

**NUMERICAL SIMULATION
OF
SPOILER FLOWS**

by

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SUMMARY

The unsteady separated flow over a fixed and a moving spoiler fitted on the upper surface of an aerofoil has been simulated numerically.

Initially, an analytic conformal transformation is developed, which transforms an aerofoil with a spoiler into a unit circle. The flow separating from the spoiler tip and the aerofoil trailing edge is simulated using the Discrete Vortex Method. The Biot-Savart Law is employed to convect the shed vortices in the circle (transformed) plane and a large number of vortices is used for a good representation of the wake. The model has been developed keeping empirical inputs to a minimum.

The motion of the spoiler is modelled using a distribution of singularities along its surface, namely sources and sinks. The pressure distribution over the aerofoil and spoiler is calculated and force coefficients, such as lift and drag coefficients, are obtained using pressure integration and a Momentum method.

The results obtained from the method when applied to an aerofoil with a spoiler in an impulsively started flow are compared, where possible, with existing experimental results. The lift follows the experimentally observed behaviour, i.e. an initial increase to a peak followed by a subsequent drop. The peak in lift is seen to occur when the main vortex formed from the spoiler tip passes the aerofoil trailing edge. For the 'fixed' spoiler case, final lift coefficients are over-predicted when compared with experimental results. Good agreement is found for the pressure distribution over the aerofoil. For the moving spoiler case, good agreement is found for the delay times for transient response with experimental results. The present model enables the calculation of delay times to maximum adverse lift at very high spoiler deployment rates, as well as the calculation of forces on the aerofoil and spoiler separately. Both these are very difficult to predict experimentally.

In general, the numerical model is found to be in good qualitative agreement with experiment.

*To the Memory
of
my Mother*

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LIST OF SYMBOLS

C_l	Lift coefficient
C_d	Drag coefficient
C_p	Pressure coefficient
C_f	Force coefficient
c	Aerofoil chord
i	$=\sqrt{-1}$
l	spoiler length
m	sources/sink strength in aerofoil plane
m'	source/sink strength in 'straight-line' line plane
N	Total number of vortices
NP	Number of control points on the body
NSP	Number of control points on spoiler
q	Surface velocity
R	Radius of cylinder
R_T	Routh's term
SP	Spoiler position along the aerofoil surface
t_a'	Non-dimensional time to maximum adverse lift
t_o	Time delay to onset of adverse lift
t_f	Time delay to maximum adverse lift
T_o	Time to final spoiler position
u_t	Spoiler tip velocity
U_∞	Free stream velocity
$U_\infty T/c$	Non-dimensional time
u,v	Horizontal and vertical components of velocity
V_∞	Free stream velocity in circle plane
x,y	Cartesian coordinates

z	Complex position in the aerofoil plane
Z_f	Complex force
W	Complex potential
α	aerofoil incidence
Γ	Circulation (convention adopted +ve anticlockwise)
δ	spoiler angle
δ_o	initial spoiler angle
δ_f	Final spoiler angle
Δt	Time step
ζ	Complex position in the circle plane
μ	amplification factor of the free stream velocity when in the circle plane
σ	Core vortex radius
Φ	$\text{Re}(W)$
Ψ	$\text{Im}(W)$, streamfunction
ω	Spoiler angular velocity
Ω	vorticity
$*$	Defines a complex conjugate
$\bullet$	Defines a negative vortex
$\circ$	Defines a positive vortex

CHAPTER ONE

INTRODUCTION

1.1 The use of Spoilers as Active Control Devices.

Spoilers, as the name implies, are devices which, when operated, 'spoil' or separate the flow around a wing or an aerofoil. As a result, overall aerodynamic loads may be modified and controlled.

Spoilers are positioned at different places on the wings of an aircraft (figure 1.1) so that their effectiveness may be used to a maximum, during take-off, landing or cruising configurations of the aircraft. Inboard and outboard spoilers are used as pure air brakes during landing or for symmetric lift control, while outboard spoilers are normally used for asymmetric roll control. Spoilers have become widely used to provide roll control on high manoeuvrability combat aircraft because of the reduced efficiency of conventional aileron controls at high speeds. In all these applications, spoilers are deflected or retracted relatively slowly so that during their movement their effects can be characterised as quasi-steady.

Current interest is centred around the question of how far spoilers can be used as fast aerodynamic devices in active control technology (ACT) applications to improve the efficiency and cost effectiveness of military and civil aircraft. For example, they may be used to raise flutter margins or for load alleviation due to gusts. To achieve such effects, spoilers must be deflected at rates of the order of 400 deg/sec. At these high rates of spoiler deployment, their effects are aerodynamically unsteady and very much different to static spoiler characteristics.

Present research is aimed at understanding the unsteady loads induced on wing-spoiler configurations for arbitrary transient motions of the spoiler, and particularly the two actions of deploying and retracting the spoiler. Also, an understanding of the wake they produce and its effects on trailing edge flaps and the tail

plane, is necessary if their design is to improve further and their application is to be extended.

1.2 Experimental Work.

Early experimental work on spoilers showed that forward positions of a spoiler on a wing were unsuitable because of unacceptably slow response to roll commands, while more aft spoiler locations gave much more reasonable response characteristics.

In the early 1950's, work done by DeYoung (1951) and Franks (1954) gave the USAF Stability and Control DATCOM the basis to suggest a preliminary design method for spoiler effectiveness.

However, although spoilers have been used for over forty years, their characteristics are the most difficult to predict of any of the other control surfaces used. Experimental work carried out by Boeing (MACK et al (1979)), outlines the difficulties associated with spoiler design. Their aerodynamic characteristics together with the need to use them with trailing edge control surfaces (flaps), restrict the spoilers' geometry, location and maximum deflection.

This experimental program showed that when the spoilers are raised, they cause flow separation over the wing and create a wake, which is highly turbulent and has regions of reversed flow behind the spoiler. It also indicated that persistent concentrations of energy exist in this wake, at characteristic frequencies. This spoiler's wake affects the trailing edge flaps (figure 1.3) and the tail plane and may cause buffet at large spoiler deflections. As a result, some of the spoiler panels along the wing of an aircraft can only be used as air brakes on the ground and other panels are limited in deflection to reduce buffet and wake effects on the tail.

Boeing's experimental program also revealed the highly nonlinear spoiler characteristics for take off, landing and cruise, which complicate further the gearing problem, as well as the autopilot and control system design. Figure 1.2 shows:

- a) A large change in maximum control power with flap deflection, for landing flaps typically four to six times the flap up level.
- b) Low effectiveness at small spoiler angles.
- c) The 'S' shape of the rolling moment vs spoiler deflection for landing flaps configuration.

WENTZ and OSTOWARI (1981) conducted wind tunnel tests to determine effects of certain design variables on spoiler performance and spoiler flow field characteristics. They found that for low and moderate angles of attack control response is nearly linear with spoiler projection height. As the angle of attack increases to near stall values, control effectiveness is greatly reduced, with a 'dead' band or in some instances a slight control reversal appearing for small spoiler deflections. In their efforts to reduce this spoiler 'dead' band tendency they employed hingeline gap and porosity on the surface of the spoiler. This reduced the 'dead' band but also showed some drop in spoiler control effectiveness. The fact that their experiments showed that spoiler effectiveness is nearly proportional to spoiler span, indicated that perhaps two-dimensional test data could be applied to three-dimensional wings, at least for zero sweep cases.

During the same series of experiments they also investigated wake turbulence generated by certain spoiler configurations, using a dual split-film anemometer probe. It was found that the turbulent energy increases with increasing spoiler deflection and that strongly dominant frequencies appear as the spoiler is deflected. Using the distance from the spoiler tip to the trailing edge of the aerofoil as the characteristic dimension, it was found that the dominant frequency of the wake turbulence for the basic spoiler results in Strouhal numbers ranging from 0.19 to 0.26. The Strouhal number is defined as $St=f.c/U$, where, f , is the vortex shedding frequency, c , is a characteristic body dimension and, U , is the free stream velocity.

MACK et al (1979) also recorded the existence of dominant frequencies in the separated wake, which, they believed, were associated with vortex shedding from

the spoiler tip and the flap. These frequencies yielded Strouhal numbers ranging from 0.17 to 0.42. At high Reynolds numbers, a predominant Strouhal number becomes rather well defined. This stresses the need for tests at high Reynolds numbers to understand the wake's characteristics, which, in turn, would enable the understanding of the dynamics of tail plane buffet, caused by the spoiler's wake.

1.2.1 Steady spoiler characteristics.

Before proceeding to discuss the unsteady flow over a spoiler, it would be very useful to look at the effects that a steady spoiler has on an otherwise undisturbed flow over an aerofoil.

Experimental research on spoilers normal to aerofoil surfaces dates back to the work of WOODS (1956). PARKINSON et al (1974) and LANG (1976) carried out experiments to obtain surface pressure distributions with a normal spoiler on a single element aerofoil and their work revived interest in spoiler aerodynamics.

SIDDALINGAPPA and HANCOCK (1979) conducted experiments on a two dimensional spoiler placed on the floor of a small blower tunnel. Their spoiler had a gap at the hinge line and their experiments concentrated on obtaining pressure distributions along the tunnel floor near the spoiler, for various spoiler angles and gap sizes.

The 'simplest' spoiler is a flat plate placed at an angle on a flat surface in a subsonic free stream. AHMED and HANCOCK (1983) measured the pressures on the tunnel floor near a spoiler, for a range of spoiler angles. Generally, the chord of the spoiler is much larger than the shear layer on the wind tunnel floor in the absence of the spoiler.

As the spoiler deflection increases, the flow must turn through a progressively larger angle at the hinge of the spoiler. This slows the flow down and the adverse pressure gradients cause a separation bubble ahead of the spoiler. The flow reattaches on the spoiler's front surface. Then it separates again from the spoiler tip as a

thin shear layer, which entrains fluid on both its sides. The width of the shear layer increases until the flow reattaches on the tunnel floor, forming a bubble of slowly recirculating fluid (figure 1.4). The length of the bubble increases with increasing spoiler deflection, and for very small spoiler deflections, the boundary layer may just thicken without separation.

Looking at the pressures, there is a compression ahead of the spoiler, which causes the separation already mentioned. Behind the spoiler, there is a region of suction pressure, corresponding to the bubble, followed by a pressure recovery as the flow reattaches on the tunnel floor.

Oil flow studies carried out by WENTZ and OSTOWARI (1981), illustrated the extent and nature of the spoiler wake. Typical photos show two standing eddies which serve to redirect the flow toward closing the wake and forming the bubble mentioned above. The lengths of the oil streaks give an indication of the magnitude of the velocities within the wake. The bubble appears to be a near dead water region with high velocities round the edges.

Although the basic characteristics of the flow around the spoiler are the same, the whole problem becomes more complicated when a spoiler is placed on an aerofoil. For example, the separation bubble behind the spoiler modifies the pressure distribution and circulation of the aerofoil and affects hinge moments, lift and pitching moments. Depending on spoiler deflection and position on the aerofoil, the separated shear layer from the tip of the spoiler may reattach on the aerofoil's surface or join the separated layer from the trailing edge forming a highly turbulent wake. Experiments carried out by MACK et al (1979) have shown how the wake behind a spoiler on the wing of one of today's airliners is going to interfere with the flaps, which for large spoiler deflections may become ineffective.

Experiments carried out by CONSIGNY et al (1984) show the effects that the free stream Mach number has on the pressure distribution round an aerofoil with a spoiler and also on the aerodynamic coefficients (lift, pitching moment and spoiler

hinge moment), for different spoiler angles. They measured the pressure distribution round an aerofoil/spoiler combination for free stream Mach numbers ranging from 0.3 to 0.8. Their experiments showed that for a free stream Mach number of 0.73, the separation bubble formed behind the spoiler reaches the trailing edge, and changes in both upper and lower surface pressure distributions are obtained for lower values of the spoiler deflection when compared to the low speed case.

1.2.2 Unsteady spoiler characteristics.

The actuation of a spoiler on a wing surface results in local flow changes in the neighbourhood of the spoiler and global flow changes, involving modifications to the overall circulation around the wing and spoiler combination.

An investigation of the global vorticity field generated by high frequency oscillations of a fence-type spoiler (i.e. perpendicular to the surface) located on one surface of an aerofoil has been carried out by FRANCIS et al (1979). It has revealed the formation and growth of an energetic and tightly wound vortex immediately downstream of the spoiler. This vortex convects downstream and is responsible for the time delay for the downstream conditions to become close to the final steady state values.

Experiments carried out by MABEY et al (1982), indicate that rapidly deployed spoilers do decrease lift but, depending on deployment rate, the final lift is achieved some time after the spoiler comes to rest. For example, for very fast rates peak adverse lift would occur after the spoiler had stopped. In the early stages of the spoiler extension the lift can increase due to the development of a strong vortex immediately downstream of the spoiler. This adverse lift effect could increase the gust load, which the spoiler is intended to reduce. A similar result was obtained by CONSIGNY et al (1984), who carried out experiments with a spoiler performing simple harmonic oscillations on an aerofoil. They observed that the magnitude of the oscillatory lift first increases rapidly, reaches a maximum, and then decreases gradually during a cycle of spoiler movement.

The generation of the vortex and its convection downstream affects the pressure distribution on the aerofoil surface in the immediate vicinity of the spoiler and consequently the aerodynamic forces. AHMED and HANCOCK (1983) measured the transient pressure response at stations positioned along a straight line on the floor of a tunnel, immediately after the spoiler root. The spoiler was deflected from 0° to 45° in 0.003 s and then kept steady at the final angle. Figure 1.5 shows a typical pressure response at these stations.

The undesirable effect of lift increase in the early stages of rapid spoiler deflection, may be reduced either by modifications to the spoiler (i.e. hinge gap, surface porosity) or by altering the shape of the spoiler displacement time curve, as shown by experiments carried out by MABEY et al (1982) and KALLIGAS (1986).

When the spoiler is retracted rapidly, the lift increases very quickly without any initial decrease (MABEY et al (1982)). This is due to the convection downstream of a large separation bubble (in the cases when the flow reattaches behind the spoiler), which results in a monotonic increase in lift. A similar observation was also made by SIDDALINGAPPA and HANCOCK (1979).

1.2.3 The need for numerical methods.

Experimental results demonstrate the complexity of the flow pattern associated with the motion of spoilers. Experience has also shown that it is difficult to accurately predict spoiler effectiveness from wind tunnel tests, due to tunnel interference and high spoiler rates of deployment. MACK et al (1979) comes to the conclusion that experiments must be combined with computational methods. The latter, would aid the understanding of the physics involved and look at quantities difficult to measure experimentally, for example vortex strength.

Numerical methods are becoming an extremely important tool in the study of complex separated flows around vortex shedding bodies, following the fact that computing cost has been decreasing over the years and that computers have become

faster and more powerful machines. In the case of a rapidly moving spoiler, a computational fluid dynamics (CFD) code would help to understand the effects that the motion of the spoiler has on the flow and consequently on the aerofoil. Also, numerical methods may be used alongside an experimental programme, to give a preliminary selection of design and to reduce the measurement efforts.

Analysing the flow past an aerofoil/spoiler combination with a numerical method has other distinct advantages too, considering that flow parameters may be varied individually, thus showing the dependence of the aerofoil performance on them. Physical parameters of the aerofoil and spoiler may be changed very simply in a numerical code without having to build a series of different models, which an experimental study would require. With the aid of the numerical model used in this study, velocities and pressures are obtained on positions like the surface of the moving spoiler. This is an extremely difficult, if not impossible, task experimentally.

The full description of the wake of a bluff body requires, strictly, a 3-D calculation. However, the use of a 2-D scheme has produced 'qualitatively' good results for steady unconfined (CLEMENTS (1973)) and confined flow (FELIX (1987)) over low and high aspect ratio bluff bodies and also, for the separated flow over a 2-D Joukowski aerofoil (BASUKI (1983)), but forces are overpredicted. Hopefully, with the present fast advances in computer design and development, computer time and storage problems will soon be eliminated.

The main aim of this present research is to apply a numerical method, namely the 'Discrete Vortex Method', to the problem of a rapidly moving spoiler on an aerofoil (comparison of the method with finite difference methods is presented in section 1.4.5). Its detailed description, applicability and limitations will be discussed later in this chapter.

1.3 Numerical methods of modelling the flow past aerofoils and aerofoils with spoilers.

In this section, certain mathematical methods of transforming fairly complicated shapes into simpler ones (mainly a circle) are discussed. Following that, different potential-flow methods of modelling the flow past bluff bodies, aerofoils and aerofoils with spoilers are also discussed.

1.3.1 Numerical mapping of exterior domains.

The flow around real shapes of bodies, bluff or streamlined, is normally complicated to solve numerically in the physical plane, because of the boundary conditions. To deal with that problem, mathematical methods have been developed, which transform a complicated shape in the physical plane, into a much simpler shape in the transformed or working plane. This way, the task of satisfying surface boundary conditions is greatly simplified. A very commonly used transformed cross section has been the circle, because of its simple geometry and the fact that it has a simple potential flow representation.

One of the earlier conformal transformation methods is that of THEODORSEN (1932), which transformed shapes, close to Joukowski aerofoil profiles, into a circle. NAYLOR (1982) used the Joukowski transformation to transform an extremely thin plate into a circle. This enabled him to solve for the separated flow over the plate, in the transformed plane. BASUKI (1983) also used a similar transformation to map a Joukowski aerofoil into a circle, and then to solve for the stalled 2-D flow over the aerofoil.

The circle has not been the only shape to which bodies in the physical plane have been transformed. JAROCH (1986) solved for the flow past a flat plate normal to a long splitter plate, by mapping the flow in the physical plane into the flow in the upper half of the transformed plane. A Schwarz-Christoffel transformation was employed. Also using conformal mapping, EVANS and BLOOR (1976) solved for the separated flow over a knife-edge situated in a duct. They used a Schwarz-Christoffel transformation, which mapped the interior of the duct in the physical plane into the upper

half of the transformed plane. It is true, however, that analytical solutions for conformal transformations are generally only possible for special geometries.

SYMM (1967) developed a numerical conformal mapping technique for arbitrary exterior domains, which he later extended to include doubly connected regions (SYMM (1969)). This method, in general, transforms a two-dimensional body inside a duct into an annulus. FELIX (1987) applied this method successfully, to solve for the unsteady, confined flow around two-dimensional bluff body geometries (rectangular and triangular prisms). This method seems also to be limited to the shape of bodies to which it can be applied (it does not favour sharp corners). However, in some cases pretransformation of the sharp edges can be applied before Symm's method is employed.

In this research project, an analytic conformal transformation is employed to map a Joukowski aerofoil with an arbitrarily positioned spoiler into a circle. The transformation is discussed in detail in Chapter Two.

1.3.2 Steady flow over aerofoils.

The steady, inviscid, attached flow over a 2-D aerofoil section, may be calculated analytically by transforming the aerofoil and the flow in the physical plane into the flow over a circle in the transformed plane. A very simple conformal transformation, is the Joukowski transformation:

$$z = \zeta + \frac{a}{\zeta} \quad (1.1)$$

This equation transforms an infinitely thin plate or a symmetric aerofoil with a cusped trailing edge and variable thickness (Joukowski aerofoil), into a circle. Equation 1.1 may be modified to take into account camber too. In the circle plane, the flow field is obtained from the complex potential for attached flow over the circle, including any required flow

incidence. Normally, the Kutta-Joukowski condition is satisfied at the trailing edge by including a bound vortex at the centre of the circle. The strength of this vortex is such that the tangential velocity at the point on the circle corresponding to the trailing edge of the aerofoil, is zero.

An aerofoil with a non-zero trailing edge angle may be transformed to a circle using the Karman-Trefftz transformation. More realistic aerofoils may be obtained using Theodorsen's method, as discussed above.

1.3.3 Unsteady flow over aerofoils.

The flow over an aerofoil may be unsteady because the free stream is a function of time or because of oscillatory wake formation. The calculation then becomes time dependent. Most calculations employ a starting flow with a constant free stream.

GIESING (1968) and BASU and HANCOCK (1977) studied the unsteady attached flow over aerofoils undergoing high frequency oscillations. They used the Hess and Smith surface singularity method, as applied to the steady flow calculation. The additional complication for unsteady flow is the vorticity shed from the trailing edge of the aerofoil, which has to satisfy the Kutta condition at successive time intervals.

The repeated application of a panel method has been applied by HENDERSON (1978), to compute the lift of separating multielement aerofoils in incompressible flow. This model solves for the separated wake displacement surface using entirely inviscid boundary conditions. The initial shape of the wake is guessed and then a local surface angle correction, based on the local value of normal and tangential wake surface velocity, is applied to force the wake geometry to approximate more closely a streamline.

Fully separated flows, however, cannot be handled satisfactorily by boundary layer and potential flow theories. This is because if separated regions are included, iterations are needed between potential flow, boundary layer flow and separated flow regions, which would have been continuously matched. Instead,

MEHTA and LAVAN (1975) solved the full Navier-Stokes equations numerically, in the whole flow field around the aerofoil. They used a thin symmetrical aerofoil at 15° incidence, placed in a low Reynolds number laminar flow. The governing equations in terms of the vorticity and stream function were solved using an implicit finite difference scheme. They recorded, for the separated aerofoil, an initially large value in lift coefficient due to the impulsive start, followed by a rapid drop. However, the predicted forces on the aerofoil were higher than those obtained from experiments. Their method was later extended by MEHTA (1977), to investigate the dynamic stall of an oscillating aerofoil.

A similar problem of the separated flow over an aerofoil was investigated by WU et al (1977). An integro-differential formulation was employed and was confined to the vortical region. This was achieved by computing the vorticity on a grid, but only the cells that contained vorticity were active. This way, the computational effort, compared to that required for the solution of the full Navier-Stokes equations, was much reduced. The magnitudes of the forces obtained were realistic but the pressure distributions showed errors as in the numerical study of MEHTA and LAVAN (1975).

HEGNA (1981) used the time dependent Reynolds averaged Navier-Stokes equations to solve for 2-D incompressible, turbulent, viscous, near-stall flow over a NACA 0012 aerofoil. The Reynolds number was 1.7 million. Turbulence was modelled with an algebraic eddy viscosity technique, modified for separated adverse pressure gradient flows. Their computed lift and drag coefficients compared well with experimental results.

1.3.4 Steady attached flow over aerofoils with spoilers.

The solution of the steady, inviscid, attached flow over an aerofoil with a spoiler using numerical methods, is not going to show most of the spoiler characteristics, since the flow past the aerofoil always separates at the spoiler tip. The main features of this solution are the two stagnation points at either side of the spoiler

root, the infinite velocity at the spoiler tip and the Kutta condition at the trailing edge. However, the inviscid solution may be combined with several other methods to model the separated flow over the spoiler. In this project, the attached flow was first solved using a panel method developed by KENNEDY and MARSDEN (1976). Also, the attached flow over a Joukowski aerofoil with a normal spoiler and a flat plate with a normal spoiler, was solved using two different conformal mappings to transform the flow in the physical plane into the flow over a circle in the transformed plane. The results obtained were mainly used to check the validity of the conformal transformation, which was developed in the beginning of the research.

1.3.5 Unsteady separated flow over aerofoils with spoilers.

Although upper surface spoilers on lifting aerofoils have been used extensively over the years, there has been relatively little theoretical information available on their performance characteristics, particularly the transient characteristics. The reasons lie in the complexity of the wake dynamics and the general inability to predict wake properties of separated flows. However, the separation points are fixed at the spoiler tip and the trailing edge, and the separating shear layers are thin and well defined near the aerofoil. It may then be argued that an irrotational free streamline model of the flow outside the wake should be capable of producing accurate results, except for any boundary-layer separation bubble, formed due to an adverse pressure gradient ahead of the spoiler. To complete such an irrotational model, some empirical data of the flow conditions at the edges of the separated layers and the wake itself are required, since the vortex formations inside the wake are not modelled.

One of the first works to be published along these lines is that of WOODS (1953). His linearised 2-D model assumed an infinite wake behind the spoiler and the trailing edge, which was characterised by a constant pressure change caused by the presence of the spoiler. The constant, together with a symmetrical boundary pressure distribution representing the wake, were empirical inputs. With this free-streamline

model it was possible to calculate the loading on the aerofoil.

BARNES (1965) carried out experiments on a symmetrical aerofoil fitted with a spoiler, to measure the displacement thickness of the boundary layer at the spoiler position and the pressure in the separated wake behind the spoiler. These results were used to modify Woods's theory for normal spoilers and also to provide an empirical formula for the incremental spoiler base pressure.

Two-dimensional irrotational flow has also been applied to partially separated flows over cavitating hydrofoils. In these models, the empirical input is normally the constant pressure inside the wake and the nature of the downstream closure of the separation bubble. Such a model, in different forms, was applied by PARKIN (1959), FABULA (1962) and SONG (1965), to solve various hydrofoil problems.

The theoretical models mentioned above are all linearised and restricted to thin aerofoils at low incidence with small spoilers. As a result, they might be expected to predict forces and moments but not surface pressure distributions. This is because in conventional thin aerofoil theory without separation, thickness has no effect on the lift and moment. However, the presence of the spoiler removes the upper surface of the aerofoil behind the spoiler, from the effective flowfield. Consequently, the effective thickness envelope of the aerofoil and spoiler becomes asymmetric. Therefore, it now affects directly the spoiler and aerofoil incidence and camber.

For a model to be capable of predicting pressures relatively correctly, it must include thickness. JANDALI and PARKINSON (1970) took that into consideration in their theory (an extension of that by PARKINSON and JANDALI (1970)) for the calculation of the pressure distribution in 2-D incompressible potential flow on Joukowski aerofoils, with normal upper surface spoilers. Aerofoil camber, thickness and incidence were unrestricted. They used a series of conformal transformations from a basic flow past a circle. One or two sources were placed on the part of the circle corresponding to the surface of the aerofoil and spoiler, exposed to the wake. The presence of the sources allow for the satisfaction of the Kutta conditions at

the spoiler tip and trailing edge, with the desired pressure. The wake pressure was an empirical input. JANDALI (1970) extended this theory to apply to solid aerofoils of arbitrary profile with normal spoilers, using an adaptation of Theodorsen's method. Furthermore, BROWN (1971) extended this method to apply to aerofoils with inclined spoilers and slotted flaps. He succeeded in that by combining the surface source distribution method of Hess and Smith and the wake source model. This theory was extended by BROWN and PARKINSON (1972) (also PARKINSON et al (1974)), to solve for the steady-state lift and the transient lift after spoiler actuation on aerofoils of arbitrary camber, thickness and incidence. The wake was still modelled as a cavity of empirically constant pressure, and the complex acceleration potential was used. A series of conformal transformations was employed to map the linearised physical plane, with a slit on the real axis representing the aerofoil plus cavity, onto the upper half of the plane exterior to the unit circle. However, they had to assume a cavity pressure equal to that of the free stream for the transient lift solutions, which increased the empiricism and possible errors.

Recently, PARKINSON and YEUNG (1987) extended one of Parkinson's earlier potential flow theory models, by incorporating new conformal mapping sequences to solve for the inviscid separated flow over an aerofoil with a spoiler or a split flap. These are placed at an arbitrary position and angle on the aerofoil. Still, though, the base pressure coefficient was an empirical input.

All the above methods require a specified experimental base pressure to be fed into the calculation. This makes their application rather inconvenient, because base pressures vary with aerofoil and spoiler configuration and angle of aerofoil incidence relative to the free stream.

An important advance has been made by PFEIFFER and ZUMWALT (1981), who developed a model quite different to those mentioned above; they employed a turbulent jet mixing analysis and conservation of mass and momentum to simulate the time average flow within the wake. In addition, they solved for an effective closed-wake

body, formed by adding to the original aerofoil/spoiler combination: the boundary layer displacement thickness, a closed wake behind the spoiler, and a trapped vortex at the hinge of the spoiler. They found that pressure distributions and forces and moments results correlated well, but results for extreme cases were only 'reasonable'.

In an attempt to remove the empirical constant-pressure input in the separated region of the 'wake source models' mentioned above, TOU and HANCOCK (1983) developed a different inviscid panel model to predict 2-D spoiler characteristics at low speeds. They modelled the surface of the aerofoil and spoiler with elements of linear piecewise continuous vorticity, while the separating thin shear layers from the spoiler tip and trailing edge were modelled with elements of constant vorticity. The separation lines extended for a finite distance and the wake was closed by two vortices. They assumed a uniform total head inside the wake, different to the uniform total head of the outer flow, and vorticity on the separated lines was related to the difference in total head across them. It was found that there was a minimum spoiler angle below which solution was not possible. Empiricism, however, has not been avoided here either, since the length of the separated lines and the strengths of the closing vortices are empirical inputs (and also interrelated). This model was later modified (TOU and HANCOCK (1985)) to solve for the inviscid separated flow past a steady 2-D aerofoil fitted with a spoiler and a slotted flap at low speeds. For the cases where the flow over the flap was separated, the position of the separation point was assumed so that it fitted experimental data. The model was extended even further to account for the spoiler performing small amplitude oscillations. Although both models were crude, encouraging results were claimed.

Nearly all the methods discussed above about solving the separated flow over an aerofoil with a spoiler, have a linearised form. As a result, they can only be applied to relatively thin aerofoils and they are unable to predict the adverse lift effects associated with transient spoiler characteristics observed in experiments. Also, they do not forecast the correct time scale for the development of lift around a rapidly deployed spoiler. In order to make it possible to solve computationally the steady and transient

spoiler characteristics, a numerical method must incorporate the modelling of the vortical structure of the wake behind the spoiler. In this project, the Vortex Method is combined with the potential flow over the aerofoil.

1.4 The Discrete Vortex Method (DVM).

In flows past bluff bodies or bodies at high incidence to the free stream (stalled aerofoils), the shear layers leaving the separation points tend to roll into vortices of dimensions large compared to that of the boundary layers before separation. Flow visualisations carried out by PIERCE (1961) and PULLIN and PERRY (1980) give evidence of such rolled shear layers. Experiments have also demonstrated that the dominant feature of separated bluff body flows is the convection of large scale eddies. This convection could be represented by the movement of inviscid vortices, which would provide a more natural and efficient description of the eddies and of the vorticity they carry.

The Discrete Vortex Method represents the cross-section of separated, 2-D shear layers by an array of discrete point vortices. Its major strength lies in its ability to simulate vortex dynamics in high Reynolds number flows, since in such flows a free shear layer is infinitely thin. It may be adapted, however, to low Reynolds number flows by including viscous effects. Also, this method appears to be particularly suited to flows around bodies with moving boundaries or bodies in oscillating flows. Computationally, DVM is very attractive because when external flows are treated, vortices are not needed in the large irrotational region of the flow (since the rolled-up shear layers are the only regions where transport, production and diffusion of vorticity are of significant importance) but they are concentrated in the wake, where high resolution is required. This way, large amounts of computer storage are saved. The vortices can be either freely convected in the flow field under the velocity field they generate, the so called Biot-Savart method, or associated with a mesh in the Cloud-in-Cell method.

Before the applications of DVM in flows past different geometries are

presented, its most important features will be discussed in the next section.

1.4.1 Vortex sheets represented by discrete vortices.

A separating shear layer is represented with an array of mobile discrete point vortices. These vortices follow the fluid like particles (the Lagrangian description). They retain their circulation in time, thus conserving total vorticity in the flow field. However, the method is only applied to a fluid which is assumed to be incompressible and inviscid. The incompressibility restriction is necessary because the Biot-Savart law depends on it. This means that it can be applied realistically to air flows of low Mach numbers.

On the inviscid restriction, SMITH (1966) pointed out that in separated incompressible high Reynolds number flows, viscosity is important mainly in the boundary layers before separation, as well as in the initial development of the shear layers and in the centers (sub-cores) of the individual vortices representing the shear layers. He stated, also, that the diameter of these sub-cores is only 5% of the typical diameter of a vortex. Therefore, the diffusion of vorticity for high Reynolds number flows is negligible. In addition, when the separated layer originates from a sharp point on a bluff body, then the separation point is fixed and independent of Reynolds number and hence of viscosity (MAULL (1980)). Explicitly adding the viscous term, $\nu \Delta \omega$, is not convenient in a Lagrangian frame of reference because it involves derivatives with respect to the Eulerian coordinates (SPALART et al (1983)).

Early investigations of the method concentrated on the validity of the idea to represent a separated layer with discrete vortices. ROSENHEAD (1932) was the first to introduce this concept. In an attempt to describe the time dependent roll-up of a free shear layer undergoing sinusoidal instability, he replaced the vortex sheet by a distribution of discrete elemental vortices, spaced evenly along the sheet. Both ROSENHEAD (1932) and WESTWATER (1935) demonstrated the roll-up of a vortex sheet.

Later, ABERNATHY and KRONAUER (1962) used discrete vortices to represent a vortex street wake by employing the initial perturbation of Rosenhead on two initially parallel shear layers emanating from a bluff body. It was shown that through the non linear interaction between the two sheets, cancellation of vorticity and broadening of the wake occurred. This result reaffirmed the experimental result by FAGE and JOHANSEN (1927) i.e. the reduction of strength of the vortex clusters.

During the rolling-up of vortex sheets, randomisation of the vortex positions occurs due to mutually induced erroneous velocities (MOORE (1974)). As a result of the simple model of the velocity field of a point vortex, large velocities are induced on vortices being close to each other, and this may lead to crossing over of the paths of the individual vortices. Computationally, short wavelength perturbations are introduced spuriously by roundoff error and may grow, leading to the destruction of the accuracy of the calculation (KRASNY (1986)).

A large number of different methods have been applied to overcome the instability problem. CHORIN (1973) introduced a core around the centre of the vortices so that the velocity field within the core was not unrealistically large. Chorin also suggested that this technique could be analogous to the introduction of a small viscosity which, by allowing the core to increase its radius, would diffuse the concentrated vorticity of the point vortex. This idea was also applied by CHORIN and BERNARD (1973) to the study of the roll-up of a vortex sheet induced by an elliptically loaded wing.

A different technique of stabilising the roll-up of separating shear layers is the Cloud-in-Cell method. Stability is achieved by distributing the point vortex representation of the flow field, onto the grid points of a fixed Eulirean grid system, which effectively diffuses the vorticity of the point vortices over a cell of the grid. Hence, stability similar to that obtained by Chorin's core vortices, is achieved. This method has been used successfully by a number of researchers, for example CHRISTIANSEN (1973), BASUKI (1983).

Amalgamation of vortices that come close together has also been used by SARPKEYA (1975) and STANSBY (1977), to reduce instabilities. However, amalgamation of vortices, especially near the surface of the body, may cause sudden changes in the motion of vortices. A rediscritisation method, suggested by FINK and SOH (1974), ensured that the point vortices were always located at the mid points of segments that represented the sheet. This way the vortices were kept at a constant distance apart. An other remedy that has been investigated by MOORE (1981), is to dampen the growth of small scales of instability by a local averaging of the solution in physical space.

Quite recently, KRASNY (1986) attempted to desingularise the equations governing periodic vortex sheet roll-up, by modifying the Biot-Savart law. He added a smoothing constant in the velocity equations so that the velocity never becomes infinite when two vortices get extremely close together. His results show that this smoothing factor diminishes the short wavelength instability of the vortex sheet model.

In the following sections, applications of DVM to separated flows past different geometries will be discussed.

1.4.2 Flow round non-lifting bodies.

The Discrete Vortex Method has been applied extensively to circular cylinders and bodies with sharp corners that may be easily transformed to a circle or an upper half plane. The advantage of a bluff body with sharp corners is that the separation points are fixed at the sharp corners, whereas on a smooth surface the location of separation may change depending on flow conditions and Reynolds number. In applying DVM to model these separating layers, it is necessary to establish the position of their origin on the body surface (separation point) as well as the vortex strength and the vortex convection velocity. Since the problem is a time dependent one, vortices are released in time and they constitute the separated layer, its growth, and their velocity its motion in the fluid. This process is naturally complicated and it imposes significant difficulties for

the numerical accuracy. Also, it dictates the manner in which various schemes are developed to meet individual problems.

GERRARD (1967) was the first to study the wake behind a cylinder in an impulsively started flow, using the Discrete Vortex method. Following his work, SARPKEYA (1968) and BELLAMY-KNIGHTS (1967) employed a method according to which the position and strength of the nascent vortices would respond to changes in the flow field downstream of the cylinder. In these studies the flow was forced to remain symmetric and the results showed the vortices rolling up and producing secondary vortex rolling up, as observed in experiments performed by PIERCE (1961). A number of techniques have been devised by researchers (DEFFENBAUGH and MARSHALL (1976), STANSBY (1977, 1981), SARPKEYA and SHOAF (1979), STANSBY and DIXON (1982)) to predict the position of the separation point on the surface of a cylinder and the strength of the nascent vortices, including the satisfaction of the no-slip condition at the separation points as well as assumptions based on experimental correlation of pressure distribution and separation position.

A different approach was proposed by LAIRD (1971). At the start of the impulsive calculation there was an asymmetric distribution of bound vortices on the surface of the cylinder, representing the boundary layer. The strength of the bound vortices was determined from boundary layer theory, and vortices were introduced over the whole cylinder surface at each time step, while their strength satisfied the no-slip condition. Similar work was carried out by CHAPLIN (1973), who used Rankine vortices rather than point vortices, avoiding this way instabilities from vortices coming too close to each other, and also by DOWNIE (1981), whose model allowed the introduction of vortices from both the primary and secondary separation points.

Although bodies with sharp edges do not present the complication of locating the separation point, since this is fixed at each sharp edge, the position and strength of the nascent vortices still have to be calculated. A considerable amount of work has been carried out by CLEMENTS (1973) and CLEMENTS and MAULL

(1975), who applied DVM to a semi-infinite body with right angle corners, base cavity and the flow down a step. They incorporated the Kutta condition to calculate the rate of shedding of circulation from the separation points. An extensive review of the development and applications of the method is presented by CLEMENTS and MAULL (1975).

GRAHAM (1980) used the discrete vortex method to analyse the forces induced by separation and vortex shedding from sharp-edged bodies in oscillatory flow at high Reynolds number. Comparison of the results obtained from this model with experimental results was satisfactory. However, forces tended to be overestimated.

The separated layers behind a bluff body in a numerical calculation, especially one with sharp edges, tend to roll-up near the body. The strong vorticity close to the body surface causes the forces to be overestimated compared with experimental results, although the Strouhal number is predicted correctly. SARPKEYA (1975) and KIYA and ARIE (1977) noted that cancellation of vorticity when using DVM to model the wake, does not reduce the total circulation to levels observed in experiments. Several methods have been employed to solve this problem. For example, vortices that get close to the body surface may be removed from the calculation, since for a very small time step, a vortex moving towards the surface would coincide with its image on the surface and cancel. But if vortices are allowed to get very close to the surface, then unrealistically high velocities are induced on them by their images. Also, in some investigations, vortices of opposite sign are cancelled, if they get closer than a specified distance to each other. SARPKEYA and SHOAF (1979), who employed the rediscritisation of the vortex sheet suggested by FINK and SOH (1974), used a vorticity reduction technique, such that every vortex in the wake loses its strength by an amount proportional to its current strength and position after the rediscritisation of the sheet. This technique gave force results in closer agreement to experimental values, than other methods that did not employ vorticity reduction. However, the loss of vorticity is not in agreement with Kelvin's theorem of conservation of circulation, unless the vorticity loss is due to

mixing. Also, the Blasius' Theorem cannot be applied since it is based on the conservation of momentum in the flow field. Nevertheless, in certain applications, vorticity reduction appears to be necessary in order to obtain results comparable to experimental ones (KIYA et al (1979), BASUKI (1983)). NAGANO et al (1980) calculated the flow past a rectangular prism. One of the difficulties associated with this type of flow is that the vortices shed from the front separation points lie in shear layers close to the surface of the body. Therefore, it is not only necessary to represent the vortex sheets in the downstream wake, but also to simulate the vortex interaction with the body surface. They employed vortex amalgamation and Chorin's vortex core to reduce instabilities and obtain fairly accurate results.

SAKATA et al (1983) developed a new DVM to study the unsteady separated flow around a square prism. They represented the body and the free shear layers with discrete vortices, so that conformal mapping was not required.

1.4.3 Flow round lifting bodies.

A flat plate placed at a positive incidence to the free stream produces lift, while placed normal to the free stream becomes a non-lifting body. The Discrete Vortex Method has been applied extensively to study the separated flow behind a flat plate, as it has been applied to study the separated flow behind a circular cylinder. This is due to the simplicity with which the flat plate may be transformed into a circle, on which the surface boundary conditions may be satisfied by means of vortex images.

The impulsively started, separated flow past a flat plate was first studied by KUWAHARA (1973). The plate was at an incidence to the free stream (30° to 89°) and a small time step was used to prevent the vortices from moving through the flat plate, especially on the top surface of the plate near the leading edge. There, the separated layer forms very close to the plate. In principle, vortices can not cross the body surface since they are supposed to cancel with their image as soon as they coincide with the solid boundary. This requires a very small time step, which can be computationally

expensive. If a vortex is very close to the surface at the beginning of a time step and the time step is not small enough, then at the end of the time step the vortex may cross the body surface. They employed the Kutta condition to determine the strength of the nascent vortices released at the two plate edges. Their results showed that DVM could be used to calculate the unsteady flow past a flat plate. SARPKAYA (1975) also studied the same problem, but the strength of the vortices was determined by using

$$\frac{d\Gamma}{dt} = \frac{1}{2} U_2^2 \quad (1.2)$$

where U_2 was interpreted as the average velocity of the last four shed vortices. The forces on the plate were calculated using the Blasius theorem and overestimated experimental results by about 20%. Rediscretisation of the vortex sheets was also used but produced identical results.

KIYA and ARIE (1977) and KIYA et al (1979) studied the separated flow past a flat plate using a fixed point for introducing vortices into the flow and equation 1.2 respectively. The first study focused on the variation of the distance of the nascent vortices from the separation points. Both studies showed regular shedding of vortex streets, while it was shown that the Strouhal number was not very sensitive to the position of the nascent vortices. KIYA et al (1979) also used vorticity reduction as a function of the age of the vortices. They came to the conclusion that absence of sufficient cancellation of vorticity in these regions is one of the most important shortcomings in the discrete vortex approximation, especially when applied to bluff bodies. The incorporation of vorticity reduction in the calculation produced results for the normal force coefficient, which were in good agreement with experimental results obtained by FAGE and JOHANSEN (1927). KAMEMOTO and BEARMAN (1978) studied the influence of the distance of the nascent vortices from the edges of the plate. They determined a parameter based on this distance, time step and free stream and they

showed that cases with the same values of this parameter had similar flow features. NAYLOR (1982) used DVM together with the Cloud-in-Cell method to study the flow past a flat plate in a steady and oscillatory free stream. His results showed good agreement with experiment, but the steady results slightly overestimated the forces.

LEWIS (1981) introduced an image-free form of the Vortex Method. He used the surface vorticity technique, originated by MARTENSEN (1959), to represent a two-dimensional body. This removed the need of mapping the body into a simpler shape. Only a small number of vortices was introduced at two separation points and the boundary layer was not treated independently. Subsequently, PORTHOUSE and LEWIS (1983) used a random walk model to account for the effects of viscosity, and their results showed that a large number of vortices and a very small time step would be needed for the random walk effect to be meaningful at practical values of the Reynolds number.

Aerofoils are true lifting devices and Vortex Methods have been applied extensively to solve for the attached and separated flow past them. GRAHAM (1983), for example, used discrete point vortices to study the initial development of lift on an aerofoil in inviscid starting flow. It was shown through this analysis that because of the spiral shape of the sheet shed initially from the trailing edge, the lift and drag were both singular at the start of the impulsive motion.

Fully separated flow over an aerofoil introduces the additional complication of the separated boundary layer from the leading edge. This is complicated by the need to predict the location of the separation point and the direction of convection of the shear layer, which may vary with Reynolds number, aerofoil incidence and aerofoil profile. Like in the separated flow past a flat plate at positive incidence mentioned above, the separated layer from the leading edge of the aerofoil lies close to the upper surface of the aerofoil. This makes the application of DVM more complex, since the induced velocities by the images of the vortices coming near the surface of the aerofoil, become unrealistically high.

KATZ (1981) used a discrete vortex method to solve the separated flow past a thin aerofoil. The separation point near the leading edge of the aerofoil was assumed to be known from experiments and flow visualisation. The rate of shedding of vorticity was calculated by setting it equal to half the difference of the squares of the average velocities at the upper and lower edges of the separated shear layer. Results obtained by this method were in good agreement with experimental values. However, two numerical parameters were adjusted to "calibrate the model".

CHORIN (1978) introduced the Vortex Sheet Method, in an attempt to model more accurately the widely different scales in the tangential and perpendicular directions of the boundary layer. This is a hybrid method, where the region exterior to the boundary layer is treated by discrete vortices incorporating a core, with an exchange of vortex elements, i.e. sheets become concentrated core vortices and vice versa. Effort was directed towards producing a good transition between the vortex sheets and the core vortices. This method was applied by CHEER (1983) to the flow past a cylinder and a stalled aerofoil. The results obtained from this method showed good agreement with experimental results, but only short computer runs were carried out to try and keep computer cost under control.

1.4.4 Separated flow over aerofoils with spoilers.

As mentioned earlier in this chapter, solutions of the flow past an aerofoil fitted with a spoiler have been restricted to wake source models. The application of DVM to study the characteristics of a fixed or rapidly moving spoiler on an aerofoil has not been investigated. In addition, very few experimental results exist, which show that steady and transient spoiler characteristics are very complex and not yet fully understood.

In experiments carried out by FRANCIS et al (1979), who used an oscillating fence type spoiler to disturb the flow over an aerofoil, and VIETS et al (1979), who used a rotating cam-shaped rotor to disturb the flow, organised vortex

structures were observed to form periodically behind the moving mechanism.

The impulsively started flow past a normal, steady spoiler on an aerofoil, may resemble the impulsively started flow over a normal flat plate on a long horizontal plate, if the aerofoil is adequately long and thin. One of the problems associated with the application of DVM to such flows, would be the fact that the separated layers stay close to the body surface. EVANS and BLOOR (1977) used a vortex discretisation method to study the flow past a normal, infinitely thin plate situated on the floor of a duct. The strength of the nascent vortices was determined by satisfying the Kutta condition at the edge of the flat plate and the vortices were always positioned at the centres of vorticity elements, in order to avoid instabilities due to vortices coming close together. LEWIS (1981) also applied a surface singularity method and discrete vortices to model the flow past a normal plate on a long plate.

Recently, JAROCH (1986) applied DVM to model the flow past a normal flat plate with a long wake-splitter plate. The initial application of a simple point vortex method, limited calculations to estimating the drag on the plate. However, the expansion of the model to include Rankine type vortices and vorticity reduction, gave results of good qualitative agreement with experimental results carried out by RUDERICH and FERNHOLZ (1986). Quantitative agreement was acceptable only for the time mean values. There was a buffer region behind the plate and very near the surface of the splitter plate, so that vortices that crossed that region were removed from the calculation.

The modelling of a rapidly moving spoiler on an aerofoil complicates the problem even further, since it should represent correctly the dynamic response of the flow to the actuation of the spoiler. A simplified case of a spoiler moving on an aerofoil would be a flat plate rotating about one of its edges, hinged on the real axis of the physical plane. This simple model has been used in the past by researchers to model the 'clap and fling' mechanism of lift generation over insect wings, postulated experimentally and theoretically by WEIS-FOGH (1973). Of particular importance are

MAXWORTHY's (1979) experimental results, which showed that the production and motion of a leading edge separation vortex accounted for virtually all of the circulation generated during the initial phase of the 'fling' process.

ZANDJANI (1983) modelled the Weis-Fogh mechanism using a discrete vortex method. He used several techniques, including Chorin's cores and amalgamation, to stabilise the separated shear layer from the leading edge of a rotating flat plate, which was representing the wing of an insect. The BROWN and MICHAEL (1955) method was employed to find the position and strength of the nascent vortices. This method is employed in the present research too, and it will be discussed in detail later.

Closer to the problem of a moving spoiler disturbing the flow over an aerofoil, is a recent work by CHOW and CHIU (1986). They released vortices from the upper surface of an aerofoil intermittently, in an attempt to simulate the flow observed in experiments, which was perturbed by an oscillating spoiler. Results from this model showed that the aerofoil lift, which oscillates, generally increases with time and it seemed that it would approach an asymptotic value as time increased indefinitely. The behaviour of the drag was similar to that of the lift but two orders smaller in magnitude.

In this present research, an aerofoil with a spoiler arbitrarily positioned on its surface is transformed into a circle, and DVM is applied to simulate the separated layers from the spoiler tip and the trailing edge, for both steady and transient spoiler cases. The mathematical formulation of the method is discussed later.

1.4.5 Vortex Methods versus Finite Difference Methods (FDM).

In general, it appears that at present neither of the methods can model high Reynolds number flows with the accuracy that is required in engineering. Very often, the results obtained from these methods offer only qualitative agreement, for example with flow visualisations, and quantitative agreement is restricted to few numbers that characterise the flow, such as drag or shedding frequency. There are three

main reasons for this:

a) Very often, quantitative and verified experimental results are not available for these flows, simply because it appears to be as difficult to measure these flows as it is to compute them.

b) Numerical modelling is still two dimensional, while experiments, although carried out about two-dimensional geometries, very often have significant three-dimensional effects.

c) FDM are unstable when used for incompressible Euler equations, unless artificial viscosity is used.

For Reynolds number less than 1000, FDM give good results. However, Reynolds numbers of aeronautical interest are one million or more. In these high Reynolds number flows, the vortical structures in the wake become so small that an extremely fine grid is needed, which requires a very small time step and, effectively, a very large memory. An alternative would be to use a coarser grid but in this case numerical diffusion and dispersion could easily dominate physical diffusion (SPALART et al (1983)). A coarse mesh is acceptable far away from the body surface.

In general, the main advantages of the Finite Difference Methods over the Vortex Methods may be summarised as follows:

a) FDM have a well established theory of convergence (mainly for bounded or periodic geometries since infinite domains are not treated in a fully satisfactory way). This is not the case for the Vortex methods, especially when viscosity and boundaries are involved.

b) They can be extended to compressible flows without any major changes, while their extension to three-dimensions is simpler, in principle, than for the Vortex methods.

c) Boundary layer assumptions are not involved in FDM and therefore singularities are eliminated. The transition from a viscous treatment, represented by a fine mesh near the body surface, to an effectively inviscid treatment, represented by a

coarser mesh away from the body surface, is smooth.

However, although FDM model the stresses in the boundary layer with relative ease, this is not the case for the wake, where modelling the stresses is extremely difficult. Some models include turbulent stresses (for example SUGAVANAM and WU (1980)) but these are evaluated with so much uncertainty that the benefit is not so obvious in terms of accuracy.

An obvious advantage of the Vortex method over the Finite Difference Methods is that it is mesh-free (not true for the Cloud-in-cell method), since it is not always easy to generate a grid for a complicated shape. Therefore, they are particularly attractive for solving flows past multiple bodies (for example, cascades of aerofoils). Also, the Vortex Method effectively includes an infinite flow domain, in contrast with Finite Difference methods which model a finite domain only. This means that certain boundary conditions must be applied at a certain distance from the body and possible inaccuracies in these conditions hide the danger of restricting the solution. Both these factors are absent from a Vortex Method and therefore empiricism is greatly minimised. It is also relatively easy for a Vortex Method to incorporate wind tunnel effects, provided that the flow does not separate from the tunnel walls (FELIX (1987)).

Vortex Methods model the vortical regions of the flow only and therefore, they can be faster to compute than solving the full Navier-Stokes equations using a grid method. They are also more efficient in terms of memory storage, especially for short computational runs.

In the Vortex method, N_v interactions are calculated at every time step, where N_v is the number of vortices. Many FDM, however, only require N_m operations, where N_m is the number of mesh points. The combined application of DVM with the Cloud-in-Cell method solves this problem to a certain extent but introduces a mesh. Although FDM may look more attractive from that point of view, in practical terms the values of N_m and N_v are limited and the question is what values of these parameters are needed to achieve a certain degree of accuracy.

The problem of the separated flow past an aerofoil fitted with a fixed or rapidly moving spoiler is modelled here by applying DVM, and a mathematical description of the method and its application is given in the chapters that follow.

CHAPTER TWO

ATTACHED FLOW

2.1 Attached flow over aerofoils with a spoiler using a surface singularity method.

Modelling the attached flow can form the basis of a method which may then be extended to include separation. However, the solution of the attached flow over an aerofoil with a spoiler is not going to show many of the spoiler characteristics, since the real flow always separates at the spoiler tip. In the attached flow solution, there will be an infinite velocity at the spoiler tip and stagnation points on both sides of the spoiler root. Also, the flow leaves the trailing edge smoothly while satisfying the Kutta-Joukowski condition, i.e. $dW/dz=0$ at the trailing edge of the Joukowski aerofoil.

Body surface velocity and pressure are important parameters to be determined for a fluid problem. Most analytical solutions are restricted to certain body shapes and for more complex problems recourse is made to numerical methods. Such a numerical method is the panel method, according to which the body surface is divided into surface elements with a distribution of singularities, ideally with a more dense distribution of elements around regions with large velocity gradients.

The most widely used method is that of HESS and SMITH (1967). This method uses a distribution of sources and sinks on the surface of the aerofoil section combined with a vorticity distribution to generate circulation. MARTENSEN (1959) and WILKINSON (1967) used a vorticity distribution to represent the body. The vorticity distribution satisfied a zero internal tangential velocity condition. This method has the advantage that it gives directly the surface velocity on the body, which is equal to the local vorticity density.

KENNEDY and MARSDEN (1976) developed a method which uses a surface vorticity technique and a constant stream function boundary condition. In the early stages of this work, the method of Kennedy and Marsden was applied to a

symmetric aerofoil section (NACA 0012) fitted with a spoiler of very small but finite thickness, and also modified to take into account the motion of the spoiler, in order to calculate the attached flow over the aerofoil/spoiler combination.

The solution of the attached flow over the aerofoil and spoiler using this singularity method has the disadvantage that the accuracy of the solution depends on the number of elements around the aerofoil. If the solution requires a large number of elements to converge, then the singularity method may prove to be expensive. Also, the application of the method by BASUKI (1985) to solve the attached and fully separated flow over an aerofoil and a cascade of aerofoils, has shown that although the method is efficient, it is sensitive to the way the surface is divided into elements. The size of the neighbouring elements must not change abruptly, and small elements must be used in the region where velocity gradients are high. It is certainly extremely useful when an analytic transformation is not available.

Nevertheless the application of Kennedy and Marsden's method to the aerofoil and spoiler problem demonstrated that it can give an accurate solution with little computer effort. It gave reasonable results, taking into consideration the fact that regions of large velocity gradients, such as the spoiler root and tip, were not given an extremely fine distribution of vorticity elements.

The investigation of the application of the surface singularity method was not carried any further, mainly because an analytic method was developed which would transform an aerofoil with an arbitrary spoiler, in the physical plane, into a circle in the transformed plane and this method was felt to be more accurate. On the circle, boundary conditions at points such as the spoiler tip and the trailing edge may be easily applied, and the solution can be extended with relative ease to include flow separation.

The development of a conformal transformation for the aerofoil/spoiler combination and its application to solve the attached flow over the aerofoil and spoiler are presented in the following sections.

2.2 Attached flow over aerofoils with spoilers using a Conformal Transformation method.

In the beginning of the research it was not realised that, a conformal transformation method based on simple functions was available, which would transform an aerofoil with a spoiler into a circle. Therefore, certain existing numerical mapping methods that would map a general shape into a circle were applied. Although these methods did not prove to be successful, they are briefly described here, and a possible explanation of their failure is given.

SYMM (1967) developed a numerical method which computes the conformal mapping of the exterior of a given simply-connected region onto the exterior of a unit circle. This method was extended later (SYMM (1969)) to map a doubly-connected domain onto a circular annulus. Both methods have been applied successfully by FELIX (1987) to solve the separated flow over rectangular prisms, in confined flow.

Here the first method is applied to a symmetric aerofoil with a finite trailing edge (NACA 0012), and is described as follows:

Given a simply-connected region D_1 bounded by a Jordan (i.e. simple closed) curve L , and D is let to be the domain complementary to D_1+L in the real plane, a general mapping function between exterior domains can be written as:

$$W(z) = e^{(\ln(z)+f+g(x,y)+ih(x,y))} \quad (2.1)$$

where, 'z' is the physical plane, 'W' is the transformed plane and $g(x,y)$ and $h(x,y)$ tend to zero as 'z' tends to infinity (SYMM (1967)). These two functions can be expressed in terms of a surface singularity distribution, σ , as:

$$\begin{aligned}
 g(x,y) &= \sum_{r=1}^Q \int_I \ln |z-\zeta| |\mathrm{d}\zeta| \sigma_r \\
 h(x,y) &= \sum_{r=1}^Q \int_I \operatorname{Arg}(z-\zeta) |\mathrm{d}\zeta| \sigma_r
 \end{aligned}
 \quad z \in D+L \quad (2.2)$$

from which the mapping function $W=u+iv$ is readily evaluated (D and L define the boundary of the domain, as described above). The same method, but modified as an inverse method from the transformed to the real plane (FELIX (1987)), correctly transforms a circle (ζ -plane) into a trapezium with angles as small as 30° . However, when applied to an aerofoil with a 12° trailing edge, it mapped points near the trailing edge incorrectly, indicating that the method cannot, so far, cope with corners beyond a certain sharpness.

If a sharp corner is removed by a preliminary transformation, then Symm's method may be applied. This requires the use of a Karman-Trefftz, or similar transformation, to open up the sharp corner. Here, the trailing edge of the aerofoil was opened up using the following transformation:

$$\frac{z - z_0}{z - z_1} = \left[\frac{\zeta - \zeta_0}{\zeta - \zeta_1} \right]^\lambda \quad (2.3)$$

where ' z ' is the physical plane and ' ζ ' is the transformed plane, $\lambda=2-\delta/\pi$, and ' δ ' being the trailing edge angle. For large z and ζ , then $\lambda(\zeta_0 - \zeta_1) = (z_0 - z_1)$.

After applying the Karman-Trefftz transformation the aerofoil was transformed to a smooth, near circular shape. This shape was then transformed to a circle using Symm's method. However, when the spoiler is added to the surface of the aerofoil the problem is complicated even further with an additional sharp edge.

This method was not investigated any further, because of the

complexity involved. Instead, an analytic conformal transformation was developed, based on an initial idea by PARKINSON and YEUNG (1987), which would map an aerofoil with an arbitrarily positioned spoiler on its surface into a circle, through a series of transformations. The main disadvantage of the method is that the spoiler is not straight but slightly curved away from the aerofoil surface. This causes some problems for small spoiler angles. The development of this complicated series of transformations is analysed in the next section.

2.2.1 The transformation.

In this section the sequence of conformal transformations is described, which map the field outside a Joukowski aerofoil fitted with an upper-surface spoiler of arbitrary size, inclination to the surface and chordwise position, into the field outside the unit circle. The sequence of transformations is shown in figure (2.1).

The aerofoil in the z_1 -plane is obtained by a Joukowski transformation (equation 2.4) applied to the circle and spoiler in the z_2 -plane:

$$z_1 = (z_2 - \beta_s) + \frac{\beta_2}{(z_2 - \beta_s)} \quad (2.4)$$

where $\beta_2 = (R/(1+e))^2$ and $\beta_s = e.R/(1+e)$. The constant 'R' defines the radius of the circle and 'e' determines the thickness of the aerofoil. For the 11% thick aerofoil used here, 'e' is equal to 0.092376.

The circle in the z_2 -plane is rotated and translated according to equation

$$z_3 = e^{i(\frac{\pi}{2} - \delta)} (e^{-i\phi} z_2 - R) + c \quad (2.5)$$

so that the spoiler is aligned with the x-axis in the z_3 -plane. Proceeding forward from the

z_3 -plane, use is made of the fact that the circle and spoiler are on coordinate curves in a bipolar coordinate system. Therefore, the field exterior to the circle and spoiler in the z_3 -plane may be mapped to the interior of a channel with sides and spoiler extending to infinity, and the spoiler positioned along the imaginary axis in the z_4 -plane. This is achieved by using the following equation:

$$z_4 = -i \ln \left[\frac{z_3 - c}{z_3 + c} \right] \quad (2.6)$$

The segments of the circle above and below the real axis in the z_3 -plane transform to the right and left hand sides of the infinite channel respectively in the z_4 -plane. Also, the point at infinity in the z_3 -plane and the physical plane becomes the origin of the axes in the z_4 -plane. The infinite channel and spoiler represent a degenerate polygon and a suitable Schwartz-Christoffel transformation maps it into the upper half of the z_5 -plane. The transformation is given by the following equation:

$$z_4 = ik (B \ln(z_5 - B) - D \ln(z_5 - D)) + \beta \quad (2.7)$$

where, $k = 1/\pi e^{\pi\gamma/(\alpha-\beta)} \cdot \alpha^{\alpha/(\alpha-\beta)} \cdot (-\beta)^{-\beta/(\alpha-\beta)}$

$$B = -\alpha/\pi k,$$

$$\text{and } D = -\beta/\pi k$$

In this plane, the tip of the spoiler transforms to the origin of the axes system and the spoiler roots on either side, as shown in figure (2.1).

Applying a shift by ' ϵ ' and dividing by ' ρ ' the origin in the z_5 -plane is shifted to $z_5' = i$, so that the bilinear transformation

$$z_5' = i \frac{z_6 - 1}{z_6 + 1} \quad (2.8)$$

transforms the straight line into a circle and the origin goes to infinity.

Finally, the circle in the z_6 -plane is rotated by ' α_0 ' so that the free stream, which through the series of transformations is rotated and amplified, is along the real axis.

It is important to note that starting from the aerofoil with a spoiler, the transformation can be taken forward as far as the z_4 -plane. Then, equation (2.7) can only be written as:

$$z_5 = B + (z_4 - D)^{D/B} \cdot e^{-\left(\frac{z_4 - \beta}{kB}\right)i} \quad (2.9)$$

which is a transcendental equation and difficult to solve rapidly for the correct root in all cases. However, equation (2.9) has been used successfully here to find where the origin in the z_4 -plane maps to in the z_5 -plane. This was very important, since the free stream in the physical plane is represented in the channel plane by a dipole positioned at the origin. Therefore, the transformed position of the dipole must be known in the z_5 -plane.

The difficulties involved in solving equation (2.9) restrict the transformation in the sense that the sequence of transformations must be taken backwards from the z_7 -plane, i.e. the final circle plane. A circle of unit radius is defined in this plane, and the transformations mentioned above transform the circle into an aerofoil with a spoiler on its upper surface. The disadvantage here is that points on the aerofoil and spoiler cannot be defined a priori but depend on the positions of corresponding points chosen on the final circle. However, the distribution can be improved (for example to have more points on the spoiler) by increasing the number of points on the initial circle or by using a suitable interpolation scheme.

The resulting spoiler on the aerofoil is not straight but slightly curved, away from the aerofoil surface. The spoiler curvature increases as the spoiler angle decreases. It is therefore impossible to use the present transformation to completely close a spoiler of finite length.

In this investigation, a symmetric Joukowski aerofoil of 11% nominal thickness is employed, with a spoiler of length equal to 10% of the aerofoil's chord, positioned at different locations along the upper surface of the aerofoil (figure 2.2).

Although in this study a symmetric Joukowski aerofoil fitted with a spoiler is investigated, the transformation method described above may be applied, with suitable modification, to any single element aerofoil profile. This is achieved by employing the method of THEODORSEN (1931) to map the aerofoil into a circle in the transformed plane (PARKINSON and YEUNG (1987)). When Theodorsen's method is applied to an aerofoil with a spoiler, the aerofoil is suitably positioned in the physical plane so that it can be transformed into a shape which is nearly circular with a spoiler, using a Joukowski transformation. The series of transformations in Theodorsen's method could then be applied to transform the near circle into a true circle with the spoiler along the real axis of the z_3 -plane seen above. The sequence of transformations already used here would then transform the circle and spoiler into the final circle.

It is important to note that through the transformation, separation points in the physical plane become stagnation points in the transformed plane.

Having transformed the aerofoil and spoiler into a circle, the boundary condition of zero normal velocity through the body surface, when it is placed in a uniform stream of homogeneous fluid, is satisfied in the circle plane by introducing a suitable strength doublet. Vortices may be now introduced into the flow from the points on the circle representing the separation points in the physical plane. The condition of zero normal velocity through the surface in the presence of the vortices is satisfied on the circle by using an image for each vortex that has been shed, as shown in figure 2.3.

The validity and applicability of the transformation method described above has been checked in the following way: by setting a spoiler normal to the surface of a symmetric Joukowski aerofoil, attached flow results for the pressure distribution can be compared with results obtained by a conformal transformation applicable only to a

Joukowski aerofoil with a normal spoiler, described by JANDALI and PARKINSON (1970) (see section 2.2.3).

2.2.2 Attached flow over Joukowski aerofoil with spoiler at an arbitrary angle and position.

In order to solve for the attached flow over a Joukowski aerofoil with a spoiler at an arbitrary angle and position on its surface, the transformation developed in section 2.2.1 is employed. However, the geometric nature of the transformation has two effects on the free stream as it takes it through the different stages of the transformation, in that the free stream direction is rotated and its strength is amplified. The rotation and amplification of the free stream may be found by evaluating the behaviour of the individual equations of the transformation at infinity.

Equations (2.4) and (2.5) at infinity, simply become:

$$z_1 = z_2 \quad (2.10)$$

$$z_3 = e^{i(\pi/2 - \delta - \phi)} \quad (2.11)$$

Expanding equation (2.6) for large z_3 and provided that $-1 < c/z_3 < 1$:

$$z_4 = 2i \left(\frac{c}{z_3} + \frac{1}{3} \left(\frac{c}{z_3} \right)^3 + \dots \right) \approx 2i \frac{c}{z_3} \quad (2.12)$$

From equation (2.7) it can be seen that position $z_5 = \epsilon + i\phi$ corresponds to the origin ($z_4 = 0$) in the z_4 -plane. Therefore

$$z_4 = 0 = ik (B \ln(\epsilon - B + i\phi) - D \ln(\epsilon - D + i\phi)) + \beta \quad (2.13)$$

Assuming that $z_5 = \zeta + \varepsilon + i\rho$, $\zeta \rightarrow 0$, then:

$$\begin{aligned}
 z_4 &= ik (B \ln(\varepsilon - B + i\rho + \zeta) - D \ln(\varepsilon - D + i\rho + \zeta)) + \beta \\
 &= ik (B (\ln(\varepsilon - B + i\rho + \frac{\zeta}{\varepsilon - B + i\rho} \dots\dots\dots)) - \\
 &\quad D (\ln(\varepsilon - D + i\rho + \frac{\zeta}{\varepsilon - D + i\rho} \dots\dots\dots)) + \beta \\
 &= ik \left[\frac{B}{\varepsilon - B + i\rho} - \frac{D}{\varepsilon - D + i\rho} \right] \zeta
 \end{aligned}$$

Therefore:

$$z_4 = ik \left[\frac{B}{\varepsilon - B + i\rho} - \frac{D}{\varepsilon - D + i\rho} \right] (z_5 - (\varepsilon + i\rho)) \quad (2.14)$$

Expanding equation (2.8) for large z_6 :

$$\begin{aligned}
 z'_5 &= i \frac{z_6 - 1}{z_6 + 1} \\
 &= i \left(1 - \frac{1}{z_6}\right) \left(1 - \frac{1}{z_6}\right) \approx i \left(1 - \frac{2}{z_6}\right)
 \end{aligned}$$

Hence,

$$z_5 = i\rho \left(1 - \frac{2}{z_6}\right) + \varepsilon \quad (2.15)$$

Combining the expanded equations together:

$$z_1 = \frac{2ce^{-i(\frac{\pi}{2} - \delta - \phi)}}{(k\chi(\frac{-2\rho i}{z_6}))} \quad (2.16)$$

where,

$$\chi = \frac{B}{\epsilon - B + i\rho} - \frac{D}{\epsilon - D + i\rho}$$

But ' χ ' may be written as:

$$\chi = \left[\frac{B(\epsilon - B)}{(\epsilon - B)^2 + \rho^2} - \frac{D(\epsilon - D)}{(\epsilon - D)^2 + \rho^2} \right] + \left[\frac{D\rho}{(\epsilon - D)^2 + \rho^2} - \frac{B\rho}{(\epsilon - B)^2 + \rho^2} \right] i$$

which is equal to $\chi = |\chi| e^{i\theta_s'}$.

Finally,

$$z_1 = \frac{c}{k|\chi|\rho} e^{i(\delta + \phi - \theta_s')} \cdot z_6 \quad (2.17)$$

Equation (2.17) shows that the free stream in the z_6 -plane has been amplified by $c/(2k|\chi|\rho)$ and rotated by an angle $\alpha_0 = (\delta + \phi - \theta_s')$. Therefore, the final circle is rotated through an angle ' α_0 ' so that the free stream is along the real axis. The magnitude of the free stream is now $U_\infty c/(2k|\chi|\rho)$.

The velocity field in the circle plane is given by

$$w(z_7) = V_\infty \left(1 - \frac{R^2}{z_7^2} \right) + \frac{i\Gamma}{2\pi} \frac{1}{z_7} \quad (2.18)$$

and the velocity in the physical plane is given by:

$$W(z_1) = \frac{W(z_\gamma)}{(dz_1/dz_\gamma)}$$

while the pressure coefficient is obtained by substituting $W(z_1)$ in:

$$C_p = 1 - \frac{|W(z_1)|^2}{U_\infty^2} \quad (2.20)$$

The term ' Γ ' in equation (2.18) is the bound circulation that has to be added at the centre of the circle, so that the Kutta condition is satisfied at the trailing edge, and is given by:

$$\Gamma = 2\pi i V_\infty (e^{i\phi_{te}} - e^{-i\phi_{te}}) \quad (2.21)$$

where, ' ϕ_{te} ' is the angular position of the trailing edge on the circle and V_∞ is the free stream velocity in the circle plane. Since the strength of the bound vortex is known, the lift coefficient may be evaluated for the aerofoil at zero incidence (assuming attached flow with only the Kutta-Joukowski condition satisfied at the trailing edge):

$$\begin{aligned} Cl &= \frac{2\Gamma}{U_\infty c} \\ &= \frac{8\pi V_\infty \sin \phi_{te}}{U_\infty c} \end{aligned} \quad (2.22)$$

where ϕ_{te} is very small.

Pressure distributions obtained using the transformation and the

potential solution are presented in the following section for different spoiler angles.

2.2.3 Discussion of results of the pressure distribution for attached flow.

The set-up of the transformation to map a Joukowski aerofoil with an arbitrarily positioned spoiler on its upper surface was both complicated and time consuming. It was, therefore, thought appropriate to have some results available as a reference to check the transformation. These were obtained by solving for the attached flow over a Joukowski aerofoil with a spoiler at 90° to the surface, using a method developed by JANDALI and PARKINSON (1970). This method transforms a Joukowski aerofoil with a spoiler only at 90° to the surface into a circle. Their potential solution and transformation was used here to find the C_p distribution over a symmetric, 11% thick Joukowski aerofoil with a spoiler normal to the surface and at 50% chord.

Comparing the C_p distribution calculated over the aerofoil with the spoiler set at 90° to the surface (figure 2.5) and that obtained by Jandali and Parkinson's method (figure 2.4), it can be seen that they are identical (note that the C_p distribution over the aerofoil and spoiler for this particular case, are presented separately in figures 2.6 and 2.7). This shows that the sequence of transformations that map the Joukowski aerofoil with a spoiler on its upper surface at an arbitrary inclination and position works correctly.

The C_p distributions over the aerofoil and spoiler, with the spoiler at 90° , 60° , 45° , and 15° , are presented separately for the aerofoil and spoiler in figures 2.6 and 2.7. All pressure distributions give stagnation points at either side of the spoiler root and the leading edge of the aerofoil, while the flow leaves the trailing edge smoothly. The velocity, and hence C_p , at the spoiler tip is infinite. The very large C_p value at the spoiler tip is not included in the plots of the C_p distributions for different spoiler angles, so that greater detail of the C_p distribution over the rest of the aerofoil and spoiler can be obtained.

Figure 2.6 shows the C_p distribution around the aerofoil with a spoiler

at 50% chord, for different spoiler angles. It can be seen that the lower surface is not affected by the spoiler angle. At the upper surface, there is a compression ahead of the spoiler, which increases with increasing spoiler angle since the flow has to turn through a smaller angle at the front root of the spoiler. The flow stagnates at the spoiler root.

Figure 2.7 shows the C_p distribution over the length of the spoiler, for various spoiler angles. There are stagnation points either side of the spoiler root. The flow accelerates over the spoiler and the pressure at the tip of the spoiler shows an infinite suction.

CHAPTER THREE

SEPARATED FLOW

3.1 Discrete Vortex Method (DVM) - flow features.

In this chapter, a numerical model based on the DVM is formulated to solve the unsteady separated flow past a 2-D aerofoil fitted with a spoiler on its upper surface. The aerofoil/spoiler combination is transformed into a circle by the sequence of conformal transformations, shown in section 2.2.1. In the circle plane, the flow is represented by the potential flow around the circle and discrete vortices, which model the separated shear layers. The aerofoil and spoiler are immersed in an impulsively started, steady incident flow. The initial condition of the solution is the attached flow over the aerofoil without the spoiler (at $t < 0$). The spoiler suddenly appears at $t = 0$, and may be either 'fixed' or moving. The term 'fixed' really means that the spoiler is impulsively raised, so that it suddenly appears at its final deflection on the surface of the aerofoil, as the flow is started ($t = 0$; see also figure 3.1).

In incompressible, very large Reynolds number flows the separated layers are initially very thin compared to body width. Therefore, it may be argued that in the limit of the Reynolds number going to infinity and ignoring diffusion due to turbulence, the shear layers may be represented by infinitely thin sheets, which in turn may be replaced by an array of discretised point vortices. Such a representation gives a good approximation to the velocity field due to the vortex sheet, provided that the sheet does not vary in strength very rapidly and that it does not have too large a curvature (VAN der VOOREN (1980)). The separated sheets may roll up to form large vortex structures.

In the present case of the aerofoil and spoiler, the points from which the vortex sheets are shed are assumed to be the tip of the spoiler and the trailing edge of the aerofoil. The separation points are independent of time, Reynolds number and local

pressure fields. This is a significant advantage since separation points on continuous curved surfaces are very difficult to predict analytically and some form of empiricism is required (KATZ (1981)).

The evolution and formation of the shear layers behind a bluff body is a complex process. Here, once the rolled up vortex sheet from the spoiler tip has grown to a certain size, it is strong enough to draw the opposite (trailing edge) shear layer across the wake, so that under the influence of oppositely signed vorticity it breaks away from the separation point and moves downstream under the influence of the other vortices in the field and the free stream. Vorticity continues to be generated at the separation point and subsequently this rolls up to form a new large vortex structure. This is shown schematically in figure 3.2.

3.1.1 Point vortices and vortex sheets.

A vortex sheet, which is assumed to be infinitely thin, has circulation density $\gamma(s)$, where s is distance along the sheet. Under the assumption that the flow outside the vortex sheet is irrotational, and taking clockwise vorticity to be positive, then the Biot-Savart law may be employed to give the velocity at a point ζ' in the flow field induced by the vortex sheet:

$$u' - iv' = \frac{i}{2\pi} \int \frac{\gamma(s)}{\zeta' - \zeta(s)} ds \quad (3.1)$$

where ζ' is the complex position of a point in the flow field.

Equation 3.1 can be written in discretised form as:

$$u' - iv' = \frac{i}{2\pi} \sum_j \int_{\Delta s_j} \frac{\gamma(s)}{\zeta' - \zeta(s)} ds \quad (3.2)$$

The discretisation of a vortex sheet in two dimensions can be done by using point vortices, as shown in figure 3.3. Each of the point vortices represents the concentrated strength of a small segment of the vorticity sheet, in which case equation 3.2 can be written as:

$$u' - iv' = i \sum_j \frac{\Gamma_j}{\zeta' - \zeta_j}$$

where:

$$\Gamma_j = \int_{\Delta s_j} \gamma(s) ds$$

The vorticity is defined as

$$\omega = \nabla \times \mathbf{q} \quad (3.3)$$

where $\mathbf{q}$ is the velocity vector. For inviscid flow Euler's equation gives:

$$\frac{D\omega}{Dt} = \omega \cdot \nabla \mathbf{q} \quad (3.4)$$

However, in two dimensions, ω , is a vector perpendicular to the (x,y) plane and has magnitude:

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (3.5)$$

Hence, the term $\omega \cdot \nabla \mathbf{q}$ is zero and equation (3.4) becomes:

$$\frac{D\omega}{Dt} = 0 \quad (3.6)$$

Therefore, the vorticity of a fluid element remains constant in time. Since $Da/Dt=0$ (where 'a' is the cross sectional area of a fluid element) for continuity and $\Gamma=\omega.a$, then $D\Gamma/Dt=0$ for a fluid element. Hence dynamically, point vortices follow the fluid like particles (Lagrangian description) and they retain their circulation in time, in order to conserve total circulation in the flow field.

3.2 Complex potential flow.

In this problem, the aerofoil/spoiler combination is transformed into a circle in the transformed plane, where the boundary conditions on the solid surface may be satisfied by employing the vortex-image system according to the Milne-Thomson Theorem.

The complex potential, W , of a single point vortex of strength Γ_v , located at a point ζ_v , is given by:

$$W(\zeta) = \frac{i\Gamma_v}{2\pi} \ln(\zeta - \zeta_v) \quad (3.7)$$

and by differentiating and summing over all the vortices in the flow field the velocity at a point due to all the vortices in the field is given by:

$$\frac{dW(\zeta)}{d\zeta} = u - iv = \frac{i}{2\pi} \sum_{v=1}^N \frac{\Gamma_v}{\zeta - \zeta_v} \quad (3.8)$$

The complex potential in the transformed plane may be written as (figure 3.4):

$$\begin{aligned}
W(\zeta) = V_{\infty} \left(\zeta e^{-i\alpha} + \frac{R^2}{\zeta} e^{i\alpha} \right) + i \sum_{v=1}^N \frac{\Gamma_v}{2\pi} \ln(\zeta - \zeta_v) \\
- i \sum_{v=1}^N \frac{\Gamma_v}{2\pi} \ln \left(\zeta - \frac{R^2}{\zeta_v^*} \right) + \frac{i\Gamma_b}{2\pi} \ln(\zeta) \quad (3.9)
\end{aligned}$$

where, the first term on the right hand side represents the potential flow past the circle, the second term the vortices in the flow field and the third term the image vortices. The last term is the circulation which the aerofoil may have, prior to the spoiler being raised (i.e. the initial condition ($t < 0$) for attached flow past an aerofoil without a spoiler). It is the image (at the centre of the circle) of the 'starting vortex' which is now far downstream, moving at the free stream velocity. The calculation then for the 'fixed' spoiler case starts with the spoiler fixed at its final position having in effect moved out of the surface at an infinite velocity, i.e. at one instant the spoiler is not on the surface and at the next instant it appears to be fixed on the aerofoil at a prescribed angle to its surface.

The free stream velocity is amplified in the circle plane, due to the transformation (see section 2.2.1), so that $V_{\infty} = \mu U_{\infty}$, where μ is the amplification factor. The value of μ is very nearly 1.0 and varies with spoiler angle and spoiler length. For example, $\mu = 1.008, 1.005, 1.003, 1.001$ for $\delta = 90^\circ, 45^\circ, 30^\circ$ and 10° respectively (it also decreases with decreasing spoiler length).

By differentiating equation 3.9 the velocity field is given by:

$$\begin{aligned}
\frac{dW(\zeta)}{d\zeta} = V_{\infty} \left(e^{-i\alpha} - \frac{R^2}{\zeta^2} e^{i\alpha} \right) + i \sum_{v=1}^N \frac{\Gamma_v}{2\pi} \frac{1}{\zeta - \zeta_v} \\
- i \sum_{v=1}^N \frac{\Gamma_v}{2\pi} \frac{1}{\left(\zeta - \frac{R^2}{\zeta_v^*} \right)} + \frac{i\Gamma_b}{2\pi} \frac{1}{\zeta} \quad (3.10)
\end{aligned}$$

This equation is used to find the velocity induced at a vortex position due to the free stream and all the other vortices in the flow field (Biot-Savart).

The application of equation 3.10, although it is relatively straight forward, suffers from high computation cost (see section 1.4.5).

3.3 Vortex shedding mechanism.

In modelling the impulsively started flow over an aerofoil fitted with an infinitely thin spoiler by DVM, it is important to represent realistically the shedding of vorticity into the flow field, from the shedding edges i.e. the spoiler tip and the trailing edge. The positions of the separation points on the circle are known in the transformed plane, through the analytic transformation.

The flow of a real fluid around a sharp edge is characterised by the formation of a vortex sheet due to flow separation, so that an infinite velocity is prevented at the tip of the sharp edge. The strength of the sheet is determined by the Kutta condition. For this condition to hold, the velocity tangential to the surface at the separation point must be finite, and the flow must leave the edge tangentially (GIESING (1969)). In this problem it is easier to satisfy the Kutta condition by a stagnation point at the corresponding point in the circle plane, and it may be implemented in a number of different ways, but generally two types of method are employed:

(i) the position of the new vortex to be shed (the nascent vortex) is chosen and is fixed in advance, while the vortex circulation is determined so that the condition of zero tangential velocity at the point in the circle plane, corresponding to separation in the physical plane, is satisfied (KAMEMOTO and BEARMAN (1978)).

(ii) the circulation Γ_n of the nascent vortex is determined (CLEMENTS (1973), CLEMENTS and MAULL (1975) and MAULL (1980)) by:

$$\Gamma_n = \frac{1}{2} U_s^2 t \quad (3.11)$$

where U_s represents the velocity on the body surface at the point of separation. The position of the nascent vortex is obtained by satisfying $dW(\zeta)/d\zeta=0$ at the separation point.

A different approach to vortex shedding from an edge is given by the Brown and Michael method. According to this method the strength and position of nascent vortices is derived by solving simultaneous equations which satisfy the Kutta condition at the edge and a zero net force condition on the 'cut' between the nascent vortex and the edge. This removes the empirical choice of a 'release height' and aids the initial sheet formation by generating a strong 'core' vortex at the start of the calculation (see section 3.5).

In the initial stages of the research, the nascent vortices were positioned at fixed distances above the separation points in the circle plane, as shown in figure 3.5. The positions were decided following the studies of KAMEMOTO and BEARMAN (1978). Later, the Brown and Michael method was applied to the problem, as described in detail in section 3.5.

3.4 Convection of vortices and Routh's velocity correction.

The flow due to a vortex in the physical plane is transformed into that due to a vortex of equal strength at the transformed position of the vortex, in this case in the circle plane. Hence, vortices may be released from points in the circle plane, corresponding to separation points in the physical plane, so that their convection velocity and hence position in time, may be found in the circle plane, using the complex velocity (eq. 3.10) and the $ldz/d\zeta|^2$ term, and then returned to the physical plane through the conformal transformation between the two planes (in doing so, the convection velocity must include the Routh's term, which is described later in this section). This greatly

increases the efficiency of the calculation of the flow round the aerofoil and spoiler. Convecting vortices in the circle or 'working plane' has been employed by many researchers (KUWAHARA (1973), NAYLOR (1982), BASUKI (1983)).

If z_v is a vortex position in the physical plane and $W(z)$ the complex potential in that plane, then if $W(z)$ is used to calculate the convection velocity of this point vortex in the physical plane it must exclude this vortex's own velocity contribution. According to CLEMENTS (1973):

$$W_{z_v}(z) + \frac{i\Gamma_v}{2\pi} \ln(z - z_v) = W_{\zeta_v}(\zeta) + \frac{i\Gamma_v}{2\pi} \ln(\zeta - \zeta_v) \quad (3.14)$$

where $W_{z_v}(z)$ and $W_{\zeta_v}(\zeta)$ are the potentials at z_v and ζ_v due to all causes except the vortex of strength Γ_v at ζ_v . Thus,

$$W_{z_v}(z) = W_{\zeta_v}(\zeta) + \frac{i\Gamma_v}{2\pi} \ln\left(\frac{\zeta - \zeta_v}{z - z_v}\right) \quad (3.15)$$

If a first order difference scheme is employed for the displacement of a vortex at time 't', then the position of the vortex in the circle plane at the end of a time step Δt is given by:

$$\begin{aligned} \zeta(t+\Delta t) &= \zeta(t) + dz \frac{d\zeta}{dz} \\ &= \zeta(t) + \left(\frac{dW}{dz} \right)^* \Delta t \left(\frac{d\zeta}{dz} \right) \\ &= \zeta(t) + \left(\frac{dW}{d\zeta} \right)^* \Delta t \left| \frac{d\zeta}{dz} \right|^2 \end{aligned} \quad (3.16)$$

where '*' defines the complex conjugate and ' Δt ' is the time step. The above expression

implies that vortices in the circle plane are convected with a velocity of $(dW/d\zeta)^*$. $|dz/d\zeta|^2$. However, the velocity field of a point vortex in the transformed plane does not necessarily remain the same when transformed to the physical plane, since the velocity field of the vortex may change its shape, for example stretch in the x-direction. This is more evident near sharp edges, where mesh distortion is high. Therefore, in calculating the velocity at a vortex position, its return to the physical plane must be corrected to include the above effect.

Now, if Taylor's expansion is used to expand $f(z)$ about z_v , then:

$$\zeta=f(z)=f(z_v)+(z-z_v)f'(z_v)+1/2 (z-z_v)^2 f''(z_v)+O(z-z_v)^3$$

and,

$$\zeta-\zeta_v=(z-z_v)f'(z_v)+1/2 (z-z_v)^2 f''(z_v)+O(z-z_v)^3$$

Substituting for $(\zeta-\zeta_v)$ in equation 3.15:

$$W_{z_v}(z) = W_{\zeta_v}(\zeta) + \frac{i\Gamma_v}{2\pi} \ln \left[f'(z_v) + \frac{1}{2} (z-z_v)f''(z_v) + O(z-z_v)^2 \right]$$

and by differentiating:

$$\begin{aligned} \frac{dW_{z_v}}{dz} &= \frac{dW_{\zeta_v}}{d\zeta} \frac{d\zeta}{dz} + \frac{i\Gamma_v}{2\pi} \left[\frac{\frac{1}{2} f''(z_v) + o(z-z_v)}{f'(z_v) + o(z-z_v)} \right] \\ &= \frac{dW_{\zeta_v}}{d\zeta} \frac{d\zeta}{dz} + \frac{i\Gamma_v}{2\pi} \left[\frac{1}{2} f''(z_v) + o(z-z_v) \right] \cdot \\ &\quad \frac{1}{f'(z_v)} \left[1 + o(z-z_v) \right] \end{aligned} \quad (3.17)$$

which, as $z \rightarrow z_v$ and $\zeta \rightarrow \zeta_v$ gives:

$$\frac{dW_{z_v}}{dz} = \left[\frac{dW_{\zeta_v}}{d\zeta} \right] \left[\frac{d\zeta}{dz} \right] + \frac{i\Gamma_v}{4\pi} \frac{f'(z_v)}{f(z_v)} \quad (3.18)$$

where the expression $dW_{\zeta_v}/d\zeta$ is composed of the sum of the velocities induced at ζ_v by all the other vortices and the free stream in the circle plane. Therefore, the velocity at point ζ_v is given by:

$$\begin{aligned} \frac{d\zeta_v}{dt} &= \left[\frac{dW}{d\zeta} \right]_{\zeta_v}^* \left| \frac{d\zeta}{dz} \right|_{\zeta_v}^2 - \frac{i\Gamma_v}{4\pi} \left(\frac{d^2z/d\zeta^2}{dz/d\zeta} \right)_{\zeta_v}^* \left| \frac{d\zeta}{dz} \right|_{\zeta_v}^2 \\ &= \left[\frac{dW}{d\zeta} + \frac{i\Gamma_v}{4\pi} \left(\frac{d^2z/d\zeta^2}{dz/d\zeta} \right) \right]_{\zeta_v}^* \left| \frac{d\zeta}{dz} \right|_{\zeta_v}^2 \end{aligned} \quad (3.19)$$

where the second term in the parenthesis is known as the Routh's velocity correction.

This term was introduced by CLEMENTS (1973) in a 2-D study of the vortex shedding behind a square-based section in inviscid flow, using DVM. The Routh's velocity correction term was also used by SARPKAYA (1975) in his study of the inviscid separated flow past an inclined flat plate. For the aerofoil/spoiler case, a local expression for the Routh's term is derived, as is discussed later.

3.5 The Brown and Michael Method.

The Brown and Michael method is used in the aerofoil/spoiler problem, both as a way of shedding point vortices sequentially to avoid the empirical choice of

'release height' in the method described in section 3.3, and as a way of generating stronger 'core' (but still point) vortices at the start of the calculation to aid the initial sheet formation.

3.5.1 Single-vortex shedding.

In an impulsively started flow past a sharp edge, a vortex sheet separates tangentially from the edge (GIESING (1969)) and develops into a vortex spiral. In the computation of the inner part of a spiral it is very important for stability, to maintain a smaller separation between succeeding vortices along the sheet than the gap between turns of the spiral (MOORE (1974)). For this reason, the inner turns may be represented by a single concentrated vortex. A simplified model of the whole of the spiral vortex sheet may also be used, replacing the sheet by a concentrated point vortex. In this model the point vortex is joined to the shedding edge by a cut in the plane, across which the velocity potential is discontinuous (BROWN and MICHAEL (1954)). The presence of the cut is necessary since it represents a feeding vortex sheet, feeding the concentrated vortex with circulation from the separation point.

By representing the spiral by a concentrated point vortex with changing strength, an unbalanced pressure jump will be found across any line connecting the edge and the vortex. To solve this problem for slender wings, BROWN and MICHAEL (1955) proposed that the single vortex should not be free but subject to a Joukowski Force, which cancels the unbalanced force on the cut.

3.5.2 Multi-vortex shedding.

It is possible (GRAHAM (1977)) to use the Brown and Michael equations to calculate the position and strength of each new vortex shed in a multi-vortex representation of the sheet. The new vortices leave along the bisector of the sharp edge, since the dominant component of velocity is the U-component and not V, where U and V are real constants describing the symmetrical and asymmetrical parts of the flow

(GRAHAM (1985)), as shown in figure 3.11. Therefore, the strength and position of a new vortex can be determined at the end of the time step at which it was introduced.

Derivation of Brown and Michael equations for single or multi-vortex shedding.

The pressure jump on any line connecting the shedding edge and a vortex, whose strength is changing due to shedding from the edge, is related to the rate of change of the jump in Φ across the line with time, which is $-\rho(d\Phi/dt)$, since Γ is the difference in the values of Φ at the two sides of the edge. Due to the 'hydrostatic' character of this jump in pressure, the resultant force is the same on any line connecting the edge z_e and the vortex z_o and is equal to:

$$-(z_o - z_e) \rho d\Gamma/dt$$

with its direction perpendicular to the length $(z_o - z_e)$. The force applied on the cut is equal to :

$$F_1 = -i\rho(z_o - z_e)(d\Gamma/dt) \quad (3.20)$$

The Joukowski Force on the vortex is given by:

$$F_2 = i\rho V_L \Gamma \quad (3.21)$$

where, V_L is the velocity of the local flow relative to the vortex.

Velocity V_L is given by:

$$V_L = \left[\frac{dW}{dz} - \frac{i\Gamma}{2\pi(z - z_o)} \right]_{z \rightarrow z_o}^* - \frac{dz_o}{dt} \quad (3.22)$$

so that the zero net force condition leads to:

$$\frac{1}{\Gamma}(z_o - z_e) \frac{d\Gamma}{dt} + \frac{dz_o}{dt} = \left[\frac{dW}{dz} - \frac{i\Gamma}{2\pi(z - z_o)} \right]_{z \rightarrow z_o}^* \quad (3.23)$$

Taking the conjugate of both sides gives:

$$\frac{1}{\Gamma}(z_o^* - z_e^*) \frac{d\Gamma}{dt} + \frac{dz_o^*}{dt} = \left[\frac{dW}{dz} - \frac{i\Gamma}{2\pi(z - z_o)} \right]_{z \rightarrow z_o} \quad (3.24)$$

or,

$$\frac{1}{\Gamma}(z_o^* - z_e^*) \frac{d\Gamma}{dt} + \frac{dz_o^*}{dt} = \left[\frac{dW}{dz} + \frac{i\Gamma}{2\pi} \left(\frac{d^2\zeta/dz^2}{2(d\zeta/dz)^2} - \frac{\zeta}{\zeta_o + \zeta_o^*} \right) \frac{d\zeta}{dz} \right]_{z \rightarrow z_o} \quad (3.25)$$

The case is now considered where a new point vortex in a sequence of vortices representing a sheet is being shed between time t_o and $t_o + \Delta t$:

In this particular problem of the flow separating from the spoiler tip and the trailing edge of the aerofoil, local axes systems are defined at the separation points for convenience, as shown in figure 3.6. The transformation may be written locally for these axes as:

$$\hat{z} = k_a \hat{\zeta}^2 + \text{higher order terms (for spoiler tip)}$$

$$\hat{z} = k_b \hat{\zeta}^2 + \text{higher order terms (for trailing edge)}$$

where $\hat{\zeta}$'s refer to the local axes systems.

Supposing that the Kutta condition is satisfied at $\hat{\zeta}=0$ by the previously shed vortices and the free stream at $t=t_0$, then $W(\hat{\zeta}, t_0) = \hat{W}_0 \hat{\zeta}^2 + \dots$, giving $(dW/d\hat{\zeta})_{\hat{\zeta}=0} = 0$ at $t=t_0$. Also, since $\hat{\zeta}=i\eta$ is a streamline (fig. 3.7), then $\text{Im}(\hat{W}_0(i\eta)^2) = 0$, as $\Psi=0$ on the surface and therefore $\hat{W}_0$ is real.

At time $t > t_0$ the Kutta condition is no longer satisfied since the vortices have moved. Therefore:

$$W(\hat{\zeta}, t) = \hat{W}_1 \hat{\zeta} + \hat{W}_2 \hat{\zeta}^2 + \dots \quad (3.27)$$

where $\hat{W}_2$ and $\hat{W}_1$ are the symmetric and asymmetric parts of the flow respectively (since $\Psi=0$ on $\hat{\zeta}=i\eta$, $\hat{W}_1$ is imaginary and $\hat{W}_2$ is real). So for small $t-t_0$,

$$W(\hat{\zeta}, t) = W(\hat{\zeta}, t_0) + \left[\frac{dW}{dt} \right] (t - t_0) \quad (3.28)$$

which may be written as,

$$W(\hat{\zeta}, t_0) = i\hat{W}_1' \hat{\zeta}(t-t_0) + (\hat{W}_2'(t-t_0) + \hat{W}_0) \hat{\zeta}^2 + \dots \quad (3.29)$$

For the Kutta condition to hold for all times, t , a new vortex is shed to cancel the edge velocity. Its strength is given by:

$$\left(\frac{\partial W}{\partial \hat{\zeta}} \right)_{\hat{\zeta}=0} + \frac{i\Gamma}{2\pi} \left(-\frac{1}{\hat{\zeta}_0} \right) - \frac{i\Gamma}{2\pi} \left(\frac{1}{\hat{\zeta}_0^*} \right) = 0 \quad (3.30)$$

and equation 3.30 may be written as (see also figure 3.7):

$$\hat{W}_1(t-t_0) = \frac{\Gamma}{2\pi} \left(\frac{1}{\hat{\zeta}_0} + \frac{1}{\hat{\zeta}_0^*} \right) \quad (3.31)$$

It has been shown by GRAHAM (1977) that the strength and position of a newly shed vortex are given by:

$$\begin{aligned} \Gamma &= \sqrt{\frac{2\hat{W}_0}{5}} \frac{\pi\hat{W}_1}{k} (\Delta t)^{3/2} \\ \hat{z}_0 &= \frac{2}{5} \frac{\hat{W}_0}{k} \Delta t \end{aligned} \quad (3.32)$$

where $\Delta t = t - t_0$ and $\hat{z}_0$ gives the vortex position relative to the local axis system.

These equations hold for a general starting flow and are the required solutions for new vortex strength and position at the end of the time step, Δt , in an axis system centred on the shedding edge at the start of the time step.

In the case of the moving spoiler, the local axes system at the spoiler tip may be defined to be moving with the spoiler so that the real axis is always aligned with the spoiler, as shown in figure 3.6. In this case the above equations for the strength and position of a nascent vortex are valid for a 'fixed' and a moving spoiler, with the exception that in the case of a moving spoiler the complex potential includes the effect of the sources and sinks, which are distributed along the surface of the spoiler.

The first time step of impulsively started flow is the result for the case when a flow already exists. However, for starting flow $\hat{W}_0 = 0$ and $\hat{W}_2 = 0$. The solution of the Brown and Michael equations for the growth and motion of the first shed vortex for starting flow is:

$$\Gamma_o(\Delta t) = \frac{\pi \hat{W}_1'}{6^{1/3} k^{2/3}} (\Delta t)^{5/3} \quad (3.33)$$

$$\hat{z}_o = - \frac{i \hat{W}_1'^{2/3}}{2 \cdot 6^{2/3} \cdot k^{1/3}} (\Delta t)^{4/3}$$

where Δt is the time step and $\hat{z}_o$ refers to the local axis system. In this case the vortex lies on a line perpendicular to the spoiler or the trailing edge surface.

The way the constants $\hat{W}_o$, $\hat{W}_1'$, k_a and k_b , are calculated is shown in Appendix I.

3.6 Local convection scheme.

When the Biot-Savart law is used to convect point vortices, certain instabilities appear as vortices come close together or approach a solid surface. Similarly very close to the trailing edge the $dz/d\zeta$ term is very small and hence vortices close to the edge have high convection speeds in the circle plane. This in turn necessitates very small, and uneconomical, time steps so that the first order integration scheme does not introduce large errors.

A local convection scheme has been derived, which allows first order integration of the trajectories to retain greater accuracy near the edge. This convection scheme is applied within a certain radius (equal to $0.2R$, where R is the radius of the circle) around the two edges.

Near an edge the local transformation may be written locally as:

$$z - z_e = k(\zeta - \zeta_e)^2 + \dots \quad (3.34)$$

Then, instead of using equation 3.19 to convect vortices in the circle plane, an alternative equation may be written in terms of the velocity of a vortex in the real plane:

$$\frac{dz_v}{dt} = \left(\frac{dW}{d\zeta_v} \right)^* \left(\frac{d\zeta_v}{dz_v} \right)^* \quad (3.35)$$

This is finite at the shedding edge because of the Kutta condition. The new position of a vortex in the real plane is given by:

$$z_{vn} = z_{vo} + \frac{dz_v}{dt} \Delta t \quad (3.36)$$

where subscripts 'o' and 'n' define the old and new position of the vortex respectively. Taking into account equation 3.36 and transforming the vortex position from the real to the circle plane, the new position of the vortex in the transformed plane is given by:

$$\begin{aligned} \zeta_{vn} &= \left[\frac{z_{vn}}{k} \right]^{1/2} = \left[\frac{z_{vo}}{k} + \left(\frac{dW}{d\zeta_v} \right)^* \left(\frac{d\zeta_v}{dt} \right)^* \frac{\Delta t}{k} \right]^{1/2} \\ &= \left[\zeta_{vo}^2 + \left(\frac{dW}{d\zeta_v} \right)^* \left(\frac{d\zeta_v}{dz_v} \right)^* \frac{\Delta t}{k} \right]^{1/2} \end{aligned} \quad (3.37)$$

But, $k=1/(2\zeta_v(d\zeta_v/dz_v))$, from the definition of $d\zeta/dz$. Therefore, equation 3.37 becomes:

$$\zeta_{vn} = \zeta_e + \left[(\zeta_{vo} - \zeta_e)^2 + 2 (\zeta_{vo} - \zeta_e) \left(\frac{dW}{d\zeta_v} \right)^* \left| \frac{d\zeta_v}{dz_v} \right|^2 \Delta t \right]^{1/2} \quad (3.38)$$

If this equation is expanded for small Δt and $d\zeta/dz$ not too large, it gives:

$$\zeta_{vn} = \zeta_{vo} + \left(\frac{dW}{d\zeta_v} \right)^* \left| \frac{d\zeta_v}{dz_v} \right|^2 \Delta t + \dots \quad (3.39)$$

which is the equation used generally for convecting vortices in the circle plane. However, near an edge, where $d\zeta/dz$ tends to infinity, the binomial expansion 3.39 of 3.38 is not accurate, since the second term is no longer small. Therefore, for this case equation 3.38 shall be used to convect the vortices.

3.7 Use of a local Routh's velocity correction.

The magnitude of the Routh's term in the near wake represents only a small percentage (less than 1%) of the free stream velocity. However, near sharp edges, like the spoiler tip and the cusped trailing edge, Routh's velocity correction may be of the same order as the free stream, since the mesh is substantially distorted from the physical to the transformed plane.

In this study, seven stages are involved in the transformation which maps the aerofoil/spoiler plane into the circle plane and $dz/d\zeta$ and $d^2z/d\zeta^2$ terms become very complicated. For this reason and also because the Routh's term gets very small away from the edges, a local expression for the Routh's term is derived that is accurate near the spoiler tip and the trailing edge and tends to zero away from the regions where the correct term would be very small but difficult to calculate.

The local transformation may be written in the neighbourhood of the edges as:

$$z - z_e = k(\zeta - \zeta_e)^2 + \dots,$$

where z_e is the edge position in the physical plane and ζ_e is the transformed edge

position, so that

$$dz/d\zeta = 2k(\zeta - \zeta_e),$$

and

$$d^2z/d\zeta^2 = 2k$$

Then, the Routh's term is given by:

$$\begin{aligned} R_T &= \frac{i\Gamma}{4\pi} \left[\frac{d^2z/d\zeta^2}{dz/d\zeta} \right]^* \\ &= \frac{i\Gamma}{2\pi} \frac{1}{(\zeta - \zeta_e)} \end{aligned} \quad (3.41)$$

The convection equation including the Routh's velocity correction may be written locally as:

$$\frac{d\zeta}{dt} = \left[\left(\frac{dW}{d\zeta} \right)^* + \frac{i\Gamma}{2\pi(\zeta - \zeta_e)^*} \right] \left| \frac{d\zeta}{dt} \right|^2 \quad (3.42)$$

It may be seen that the Routh's term decreases rapidly as $(\zeta - \zeta_e)^*$ increases, i.e. for points which are not close to the shedding edge.

Equation 3.42 is only applied locally near the spoiler tip and the trailing edge, within two circular regions of radius $0.2R$, centred at the shedding points in the circle plane (figure 3.9). Initial tests showed that Routh's term was indeed very small away from the spoiler tip and the trailing edge. However, very near the spoiler tip, the

separated shear layer was slightly displaced away from the spoiler tip in the downstream direction, if no Routh's term was included.

3.8 Time Integration.

A first order Eulerian integration scheme is employed here for time integration, for reasons of simplicity and to aid stability. Errors in the integration of the vortex paths are mainly caused by strong velocity gradients.

CLEMENTS (1973) in his investigation of the vortex shedding behind a semi-infinite body with rectangular base used a first and a second order integration scheme and it was found that for a relatively small time step the first order scheme gave good enough results. SARPKAYA (1975) tested three different schemes, a first order Eulerian, a second order Runge-Kutta and a centred-difference, and found that the computed results were comparable for a non-dimensional time step equal to 0.02.

The new position of a vortex in the circle plane at the end of the time step Δt according to a first order integration scheme is given by,

$$\zeta(t + \Delta t) = \zeta(t) + \frac{d\zeta}{dt} \Delta t \quad (3.43)$$

where $d\zeta/dt$ is the complex convection velocity in the circle plane (given by equation 3.50) and Δt is the time step. The choice of the size of the time step is restricted by the fact that the numerical integration must convect vortices along the true streamlines of the flow (CLEMENTS 1973)). This is more of a problem near the separation points, where streamlines are highly curved and strong velocity gradients are present. Ideally, a very small time step would be desirable, but using the Biot-Savart law to calculate the induced velocity at a vortex due to all the other vortices in the flow field increases the cost of the computation dramatically, since the number of operations is of order (N^2) , where N is the number of vortices in the flow field. At the same time, it is required to have a large

number of vortices in the computation for a correct representation of the wake. In most cases, the time step size is a compromise between cost efficiency and accuracy of integration.

In an effort to keep the time step small but have a reasonable limit on the number of vortices in the flow field, velocities at vortex positions and subsequent vortex movement were calculated twice before new vortices were released i.e.

$$\Delta t_i = 2\Delta t \quad (3.44)$$

where Δt_i is the time interval between the introduction of nascent vortices. The same method has been employed, among others, by CLEMENTS and MAULL (1975), KAMEMOTO and BEARMAN (1978) and KIYA and ARIE (1977).

In this work, after preliminary tests, a non-dimensional time step value of $\Delta t' = U_\infty \Delta t / R = 0.02$ was chosen as a compromise between accuracy and computational efficiency. Although this time step size was appropriate for the 'fixed' spoiler case, it proved to be too large for the moving spoiler case, especially for small starting spoiler angles and high rates of spoiler deployment, so that a much smaller time step had to be chosen, as will be discussed later. Also, for the moving spoiler $\Delta t_i = \Delta t$, i.e. vortices were released at every time step.

3.9 Force coefficients using the Momentum Theorem.

The calculation of forces on the aerofoil and spoiler is of prime importance. Since the shed vorticity in the wake is represented by inviscid point vortices the complex force, Z_f , induced on the body by the free stream and the vortices may be calculated using Blasius' equation:

$$Z_f = -i\rho \oint_s \frac{\partial}{\partial t} W dz - \frac{i\rho}{2} \oint_s \left[\frac{dW}{dz} \right]^* dz \quad (3.45)$$

where, W , is the complex potential in the physical plane, s , defines the perimeter of the body and, $*$, indicates a complex conjugate. The complex potential in the transformed plane in this problem is given by equation 3.15. GRAHAM (1980) showed, using the residue theorem, that the complex force given by equation 3.45 may be given by considering the rate of change of momentum across a circuit at infinity:

$$Z_f = -i\rho \frac{\partial}{\partial t} \left\{ \oint_{\infty} W dz \right\} \quad (3.46)$$

Expanding, W , and $dz/d\zeta$ for large ζ and using the residue theorem, the force due to vortex shedding is then:

$$Z_f = -i\rho \frac{\partial}{\partial t} \left\{ \sum_1^N \Gamma_v \left(\zeta_v - \frac{R^2}{\zeta_v^*} \right) \right\} \quad (3.47)$$

where ζ_v are the vortex positions in the circle plane, N is the number of vortices and R is the radius of the circle. Therefore, the force exerted on the body depends on the rate of change of the distance between vortex positions and their images, and on the rate of change of circulation in the flow.

Carrying out the differentiation in equation 3.47 with respect to time gives:

$$\begin{aligned} Z_f = & -i\rho \sum_1^N \Gamma_v \left[\frac{\partial \zeta_v}{\partial t} + \frac{R^2}{\zeta_v^{*2}} \left(\frac{\partial \zeta_v}{\partial t} \right)^* \right] \\ & -i\rho \sum_1^N \frac{\partial \Gamma_v}{\partial t} \left[\zeta_v - \frac{R^2}{\zeta_v^*} \right] \end{aligned} \quad (3.48)$$

In this particular application, the vortices shed from the spoiler tip and the trailing edge have a constant circulation, which they retain as they are convected downstream. Therefore, the second term of the R.H.S in equation 3.48 represents only the newly shed vortices in the flow field. Equation 3.48 may then be written as:

$$Z_f = -ip \sum_1^N \Gamma_v \left[\frac{\partial \zeta_v}{\partial t} + \frac{R^2}{\zeta_v^{*2}} \left(\frac{\partial \zeta_v}{\partial t} \right)^* \right] - ip \frac{\Gamma_{ev}}{\Delta t} \left[\zeta_{ev} - \frac{R^2}{\zeta_{ev}^*} \right] - ip \frac{\Gamma_{tv}}{\Delta t} \left[\zeta_{tv} - \frac{R^2}{\zeta_{tv}^*} \right] \quad (3.49)$$

where, ζ_{ev} , and, ζ_{tv} , are the positions of the nascent vortices near the trailing edge and the spoiler tip respectively. The method of finite-differencing equation 3.47 does include this contribution. Initial tests showed that their inclusion in equation 3.49 was necessary to bring equations 3.47 and 3.49 into agreement.

The advantage of using equation 3.49 is that the vortex convection velocities are readily available, since they have been calculated already in the program, and may be used directly in equation 3.49. However, there is a difference in the two equations, as shown below.

If equation 3.47 is differenced over two time steps:

$$Z_f = -ip \frac{1}{\Delta t} \sum_1^{N+2} \Gamma_v \left[\zeta_v - \frac{R^2}{\zeta_v^*} \right] + ip \frac{1}{\Delta t} \sum_1^N \Gamma_v \left[\zeta_o - \frac{R^2}{\zeta_o^*} \right] = -ip \frac{1}{\Delta t} \sum_1^N \Gamma_v \left[\frac{\zeta_v - \zeta_o}{\Delta t} - \frac{R^2}{\zeta_v^* \zeta_o^*} \left(\frac{\zeta_o - \zeta_v}{\Delta t} \right)^* \right] + \text{'term'}$$
(3.50)

where subscript 'o' denotes the old vortex positions at the end of the previous time step and 'term' is equal to the second and third terms of the R.H.S in equation 3.49.

Comparing equations 3.49 and 3.50, there is a difference in the second term in the parenthesis (which is 2nd order small and results from having carried out a time differentiation analytically), i.e. $R^2/(\zeta_v^*)^2$ compared to $R^2/(\zeta_v \zeta_o)^*$, and since

$$\zeta_o = \zeta_v - \zeta_v \left[\frac{\partial \zeta_v}{\partial t} \right] \Delta t$$

then,

$$\frac{R^2}{\zeta_v^* \zeta_o^*} = \frac{R^2}{\zeta_v^{*2} - \zeta_v^* \frac{\partial \zeta_v^*}{\partial t} \Delta t} \quad (3.51)$$

The difference between this and $R^2/(\zeta_v^*)^2$ was calculated and found to be very small but not zero.

Provided the free stream is constant and the aerofoil is not changing shape (i.e. fixed spoiler), the overall complex force coefficient is given by:

$$C_f = -\frac{2i}{U_\infty^2 c} \sum_1^N \Gamma_v \left[\zeta_v - \frac{R^2}{\zeta_v^*} \right] \quad (3.52)$$

where, c , is the aerofoil chord. The lift and drag force components along the y-axis and x-axis respectively (figure 3.13) are equal to the imaginary and real part of equation 3.52. For a free stream at incidence, α , to the chord of the aerofoil:

$$\begin{aligned}
C_l &= C_y \cos \alpha - C_x \sin \alpha \\
C_d &= C_x \cos \alpha + C_y \sin \alpha
\end{aligned} \tag{3.53}$$

where,

$$C_x = \text{Re}(C_f) \text{ and } C_y = \text{Im}(C_f).$$

3.10 Pressure distribution.

As has been shown above, the forces on the aerofoil fitted with a 'fixed' or moving spoiler may be calculated from the shed point vortices in the wake. However, it is important to know the pressure distribution on the body, so that it is possible to calculate the forces acting on it. In the case of an aerofoil/spoiler combination, surface pressure integration gives the forces on the aerofoil and spoiler separately. This is an advantage in order to compare with some experimental results for the moving spoiler case, where forces only on the aerofoil were measured.

The pressure distribution over the aerofoil and spoiler is given by:

$$P_\infty + \frac{1}{2} \rho U_\infty^2 + \rho \left(\frac{\partial \Phi}{\partial t} \right)_\infty = P + \frac{1}{2} \rho U_s^2 + \rho \left(\frac{\partial \Phi}{\partial t} \right)_s \tag{3.54}$$

where P is the static pressure, U is the velocity, Φ is the velocity potential and subscripts ∞ and s define undisturbed conditions and the body surface respectively.

From the definition of the pressure coefficient i.e.

$$C_p = \frac{P - P_\infty}{\frac{1}{2} \rho U_\infty^2} \tag{3.55}$$

and equation 3.54, the unsteady pressure coefficient at any instant is given by:

$$C_p = 1 - \left[\frac{U_s}{U_\infty} \right]^2 - \frac{2}{U_\infty^2} \left[\frac{\partial \Phi}{\partial t} \right]_s \quad (3.56)$$

where U_s is the surface velocity in the physical plane.

It should be noted that $(\partial \Phi / \partial t)_\infty$ tends to zero at positions far from the body, since at infinity the free stream conditions are steady.

The surface velocity is calculated by taking into account the free stream and the effect of the vortices on the body, while in the moving spoiler case the effect of sources and sinks representing movement of the body surface must also be taken into account.

The unsteady term $(\partial \Phi / \partial t)_s$ in equation 3.56 may be determined in two ways: first by direct differentiation of the complex potential with respect to time (method A) and secondly by time-differencing and integrating the surface velocity around the circle in the transformed plane (method B).

The complex potential is given by equation 3.9, since $\Phi = \text{Re}(W)$. Differentiating equation 3.9 with respect to time:

$$\left[\frac{\partial \Phi}{\partial t} \right]_s = \text{Re} \left[\frac{i}{2\pi} \sum_1^N \frac{\Gamma_v}{\zeta - \zeta_v} \left[-\frac{d\zeta_v}{dt} \right] - \frac{i}{2\pi} \sum_1^N \frac{\Gamma_v}{\zeta - \frac{R^2}{\zeta_v^*}} \left[\frac{R^2}{\zeta_v^{*2}} \right] \left[\frac{d\zeta_v}{dt} \right]^* \right] - \frac{1}{2\pi} \frac{\Gamma_{ev}}{\Delta t} \mu_{ev} - \frac{1}{2\pi} \frac{\Gamma_{tv}}{\Delta t} \mu_{tv} \quad (3.57)$$

where, Γ_{ev} , Γ_{tv} , are the strengths of the last two vortices shed at the separation points, μ_{ev} , μ_{tv} , are angles defined as shown in figure 3.8 and ζ are complex positions of control points on the body surface. The last two terms in equation 3.57 are due to the

jumps in the complex potential at the shedding points. $\partial\Phi/\partial t$ calculated using equation 3.57 is substituted into equation 3.56, to give the pressure coefficient.

Alternatively, $\partial\Phi/\partial t$ may be calculated by evaluating Φ over two time steps. Since the velocity potential is defined as the integral of the tangential velocity component of fluid flow along a line of integration (figure 3.10), then for the closed surface of the aerofoil/spoiler combination Φ is given by:

$$\Phi = \Phi_{\text{START}} + \int_s q \, ds \quad (3.58)$$

where s defines the surface of the body and Φ_{START} is the starting value of integration. Φ_{START} is calculated at or near the stagnation point analytically using equation 3.9 since $\Phi = \text{Re}(W)$. The sign of the integral part in equation 3.58 depends on the direction of integration and the direction of the tangential velocity. The convention employed here is that if q (where q is the tangential velocity on the circle surface) has the same direction as the direction of integration, then $\int_s q \, ds$ is taken to be positive.

By definition, circulation is the line integral of the tangential velocity component around any circuit. Therefore, Φ is discontinuous across the separation points (i.e. spoiler tip and trailing edge) and equation 3.58 for separated flow should be:

$$\Phi = \Phi_{\text{START}} + \int_s q \, ds + \frac{\text{TOTAL CIRCULATION SHED FROM ANY POINTS ON THE PATH 's'}}{\quad} \quad (3.59)$$

After taking the velocity integral around the circle back to the starting point and adding all the shed circulation from the separation points, it should be expected that the complex potential at the end of the integration, Φ_{END} , would be equal to Φ_{START} , so that:

$$\Phi_{\text{END}} - \Phi_{\text{START}} = 0$$

However, trial calculations showed that there was a difference between Φ_{END} and, Φ_{START} , i.e.

$$\Delta\Phi_{\text{INTG}} = \Phi_{\text{END}} - \Phi_{\text{START}} \quad (3.60)$$

This difference comes from errors in the velocity integration, especially over the separated flow region (here, the trapezoidal rule is used to integrate over the circle in the transformed plane).

To eliminate the integration error in Φ , a linear correction procedure was applied, also used to correct the complex potential by BASUKI (1983), who studied the fully separated flow over an aerofoil.

As shown in figure 3.12, integration is carried out in the anticlockwise sense, starting from a point near or at the stagnation point at the leading edge of the aerofoil. According to the linear correction scheme,

$$\Delta\Phi_{\text{AVG}} = \frac{\Delta\Phi_{\text{INTG}}}{M_0} \quad (3.61)$$

is linearly distributed over the ' M_0 ' points of the separation region between the trailing edge and the spoiler tip, and finally $\Delta\Phi_{\text{INTG}}$ is added to the rest of the points from the second separation point (the spoiler tip) to the point at which integration started. Therefore, the corrected Φ in the separated region (region A) is given by:

$$\Phi_{\text{J, CORR}} = \Phi_{\text{J, CALC}} + M \Delta\Phi_{\text{AVG}} \quad (3.62)$$

where the second term in the R.H.S is the calculated value of Φ at a point inside the separation region and ' M ' is the number of points from the first separation point to the point at which Φ is being calculated (see also figure 3.12). The corrected Φ in the region

further, the pressure distribution from method A was found to fluctuate strongly on the upper surface of the aerofoil near the trailing edge. This was thought to be due to the large convection velocities of vortices coming close to the surface of the aerofoil, and being influenced by their images. This finding is in agreement with the results of STANSBY and DIXON (1982), who found that the velocity potential on the body surface was very sensitive to vortices coming close to the surface. They also had difficulties when dealing with the flow just downstream of the separation points. Similar difficulties were found by BASUKI (1983). Therefore, method B was mainly used here, especially for long runs and to obtain results for the rapidly deployed spoiler.

3.10.1 Force coefficients by surface pressure integration.

In order to find the lift and drag coefficients over the aerofoil and spoiler, the pressure distribution may be integrated over the aerofoil and spoiler:

$$\begin{aligned}
 C_f &= -\frac{1}{c} \oint C_p dy + \frac{i}{c} \oint C_p dx \\
 &= \frac{i}{c} \oint C_p (dx + i dy) \\
 &= \frac{i}{c} \oint C_p dz
 \end{aligned} \tag{3.64}$$

where C_x , C_y are the force coefficients along the x-axis and y-axis respectively, dz is the complex distance between two points on the body in the physical plane and c is the aerofoil's chord (see also figure 3.13). C_x and C_y may also be calculated by integrating C_p (from the physical plane) on the circle in the transformed plane, since $\zeta = re^{i\theta}$, $d\zeta = ire^{i\theta} d\theta$ and $rd\theta = ds$ in the circle plane. Equation 3.64 may be written as:

$$C_f = -\frac{1}{c} \oint C_p e^{i\theta} \left(\frac{dz}{d\zeta} \right) ds \tag{3.65}$$

between the second separation point and the starting integration point (region B) is given by:

$$\Phi_{J, \text{ CORR}} = \Phi_{J, \text{ CALC}} + \Delta\Phi_{\text{INTG}} \quad (3.63)$$

It should be noted that the calculated values of Φ in the two equations above, include the circulation shed from the separation points.

This correction procedure eliminates the error in Φ at the starting point, and allows a stable calculation of $d\Phi/dt$ on the surface of the aerofoil and spoiler, and hence the pressure coefficient. $\Phi_{\text{END}} - \Phi_{\text{START}}$ is mainly due to errors in the integration technique employed here, which may deteriorate as vortices passing very near the upper surface of the aerofoil may cause surface velocity fluctuations. Hence the correction is applied only to the separation region.

The size of the error before correction due to integration errors ($\Delta\Phi_{\text{INTG}}$), depends on spoiler position, spoiler angular velocity and aerofoil incidence. For example, for a zero incidence aerofoil with a 'fixed' spoiler normal to the surface at 70% chord, $\Delta\Phi_{\text{INTG}}/U_{\infty}c = 2.6 \times 10^{-4}$, while for a rapidly deployed spoiler ($u_t/U_{\infty} = 0.37$), $\Delta\Phi_{\text{INTG}}/U_{\infty}c = 50 \times 10^{-4}$ (compared to $\Gamma_{\text{SHED}}/U_{\infty}c = 0.77$). Similar values were found by BASUKI (1983) in integrating Φ round an aerofoil in fully separated flow. The value of $\Delta\Phi_{\text{INTG}}/U_{\infty}c$ also depends on the proximity of the vortices to the aerofoil surface, but it always lies within the values mentioned above.

The advantage of using equation 3.57 (method A) to calculate the surface pressure distribution is that the vortex convection velocity, which has already been calculated, may be substituted directly and the surface pressure distribution obtained at every time step. It was found here that for the initial development of the flow, when vortices from the spoiler tip were still not very close to the surface or the trailing edge, the pressure distribution and the force coefficients obtained from it were in close agreement with the results obtained from method B. However, as the flow developed

where θ is the angular position of a point on the circle (figure 3.12).

When the aerofoil is at incidence to the free stream, then the lift and drag coefficients are given as in equation 3.53. The magnitude of C_x is small compared to C_y and it is more difficult to calculate accurately.

Both equations 3.64 and 3.65 may be used to calculate lift and drag coefficients. Initially, equation 3.65 was used because of the simplicity of integrating over the circle, and because the control points on the circle are not fixed (as will be discussed later). However, $(dz/d\zeta)$ terms on the surface take large values when close to the trailing edge or the spoiler tip. In order to avoid this, equation 3.64 was used to integrate pressures over the aerofoil and spoiler in the physical plane. Lift and drag coefficients are obtained separately on the aerofoil and spoiler surface, so that the force contribution of the spoiler may be evaluated.

CHAPTER FOUR

STABILITY AND VORTICITY REDUCTION TECHNIQUES

4.1 Stability of the Biot-Savart method.

The velocity field generated by a point vortex is given by:

$$u - iv = \frac{i\Gamma_j}{2\pi} \cdot \frac{1}{\zeta - \zeta_j} \quad (4.1)$$

where, ζ , is a point in the flow field and, ζ_j , is the vortex position. If the absolute value is taken on both sides of the equation, then it can be written as $q = \Gamma_j / 2\pi r$ where r is the distance between the vortex and a point, and q is the resultant velocity at that point. According to equation 4.1 the velocity field of the point vortex becomes infinite at the vortex centre. Therefore, large velocities are induced on vortices close to each other, which during the rolling-up of a vortex sheet results in a less accurate representation of the motion of the vortices (due to unrealistically large convection velocities) and to increasing randomisation of the vortex positions.

This can cause large amplitude fluctuations in the force coefficients. Similarly, vortices that come very close to a solid surface have large induced convection velocities due to their images and also induce large surface velocities. These extreme velocities on the surface cause very low pressures on the body, which can lead to high forces. This is a failure of the point vortex method. Sheet elements would, at greater computational cost, avoid some but not all of these problems.

In this particular study, the vortices of the separating layer from the spoiler tip come close to the aerofoil surface behind the spoiler, as the layer rolls-up and vortices convect downstream. An additional complication is the increasing influence of the spoiler tip vortices on the trailing edge of the aerofoil, as they approach the trailing

edge and finally convect past it.

A number of different techniques have been devised over the years, in an attempt to eliminate the instabilities of a rolling-up sheet and prevent the randomisation of the vortex positions. Some of these techniques have been applied in this study and are discussed in the following sections.

4.2 Cut-off Radius.

In principle, if a small enough time step is used, the displacement is more correctly calculated and vortices, which have a velocity component towards the body, must approach the surface asymptotically. How correctly the path of a vortex is calculated depends, as discussed in Chapter Three, on the accuracy of the integration scheme and the time step size. However, a very small time step is computationally very expensive. To improve cost efficiency, a bigger time step has to be used in which case vortices that are very close to the surface at the end of a time step may be given displacements which cross the surface at the end of the next time step. Since this is incorrect, the present study employs a cut-off radius (or buffer region) around the body in the transformed plane (a concentric annulus to the solid circle; see figure 4.1). Vortices entering that region are stopped from moving further inward, at the boundary of the region (see figure 4.1).

JARROCH (1986) applied a buffer region round the body in the circle plane, in his study of the flow past a plate normal to a long wake splitter plate. Vortices that came within a distance of $0.1R$ (where R is the radius of the circle in the transformed plane) from the body surface were removed from the calculation. This was done to stop vortices from coming very close to the body surface, and also partly to represent the cancellation of vorticity by the creation of vorticity of opposite sign at the walls. Vortices were also removed from the calculation if they came within a certain distance from the body in studies by STANSBY (1977) and KIYA et al (1982). CLEMENTS (1973), in his study of the shedding from the edges of a square-based section, removed vortices

that came within a certain distance from the base to avoid them having unduly high velocities along the body.

Removing vortices that come near the body may cause large changes in the force on the body, since this force strongly depends upon the vortices in the vicinity of the body surface. Also, it is incorrect to remove vortices from an inviscid flow field because of Kelvin's theorem.

In the case of the aerofoil and spoiler, the thickness of the buffer region has to be kept small because the circular annulus in the circle plane transforms to a region around the aerofoil that is very thin at the leading edge, trailing edge and spoiler tip, but much thicker on either side of the spoiler root (figure 4.3). For this reason and also because the buffer region interferes with the path the vortices want to follow, it was chosen to be of radius $1.001R$. Therefore, the buffer region is mainly introduced here to stop the vortices from moving through the surface but partly to prevent them from coming very close to the surface.

Using a buffer region is strictly incorrect since vortices are not allowed to follow the path dictated by their velocity near the body surface. This may affect the flow field locally and constrain the accurate calculation of the forces on the body. However, the error introduced is much smaller than it would be in the absence of the buffer region. It was found during initial tests, that only a small number of vortices were stopped by the buffer region (compared to the number of vortices in the flow field). Also, a vortex close to the surface may cross the buffer region more than once.

4.3 Core Vortices.

As seen earlier, when point vortices get close together, strong convection velocity gradients are generated. To overcome this problem, core vortices may be used. These are vortices whose vorticity is spread over a finite core leading to a reduced velocity field inside a certain radius round the centre, compared with the point vortex. Such vortices were first employed by CHORIN (1973) and later applied to the

rolling-up of wing tip vortices by CHORIN and BERNARD (1973).

In this study, when core vortices are used, their velocity field is given by:

$$u - iv = \begin{cases} \frac{i\Gamma_j}{2\pi} \cdot \frac{1}{\zeta - \zeta_j} & r \geq \sigma \\ \frac{i\Gamma_j}{2\pi} \cdot \frac{(\zeta - \zeta_j)^*}{\sigma^2} & r < \sigma \end{cases} \quad (4.2)$$

where, $r = |\zeta - \zeta_j|$, is the distance from the centre and, σ , is the core size.

The core radius does not influence computing cost significantly and a form of optimum value exists (unrelated to cost). SPALART et al (1983) found that if the core is large, the velocity is very smooth locally and the 'noise' (i.e. fluctuations in, for example, C_l vs time curves) is low. As a result vortices do not scatter much. The application of DVM to simple problems with known exact solutions by NAKAMURA et al (1982), showed second order convergence in terms of the core radius. However, a large core radius can suppress velocity gradients that are physically significant and 'freeze' a coherent structure that would be better represented if the cores were small enough to allow it to evolve correctly. SPALART et al (1983) computed the separated flow over a square cylinder using values of σ equal to 0.005 and 0.05. Changing the core radius by a factor of 10 did not cause a striking difference. In the same study the core radius is taken to be of the order, $\Delta s/2$, where Δs is the spacing of control points on the solid surface. However, the value of σ must be less than or equal to the distance from the surface of the point at which nascent vortices are released. Following these arguments, a value of σ/c was chosen here to be equal to 0.007, where c is the chord of the aerofoil.

4.4 Vortex Amalgamation.

In general, vortex amalgamation (or merging) is applied, in an effort to limit the number of vortices in the flow. Otherwise, only flows of relatively short duration could be computed before the number of vortices, and hence the associated computing cost, became excessive. The continuous addition of new vortices may be balanced by amalgamating pairs of vortices (or merging them) far downstream. This way, there is a larger number of vortices near the body, where high resolution is required, and vortices become sparser further away from it.

CLEMENTS (1973) amalgamated clusters of vortices downstream of the square-based body into single vortices placed at the centre of vorticity of the cluster. The strength of the amalgamated vortices was equivalent to the sum of the individual vortices of the original cluster (i.e. total circulation and first moment of vorticity are preserved).

In this work, amalgamation is sometimes applied to pairs of vortices of the same sign, so that the resulting vortex is placed between the two original vortices (figure 4.4). The circulation and position of the amalgamated vortex are:

$$\begin{aligned}\Gamma_{\text{NEW}} &= \Gamma_1 + \Gamma_2 \\ z_{\text{NEW}} &= \frac{\Gamma_1 z_1 + \Gamma_2 z_2}{\Gamma_1 + \Gamma_2}\end{aligned}\tag{4.3}$$

Therefore, the total circulation is preserved and so is the first moment of vorticity, which is equal to the impulse of the flow. If amalgamation is applied downstream of the body, then the calculation of the forces on the body is not affected, since the effect of the amalgamated vortices on the body diminishes as they get further downstream. Amalgamation of vortices very near the body surface may cause erratic changes in the force calculations. Here, equation 4.3 is applied only to pairs of like signed vortices

some distance downstream i.e: $x > 2c$, where c is the aerofoil chord.

4.5 Calculation of the trailing edge velocity.

One of the most difficult and time consuming tasks in this work was the calculation of forces by surface pressure integration. It was found that initially, the C_l vs TU_∞/c curve was 'smooth' and in good agreement with that predicted by the Momentum theorem. However, as the influence of the spoiler tip vortices on the trailing edge vortices increased (this will be discussed in detail later), unrealistically large fluctuations in C_l were obtained.

A number of different tests were devised to understand the cause of the fluctuations, and different techniques were employed to reduce them, including amalgamation of vortices near the trailing edge. It was finally found that fluctuations in C_l were reduced if the velocity at the trailing edge (which was not necessarily a control point of the transformation) was averaged in terms of the velocities at two (rather than just one) neighbouring points on either side in the following way:

$$q_{t.e.} = \frac{1}{4} (q_{j-2} + q_{j-1} + q_{j+1} + q_{j+2}) \quad (4.4)$$

where q_{j-2} , q_{j-1} , q_{j+1} and q_{j+2} are velocities on the cylinder in the transformed plane, as shown in figure 4.2 (taking the weighted average for the two points closer to the edge did not make any significant difference). Using equation 4.4, agreement of C_l calculated by pressure integration and the Momentum theorem was greatly improved (as will be seen in Chapter Seven).

In this study, the Brown and Michael method of releasing vortices into the flow field has been adopted (see Chapter Three). According to this method, both the strength and position of nascent vortices varies for every time step. When vortices shed earlier from the spoiler eventually interact with the trailing edge vortex sheet (near the

trailing edge) they cause the strength and position of trailing edge nascent vortices to fluctuate considerably with a feed back effect on the induced velocity at the trailing edge; the result is amplification of the initial disturbance causing large fluctuations in C_l . Equation 4.4 offers a way of smoothing-out these fluctuations in the velocity at the trailing edge. BASUKI (1983) similarly used equation 4.4 in the real plane, to calculate the velocity at the trailing edge of a symmetric Joukowski aerofoil in fully separated flow.

CHAPTER FIVE

THE MOVING SPOILER

5.1 Modelling the moving spoiler.

As seen earlier, a 'fixed' spoiler is defined here as a spoiler which suddenly appears on the surface of the aerofoil at $t=0$. Applying the Discrete Vortex Method to a 'fixed' spoiler is important, since the effects of its sudden appearance on the aerodynamic forces exerted on the aerofoil (and spoiler) can be investigated. However, modelling the moving (rotating) spoiler is closer to real applications and includes the additional effect of its motion, but it is more complicated, since it involves a time dependent flow around a body with a moving boundary. In real applications, it appears that the rate of spoiler deployment determines the maximum adverse lift, as well as the time it takes before it is reached. Both these are very important in the design and use of spoilers. Therefore, a numerical model of the moving spoiler would aim to analyse the effects of spoiler deployment on the aerodynamic performance of the aerofoil.

In the case of a moving spoiler $\partial\Phi/\partial n$ (or $\mathbf{U} \cdot \mathbf{n}$) $\neq 0$, where $\mathbf{n}$ is a vector perpendicular to the spoiler surface, since there must be no flow through the solid surface. This effect is modelled using a distribution of sources and sinks on the spoiler. The source/sink distribution is defined for convenience in the straight-line plane (z_5 -plane), as seen in figure 5.1. In the z_5 -plane, the spoiler is opened up with the origin of the axes positioned at the spoiler tip (figure 5.1).

In the aerofoil plane (z_1 -plane), the velocity perpendicular to the spoiler due to its rotation is given by

$$q = \Omega \cdot r \quad (5.1)$$

where Ω is the angular velocity of the spoiler and 'r' is the distance measured from the

spoiler root (figure 5.2). In the z_5 -plane the corresponding velocity is given by the velocity in the aerofoil plane multiplied by the derivative of the transformation between the two planes, i.e:

$$q = \Omega r \left[\frac{dz_1}{dz_5} \right] \quad (5.2)$$

In the z_5 -plane the source/sink elements are straight. Hence, the source/sink strength per unit length along the spoiler in the z_5 -plane is given by:

$$m' = \pm 2\Omega r \left[\frac{dz_1}{dz_5} \right] \quad (5.3)$$

The complex potential due to this source/sink distribution, satisfying zero flow through the surface, can be written as:

$$W(z_5) = \int_B^D \frac{m'(t)}{2\pi} \ln |z_5 - t| dt \quad (5.4)$$

and the complex velocity is given by:

$$\frac{dW(z_5)}{dz_5} = \int_B^D \frac{1}{2\pi} \frac{m'(t)}{(z_5 - t)} dt \quad (5.5)$$

The segment BCD in figure 5.1 represents the two surfaces of the opened-up spoiler. BC and CD are divided into an equal number of elements with the source/sink strength distributed over each element but evaluated at its mid-point. Since

BC is longer than CD, if the spoiler angle is less than 90° to the surface, the elements on CD are of smaller length than those on BC. On a rising spoiler, segment BC has a distribution of sources and segment CD a distribution of sinks, and vice versa to represent the spoiler's retraction.

A distribution of sources and sinks has been employed by CHENG and EDWARDS (1982) and ZANJDANI (1983) to model the moving wings of an insect, to investigate the Weis-Fogh lift-generation mechanism.

It should be noted that the positions of sources and sinks along BCD are in general different to those control points which lie on BCD in the circle plane and are mapped on BCD in the straight line plane (figure 5.3). Equations 5.4 and 5.5 may be used to calculate the complex potential and the velocity at any point on the real axis outside BCD in figure 5.1, and off the real axis. However, on BCD they can only be used at the mid points of the source/sink elements, and they may introduce small integration errors depending on the form of the source/sink distribution. In order to improve the accuracy of the calculation at the mid points of elements on BCD, an analytic expression of the complex velocity is found by applying equation (5.5) over an element, i.e.:

$$\frac{dW(z_5)}{dz_5} = - \frac{m'(t)}{2\pi} \ln \left[\frac{z_5 - M}{z_5 - L} \right] \quad (5.6)$$

considering that $m'(t)$ is constant over the element, L and M are the end points of the element (figure 5.1), and z_5 is the mid point of an other element. The complex velocity at the mid point of an element inside BCD, may be calculated by summing equation 5.6 over all the source/sink elements.

The velocity due to the source/sink distribution at the control points inside BCD is calculated by interpolation, once the velocity at the mid points of the source/sink elements has been evaluated. Equation 5.6 may also be used to calculate the

velocity at any point outside BCD. However, this is not very efficient computationally, and for points outside BCD equation 5.5 is both adequately accurate and efficient to compute. Therefore, equation 3.10 (for moving vortices) becomes:

$$\left[\frac{dW(\zeta)}{d\zeta} \right]_{\text{MOVING SPOILER}} = \frac{dW(\zeta)}{d\zeta} + \left[\int_B^D \frac{1}{2\pi} \frac{m'(t)}{z_5 - t} dt \right] \left[\frac{dz_5}{d\zeta} \right] \quad (5.7)$$

5.2 The Momentum Theorem applied to the moving spoiler.

The Blasius Theorem, giving the force on a body with fixed boundaries, was discussed in section 3.9.

However, when the spoiler on the aerofoil is deployed, the body surface changes with time and in effect, the stream function over the spoiler is not constant any more. Taking into account the variation of the stream function over the moving spoiler a new expression may be derived for equation 3.45, as shown below.

If the unsteady form of Bernoulli's equation is considered (equation 3.54), the total complex force on the body is obtained by integrating the pressure over the body surface i.e.:

$$Z_f = i \oint_s \left[P - \frac{1}{2} \rho q^2 - \rho \frac{\partial \Phi}{\partial t} \right] dz \quad (5.8)$$

Since the spoiler is moving, $\partial \Psi / \partial t \neq 0$ on its surface and therefore,

$$\frac{\partial \Phi}{\partial t} = \frac{\partial W}{\partial t} - i \frac{\partial \Psi}{\partial t}$$

which when substituted in equation 5.8 yields:

$$Z_f = -i\rho \frac{\partial}{\partial t} \oint_s W dz - \rho \frac{\partial}{\partial t} \oint_s \Psi dz - \frac{i\rho}{2} \oint_s \left[\frac{\partial W}{\partial z} \right]^* dz \quad (5.9)$$

By considering a circuit at infinity to be shrunk around the body plus the circuits round each vortex in the flowfield and the cuts joining each vortex to the edge from which it was shed, GRAHAM (1980) has shown that

$$Z_f = -i\rho \frac{\partial}{\partial t} \oint_s W dz - \frac{i\rho}{2} \oint_s \left[\frac{\partial W}{\partial z} \right]^* dz = -i\rho \frac{\partial}{\partial t} \oint_\infty W dz \quad (5.10)$$

This result has been derived without assuming that $\text{Im}(W)=\Psi=0$ anywhere. In addition,

$$\Psi = -\frac{1}{2} \Omega (z e^{-i\delta})^2$$

and

$$\frac{\partial}{\partial t} \oint_{S_p} i \Psi dz = -\frac{i\Omega}{2} \frac{\partial}{\partial t} \left[\oint_{S_p} (z e^{-i\delta})^2 dz \right] = 0$$

where S_p denotes the spoiler surface. Therefore equation 5.10 is valid for the moving spoiler case. Using equation 5.10, equation 5.9 can be written as:

$$Z_f = -\rho \frac{\partial}{\partial t} \oint_s \Psi dz - i\rho \frac{\partial}{\partial t} \oint_\infty W dz \quad (5.11)$$

Consider that the transformation between the two planes is $z=f(\zeta)$, where

$$f(\zeta) = \mu(t)\zeta + b_o(t) + \sum_{n=1}^{\infty} b_n(t)\zeta^{-n} \quad (5.12)$$

and $\mu(t)$ is the amplification factor discussed in Chapter Two. The complex potential in the circle plane including the sources and sinks on the spoiler may be written as:

$$\begin{aligned} W(\zeta) = V_{\infty} \left[\zeta + \frac{R^2}{\zeta} \right] + \frac{i}{2\pi} \sum_1^N \Gamma_v \ln(\zeta - \zeta_v) - \\ \frac{i}{2\pi} \sum_1^N \Gamma_v \ln\left(\zeta - \frac{R^2}{\zeta_v^*}\right) + \\ \sum_1^{NSP} \frac{m_q}{2\pi} \ln(\zeta - \zeta_q) - \sum_1^{NSP} \frac{m_q}{2\pi} \ln(\zeta - \zeta_q') \end{aligned} \quad (5.13)$$

where m_q is the source/sink strength and ζ_q, ζ_q' refer to points on the front and back of the spoiler. Expanding $\ln(\zeta - \zeta_{v,q})$ for large ζ in equation 5.13, 'W' at infinity is given as:

$$\begin{aligned} W(\zeta) \approx V_{\infty} \left[\zeta + \frac{R^2}{\zeta} \right] + \frac{i}{2\pi} \sum_1^N \Gamma_v \left[-\frac{\zeta_v}{\zeta} + \frac{R^2}{\zeta \zeta_v^*} \right] + \\ \frac{1}{2\pi} \sum_1^{NSP} |m_q| \frac{(\zeta_q + \zeta_q')}{\zeta} \end{aligned} \quad (5.14)$$

since $\sum |m_q| = 0$ over the spoiler. Also, by writing dz as $(dz/d\zeta)d\zeta$, and taking into account that:

$$\frac{dz}{d\zeta} = \frac{df}{d\zeta} = \mu(t) - \sum_{n=1}^{\infty} \frac{nb_n(t)}{\zeta^{n+1}} \approx \mu(t) - \frac{b_1(t)}{\zeta^2}$$

for large ζ , then:

$$\begin{aligned} \frac{\partial}{\partial t} \oint_{\infty} W(\zeta) \frac{dz}{d\zeta} d\zeta &= \frac{\partial}{\partial t} \oint_{\infty} \left\{ \mu(t) \left(\frac{U_{\infty} R^2}{\zeta} - \frac{i}{2\pi} \sum_1^N \Gamma_v \frac{1}{\zeta} \left(\zeta_v - \frac{R^2}{\zeta_v^*} \right) - \right. \right. \\ &\quad \left. \left. \frac{1}{2\pi} \sum_1^{NSP} |m_q| \frac{(\zeta_q - \zeta'_q)}{\zeta} \right) - \frac{b_1(t) V_{\infty}}{\zeta} + \right. \\ &\quad \left. \text{terms in } \left(\zeta, \frac{1}{\zeta^2}, \frac{1}{\zeta^3}, \dots \right) \right\} d\zeta \end{aligned} \quad (5.15)$$

Using the Residue Theorem,

$$\begin{aligned} \oint_{\infty} W dz &= 2\pi i V_{\infty} (R^2 \mu(t) - b_1(t)) + \\ &\quad \sum_1^N \Gamma_v \left[\zeta_v - \frac{R^2}{\zeta_v^*} \right] - i \sum_1^{NSP} |m_q| (\zeta - \zeta'_q) \end{aligned} \quad (5.16)$$

Therefore:

$$\begin{aligned} Z_f &= -\rho \frac{\partial}{\partial t} \int_{S_p} \Psi dz - \rho \frac{\partial}{\partial t} \sum_1^{NSP} |m_q| (\zeta_q - \zeta'_q) + \\ &\quad 2\pi \rho \frac{\partial}{\partial t} \left[V_{\infty} (R^2 \mu(t) - b_1(t)) \right] - i \rho \frac{\partial}{\partial t} \sum_1^N \Gamma_v \left[\zeta_v - \frac{R^2}{\zeta_v^*} \right] \end{aligned} \quad (5.17)$$

The constant b_1 is obtained in the following way:

$$2\pi i b_1 = \oint_{\text{CIRCLE}} z d\zeta = \sum_j^{NP} i z_j R e^{i\theta_j} \delta\theta_j$$

where NP defines the number of control points on the body, and hence,

$$b_1 = \frac{1}{2\pi} \sum_j^{NP} z_j R e^{i\theta_j} \delta\theta_j \quad (5.18)$$

Also, U_∞ is constant for $t > 0$.

Compared to the original Blasius equation, equation 5.17 has three extra terms on the R.H.S, which when calculated are small (their inclusion in equation 5.17 gives fluctuations in Cl of less than $\pm 5\%$), and they get smaller as the spoiler angle increases and the spoiler deployment rate decreases.

Figure 5.6 shows the lift coefficient obtained from equations 3.47 and 5.17, for a spoiler of 10% chord in length moving from 3° to 32° so that $u_t/U_\infty = 0.37$ (where u_t is the spoiler tip velocity). The final angle is reached at $U_\infty T/c = 0.149$. The Cl variation with time from the two equations is very similar, and the small oscillations observed in the curve obtained by the modified Blasius equation (i.e. equation 5.17) are attributed to errors in the integration over the spoiler. This is due to the transformation which tends, especially for small spoiler angles, to transform evenly spaced points in the circle plane corresponding to the back of the spoiler very near the spoiler tip. These oscillations become almost extinct for large spoiler angles, for example greater than 40° , and lower spoiler deployment rates.

The modified Blasius equation gives results similar to those obtained from the original Blasius equation, and therefore the unmodified Blasius equation is an adequate approximation to use. This is, therefore, used here to calculate force

coefficients, since it is easier to implement in the code and more efficient computationally.

5.3 Starting the spoiler at small angles.

One of the difficulties associated with the moving spoiler is to start it from a very small (near zero) angle relative to the aerofoil surface. A prime reason for this is that theoretically for inviscid flow, as the spoiler leaves the surface at $t=0$, the problem is singular. Physically, as the spoiler leaves the surface, there is a violent inrush of fluid trying to fill the region (gap) between the spoiler and the aerofoil surface. Locally, the flow around the moving spoiler is similar to the flow around a flat plate rotating as shown in figure 5.5. This has been used (LIGHTHILL (1973), CHENG and EDWARDS (1982)) to model the Weis-Fogh mechanism of lift generation. MAXWORTHY (1979) in his experiments to investigate the flow about two such plates hinged at one end and opening suddenly, found it difficult to start the motion from the position where the plates were closed, because they would not open and also remain stable, within a reasonable time. Therefore, the flow configuration in this region is characterised by large velocity gradients. It follows that an accurate mathematical modelling of this stage of the flow is likely to be very difficult.

In an attempt to start the spoiler from a very small angle relative to the aerofoil surface, a solution given by CHENG and EDWARDS (1982) was employed here, which gave the position (relative to the spoiler tip) and strength of the first (starting) vortex, between the spoiler tip and aerofoil surface, in the physical plane:

$$z_o = \frac{i\delta l}{2} + \frac{\delta l}{2\pi} \ln \left(\frac{\delta}{2\pi} \right) \quad (5.19)$$

$$\Gamma_o \approx l^2 \sqrt{\pi \beta}$$

where δ is the spoiler angle, l is the spoiler length and β is a constant determined by δ and the angular velocity. The spoiler was started at the small angle suggested by Cheng and Edwards (see also figure 5.4).

It was found that the above equations applied to the spoiler case did not exactly satisfy the Kutta condition at the spoiler tip. One of the reasons may be that equations 5.19 have been derived for the limiting case of the starting spoiler angle being equal to zero. However, it is not possible to start the spoiler from a zero angle in the present application, since due to the form of the transformation used, the spoiler for small angles is curved away from the aerofoil surface. This curvature increases with decreasing spoiler angle (figure 5.4). In addition, the aerofoil surface may also be curved either way. Therefore, it is impossible to use the present transformation to completely close a spoiler of finite length. Also ZANDJANI (1983), who studied the same problem as Cheng and Edwards but with sequential vortex shedding from the spoiler tip, found that equations 5.19, which he used directly, did not stabilise his solution for small angles of the flat plate.

In the present case, with the spoiler at 50% chord and rotating so that $u_t/U_\infty=0.37$ (where u_t is the spoiler tip velocity), a position and strength was found (by trial and error) for a starting vortex, such that the Kutta condition was satisfied at the spoiler tip and the convection velocity of the vortex was not large. This position of the starting vortex is extremely sensitive, mainly because it is very near the aerofoil surface, so that its image induces very high velocities at that point.

The strength and position of this vortex changes with different spoiler locations on the aerofoil and angular velocity. Therefore, since equations 5.19 do not apply here, an iterative scheme was needed to find the strength and position of the spoiler starting vortex in the physical plane. Also, an iterative scheme was needed to transform this position to the circle plane, since the transformation can only be written algebraically in terms of circle plane variables. The interpolation involved in the iterative scheme had to be extremely accurate, since the mesh is highly distorted in the region

between the spoiler and the aerofoil surface.

Due to the complications involved and the uncertainty of modelling accurately the 'opening up' of the spoiler, it was decided to allow a 'core' vortex to grow for a few time steps, before it was released, and to ignore shed vorticity in the early stages of the motion of the spoiler, assuming that vortex shedding only commences when the spoiler has reached an angle of 10° . The position and strength of the starting vortex are given by the Brown and Michael method, discussed in Chapter Three. Therefore, an interpolation scheme is not needed any more and large velocity gradients on small vortices subsequently shed are avoided. A similar technique was employed by ZANDJANI (1983).

CHAPTER SIX

DESCRIPTION OF THE PROGRAM

6.1 Description of the program for the 'fixed' spoiler.

A numerical code was initially developed to calculate the two-dimensional time dependent solution of separated flow past an aerofoil with a 'fixed' spoiler on its upper surface. This was later modified to cope with the deployment of the spoiler over a finite time, as is discussed later.

In the beginning, the code was written to be able to transform the aerofoil and spoiler into a circle. Then, it was gradually extended to include the effects of shed vortices, and also to calculate forces and pressures on the aerofoil and spoiler.

All the calculations were carried out in the circle plane and in cartesian coordinates, with the origin of the axes positioned at the centre of the circle. The free stream was rotated relative to the circle, to give the desired incidence of the flow relative to the body.

The $dz/d\zeta$ terms may be calculated analytically for any point in the flow field or on the body. Because certain intermediate stages of the transformation are complicated, the $dz/d\zeta$ term is calculated in the following way:

$$\frac{dz}{d\zeta} = \frac{dz_1}{dz_7} = \frac{dz_1}{dz_2} \frac{dz_2}{dz_3} \frac{dz_3}{dz_4} \frac{dz_4}{dz_5} \frac{dz_5}{dz_6} \frac{dz_6}{dz_7} \quad (6.1)$$

where subscripts 1 to 7 denote the different planes, from the aerofoil (1) to the circle (7).

The velocity in the physical plane is given by:

$$\frac{dW}{dz} = \frac{dW}{d\zeta} \frac{d\zeta}{dz} = u(z) - iv(z) \quad (6.2)$$

where z and ζ are corresponding complex positions of a point in the physical and transformed plane respectively. Using equation 6.1, an expression can be found for the term $|dz/d\zeta|^2$, which is used to calculate the vortex convection velocities in the circle plane.

The flow chart in figure 6.1 gives the main structure of the program. The program sequence together with a description of the main subroutines is as follows:

I. The aerofoil geometry (thickness, chord length), spoiler size and inclination, flow incidence, etc. are defined in subroutine <PARAMT>.

II. The transformation of the aerofoil is carried out in <ASMOVE>. Here, the corresponding positions of the fore and aft roots of the spoiler and the spoiler tip are first found on the circle. This way, equal numbers of control points (i.e points on the body where properties such as pressure, velocity, etc.) may be defined on an arc of a unit circle, in the transformed plane, corresponding to the spoiler in the physical plane, and the rest of the circle. Following the sequence of transformations described in Chapter Two, the circle is transformed into an aerofoil with a spoiler at an arbitrary angle on its upper surface.

III. Having found the positions of the control points in the different planes of the intermediate transformations, $dz/d\zeta$ and $|dz/d\zeta|^2$ are calculated at these points in subroutine <SURFVE> and are stored to be used later.

IV. Subroutine <BROWN> solves the Brown and Michael equations for vortex shedding into the flow field. This is composed of two parts:

a) For the starting flow, the initial vortex sheet is represented by a 'core' (starting) vortex. The starting vortex grows for five time steps and is then released in the flow

field, where it is free to convect.

b) This is followed by sequential vortex shedding. The position and strength of the subsequent point vortices is also determined by solving the Brown and Michael equations.

V. Once a vortex is released in the circle plane, it is influenced by the free stream and other vortices that are present in the flow field. Subroutine <VELOC> calculates the induced velocity on a vortex due to the free stream, all other vortices and all vortex images. If time integration is to be performed in the circle plane, the velocity must take the conformal transformation into account, i.e:

$$(u - iv)_\zeta = \frac{(\partial W / \partial \zeta)^*}{|dz / d\zeta|^2} \quad (6.3)$$

where the denominator has to be calculated at every vortex position. This is carried out in <MODVOR>.

VI. Time integration is carried out in <TSTEP>, using the convection velocity already calculated. In this subroutine, the local convection scheme and the local Routh's correction velocity are applied. A first order integration scheme is employed, so that the new position of a vortex is given as:

$$NEW(x,y) = OLD(x,y) + VELOCITY * TIME$$

Vortices in the flow field are convected twice, for the 'fixed' spoiler, so steps V and VI are repeated before two new vortices are released by <BROWN>.

VII. After the end of each time step the positions of vortices relative to the solid

boundary are checked in <TRAIL>. If vortices cross a pre-defined buffer region around the body, they are placed just outside this region along the radial direction (i.e. along the line connecting the centre of the circle to the vortex position inside the buffer region).

VIII. Once the new vortex positions have been calculated, C_l and C_d coefficients using the Momentum theorem are found in <BLASIUS>.

IX. Forces may also be calculated by pressure integration. Subroutine <BODVEL> calculates the velocity on the circle due to the free stream, the vortices and their images. This is needed in order to calculate Φ , in <CPDPHI>.

X. The surface velocity and Φ , are used by <DPDT> to calculate the C_p distribution over the aerofoil and spoiler.

XI. Integrating C_p in <FORCP> gives C_l and C_d over the aerofoil. Integrating C_p in <FORSP0> gives C_l and C_p over the spoiler.

XII. Return to step IV for the next time step.

At these times when a flow picture is required, the coordinates of the vortex positions in the circle plane are transformed to their corresponding positions in the aerofoil plane using subroutine <TRANSF>.

6.2 Description of the program for the moving spoiler.

The moving spoiler program is derived from the 'fixed' spoiler one and most steps are identical. Figure 6.2 shows a flow chart for the moving spoiler program.

The main difference between the two programs is that the aerofoil/spoiler geometry does not remain fixed, since the spoiler is moving. Therefore

<ASMOVE> has to be used at every time step to calculate the new spoiler positions. Also, the distribution of sources and sinks along the spoiler, calculated in <SOSINK>, induce an additional velocity at each vortex and on the body, which is calculated in <VELSOR>. Because of the simplicity offered by the shape of the spoiler in the straight-line plane, the vortex positions are transformed to this plane and the induced velocities on the vortices due to the sources and sinks are calculated. Subroutine <W7W5> transforms the induced velocities, from the z_s -plane to the circle plane.

An additional complication exists in general, and specifically in the calculation of $d\Phi/dt$, when the spoiler moves. This is introduced by the fact that the control points on the circle, including spoiler roots and spoiler tip, change position as the spoiler moves. Therefore, in order to calculate $d\Phi/dt$ over two time steps, Φ at time step I has to be linearly interpolated on to the control points at time step I-1, in the real plane (subroutine <INTPHI>). This introduces inaccuracies, as discussed later, and slows down the program.

The spoiler moves at a specified angular velocity, and is at a new position at every time step. Two vortices are released, one from the spoiler tip and one from the trailing edge at the end of every time step, at the new spoiler position. Therefore <VELOC1>, <TSTEPS> and <TRAIL> are used once for every time step. All the other extra subroutines mentioned in this section, are called in the program to calculate the source/sink distribution and its effect on the vortices and the body surface.

CHAPTER SEVEN

RESULTS OF THE NUMERICAL METHOD AND DISCUSSION

This work is primarily concerned with the unsteady loads experienced by an aerofoil and spoiler, while the spoiler either appears suddenly on the aerofoil surface at the start of the impulsive flow (defined as the 'fixed' spoiler case) or it rotates, from a small starting angle, at different angular velocities. Therefore, the flow characteristics and aerodynamic forces during the unsteady parts of the flow are of prime interest and an effort is made to understand them through the numerical model developed here. For the 'fixed' spoiler, where possible from a computational efficiency point of view, the computation has been carried further in an attempt to reach a steady state and compare the numerical results with existing experimental findings. An attempt is also made to compare numerical results for the moving spoiler with experimental results obtained by KALLIGAS (1986), although this proved to be difficult as will be discussed later.

7.1 The 'fixed' spoiler - test cases.

The aerofoil used for the flow computations, is an 11% thick, symmetric Joukowski aerofoil, with an infinitely thin (nominally straight) spoiler of length equal to 10% of the aerofoil's chord (c), positioned on its upper surface.

Flow calculations were carried out for different spoiler positions (SP) along the upper surface of the aerofoil, spoiler angles (δ) and free stream incidences (α). The test cases are as follows:

Test A: $\alpha = 0.0^\circ$ $\delta = 90.0^\circ$ $SP = 70\%c$.

Test B: $\alpha = 0.0^\circ$ $\delta = 45.0^\circ$ $SP = 70\%c$.

Test C: $\alpha=12.0^\circ$ $\delta=90.0^\circ$ SP=70%c.

Test D: $\alpha=0.0^\circ$ $\delta=90.0^\circ$ SP=50%c.

Test E: $\alpha=0.0^\circ$ $\delta=45.0^\circ$ SP=50%c.

Test F: $\alpha=12.0^\circ$ $\delta=30.0^\circ$ SP=70%c.

Test G: $\alpha=6.0^\circ$ $\delta=30.0^\circ$ SP=70%c.

The time step used in all the above cases was $U_\infty \Delta t / R = 0.02$, and vortices were convected twice before two new vortices were shed from the spoiler tip and the trailing edge, i.e. $\Delta t_i = 2\Delta t$. During calculating Test A, it was found that after 200 time steps ($TU_\infty / c = 2.17$), the execution of the program became very slow and expensive due to the large number of vortices (this was expected since the Biot-Savart law was used to calculate the velocity field). Initial tests showed agreement in force coefficients obtained by the Momentum method and surface pressure integration. Therefore, in order to improve the computational efficiency of the program, it was decided to calculate force coefficients by both methods for the first 200 time steps only. After that the computation was continued until $U_\infty T / c = 4.0$, while force coefficients were calculated using only the Momentum method.

In the test cases mentioned above, the wake contains approximately 700 vortices, when the spoiler is at 70%c, and 1000 vortices when the spoiler is at 50%c.

In the following sections the results obtained from the above computations are presented, discussed and compared, where possible, with existing experimental results.

7.1.1 Vortex shedding.

The flow past the aerofoil with the spoiler is impulsively started, and the Brown and Michael method is employed to shed vortices from the spoiler tip and the trailing edge. For the starting flow, a vortex is allowed to grow for five time steps before it is released. This builds up a strong initial 'core' vortex, which helps to initiate the

roll-up of the sheet.

Figures 7.1.a to 7.1.d show different stages (at 20 time step intervals) in the vortex shedding from the spoiler tip and the trailing edge, for Test A. It can be seen that at the end of the first 20 time steps, there is little sign of instability in the shear layers from the two separation points. As the flow progresses, the shear layer from the spoiler tip rolls up further and a vortex cluster is formed. At the same time, the regular rolling-up of the spoiler shear layer begins to 'break down' due to point vortices in successive arms of the shear layer coming close together. The growth of this positive vortex starts to affect the trailing edge shedding by inducing, at points near the trailing edge, a velocity with a horizontal component opposite to the usual (streamwise) convection velocity. As the size of the spoiler vortex increases, it disturbs unevenly the continuous vortex sheet shed from the trailing edge, near the edge, and results in the initiation of a randomised back-flow at the trailing edge, i.e. the upstream convection of trailing edge vortices over the upper surface of the aerofoil (figure 7.1.c).

It is not known if this phenomenon occurs in reality, but it is assumed that the interaction between the spoiler wake and the trailing edge is likely to be weaker in practice, due to diffusion and three-dimensional effects.

The back-flow results in the generation of a negative cluster of randomly moving vortices, very near the upper surface of the aerofoil. Some weak negative point vortices are caught in the recirculating region of the spoiler wake and are convected upstream over the upper surface of the aerofoil (figure 7.1.d).

As the flow develops further, the negative vortex cluster which started to form at the trailing edge grows and, when it becomes strong enough, it breaks away and convects downstream, while a new one begins to form at the trailing edge. This process is then repeated. Figure 7.1.e shows the wake behind the aerofoil in Test A. Negative clusters that have been shed from the trailing edge may be seen downstream. Also, weak negative vortices from the trailing edge are trapped in the spoiler near wake. These vortices can reach the spoiler and may disturb the vortex shedding from the tip

(figure 7.1.e). This can be seen in figure 7.6, where the strengths of the vortices shed from the spoiler tip and the trailing edge are shown. Both curves are smooth until back-flow begins at the trailing edge. The spoiler tip shedding remains fairly smooth, the small disturbances observed being due to trailing edge vortices getting very near the spoiler tip.

The negative cluster that forms at the trailing edge, affects the vortex shedding locally. Vortices may get very close to the surface and can be convected with large velocities under the influence of their images. Therefore, the velocity induced at the trailing edge by the vortices in the flow field at the end of a time step fluctuates, and so does the strength of the vortices that have to be released to satisfy the Kutta condition at the edge (figure 7.6). This phenomenon has an effect on pressures and forces, as will be discussed later.

The strength of the trailing edge vortices depends on the strength of the spoiler vortices and also on the length of time before back-flow begins at the trailing edge. For a given spoiler position and free stream incidence, this depends on the spoiler angle. Figure 7.7 shows the strength of vortex shedding for Test B. Since the spoiler is at 45° to the surface, the tip is closer to the trailing edge than when the spoiler is at 90° . Therefore, interaction with the trailing edge vortices starts earlier. Also, the fluctuations in the strength of the trailing edge vortices are weaker for this case (figures 7.6 and 7.7).

In Test A, the aerofoil is at 0° incidence and the trailing edge vortices are weak. Therefore, under the influence of the spoiler vortices, their velocity near the trailing edge becomes close to zero and hence their behaviour is unstable. However, if the aerofoil is set at an incidence to the free stream (figure 7.2.a to 7.2.e), then the negative cluster which is formed at the trailing edge is more orderly than when the aerofoil is at 0° incidence. This is because the trailing edge is now effectively at incidence to the free stream. The cluster is also spread over a larger area, while it grows at the trailing edge (figure 7.3). Both these phenomena lead to a weaker interaction of the trailing edge vortices and the edge itself, and hence a smoother lift variation with time.

Also, when the aerofoil is at incidence, the wake is more spread out downstream (figures 7.4.a and 7.4.b).

With the spoiler positioned at $70\%c$, a full recirculating region (bubble) does not form because it is interrupted by the back-flow of trailing edge vortices. Hence, some spoiler vortices convecting past the trailing edge cannot move back upstream. Instead, they are cut-off from the main cluster and flow downstream i.e. reattachment of the reversed trailing edge flow occurs (for example, figures 7.1.d and 7.2.d). On the contrary, when the spoiler is positioned at $50\%c$, a full bubble is formed behind the spoiler, as shown in figures 7.5.a and 7.5.b, before there is any significant interaction between spoiler vortices and the trailing edge (non-reattachment). Therefore, vortex shedding at the trailing edge remains smooth for a much longer time (figure 7.8).

In the following sections the effects of vortex shedding on pressures and forces are discussed.

7.1.2 Pressure distribution.

The pressure distribution over the aerofoil and spoiler may be calculated by considering the effect of the free stream and vortices on the body. Here, the unsteady form of Bernoulli's equation is employed to calculate the pressure distribution over the aerofoil and spoiler, as discussed earlier.

The impulsively started flow over the aerofoil/spoiler combination results in the generation of a strong positive vortex behind the spoiler, which grows and slowly moves downstream over the upper surface of the aerofoil. Figure 7.11 shows the instantaneous pressure distribution over the aerofoil for Test D (spoiler at $50\%c$) at different times, as the flow progresses. It can be seen that there is a stagnation point near the leading edge and at the upstream foot of the spoiler. Over the upper surface of the aerofoil behind the spoiler, there is a suction peak associated with the spoiler positive vortex cluster. Initially, the suction peak is immediately behind the spoiler. As the positive cluster elongates, its centre moves downstream and so does the suction peak

over the upper surface of the aerofoil (figure 7.11, 7.14), while the suction region spreads to the trailing edge. At the same time, back-flow at the trailing edge has started. Hence, as negative vortices from the trailing edge are convected upstream over the aerofoil surface, they can get very close to the surface and induce high velocities there, causing fluctuations (oscillations) in the pressure distribution near the trailing edge (figure 7.12). A similar effect (figure 7.12) is caused by positive vortices of the lower part of the spoiler vortex cluster, as they convect back towards the spoiler over the aerofoil surface (figure 7.5.a).

If the spoiler is at 70%c, interaction between spoiler and trailing edge vortices starts earlier. Hence, although initially the pressure distribution is similar to that with the spoiler at 50%c, after a short time, the negative vortex cluster forming over the aerofoil surface near the trailing edge, causes large pressure fluctuations locally (figures 7.16,17,18). The pressure distributions at $TU_{\infty}/c=0.868$ in figures 7.16 and 7.17 corresponding to the wake formations in figures 7.1.d and 7.2.d respectively, show a drop in suction approximately half way between the spoiler root and the trailing edge. This is caused by negative vortices convecting upstream very close to the upper surface of the aerofoil (see figure 7.1.d, 7.2.d).

The spoiler angle determines the strength of the vortices that leave the spoiler tip. Therefore, the higher the spoiler angle, the stronger the spoiler vortices and hence, the higher the suction peak over the surface of the aerofoil behind the spoiler. This is demonstrated by figures 7.11 and 7.12 (which show the spoiler at 50%c and at 90° and 45° to the surface, respectively) and also figures 7.16 and 7.20.

Figures 7.17 and 7.18 (corresponding to Test C and Test F) show the same effect of spoiler angle on pressure distribution, together with a suction peak on the upper surface of the aerofoil near the leading edge, due to the incidence of the aerofoil to the free stream.

As the flow develops, the pressure coefficient in the wake behind the spoiler tends to a steady value, while the suction near the leading edge drops. This

steady state is reached much later, when the spoiler is at 50% c than when it is at 70% c . Figure 7.19 shows the C_p distribution over the aerofoil at $TU_\infty/c=2.0$, and it can be seen that the suction peak near the leading edge has dropped, compared to figure 7.18. It is still, however, overestimated by the numerical model.

Experimental results for the mean C_p distribution obtained by PARKINSON and YEUNG (1987) over a Joukowski aerofoil are also shown in figure 7.19. The spoiler position and angle, in their experiment, is identical to Test F, but the aerofoil has 2.5% camber. Although the computation has not yet reached a steady state and the aerofoils are not identical, figure 7.19 shows good qualitative agreement between the experimental (mean C_p) and the numerical ($C_p(t)$) results in the wake region.

In all cases (for example figures 7.16 and 7.20) there is a compression region ahead of the spoiler, with its size (for a given spoiler length) increasing with increasing spoiler angle, as the flow has to turn through a larger angle at the root of the spoiler. The adverse pressure gradient associated with this compression region is the cause of the separation bubble found ahead of spoilers in practice. This result is in agreement with experiment (MACK et al (1979), CONSIGNY et al (1984) etc) and other numerical models (PARKINSON and YEUNG 1987).

$C_p(t)$ distributions over the spoiler at different angles and positions along the aerofoil are presented here (for example figure 7.15), where the origin of the axes is at the tip of the spoiler and the distance of the control points (i.e. where properties of the flow are calculated) is measured from the tip to either side of the roots of the spoiler. The distance from the tip to the upstream side of the root is taken as negative and to the downstream side of the root as positive.

The pressure distribution over the spoiler in all cases (see, for example, figure 7.15), follows a similar trend: it is near stagnation at the upstream side of the root of the spoiler and then it decreases and becomes negative near the spoiler tip. The back surface of the spoiler, is characterised by suction of nearly constant strength, for a particular time. As the flow develops further, the suction at the back of the spoiler drops,

and this is associated with the growth and convection of the positive vortex cluster behind the spoiler. The level of suction depends initially ($TU_{\infty}/c=0.325$, see figures 7.10, 7.13) on the spoiler angle. It is higher when the spoiler is at 90° than when it is at 45° to the surface of the aerofoil. This is associated with the strength of the 'core' vortex shed at the start.

The spoiler angle determines the acceleration of the flow over the spoiler's surface. Comparing figures 7.10 and 7.13, it can be seen that when the spoiler is at 90° the pressure drops very slowly with distance from the front root of the spoiler, and near the spoiler tip it drops rapidly and becomes negative. This process is much slower when the spoiler is at 45° (figure 7.13).

All C_p distributions over the spoiler presented here, indicate that there is a rapid change of the velocity field near the spoiler tip. However, prediction of this velocity, and hence pressure, at the tip or at points extremely close to it is not very accurate, due to very large values of the $1/(dz/d\zeta)$ and $1/|dz/d\zeta|^2$ terms of the transformation at these points, and C_p values there are $O(10^4)$ near the tip and $O(10^{10})$ at the tip. Therefore, these terms are evaluated at the spoiler tip (say, j) and two control points on either side of the spoiler tip ($j \pm 1, j \pm 2$), by linearly extrapolating the value of $(dz/d\zeta)$ and $|dz/d\zeta|^2$ from the control points ($j \pm 3, j \pm 4$). This does improve the C_p distribution near the tip. However, the contribution of the spoiler surface near the tip to Cl is not significant, since the distance over which this interpolation is used is very small i.e. $\approx 5\%c$.

7.1.3 Lift and Drag coefficients

Forces have been calculated by two different methods: surface pressure integration and a momentum method. By integrating pressures over the aerofoil and spoiler, the forces can be obtained separately on the aerofoil and spoiler. This is important since the load which the spoiler carries is of interest and can not be provided by the Momentum formula. It was thought important to use both surface pressure

integration and the Momentum method, so that agreement of the two methods would indicate that either could be used, depending on which was more efficient computationally and what it was expected to calculate (i.e. pressures or only overall forces).

The lift variation with time is closely associated with the motion of the point vortices and the wake they form. Figure 7.21 shows the Cl variation with TU_{∞}/c for Test A, and results by pressure integration and the Momentum method are presented. It can be seen that there is a rapid increase in lift, due to the generation of a strong positive vortex behind the spoiler, which reaches a peak value before it starts to drop. During this stage, the Cl curve is smooth. The results of the two methods of calculating Cl differ initially. However, the two methods show increasing convergence with each other as peak lift is reached. It was found during initial tests that this difference was due to the sparse control point distribution on the downstream spoiler surface and on the aerofoil surface either side of the spoiler root. This would introduce errors in the velocity integration to calculate Φ and hence in the pressure integration in that region.

The initially smooth part of the curve (figure 7.21) is followed by fluctuations, which start as the positive spoiler vortex starts to affect the trailing edge by causing a disturbance in the continuous trailing edge shear layer. This is finally cut-off and starts rolling back over the upper surface of the aerofoil (it corresponds to the position of vortices shown in figure 7.1.c). As a negative vortex cluster with randomly moving vortices is formed at the trailing edge, fluctuations increase and the lift decreases. This is associated with the growth of the negative vortex at the trailing edge, which reduces the effect of the spoiler positive vortex on the aerofoil.

When the negative cluster has become strong enough, it breaks way and convects downstream. At this point, Cl has reached a minimum and is followed by an increase, which shows no large fluctuations due to the absence of a negative cluster near the trailing edge. While Cl increases again, a negative cluster begins to form and the same process is repeated, so that Cl oscillates with time (figure 7.21).

The initial increase in Cl in impulsively started separated flows due to the generation of a strong positive vortex has been predicted by Discrete Vortex Methods and Finite Difference Methods applied to stalled aerofoils. KATZ (1981) and BASUKI (1983) investigated such flows and found that there was an initial increase in lift due to the formation of a positive vortex cluster by the separated shear layer leaving the upper surface of the aerofoil near the leading edge.

The Momentum method calculates forces by evaluating the rate of change of position of vortices in the flowfield. Therefore, vortices at the trailing edge that get close to the surface and acquire large velocities under the influence of their images may experience large displacements over a time step, thus causing fluctuations in Cl . This could be avoided if an infinitely small time step could be used and the vortex paths were more accurately calculated near the surface of the aerofoil, in which case vortices would follow the true streamlines of the flow and would not cross the solid surface. Fluctuations in Cl are also caused by vortices coming close together. This is a consequence of the discretisation of the vortex sheet equations and shortest wavelength Kelvin-Helmholtz instability. Short wavelength perturbations are introduced, in an actual computation, by round-off errors and they may grow fast enough to destroy the calculation's accuracy (KRASNY (1986)). Once point vortices in the adjacent arms of a shear layer spiral become unstable, they may come close together and induce upon each other large velocities due to their singular velocity fields near their centres. This does not represent real flows in this region, which consists of vortex sheets diffusing into each other.

As far as fluctuations in Cl obtained by surface pressure integration are concerned, these can be caused by high velocities associated with the singular nature of point vortices that get very close to 'control' (i.e. measurement) points on the body surface, between the spoiler and the trailing edge. The buffer region employed here, as discussed in Chapter Four, is mainly to prevent vortices from convecting through the body and still permits them to come quite close to the surface. These fluctuations appear

as 'noise' of high frequency, but very low power, as opposed to vortex shedding, which is of much lower frequency.

Removal of the 'noise' element gives a clearer picture of the C_l variation with time, and it was decided to apply a three-point centre-weight time averaging scheme to 'smooth' the C_l curves, as shown in figure 7.22. Compared with the unsmoothed results in 7.21, in order to ensure that this scheme was not removing important information from the curve, part of the computation for Test A was repeated at half the time step and it was found that the two curves were identical after the 'noise' had been removed.

Figure 7.23 shows the C_d variation with time for Test A, calculated by the two methods mentioned above. There is an initial drop in C_d , followed by a very slow rate of decrease. A similar behaviour of C_d has been found by JAROCH (1986), who applied DVM to the unsteady flow past a flat plate normal to a long wake splitter plate. The initial disagreement between the two methods is more significant than found when calculating C_l (figure 7.21). This is due to the coarse distribution of control points near the root of the spoiler, which becomes coarser as the spoiler angle decreases. This effect, which was due to the transformation, was difficult to prevent and its effect on C_d is more pronounced because C_d is more sensitive to integration errors. However, the two methods seem to converge in Test A for $TU_\infty/c > 2.0$.

Fluctuations in C_d calculated by surface pressure integration are of high frequency, while the Momentum method gives a smooth C_d variation with time (figure 7.23). As in calculating C_l by integrating pressures, C_d is affected by vortices coming close to the body and inducing, due to their singular nature, large velocities which cause fluctuations in pressure.

In this section, the general behaviour of C_l and C_d with time has been discussed in relation to vortex convection and wake formation. The following sections deal with the effects of spoiler position, angle and aerofoil incidence on forces.

7.1.4 Effects of spoiler position on forces.

The spoiler position on the surface of the aerofoil influences the wake formation and hence the control response and aerodynamic effectiveness of the spoiler. It was found, in the early years of using spoilers on aircraft wings, that very forward locations were unsuitable for roll control because of unacceptable lag effects following roll commands. However, more aft locations ($>50\%c$) gave more reasonable response characteristics. In this work, computations were carried out for two spoiler positions; $50\%c$ and $70\%c$, measured from the leading edge of the aerofoil.

Figures 7.22 and 7.24 show the Cl variation with time for Test A and Test D respectively. It can be seen that maximum adverse lift coefficient, when the spoiler is at $50\%c$, is approximately 0.5 and is reached at $t'_a = TU_\infty/c = 1.3$. These values are 0.16 and 0.5 respectively, when the spoiler is at $70\%c$ (figure 7.22). Therefore, it takes more than twice as long for maximum adverse lift to be reached when the spoiler is at $50\%c$. Similar results are found for the two different spoiler positions with the spoiler at 45° , as shown in figures 7.25 and 7.26. In all cases tested here, maximum adverse lift is reached just before back-flow at the trailing edge becomes extensive enough to initiate fluctuations in Cl .

As discussed in section 7.1.2, when the spoiler is at $50\%c$ a large recirculating region is formed over the surface of the aerofoil, between the spoiler and the trailing edge (figure 7.5.a). Therefore, 50% of the aerofoil's upper surface is under additional suction due to the influence of this positive vortex cluster. However, when the spoiler is at $70\%c$, interaction with the trailing edge begins much earlier, and a full recirculating region has no time to form before it is interrupted by negative vortices from the trailing edge (figure 7.1.d). This interaction between the positive and negative vortex clusters determines the amplitude and frequency of oscillations in Cl (see figures 7.24, 7.25); i.e. with the spoiler at $50\%c$ the frequency is lower and the amplitude is higher than when the spoiler is at $70\%c$. The higher maximum adverse lift, when the spoiler is at $50\%c$, is also due to larger positive vortex cluster formed behind the spoiler. A

Strouhal number can be formed based on a cross-stream dimension of the spoiler tip above the trailing edge as $S_t = fd/U_\infty$, where $d = \beta c \sin \alpha + \varepsilon \cos \alpha + 0.1 c \sin(\delta - \alpha)$. Constants β and ε define the position of the spoiler from the trailing edge and the thickness of the aerofoil above the chord-line respectively. Therefore, $S_t = 0.21$ for Test A and $S_t = 0.19$ for Test D (i.e. when the spoiler is at 70%c and 50%c respectively, and the aerofoil at 0° incidence). So, the Strouhal number decreases as the spoiler is positioned further upstream on the aerofoil surface.

All test cases with the spoiler at 70%c, show that C_l performs oscillations of decreasing amplitude about a steady value, after TU_∞/c is about 4.0 (see, for example, figure 7.25). This is due to the faster interaction between spoiler and trailing edge vortices, when the spoiler is nearer the trailing edge. Also, the mixing of positive and negative trailing edge vortices behind the spoiler (figure 7.27.b) causes the damping of oscillations in C_l , since it weakens the overall effect of the spoiler vortex cluster. On the contrary, for the 50%c position, C_l is oscillating strongly for the whole length of the calculation ($TU_\infty/c = 5.5$). The computation in this case was carried out for 500 "vortex shedding" time steps (1000 vortices), above which computing cost was extremely high.

It is expected that for long enough computer runs, these oscillations will gradually decrease and a steady state will be reached when equal amounts (when averaged over time) of circulation are shed from the trailing edge and the spoiler tip. However, oscillatory vortex shedding will occur from the two separation points at all times. Figures 7.28 and 7.29 show the variation of $\sum d\Gamma/(dt.U_\infty^2)$ with time for Test A and Test B respectively, and it can be seen that the total vorticity shed oscillates (after $TU_\infty/c > 0.7$) about zero.

The variation of C_d with time for zero incidence, and its dependence on spoiler position is summarised in figure 7.30. It can be seen that all curves follow a similar trend, in the sense that initially there is a rapid drop in C_d , after which C_d asymptotes to a steady value. If the spoiler angle is kept constant, then it can be seen that

C_d has a higher starting value, when the spoiler is at 50%c and it also tends to a higher steady value, as the flow progresses. This is true for both spoiler angles tested here (figure 7.30) and it is due to the higher spoiler position on the aerofoil, which causes a much longer and slightly broader 'bubble' behind the spoiler, compared to that formed when the spoiler is nearer the trailing edge (see figures 7.1.e and 7.32.a,b).

7.1.5 Effects of spoiler angle on forces.

Figure 3.34 shows that steady final Cl decreases with increasing spoiler angle. Experimental measurements of steady state, time mean Cl vs δ by TOU and HANCOCK (1983) are also shown in figure 7.34, for a CLARK Y-14 aerofoil fitted with a 10%c spoiler at 6° incidence to the free stream. Although the numerical results obtained here cannot be compared directly because of the camber of the CLARK Y section, they follow a similar trend to that predicted by experiment.

Figures 7.22 and 7.25 show the Cl variation with time for Test A and Test B. It can be seen that time (t'_a) to maximum adverse lift due to the shorter distance from tip to trailing edge is shorter for a smaller spoiler angle. Also, maximum adverse lift is slightly higher for the larger spoiler angle (0.17 compared to 0.15).

Oscillations of Cl with time are of higher frequency and lower amplitude, when the spoiler is at a lower angle (figures 7.22 and 7.25). The corresponding S_t for $\delta=45^\circ$ and 90° is 0.33 and 0.21 respectively. Therefore, the Strouhal number decreases with increasing spoiler angle and this is in qualitative agreement with experimental measurements of S_t carried out by WENTZ and OSTOWARI (1981). This is due to the narrower wake caused by the spoiler at 45° (compare figures 7.1.e and 7.33.a,b) and the faster interaction between spoiler and trailing edge vortex clusters.

The influence of spoiler angle on C_d is shown in figure 7.30. If α and spoiler position are kept the same, then it can be seen that C_d is lower when the spoiler angle is lower. Obviously this happens since the overall frontal area of the

aerofoil/spoiler combination decreases (spoiler projection height, $h=l.\sin\delta$, where l is the length of the spoiler) as the spoiler angle decreases. The 'bubble' behind the spoiler at 45° is smaller in size than that of the spoiler at 90° , which results in a higher base pressure and hence lower drag.

7.1.6 Aerofoil incidence.

The first observation of increasing the aerofoil incidence relative to the free stream is that the overall level in C_l increases (compare, for example, figures 7.36 and 7.37), as shown in figures 7.39 and 7.40, where the variation of C_l with α is plotted. This is because the initial ($t < 0$) C_l on the aerofoil, for attached flow and with the spoiler retracted, is higher. This is shown in figures 7.39 and 7.40, where the variation of C_l with α is plotted.

Steady experimental results for C_l by TOU and HANCOCK (1983) and PARKINSON and YEUNG (1987) are also presented in figures 7.38 and 7.39. They carried out experiments on a CLARK Y-14 aerofoil and a cambered (2.5%) Joukowski aerofoil respectively, both fitted with a 10% c spoiler, located at 70% c on the upper surface. Figures 7.36 and 7.37 show how the unsteady lift computed by the present method tends to a steady value with time. Quantitative comparison between numerical and experimental results is not possible since both aerofoils involved in the experiments are cambered. Therefore, at zero incidence they both have a positive C_l , while the symmetric aerofoil employed in the numerical method has zero C_l . Consequently, the present model overestimates the lift coefficient. However, the variation of C_l with α is in good qualitative agreement with the experimental results, i.e. the C_l vs α curves follow similar trends with the experimental ones in figures 7.38 and 7.39.

The DVM is normally found to overestimate the forces experienced by a vortex shedding body, due to the tendency of the vortex clusters to form very close to the body, so that high velocities are induced on the solid surface. This is partly due to the singular nature of point vortices close to a control point. Also, due to insufficient

cancellation of vorticity, the vortex clusters are stronger than they are found to be in experiments (FAGE and JOHANSEN 1927), thus causing a large suction on the surface which results in high forces.

KIYA, ARIE and HARIGANE (1977) calculated the flow behind an inclined flat plate using DVM and found that the drag force, time-averaged velocity and root-mean-square values of the fluctuating velocities in the near wake were overestimated. KATZ (1981) and BASUKI (1983) calculated the flow over a stalled aerofoil using DVM. They also found that lift on the aerofoil was overestimated and pressures on the surface were unrealistic. Vortex decay was employed in both cases, in order to obtain results comparable to experiment. The loss of circulation of a vortex with time was introduced to represent the cancellation and diffusion of vorticity in the formation region of the rolled-up vortices.

In the present work, the effect of a strong positive cluster near the body surface is more pronounced when the spoiler is at 50%c (figure 7.5.a,b). This leads to a high adverse lift and high amplitude oscillations of Cl with time, as seen earlier. When the spoiler is at 70%c, the formation of a recirculation region behind the spoiler is interrupted by negative vortices which are entrained in this region, thus enhancing the mixing of vortices (figure 7.33.b). This is probably why reasonably good agreement was obtained for the pressure distribution, as seen in section 7.1.2. It was decided not to employ vortex decay in this work since it cannot be justified mathematically. Also, the use of decay would introduce an additional empirical input - the decay rate with time.

As the aerofoil incidence increases, the spoiler becomes less effective, as far as its unsteady performance in Cl is concerned. Comparing figures 7.22 and 7.35, it can be seen that the change in Cl from the start of the flow to maximum adverse Cl , ΔCl_a , is 0.36 when $\alpha=0^\circ$ and it is reduced to 0.30 when $\alpha=12^\circ$. Also, when $\delta=30^\circ$ figures 7.36 and 7.37 show a drop in ΔCl_a from 0.25 ($\alpha=6^\circ$) to 0.22 ($\alpha=12^\circ$). In the present model, the drop in ΔCl_a is small for the test cases examined. However, in reality this effect is more pronounced since as α increases, the boundary layer increases its

thickness near the spoiler root and over the lower part of the spoiler, so that less of the spoiler is effective (the flow is separated at the spoiler root and is controlled locally by a parameter δ^*/l , where δ^* is the boundary layer thickness and l is the spoiler length). Therefore, the spoiler becomes less efficient in 'spoiling' the lift. If α is increased even more so that stalling occurs, then the spoiler becomes ineffective and may even cause control reversal (MACK et al (1979)). The numerical model, although it assumes that the boundary layer over the aerofoil is extremely thin and does not model separation from the 'nose' of the aerofoil, does predict a drop in ΔC_{l_a} with increasing α . This is because the projected height of the spoiler normal to the free stream is reduced as the aerofoil is set at incidence.

Figure 7.31 shows the variation of C_d with time, when the aerofoil is at incidence. It can be seen that for a fixed spoiler angle, the main effect that incidence has on the drag force is that C_d oscillations with time become stronger with incidence. This is because as α increases, the formation of a vortex cluster at the trailing edge is more orderly than it is when the aerofoil is at zero incidence (also discussed in section 7.1.1). Therefore, there is a stronger interaction between the spoiler and trailing edge vortices, which affects the pressure in the near wake behind the spoiler and causes oscillations in C_d .

Figures 7.40.a,b show the positions of positive and negative vortices in the near wake at $TU_\infty/c \approx 4.0$, together with the velocity field around the aerofoil and spoiler. It can be seen that the negative vortex cluster is quite strong and influences the spoiler vortices. Its growth, interaction with the spoiler vortices and convection causes the oscillations in C_d shown in figure 7.31. The frequency of oscillations in C_d is higher for lower spoiler angles (figure 7.31), due to the faster interaction between spoiler and trailing edge shear layers. This fast interaction is also represented by a rapid change in the strength of trailing edge vortices (figure 7.41), while $\sum d\Gamma/(dt.U_\infty^2)$ oscillates at a high frequency, with time, about zero (figure 7.42).

7.1.7 Lift on aerofoil and spoiler separately.

It was shown in section 7.1.3 that there was good agreement in C_l , obtained by the Momentum method and surface pressure integration. So far, all the force results that have been presented, refer to the aerofoil/spoiler combination. However, in many experimental results, the overall forces on the body have been obtained by integrating pressures measured in the wind tunnel on the surface of the aerofoil only. One advantage of the numerical method developed here is that forces may be obtained separately on the aerofoil and spoiler.

Figures 7.43 and 7.44 show the variation of the total lift and the lift separately on the aerofoil and spoiler for Test E and Test B respectively. It can be seen that the spoiler carries a negative lift force, which drops as the flow develops and tends to reach a steady negative value. This is expected since the spoiler experiences a force backwards and downwards towards the aerofoil surface. This contributes a negative lift. Consequently, the lift on the aerofoil only is higher than the total lift. The negative lift decreases as the flow develops. Therefore, if the contribution of the spoiler is not included, the lift is overestimated by about 10% and 15% for Test B and Test E respectively, at least in the initial stages of the flow.

When the aerofoil is at zero incidence and the spoiler at 90° to the surface, the lift on the spoiler is obviously zero or extremely small, and its inclusion in the calculation is not significant. However, once the aerofoil with a 90° -spoiler is placed at incidence, then the spoiler contributes negative lift and there is initially a difference of about 10% (which depends on the value of α) between the total lift and the lift on the aerofoil, as shown in figure 7.45.

It may be concluded that, in experiments on spoilers, where the flow has reached a steady state before measurements are taken, lift forces are overestimated if the spoiler contribution is not included. This is more significant when forces are measured during the initial stages of the impulsive flow and before the flow has reached a steady state.

7.2 The moving spoiler - test cases.

Published experimental results on pressures and forces for a rapidly moving spoiler are very limited, probably due to the high level of complexity involved in measuring transient properties of the flow.

An experimental investigation on transient loads and pressures on an aerofoil fitted with a moving, 10%c long spoiler has been carried out recently by KALLIGAS (1986), using a blower tunnel at Bristol University, and an effort has been made here to compare the numerical results with some of his measurements. However, and as often is the case, the numerical results obtained here cannot be quantitatively compared with the experiment for many different reasons. The most important reason is that in all the test cases of the above experimental programme, the blower tunnel was started and a steady flow was established around the aerofoil and spoiler, before the spoiler was deployed from a fixed initial angle. It is possible to compute such a case with the present numerical method, but as was seen earlier, a very long computation is needed before the flow around the aerofoil/spoiler reaches a steady state. It is therefore due to computational cost that the numerical calculation is started only as soon as the spoiler is activated.

Another important reason is that the experimental results available for comparison here, have been obtained either with spoiler perforations or a hinge gap present, or both. These are features that the numerical model does not include, making a direct comparison of results even more difficult.

A long run ($TU_{\infty}/c=2.7$) was carried out here (Test M), to enable as close as possible a comparison with some of the experimental results mentioned above. Although the computation was not long enough for the spoiler to reach the final position obtained in the experiment nor for a new mean lift to be established, it was found that maximum adverse lift was reached well within that time. That was satisfactory, since the transient effect of spoiler deployment on the lift force is of prime importance in this

study. Shorter computations were performed at different spoiler deployment rates, to investigate the delay times for transient spoiler response and compare them with experimental results (see section 7.2.4).

For all these test runs, for which forces were calculated, the aerofoil was at zero incidence and the spoiler positioned at 70% chord. The spoiler was deployed at a steady rate from an initial angle (δ_0) of 10° to a final deflection (δ_f) of 50° . Since a smaller time step is needed when the spoiler starts from a small angle and moves at a high rate (see section 7.2.2), three different time steps were used here, as seen below. During the motion of the spoiler, vortices are released at every time step i.e. $\Delta t_i = \Delta t$. However, for $u_t/U_\infty > 0.18$ the spoiler reaches its final angle early in the computation and is then treated as a 'fixed' spoiler. Therefore, Δt is increased gradually to 0.02 and vortices are released at every other time step i.e. $\Delta t_i = 2\Delta t$.

The test cases are summarised as follows:

Test M:	$u_t/U_\infty = 0.0174$	$\Delta t = 0.020$.
Test P:	$u_t/U_\infty = 0.0300$	$\Delta t = 0.020$.
Test Q:	$u_t/U_\infty = 0.1900$	$\Delta t = 0.010$.
Test R:	$u_t/U_\infty = 0.3700$	$\Delta t = 0.008$.
Test L:	$u_t/U_\infty = 0.7400$	$\Delta t = 0.008$.

7.2.1 Vortex shedding.

For the moving spoiler, the choice of time step is more complicated than it is when the spoiler suddenly appears on the aerofoil at $t=0$, since it depends on spoiler angle and deployment rate.

When the spoiler is activated from a very small angle to the surface of the aerofoil, due to a sudden inrush of fluid into the expanding region between the spoiler and the aerofoil surface vortices have high convection velocities, so that a very small time step is required to convect them as closely as possible along their true paths.

As the spoiler angle increases, vortices in the gap between the spoiler and the aerofoil surface have lower convection velocities, so that the time step can be reasonably increased. CHENG and EDWARDS (1982) and ZANDJANI (1983), who modelled the Weis-Fogh lift-generation mechanism, had to use very small time steps during the initial opening of their flat plate from small angles to ensure stability.

However, time step size and angular velocity are interrelated. For example, if the spoiler is opening at a fast rate, then a small time step must be employed so that the spoiler's motion is as continuous as possible (i.e. it moves through small angle increments).

The Brown and Michael equations described in Chapter Three are employed to shed vortices from the moving spoiler tip and the trailing edge. Due to the difficulties present in modelling the opening of the spoiler from small angles (see Chapter Five), a growing 'core' vortex is shed initially as the spoiler is deployed, and is allowed to convect as the spoiler starts to move.

Figures 7.46.a to 7.46.d show the roll-up of the vortex sheet separating from the tip of the moving spoiler at 10° intervals (starting from an angle of 40°). The spoiler is deployed so that $u_t/U_\infty=0.19$.

Figure 7.46.a shows the position of the spoiler vortex in the initial stages of the motion. This vortex would be expected, physically, to be found nearer to the 'entrance' of the gap between the spoiler tip and the aerofoil surface, since the initial inrush of flow draws the generated vorticity inside the gap, some of it being destroyed by diffusive action (LIGHTHILL (1973)). The position of the initial vortex is found to be further away from the 'entrance', and this is mainly caused by the inability of the present method to model accurately the 'opening up' of the spoiler in the initial stages.

Also, the spoiler is curved away from the aerofoil surface, for small spoiler angles, due to the conformal transformation and the fact that the spoiler is assumed to be straight in the z_5 plane. However, the Brown and Michael equations for the starting flow give the position z_0 of the 'core' vortex (i.e. the vortex which has

grown for a few time steps) perpendicular to the spoiler at the tip, provided that the vortex is much nearer to the tip than it is to the aerofoil surface. Therefore, the position of the 'core' vortex is bound to be further away from the gap for a curved spoiler, than it would be if the spoiler were straight, as shown schematically in figure 7.61 (this effect, however, is very small).

As the spoiler motion continues, the regular vortex roll-up breaks down due to point vortices in successive arms of the shear layer coming close together, as it rolls-up away from the spoiler tip. If the calculation is carried on further, as in Test M (figures 7.49.a,b), the spoiler vortex interacts with the trailing edge shear layer in a way similar to the 'fixed' spoiler examples discussed earlier.

In an effort to stop the randomisation of the rolling-up of the shear layer in his modelling of the Weis-Fogh mechanism of lift generation, ZANDJANI (1983) released vortices from the tip of the flat plate at 5° intervals of its motion, and employed different techniques to amalgamate point vortices of the inner part of the rolling-up sheet with the 'core' vortex. One of them was gradual absorption of vortices of the shear layer with the 'core' vortex during a time interval, satisfying at all times the Brown and Michael equations. Straight forward amalgamation of point vortices with the 'core' vortex was applied here during initial tests, and did help the initial roll-up for small spoiler starting angles. However, the conditions under which amalgamation was applied (for example, the time at which amalgamation was started) tended to vary with spoiler starting angle, time step and rate of deployment. Finally, it was decided not to employ amalgamation since it did not greatly improve the modelling of the flow during the 'opening-up' stage, and involved some form of empiricism.

7.2.2 Pressure Distribution.

Calculating the pressure distribution over the aerofoil and moving spoiler proved to be a very complicated and difficult task, mainly due to the continuous shifting of the position of the control points on the aerofoil and spoiler surface.

Initial tests showed that the accuracy of calculating pressure over the aerofoil, was severely affected by errors in integrating surface velocity, a) due to the distribution and shifting of control points and b) due to vortices convecting very close to the surface of the aerofoil and the trailing edge, for small spoiler angles:

a) When the spoiler moves, the control points on the surface change position from one time step to the other. Therefore, in order to calculate $d\Phi/dt$, Φ at time step i had to be linearly interpolated on to the control points at time step $i-1$, in the aerofoil plane. To achieve this, a continuous search was needed to locate the control points at i relative to the control points at $i-1$. This operation slowed down the program considerably and had to be carried out carefully, especially in areas near the spoiler root and tip as well as the trailing edge.

Although the interpolation was applied correctly, for small spoiler angles the distribution of control points on the spoiler's back surface and on the surface of the aerofoil immediately behind the spoiler root is very poor, since the transformation tends to concentrate the control points on the spoiler (corresponding to evenly distributed points in the circle plane), near the tip. At the same time, the sink strength representing spoiler motion becomes extremely large in the circle plane towards the position of the downstream root of the spoiler. It is possible that the linear interpolation of Φ in that region is not adequate. This is probably one of the causes of large fluctuations in pressure found during initial tests, especially for large u_t/U_∞ values when a smaller time step was used.

The error in linear interpolation is of order $\Delta s^2/l^2$ (where Δs is the distance between two control points). If for example $u_t/U_\infty=0.37$ and Φ is needed to be 0.1% accurate, then $\Delta s \ll 1 \times O(\Delta t)^{1/2}$, where a typical time step value for the moving spoiler in the initial opening stage is 0.008. Consequently, at least 30 points are needed on each side of the spoiler and the distance between two adjacent points must not be larger than 1/30th of the spoiler length.

b) When the spoiler is deployed from a small angle and at a low angular velocity (Test M), the gap between the spoiler and the aerofoil surface remains small (angle δ less than 30°) during the calculation, since the spoiler moves slowly. Therefore, the shear layer separating from the spoiler tip remains very close to the body surface, in some cases (for example in Test M) throughout the motion of the spoiler. Also, vortices from the trailing edge convect upstream very close to the surface of the aerofoil, once interaction between the spoiler vortex and trailing edge shear layer has started. As a result, very large velocities are induced on the surface of the aerofoil by these vortices, contributing to the errors in integrating surface velocities discussed above.

However, vortices do not get very close to the back surface of the spoiler and initial tests showed that the pressure distribution obtained on it is similar to the 'fixed' spoiler case.

If the deployment rate is high, so that the spoiler moves away from the aerofoil surface and comes to rest at a relatively large angle ($>30^\circ$) in a short time, then since the onset of adverse lift begins after the spoiler has come to rest (see section 7.2.4), the difficulties discussed above are eliminated, and pressure distributions are similar to those obtained for a 'fixed' spoiler. This is because after the spoiler has come to rest the control points stop changing position, and the gap between the spoiler and the aerofoil surface is large.

7.2.3 Lift coefficients on aerofoil and spoiler.

As discussed in the above section, pressures calculated on the aerofoil surface were fluctuating strongly. Lift forces obtained by surface pressure integration were very much higher than those predicted by the Momentum method, especially during the initial stages of the spoiler motion. It was, therefore, decided to use the Momentum method to calculate the total force on the body. However, it is important (in general) to be able to calculate the force on the aerofoil and spoiler separately, first for comparison

reasons, since in many experiments forces are calculated by integrating pressures on the aerofoil surface only, and second for design purposes, since the aerodynamic loads on the spoiler would determine its structural stiffness and dictate the gearing system. Here, forces on spoiler and aerofoil can be evaluated separately by integrating the pressure over the spoiler surface to calculate the spoiler C_l , and then subtracting it from the total C_l to obtain the lift coefficient over the aerofoil only (figure 7.50). Therefore, the C_l variation with time over the aerofoil only can be compared with experimental results obtained by KALLIGAS (1986).

Figure 7.48 shows the variation of C_l with time for Test M, where the spoiler is displaced linearly as shown in figure 7.47. It can be seen that there is a rapid increase in C_l to a peak value caused by the positive vortex cluster forming behind the spoiler. This is followed by a drop in C_l . As the flow develops further and interaction between the spoiler and trailing edge vortices is initiated, there is a second peak in C_l at $TU_\infty/c=1.0$, corresponding to the shedding of a negative vortex cluster from the trailing edge (this has been explained for the 'fixed' spoiler in section 7.1.1). A smooth variation of C_l with time is followed by increasing fluctuations in C_l (figure 7.48). This is mainly due to the increasing number of vortices in the near wake, which results in increasing close interactions between them leading to some instability, and also between vortices and the aerofoil surface, as vortices from the spoiler tip and the trailing edge convect very near the body. Both these phenomena are more intense here, because the spoiler starts at a small angle to the surface and moves at a low angular velocity ($u_t/U_\infty=0.0174$), hence limiting the space in which vortices can move freely.

The spoiler is not greatly affected by vortices close to its surface, except near the spoiler tip (figure 7.49.b). It can be seen in figure 7.50 that C_l on the spoiler (calculated by surface pressure integration) starts at a negative value due to suction caused by the initial roll-up of positive point vortices. This is followed by a gradual increase in C_l , as the positive vortex cluster grows and slowly moves away from the spoiler tip. The final value of C_l on the spoiler depends on the steady angle that the

spoiler attains.

The Cl vs time results calculated in Test M are compared with a set of experimental results obtained by KALLIGAS (1986). During the experiment the spoiler was started from 10° , after the flow had reached a steady state around the aerofoil and spoiler, and came to rest at an angle of 50° . Also, there was a gap at the hinge of the spoiler, which normally reduces the adverse effect of spoiler deployment on Cl .

Figure 7.48 shows the Cl variation with time obtained from the numerical method and the experiment. The numerical Cl is the total Cl over the aerofoil and spoiler, while the experimental Cl is that obtained by integrating pressures on the aerofoil only. Therefore, the two curves cannot be compared directly. However, they both show an increase in Cl to a peak, followed by a drop in Cl . Figure 7.51 shows the numerical total Cl and Cl on the aerofoil only, as well as the experimental result. As far as the two numerical results are concerned, there is a difference until maximum adverse lift is reached. This difference is reduced as the flow develops further. Comparing the experimental Cl with the numerical Cl on the aerofoil only, the first observation is that the levels are much different. This is mainly due to the fact that in the experiment the flow has reached a steady state with the spoiler fixed at 10° and hence Cl on the aerofoil has a negative steady value, before the spoiler is deployed. On the other hand, in the numerical model the spoiler is deployed as soon as the flow is impulsively started. Therefore, Cl is expected to include this effect. Also, the high suction on the spoiler and the surface of the aerofoil (under the spoiler tip), due to the 'starting' vortex generated by the combined effect of the impulsive incident flow and the motion of the spoiler, contributes to the high initial Cl on the aerofoil. However, the incremental increase in Cl from the start of the calculation to maximum adverse lift (ΔCl_a) predicted by the numerical method, is in good agreement with that predicted experimentally i.e. $\Delta Cl_a = 0.26$ and 0.25 respectively.

7.2.4 Effects of spoiler deployment rate on delay times for transient response.

Two delay times are defined here:

t_a : the time to maximum adverse lift

t_o : the time to onset of lift change

Also, T_o is defined as the time to final spoiler deflection. MABEY et al (1982) found that a good correlation with the parameter $T_o U_\infty / c$ was obtained with both non-dimensional time delays. This parameter was also used by KALLIGAS (1986) and was employed here, to compare numerical results with experiment.

The Cl and spoiler angle variation with time, for different spoiler deployment rates, are shown in figures 7.52 to 7.59. From these results, t_a and t_o can be obtained and they are plotted against $T_o U_\infty / c$ in figure 7.60.

All Cl vs time curves show that the rapid spoiler extension is dominated by the formation of a starting vortex, which causes an increase in Cl followed by a rapid fall in Cl. It is important to note that this is a dynamic effect which is quite different from the static characteristics.

For large values of T_o , i.e. low deployment rates, the onset of adverse lift is very short, while maximum adverse lift is reached in the early stages of the spoiler motion (figure 7.48 and 7.53). Final lift is reached shortly after the spoiler has stopped. This is not very clear from figure 7.48, since the spoiler has not yet stopped, but it can be seen that Cl falls, as the spoiler moves to its final position. These results are in agreement with experimental observations by MABEY et al (1982).

For higher deployment rates the delay times t_o/T_o for the start of the lift change are appreciably longer (for example figure 7.57). This indicates that the separation immediately behind the spoiler is delayed until higher spoiler deflections are reached (MABEY et al (1982)). Also, the time delay for adverse lift increases for higher

deployment rates, and this is due to the delay of the extension of the flow separation to the trailing edge, as was found to be the case by KALLIGAS (1986).

Time to Cl_{max} is closely related to the generation and growth of the vortex behind the spoiler. For all deployment rates, Cl_{max} is reached when the vortex behind the spoiler has reached its maximum strength and just before backflow from the trailing edge begins to largely reduce the suction on the aerofoil surface, caused by the positive spoiler vortex. Time to Cl_{max} is also associated with the position of the vortex relative to the trailing edge. For slow deployment rates, the spoiler vortex convects faster towards the trailing edge, but for fast deployment rates this convection is delayed and the spoiler vortex stays close to the spoiler tip for a longer time. Figure 7.57' shows the position of the spoiler vortex (and its interaction with the trailing edge shear layer) for different times during the motion of the spoiler, shown in figure 7.57 at A,B,C and D. It can be seen that at A and B the spoiler vortex is tightly wound near the spoiler tip. At C, Cl_{max} has been reached and the centre of the spoiler vortex is seen to be convecting near the trailing edge. For slow deployment rates (or even for zero deployment rates, as seen for the 'fixed' spoiler case), the spoiler vortex starts to convect towards the trailing edge from the beginning of its formation.

Figure 7.60 summarises the delay times for different spoiler deployment rates showing a comparison with some experimental results. Good agreement is found between the numerical and experimental results, although the numerical method does not exactly model all the experimental conditions, i.e. the gap at the spoiler hinge and the steady flow conditions in the tunnel before the spoiler is deployed. In the limit of $T_0 \rightarrow \infty$ the static and dynamic calculations are identical. On the contrary, as $T_0 \rightarrow 0$, the increase in lift begins as soon as the spoiler has reached its final angle, i.e. $t_0/T_0 = 1.0$. The advantage of the numerical model is obvious in investigating the time delays for $T_0 U_\infty / c < 1.0$, since this region corresponds to very high spoiler deployment rates, which are extremely difficult to achieve experimentally or indeed on an aircraft.

CHAPTER EIGHT

CONCLUSIONS AND RECOMMENDATIONS

A study into the application of the Discrete Vortex Method to model numerically the separated flow past an aerofoil fitted with a 'fixed' and a moving spoiler has been presented. The steady and unsteady spoiler characteristics have been investigated using the numerical method, while forces and surface pressures have also been calculated.

The Discrete Vortex Method was applied to model the formation and shedding of vortices from the spoiler tip and the aerofoil trailing edge. In order to keep the use of empirical parameters to a minimum, the Biot-Savart method was employed, instead of a mesh method (e.g. the Cloud-in-Cell), which ensured that the model was mesh-size independent, but also meant that long computations were expensive.

Criteria for comparison with experimental results involved the calculation of lift and drag forces on the body and also surface pressures. For the moving spoiler case delay times for transient response were calculated. The numerical code was initially developed for the 'fixed' spoiler case and was then modified to take into account the opening of the spoiler. Direct comparison with experimental results was complicated by the fact that different aerofoils were used in the experiments, and a hinge-gap and sometimes surface porosity were present. The numerical model did not incorporate these features.

'Fixed' spoiler.

The numerical method predicted correctly the variation of C_l with time observed experimentally, i.e. an initial increase in lift to a peak value followed by a drop in C_l to a steady value. Since the spoiler suddenly appeared at its final position in the

beginning of the impulsive flow, the increase in adverse lift was initiated right from the start of the flow. Comparison of the calculated steady Cl with experimental results indicated that the numerical method overestimated the force coefficients.

The variation of Cl with aerofoil incidence was found to be linear, for the range examined, and showed good qualitative agreement with experimental results and results obtained by Wake-Source models. Good qualitative agreement was also found for the variation of Cl with spoiler angle, which drops with increasing spoiler angle.

By integrating pressures over the aerofoil and spoiler it has been possible to calculate the force experienced by the spoiler and the aerofoil separately. This is an advantage of the numerical method over experimental methods, where the total force on the aerofoil/spoiler combination is sometimes estimated by integrating measured pressures on the aerofoil surface only. It was found that forces on the aerofoil are overestimated if the spoiler contribution is not included.

The pressure distribution over the aerofoil was found to be in good agreement with experimental results, but it was overestimated near the upper surface of the leading edge.

Moving spoiler.

The motion of the spoiler was modelled using a source/sink distribution along its surface. Body forces and surface pressures were calculated, as well as time delays for transient response. The opening of the spoiler from a zero angle to the surface was not modelled successfully, due to high velocity gradients in the gap between the spoiler and the aerofoil surface. These gradients represent the inrush of fluid into the gap during the initial opening of the spoiler, observed in experiments.

The pressure distribution calculated over the aerofoil was found to be unrealistically high due to vortices being very near the aerofoil surface, during the beginning of the spoiler's actuation, and also due to the continuous change of the

position of the aerofoil and spoiler surface control points with spoiler angle.

It has been possible to calculate the Cl response using a Momentum method, and hence to calculate the delay times for the initiation of adverse lift increase and maximum adverse lift.

In the early stages of spoiler extension the lift increases, owing to a strong positive vortex forming immediately behind the spoiler. This adverse lift would increase the gust loads, which the spoiler extension is intended to eliminate. For rapid enough spoiler deployment rates the onset of adverse lift occurs as soon as the spoiler has stopped moving, and maximum adverse lift is reached shortly afterwards. For low deployment rates, the onset of adverse lift is very short and maximum adverse lift is reached in the early stages of the spoiler motion, with final lift being reached shortly after the spoiler has stopped. This is in agreement with experimental observations.

Comparison of delay times for transient response obtained from the numerical model shows very good agreement with experimental results. This indicates that the present inability of the method to model correctly the opening-up of the spoiler in the initial stages does not greatly affect the results. The present model can be used to calculate delay times for transient response at spoiler deployment rates which are difficult to be achieved experimentally.

In general, the model has shown that it is possible to simulate the separated flow past an aerofoil/spoiler combination with the spoiler suddenly appearing on the surface of the aerofoil at the start of the calculation, or moving. However, it must be kept in mind that vorticity in real vortices is not concentrated in points, as the numerical model suggests. The vorticity in reality occurs in thin diffusing sheets. Vorticity diffuses and is swept across the wake, while being dissipated by turbulence. Vortices in a real flow are subjected to strain fields imposed by nearby vortices, and the resulting patterns are of continuously changing geometry. The complexity of the interaction between strained and distorting vortices is complicated further by the addition

of turbulence to the wake. This is likely to produce a more diffusive vorticity distribution and thus an additional shear field. In this present model the only reduction of vorticity is due to mixing of vortices of opposite sign, and this is a reason for overestimating the body forces. Therefore, any numerical method on its own would give an inadequate account of what really happens. However, the present model offers a good qualitative account of the separated flow past 'fixed' and rapidly moving spoilers on aerofoils.

The model for the moving spoiler case could be improved by a local analysis of the flow during the opening-up stages. This would give an initially more realistic representation of the flow. An investigation into the more accurate calculation of $\partial\Phi/\partial t$ during the motion of the spoiler would give more reliable results for the surface pressure distribution. This could also be improved by introducing an iterative scheme to obtain a finer distribution of points over the downstream surface of the spoiler and on the aerofoil surface, either side of the spoiler root.

The program could easily be modified to investigate the effects of spoiler retraction on transient response. Also, by introducing Theodorsen's method, different aerofoil profiles could be tested.

If longer calculations are required, then the Cloud-in-Cell method would be more efficient to use, from the point of view of computing cost. For that purpose, the present program can generate a mesh of variable size very efficiently. This way vorticity diffusion in the flowfield could be implemented, which would introduce a finite Reynolds number and add to the realism of the model.

Finally, since the rapidly moving spoiler would find application in Active Control Technology, where high 'alpha' flight is essential, the model could be complicated even further by modelling the separation from the leading edge of the aerofoil, for high enough incidences.

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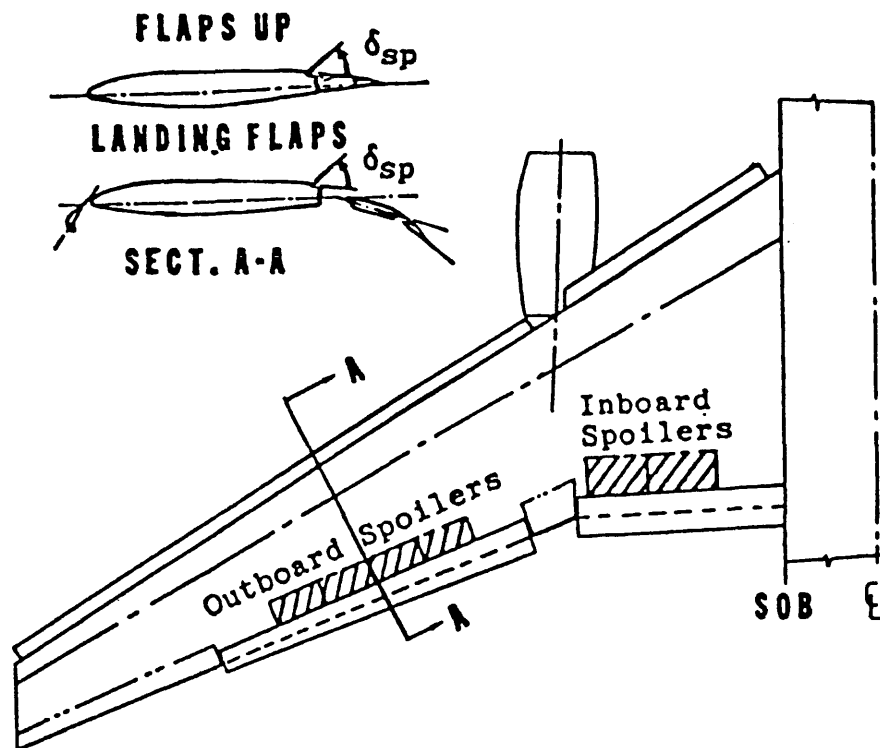


Figure 1.1: Typical Transport Airplane Spoiler Configuration.

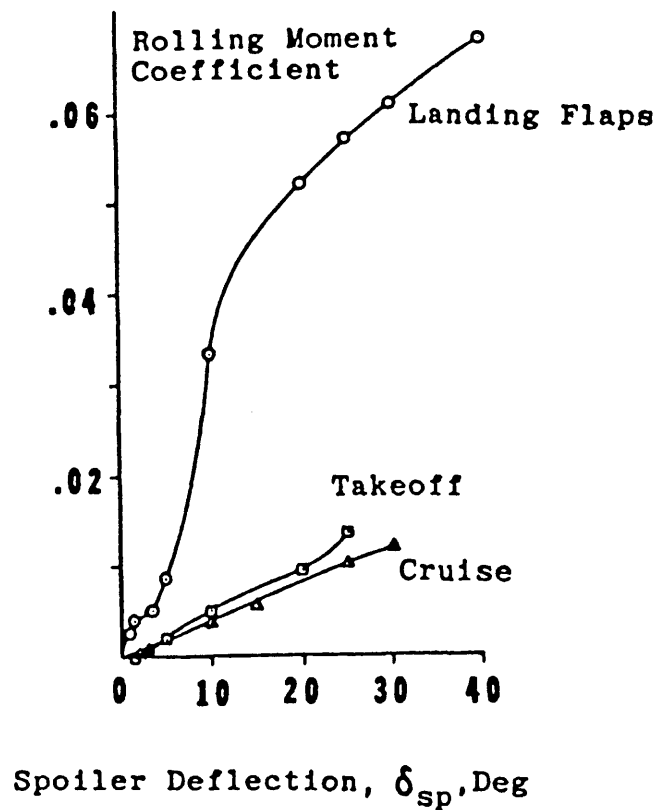


Figure 1.2: Typical Spoiler Effectiveness, Wind Tunnel; MACK et al (1979).

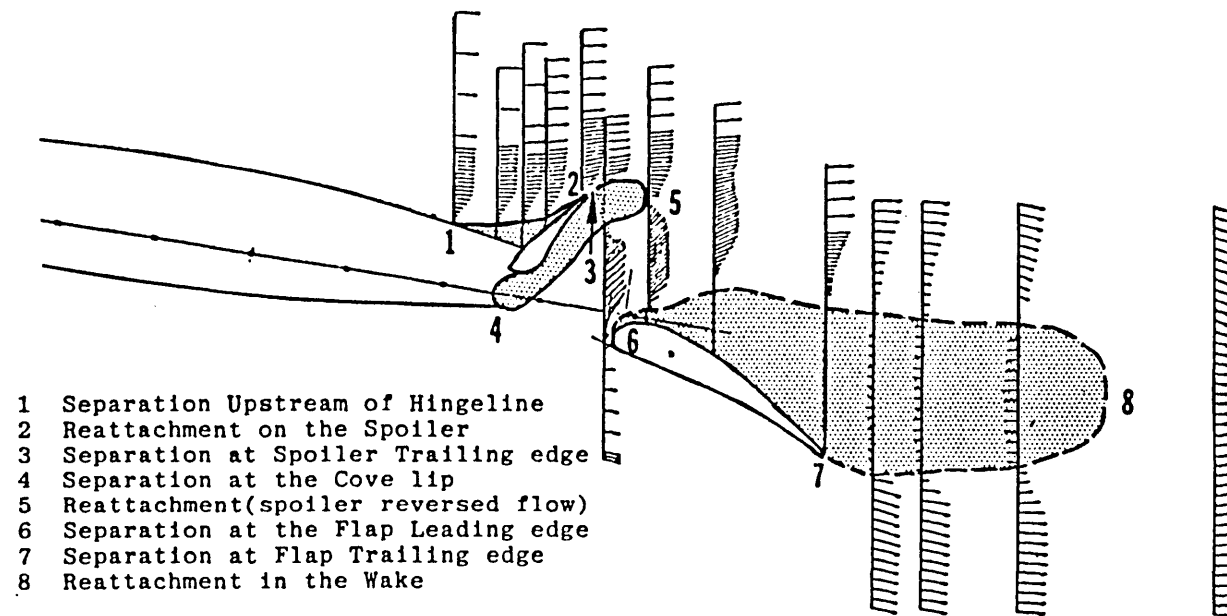
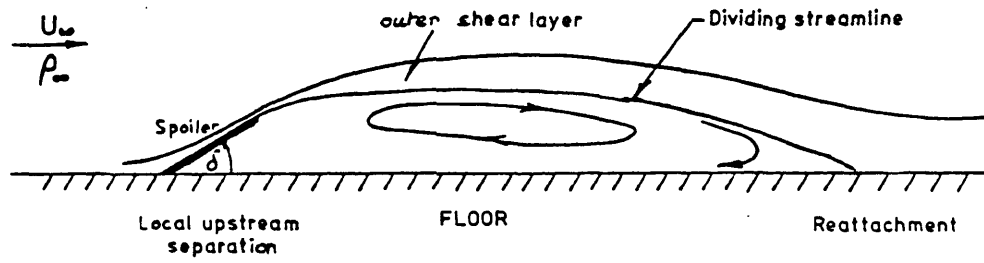
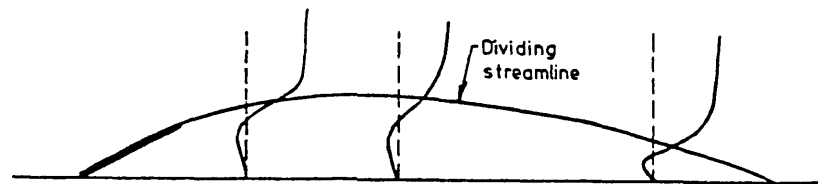


Figure 1.3: Separated Flow patterns: Flap down, deflected spoiler; MACK et al (1979).



a) GENERAL FLOW FIELD



b) MEAN VELOCITY PROFILES

Figure 1.4: Steady flow characteristics about spoiler on tunnel floor.

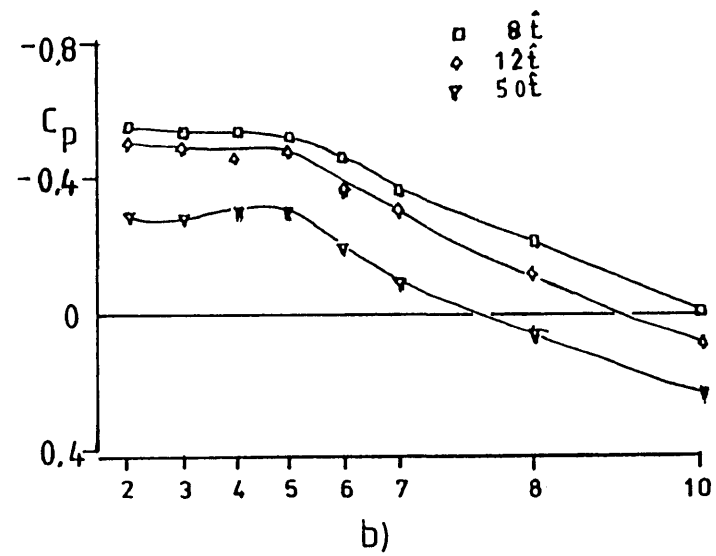
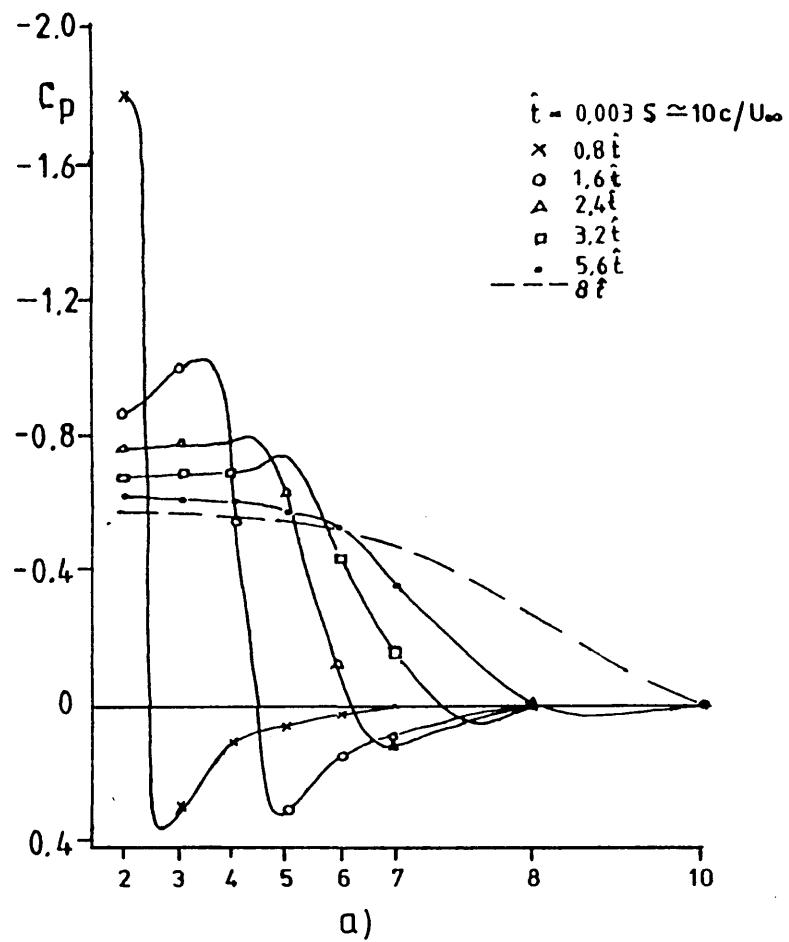


Figure 1.5: Unsteady pressures for moving spoiler from 0 deg. to 45 deg. in 0.003 sec. at $M_\infty = 0.26$; AHMED and HANCOCK (1983).

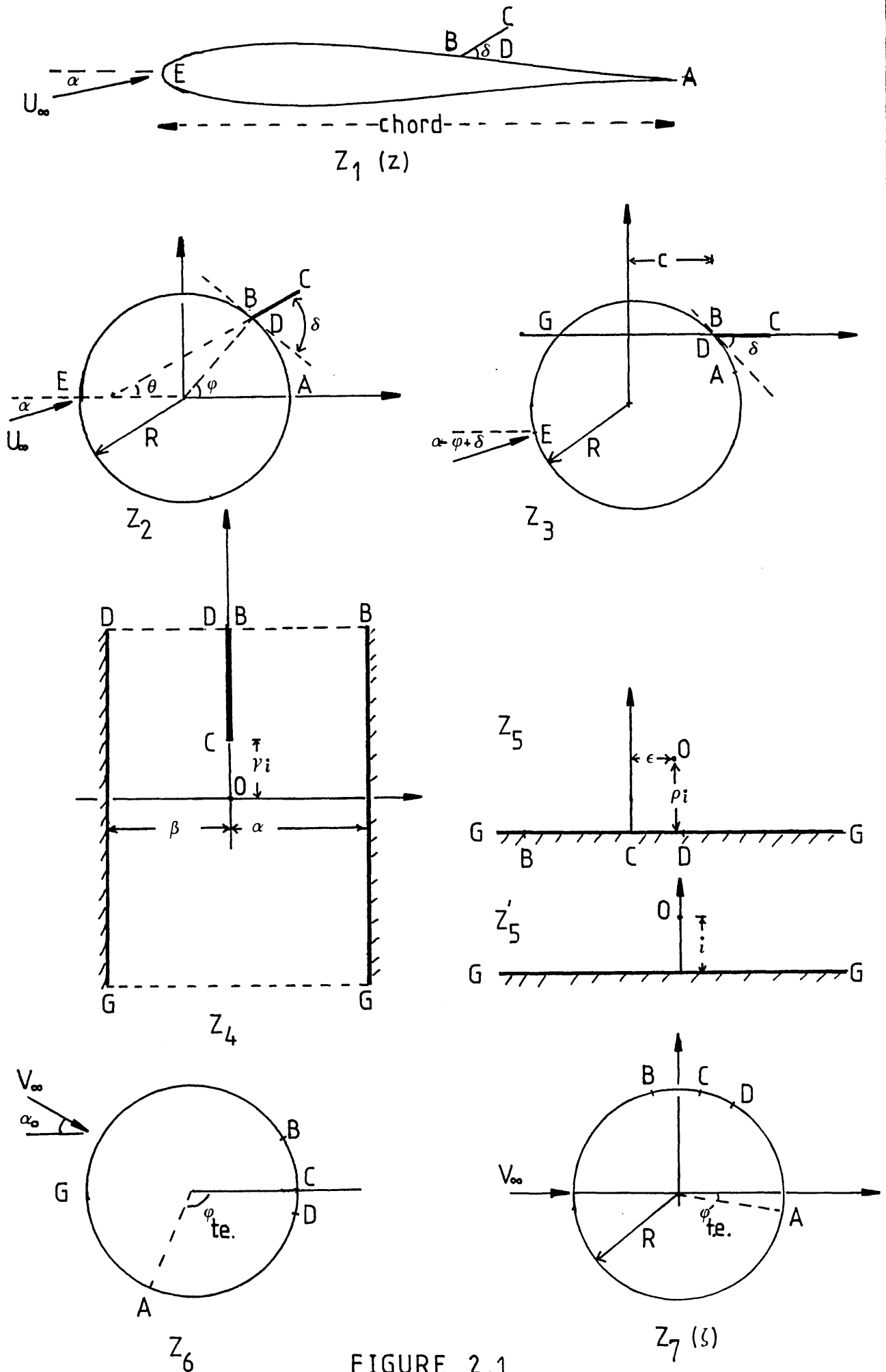


FIGURE 2.1

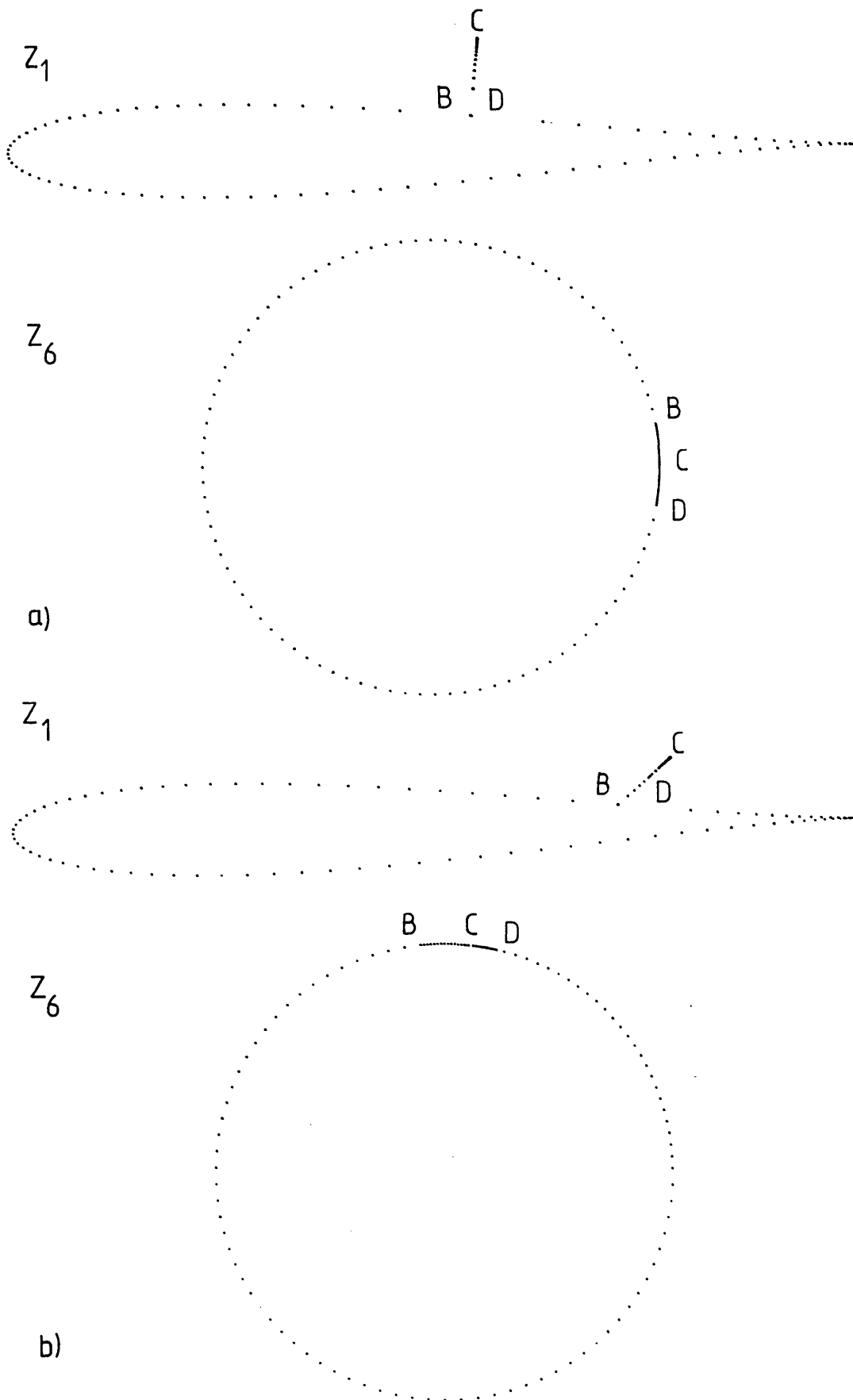


Figure 2.2: a) Spoiler at 90 deg. and 50% chord.
 b) Spoiler at 45 deg. and 70% chord.

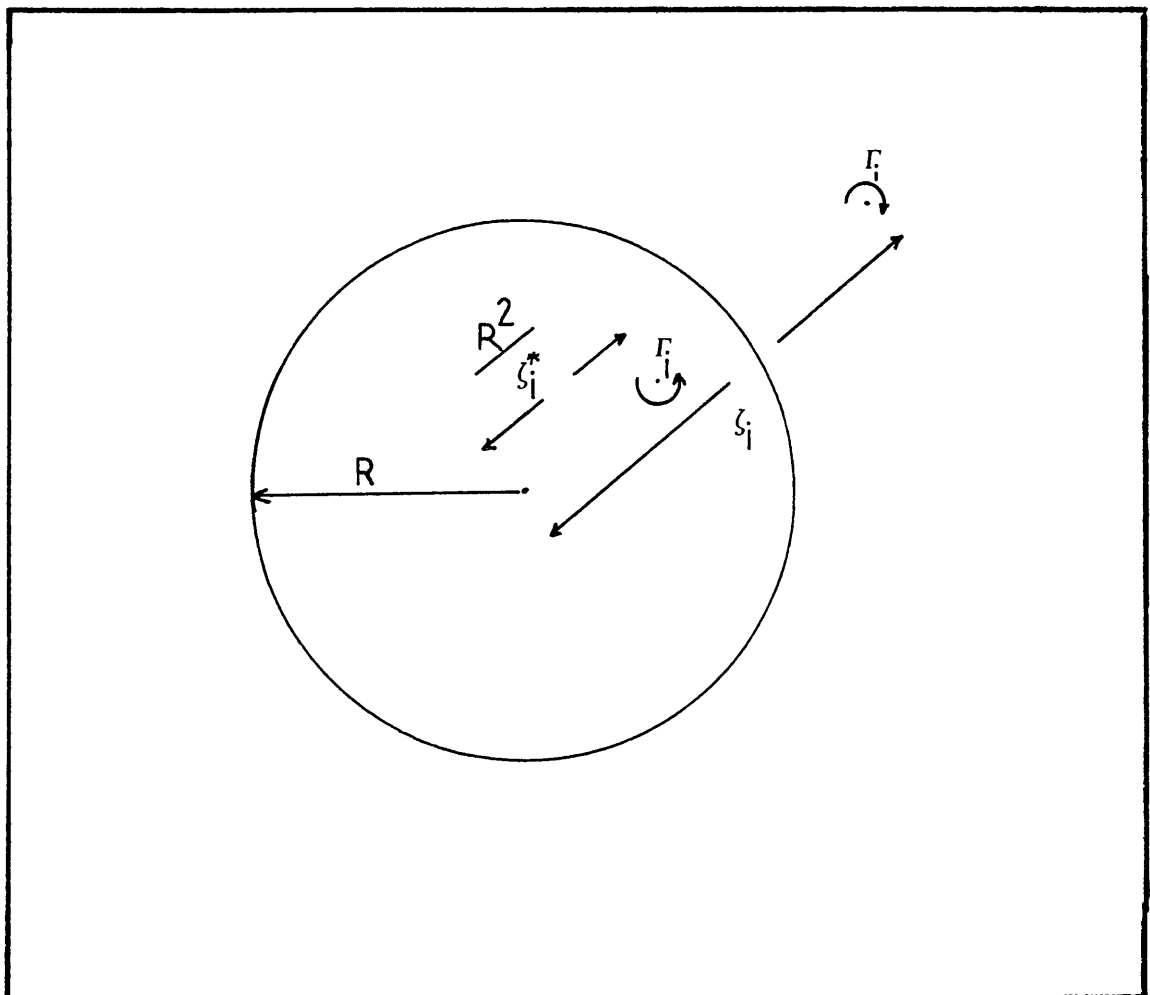


Figure 2.3: Vortex and image system.

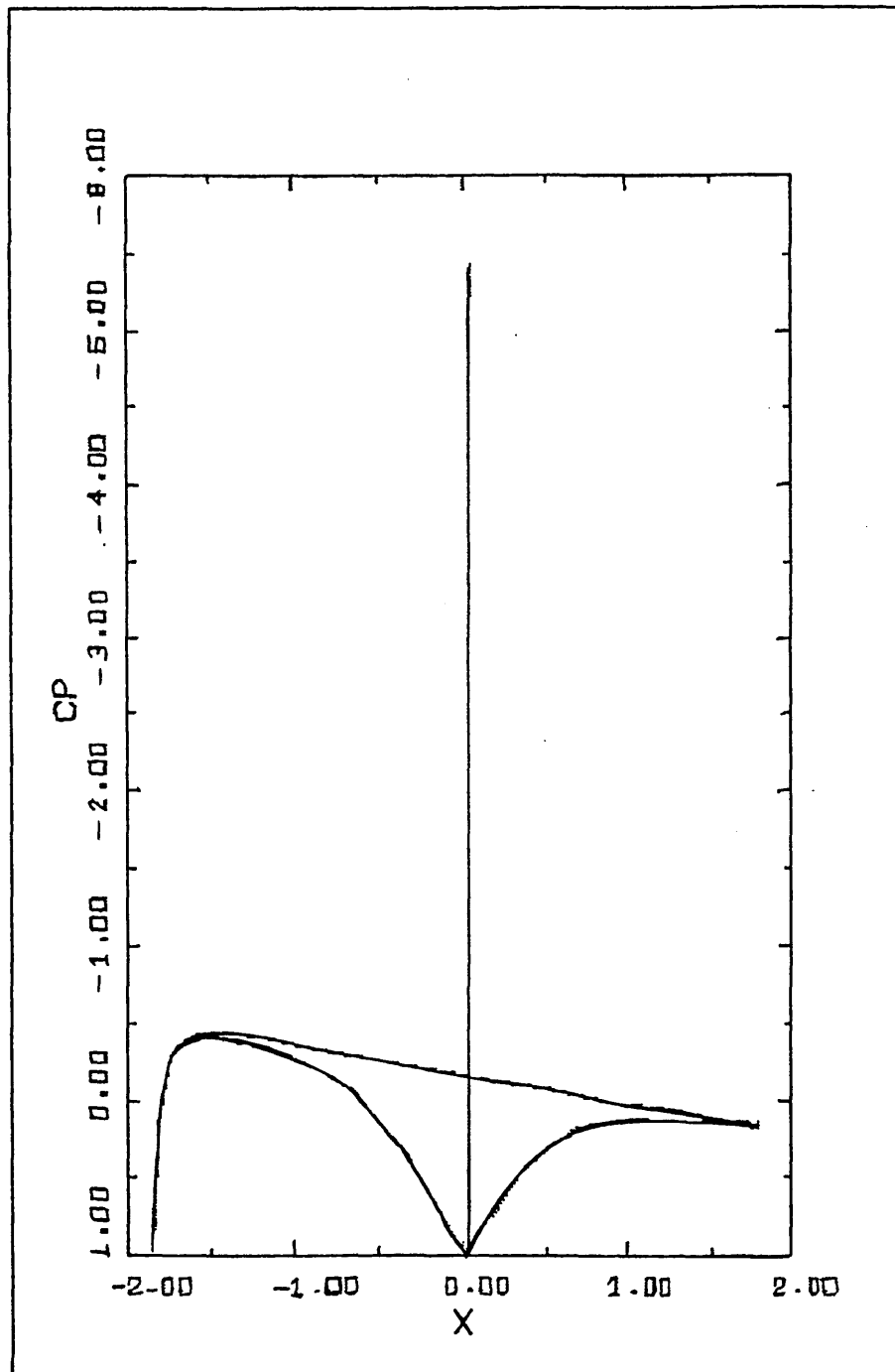


Figure 2.4: Attached flow C_p distribution over a 11% thick Joukowski aerofoil with a spoiler at 90 deg..

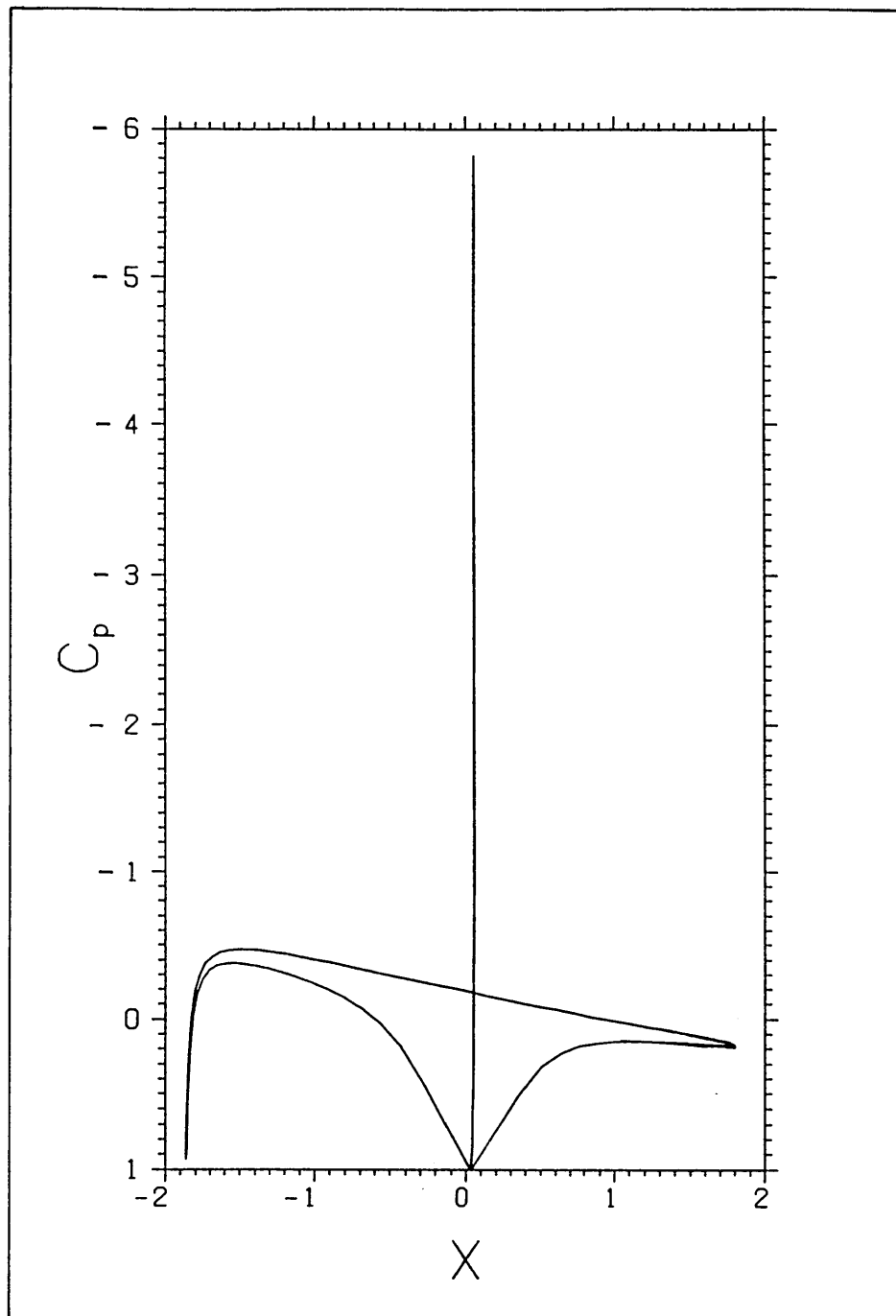
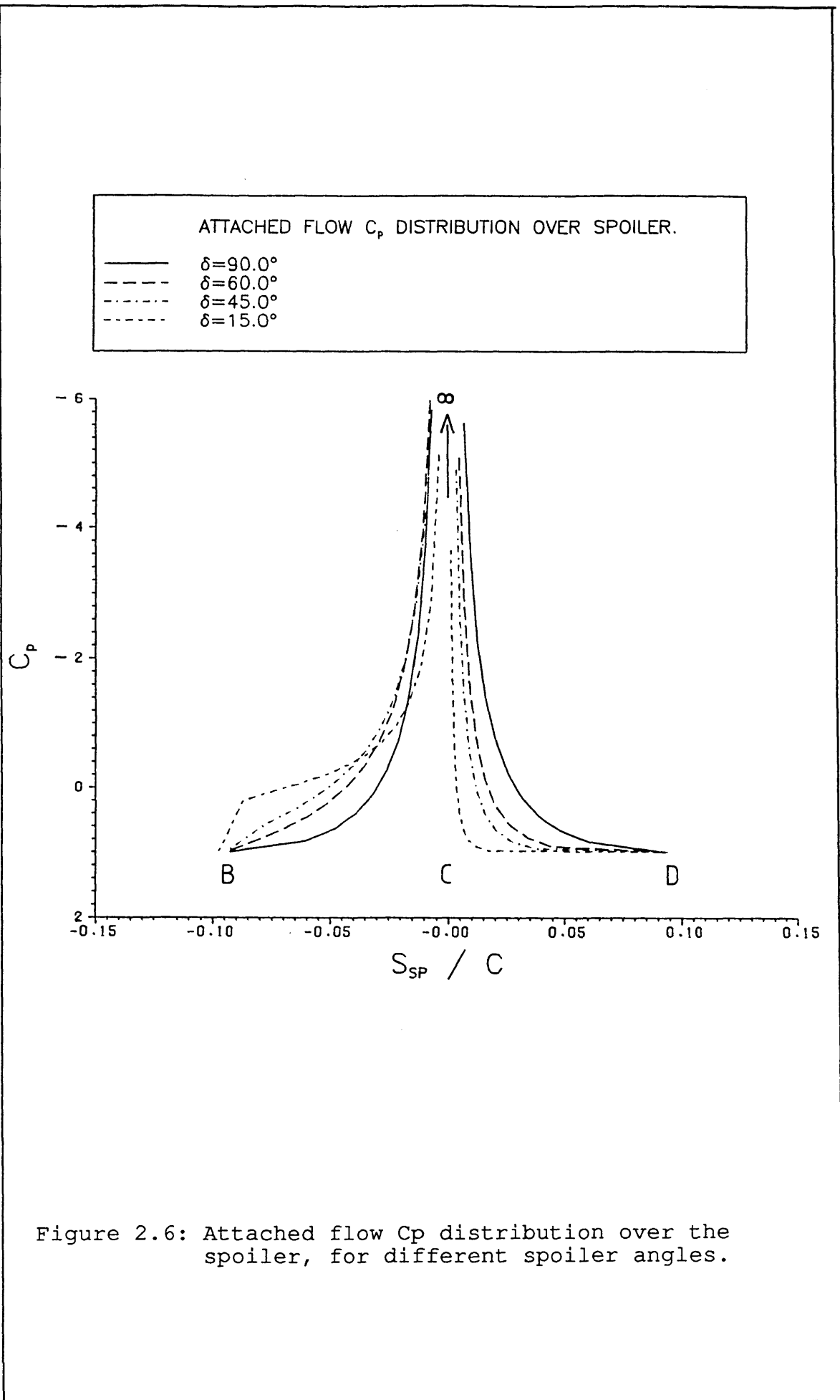


Figure 2.5: Attached flow C_p distribution over a 11% thick Joukowski aerofoil with an arbitrary spoiler set at 90 deg..



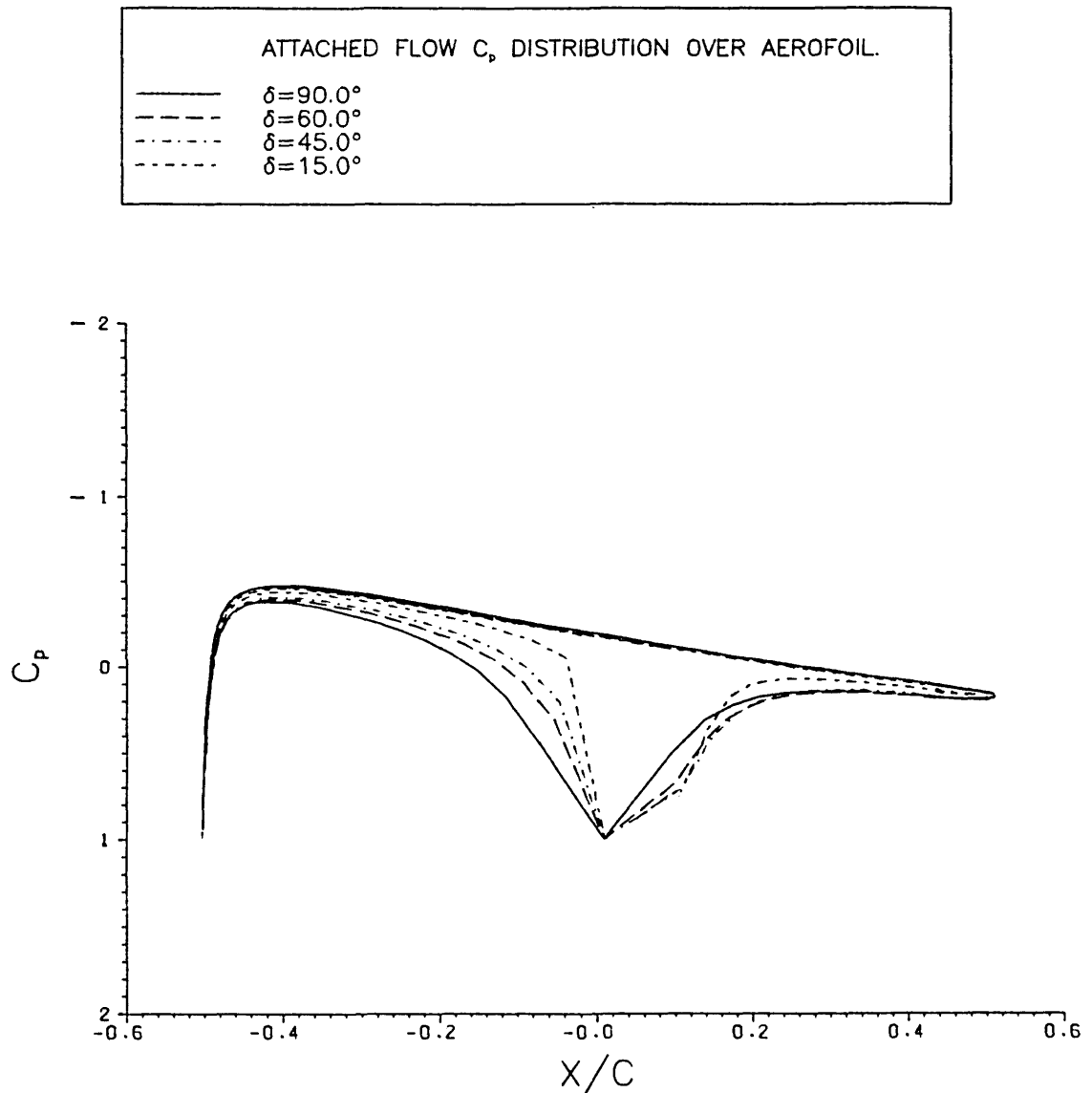


Figure 2.7: Attached flow C_p distribution over the aerofoil only, for different spoiler angles.

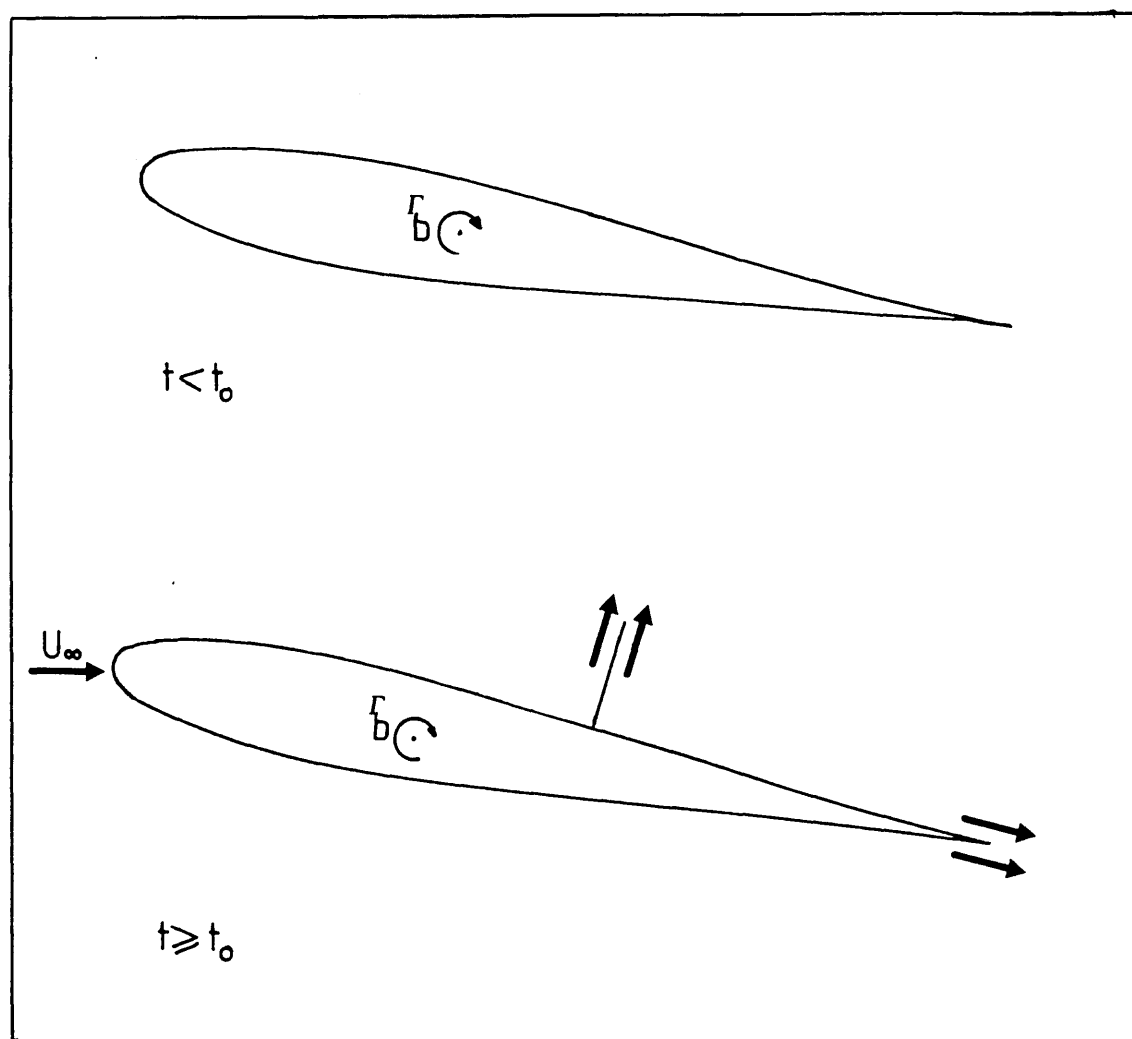


Figure 3.1: Spoiler raised impulsively on the aerofoil surface.

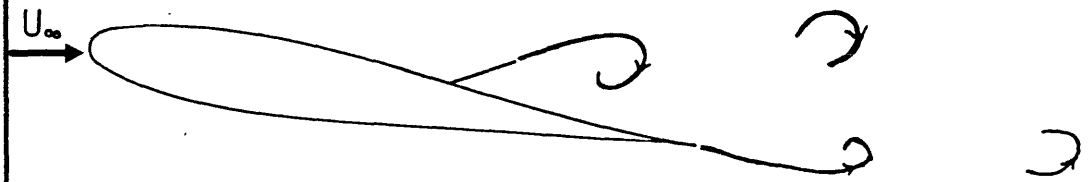


Figure 3.2: Schematic representation of shear layer interaction.

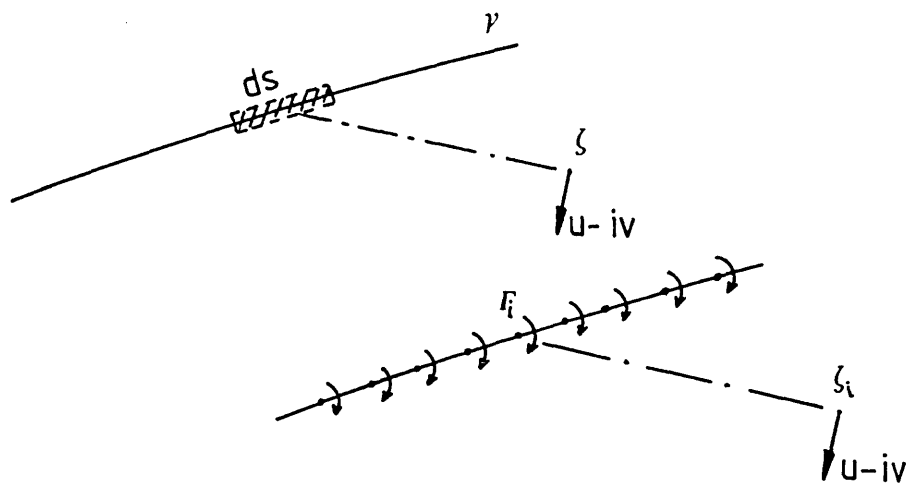


Figure 3.3: Vortex sheet approximated by discrete vortices.

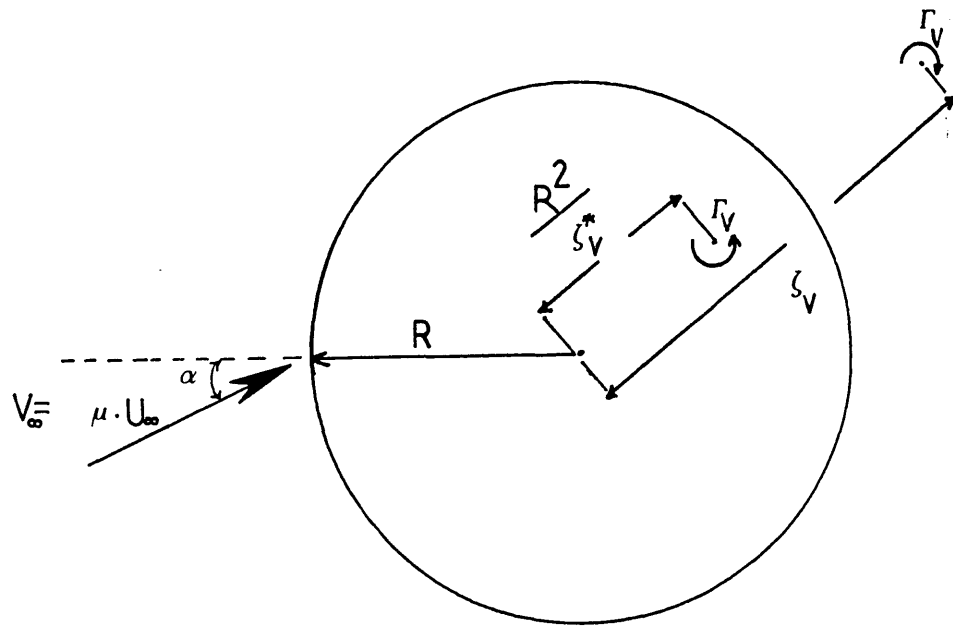


Figure 3.4: Vortex-Image system and free stream in circle plane.

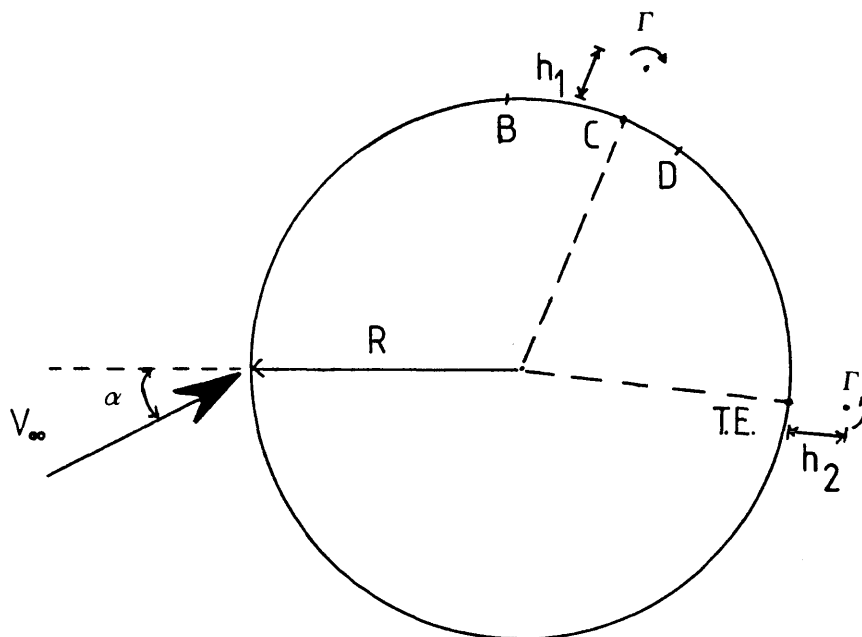


Figure 3.5: Release hight for nascent vortices.

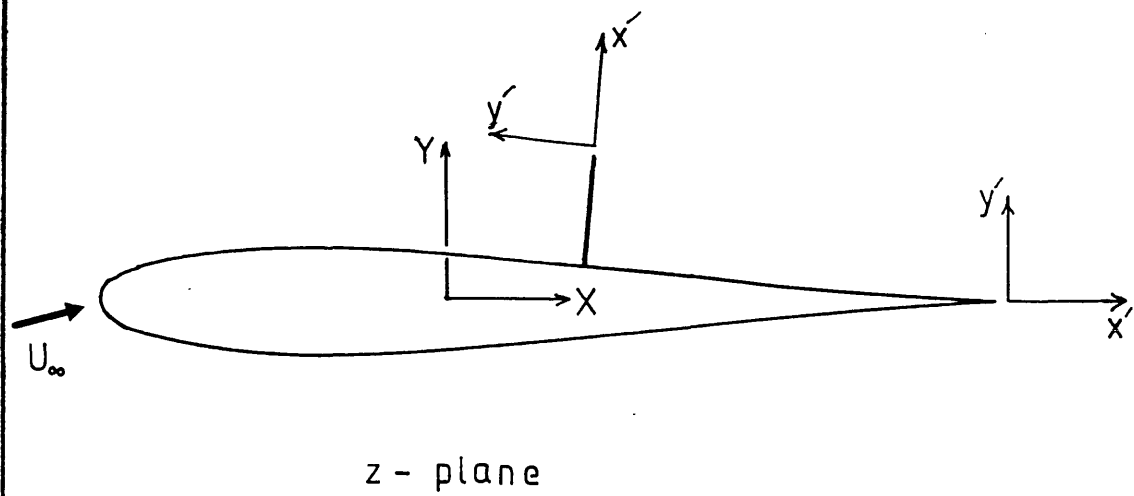


Figure 3.6: Local axis systems.

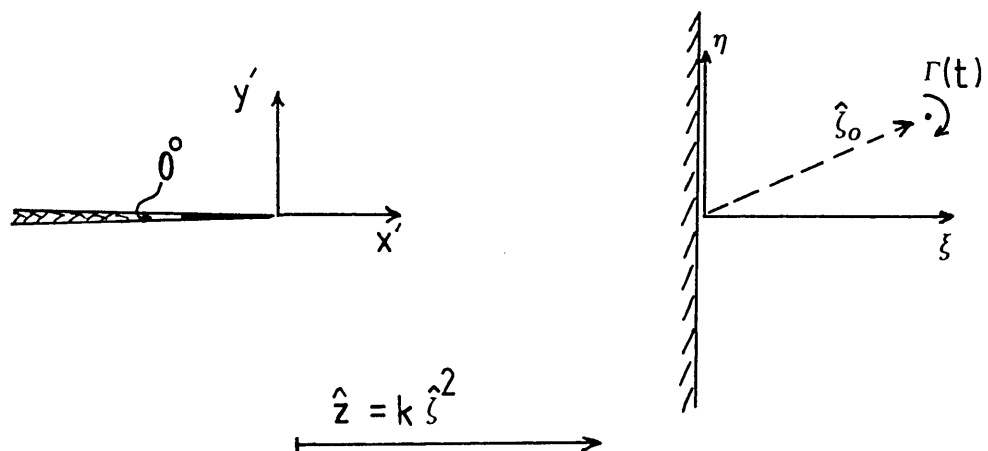


Figure 3.7: Local transformation of sharp edge.

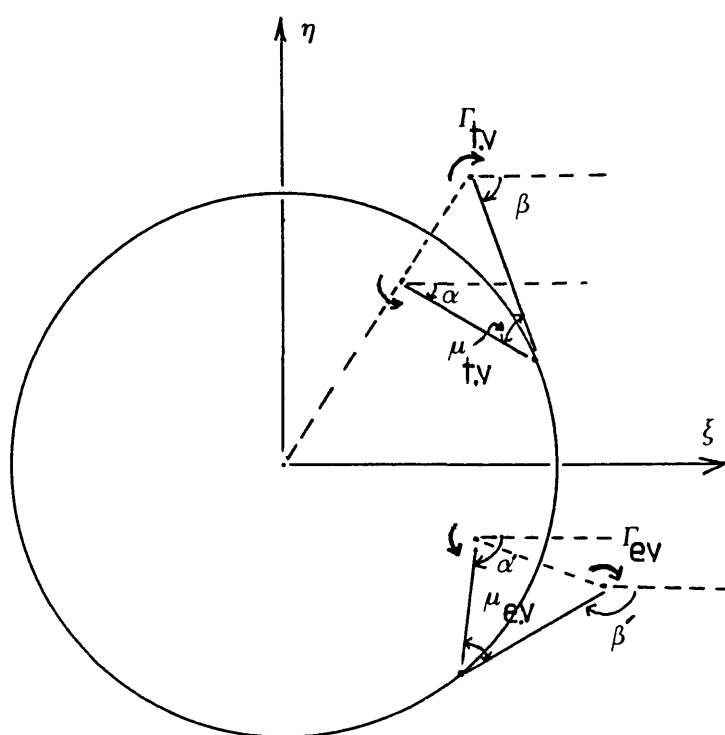


Figure 3.8: Region of application of local velocity scheme.

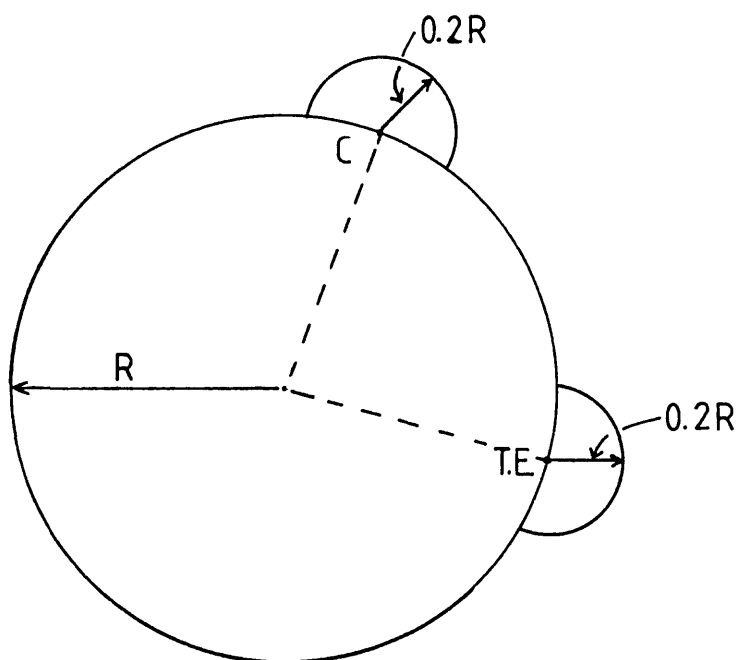


Figure 3.9: Definition of angles to calculate $d\Phi/dt$ analytically.

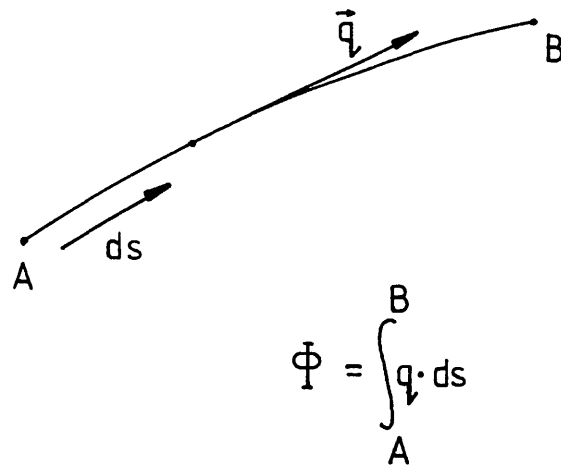


Figure 3.10: Definition of Φ .

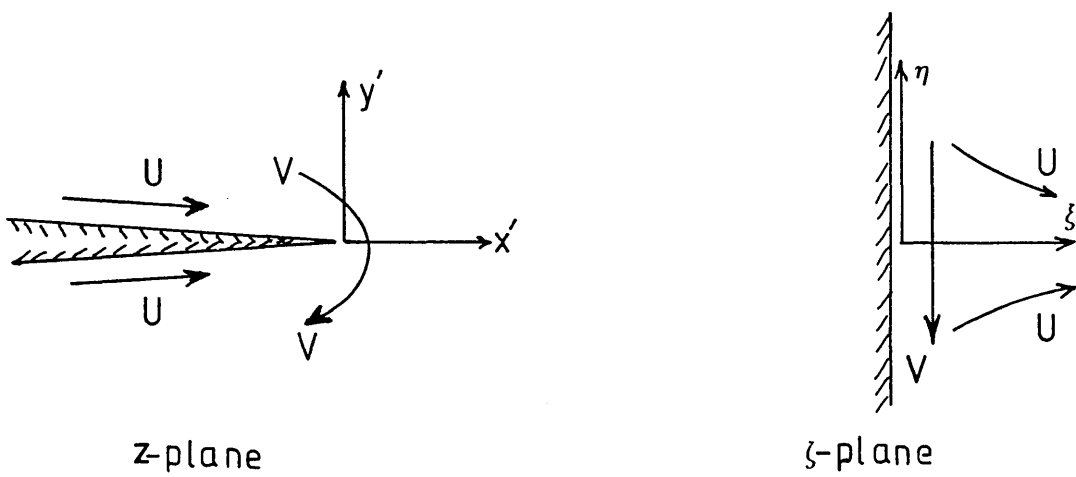
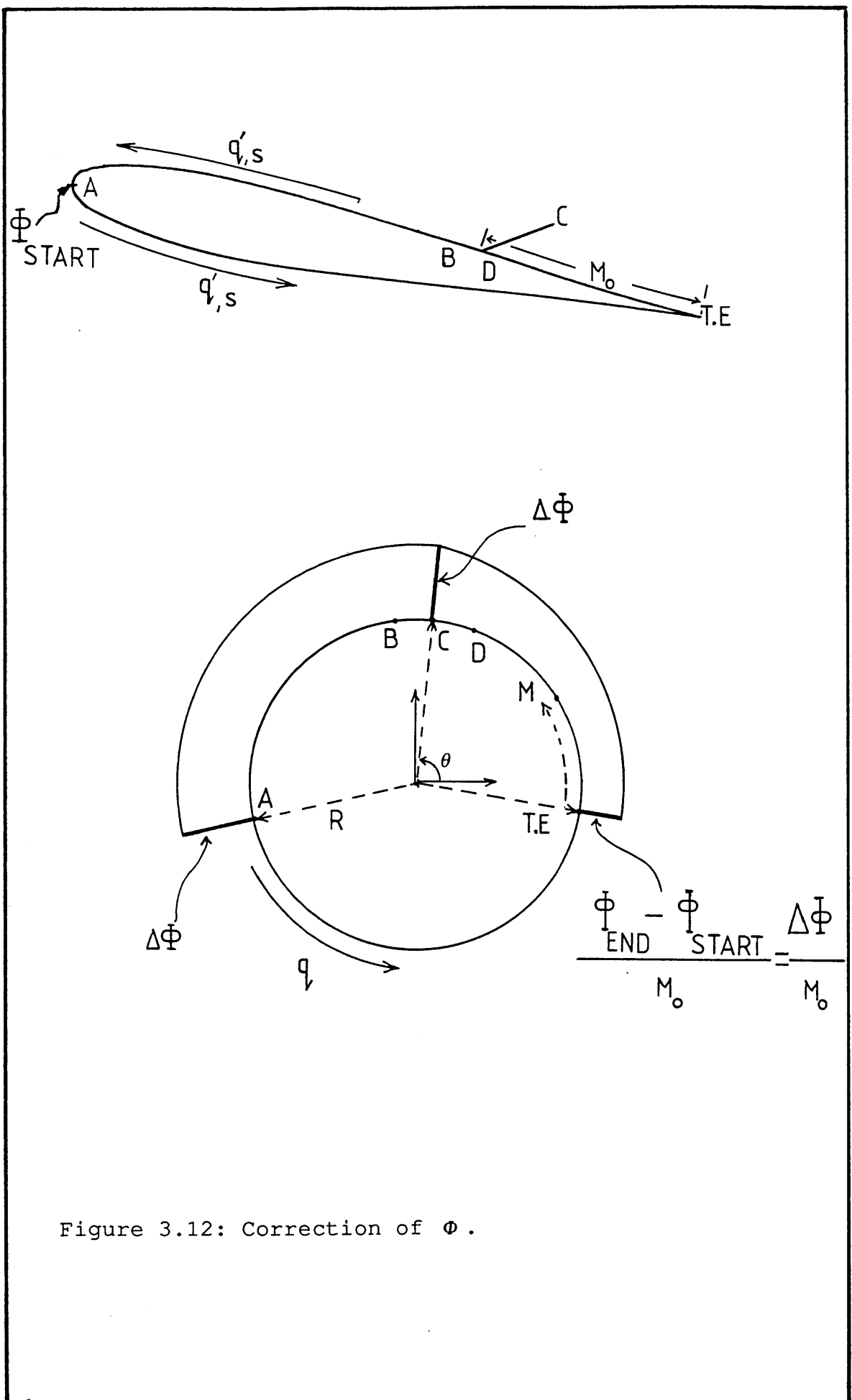


Figure 3.11: Symmetric and asymmetric components of velocity.



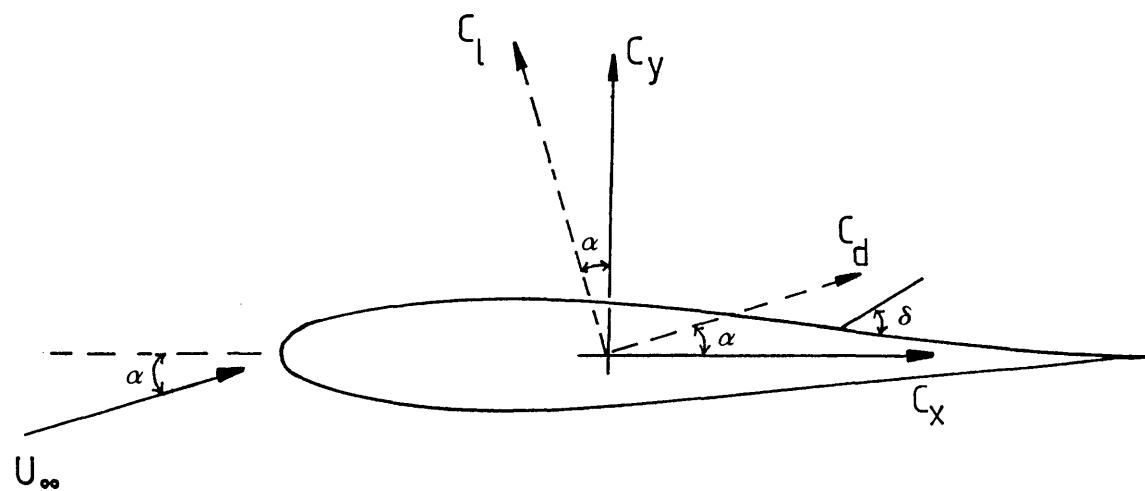


Figure 3.13: Axes for C_l and C_d .

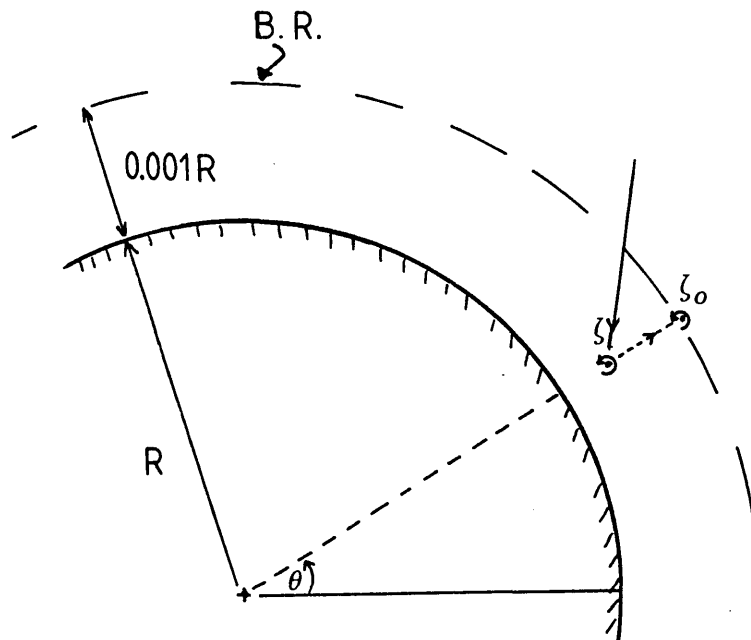


Figure 4.1: Buffer Region (B.R.); vortex position inside B.R and displaced position outside B.R.

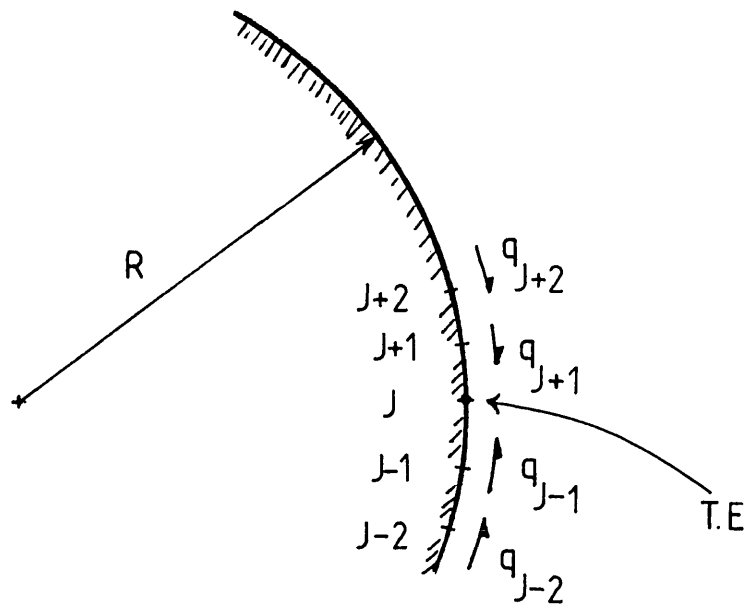
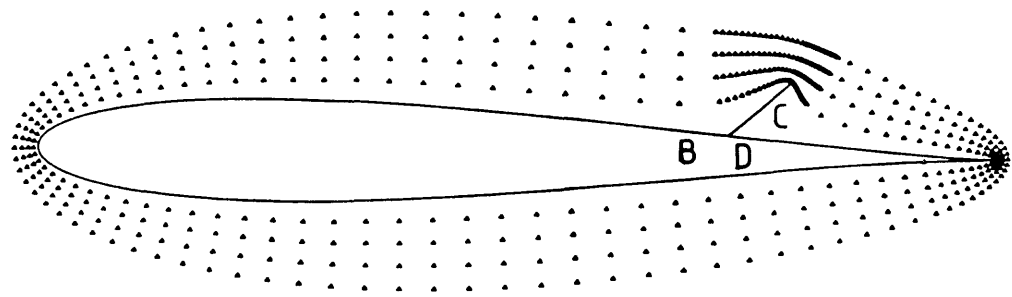
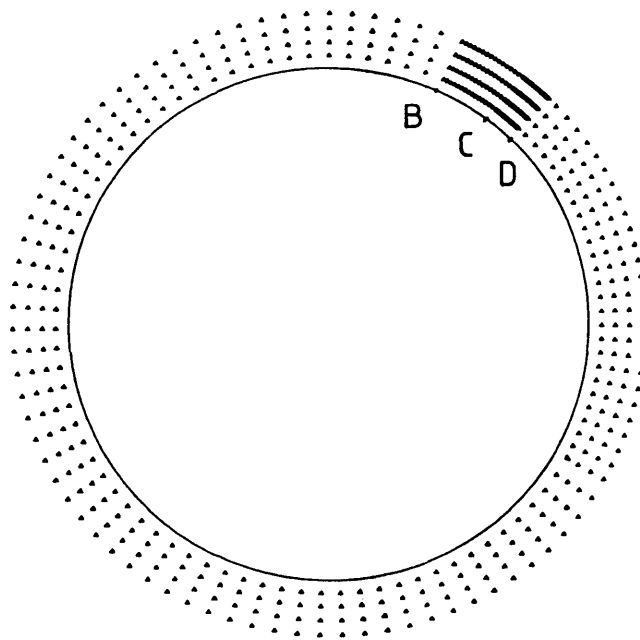


Figure 4.2: Points near the trailing edge used to calculate the trailing edge velocity.



z-plane



ζ -plane

Figure 4.3: Mesh points in the circle and aerofoil plane; $\delta=45.0$ deg..

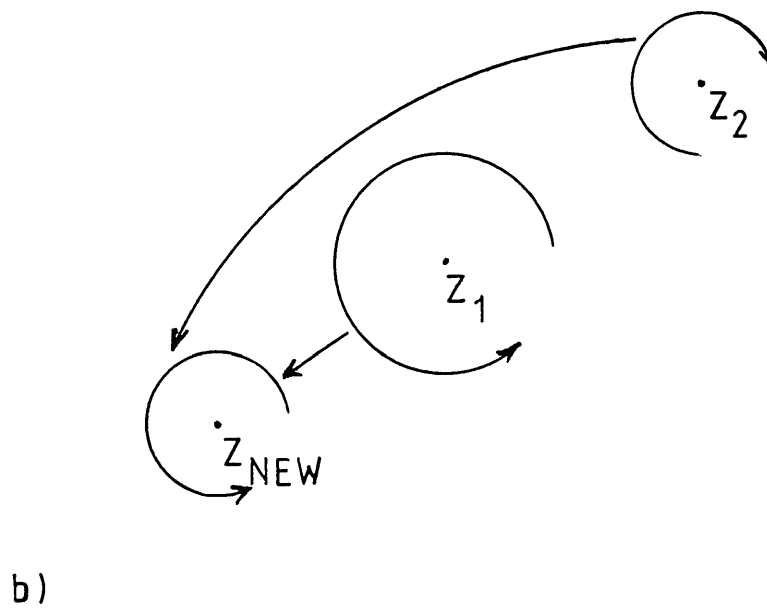
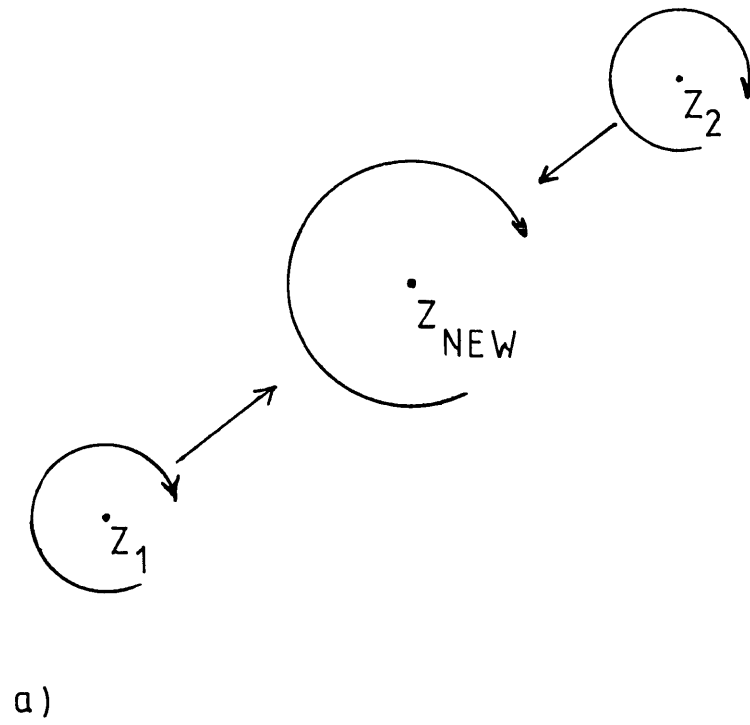


Figure 4.4: Amalgamation of vortex pairs.

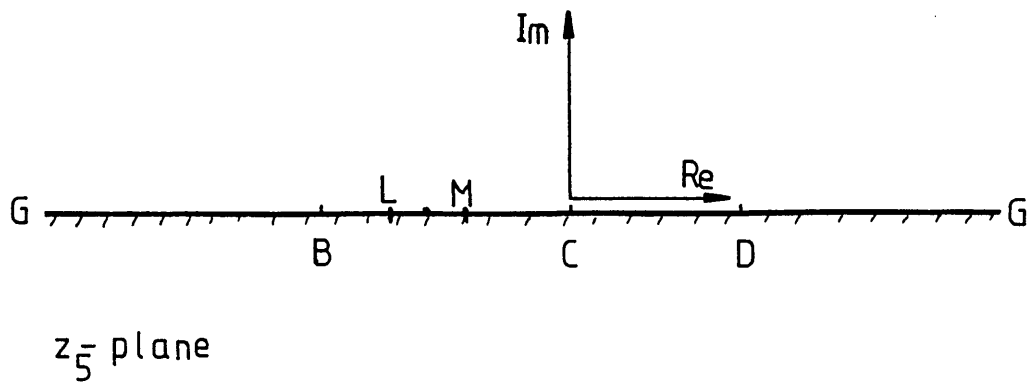


Figure 5.1: Opened-up spoiler in the straight-line plane, and source/sink element.

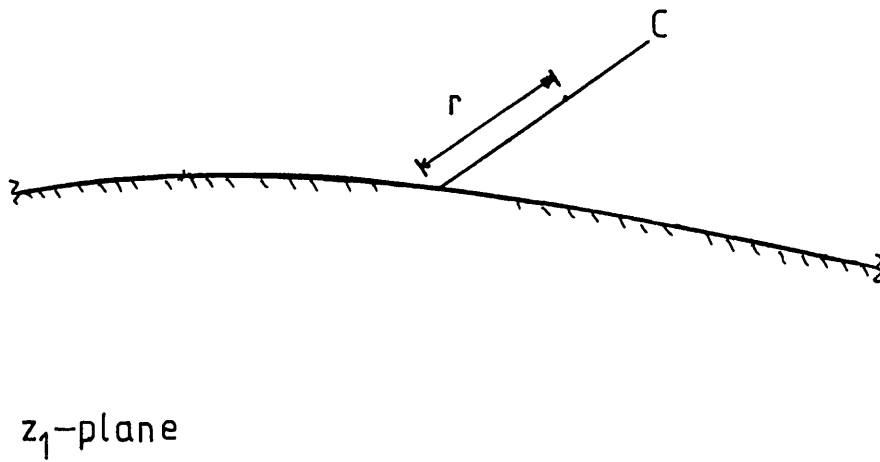


Figure 5.2: Distance 'r' of a control point on the spoiler surface from the spoiler root.

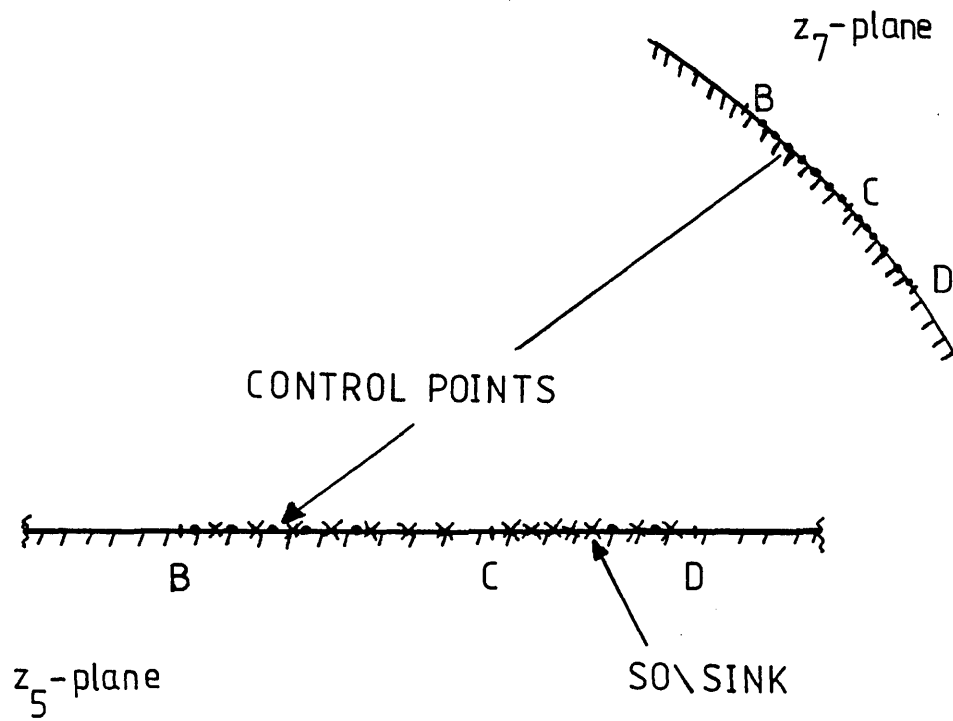


Figure 5.3: Control points on BCD; sources and sinks on BCD.

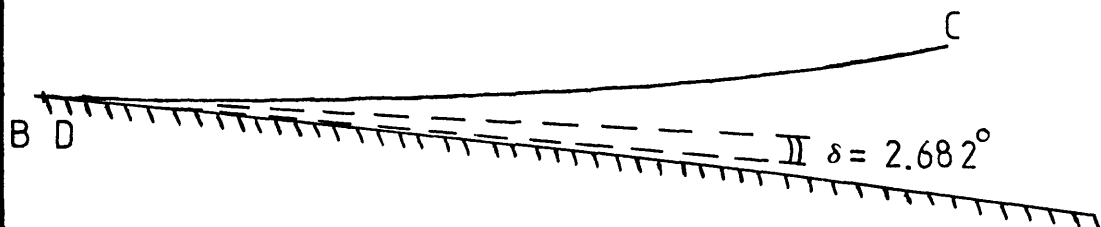


Figure 5.4: Spoiler curvature at a small spoiler angle.

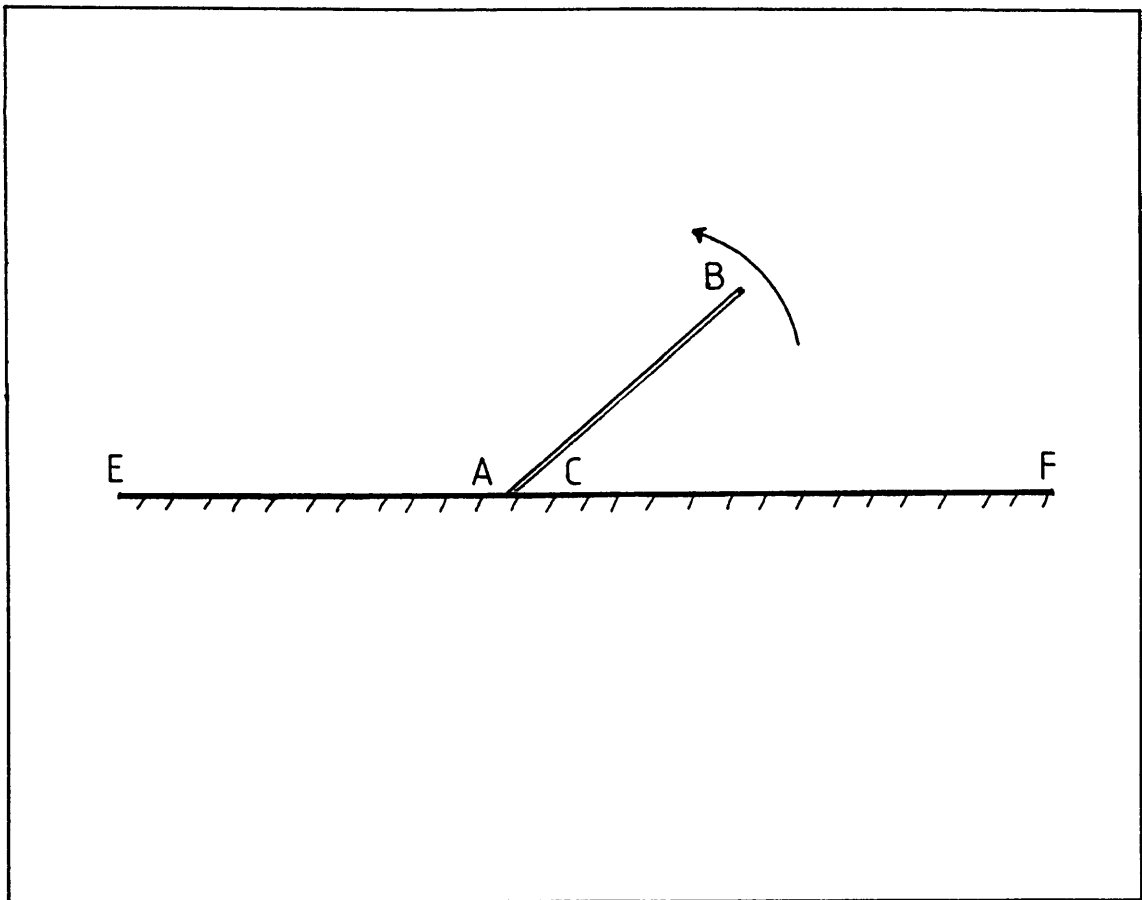


Figure 5.5: Model used by CHENG and EDWARDS (1982).

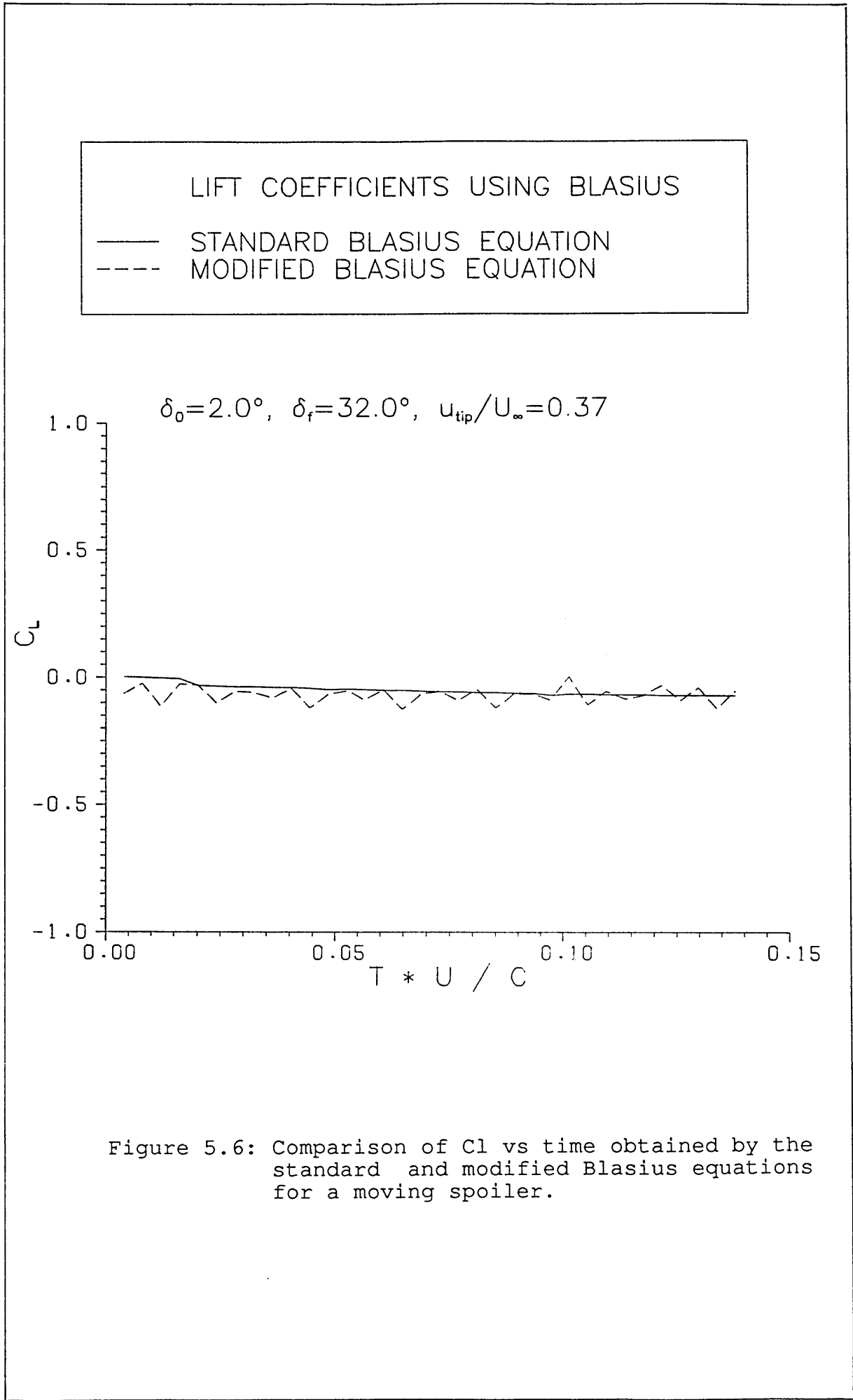


Figure 5.6: Comparison of C_l vs time obtained by the standard and modified Blasius equations for a moving spoiler.

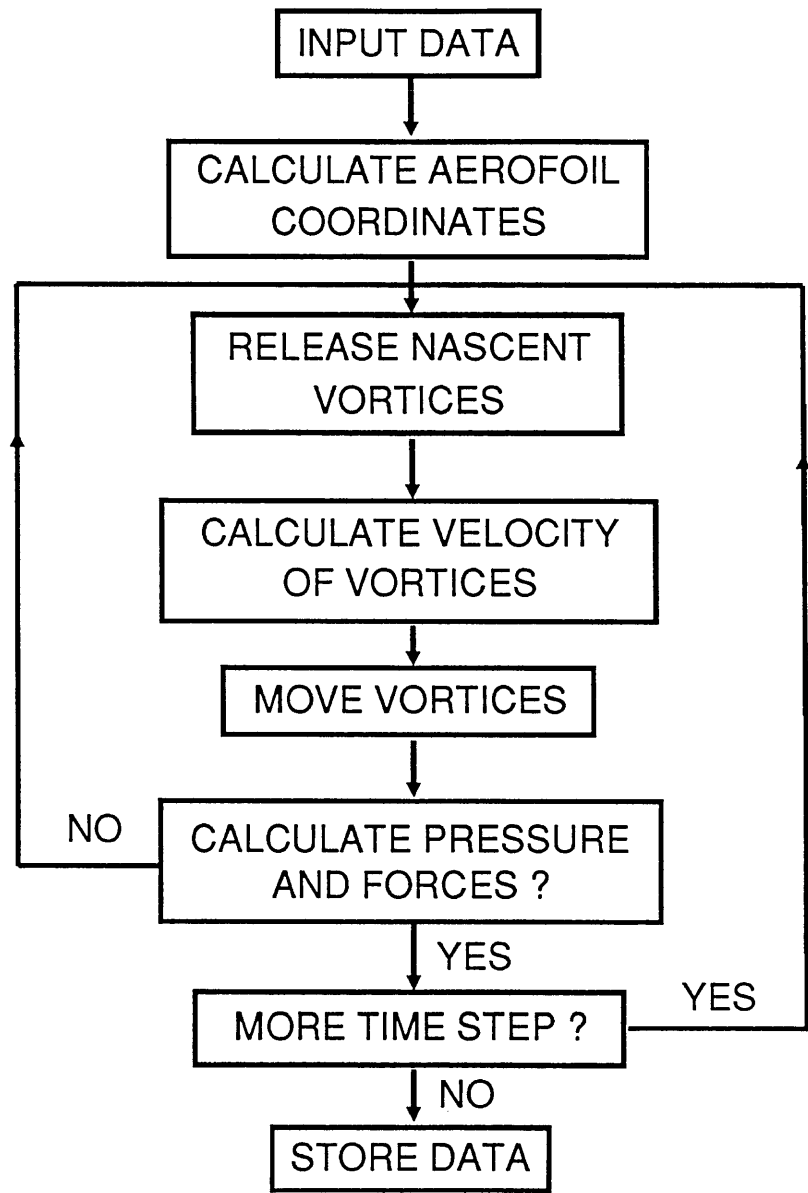


Figure 6.1: 'Fixed' Spoiler program flow chart.

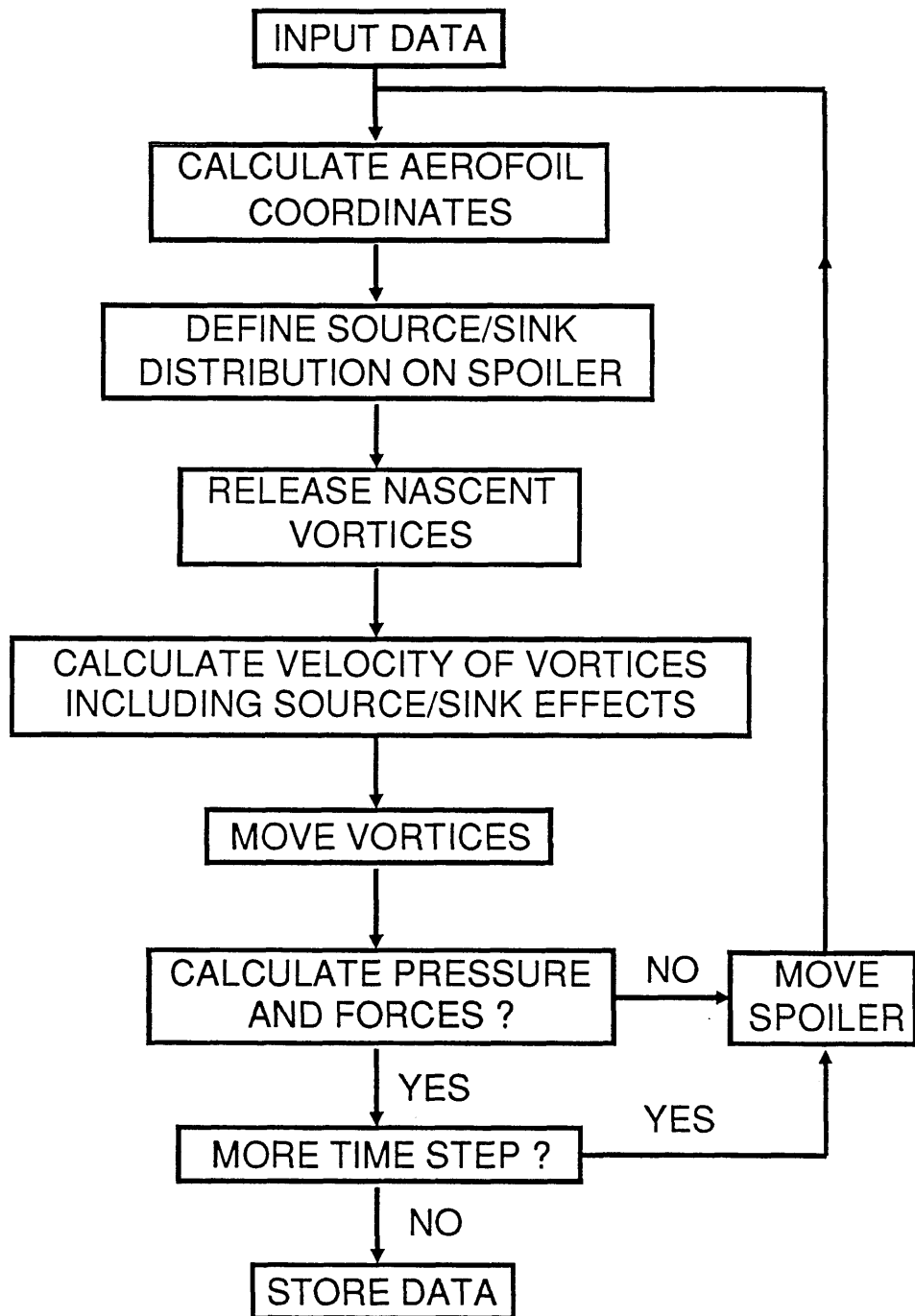


Figure 6.2: Moving Spoiler program flow chart.

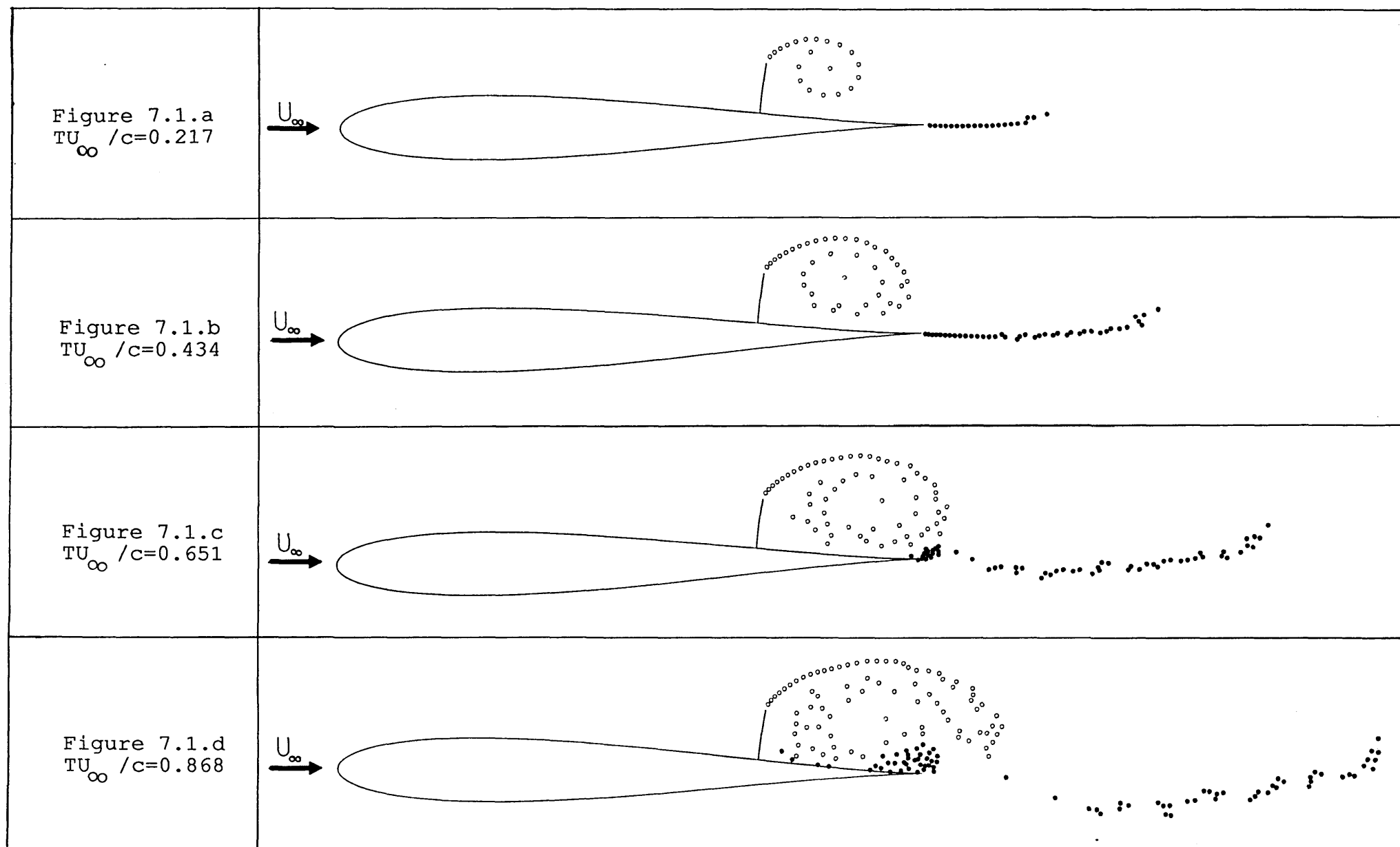




Figure 7.1.e: $TU_\infty / c = 3.62$

Figure 7.1: $\alpha = 0$ deg., $\delta = 90$ deg., $SP = 70\%c$.

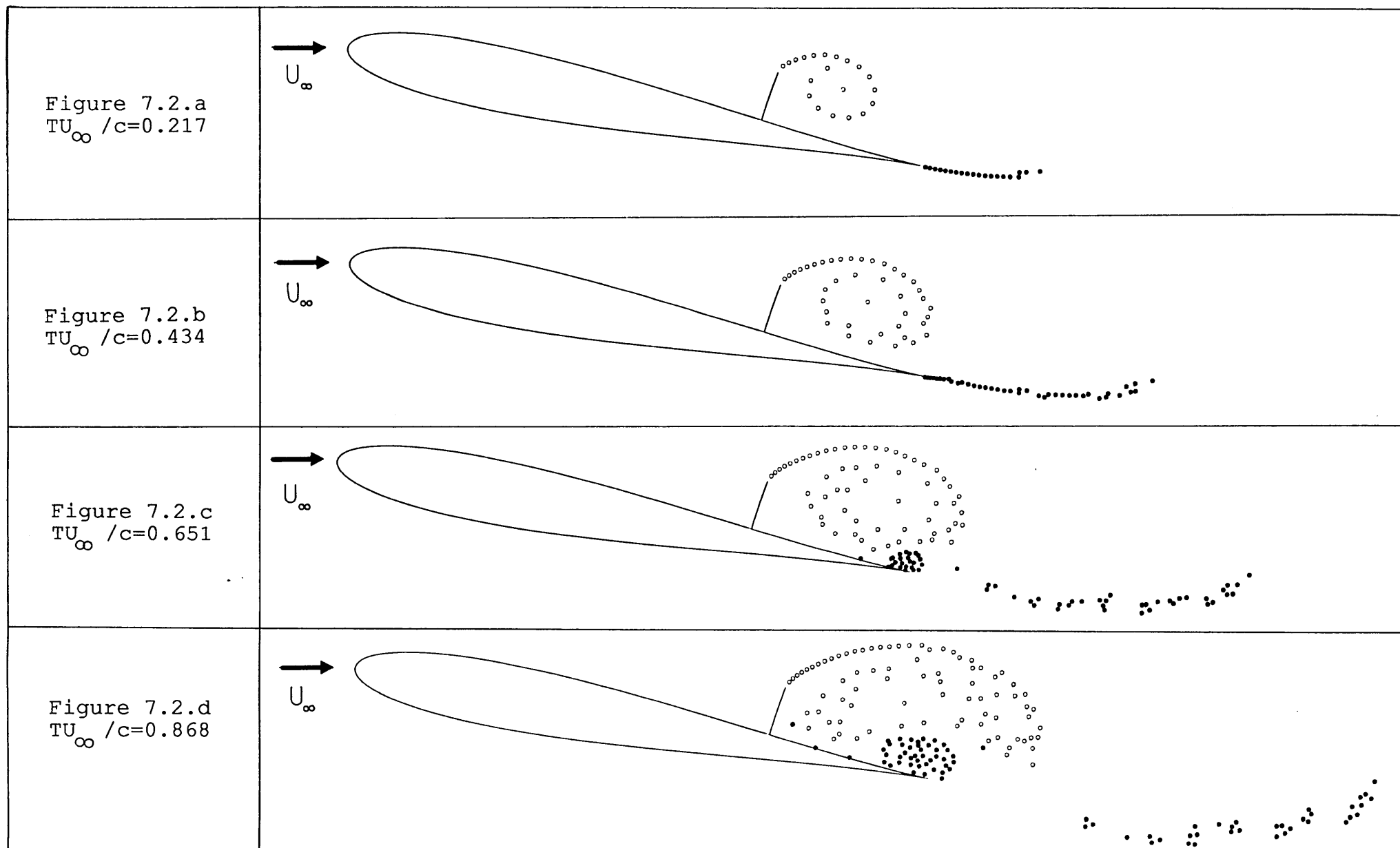




Figure 7.2.e
 $TU_\infty / c = 1.58$

Figure 7.2: $\alpha = 12$ deg., $\delta = 90$ deg., $SP = 70\%c$.

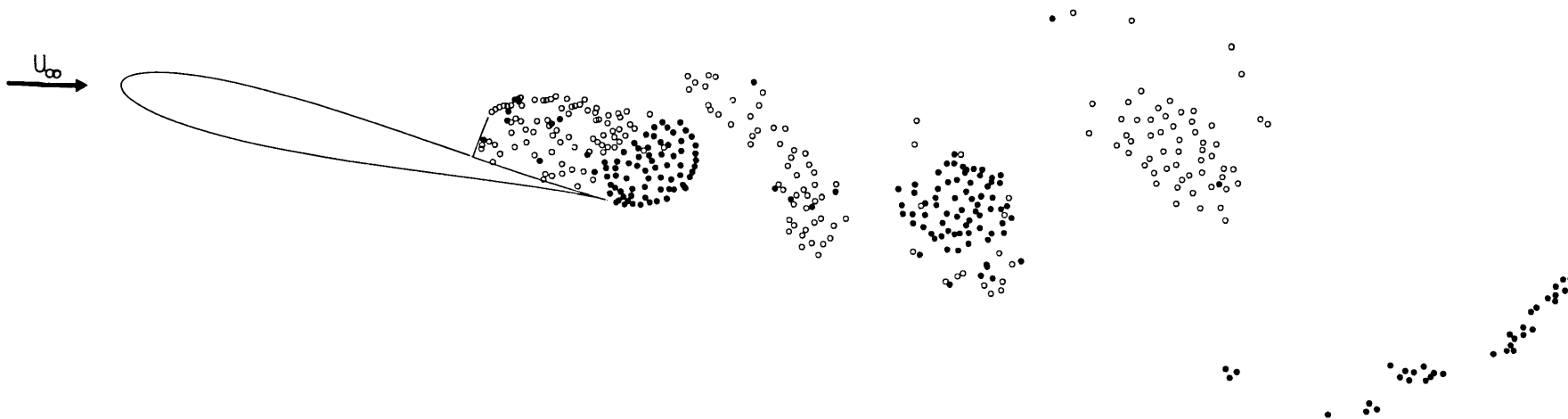


Figure 7.3: $\alpha=12$ deg., $\delta=90$ deg., $SP=70\%c$, at $TU_\infty/c=1.63$.



Figure 7.4.a: Numerical wake-flow visualisation.



Figure 7.4.b: $\alpha=12$ deg., $\delta=90$ deg., $SP=70\%c$, at $TU_\infty/c=3.8$.

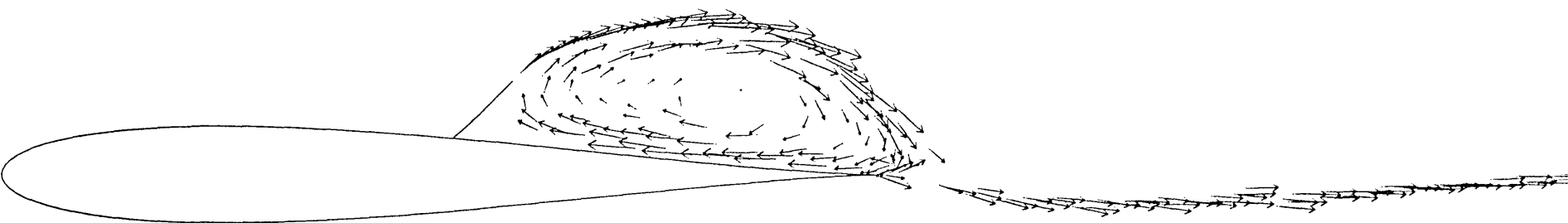


Figure 7.5.a: Numerical wake-flow visualisation.

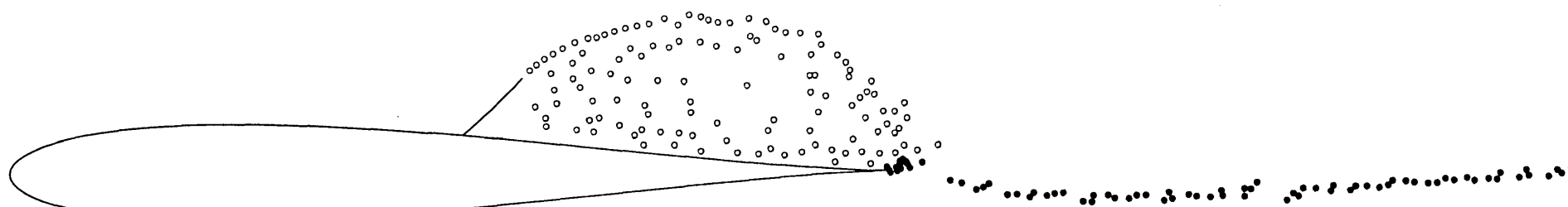


Figure 7.5.b: $\alpha=0$ deg., $\delta=45$ deg., SP=50%c, at $TU_{\infty}/c=0.50$.

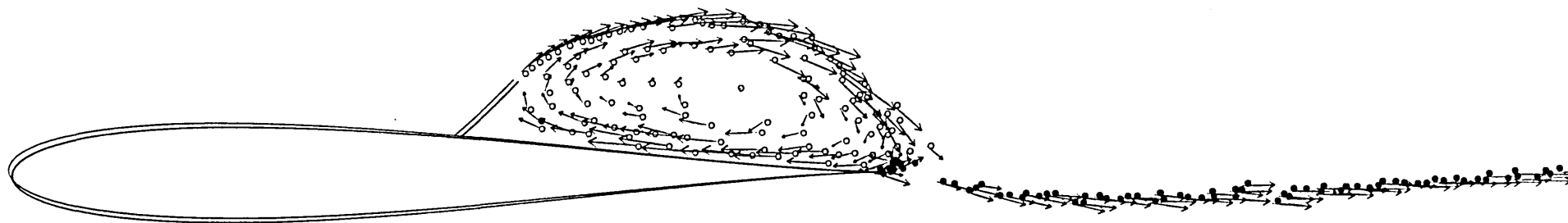


Figure 7.5.a: Numerical wake-flow visualisation.
 Figure 7.5.b: $\alpha=0$ deg., $\delta=45$ deg., $SP=50\%c$, at $TU_{\infty}/c=0.50$.

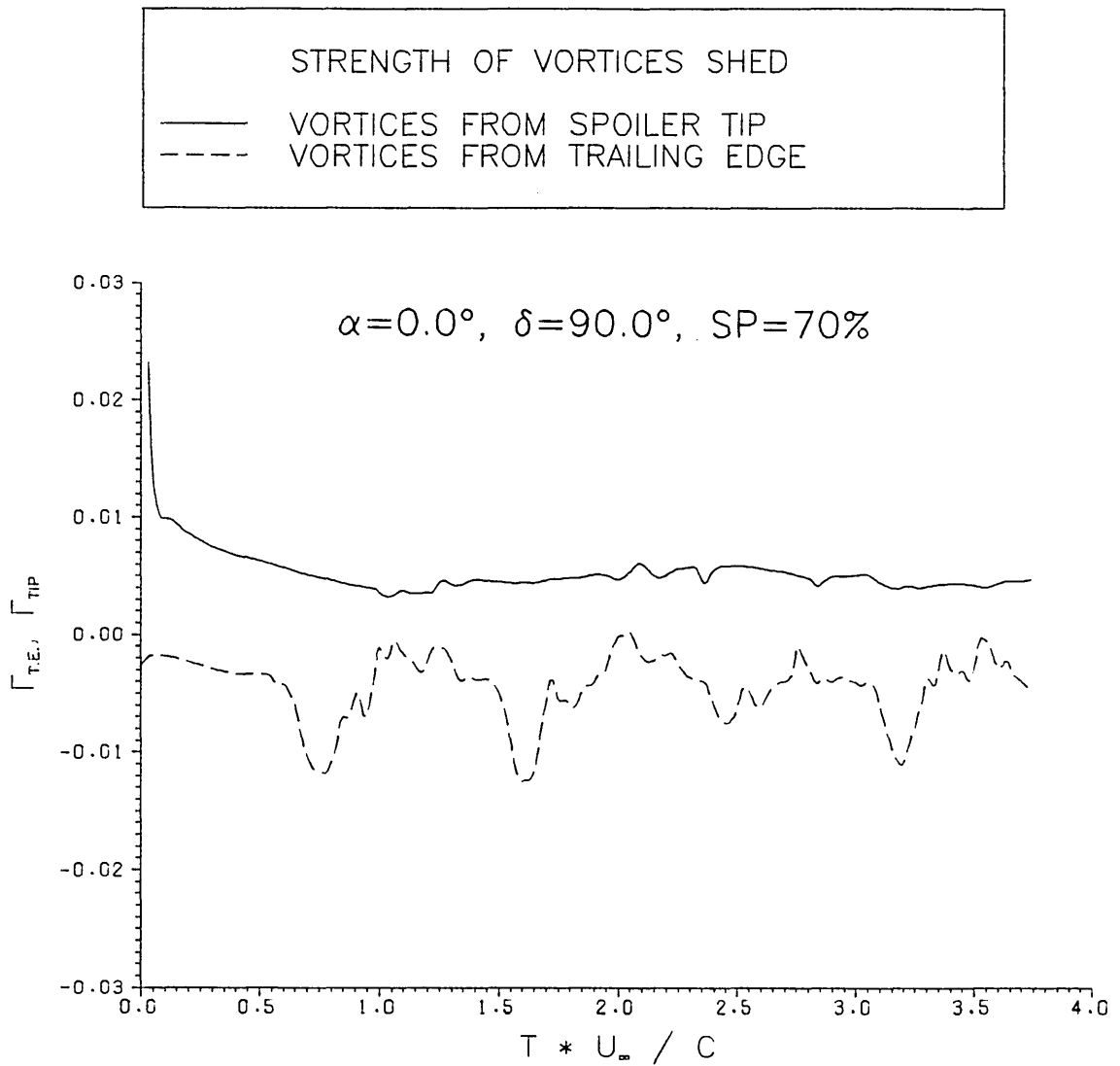


Figure 7.6: Vortex strength vs time.

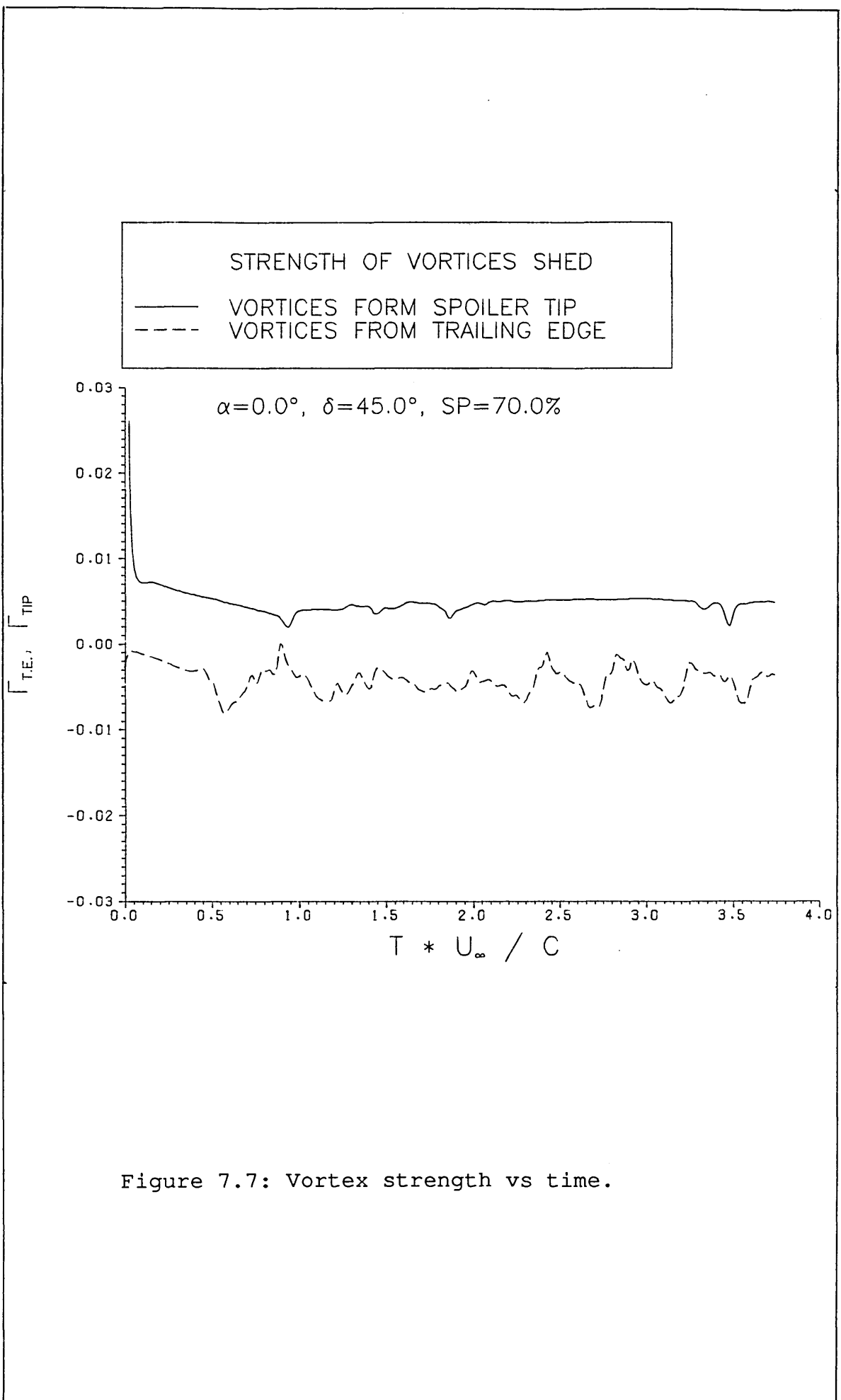


Figure 7.7: Vortex strength vs time.

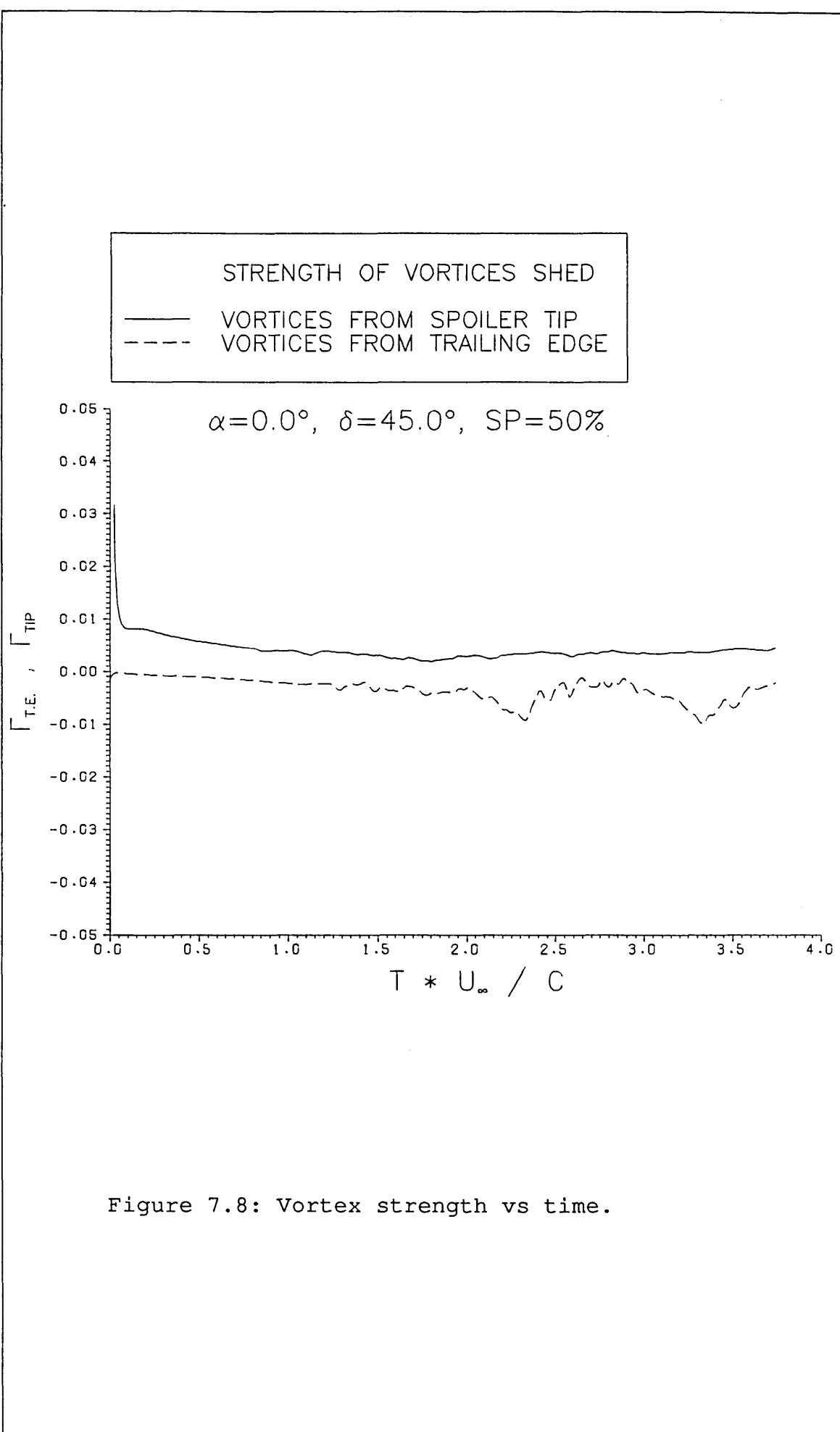


Figure 7.8: Vortex strength vs time.

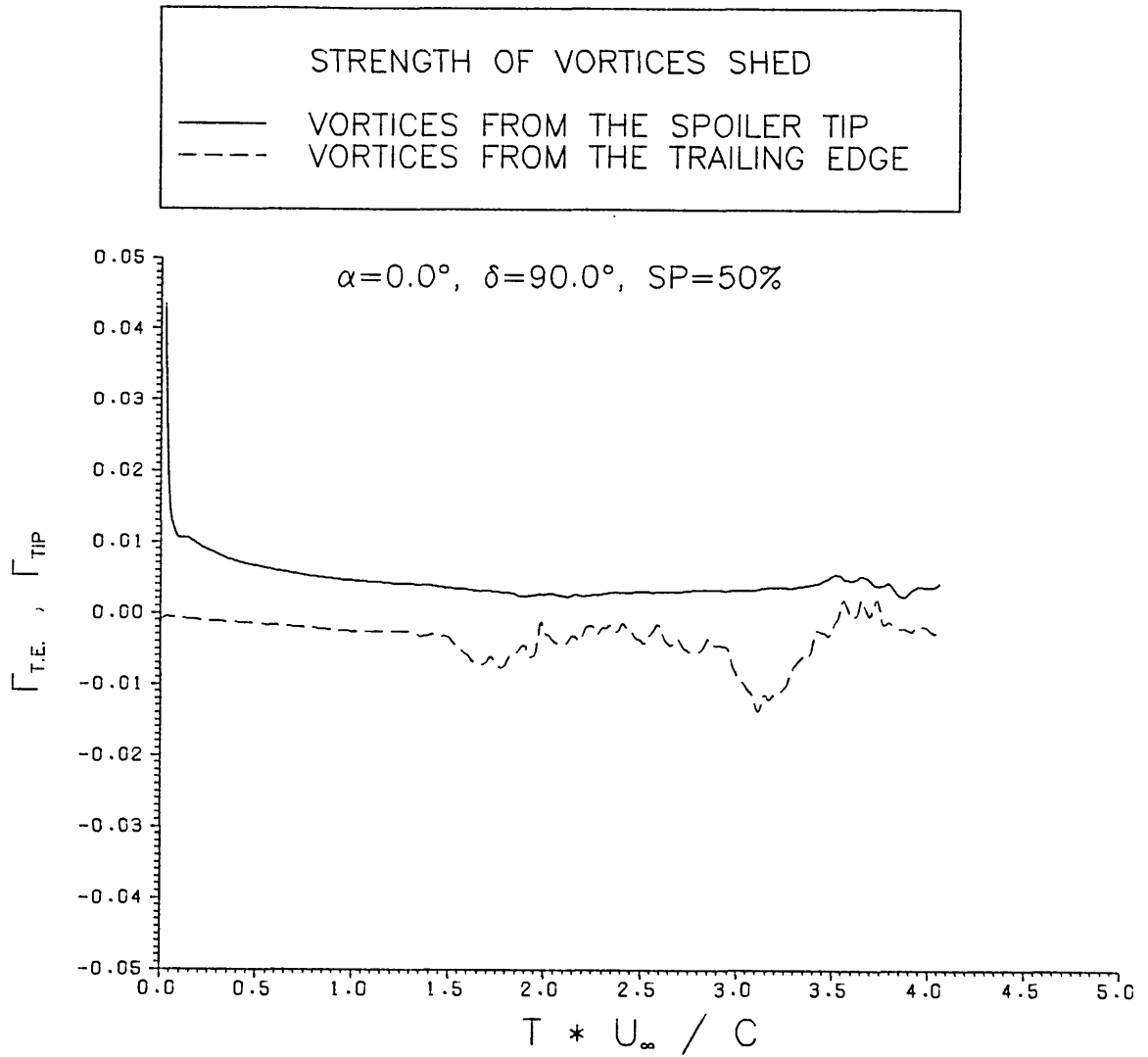


Figure 7.9: Vortex strength vs time.

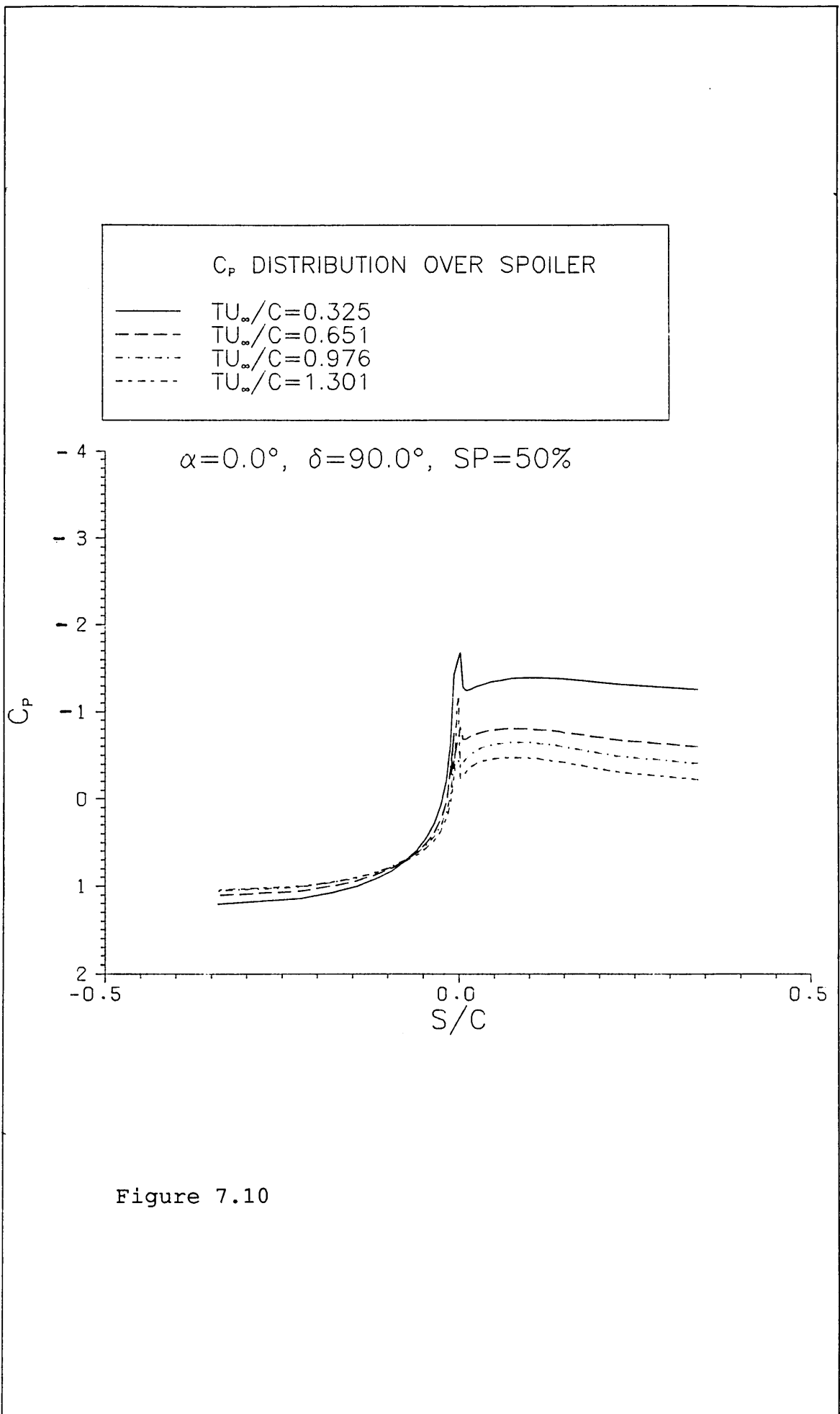


Figure 7.10

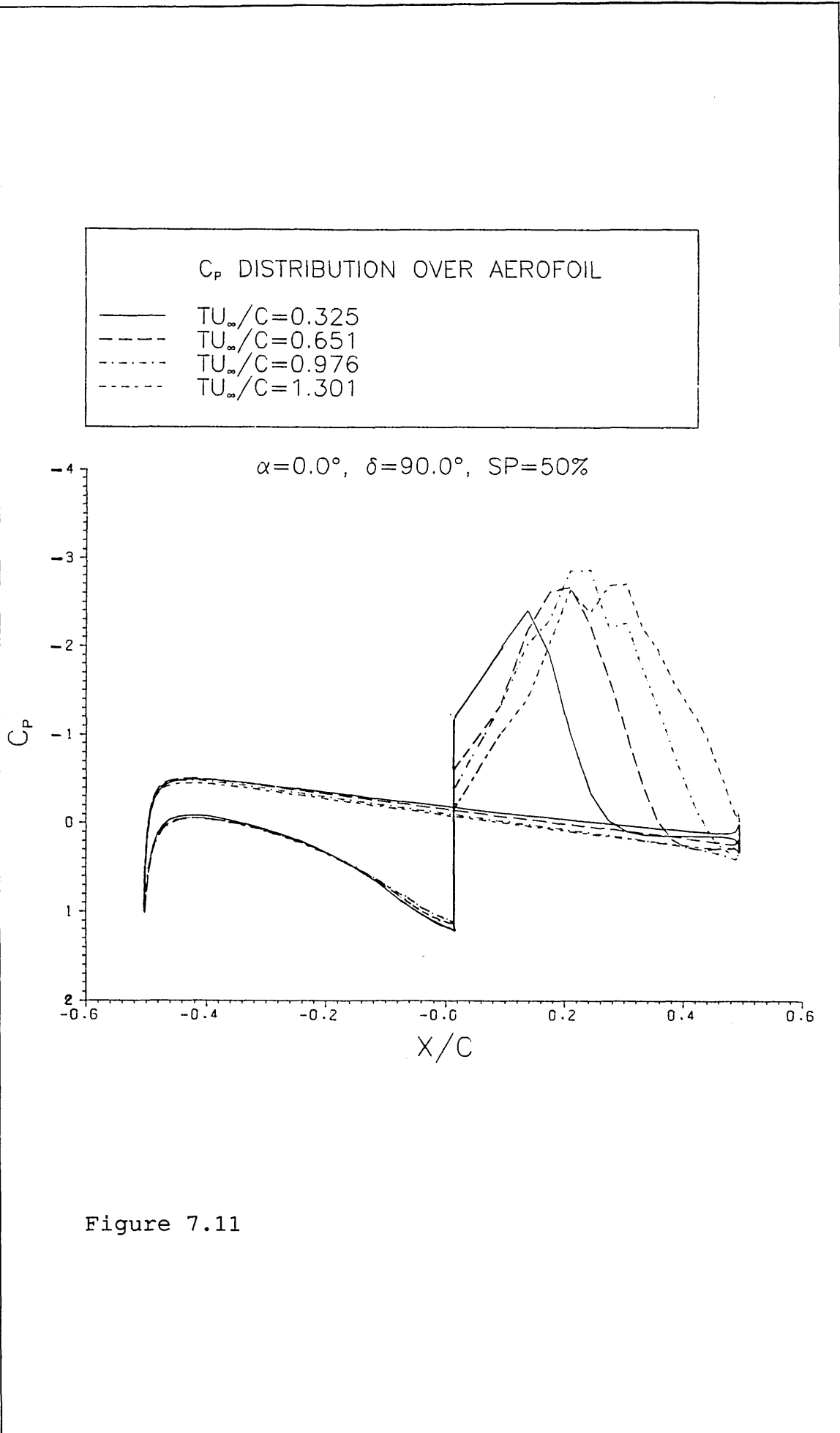


Figure 7.11

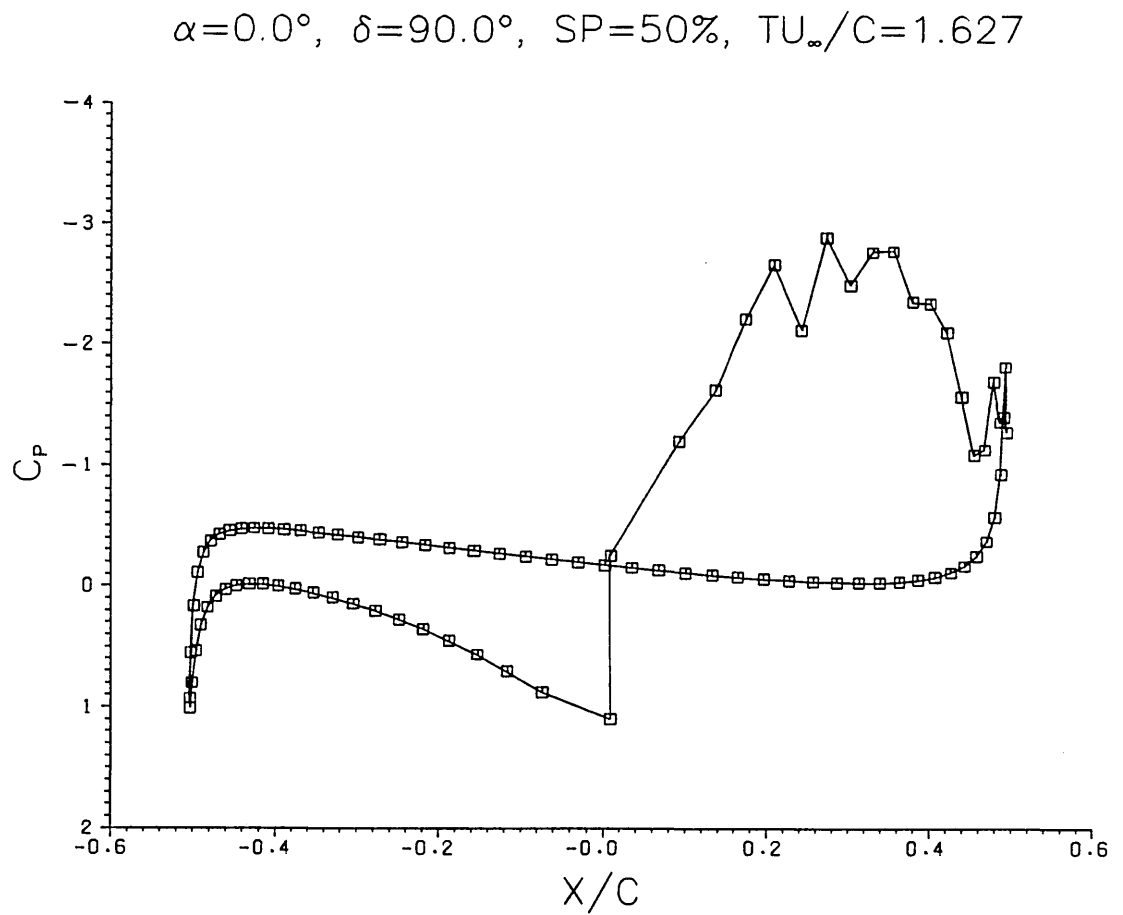


Figure 7.12: C_p distribution over aerofoil only.

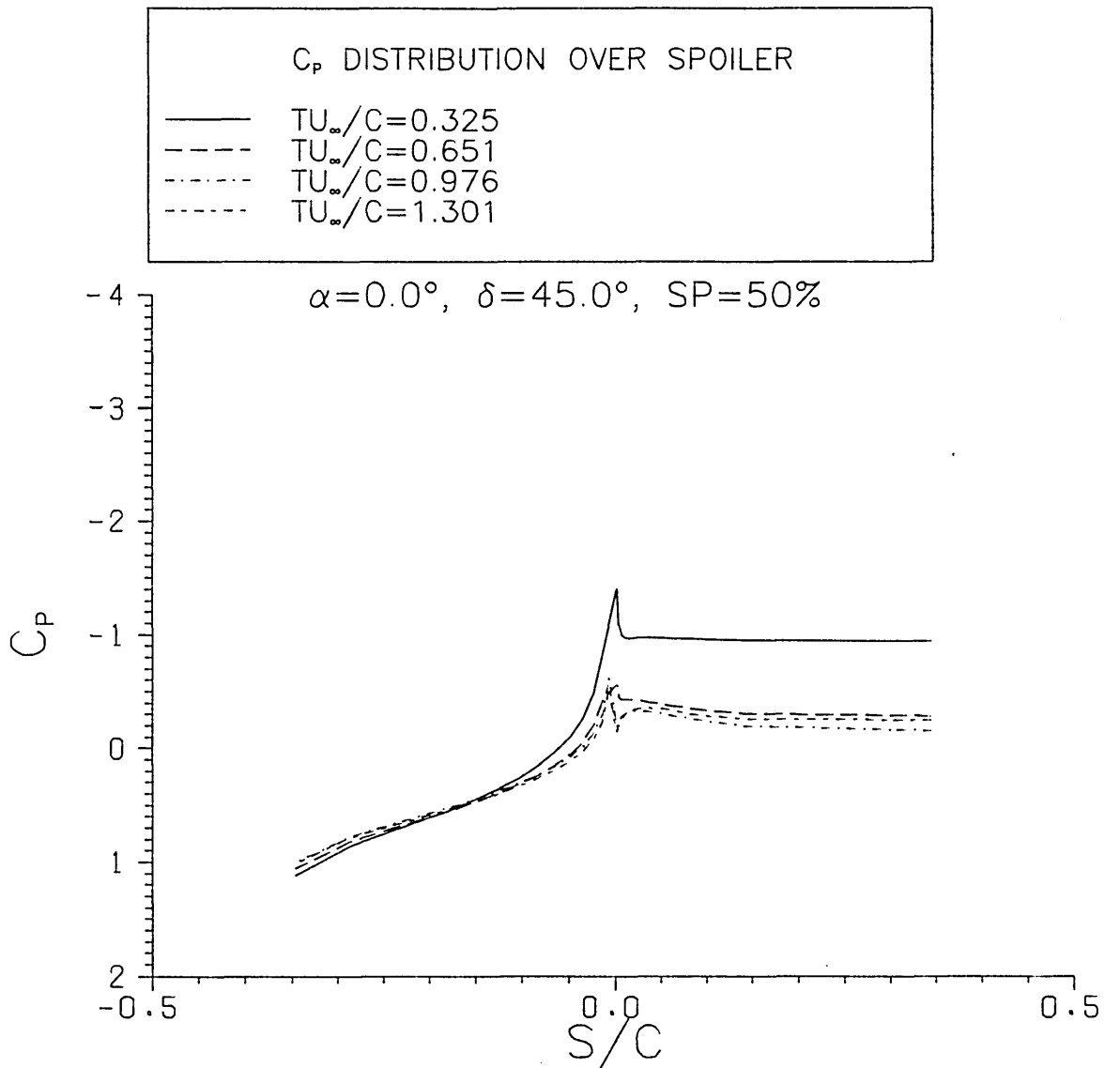
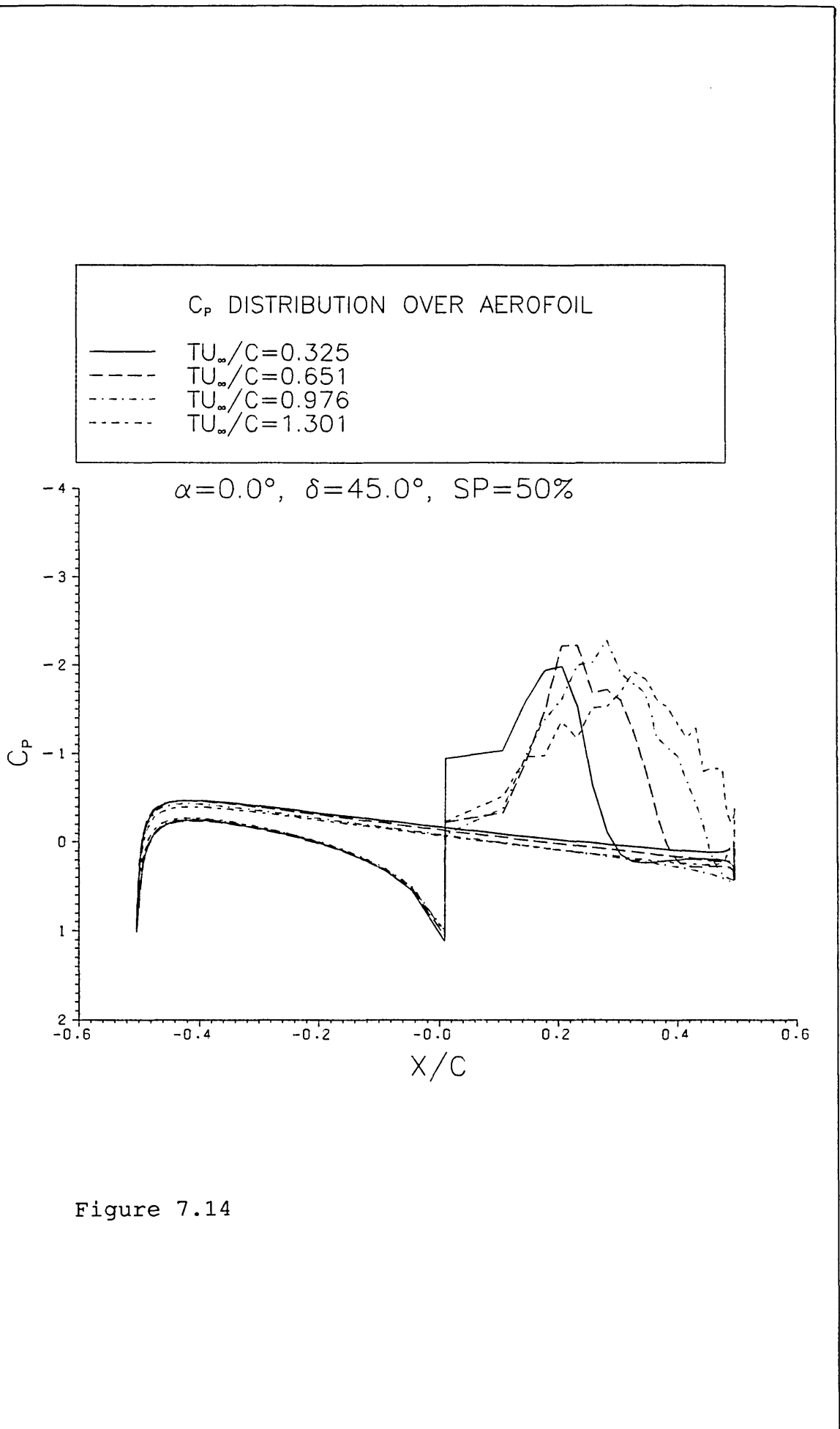


Figure 7.13



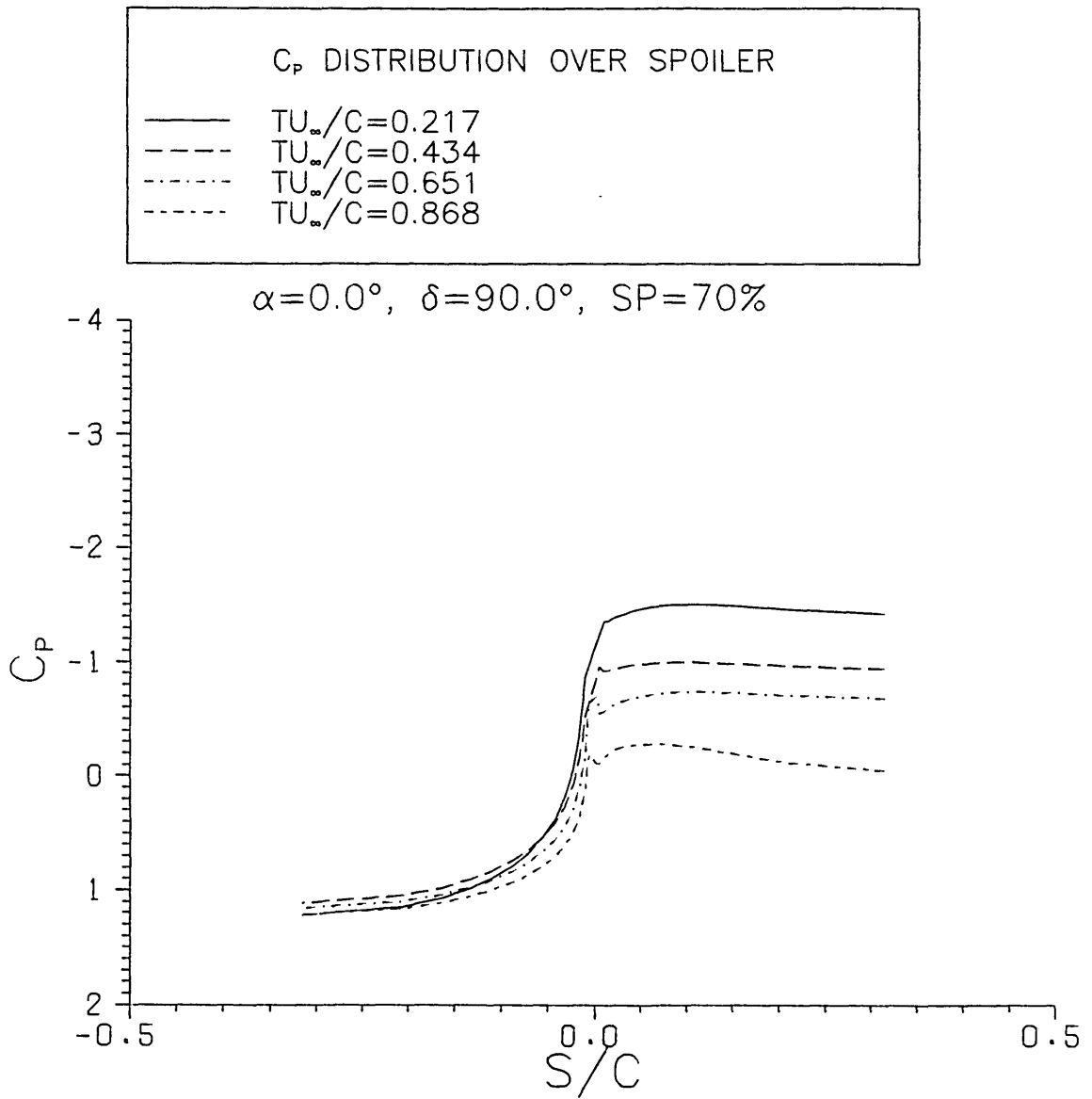


Figure 7.15

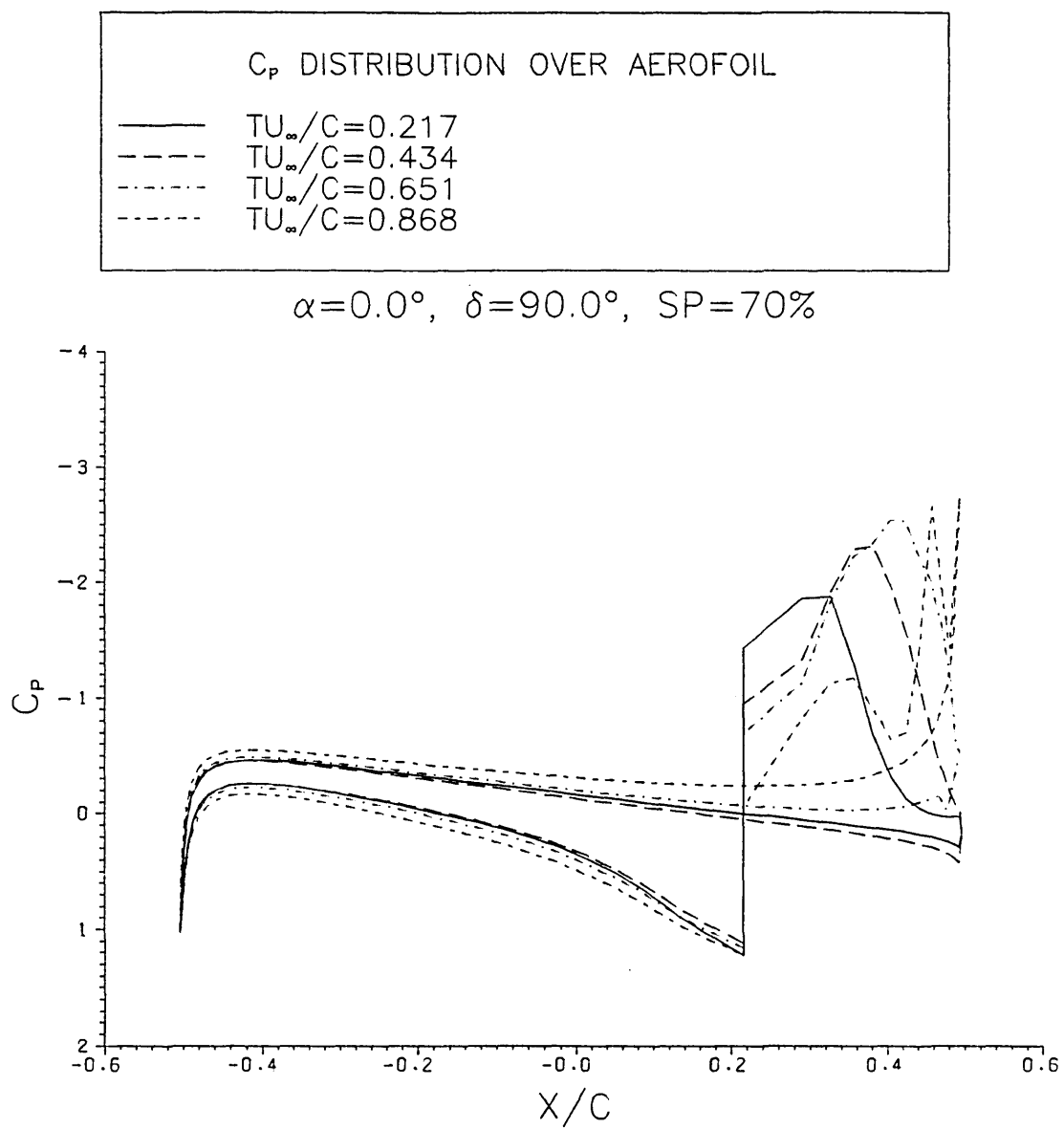
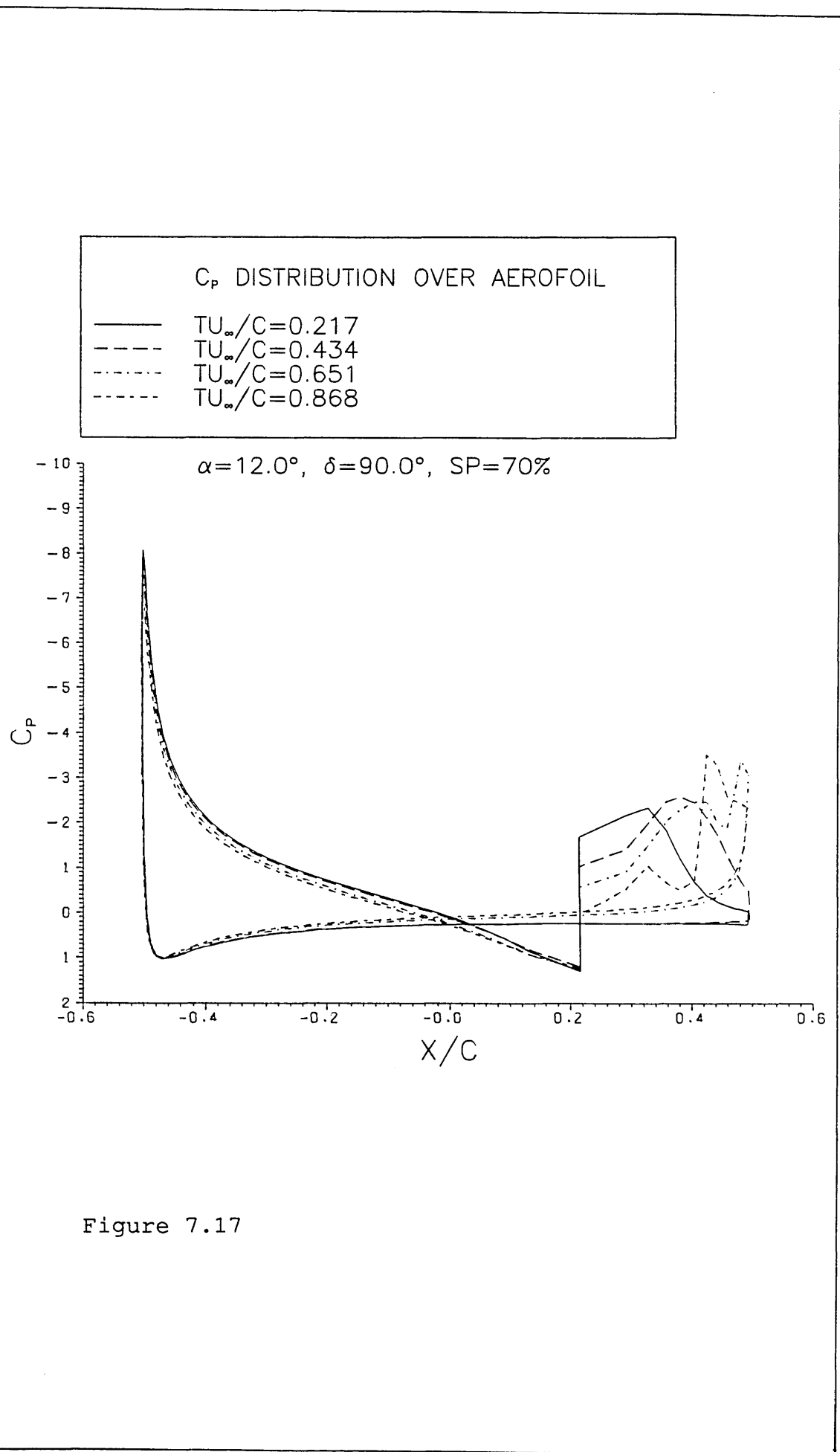


Figure 7.16



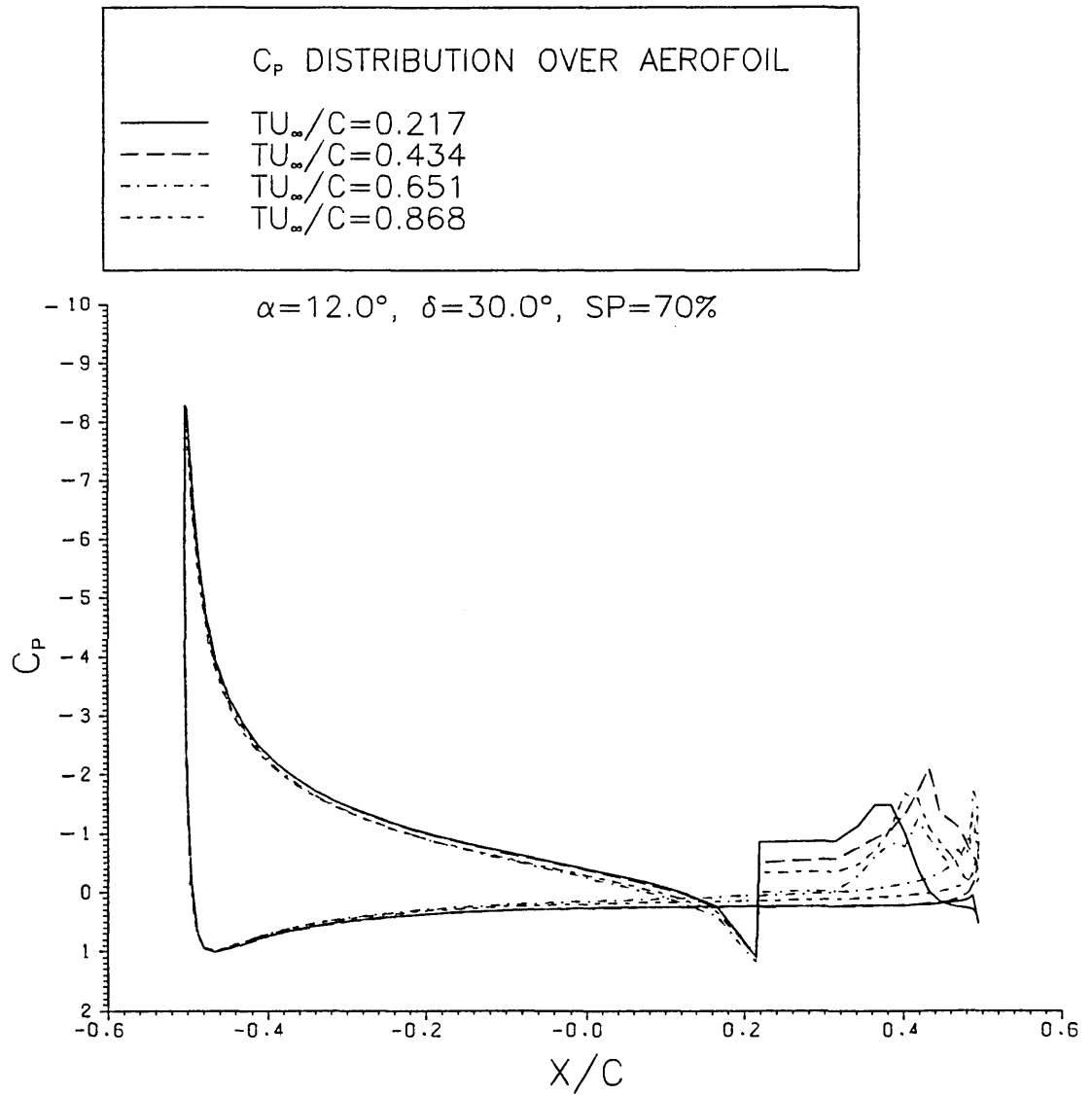


Figure 7.18

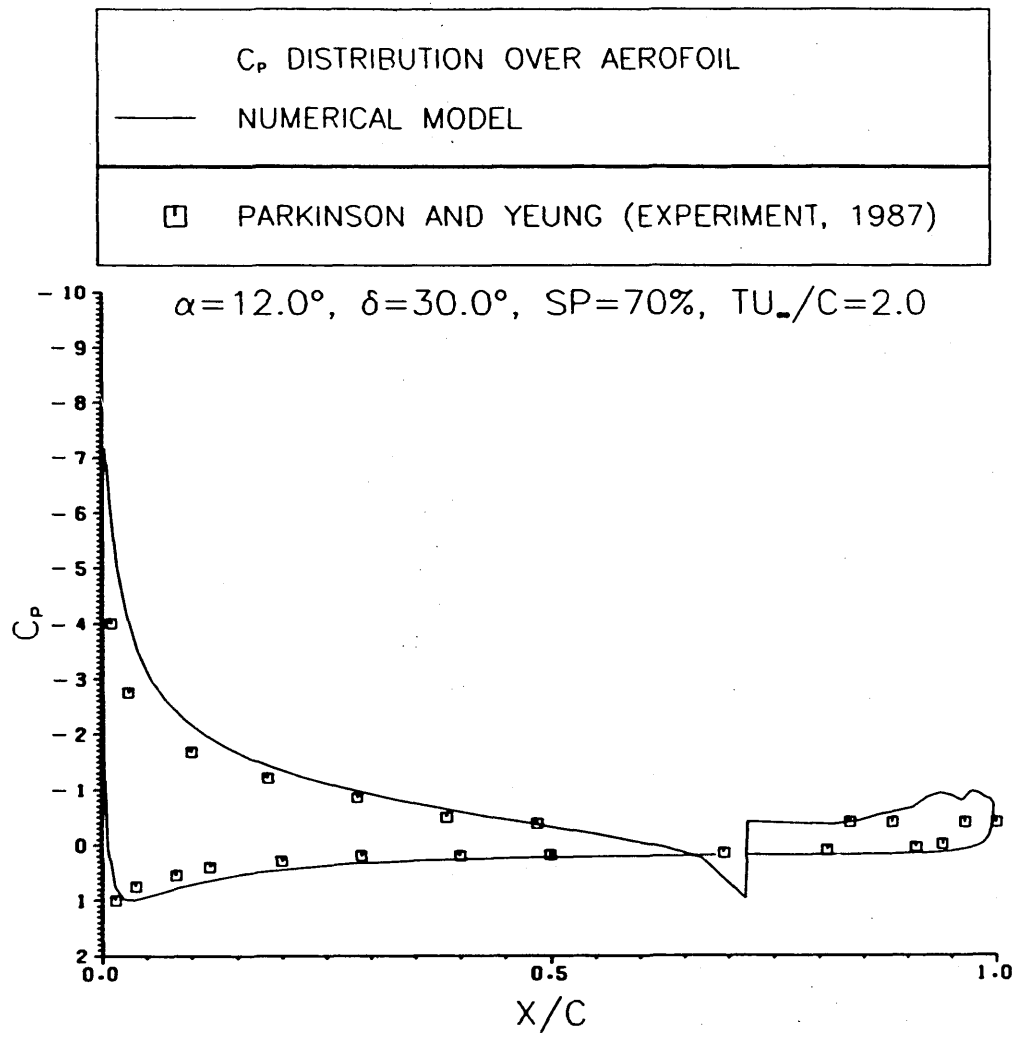


Figure 7.19

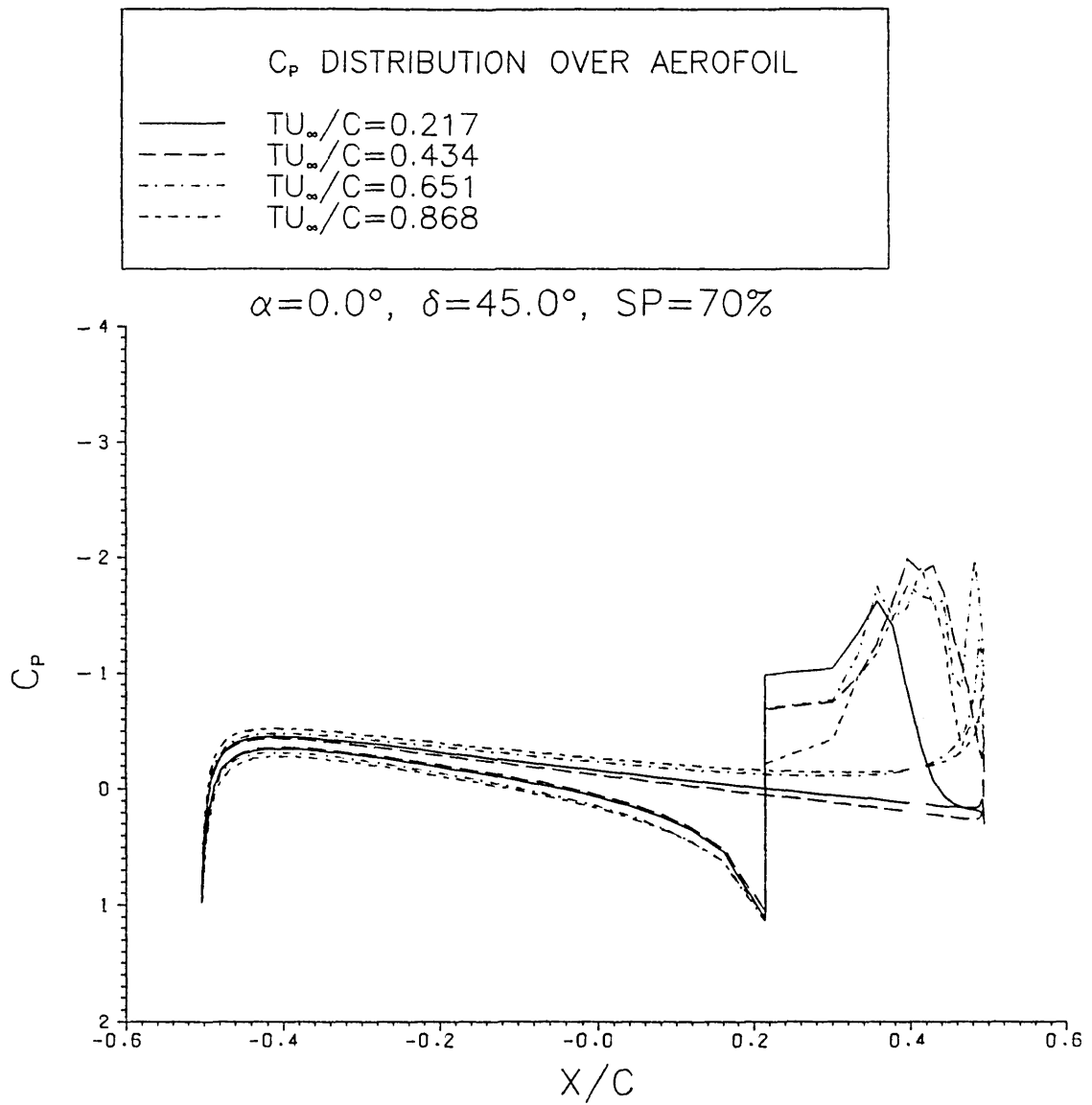
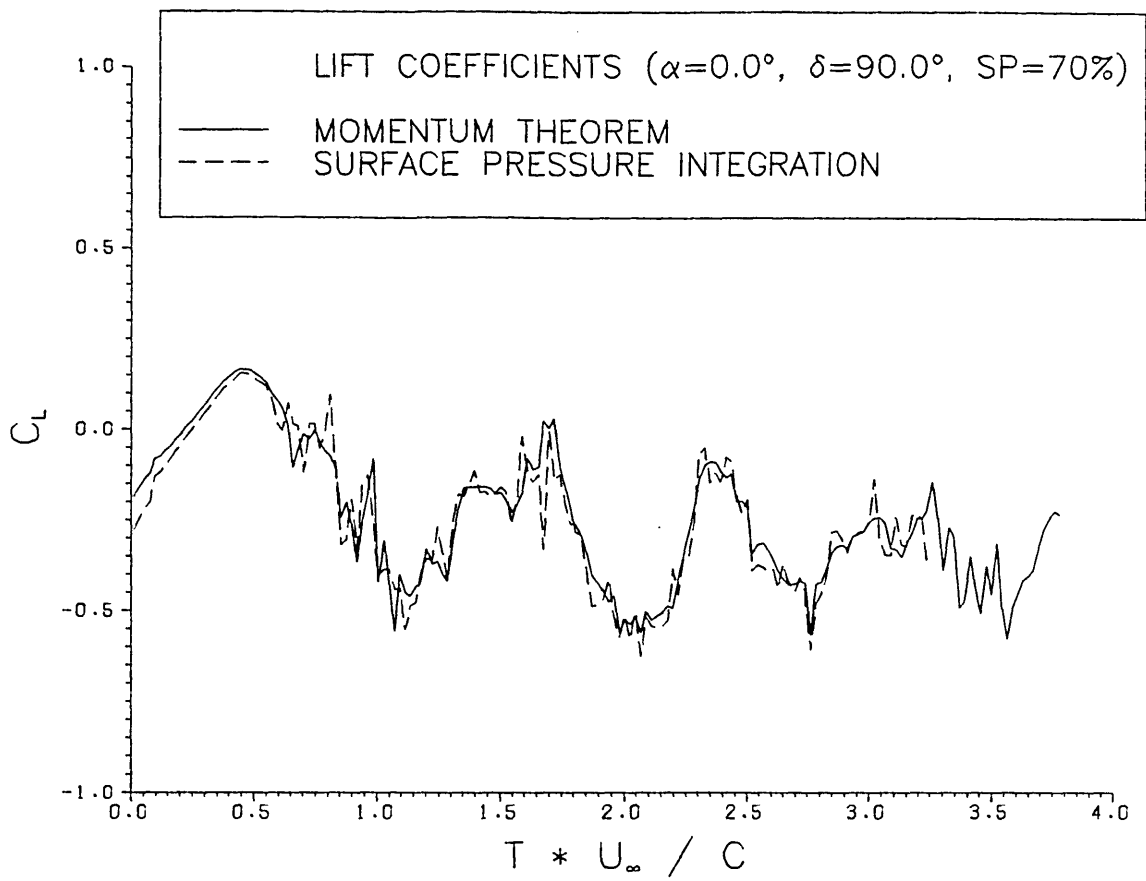
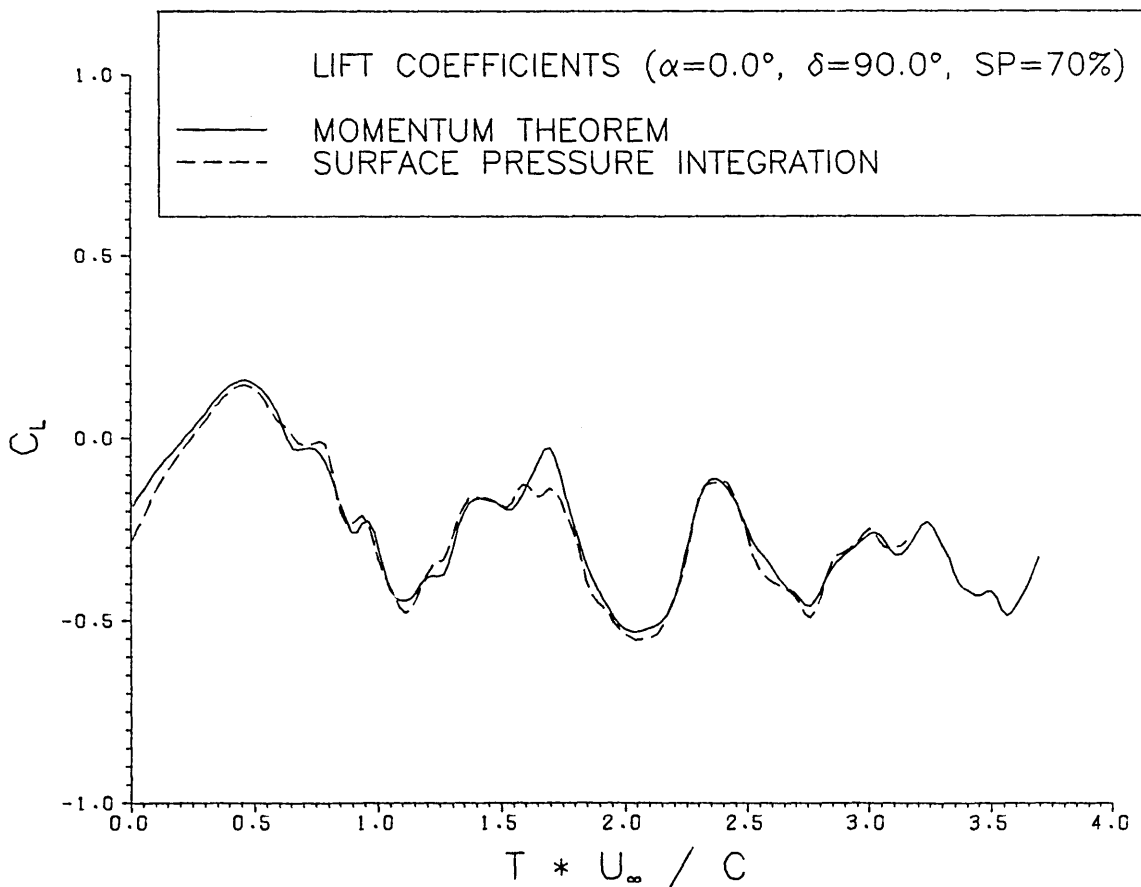


Figure 7.20

Figure 7.21: C_L vs time with 'noise'.Figure 7.22: C_L vs time; 'noise' removed.

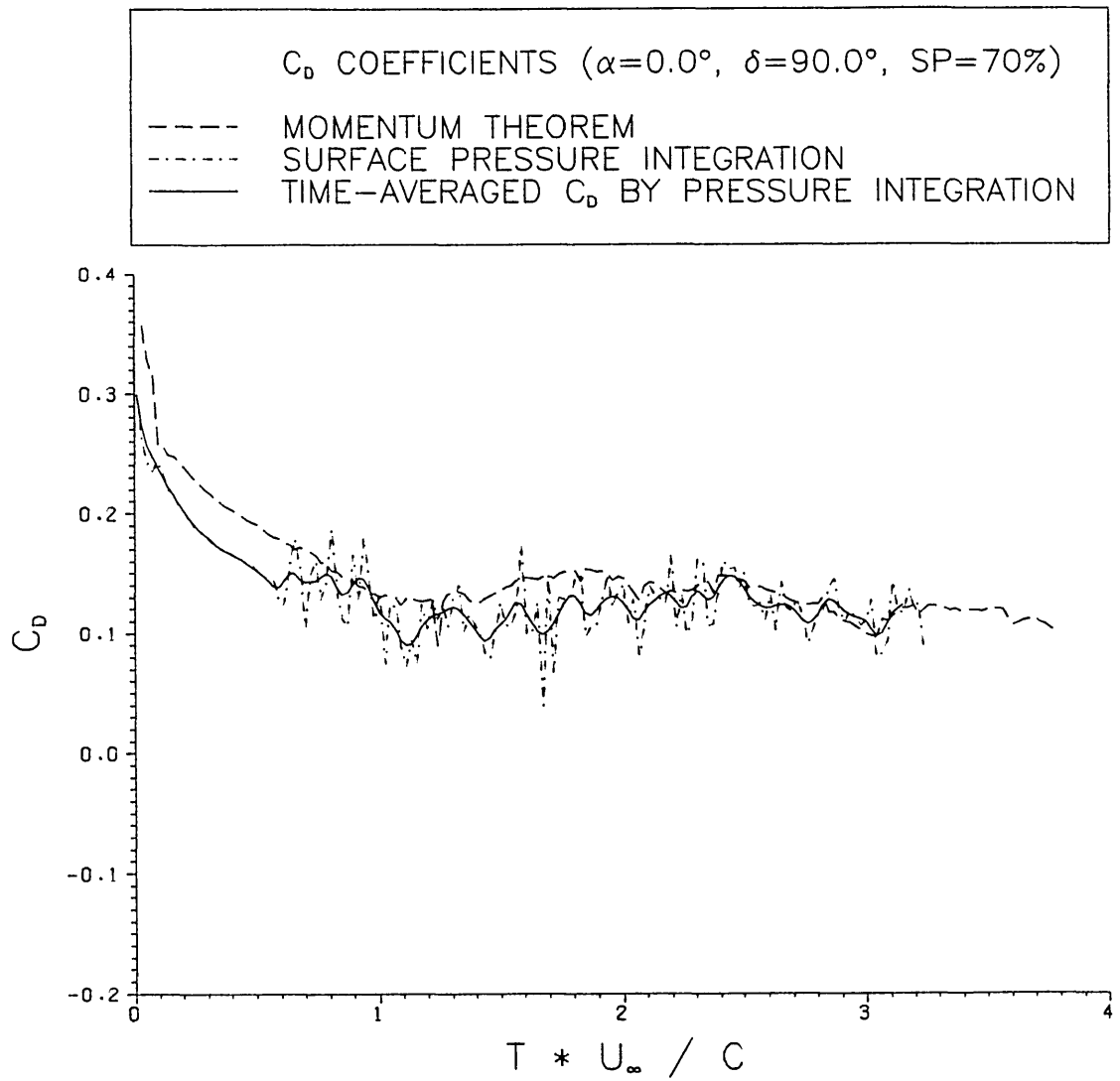
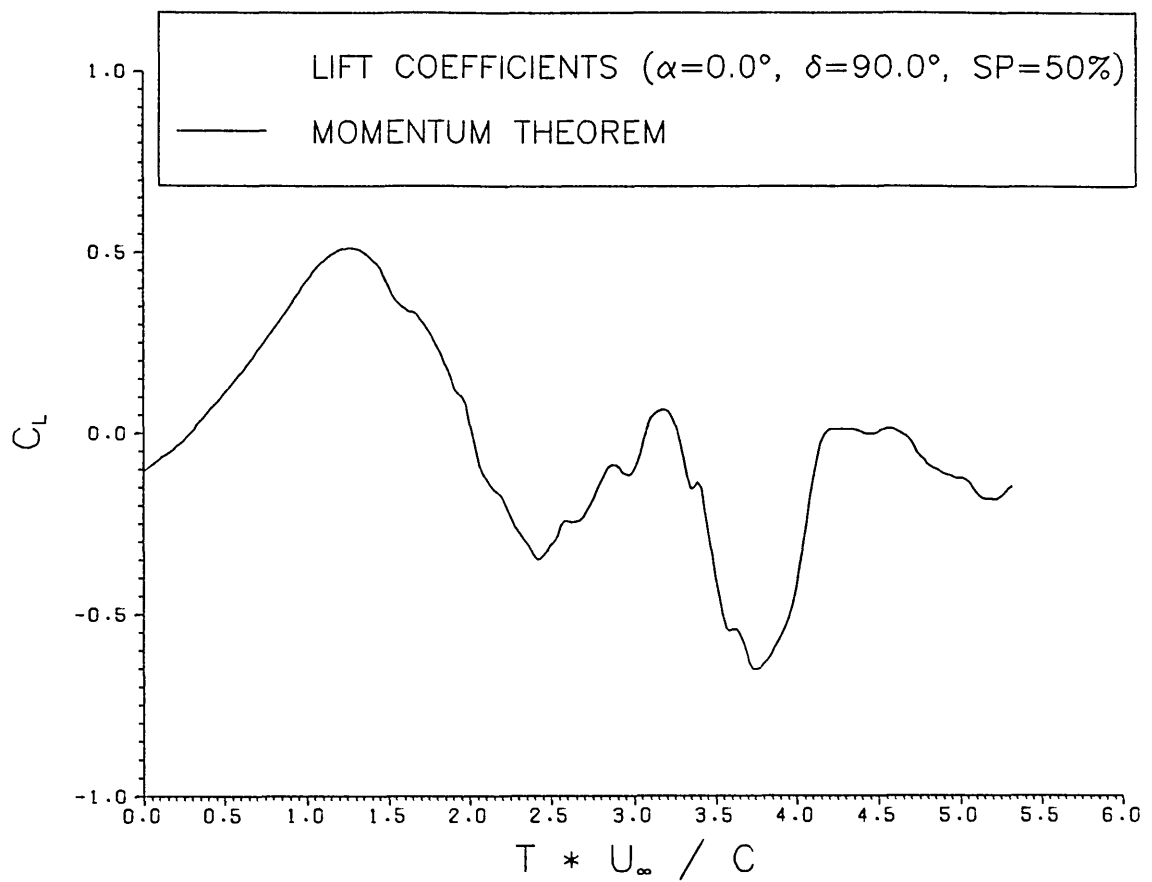
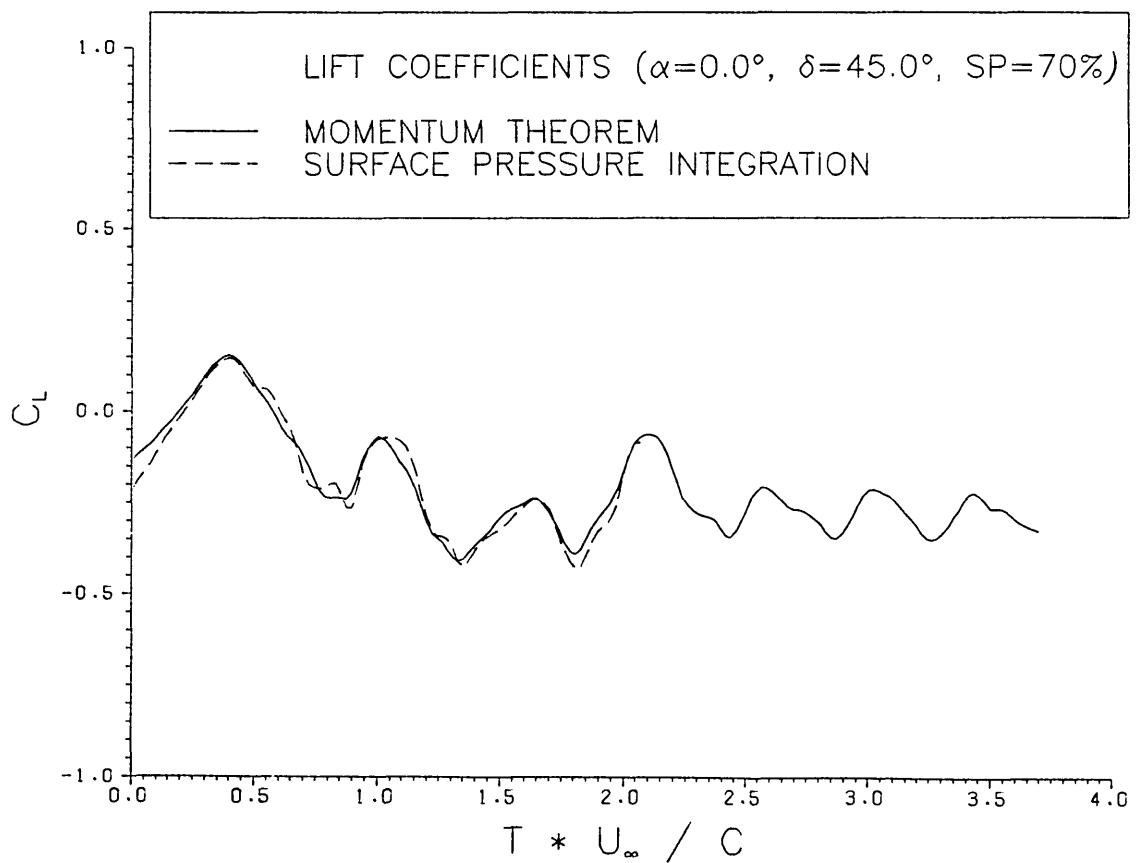


Figure 7.23: C_D vs time.

Figure 7.24: C_L vs time.Figure 7.25: C_L vs time.

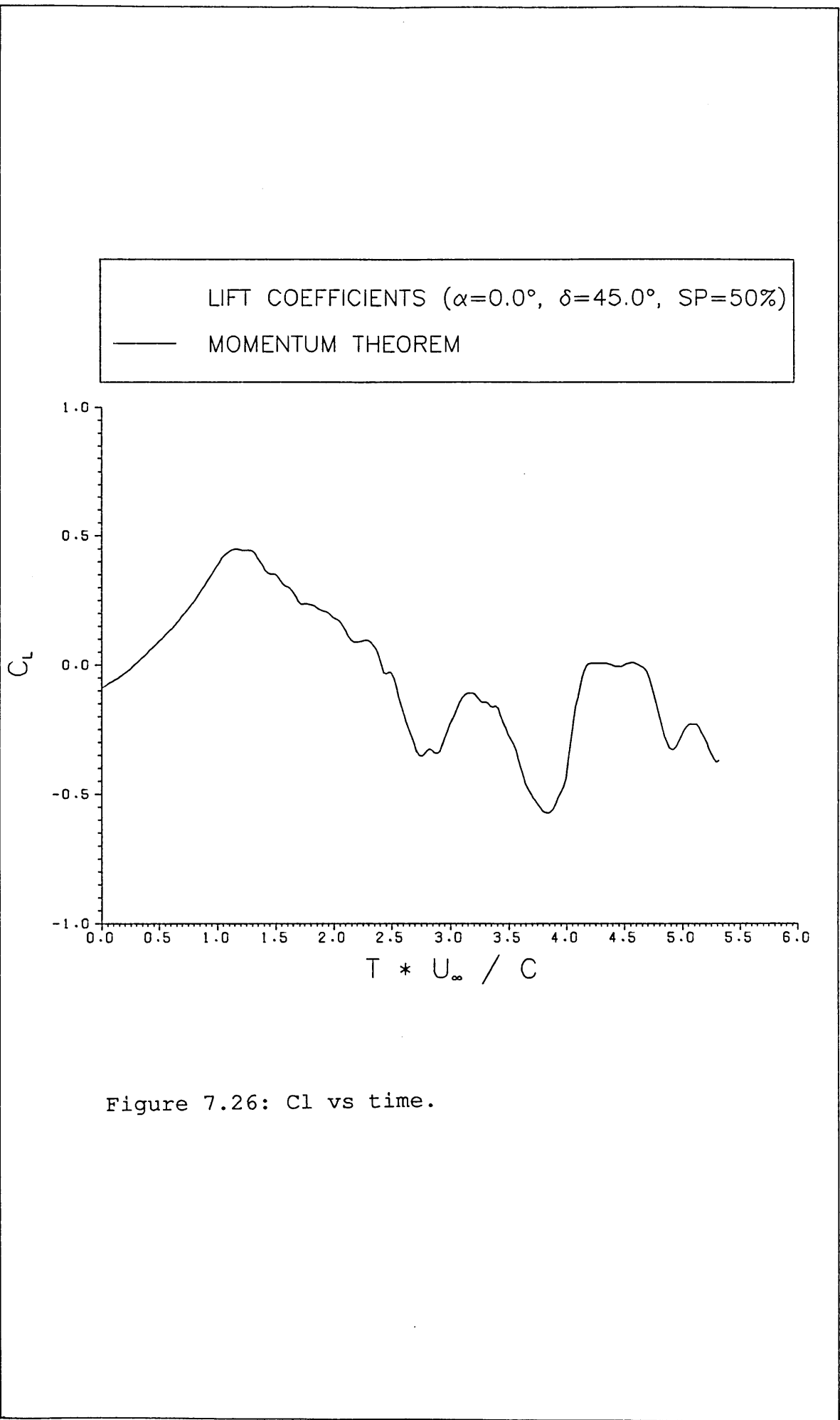


Figure 7.26: C_l vs time.

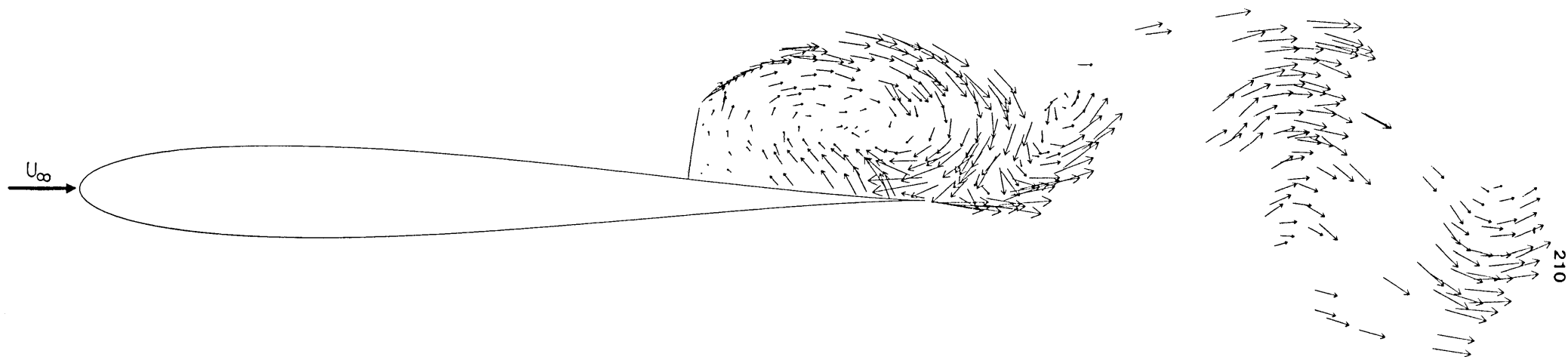


Figure 7.27.a: Numerical near-wake flow visualisation.

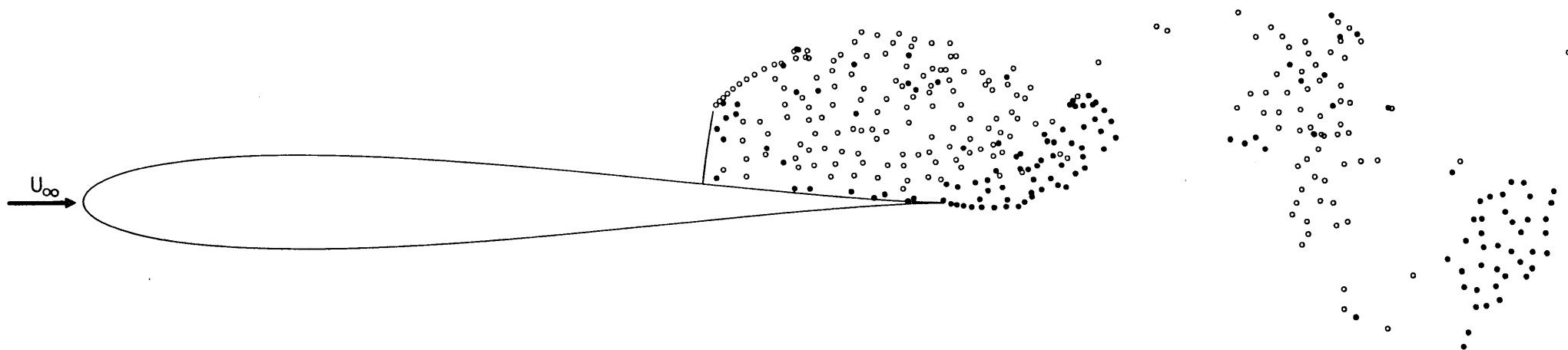


Figure 7.27.b: $\alpha=0$ deg., $\delta=90$ deg., SP=70%c, at $TU_\infty/c=3.62$.

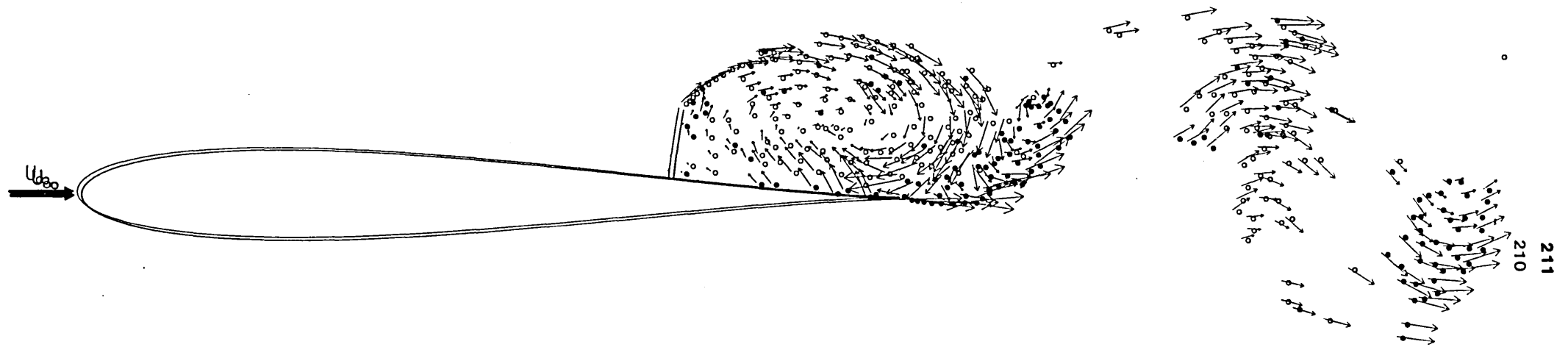


Figure 7.27.a: Numerical near-wake flow
 Figure 7.27.b: $\alpha = 0$ deg, $\alpha = 90$ deg., SP=70%c, at $TU_{\infty}/c=3.62$.

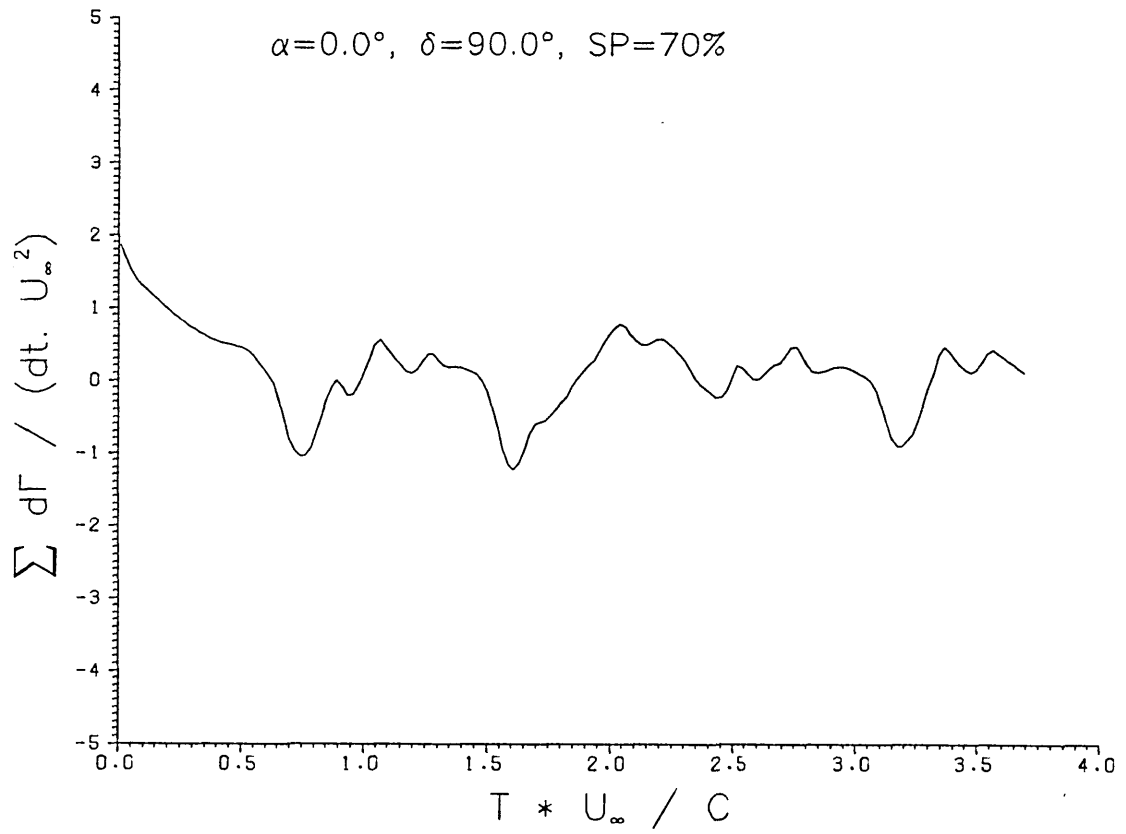


Figure 7.28

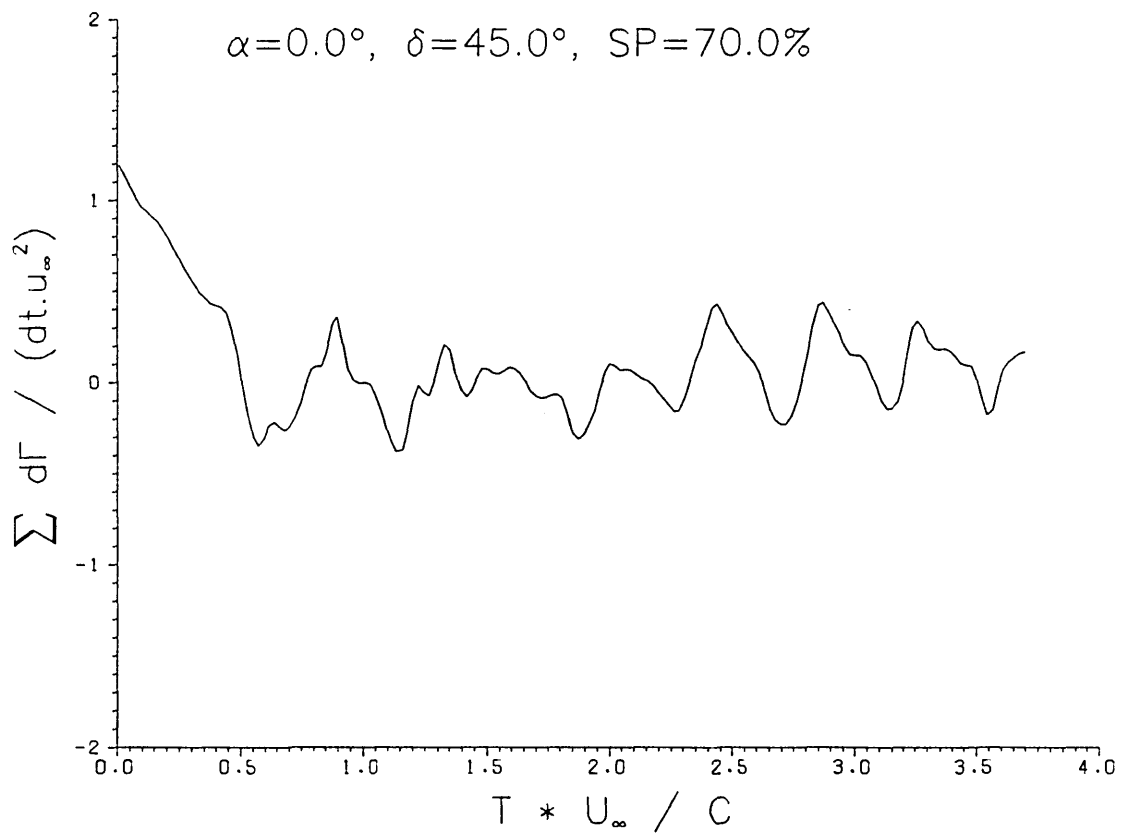


Figure 7.29

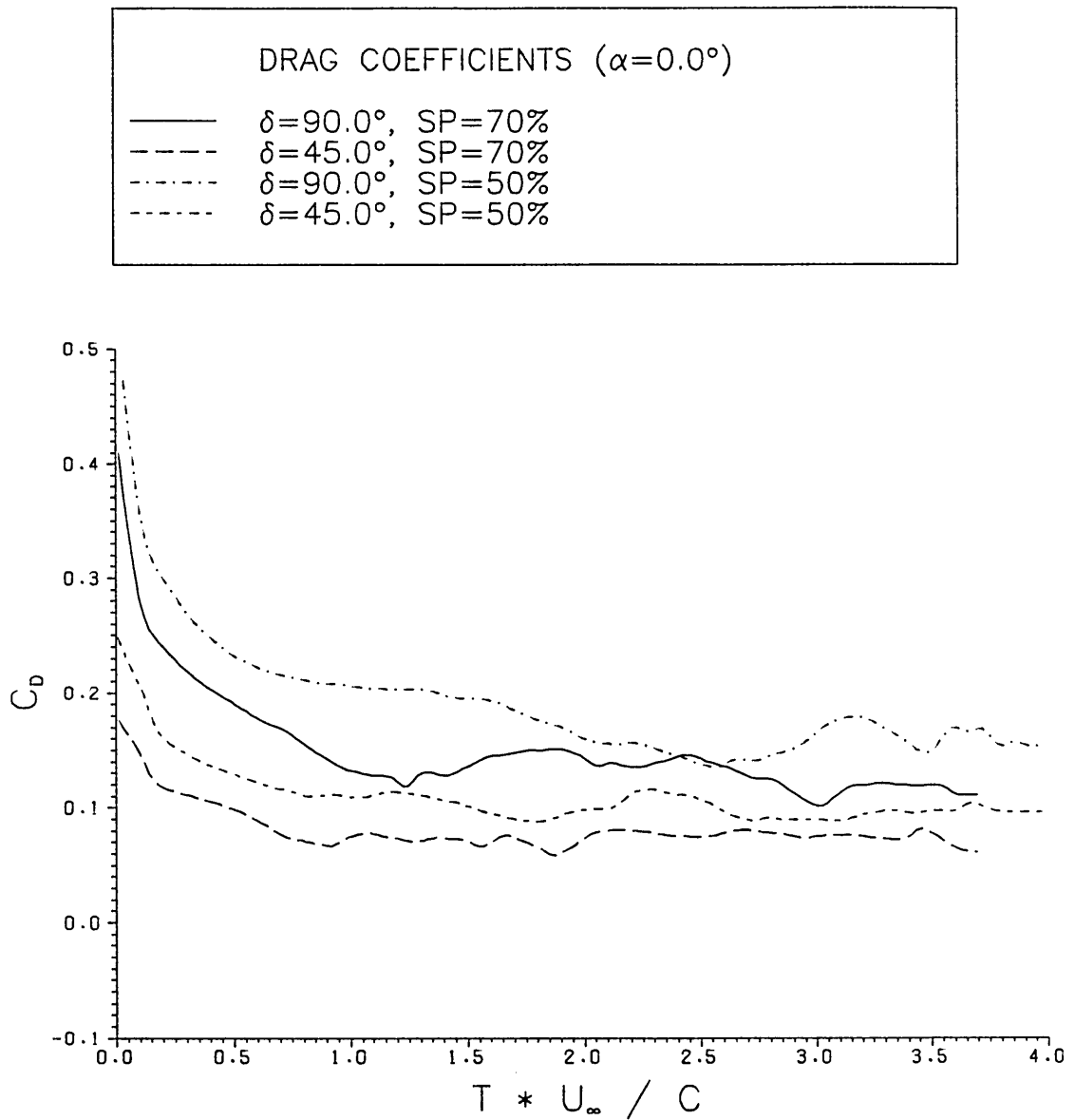


Figure 7.30: C_d vs time for varying spoiler angles and spoiler positions.

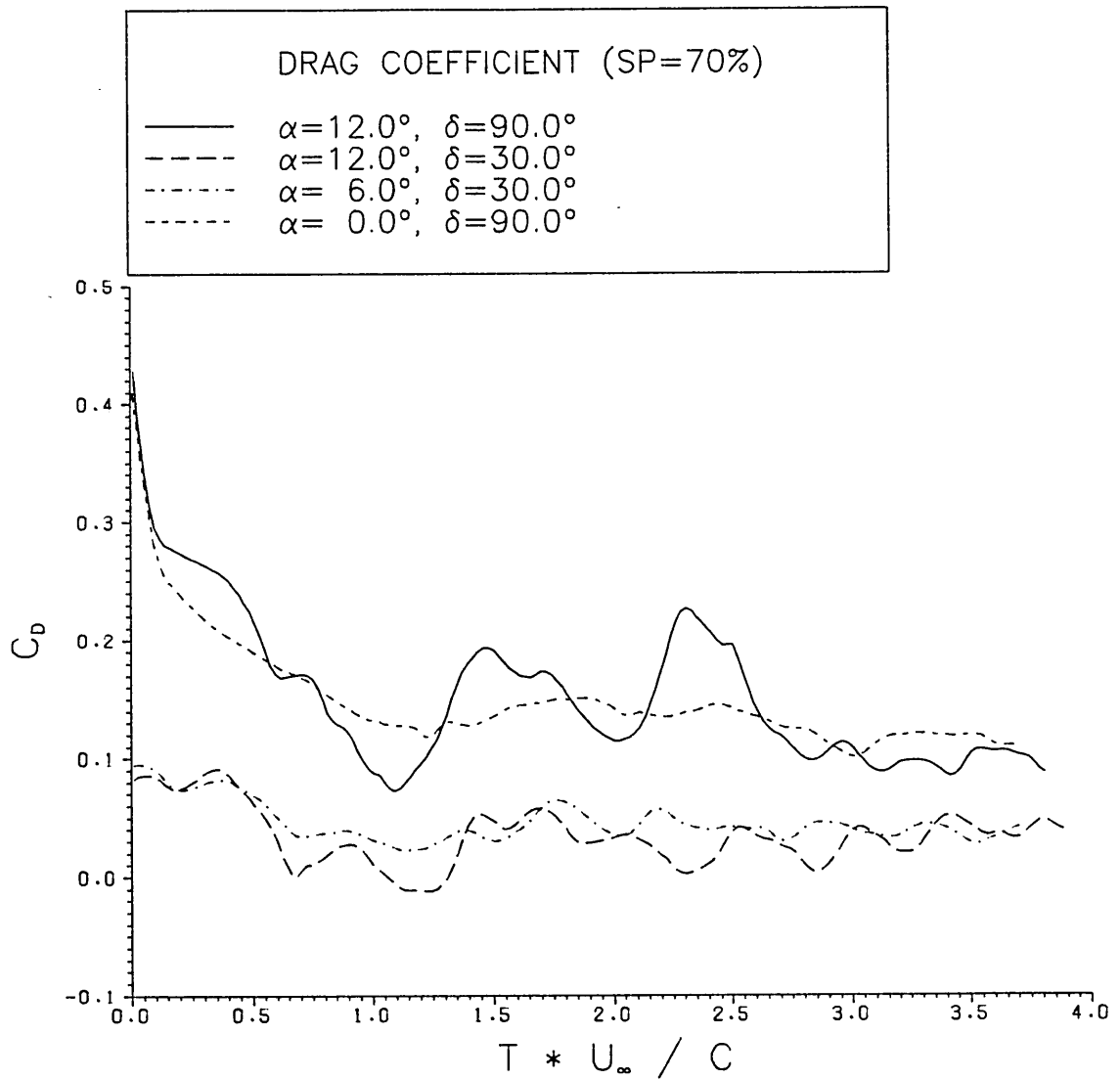


Figure 7.31: C_d vs time for varying aerofoil incidence.



Figure 7.32.a: Numerical wake-flow visualisation.



Figure 7.32.b: $\alpha = 0$ deg., $\delta = 90$ deg., SP=50%c, at $TU_\infty / c = 5.5$.



Figure 7.32.b: $\alpha = 0$ deg., $\delta = 90$ deg., $SP = 50\%c$, at $TU_\infty / c = 5.5$.
 Figure 7.32.a: Numerical wake-flow visualisation.

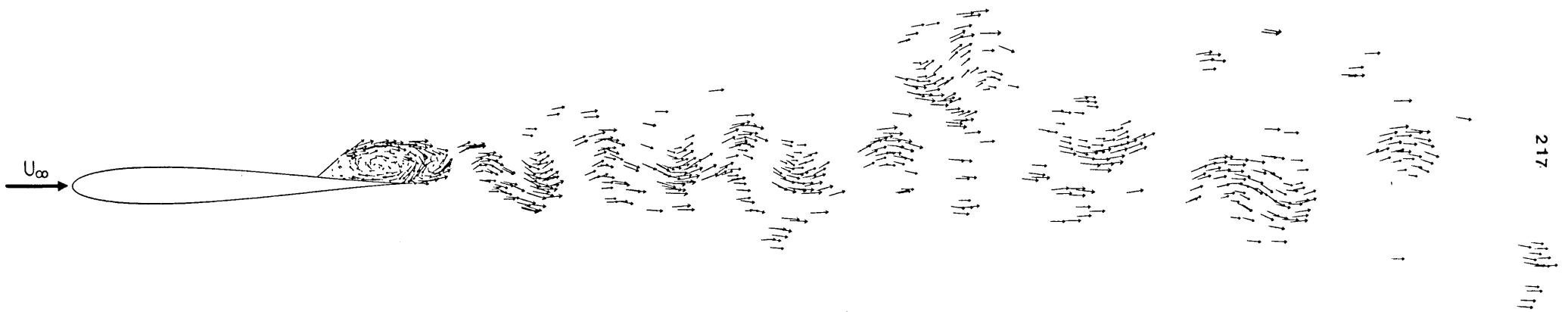


Figure 7.33.a: Numerical wake-flow visualisation.

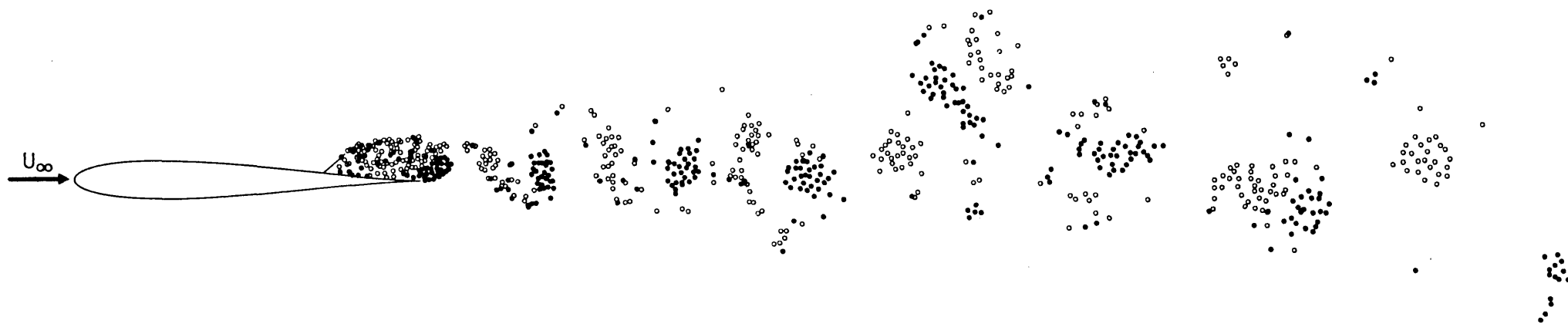


Figure 7.33.b: $\alpha=0$ deg., $\delta=45$ deg., $SP=70\%c$, at $TU_\infty/c=3.8$.

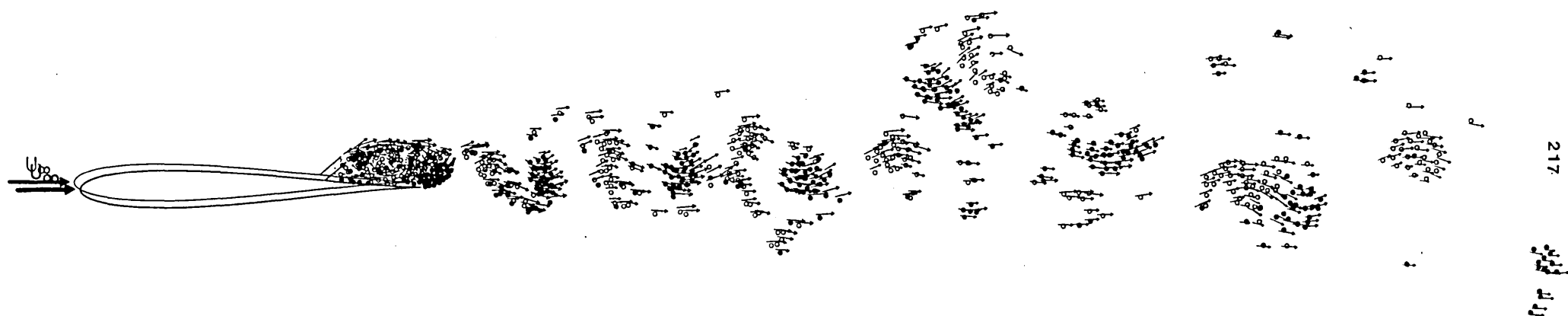


Figure 7.33.b.a: Numerical wake flow visualisation. $\delta = 45^\circ$, $SP = 70\%$ at $TU_\infty / c = 3.8$.

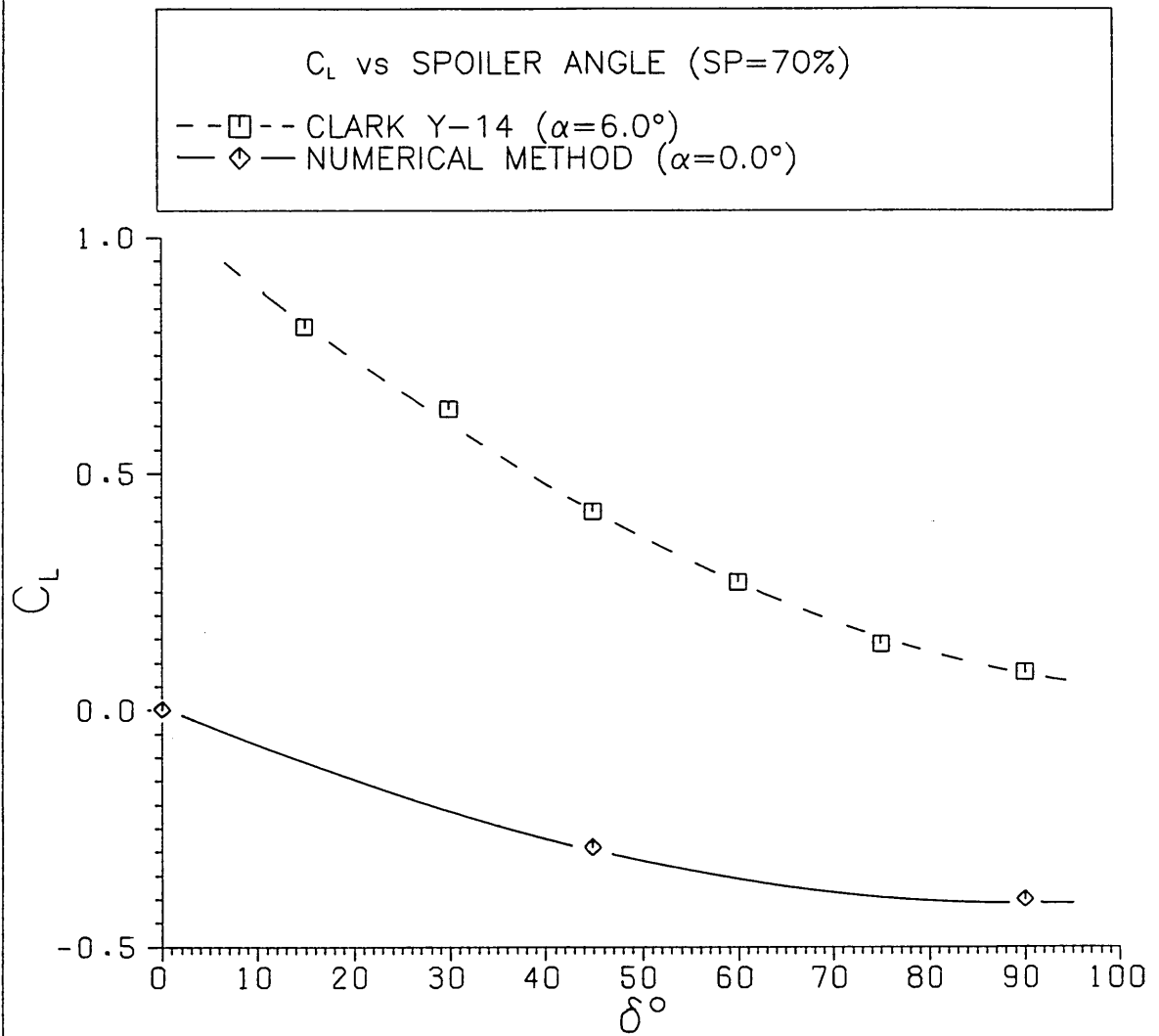
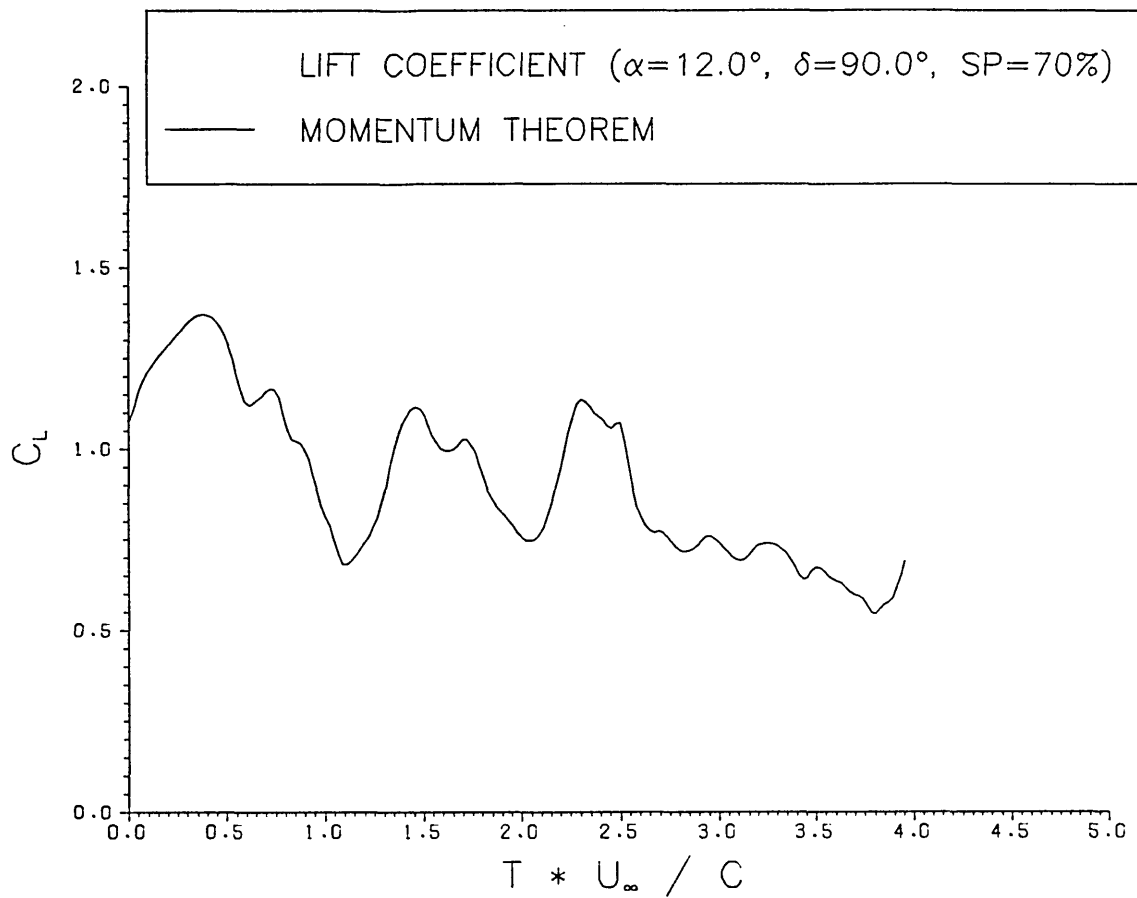
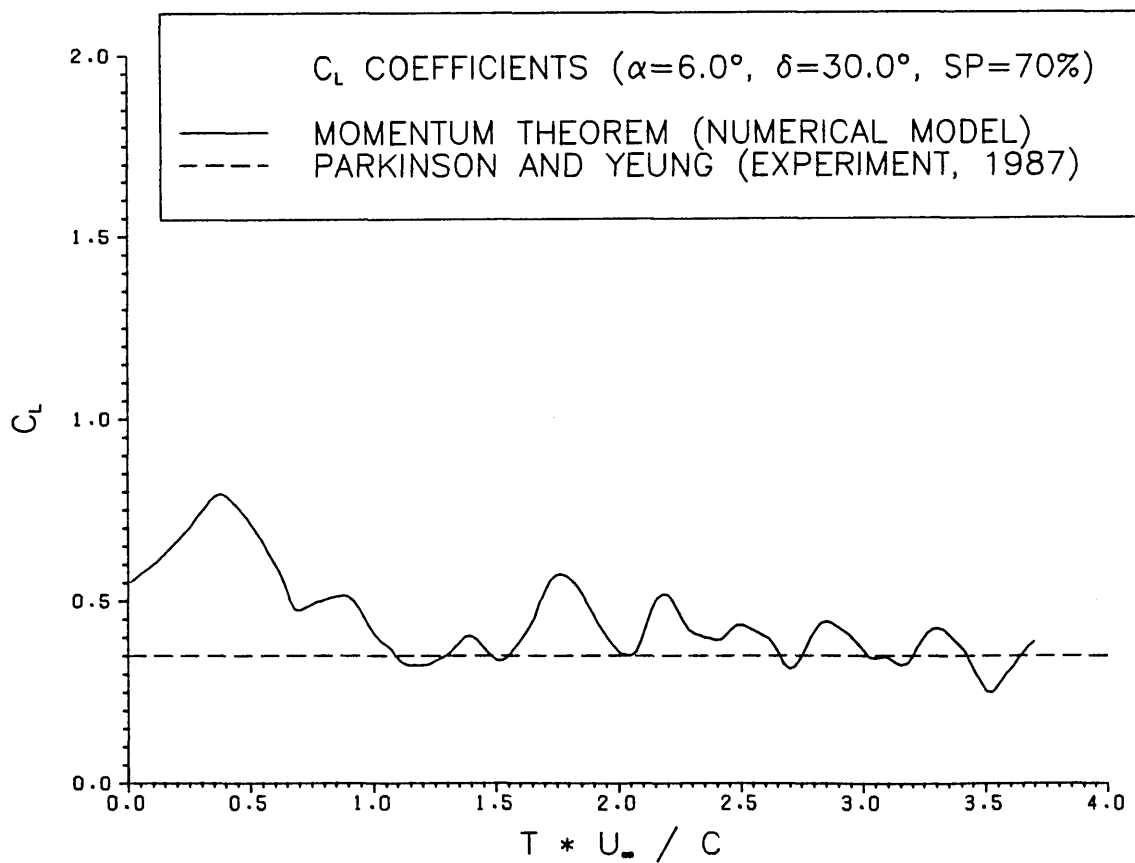


Figure 7.34

Figure 7.35: C_L vs time.Figure 7.36: C_L vs time.

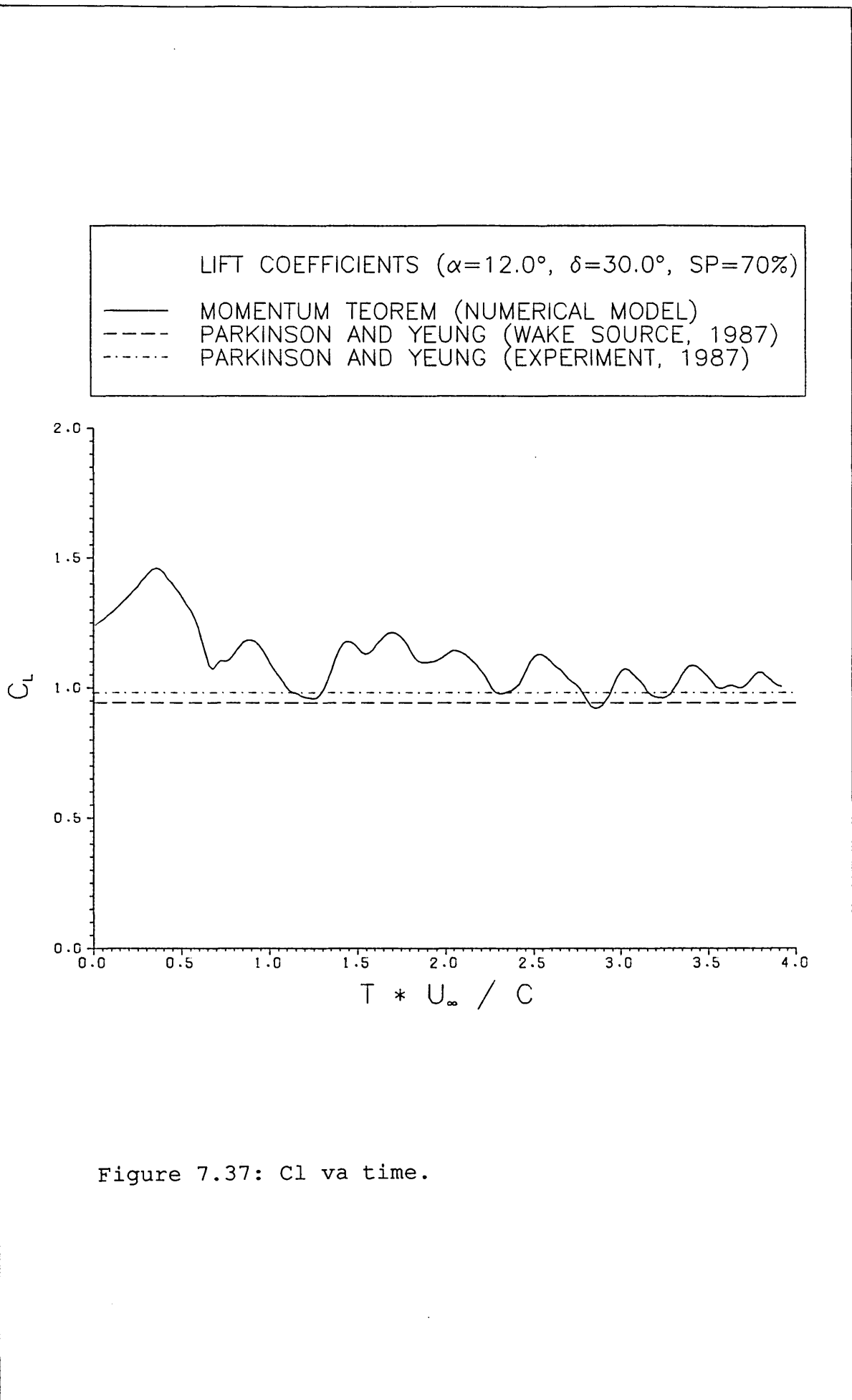


Figure 7.37: C_l vs time.

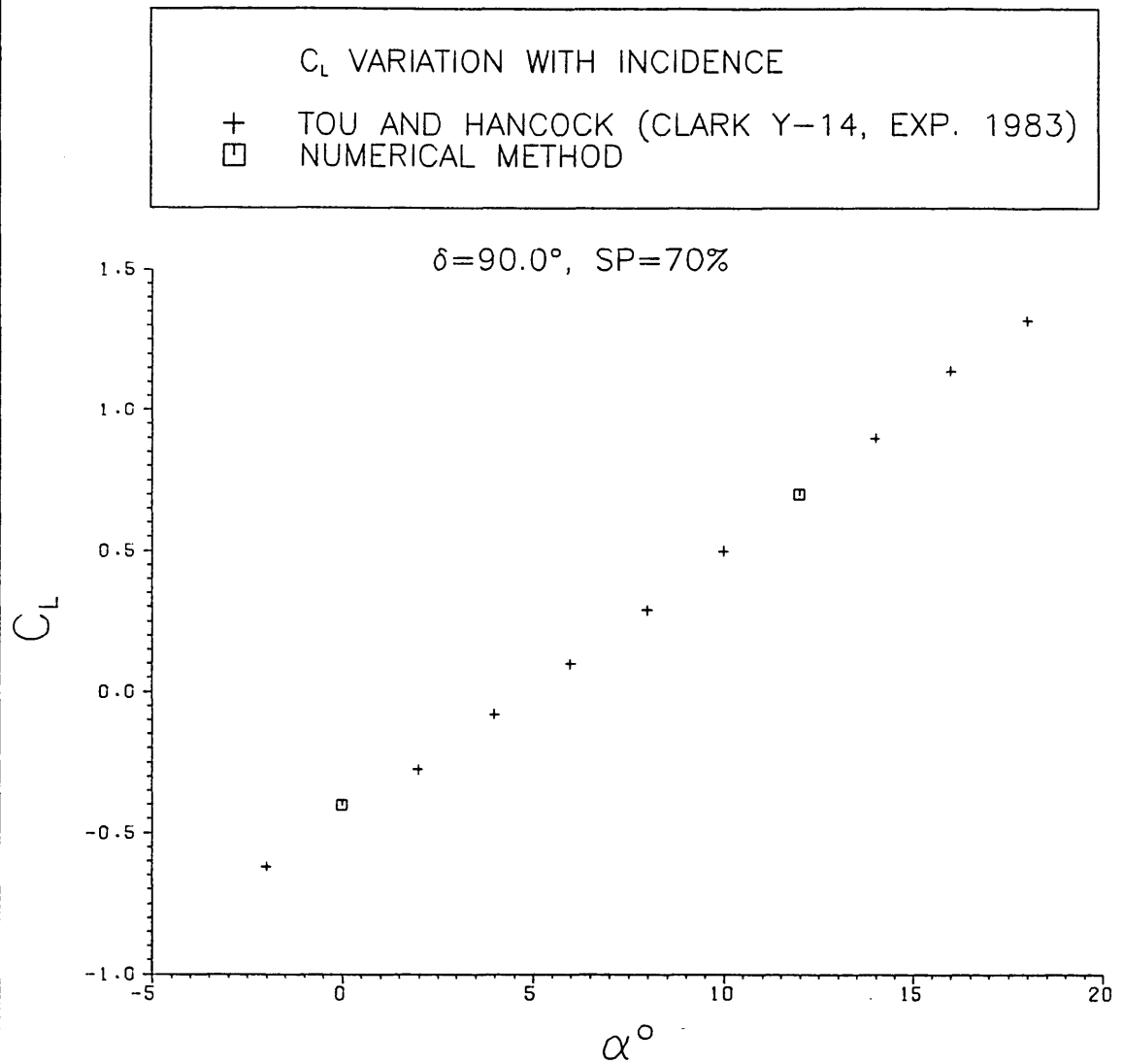


Figure 7.38: C_L variation with incidence.

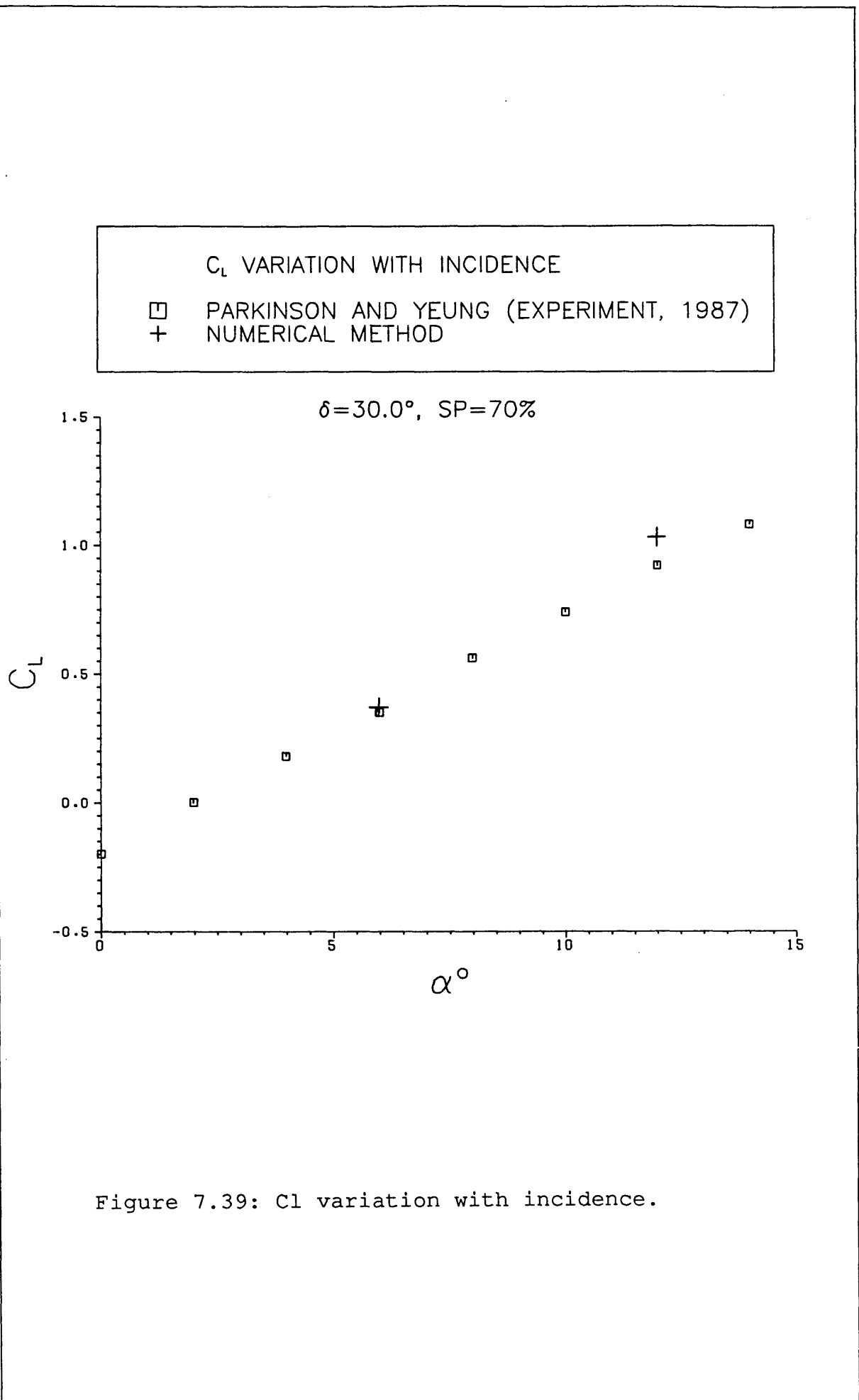


Figure 7.39: C_L variation with incidence.

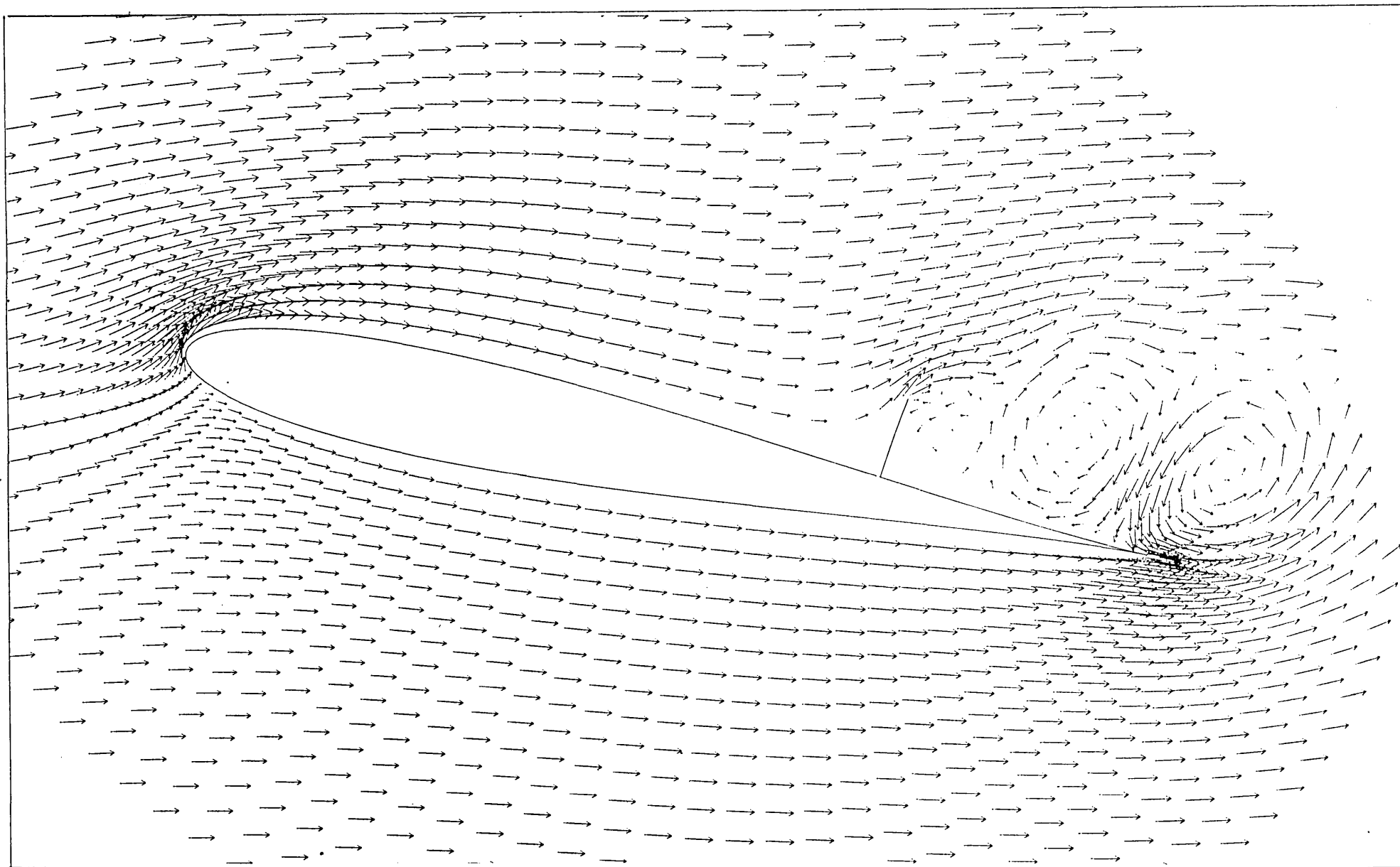


Figure 7.40.a: Numerical visualisation of the flow field over the aerofoil.

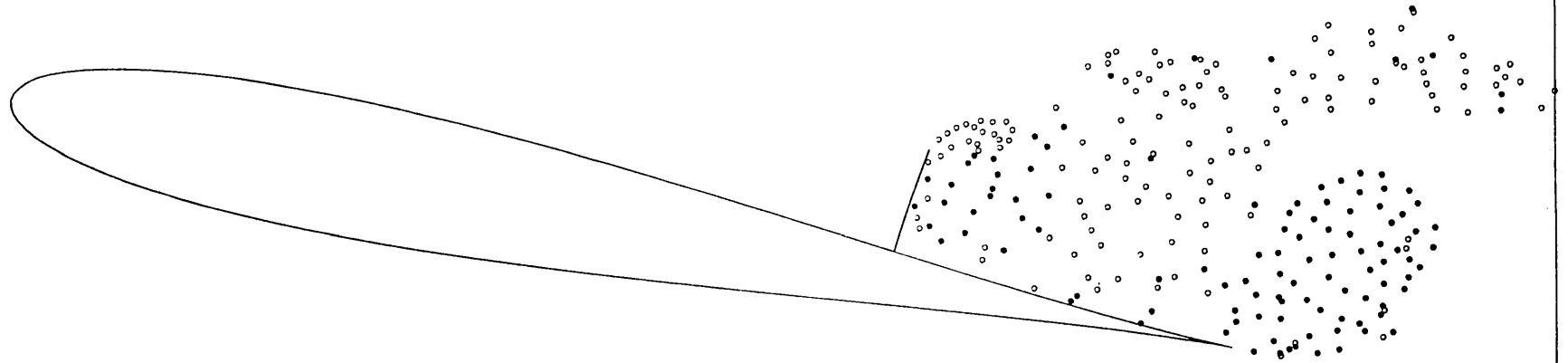


Figure 7.40.b: $\alpha=12$ deg., $\delta=90$ deg., SP=70%C, at $TU_{\infty}/c=4.0$.

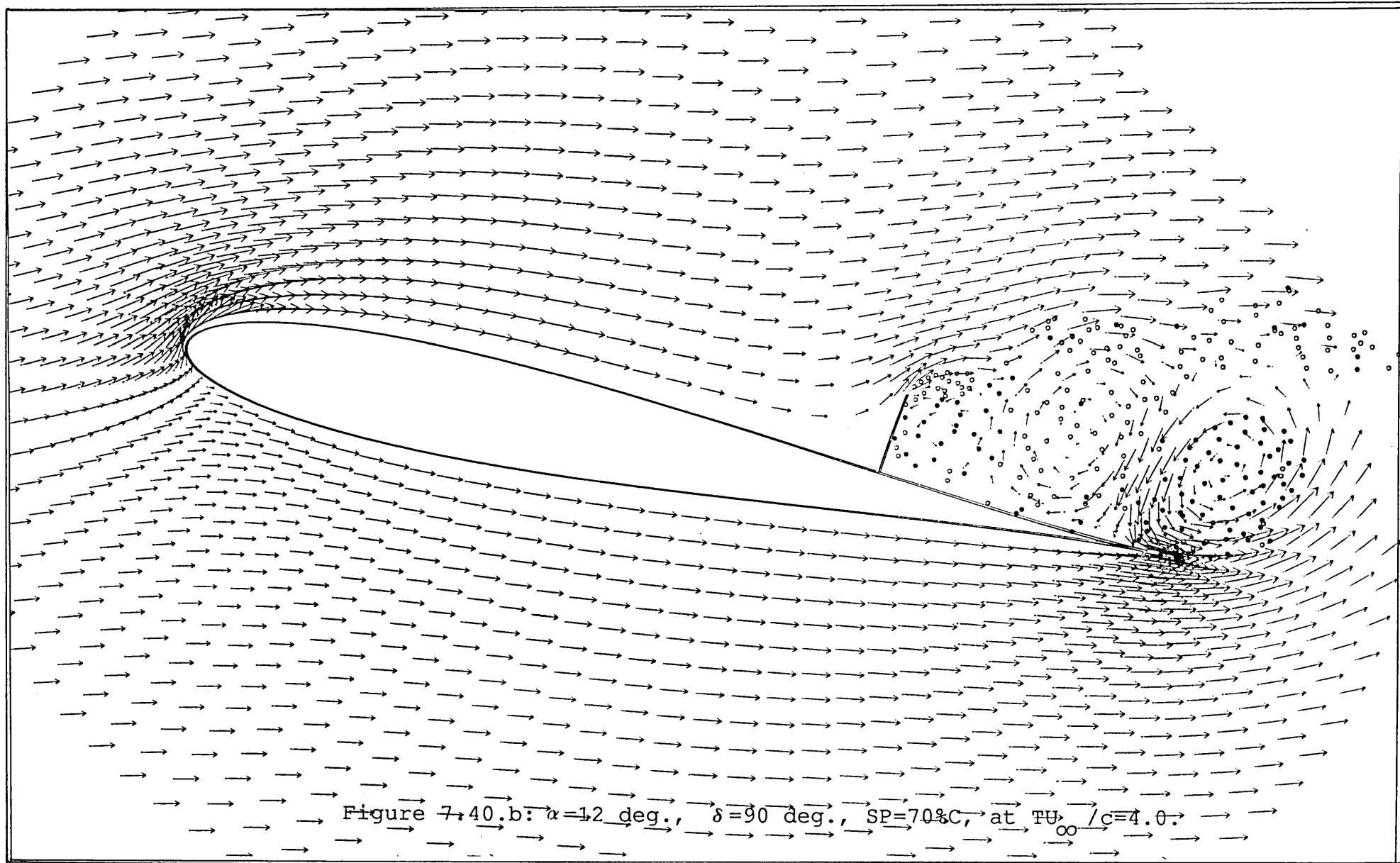


Figure 7.40.a: Numerical visualisation of the flow field over the aerofoil.

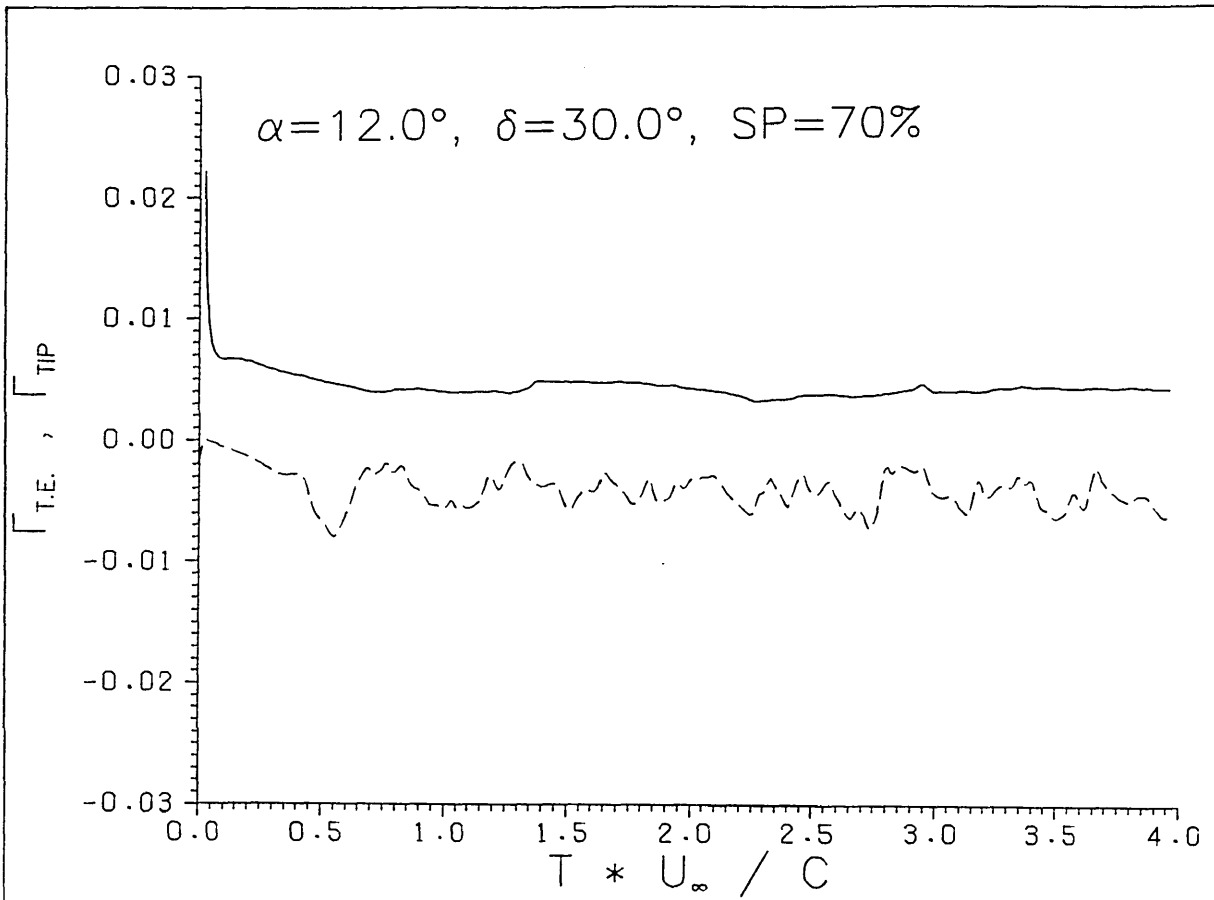


Figure 7.41: Vortex strength vs time.

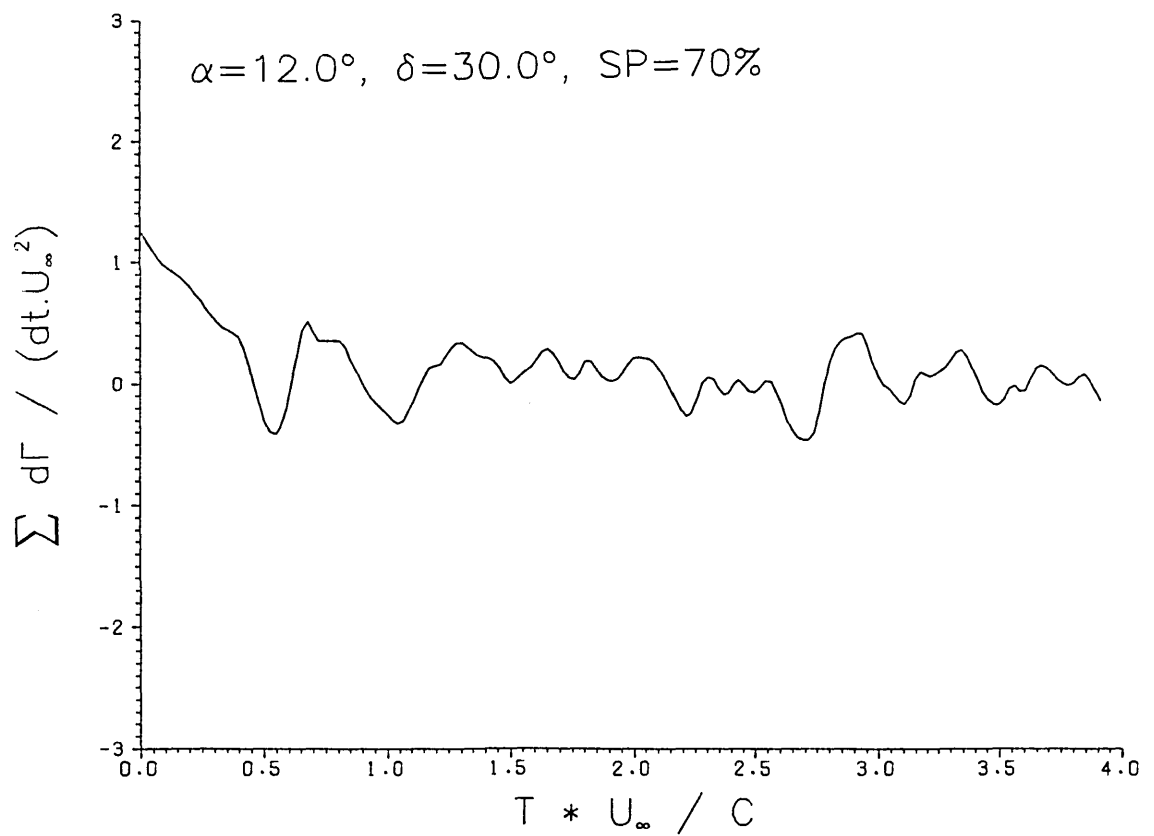


Figure 7.42

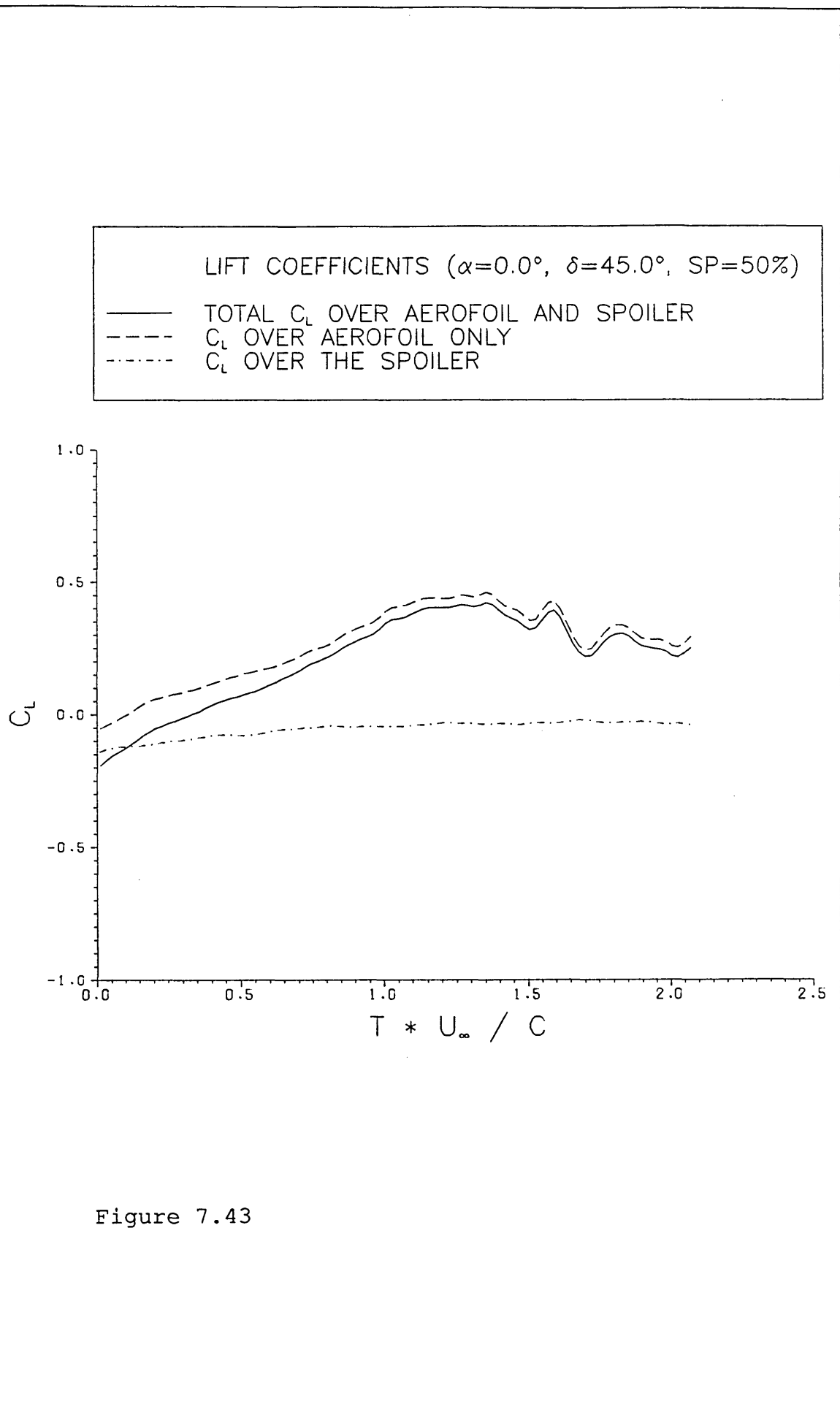


Figure 7.43

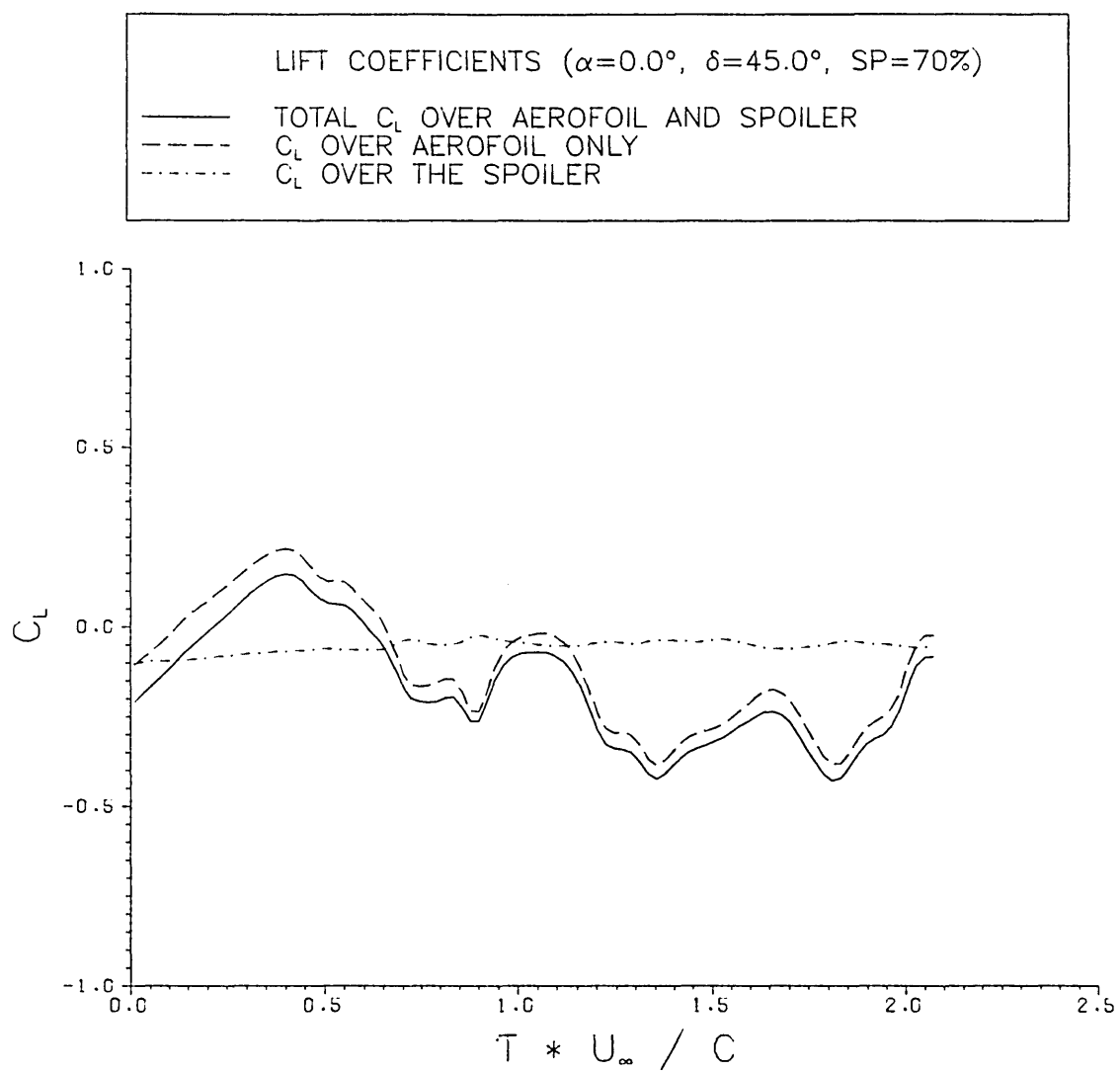


Figure 7.44

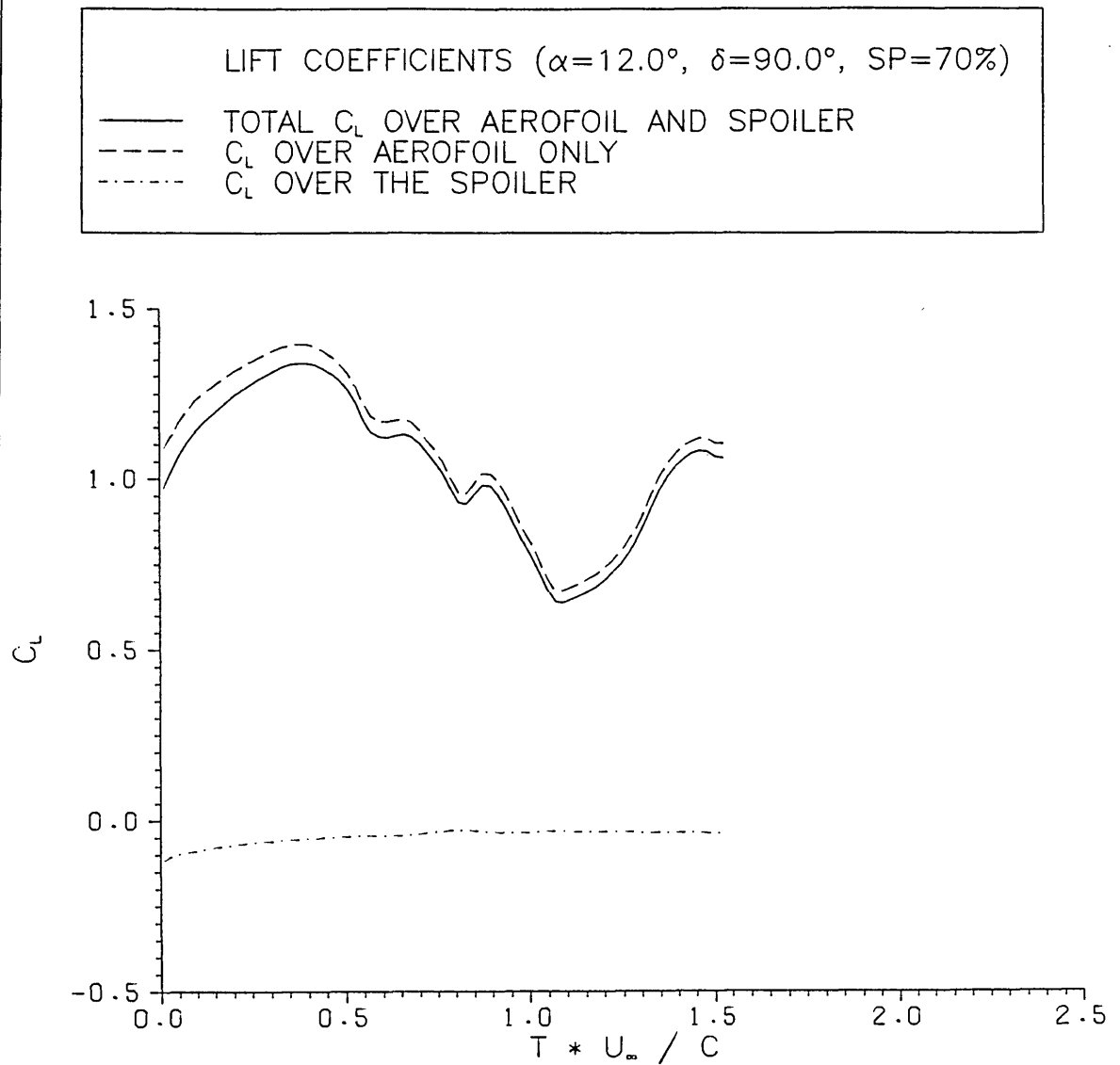


Figure 7.45

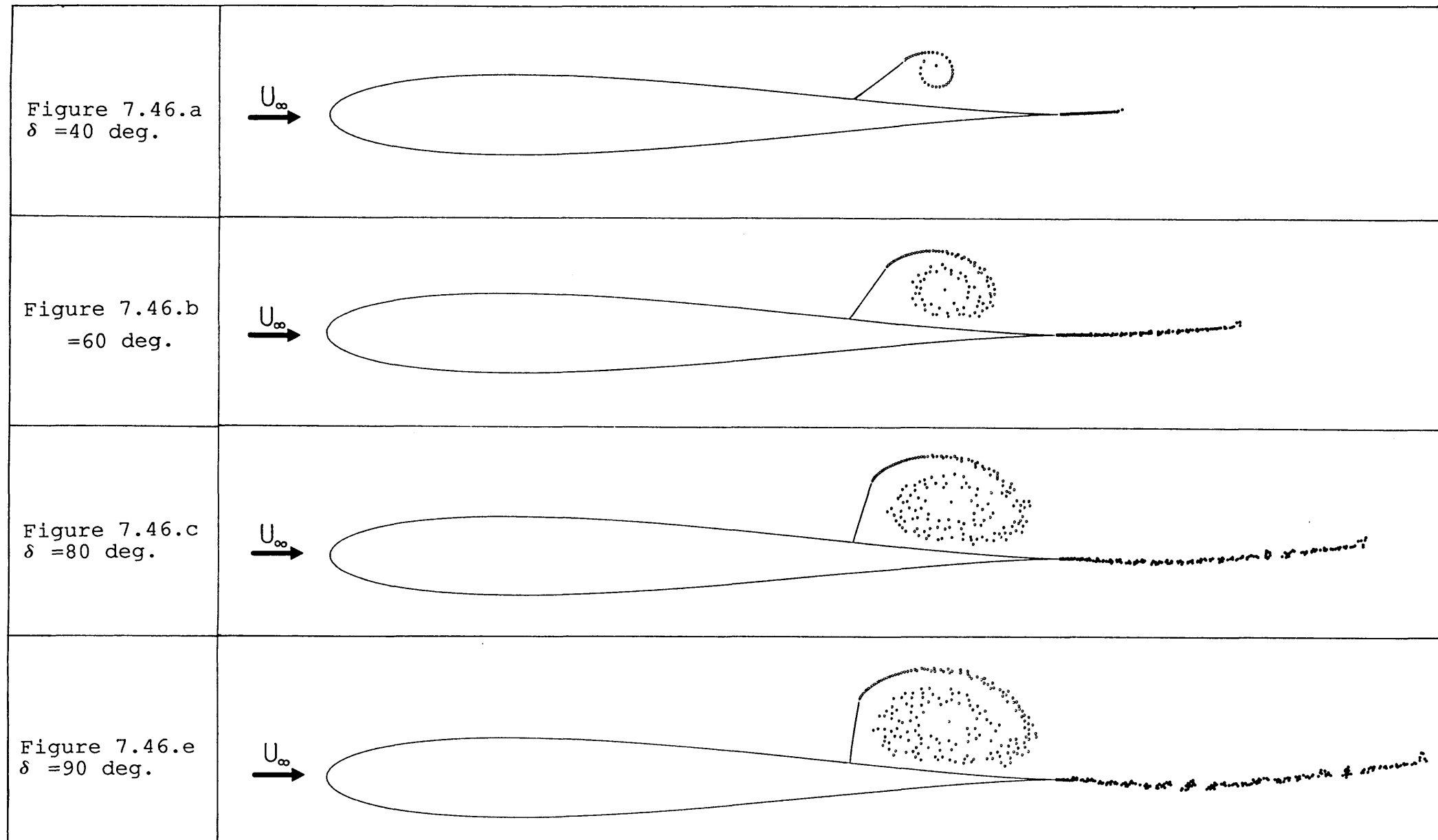


Figure 7.46: Moving spoiler at $u_t / U_\infty = 0.19$, from a starting angle of 30 deg..

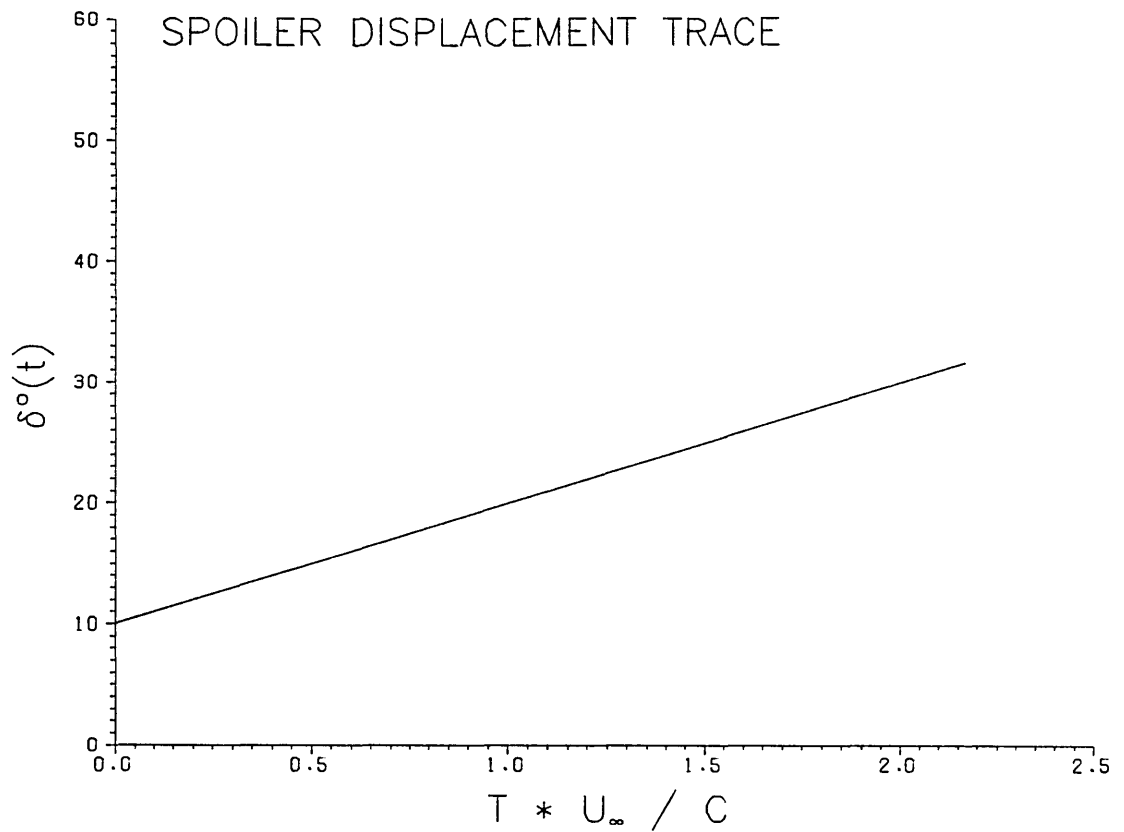


Figure 7.47

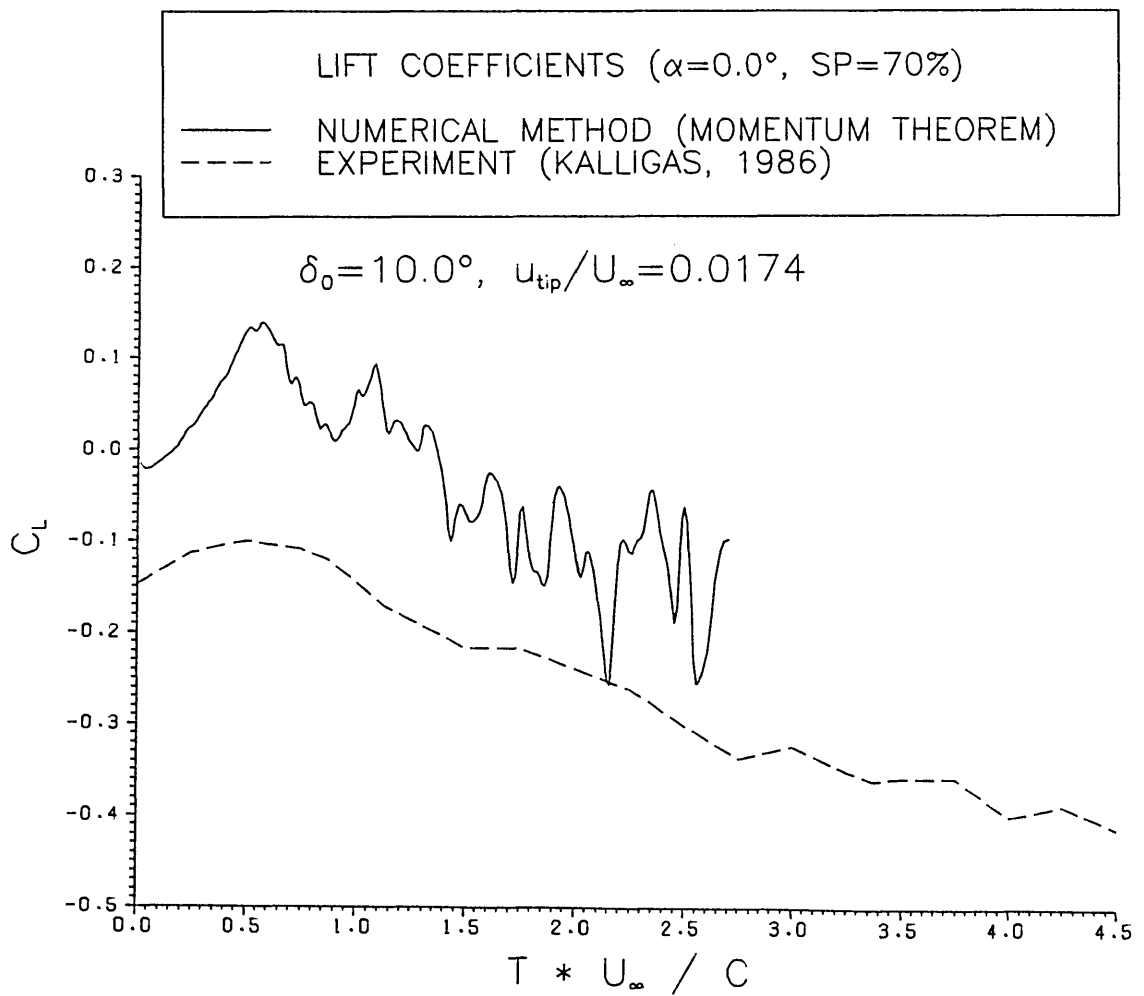


Figure 7.48

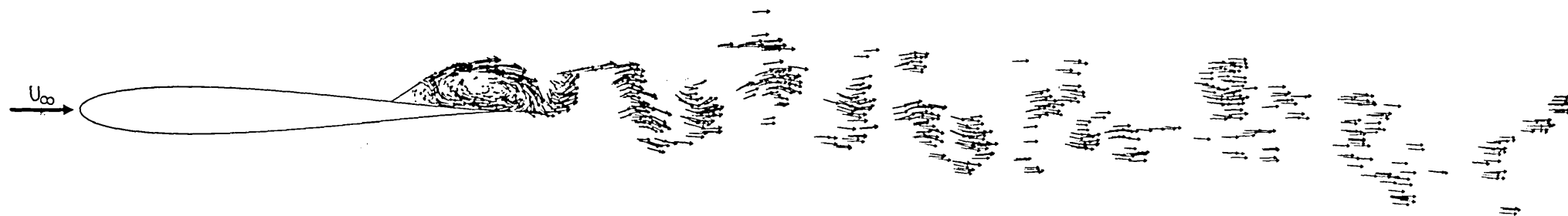


Figure 7.49.a: Numerical wake-flow visualisation; moving spoiler.

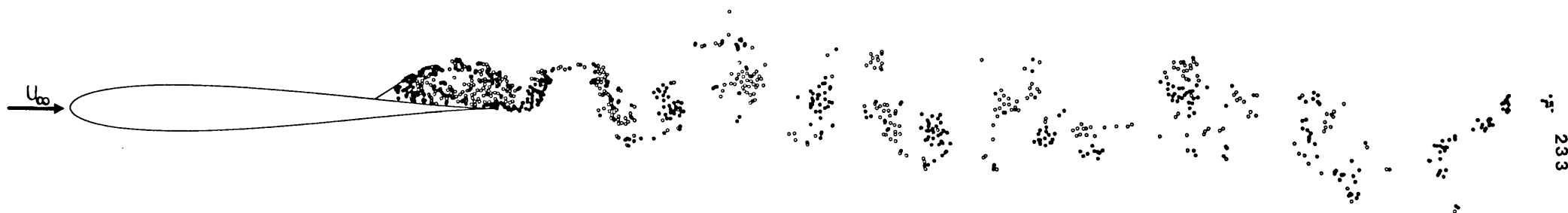


Figure 7.49.b: Spoiler moving at $u_t / U_\infty = 0.0174$, $\alpha = 0$ deg.,
SP=70%c.

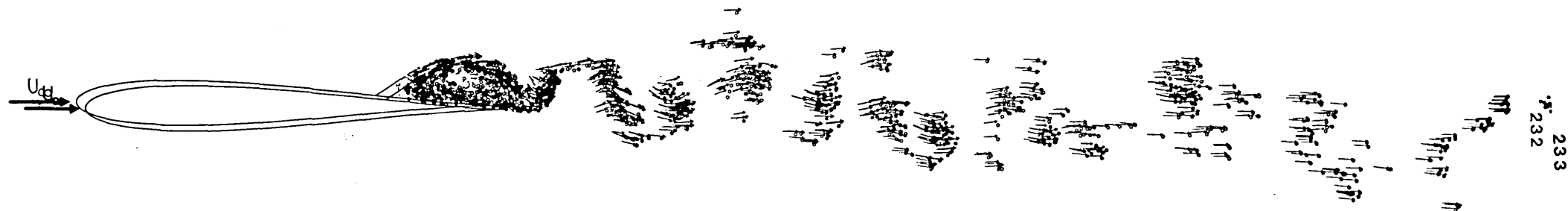


Figure 77499ba: Sponner moving wake flow visualization, $\alpha = 0$ deg, $U_0 = 0.174$, $SP = 30\%$ cler.

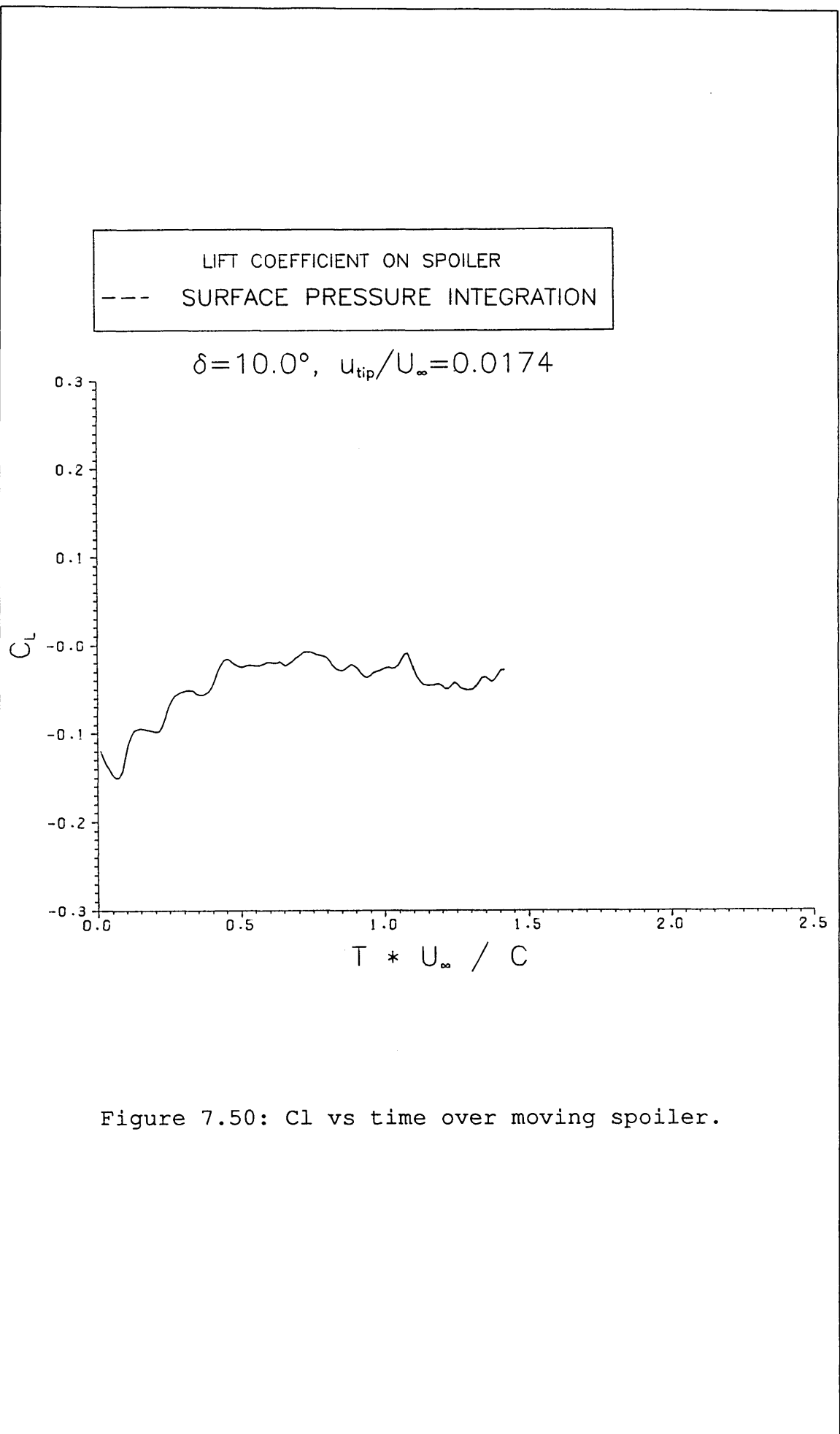


Figure 7.50: C_l vs time over moving spoiler.

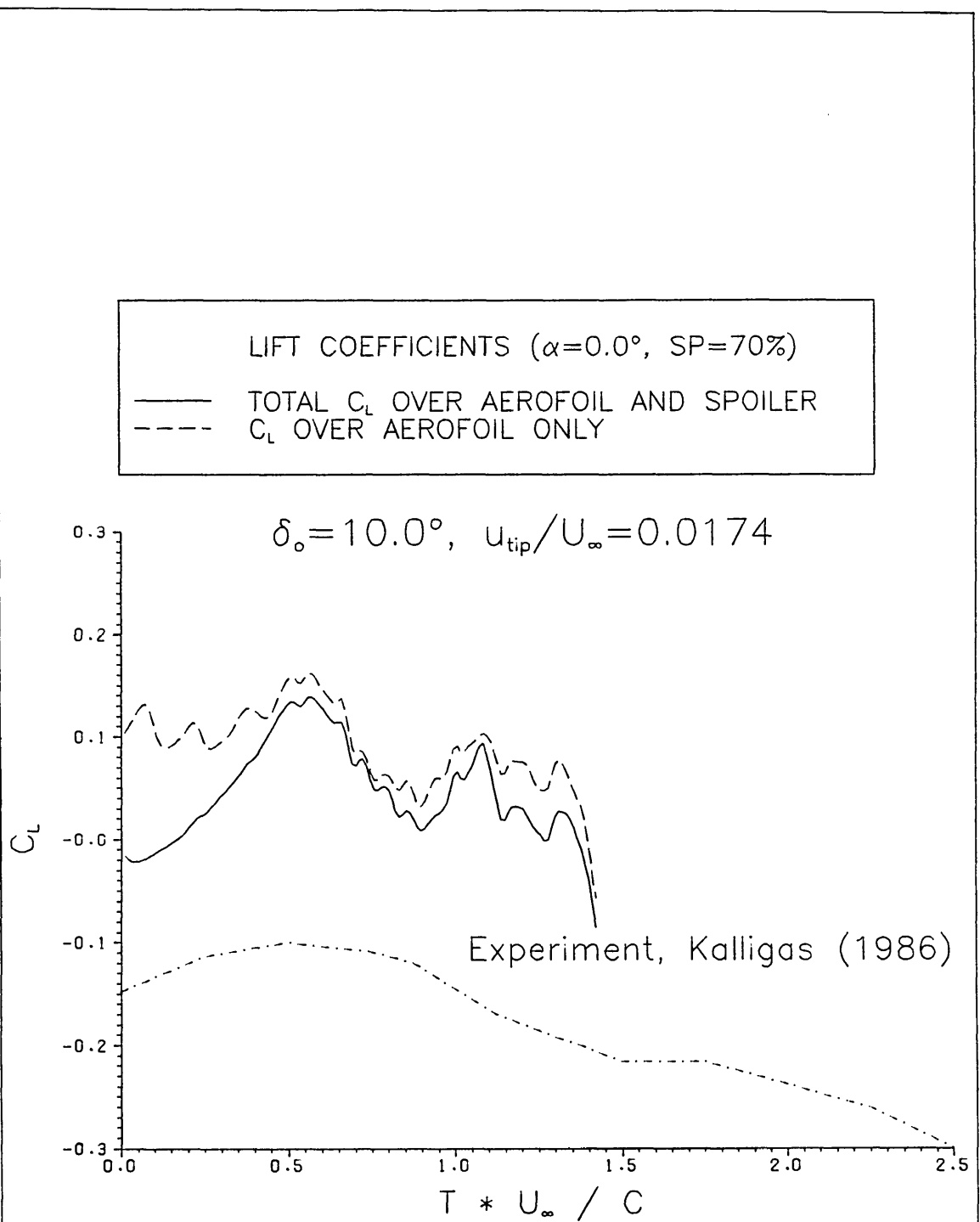


Figure 7.51

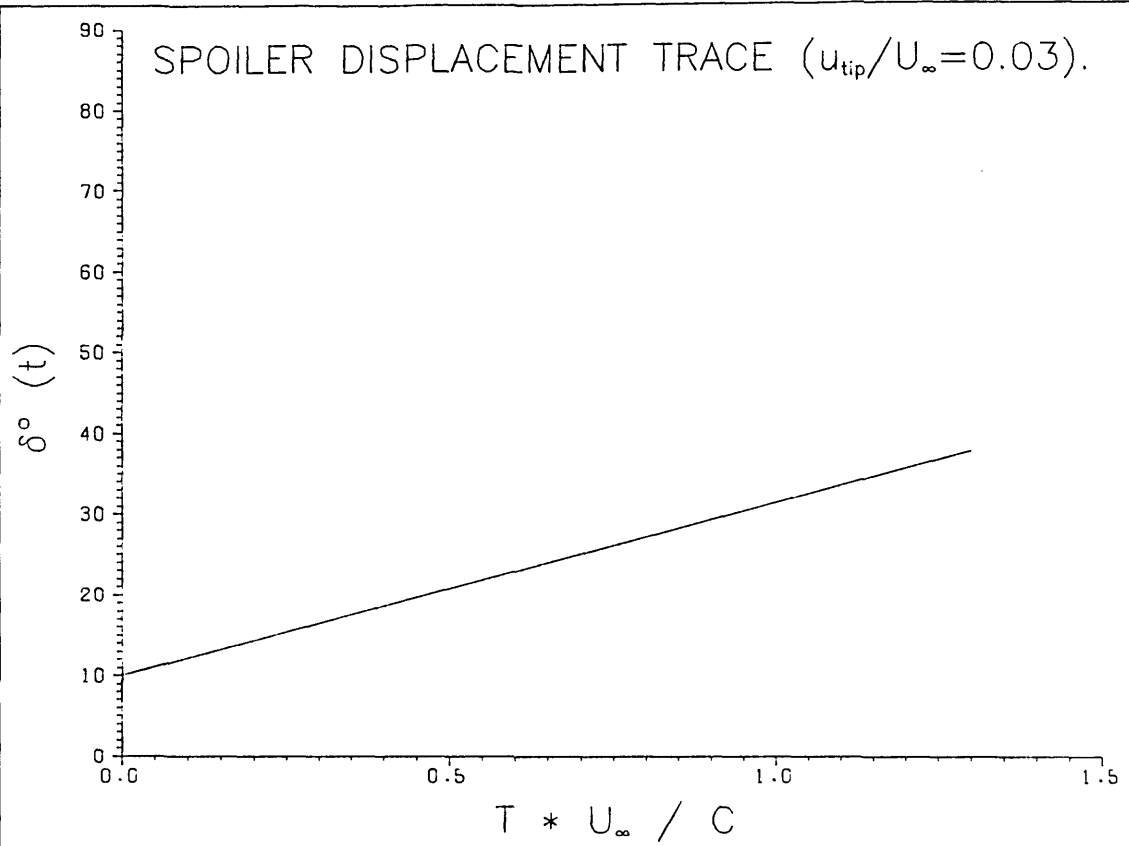


Figure 7.52

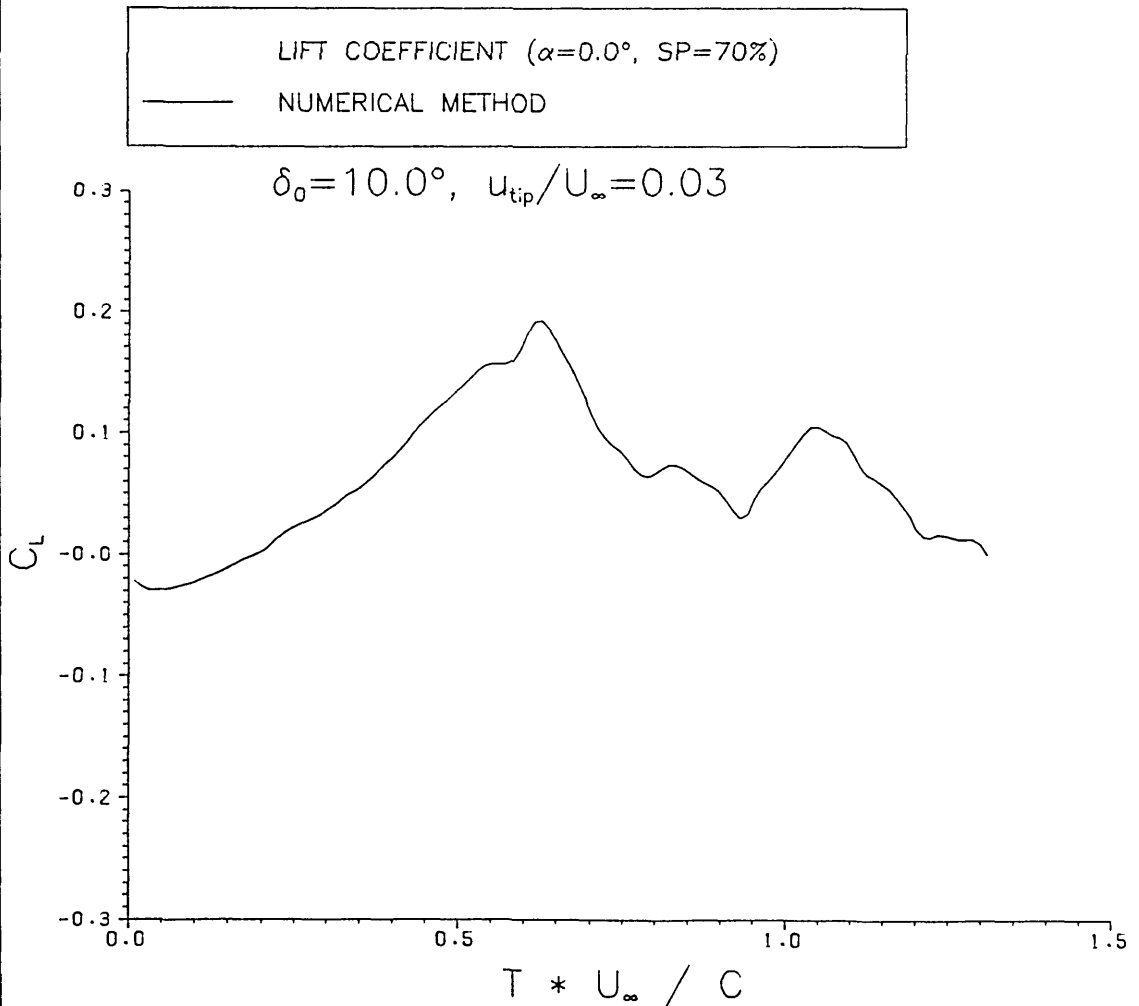


Figure 7.53

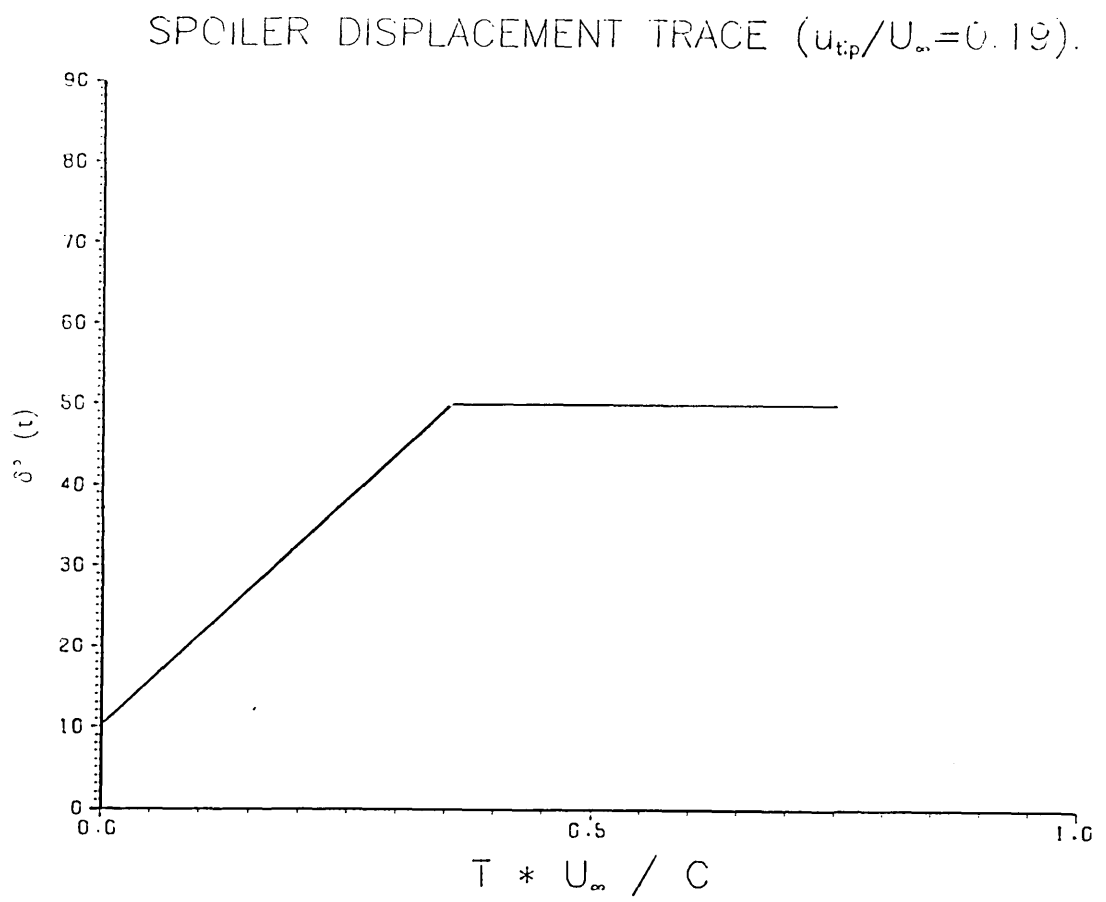
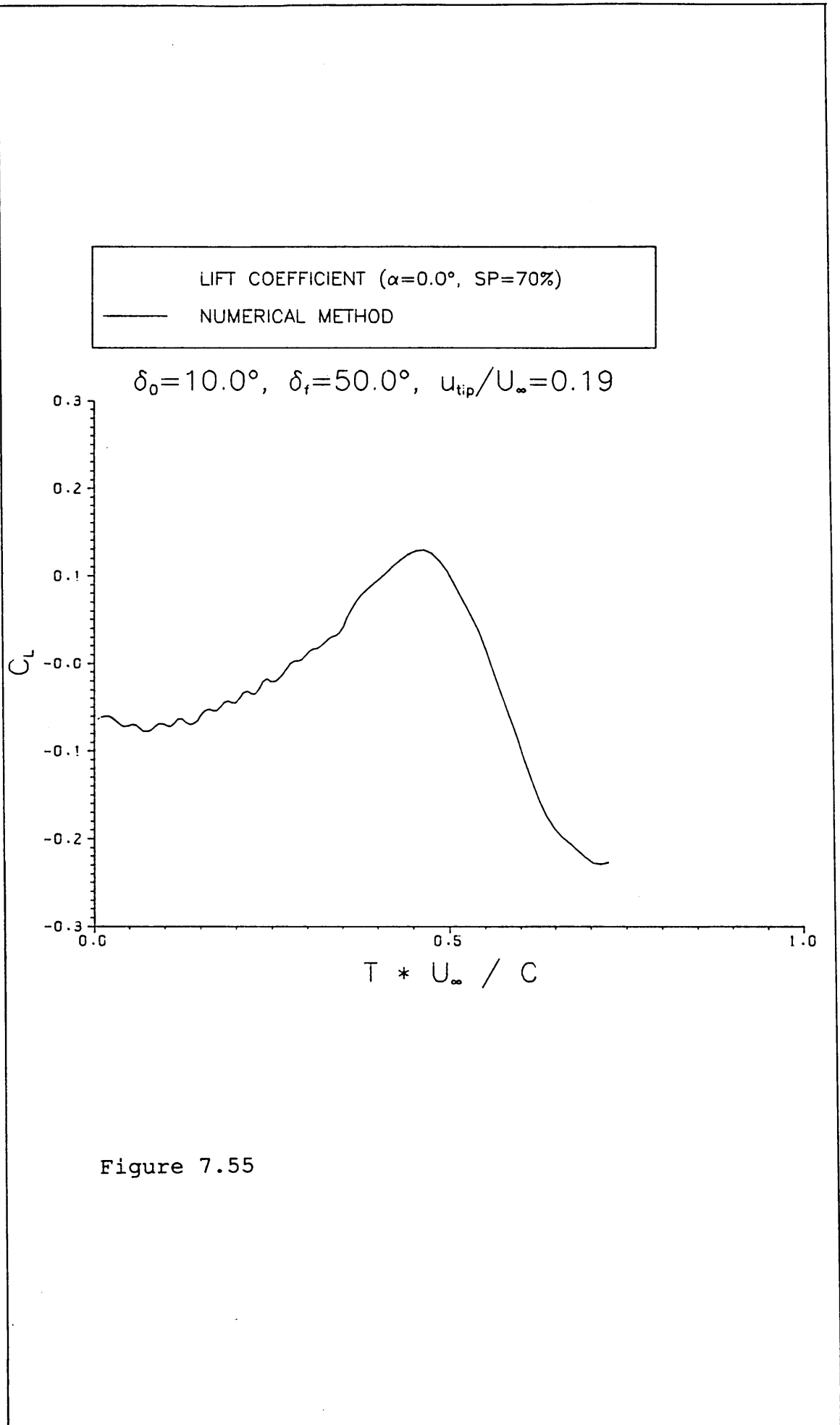


Figure 7.54



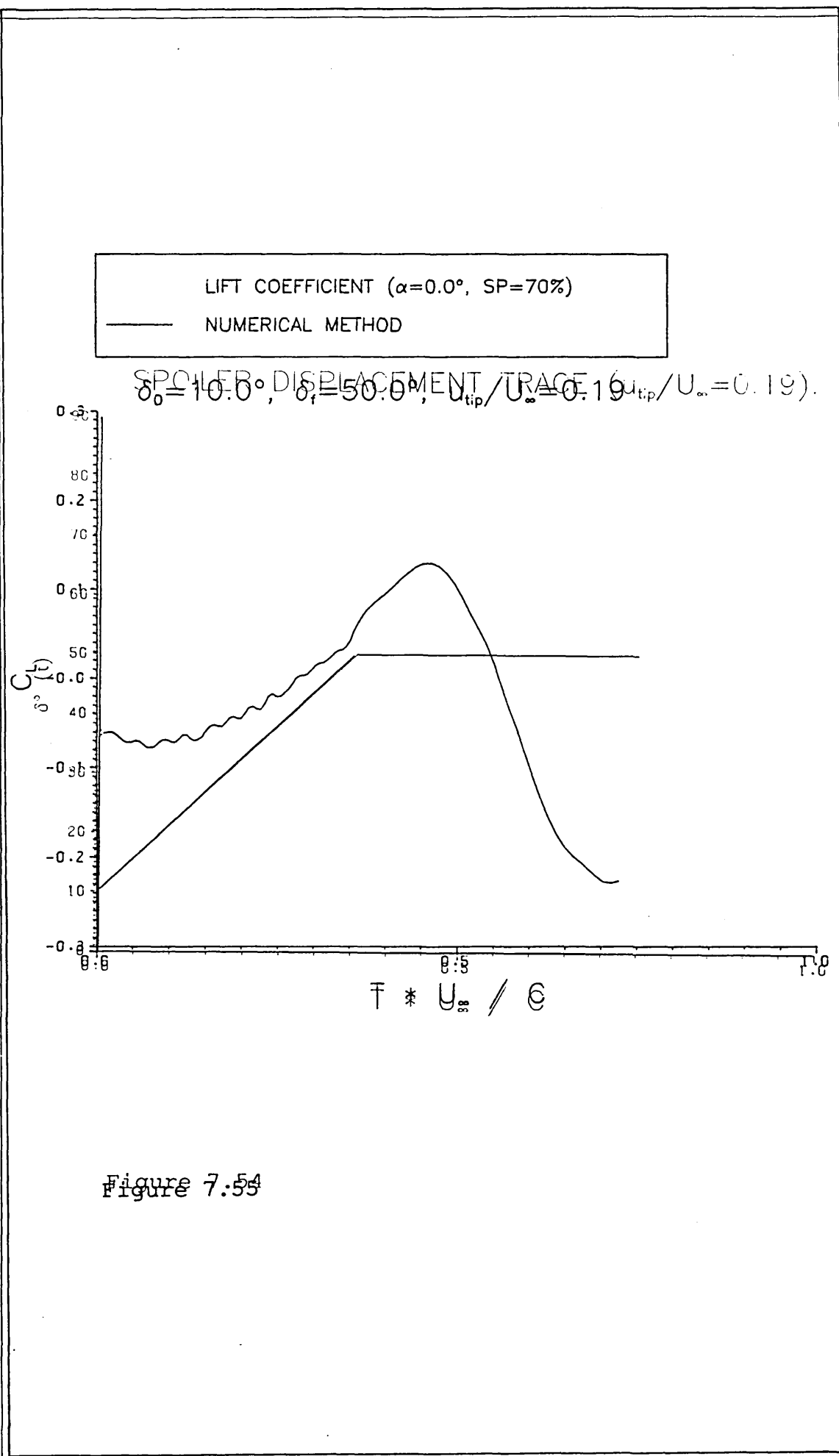


Figure 7.54

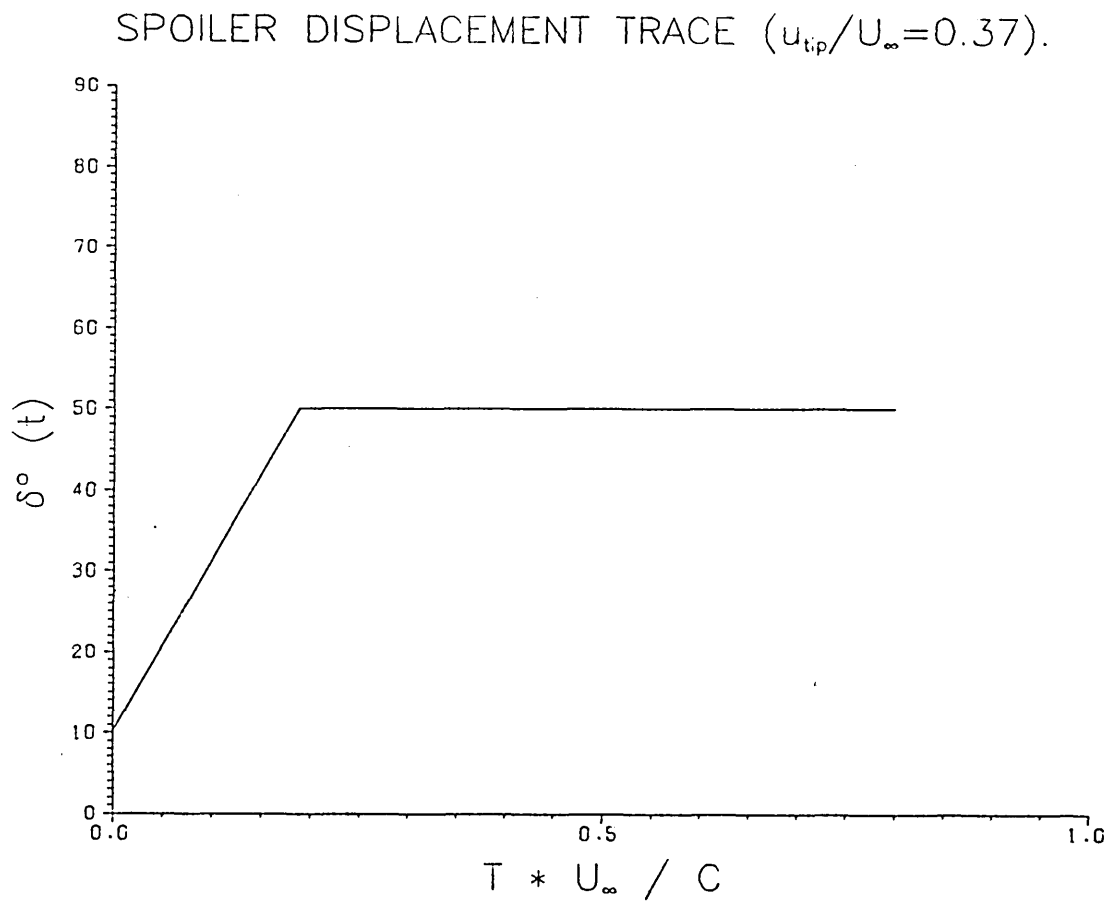


Figure 7.56

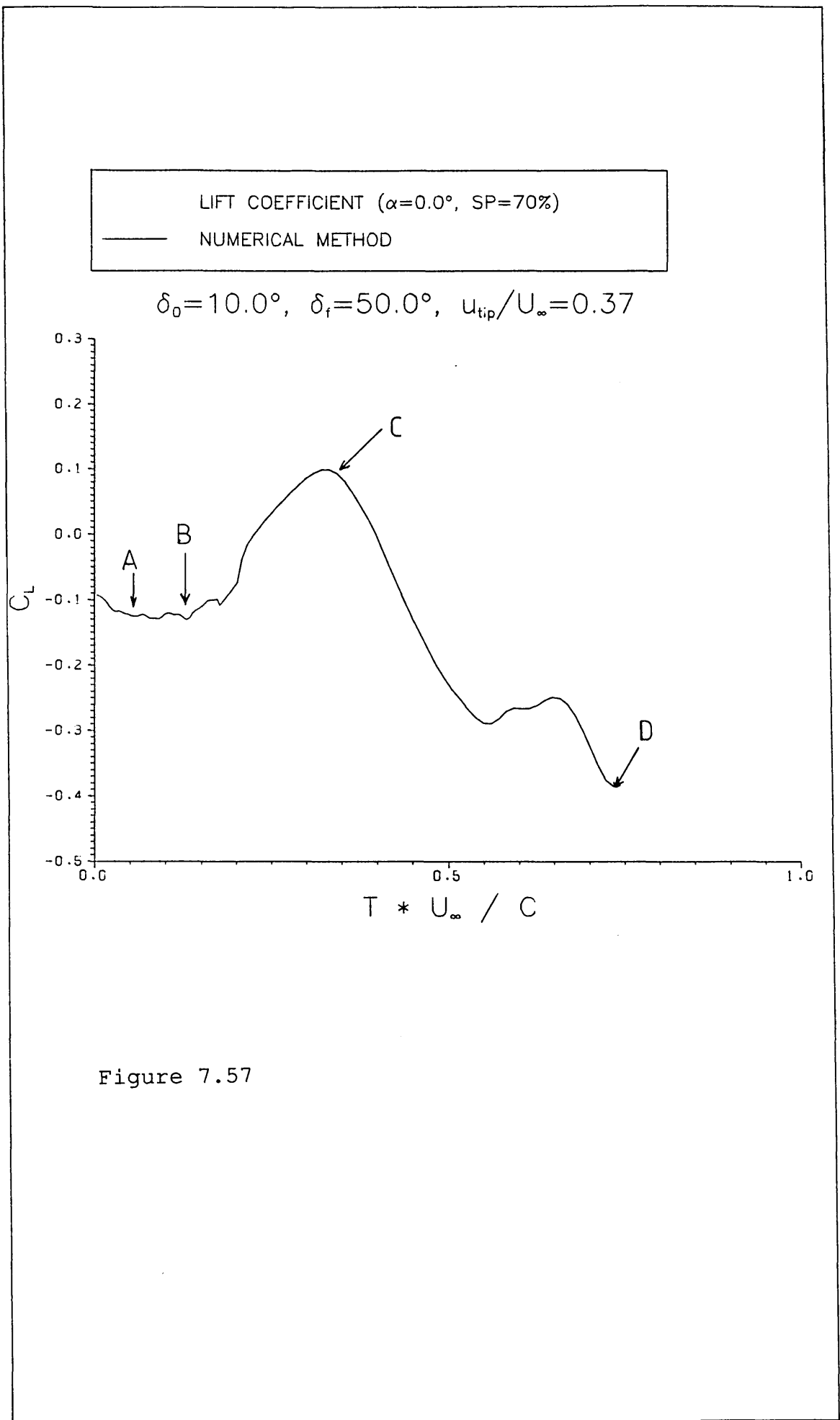


Figure 7.57

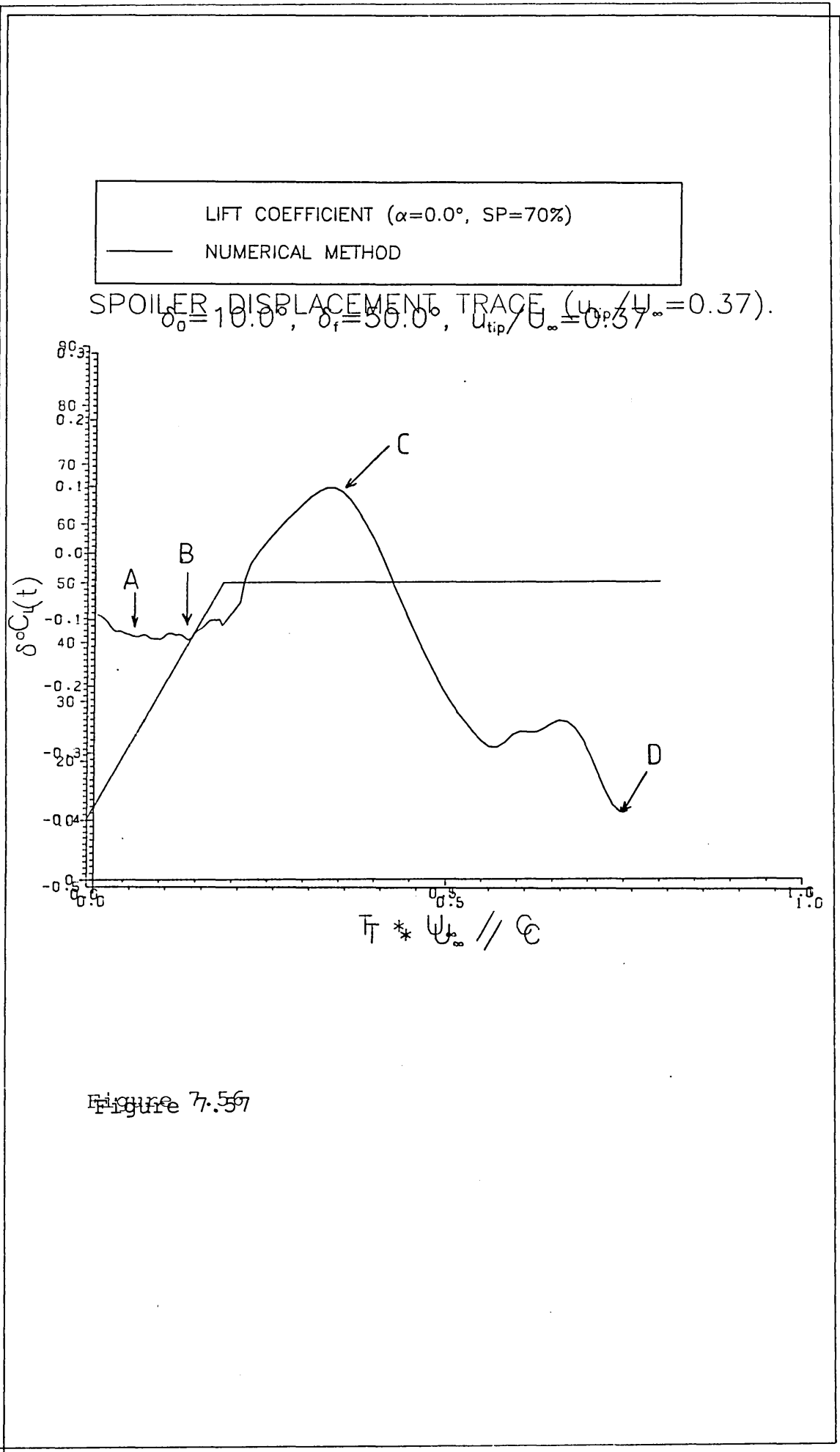


Figure 7.56
Figure 7.57

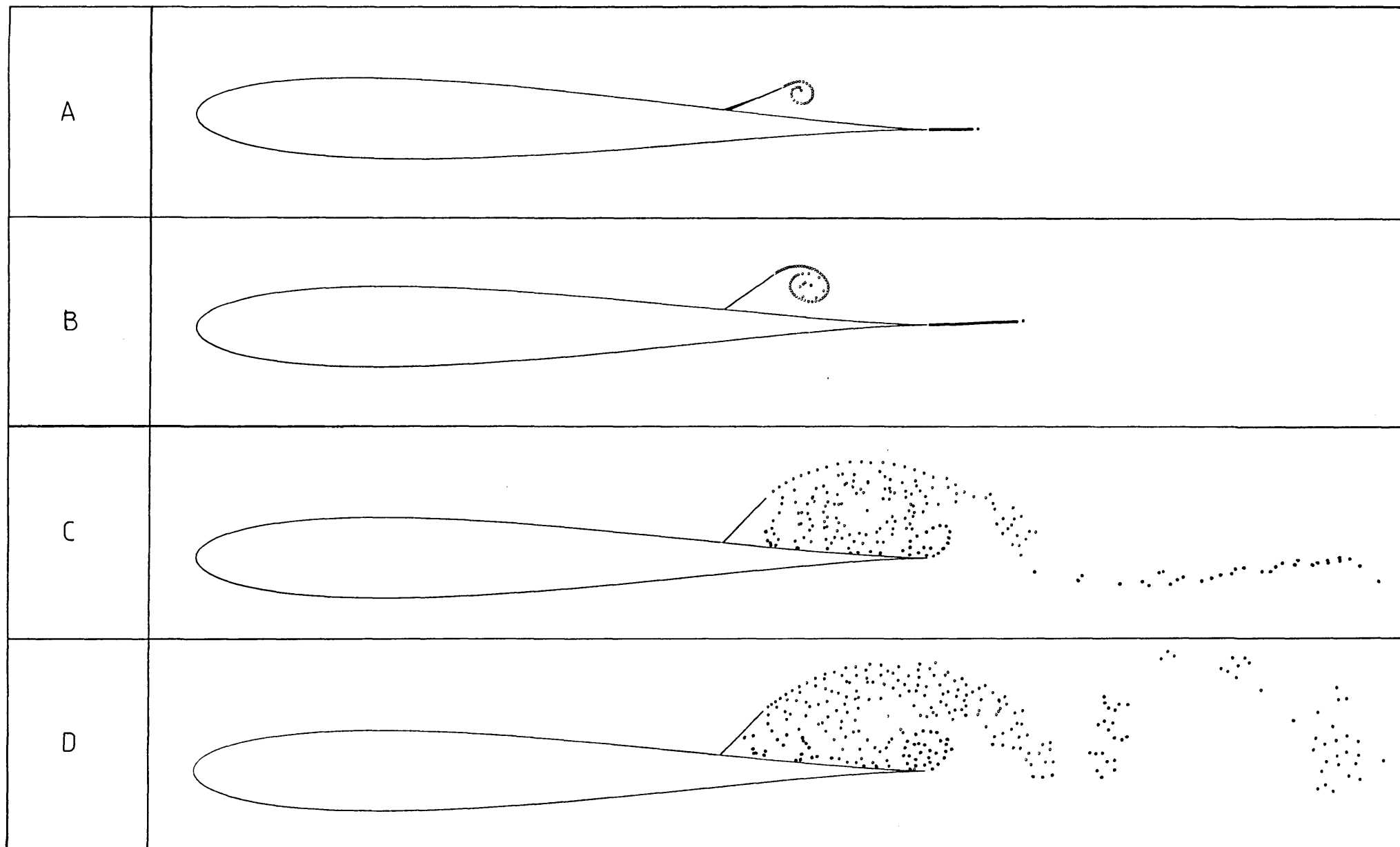


Figure 7.57': Flow visualisations at times A,B,C,D as shown on figure 7.57.

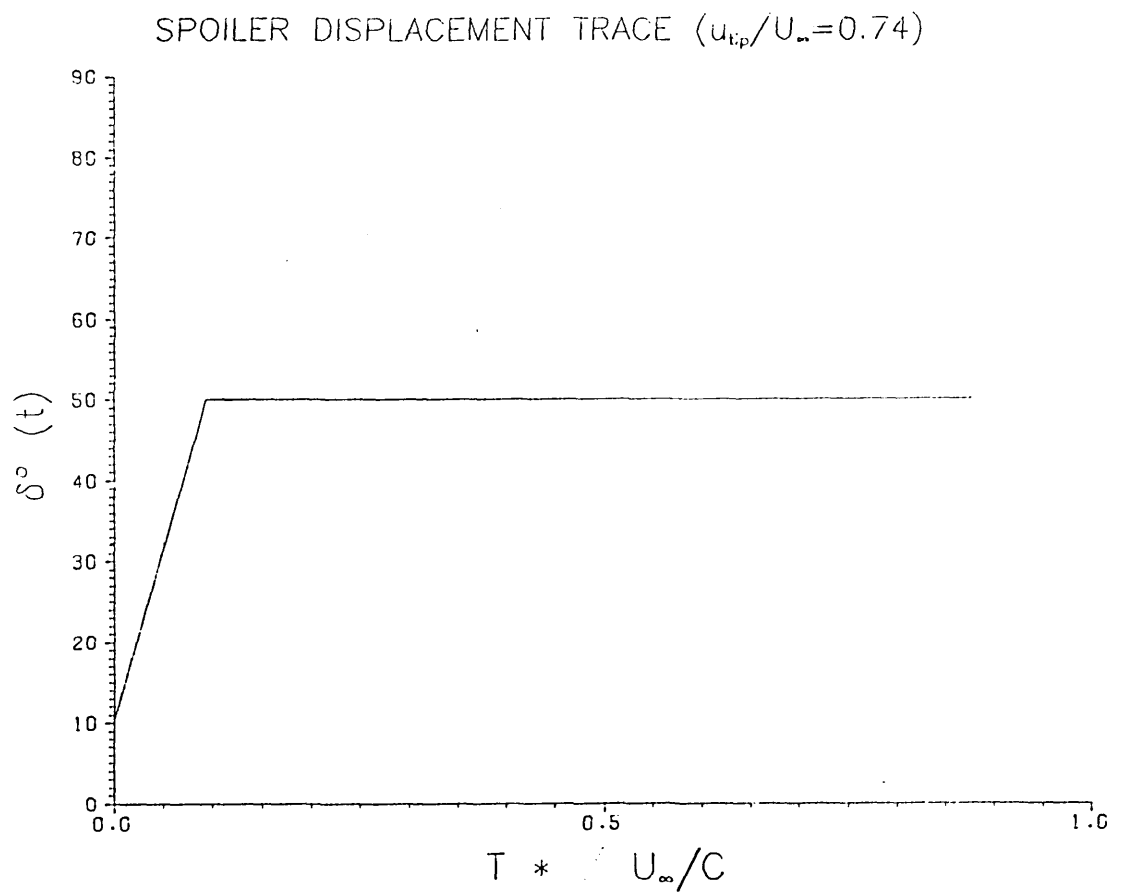


Figure 7.58

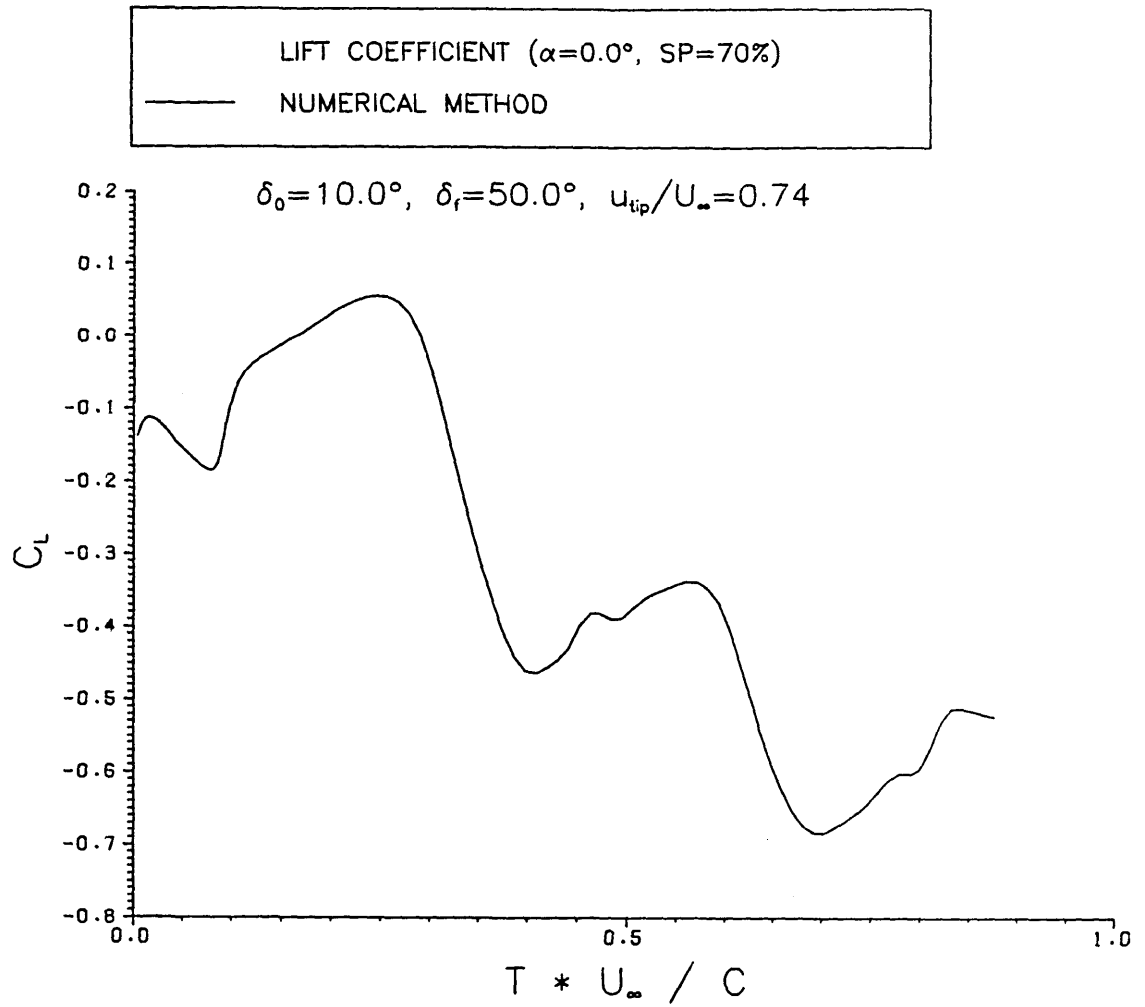


Figure 7.59

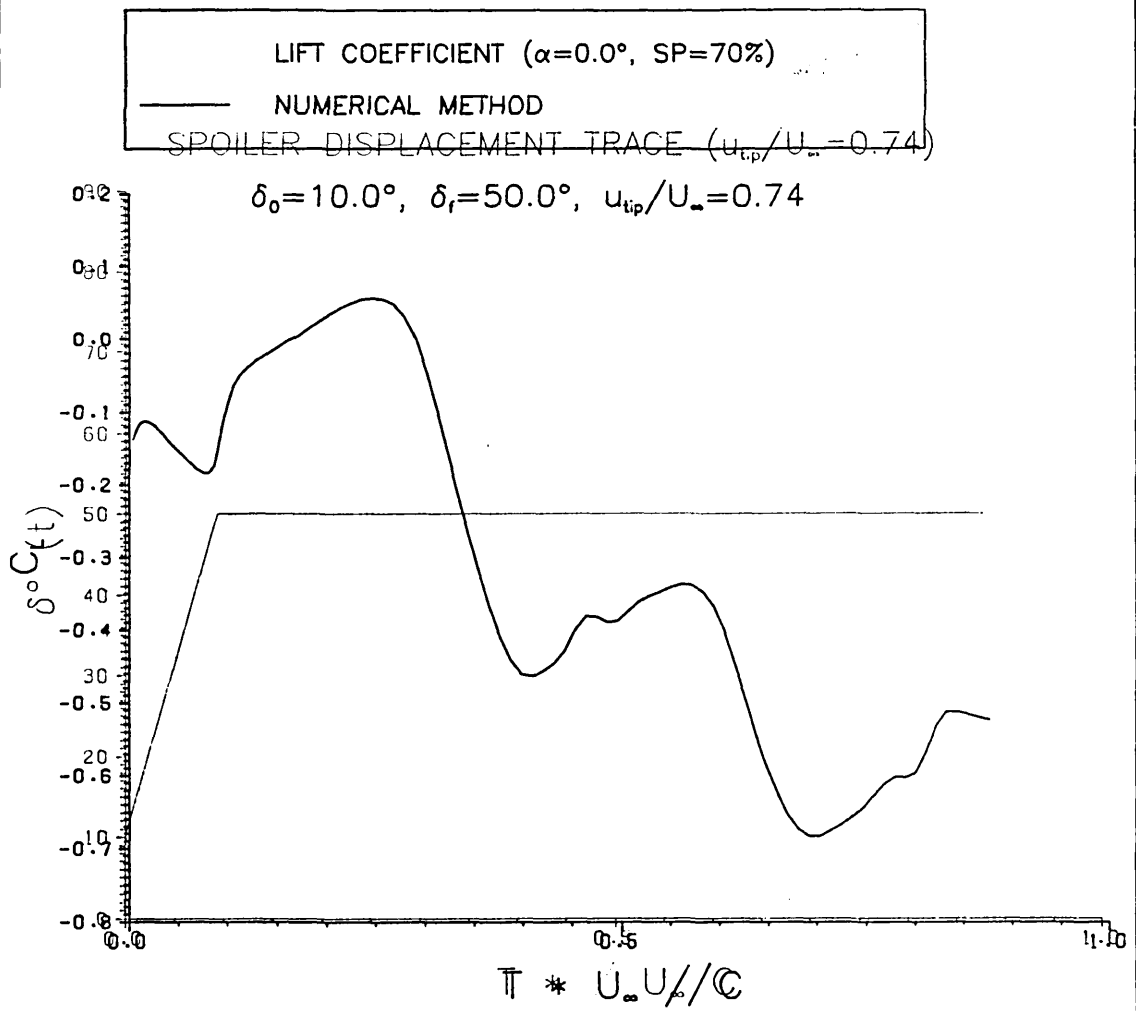


Figure 7.58
Figure 7.59

DELAY TIMES FOR SPOILER EXTENSION

- + t_a/T_0 , NUMERICAL METHOD
- t_a/T_0 , EXPERIMENT (KALLIGAS, 1986)
- ⋈ t_0/T_0 , NUMERICAL METHOD
- Z t_0/T_0 , EXPERIMENT (KALLIGAS, 1986)

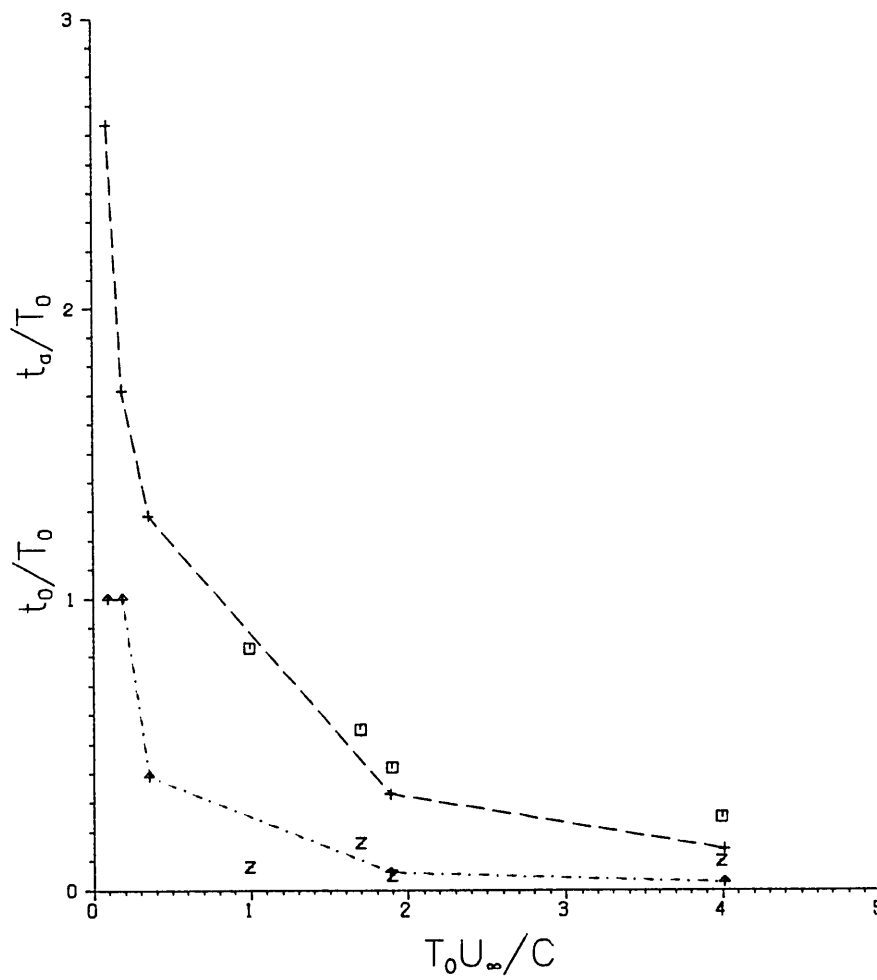


Figure 7.60: Delay times for spoiler extension.

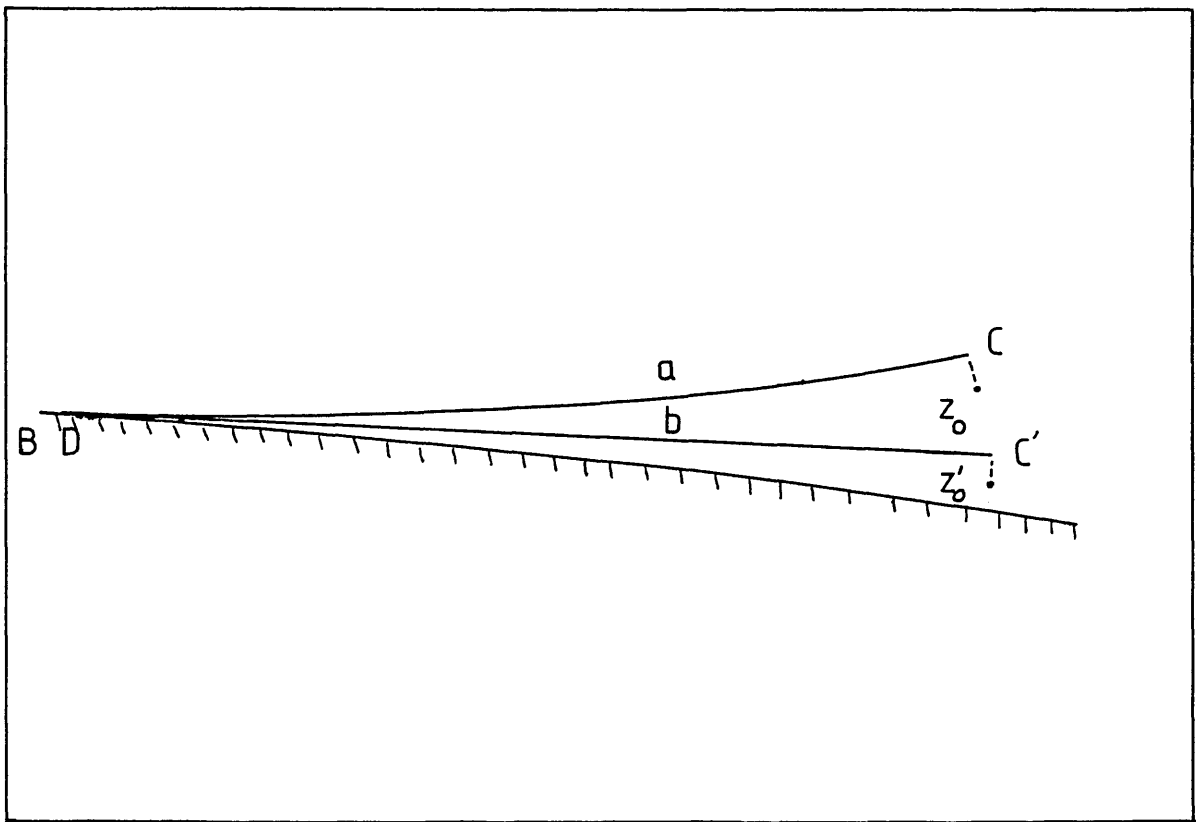


Figure 7.61: Position of the 'core' vortex for a curved and a straight spoiler.

APPENDIX I

1. The Brown and Michael method - Estimation of $\hat{W}_0, \hat{W}_1'$ and k .

i) $\hat{W}_0$ and $\hat{W}_1'$.

In the circle plane, the velocity q is evaluated on the surface at the end of a time step, after convecting shed vortices but before shedding a new vortex. Let the velocity at the edge be q_e , and q_1 and q_2 at the nearest points on either side of the edge, as shown in figure I.1. Let points be at distances σ_1 and σ_2 from the edge (both taken as positive) along the arc of the surface at $t = \Delta t$. Then,

$$\frac{dW}{d\hat{\zeta}} = i \hat{W}_1' \Delta t + 2(\hat{W}_0 + \hat{W}_2' \Delta t) \hat{\zeta} \quad (I.1)$$

where, W_2' is of lower order. Therefore:

$$\begin{aligned} q_e &= \hat{W}_1' \Delta t \\ q_1 &= \hat{W}_1' \Delta t + 2\hat{W}_0 \sigma_1 \\ q_2 &= \hat{W}_1' \Delta t - 2\hat{W}_0 \sigma_2 \end{aligned} \quad (I.2)$$

Then:

$$\begin{aligned} \hat{W}_0 &= \frac{-(q_1 - q_2)}{2(\sigma_1 + \sigma_2)} \\ \hat{W}_1' &= \frac{1}{\Delta t} q_e \end{aligned} \quad (I.3)$$

ii) Evaluation of 'k'.

The constant 'k' is a scaling factor between the physical plane and the final (circle) plane, where $\hat{W}_0$ and $\hat{W}_1$ are to be evaluated.

Consider the same three corresponding points (the edge and one point on either side in the two planes) as shown in figure I.2. Distances S_1 and S_2 are as shown.

Then:

$$S_1 = k \sigma_1^2 \quad (I.4)$$

$$S_2 = k \sigma_2^2$$

from which 'k' is obtained:

$$k = \frac{1}{2} \left[\frac{S_1}{\sigma_1^2} + \frac{S_2}{\sigma_2^2} \right] \quad (I.5)$$

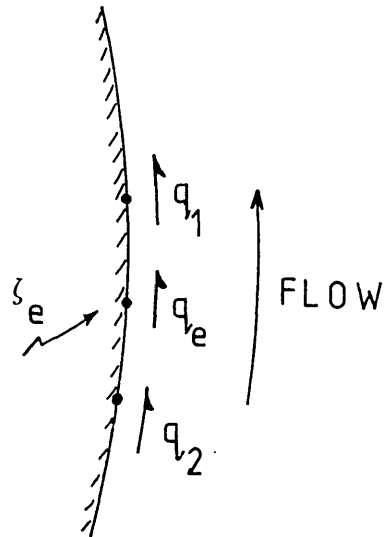


Figure I.1: Definition of velocities near the edge.

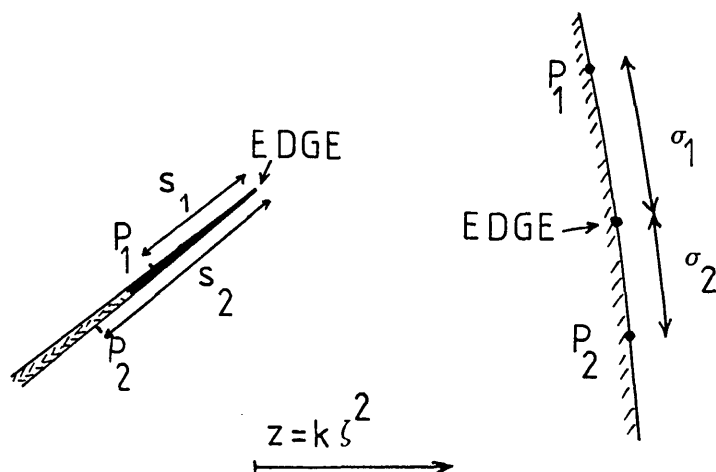


Figure I.2: Distances of points near the edge.