

UNIVERSITY OF LONDON THESIS

Degree PhD

Year 1992

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**LOCAL WEB BUCKLING IN
TAPERED COMPOSITE BEAMS**

by

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Thesis submitted to the University of London

for the Degree of Doctor of Philosophy

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FEBRUARY 1992

To my loving and caring wife **Marley**

ACKNOWLEDGEMENTS

I wish to express my indebtedness to my supervisor, Prof. R. E. Hobbs, for his endless support, friendship, patience and guidance throughout this research project. I would also like to thank Prof. P. J. Dowling for his constant encouragement and most valuable advice during these past few years.

I would like to thank CNPq (Brazilian Research Council) for sponsoring my postgraduate studies here at Imperial College. The laboratory work was supported by SERC (Science and Engineering Research Council) and Ward Brothers Sherburn Ltd. to whom I am most grateful. I would also like to thank Mr. A. Pottage and Mr. G. Raven for their indubitable contributions to the laboratory specimens.

I would like to thank Mr. J. Finn, Mr J. Mullord, Mr. T. Secker, Mr. J. Neale, Mr. A. Boxall, Mr G. Scopes, Mr A. Roberts, Mr. R. Day, and Mr. A. Laidlaw for their constant support and friendship during my stay in the Structures Laboratory. Many thanks also to Mr. A. Chipling and Mr. R. Packer for their most valuable photographic support. I would also like to thank all my colleagues at Imperial College for their continuous support and encouragement over these years.

I would like to thank: Eliseu & Eliana Lucena, Paulo & Gabi Costa, Luiz & Andréa Palmier, Marco Pacheco, Ageu & Vera Pacheco, Wanderley & Terezinha Lobianco, Rodolfo Suanno & Deane Roehl, and most specially: Sérgio Koide & Cristina Brandão, Marcos & Vanessa Von Sperling, Meyer & Debinha Nigri, Icléa Ortiz, Bruno & Carmem Feijó, João & Rita Bento, and many others for their continuous friendship, encouragement and support all over these years.

I would like to thank my parents Colmar and Maria Laura, for their example, love, care and confidence that made this work possible. Many thanks also to my brother João for his constant encouragement and friendship. Finally I would like to thank my grandmothers Rachel and Bella for having filled with their love and affection a tremendous gap in my life.

ABSTRACT

Recent demand for very heavily serviced widespan office floors has led, as a natural development of the concept of haunched and tapered frames, to the use of composite flooring systems supported by tapered (varying web depth) beams. These systems are very attractive from an economic point of view, combining wide spans, rapid erection and easy access to services between the shallow parts of the beam and the ceiling below.

One of the most pressing needs is for a better understanding of the local buckling behaviour of the relatively deep slender web at changes of slope in the lower flange, in particular at the slope change which occurs at the centre of a beam whose depth increases linearly from each support. In the past, with plate girders, this failure mechanism was less relevant than lateral torsional buckling. With the substantial top flange of a composite section, lateral buckling is less likely, and local buckling more likely to govern the design.

The tapered beam sections are fabricated from plates by welding and are susceptible to imperfection effects and residual stresses. These phenomena may interact with the localized compressive stress field which is generated in the web at any slope change in the flange to cause local web buckling. The provision of local stiffening is technically feasible, but very expensive. The beam itself is produced fully automatically and the subsequent hand installation of stiffeners consumes man-hours out of all proportion to the weight of steel involved.

The prime objective of the present study was to identify the relative importance of several parameters that influence the phenomenon, such as angle of taper, web thickness, flange area and others.

For practical reasons, the experiments were executed at half-scale and the models simulated the concrete slab of the composite beam by a heavy steel top flange. Three tapered models have been tested with different web thicknesses i.e, 3, 5, and 7 millimetres. Three more specimens with different plastic neutral axis positions were also tested.

An extensive parametric study has been made using a non-linear elasto-plastic finite element program. This study covered practical ranges of the important parameters including the area of the tension flange and the position of the plastic neutral axis. Moment-rotation relations, peak moments and failure mechanisms have been predicted.

Very good correlation was obtained between the results of the parametric study and the experiments. It has become apparent that in the great majority of practical cases it is possible to avoid the use of the expensive hand welded stiffeners. A design formulation has been developed to produce a very simple test of the need for stiffeners.

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LIST OF SYMBOLS.

a	Distance between two consecutive vertical stiffeners, width of the web panel.
b_e	factor (function of h , t_f and K_w).
b_{eff}	Effective breadth of the web.
b_f	Width of the flange.
c	Patch length, distance defined in figure 2.5, page 40.
d	Depth of the girder.
$d\theta$	Infinitesimal angle of curvature.
dx	Infinitesimal longitudinal distance.
e	Distance between stiffener and point of applied load.
f^*	Correction factor for patch load defined by equation 2.32, page 52.
f_c	Correction factor for patch load based on patch length.
f_h	Correction factor for patch load based on the height of the web.
f_{hs}	Correction factor for patch load based on the presence of horizontal stiffeners.
f_r	Reduction stress factor defined by equation 2.58, page 66.
f_{vs}	Correction factor for patch load based on the presence of vertical stiffeners.
f_δ	Correction factor for patch load based on the imperfections.
f_{σ_b}	Correction factor for patch load based on the coexisting bending stress.
$f_{\sigma_y^w}$	Correction factor for patch load based on the yield stress of the web.
h	Height of the web.
h_{st}	Height of the stiffener.
j	Factor defined by equation 2.48, page 59.
k	Factor (function of η , t_f and t_w).
p_c	Compressive strength resistance.
p_j^f	Load contribution of the flanges to the ultimate load P_j .
p_j^{st}	Load contribution of the stiffener to the ultimate load P_j .
p_j^w	Load contribution of the unstiffened web to the ultimate load P_j .
r	root radius.
t_e	Thickness of the plate that applies the patch load.
t_f	Thickness of the flange.
t_i	Equivalent thickness (Function of b_f and t_f).
t_{sf}	Thickness of the stiffener.
t_w	Thickness of the web.
z	Factor defined by equation 2.19, page 47.
A_Π	Area of the flange.
A_p	Effective area of the web defined by equation 2.43, page 58.
A_{st}	Area of the stiffener.
A_w	Area of the web.
D	Plate stiffness.

E	Young's modulus.
F_s	Factor of safety.
F_{st}	Strength resistance of the stiffener.
I_{fl}	Moment of inertia of the flange.
I_{st}	Moment of inertia of the stiffener.
I_w	Moment of inertia of the web.
K	Factor (function of r and t_f).
K'	Correction factor for the critical load.
K_c	Coefficient of web strength.
K_p	Factor (function of k).
K_w	Factor (function of h , a and t_f).
K_1	Effective width of the web.
K_2	Square of the effective length of the web.
L_e	Effective length of the web.
L_{eff}	Distributed length for the dispersion of the patch load.
L_p	Effective width defined by equation 2.37, page 55.
L_{p1}	Effective width defined by equation 2.37, page 55.
L_{p2}	Effective width defined by equation 2.37, page 55.
M	Applied moment.
M_{fl}	Moment capacity of the flange.
M_{pl}	Plastic moment capacity of the beam.
M_w	Moment capacity of the web.
M_u	Ultimate moment.
P	Applied patch load.
P_{cr}	Elastic critical load.
P_{cy}	Load defined by equation 2.38, page 55.
P_e	Elastic load.
P_{exp}	Experimental load.
P_f	Ultimate load.
P_{fl}	Part of the load supported by the flanges.
P_j	Ultimate load for end-post stiffener.
P_{pl}	Plastic load.
P_w	Part of the load supported by the web.
Q	Applied shear force.
Q_f	Ultimate shear resistance.
R	Applied load.
R_{fl}	Design value of allowable stress for the compressive flange.
S_s	Distance defined in figure 2.18.a, page 57.
S_y	Coefficient of yield load of the web.
W	Factor that takes into account initial imperfections and post-buckling behaviour.
α	Angle of slope, angle defined in figure 2.5, page 40, factor defined by equation 2.19, page 47.
β	Angle defined in figure 2.5, page 40, factor defined by equation 2.19, page 47.
ε	Distance defined in figure 2.22, page 65.
ε_f	Longitudinal deformation.

ϕ	Angle of inclination of the web to the vertical.
ϕ_f	Factor defined by equation 2.43, page 58.
γ	Factor (function of c and t_f), factor defined by equation 2.19, page 47.
γ_{j2}	Factor of safety.
γ_m	Factor of safety.
θ	Angle defined in figure 2.5, page 40.
Ω	Factor (function of α , β , and others).
ν	Poisson's ratio.
λ	Factor (function of P_{cr} and P_{cy}), factor defined by equation 2.48, page 59, factor (function of b_e , σ_y^w and t_w).
η	Factor that varies with the ratio t_f/h , factor that varies with the ratio t_f/t_w , factor defined by equation 2.43, page 58.
ψ^*	Relative rigidity of the stiffener.
ψ_0	Ideal rigidity factor.
σ	Stress.
σ_b	Bending stress.
σ_{cr}	Elastic plate buckling stress.
σ_{crip}	Critical stress.
σ_r	Residual stress.
σ_r^{fl}	Residual stress of the flange.
σ_y	Yield stress.
σ_y^{fl}	Yield stress of the flange.
$\sigma_y^{fl'}$	Equivalent yield stress of the flange to take into account coexisting bending stress.
σ_y^{st}	Yield stress of the stiffener.
σ_y^w	Yield stress of the web.
$\sigma_y^{w'}$	Equivalent yield stress of the web to take into account coexisting bending stress.
τ_{cr}	Critical shear stress.
τ_y	Yield shear stress.
θ	Factor defined by equation 2.20, page 47.

CHAPTER 1

INTRODUCTION.

1.1. MOTIVATIONS AND GOALS.

Recent demand for very heavily serviced widespan office floors has led, as a natural development of the concept of haunched and tapered frames, to the use of composite flooring systems supported by tapered (varying web depth) beams. These systems are very attractive from an economic point of view, combining wide spans, rapid erection and easy access to services between the shallow parts of the beam and the ceiling below.

The potential of this kind of structural section has been demonstrated by its incorporation in several major projects. However, full economic exploitation is inhibited by the limited understanding of certain aspects of the behaviour of such sections.

One of the most pressing needs is for a better understanding of the local buckling behaviour of the relatively deep slender web at changes of slope in the lower flange, in particular at the slope change which occurs at the centre of a beam whose depth increases linearly from each support. In the past, with plate girders, this failure mechanism was less relevant than lateral torsional buckling. With the substantial top flange of a composite section, lateral buckling is less likely, and local buckling more likely to govern the design.

The tapered beam sections are fabricated from plates by welding and are susceptible to imperfection effects and residual stresses. These phenomena may interact with the localized compressive stress field which is generated in the web at any slope change in the flange to cause local web buckling. The provision of local stiffening is technically feasible, but very expensive. The beam itself is produced fully automatically and the subsequent hand installation of stiffeners consumes man-hours out of all proportion to the weight of steel involved.

The prime objective of the present study was to identify the relative importance of several parameters that influence the phenomenon, such as angle of taper, web thickness, flange area and others.

Experiments were then conceived in order to investigate the influence of these parameters. An extensive parametric study has also been made using a non-linear elasto-plastic finite element program. This study covered practical ranges of the important parameters including the area of the tension flange and the position of the plastic neutral axis.

Another goal was the development of a simple design model that could enable steel designers to predict/avoid occurrence of the local web buckling. The model will then provide a minimum web thickness necessary to resist the buckling. If a girder has a web thickness less than this specified value, then a vertical stiffener should be used on both sides of the web at the slope change.

1.2. THESIS ORGANIZATION.

The chapters are organized as follows:

Chapter 2 is a literature review and consists of five sections. The first covers the work done on composite tapered beams, while the second deals with aspects of tapered and haunched beams. The third and fourth parts relate to composite beams and a patch load study respectively. The last section considers the design of bearing and end-post stiffeners.

The patch load section was divided into four sub-sections. The first of these covers theoretical work and some numerical studies, while the second deals with finite element simulations. The third concerns the experimental work done in this area and the last considers the approach taken by design standards and some state of the art surveys.

Chapter 3 presents in detail the experiments performed. A brief introduction to the equipment and materials used is followed by a description of the experimental rig and the data acquisition process. Lastly, the results are presented and discussed.

For practical reasons, the experiments were executed at half-scale and the models simulated the concrete slab of the composite beam by a heavy steel top flange. Three tapered models have been tested with different web thicknesses, i.e, 3, 5, and 7 millimetres. Three more specimens with different plastic neutral axis positions were also tested.

Chapter 4 comprises the description of the finite element model used to simulate the local web buckling problem, and its comparison with the experiments discussed in the previous chapter. Using the program FINAS, a simulation of a panel of a tapered beam was developed, and validated against both series of experiments and some additional runs using different parameters to isolate the buckling effect.

Chapter 5 describes the subsequent finite element parametric study. The investigation has focused on three main parameters. A study of the angle of taper and web thickness was first conducted. This was followed by an examination of the influence of the tension flange area on the load carrying capacity of the girders. Moment-rotation relations, peak moments and failure mechanisms have been predicted. With the results in hand, conclusions were drawn on the influence of the variables on the beam's performance.

Chapter 6 describes the design model developed to predict the occurrence of local web buckling in tapered composite beams. A quick review of the relevant design codes for parallel beams is presented first. This is followed by the derivation and behaviour of the model. With the completion of this phase, the calibration of the model against the experiments and parametric study is given. Finally, some conclusions about the performance of the design model are drawn.

Finally, chapter 7 is divided into two main sections. The first part describes the main conclusions that are reached in the thesis. The second part suggests some further steps that can be taken to improve the understanding of local web buckling in tapered composite beams.

CHAPTER 2

LITERATURE REVIEW.

2.1. INTRODUCTION.

The initial objective of this review was to provide a survey of the work done in the field of composite tapered beams, particularly related to the problem of localized buckling in the web region. In the event, very few papers were found on composite tapered beams and only one reference was found on the specific question of local buckling.

In view of this fact, it was decided that a broader survey should be made in order to include other aspects directly related to the study. This chapter now consists of five sections. The first covers the work done on composite tapered beams, while the second deals with aspects of tapered and haunched beams. The third and fourth relate to composite beams and a patch load study respectively. The last one considers the design of bearing and end-post stiffeners.

The patch load topic seemed to be particularly relevant to the problem and for this reason its section of the review is wider than others. It covers theoretical solutions, finite element idealizations, experimental work and state of the art surveys made in this area in recent years.

2.2. COMPOSITE BEAMS.

Over the past few years, continuous composite structures have been used more frequently in commercial buildings because of their economic advantages. The work of Brett et al, [BRET87], provided some special procedures to improve the performance of composite beams: the minimisation of bracing requirements in the hogging moment regions, simple methods for the design of composite floors and an easy process for the estimation of the effective length of columns. Lawson, [LAWS89], described a method for the design of slabs and beams with steel decking and produced a series of design tables. The method was based on a plastic composite section analysis at ultimate load and on the elastic behaviour at serviceability loads. Johnson & Fan, [JOHN88], revised the Eurocode 4 procedure for the semi-compact composite continuous section design. They made a parametric study of the design procedure and compared its results with those obtained from a series of tests performed on cantilevers to examine their performance in the post-yield region. The principal concern of the work was the redistribution of moments between the hogging and sagging regions: the code was found to be on the safe side. Smith, [SMIT89], made a useful comparison between the existing design codes, including Eurocodes 4 and 8, the British BS5950 parts 3.1 and 4, and BS5400 part 5. It covered topics like limit state design, shear connectors, serviceability limit states and others. Another comparison between BS5950 and the Eurocode 4 was made by Lawson, [LAWS90]. He concluded that both codes achieved very similar ultimate loads for simply supported beams with full shear connection. On the other hand when a continuous beam or the partial interaction technique was used, BS5950 predicted a higher load carrying capacity. It also stated that in several practical cases the serviceability limit controlled the design.

Earlier, Hope-Gill & Johnson, [HOPE76], tested three -span continuous composite girders in order to verify the plastic design rules present in the British Code CP-117. They concluded that they were conservative in the majority of the cases and that the causes of serviceability failure were yielding or cracking. Wright and Francis, [WRIG90], investigated the behaviour of composite beams with low levels of shear connection. They tested four full-scale composite beams with profiled steel decking and universal steel beam sections up to failure. They concluded that BS5950 is conservative in the matter of ultimate moment capacity and that the stiffness of the beams is related to the degree of shear connection present.

Another topic that deserves consideration is the local instability of composite girders. Bradford, [BRAD86], and Johnson [JOHN83], performed an investigation in this area using a finite strip analysis. They looked into the local buckling problems that could occur in negative bending regions taking into account residual stresses and the inelastic properties of the materials. The comparison of the computational simulation with some experimental results proved that the method was accurate. The phenomenon of lateral buckling in internal support regions of continuous beams was examined by Weston, Nethercot and Crisfield, [WEST91]. By the use of an elasto-plastic finite element analysis they discovered some very conservative rules for design still present in BS5400. They noted that for compact or semi-compact beams with the ratio of depth to thickness of the web lower than sixty, the mode of failure was controlled by elastic local buckling near the internal supports. For the other cases lateral buckling governed the design but the loads predicted by BS5400 were four to five times lower than the one found in the analysis. Considering those facts, they recommended a simple modification to the design rules that improves the failure prediction considerably.

Climenhaga & Johnson, [CLIM72], studied the rotation capacity needed to permit a plastic analysis of a composite girder. They proved by experiments that some beams that satisfied the then current design rules could achieve the plastic moment but could not redistribute the moments because they lacked rotation capacity. In view of this conclusion they proposed some new slenderness limitations based on experimental data. They also stated the benefits of transverse stiffeners on the load capacity of the beam. Finally they concluded that a half-depth stiffener would have almost the same effect on the rotation capacity as a full-depth one. Kemp & Decker, [KEMP91], tested 44 beams subjected to a moment gradient to assess their inelastic rotation capacity. They discovered two different failure modes. In the beams with a low lateral slenderness ratio, lateral buckling occurred first followed by a local buckling of the compressive flange. In the other cases the local buckling of the flange was predominant and was succeeded by lateral and local web buckling. Their main conclusion was that no distinction was necessary in respect of the local buckling provisions between composite and non-composite sections.

2.3. TAPERED AND HAUNCHED COMPOSITE BEAMS.

The search for an economic answer to the problem of long span building construction has led to the composite tapered beam solution. One of the first investigations in this area was done by Owens, [OWEN86]. He studied eight composite structural forms in order to demonstrate the advantages and disadvantages of each solution. These were respectively: light plate girders, tapered beams, haunched beams, trusses, stub girders, castellated sections, dual plane grillages and simple universal beams. His results demonstrated that the tapered and haunched

solutions achieved the best results, combining a good flexibility for services and a low ratio of steel to area of construction, for a span in the order of 15 metres.

In the light of these figures a composite tapered beam was tested at Imperial College in 1986, [HARG86]. This study was commissioned by Wards Building Systems Ltd who provided the materials, steelwork and design, [WARD87]. The beam tested achieved its working load without distress and showed a good reserve of strength. The beam was simply supported at both ends and had the shape shown in figure 2.1, page 34.

The beam was initially designed without any web stiffeners. Under the working load a localized web buckle appeared in the region where a change of slope of the flanges occurred, figure 2.2.a, page 34. It was then decided to weld a small stiffener at that point and to continue the test, figure 2.3, page 34. The results proved that the ultimate capacity of the beam was 1.98 times its working load, well above the design factor of 1.6.

The localized buckle can be explained by a simple equilibrium model (figure 2.2.b, page 34). The tension forces carried by the bottom flange induce a compression force in the web which causes the buckling. This phenomenon is the primary object of study in this work, as has been explained in the first chapter of this thesis.

A linear elastic finite element analysis of the composite tapered beam was performed by Nilsson, [NILS87]. The results proved to be close to those of the experimental tests mentioned previously, [HARG86]. The formulation assumed full interaction at the interface between steel and concrete, and linear elastic behaviour of the concrete deck and steel beam. The second and third hypotheses, at least, are not in strict accordance to reality, since problems like the initial buckling in the web region and cracking of the concrete slab were noticed in the experiment.

Raven, [RAVE87], [RAVE87a]. produced a list of advantages for composite tapered beams. It included: planning flexibility, fewer foundations, beam depth similar to that of rolled steel beam solutions, good integration of services, easier fit out of floors and ceilings and some others less significant factors. He pointed out the relative simplicity of the design process and compared some of the results obtained with the experimental ones previously described, [HARG86]. He also called attention to the web buckling problem, stating that a stiffener was necessary in the change of slope region to avoid premature failure.

Finally it is appropriate to mention the work done by Lawson & Rackham, [LAWS89a]. They developed a design method for haunched composite beams based on the plastic capacity of the beam section. They also proposed two ways of calculating the global moments present in the structure based on an elastic or a plastic analysis. Serviceability tests for deflections, stresses and vibrations were suggested and like Owens, [OWEN86], they made a comparison of the different options available for composite construction. In respect of the tapered composite beam they implied that this type of section was economic for spans between 13 and 20 metres. The presence of a web stiffener at the point where the flange slope changes was only regarded as necessary when the change in slope exceeded 6° ($\cong 1/10$). With hindsight, this appears to be an oversimplified requirement because web thickness, tension flange area and other parameters are equally important. Lawson & Rackham gave no reason nor supporting evidence for their criterion.

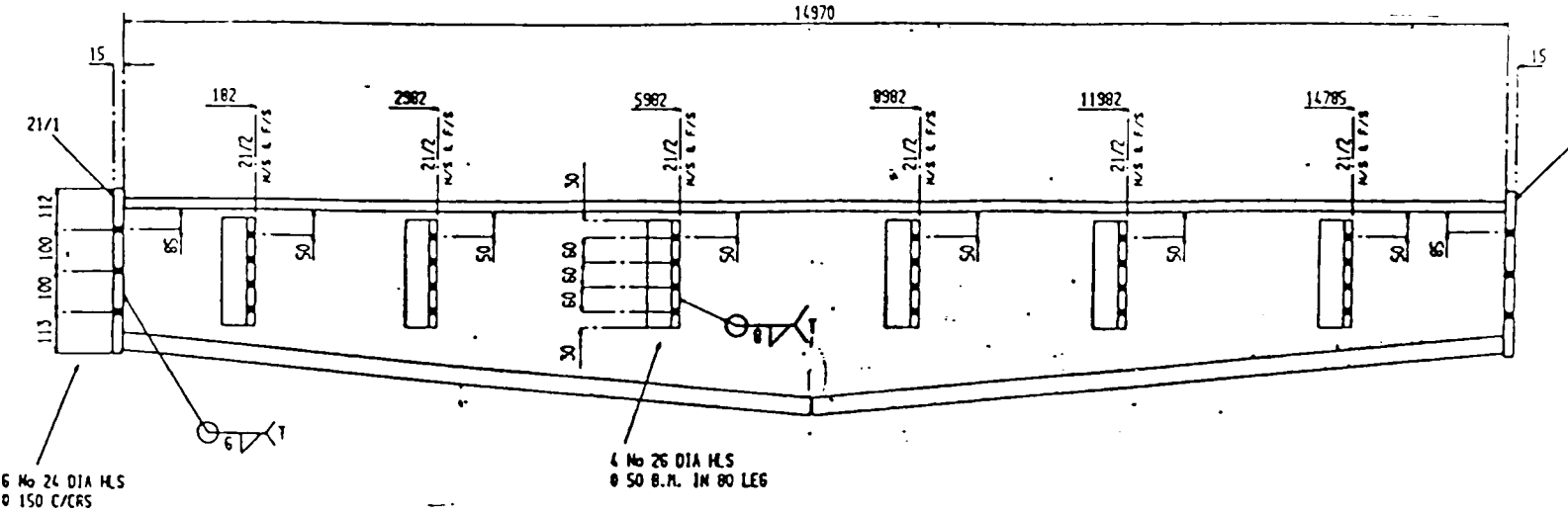


Figure 2.1 - Configuration of the Composite Tapered Beam.
(after [HARG86])

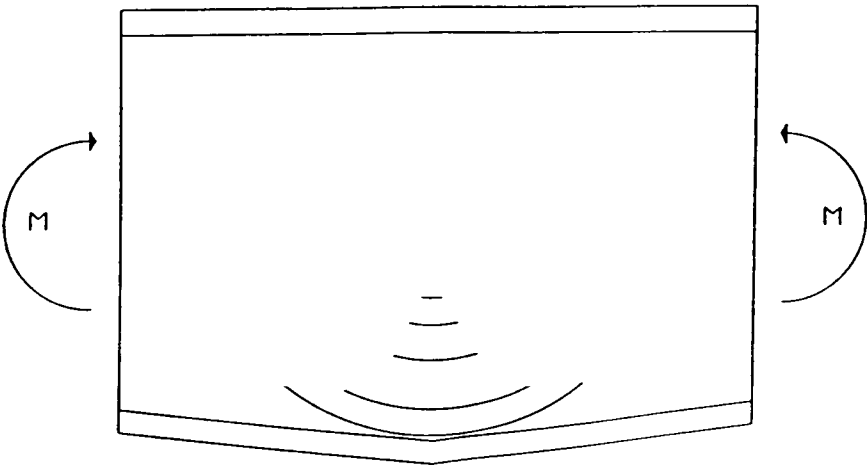


Figure 2.2.a - Buckling Model.

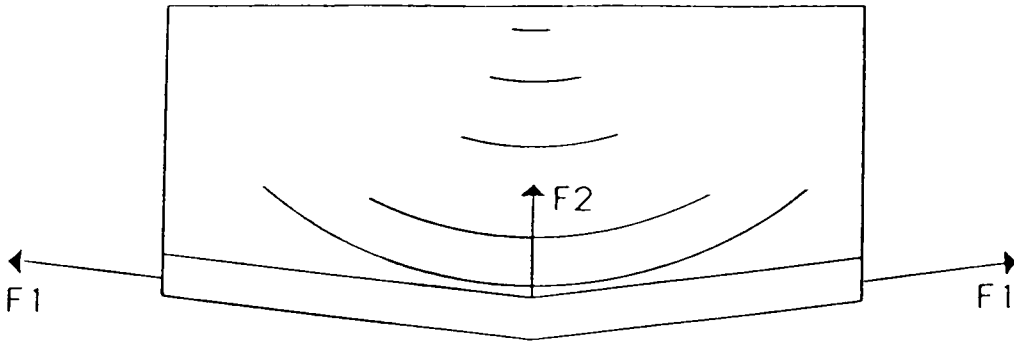


Figure 2.2.b - Buckling Model Detail.

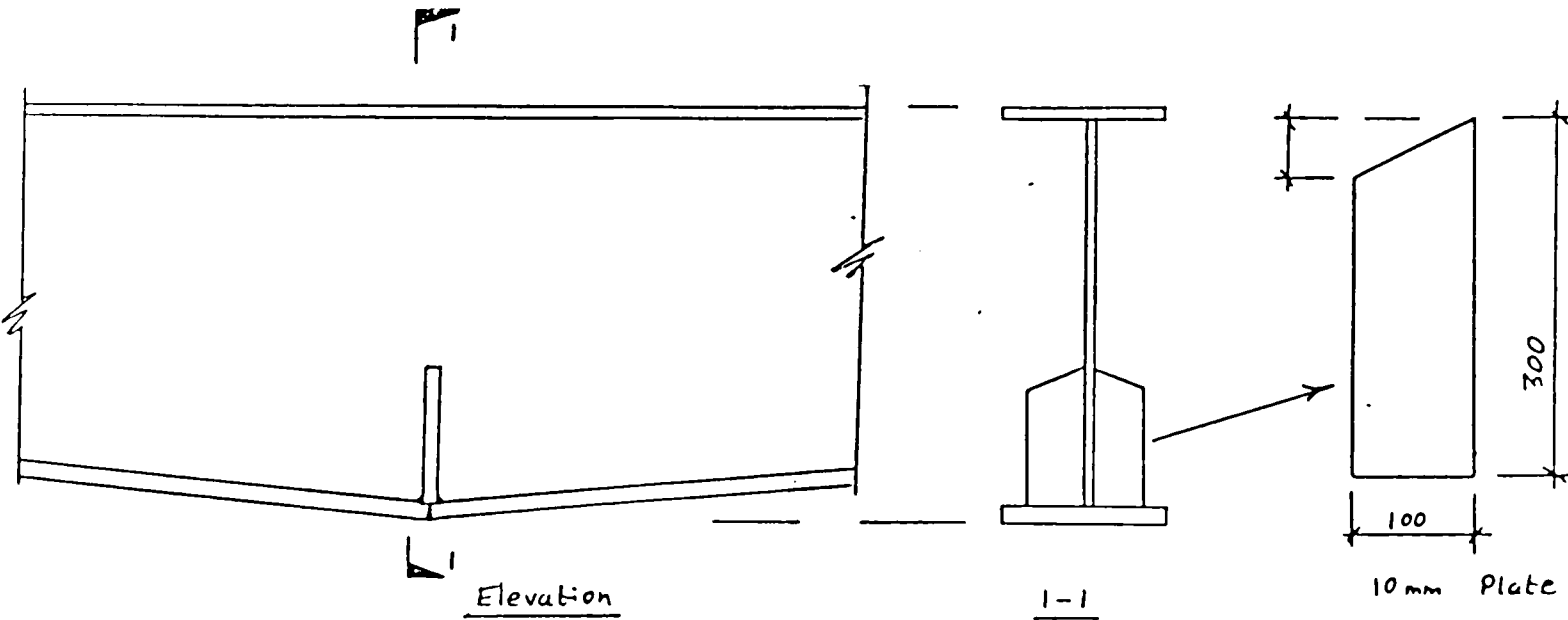


Figure 2.3 - Stiffener Detail. (after[HARG86])

2.4. TAPERED AND HAUNCHED BEAMS.

Investigations in the area of tapered/haunched structures could be found relatively easily in the literature. However, as they proved to have only a limited relevance to the present study, they will only be described briefly here.

One of the major studies on tapered and haunched beams was developed by Horne, [HORN79], [HORN79a]. It comprised a theoretical study of the stability of tapered and haunched beams and some experimental tests to calibrate the formulae proposed for design. It pointed out some of the deficiencies of the existing British design rules and offered new simplified models. It covered in detail the lateral torsional buckling performance of this type of structure and suggested some ideas to deal with points such as the effectiveness of lateral supports, achievement of plastic moment capacity and several others. One of the main conclusions of the experimental tests was that if it is intended to develop the plastic moment capacity of a tapered beam, it is necessary to provide a web stiffener at slope changes in the web. This stiffener was responsible for taking the compressive force that came from the sloping flange. Unfortunately, this study did not provide any details of the value of that force, its effect on the plastic capacity of the section or the method to design the stiffener.

Looking at haunched portal frames and their applications in industrial buildings, it is important to mention the work of Morris & Nakane, [MORR85], [MORR83], Dowling et al, [DOWL82], Morris, [MORR81], and Davies et al [DAVI90]. Morris & Nakane, [MORR85], organized a very useful survey in this area, covering in detail the member stability aspects of the problem. The main part of the study concerned the lateral torsional buckling phenomenon, but they also provided guide-lines on the design of lateral supports. Their earlier paper, [MORR83], described the experimental work on haunched members. They concluded that one of the most difficult tasks to handle was the assurance that instability will not occur prior to the formation of a plastic hinge in the haunch/rafter intersection. The third and fourth papers [DOWL82], [MORR81], dealt with various issues relevant to the design of haunched frames, such as lateral restraints, beam/column connection, adequacy of column web panel, interaction between local and overall buckling and some others. Davies et al, [DAVI90], performed three full-scale portal frame tests with spans that varied between 12 and 24 metres. They also developed a finite element program to simulate the behaviour of the portal frames and demonstrated a very good agreement with the experiments. They observed that the main cause of failure was the yielding of the column web. The load corresponding to that failure was also found to be lower than the one predicted by the code rules. They stated that this happened because no shear or bearing stiffener was present in the column.

Lee & Morrell, [LEE75], studied the American design rules for tapered members in order to clarify their use. Topics like local buckling limitation and tension flange support requirements were covered and a series of design examples were produced. Falby, [FALB76], proposed some modifications to the tension field design method to adjust its use in tapered beams. He stated that for girders with an angle of taper less than $1/15$, the average depth of the beam could be taken for design purposes with little error. He proposed a formula to cope with situations outside this limit and pointed out the necessity of studying the influence of stiffeners on the performance of that specific type of structure.

To conclude this section, the work of Hendry, [HEND47], Wright, [WRIG57], and Morris, [MORR84], in the field of portal frame knees will be described. Hendry, [HEND47], divided his work into three main phases. First a series

of tests of several knees with different slopes was performed. This was followed by some mathematical solutions to the problem and by some photo-elastic experiments. He concluded that curved knees were less critical than sharp ones and that if a knee of 90° was to be used it was necessary to provide a diagonal stiffener. Wright, [WRIG57], made a state of the art survey of this area and proposed some modifications to the existing design formulae based on the experimental results reviewed by him. Lastly Morris, [MORR84], produced an updated literature review followed by tests that resulted in suggestions for the modification of the existing design formulae. He stated that one of the major problems was concerned with the lateral instability of knees when no, or inadequate, lateral restraint was provided. He also proposed a new way of bracing the frame knee against lateral buckling and concluded that the region where the plastic hinge appears should be effectively braced to avoid a reduction in the load capacity of the member.

2.5. PATCH LOAD.

The subject of patch load has been a topic of great interest over the last few years. To organize this section in a way that could make a broader view of the matter possible, four separate sub-sections have been created. The first one covers the theoretical work and some numerical studies, while the second deals with finite element simulations. The third concerns the experimental work done in this area and the last considers the approach taken by design standards and some state of the art surveys.

2.5.1. Theoretical and Numerical Works.

Several analytical methods have been proposed to solve the problem of a plate with edge loads. They involve mathematical and physical models to simulate, as closely as possible, the conditions found in reality. Some of the solutions will be covered in this section to give an idea of the present state of theoretical research in the area.

Zetlin, [ZETL55], studied the buckling of rectangular plates by the energy method. He looked into the partial differential equations that govern the phenomenon in order to generate a simple solution based on some graphs previously developed. The results were then set against experimental data and some faults were detected which were attributed to the fact that the post-buckling strength of the plate was not taken into account in the analysis and also to the differences between the tests and the theoretical cases. White & Cottingham, [WHIT62], used a finite difference program to calculate the elastic buckling loads of rectangular plates with reasonable results. Dogaki et al, [DOGA91], elaborated a finite difference procedure to model the patch load behaviour and compared the results with experiments, [ROBE79]. They investigated the influence of the length of the patch load, the ratio of depth to thickness of the web, residual stresses and initial imperfections on the ultimate strength and post-buckling behaviour of the beams. Steup, [STEU77], tried to look into the problem by minimizing the buckling stress obtained from a Fourier-Series type solution. Few examples were executed and no calibration with experimental or other results was made.

Khan & Walker, [KHAN72], examined the buckling of rectangular plates due to patch load by using a simplified solution based on the energy method. A comparison was made with finite element simulations and it was shown that the

difference between these approaches was no more than 5%. They also tested some plates to investigate the influence of imperfections on the buckling load. Khan et al, [KHAN77], modified the method to predict the elastic buckling load of the plates examined previously, in order to make a more precise parametric study.

Graves-Smith & Gierlinski, [SMIT82], studied (by a finite strip method) the influence of longitudinal stiffeners on the resistance of plates subjected to edge loads. They considered the influence of the bending and torsional resistance of the stiffener on the collapse load of the plate by proposing two separate formulae for design. The first one was a lower bound on the stiffener bending rigidity for it to continue to be straight after the buckling of the panel. The second related the buckling strength of the web to the torsional rigidity of the stiffener. Bergfelt, [BERG84], proved by a comparison with some experimental results that the formulae developed overestimated the resistance of the plates by up to 300%. Shimizu et al, [SHIM90], used the dynamic relaxation method to investigate the problem. They discovered that for beams with smaller patch lengths and flexible loaded flanges a three yield line model could be used. In the other cases a behaviour similar to experiments, [ROBE79], [SHIM87], was observed.

Bazile, [BAZI73], made a comparison among three different ways of treating the edge load question. The first concerned a study based on theory of elasticity. The second comprised an approach based on elastic buckling which was used at that time in the Russian, German and Czechoslovakian design rules. The last was a practical approximation for the phenomenon. Unfortunately no comparison was made with experimental results giving less credibility to the work.

Thornton & Conway, [THOR80], used two approaches to investigate a rectangular homogeneous isotropic plate with two sides free and two simply supported, subjected to a concentrated load on one of the free edges. The first involved a Levy-type analysis and the second a superposition method. The results showed a good agreement with some experiments performed on a square plate.

2.5.2. Finite Element Solutions.

One of the first attempts to use the finite element method to model the behaviour of webs subjected to patch load was by Bagchi & Rockey, [BAGC68], [BAGC75]. They provided a brief description of the method and suggested a non-dimensional formula to calculate the buckling load P_{cr} :

$$\frac{P_{cr}}{a t_w} = K' \frac{\pi D}{h^2 t_w} \quad \text{where:} \quad D = \frac{E t_w^3}{12 (1 - \nu^2)} \quad (2.1)$$

In this equation E is Young's modulus, ν is Poisson's ratio and t_w is the thickness of the web. The factor K' is a function of the aspect ratio of the web panel a/h and the ratio c/h where c is the length of the patch load, a is the width and h is the height of the web's panel. A series of graphs was provided to determine K' . Later, [ROCK70], they investigated the influence of the flange/web interaction. They discovered that the flange can influence the buckling load of the beam and the web stress distribution. No experimental calibration was made in either case.

Rockey & Elgaaly, [ROCK71], proposed a design formula for the ultimate load based on the elastic critical buckling load of the web. They developed this theory following some experiments that were performed in order to investigate the influence

of parameters like c/a , a/h and h/t_w . The relation between the failure load P_f and P_{cr} was defined as being linear and is given by equation 2.2:

$$P_f = P_{cr} (4.5 + 6.4 (c/a)) (h/t_w) 10^{-3} \quad (2.2)$$

Rockey et al, [ROCK79], looked into the problem of longitudinal stiffening of web plates subjected to patch loads. They used the finite element method and provided a relationship between the buckling load and the flexural rigidity of the stiffener. This formula was only applicable to cases where the stiffener was positioned at one-fifth of the depth of the girder, near to the loaded flange. They limited the value of K' to 11.4 corresponding to an ideal value of the ratio of width to thickness of the stiffener, b_s/t_s equal to 17.

Mukhopadhyay, [MUKH83], used an isoparametric quadratic finite element to simulate the problem. He developed an explicit value for the element loading matrix for a patch load and examined plates with different aspect ratios. No buckling load was suggested nor was any comparison with other methods or experimental tests made.

Weimar, [WEIM87], performed a parametric study of the patch problem utilizing non-linear shell elements. He concluded that the method was able to produce good results in terms of ultimate load prediction. He also confirmed that the most important parameter related to the patch problem was the thickness of the web.

Raoul et al, [RAOU91], developed a non-linear finite element program with triangular elements to make a parametric study of the patch load problem. They compared the results obtained with 118 experiments, [GALE87], and achieved good agreement. They also developed a design formula to predict the ultimate load capacity of the beams, equation 2.3.

$$P_f = 1.38 (E)^{0.3} (\sigma_y^w)^{0.7} (t_w)^{1.53} (I_{ff})^{0.06} (c)^{0.23} \quad \text{with } I_{ff} = b_f t_f^3 / 12 \quad (2.3)$$

in which: σ_y^w is the yield stress of the web, b_f is the width of the flange and t_f is the thickness of the flange. This equation overestimated the failure load by approximately 35% when compared with the experiments.

2.5.3. Experimental Investigations.

Some of the pioneer experimental efforts on the subject of patch load can be found in the work of Lyse and Godfrey, Moore, and Hendry. Lyse and Godfrey, [LYSE35], and Moore [MOOR35], identified the reasons for the web failure and, more specifically, looked into the problem of web crippling by testing six rolled beams with h/t_w of the order of 52. They identified the problem as being a localized effect based on the high local stresses obtained in the test series. Finally they proposed a formula (eq. 2.4) which is still the basis for bearing design in several current national standards including the American and Canadian ones.

$$\sigma_{crip} = \frac{R}{t_w (c + 2 t_f)} \leq \sigma_y \quad (2.4)$$

where c is the bearing length, R the applied force and σ_y the yield stress.

Hendry, [HEND49], made a photoelastic analysis and conducted a full scale test series on the patch problem. He recognized that the phenomenon was restricted to a small area in the vicinity of the load and stated the benefits of the extension of the bearing plate. He noticed that yield zones were appearing in the web prior to the formation of plastic zones in the flanges and suggested that the conventional theory for the calculation of the shear stress was only valid at a distance greater than $1.5 h$ from the point where the load was applied.

One of the most active investigators in the patch load field is Bergfelt. He has made several studies in this area and published many papers concerning diverse aspects of the field. In 1968, [BERG68], he examined the patch load in unstiffened webs. He pointed out some of the advantages of that design philosophy such as less painting and future maintenance, and focused his interest on slender webs. He also investigated the influence of intermediate stiffeners on the ultimate capacity of the beam and showed some interesting results concerning the detailing of the stiffener. He produced a series of tests involving the web crippling phenomenon and from the results concluded that some factors such as height of the beam, dimension of the flanges and the span had a minor influence on the failure load. He confirmed the research results obtained by Granholm, [GRAN60], which demonstrated the major influence of the thickness of the web on the ultimate load capacity of the beam and proposed the following design formula:

$$P_f = 0.045 E t_w^2 \quad (2.5)$$

Bergfelt discovered that the height of the stiffener was an important factor in the design proving that if the total depth of the beam was reached, the maximum capacity was achieved. He also concluded that the girder had a large post-critical reserve under patch loading. That capacity could be reached provided that the beam as a whole was able to resist other types of failure such as local buckling of the flange and lateral torsional buckling prior to the occurrence of the main patch failure.

In 1971, [BERG71], Bergfelt described a new series of tests on patch loading. He divided the failure behaviour into three different cases: yielding, web buckling, and web crippling, and examined the last case in more detail. The tests were performed on beams with the ratio h/t_w between 150 and 350 and with the thickness of the web t_w varying between 2 and 6 mm. The influence of t_w on the ultimate load of the beam can be better understood from figure 2.4, page 40. The experimental investigations of Granholm, [GRAN60], Zetlin, [ZETL55], and Bergfelt & Hovik, [BERG68], are compared with equation 2.5, and a reasonable agreement shown. He also suggested that for beams with thicker flanges where the crippling behaviour governed, the following equation should be used:

$$P_f = 0.045 E t_w^2 (0.55 + 0.22 t_f/t_w) \quad (2.6)$$

Subsequently, [BERG76], he described the failure process of a beam subjected to a patch load in three major phases. The first comprised the path until some yielding was detected in the surface of the web (by the brittle lacquer technique). The second phase ran to the moment where some small folds could be detected in the web and the last phase went to the point of failure. The collapse could be achieved by a overall buckling of the web or by a localized zone under the load where bigger folds and high stress concentrations were present. He suggested a modification to equation 2.5 to include other parameters such as the yield stress and

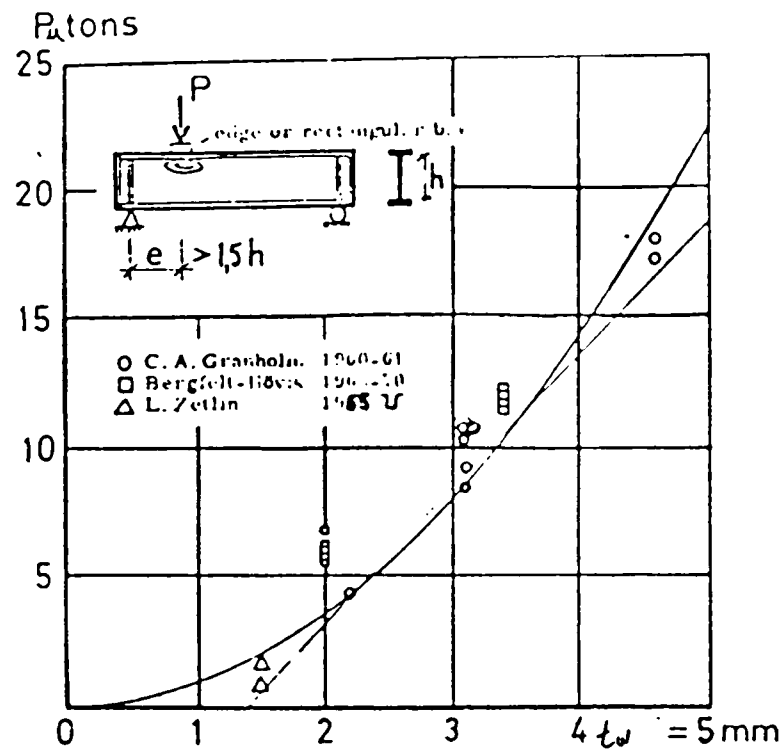


Figure 2.4 - Influence of the Thickness of the Web on the Failure Load. (after [BERG71])

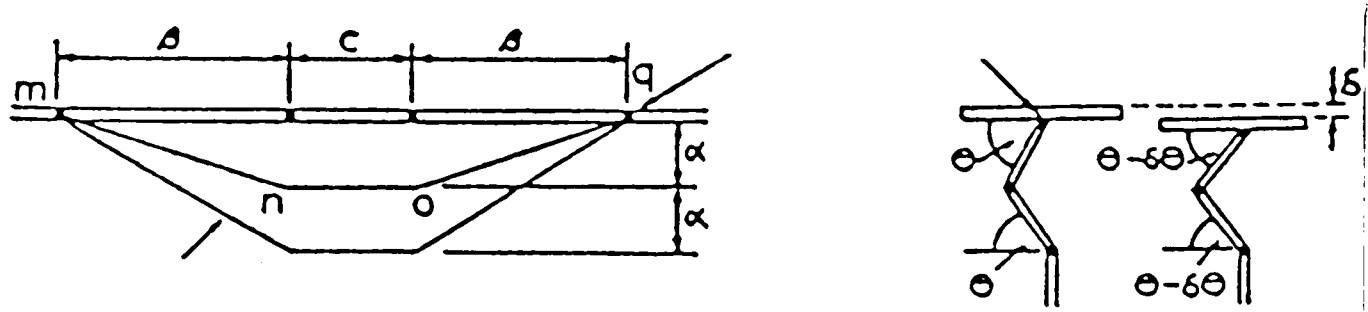


Figure 2.5 - Collapse Mechanism. (after [ROBE78])

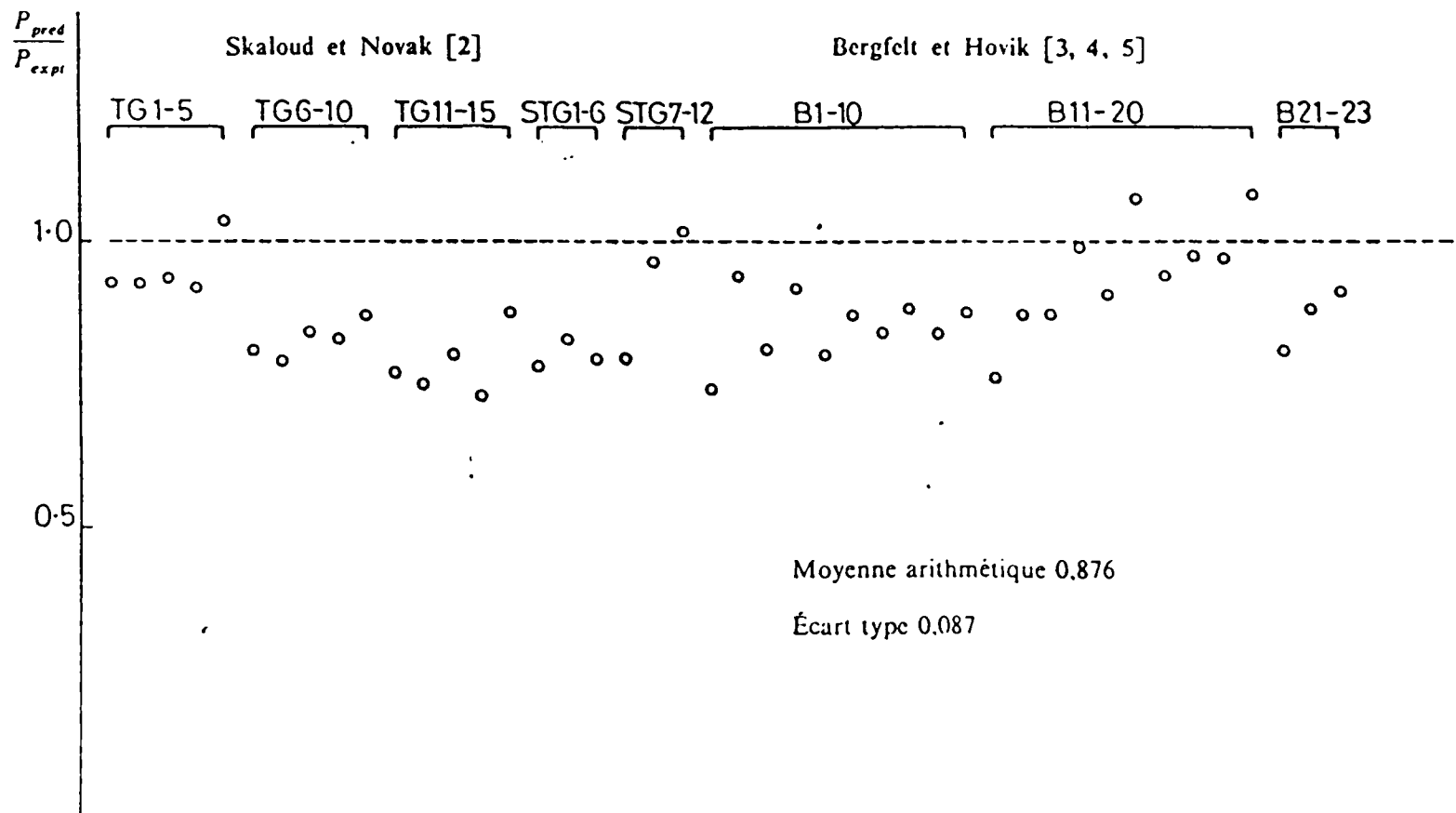


Figure 2.6 - Comparison of Equation 2.12 with Experimental Results. (after [ROBE78])

the flange dimensions that now proved to influence the ultimate load capacity despite his earlier conclusion about t_w .

$$P_f = 13 \eta t_i t_w \sigma_y \quad (2.7)$$

where $t_i = t_f \sqrt[4]{\frac{b_f}{25 t_f}}$, and η is a factor that varies with the ratio t_i/h . Another modification to equation 2.6 to include a similar range of parameters produced equation 2.8:

$$P_f = 0.6 t_w^2 \sqrt{E \sigma_y} (1 + 0.4 t_i / t_w) \quad (2.8)$$

This equation is valid for values of t_f/t_w between 1 and 2. He proposed a design method that used the smaller of the results of equations 2.7 and 2.8. Equation 2.7 controlled the design in situations where the flanges were weak and failure by yielding was likely to occur. On the other hand, when the flanges were stiff, equation 2.8 should be used, when failure would occur by web buckling. This procedure was developed with the aid of some experiments on beams with t_f/t_w varying between 1.5 and 5 and was proved to be satisfactory.

For a load distributed over a finite length rather than applied to a single point, he introduced a factor $f(c)$. This factor, based on the theory of a beam on an elastic foundation, is applied to the value of P_f found above.

$$f(c) = \frac{\gamma}{1 - e^{-\gamma} \cos \gamma} \quad ; \quad \gamma = \frac{c}{13.4 t_f} \quad (2.9)$$

where c is the distance over which the load is distributed. To deal with cases where a combination of yielding and buckling occurred he proposed the following formulae (equations 2.10.a and 2.10.b) as alternatives to equation 2.8.

$$P_f = 0.75 t_w^2 \sqrt{E \sigma_y t_f/t_w} \quad (2.10.a)$$

$$P_f = 0.68 t_w^2 \sqrt{E \sigma_y} (t_f/t_w)^{0.6} \quad (2.10.b)$$

The interaction between moment and patch load has also been studied and proved to be important only in cases where the ratio M/M_u was larger than 0.6. Formulae 2.11.a and 2.11.b were then suggested in order to model this interaction.

$$P/P_f = \sqrt{1 - (M/M_u)^2} \quad (2.11.a)$$

$$P/P_f = \sqrt[8]{1 - (M/M_u)^2} \quad (2.11.b)$$

where P and M are the patch load and moment present in the beam. M_u and P_f are the individual moment and patch load resistances respectively. The second of these formulae seemed to be more precise when compared to the series of tests performed.

Roberts & Rockey, [ROBE78], created a method to predict the ultimate load resistance based on the plastic hinge mechanism shown in figure 2.5, page 40. Using the theorem of virtual work they developed the following equation:

$$P_f = \frac{4 M_{fl}}{\beta} + \frac{4 \beta M_w}{\alpha \cos \theta} + \frac{2 c M_w}{\alpha \cos \theta} - \frac{2 \Omega M_w}{\alpha \cos \theta} \quad (2.12)$$

$$\text{in which } M_w = \frac{\sigma_y^w t_w^2}{4} \quad ; \quad M_{fl} = \frac{\sigma_y^{fl} b_f t_f^2}{4} \quad ; \quad \text{and}$$

$$\Omega = \frac{(4 \beta + 2 c) M_w}{\left[\frac{2 M_w}{\alpha \cos \theta} + \sigma_y^w t_w \right] \alpha \cos \theta}$$

α , β , c and θ are defined in figure 2.5, and σ_y^{fl} is the flange's yield stress.

This expression proved to be close to experimental results provided by Bergfelt & Hovik, [BERG68], and Skaloud & Novak, [SKAL71]. This can be seen in figure 2.6, page 40 where the test results for beams submitted to patch load with h/t_w varying between 150 and 400, are compared to the results of equation 2.12. They also concluded that b_f had little influence on the ultimate load, although this view was later discovered to be wrong, [ROBE81].

In 1979 Roberts & Rockey, [ROBE79], compared their method with the experiments performed by Skaloud & Novak, [SKAL72], Bergfelt & Hovik, [BERG76], Bagchi & Rockey, [BAGC75], and Drdacky & Novotny, [DRDA77]. Other calculation methods such as the ones developed by Granholm, [GRAN60], Bergfelt, [BERG76], Dubas & Gehri, [DUBA86a], Skaloud & Novak, [SKAL75], and Herzog, [HERZ74], were compared with the same four experimental data sets and the results are presented in figure 2.7, page 43. Apparently the authors' method proved to be the best, with the smallest coefficient of variation. This conclusion was later disproved following a modification of the expression for the calculation of the ultimate load, as can be seen in their later paper, [ROBE83].

Roberts, [ROBE81], made a new series of tests in order to study the influence of b_f , t_f and h on the ultimate load carrying capacity of the girder. They confirmed that P_f was proportional to the square of the thickness of the web and was, less strongly, proportional to the thickness of the flange. They also stated that the height of the web had little or no influence on the load P_f . They proposed a new design method that consisted in testing two equations and taking the smaller value. The first one was a modification of equation 2.12 and represented an ultimate limit state failure by buckling of the web:

$$P_f = 0.5 t_w^2 \sqrt{E \sigma_y^w t_f/t_w} [1 + (3 c/h) (t_w/t_f)^{3/2}] \quad (2.13)$$

This equation incorporated the influence of the length of the patch load by the presence of the variable c . They recommended that the value of c/h should be limited to 0.2. They also stated that this formula tended to underestimate the value of the collapse load of girders with very thin webs and flanges. In the view of this fact they established a minimum value of three to be used for t_f/t_w .

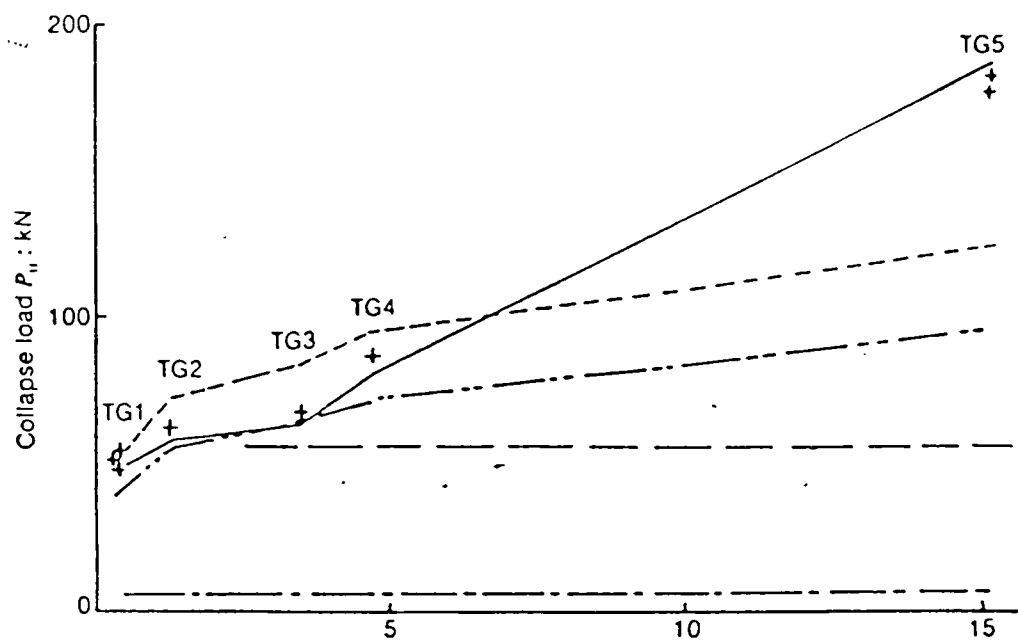


Figure 2.7 - Comparison of Equation 2.12 with Experimental Results.
(after [ROBE79])

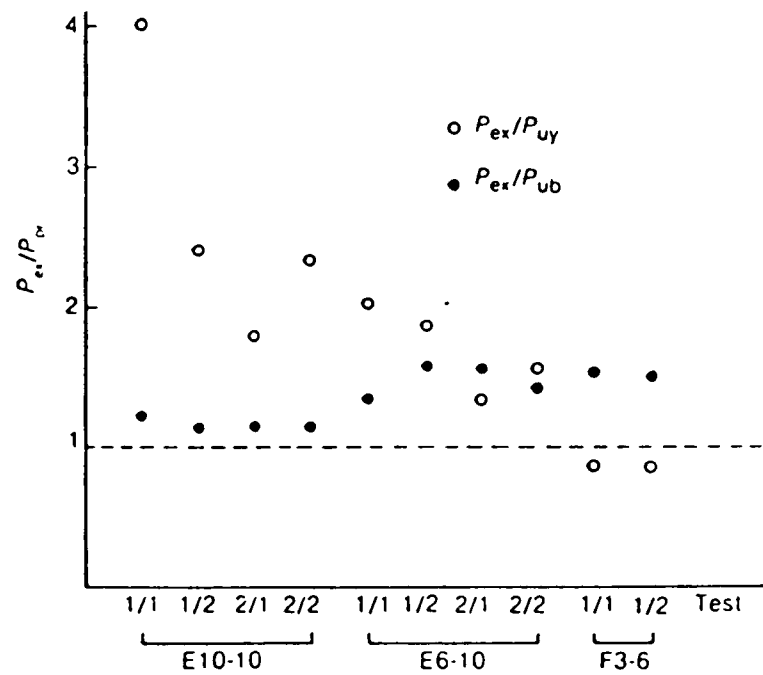


Figure 2.8.a - Comparison of Equation 2.13 with Experimental Results.
(after [ROBE83])

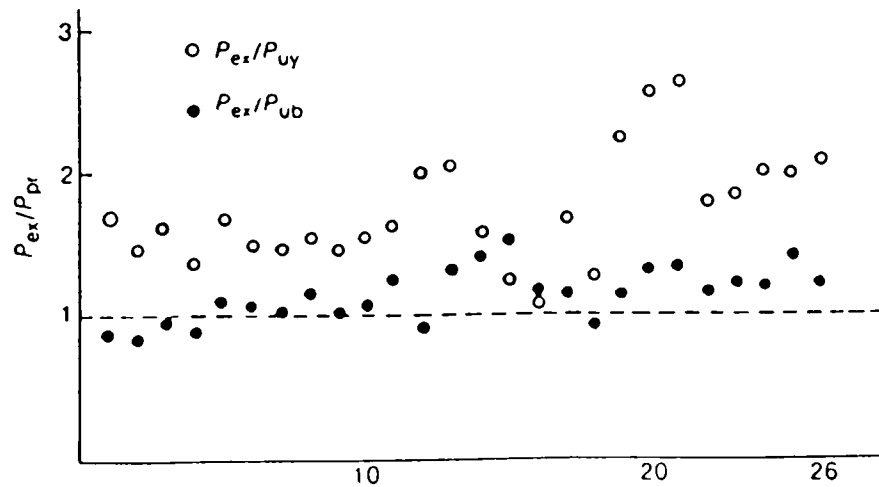


Figure 2.8.b - Comparison of Equation 2.15 with Experimental Results.
(after [ROBE83])

For girders with thicker webs, another limit state controlled the process of failure. In these cases a direct yielding of the web predominated and the mechanism of collapse changed. Using the theorem of the virtual work again gave equation 2.14:

$$P_f = (4 M_f / \beta) + \sigma_y^w t_w (\beta + c) \quad (2.14)$$

By a minimisation of P_f , the expression $\beta^2 = 4(M_f / \sigma_y^w) t_w$ was obtained. If this value is substituted in equation 2.14 the final expression 2.15 for the collapse load is achieved:

$$P_f = 4 \sqrt{M_f \sigma_y^w t_w} + \sigma_y^w t_w c \quad (2.15)$$

A comparison was made between this design method and experimental test results by Skaloud & Novak, [SKAL72], Bergfelt & Hovik, [BERG76], Drdacky & Novotny, [DRDA77], and Bergfelt, [BERG79]. The arithmetic mean for the values of P_{exp}/P_f was 1.42 with a standard deviation of 15.3% showing that the design procedure was conservative.

Another series of plate girder tests was made by Roberts, [ROBE83], and comprised webs with thicknesses that varied between 3 and 10 mm and h/t_w values that ranged between 50 and 166. These tests, together with some others performed by Dubas & Gehri, [DUBA86a], were then compared with Roberts' method and with the ones proposed by Granholm, [GRAN60], Bergfelt, [BERG76], Dubas & Gehri, [DUBA86a], and Skaloud & Drdacky, [SKAL75]. He claimed that the results of the last three were very similar to the results of equation 2.13. In the light of this fact, he only compared his method to the experimental results. The comparison can be seen in figures 2.8.a and 2.8.b, page 43. The arithmetic mean and the standard deviation were equal to the ones obtained in his previous work [ROBE81]. Lastly he concluded that his method was too conservative in respect of thicker beams. He suggested that instead of using the lesser of equations 2.13 and 2.15, equation 2.13 alone should be used in all situations. He explained that equation 2.15 represented only a serviceability limit for the beam when it was in the non-linear path with significant membrane yielding.

Roberts & Coric, [ROBE88], continued their work on the yield-line patch model and suggested another modification to equation 2.13. They pointed out that if a factor of 0.77 was used instead of the 0.5 present in that equation, the results would be closer to the experimental ones. They also investigated the influence of patch/moment interaction and recommended a formula very similar to equation 2.11.a.

Shimizu et al, [SHIM89], suggested another yield line model of the same type as that proposed by Roberts & Rockey, [ROBE78]. Their method comprised a mechanism composed of two yield lines in the web and four plastic hinges in the loaded flange. They also recommended a new expression for the prediction of the failure load, very similar to equation 2.12. They concluded that their expression was good when compared with the experimental data available, [SHIM87], but some new tests were necessary to give a reliable calibration to their design formula.

Skaloud & Novak, [SKAL71], studied the influence of the flange stiffness on the load carrying capacity of a beam subjected to patch load. They investigated static and cyclic loading conditions and concluded that the beams possessed a large post-buckling reserve of strength. They also arrived at the fact that the flange stiffness

had a big influence on the ultimate load. This influence could be noticed in the tests performed by the extra resistance which was achieved with stiffer flanges. On the same lines, they tested another series of specimens, [SKAL72], varying the thickness and the width of the flanges. They noticed that the flange stiffness changed the collapse pattern of the beam. For less stiff beams the failure was localized over the application point of the load and for the stiffer ones the collapse was spread all over the web panel in a more uniform way. They plotted the influence of this stiffness (figure 2.9, page 46) for two beams with different h/t_w ratios, where the ratio of P_f/P_{cr} is drawn against the flange stiffness factor $I_{ff}/a^3 t_w$. They stated that the post-buckling strength of the beams increased as h/t_w or I_{ff} increased.

Drdacky & Novotny, [DRDA77], [DRDA77a], tested some beams with h/t_w varying from 75 to 200 in order to complement the other series of experiments made by Skaloud & Drdacky, [SKAL75] that ranged from 200 to 400. The aim of both series was to clarify a procedure for the prediction of the load capacity of a beam with a patch load condition. They found that the collapse occurred in the majority of the cases in the upper third of the girder depth, near to the loaded flange. To predict the ultimate capacity they proposed a design formula, equation 2.16, based on tests:

$$P_f = 0.55 t_w^2 (0.9 + 1.5 c/h) \sqrt{E \sigma_y t_f/t_w} \quad (2.16)$$

The maximum and minimum values for the ratio P_{exp}/P_f were found to be 1.52 and 1.07 respectively. From these results and the observation that the buckling pattern of the web had not changed for beams with low h/t_w ratios they concluded that equation 2.16 was valid for a range of h/t_w from 75 to 400.

In 1986, [DRDA86], Drdacky described another series of experiments with approximately 170 tests in order to look into the mechanism of collapse of beams subjected to patch load. He discovered that a smooth transition occurred between the stages of pre - and post-buckling and proposed another expression to predict the failure load:

$$P_f = 19.54 t_w^2 \sigma_y^w (1 + 0.004 c/t_w) (I_{ff} / t_w^4)^{0.1} \quad (2.17)$$

where I_{ff} is the moment of inertia of the flange. The performance of this equation is presented in figure 2.10, page 46, where the ratio P_{exp}/P_f is plotted against h/t_w . The results proved to be very reasonable. Note the high concentration of points in the region near the horizontal line $P_{exp}/P_f = 1$.

Moriwaki et al, [MORI83], suggested another formula for the failure load based on a series of tests performed on I-section beams and box girders:

$$P_f = P_{cr} + P_w + P_{ff} \quad (2.18)$$

Their expression is composed of three separate parts. The first one, P_{cr} , is the elastic buckling strength of the web. The second, P_w , is the post-buckling strength of the web and the third, P_{ff} , is the flange strength. This formula was a modification of equation 2.12, and its philosophy was still based on the plastic collapse mechanism of the beam. A complete flow chart explaining the several steps to calculate P_{cr} , P_w and P_{ff} is presented in the paper, but it is too complicated to use. The arithmetic mean

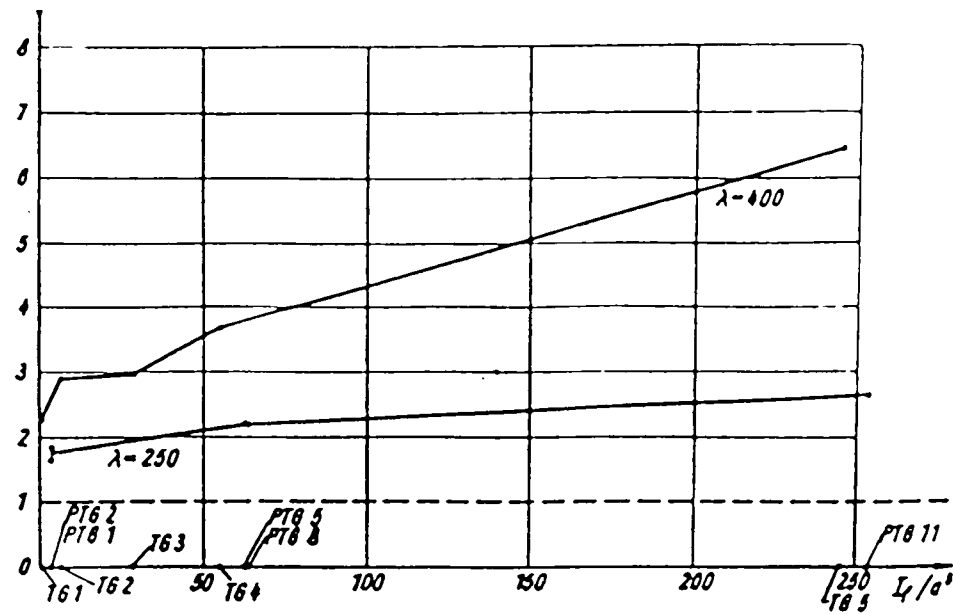


Figure 2.9 - Influence of the Flange Stiffness on the Failure Load.
(after [SKAL71])

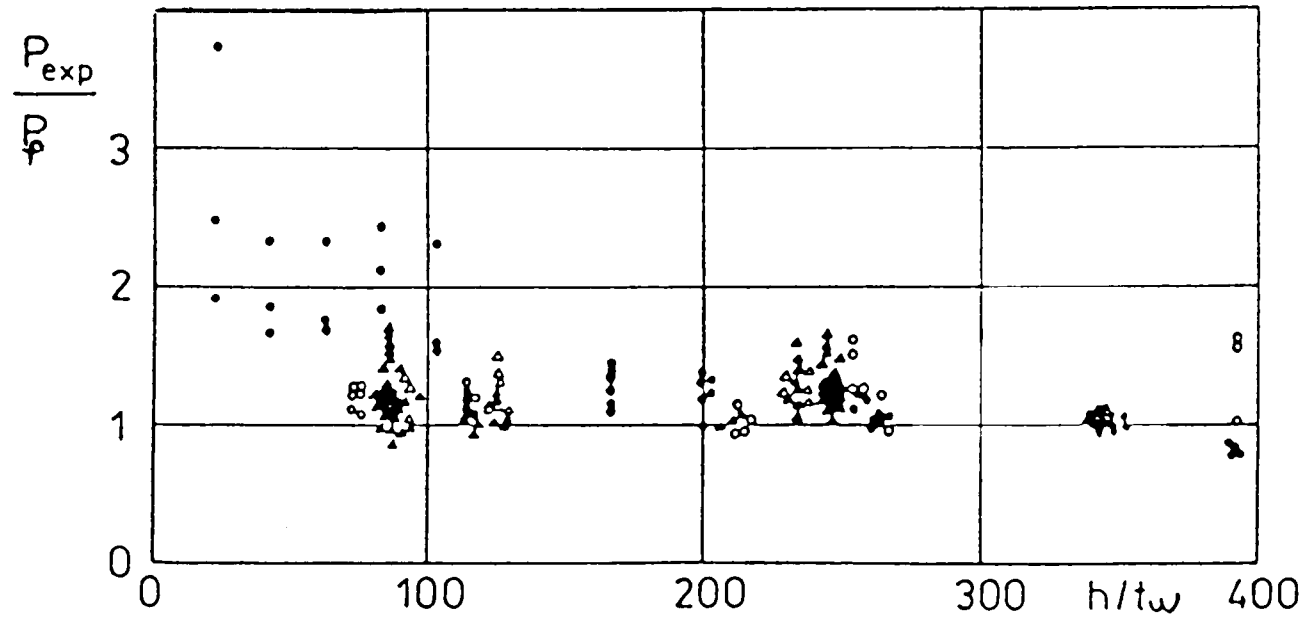


Figure 2.10 - Comparison of Equation 2.17 with Experimental Results.
(after [DRDA86])

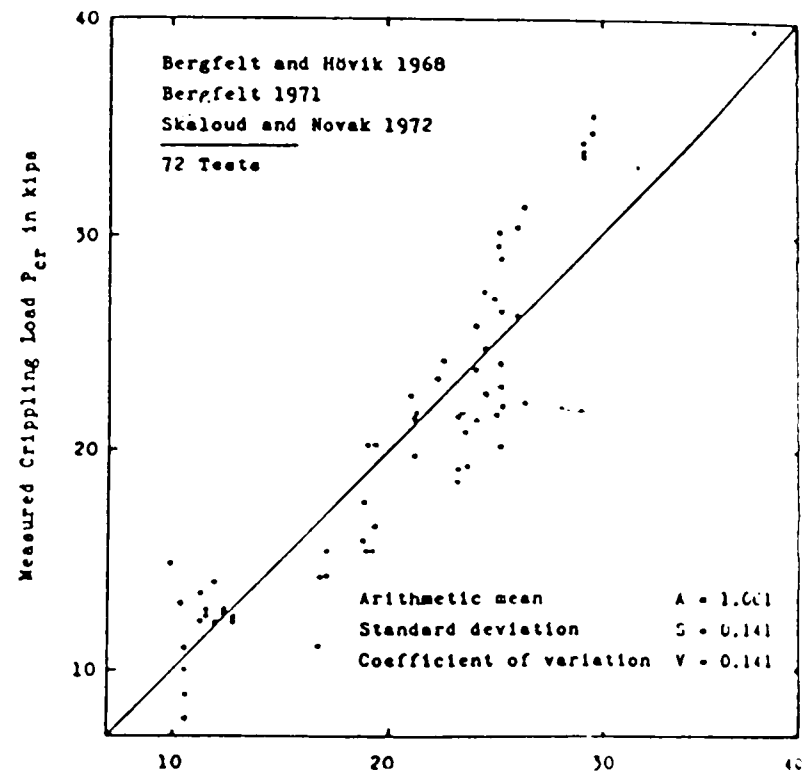


Figure 2.11 - Comparison of Equation 2.24 with Experimental Results.
(after [HERZ74])

value of P_{exp}/P_f was found to be equal to one and the standard deviation to be equal to 9%. Another solution for the failure load P_f based on a collapse mechanism was suggested by Coric, [CORI86]:

$$P_f = \frac{4 M_{\Omega}}{\beta} + 2 M_w \beta \frac{\tan \theta}{1 + 1/z} + M_w c \left[\frac{\tan \theta (2 + 1/z)}{\alpha (1 + 1/z)} \right] + 2 M_w \gamma \frac{\tan \theta}{\alpha} \quad (2.19)$$

where:

$$\begin{aligned} M_{\Omega} &= 0.25 \sigma_y^{\Omega} b_f t_f^2 & ; & & M_w &= 0.25 \sigma_y^w t_w^2 & ; \\ \beta &= \left[\frac{2 M_{\Omega} \alpha (1 + 1/z)}{M_w \tan \theta} \right] & ; & & \alpha &= \frac{c}{1.25 t_w} & ; \\ \gamma &= \frac{t' \sigma'}{10 t_w \sigma_y^w} & ; & & z &= \gamma/h (0.6 h/\gamma - 6.5 \gamma/h) & ; \end{aligned}$$

$t' = 2.0 \text{ mm}^2$; $\sigma' = 300 \text{ N/mm}^2$; and θ could be found with the use of the interactive equation 2.20.

$$\left[\frac{1}{\sin \theta} + \frac{z}{\cos \left[\frac{90^\circ - \theta}{z} \right]} \right] - (1 + z) = \frac{M_{\Omega}^2}{E M_w b_f t_f^3} (1 + 1/z) (1/\tan \theta) \quad (2.20)$$

The arithmetic mean and the standard deviation values for the ratio P_{exp}/P_f were 1.42 and 17.8% respectively. He produced a new set of experiments, [CORI91], for beams with long spans and narrow flanges and concluded that his method was suitable to model the phenomenon quantitatively and qualitatively. However, this method is, in the present author's view, probably too complicated to use in daily practice.

Dubas, [DUBA86], [DUBA88], investigated the interaction between patch load and moment resistance with the aid of equations 2.11.a and 2.11.b. He pointed out that the two approaches gave very different results. He also reported briefly the advantages of using longitudinal stiffeners in the girder and stated that they could only be effective if they were placed at a distance no greater than $80 t_w$ from the loaded flange.

Dubas & Tschampeer, [DUBA90], conducted a large number of tests in order to study further the patch/moment interaction. They made some tests with only patch load, some with only bending moment and some with both present. They arrived at a formula to calculate the ultimate resistance to patch load based on a mechanism solution calibrated against the tests:

$$P_f = 0.11 t_w^2 \sqrt{E \sigma_y^w \frac{(c + 2t_f)}{t_w}} + \sqrt{\frac{b_f t_f}{100 t_w^2}} \sqrt[3]{E \sigma_y^{\Omega} t_w^5 (c + 2t_f)} \quad (2.21)$$

This equation consists of two independent terms. The first represents the web's contribution and the second the influence of the flange in the total patch load resistance of the beam. To take into account the patch/moment interaction they recommended equation 2.22:

$$\left[\frac{1.2P}{P_f} \right]^2 + \left[\frac{M}{M_f} \right]^2 \leq 1.15 \quad (2.22)$$

Herzog, [HERZ74a], studied the influence of patch load in cold-formed steel sections and found that the design formula presented in the German Standard for cold-formed construction, [DINN64], equation 2.23, was very inaccurate.

$$P_f = 7.03t_w^2 (3050 + 23c/t_w - 0.09ch/t_w^2 - 5h/t_w) \sigma_y/1.4 \quad (2.23)$$

When compared with experimental results, the ratio P_f/P_{exp} for this equation ranged from 0.49 to 7.37! In view of this fact he proposed a new formula, equation 2.24, that when compared with the results of the tests gave, for the ratio P_f/P_{exp} , an arithmetic mean and a standard deviation equal to 0.997 and 8.2% respectively.

$$P_f = 15t_w^2 (1 + c/a)^2 (1 - L/100h) (1 - \sigma_b/\sigma_y) \sin\phi \quad (2.24)$$

In this equation, L represents the free span of the beam, c/a the loading length ratio, σ_b the longitudinal bending stress, and ϕ the angle of inclination of the web to the vertical. He also investigated the patch load behaviour in plate girders and rolled beams, [HERZ74b]. He suggested an empirical formula based on some test observations, equation 2.25, to predict the failure load of the beam:

$$P_f = 20\sigma_y t_w^2 \sqrt{(t_f/t_w) (0.4 + h/a)} \quad (2.25)$$

This equation was valid up to $\sqrt{(0.4 + h/a)} = 1$. In subsequent work, he improved equation 2.25 arriving at the more accurate equation 2.26:

$$P_f = 981t_w^2 [1.2 + 1.25 I_{ff}/I_w h/t_w (1 + c/h)^2 (0.85a/100h)] \sqrt[8]{1 - (\sigma_b/\sigma_y)^2} \quad (2.26)$$

This equation was based on experimental tests that ranged from $150 < h/t_w < 400$. The performance of this formula can be easily seen in figure 2.11 where its results are compared with experimental data from [BERG71], [BERG68], and [SKAL72]. Finally, [HERZ86], he arrived at the latest version of his design formula for beams subjected to patch load, equation 2.27:

$$P_f = 1000t_w^2 [1.2 + 1.25 I_{ff}/I_w h/t_w (1 + c/h)^2 (0.85a/100h)] \sqrt[8]{1 - (\sigma_b/\sigma_y)^2} \quad (2.27)$$

Roberts & Chong, [ROBE81a], investigated the effect of a uniform distributed edge load on the resistance of plate girders. They developed a new form of mechanism in order to try to model the failure behaviour of that kind of structure. Again utilizing the theorem of virtual work they derived equation 2.28:

$$P_f = \frac{8 M_{\eta}}{\beta} + \frac{4 \beta M_w}{\alpha \cos\theta} - \sqrt[8]{1 - \left[\frac{4 M_w}{\alpha \cos\theta \sigma_y^w t_w} \right]^2} \quad (2.28)$$

$$\text{where:} \quad \sin\theta = 1 - \frac{M_f \beta^2}{12 E \alpha I_f} \quad \text{and} \quad \alpha = 25 t_w \leq h/2$$

They suggested another expression, equation 2.29, in order to cover the case where the failure occurred by excessive yielding of the web.

$$P_f = 8 M_f / \beta + 2 \beta \sigma_y^w t_w \quad (2.29)$$

The verification of the accuracy of these formulae was attempted by a comparison with some tests done by Bossert & Ostapenko, [BOSS67], in Lehigh university. It was noted that the results of equations 2.28 and 2.29 were very high when compared to the experimental ones, giving poor credibility to the proposed method.

In a discussion of a paper presented by Novak & Skaloud, [NOVA73], Bergfelt presented some interesting remarks on the influence of the type of steel on the failure load of beams subjected to patch load. He had discovered that the failure load was proportional to $(\sigma_y)^{2/3}$ or $(\sigma_y)^{1/2}$, instead of the simple value of σ_y initially expected. He also stated that the flange thickness should be corrected to

$t_i = t_f \sqrt[4]{b_f / (25 t_f)}$ in order to adjust the experimental formula 2.8 proposed in 1976, [BERG76].

In 1979 Bergfelt, [BERG79], continued his work on patch load phenomena and looked into the influence of horizontal and vertical stiffeners in the failure load of plate girders. He developed a three hinge failure theory in order to predict, more consistently, the load carrying capacity of beams under that kind of loading. For very thin webs the results are described by equation 2.30 and for normal webs, by equation 2.31:

$$P_f = 10 t_f t_w \sqrt{t_f / t_w \sigma_y^f \sigma_y^w} \quad (2.30)$$

$$P_f = K_p t_w^2 \sqrt{E \sigma_y^w} ; \quad K_p = \sqrt{\frac{1 + \sqrt{1 + k}}{2}} \quad (2.31)$$

where $k = 7.51 \times 10^{-4} \frac{1}{\eta^2} (t_f / t_w)^6$ and η is a parameter that varies with the ratio t_f / t_w . The variable K assumes a value of one when t_f / t_w approaches zero and a maximum of 1.405 when t_f / t_w is equal to six or more.

He also continued his study of the influence of the quality of the steel on the load capacity as shown in figure 2.12, page 50. By a new series of tests (described in this graph) he confirmed his earlier hypothesis that the increase of load was proportional to $\sqrt{E \sigma_y^w}$. The influence of the spacing of vertical stiffeners was also investigated by Bergfelt and his conclusions can be seen in figure 2.13, page 50. He pointed out that for beams that possessed thicker flanges, the increase in the failure load varied between 10 and 20%. On the other hand, for girders with thin flanges, the

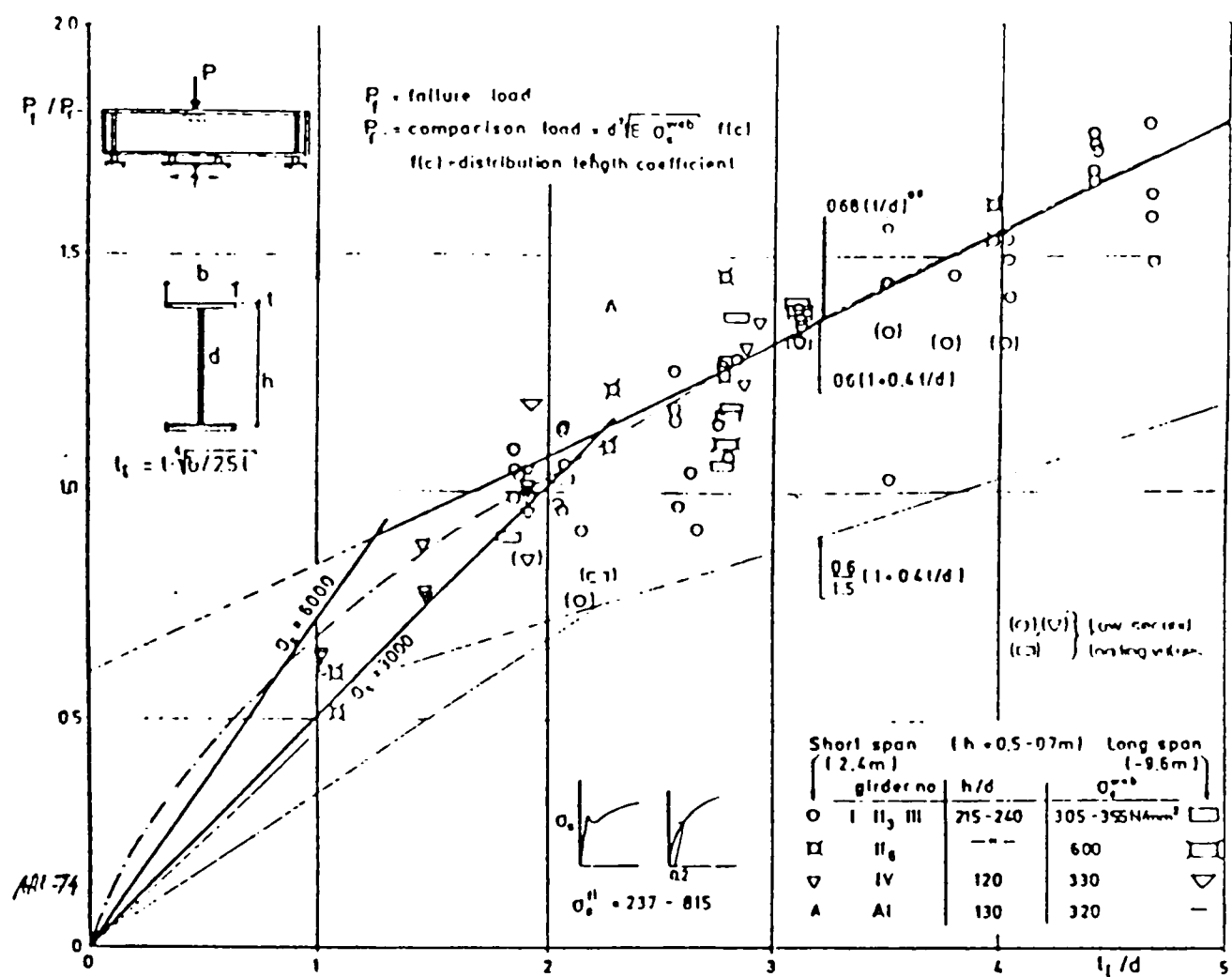


Figure 2.12 - Influence of the Steel Quality on the Failure Load. (after [BERG79])

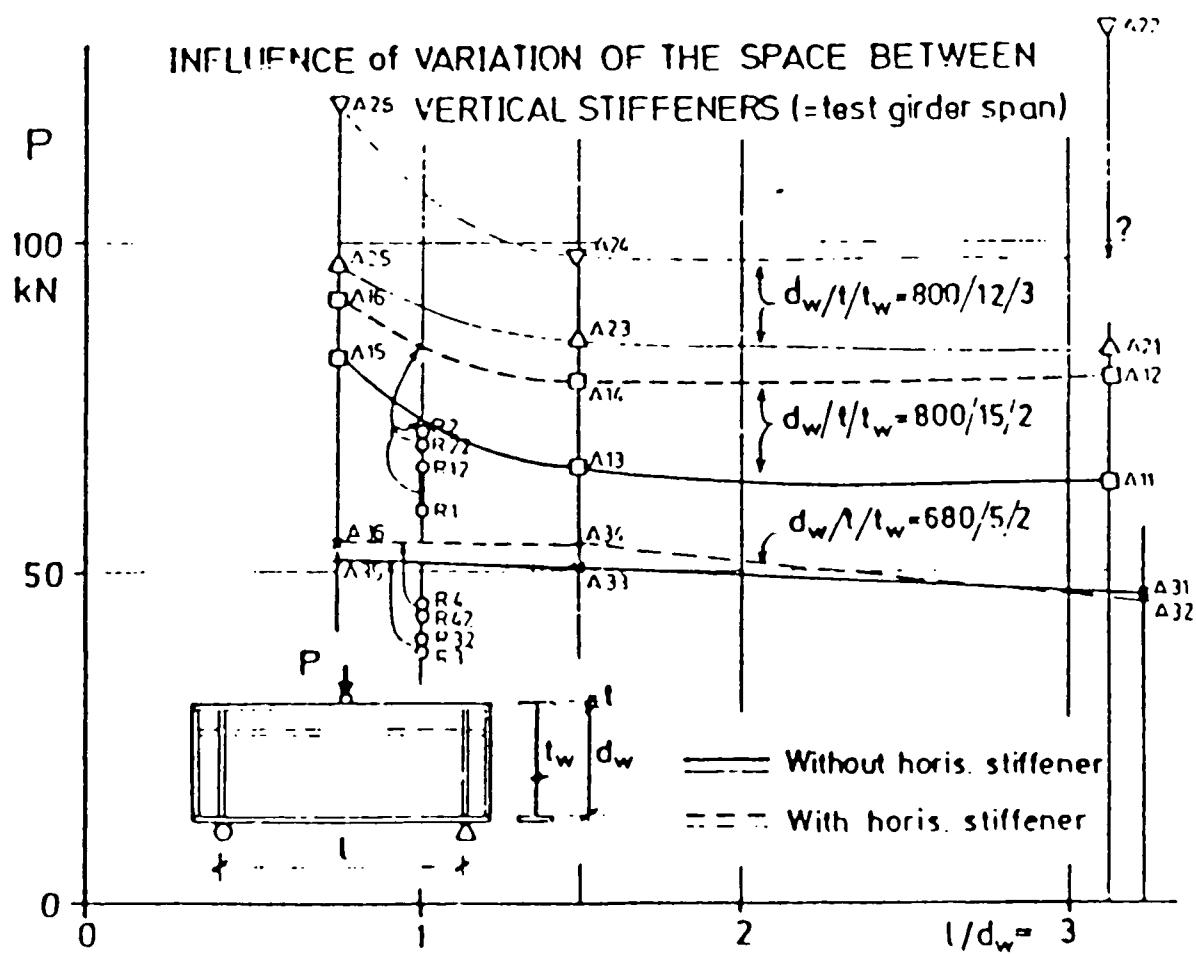


Figure 2.13 - Influence of the Variation of the Space Between Vertical Stiffeners on the Failure Load. (after [BERG79])

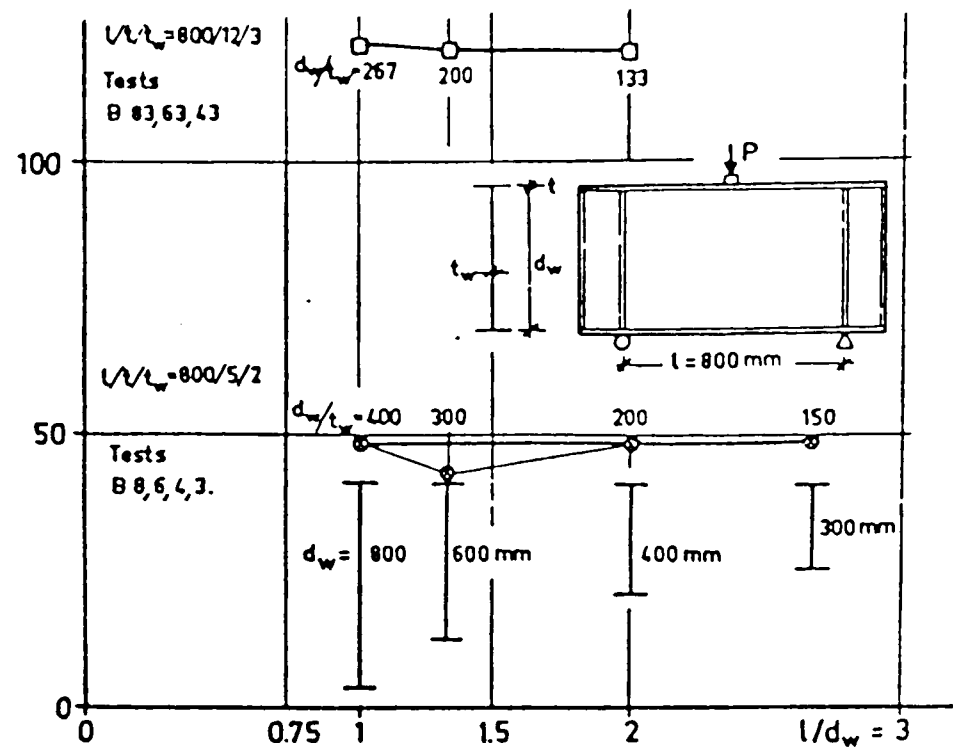


Figure 2.14 - Influence of the Height of the Beam on the Failure Load. (after [BERG79])

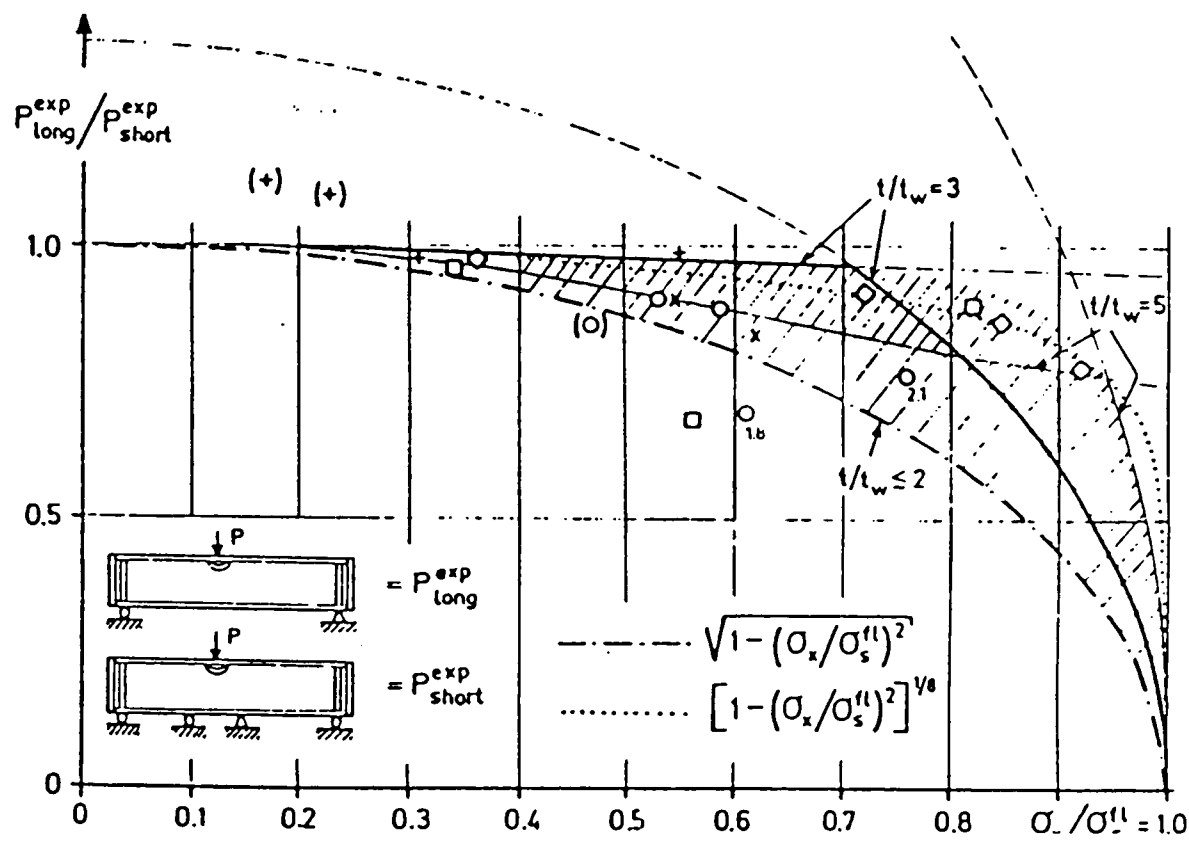


Figure 2.15 - Interaction Patch/Bending. (after [BERG79])

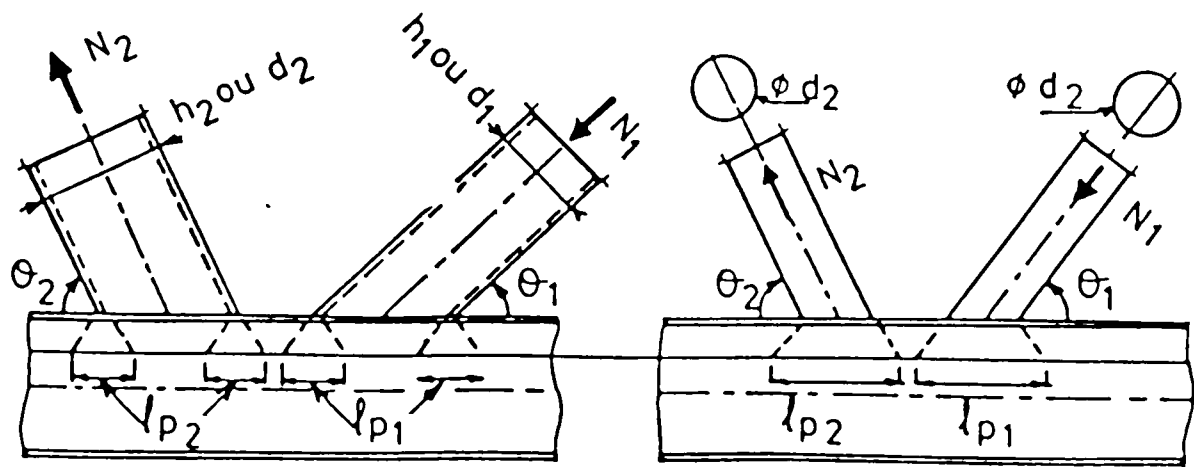


Figure 2.16 - Loading Configuration. (after [ARIB88])

increase in the load capacity was only about 7%. The influence of the height of the web on the failure capacity was another topic that attracted Bergfelt's attention. The results of that analysis are presented in figure 2.14, page 51. An increase of web depth produces no significant change in the ultimate load P_f . The interaction of patch load and bending was also studied and the conclusions are displayed in figure 2.15, page 51. Equations 2.11.a and 2.11.b are plotted together with the results of several tests, proving that 2.11.b is the more accurate.

He mentioned that the limitation of the web area that may buckle and the support given to the loaded flange were the main advantages of reducing the space between vertical stiffeners. In respect to the advantages of using longitudinal stiffeners he pointed out that the longitudinal stiffener makes the portion of the web between its position and the loaded flange straighter. It also changes the possible buckling pattern and, indeed, prohibits buckles in its neighbourhood. Lastly it can be viewed as a reduction in the web's depth. Based on the test results he proposed a different design formula, equation 2.32:

$$P_f = 0.8 t_w^2 \sqrt{E \sigma_y^w (t_i/t_w)} f^* \quad (2.32)$$

$$\text{where } t_i = t_f \sqrt[4]{b_f/(25t_f)} \quad \text{and } f^* = f_c f_h f_{\sigma_y^w} f_{\sigma_b} f_{\delta} f_{vs} f_{hs}$$

where the influence of the portion of the length that was effectively loaded, c , on the ultimate load of the beam is described by equation 2.33.a:

$$f_c = 1 + \frac{40 c t_w}{a h} \quad (2.33.a)$$

The influence of the web's depth was, in the majority of cases, equal to one, but for beams in which the ratio h/t_w falls between 200 and 400 it can be calculated with the aid of equation 2.33.b:

$$f_h = \sqrt{\frac{350 t_w}{h}} \quad (2.33.b)$$

The influence of the yield stress of the web σ_y^w is taken into account by equation 2.33.c:

$$f_{\sigma_y^w} = \sqrt[6]{\frac{700 \sigma_y^w}{E}} \quad (2.33.c)$$

The reduction in the failure load, due to the presence of bending stresses, f_{σ_b} , is expressed by equation 2.33.d, which is a direct utilization of the more accurate formula 2.11.b already mentioned.

$$f_{\sigma_b} = \sqrt[8]{1 - \left(\frac{\sigma_b}{\sigma_y^n} \right)^2} \quad (2.33.d)$$

The initial web imperfection was found to be an important parameter for beams that possessed a ratio t_f/t_w greater than two. The behaviour of a beam subjected to patch load could be interpreted by a column model in which the imperfections played a big role in the total reduction of the failure load. Some of the important parameters were identified, such as the magnitude of the initial deformation, the slope of the deformation along the structural element, the slenderness of the element, and the relative flange to web stiffness. This study showed a significant decrease in the load carrying capacity of the beam as the initial imperfection increased. To model this behaviour equation 2.33 was developed:

$$f_\delta = 1 - 15(\delta_i - 0.001 t_f h/t_w)/h ; \quad 0.8 \leq f_\delta \leq 1. \quad (2.33.e)$$

The contribution of a change in the spacing of vertical stiffeners, f_{vs} is described by equation 2.33.f:

$$f_{vs} = \frac{2a}{L} \sqrt{\frac{1 + \sqrt{1 + K (L/2a)^4}}{1 + \sqrt{1 + K}}} \quad (2.33.f)$$

where K is given by equation 2.31. Finally the influence of longitudinal stiffeners on the load capacity of the beam is expressed by equation 2.33.g:

$$f_{hs} \cong 1 + (1/3 - s/h) \sqrt{\frac{a}{3s}} \quad (2.33.g)$$

One of the interesting points of the publication was the clear distinction suggested by Bergfelt between the web crippling theory that he proposed and the folded web theory developed by Roberts & Rockey, [ROBE79]. He stated that in the web crippling theory, yield hinges form in the flange due to a negative moment on both sides of the load. This is followed by either yielding or a buckle of the web. In the folded web theory the collapse load is determined by the more advantageous flange-hinge distance determined by the application of the theorem of virtual work.

In a subsequent paper, [BERG83], Bergfelt confirmed that the height of the beam, h , had little influence on the load P_f for girders with the ratio h/t_w greater than 150. He stated that for the beams analyzed that possessed a ratio b_f/t_f greater than 12, the width of the flange b_f played only a minor role in the failure load P_f . He studied the effects of horizontal stiffeners on the load capacity of the beam and concluded that the major advantage of their use was that they modified the buckling pattern of the web as already stated in his previous work, [BERG79]. The increase in the load P_f varied between 6 to 21% and the best results happened for the case where the distance between the stiffener and the loaded flange was equal to 0.2. He also investigated the use of a thinner plate in the lower part of the web and found that it did not affect the load resistance of the structural member.

He compared the results of his design equation 2.32 with the experimental ones obtained by the test series and noticed that the vast majority of the points fell into the range of $P_f(1 \pm 0.1)$. Finally he concluded that an increase in the length of the loading c will definitely increase the ultimate load P_f . Although the factor f_c , equation 2.33.a, showed an increase of the load of 1 to 4%, the experimental tests

produced some more optimistic results that varied between 2 to 15% improvement in P_f .

Karnikova & Skaloud, [KARN86], [KARN86a], and Janus et al, [JANU88], also looked into the use of longitudinal stiffeners. They concluded that for beams with the ratio s/h less or equal to 0.25, the load P_f was strongly affected. It increased by up to 20% when s/h was equal to 0.1. They made several tests with different aspect ratios $\alpha = a/h$, but discovered that this factor, along with the use of a single or double side longitudinal stiffener, had little influence on the failure load. The size of the stiffener was also investigated and showed little or no influence on the ultimate load of the beam.

Kristek & Skaloud, [KRIS91], described a series of tests to assess the increase in patch load resistance due to the use of stiffeners. Longitudinal stiffeners were found to increase the resistance by up to 85% of the unstiffened strength. Corner stiffened flanges increased the load carrying capacity by 35%

Galea et al, [GALE87], investigated the beneficial aspects of the use of longitudinal stiffeners on the ultimate patch load capacity. They made a series of experiments combining different positions of the stiffener with and without the presence of a bending moment. It was concluded that the presence of stiffeners increased the load capacity by up to 37% and that the position of the stiffener had little influence on the ultimate load. The results were compared with the ones developed by Bergfelt, [BERG79], and Karnikova, [KARN86], with a good agreement. They also stated that even small bending moments had a great influence on the failure load in direct contradiction to Bergfelt, Roberts and others.

Elgaaly & Nunan, [ELGA89], studied the influence of small eccentricities of patch load in relation to the plane of the web. They tested 22 beams with h/t_w ratios equal to fifty and load eccentricities lower than 1/8 of the total flange width. They also developed a linear finite element model that produced good results in the initial loading stages. They applied the load by two means. The first used a thick patch plate and the second a cylindrical bar. With the thick patch plate the difference in the ultimate load was found to be negligible. This was due to the fact that as the test progressed the flange rotated and made the eccentricities equal to zero. The cylinder however, produced a load decrease of up to a half of the capacity of a beam without eccentricities.

Aribert & Lachal, [ARIB77], studied the patch load phenomenon in rolled members as a particular case of a beam-column connection. They identified the patch load as the compressive force of one of the beam's flanges and the column as the structural member that resists its action. They began their investigation by an analysis of the existing design formulae in the French code, equation 2.34:

$$P_f = \sigma_y t_w (t_f + 5K) ; \quad K = t_f + r \quad (2.34)$$

in which r is the root radius that exists in rolled beams between the flanges and the web. They developed a method that was based on an elasto-plastic model of a beam and subsequently calibrated it with some experimental tests. The results of that study produced equations: 2.35.a, 2.35.b and 2.35.c, that represents the elastic, the plastic and the failure load:

$$P_e = 2.3 K t_w \sigma_y \quad (2.35.a)$$

$$P_{pl} = 5.0 K t_w \sigma_y \quad (2.35.b)$$

$$P_f = 7.0 K t_w \sigma_y \quad (2.35.c)$$

In a subsequent paper, [ARIB81], they refined their initial model by using a method based on plastic hinge formation in the flanges. They also studied the influence of a transverse stiffener located at a distance e from the load application point on the load capacity of the structure. Their results were developed with the support of experimental data and produced equations 2.36.a, 2.36.b and 2.36.c:

$$P_e = 2.3 K t_w \sigma_y^w + 0.5 (1 - 0.5 e/K) A_{st} \sigma_y^{st} \quad (2.36.a)$$

with $(1 - 0.5 e/K) > 0$.

$$P_{pl} = 5.0 K t_w \sigma_y^w + 0.5 (1 - 0.5 e/K) A_{st} \sigma_y^{st} \quad (2.36.b)$$

with $(1 - 0.5 e/K) > 0$.

$$P_f = 7.0 K t_w \sigma_y^w + 0.5 (1 - 0.5 e/K) A_{st} \sigma_y^{st} \quad (2.36.c)$$

with $(1 - 0.5 e/K) > 0$.

in which A_{st} and σ_y^{st} are the area and the yield stress of the stiffener. They proceeded with their studies in this field and looked into other aspects that were related to those previously considered, [ARIB88]. One of these situations was the one shown in figure 2.16, page 51 where the contact between the beam and the column is not perpendicular. They modified equation 2.35.b and arrived at the plastic load of the member, equation 2.37:

$$P_{pl} = \frac{L_p t_w \sigma_y}{\sin \theta} \quad (2.37)$$

where L_p is the lesser of L_{p1} and L_{p2} :

$$L_{p1} = 2[t_{fb} + 5(t_f + r)] \quad ; \quad L_{p2} = \frac{h_b}{\sin \theta_1} + 5(t_f + r)$$

Recently Aribert et al, [ARIB90], developed another approach to tackle the patch load on two sides present in the majority of column/beam connections. The method was based in a Von Karman type formulation present in the Eurocode 3, [EURO91a], and was calibrated against a range of 59 tests. The results are described in equation 2.38 below:

$$P_f = P_{cy} \left[\frac{1}{\lambda} \left[1 - \frac{0.22}{\lambda} \right] \right] \leq P_{cy} \quad (2.38)$$

$$\text{where:} \quad \lambda = \sqrt{\frac{P_{cr}}{P_{cy}}} ; \quad P_{cy} = \sigma_y^w t_w (c + 2t_c + 5K) ;$$

$$P_{cr} = \frac{\pi E t_w^3}{3 h (1 - \nu^2)}$$

and t_e is the thickness of the plate that applies the patch load. This formula was compared with Roberts approach, [ROBE81], and the one present in the Eurocode 3, [EURO91a], with better results in terms of agreement with experimental data.

In the area of cold-formed members subjected to patch load it is appropriate to mention the works of Rockey et al, [ROCK72], Elgaaly, [ELGA73], [ELGA75], Hetrakul & Yu, [HETR79], and Santaputra et al, [SANT89]. Rockey et al, [ROCK72], made a series of experiments to study the influence of the ratios a/h , h/t_w and c/a on the failure load P_f . Elgaaly, [ELGA73], [ELGA75], investigated the influence of the ratio h/t_w on the load P_f in cases where some moment was present. He recommended the interaction formula, equation 2.39:

$$\left[\frac{P}{P_f} \right]^3 + \left[\frac{M}{M_f} \right]^3 \leq 1.0 \quad (2.39)$$

He also looked at the interaction of patch load and shear as seen in figure 2.17, page 57 and suggested equation 2.40:

$$\sqrt[8]{\frac{P}{P_f}} + \sqrt[8]{\frac{Q}{Q_f}} \leq 1.0 \quad (2.40)$$

where Q and Q_f represent the applied shear and the individual shear resistance.

Hetrakul & Yu, [HETR79], also investigated the moment/patch load interaction mentioned previously. They suggested a formula, equation 2.41, to predict the failure load P_f of the beam when subjected to patch load only:

$$P_f = t_w^2 \sigma_y [1.49 - 0.53(\sigma_y/33)] [0.88 + 0.12(t_w/0.075)] (15 + 3.25 \sqrt{c/t_w}) \quad (2.41)$$

For the bending/patch load interaction they suggested two design equations, 2.42.a and 2.42.b. These equations were also compared with test data with a reasonable agreement.

$$0.56(P/P_f) + (M/M_f) \leq 1.38 \quad ; \quad \text{for } h/t_w \leq 400 \quad (2.42.a)$$

$$0.61(P/P_f) + (M/M_f) \leq 1.18 \quad ; \quad \text{for } h/t_w > 400 \quad (2.42.b)$$

Santaputra et al, [SANT89], developed a series of empirical formulae to model the behaviour of hat and I cold-formed sections subjected to patch load. They also recommended a new design formula to simulate the patch/moment interaction. The formulae were based on a large series of tests performed at Missouri-Rolla University and revealed that the requirements of the current design codes are not valid for members with yield stress greater than 552 MPa. Two modes of failure controlled the behaviour under patch load. They were bearing and buckling modes of failure and were represented by distinct design equations.

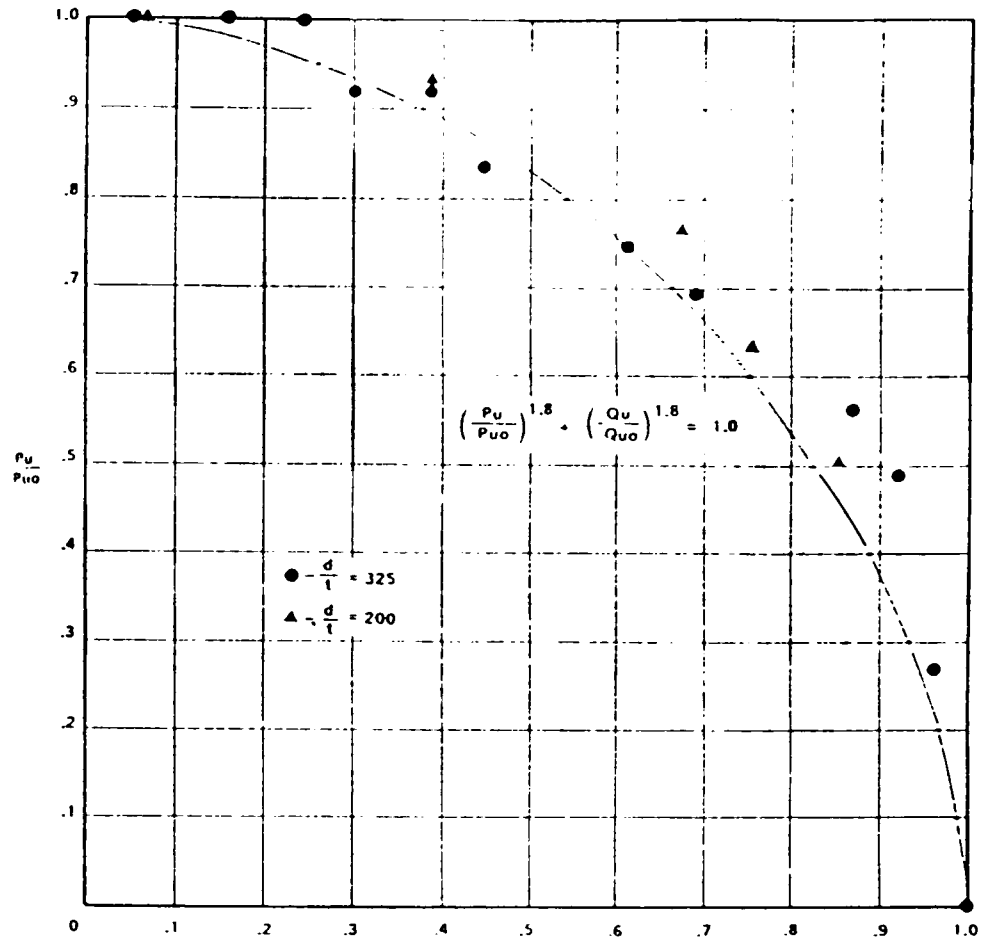


Figure 2.17 - Interaction Patch/Shear. (after [ELGA75])

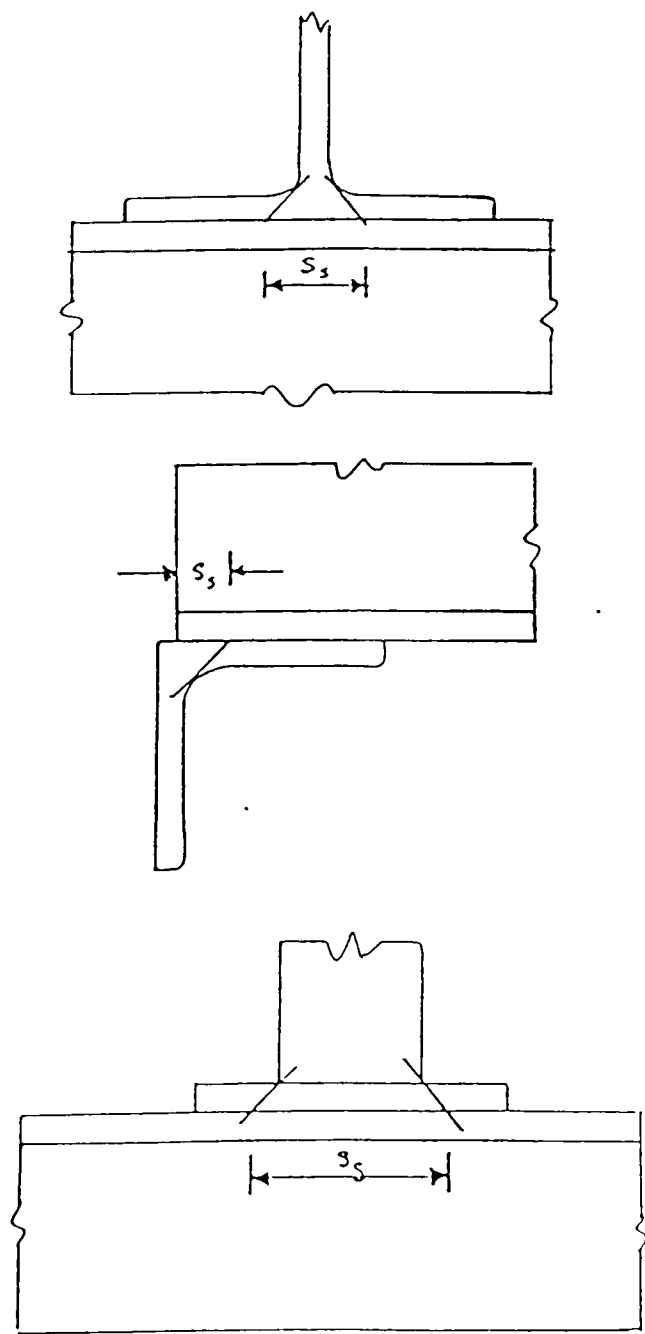


Figure 2.18.a - Values of S_s for Three Different Configurations. (after [EURO88])

2.5.4. Design Standards and State of the Art Reviews.

An interesting approach to the problem of patch load is presented in the design standard of Czechoslovakia, [AISC82]. In this standard, the resistance to patch load is considered to be a limit state and its ultimate value P_f is based on an investigation that took into account effects such as initial imperfections and residual stresses. It also considers the post-buckling strength provided by slender webs as can be seen in equation 2.43:

$$P_f = \phi_f A_p \sigma_y / \gamma_m \quad (2.43)$$

where: $\gamma_m = 1.1$; $\phi_f = (1.46 + 24.4 c/h) \eta \frac{\sqrt{t_f t_w}}{c} \sqrt{1 - \sigma_b \gamma_m / \sigma_y}$

$$\eta = \sqrt{\frac{240}{\sigma_y}} \quad \text{and} \quad A_p = c t_w$$

In the German standard, [DASE88], the ultimate resistance of unstiffened webs subjected to patch load is described by equation 2.44. The interaction between patch load, bending moment and shear force is taken into account by equation 2.45:

$$P_f = 20 \sigma_y t_w \sqrt{\frac{t_f}{t_w} (0.4 + h/a)} \quad (2.44)$$

for $Q \leq Q_f$ then:

$$(P/P_f) \leq 10/3 (1 - M/M_f) \quad (2.45)$$

In the British standard BS5950, [BRIT85], the resistance to patch load is expressed by equation 2.46:

$$P_f = p_c t_w (c + h) \quad (2.46)$$

in which p_c is the compressive strength resistance. This compressive resistance is calculated based on the value of the yield stress σ_y and the web slenderness $\lambda = 2.5 h/t_w$. In the British design standard for steel bridges BS5400, [BRIT78], two different methods are suggested. The first is a "simplified" one and is described by equation 2.47:

$$\sigma_c \cong \frac{P}{c + h} \leq \frac{3 \sigma_y^w t_w}{\sqrt{c h}} \sqrt{\frac{355}{\sigma_y^w}} \quad (2.47)$$

The second approach, which is presented in the appendix D, is more elaborate, equations 2.48.a and 2.48.b:

$$\text{if } t_f/t_w \leq 1.2 \sqrt{\frac{\sigma_y^{w'} (c + 2j)}{b_f \sigma_y^{fl'}}}$$

$$P_f = \frac{1}{\gamma_m \gamma_{j2}} \left[\frac{b_j t_f^2 \sigma_y^j}{j} + K_c t_w \sigma_y^{w'} (2j + c) \right] \quad (2.48.a)$$

$$\text{otherwise, } P_f = \frac{K_c t_w \sigma_y^{w'} c}{\gamma_m \gamma_{j3}} \quad (2.48.b)$$

$$\text{where: } \sigma_y^{fl'} = \sigma_y^{fl} [1 - (\sigma_b/\sigma_y^{fl})^2]; \quad \sigma_y^{w'} = \sigma_y^w [1 - (\sigma_b/\sigma_y^w)^2]$$

K_c is the web strength for transverse compression determined by a coefficient of slenderness $\lambda = b_e/t_w \sqrt{\sigma_y^w/355}$ and the aid of a table present in the standard ; $b_e = 1.9 \sqrt{c h/K_w}$; $\gamma_m = 1.05$, $\gamma_{j2} = 1.1$;

$$K_w = (3.4 + 2.2 h/a) (0.4 + c/2a) \quad \text{and } j = \sqrt{(b_j/2K_c) (1/t_w) (\sigma_y^{fl'}/\sigma_y^{w'})} .$$

The value of j should be less than $a - c/2$ for a symmetrically placed patch, or the larger of the two unloaded lengths between the patch load and the adjacent transverse stiffeners, for the unsymmetrical cases.

Dubas & Gehri, [DUBA86a], identified three different types of failure due to the presence of patch load: yielding, web buckling and web crippling. One of the most important factors of influence on this choice of failure was found to be the depth h . The three regions of failure previously mentioned correspond to stocky, intermediate and deep beams respectively. They also made a review of the existing methods of design, from the first ones which assumed a distributed length L_{eff} for the dispersion of the load (equations 2.49.a and 2.49.b) to the more recent ones based on experimental results proposed by Bergfelt and Skaloud.

$$L_{eff} = c + 5(t_f + r) \quad \text{for rolled beams} \quad (2.49.a)$$

$$L_{eff} = c + 1.4 t_f \quad \text{for welded beams} \quad (2.49.b)$$

In the new draft of the Eurocode, [EURO91a], patch load failure is governed by two equations 2.50 and 2.51, that represent the yielding and the buckling of the web:

$$P_f = (S_s + S_y) t_w \sigma_y^w / \gamma_m \quad (2.50)$$

$$\text{where: } S_y = 2t_f \sqrt{\frac{b_f \sigma_y^{fl}}{t_w \sigma_y^w} \left[1 - \left(\frac{\gamma_m \sigma_b}{\sigma_y^{fl}} \right)^2 \right]} \quad \text{for: } b_f \leq 25t_f \text{ and,}$$

$$S_y = \frac{2.5 (h - d) \sqrt{1 - \left[\frac{\gamma_m \sigma_b}{\sigma_y^{fl}} \right]^2}}{\left[1 + \frac{0.8 S_s}{h - d} \right]} \quad \text{for rolled beams.}$$

and the value of the distance S_s is shown in figure 2.18.a, page 57 for three different patch load situations. To take into account web buckling they suggest that for a single patch load:

$$P_f = 0.5 t_w^2 \sqrt{E \sigma_y^w} \left[\sqrt{\frac{t_f}{t_w}} + \frac{3 S_s t_w}{h t_f \gamma_m} \right] \quad (2.51)$$

where S_s/h should not be taken greater than 0.2.

When a double patch load is present it was suggested that an additional check should be made. It consists of calculating the buckling resistance of the section by considering the web as a virtual compressive member with an effective breadth b_{eff} obtained from equation 2.52:

$$b_{eff} = \sqrt{h^2 + S_s^2} \quad (2.52)$$

or from the situations described in figure 2.18.b, page 61. The buckling resistance should be determined by using the buckling curve "c" present in the standard and $\beta_a = 1$. For the bending/patch load interaction they suggest equation 2.53:

$$P/P_f + M/M_f \leq 1.5; \quad P/P_f \leq 1.0; \quad M/M_f \leq 1.0 \quad (2.53)$$

The performance of equations 2.50 and 2.51 can be seen in figures 2.19.a and 2.19.b, page 62, where the ratio P_{exp}/P_f is plotted against the web slenderness h/t_w . It is noted that these two equations, when compared with experiments, behave in a very poor way with arithmetic means equal to 0.82 and 1.558, and standard deviations of 64% and 28%. These equations were found to be simple enough for design use, when compared with the more complex ones described by equations 2.54.a and 2.54.b:

$$\left[\frac{P}{P_f} \right]^2 + \left[\frac{M}{M_f} \right]^2 \leq 1.0 \quad (2.54.a)$$

$$\left[\frac{P}{P_f} \right]^8 + \left[\frac{M}{M_f} \right]^8 \leq 1.0 \quad (2.54.b)$$

In 1975 Skaloud and Drdacky, [SKAL75], made a survey of the most significant work in patch load design, and proposed a formula for the prediction of the ultimate load. That formula was incorporated in the Czechoslovakian standards and was previously described as equation 2.16. This equation was then compared with experimental evidence, as can be seen in figures 2.20.a and 2.20.b, page 63. In those figures the ratio P_{exp}/P_f is plotted against the ratios t_f/t_w and h/t_w respectively, with

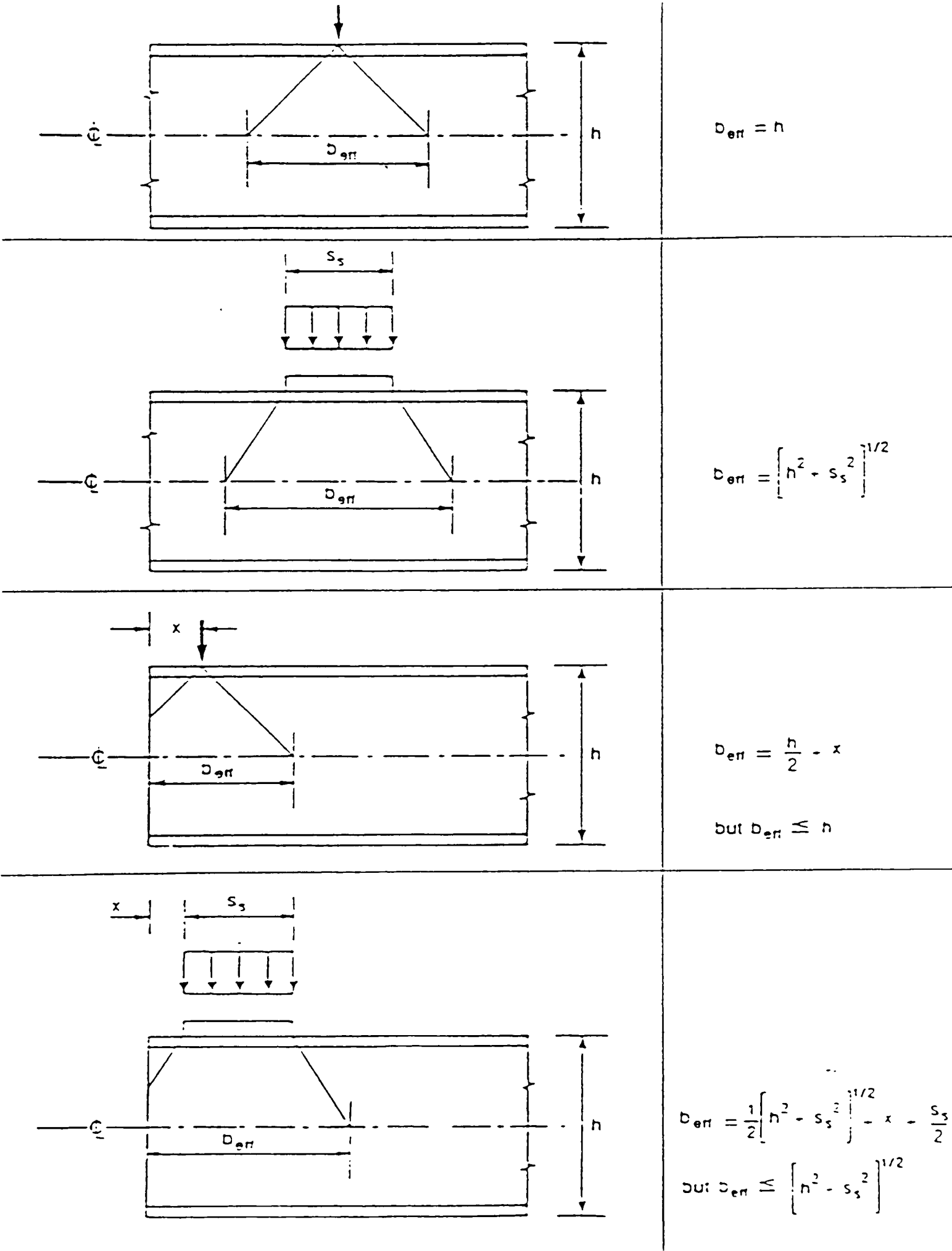


Figure 2.18.b Effective Breadth for Web Buckling Resistance.
(after [EURO91a])

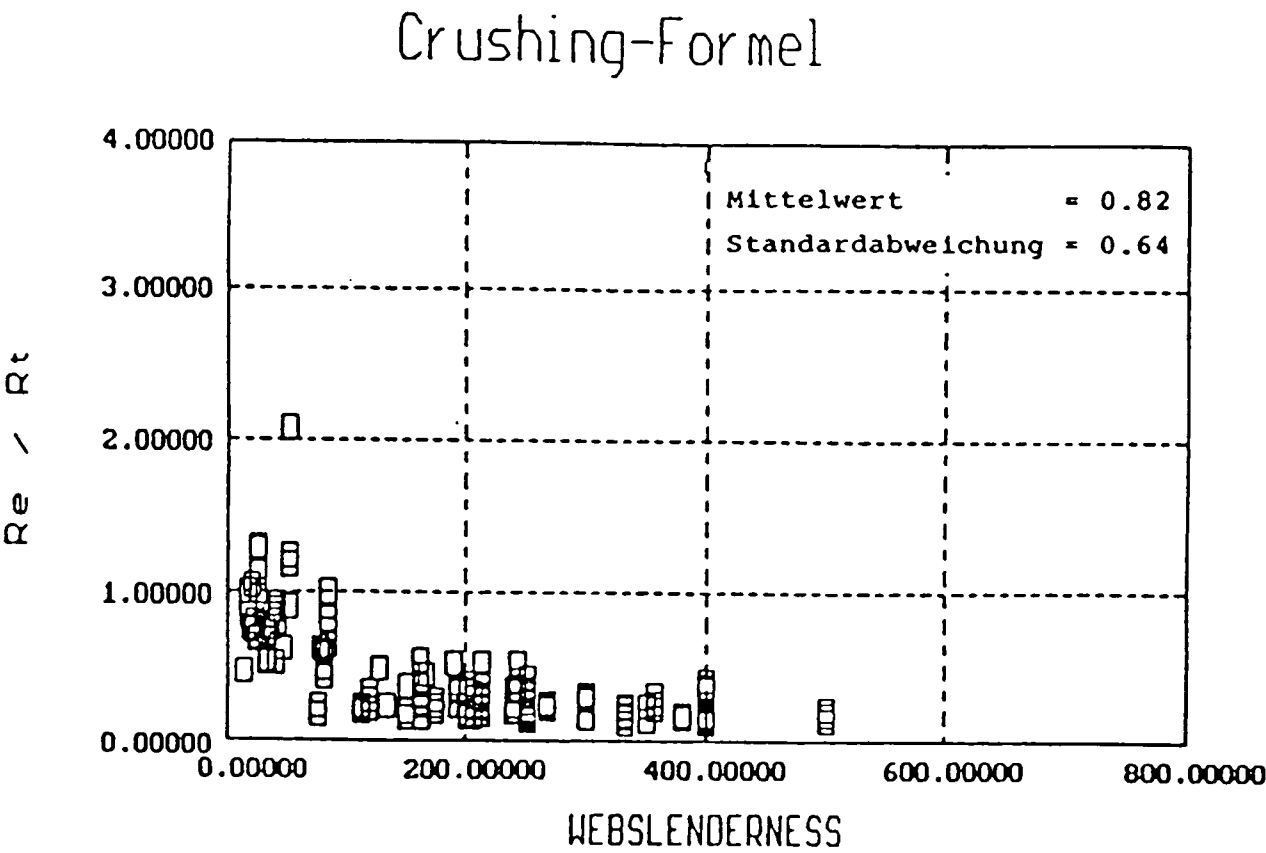


Figure 2.19.a - Comparison of Equation 2.50 with Experimental Results.
(after [EURO91])

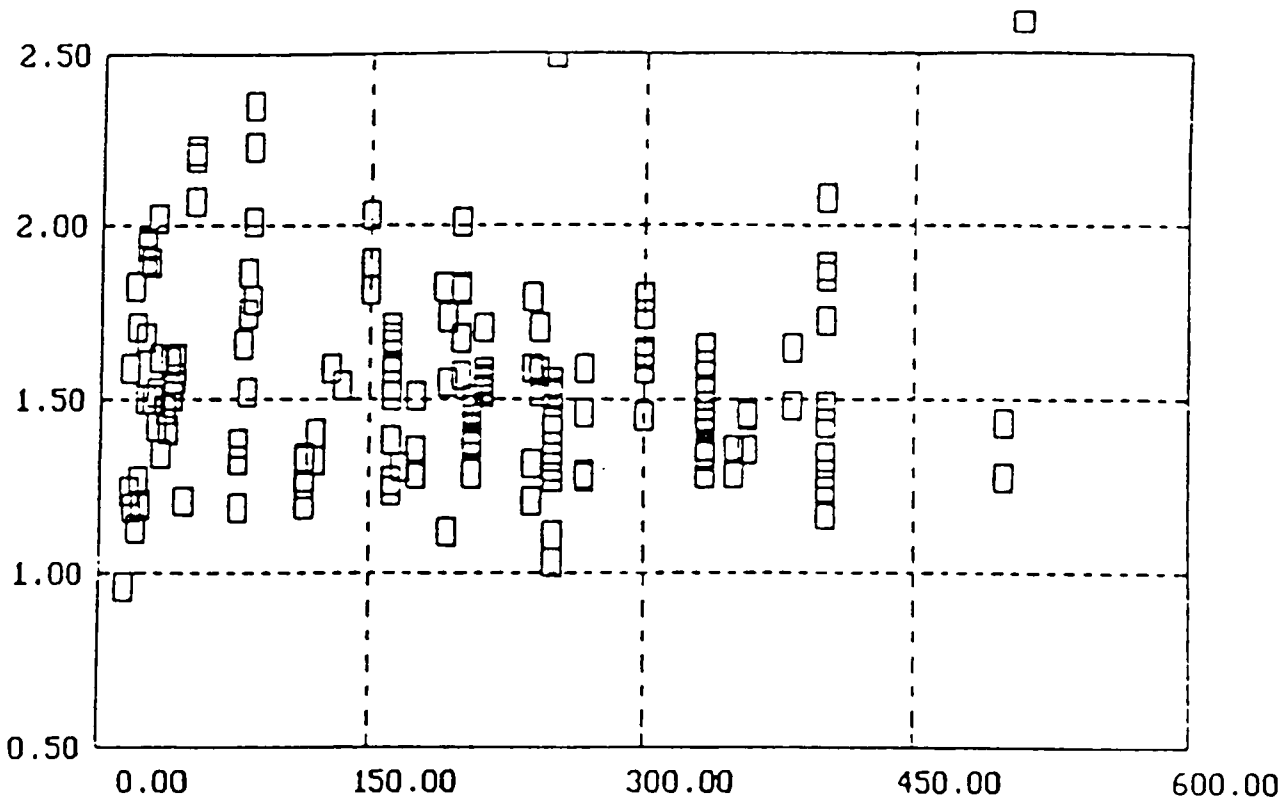
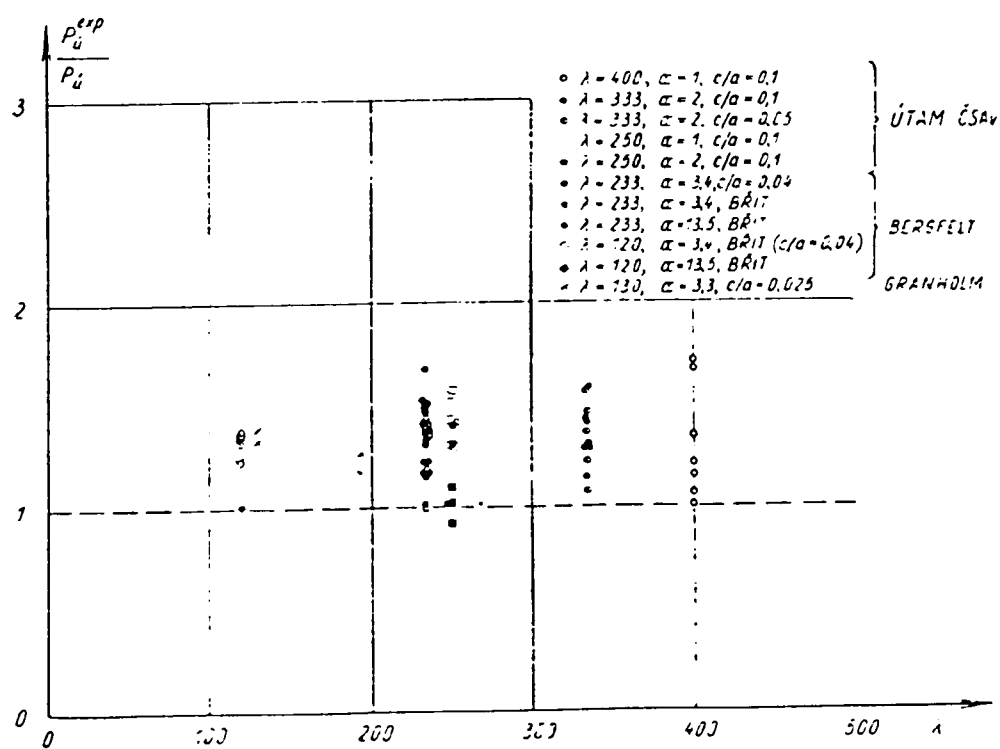
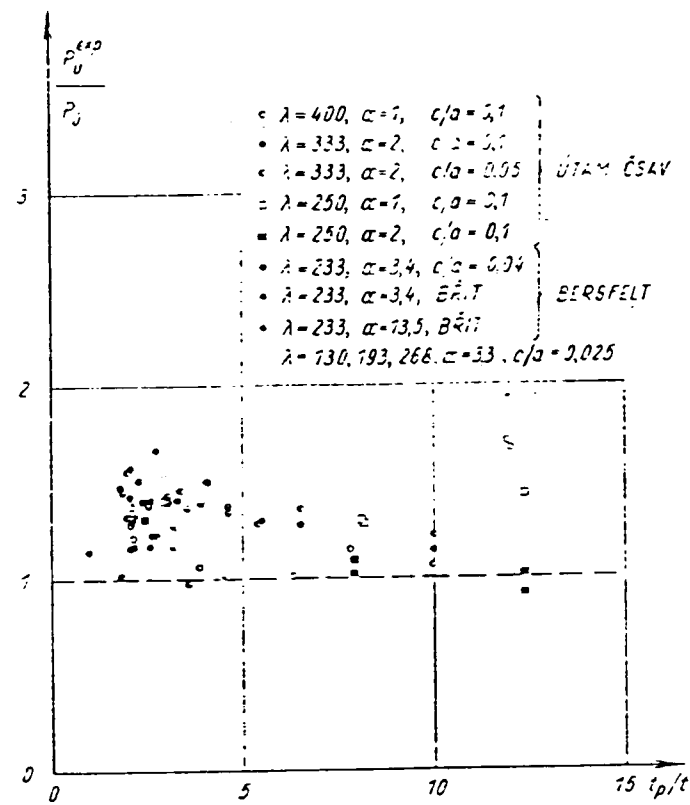


Figure 2.19.b - Comparison of Equation 2.51 with Experimental Results.
(after [EURO91])



Figures 2.20.a and 2.20.b - Comparison of Equation 2.16 with Experimental Results.
(after [SKAL75])

acceptable results. They also confirmed that equation 2.11.b is a good interaction equation for describing the combined action of moment and patch load.

In a recent SSRC publication, [SSRC88], another useful survey of the patch load field was made. They pointed out the poor results provided by the dispersion formula 2.4 when compared with experimental evidence. They also mentioned that the method suggested by Roberts & Rockey, [ROBE79], overestimated the failure load for large values of the patch length c .

In 1983 Elgaaly, [ELGA83], made an excellent survey of the patch load problem related to rolled and welded beams. He stated that all the failures which are caused by the application of a patch load occurred by web crippling. The difference that could occur in beams with different web slenderness was linked to the way that crippling started. He affirmed that in the case of stocky beams, failure will begin by yielding in the web. That would be followed by the crippling action that was prevented from occurring before by the post-buckling resistance of the web. On the other hand when the beam is relatively slender the trigger of the process would be the buckling of the web. That would be followed by a different type of crippling effect. He covered several aspects of the problem such as analytical and theoretical elastic buckling predictions, and ultimate strength resistance that involved experimental, analytical and design methods, and the possible interactions with moment and shear. One interesting point of the interaction between patch load and moment is described in figure 2.21, page 65 where a reduction factor for the patch load resistance was plotted against the percentage of moment present in the member. This comparison was made for the equations 2.11.a and 2.11.b and another suggestion, equation 2.55:

$$f_b = \sqrt{1 - \left[\frac{M}{M_u} \right]^2} \quad (2.55)$$

He concluded that for safety in the absence of sufficient experimental evidence, the more conservative formula 2.11.a should be adopted.

He also made a thorough review of the American codes of practice in this area. The AISC proposed that the value of the compressive stress obtained by the use of equation 2.4 should not be greater than 75% of the value of the yield stress of the steel used, or:

$$\sigma = \frac{P}{t_w (2K + c)} \leq 0.75 \sigma_y \quad (2.56)$$

The same stress should not be greater than the value obtained by equation 2.57.a, for rotation-restrained flanges, or by equation 2.57.b for other cases:

$$\sigma = \frac{10000 [5.5 + 4/(b_f/h)^2]}{(h/t_w)^2} \quad (2.57.a)$$

$$\sigma = \frac{10000 [2.0 + 4/(b_f/h)^2]}{(h/t_w)^2} \quad (2.57.b)$$

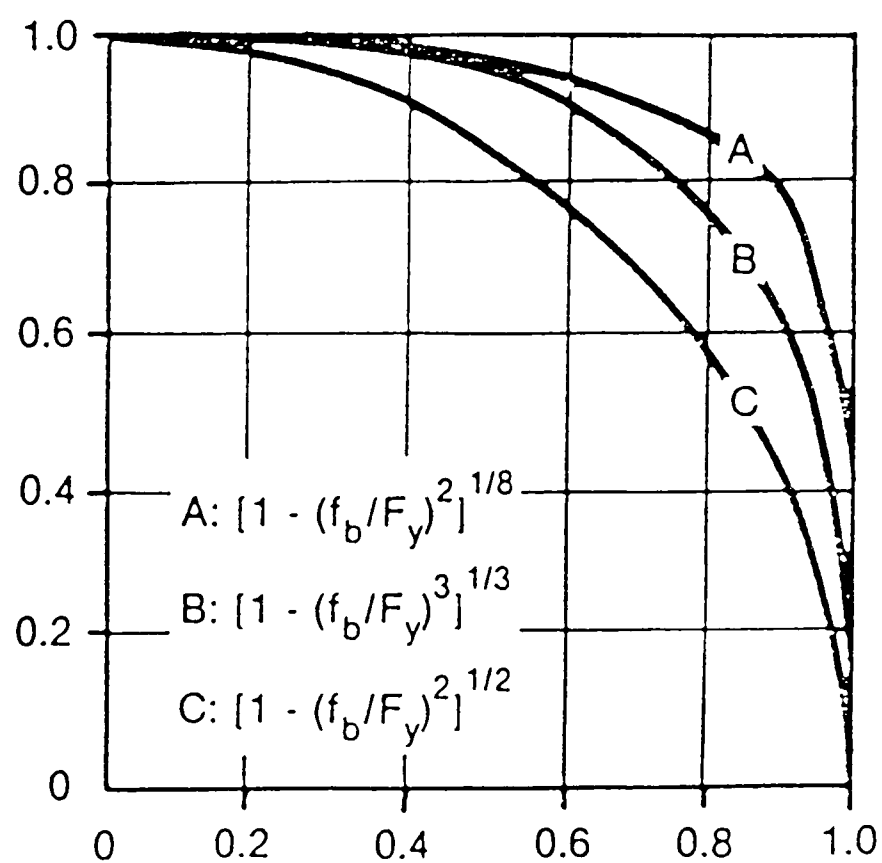


Figure 2.21 - Reduction Factor for the Interaction Patch/Bending.
(after [ELGA 83])

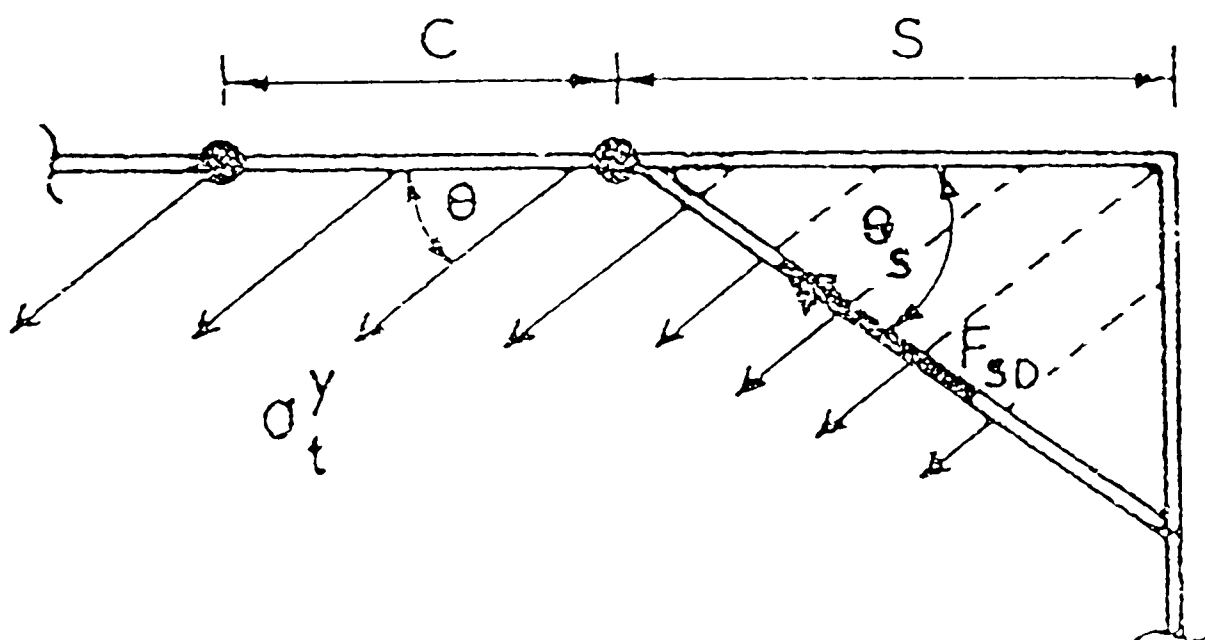


Figure 2.22 - Inclined Stiffener Design Model.
(after [PORT86])

A comparison was made of equations 2.57.a and 2.57.b with experimental results, which demonstrated that they can only be regarded as very poor approximations to the test results. Equation 2.56 overestimated the failure load in cases where $(h/t_w) (c/t_f)$ was less than 900. To overcome this difficulty a correction factor f_r was proposed in order to modify equation 2.56. This modification can be seen in equation 2.58:

$$\sigma = \frac{P}{t_w (2K + c)} \leq f_r \sigma_y \quad (2.58)$$

in which:

$$f_r = 1.7 - 1.2 \times 10^{-3} (h/t_w) (c/t_f) \text{ for } (h/t_w) (c/t_f) \leq 1000 \quad \text{or:}$$

$$f_r = 0.525 - 0.025 \times 10^{-2} (h/t_w) (c/t_f) \text{ for } (h/t_w) (c/t_f) > 1000$$

He suggested a modification in the factor of safety, equation 2.59, to be applied to equations 2.57.a and 2.57.b in order to improve their performance:

$$F_s = 1.0 + 0.003 (h/t_w) (t_f/t_w) \quad (2.59)$$

Despite all this effort he recommended yet another design formula, equation 2.60:

$$P_f = 0.5 t_w^2 \sqrt{\frac{E \sigma_y^w t_f}{t_w}} \quad (2.60)$$

The AISC code, [AISC82], based on load and resistance factor design, LRFD, proposed other expressions for the failure load, equations 2.61 and 2.62:

$$P_f = 108 t_w^2 \sqrt{\sigma_y} \quad (2.61)$$

$$P_f = (0.5c + 5K) \sigma_y^w t_w \quad (2.62)$$

Rockey et al, [ROCK81], made a good state of the art survey of the design of stiffened welded plates. They summarized the available work in that area, concentrating their efforts on the design of longitudinal and transverse stiffeners, end-post stiffeners and patch load. In the patch load field they covered a good part of the theoretical and analytical work. They also reviewed the investigations of the ultimate load capacity of girders and made an interesting resume of the proposed design formulae to model the phenomenon.

On the same line Chatterjee, [CHAT81], looked into the design of stiffened webs. He clarified the mechanism of collapse proposed by Roberts & Rockey, [ROBE79], and by using it, demonstrated how the formulae 2.48.a and 2.48.b were developed. Dubas, [DUBA87], suggested another expression, equation 2.63, to model the ultimate load behaviour of girders subjected to patch load. He also made more tests to study the interaction of patch load and bending and discovered that equations

2.11.a and 2.11.b represented, in fact, upper and lower bounds for the specified interaction.

$$P_f = 0.5 \sigma_y^w t_w^2 \sqrt{\frac{E t_f}{\sigma_y^w t_w}}$$

(2.63)

Lastly Roberts, [ROBE84], produced a very good state of the art review of patch loading on plate girders. He concluded that all the formulae that had been suggested for the patch load failure had an average error of $\pm 20\%$ when compared with the experimental values.

2.5.5. Summary.

To summarise the patch load research over the past forty years, Table 2.1 was generated.

Theoretical and Numerical Works	Experimental Works	Experimental Works	Design Standards and State of the Art Reviews
BAGC68	ARIB77	HERZ74	AISC82
BAGC75	ARIB81	HERZ74a	BRIT85
BAZI73	ARIB88	HERZ74b	BRIT78
DOGA91	ARIB90	HERZ86	CHAT81
KHAN72	BERG68	HETR79	DASE88
KHAN77	BERG71	JANU88	DUBA87
MUCK83	BERG76	KARN86	ELGA83
RAOU91	BERG79	KARN86a	EURO91
ROCK70	BERG83	KRIS91	EURO91a
ROCK71	BOSS67	LYSE35	ROBE79
ROCK79	CORI86	MOOR35	ROBE84
SHIM90	CORI91	MORI83	ROCK81
SMIT82	DRDA77	NOVA73	SKAL75
STEU77	DRDA77a	ROBE78	SSRC88
THOR80	DRDA86	ROBE81	
WEIM87	DUBA86	ROBE83	
WHIT62	DUBA86a	ROBE88	
ZETL55	DUBA88	ROCK72	
	DUBA90	SANT89	
	ELGA75	SHIM87	
	ELGA89	SHIM89	
	GALE87	SKAL71	
	GRAN60	SKAL72	
	HEND49		

Table 2.1 Patch Load Summary.

Some eighty references have been found to a major international effort on patch load effects. Painstaking experimental studies have been combined with a variety of analytical and numerical approaches to derive (and later to criticise) the design formulations in the national design codes of many countries.

The only conclusion that can be reached is that the question of patch loading is still to some extent an open one, one which continues to attract the attention of many research teams worldwide. Roberts', [ROBE84], view that a difference of $\pm 20\%$ exists between experiment and code formula probably still stands. If this difference is to be reduced, a significant set of experiments, backed by state of the art numerical analysis including the effects of initial imperfections, eccentricity, and residual stresses will be needed. Given a validated computer model, a properly considered parametric study might give rise to improved design rules if practical factors are also taken into account.

2.6. STIFFENER DESIGN.

A very important study of the optimization of the rigidity of transverse and longitudinal stiffeners was performed by Skaloud, [SKAL84]. He based his studies on two different parameters: ψ^* and W . The relative rigidity of the stiffener ψ^* is first chosen using a linear web buckling theory, in order to prevent it being too flexible. After that, the value ψ^* is modified by another variable W that takes into account factors like initial imperfections and post-buckling behaviour. The value of the parameter ψ^* is given by:

$$\psi^* = \frac{E I_{st}}{D h} \quad ; \text{ where: } \quad D = \frac{E t_w^3}{12(1 - \nu^2)} \quad (2.64)$$

The value of the linear buckling parameter ψ^* is described by some tables presented in Skaloud's work, and takes into account factors such as type of loading and stiffener configuration. The ideal rigidity factor ψ_0 is given by:

$$\psi_0 = W \psi^* \quad ; \quad W > 1.0 \quad (2.65)$$

where the value of the parameter W is a function of the type of stiffener (longitudinal or transverse), type of its cross section (open or closed), and of the ratio h/t_w .

Mele & Puhali, [MELE87], investigated some very interesting aspects of the design of stiffeners. They covered the effectiveness of the cross section and how the tension field forces are distributed on the stiffeners. To cope with these difficulties they proposed an effective cross section breadth, equation 2.66:

$$b_{eff} = 35 t_w (1 - \tau_{cr}/\tau_y) \quad (2.66)$$

in which τ_{cr} and τ_y were the critical and the yield shear stress respectively. They also suggested that an effective length L_{eff} should be used in the buckling analysis. They

proposed it to be equal to the full depth of the web, if the stiffener supported a direct load, and to be equal to half the web depth in other cases.

They recommended that the stiffeners should be designed using an beam-column interaction formula, equation 2.67. The compressive load P is the force that corresponds to the diagonal shear of the collapse mechanism in the post-critical phase. The moment M should be taken as the product of the compressive force and the eccentricity of this force with respect to the effective centroid of the stiffener.

$$\frac{P}{P_{cr}} + \frac{M}{M_{pl} (1 - P/P_{cr})} \leq 1.0 \quad (2.67)$$

Elgaaly, [ELGA83], made an interesting survey of the design formulae proposed by AISC for the cases of end stiffeners, which state that if the condition expressed in equation 2.68 is not satisfied, an end stiffener should be used.

$$\frac{P}{t_w (K + c)} \leq 0.75 \sigma_y^w \quad (2.68)$$

He proved that its use is only reasonable in the region where the parameter $(h/t_w) (c/t_f)$ is greater than 600. In the other zones it overestimates the ultimate load. In a later standard, AISC LRFD, it is suggested that the smaller value of those given by equations 2.69.a and 2.69.b should be used:

$$P_f = 54 t_w^2 \sqrt{\sigma_y} \quad (2.69.a)$$

$$P_f = (0.5c + 2.5K) \sigma_y^w t_w \quad (2.69.b)$$

Elgaaly recommended the use of equation 2.70 based on his own tests, [ELGA83].

$$P_f = 0.25 t_w^2 \sqrt{\frac{E \sigma_y^w t_f}{t_w}} \quad (2.70)$$

Porter, [PORT86], studied the benefits of the use of corner stiffeners on the ultimate load of webs. The web slenderness factor was also taken into account and it was shown that the more slender the beam was, the greater the increase in the ultimate load that could be obtained. The design of the corner stiffener is based on the model described in figure 2.22, page 65, and is given by:

$$F_{st} = 0.7 \sigma_y^w t_w (c + \varepsilon/2) \quad (2.71)$$

He also proposed some limitations on the area, depth and thickness of the stiffener, equations 2.72.a, 2.72.b and 2.72.c:

$$A_{st} = F_{st} \sigma_y^{st} \quad (2.72.a)$$

$$h_{st} = 0.23 s \sqrt{\frac{\sigma_y^{st}}{355}} \quad (2.72.b)$$

$$t_{st} = \frac{h_{st}}{10} \sqrt{\frac{\sigma_y^{st}}{355}} \quad (2.72.c)$$

Voss et al, [VOSS87], [VOSS88], [BAMM88], tested several beams to develop a procedure for end-post stiffener design. The main concern of the matter is displayed in equation 2.73. The ultimate loads: p_j^{fl} , p_j^w and p_j^{st} corresponded to the contribution of the flanges, the unstiffened web and the stiffener respectively. The factors R_2 and R_3 are magnification variables that take into account other important aspects of the problem.

$$P_j = p_j^{fl} + R_2 p_j^w + R_3 p_j^{st} \quad (2.73)$$

The details of the rather complex terms of this equation can be found in [VOSS88]. He also recommended that if end-post stiffeners were necessary, at least one should run the full depth of the web.

Finally Nakai & Sakano, [NAKA85], have also looked into the problem of end-post stiffeners. They concentrated their efforts in studying the interaction between the compressive force from the support and the moment caused by the lateral force from the tension field. They proposed a design formula very similar to the one described by equation 2.67. Their main difference was that they suggested that the effective length of the stiffener should be taken as 71 % of the full depth of the girder.

This section forms an intentionally brief review of the use and design of web stiffeners. The extensive requirements of the various design codes have not been touched upon here. The brevity of this section is related to the way in which the present research developed: initially stiffener design was expected to be more relevant than in the event it proved to be.

2.7. CONCLUSIONS.

Very few references to the problem of vertical web buckling at the central point of tapered composite beams were found in the literature. None contained any significant information about experiments or numerical simulations developed to understand its behaviour. In the view of that fact a broader survey was made involving similar problems such as the patch load resistance.

The patch load review was very useful in identifying the most important variables to be studied. The thickness of the web was considered the major parameter by the great majority of the researchers. The significant effect of a combination of bending moment and patch load on the ultimate capacity was also emphasised. The material and geometric characteristics of the flanges were also found to influence the patch load behaviour but with less severe consequences for the ultimate resistance.

With these results in hand, the next stage of the work will be the description of the laboratory tests. Their choice and performance will be reported in full detail in the next chapter of this thesis.

CHAPTER 3

EXPERIMENTAL WORK.

3.1. INTRODUCTION.

In this chapter the experiments performed will be described in detail. A brief introduction to the equipment and materials used will be followed by a description of the experimental rig and the data acquisition process. Lastly, the results themselves will be presented and discussed.

The primary objective of the experiments was to determine whether vertical stiffeners were really necessary in the web at the point where the bottom flange changes direction. An assessment of the most important parameters that could influence the buckling behaviour of the web, which was based on the patch load literature described in the previous chapter, revealed that the research had to be focused on the thickness, or perhaps the depth to thickness ratio of the web. Bearing this fact in mind, a first series of tests was developed to study the influence of web thickness on the load-carrying capacity of the beams. The plastic neutral axes of these beams were in or close to the top flange.

Once these results were to hand, a second series was conducted in which the plastic neutral axis was lowered progressively partly to assess the influence of neutral axis position, including the construction case where the steel beam has to carry the freshly cast slab. Another purpose was to explore a significant difference between classical patch loading and the concentrated load induced by the slope change in the lower flange, namely that patch loads normally act on the compression flange and induce biaxial compression in the adjacent web while the slope change creates vertical compression in part of the web which is under horizontal tension. This difference significantly increases the buckling load of the web, and makes the slope change less critical than conventional patch loading.

3.2. MATERIALS AND METHODS.

This section comprises a complete description of the experimental rig and the process of data acquisition. To begin the materials used will be described and details of the spans, jacks, supports and others relevant aspects given. This section will be followed by a description of the instrumentation used to monitor the behaviour of the beams during the test.

The experiments had the support of the Science and Engineering Research Council (SERC), and Ward Brothers Sherburn Ltd. SERC provided funds for materials and technician time, while Wards fabricated the beams at cost price. Because of the obvious value of keeping fabrication in line with their usual commercial practice, discussions with Wards played a large part in determining the size and layout of the specimens. Initially, specimens of about one quarter full size were envisaged, but this scale would have given specimens too small, and (importantly) too thin for Wards' automated processes and standard plate thicknesses. As a result beams of approximately half full size were selected.

Again, for strictly practical reasons it was decided to model the concrete slab of the composite beams by using a heavy steel top flange (sized to give the same plastic neutral axis position in the flange) to the fabricated beam. The properties of the

concrete deck, and of its interaction with the steel section, were not of great interest, and a great deal of time and expense was saved by this decision. An additional advantage was the greater control of model properties achieved by employing a homogeneous top flange. The only drawback to the use of the steel top flange was the reduced lateral torsional restraint which it gave for the same axial stiffness. It was relatively easy to provide external lateral restraints to overcome this problem: details are given below.

Since flange slope changes inducing web compression normally occur at the centre of the beam, the loading chosen was a pure moment with zero shear, a choice simply achieved in the lab by a four point loading system.

3.2.1. Rig Description.

The test rig consisted of a simply supported beam 11.46m long subjected to two symmetrically disposed concentrated loads. This layout can be seen in plate 1, page 73, and in more detail in figure 3.1, page 74. The beam was composed of three distinct parts: the central "specimen" and two "loading arms". The loading arms were reused in all of the tests, and their purpose was just to receive the load from the hydraulic jacks and transmit it effectively as a pure moment to the specimen. The specimen was changed every test and was the main object of the experiments.

The loads were applied monotonically by two 1MN Amsler hydraulic jacks supported on loading frames. The simple supports were provided by steel rollers (plate 2, page 73). Everything was arranged on, and firmly bolted to, the structures laboratory strong floor. The horizontal movement of the steel rollers during the tests did not exceed 15mm and their performance was considered to be satisfactory.

Since the loading arms were shallower than the centre of the specimen, but had to carry the same moment with no risk of failure, they were constructed in grade 55 steel, while the specimens were assembled from grade 43 plate. Each loading arm was 3.1m long with an effective lever arm between the roller support and the jack centre line of 2.15m. The total height of the loading arms was 500mm, the top and bottom flanges were 25mm thick and 300mm wide, and the web was 12mm thick. The specimens were 5.26m long, of which the central 3m formed a tapered test panel with minimum and maximum heights of 500 and 650mm respectively, and the desired web thickness. These dimensions correspond to a taper angle of 1:10 or $\cong 5^\circ$, the maximum accepted by Wards machinery. The top and bottom flanges were 25 and 12mm thick and 300mm wide. The outer ends of the specimens had to match the loading arms, and were thus 500mm deep with 300mm by 25mm flanges and a 12mm web thickness. Due to restrictions imposed by the equipment that built the beams, a progressive reduction of the specimens' web thickness was necessary. From the connection point, the web thickness of 12mm was reduced by stages in the parallel part of the specimen to the required value at the tapered panel. Small web stiffeners were provided at the thickness changes.

The moment was transmitted from the loading arms to the specimens by means of the full moment friction-grip connection which can be seen in plate 3, page 75. It used 20 M20 bolts in the web and 16 M24 bolts in each flange. The bolt torque to ensure friction resistance was applied according to the part-turn method. In each flange connection, an outside plate 500 by 650 by 20mm and two inside plates 120 by 650 by 20mm were used. The difference in bottom flange thickness between the specimen and loading arms made also necessary the use of four extra 320 by 120 by

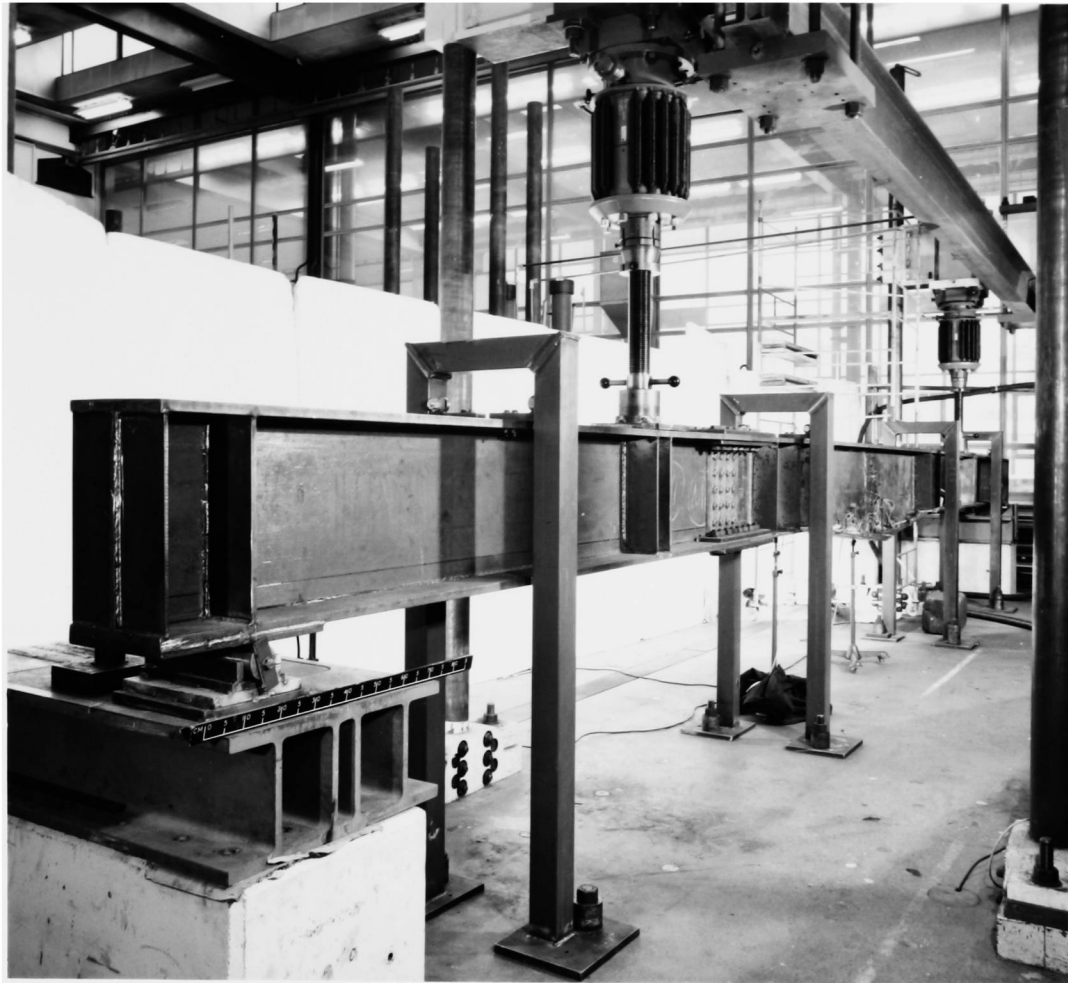


Plate 1 - General arrangement of loading rig.

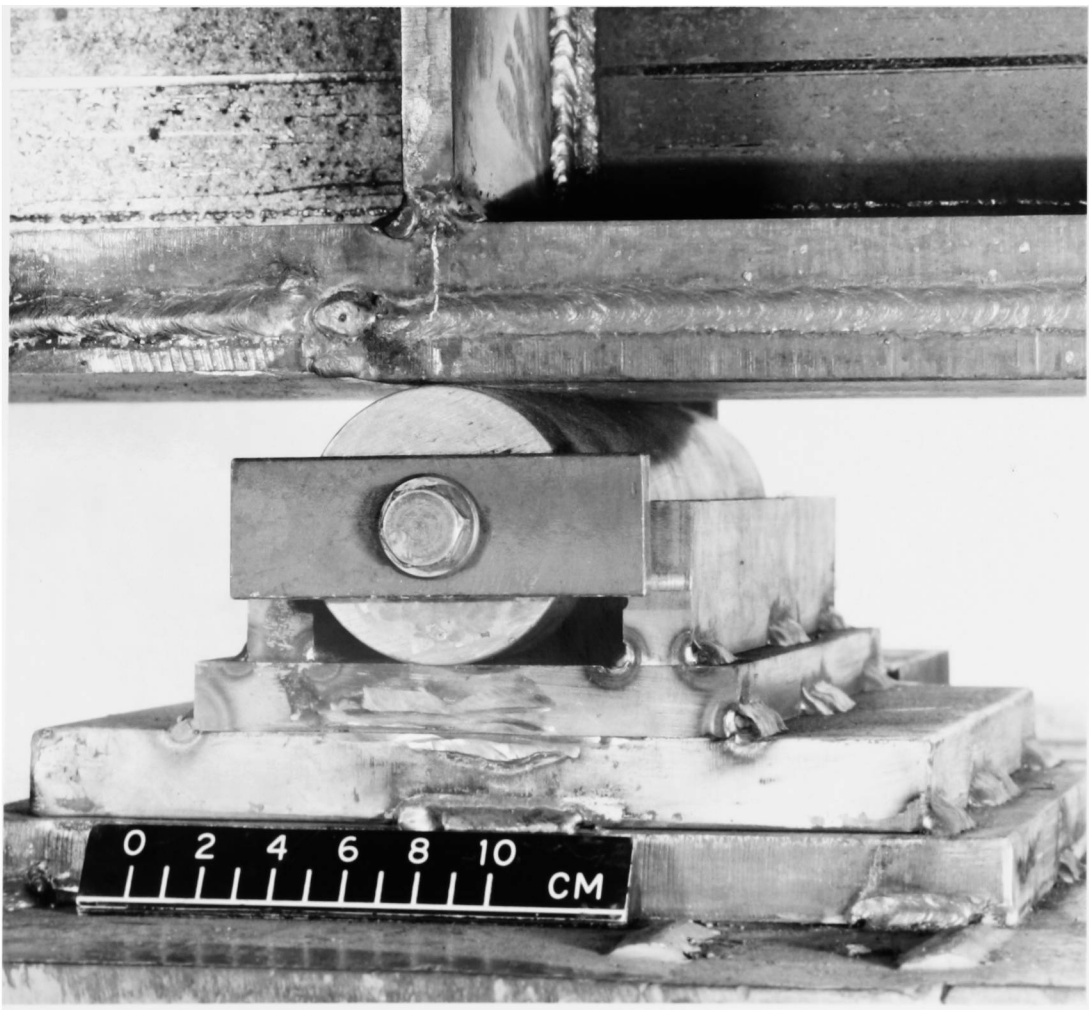


Plate 2 - Support detail.

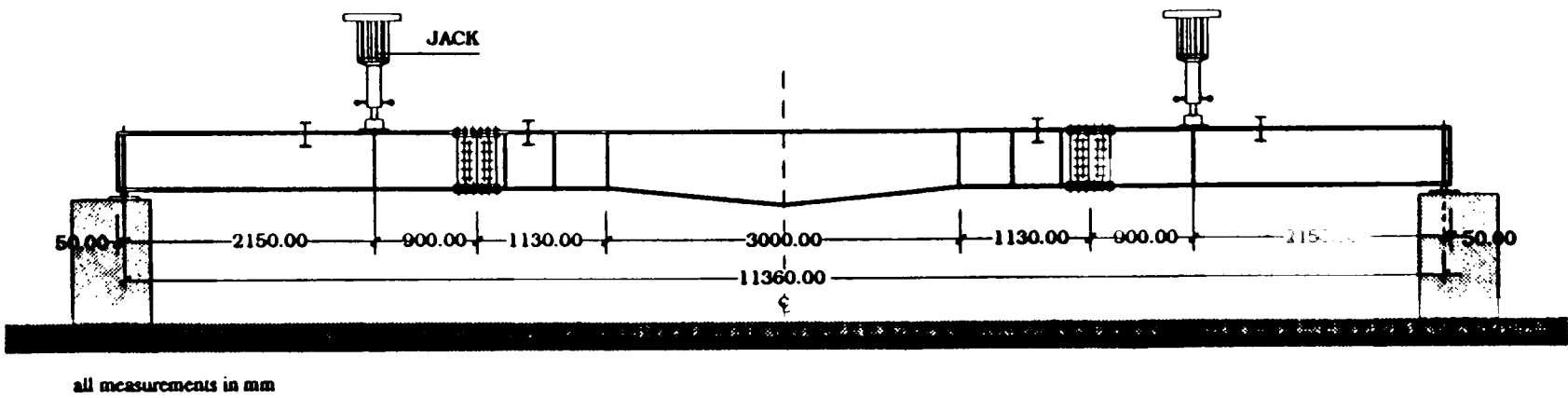


Figure 3.1 - Test Rig Dimensions.

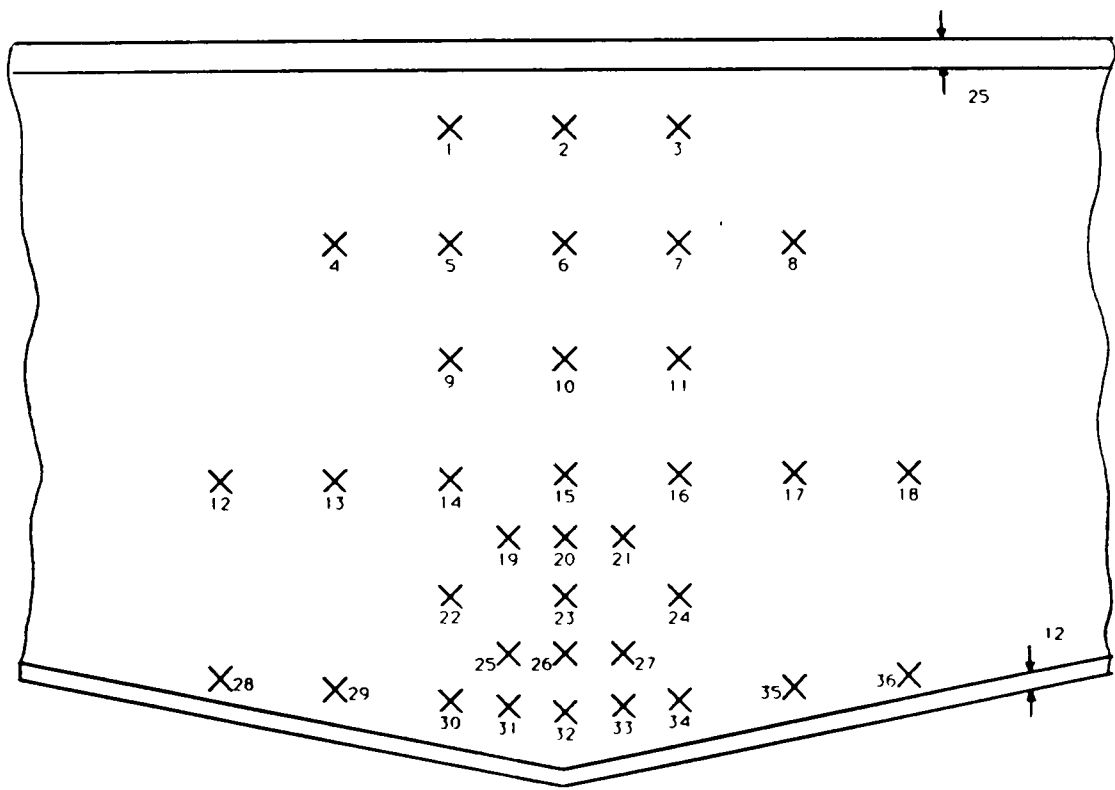


Figure 3.2 - Lateral Transducer Layout (dimensions in mm).

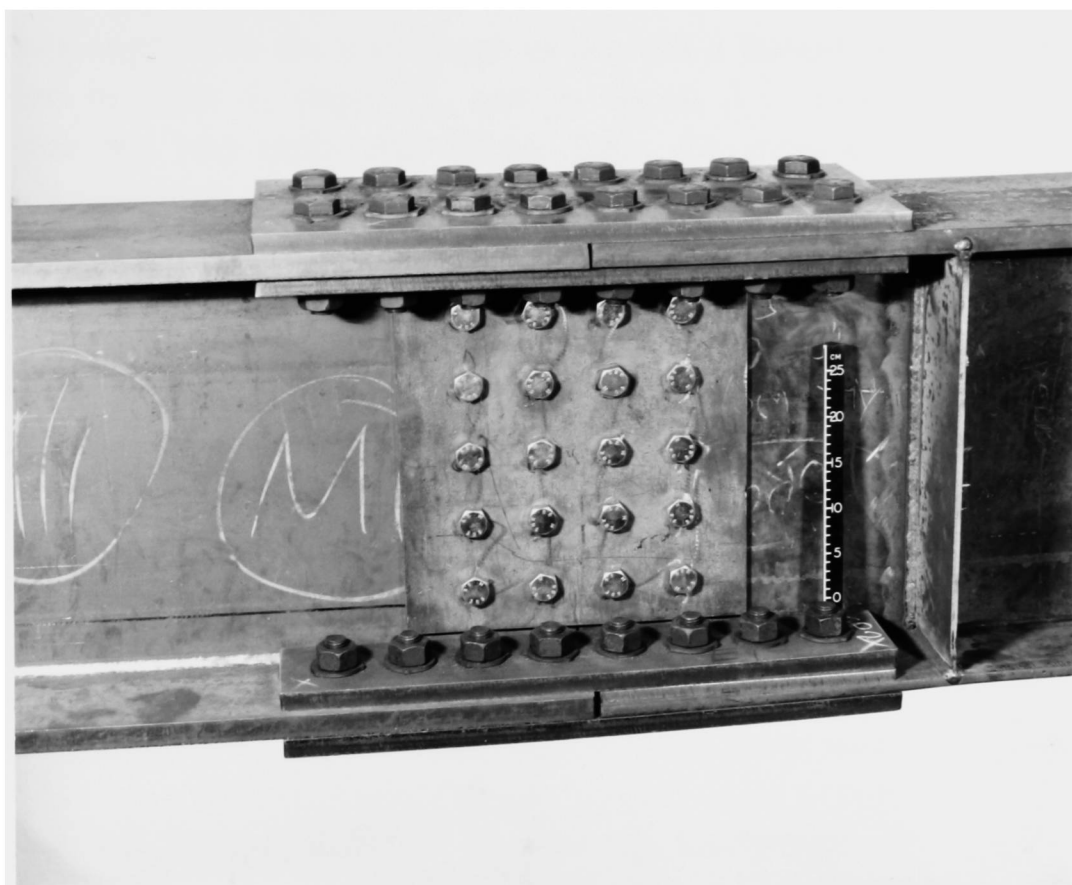


Plate 3 - Full moment connection detail.



Plate 4 - Lateral restraint frame detail.

13mm plates. In the web two plates 400 by 375 by 10mm were used. All plates were grade 55 steel.

As explained previously, four lateral restraint frames were used to provide the necessary support to the top flange to prevent a lateral buckling failure. They can be identified in plate 1, page 73, and in figure 3.1, page 74. They were located 500mm from the two jacks and 50mm from the centres of the connections. The restraint was provided by guide rollers (plate 4, page 75) carried on the top flange of the beam acting on running surfaces mounted on robust portals bolted to the strong floor of the laboratory.

During the tests it was noted that the jacks could run out of stroke due to the large vertical deflections developed by the beams. To overcome this problem, two additional thick plates which were connected to the laboratory strong floor by long floor bolts were introduced to the rig. When it was noticed that the jacks were reaching the end of their range, these plates, which were located on the top of the loading arm upper flange, were firmly bolted to the strong floor. This maintained the load on the specimen while the pressure in the jacks was decreased to permit them to be mechanically extended to a new position. After doing that, the jack pressure was increased to the point where the floor bolts became loose and the test continued.

3.2.2. Data Acquisition Process and Equipment.

To monitor the behaviour of the beam during the test, transducers and strain-gauges were used. They were linked to a Schlumberger computer data acquisition system and all the information obtained was kept on floppy disks for post-processing. Values were recorded at the various loading stages during the test. The value of the load in the jacks was obtained through a pressure transducer located at the return tube to the loading cabinet, and was frequently checked against the pressure indicated in the control cabinet. A multiplication of this pressure by the jack's effective cross section area and by the moment lever arm, gave the value of the applied moment at every stage of the test.

A plate carrying 36 displacement transducers was built to monitor the lateral deflections of the web at its critical central area. Their location and arrangement can be seen in detail in figure 3.2, page 74, and in plate 5, page 77. The purpose of this transducer array was to map the vertical buckling pattern for later comparison with the finite element simulations, a comparison which will be described in Chapter 4. Vertical displacement transducers were positioned on the top flange at the ends and centre of the tapered beam section.

To measure the values of the strains and hence stresses present in the beam, linear and rosette strain-gauges were employed. The linear strain gauges were used in the flanges and at points near the edges of the tapered panels. Rosettes were utilised at the centre of the tapered beam, as illustrated in plate 6, page 77. A typical strain gauge arrangement can be found in figures 3.3 and 3.4, page 78.

3.3. INFLUENCE OF THE THICKNESS OF THE WEB.

To assess the influence of one of the most important parameters in the vertical buckling problem, the thickness of the web, three tests were executed in phase 1. They were planned to cover the largest possible range of situations that might be found in design practice. The values of the web thicknesses chosen were equal to 7, 5

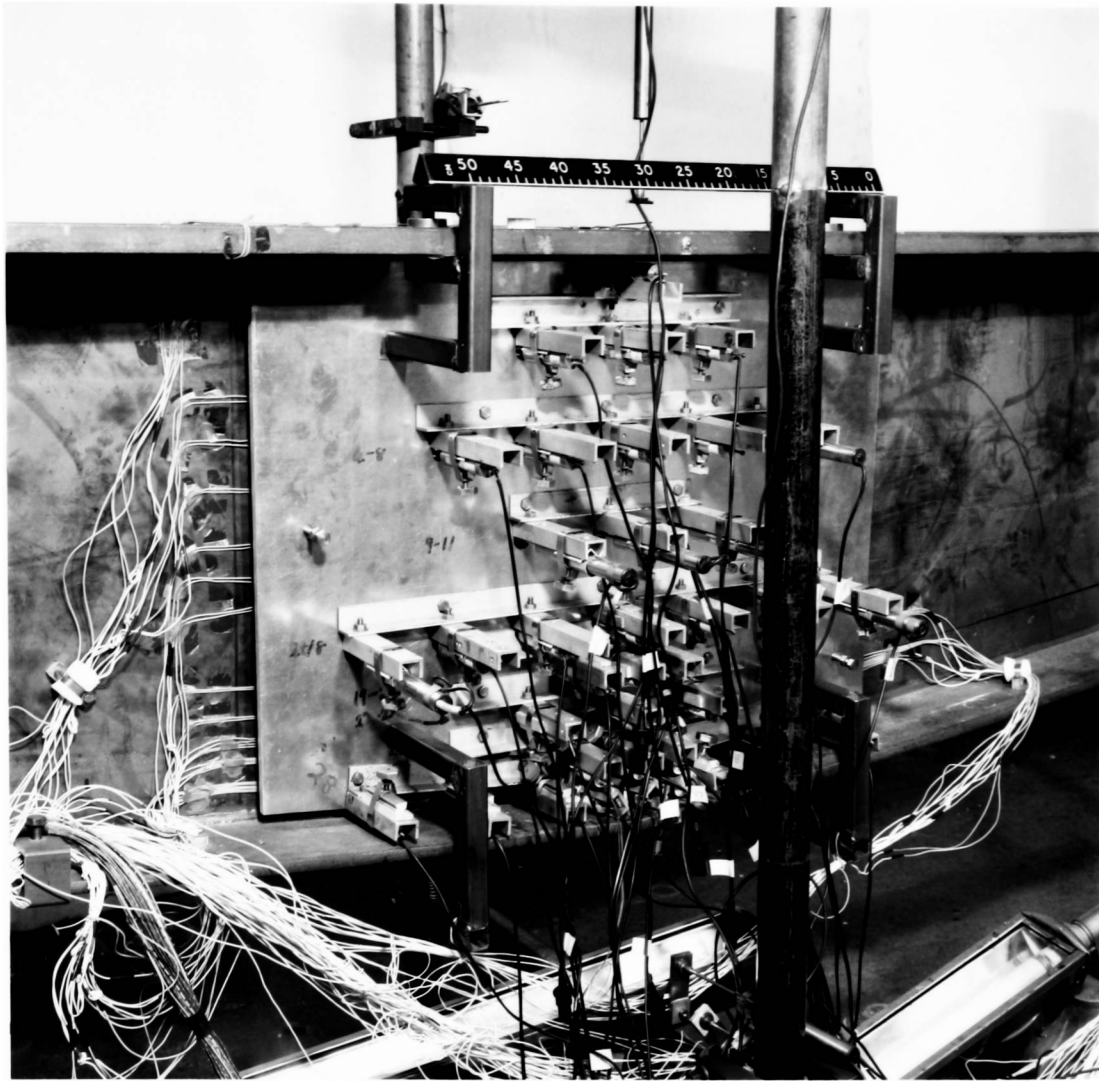


Plate 5 - Lateral transducer plate layout.

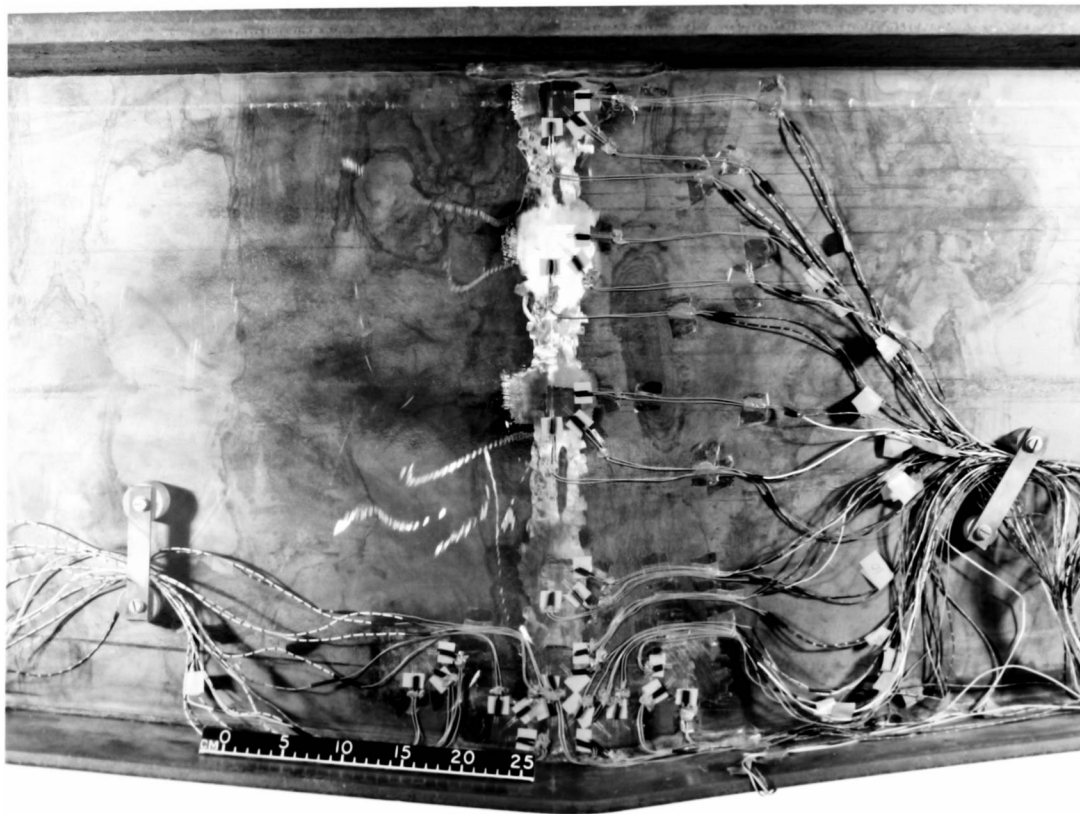


Plate 6 - Typical strain gauge arrangement.

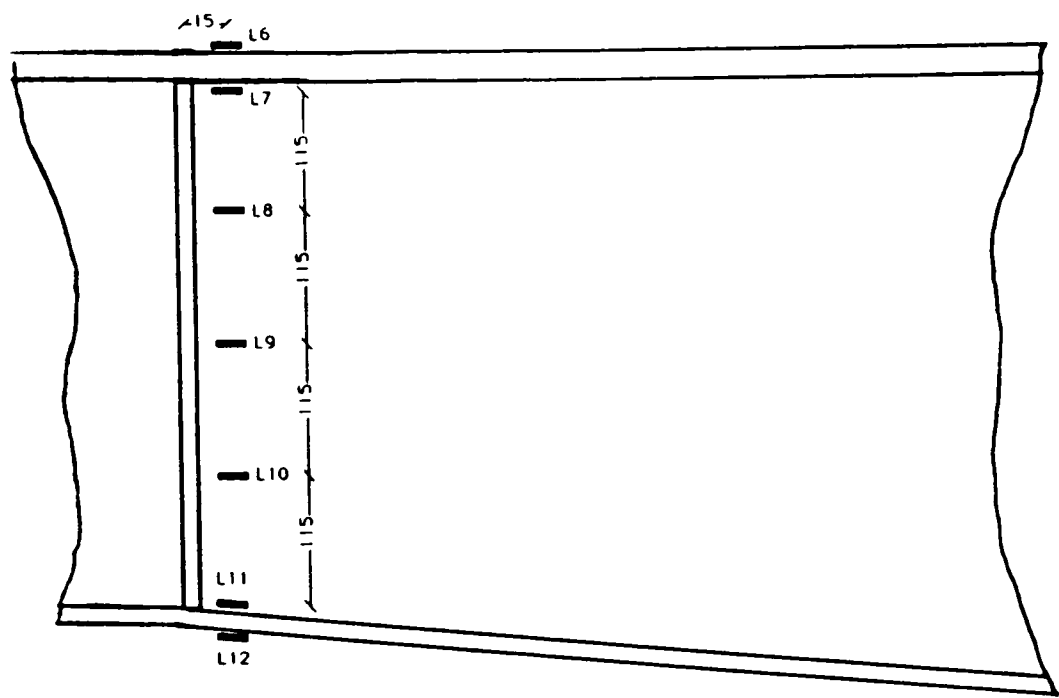


Figure 3.3 - Typical Side Section Linear Strain Gauge Positions (dimensions in mm).

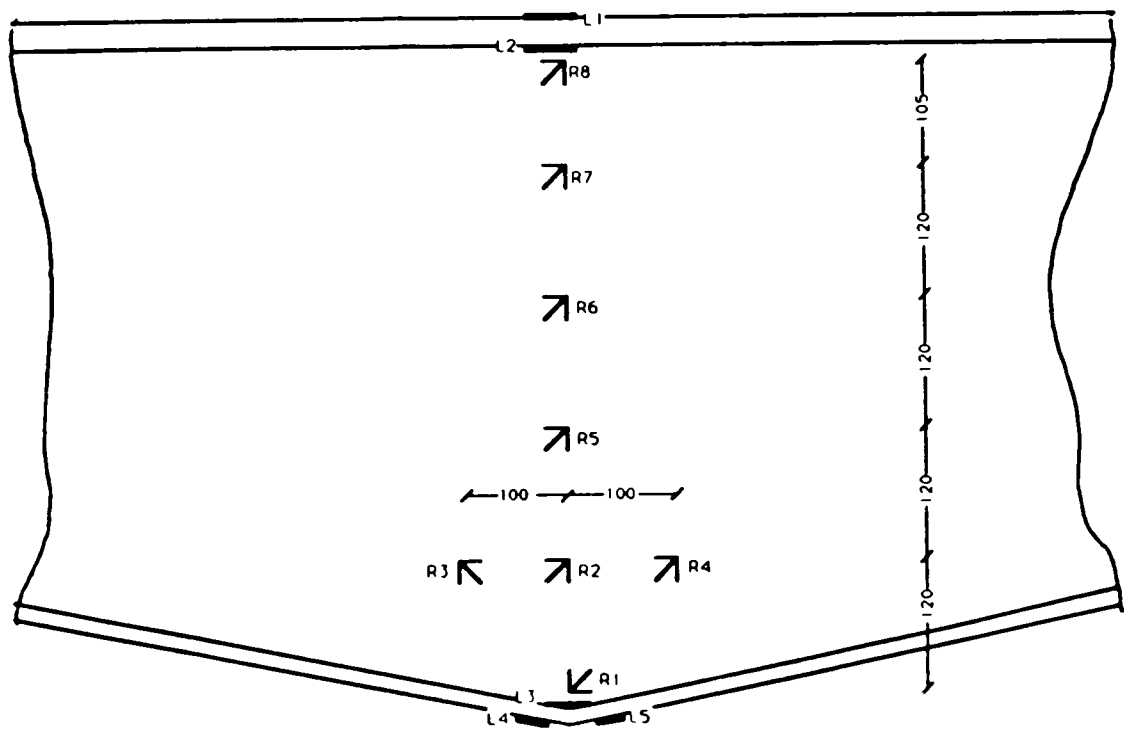


Figure 3.4 - Typical Central Section Strain-Gauge Positions. (dimensions in mm).

and 3mm corresponding to web height to thickness ratios, h_{max}/t_w , of: 92.86, 130. and 216.67, respectively.

To determine the values of the yield stress, σ_y , and the ultimate stress , σ_u , for the plates used in the fabrication of the beams, a series of coupon specimens was tested. The average yield and ultimate stress results for these plates are presented in table 3.1 below. Unfortunately, the coupons for these three beams had to be taken from plates of the tested specimens and indicated perhaps unreliable values well above the minima established for grade 43 steel.

Plate Thickness (mm)	3	5	7	12	25
σ_y (MPa)	322.97	376.75	398.53	309.10	282.58
σ_u (MPa)	492.35	497.52	461.38	434.89	454.96

Table 3.1 - Yield and Ultimate Stress Results: First Test Series.

The plastic neutral axis positions (measured from the top of the compressive steel flange) and the plastic moment for the 7, 5 and 3mm beams at the central point, $h = 650\text{mm}$, are given in table 3.2 below. All the figures presented in this table were calculated using the measured steel properties of table 3.1. The 7 and 5mm beams had their plastic neutral axes inside the top flange while the 3mm beam had it 74.18mm inside the web.

Beam Number	1	2	3
Web Thickness (mm)	7	5	3
Neutral Axis Position (mm)	99.18	24.21	21.19
Plastic Moment (kNm)	1169.34	1012.54	839.37

Table 3.2 - Geometric Properties: First Test Series.

Having established the dimensions and the mechanical and material properties for the specimens tested, a full description of the experimental behaviour, with the results and conclusions, will be given in the next sections.

3.3.1. Beam 1 ($t_w = 7\text{mm}$).

3.3.1.1. *Description of the Test.*

The beam sustained the load in the elastic range up to a value corresponding to 30% of its plastic capacity at the central section. After that, some plasticity began to develop and the beam also tended to come into contact with the lateral support frames. The failure mode corresponded to the formation of a plastic hinge at the edge of the tapered panel where the web had the minimum height, h_{min} . The plastic moment recorded in the test was equal to 84.13% of the plastic capacity at the point of minimum depth. This moment capacity was calculated with the yield stress obtained from the coupon tests. As explained before, the coupons were made from the plates of the tested specimens, giving rise to a higher nominal yield stress and consequently to a higher plastic capacity. The maximum moment recorded in the test corresponded to 81.1 % of the plastic capacity at the point of maximum height.

The general failure mode of the beam can be seen in plate 7, page 81. Plate 8, page 81, is a photograph taken after the test indicating the tendency to lateral movement described previously. It also demonstrates the lateral frames' efficiency in preventing a lateral buckling failure.

The decision to stop the test was taken when the central vertical deflection reached approximately 450mm, which was certainly sufficient to represent a serviceability limit state, as it was well over the limit value of $\text{span}/50$, i. e., 220mm. The final deformed shape of the beam is presented in figure 3.6, page 83, where it is possible to note the large vertical deflection attained by the beam and the high localized curvatures at the sections in line with the edges of the tapered panel.

At large deflections a crack 20mm long developed in the butt weld that connected the two halves of the tapered web plate, shown in plate 9, page 82. The web had very little lateral displacement and no vertical or any other kind of buckling could be seen. It remained straight, but had yielded to a very considerable degree.

3.3.1.2. Results.

A load against vertical deflection diagram for the central point of the beam is shown in figure 3.5, page 83. The final lateral displacements measured are shown in figure 3.7, page 84. The maximum lateral deflection measured in the web was in the order of 6mm, which can be considered almost negligible when compared to the other dimensions of the beam. It can be partially explained by the twist that occurred in the beam due to its tendency to buckle laterally. These very small lateral displacements can also be seen in figures 3.8.a and 3.8.b, page 84, where the deflection at some lateral transducers is plotted against the applied moment. The correlation between the number of the transducer and its location is given in figure 3.2, page 74.

The maximum vertical compressive force available to cause vertical buckling of the web occurs when the bottom flange has completely yielded. To prove that the vertical compressive force had achieved its maximum value prior to the end of the test, strain-gauges were positioned on the top and bottom flanges at the centre of the beam. Their readings are shown in figure 3.9, page 85, where the corresponding stresses are plotted against the applied moment. It is very clear that at the gauges L3e, L3w, L4 and L5, located in the bottom flange, the yield stress was achieved very early in the test, proving that the flange was fully plastified well before the end of the experiment.

The presence of the plastic hinge at the edge of the tapered panel can be deduced from figure 3.10, page 85. The correlation between the number of the strain gauge and its location is given in figure 3.3, page 78. Almost all the strain-gauges have reached the yield stress, proving that this section of the beam is almost fully plastified, corresponding to the formation of the plastic hinge.

To give a fuller picture of the stress pattern in the web, contour plots of the average in-plane stresses determined by the rosettes will be presented. The non-dimensionalised longitudinal stresses σ_{xx}/σ_y and vertical stresses σ_{yy}/σ_y present in the web during the final loading stage of the beam are presented in figures 3.11.a and 3.11.b (page 86), respectively. The growth of the equivalent (Von Mises) stress σ_{eq}/σ_y is shown in figures 3.12.a, 3.12.b, page 86, and 3.13.a, 3.13.b, page 87. These four figures correspond to loading stages equivalent to 18.9%, 39.8%, 60.2% and 80.5% of the plastic moment capacity of the beam at the central section.



Plate 7 - Failure of Beam 1 ($t_w = 7\text{mm}$).



Plate 8 - Detail of the failure of Beam 1 ($t_w = 7\text{mm}$).

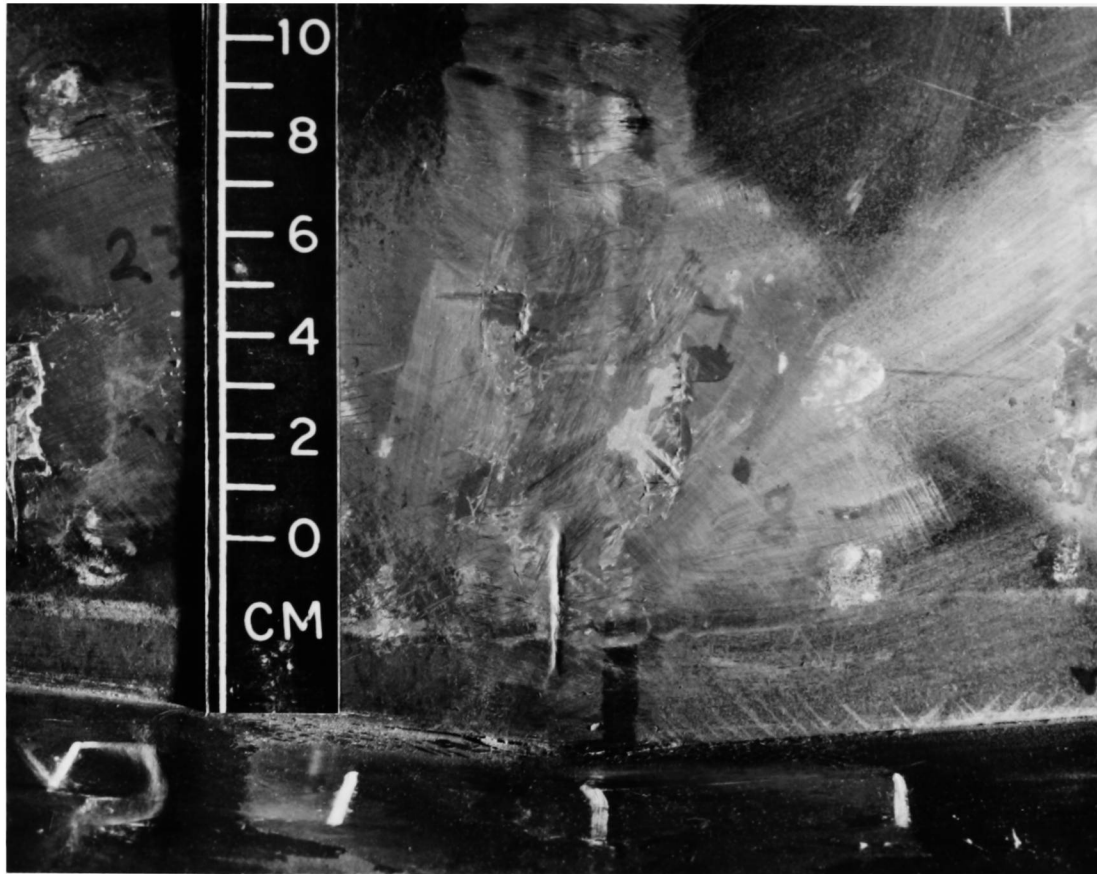


Plate 9 - Weld crack in Beam 1 at very large strains.

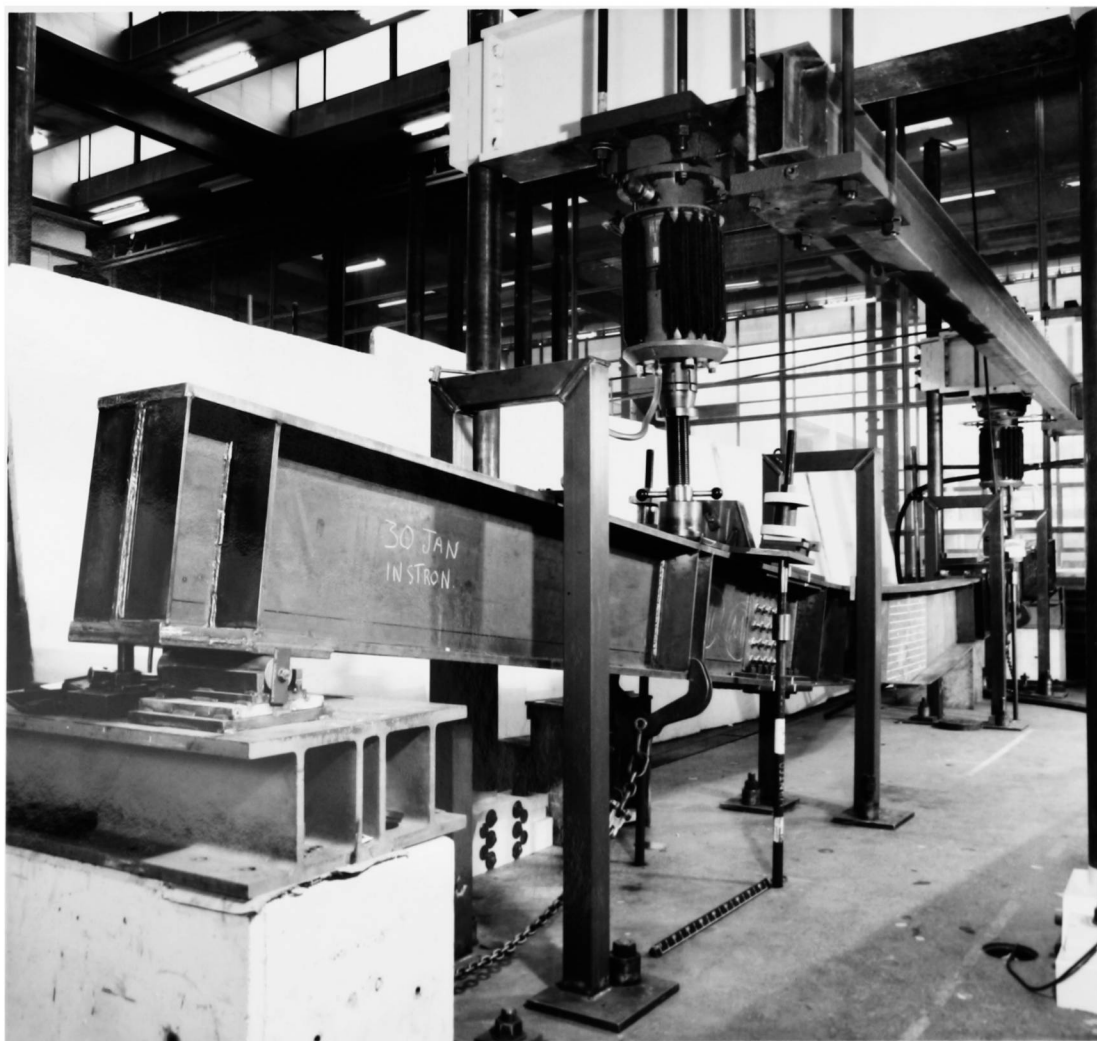


Plate 10 - Failure of Beam 2 ($t_w = 5\text{mm}$).

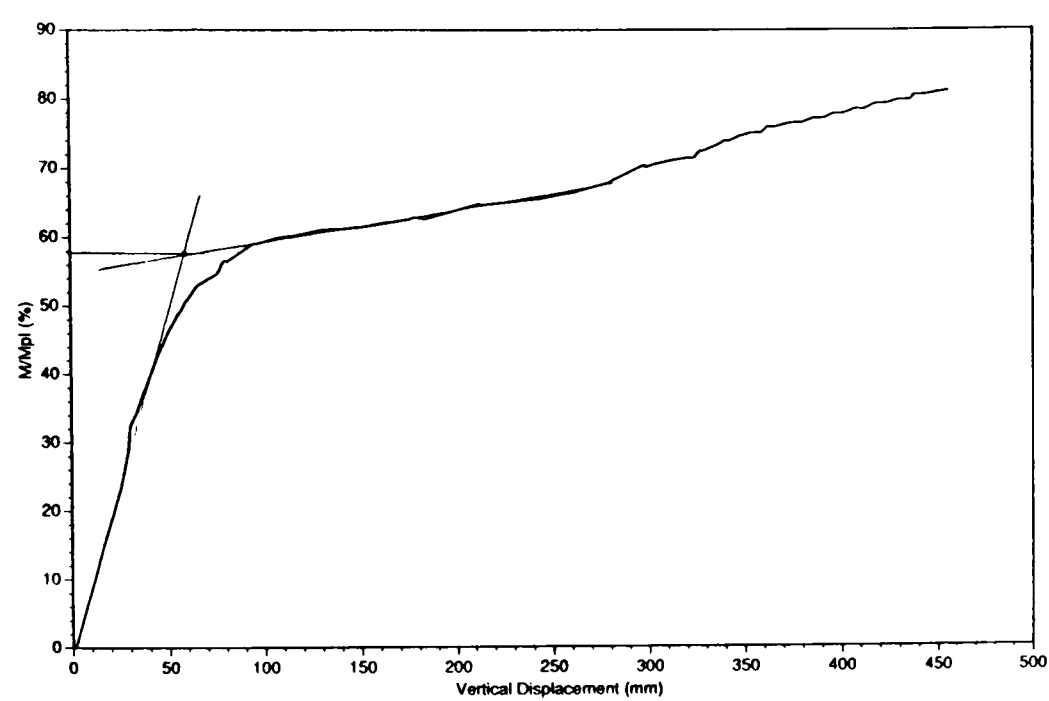


Figure 3.5 - Load vs. Central Vertical Deflection for Beam 1.

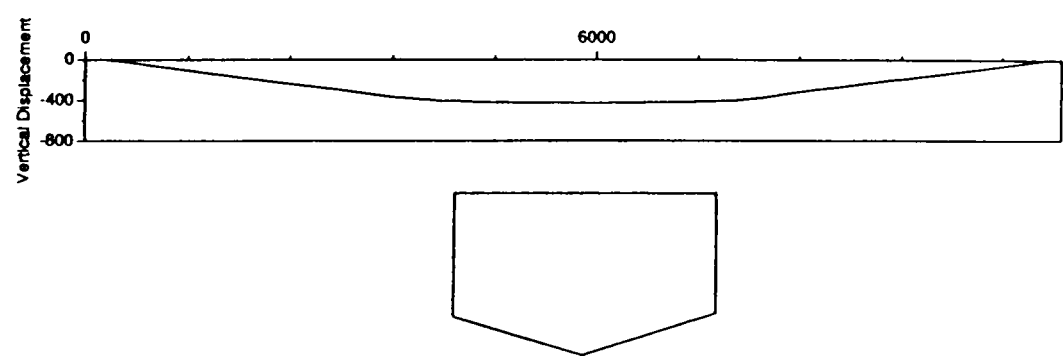


Figure 3.6 Final Deformed Shape of Beam 1 (mm).

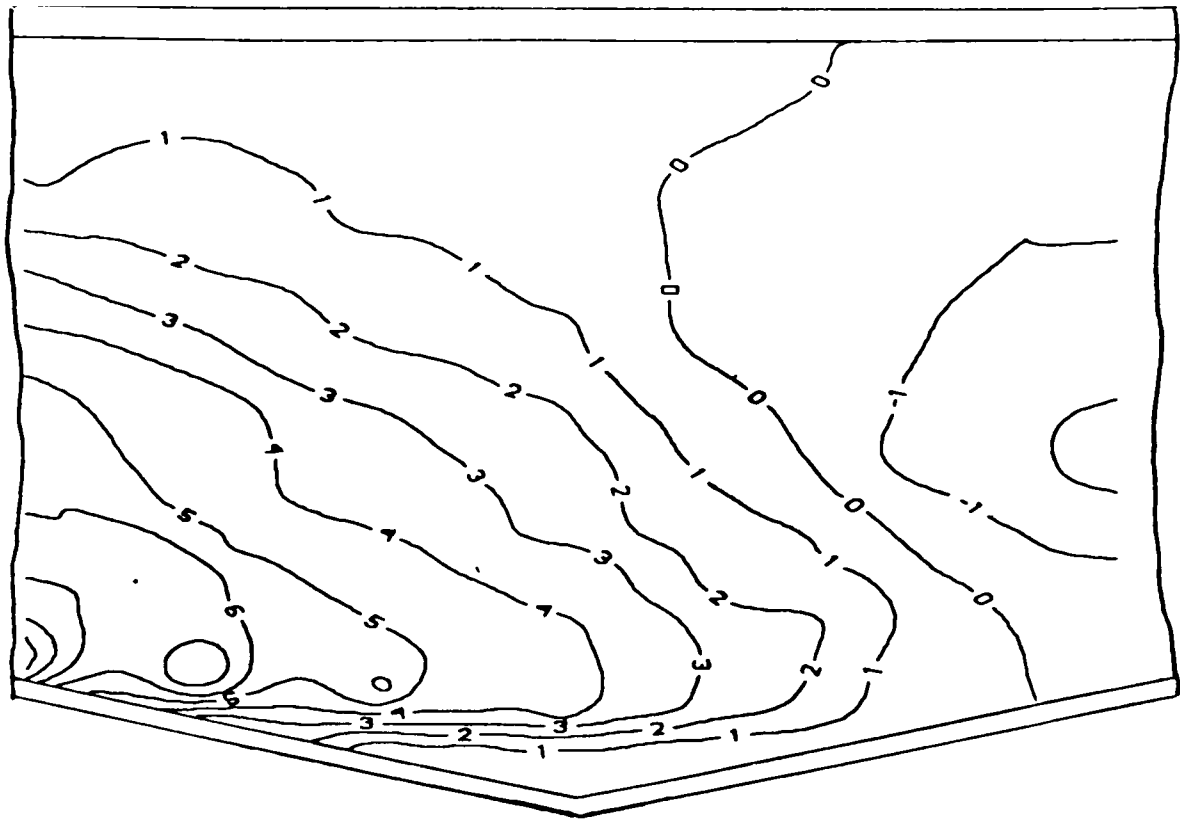


Figure 3.7 - Final Lateral Deflection Pattern of Beam 1 (mm).

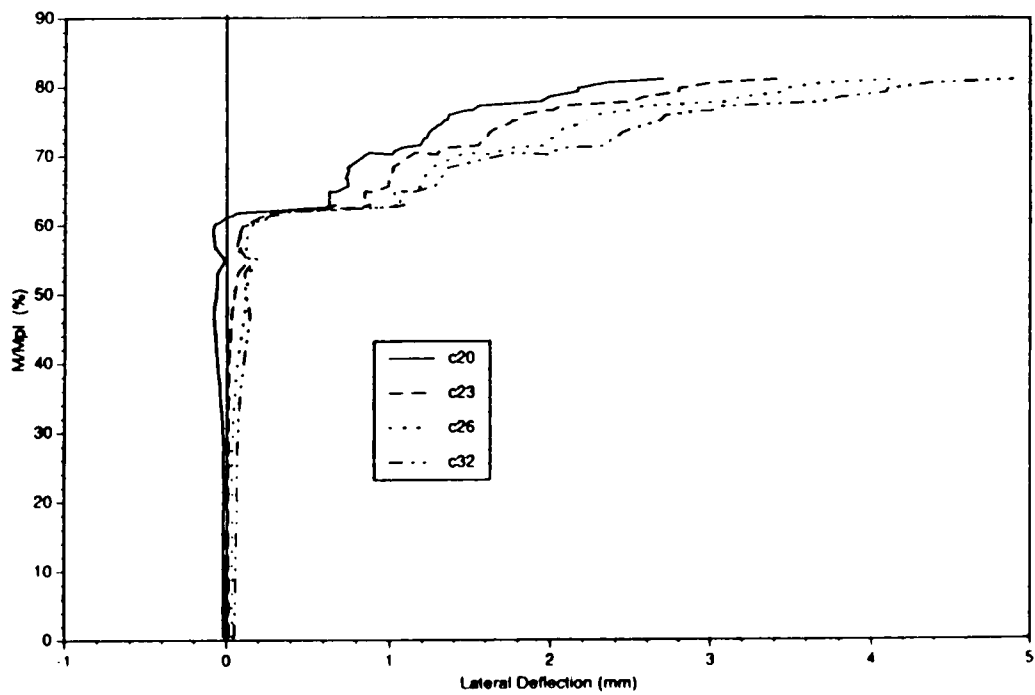
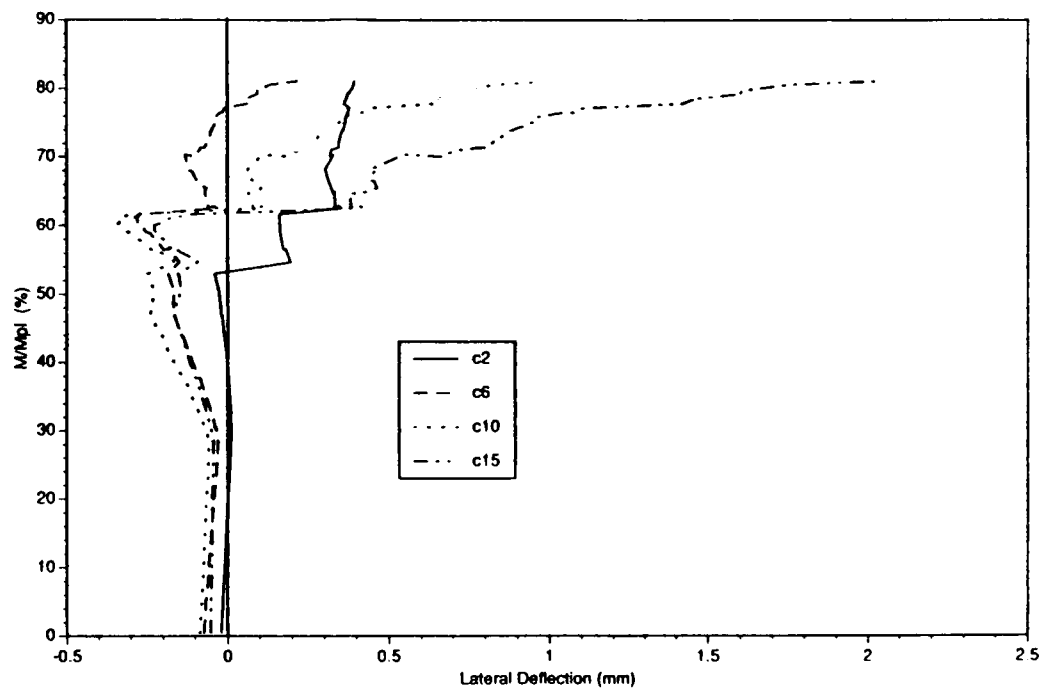


Figure 3.8.a - Load vs. Lateral Displacement of Points 2, 6, 10 and 15 (Beam 1).

Figure 3.8.b - Load vs. Lateral Displacement of Points 20, 23, 26 and 32 (Beam 1).

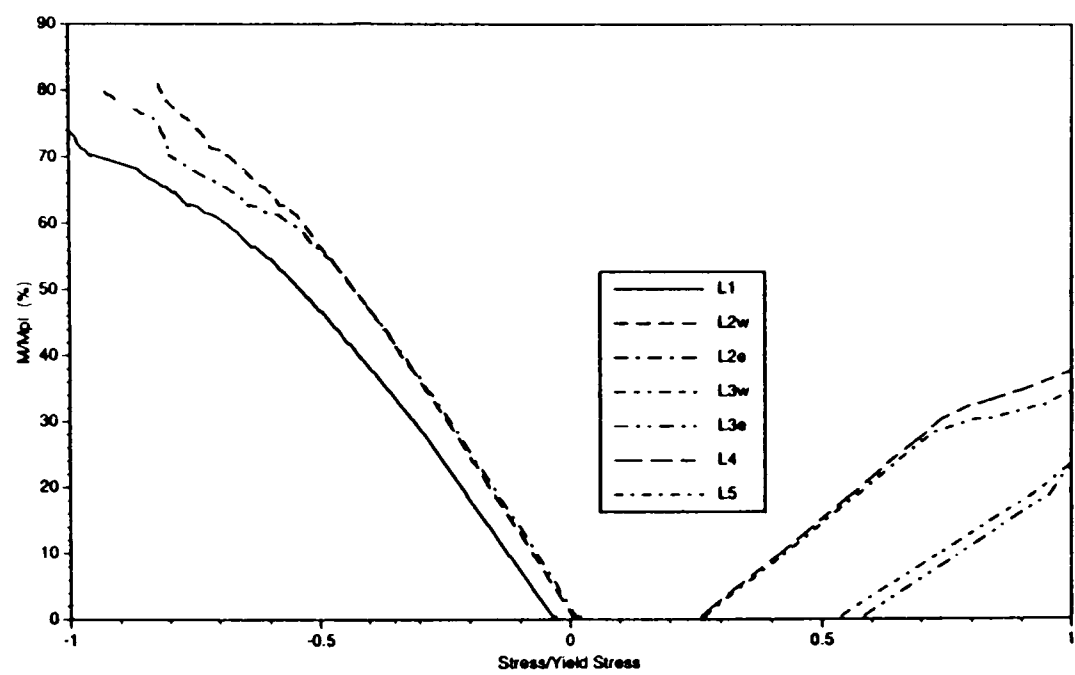


Figure 3.9 - Central Linear Strain Gauge Results (Beam 1).

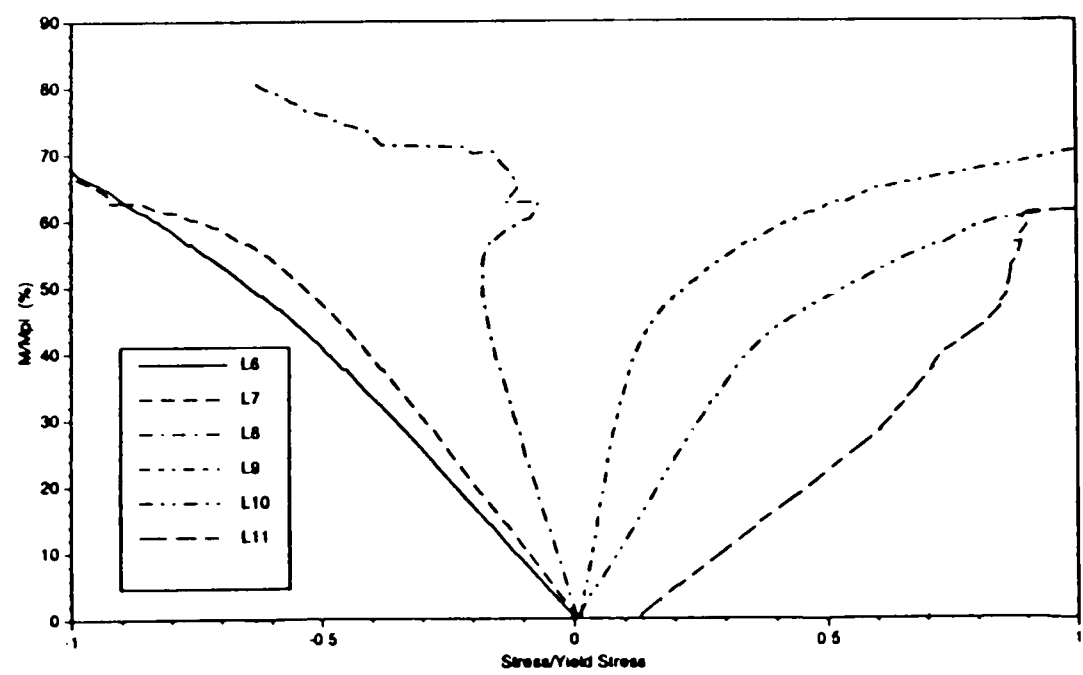
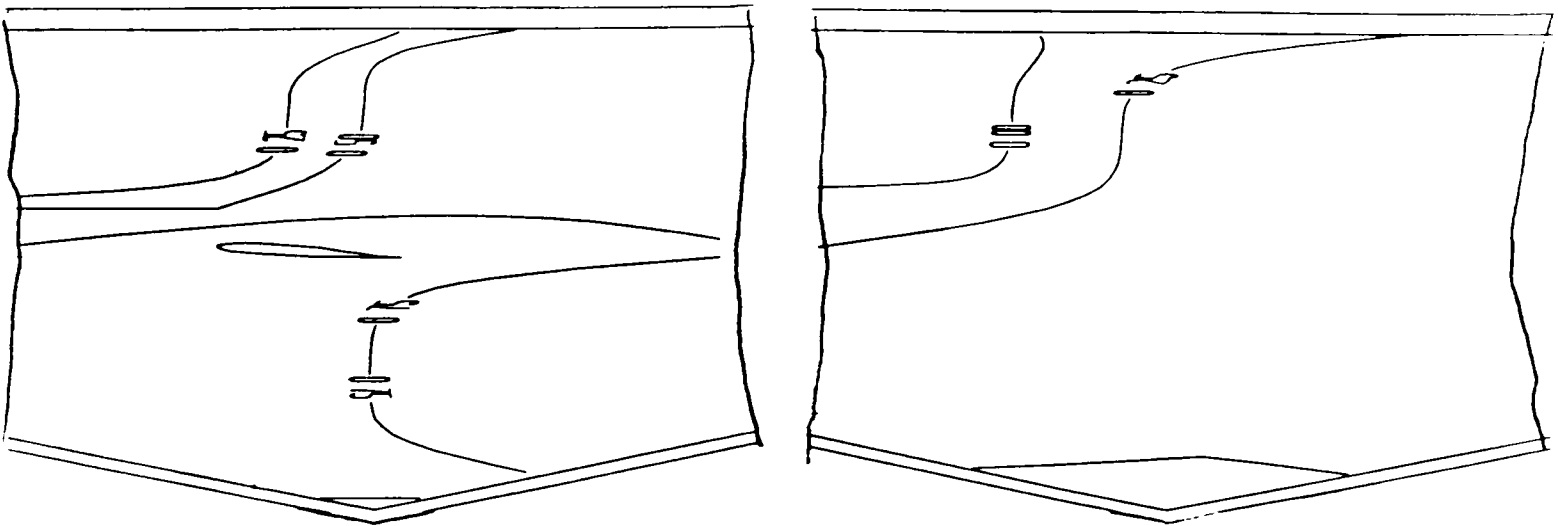
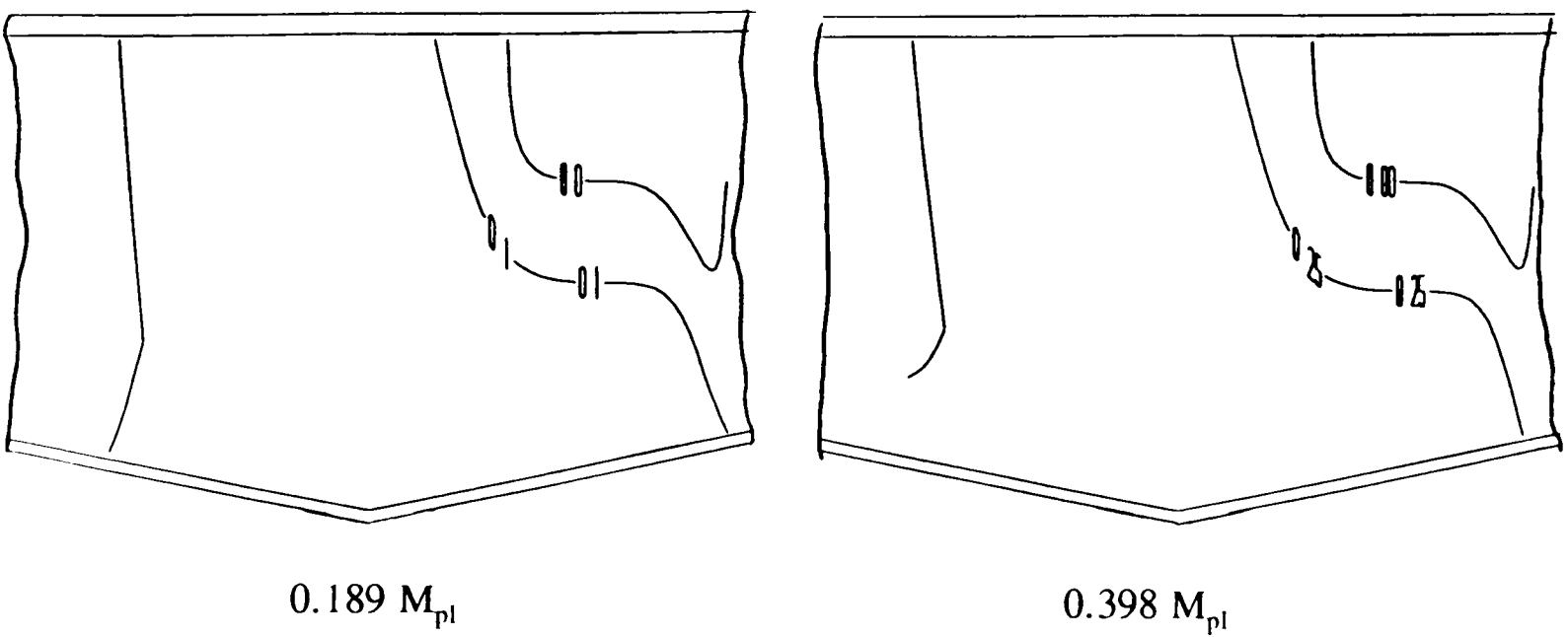


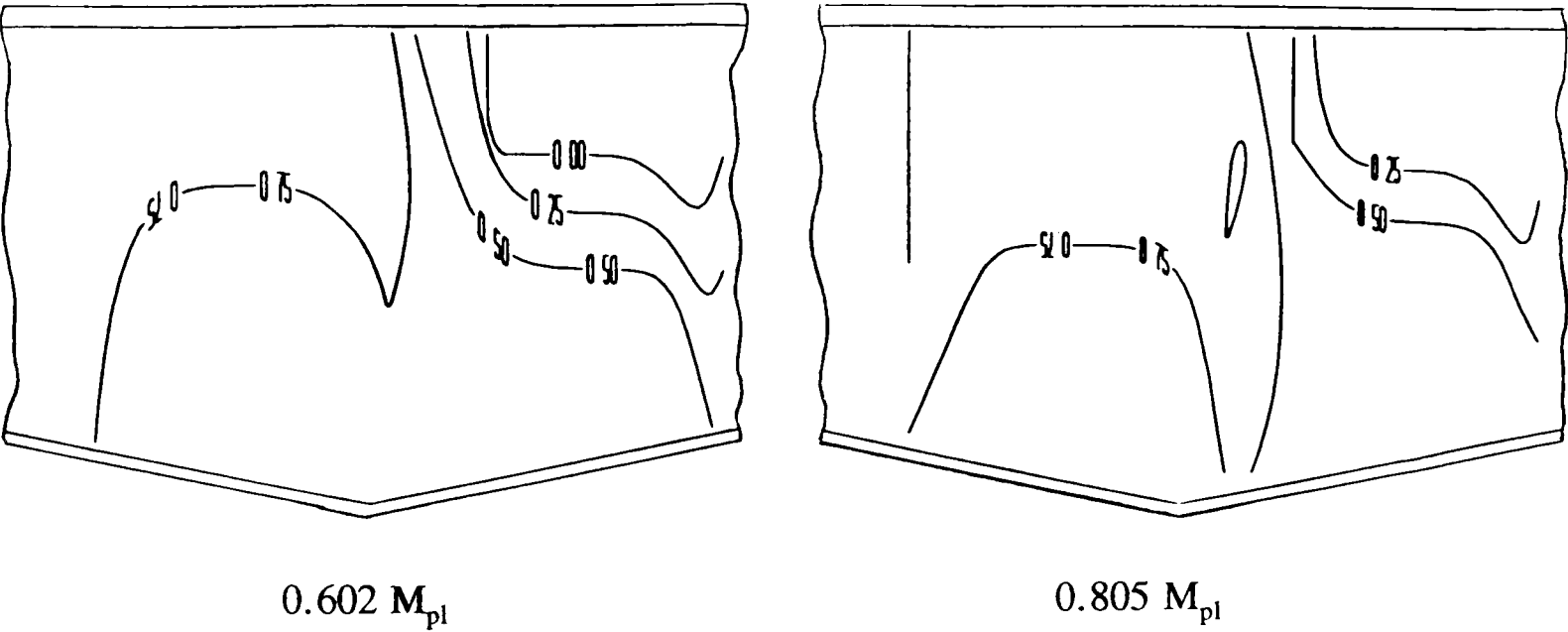
Figure 3.10 - Panel Edge Linear Strain Gauge Results (Beam 1).



Figures 3.11.a and 3.11.b - Normalized Longitudinal and Vertical Stresses in the Web (Beam 1).



Figures 3.12.a and 3.12.b - Normalized Equivalent Stresses in the Web (Beam 1).



Figures 3.13.a and 3.13.b - Normalized Equivalent Stresses in the Web (Beam 1).

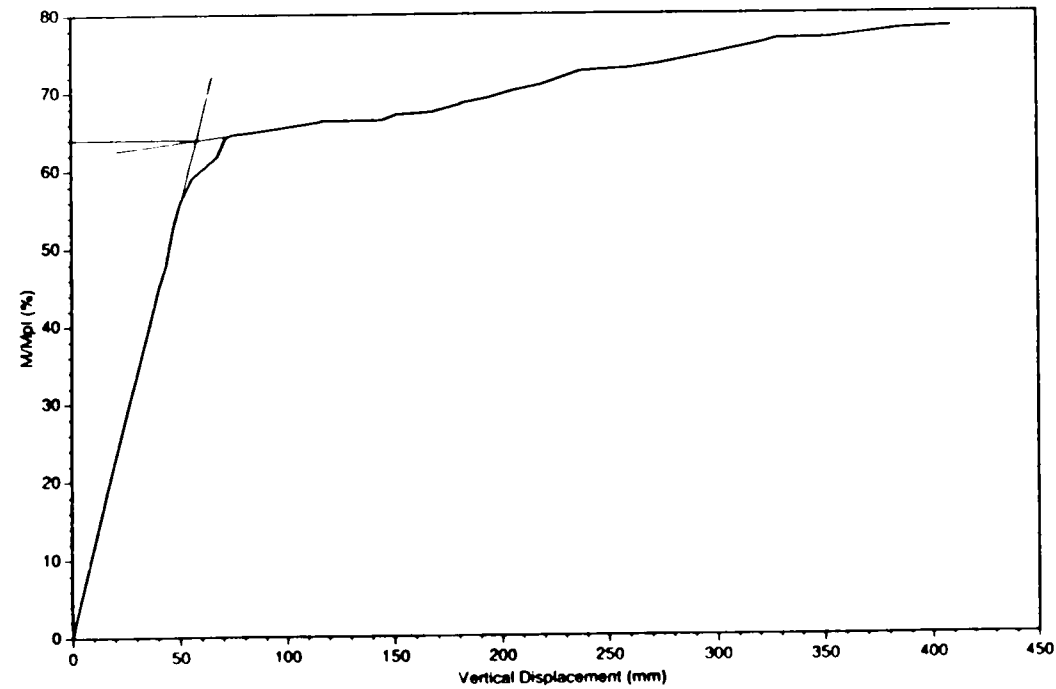


Figure 3.14 - Load vs. Central Vertical Deflection for Beam 2.

3.3.2. Beam 2 ($t_w = 5\text{mm}$).

3.3.2.1. *Description of the Test.*

The beam remained elastic up to a value of 46% of the plastic moment at the central section. Very large vertical deflections followed from about $0.6 M_{pl}$ at the centre section, showing the decrease of stiffness imposed by the growth of plasticity into web of the beam. The plastic moment recorded in the test was equal to 91.6% of the nominal plastic capacity at the point of minimum depth, again calculated with the yield stress obtained with the coupon tests which almost certainly overestimated the true figure. The maximum moment recorded in the test corresponded to 78.11% of the plastic capacity at the point of maximum height. The failure mode was again related to the appearance of a plastic hinge at the edge of the tapered web. The failure can be observed in plate 10, page 82.

The decision to stop the test was again based on a serviceability condition: when the beam's central vertical deflection reached 420mm the test was ended. Again this final vertical deflection was greater than the span/50 limit of 220mm. The residual vertical deflection of the beam after the load was taken off is presented in figure 3.15, page 89. It is again evident that the failure mode was associated with very large plastic deflections which remained on unloading.

The 5mm web also remained straight and showed no signs of even a small buckle. No cracks were detected in the butt weld on the centre line of the tapered web junction. In the final moments of the test, it was possible to hear and see the falling of mill-scale from the web and flanges of the tapered beam. This confirmed the existence of large plastic strains/stresses in the surface region of the girder.

3.3.2.2. *Results.*

A load against central vertical deflection curve for the beam is plotted in figure 3.14, page 87. The last lateral displacement pattern recorded is shown in figure 3.16, page 89. The lateral transducer plate was taken off the web before the very end of the test to avoid damage to the instruments. The maximum lateral deflection of the web was found to be in the order of 5mm, which could again be considered very small when compared to the beam's other dimensions. No sign of lateral torsional buckling was noted during the test. A selection of key lateral displacements plotted against the applied moment can be seen in figures 3.17.a and 3.17.b, page 90.

The vertical compressive force, that appears in the web to cause vertical buckling, also reached its maximum value well before this test was declared finished. This can again be explained with the aid of the linear strain-gauge results for the bottom flange, at the centre of the beam. Strain-gauges L4 and L5 reached the yield stress when the applied moment was about 55% of the plastic moment at the central section, as can be observed from figure 3.18, page 90. The plastic hinge formation at the edge of the tapered panel can be visualized with the aid of figure 3.19, page 91. It is clear that this section of the tapered girder was almost completely plastified before the end of the test.

The non-dimensional longitudinal and vertical stress contour patterns in the central portion of the web during the last loading stage are presented in figures 3.20.a and 3.20.b, page 91. The non-dimensional equivalent (Von Mises) contour stress pattern for the central part of the tapered web is displayed in figures 3.21.a,

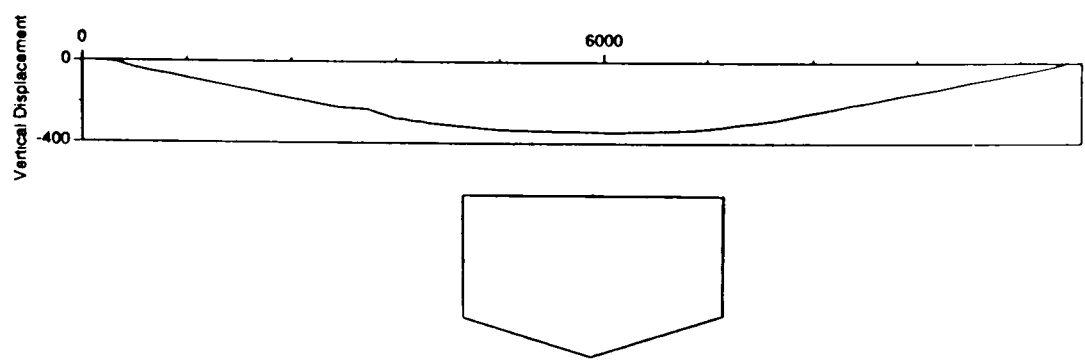


Figure 3.15 - Final Deformed Shape of Beam 2 (mm).

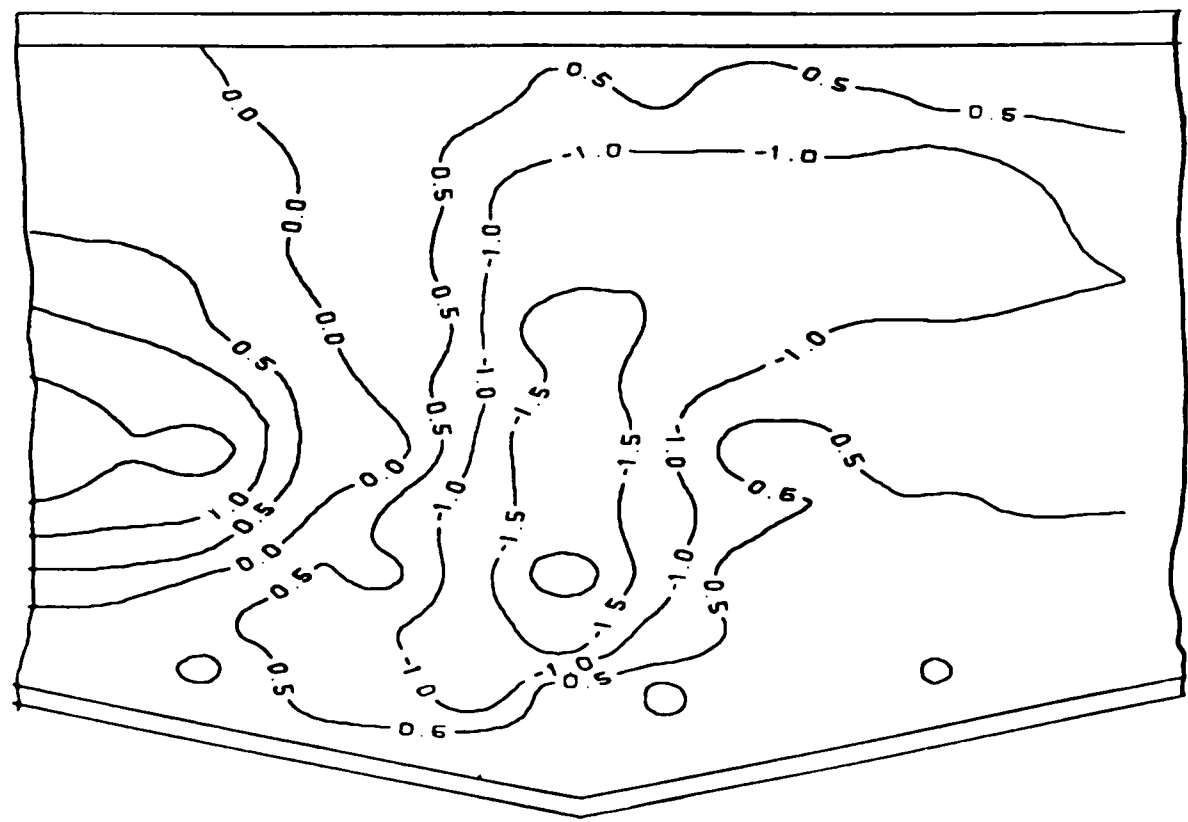


Figure 3.16 Final Lateral Deflection Pattern of Beam 2 (mm).

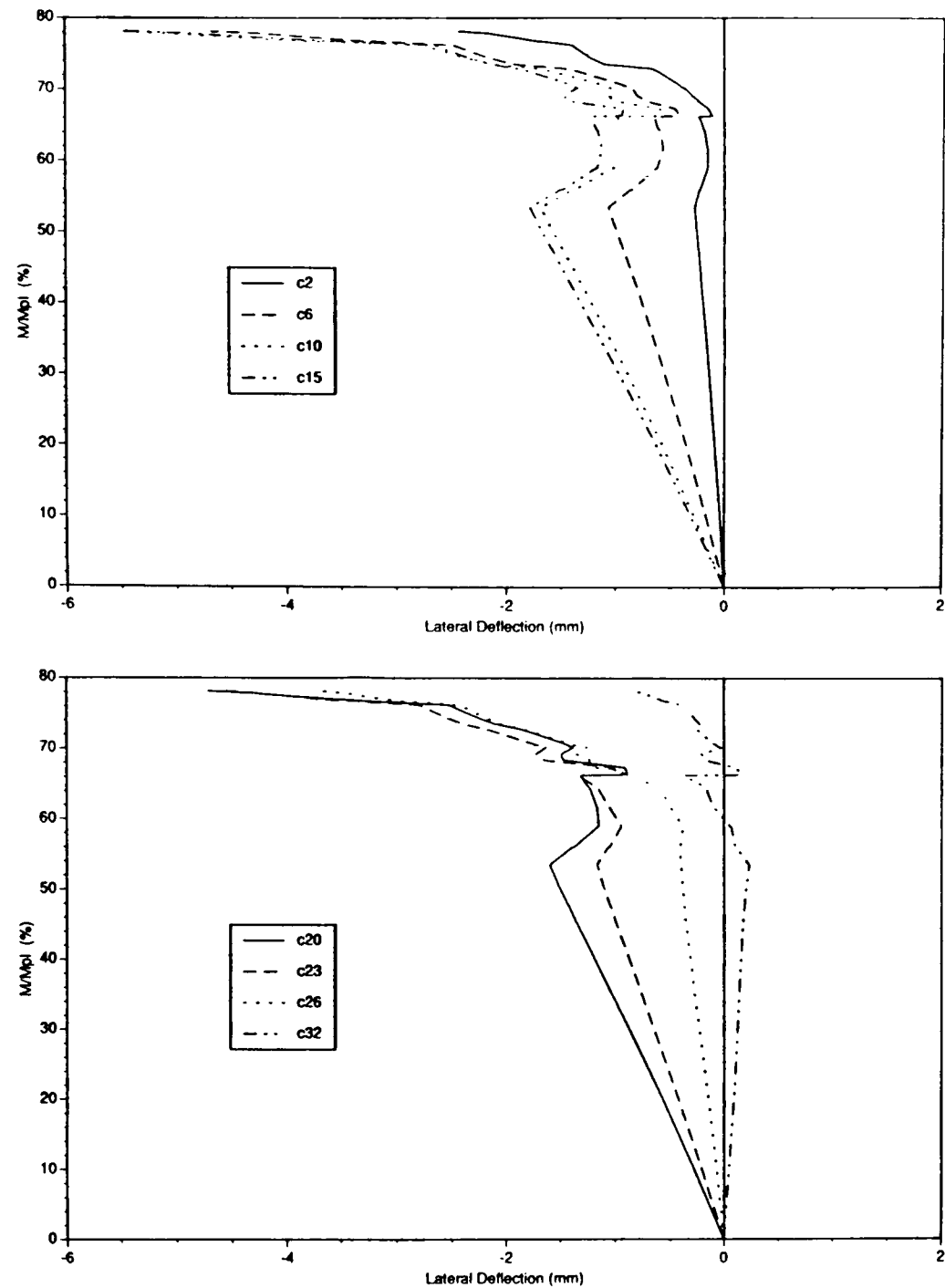


Figure 3.17.a - Load vs. Lateral Displacement of Points 2, 6, 10 and 15.
Figure 3.17.b - Load vs. Lateral Displacement of Points 20, 23, 26 and 32.

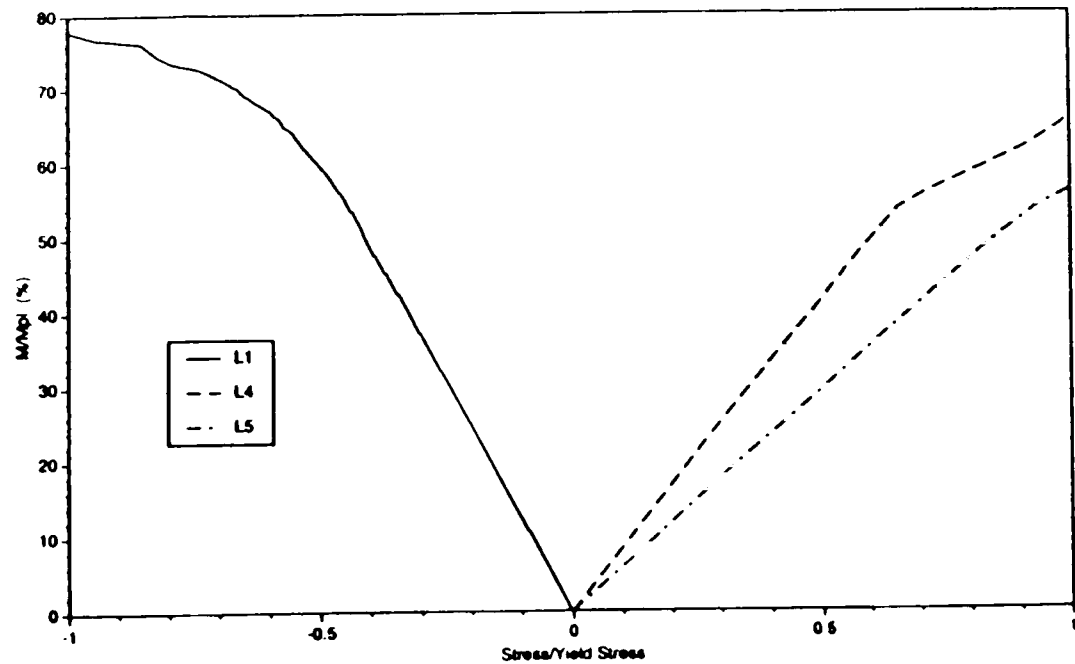


Figure 3.18 - Central Linear Strain Gauge Results (Beam 2).

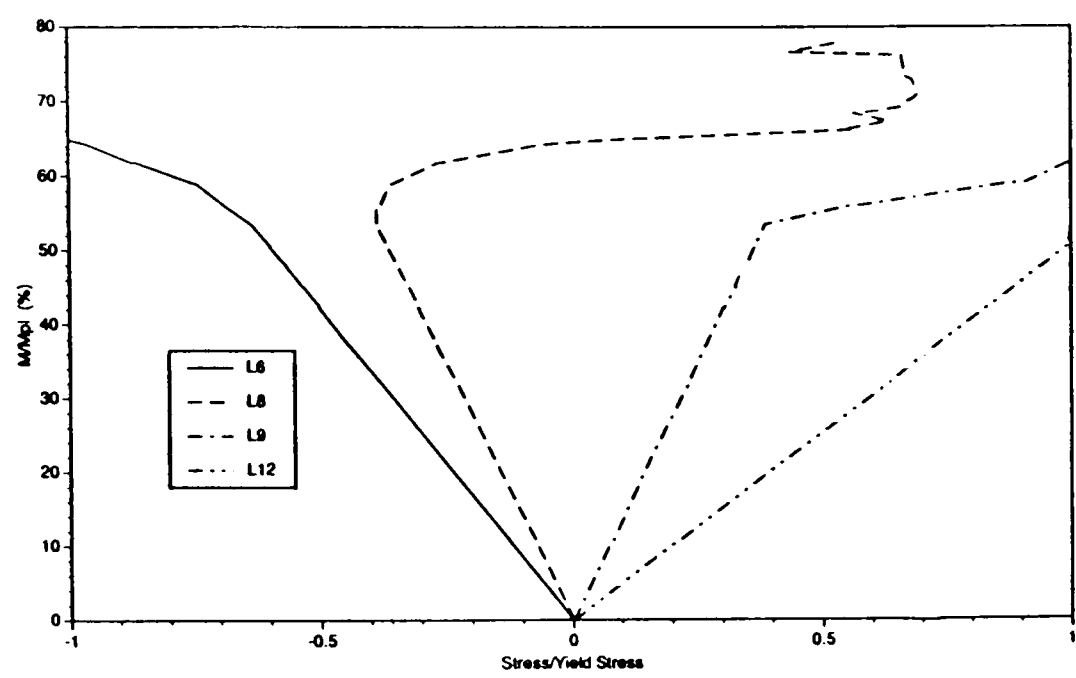
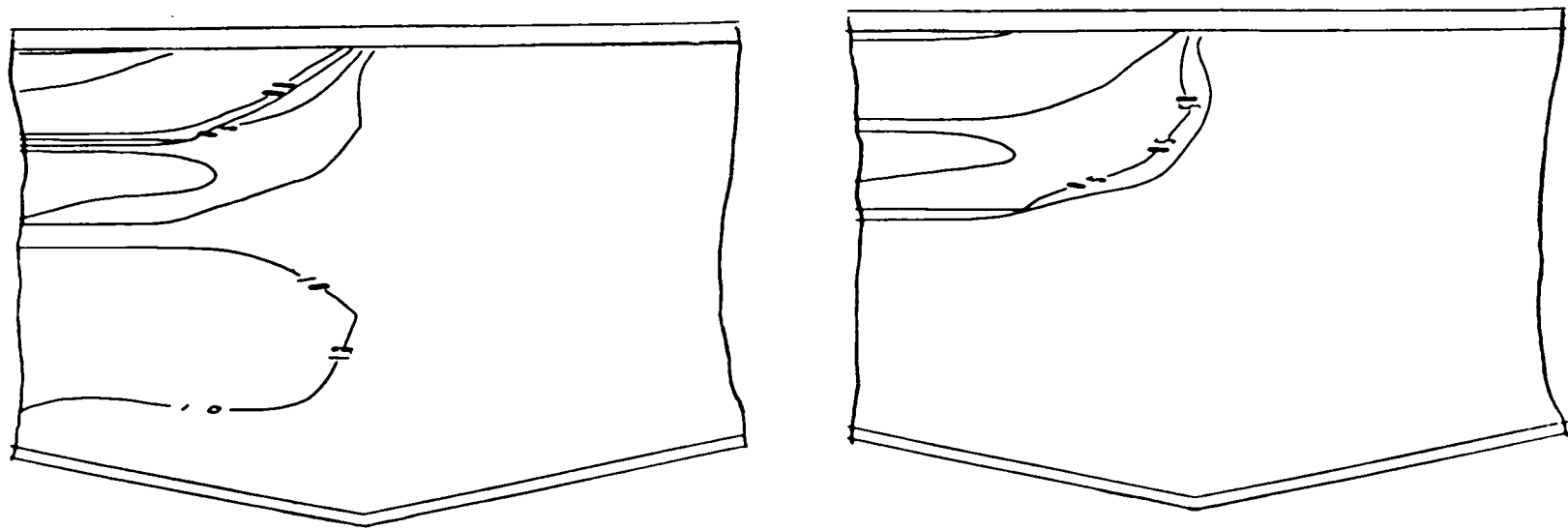


Figure 3.19 - Panel Edge Linear Strain Gauge Results (Beam 2).



Figures 3.20.a and 3.20.b - Normalized Longitudinal and Vertical Stresses in the Web (Beam 2).

3.21.b, 3.22.a and 3.22.b, page 93. These four figures correspond to loading stages equivalent to 21.4%, 50.6%, 64.9% and 78.1% of the beam's central plastic moment capacity.

3.3.3. Beam 3 ($t_w = 3\text{mm}$).

3.3.3.1. *Description of the Test.*

From the very early stages of the test, it was noted that the web did not stay straight, buckling adjacent to the slope change in the lower flange. Nevertheless, the beam remained elastic up to a value corresponding to 35% of its plastic capacity at the central section. The vertical buckle continued to grow, changing significantly the shape of the web. The web response was very sensitive to every load increase, displacing more and more in the lateral direction. When the load reached 59% of M_{pl} , the beam almost completely lost its capacity to sustain further load. In addition, the vertical displacements also increased significantly, together with a continuous progress of the web buckling. Around 69% of M_{pl} at the point of maximum height, the tapered girder reached its failure point. At this stage, the flanges started to act as two separate plates, with no effective connection between them. The web was completely buckled, and the load dropped as the lever arm between top and bottom flanges fell.

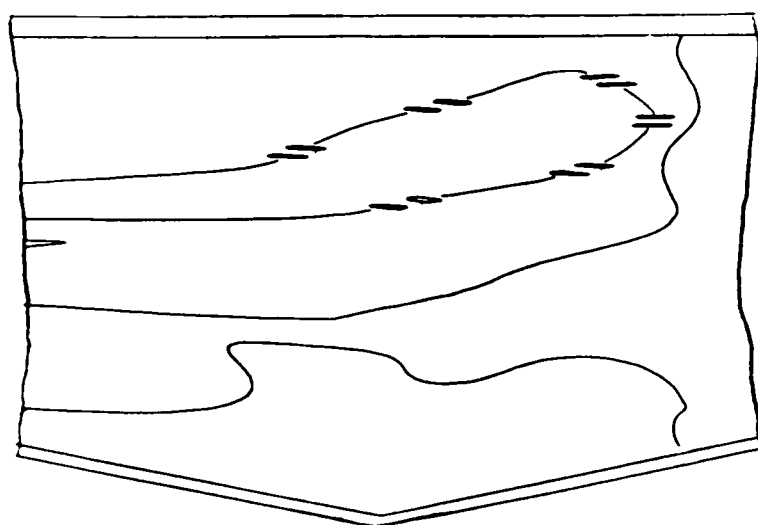
A general view of the failure mode of the beam is presented in plate 11, page 94. A detailed view of the east and west faces of the deformed girder is displayed in plate 12, page 94 and plate 16, page 96. It is also interesting to observe the contrast between the undeformed and deformed east and west sides, displayed in plate 13, plate 14, and plate 15, page 95. In the centre of the beam, the buckling mode in the vertical direction was characterized by the shape associated with 1.5 cycles of a sine function. In the longitudinal direction the failure shape was equivalent to just one half of a sine wave.

The final vertical displacement of the whole beam is summarised in figure 3.24, page 97. In this diagram only one "hinge", at the central section, is present instead of the two hinges, at the edges of the tapered panel, noted in the first two beams. The bottom flange buckled into the web, as can be observed in figures 3.25.a and 3.25.b, page 98. These figures represent the east and west faces of the tapered beam.

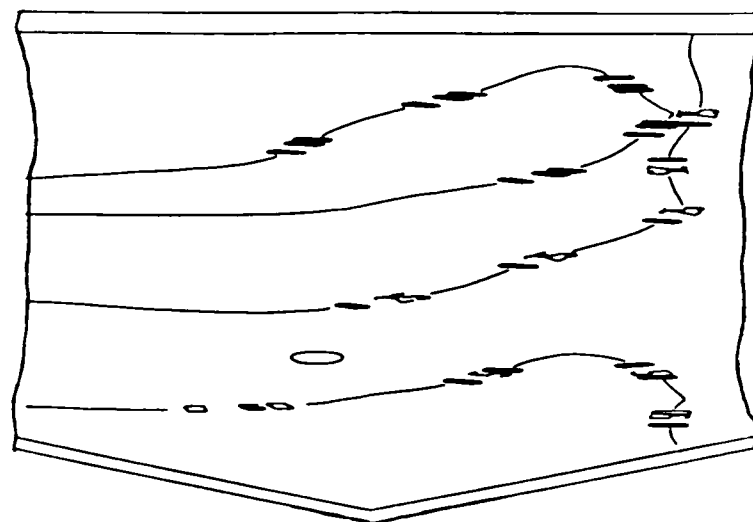
In the final stages of the test another crack, similar to the one present in beam 1, appeared in the butt weld that connected the plates which formed the tapered panel. The main difference was in its size and speed of growth (plate 17, page 96). At the end of the test it was more than 200mm long and 10mm wide. If the test had continued, the crack would probably have separated the two halves of the web plate completely.

3.3.3.2. *Results.*

The load against central vertical deflection plot for the beam is shown in figure 3.23, page 97. The maximum vertical displacement of the girder was $\cong 240\text{mm}$. The configuration of the tapered web at a very late stage is exhibited in figure 3.26.a, page 99. To make the visualization of the deformed shape clearer, a perspective view at this point is shown in figure 3.26.b, page 99. As in the case of beam 2, the transducer plate was taken off the web before the test was finished to avoid damage. The very large displacements recorded by the lateral transducers can

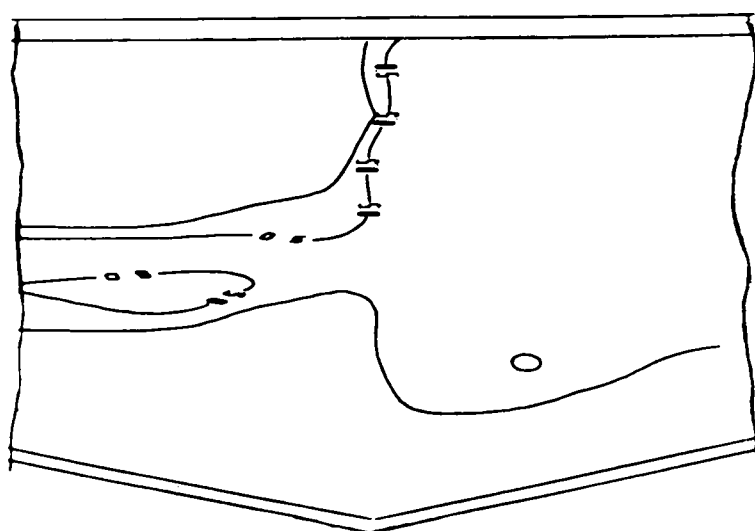


$0.214 M_{pl}$

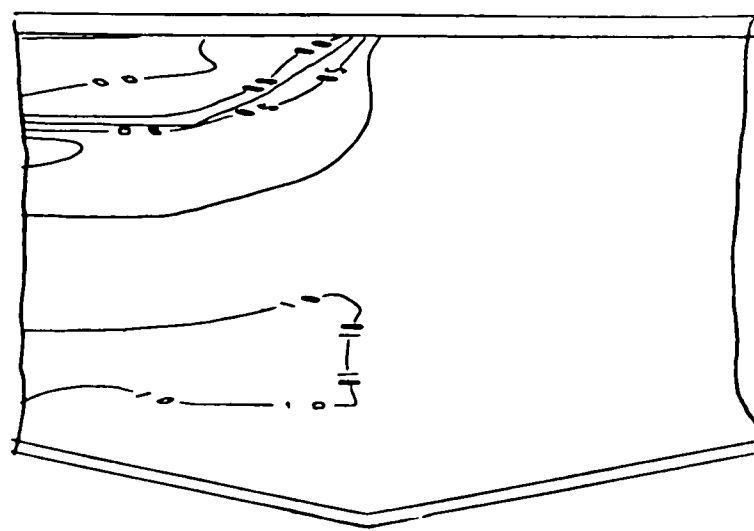


$0.506 M_{pl}$

Figures 3.21.a and 3.21.b - Normalized Equivalent Stresses in the Web (Beam 2).



$0.649 M_{pl}$



$0.781 M_{pl}$

Figures 3.22.a and 3.22.b - Normalized Equivalent Stresses in the Web (Beam 2).



Plate 11 - Failure of Beam 3 ($t_w = 3\text{mm}$).

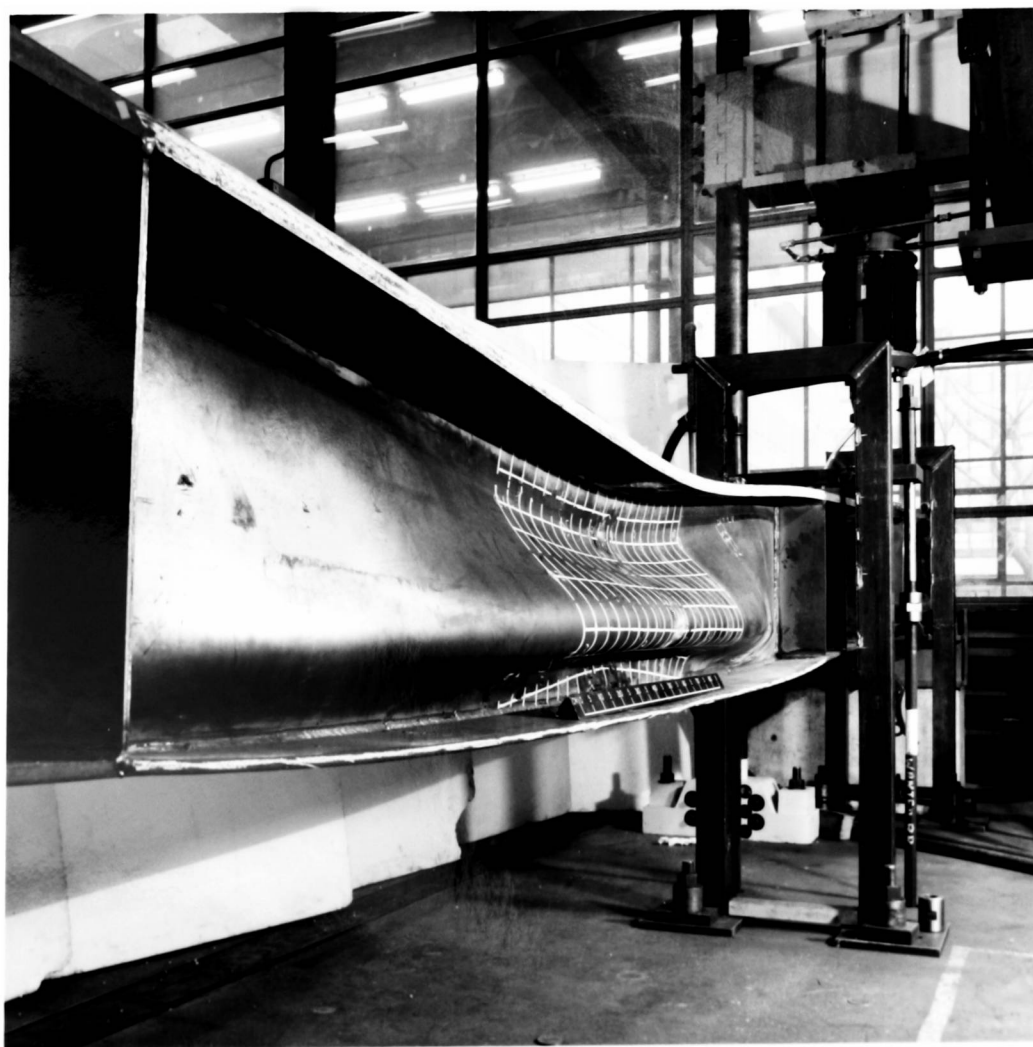


Plate 12 - Beam 3: detail of the failure (view from East).

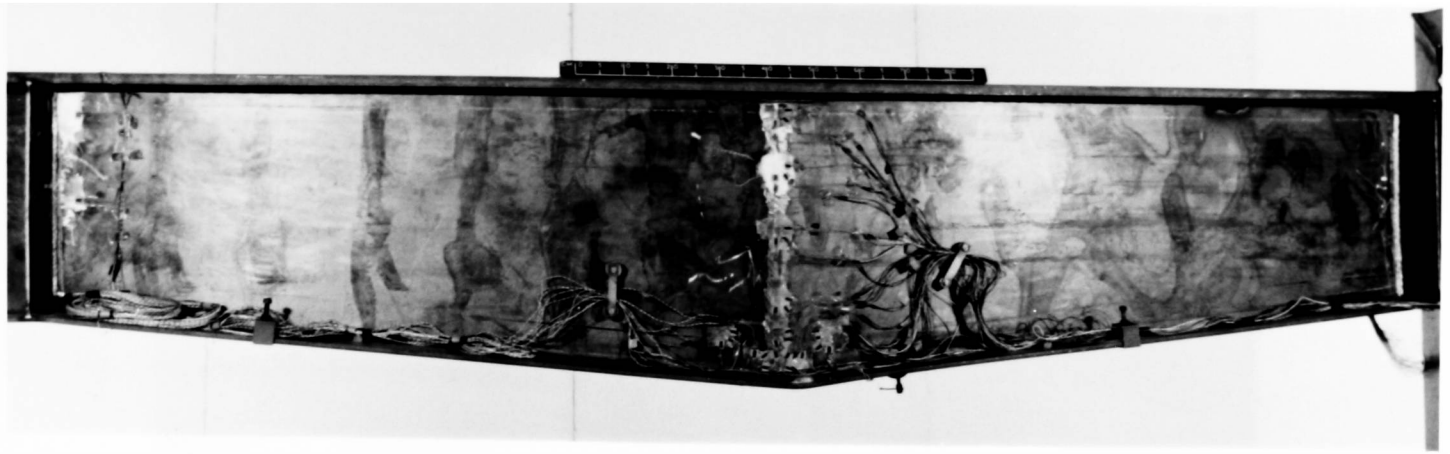


Plate 13 - Tapered section of Beam 3 - Initial.

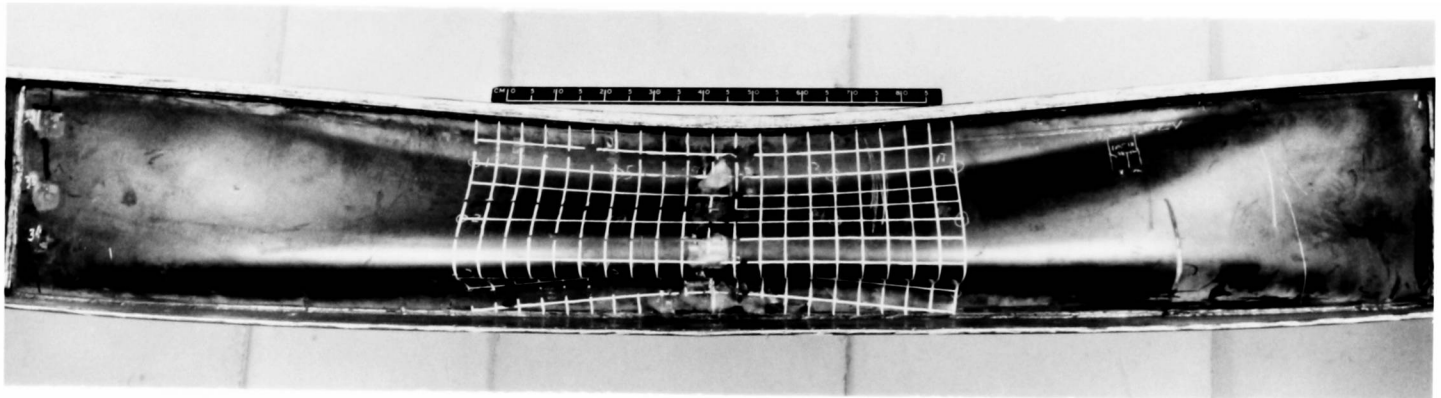


Plate 14 - Tapered section of Beam 3 - View of failure from East.

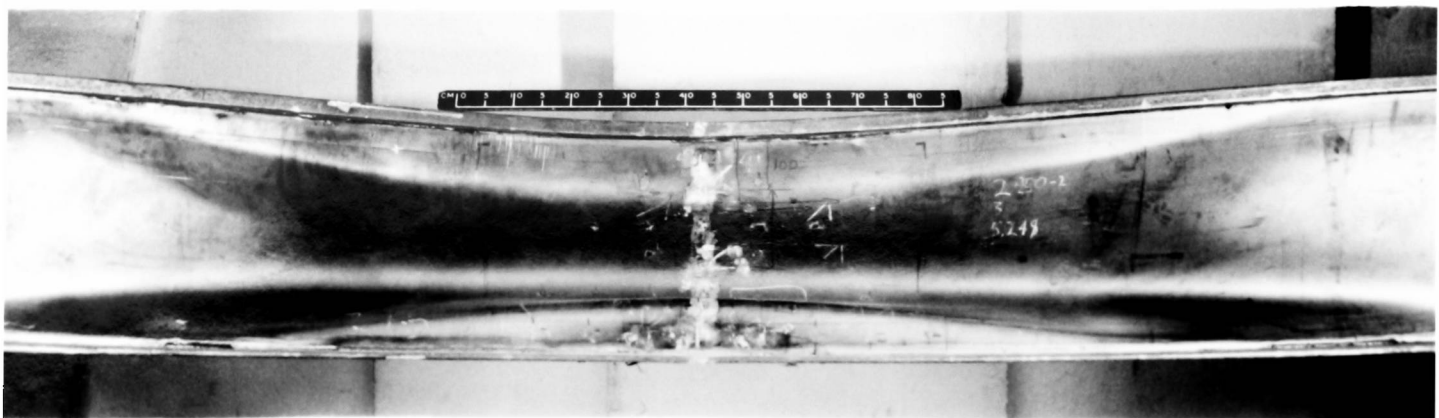


Plate 15 - Tapered section of Beam 3 - View of failure from West.

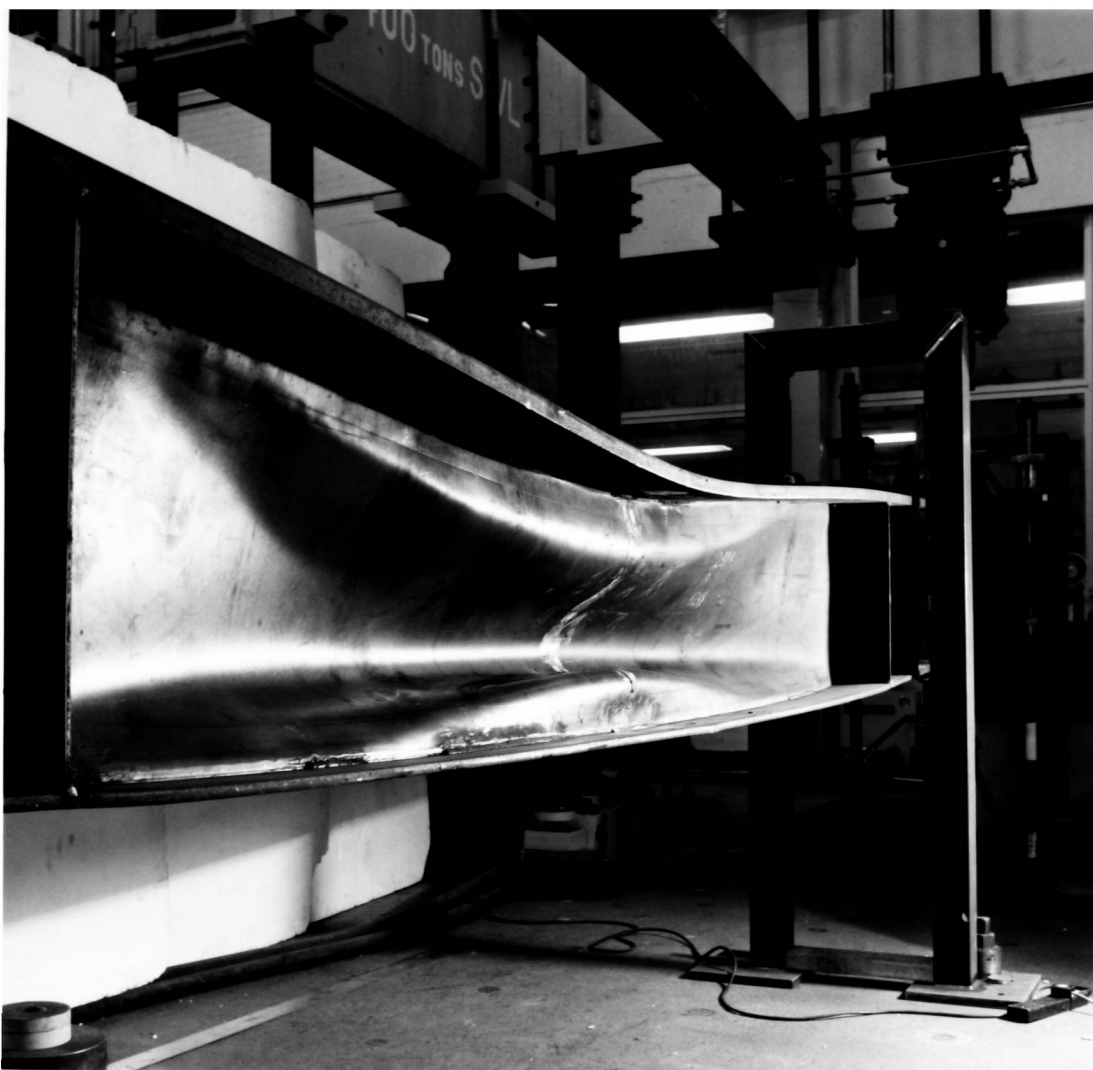


Plate 16 - Beam 3: detail of the failure (view from West).

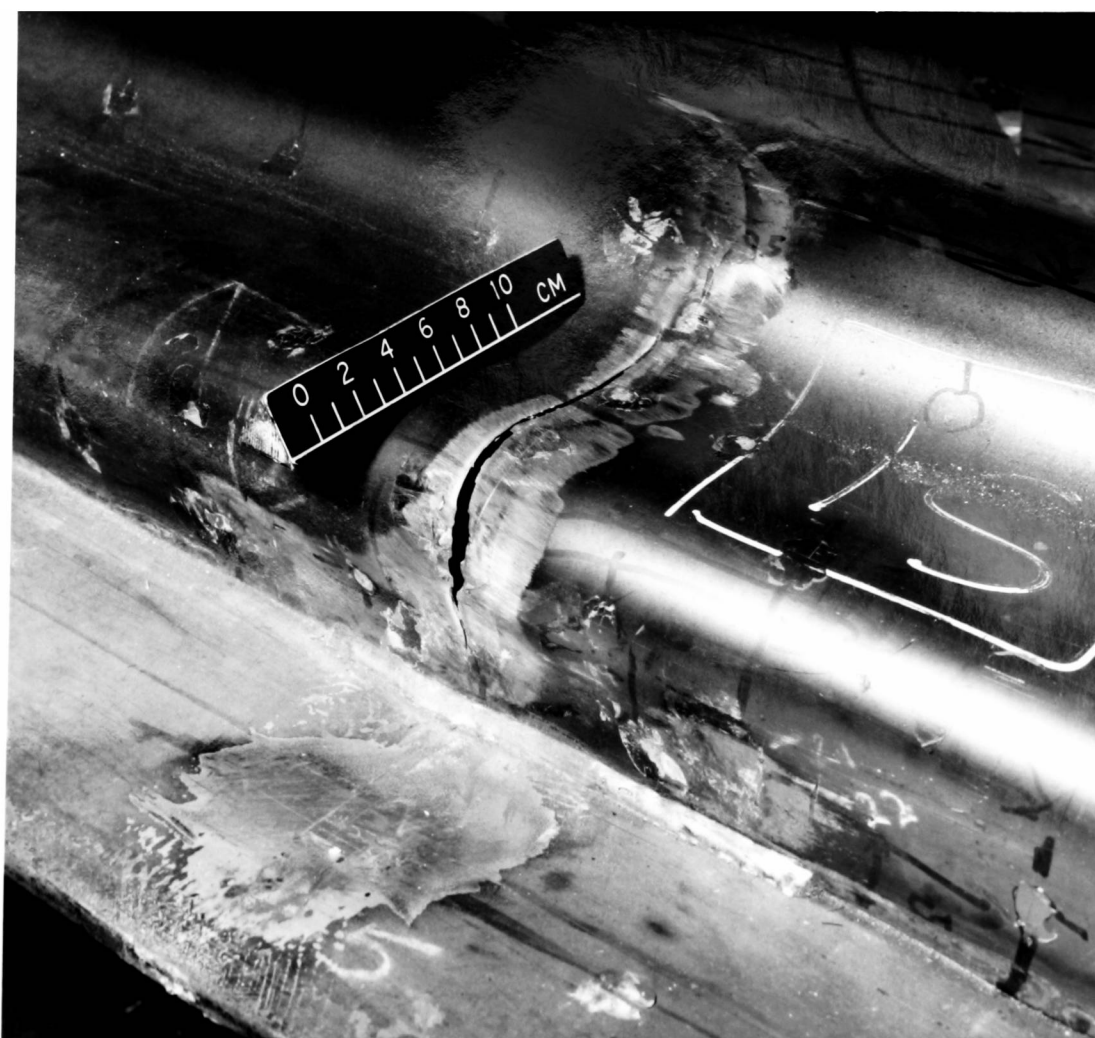


Plate 17 - Weld crack in Beam 3 at very large strains.

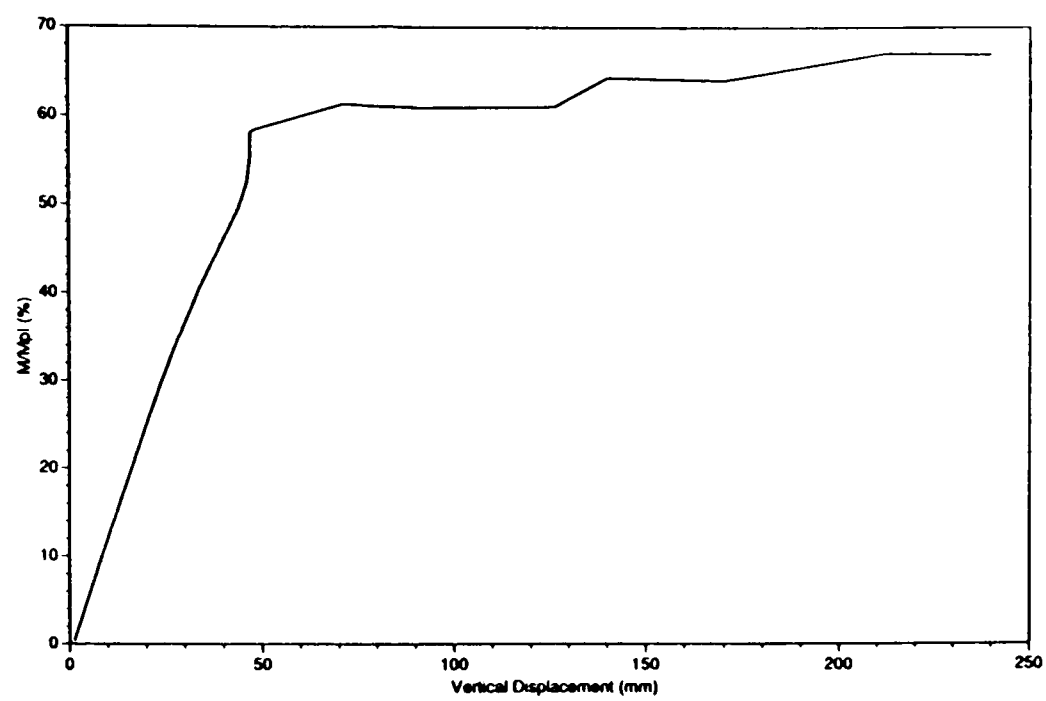


Figure 3.23 - Load vs. Central Vertical Deflection for Beam 3.

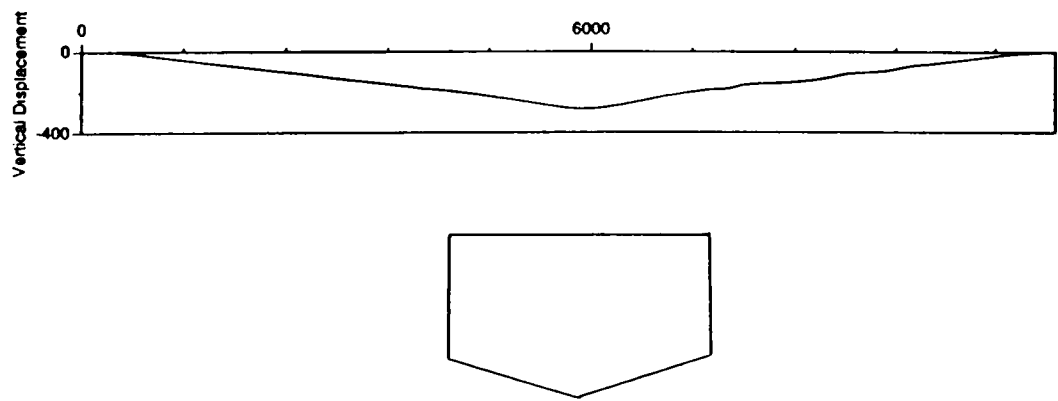


Figure 3.24 Final Deformed Shape Pattern of Beam 3 (mm).

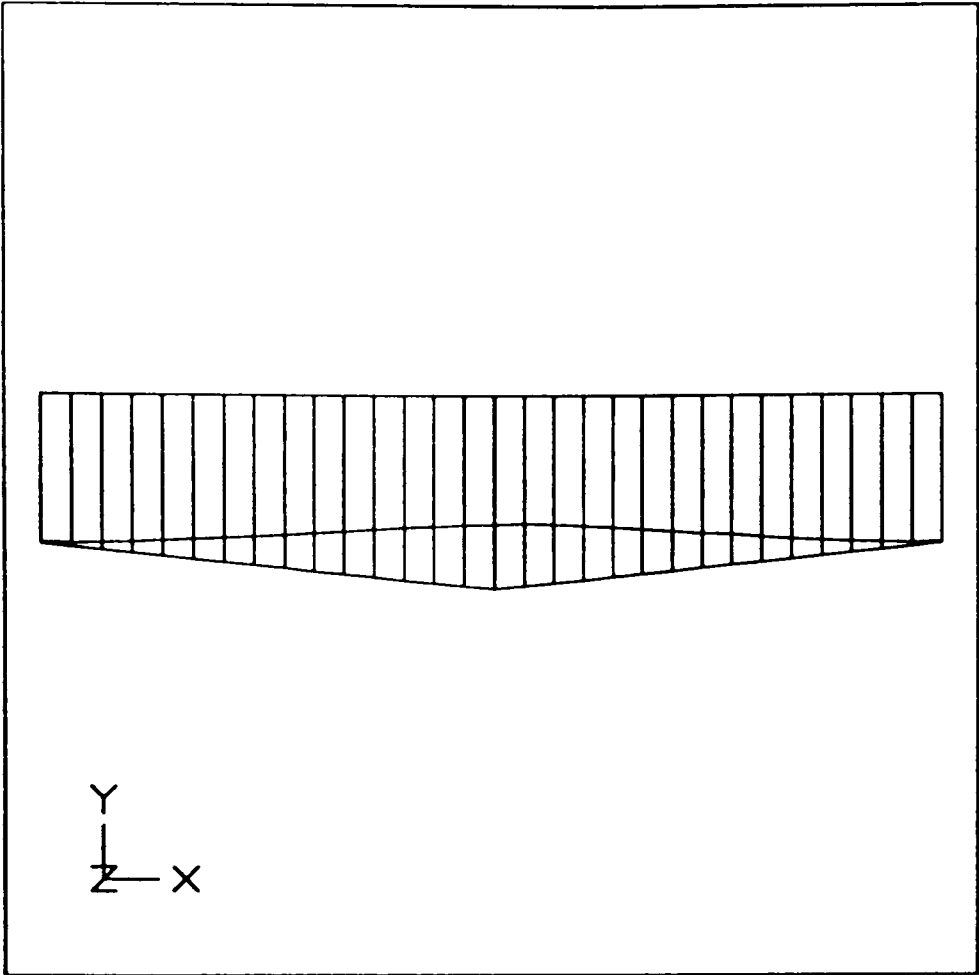


Figure 3.25.a - Displaced Shape of Flange - East Side (Beam 3).

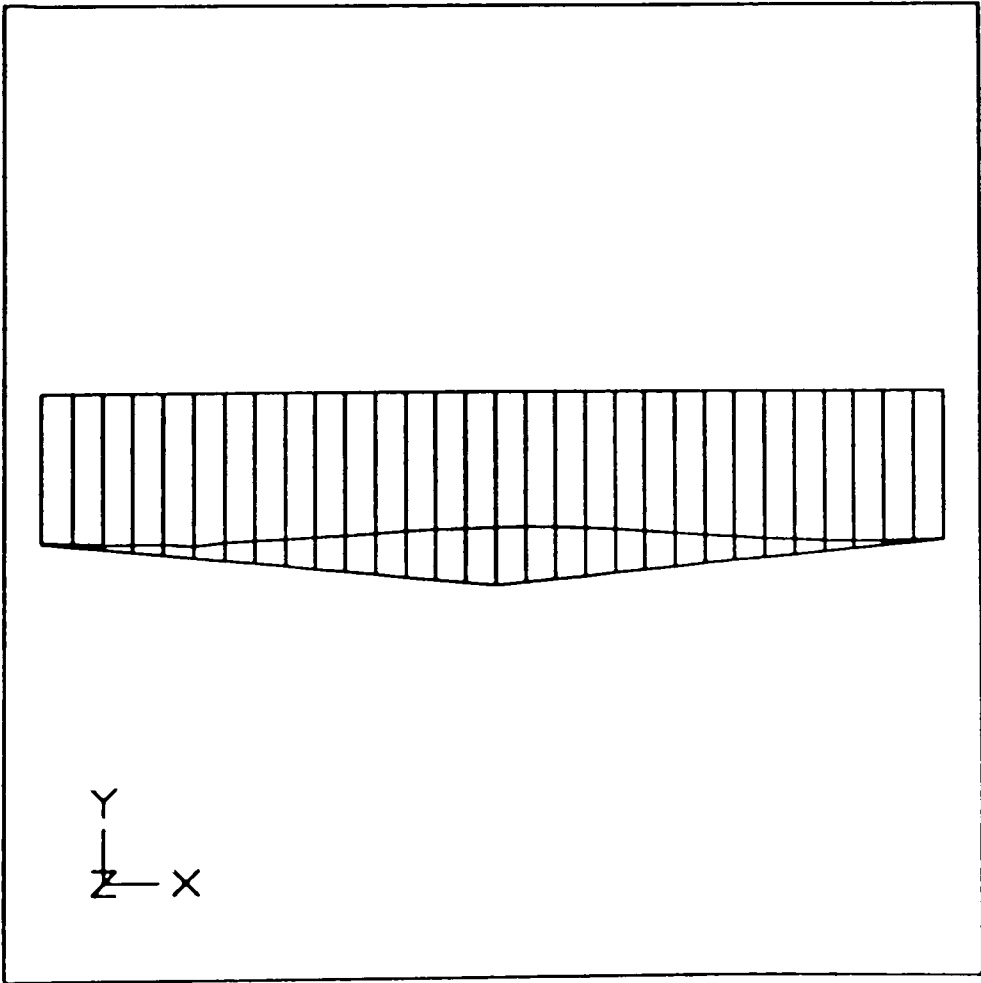


Figure 3.25.b - Displaced Shape of Flange - West Side (Beam 3).

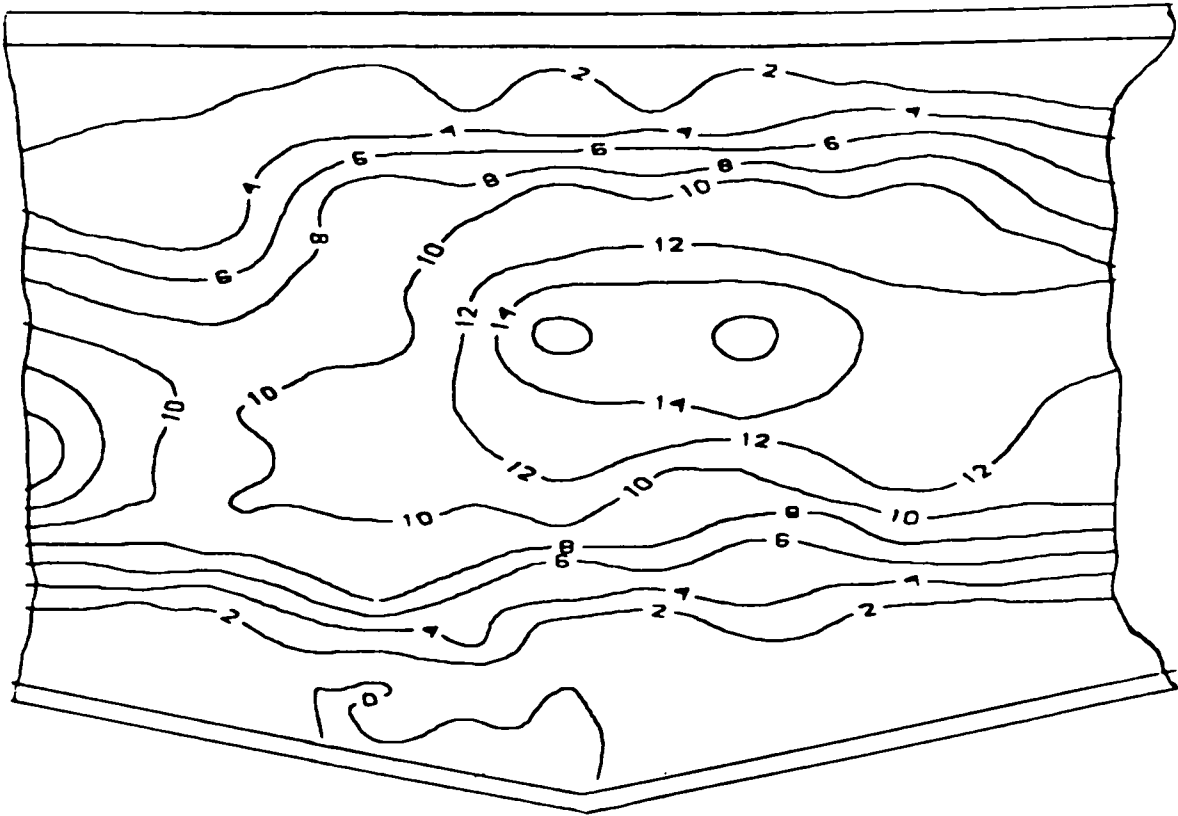


Figure 3.26.a - Late Lateral Deflection for Beam 3 (mm).

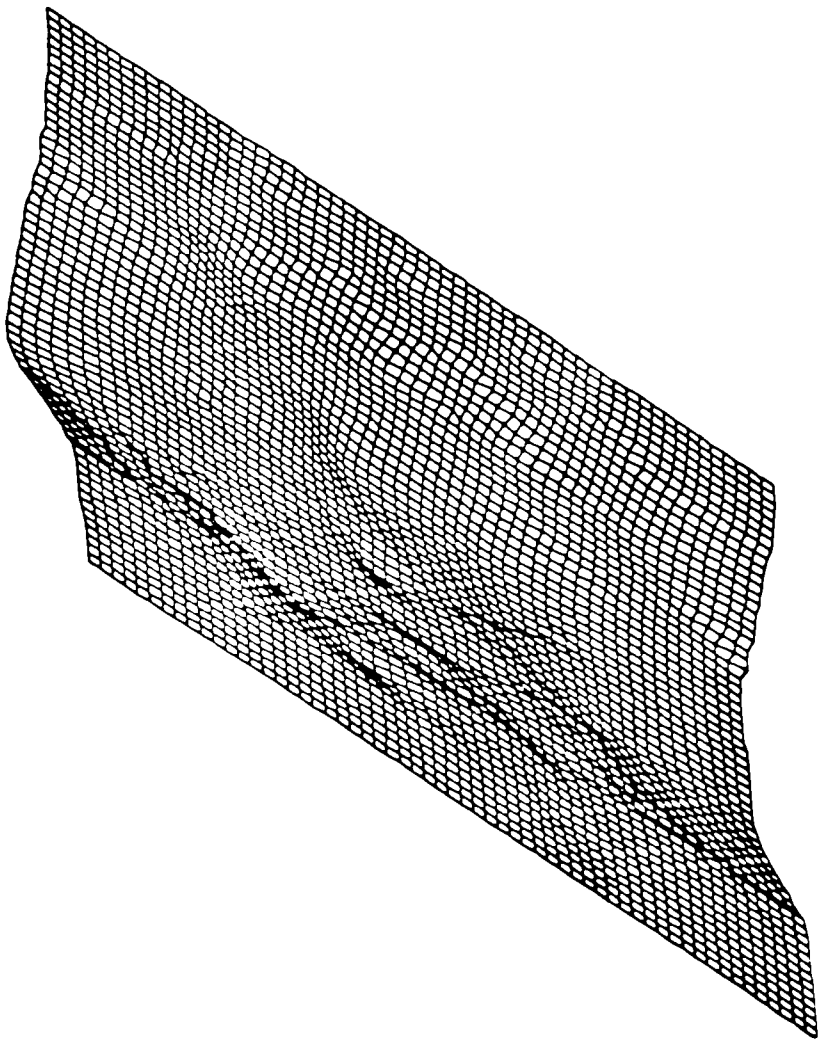


Figure 3.26.b - Final Lateral Deflection Perspective for Beam 3.

be seen in figures 3.27.a and 3.27.b, page 101, where they are plotted against the applied moment. After the end of the test, the residual plastic deformation of the web was measured. A full picture of the tapered part of the beam is presented in figure 3.28, page 101. A view of the buckled shape near the slope change, which used more points to map the deformed region, is displayed in figures 3.29 and 3.30, page 102.

The value of the force that caused the buckling of the web can be estimated with the aid of the linear strain-gauges positioned in the tension flange, at the centre of the tapered girder. Their behaviour is presented in figure 3.31, page 103, where the values are plotted against the applied moment. Gauges L3e and L3w, positioned on the inner part of the flange, reached the yield stress around a value of 40% of the plastic moment. The external gauges L4 and L5 behaved in a different way. Up to a value of 56% of the plastic moment, they deformed in tension. After that, they were destressed by the enormous change in shape that began to occur in the web. Finally, after a new deformed web configuration reached around 60% of M_{pl} , they again showed an increasing tensile strain up to the point of failure. As the external gauges did not reach the yield stress, the vertical force did not attain its maximum possible value in this test. X

The contour plots of the longitudinal and vertical stresses in the web, produced by the rosette strain-gauges, are displayed in figures 3.32.a and 3.32.b, page 103. It can be observed that although the buckling occurred in the elastic range, the web has reached very large values of stress in both directions. The bottom part of the web is the most plastified. This has occurred due to the presence of the large vertical force explained previously.

3.4. INFLUENCE OF THE PLASTIC NEUTRAL AXIS POSITION.

The most important difference between a classical patch load situation and the vertical buckling problem is the nature of the biaxial stress distribution in the web. As stated before, the vertical buckling of tapered composite beams is, in the great majority of cases, associated with compressive stresses in the vertical direction and tension stresses in the longitudinal direction. This happens due to the very localized characteristic of the phenomenon together with the typical position of the plastic neutral axis of the girders. With a low enough plastic neutral axis, the longitudinal component might change to compression and the response be closer to the patch load case.

To cover cases where the neutral axis might be lower in the web, the second series of tests was planned. To give continuity with the first batch of experiments, beams with similar proportions were chosen as far as possible. A 5mm web was used in all the new tests because it had not buckled in the first series. If any disadvantageous change in behaviour should occur due to the neutral axis effect, it was expected that the 5mm web would show it.

The new specimens were also made with grade 43 steel. They had the thickness of the web, t_w , equal to 5mm, the width of both flanges, b_f , equal to 300mm, and the maximum and minimum heights equal to 650 and 500mm respectively. To alter the position of the neutral axis, the thickness of the flanges was altered. The fourth beam (the first in the new series) had the thickness of the top flange equal to 20mm and the thickness of the bottom flange equal to 12mm. The fifth beam had the top and bottom flange thicknesses of 15mm and 12mm respectively, while the sixth specimen had the top and bottom flange thicknesses of 15mm and 25mm.

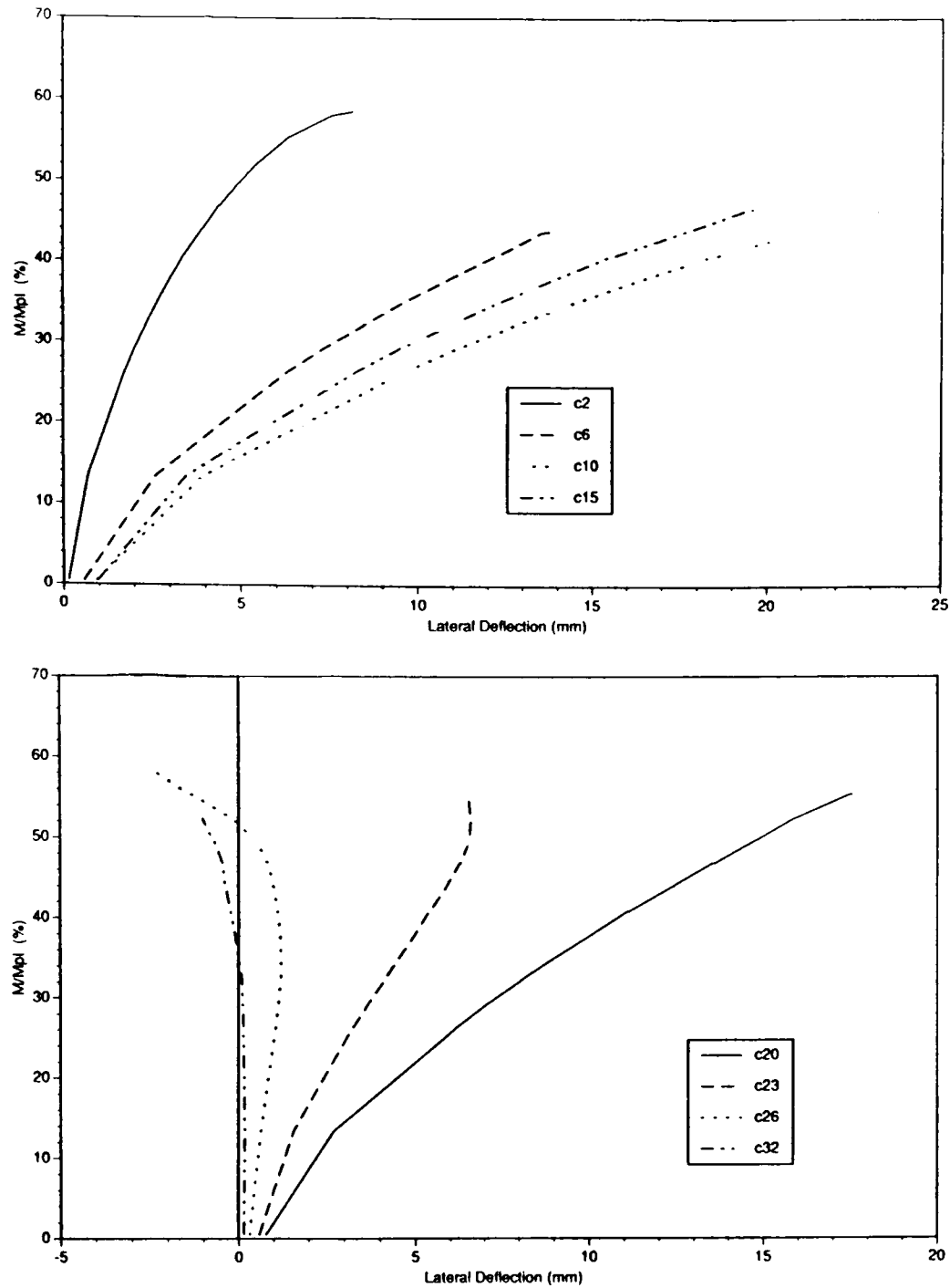


Figure 3.27.a - Load vs. Lateral Displacement of Points 2, 6, 10 and 15.
Figure 3.27.b - Load vs. Lateral Displacement of Points 20, 23, 26 and 32.

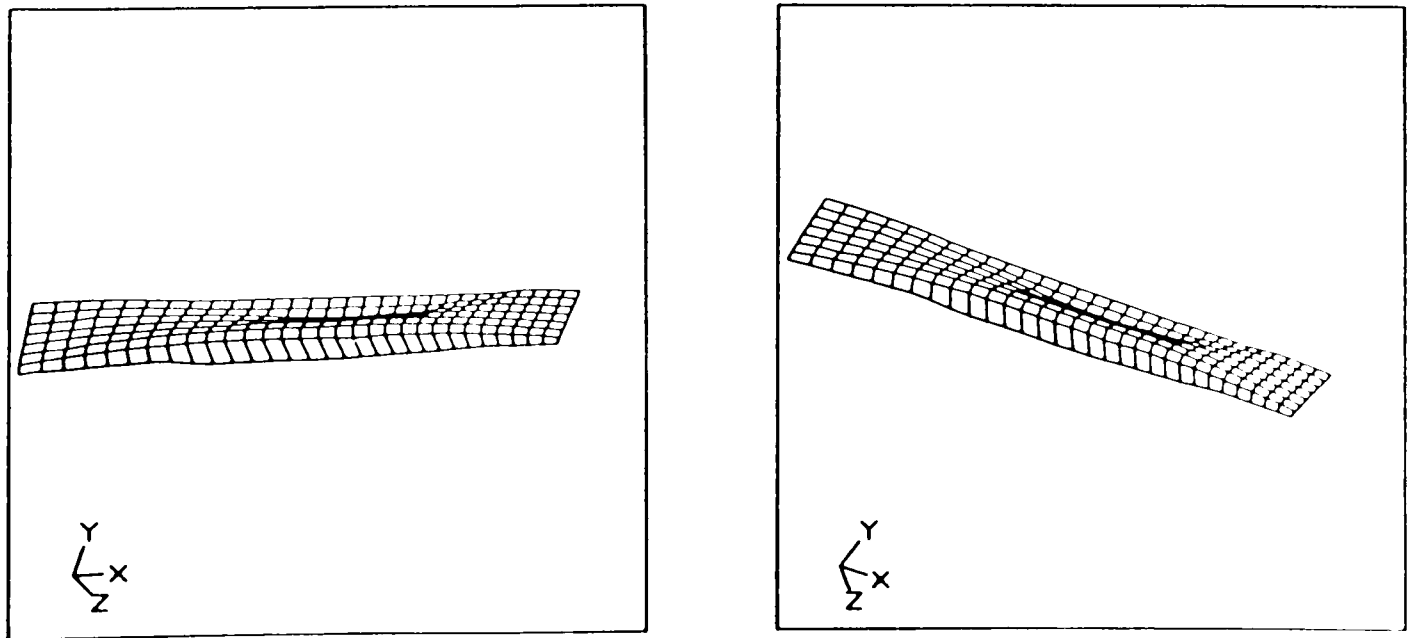


Figure 3.28 - Buckled Shape: Overall View (Beam 3).

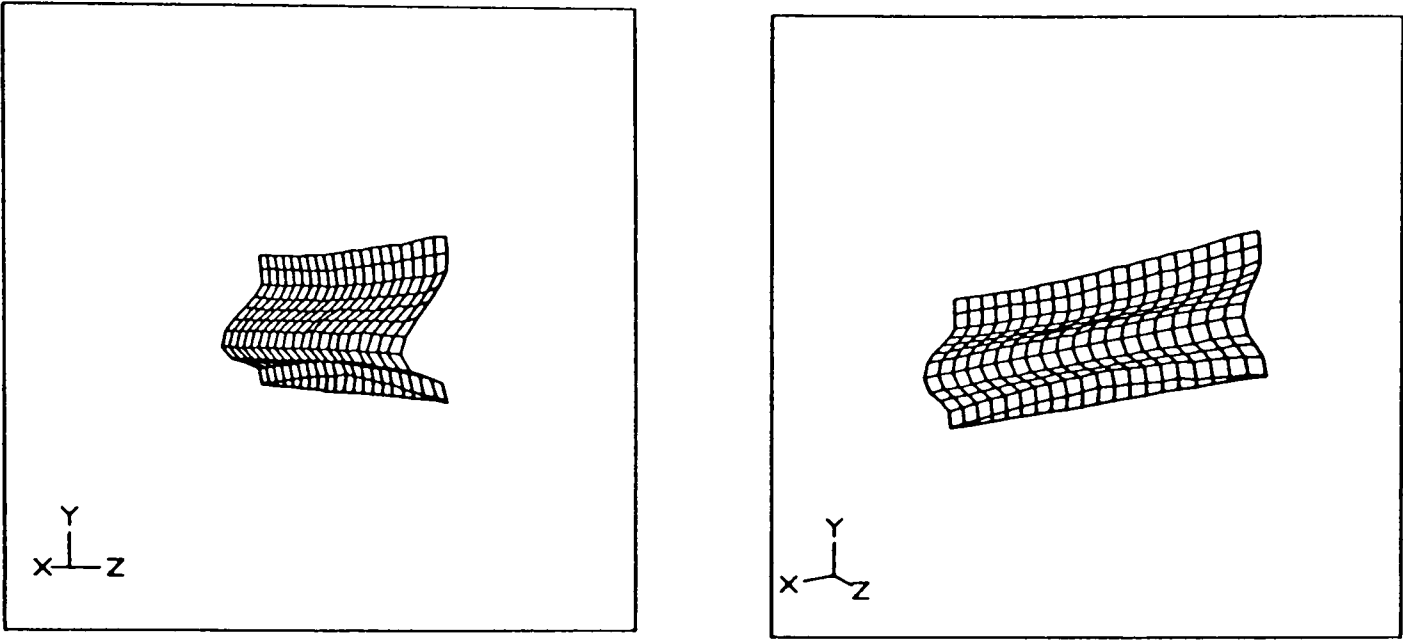


Figure 3.29 - Buckled Shape: Views at Slope Change (Beam 3).

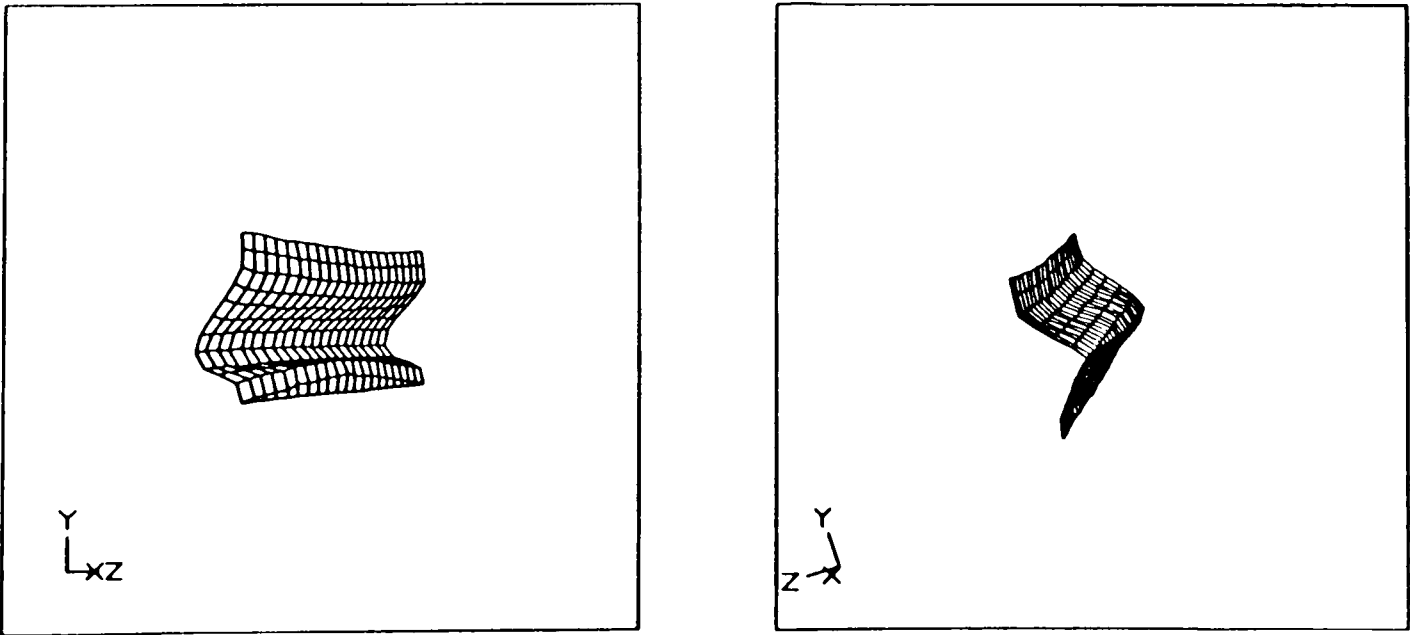


Figure 3.30 - Buckled Shape: Views at Slope Change (Beam 3).

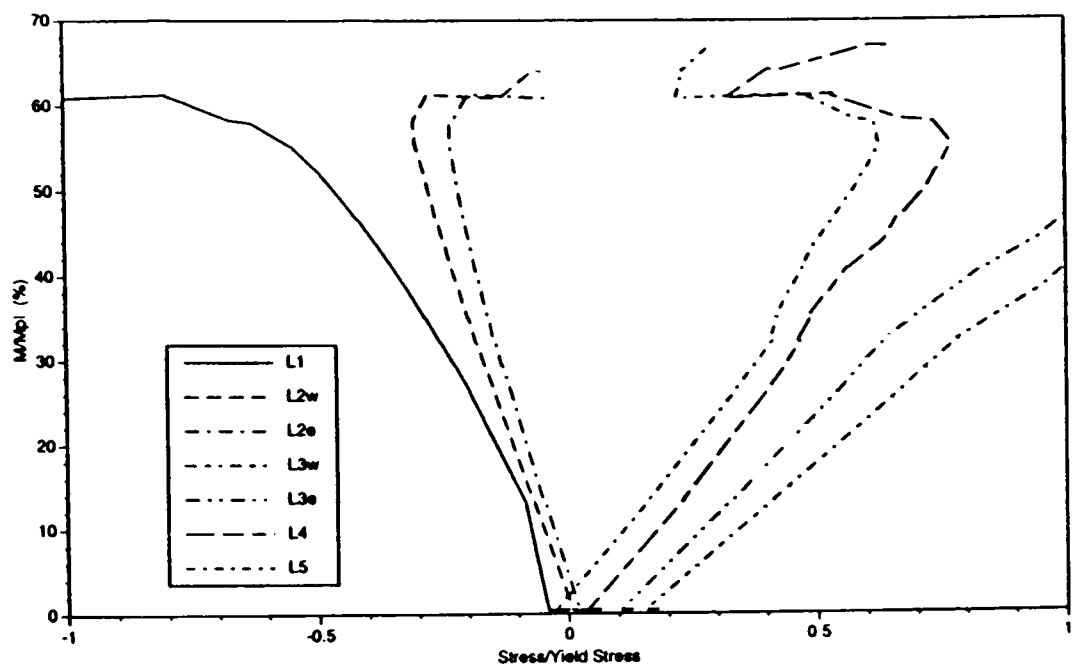
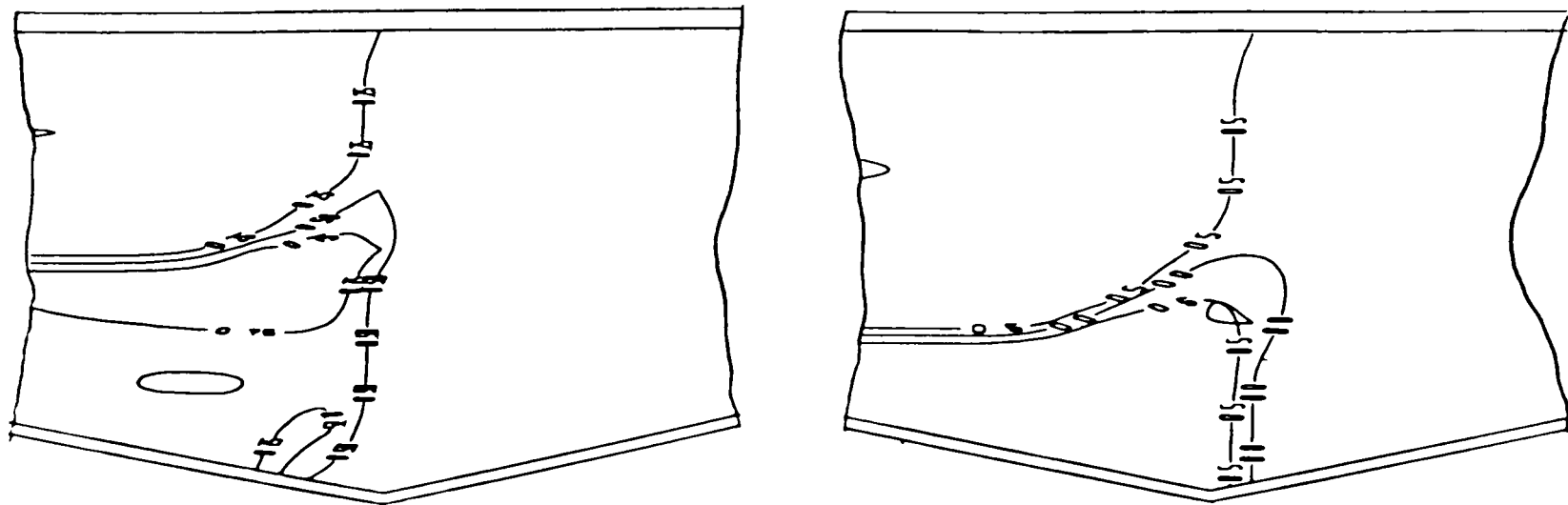


Figure 3.31 - Central Linear Strain Gauge Results (Beam 3).



Figures 3.32.a and 3.32.b - Normalized Longitudinal and Vertical Stresses in the Web (Beam 3).

The sixth beam's proportions were chosen to simulate a case where unpropped construction was used and the beam had only the bare steel to sustain the construction loads.

With top flange thicknesses of 15mm, beams 5 and 6 lay within the semi-compact class of BS5950. The flange can , in principle, achieve the yield stress at the extreme compression fibre but cannot be fully plastified. However, in view of the experience acquired in the other tests, this was not expected to affect the experiment, because the maximum tension flange force that might cause vertical buckling was expected to be achieved with relatively low values of the applied moment.

The values of the yield and ultimate stresses, σ_y and σ_u , for the plates used in the fabrication of beams 4, 5 and 6 were determined in a series of coupon tests using virgin material. The average yield and ultimate stress results for these plates are presented in table 3.3 below, while these average stress values are still well above the specification for grade 43 steel, the yield values are not as high as those for the first test series (table 3.1, page 79).

Web Thickness (mm)	5	12	15	20	25
σ_y (MPa)	336.20	250.05	286.32	278.54	267.45
σ_u (MPa)	477.33	442.42	446.82	442.87	416.51

Table 3.3 - Yield and Ultimate Stress Results: Second Test Series.

The plastic neutral axes positions, measured from the top of the compressive steel flange, and the plastic moments at the central point, $h = 650\text{mm}$, for the second series of beams, are given in table 3.4 below. The figures presented in this table were calculated using the measured steel properties of table 3.3. The sixth beam had the web fully in compression with the plastic neutral axis 0.7mm inside the bottom flange. With these values established, a full description of the results of the new series of tests will follow.

Beam Number	4	5	6
Neutral Axis Position (mm)	171.27	298.40	625.70
Plastic Moment (kNm)	918.18	848.02	1081.83

Table 3.4 - Geometric Properties: Second Test Series.

3.4.1. Beam 4 ($t_{fc} = 20\text{mm}$).

3.4.1.1. *Description of the Test.*

The tapered panel remained elastic up to 50% of M_{pl} at the central section. After that, the beam tended to displace towards the lateral supports and plasticity began to spread over the web. The plastic moment recorded in the test was equal to 95.7% of the plastic capacity at the point of minimum depth. This moment capacity was again calculated with the yield stress obtained with the coupon tests. The maximum moment recorded in the test corresponded to 89.04% of the plastic capacity at the point of maximum height. The failure mode pictured in plate 3.18, page 106, corresponded to plastic hinge formation at the edge of the tapered section. A slight

buckle of the top flange at the plastic hinge location and close to the web stiffener was also observed and is seen in plate 3.19, page 106.

The test was stopped (on a serviceability criterion) when the central vertical deflection of the beam reached 360mm (greater than $\text{span}/50$ or 220mm). The final deformed shape of the beam is presented in figure 3.34, page 107, where the large vertical deflections of the beam, and the plastic hinges at the edges of the tapered panel, can be more clearly visualized.

3.4.1.2. *Results.*

The load against vertical deflection behaviour of the beam is shown in figure 3.33, page 107. Channels 40 and 42 displayed in that graph correspond to the vertical deflection measured at the centre and the edge of tapered panel, respectively. A contour plot of the final lateral displacements of the web is given in figure 3.35, page 108. As can be observed, the maximum lateral displacements in the web were very small and were due to the twist induced by the tendency to lateral torsional buckling. Some deflection readings were plotted against the applied moment and are depicted in figures 3.36.a and 3.36.b, page 108.

The achievement of the maximum compressive force was again proved by the linear strain-gauges located at the bottom flange, as illustrated in figure 3.37, page 109. Gauges L3w, L3e, L4 and L5 reached the yield stress at load levels between 42 and 55% of the central plastic moment capacity. All the linear gauges placed in the web and flanges at the edge of the panel reached the yield stress (figure 3.38, page 109).

The longitudinal and vertical web stress contours at the final stage of the test are presented in figure 3.39.a and 3.39.b, page, 110. They characterize a web with large longitudinal but small vertical stresses. The equivalent (Von Mises) stress contours for load levels corresponding to 25.1%, 50.3%, 78.9% and 89.1% of M_{pl} , are exhibited in figures 3.40, page 110 and 3.41, page 111.

3.4.2. Beam 5 ($t_{fc} = 15\text{mm}$).

3.4.2.1. *Description of the Test.*

The tapered girder sustained the load in the elastic range up to a value corresponding to 35% of its plastic capacity at the central section. After that, plasticity began to play a substantial role in the load-response behaviour. Around 70% of M_{pl} , the upper flange began to buckle at the edge of the tapered panel. The buckling happened very quickly, and at this point the beam completely lost its capacity to sustain the applied moment. The beam failed at $\cong 73\%$ of M_{pl} at the central section, due to a combination of a plastic hinge and top flange buckling, as shown in plate 20, page 112. A closer view of this failure, viewed from above of the girder, is displayed in plate 21, page 112. In this picture the buckle, and the associated twist of the top flange, are very plain and characterize very well this type of failure.

The final deflected shape of the beam is presented in figure 3.43, page 113. The high local curvature at the left hand edge of the tapered panel, associated with the hinge and the flange buckle, is obvious. The final vertical deflections of the girder can be considered small when compared to the previous tests, because the sudden buckling failure of the top flange brought the test to an abrupt end.



Plate 18 - Failure of Beam 4 ($t_{fc} = 20\text{mm}$).

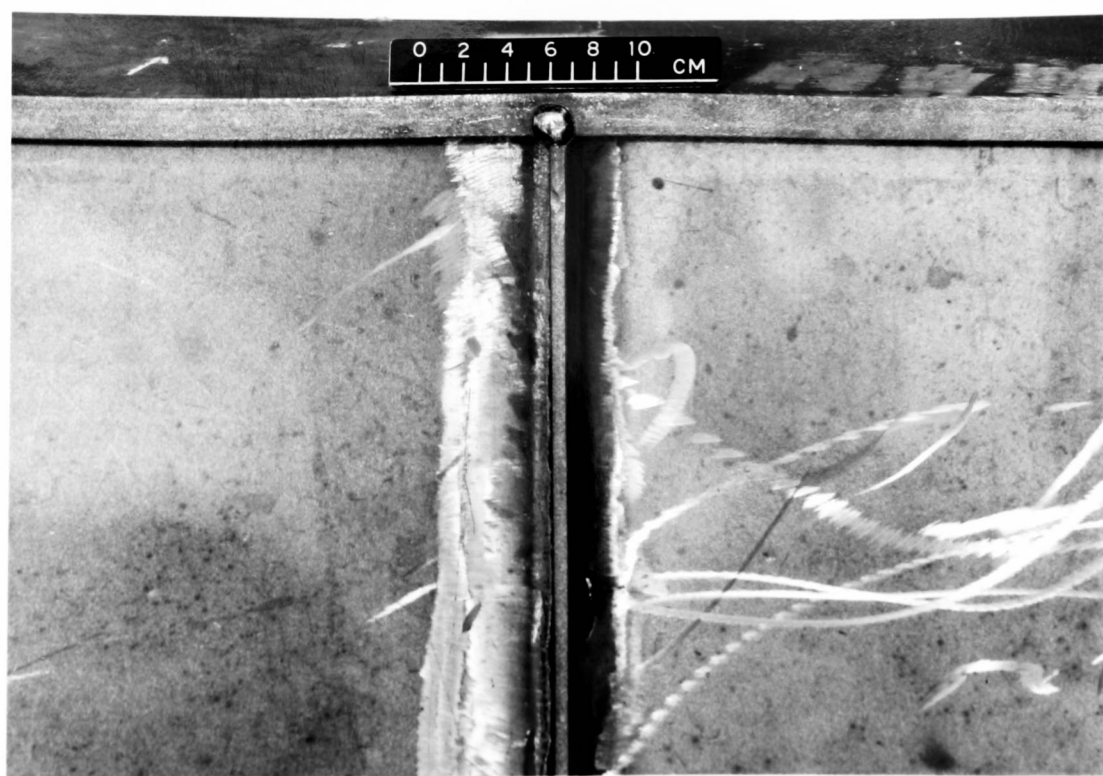


Plate 19 - Beam 4: buckling of the top flange.

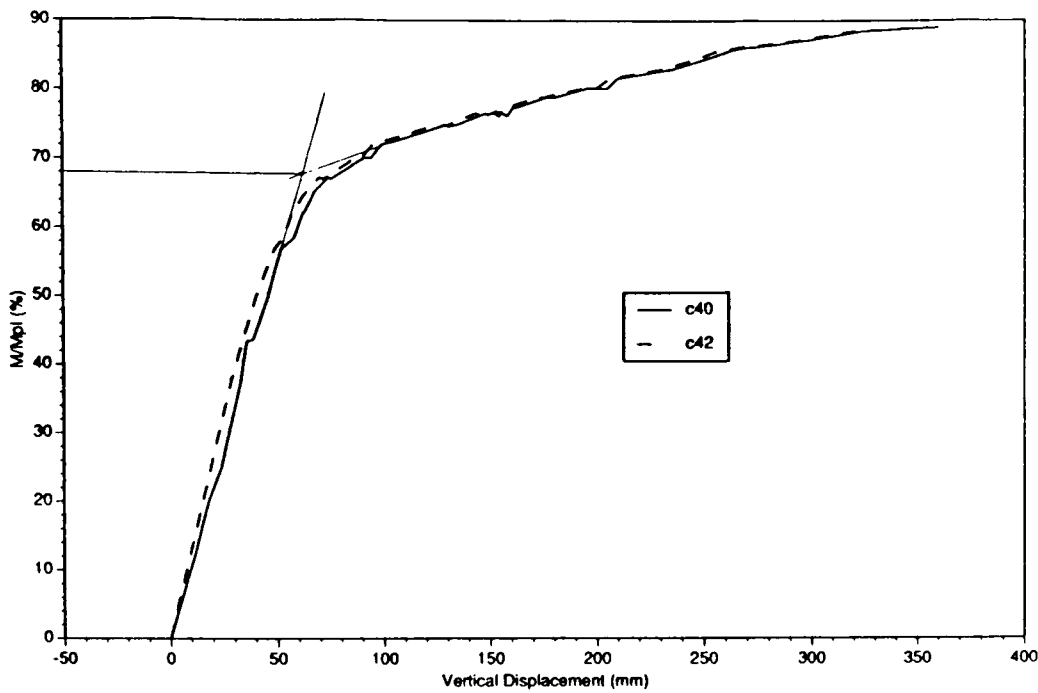


Figure 3.33 - Load vs. Vertical Deflection for Beam 4 at Centre and Edge of the Tapered Panel.

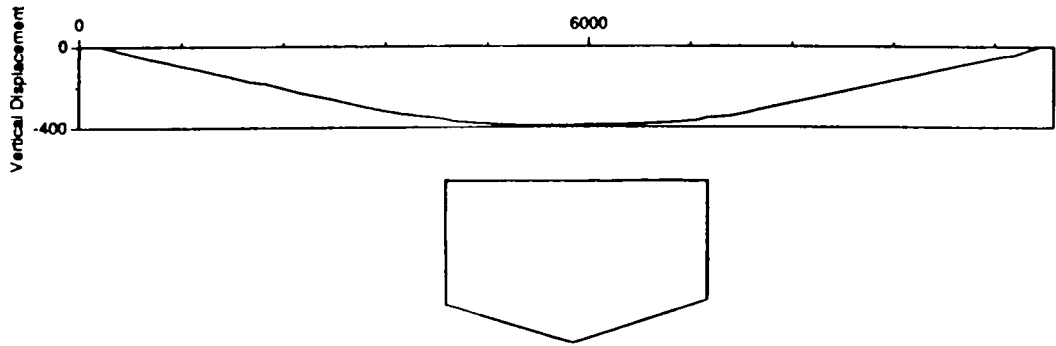


Figure 3.34 - Final Deformed Shape of Beam 4 (mm).

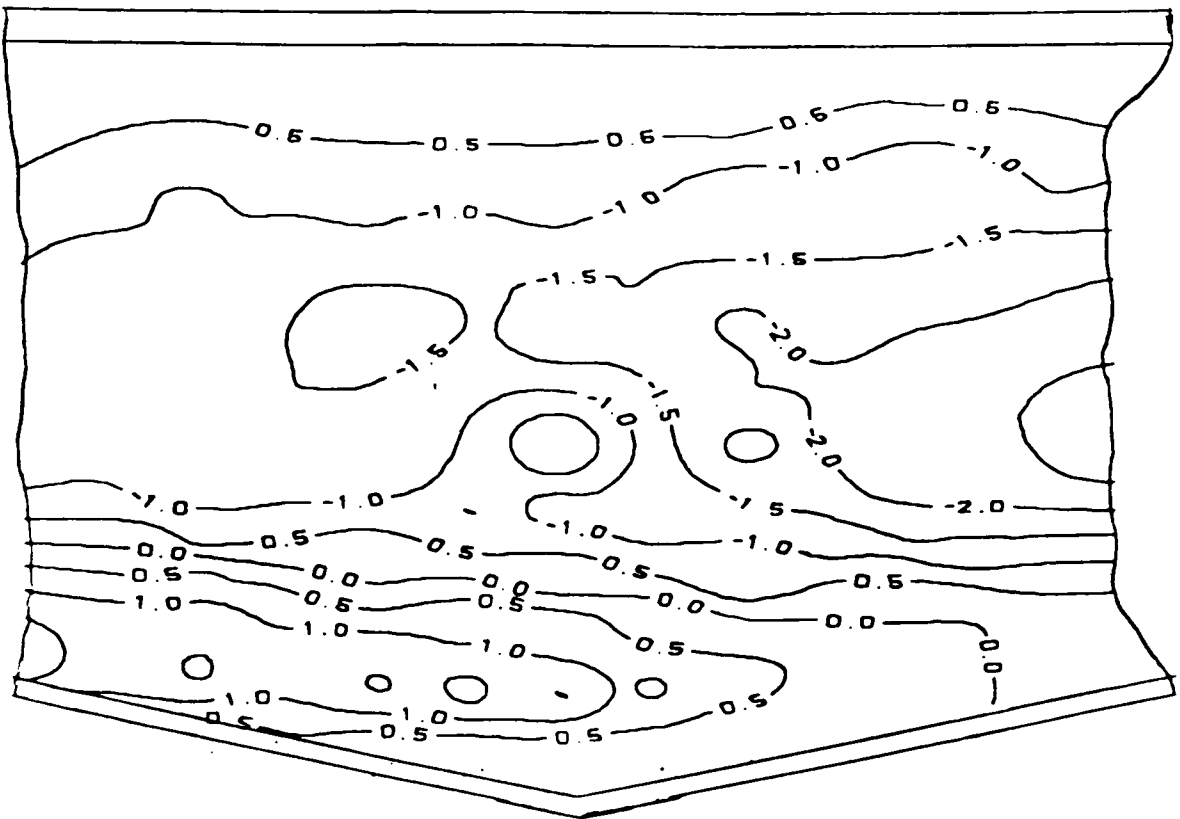


Figure 3.35 - Final Lateral Deflection Pattern of Beam 4 (mm).

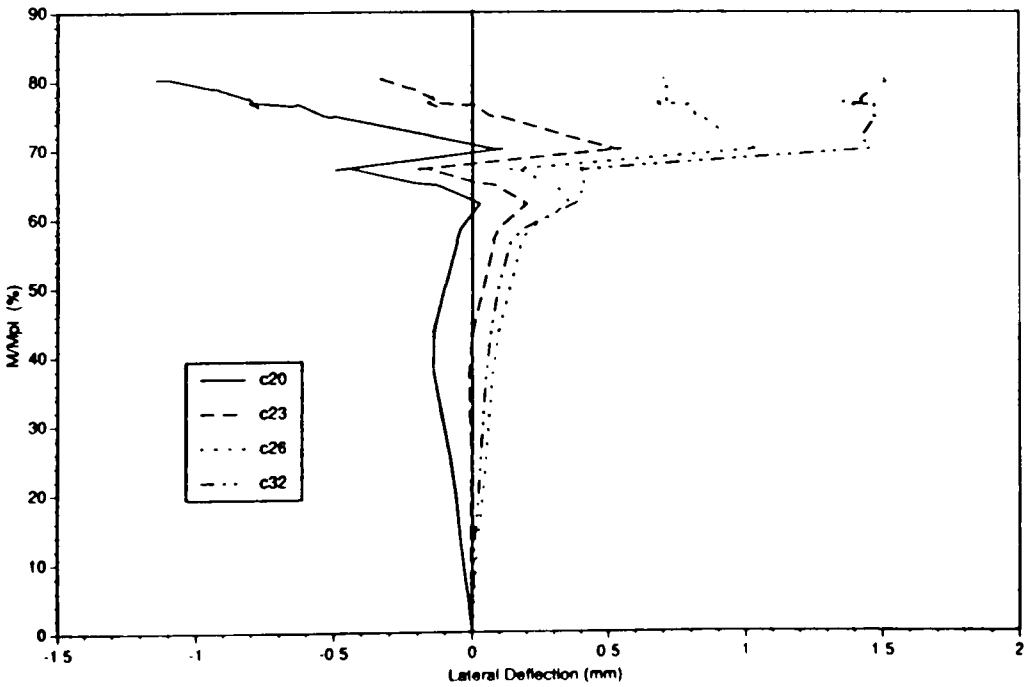
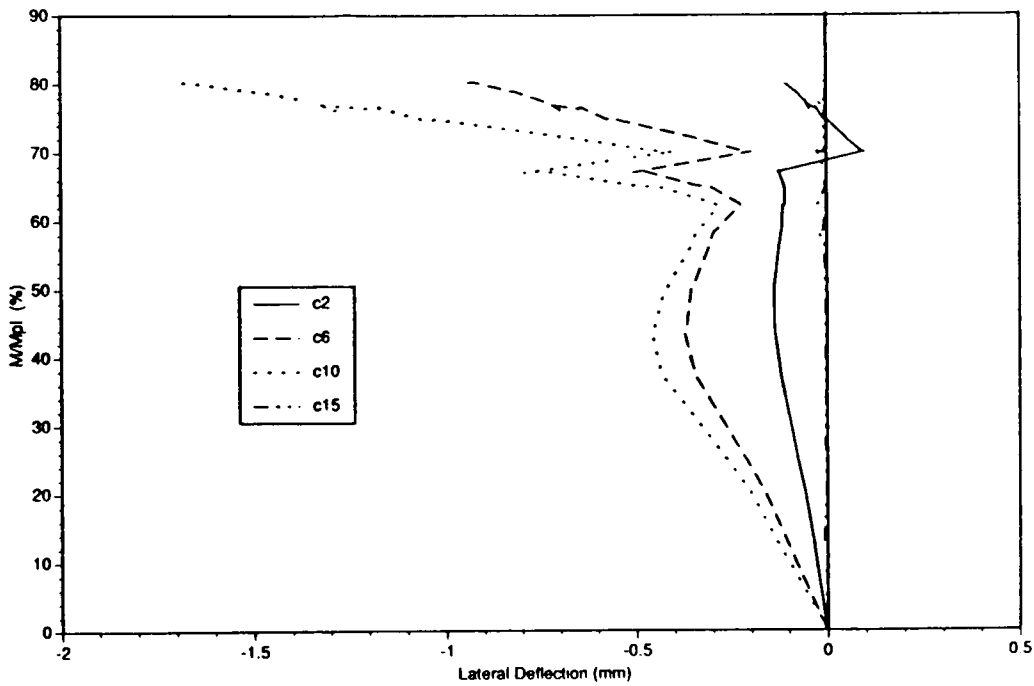


Figure 3.36.a - Load vs. Lateral Displacement of Points 2, 6, 10 and 15.
Figure 3.36.b - Load vs. Lateral Displacement of Points 20, 23, 26 and 32.

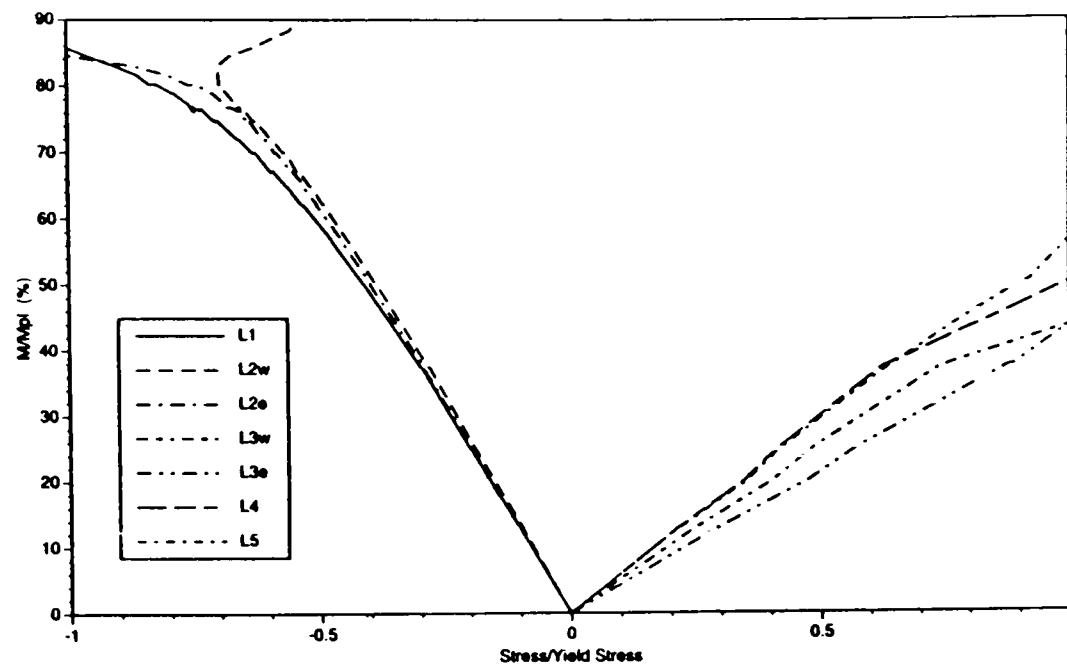


Figure 3.37 - Central Linear Strain Gauge Results (Beam 4).

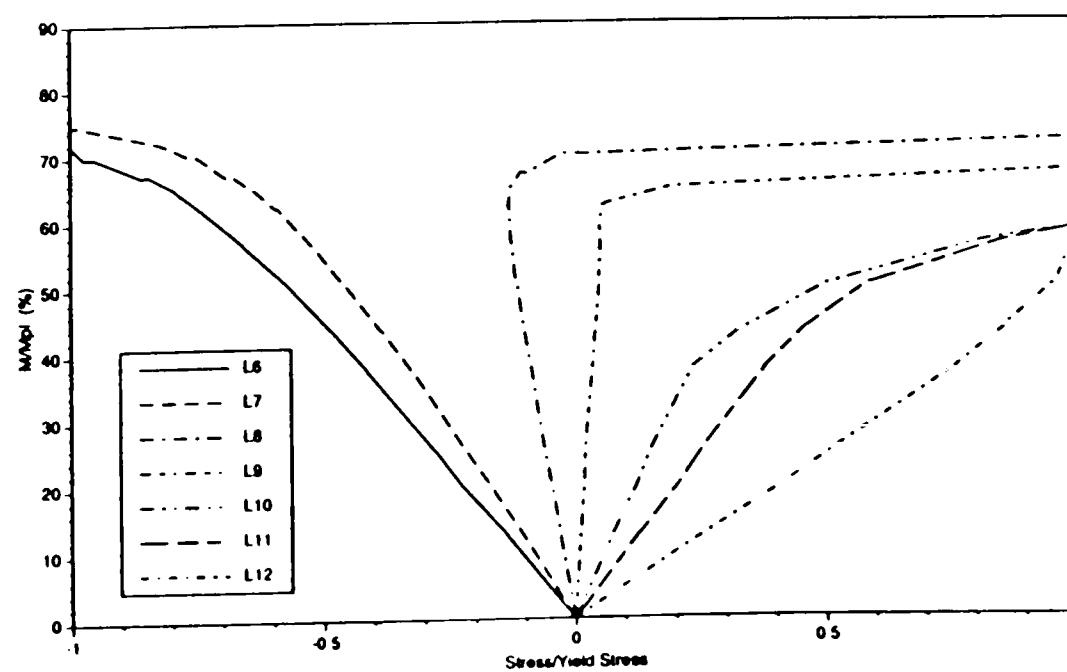
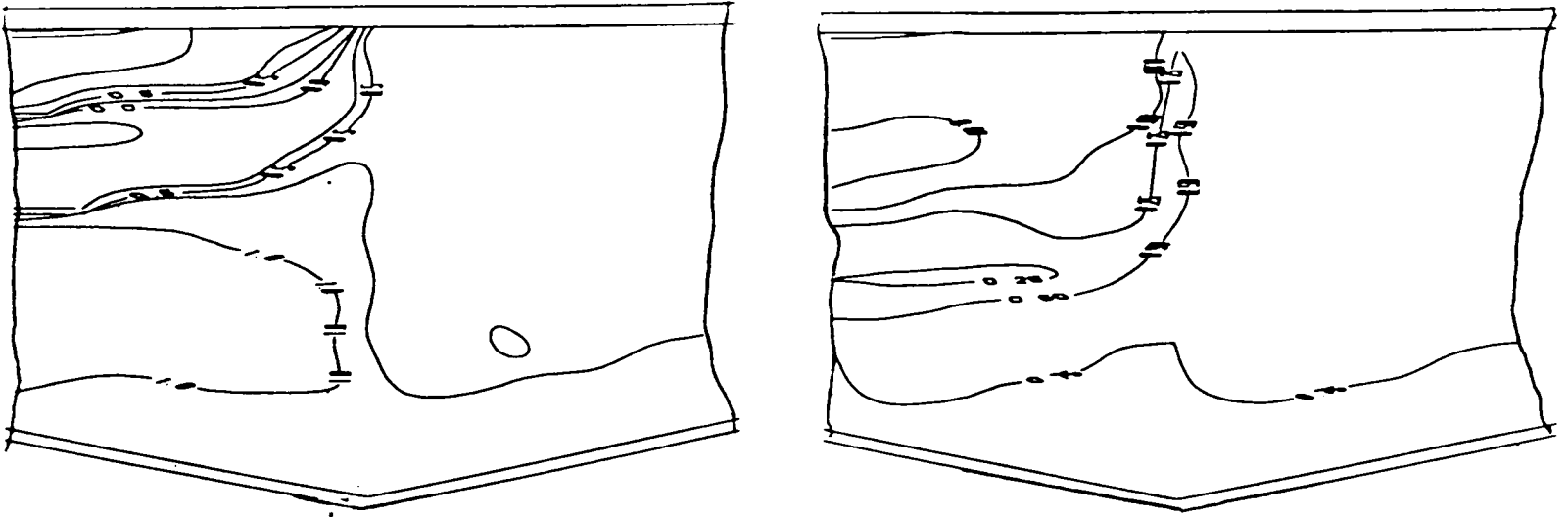
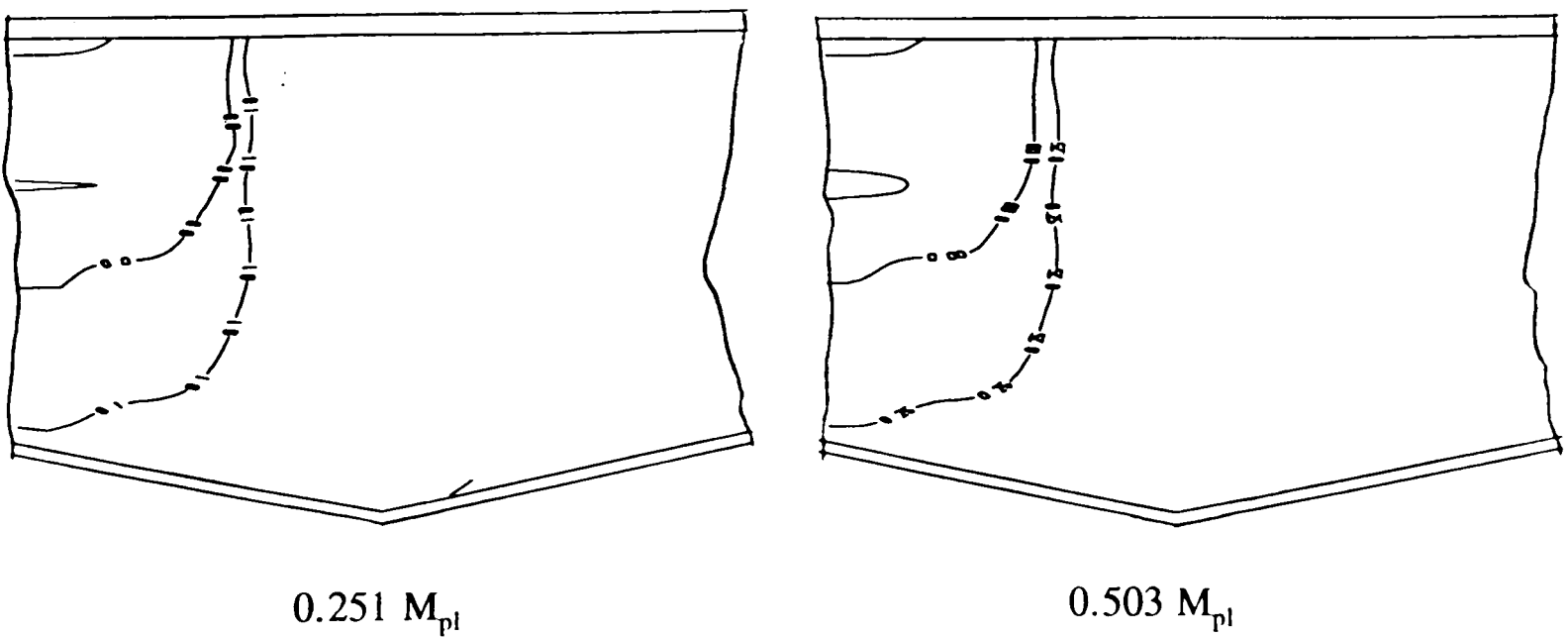


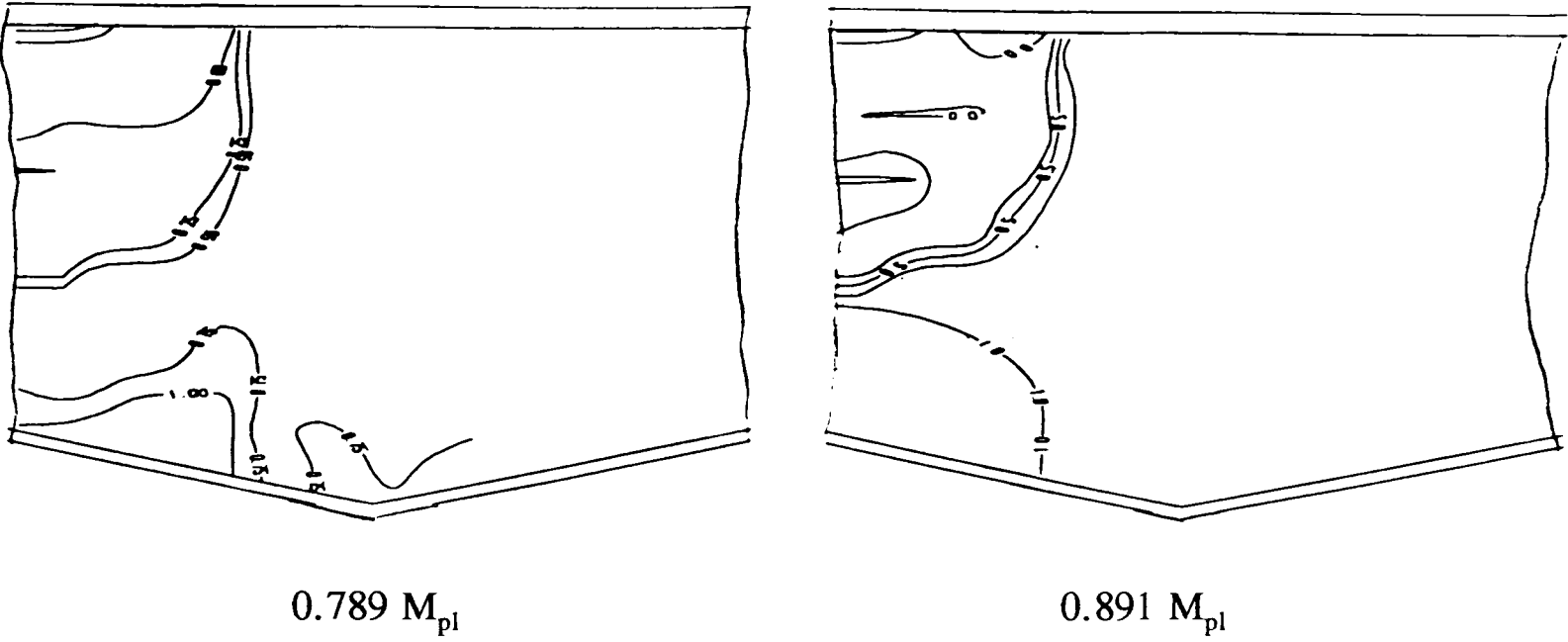
Figure 3.38 - Panel Edge Linear Strain Gauge Results (Beam 4).



Figures 3.39.a and 3.39.b - Normalized Longitudinal and Vertical Stresses in the Web (Beam 4).



Figures 3.40.a and 3.40.b - Normalized Equivalent Stresses in the Web (Beam 4).



Figures 3.41.a and 3.41.b - Normalized Equivalent Stresses in the Web (Beam 4).

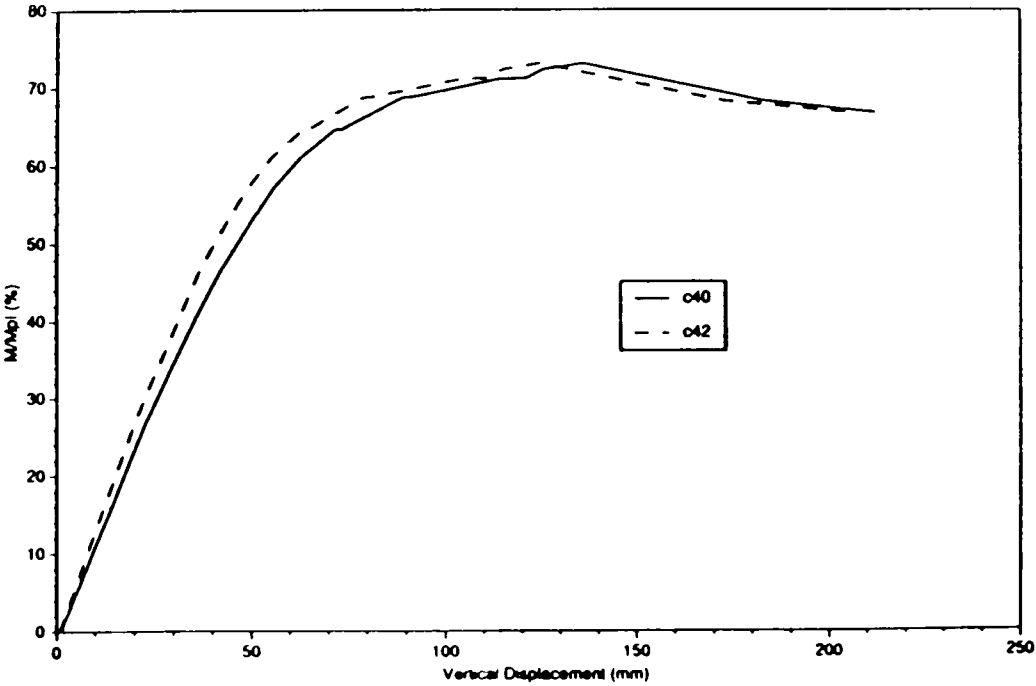


Figure 3.42 - Load vs. Vertical Deflection for Beam 5 at Centre and Edge of the Tapered Panel.



Plate 20 - Failure of Beam 5 ($t_{fc} = 15\text{mm}$).

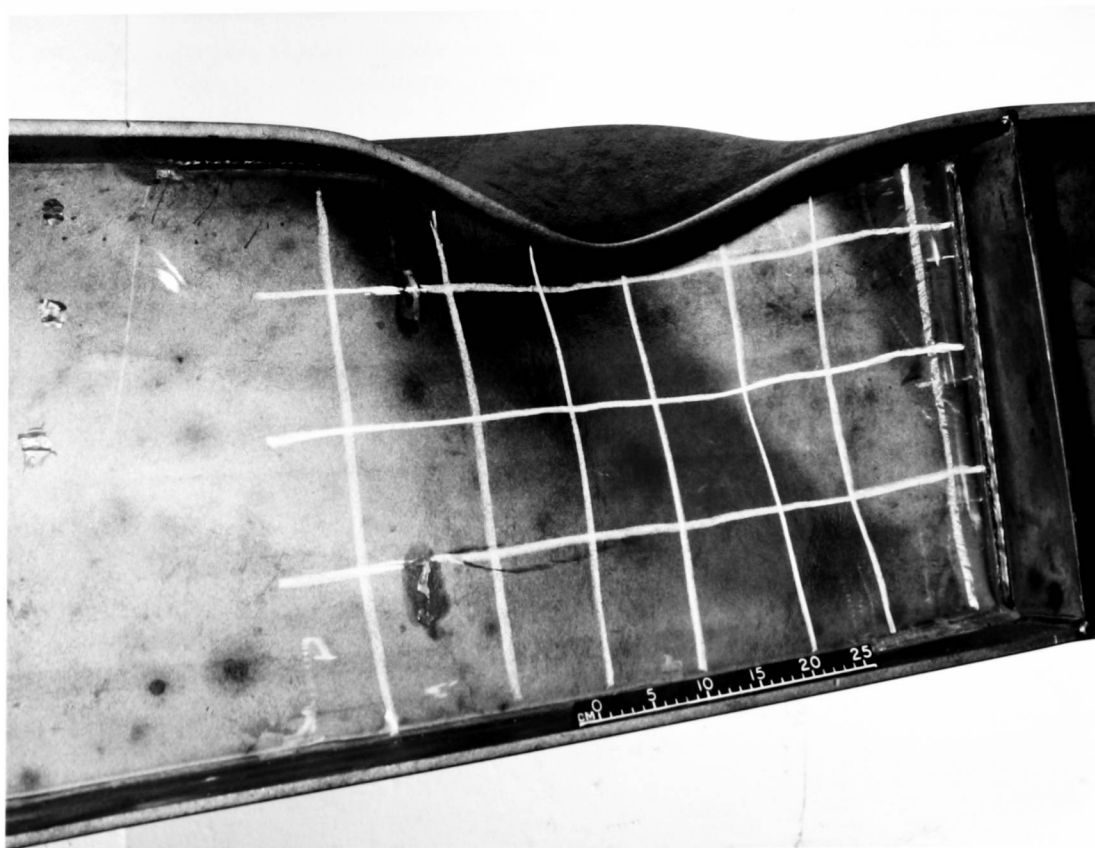


Plate 21 - Beam 5: buckling of the top flange.

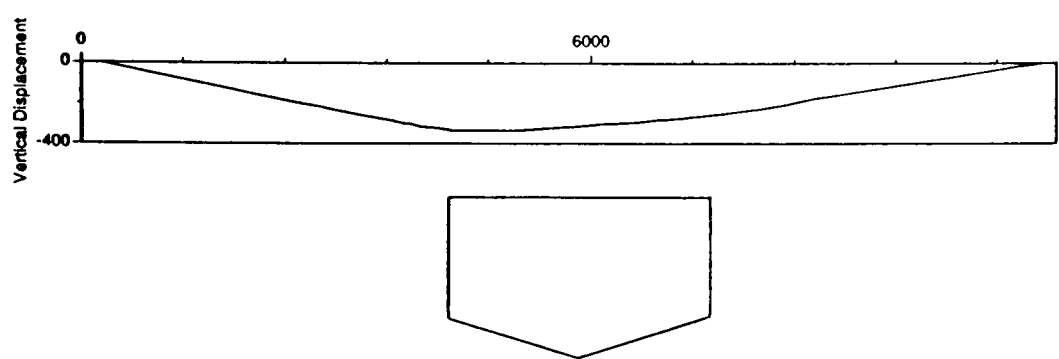


Figure 3.43 - Final Deformed Shape of Beam 5 (mm).

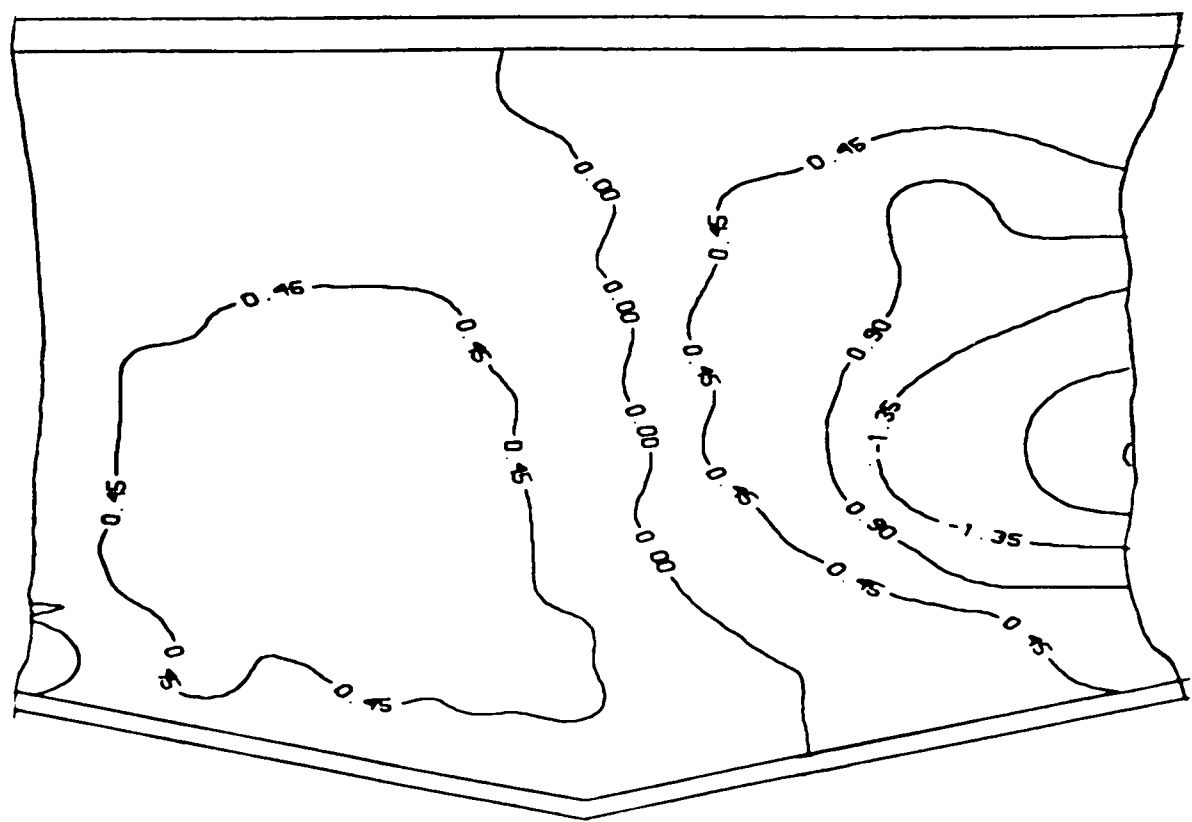


Figure 3.44 - Final Lateral Deflection Pattern of Beam 5 (mm).

3.4.2.2. Results.

The load versus vertical deflection curve for the test is shown in figure 3.42, page 111. The two channels displayed are located in the same positions as those described in test number 4. From the graph, the maximum vertical deflection was around 230mm. The central portion of the web remained virtually undeformed during the test. This is clear from the results from the lateral transducers portrayed in figure 3.44, page 113. The individual responses of some of the channels have been plotted against the applied moment and are presented in figures 3.45.a and 3.45.b, page 115. The correlation between the number of the transducer and its corresponding location on the transducer plate is presented in figure 3.2, page 74.

The maximum possible vertical force on the web was again reached, as shown by the linear strain-gauges located at the centre of the beam's tension flange, illustrated in figure 3.46, page 115. Gauges L3e, L3w, L4 and L5 reached the yield stress between 40% and 50% of M_{pl} , well before the failure. Plastic hinge formation, at the point where the buckling of the top flange occurred, can be seen in figure 3.47, page 116. All the linear gauges at that section have reached the yield stress.

The longitudinal and vertical contour stress plots of the central portion of the web are presented in figures 3.48.a and 3.48.b, page 116, for the last load step of the test. The equivalent (Von Mises) contour stress plots for load stages corresponding to 13.3%, 40.5%, 64.5% and 73.1% of M_{pl} are shown in figure 3.49.a, 3.49.b, 3.50.a and 3.50.b, page 117. The maximum stresses were located in the bottom part of the web, as expected, although they were less significant than the ones present in the previous beams. This is again related to the sudden failure relatively early in the load history.

3.4.3. Beam 6 ($t_f = 25\text{mm}$).

3.4.3.1. Description of the Test.

The beam remained elastic up to a level corresponding to 38% of the plastic capacity of the central section. After that point, the material and geometric non-linearities began to affect the load carrying capacity of the beam and, allied with that, the displacements grew significantly. The maximum moment recorded in the test corresponded to 68.96% of the plastic capacity at the point of maximum height. As in the case of the fifth beam, the failure was due to the buckling of the top steel flange at the edge of the tapered panel. This collapse was also very fast and the load dropped very quickly. The general failure of the beam is shown in plate 22, page 118. A more detailed view of the plastic collapse, which clearly shows the buckle and twist of the top flange, is portrayed in plate 23, page 118. The web was also very much affected by the buckling of the flange. The final vertical deflection of the whole beam is displayed in figure 3.52, page 119: the sharp curvature at the left end of the test panel is again noteworthy.

3.4.3.2. Results.

The vertical displacements were measured at the same points as for the fourth and fifth beams. Their results were plotted against the applied moment and are presented in figure 3.51, page 119. The maximum displacement, as in the previous

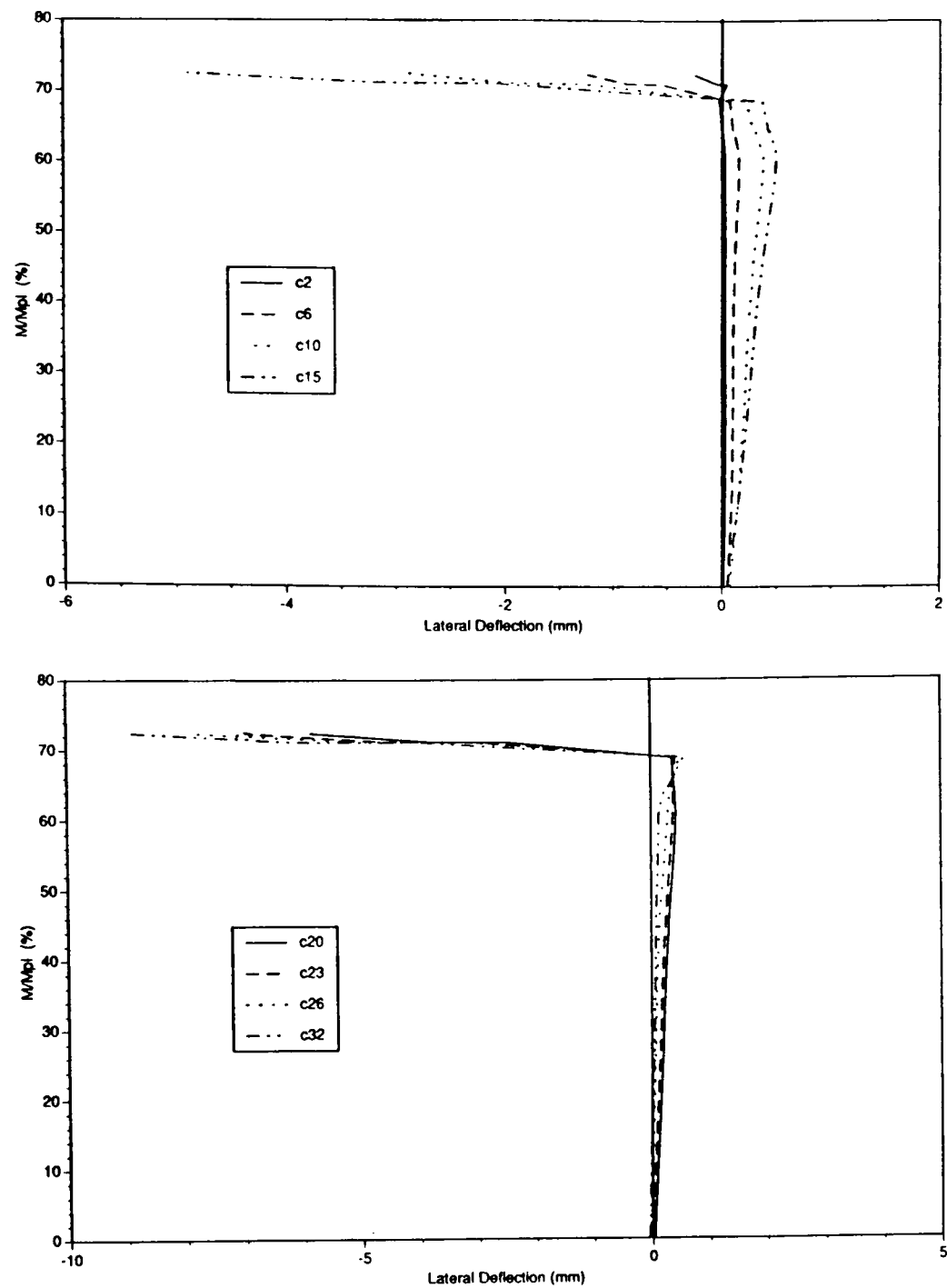


Figure 3.45.a - Load vs. Lateral Displacement of Points 2, 6, 10 and 15.
Figure 3.45.b - Load vs. Lateral Displacement of Points 20, 23, 26 and 32.

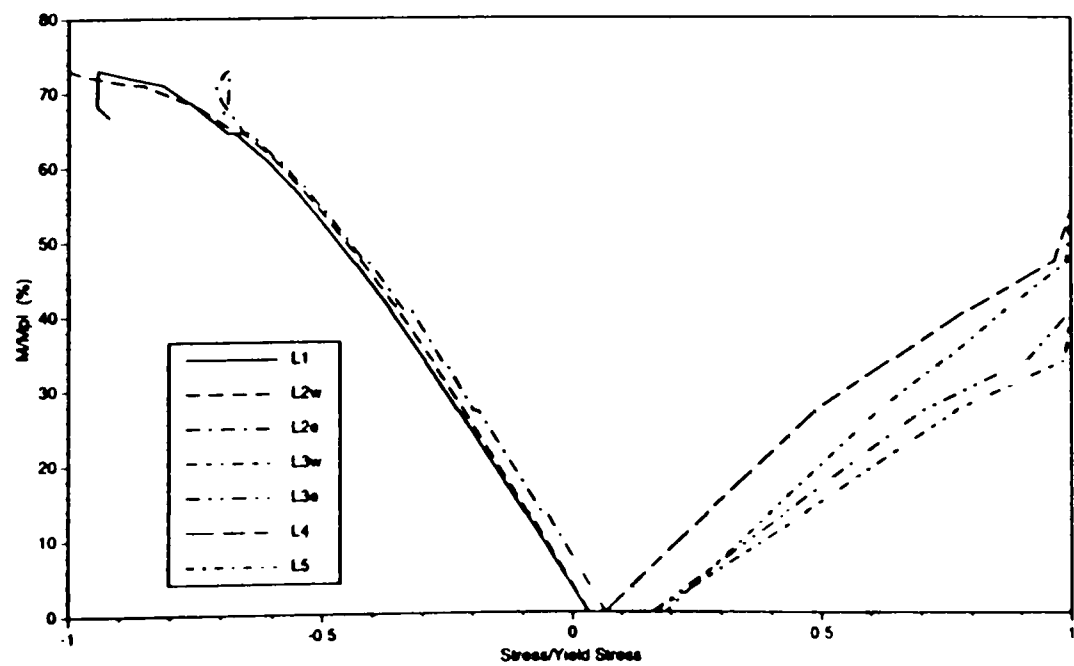


Figure 3.46 Central Linear Strain Gauge Results (Beam 5).

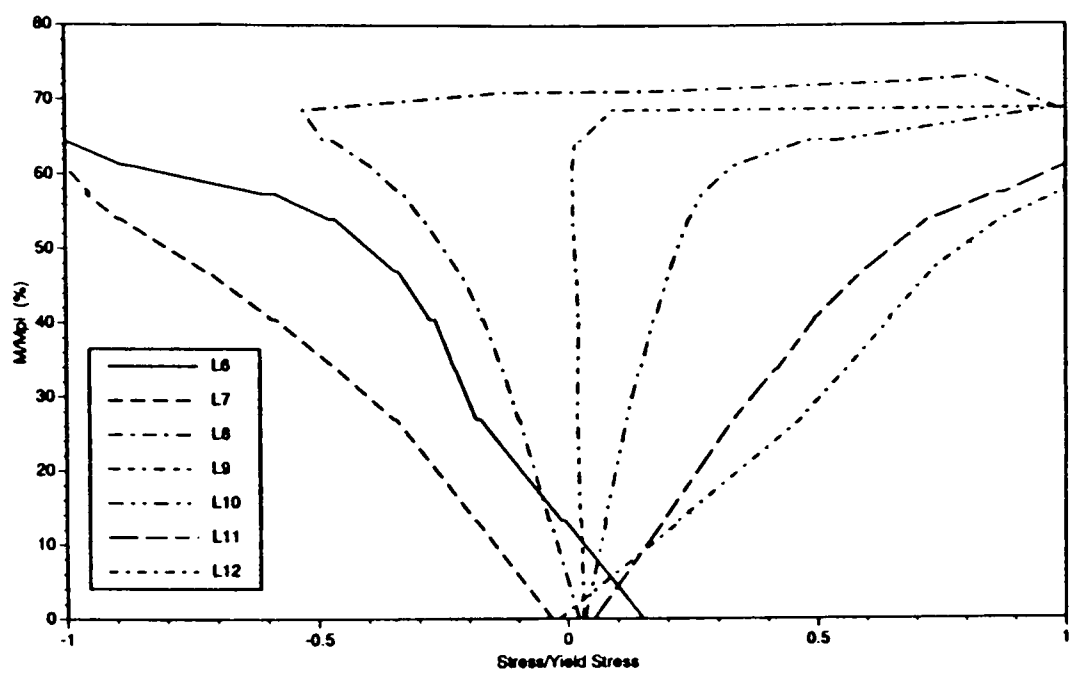
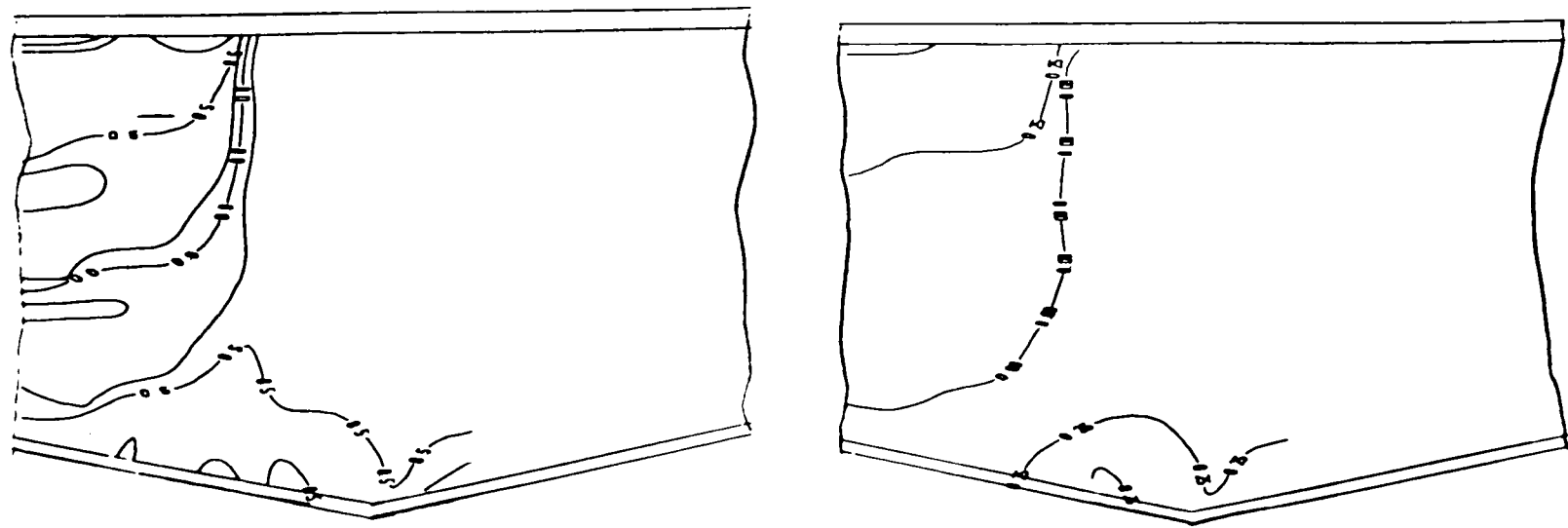
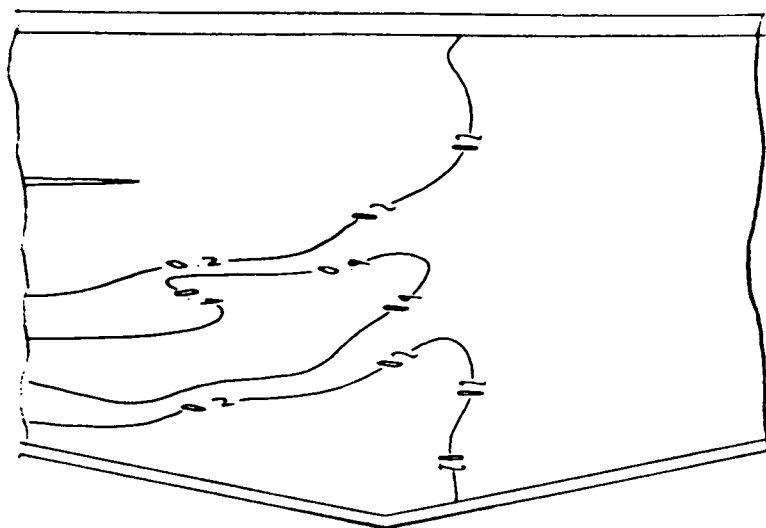


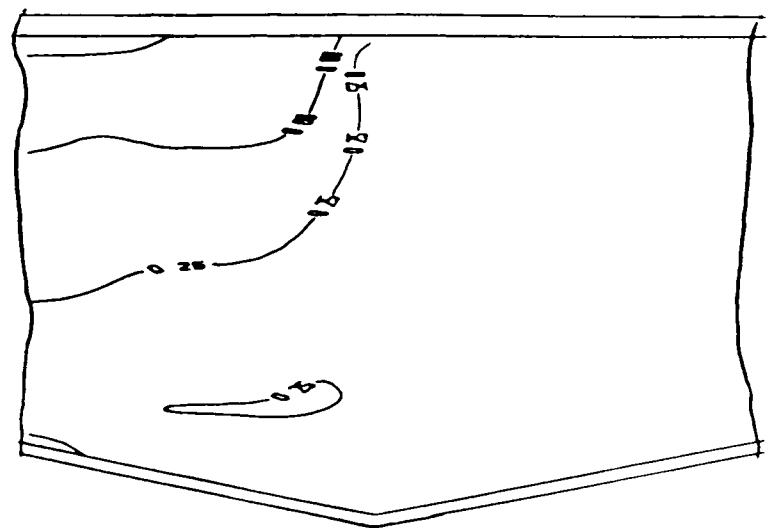
Figure 3.47 - Panel Edge Linear Strain Gauge Results (Beam 5).



Figures 3.48.a and 3.48.b - Normalized Longitudinal and Vertical Stresses in the Web (Beam 5).

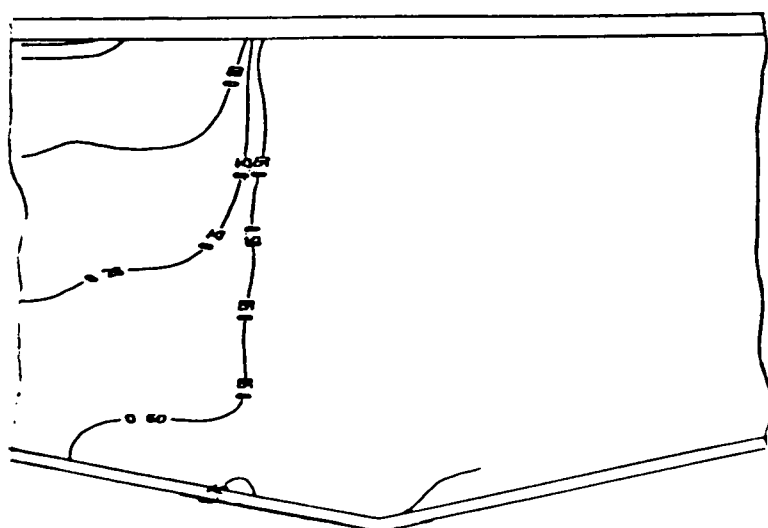


$0.133 M_{pl}$

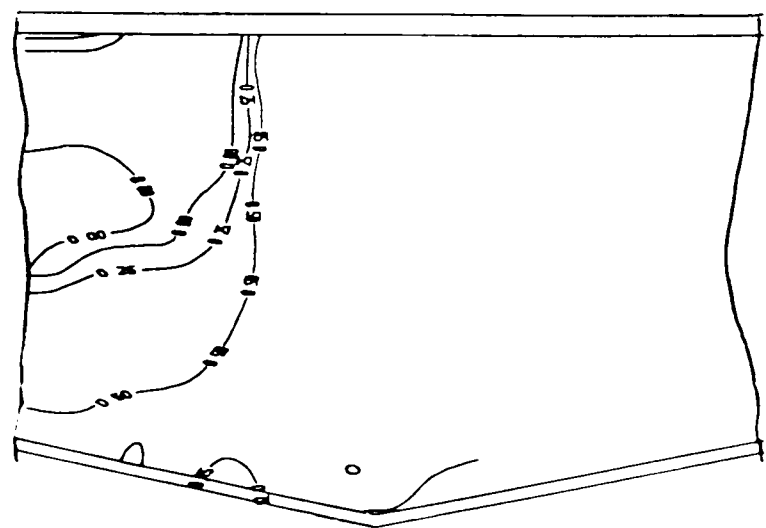


$0.405 M_{pl}$

Figures 3.49.a and 3.49.b - Normalized Equivalent Stresses in the Web (Beam 5).



$0.645 M_{pl}$



$0.731 M_{pl}$

Figures 3.50.a and 3.50.b - Normalized Equivalent Stresses in the Web (Beam 5).



Plate 22 - Failure of Beam 6 ($t_f = 25\text{mm}$).

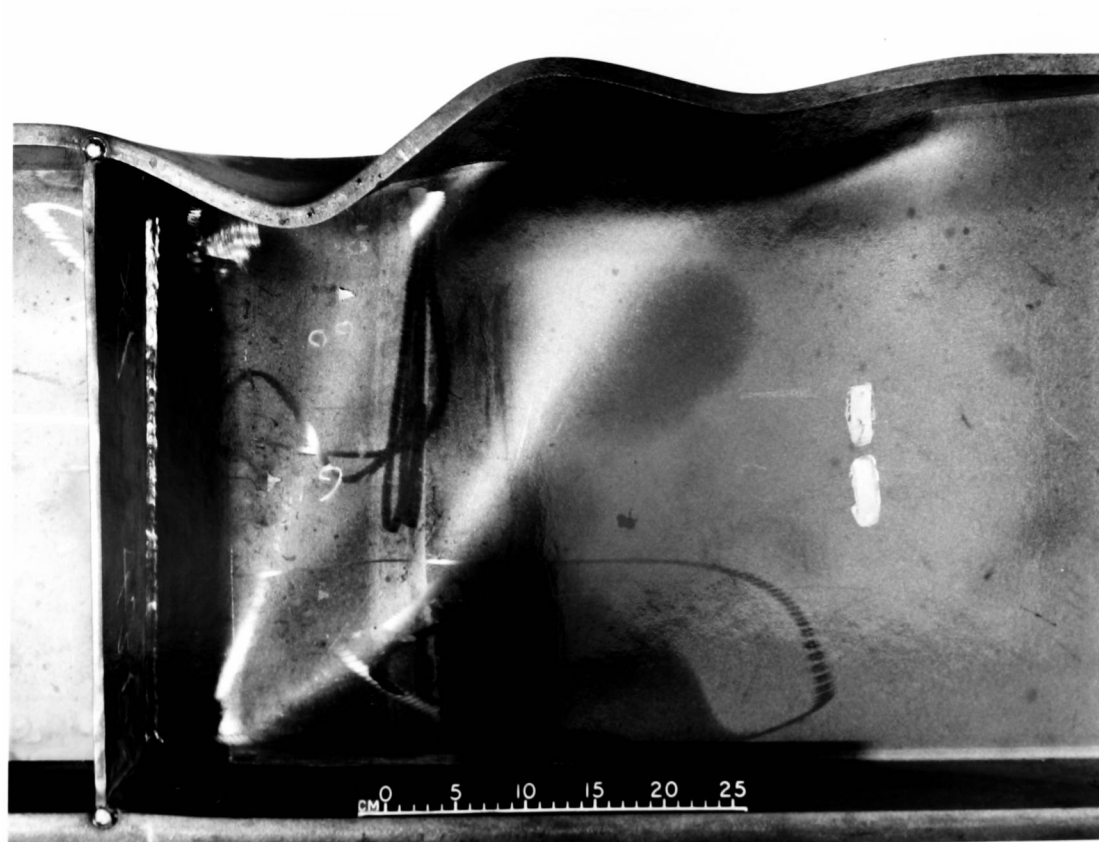


Plate 23 - Beam 6: buckling of the top flange.

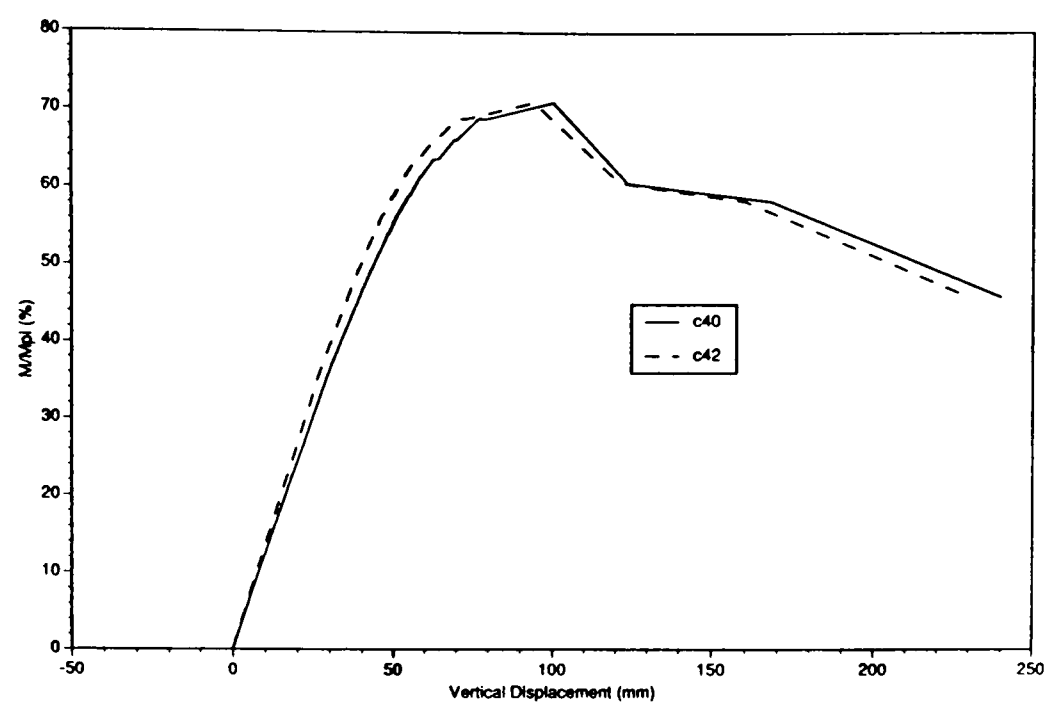


Figure 3.51 - Load vs. Vertical Deflection for Beam 6 at Centre and Edge of the Tapered Panel.

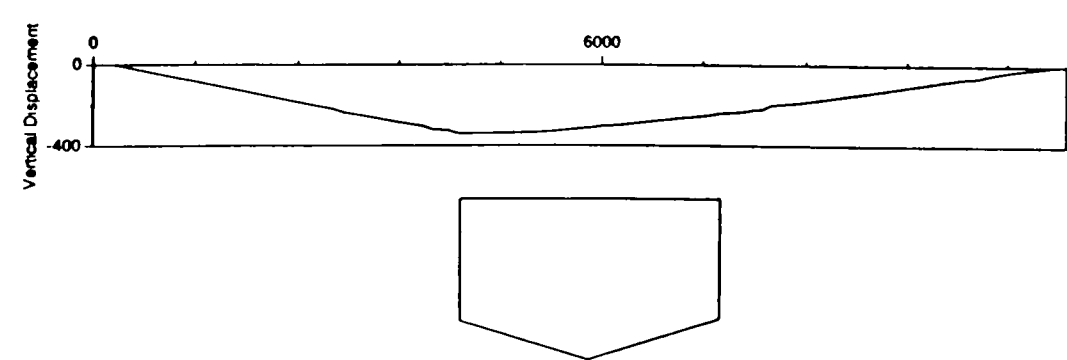


Figure 3.52 - Final Deformed Shape of Beam 6 (mm).

test, was in the order of 250mm and it can be considered small when compared with the first four cases tested.

The web in the central part of the tapered beam remained undeformed during the entire test. This can be observed in the contour plot of the lateral displacements, figure 3.53, page 121. The individual responses of some of the transducers are plotted in figures 3.54.a and 3.54.b, page 121. It is clear that the lateral displacements can be neglected since they have reached values less than 2mm.

The linear strain-gauges located at the centre of the bottom flange produced the graph shown in figure 3.55, page 122. Gauges L3e and L3w achieved the yield stress at around 60% of M_{pl} . Gauges L4 and L5 had only reached between 60% and 70% of the yield stress when the test was ended, showing that the maximum possible vertical force had not been reached. This is not very relevant because this case, which had the plastic neutral axis in the bottom flange, was conceived in order to test the beam at construction load levels. These load levels were clearly exceeded when the test was finished, and the objective of proving that no harm would occur to the central part of the girder was achieved. The results from the linear gauges at the edge of the tapered panel are displayed in figure 3.56, page 122. They show the beginning of plastic hinge formation that was suddenly stopped by the buckling of the top flange.

The contour plots of the longitudinal and vertical stresses in the final load step are shown in figures 3.57.a and 3.57.b, page 123. The equivalent (Von Mises) contour plot stresses are displayed in figures 3.58.a, 3.58.b on page 123, and 3.59.a, 3.59.b and 3.60 on page 124. They correspond to load stages equivalent to 21.9%, 39.8%, 58.9%, 68.9% and 60.7% of the plastic moment capacity at the central section. As in the previous test, the stresses in this part of the web could be considered small when compared to those of the first four cases.

3.5. CONCLUSIONS.

The results of the first series of test confirmed that the thickness of the web is one of the most important parameters in the vertical buckling problem. The 7 and 5mm beams reached their ultimate load with no sign of web buckling. They failed by the development of a plastic hinge at the edge of the tapered panel. The beam with the 3mm web did fail by vertical buckling. It was a very comforting demonstration that the postulated phenomenon could occur in reality.

A graph that summarizes the maximum moments (normalized by M_{pl}) achieved by the three specimens is presented as figure 3.61, page 125. It is very clear from that diagram that the reduction in resistance between the 5 and 3mm web specimens was very significant. For beams with the present geometric and material characteristics, it is possible to say that the limit for the vertical buckling of the web lies between the values of 3 and 5mm for the thickness of the web, which corresponds to h/t_w limits of 130 and 216.67.

The plastic neutral axis position (in contrast to what was initially thought) had no significant influence on the vertical buckling. This was demonstrated with the aid of the second series of tests summarized in figure 3.62, page 125. Here the ultimate moments (normalized by M_{pl}) for beam 2, and beams 4 to 6 are plotted against the plastic neutral axis position.

The fifth and sixth beams failed by local buckling of the top flange. Of course, in a real composite beam with the section properties of beam 5, the steel deck and the concrete slab might have prevented flange buckling so that local web buckling

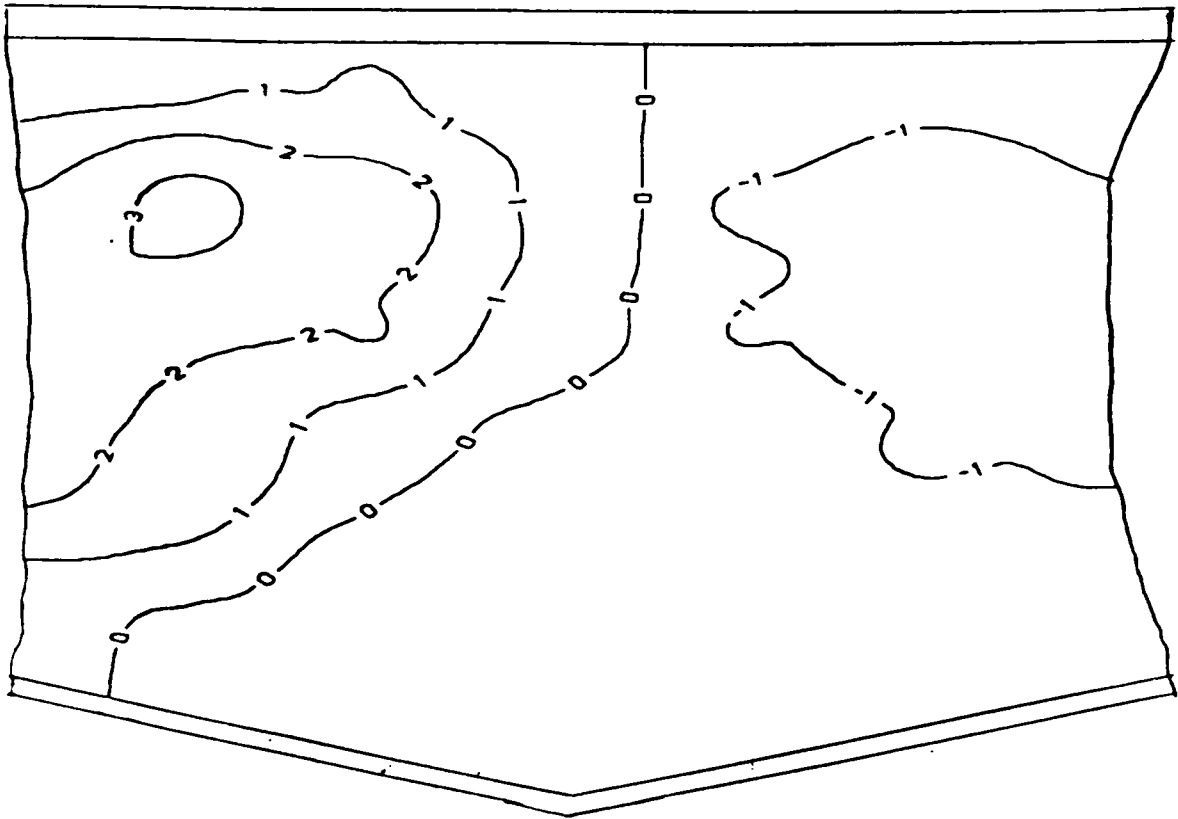


Figure 3.53 - Final Lateral Deflection Pattern of Beam 6 (mm).

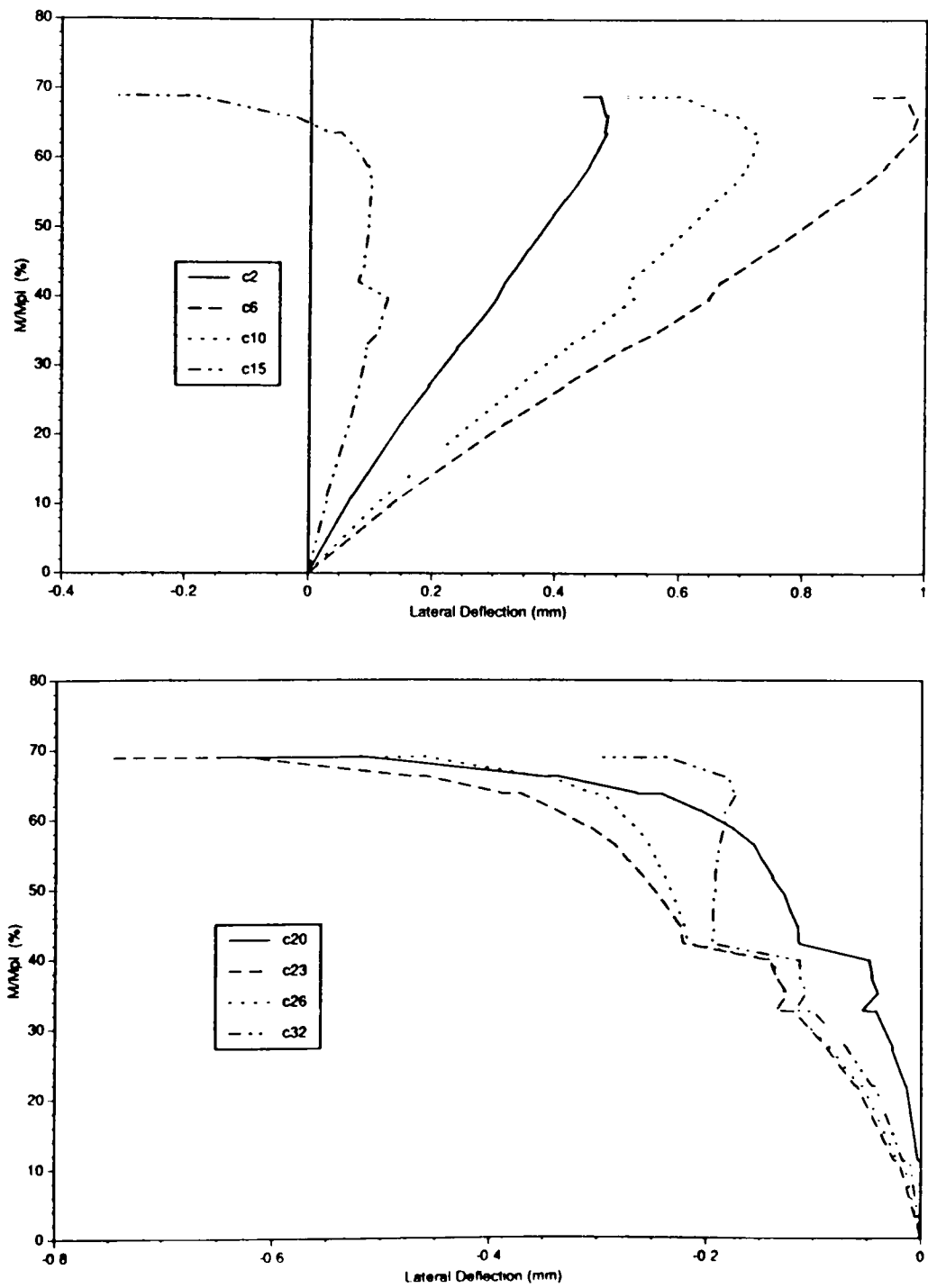


Figure 3.54.a - Load vs. Lateral Displacement of Points 2, 6, 10 and 15.

Figure 3.54.b - Load vs. Lateral Displacement of Points 20, 23, 26 and 32.

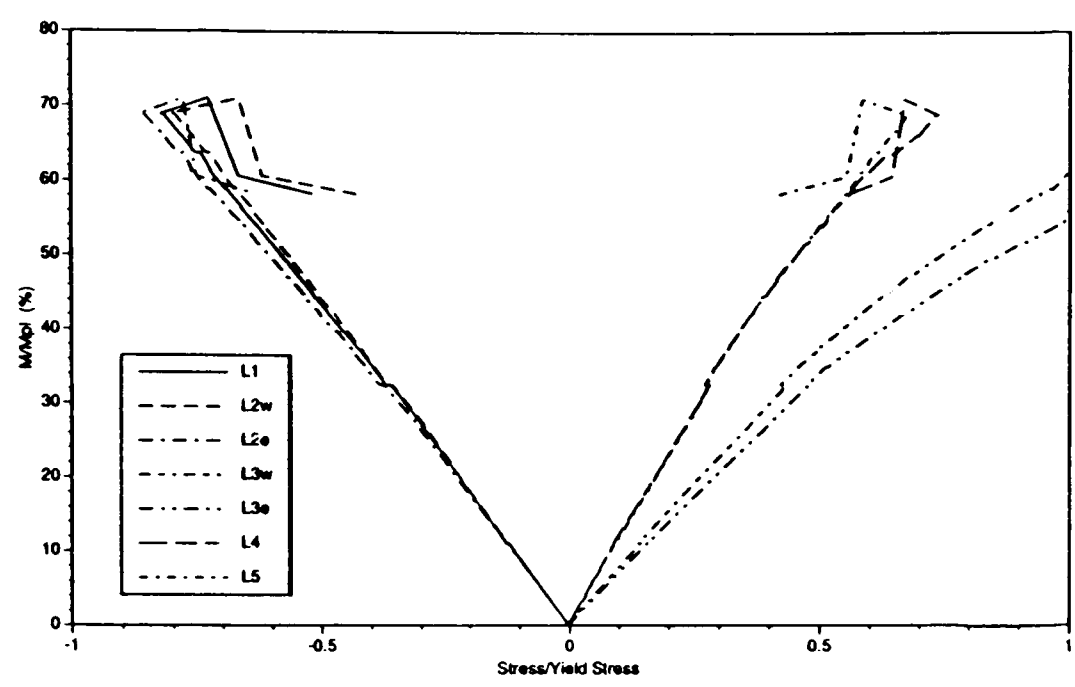


Figure 3.55 - Central Linear Strain Gauge Results (Beam 6).

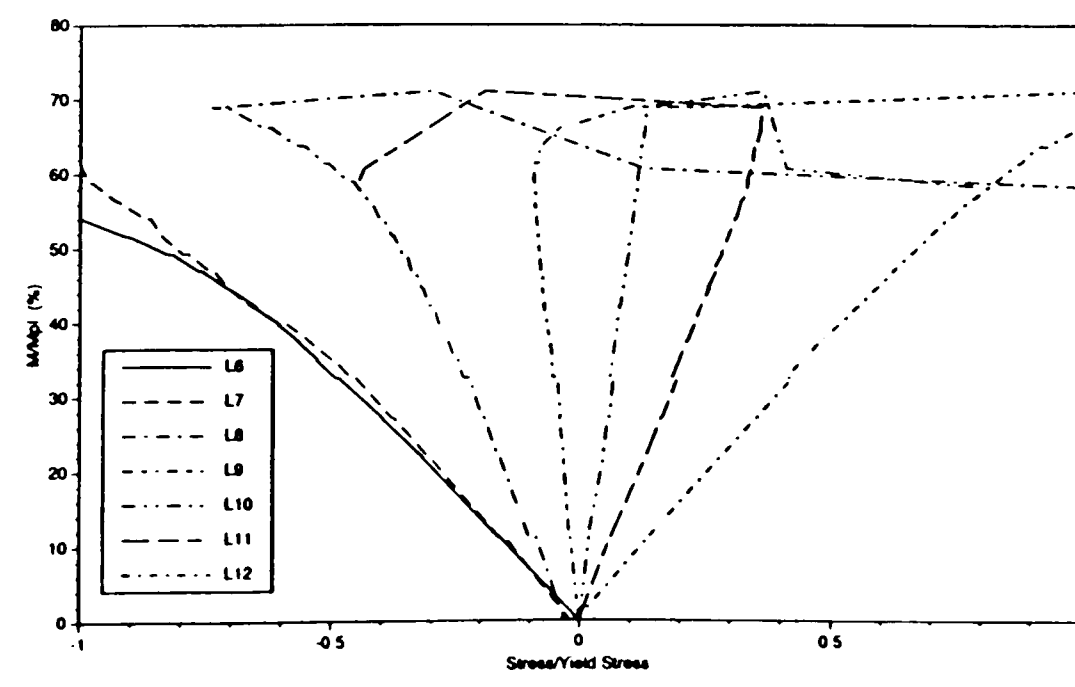
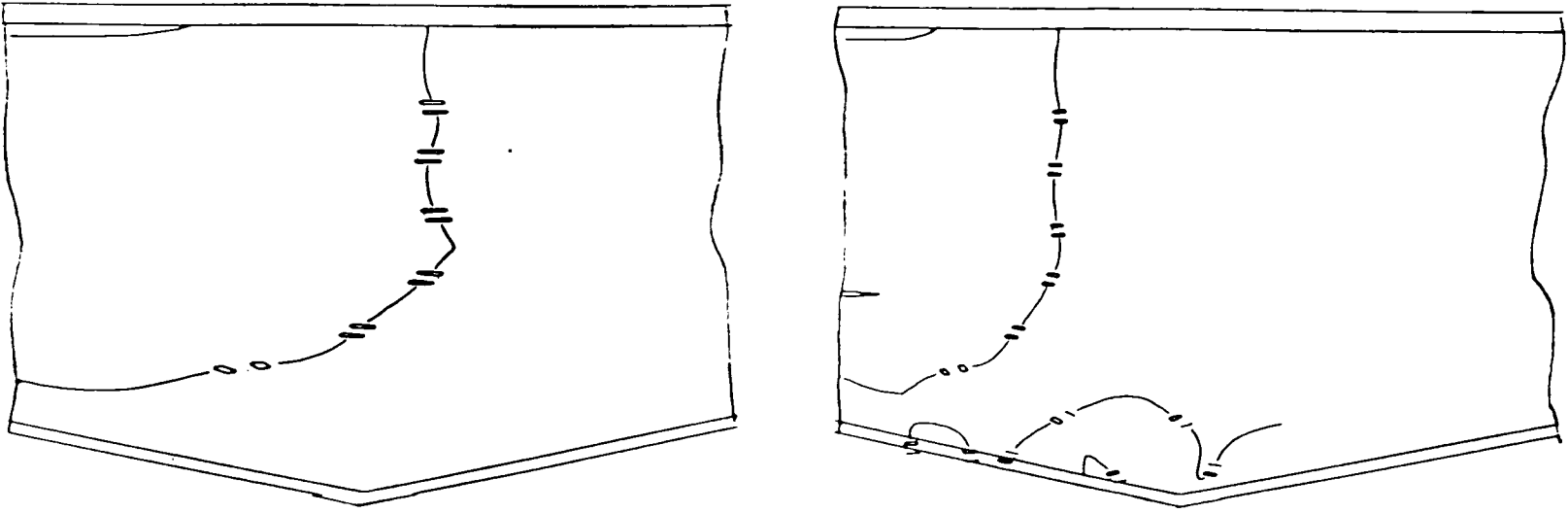
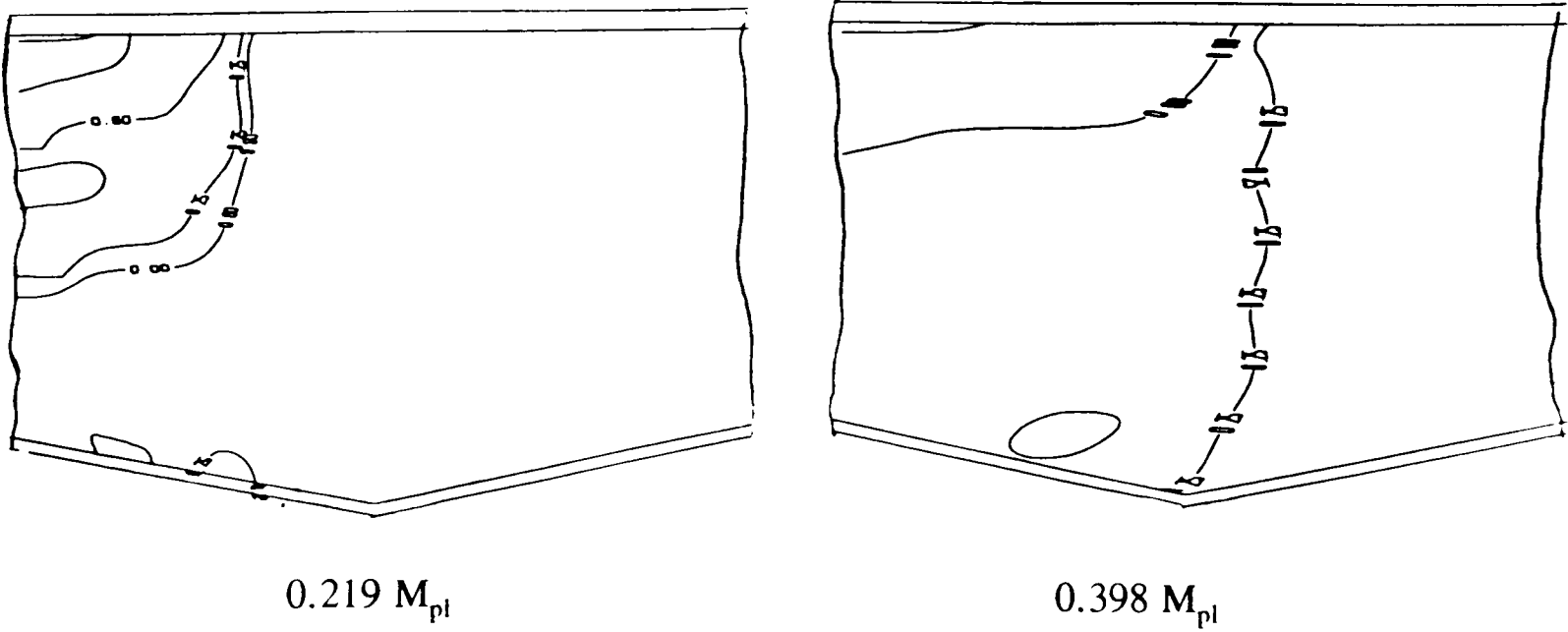


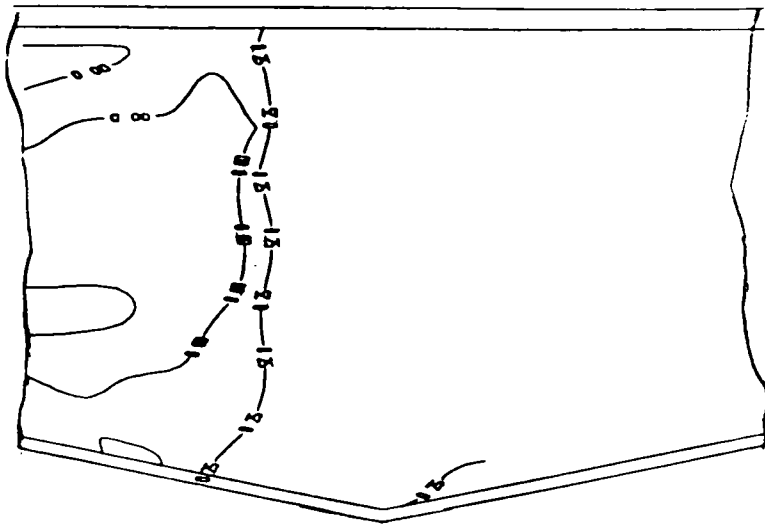
Figure 3.56 - Panel Edge Linear Strain Gauge Results (Beam 6).



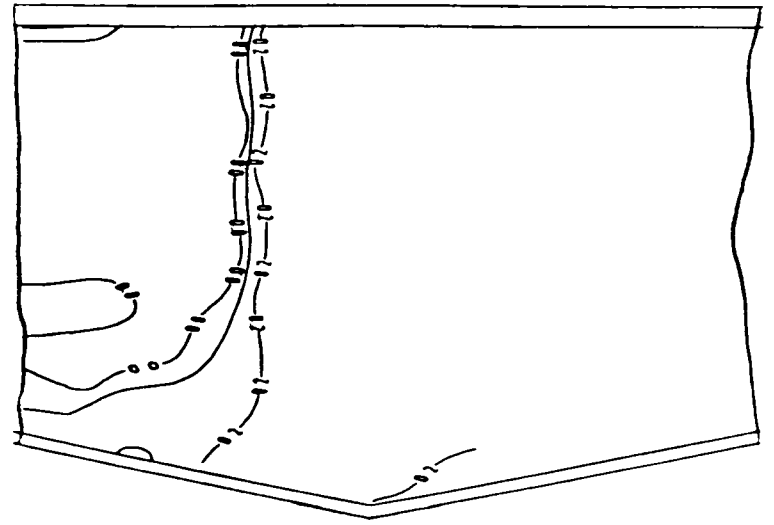
Figures 3.57.a and 3.57.b - Normalized Longitudinal and Vertical Stresses in the Web (Beam 6).



Figures 3.58.a and 3.58.b - Normalized Equivalent Stresses in the Web (Beam 6).

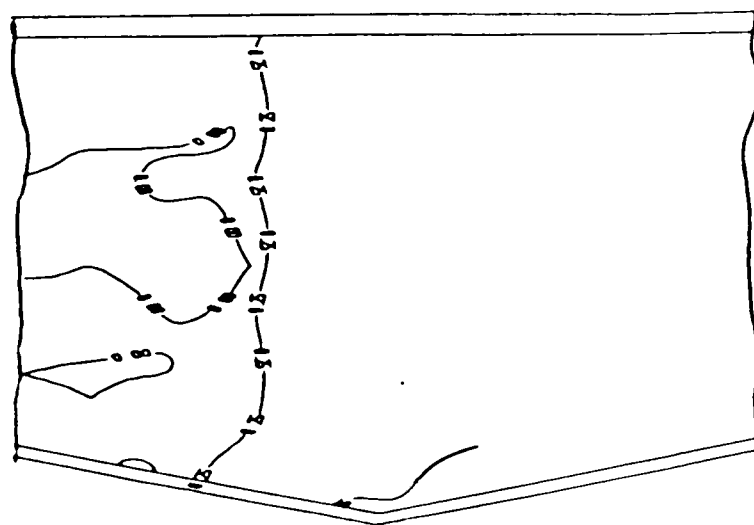


0.589 M_{pl}



0.689 M_{pl}

Figures 3.59.a and 3.59.b - Normalized Equivalent Stresses in the Web (Beam 6).



0.607 M_{pl}

Figure 3.60 - Normalized Equivalent Stresses in the Web (Beam 6).

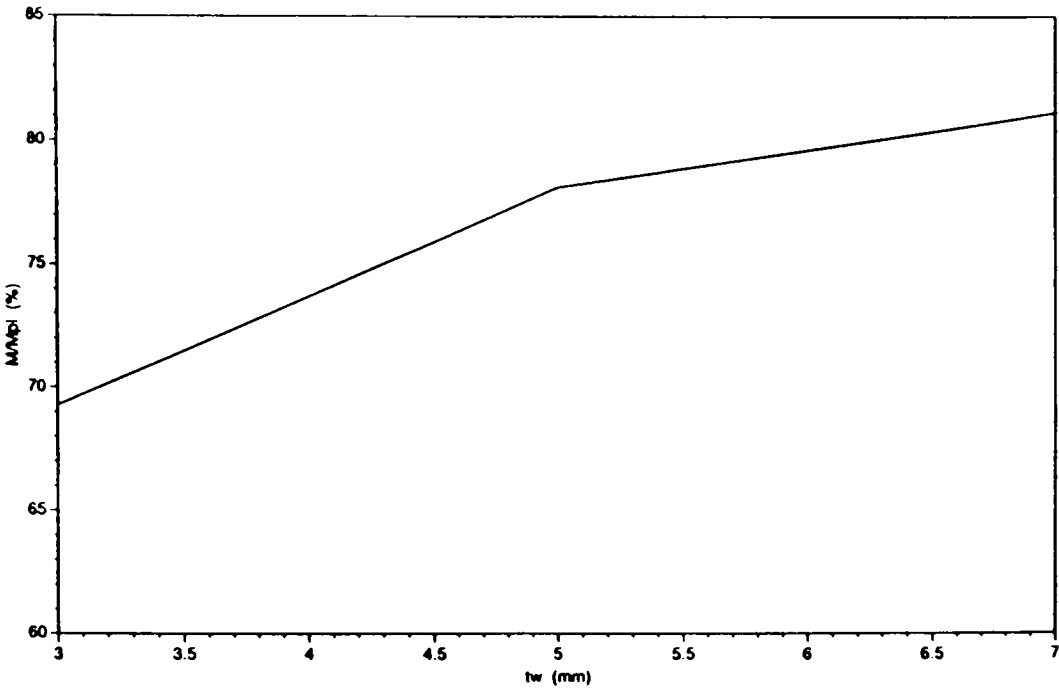


Figure 3.61 - Influence of the Thickness of the Web on the Failure Load.

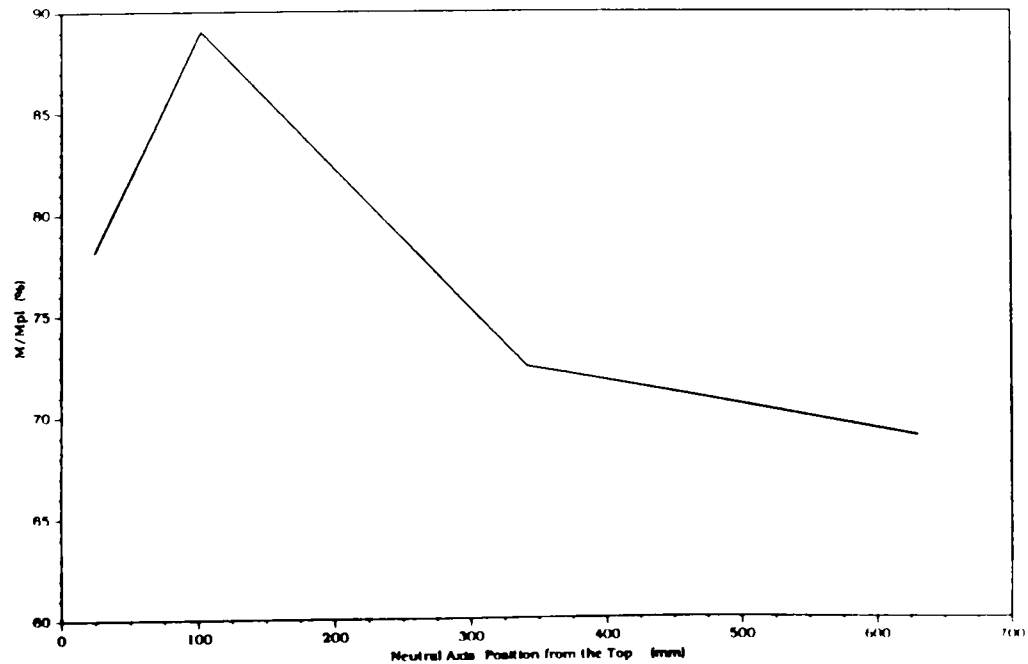


Figure 3.62 - Influence of the Plastic Neutral Axis Position on the Failure Load.

governed. However, the failure of beam 6, modelling unpropped construction, is not open to this objection. Even with its neutral axis in the lower flange, top flange buckling rather than web buckling governed the behaviour. This strongly suggests that plasticity rather than web buckling would govern the behaviour of a steel/concrete composite beam with the section properties of beam 5.

The experiments showed that vertical stiffeners in the web would only be needed in extreme situations. Most of the cases found in practice will not need stiffeners. Given the high cost of fitted stiffeners the approach of slightly increasing the thickness of the web to avoid the vertical buckling seemed likely to be the most satisfactory design expedient in limiting cases. With the completion of the description of the experiments, the next phase of the present work is the development and calibration of the finite element model used later to simulate other cases not studied in the laboratory. This will be described in the next chapter, together with a full comparison between the experimental and finite element results for the models described in this chapter.

CHAPTER 4

FINITE ELEMENT MODELLING.

4.1. INTRODUCTION.

This chapter comprises a description of the finite element model used to simulate the local web buckling problem, and its comparison with the experiments reported in the previous chapter. It covers both series of tests and some other examples to try to isolate the web buckling effect.

The final goal of using the finite element method was the production of a comprehensive set of results, covering the full practical ranges of a number of different variables, a so-called "parametric study", which could be used for design purposes or to validate any proposed simplified design rules. This parametric study forms the subject of chapter 5.

Preliminary finite element simulations were run prior to the experiments to obtain experience and develop ideas on the best way to model the phenomena physically and in the computer, and ideas on the design and outcome of the experiments. The parametric study took place once the application of the computer program to the tapered beam problem had been validated.

The obvious way to carry out this validation was by comparisons with the experiments. These comparisons provided the ideal means to refine the calculation model, in respect of the mesh arrangement, boundary conditions and the means of applying the loading.

4.2. DESCRIPTION OF THE PROGRAM 'FINAS'.

FINAS is a non-linear finite element program developed at Imperial College which can cope with material and geometric non-linearities, with a particular emphasis on the analysis of plate and shell structures. It was originally developed by Trueb, [TRUE83], (starting from a version of NONSAP) and extended by Bates, [BATE86], by the addition of further shell and eccentric beam (plate stiffener) elements.

The program uses a Gauss elimination technique to solve the stiffness equations. Full and modified Newton-Raphson incremental methods are provided to cope with the non-linear characteristics of the solutions, and convergence strategies including load, displacement and arc-length control are also available.

The finite element used in the present modelling is a ¹eight-node doubly curved shell with six degrees of freedom per node. The displacement relationship is governed by an updated Lagrangian technique. The material non-linearity is treated by a Von Mises analysis through two layers in the thickness of the shell. A 2 by 2 Gauss numerical integration is used and proved to be effective in avoiding shear-locking problems, [BATE86].

4.3. DESCRIPTION OF THE CALCULATION MODEL.

Because of the symmetry of the beam examined, it was possible to include only half of its length in the calculation model. To impose the conditions of symmetry on the model, the displacements in the longitudinal direction x , and the rotations

about the axes y and z , were made zero along the axis of symmetry located at the centre of the beam.

The calculation model was composed of 112 shell elements as can be seen in figures 4.1 and 4.2, page 129. The length of the specimen panel was equal to 1.3m and the total length was 1.95m. The other dimensions were set up to match those of the experimental models described in the previous chapter. The position of the load, and the supports in the y direction are represented by six vertical arrows in figure 4.1, page 129. The load was applied by three concentrated forces at the top of the upper flange and was resisted by three vertical supports on the bottom flange. The force applied directly over the web was 50% of the total load. The numbering of some important nodes in the web is also presented in figure 4.2, page 129.

Web stiffeners with thickness, t_{st} , equal to 50mm, were used at the loaded points and the supports. They prevented local failure of the web at those points and contributed to the loading scheme in a very efficient way. The flanges and the web between the stiffeners were also designed not to buckle or yield, using very high yield stresses and substantial thicknesses to avoid any adverse influence on the load carrying capacity of the specimen panels.

Lateral buckling of the compression flange was prevented by restraining the movement in the out-of-plane z direction of all the nodes on the line that joined the web and the top flange. This was necessary to simulate the lateral restraint contribution of the concrete deck to the top steel flange.

To standardize the analysis, an imperfection shape in the form of a half-sine wave in the longitudinal and vertical directions was adopted. The maximum amplitude was equal to 2.5mm approximately, which is equal to $h/250$. A yield stress of 265MPa was assumed for the flanges and the web following BS5950, [BRIT85], for a grade 43 steel. In a subsequent stage, some runs were executed including the experimental measured imperfections and true yield stresses for calibration and comparisons with the assumed values.

The number of load steps to achieve and pass the peak load varied from case to case. Due to the great non-linearity of the problem, the full Newton-Raphson method had to be used in some cases from very early stages of loading, and the number of increments reached up to 400. This made the process very time consuming. There were cases that ran alone in a Sun Spark 2 machine for more than one week to reach the final value. If other strategies were used or a larger load step tried, the problem diverged before reaching the ultimate load. On the other hand, in a few cases the maximum load was very quickly achieved with the modified Newton-Raphson technique in 20 or 30 load steps. Other problems encountered in the analysis will be discussed in more detail in the last section of this chapter.

4.4. INFLUENCE OF THE THICKNESS OF THE WEB.

In this section the finite element results using standard imperfections and yield stresses for the first three beams tested will be presented alongside their comparison with the experimental data. After that, the runs which used the measured values of the imperfections and yield stresses will be considered. Subsequently, some additional cases to identify even closer boundaries for the local web buckling problem will be outlined. Lastly, the main conclusions will be drawn and a final summary made.

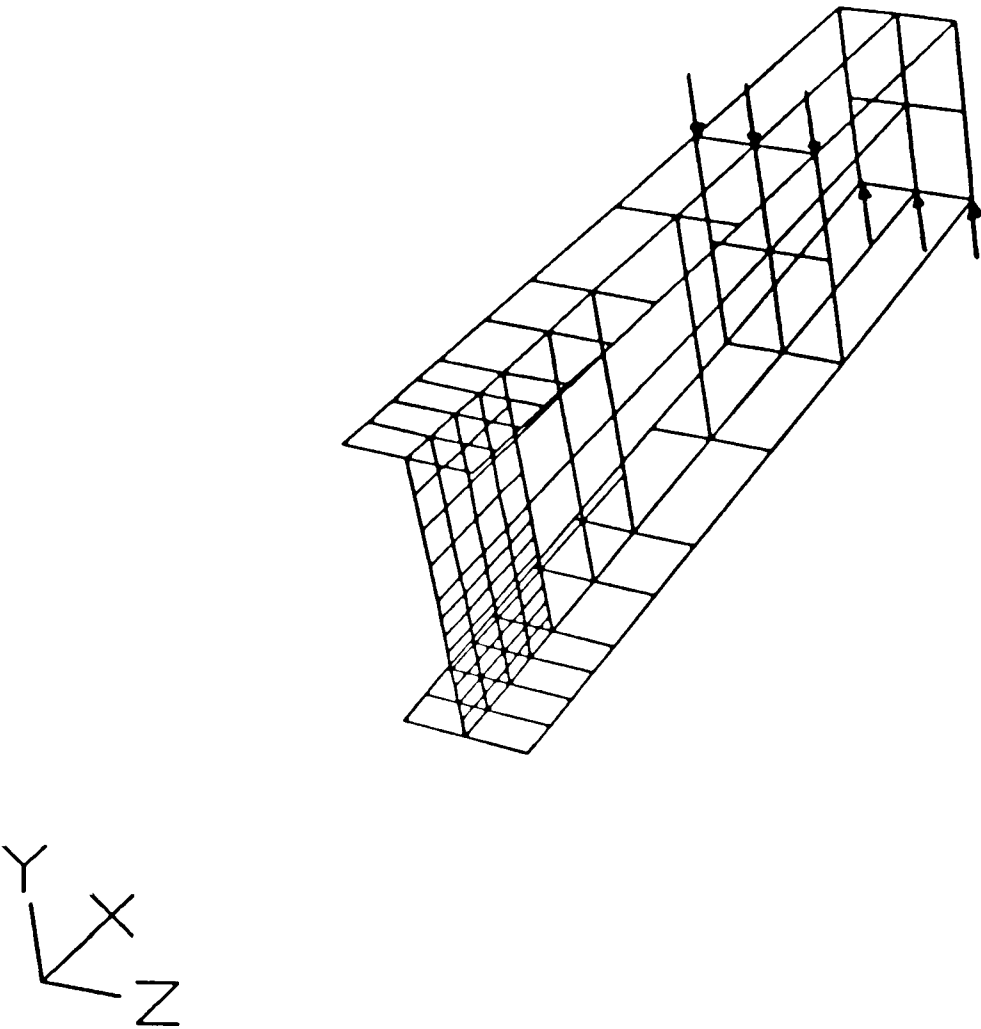


Figure 4.1 - Finite Element Mesh: Perspective.

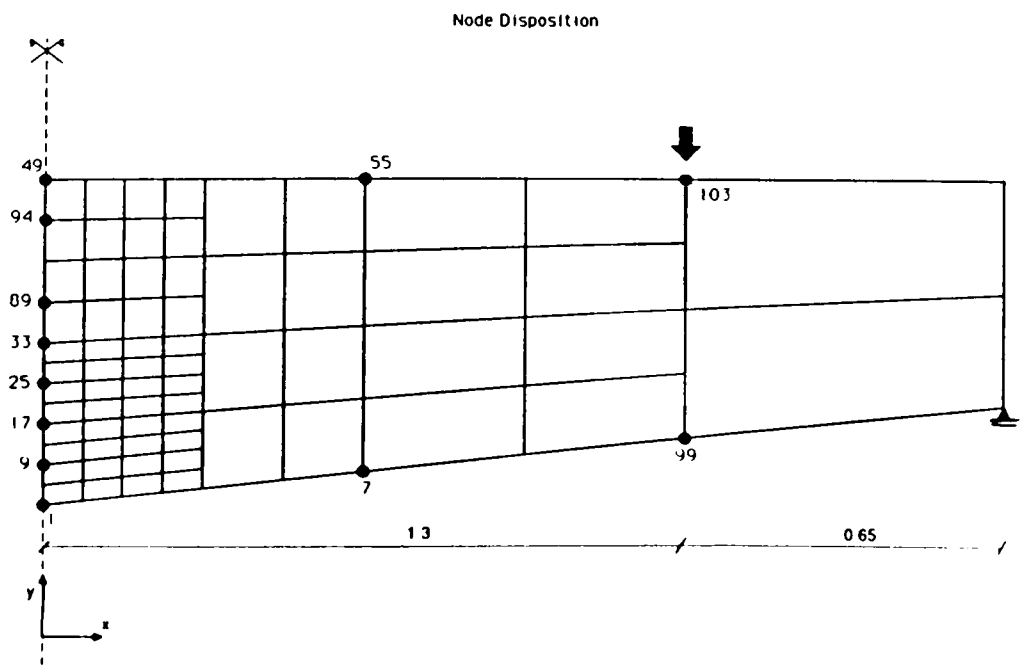


Figure 4.2 - Web Finite Element Mesh: Node Configuration.

4.4.1. Beam 1. ($t_w = 7\text{mm}$).

A perspective of the deformed configuration of beam 1, at the point of maximum load, magnified by a factor of five, is presented in figure 4.3, page 131. It is clear that the web remained unbuckled. There was a large rotation close to the stiffener, showing the position of the plastic hinge which was the cause of the girder's failure. The transverse curvature achieved by the bottom flange at the centre of the specimen is also very visible. This final shape was very similar to the one obtained in the laboratory and was considered very satisfactory.

Figures 4.4.a and 4.4.b, page 131, represent the vertical and equivalent (Von Mises) stress patterns in the web at the point of maximum load. The units are kN/mm^2 and the maximum stress is equal to the yield value. There is a very concentrated vertical stress region near the bottom flange, at the centre of the beam. However, the most plastified region of the beam was near to the top flange due to the longitudinal stress contribution (biaxial compression).

A comparison between the graphs of load against vertical displacement for the finite element simulation and the experimental data is shown in figure 4.5, page 132. The values of the vertical displacement for the finite element solution had to be multiplied by a correction factor to take into account the differences of inertia, total span, and moment lever arm, existing in the two different approaches. It is clear that the experiment was stopped before achieving the ultimate load, due to excessive vertical deflections.

Load against lateral displacement graphs for channels 2, 10, 20 and 26 are presented in figures 4.6.a, 4.6.b, 4.7.a and 4.7.b, page 132 and 133, with their respective finite element counterparts. They demonstrate that both methods gave similar trends of lateral web displacements. They also characterize, very effectively, the small web lateral displacements that occurred.

4.4.2. Beam 2. ($t_w = 5\text{mm}$).

A perspective of the deformed configuration of beam 2, amplified by the same factor as in the previous case, is presented in figure 4.8, page 133. Again, the finite element results coincide with the experiment's final configuration. The same pattern of failure by plastic hinge formation appeared. The web remained unbuckled with very small lateral deflections. The transverse curvature of the bottom flange also existed but was less severe than in the 7mm case.

The web stress pattern is shown in figures 4.9.a and 4.9.b, page 134, for the stage of maximum load. The vertical stress distribution is more concentrated in the lower part of the web. The maximum equivalent stresses have remained near the top flange for the reason stated in the previous case.

A graph of load against vertical displacement is drawn in figure 4.10, page 134. It shows the tendency present in the first beam. The lateral web displacements for channels 2, 15, 23 and 26 are presented in figures 4.11.a, 4.11.b, 4.12.a and 4.12.b, pages 135 and 136. They had again the same tendency to displace laterally by only modest amounts.

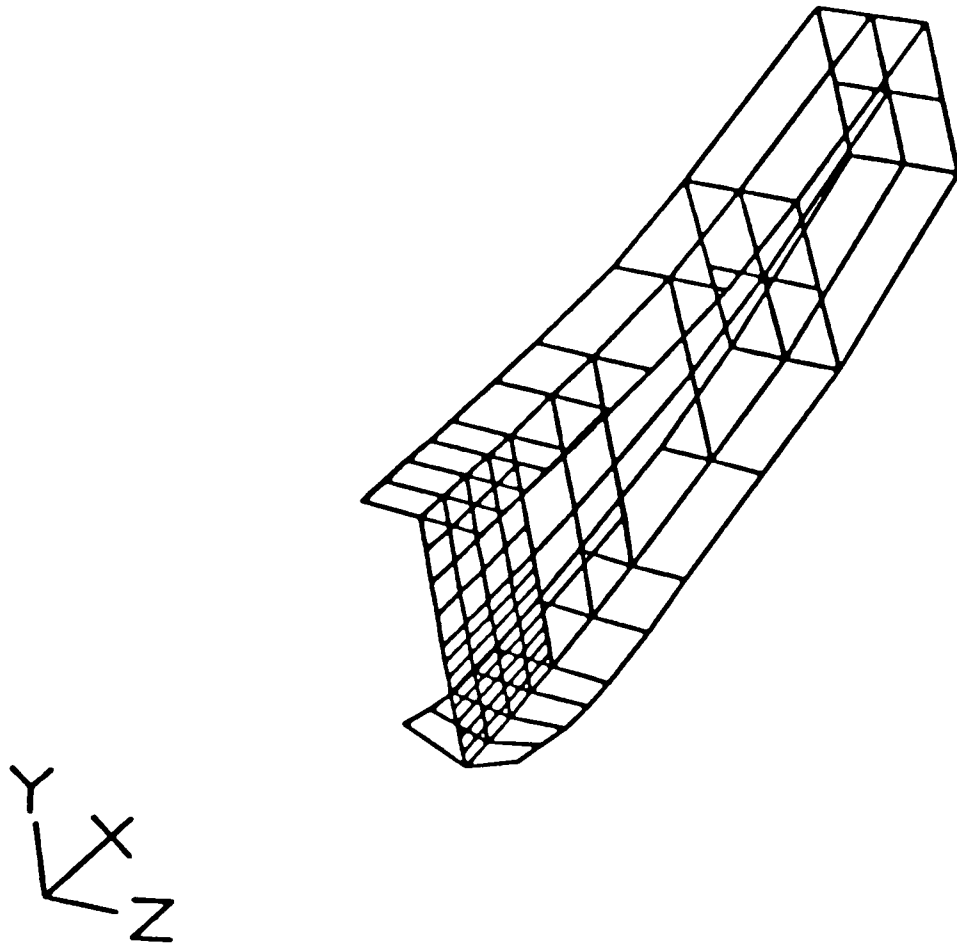
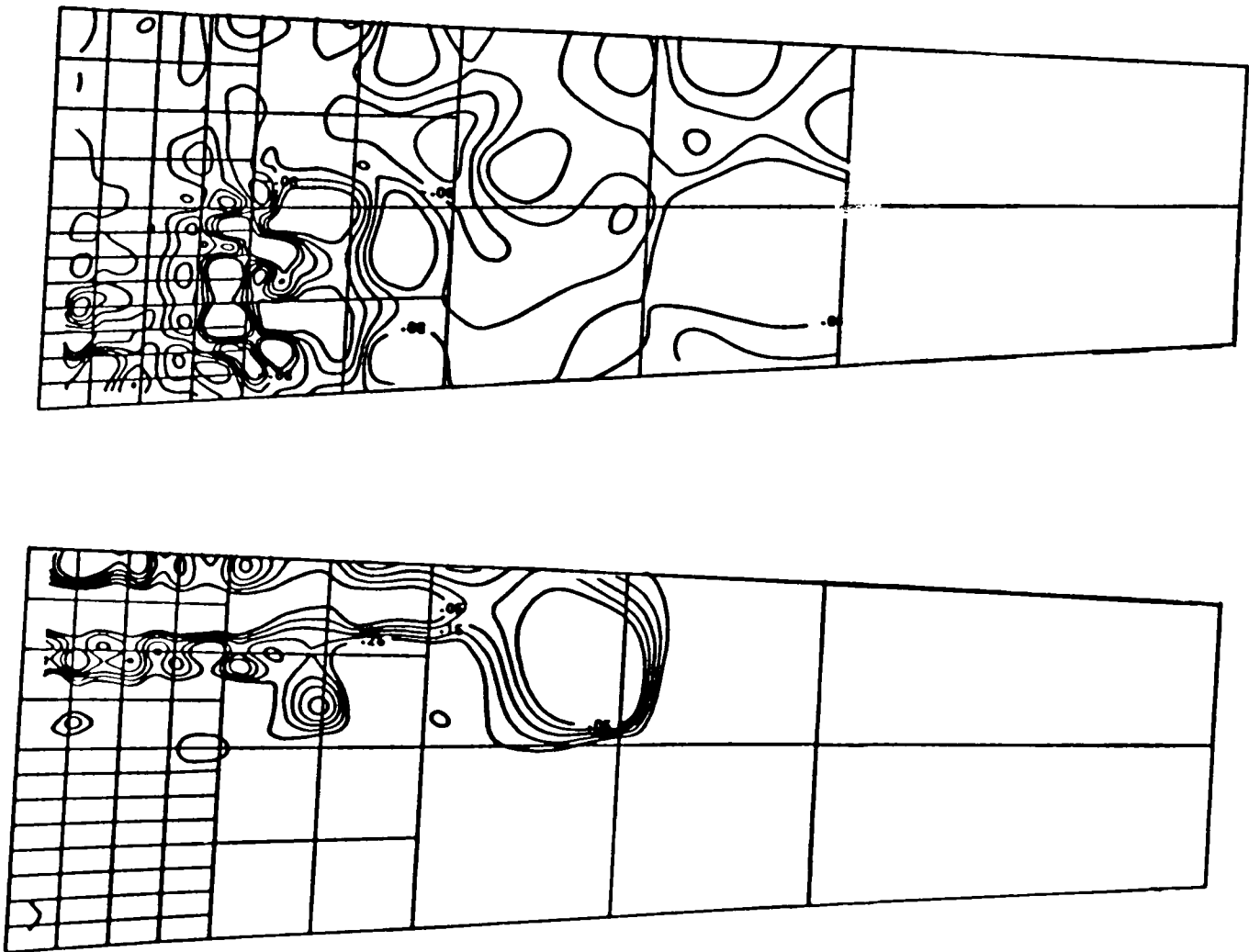


Figure 4.3 - Deformed Configuration of Beam 1 ($t_w = 7\text{mm}$).



Figures 4.4.a and 4.4.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam 1 ($t_w = 7\text{mm}$).

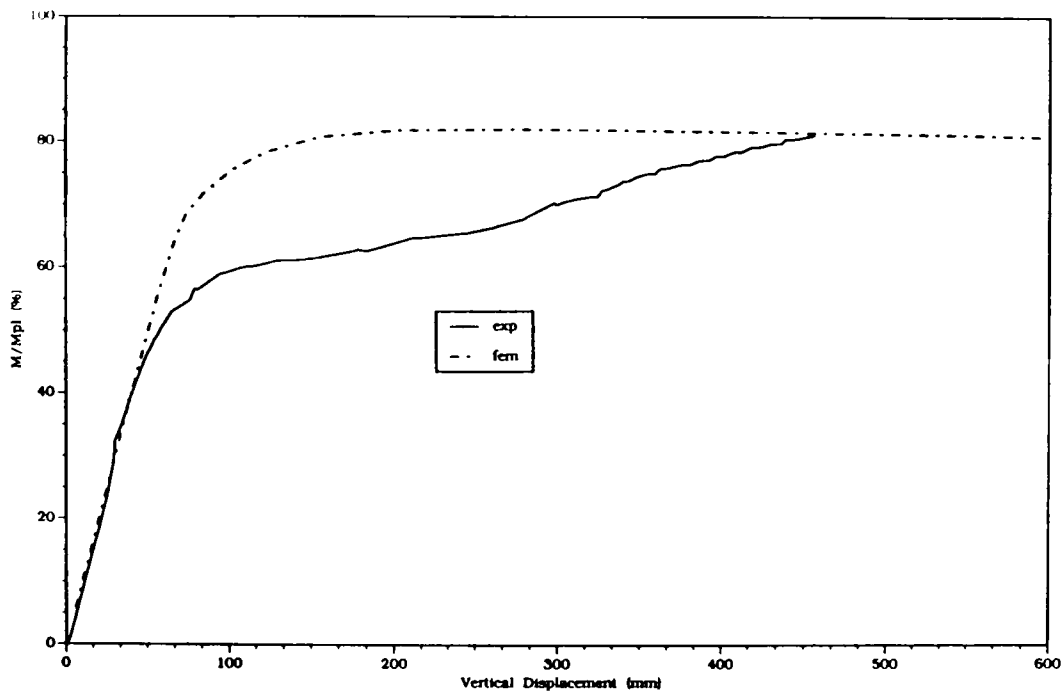
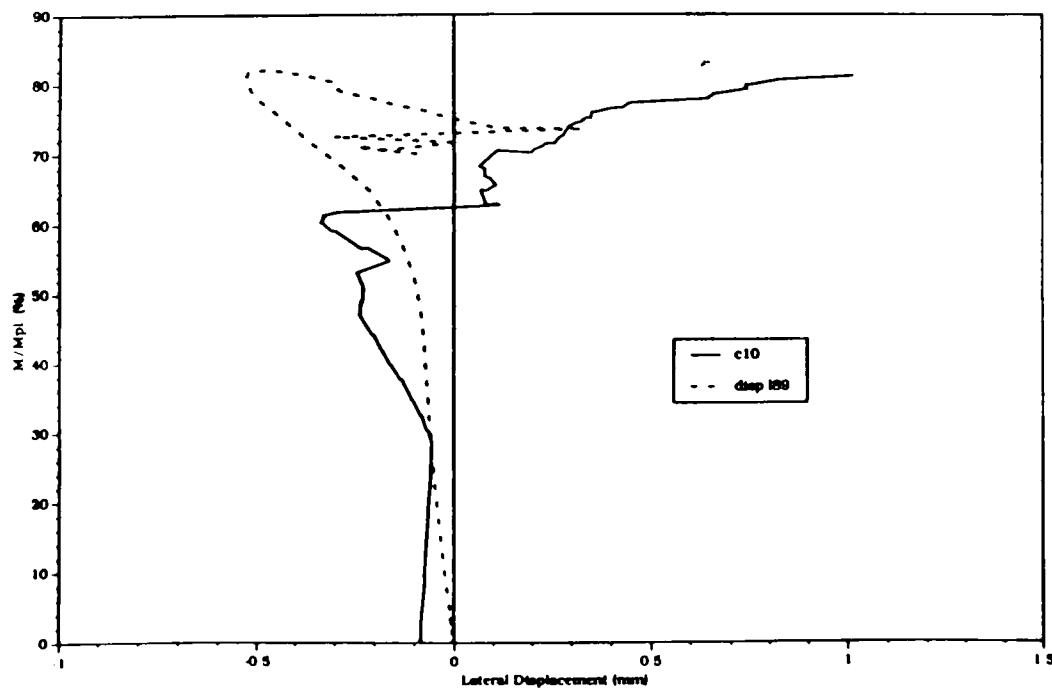
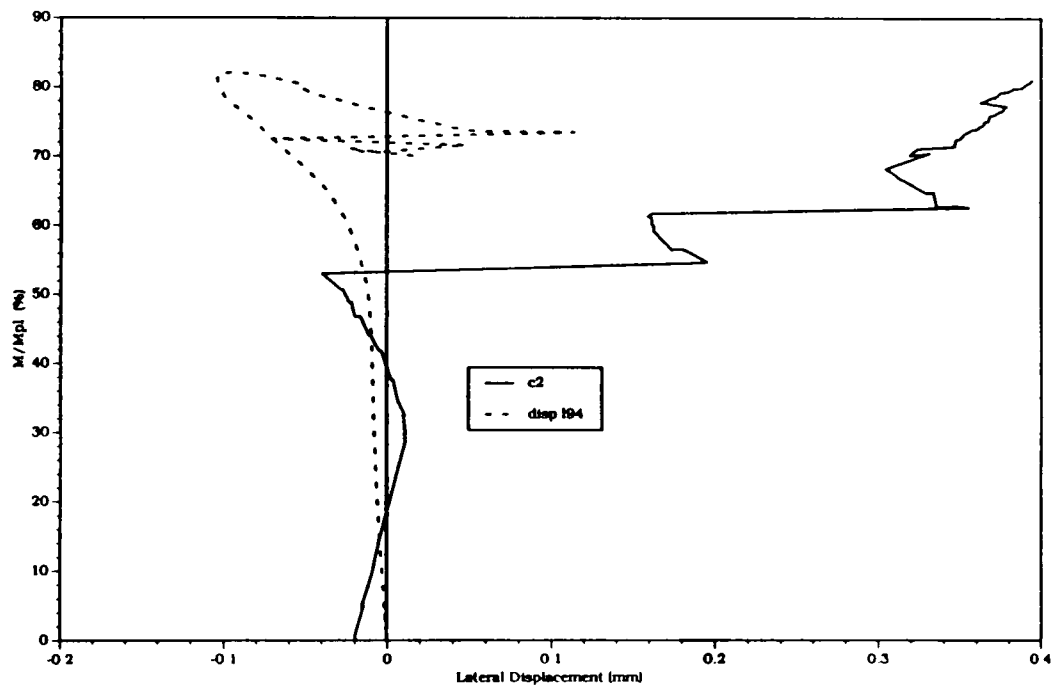
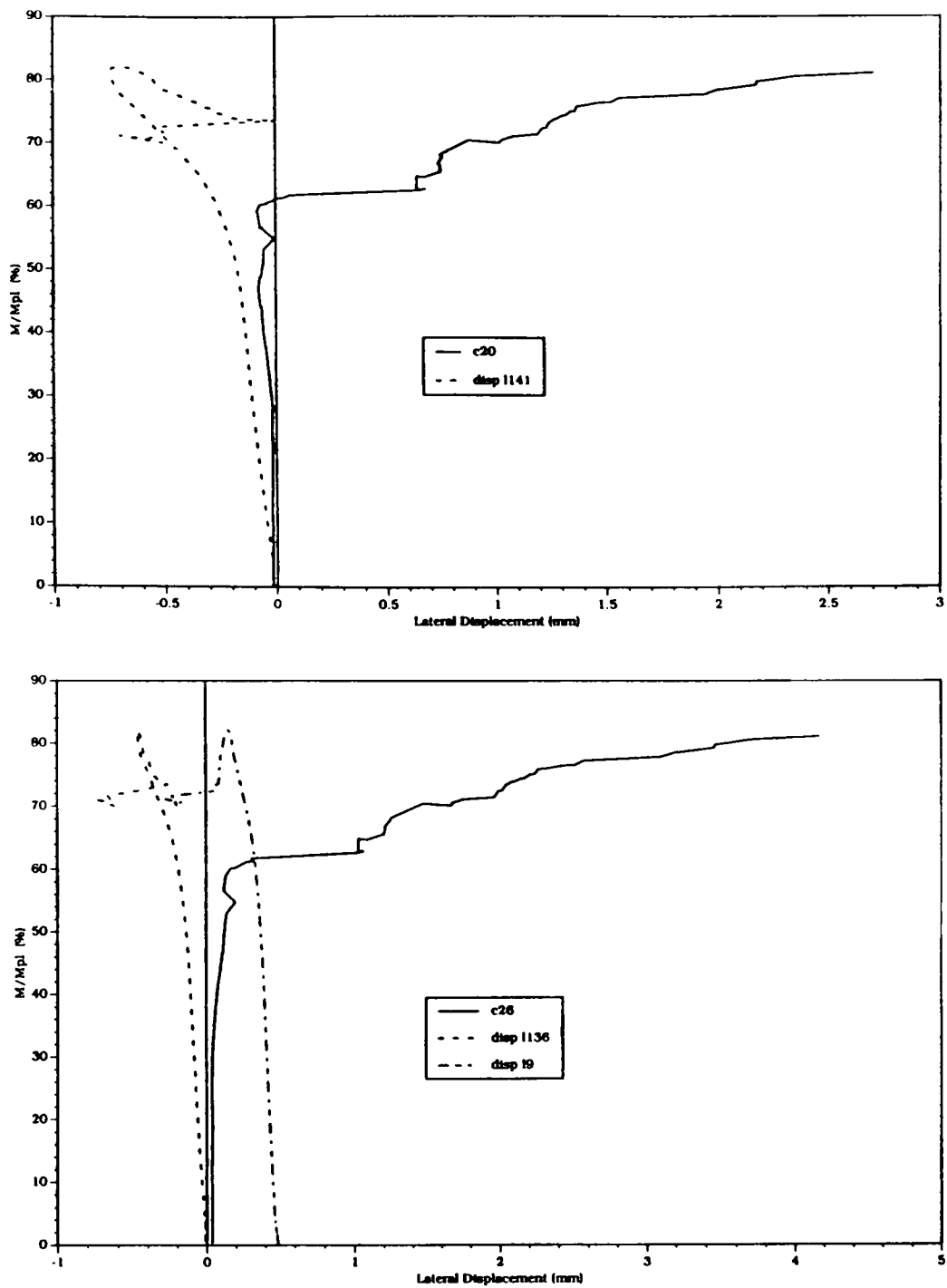


Figure 4.5 - Load vs. Vertical Displacement Comparison (Beam 1).



Figures 4.6.a and 4.6.b - Load vs. Lateral Displacement Comparisons for Channels 2 and 10, Beam 1.



Figures 4.7.a and 4.7.b Load vs. Lateral Displacement Comparisons for Channels 20 and 26, Beam 1.

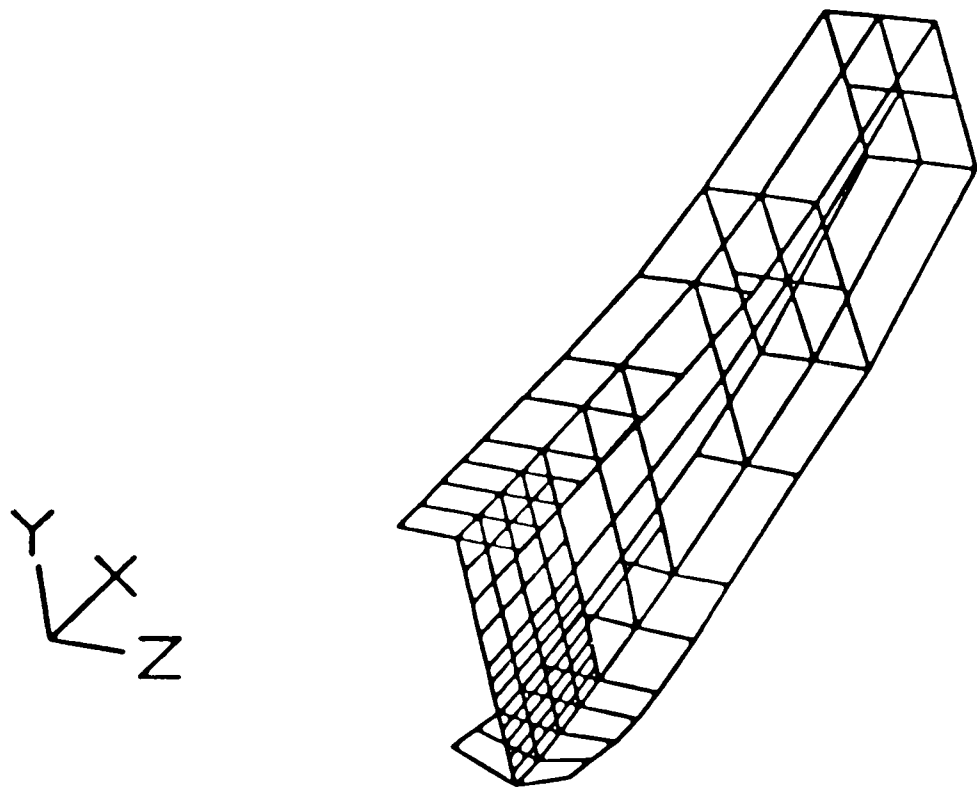
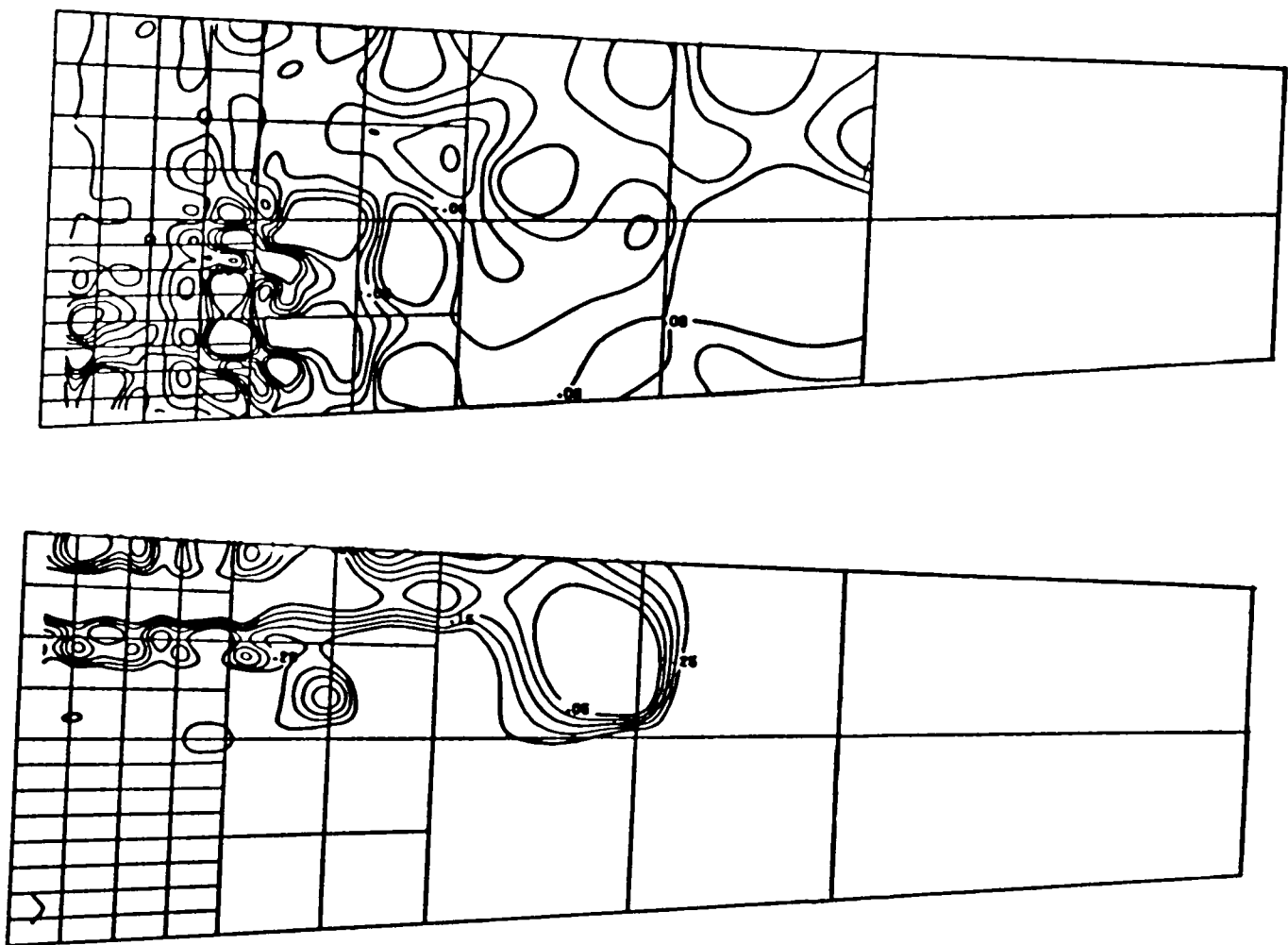


Figure 4.8 - Deformed Configuration of Beam 2 ($t_w = 5\text{mm}$).



Figures 4.9.a and 4.9.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam 2 ($t_w = 5\text{mm}$).

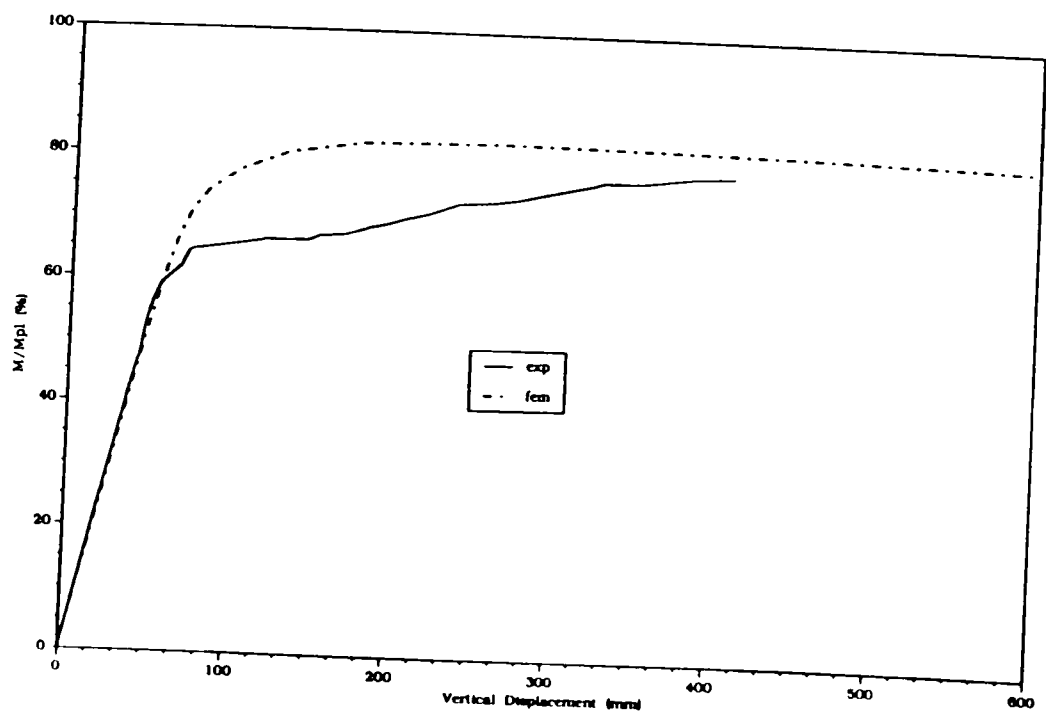
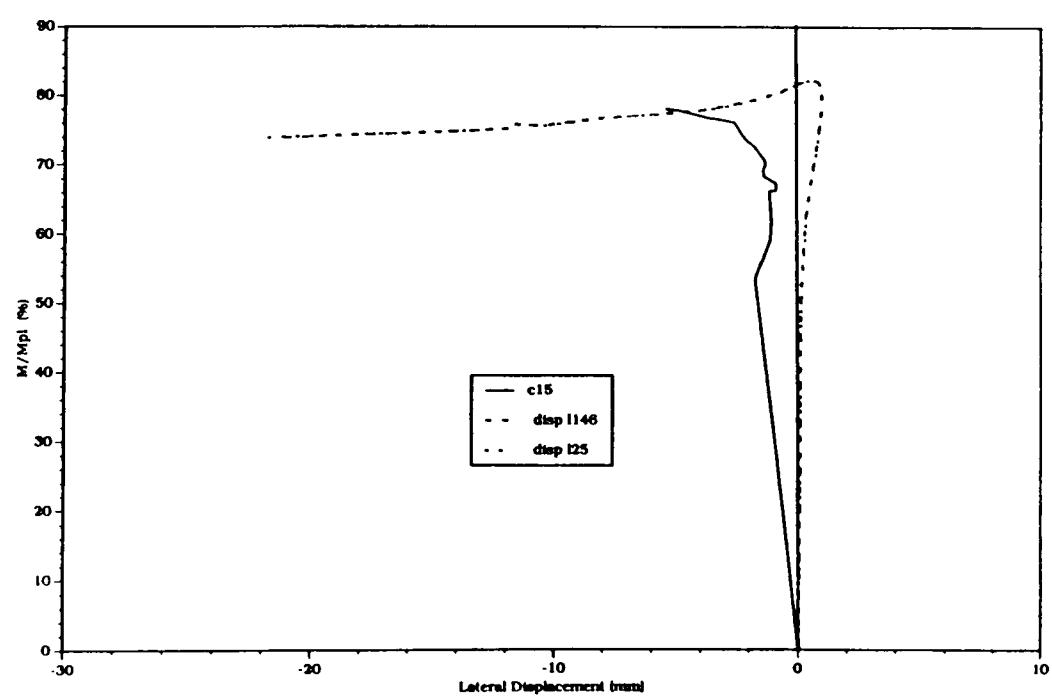
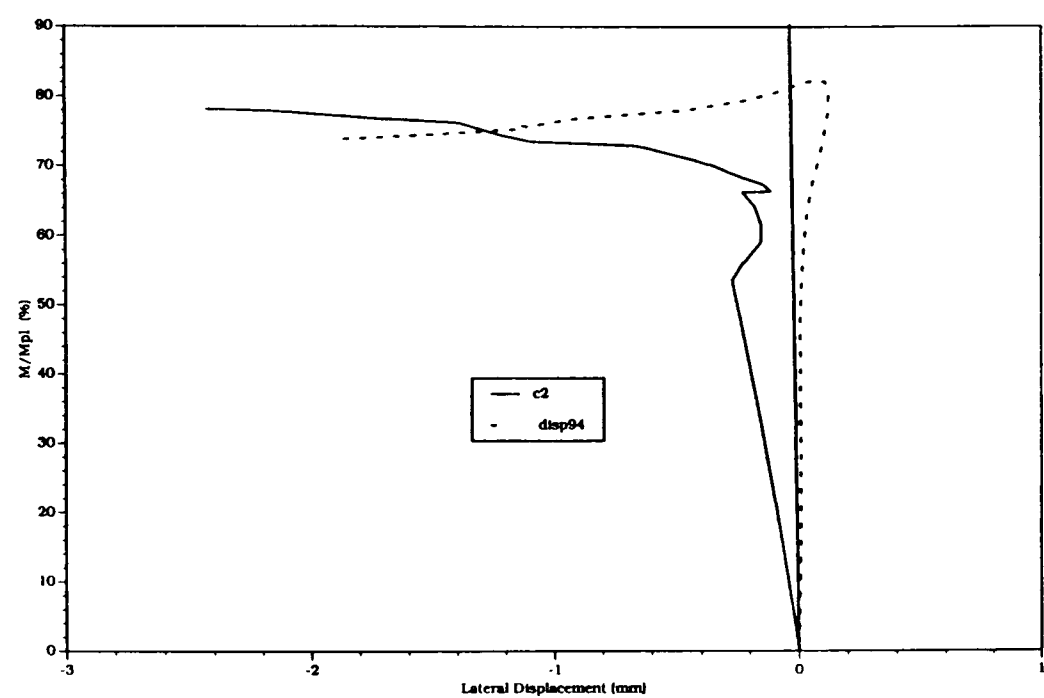
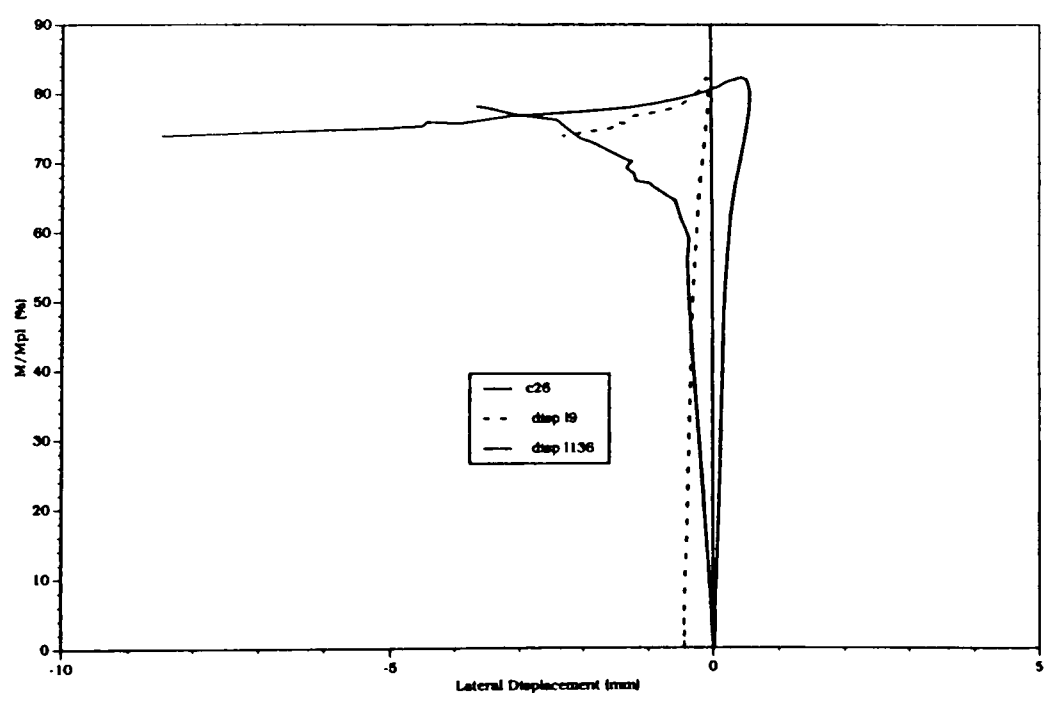
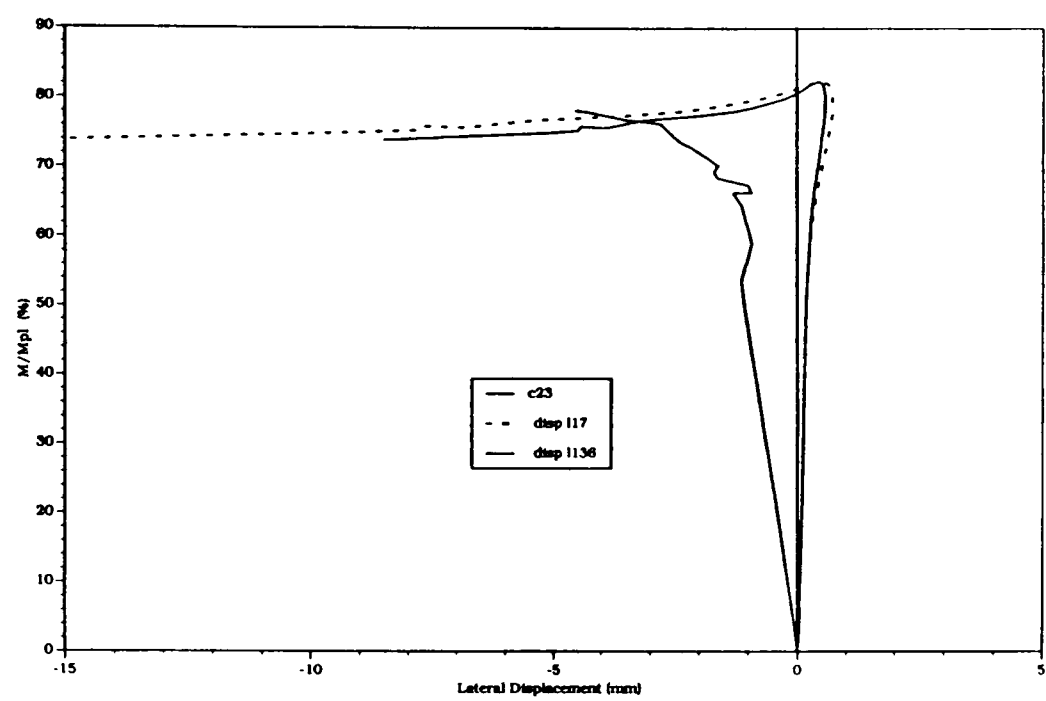


Figure 4.10 - Load vs. Vertical Displacement Comparison (Beam 2).



Figures 4.11.a and 4.11.b - Load vs. Lateral Displacement Comparisons for Channels 2 and 15, Beam 2.



Figures 4.12.a and 4.12.b - Load vs. Lateral Displacement Comparisons for Channels 23 and 26, Beam 2.

4.4.3. Beam 3. ($t_w = 3\text{mm}$).

The same amplification factor of five has been used for the deformed configurations presented in figures 4.13.a and 4.13.b, page 138. For this run, the web has buckled in the vertical direction with the same pattern of deformation present in the experiment. It was also an elastic type of failure and was strongly localized at the centre of the specimen.

The stress field in the web for the ultimate load stage is presented in figures 4.14.a and 4.14.b, page 138. In the vertical direction the stress distribution is very concentrated around the bottom half of the beam but the maximum equivalent stresses are still close to the top flange. This happened again due to the large contribution from, and compressive nature of, the longitudinal stresses in that particular region.

The vertical displacement comparison is shown in figure 4.15, page 139. One possible reason for the difference between predicted and measured ultimate load may have been the difficulty in simulating the flange web junction. In the end, the default option of taking the stiffness the shell element connection gave automatically was followed, after some numerical experimentation had shown that no significant sensitivity existed.

The lateral displacement comparison, for channels 2, 15, 26 and 32 is delineated in figures 4.16.a, 4.16.b, 4.17.a and 4.17.b, pages 139 and 140. It is important to remember that the final experimental values do not represent the ultimate loading stage, because the transducer plate had to be taken out of the web to avoid being crushed by the flanges. The same tendency to displace certainly occurred and the web lateral behaviour was represented very well by the calculations.

The next section portrays a second round of finite element simulations which used the measured experimental values of imperfections and yield stresses. Their results will be compared with those just presented for the assumed standard values of imperfections and yield stresses for calibration and accuracy check purposes. The finite element results for the second series of tests will be then described in section 4.5.

4.4.4. Assumed Versus Measured Values Comparison.

Three further finite element simulations, using the measured imperfections and yield stresses of the beams tested, were executed. Their results were then compared with those for the values assumed in section 4.3. Figures 4.18.a and 4.18.b, page 141, present two of these comparisons in respect of the vertical deflections. They are related to points 1 and 49 of the finite element mesh shown in figure 4.2, page 129. The differences that occurred in respect to vertical deflections or ultimate load can be considered negligible.

The lateral behaviour of the web varied a certain amount, as can be confirmed by figures 4.19.a and 4.19.b, page 142, where the lateral displacements of points 33 and 89 are depicted. In spite of the difference in the lateral behaviour, the close proximity of the other results (peak load, vertical deflection) suggested that there was little point in repeating the comparison between nominal and actual initial deformations and yield stresses for beams 4 to 6. Another very good reason for keeping the simulation with the assumed values for the imperfections as the standard, was that in the parametric study, to be reported in the next chapter, no values were

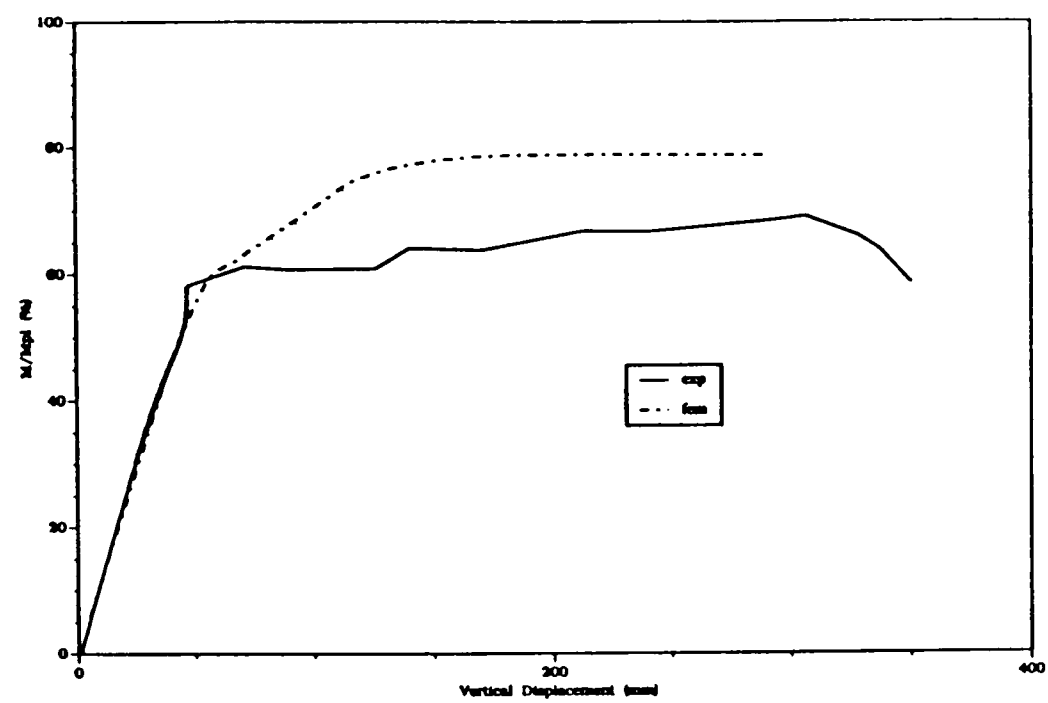
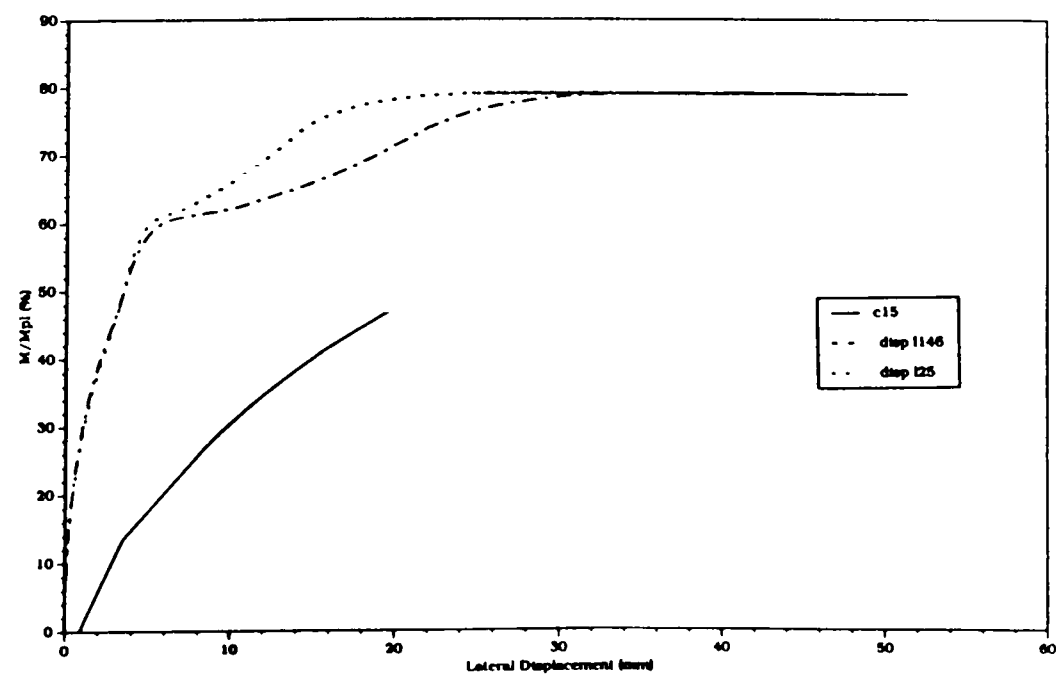
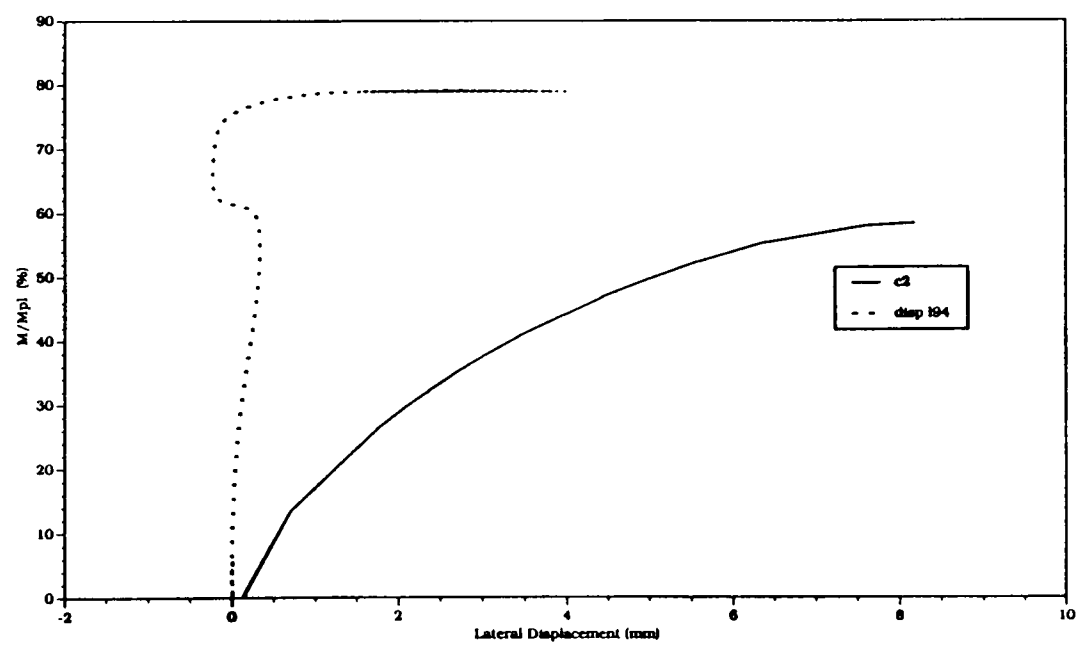
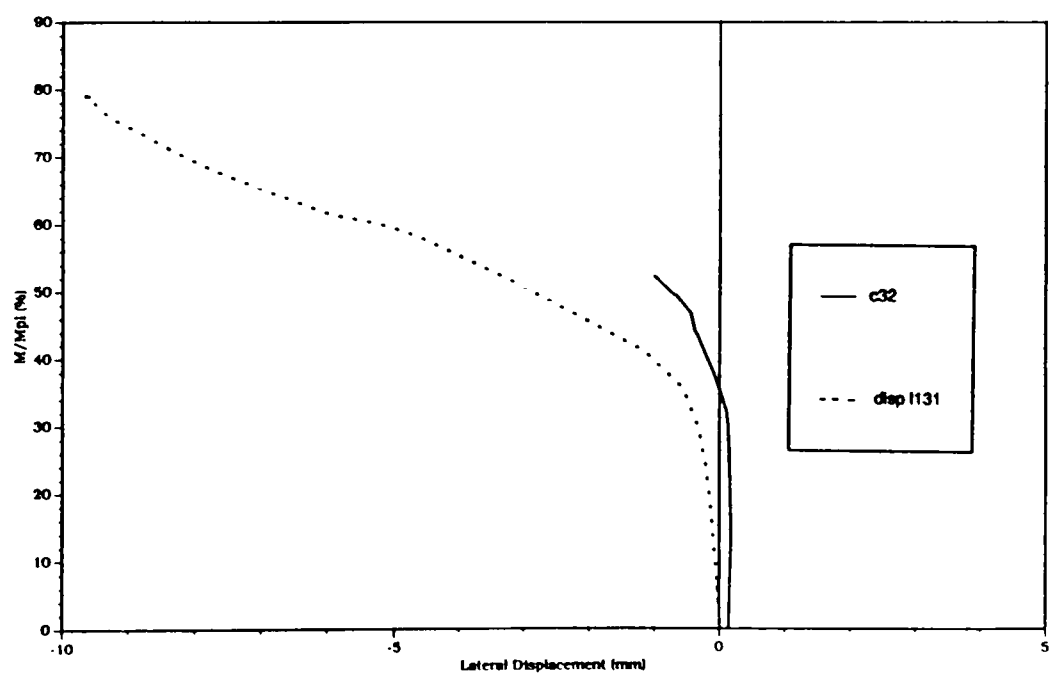
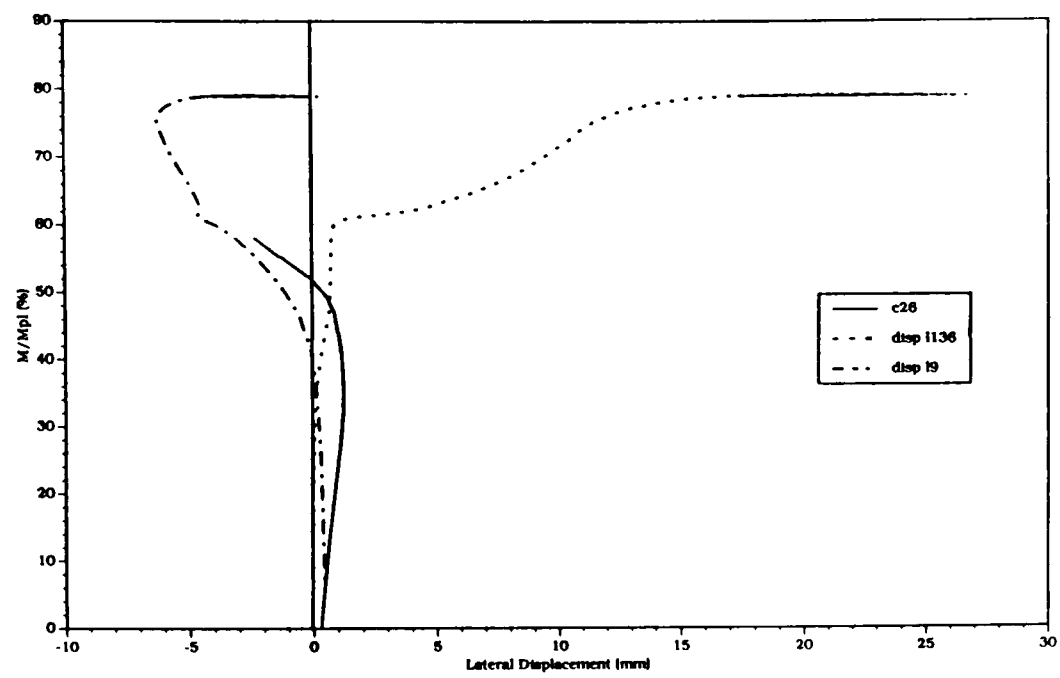


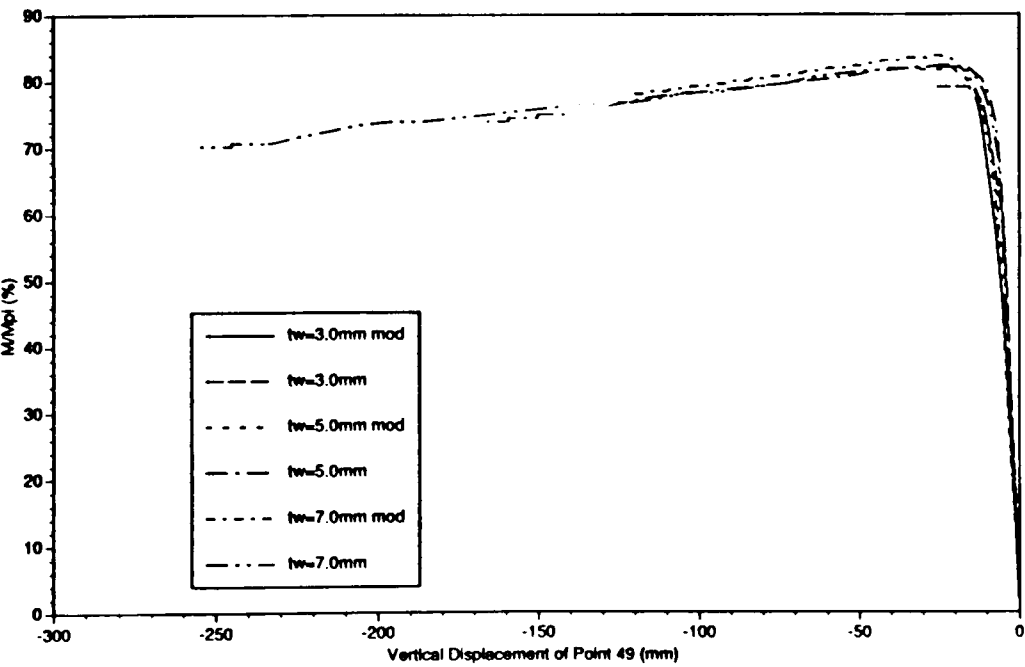
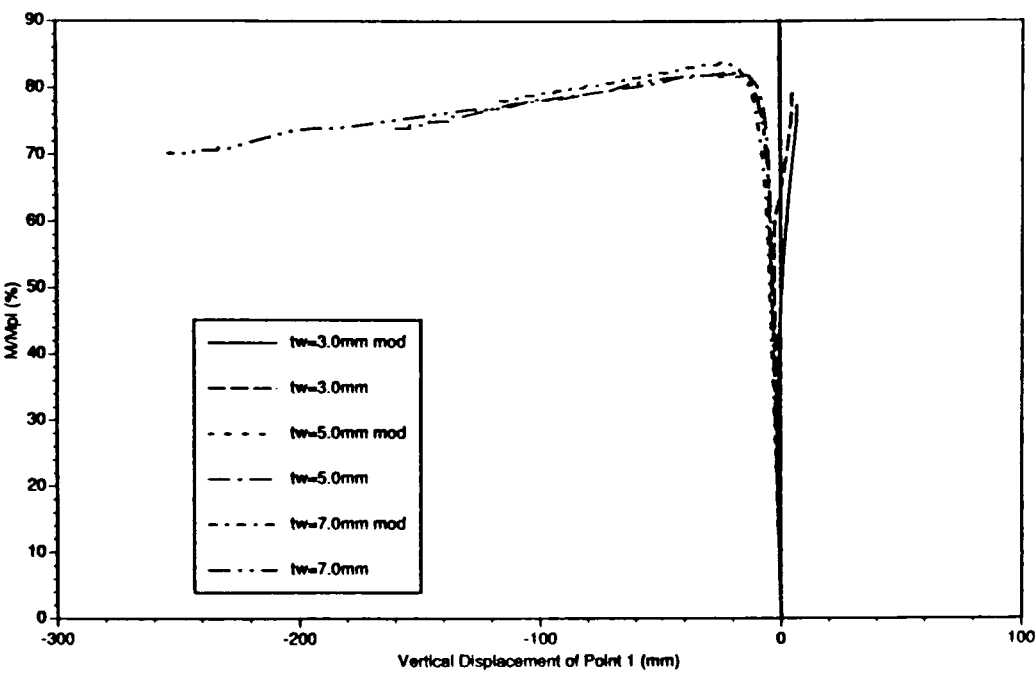
Figure 4.15 - Load vs. Vertical Displacement Comparison (Beam 3).



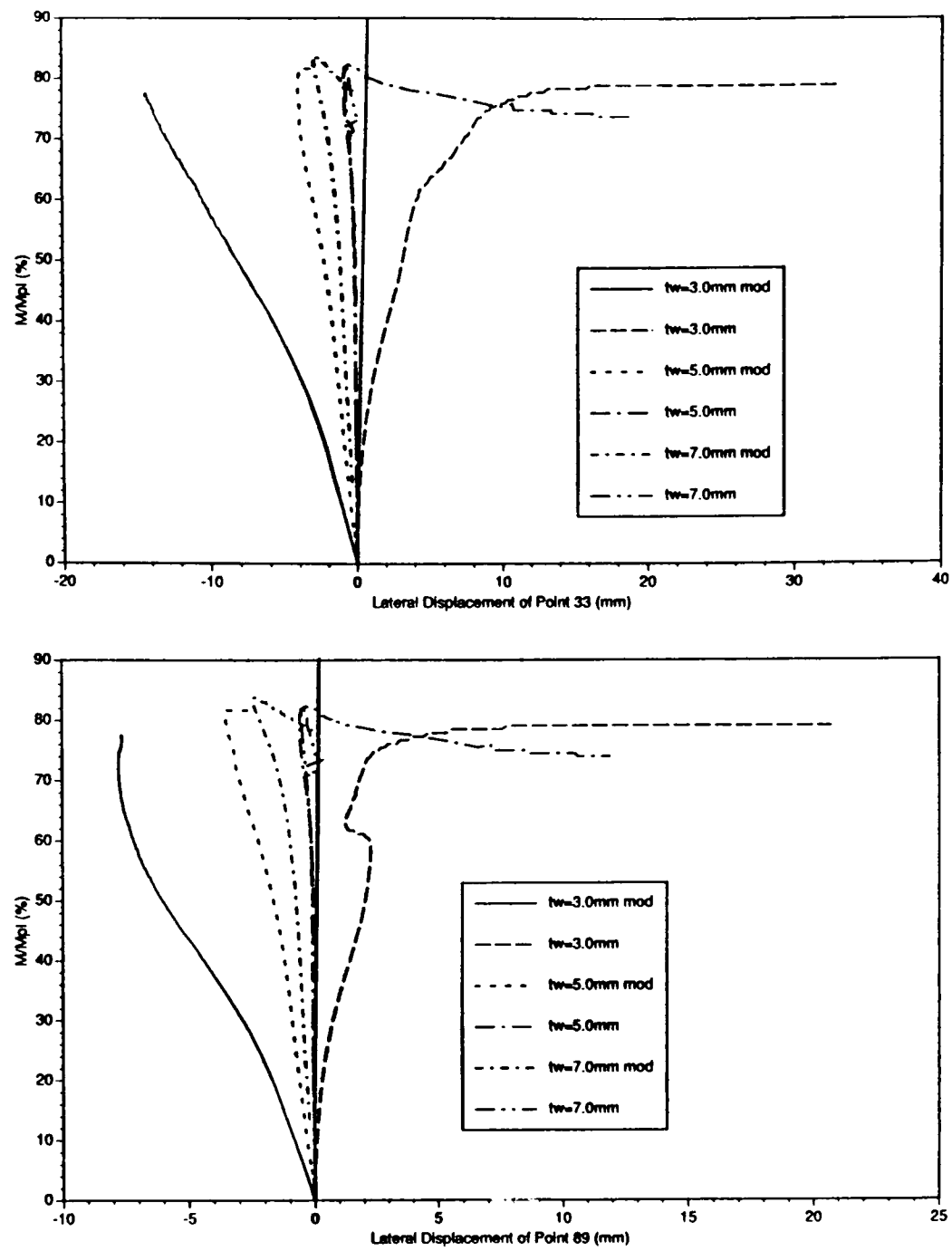
Figures 4.16.a and 4.16.b - Load vs. Lateral Displacement Comparisons for Channels 2 and 15, Beam 3.



Figures 4.17.a and 4.17.b - Load vs. Lateral Displacement Comparisons for Channels 26 and 32, Beam 3.



Figures 4.18.a and 4.18.b - Vertical Displacement Comparison for Points 1 and 49.



Figures 4.19.a and 4.19.b - Lateral Displacement Comparison for Points 33 and 89.

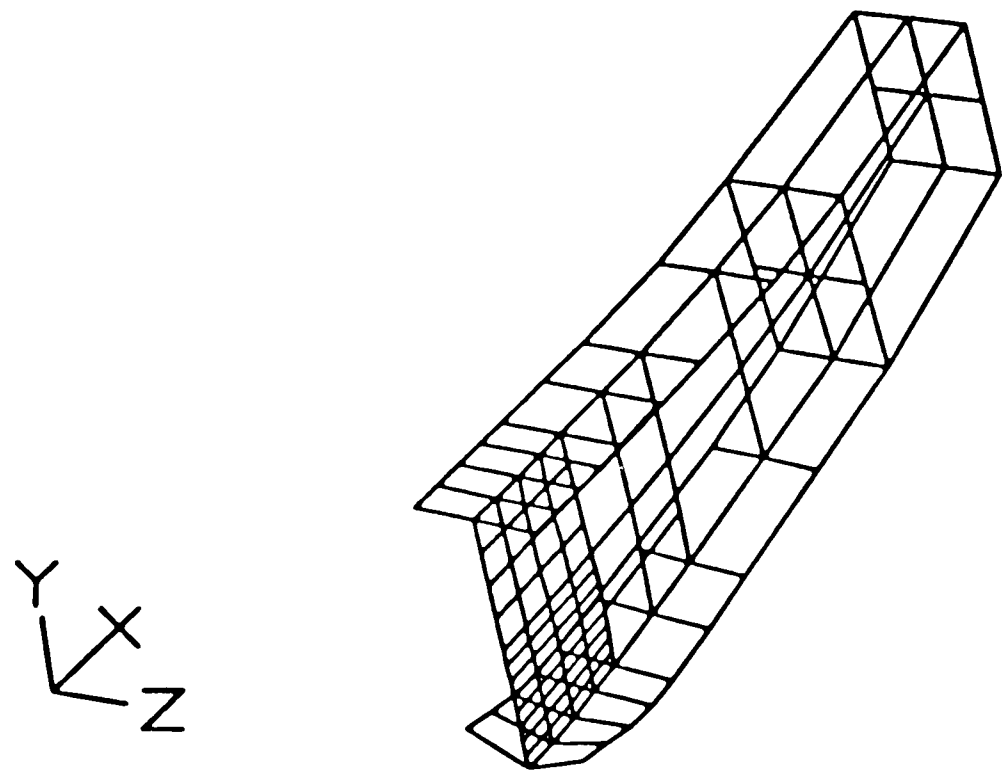


Figure 4.20 - Deformed Configuration of Beam A ($t_w = 4.5\text{mm}$).

available from experiments and one had to be assumed. With this argument in mind, the finite element study proceeded with the assumed values.

4.4.5. New Simulations.

The finite element simulations confirmed the experimental result which placed the thickness limit for buckling of the web between 3 and 5mm. In order to sharpen that boundary, runs with thickness of the web equal to 4.5, 4 and 3.5mm were also performed. The results of those simulations will be discussed in the following sub-sections.

4.4.5.1 *Beam A* ($t_w = 4.5\text{mm}$).

The 4.5mm beam failure behaviour (very similar to the 5 and 7mm beams) can be observed in figure 4.20, page 142, with an amplification factor of five. It has failed by the plastic hinge mechanism near the stiffener point and had the bottom flange slightly bent transversely. The web remained undeformed, proving that no web stiffeners were required for this web thickness. The web's vertical and equivalent (Von Mises) stress distributions are shown in figures 4.21.a and 4.21.b, page 144. They possess the characteristics described in the previous sections for the 5mm beam.

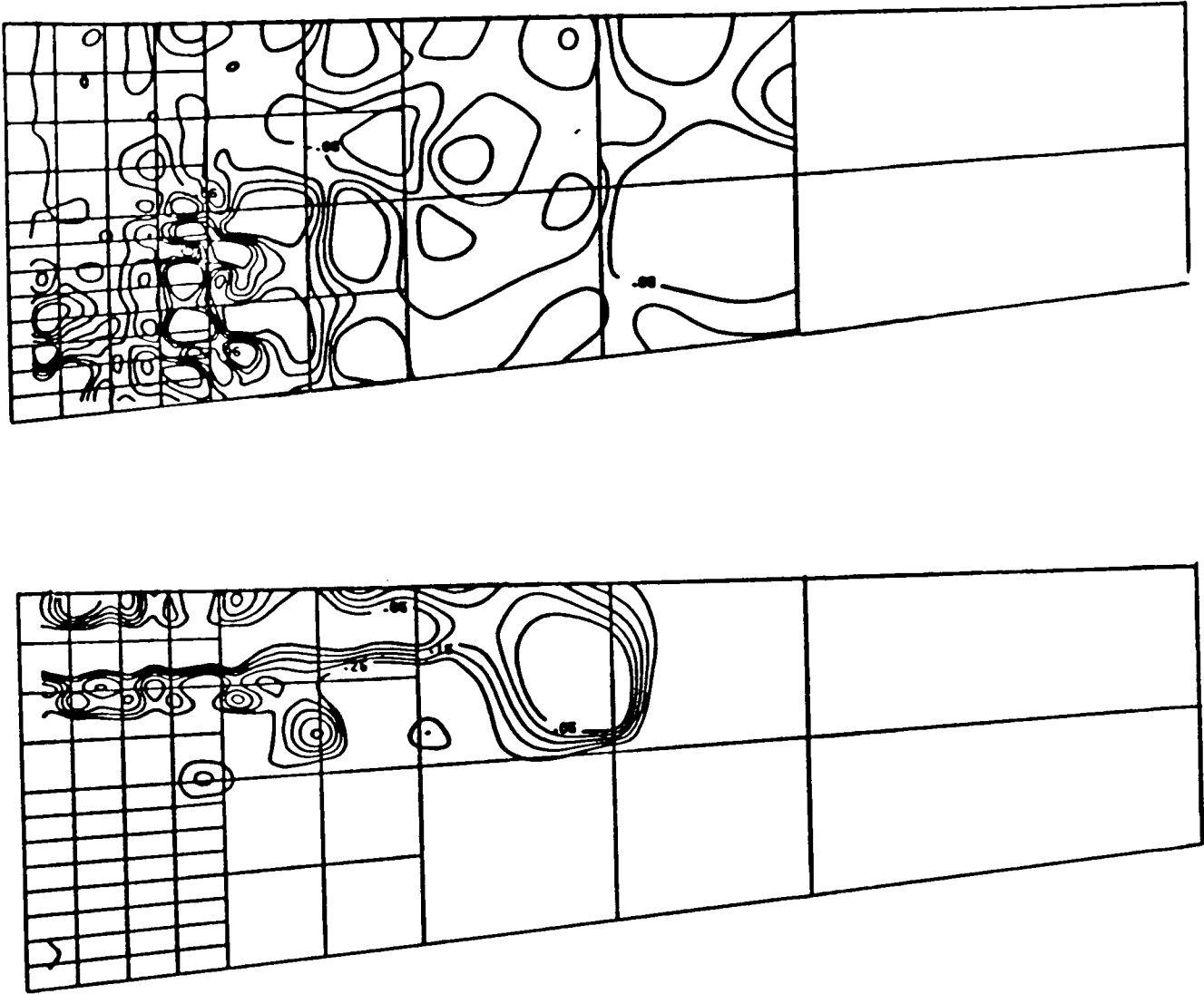
4.4.5.2 *Beam B* ($t_w = 4\text{mm}$).

The final deformed shape of this beam is presented in figure 4.22, page 144, using the usual factor of amplification. It can be assumed to be an intermediate case between the vertical buckling and the plastic hinge failures. The web was beginning to buckle in the vertical direction, but the amount of rotation and vertical displacement around the vertical stiffener cannot be ignored. The vertical and equivalent (Von Mises) stress patterns, for the ultimate load stage, are shown in figures 4.23.a and 4.23.b, page 145. These also followed the same pattern as the previous specimens. This web thickness can be considered the limit for local web buckling.

4.4.5.3 *Beam C* ($t_w = 3.5\text{mm}$).

Two different views of the beam deformed shape at the ultimate load stage are shown in figures 4.24.a and 4.24.b, page 145. The web has buckled in the vertical direction, but the two flanges have not rotated as much as in the 3mm case. This type of failure mode was the same as in the 3mm beam, which confirms the limit of 4mm for the web thickness noted previously. The web stress distribution is presented in figures 4.25.a and 4.25.b, page 146, where the vertical and equivalent (Von Mises) stress contours are given. The same pattern of stress noted previously applied for this beam.

With the completion of this analysis, the next section will present some comparisons between the results of the six beams studied so far.



Figures 4.21.a and 4.21.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam A ($t_w = 4.5\text{mm}$).

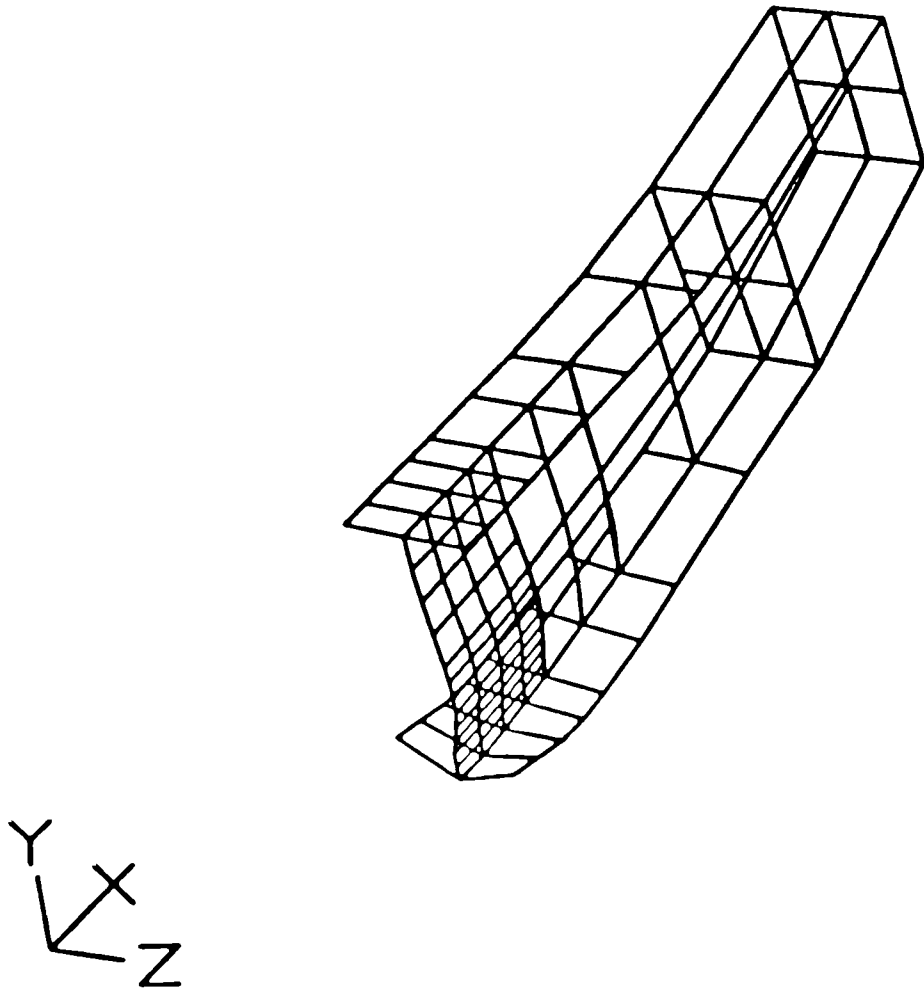
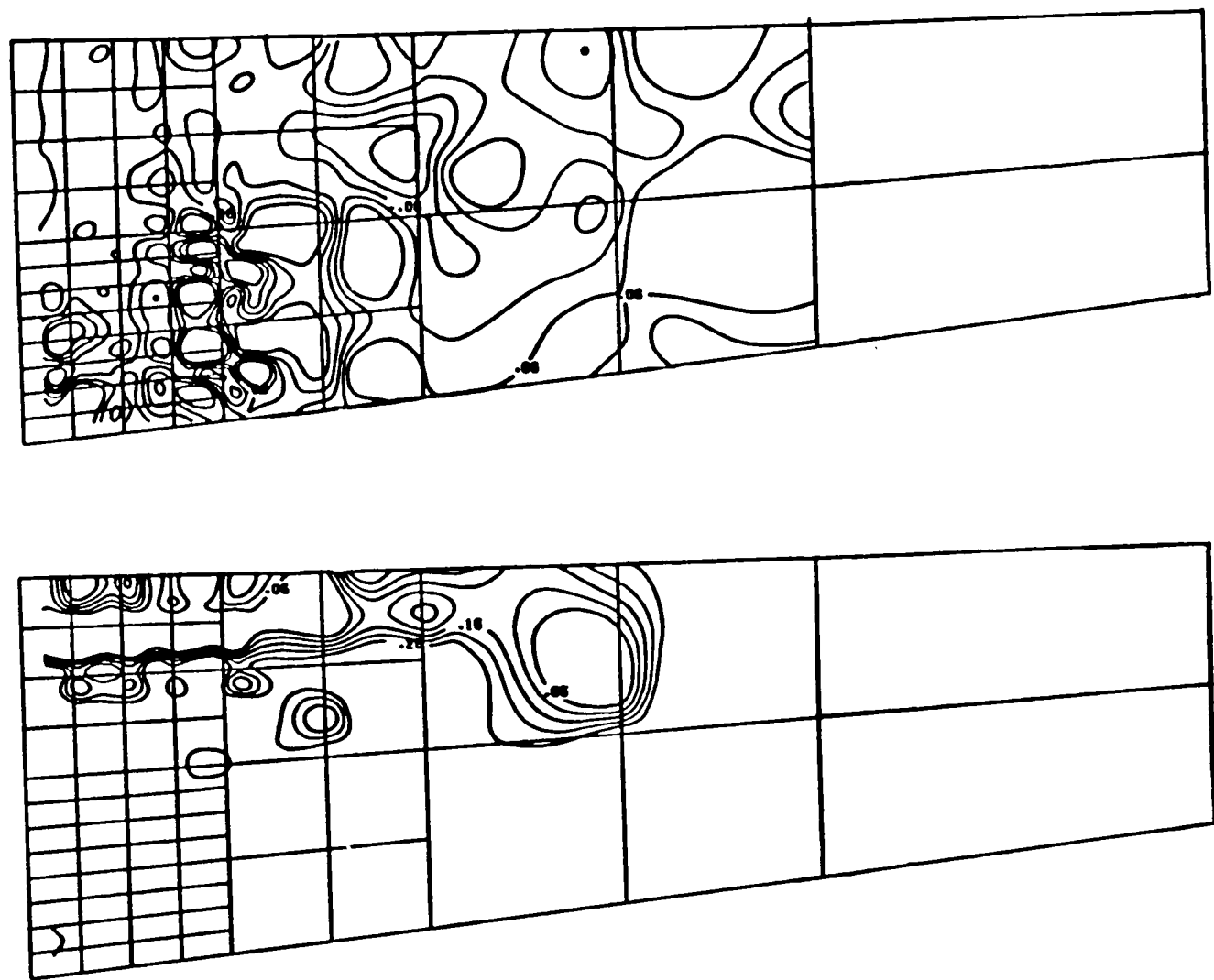
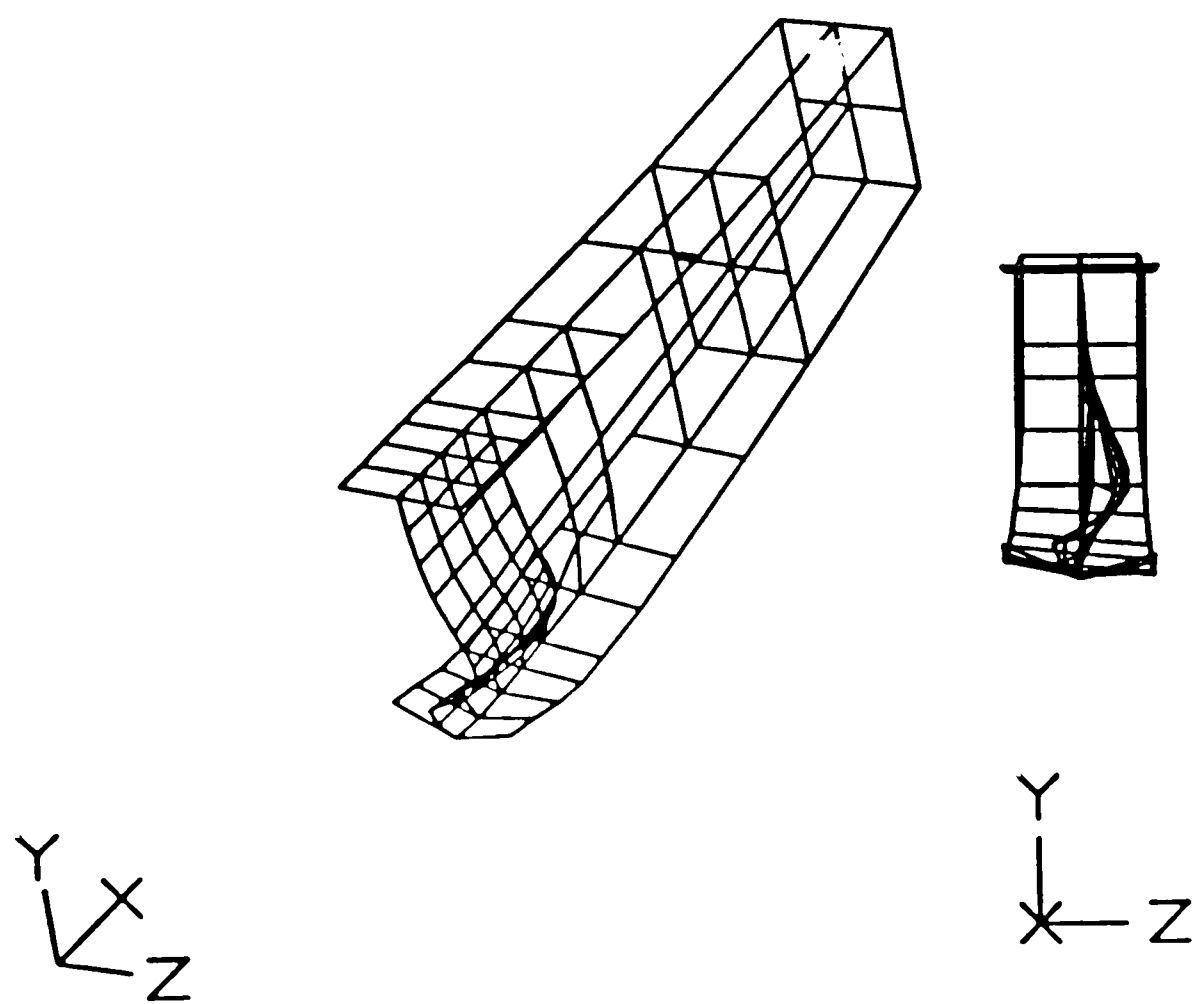


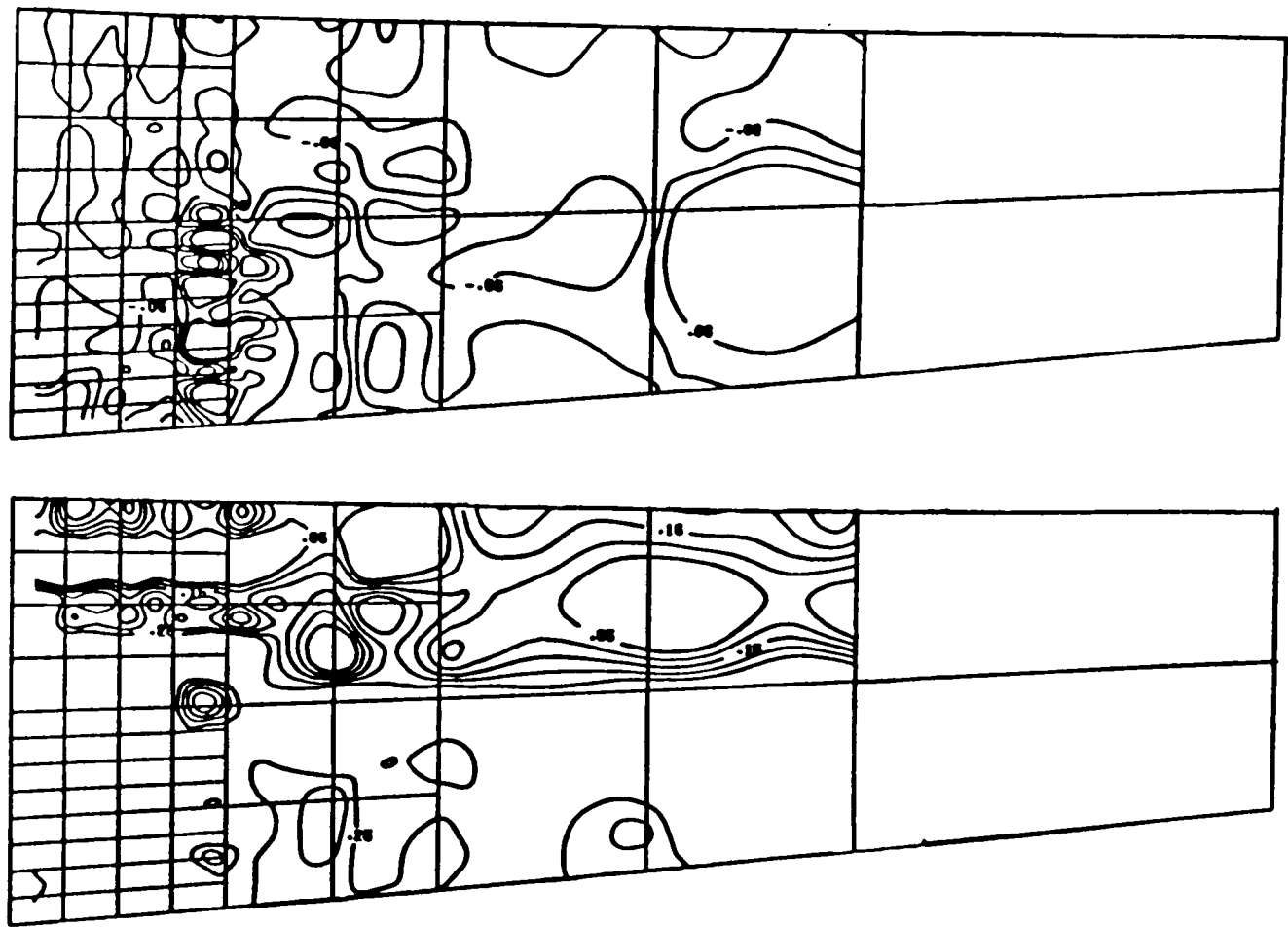
Figure 4.22 Deformed Configuration of Beam B ($t_w = 4\text{mm}$).



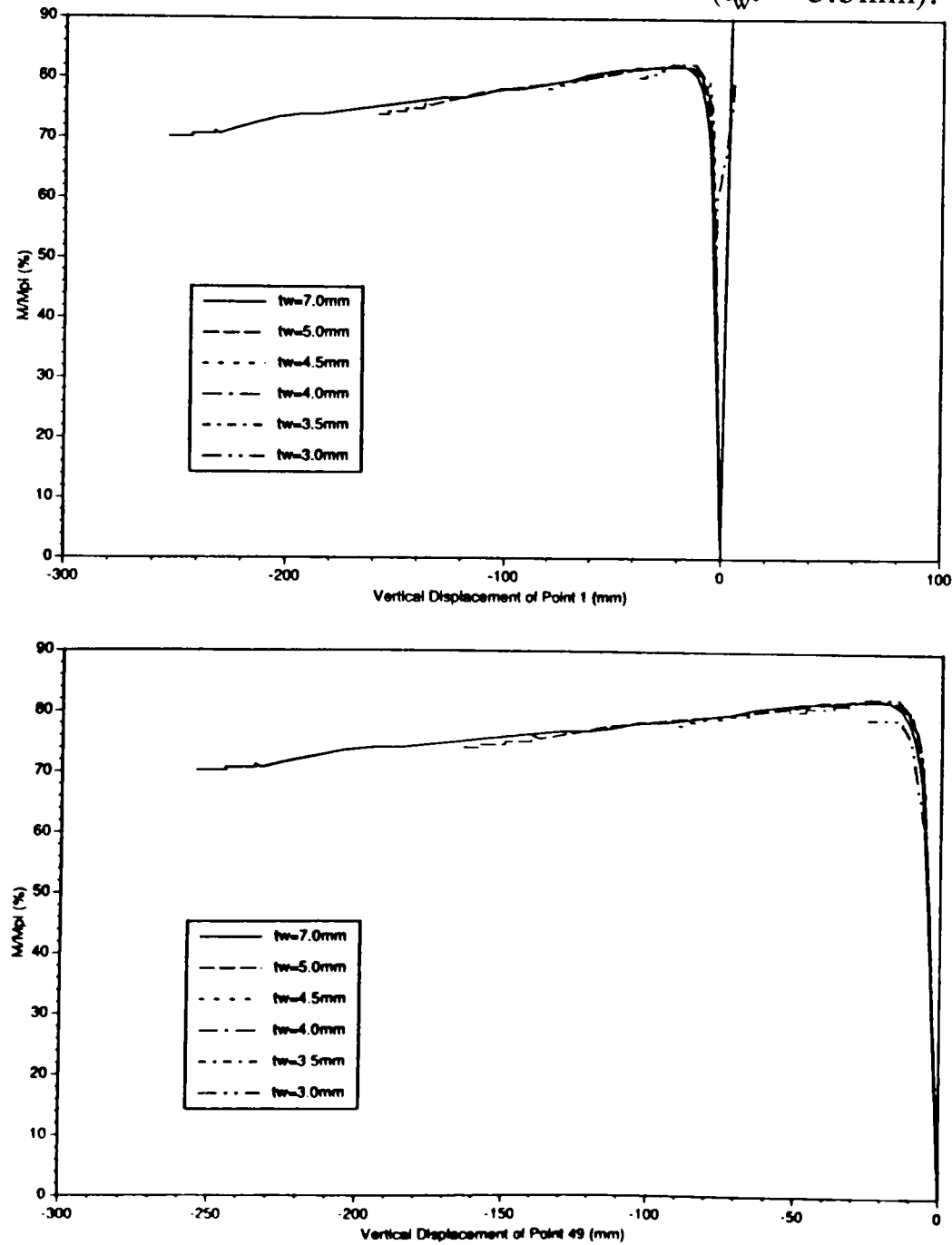
Figures 4.23.a and 4.23.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam B ($t_w = 4\text{mm}$).



Figures 4.24.a and 4.24.b - Deformed Configurations of Beam C ($t_w = 3.5\text{mm}$).



Figures 4.25.a and 4.25.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam C ($t_w = 3.5\text{mm}$).



Figures 4.26.a and 4.26.b - Load vs. Vertical Displacement Comparison for Points 1 and 49 of All the Six Thicknesses Studied.

4.4.6. A Discussion and Summary.

Table 4.1, page 147, presents a comparative summary of the maximum and plastic loads reached in terms of the central and edge plastic moment percentage. The difference between the finite element method and the experiments varied between 14.2% and 31.4%.

TYPE OF ANALYSIS	THICKNESS OF THE WEB (mm)					
	3	3.5	4	4.5	5	7
F.E.M. $M_{max}/M_{plcentre}$ (%)	79.11	79.54	82.37			
F.E.M. M_{max}/M_{pledge} (%)			108.35	108.97	109.54	110.57
EXP. M_{pl}/M_{pledge} (%)					91.60	84.13
EXP. $M_{max}/M_{plcentre}$ (%)	69.26					
EXP M_{pl}/M_{pledge} (%) ($\sigma_y=275\text{MPa}$)					105.59	101.80

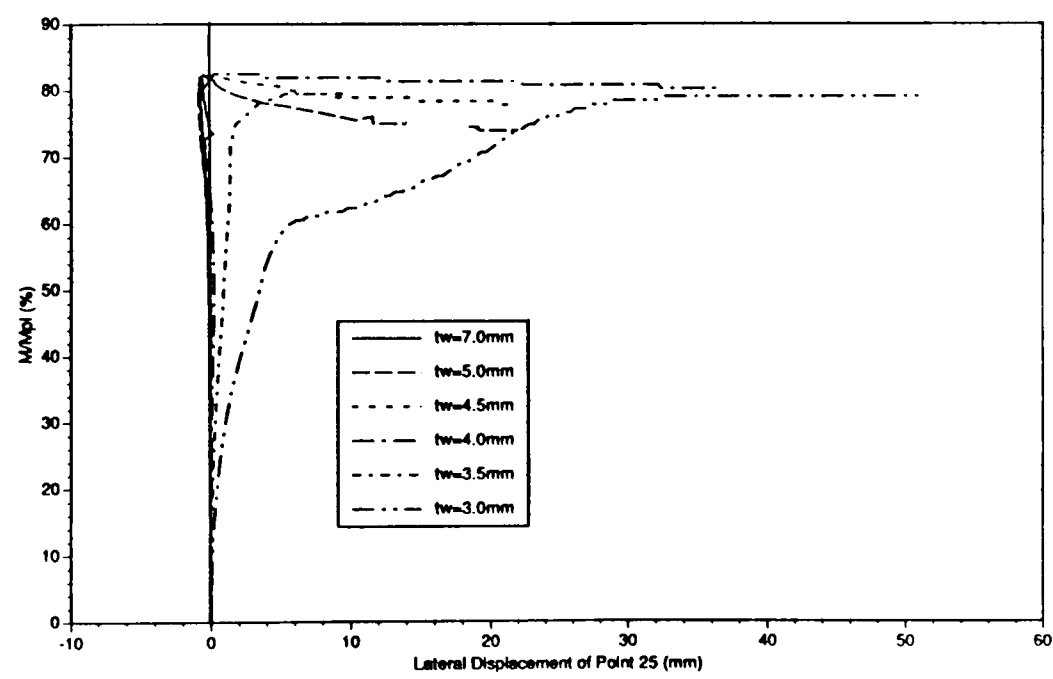
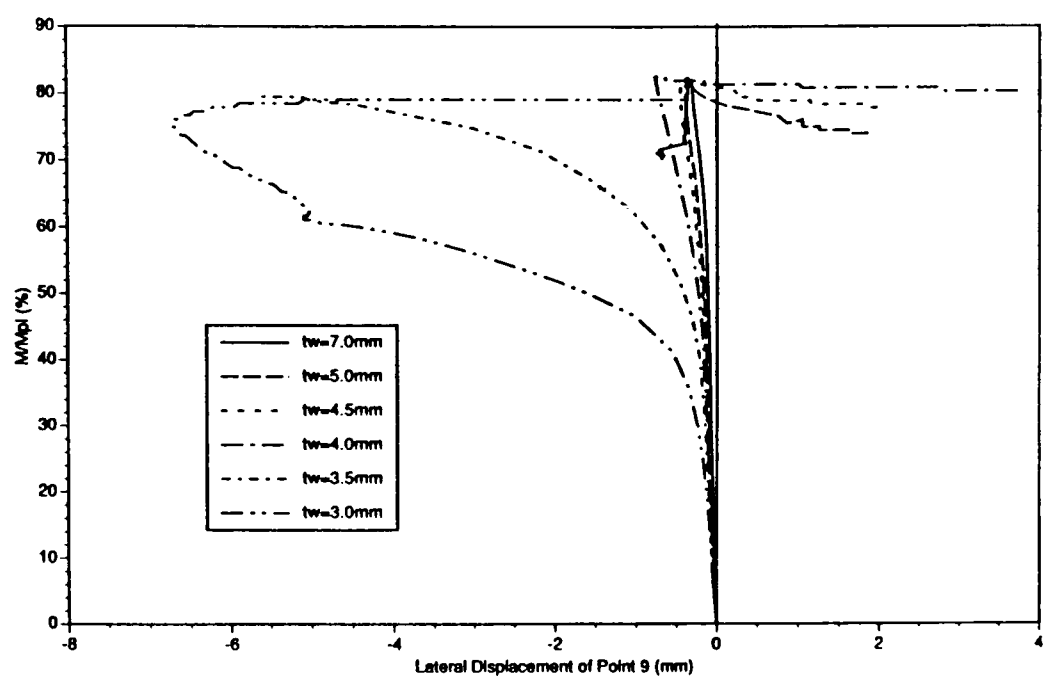
Table 4.1 - Ultimate Load Comparison.

The failure by the plastic hinge mechanism was predicted to occur at around 110% of the plastic capacity at the edge of the taper panel. This relatively high strength capacity was reached because the plastic hinge did not develop right at the edge of the tapered panel. The presence of the very thick stiffener shifted the hinge location to a point further inside the specimen and consequently increased the beam's plastic resistance. The experimental values for the plastic moment for the 7 and 5mm beams were determined by the intersection of two tangents to the curves presented in their respective load vs. vertical deflection diagrams (figures 3.5 and 3.14, pages 83 and 87). They were found to be around 85% - 90% of the plastic capacity at the edge of the tapered panel calculated with the coupon test results for yield stress. However, the material tests were (as stated previously) made on coupons cut from the beams after they had been tested, and may well have overestimated the true yield stresses. If, on the other hand, the guaranteed minimum yield stress for grade 43 material of 275N/mm² is used to calculate the plastic capacity, then the figures presented in the last line of table 4.1 are obtained. This new set of values would consequently change the maximum error from 31.4% to just 9.4%.

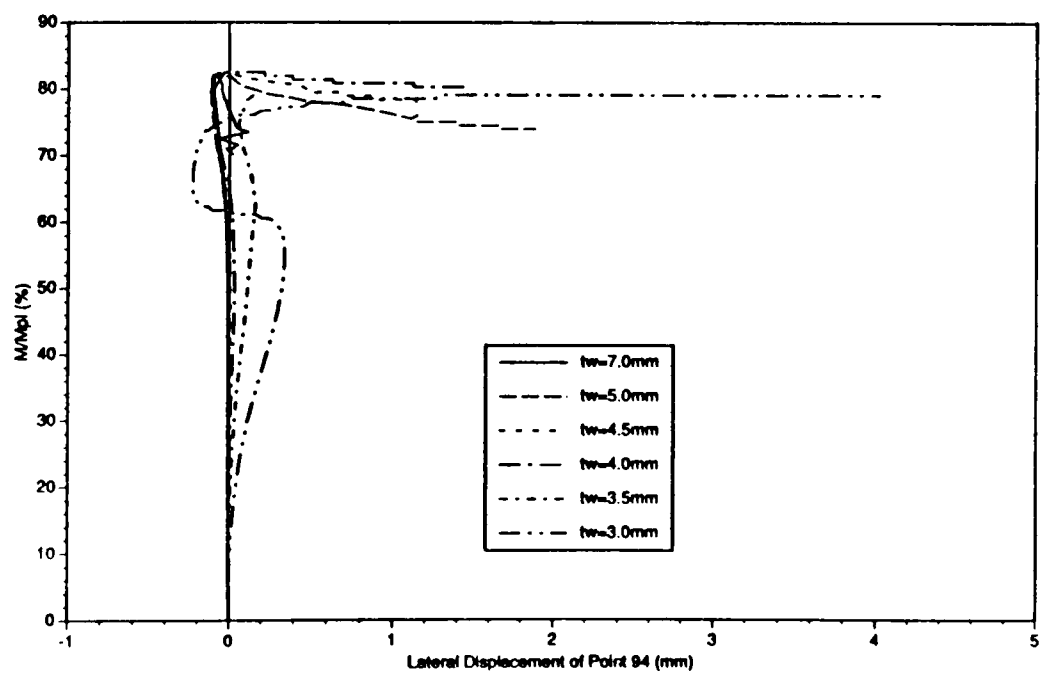
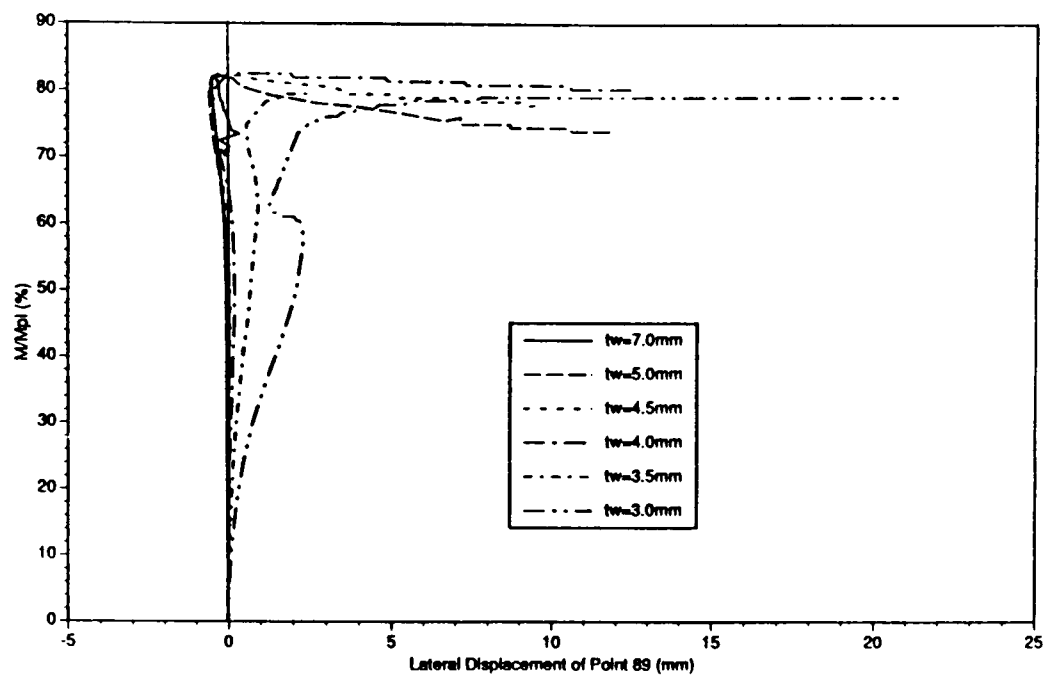
The vertical buckling failure was predicted to occur in the region of 79% of the plastic capacity at the central section but the experimental failure happened around 69%. That disparity of results can be considered small when all differences existing between the two methods are taken into account.

Load against vertical displacement comparisons for points 1 and 49, for the six cases studied, are shown in figure 4.26.a and 4.26.b, page 146. The slope change in the vertical displacement at point 1, for the 3 and 3.5mm beams, is the proof of the occurrence of vertical buckling, as the bottom flange starts to displace upwards towards the top flange.

Some other interesting comparisons are presented in figures 4.27.a, 4.27.b, 4.28.a and 4.28.b, pages 148 and 149. They represent the lateral displacement behaviour of points 9, 25, 89 and 94 , for the six simulated cases. The difference in the behaviour of the beams is very evident and it is also possible to



Figures 4.27.a and 4.27.b - Load vs. Lateral Displacement Comparison for Points 9 and 25 of All the Six Thicknesses Studied.



Figures 4.28.a and 4.28.b - Load vs. Lateral Displacement Comparison for Points 89 and 94 of All the Six Thicknesses Studied.

establish the transition between the two modes of collapse. Point 9 clarifies the distinctive ultimate load stages for the two types of collapse achieved.

4.5. INFLUENCE OF THE PLASTIC NEUTRAL AXIS POSITION.

This section will report the finite element simulations that were executed in order to cover the second series of laboratory tests. Again, comparisons between the two different approaches will be drawn for the verification of the finite element model. Subsequently, new cases will be introduced to fill the gaps of data that could not be covered in the experiments. At the end of this section, a brief summary of all the cases considered will be made.

4.5.1. Beam 4. ($t_{fc} = 20\text{mm}$).

The final deformed shape of beam 4, amplified by a factor of five, is portrayed in figure 4.29, page 151. Again the failure mode was related to a plastic hinge mechanism near the panel edge vertical web stiffeners. At the centre of the specimen, as in the first two beams, the bottom flange has bent slightly inwards.

The vertical and equivalent (Von Mises) web stress contours, for the ultimate loading stage, are presented in figures 4.30, page 151. The vertical stresses are located predominantly at the bottom part of the web, as was expected in the presence of the vertical force that comes from the bottom flange. The maximum equivalent stresses are located in the top part of the web, as in the first series, due to the large compressive longitudinal stresses present in that region.

A load vs. vertical displacement comparison, for the positions of channels 40 and 42 of the experiments, is shown in figure 4.31, page 152. In this graph it is possible to observe that the transition towards the plastic phase has happened in both approaches at around 66% of the plastic capacity. It also demonstrated that the finite element model is more rigid than the experiments.

The very small web lateral deflection can be seen in figures 4.32.a, 4.32.b, 4.33.a and 4.33.b, pages 152 and 153. In these diagrams channels 2, 6, 23 and 32 are compared with their finite element counterparts, showing the same trend.

4.5.2. Beam 5. ($t_{fc} = 15\text{mm}$).

The deformed configuration of the beam at the ultimate loading stage is presented in figure 4.34, page 153. The initial buckling of the top flange, coupled with some rotation, is visible near the vertical stiffener. There is also a transverse curvature of the bottom flange, as in the previous beams. Large vertical displacements in the whole beam were noted. The web remained straight, confirming the experimental predictions.

The vertical and equivalent (Von Mises) web stress contours are presented in figures 4.35.a and 4.35.b, page 154. The influence of the top flange buckling is clearly visible in both diagrams. It has created a concentration of vertical compressive stresses in the web and modified the equivalent stress configuration.

The load vs. vertical displacement comparison, for the two methods, is portrayed in figure 4.36, page 154. They had the same pattern of displacement but reached different ultimate loads. This has happened due to the boundary conditions imposed to the top of the web in the finite element model inhibiting the first onset of buckling, creating a higher ultimate load. Very small lateral displacements were

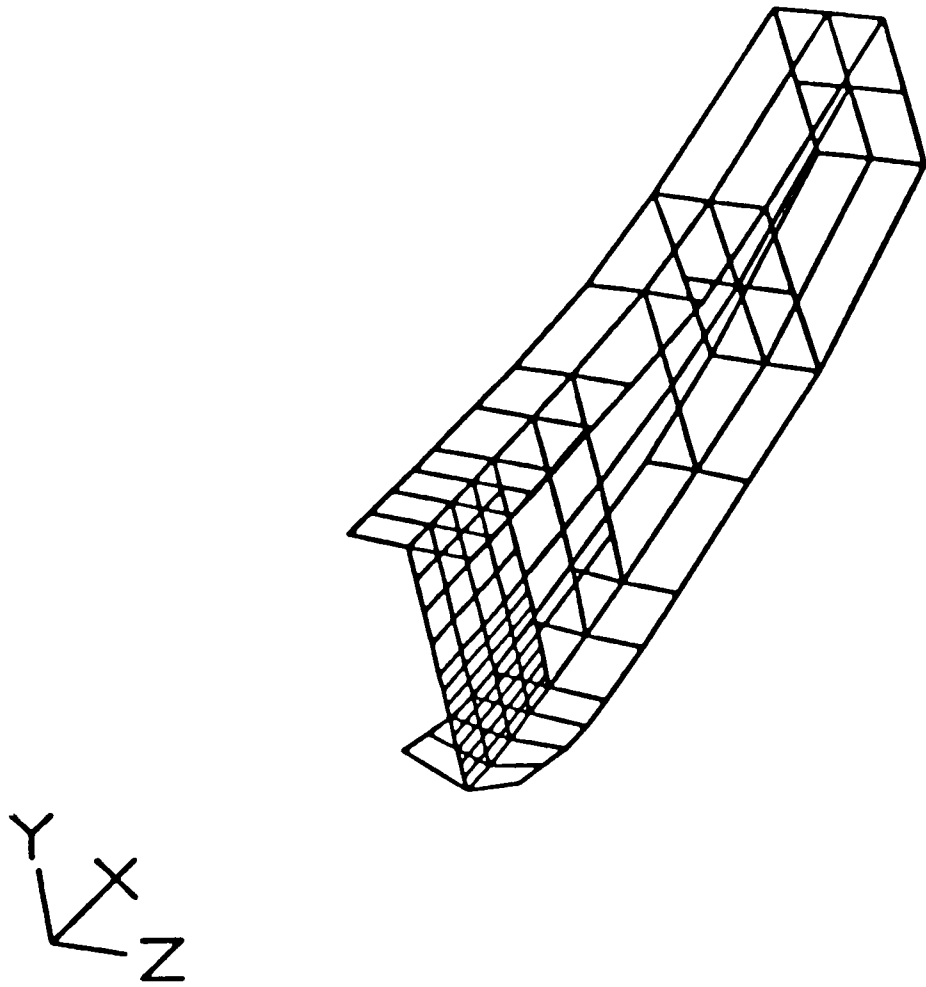
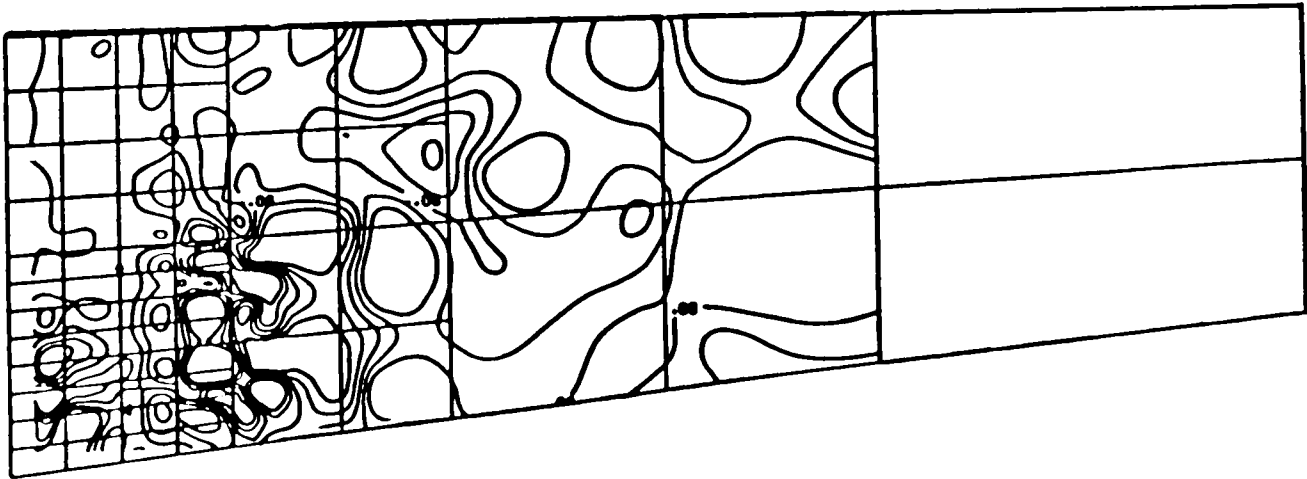
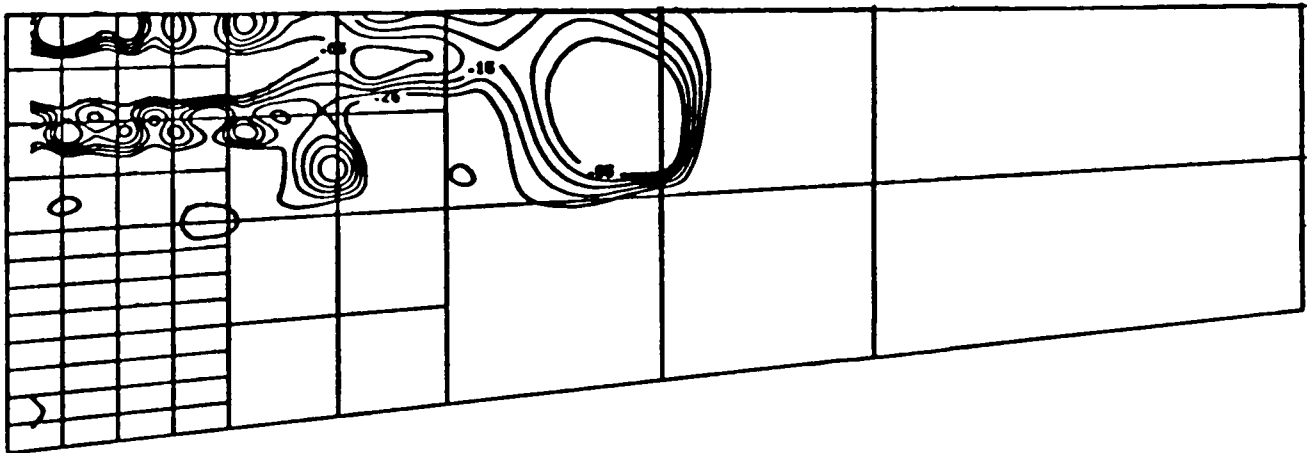


Figure 4.29 - Deformed Configuration of Beam 4 ($t_{fc} = 20\text{mm}$, $t_{ft} = 12\text{mm}$).



Figures 4.30.a and 4.30.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam 4 ($t_{fc} = 20\text{mm}$, $t_{ft} = 12\text{mm}$).

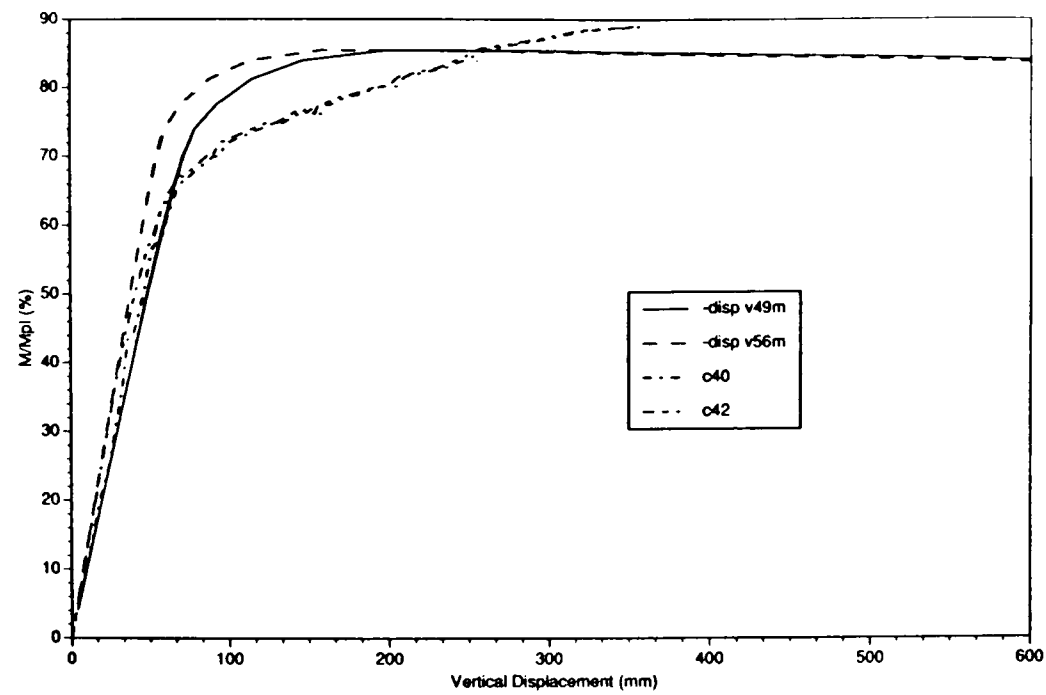
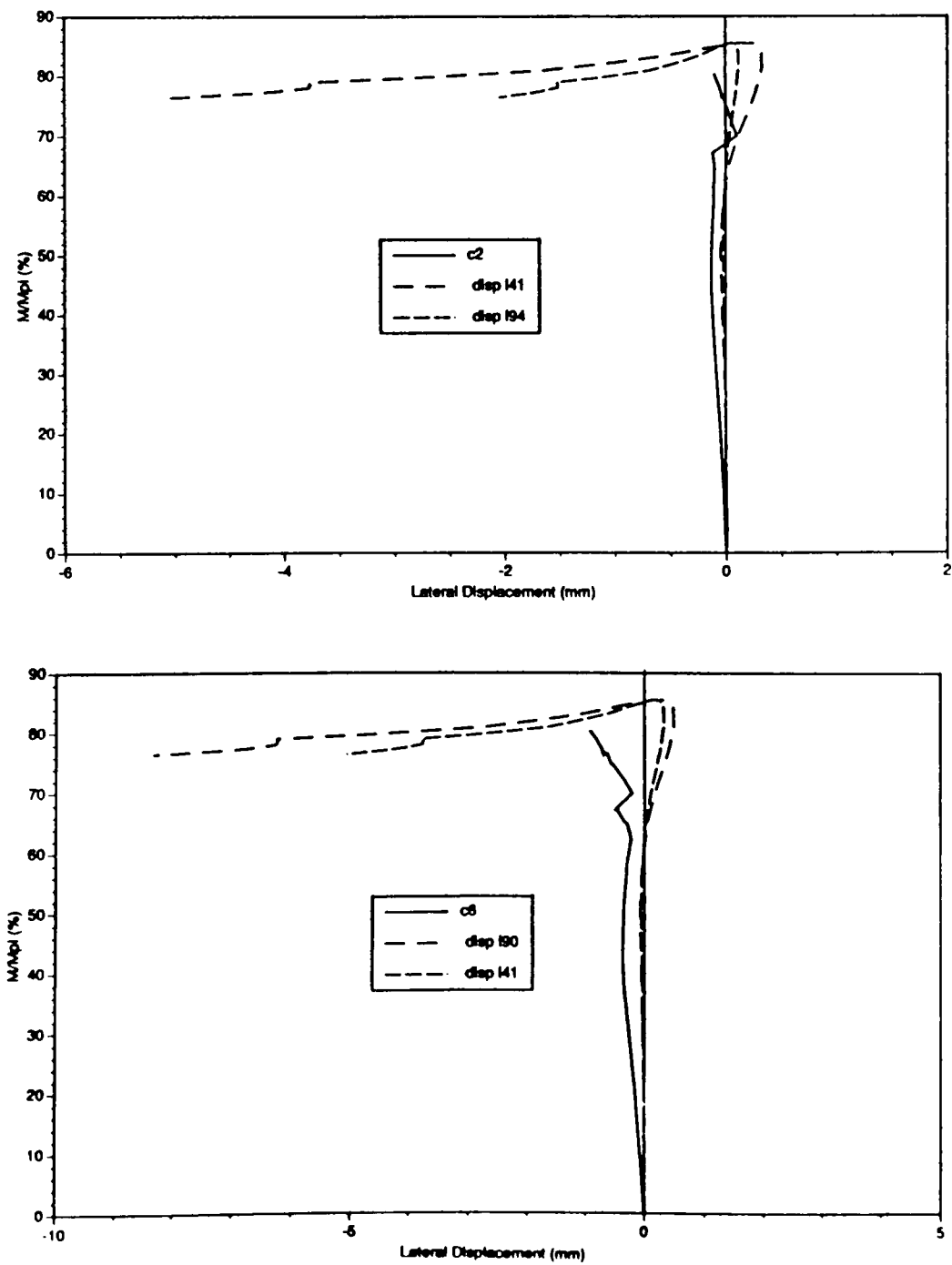
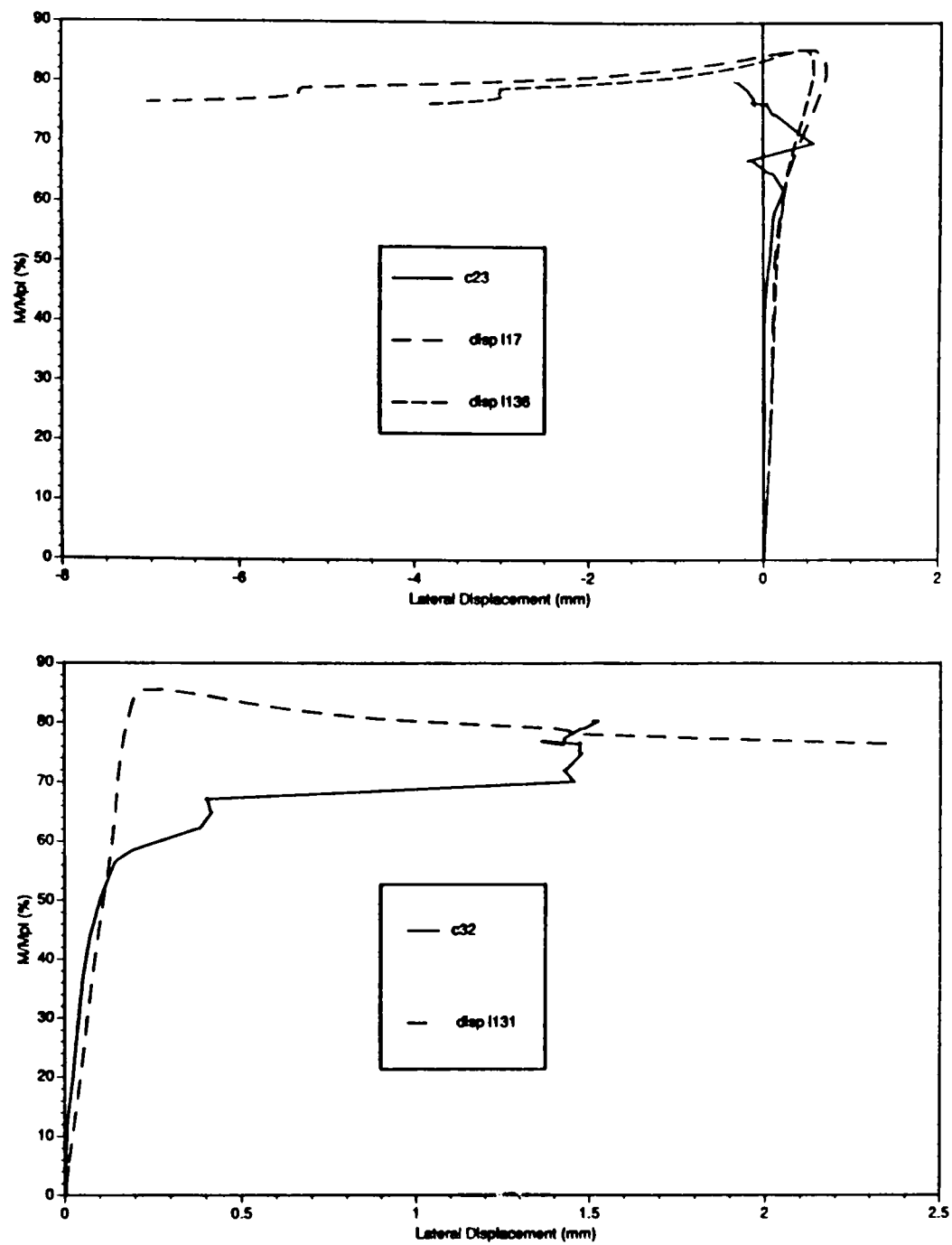


Figure 4.31 - Load vs. Vertical Displacement Comparison (Beam 4).



Figures 4.32.a and 4.32.b - Load vs. Lateral Displacement Comparisons for Channels 2 and 6, Beam 4.



Figures 4.33.a and 4.33.b - Load vs. Lateral Displacement Comparisons for Channels 23 and 32, Beam 4.

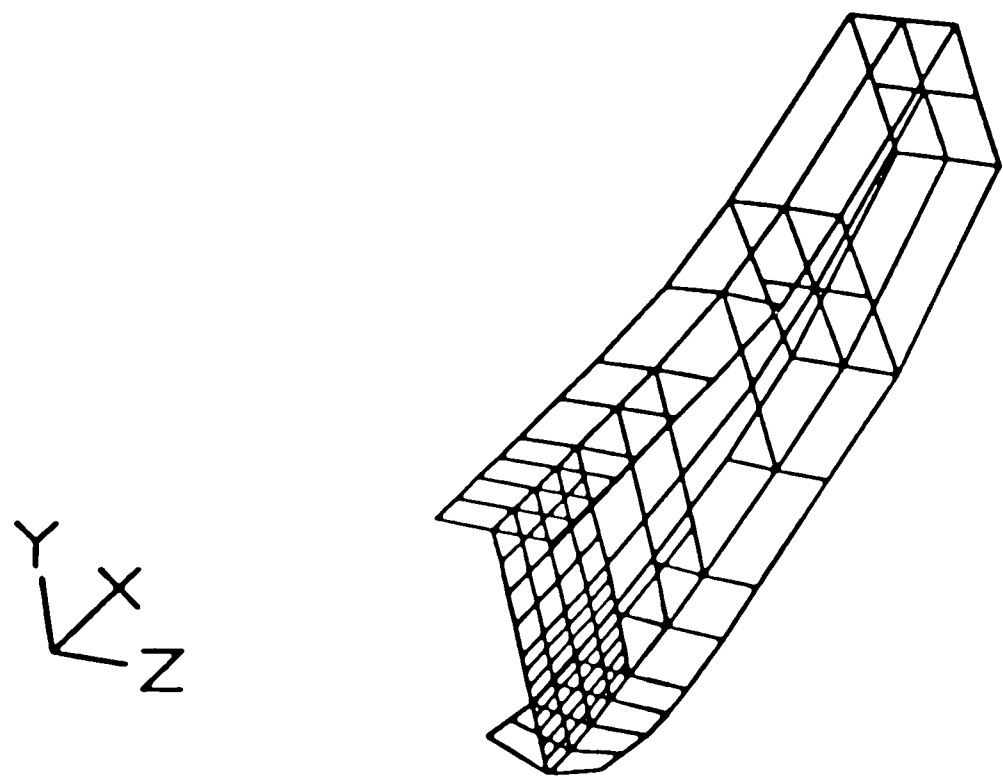
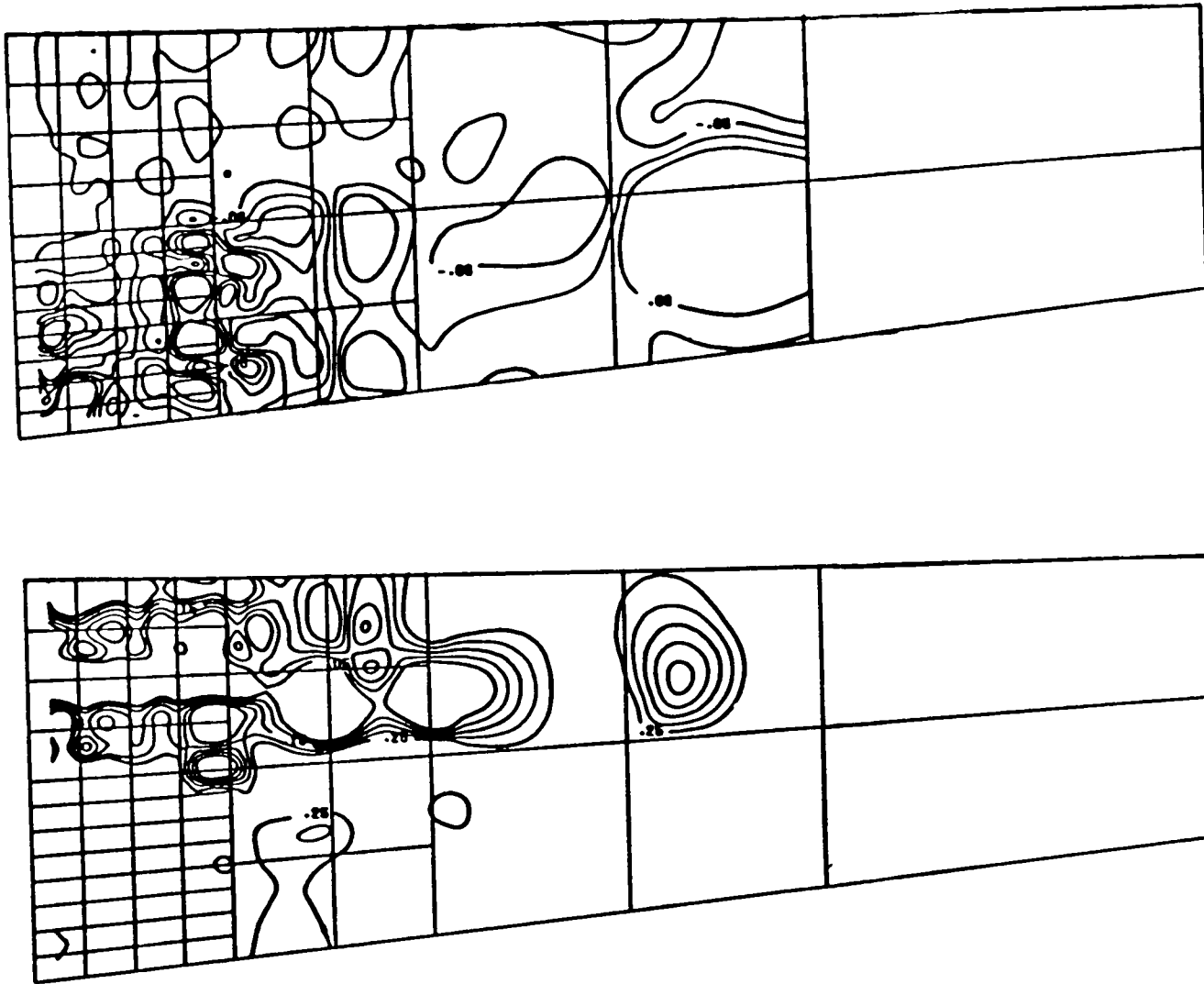


Figure 4.34 - Deformed Configuration of Beam 5 ($t_{fc} = 15\text{mm}$, $t_{ft} = 12\text{mm}$).



Figures 4.35.a and 4.35.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam 5 ($t_{fc} = 15\text{mm}$, $t_{ft} = 12\text{mm}$).

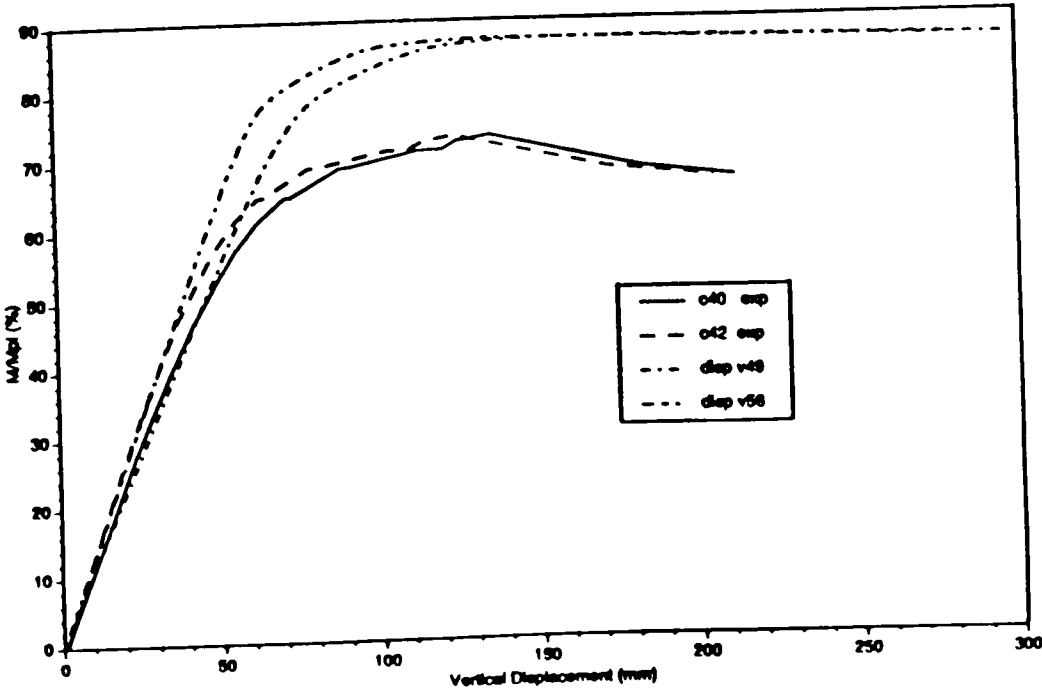


Figure 4.36 - Load vs. Vertical Displacement Comparison (Beam 5).

registered in the web region, as can be observed in figures 4.37.a, 4.37.b, 4.38.a and 4.38.b, pages 156 and 157. They represent the values of channels 2, 10, 23 and 32 recorded in the experiments and compared with their finite element counterparts. Again a similar form of displacement was reached by the two approaches.

4.5.3. Beam 6. ($t_f = 25\text{mm}$).

The top flange buckling of beam 6 is clearly present in the finite element results, figure 4.39, page 158. This drawing corresponds to the ultimate loading with the deflections magnified by a factor of 5. It is important to observe that the finite element mesh in the top flange is too coarse to model quantitatively the flange buckling. Despite that, the answers that have been produced can be considered qualitatively very interesting.

The flange rotation has introduced very large web lateral displacements near the vertical stiffener, as were seen in the experiments described in chapter 3. The buckle has also affected the centre section lateral displacement behaviour, although no vertical web stiffeners were judged to be necessary. It is important to recall that if the concrete deck were used, the top flange buckling would not have occurred.

The vertical and equivalent (Von Mises) web stress patterns are delineated in figures 4.40, page 158. As in the previous beam, the top flange buckling has affected the web stress distribution in the vicinity of the vertical web stiffener. It has induced compressive vertical stresses that, combined with the longitudinal stresses produced a very concentrated region of equivalent stress. It also has distributed the equivalent stress more evenly at the centre of the beam.

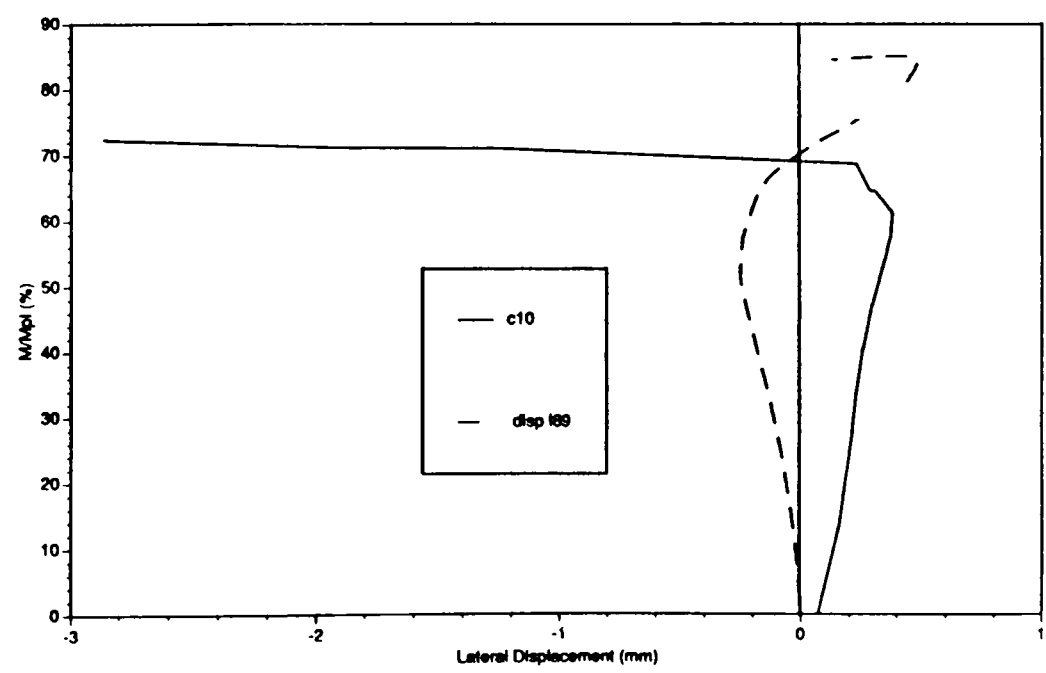
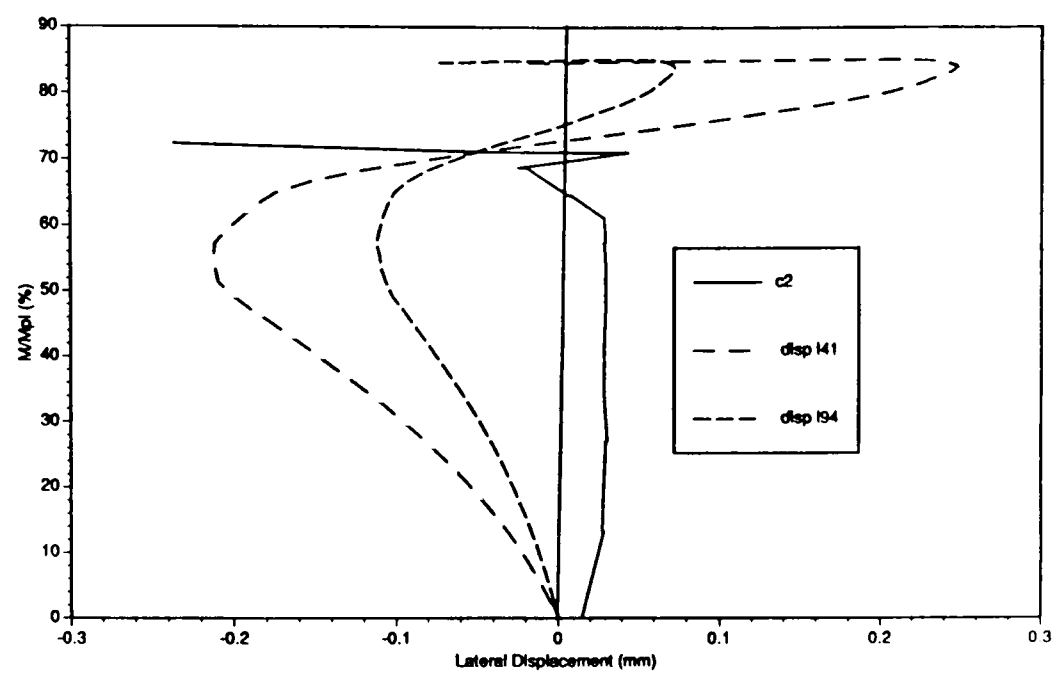
The vertical displacement behaviour of points 49 and 56 from the finite element simulation is shown in figure 4.41, page 159. Again, their values are plotted against the applied moment and compared with the experimental values. The ultimate load difference existing between the two approaches occurred because of the reasons discussed for the previous beam. However, the qualitative behaviour is very similar. Very small web lateral displacements were achieved, as can be seen from figures 4.42.a, 4.42.b, 4.43.a and 4.43.b, pages 159 and 160, which represent the lateral displacements of nodes 2, 10, 23 and 32 from the finite element mesh (figure 4.2, page 129). The same lateral displacement pattern was also common to both methods.

4.5.4 New Simulations.

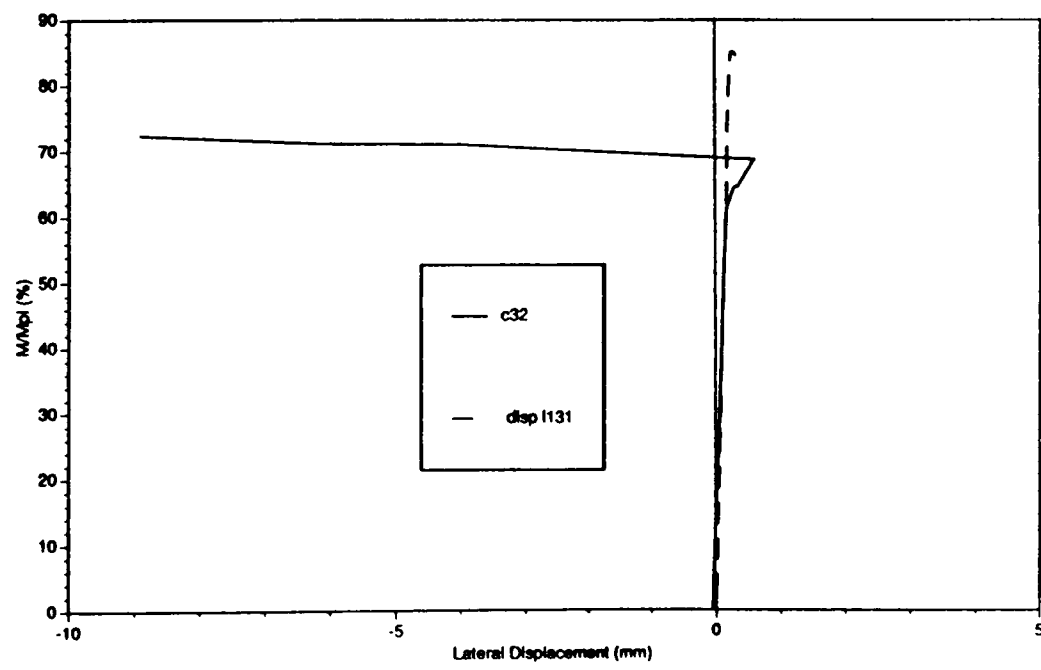
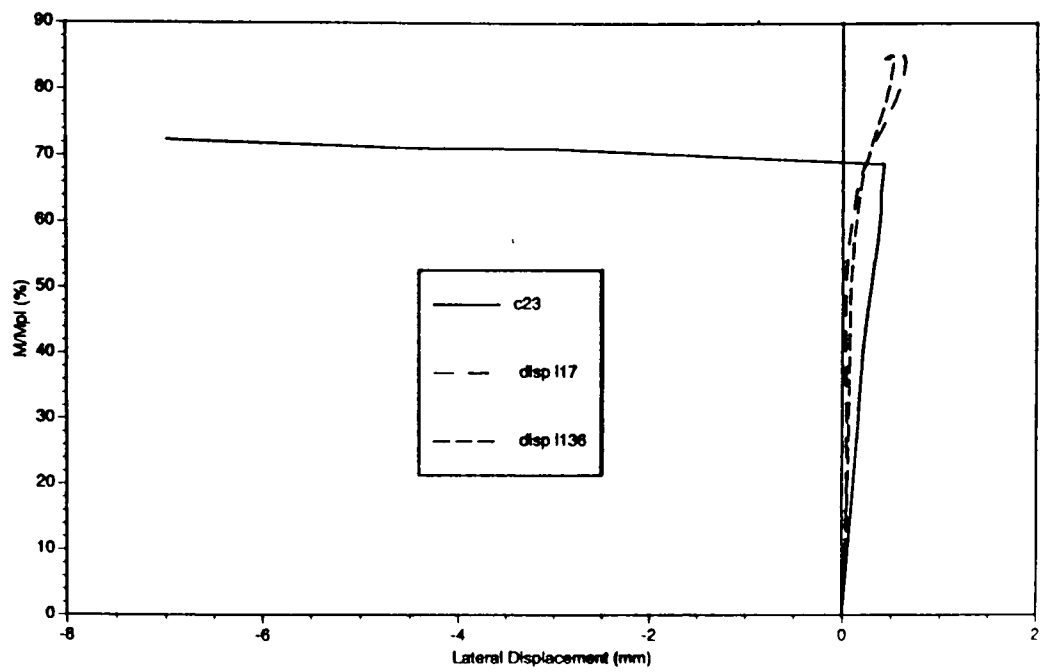
Three further finite element simulations were performed to cover intermediate cases of the neutral axis position, chosen to fill the gaps left by the experiments and make the study more comprehensive. In the first three specimens, the plastic neutral axes were situated at 77mm, 410mm and 630mm from the top of the compressive flange. The new cases depict situations where the plastic neutral axis is located at 168mm, 317mm and 533mm.

4.5.4.1 Beam D ($t_{fc} = 17\text{mm}$, $t_f = 12\text{mm}$).

The collapse characteristic described in beam 4 applied in this example as well, with the beam failing by a plastic hinge mechanism. Its deflected shape at the failure point is shown in figure 4.44, page 160. The web stress distribution at the ultimate loading stage is presented in figures 4.45.a and 4.45.b, page 161. The vertical stresses are mainly located at the bottom part of the web, although some



Figures 4.37.a and 4.37.b - Load vs. Lateral Displacement Comparisons for Channels 2 and 10, Beam 5.



Figures 4.38.a and 4.38.b - Load vs. Lateral Displacement Comparisons for Channels 23 and 32, Beam 5.

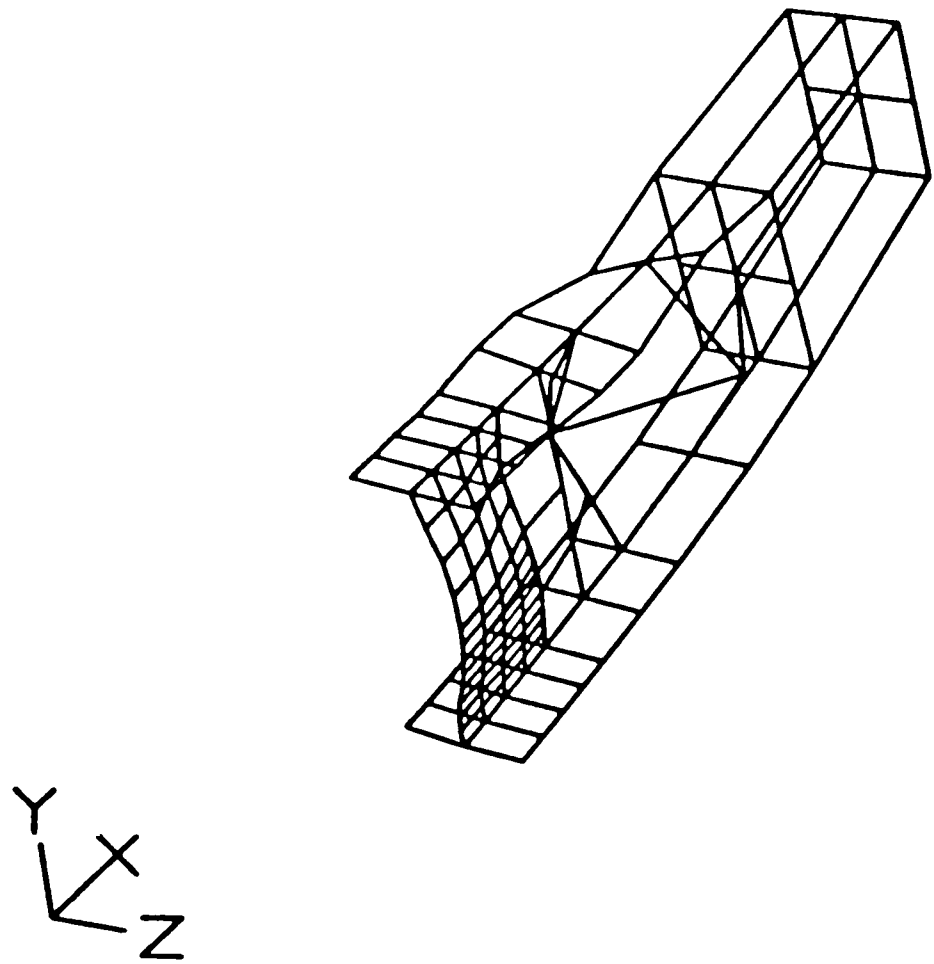
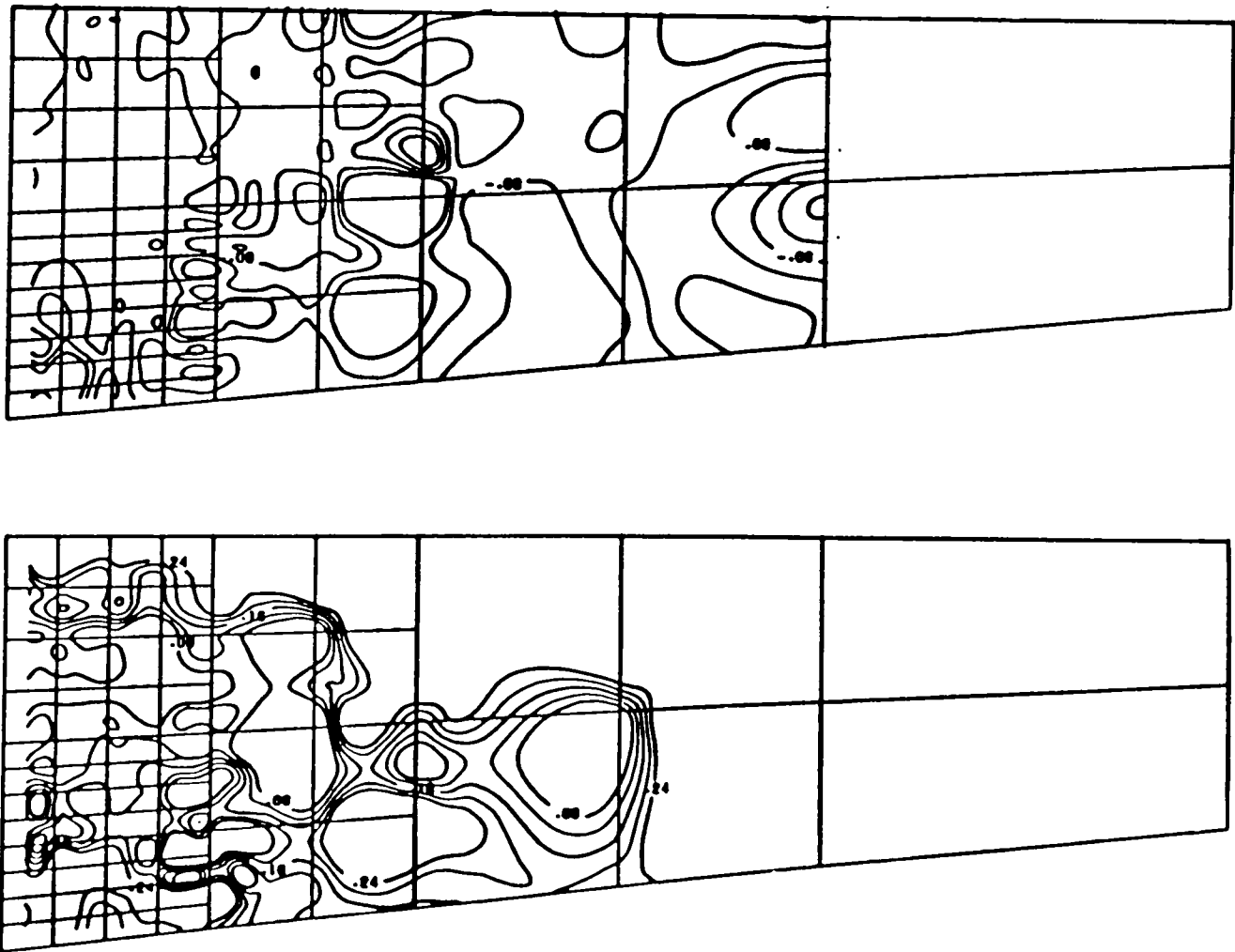


Figure 4.39 - Deformed Configuration of Beam 6 ($t_{fc} = 15\text{mm}$, $t_{ft} = 25\text{mm}$).



Figures 4.40.a and 4.40.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam 6 ($t_{fc} = 15\text{mm}$, $t_{ft} = 25\text{mm}$).

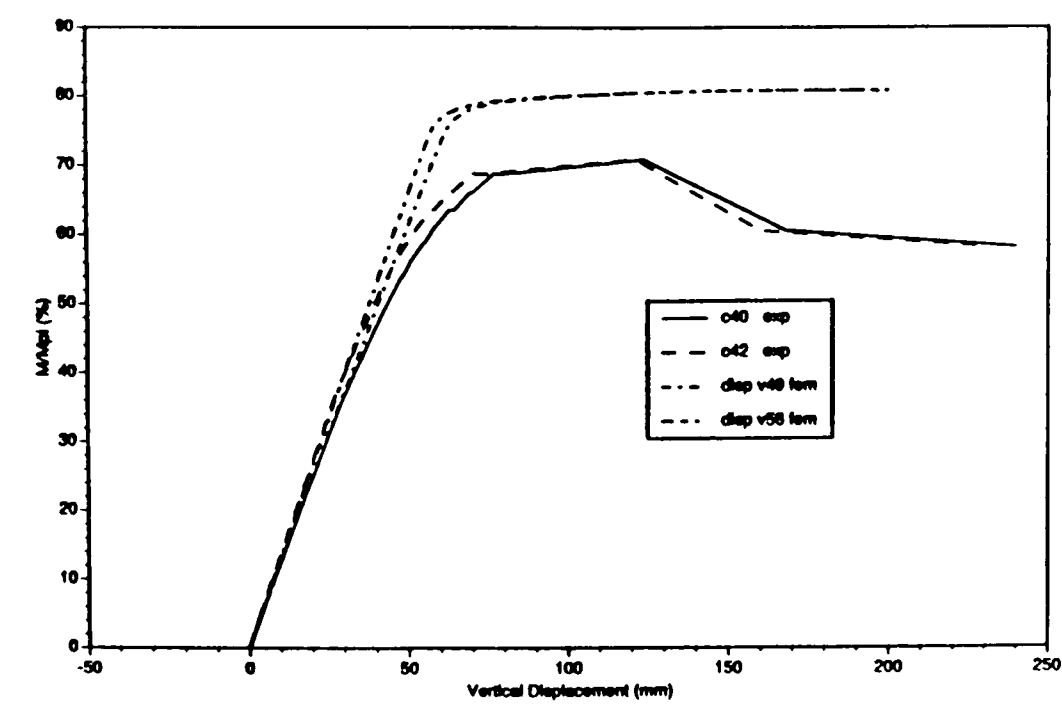
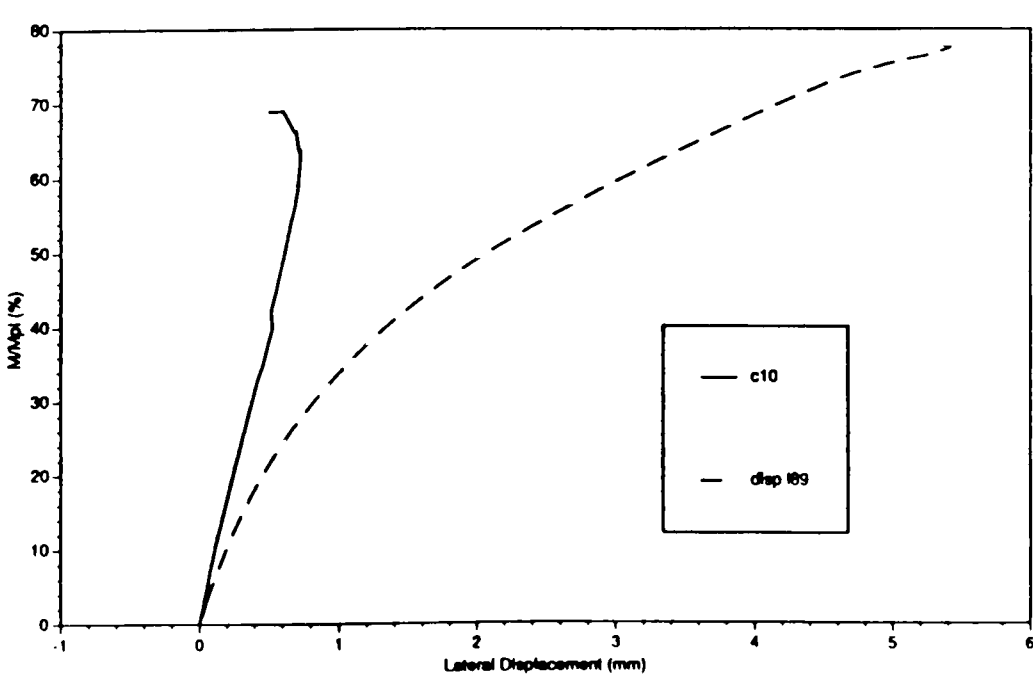
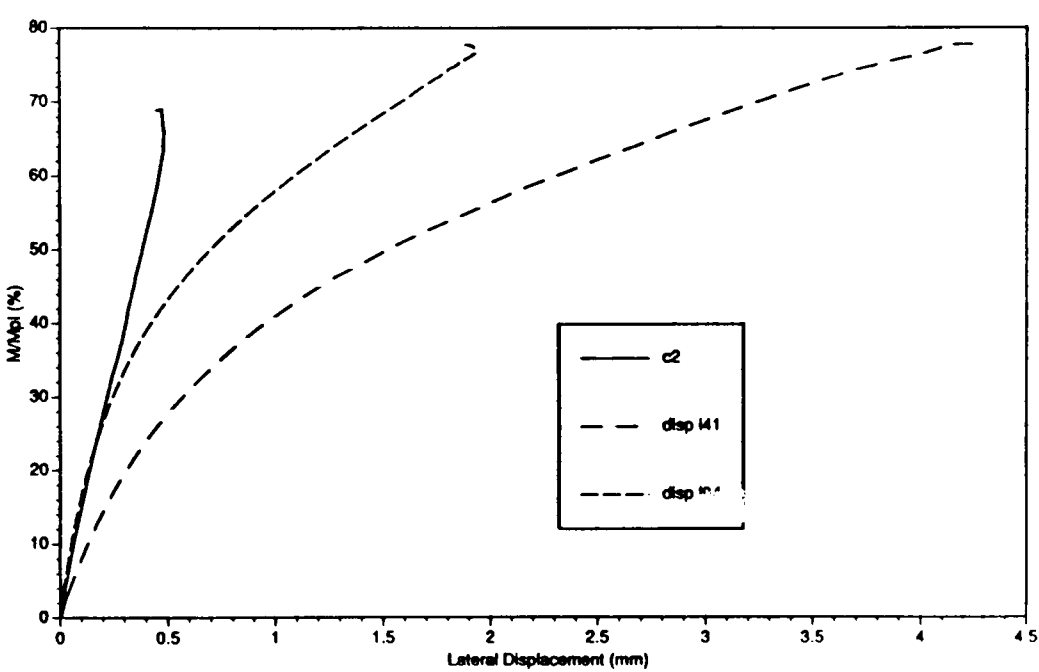
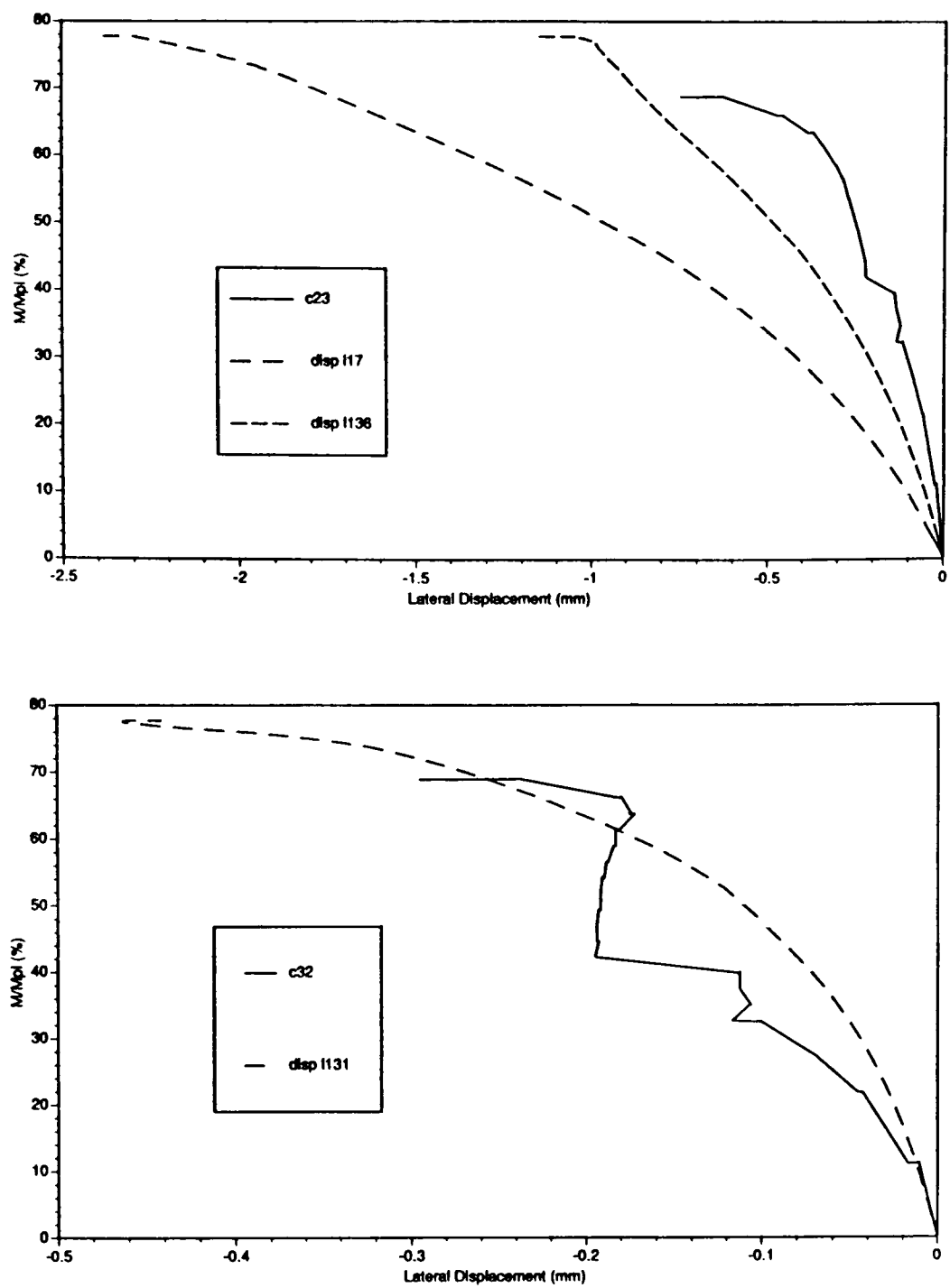


Figure 4.41 - Load vs. Vertical Displacement Comparison (Beam 6).



Figures 4.42.a and 4.42.b - Load vs. Lateral Displacement Comparisons for Channels 2 and 10, Beam 6.



Figures 4.43.a and 4.43.b - Load vs. Lateral Displacement Comparisons for Channels 23 and 32, Beam 6.

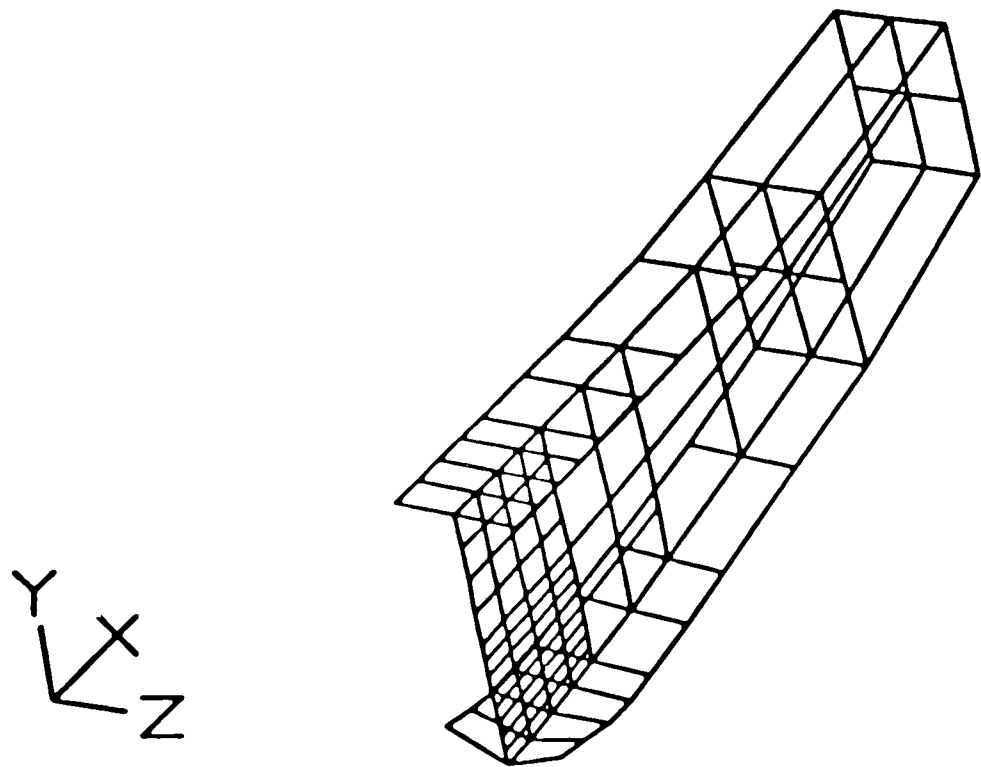
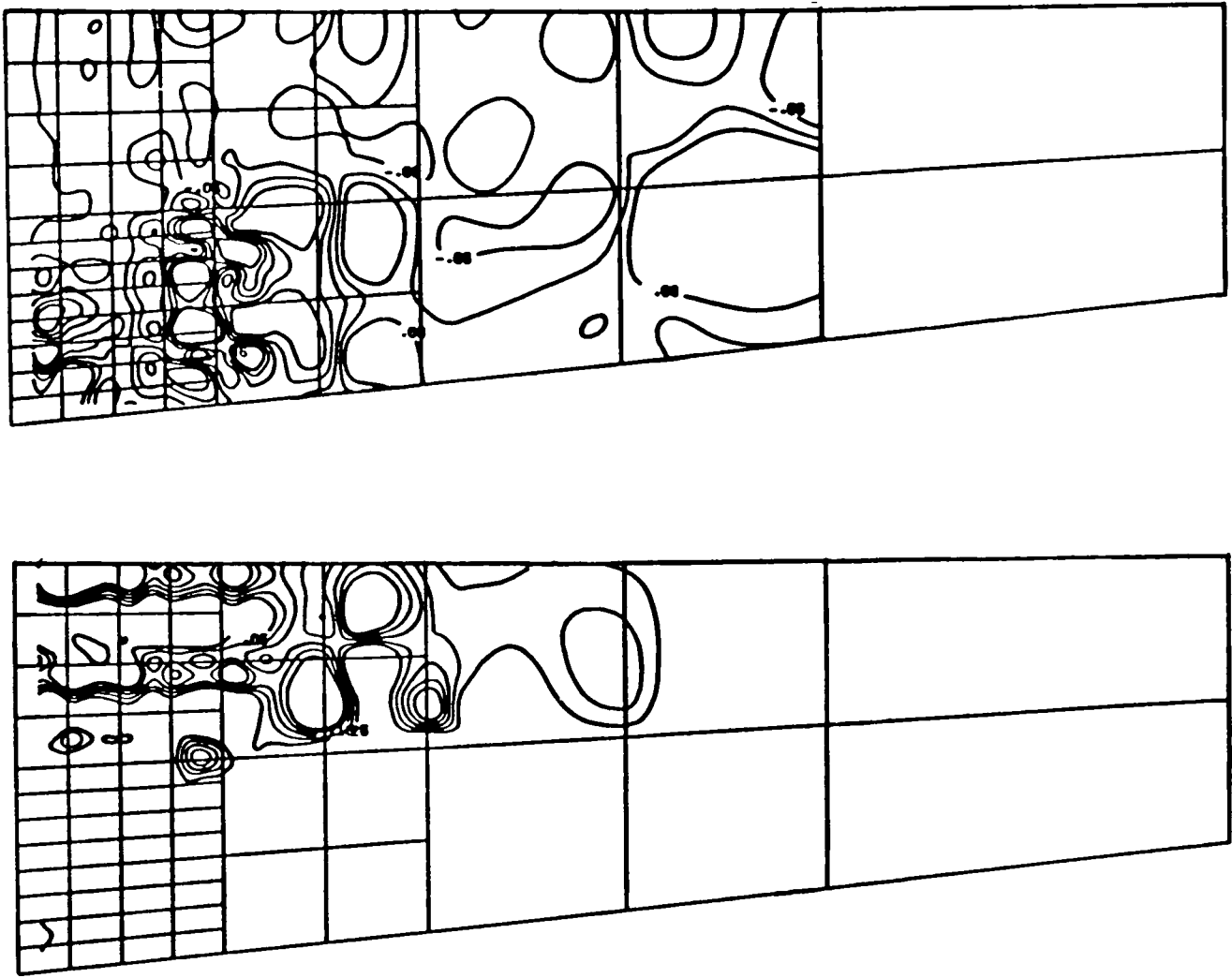


Figure 4.44 - Deformed Configuration of Beam D ($t_{fc} = 17\text{mm}$, $t_{ft} = 12\text{mm}$).



Figures 4.45.a and 4.45.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam D ($t_{fc} = 17\text{mm}$, $t_{ft} = 12\text{mm}$).

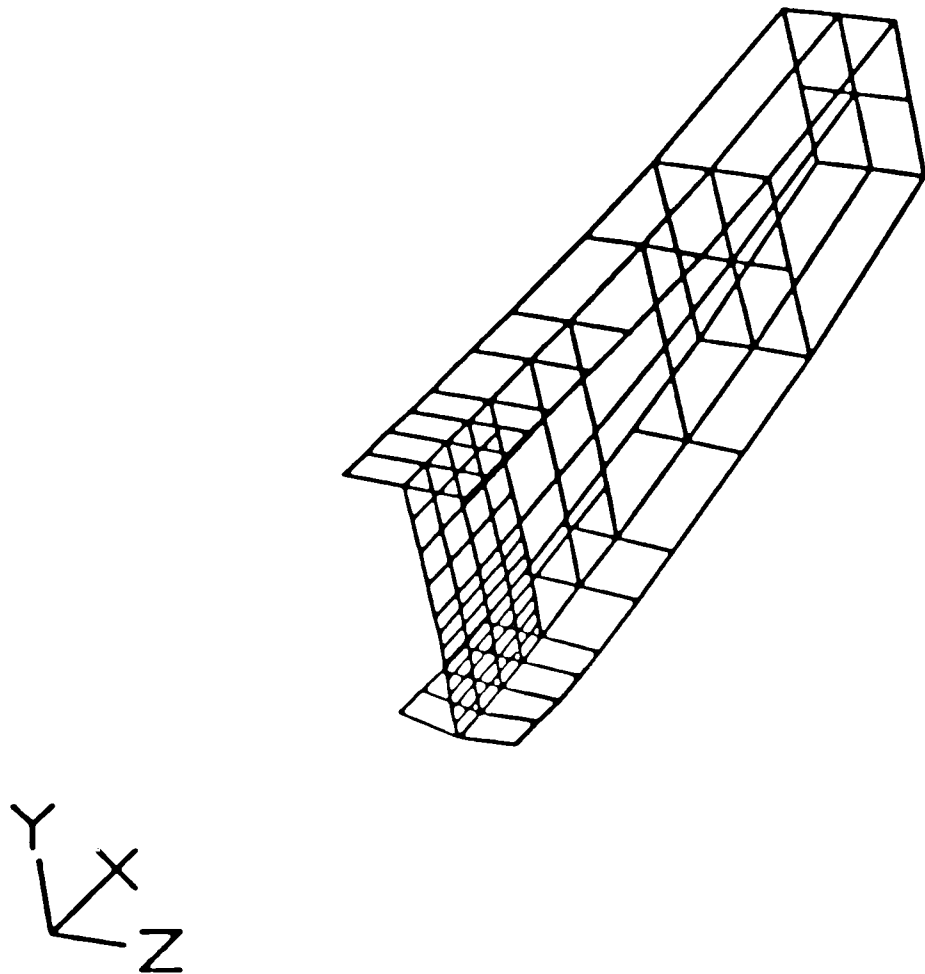


Figure 4.46 - Deformed Configuration of Beam E ($t_{fc} = 17\text{mm}$, $t_{ft} = 17\text{mm}$).

compressive action can be felt in the region close to the web stiffener. The maximum equivalent (Von Mises) stresses are situated predominantly in the top part of the web. However, their presence is also seen in the vicinity of the stiffener.

4.5.4.2 Beam E ($t_{fc} = 17mm$, $t_{ft} = 17mm$).

The initiation of buckling of the top steel flange, near the vertical stiffener, can be observed in figure 4.46, page 161. A careful inspection of this drawing shows an initial rotation of the top flange in that region. The web has remained straight at the centre of the beam, showing no need for web stiffeners. The stresses in the web are delineated as figures 4.47.a and 4.47.b, page 163, and follow the pattern described the beam 5. In the region close to the stiffener, compressive vertical stresses are very concentrated.

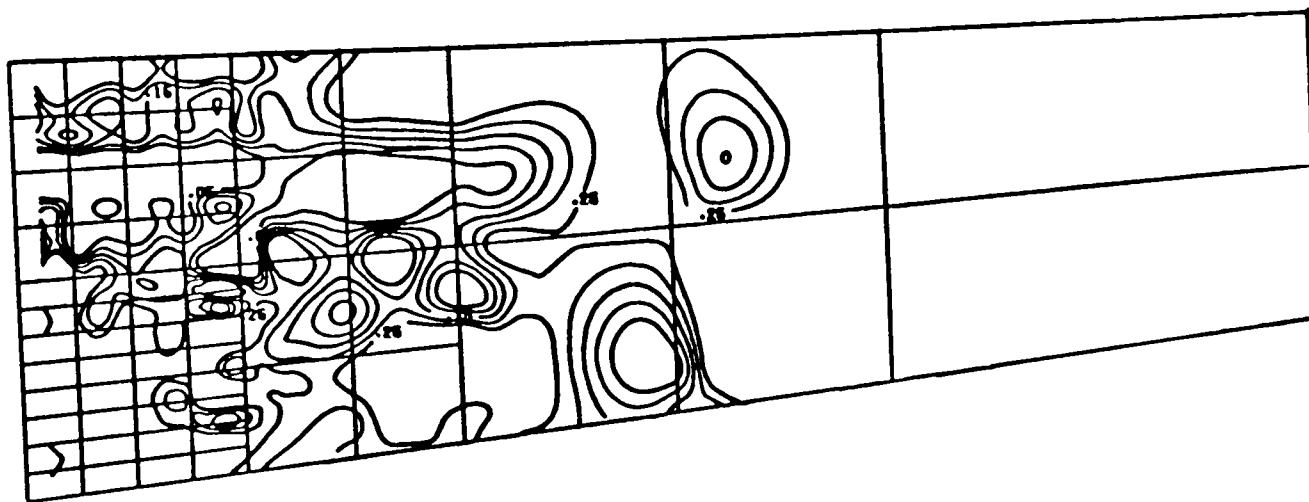
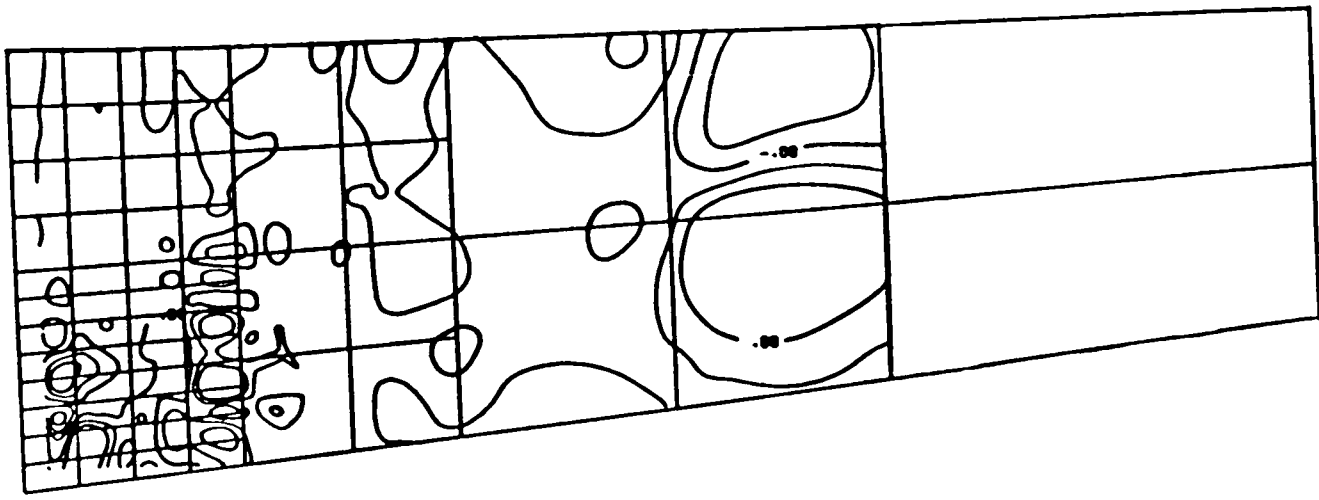
4.5.4.3 Beam F ($t_{fc} = 17mm$, $t_{ft} = 25mm$).

Top flange buckling is clearly visible in figure 4.48, page 163. In this picture, the deformed shape of the beam is plotted with an amplification factor of 5 for the ultimate loading stage. The web's lateral movement has occurred due to the rotation of the top flange. No stiffeners were found to be required at the centre of the beam. The vertical and equivalent (Von Mises) stresses in the web are presented in figures 4.49.a and 4.49.b, page 164. They have the same characteristics as those seen for beam 6.

4.5.5. Discussion and Summary.

The ultimate loads reached by the second series of specimens, in terms of percentage of the plastic moment capacity is outlined in table 4.2, page 165. The difference in terms of ultimate load reached by the two approaches varied between 12.3% and 17.4%. For beam 4, with the highest neutral axis, failure by the plastic hinge mechanism was again predicted to occur at around 110% of the plastic capacity at the edge of the taper panel. This apparently high strength capacity was predicted because the plastic hinge did not develop right at the edge of the tapered panel, as stated before. The experimental value for the plastic moment for this beam was determined, once more, by the intersection of two tangents to the curve presented in its load vs. vertical deflection diagram (figure 3.33, page 107), and found to be 95% of the plastic capacity at the edge of the tapered panel. The yield stress used came from independent coupons for the test.

The flange buckling failures were predicted to be in the region of 80% of the plastic capacity at the central section but the experimental failures happened at around 70%. That disparity can also be considered small when all the unknown factors of an experimental research are taken into account. In these cases, the finite element simulation can be considered successful due to the fact that the same pattern of final displacement and stresses was reached by both methods. However, it is not surprising that the finite element analysis gave an overestimate because the coarse top flange mesh would be too stiff to model the flange buckling precisely.



Figures 4.47.a and 4.47.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam E ($t_{fc} = 17\text{mm}$, $t_{fl} = 17\text{mm}$).

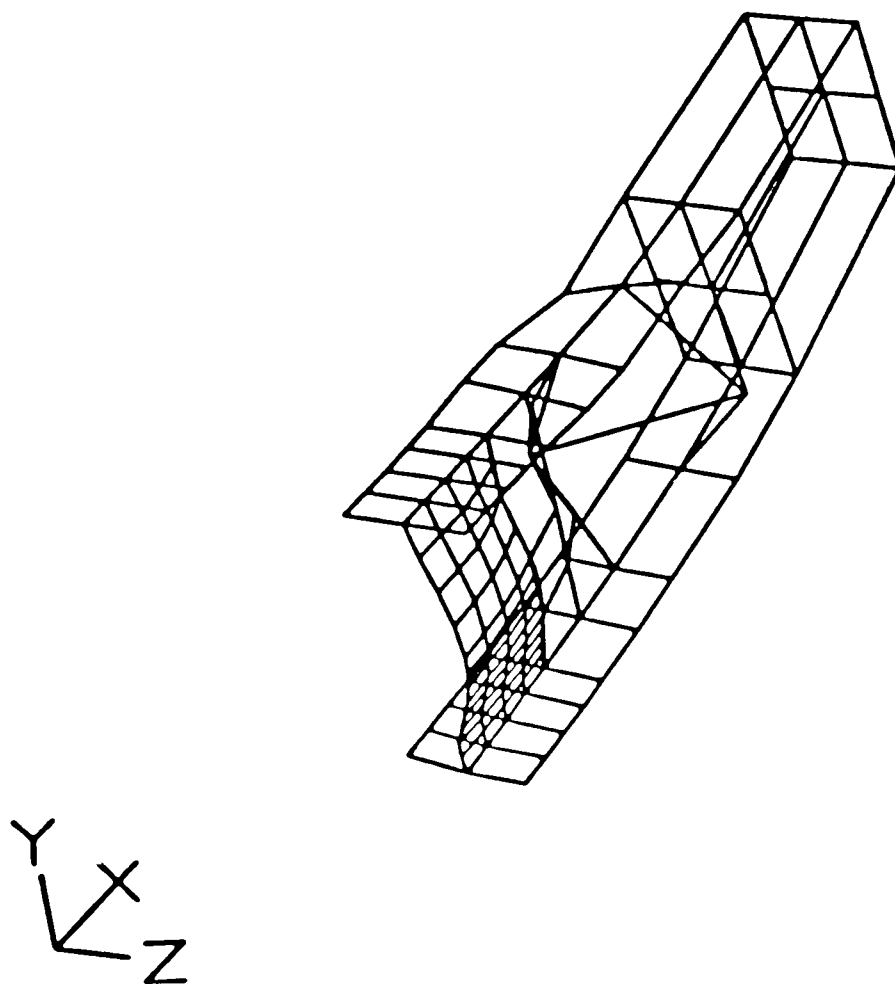
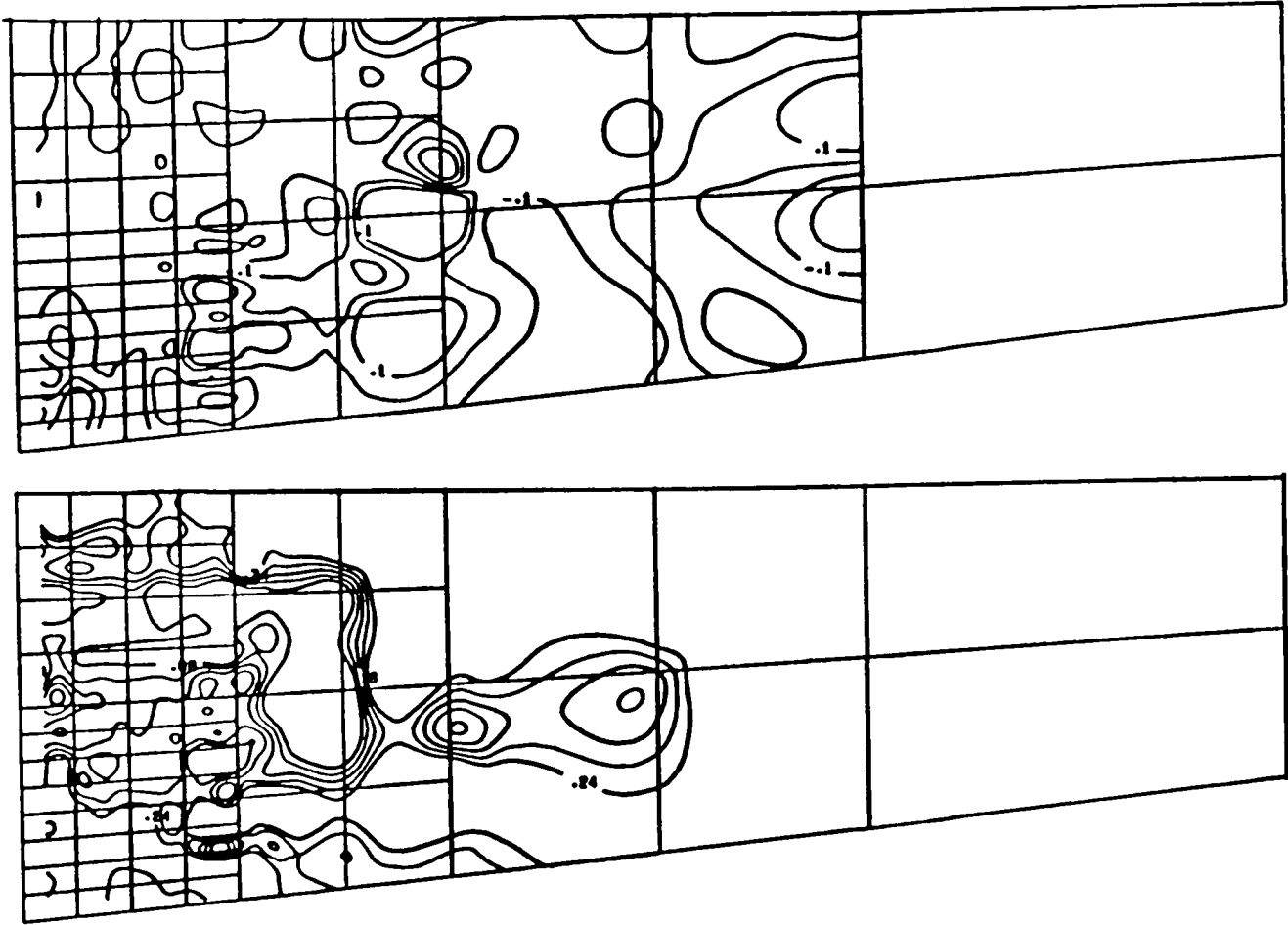
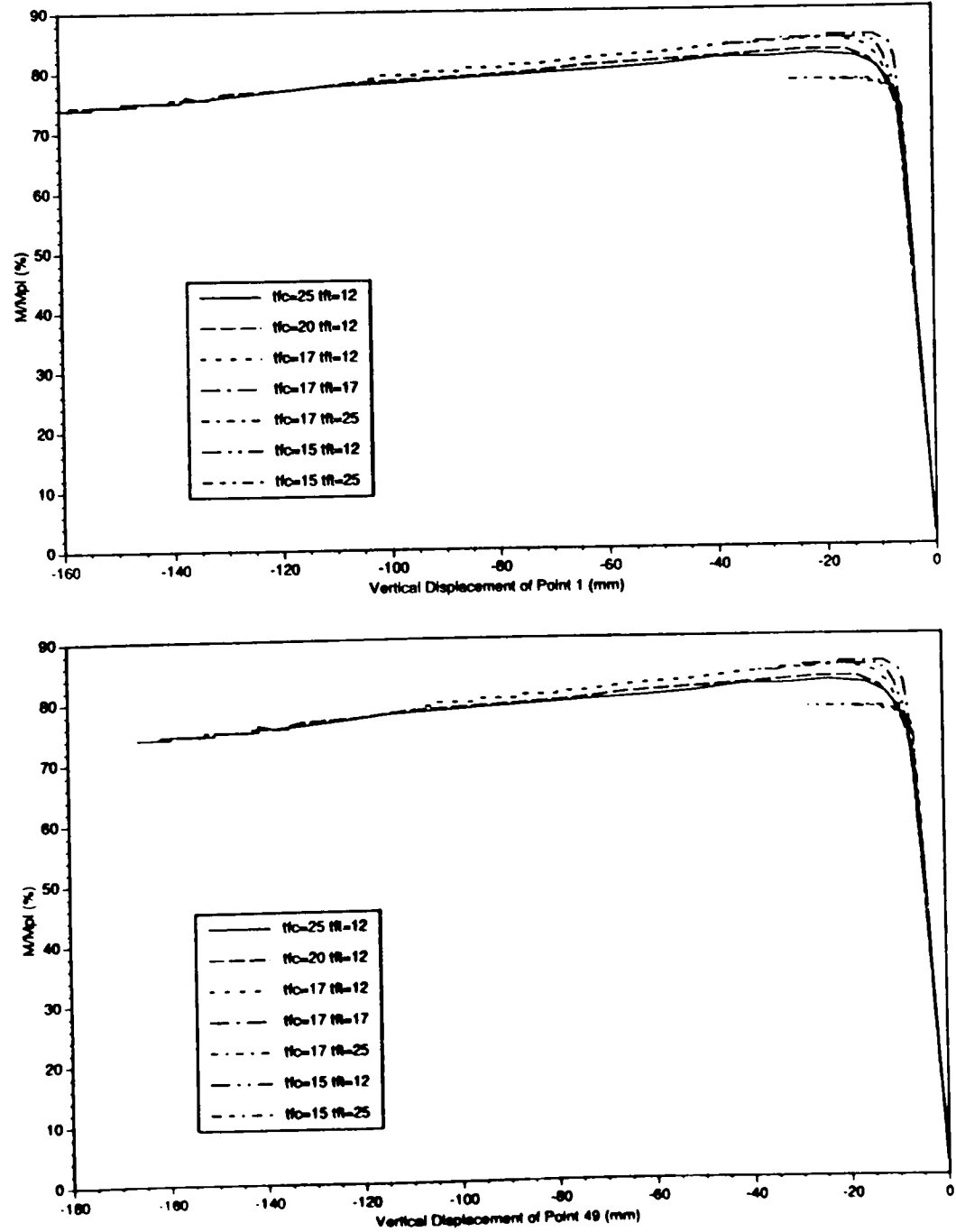


Figure 4.48 - Deformed Configuration of Beam F ($t_{fc} = 17\text{mm}$, $t_{fl} = 25\text{mm}$).



Figures 4.49.a and 4.49.b - Vertical and Equivalent (Von Mises) Stresses in the Web of Beam F ($t_{fc} = 17\text{mm}$, $t_{ft} = 25\text{mm}$).



Figures 4.50.a and 4.50.b - Load vs. Vertical Displacement Comparison for Points 1 and 49 of All the Six Cases Studied.

TYPE OF ANALYSIS	NEUTRAL AXIS POSITION (mm)					
	77.	168.	317.	410.	533	630.
F.E.M. $M_{max}/M_{plcentre}$ (%)		84.98	85.38	85.05	78.05	77.45
F.E.M. M_{max}/M_{pledge} (%)	109.39					
EXP. M_{pl}/M_{pledge} (%)	95.7					
EXP. $M_{max}/M_{plcentre}$ (%)				72.41		68.96

Table 4.2 - Ultimate Load Comparison.

The vertical displacements of points 1 and 49, for all the finite element cases reviewed in this section, are presented in figures 4.50.a and 4.50.b, page 164. Two distinctive ultimate failure patterns can be identified in those diagrams. The first corresponds to the collapse by the plastic hinge mechanism and possessed higher ultimate loads. The second conforms with the beams that failed by top flange buckling. The lateral displacements of points 9, 25, 89 and 94, of all the girders covered in this series, are shown in figures 4.51.a, 4.51.b, 4.52.a and 4.52.b, pages 166 and 167. Again the difference in the collapse mode can be clearly identified.

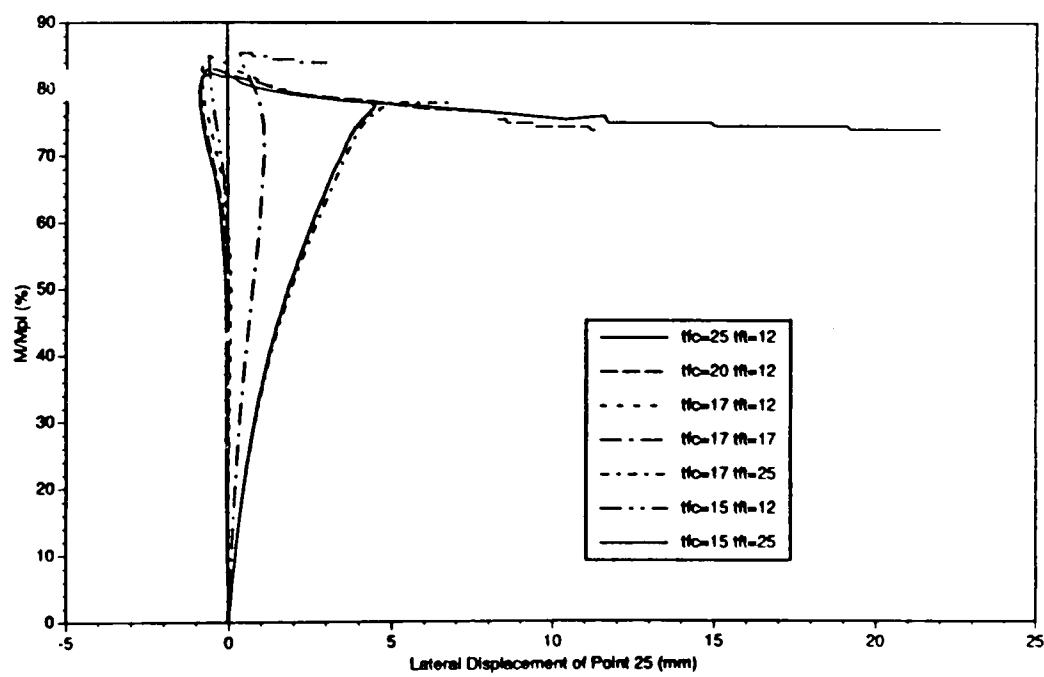
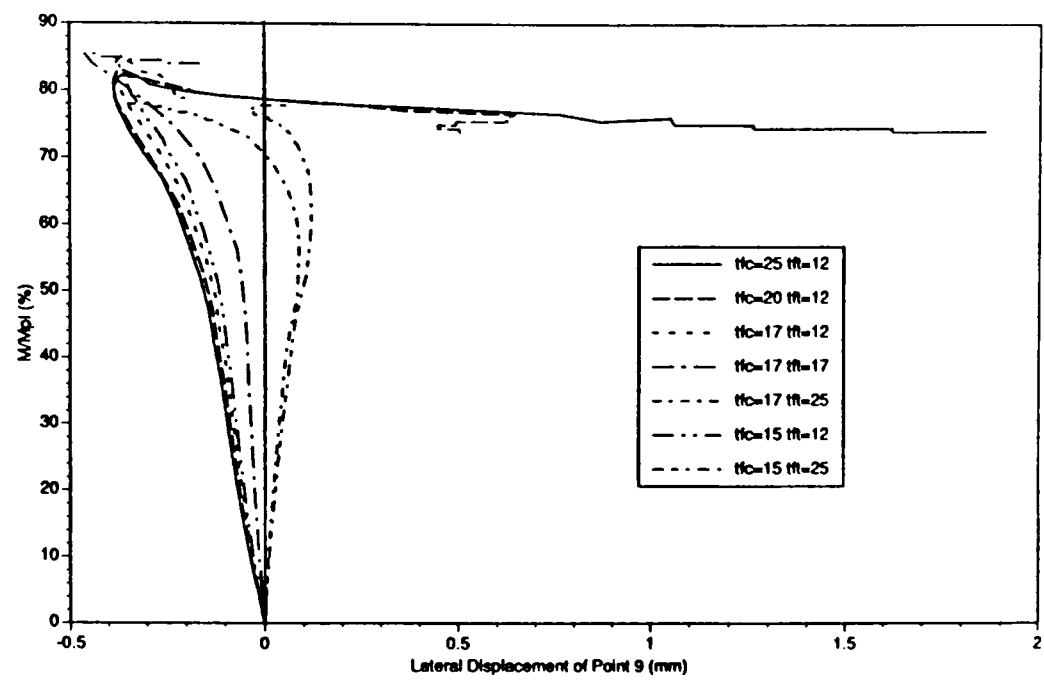
4.6. GENERAL CONCLUSIONS.

The finite element simulations have reached a reasonable agreement with the experiments. The numerical method predicted the final deformed shape and the web stress patterns reached by the girder at the ultimate loading stage. In the first series, web stiffeners were only found to be necessary for webs that had thicknesses less than 4mm. No stiffeners were required in the second series of beams.

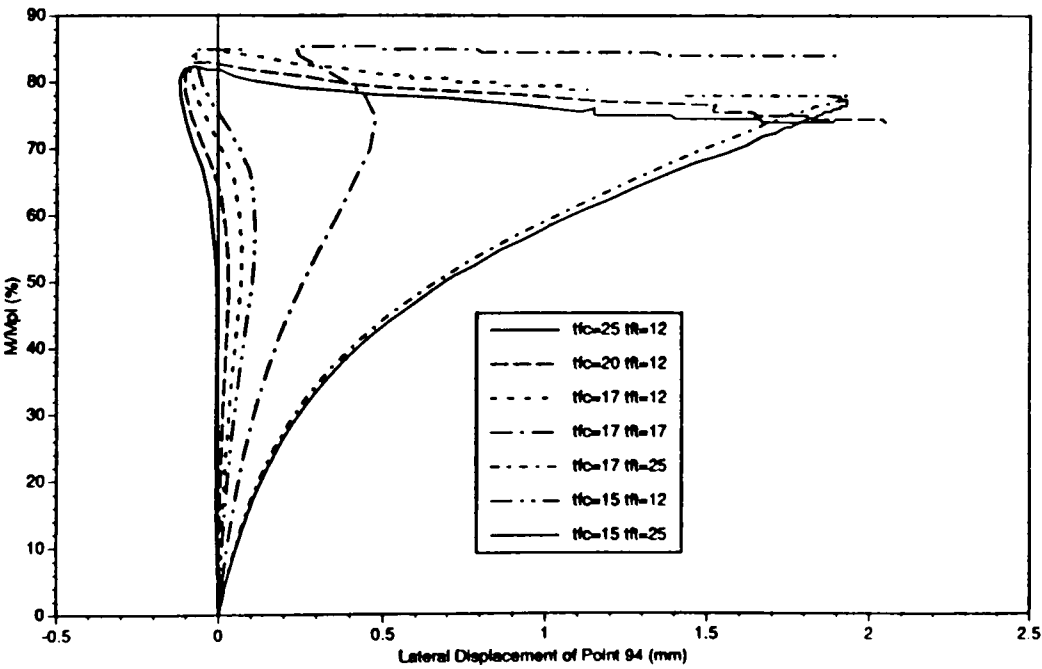
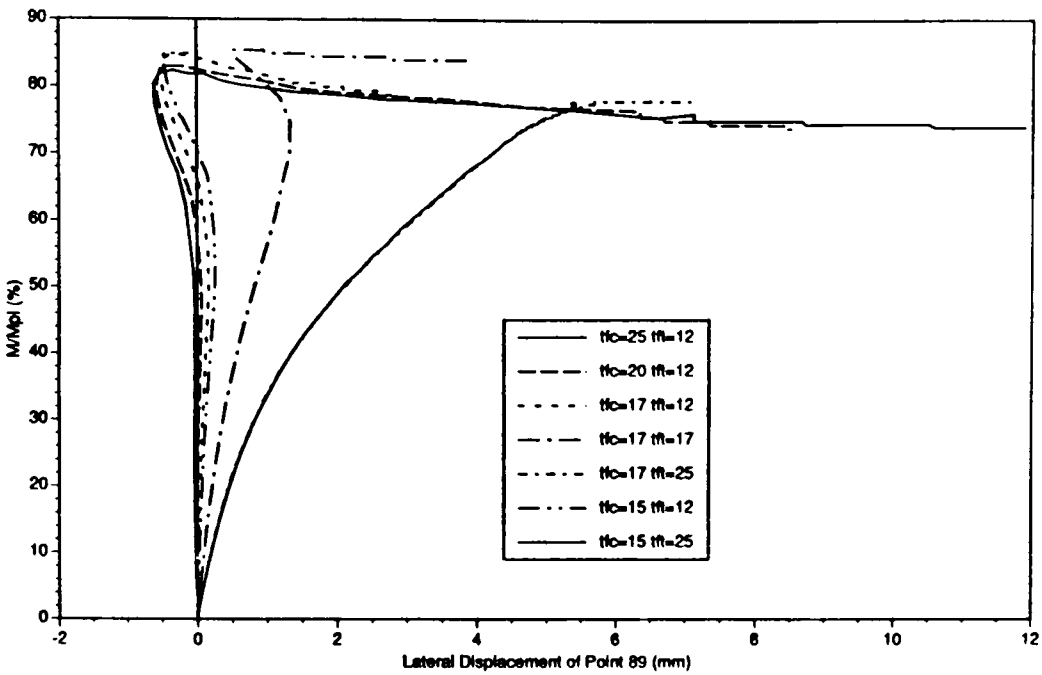
The influence of the thickness of the web on the resistance of the beams studied is portrayed in figure 4.53, page 168. In those cases, a limit of 4mm for the web thickness is recommended to avoid local web buckling. The neutral axis influence on the failure load of the beams is presented in figure 4.54, page 168. The significant decrease in resistance for beams with the neutral axis depth greater than 300mm is explained by the sudden top flange buckling failure.

The finite element program, FINAS, employed in these simulations, presented some problems for the user. No restart facility is available in the program, which leads to a large increase in computation time and user effort to surpass the maximum load point. The control over the solution strategies could also be improved. Divergence problems were very common and difficult to overcome. There is a very restrictive option in terms of shell elements. No pre or post processor of data was available at the time of use. The great majority of graphs contained in this thesis has been produced with the aid of programs developed by the author, although a post processor has recently been implemented. A considerable amount of time and effort was also spent in the non-trivial task of finding the most suitable mesh for the problem.

The next chapter will describe the finite element parametric study executed to cover the most important variables of the local web buckling problem. The thickness of the web and the angle of taper slope will be the object of a comprehensive investigation. This will be followed by an examination of the influence of the tension flange area on the beam's final behaviour.



Figures 4.51.a and 4.51.b - Load vs. Lateral Displacement Comparison for Points 9 and 25 of All the Six Thicknesses Studied.



Figures 4.52.a and 4.52.b - Load vs. Lateral Displacement Comparison for Points 89 and 94 of All the Six Thicknesses Studied.

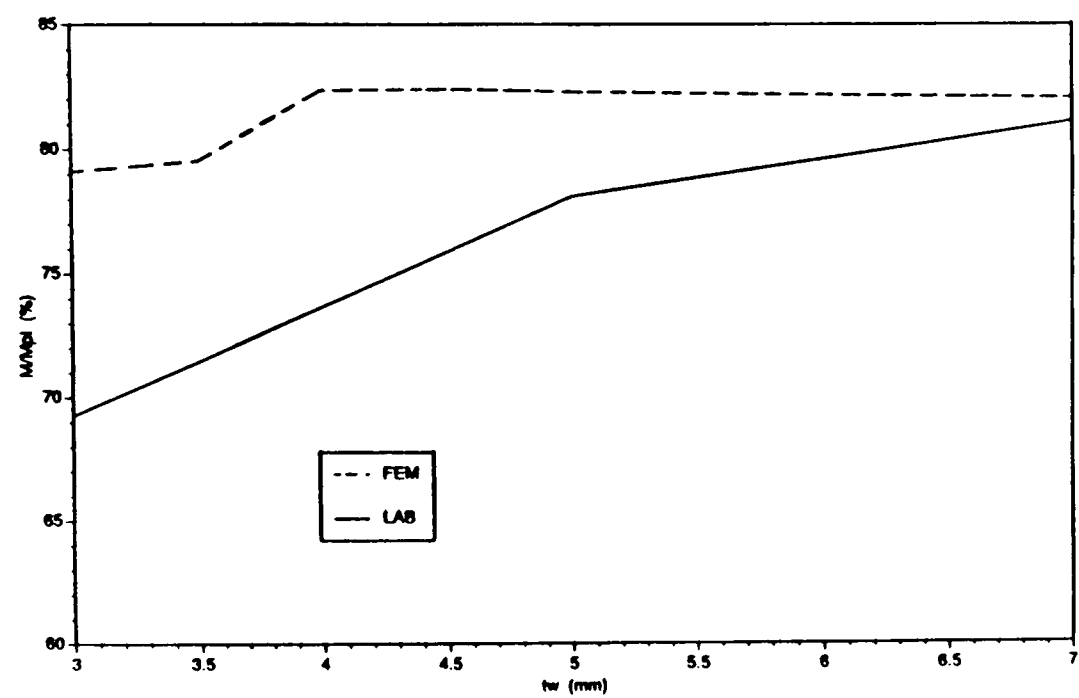


Figure 4.53 - Influence of the Thickness of the Web on the Failure Load.

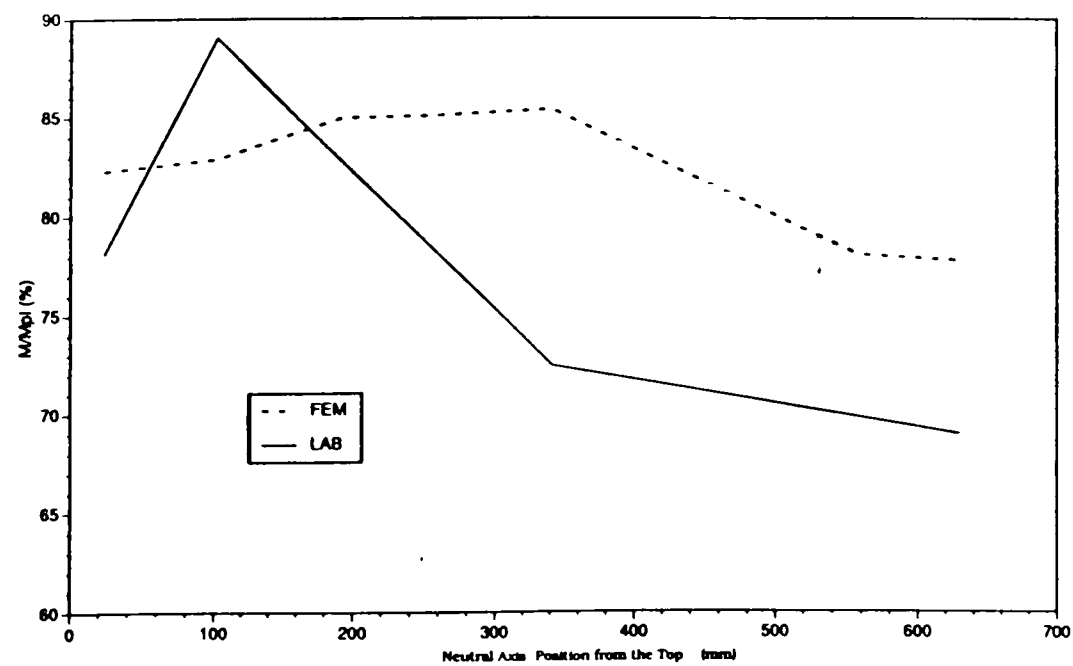


Figure 4.54 - Influence of the Neutral Axis Position on the Failure Load.

CHAPTER 5

PARAMETRIC STUDY.

5.1. INTRODUCTION.

This chapter describes the finite element parametric study of local web buckling at the point where a change in flange slope occurs. A parametric study is very useful to cover the large range of situations in which this problem may affect the behaviour of the beam. Properly calibrated, it proves to be much more economic than an experimental investigation covering a similar range. A complete validation of the finite element model was presented in the previous chapter. The calculation model and the FINAS program, as described in chapter four, were used throughout the parametric study.

The investigation has focused on three main parameters. A study of the angle of taper and web thickness is first reported. This is followed by an examination of the influence of the tension flange area on the load carrying capacity of the girder. With the results to hand, conclusions can be drawn on the influence of these variables on the beam performance.

5.2. PARAMETERS STUDIED.

The literature review on patch loading suggested that one of the most important parameters was the thickness of the web. The results obtained from the experimental investigation (described in detail in chapter three) confirmed that view. These two strong arguments indicated that if a complete picture of the beam's behaviour was required, the influence of the thickness of the web should be properly investigated.

Consideration of the vertical web buckling phenomenon can easily identify two other pertinent variables. If the influence of magnitude of the force that may induce buckling is addressed, the importance of the area of the tension flange and the change in taper are immediately identified.

Other less relevant parameters, such as the beam's depth and the yield stress, could have been taken into account. The enormous increase in possible cases that could be studied, and the computational effort associated, prevented their inclusion in this investigation.

5.3. INFLUENCE OF ANGLE OF TAPER AND WEB THICKNESS.

The finite element model used in this simulation is similar to those described in chapter four. The loading and support configurations, initial imperfections, and boundary conditions are the same. The main differences are related to the geometric characteristics of the beam's cross section, while the total length and moment lever arm were based on a composite tapered beam tested in the structures laboratory earlier, described in the literature review, [HARG86].

All the beam models in this section had a total span of 4.8m and a moment lever arm of 800mm. The widths of the top and bottom flanges were 250mm and their thicknesses were 25 and 12mm respectively. The maximum depth at the centre of the beam was 800mm. The yield stresses of the web, top flange and bottom flange were

taken as 420MPa, 391MPa and 398MPa respectively. These high values (based on [HARG86]) were chosen to induce the most critical situation for local web buckling. If the buckling did not manifest its presence with these values, it will certainly not occur with lower yield stresses.

The four web thicknesses used were 3, 5, 7 and 10mm. The angles of taper varied from 0° to 14.84° ($0 \leq \alpha \leq 1/4$). The parametric investigation can be better understood with the aid of Table 5.1 below, where a summary of the cases studied is made. The variation of the slope in some of these beams can be seen in figures 5.1.a, 5.1.b, 5.2.a and 5.2.b, page 171. The cases with a zero angle, which are related to a beam with parallel flanges, were considered because they represent an upper bound to the problem. They can also be used to calibrate the model against well-known situations in the literature.

t_w / α	0°	1.15°	1.43°	1.91°	2.86°	8.53°	11.31°	14.84°
10mm	X	X	X	X	X	X	X	X
7mm	X	X	X	X	X	X	X	X
5mm	X	X	X		X	X	X	X
3mm	X		X		X	X	X	X

Table 5.1 - Finite Element Cases Studied in Section 5.3.

The rest of this section is organized as five different subsections, in order to present a broader view of the study. Each of the first four subsections corresponds to a different thickness of the web and a range of taper angles. The last subsection summarizes the investigations and presents some interesting comparisons.

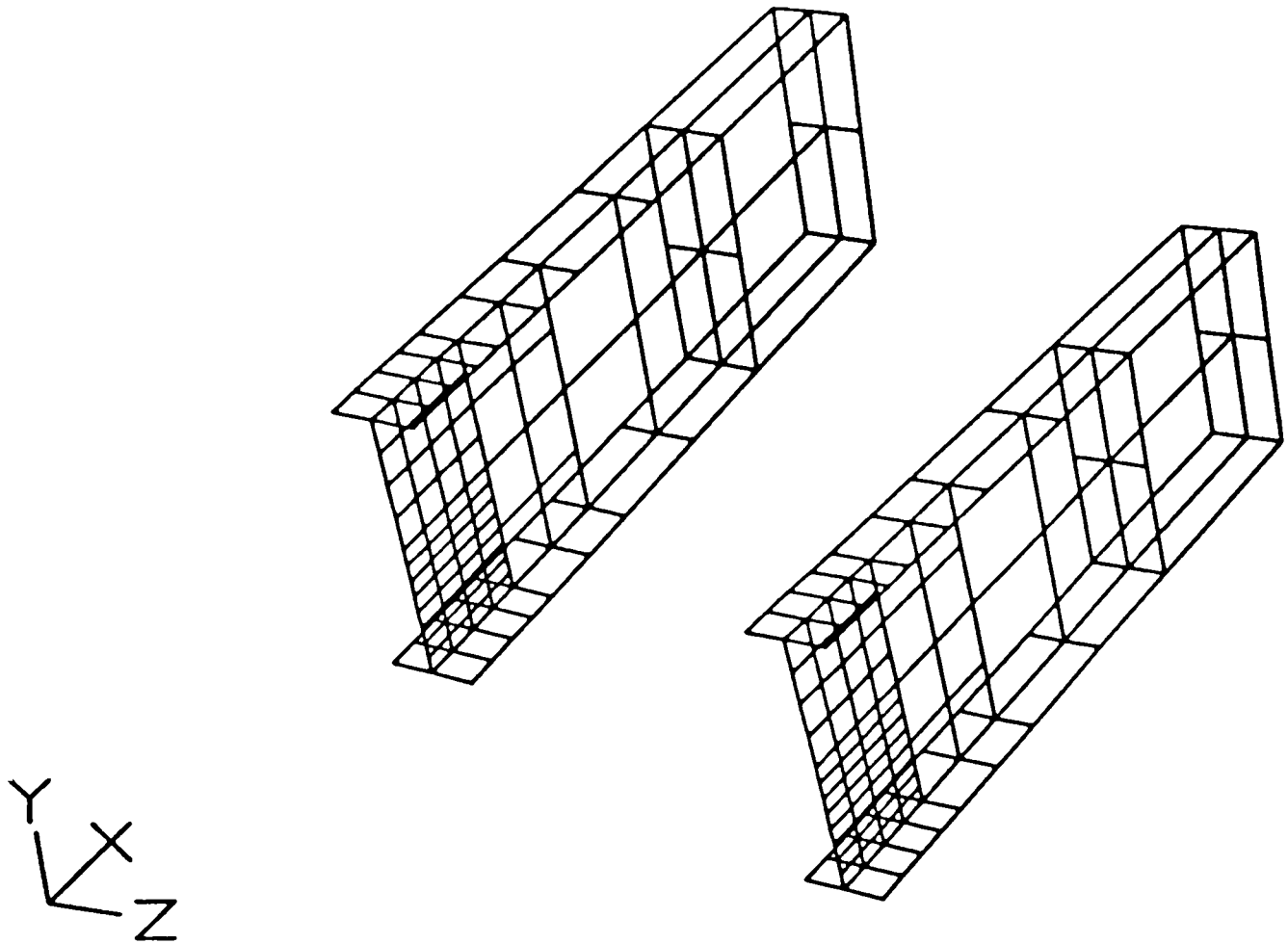
5.3.1. Web Thickness Equal to 10mm.

All the 10mm beams failed by the development of a plastic hinge. The exact position of this plastic hinge changed from beam to beam, moving towards the side vertical stiffener as the taper angle increased. This is explained by the fact that sections closer to the side stiffener had less web depth (and consequently less inertia) than the points at the centre of the girder. The web at the centre of the girder remained undeformed, showing no need for a vertical stiffener even with the highest slope.

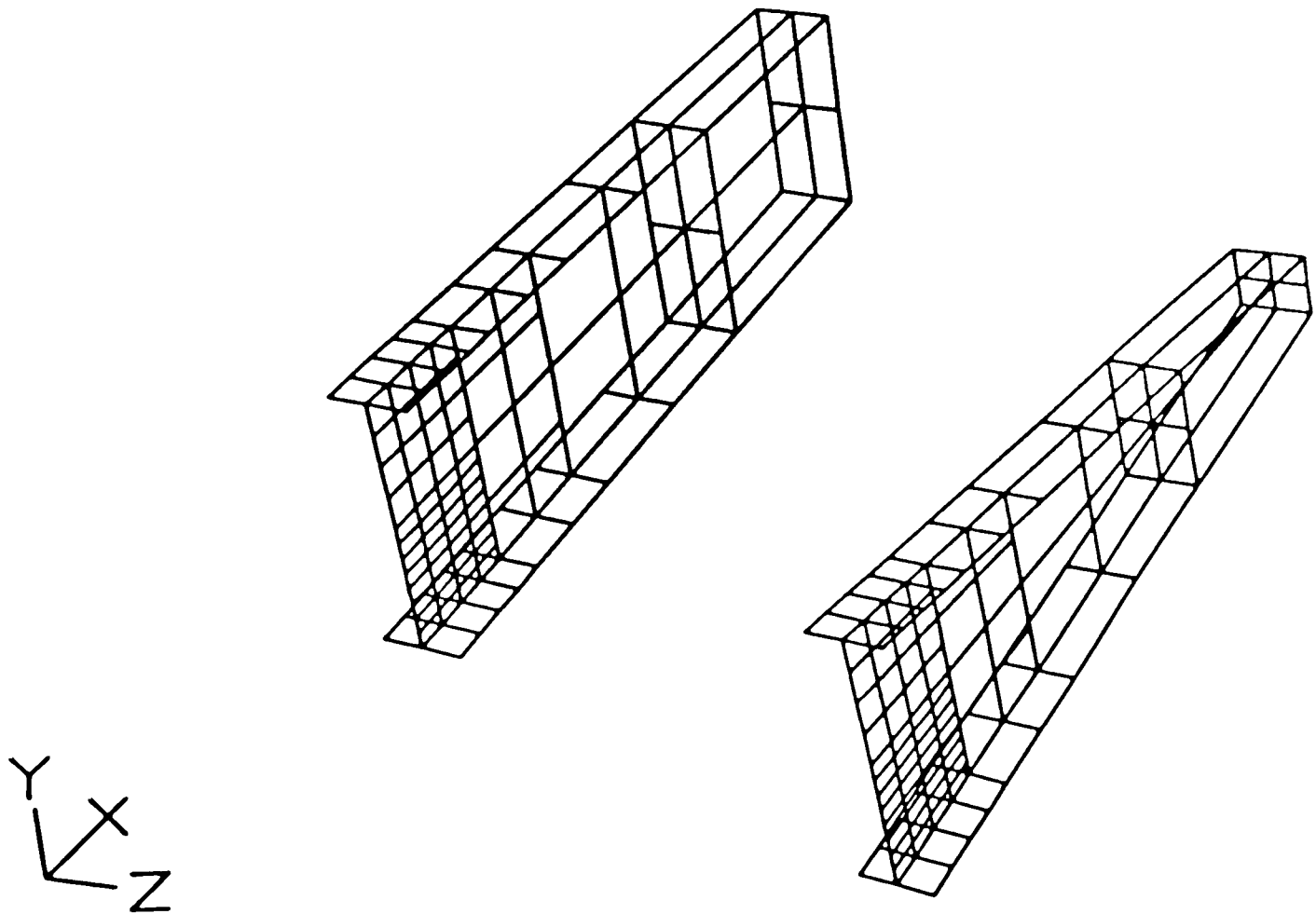
The deformed configuration of the beam with parallel flanges ($\alpha = 0^\circ$) is shown in figure 5.3, page 172. The vertical and equivalent (Von Mises) stresses at the ultimate loading stage are presented as figures 5.4.a and 5.4.b, page 172. From the equivalent stress graph it is possible to note that the bottom part of the web, at the centre of the beam, has not developed a high stress concentration.

For the beam with an angle of 1.15° ($\alpha = 1/50$), the behaviour was very similar, as can be seen from figures 5.5, 5.6.a and 5.6.b, page 173. The only difference in behaviour was the presence of small equivalent stresses in the bottom part of the web, at the centre of the girder.

The beam with an angle of 1.43° ($\alpha = 1/40$) has deformed as portrayed in figure 5.7, page 174. The final stress representation is shown in figures 5.8.a and 5.8.b, page 174. No significant change in the behaviour has occurred.



Figures 5.1.a and 5.1.b - Undeformed models with Slope angles of 0° and 1.43° (0 and 1/40).



Figures 5.2.a and 5.2.b - Undeformed models with Slope angles of 2.87° and 14.84° (1/20 and 1/4).

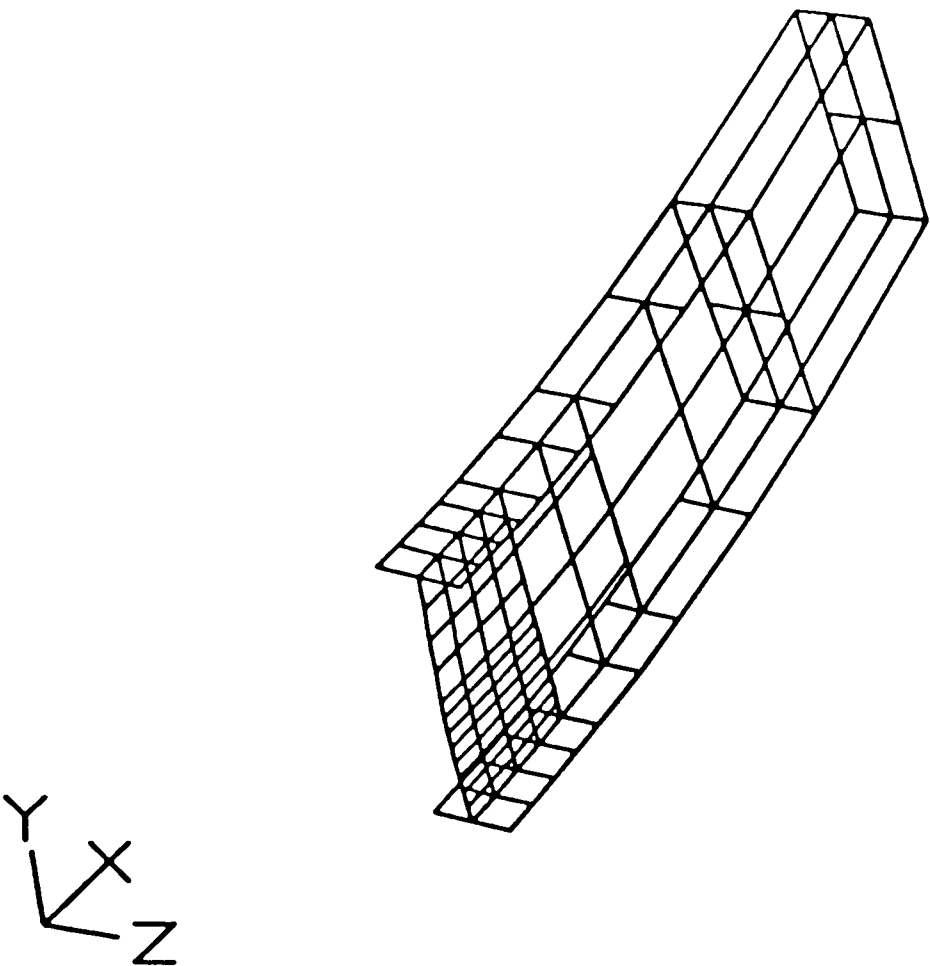
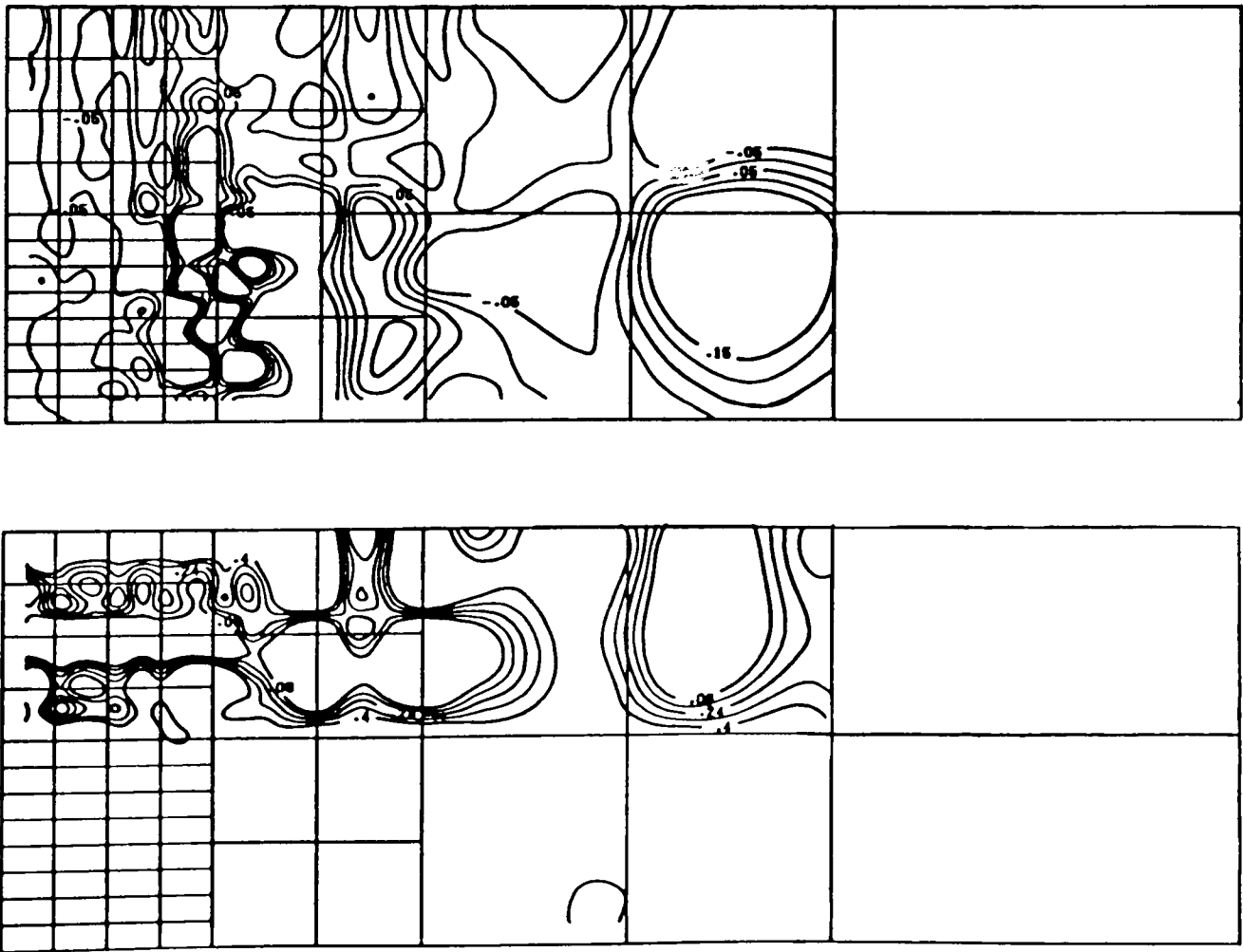


Figure 5.3 - Deformed Configuration of the Beam with $\alpha = 0^\circ$ ($t_w = 10\text{mm}$).



Figures 5.4.a and 5.4.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 0^\circ$.

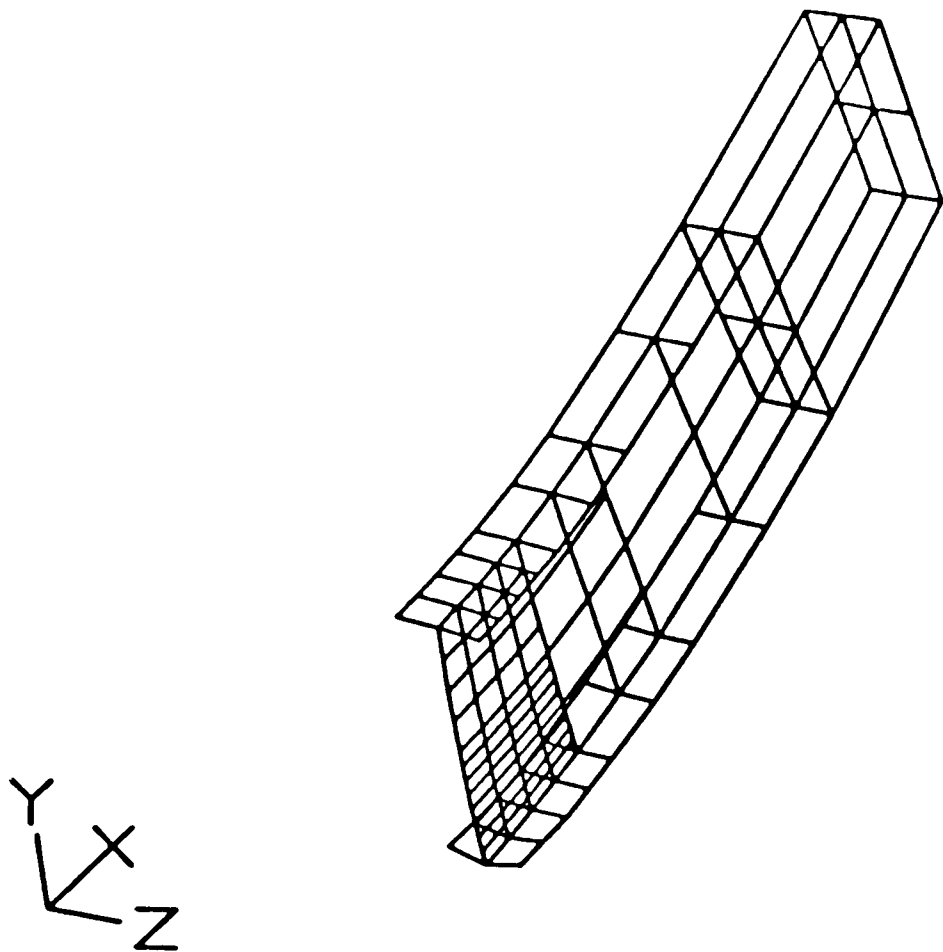
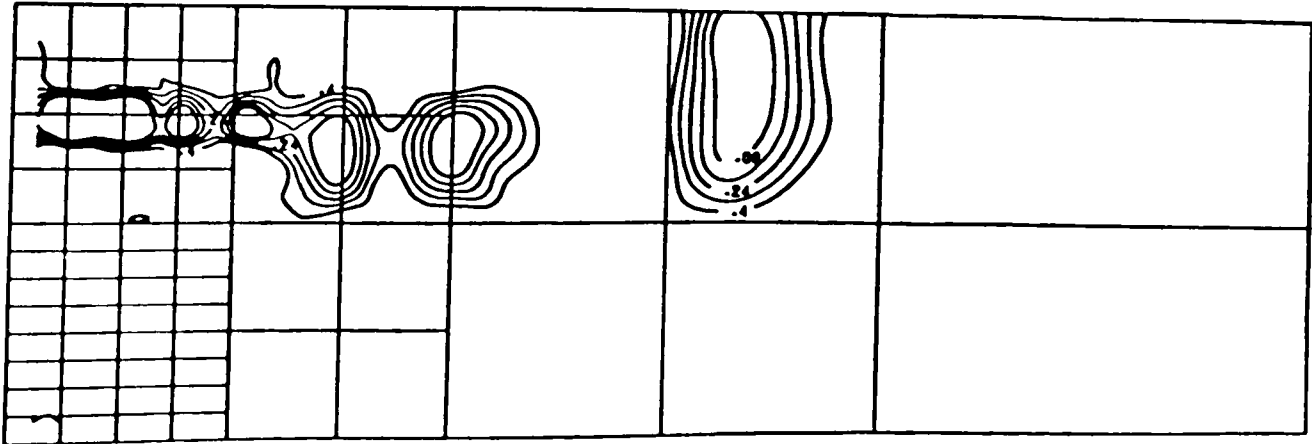
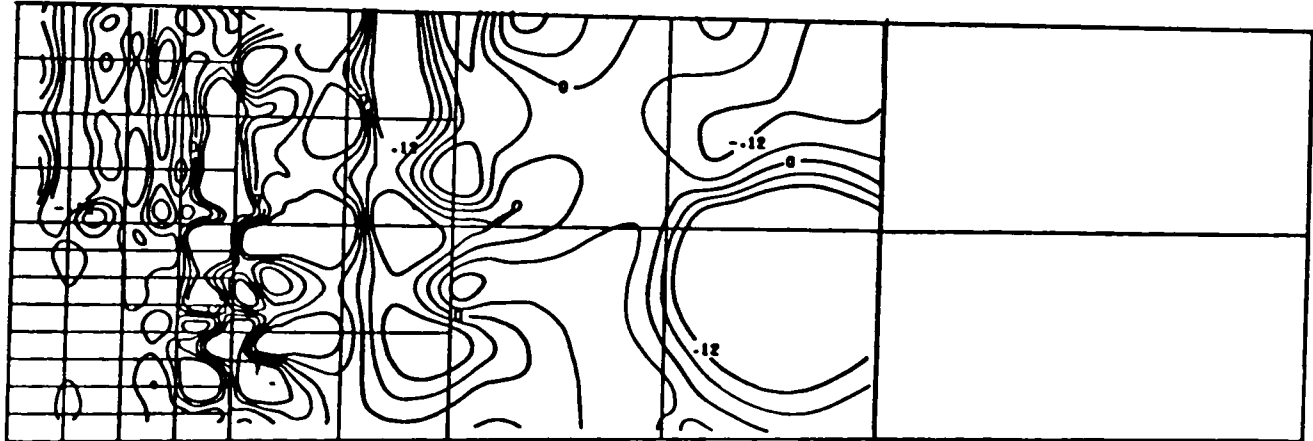


Figure 5.5 - Deformed Configuration of the Beam with $\alpha = 1.15^\circ$ ($t_w = 10\text{mm}$).



Figures 5.6.a and 5.6.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 1.15^\circ$.

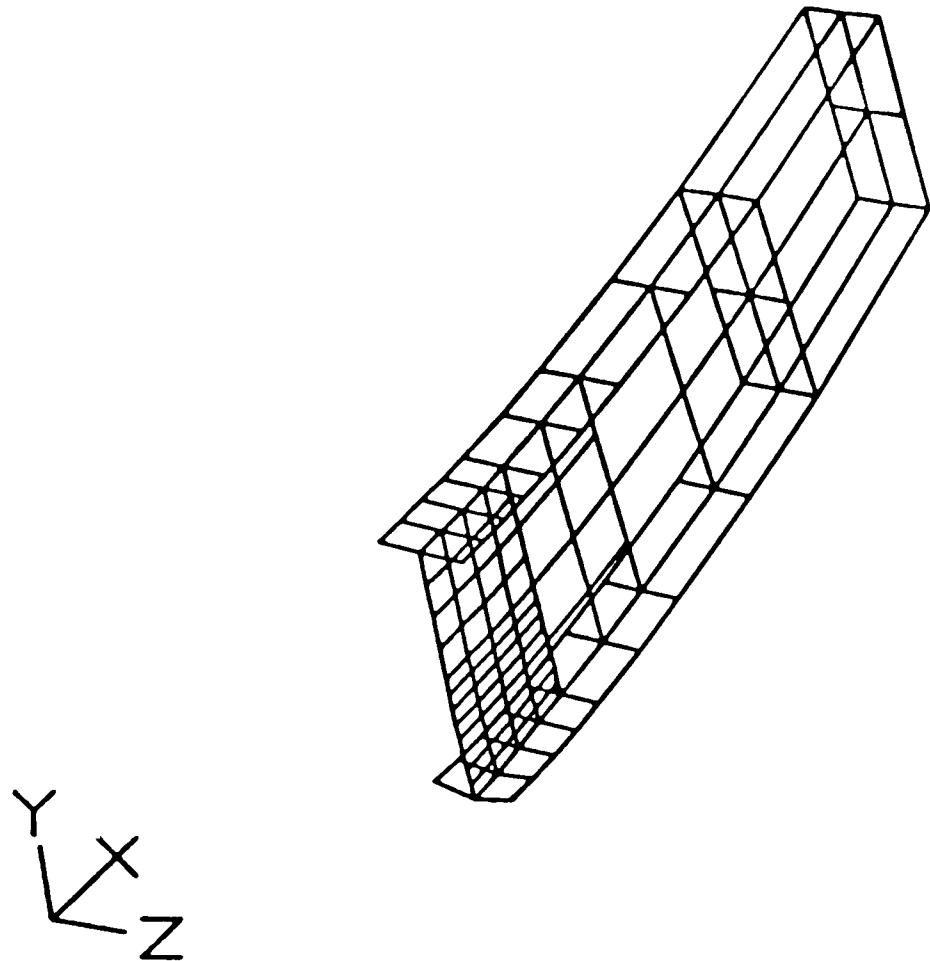
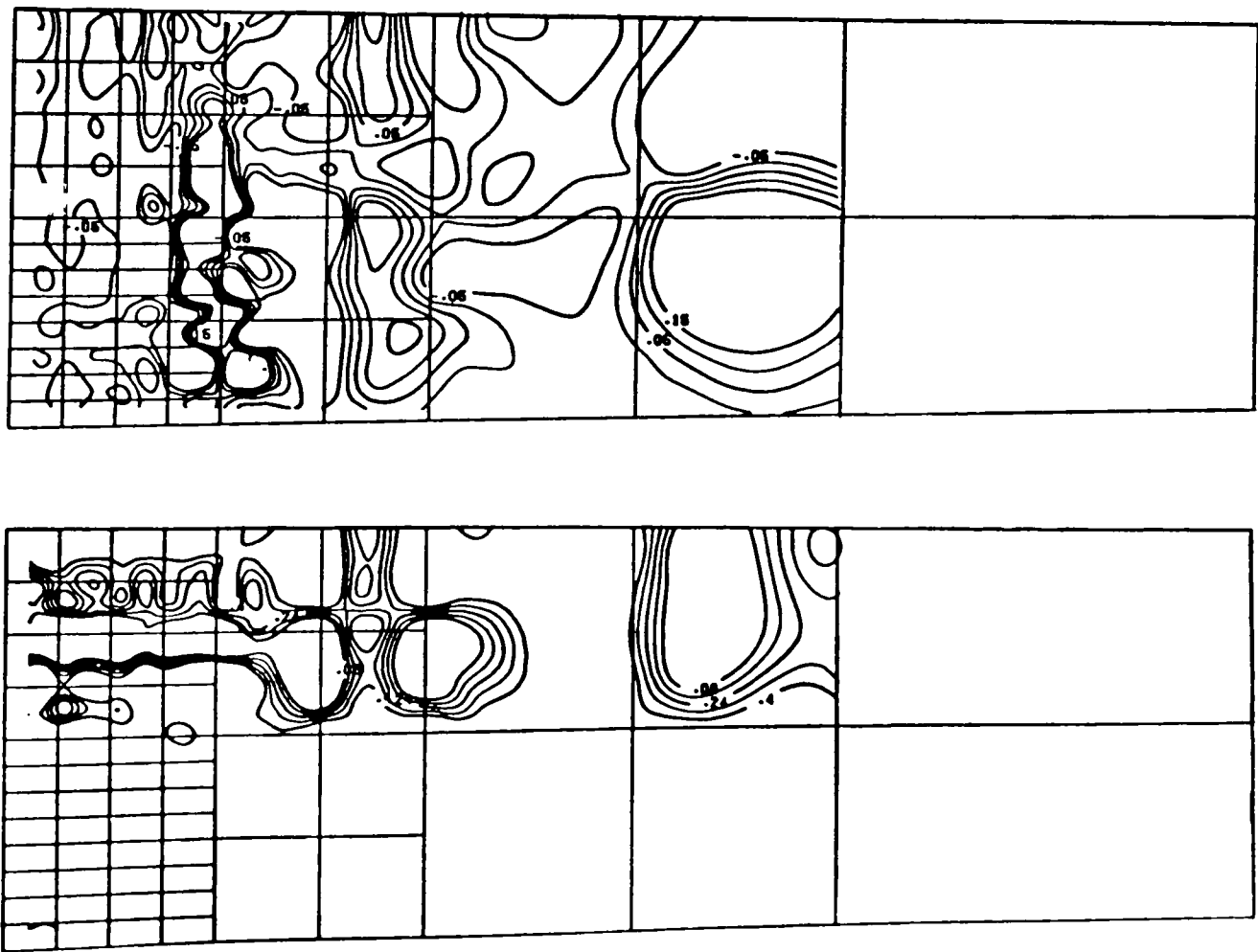


Figure 5.7 - Deformed Configuration of the Beam with $\alpha = 1.43^\circ$ ($t_w = 10\text{mm}$).



Figures 5.8.a and 5.8.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 1.43^\circ$.

Again, only very small changes have developed in the performance of the beam with an angle of slope equal to 1.91° ($\alpha = 1/30$). The deformed shape at the ultimate load is presented in figure 5.9, page 176. The stress contours are delineated in figures 5.10.a and 5.10.b, page 176. For the beam with an angle of 2.86° ($\alpha = 1/20$), the behaviour remained unchanged, as can be seen in figures 5.11, 5.12.a and 5.12.b, page 177.

The girder with an angle of 8.53° , ($\alpha = 1/6.67$) had the failure configuration described by figure 5.13, page 178. There is a significant change in the position of the plastic hinge in relation to the previous cases. At the centre of the beam the bottom flange has bent transversely. At the same point of the web, near the bottom flange, higher stresses have appeared, as shown in figures 5.14.a and 5.14.b, page 178.

The beam with an angle of 11.31° ($\alpha = 1/5$) has deformed in the shape delineated by figure 5.15, page 179. Again, the collapse was very similar to the previous case, with just some minor differences. The web's stress configuration at the final loading stage is depicted in figures 5.16.a and 5.16.b, page 179. There was a significant increase on the equivalent (Von Mises) Stress in the bottom part of the web, at the centre of the girder.

Finally, the angle of 14.84° ($\alpha = 1/4$) has produced the collapse form described by figure 5.17, page 180. In this case the plastic hinge has moved once more towards the vertical stiffener. The bottom part of the web, at the centre of the beam, is under even higher stresses, as can be observed in figures 5.18.a and 5.18.b, page 180.

A summary of the ultimate loads achieved by the beams with 10mm webs, in terms of percentage of the central plastic moment capacity, is delineated in table 5.2. As the angle increases, there is a drop in the maximum load. This was due to the type of plastic collapse developed by the beams, with the plastic hinges located close to the vertical stiffener. With an increase in the taper angle, the cross section depth, h , and the plastic section modulus decrease at the critical section, explaining the loss in moment capacity.

Angle	0°	1.14°	1.43°	1.91°	2.86°	8.53°	11.31°	14.84°
Final Load	97.50	96.22	96.04	95.17	93.05	73.99	63.26	55.36

Table 5.2 - Final Loads (% Central M_{pl}) for Beams with $t_w = 10\text{mm}$.

Figures 5.19 and 5.20, page 181, portray an interesting comparison in terms of vertical displacement (the position of the points in the finite element mesh is shown in figure 4.2, page 129). Both points have shown the same tendency to displace, which characterises very well the behaviour of this group of beams. The lateral displacements of web points 9, 25, 33 and 94 are presented in figures 5.21.a, 5.21.b, 5.22.a and 5.22.b, pages 182 and 183. These graphs confirm that the web had very small lateral deflections.

5.3.2. Web Thickness Equal to 7mm.

The beam with parallel flanges ($\alpha = 0^\circ$) failed by the plastic hinge collapse described in the previous section. Its deformed shape at the ultimate loading stage is presented in figure 5.23, page 184. At the centre of the beam the web has

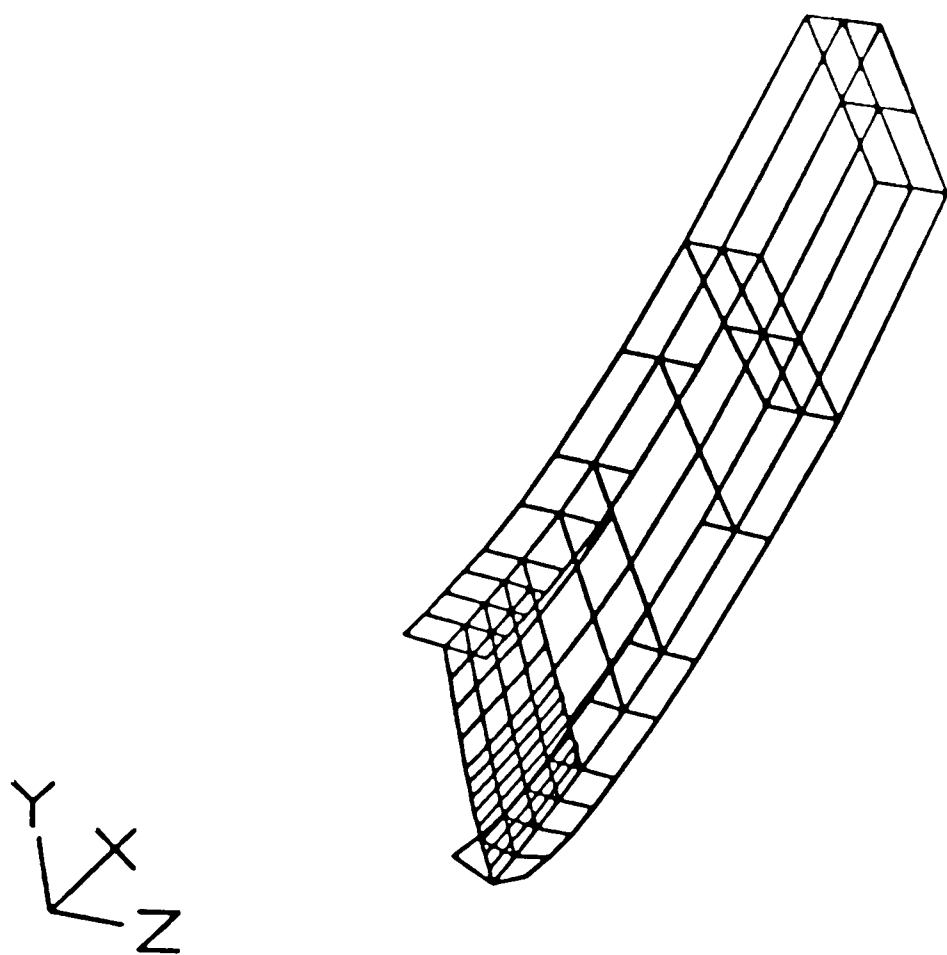
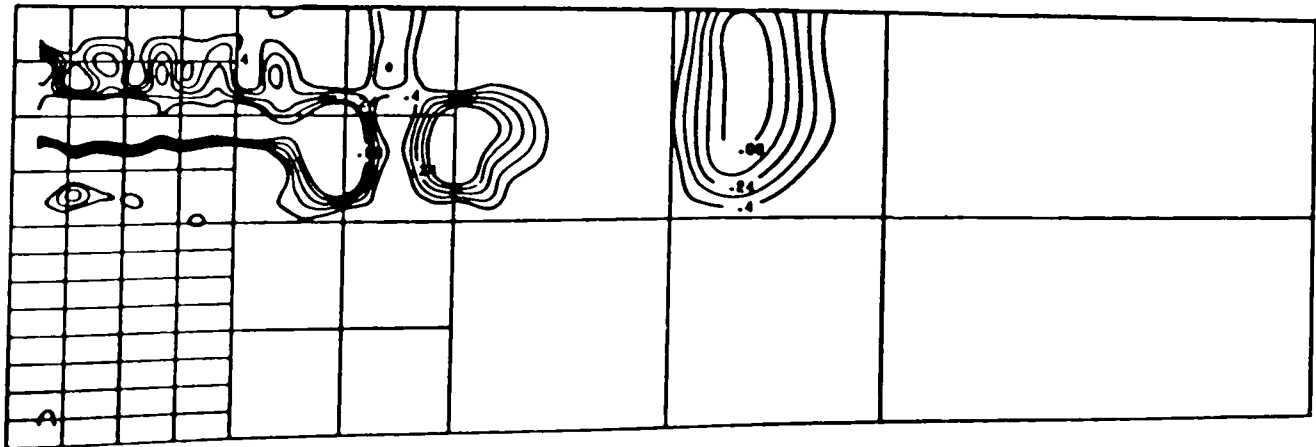
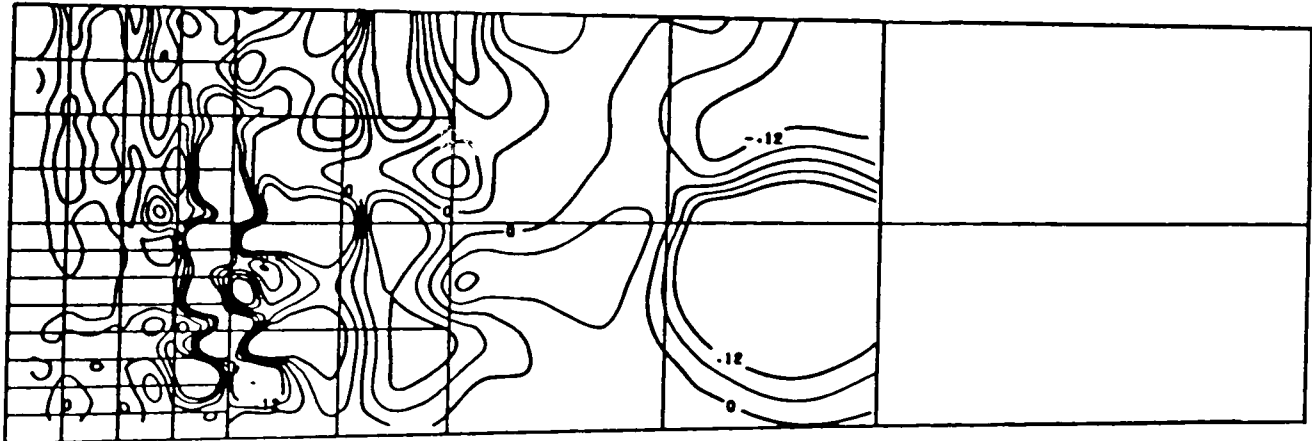


Figure 5.9 - Deformed Configuration of the Beam with $\alpha = 1.91^\circ$ ($t_w = 10\text{mm}$).



Figures 5.10.a and 5.10.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 1.91^\circ$.

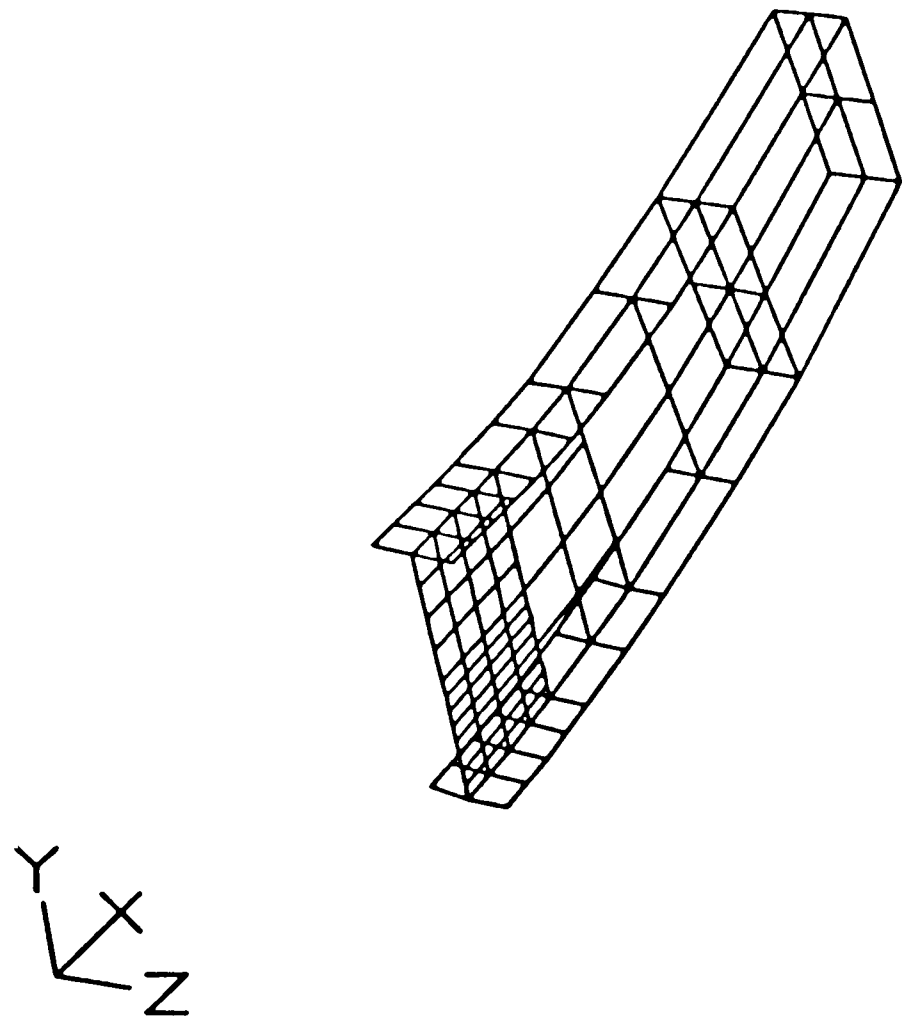
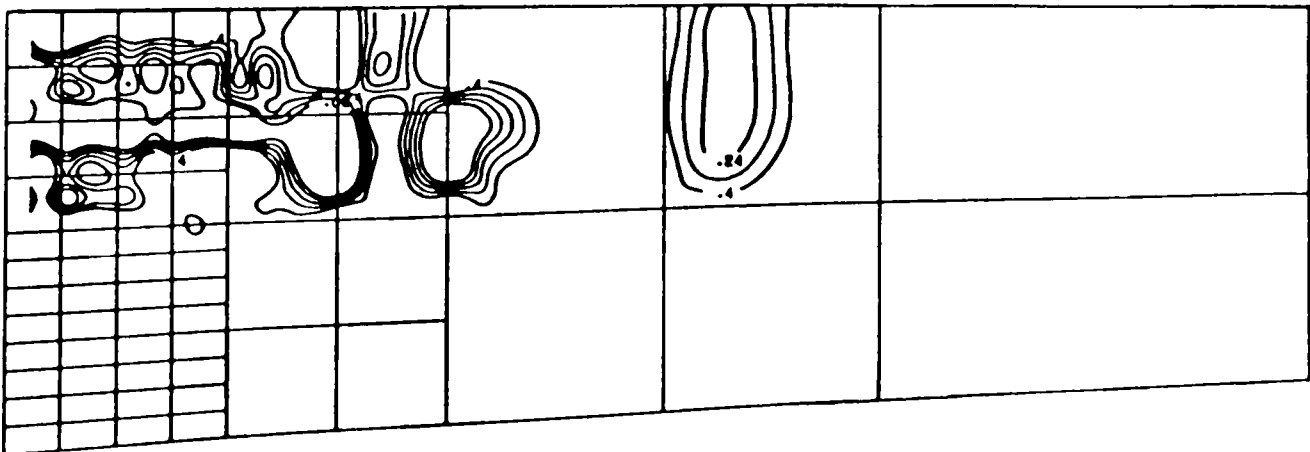
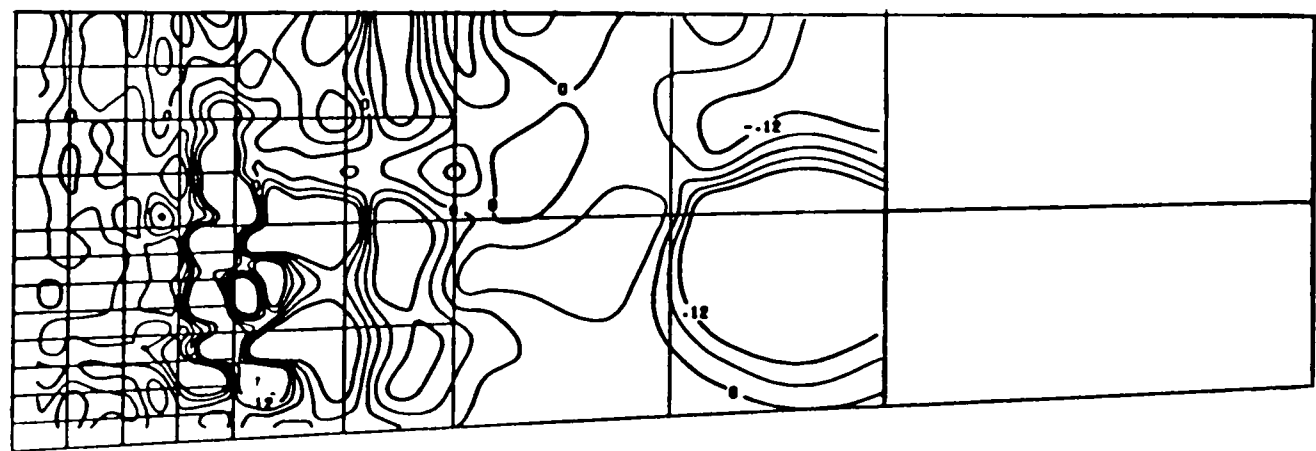


Figure 5.11 - Deformed Configuration of the Beam with $\alpha = 2.86^\circ$ ($t_w = 10\text{mm}$).



Figures 5.12.a and 5.12.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 2.86^\circ$.

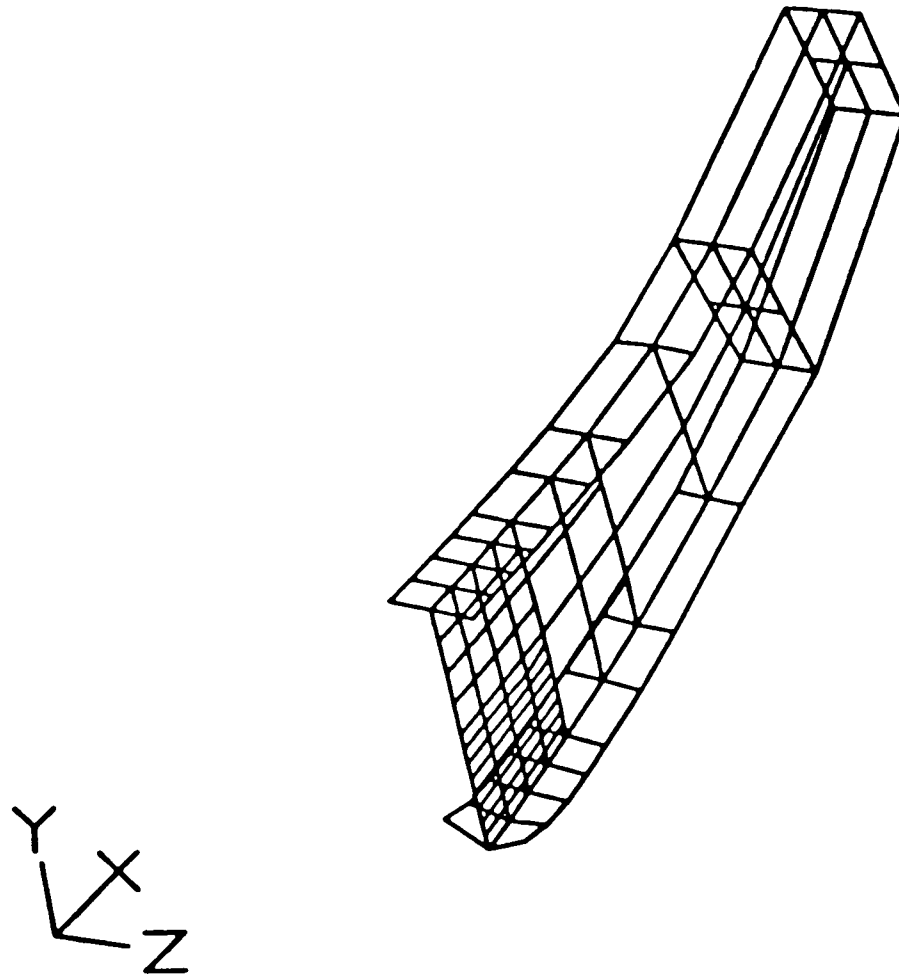
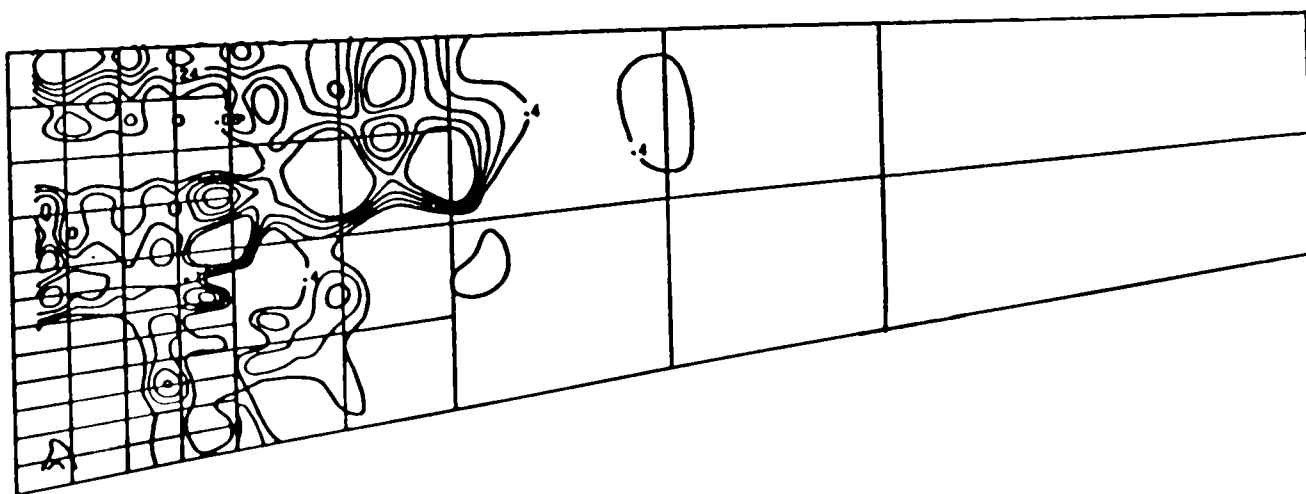
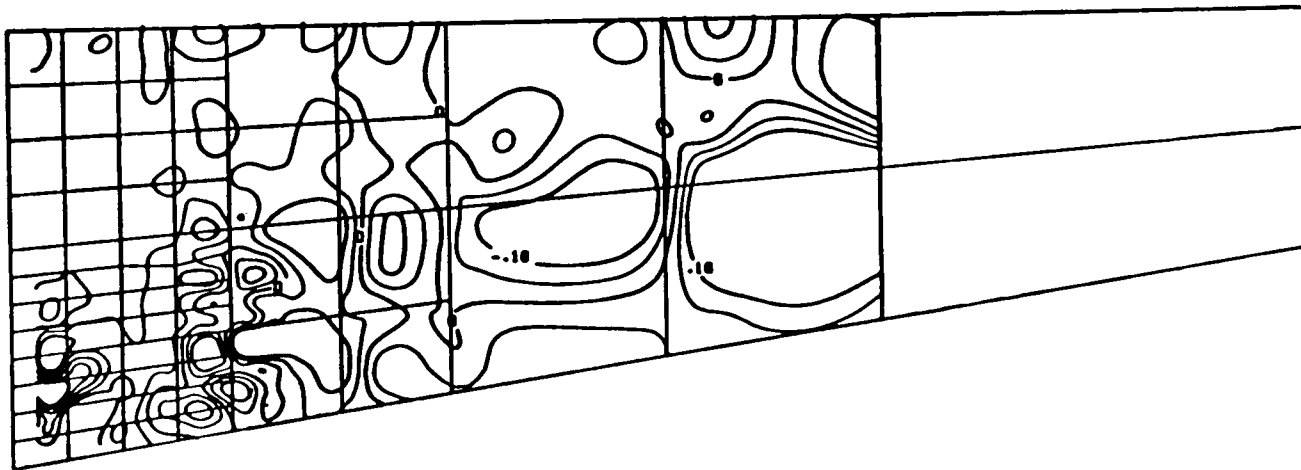


Figure 5.13 - Deformed Configuration of the Beam with $\alpha = 8.53^\circ$. ($t_w = 10\text{mm}$)



Figures 5.14.a and 5.14.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 8.53^\circ$.

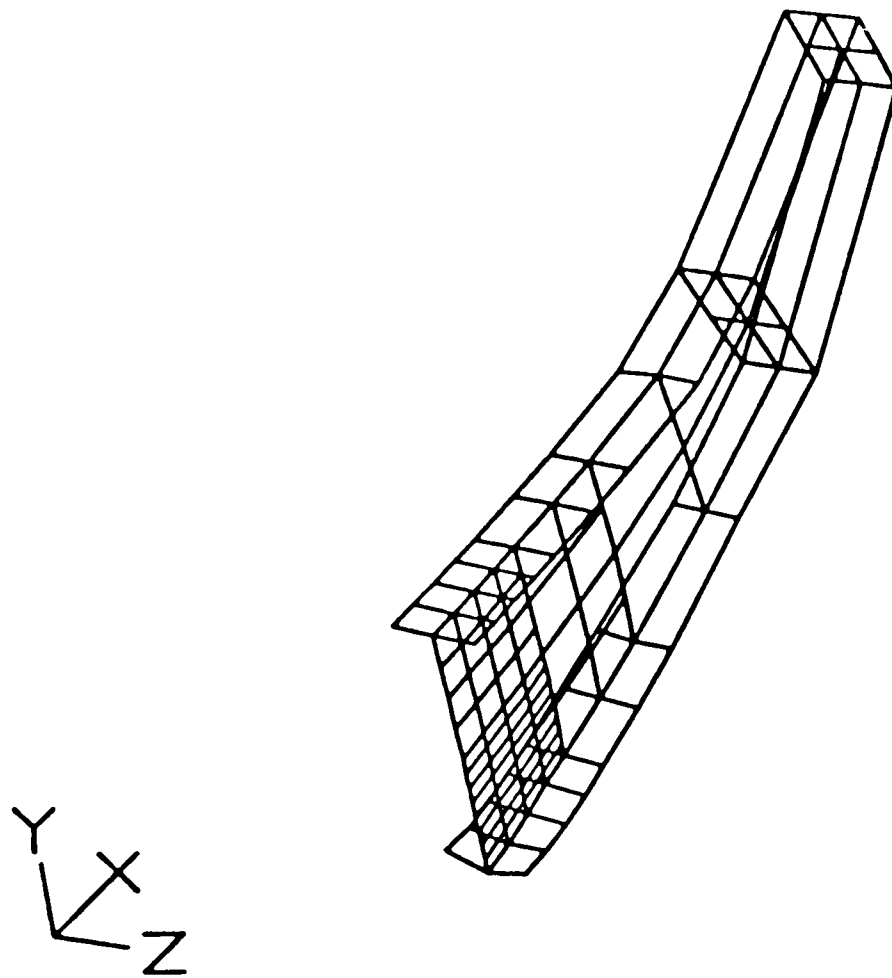
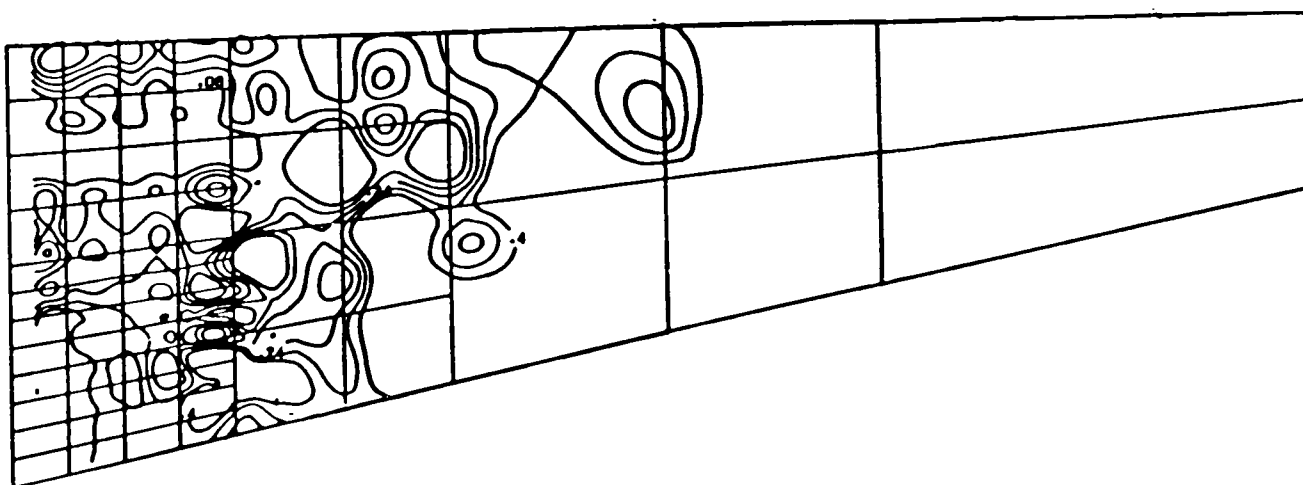
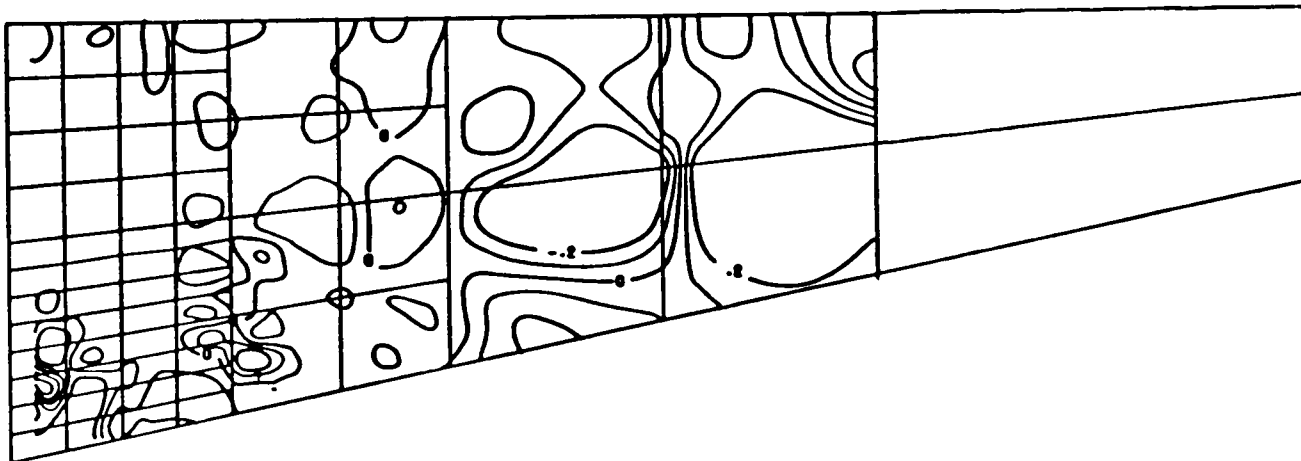


Figure 5.15 - Deformed Configuration of the Beam with $\alpha = 11.31^\circ$ ($t_w = 10\text{mm}$).



Figures 5.16.a and 5.16.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 11.31^\circ$.

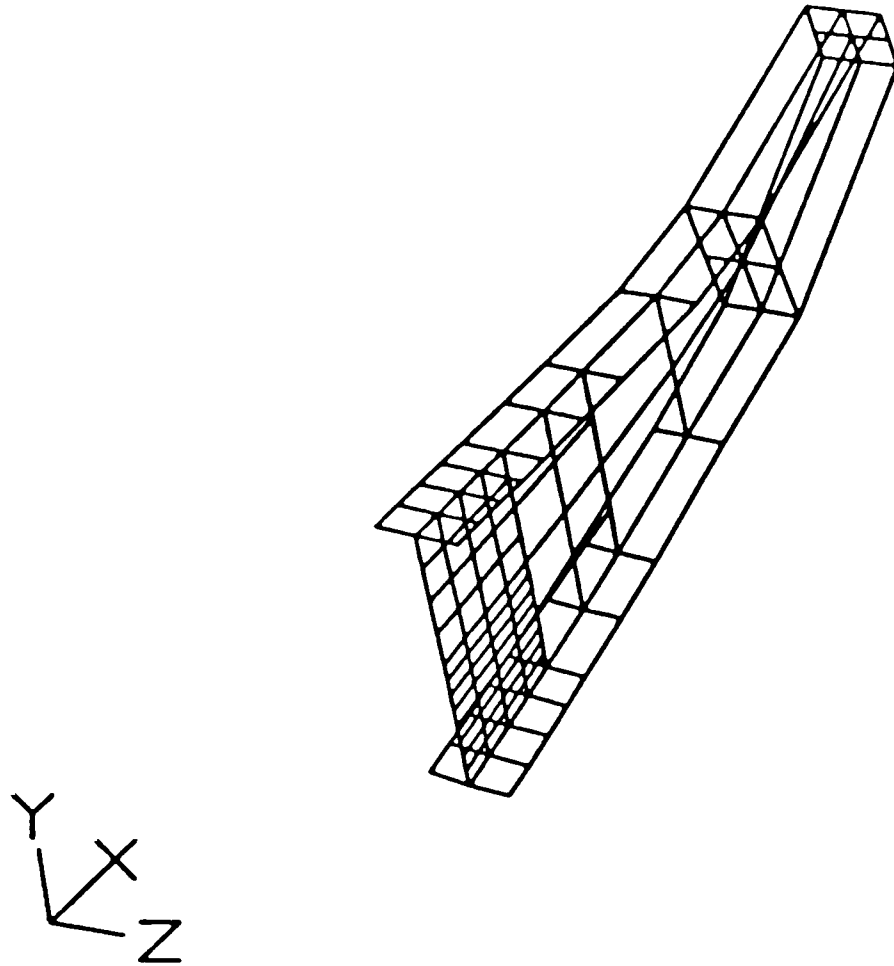
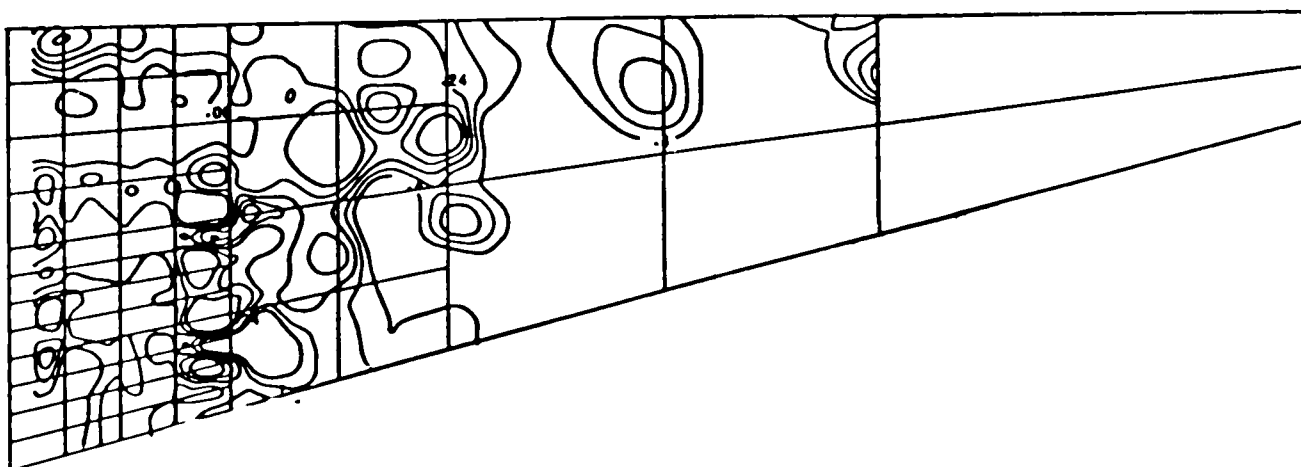
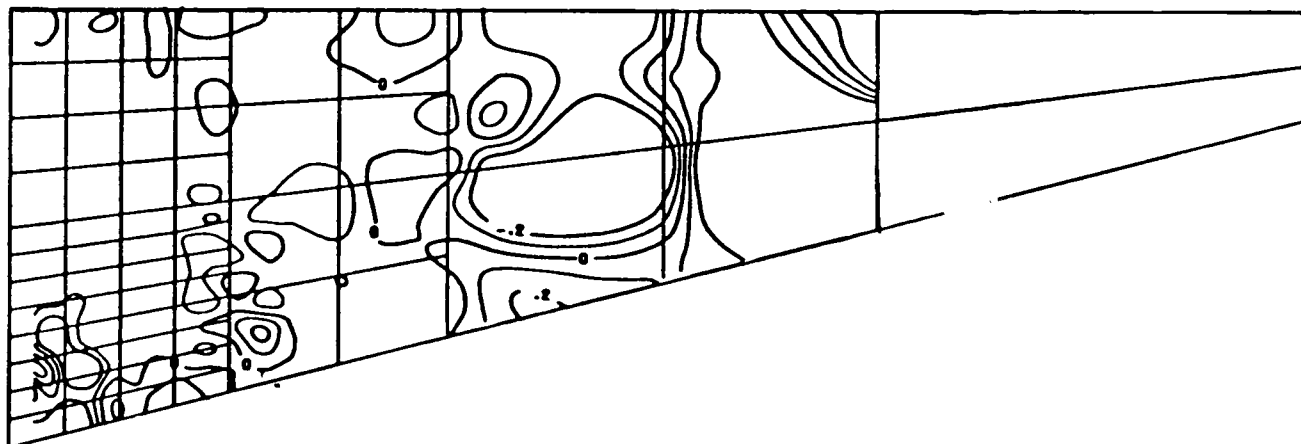


Figure 5.17 - Deformed Configuration of the Beam with $\alpha = 14.84^\circ$ ($t_w = 10\text{mm}$).



Figures 5.18.a and 5.18.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 14.84^\circ$.

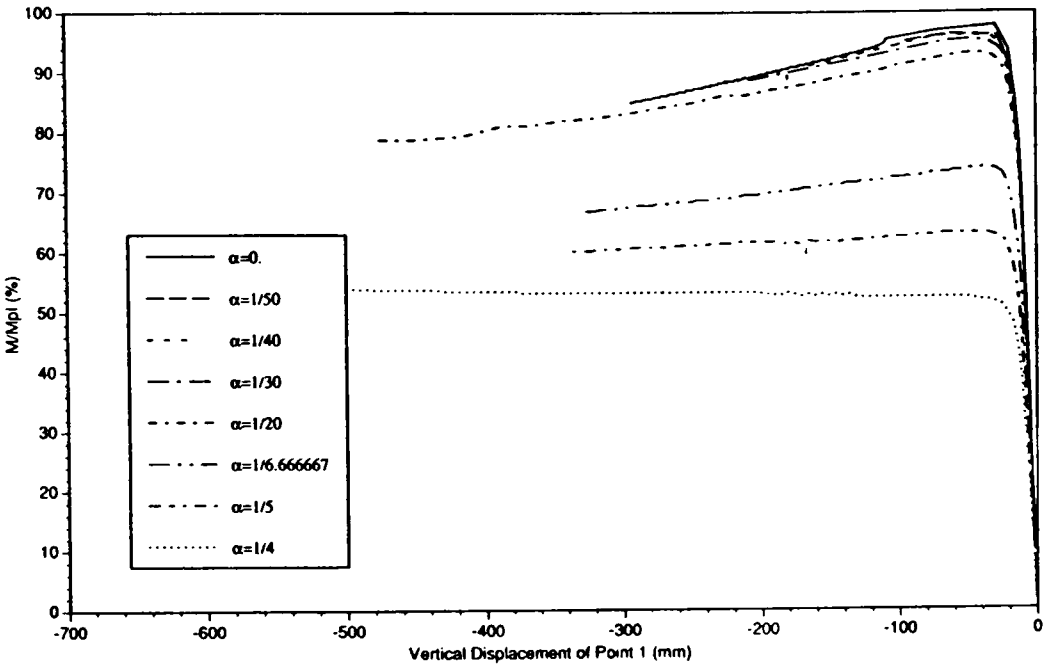


Figure 5.19 - Vertical Displacement Comparison for Point 1 ($t_w = 10\text{mm}$).

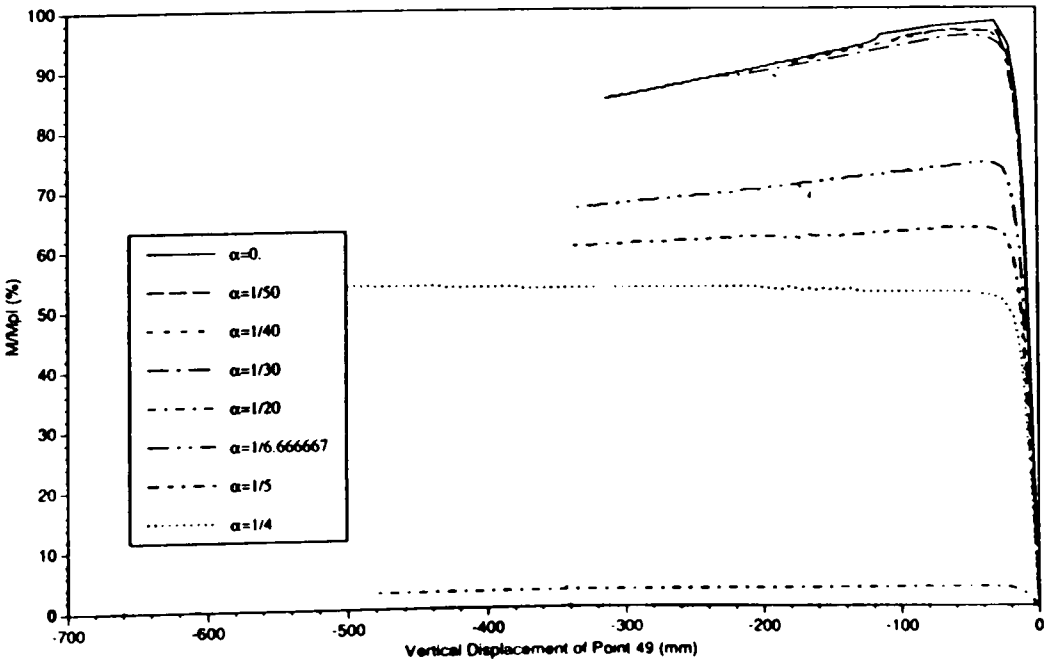
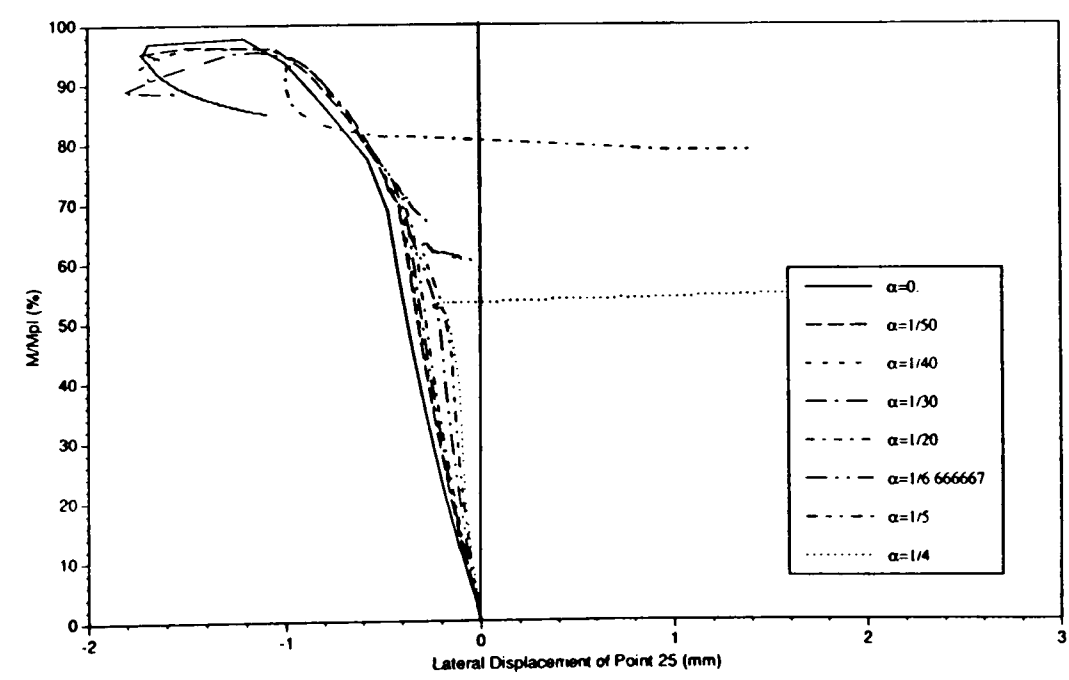
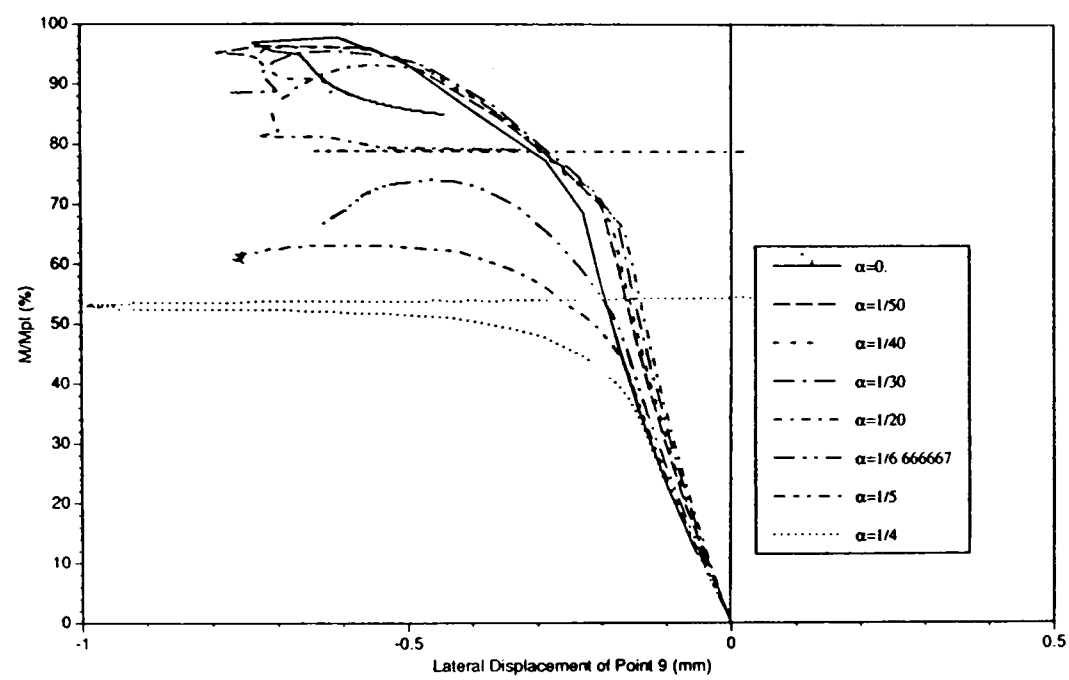
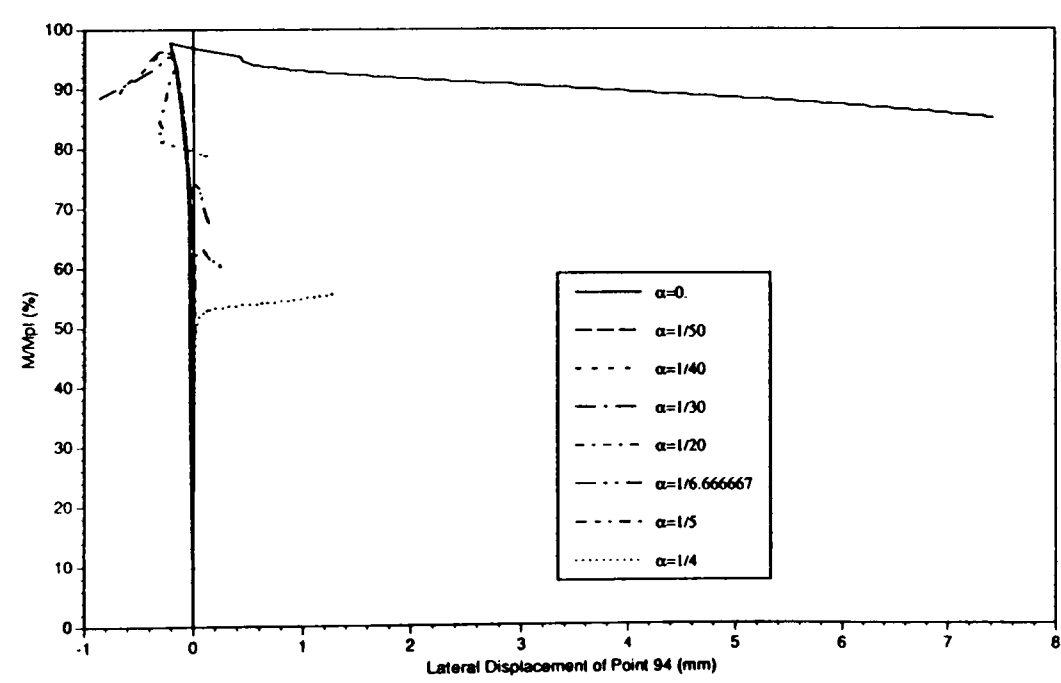
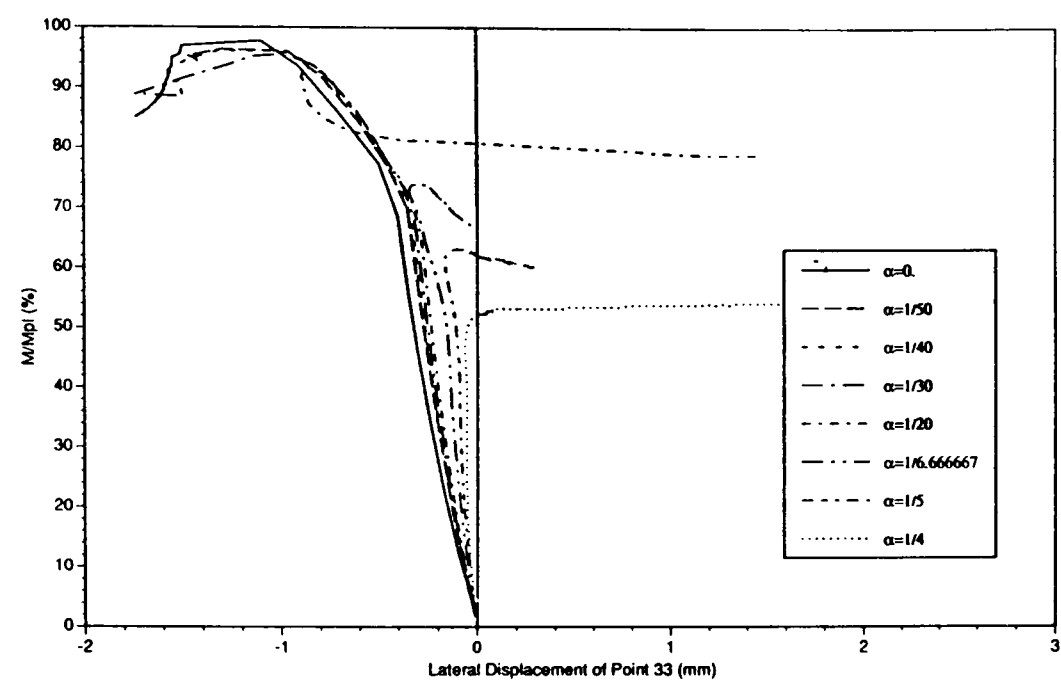


Figure 5.20 - Vertical Displacement Comparison for Point 49 ($t_w = 10\text{mm}$).



Figures 5.21.a and 5.21.b - Lateral Displacement Comparison for Points 9 and 25 ($t_w = 10\text{mm}$).



Figures 5.22.a and 5.22.b - Lateral Displacement Comparison for Points 33 and 94 ($t_w = 10\text{mm}$).

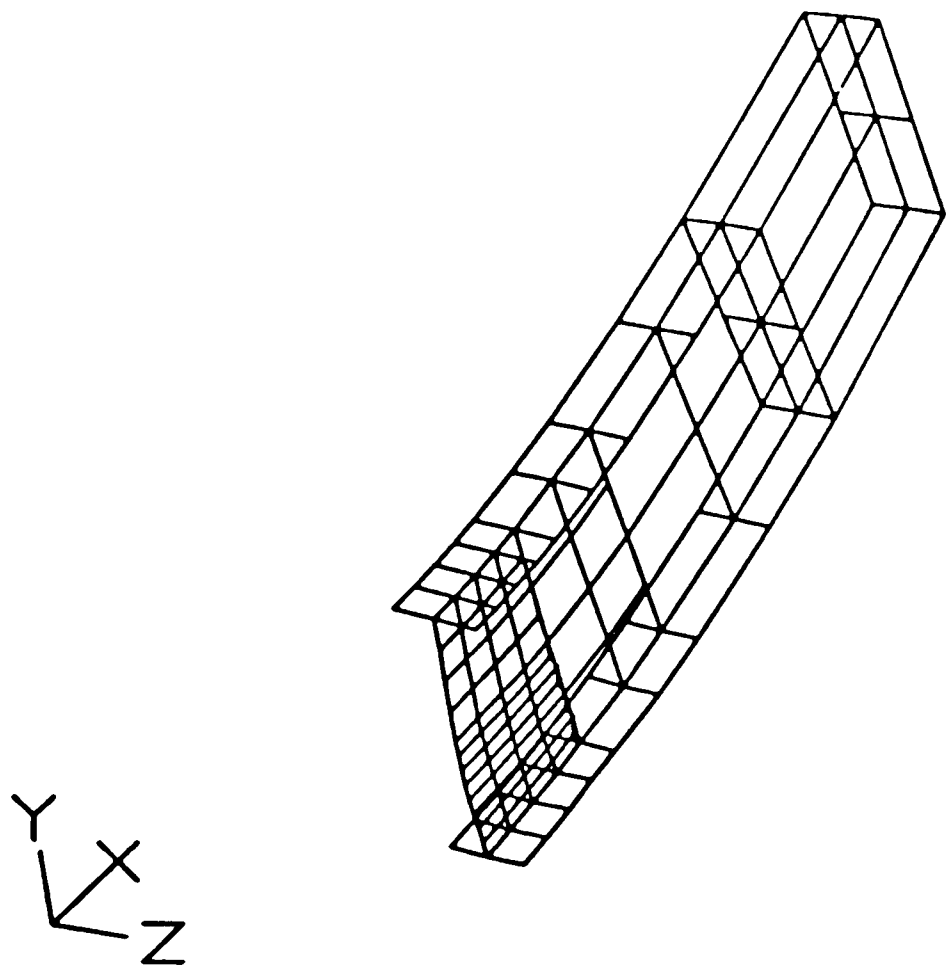
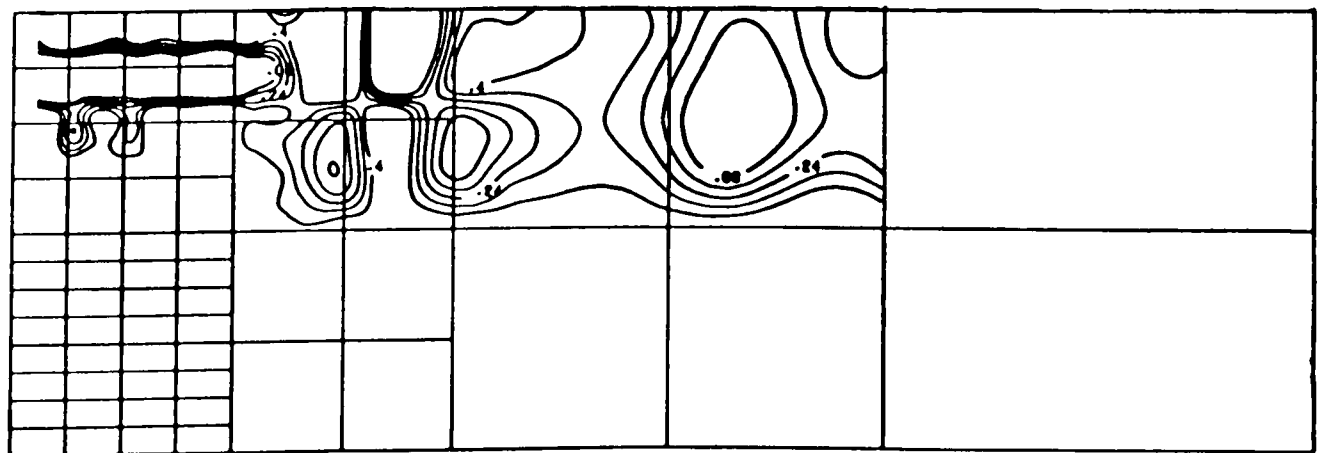
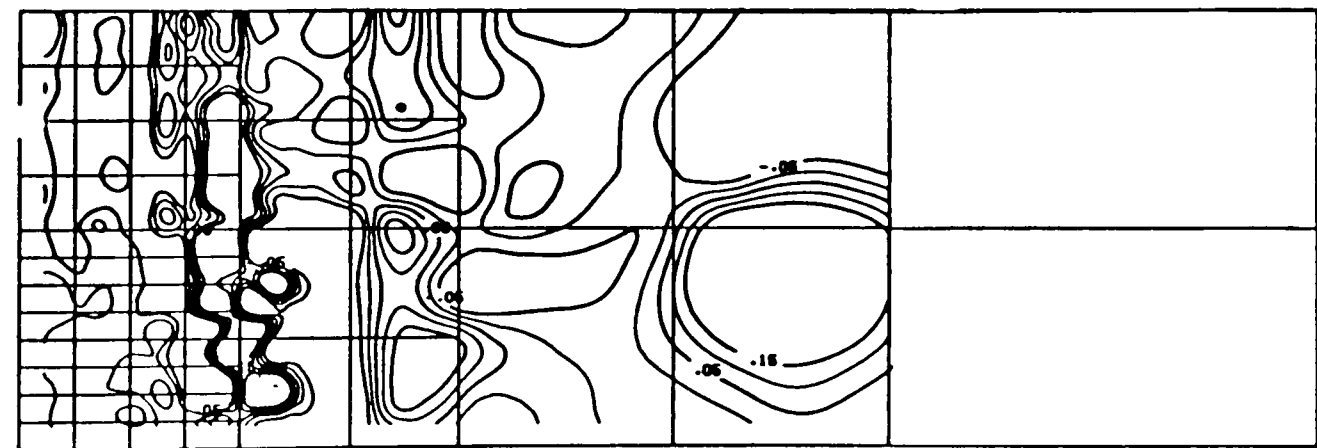


Figure 5.23 - Deformed Configuration of the Beam with $\alpha = 0^\circ$ ($t_w = 7\text{mm}$).



Figures 5.24.a and 5.24.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 0^\circ$.

deformed laterally. These deflections were associated with the rotation of the top flange. The vertical and equivalent (Von Mises) web stress patterns are shown in figures 5.24.a and 5.24.b, page 184.

The same type of failure occurred for the beams with slopes of 1.15° and 1.43° ($\alpha = 1/50, \alpha = 1/40$). The failure mode and the web stress distribution for the beam with an angle of 1.15° are indicated in figures 5.25, 5.26.a and 5.26.b, page 186. The angle of 1.43° resulted in the failure and stress patterns described in figures 5.27, 5.28.a and 5.28.b, page 187. In these two cases it is possible to note that at the centre of the beam, near the bottom flange, compression stresses have appeared in the equivalent (Von Mises) stress diagrams.

The angles of 1.91° and 2.86° ($\alpha = 1/30, \alpha = 1/20$) were associated with the same collapse pattern. The deformed configuration and the web stress distribution for the angle of 1.91° are depicted in figures 5.29, 5.30.a and 5.30.b, page 188. For the angle equal to 2.86°, the same types of diagrams are presented in figures 5.31, 5.32.a and 5.32.b, page 189. The contribution of the vertical compressive component, that arises from the bottom flange, has increased substantially but not by the amount necessary to make the web buckle.

The girder with an angle of 8.53° ($\alpha = 1/6.67$) had the failure configuration shown by figure 5.33, page 190. The plastic hinge position has moved towards the vertical stiffener. At the centre of the web, near the bottom flange, higher stresses have appeared, as displayed in figures 5.34.a and 5.34.b, page 190. The web has still remained undeformed for the angle of 11.31° ($\alpha = 1/5$). The ultimate deformed shape and the stress pattern for the ultimate loading stage of that beam are shown in figures 5.35, 5.36.a and 5.36.b, page 191.

A very slightly deformed configuration of the web is presented in figure 5.37, page 192. It represents the taper slope angle of 14.84° ($\alpha = 1/4$) and can be viewed as one of the buckling boundaries for the problem. The web stress configuration is depicted in figures 5.38.a and 5.38.b, page 192. In these figures it is very clear that the bottom part of the web, at the centre of the beam, was subjected to very high compressive vertical stresses. They were the main causes of the failure. The collapse happened by a combination of web buckling and the development of a plastic hinge near the vertical stiffener.

A summary of the final loads achieved by the beams with a 7mm web is displayed in table 5.3 below. The ultimate load drops as the taper angle increases. With the exception of the largest angle of 14.84°, the beams have failed by the plastic hinge mechanism described in the previous section. The last beam ($\alpha = 1/4$) has failed by a combination of plasticity and buckling of the web.

Angle	0°	1.15°	1.43°	1.91°	2.86°	8.53°	11.31°	14.84°
Final Load	97.14	96.25	95.56	94.54	92.20	72.64	61.93	51.68

Table 5.3 - Final Loads (% Central M_{pl}) for Beams with $t_w = 7\text{mm}$.

A vertical displacement comparison for points 1 and 49 of all the beams studied in this subsection is portrayed in figure 5.39 and 5.40, page 193. Two different regions can be seen in these figures. The first corresponds to the beams with small angles of taper (up to 2.86°). The other represents the higher slopes in which the failure is associated with the substantial loss of height in the region near the vertical stiffener. A lateral displacement comparison for points 9, 25, 33 and 94, for the same beams, is presented in figures 5.41.a, 5.41.b, 5.42.a and 5.42.b, pages 194

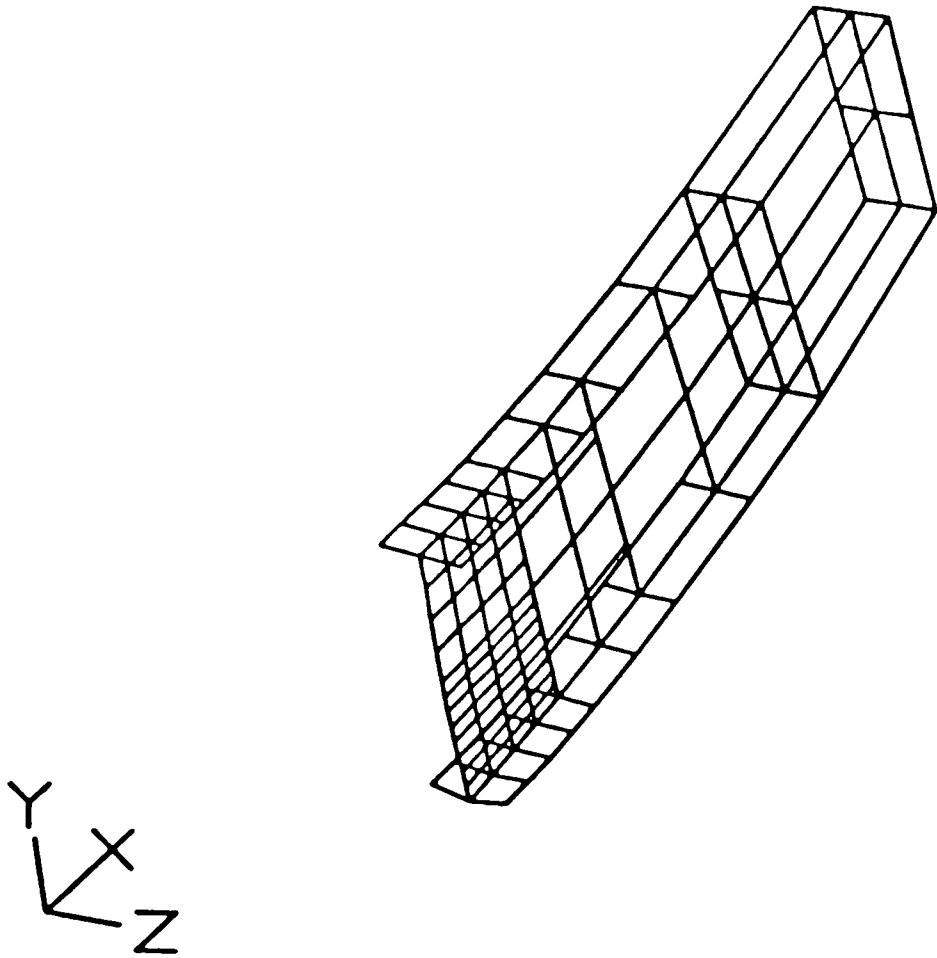
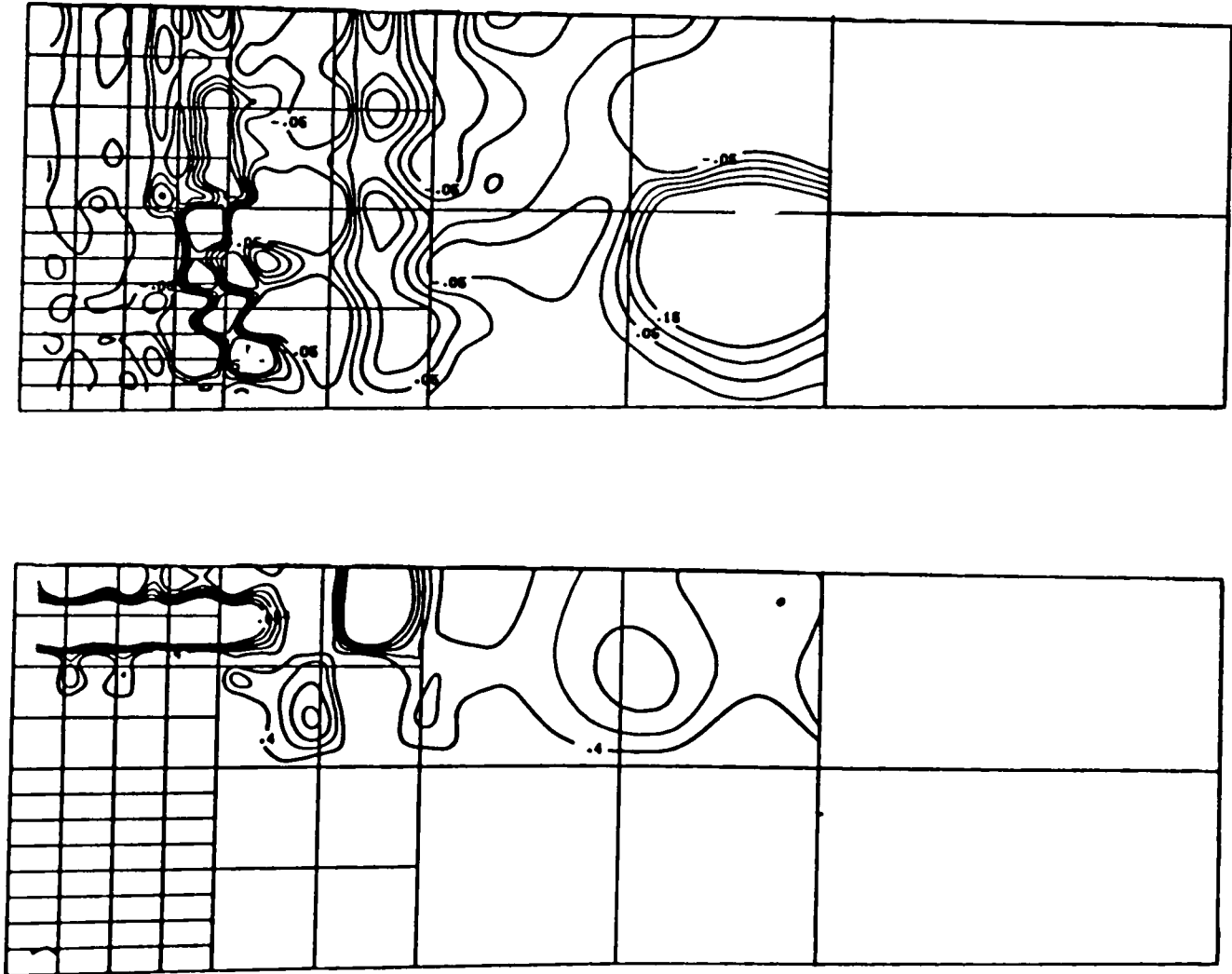


Figure 5.25 - Deformed Configuration of the Beam with $\alpha = 1.15^\circ$ ($t_w = 7\text{mm}$).



Figures 5.26.a and 5.26.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 1.15^\circ$.

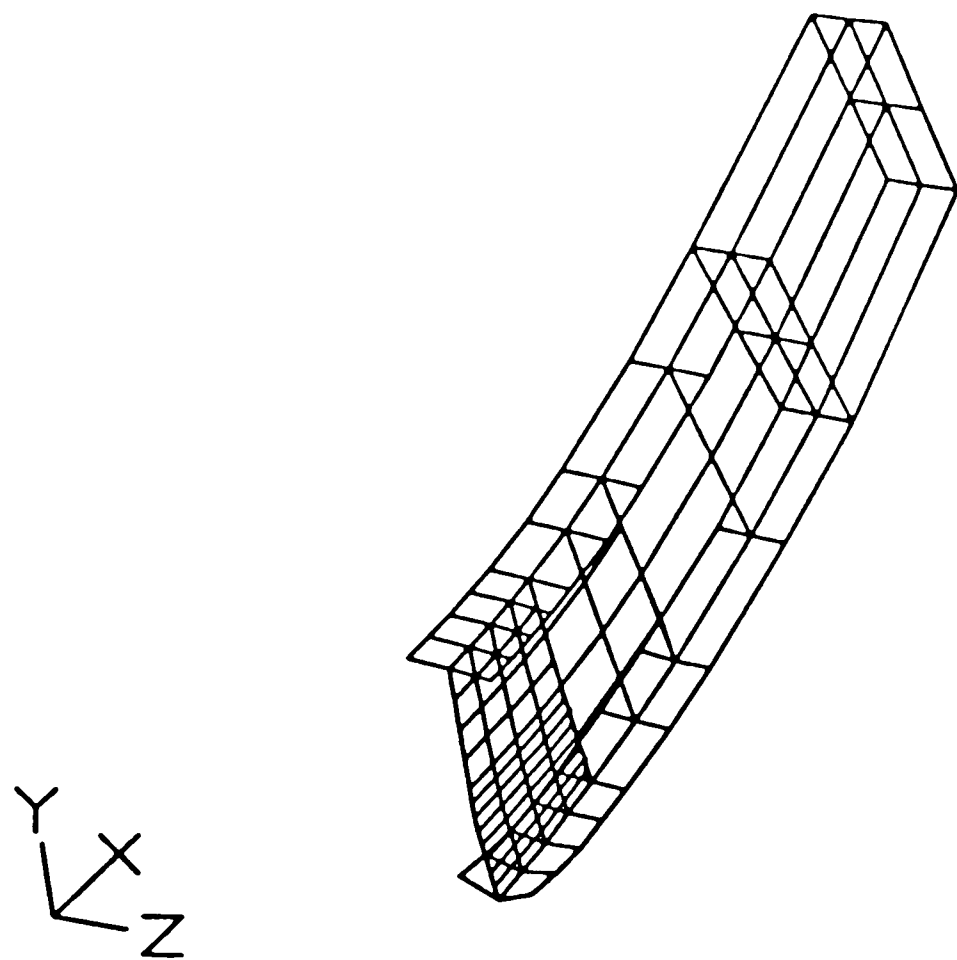


Figure 5.27 - Deformed Configuration of the Beam with $\alpha = 1.43^\circ$ ($t_w = 7\text{mm}$).



Figures 5.28.a and 5.28.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 1.43^\circ$.

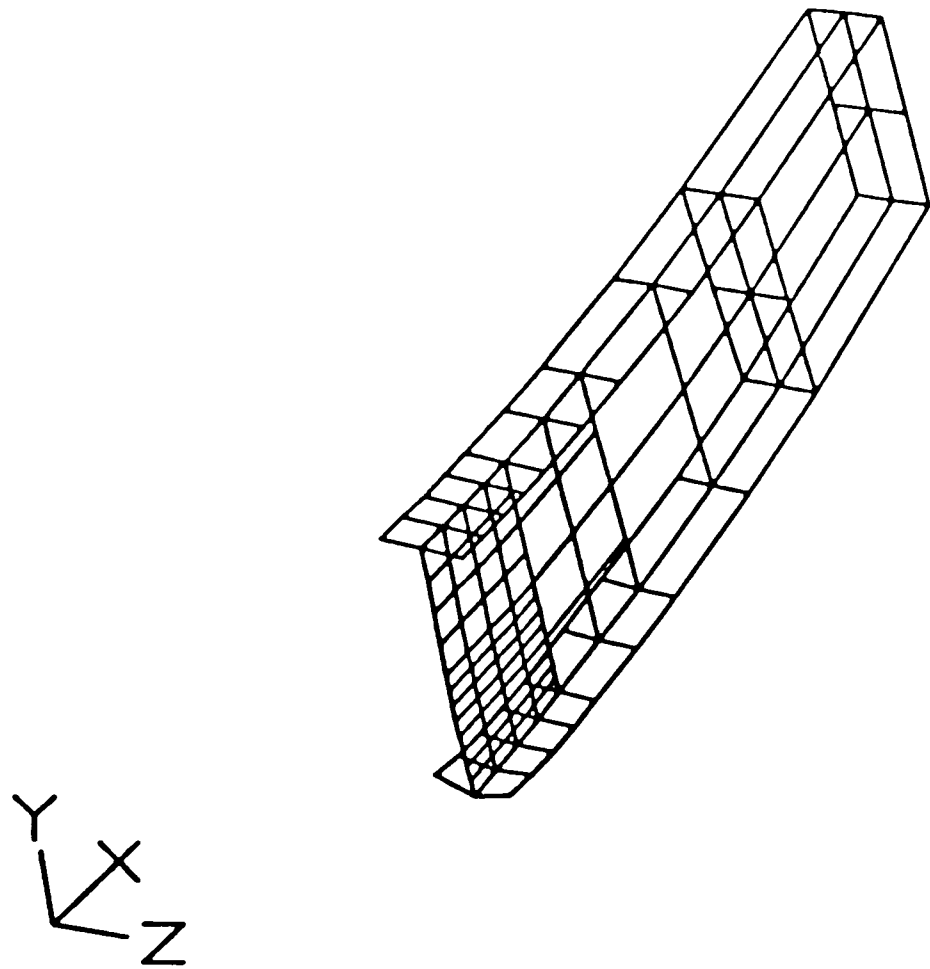
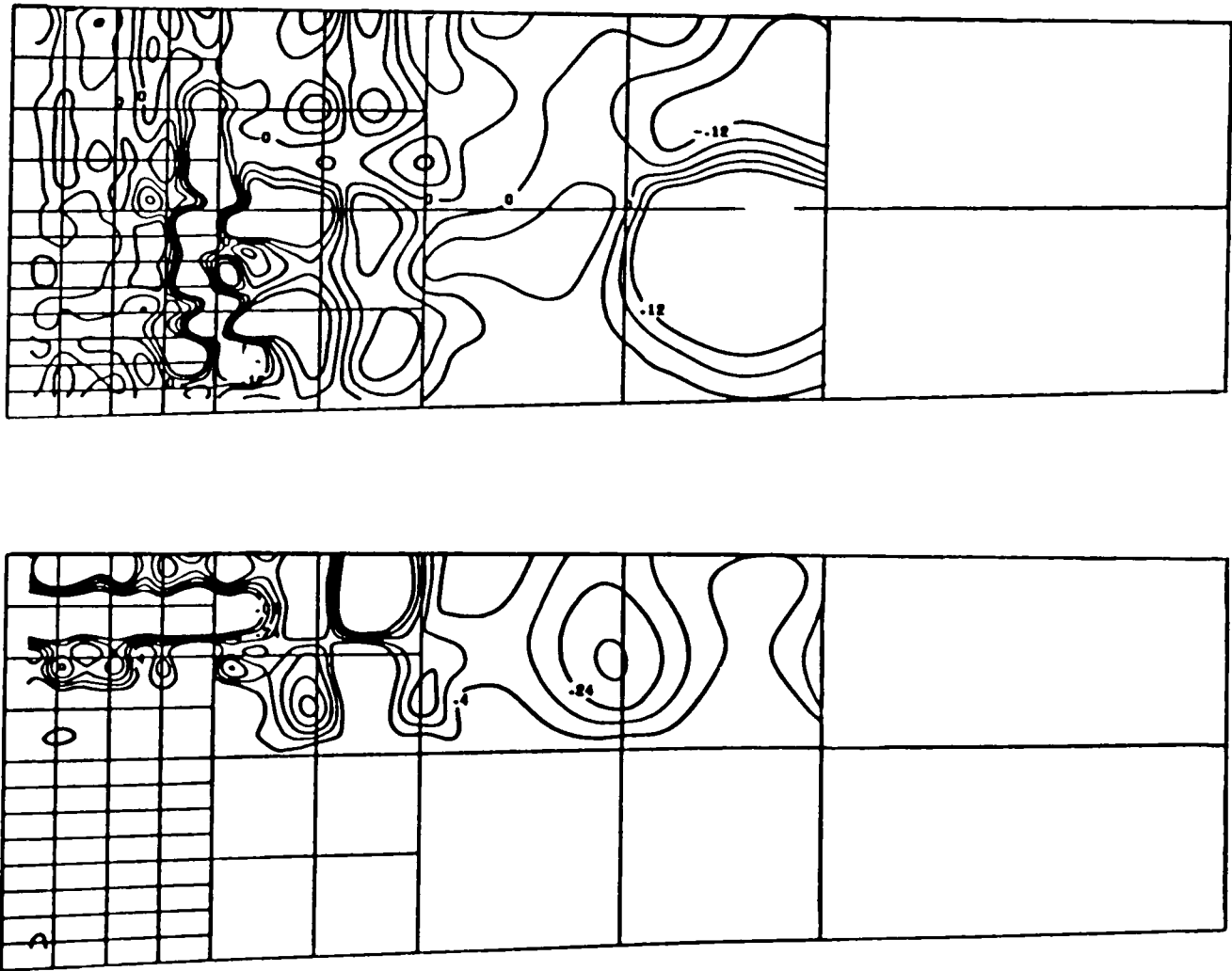


Figure 5.29 - Deformed Configuration of the Beam with $\alpha = 1.91^\circ$ ($t_w = 7\text{mm}$).



Figures 5.30.a and 5.30.b Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 1.91^\circ$.

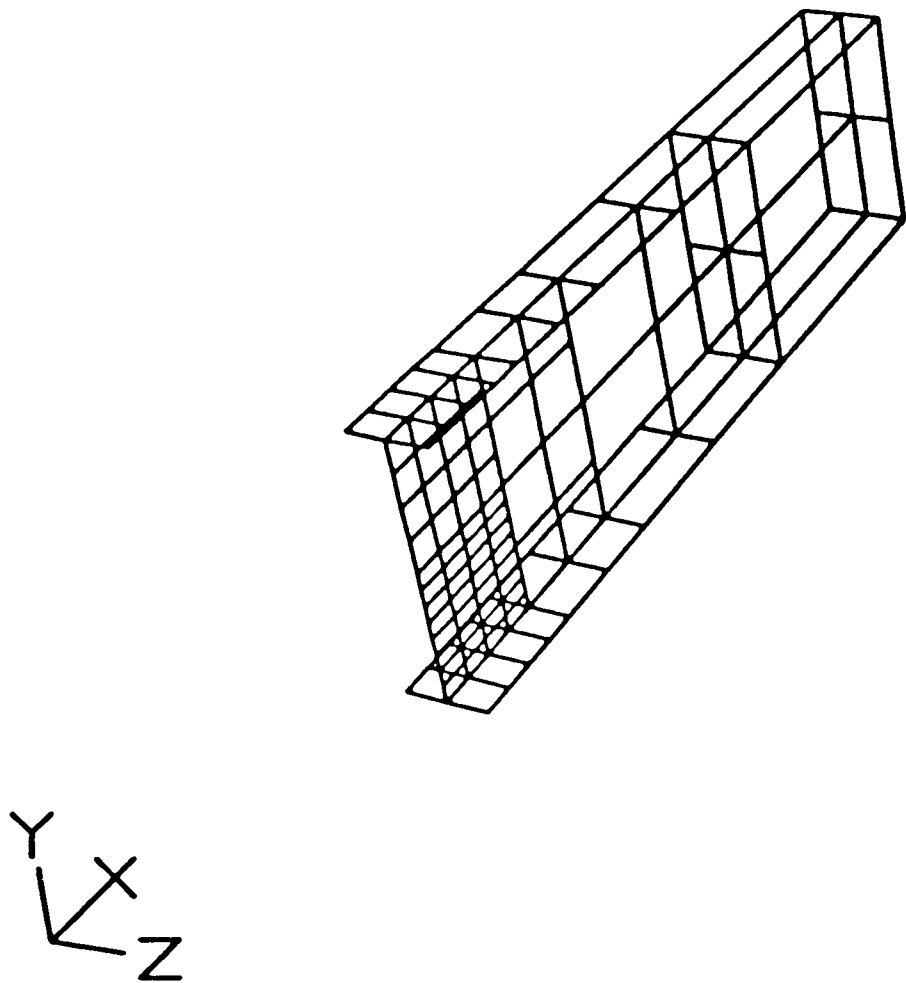
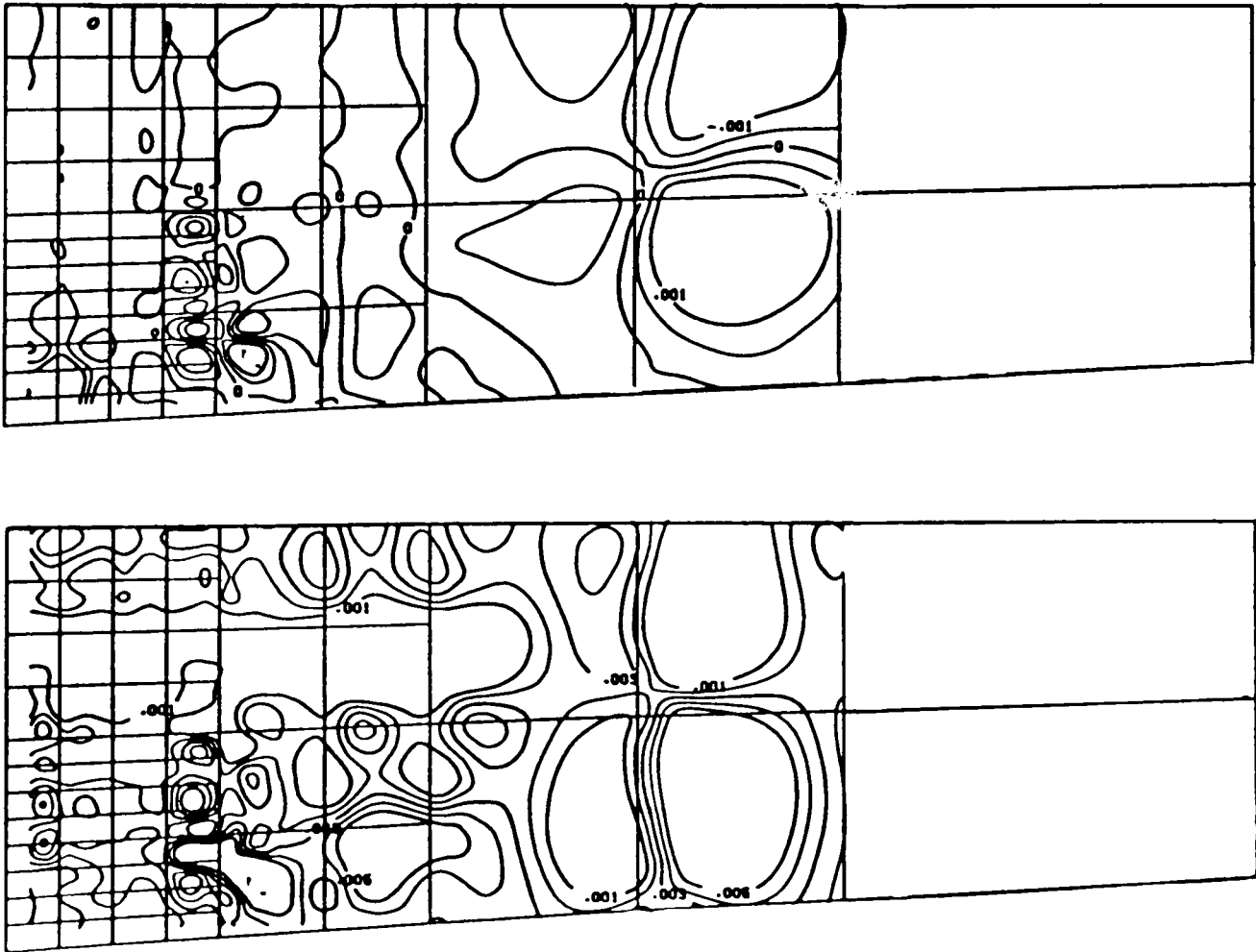


Figure 5.31 - Deformed Configuration of the Beam with $\alpha = 2.86^\circ$ ($t_w = 7\text{mm}$).



Figures 5.32.a and 5.32.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 2.86^\circ$.

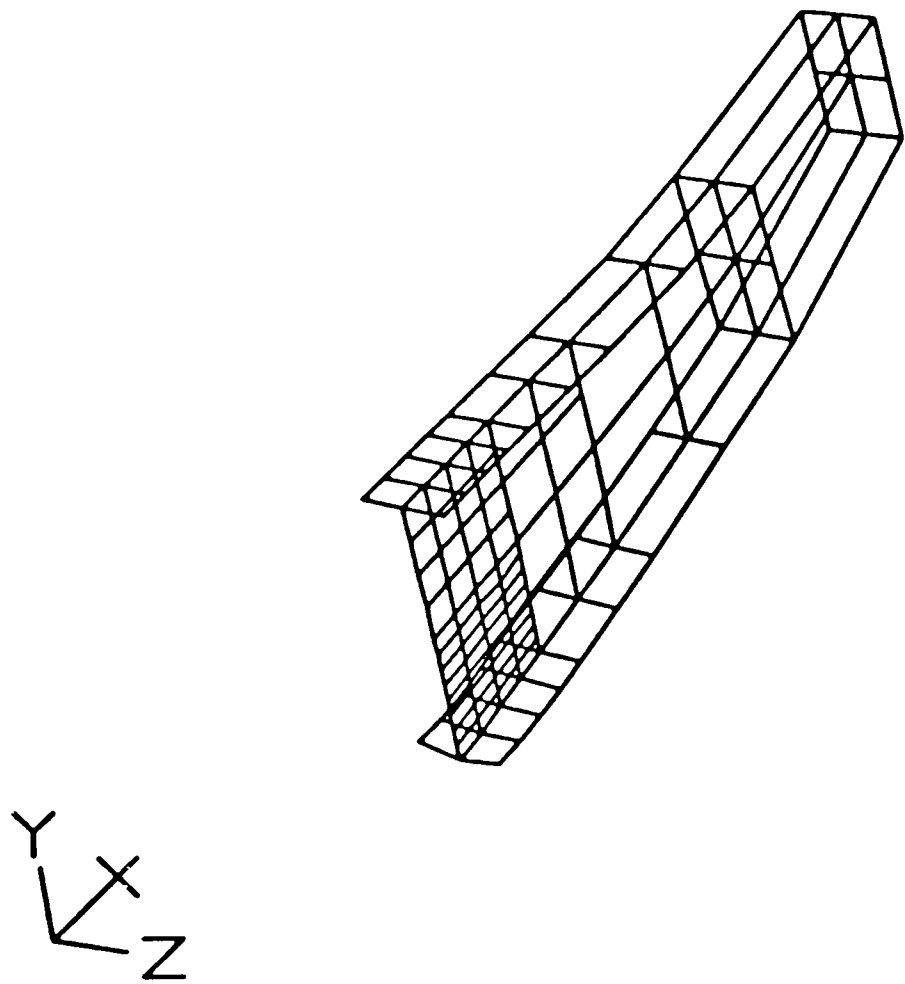
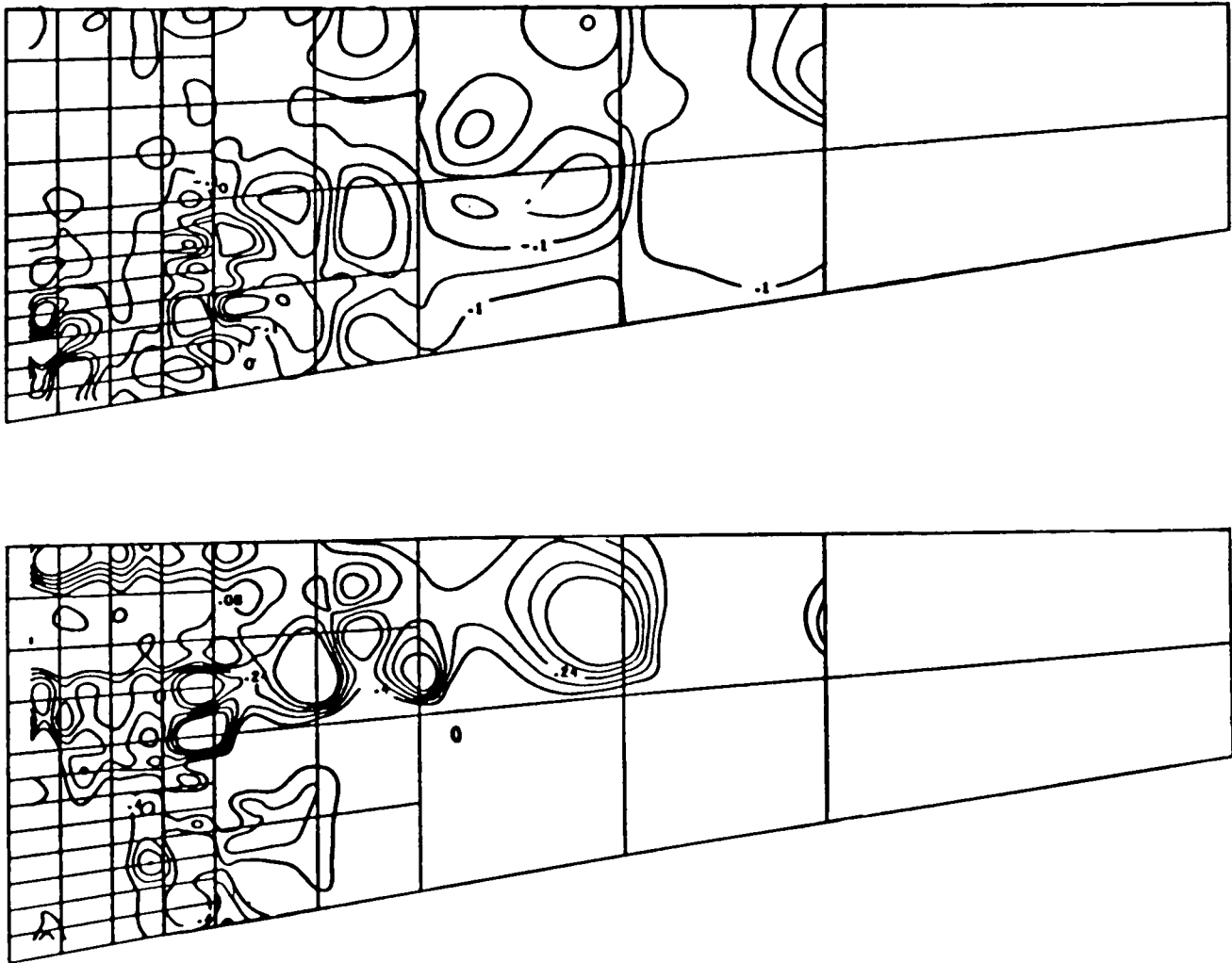


Figure 5.33 - Deformed Configuration of the Beam with $\alpha = 8.53^\circ$ ($t_w = 7\text{mm}$).



Figures 5.34.a and 5.34.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 8.53^\circ$.

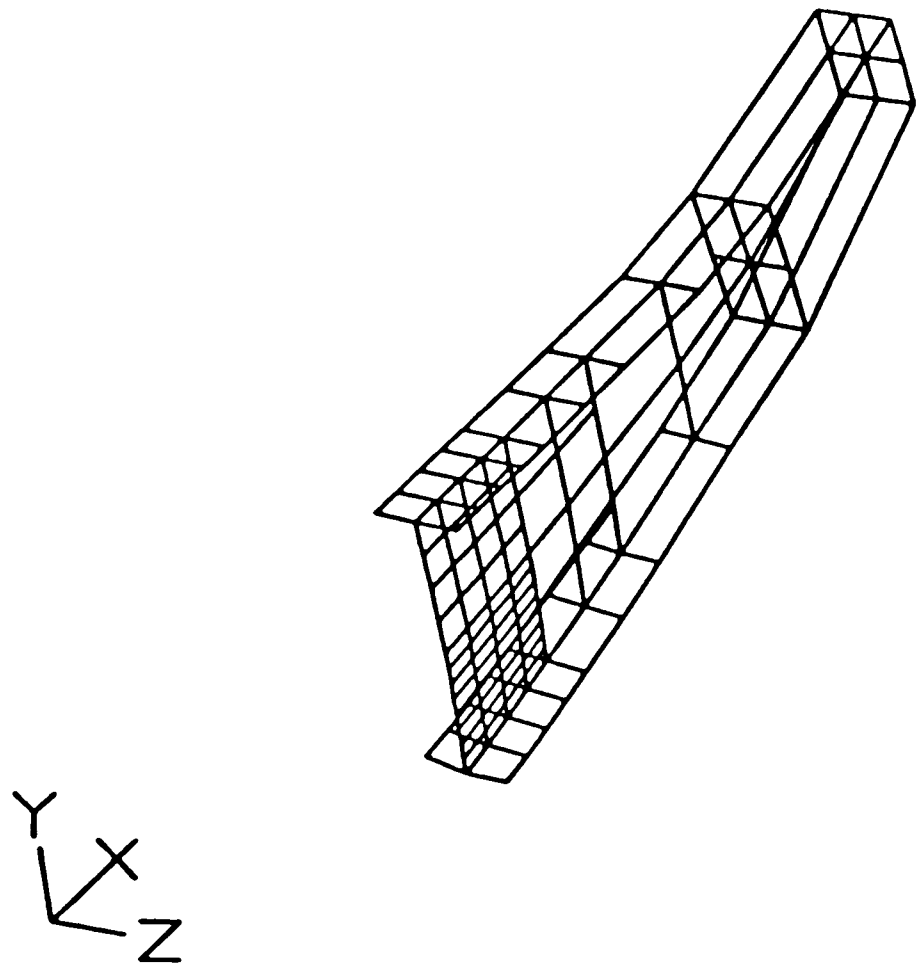
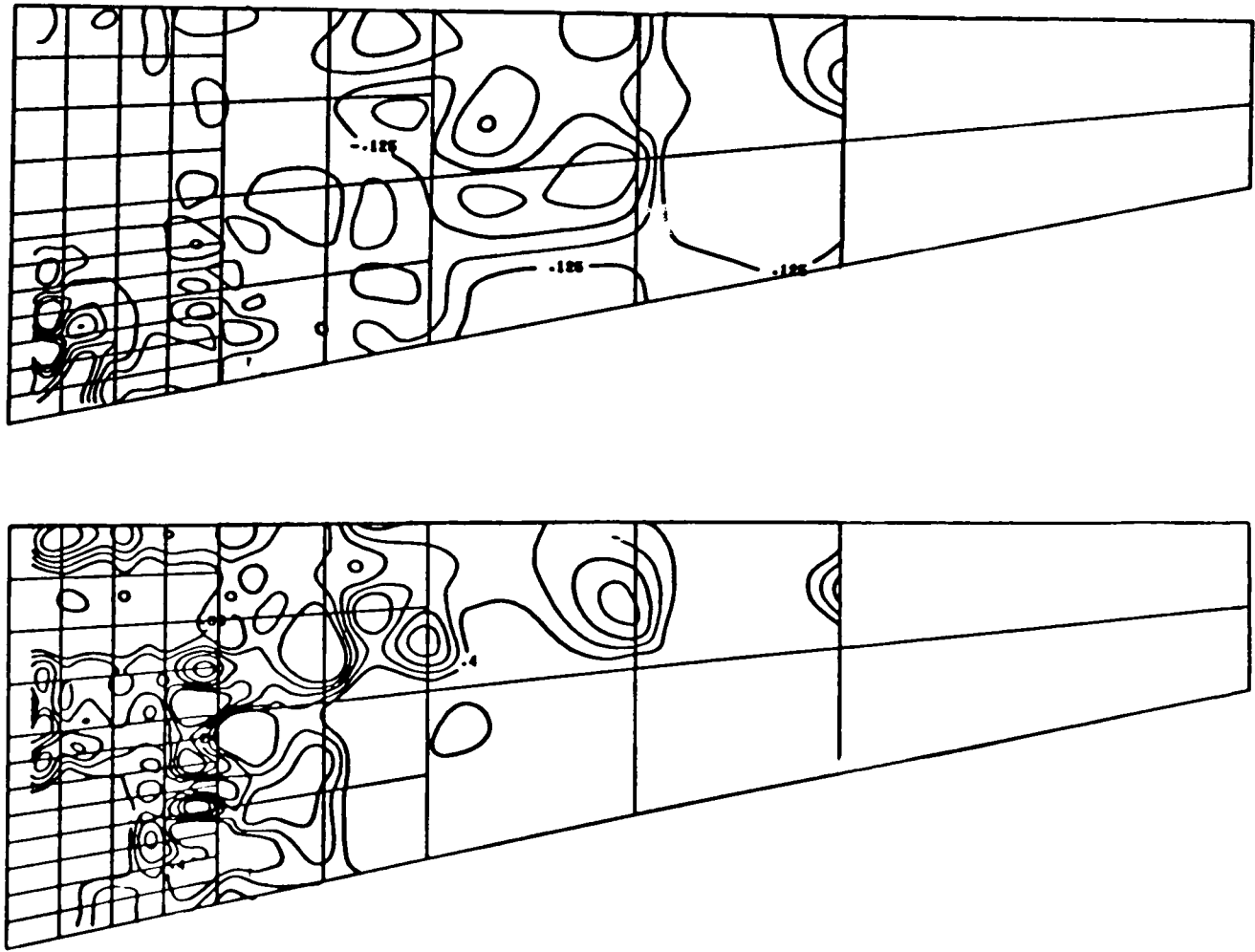


Figure 5.35 - Deformed Configuration of the Beam with $\alpha = 11.31^\circ$ ($t_w = 7\text{mm}$).



Figures 5.36.a and 5.36.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 11.31^\circ$.

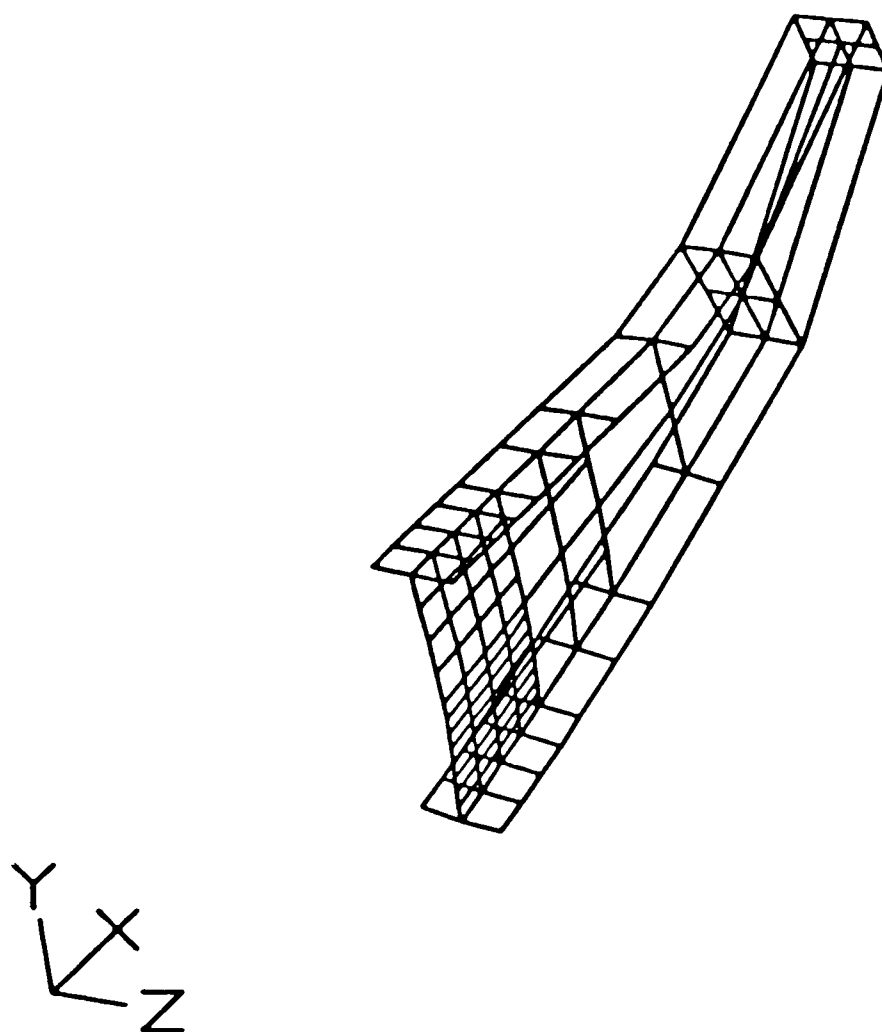
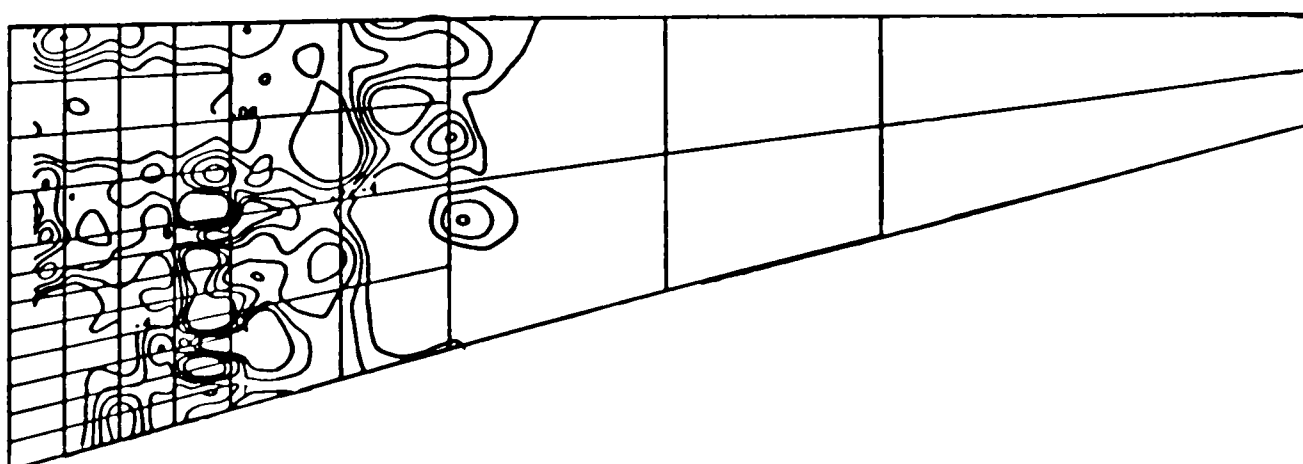
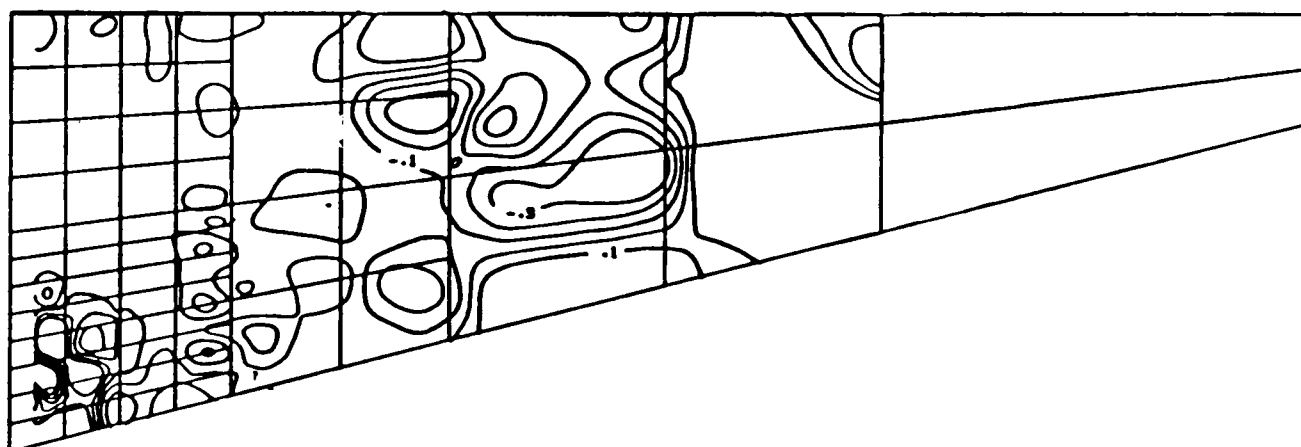


Figure 5.37 - Deformed Configuration of the Beam with $\alpha = 14.84^\circ$ ($t_w = 7\text{mm}$).



Figures 5.38.a and 5.38.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 14.84^\circ$.

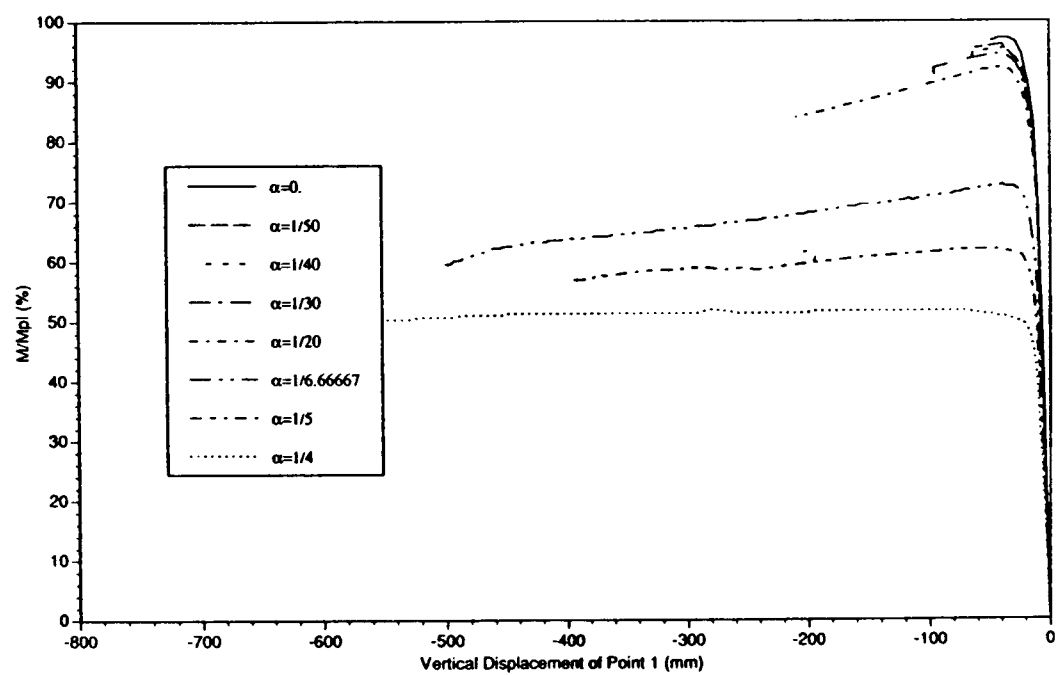


Figure 5.39 - Vertical Displacement Comparison for Point 1 ($t_w = 7\text{mm}$).

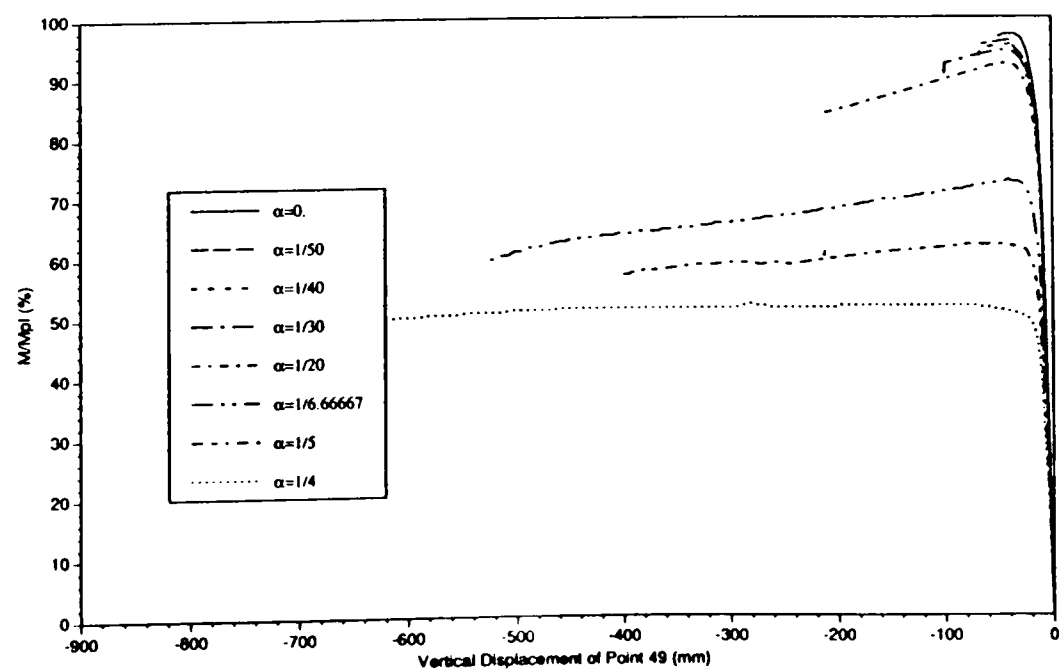
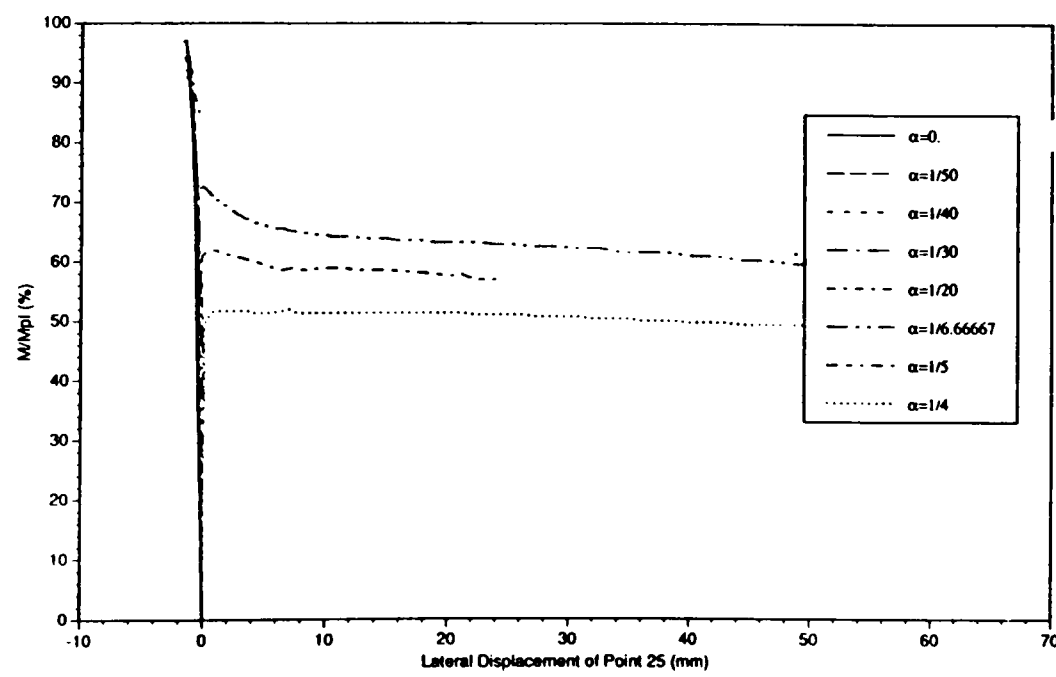
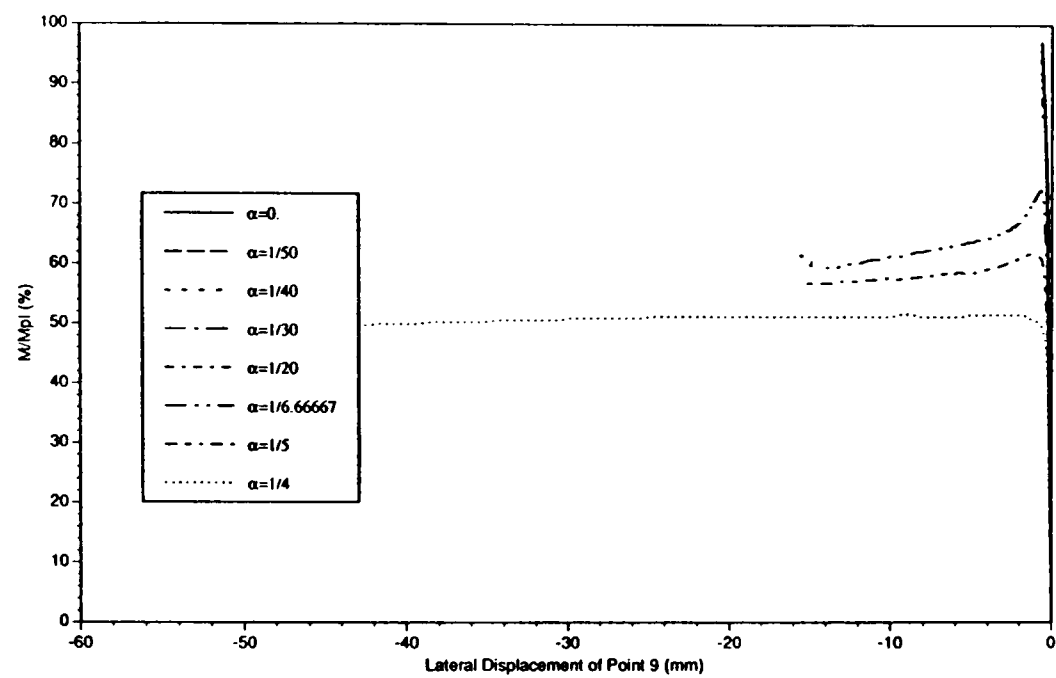
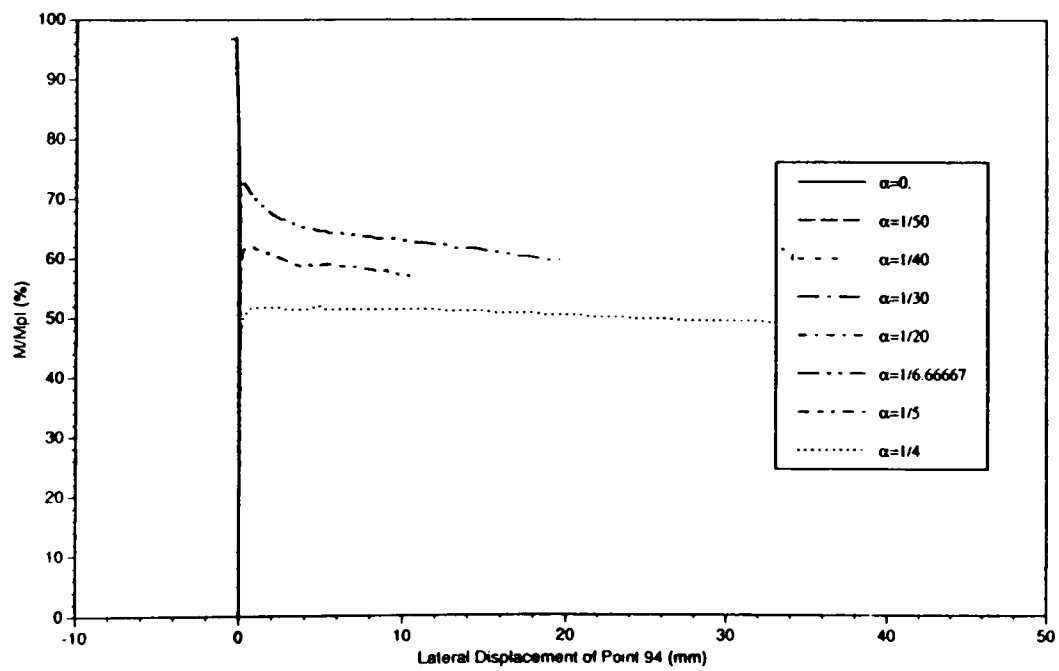
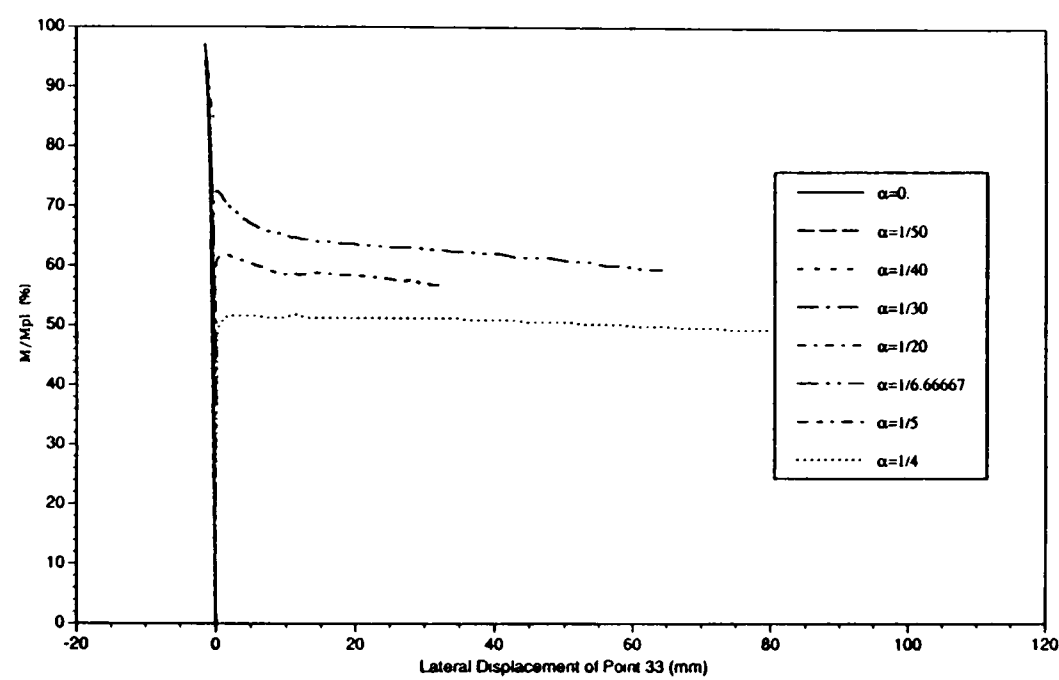


Figure 5.40 - Vertical Displacement Comparison for Point 49 ($t_w = 7\text{mm}$).



Figures 5.41.a and 5.41.b - Lateral Displacement Comparison for Points 9 and 25 ($t_w = 7\text{mm}$).



Figures 5.42.a and 5.42.b - Lateral Displacement Comparison for Points 33 and 94 ($t_w = 7\text{mm}$).

and 195. The main conclusion for this section is that, with a 7mm web, only the angle of 14.84° ($\alpha = 1/4$) shows even a very slight web buckling tendency.

5.3.3. Web Thickness Equal to 5mm.

The results of the two previous sections indicated that there were too many simulations with angles around 2°. They all presented a very similar load response and failure behaviours. For that reason, the case with a slope angle of 1.91° ($\alpha = 1/30$) was excluded for the 5mm web.

The beam with parallel flanges collapsed following plastic hinge formation near the vertical stiffener. Its deflected shape at the failure point is presented in figure 5.43, page 197. At the centre of the beam, the web deformed laterally associated with the rotation of the top flange. The web stress distribution is shown in figures 5.44.a and 5.44.b, page 197. No significant equivalent stresses were found near the bottom flange, at the centre of the beam.

The girder with an angle of 1.15° ($\alpha = 1/50$) failed by a plastic vertical buckling of the web. Its deformed configuration at the point of maximum load is portrayed in figure 5.45.a, page 198. Very little web deformation is noticeable. However, at a further stage in the unloading path, the web has become very distorted, as can be seen in figure 5.45.b, page 198. The stress pattern in the web is shown in figures 5.46.a and 5.46.b, page 198. At this stage, the stresses at the critical point at the centre of the beam near the bottom flange began to be visible.

The same type of failure occurred for the beams with angles of 1.43° and 2.86° ($\alpha = 1/40$, $\alpha = 1/20$). Both cases had very large associated web deformations. They only appeared at points on the unloading path far from the maximum load, confirming the inelastic characteristic of the collapse. The beam with a taper angle of slope equal to 1.43° had the deformed shape and the stress pattern at ultimate loading stage shown in figures 5.47, 5.48.a and 5.48.b, page 199. The angle of 2.86° has produced the deflected configuration and web stresses depicted in figures 5.49, 5.50.a and 5.50.b, page 200.

The angles of 8.53°, 11.31° and 14.84° ($\alpha = 1/1.67$, $\alpha = 1/5$ and $\alpha = 1/4$) produced an elasto-plastic type of failure. The failure shape and the web stress configuration associated with the angle of 8.53° are delineated in figures 5.51, 5.52.a and 5.52.b, page 201. The beam with a taper angle of slope equal to 11.31° had the deformed shape and the stress pattern at the ultimate loading stage, as shown in figures 5.53, 5.54.a and 5.54.b, page 202. The angle of 14.84° has produced the deflected configuration and web stresses depicted in figures 5.55, 5.56.a and 5.56.b, page 203. It is easy to note that the stresses near the bottom flange have become more concentrated and the buckling shape more visible, as the angle of slope increased.

A summary of the ultimate loads achieved by the beams with 5mm webs is given in table 5.4 below. The parallel beam failed by the formation of a plastic hinge. All the other girders had a vertical web buckling failure in the elasto-plastic and elastic ranges.

Angle	0°	1.15°	1.43°	2.86°	8.53°	11.31°	14.84°
Final Load	95.40	94.03	93.37	90.02	70.68	60.04	49.53

Table 5.4 - Final Loads (% Central M_{pl}) for Beams with $t_w = 5\text{mm}$.

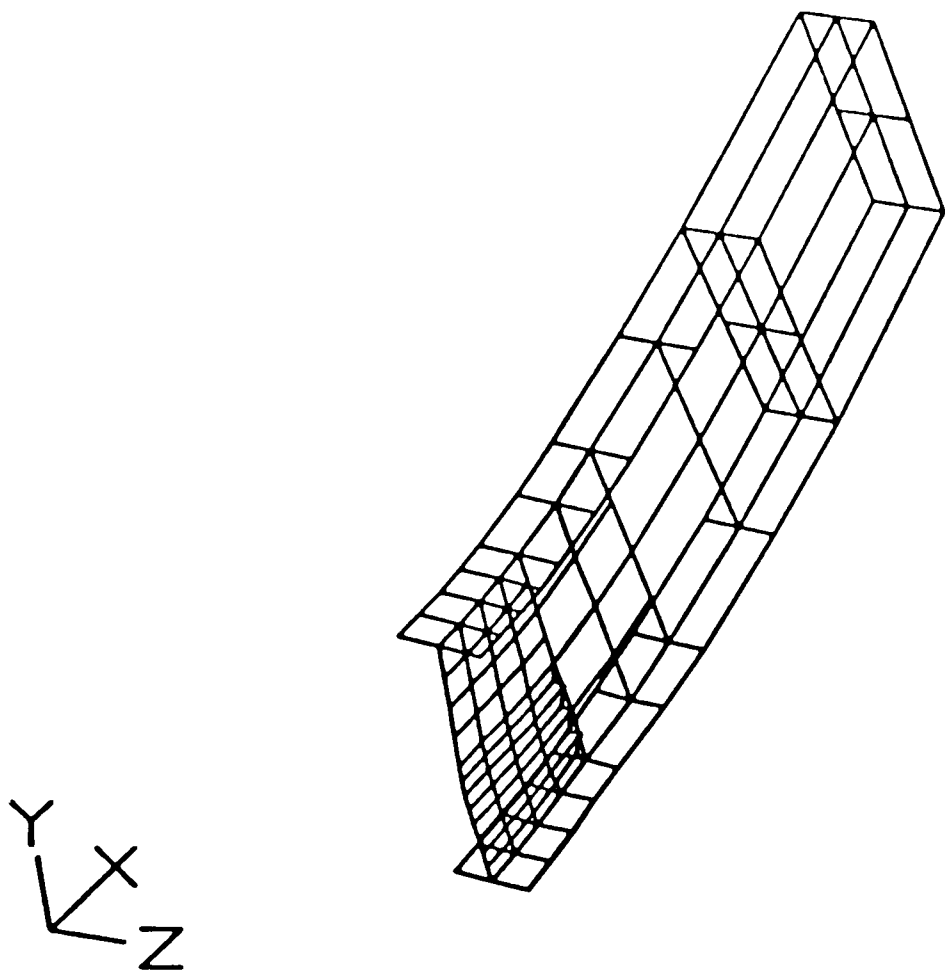
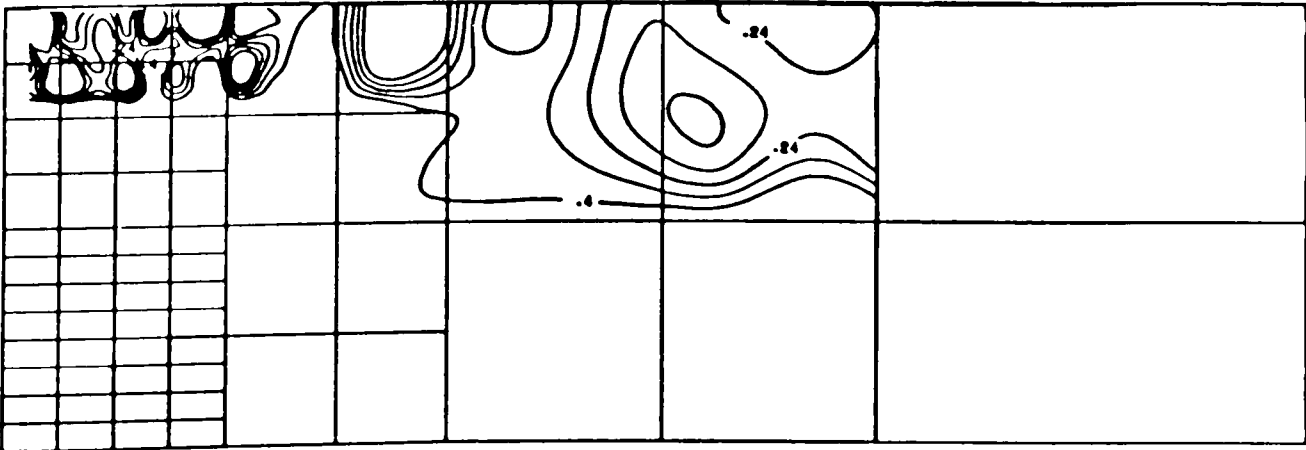
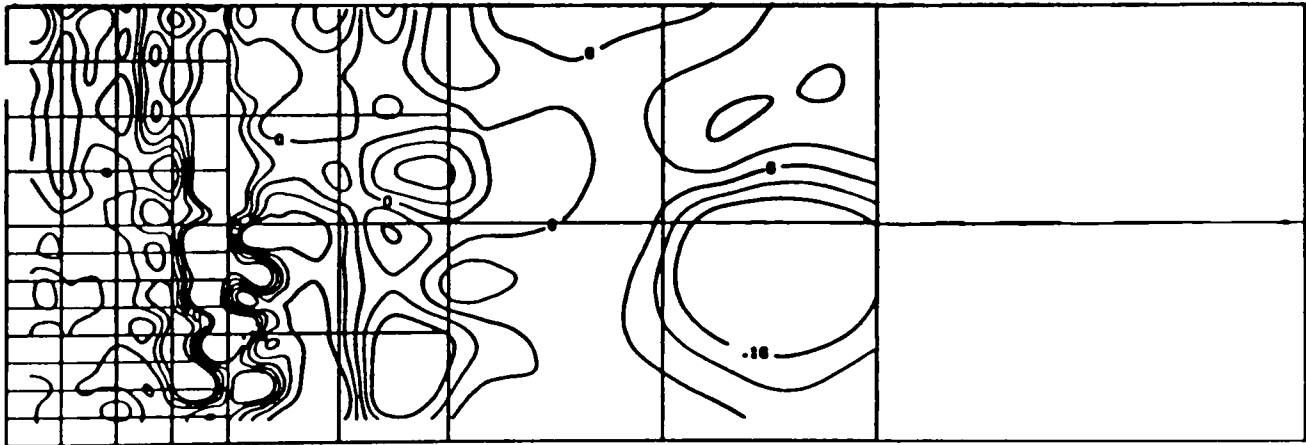
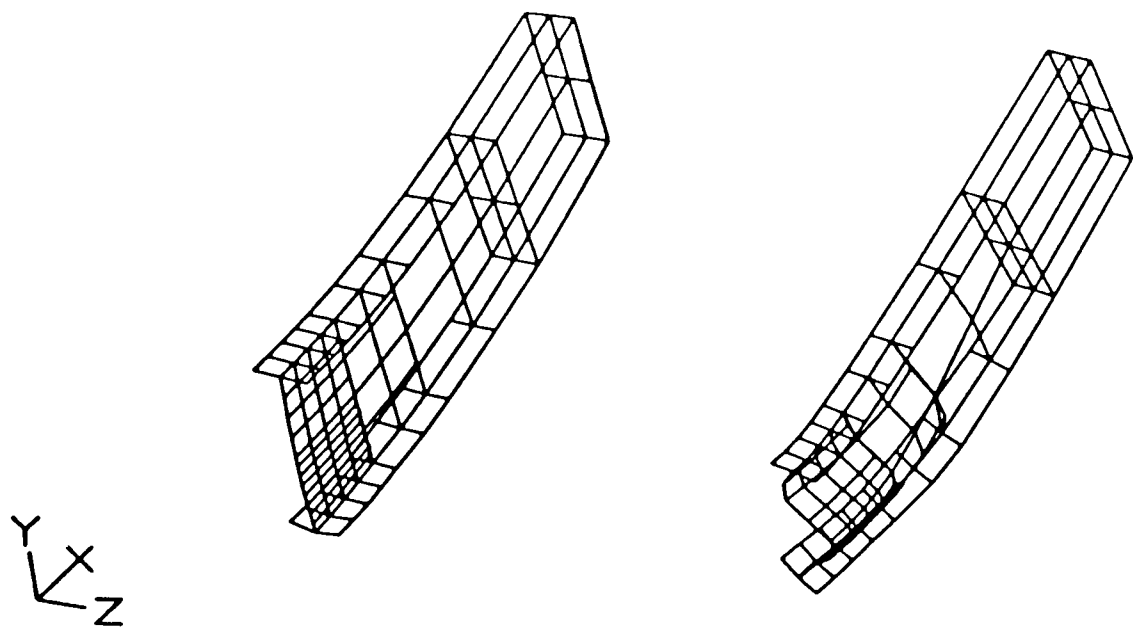


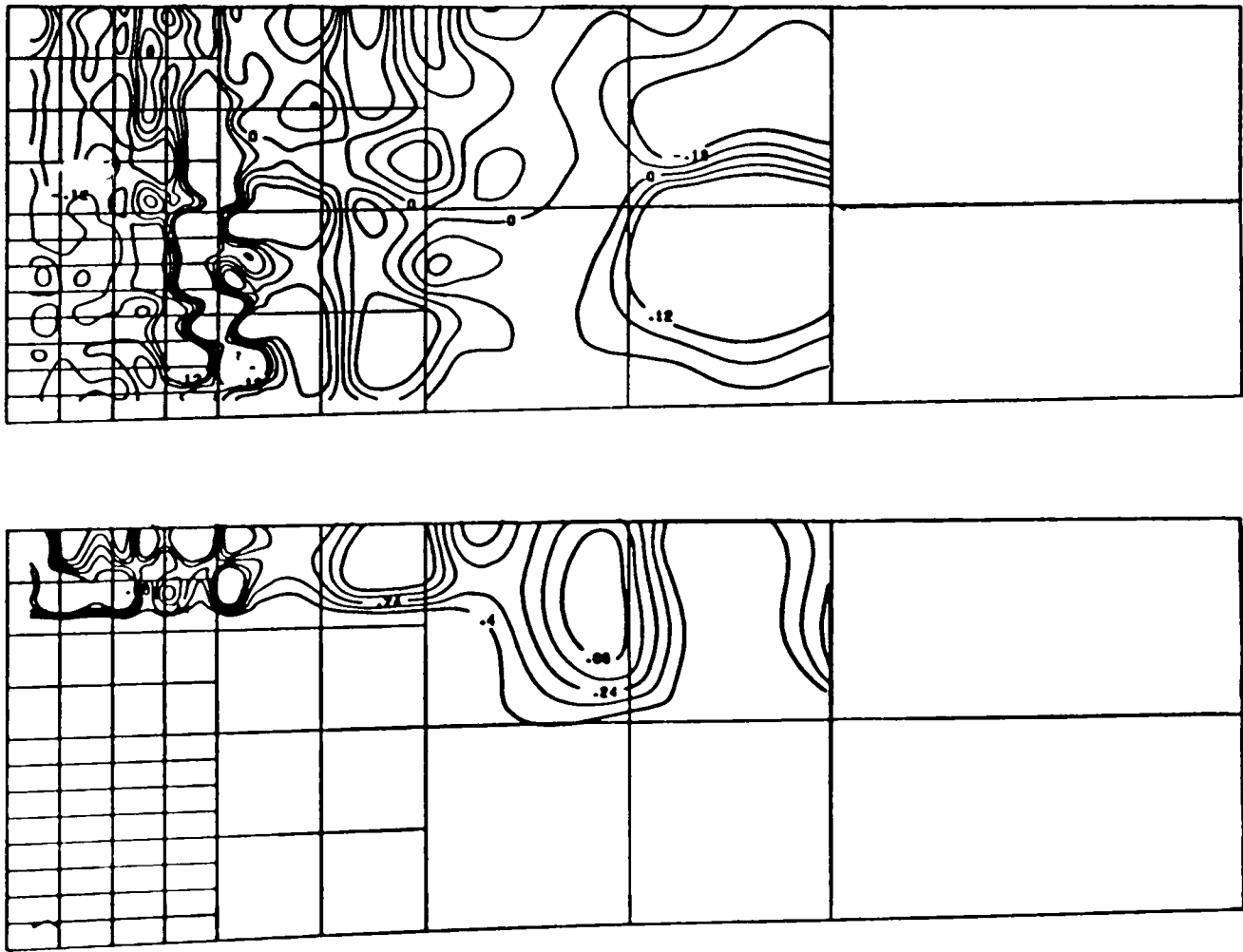
Figure 5.43 - Deformed Configuration of the Beam with $\alpha = 0^\circ$ ($t_w = 5\text{mm}$).



Figures 5.44.a and 5.44.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 0^\circ$.



Figures 5.45.a and 5.45.b - Deformed Configurations of the Beam with $\alpha = 1.15^\circ$ at Ultimate and Last Loading Stages ($t_w = 5\text{mm}$).



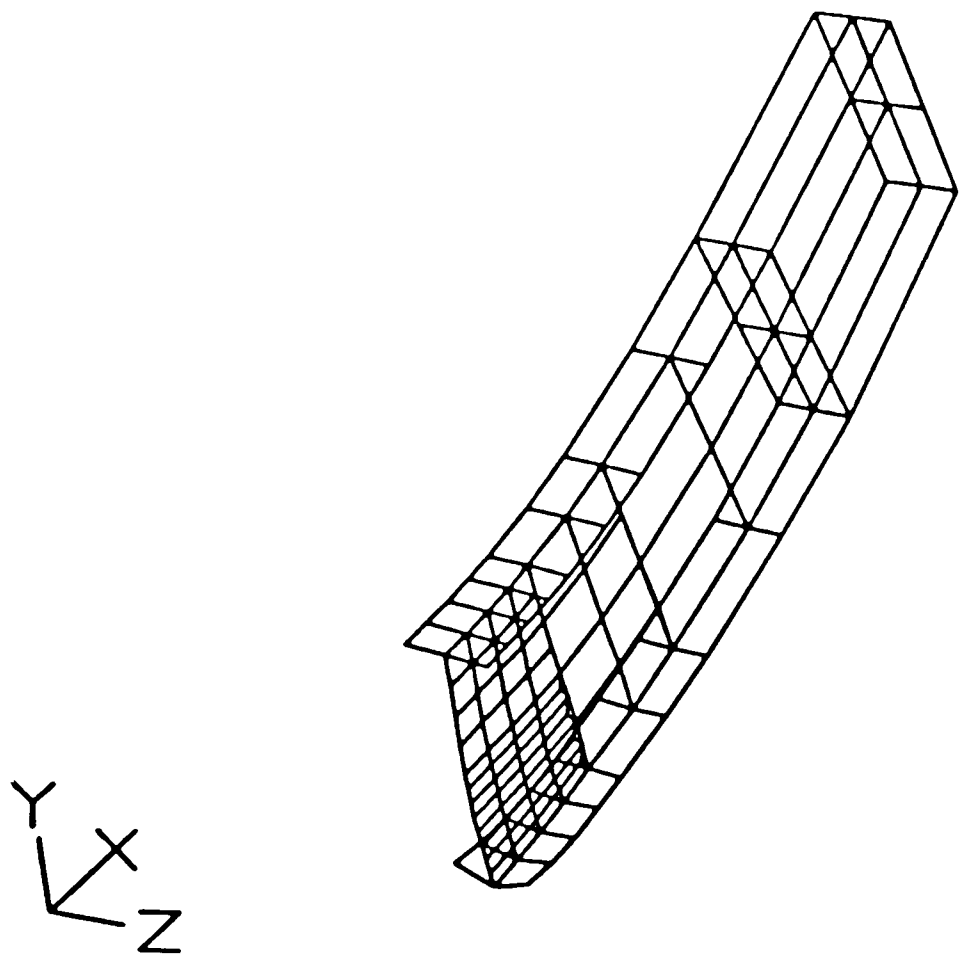
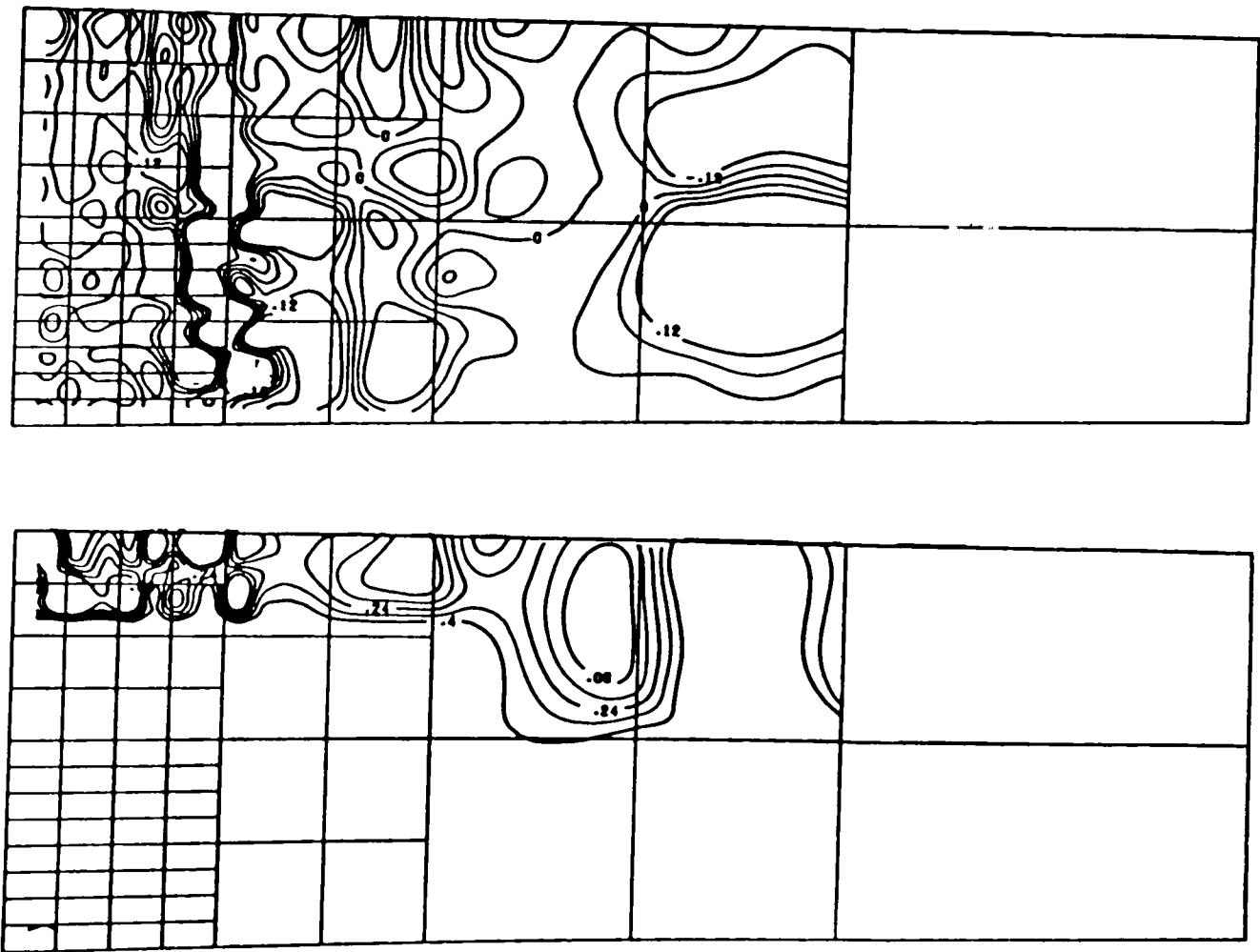


Figure 5.47 - Deformed Configuration of the Beam with $\alpha = 1.43^\circ$ ($t_w = 5\text{mm}$).



Figures 5.48.a and 5.48.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 1.43^\circ$.

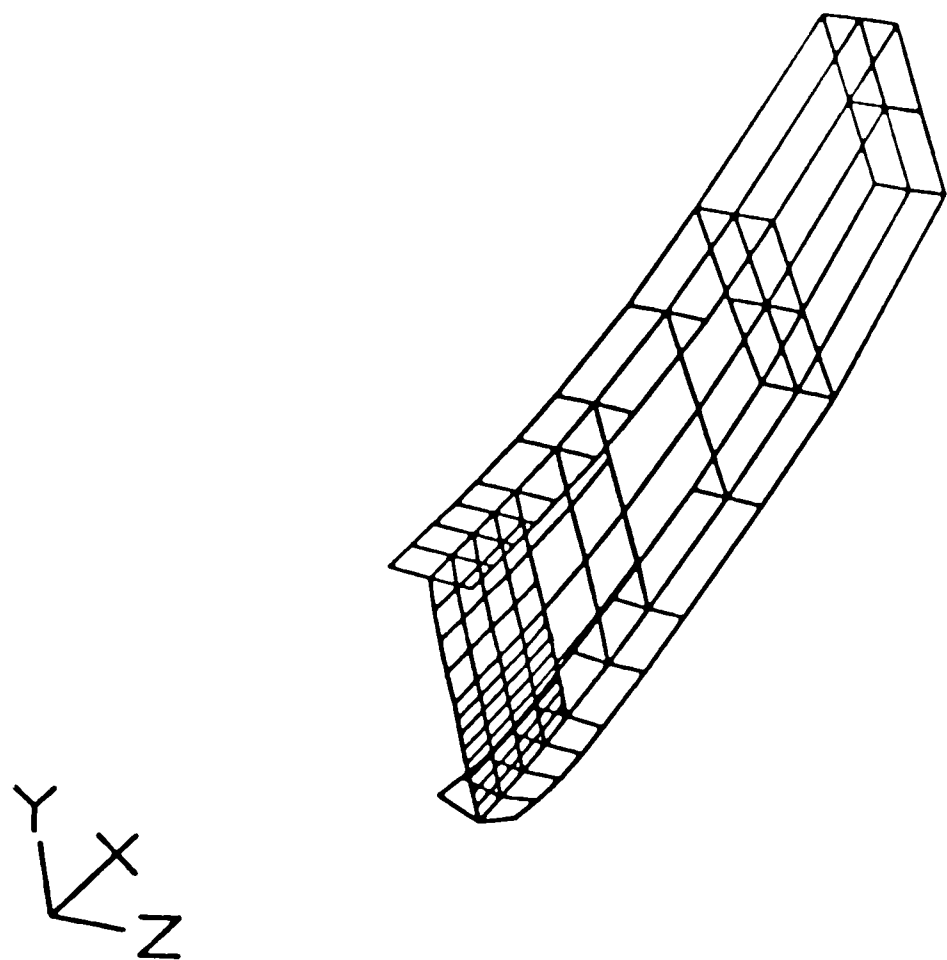


Figure 5.49 - Deformed Configuration of the Beam with $\alpha = 2.86^\circ$ ($t_w = 5\text{mm}$).



Figures 5.50.a and 5.50.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 2.86^\circ$.

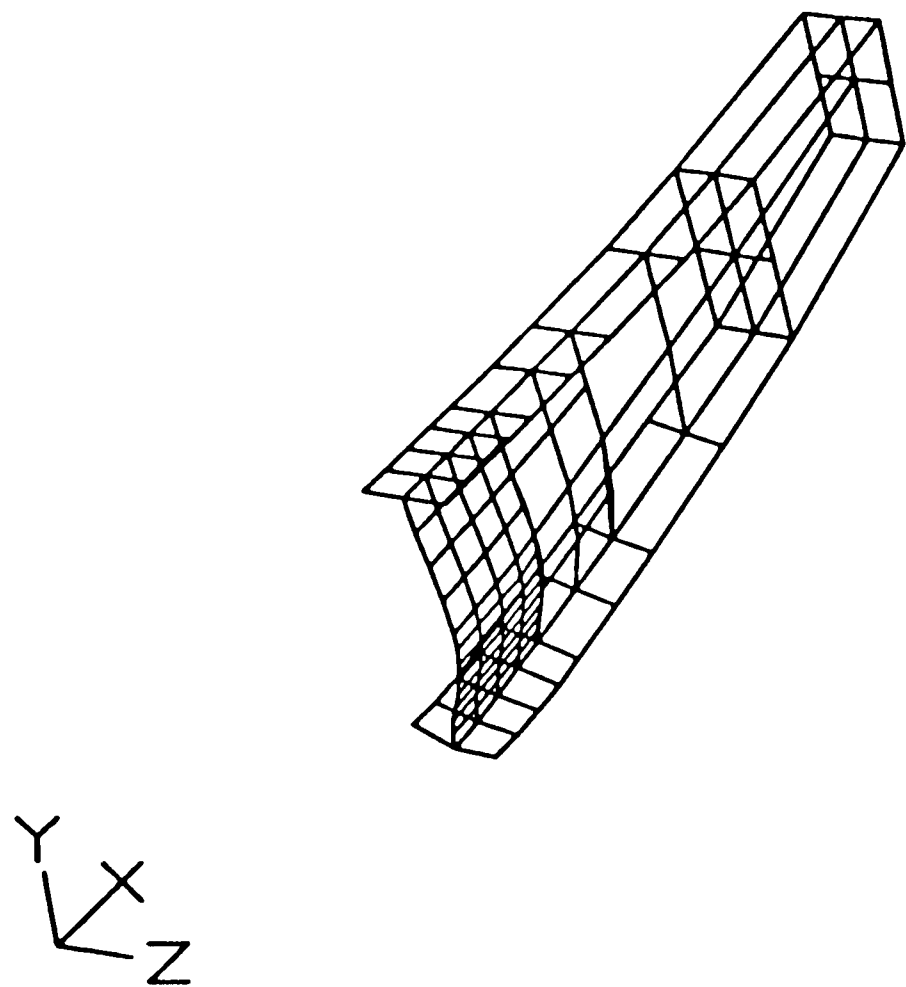
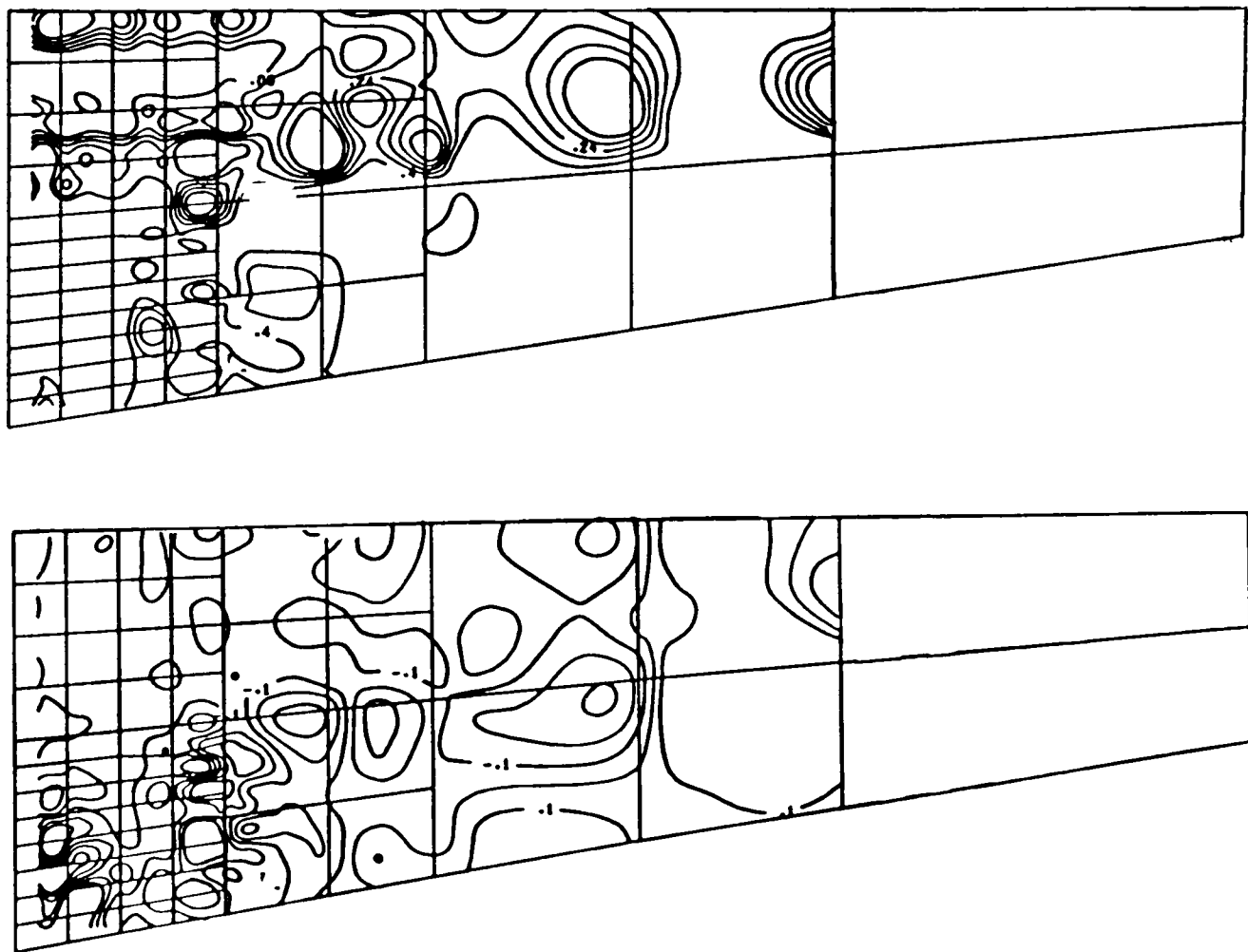


Figure 5.51 - Deformed Configuration of the Beam with $\alpha = 8.53^\circ$ ($t_w = 5\text{mm}$).



Figures 5.52.a and 5.52.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 8.53^\circ$.

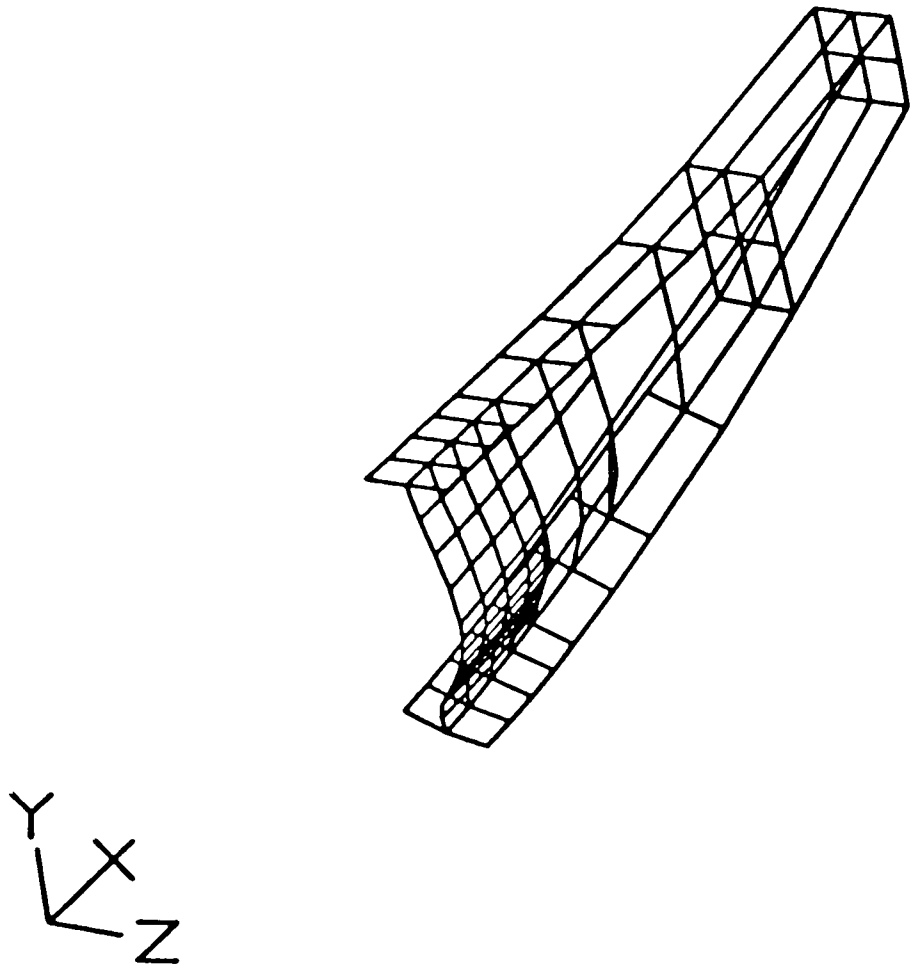
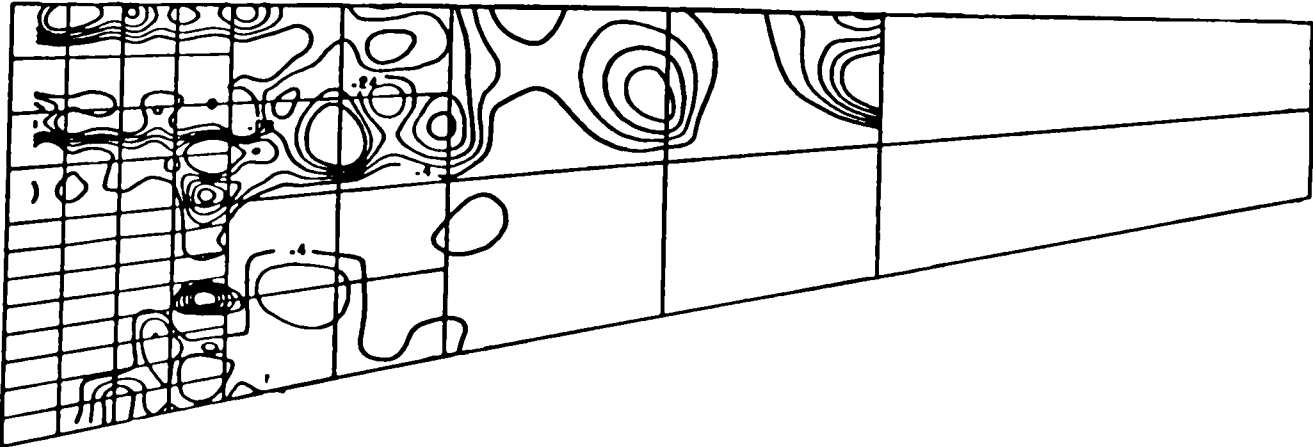
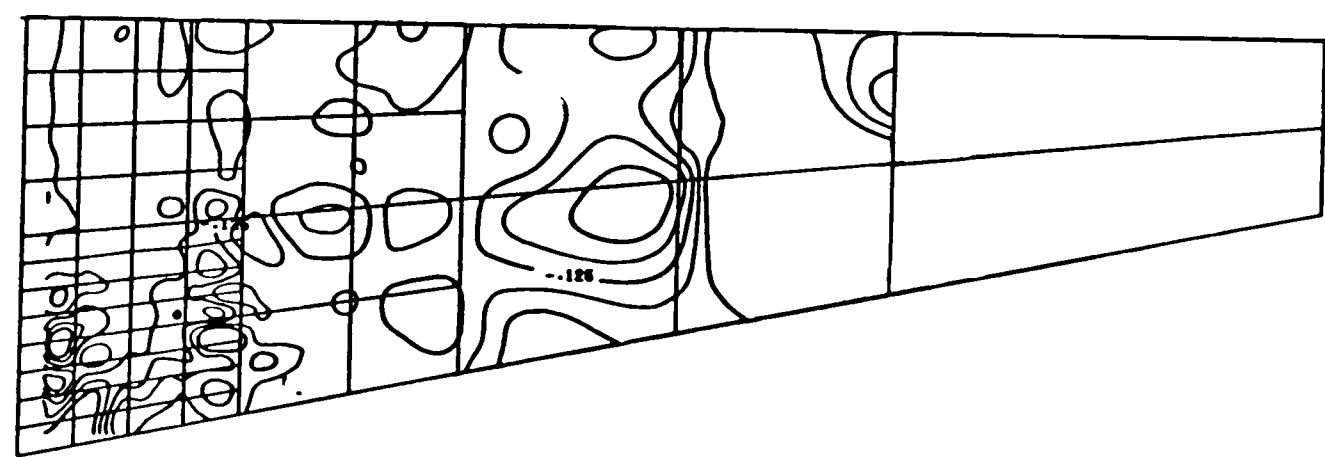


Figure 5.53 - Deformed Configuration of the Beam with $\alpha = 11.31^\circ$ ($t_w = 5\text{mm}$).



Figures 5.54.a and 5.54.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 11.31^\circ$.

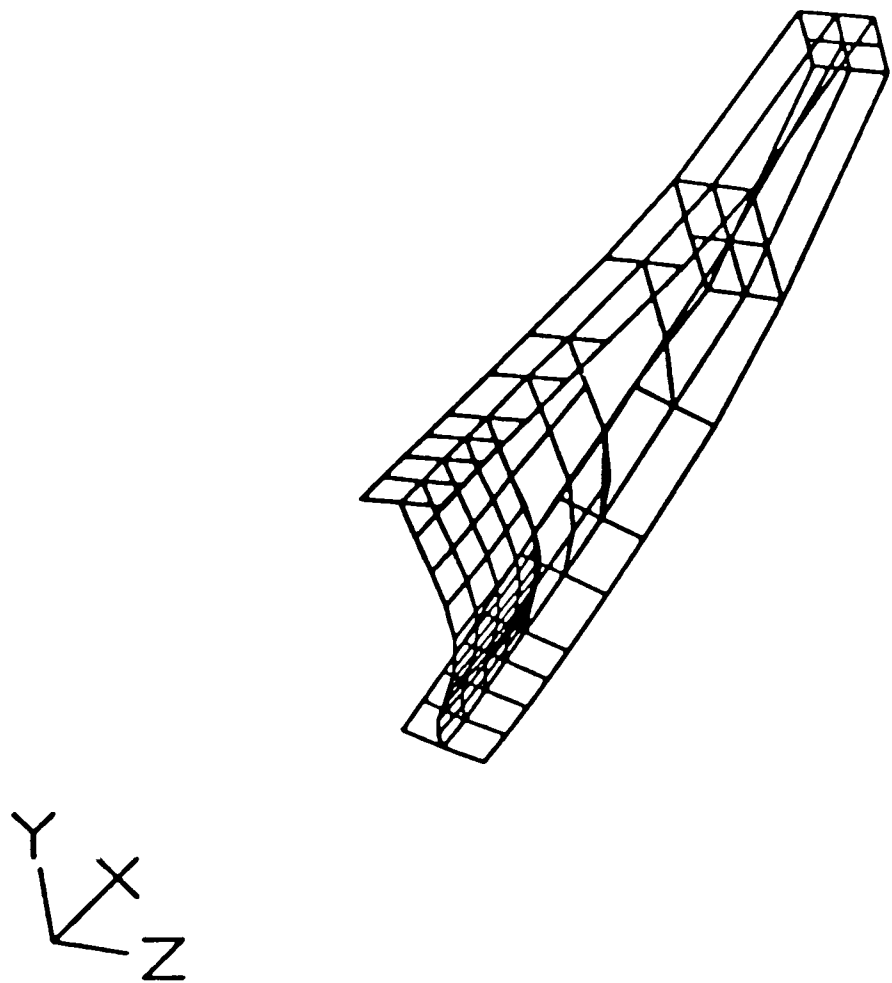
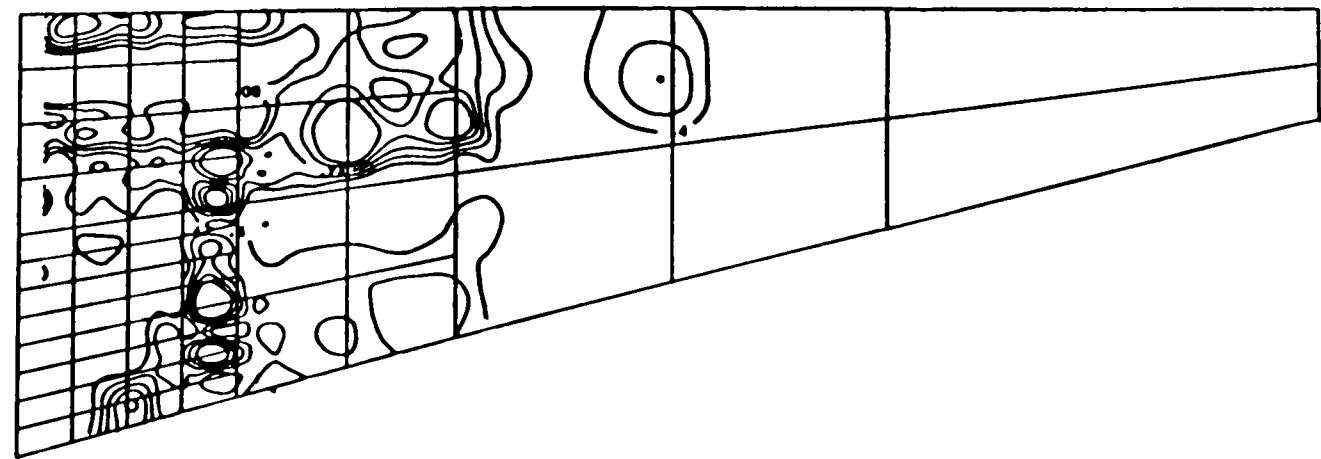
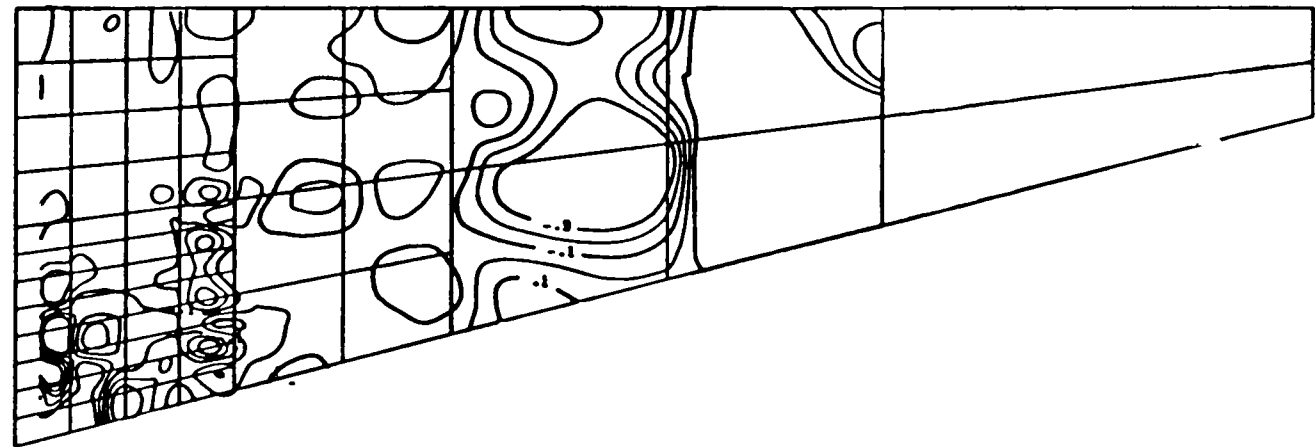


Figure 5.55 - Deformed Configuration of the Beam with $\alpha = 14.84^\circ$ ($t_w = 5\text{mm}$).



Figures 5.56.a and 5.56.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 14.84^\circ$.

A vertical displacement comparison for points 1 and 49 of all the beams investigated in this section is presented in figures 5.57 and 5.58, page 205. The significant drop in ultimate load, as the angle of slope increased, is very easy to observe. The two regions of plastic and elasto-plastic failure are also clearly identifiable. A web lateral displacement comparison for points 9, 25, 33 and 94 is depicted in figures 5.59.a, 5.59.b, 5.60.a and 5.60.b, pages 206 and 207. The same lateral displacement tendency was observed in these points for all the beams studied, with the exception of the one with parallel flanges. With these results in hand, it is possible to state that for the 5mm web all the slopes required a vertical stiffener at the centre bottom part of the girder to avoid local buckling.

5.3.4. Web Thickness Equal to 3mm.

The previous subsection showed that the number of simulations with a slope angle around 2° was still excessive. In view of that, the case of the taper slope angle equal to 1.15° ($\alpha = 1/50$) was not included in the finite element simulations for a 3mm web.

The beam with parallel flanges buckled in the longitudinal direction, because of the high longitudinal stresses. The collapse at the ultimate loading stage is described in figure 5.61, page 208. At the same final point, the stresses in the web are portrayed in figures 5.62.a and 5.62.b, page 208. No significant equivalent (Von Mises) stresses were found near the bottom flange.

The girder with an angle of 1.43° ($\alpha = 1/40$) failed by a combined longitudinal and vertical buckling of the web. The deformed shape of that beam is depicted in figure 5.63, page 209. The web stress pattern has changed greatly when compared to the first case, as illustrated in figures 5.64.a and 5.64.b, page 209. High compressive vertical stresses, in the region near to the bottom flange at the centre of the beam, have modified the equivalent stress diagram, generating a larger area of web yield. A vertical buckling of the web in the inelastic region was the main cause of failure for the beam with an angle of 2.86° ($\alpha = 1/20$). This beam had the failure shape and the stress patterns at the ultimate loading stage as presented in figures 5.65, 5.66.a and 5.66.b, page 210.

The angles of 8.53°, 11.31° and 14.84° ($\alpha = 1/6.67$, $\alpha = 1/5$ and $\alpha = 1/4$) induced an elastic vertical web buckling collapse. The beam with an angle of 8.53° has produced the deflected configuration and the web stresses depicted in figures 5.67, 5.68.a and 5.68.b, page 211. The deflected shape and web stress patterns, for the angle of 11.31°, are shown in figures 5.69, 5.70.a and 5.70.b, page 212. The last simulation with an angle of taper slope equal to 14.84° had the deformed configuration and the stress patterns at the ultimate loading stage depicted in figures 5.71, 5.72.a and 5.72.b, page 213.

A summary of the final loads achieved by the 3mm beams is presented in table 5.5. There is a significant loss of resistance and change in the type of failure as the angle increases. With a zero angle, the failure was caused by a longitudinal web buckling. As the angle increased, the vertical stresses began to influence the failure in a more significant way, changing completely the form of collapse.

Angle	0°	1.43°	2.86°	8.53°	11.31°	14.84°
Final Load	95.53	93.31	88.79	60.57	49.16	38.14

Table 5.5 - Final Loads (% Central M_{pl}) for Beams with $t_w = 3\text{mm}$.

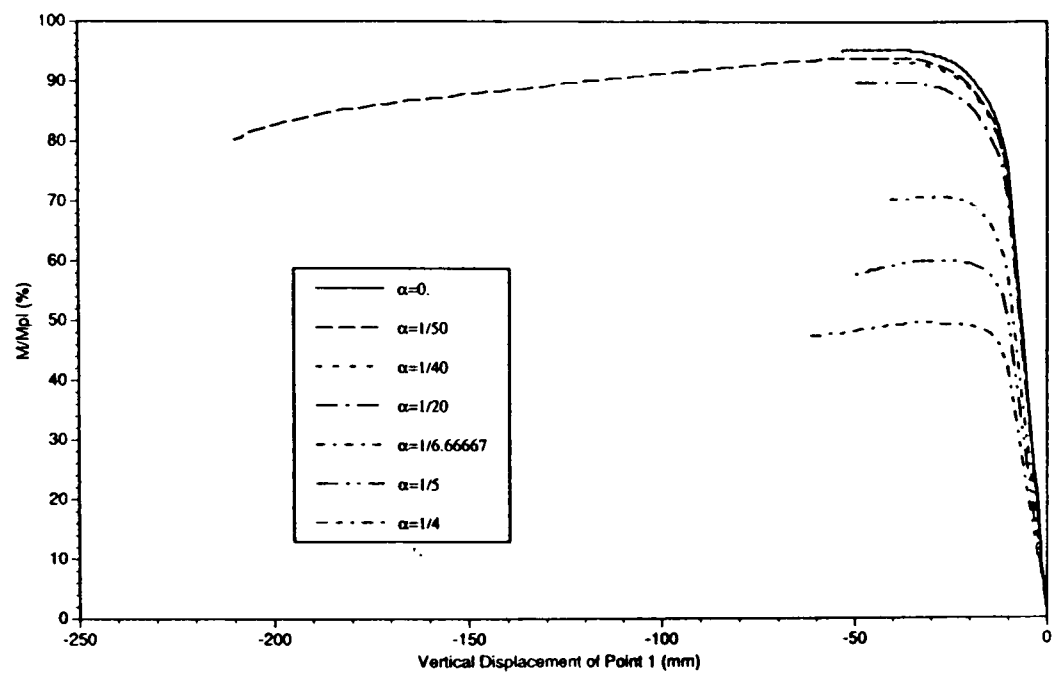


Figure 5.57 - Vertical Displacement Comparison for Point 1 ($t_w = 5\text{mm}$).

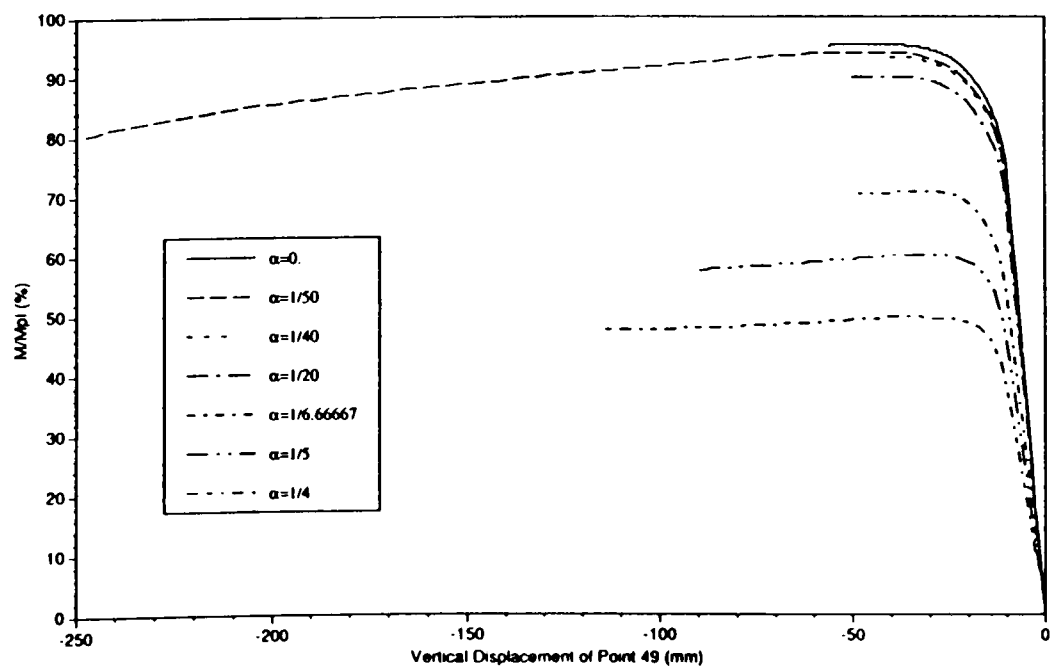
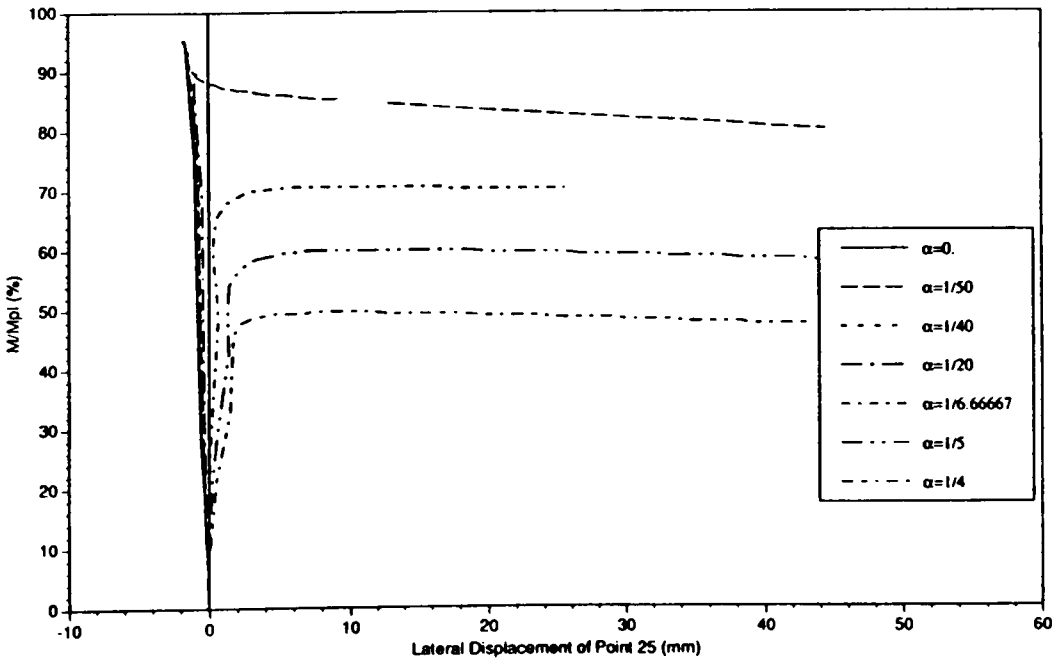
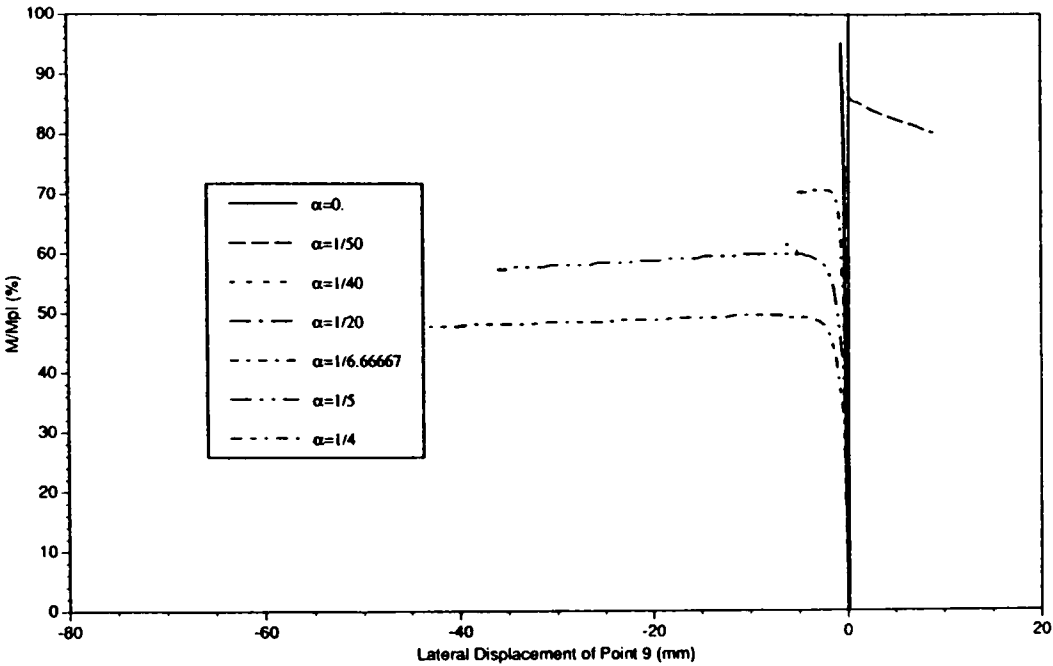
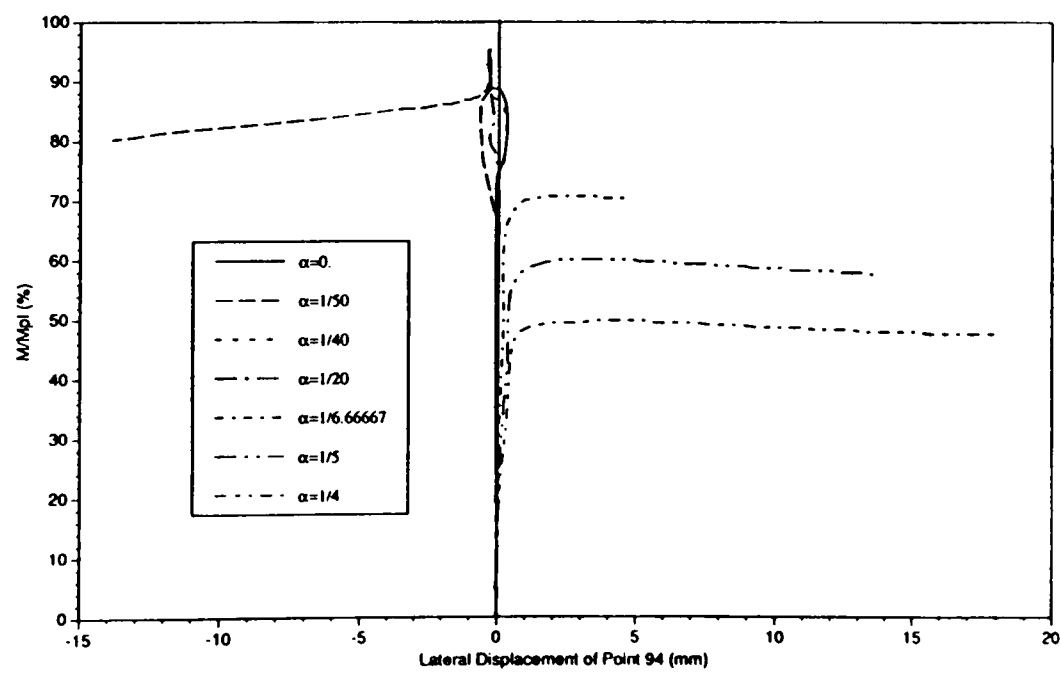
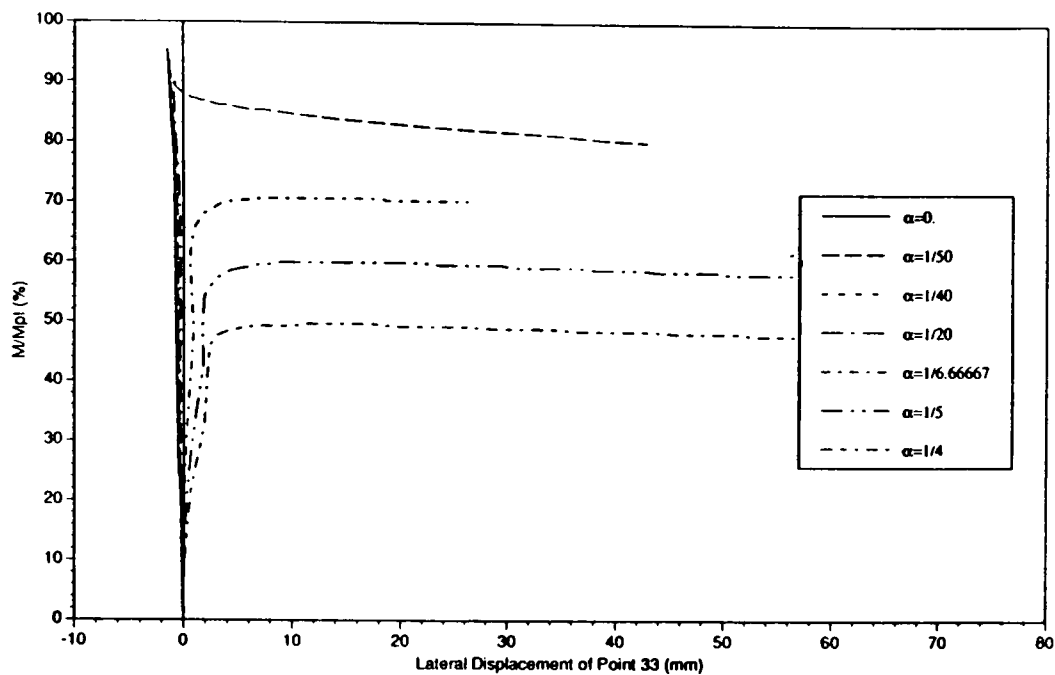


Figure 5.58 - Vertical Displacement Comparison for Point 49 ($t_w = 5\text{mm}$).



Figures 5.59.a and 5.59.b - Lateral Displacement Comparison for Points 9 and 25 ($t_w = 5\text{mm}$).



Figures 5.60.a and 5.60.b - Lateral Displacement Comparison for Points 33 and 94 ($t_w = 5\text{mm}$).

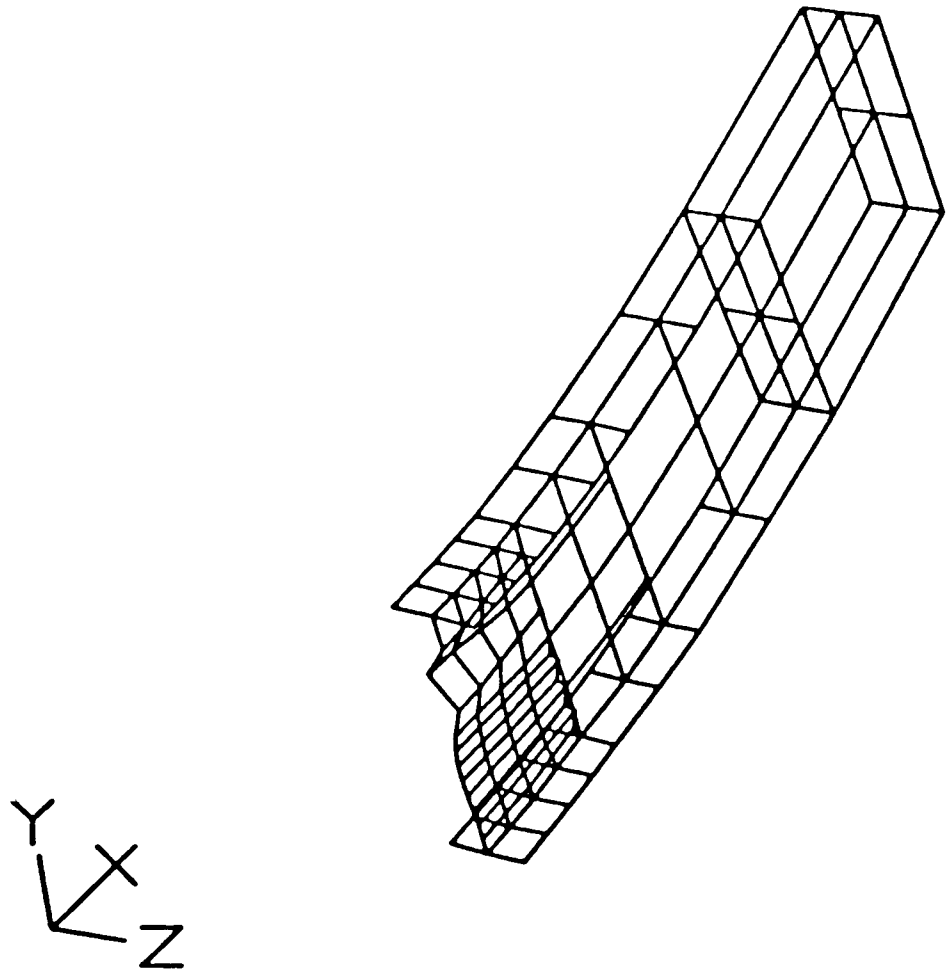
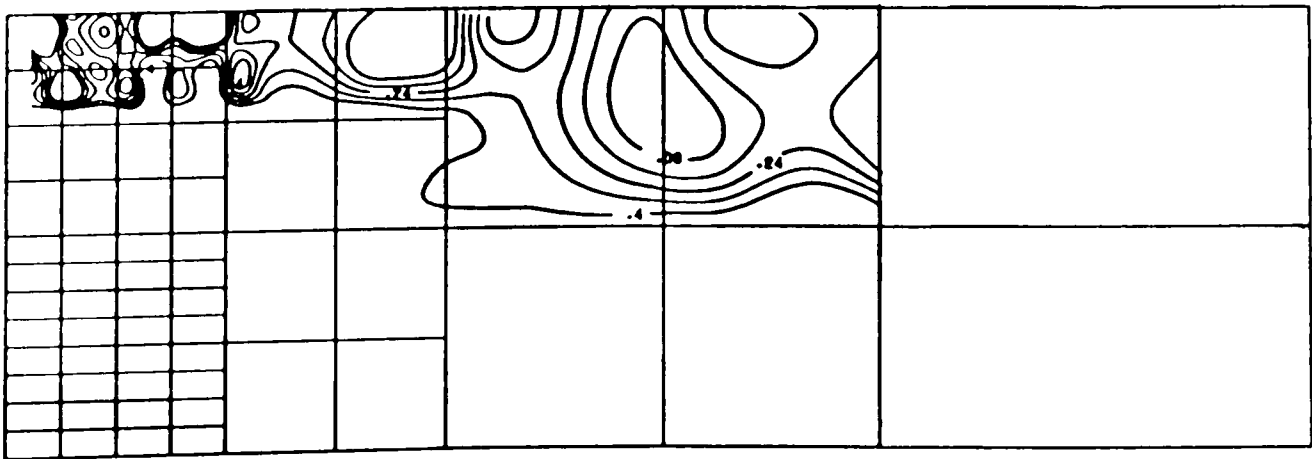
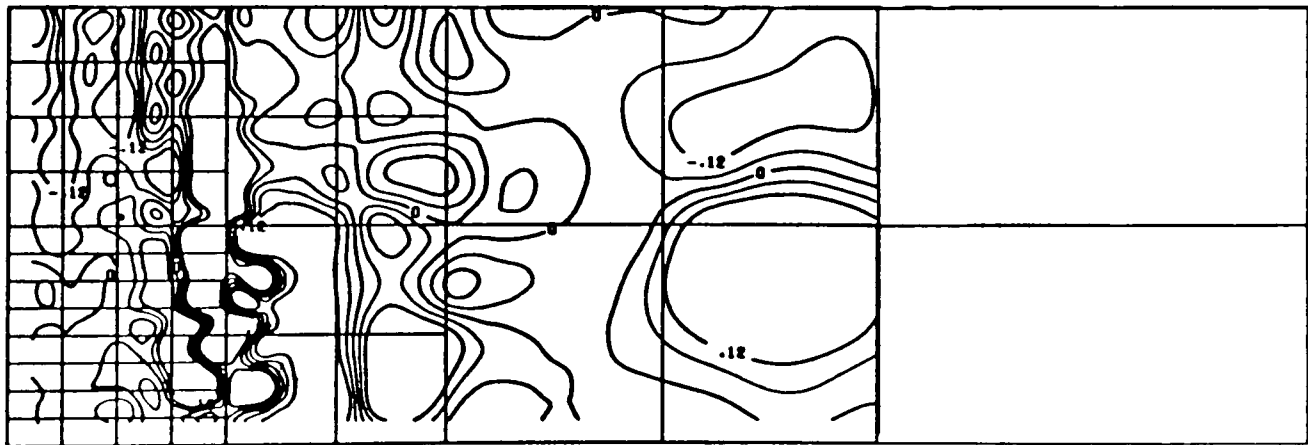


Figure 5.61 - Deformed Configuration of the Beam with $\alpha = 0^\circ$ ($t_w = 3\text{mm}$).



Figures 5.62.a and 5.62.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 0^\circ$.

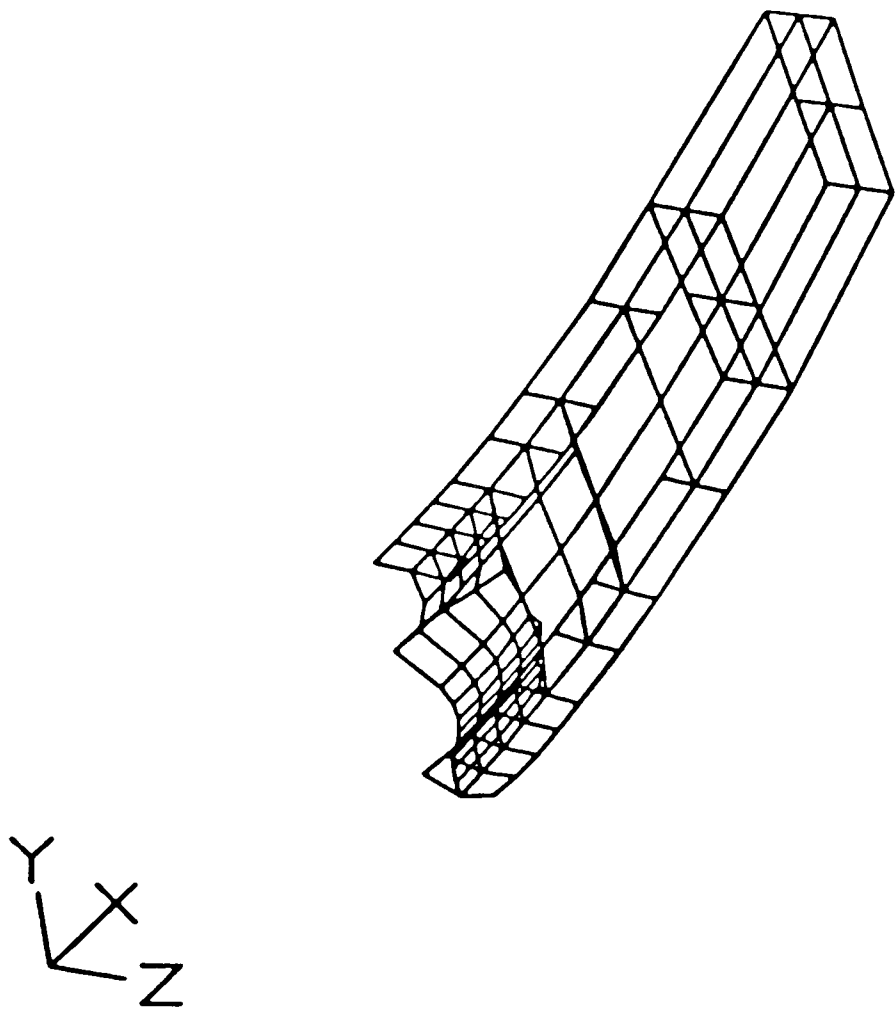
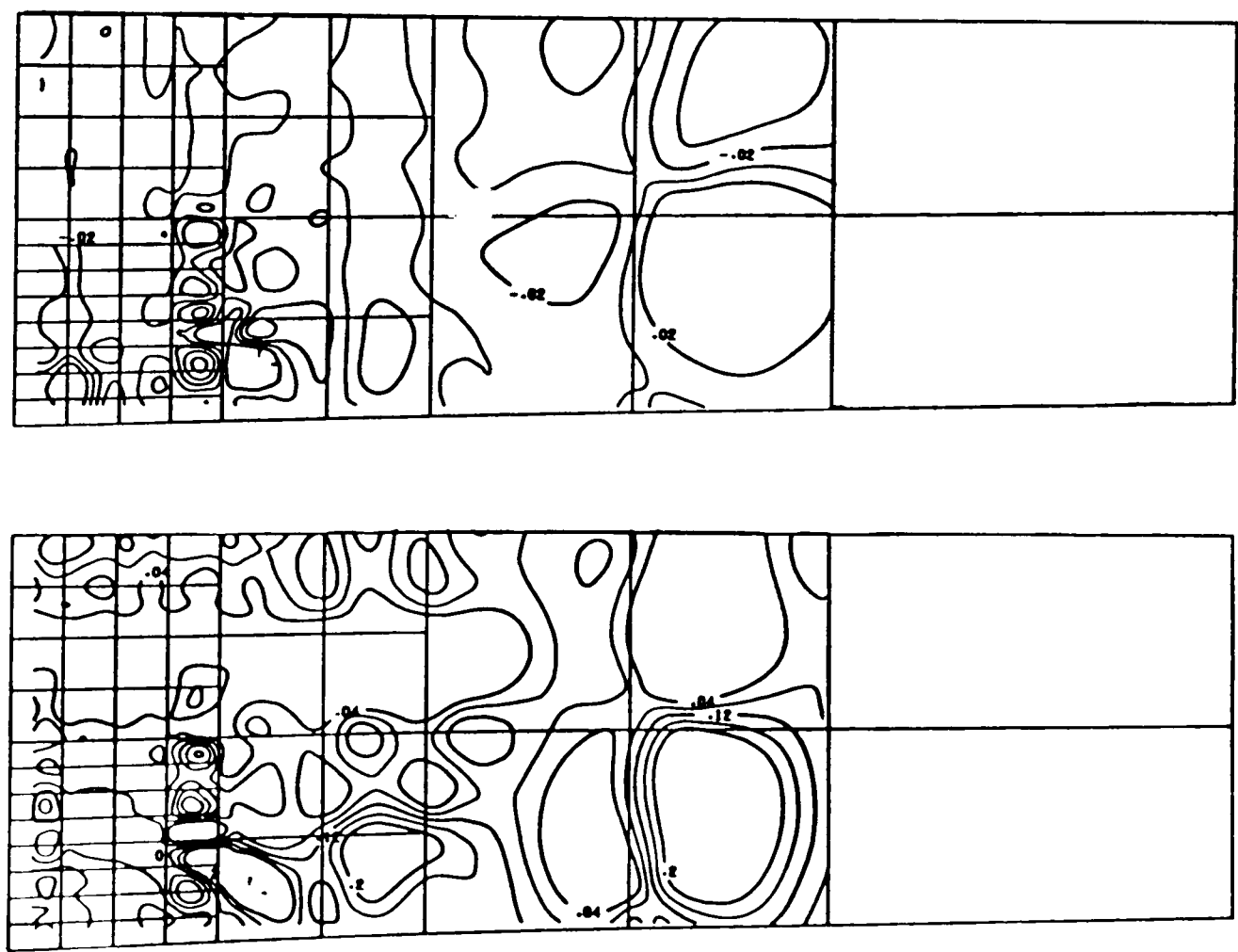


Figure 5.63 - Deformed Configuration of the Beam with $\alpha = 1.43^\circ$ ($t_w = 3\text{mm}$).



Figures 5.64.a and 5.64.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 1.43^\circ$.

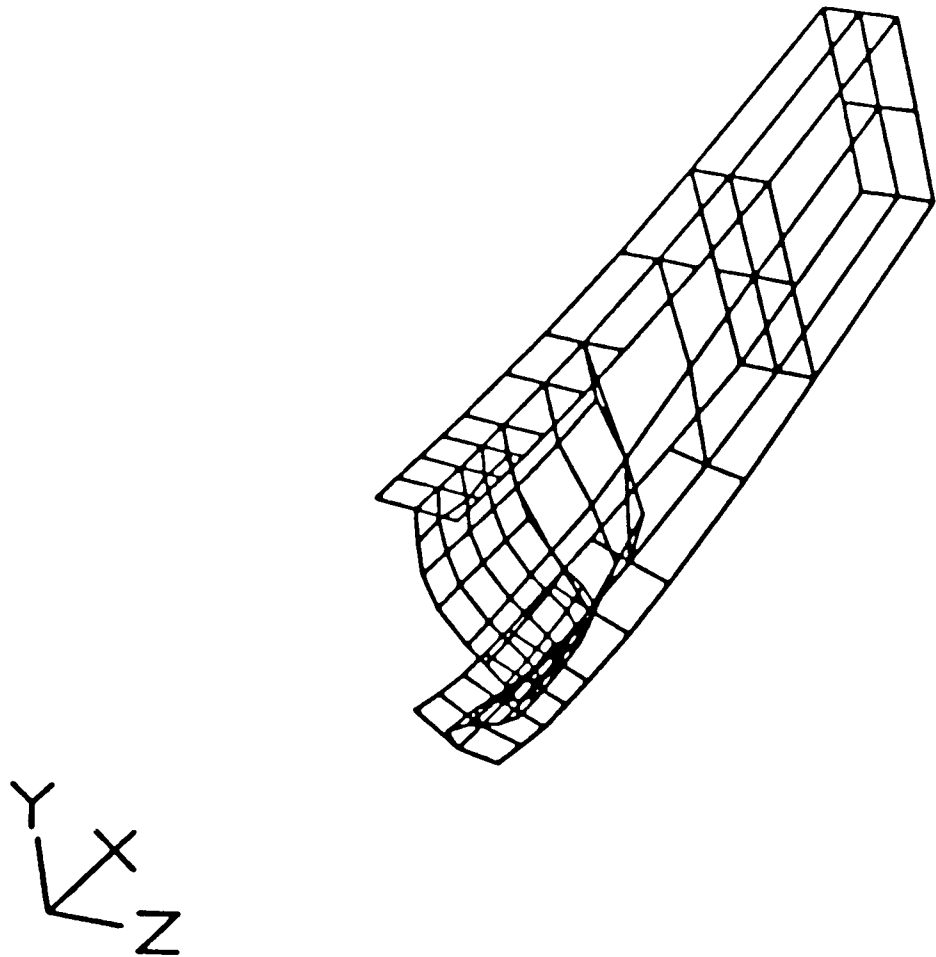
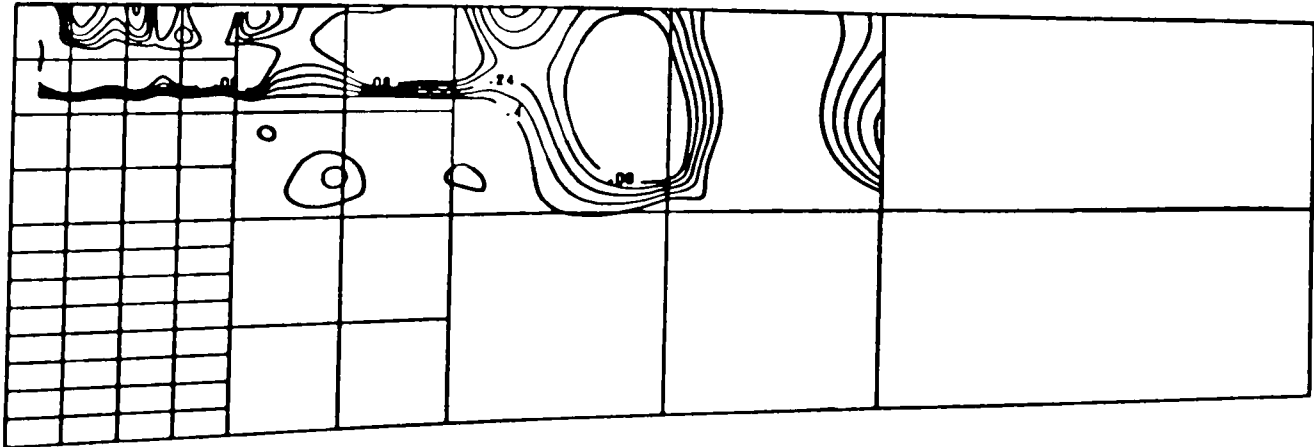
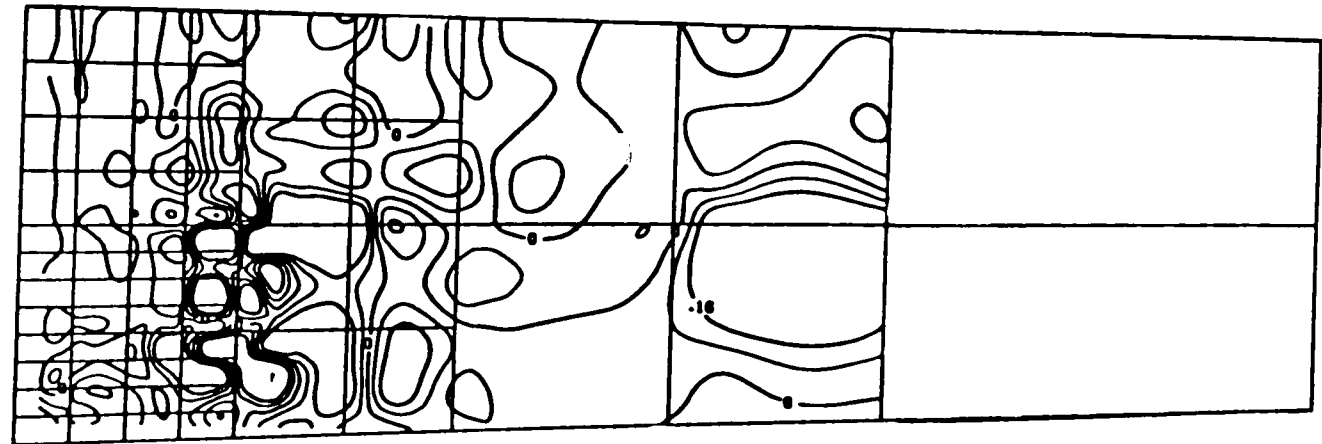


Figure 5.65 - Deformed Configuration of the Beam with $\alpha = 2.86^\circ$ ($t_w = 3\text{mm}$).



Figures 5.66.a and 5.66.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 2.86^\circ$.

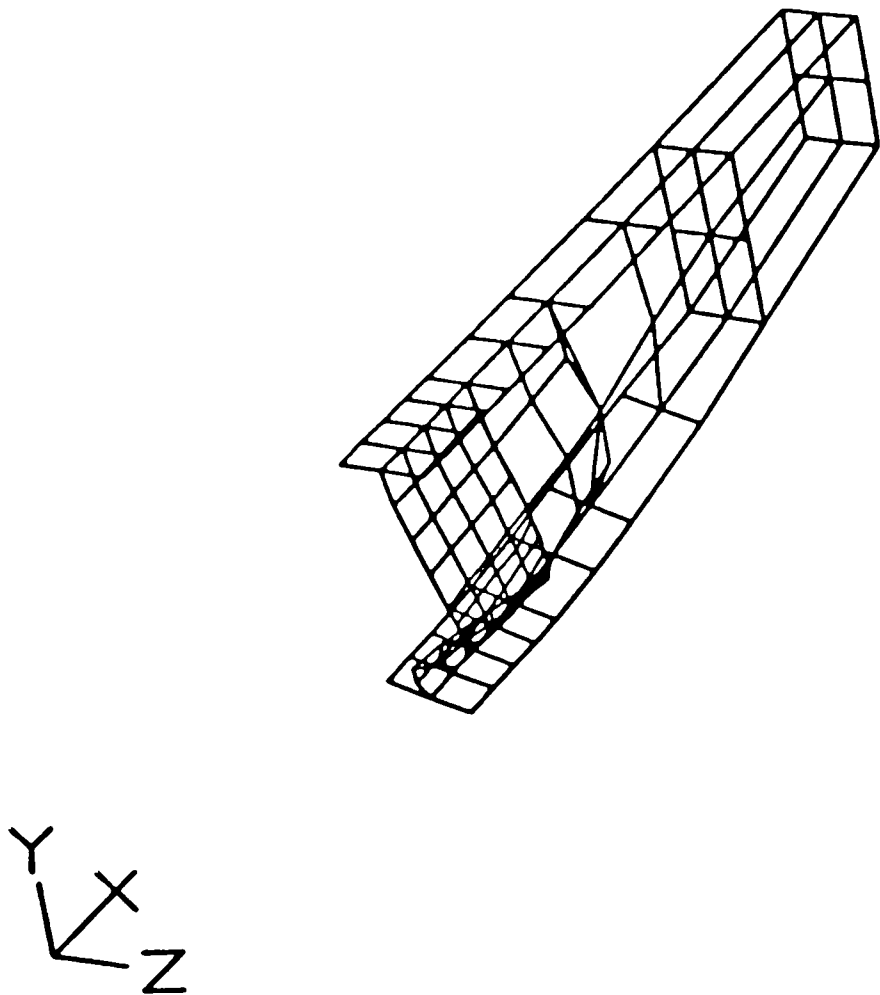
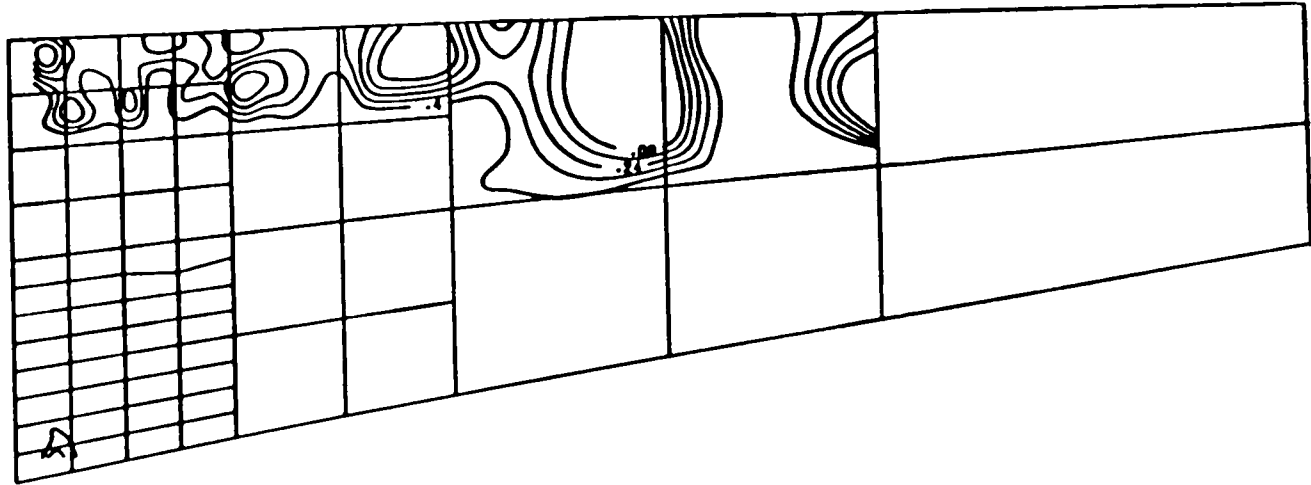
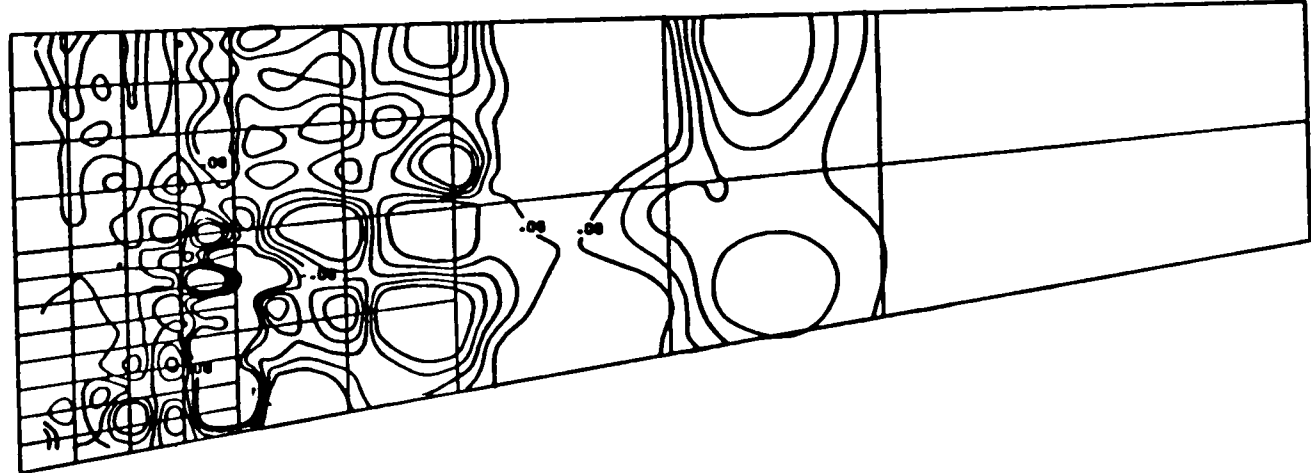


Figure 5.67 - Deformed Configuration of the Beam with $\alpha = 8.53^\circ$ ($t_w = 3\text{mm}$).



Figures 5.68.a and 5.68.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 8.53^\circ$.

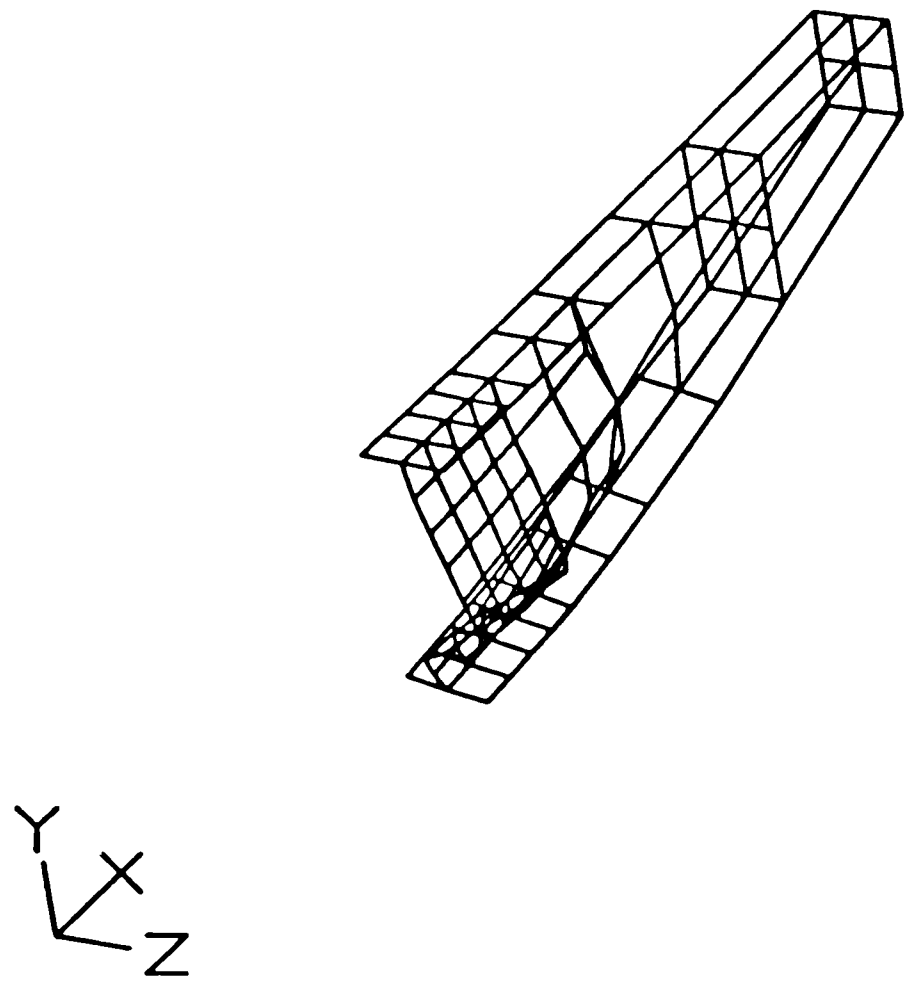
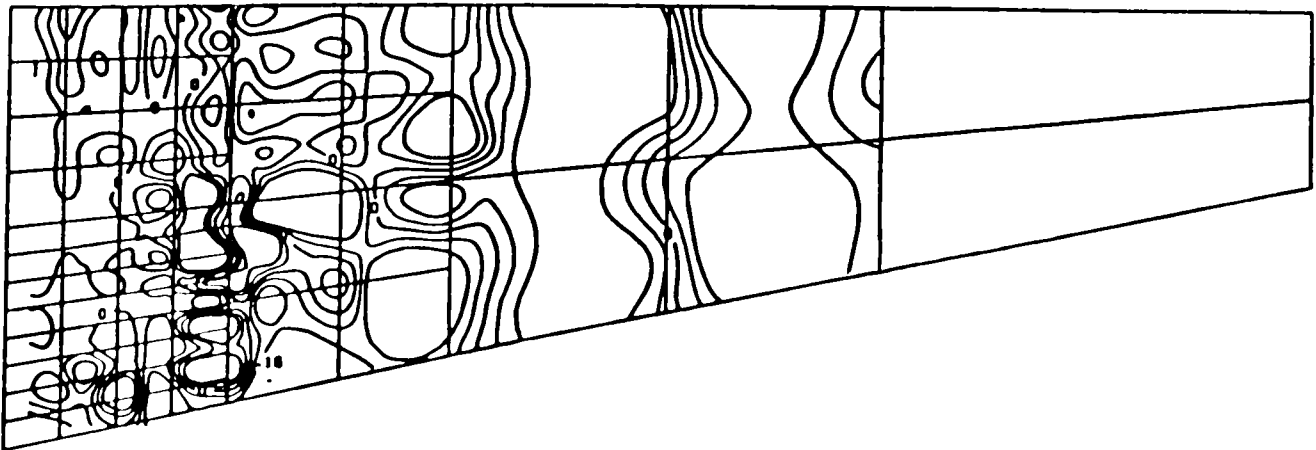
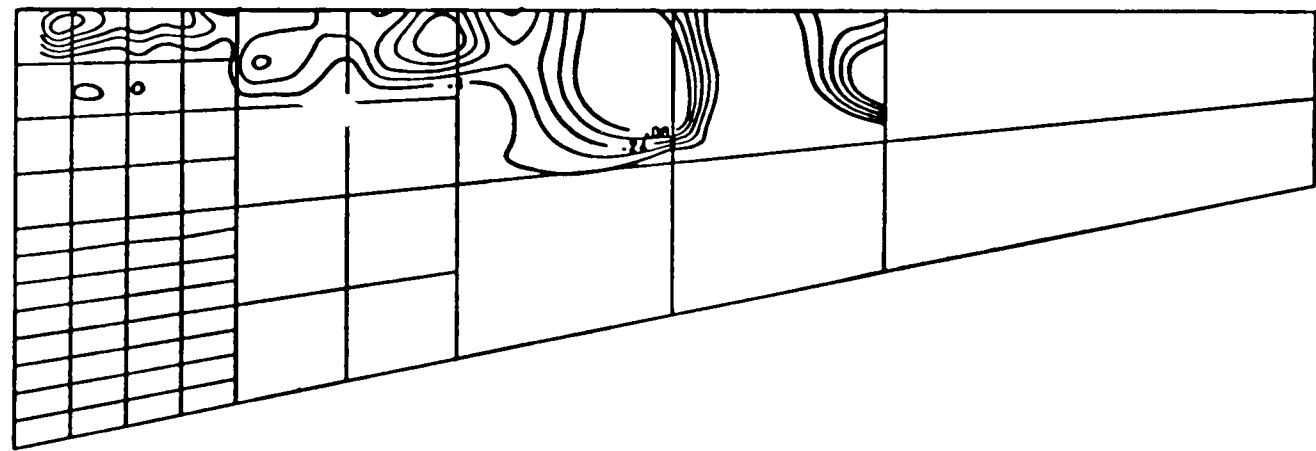


Figure 5.69 - Deformed Configuration of the Beam with $\alpha = 11.31^\circ$ ($t_w = 3\text{mm}$).



Figures 5.70.a and 5.70.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 11.31^\circ$.

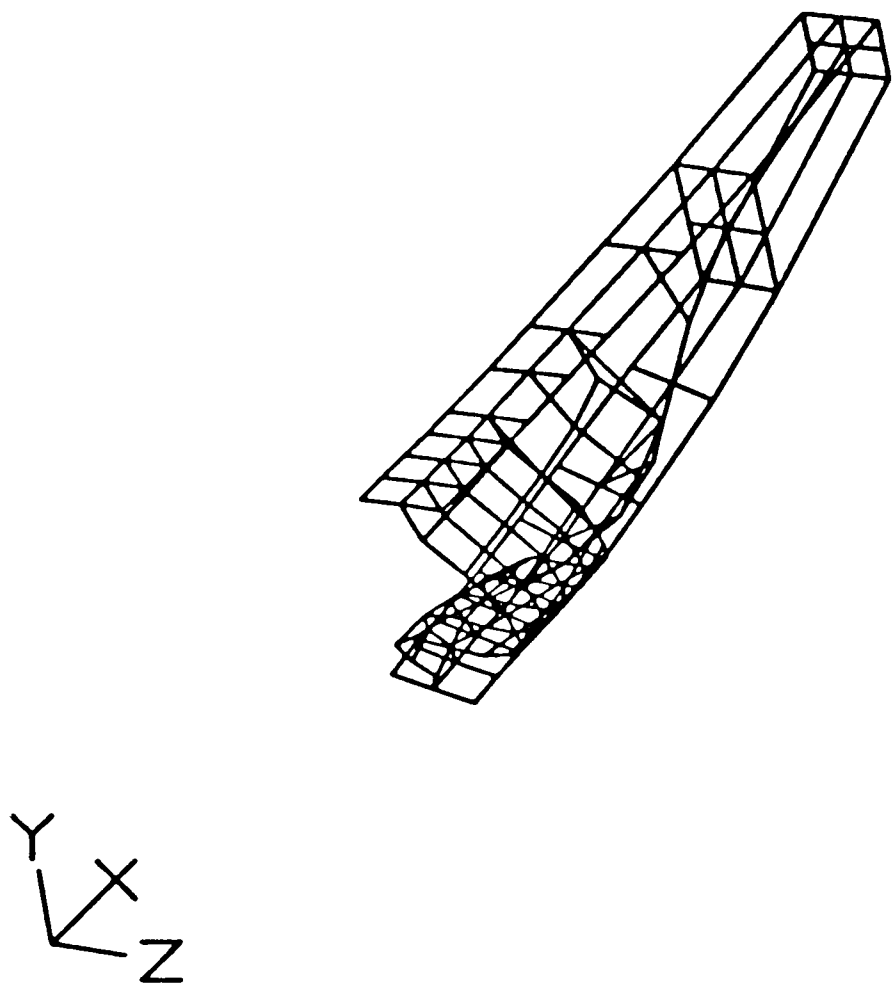
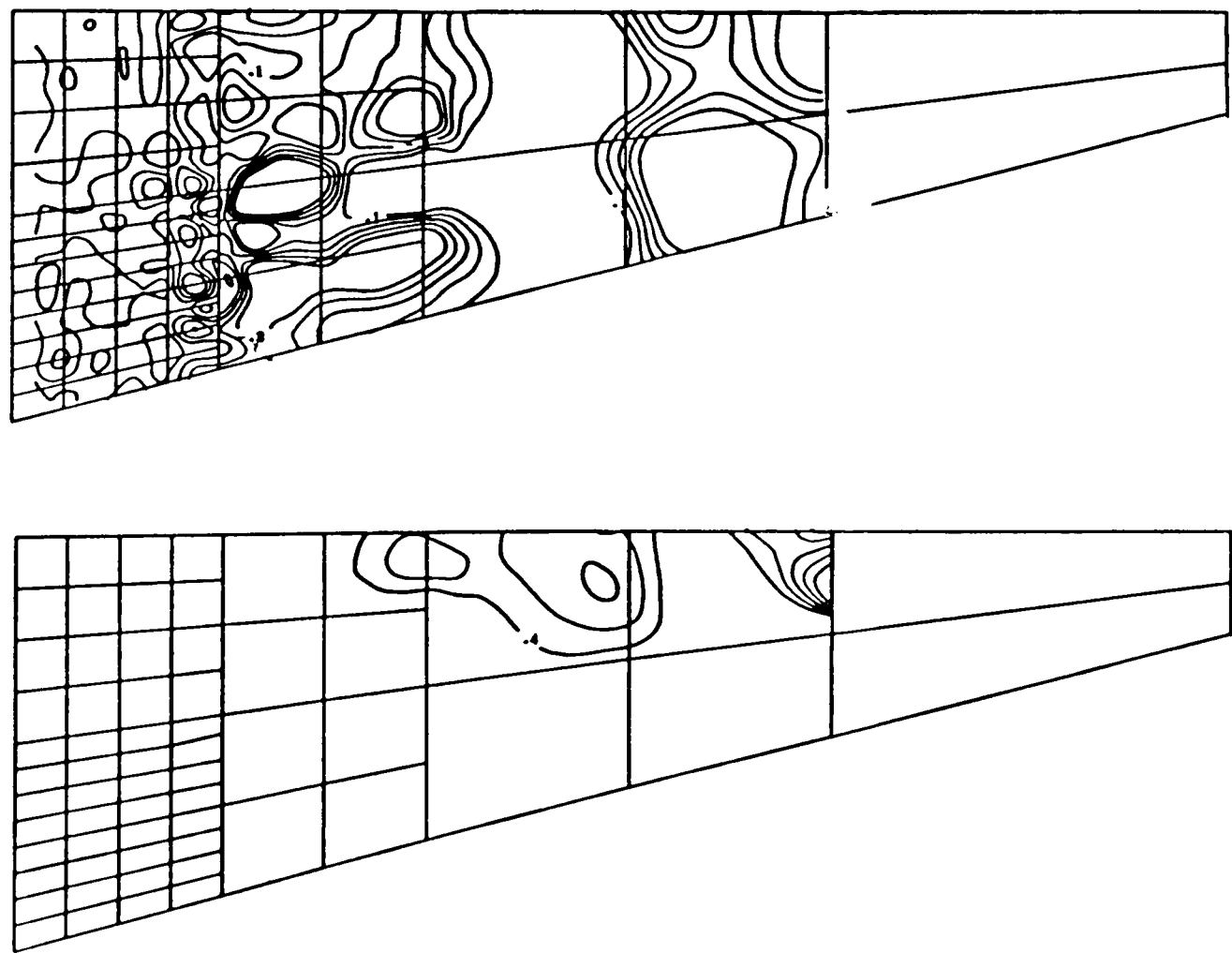


Figure 5.71 - Deformed Configuration of the Beam with $\alpha = 14.84^\circ$ ($t_w = 3\text{mm}$).



Figures 5.72.a and 5.72.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $\alpha = 14.84^\circ$.

Figures 5.73 and 5.74, page 215, delineate an interesting vertical displacement comparison for point 1 and 49 of all the beams investigated in this section. A lateral displacement comparison is portrayed in figures 5.75.a, 5.75.b, 5.76.a and 5.76.b, pages 216 and 217. Again, the different forms of collapse are well characterized by these diagrams. From these results it is evident that if a 3mm web is to be used, vertical stiffeners are required at the centre of the web for any angle of taper.

5.3.4. Summary.

It is interesting to examine the influence of web thickness for a given angle of taper. The thicknesses correspond to central h/t_w of 260.5, 156.3, 111.7 and 78.2. This type of analysis is also very useful to isolate the influence of the angle of slope on the load carrying capacity of the girders.

The results obtained for the beams with parallel flanges ($\alpha = 0^\circ$) can be better understood with the help of the vertical and lateral displacement comparisons delineated in figures 5.77, 5.78.a and 5.78.b, page 218, for point 1, 25 and 94. The girders with 10 mm and 7 mm webs have failed by the development of a plastic hinge mechanism near the vertical loaded stiffener. At the centre of the beam, the web has remained undeformed, showing no necessity for a vertical stiffener. The 3mm beam had a longitudinal buckling collapse.

The angle of 1.15° ($\alpha = 1/50$) produced the comparisons shown in figures 5.79, 5.80.a and 5.80.b, page 219. The beams with thicknesses equal to 7 and 10mm have failed by the plastic hinge mechanism. The beam with a 5mm web had a collapse associated with an inelastic vertical web buckling at the centre of the beam, while the beam with a 3mm web failed by elastic vertical web buckling. These last two beams require a vertical stiffener to improve their loading performance.

The taper angle of 1.43° ($\alpha = 1/40$) generated the deflection diagrams displayed in figures 5.81, 5.82.a and 5.82.b, page 220. As in the previous case, the 7 and 10mm beams failed by plastic hinge collapse as already mentioned. The 5 and 3mm beams have collapsed under an inelastic vertical web buckling failure. They both require vertical stiffeners at the centre part of the beam. The angle of 1.91° ($\alpha = 1/30$) was only investigated with 10 and 7mm, as shown in figures 5.83, 5.84.a and 5.84.b, page 221. They have failed due to the plastic mechanism developed near the panel edge.

The same pattern of failure was developed by the beam with a slope angle of 2.86° ($\alpha = 1/20$). Their results are displayed in figures 5.85, 5.86.a and 5.86.b, page 222. The two higher thicknesses have failed by plastic hinge collapse. The other two had their web buckled inelastically in the vertical direction, showing the need for vertical stiffeners at the centre of the beam.

The angle of 8.53° ($\alpha = 1/6.67$) produced the comparisons displayed in figures 5.87, 5.88.a and 5.88.b, page 223. The 10 and 7mm beam have still failed due to the formation of a plastic hinge mechanism near the loaded vertical stiffener. On the other hand, the 5 and 3mm beams developed the elastic vertical buckling failure explained previously. These two last beams require a vertical stiffener to improve their performance.

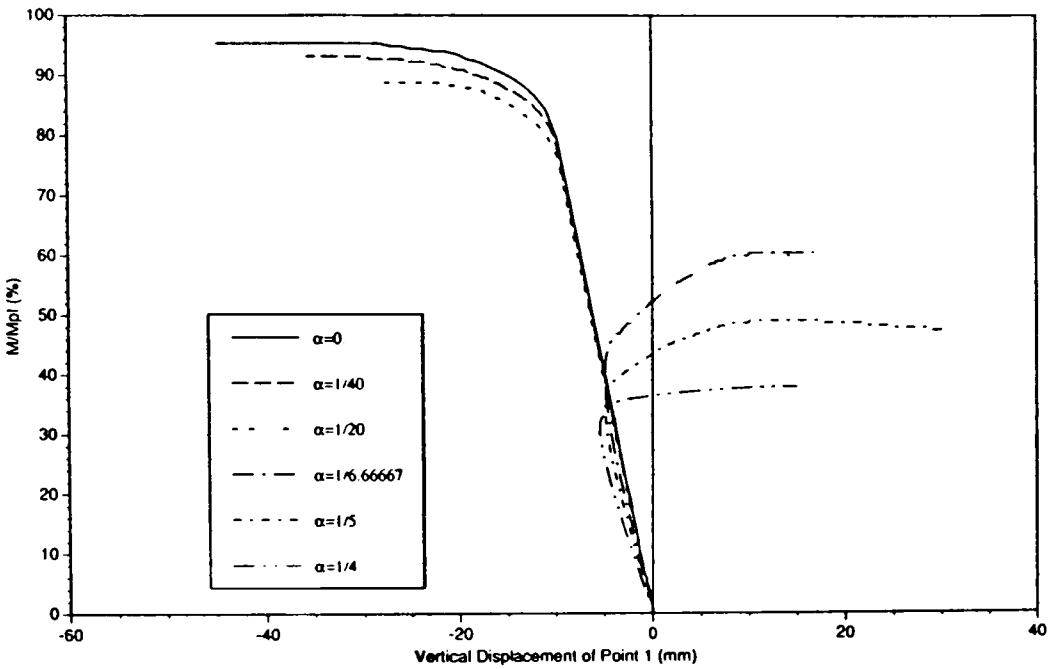


Figure 5.73 - Vertical Displacement Comparison for Point 1 ($t_w = 3\text{mm}$).

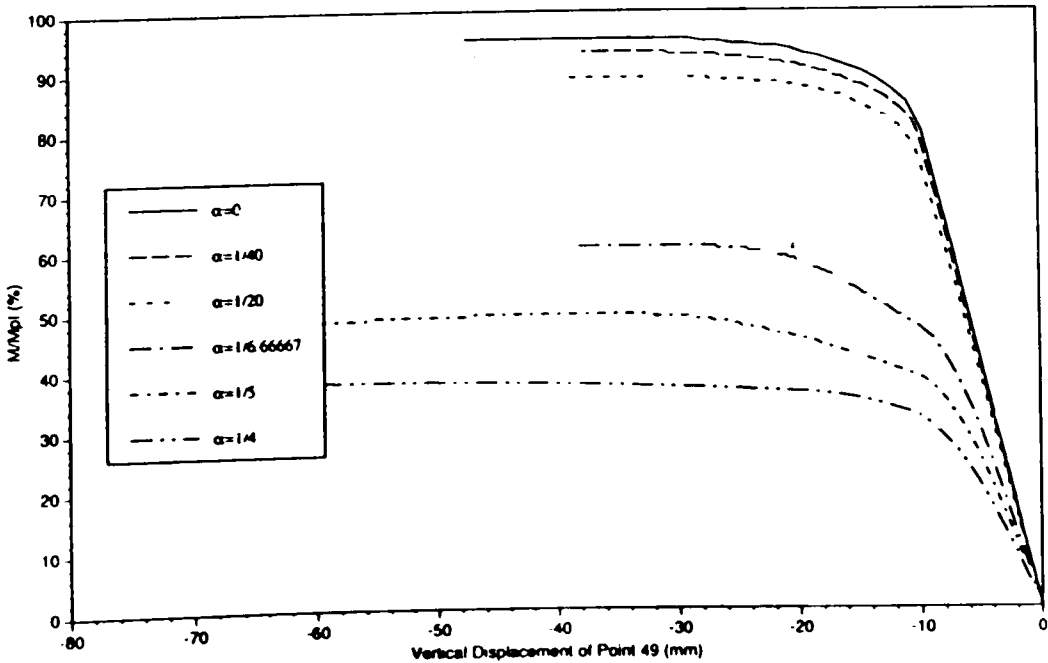
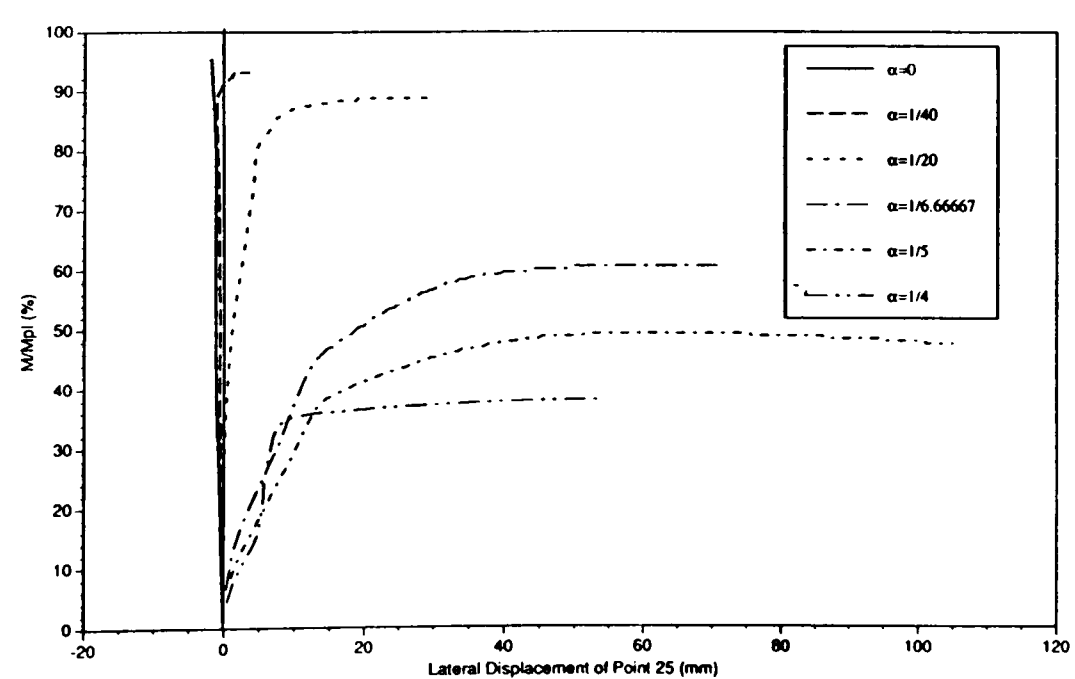
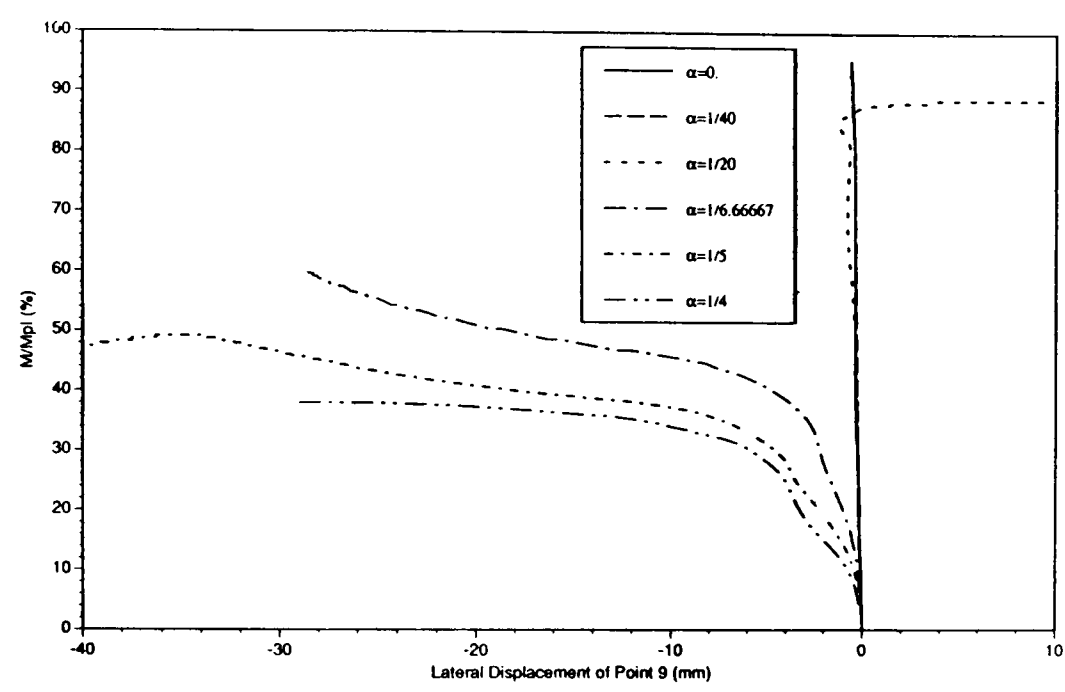
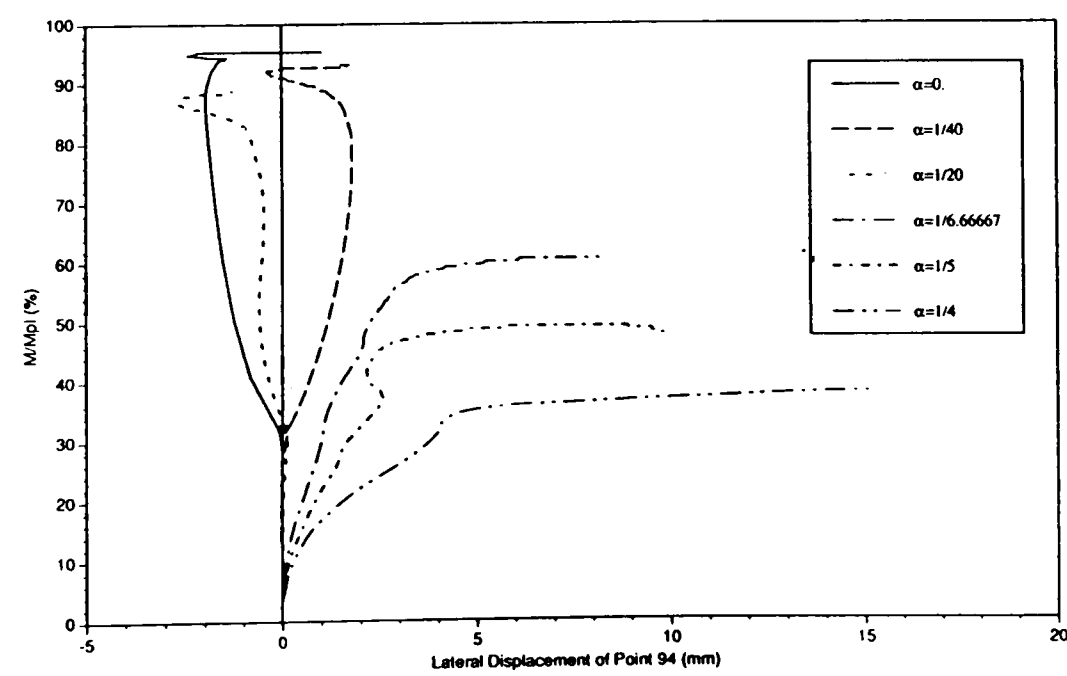
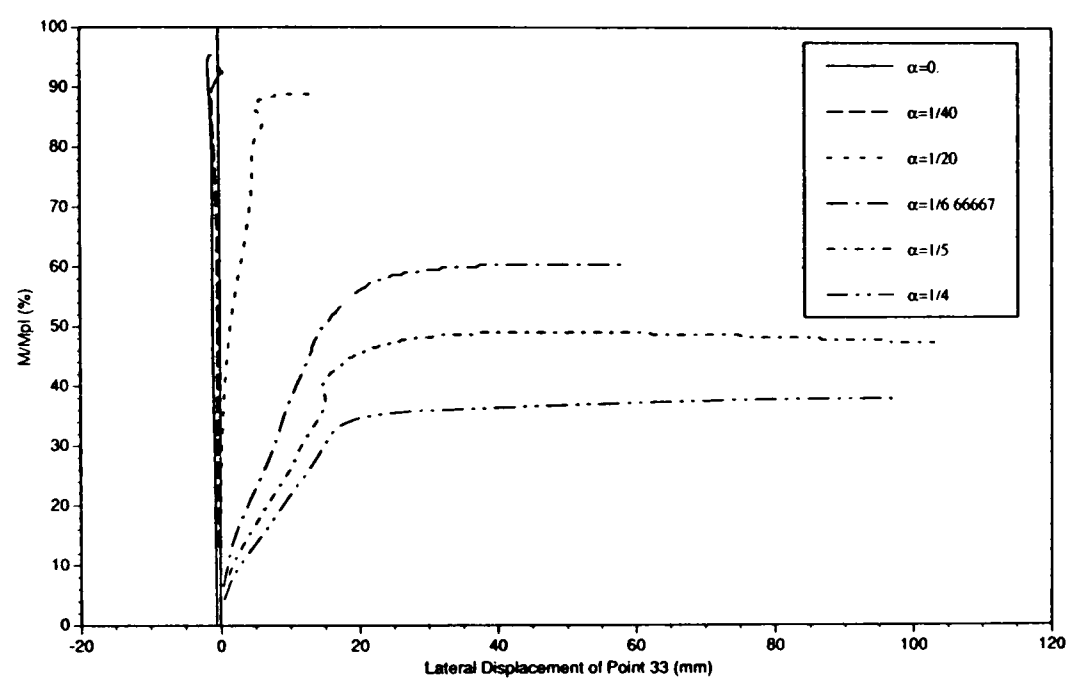


Figure 5.74 - Vertical Displacement Comparison for Point 49 ($t_w = 3\text{mm}$).



Figures 5.75.a and 5.75.b - Lateral Displacement Comparison for Points 9 and 25 ($t_w = 3\text{mm}$).



Figures 5.76.a and 5.76.b - Lateral Displacement Comparison for Points 33 and 94 ($t_w = 3\text{mm}$).

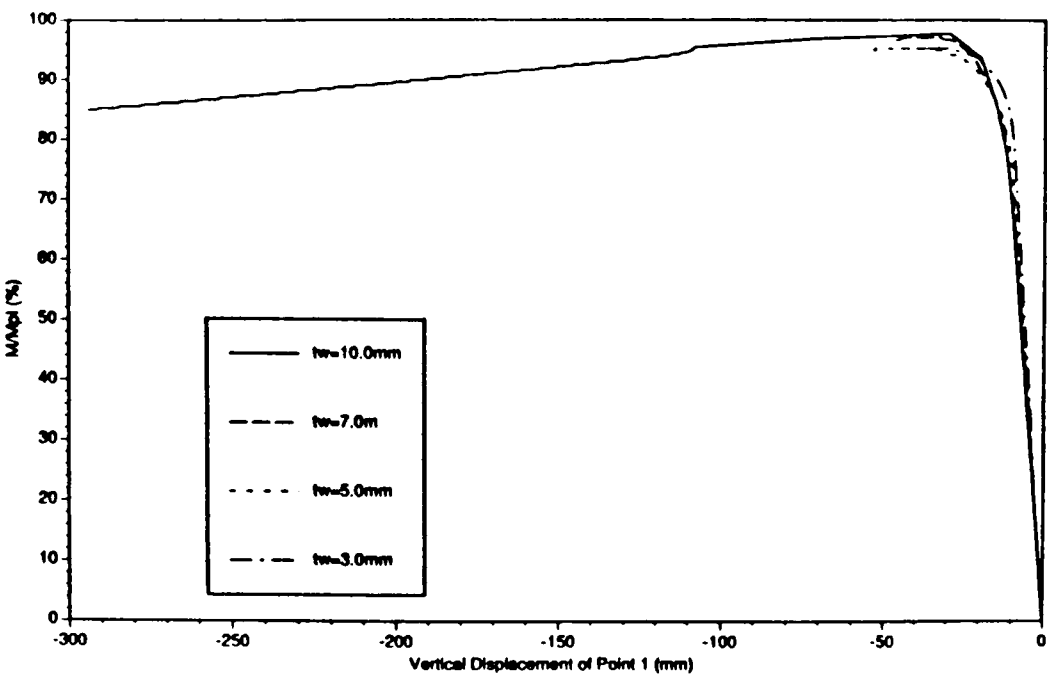
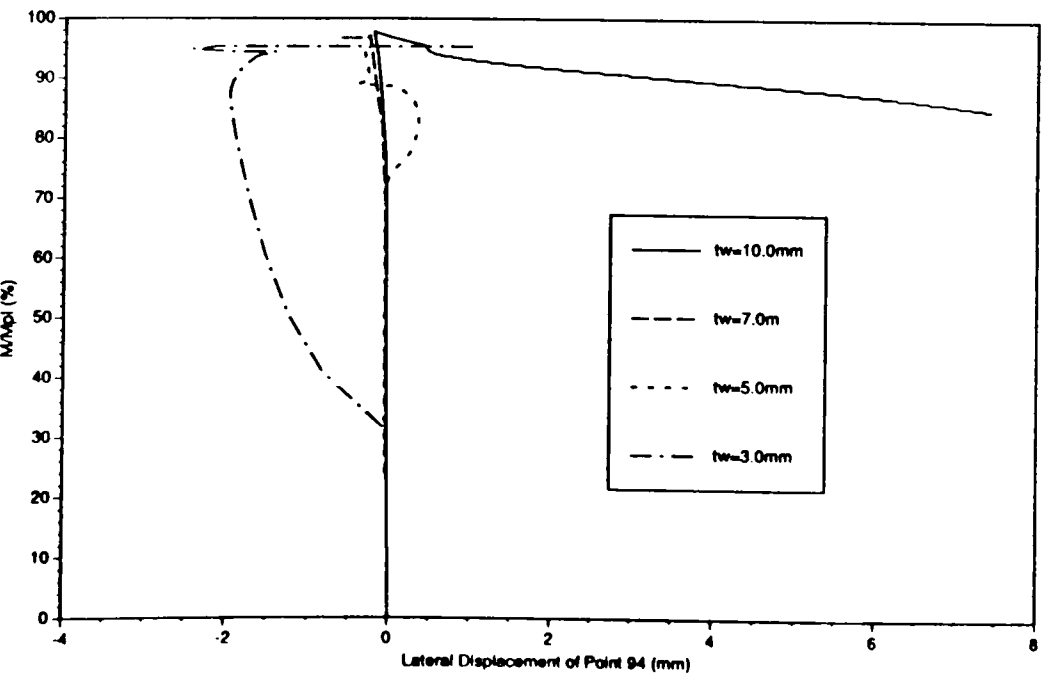
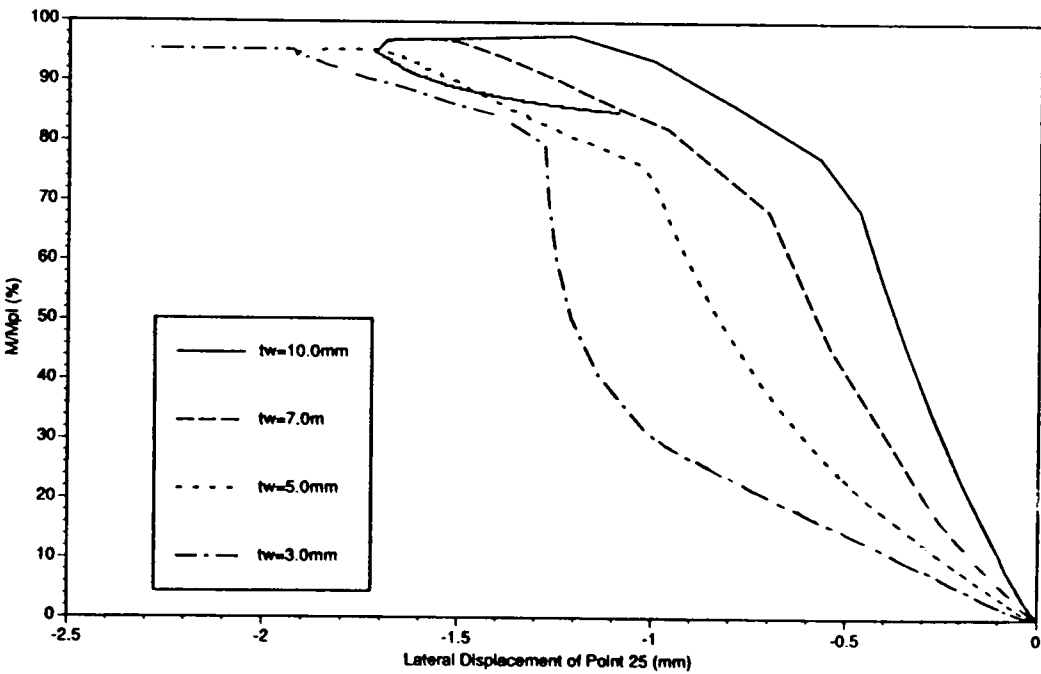


Figure 5.77 - Vertical Displacement Comparison for Point 1 ($\alpha = 0^\circ$).



Figures 5.78.a and 5.78.b - Lateral Displacement Comparison for Points 25 and 94 ($\alpha = 0^\circ$).

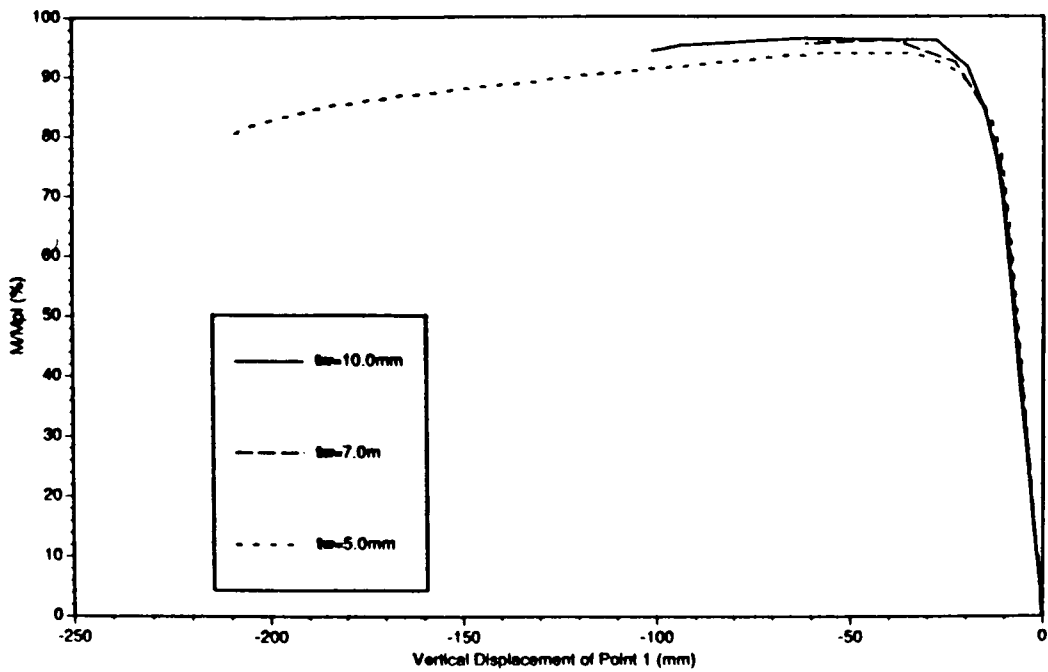
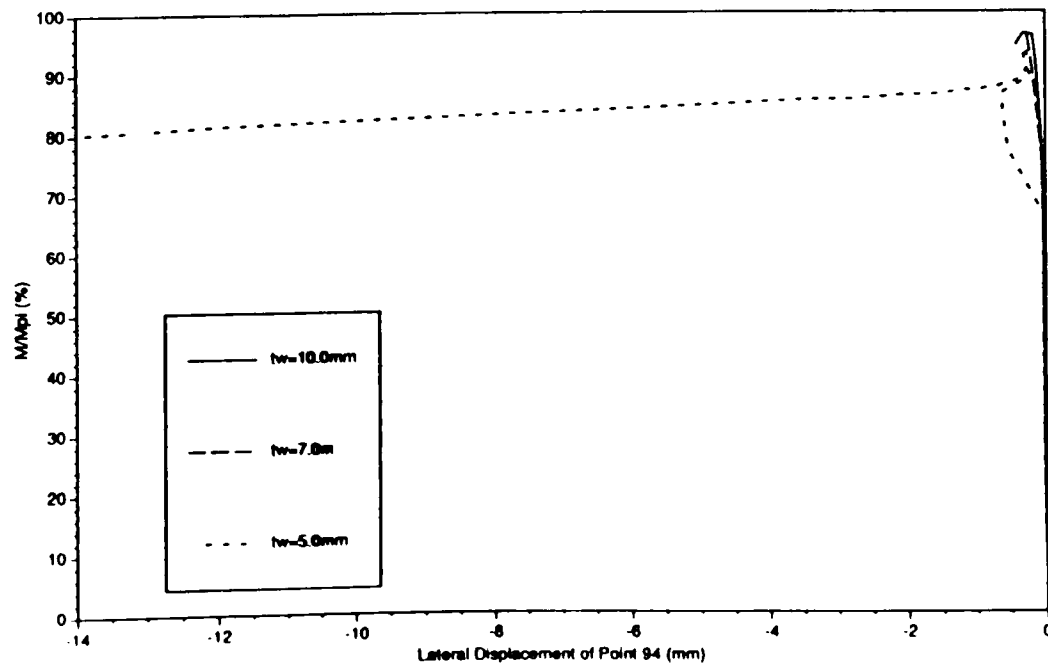
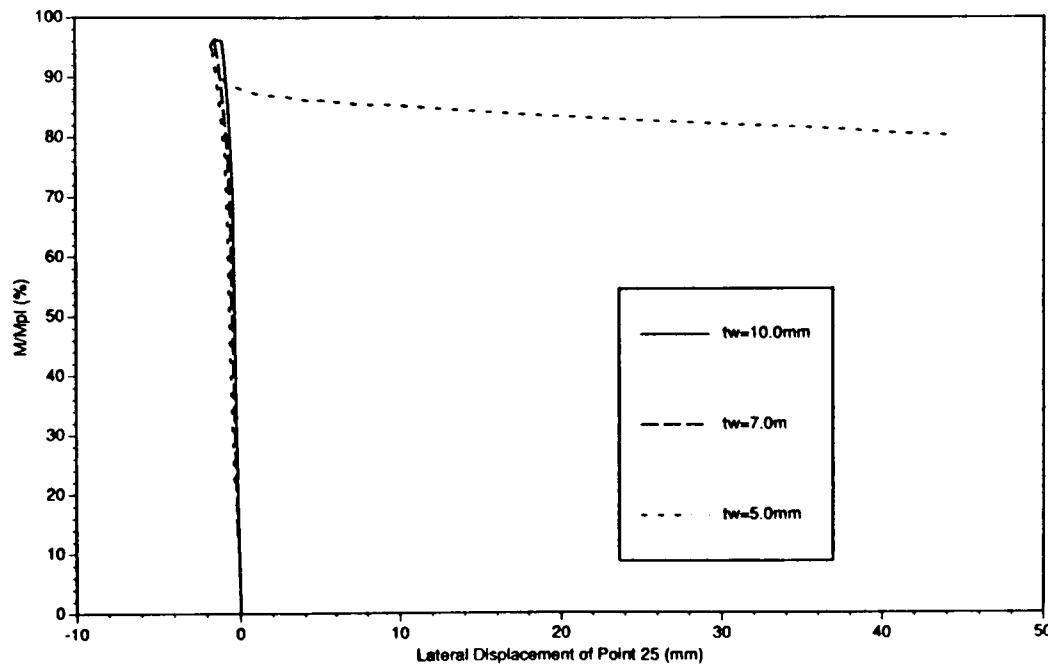


Figure 5.79 - Vertical Displacement Comparison for Point 1 ($\alpha = 1.14^\circ$).



Figures 5.80.a and 5.80.b - Lateral Displacement Comparison for Points 25 and 94 ($\alpha = 1.14^\circ$).

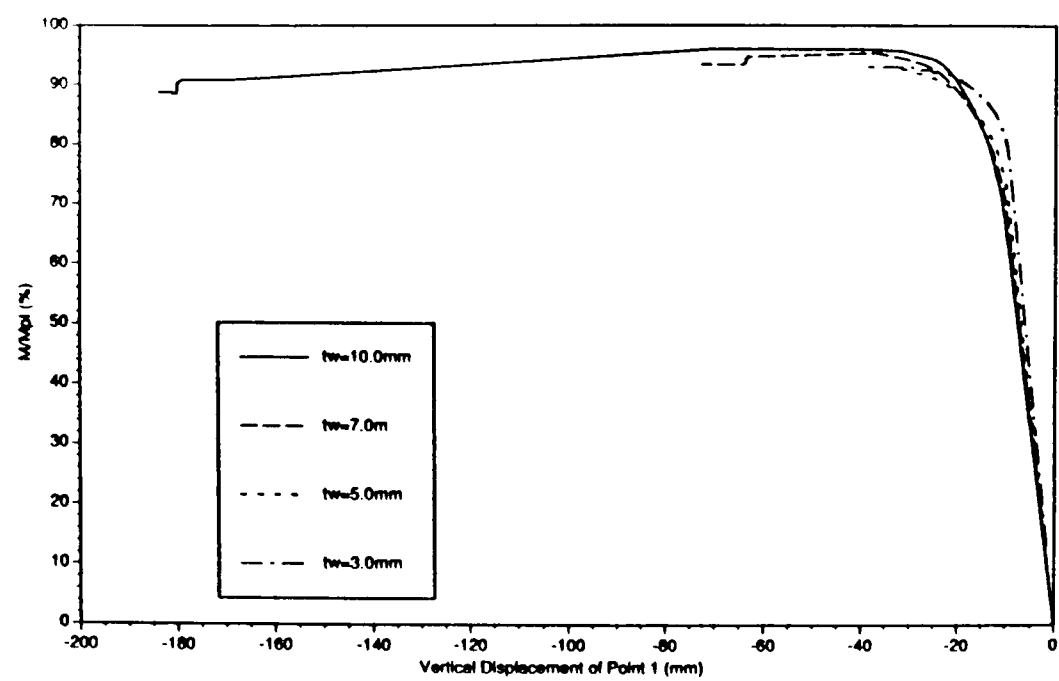
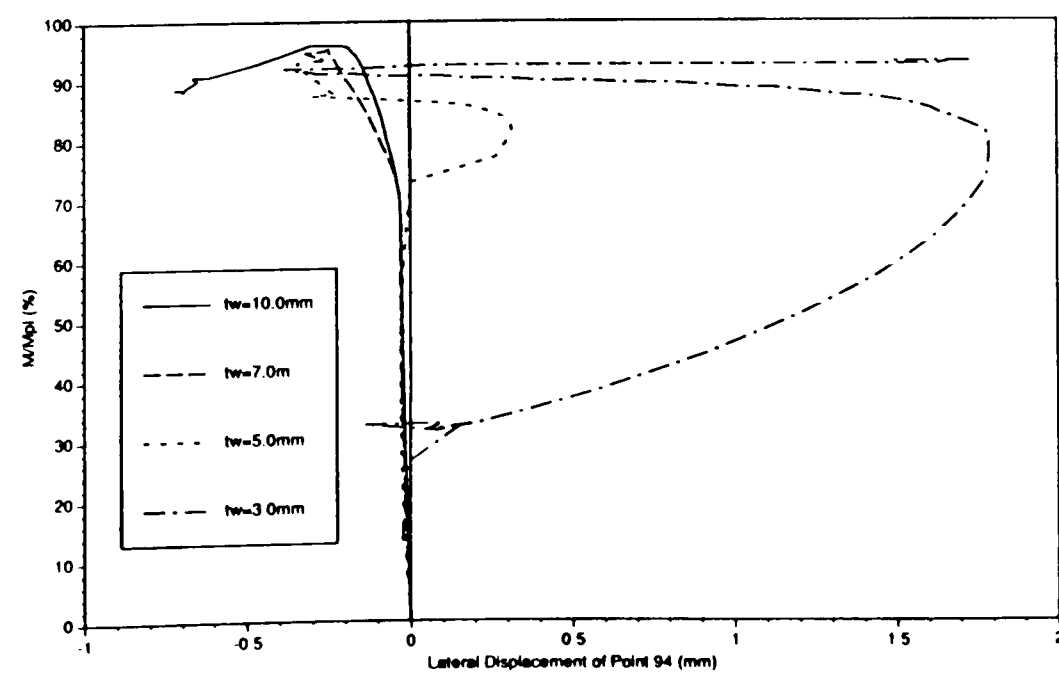
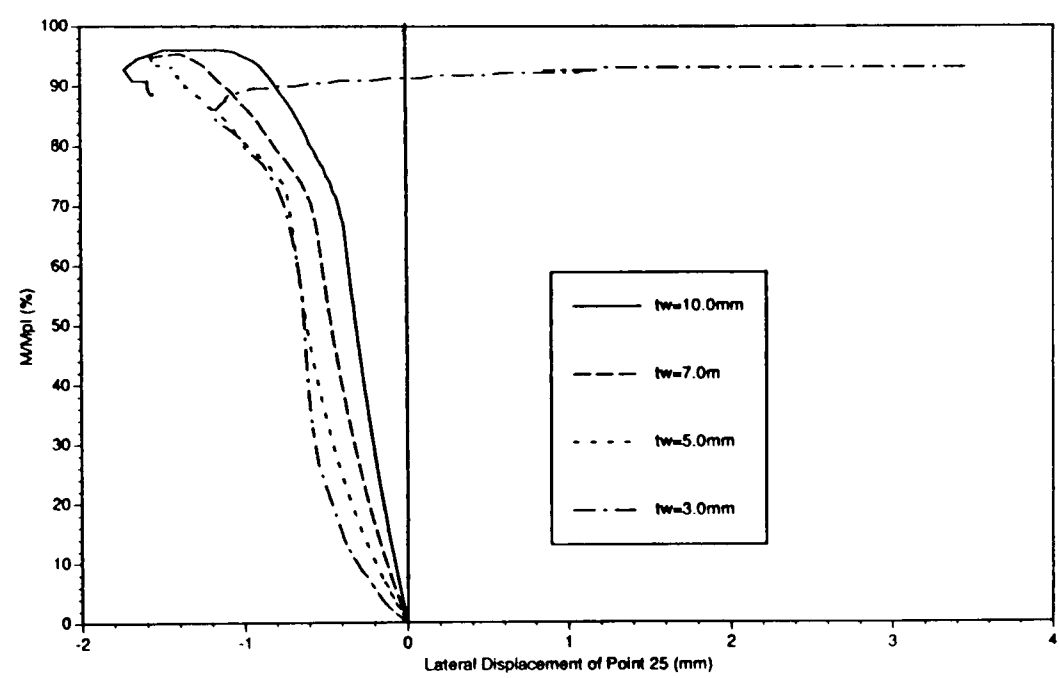


Figure 5.81 - Vertical Displacement Comparison for Point 1 ($\alpha = 1.43^\circ$).



Figures 5.82.a and 5.82.b - Lateral Displacement Comparison for Points 25 and 94 ($\alpha = 1.43^\circ$).

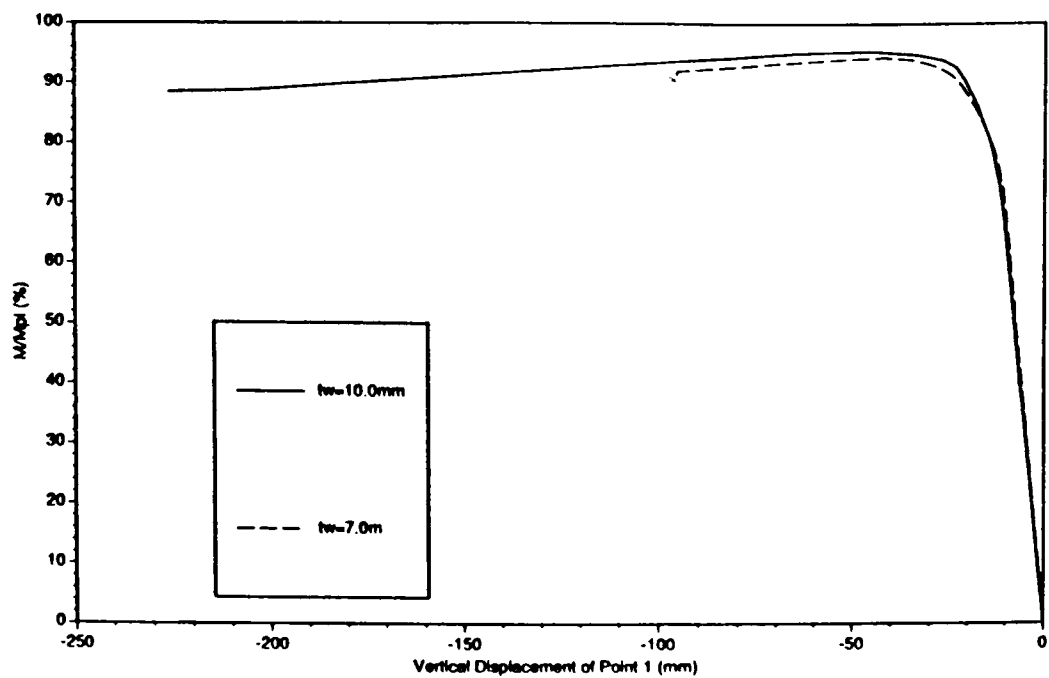
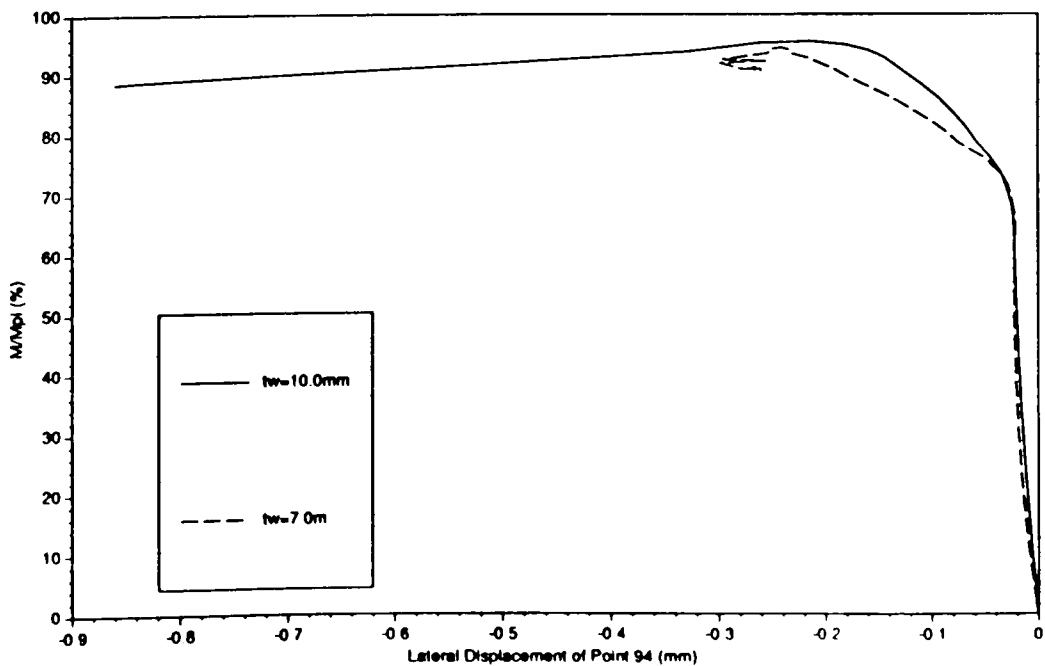
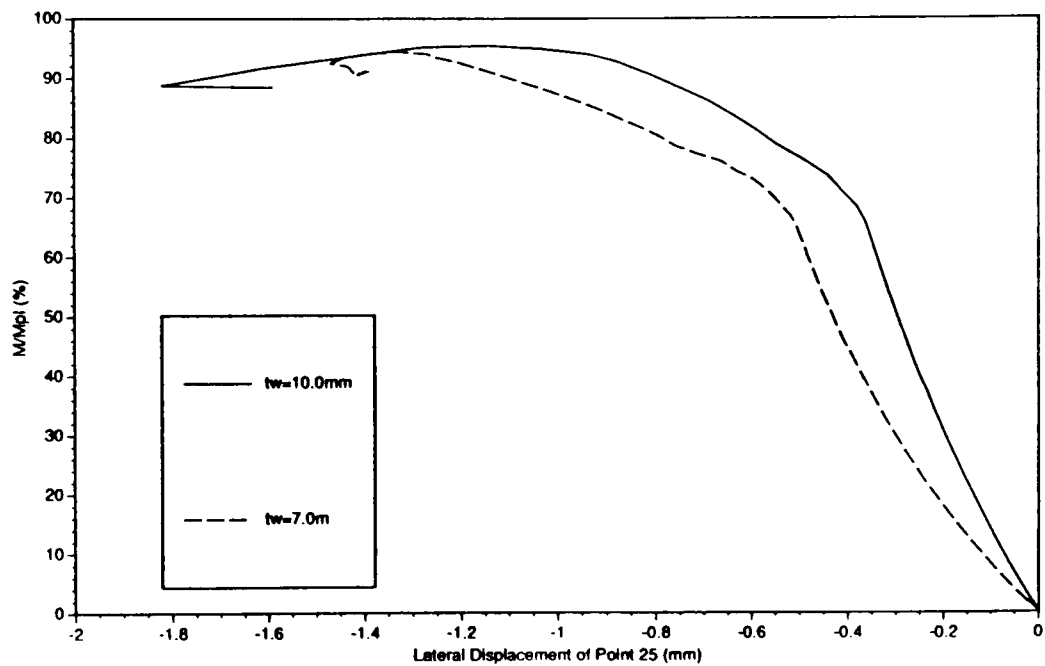


Figure 5.83 - Vertical Displacement Comparison for Point 1 ($\alpha = 1.91^\circ$).



Figures 5.84.a and 5.84.b - Lateral Displacement Comparison for Points 25 and 94 ($\alpha = 1.91^\circ$).

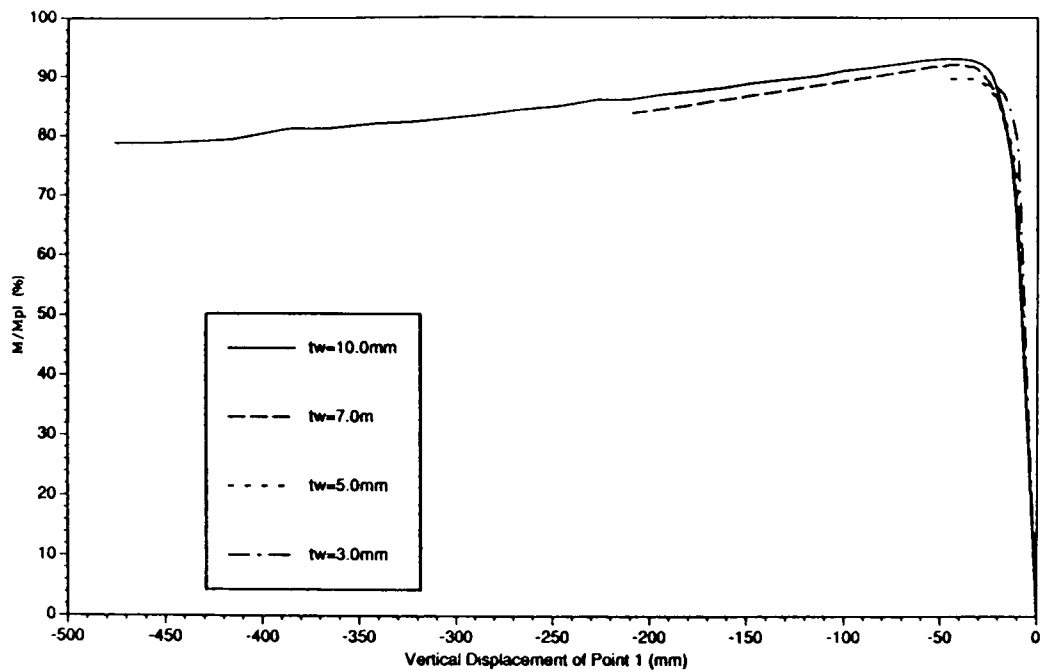
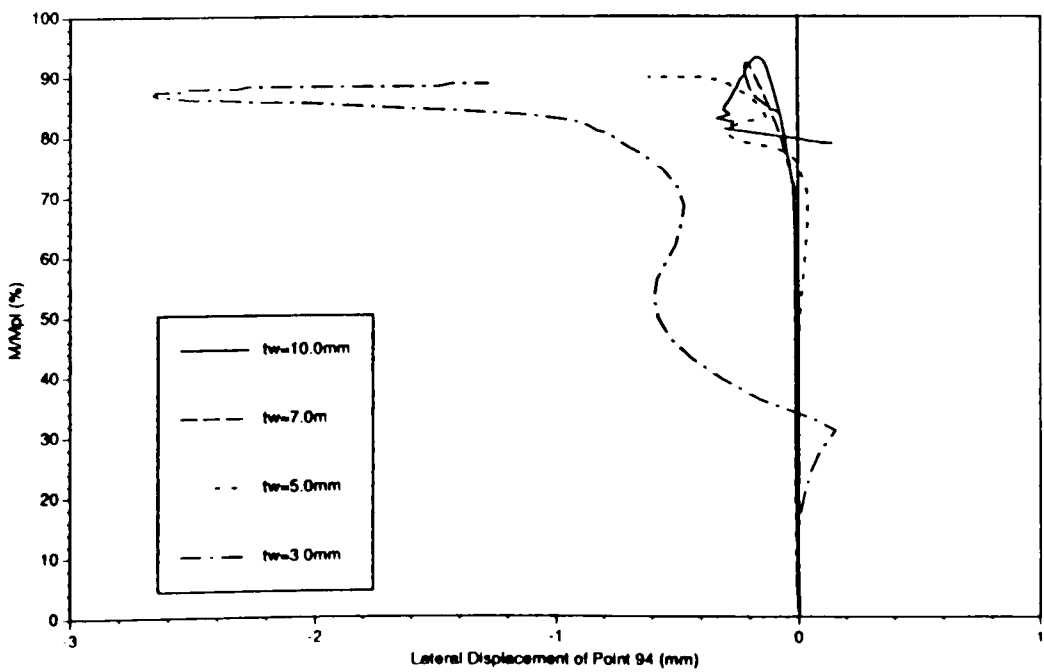
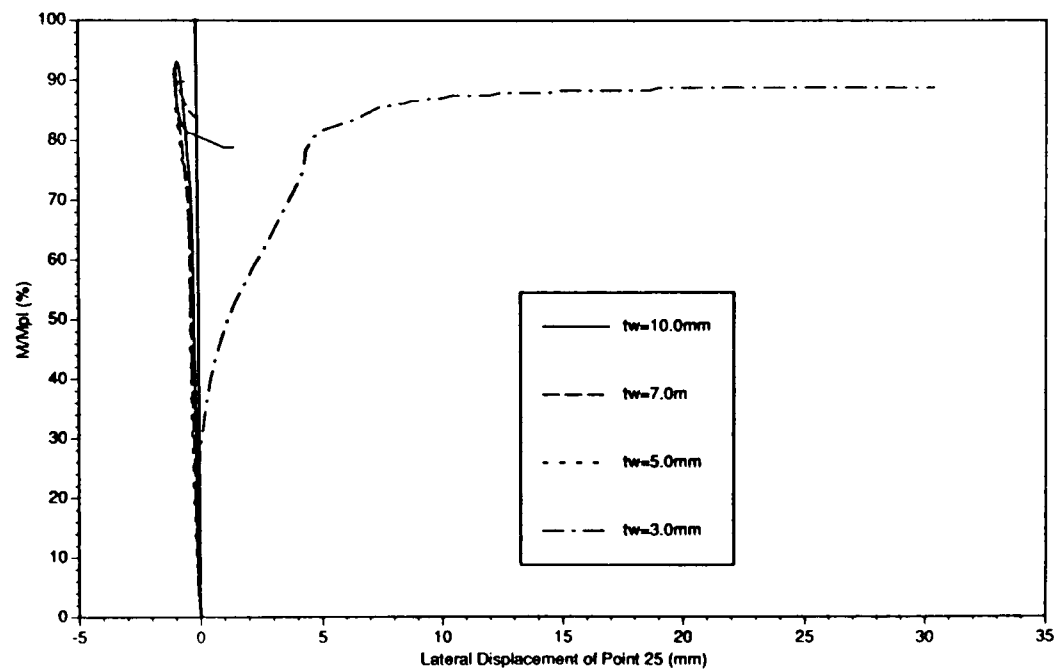


Figure 5.85 - Vertical Displacement Comparison for Point 1 ($\alpha = 2.86^\circ$).



Figures 5.86.a and 5.86.b - Lateral Displacement Comparison for Points 25 and 94 ($\alpha = 2.86^\circ$).

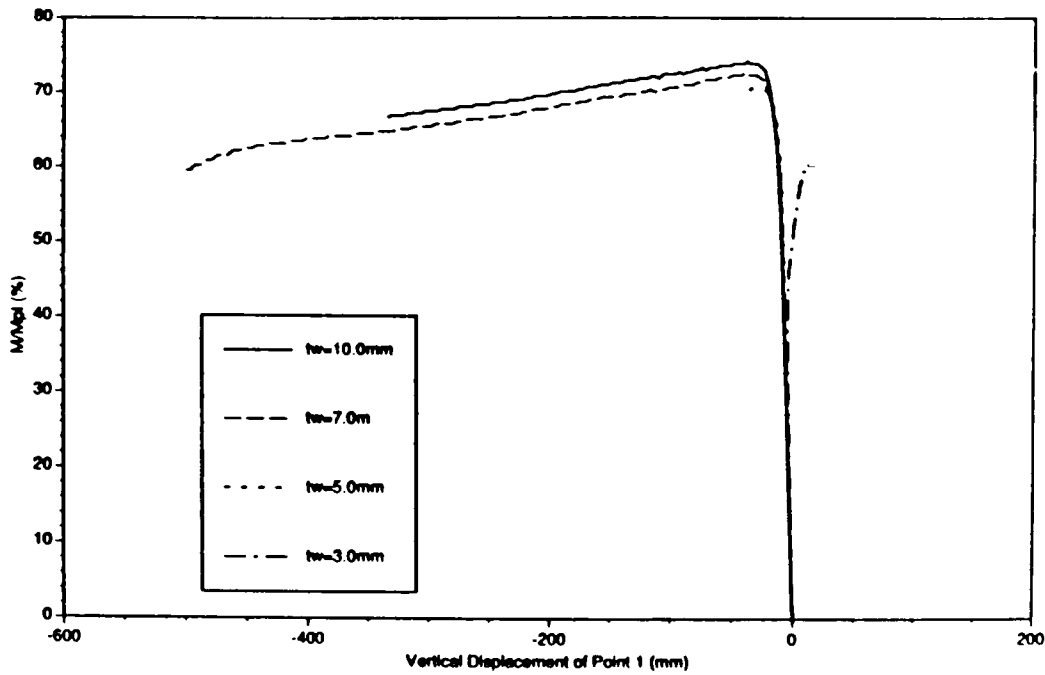
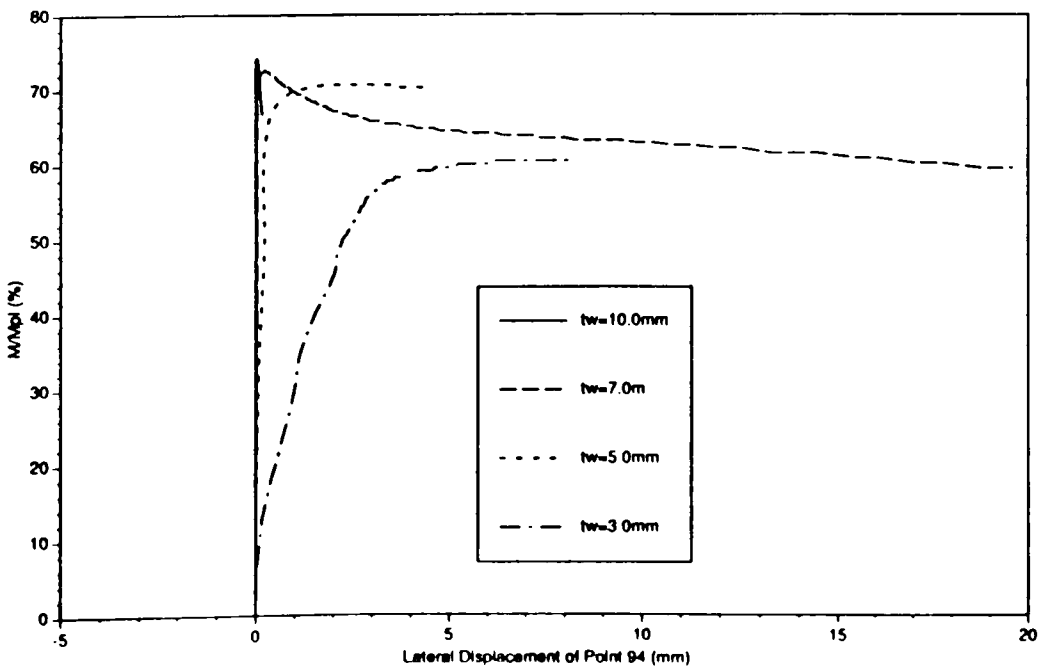
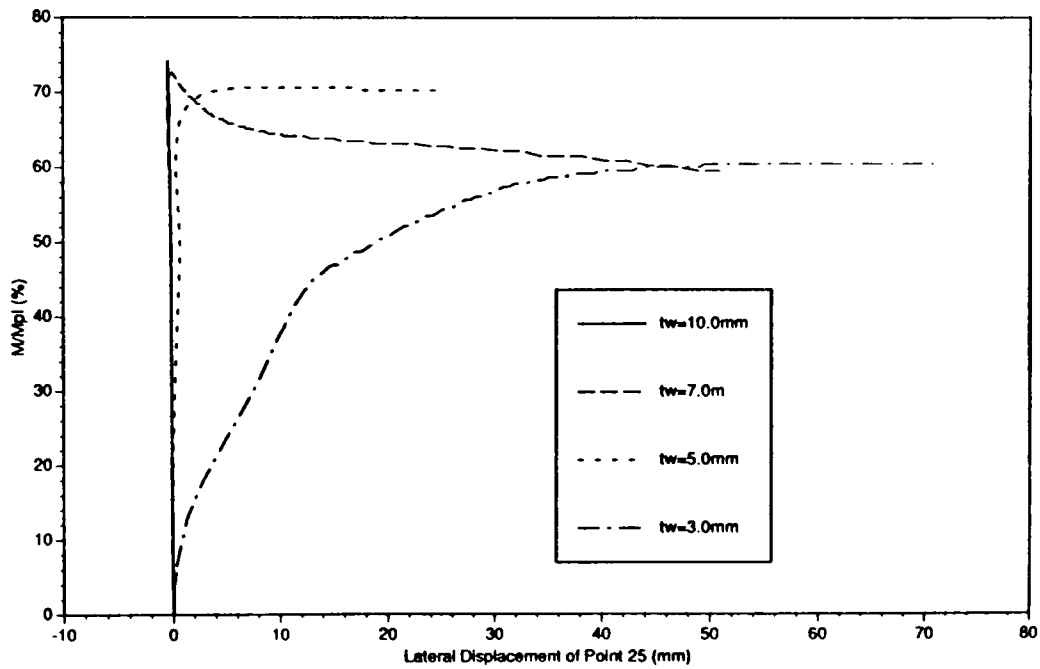


Figure 5.87 - Vertical Displacement Comparison for Point 1 ($\alpha = 8.53^\circ$).



Figures 5.88.a and 5.88.b - Lateral Displacement Comparison for Points 25 and 94 ($\alpha = 8.53^\circ$).

Very similar kinds of failures were reached by the beams with a taper of 11.31° ($\alpha = 1/5$). Their results are portrayed in figures 5.89, 5.90.a and 5.90.b, page 225, where some interesting vertical and lateral displacement comparisons are drawn. Again, the same plastic hinge failure was the cause of collapse for the 10 and 7mm beams. Vertical stiffeners were also required to minimize the effects of the elastic vertical web buckling failure developed by the other two beams.

Finally, figures 5.91, 5.92.a and 5.92.b, page 226, delineate the vertical and lateral displacement comparisons for the girders with an angle of slope of 14.84° ($\alpha = 1/4$). The 10mm beam has collapsed by the plastic hinge mechanism, as in the previous cases. The 7mm beam has buckled inelastically in the vertical direction. The last two beams have elastically buckled in the vertical direction. All the girders that developed the vertical buckling would have their performance significantly enhanced if a vertical web stiffener was used at the critical centre point of the beam.

5.4. INFLUENCE OF THE TENSION FLANGE AREA.

The finite element simulations of this section are very similar to the examples studied in the previous section. They possess the same loading and support configurations, initial imperfections and material properties. The four thicknesses of the web used were 3, 5, 7 and 10mm, while the angle of slope was fixed at 5.71° ($\alpha = 1/10$).

The beams investigated in this section again had a total span and moment lever arm of 4800 and 800mm. The widths of the top and bottom flanges were 250mm. The maximum height at the centre of the beam was 800mm. The yield stresses of the web, top flange and bottom flange were again taken as: 420MPa, 391MPa and 398MPa respectively. The only geometric change from the beams of the previous section was the thickness of the top and bottom flanges, increased to 50 and 25mm respectively.

This section is organized as follows. First the results of the four beams studied in this section will be described, then a comparison with the interpolated results taken from the conclusions of the previous section will be presented to assess the influence of the area of the tension flange.

5.4.1 Tension Flange Area Equal to 6250mm².

The mode of failure associated with the beam with the 10mm web is illustrated in figure 5.93, page 227. A quick inspection of this diagram indicates that the collapse was due to the development of a plastic hinge mechanism near the edge vertical stiffener. For the point of maximum load, the web stress configuration is depicted in figures 5.94.a and 5.94.b, page 227. Despite the presence of very high compressive stresses near the bottom flange at the centre of the beam, only very small lateral deflections developed in the web. With that in mind, it is possible to conclude that stiffeners were not required in this case.

The girder with a 7mm web has failed by a plastic vertical buckling of the web. Its deformed configuration at the point of maximum load is portrayed in figure 5.95.a, page 228. In this diagram, only very small lateral web deflections are noticeable. However, at a further loading stage in the unloading path, the web has become very distorted, as can be seen in figure 5.95.b, page 228. The stress pattern in the web is shown in figures 5.96.a and 5.96.b, page 228. The compressive stresses at the critical point, near the bottom flange at the centre of the beam, increased

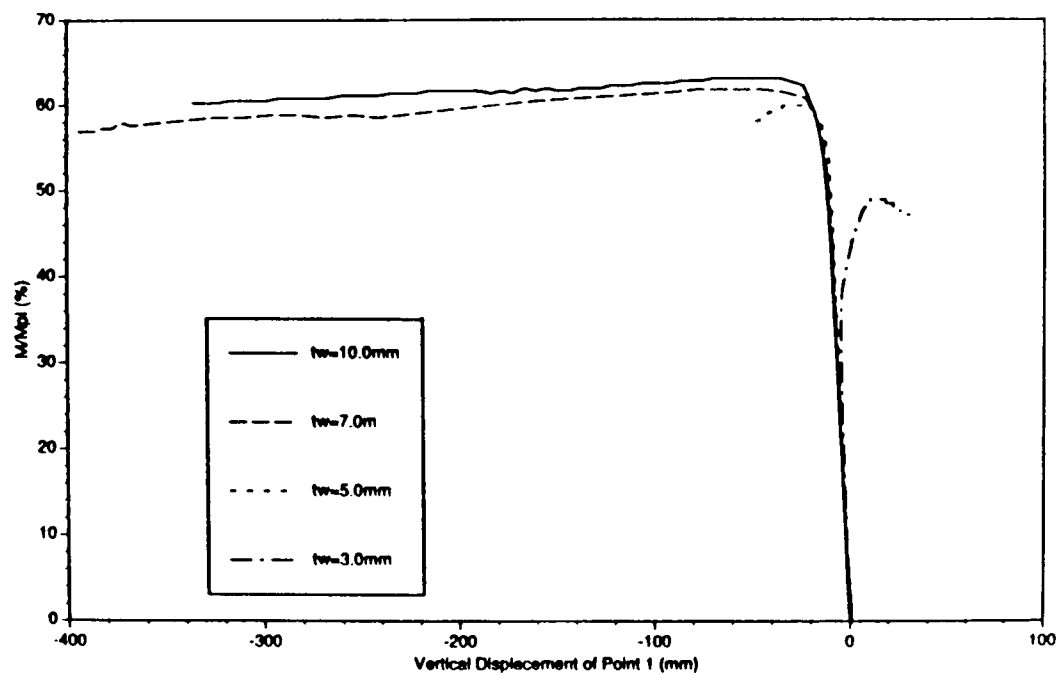
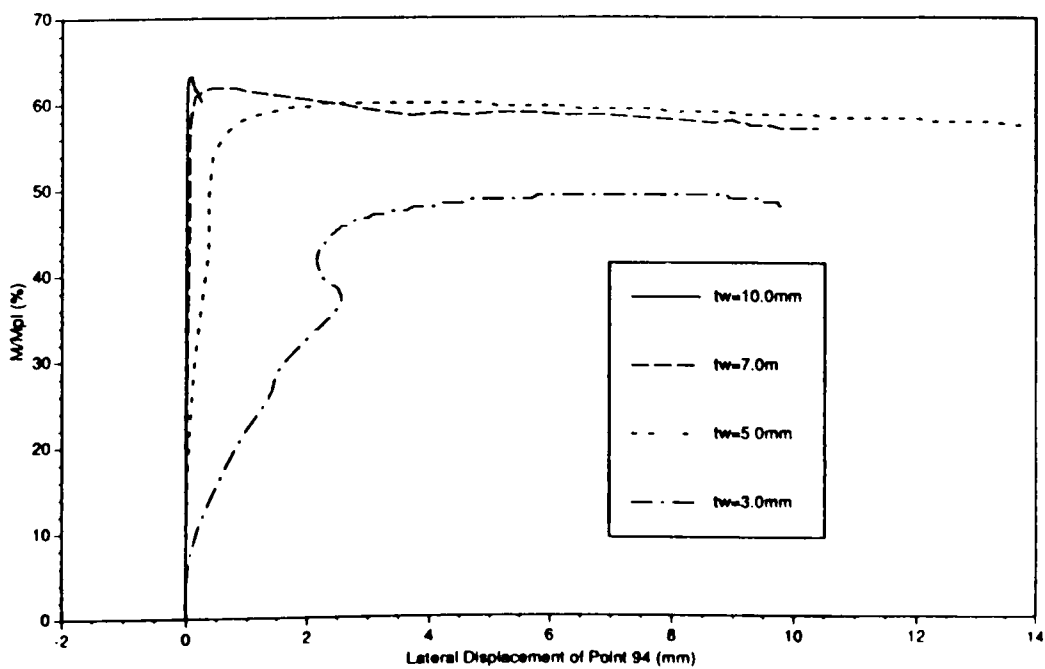
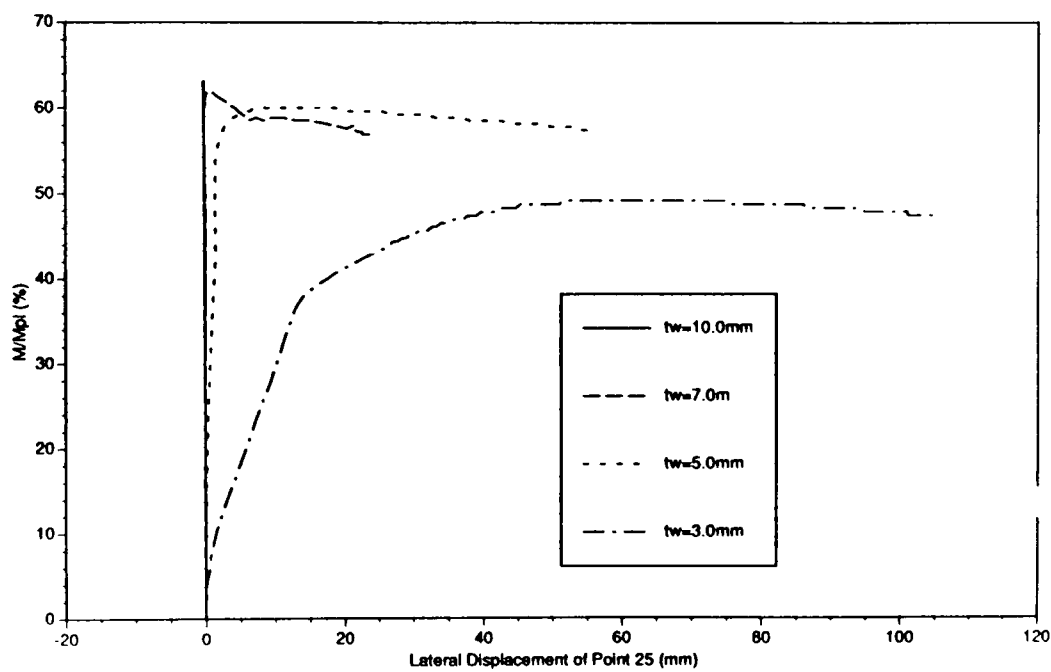


Figure 5.89 - Vertical Displacement Comparison for Point 1 ($\alpha = 11.31^\circ$).



Figures 5.90.a and 5.90.b - Lateral Displacement Comparison for Points 25 and 94 ($\alpha = 11.31^\circ$).

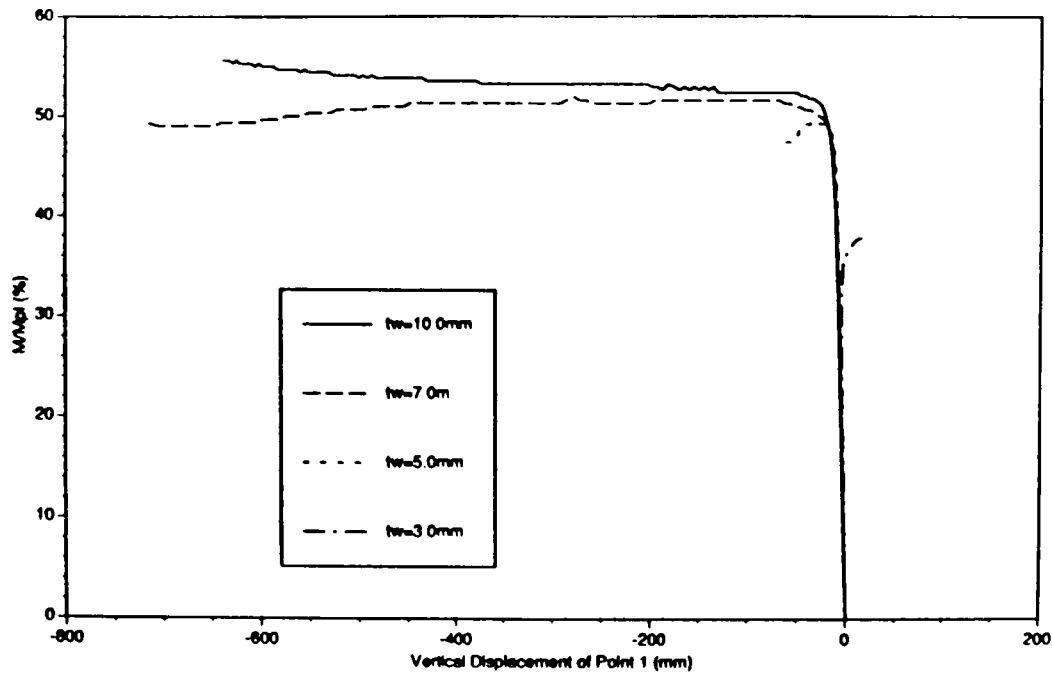
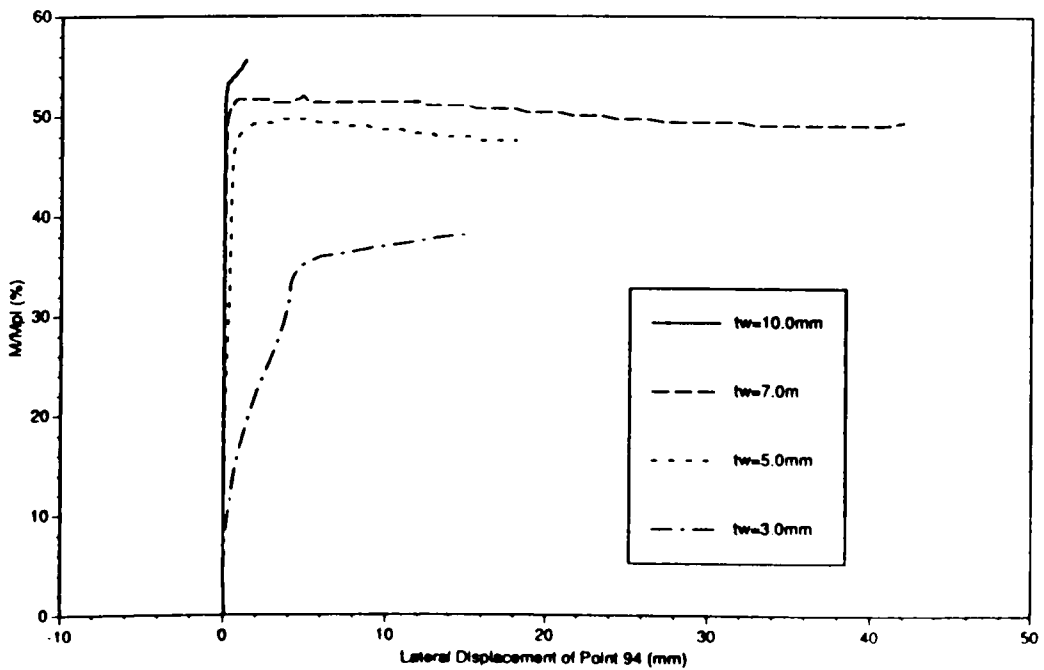
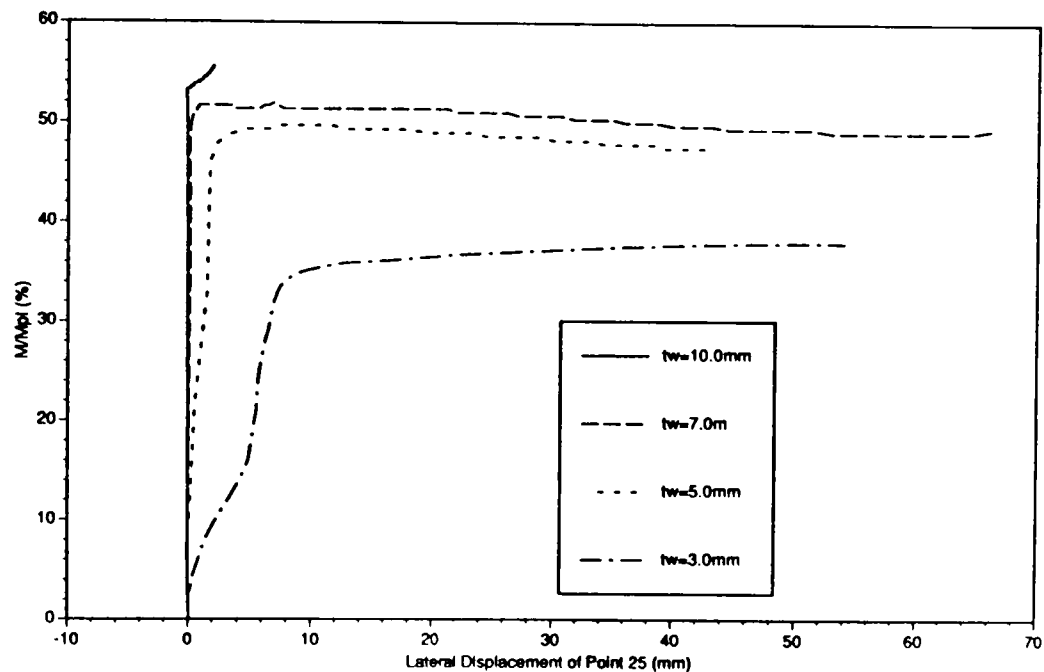


Figure 5.91 - Vertical Displacement Comparison for Point 1 ($\alpha = 14.84^\circ$).



Figures 5.92.a and 5.92.b - Lateral Displacement Comparison for Points 25 and 94 ($\alpha = 14.84^\circ$).

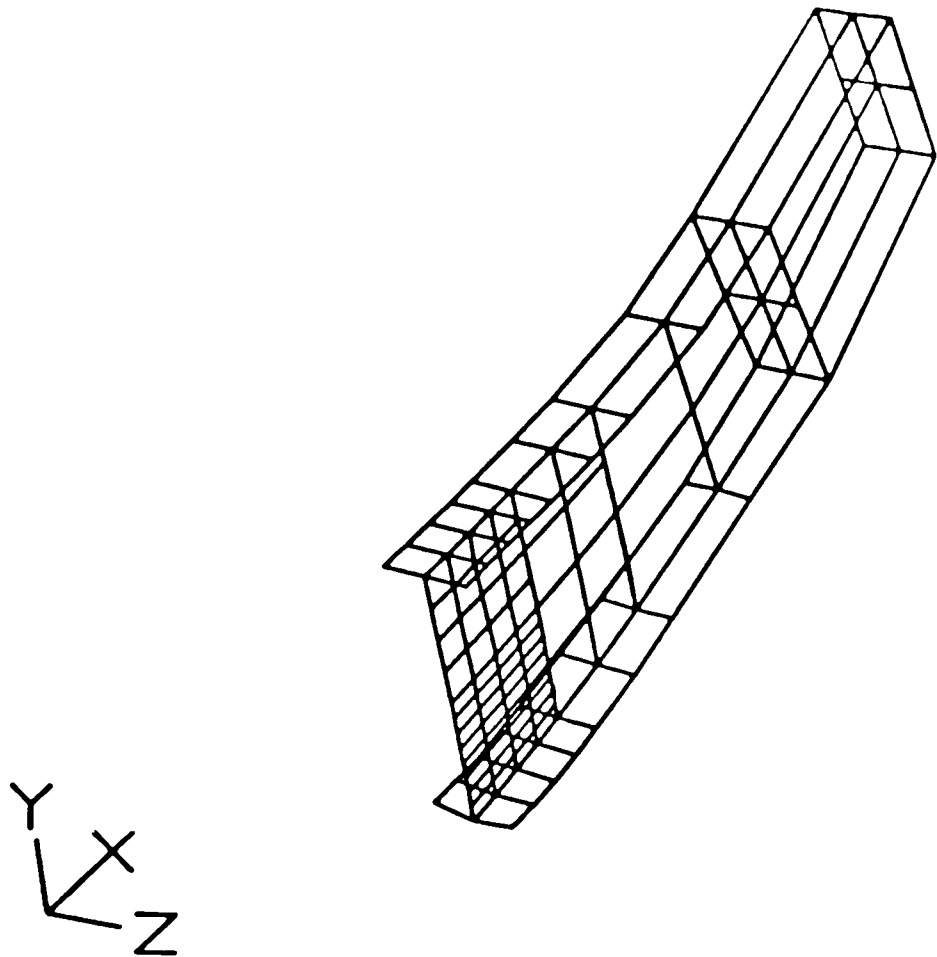
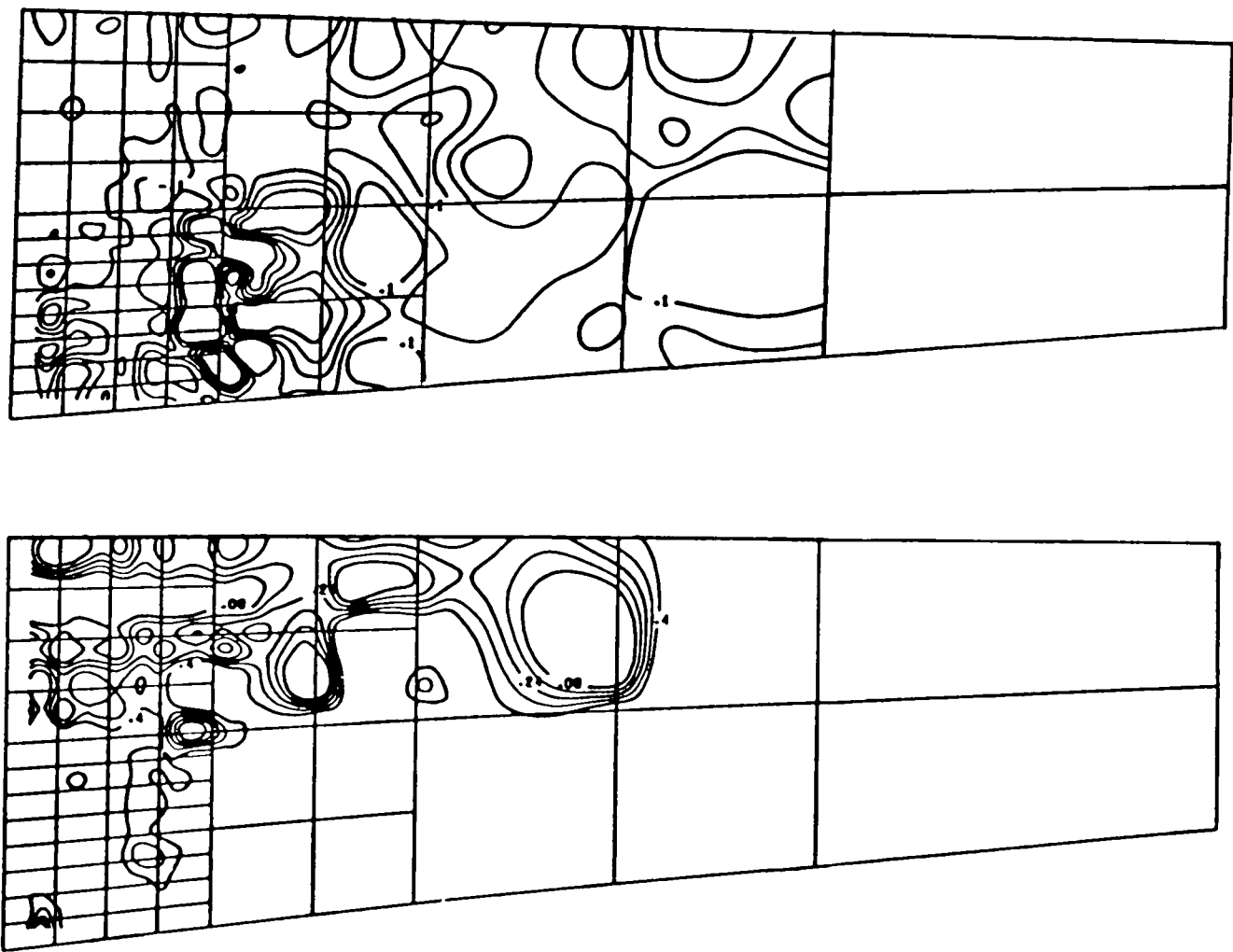


Figure 5.93 - Deformed Configuration of the Beam with $t_w = 10\text{mm}$.



Figures 5.94.a and 5.94.b Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $t_w = 10\text{mm}$.

significantly when compared to the previous case. The use of a vertical stiffener at this particular point would substantially improve the beam's load carrying capacity.

The last two beams investigated failed by a plastic and an elastic vertical web buckling respectively. The beam with a 5mm web has produced the plastic deflected configuration and the web stresses depicted in figures 5.97, 5.98.a and 5.98.b, page 230. The elastic deflected shape and web stress patterns for the 3mm web are shown in figures 5.99, 5.100.a and 5.100.b, page 231. The compressive stresses near the bottom flange, at the centre of the beam, continued to increase as the thickness of the web decreased. Both cases required the presence of a vertical stiffener to improve their performance.

5.4.2. Summary.

Table 5.6 below describes the maximum loads reached by the four simulations performed in this section together with some results taken from section 5.3. The percentage moment resistance capacity was substantially reduced when the area of the bottom flange was increased. This is explained by the growth in the force created by the tension area increase.

In both cases, the 3mm beam failed by an elastic vertical buckling of the web. The 5mm beams have their collapse associated with a plastic vertical buckling of the web. The 10mm beams would fail by the plastic hinge mechanism developed at a point near the panel edge. The main difference of the cases should occur in the beams with a 7mm web. The beam with a 3000mm² tension flange area was expected to fail by the plastic mechanism described above, while the other case failed by a plastic vertical buckling of the web.

t_w (mm) A_{fl} (mm ²)	3	5	7	10
3000	85.95	88.60	90.40	91.45
6250	50.99	54.67	56.30	56.41

Table 5.6 - Maximum Loads (% Central M_{pl}) for Beams
with $A_{fl} = 3000$ and 6250mm^2 .

Figures 5.101 and 5.102, page 232, portray an interesting vertical displacement comparison for points 1 and 49 of all the beams investigated in this segment. A lateral displacement comparison is portrayed in figures 5.103.a, 5.103.b, 5.104.a and 5.104.b, pages 233 and 234. Again, the different forms of collapse are very well characterized by these diagrams. From these results it is possible to conclude that vertical stiffeners at the centre of the web are required for the thickness of the web less or equal to: 5mm in the 3000mm² cases, and 7mm in the 6250mm² specimens.

5.5. GENERAL CONCLUSIONS.

The influence of the angle of taper and the thickness of the web can be better visualized with the aid of figure 5.105, page 235. From that diagram it is evident that the differences in moment resistance for beams with different web thicknesses, with the same slope, tended to increase as the slope becomes steeper.

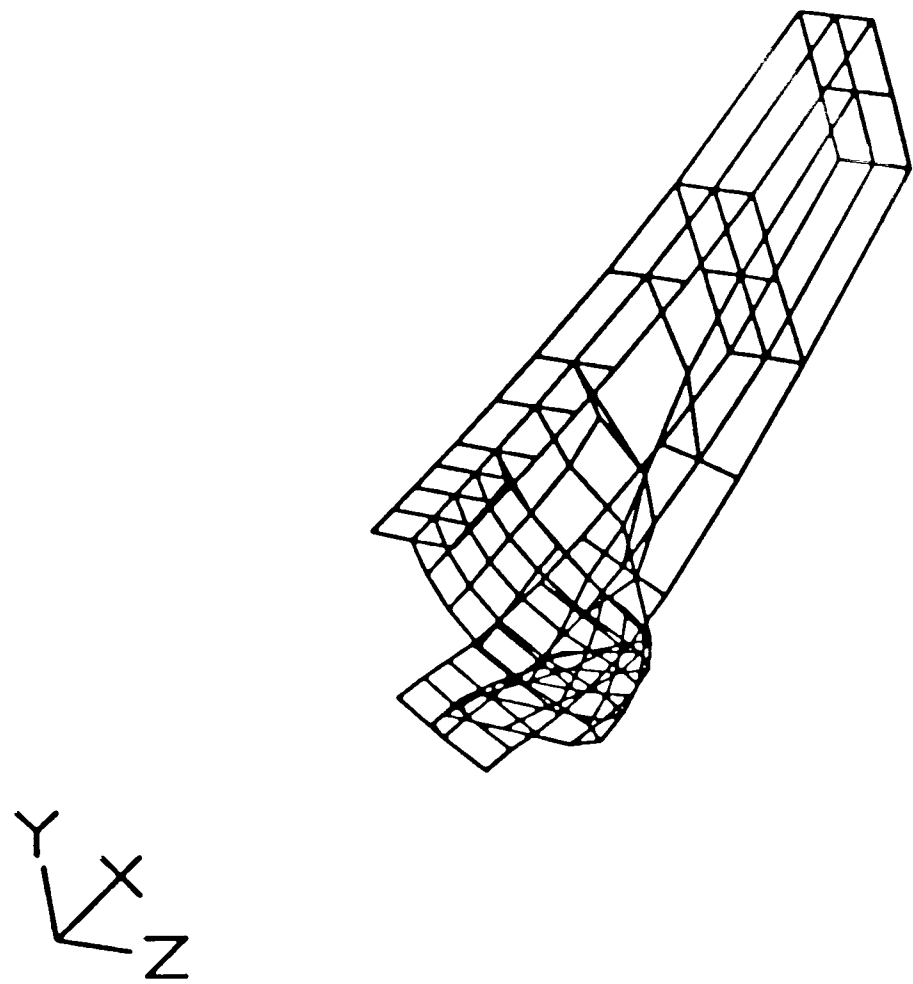
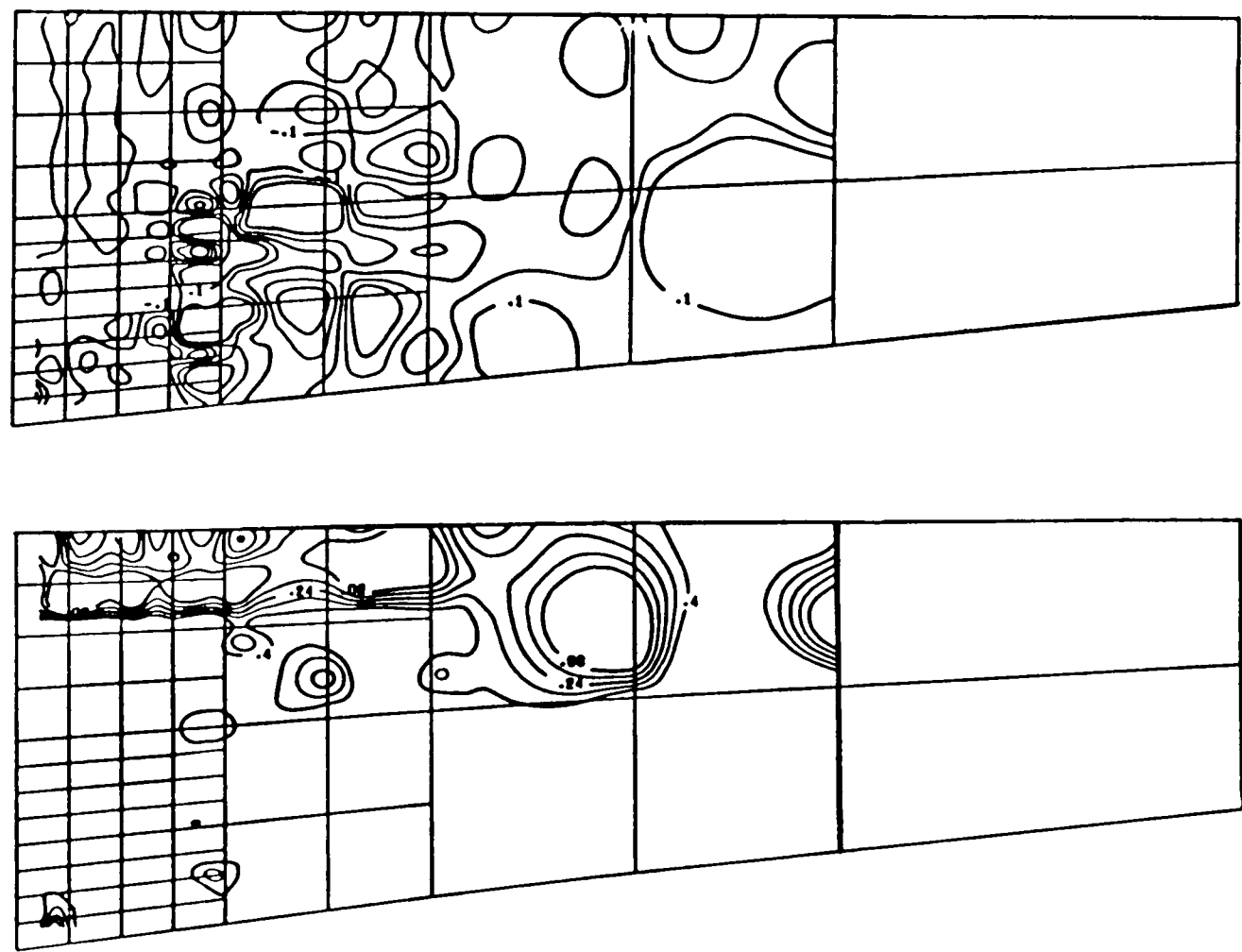


Figure 5.97 - Deformed Configuration of the Beam with $t_w = 5\text{mm}$.



Figures 5.98.a and 5.98.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $t_w = 5\text{mm}$.

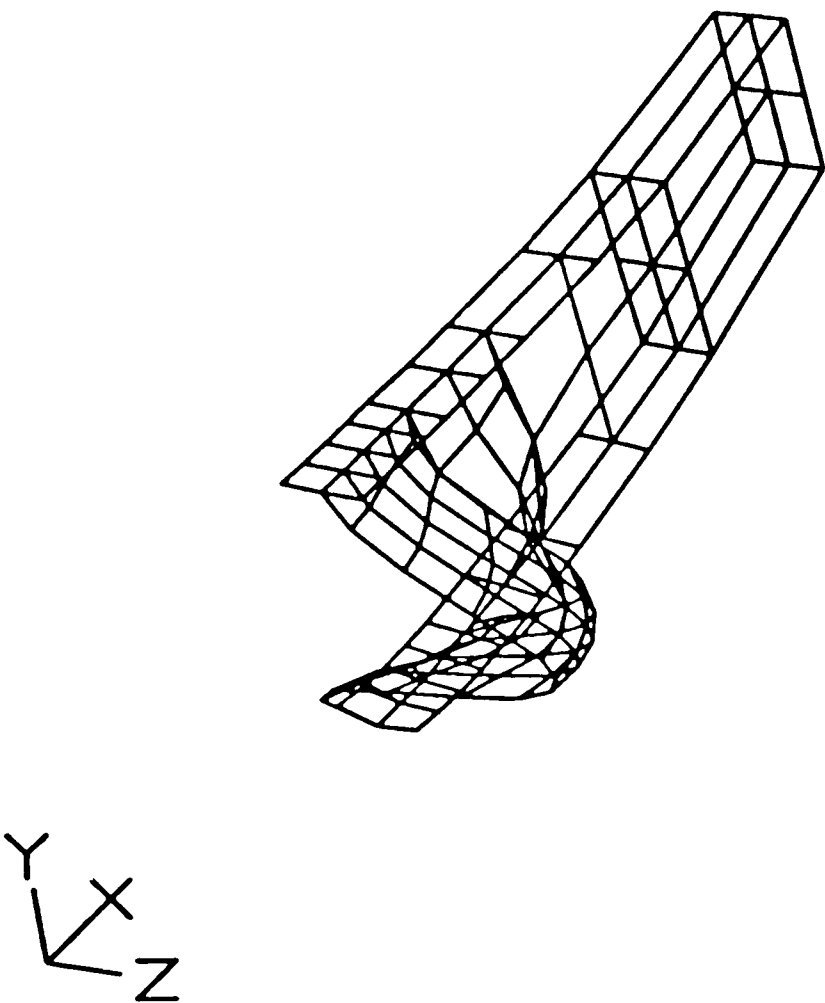


Figure 5.99 - Deformed Configuration of the Beam with $t_w = 3\text{mm}$.



Figures 5.100.a and 5.100.b - Vertical and Equivalent (Von Mises) Stresses in the Web of the Beam with $t_w = 3\text{mm}$.

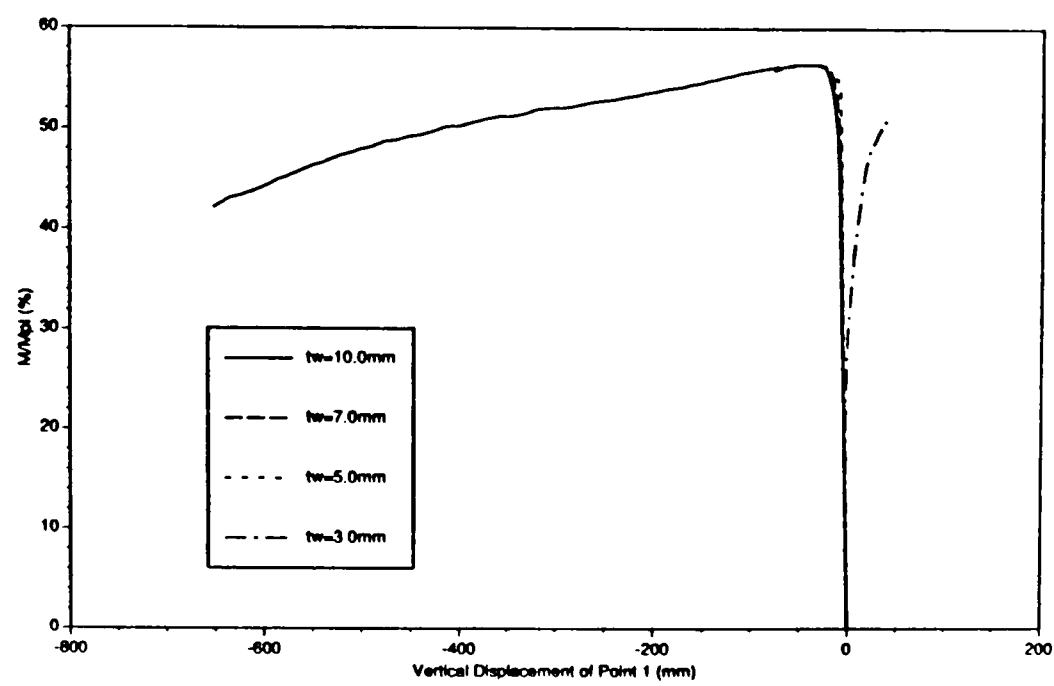


Figure 5.101 - Vertical Displacement Comparison for Point 1.

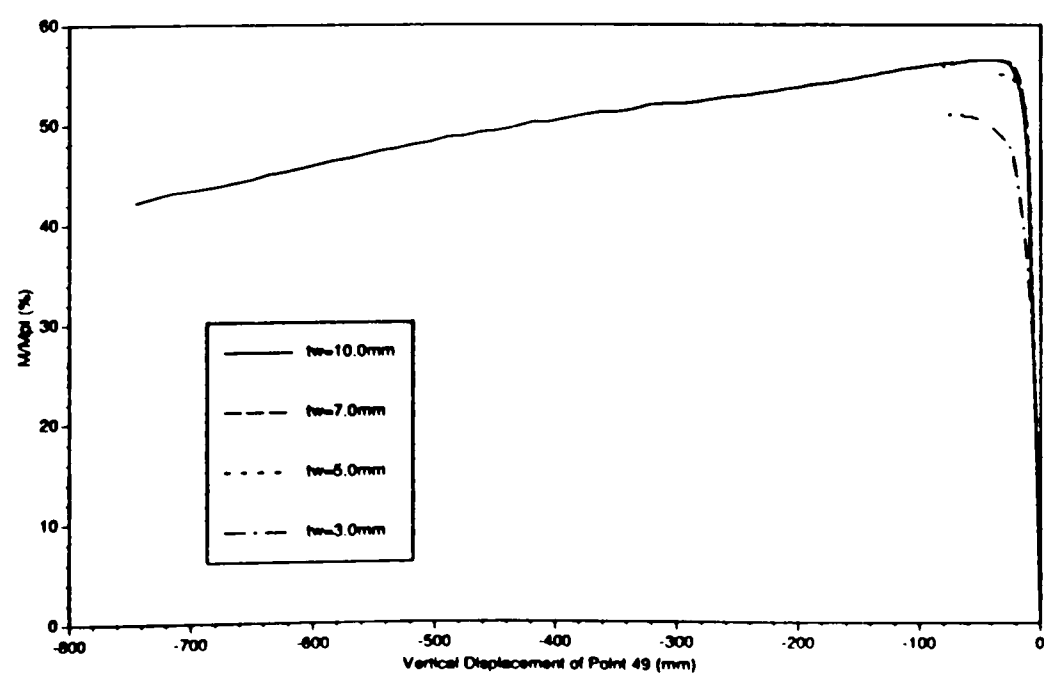
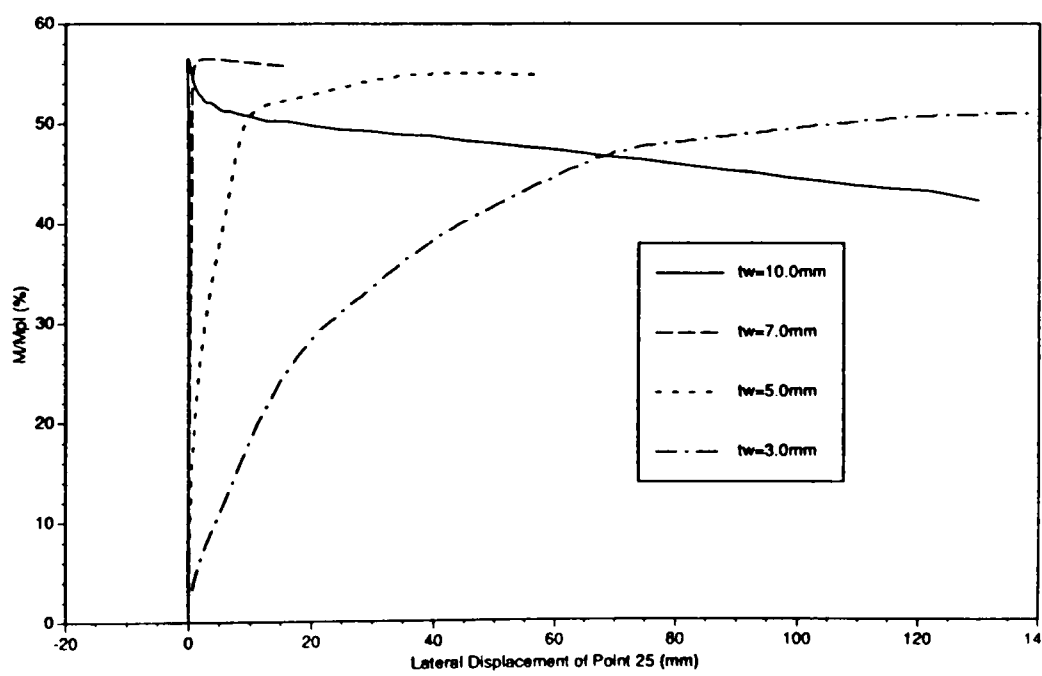
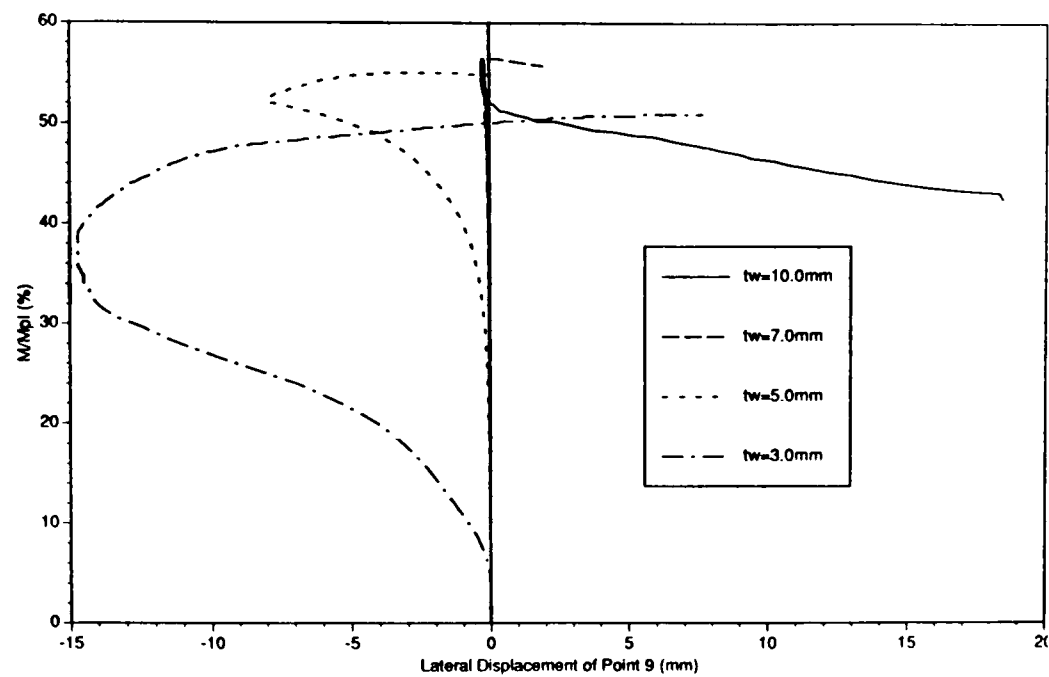
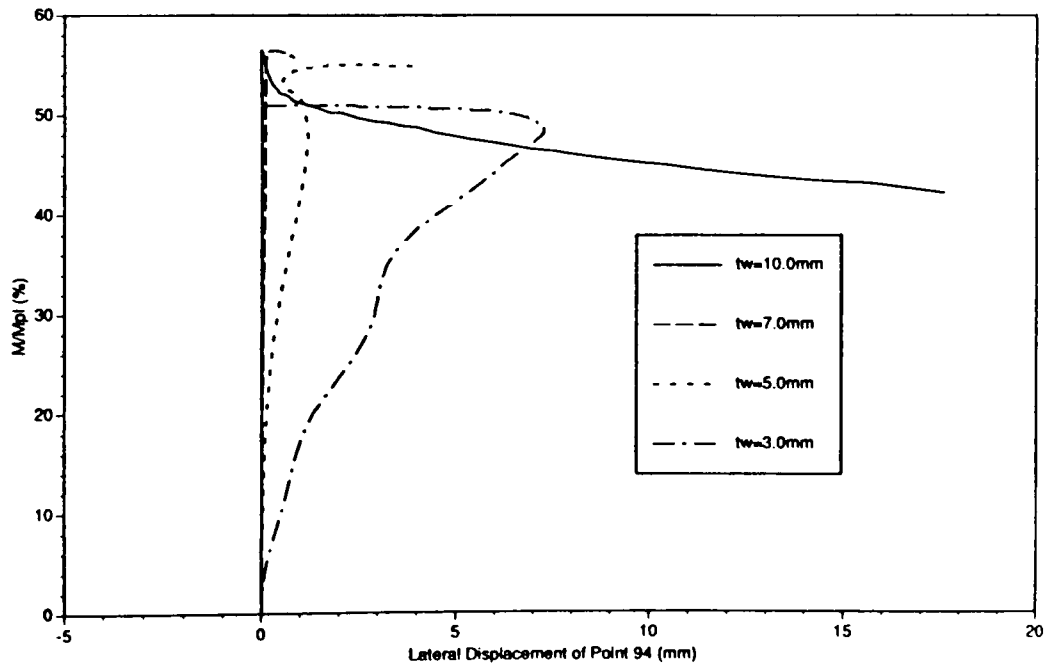
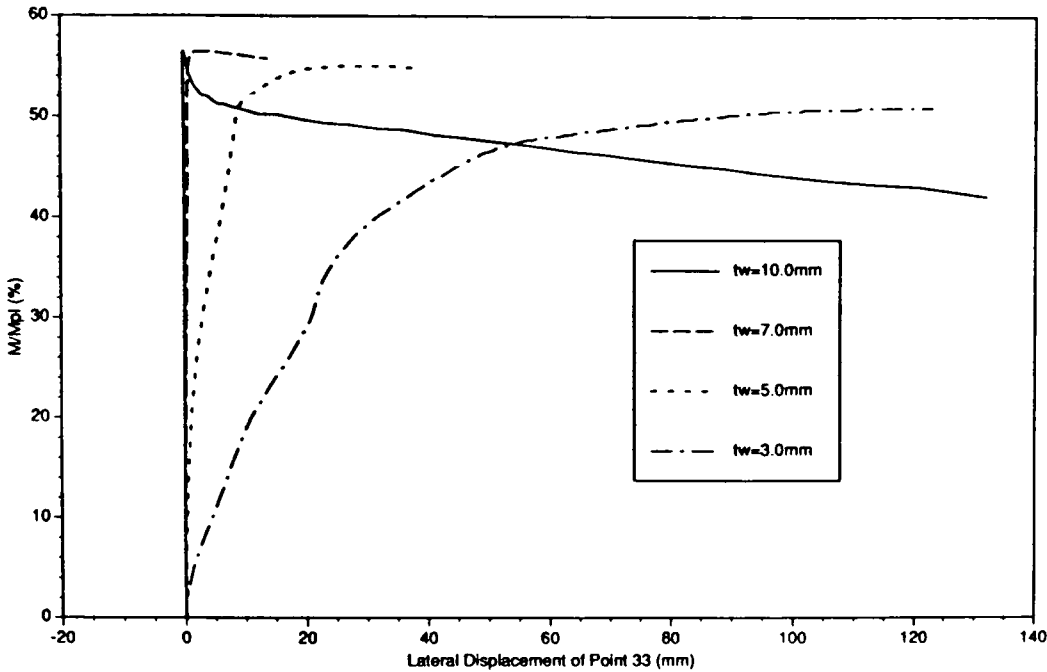


Figure 5.102 - Vertical Displacement Comparison for Point 49.



Figures 5.103.a and 5.103.b - Lateral Displacement Comparison for Points 9 and 25.



Figures 5.104.a and 5.104.b - Lateral Displacement Comparison for Points 33 and 94.

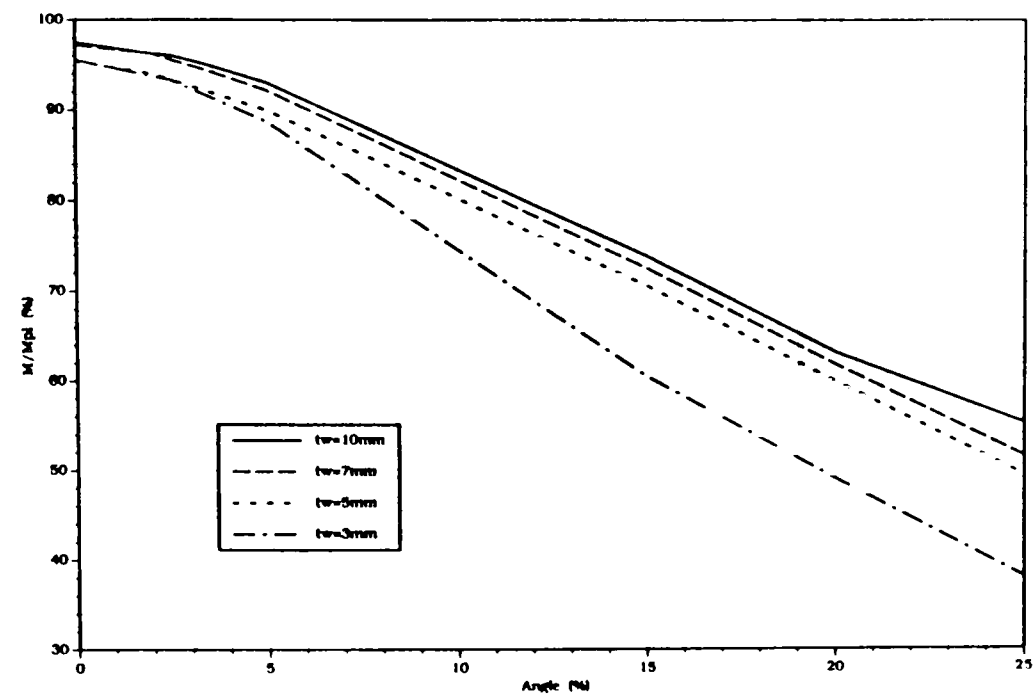


Figure 5.105 - Influence of the Angle of Taper and the Thickness of the Web on the Failure Load.

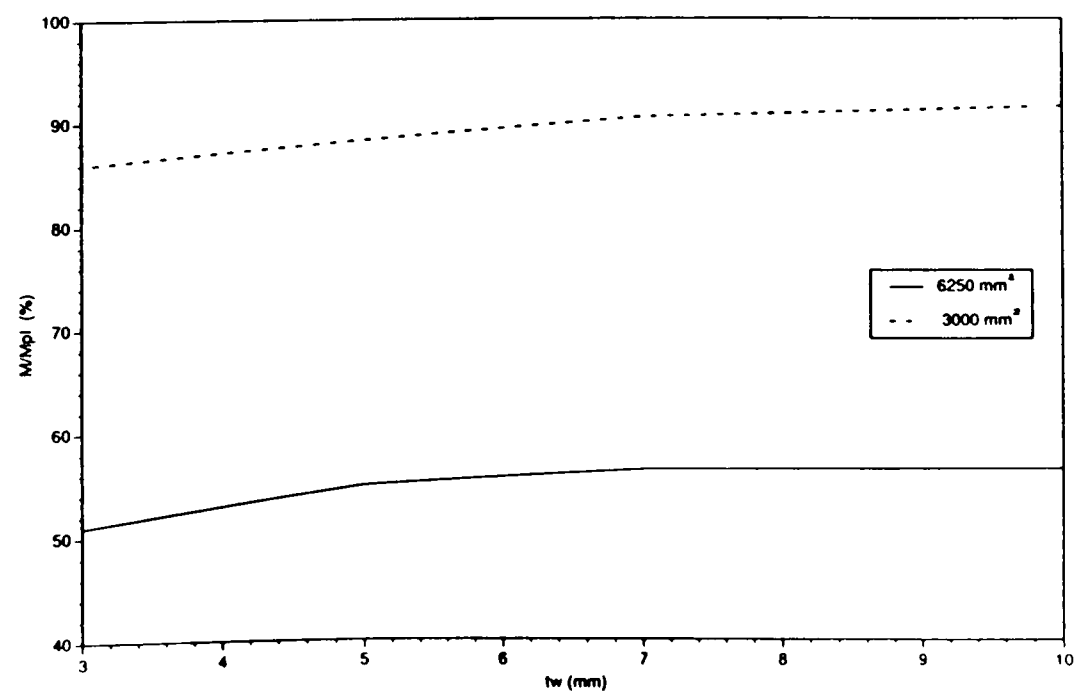


Figure 5.106 - Influence of the Area of the Bottom Flange on the Failure Load.

These results proved that the angle of slope is a very significant variable in the load carrying capacity of the beam and should be taken into the design formulation.

Vertical web buckling only occurred in one case for the standard flanges for a h/t_w ratio less than 155. This happened when the 7mm beam was subjected to a very large angle of 14.84° . Situations with these high h/t_w ratios are rarely found in design practice. A small increase in the thickness of the web would be the most effective and least costly solution to avoid this kind of collapse.

The effect of the tension flange area on the moment capacity of the girders is very well illustrated by figure 5.106, page 235. When the area of the tension flange was doubled the decrease in moment resistance as a proportion of the fully plastic moment at the centre was up to 45%. These results also proved the indubitable influence of this parameter on the buckling phenomena, and, by that token, should also be taken into account in the design formulation.

The next chapter will present a simple design method to avoid vertical web buckling. This design formulation will be based on the results of the experiments described in chapter 3, and on the finite element simulations delineated in chapters 4 and 5.

CHAPTER 6

DESIGN MODEL

6.1. INTRODUCTION.

This chapter describes the design model developed to predict the occurrence of local web buckling at slope changes in the bottom flange of tapered beams. The essence of the model is its simplicity of use. It will enable steel designers to predict/avoid local web buckling, just with the aid of a pocket calculator.

In order to give a general view of the subject, a resumé of the relevant design codes for parallel beams will be presented first. This will be followed by the derivation and behaviour of the new model. With the completion of this phase, the calibration of the model against the experiments and parametric study described in the previous chapters will be given. Finally, some conclusions will be drawn about its performance.

6.2. REVIEW OF DESIGN CODES FOR PARALLEL BEAMS.

In the limit, the proposed design model should cover cases with a zero taper angle (parallel beams). For this reason a review of some current steel design codes is important for a verification of the behaviour of the model.

The American codes, [AISC69], [AISC86], which were the main basis of the proposed design model will be presented first. The British code BS5950, [BRIT85], and the European code EC3, [EURO91], [EURO91a], will then be described.

6.2.1. American Codes (AISC and AISC-LFRD).

Both the Load and Design Resistance Factor Code, AISC-LFRD, [AISC86], based on a limit state approach, and the old allowable stress design code, [AISC69], have the same limits to prevent vertical web buckling. One very interesting derivation of these limits can be found in the book of Salmon & Johnson, [SALM90], based on previous work by Basler & Thurlimann, [BASL61]. After some data manipulation related to simple plate elastic buckling theory, they concluded that the ratio h/t_w should obey the following requirement:

$$\frac{h}{t_w} \leq \sqrt{\frac{\pi^2 E^2 h t_w}{24 (1 - \nu^2) b_f t_f \sigma_y^{\text{fl}} (\sigma_y^{\text{fl}} + \sigma_r)}} \quad (6.1)$$

Using $\nu = 0.3$, the residual stress $\sigma_r = 114$ MPa, $E = 205000$ MPa, their suggestion that the ratio $\frac{ht_w}{b_ft_f}$ is rarely below (and can safely be taken as) 0.5, and some rounding off, equation 6.1 becomes:

$$\frac{h}{t_w} \leq \frac{96500}{\sqrt{\sigma_y^{\text{fl}} (\sigma_y^{\text{fl}} + 114)}} \quad ; \quad (\sigma_y^{\text{fl}} \text{ in MPa}) \quad (6.2)$$

If transverse stiffeners are used and the ratio $a/h \leq 1.5$, in which a is the distance between vertical stiffeners, then the following formula was proposed by a joint ASCE-AASHTO committee, [ASCE68]:

$$\frac{h}{t_w} \leq \frac{5250}{\sqrt{\sigma_y^fl}} \qquad ; \qquad (\sigma_y^fl \text{ in MPa}) \qquad (6.3)$$

Their proposal was based on hybrid (different yield stresses for web and flanges) beam experiments by Carskaddan, [CARS69], and Frost & Schilling, [FROS64]. Frost & Schilling also suggested that the limit for the ratio h/t_w can be conservatively taken as 250, if $a/h \leq 1.0$, or 200, if $1.0 \leq a/h \leq 1.5$. With the results obtained from equations 6.2 and 6.3, Salmon & Johnson, [SALM90], constructed the following comparative table:

Yield Stress σ_y (MPa)	Limiting h/t_w	
	Equation 6.2	Equation 6.3
248	322	333
290	282	309
310	266	298
345	243	283
379	223	270
414	207	258
448	192	248
689	130	200

Table 6.1 - Comparison of Equations 6.2 and 6.3. After [SALM90]

One of the apparent deficiencies of equations 6.2 and 6.3 is the assumption that the ratio $\frac{h t_w}{b_f t_f}$ should be taken as 0.5, when with a simple mathematical manipulation, h and t_w could have been moved to the other side of the equation 6.1. If this is done, it leads to a new formula:

$$t_w = \frac{1}{2668} \sqrt[3]{h b_f t_f \sigma_y (\sigma_y + 114)} \qquad (6.4)$$

6.2.2. British Code (BS5950).

The British code BS5950, [BRIT85], has a similar (but simpler) approach to the two previous standards. In clause 4.4.2.3 it recommends the following condition to avoid vertical web buckling:

$$\frac{h}{t_w} \leq 250 \frac{345}{R_n} \qquad (6.5)$$

in which R_{fl} is the design strength of the compression flange. This equation is valid for beams with no stiffening, or with an a/h ratio of stiffener spacing greater than 1.5. Otherwise:

$$\frac{h}{t_w} \leq 250 \sqrt{\frac{455}{R_{fl}}} \quad (6.6)$$

for girders with the ratio a/h less than 1.5.

This approach is also very conservative and includes the $ht_w/A_{fl} = 0.5$ simplification. It is also very close to the Frost & Schilling, [FROS64] approach mentioned previously.

6.2.3. European Code (EUROCODE 3).

The European code, [EURO91], [EURO91a], was based on the same work by Basler, [BASL61]. Elastic plate buckling theory was again used to arrive at the following equation:

$$\frac{h}{t_w} \leq 0.67 \frac{E}{\sigma_y^{fl}} \sqrt{\frac{A_w}{A_{fl}}} \quad ; A_w = h t_w, A_{fl} = b_f t_f \quad (6.7)$$

To take into account the effect of imperfections and residual stresses, they modified equation 6.7 using a simple second order buckling theory, leading to equation 6.8:

$$\frac{h}{t_w} \leq 0.55 \frac{E}{\sigma_y^{fl}} \sqrt{\frac{A_w}{A_{fl}}} \quad ; A_w = h t_w, A_{fl} = b_f t_f \quad (6.8)$$

A reduction of the factor of 0.55 to 0.4 if the flange was classified as class 2, and to 0.3 if the flange belonged to class 1, was also suggested, which made the equation more conservative.

This formula is used in the code to predict a maximum for the ratio h/t_w to avoid vertical web buckling. It is again very strange that the term A_w , which is a function of h and t_w , was still kept on the right hand side of the formula. This makes the design process difficult since, to discover the maximum h/t_w , it is necessary to make several iterations before the equation is satisfied. Again, if the term A_w had been transferred to the other side of the equation it would have made the discovery of the maximum h/t_w or, in practical terms, the minimum t_w for a given h , easier.

Having completed this review of some important design standards in respect to vertical web buckling, the derivation of the model and its behaviour will be described in the next section.

6.3. DESCRIPTION OF THE PROPOSED DESIGN MODEL.

The main basis of the proposed design model is the work of Basler & Thurlimann, [BASL61], described in the previous section. The model described here was conceived from their experience on beams with parallel flanges, and has

incorporated the main changes in behaviour that occur when a tapered girder is considered.

Unlike the existing codes, the proposed design model will directly and explicitly suggest the minimum web thickness required to avoid the local web buckling problem. As noted above, the codes suggest a maximum for the ratio h/t_w as a function of the area of the web often as a result of the dubious assumption that the ratio between the flange and web areas could be taken as two.

6.3.1. Derivation of the Model.

Following Basler & Thurlimann, [BASL61], it is possible to express the curvature $d\theta/dx$ for a beam with equal flanges as:

$$\frac{d^2y}{dx^2} = \frac{d\theta}{dx} = \frac{2 \mathcal{E}_f}{h} \quad (6.9)$$

where $\mathcal{E}_f$ is the flange strain. If $d\theta$ is considered small when compared to the angle of slope α then:

$$\sin(d\theta) = \tan(d\theta) = d\theta$$

At flange yield the mean compressive stress acting vertically in the web is now a combination of the effects of the curvature of the beam and of the slope change in the lower flange:

$$\sigma_c = \frac{\sigma_y^{fl} A_{fl} d\theta}{t_w dx} + \frac{2 \sigma_y^{fl} A_{fl} \sin\alpha}{K_1 t_w h} \quad (6.10)$$

in which $K_1 h$ represents the effective width (figure 6.1, page 241) that is subjected to the vertical force. The limiting longitudinal flange strain can be expressed as:

$$\mathcal{E}_f = \frac{(\sigma_y^{fl} + \sigma_r^{fl})}{E} ; \text{ taking the residual stress, } \sigma_r^{fl}, \text{ as } 0.3 \sigma_y^{fl}, \therefore \mathcal{E}_f = \frac{1.3 \sigma_y^{fl}}{E} \quad (6.11)$$

If formula 6.11 is substituted in equation 6.9, and the result used in equation 6.10 then:

$$\sigma_c = \frac{2 \sigma_y^{fl} A_{fl}}{t_w h} \left[\frac{1.3 \sigma_y^{fl}}{E} + \frac{\sin\alpha}{K_1} \right] \quad (6.12)$$

Thus at this point, the Basler & Thurlimann formula has been modified by the $\frac{\sin\alpha}{K_1}$ term for flange slope change. The elastic plate buckling stress, σ_{cr} , can be written as:

$$\sigma_{cr} = \frac{K_2 \pi^2 E}{12 (1 - \nu^2) (h/t_w)^2} \quad (6.13)$$

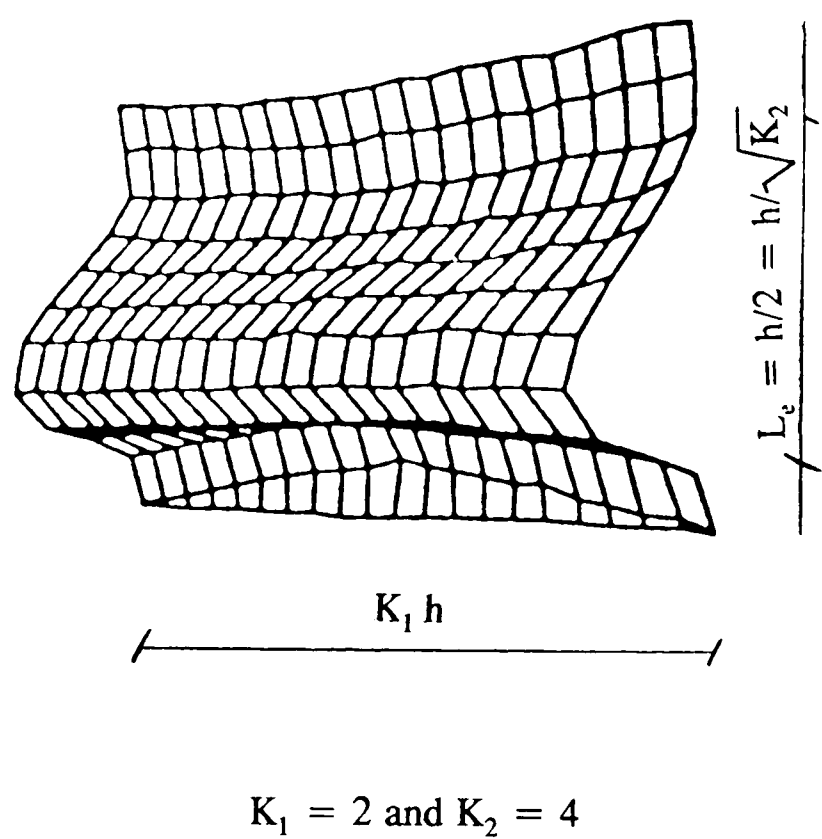


Figure 6.1 - Values of K_1 and K_2 for the Tapered Beam Model.

in which K_2 is related to the effective length of the plate (figure 6.1, page 241) by $K_2 = \left(\frac{h}{L_e}\right)^2$. Equating the buckling stress σ_{cr} and the compressive stress σ_c then:

$$\frac{2 \sigma_y^{fl} A_{fl}}{t_w h} \left[\frac{1.3 \sigma_y^{fl}}{E} + \frac{\sin \alpha}{K_1} \right] = \frac{K_2 \pi^2 E}{12 (1 - \nu^2) (h/t_w)^2} \quad (6.14)$$

Substituting $A_{fl} = b_f t_f$ in equation 6.14 and rearranging, it is possible to arrive at an expression for t_w :

$$t_w^3 = \frac{24 \sigma_y^{fl} b_f t_f h (1 - \nu^2)}{K_2 \pi^2 E} \left[\frac{1.3 \sigma_y^{fl}}{E} + \frac{\sin \alpha}{K_1} \right] \quad (6.15)$$

If the values of π , $E = 205000$ MPa, and $\nu = 0.3$ are substituted in 6.15 then:

$$t_w = \sqrt[3]{\frac{2.213 \sigma_y^{fl} b_f t_f h}{205000 K_2} \left[\frac{1.3 \sigma_y^{fl}}{205000} + \frac{\sin \alpha}{K_1} \right]} \quad ; (\sigma_y^{fl} \text{ in MPa}) \quad (6.16)$$

The values chosen for K_1 and K_2 clearly control the results of equation 6.16, but for any reasonable K_1 they are dominated by the slope change term in $\sin \alpha$ even for small slopes. For non-zero α , the values of K_1 and K_2 were chosen on a basis of the buckle observed in test 3, described in chapter 3 but reproduced here as figure 6.1, page 241. The most suitable value for the effective width coefficient K_1 appears to be 2, while the vertical wavelength suggests $K_2 = 4$. If these values are substituted in equation 6.16 then:

$$t_w = \sqrt[3]{\frac{0.553 \sigma_y^{fl} b_f t_f h}{205000} \left[\frac{1.3 \sigma_y^{fl}}{205000} + \frac{\sin \alpha}{2} \right]} \quad ; (\sigma_y^{fl} \text{ in MPa}) \quad (6.17)$$

This equation for the minimum web thickness to avoid vertical web buckling needs to be used with care, and it may be appropriate to modify K_1 and/or K_2 to achieve a more conservative result for design use or to reflect variations in behaviour. In particular, if the slope is zero, the effective length observed in practice is much closer to the full web depth, implying that K_2 should be taken as 1 conservatively, [BASL61]. Using these values and $\alpha = 0$ a version of equation 6.4 is obtained:

$$t_w = \frac{1}{2444.52} \sqrt[3]{h b_f t_f \sigma_y^{fl^2}} \quad (\sigma_y^{fl} \text{ in MPa}) \quad (6.18)$$

The next section will delineate the behaviour of equation 6.17 for some practical situations.

6.3.2. Behaviour of the Model.

The behaviour of the design equation 6.17 was assessed by using it in a series of practical design situations. These examples covered cases where the geometric properties varied as follows: flange area, A_f , between 2000 and 26000mm²; the angle of slope, α , between 0 and 30% (0° to 16.70°); and the depth of the beam, h , between 250 and 1550mm. The yield stress of the flange was varied using the values for grades 43, 50 and 55 (265, 340 and 430MPa).

The results of equation 6.17 using the values shown above were then plotted in a series of graphs. The effects of the angle α and the depth h on the minimum web thickness, and maximum h/t_w ratio are presented in figures 6.2.a and 6.2.b, page 244. As the angle α increases, the influence of the depth becomes more significant, producing higher values for the minimum web thickness. The variation in the minimum web thickness between the angles of 0 and 5% is also more drastic.

The effects of the angle α and the flange yield stress σ_y^f on the minimum web thickness and maximum h/t_w are depicted in figures 6.3.a and 6.3.b, page 245. As the angle α increases, the influence of the yield stress becomes more important. Figures 6.4.a and 6.4.b, page 246, where the influences of the angle α and the area of the tension flange are demonstrated, display a similar trend.

The effects of flange yield stress σ_y^f and the depth h are portrayed in figures 6.5.a and 6.5.b, page 247. As the depth increases, the higher yield stresses tend to produce higher values for the minimum web thickness. It is interesting to note that the maximum h/t_w ratio also increases with higher h values. Similar conclusions can be drawn from figures 6.6.a and 6.6.b, page 248, where the influence of the depth and the tension flange area are assessed. Finally, figures 6.7.a and 6.7.b, page 249, delineate the influence of the flange yield stress and the area of the tension flange on the minimum web thickness and the maximum h/t_w ratio. Their effects had similar characteristics to the previous graphs.

A summary of the influence of all the parameters described above can be found in figure 6.8, page 250. In that graph, the minimum thickness t_w can be determined by the following method. First the flange yield stress will establish the curve to be used. With the correct curve and the angle of slope, it is possible to determine the equivalent non-dimensional value in the y axis. If this value is multiplied by $(10000 A_f h)$ and then a cubic root is taken, the minimum thickness value is found.

The next section will present the calibration of the design equations against the experiments and the parametric study described in the previous chapters.

6.4. CALIBRATION OF THE MODEL.

The calibration will be made in two distinct phases. First, the model will be applied to the laboratory tests described in detail in chapter 3. Then, it will be used for the girders that formed the finite element parametric study presented in chapter 5.

6.4.1. Comparison with Laboratory Tests.

In order to make the calibration of the model against the laboratory tests more effective, a table with the results for the minimum web thickness was prepared.

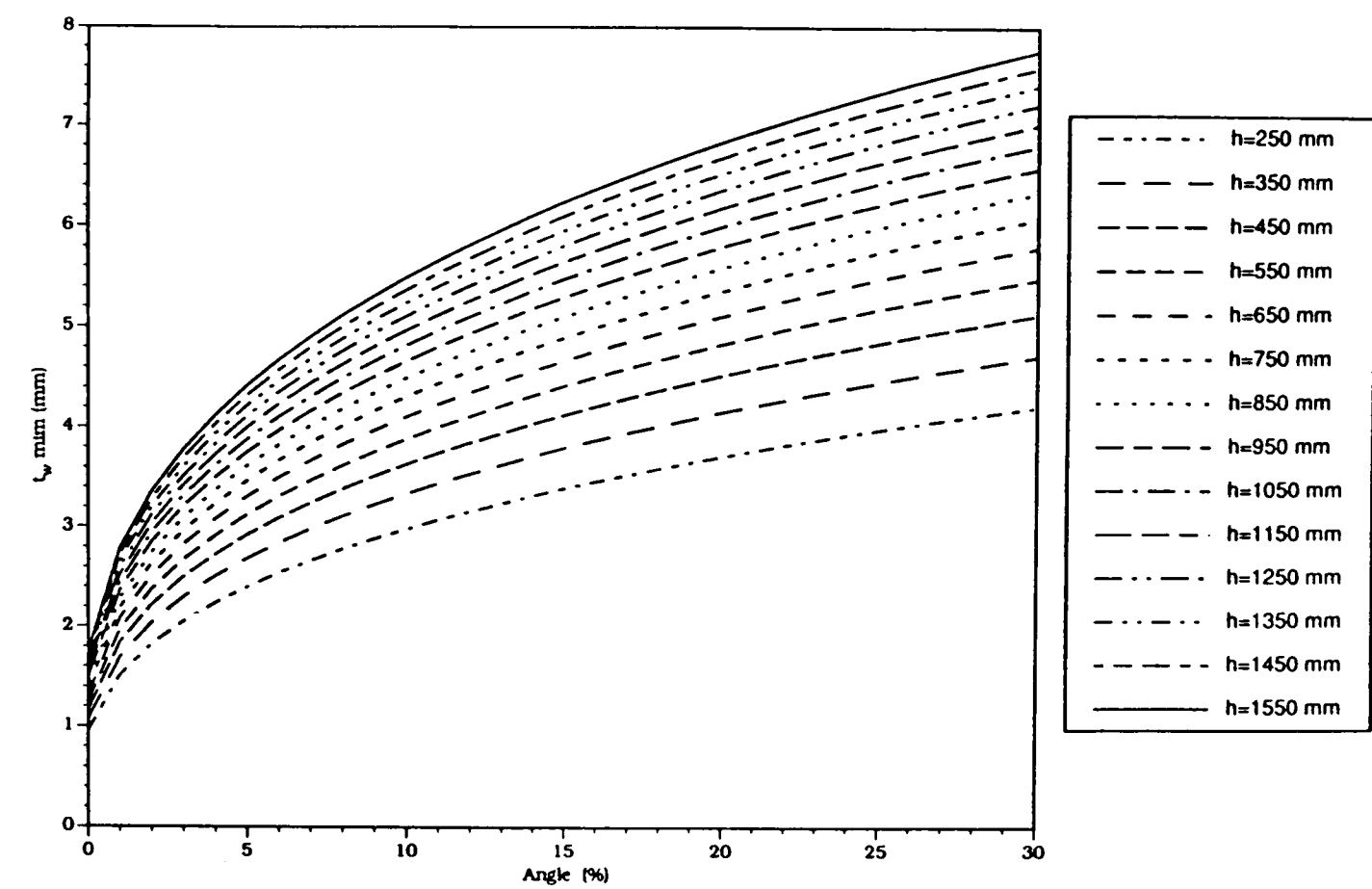


Figure 6.2.a - Effect of h and α on the Minimum Web Thickness t_w .

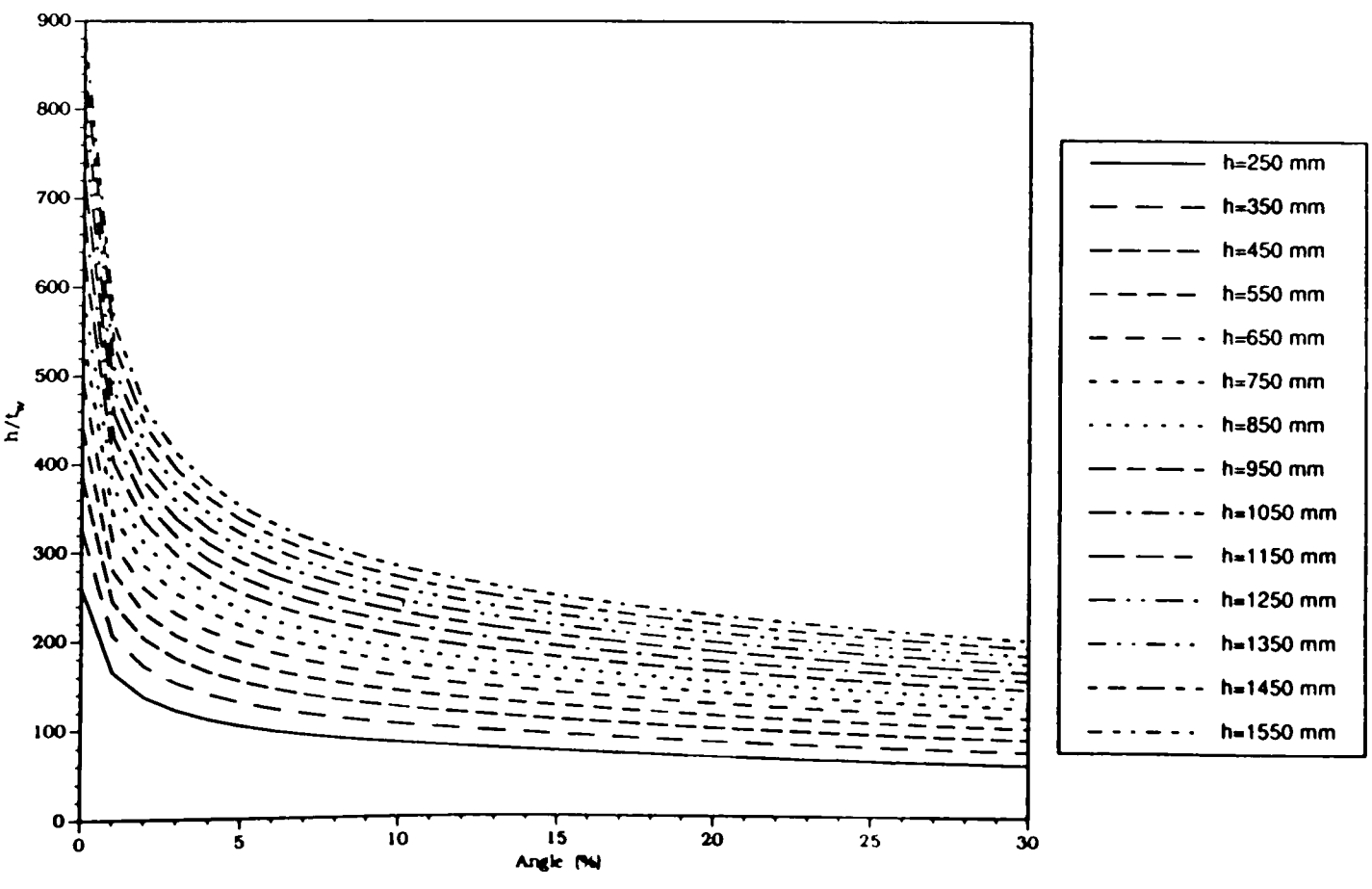


Figure 6.2.b - Effect of h and α on the Maximum h/t_w Ratio.

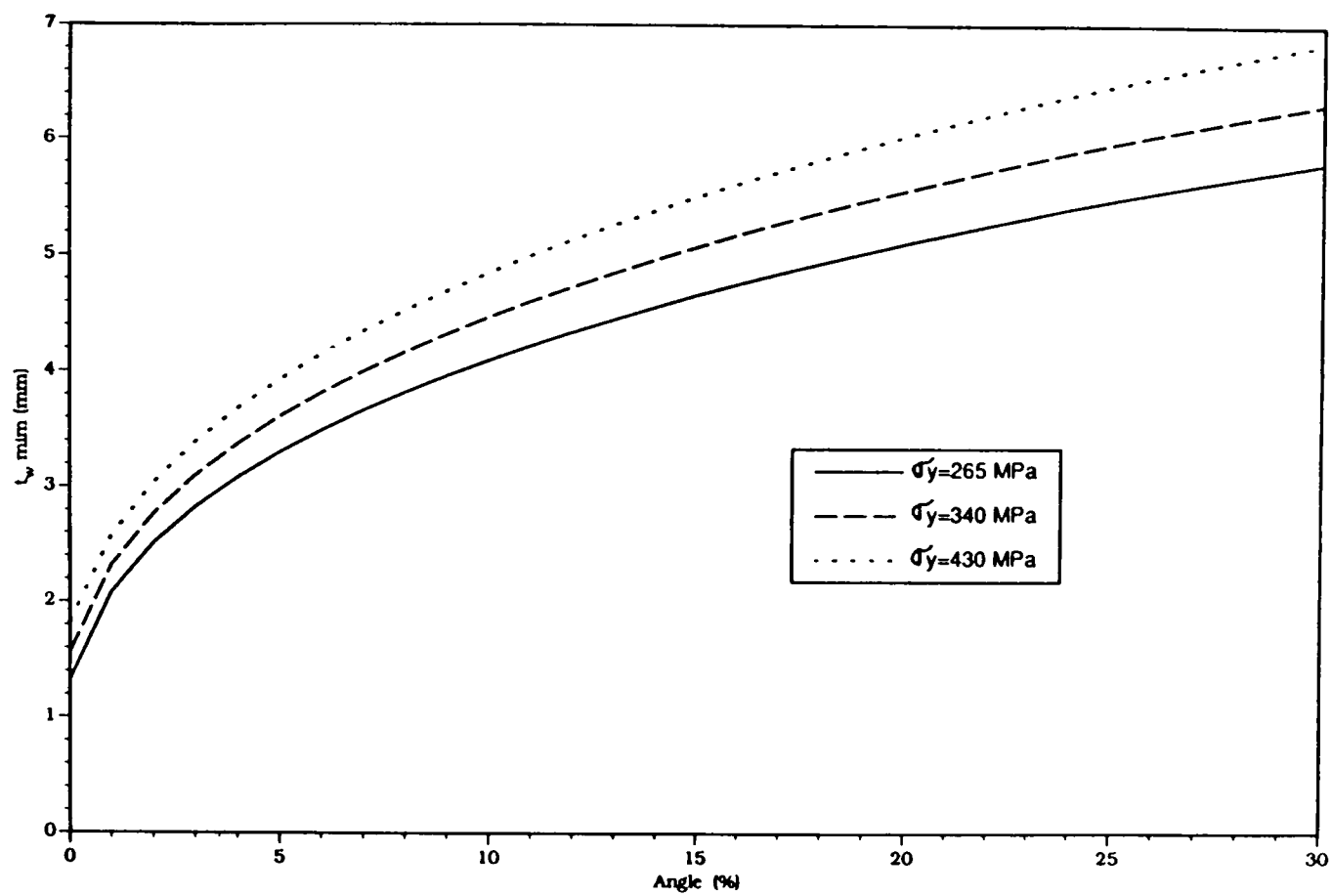


Figure 6.3.a - Effect of σ_y^{fl} and α on the Minimum Web Thickness t_w .

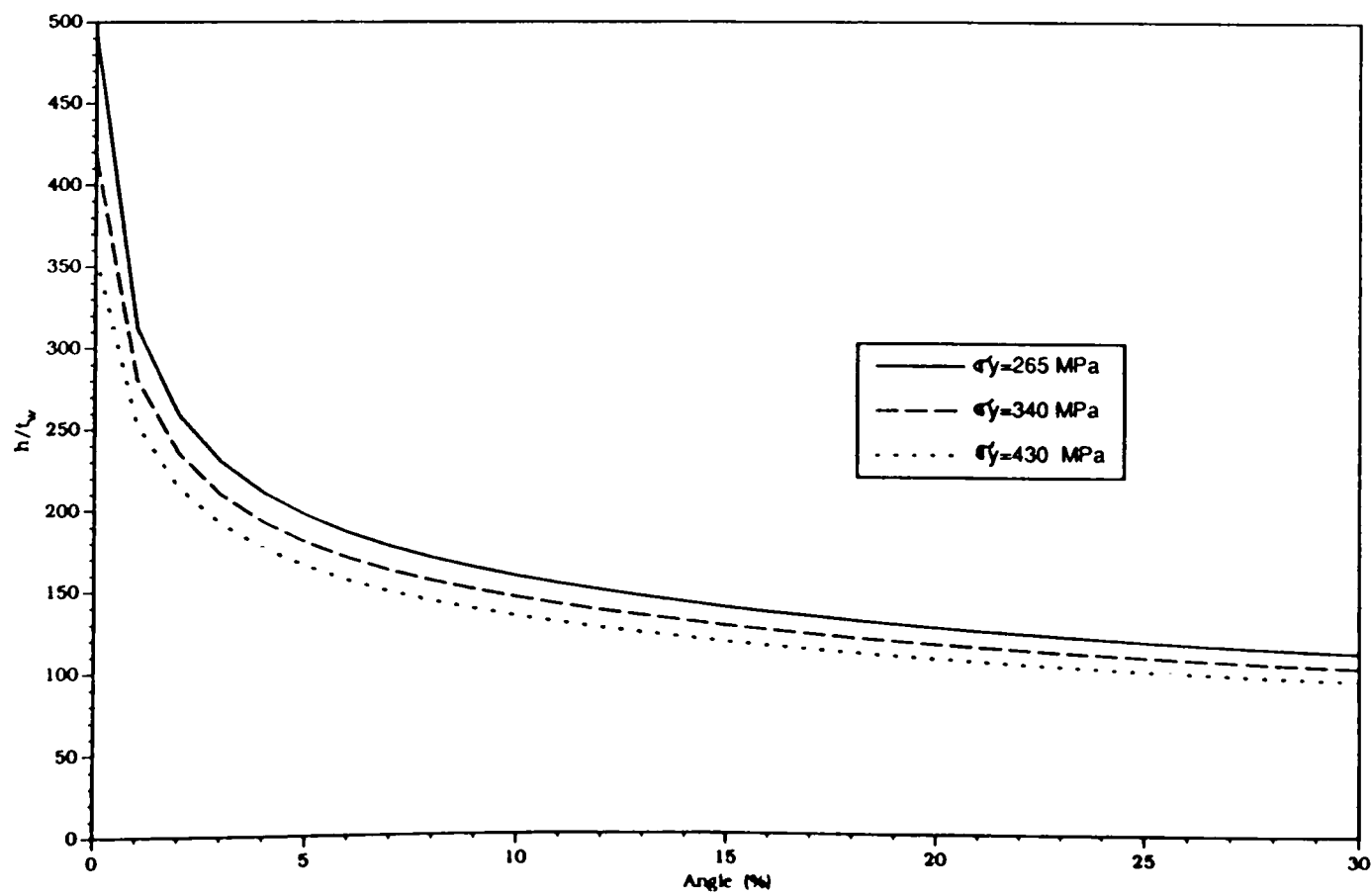


Figure 6.3.b - Effect of σ_y^{fl} and α on the Maximum h/t_w Ratio.

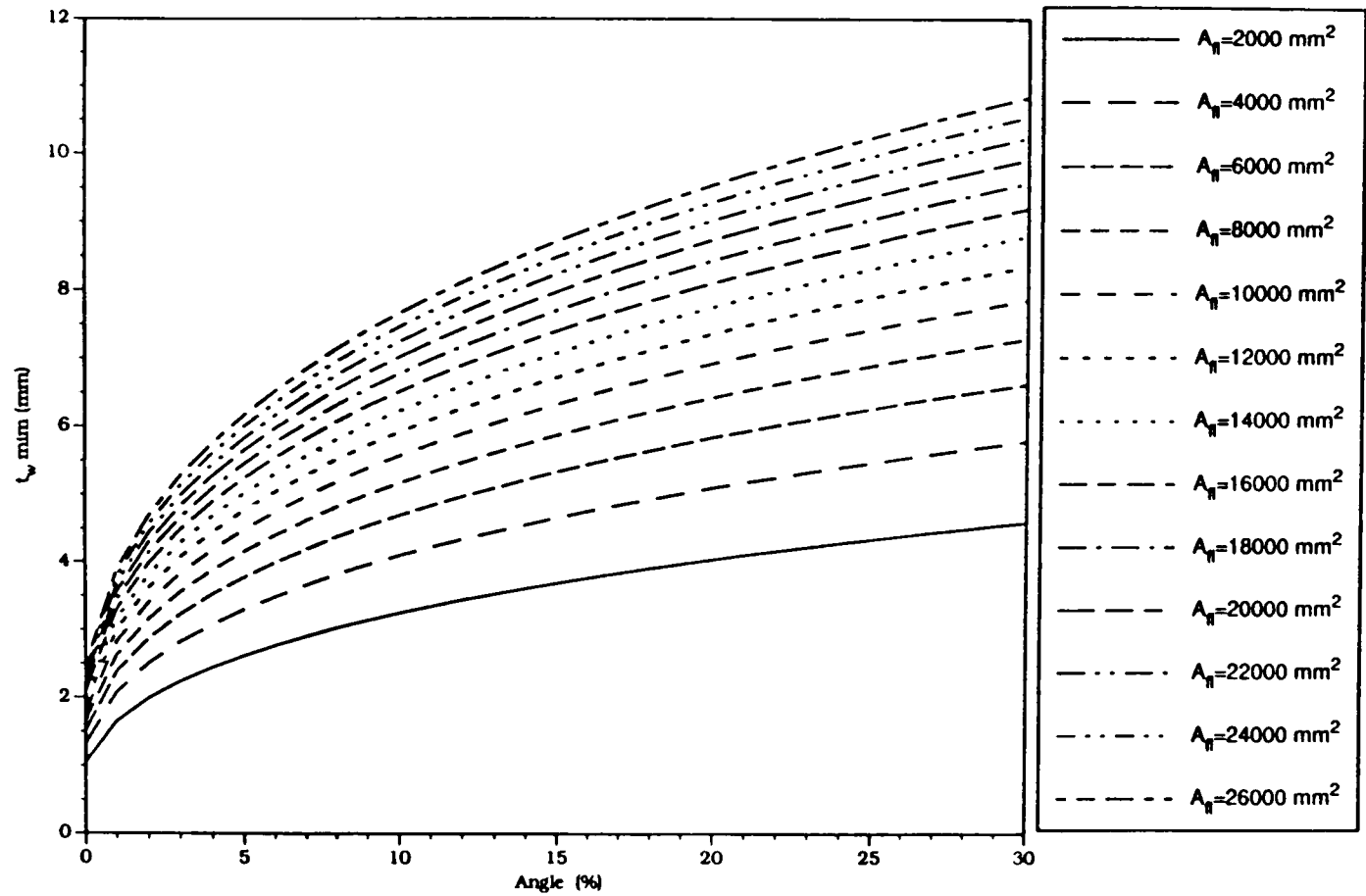


Figure 6.4.a - Effect of A_{fl} and α on the Minimum Web Thickness t_w .

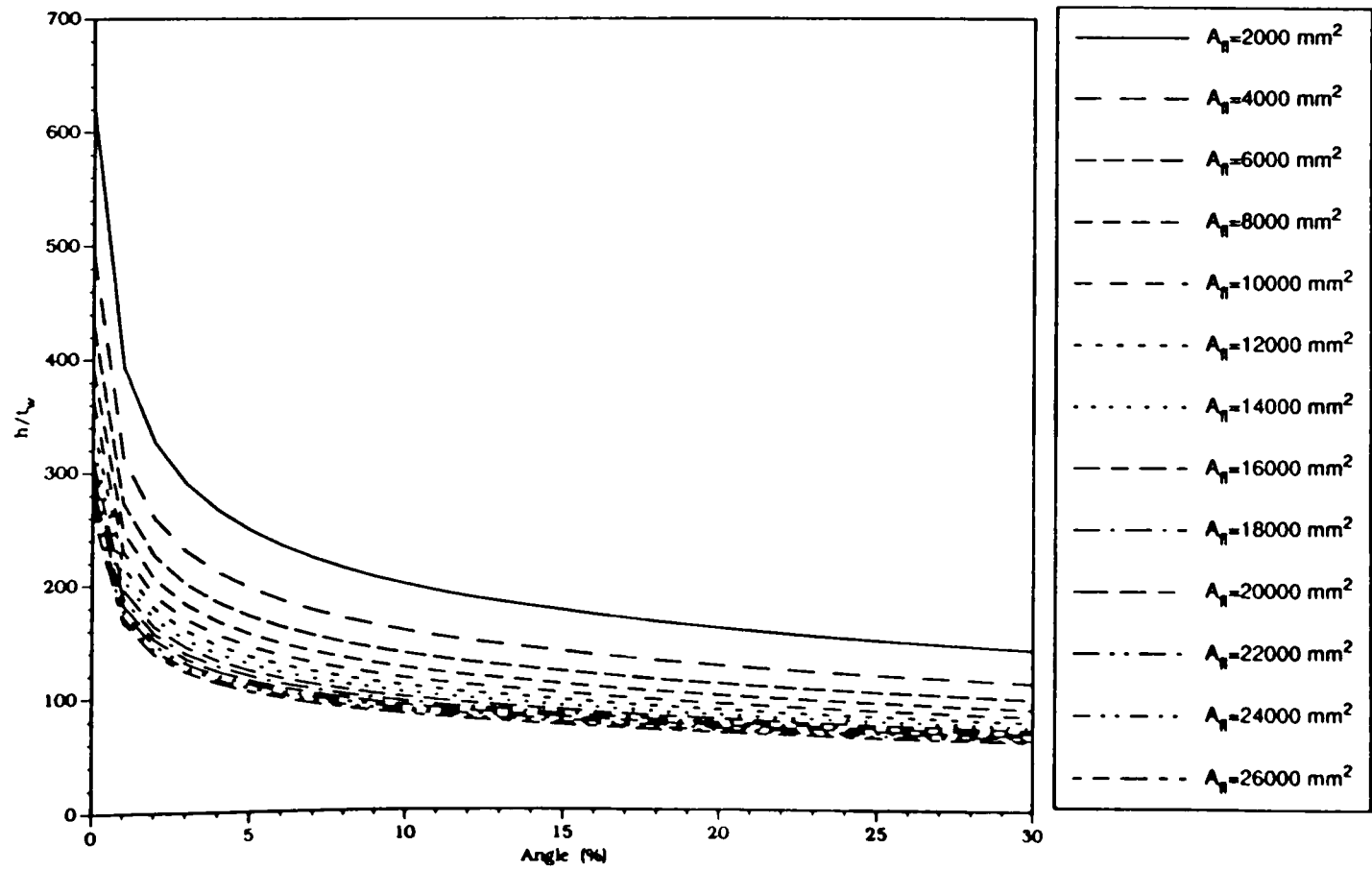


Figure 6.4.b - Effect of A_{fl} and α on the Maximum h/t_w Ratio.

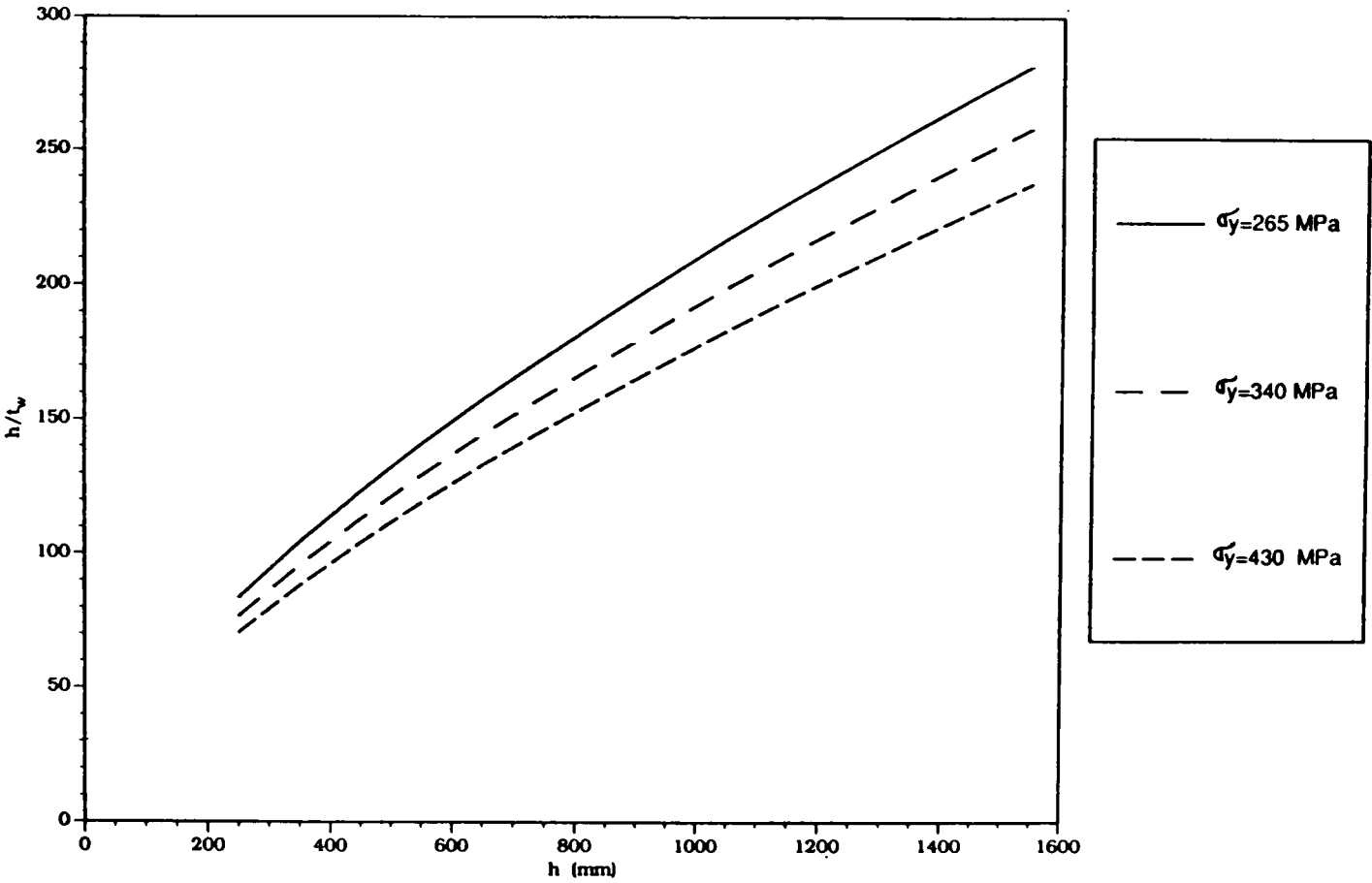


Figure 6.5.a - Effect of σ_y^{fl} and h on the Minimum Web Thickness t_w .

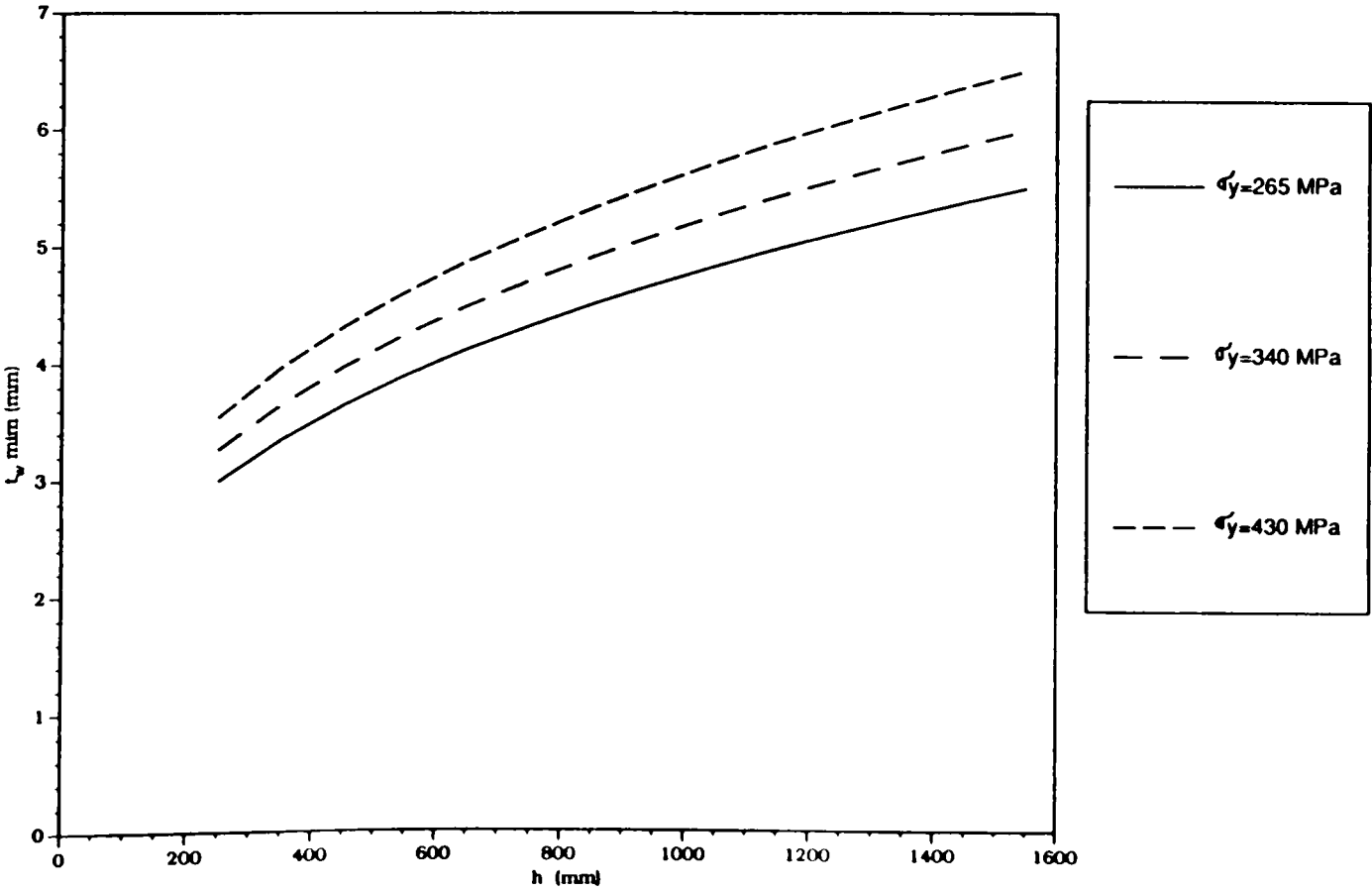


Figure 6.5.b Effect of σ_y^{fl} and h on the Maximum h/t_w Ratio.

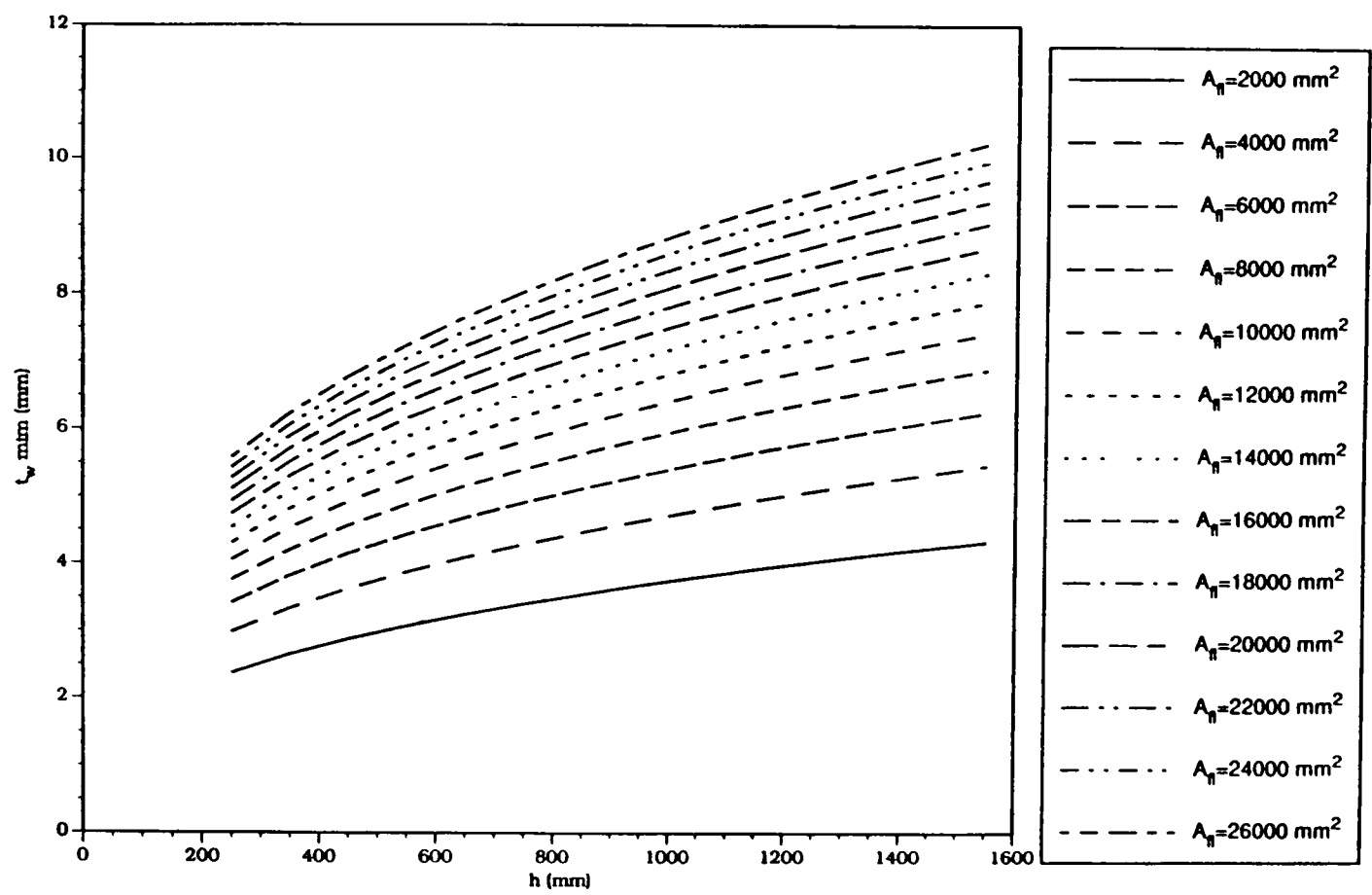


Figure 6.6.a - Effect of A_n and h on the Minimum Web Thickness t_w .

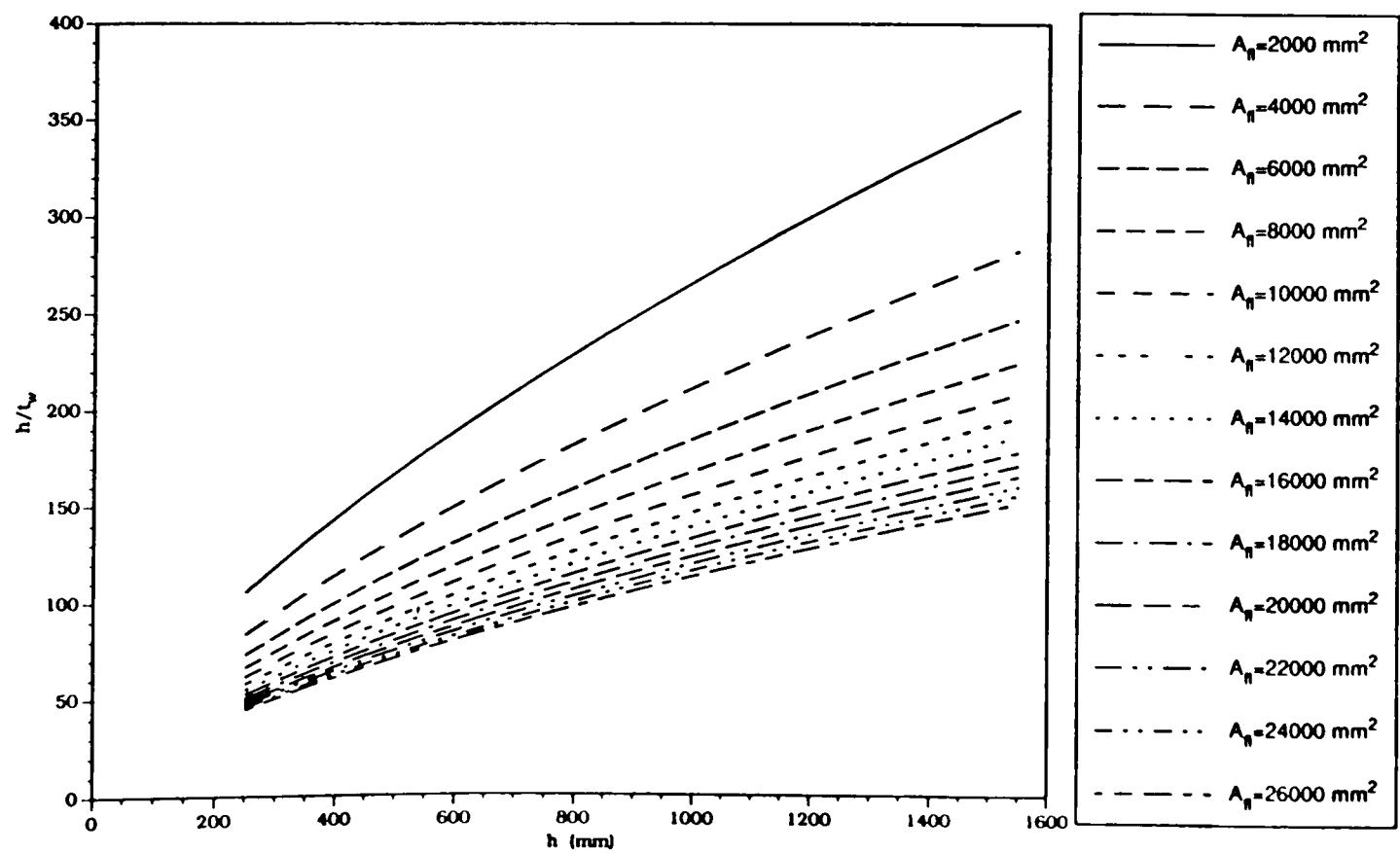


Figure 6.6.b - Effect of A_n and h on the Maximum h/t_w Ratio.

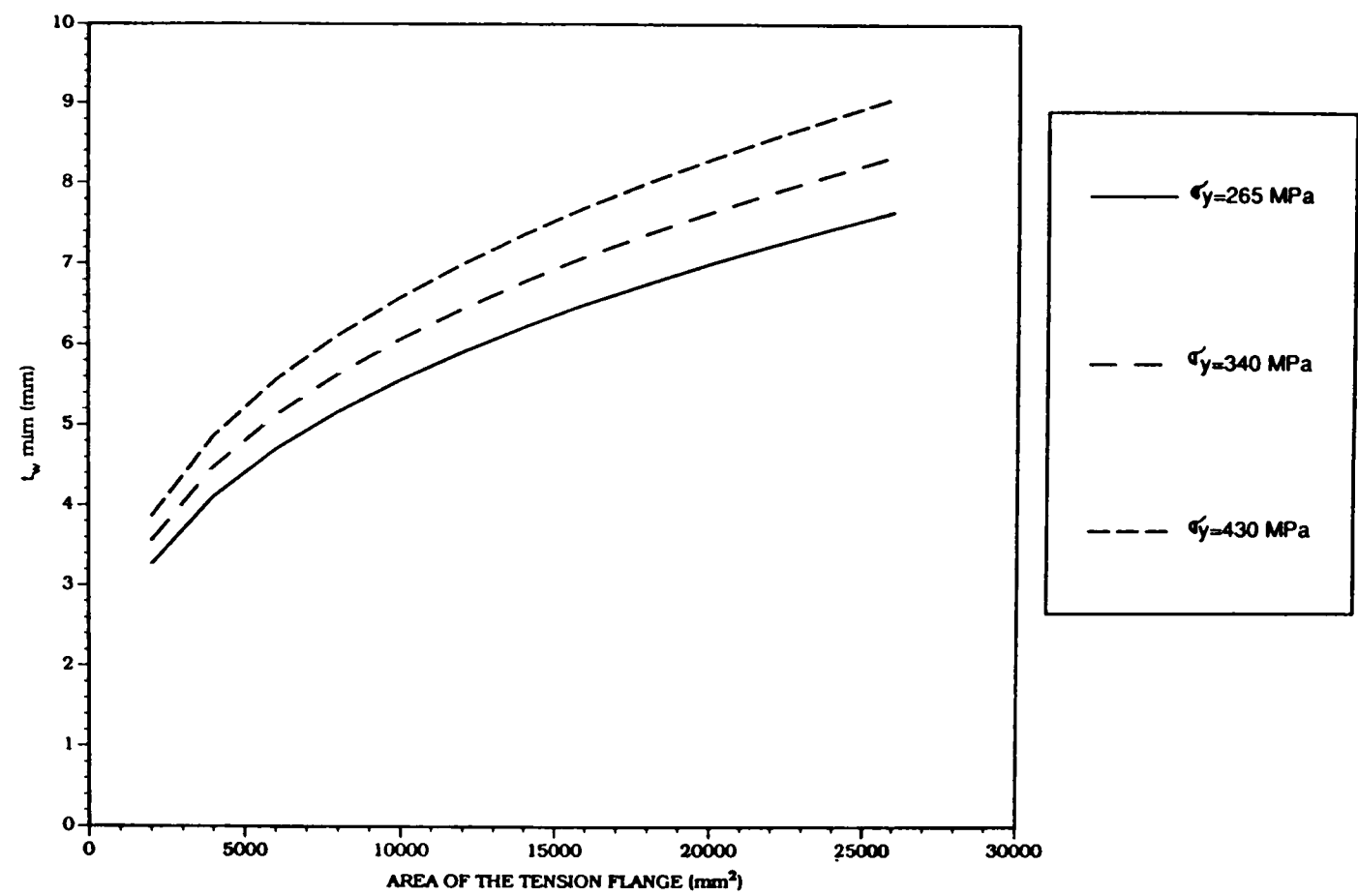


Figure 6.7.a - Effect of σ_y^Π and A_Π on the Minimum Web Thickness t_w .

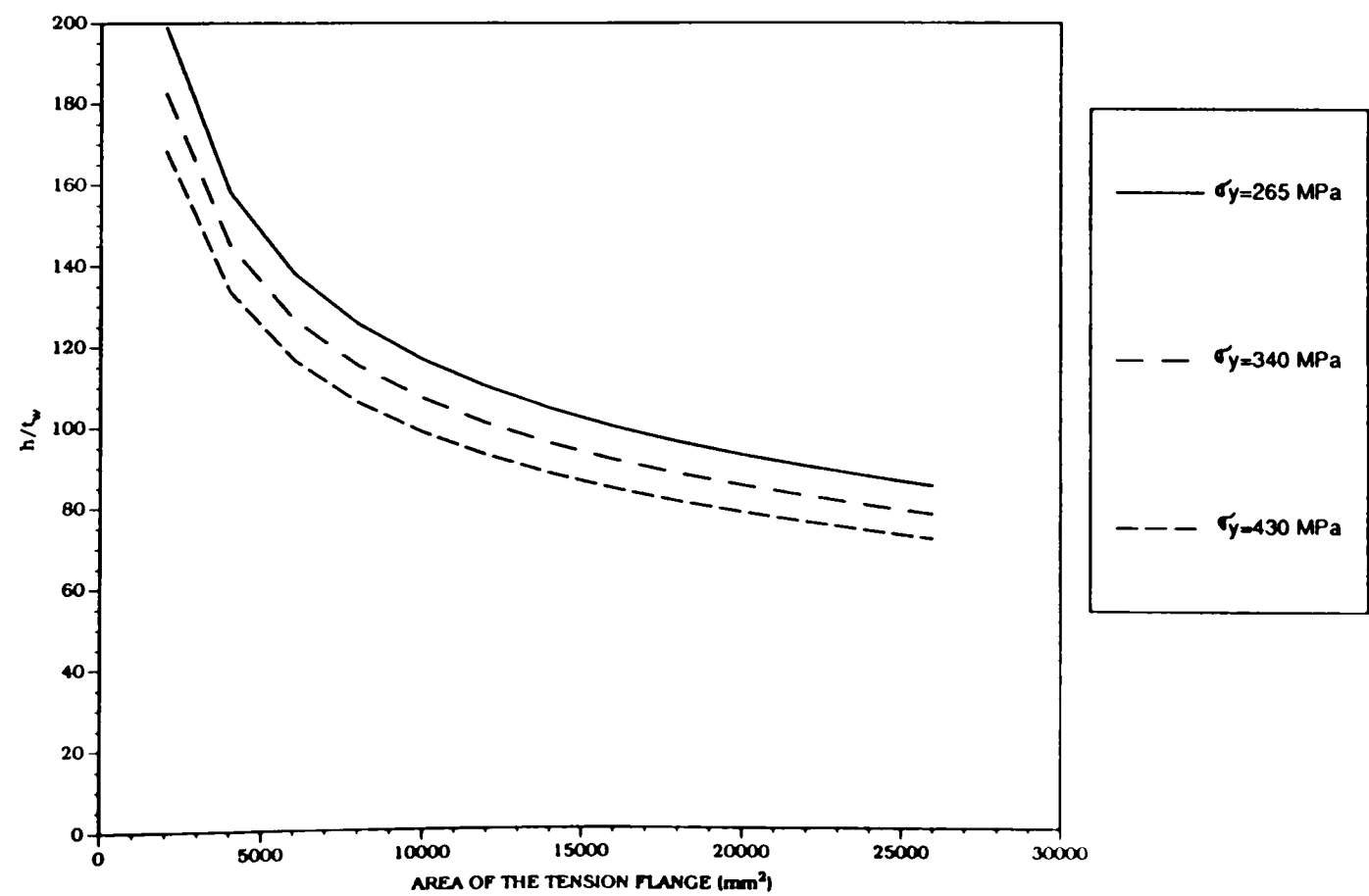


Figure 6.7.b - Effect of σ_y^Π and A_Π on the Maximum h/t_w Ratio.

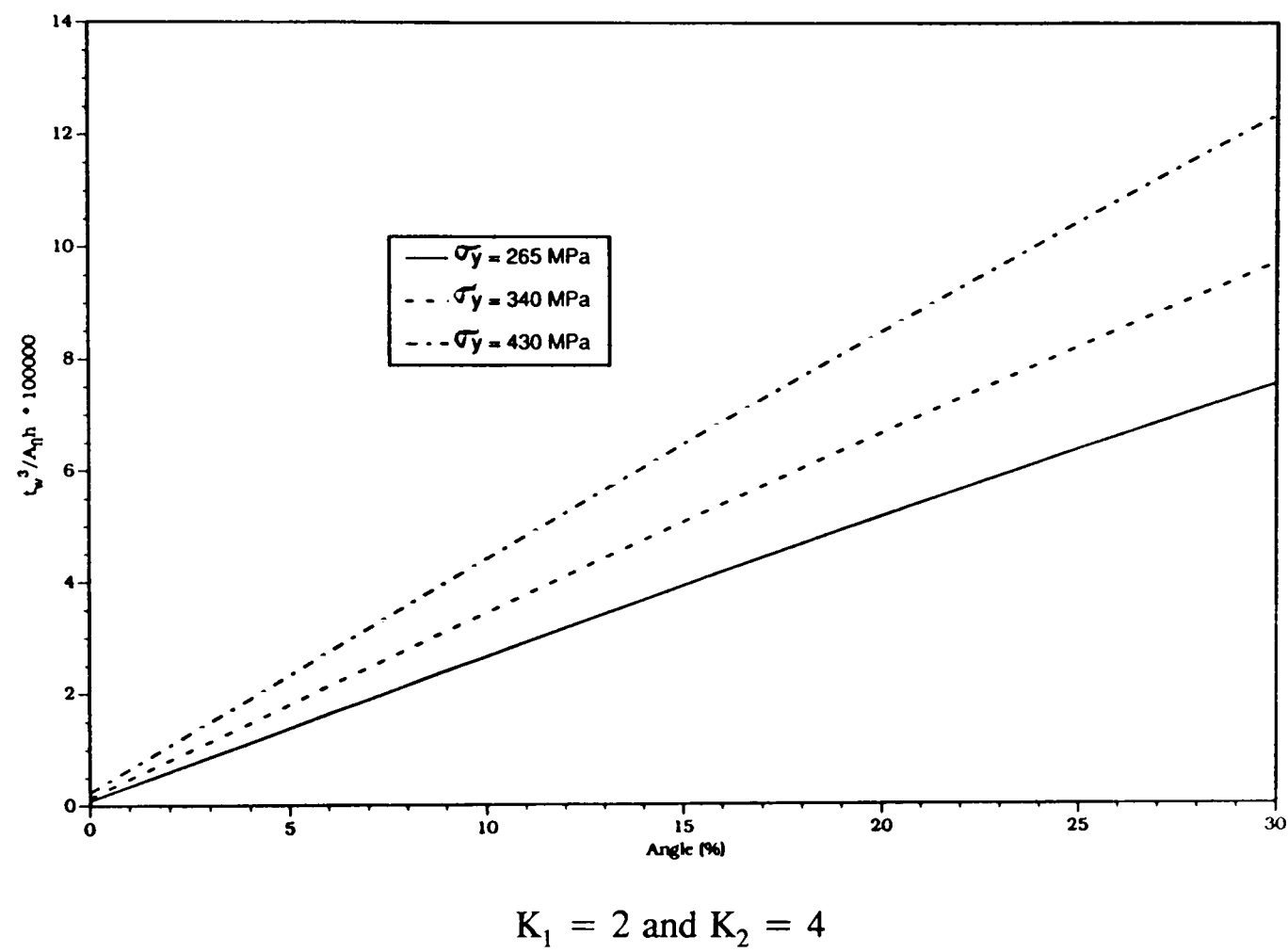


Figure 6.8 - Effect of σ_y^{fl} and α on the Non-Dimensional Factor $\left[\frac{100000 t_w^3}{A_n h} \right]$.

Table 6.2 presents the results of equations 6.2, 6.5, 6.8, 6.16, and 6.17 for the six beams tested. The results of equation 6.16 were obtained using $K_1 = 1.0$ and $K_2 = 4.0$, being then a more conservative version of equation 6.17.

Beam Numbers	Minimum Web Thickness (mm)				
	Eq. 6.2	Eq. 6.5	Eq. 6.8	Eq. 6.16	Eq. 6.17
1 to 5	2.13	1.98	2.95	5.53	4.42
6	2.13	1.98	2.95	7.07	5.64

Table 6.2 - Experimental Data Equation Comparison.

The first five beams are grouped as one because their material and geometric characteristics affect the equations in the same way. The first three equations gave results in which the minimum web thickness could be taken less than 3 mm. The new design model produced a value above 3mm and below 5mm which confirms what was achieved experimentally. The finite element runs also indicated a limit of 4mm for the web buckling.

The sixth beam had a thicker bottom flange which modified the predictions of the last two equations. They have suggested values greater than 5 mm. The laboratory beam did not develop a vertical web buckle although it had a 5 mm web because the top flange failed prematurely. The same behaviour occurred in the finite element simulation.

These limited results confirmed that the proposed model represents the behaviour of the tested specimens and can therefore be used to predict elastic vertical web buckling.

6.4.2. Comparison with Parametric Study.

The finite element parametric study cases were then checked with the same equations. To begin, the first series of runs was tested with the parallel flange case being examined first, table 6.3.a below:

Angle of Taper	Minimum Web Thickness (mm)					
	Eq. 6.2	Eq. 6.5	Eq. 6.8	Eq. 6.16	Eq. 6.17	Param. Study
0	3.68	3.63	3.89	1.87	1.87	5.00

Table 6.3.a - First Parametric Series Equation Comparison: parallel flanges.

For the beams with parallel flanges ($\alpha = 0^\circ$) the results of the first three equations agreed with the parametric study which predicted that a 3mm web would buckle while a 5mm web would not. Using a factor $K_2 = 4.0$, equations 6.16 and 6.17 suggested an unsafe thickness of 1.87mm. As noted earlier, the greater vertical wavelength of the buckle with parallel flanges suggests that $K_2 = 1.0$ would be more accurate, and equation 6.16 (or equivalently equation 6.18) with this value of K_2 suggests a thickness of 2.93mm. This value is marginally less than the 3mm thickness which buckled in the parametric study, and the difference between it and the predictions of equations 6.2, 6.5, and 6.8 is some indication of the size of the conservative approximations built into the code equations.

All the other slope cases for the first test series are presented in table 6.3.b below, where equation 6.16 used $K_1 = 1.0$ and $K_2 = 4.0$, while equation 6.17 incorporates $K_1 = 2.0$ and $K_2 = 4.0$.

Angle of Taper	Minimum Web Thickness (mm)		
	Eq. 6.16	Eq. 6.17	Parametric Study
1/50	3.87	3.18	5.00
1/40	4.14	3.38	5.00
1/30	4.52	3.67	5.00
1/20	5.13	4.14	5.00
1/10	6.41	5.13	5.00
1/6.667	7.30	5.83	7.00
1/5	8.00	6.38	7.00
1/4	8.58	6.83	10.00

Table 6.3.b - First Parametric Series Equation Comparison.

Both equations 6.16 and 6.17 required a web thickness greater than 3mm for the angle of 1/50. The parametric study showed a plastic vertical buckling in the unloading path of the beam even with a five millimetre web. However, this buckle, as explained in chapter 5, did not appear before peak load, making the predictions safe. This is especially true if effects like strain-hardening, the stiffening effect of the steel deck and concrete slab, and others are taken into account. The same conclusion can be reached for the angles of 1/40, 1/30, 1/20, and 1/10.

The angles of 1/6.667 and 1/5 produced an elasto-plastic vertical buckling for the 5mm beams in the parametric study. Both equations 6.16 and 6.17 recommended a thickness greater than 5 mm for this angle, in accordance with the finite element predictions.

Lastly, the angle of 1/4 was analysed using the design equations. The parametric study predicted plastic vertical buckling for the 7 mm beam in the unloading path. Both proposed equations had a good agreement with this result.

The calibration of the model had as its final test the case with the larger area of the tension flange and $\alpha = 1/10$. The results of the design equations are presented in table 6.4 below, with the code parallel flange values for comparison.

Area of the Flange	Minimum Web Thickness (mm)					
	Eq. 6.2	Eq. 6.5	Eq. 6.8	Eq. 6.16	Eq. 6.17	Parametric Study
3000 mm ²	3.68	3.63	3.89	6.41	5.13	5.00
6250 mm ²	3.68	3.63	4.91	8.18	6.55	7.00

Table 6.4 - Second Parametric Series Equation Comparison.

The first case of this series was just inserted in this table as a mean of comparison: is the case already mentioned in table 6.3, for the angle of 1/10. The other case with an area of the tension flange equal to 6250 mm² had the limit (from the finite element study) of 7 mm for the web with a plastic vertical buckling in the unloading phase. Both proposed models had a good agreement with the parametric study, recommending limiting web thicknesses in the order of 7 mm.

The next section will present the main conclusions reached about the proposed design model.

6.5. GENERAL CONCLUSIONS.

A review of three steel design codes has been made in respect of the prediction of the vertical web buckling phenomenon. The American code used an unnecessary simplification of the A_w/A_f ratio. The European formula had the terms h and t_w in both sides of the equation, making the design process longer. The British code used a very simplified version of the American code. All three codes only contained predictions for beams with parallel flanges and did not cover the buckling in tapered girders.

The proposed design model is based, like the codes, on the previous work of Basler & Thurlimann, [BASL61]. It predicts a minimum thickness to avoid local web buckling. If a thickness greater than this value is used, no stiffeners are necessary at the slope change. Otherwise, a vertical stiffener on both sides of the web, with approximately one third of the web's depth, should be used at this point. The design of that stiffener should be made according to the requirements of any steel design standard as a load bearing stiffener. The magnitude of the design force should be equal to the vertical component that is introduced to the web by the flanges: $2 A_f \sigma_y^f \sin \alpha$.

The proposed design model, equation 6.16, reached a good agreement with the laboratory tests described in chapter 3 using $K_1 = 2$, $K_2 = 4$. It also produced good results when compared with the finite element parametric study described in chapter 5.

For design purposes for the parallel flange case, the parameters K_1 and K_2 should be set to one in the proposed design equation. Another option is to use any code formula for parallel beams. For tapered webs, equation 6.16 should be used (with the more conservative values of $K_1 = 1$ and $K_2 = 4$) to predict a minimum web thickness, for use if it is greater than the parallel flange provision. The less conservative approach of having $K_1 = 2$ and $K_2 = 4$ needs more experimental background evidence before it can be recommended for design application.

The proposed design model produced more consistent results than the Lawson proposal, [LAWS89a], of using stiffeners for every beam with a slope angle greater than $1/10$, since the present study has shown that some beams with slopes less than $1/10$ can develop vertical buckling, and that beams with slope greater than $1/10$ do not need stiffeners if at least the proposed minimum web thickness is used.

The next chapter will present the main conclusions of this research, together with some suggestions for future developments in this area.

CHAPTER 7

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK.

7.1. INTRODUCTION.

The main purposes of this chapter are to summarize the conclusions of the research and to provide a set of useful suggestions for future work.

7.2. CONCLUSIONS.

Very few references to the problem of vertical web buckling at the central point of tapered composite beams were found in the literature. They contained no significant information about experiments or numerical simulations developed to understand its behaviour. A broader survey was then made involving related problems such as the patch load resistance.

This patch load review was very useful in identifying the most important variables to be studied. The thickness of the web was considered the major parameter by the great majority of the researchers. The material and geometric characteristics of the flanges were also found to influence the patch load behaviour, but with less severe consequences for the ultimate resistance. The significant effect of a combination of bending moment and patch load on the ultimate capacity was also emphasised in the review.

Another important conclusion was that a significant difference exists between classical patch loading and the concentrated load induced by the slope change in the lower flange. Patch loads act on the compression flange and induce biaxial compression in the adjacent web, while the slope change creates vertical compression in part of the web which is under horizontal tension. This difference significantly increases the buckling load of the web, and makes the slope change less critical than conventional patch loading.

The results of the first series of tests confirmed the early conclusion that the thickness of the web was one of the most important parameters in the local web buckling problem. The 7 and the 5 mm beams reached their ultimate load with no sign of web buckling. They failed by the development of a plastic hinge at the point where the tapered beam had its minimum height. The 3 mm web failed by vertical buckling, being very useful to prove that the phenomenon really happens. The reduction in resistance between the 5 and the 3 mm web specimens was very significant. For beams with the present geometric and material characteristics, it was possible to say that the physical limit for the vertical buckling of the web lay between the 3 and 5mm thicknesses.

The plastic neutral axis position (in contrast to what was initially thought) did not have a major influence on the vertical buckling. This was proved with the aid of the second series of tests. The fourth beam failed by excessive deflection caused by the formation of a plastic hinge. The substantial decrease in the load carrying capacity, demonstrated by the fifth and sixth beams, was due to their premature failure by local buckling of the top flange. Flange buckling itself was not particularly

relevant to the present study because in composite beams the steel deck/concrete slab would prevent its occurrence. The sixth beam served to demonstrate that, even at the construction stage where the plastic neutral axis position is on the bottom flange, the web was not affected by vertical buckling.

The finite element simulations have reached a reasonable agreement with the experiments. The numerical method predicted the final deformed shape and the web stress patterns reached by the girder at the ultimate loading stage. In the first series, web stiffeners were only found to be necessary for webs that had thickness less than four millimetres. No stiffeners were required in the second series of beams.

The finite element program, Finas, employed in these simulations, presented some problems for the user. No restart facility was available in the program, which led to a large increase in computation time and user effort to reach and pass the maximum load point. No real control over the solution strategies was given. Divergence problems were very common and difficult to overcome. There was a very restricted menu of shell elements. No pre or post processor was available at the time of use. The great majority of graphs contained in this thesis have been produced with the aid of programs developed by the author, although a post processor for FINAS has recently been implemented. A considerable amount of time and effort was also spent in the non-trivial task of finding the most suitable mesh for the problem.

The finite element parametric investigation has focused on three main parameters. A study of the angle of taper and web thickness was first reported. This was followed by an examination of the influence of the tension flange area on the load carrying capacity of the girders. The differences in moment resistance, for a certain angle of slope, tended to increase as the slope became steeper. These results proved that the slope was a very significant variable in the load carrying capacity of the beam. When the area of the tension flange was doubled, the decrease in moment resistance reached values of up to 45%.

The vertical web buckling only occurred in one case for the standard flange for a h/t_w ratio less than 155. This happened when the seven millimetre beam was subjected to a very large angle of 14.84° . Situations with these high angles are rarely found in design practice. A small increase in the thickness of the web would be the most effective and least costly solution to avoid this kind of collapse.

A review of three steel design codes has been made in respect of the prediction of the vertical web buckling phenomenon. All three codes only contained predictions for beams with parallel flanges and did not cover the buckling in tapered girders.

The proposed design model predicts an explicit minimum web thickness to avoid vertical buckling. It has reached agreement with the laboratory tests. It also produced good results when compared with the finite element parametric study.

For design purposes for the parallel flange case, the parameters K_1 and K_2 should be set to one in the proposed design equation. Another option is to use any code formula for parallel beams. For tapered webs, equation 6.16 should be used (with the more conservative values of $K_1 = 1$ and $K_2 = 4$) to predict a minimum web thickness for use if it is greater than the parallel flange provision. The less conservative approach of having $K_1 = 2$ and $K_2 = 4$ needs more experimental background evidence before it can be recommended.

The proposed design model produced more consistent results than the Lawson proposal, [LAWS89a], of using stiffeners for every beam with a slope angle greater than $1/10$, since the present study has shown that some beams with slopes less

than 1/10 can develop vertical buckling, and that beams with slopes greater than 1/10 do not need stiffeners if at least the proposed minimum web thickness is used.

Vertical stiffeners would only be needed in the web in extreme situations. Most of the cases found in practice will not need stiffeners. Given the high cost of fitted stiffeners, the approach of slightly increasing the thickness of the web to avoid vertical buckling seems likely to be the most satisfactory design expedient in limiting cases.

If the thickness of the web cannot be increased, then a vertical stiffener on both sides of the web with approximately one third of the web's depth should be used. The design of that stiffener should be made according to the requirements of any steel design standard as a load bearing stiffener. The magnitude of the design force should be equal to the vertical component that is introduced into the web by the flanges, i.e., $2 A_f \sigma_y^f \sin \alpha$.

7.3. SUGGESTIONS FOR FURTHER WORK.

One of the most pressing needs in a continuation of this line of research is the execution of more experiments. The new set of experiments could corroborate the parametric study in respect of the influence of parameters like: the angle of taper, the tension flange area and yield stresses of the web and flanges.

Another interesting set of experiments could be related to the use of a full composite tapered beam with a concrete slab, steel decking and the tapered steel beam. This will be important to assess the real influence of the concrete slab and steel deck on vertical web buckling and the subsequent load-carrying capacity of this type of beam.

A beam with the same geometric characteristics as the 3 mm beam tested in this study but with a vertical stiffener present at the slope change would also be of interest. The investigation of the effectiveness of that type of stiffener should also make clear the exact gain in structural stability that its use will provide.

The less conservative model ($K_1 = 2$) could then be checked against the new cases, which could provide more reliability to its full use as a design tool. The model could also be improved by the introduction of effects like imperfection and residual stress.

The parametric study could also be extended to cover a new set of parameters that can also influence the local web buckling such as: the rotation restraint provided by the flange to the web, residual stresses, hybrid (different yield stress for web and flanges) tapered girders and others. In order to make this study possible, several major changes will have to be introduced in the finite element program to make it more cost effective and user-friendly.

Another point that can be raised is in respect to the interaction of this type of buckling with shear and normal stresses. The buckling resistance could be seriously affected by these extra stresses and its full behaviour is not completely understood. Possible design interaction curves could be suggested based on future tests and parametric studies in that area.

Finally, a new less conservative approach for the stiffener design could be developed based on the new test with stiffeners. The aim would be to exploit any differences between a normal load bearing stiffener, located in part of the web under biaxial compression, and the slope-change stiffener in part of the web under horizontal tension and vertical compression.

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