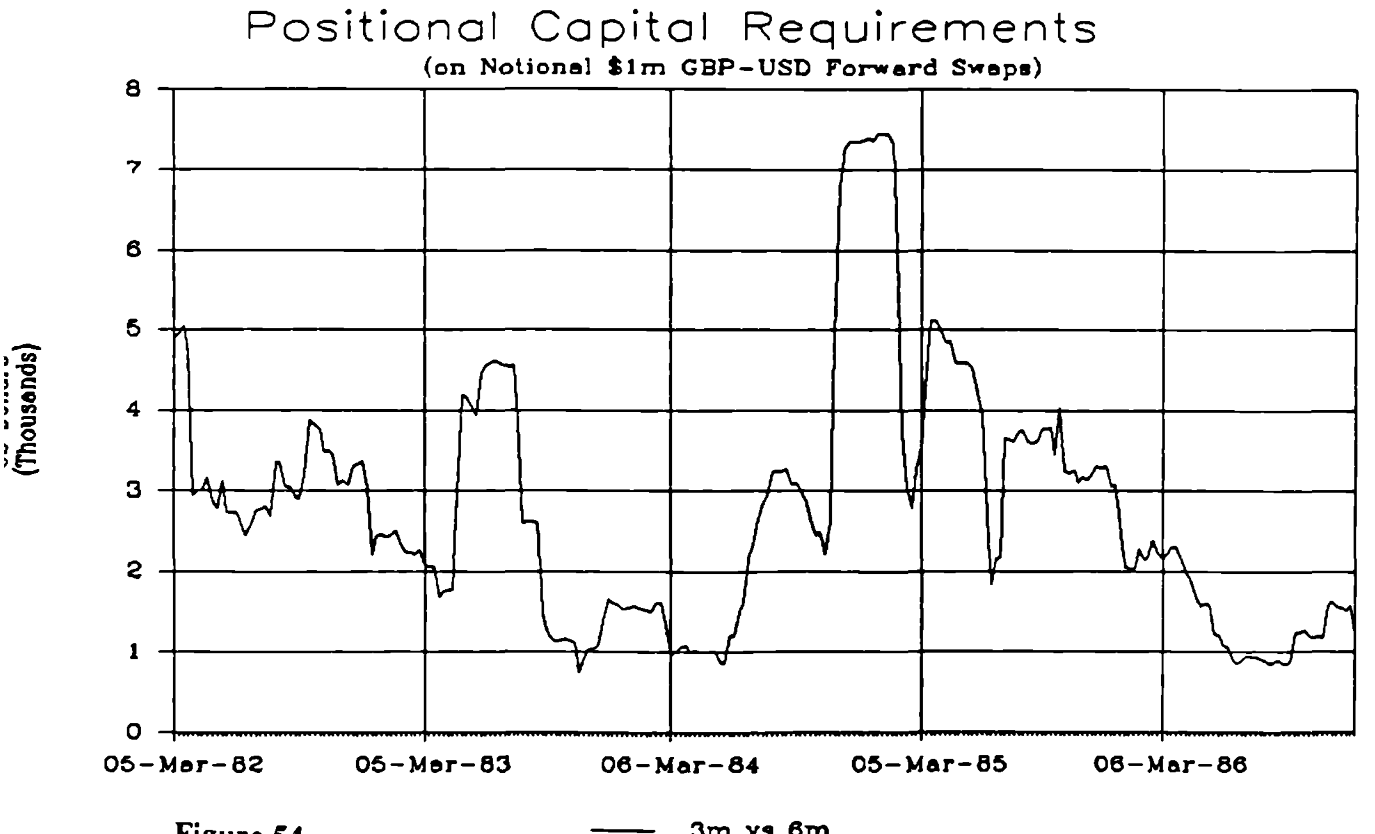


Figure 54

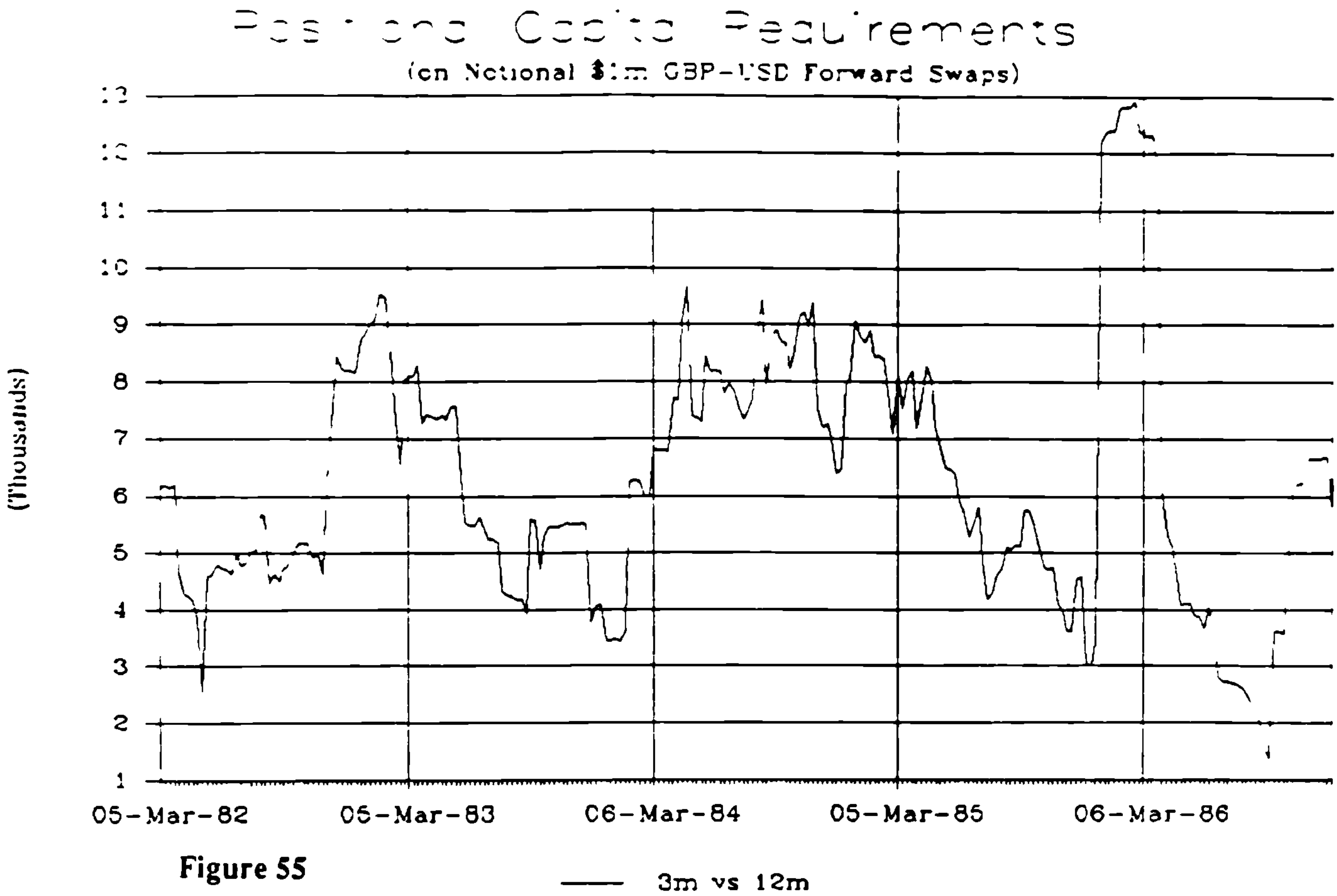


Figure 55

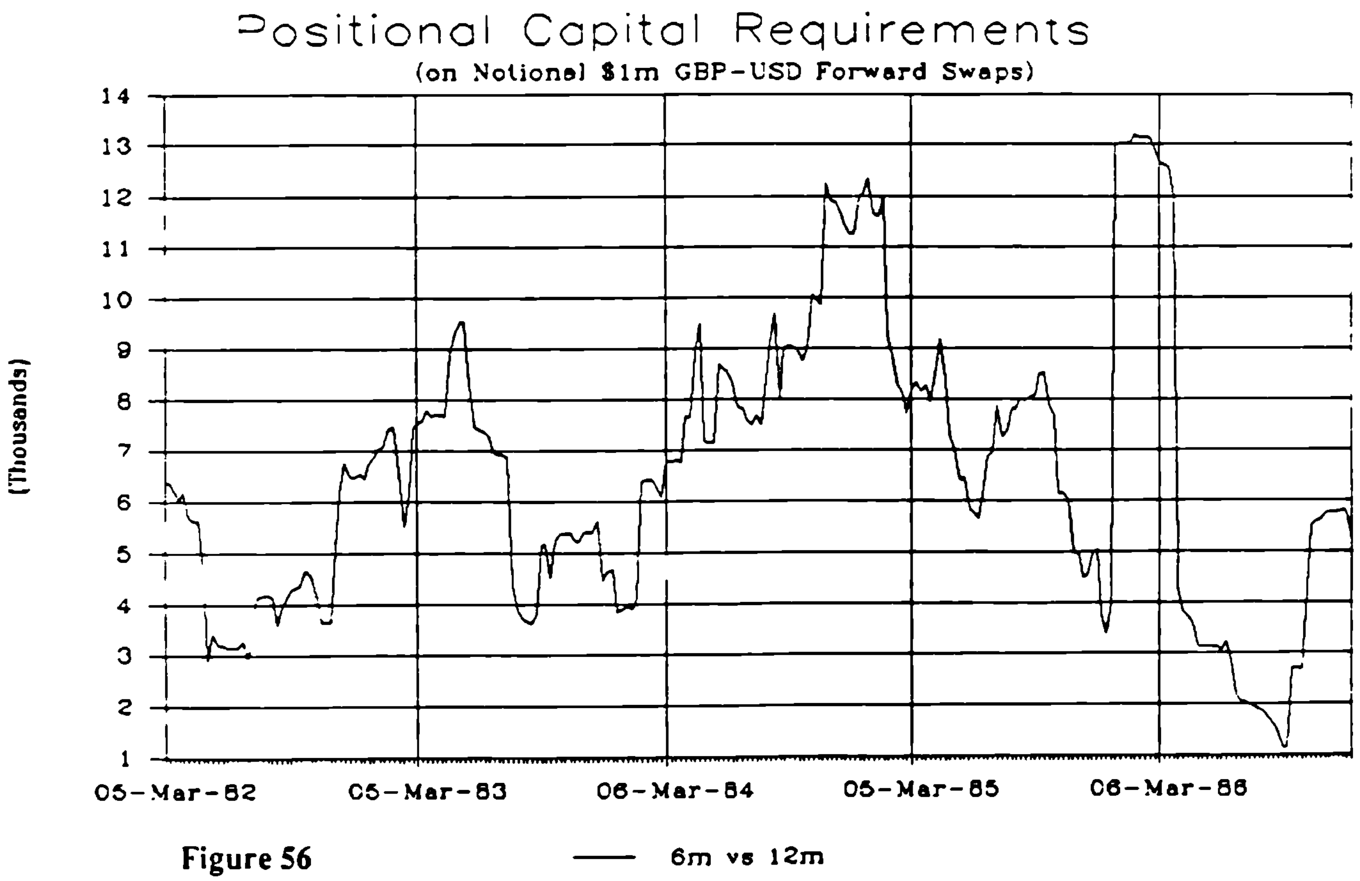


Figure 56

Chapter 4

The Use of Options in Managing Foreign Exchange Risk

4.1 Introduction

The market for foreign currency options has grown dramatically in the last five to ten years. In particular, the OTC market is now open 24 hours a day operating alongside the underlying spot market, with options on most currency combinations up to one year maturity, and sometimes beyond, now trading. This expansion can be attributed to three main causes. Firstly, the general expansion in the foreign exchange market which has led to an increasing demand for derivative instruments such as forwards, futures and options. Secondly, the opportunities provided by options for leverage and to design specific payoff functions has attracted interest from both corporate hedgers and speculators. This has been additionally fuelled by the extensive marketing by international banks and securities firms attracted to the new market by the wider profit margins available. Thirdly, and perhaps most importantly, the growth in the market has been based on the widespread acceptance by market practitioners, of the Black and Scholes option valuation model. The foreign currency version of this model[1] has become the standard tool both for valuation and deriving hedge positions. This has encouraged the development of a liquid market since traders can be sure of dealing at a known price once all the parameters of the model are fixed. In particular the volatility parameter, as the only unobservable parameter, has become traded as if it was an asset price. Thus traders can take positions in volatility and unwind previous positions at a known price. The model simply provides a means of converting volatility into price which is acceptable to all players in the market.

1 See Garman and Kohlhagen (1983), Grabbe (1983) and Biger and Hull (1983)

The importance of this standardisation can be appreciated by comparing the size of the currency options market to that of the interest rate options market. The later, whilst fairly liquid at the shorter maturity end, suffers from the lack of a standard model especially for evaluating longer dated instruments. This has led to a reluctance on the part of some institutions to develop large portfolios in these products which in turn has hampered the development of the market.

From the corporate viewpoint the advent of currency options appears to provide an alternative to the use of the spot or forward markets for hedging their exposures. The classic approach has been to suggest that by purchasing an option the hedger can lock into a fixed exchange rate but still allow himself to gain from any favourable movements which may occur later. More sophisticated strategies may involve writing, that is selling, options as well as buying them to create alternative payoffs for a reduced upfront cost. Two important issues however must be addressed when examining the use of options as an alternative to the spot or forward markets. The first concerns the assumption in the valuation model, of unbiased forward prices. As we discussed in the previous chapter, when forward prices are unbiased forward contracts are always the optimal hedge for transactional exposure since risk can be eliminated without loss of expected return. Thus for these exposures an option would only be appropriate when the forward is biased. As we mentioned before there is a reasonable amount of evidence to suggest that forward prices are not unbiased and so there may well be some justification for using options. The exact options to choose however will depend on the direction and degree of this perceived bias.

The second issue concerns the continuous trading assumption. Since one of the basic arguments behind the Black and Scholes option valuation model is that of the replicating portfolio, if continuous trading is assumed then options are redundant to a hedging strategy since they can be replicated by a continuously adjusted portfolio of cash and forwards. In practice of course, trading takes place at discrete points in time and there are transaction costs so that perfect replication is not possible. It is interesting to note however that there has been an increasing growth in the use of synthetic option models by treasury managers. These models provide trading rules which attempt to guarantee the same payoff as a conventional option but with less overall cost than the premium on an OTC option. The suppliers of these models argue that conventional options are redundant even with discrete trading and transaction costs since they are in general overpriced. We shall discuss the empirical work that has been done on testing for biases in the Black and Scholes model in the review of previous work.

As the OTC market has become more established a new market in so called derivative or complex option products has begun to develop. These products are in general, contingent claims whose payoff is a complex function of one or more underlying currencies sometimes based on the price history of the particular exchange rate(s). Examples of these include Dual Currency options, Portfolio options, Average Rate options, Barrier options, etc. The value of these instruments to the treasury manager is that they allow for a more exact hedge to be constructed for specific complex exposures which could not be provided previously. Thus in particular situations the use of a derivative option may be appropriate for an unusual or complicated risk.

From the banks point of view these instruments are attractive because of the wider profit margins involved, especially since the OTC options market has become more liquid and the spreads narrowed, and also because of their relationship to the original Black and Scholes model. This is important since it is the fact that the Black and Scholes model has been so successful that the banks have been willing to offer these more complicated and potentially risky products. Although each product has a different payoff, the underlying assumptions regarding the behaviour of the market and the valuation and hedging arguments are the same as in the Black and Scholes model. Thus the validity of these assumptions has to some extent been proved, in a practical sense, by the successful performance of the standard model under real market conditions. Furthermore, the availability, at a fairly low cost, of standard currency options for use in hedging these derivative options, has been an important factor in the later's development.

This chapter deals specifically with this growing market and the opportunities offered by these new instruments for hedging complex exchange rate exposures. As before the problems inherent in using unrealistic distributional assumptions are discussed and their implications from the treasurers point of view are analysed. The rest of the chapter is divided into four parts. The first gives a brief overview of the theory behind financial option valuation and its application to foreign exchange. This is important since the arguments presented here will be referred to later in the valuation and analysis of the more complex options. The next section is a survey of previous work. Although a relatively large amount of work has been done on the implications of different market models for option pricing and hedging, little work on the use of the new derivative options in hedging foreign exchange exposures has been done. In section 4.4 we define several of the derivative options, present valuation formulas where they are known, and discuss their uses in hedging particular exposures. Finally section 4.5 contains a summary and conclusions.

4.2 Basic Option Pricing Theory

In this section we present the basic theory used in the valuation of financial options and in particular, options on foreign currency. The assumptions behind the standard Black and Scholes equation are described and the risk neutral valuation technique, which is an important tool in valuing the more complex contingent claims, is explained. This procedure is illustrated with an example. Some of the most popular approximation techniques are then described since in practice they often offer the only way of calculating a value for some of the new derivative options when no exact closed form solution is possible.

The first assumption needed to derive the Black and Scholes equation is that the underlying security follows an ito process such that;

$$dS = \mu S\,dt + \sigma S\,dz \qquad 4.1$$

where S is the price of the underlying asset, μ is the expected drift, σ^2 is the instantaneous variance and dz is a Weiner process ie $dz = \varepsilon\sqrt{dt}$ where ε is a random sample from a standardised normal distribution. In addition μ and σ are assumed to be constant although the Black and Scholes equation is still correct if they are known functions of S and t. The process described in equation 4.1 means that changes in dS/S are normally distributed with mean μ and variance σ^2 and are iid. Thus if we define S_T as the underlying asset price at some time T in the future, then;

$$\ln(S_T) \sim N(\mu T, \sigma^2 T)$$

that is, the log of the price is normally distributed ie prices are lognormally distributed. This is the same assumption we tested for in the previous empirical work. Now from the properties of normal variables we can write,

$$E(S_T) = e^{\mu T + \frac{\sigma^2}{2}T} \qquad 4.2$$

but since we also make the efficient market assumption we know that the expected underlying price is the forward price which in the case of exchange rates is given by the interest rate parity condition;

$$E(S_T) = F = S e^{(r_D - r_F)T} \qquad 4.3$$

where S is the current spot price in terms of domestic currency per one unit of foreign currency, r_D and r_F are the domestic and foreign risk free interest rates and T is the time period as a proportion of a year. Note that this formula is different from the one used in the previous empirical work (Chapter 2, equation 2.4) since these interest rates are continuous compounding rates rather than money market rates. From equations 4.2 and 4.3 then;

$$e^{\mu T + \frac{\sigma^2}{2}T} = S e^{(r_D - r_F)T}$$

$$\mu T + \frac{\sigma^2}{2}T = \ln(S) + (r_D - r_F)T \qquad 4.4$$

$$\mu T = \ln(S) + \left(r_D - r_F - \frac{\sigma^2}{2}\right)T$$

The second important assumption is that the risk free interest rates are constant and the same for all maturities (ie the yield curve is flat). Again it can be shown that if the riskless rates are known functions of S and T the Black and Scholes equation is still correct. The remaining assumptions are the usual perfect markets assumptions ie short selling is allowed, no transaction costs or taxes, no riskless arbitrage and continuous trading. These assumptions are essentially the same as the ones used in the portfolio model described in the previous chapter and so our reservations with regard to the distributional specification in particular will also carry over.

Now if we define f to be some function of S and t, ie the price of a derivative security contingent on S, then we can use ito's lemma to write down;

$$df = \left(\frac{\partial f}{\partial S} \mu S + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial f}{\partial S} \sigma S \, dz \tag{4.5}$$

where dz is the same Weiner process as in equation 4.1 (that is the derivative and the underlying are perfectly correlated). In order to derive a non-stochastic partial differential equation for the derivative security we have to eliminate the Weiner process by choosing an appropriate portfolio of S and f. Suppose then that we have a portfolio P which is short of one derivative security and long of an amount $\frac{\partial f}{\partial S}$ of the underlying asset. The change in value of this portfolio in a time interval dt is given by;

$$dP = -df + \frac{\partial f}{\partial S} dS \tag{4.6}$$

so substituting for dS and df from equations 4.1 and 4.5 we get;

$$dP = \left(-\frac{\partial f}{\partial t} - \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) dt \tag{4.7}$$

Since this equation does not involve dz it must be riskless over the interval dt and so from our assumptions, it must earn the riskless return ie;

$$dP = -fr_D dt + \frac{\partial f}{\partial S} S(r_D - r_F)dt \qquad 4.8$$

Note that the riskless return on the spot rate is the interest rate differential from equation 4.3. Substituting from 4.7 into 4.8 gives;

$$\left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt = \left(r_D f - (r_D - r_F)S\frac{\partial f}{\partial S}\right)dt$$

$$\frac{\partial f}{\partial t} + (r_D - r_F)S\frac{\partial f}{\partial S} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 = r_D f \qquad 4.9$$

Equation 4.9 is known as the Black and Scholes differential equation which describes the process for any derivative security contingent on a foreign currency. The solution for a particular derivative is obviously a function of its boundary conditions and for some of the more complex securities which we describe later, no closed form solution exists.

We now describe one of the most useful techniques for option valuation, risk neutral valuation[2]. The basis for this approach is that the Black and Scholes equation is independent of risk preferences ie μ is not present. In this case the argument is that since risk preferences can not effect the solution, any set of risk preferences can be used to evaluate f, in particular the simple assumption of risk neutrality. This is convenient since in a risk neutral world all securities are expected to earn the risk free rate and the present value of any cashflow is obtained by discounting its expected value by the riskless rate. This argument enables complex option payoffs to be valued by calculating the expected terminal payoff assuming that the return on the underlying asset(s) is equal to the riskless rate and then discounting this at the same riskless rate. This device is especially useful for valuing contingent claims when the distribution of the underlying asset(s) is complicated.

2 See Cox and Ross (1976)

As an example of the application of this technique we derive below the value of a conventional European style (ie only exercisable at maturity) call option on a foreign currency.

The payoff of a call option on a unit of foreign currency with maturity T, is given by;

$$C_T = \text{Max}[S_T - K, 0]$$

where S_T is the spot rate at maturity T and K is the strike price of the option. Both prices are expressed as domestic currency per unit of foreign currency. Using the risk neutral valuation approach, the value of this option today is just;

$$C = e^{-r_D T} E[\text{Max}[S_T - K, 0]] \qquad 4.10$$

where E() is the expectation operator. If $g(S_T)$ is the probability distribution of S_T in a risk neutral world then we can write equation 4.10 as;

$$C = e^{-r_D T} \int_K^{\infty} (S_T - K) g(S_T) dS_T \qquad 4.11$$

since S_T is lognormally distributed its probability distribution is given by;

$$g(S_T) = \frac{1}{\sqrt{2\pi}\,\sigma\sqrt{T}\,S_T} e^{\left[-\frac{(\ln(S_T)-\mu T)^2}{2\sigma^2 T}\right]}$$

Expanding 4.11,

$$C = e^{-r_D T}\int_K^{\infty} S_T g(S_T)dS_T - e^{-r_D T}\int_K^{\infty} K g(S_T)dS_T \qquad 4.12$$

Let the first term in 4.12 be A and the second term B. Solving for A first and substituting for $g(S_T)$ we get;

$$A = \frac{e^{-r_D T}}{\sqrt{2\pi}\sigma\sqrt{T}}\int_K^{\infty} e^{\left[-\frac{(\ln(S_T)-\mu T)^2}{2\sigma^2 T}\right]}dS_T$$

Let $S_T = e^W \;\; \Rightarrow \;\; dS_T = e^W dW$ and when $S_T = K, \;\; W = \ln(K)$ so;

$$A = \frac{e^{-r_D T}}{\sqrt{2\pi}\sigma\sqrt{T}}\int_{\ln(K)}^{\infty} e^{\left[W-\frac{(W-\mu T)^2}{2\sigma^2 T}\right]}dW$$

expanding the exponent term and completing the square yields;

$$A = \frac{e^{-r_D T}}{\sqrt{2\pi}\sigma\sqrt{T}}e^{\mu T+\frac{\sigma^2}{2}T}\int_{\ln(K)}^{\infty} e^{\left[-\frac{(W-(\mu T+\sigma^2 T))^2}{2\sigma^2 T}\right]}dW \qquad 4.13$$

but from equations 4.4 we can write;

$$A = \frac{Se^{-r_F T}}{\sqrt{2\pi}\sigma\sqrt{T}}\int_{\ln(K)}^{\infty} e^{\left[-\frac{(W-(\mu T+\sigma^2 T))^2}{2\sigma^2 T}\right]}dW$$

Let $y = \frac{W-(\mu T+\sigma^2 T)}{\sigma\sqrt{T}} \;\; \Rightarrow \;\; dy = \frac{dW}{\sigma\sqrt{T}}$ and when W = ln(K), $y = \frac{\ln(K)-(\mu T+\sigma^2 T)}{\sigma\sqrt{T}}$ so;

$$A = Se^{-r_F T}\frac{1}{\sqrt{2\pi}}\int_{\frac{\ln(K)-(\mu T+\sigma^2 T)}{\sigma\sqrt{T}}}^{\infty} e^{-\frac{y^2}{2}}dy$$

But,

$$N(a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-\frac{x^2}{2}}dx$$

where N() is the cumulative normal ie N(a) is the probability that a normally distributed random variable x is less than or equal to a. We can therefore rewrite A as;

$$A=Se^{-r_F T}N\left(\frac{\mu T+\sigma^2 T-\ln(K)}{\sigma\sqrt{T}}\right)$$

using equation 4.4 again this can be written as;

$$A=Se^{-r_F T}N\left(\frac{\ln\left(\frac{S}{K}\right)+\left(r_D-r_F+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

The second term B, can be expanded using the expression for $g(S_T)$ giving;

$$B=\frac{e^{-r_D T}}{\sqrt{2\pi}\sigma\sqrt{T}}\int_{K}^{\infty}\frac{K}{S_T}e^{\left[-\frac{(\ln(S_T)-\mu T)^2}{2\sigma^2 T}\right]}dS_T$$

Let $y=\frac{\ln(S_T)-\mu T}{\sigma\sqrt{T}} \quad\Rightarrow\quad dy=\frac{dS_T}{S_T\sigma\sqrt{T}}$ and when $S_T=K,\quad y=\frac{\ln(K)-\mu T}{\sigma\sqrt{T}}$ so;

$$B=Ke^{-r_D T}\frac{1}{\sqrt{2\pi}}\int_{\frac{\ln(K)-\mu T}{\sigma\sqrt{T}}}^{\infty} e^{-\frac{y^2}{2}}dy$$

$$=Ke^{-r_D T}N\left(\frac{\mu T-\ln(K)}{\sigma\sqrt{T}}\right)$$

$$=Ke^{-r_D T}N\left(\frac{\ln\left(\frac{S}{K}\right)+\left(r_D-r_F-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

The solution for the value of the option is then just A - B ie;

$$C = Se^{-r_F T} N\left(\frac{\ln\left(\frac{S}{K}\right) + \left(r_D - r_F + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right) - Ke^{-r_D T} N\left(\frac{\ln\left(\frac{S}{K}\right) + \left(r_D - r_F - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

It is often useful to write the solution in terms of the means and variances of the underlying spot price since this can be used to easily derive the value of an option for different lognormally distributed underlying assets as long as the mean and variance can be found. We shall demonstrate an application of this procedure in deriving the value of one of the complex options later. The general solution is simply;

$$C = e^{-r_D T}\left[e^{\mu T + \frac{\sigma^2}{2}T} N(d_1) - KN(d_2)\right] \qquad 4.17$$

where,

$$d_1 = \frac{(\mu T + \sigma^2 T - \ln(K))}{\sigma\sqrt{T}}$$

$$d_2 = \frac{(\mu T - \ln(K))}{\sigma\sqrt{T}}$$

In some of the derivative options which we describe later the payoff of the option involves two underlying variables and is of the form,

$$C_T = \text{Max}[S1_T - S2_T, 0]$$

where S1 and S2 are both lognormally distributed following the process in equation 4.1 and having a covariance $\sigma_{12} = \sigma_1\sigma_2\rho_{12}$ where ρ_{12} is the correlation between S1 and S2. Using the same procedure as that described above, the general solution to this can be found and is given by,

$$C = e^{-r_D T}\left[e^{\mu_1 T + \frac{\sigma_1^2}{2}T} N(d_1) - e^{\mu_2 T + \frac{\sigma_2^2}{2}T} N(d_2) \right] \qquad 4.18$$

where,

$$d_1 = \frac{(\mu_1 T + \sigma_1^2 T) - (\mu_2 T + \sigma_{12} T)}{\sqrt{\sigma_1^2 T + \sigma_2^2 T - 2\sigma_{12} T}}$$

$$d_2 = \frac{(\mu_1 T + \sigma_{12} T) - (\mu_2 T + \sigma_2^2 T)}{\sqrt{\sigma_1^2 T + \sigma_2^2 T - 2\sigma_{12} T}}$$

Again this solution will be used later to value some of the derivative options which we describe then[3].

As we have seen the risk neutral valuation technique provides an extremely useful tool for obtaining closed form solutions for many complex contingent claims. Two questions arise in the use of this approach however which are relevant to our research. Firstly, are there any types of underlying process for which the technique can not be used. This is important since we have found that the normal distribution is a poor approximation to the actual distribution of exchange rates and several alternative processes have been put forward. It may be that the risk neutral valuation technique can not be applied when the underlying process is one of the more complex processes which we described before. In fact for most processes the approach will work, the only exceptions being ones where the underlying process and the risk neutralised process do not have the same support. This happens if the drift of the original process has a singularity or the variances of the two processes have zeros at a point where the true and risk neutralised drifts have different signs[4]. Fortunately such examples are rare so that in practical terms

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3 This is the general solution to the problem examined by Magrabe (1978).

4 See Ingersoll (1987) pp 384-385 for an explanation of these problems.

there are a wide range of possible processes which can be applied. In the survey of previous work we describe many of the tests which have been done on these alternative processes.

The second issue arises when no solution to the differential equation exists for a particular set of boundary conditions. The question is whether the risk neutral valuation procedure can be used to find an approximate closed form solution. The answer is yes, and this is in fact one of the most useful applications of the technique. As long as the underlying distribution can be specified, possibly by some approximation, then the value of the derivative security can be found (using equations similar to 4.17 and 4.18). In this way an approximate closed form solution can be found relatively easily when complex underlying processes, often involving several assets, are present. The benefit of a closed form approximation is that it has a fast calculation time. For many derivative options, approximation techniques must be used to value the instrument and since most of these procedures are fairly computer intensive a fast calculating answer is obviously preferable. In some cases however, these alternative procedures have to be used because of the nature of the particular boundary conditions. It is therefore useful to understand how these procedures work if one is to use these instruments so that a better understanding of the assumptions behind them and the nature of the approximation can be gained. Here we will briefly outline the most common techniques.

The most popular approximation technique is the binomial lattice[5], or in general the multinomial lattice[6], valuation method. In this approach the path of the underlying asset(s) are modelled directly by creating a lattice or tree structure, so that from the initial value it can move, say, either up or down and then up or down again at the next step and

5 See Cox, Ross and Rubinstein (1979) for the seminal work on binomial lattices

6 See Boyle (1988), Omberg (1988), Cheyette (1988) and Boyle, Evnine and Gibbs (1989) for examples of multinomial lattice models.

so on. The idea is to choose these movements, and their probabilities, so that they converge to the correct underlying distribution. The value of the derivative is then obtained by applying its payoff formula to this lattice of underlying prices and using an iterative procedure, working backwards from the final prices, to find the price of the derivative today. This approach is especially useful for valuing American style options, that is options which can be exercised at any time up to their maturity date, since exact closed form solutions are usually not possible in these cases[7]. For multi-asset problems the lattice approach is also useful when the underlying distribution is complicated.

Many of the derivative options have payoffs which are a function of the history of the underlying spot price. These so called path dependant contingent claims sometimes have no exact closed form solution and so an approximation procedure is needed. In these cases the most useful technique is the Monte Carlo method[8]. In this technique samples are drawn from the normal distribution successively to create a sample path for the underlying exchange rate. Using this sample path the value of the derivative is calculated. The procedure is then repeated, usually many thousands of times, and the average value of the derivative is found. This is then used as the estimate for the true value of the derivative and the procedure also provides information on the accuracy of this value from the standard deviation of the derivative prices. The technique can be used for multi-asset problems but not for valuing American style options.

Finally, the finite difference method can be used to solve the differential equation numerically by converting it to a difference equation. Although this technique is often more complicated to apply, Geske and Shastri (1985) have suggested that it may be preferable when evaluating a large number of option prices simultaneously.

7 Although see Shastri and Tandon (1987) for an analytical approximation technique for standard American style foreign currency options originally suggested by Geske and Johnson (1984)

8 See Boyle (1977) and Hull and White (1988) for a useful control variate technique.

The next section presents a survey of previous work in the two main areas of options research which are of interest to our research. Firstly, work on the effects of misspecification of the underlying distribution on the option price, and some of the alternative distributions which have been used as the basis for option models, are discussed. Secondly, the relatively small amount of work on applying the new derivative option products to complex foreign exchange exposure management is examined.

4.3 Review of Previous Work

In this section we review two strands from the large body of research on the theoretical valuation and empirical performance of alternative option pricing models. The first area of work concerns the biases in the Black and Scholes model, and its foreign currency derivative, which arise because of non-normality in the distribution of underlying asset returns. This work is closely related to research into alternative option pricing models based on more realistic distributions or stochastic processes for the underlying asset. Unfortunately, most of this work has been developed in relation to stock options and so is perhaps not directly applicable to foreign exchange options. The work does however provide pointers to future work which may be carried out in the currency market. Three particular studies of option models in the foreign exchange market are reviewed and we examine these in somewhat more detail.

The second area of work is on the application of derivative option products to exposure management. In fact very little work on this has been published apart from the original valuation models which have sometimes described examples of possible applications. In terms of foreign exchange exposure, we have found one paper which indirectly studies one of the derivative options and we describe these results later.

The most important determinant of option value is the distribution of the underlying security price. For American style options and the path dependant options we describe later, the distribution of the rates in time is also critical. Most of the work on examining biases in the Black and Scholes model has focussed on this distribution and in particular the variance of the stochastic process described in equation 4.1. The main results from this work have been presented in Jarrow and Rudd (1982) where they demonstrate that the shape of the distribution and in particular fat-tails ie leptokurtosis, gives rise to systematic underpricing of in and out-of-the-money puts and calls. This is due in essence to the higher probability mass in the true distribution than the assumed normal distribution so that there is a higher probability of exercise than that assumed in the model.

Various alternative models have been put forward which better reflect the true underlying distribution and so eliminate some of these biases. These range from Displaced Diffusion models (Rubinstein(1983)), Constant Elasticity of Variance (CEV) models (Cox and Ross (1976)), Pure Jump models (Cox, Ross and Rubinstein (1979)), Jump Diffusion models (Merton (1976)) and models based on stochastic volatility (Wiggins (1986), Hull and White (1987a), Johnson and Shanno (1987) and Scott (1987)). More recently, a variance gamma model (Madan and Senata (1990) has been suggested and the effects of non-linear dependence in the underlying price process has been examined (Savit (1989)). None of the models has been shown to significantly outperform the Black and Scholes model in terms of eliminating all the biases all the time. As we explain later, one of the main problems has been the difficulty of designing accurate empirical tests.

In terms of the foreign exchange market, the empirical work presented in Chapter 2 suggests that observed leptokurtosis is probably due to a changing variance in the underlying process. This would imply that a stochastic, or even deterministic, changing variance model might be a better model for foreign currency options. Mellino and

Turnball (1987a) have examined currency option pricing for a particular set of processes where the variance is a function of the level of the exchange rate, in a similar way to the CEV models used for stocks. They found however that these models did not perform significantly better than the Black and Scholes model in explaining observed prices. More recently they have also begun to examine the effect of stochastic volatility (Mellino and Turnbull (1987b)). One of the important implications of a stochastic volatility model is that the biases described earlier become larger as a percentage of the option price as the maturity increases. On the other hand, if the biases are caused by jumps then the effects become less important as the maturity increases. This obviously has important implications for long dated option pricing. Shastri and Wethyavivorn (1988) have developed a generalised currency option pricing model using Edgeworth series expansion. They report however that the improvement in pricing is less than 0.5 percent beyond that exhibited by a model based on pure diffusion. Finally Tucker, Peterson and Scott (1988) have confirmed the results of Mellino and Turnbull (1987a) in tests of a CEV model for currency options. They find that the CEV model has no greater pricing accuracy than a pure diffusion model for predictive intervals of more than three trading days.

In order to test the performance of an option model the first problem is that any test must be a joint test of the model and the EMH[9]. In the case of currency options the assumption of the validity of the UIP condition is also often necessary. In our survey of the empirical work on the foreign exchange market we found that the concensus appears to be that the EMH does not hold, maybe because of the failure of the rational expectations part of the hypothesis. In this case any inferences drawn by the failure of tests on option pricing models should perhaps be viewed with this in mind.

9 See for example Black and Scholes (1972) and Galai (1977)

A second problem is that the volatility parameter is unobservable and so must be estimated in some way. Several researchers have examined the problem of estimation in option models (see for example Boyle and Ananthanarayanan (1977) and Butler and Schacter (1986) for a review of previous work). The problem is that even if the variance estimate is unbiased, it may produce a biased option price because the Black and Scholes model is non-linear in the variance. New estimation procedures have been suggested as alternatives to the sample variance, incluuing Bayesian procedures or the use of implied volatilities from option prices. It has been suggested that the size of the bias however is small. Unbiased estimates are important though when testing alternative option models since the biases induced may effect the models in different ways.

Thirdly, when actual option prices are used, the values of the underlying asset and interest rates must be sampled synchroneously. Thus any empirical examination of the OTC currency options market is impossible since no central record of option prices and trade time exists.

Shastri and Tandon (1987), Bodurtha and Courtadon (1987) and Mellino and Turnbull (1987a) have all used prices of currency options traded on the Philadelphia Stock Exchange to test models of foreign exchange option valuation. The first study used transactions data on the five main currencies (GBP, DEM, CAD, CHF and JPY) versus the USD from March 1983 to August 1984. Estimates of volatility were made both from historic spot prices and implied from the prices of at-the-money options. An American option pricing model based on the analytical model of Geske and Johnson (1984) was used to test for market efficiency (currency options on the Philadelphia exchange are all American style). In both cases mispricing relative to observed prices was found, in particular, out-of-the-money options are found to be underpriced and in-the-money options overpriced. In addition longer maturity options are more mispriced than shorter maturity options. From this they conclude that either the model is mis-specified or the market inefficient. They go on to test for efficiency by examining if a trading rule can generate excess returns using the pricing model they have chosen. They find that it can but only if the bid ask spread is removed so that the market is probably efficient at least for outside players who have to pay the spread.

The second study by Bodurtha and Courtadon (1987) used an American pricing model due to Parkinson (1977) and compared it to a standard Black and Scholes type European model. They used volatility estimates derived from a selection of option prices from the previous day using the approach suggested by Whaley (1982). Their data consisted of all the option trades for the five main currencies (DEM, GBP, CAD, CHF and JPY) against the USD from February 1983 to March 1985. Interest rates were taken from US Treasury bill prices and the futures contracts trading on the International Money Market using the UIP condition. The results suggested that the model underpriced out-of-the-money options relative to at-the-money or in-the-money options especially for shorter maturities. They conclude that this is consistent with a Jump Diffusion process but it could also in fact be explained by heteroskedasticity.

Mellino and Turnbull (1987a) point out that the model used by Bodurtha and Courtadon is misspecified if the distribution is not lognormal. In fact the same is true for the Shastri and Tandon study. In their study they develop a pricing model based on a general CEV process and then test directly the performance of this model over actual data (the same data set as that used by Bodurtha and Courtadon). They found that for a variety of parameter values, including those which give a lognormal distribution, all the models consistently undervalued the options relative to the observed prices. They conclude that the problem seems to be that the volatility estimates from the actual data are significantly smaller than those implied in the option prices. They suggest that a stochastic variance model may be more appropriate.

Finally, Ogden and Tucker (1988) have performed some empirical tests on the prices of currency options trading on the Philadelphia Exchange in comparing these prices with those of options on currency futures trading on the Chicago Mercantile Exchange. They found that significant riskless arbitrage profits could be made during 1986 using conventional pricing models for these options by dealing across the exchanges. They are unable to conclude however whether this is because of market inefficiency or the failure of the models to incorporate stochastic interest rates which could also have explained the results.

Studies which have tested a stochastic variance model on stocks have found in general that the difference in price between the Black and Scholes model and the more complicated model are small. Thus the observed mispricing found by Mellino and Turnbull may be due to factors which are not captured in these models. We discuss some of these potential factors below. It may be that the impact of a changing variance though is more important for hedging[10]. Certainly the creation of a riskless hedge portfolio when the variance is changing is problematic since the standard deviation is not a traded asset (although for currency options this is almost the case).

The relatively poor performance of the alternative models to describe the observed biases produced by the Black and Scholes model is perhaps not so surprising. If one observes the way in which currency options are priced in practice one can appreciate the difficulty in developing a better alternative to Black and Scholes. In reality when a trader is asked for an option price he uses the Black and Scholes model, with an estimate of implied volatility obtained from the broker market, as a first approximation. The actual price he quotes is this theoretical price adjusted to take into account several factors which

10 A useful study of the ways in which currency options can be hedged can be found in Hull and White (1987b). Gilster (1990) examines the hedging problem in general from the point of view of discrete rather than continuous rebalancing.

it would be difficult to model externally in a consistent way. Such factors include the option delta (ie the extent it is in or out-of-the-money), the liquidity in the option and underlying spot or forward market, his own position and that of the options market as a whole, and even the day of the week. It has been shown, for example, that currency option prices are lower on Friday since traders mark their prices down in order to sell more options and therefore earn the time value over the weekend. Implicitly they are adjusting the model to allow for the fact that the potential movement in the spot price over the weekend is less than three times that over a weekday. In a similar adjustment, the in and out-of-the-money biases are mitigated by raising the estimate of volatility used in the model for these options giving rise to the well known "smile effect" in the relationship between strike price and volatility.

It is not surprising then that empirical tests find that all models perform roughly the same, including Black and Scholes, and all have biases of one form or another. Given these results, the fact that the Black and Scholes model is so simple to use, only requiring one unobservable parameter to be estimated, is probably the most compelling reason for its continued usage. That and the fact that it works well enough for hedging to be reasonably successful under most market conditions.

To summarise the findings of the research into alternative option pricing models, the results appear to suggest that although the Black and Scholes model has many inadequacies, especially in its assumption of lognormality, sophisticated models based on more realistic processes do not appear to significantly outperform the model in empirical tests. In the foreign exchange market in particular, no satisfactory alternative has yet been found and the continued use of the model by market practitioners is perhaps the strongest indication that the assumptions underlying it are at least approximately valid.

Work on the application of new derivative options to exposure management has so far been largely restricted to examples in the papers which have described the valuation of these instruments. We shall cover this work in detail in our analysis of the individual options in the next section. One study which is relevant to our research however has been carried out by Ware and Winter (1988). In this paper they examined economic exposure in the foreign exchange market and demonstrated how standard currency options could be used to hedge this type of exposure when a firm has ex post production flexibilty in a single exchange rate environment. In the case of multiple currencies however they show that more complex options are needed. Such a complex option would "...replicate, for the option holder, exactly the choice set faced by the firm." Thus the firm writes an option, ie sells its opportunity set, to the market in order to eliminate its risk. This highlights one of the important arguments presented in the paper, that the hedging of economic exposure through options involves a firm in writing rather than buying options. They argue that "The flexibility to respond to realised exchange rates can be viewed as an option (in general, a complex option). The firm can eliminate risk only by transfering this option to the market through the writer's position". This argument is contrary to the traditional view expounded by the banks in particular and which we described earlier, which has suggested that firms wishing to hedge using options should buy rather than sell them.

In their paper they examine the payoff required by a firm when it has flexibility in input/output and in the choice of production technique. They show how multiple options (ie complex options whose payoff is a particular function of several exchange rates) can replicate the desired payoff under certain circumstances. Unfortunately, they are apparently unaware of some of the newer derivative options when they state that in cases involving more than two currencies the required complex options do not exist. We show later that the payoffs which they require can indeed be created using these new derivative option products. The exposures which they study are in fact analogous to the competitive exposures which arise when a firm faces competition from foreign companies or domestic companies which source inputs overseas. These risks have been analysed by Benson and Levy (1990) where they describe the use of a particular derivative option to hedge these exposures for any number of currencies. In the next section we examine in more detail the use of multiple currency options (in particular so called max-min options or Dual Currency options) and also describe the relationship between these options and options which hedge portfolios of currencies.

4.4 Complex Options and Applications to Risk Management

In this section we present several complex foreign currency options which have become available as traded instruments in the OTC options market in recent years. The payoff functions are described and valuation formulas presented (where available). The uses of the options in managing particular complex exposures are then described and new results on the relationship between different complex options are derived.

4.4.1 Max-Min Multicurrency Options

4.4.1.A Definition

A max-min option (sometimes called a Dual Currency Option in the case of two currency pairs) is defined as an option which pays the holder at maturity the difference between the maximum (or minimum) of a set of exchange rates and the strike price. Thus a call on the maximum of n exchange rates $S_1, \ldots, S_n$ with strike price K, has a payoff;

$$C_{\text{Max}} = \text{Max}[\text{Max}[S_1, S_2, \ldots, S_n] - K, 0]$$

and similarly a call on the minimum has a payoff given by;

$$C_{\mathrm{Min}} = \mathrm{Max}[\mathrm{Min}[S_1, S_2, \ldots, S_n] - K, 0]$$

One can also define puts on the maximum and minimum as;

$$P_{\mathrm{Max}} = \mathrm{Max}[K - \mathrm{Max}[S_1, S_2, \ldots, S_n], 0]$$

$$P_{\mathrm{Min}} = \mathrm{Max}[K - \mathrm{Min}[S_1, S_2, \ldots, S_n], 0]$$

We shall concern ourselves only with European style max-min options but obviously American style options are also possible. Unfortunately no closed form solutions are available for these in general[11]

It should be noted that in the case of currencies care should be taken to ensure that all the exchange rates have a currency in common ie the domestic currency, and are all expressed as units of domestic currency per unit of foreign currency. Therefore for example one can have options on the maximum or minimum of USD/GBP and USD/DEM but not USD/GBP and DEM/JPY. Another problem is the specification of the strike price particularly when the value of individual rates can vary considerably in absolute size. A solution to this problem can be found by observing that the payoff on a call on the maximum for example, can be written as;

$$C_{\mathrm{Max}} = \mathrm{Max}[\mathrm{Max}[(S_1 - K), (S_2 - K), \ldots, (S_n - K)], 0]$$

11 See Boyle (1988), Omberg (1988), Cheyette (1988) and Boyle Evnine and Gibbs (1989) for alternative lattice approaches to the American valuation problem. Boyle and Tse (1990) have presented an analytical approximation method which is efficient for particular parameter values.

$$= K \operatorname{Max}\left[\operatorname{Max}\left[\left(\frac{S_1}{K} - 1\right), \left(\frac{S_2}{K} - 1\right), \ldots, \left(\frac{S_n}{K} - 1\right)\right], 0\right]$$

Now if we define individual strike prices $K_1, \ldots, K_n$ for each exchange rate, we can write;

$$C_{\text{Max}} = K^* \operatorname{Max}\left[\operatorname{Max}\left[\left(\frac{S_1}{K_1} - 1\right), \left(\frac{S_2}{K_2} - 1\right), \ldots, \left(\frac{S_n}{K_n} - 1\right)\right], 0\right]$$

where K^* is the strike associated *with the exchange rate S^* of the foreign currency* which is exercised ie

$$\left(\frac{S^*}{K^*}\right) = \operatorname{Max}\left[\left(\frac{S_1}{K_1}\right), \left(\frac{S_2}{K_2}\right), \ldots, \left(\frac{S_n}{K_n}\right)\right]$$

The price of the option expressed as a percentage of the domestic currency is given by $\frac{C_{\text{Max}}}{K^*}$ and can be found by using the standard valuation formula and substituting the spot rates by the spot rate divided by its respective strike price and setting the actual strike equal to one. This allows different strikes to be incorporated thus overcoming the potential scaling problem which could occur in foreign currency applications. By writing the payoff in terms of individual strike prices for each currency pair the relationship between these complex options and conventional options is easily seen. Thus the payoff can be compared to that on a portfolio of options where only the option which is most in-the-money at maturity is allowed to be exercised. Later we demonstrate the relationship between these complex options and options on a portfolio of currencies.

4.4.1.B Valuation

The valuation of max-min options on stocks was originally derived by Stulz (1982) and Johnson (1982) when only two stocks were involved. Later Johnson (1987) extended these results for n assets. Stapelton and Subrahmanyan (1984) and Babbs and Salkin

(1989) have also presented general formulas for valuing multivariate contingent claims which can be used to solve for the value of max-min options. The adapted formulas for foreign currency max-min options which incorporate several strikes were presented by Benson and Salkin (1989) and are shown below;

$$
\begin{aligned}
C_{Max} = & \left(\frac{S_1}{K_1}\right) e^{-r_{F1}T} N_n(d_1(S_1,K_1,\sigma_1^2), d_1^*(S_1,S_2,\sigma_{12}^2), \ldots, d_1^*(S_1,S_n,\sigma_{1n}^2), \rho_{112}, \rho_{113}, \ldots) \\
& + \ \ldots \\
& + \left(\frac{S_n}{K_n}\right) e^{-r_{Fn}T} N_n(d_1(S_n,K_n,\sigma_n^2), d_1^*(S_n,S_1,\sigma_{1n}^2), \ldots, d_1^*(S_n,S_{n-1},\sigma_{(n-1)n}^2), \rho_{n1n}, \rho_{n2n}, \ldots) \\
& - e^{-r_D T}(1 - N_n(-d_2(S_1,K_1,\sigma_1^2), -d_2(S_2,K_2,\sigma_2^2), \ldots, -d_2(S_n,K_n,\sigma_n^2), \rho_{12}, \rho_{13}, \ldots)
\end{aligned}
\tag{4.19}
$$

where,

$$d_2(S_i,K_i,\sigma_i^2) = \frac{\left[\ln\left(\frac{S_i}{K_i}\right) + \left(r_D - r_{Fi} - \frac{1}{2}\sigma_i^2\right)T\right]}{\sigma_i\sqrt{T}}$$

$$d_1(S_i,K_i,\sigma_i^2) = d_2 + \sigma_i\sqrt{T}$$

$$d_1^*(S_i,S_j,\sigma_{ij}^2) = \frac{\left[\ln\left(\frac{S_iK_j}{S_jK_i}\right) + \left(r_{Fi} - r_{Fj} + \frac{1}{2}\sigma_{ij}^2\right)T\right]}{\sigma_{ij}\sqrt{T}}$$

$$\sigma_{ij} = \sigma_i^2 + \sigma_j^2 - 2\sigma_i\sigma_j\rho_{ij}$$

$$\rho_{iij} = \frac{[\sigma_i - \rho_{ij}\sigma_j]}{\sigma_{ij}}$$

$$\rho_{ijk} = \frac{[\sigma_i^2 - \rho_{ij}\sigma_i\sigma_j - \rho_{ik}\sigma_i\sigma_k + \rho_{jk}\sigma_j\sigma_k]}{\sigma_{ij}\sigma_{ik}}$$

and T is the time to maturity, N_i is the i-variate standard cumulative normal[12], n is the number of exchange rates, ρ_{ij} the correlation between the returns on currencies i and j, and r_{F_i} the risk free interest rate for foreign currency i.

Note that all the same assumptions made in deriving the Black and Scholes equation are used in the valuation of these options so all the spot rates are assumed to follow jointly lognormal diffusion processes (as in 4.1).

The values of C_{Min}, P_{Max} and P_{Min} can also be derived from solving the multi-variate Black and Scholes equation to give solutions similar to equation 4.19 above or else they can be found from the arbitrage conditions;

$$C_{\text{Max}} + C_{\text{Min}} = \sum_{i=1}^{n} C_{BS}(S_i, K_i) - \sum_{i=2}^{n-1} C_{iBest} \qquad 4.20a$$

$$P_{\text{Max}} + P_{\text{Min}} = \sum_{i=1}^{n} P_{BS}(S_i, K_i) - \sum_{i=2}^{n-1} P_{iBest} \qquad 4.20b$$

where $C_{BS}(S_i,K_i)$ and $P_{BS}(S_i,K_i)$ are the values of standard Black and Scholes European style currency options on spot rate S_i with strike price K_i and C_{iBest} (P_{iBest}) is a call (put) on the i-th best currency ie the second best, third best, etc. So $C_{\text{Max}} = C_{1Best}$ and $C_{\text{Min}} = C_{nBest}$. We can therefore write equations 4.20 in general as;

$$\sum_{i=1}^{n} C_{iBest} = \sum_{i=1}^{n} C_{BS}(S_i, K_i) \qquad 4.21a$$

12 Routines for valuing multi-variate normal distributions can be found in Schervish (1985) and are referred to in Geske (1977) and Geske and Johnson (1984)

$$\sum_{i=1}^{n} P_{iBest} = \sum_{i=1}^{n} P_{BS}(S_i, K_i) \qquad 4.21a$$

In words this means that the sum of options on the best, second best, third best, and so on, of n exchange rates, is equal to the sum of standard options on each currency. This suggests a relationship between options on the maximum and minimum, and options on portfolios of currencies which we will demonstrate with an example later. First though we derive the put call parity condition for these options using equations 4.21.

For each European currency option we know the put call parity condition is just;

$$C_{BS}(S_i, K_i) = P_{BS}(S_i, K_i) + S_i e^{-r_{Fi}T} - K_i e^{-r_D T}$$

so for a portfolio of options we can write;

$$\sum_{i=1}^{n} C_{BS}(S_i, K_i) = \sum_{i=1}^{n} P_{BS}(S_i, K_i) + \sum_{i=1}^{n} S_i e^{-r_{Fi}T} - \sum_{i=1}^{n} K_i e^{-r_D T} \qquad 4.22$$

Now substituting from 4.22 into 4.21 we get;

$$\sum_{i=1}^{n} C_{iBest} = \sum_{i=1}^{n} P_{iBest} + \sum_{i=1}^{n} S_i e^{-r_{Fi}T} - \sum_{i=1}^{n} K_i e^{-r_D T} \qquad 4.23$$

This is the put-call parity condition for max-min options which is obviously very similar to that for portfolios of options. Note that when n = 1 equation 4.23 collapses into the put-call parity condition for a single underlying exchange rate.

Finally the hedging of these options is of interest to the treasury manager since the procedure used by the market maker will be reflected in the price of the option. The riskless portfolio, obviously consists of positions in each of the underlying currencies given by the partial derivatives ie $\frac{\partial C_{Max}}{\partial S_i}$. However in practice the trader will use standard currency options as his principal hedging instruments with only residual hedging in the underlying spot or forward market. This is because by using options he will partially protect himself against spot and forward movements simultaneously and more importantly, give himself a volatility hedge and gamma protection. The options he will use will be those with the same maturity, and strike price as the max-min option. The hedge ratios with respect to the underlying options are simply calculated from the ratios of the deltas on the max-min option and those of the underlying options ie;

$$\frac{\partial C_{\mathrm{Max}}}{\partial C_{BS}(S_i, K_i)} = \frac{\partial C_{\mathrm{Max}}}{\partial S_i} \quad \frac{\partial S_i}{\partial C_{BS}(S_i, K_i)}$$

The price of the max-min option should therefore reflect the market volatility for the individual options which it is equivalent to. The most important parameters however are the correlations which in practice are either calculated form historic data or implied from cross currency option volatilities as explained in Chapter 3.

4.4.1.C Applications

Several possible applications for these options have been put forward in the valuation papers mentioned earlier. Stulz looks at option-bonds, that is bonds which payout in either of two foreign currencies. The delivery option on futures contracts has been suggested by Boyle and Tse amongst others and other embedded options, ie commodity linked bonds, have also been put forward. No direct applications to exposure management in foreign exchange have however been looked at although the Ware and Winter study does examine exposures which could be hedged with these options.

Before looking in detail at the problem suggested by Ware and Winter though we describe another exposure which can be hedged using max-min options which we show later is in fact analogous to the risks which they examine. We then go on to derive the relationship between these options and other complex options, in particular options on portfolios, which have been suggested by the arbitrage and parity conditions derived above. For convenience we restrict the analysis to the case of two foreign countries although we point out how the results would be extended to allow more than two countries.

Example 1

Consider the case of a domestic producer who can only sell in its home market, but faces competition from two foreign firms based in different countries exporting into that market. Assume that the size of the market is constant and the only determinant of the relative sales of the three companies is price (ie product specification, marketing expense, etc are all identical). Each firm aims to increase market share but can only increase sales at the expense of the other firms. All the firms are operating efficiently so that they cannot reduce production costs to increase sales. Since price is the only determinant of market share, and is itself only a function of the exchange rates, market share will only change as a function of exchange rate movements. In addition we assume that this function is linear[13].

The domestic company obviously has an exposure to an appreciation in its currency relative to that of the two foreign currencies. This is because an appreciation in the domestic currency will encourage the foreign firms to reduce their prices since they can generate the same unit profit in *their* home currency at a lower price and hence increase market share. The sales of the domestic company therefore suffer as its currency appreciates. We represent this exposure diagrammatically in Figure 57 where S_1^* and S_2^* are equilibrium exchange rates at which the domestic company sells at the same price

13 This assumption is made in order to restrict the exposure profile to be piecewise linear and therefore hedgeable with options.

as its competitors in foreign country 1 and 2 respectively. All exchange rates are expressed as domestic currency per unit of foreign currency, as before, and so an appreciation in the domestic currency causes the exchange rate to fall, ie less domestic currency is needed to buy the same amount of foreign currency. Notice that the domestic company only ever loses sales to one of its competitors, that is the one whose currency deppreciates furthest relative to the domestic currency. This is because customers will change their brand only to buy the cheapest alternative so that even if both foreign products are cheaper than the domestic product, sales are lost only to the cheaper of the competitors products.

Now consider a put option P_{Max} on the maximum of S_1 and S_2 with strike prices K_1 and K_2. The payoff of this option can be represented in the same way and is shown in Figure 58. As can be seen the boundaries of the payoff on this option will exactly match those of the exposure profile in Figure 57 if $K_1 = S_1^*$ and $K_2 = S_2^*$. Therefore a long position in this option, with these strike prices would pay out in exactly the situations where the domestic company loses sales and so could be used as a hedge in this area of the exchange rate space. However this would not be a complete hedge since in the area where the company gains, the payoff of the option is zero. Thus the company would still have exposure to foreign exchange movements in this region although the exposure will always be beneficial. In order to be fully hedged though the company must eliminate entirely the impact of exchange rates on its profitability.

Consider a call option C_{Max} on the maximum of $S_1$and S_2. Its payoff is shown in Figure 59. This option pays out only in the area where the company gains from foreign exchange movements. A complete hedge would be therefore to purchase a P_{Max} option and sell a C_{Max} option both with strike prices $K_1 = S_1^*$ and $K_2 = S_2^*$. The combined position, shown in Figure 60, will exactly replicate the exposure profile in Figure 57.

Notice that the payoff divides the exchange rate space into just two regions and that it can be both positive and negative, unlike an option payoff which is only positive or zero. This suggests that the position can be interpreted as a kind of 2-dimensional forward position. Consider then an instrument H_{Max} which at maturity settles as a forward contract against the maximum of two currencies. The contract specifies the two forward rates (in our case K_1 and K_2) at which the parties are *obliged* to deal. Thus if $S_1 > S_2$ then a long position in H_{Max} pays out $S_1 - K_1$ and if $S_2 > S_1$ it pays out $S_2 - K_2$. Note that the payoffs can be negative if $K_1 > S_1 > S_2$ or $K_2 > S_2 > S_1$. A short position in this instrument will have a payoff as shown in Figure 60.

This result is intuitive in that for conventional options, a long put position combined with a short call position with the same strikes, is equivalent to a short forward outright position. It is not surprising that a long position in P_{Max} combined with a short position in C_{Max} is the same as an outright forward position in a kind of multi-currency derivative forward agreement.

The result also provides a way of linking the example of competitive exposure we are examining with the import/export flexibility problem analysed by Ware and Winter. In their study the first result they present is in the form of a proposition which states;

"For a price taking firm facing known local currency prices, exchange rate uncertainty can be hedged completely in the forward exchange market if and only if the is no flexibilty in production or import/export decisions."

The important point is that when there is no flexibilty a complete hedge can be constructed using only forwards. Since they do not consider competitive exposures however, the no flexibility problem reduces to a single exchange rate problem because the firm is constrained to import/export to one country. When the firm faces competition though, as we have seen, even if it has no flexibility to change its market (remember our company could only sell domestically) it can still have a multi-currency exposure. We have shown however that in this case too a complete hedge can be constructed using forwards alone. When competition induces this multi-currency exposure though the required forward contracts are max-min or derivative forward rather than conventional forward agreements. We can therefore extend the proposition stated above by writing;

Proposition:

"For a price taking firm facing known local currency prices, exchange rate uncertainty can be hedged completely in the forward exchange market if and only if there is no flexibilty in production or import/export decisions. When this exposure is multi-currency however the appropriate hedging instruments are derivative rather than conventional forward agreements."

Using the arbitrage condition for max-min options given in equation 4.20 we can write for n = 2;

$$C_{\mathrm{Max}} + C_{\mathrm{Min}} = C_{BS}(S_1, K_1) + C_{BS}(S_2, K_2)$$

$$P_{\mathrm{Max}} + P_{\mathrm{Min}} = P_{BS}(S_1, K_1) + P_{BS}(S_2, K_2)$$

so that;

$$C_{\mathrm{Max}} - P_{\mathrm{Max}} = P_{\mathrm{Min}} - C_{\mathrm{Min}} + C_{BS}(S_1, K_1) - P_{BS}(S_1, K_1) + C_{BS}(S_2, K_2) - P_{BS}(S_2, K_2) \quad 4.24$$

Now if we define H_{Min} such that if $S_1 > S_2$then a long position in H_{Min} pays out $S_2 - K_2$ and if $S_2 > S_1$ it pays out $S_1 - K_1$ then we can write equation 4.24 as;

$$H_{Max} = -H_{Min} + H_1 + H_2 \qquad 4.25$$

where H_i are conventional forward contracts on currency i settling at forward prices K_i. Thus the hedge we described earlier could also be achieved by going short of each currency using conventional forwards and also going long of an H_{Min} derivative forward contract.

We can easily extend the result for more than two foreign currencies. Since the domestic company is only ever exposed to the competitor whose currency moves furthest relative to the domestic currency, a forward contract on the maximum of all the currencies would provide a complete hedge for any number of currencies. Thus we can state a general proposition with regard to these exposures ie,

A General Proposition;

"A firm facing n foreign competitors and with no flexibility in operating decisions has an optimal hedge using forwards on the maximum of the n foreign currencies."

Finally we can extend equation 4.25 for n > 2, using the same approach as for the max-min options in equation 4.21 ie;

$$\sum_{i=1}^{n} H_{iBest} = \sum_{i=1}^{n} H_i$$

This example has extended the previous results on the hedging of foreign exchange exposure when there is no flexibility in decision making. In the multi-currency environment created by competition we have shown that forward contracts are still sufficient to completely hedge the exposure although max-min type derivative forwards rather than conventional forwards are now needed. In order to examine situations when options rather than forwards provide the better hedge it is necessary to allow some flexibility in decision making. This highlights the point that since options by definition provide a choice, ie to exercise or not, their use is only appropriate in economic exposure situations where choice, that is flexibility, is present. This is important since the traditional approach to exposure management using options has usually failed to make this clear. We now examine a situation in which flexibility is present.

Example 2

Consider a domestic producer which can sell its product at home or in one of two foreign countries. There is no competition in any of the countries so the same amount of product can be sold in each market. The decision to switch sales from one country to another is based only on the increase in revenue, in domestic currency terms, which is generated from these sales and so is a function only of the exchange rates between the domestic currency and the two foreign currencies. This is the problem examined by Ware and Winter and they show the decision profile of the domestic company in their Figure 4 (pg 299) which we reproduce in our Figure 61. They state that such a payoff cannot be hedged using "binary options" since they would create a payoff boundary along the dotted line. Unfortunately they do not define exactly what a "binary option" is although it appears to be a Magrabe type option ie an option to exchange one asset for another, in this case currencies. The payoff of this option is given by;

$$C_{EX} = \text{Max}[S_1 - S_2, 0]$$

The problem is that this option has no explicit strike prices and so, as they point out, cannot produce the required payoff. The option is however related to max-min options, as Stulz has shown, through the arbitrage relationship;

$$\text{Max}[S_1 - S_2, 0] = S_1 - \text{Min}[S_1, S_2]$$

Thus C_{EX} is equivalent to a long position in currency 1 and a short position in a call option on the minimum of the two currencies, with a zero strike price. This explains why the "binary option" payoff boundary goes through the origin.

The exposure in Figure 61 can however be hedged using a max-min option with non zero strike prices. The exposure here arises because if the company chooses to export then it has a risk to the exchange rate at which it converts its foreign currency income. From Figure 58 it is easy to see that the required hedge is an option on the maximum of currency 1 and 2 since this only pays out in the region in which the company has exposure. It is also clear that the required position is a short position since the company gains more, if unhedged, the further its currency depreciates against either of the foreign currencies. By selling a C_{Max} option with strikes at the correct equilibrium boundaries, the company will give up this potential excess profit and leave itself with no exposure to exchange rates at all.

The exposure examined by Ware and Winter then is in fact hedgeable completely albeit with a kind of complex option which they did not consider. It is possible again to extend this result to more than two foreign currencies since the firm will only ever require one foreign currency, the one which appreciates the most relative to the domestic currency, to be converted. Therefore a short position in a C_{Max} option written on *all* the foreign currencies will provide a complete hedge. From this example then we can state another general proposition in relation to fexibility in export/import decisions ie,

A General Proposition;

" A firm with flexibility amongst n markets for its product or n sources for its supplies can hedge this exposure by *writing* a complex option whose payoff is related to the maximum or minimum of the n foreign currencies."

It is interesting to note from the results of these two examples that the impact of having flexibility to export is equivalent to the competitive exposure which arose in the first example when the domestic currency appreciates. This is because the presence of foreign competition gives rise to the same type of exposure as that which arises from having the flexibility to export. This flexibility is equivalent to a long position in C_{Max}, which is why a short position was needed to hedge the exposure, and the firm facing competition in the first example also needed to sell this option[1] to hedge its gain in sales from its foreign competition. By having the flexibility to export the firm effectively creates the same exposure to exchange rate rises as would the arrival of foreign competition. This highlights the inter-relationship between the options generated by production and operating flexibility and the options which are required to hedge exposures. Analysing the decision making process in terms of complex options provides a way of relating these decisions to the underlying exposures. It also allows the company to value quantitatively the benefit of increasing flexibility in its operating processes. Thus, for example, a firm may be considering setting up a dealer network so that it can begin exporting its products. The cost is known but the benefits are less easy to quantify. One way of valuing the benefits though would be to price the appropriate C_{Max} option since this represents the value which arises from this new opportunity[2].

Finally the arbitrage condition presented in equation 4.21 suggested a possible relationship between max-min options and portfolio options ie options on portfolios of assets, in our case currencies[3]. The payoff of a call option on a portfolio with strike price K then in general is given by;

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1 Remember that the hedge in the first example was to go short of F_{Max} which is equivalent to buying P_{max} and selling C_{Max}.

2 The flexibility to switch between different inputs and outputs in a single currency environment has been discussed by Mason and Merton (1985).

3 An example of a portfolio option is an equity index option ie an option on the FTSE100 index. Typically index options are evaluated by assuming that the index is a traded asset which is itself lognormally distributed.

$$C_{Port} = \mathrm{Max}[P - K, 0]$$

where,

$$P = \sum_{i=1}^{n} S_i$$

and S_i is the exchange rate for foreign country i as before and so P is the domestic currency value of the portfolio per unit of foreign currency (Note we have assumed equal weightings for simplicity). For n = 2 then the payoff is;

$$C_{Port} = \mathrm{Max}[S_1 + S_2 - K, 0]$$

which can be re-written as;

$$C_{Port} = \mathrm{Max}\left[\left(S_1 - \frac{K}{2}\right) + \left(S_2 - \frac{K}{2}\right), 0\right]$$

Now this way of writing the payoff suggests that the value of C_{Port} might be written as;

$$C_{Port} = C_{BS}\left(S_1, \frac{K}{2}\right) + C_{BS}\left(S_2, \frac{K}{2}\right) - L \qquad 4.26$$

where C_{BS} is a Black and Scholes type European style currency call option as before and L is some type of instrument whose payoff equates both sides of the equation.

Now the payoff of C_{Port} is shown diagrammatically in Figure 62 and those of the European options in Figures 63 and 64. Using these payoffs we can derive the payoff on L and this is shown in Figure 65. As can be seen this is a complex payoff function with several boundaries. The first observation is that no area in its payoff is negative which implies that it may be some form of option. Secondly there are two areas of zero payoff, diagonally opposite each other, which might suggest that its payoff can be created from two other options. Consider the payoffs on calls and puts on the maximum and minimum of S_1 and S_2 all with strikes equal to $\frac{K}{2}$. These payoffs are shown in Figures 66-69. Close observation of these diagrams will confirm that the payoff on option L is equal to the minimum of the payoff on C_{Max} and P_{Min}, ie we can write;

$$L = \text{Min}[C_{\text{Max}}, P_{\text{Min}}] \qquad 4.27$$

The value of the portfolio option then can be interpreted as the sum of individual options on each currency in the portfolio, with strike prices equal to the portfolio option strike price divided by the number of currencies, less an amount which is a function of max-min options with the same strikes. The complex option L can be thought of as the lost opportunity which arises when the portfolio option finishes out-of-the-money but some of the individual options are in-the-money. This lost opportunity has a payoff which is related to the maximum and minimum currencies in the portfolio, and in general the second best, third best, etc as well. We can also prove that the option L is the same for put options on portfolios ie;

$$P_{Port} = P_1 + P_2 - L \qquad 4.28$$

where the payoff on L is still defined by equation 4.27. Thus we can combine equations 4.26 and 4.28 with the put-call parity condition in 4.22 to derive a put-call parity condition for portfolio options ie;

$$C_{Port} = P_{Port} + P^{*} - Ke^{-r_D T}$$

where,

$$P^* = \sum_{i=1}^{n} S_i e^{-r_{F_i} T}$$

ie P^* is the current value of the portfolio discounted at the foreign rates. As with equation 4.23 this collapses into the standard put-call parity condition when n = 1.

Unfortunately evaluation of L is difficult since it involves a complex function of several assets which are jointly lognormally distributed such that the distribution of the required combination is not lognormal. This is essentially because the distribution of the sum of lognormally distributed variables is not itself lognormal. The same problem occurs in the valuation of Average Rate and Swap Point options although in those cases an analytical approximation can be used[4]. In the case of L, because the underlying assets can in general have any correlation with one another a different approximation must be used, the most appropriate being either a multinomial lattice or Monte Carlo simulation.

The significance of these results is that they show that different types of multivariate options are related, although in a complicated way. Portfolios of conventional options and max-min options held in the appropriate positions, then will produce payoffs identical to those of an option on a portfolio for particular outcomes of the underlying assets. The use of conventional options and max-min options may therefore be viewed as complementary in creating an overall option hedge for a portfolio of currencies. In this sense the max-min options can be thought of as "filling-in" the parts of the portfolio risk which

4 This is because the particular correlation structure of the underlying assets in these problems means that an analytical approximation produces very good answers for typical parameter values.

are overhedged when individual options are used. This is analogous to their use in hedging flexibility in operating decisions, where the max-min options were also used to sell off this excess exposure.

4.4.1.D Implications of the Distributional Assumption and Conclusions

Having identified the required complex option the main consideration for the treasury manager is that the valuation model will calculate a "fair" value for the flexibility being traded. In other words the question is whether any biases in the model prices may occur and in what form they will be present. Obviously this depends on the assumptions underlying the model, which as we mentioned earlier are essentially the same as those in the Black and Scholes model. The biases created by the assumption of a lognormal distribution which occur in that model will also therefore be present in the complex option prices. In particular for in and out-of-the-money strike prices we would expect the model to undervalue the option. Since the treasury manager will in many cases be selling these options this is obviously a cause for concern.

The crucial determinant of max-min option prices is the correlation between the returns on the two currencies. As we saw in the empirical work, these correlations are unstable and so we might expect that biases in valuation may occur since the model assumes they are constant. This problem is analogous to that of using the standard Black and Scholes model when the variance is in reality non-stationary. The results of tests on models which incorporate stochastic variance though as we mentioned before have found that the biases induced by the constant variance assumption are in general small. The impact of a changing variance is felt more from the hedging point of view than in the price[5].

An unstable correlation structure then may be expected to cause more problems in the maintenance of a dynamic hedge portfolio than in the pricing itself. For the treasury manager this instability in the covariance matrix of currency returns should be less of a concern than the fact that the individual distributions are leptokurtotic since the effect of the later on the actual price is probably greater. This emphasises the point that as a price taker rather than an option hedger, the treasury manager is only concerned with the assumptions underlying the valuation model to the extent that they effect the option price. Since stochastic variance, and by implication covariance, has not been found to systematically bias option prices, the error caused by this assumption from the treasury managers viewpoint is not critical.

To conclude, in this section we have described a type of multivariate contingent claim known as a max-min option. We have demonstrated its application to two types of foreign exchange exposure and have extended the results of some previous work in this area. We have also shown how this type of option is related to another multivariate option, the portfolio option, in particular for the case of two foreign currencies. Finally we have pointed out some of the practical issues related to the distributional assumptions in the pricing model which have implications for the actual use of these instruments.

5 See Hull and White (1987b)

4.4.2 Foreign Currency Barrier Options[6]

4.4.2.A Definition

A foreign currency barrier option is a type of path dependant contingent claim which appears (or disappears) if the underlying exchange rate reaches the barrier level during the life of the option. If the option exists at maturity then its payoff is identical to a conventional option. At the time the spot rate reaches the barrier level a fixed payment may be made.

These types of options are similar to American style options in that their value depends not only on the value of the underlying exchange rate at maturity but also on its path in getting there. Unlike American options however the critical boundary is known in advance which is why closed form solutions can be found. Barrier options which appear at the barrier level can also be regarded as a type of compound option. This is because with a compound option the holder has the right to sell or buy an underlying conventional option over some time period. Thus if volatility and interest rates are constant, the exercise decision is based purely on the spot rate and so one can regard the underlying option as a form of barrier option which will appear, ie be exercised into, if the spot rate reaches the exercise boundary during the life of the compound option. With the compound option however this boundary is time dependant whereas barrier options in practice tend to have constant barrier levels. Also with a compound option the holder is not obliged to exercise at this level but the barrier option will always by definition appear when the underlying spot rate is at that rate.

6 These options are sometimes called Down-and-out options, Knockout options, Dropout options or Stoptions.

We can define eight types of barrier options, puts and calls, which appear or disappear as the spot rate moves to an upper or lower barrier and when the strike price is above or below this barrier level. The full set of possible combinations is;

1. C_{B1}, P_{B1} - Disappears when S = B, exists if S > B, K > B, $S_0 > B$
2. C_{B2}, P_{B2} - Appears when S = B, exists if S < B, K > B, $S_0 > B$
3. C_{B3}, P_{B3} - Disappears when S = B, exists if S < B, K > B, $S_0 < B$
4. C_{B4}, P_{B4} - Appears when S = B, exists if S > B, K > B, $S_0 < B$
5. C_{B5}, P_{B5} - Disappears when S = B, exists if S > B, K < B, $S_0 > B$
6. C_{B6}, P_{B6} - Appears when S = B, exists if S < B, K < B, $S_0 > B$
7. C_{B7}, P_{B7} - Disappears when S = B, exists if S < B, K < B, $S_0 < B$
8. C_{B8}, P_{B8} - Appears when S = B, exists if S > B, K < B, $S_0 < B$

where C_{Bi}, P_{Bi} are barrier call and put options, S is the spot exchange rate, S_0 is the initial spot rate, B is the barrier level and K is the strike price, all expressed as domestic currency per unit of foreign currency. In general a barrier option could have two barriers, one above and one below the current spot level. Such options are rare however and so we do not consider them here. Also for simplicity we consider only European style barrier options. As we mentioned, any of these options may also have one or more rebates attached such that a fixed payment is made, or received, if the spot rate reaches a pre-determined level(s) during the life of the option. The valuation of rebates can be regarded as a separate problem however and we cover this in the next section.

First it is useful to examine the payoffs of the above options in order to identify which may have possible applications to exposure management.

C_{B1} - This is the most common type of barrier call option which disappears as it goes out-of-the-money and at the point it disappears it would have no intrinsic value as a conventional option. If it does not disappear then it has the same limitless upside as a conventional option.

P_{B1} - This option disappears as it goes in-the-money and at that point has intrinsic value. The payoff, at maturity, therefore has a vertical payoff line since the option goes from having value to zero immediately at the barrier level. This makes it very difficult to hedge, as we discuss later. The option no longer has a limitless upside and if the strike is close to the barrier the possibility of exercise is very small.

C_{B2} - This is the counterpart to C_{B1} in that it is a call which appears as it goes out-of-the-money and at that point has no intrinsic value. Once it has appeared it has exactly the same payoff as a conventional option.

P_{B2} - This is the counterpart to P_{B1} and so appears as it goes in-the-money and when it appears it has intrinsic value. The payoff at maturity, if it has not already appeared, obviously has the same vertical payoff line.

C_{B3} - This is a redundant combination since the option can never be exercised ie it will always disappear before becoming in-the-money.

P_{B3} - This option will always be exercised as long as it does not disappear. If it reaches maturity then it has a vertical payoff as with P_{B1} and P_{B2} and so will be difficult to hedge.

C_{B4} - This option appears as it goes in-the-money but at the point it appears it has no intrinsic value. Once it appears it will payoff exactly like a conventional option. In fact there is no situation where this option will payout any differently to a conventional option and so it should have the same value.

P_{B4} - This option appears as it goes out-of-the-money and at that point has intrinsic value. It is the counterpart to $P_{B3}$and therefore has a vertical payoff line at maturity if it has not already appeared.

C_{B5} - This option is similar to C_{B1} in that it is a call that disappears going out-of-the-money except that in this case it has intrinsic value at that point. It therefore has the similar characteristics to P_{B3}.

P_{B5} - As with C_{B3} this option is redundant in that it will never be exercised.

C_{B6} - This call option appears when going out-of-the-money and has intrinsic value at that point. It is therefore the counterpart to C_{B5}.

P_{B6} - This option appears as it goes in-the-money but at that point has no intrinsic value. It has similar characteristics to C_{B4} ie it is essentially a conventional European style option.

C_{B7} - This option disappears as it goes in-the-money and at that point has intrinsic value. It has similar characteristics to P_{B1}.

P_{B7} - This is the most common type of barrier put option since it disappears as it goes out-of-the-money and at that point has no intrinsic value as with C_{B1}. This makes it easier to hedge since there are no potential vertical payoffs.

C_{B8} - This option appears as it goes in-the-money and at that point has intrinsic value. It is therefore the counterpart to C_{B7} and has the same characteristics as P_{B2}.

P_{B8} - This is the counterpart to P_{B7} in that it appears as it goes out-of-the-money and at that point has no intrinsic value.

Using these definitions it is possible to derive arbitrage relationships relating the value of combinations of these options to standard European options with the same maturity and strike price. For example consider a portfolio containing a long position in C_{B1} and a long position in C_{B2} both with the same barrier level B. If the lowest spot rate S_{Min} during the life of the options is greater than B then C_{B1} will payoff as a conventional option, since it will not have disappeared, and C_{B2} will pay zero since it has not appeared. If S_{Min} < B then C_{B1} will have disappeared and so will have zero payoff, but C_{B2} will have appeared and payoff identically to a European option. Therefore in no circumstances will the portfolio payoff be different to that of a conventional option and so to avoid arbitrage it must sell for the same value ie

$$C_{B1} + C_{B2} = C_{BS}$$

In fact we can prove using the same arbitrage arguments that in general,

$$C_{Bi} + C_{B(i+1)} = C_{BS}$$

$$P_{Bi} + P_{B(i+1)} = P_{BS} \qquad 4.29$$

for i = 1,3,5,7 and each pair of barrier options has the same barrier level. We can therefore state a proposition such that, Proposition;

"For every barrier option which *disappears* at a particular barrier level there is an equivalent barrier option which *appears* at the same level such that the sum of the values of the two barrier options is equal to the value of a conventional option with the same strike price and maturity."

It is interesting to note that the relationship is true even for the redundant options and their counterparts. Since the redundant options must have zero value, this confirms the fact that for certain combinations of barrier level and strike the value of a barrier option is equal to that of a conventional option. In fact as the difference between the barrier level and the spot price increases the options should converge to the value of standard options (for options which disappear) or zero (for options which appear). Actually for those options which disappear at the barrier level when they have intrinsic value then this is only true when the option has a rebate attached which pays this intrinsic value when the option disappears. For some of these options, specifically C_{B7} and P_{B1} this convergence may not always be strictly from below ie for some barrier levels the options may have a higher value than the corresponding European option. This occurs when the underlying foreign currency has a higher interest rate for the call or a lower interest rate for the put, than the domestic currency ie where the American option has a higher value than the European option. Remember that these options disappear as they go in-the-money so in these situations the holder may prefer the option to disappear so that he can receive the intrinsic value now since this may be worth more than waiting to exercise the option at maturity. This suggests that the valuation of these options may provide an alternative approach to finding the value of American style options.

Finally, equation 4.29 suggests the a put call parity relationship for barrier options ie

$$C_{Bi} - P_{Bi} + C_{B(i+1)} - P_{B(i+1)} = C_{BS} - P_{BS}$$

Let $H_{Bi} = C_{Bi} - P_{Bi}$ for i = 1,...,8 and using the put call parity relationship for conventional options we have;

$$H_{Bi} + H_{B(i+1)} = Se^{-r_F T} - Ke^{-r_D T}$$

for i = 1,3,5 and 7. We can interpret H_{Bi} ($H_{B(i+1)}$) as a forward contract to exchange the foreign currency for domestic currency at a rate equal to the strike price K, which disappears (appears) if the spot rate reaches the barrier level B during the life of the instrument. We call this a barrier forward contract and demonstrate its use in a practical example later.

Although as we have shown, in some cases the value of a barrier option is the same as a standard European option, in general the valuation involves both the distribution of the underlying asset at maturity and the first passage time distribution ie the time to reach the barrier.

4.4.2.B Valuation

The original valuation equations for some specific barrier options, were presented in Merton (1976) in which he derived the value of C_{B1} when the barrier was in general a function of time and the underlying asset was a stock paying no dividend. As he pointed out the value turned out to be equal to the value of a standard European option less a "..."discount" due to the down-and-out feature". As we can see from equation 4.29 this discount is in fact equal to the value of C_{B2} ie in Merton's terminology a "down-and-in" option. Ingersoll (1987, pg 371) presents the general solution to the partial differential equation for barrier options with barriers above and below the current asset price. This solution is;

$$H(S,t) = e^{-r_D(T-t)} \int h(S_T) f(S_T, T) dS_T + \int_t^T e^{-r_D(u-t)} [\overline{h}(u)\overline{g}(u) + \underline{h}(u)\underline{g}(u)] du \quad 4.30$$

where H(S,t) is the option value, r_D is the riskless interest rate, S_T is the final security price, $h(S_T)$ is the terminal option value (ie payoff function), $f()$ is the risk neutral density of the underlying price at maturity with an absorbing barrier, $h(u), \underline{h}(u)$ are the contractually specified payouts at the upper and lower barriers respectively, and $\overline{g}(u)$ and $\underline{g}(u)$ are the first passage time densities to the upper and lower barriers respectively. For particular payoff functions then we can derive the option value using this equation and substituting for the appropriate boundary conditions and the density functions f and g. For the most common barrier options described above the valuation formulas are;

$$C_{B1} = C_{BS} - \lambda C_{BS}(S = S^*)$$

$$P_{B7} = P_{BS} - \lambda P_{BS}(S = S^*)$$

where,

$$\lambda = \left(\frac{S}{B}\right)^{1-\delta}$$

$$\delta = \frac{2}{\sigma^2 T} \ln\left(\frac{F}{S}\right)$$

$$S^* = \frac{B^2}{S}$$

Thus we can write down the value of these barrier options in terms of standard Black and Scholes option values with a change of variables. If the option has a rebate feature which is a fixed payment M, if the spot rate beaches a level B, then the value of this is given by;

$$R = M\left(\frac{S}{B}\right)\left[\left(\frac{S}{B}\right)^{-\delta-1} N(\xi d_1) + N(\xi d_2)\right]$$

where,

$$d_1 = \frac{\ln\left(\frac{B}{S}\right) + \left(r_D + r_F + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and $\xi = 1$ if $S_0 > B$ or -1 if $S_0 < B$.

We now examine some possible applications of these options in hedging specific foreign exchange exposures.

4.4.2.C Applications

No actual application of the barrier option considered by Merton was given in his paper. In the marketing literature produced by the banks and security houses offering these products the main applications suggested have been as a tool for speculation. A typical strategy is to sell barrier option C_{B1} if one has a view that the rate will fall to the when the forward is biased. In the second example we examine economic exposure and look at a situation where barrier options can be used as alternative options even when the forward is unbiased.

Example 1

Consider a company expecting a foreign currency receivable at some time T in the future. The forward rate F_T at which the firm can sell this currency for its domestic currency is believed to be biased at the moment so that the expected spot rate is $F_T - X$. The manager responsible for hedging this exposure is reluctant to wait until time T to sell the foreign currency at this better rate since he is worried that there is a small chance that it may be higher than $F_T - X$ but would be willing to lock into the forward rate as soon as the bias is removed. Although he could buy a call option with a strike at $F_T - X$ this is expensive, since it is in-the-money relative to the current forward, and his budget is not large enough to cover the cost. Additionally, he does not want to write a conventional put option with the same strike against the call because this will just cost him the discounted value of the bias ie $Xe^{-r_D T}$ which he expects to disappear. Ideally he would like to have some protection up to the time at which the bias disappears at which point he would hedge forward.

One solution to this problem would be to use the barrier option C_{B1} or C_{B5} combined with a barrier forward H_{B2} or H_{B6}. The strategy would be to buy the barrier option with barrier level set at a distance equal to X below the current spot rate ie $B = S - X$ and also buy the barrier forward which will appear at the same level and which has a strike rate equal to $F_T - X$. Thus the company is still protected at a fixed rate, the strike price of the option, if the spot rate rises, but if it falls such that the forward rate equals the expected spot rate at which he is happy to lock into, then he does so. The assumption is made that the interest rate differential does not move, or at least its movement is not significant compared to the movement in the spot rate. Thus when $S \Rightarrow S - X$, $F_T \Rightarrow F_T - X$. If the perceived bias X is large enough then the movement in the spot rate should be much larger than the movement in the forward points.

Whether or not C_{B1} and H_{B2} or C_{B5} and H_{B6} are bought depends on the level of the strike rate relative to the barrier. If the strike was chosen to be the expected spot rate, $F_T - X$, then if the current forward rate was lower than current the spot rate the hedge would be $C_{B5} + H_{B2}$ otherwise $C_{B1} + H_{B6}$. Note that the put call parity condition and arbitrage condition in equation 4.29 above means that an alternative strategy exists in each case ie;

$$C_{B1} + H_{B2} = C_{B1} + C_{B2} - P_{B2} = C_{BS} - P_{B2}$$

$$C_{B5} + H_{B6} = C_{B5} + C_{B6} - P_{B6} = C_{BS} - P_{B6}$$

so the same payoff can be created from a long position in a conventional call option and a short position in a barrier put option. This highlights the similarity between this strategy and a conventional collar, ie long call, short put. Both strategies involve buying the call and writing a put against it which reduces the upside potential and also the cost of the strategy. In the barrier approach however the option written is a barrier option and so the payoff depends on the path of the exchange rate as well as the final value. If the treasury manager believes that the rate will move in his favour at some time during the period but is unsure whether it will remain in his favour at maturity, the barrier option strategy provides a more efficient hedge. Also an analogous strategy obviously exists for hedging a foreign currency payable, this time combining $P_{B3} - H_{B4} \equiv P_{BS} - C_{B4}$ or $P_{B7} - H_{B8} \equiv P_{BS} - C_{B8}$.

This example demonstrates a possible use of barrier options for hedging a transactions exposure when the forward rate is believed to be temporarily biased. By providing an option hedge only up to the time at which the perceived bias disappears these options provide a link between the use of forward contracts and conventional options as forward prices move with respect to expected future spot prices. Given the instability in this bias, or risk premium, that has been found from the empirical work, payoffs such as these can have a useful role in many practical hedging decisions. From the example we can state a general proposition of the form,

Proposition;

"When forward prices are temporarily biased with respect to expected future spot rates, exposure can be effectively hedged using combinations of barrier options and barrier forwards or conventional options and barrier options."

We now examine the use of these options, again as alternatives to standard options, in hedging economic exposure.

Example 2

Consider a firm which can buy its raw material either on the domestic market or in one foreign country. The decision to switch is based purely on the relative domestic currency cost of the input ie the firm will import if $P_F S < P_D$ where P_F is the foreign currency unit cost of the raw material, P_D is the domestic currency unit cost and S is the spot exchange rate. In the study referred to earlier, Ware and Winter have shown that the required hedge for the company in this case is to sell a put option on the foreign currency with a strike price equal to the ratio $\frac{P_D}{P_F}$. Thus if the firm buys domestically, the option will not be exercised, and if it buys abroad then the option will be exercised and the company will receive the necessary amount of foreign currency at the required rate. This is an example of economic exposure, which as with the max-min options before, is hedged by selling options, ie flexibility, to the market.

Remember that in hedging economic exposure when the firm has flexibility, options have a role even if the forward rate is unbiased, since forward contracts alone cannot produce the required payoff function. However suppose now that the treasury manager has a view that the spot rate will probably rise during the period. This may or may not be consistent with the expected movement in the spot rate implied by the forward rate ie the forward may still be unbiased. As long as he believes that the spot rate may increase, an alternative strategy using barrier options exists.

Instead of selling a conventional put option the firm would sell a barrier option type P_{B3} or P_{B7} depending on whether the strike price $K = \frac{P_D}{P_F}$ is above or below the barrier level B. If the option disappears then the firm could sell a conventional put option or maybe another barrier option with a higher barrier level. The idea is to maintain the correct hedge position, ie short of a put, but to benefit from any movement in the exchange rate which is consistent with a prior view. The benefit obviously arises when the premiums received are greater than the original premium which would have been received if the conventional put had been sold. Obviously if the exchange rate falls then firm will not have received sufficient compensation, in the form of premium, for the flexibility it has given up. However, the reason for entering into the strategy is to benefit from a view on the direction of the spot rate. If this view turns out to be incorrect then the strategy is expected to be sub-optimal.

One of the benefits of this strategy is that it can be used to exploit the different vega or volatility sensitivities of conventional and barrier options. In general for a conventional option the partial derivative $\frac{\partial H_{BS}}{\partial \sigma} > 0$ where H_{BS} is a conventional derivative security. For barrier options this is not always the case and in fact the sign of the partial derivative may be positive or negative depending on the level of volatility, σ. For barrier options which disappear in particular, as volatility rises the option value will usually increase in the normal way but any further increases result in lower option prices since the probability of the option disappearing and therefore having zero value, increases. In Figure 70 this effect can be clearly seen for a one month maturity European call and barrier option C_{B1} where the price (in domestic currency per foreign currency) is shown and K = S = 2.7550 and B = 2.7500. The strategy described above can therefore be used to take advantage of these effects. For example at times when volatility is low the difference between the conventional put and the barrier put will be lower than when volatility is high. Therefore the marginal cost of using the barrier strategy, ie the potential loss in premium if the view is wrong, is smaller. In this situation then the barrier strategy may be preferable, whereas when volatility is high the conventional strategy would be preferred.

The use of a barrier option as an alternative to a conventional option in this situation again points out the opportunities offered by these instruments to take advantage of expected movements in the spot rate in an efficient way, whilst maintaining the required hedge position. It is important to remember that in the second example though the view need not be different from that implied by the forward rate. Thus as with conventional options, barrier options are an efficient hedge for economic exposure even when the forward rate is unbiased.

4.4.2.D Implications of the Distributional Assumption and Conclusions

Since the assumptions, and particularly the distributional assumption, used in deriving the valuation models presented here are the same as those used in the Black and Scholes model, the same pricing biases with regard to in-the-money and out-of-the-money option prices also apply. The most important concern however in the valuation and hedging of these options is the assumption of a pure diffusion process and the related assumption of continuous trading. Since the underlying exchange rate is assumed to follow a pure diffusion process, no jumps in the price are possible and so trading can be done at every level in any amount. In reality we know however that the spot price can and does jump, for example after major political events or economic news. Even in normal markets there is a bid offer spread which causes problems in specifying when a barrier level has been reached.

The problem with assuming a pure diffusion process as opposed to a mixed jump or pure jump process is that it causes significant difficulties in the hedging argument. In order to examine the effect of the assumption on the prices of barrier options it is useful to analyse the hedging problem in detail. This approach is useful in general since in practice the price of any option will reflect the perceived cost of hedging ie market makers will induce a bias in their prices by adjusting the prices they quote away from the values produced by the model to take into account the problems involved in hedging.

For barrier options these induced biases will reflect the problems associated with hedging the particular option being quoted which are primarily a function of the position the trader will have if and when the option reaches the barrier. For example in the case of C_{B1} a short position in the option requires the hedge portfolio to be long of the underlying currency, as with a conventional call. Thus when the option disappears the trader will have to sell this hedge position. If the spot price jumps however so that it never trades at the barrier level but the next price is below the level, the trader must sell his hedge at a lower level, therefore losing money. Alternatively if the trader is long of the option his hedge will be to short the currency in which case if there is a jump through the barrier he will make money. When pricing a barrier option type C_{B1} then the trader will, all else being equal, increase his price above the model price to take in more premium if he sells the option and pay out more if he buys. The induced bias because of jumps in the underlying exchange rate in the price of this option is therefore downward ie the model undervalues.

Using the same procedure the biases for each of the barrier options considered above can be deduced ie,

C_{B1}, C_{B5}, C_{B7}, P_{B1}, P_{B3} and P_{B7} are undervalued.

C_{B2}, C_{B6}, C_{B8}, P_{B2}, P_{B4} and P_{B8} are overvalued.

C_{B3}, C_{B4}, P_{B5}, P_{B6} are unaffected ie these are redundant options and their European option counterparts.

Note that the barrier options which disappear are all undervalued by the model and the ones which appear are overvalued. The arbitrage relationship in equation 4.29 therefore is not effected by these biases since the sum of the barrier option prices will equal the conventional option value even when the barrier options are individually biased. Also the fact that liquidity in the market varies, exacerbates these biases by effectively increasing the size of the jump. This is because when the market is illiquid the next price at which one can execute the required amount may be even further from the barrier than the first price quoted after the jump. Thus illiquidity increases, from a practical viewpoint, the magnitude of the jump.

A second problem concerning hedging for some of the barrier options is the vertical payoff line which occurs when the strike price is on the in-the-money side of the barrier. The traditional approach to contingent claims pricing assumes that the terminal payoff is piecewise linear and there are no discontinuities. For these options however this is not the case and so although they can be priced using the risk neutral valuation procedure, from a practical viewpoint they are almost impossible to hedge, other than with another barrier option or a pure rebate instrument. This is because the required hedge position at the point the option reaches the barrier could potentially be very large ie $\lim_{S \to B} \frac{\partial H_{Bt}}{\partial S} = \infty$. Thus although a theoretical value may be calculable for these types of barrier options it is unlikely that any institution would agree to deal on one and even if they did it would be unlikely to be at this price.

Further pricing biases will also occur because of the assumption of constant interest rates. Stochastic interest rates will tend to increase the volatility of the forward rate, depending on the correlation between interest rates and the exchange rate[7], and so all options models based on this assumption will tend to underprice. For longer dated options this underpricing is exaggerated as these options are most sensitive to volatility and interest rate movements. For barrier options however the problem is compounded by the changing sensitivity of the option to the interest rates as the spot rate moves close to the barrier. Thus for options which disappear at the barrier, when the spot rate is far from this level the option behaves like a conventional option with the appropriate level of sensitivity, but when the spot rate is close to the barrier the option will have very little forward exposure since it is close to disappearing. This adds an additional difficulty for hedging which cannot be covered with alternative instruments and so there will be a "premium" to pay for buying or selling a long dated barrier option.

7 In the equilibrium monetary models the correlation between the spot rate and the interest rate differential is positive since an appreciation in the domestic currency causes domestic rates to fall relative to foreign rates resulting in a larger movement in the forward rate.

The options which are traded actively though can still offer the treasury manager useful additional tools in managing his exposure, as we have shown in the examples. The effect of the assumption of pure diffusion will tend however to increase the biases already present in the price of the option due to the leptokurtotic terminal distribution. It is therefore important that these effects are bourne in mind when evaluating alternative strategies, although as we have shown some of the differential sensitivities between these options and conventional options may provide additional profit opportunities.

4.4.3 Options on Forward Swap Points

4.4.3.A Definition

A call option on the forward swap points C_{FSP} is an option which gives the holder the right to enter into a forward swap in which he buys the foreign currency on the forward date and hence sells it on the spot date, in exchange for the domestic currency. These options can in general be both European and American style but we will consider only European options in which case the maturity date of the option is the same as the near date of the swap. The payoff on an option with strike price K, is given by;

$$C_{FSP}(T) = \text{Max}[SP(T) - K, 0] \qquad 4.31$$

where $SP(T)$ is the swap points at maturity of the option T. For a put obviously the payoff is just,

$$P_{FSP}(T) = \text{Max}[K - SP(T), 0]$$

As an example consider a three month call on a six month swap in USD/GBP with a nominal amount of £10m. Suppose the spot rate is 1.6000 USD/GBP and the current forward rates for three, six and nine months are 1.5800, 1.5600 and 1.5300 respectively. The forward swap points for three, six and nine months are therefore -0.0200, -0.0400 and -0.0700. Now although the current six month swap points are -0.0400 the at-the-money strike of the option must be the forward/forward swap points between three and nine months. This is because these dates will be the spot and six month forward dates when the option matures in three months time. Thus the actual underlying position is a forward forward swap, three months against nine months. Suppose then that the strike is chosen to be at-the-money ie -0.0500 points[1], and in three months the option expires with the current six month swap points at -0.0400. The value of the option is therefore given by;

$$C_{FSP}(T) = \mathrm{Max}[-0.0400 - (-0.0500), 0] = 0.0100$$

ie it will be exercised. Thus the buyer will enter into a six month swap selling £10m spot and buying £10m on the six month date at a difference of -0.0500 points. By reversing the swap immediately in the market at the actual swap rate of -0.0400 he would generate a profit of 0.0100 dollars per pound ie $100,000. Note that this profit will be realised on the six month date and so if the option were to be cash settled, as is often the case, then the payoff would have to be discounted at the appropriate rate.

The instrument is essentially an option on the interest rate differential since any change in the swap points is predominantly due to changes in this differential. Changes in the spot rate will have some effect though since by definition $\frac{\partial SP}{\partial S} > 0$ except when $r_D = r_F$. This effect will usually be small however compared to the effect of changes in the interest rates. As such the option is analogous to options on cross currency interest rates swaps, and in fact could be defined as an option on a zero coupon cross currency swap. Options on interest rates differentials are also beginning to trade in the form of options on the difference between interest rate futures prices where the futures are on different currency interest rates, and also options on the differences between bond prices

1 The forward/forward swap points between two dates T_1 and T_2 in the future, is just given by,

$$SP(T_1, T_2) = F(T_0, T_2) - F(T_0, T_1)$$

where T_0 is the current date.

in different countries. The swap point option therefore can be seen to fit into a family of new products all of which are concerned with the relative movement in interest rates between currencies.

4.4.3.B Valuation

Although these options have been trading for some time there have been no published articles on their valuation. It is possible to show however that an exact closed form solution which is consistent with the assumption of lognormality in the spot rate, cannot be obtained. This is because the particular payoff function for this option requires that the distribution of the difference between two lognormal variables be evaluated. Unfortunately this can only be done numerically and so the solution will only be approximate to the degree of accuracy of the numerical procedure used. To see this let us write the payoff in equation 4.31 as,

$$C_{FSP}(T) = \text{Max}[F(T) - S(T) - K, 0] \qquad 4.32$$

where $F(T)$ is the forward rate for the far date of the swap at maturity of the option and $S(T)$ is the spot rate on maturity of the option. Now if the spot rate is lognormally distributed then if we maintain the assumption of constant interest rates for the moment, the forward rate must also be lognormally distributed. The payoff will therefore involve the difference between two lognormally distributed variables. Now if the strike price K = 0 then the problem actually reduces to the Magrabe problem described earlier and so an exact closed form solution exists (equation 4.18, section 4.2). In general however the solution requires the integral over the distribution of the difference between two lognormal variables which as we said can only be evaluated numerically. Of course if we assume that the swap points or rather changes in swap points, follow some plausible distribution, perhaps lognormal, then we could evaluate equation 4.31 directly using the risk neutral approach. In this case the solution would be equivalent to the standard option value formula and we would have to specify the volatility of the swap points to calculate a price. The first problem with this is that the assumed distribution would not necessarily be consistent with the assumed lognormal distributions for the spot and forward rates, certainly not if we chose lognormality. Secondly, the distributions of changes in swap points, as we have seen in Chapter 2, are significantly non-lognormal and so the use this assumption in a redefined Black and Scholes type formula would be even more inaccurate than for standard currency options. Thirdly, since swap points can be both positive and negative the lognormal assumption could not be used in any case. Of course we could use an entirely different distribution assumption but in order to be tractable this would also lead to inconsistencies in valuing these options when compared to the valuation of conventional options. This could cause particular problems when hedging with standard or derivative options based on the assumption of lognormal spot rates.

Of course in reality the interest rates are not constant, in fact if they were then there would be no need for this option. An alternative valuation procedure could therefore model the individual interest rates as stochastic variables perhaps following a process as in equation 4.1 or more likely involving mean reversion and maybe jumps. The problem with this is that since the swap points are also a function of the spot rate, the model would need to incorporate its movements also and this would lead to a complex tri-variate distribution for the underlying swap points. Evaluation of this distribution would almost certainly be just as difficult as the distribution of the sum of lognormal variables needed before and an additional problem would be in specifying the covariance matrix for the two interest rates and the spot rate.

It is not necessary however to model each rate separately since stochastic interest rates could still produce an approximately lognormal forward rate since as we noted earlier movements in the forward rate are dominated by movements in the spot rate. Thus we would expect that the lognormal assumption would be equally plausible for the forward rate as it is for the spot rate. Indeed this is why we analysed the forward swap points rather than the outright forward rate in the empirical tests in Chapter 2. Movements in the interest rates could be reflected in differential volatilities of the spot and forward rates and a non-perfect correlation between them. In valuing these options then we would like to preserve the lognormal assumption for spot and forward rates but avoid the use of a numerical integration procedure for evaluating the option price since this could involve excessive calculation time.

An approach which satisfies these conditions is based on a technique used by Levy (1990) in evaluating Average Rate options. The technique is essentially an application of the method of moments which is used to solve for the first and second moments of a lognormal distribution so that they are equal to those of the actual complex multivariate distribution. The approach relies on the so-called "Wilkinson approximation" which states that the sum of n lognormal distributions can be approximated by another lognormal distribution with the same moments to a high level of accuracy as long as the correlations between the individual distributions and their variances are within certain ranges. If we set up the payoff function of the option in an appropriate way then the range of values of correlation and volatilities of our adjusted variables fall within the acceptable limits. Careful manipulation of the payoff function also avoids the problem associated with negative values for the swap points or the strike price. The method provides an accurate approximation for the complex distribution in the form of another lognormal distribution. Once this distribution has been specified, the normal risk neutral valuation procedure can be used to find the option value giving an approximate closed form solution which is very fast to calculate.

First we make the following definitions; $t_1 = T$ = the maturity of the option, t_2 = the time to the far date of the underlying swap if exercised, t_3 = the time to the far date of the underlying swap from today. Thus for example for a three month option on a six month swap, $t_1 = T$ = 3 months, t_2 = 9 months (ie three plus six), and t_3 = six months. Note that t_3 is fixed but the others vary. Also define F_i as the current forward rate for t_i for i = 1,2,3.

To solve for the value of a call option C_{FSP} let us define a variable $S^* = S + K$ so that we can write the payoff in equation 4.32 as;

$$C_{FSP}(T) = \text{Max}[F_3(T) - S(T)^*, 0] \qquad 4.33$$

The value of the option at any time t < T using risk neutral valuation is therefore just;

$$C_{FSP}(t) = e^{-r_D T} E[\text{Max}[F_3(T) - S^*(T), 0]] \qquad 4.34$$

Now we assume S and F_3 follow geometric diffusion processes as in equation 4.1 ie;

$$dS = \mu_S S\,dt + \sigma_S S\,dz_S$$

$$dF_3 = \mu_{F_3} F_3 dt + \sigma_{F_3} F_3 dz_{F_3}$$

and z_S and z_{F_3} are Weiner processes with correlation ρ_{SF_3}. Since both rates are assumed to be lognormally distributed and we use the same efficient market assumptions as in Black and Scholes, their means can be written down using equation 4.4 as;

$$\mu_S T = \ln(S) + \left(r_{D1} - r_{F1} - \frac{1}{2}\sigma_S^2\right)T \qquad 4.35a$$

$$\mu_{F_3} T = \ln(F_3) + \left(r_{D32} - r_{F32} - \frac{1}{2}\sigma_{F_3}^2\right)T \qquad 4.35b$$

where r_{D32} and r_{F32} are the forward forward domestic and foreign interest rates respectively between t_3 and t_2. Equation 4.35b can be derived from the efficient market assumption with regard to forward rates ie;

$$E(F_3) = F_2$$

that is the expected value of the underlying forward rate at the maturity of the option is the current forward rate for the far date of the swap if it were exercised. This requirement ensures that the drift in the forward rate F_3 is such that it converges correctly to F_2 when the option expires. The forward forward interest rates can be found from the arbitrage condition $r_{ij}t_{ij} = r_j t_j - r_i t_i$ for $t_i < t_j$ and so we can write F_2 as;

$$F_2 = S e^{(r_{D2} - r_{F2})t_2}$$

$$= S e^{(r_{D3} - r_{F3})t_3 + (r_{D32} - r_{F32})t_{32}}$$

$$= F_3 e^{(r_{D32} - r_{F32})t_{32}}$$

Thus we have an analogous expression to equation 4.3 for the forward rate ie;

$$E(F_3) = F_3 e^{(r_{D32} - r_{F32})T}$$

Since the forward forward maturity period is always equal to the option maturity T we get the expression for the mean of F_3 shown above.

Now S^{*} is not lognormally distributed but since K is a constant we will assume that its true distribution can be well approximated by a lognormal distribution. The problem is then to find the appropriate lognormal distribution which provides the best fit for the true distribution. To determine the parameters of the approximating distribution we use the moment generating function for $\ln(S^{*})$ given by;

$$M(n) = E([S^{*}]^{n}) = e^{n\mu_{s^*}.T + \frac{1}{2}n^2\sigma_{s^*}^2.T} \qquad 4.36$$

By substituting for n = 1, 2 for the first and second moments, we have two simultaneous equations from which we can solve for $\mu_{s^*}.T$ and $\sigma_{s^*}^2.T$ to get;

$$\mu_{s^*}.T = \ln(E([S^{*}]) - \frac{1}{2}\sigma_{s^*}^2.T \qquad 4.37a$$

$$\sigma_{s^*}^2.T = \ln(E([S^{*}]^2) - 2\ln(E[S^{*}]) \qquad 4.37b$$

To evaluate the moments then we just use the expression for $E(S^{*})$ and $E(S^{*2})$ derived by equating the first two moments to those of the actual distribution[2] ie;

$$E([S^{*}]) = E(S + K)$$

$$= e^{\mu_S T + \frac{1}{2}\sigma_S^2 T} + e^{\mu_K T + \frac{1}{2}\sigma_K^2 T} \qquad 4.38$$

$$E([S^{*}]^2) = E([S + K]^2)$$

2 Since we assume $E[\ln(S), \ln(K)]$ is bivariate normal we make use of the moment generating function for bivariate normal distributions ie

$$M(m,n) = E[(S)^m, (K)^n] = e^{m\mu_S T + n\mu_K T + \frac{1}{2}\left(m^2\sigma_S^2 + 2\rho_{SK}\sigma_S\sigma_K mn + n^2\sigma_K^2\right)}$$

$$= E[(S)^2] + E[(K)^2] + 2E[(S),(K)]$$

$$= e^{2\mu_S T + 2\sigma_S^2 T} + e^{2\mu_K T + 2\sigma_K^2 T} + 2e^{\mu_S T + \mu_K T + \frac{1}{2}(\sigma_S^2 + \sigma_K^2 + 2\rho_{SK}\sigma_S\sigma_K)T} \qquad 4.39$$

Note that since K is a constant,

$$\mu_K T = \ln(K)$$

$$\sigma_K^2 T = \rho_{SK} = 0$$

By substituting for μ_S, σ_S, μ_K, σ_K and ρ_{SK} into equations 4.38 and 4.39 and simplifying we get;

$$E([S^*]) = F_1 + K$$

$$E([S^*]^2) = F_1^2 e^{\sigma_S^2 T} + K^2 + 2KF_1$$

where $F_1 = Se^{(r_{D1} - r_{F1})T}$. Finally substituting into equations 4.37a and 4.37b yields the solution for the mean and variance of the approximating distribution. Having evaluated these parameters the option can be valued using the generalised Magrabe solution in equation 4.18 substituting for μ_{F_3}, σ_{F_3}, μ_{S^*}, σ_{S^*} and $\sigma_{F_3S^*}$. The covariance can be found from $\sigma_{F_3S^*} = \sigma_{F_3}\sigma_{S^*}\rho_{F_3S^*}$ where the correlation must be estimated from market prices. The solution for the value of C_{FSP} is given by;

$$C_{FSP} = e^{-r_{D1}T}\left[F_3 e^{(r_{D32} - r_{F32})T} N(d_1) - (F_1 + K)N(d_2)\right]$$

where,

$$d_1 = \frac{\ln(F_3) + \left(r_{D32} - r_{F32} - \mu_{S^*} + \sigma_{F_3}\left(\frac{1}{2}\sigma_{F_3} - \sigma_{S^*}\rho_{F_3S^*}\right)\right)T}{\sqrt{\sigma_{F_3}T + \sigma_{S^*}T - 2\sigma_{F_3}\sigma_{S^*}\rho_{F_3S^*}T}}$$

$$d_2 = \frac{\ln(F_3) + \left(r_{D32} - r_{F32} - \mu_{S^\bullet} - \frac{1}{2}\sigma_{F_3}^2 + \sigma_{S^\bullet}\left(\sigma_{F_3}\rho_{F_3S^\bullet} - \sigma_{S^\bullet}\right)\right)T}{\sqrt{\sigma_{F_3}T + \sigma_{S^\bullet}T - 2\sigma_{F_3}\sigma_{S^\bullet}\rho_{F_3S^\bullet}T}}$$

$$\mu_{S^\bullet} = \frac{1}{T}\ln\left[\frac{(F_1+K)^2}{\sqrt{F_1^2 e^{\sigma_S^2 T} + K^2 + 2KF_1}}\right]$$

$$\sigma_{S^\bullet}^2 = \frac{1}{T}\ln\left[\frac{F_1^2 e^{\sigma_S^2 T} + K^2 + 2KF_1}{(F_1+K)^2}\right]$$

Note that the solution is exact for K = 0, since the option becomes the Magrabe exchange of assets option. Also for K < 0 a solution is not possible because ln(K) is not defined. However by manipulating the payoff so that we define $C_{FSP} = \mathrm{Max}[F_3^*(T) - S(T), 0]$ where $F_3^* = F_3 + K^*$ and $K^* = -K$ we can proceed as above to derive an analogous solution which works in this case.

Also we can derive the value of a put option using the same procedure or through the put-call parity relationship;

$$C_{FSP} = P_{FSP} - e^{-r_{D1}T}\left[F_1 - F_3 e^{(r_{D32} - r_{F32})T} + K\right]$$

$$= P_{FSP} - e^{-r_{D1}T}[F_1 - F_2 - K]$$

$$= P_{FSP} - e^{-r_{D1}T}[SP_{12} - K]$$

Note that the call and put values are equal when the strike is equal to the forward forward swap points ie $K = SP_{12}$. This is assured by the selection of the drifts for S and F_3 which were chosen so that they converged to F_1 and F_2 respectively. This is obviously important to avoid arbitrage.

We have tested the model over a the range of volatilities and correlations encountered in practice and have found that the approximating distribution gives very good answers for the option value in tests against a Monte Carlo simulation. The reason for this success is the fact that since K is a constant. the distribution of S^* is very close to a lognormal distribution as S is itself lognormal. Also the close correlation between S^* and F_3 is in practice usually above 0.90 which means that any errors in the approximation are minimised.

4.4.3.C Applications

As with the valuation of these instruments there has been no published work on the uses of these options in exposure management. The portfolio model which we developed in Chapter 3 however analysed a portfolio of spot and forward foreign currency cashflows in terms of spot and swap exposures. As we mentioned the reason for this was to isolate the exposure to interest rate differentials from that due to spot movements. Having split the risk in this way then the Swap Point option provides an additional tool with which to manage this exposure. As with conventional options however the management of this transactional exposure with Swap Point options is only appropriate in situations where the forward prices, in this case the forward swap points, are believed to be biased. If the forwards are unbiased then obviously the swap points are also unbiased and so the optimal hedge would again be to do a swap. In some common situations however these options may be useful in short term cash management as the following example demonstrates.

Example 1

When there are timing mismatches in cashflows the Swap Point option has a practical use in switching currency from one date to another at particular rates. In practice, when managing a large portfolio there will often be mismatch exposures which are not managed actively and instead positions are rolled on a daily or weekly basis at market rates. Writing Swap Point options, with strikes at the money, provides a way of generating potential excess profit from these positions whilst taking little additional risk.

Suppose a company has a short term mismatch between an incoming foreign cashflow in say, one week and a outflow in one month. In may firms this would probably be funded on a daily or perhaps weekly basis. This would give rise to an ongoing exposure to the overnight rates. An alternative hedge would be sell a one week P_{FSP} option with a strike at the current forward forward rate between one week and one month. Let the company be UK based with the foreign cashflows in US Dollars. The spot rate is 1.7000 USD/GBP and the 1, 3 and 4 week swap points are -0.0010, -0.0030 and -0.0040 respectively. The sterling yield curve is flat at 10%, spot and forward volatility is 12% and the correlation between the spot and forward rates is 0.98. The at-the-money strike is therefore -0.0030 giving an option price of 0.0022 USD/GBP or 0.13% of GBP. Thus for a nominal of £10m equivalent for each date, the premium received would be £13,000. If the option is exercised then the firm has the same position as if it had hedged with a forward forward in the first place. If the option is not exercised the firm can lock into a straightforward three week deposit or forward swap. Since the position would not have been managed actively, any profits or losses resulting from daily rolling would in general fluctuate and so add to the volatility in the portfolio return. Using Swap Point options in this way to hedge short term natural cashflows, forces the manager to lock into the forward rates or else only benefit from any changes.

Another potential use of the option is in situations where there is some flexibility in the timing of payments or receipts. This sometimes happens on long term projects or in situations where a payments to suppliers can be delayed. A common product offered by banks in these situations is a so-called option-forward, where the counterparty can exercise his forward contract at any time between two dates. The contract however must be exercised at some time so it is not a currency option. Typically these are valued using the worst rate for the period, ie it is assumed that the counterparty will exercise only when it is least advantageous to the Bank.

This type of exposure is similar to the economic exposure analysed in section 4.4.1 in that the firm has some flexibility, in this case in the timing of its cashflows, rather than the currency. We might therefore suspect that the risk could be hedged with an appropriate option since we know that in these situations the use of forward, or in this case forward forward, contracts will probably not provide the correct solution. In fact Swap Point options can be used to hedge this risk and as we demonstrate in the example below, they appear as short positions as with the max-min options before.

Example 2

Consider a firm importing materials from a foreign country. The import agreement states that the company must pay for the goods in the foreign currency but has the option to pay on either of two dates t_1 or t_2 in the future. Assume that the market is efficient in the sense that forward prices are equal to expected future spot prices. The exposure faced by the company then is to buy the required foreign currency at the lowest price.

Now let F_1, F_2 be the current forward rates for t_1 and t_2 respectively, so that the company must solve for the $\mathrm{Min}(F_1, F_2)$ at time t_1. Notice that since the company has the option to switch its payment date it would not be hedged by simply buying the currency at the lowest rate now since during the time to t_1 it might became profitable to switch out of the original deal and lock into the other date. As doing so would always give rise to a profit it is clear that the company is long of some form of option.

To see what this option is it is useful to use the relationship shown earlier relating options to exchange assets to max-min options (pg 184). Using this the exposure can be written as,

$$\mathrm{Min}(F_1, F_2) = F_1 - \mathrm{Max}(F_1 - F_2, 0)$$

Thus the firm's payoff can be represented as a long position in a forward contract expiring at t_1 and a short position in an option to exchange a forward contract expiring at t_1 and one expiring at t_2. However $F_1 - F_2$ is equal to $-SP_{12}$ ie minus the forward swap points between t_1 and t_2. Thus the option is equivalent to a P_{FSP} option with a zero strike price.

In order to hedge this exposure then the firm would buy the currency forward for delivery on t_1 at a rate of F_1 and write a put on the swap points between t_1 and t_2 with a strike price of zero. Using put-call parity for Swap Point options an alternative strategy would be to buy forward on t_2 and sell a call option also with zero strike.

Note that using the lognormal assumption for spot and forward rates the model describe in section 4.4.3.B gives an exact solution in this case since the strike is zero and so the problem is analogous to Magrabe's option.

This example illustrates again that flexibility in operating decisions is equivalent to being long of, usually complex, options. The hedge for this economic exposure is once again to sell the appropriate option to the market thus transferring the opportunity given by the flexibility. The firm providing such delivery flexibility, or delivery "options", can also use the valuation model to put a quantitative value on the agreement. Without the model the firm may have justified the granting of the option on the grounds of client relations or perhaps as a technique necessary to win the order. It can now however put a value on this option and so be in a better position to decide whether this cost is indeed justified.

In the next section we discuss some of the biases present in the pricing model as a result of the underlying assumptions and the approximation technique we have used.

4.4.3.D Implications of the Distributional Assumption and Conclusions

The valuation model presented above relies on the lognormality of both the spot rate and the forward outright rate. In reality, as we have seen in our empirical studies in Chapter 2, spot rates are significantly leptokurtotic and in other studies[3] forward rates have been shown to follow very similar distributions. This is not surprising since spot and forward rates are very highly correlated. In addition the valuation is crucially dependant on the estimate for the correlation between the forward rate and the adjusted spot rate, S^*. Since the model assumes that this is constant we would expect that biases might result from this assumption if the correlation is stochastic. As with the other complex options discussed above therefore, the treasury manager needs to be aware of any biases these assumptions may introduce in the pricing of the instrument and in addition for these options, the inaccuracies which may result from the approximation technique used.

Considering first the lognormality assumption, we are concerned with the effect of excess kurtosis in the individual distributions of the spot and forward rates on the option price. In addition we found in the empirical study that the degree of excess kurtosis in the swap point distributions was higher than that of the spot rate distributions for all

3 For example Westerfield (1977)

the currencies, except the JPY (see Table 2.3 of Chapter 2). We know however that even if the spot and forward rates were lognormally distributed, the swap points distribution would have a more complex density function since it can only be evaluated numerically. Thus the excess kurtosis exhibited by the swap points may just be a reflection of this inherent non-lognormality. In general the degree of this excess non-lognormality will be a function of the individual variances and correlation between the spot and forward rates. This is suggested from the results of other researchers who have shown[4] that the distribution of the difference between two lognormal variables can be well-approximated by a lognormal distribution particularly when the correlation between the two variables is high or the variances low. The extent of the error arising from the lognormal assumption then will probably be a function of the correlation between the spot and forward rates, which in reality is very high. We would expect therefore that the lognormal assumption will give rise to only small biases under most circumstances but in situations where the correlation is low, for example for short dated options on long dated swaps, the biases would be higher.

The fact that the correlation is probably also unstable is another potential problem. In this respect the model is analogous to the max-min option model described earlier since the crucial determinant of price is again a correlation. We suggested however that for max-min options instability in the covariance matrix between the assets created more of a problem for hedging than pricing. This was due to the fact that previous work on option models which incorporated stochastic volatility had not in general produced significantly different prices to the standard Black and Scholes model. Thus from a price-taker's viewpoint rather than from a hedger's, unstable variances and covariances

4 See for example Levy (1990) and references therein

were not a significant problem[5]. Since this option model is also based on the same theoretical model of the dynamics of the underlying assets the effect on this model will also be on the hedging rather than the price. Indeed one of the explanations for this effect is that for non-path dependant European options the price is only a function of the terminal distribution whereas the hedging involves continuous rebalancing. Thus stochastic volatility, or in this case correlation, has more of an effect on the hedger since he has to manage the position as volatility changes through time. The price on the other hand is only effected by the shape of the final distribution. Since this option is non-path dependant and also European style the effect will be the same.

Finally, the approximation technique used relies on the goodness-of-fit between the true distribution of S^* and the assumed lognormal distribution. As we mentioned earlier, in tests using Monte Carlo simulation the approximation was found to give very close answers for the option value compared to the Monte Carlo option values using 10,000 simulations. Errors however do begin to occur as the volatility rises and as the correlation between the two variables falls. For most practical values however, that is up to about 20% annualised volatility and 0.8 correlation, the results are extremely good and no consistent systematic biases were found.

To conclude then, this section has presented a new complex option product whose payoff is a function of the forward swap points between two future dates. A valuation model has been developed which uses an analytical approximation method to derive a closed form solution. The model appears to perform well for most practical parameter values when tested against a Monte Carlo model. We have demonstrated some

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5 Obviously the hedger's problem becomes the price-taker's when it is reflected in the price, but the previous research suggests that this particular problem only effects the price in a small way ie it effects the *way* the trader maintains his hedge rather than the cost of maintaining it.

applications of the option to typical foreign exchange risks for both transactional and economic exposure and have briefly discussed some of the biases which may be present in the model as a result of the distributional assumptions underlying it.

4.4.4 Average Rate Currency Options[6]

4.4.4.A Definition

An average rate foreign currency call option with strike price K and maturity T, is a path dependant contingent claim defined by the terminal payoff function;

$$C_{AV} = \mathrm{Max}[A(T) - K, 0]$$

where A(T) is the arithmetic average exchange rate calculated from discrete samples taken at regular intervals (daily, weekly, etc) over a period T_0, T. A(T) is therefore defined by;

$$A(T) = \frac{1}{N} \sum_{i=1}^{N} S_i$$

where N is the number of rates in the average. In general T_0 can be equal to the original date the option is dealt or some date in the future. In addition another less common type of average rate option exists which has a payoff given by;

6 These options are sometimes called Asian options

$$C^{*}_{AV} = \mathrm{Max}[S(T) - A(T), 0]$$

where S(T) is the spot rate at time T. This option therefore has no explicit strike price, ie the strike price is the average which is only known for certain on the maturity date. The two types of average rate options are related to conventional options through the arbitrage condition;

$$C^{*}_{AV} + C_{AV} \geq C_{BS}$$

Proof:

Let us write the above expression in terms of the payoffs of the options at maturity. We therefore need to prove that,

$$\mathrm{Max}[S(T) - A(T), 0] + \mathrm{Max}[A(T) - K, 0] \geq \mathrm{Max}[S(T) - K, 0]$$

There are six possible combinations of outcomes for the relationship between S(t), A(T) and K. Examining each of them in turn,

1. S(T) > A(T) > K

L.H.S. : S(T) - A(T) + A(T) - K = S(T) - K

R.H.S. : S(T) - K

So L.H.S. = R.H.S.

2. $S(T) > K > A(T)$

L.H.S. : $S(T) - A(T)$

R.H.S. : $S(T) - K$

So L.H.S. > R.H.S.

3. $K > S(T) > A(T)$

L.H.S. : $S(T) - A(T)$

R.H.S. : 0

So L.H.S. > R.H.S.

4. $K > A(T) > S(T)$

L.H.S. : 0

R.H.S. : 0

So L.H.S. = R.H.S.

5. $A(T) > K > S(T)$

L.H.S. : $A(T) - K$

R.H.S. : 0

So L.H.S. > R.H.S.

6. $A(T) > S(T) > K$

L.H.S. : $A(T) - K$

R.H.S. : $S(T) - K$

So L.H.S. > R.H.S.

In all cases then the L.H.S. is greater than or equal to the R.H.S.

Q.E.D.

Note that from these definitions it is clear that as the average becomes more certain, as $t \rightarrow T$, the value of C_{AV} becomes more fixed and so its characteristics will become less option-like. On the other hand C^{*}_{AV} becomes more like a conventional option since its strike price becomes more fixed. The two instruments therefore have in this sense opposite characteristics as time goes on. Obviously put options exist for both types of options and have payoff functions;

$$P_{AV} = \mathrm{Max}[K - A(T), 0]$$

$$P^{*}_{AV} = \mathrm{Max}[A(T) - S(T), 0]$$

For simplicity we shall concern ourselves only with European style average rate options and in particular the more common type C_{AV}.

4.4.4.A Valuation

There has been a large amount of interest in this product over the last two years in particular, and there have been several alternative valuation models produced for valuing C_{AV}. The reason for this range of models is that no exact closed form solution exists for the option because if the usual assumption for the spot rate process is maintained (equation 4.1) then the distribution of the final average is not Gaussian and its density function can only be evaluated numerically. This has led to a variety of different approximation procedures being suggested in an attempt to find an accurate solution for the range of volatility experienced in the market, without requiring a large amount of computational effort. The principal models which have been suggested so far are Kemna and Vorst (1990) which uses Monte Carlo valuation, Levy (1990) who uses the Wilkinson approximation idea which we used in valuing the swap point option, Turnbull and

Wakeman (1990) who use Edgeworth series expansion in a similar approach to approximating the distribution of A(T) as Levy, and Vorst (1990) and Ruttiens (1989) who both use an adapted geometric average rate option model for which there is a closed form solution[7].

Since there is no generally accepted pricing model for these instruments the user must compare the alternative models in order to judge which he feels is the most appropriate in the particular circumstances. However there are some relationships which any model should satisfy which are important, in particular put call parity. As Levy has shown the put call parity relationship for average rate options is given by;

$$C_{AV} = P_{AV} + e^{-r_D\tau}(A(t) - K) + \frac{S(e^{-r_F\tau} - e^{-r_D\tau})}{(T - T_0)(r_D - r_F)}$$

where $\tau = T - t$, m is the number of rates collected up to time t. N is the total number of rates in the final average and A(t) is defined by;

$$A(t) = \frac{1}{(T - T_0)} \int_{T_0}^{t} S(w)dw$$

ie A(t) converges to being the final average A(T) as $t \rightarrow T$[8].

All the analytical models, except Rutteins, satisfy this condition but the Monte Carlo models cannot be relied on to do so. However when using Monte Carlo in practice

7 The geometric average rate option has an exact solution under the assumption of geometric Brownian motion for the spot rate because it involves the product rather than the sum of lognormally distributed variables. Since the product is also lognormal the equivalent Black and Scholes equation for average rate options can be solved to give an exact solution.

8 Note that Bergman (1985) derived a similar formula in valuing an "averaging claim" which was essentially an average rate option with zero strike price.

the put call parity condition is often used to find the put value once the call has been found from simulation so that arbitrage is avoided. The intuition behind the put call parity condition is that the payoff on a portfolio short a put and long a call at the same strike can be replicated by a strip of forward contracts coinciding with the dates on which the spot rates used in the average are sampled. If we let the running average equal the current spot price, ie the averaging has just started, then the call price equals the put price when the strike rate is set to be the average of these forward rates ie from the EMH this is the expected average rate. Thus the relationship is analogous to the conventional put call parity condition where $C_{BS} = P_{BS}$ when the strike is the current forward rate.

The Monte Carlo model is used by all the researchers to test the accuracy of their models and so can be regarded as providing a benchmark value for these types of options. The drawback with the approach however, as we mentioned in section 4.3, is that it is very computer intensive since a large number of simulations are needed to reduce the variance of the option price. One very useful aid to variance reduction which has been used for these options however is to use the value of an equivalent geometric average rate option as a control variate. This was originally suggested by Kemna and Vorst and has been found to reduce the number of simulations required to achieve the same degree of accuracy by as much as a factor of 10. A Monte Carlo valuation model using this control variate then becomes more feasible although the task of calculating hedge ratios and other sensitivities can still be time consuming.

The models which attempt to approximate the true non-Gaussian distribution for the average by using the known moments of the distribution (Levy, and Turnbull and Wakeman) produce closed form solutions for the option value. These models are therefore very fast to calculate and can be used to derive sensitivities more easily. Care however must be taken when the parameters, in particular the volatility of the underlying spot rate, is outside the acceptable range for the approximation method being used. Levy reports that for up to 20 % annualised volatility his model produces very good results but for higher levels a model which matches the higher moments of the true distribution maybe needed. Turnbull and Wakeman report high levels of accuracy up to 30% volatility but slight inaccuracy for deep in the money options. Turnbull and Wakeman's model however is tested for stocks rather than foreign currency as Levy's was and it must be remembered that both models are tested against a Monte Carlo model which is itself only an approximation. The inaccuracies for higher volatility also apply for longer maturity since the both models in fact deteriorate as σt increases.

Finally the models based on the geometric average rate option also provide closed form solutions. These models rely on the close relationship between arithmetic and geometric averages and the fact that an arithmetic average is always greater than a geometric average. The geomtric average rate option then, which can be solved to provide an exact closed form solution, gives a close approximation for the arithmetic average rate option and is also known to be a lower bound. Ruttiens simply uses the geometric option as a straightforward approximation. Vorst however adapts the strike price of the option to take into account the systematic bias in the geometric price ie that it is lower than the arithmetic price. The results he presents show a good level of accuracy again compared to a Monte Carlo model with inaccuracies in most cases being no greater than the errors in specifying the implied volatility for OTC options in practice.

Of the published models the most appropriate for average rate currency option valuation are those of Levy, Turnbull and Wakeman and Vorst since for spot rates, volatility is usually below 20%. Of these the only one which we are aware of being used in practice to manage a large portfolio of these options is the Levy model. Given the apparent success of this model under actual market conditions does perhaps suggest that this model is the preferred alternative. For this reason we present below the valuation formulas for C_{AV} derived from this model;

$$C_{AV} = e^{-r_D\tau}[E[A(T)]N(d_1) - KN(d_2)] \qquad 4.40$$

where,

$$d_1 = \frac{\frac{1}{2}\ln(E[A(T)^2]) - \ln(K)}{v}$$

$$d_2 = d_1 - v$$

$$v^2 = logE[A(T)^2] - 2logE[A(T)]$$

$$E[A(T)] = A(t) + \frac{S}{(T - T_0)g}(e^{g\tau} - 1)$$

$$E[A(T)^2] = A(t)^2 + \frac{2A(t)S}{(T - T_0)g}(e^{g\tau} - 1) + \frac{2S^2}{(T - T_0)^2(g + \sigma^2)}\left[\frac{e^{(2g+\sigma^2)\tau} - 1}{(2g + \sigma^2)} - \frac{e^{g\tau} - 1}{g}\right]$$

and $g = r_D - r_F$ and σ is the volatility of the spot exchange rate. Note that this formula is based on continuous sampling of the spot rate. Of course in practice rates are sampled at discrete intervals, however the formula works well for daily intervals. For wider intervals it is suggested that a discrete time formulation be used which is done by using discrete time formulas for $E[A(T)]$ and $E[A(T)^2]$ (see Levy Appendix A).

We now look at some examples of applications for this type of option in hedging common types of transactional and translational exposure.

4.4.4.C Applications

The growth in the market for these types of options, by far the most successful of the complex options we have looked at, suggests that they have found a useful complementary role to conventional options. In particular since this growth has been largely in the corporate rather than interbank market implies that they have been used more for hedging than for speculation. It has been suggested also that the growth in this particular market has been due to the demand from less sophisticated smaller corporates who are attracted by the lower option prices for these instruments compared to conventional options.

In the literature, only Kemna and Vorst put forward any possible uses for the options. They site their use however in commodity linked bonds particularly for crude oil and suggest that this application is a result of the illiquidity of the underlying market. An average rate option, with the averaging over the last period of its life reduces the possibility of manipulation in the price of the underlying asset which might occur if settlement was against a rate fixed on just one day. Thus the use of this type of payoff provides a way of reassuring investors that they will be protected against unwelcome price fixing. Kemna and Vorst also suggest that bonds with these options attached provide a way for investors to share in the general prosperity of a firm and avoid the issuer the problem of having the price of the underlying commodity at an atypical level at option maturity.

The use of average rate options in hedging foreign exchange exposure can be illustrated with examples of hedging two types of exposure, one transactional and one translational. We examine these two uses in the examples below.

Example 1

The first use for these options is in hedging a stream of foreign currency cashflows. Consider a domestic company importing raw materials on a regular basis from overseas. It therefore has a regular, say monthly, demand for foreign currency of equal amount. Now this type of transactional exposure can of course be completely hedged in the forward market when forward prices are unbiased. If they are not then conventional options or a combination of options and forwards may be required depending on the nature of the biases in each of the forward prices. As we suggested earlier barrier options may also be appropriate when the biases in the forward prices are believed to be temporary.

The average rate option however provides another tool for managing the risk in this situation since the company can regard its exposure as a risk against the average rate at which it buys the total amount of foreign currency. It is therefore concerned about whether the average forward rate is biased or not rather than the individual rates. Obviously it may be that the average is unbiased when individual rates are biased. By analysing the exposure in this way the company might not hedge with options, even though some or all of the forward prices were biased as long as the average forward was unbiased. If the average is unbiased then the optimal hedge is to buy the currency for each date in the forward market. A practical problem with this however is that the transaction costs in the forward market may be high compared to those in the spot market. An alternative however would be to create a synthetic forward strip of forwards using average rate options, ie buy a call and sell a put with strikes at the current average forward rate. The company would then buy its currency in the spot market on each date as required at the current rate but would be compensated through the options for the difference between the actual average rate it buys the currency and the initial average forward rate. In both cases the company is hedged completely at the initial average forward rate and so the choice between the strategies therefore depends on the relative transaction costs. Since the cost to the bank which offers the options of dealing in the forward market should be less than that of the company, the firm should benefit from effectively letting the bank buy the strip of forward contracts for it. For a conventional currency options the spreads on the options usually prevent the use of synthetic forwards being efficient, but since a single average rate option is equivalent to a strip of forwards the spread on this option should be less than the combined spreads on the forwards. An average rate forward may therefore be a more efficient hedge when forward prices are unbiased if transaction costs are significant.

If the average forward rate was biased rather than creating a portfolio of options and forwards for each date, the company could hedge the entire cashflow with combinations of average rate options. Thus individual average rate options might be bought or sold or combinations used in order to take advantage of the perceived bias. This strategy has the added advantage as before that it is cheaper in terms of transactions costs than using a portfolio of conventional options and forwards since less instruments are used. The fact that the instrument is net cash settled also has advantages since the company is not required to exchange principal amounts as with a conventional option or forwards but simply pays or receives the domestic currency value of the options at maturity. This again reduces transaction costs.

For hedging certain types of common transactional exposures then the average rate option has some benefits over conventional options and forwards, in particular with regard to transaction costs and ease of use. If fact it is probably the simplicity of the instrument from a corporate point of view, together with its relative cheapness, which has been the main reason for the rapid growth in the market.

Example 2

The second use to which this option has been put is in hedging a common form of translational exposure. In many companies foreign assets and liabilities are converted into the domestic currency for year end accounting purposes at the average exchange rate prevailing over the year. In this situation the company has a direct exposure to this average rate even though it does not have any actual foreign currency cashflows to hedge. Since this is a notional exposure the use of forwards is not usually felt to be appropriate especially since their use gives rise to physical cashflows, with inherent transactions costs. In addition the average is usually calculated from particular fixing rates, ie Bank of England rates or Financial Times rates taken say at the end of each month. This presents timing problems when unwinding the forward positions against the fixing rate which is sometimes not published until the following day.

In these situations many firms have chosen to use average rate options to allow them to budget for the effective average rate used at the year end and to avoid the problems with timing and rate sources. Since the OTC average rate options which are offered by the banks can use the same fixing sources the company has a perfect hedge for this problem. In addition as we mentioned above the option is cash settled so that the firm has no foreign currency cashflows to deal with. This again adds to the attraction of the instrument from a practical point of view. The strategy then would involve buying or selling the option depending again on the perceived bias in the forwards, or even constructing an average forward position when forwards are unbiased, by buying a put and selling a call or vice versa. Even though such a position could be achieved in the forward market the use of options may be more efficient for reasons related to the timing and rates source problem discussed above as well as transactions costs.

The average rate option therefore provides a simple and efficient way of hedging a common form of translational exposure. The cash settlement system and use of fixings in particular suit this type of notional exposure well so that even with an unbiased forward market this particular option can have significant advantages.

4.4.4.D Implications of the Distributional Assumption and Conclusions

The problem with valuing these options is that the assumed diffusion process for the spot rate leads to an complicated density function for the underlying arithmetic average. This leads to option valuation which is necessarily based on one form of approximation or another. When the spot rate process is in fact itself an approximation of a true underlying process which probably has a stochastic variance and certainly exhibits a leptokurtotic distribution then the effect on the option price would be expected to be compounded. The true distribution of the sums of exchange rates, or exchange rate averages has not been studied although averages are used in the analysis of exchange rate movements carried out by technical analysts. Observations of this work show that the extreme changes exhibited by the underlying spot rate are dampened out by the averaging. This suggests that the distribution of the average might be more compact.

If this is the case then it may be that the lognormal assumption for the average rate distribution used by Levy and Turnbull might in fact be a better representation of the true distribution than expected. Thus it may be that the two approximation processes in some sense act in opposite directions so that the outcome is a reasonably realistic distributional assumption for the underlying asset. In order to confirm this however it would be necessary to carry out tests of the distribution of average exchange rates and examine the goodness of fit to a lognormal or alternative distributions. If the underlying spot rate does have a changing variance though the average will also have an unstable variance since the two are related by the expression for $E[A(T)^2]$ in the option formula equation 4.40. We would therefore expect that some problems with hedging this option as with conventional options, will occur.

The fact that the average becomes effectively less volatile as it accumulates will to some extent mitigate this problem since movements in the spot rate will in general become less and less important for the value of the option as time goes on. This effect may in fact make the distributional assumptions less important for this option than for others. In particular, although the option is path dependant like the barrier option, the problem of assuming a pure diffusion process as opposed to one with jumps is not as critical. This is because the dampening effect of the averaging process makes the option less sensitive to sudden large movements in the underlying rate as time goes on. In fact after about one third of its averaging period the option usually stabilises so that the gamma risk becomes less important.

A more important problem for this option is the assumption of constant flat yield curves. The problem here is not so much that interest rates are changing but that in reality the yield curves are not flat. This means that the average forward rate given by $E[A(T)]$ might not be the actual average rate in the market if the rates for each fixing date are different. The assumption of constant yield curves does not require all the individual rates for each maturity to be specified since they are assumed to be equal. The expression for $E[A(T)]$ is therefore that given in equation 4.40. In reality since the yield curves are not flat this value may be far from the actual value necessary to avoid arbitrage. In this situation the correct expression for $E[A(T)]$ would be;

$$E[A_N(T)] = A_N(t) + \frac{1}{N}\sum_{i=m+1}^{N} E[S(T_i)]$$

$$= A_N(t) + \frac{S}{N}\sum_{i=m+1}^{N} e^{(r_{D_i} - r_{F_i})(T_i - t)}$$

where $A_N(t)$ is the discrete version of A(t) defined above, m is the number of rates known and r_{D_i} and r_{F_i} are the domestic and foreign interest rates for T_i respectively. Thus care must be taken when evaluating these options when the assumed flat yield curves are far from those in the actual market.

To conclude then this section has presented an increasingly common type of complex option called the average rate option. Although there is no generally accepted valuation formula, because some form of approximation technique is usually needed to value the instrument, a formula derived from one of the models which has performed particularly well in practice is presented. Two examples of common applications of the option have been described and we have also discussed some of the implications of the distributional and particularly the constant interest rate assumption for the value of the option.

4.5 Summary and Final Conclusions

In this chapter we have examined the use of options and specifically some of the new complex derivative options, in managing foreign exchange exposures. We have shown that the Black and Scholes model which is widely used in valuing conventional options, has biases as a result of the distributional assumption. However more complicated conventional option valuation models have failed to provide significant improvements in empirical tests and the simpler model continues to be used in practice. We have suggested that the problem of specifying a more accurate model is that in practice the Black and Scholes model is the basis for actual prices but that adjustments are made because of factors which are difficult to model consistently since many of them are unstable over time. Since the more complicated models, which are more empirically appealing, usually require the estimation of extra parameters, the instability of the additional factors, which it is hoped are being collected by these parameters, leads to difficulties in testing and indeed applying the models in practice. Thus the fact that the biases in the Black and Scholes model are well known and can be adjusted for satisfactorily in an almost ad hoc way, has meant that it has continued to be used by practitioners. Of course there is an additional effect which acts as a virtuous circle, in that as the market becomes more liquid and the model more widespread, the underlying assumptions become to an extent less important. This is because many of the biases can be offset by taking opposite positions in other options leaving only residual risks to be hedged in the underlying spot or forward market. Assumptions about the spot rate process obviously become less important when only small residual risks are being managed in that market. In a sense then much of the risk due to the incorrect assumptions can be hedged using other options. This is particularly true of the constant volatility assumption which from the results of the empirical work in Chapter 2, is perhaps the most important error in the model. The more widely used the model becomes however, the more liquid the market becomes. The more liquid the market, the easier it is to hedge the stochastic volatility risk using other options and therefore the inaccuracy of this underlying assumption become less important. It is perhaps ironic that in reality market practitioners talk of trading volatility, whereas they use a valuation model which assumes it is constant.

The increasing growth of the market in conventional currency options has given an impetus to the growth of a new market in more complex options. We have analysed four such options, defining each one, giving valuation formulas and applications, and discussed some of the additional biases which may be present in their prices as a result of the new boundary conditions. We have found that each option has a payoff which provides a more efficient hedge to a particular exposure than that available with either conventional options or forwards. Thus each has a role to play in hedging a particular complex exposure or view of the underlying market. The valuation of these new instruments is generally based on the same assumptions as are used in the Black and Scholes model. Thus the biases which are present in that model are carried over into the prices of the more complex options. Each option also has additional specific risks due to these assumptions, which provide additional biases to the option price. The effect of the virtuous circle mentioned above however has implications for the biases in the prices of these options too. Since all of the assumptions used are the same as for conventional options they can also to some degree be hedged in that market. The problem becomes one of managing the differential exposures of the different types of options to these assumptions. The implications of the errors in the underlying assumptions are only relevant to the extent that they effect the residual positions which must be hedged in the underlying market. This is an important effect since as we mentioned before, the ability of the banks to offer these new products has been based to a large extent on the liquidity in the market for conventional options. It is clear that this market is used to hedge much of the risk associated with the distributional assumptions and so leave only residual risks specific to that product to be hedged in the underlying market. Of course the differential sensitivities of each type of option make a perfect hedge impossible, especially when these sensitivities change through time. However it is certain that without access to a liquid market in conventional options the newer derivatives would be unlikely to have been offered.

From the treasury managers point of view the importance of the inaccuracy in the distributional assumptions is then perhaps less significant than might be expected. His first concern is with the payoff offered by the instrument, and whether or not it can be used to hedge some specific exposure. That the valuation of the instrument is "fair" given the inaccuracy of the distributional assumption is a secondary concern. However if it is possible for the banks to offer the instrument at close to "fair" value, because the errors in the assumptions can be partially hedged in another market, then the treasury manager will achieve a satisfactory price for the exposures he buys or sells to the market. Of course the additional implications of the assumptions which are specific to the particular product cannot be completely hedged and so it is important that he be aware of these specific problems. However it is also possible that he can take advantage of these risks in designing hedging strategies, as we have shown in previous examples, so that he benefits from the differential exposures of the conventional and complex options.

The treasury manager's primary concern is with the payoff of the option. From the examples of the applications of the options we have studied we can summarise the results into three general propositions on the uses of options in exposure management problems.

Proposition 1:

When forward prices are unbiased, risk can be completely hedged with forward contracts, synthetic forwards or derivative forwards, as long as there is no flexibility in operating decisions.

Proposition 2:

When forward prices are unbiased, and there is flexibility in operating decisions, risk can be only be completely hedged with conventional or derivative options.

Proposition 3:

When forward prices are biased, risk can only be hedged with combinations of conventional and/or derivative options and/or forwards.

The key result is that options are associated only with situations where some form of flexibility is present, either in currency or timing, or else when forward prices are biased. The particular options which are required depends on the nature of the flexibility and bias as we have shown with the examples. In the final chapter we shall expand on these results and incorporate the earlier work on the underlying process for exchange rate movements and the use of portfolio analysis, to develop an integrated approach to exposure management in foreign exchange.

Figure 57 : Exposure Profile for Max-Min Option Example 1

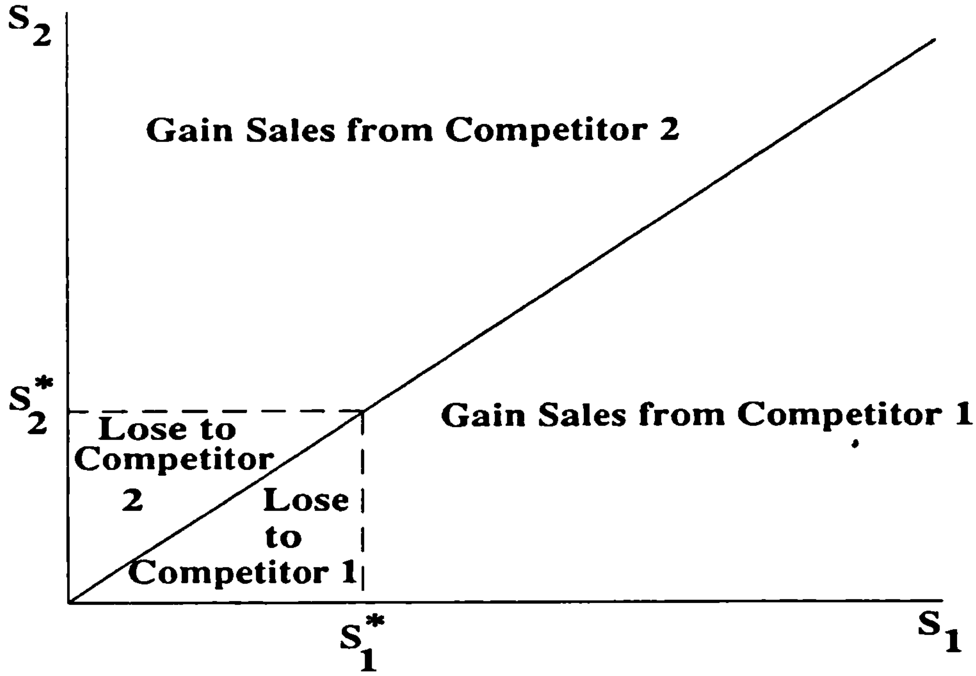


Figure 58 : Payoff on P_{Max}

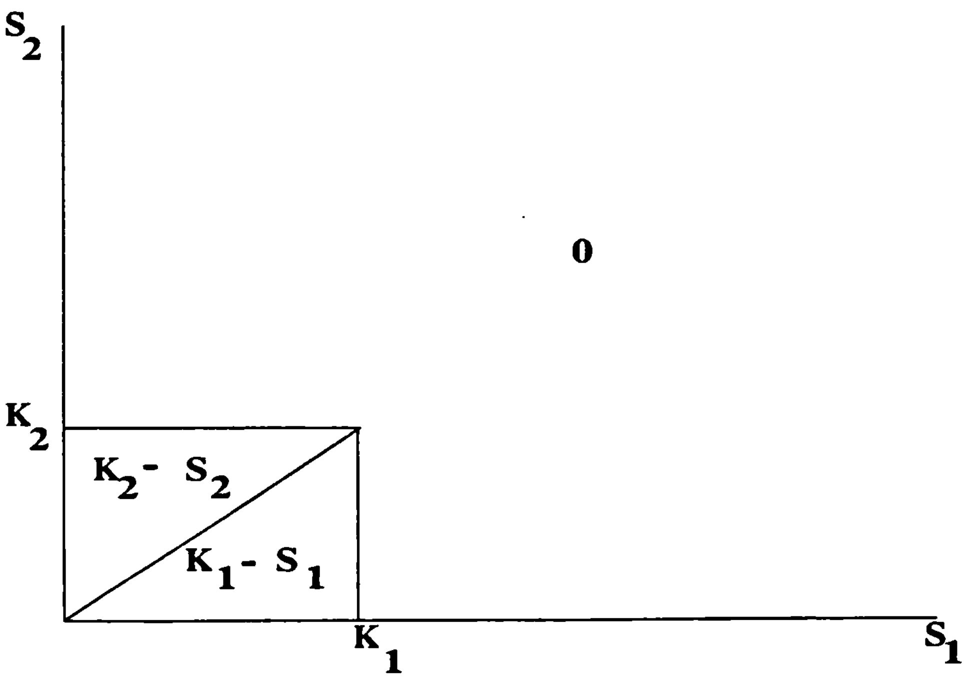


Figure 59 : Payoff on C_{Max}

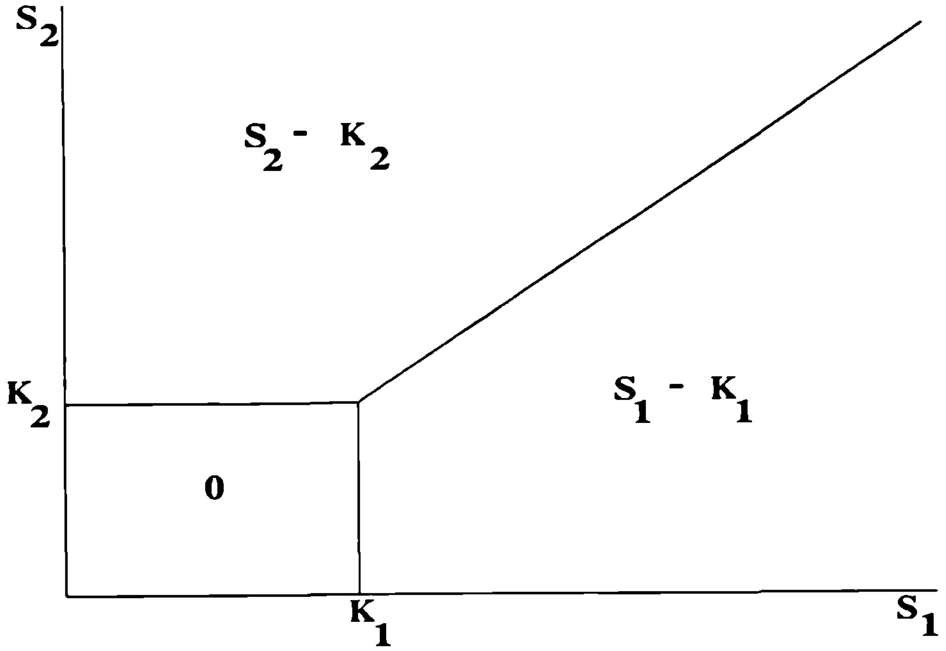


Figure 60 : Combined Position $P_{Max} - C_{Max}$

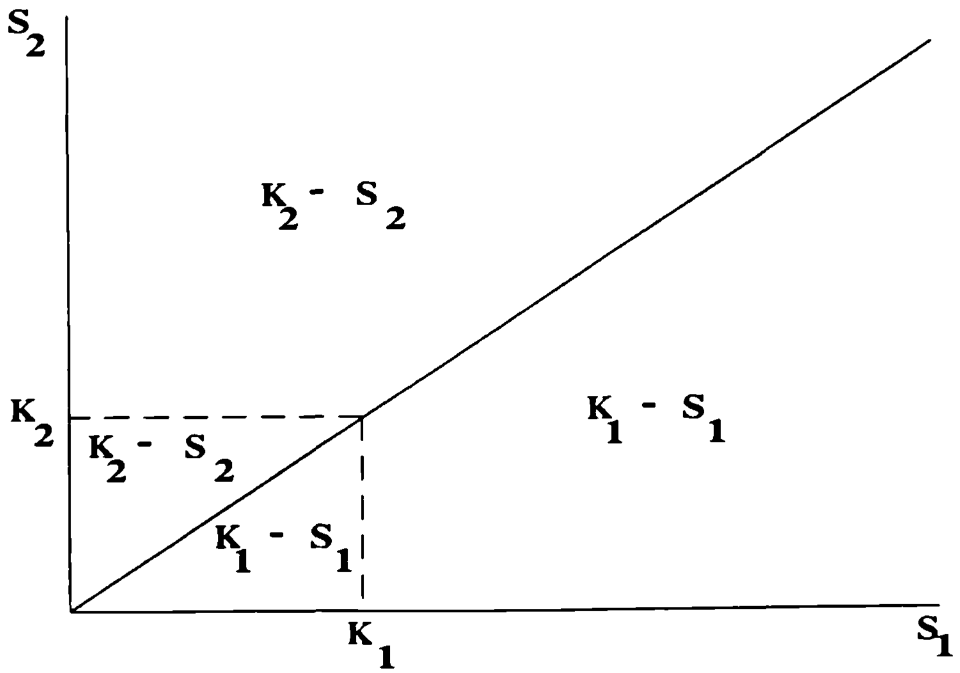


Figure 61 : Decision Profile from Max-Min Option Example 2 (From Ware and Winter, Figure 4, pg 299)

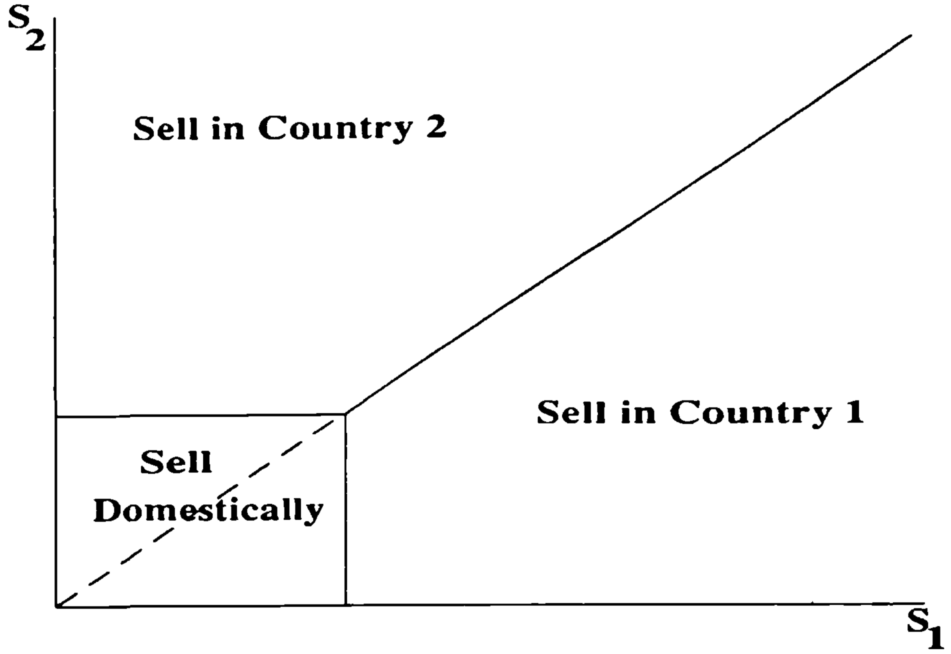


Figure 62 : Payoff on C_{Port}

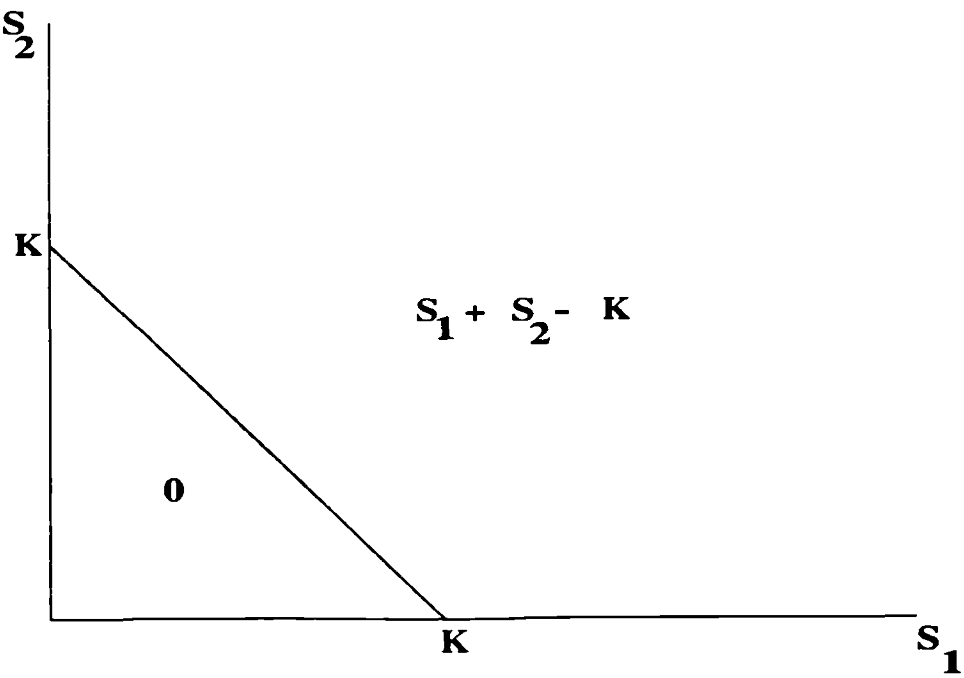


Figure 63 : Payoff on $C_{BS}(S_1)$

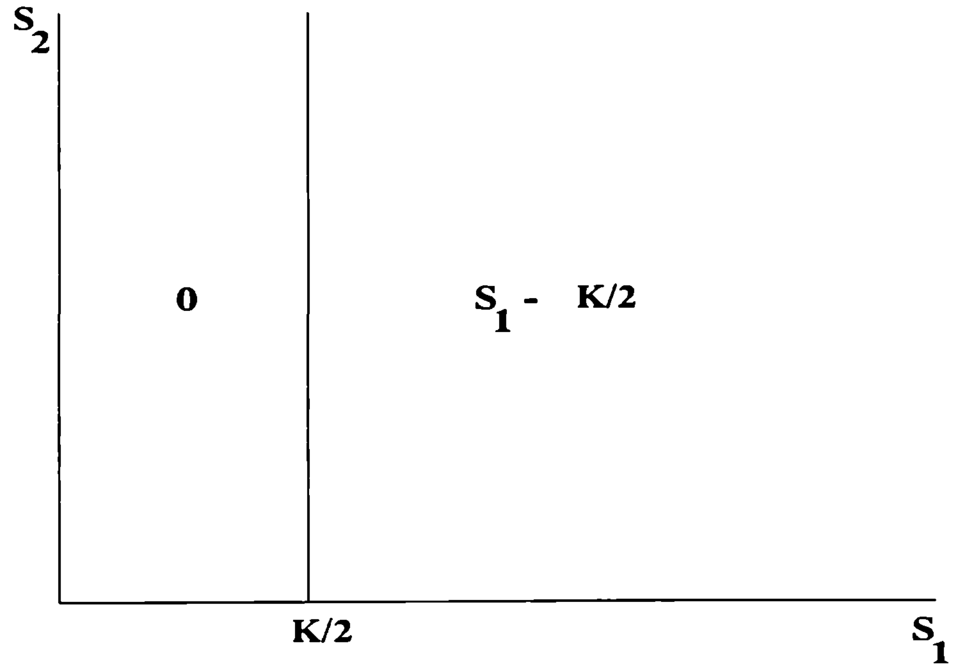


Figure 64 : Payoff on $C_{BS}(S_2)$

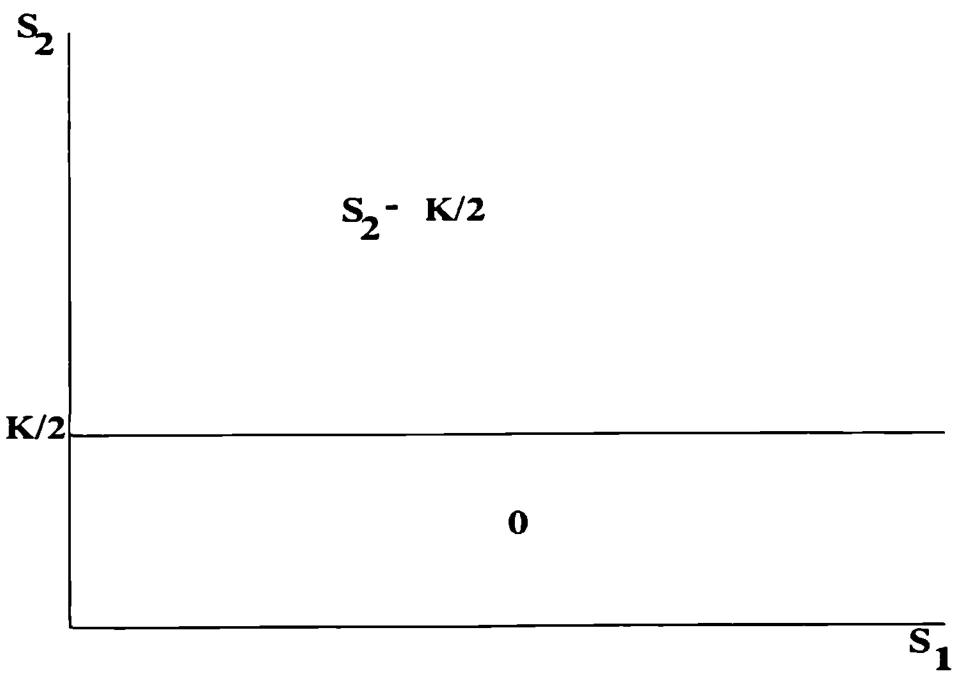


Figure 65 : Payoff on Complex Option L

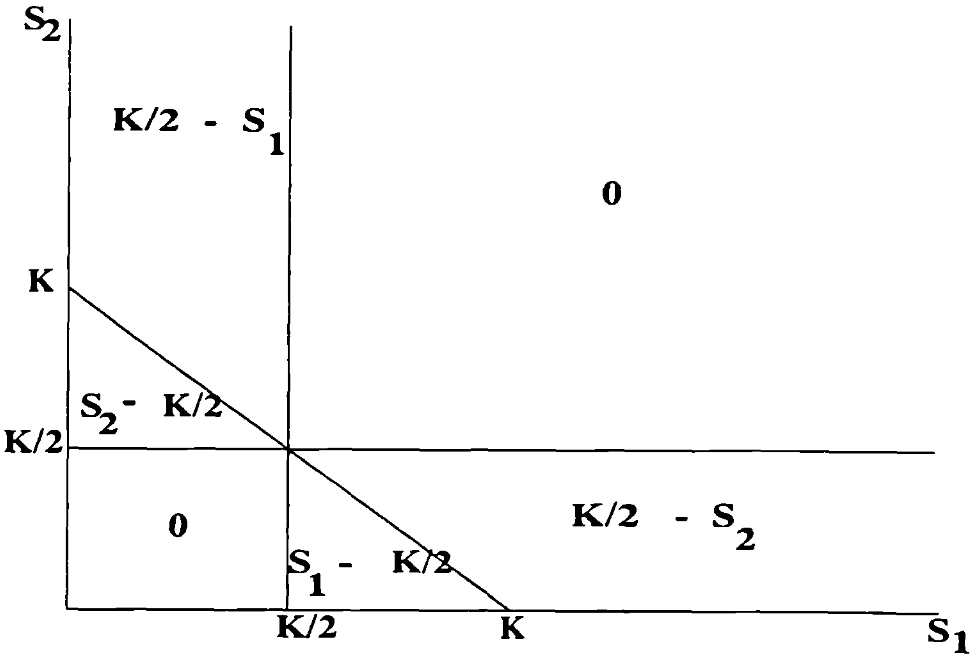


Figure 66 : Payoff on C_{Max}

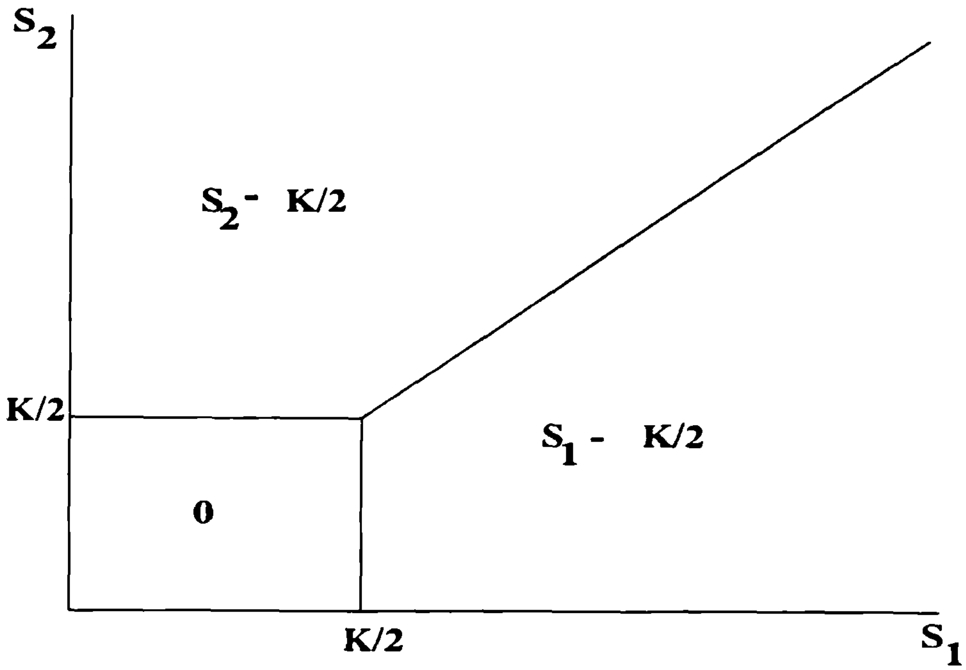


Figure 67: Payoff on C_{Min}

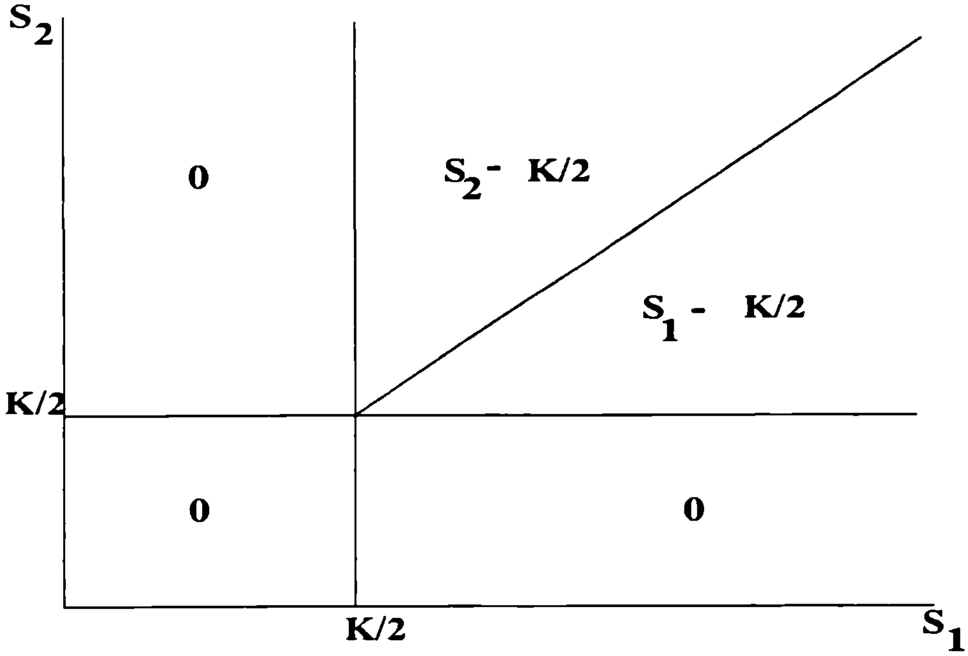


Figure 68 : Payoff on P_{Max}

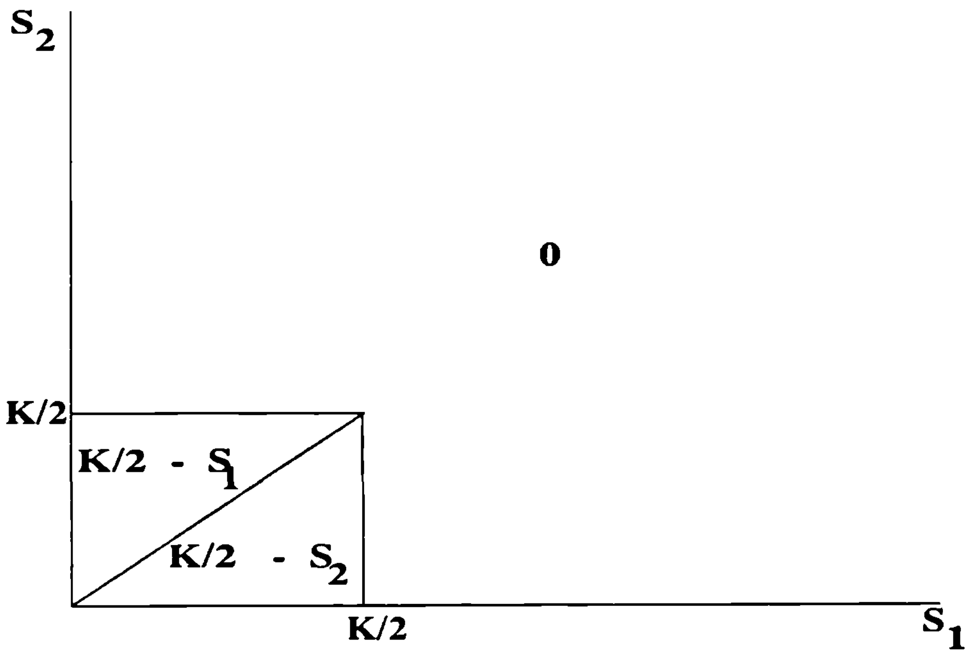


Figure 69 : Payoff on P_{Min}

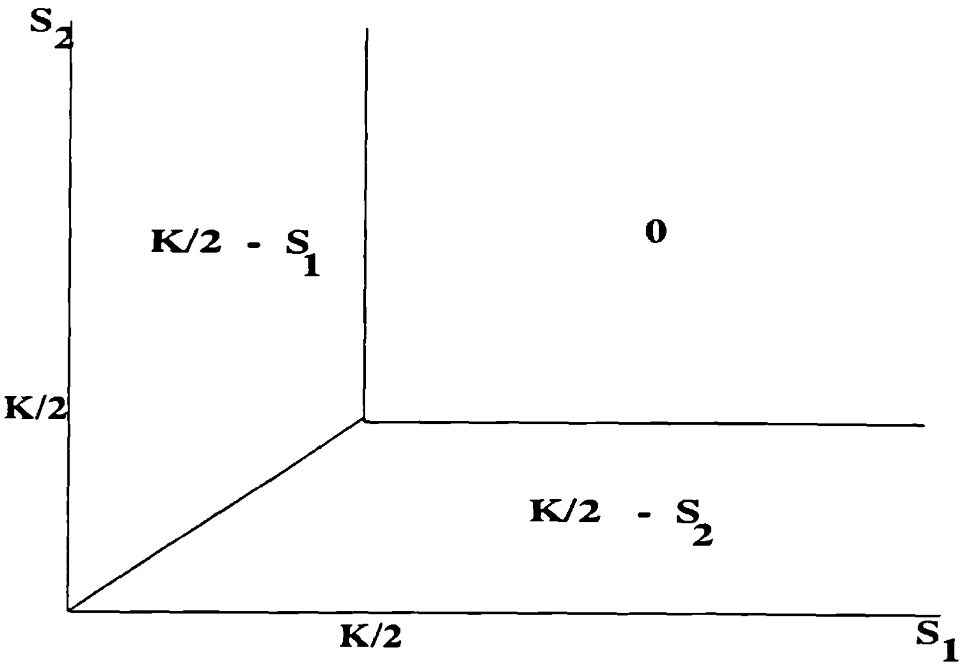
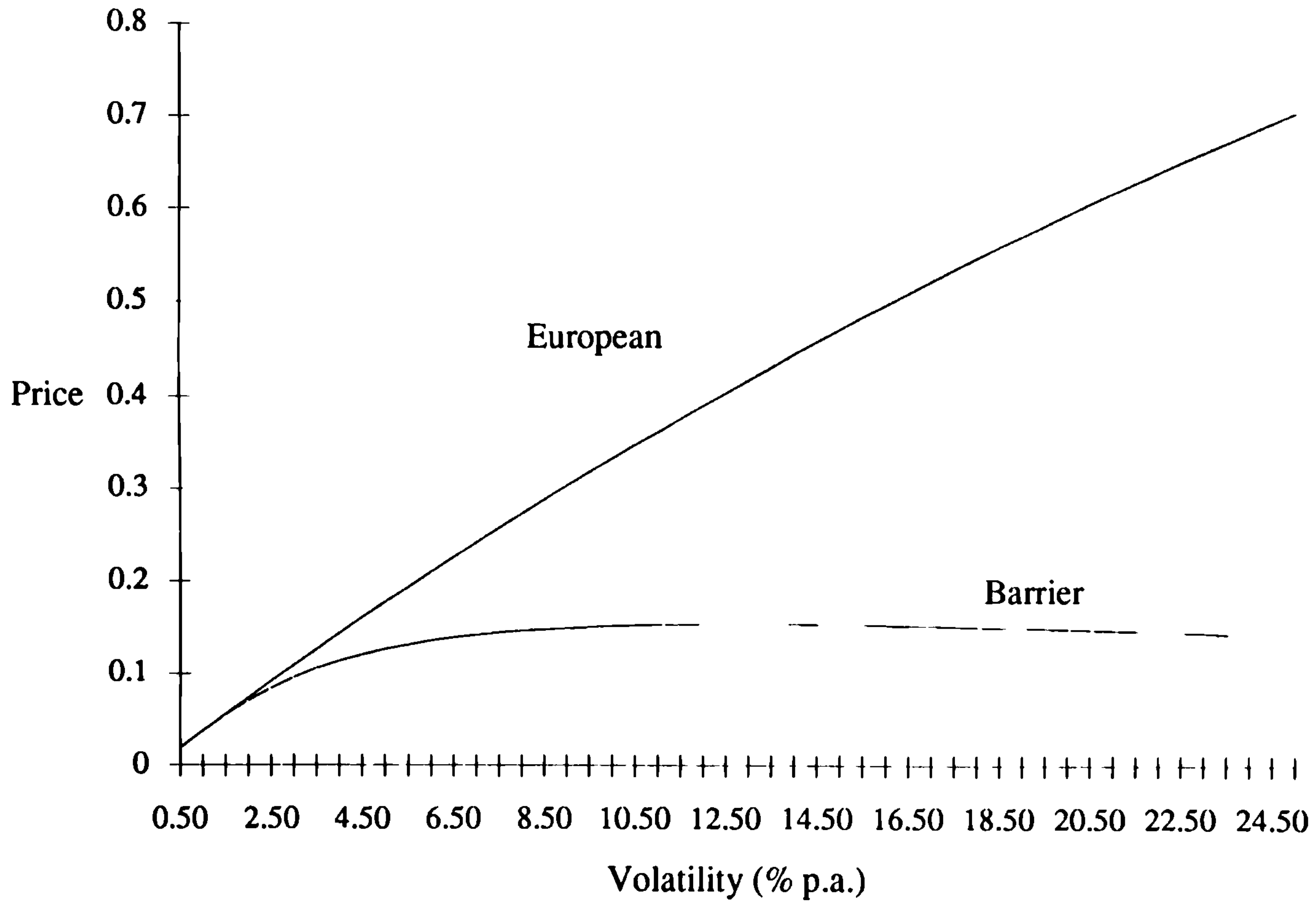


Figure 70 : Volatility Sensitivity of European and Barrier Option Prices



Chapter 5

An Integrated Approach to Exposure Management in Foreign Exchange

5.1 Introduction

In this final chapter we aim to synthesise the results of the empirical tests and the theoretical models which we presented earlier, to derive an integrated approach to exposure management. The chapter is divided into three sections. The first summarises the results of the previous three chapters and discusses the requirements of an exposure management strategy based on the available instruments, the nature of the market and the effect of the model assumptions on hedging performance.

The second section presents an integrated approach to exposure management by which we mean a strategy which combines an efficient use of available hedging instruments and a knowledge of the actual process for market prices, to produce a structured portfolio of hedging transactions. We define the strategy by deriving the steps which should be followed in such a structured approach, from analysis of the types of exposure through to the implementation of hedging transactions. The aim is to develop a logical sequence of steps in the decision making process that is consistent with an efficient use of available instruments and an application of knowledge about the effects on hedging performance of the assumptions behind the models being used.

The last section presents some final conclusions on the application of such an approach in practice in the context of the existing strategies being used for exposure management. We also discuss the scope for further work which could expand on this research.

5.2 Synthesis of Results from Empirical and Theoretical Work

In the second chapter we examined the actual movement of exchange rates, both spot rates and forward swap rates, and in particular the distribution of changes in the rates over a daily or weekly interval. We found that the process appeared to be random, ie there was no significant serial dependence. However, further tests showed that there may be non-linear dependence in the spot rate data of a form which gives rise to an unstable variance. The distribution of changes in both spot rates and swap points is certainly leptokurtotic. It is likely that the excess kurtosis is the symptom of this changing variance rather than the function of a stable process with a non-normal density.

If this is the case then the implication of these results is that instability in the parameters of the underlying process should be incorporated into any theoretical model which is used to design hedging strategies. The availability of tractable models based on realistic assumptions about the spot rate process, consistent with the empirical evidence, however is low. A treasury manager therefore, who wants to apply a more quantitative or objective approach to the management of his exposures, is forced to use models based on simpler assumptions and so needs to understand the implications of these simpler assumptions on the performance of the models in practice. This problem is the central concern of this research, that is, how to design an efficient exposure management strategy, using appropriate instruments and valuation models, in a way which is consistent with the empirical evidence on exchange rate movements.

The first model we examined was a mean-variance Portfolio model which appears to be particularly applicable to transactional type exposures. The model could be used as a way of both measuring and managing the exposure of a large number of foreign currency cashflows in both spot and forward markets. The implications of parameter instability, especially in the covariance matrix, were examined by simulation. We found that a large amount of fluctuation in the calculated risk could result for even a static portfolio, when the covariance matrix was calculated over a rolling historic period. However, the risks on typical swap positions were found to exhibit similar time series properties which suggested hedging strategies which might mitigate these problems. Nevertheless problems with parameter instability and changing positions, which occurs because of a changing business environment, force the adoption of a more flexible approach to the application of the model than that used typically in investment portfolios. The approach is backed up though by a more thorough analysis of the constituents of the risk within the portfolio. This is particularly important when the model is not being used to generate specific hedge transactions. Perhaps the most useful application of the model however is as a way of analysing the effect of new or changing positions in a consistent way. Since one of the most difficult problems facing treasury managers is deciding how to react to new exposures as they arise, the model is an important additional tool. The model provides a framework in which risk can be analysed however without providing a specific prescription in terms of particular hedging transactions. Analysing the risks inherent in a portfolio of foreign currency cashflows using a mean-variance is certainly an improvement on the selective hedging approach. The application of the model however cannot ignore the underlying assumptions or the problems of operating in an ongoing business environment. Consideration of these problems necessarily leads to a more flexible approach, but should not lose the structure which the model provides.

The next group of models we examined were a selection of the new derivative option valuation models which have become available in the foreign exchange market in recent years. Four such options were considered and each was found to offer a payoff which more efficiently hedged particular, usually complex, exposures than those provided by conventional instruments. From several examples of the application of these options we were able to derive three general propositions on the use of options in hedging. The crucial result we found was that option type payoffs are only appropriate when flexibility in operating decisions is present of else forward prices are biased. Indeed, flexibility can itself be regarded as an option so that in these situations the firm finds itself selling a complex currency option to the market in order to be hedged. This result supports some previous research in contradicting the perceived wisdom about options, which proposes the *buying* of options for hedging purposes. We found that this is not necessarily the case and in fact is only the case when forward prices are biased.

For the treasury manager then options, and in particular these complex options, provide important additional hedging instruments for particular complex risks often arising from economic exposure. Once the appropriate option is selected, analysis of the instrument focuses on its valuation, and the assumptions underlying the valuation model. The treasury manager as a price taker however, is only concerned with a "fair" price for the risk he trades. The inaccuracy of the underlying assumptions and the problems of hedging the option therefore are only important to the extent that they effect this price. We suggest though that where liquid markets exist in other instruments priced on the same assumptions, in this case conventional currency options, problems caused by simplification in the assumptions can to some extent be mitigated. This is because the risks inherent in the assumptions can be partially hedged away in that other market so reducing their effect on the option price. The development of complex option products only in markets where conventional options are liquid provides some evidence for the validity of this hypothesis[1].

One implication of the treasury managers role as a price taker is, as we mentioned in the previous chapter, that the effects of the underlying assumptions are less important for the options since this risk can be hedged in the conventional options market. This is

1 The interest rate option market for example has not seen a rapid growth in the trading of the derivative option products.

not the case for the portfolio model however. This is because applying this model involves dealing directly with the underlying market and so relying more directly on the effects of the distributional assumptions on hedging performance. With options, as long as the initial price is "fair", the exposure is correctly hedged for the entire period. Managing a portfolio of positions in the underlying assets on the other hand is more akin to the dynamic replication involved in hedging options, so the implications of the assumptions and in particular the constant variance assumption are more important.

It is nevertheless necessary that the treasury manger understand the *differential* exposures of conventional and complex options since it is these risks which will in fact be reflected in the price. In addition, knowledge about the differential effects of the underlying assumptions can lead to better tactical decision making on the size or timing of hedging transactions. We have illustrated one example of this in analysing the choice between conventional and barrier options in different volatility environments.

The fact that the treasury manger is a price taker leads to an argument often made which suggests that it is irrelevant whether the assumptions used in the pricing model are realistic, as long as there is competition. The idea is that if there is competition amongst option providers they will forced to cut their profit margin to the minimum and so the lowest price is therefore "fair" in that it reflects the real perceived risk of hedging the option. Whilst the argument is appealing in theory, there are practical problems in its application, especially in the new derivative markets. For these newer products there is much less competition and so the implication of the argument is that the treasury manager should not deal them since the market is not liquid. In order to hedge then he is forced to use other instruments which are less efficient. The result is a potentially much worse overall hedge than that which would have been available using the more efficient product even if it was mispriced. The point is that whereas there is certainly a case for being reluctant to deal in illiquid markets, when the product required is the most efficient available the solution is not to ignore it but to understand the valuation. It is therefore even more important with these products that the treasury manager can value the instrument and understands the implications of the underlying assumptions. In this way he can feel more confident of using the product and he opens up the opportunity of improved hedging performance without risking trading at unfavourable prices.

A structured approach to exposure management then requires firstly, an analysis of the risks being run. This involves an examination not only of known foreign currency cashflows, but competitive and other economic as well as accounting type exposures. Secondly, a knowledge of available hedging instruments and the conditions under which their use is appropriate. The models we have described are only a selection of the new option products now available. The firm should be aware of all the possibilities and in addition by analysis of his exposures be able to define new payoffs which are perhaps not yet being offered. Thirdly, an understanding of the models on which the theories or instruments are based and in particular the assumptions underlying them. This necessarily leads to an appreciation of the effects of these assumptions on the performance of the model in practice so that appropriate hedging transactions can be made. In the next section we describe an approach which satisfies these general requirements, and makes use of the models we have presented earlier.

5.3 An Integrated Approach to Exposure Management

In designing any business strategy a common approach is to separate the decisions to be made into two broad areas, strategic decisions and tactical decisions. In the context of an exposure management strategy then strategic decisions would be related to what types of exposure are present and which need to be hedged, what market conditions are being faced, the economic climate and more particularly analysis of expected exchange rate movements over some, perhaps several, time horizons. Tactical decisions on the other hand are those concerned with the actual transactions to be performed, the timing of deals, in what currency and in what amount. In order to relate the two decision areas we need to find a process which links them in a structured way and incorporates the information about exposure management tools and market processes.

The approach we suggest is shown in the schematic diagram in figure 71. The idea is that strategic decisions lead to a specific subset of appropriate and efficient instruments which based on these decisions can fully hedge the exposures being managed. Thus in a simple case, a firm facing only a single foreign currency cashflow at a known time in a known size, which believed forward prices were unbiased, would have only forward contracts in the subset of efficient instruments. For a firm with a more complex multi-currency competitive exposure, and where forward prices were believed to be biased, on the other hand would have a much larger subset of instruments including both conventional and complex options and forwards. By relating strategic decisions to the appropriate instruments in a logical way, the treasury manager is able to eliminate inefficient or redundant instruments and so focus on those required to provide the best hedge.

Once the choice of instruments has been made, the next step is to decide which models, either exposure management models or valuation models are appropriate. This will obviously depend on the instruments needed, so for example if only forward contracts are required, a portfolio approach would be the only model which might be appropriate. If a complex option is needed then the particular valuation model must be used to price the instrument. The selection of the model though leads to an analysis of the assumptions underlying the model which ultimately impact on the tactical decisions made before the hedging transaction is carried out.

The final step is therefore the tactical decision-making about the actual deals to be performed. These decisions however do not relate to the choice of instrument but to the timing, size, or perhaps the strike price of the instrument being traded. These decisions are crucially based on the effects of the assumptions underlying the models used and so it is at this step that the implications of say, distributional assumptions, and the biases which they may create, are incorporated. The rationale behind the approach is that the first consideration of the treasury manager is in defining his risks and in selecting the most efficient portfolio of hedging instruments. It is only at the tactical level, when decisions are made on the actual transactions to be performed, that the effects of the underlying assumptions in the theoretical models are relevant. This does not imply that these effects are less important, in fact they are equally so, but they are secondary to the analysis of risk and selection of appropriate instruments.

Let us examine in more detail the actual steps which would be followed in this approach. A decision tree representing the application of this exposure management approach, using the instruments we have considered, is shown in Figure 72. The diagram is aimed at providing a plan or map of the decision making process suggested by the approach and provides a visual representation of the discussion below.

Step 1: Strategic Decisions

The first step is to decide what risks are being faced and whether or not a hedge is required. As we discussed in Chapter 1 there are situations when hedging is often undesirable if the exposure can be passed on to the customer and in particular when competitors are all acting in the same way. Another example where hedging may be inappropriate would be for example an oil producing company considering hedging the price it gets for its output. One issue it might consider is that its shareholders are investing in the company as a way of gaining exposure to the future price of oil. If the company hedges this risk then it is perhaps acting against its shareholders interests. In this situation therefore the company may decide not to hedge this exposure but let it impact the profitability directly. In practice as we discussed in Chapter 1 although hedging can be done at the investor level, most companies possess better information on their positions and can access the markets more efficiently. They therefore take on the hedging on their shareholders behalf.

Other factors to consider might be whether to include funding positions, ie foreign currency loans or deposits, of foreign currency investments in bonds, etc. We discussed this in Chapter 3 and pointed out that regulatory and tax issues may effect the inclusion of these positions. The analysis should however examine the competitive position faced by the company, even if it is wholly domestic in terms of inputs, outputs and competition. As we mentioned before foreign competition gives rise to currency exposure, but even domestic competition can produce currency exposure if the competing firms for example source their inputs overseas. A thorough examination is therefore required not just of the explicit currency positions, and the internal exposures arising from operating decisions, but also of the exposures faced by the other players in the market. Although it might be a complex and time consuming process to assemble this information, the results of such a detailed analysis of the business, the market and the competition, could also be useful for decisions related to other areas of the business.

Having decided what risks are present and which of them require hedging, the next step is to categorise the risks into the three main types, ie transactional, economic and translational. Although there are other ways of categorising risk, we use these three broad types to illustrate the application of the approach. Each type of risk will generate its own set of further strategic considerations and hence its own set of instruments, models and ultimately, hedging transactions. The next decision though is common to all types of risk and concerns the analysis of the foreign exchange market and in particular, whether or not forward prices are believed to be biased. Now this is of course one of the most difficult decisions, since it will fundamentally effect the rest of the strategy but cannot be made on a purely objective basis. Although it has not been the aim of this research to examine biases in forward prices, we have discussed some of the previous work in this area. The conclusion on the EMH appears to be that the failure of the hypothesis is due more to the rational expectations aspect rather than risk neutrality and the lack of any firm evidence for risk premia suggests that forward prices are perhaps unbiased. Other research however has also found that forward prices are not optimal predictors of future spot rates which suggests that forward prices are biased. Empirical evidence can in fact be found to support both views but perhaps most importantly, in practice, many firms may feel that over certain intervals and especially when important economic or political events, for example elections, are expected, forward prices do not correctly reflect the expected spot rate. It should be remembered that the IRP condition has been found to be valid in most cases, indeed the forward markets are normally free from arbitrage. The forward rate then is simply a deterministic function of the spot rate and the domestic and foreign interest rates. When interest rates are constrained, for example because of political reasons, then the implied forward rate may appear to be unrealistic. Instances of examples of this can be found in the EMS system when the forward rate for some currencies is sometimes outside the permitted fluctuation band, suggesting that a realignment will take place. In fact the perceived chance of realignment may be small and so the forward rate in this situation is not equal to the expected spot rate. Often in these situations one finds that speculators buy or sell the currency at levels outside the bands in order to profit from this anomaly and contribute to moving the rate back inside the band. Many other examples can be found and indeed the widespread use of forecasting agencies is perhaps the strongest evidence that firms, and banks, continue to regard the forward price as potentially biased at least for temporary periods.

The question then of deciding whether forward prices are biased or not is probably the most difficult to make. However such a decision must be made because it critically effects subsequent actions. In the final section we discuss some of the practical approaches used to, in a sense, mitigate this problem, but for now we assume that the decision has been made.

Having decided whether forward prices are biased or not, and one could decide that for some currencies they are and others not, the next step, is dependant on the particular risk being hedged. For transactional exposure we would need to examine whether the biases are considered to be temporary or persistent, since this would lead to the use of conventional or barrier type option strategies as we discussed before. For economic exposure, we would need to examine whether there was any flexibility in the firm's operating procedures, either in timing of cashflows or in the sources and therefore currency of inputs and outputs. These decisions would determine what type of instrument was needed, since the presence of flexibility would effectively generate option type positions with or without biased forward prices. Of course many more considerations and complexities could be added, for example we might consider the existence of temporary biases when hedging economic exposure in the presence of flexibility in the timing of cashflows. This example would lead to the use of barrier type swap point options which unfortunately we have not considered, and are in any case as yet unavailable. We limit ourselves therefore to particular examples of the types of strategic decisions which must be made in order to determine the particular set of instruments amongst the ones we have described which are necessary to provide the hedge. As we mentioned earlier an important element in the application of the approach is a knowledge of available instruments, and perhaps more importantly an understanding of the risks being faced and the payoffs which would be required to hedge them. Such an analysis would lead perhaps to the need for a new instrument, which could be offered to the banks.

Once these strategic decisions have been made the subset of sufficient hedging instruments must be defined. The decisions which have been made however become boundary conditions for the inclusion or exclusion of particular instruments.

Step 2: Instruments

As we mentioned above we limit our choice of instruments to those which we have described previously. This set however provides enough choice to demonstrate the approach. The available instruments are firstly conventional and derivative forward contracts, where the derivative forwards are barrier type and max-min type forwards. Secondly, we have conventional and derivative options, where the derivative options are max-min, barrier, swap point and average rate options.

Consider first the case of transactional exposure where forward prices are unbiased. The only instrument required in this case is a conventional forward contract or contracts since as we have demonstrated before this will fully hedge this exposure. Options are not required since the forward prices are unbiased and there is no flexibility by definition ie the cashflow is fixed, in currency and timing. Consider now the same exposure when forward prices are temporarily biased. In this situation a temporary option is required until the forward returns to the expected spot rate, after which a forward contract is needed. As we demonstrated earlier a barrier option combined with a barrier forward can achieve this position and are therefore the only instruments required. Finally consider the same exposure when forward prices are persistently biased ie they are believed to be biased over the entire period being hedged. In this situation conventional options are needed maybe in the form of combinations, in order to develop a strategy consistent with the perceived bias. There are thus three distinct subsets of instruments needed to hedge the same exposure under different strategic decisions. This illustrates the importance of not just defining the type of risk, but deciding the nature of market prices.

Now let us examine economic exposure. Remember that the first strategic decision which we considered was whether forward prices were biased or not (see Figure 72). If prices are biased then the subset of instruments contains a large set of conventional and complex options. This is because when forward prices are biased options of some form are required, but the potentially multi-currency and multi-period nature of the economic exposure implies that more complex options, specifically max-min or swap point options, might also be needed. Once again the appropriate mix of instruments will depend on the nature of the bias, and for example, temporary biases would require barrier type options and forwards although we have not considered max-min or swap point options with barriers attached. If the forward rates are unbiased then the range instruments depends on whether there is flexibility and what type of flexibility is present. With no flexibility, conventional or complex forwards would be needed ie max-min type forwards, since the lack of flexibility rules out options. The complex forwards would only be needed when there was exposure to more that one foreign currency. When there is flexibility in the currency decision then conventional or max-min options are sufficient, with the complex options only needed when more that two foreign currencies are present. Finally when the flexibility is in the timing of foreign currency cashflows only a complex option, the swap point option, is required. Economic exposure then generates four subsets of hedging instruments depending on the strategic decisions made concerning forward prices and flexibility.

Finally given the choice of available instruments, translational exposure is the most straightforward to analyse. The strategic decision is simply whether forward prices are biased or not and so only two subsets of instruments are created. Unbiased forwards lead to conventional or synthetic forwards and biased forwards to complex options ie average rate options. In general translational exposure may not be based on the average rate so another set of instruments may be appropriate. We have just used this as an example, since it is common in practice, to illustrate the methodology. The inclusion of synthetic forwards in this example is to take into account the practical issues we discussed earlier related to hedging a notional exposure.

Once the subsets of instruments have been defined the next step is to select the appropriate exposure management or valuation model or models for those instruments.

Step 3: Models

The only model which we have considered for the management of forward positions is the portfolio model and so its use is relevant for transactional and translational exposure when forward prices are unbiased since in these situations only forward contracts are effectively needed. Some subsets include complex forwards and so a portfolio approach may be appropriate. Its application for these derivative forwards however is problematic. Barrier forwards for example could be treated as conventional forwards as long as they existed and taken out of the portfolio when they disappeared, or before they appeared. Another approach would be to use the delta of the instrument, to derive the proportion of the position which would be incorporated into the portfolio model. Since these deltas would fluctuate however this might make the management of the portfolio difficult.

For the other subsets which include options, selection of the appropriate valuation model is required. These models are shown in Figure 72, the only issue being the decision on which approximation to use in the cases where no exact solutions are available. In general, different models might be used for the same instrument and in these situations a decision on the best model in the circumstances must be made. For average rate options for example, if the exposure was to high volatility currencies then the Levy (1990) model we have described may not be appropriate. One of the other models we described or some other valuation technique could be used instead. The choice of model however is important to the extent that it will bring with it assumptions about the underlying market which will in general be simplifications. The final step in our approach is to examine the effects of these assumptions, before making the tactical decisions on hedging transactions to be performed.

Step 4: Tactical Decisions

The final step in this exposure management approach is the implementation of actual hedging transactions based on the tactical decisions on the size, timing, etc of individual trades. In the context of the portfolio model then for example analysis would be made of the effect of individual deals on the portfolio risk based on the current covariance structure. However the instability which we have found in the correlations and variances of spot and forward rates, will effect the actual deals which are done. Thus as we saw, particular swap deals with different maturities have similar exposures through time as the parameters of the covariance matrix change. This might suggest that a six month versus twelve month swap may be preferable to a spot versus twelve month swap if its exposure to changing variances and correlation more closely followed that of the rest of the portfolio. More generally, the application of the portfolio model which we have presented, is based on a flexible approach to hedge deals which incorporates the inadequacies in the assumptions by not requiring the treasury manager to position his portfolio always at the efficient boundary. The effects of the assumptions, combined with practical restrictions on deal size, etc, dictate a more actively managed approach to the management of the forward risk. Thus tactical decisions on the timing, size and type of spot or forward deal are based on this more flexible approach, which by definition incorporates an appreciation of the effects of the distribution assumptions.

For the option deals, the effects of the distributional assumptions used in the valuation model will ultimately effect the type, size, timing and maybe strike price of the option dealt. In the case of conventional versus barrier options for hedging economic exposure with flexibility in one currency, the ultimate choice may depend on whether the distributional assumptions affect the price of one option more than another and therefore lead to larger pricing biases. For example, it may be that the appropriate barrier option is one which is difficult to hedge due to the position of the required strike relative to the boundary, ie it may have a vertical payoff line at the barrier level. In this situation, the underlying assumptions are particularly inadequate in valuing the option and so a considerable bias might be expected in the price. If this bias worked against the deal, ie the option was expensive but needed to be bought, then a tactical decision would be made to use the conventional option. Of course situations may arise in which these biases work in favour of the deal, for example if the purchase of out-of-the-money max-min options was needed as part of possible hedge for economic exposure when forward prices are biased. These options would be expected to be underpriced by the model and so there may be a tactical advantage in using them rather than another option as part of an alternative hedge.

The implications of the underlying assumptions, and indeed the practical problems associated with any transaction, form a vital part of the final stage in deciding upon the actual deals to be performed. The approach we propose then provides a logical sequence of decisions, from analysis of exposure through to hedging deals, which provides a framework for managing these types of exposures in a systematic way. The approach demonstrates how the results on the actual movement of exchange rates can be integrated with the exposure management and valuation models into the decision making process. In the last section we examine such an approach in comparison to the strategies commonly used, and suggest some areas of future research.

5.4 Final Conclusions and Possible Future Research

The actual use of the approach which we have described is based to a large extent on both the perceived effort involved in its application and the benefits which would be derived. It is therefore useful to briefly examine some of the common strategies which are currently used so that a comparison of the costs/benefits might be appreciated and therefore the likelihood of such an approach being used.

One of the typical strategies which corporates use is the setting of ratios for the proportion of their total exposure which they will hedge. Sometimes a ratio between the proportion of forward and option hedging is also set. The idea behind these strategies, which are mostly used for transactional exposure, is that it diversifies the risk by allocating it amongst different instruments. The strategy is also aimed at the problem of forecasting, that is in our sense deciding whether the forward price is biased. Taking one position in an instrument which is optimal when forward prices are unbiased, and another in an instrument which is optimal when they are biased, is in some sense hedging the risk of an incorrect forecast. Although such strategies may seem illogical from a theoretical viewpoint, since it implies that two opposite views are held simultaneously, the widespread use of these strategies points to the extreme difficulty in making the decision on forward prices.

Given this significant practical problem the approach we have proposed may appear therefore to be unworkable, since we require the decision to be made early on in the hedging process. However the approach could be adapted so that a proportion of the exposure was hedged based on unbiased forward prices and the rest on biased forwards. Thus the model could still be used to provide a structured approach ensuring that in both cases the most efficient hedges are performed. The main problem is the ratio to be used which is in practice fairly arbitrary. As long as their is some difference between forward rates and expected spot rates though the natural conservatism of most corporate treasuries will incline them towards the use of some kind of ratio hedging.

Although there has been as yet no significant results on finding consistent biases in forward prices, we feel that given the lack of support for the unbiased estimator theory by those who operate in the market, further research into this question would be extremely valuable. This is especially important given the crucial role of forward prices in any hedging strategy. The other area of future research which is important to extend this research, is analysis of the effects of portfolios of options on hedging performance. The interaction amongst different option payoffs and their effects on other positions and the portfolio of exposures as a whole, is an important area of study especially as the use of options is growing and a much larger proportion of risk will in future be hedged with these instruments.

Figure 71 : Schematic of Integrated Approach

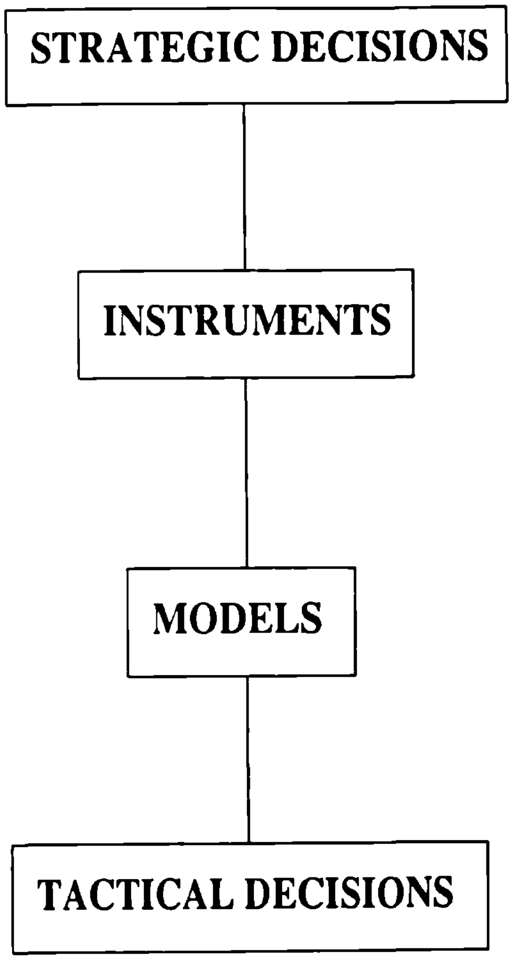
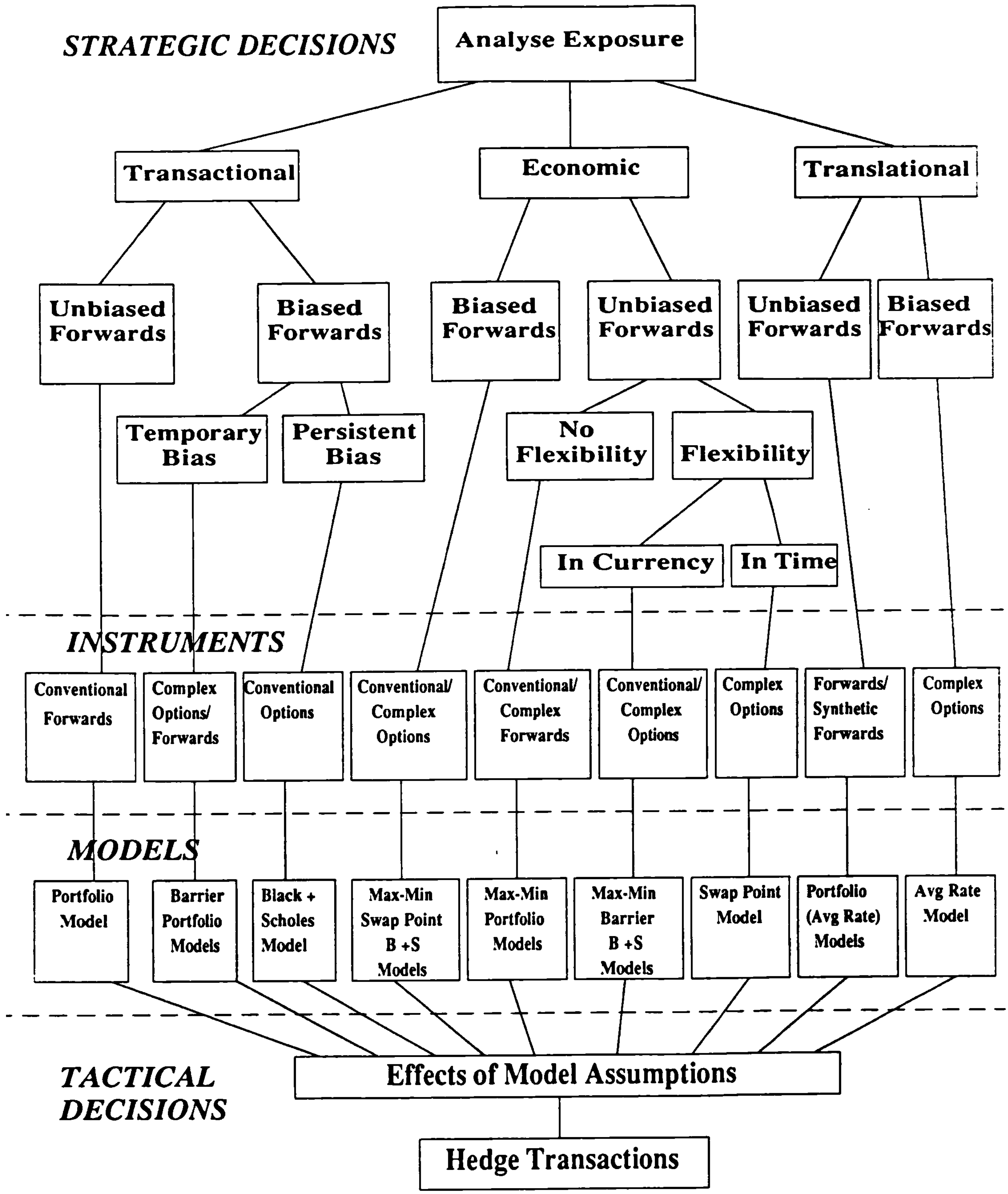


Figure 72 : Exposure Management Decison Tree



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