A Thesis submitted for the Degree of Doctor of Philosophy in the Faculty of Engineering of the University of London.

AN EXPERIMENTAL INVESTIGATION INTO THE TORSIONAL RIGIDITY OF PLAIN AND REINFORCED CONCRETE AND THE APPLICATION IN PROBLEMS OF DESIGN.

London.
June, 1939.

W.T. Marshall, B.Sc., A.C.G.I., D.I.C., Assoc.M.Inst.C.E.

CITY AND GUILDS COLLEGE
Department of Civil Engineering

TABLE OF CONTENTS.

Chapter.		Page.
	Preface	1
I.	Introduction - Importance and Reasons for work.....	5
II.	Theory of Torsion - ~~for~~ Circular & Rectangular Sections	23
III.	Previous Investigations.	31
IV.	Method of Carrying out the Experiments.	56
	(a) Manufacture of Test Pieces	57
	(b) The Test Machine.....	68
	(c) Method of Testing.....	76
V.	Tests on plain concrete specimens.	81
	(a) Circular.....	82
	(b) Rectangular.....	90
	(c) Tee and Ell sections.....	99
VI.	Tests on reinforced concrete specimens.	115
	(a) Longitudinal bars only	117
	(b) Longitudinal bars and stirrups .	122
	(c) Spirals	134
	(d) Reinforced Tee Sections	139
VII.	Conclusions from experiments	158
VIII.	Application to design of bow girders in reinforced concrete.....	167
IX.	Geological significance of plain concrete tests	178
	Appendix I- Design of Test Machine	183
	- II - Strain Energy methods of Analysis	188

PREFACE.

The experimental work described in this thesis was carried out in the Civil Engineering Laboratories of the City and Guilds College under the supervision of Professor A.J. Sutton Pippard to whom the author is greatly indebted.

The actual experiments were carried out by Mr. N.R. Tembe, M.Sc., B.Eng. working in conjunction with the author, and thanks are due to Mr. Tembe for the valuable assistance rendered.

CITY AND GUILDS
Department
of
Civil Engineering
COLLEGE

BIBLIOGRAPHY.

(1). Le Génie Civil, 1936, 108(6).

(2). Concrete and Constructional Engineering, November, 1934.

(3). Explanatory Handbook on the Recommendations for a New Code of Practice in reinforced concrete - Scott and Glanville.

(4). Die Theorie Elasticher Gwebe - Springer.

(5). Plain and Reinforced Concrete Construction - Probst.

(6). Plain and Reinforced Concrete in torsion - Institution of Civil Engineers - Selected Engineering Paper No.165.

(7). Concrete Steel Construction - Enid Morsch.

(8). Applied Elasticity - Timoshenko and Lessells.

(9). Advanced Strength of Materials - Seely.

(10). Deutsche Ausschus fur Eisenbeton, Vol. 16.

(11). Verdreckuagsversuche zur Klarung der Schubfestighet Von Eisonbeton - 1921.

(12). Concrete and Constructional Engineering - November, 1927.

(13). University of Toronto - Bulletin No.3 - 1922.

(14). Proceedings of The American Society of Civil Engineers - May, 1934.

(15). Proceedings of The American Society of Civil Engineers - October, 1937.

(16). Journal of The American Concrete Institute - September, 1937.

(17). A Study of the Circular Arc Bow Girder - Gibson and Ritchie.

(18). The Stress Analysis of Bow Girders - Pippard and Barrow.

(19). Reinforced Concrete Designers Handbook - Reynolds.

(20). Proceedings Institution of Mechanical Engineers - 1917.

(21). The Analysis of Engineering Structures - Pippard and Baker.

(22). The Mathematical Theory of Elasticity - Love.

NOTATION.

b = short side of rectangle.

d = long side of rectangle.

l = gauge length on which angle of twist is measured.

A_t = area of steel used as tension reinforcement.

A_c = area of steel used as compression reinforcement.

A = area of concrete section.

θ = angle of twist.

ϕ = angle of shear strain.

q = maximum shear stress in circular section.

s_1 = maximum shear stress in rectangular section.

s_x, s_y = shear stresses in directions parallel to x and y axes at points whose co-ordinates are x and y.

N = modulus of rigidity.

J = polar second moment of area, or effective polar moment.

NJ = torsional rigidity of section.

E = modulus of elasticity.

I = Second moment of area.

EI = flexural rigidity of section.

T = twisting moment.

M = bending moment.

Where notation other than this is used the symbols adopted will be explained in the text.

CHAPTER I.

INTRODUCTION.

CHAPTER I.

INTRODUCTION.

Research work can be divided generally into two main classes, firstly, that of a purely academic nature and secondly, the investigation of a practical problem with the object of producing data to solve this problem. Each of these classes has its part to play in the acquisition of knowledge and it often occurs that the work done from the purely academic standpoint is of great assistance to the person endeavouring to produce data to solve a practical problem.

When the author began this research on the torsional rigidity of plain and reinforced concrete some criticism was levelled against it on the grounds that concrete was never used in torsion, in other words, that the research was academic and of no practical value. Whilst to the person who has had little experience of reinforced concrete design it may appear that concrete is never used in torsion, the author having found himself (on more than one occasion) faced with the problem of designing reinforced concrete members to resist a torsional moment realises how valuable some information on this problem would be.

The first occasion when the author was faced with the problem occurred when he had to design a

cantilever loading platform which formed part of a floor in which there were ducts for pipes, etc. The floor is shown on Fig 1: due to the fact that no information was available for the design of the beam "A"

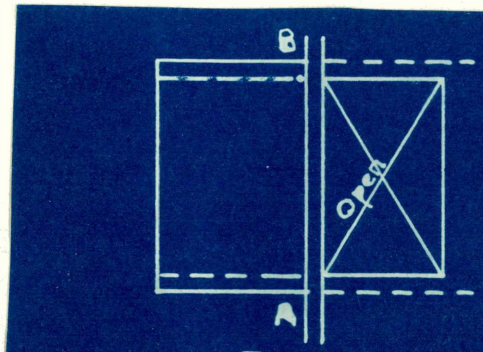


FIG. 1

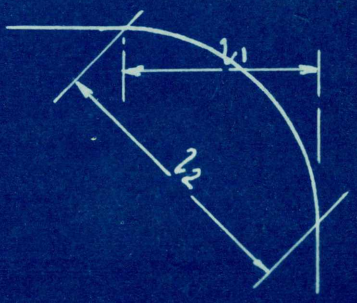
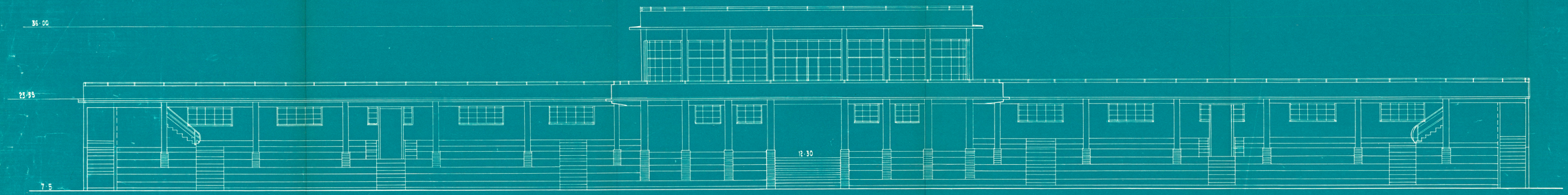


FIG. 2

which was subjected to torsional moment, the problem was overcome by the insertion of beams"B" and the slab taken as spanning between these beams instead of cantilevering from beam "A". Whilst this solved the problem it did so in an uneconomic and unsatisfactory way, and also spoilt the architectural features of the building. It also showed that reinforced concrete designers were definitely afraid of any member subject to torsion because of a lack of information about the problem.

It was this occasion which first brought the problem to the author's notice, but the importance was emphasised when he found himself having to design three structures in succession in each of which there were members subjected to torsional loading. These three structures are shown on pages 8, 9 10 and they are:-



ELEVATION TO SEA.



ELEVATION TO ROADWAY.



SCALE:- 8 FEET TO 1 INCH.

BURGH OF DUNOON. ELEVATIONS OF NEW BATHING STATION.

SCALE 1/8" = 1'-0".

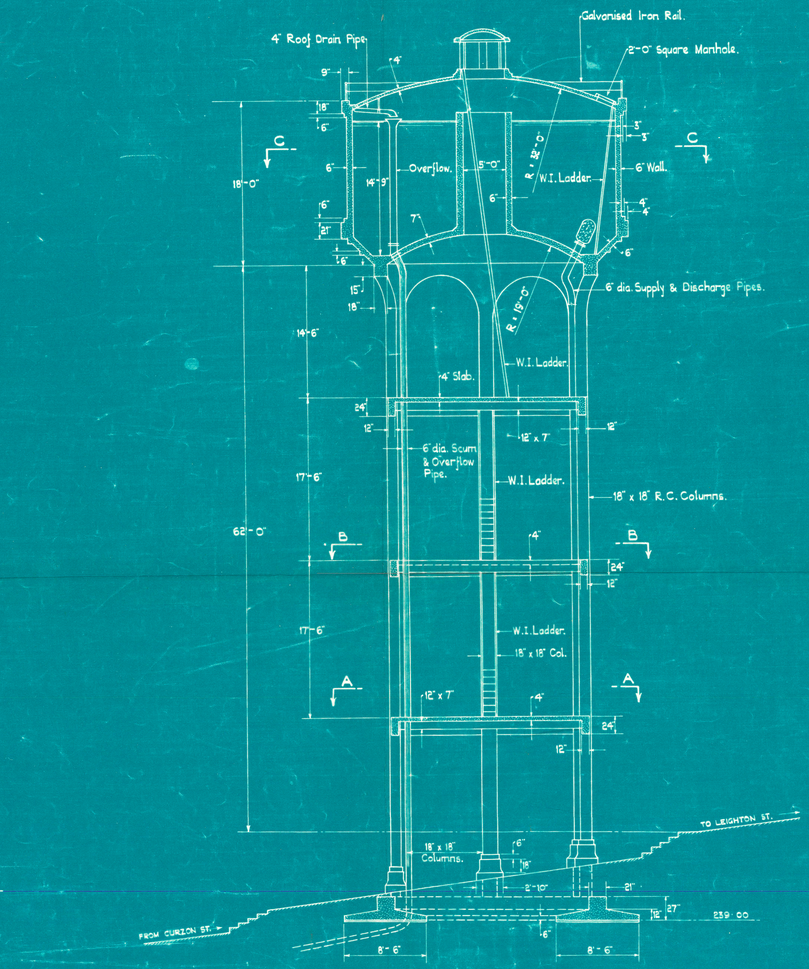
F.A. MACDONALD & PTNS. (GLASGOW) LTD.
135, WELLINGTON STREET, GLASGOW, C2.

DRG. NO. 571 - 2.

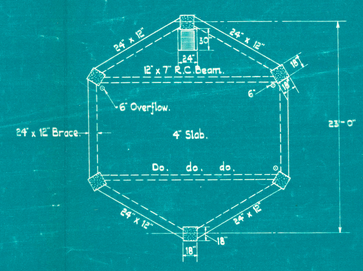
N.B. 23.12.35



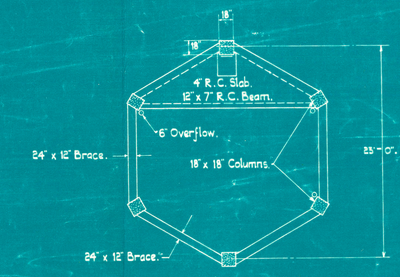
ELEVATION.



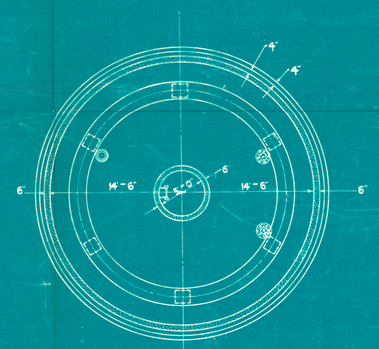
SECTION D - D.



SECTIONAL PLAN A - A.

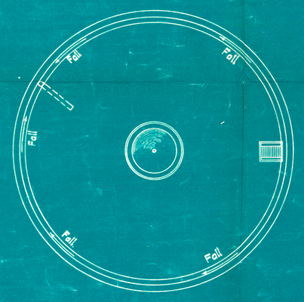


SECTIONAL PLAN B - B.

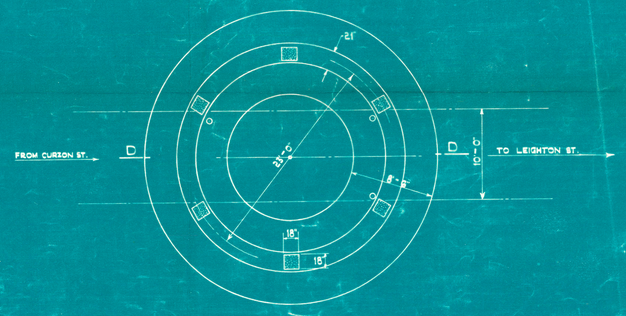


SECTIONAL PLAN C - C.

NOTE:- CONCRETE MIX TO BE 1:2:4 LBS. CEMENT, 2½ CUB. FT. SAND, 2½ CUB. FT. AGGREGATE FOR TANK WALLS, DOME, RING & ARCH BEAMS.
CONCRETE MIX TO BE 1:2:4 LBS. CEMENT, 2½ CUB. FT. SAND, 3¾ CUB. FT. AGGREGATE FOR REMAINDER OF WORK.
SAFE BEARING PRESSURE OF GROUND ASSUMED AS 1 TON PER SQ. FT.



ROOF PLAN.



FOUNDATION PLAN.

GLASGOW CORPORATION WATER WORKS.

PROPOSED REINFORCED CONCRETE WATER TOWER.

CAPACITY 50,000 GALLONS.

SCALE:- 1/8" = 1'-0".

JOHN COCHRANE M.INST.M.&CY.E.,
WATER ENGINEER ~ GLASGOW.

F. A. MACDONALD & PTNS. (GLASGOW) LTD.
135, WELLINGTON STREET, GLASGOW.

WKG. DRAWING No.:- 609-1.

DRAWN BY W.A.A. CHECKED BY
TRACED BY E.C.S. 3/4/36. APPROVED BY

CITY AND GUILDS
Department
of
Civil Engineering
COLLEGE

ABSTRACT OF THESIS

entitled

"AN EXPERIMENTAL INVESTIGATION INTO THE TORSIONAL RIGIDITY OF PLAIN AND REINFORCED CONCRETE AND THE APPLICATION TO PROBLEMS OF DESIGN."

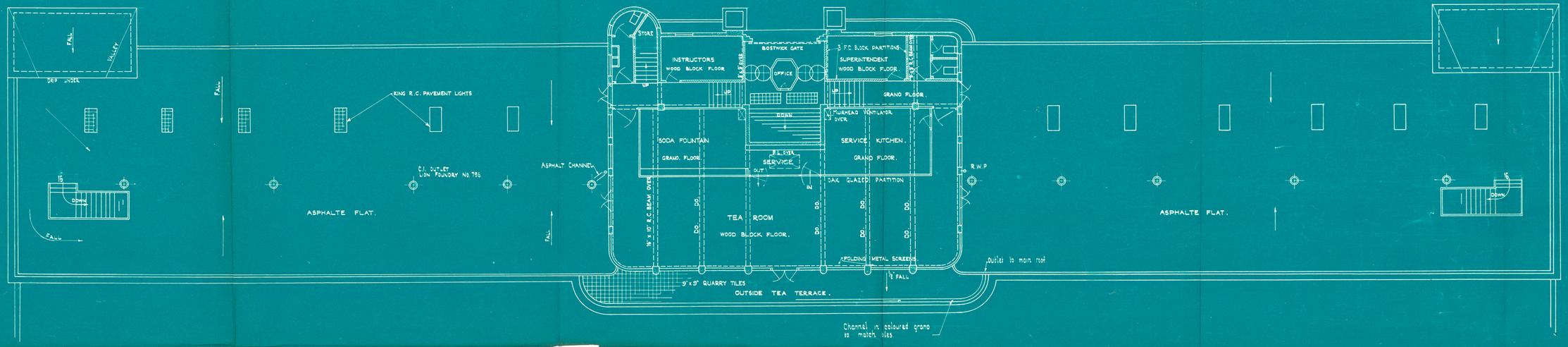
In the introductory chapters the importance of a knowledge of the behaviour of concrete when subject to torsion is stressed, existing theories are discussed and a survey of previous investigations given.

Tests were carried out on plain and reinforced concrete members of circular, rectangular, Tee and Ell sections. From the experiments it was possible to determine,

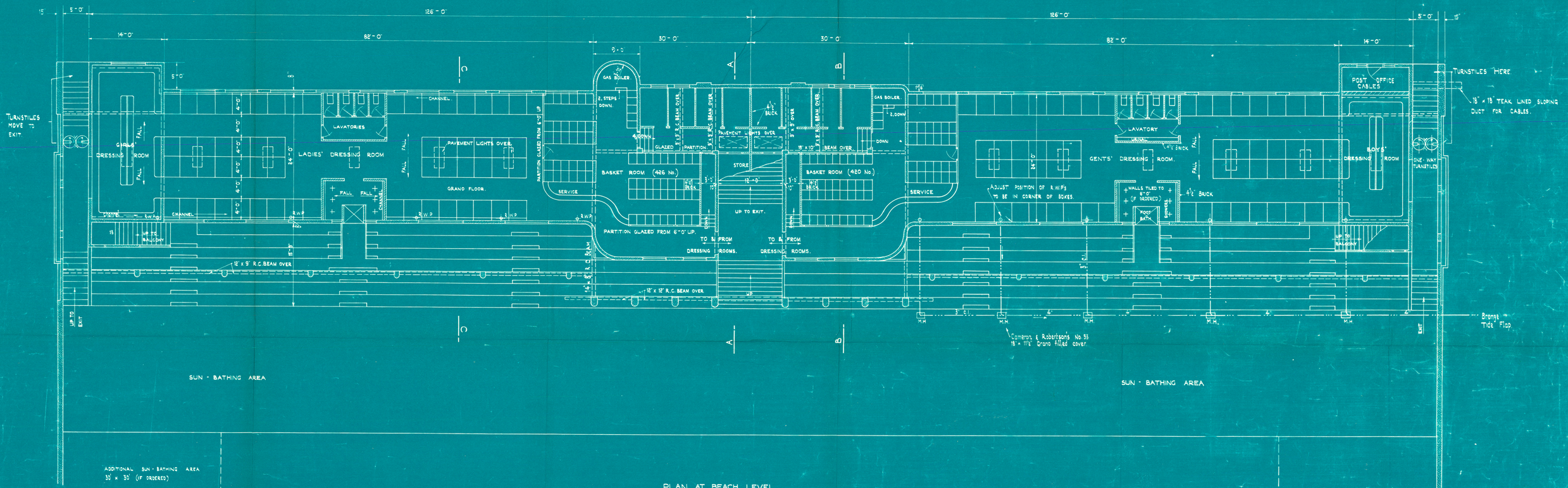
1) the value of the ultimate torsional stress in plain concrete,
2) the torsional rigidity of rectangular, Tee and Ell sections
3) the increase in torsional strength obtained by the use of different systems of reinforcement.

A simple analysis of a bow girder in reinforced concrete is given and the experimental data used to design such a member.

The tests on plain concrete members are of interest to the geologist as they give a new explanation of reversed faulting.



PLAN AT BALCONY & MAIN ENTRANCE LEVEL.



PLAN AT BEACH LEVEL.

BURGH OF DUNOON ~ 1/8" SCALE PLANS OF NEW BATHING STATION.

F. A. MACDONALD & PTNS. (GLASGOW) LTD
135, WELLINGTON STREET, GLASGOW

WKG. DRAWING No.:- 571 - 1

DRAWN BY
TRACED BY 27/12/35.
CHECKED BY
APPROVED BY

REVISED 20/2/36.

1. Bathing Station at Dunoon.
2. Pit Head Baths at Arniston, Fifeshire.
3. Water Tower at Ruchill for Glasgow Corporation.

The problem which was common to all these structures was the design of a circular arc bow girder in reinforced concrete. This problem could not be solved in the simple way that the cantilever balcony was solved and the method of design adopted was as follows:-

1. For bending moments: design the beam (a) as a cantilever of length "l_1" as shown in Fig 2., and (b) as a beam of fixed ends of span "l_2": the steel required at the support was then taken as that given by (a) and the steel at midspan that given by (b).
2. For shearing forces: design the beam as a simply supported beam of span "l_2" and put in twice the amount of shear reinforcement required at the supports.

On the somewhat well worn civil engineering design principle, that if the structure stands up the design must be satisfactory, it could be said that this method of design is in order, because the structures in question are still standing and show no cracks. Unfortunately this well worn principle does not take into account the economics of design, and the correct principle should surely be to design a structure such that with the

minimum quantity of material it is capable of resisting the applied load.

In these days of keen competition the economics of design are very important and it will be shown later in this thesis how uneconomical the design adopted was when compared with a design based on mathematical theory combined with experimental observation.

In addition to cantilever balconies and beams curved in plan, another type of member which must be designed to resist torsional stresses is the screw pile.

The advantages of screw piles for certain forms of construction have been enumerated by Col. Guillet, Chief Engineer of Nantes in a paper given to "L'Association pour l'advancement des Sciences" in 1935.

M. Grimaud (1) has invented a screw pile in reinforced concrete which has been used successfully on different types of engineering construction. The pile is octagonal in cross section with a screw shaped cutting edge at the toe. The reinforcement consists of longitudinal bars and with 45^{o} spirals, the latter being designed to resist the torsional stresses set up during driving. In cases where driving is likely to be hard, a pipe runs longitudinally through the pile to enable the use of a water jet to assist driving.

Unfortunately Grimaud publishes no details of how the torsional stresses are calculated nor does he state any theory on which the design of his section is based, but as his pile is patented this is doubtless information which he would rather not divulge.

In addition to those members already mentioned in which torsion plays such an obvious part that its presence cannot be neglected, there are many members where it would appear at first sight that torsion does not occur, but a closer examination shows that this is incorrect. A few common cases of this type of member will now be given.

1. Beam and Slab floor with alternate panels loaded (Fig 3). The effect of this loading is to cause the slab to deflect as shown, producing a torsion on the beams. Whilst loading of this form is always considered when designing the slab (since it produces the maximum positive and negative moments at mid span) the torsional effect is always neglected when the design of the beams is considered.

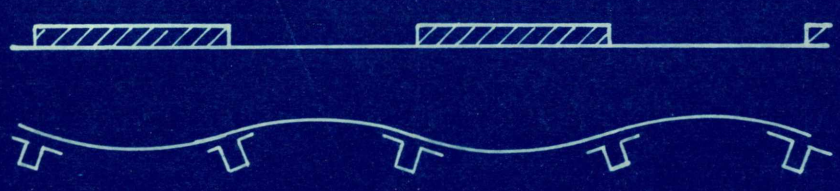


FIG. 3



FIG. 4

2. Exterior Floor Beams (Fig 4). In this case even the effect of the dead loading alone is sufficient to cause a deflection of the slab (into a position somewhere between that of a freely supported and totally fixed slab) which in turn tends to twist the beam. Friedmann and Germansky(2) have studied the effect of this on the design of the slab, but they make no mention of the effect on the design of the beam. The general practice in this country is to allow for a fixing moment of $\frac{wL^2}{24}$ in the slab where it runs into an exterior beam, but no account is taken of the torsion produced on the beam by this moment.

3. Framed Structures(see Fig 5). Consider the cases of an arrangement of beams and columns (a) where the columns are rigid (b) where they are flexible. In the first case if a load is applied to the secondary beam this member will tend to deflect as shown causing the main beam to twist; the rigidity of the column, however, restrains it and consequently the main beam is subjected to torsion. In the second case if a load is applied to the secondary beam a deflection of the column is produced which in turn imposes a torque on the beam. Obviously in a large framed structure one could magnify this torsion considerably by placing loads at selected points, although the fact that a column is never perfectly rigid nor perfectly flexible means that the

condition would not reach either of the extremes shown.

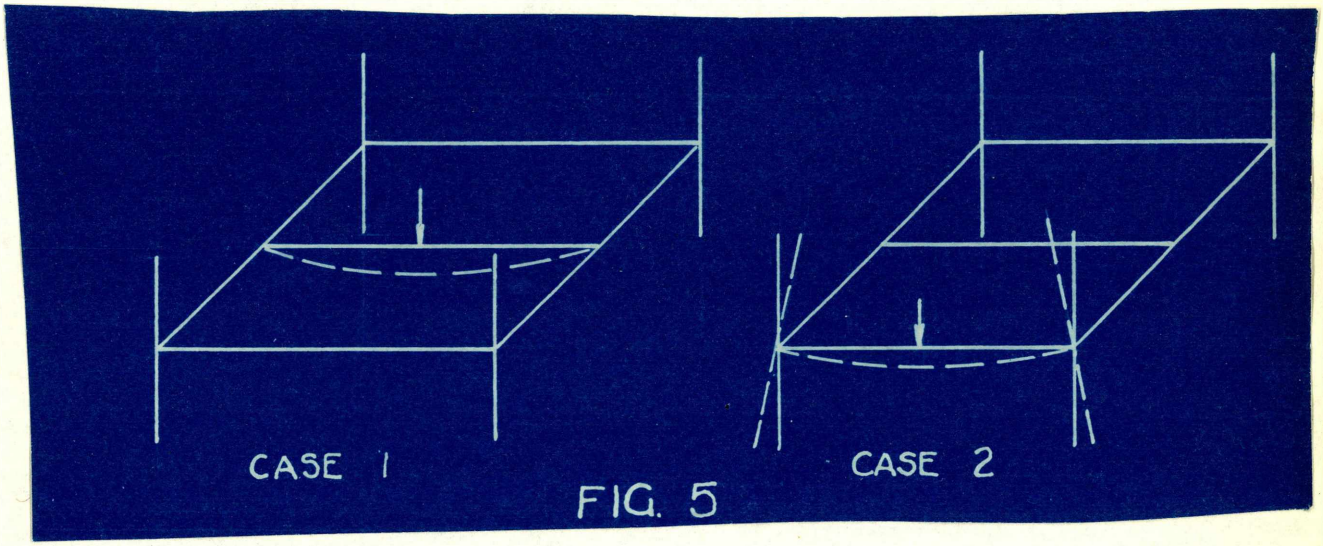


FIG. 5

4. Square Slabs

In the Recommendations for a New Code of Practice Clause 602(c) paragraph ii, the presence of torsion at the corners of a slab simply supported on all four sides is recognised and it is stated that when reinforcement to resist this force is provided the bending moments in the centre of the spam may be reduced. No mention is made of what constitutes adequate reinforcement, and Scott and Glanville(3) point out that there is little experimental evidence available to show the best type of reinforcement to use. Dr. Marcus (4) has given a method of slab design which allows for the assistance given by torsion and research work done in his country has indicated a suitable method of reinforcing the slab to resist the torsional loading.

The three first-mentioned cases are everyday occurrences in reinforced concrete design yet in no case is the torsional moment produced taken into account in the design of the various members. Whilst it may be argued that in spite of this neglect the structures still stand therefore why take the torsion into account, it has to be remembered that the tendency in modern design is to use higher stresses and to take advantage of the monolithic nature (of a reinforced concrete building by reducing the bending) moments. The older structures used lower stresses and more conservative bending moments and it does seem that an examination of the stresses produced by the torsional loading introduced by considering the monolithic effect would make modern design more consistent.

Many cases occur of cracking in a reinforced concrete structure: these cracks are in most cases not dangerous but nevertheless their prevention is to be desire in most cases the cracks are ascribed to secondary stresses due to shrinkage, etc. but it is at least possible that one of the causes of the cracks which occur in beams is the neglected torsional stresses.

It is evident, therefore, that the problem of the design of a reinforced concrete section to resist a torsion moment is a very practical one, and its solution which (as far as the author knows) is not given in any text book on reinforced concrete design would provide a useful contribution to the knowledge of that subject.

In Chapter 3 the results obtained by previous investigators of this problem have been collected and discussed, and it will be seen that many people have investigated the problem and some have suggested formulae for the design of a section to resist a torsional moment. The objection to these formulae is that the only type of reinforcement employed is the 45° spiral, and the use of such reinforcement in a framed structure of reinforced concrete is not practicable. In the author's opinion the only formulae of practical value are those in which any torsion which cannot be resisted by the plain concrete is taken by reinforcement in the form of longitudinal bars and stirrups. One of the motives prompting the research described in this thesis has been the production of such formulae.

The 45° spiral formulae are in all cases applied either to circular or square sections, and no information is given as to how they can be applied to rectangular sections. Circular beams are never used and the only useful application of formulae thus given would be to screw piles. Square beams are very unlikely to occur and their use is generally uneconomical when considered from bending moment considerations. Consequently one would seldom be able to apply the formulae as they stand, and further work is necessary and new formulae must be

introduced before a rectangular reinforced concrete sectio: can be designed to resist a given torsional moment.

The objections to the 45° spiral are:-

1. Longitudinal bars and vertical stirrups would have to be provided in all cases to resist the bending moment and shearing forces, and a third type of reinforcement would produce a very complicated system especially as the spirals would have to be introduced near the supports where the shearing forces are greatest, and therefore where the vertical stirrups are most closely spaced. Scott and Glanville have pointed that one of the advantages of Clause 602,(a) of the Recommendations for a New Code of Practice is that it allows the designer to use a B.M. at the support which is less than the theoretical, and thereby to reduce the congestion of the reinforcement at the supports where the satisfactory consolidation of the concrete is so often a practical difficulty. The introduction of spiral reinforcement at the supports would add greatly to this difficulty and would be a retrograde step.

2. In the case of beams designed to resist a torsional moment which could be applied in a clockwise or anti-clockwise direction two opposing spirals would be necessary, thus making four systems of reinforcement.

3. The fabrication of reinforcement containing spirals is very difficult. The small beam 6" x 4" x 4' 0" forming beam K of the author's tests took nearly a day to fabricate and the difficulty involved in a longer beam with heavier reinforcement and with stirrups in addition would be considerable. It has been suggested to the Author that the winding of the binding around helically bound columns which is usually carried out by uncoiling wire does not present much difficulty and neither should the spiral reinforcement for resisting torsion. In helically bound columns, however, the core is either octagonal or circular, and bending wire round either of these forms is easier than bending round a square or rectangle; moreover, the pitch of the spiral used in columns is about 80°, which is easier to place than the 45° spiral used for torsion reinforcement; finally, only one spiral is used for columns whereas for torsional resistance four or more spirals might be necessary

4. The presence of a spiral around the main bars and stirrups necessitates additional cover for these bars in order to provide adequate cover for the spiral. The effective depth of the beam will be reduced and additional longitudinal reinforcement will be required.

The author has dealt fully with the objections to spiral reinforcement in order to emphasise the importance of

obtaining formulae which enable longitudinal bars and vertical stirrups alone to be used. Of the arguments again 45° spirals the only one which can be applied to the use of bars and stirrups is that since the greatest torsion usually occurs near to the supports the use of additional bars and stirrups will tend to increase the congestion of reinforcement at that point. This is true but in the author's opinion the congestion thereby caused is much less than that introduced by the use of spiral reinforcement.

To investigate the behaviour of plain and reinforced concrete in torsion with the object of producing data on which a design could be based experiments were carried out on 46 specimens which were classified as follows

1. Circular specimens of plain concrete. The reasons for testing specimens of a circular cross section were, firstly, to determine a true value for the torsional strength of plain concrete since the circular section is the only one for which the behaviour under torsion can be easily analysed mathematically and, secondly, to determine the variation in torsional strength with age. The specimens were tested after 7, 28, 60 and 90 days; the results of the tests are given on pages 86 and discussed on pages 159

2. Plain rectangular specimens. Tests were carried out on rectangular specimens of the following cross-sectional

dimensions 6 ins. x 4 ins; 6 ins. x 2 ins; 6 ins. x 3 ins; 6 ins. x $3\frac{1}{2}$ ins; 4 ins. x 4 ins. These cover a ratio of depth to breadth from 1 to 1 to 3 to 1: in practice the cross section of a beam would generally lie between these two extremes. These tests appear to form the most complete series of rectangular plain concrete specimens ever tested (no previous investigator tested more than three different cross sections of rectangular specimen). They therefore provide information for checking the various formulae which can be used to calculate the stress in a rectangular section submitted to torsion.

3. Reinforced Rectangular specimens. These really form the most practical of the various tests. The cross section of specimens used in these tests was a 6 in. x 4 in and the reinforcement provided to resist the torsion was as follows (a) longitudinal bars only B1-B3 and C1-C3, (b) longitudinal bars and stirrups (D1-D3, E1-E2, F1-F2, G1-G2); the total amount of reinforcement provided varied for each class, the variation being from 1.68% in the case of specimens G to 5.0% in the case of specimens E. In all cases the total quantity of vertical reinforcement was equal to the total quantity of longitudinal reinforcement. (c) longitudinal bars and 45° spirals H, J and K.

These tests scarcely provide sufficient data on which to base formulae for design and it is hoped to carry out a further series of tests of class (b) to

check the formulae which have been deduced.

4. Tee and Ell section specimens. Plain and reinforced Tee sections were tested. The plain specimens were of differing cross sections, T1-T2 having flanges 8 ins. wide, and T5-T6 having flanges 9 ins. wide: specimens T3 and T4 were similar in cross section to T1 but were reinforced.

Plain Ell sections only were tested.

In addition cubes and tension specimens were tested to obtain the compressive and tensile strengths of the concrete used.

From these tests it has been possible to determine

1. The torsional strength and modulus of rigidity of plain concrete.
2. The effect of the age of the concrete on these values.
3. The relationship between the torsional, compressive and tensile strengths of concrete.
4. The relationship between the moduliu of rigidity and elasticity of plain concrete and the value of Poissons ratio.
5. The torsional rigidity of reinforced concrete.
6. The increase in strength due to various methods and percentages of reinforcement.
7. The behaviour of Tee and Ell beams in torsion.
8. The relationship between the strength and the percentage of longitudinal and vertical reinforcement

CHAPTER II.

THEORIES OF TORSION FOR CIRCULAR AND RECTANGULAR SECTIONS.

CHAPTER II.

THEORIES OF TORSION FOR CIRCULAR AND RECTANGULAR SECTIONS.

If a prism is subjected to torque the motion of any particle in that prism consists of (1) a rotation about the axis of twist in a plane at right angles to the axis, and (2) a movement in a direction parallel to the axis of twist. Thus plane sections will not generally remain plane and the problem of calculating the stresses and strains in terms of the twisting moment is in the general case extremely difficult.

In the particular case of a circular cross section subjected to torque plane sections will remain plane after twisting and the problem becomes simple, the formula for the stress and strain in terms of the torque being in the case of an elastic material $\frac{T}{J} = \frac{q}{r} = \frac{N}{l}$ (1) (for notation see page 4).

In a rectangular cross section, however, plane sections will not remain plane and the problem cannot be solved in a simple manner as with a circular section.

The classic solution of the general case is that of Saint-Venant. This was published in 1855 in the "Memoirs des Savants étrangers". He has used what is described as a "semi inverse method of solution" and he makes certain assumptions with regard to the application of the load. Because of these assumptions his theory is not true in the

immediate neighbourhood of the point of application of the load.

Applying Saint - Venant's theory (22) to the case of a rectangular section of breadth b. and depth d. the shear stresses at a point in the area whose co-ordinates are x and y in the case of an elastic material given from the formulae

$$S_x = \frac{2N\theta}{l}x - \frac{8NJd}{\pi^2}\left\{\frac{\cosh\frac{\pi y}{d}}{\cosh\frac{\pi b}{d}}\sin\frac{\pi x}{d} - \frac{1}{3^2}\cdot\frac{\cosh\frac{3\pi y}{d}}{\cosh\frac{3\pi b}{d}}\sin\frac{3\pi x}{d} + \text{etc.}\right.$$

$$S_y = \frac{2N\theta}{l}y - \frac{8NJb}{\pi^2}\left\{\frac{\cosh\frac{\pi x}{b}}{\cosh\frac{\pi d}{b}}\sin\frac{\pi y}{b} - \frac{1}{3^2}\cdot\frac{\cosh\frac{3\pi x}{b}}{\cosh\frac{3\pi d}{B}}\sin\frac{3\pi y}{b} + \text{etc.}\right\}$$

where S_x and S_y are the stresses parallel to the x and y axes respectively.

It can be shown that the maximum value of S_x will occur when $y = 0$ and $x = \frac{d}{2}$; also that the maximum value of S_y will occur when $x = 0$ and $y = \frac{b}{2}$, also if d is greater than b then the maximum value of S_y will be greater than the maximum value of S_x: in other words the maximum stress occurs at the mid point of the longer side.

The value of the torque carried by the section is given by the formula -

$$T = \frac{b^3 d}{3} \cdot \frac{N\theta}{l} - \frac{64b^4}{\pi^5} \cdot \frac{N\theta}{l} \left\{ \tanh r + \frac{1}{3^5} \tanh 3r + \text{etc.} \right\}$$

where $r = \frac{d}{2b}$

Eliminating $\frac{N\theta}{l}$ from these equations gives -

$$T = S_1 \times \left\{ \frac{\frac{bd^3}{3} - \frac{64b^4}{\pi^5} \left(\tanh r + \frac{1}{3^5} \tanh 3r \right)}{b - \frac{Sb}{\pi^2} \left(\operatorname{sech} r + \frac{1}{3^2} \operatorname{sech} 3r + \text{etc.} \right)} \right\}$$

In addition to these somewhat complicated formulae Saint-Venant has given a simpler formula in which he states that the torque is given by,

$$T = \frac{b^2 d^2}{1.8b + 3d.} S_1 .$$

Where S_1 = the maximum stress (i.e. the stress at the mid point of the longer side).

This empirical formula gives results which agree to within 5 per cent of the exact formula.

Timoshenko (8) has also given a simple formula for the torque carried by a rectangular section. He states that,

$$T = k_1 b^2 d\, S_1 .$$

Where K_1 is constant for any given ratio of d to b, the value for varying ratios being as given in Table 1, and S_1 has the same value as in Saint-Venant's formula.

Timoshenko also gives the relationship between the torque and the angle of twist as follows:-

$$\frac{\theta}{l} = \frac{T}{K_2 N d b^3} .$$

where K_2 has the values given in Table 1 for varying ratios of d to b.

Table 1. Values of K_1 and K_2 in Timoshenko's formulae.

$\frac{d}{b}$	1.00	1.50	1.75	2.00	2.50	3.00	4.00.
K_1	.208	.231	.239	.246	.258	.267	.282
K_2	.141	.196	.214	.229	.249	.263	.281

Bach has suggested the following formula:-

$$T = \frac{db^2}{3 + \frac{2.6}{0.45+\frac{d}{b}}} \times S_1.$$

Seely (9) in his "Advanced Strength of Materials" gives another solution, due to Bach, of the stresses in a rectangular section subjected to torque. He has assumed, firstly, that the distribution of shear stress along a side is parabolic, having maximum and zero values at the centre edges respectively; secondly, he has assumed that the stress component in direction of the x axis varies directly as the x co-ordinate of the point under consideration and similarly with the stress component in the direction of the y axis.

Thus at any point (x-y), the stresses are,

$$S_x = \frac{2y}{b}\left[1 - \left(\frac{2x}{d}\right)^2\right]S_1$$

$$S_y = \frac{2x}{d}\left[1 - \left(\frac{2y}{b}\right)^2\right]S_1^1$$

Where $S_1^1 = \frac{b}{d} S_1$.

The value of the torque is then

$$T = \frac{2}{9} b^2 d S_1.$$

This method was used by Young, Sagar and Hughes in the Toronto tests described on page

These formulae give different results when applied to the same cross section. In Table 2 a comparison is given of the approximate formulae with the exact solution of Saint-Venant.

In preparing this Table it has been assumed that unit torque will produce a maximum stress S in a section when Saint-Venant's formula is used in the calculations. The values given in the table are those of the torque required to produce the same maximum stress when other formulae are used in the calculations.

Table 2. Comparison of Formulae.

$\frac{b}{d}$ ratio	1:1	1:1.5	1:2	1:3
Saint-Venant exact	1.00	1.00	1.00	1.00
Do. approximate	1.00	1.03	1.05	1.05
Timoshenko	1.00	1.00	1.00	1.01
Bach	1.00	1.00	1.00	1.01
Seely	1.07	0.96	0.89	0.84

It is seen that Timoshenko's and Bach's formulae show almost exact agreement with Saint-Venant over the range of $\frac{b}{d}$ ratios taken. Since the former is the simpler it will be used throughout the tests on rectangular specimens described in this thesis.

In considering the effect of the application of a twisting moment to a circular section of concrete allowance has to be made for the fact that it is a brittle material, i.e. one in which the ultimate tensile strength is about a tenth of the ultimate compressive strength.

The effect of torsion is to produce on an element shear stresses q, a tension stress t, and a compressive stress c, t and c being complimentary, and each equal and inclined at 45° to q.

The failure of any material under torsion will be governed by its ability to resist tension, shear and compression. In a material which has less resistance to shear than tension or compression, the fracture will be in a plane at right angles to the axis of twist: such a fracture occurs in the case of mild steel. In a brittle material failure will occur in tension along a helix inclined at 45° to the axis of twist: this type of failure is seen in materials such as concrete or cast iron.

If the simple theory of torsion were true for concrete, the value obtained for the ultimate shear stress should be equal to the ultimate tensile stress. It is found, however, that this is not the case (see Chapter V).

Morsch (7), Bach and Graf (5) tested annular sections of concrete and found that the ultimate shear stress was approximately equal to the ultimate tensile strength. It would appear, therefore, that the difference between the ultimate shear stress and the ultimate tension stress referred to in Chapter V is partly due to the fact that concrete does not behave as a perfectly elastic material.

If the material is perfectly plastic then the standard formula for a circular section of elastic material given on

page 24 becomes

$$T = \frac{2\pi r^3}{3} \times q \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If the material is obeying an exponential stress-strain law then in a circular section the stress q_1 on any element at a distance x from the centre can be expressed by the formula

$$\frac{x}{r} = \left(\frac{q_1}{q}\right)^k$$

where k is a constant for any given material. For a circular section the relationship between torque and stress then becomes

$$T = \frac{2\pi kr^3}{3k+1} q \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Probst (5) suggests that concrete obeys an exponential stress-strain law and gives the value of k as varying between 1.11 and 1.16.

If K = 1.11 then $T = 1.61\ r^3 q$.

If K = 1.16 then $T = 1.628\ r^3 q$.

Leslie Turner and Davies (6) suggest that the stress at any section distant x from the centre of a circle subjected to torsion is given by

$$q_1 = q - \alpha(r-x)^{1.5}$$

where α is a co-efficient. This relationship gives the following formula for T in terms of q

$$T = 1.84\ q\ r^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

When dealing with a circular section concrete in torsion it is difficult to say which of the equations 1,2,3 or 4 should be applied. In the chapter dealing with these tests all of them have been used and the comparison is interesting.

CHAPTER III.

PREVIOUS INVESTIGATIONS.

CHAPTER III.

PREVIOUS INVESTIGATIONS.

Since the main object of this thesis is to present a method of designing a bow girder in reinforced concrete attention is devoted in this chapter not only to investigations of the torsional strength of plain and reinforced concrete but also to those dealing with the design of bow girders.

Previous investigations of the torsional strength of plain and reinforced concrete appear to have had two objectives:

1. To discover the ultimate shearing strength of concrete. Torsional loading is the only way of producing a pure shear stress; in all other methods the shear force is combined with a certain amount of bending and the test results are difficult to analyse.
2. To study the behaviour of plain and reinforced concrete under torsional loading.

The earlier tests were carried out with the first objective in view.

The earliest investigations were made in 1903, the next did not appear until 1911, but from 1921 onwards a considerable number of investigations have been reported.

The investigations will be considered in chronological order. It will be noticed that whilst British

and American investigators have concentrated on some form of rectangular section; Continental investigators have used circular sections. It is also of interest to note that this problem has attracted little or no attention in France in spite of the popularity of concrete construction in that country.

Tests by Professor Emil Morsch (7) (Stuttgart) - 1903.

These were the earliest tests carried out. The specimens were solid and hollow cylinders of plain concrete. Both types were 10.25 centimetres diameter and 34 centimetres long, the internal diameter of the hollow cylinders being 5.9 centimetres. The concrete was a 1:3 mix with 14% water. The tests on solid cylinders gave an ultimate shear stress of 243 lbs. per square inch after 90 days, whilst the hollow cylinders gave a stress of 126 lbs. per square inch after 55 days. The tensile strength of concrete of the same mix was 113 lbs. per square inch.

It appears that these tests were made to compare the value of the ultimate shear strength with the value of 440 lbs. per square inch at 45 days obtained from the same experimenter's punching shear tests (Probst (5) page 235).

Morsch also developed a formula for the stress in 45 degrees spiral used as torsion reinforcement. He assumed that the excess twisting moment carried by a reinforced section over the corresponding plain section is divided equally between the tension in the spirals and the diagonal compression in the concrete. Thus if T^1 is the excess moment of the reinforced section over the plain, N the number of spirals each of area A, at a radius R (measured to the centre of the wire) and S the stress in the spiral, then

$$\frac{T^1}{2} = \frac{A.S.R.N.}{\sqrt{2}.}$$

and

$$S = \frac{T^1}{A.R.N.\sqrt{2}}$$

The objection to this formula is that it is only applicable to circular sections and does not help to solve the problem of torsion in rectangular beams. It could, however, be used for the design of screw piles.

Tests by Bach and Graf (Sluttgart) (5) and (10) - 1911.

These were more complete than those of Morsch. They were carried out on specimens 6.5 feet long with the following cross sections:-

(a) square up to 10 inches.

(b) rectangular up to 17 inches by 8.5 inches.

(c) circular up to 16 inches diameter.

(d) annular up to 16 inches external diameter and 10 inches internal diameter.

The specimens were made from a 1:2:3 mix concrete with 9 per cent by weight of water. The aggregate used was Rhine sand and gravel. The specimens were tested after 45 days.

The ultimate stresses were calculated from the formulae based on the elastic theory and the values for the different cross sections are given in Table 3.

The calculated value of the ultimate shear stress varied with the cross section being greatest for the rectangle and least for the annular.

Tests were also made on reinforced square sections and the following summary is given by Probst:

"For the unreinforced square sections the cracking moment amounted to 12840 lbs. per foot and when reinforced with 4 bars was 13600 lbs per foot and with 8, 14,200 lbs per foot. When spirals

were used as well as 8 longitudinal bars the cracking moment was increased to 18,900 lbs. feet and the breaking moment to 29,400 lbs. feet".

This statement whilst giving some information about the effect of reinforcement is of little value since it does not state the diameter of the bars used as reinforcement nor any information as to the amount of spiralling.

Table 3 - Tests by Bach and Graf.

Specimen.	Square.	Rectangle.	Circle.	Annular.
Shear Stress. lb. per sq. in.	425	450	360	200
Modulus of Rigidity lb. per sq. in.	1.93	2.02	2.00	1.86

Tests By Bach (5).

In addition to the tests made in collaboration with Graf, Bach carried out a similar series of tests on unreinforced specimens.

The results, which agree very closely with his other experiments, are given in Table 4.

Table 4 - Tests by Bach.

Specimen.	Square.	Rectangle.	Round.	Annular.
Ultimate Shear Stress lb. per sq. in.	430.	460	360	240
Ratio: $\frac{\text{Torsion Strength}}{\text{Torsion Strength of Annular.}}$	1.78	1.90	1.50	1.00
Ratio: $\frac{\text{Torsion Strength.}}{\text{Tensile Strength}}$	1.63	1.75	1.38	0.92
Ratio: $\frac{\text{Torsion Strength}}{\text{Cube Strength}}$	0.12	0.13	0.10	0.07

Tests by Professors Graf and Morsch (11).

These tests were made on circular specimens only. The outside diameter was 40 centimetres in each case and the following types of specimens were tested:

(a) Hollow cylinder of plain concrete 25 centimetres internal diameter.

(b) Solid cylinder of plain concrete.

(c) Similar to (b) but with ten longitudinal bars 10 millimetres diameter.

(d) Similar to (b) but with rings of 10 millimetres diameter bars at 10.8 centimetres pitch, each ring being 34 centimetres in diameter.

(e) Combination of (c) and (d).

(f) Similar to (b) but with seven spirals at 45^0 of 10 millimetres diameter bars.

The concrete used was a 1:2:5 mix with 9.3 per cent by weight of water. The values of the torque at the appearance of the first crack and the maximum torque are given in Table 5.

The tensile strength of the concrete was 11.5 kilometres per square centimetre (164 lbs. per square inch) whilst the compressive strength obtained from prisms was 125 kilometres per square centimetre (1780 lbs per square inch)

Table 5 - Graf and Mörsch Tests.

Specimen.	A.	B.	C.	D.	E.	F.
% of reinforcement	0	0	0.625	0.617	1.242	0.617
Torque at first crack kilo. cms	146.0	233.3	233.3	250	266.6	270.0
lb. ins.	126.3	202.0	202.0	216.5	231.0	233.8
Torque at rupture kilo. cms	146.0	233.3	238.3	250	378.3	700.0
lb. ins.	126.3	202.0	206.5	216.5	328.0	606.0
Ratio to plain circle	0.63	1.00	1.04	1.07	1.62	3.00

These tests do not materially assist in the design of a section to resist a given torsional moment. They do, however, show the advantage of combining longitudinal and vertical reinforcement and the comparative uselessness of one of these forms when used without the other. The values given for the torque at first crack and ultimate torque appear very small, even allowing for the poor concrete used when compared with the values obtained in the Author's tests.

Tests by Foppl -(1911)

Foppl carried out tests on plain and reinforced concrete circular specimens. He made his specimens from a mortar instead of from ordinary concrete. The details are given in Exemplarder Mittkilungh des Mechanisch-technischen Laboratoriums, Heft 32a paper which the Author has been unable to procure, but Probst (5) gives the following summary of his results:

1. The ultimate shear strength calculated on the elastic theory was 370 lbs. per square inch at 112 days and 425 lbs. per square inch at 210 days, from which Foppl concluded that the older specimens showed the greatest resistance to torsion.

2. The modulus of rigidity was almost constant at a value of 2×10^6 lbs. per square inch for dry mixes and 1.6×10^6 lbs. per square inch for wet mixes.

3. The breaking load was increased by the presence of reinforcement, the increase being greater in the more heavily reinforced specimens. The reinforcement also made the specimens more resistant to shocks. The 45° spiral is recommended as the most suitable.

4. Foppl assumed that an equal action and reaction took place between the steel and the concrete and from this deduced a formula for the shear stress at the surface of

the steel at the moment when slipping was about to occur. By application of this formula he showed that the strength of a torsion specimen may be reduced if the reinforcement is not prevented from moving by anchoring at the ends.

This formula is of interest and Foppl is the only investigator who makes any mention of it, but unfortunately he only gives it for reinforced circular sections.

Tokio Tests (12).

These tests were carried out by T. Mujamoto in the Home Department, Tokio. They form the most complete series for circular specimens both plain and reinforced. The specimens were 30 centimetres diameter and 1.5 metres long, made of 1:2:4 concrete with a water content of about 9%, which had strengths of 1780 lbs. per square inch and 175 lbs. per square inch in compression and tension respectively after 45 days.

The reinforced specimens were divided into three groups:-

1. Spiral reinforcement at 30^{0}, 45^{0} and 60^{0} respectively to the axis of twist.
2. Combinations of reinforcement consisting of:-
   (a) Spirals and longitudinal bars.
   (b) Spirals and rings.
   (c) Longitudinal bars and rings.
3. Double spiral reinforcement.

The conclusions drawn were:-

(1) Failure of plain concrete and cracks of reinforced concrete follow without exception a direction of 45^{0}.

(2) Of the three forms of spiral tested (see 1) above), the 45^{0} was the most effective and the 30^{0} the least effective.

(3) Longitudinal bars or rings added to the 45^{0} spiral increased the torsional resistance: the rings were more effective than the longitudinal bars.

(4) The additional of longitudinal bars to the 60° spiral and rings to the 30° spiral made them as strong as the 45° spiral.

(5) A combination of longitudinal bars and rings has no influence upon torsional resistance.

Summary of the tests on Circular Specimens.

The tests so far considered were on circular specimens and were in close agreement. The general conclusions are:-

1. Failure is due to diagonal tension, the cracks running in a 45° spiral.

2. The value of the ultimate shear stress on the elastic theory is different in solid and hollow cylinders. In the latter the stress is approximately the same as that obtained from direct tension tests.

3. Longitudinal reinforcement alone has little effect on the ultimate torsional resistance (about 5 per cent increase with 1 per cent reinforcement).

4. Lateral reinforcement alone has a greater effect than longitudinal reinforcement (about 10 per cent increase with 1 per cent reinforcement).

5. The effect of combining longitudinal and lateral reinforcement is to give an increase in torsional resistance of about 40 per cent with 1 per cent reinforcement. (The Tokio tests and Professor Graf's tests are in definite disagreement upon this point).

6. The 45° spiral is the most effective and economical type of reinforcement against torsion.

These tests are mainly of academic interest, the only possible application to design being for screw piles and torsion in circular columns, both of which

are very rare and unlikely problems.

Some consideration will now be given to the various tests carried out on rectangular sections.

Toronto Tests (13) 1922.

These tests were carried out by Professor C.R. Young in the University of Toronto. The specimens tested were plain and reinforced concrete rectangular beams of three different cross sections, 5 in. x 5 in, 5 in. x $7\frac{1}{2}$ in. and 5 in. x 10 in, the b to h ratios being 1:1, 1:1.5 and 1:2. The concrete used was a 1:2:4 mix, the aggregates being bank sand and limestone. The specimens were tested after 28 days. The reinforced specimens were divided into three groups:-

1. Longitudinal reinforcement only.
2. Longitudinal reinforcement with light 45° spirals.
3. Longitudinal reinforcement with close 45° spiralling.

The complete results of the tests are given in Table 6.

These tests which were the first to be made on plain and reinforced rectangular sections show interesting features but have the following disadvantages.

(1) Only one specimen of each type was tested and it is unwise to draw definite conclusions from the experiments. Three specimens at least of each type should be used in dealing with a material such as concrete, which seldom gives consistent experimental results.

(2) No experiments were done to show the effect of vertical stirrups.

(3) No design formulae has been deduced from the results.

Table 6 - Toronto Tests on Rectangular Sections.

Specimen.	Nominal Size.	Longitudinal Reinforcement.	Lateral Reinforcement.	Ultimate torsional strength, lbs. per square inch.	Ratio of strength to that of plain concrete.
A 1.	5 in. x 5 in.			539	
A 2.	5 in. x 7.5 in.	None.	None.	467	-
A 3.	5 in. x 10 in.			602	
B 1.	5 in. x 5 in.	4 - $\frac{1}{4}$ in. diameter		532	0.99
B 2.	5 in. x 7.5 in.	4 - $\frac{5}{16}$ in. diameter	None.	506	0.95
B 3.	5 in. x 10 in.	4 - $\frac{3}{8}$ in. diameter		584	1.09
C 1.	5 in. x 5 in.	4 - $\frac{1}{4}$ in. diameter	4 - 45° spirals	668	1.25
C 2.	5 in. x 7.5 in.	4 - $\frac{5}{16}$ in. diameter	of $\frac{3}{16}$ in.	621	1.16
C 3.	5 in. x 10 in.	4 - $\frac{3}{8}$ in. diameter	diameter.	530	0.99
D 1.	5 in. x 5 in.	4 - $\frac{1}{4}$ in. diameter	8 - 45° spirals	838	1.56
D 2.	5 in. x 7.5 in.	4 - $\frac{1}{4}$ in. diameter	of $\frac{3}{16}$ in.	740	1.38
D 3.	5 in. x 10 in.	4 - $\frac{1}{4}$ in. diameter	diameter.	790	1.47

Tests by Turner and Davies (6) - 1934.

These tests which are the only previous tests made in this country were carried out by Messrs L. Turner and V. Davies at the Battersea Polytechnic.

The specimens were plain and reinforced concrete of square, rectangular and Tee section. They were made of a 1:2:4 mix concrete, the aggregates being Thames Valley sand and Thames ballast.

Details of the specimens, reinforcement provided and the results of the tests are given in Table 7. All the specimens were tested after 35 days except S2 which was tested after 25 days.

The conclusions drawn by these investigators are in close agreement with those of others, although their work has special interest since they suggested formulae for the behaviour of plain concrete in tension to allow for the effects of plasticity, etc., and also for the design of a reinforced concrete section in torsion. It should be noted, however, that this latter formula as published is in error. The tests appear to be the first on a combination of long and lateral reinforcement in a rectangular section, also effect of splays.

These tests are open to criticism on the following ground:

1. The control of the concrete could have been imporved: it is difficult to draw legitimate deductions from the

experimental results as the tension tests give results varying from 156 to 356 lbs. per square inch.

2. No attempt was made to check the formula given for design with the experimental results.

Table 7 - TURNER AND DAVIES TESTS.

Specimen.	Size.	Details of Reinforcement.	Per cent of Reinforcement.	Max.Torque lb. ins.	Strength of Tension Specimens	Remarks
S1.	5.15 in.sq.	None	-	11,000	301	
S2.	5.10 in.sq.	None	-	11,750	-	
S3.	5.15 in.sq.	4 - $\frac{3}{8}$ in. diameter bars	1.66	12,500	222	
S4.	5.10 in.sq.	4 - $\frac{3}{8}$ in. diameter bars, $\frac{1}{8}$ in. diameter stirrups, $3\frac{3}{4}$ in. pitch.	1.87	10,550	211	Poor result, dry mix used.
S5.	5.00 in. sq.	4 - $\frac{3}{8}$ in. diameter bars, $\frac{1}{8}$ in. diameter spiral at 45°	1.95	13,250	255.	
S6.	5.00 in. sq.	As S4.	1.87	12,250	255	
S7.	5.00 in. sq.	As S3.	1.66	12,000	212	
S8.	5,00 in. sq.	4 - $\frac{3}{8}$ in diameter bars, $\frac{1}{8}$ in. stirrups, $1\frac{7}{8}$ in. pitch	2.08	16,000	356	
R1.	8.2 in. x 3.99 in.	None.	-	17,000	-	
R2.	8.0 in. x 4.0 in.	None.	-	12,000	217	Poor Concrete
T1.	8 in. x 8 in. O.A. Tee, flange.	None.	-	18,500	-	
T2.	2 in. thick, rib 4 in. wide.	None.	-	16,000	156	Poor Concrete

Tests by Paul Anderson (14) and (16) - 1934 and 1937.

These tests were made by R.L. Brown and P. Anderson in the laboratories of the University of Illinois.

The first series of tests (1934) was carried out on six different types of plain and reinforced concrete specimens. The concrete was a 1:2:4 mix and the specimens were tested after 28 days. A novel feature was the measurement, by means of strain gauges set at 45° on the sides of the specimen, of the principal compressive and tensile strains. The reinforced specimens were square in cross section, the reinforcement consisting of longitudinal rods, longitudinal rods and vertical stirrups, or longitudinal rods and spirals.

The second series of tests (1937) was carried out using the same mix of concrete but testing after 21 days. Plain and reinforced specimens were again tested, the reinforcement being in the form of longitudinal rods alone or longitudinal rods and spirals.

In the paper describing these tests Mr. Anderson gave a formula for the design of a section, reinforced with 45° spirals, to resist a given torsional moment. He also discussed the torsional moments which are likely to occur in framed structures.

The general conclusions as to the torsional resistance of the different types of reinforcement agree fairly closely with those of other investigators.

One must again criticise the lack of control in the manufacture of the concrete. The test cylinders gave results varying from 3900 to 7000 lbs. per square inch which is too large a variation to inspire confidence in the deductions.

Summary of Tests on Rectangular Specimens.

The conclusions drawn from the foregoing tests are similar in all respects to those drawn from the earlier tests on circular sections (see pages 45). The Toronto tests also show that the values of the ultimate torsional stress calculated on the elastic theory depend on the geometry of the section. The tests all show the usefulness of combining longitudinal and lateral reinforcement to resist torsion and prove the superiority of the 45° spiral reinforcement.

The investigators gave formulae for the design of sections using such reinforcement, but in only one instance (T and D) was a formula given for use with a combination of longitudinal and lateral reinforcement without spirals, and the formula is of doubtful value.

Previous Investigations of the Bow Girder.

Experimental work to verify the existing theories of design is at present being carried out in the Civil Engineering laboratories of the City and Guilds College. Mathematical analyses of the problem have been made in this country by Gibson and Ritchie (17) and Pippard and Barrow (18).

Gibson and Ritchie gave a solution of the problem of the circular arc bow girder in their book published in 1914, the former having previously presented a paper to the Royal Society of Edinburgh in 1912 containing the mathematical solutions for the case. They did a considerable amount of experimental work on the torsional rigidity of steel sections but do not attempt any solution of the bow girder of reinforced concrete.

Pippard and Barrow in 1926 gave a more general mathematical solution for any shape bow girder using strain energy methods of stress analysis. The formula given in the papers referred to will be used later when they will be applied to the design of a bow girder in reinforced concrete.

CHAPTER IV.

METHOD OF CARRYING OUT THE EXPERIMENTS.

Manufacture of Test Pieces.

The different test pieces used for the present investigations can be divided into four main groups. (1) unreinforced circular specimens; (2) unreinforced rectangular specimens; (3) reinforced rectangular specimens; (4) specimens of Tee and Ell section both plain and reinforced.

In all cases the concrete mix and the method of curing were kept constant.

The Concrete Mix.

The proportions used throughout were one part of cement to two parts sand and four parts of aggregate. The proportions were obtained by weight.

The cement used was Ferrocrete portland cement which satisfied the requirements of the British Standard specification.

The sand was a well graded and washed river sand and the aggregate was $\frac{3}{4}$ in. Thames Ballast.

The water-cement ratio used throughout the experiments was 55 per cent.

The concrete was machine mixed in a "Liner" concrete mixer, the time of mixing for each batch being 3 minutes.

The concrete was in all cases consolidated by vibration by a Sinex Electric vibrator giving 6000 vibrations per minute.

Method of Curing.

The forms were, except where otherwise mentioned, stripped after 24 hours and the specimens were then totally immersed in water until the time of testing.in the case of specimens tested after 7 days. In those cases where tests were made of specimens that were more than 7 days old the specimen was removed from the water after 6 days immersion and then stored in the atmosphere until the time of test. The temperature of the laboratory where the specimens were made and cured remained fairly constant during the whole series of experiments.

Circular Specimens.

These were 3 ft. 6 ins. long and 5 ins. diameter. The forms for the specimens consisted of a wooden framework which was lined with sheet metal to give the circular cross section. For a distance of 6 inches from each end the forms were made so as to produce a concrete section 6 inches square. The forms were constructed in two pieces to facilitate their removal and on reassembling care had to be taken to produce the correct alignment in order to give a perfectly shaped section. Details of the forms are shown in figure 8.

The specimens were cast vertically. Some difficulty was experienced in connection with the specimens which were made first because they showed a weakness at the junction of the circular section and the square ends. To overcome

this a ring of expanded metal 3½ ins. diameter and 9 ins. long was placed in the concrete at that section. The expanded metal used was No.22 gauge thick, at ⅜ in. centres short-way of mesh and 1 11/16 in. centres long-way of mesh. One of the earlier specimens broke when being removed from the forms after 24 hours and it was therefore decided to keep all future specimens in the forms for 48 hours before stripping.

The vibrating clamp was placed at mid-height of the forms when the latter were vertical.

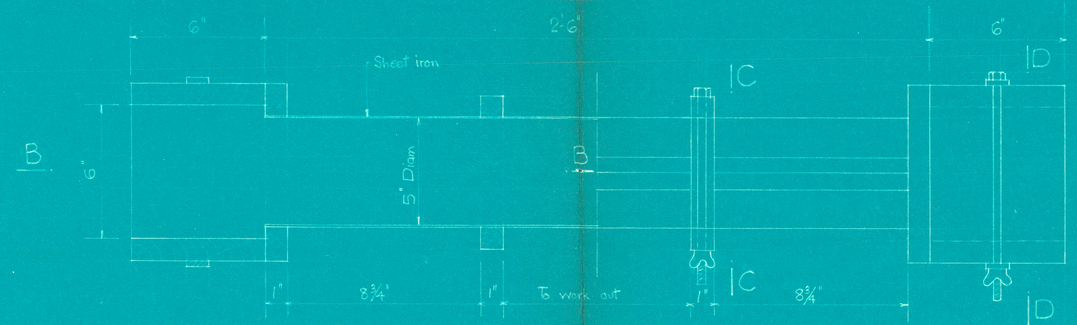
Unreinforced rectangular sections.

These were 4 ft. 0 ins. long generally, and 6 ins. deep, and made in the following widths, 2 inches, 3 inches, 3½ inches and 4 inches. One series of specimens were made of 4 inch square cross section. For a distance of 6 inches from each end they were thickened out to a section 2 inches greater in width than that of the specimen, e.g. in the case of the 6 in. x 4 in. specimens the ends were made 6 ins. square.

Details of the forms are shown in figure 9.

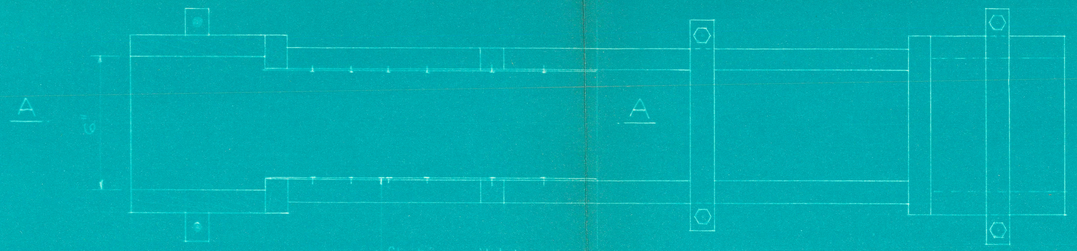
The specimens were cast horizontally and no difficulty was experienced at the junction of the two cross sections.

In the case of the 4 in. square specimens a platform 2 inches deep was placed in the bottom of the forms thus giving a specimen 4 ins. square over the central



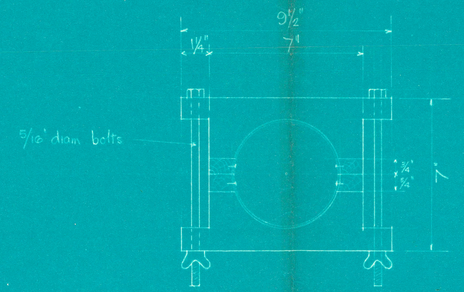
SECTION A-A

ELEVATION

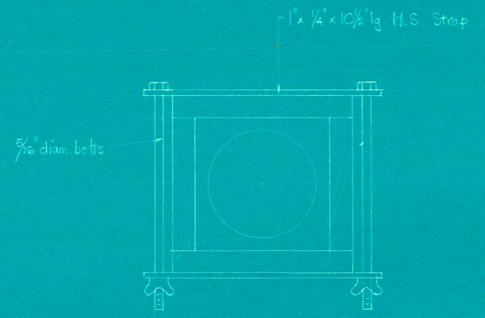


SECTION B-B

PLAN



SECTION C-C



SECTION D-D

DETAILS OF MOULDS FOR CIRCULAR TORSION SPECIMENS.

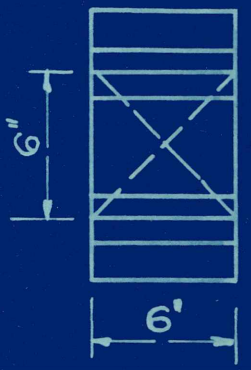
SCALE :- ¼" F.S.

FIG. 8

5/6/56



PLAN



SECTION

Note : These end pieces vary in width to suit the width of the specimen being cast.

DETAILS OF FORMS FOR RECTANGULAR SPECIMENS

FIG. 9

SCALE : 1½" TO 1'-0"

Table 8 - Details of Reinforcement in Rectangular Specimens.
(nominal size 6 in. x 4 in.)

Specimen	Longitudinal Reinforcement		Lateral Reinforcement		Spiral Reinforcement		Per cent of total reinforcement
	Details	Per Cent	Details	Per Cent	Details	Per Cent	
Series B	4 - 5/16 in. diameter bars in corners.	1.28	None	-	None	-	1.28
Series C	4 - 5/16 in. diameter bars in centre of sides.	1.28	None	-	None	-	1.28
Series D	4 - 5/16 in. diameter bars in corners	1.28	18 - 1/4 in. diameter stirrups at 2.54 in. pitch	1.28	None	-	2.56
Series E	4 - 7/16 in. diameter bars in corners.	2.50	15 - 3/8 in. diameter stirrups at 2.90 in. pitch	2.50	None	-	5.00
Series F	4 - 3/8 in. diameter bars in corners	1.80	17 - 5/16 in. diameter stirrups at 2.80 in. pitch	1.80	None	-	3.60.
Series G	4 - 1/4 in. diameter bars in corners	0.82	22 - 3/16 in. diameter stirrups at 2.22 in. pitch	0.82	None	-	1.64.
Series H	do.	0.82	None	-	1 - 3/16 in.diam. spiral at 45°	0.165	0.985.
Series J	do.	0.82	do.	-	3 - 3/16 in.diam. spirals at 45°	0.495	1.315
Series K	do	0.82	do.	-	5 - 3/16 in.diam. spirals at 45°	0.825	1.645

portion but with ends 4 ins. x 6 ins.

The vibrating clamp was placed at the centre of the forms.

Reinforced Rectangular Specimens.

These were 4 ft. 0 ins. long and 6 ins. x 4 ins. in cross section. The same forms were used as for the 6 ins. x 4 ins. unreinforced specimens, the section being thickened out at the end to 6 ins. square. The various types of reinforcement were as shown in Table 8.

Details of the various reinforcement "cages" are as shown in figure 10.

The bars were bent cold to the required shape and fixed in position relative to each other by the use of soft iron tying wire.

The specimen in Series K caused great trouble in fabrication. Due to the interlocking of the various spirals it was found necessary to cut them in places and the weld up in order to keep continuity. It was also found very difficult to bend the spirals to the shape required. The spirals were placed so as to be in tension when the specimens were twisted in the test machine.

The reinforcement for the specimens in series B and C consisted of longitudinal bars only and these were kept in position by two stirrups at each end of the cage.

When casting, a layer of concrete of thickness equal to the cover to the main bars was first placed in the

forms and well consolidated, the reinforcement "cage" was then placed in position and the concreting of the specimen finished off. It is realised that this method tends to produce slightly less cover of concrete on the bottom than on the top but the inaccuracy produced can be neglected when dealing with a material such as concrete which even when tested under the most ideal conditions seldom gives consistent results.

Tee and Ell section specimens.

The specimens in this series were as follows:-

Series T1 - T2 - Tee section 18 ins. x 6 ins. overall, flange $2\frac{1}{4}$ ins. deep, rib $3\frac{1}{2}$ ins. wide

Series T3 - T4 - As T1 but reinforced.

Series T5 - T6 - Tee section 19 ins. x 6 ins. overall, otherwise as T1.

Series L1 - L2 - Ell section, 9 ins. x 6 ins. overall, flange and rib as for T1.

Details of the forms used are shown in figure 11. Special attention had to be paid to the design of the forms in this series in order to give (1) a section at the end of greater torsional rigidity than that at mid span, and, (2) a gradual change in rigidity where the end section joined the main section.

The forms as designed gave fairly satisfactory results in both these respects.

Specimens T5 and T6 were reinforced with expanded metal at the end splays to attempt to avoid cracking at

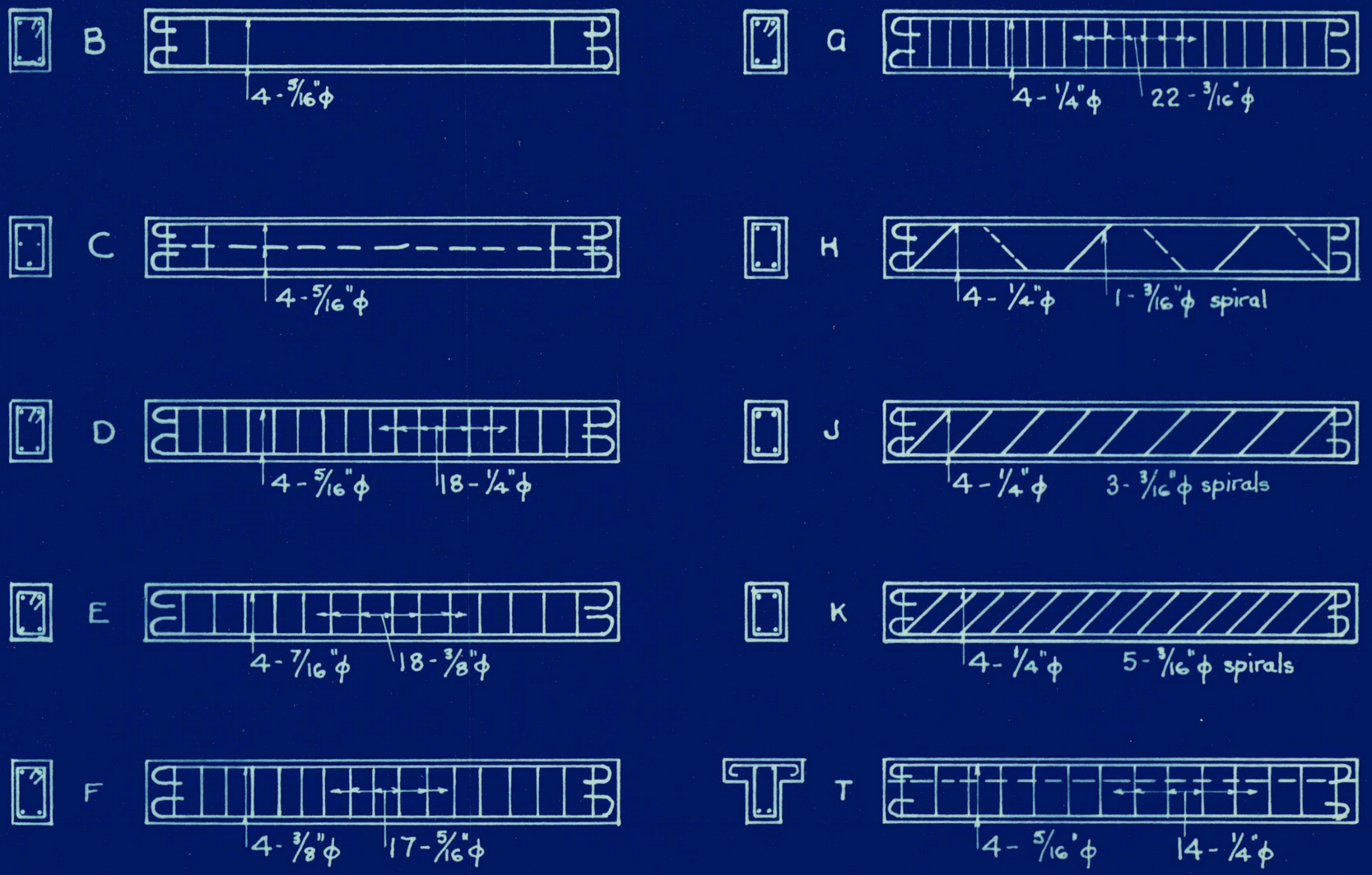
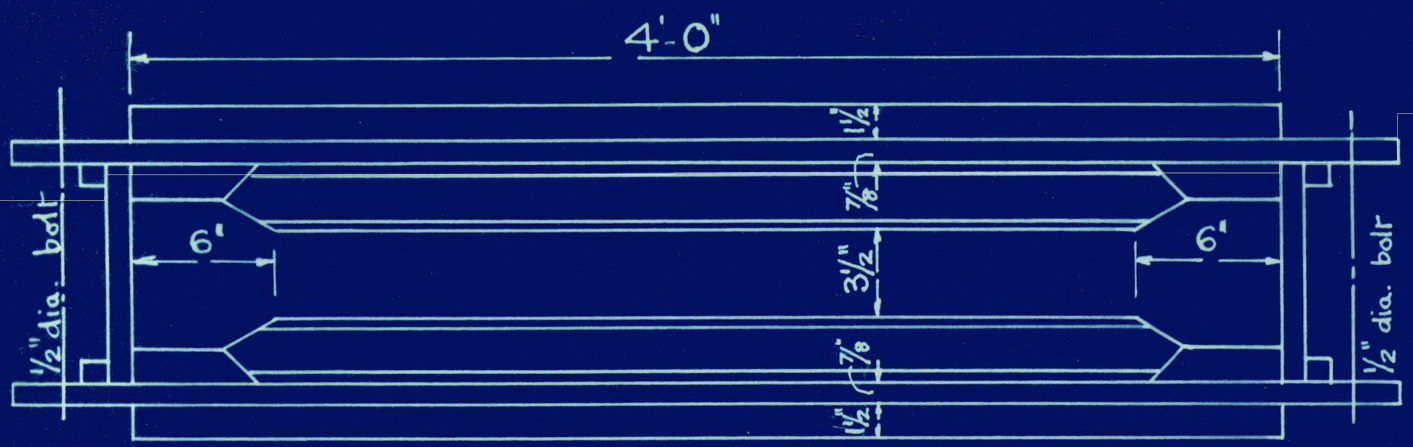
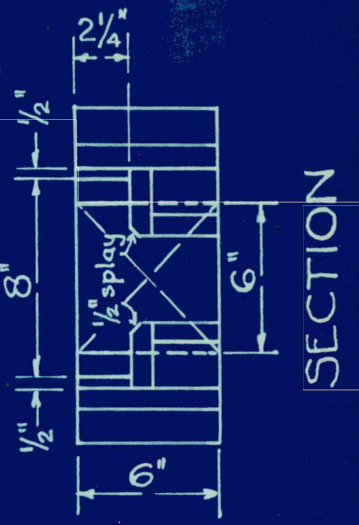


FIG. 10 - DETAILS OF REINFORCEMENT

SCALE : 3/4" TO 1'-0"

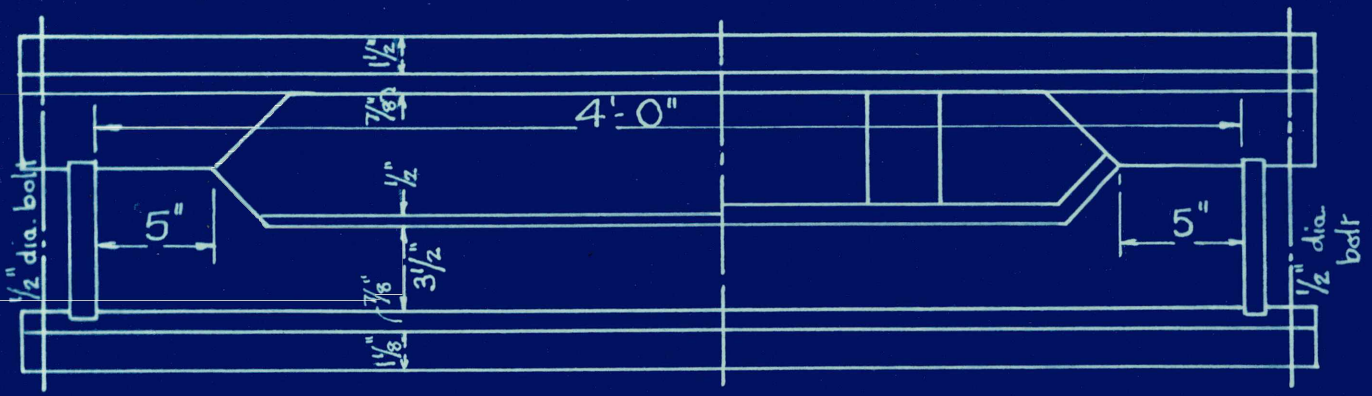


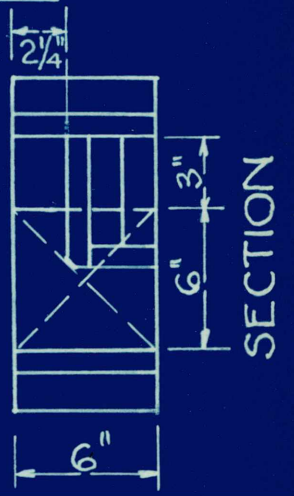


PLAN

SECTION

DETAILS OF FORMS FOR TEE BEAMS





TOP PLAN

SECTIONAL SECTION

DETAILS OF FORMS FOR ELL BEAMS

FIG. 11

SCALE : 1½" TO 1'-0"

this section. Specimens L1 and L2 were also reinforced at the ends for the same purpose, the reinforcement consisting of a short cage of five 5/16 in. diameter bars with ¼ in. diameter stirrups.

The reinforced specimens were made in a similar way to that described for reinforced rectangular specimens.

The vibrating clamp was placed at the centre of the specimen.

Cubes for Compressive Strength.

Throughout the series of tests cubes were made to determine the compressive strength of the concrete. This served two purposes:-

1. To compare torsional strength with compressive strength under similar conditions.
2. To check up that the quality of cement and other materials used throughout the tests had no appreciable variation.

The cubes were 6 in. x 6 in. x 6 in. and were made in steel moulds on the vibrating table. They were removed from the moulds after 24 hours, immersed in water for 6 days and then stored in air until time of testing.

The Test Machine.

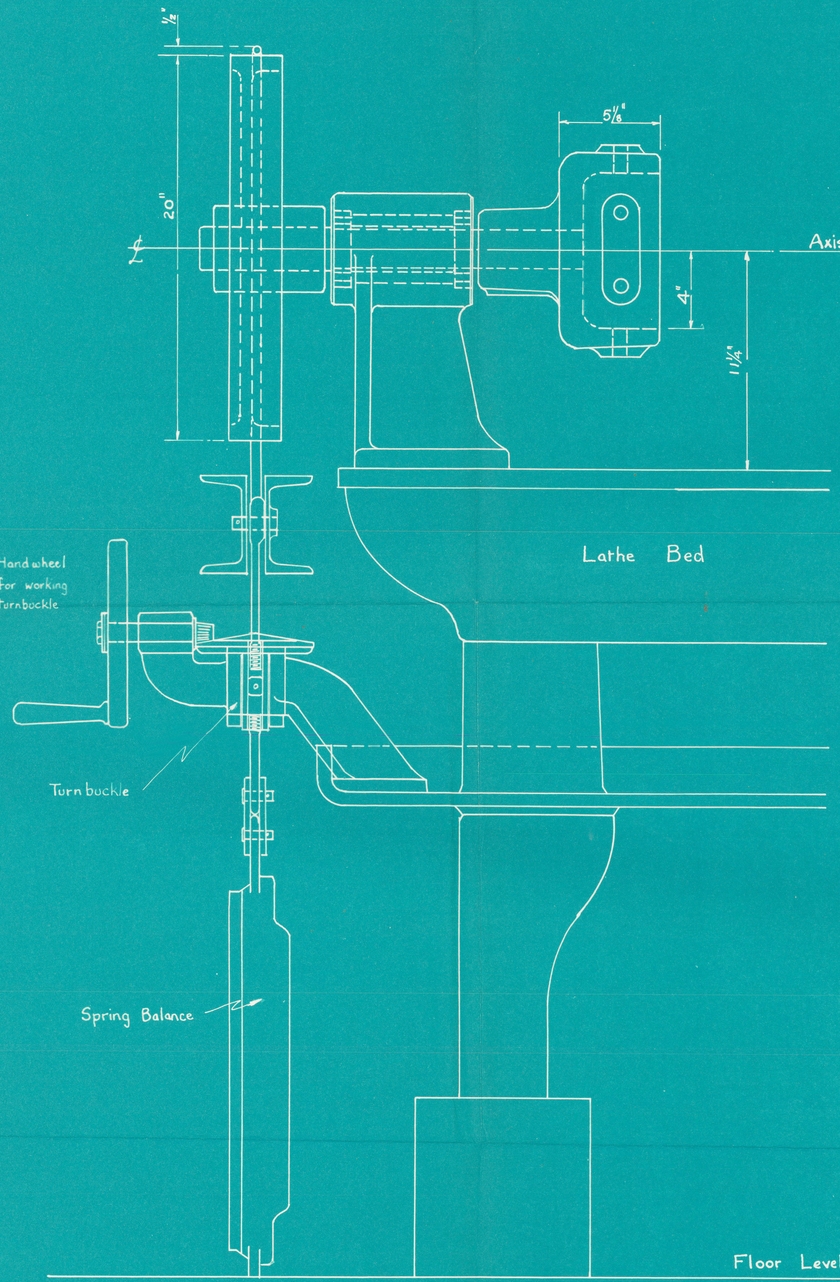
The tests were carried out in a torsion testing machine designed by the Author in collaboration with Professor A.J.S. Pippard and members of his staff. The machine was assembled in the workshops of the Department of Civil Engineering of the City and Guilds College.

Details of the machine are shown in figure 12 and plates 1 and 2.

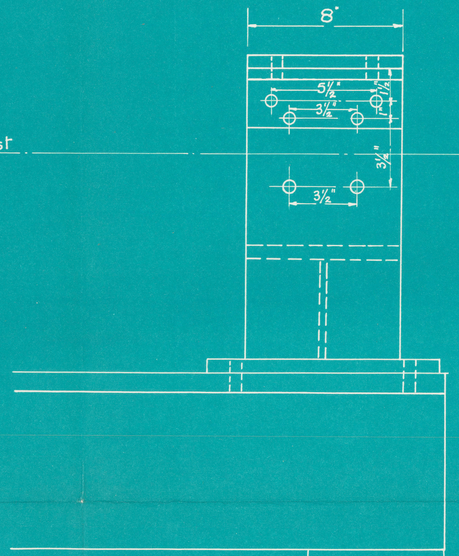
The structural calculations required for the various parts of the machine are given in Appendix 1.

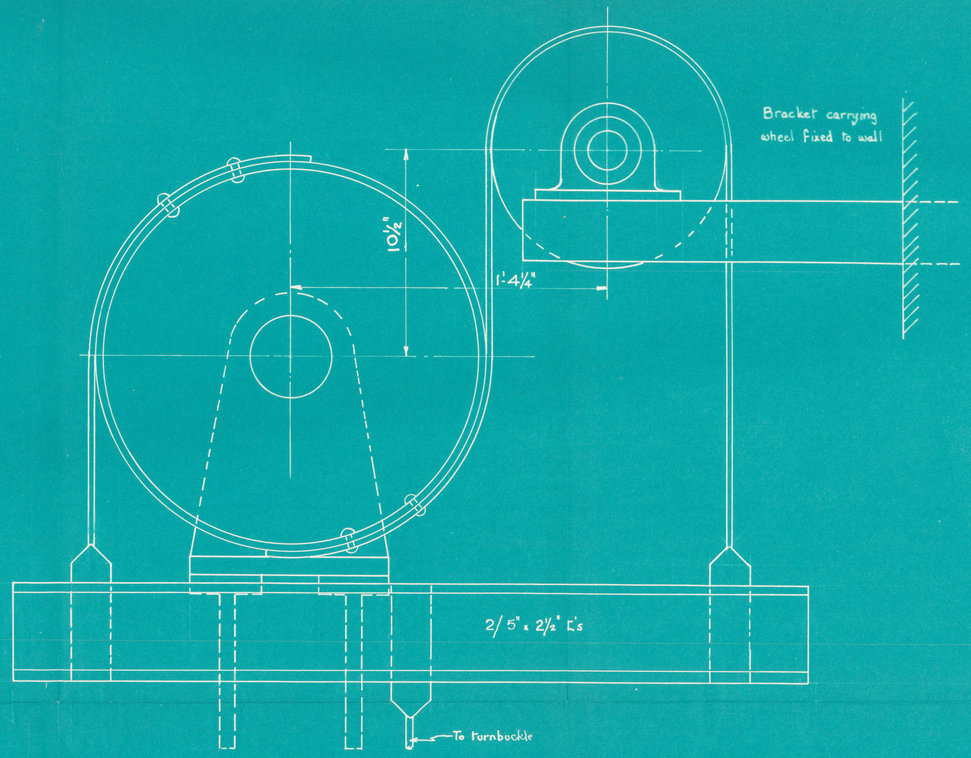
The machine consisted of a lathe bed A mounted on two concrete foundation blocks B. On the lathe bed were mounted the fixed "chuck" and head C and the "chuck" D. The fixed chuck was cut out of a 10 in. x 8 in. x 55 lb. B.S.B. whilst the movable chuck was a casting.

Through the top of the chuck D was placed a $2\frac{1}{4}$ in diameter shaft which ran in a self centering bearing. To one end of this shaft the rotating head E was keyed and to the other a wheel F, 20 ins. in diameter. The movable head E was of cast steel. The horizontal line running through the centre of the $2\frac{1}{4}$ in. diameter shaft was the axis of twist.

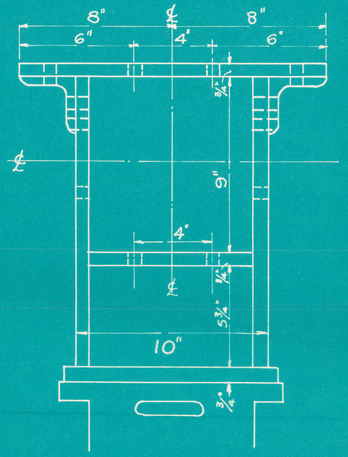


SIDE ELEVATION

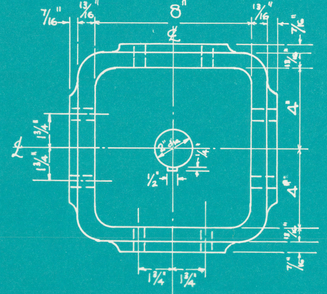




FRONT ELEVATION



GRIP AT FIXED END



GRIP AT ROTATING END

FIG. 12 - GENERAL ARRANGEMENT OF TORSION MACHINE

SCALE : ¼ F.S.

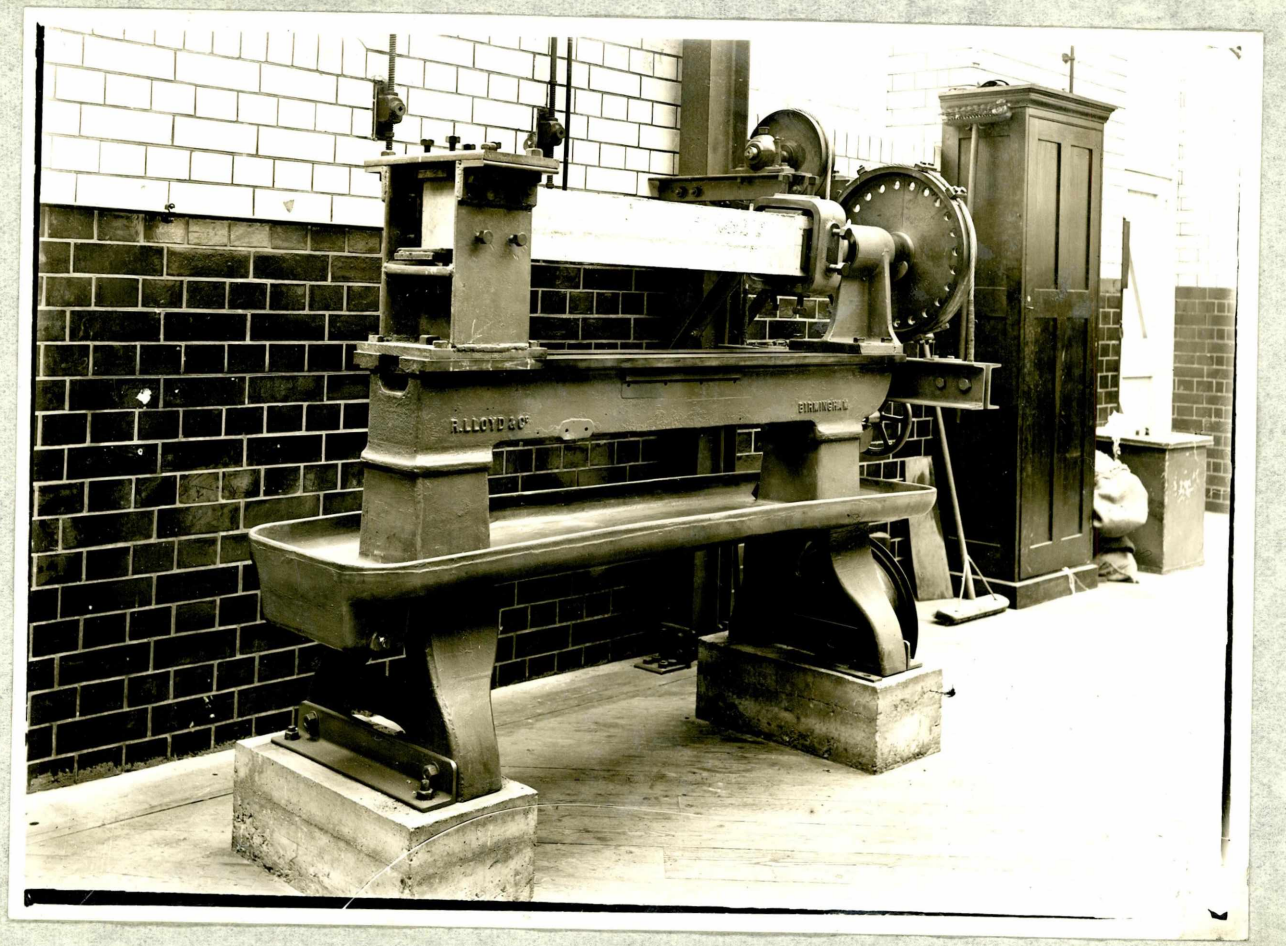


Plate 1 - Test Machine showing fixed end



Plate 2 - Test Machine showing loading device

The horizontal plane through the axis of twist was marked by four indentations G, one on each side of the two heads. The vertical plane was marked in a similar way by indentations H.

A pointer R was attached firmly to a stationary bracket, the pointer giving a reading on to a scale S fixed to the wheel trough which the torque was applied. This scale was divided up into degrees and it served two purposes firstly, it enabled the angle of twist to be measured when this was large and,secondly, it enabled one to know when the top of the moving head was horizontal.

To enable the specimens to be securely held in the heads and also to be accurately centred each head contained packing pieces J which were held in position by means of screws K. A layer of felt paper was placed between the steel packing pieces and the concrete specimens to eliminate uneven pressure distribution due to surface irregularities.

Saint-Venant has shown in his theory of rectangular sections to which a torque is applied that in addition to a rotation in a plane at right angles to the axis of twist, a particle also moved in a direction parallel to the axis. Since the specimens in the machine were rigidly held in a direction parallel to the axis no movement in this direction was possible and as a result tension stresses were set up in the specimen. The magnitude of these stresses is, however, very small and can be neglected. The 45 degrees

spiral failures given by the unreinforced specimens show that these tensile stresses are negligable.

The torque was applied to the specimen by loading two $\frac{1}{2}$ in. diameter wire ropes which were securely fastened to the wheel F. One of these ropes ran direct to one end of a pair of 5 in. x $2\frac{1}{2}$ in. x 10.22 lb. B.S. Channels L (placed back to back). The other rope ran over a wheel M 12 in. diameter which ran in a bearing supported by a bracket from the wall of the laboratory, and then to the other end of the channels.

The load was applied at the mid point of the channels through a turnbuckle N worked by means of a hand wheel P. The value of applied load was read on the spring balance Q.

The advantage of this type of test machine over the types more commonly seen in test laboratories is that the load is applied in opposite directions to both sides of the wheel F thus no reaction is necessary at the centre spindle to balance the applied load. In the usual type of test machine the load is applied (by means of a screw and bevel device) to one side of a wheel similar to wheel F and to balance this force a reactive force is required at the spindle. There is bound to be a certain amount of friction between the spindle and its bearings and the effect of introducing a reaction at the bearings means that additional

torque will have to be applied in order to overcome the frictional component of the reactive force. In the type of machine designed for these tests no torque is required to overcome any resistance caused by the frictional component of the reaction as the latter does not exist.

It was noticed that when there was no specimen in the machine only a touch on the channels was necessary to revolve the head.

The distance between the centres of the wire ropes running round wheel F was $20\frac{1}{2}$ ins and as half the applied load is taken by each rope the torque applied to the specimen when a load P is indicated by the spring balance is $\frac{P}{2}$ x 20.5, i.e. 10.25 x P.

In order to test the efficiency of the machine and the method to be used for the tests four preliminary experiments were carried out on a hollow steel cylinder 5 in. diameter and 0.16 ins. thick. Two steel plates 6 ins. square and 1 in. thick were shrunk on to each end and welded. The square ends thus formed were placed in the heads of the machine.

Two typical torque-twist curves are given in figure 13 and the average value of N, the modulus of rigidity, obtained from the four tests was 12.08 x 10^6 lbs. per sq. in., a very satisfactory figure.

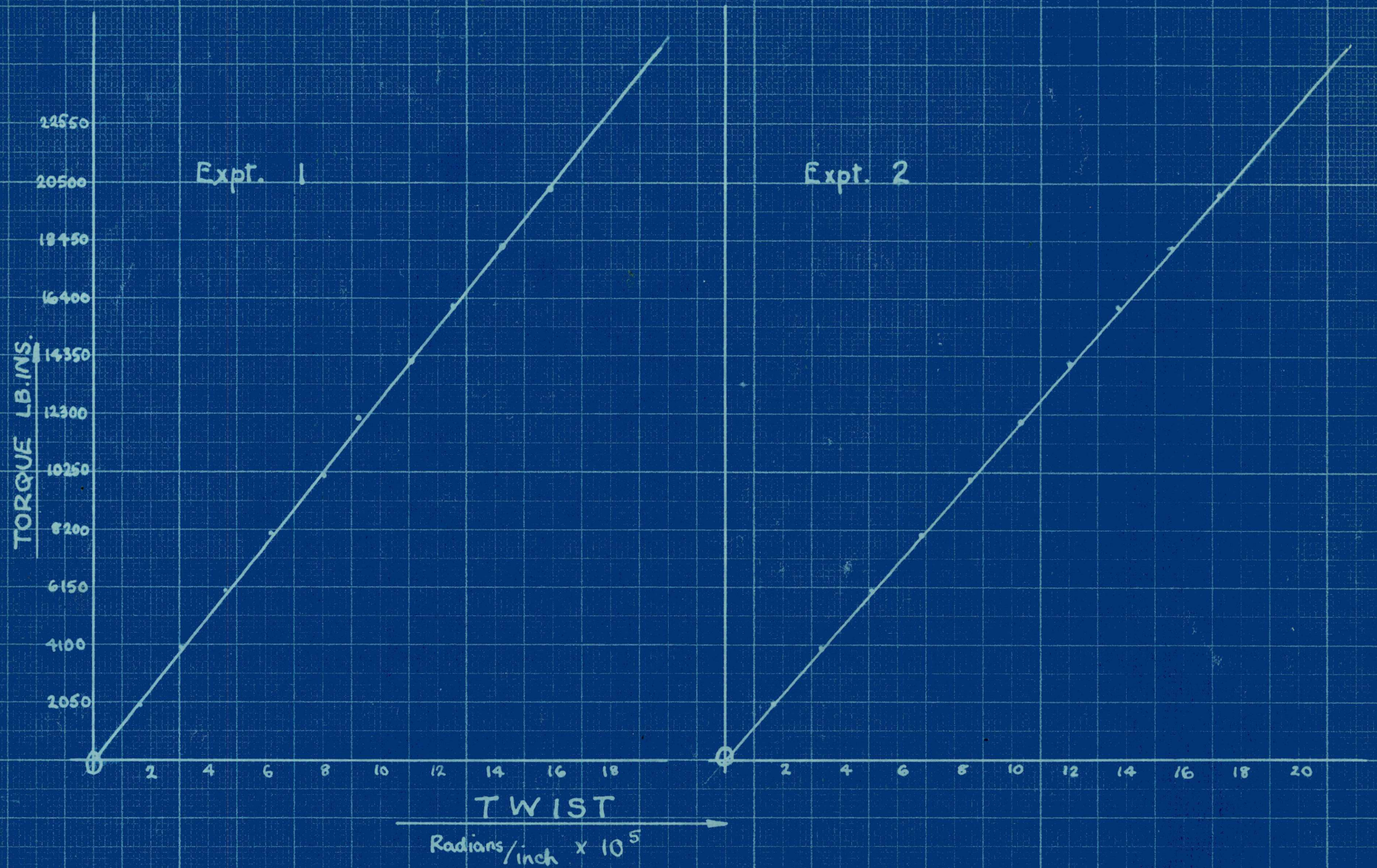


FIG. 13 - TORQUE - TWIST FOR STEEL CYLINDER

SL

Method of Carrying out Tests.

Before placing the test members in the machine the horizontal and vertical planes through the centre of gravity of the cross section were marked on the specimens by lines drawn on the surface of the concrete. The member was then placed in the test machine such that these horizontal and vertical planes coincided with the horizontal and vertical planes through the axis of twist, the positions of the latter being given by the indentations previously mentioned. When adjusting the member to make the lines on it coincident with the indentations on the head of the machine care had to be taken that the top of the moving head was horizontal.

In the case of the Tee and Ell section specimens it was not possible to place them in the machine with the horizontal plane through the centre of gravity of the specimen coincident with the horizontal plane through the axis of twist.

The Tee and Ell specimens were placed with the vertical plane through their centre of gravity coincident with the vertical plane through the axis of twist. The fact that the two axes do not coincide does not affect the results as Saint-Venant has shown that the torsional stresses produced are independant of the axis of twist provided the latter does not occur about a scew axis.

When the specimen had been correctly fixed in the machine mirrors for measuring the angle of twist were then fixed on the vertical plane through the centre of gravity of the section. The mirrors were placed equidistant from each end of the specimen the distance between them being 30 ins. for the rectangular specimens, 24 ins. for the Tee and Ell, and 20 inches for the circular.

The mirrors were fixed to the concrete by means of plasticine: this material was very effective as it gave good adhesion between the different materials and also allowed the mirrors to be rotated as required when fixing them so that the rays of light from the scale were reflected into the telescope.

The load was applied in stages of 50 lb. increments and readings were taken of the angle of twist after each increment of loading. The loading was carried on until the specimen failed.

The time taken for the test varied but in no case was it greater than 15 minutes, consequently there is little possibility of any errors arising from temperature or humidity changes during the test.

It should be pointed out that preliminary tests on the machine showed that as the torque was applied the machine itself twisted slightly on its foundations the amount of twist being dependant on the part of the machine

under consideration. The twist was greatest at the end nearer to the spring balance and least at the further end. (The amount of twist was measured by fixing mirrors to the "fixed" and"movable" chucks).

The effect of this differential twist of the machine was to cause the axis of twist of the specimen to be slightly on the scew. The amount of scew was so small however that any error thereby produced can be neglected.

Measurement of Angle of Twist.

In order to calculate the torsional rigidity of the specimen it is necessary to know -

1. The torque applied.
2. The angle of twist per unit length produced by this torque.

The applied torque was measured from the spring balance reading as previously described and the method of measuring the angle of twist will now be described. Since the angle of twist per unit length is small the usual meth is to measure the twist over a longer length, known as the gauge length, and from this to calculate the twist per unit length. The gauge length taken varied with the specimens (see page).

A pair of mirrors was placed at each end of the gauge length, one on the upper and the other on the lower side of the specimen, both mirrors being originally on

the vertical plane through the axis of twist.

The rays of light from an illuminated centimetre scale A placed about 10 feet away from the mirrors were reflected by the mirrors into a telescope B.

As the specimen rotates under the applied torque the mirror moves in three directions as shown in Figure 14.

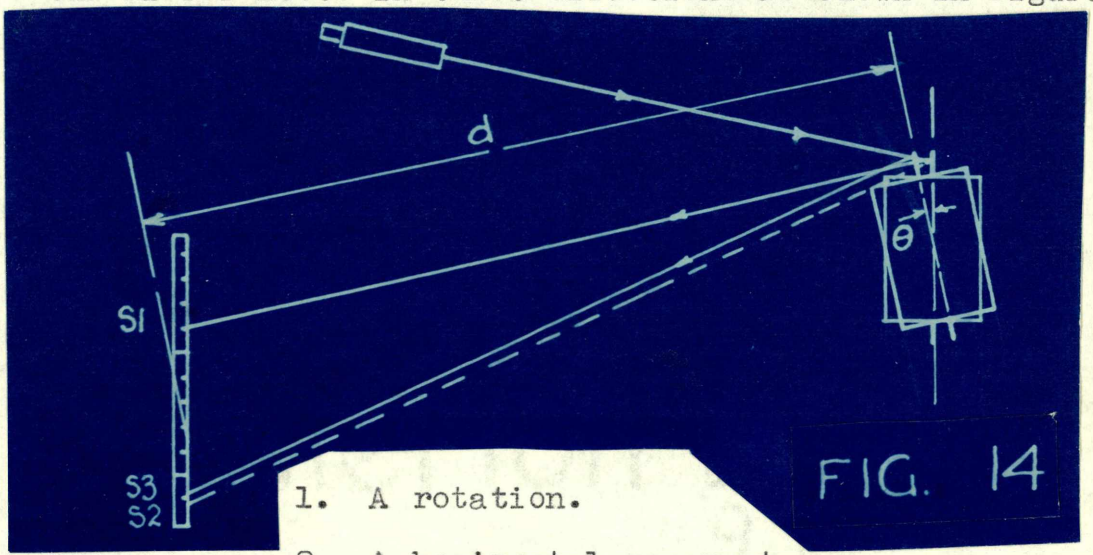


FIG. 14

1. A rotation.
2. A horizontal movement.
3. A vertical movement.

The effect of the rotation of the mirror is to cause the ray of light to rotate through an amount shown up by a movement on the scale from S1 to S2. If the scale is a distance "d" away from the mirror then for small values of θ,

$$\theta = \frac{S2 - S1}{2d}$$ where θ is in radiams.

The rotation of the specimen through the angle θ causes the mirror placed on the upper side to move forward horizontally, the effect of this movement being to <u>decrease</u>

the reading on the scale from S2 to S3. Correspondingly the mirror on the lower side moves backward horizontally and the effect of this is to <u>increase</u> the scale reading from S2 to S4 where -

$$S4 - S2 = S2 - S3.$$

Therefore $$S2 = \frac{S3 + S4}{2}.$$

Now the actual scale readings observed in the telescope are S3 and S4, therefore to get the correct value of the angle of twist the mean of the two values given by the upper and lower mirrors is taken.

The vertical movement of the mirror has no effect on the scale readings.

The angles of twist given by each pair of mirrors were calculated for each value of the torque and th angle of twist per unit length, θ, was given from the formula:

$$\theta = \frac{S2 - S1}{l}$$

where S2 = angle of twist at movable end.

S1 = angle of twist at fixed end.

l = gauge length.

CHAPTER V.

TESTS ON PLAIN CONCRETE SPECIMENS.

CHAPTER V.

TESTS ON PLAIN CONCRETE.

Part I - Circular Sections.

The objects of this series of tests were:-

1. To determine the ultimate torsional strength of plain concrete.
2. To determine the increase in torsional strength with age.
3. To determine the relationship between the ultimate strength in torsion, tension and compression.
4. To determine which of the equations mentioned on page represents the true behaviour of concrete when subjected to torsion.

In this series of tests 10 specimens were made as described on page 58. Specimens 01 - 04 were tested after 7 days, 05 and 06 after 28 days, 07 and 08 after 60 days and 09 and 10 after 90 days. Test cubes were also made from each mix and these were tested at the same time as the torsion specimens.

The tests were carried out as described on pages 76 onward. A summary of the results of the whole series is given in Table 9. In addition to this summary of results graphs showing the relationship between the applied torque and the angle of twist are shown on pages 106-110. From these graphs the values of the modulus of rigidity given in Table 9 have been calculated.

It is noticed that these graphs are more or less straight lines which bend over towards the "angle of twist" axis at the point of failure. This would seem to indicate

that concrete behaved elastically under load, the curving off near the point of failure being caused by a certain amount of plasticity.

The failure of the specimen came suddenly as in the case of an ordinary cement mortar brickette when tested. The failure took the form of a spiral tension crack at approximately 45 degrees to the angle of twist as shown in Plate 3 and as explained on page 29

The figures given by the tests were fairly consistent apart from 06 which gave an excessively high result and was therefore rejected. In the 7 day tests the two worst results differ by less than 10 per cent from the mean result.

The tests showed that the torsional strength of concrete increased with age, the increase after 28 days being 15 per cent, that after 60 days being 20 per cent, and that after 90 days being 22 per cent: in each case the strength is compared with the 7 day strength in torsion. (See page 37 for graph showing this).

The modulus of rigidity also increases with age, the increase on the value at 7 days being 4 per cent at 28 days, 10 per cent at 60 days and 20 per cent at 90 days: the last value is however rather exceptional and seems on the high side. It is noticed that the increase in the modulus of rigidity is not so great as the increase in torsional strength.

The increase in torsional strength does not correspond with the increase in compressive strength, the ratio becoming smaller as the age increased as shown on the graph on page 87 Why this should be so is difficult to explain as both tests are virtually a test of the quality of the concrete mix and of the binding powers of the cement used: the difference is probably due to the fact that the two tests place different types of loading on the binding material. It is interesting to note that the value of m = modular ratio as suggested by the Recommendations for a New Code of Practice would be $\frac{40000}{4500}$ = 8.9. In this case E for concrete would be 3.37 x 10^6 lb. per square inch. Using this value of E and substituting in the well known relationship

$$E = 2N(1 + \tfrac{1}{m})$$

where $\frac{1}{m}$ = Poisson's ratio.

gives a value for m which is negative; the reason for this is that the value of Youngs Modulus for concrete as obtained from the empirical formula mentioned above allows for plasticity effects after much loading whilst the value for the torsional modulus obtained from the experiments was obtained almost instantaneously so that the plastic yield had little time in which to operate. In connection with the tension tests about to be described the value of Youngs Modulus obtained instantaneously was 5.0 x 10^6 lb. per

square inch and this value gives Poisson's ratio for concrete as 1:8.4.

In order to determine the relationship between the tensile stresses as calculated from the failure by torsion and those when failure was by direct tension a number of tests were carried out on tension specimens. (For a description of the method by which these tests were carried out see the Author's thesis on "The strength of thin section isolated column bases in reinforced concrete"). The value of the tensile strength thus obtained was 282 lb. per square inch after 7 days. This gives a ratio between the torsional and tensile stresses of 1.15 which is much less than that obtained by Bach in the tests referred to on page

It is of interest to compare the values of the ultimate stress obtained by the use of the various equations given on page The average values obtained for the strength from the 7 day tests are as follows:-

Elastic Theory	325	lb.	per	square	inch.
Plastic Theory	243	"	"	"	"
Exponential Theory (K=1.11)..	316	"	"	"	"
(K=1.16)..	312	"	"	"	"
Turner and Davis Theory	276	"	"	"	"
Pure tension	282	"	"	"	"
Mean of elastic and plastic.	284	"	"	"	"

On comparing these values it will be noticed that the value as calculated from the theory put forward by Turner and Davis (6) agrees most closely with the value obtained with the direct ultimate tension strength. The Author

suggests however that the closest approximation is obtained by taking the mean of the values obtained by the elastic and plastic theories. This latter result is interesting and possibly suggests how concrete behaves when loaded.

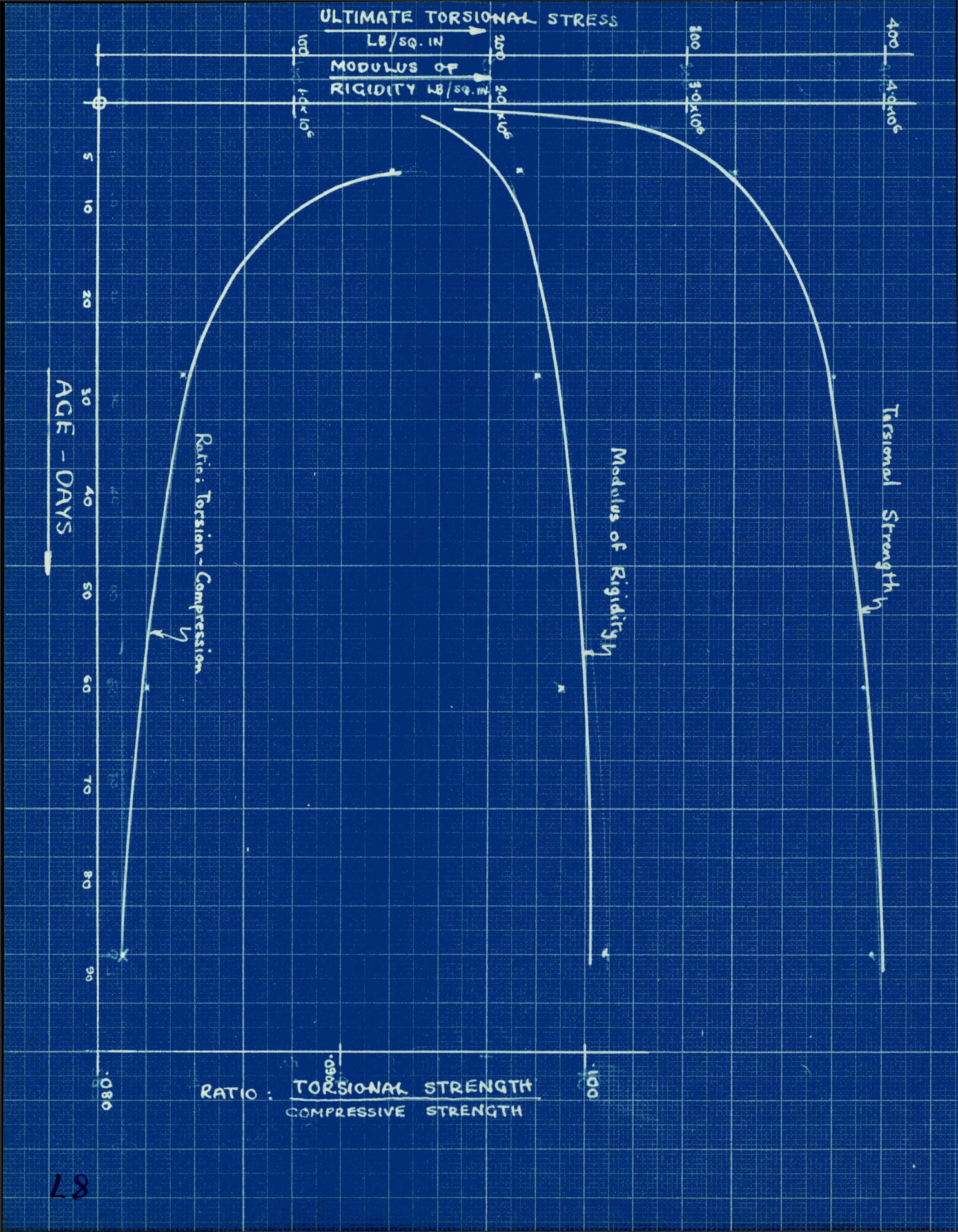

ULTIMATE TORSIONAL STRESS
LB/SQ. IN
100
200
300
400
MODULUS OF
RIGIDITY LB/SQ. IN
1·0 x 10⁶
2·0 x 10⁶
3·0 x 10⁶
4·0 x 10⁶
AGE - DAYS
5
10
20
30
40
50
60
70
80
90
Ratio: Torsion-Compression
Modulus of Rigidity
Torsional Strength
·060
·080
·100
RATIO: TORSIONAL STRENGTH / COMPRESSIVE STRENGTH

Table 9 - RESULTS OF TESTS ON CIRCULAR SECTIONS.

Specimen	Actual Diameter(in.)	Age at testing (days).	Maximum Torque lb. in.	Modulus of Rigidity lb.per sq.in.	Maximum Stress by Elastic formula lb. per sq. in.	Ratio of torsional strength to compressive strength.
01	4.8	7	7590	2.36×10^6	356	
02	4.88	7	7280	2.26 "	320	
03	4.94	7	6910	2.18 "	294	
04	4.92	7	7690	2.16 "	330	
Average				2.24 "	325	.0920
05	4.91	28	8700	2.17 "	374	
06	4.90	30	10560*	2.27 "	450*	
Average				2.24 "	374	.0835
07	4.90	60	9240	2.28 "	400	
08	4.86	60	8700	2.48 "	386	
Average				2.38 "	392	.0820
09	4.94	90	9200	2.30 "	390	
010	4.90	92	9220	2.90 "	400	
Average				2.60	395	.810



Plate 3 - Spiral Crack in Circular Specimen

Part II - Rectangular Sections.

The objects of this series were:-

1. To determine the torsional rigidity of beams of plain concrete with varying ratios of depth to breadth.
2. To compare the values of the ultimate torsional stress in various beams with that obtained from tests on circular sections.

In this series of tests 12 specimens were made as described on page 59 . Specimens A1 - A4 were 6in. x 4in A5 and A6 were 6 in. x 2 in., A7 and A8 were 6 in. x 3 in., A9 and A10 were 6 in. x $3\frac{1}{2}$ in. and A11 and A12 were 4 in. x 4in. This gave ratios of depth to breadth varying from 3:1 to 1:1. Reynolds (19) has pointed out that the dimensions of reinforced concrete beams are such that the ratio of depth to breadth lies between these limits in almost every case. Other experimenters have carried out tests on rectangular sections of plain concrete, notably Anderson, (14) and (16), Young, Sagar and Hughes (13), and Turner and Davies(6); of these, Anderson used three different values of the d/b ratio (1:1, 1.25:1 and 1.5:1), Young, etc. used three values (1:1, 1.5:1 and 2:1) and Turner used two values (1:1 and 2:1). Thus it will be seen that the Author's tests form the most complete series of tests carried out as he has used five different values of the ratio which cover a much greater field than that covered by other experimenters.

The specimens were in all cases tested after 7 days in the manner previously described. The ultimate stress

was calculated from the torque at failure by the use of Timoshenko's formula, the appropriate co-efficient being taken from those given in Table 1 . A summary of the results of the whole series is given in Table 10 whilst the graphs showing the relationship between the applied torque and the angle of twist are given on pages 111-114, from these graphs the values of the torsional rigidity are calculated and then the modulus of rigidity as given in Table 10.

It is noticed that these graphs are also straight lines, which would seem to indicate elastic behaviour under the test.

The failure of the specimen occurred suddenly as with the circular specimens by the development of a tension crack in a plane inclined at approximately 45 degrees to the axis of twist. Plate 4 (96) shows the type of fracture produced at failure. The line of fracture on the long side was inclined at 45 degrees to the axis of twist but apart from specimens A5 and A6 it did not take on the spiral form which was observed in the circular specimens.

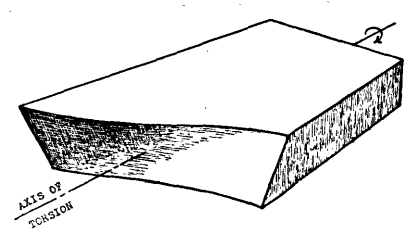


Fig. 15
CURVED FRACTURE-SURFACE.

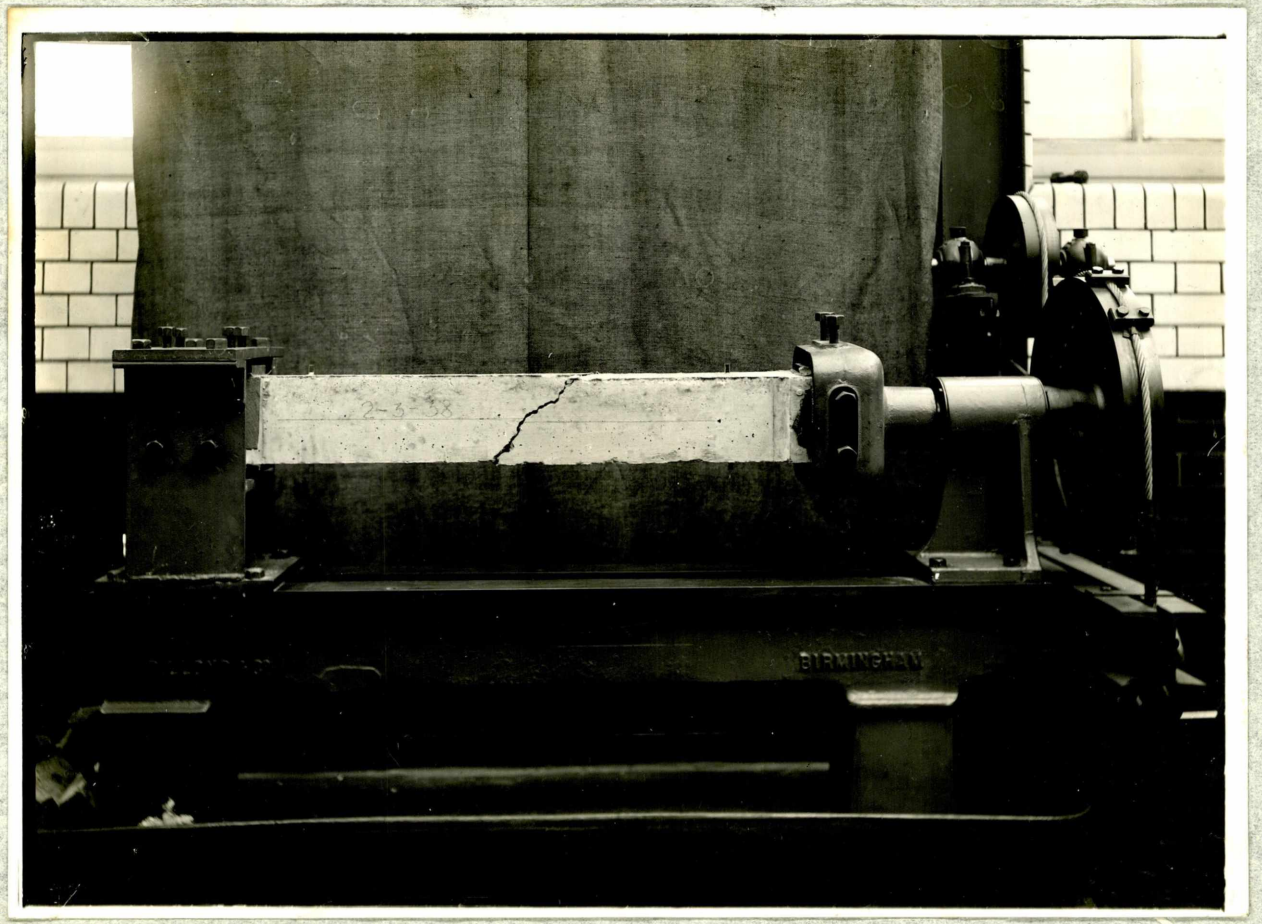


Plate 4 - Failure of Plain Rectangular Section



Plate 6 - Concrete Member after Test

The fractured surface was however inclined in varying directions to the axis of twist as shown in Figure 15. This shows that the fracture produced tended to be spiral in form. The dead weight of the member produced a tension stress on the underside and this stress combined with the stress due to torsion caused the simple plane of fracture instead of the spiral plane as in the case of the circular specimens.

In the case of the narrow specimens A5 and A6 the trace of the fractured surface was a definite spiral similar to that observed in the case of the circular specimens. The reason suggested for this is that as the specimen was so narrow the maximum stress at the middle of the long side was so much greater than the stress on the short side that even the additional stress imposed by bending was unable to cause a failure on the short side.

According to the elastic theory the point of maximum stress in a rectangular section subjected to torsion is at the mid point of the longer side. It would therefore be expected that a crack would appear at this point before cracks appeared elsewhere on the section. As already mentioned, failure was in all cases sudden and no crack was visible anywhere before failure took place. To assist in the observation of cracks a narrow strip of plaster was placed along the centre of the longer sides as shown in Plate 6; even with the help afforded by this device

it was not possible to observe any cracks on the member before the one which produced the instanteous failure.

The tests apart from A5 and A6 gave extremely consistent results, the greatest range of variation of the extreme value from the mean value being just under 3 per cent in the case of specimens sized 6 in. x 4 in. The narrow specimens A5 and A6 showed a variation of just over 5 per cent.

The tests show that apparently concrete is much stronger in torsion when placed in a rectangular section than it is when placed in a circular section, the ratio of the maximum stress for the two sections varying from 1.78, in the case of a rectangular section with a ratio of sides of 3 to 1, to 1.45 in the case of a section with a ratio of 1.5 to 1. The mean value of the ratio of the maximum stresses for a rectangular section and a circle was 1.60.

The modulus of rigidity obtained from the various sections varied from 2.20×10^6 to 2.52×10^6, having a mean value of 2.40×10^6 lb. per square inch. It is interesting to notice that there is no apparent relationship between the maximum stress and the modulus of rigidity for the section in which the concrete develops the greatest stress does not give the greatest value for the modulus of rigidity neither does the section giving the lowest stress give the lowest modulus.

The value of the modulus of rigidity obtained from the circular specimens was 2.24 x 10^6 lb. per square inch thus giving the ratio between the values obtained from rectangular and circular sections as 1.07 to 1 which shows a very much smaller variation than the ratio of the maximum stresses.

It is of interest to compare the values obtained by other experiments with those obtained by the Author. This comparison is shown in the Table below:

Test	Ratio of sides. 1:1	1.25:1	1.5:1	1.7:1	2:1	3:1
Anderson I.	539	505	575	-	-	-
Anderson II.	640	609	658	-	-	-
Young, etc.	539	-	467	-	602	-
Turner & Davis	414	-	-	-	260	-
Author	508	-	472	530	516	580

Apart from the tests carried out by Turner and Davis, the results of other experiments seem to agree in principle with those of the Author. They all show that the value of the maximum stress depends upon the ratio of two sides of the rectangle, and it is observed that the value of the stress shows a slight increase as the ratio of the sides increases. The majority of the experiments also agree that the ratio of the maximum stress in a rectangular section to that obtained in a circular section is about 1.5:1.

All the above figures for maximum stresses have been worked out on the elastic theory using the co-efficient: given by Timoshenko as previously given on page

It is of interest to work out the values of the maximum stress obtained by the use of the plastic theory on the assumption that concrete behaves as a perfectly plastic material. The values are given in the Table below:

	Ratio of sides.				
	1:1	1.5:1	1.7:1	2:1	3:1
Elastic Theory	508	472	530	516	580
Plastic Theory	316	278	306	304	348

Table 10 - RESULTS OF TESTS ON PLAIN CONCRETE RECTANGULAR SECTIONS.

Specimen	Nominal Size.	Actual Size.	Maximum Torque.	Modulus of Rigidity	Maximum Stress on Elastic Theory.	Ratio of maximum stress to that for plain circle.
	inches	inches	lb. in.	lb.per sq.in.	lb. per sq. in.	
A1	6 x 4	6.00 x 4.00	10,250	2.36×10^6	460	
A2		6.00 x 3.90	9,740	2.36 "	460	
A3		6.10 x 3.90	10,250	2.16 "	480	
A4		6.00 x 3.90	10,350	2.34 "	485	
Average of 4 specimens				2.30 "	471	1.45
A5	6 x 2	6.10 x 1.90	3,590	2.60 "	610	
A6		6.10 x 1.85	3,080	2.36 "	550	
Average of 2 specimens				2.48 "	580	1.78
A7	6 x 3	6.10 x 2.80	6,150	2.94 "	520	
A8		6.08 x 3.05	7,180	2.10 "	512	
Average of 2 specimens				2.52 "	516	1.59
A9	6 x $3\frac{1}{2}$	6.10 x 3.55	9,520	2.20 "	540	
A10		6.10 x 3.50	9,220	2.20 "	520	
Average of 2 specimens				2.20 "	530	1.63
A11	4 x 4	4.10 x 3.95	6,860	2.56 "	516	
A12		4.25 x 4.00	7,060	2.42 "	500	
Average of 2 specimens				2.49 "	508	1.56

Part III - Tests on Tee and Ell Sections.

The objects of this series of tests were:-

1. To determine the torsional rigidity of members of Tee and Ell sections in plain concrete.
2. To determine the additional strength given by the flanges of such members.
3. To investigate a simple method of calculating the ultimate torque taken by these irregular sections.

The study of members of irregular section when subjected to torsion involves difficulties where re-entrant angles occur. This is because if the section is analysed on the elastic theory the stress at any re-entrant angle is infinite. Griffith and Taylor (20) have suggested a method for estimating the torsional rigidity of complex sections but their rules have the objection that when applied to Tee and Ell sections they give a value for the rigidity of these sections which is less than that of the rectangular rib: such a result seems absurd and the Author's experiments show it to be untrue. This is probably due to the fact that concrete is certainly not a perfectly elastic material and the formula given by Griffith and Taylor is only applicable to elastic materials.

Turner and Davies (6) have suggested a method whereby the section is split up into rectangles, the torque taken by each rectangle is calculated and the torque of the section is taken as the sum of that taken by its component rectangles multiplied by a correction factor. This correction factor is taken as $\frac{4A}{PD}$ where A is the area of the

specimen, P is the perimeter and D is the diameter of the largest inscribed circle that can be placed in the section. This method will be adopted for the specimens used in this series of tests.

Since the stress at the re-entrant angle is infinite for an elastic material it is obvious that the insertion of a fillet between the flange and the rib will prevent the occurence of a point of infinite stress. On reference back to figure 11 it will be noticed that in the members under test a $\frac{1}{2}$ inch fillet has been inserted between the flange and the rib. Turner and Davies(6) have, however, shown that the difference between the ultimate torque when there is no fillet and that when there is a small fillet is practically negligable, this is doubtless due to the plastic nature of concrete.

In this series of tests 6 specimens, made as described on page 64 were tested in the usual manner. The results for the series are given below and graphs showing the torque-twist relationship are given on pages 153-157

It is seen that the latter shows a linear relationship seemingly indicating elastic behaviour.

Tests on Specimens T1 and T2.

Ultimate torque taken by T1 = 14880 in lb.

" " " " T2 = 15400 "

Average = 15140 "

Increase over 6 in. x $3\frac{1}{2}$ in. rectangle = 5770 "

The failure of the specimen was sudden and occurred by the formation of a diagonal tension crack as shown in plate 4 . Although in theory the stress at the re-entrant angle is much greater than at other parts of the section no cracks were observed along this angle before the crack causing failure occurred.

Calculation of Torsional Resistance of T1 and T2.

These sections can be split into three rectangles the rib being one sized 6 in. x $3\frac{1}{2}$ in. and the flanges sized $2\frac{1}{4}$ in. square being the other two.

From Table 10, page 98 torque taken by

6 in. x $3\frac{1}{2}$ in. rectangle = 9,370 in lb.

Allowing for an ultimate stress of 508 lb. per square inch torque taken by 2 - $2\frac{1}{4}$ in. squares

$= 2 \times 508 \times 2.25^3 \times .208 = 2400$ in lb.

Diameter of inscribed circle = 3.62 in.

Area of section = 31.12 sq. in. Perimeter = 28 in.

Correction factor = $\frac{4 \times 31.12}{3.90 \times 28} = 1.15$.

Therefore taken by section = 1.15(9,370+ 2,400) =

13550 in lb.

This agrees to within 10 per cent of the experimental value.

Tests on specimens T5 and T6.

Ultimate torque taken by T5 = 17410 in lb.

" " " " T6 = 15400 in lb.

Average = 16405 in lb.

Increase over 6 in. x 3½ in. rectangle = 7035 in lb.

The failure of the specimen was sudden and occurred by cracking at the ends of the specimen. Thus the results given by the test are not satisfactory. Specimen T5 gave a very poor torque-twist curve and this has therefore been omitted from the results.

Calculation of Torsional resistance of T5 and T6.

The section is split into 3 rectangles, one sized 6 in. x 3½ in. and the other two sized 2¼ in. x 2¾ in. Using Timoshenko's co-efficients and taking a value of 500 lb. per sq. in. for the ultimate stress in the 2¼ in. x 2¾ in. sections the value of the ultimate torque for these latter becomes

$$2 \times 500 \times 2.25 \times 2.75^2 \times .22 = 3730 \text{ in lb.}$$

The "Turner - Davies" correction factor

$$= \frac{4 \times 33.35}{3.90 \times 30} = 1.15$$

Therefore Torque taken = 1.15(9,370 + 3,730) = 15100 in lb.

This agrees to within 8 per cent of the experimental value.

Tests on specimens L1 and L2.

Ultimate torque taken by L1 = 12800 in lb.

" " " " L2 = 13820 "

Average = 13310 "

Increase over 6 in. x $3\frac{1}{2}$ in. rectangle = 3940 in lb.

The failure in these specimens was also sudden and caused by the development of a diagonal tension crack. In specimen L1 this crack unfortunately occurred at the end of the specimen but in L2 it occurred in the centre.

Calculation of the Torsional Resistance of L1 and L2.

In this case the section can be split into two rectangles, one sized 6 in. x $3\frac{1}{2}$ in. and the other sized $5\frac{1}{2}$ in. x $2\frac{1}{4}$ in. Using Timoshenko's co-efficients and taking a value of 500 lb. per square inch for the ultimate stress in a $5\frac{1}{2}$ in. x $2\frac{1}{4}$ in. section the value of the torque taken by this latter section becomes

$$500 \times 5.5^2 \times 2.25 \times .258 = 8710 \text{ in lb.}$$

The Turner Davies correction factor

$$= \frac{4 \times 33.4}{3.68 \times 30} = 1.21.$$

Therefore Torque taken = 1.21 (8710 + 9370) = 20600 in lb. It is seen that this shows no appreciable agreement with the experimental values.

Thus from the complete series it is seen that whilst the torque of the component rectangles multiplied by the "Turner-Davis" correction factor gives values which are

approximately true for Tee sections it is useless to apply this method to Ell sections. The reason for this is surely the lack of symmetry in an Ell section for it seems obvious that a symmetrical Tee section will be stronger than that of an Ell section of the same overall dimensions.

The Author suggests a simple method of calculating the ultimate torque of an Ell section which for the case in question agrees fairly well. The projecting flange is divided into two equal parts as shown in Figure 16 and one of these parts is taken as assisting the torsion and the other is neglected: the torque taken by the component rectangles is then multiplied by the "Turner-Davies" correction factor.

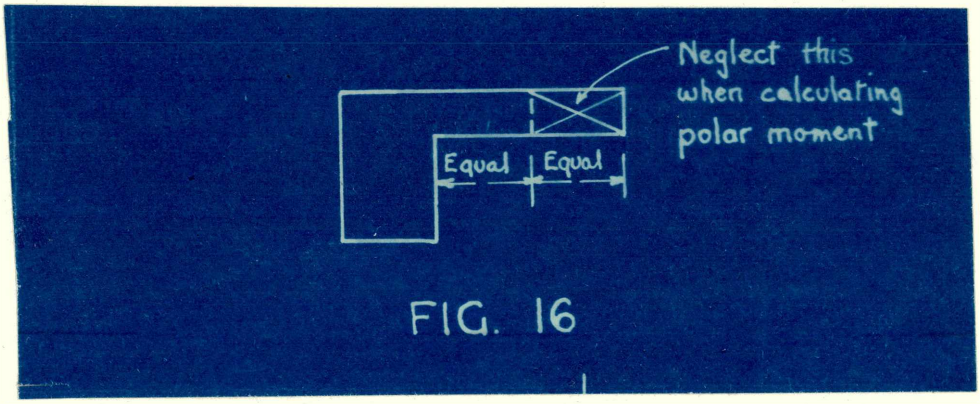


FIG. 16

Applying this to the case under consideration the value of the torque taken by the Ell section becomes

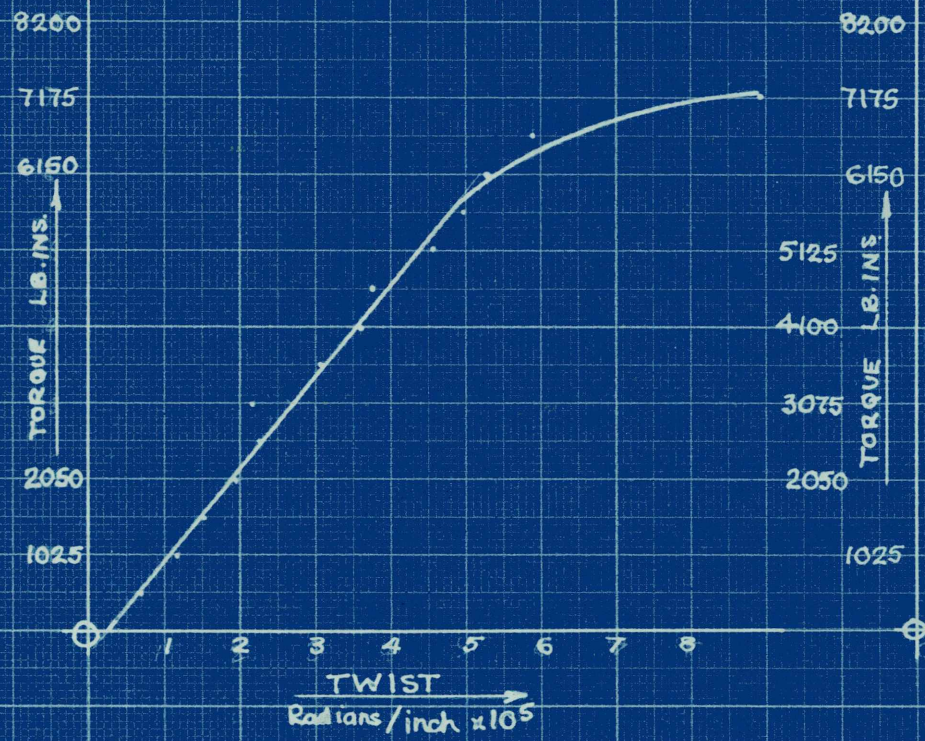
$$1.21\ (9370 + 1865) = 13600 \text{ in lb.}$$

This value agrees to within 2 per cent of the experimental value.

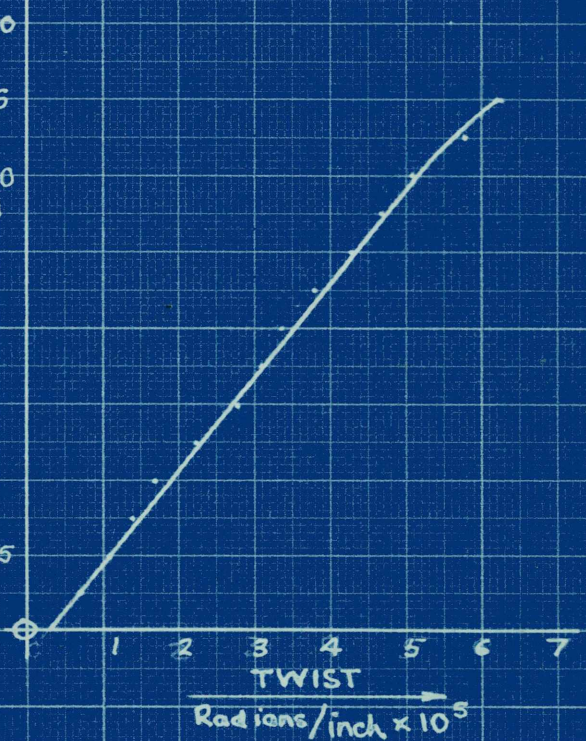
The graphs shown on pages 153-157 are not very helpful in giving a comparison of the torsional

rigidities of the different types of sections used as the graphs show that T1, T6 and L2 have the same rigidity. It is noticed however that the sections are much more rigid than the ordinary rectangular sections. A comparison of the graphs with those for specimens A9 and A10 shows that the Tee and E11 sections are about 66 per cent more rigid than the 6 in. x $3\frac{1}{2}$ in. rib alone.

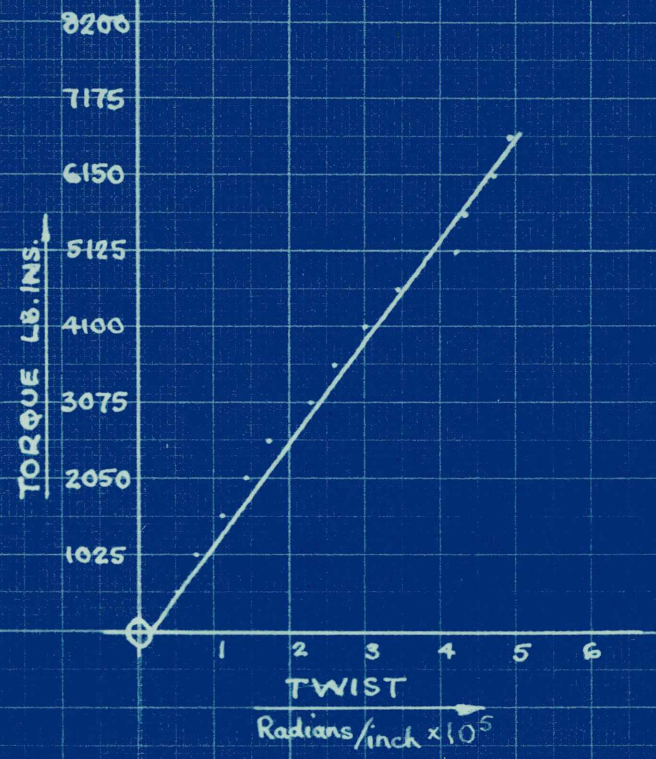
Specimen 01



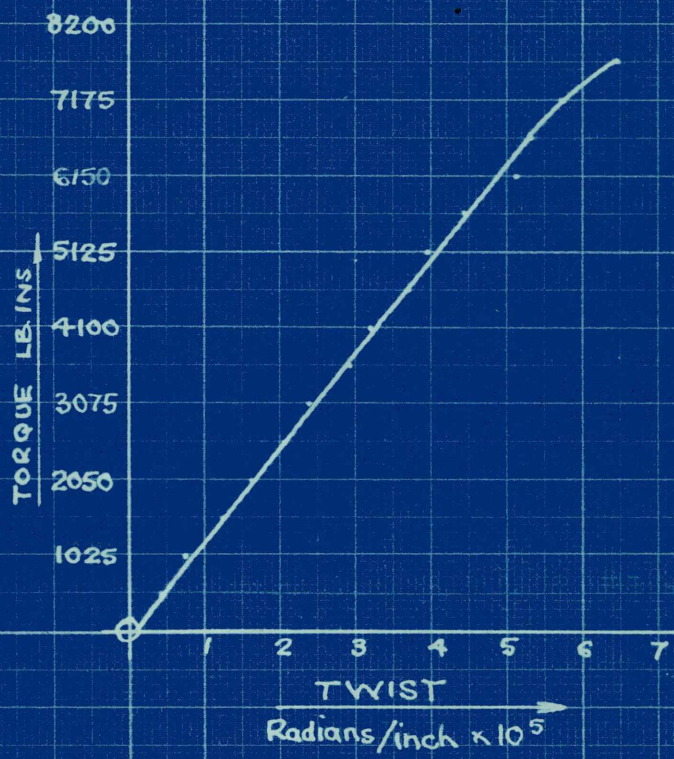
Specimen 02



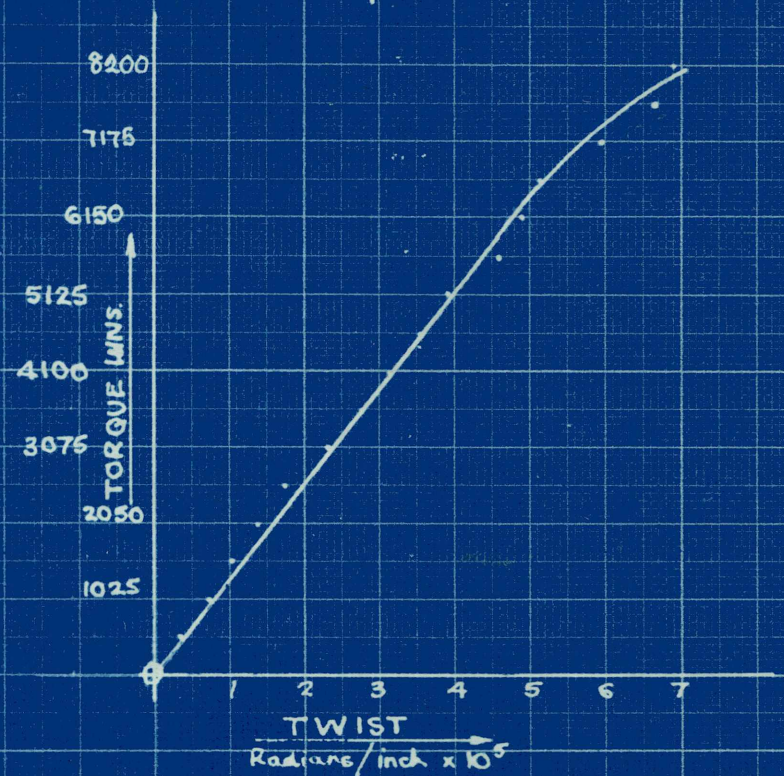
Specimen 03



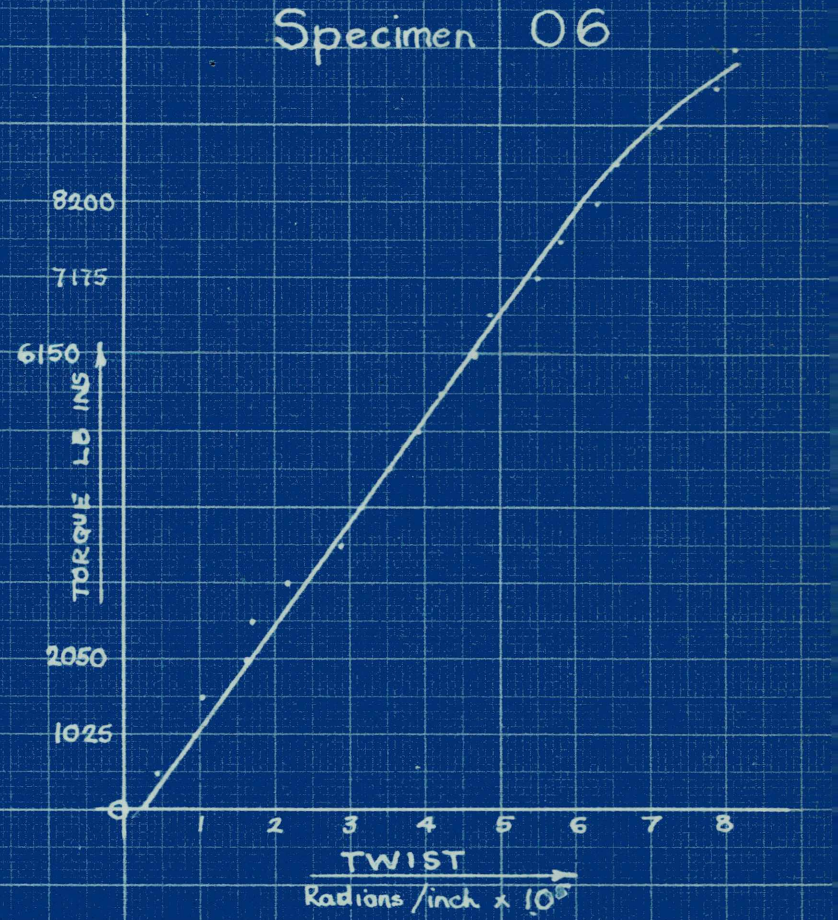
Specimen 04



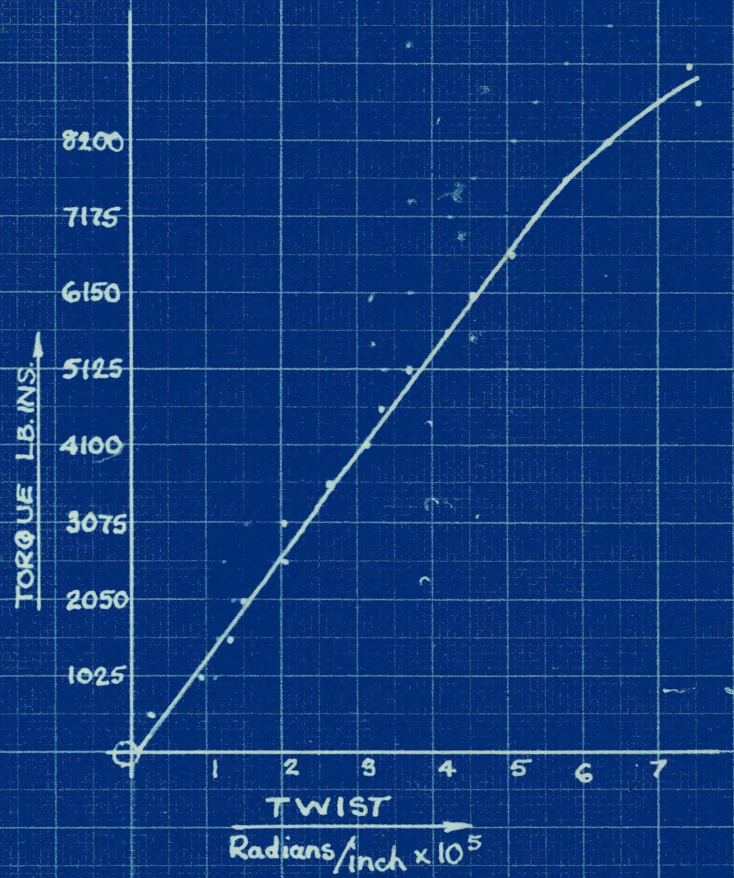
Specimen 05



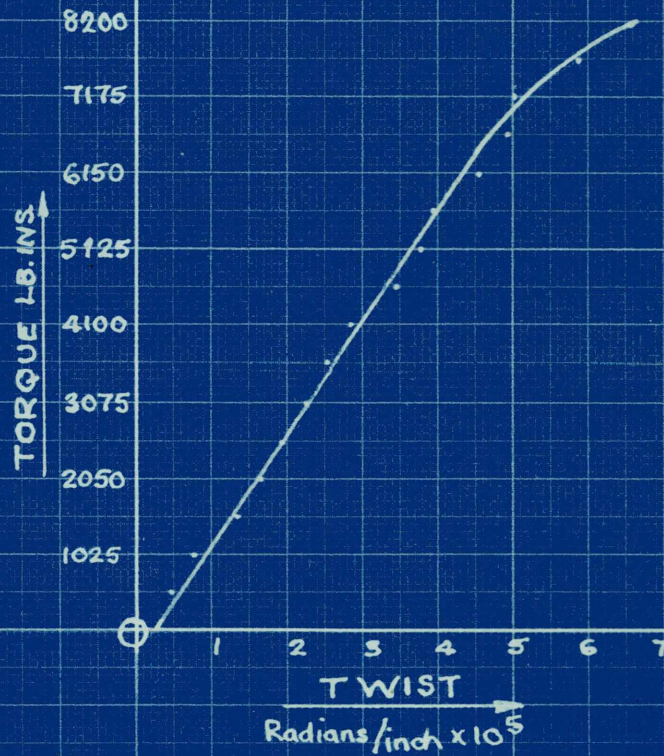
Specimen 06

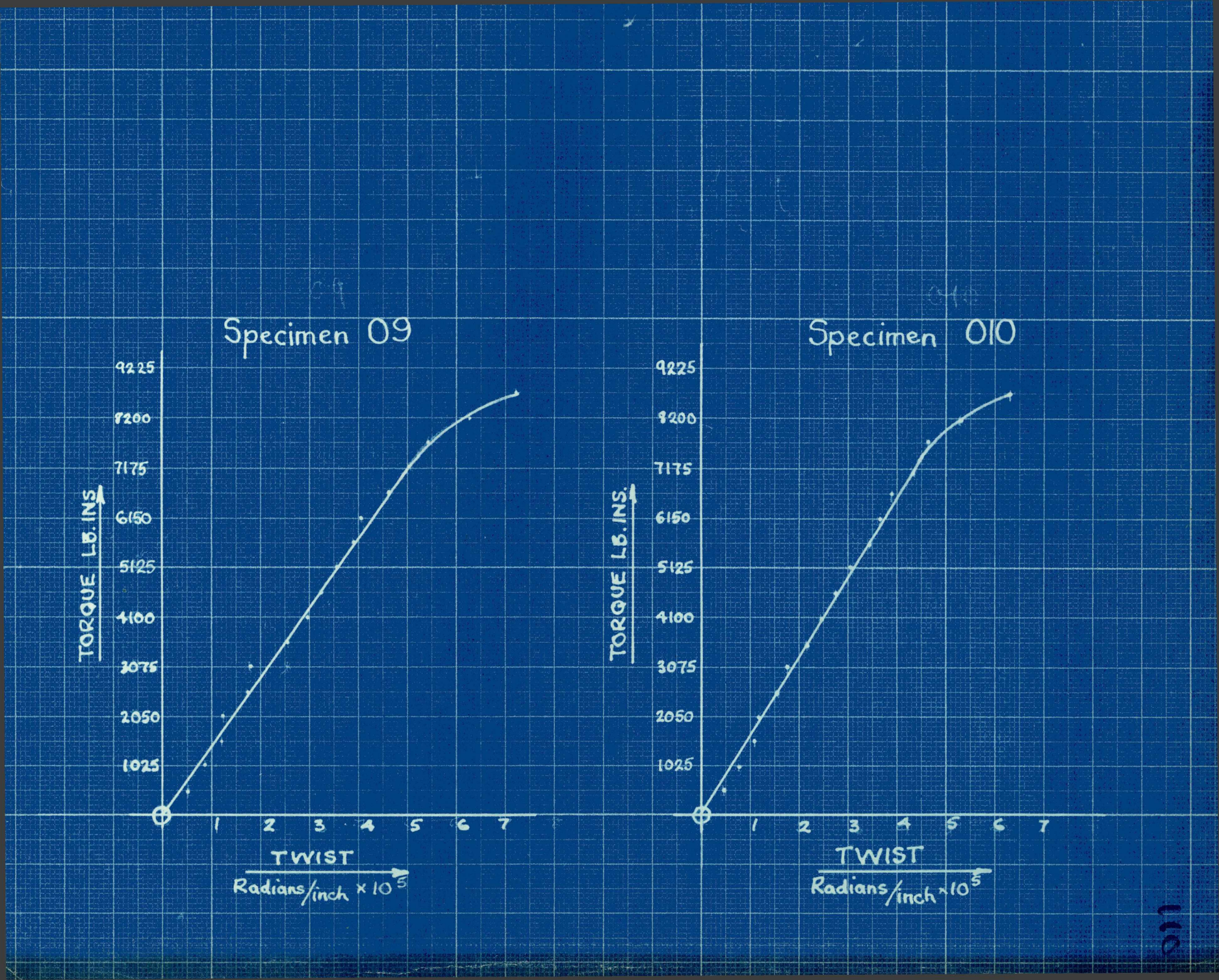


Specimen 07

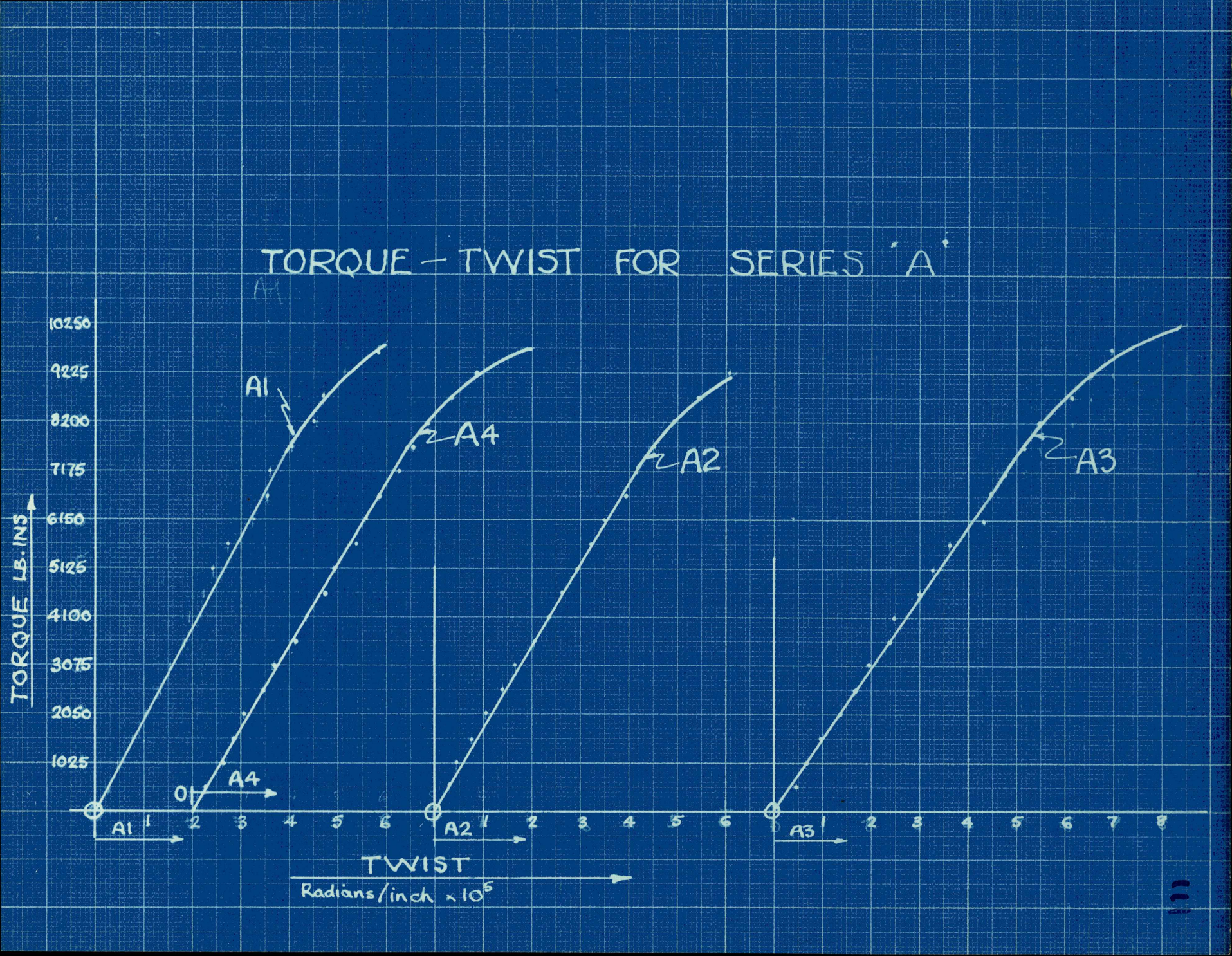


Specimen 08

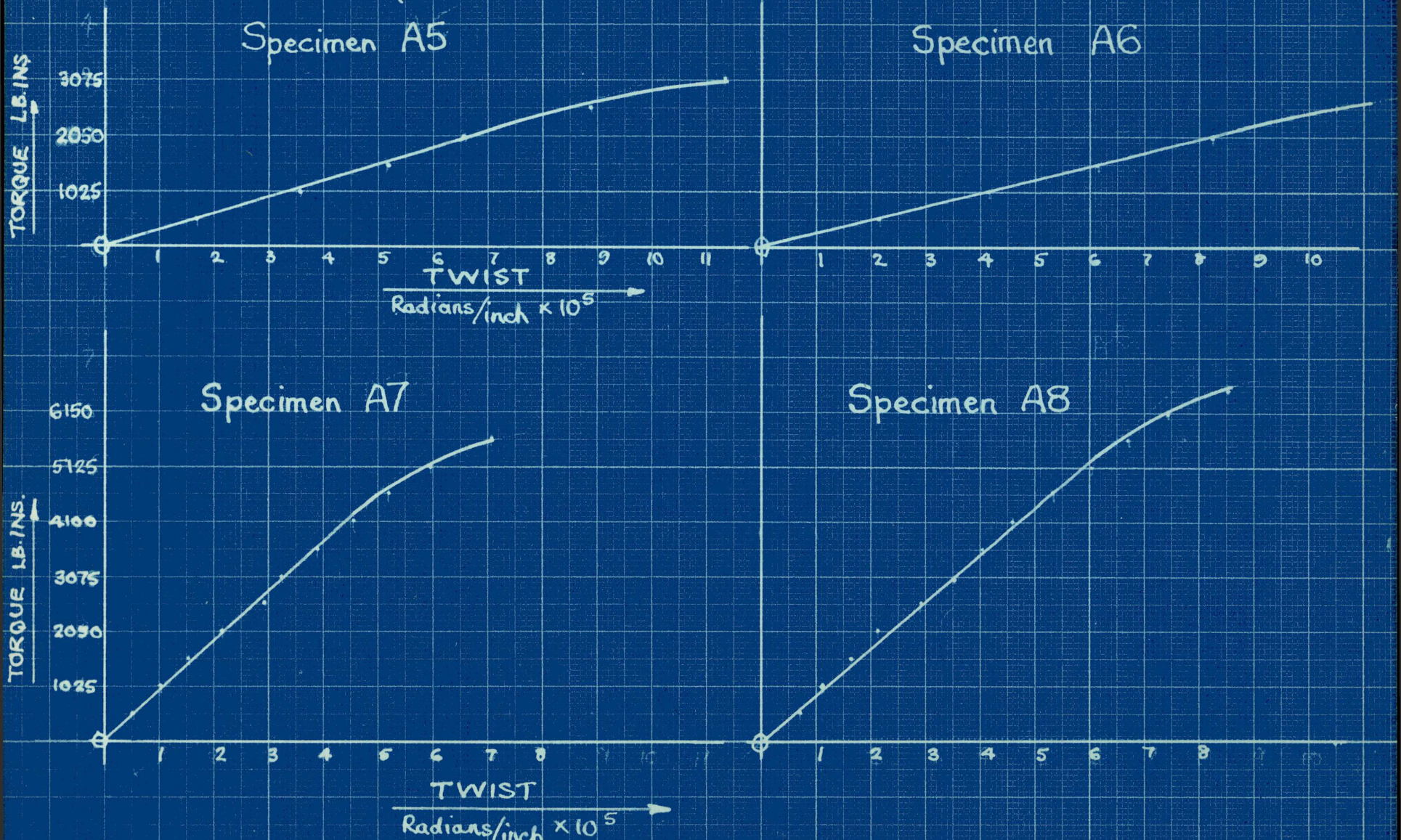


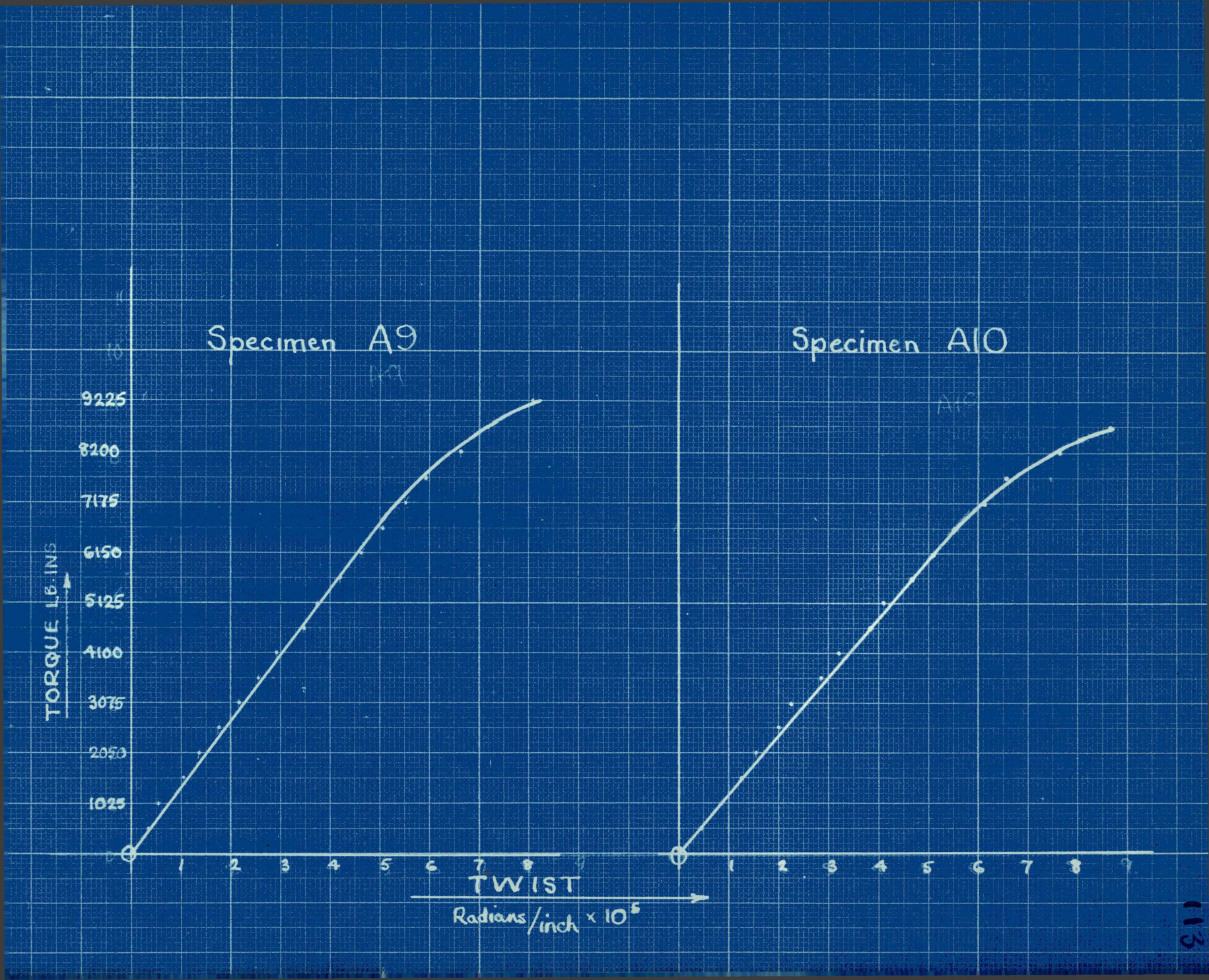

Specimen 09
TORQUE LB. INS.
9225
8200
7175
6150
5125
4100
3075
2050
1025
1 2 3 4 5 6 7
TWIST
Radians/inch × 10⁵
Specimen 010
TORQUE LB. INS.
9225
8200
7175
6150
5125
4100
3075
2050
1025
1 2 3 4 5 6 7
TWIST
Radians/inch × 10⁵

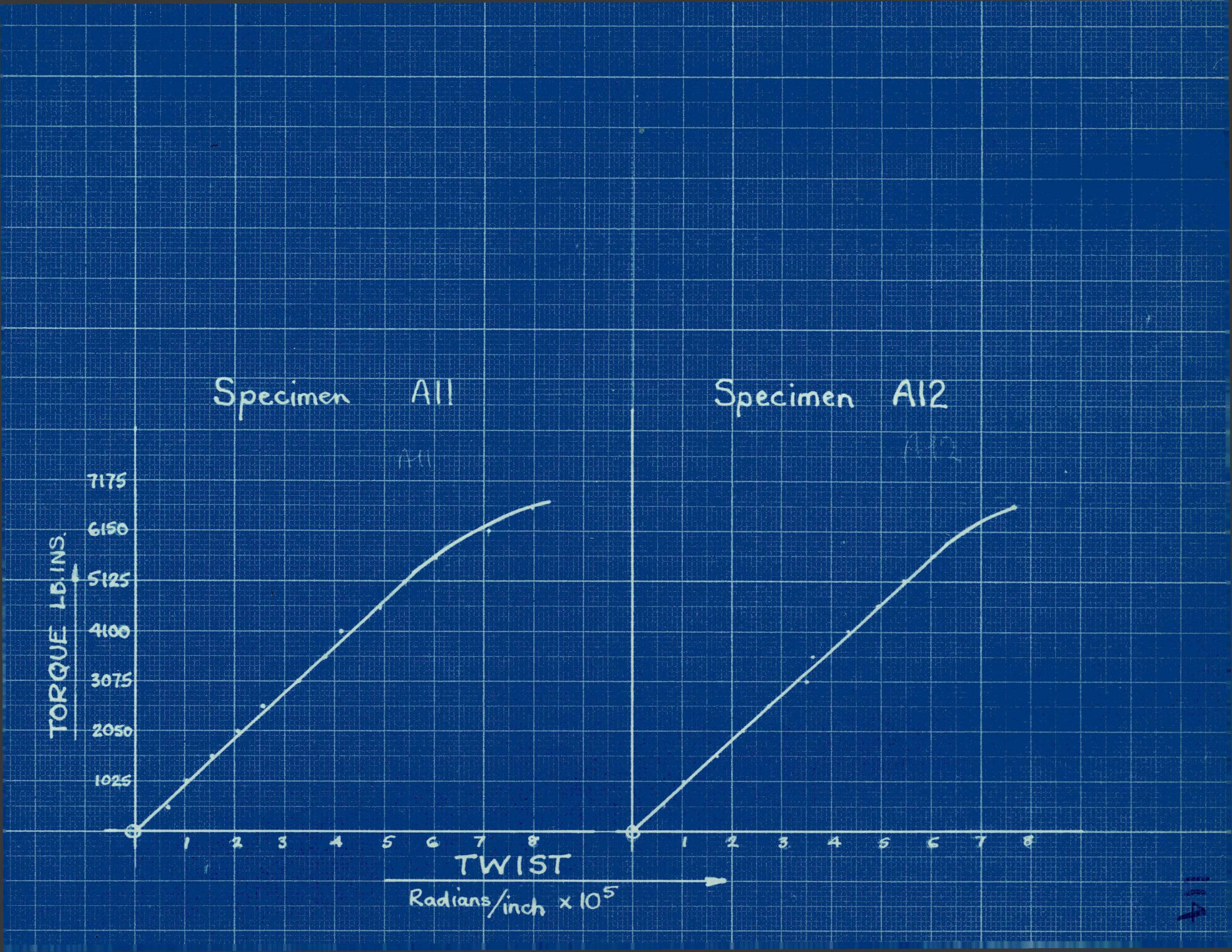
TORQUE – TWIST FOR SERIES 'A'



TORQUE – TWIST FOR SERIES "A"




Specimen A9
Specimen A10
9225
8200
7175
6150
5125
4100
3075
2050
1025
TORQUE LB. INS
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
TWIST
Radians/inch × 10⁵


Specimen A11
Specimen A12
TORQUE LB.INS.
7175
6150
5125
4100
3075
2050
1025
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8
TWIST
Radians/inch x 10^5

CHAPTER VI.

TESTS ON REINFORCED SPECIMENS.

CHAPTER VI.

TESTS ON REINFORCED SPECIMENS.

The objects of this series of tests were as follows:

1. To study the behaviour of a reinforced concrete member when subjected to torsion.
2. To determine the increase in strength over that of plain concrete gained by the addition of reinforcement.
3. To determine the effeiency of the following different types of reinforcement in resisting torsion:
   (a) Longitudinal bars alone.
   (b) Longitudinal bars and stirrups.
   (c) Spiral reinforcement.
4. To check where possible existing theories of design.
5. To obtain a formula giving the resistance to torsion of a section reinforced with longitudinal bars and stirrups

To study these various points the various specimens given in Table 8 on page 62 were made and tested in the usual manner after 7 days. In testing the torque at the first crack was observed in addition to the ultimate torque.

The complete results for the series are given in Table 11 which in addition to indicating the torque at firs crack and failure also indicates the increase in strength of the reinforced section over a plain section. The strength of the latter is taken as the mean of that of the

6 in. x 4 in. rectangles, the results of which are given in Table 10 and has a value of 10,450 in. lb.

Since the dimensions of each section vary slightly the value of the additional torque is worked out on that which a 6 in. x 4 in. reinforced specimen would take. The torque-twist curves for the complete series are given on pages 141-157.

Since the theories for the three different types of reinforcement used are different it is proposed to divide the remainder of this chapter into three parts and discuss the results separately.

Part I - Longitudinal bars only.

In this section six specimens were tested, being those marked B and C in Table 11. The results indicate that bars in the corners do not increase the resistance as much as bars placed at the centres of the sides, the increase in the former case being 9 per cent and in the latter 15 per cent. In each case the reinforcement used was 1.28 per cent of the cross sectional area of the specimen.

Failure occurred by the development of tension cracks at 45 degrees to the axis of twist. Failure was not suddent there being considerably creep near the ultimate load. The cracks in the series B specimens were more widely distributed than in the series C tests and in

addition there were longitudinal cracks along the line of the bars which generally indicate a failure in bond.

On examining the torque-twist curves it is noticed that both specimens show similar behaviour. The graph is a straight line until about two-thirds of the ultimate load is reached and after this point the creeping effect becomes apparent. The average value of the torsional rigidity obtained from these curves is 2.46×10^6 lb./sq.in. which shows very little increase over the average for the 6 in. x 4 in. plain specimens.

The theory which it is intended to apply to these tests is that suggested by Turner and Davies whereby the torque taken by the rods is added to that taken by the concrete. Instead, however, of calculating the stress in the rods by the St. Venant formula as they have done the simple formula taken from Seely and given on page will be used.

The value of the modulus of rigidity for steel will be taken as 12×10^6 lb. per sq. in. and that for concrete as 2.30×10^6 giving a modular ratio of 5.2

Calculation of Strength of Series B Specimens.

The dimensions to the centre of the reinforcing bars are as given on Figure 17.

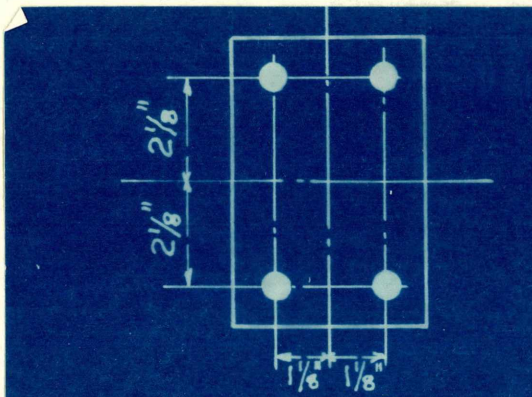


FIG. 17

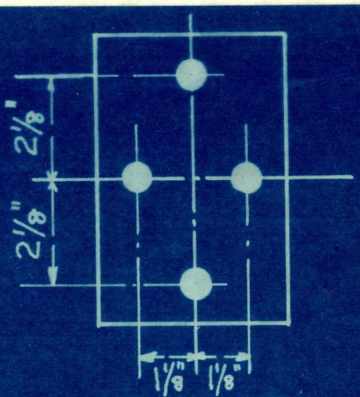


FIG. 18

If S denoted the maximum stress (i.e. stress at centre of long side then the stress on the bars given from the formulae:

$$S_x = \frac{2y}{b}\left[1 - \left(\frac{2x}{d}\right)^2\right] S_1$$

and $$S_y = \frac{2x}{d}\left[1 - \left(\frac{2y}{b}\right)^2\right] \frac{b}{d} \cdot S.$$

is $S_x = 0.282\ S.$

and $S_y = 0.322\ S.$

Giving a resultant stress $= \sqrt{(.282^2 + .322^2)}\ S$

$= .43\ S.$

Lever Arm $= \sqrt{(2\frac{1}{8})^2 + (1\frac{1}{8})^2}$

$= 2.4$ in.

Total area of bars = 0.3065 sq. in.

Modular ratio $= 5.2\ (M^1)$

$M^1 - 1 = 4.2$

Therefore Additional torque due to reinforcement

$= .43S \times 2.4 \times 0.3065 \times 4.2$

$= 1.33\ S.$

Torque taken by plain section

$$= \frac{2}{9} \times 4^2 \times 6 \times S$$

$$= 21.2S.$$

Therefore percentage increase $= \frac{1.33}{21.2} = 6.3$ per cent.

Actual experimental value obtained = 9 per cent.

Calculation of the strength of series C specimens.

The dimensions to the centre of the reinforcemer are as shown in Figure 18.

The stress in the bars is

$S_m = .645\ S \qquad S_n = .462\ S.$

Lever arm for S_m = 1.125 in. and for S_n = 2.125 in.

Therefore additional torque due to reinforcement

$= 2 \times 4.2 \times .076 \times S(.645 \times 1.125 + .462 \times 2.125)$

$= 1.090\ S.$

Torque taken by plain section = 21.2 S.

Therefore percentage increase $= \frac{1.090}{21.2} = 5.2\%.$

Actual experimental increase = 15%.

Thus it is seen that whilst this method of calculation gives reasonably accurate results when applied to the case of bars in the corners, it is useless when applied to bars in the centres of the sides. If this theory were true then series C should be weaker than series B. Experiment shows the reverse to the case and

the explanation given the Author is that since the position of greatest stress is at the centre of the sides the bars in series C are placed in the position of greatest stress and therefore give greater assistance to the concrete to resist the development of a tension crack at these critical points.

Part II - Longitudinal Bars and Stirrups.

In this section 9 specimens were tested, being those marked D, E, F and G in Table II.

In order to test a theory given by Turner and Davies (6) the reinforcement was so arranged as to give an equal percentage longitudinally and laterally. Whilst this does not affect experiments carried out with a view to deriving a general formula for the strength of a specimen in torsion in terms of the areas of longitudinal and lateral reinforcement and of the concrete section, it is in the Author's opinion a rather empirical attempt to tackle the problem. The Author realises that the few experiments performed are grossly inadequate on which to base a new theory of design but hopes to be able to perform further experiments to obtain a general expression for the strength of a reinforced specimen in torsion.

All the specimens behaved in a similar manner under test. Failure was in all cases caused by the development of tension cracks inclined at 45 degrees to the axis of twist: these cracks were evenly distributed and quite numerous as is seen from Plate 1. The longitudinal cracks noticed in series B were scarcely noticable in the specimens described in this series.

The most interesting feature of the tests is obtained by an examination of the various torque twist curves. A typical torque-twist curve (see pages 143-151

shows three distinct parts:

Stage I - A straight line from zero load up to about half of the failing load.

Stage II- A curved portion joining this straight line to the straight line mentioned in part III.

Stage III-A straight line which started at a value of the torque just below failing load and continued as far as it was possible to obtain readings on the scales.

During that stage of the test covered by this part of the graph and beyond the specimen twisted so rapidly that it was impossible to take readings thus showing that the specimen possessed very little torsional rigidity.

An examination of Stage I shows that the slope of this line is almost the same as that for the 6 in. x 4 in. plain specimens (see pages [illegible]): this seems to indicate that at the early stages the torque is resisted by the concrete alone and also gives the very useful information that the maximum torsional rigidity of a reinforced section is the same as that of a plain one of the same dimensions. The fact that the reinforcement makes no appreciable difference in the value of the torsional rigidity is seen by comparing the slopes of the straight line portions of the graphs for the different percentages of reinforcement used in this series of tests. Although each series D, E, F and G has varying amounts

of reinforcement yet the "Stage I" line has the same slope in each case.

The "Stage III" line shows that during the later stages of the test the beam possesses very little torsional rigidity. During this stage it is obvious that the concrete is taking very little load and that most of the load is being taken by the reinforcement. An examination of the reinforcement (as shown on page 65) shows the very obvious fact that the reinforcement "cages" alone possess very little torsional rigidity and this accounts for the fact that the line in "Stage III" is almost parallel to the "angle of twist" axis.

An examination ofthe reinforcement after failure showed that the stress in it had in no case reached its ultimate value, consequently failure of the specimen was not caused by failure of the reinforcement. It would seem therefore that during the "Stage III" section of the test the reinforcement was in some way bound together by the concrete in such a way as to make the "cage" stiff enough to resist the applied torque. As the load was further applied this "bond" broke down and failure was caused by this "bond" failure. The exact nature of this bond is difficult to understand and further work on the subject will be necessary before it can be definitely established. It is certainly not a "bond" caused solely by bond stresses between the

reinforcement and the surrounding concrete but seems to be caused by internal forces in the member which have the effect of producing the extra stiffness to the reinforcement cage.

That part of the torque - twist curve which has been referred to as Stage II shows that stage of the test where the torque is being resisted partly by the plain concrete and partly by the reinforcement. It is noticed that in every case this portion of the curve has ended before the first visible crack was observed.

Calculation of Strength of Series D, E, F and G. Specimens.

The only existing method for calculating the strength of such specimens is that given by Turner and Davies (6). They assume that the tensile torsional component at 45 degrees is resolved into two components (inclined to each other at 90 degrees), one running along the longitudinal steel and the other running along the binders: by this means they deduce the following formula:

T reinforced section = T plain (1+ 0.25p)

where p is the total percentage of reinforcement. They state that this formula only holds for values of p up to 1½ to 2 per cent. It has been applied to the tests in the series and the results are given in the table below. In calculating these results the torque for a plain section has been taken as 10,450 in. lb.

Series.	Percentage of Reinforcement.	"Turner-Davies" Torque.	Experimental.
D	2.56	17,100 in.lb.	17,300 in. lb.
E	5.00	23,500 "	19,500 "
F	3.60	19,800 "	17,700 "
G	1.64	14,700 "	15,700 "

Thus it is seen that over the small range of values from 0 up to 1½ to 2 per cent the Turner and Davies formula gives results agreeing to within approximately 5 per cent of those obtained in the Author's experiments. For values of p greater than 2½ per cent the formula is obviously of little use.

To endeavour to produce a formula which is valid for higher percentages of p the Author has plotted a graph of increase in torque against percentage of total reinforcement and this is shown on page 132. From this graph the Author deduces the following simple formula which he suggests can be applied to cases where p varies from 1½ to 5 per cent.

$$T_{reinforced} = T_{plain} \ (1.33 + 0.11p)$$

The graphs on page 133 although not affording any information on which to base a formula for design do show a very interesting feature, namely, that the additional torque is directly proportional to the diameter of the longitudinal bar. It is seen, however, that the value of the crushing stress between the bars and the surrounding concrete is very small, consequently failure cannot be caused solely by this. It is, however, a

feature which might well be studied in future work on this subject.

Analysis of Member by Strain-Energy Methods.

It has already been stated that there seemed to exist some bond between the reinforcement and the concrete which was caused by internal forces. In analysing the problem by strain-energy methods it is assumed, firstly, that this internal force is a diagonal compression taken by a bond in the concrete and, secondly, that the AE for this band is so great that the strain energy in it can be neglected.

The problem then resolves itself into one similar to the aeroplane fuselage, the members being as shown in Figure 19.

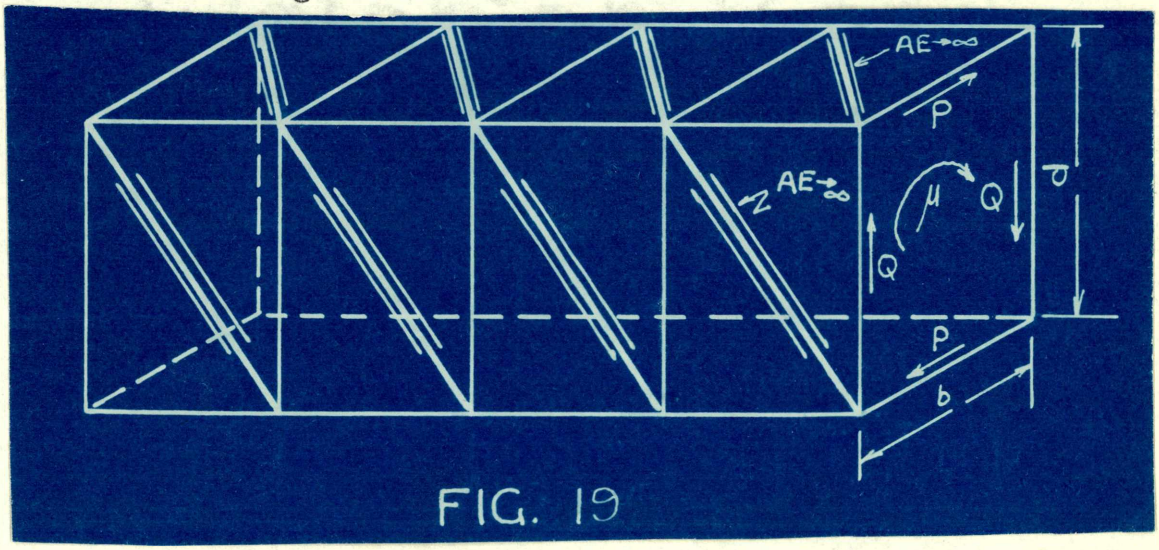


FIG. 19

Then if μ = applied torque,

we have $Qb + Pd = \mu$

or $Q = \frac{1}{b}(\mu - Pd)$

But $\frac{dU}{dP} = 0$ hence we find P and Q

Knowing P and Q the loads in all the other members can be determined.

Also $\frac{dU}{d\mu} = \theta$

$$NJ = \frac{\mu l}{\theta} = \frac{\mu l}{\frac{dU}{d\mu}}$$

Hence NJ for the composite structure can be calculated.

The calculations for this are given in Appendix II which is divided into the following sections:

Pages 188 to 192 which give a general formula for the loads in members and the torsional rigidity of the structure in terms of the areas of longitudinal and laterial reinforcement and the pitch of the binders.

Pages 193 to 194 which give the torsional rigidity of the composite sections calculated from the formula given on page 191

Pages 194 to 196 which give the loads in the various members calculated from the formula given on page

Notes on the strain energy method.

It is noticed that the values of the NJ for the composite sections calculated by this method agree reasonably well with the values obtained by taking the slope of the line in the Stage III portion of the graphs already referred to. It is unwise, however, to place too much reliance on this fact as it was impossible to take readings of the angle of twist except in the very early stages of Stage III.

On analysing the values of the loads in the members given in pages 194 to 196 it is seen that at failure the stresses in the steel vary considerably with the different series, and that whilst for series E and F the stresses are within reasonable limits for series D the stress in the binders is almost equal to the ultimate stress in mild steel whilst for series G it greatly exceeds the value for the ultimate stress in mild steel. It is obvious therefore that this method of approach is useless for calculating the stresses in the reinforcement.

An interesting feature is observed from the loads at failure in the imaginary concrete "bond-member" which is assumed (see page 127) to bind the reinforcement cage together. Whilst the loads in this concrete member vary considerably for each series it is found that the value P x sin (θ - 45), where P is the load in the member and θ the angle it makes with horizontal, is reasonably constant, its value being 992 lb. This

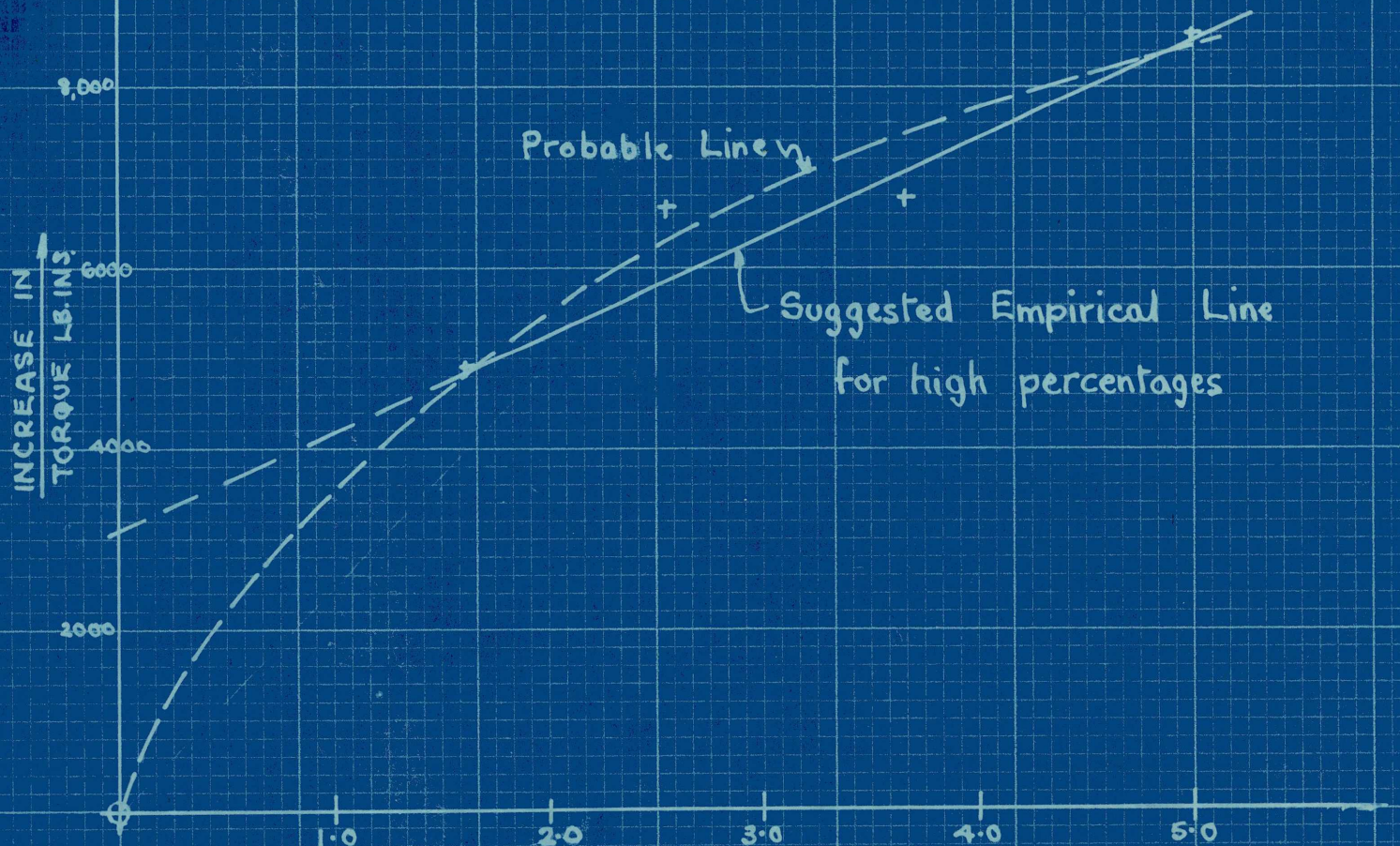
component of P is the force which would cause a tension crack at 45 degrees and the failure was in all cases accompanied by a series of tension cracks at this angle. It has already been mentioned that failure of the reinforced section was due to a breakdown in "bond" between the steel and the concrete and it is of interest to note that the strain energy method gives some interesting figures showing how this breakdown may occur.

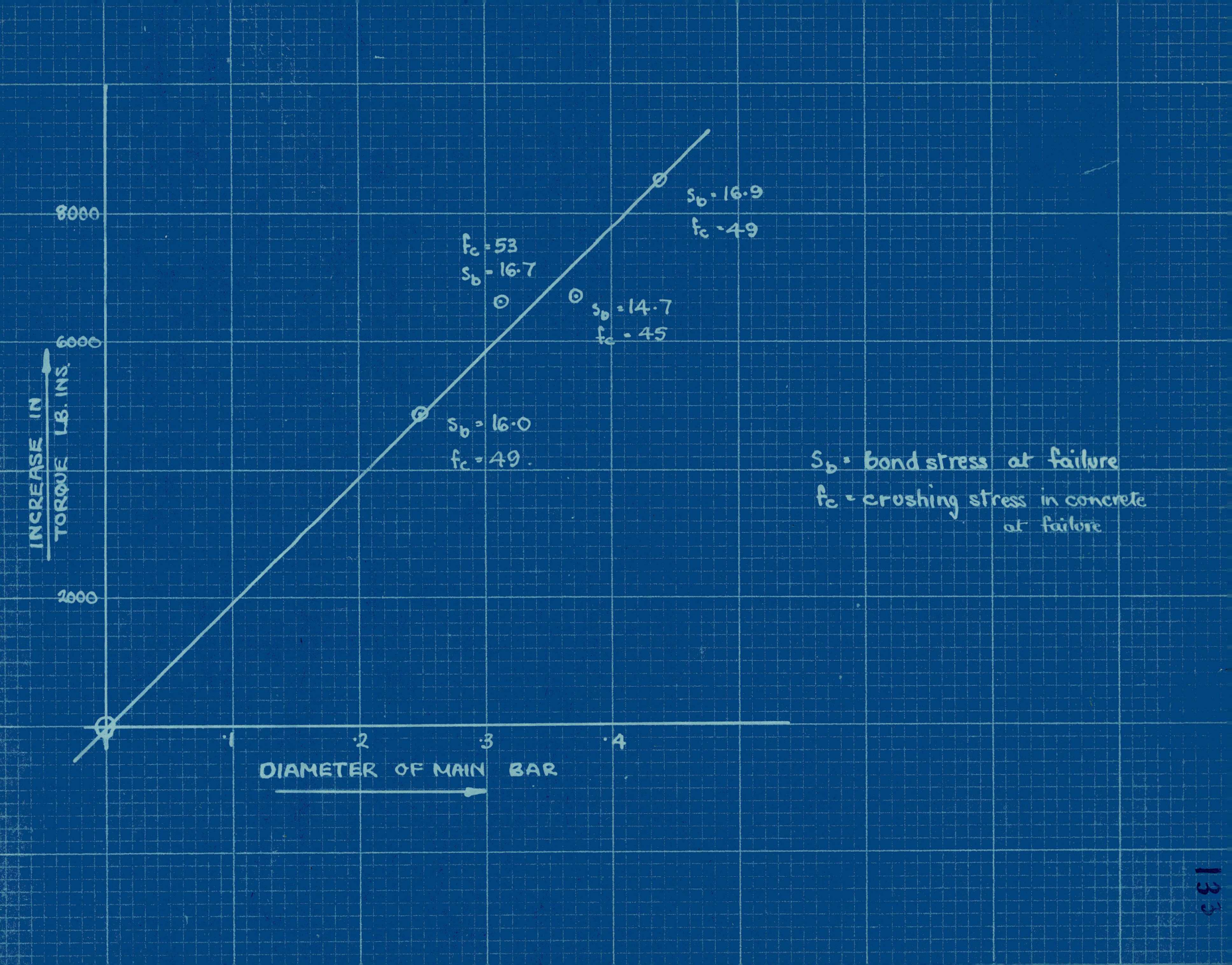
It is of course realised that the evidence upon which this is based is extremely slender but the Author mentions it as a possible line of approach for future investigators.

Table 11 - Tests on Reinforced Rectangular Specimens
(To be read in conjunction with Table 8).

Specimen	Size ins.	Torque at 1st crack lb.in.	Maximum Torque lb.in.	Maximum Torque taken by 6in. x 4in. section.	Maximum Stress at First Crack.	Maximum Stress at Failure	Percentage increase At first crack	Percentage increase At failure	Modulus of Rigidity lb./sq.in. x 10-6
B1	6.09 x 4.02	11800	11800		515	515			2.33
B2	6.18 x 4.0	11280	11280		495	495			2.40
B3	6.15 x 3.9	11800	11800		530	530			2.64
Average				11350	513	513	9%	9%	2.46
C1	6.15 x 3.8	11280	11280		550	550			2.58
C2	6.12 x 3.9	11850	11850		550	550			2.52
C3	6.10 x 3.8	10760	10760		530	530			2.54
Average				12000	543	543	15%	15%	2.55
D1	6.1 x 3.94	14360	17410		655	790			2.40
D2	6.1 x 3.92	13320	17200		610	792			2.60
D3	6.14 x 3.9	13320	16400		620	760			2.52
Average				17300	628	781	34%	66%	2.51
E1	6.1 x 3.9	13820	18240		645	855			2.53
E2	6.1 x 3.9	15400	19500		710	910			2.68
Average				19500	678	883	42%	88%	2.60
F1	6.1 x 3.9	13320	17420		620	810			2.48
F2	6.1 x 3.9	14100	16800		662	790			2.42
Average				17700	641	800	37%	70%	2.45
G1	6.1 x 3.9	12550	15160		585	706			2.67
G2	6.1 x 3.9	12550	15290		585	712			2.50
Average				15700	585	709	24%	51%	2.58
H	6.1 x 3.9	10250	10760	11200	480	506	2%	8%	2.24
J	6.1 x 3.9	12800	19490	19900	590	900	26%	92%	2.44
K	6.1 x 3.9	17410	28700	29900	815	1350	73%	188%	2.90

INCREASE - PERCENTAGE REINFORCEMENT




8000
6000
2000
INCREASE IN TORQUE LB. INS.
$s_b = 16.9$
$f_c = 49$
$f_c = 53$
$s_b = 16.7$
$s_b = 14.7$
$f_c = 45$
$s_b = 16.0$
$f_c = 49$
s_b = bond stress at failure
f_c = crushing stress in concrete at failure
·1
·2
·3
·4
DIAMETER OF MAIN BAR

Part III - Spiral Reinforcement.

In this short series of tests three specimens marked H, J and K were tested. The reinforcement in them consisted of 4 - ¼ in. diameter longitudinal bars with one, three and five 3/16 in. diameter spirals respectively. The results of the tests are given in Table 11 and graphically on page 152. (For comparison with a similar section reinforced laterally with binders instead of spirals see details of specimen G ~~are also~~ given on the graph p.150) The specimens failed in a similar way to the other reinforced specimens, namely, tension cracks at 45 degrees.

The torque twist curves show the same three "stages" as those for the specimens in D, E, F and G series, but it is noticed in the case of the spirally reinforced sections that the slope of the stage III line is somewhat greater. This indicates that these sections have a greater torsional rigidity than those where the reinforcement consists of binders and longitudinal bars. This fact can be easily understood as it is obvious that a spiral is stiffer in torsion than any other type of reinforcement.

A glance at the values of the ultimate torque taken by the different sections shows the superiority of the spiral as a form of reinforcement against torsion. Specimen J which has a total percentage of reinforcement of 1.3 per cent gives an increase of approximately 90 per

cent to the ultimate torque which is almost the same as that given by specimen E which contains 5.0 per cent of reinforcement in the form of longitudinal bars and stirrups. Specimen K is even more noticable as in this case with only 1.6 per cent of reinforcement the increase is almost 200 per cent.

Calculations of theoretical strength of specimens J and K.

The earliest formula for the strength of concrete with spiral reinforcement was given by Morsch and has been already mentioned in the chapter on previous investigations. This formula,as given, only applies to circular sections and a formula on the same lines has also been given by Turner and Davies. The only formula given for non-circular sections is one which has been given by Paul Anderson. He develops a formula for a circular section and then by introducing a "reinforcement efficiency co-efficient" applies this formula to a square section reinforced with spirals. In developing this formula he has assumed that the reinforcement takes all the tension stresses over and above those which could be taken by a plain concrete section. The actual formula he gives for a square section is as follows:

$$\frac{A_s}{p} = (V_e - S_c)^2 (3V_e^2 + 2V_eS_c + S_c^2) \quad \frac{\pi \rho_e^3}{32V_e^3 \rho_s^2 S_s}$$

where A_s = area of one spiral rod.

p = pitch of spirals.

V_e = shearing stress produced by torque T.

S_c = permissible unit tension for plain concrete.

ρ_e = radius of circle or side of square.

ρ_s = distance from centre of circle to centre of spiral reinforcement.

S_s = stress in steel.

Although Anderson has not applied this formula to rectangular reinforced sections the Author intends to do so by taking for ρ_e the mean of the greatest and least distances from the centre of the section to the edges and for ρ_s the mean of the greatest and least distances from centre of the section to the centre of the reinforcing spiral.

Taking a value of 500 lb. per sq. in. for S_c it is intended to apply the formula to specimens J and K to calculate the ultimate stress in the steel: V_e in each case being calculated from the ultimate torque by Timoshenko's formula.

Specimen J.

A_s = .027 sq. in. p = 4 in. V_e = 900 lb. per sq. in.

ρ_e (min) = 2 in.: ρ_e (max) = 3.61 in. mean = 2.8 in.

ρ_s (min) = 1.45 in.: max = 2.85 in. mean = 2.15 in.

Hence $S_s = \dfrac{(V_e - S_c)^2 (3V_e^2 + 2V_eS_c + S_c^2) \pi \rho_e^2 \times P.}{32 V_e^3 \rho_s^2 A_s.}$

$= \dfrac{400^2 (3 \times 900^2 + 2 \times 900 \times 500 + 500^2) \pi \times 2.8^3 \times 4.}{.027 \times 32 \times 900^3 \times 2.15^3}$

= 54,400 lb. per sq. in.

Specimen K.

p = 2.7 in. V_e = 1350 lb. per sq. in.

Other data as for Specimen J.

$$S_s = \frac{850^2(3 \times 1350^2 + 2 \times 1350 \times 500 + 500^2)\pi \times 2.8^3 \times 2.7.}{.027 \times 32 \times 1350^3 \times 2.15^3}$$

= 45,300 lb. per sq. in.

Thus it will be seen that the steel stress calculated from Anderson's formula is greater than the yield point stress. The formula as derived only applies to sections which have a large number of spirals, i.e. similar to specimen K and it would certainly appear to give very reasonable results.

It is of interest to note that the formula can be applied reasonably accurately to a rectangular section with a ratio of breadth to depth of 1.5 to 1. Presumably as this ratio increases the formula becomes less accurate.

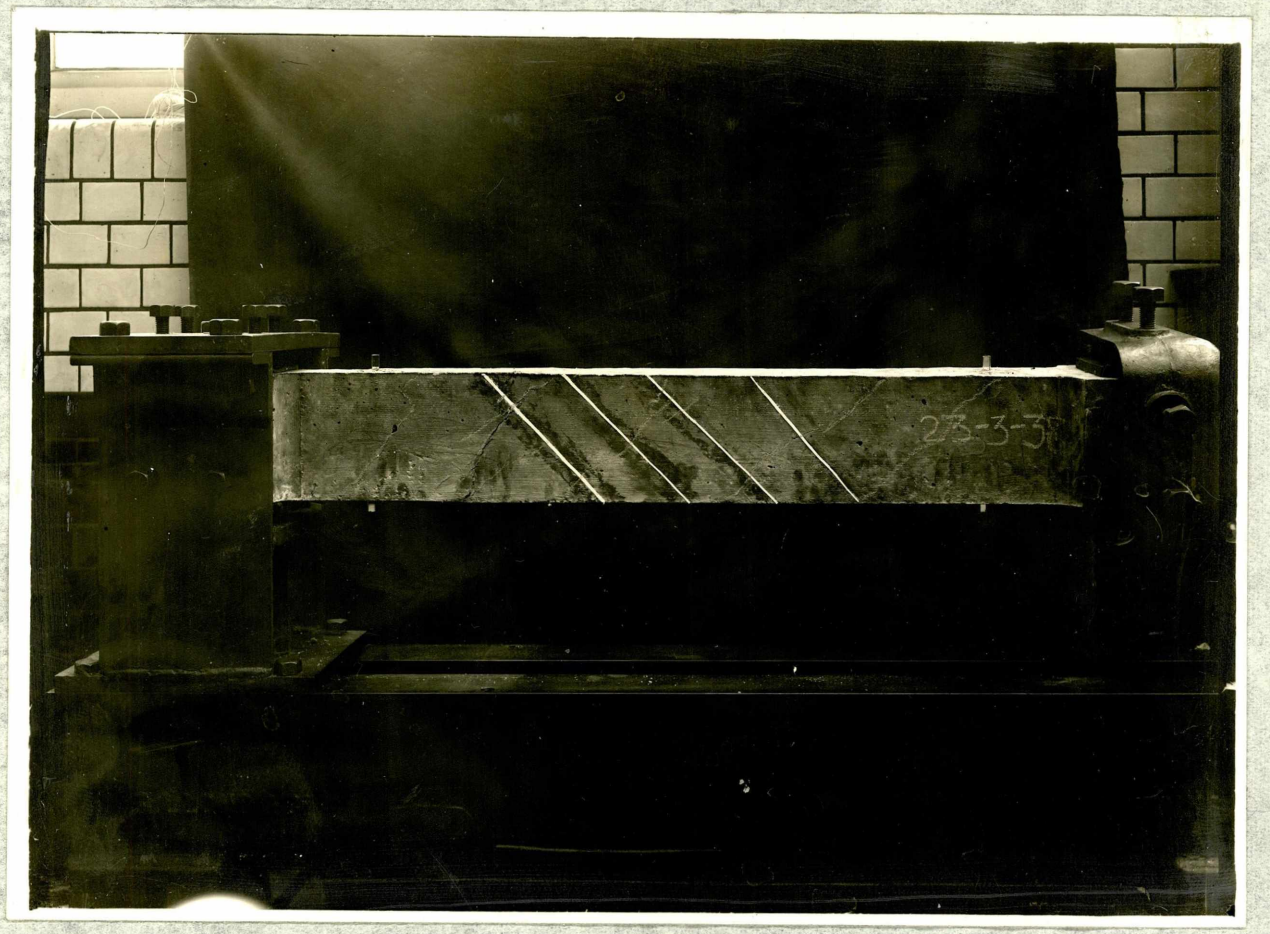


Plate 5 – Failure of Specimen reinforced with Spirals
White lines show position of spirals

Part IV - Reinforced Tee Sections.

Only one series of tests were made on reinforced Tee sections. The specimens T3 and T4 which were tested were of the same cross section as T1 and T2 but were reinforced with 4 - 5/16 in. diameter bars at the corners of the web and had 14 - $\frac{1}{4}$ in. diameter stirrups throughout the section. To reproduce the conditions actually occuring in practice to comply with the New Code of Practice, Clause 601(g) the binders were extended at the top into the width of the flange.

The behaviour during test was similar to all the other specimens tested; failure occurred by the development of tension cracks inclined at 45 degrees to the axis of twist. The torque-twist curves shown on pages 154-156 also exhibit the three stages which are characteristic of all the reinforced specimens.

The average value for the ultimate torque was 19900 in. lb. For the plain beams T1 and T2 the average value for the ultimate torque was 15100 in. lb. thus the effect of the reinforcement was to increase the ultimate torque by 32 per cent.

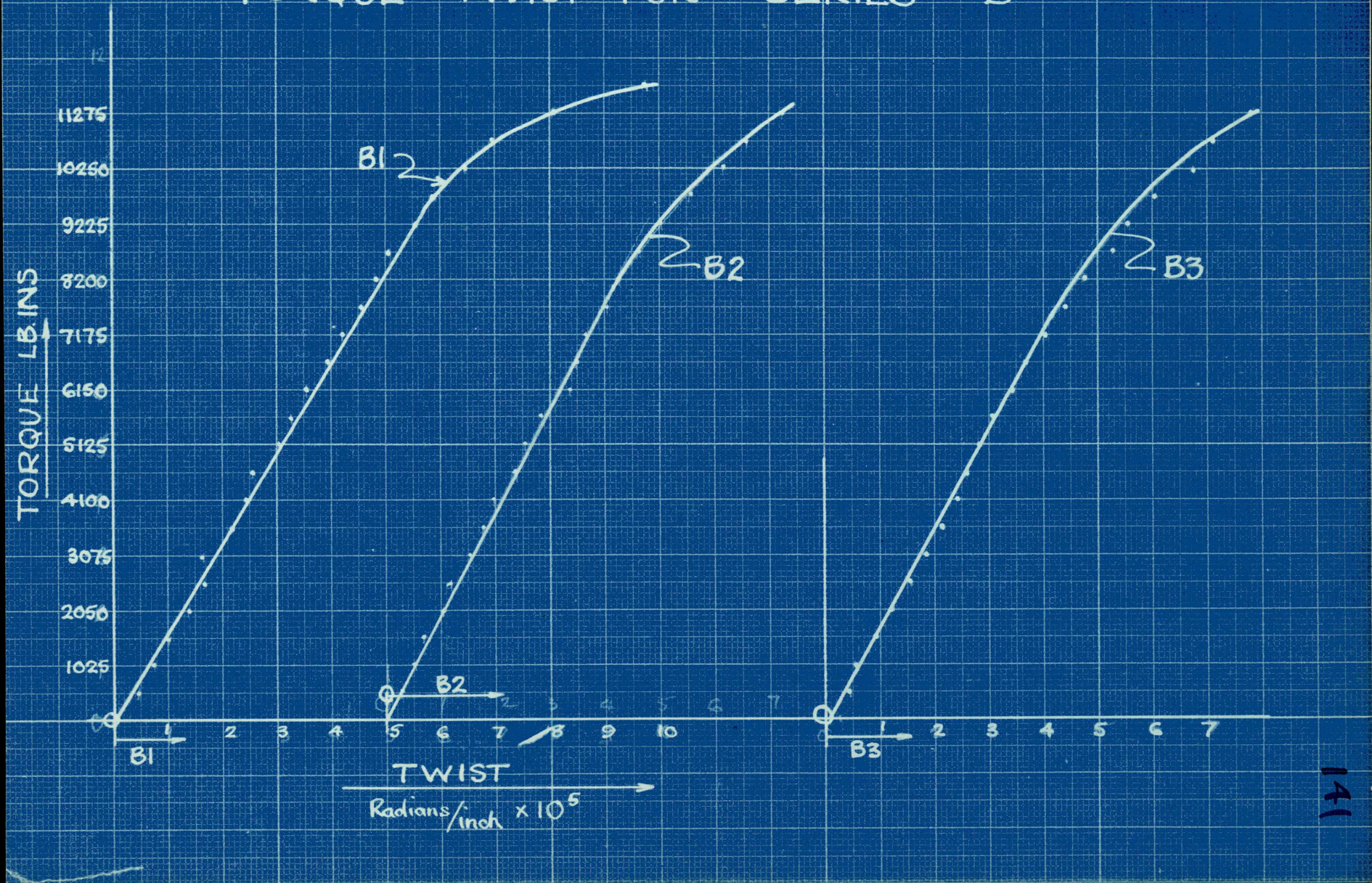
The total percentage of reinforcement used was 1.96 per cent (expressed as a percentage of the actual area of the cross section). On comparing this result with specimen G, a 6 in. x 4 in. rectangular specimen with 1.64 per cent reinforcement which increased the

ultimate torque by 50 per cent, it is seen that in a Tee section reinforcement is not so effective as in a rectangular one. Expressing by means of the simple percentage formula it might be said that for a reinforced Tee section the ultimate torque T is given from the formula -

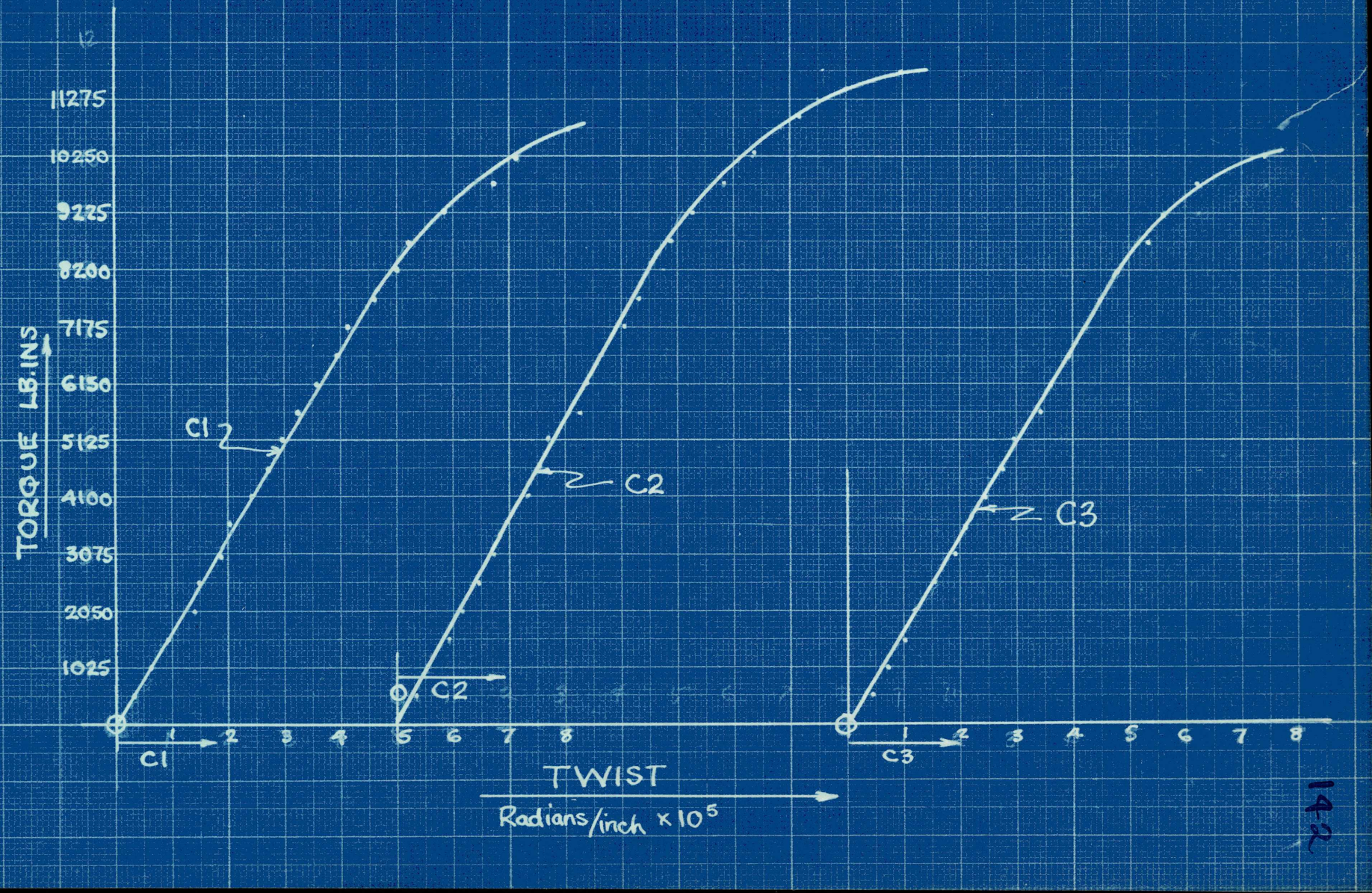
$$T = T_{plain} (1 + 0.16p).$$

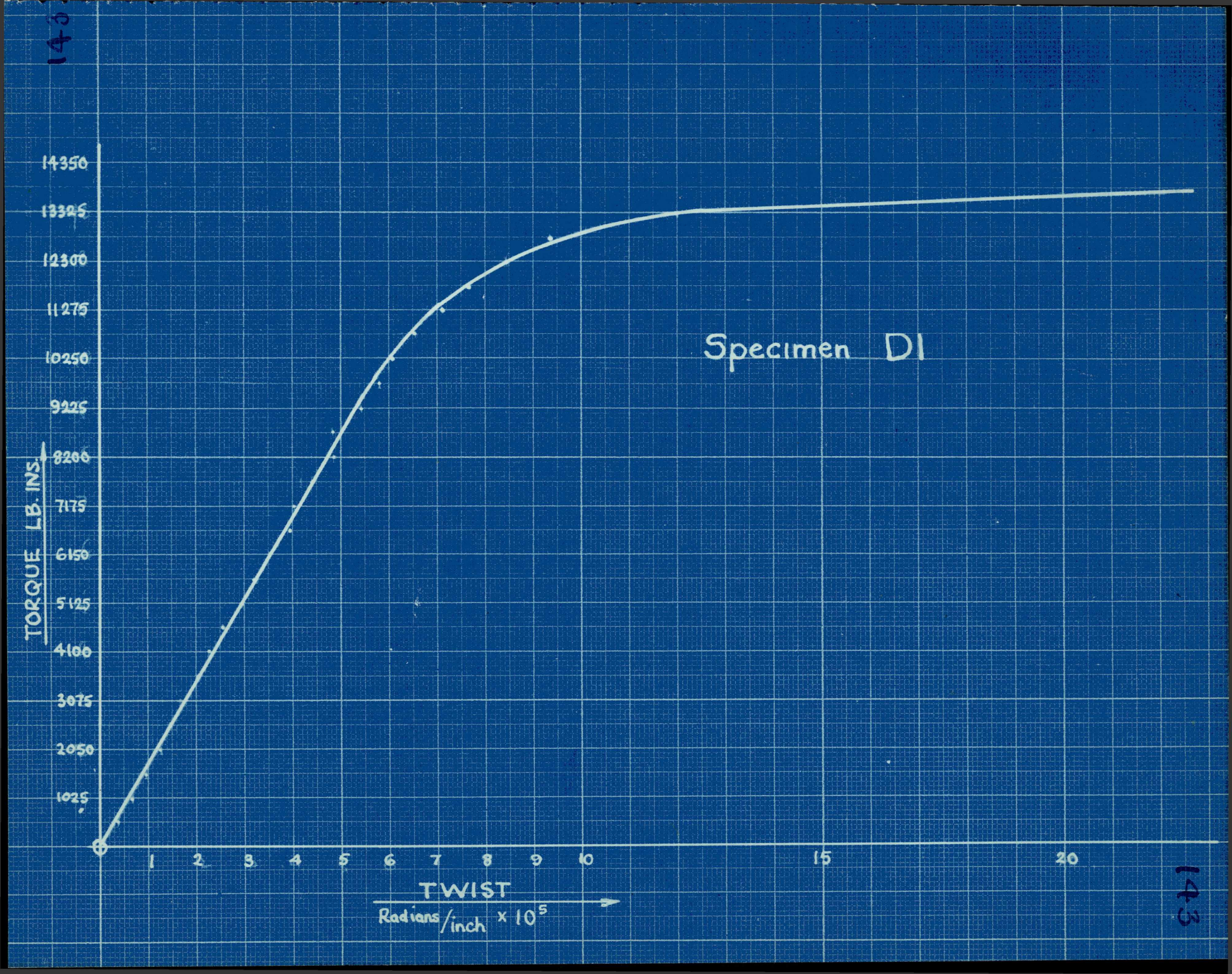
which compares very unfavourably with the Turner - Davies formula given on page 125

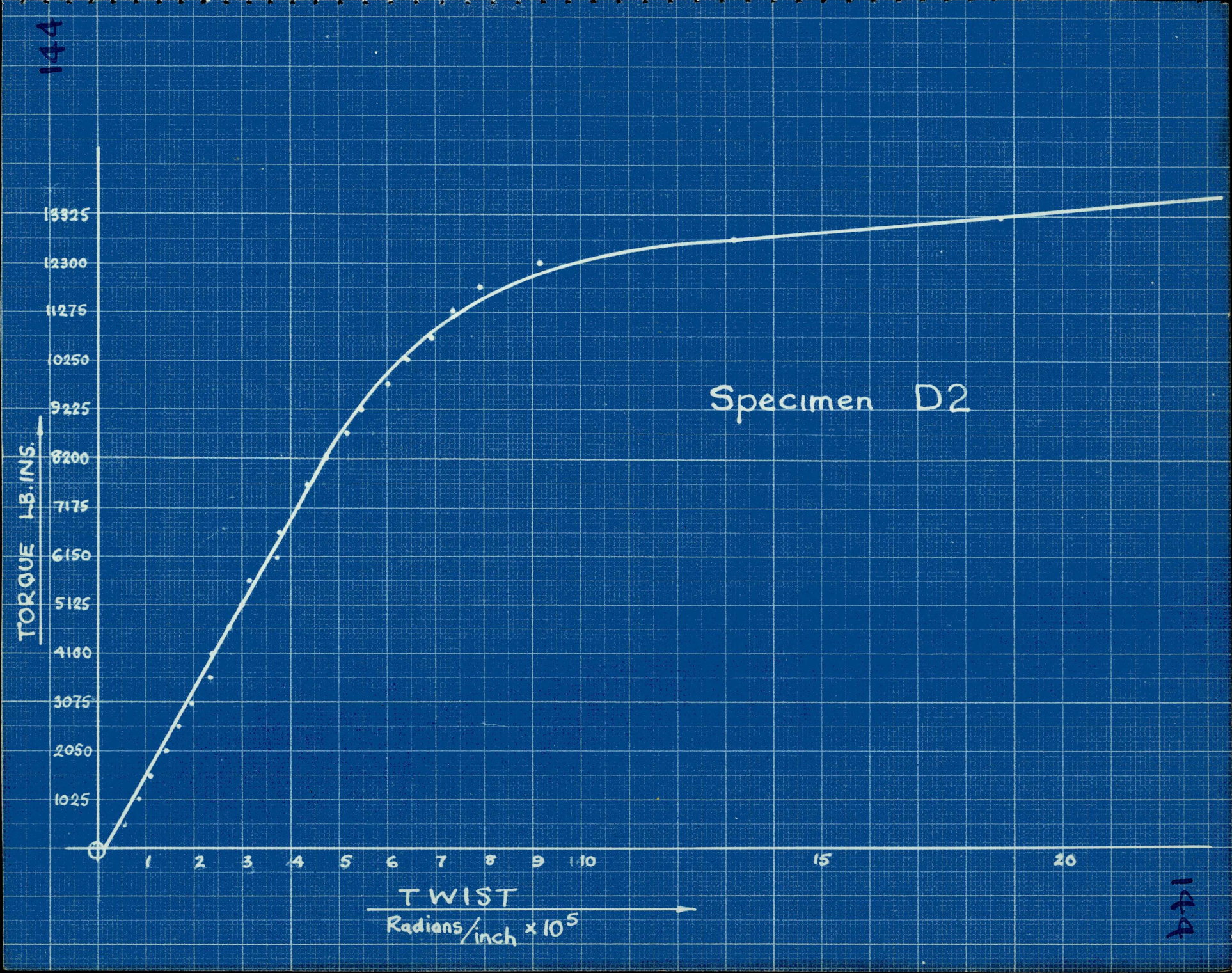
TORQUE - TWIST FOR SERIES "B"

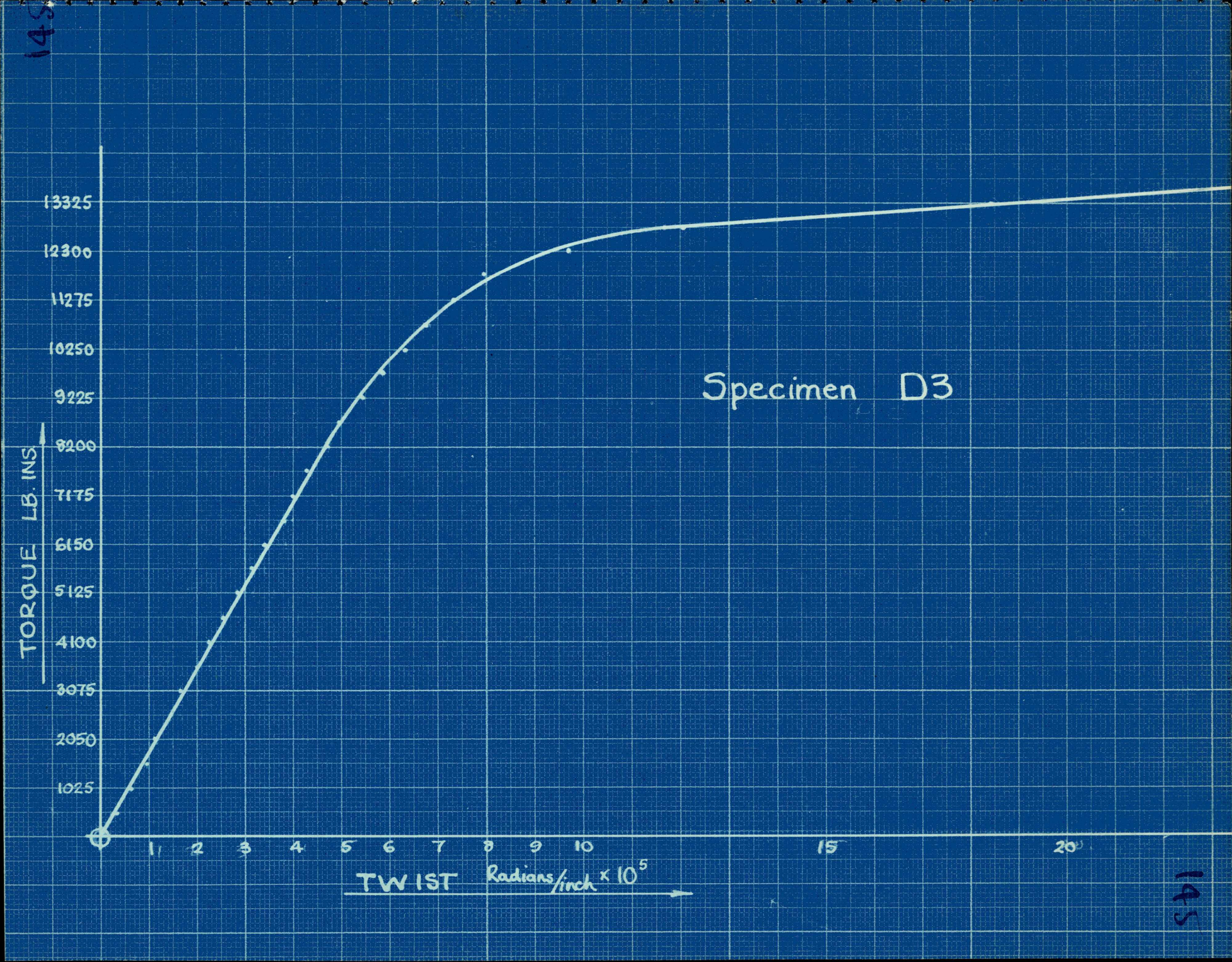


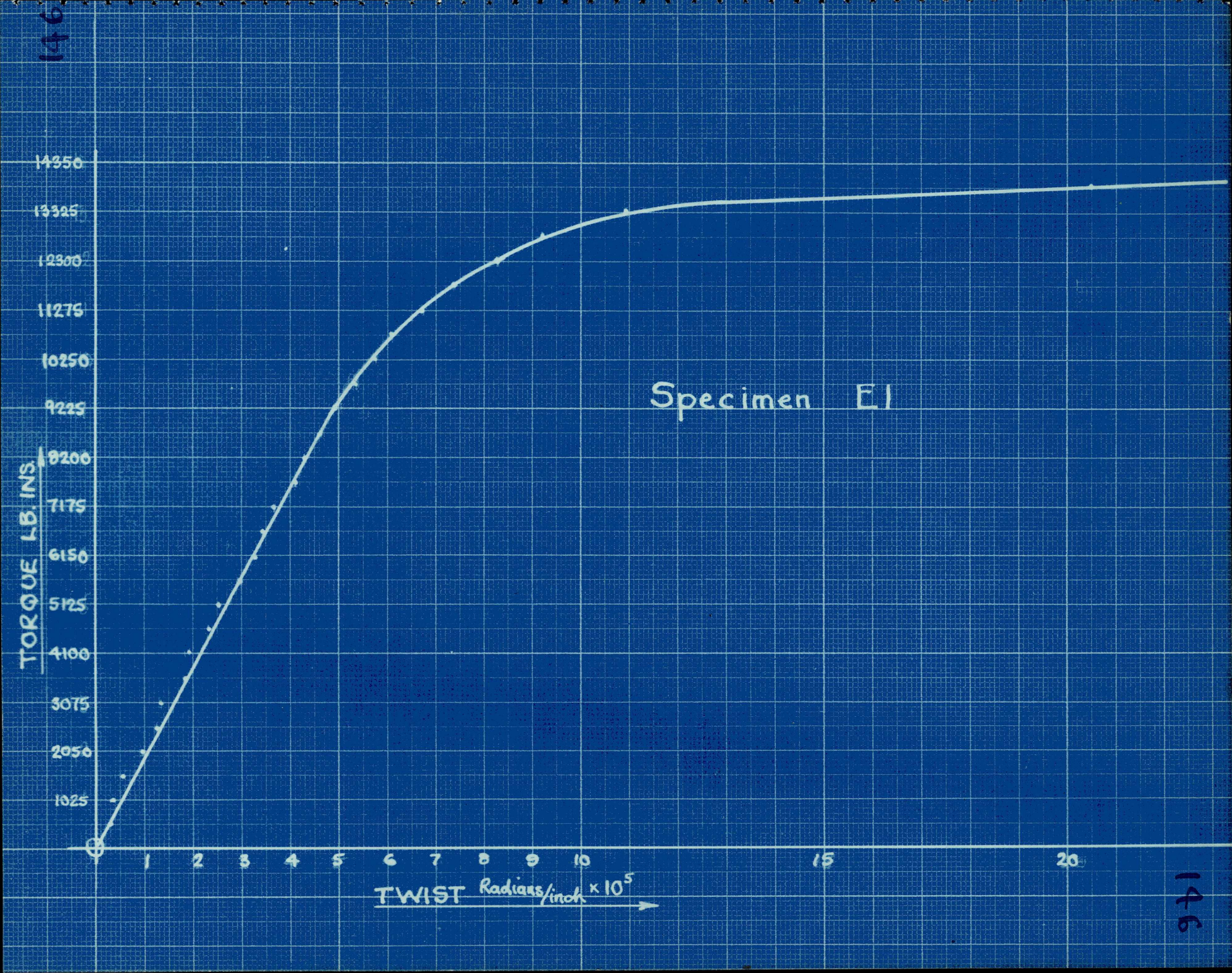
TORQUE – TWIST FOR SERIES "C"

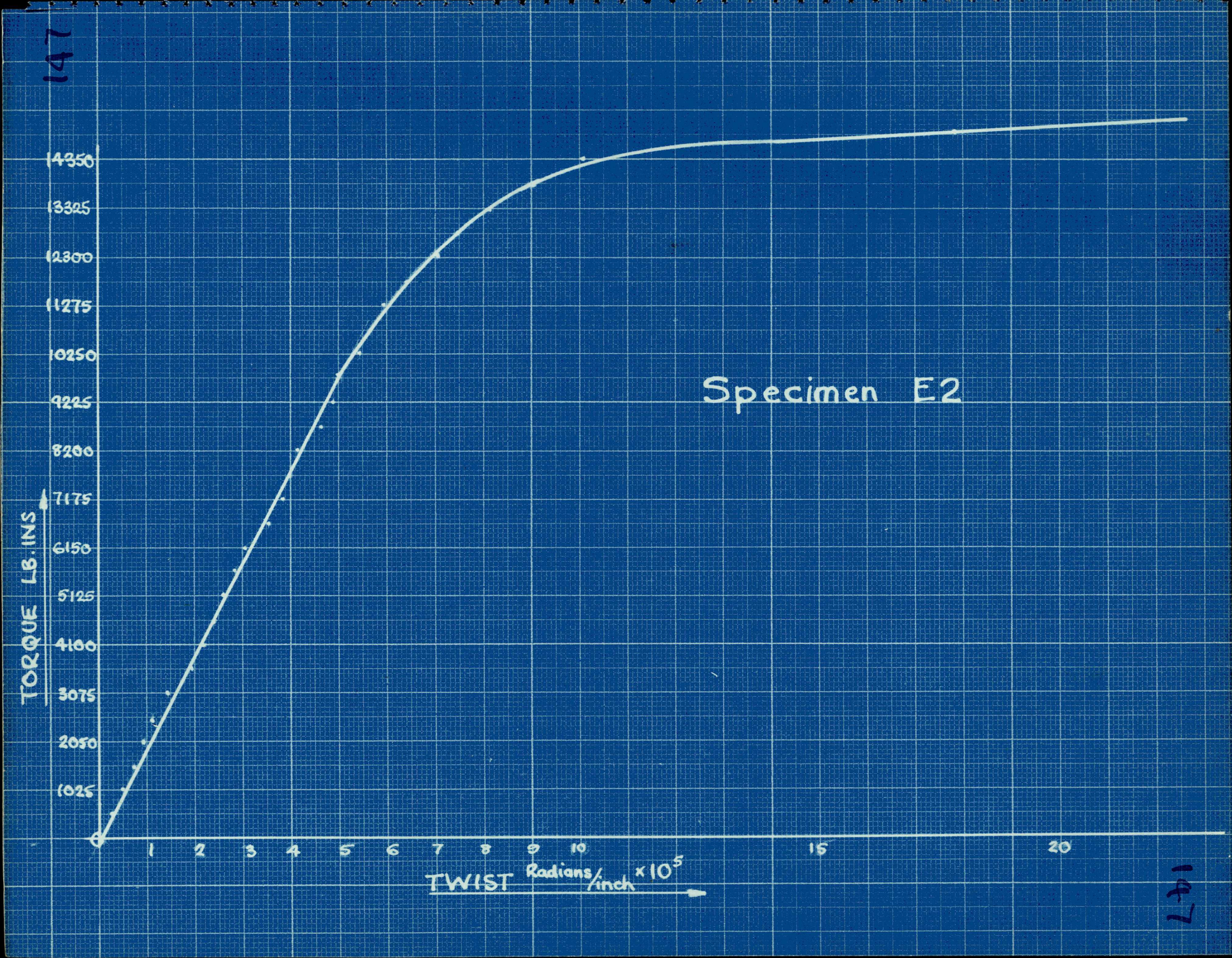


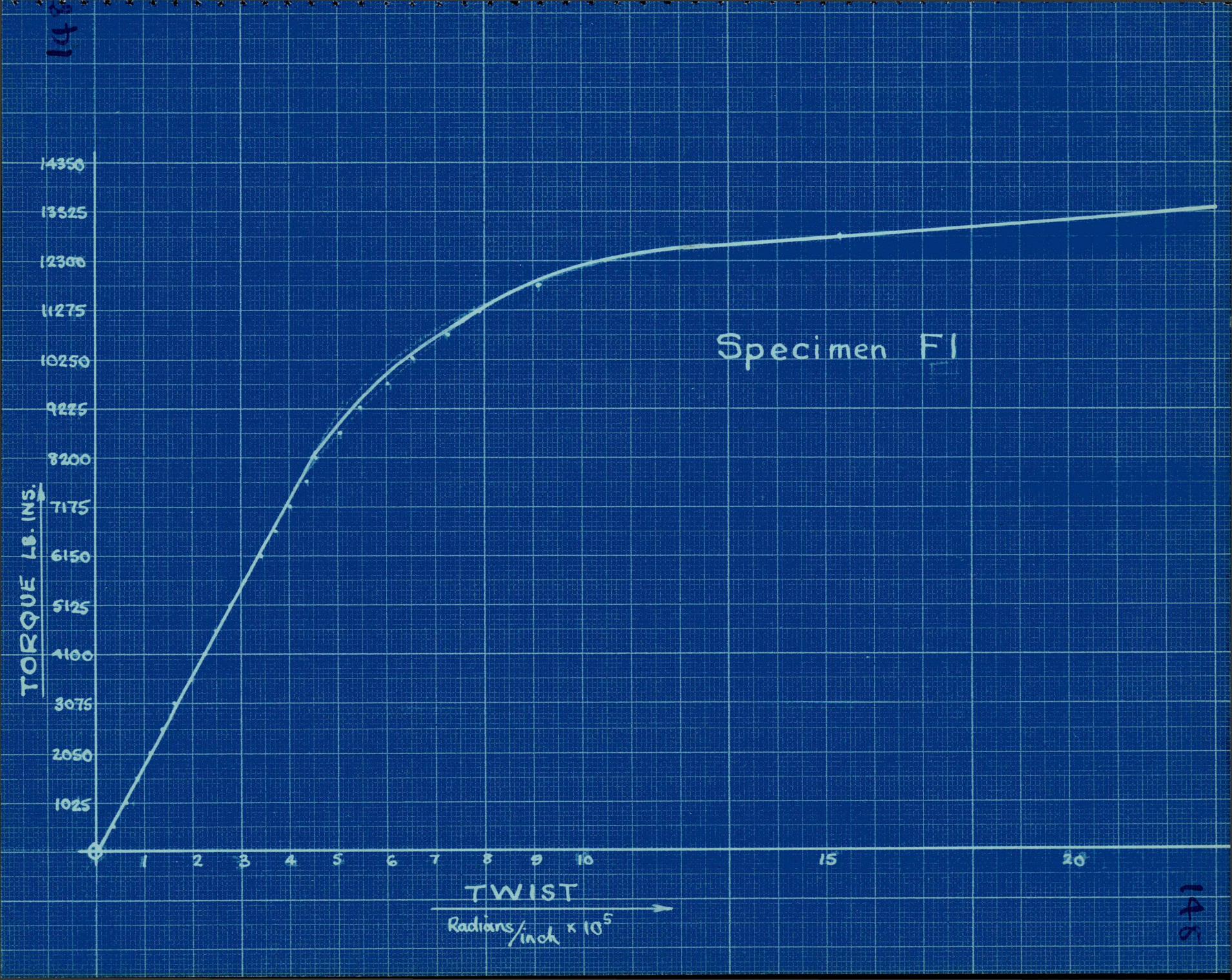

Specimen D1
TORQUE LB. INS.
14350
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
0
1
2
3
4
5
6
7
8
9
10
15
20
TWIST
Radians/inch x 10^5

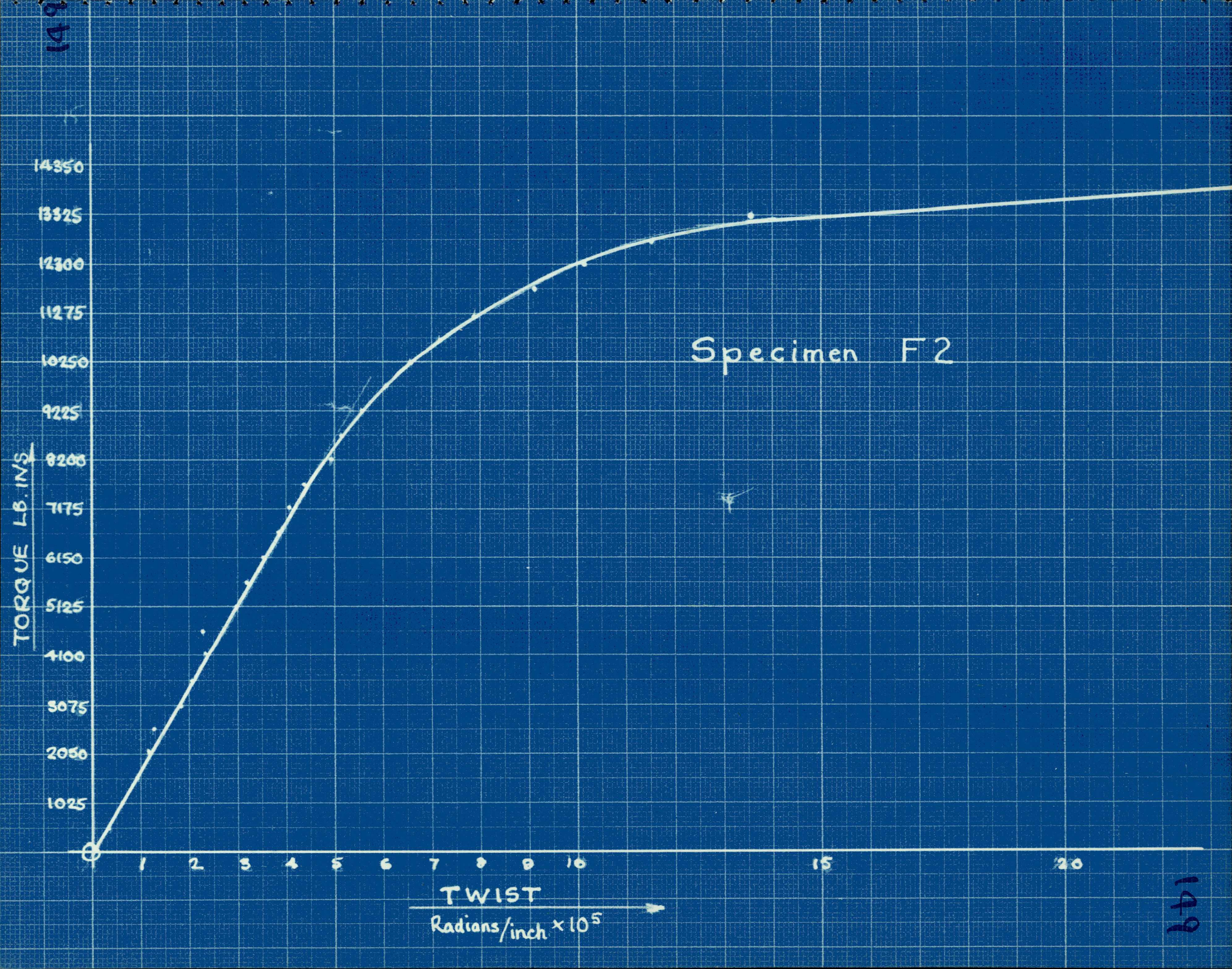

13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
TORQUE LB. INS.
1
2
3
4
5
6
7
8
9
10
15
20
TWIST
Radians/inch x 10^5
Specimen D2

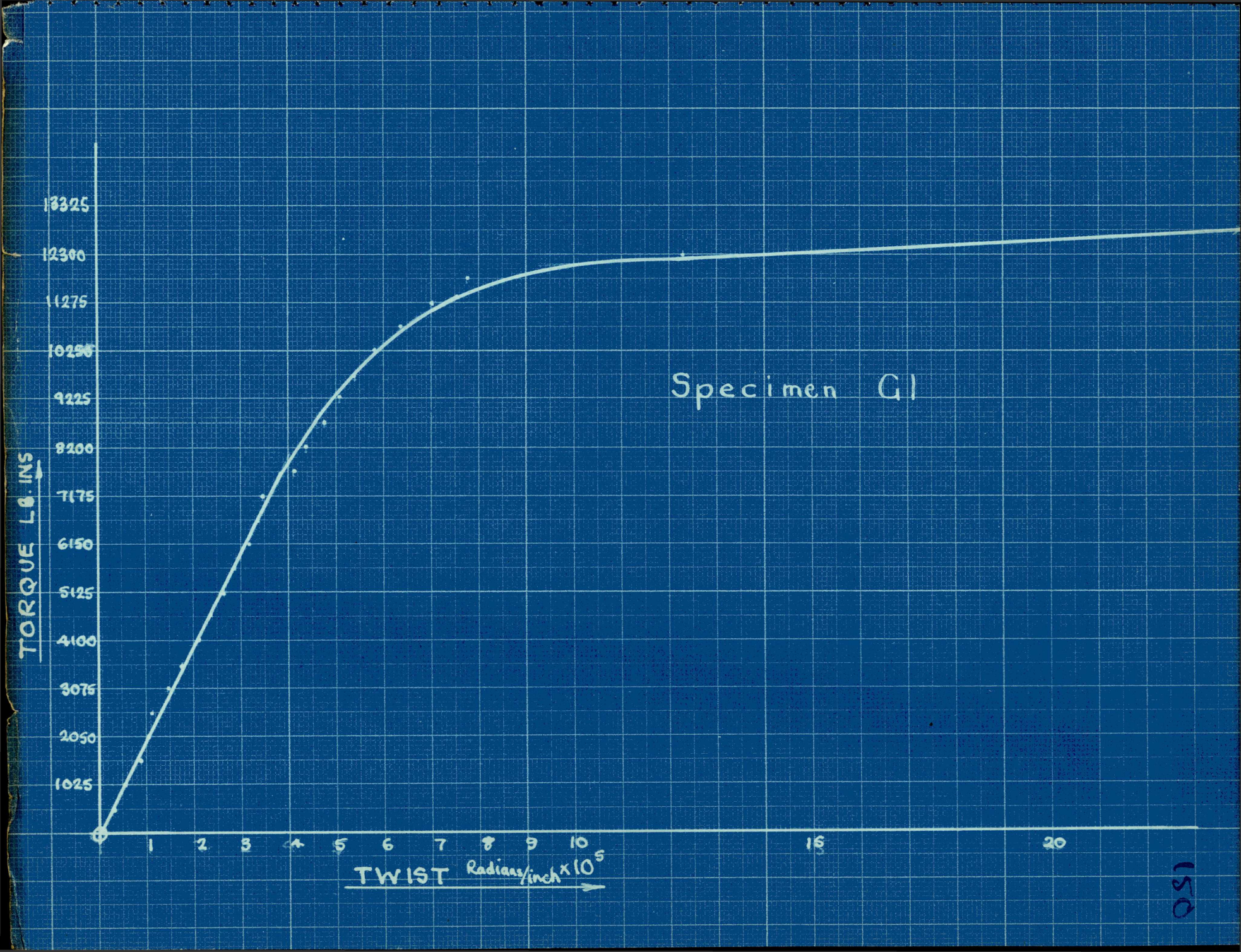

13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
TORQUE LB. INS
1
2
3
4
5
6
7
8
9
10
15
20
TWIST Radians/inch x 10^5
Specimen D3

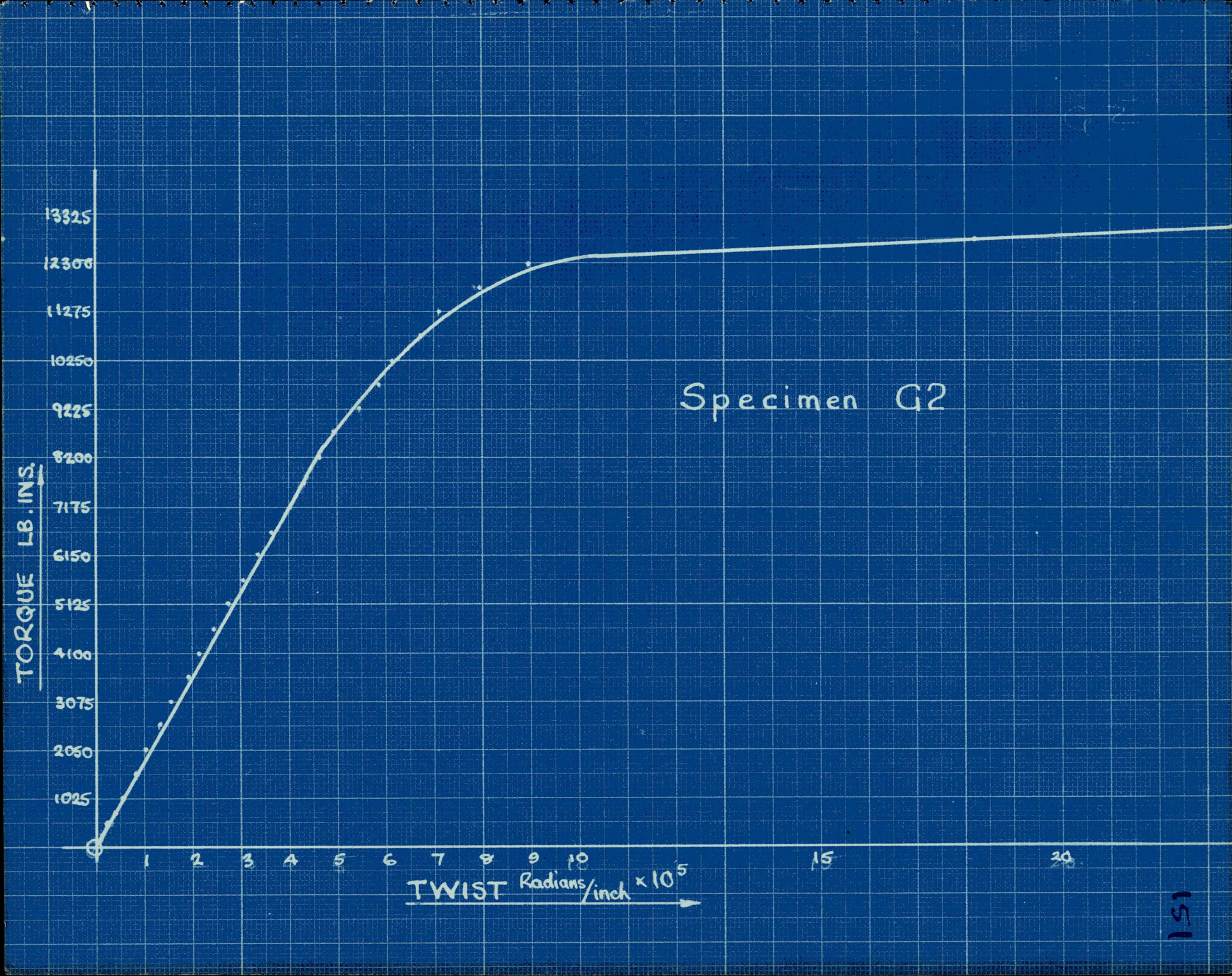

14350
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
TORQUE LB. INS.
Specimen E1
0
1
2
3
4
5
6
7
8
9
10
15
20
TWIST Radians/inch × 10^5

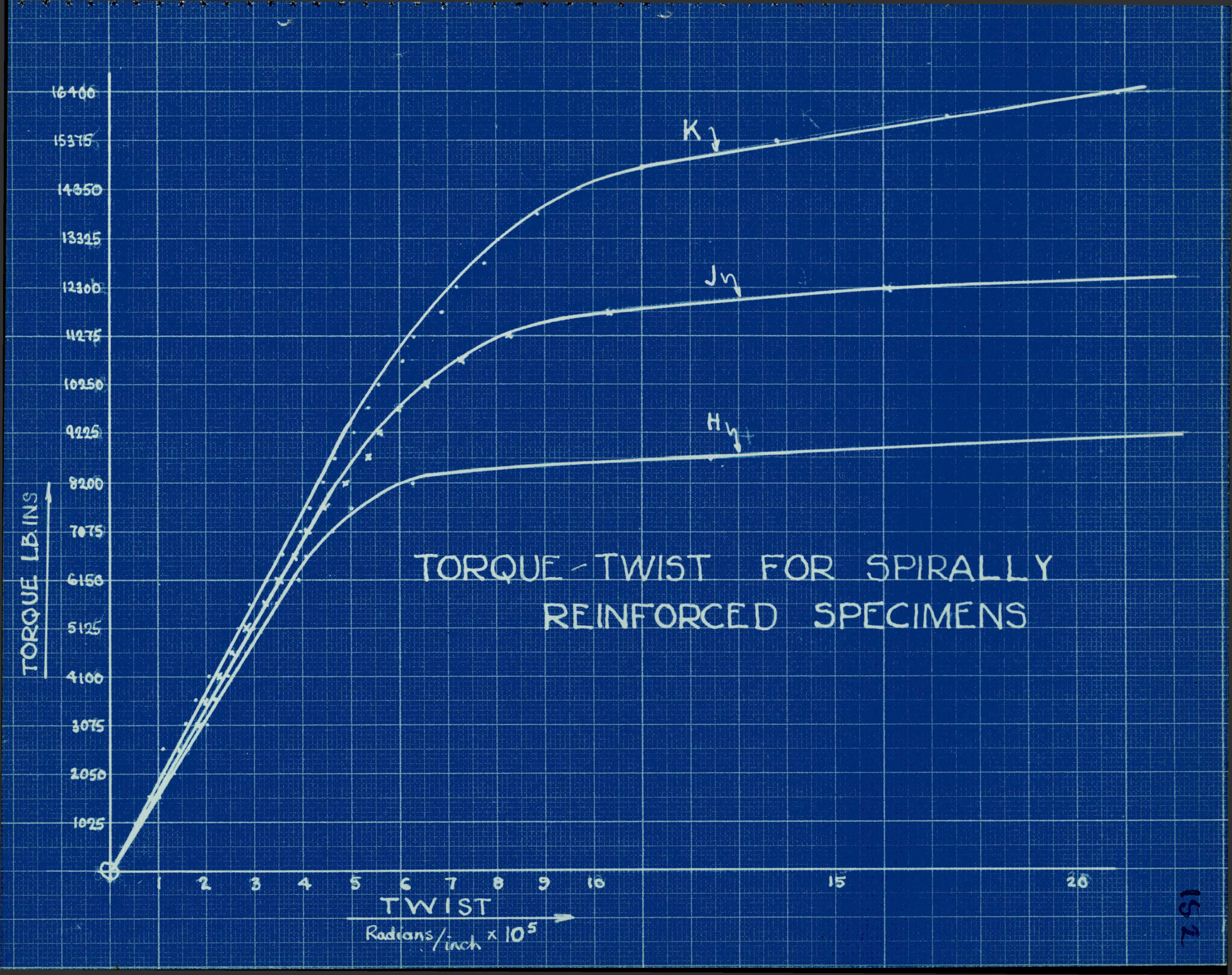

14350
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
TORQUE LB. INS
1
2
3
4
5
6
7
8
9
10
15
20
TWIST Radians/inch x 10^5
Specimen E2

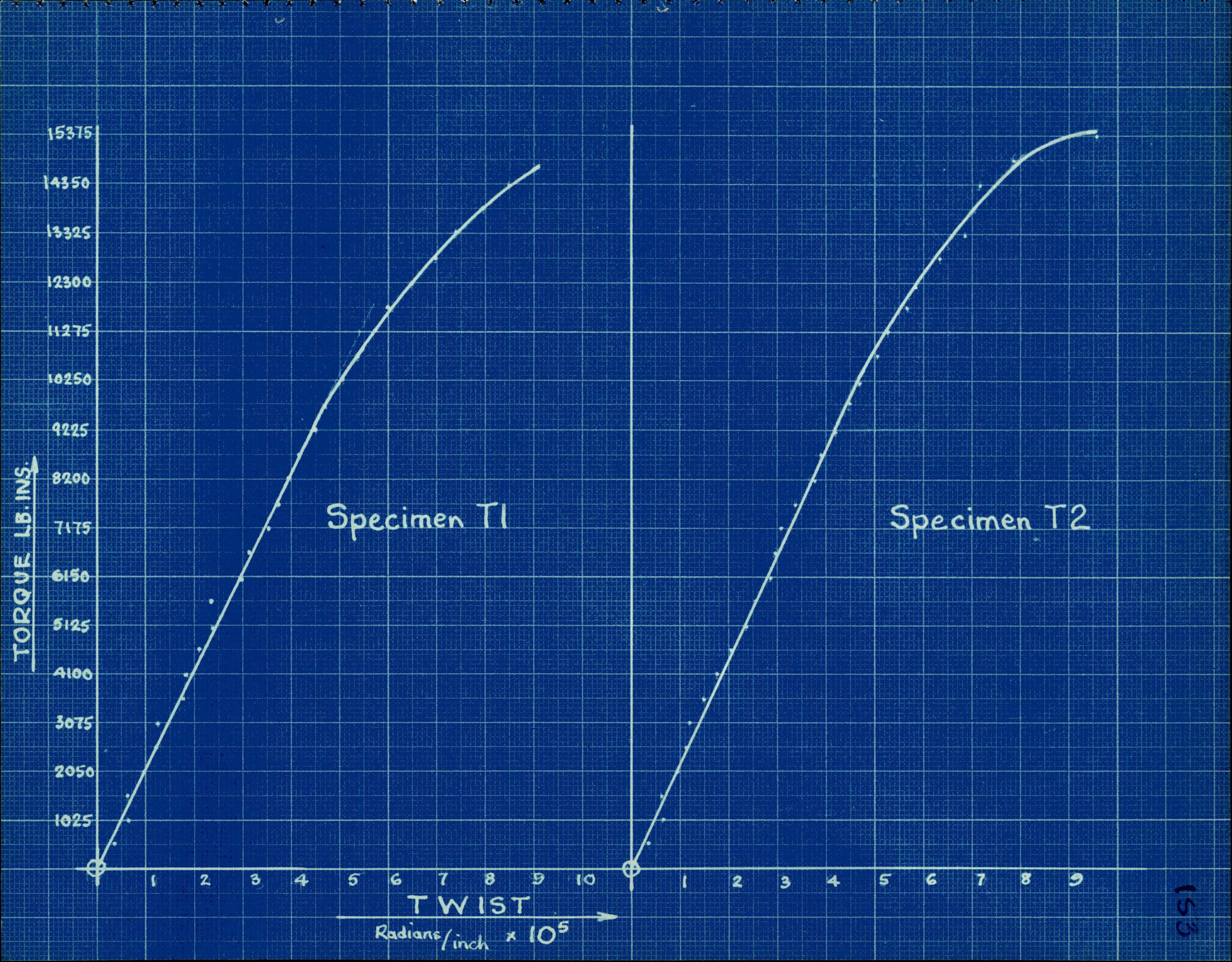

14350
13525
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
TORQUE LB. INS.
0
1
2
3
4
5
6
7
8
9
10
15
20
TWIST
Radians/inch x 10^5
Specimen F1

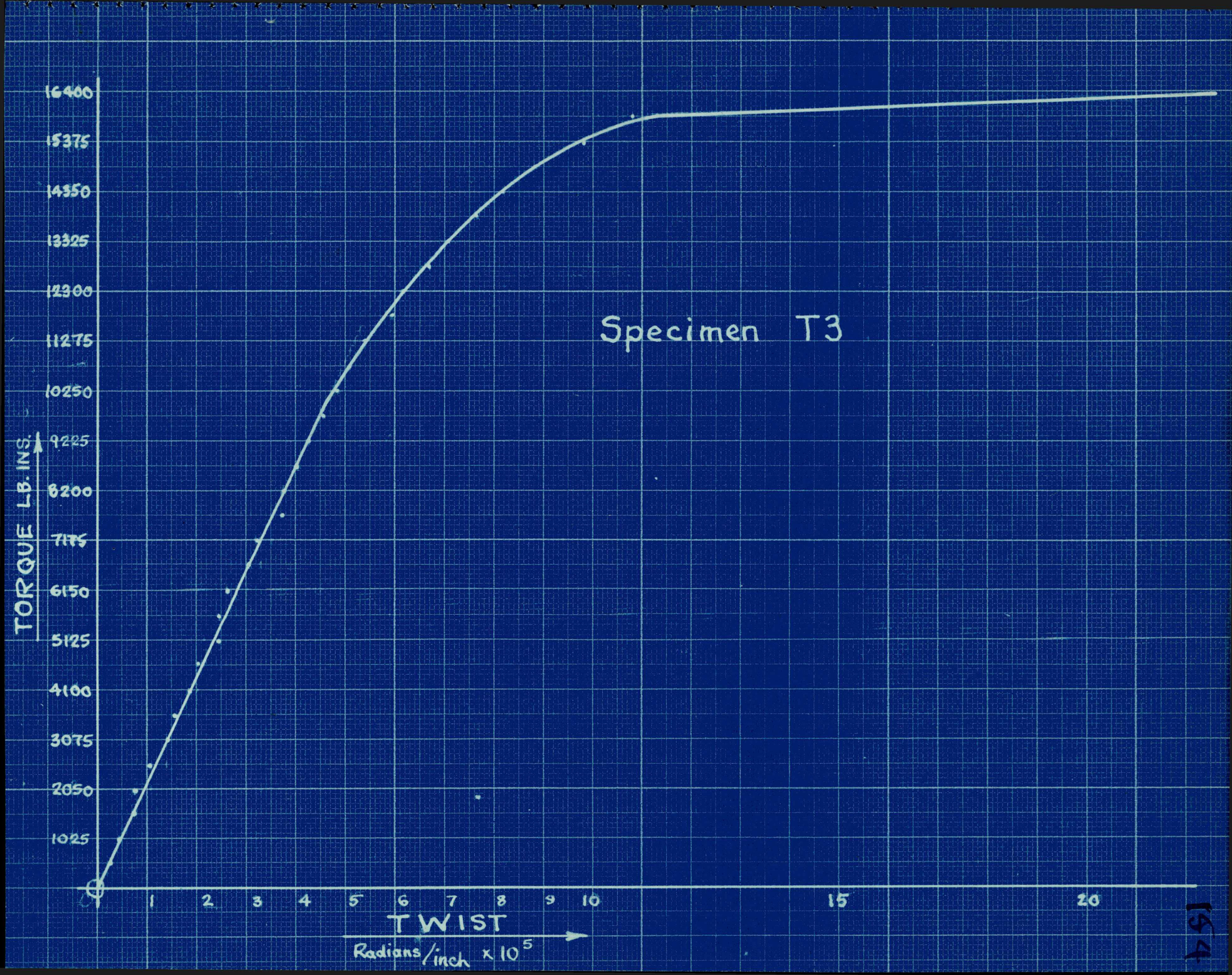

14350
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
TORQUE LB. IN'S
Specimen F2
1
2
3
4
5
6
7
8
9
10
15
20
TWIST
Radians/inch × 10⁵

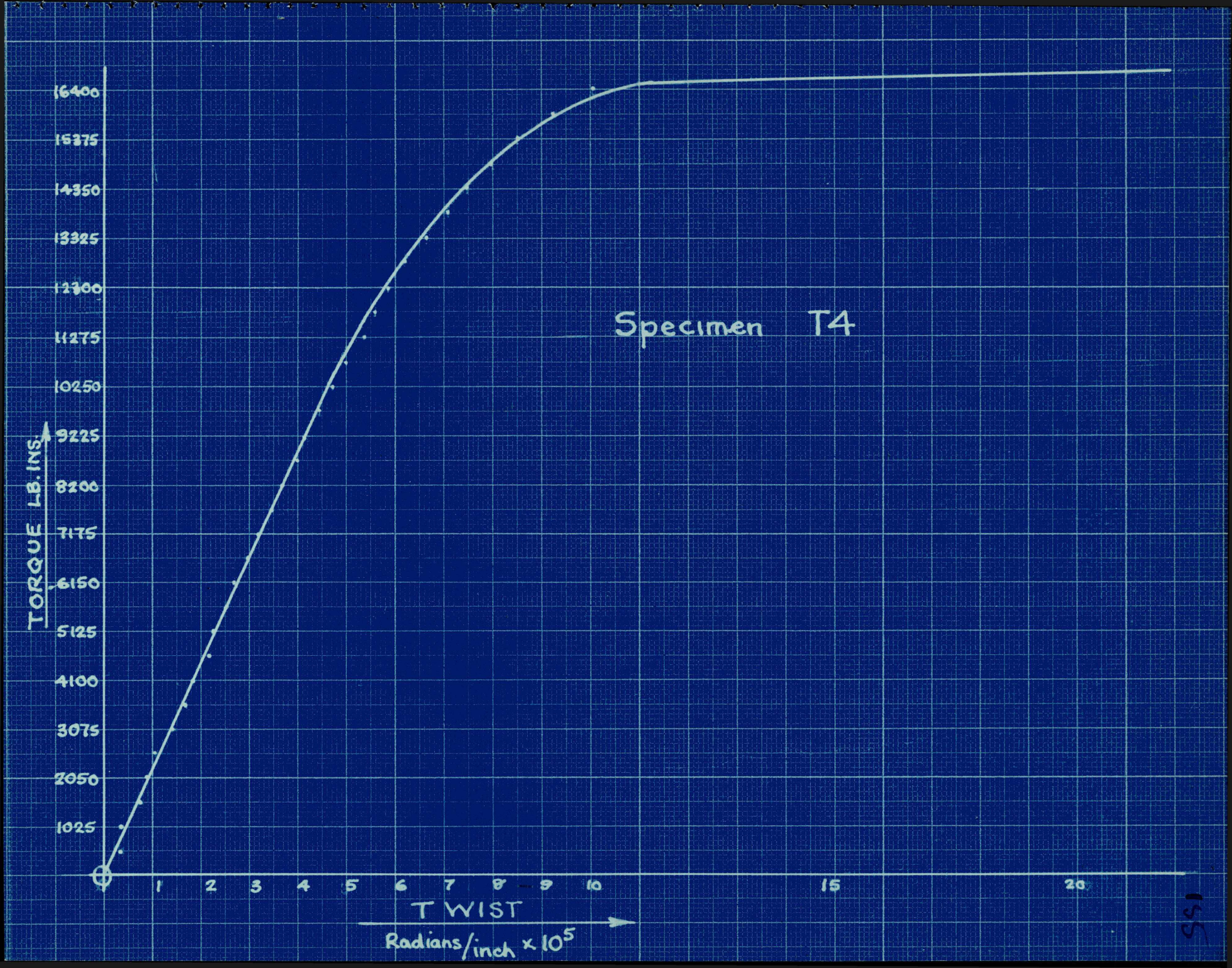

Specimen G1
TORQUE Lb. INS
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
1
2
3
4
5
6
7
8
9
10
15
20
TWIST Radians/inch × 10^5

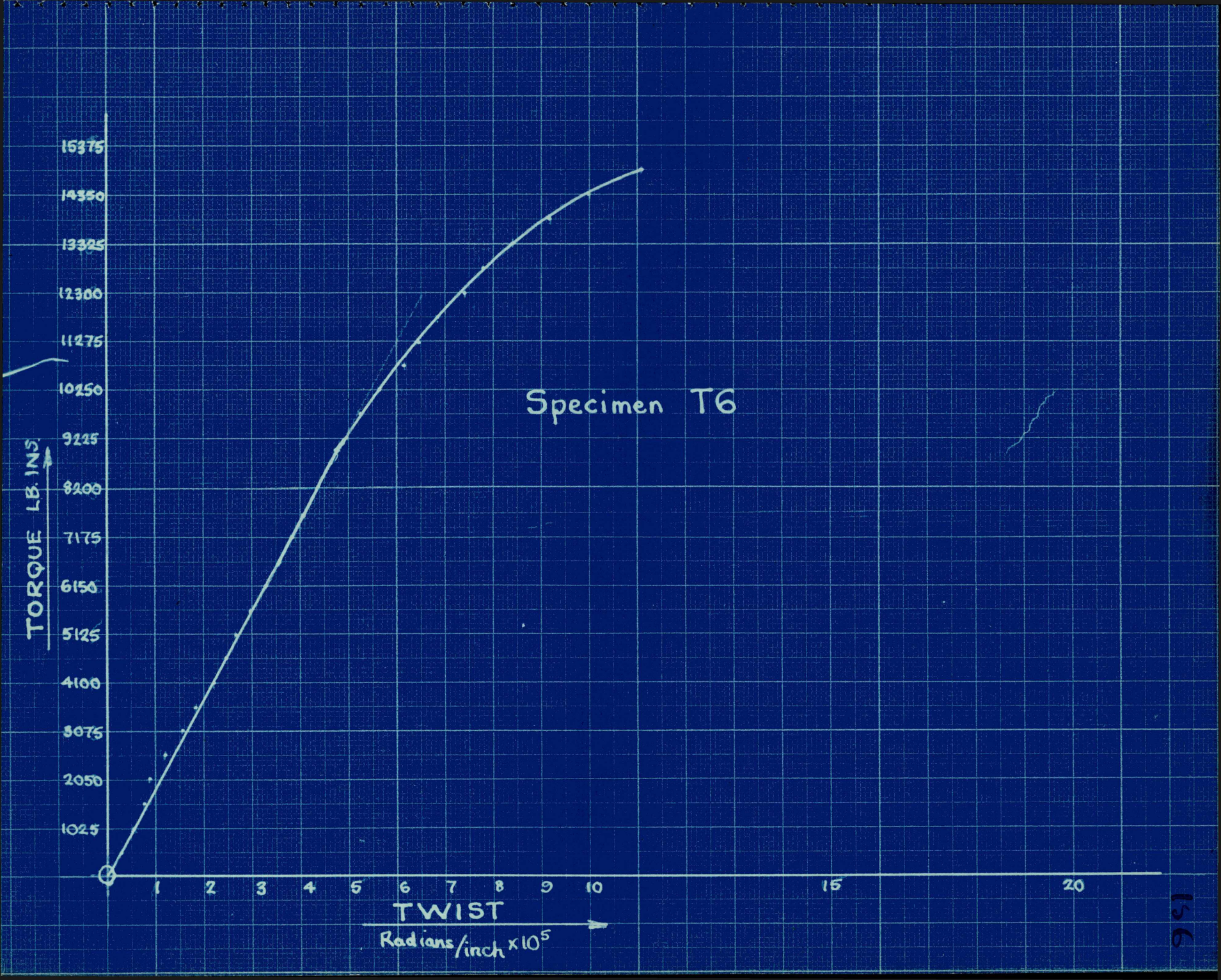

Specimen G2
TORQUE LB. INS.
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
1
2
3
4
5
6
7
8
9
10
15
20
TWIST Radians/inch × 10^5

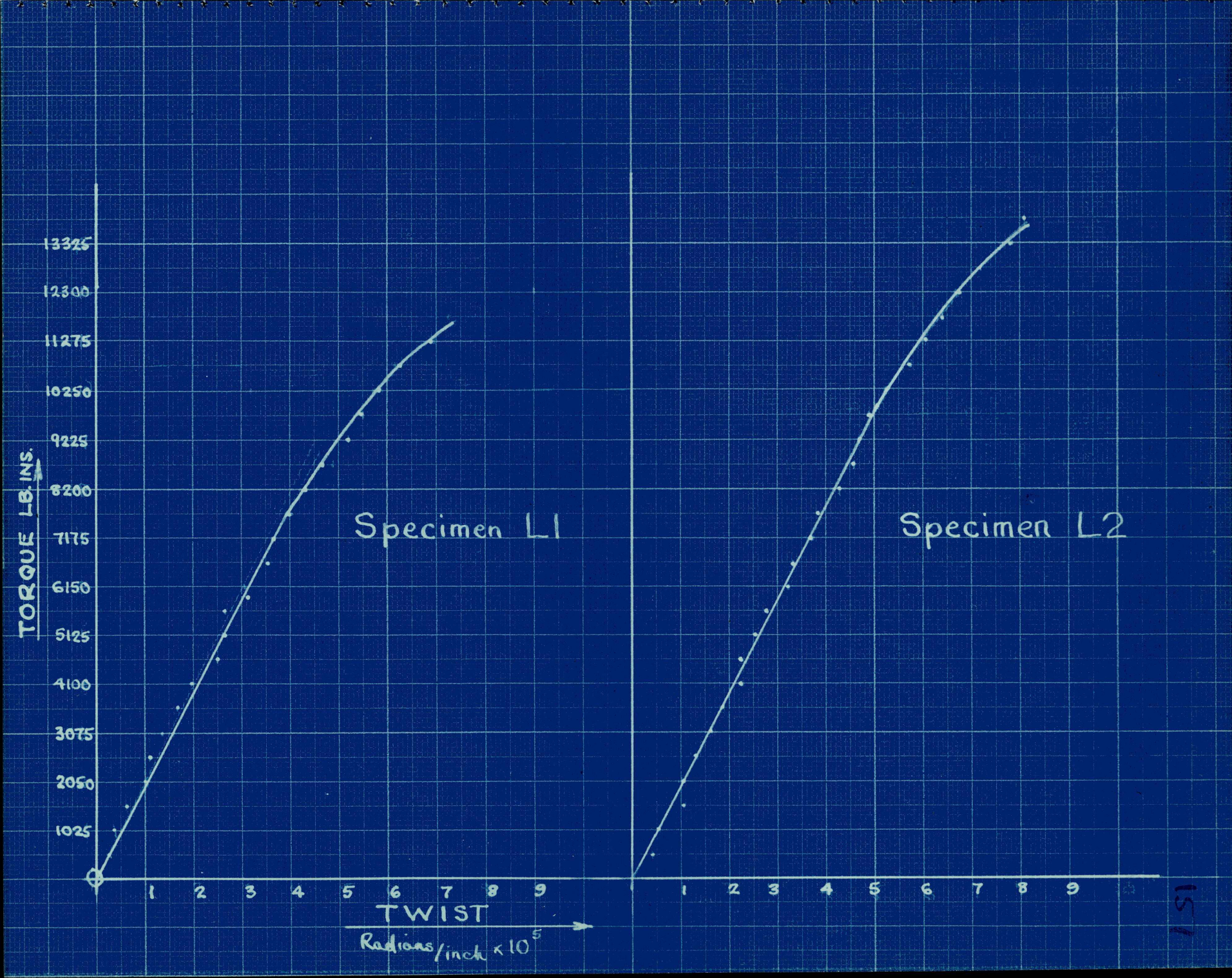

TORQUE - TWIST FOR SPIRALLY REINFORCED SPECIMENS
TORQUE LB INS
16400
15375
14350
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
K
J
H
1
2
3
4
5
6
7
8
9
10
15
20
TWIST
Radians/inch x 10^5


15375
14350
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
TORQUE LB. INS.
Specimen T1
Specimen T2
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9
TWIST
Radians/inch x 10^5
151


Specimen T3
TORQUE LB. INS.
16400
15375
14350
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
1
2
3
4
5
6
7
8
9
10
15
20
TWIST
Radians / inch x 10^5


Specimen T4
TORQUE LB. INS.
16400
15375
14350
13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
0
1
2
3
4
5
6
7
8
9
10
15
20
TWIST
Radians/inch x 10^5


Specimen T6
TORQUE LB INS
15375
14350
13325
12300
11275
10250
9215
8200
7175
6150
5125
4100
3075
2050
1025
1
2
3
4
5
6
7
8
9
10
15
20
TWIST
Radians/inch x10^5


13325
12300
11275
10250
9225
8200
7175
6150
5125
4100
3075
2050
1025
TORQUE LB. INS.
Specimen L1
Specimen L2
1 2 3 4 5 6 7 8 9
TWIST
Radians/inch × 10^5
1 2 3 4 5 6 7 8 9

CHAPTER VII.

CONCLUSIONS FROM THE EXPERIMENTS.

CHAPTER VII.

CONCLUSIONS FROM THE EXPERIMENTS.

It is proposed in this chapter to summarise the deductions from the results of the experiments mentioned in the two previous chapters. Much of the reading in this chapter is therefore in some ways a repetition of the material already given in these chapters, but it is suggested that a somewhat brief summary of the conclusions which can be drawn from the tests would prove useful.

Part I - Plain Concrete Tests (a) Circular Specimens.

1. Failure under torsion is by the development of tension cracks of spiral form running along a helix inclined at 45 degrees to the axis of twist.

2. The torque-twist graph was substantially a straight line for most of the test, indicating elastic behaviour: near to the failing load the graph curved over towards the twist axis indicating a small amount of plasticity.

3. The torsional strength of concrete increases with age but the increase after a given time is not as great as the corresponding increase in compressive strength.

4. The modulus of rigidity of concrete also increases with age.

5. The value for the modulus of rigidity of a 1:2:4 mix concrete after 7 days is 2.25×10^6 lb. per sq. in.

6. Poissons ratio for concrete as worked out from the tests is 1:8:4.

7. The torsional strength of plain concrete calculated from the elastic theory is 325 lb. per sq. in. for a 1:2:4 mix concrete 7 days old, this value showing a 33 per cent increase over the strength in pure tension.

8. The torsional strength of plain concrete when calculated by the "Turner - Davies" (6) theory is approximately equal to the strength in pure tension.

9. The mean value of the ultimate torsional strengths given (a) by the elastic theory and (b) by the plastic theory is also approximately equal to the strength in pure tension.

Part I - (b) Plain Concrete Rectangular Specimens.

1. Failure of rectangular specimens was also by the development of tension cracks at 45 degrees.

2. The torque-twist graphs were similar to those for circular sections indicating elastic behaviour during the major part of the test.

3. The mean value of the modulus of rigidity from the rectangular sections for a 1:2:4 mix concrete 7 days old was 2.40×10^6 lb. per sq. in.

This value has been obtained in an experiment where the plastic yield and creep effects of concrete were allowed to play little part. In practice these effects will play a big part and just as their effect on E is to reduce its value considerably on N will be similar. Hence for design purposes in practice it is recommended that the value taken for N is 1.2×10^6 lb. per sq. in., i.e. half the value obtained under the elastic test conditions.

4. The torsional strength of plain concrete calculated from Timoshenko's formula based on the elastic theory is much greater for rectangular than for circular sections, an average value being about 500 lb. per sq. in.

In design the factor of safety usually aimed at is four. Based on the experimental results the following suggestion is made for design purposes:

"In the case of shear stresses caused by twisting moment no reinforcement is necessary unless these stresses exceed 125 lb. per sq. in."

5. The value of the torsional stress calculated from the elastic theory shows a tendency to increase as the ratio of depth to breadth of the sides of the rectangle increases

6. Although according to the elastic theory the maximum stress is developed at the centre of the longer side the experiments showed that at failure the stress was almost uniformly distributed along the longer side.

7. It would seem that under torsion concrete behaves elastically up to a certain point (presumably that where the ultimate tension strength is reached) after which it behaves plastically, with a redistribution of stress, until failure occurs.

Part I - (c) Tee and Ell Plain Sections.

1. The general behaviour under test was similar to the other members in plain concrete.

2. The elastic theory for the distribution of stress in irregularly shaped figures subjected to torsion does not hold for concrete members since the failure was in no case caused by excessive stresses being developed at the re-entrant angle.

3. The value of the torque taken by a Tee shaped section can be obtained with reasonable accuracy by adding the torque taken by the two flanges to that taken by the rib and multiplying by a correction factor of $\frac{4A}{PD}$.

4. The value for the torque taken by an Ell section can be obtained with reasonable accuracy by adding the torque taken by half the flange (N.B. not half the torque taken by the flange) to that taken by the rib and multiplying by a similar correction factor to that used for Tee sections.

Part II - Tests on Reinforced Sections.

1. The failure of all classes of reinforced specimens was by the development of spiral tension cracks inclined on a helix at 45 degrees to the axis of twist.

2. Longitudinal bars placed in the corners of the member do not add greatly to its torsional resistance, the increase being about 9 per cent for 1.28 per cent reinforcement.

3. Longitudinal bars placed at the centres of the sides are more efficient but only give an increase of 15 per cent in the torsional resistance for 1.28 per cent reinforcement.

4. The only existing theory for calculating the torsional strength of rectangular members reinforced with longitudinal bars only is useless.

5. Sections when reinforced with longitudinal bars and stirrups and also spirally reinforced sections show three distinct stages under test, firstly, elastic behaviour similar to that of a plain concrete specimen, secondly, a transition period and thirdly, a long period where the angle of twist increases very rapidly with little increase in torque: it is clear that during this latter stage the reinforcement is taking most of the load. With spirally reinforced sections the section is considerably more rigid during this latter stage.

6. The value of the modulus of rigidity during the first stage previously mentioned shows little variation with

different amounts of reinforcement and does not differ substantially from that of a plain section. This fact is suggested as one of the most helpful discoveries of the whole series of tests.

7. During the third stage of the tests it would appear that since the reinforcement cages alone could carry little torque internal forces in the concrete are brought into play which have the effect of stiffening up the section. The exact nature of these internal forces is not known but possibly "strain-energy" methods of analysis may help to solve this part of the problem.

8. For reinforced sections which have an equal percentage of steel lateral and longitudinally the increase in the ratio of the torsional resistance of the reinforced section to that of a plain section is equal to 0.25p, where p is the total percentage of steel, for values of p up to about 2 per cent and that for values of p greater than this the increase is 1.33 + 0.11 p.

It is realised that this formula is very empirical since it only takes into account the total quantity of reinforcement present and assumes that it is divided equally longitudinally and laterally. It takes no account of the positions and sizes of the longitudinal bars, is unaffected by the amount of the cover given to them and takes no account of the spacing of the lateral ties (it seems obvious that the closer

these are spaced the greater their effect must be). It is realised that much more experimental work will have to be done before any really mathematical formula can be given.

9. The spiral is by far the most efficient of reinforcement against torsion being roughly 5 times more efficient than an equal percentage of reinforcement used in the form of longitudinal bars and binders.

10. The formula given by Paul Anderson (15) for the strength of a spirally reinforced section can be applied with reasonable accuracy to a rectangular as well as a square section.

11. The reinforcement in a Tee shaped member is not so efficient as an equal quantity of reinforcement in a rectangular member.

CHAPTER VIII.

APPLICATION TO DESIGN OF BOW GIRDERS IN REINFORCED CONCRETE.

CHAPTER VIII.

Application to design of bow girders in reinforced concrete.

It has already been mentioned in the introduction that one of the objects of this research was to present a method whereby bow girders could be designed in reinforced concrete. It is realised that very little information has been obtained with which to carry out such a design but an attempt will be made to tackle a simple problem of bow girder design.

The problem is as follows:

"Design a bow girder in reinforced concrete to carry a load of 3000 lb. per foot run uniformly distributed on a circular arc subtending an angle of 90 degrees and wit a radius of 10 feet".

The maximum permissible stresses in the concrete and steel are 750 and 18000 lb. per sq. in. respectively and generally the recommendations for a new code of practice will be adopted.

If Fo, Mo and To are the shearing force, bending moment and twisting moment respectively at the ends of the girder then on substituting the value for $\phi = \frac{\pi}{2}$ and $\psi = \frac{\pi}{4}$ in the equations given by Pippard and Baker (21) the following equations are obtained from which Fo, Mo and To can be obtained.

Since the loading is symmetrical

$$Fo = \frac{\text{total load}}{2} = \frac{\omega R \pi}{4}$$

This only leaves two unknowns and to solve these the first and third of the equations given on page 401 of Pippard and Baker will be used, the value $\frac{\pi}{2}$ being substituted for ϕ. This gives

$$Mo\left[\frac{\pi}{EI}+\frac{\pi}{NJ}\right]+To\left[\frac{2}{EI}-\frac{2}{NJ}\right]-\frac{\omega R^2\pi}{4}\left[\frac{2}{EI}+\frac{2}{NJ}\right]$$

$$+\omega R^2\left(\frac{4-\pi}{EI}+\frac{4-\pi}{NJ}\right)=0 \quad \ldots\ldots(1).$$

$$Mo-To-\frac{\omega R^2\pi}{4}\left(\frac{\pi}{2}-1\right)+\omega R^2\left(\frac{\pi^2}{8}-1\right)=0 \quad \ldots\ldots(2).$$

From (2) $Mo = To + \omega R^2\left(1-\frac{\pi}{4}\right)$ .....(3).

Substituting in (1)

$$\pi\left\{To+\frac{\omega R^2}{4}(4-\pi)\right\}\left(\frac{1}{EI}+\frac{1}{NJ}\right)+2To\left(\frac{1}{EI}-\frac{1}{NJ}\right)+\omega R^2\left(4-\frac{3\pi}{2}\right)\left(\frac{1}{EI}+\frac{1}{NJ}\right)=0$$

leading to $To = \frac{\omega R^2}{4}\cdot\frac{(EI+NJ)(\pi^2+2\pi-16)}{(\pi+2)NJ+(\pi-2)EI}$ .....(4).

Substituting in (3) gives

$$Mo = \omega R^2\cdot\frac{(2\pi-6)EI+(\pi-2)NJ}{(\pi+2)NJ+(\pi-2)EI}.$$

It is seen that in order to calculate the values of the bending and twisting moments it is necessary to know the values of EI and NJ. These of course cannot be known until the sizes of the member are known and it will therefore be necessary to estimate this.

Since the beam is rectangular in cross section to design for the economic percentage of steel would not be usual in practice as this would give a very deep beam which would seriously

hinder the problem of headroom. It is suggested that a beam sized 18 inches deep by 10 inches wide is of the type that would be used on such a span and these sizes will be taken.

To calculate the flexural rigidity El of the beam the method recommended in Clause 506 of the new code will be adopted. This method gives El by multiplying E for plain concrete by the moment of inertia of the concrete section about the neutral axis.

In the case under consideration:

Depth of neutral axis

$= .42 \times 16.5 = 6.94$ inches.

Hence $I = \frac{1}{12} \times 10 \times 18^3 + 10 \times 18 \times 2.06^2$.

$= 5625$ in.^{4} units.

$M = 17.78$ hence E concrete $= \frac{30}{17.78} = 1.69 \times 10^6$ lb. per sq. in

Hence $El = 5625 \times 1.69 \times 10^6$

$= 9.5 \times 10^9$.

In calculating NJ use will be made of Timoshenko's formula for rectangular sections (see page 27 and the fact established from the experiments that N for design purposes under conditions where plastic flow, etc. take place can be taken as 1.2×10^6 lb. per sq. in.

$J = .214 \times 10^3 \times 18 = 3.86 \times 10^3$ in.^{4} units.

Therefore $NJ = 4.63 \times 10^9$.

The values Fo, To and Mo can now be evaluated

$$Fo = \frac{\omega R \pi}{4} = \frac{3000 \times 10 \times \pi}{4} = 23{,}600 \text{ lb.}$$

$$To = \frac{\omega R^2}{4}\left[\frac{(El+NJ)(\pi^2 + 2\pi - 16)}{NJ(\pi + 2)+El(\pi - 2)}\right]$$

$$= \frac{1000 \times 10^2}{4}\left[\frac{(4.63+9.5)(9.86+6.28 - 16)}{4.63 \times 5.14 + 9.5 \times 1.14}\right]$$

$$= \frac{3000 \times 10^2}{4} \times \frac{1.98}{34.6}$$

$$= 1.43 \times 10^3 \text{ ft. lb.} = 51{,}600 \text{ in lb.}$$

$$Mo = \omega R^2\left[\frac{El(2\pi - 6)+NJ(\pi - 2)}{NJ(\pi + 2)+El(\pi - 2)}\right]$$

$$= 3000 \times 10^2\left[\frac{9.5 \times .28 + 4.63 \times 1.14}{4.63 \times 5.14 + 9.5 \times 1.14}\right]$$

$$= 3000 \times 10^2 \times \frac{7.9}{34.6}$$

$$= 6.85 \times 10^4 \text{ ft. lb.} = 823{,}000 \text{ in lb.}$$

This is the value of the bending moment at the support and its sign being positive it is, according to the convention adopted by Pippard and Baker(21), a "hogging" moment producing tension in the top of the beam. At mid span the moment will be such as to produce tension in the bottom of the beam. It is suggested that the value of this moment can be obtained as follows:

Free Moment on a span of $10 \times \frac{\pi}{2}$ ft.

$$= 3000 \times 10^2 \times \frac{\pi^2}{4} \times 1.5 = 1{,}010{,}000 \text{ in. lb.}$$

Fixing moment at support = 823,000 in.lb.

Therefore at mid span = 187,000 in. lb.

Thus it is seen that at the support, which is the critical section the beam must be strong enough to resist a bending moment of 823,000 in lb., a torque of 51,600 in lb. and a shear force of 23,600 lb.

The reinforcement required to resist the bending moment and shear force presents little difficulty in calculation.

For bending moment design on "steel beam" theory giving,

$$A_t = A_c = \frac{823,000}{18000 \times 15} = 3.05 \text{ sq. in.}$$

Use 4 - 1 in. diameter = 3.14 sq. in.

At mid span the moment is much less and can be resisted without introducing compression reinforcement

$$A_t = \frac{187,000}{18000 \times .86 \times 16.5} = 0.73 \text{ sq. in.}$$

Use 2 - 1 in. diameter = 1.57 sq. in.

For shear stress due to shear force,

$$\text{Unit shear} = \frac{23,600}{10 \times 16.5 \times .86.} = 157 \text{ lb. per sq.in}$$

Since this is greater than 75 lb. per sq. in. reinforcement will have to be provided. Using $\frac{3}{8}$ in. diameter stirrups in pairs

$$\text{Pitch} = \frac{18000 \times 4 \times .11 \times .86 \times 16.5}{23600} = 4.8 \text{ in.}$$

Use $4\frac{1}{2}$ in. pitch.

The problem of designing the section for the twisting moment has now to be faced and this causes

difficulty because of the difference of opinion between American and British engineers regarding reinforcement required for shear where the unit shear stress exceeds the permissible.British practice states that in such a case reinforcement must be provided to take the whole of the shear force: American practice states that reinforcement must be provided to take the shear force in excess of that taken by the plain concrete. In the case in point the shear stress due to the torque calculated from Timoshenko's formula is

$$\text{Unit shear} = \frac{51{,}600}{.24 \times 10^2 \times 18} = 120 \text{ lb. per sq. in.}$$

It has been suggested (page 162) that the maximum permissible shear in plain concrete subjected to torque is 125 lb. per sq. in, therefore in the case in question no reinforcement is required to enable the plain concrete to resist the twisting moment.

If, however, the British view on the subject is taken then since the shear stress due to the shearing force already exceeds the permissible shear in plain concrete reinforcement must be provided to take all the shear.

In this case stirrups will have to be used to resist a unit shear of 157 + 120 = 277 lb. per sq. in. Using $\frac{3}{8}$ in. diameter stirrups in pairs as before

$$\text{Pitch} = \frac{157}{277} \times 4.8 = 2.74 \text{ inch say 3 inch pitch.}$$

Thus it will be seen that the reinforcement

required by the British system is much more than that required by the American. Details of the beams showing the reinforcement for both systems are given in Figure 20.

The 90 degrees circular arc bow girder is the commonest type met with in concrete construction and the following points of interest arise from the design carried out above.

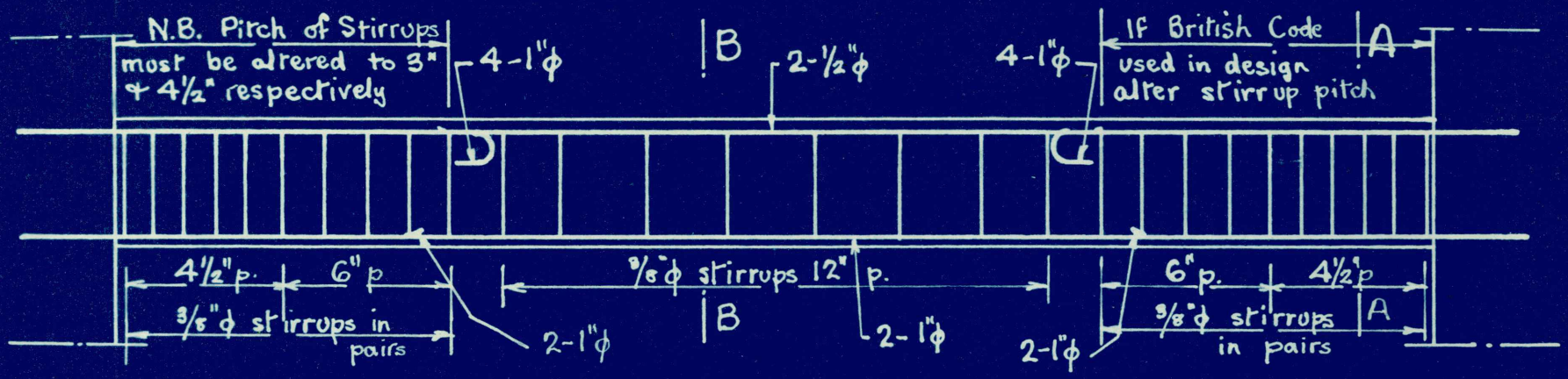
1. From the formula given on page 71 it is seen that the expression for the torque To contains in the numerator the term $\pi^2 + 2\pi - 16$; this expression is very small, hence the twisting moment is small in comparison to the other factors. In the case under consideration its value was within the permissible that could be taken for a plain section. In practice a load of 3000 lb. per ft. run is as much as a beam of the dimensions chosen is likely to be called upon to carry and it is of interest to know that this load produces a torque which can be resisted by the plain section. It must also be remembered that the section taken was a rectangle and in practice the most common section which is used in the design of bow girders is the Ell section which has been shown to be appreciably stronger than the rectangle. Therefore one concludes that in design according to the American code the torque can be neglected provided sufficient shear reinforcement is put in to take the whole of the shear due to the shearing force alone.

2. The bending moment at the ends is approximately equal, for the case under consideration, $\frac{\omega L^2}{11}$ where L = length of arc of the girder. In the expression given for the bending moment on page it is seen that this moment depends on the values of El and NJ. On page 177 a curve is given showing the relationship between El and NJ for varying values of the b:d ratio for the beam. The Author has calculated the value of the bending moment at the end for many values of the El to NJ ratio and has arrived at the interesting fact that the value of the bending moment is the same for all values of this ratio: this fact is of great use in calculating the bending moments at the support and it can be conclusively stated that for all values of El and NJ within practical limits the value of this moment is equal to $\frac{\omega L^2}{11}$ where ω = uniformly distributed load, L = length measured along circular arc.

3. In the calculation for the twisting moment it can be shown that unlike the bending moment this does vary with the relative values of El and NJ. To facilitate work for future calculators of this moment the Author has prepared a graph showing the relationship between the NJ-El ratio and the depth-width of beam ratio. From this table the twisting moment can be calculated as follows. If r = ratio NJ-El then

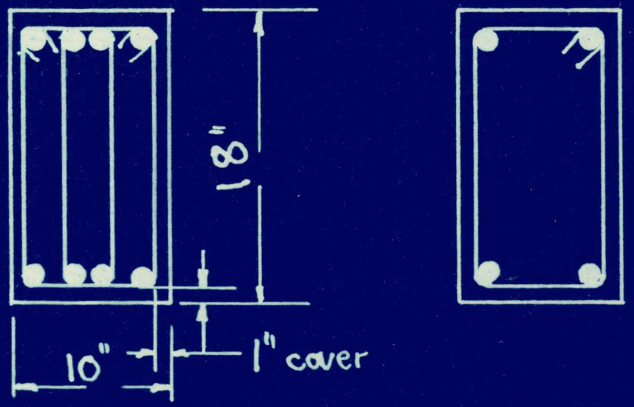
$$To = \frac{(1 + r)(\pi^2 + 2\pi - 16)}{r(\pi + 2) + \pi - 2.} \omega R^2$$

$$= \frac{.14\ (r + 1)}{5.14r + 1.14} \omega R^2$$



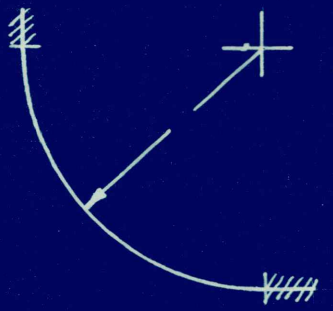
DEVELOPED ELEVATION OF BOW GIRDER

½" to 1'-0"



SECTION A-A SECTION B-B

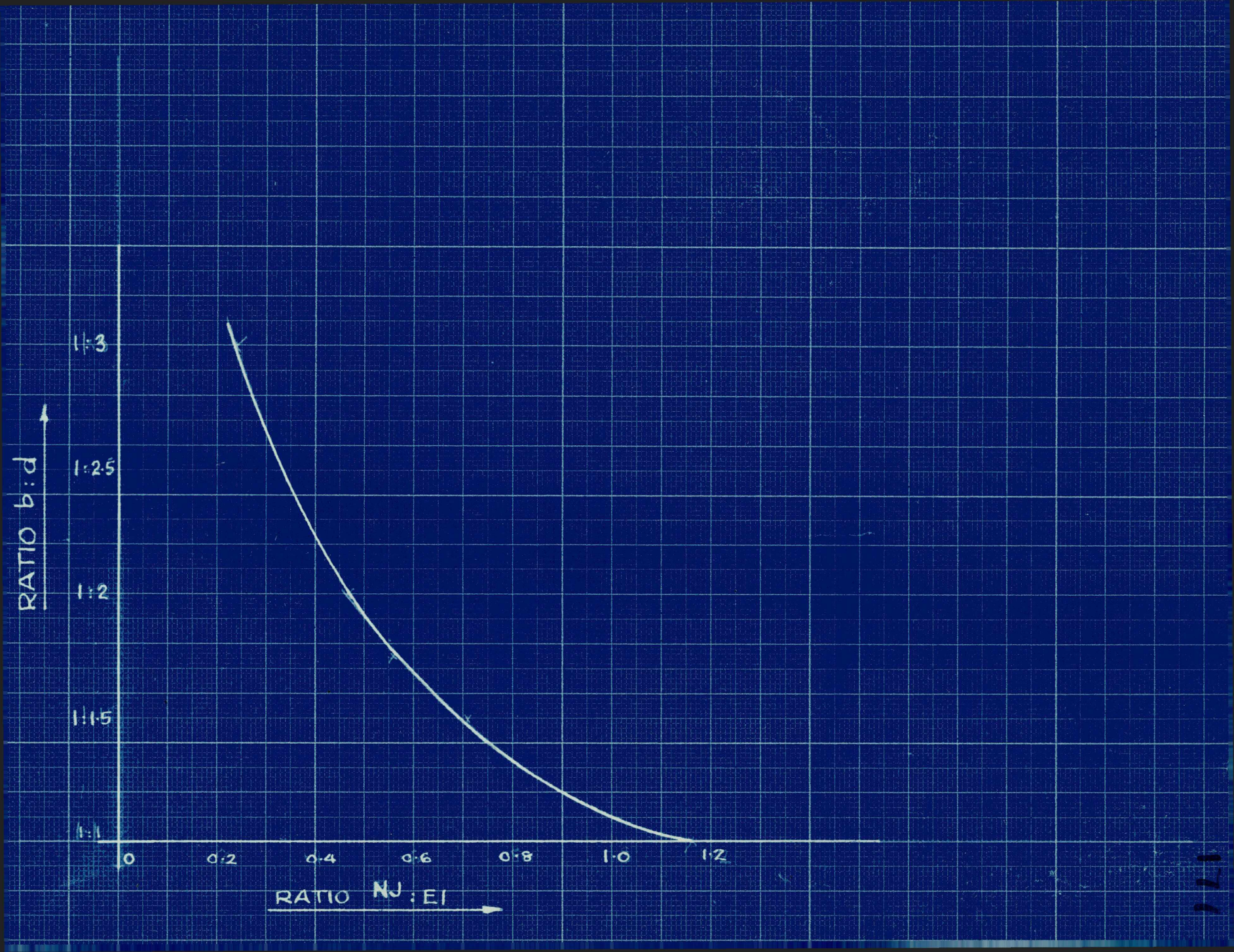
1" to 1'-0"



PLAN

⅛" to 1'-0"

FIG. 20 - DETAILS OF BOW GIRDER


RATIO b:d
1:3
1:2.5
1:2
1:1.5
1:1
0
0·2
0·4
0·6
0·8
1·0
1·2
RATIO NJ : EI

CHAPTER IX.

GEOLOGICAL SIGNIFICANCE OF THE PLAIN CONCRETE TESTS

CHAPTER IX.

GEOLOGICAL SIGNIFICANCE OF THE PLAIN CONCRETE TESTS.

Experimental tests on concrete are of interest to the geologist since concrete is a brittle material and possesses physical characteristics similar to those of certain rocks. Its behaviour under load and in particular under breaking load may therefore throw light on the interpretation of structural phenomena in geoglogy.

In addition to the plain concrete rectangular specimens already mentioned in Chapters IV and V, two special members of size 6 in. x 4 in. were made to assist in the investigation of geological phenomena: these members were made of a mortar consisting of one part by weight of cement to three parts of Stone Court sand thus producing a mortar approximating to a natural sandstone or grit in composition. These specimens were stored in the manner previously described and tested after seven days. Their behaviour under load and at failure was similar to the plain concrete rectangular specimens.

Two features were noticed at failure in all the tests on the plain concrete and mortar rectangular members

1. The production of a tensile crack of roughly spiral form inclined at roughly 45 degrees to the plane of twisting (this has already been commented on).

2. Immediately after failure there was a relative angular movement of the two separated sections of about

$2\frac{1}{2}$ degrees as shown in plate 7.

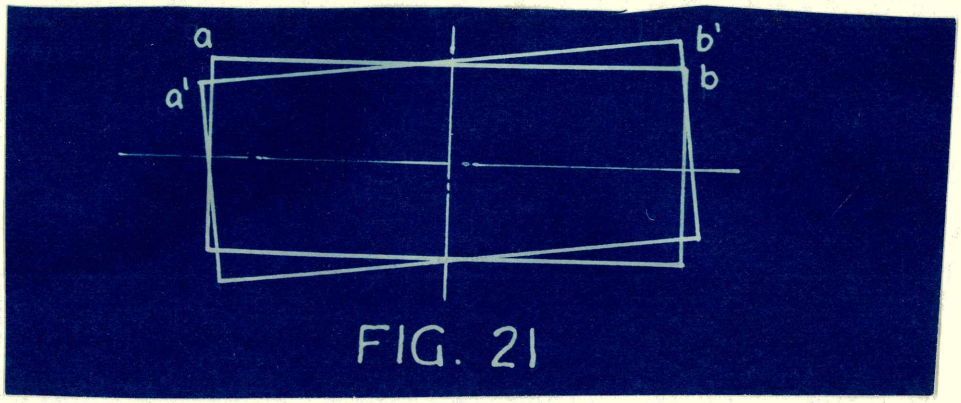


FIG. 21

Figure 21 also shows in end view the type of displacement, the side a-b being moved to a 'b'. The curved fracture-surface is shown on Figure 15, page . It is this relative angular movement at failure which makes the tests of interest to the geologist.

The surface of the fracture was not normal to the face but had a varying inclination and sloped in opposite directions on either side of the centre line of the face (see Figure 15). Thus in a broken member each of the fracture-surfaces on account of its curvature consisted of a "hanging wall" of one side of the axis of torsion and a "footwall" on the other side to use the terminology applied to faults. When fracture took place the "hanging" wall was in each case raised with respect to the "footwall" thus producing a reversed fault.

The concrete broke in tension but owing to the fact that this was produced by torsional loading, the separation of the two parts involved the motion described above which would correspond to that of a pivotal reversed fault if it occurred under geological conditions.

In text books of geology, reversed faulting has generally been attributed to compressive forces. A brittle stratum or series of strata in nature however might be subjected to torsional loading, for example, through unequal settlement over an area; the torsional axis in this case would be horizontal (or nearly so). It is suggested that such a loaded stratum would then be likely to break in tension in a similar way to that seen in the tests described

The Author in conjunction with Mr. F.G.H. Blyth, M.Sc. exhibited plain concrete beams which had been subjected to torsion and which showed the failure described at the Royal Society Conversatione in May 1938, and the exhibit aroused considerable interest amongst geologists. A brief description of the phenomena were also published in Nature, Volume 143, No.3612.



Plate 7. Angular Movement at Failure

APPENDIX.I

DESIGN OF THE TEST MACHINE.

APPENDIX. I

Design of the Test Machine.

It is proposed in this appendix to give the calculations prepared for the design of the torsion machine described on pages 68 onward.

The various components of the machine will be dealt with in order. Except where otherwise mentioned the parts were made from ordinary mild steel and the following values were taken for the working stresses.

In bending and direct tension - 7.5 tons per sq. inch.

In shear - 5 tons per sq. inch.

For description and reference to the various parts see Plates 1 and 2.

Spring Balance "S"

This was designed to read up to two tons. This gave the maximum load which could be applied through the turnbuckle. Hence the maximum torque which could be applied was 10.25 x 4480 = 46,000 in lb.

Turnbuckle (High tensile steel safe stress = 15 tons per sq. inch).

Least Diameter of rod = $\frac{3}{8}$ inch.

Load = 2 tons.

Therefore Stress = $\frac{2}{.11}$ = 18.1 tons per sq. inch.

This is slightly in excess of the safe working stress but since the material will always be in tension and no reversal of stress will take place a stress greater than the permissible can be permitted.

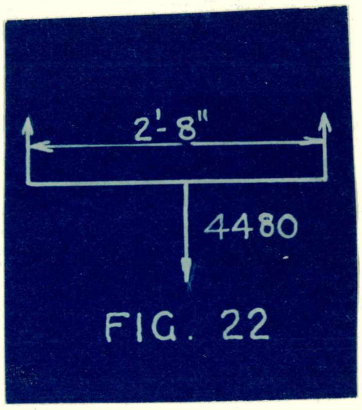


FIG. 22

Channels "L".

For loading see Figure 22.

$M = \frac{WL}{4} = 4480 \times 2.67 \times 3.$

$= 35800$ in. lb.

$M = fZ$

Therefore $Z = \frac{35800}{7.5 \times 2240} = 2.14$ in ^{3} units.

Section Modulus of 2 - 5 in. x $2\frac{1}{2}$ in. x 10.22 lb. channels $= 2 \times 4.749$

$= 9.498$ in ^{3} units.

Since this is less than the 2.14 in ^{3} units required the design is safe.

Rapes transferring load

Load = 1 ton.

Area required $= \frac{1}{7.5} = .133$ square inch.

Area provided $= \frac{\pi}{4} \times (\frac{1}{2})^2 = .196$ square inch.

Shaft to which wheel F is attached.

Torque to be transmitted = 46,000 in. lb.

Net diameter allowing for keyway = 2 inches.

Shear stress $q = \frac{16\ T}{\pi\ d^3}$

$= \frac{16 \times 46000}{\pi \times 2.00^3}$

$= 29300$ lb. per sq. inch.

This of course is well in excess of the permissible shear stress but is below the elastic limit of stress. Unfortunately it was impossible to increase the

diameter of the shaft without adding considerably to the cost of the machine and as this was not desired it was decided to leave the 2 inch diameter shaft and cut down the factor of safety by about 50 per cent.

Rotating Head E.

The dimensions of this head were as shown in Figure 12.

It virtually consists of a square of sides $9\frac{3}{4}$ inches from which a square of sides $8\frac{1}{4}$ inches has been cut.

By the use of Timoshenko's formula

$T_1 = .208 \times 5 \times 2240 \times 9.75^3$

$T_2 = .208 \times 5 \times 2240 \times 8.25^3$

Therefore Torque resisted by section

$= .208 \times 5 \times 2240 \times (9.75^3 - 8.25^3)$

$= 842{,}000$ in. lb.

This greatly exceeds the applied torque, hence the section is safe.

Fixed Head C.

The dimensions of this head were as shown in Figure 12.

In the design it was assumed that the torque on the head would have to be carried as a bending moment in the joist. The 10 inch dimension of the joist was fixed from two considerations,

1. The width of the lathe bed.
2. The size of the specimens to be inserted in the chuck.

Applied M = 46,000 in. lb.

Section Modulus of 10in. x 8in. x 55 lb. B.S.B. = 57.74.

Therefore Stress = $\frac{46,000}{57.74}$ = 800 lb. per sq. inch, which is safe.

It was further assumed that the torque was transmitted to the joist by means of direct tension and compression on the 4 bolts at the top of the machine and then by shear on the 4 bolts at the side.

The net cross section of the bolt below the thread was approximately $\frac{5}{8}$ inch diameter.

Bolts in tension and compression.

Distance centre to centre = $13\frac{1}{2}$ inches.

Area of bolts = $2 \times \frac{\pi}{4} \times (\frac{5}{8})^2$

Stress in bolt = $\frac{46,000}{13.5 \times .60.}$

= 5,700 lb. per sq. inch, which is safe

Bolts in shear.

Distance centre to centre = 10 inches.

Area of bolts = .60 sq. inch.

Stress in bolt = $\frac{46,000}{10 \times .6}$

= 7,680 lb. per sq. inch, which is saf

Appendix 2 - Analysis by Strain - Energy Methods.

In this analysis the reinforcement is assumed to be stiffened by bands of concrete of infinite stiffness as shown in fig. 19.

Since such a structure is redundant, it is necessary to apply firstly Castigliano's 2nd theorem to calculate the loads in the members and then his 1st theorem can be applied to calculate the torsional rigidity.

The analysis will be worked out for the general case of a member of length L having (n-1) intermediate binders. The area of each of the main bars is A, and that of the binders is "a". The member is b by d in size.

The loads in the members are as shown in fig.23 and the loads, partial differential coefficients and products for strain energy calculations for the redundant member are given in Table S1.

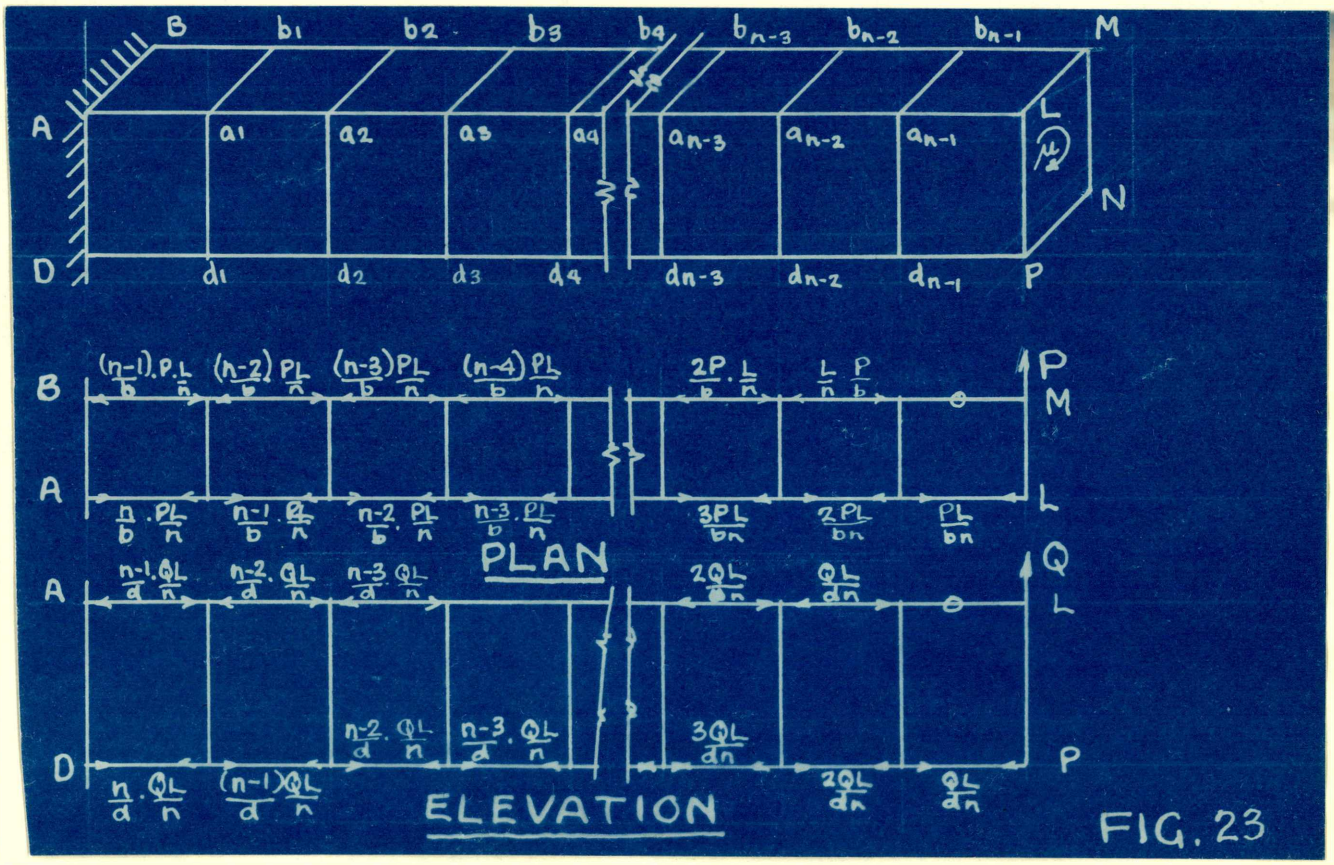


FIG. 23

TABLE SI

	Member	Length	Area	Load = P_o	$\partial P_o/\partial P$	$P_o\,\partial P_o/\partial P$
Main bars	$A a_1$	$\frac{L}{n}$	A	$\frac{(n-1)L}{ndb}(\mu - Pd) - \frac{nL}{nb}\cdot P$	$-\{n+(n-1)\}\frac{L}{nb}$	$(2n-1)\frac{L}{nb}\left\{\frac{(2n-1)L}{nb}P - \frac{(n-1)L}{ndb}\mu\right\}$
	$a_1 a_2$	"	"	$\frac{(n-2)L}{ndb}(\mu - Pd) - \frac{(n-1)L}{nb}\cdot P$	$-\{(n-1)+(n-2)\}\frac{L}{nb}$	$(2n-3)\frac{L}{nb}\left\{\frac{(2n-3)L}{nb}P - \frac{(n-2)L}{ndb}\mu\right\}$
	$a_{n-2} a_{n-1}$	"	"	$\frac{L}{ndb}(\mu - Pd) - \frac{2L}{nb}\cdot P$	$-\{1+2\}\frac{L}{nb}$	$3\frac{L}{nb}\left\{\frac{3LP}{nb} - \frac{L}{ndb}\mu\right\}$
	$a_{n-1} L$	"	"	$-\frac{L}{nb}\cdot P$	$-\frac{L}{nb}$	$\left(\frac{L}{nb}\right)^2 P$
	$B b_1$	$\frac{L}{n}$	A	$\frac{(n-1)L}{bn}\cdot P - \frac{nL}{nbd}(\mu - Pd)$	$\left\{\frac{(n-1)L}{bn} + \frac{nLd}{nbd}\right\}$	$(2n-1)\frac{L}{nb}\left\{\frac{(2n-1)L}{nb}P - \frac{nL}{nbd}\mu\right\}$
	$b_1 b_2$	"	"	$\frac{(n-2)L}{bn}P - \frac{(n-1)L}{nbd}(\mu - Pd)$	$\{(n-2)+(n-1)\}\frac{L}{nb}$	$(2n-3)\frac{L}{nb}\left\{\frac{(2n-3)L}{nb}P - \frac{(n-1)L}{nbd}\mu\right\}$
	$b_{n-2} b_{n-1}$	"	"	$\frac{L}{bn}\cdot P - \frac{2L}{nbd}(\mu - Pd)$	$(1+2)\frac{L}{nb}$	$\frac{3L}{nb}\left\{\frac{3}{nb}PL - \frac{2L}{nbd}\mu\right\}$
	$b_{n-1} M$	"	"	$-\frac{L}{nbd}(\mu - Pd)$	$\frac{L}{nb}$	$\frac{L}{nb}\left\{\frac{L}{nb}P - \frac{L}{nbd}\mu\right\}$
Binders	$a_1 d_1$	d	a	$-\frac{1}{b}(\mu - Pd)$	$+\frac{d}{b}$	$\frac{\partial U}{\partial P} = \frac{L}{AE}\sum P_o\frac{\partial P_o}{\partial P}$ $= \frac{d}{aE}\cdot\frac{nd}{b^2}(Pd-\mu)$
	$a_2 d_2$	"	"	do.	$+\frac{d}{b}$	
	$a_{n-1} d_{n-1}$	"	"	do.	$+\frac{d}{b}$	
	LP	"	"	do.	$+\frac{d}{b}$	
	$b_1 a_1$	b	a	$-P$	-1	$\frac{\partial U}{\partial P} = \frac{b}{aE}\cdot nP$
	$b_2 a_2$	"	"	$-P$	-1	
	$b_{n-1} a_{n-1}$	"	"	$-P$	-1	
	ML	"	"	$-P$	-1	

To calculate the loads in the members in terms of the applied torque put $\frac{\partial U}{\partial P} = 0$ thus giving a value for P.

For the main bars $\frac{\partial U}{\partial P} \quad \frac{L}{AE}\{P_o \frac{\partial P_o}{\partial P}$

$$= \frac{L}{nAE}\left[\frac{L^2}{n^2b^2}\; P\left\{(2n-1)^2 + (2n-3)^2 \ldots\ldots\ldots + 3^2 + 1^2\right\}\right]$$

$$- \frac{L}{nAE}\left[\frac{L^2\mu}{n^2b^2d}\left\{(2n-1)(n-1) + (2n-3)(n-2) \ldots\ldots 5.2 + 3.1\right\}\right]$$

$$+ \frac{L}{nAE}\left[\frac{L^2}{n^2b^2}\; P\left\{(2n-1)^2 + (2n-3)^2 \ldots\ldots\ldots\ldots + 3^2 + 1^2\right\}\right]$$

$$- \frac{L}{nAE}\left[\frac{L^2\mu}{n^2b^2d}\left\{(2n-1)n + (2n-3)(n-1) \ldots\ldots\ldots + 3.2 + 1.1\right\}\right]$$

$$\simeq \frac{L^3P}{n^3b^2AE}\left\{\frac{2n}{3}(4n^2-1)\right\} - \frac{L^3\mu}{n^3b^2dAE}\left\{(2n-1)(2n-1) + (2n-3) + 3^2 + 1^2\right\}$$

$$= \frac{L^3P}{n^3b^2AE} \cdot \frac{2n}{3}(4n^2-1) - \frac{L^3\mu}{n^3b^2dAE} \cdot \frac{n}{3} \cdot (4n^2-1)$$

For the binders:-

$$\frac{\partial U}{\partial P} = \frac{nd^2}{b^2aE}(Pd - \mu) + \frac{b}{aE}\, nP$$

Adding the partial differentials for main bars and binders and equating their sum to zero the value for P is obtained and that for Q can be obtained by substitution. The algebra is not given here but it is found that

$$P = \frac{\mu}{d}\left\{\frac{4n^2aL^3 - aL^3 + 3n^3d^3A}{8an^2L^3 - 2aL^3 + 3n^3d^3A + 3n^3b^3A}\right\} \quad \text{and}$$

$$Q = \frac{\mu}{d}\left(\frac{4n^2aL^3 - aL^3 + 3n^3b^3A}{8an^2L^3 - 2aL^3 + 3n^3d^3A + 3n^3b^3A}\right)$$ or by writing

$P = 4n^2aL^3 - aL^3 + 3n^3d^3A$

$q = 4n^2aL^3 - aL^3 + 3n^3b^3A$

$s = 8an^2L^3 - 2aL^3 + 3n^3d^3A + 3n^3b^3A = p - q$

The values of P and Q may be written

$$P = \frac{\mu P}{ds} \quad \text{and} \quad Q = \frac{\mu q}{bs}$$

The loads in the members can now be expressed in terms of μ (this is given in Table s2 together with partial differentials etc)

Then $\frac{\delta U}{\delta \mu}$

and $$N\surd = \frac{\mu L}{\theta} = \frac{\mu L}{\frac{\delta U}{\delta \mu}}$$

The algebra involved in the summation of the various series given in Table 2 is somewhat lengthy and it is not given here. On calculating the value for θ and dividing μL by this value it is found that the torsional rigidity N is given by the following expression

$$N\surd = \frac{Eb^2d^2s^2L}{\lambda\left[\frac{L^3}{An^2}\left\{\frac{p^2+q^2}{3}(2n^2+1) - \frac{2pq}{3}(2n^2-2)\right\} + \frac{n}{a}(b^3p^2 + d^3q^2)\right]}$$

where p,q and s have the values given on page

These general formulae will now be applied to the particular cases under consideration and calculations given for,-

1) The torsional rigidity of the different specimens and,

2) The loads in the members

TABLE S2

	Member	Length	Area	Load P	Load Q	Load in terms of $\mu = P_o$	$\partial P_o / \partial \mu$
Main bars	Aa_1	$\frac{L}{n}$	A	$-\frac{nL}{nb}$	$\frac{n-1}{n}\cdot\frac{L}{d}$	$\frac{\mu L}{n}\left(-\frac{np}{bds}+\frac{(n-1)q}{bds}\right)$	$\frac{L}{n}\left\{\frac{(n-1)q-np}{bds}\right\}$
	a_1a_2	"	"	$-\frac{(n-1)L}{nb}$	$\frac{n-2}{n}\cdot\frac{L}{d}$	$\frac{\mu L}{n}\left(-\frac{(n-1)p}{bds}+\frac{(n-2)q}{bds}\right)$	$\frac{L}{n}\left\{\frac{(n-2)q-(n-1)p}{bds}\right\}$
	$a_{n-2}a_{n-1}$	"	"	$-\frac{2L}{nb}$	$\frac{1}{n}\cdot\frac{L}{d}$	$\frac{\mu L}{n}\left(-\frac{2p}{bds}+\frac{q}{bds}\right)$	$\frac{L}{n}\left\{\frac{q-2p}{bds}\right\}$
	$a_{n-1}L$	"	"	$-\frac{L}{nb}$	—	$\frac{\mu L}{n}\left(-\frac{p}{bds}\right)$	$\frac{L}{n}\cdot\frac{-p}{bds}$
	Bb_1	$\frac{L}{n}$	A	$\frac{(n-1)}{bn}L$	$-\frac{nL}{nd}$	$\frac{\mu L}{n}\left(\frac{(n-1)p}{bds}-\frac{nq}{bds}\right)$	$\frac{L}{n}\left\{\frac{(n-1)p-nq}{bds}\right\}$
	b_1b_2	"	"	$\frac{(n-2)}{bn}L$	$-\frac{(n-1)L}{nd}$	$\frac{\mu L}{n}\left(\frac{(n-2)p}{bds}-\frac{(n-1)q}{bds}\right)$	$\frac{L}{n}\left\{\frac{(n-2)p-(n-1)q}{bds}\right\}$
	$b_{n-2}b_{n-1}$	"	"	$\frac{L}{bn}$	$-\frac{2L}{nd}$	$\frac{\mu L}{n}\left(\frac{p}{dbs}-\frac{2q}{bds}\right)$	$\frac{L}{n}\left(\frac{p-2q}{bds}\right)$
	$b_{n-1}M$	"	"	—	$-\frac{L}{nd}$	$\frac{\mu L}{n}\left(-\frac{q}{bds}\right)$	$\frac{L}{n}\left(\frac{-q}{bds}\right)$
Binders	a_1d_1	d	a		-1	$-\mu\left(\frac{q}{bs}\right)$	$\frac{\partial P_o}{\partial \mu}=-\frac{nq}{bs}$
	a_2d_2	"	"		-1	$-\mu\left(\frac{q}{bs}\right)$	
	$a_{n-1}d_{n-1}$	"	"		-1	$-\mu\left(\frac{q}{bs}\right)$	
	LP	"	"		-1	$-\mu\left(\frac{q}{bs}\right)$	
	b_1a_1	b	a	-1		$-\mu\left(\frac{p}{ds}\right)$	$\frac{\partial P_o}{\partial \mu}=-\frac{np}{ds}$
	b_2a_2	"	"	-1		$-\mu\left(\frac{p}{ds}\right)$	
	$b_{n-1}a_{n-1}$	"	"	-1		$-\mu\left(\frac{p}{ds}\right)$	
	ML	"	"	-1		$-\mu\left(\frac{p}{ds}\right)$	

Torsional - Rigidity of the Specimens.

Series D:- The values for this series are,

$b = 2.38"$: $d = 4.38"$: $n = 17$: $L = 43.5"$

$a = .049$ sq.in. $A = .076$ sq. in.

These values give

$p = 4.763 \times 10^6$: $q = 4.685 \times 10^6$: $s = 9.448 \times 10^6$

On evaluating the expression given it is found that

$N\surd = 8.31 \times 10^6$ lb/sq.in.

The lowest experimental value obtained for $N\surd$ was 18×10^6

Series E:- The values for this series are

$b = 2.38"$: $d = 4.38"$: $n = 14$: $L = 41"$

$a = .11$ sq. in. $A = .15$ sq.in.

These values give

$P = 6.07 \times 10^6$: $q = 5.983 \times 10^6$: $s = 12.05 \times 10^6$

On evaluating it is found that

$N\surd = 21.6 \times 10^6$ lb/sq.in.

The lowest experimental value obtained for $N\surd$ was 21.0×10^6 lb/sq. in.

Series F:- The values for this series are

$b = 2.38"$: $d = 4.38"$: $n = 16$: $L = 44.5"$

$a = .076$ sq. in. $A = .11$ sq.in.

These values give

$P = 6.99 \times 10^6$: $q = 6.914 \times 10^6$: $s = 13.91 \times 10^6$

On evaluating it is found that

$N\surd = 14.1 \times 10^6$ lb/sq.in.

The lowest experimental value obtained for $N\surd$ was 20×10^6 lb/sq. in.

Series G:- The values for this series are

$b = 2.38"$: $d = 4.38"$: $n = 21$: $L = 46"$

$a = \cdot 027$ sq. in. $A = \cdot 049$ sq.in.

These values give

$p = 4.746 \times 10^6$: $q = 4.65 \times 10^6$: $s = 9.397 \times 10^6$

On evaluating it is found that

$M = 3.78 \times 10^6$ lb/sq.in.

The lowest experimental value obtained for M was 19×10^6 lb/sq. in.

Loads in the Members:- (see Fig.23)

The loads in the binders are either P or Q, where

$$P = \frac{\mu}{d} \cdot \frac{p}{s} \quad \text{and} \quad Q = \frac{\mu}{b} \cdot \frac{q}{s}$$

The maximum load in the main bar occurs at the far end and can be shown to have the value of,

$$\frac{\mu L\,(nq - np - q)}{ndb\,(p + q)} \quad \text{or} \quad \frac{\mu L\,(np - nq - p)}{ndb\,(p + q)}$$

In the diagonal member the load is either

$$\frac{\sqrt{\left(\frac{L}{n}\right)^2 + b^2}}{b} \times P \quad \text{or} \quad \frac{\sqrt{\left(\frac{L}{n}\right)^2 + d^2}}{d} \times Q$$

These expressions will be used to calculate the stresses in the reinforcement in the different types of specimens used. The arithmetic work is not given for any case.

Series D:-

Ultimate Torque = 17,300 lb.ins.

The values of p,q,and s have been already given.

Binders:- Minimum load = 1988 lb.

Maximum load = 3,600 lb.

Maximum tensile stress = 73,500 lb/sq.in.

Series D:(contd)

Main bars Maximum load = 2700 lb.

Maximum tensile stress = 35,600 lb/sq.in.

Diagonals Maximum load = 4170 lb.

Load x sin (θ - 45) = 1010 lb.

Series E:-

Ultimate Torque = 19,500 lb.ins.

The values of p,q,and s have been already given.

Binders Maximum load = 4060 lb

Maximum tensile stress = 37,100 lb/sq.in.

Main bars Maximum load = 3210 lb.

Maximum tensile stress = 21,300 lb/sq.in.

Diagonals Maximum load = 4870 lb.

Load x sin (θ - 45) = 933 lb

Series F:- Ultimate Torque = 17,700 lb.ins.

The values of p,q,and s have been already given.

Binders Maximum load = 3680 lb.

Maximum stress = 48,800 lb./sq.in.

Main bars Maximum load = 2810 lb.

Maximum stress = 25,600 lb./sq.in.

Diagonals Maximum load = 4390 lb.

Load x sin (θ - 45) = 896 lb
