

Pulsed Quantum Light from Spontaneous Parametric Down Conversion in a Triply Resonant Cavity

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Declaration of Originality

I declare that I wrote this thesis entirely on my own and that it has not been previously submitted, in whole or in part, with any other application for a degree. The work presented is completely my own work, except if specifically stated by reference or acknowledgement.

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Abstract

Both single photons and squeezed light are forms of quantum light which offer a wide range of possible applications, from testing fundamental quantum theories to encoding quantum information for use in computing and communication. Of particular significance is their use within the fast developing field of photonic quantum computing, which uses photons to perform quantum algorithms and solve specific problems that are computationally difficult with classical computers.

The target of our research is to generate a highly-squeezed, spectrally factorisable quantum state. Our method is based upon parametric down conversion (PDC), a common technique in quantum optics because of its ability to operate at room temperature and its robustness in experiments involving a high power pump laser. Using PDC to prepare squeezed states often results in quantum states which are mixed, containing significant frequency correlations, or providing low levels of squeezing. We mitigate these dilemmas by introducing a pulsed pump laser and a cavity that is resonant with all the fields involved in the PDC process. Together these can improve both the spectral purity and degree of squeezing experienced by the quantum state.

This thesis presents an experimental demonstration of quantum state generation based on PDC inside a triply resonant cavity, using a pulsed pump. An initial theoretical proposal is outlined, which later motivates a description of the cavity design. The cavity tuning process is then described in detail, leading to a discussion of experimental findings. The well-tuned cavity is employed to observe nonlinear effects with a continuous-wave pump. Finally, a pulsed pump system is implemented to generate spectrally-uncorrelated photon pairs.

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Chapter 1

Introduction

Quantum computing is an exciting research area, with the potential to solve certain problems that are difficult for classical computers, such as large number factorisation, the hidden subgroup problem as well as unstructured search problems. Approaches to building a quantum computer have been explored on a number of different platforms including superconducting circuits [1–5], trapped ions [6–10], and photonics [11–15]. Each platform offers its own advantages and disadvantages. For instance, superconducting circuits are a promising platform, due to the fast quantum operation times and the ability to take advantage of modern semiconductor fabrication techniques. However, the decoherence time of superconducting qubits is relatively short compared with other platforms [4]. The trapped ion system employs the energy levels of a trapped ion to form a qubit. The use of this platform enables the preparation of the qubits and operation of single qubit rotations with high fidelity [8, 9]. A key disadvantage is the fact that, inside any given trap, only a limited number of qubits can be contained, which are difficult to interact with qubits from separate traps.

The photonics system is a promising candidate for realising quantum computing. A photon is a boson that can be described as a quantum of the electromagnetic field. In 2001, Knill et al. [16] proposed a system based on linear optical quantum computing (LOQC) which employed single photon sources, linear optical components, and photon detectors to efficiently process quantum information. While there are promising alternative photonic implementations outside of LOQC, this proposal was significant for opening the door to further research in pursuit of universal quantum computation. In LOQC, qubits are typically encoded in the degrees of freedom of single photons. Optical components, many of which are readily available and robust at room temperature, can be used to perform quantum operations. Photon detectors can be used to measure quantum information, with photon-number resolving systems offering the possibility to implement complex quantum protocols.

The photonics platform has many advantages including a long decoherence time which indicates a photonics system is more stable than alternative platforms. On a practical level, it has the capacity to operate at room temperature and integrate into existing fibre optic infrastructure, reducing the cost

to operate.

A key challenge for photonic quantum computation is to precisely prepare the necessary quantum states of light. This can be illustrated by the simple case of single photons, as in the initial LOQC proposal, but as we will outline later, similar challenges are faced by other approaches such as squeezed states.

Single photons need to be indistinguishable in order to interact with each other. An ideal single photon source generates single photons deterministically with high collection efficiency. A single photon is conventionally denoted as $|1\rangle_k$, where k is the mode of the field. In reality, single photon sources often generate a mixture of vacuum, single photon, and multi-photon states. Even though in the Fock basis a single photon state means that there is only one photon in the system, other degrees of freedom, such as frequency and polarisation, have to be considered when attempting to generate single photon states. Similarly, squeezed states, which are a different form of quantum light, generate photon pairs that may have multiple frequency modes. The multi-frequency contribution reduces the chance of interference between squeezed light generated at different times. A single frequency mode photon pair is desired to perform high fidelity quantum gates to conduct meaningful computation.

In this chapter, we will firstly introduce quantum light with non-classical behaviour. Then we focus on the importance of squeezed states in continuous-variable quantum information processing. This leads to the motivation for generating high quality squeezed states that can be applied in continuous-variable quantum teleportation and cluster states. To understand twin-beam squeezed light generation, we will start by introducing the mathematical expression for the two-mode squeezed vacuum, which generates a single mode photon pair. Then we analyse the spectral properties for multiple spectral modes of photon pairs. We also introduce some approaches used in the thesis to characterise squeezed state sources. Finally, we provide the structure of the thesis.

1.1 Quantum light

Classical light can be described using the solution of Maxwell's equations. An example of classical light is a laser, which is a coherent state described via the solutions of a classical harmonic oscillator. A coherent state has a Poissonian photon number distribution as each photon from the laser is independently emitted. A coherent state is also a minimum uncertainty state, which means the product of its quadrature variances is minimal. In its quantum description, a coherent state is a displaced vacuum state with equal quadrature in position and momentum with a non-zero average photon number.

Quantum light is the type of light that only quantum theories, not classical solutions, can adequately describe. Quantum light has some special properties that have application potentials in quantum metrology, quantum communication, and quantum information processing. In this section, two behaviours of quantum light are outlined: quadrature squeezing and the photon anti-bunching

effect. Together these offer two unique descriptions of quantum light, one highlighting wave-like behaviour, while the other describing particle-like behaviour.

1.1.1 Squeezed states

Quadrature operators play an essential role in the phase-space representation of quantum mechanics. To define the quadrature operators, we start with an optical mode. A monochromatic optical mode is equivalent to a harmonic oscillator of unit mass. The Hamiltonian of a harmonic oscillator at frequency ω is written with the creation operator $\hat{a}^\dagger$ and the annihilation operator $\hat{a}$ as

$$\hat{H} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \quad (1.1)$$

where both $\hat{a}^\dagger$ and $\hat{a}$ are dimensionless operators and they satisfy the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$. The quadrature operators are defined with $\hat{a}^\dagger$ and $\hat{a}$ as

$$\hat{X} = \frac{\hat{a} + \hat{a}^\dagger}{\sqrt{2}} \quad (1.2)$$

and

$$\hat{P} = \frac{\hat{a} - \hat{a}^\dagger}{\sqrt{2}i} \quad (1.3)$$

where $\hat{X}$ is the position quadrature and $\hat{P}$ is the momentum quadrature.

The quadrature standard deviations of any state satisfy the Heisenberg uncertainty principle: the product of any quantum system's conjugate quadrature standard deviations is larger or equal to $\hbar/2$. It can be written in natural units when $\hbar = 1$ as

$$\Delta\hat{X}\Delta\hat{P} \geq \frac{1}{2} \quad (1.4)$$

where $\Delta\hat{X}$ and $\Delta\hat{P}$ are position and momentum quadrature standard deviations.

Among classical states, a vacuum state and a coherent state (which is a displaced vacuum state) have the smallest (joint) quadrature variances that satisfy Heisenberg uncertainty principle. This type of state is known as a minimum uncertainty state. Vacuum states and coherent states have minimal and balanced quadrature variances whose product is equal to $1/4$, which means $\Delta\hat{X}^2 = \Delta\hat{P}^2 = \frac{1}{2}$. When a measurement is made for the quadrature of a classical state, the uncertainty is constrained to be greater than or equal to the vacuum quadrature variance. Squeezed states are a type of quantum state that have a quadrature variance smaller than the vacuum quadrature variance. Due to the Heisenberg uncertainty principle, squeezed states have unequal quadrature variances. For a squeezed state, we can use the squeezing parameter ζ to describe the degree of squeezing based on its unequal quadrature variances. The greater the imbalance, the greater the degree of squeezing. The squeezing

parameter is given through the ratio between its smaller quadrature variance (for example, $\Delta\hat{X}^2$) and vacuum quadrature variance as $\zeta = \frac{\Delta\hat{X}^2}{\Delta\hat{X}_{\text{vac}}^2}$. The degree of squeezing is often quantified in decibel (dB) units in literature. A decibel expression for the squeezing parameter is $-10 \times \log_{10}(\zeta)$.

To visualise the squeezing effect, we introduce the Wigner function [17] as a convenient representation for studying quantum systems. The Wigner function is a quasi-probability distribution that links the wave function to its phase space probability. It can be used to calculate expectation values in a way similar to classical probability distributions. A crucial distinction is that the Wigner function can take on negative values for certain non-classical states.

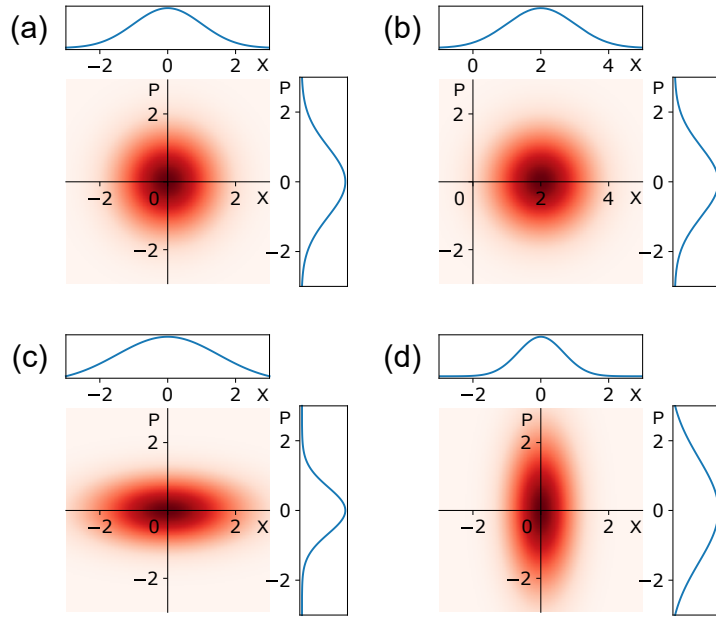


Figure 1.1: Examples of the Wigner function for different states [18]. (a) Vacuum state. (b) Coherent state. (c) Squeezed vacuum state with position quadrature squeezing. (d) Squeezed vacuum state with momentum quadrature squeezing.

Fig. 1.1 provides examples of the Wigner function for different types of states. A vacuum state is a classical state whose Wigner function is symmetric and has uncorrelated position and momentum quadratures shown in Fig. 1.1(a). A coherent state has its Wigner function shown in Fig. 1.1(b) which is similar to that of a vacuum state but displaced. A squeezed state has a Wigner function with unequal variances and presents a smaller variance on one quadrature than vacuum quadrature variances. Fig. 1.1(c) and (d) are examples of Wigner function for squeezed states.

1.1.2 Photon anti-bunching

Photon number statistics provides a particle perspective on quantum systems. The second-order correlation $g^{(2)}(\tau)$ is an important tool to evaluate the photon number statistics of a quantum state.

It is defined using the electric field operator $\hat{E}$ or the intensity operator of the light $\hat{I}$ [19]:

$$g^{(2)}(\tau) = \frac{\langle \hat{E}^-(t) \hat{E}^-(t+\tau) \hat{E}^+(t+\tau) \hat{E}^+(t) \rangle}{\langle \hat{E}^-(t) \hat{E}^+(t) \rangle \langle \hat{E}^-(t+\tau) \hat{E}^+(t+\tau) \rangle} = \frac{\langle \hat{I}(t) \hat{I}(t+\tau) \rangle}{\langle \hat{I}(t) \rangle \langle \hat{I}(t+\tau) \rangle}. \quad (1.5)$$

At zero time delay $\tau = 0$, the second order correlation is simplified as

$$g^{(2)}(0) = \frac{\langle \hat{I}(t)^2 \rangle}{\langle \hat{I}(t) \rangle^2}. \quad (1.6)$$

By analysing the second order correlations, it is possible to infer information about the photon number distribution, which is the statistical distribution from photon counting experiments. There are three categories of quantum state which can be distinguished through a measurement of $g^{(2)}(0)$. The first instance is $g^{(2)}(0) > 1$, which indicates the light source has a photon number of super-Poissonian distribution. This is referred to as photon bunching. A thermal state is an example of photon bunching, when the photons usually arrive at the photodetectors in bunches as opposed to randomly and independently.

The second scenario is $g^{(2)}(0) = 1$, which corresponds to a Poissonian photon number distribution. The Poissonian distribution has a variance equal to its mean. A coherent state obeys this relationship, which means that we can detect photons that are random and independent from one another.

The third case is $g^{(2)}(0) < 1$, whose corresponding photon number distribution is sub-Poissonian [20]. A sub-Poissonian photon number distribution has a variance that is less than its mean. This corresponds to the photon anti-bunching phenomenon [21]. As an example, this feature is a quantum property of single photon sources, which can only be explained using a full quantum formalism.

1.2 Continuous-variable quantum information

The fundamental unit of quantum information processing (QIP) is the quantum bit (qubit), a quantum superposition of bits 0 and 1 [22]. A measurement in the computational basis yields two possible results, hence they are often referred to as discrete variable (DV) systems. The main advantage of discretisation is the ability to easily employ error correction techniques. The fundamental unit for an optical mode, however, is the field quadrature amplitude, whose measurement gives a continuum of possible outcomes. Such system is called a continuous-variable (CV) system [23–25]. The CV system is able to prepare entanglement efficiently and unconditionally using squeezed light, but the quality of the entanglement protocols rely on the amount of squeezing (assuming the operations are lossless). More advanced quantum information protocols, such as universal quantum computation, demand non-Gaussian operations like photon counting. This necessitates that the light source be both highly squeezed and photon counting detection compatible. In this section, we introduce two important examples of CV QIP that require a highly squeezed and separable light source: CV quantum

teleportation and CV cluster states.

1.2.1 Quantum teleportation

Suppose Alice wants to send Bob an unknown state $|\psi\rangle$, but Alice is not able to describe the state by measuring the state multiple times to extract the information, as the state would be destroyed after one measurement. Nor can she replicate the state due to the no-cloning theorem [26]. With the help of a pair of entangled states and a classical communication channel, Alice can transport the state $|\psi\rangle$ to Bob using quantum teleportation.

Proposed by Bennett et al. in 1993 [27], quantum teleportation has been experimentally demonstrated with photons and optical fields [28–31]. It plays an important role in quantum information processing [32]. Quantum teleportation is a technique to transferring quantum information that can be applied in both discrete-variable and continuous-variable systems. Here we mainly discuss the continuous-variable quantum teleportation. It can be realised with a simple quantum circuit shown in Fig. 1.2. The prerequisite condition for quantum teleportation is that Alice and Bob share a pair of entangled states and have access to a classical communication channel. When Alice intends to send an unknown quantum state $|\psi\rangle$ to Bob, the state needs to interact with the entangled state that Alice holds. Then Alice measure the states after the interference with results m_1 and m_2 . The measured results are classical information, sent from Alice to Bob through classical channels in order to complete the quantum teleportation. Upon receiving the results, Bob needs to perform two displacement operations on the other part of the entangled state. These operations recover the state $|\psi\rangle$ on Bob's side.

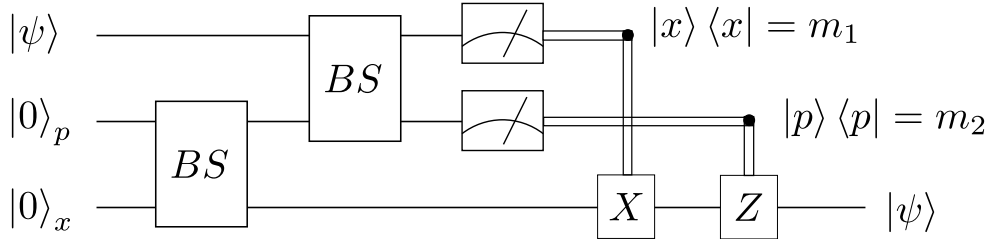


Figure 1.2: Diagram of continuous-variable quantum teleportation. The squeezed vacuum state in momentum $|0\rangle_p$ and the squeezed vacuum in position $|0\rangle_s$ interfere at the beam splitter (BS) and become an entangled state. The state $|\psi\rangle$ interferes with part of an entangled state to get projections of measurements after the interference. Based on the measurement results, the other part of the entangled state goes through X and Z gates to recover the state $|\psi\rangle$ [32].

With quantum teleportation, we are able to pass the quantum information on to other qubits as long as the states are entangled. When there are multiple states entangled, by interfering the states, measuring the outcome, and applying gates conditioned on this outcome, the information of the initial state passes through a network. This forms the basis for a framework known as measurement based

quantum computation. Next, we will introduce the notion of cluster states, which are large entangled states.

1.2.2 Cluster states

Photonic quantum computing is a promising candidate because of photons' long coherence time. On the other hand, photonics faces the challenge of scalability. Initialising qubits, as well as interacting and manipulating them are difficult, especially when large numbers of qubits are involved. Measurement-based quantum computation (MBQC) is an important proposal to implement photonic universal quantum computation [33, 34]. It involves the preparation of highly entangled photonic states and sequenced measurements. The logic gates of a quantum algorithm can be controlled by the measurements from single qubits. Because each measurement destroys a qubit, the cluster state needs to have large number of qubits upon initialisation and the ability to replenish as the measurements progress. This removes the need for complex interactions and manipulations during an experiment [33, 35].

In this thesis, we focus on the continuous-variable cluster states. Continuous-variable cluster states can be achieved with optical implementations using squeezed light sources as nodes of the cluster [12, 36, 37]. It has the advantage of deterministic computation and unconditional preparation of the cluster state. In 2019, Asavanant et al. [38] proposed and built a large scale two-dimensional temporal-mode continuous-variable cluster state. It employs the idea of time-multiplexing, which utilises extended lengths of fibre to delay and synchronise multiple modes. Fig. 1.3 illustrates the steps to entangle squeezed states from indistinguishable sources. The squeezed states are generated in pulses separated by time T . Pulses from neighbouring sources interfere with one another on a 50:50 beam splitter, creating the initial entanglement. Adding fibre loops on one beam to introduce a time delay of T , the k th pulse will be matched to the $(k+1)$ th pulse from the other beam. Afterwards, the pulsed squeezed states interfere again with its previous and successive states on another beam splitter, creating entanglement among the squeezed states.

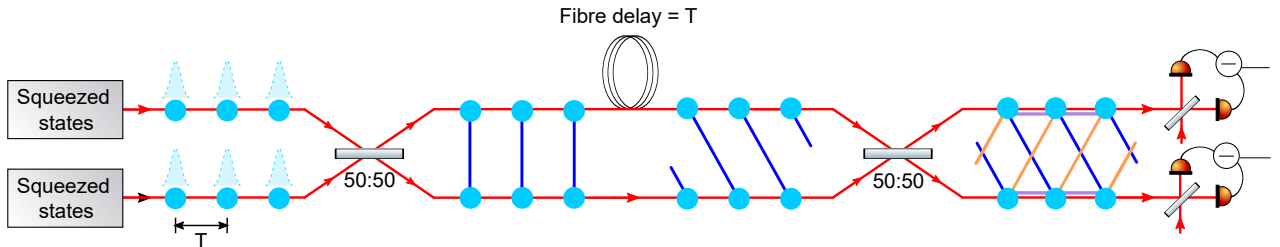


Figure 1.3: Diagram of the experimental setup to produce a cluster state [36–38]. The two squeezed light sources generate indistinguishable pulsed squeezed states with a time separation of T . The parallel squeezed states are entangled by interference on a 50:50 beam splitter. The fibre delay offsets the top rail by T . Upon second interference, the squeezed states are entangled in a cluster.

The steps above demonstrate how to use interference and fibre delays to build a CV cluster state. In

reality, the CV cluster state suffers from both intrinsic and gate operation noise. When the physical gate error rate is below a certain threshold, fault-tolerant quantum computation [39–41] is able to minimise the overall error rate by implementing quantum error correction. Because one can choose any two orthogonal wavefunctions to encode discrete-variable information into the continuous-variable states, it is advantageous to implement a code to protect the encoded quantum states from CV noise. Introduced by Gottesman, Kitaev, and Preskill in 2001 [42], a qubit can be encoded into an oscillator using the GKP state to correct small shift errors in position and momentum. The Wigner function of a GKP state is a sum of delta functions located at the points of a lattice in phase space. The ideal GKP state is a superposition of infinitely many, infinitely-squeezed states spaced by $2\sqrt{\pi}$ in the position quadrature. We can write the logical zero state $|0_L\rangle$ of the GKP as [43]

$$|0_L\rangle = \sum_{n=-\infty}^{\infty} |2n\sqrt{\pi}\rangle_x \quad (1.7)$$

where $|n\rangle_x$ is an eigenstate of position for the oscillator. Then the logical one state $|1_L\rangle$ is a position displacement of $|0_L\rangle$ by $\sqrt{\pi}$:

$$|1_L\rangle = \sum_{n=-\infty}^{\infty} |(2n+1)\sqrt{\pi}\rangle_x. \quad (1.8)$$

According to the periodicity of the Wigner function, any displacement in momentum and position smaller than $\sqrt{\pi}/2$ in absolute magnitude would allow the GKP state to be corrected without ambiguity.

An ideal GKP state with infinite delta spikes would have infinite energy. In practice, one can only generate approximate GKP states using finite energy states. Bromley et al. [44] propose a scheme to generate approximate GKP states from a Gaussian boson sampling (GBS) device as shown in Fig. 1.4. The GBS device takes a set of squeezed states to a linear interferometer, which can be decomposed into rotation and beam splitter gates. After the linear interferometer, the state is measured with photon counting detectors. When the photons in all but one mode are counted with a certain combination, the unmeasured mode is approximated to a GKP state. This process is probabilistic and is one of the reasons why GKP states are challenging to produce on demand.

Different architectures of photonic quantum computing have varying levels of accessibility and scalability. When we consider the continuous-variable (CV) approach, one way is to form a CV cluster state using GKP states. This approach demands a large number of GKP states to entangle each other. It has high tolerance of position shifting error, but suffers from the disadvantage of difficult on-demand GKP states generation. To reduce the production constrain of GKP states while implementing a fault-tolerant quantum computer, Bourassa et al. [45] present a hybrid state with squeezed states and probabilistic GKP states. It is initially built as a cluster state with squeezed states. Then GKP states are teleported into the cluster to form the hybrid state. This process is shown in Fig. 1.5. A

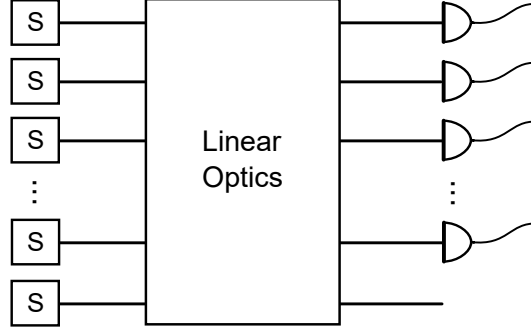


Figure 1.4: A schematic diagram for Gaussian boson sampling device [44, 45]. Multiple squeezed states are input to the device. The linear optics block consists of a sequence of rotation gates and beam splitters. The detectors at the output measure the photon numbers. When the photon counting measurement forms a certain combination, the output mode at the bottom is approximated to a GKP state.

successfully generated GKP state from a GBS device interferes with one of the squeezed states inside the cluster. Another squeezed state entangled with the interfered squeezed state is able to recover the GKP state based on the measurement results. Repeating successful GKP state generation and quantum teleportation, the hybrid state is built with the entanglement among squeezed states and GKP states. This architecture mitigates the requirement for on-demand GKP generation, thereby permitting greater scalability for photonic quantum computation.

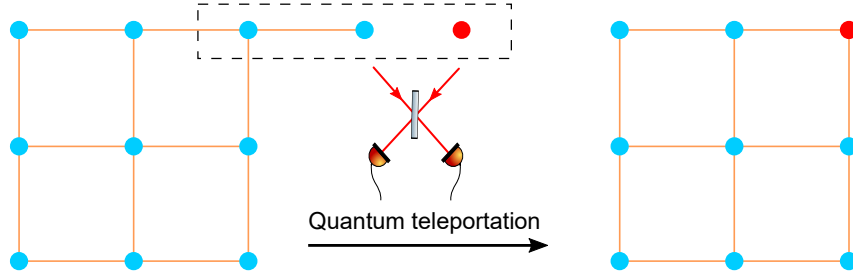


Figure 1.5: Diagram of creating a hybrid cluster state [45]. The initial cluster state is built with only squeezed states (blue dots). When a GKP state (red dot) is successfully generated from a GBS device, it will interfere with one of the squeezed state inside the cluster. Another entangled squeezed state recovers the GKP state inside the cluster based on the measurement results. The cluster state becomes hybrid after the quantum teleportation.

Implementing the hybrid cluster state architecture requires both GKP state generation and cluster state initialisation. Both processes require highly squeezed and separable light sources. A minimum of around 10 dB of squeezing is necessary to perform fault-tolerant quantum computation [45–47]. In addition, to teleport the GKP state into the cluster state, the GKP state needs to match the mode of the squeezed states. Mode matching can be achieved with highly separable squeezed states. Thus, a highly squeezed and separable state source is key to realising the potential of hybrid cluster state architecture.

1.3 Twin-beam squeezed light

The SPDC process generates two beams of squeezed light that consists of photon pairs with different frequency modes. Each single frequency mode of photon pairs is generated by a squeezing operator. This section discusses the squeezed light generation from a single mode to a product of multiple modes. We focus on the mathematical expression of the squeezed states to describe the single-mode squeezed vacuum and two-mode squeezed vacuum. Both expressions present the case of a single frequency mode generated by the squeezing parameter. In SPDC experiments, multiple frequency modes are simultaneously generated through multiple two-mode squeezing operations. We investigate spectral properties; in particular, the joint spectral intensity helps to understand correlations between the output modes, and the Schmidt decomposition indicates the mode number of the squeezed states. We also review the history of squeezed state generation, which leads to our proposal of a pulsed pump SPDC cavity source.

1.3.1 Two-mode squeezed vacuum

Squeezed states attract attention due to their reduced variance in a particular quadrature which is related to the quantum nature of light [48]. To understand squeezed states, we explore their mathematical descriptions through the perspectives of quadrature squeezing and photon number distribution. First, we present wavefunctions and quadrature variances to explain quadrature squeezing. Next, we use the squeezing operator of single-mode squeezed vacuum to link the squeezing parameter with its photon number distribution. Then we replicate the above concepts for two-mode squeezed vacuum.

We denote the eigenstate of $\hat{X}$ as $|x\rangle$ and the eigenstate of $\hat{P}$ as $|p\rangle$, where $\hat{X}|x\rangle = x|x\rangle$ and $\hat{P}|p\rangle = p|p\rangle$. The vacuum state wavefunction in the position basis is written as

$$\psi_0(x) = \frac{1}{\pi^{1/4}} e^{-x^2/2} \quad (1.9)$$

where $\psi(x) = \langle x|\psi\rangle$. Correspondingly, the wavefunction in the momentum basis is

$$\psi_0(p) = \frac{1}{\pi^{1/4}} e^{-p^2/2} \quad (1.10)$$

where $\psi(p) = \langle p|\psi\rangle$. For vacuum states, the variance of position and momentum observables are the same:

$$\langle 0|(\Delta\hat{X})^2|0\rangle = \langle 0|(\Delta\hat{P})^2|0\rangle = \frac{1}{2}. \quad (1.11)$$

A squeezed state has a quadrature amplitude which has a lower variance than that of a vacuum or a coherent state. The Heisenberg uncertainty principle limits the squeezing condition to one of the conjugate quadratures. For example, if the position quadrature is squeezed, the momentum quadrature

has to be stretched to satisfy $(\Delta\hat{X})(\Delta\hat{P}) \geq \frac{1}{2}$.

A squeezed state with squeezing factor R is obtained from vacuum state with the scaling transformation $x \rightarrow Rx$. The wavefunctions of the squeezed state can then be written as [18, 20]

$$\psi_R(x) = \frac{\sqrt{R}}{\pi^{\frac{1}{4}}} e^{-(Rx)^2/2} \quad (1.12)$$

and

$$\psi_R(p) = \frac{1}{\pi^{\frac{1}{4}}\sqrt{R}} e^{-(p/R)^2/2}. \quad (1.13)$$

With the above scaling transformations, the variance of two quadratures are

$$(\Delta\hat{X})^2 = \frac{1}{2R^2} \quad \text{and} \quad (\Delta\hat{P})^2 = \frac{R^2}{2}. \quad (1.14)$$

In the quantum optics description, the squeezing operator $\hat{S}(\zeta)$, which achieves the scaling transformation $x \rightarrow Rx$, is written as

$$\hat{S}(\zeta) = \exp\left((\zeta\hat{a}^2 - \zeta^*\hat{a}^{\dagger 2})/2\right) \quad (1.15)$$

where $\zeta = re^{i\phi}$ is the squeezing parameter with $r = \ln R$ and phase ϕ as real numbers. To simplify the discussion, we assume the phase $\phi = 0$ so that $\zeta = r$.

When the squeezing operator acts on a vacuum state, the squeezed vacuum state with wavefunctions in Eqs. (1.12) and (1.13) is generated. We can use the Fock basis to express the generated squeezed vacuum state:

$$\hat{S}(r)|0\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{n=0}^{\infty} (-\tanh r)^n \frac{\sqrt{(2n)!}}{2^n n!} |2n\rangle. \quad (1.16)$$

From the photon number statistics perspective, when the squeezed state is written in the Fock basis, it is clear that the probability of generating $2n$ photons is related to the squeezing parameter r .

Now we consider the two mode squeezed state. It is a state of two oscillators whose quadratures are correlated where $\hat{X}_a$ and $\hat{X}_b$ are position operators for optical modes a and b . The two mode vacuum state $|0\rangle \otimes |0\rangle$ has a wavefunction in position basis:

$$\Psi_{00}(x_a, x_b) = \frac{1}{\pi} e^{-x_a^2/2} e^{-x_b^2/2} = \frac{1}{\pi} e^{-(x_a - x_b)^2/4} e^{-(x_a + x_b)^2/4} \quad (1.17)$$

where $\Psi(x_a, x_b) = \langle x_a, x_b | \Psi \rangle$. Similar to the single mode vacuum state, in the two mode vacuum state Ψ_{00} , the sum $(\hat{X}_a + \hat{X}_b)/\sqrt{2}$, and the difference $(\hat{X}_a - \hat{X}_b)/\sqrt{2}$ have a Gaussian distribution with variance $1/2$. A two-mode squeezed vacuum state (TMSV) can be obtained with the scaling

transformation on the sum and difference of two mode quadratures, which has a position wavefunction:

$$\Psi_R(x_a, x_b) = \frac{1}{\pi} e^{-(x_a - x_b)^2 / (4R^2)} e^{-R^2(x_a + x_b)^2 / 4} \quad (1.18)$$

where R is the squeezing parameter as previously defined. With the help of the Fourier transform, we can obtain the correlated two-mode momentum wavefunction as

$$\Psi_R(p_a, p_b) = \frac{1}{\pi} e^{-(p_a + p_b)^2 / (4R^2)} e^{-R^2(p_a - p_b)^2 / 4} \quad (1.19)$$

where $\Psi(p_a, p_b) = \langle p_a, p_b | \Psi \rangle$. It is worth noting that the two mode squeezed state contains the correlation of the quadratures between the two modes and does not imply squeezing in individual modes.

In terms of making squeezed states, the squeezing operator for two mode squeezing is written as

$$\hat{S}_2(\zeta) = \exp(-\zeta \hat{a} \hat{b} + \zeta^* \hat{a}^\dagger \hat{b}^\dagger) \quad (1.20)$$

where $\hat{a}^\dagger$ and $\hat{b}^\dagger$ are the creation operators, and correspondingly $\hat{a}$ and $\hat{b}$ are the annihilation operators for the two modes.

Assuming $\zeta = r$, the two mode squeezed vacuum can be written with this operator in the Fock basis as

$$\hat{S}_2(r) |00\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (\tanh r)^n |nn\rangle. \quad (1.21)$$

From this expression, it is obvious that in a two-mode squeezed vacuum state, the number of photons in optical mode a is always equal to the number of photons in mode b . The coefficient of each term in the Fock basis is related to the squeezing parameter r . With a larger squeezing parameter r , there is a higher probability of generating multiple photon pairs in the two-mode squeezed vacuum. This expression links the average photon number with the squeezing parameter r as

$$\langle n \rangle = \frac{1}{(\cosh r)^2} \sum_{n=0}^{\infty} n (\tanh r)^{2n} = \frac{(\tanh r)^2}{(\cosh r)^2 (1 - (\tanh r)^2)^2} = (\sinh r)^2. \quad (1.22)$$

1.3.2 Multi-mode squeezing

While the theoretical two-mode squeezing occurs only on a single frequency mode, two-mode squeezing in experiments will occur simultaneously on multiple frequency modes of photon pairs. This process can be investigated by considering frequency correlations between a single pair of photons that are created. In the process of spontaneous parametric down conversion, during which a pump photon with frequency ω_p is converted into a pair of photons – signal and idler with frequency ω_s and ω_i respectively, we use the joint spectral intensity to describe the signal and idler spectral correlation.

For a simple model with a Gaussian shaped pump field [49], we can write the pump spectral amplitude as

$$A(\omega_p) = M \exp\left(-\frac{(\omega_p - 2\omega_0)^2}{2\sigma^2}\right) \quad (1.23)$$

where M is a normalisation constant, $2\omega_0$ is the central frequency of the pump, and σ is the pump bandwidth. For simplicity we use $\nu_p = \omega_p - 2\omega_0$ as the zero mean for pump frequency. We can also write the signal and idler frequencies as $\nu_s = \omega_s - \omega_0$ and $\nu_i = \omega_i - \omega_0$. With the energy conservation condition, the above equation can be written as

$$f(\nu_s, \nu_i) = A(\omega_p = \omega_s + \omega_i) = M \exp\left(-\frac{\nu_s^2 + \nu_i^2}{2\sigma^2} - \frac{2\nu_s\nu_i}{2\sigma^2}\right). \quad (1.24)$$

We use $f(\nu_s, \nu_i)$ as the joint spectral amplitude (JSA) and $|f(\nu_s, \nu_i)|^2$ as the joint spectral intensity (JSI) for the photon pair frequency probability distribution. The two photon state can be written as

$$|\Psi\rangle = \int \int d\nu_s d\nu_i f(\nu_s, \nu_i) |\nu_s\rangle |\nu_i\rangle = \int \int d\nu_s d\nu_i f(\nu_s, \nu_i) \hat{a}^\dagger(\nu_s) \hat{a}^\dagger(\nu_i) |0\rangle. \quad (1.25)$$

The joint temporal distribution is the 2D Fourier transform of the joint spectral distribution:

$$\tilde{f}(t_s, t_i) = \frac{1}{2\pi} \int d\nu_i \int d\nu_s f(\nu_s, \nu_i) e^{-i\nu_s t_s} e^{-i\nu_i t_i}. \quad (1.26)$$

We can write the two photon state in a temporal variable form:

$$|\Psi\rangle = \int \int dt_s dt_i \tilde{f}(t_s, t_i) |t_s\rangle |t_i\rangle = \int \int dt_s dt_i \tilde{f}(t_s, t_i) \hat{a}^\dagger(t_s) \hat{a}^\dagger(t_i) |0\rangle \quad (1.27)$$

where $\tilde{f}(t_s, t_i)$ is the joint temporal amplitude (JTA) describing the temporal correlation between the signal and idler fields. Similar to the JSI, the joint temporal intensity (JTI) is the probability density for photon pair arrival time written as $|\tilde{f}(t_s, t_i)|^2$. Both JSI and JTI can be used to characterise the correlation of a two-photon state. The JSI can be measured with tunable narrow-band filtering of the signal and idler [50]. This technique has a resolution of the filter bandwidth. The JTI can be measured with temporal filtering and coincidence detection [51]. The measurement is limited by the timing resolution of the single photon detectors. The pulse duration of the pump field is the determining factor when choosing between JSI and JTI. It is simpler to measure the JSI when the pump field is composed of short pulses. JTI is easier to record for long pulses or continuous-wave pump.

After we investigate the correlation between the signal and idler, the Schmidt decomposition can be used to study the mode number of photon pairs shown in the JSI. This information translates into the mode number of the SPDC output. A pure state $|\psi\rangle$ of a composite system AB can be decomposed

through the Schmidt decomposition as a sum of products of orthonormal states A and B as [52]

$$|\psi\rangle = \sum_i \sqrt{\lambda_i} |i_A\rangle |i_B\rangle \quad (1.28)$$

where $\sqrt{\lambda_i}$ are known as the Schmidt coefficients which are non-negative real numbers and satisfy the normalisation condition $\sum_i \lambda_i = 1$. As the coefficients would be different for a different orthonormal basis, we consider the ones that maximise the detection probability, i.e. largest $\sqrt{\lambda_n}$. The Schmidt number K is used to quantify the effective number of populated eigenstates, which is defined using Schmidt coefficients as

$$K = \frac{1}{\sum_n \lambda_n^2}. \quad (1.29)$$

The Schmidt number K quantifies the degree of correlation in the physical system. When the joint states is more separable, the Schmidt number K is closer to 1.

1.3.3 Review of experiments

Squeezed states were first generated in 1985 by Slusher [53] using four-wave mixing with Na atoms in an optical cavity. One year later, Wu generated squeezed states using parametric down conversion using a crystal in an optical cavity [54].

The four-wave mixing process is based on the $\chi^{(3)}$ nonlinear effect, which is very weak during the interaction. Efficient spontaneous four-wave mixing photon pair sources are usually implemented in guided wave systems such as integrated photonic ring resonators [55] and highly nonlinear photonic crystal fibres [56]. The three-wave mixing process is comparably stronger with the $\chi^{(2)}$ nonlinearity. It can be employed within both bulk and waveguide optical systems. Spontaneous parametric down-conversion (SPDC), a three-wave mixing process, is a common way to generate squeezed states, with a key advantage of its non-linear processes being able to operate under room temperature.

Nonlinear crystals in free space are usually used to generate squeezed states through the SPDC process. A pulsed pump field with high intensity and appropriate spectral bandwidth can be employed to increase the generation rate as well as produce more factorisable states. A pulsed parametric down conversion source was used to generate squeezing in 1987 [57] and was subsequently significantly improved by Kumar's group [58, 59]. By optimising the local oscillator and using a potassium titanyl phosphate (KTP) type II phase-matching crystal, the temporal mode of local oscillator could be nearly identical to that of the squeezed mode. The quadrature squeezing factor of 5.8 dB was measured in 1994 [59]. Experiments at telecom wavelengths have been conducted with pulsed pump and free space periodically-poled KTP (PPKTP) crystal [60, 61]. Experiments with high-gain SPDC using micro-Joule pulse energies have generated bright squeezed vacuum states with up to 10^{10} photons per mode [62]. Detailed modelling shows that maximising the photon-pair generation rate, for free-space SPDC

with co-linear Gaussian beams, reduces the spectral purity to 70 % due to the contributions of multiple spatial modes [63]. In an effort to improve the spectral purity, Lemieux et al. used interference from multiple crystals to reduce the mode number from approximately 50 to less than 2 [64]. Though this is a substantial improvement in separability over other pulsed pump SPDC, it still not fully factorisable. Detailed theoretical description of the interference approach is an ongoing area of research [65].

Crystal inside a cavity is an alternative configuration for bulk-optics-based SPDC. For many applications based on atomic quantum memories, a linewidth of tens of MHz is required to interface with atomic transitions for quantum state storage. By putting a nonlinear crystal in a cavity, the spectral bandwidth of the output photons is restricted, because the cavity acts as a filter. The squeezed states generated have a narrow linewidth from the cavity resonance. Continuous-wave squeezed states are usually generated with such configuration, which allows the continuous pump field to interact with the nonlinear crystal multiple times to enhance the down-conversion efficiency. The cavity also provides spatial mode confinement for squeezed state generation. The TEM_{00} mode is usually enhanced so that other non-squeezed transverse modes are suppressed.

Yurke was the first to realise squeezed states generation inside a cavity [66]. In his setup, the cavity had a ring configuration and the interaction is a four-wave mixing process. The early experiments generate squeezed states with squeezed factors of up to 3 dB. This approach developed further, with a squeezing factor of 15 dB achieved and applied to gravitational wave detection in 2016 [67]. For two-mode squeezing, a squeezing factor of 10 dB was measured by Eberle et al [68]. The cavity enhancement provides a large amount of squeezing and introduces spectral and temporal correlations between the two modes generated from the nonlinear process: signal and idler modes. Common cavity configurations for SPDC squeezed state generation are bow-tie cavity [69–76] and Fabry-Perot style cavity [77–83]. A bow-tie cavity consists of four mirrors and provides easier cavity length control and reduces the interference effect as the pump light does not encounter direct reflections. A Fabry-Perot (FP) cavity consists of two mirrors. The FP cavity is more compact and more resilient against mechanical vibration. To simplify the cavity components, the crystal is also usually coated on one or both sides with dielectric coatings to form the reflective surface of the cavity [84–87].

Apart from bulk optics, integrated photonics is another platform for squeezed state generation. The waveguide approach is well suited for integrated photonics applications [88]. The stronger confinement of the modes in a waveguide provides a better SPDC efficiency for photon pair generation compared with bulk optics [89–93]. Waveguide SPDC sources can be equipped with fibre-based wavelength division multiplexers and ultra-fast electro-optic modulators (EOM) to provide on-chip photon wavelength engineering for spectrally pure bi-photon states in multi-photon experiments [94, 95].

In 2002, a PPLN waveguide was used to generate squeezing by Kanter et al. [96]. Since then, there has been a growing amount of research focusing on on-chip photon pair source where Caspani et al. has reviewed the progress of the integrated photon source [97]. Squeezing were observed with both pulsed

pump [98, 99] and continuous pump [100, 101] in waveguide. Recent work using PPKTP waveguide achieved mean photon numbers up to 20 for a two-mode squeezed state in a single spatial mode [92]. However, waveguide sources suffer substantial loss from mode coupling between components and the waveguide propagation [102]. This loss limits the amount of useful squeezing from a source due to the high total loss with on-chip sources.

With various approaches to generate SPDC squeezed states, we need to find ways to characterise and describe the squeezed state source. There are two types of measurement scheme for a squeezed state: homodyne detection and photon counting detection. Homodyne detection [103] uses a balanced beam splitter, two identical photodiodes, and a homodyne local oscillator field which has an adjustable phase. The squeezed signal field and the local oscillator field interfere at the beam splitter and the outputs are sent to photodiodes. The variance of the quadratures of the squeezed state can be measured by taking the difference voltage of two photodiodes and scanning the phase of the local oscillator field, and the decibel level of squeezing can be calculated using the smaller quadrature variance. For two-mode squeezing, the squeezed variances of joint observables can also be measured using homodyne detection. It can be used for both continuous-wave states and pulsed states. Homodyne detection is constrained to the mode that matches the local oscillator, which means it is not able to fully describe a multi-mode output.

Another way to characterise squeezed light is by directly measuring the photon number distribution [104]. This approach can help to characterise a pulsed squeezed state. The photon number distribution takes contributions from multiple modes. Therefore, in order to get the photon statistics that are expected for a single squeezed state, the light source needs to isolate a single mode.

Both homodyne detection and photon counting were employed in the same experiment to measure the squeezing of a single-mode squeezed vacuum state [105]. A pulsed squeezed vacuum state which was obtained from free space KNbO_3 crystal was measured in this experiment. It provided a bound for parametric down conversion gain.

Our proposal to generate separable highly-squeezed states in the following chapters suggests a combination of the two approaches: A pulsed squeezed-states generation enhanced by a cavity, which provides both factorisable states and a large amount of squeezing. Such states could be employed in continuous-variable quantum computing as the squeezed-state source. Recently there was a publication involving a similar design by Madsen et al. from a Canadian quantum startup Xanadu [106], which demonstrated quantum advantage on the photonic platform. Their work used a doubly resonant cavity with a 10 mm type 0 phase-matched periodically poled KTP (PPKTP) crystal. In contrast, our design employs a triply resonant cavity with a 4 mm type II PPKTP crystal in an effort to increase separability. In this thesis, we will discuss in detail the design and initial characterisation of our triply resonant cavity SPDC source with a pulsed pump.

1.4 Source characteristics

The photon source can be characterised in a few approaches from the spectral relations between the two beams to estimating photon number probabilities. In this section, we will introduce some concepts and approaches to characterise the squeezed light source. We first introduce the density matrix to express the photon number probabilities. With the help of density matrix, we derive the expression for second order coherence of squeezed state sources using photon number probability. Last but not least, the photon source's indistinguishability illustrates to what degree the single mode photons can interact with one another.

1.4.1 Density matrix and photon number probabilities

Density matrix is a matrix that describes a quantum state. It enables the possibility for the probability calculations of any measurement result. The density matrix operator can be written as [20]

$$\hat{\rho} = \sum_i p_i |\psi_i\rangle \langle\psi_i| \quad (1.30)$$

where p_i is the probability of the state $|\psi_i\rangle$. When the state is pure, only one of the probability coefficient p_i is 1 and all other p_i are equal to zero. The expectation of an observable $\hat{O}$ can be expressed with the trace of the product between the density matrix and the observable:

$$\langle\hat{O}\rangle = \text{Tr}(\hat{\rho}\hat{O}). \quad (1.31)$$

The photon number probability of source emitting n photons $P(n)$ depends on the diagonal elements of the density matrix as

$$P(n) = \text{Tr}(\hat{\rho}|n\rangle\langle n|) = \langle n|\hat{\rho}|n\rangle. \quad (1.32)$$

The photon number probability is normalised as

$$\sum_{n=0}^{\infty} P(n) = \text{Tr}(\hat{\rho}) = 1. \quad (1.33)$$

When we discuss the photon probability for a pulsed source, $P(n)$ represents the probability that the source emits n photons per pulse. The average photon number per pulse μ is written as

$$\mu = \langle n \rangle = \text{Tr}(\hat{\rho}\hat{n}). \quad (1.34)$$

For a continuous-wave (CW) photon source, we can divide the continuous time into discrete bins of time intervals. It is helpful to define $P(n; T)$ as the probability of emitting n photons in the brief

time interval T .

1.4.2 Second order coherence $g^{(2)}(\tau)$

The second order coherence $g^{(2)}(\tau)$ provides information of the multi-photon emission probability. For a CW single photon source, the second-order coherence between modes measured at time t and time $t + \tau$ is expressed as [20]

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t) \hat{a}^\dagger(t + \tau) \hat{a}(t + \tau) \hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t) \hat{a}(t) \rangle^2}. \quad (1.35)$$

When the time difference τ between detecting two modes is 0, we can write

$$g^{(2)}(0) = \frac{\langle \hat{a}^\dagger(t) \hat{a}^\dagger(t) \hat{a}(t) \hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t) \hat{a}(t) \rangle^2} = \frac{\langle \hat{n}(t)(\hat{n}(t) - 1) \rangle}{\langle \hat{n}(t) \rangle^2}. \quad (1.36)$$

For a pulsed single photon source, we define a discrete version of $g^{(2)}[m]$ with square brackets:

$$g^{(2)}[m] = \frac{\langle \hat{a}^\dagger[l] \hat{a}^\dagger[l + m] \hat{a}[l + m] \hat{a}[l] \rangle}{\langle \hat{a}^\dagger[l] \hat{a}[l] \rangle \langle \hat{a}^\dagger[l + m] \hat{a}[l + m] \rangle} \quad (1.37)$$

where l and m are the integer value of pulse number. The zero delay $g^{(2)}[0]$ can be written as

$$g^{(2)}[0] = \frac{\langle \hat{n}(\hat{n} - 1) \rangle}{\langle \hat{n} \rangle^2} \quad (1.38)$$

where $\hat{n} = \hat{a}^\dagger[l] \hat{a}[l]$ measures the photon number in the l th pulse. Its expectation values can be expressed with the density matrix as

$$g^{(2)}[0] = \frac{\text{Tr}\{\hat{\rho} \hat{n}(\hat{n} - 1)\}}{(\text{Tr}\{\hat{\rho} \hat{n}\})^2} \quad (1.39)$$

where the trace operation Tr only depends on the diagonal terms of the density matrix. When we assume the photon statistics are averaged over the pulse duration, and treat each pulse as one unit, we can write the $g^{(2)}[0]$ with per-pulse photon probabilities as

$$\begin{aligned} g^{(2)}[0] &= \frac{\sum_{n=0}^{\infty} n(n-1)P(n)}{(\sum_{n=0}^{\infty} nP(n))^2} \\ &= \frac{2P(2) + 6P(3) + 12P(4) + \dots}{(P(1) + 2P(2) + 3P(3) + \dots)^2} \\ &= \frac{2P(2) + 6P(3) + 12P(4) + \dots}{\mu^2} \end{aligned} \quad (1.40)$$

where μ is the average photon number in each pulse. With a low efficiency source, we approximate the $g^{(2)}[0]$ with the assumption of $P(1) \gg P(2) \gg P(n > 2)$ [107]:

$$g^{(2)}[0] \simeq \frac{2P(2)}{(P(1))^2}. \quad (1.41)$$

For a CW source, the relationship between $g^{(2)}(0)$ and photon number probabilities can be approximated as

$$g^{(2)}(0) \simeq \frac{2P(2;T)}{(P(1;T))^2} \quad (1.42)$$

where $P(n;T)$ represents for a short time interval T the probability of having n photons, and the equation is under assumption of $P(1;T) \gg P(2;T) \gg P(n > 2;T)$.

To calculate photon statistics, for simplicity we will use a phase-averaged model described by the density matrix:

$$\rho = P(0) |0\rangle \langle 0| + P(1) |1\rangle \langle 1| + P(2) |2\rangle \langle 2| + \dots \quad (1.43)$$

where $P(i)$ represents the probability of generating i photons from the light source. To calculate the $g^{(2)}(0)$ of the squeezed state, we use the setup in Fig. 1.6 to measure the photon number distribution of a single arm. The setup employs photon resolving detector, which provides statistics on the number of photons reaching the detector. Measuring the number of photons on the signal arm indicates a photon number distribution from the SPDC process. Under the low-squeezing assumption, the output state is a superposition of $|00\rangle$ and $|11\rangle$. The second order correlation $g^{(2)}(0)$ can be calculated with the photon number distribution.

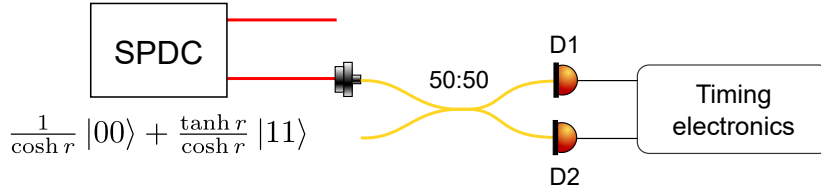


Figure 1.6: Diagram of marginal $g^{(2)}$ measurements. The SPDC source in low squeezing regime provides a superposition state of $|00\rangle$ and $|11\rangle$ with probability of $(\frac{1}{\cosh r})^2$ and $(\frac{\tanh r}{\cosh r})^2$. One arm of the SPDC is sent to a 50:50 beam splitter. Two photon counting detectors D1 and D2 capture the outputs and register their arrival time on the timing electronics. The marginal $g^{(2)}$ can be calculated with the counts registered on the timing electronics by $\frac{2P(2)}{(P(1))^2}$.

Similarly, if we add a click detector on the idler arm as shown in Fig. 1.7, we are able to perform a conditional measurement. The click detector is an on-off detector excited by non-zero photon numbers. The photon number distribution conditioned on the detection of idler arm helps calculation of heralded $g^{(2)}(0)$.

Heralded correlation function $g_{\text{Her}}^{(2)}(0)$ is expressed as photon number probabilities conditioned on heralding beam detection:

$$g_{\text{Her}}^{(2)}(0) = \frac{2P(2|\text{H})}{(P(1|\text{H}))^2} \quad (1.44)$$

where the $P(1|\text{H})$ and $P(2|\text{H})$ are the probability of generating 1 and 2 photon pairs conditioned on heralding photon detection.

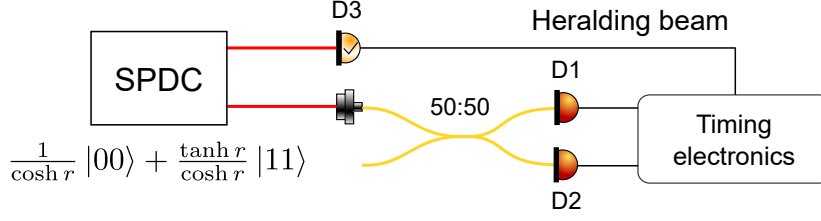


Figure 1.7: Diagram of heralded $g^{(2)}$ measurements. The SPDC source in low squeezing regime provides a superposition state of $|00\rangle$ and $|11\rangle$ with probability of $(\frac{1}{\cosh r})^2$ and $(\frac{\tanh r}{\cosh r})^2$. One arm is the heralding arm that provides the condition for the measurements. A click detector D3 is used to provide information of non-zero photon existence. The other arm of the SPDC is sent to a 50:50 beam splitter. Two photon counting detectors D1 and D2 capture the outputs based on conditioned photon existence on the heralding arm and register their arrival time on the timing electronics. The heralded $g^{(2)}$ can be calculated with the counts registered on the timing electronics by $\frac{2P(2|H)}{(P(1|H))^2}$.

1.4.3 Indistinguishability

A key challenge for lots of quantum applications is initial state preparation so that the states are able to interact with each other. For a squeezed state source, we expect the photon pairs generated within the squeezed state to fall in a single frequency mode so that photon pairs generated at different times are indistinguishable. In quantum light experiments, a common way to quantify the indistinguishability is the Hong-Ou-Mandel interference experiment [108]. Two indistinguishable photons incident on the two inputs of an ideal 50:50 beam splitter will interfere with each other and exit the beam splitter from the same output port. The beam splitter operator $\hat{B}$ can be written as

$$\hat{B} = \exp\left(\frac{\theta}{2}(\hat{a}^\dagger \hat{b} e^{i\phi} - \hat{a} \hat{b}^\dagger e^{-i\phi})\right) \quad (1.45)$$

where the $\hat{a}^\dagger$ and $\hat{b}^\dagger$ are the creation operators on the two output ports, ϕ is the phase change, and θ is related to the reflectivity r and transmissivity t of the beam splitter by

$$r = \sin \frac{\theta}{2} \quad (1.46)$$

$$t = \cos \frac{\theta}{2}. \quad (1.47)$$

Applying the beam splitter operator on the input modes, we can get

$$\hat{B} \hat{a} \hat{B}^\dagger = \hat{a} \cos \frac{\theta}{2} - \hat{b} e^{i\phi} \sin \frac{\theta}{2} = t \hat{a} - r \hat{b} e^{i\phi} \quad (1.48)$$

and

$$\hat{B} \hat{b} \hat{B}^\dagger = \hat{b} \cos \frac{\theta}{2} + \hat{a} e^{-i\phi} \sin \frac{\theta}{2} = t \hat{b} + r \hat{a} e^{-i\phi}. \quad (1.49)$$

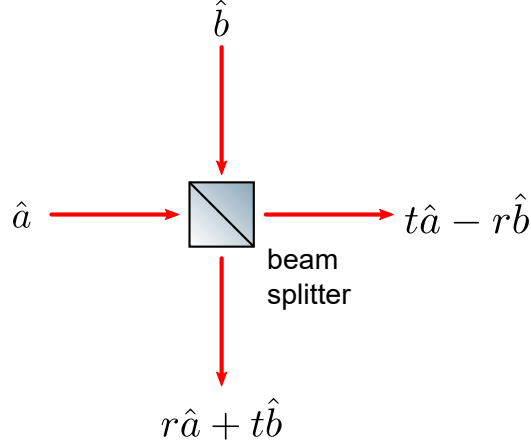


Figure 1.8: Beam splitter diagram. For the inputs $\hat{a}$ and $\hat{b}$, the beam splitter provides a transmission of t and reflection of r . The output of the beam splitter shows a combination of the two inputs $\hat{a}$ and $\hat{b}$ with a weight from transmission t and reflection r .

We take the reflection phase change $\phi = 0$, the above two equations become

$$\hat{B}\hat{a}\hat{B}^\dagger = t\hat{a} - r\hat{b} \quad (1.50)$$

$$\hat{B}\hat{b}\hat{B}^\dagger = t\hat{b} + r\hat{a} \quad (1.51)$$

as shown in Fig. 1.8.

When we consider the input state $|1, 1\rangle_{a,b} = \hat{a}^\dagger \hat{b}^\dagger |0, 0\rangle$ on the input modes a and b , we can rewrite the transformation on a 50:50 beam splitter as [20]

$$\hat{a}^\dagger \rightarrow \frac{\hat{c}^\dagger - \hat{d}^\dagger}{\sqrt{2}} \quad \text{and} \quad \hat{b}^\dagger \rightarrow \frac{\hat{c}^\dagger + \hat{d}^\dagger}{\sqrt{2}}. \quad (1.52)$$

In the classical theory, photons entered the beam splitter will independently exit in mode c or mode d . This leads to 50 % probability the photons exit in mode c and the other 50 % probability photons exit in mode d . The detection distribution of the output in classical theory suggests there are 25 % chance both photon are in mode c , 25 % chance both photon are in mode d , and the remaining 50 % probability we find one photon in mode c and one photon in mode d . However, the quantum theory proposes a different outcome:

$$\begin{aligned} |1, 1\rangle_{a,b} = \hat{a}^\dagger \hat{b}^\dagger |0, 0\rangle &\rightarrow \frac{1}{2} (\hat{c}^\dagger - \hat{d}^\dagger) (\hat{c}^\dagger + \hat{d}^\dagger) |0, 0\rangle \\ &= \frac{1}{2} (\hat{c}^{\dagger 2} - \hat{d}^{\dagger 2}) |0, 0\rangle \\ &= \frac{|2, 0\rangle_{c,d} - |0, 2\rangle_{c,d}}{\sqrt{2}}. \end{aligned} \quad (1.53)$$

We can see from this transformation for two single photon input, the term $|1, 1\rangle_{c,d}$ is absent. This

phenomenon is called Hong-Ou-Mandel (HOM) effect. If the output photon counts for mode c and d is recorded, we would observe zero coincidence count with the HOM effect. The HOM effect only appears when the two input photons are indistinguishable.

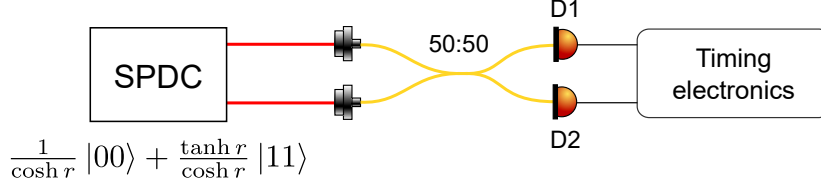


Figure 1.9: Diagram of Hong-Ou-Mandel experiment for a type I degenerated SPDC source. The SPDC source in low squeezing regime provides a superposition state of $|00\rangle$ and $|11\rangle$ with probability of $(\frac{1}{\cosh r})^2$ and $(\frac{\tanh r}{\cosh r})^2$. Both arms of the SPDC are sent to the inputs of a 50:50 beam splitter. Two photon counting detectors D1 and D2 capture the outputs of the two-photon interference and register their arrival time on the timing electronics. We expect that the detectors register 0 or 2 photons during each experiment trial due to the HOM effect.

The experimental setup for the HOM experiment using a type I degenerate SPDC source is shown in Fig. 1.9. The outputs of SPDC source are sent to a 50:50 beam splitter. The output of the interference are detected by two photon counting detectors D1 and D2. The photons' arrival time are recorded by the timing electronics to separate experimental trials. If the two photons are fully indistinguishable, the measurements from each detector in an individual experiment will only record zero or two photons.

The two photon interference can be constructed in a different way. With a polarisation beam splitter (PBS), we can perform a polarisation interference. The cube PBS transmits horizontal polarisation and reflects vertical polarisation. With a PBS setup in Fig. 1.10, we write the state at the input port as a sum of horizontal component and vertical component $\alpha|H\rangle + \beta|V\rangle = (\alpha\hat{H}_{in}^\dagger + \beta\hat{V}_{in}^\dagger)|0,0\rangle$.

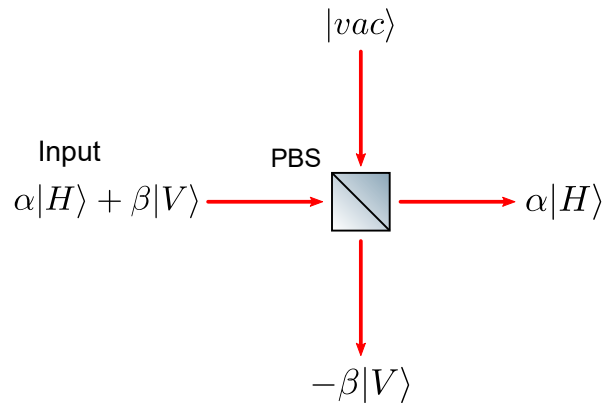


Figure 1.10: Polarisation beam splitter (PBS) diagram. The input state would have their horizontal components transmitted and vertical components reflected.

For the PBS transformation, we can write

$$\hat{H}_{in}^\dagger \rightarrow \hat{H}_c^\dagger \quad (1.54)$$

and

$$\hat{V}_{in}^\dagger \rightarrow -\hat{V}_d^\dagger \quad (1.55)$$

which means the horizontal component only exit in the horizontal output mode c , and the vertical component only exit in the vertical output mode d with a π phase change.

If we rotate the polarisation of $|H\rangle$ and $|V\rangle$ for 45 degrees so that the input states becomes $\frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $\frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$.

The transformation at the PBS can then be written as

$$\begin{aligned} \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_{in} \otimes \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)_{in} &= \frac{1}{2} (\hat{H}_{in}^\dagger + \hat{V}_{in}^\dagger) (\hat{H}_{in}^\dagger - \hat{V}_{in}^\dagger) |0, 0\rangle \\ &\rightarrow \frac{1}{2} (\hat{H}_c^\dagger - \hat{V}_d^\dagger) (\hat{H}_c^\dagger + \hat{V}_d^\dagger) |0, 0\rangle \\ &= \frac{1}{2} (\hat{H}_c^{\dagger 2} - \hat{V}_d^{\dagger 2}) |0, 0\rangle \\ &= \frac{1}{\sqrt{2}} (|2H, 0\rangle_{c,d} - |0, 2V\rangle_{c,d}). \end{aligned} \quad (1.56)$$

This indicates the output of the PBS with the rotated orthogonal polarisation input is the superposition of two photons both transmitted or reflected from the same port. By using a PBS, we are able to interfere two photons with orthogonal polarisation. A type II PDC photon pair are generated with orthogonal polarisation. Based on Eq. (1.56), we can perform a HOM measurement when the input state are from a type II PDC source whose polarisation is rotated for 45°.

1.5 Thesis outline

This thesis is composed of 7 chapters. The first chapter has introduced what is quantum light, the motivation for this project to apply the squeezed state source to continuous-variable quantum teleportation and photonic quantum computing, the concept of two-mode squeezed vacuum and its spectral properties, and methods to characterise squeezed light sources. The second chapter provides theory on parametric down conversion. It discusses the impact for squeezed state generation from a cavity configuration and a pulsed pump. The third chapter describes the cavity design in detail. It includes the optical design of the cavity, mechanical design for cavity mounts, and the electrical control to adjust and stabilise the cavity for squeezed state generation. The fourth chapter characterises the cavity optical behaviour and presents its linear response. It looks into the cavity field response to control parameters and the tuning protocol to achieve triple resonance. The fifth chapter provides the triply resonant cavity's nonlinear response including second harmonic generation, difference frequency

generation with continuous-wave lasers. It describes the cavity SPDC through photon statistics, indistinguishability, and timing correlations. The sixth chapter characterises the SPDC photon source with a pulsed pump. It presents the design of the pulsed pump system, and source characterisation with the pulsed pump SPDC. The seventh chapter concludes the thesis.

This research project began in December 2018 under the supervision of Dr. Steve Kolthammer. In the second year of my PhD, I was joined by a PhD student, Rob Gardner, to collaborate on this project. During the first two years, we focused on the derivation of the physics theory as well as conducting relevant simulations. After we gained a better understanding of the theory foundations, in July 2020, we moved to a new lab and began the experimental implementation of the project. I am the lead researcher for this project apart from the mechanical design which is led by Rob Gardner.

Chapter 2

Theory of SPDC in a triply resonant cavity

In the previous chapter, we outlined this project's motivation to apply a squeezed state source to continuous-variable quantum information processing. We also introduced twin-beam squeezed light and methods to characterise squeezed light sources.

The purpose of this chapter is to dive deeper into the generation of squeezed states: discuss the theory of parametric down conversion and how a cavity impacts squeezed states generation. Parametric down conversion (PDC) is a common method to generate squeezed states. This method converts the pump photon into a photon pair utilising the second order nonlinear interaction inside a nonlinear crystal. When a cavity is applied to the PDC process, the generated squeezed state can achieve both high intensity and narrow bandwidth. This is due to the property of a cavity to enhance the intensity of its resonant fields. When a pulsed pump is used for the PDC process, the pulse duration can be tuned to yield a more factorisable output state. By combining a pulsed pump and a cavity with PDC, we aim to generate a highly-squeezed factorisable state.

2.1 Parametric down conversion

The interaction between light and a medium can be described with physical models. When the light is present with low power, it is a simple model where the response is linear with respect to the amplitude of the electric field. However, as the intensity of the light increases, the interaction involves greater complexity and the linear model will fail to accurately predict its behaviour. The physical model can describe the light-medium interactions better by taking into account the various orders of nonlinear interaction like second harmonic generation, the Kerr effect, and stimulated Raman scattering.

In this thesis, we focus on second order nonlinear effects: second harmonic generation (SHG) which doubles the frequency of the input field, sum frequency generation (SFG) which generates a new frequency from the sum of the two frequencies, difference frequency generation (DFG) which

generates a new frequency from the difference of the two frequencies, and parametric down conversion (PDC) which converts one photon of high energy into a pair of photons (one signal photon, one idler photon) with lower energy. The main difference between DFG and PDC is the number of fields inside the crystal. Throughout the thesis, we use the term Spontaneous Parametric Down Conversion (SPDC) to specify a spontaneous conversion of one pump photon into a pair of photons rather than conversion triggered by stimulation.

In this section, we introduce a quantum description of parametric down conversion and show how it can be used to generate squeezed states. Starting with three-wave nonlinear interaction and Maxwell's equations, we derive the expressions of PDC for both classical and quantum descriptions. We also investigate the connection between squeezed state operator and the quantum description of PDC. In addition, we introduce crystal optics and phase matching in birefringent nonlinear media. Finally, we review the literature on squeezed state generation using different approaches.

2.1.1 Three-wave nonlinear interaction

When an electromagnetic field interacts with a dielectric material, the polarisation introduced can be written as a sum of different orders of nonlinear interaction in the medium [109]:

$$\mathbf{P} = \epsilon_0 \left(\chi^{(1)} \mathbf{E}_1 + \chi^{(2)} \mathbf{E}_1 \mathbf{E}_2 + \chi^{(3)} \mathbf{E}_1 \mathbf{E}_2 \mathbf{E}_3 + \dots \right) \quad (2.1)$$

where ϵ_0 is the vacuum permittivity, $\chi^{(1)}$ is the linear susceptibility, and $\chi^{(2)}, \chi^{(3)}, \dots$ are the nonlinear susceptibilities of the medium. The nonlinear interactions involves different electromagnetic fields for frequency conversion under energy conservation and momentum conservation. The $\chi^{(2)}$ term describes the three-wave nonlinear interaction.



Figure 2.1: Diagram of energy and momentum relations among the three fields during the three-wave interaction. (a) shows the energy conservation for three-wave interaction where the pump photon energy equals to the sum of signal and idler photon energy. (b) shows the momentum conservation for three-wave interaction where the wavevector of the pump photon equals to the sum of wavevectors of signal and idler photons.

Three-wave nonlinear interaction is the coupling interaction among three coherent light fields in a second-order nonlinear crystal. Fig. 2.1 presents the energy conservation and the momentum

conservation for a three-wave interaction. The frequency of the higher energy photon equals to the frequency sum of the two lower energy photons. Similarly, the wavevector of the three fields follows the momentum conservation. The pump wavevector equals to the sum of the signal and idler wavevectors. To explain the coupling interaction, we will start by introducing the classical description of the sum-frequency generation and extend into the quantum description of the parametric down conversion process.

Classical description of sum-frequency generation

Starting with the Maxwell's equations, we assume that there are no free charges in the medium before the interaction begins. The relations among the electric field $\mathbf{E}$, the magnetic field $\mathbf{B}$, and electric displacement $\mathbf{D}$ are as follows [109]:

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} \quad (2.2)$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial}{\partial t} \mathbf{D} \quad (2.3)$$

where μ_0 is the vacuum permeability.

With another curl on the electric field $\mathbf{E}$, we can get the relation

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 \frac{\partial^2}{\partial t^2} \mathbf{D}. \quad (2.4)$$

The electric displacement $\mathbf{D}$ in a nonlinear material is written as

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_0 \mathbf{E} + (\mathbf{P}_L + \mathbf{P}_{NL}) \quad (2.5)$$

where we write the polarisation $\mathbf{P}$ as the sum of the linear term $\mathbf{P}_L$ and nonlinear term $\mathbf{P}_{NL}$.

From Eq. (2.4) and Eq. (2.5), we can get

$$\nabla^2 \mathbf{E} = \mu_0 \left(\epsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{E} + \frac{\partial^2}{\partial t^2} (\mathbf{P}_L + \mathbf{P}_{NL}) \right). \quad (2.6)$$

Assuming the pump field propagates along the x-axis, the above equations can be written as

$$\frac{\partial^2}{\partial x^2} E = \mu_0 \left(\epsilon_0 \frac{\partial^2}{\partial t^2} E + \frac{\partial^2}{\partial t^2} (P_L + P_{NL}) \right). \quad (2.7)$$

By substituting the linear term of polarisation P_L with $\epsilon_0 \chi^{(1)} E$, and moving the electric field terms to the left, we can rearrange the equation as

$$\frac{\partial^2}{\partial x^2} E - \mu_0 \epsilon_0 \left(1 + \chi^{(1)} \right) \frac{\partial^2}{\partial t^2} E = \mu_0 \frac{\partial^2}{\partial t^2} P_{NL}. \quad (2.8)$$

The vacuum permittivity and permeability is related to the speed of light as $\mu_0\epsilon_0 = 1/c^2$. Also, the refractive index n of a medium is defined as $n = \sqrt{1 + \chi^{(1)}}$. Based on these definitions, we are able to rewrite the above equation as

$$\frac{\partial^2}{\partial x^2}E - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}E = \mu_0\frac{\partial^2}{\partial t^2}P_{\text{NL}}. \quad (2.9)$$

To simplify the discussion, we can assume the two incident fields to be monochromatic, meaning the fields' plane waves are expressed as

$$E_{1,2} = A_{1,2}(x)\exp(i(\omega_{1,2}t - k_{1,2}x)) \quad (2.10)$$

where A is the amplitude of the field, ω is the angular frequency of the field, and k is the wave vector of the field.

In the case of sum frequency generation, the two fields induce a new field with angular frequency $\omega_3 = \omega_1 + \omega_2$. By substituting the electric field in Eq. (2.7) with the two incident fields, we can solve for the new field as

$$\frac{dA_3}{dx} = \frac{-2i\omega_3}{n_3c}d_{\text{eff}}A_1A_2e^{i\Delta kx} \quad (2.11)$$

where n_3 is the refractive index for the field, d_{eff} is a nonlinear coefficient associated with the crystal and the Cartesian axes, and Δk is the wave vector mismatch:

$$\Delta k = k_3 - k_1 - k_2 = \frac{n_3\omega_3}{c} - \frac{n_1\omega_1}{c} - \frac{n_2\omega_2}{c}. \quad (2.12)$$

By integrating Eq. (2.11), we can get the amplitude of the generated field after propagating over a distance L :

$$A_3(L) = \frac{2d_{\text{eff}}\omega_3A_1A_2}{n_3c}\left(\frac{e^{i\Delta kL} - 1}{\Delta k}\right). \quad (2.13)$$

The intensity of the generated field is the modulus square of the amplitude:

$$I_3 = \frac{2d_{\text{eff}}^2\omega_3^2I_1I_2}{n_1n_2n_3\epsilon_0c^3}L^2\left(\frac{\sin(\Delta kL/2)}{\Delta kL/2}\right)^2 = I_3^{\text{max}} \cdot \text{sinc}^2(\Delta kL/2). \quad (2.14)$$

The efficiency of the conversion depends on the wave vector mismatch Δk . When $\Delta k = 0$, the conversion intensity is maximised. When $\Delta k \neq 0$, the generated field is out of phase by π after propagating over a distance of the coherence length L_c , which is defined as

$$L_c = \frac{\pi}{\Delta k}. \quad (2.15)$$

When the interaction distance reaches $2L_c$, destructive interference cancels out the generated field. To achieve a high conversion efficiency, the nonlinear crystal can be manufactured with phase-matching

conditions which we will discuss later in this section.

The general three-wave process involves three fields with frequencies linked by $\omega_1 + \omega_2 = \omega_3$ under the second-order nonlinearity interaction. The above discussion derived the intensity of sum frequency generation. The second harmonic generation is a special case of sum frequency generation. In the case of second harmonic generation, we use $\omega_1 = \omega_2$ for the field analysis.

Parametric down conversion (PDC) is a non-linear process in which one pump photon with frequency ω_3 is converted into two photons ω_1 and ω_2 under the conservation of energy and momentum. The PDC process is illustrated by Fig. 2.2a, where the two photons are labelled as signal photon and idler photon. Fig. 2.2b shows the energy band diagram for PDC. From the diagram, we can find the energy conservation relation $\omega_p = \omega_s + \omega_i$.

There are different types of PDC due to the birefringence of nonlinear crystals. They are categorised depending on the polarisation relationship among the three fields. Type 0 PDC generates amplified fields with the same polarisation as the pump field. Type I PDC generates both signal and idler fields with the same polarisation that is orthogonal to the pump field. Type II PDC generates orthogonally polarised signal and idler fields. The type of SPDC depends on the phase-matching of the nonlinear crystal.

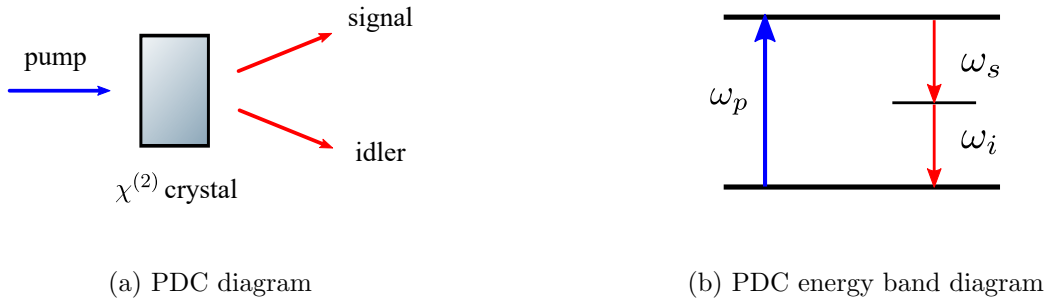


Figure 2.2: Diagram of PDC process and energy relations. (a) Process of parametric down-conversion. The nonlinear $\chi^{(2)}$ crystal converts one pump photon into two photons named signal and idler. (b) Energy band relations between pump, signal, and idler photon.

In spontaneous parametric down conversion (SPDC), the parametric gain is low and the probability of generating photons pairs is small. The above derivations can only describe interactions involving at least two fields, such as sum frequency generation. Second harmonic generation is considered as a special case of sum frequency generation when the two fields have the same descriptions. For an SPDC process, we are not able to describe the creation of two fields using a classical description because there is only one field in the nonlinear medium. Using only one field to calculate the amplitude above leads to the conclusion that A_1 , A_2 , and A_3 are all equal to zero. Therefore, we need to employ the quantum description with second-quantization formalism in order to explain the signal and idler fields.

Quantum description of PDC

The classical Hamiltonian describing the electric field interaction is

$$H \propto \int E(r, t) \cdot D(r, t) dr^3. \quad (2.16)$$

For the $\chi^{(2)}$ process, we focus on the 2nd order term in the displacement vector D :

$$H_{\chi^{(2)}} \propto \int \chi^{(2)} E_p(r, t) E_s(r, t) E_i(r, t) dr^3 \quad (2.17)$$

where p, s, and i label the pump, signal, and idler modes.

With Eq. (2.17) and quantizing electric fields, the PDC process has a quantum description as [110, 111]

$$\hat{H}_{\text{PDC}} \propto \chi^{(2)} \int_{-L/2}^{L/2} dz \hat{E}_p^{(+)}(z, t) \hat{E}_s^{(-)}(z, t) \hat{E}_i^{(-)}(z, t) + \text{h.c.} \quad (2.18)$$

where L is the crystal length along the optical axis. For mode x, the quantum electric field operators propagating towards z direction are given by

$$\hat{E}_x^{(+)} = \hat{E}_x^{(-)\dagger} = A \int d\omega_x \exp(i(k_x(\omega_x)z - \omega_x t)) \hat{a}_x(\omega_x) \quad (2.19)$$

where A is a constant which includes all constant factors, and $\hat{a}_x(\omega_x)$ is the annihilation operator for a monochromatic frequency ω_x with wavevector $k_x(\omega_x)$. Because the nonlinear interaction is weak in typical crystals, the pump field needs to be strong for the PDC to be significant. Therefore we approximated the strong pump as a classical field:

$$E_p^{(+)} = E_p^{(-)*} = \int d\omega_p \alpha_p(\omega_p) \exp(i(k_p(\omega_p)z - \omega_p t)) \quad (2.20)$$

where $\alpha_p(\omega_p)$ represents the spectral amplitude of the pump field. For a continuous-wave laser with centre frequency ω_0 , $\alpha_p(\omega_p)$ can be written as $E_p \delta(\omega_p - \omega_0)$. In contrast, $\alpha_p(\omega_p)$ has a broader distribution for a pulsed laser.

In the Schrödinger picture, the state generated by $\mathcal{H}_{\text{PDC}}$ is given by

$$|\Psi\rangle_{\text{PDC}} = \exp\left(-\frac{i}{\hbar} \int_{t_0}^t dt' \hat{H}_{\text{PDC}}(t')\right) |0\rangle. \quad (2.21)$$

In order to explore the PDC effect in the low gain regime, we take an approximation that higher orders in the perturbation form can be neglected. Therefore, the generated state with zero and first orders is

$$|\Psi\rangle_{\text{PDC}} \approx |0\rangle - \frac{i}{\hbar} \int_{t_0}^t dt' \hat{H}_{\text{PDC}}(t') |0\rangle. \quad (2.22)$$

With the above equations, we arrive at

$$\begin{aligned}
\int_{t_0}^t dt' \hat{H}_{\text{PDC}}(t') &= B \int_{t_0}^t dt' \int_{-L/2}^{L/2} dz \int \int \int d\omega_p d\omega_s d\omega_i \alpha_p(\omega_p) \exp(-i(\omega_p - \omega_s - \omega_i)t') \\
&\quad \times \exp(i(k_p(\omega_p) - k_s(\omega_s) - k_i(\omega_i))z) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) + \text{h.c.} \\
&= B \int_{t_0}^t dt' \int \int \int d\omega_p d\omega_s d\omega_i \alpha_p(\omega_p) \exp(-i(\omega_p - \omega_s - \omega_i)t') \\
&\quad \times L \text{sinc}\left((k_p(\omega_p) - k_s(\omega_s) - k_i(\omega_i))\frac{L}{2}\right) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) + \text{h.c.}
\end{aligned} \tag{2.23}$$

with an overall factor B introduced for convenience.

The z integration is evaluated along the crystal length. Because the state is regarded much longer than the interaction time in the crystal, the time interval takes the approximation that the bound is extended to infinity. Therefore, the time-dependent component integration gives $2\pi\delta(\omega_p - \omega_s - \omega_i)$:

$$\int_{-\infty}^{\infty} dt' \hat{H}_{\text{PDC}}(t') = 2\pi B \int \int d\omega_s d\omega_i \alpha(\omega_s + \omega_i) L \text{sinc}\left(\Delta k(\omega_s, \omega_i)\frac{L}{2}\right) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) + \text{h.c.} \tag{2.24}$$

where $\Delta k(\omega_s, \omega_i) = k_p(\omega_s + \omega_i) - k_s(\omega_s) - k_i(\omega_i)$, which is the difference between the pump wavevector and the generated wavevectors.

With Eq. (2.24), we can write the approximate generated state as

$$\begin{aligned}
|\Psi\rangle &= |0\rangle + B' \int \int d\omega_s d\omega_i \alpha(\omega_s + \omega_i) \text{sinc}\left(\Delta k(\omega_s, \omega_i)\frac{L}{2}\right) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) |0\rangle \\
&= |0\rangle + B' \int \int d\omega_s d\omega_i f(\omega_s, \omega_i) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) |0\rangle
\end{aligned} \tag{2.25}$$

where $B' \propto E_p L$ is the scaling factor which depends on the crystal length L and pump field amplitude E_p , and $f(\omega_s, \omega_i)$ is the joint spectral amplitude (JSA) of the PDC generated two-photon state. $f(\omega_s, \omega_i)$ is defined by the pump distribution $\alpha(\omega_s + \omega_i)$ and the phase-matching function $\text{sinc}(\Delta k(\omega_s, \omega_i)\frac{L}{2})$, which is determined by the length and dispersion of the nonlinear crystal.

2.1.2 Squeezed states from PDC

After introducing the PDC quantum description with the joint spectral amplitude, it is informative to investigate the quantum expression of squeezed states generated from the PDC process and the relation of the squeezing operator with the PDC Hamiltonian. The joint spectral amplitude $f(\omega_s, \omega_i)$ can be decomposed into a sum over products of a bi-orthogonal normalised system of $\{A_n(\omega)\}$ and $\{B_n(\omega)\}$ [110]:

$$f(\omega_s, \omega_i) = \sum_n \sqrt{\lambda_n} A_n(\omega_s) B_n(\omega_i) \tag{2.26}$$

where $\sqrt{\lambda_n}$ is the Schmidt coefficient.

In a simple theory where only monochromatic modes with photon frequency ω_{s_0} and ω_{i_0} are generated, we write the JSA as

$$f(\omega_s, \omega_i) = \delta(\omega_s - \omega_{s_0})\delta(\omega_i - \omega_{i_0}). \quad (2.27)$$

Then Eq. (2.25) can be re-written as

$$|\Psi\rangle = \exp\left(B'(\hat{a}_s^\dagger(\omega_{s_0})\hat{a}_i^\dagger(\omega_{i_0}) + \text{h.c.})\right)|\text{vac}\rangle. \quad (2.28)$$

In reality, the generated fields are usually not monochromatic. We define broadband mode operators $\hat{A}_n^\dagger$ and $\hat{B}_n^\dagger$ which shows the frequency distribution for signal and idler fields

$$\hat{A}_n^\dagger = \int d\omega A_n(\omega)\hat{a}_s^\dagger(\omega) \quad (2.29)$$

and

$$\hat{B}_n^\dagger = \int d\omega B_n(\omega)\hat{a}_i^\dagger(\omega). \quad (2.30)$$

With the broadband mode operators, we can write the generated state as

$$\begin{aligned} |\Psi\rangle &= \exp\left(B'\left(\int \int d\omega_s d\omega_i \sum_n \sqrt{\lambda_n} A_n(\omega_s) B_n(\omega_i) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) + \text{h.c.}\right)\right)|\text{vac}\rangle \\ &= \exp\left(B' \sum_n \sqrt{\lambda_n} \left(\int d\omega_s A_n(\omega_s) \hat{a}_s^\dagger(\omega_s)\right) \left(\int d\omega_i B_n(\omega_i) \hat{a}_i^\dagger(\omega_i)\right) + \text{h.c.}\right)|\text{vac}\rangle \\ &= \exp\left(B' \sum_n \sqrt{\lambda_n} (\hat{A}_n^\dagger \hat{B}_n^\dagger + \text{h.c.})\right)|\text{vac}\rangle. \end{aligned} \quad (2.31)$$

This expression has the same form as $\hat{S}_2(\zeta) = \exp(-\zeta\hat{a}\hat{b} + \zeta^*\hat{a}^\dagger\hat{b}^\dagger)$ in Eq. (1.20), where the operator is the exponential of a tensor product of the creation operators for two fields.

We can write the PDC Hamiltonian in respect of the two fields:

$$\int dt' \hat{H}_{\text{PDC}}(t') \propto \sum_n \left(\sqrt{\lambda_n} \hat{A}_n^\dagger \hat{B}_n^\dagger + \text{h.c.}\right) \quad (2.32)$$

where A and B correspond to signal and idler fields in the PDC Hamiltonian. Thus, the PDC Hamiltonian has the following relation with the squeezing operator:

$$e^{\int dt' \hat{H}_{\text{PDC}}(t')} = \bigotimes_n \hat{S}_2\left(C\sqrt{\lambda_n}\right) \quad (2.33)$$

where C is the factor that corresponds to the proportional factor in Eq. (2.32).

In an SPDC experiment using a continuous-wave pump, the most populated mode contains only a small number of total photon flux. This indicates the output state is strongly correlated and has

a large number of n . A desirable squeezed light needs to isolate the squeezing to a single mode of photon pairs so that the expression in Eq. (2.33) contains approximately one squeezing parameter. In other words, all but one λ_n are approximately equal to zero.

Is it possible to isolate squeezing to a single pair of states? U'Ren et al. suggest the factorisability condition of the state can be expressed with the crystal and pump parameters [112]:

$$\frac{4}{\sigma^2} + \gamma L^2 (k'_s - k'_p) (k'_i - k'_p) = 0 \quad (2.34)$$

where σ is the pump bandwidth, L is the crystal length, k'_p represents the first frequency derivative of k_p evaluated at $2\omega_0$, k'_μ ($\mu = s, i$) represents the first frequency derivative of k_μ evaluated at ω_0 with $\gamma = 0.193$. To fulfil the factorisability condition, the group velocity of the pump must be between the group velocity of signal and idler, i.e. one of $k'_s < k'_p < k'_i$ or $k'_i < k'_p < k'_s$ must be true. When designing the PDC squeezed state source, our objective is to use materials and frequencies that satisfy the group velocity condition.

2.1.3 Birefringent phase matching and quasi phase matching

In Eq. (2.25), the generated squeezed state's amplitude is proportional to $\text{sinc}(\Delta k(\omega_s, \omega_i) \frac{L}{2})$. This indicates that the generated state is impacted by the product of the wavevector difference Δk and the crystal length L . When $\Delta k = 0$, the second order nonlinear effect has the highest efficiency. We refer to the situation when the relative wavevector difference is equal to zero as the phase matching condition [109]. The phase matching is sensitive to the incident angle and the crystal temperature. If the phase matching is not precisely zero, the intensity of nonlinear interaction with a fixed propagation distance L would be lower than the quadratic relation under perfect phase matching. When L is larger than the coherence length, the nonlinear interaction would reverse the process and oscillate along the propagation distance.

In order to maintain a high interaction intensity, we can periodically change the second-order susceptibility by altering the orientation of the material when the light propagates for a distance of coherence length. This approach of phase matching is called quasi phase matching (QPM). The susceptibility reverses every time when the accumulated phase match reaches π along the propagation direction, so that the output power increases along the propagation axis. This process of periodic alternate orientation is called periodic poling.

The generated signal field from PDC after integration along the crystal is [113]

$$E_s \propto L d_Q \text{sinc}\left(\Delta k_Q \frac{L}{2}\right) \quad (2.35)$$

where d_Q is the effective nonlinear coefficient in QPM.

The Hamiltonian for QPM becomes

$$\int_{t_0}^t dt \hat{H}_{\text{PDC}}(t') = 2\pi B d_Q \int \int d\omega_s d\omega_i \alpha(\omega_s + \omega_i) \times L \text{sinc}\left(\Delta k_Q \frac{L}{2}\right) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) + \text{h.c.} \quad (2.36)$$

The wave vector mismatch for QPM Δk_Q is expressed as

$$\Delta k_Q = k_p - k_s - k_i - k_\Lambda = \Delta k_{\text{bulk}} - k_\Lambda \quad (2.37)$$

where the k_Λ is the grating vector which is related to the poling period Λ by

$$k_\Lambda = \frac{2\pi}{\Lambda}. \quad (2.38)$$

This equation shows that for a non-zero crystal wave vector mismatch, we can use the periodic poling to achieve a fully phase-matched interaction. The phase matching can also be tuned by changing the temperature, which results in the wavelength-dependent refractive index change. The wave vector depends on the refractive index by

$$k(\lambda, T) = \frac{n(\lambda, T)\omega}{c}. \quad (2.39)$$

The grating vector k_λ can also be affected by temperature changes as the poling period would change from the thermal expansion.

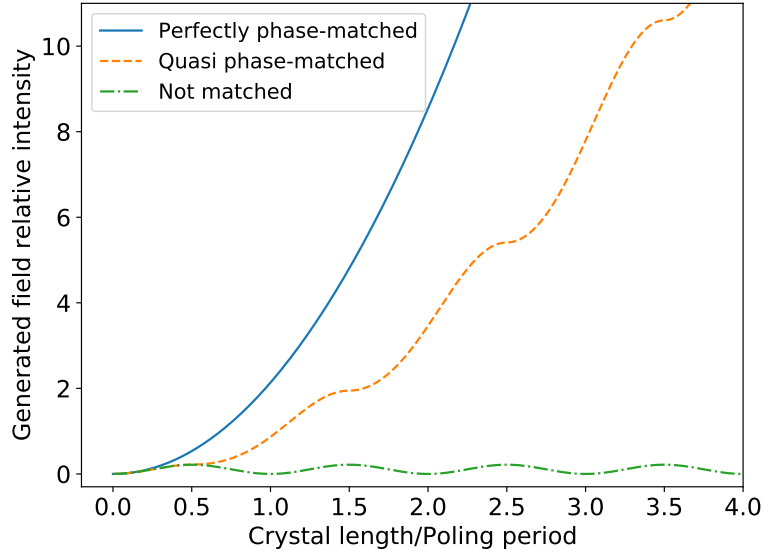


Figure 2.3: Comparison among different type of second order nonlinear crystal phase matching. The generated signal intensity along the nonlinear crystal length are impacted by the choice of phase-matching. The perfectly phase-matched crystal presents a quadratic relation along the crystal length, yet it is also the phase matching type that requires the highest accuracy. The first order quasi phase matching in dashed line provides a much larger intensity generation comparing with the not matched crystal in dash-dotted line.

Fig. 2.3 presents the phase-matching comparison between perfect birefringent phase-matching and the quasi phase-matching. The intensity of the perfect birefringent phase-matching grows quadratically

with crystal length. The quasi phase-matching grows periodically along the crystal length. Both approaches achieve much higher interaction intensity comparing with the non-phase-matched scenario. We need to choose a phase-matched crystal for our design in order to obtain a high interaction efficiency.

2.2 Cavity enhanced squeezed state generation

Although the phase-matching increases the nonlinear interaction intensity, in a free-space SPDC the pump travels only once through the crystal. The output intensity is limited by the single-pass interaction. Therefore, we introduce the optical cavity to enhance the nonlinear interactions between the pump and the crystal. We model a type II SPDC output, which includes orthogonally polarised signal and idler fields. With a birefringent crystal, the optical path lengths of the signal and idler fields are different.

Jeronimo-Moreno et al. studied the effect of cavity enhancement on SPDC photon pair generation [114], and their results are the basis for the following theory. The simple model discussed is the SPDC process with a pulsed pump in a cavity with two reflection mirrors and a nonlinear crystal sitting between the mirrors. For simplicity, we will first explore the doubly resonant cavity, in which case the signal and idler fields are resonant with the cavity, and then concentrate on the triply resonant cavity, where the signal, idler, and pump fields are resonant with the cavity.

In this section, we start with a brief introduction on the optical resonator theory. Then we investigate what we expect from the type II SPDC in a doubly resonant cavity and a triply resonant cavity adapted from the Jeronimo-Moreno paper.

2.2.1 Optical resonator

An optical resonator, also known as an optical cavity, is an arrangement of mirrors that constructs standing waves for light. The standing waves present resonant frequencies and transverse modes under the cavity configuration. The geometry of the cavity affects the cavity stability and alignment accuracy. A cavity can be characterised with free spectral range, linewidth, and finesse. The theory of optical resonators follows the presentation in [115].

Cavity modes

The light would propagate inside the cavity for multiple round-trips from mirror reflection. The interference at the surface of the mirror allows modes with certain frequency and spatial distribution to exit the cavity. There are two types of modes: longitudinal modes and transverse modes.

The longitudinal modes are the frequencies that form standing waves with constructive interference at the mirror of the cavity. A longitudinal mode is resonant with the cavity. The wavelength of the

longitudinal mode λ satisfies the relation with the optical path length of the resonator L_{opt} :

$$L_{\text{opt}} = q \times \frac{\lambda}{2} \quad (2.40)$$

where q is an integer number, which means the optical path length should be a multiple of half the wavelength.

The transverse modes in a cavity are the transverse eigenmodes that can reproduce their amplitude and phase profile with finite loss after repeated reflections inside the cavity. They describe the energy distribution of the modes in the beam cross section. The lowest-order transverse mode usually has the lowest spatial loss with the smallest diameter compared to higher order modes. Higher order modes are generated from slight misalignment of the cavity mirrors, which creates different optical path lengths.

Cavity stability and geometry

Simple cavities with a two-mirror structure are widely used in optical applications. We introduce the criteria for a stable cavity and some of the common geometries for a two-mirror cavity.

To find the cavity stability criteria, we start from two mirrors M_1 and M_2 with radius of curvature R_1 and R_2 . The optical distance between the two mirrors is L . We introduce the cavity g parameters which are helpful to define the range of stability region:

$$g_1 = 1 - \frac{L}{R_1} \quad \text{and} \quad g_2 = 1 - \frac{L}{R_2}. \quad (2.41)$$

The stability range requires

$$0 \leq g_1 g_2 \leq 1. \quad (2.42)$$

The range is shown in Fig. 2.4. Within the shaded area, the cavity structure provides a stable cavity output. Here we introduce three common geometries: confocal, concentric, and hemispherical.

A confocal cavity is a resonator that focal points of mirrors are at the centre of the other mirror. It means the cavity length is equal to the mirror's radius of curvature $L = R_1 = R_2$. The confocal cavity is insensitive to misalignment of the mirrors and has a small mode size which indicates low spatial loss inside the cavity.

A concentric cavity is a resonator with the cavity length slightly smaller than the sum of two mirrors radius of curvature. It satisfies $R_1 \approx R_2 \approx L/2 + \Delta L$. For a small-sized cavity, it has a larger mode size on the mirror and is very sensitive to misalignment. The high focusing of the pump field uses limited volume of the medium.

A hemispherical cavity is a resonator with one plane mirror and one curved mirror. The parameters are $R_1 = \infty$ and $R_2 = L + \Delta L$. The plane mirror can be formed with a coated surface on an optical

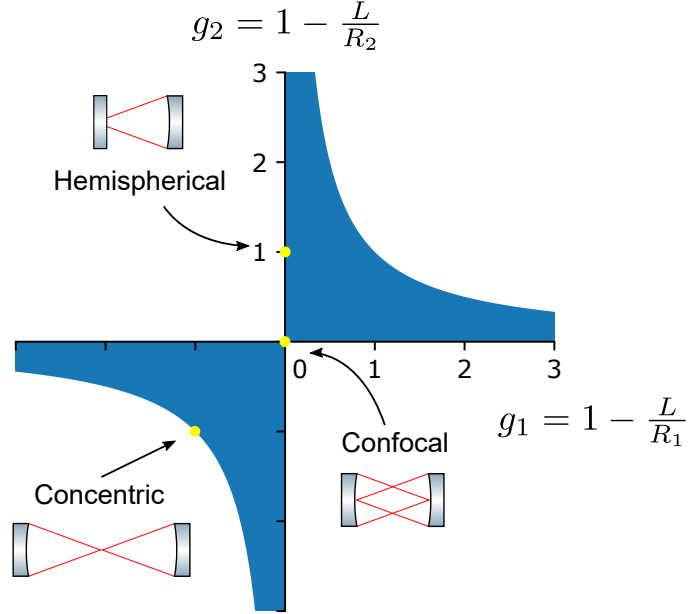


Figure 2.4: The stability diagram for a two-mirror optical resonator. The cavity is in a stable state with a repeatable mode inside the coloured region. Outside the coloured region, the cavity mode would grow too much in beam size, resulting in an unstable mode. Some typical cavity structures are marked with yellow dots in the figure: confocal, concentric, and hemispherical structures are at the edge of the stability diagram.

component. The hemispherical cavity is half of the concentric cavity but carries the benefit of being insensitive to translation and tilt misalignment.

Cavity characteristics

In order to perform SPDC inside a cavity, we consider an optical cavity with configuration shown in Fig. 2.5. The two mirrors M_1 and M_2 have reflection coefficients r_1 and r_2 , and radius of curvature R_1 and R_2 . The nonlinear crystal is between the two mirrors with length L_{cry} . The round-trip loss coefficient α_{RT} for light field includes the transmission on two mirrors ($t_1 = \sqrt{1 - r_1^2}$ and $t_2 = \sqrt{1 - r_2^2}$ respectively) and internal cavity loss α_{int} .

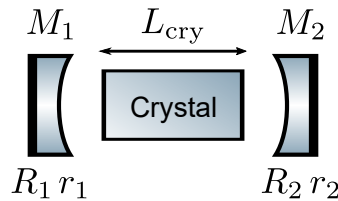


Figure 2.5: Configuration of a two-mirror optical cavity containing a crystal. The crystal length is L_{cry} . On the side, the two mirrors are M_1 and M_2 with radius of curvature R_1 and R_2 . When $R = \infty$, the mirror is flat. The total reflectivities of the mirrors are r_1 and r_2 correspondingly.

The spacing of longitudinal cavity modes in frequency is called the free spectral range (FSR). By

definition, the FSR is expressed as

$$\text{FSR} = \frac{c}{2L_{\text{OPL}}} \quad (2.43)$$

where c is the speed of light in vacuum, and L_{OPL} is the optical path length which takes the refractive index of the medium into account. The resonant modes' full-width half maximum (FWHM) in frequency is the linewidth of the cavity $\delta\nu$. Here we introduce finesse of the cavity to show how sharp the resonance curve is in respect to the FSR. The finesse is defined as the ratio between FSR and cavity linewidth $\delta\nu$:

$$\mathcal{F} = \frac{\text{FSR}}{\delta\nu}. \quad (2.44)$$

Fig. 2.6a illustrates the FSR and linewidth relationship at different finesse. The FSR is the frequency spacing between cavity modes. The cavity linewidth $\delta\nu$ marks the FWHM of the transmission peak. With a higher finesse, the cavity linewidth becomes narrower.

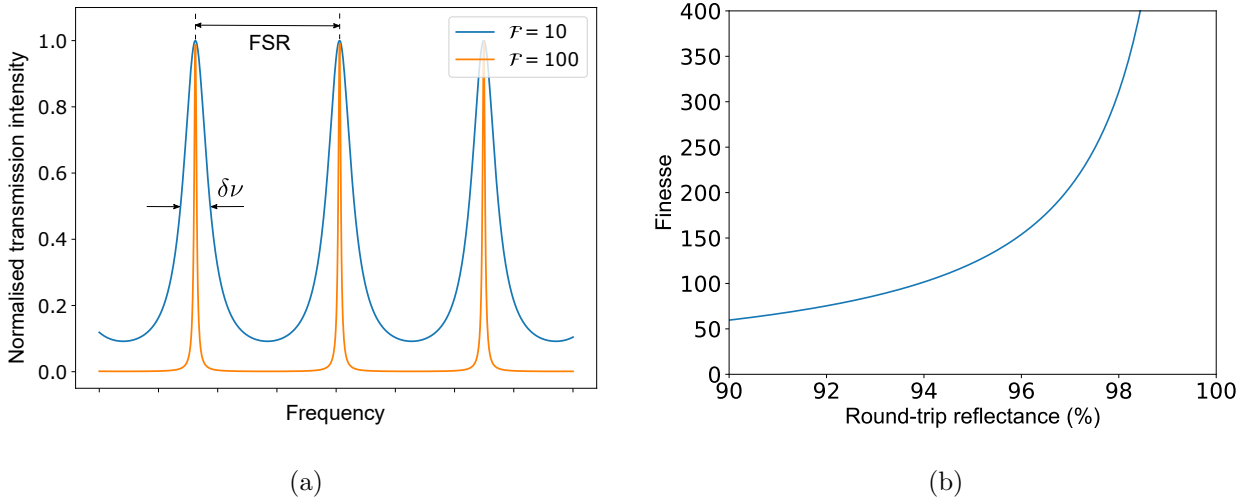


Figure 2.6: Cavity finesse definition and relation with the cavity round-trip reflectance. (a) Cavity transmission relation with frequency. The resonant frequency enables transmission of the field. The frequency difference between neighbour resonant frequencies is the free spectral range (FSR). The full-width half maximum (FWHM) is the cavity linewidth $\delta\nu$. The finesse is the ratio between FSR and cavity linewidth $\delta\nu$. With a larger cavity finesse, the transmission peak is narrower and sharper. (b) Relationship between finesse and cavity reflectance. The reflectance here is the product of reflectance of both mirrors. The higher the reflectance, the higher the finesse.

With a high finesse, the resonance curves are narrow, which also suggests the high reflectivity of the cavity. The reflectance is the modulus square of the reflection coefficient $R_i = |r_i|^2$, where $i = 1, 2$ specifying the mirror. The finesse depends on the reflectance of the mirrors as

$$\mathcal{F} \approx \frac{\pi(R_1 R_2)^{\frac{1}{4}}}{1 - (R_1 R_2)^{\frac{1}{2}}}. \quad (2.45)$$

This relationship is shown in Fig. 2.6b. The round-trip reflectance is the product of the mirrors' reflectance $R_1 R_2$. The finesse increases rapidly with higher round-trip reflectance.

2.2.2 Doubly resonant cavity

For convenience of discussion, let's assume that the mirrors are transmissive for the pump mode and each pump pulse interacts with the crystal once. After one single trip, the two-photon state can be expressed as

$$|\Psi\rangle = \int d\omega_s \int d\omega_i f(\omega_s, \omega_i) a_s^\dagger(\omega_s) a_i^\dagger(\omega_i) |\text{vac}\rangle \quad (2.46)$$

where $f(\omega_s, \omega_i)$ is the joint spectral amplitude we previously introduced and $a_\mu^\dagger(\omega_\mu)$ is a creation operator at frequency ω_μ for the cavity mode ($\mu = s, i$). The two-photon state is the same as the state that is observed without the cavity.

Jeronimo-Moreno et al. [114] show that the cavity emission is the product of the joint spectrum and the cavity transmission functions:

$$S_{\text{DR}}(\omega_s, \omega_i) = \mathcal{A}_s(\omega_s) \mathcal{A}_i(\omega_i) |f(\omega_s, \omega_i)|^2 \quad (2.47)$$

with the cavity transmission function as an Airy function $\mathcal{A}_\mu(\omega)$ ($\mu = s, i$) defined as

$$\mathcal{A}_\mu(\omega) = \frac{|t_{2\mu}|^2}{(1 - |r_{2\mu}|)^2} \frac{1}{1 + \mathcal{F}_\mu \sin^2(\Delta_\mu(\omega)/2)} \quad (2.48)$$

where $t_{2\mu}$ and $r_{2\mu}$ are the transmissivity and reflectivity of the second mirror respectively, and the coefficient of finesse $\mathcal{F}_\mu$ is given by

$$\mathcal{F}_\mu = \frac{4|r_{2\mu}|}{(1 - |r_{2\mu}|)^2} \quad (2.49)$$

and the phase factor Δ_μ is the total phase accumulated by field μ in one round-trip.

The Airy function in Eq. (2.48) represents a serie of equal-height peaks with spectral width $\delta\omega$ and resonant frequency separation $\Delta\omega$, which is also referred to as the free spectral range (FSR). The cavity is resonant for field μ at the peak of the Airy function $\mathcal{A}_\mu(\omega)$. With the assumption that the refractive index remains constant between the central peak and the subsequent peak, the spectral width and FSR can be expressed as

$$\delta\omega = \frac{2c}{n(\omega_0)l + (L - l)} \mathcal{F}^{-1/2} \quad (2.50)$$

and

$$\Delta\omega = \frac{\pi c}{n(\omega_0)l + (L - l)} \quad (2.51)$$

where $n(\omega_0)$ is the refractive index at the central SPDC frequency.

Fig. 2.7 shows the joint spectral intensity for free space, cavity supporting modes, and the cavity modified joint spectral intensity. The output joint spectral intensity is showing narrow bandwidth due to the cavity enhancement. The bandwidth of cavity-supporting modes depends on the inverse of

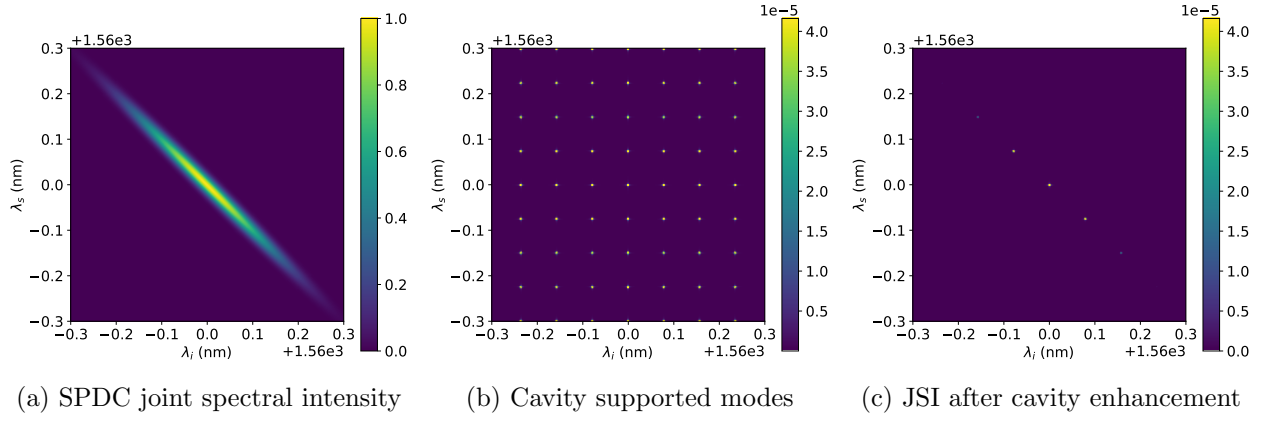


Figure 2.7: Cavity impact for the joint spectral intensity. (a) Joint spectral intensity in the free space with a phase-matched PPKTP crystal. (b) Cavity resonant modes in the joint spectrum picture. By putting the crystal inside the cavity, we combine the two joint spectrum to get a cavity enhanced JSI shown in (c). Colorbars are rescaled for (b) and (c) to better present cavity resonance.

square root of cavity finesse as $\mathcal{F}^{-1/2}$. A large finesse would lead to a narrow output bandwidth. The cavity length L is related to the FSR which characterise the resonant frequency separation $\Delta\omega$. A shorter cavity length L corresponds to a larger separation between cavity supporting modes. Given a wide enough pump bandwidth, more separated cavity resonant modes means less modes overlapping with the SPDC phase matching. This provides a higher chance to generate a more factorisable mode for SPDC output.

With the above discussions, we can see that the doubly-resonant cavity clearly changes the spectrum of the generated light. However, the photon generation rate in this case is low as the nonlinear process has low probability, especially when the pump only interacts with the crystal once every pulse [114]. Thus, we now consider a triply resonant cavity, which takes the pump mode resonance into account, to explore the joint spectral characteristics of the output mode in this case.

2.2.3 Triply resonant cavity

In a triply resonant cavity, the signal mode, idler mode, and pump mode are all resonant with the cavity. The resonance keeps the pump mode in the cavity to interact with the nonlinear crystal with sufficient times to achieve an increased generation rate. In a triply resonant cavity, Jeronimo-Moreno et al. show that the cavity-enhanced joint spectral intensity can be written as

$$S_{\text{TR}}(\omega_s, \omega_i) = \mathcal{A}_s(\omega_s) \mathcal{A}_i(\omega_i) \mathcal{A}_p(\omega_s + \omega_i) \mathcal{P}(\omega_s, \omega_i) |f(\omega_s, \omega_i)|^2 \quad (2.52)$$

where the pump frequency ω_p is set equal to $\omega_s + \omega_i$ the sum of the signal and idler frequency due to energy conservation, and $\mathcal{A}_p(\omega)$ is the pump mode Airy function given by

$$\mathcal{A}_p(\omega) = \frac{|t_{1p}|^2}{(1 - |r_{1p}r_{2p}|)^2} \frac{1}{1 + \mathcal{F}_p \sin^2(\Delta_p(\omega)/2)} \quad (2.53)$$

where the phase factor $\Delta_p(\omega)$ is the accumulated phase for the pump mode in the cavity, the pump mode finesse is

$$\mathcal{F}_p = \frac{4|r_{1p}r_{2p}|}{(1 - |r_{1p}r_{2p}|)^2} \quad (2.54)$$

and the phase balancing factor $\mathcal{P}(\omega_s, \omega_i)$ is given by

$$\mathcal{P}(\omega_s, \omega_i) = (1 + |r_{2p}|^2) \left(1 - \frac{4|r_{2p}|}{(1 + |r_{2p}|)^2} \sin^2(\Delta(\omega_s, \omega_i)/2)\right) \quad (2.55)$$

where the phase term $\Delta(\omega_s, \omega_i)$ is the total phase accumulated by the signal, idler, and pump modes during a round-trip of the pump field. Eq. (2.55) requires the following conditions to maximise the probability of photon-pair emission at given frequencies $\{\omega_{s0}, \omega_{i0}\}$: the signal and idler modes are resonant with the cavity, which we discussed in the previous section; the pump mode is resonant with the cavity; and the phase balancing term is maximised. A value of $\Delta(\omega_{s0}, \omega_{i0})$ that is an integer multiple of π maximises the phase balancing term. It occurs when the signal and idler field amplitudes created in one pump round-trip add constructively to those created in the previous round-trip.

Fig. 2.8 illustrates the impact on the JSI inside a triply resonant cavity compared with the JSI in a doubly resonant cavity. Fig. 2.8a is the doubly resonant cavity JSI, consisting of a stream of dots on the diagonal direction. Each dot is a single mode of photon pair generated from the SPDC. The JSI picture presents a mixture of multiple modes of photon pairs. The pump resonance is shown as diagonal lines in Fig. 2.8b due to energy conservation. By changing the cavity reflectance of the pump field, the pump resonance width can be adjusted to enhance different number of modes. The phase balancing in the cavity is maximised to line up with the three-field resonances. The output of the triply resonant cavity is the product of the three fields' resonances and the phase balancing function. Due to the type II SPDC's orthogonally polarised output and crystal birefringence, the free spectral range of signal and idler fields are different. This leads to a single mode of photon pair in the JSI as shown in Fig. 2.8d. Comparing the JSI from a doubly resonant cavity and a triply resonant cavity, we can find that the triple resonance provides a much better mode isolation for the output state. The main effect of mode isolation is constrained by the spectral width of the pump resonance, as a wide pump resonance covers multiple modes and a narrow pump resonance distorts the mode shape. The phase balancing factor impacts the strength of the output mode. When the three fields are in-phase, the mode achieves the maximum output.

The above equations and figures suggest that a triply resonant cavity is an approach to generate squeezed light that is separable in signal and idler frequency modes with narrow bandwidth. The joint spectral intensity suggests that the type II SPDC process provides a narrow bandwidth separable state without a spectral filter because the birefringence of the optical crystal offers different FSR for different polarisations. Thus, a type II SPDC is preferred to build the squeezed state source. In a type II SPDC cavity, the three modes have different refractive indexes, which leads to different optical

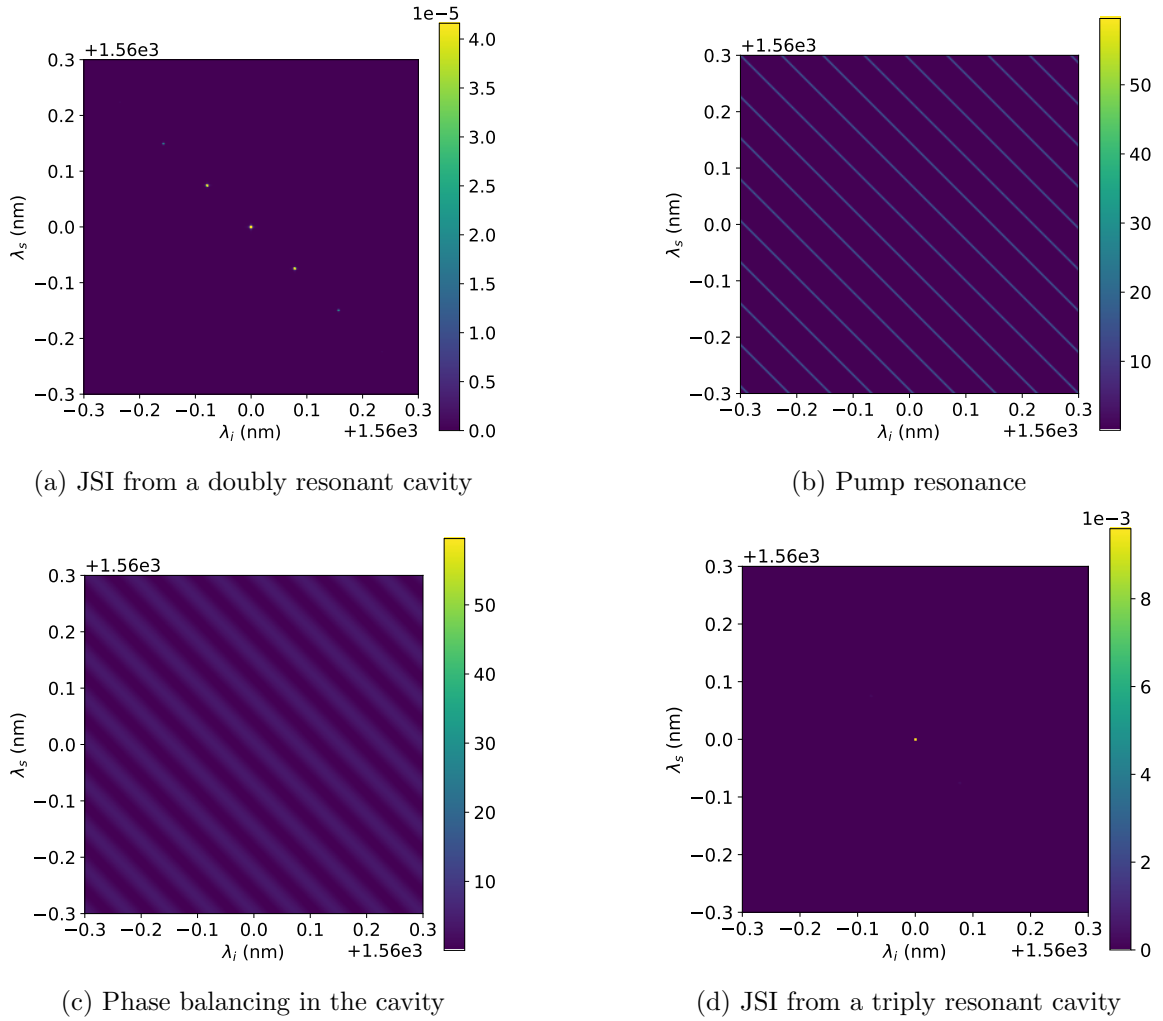


Figure 2.8: JSI modification in a triply resonant cavity. The JSI in a doubly resonant cavity shown in (a) combines with pump resonances in (b). The phase balance among pump field and signal, idler field shown in (c) plays an important role in matching the output central mode. (d) Shows the triple resonance cavity mode JSI. Colorbars are rescaled in (a) and (d) to better present cavity resonance.

path lengths. The triply resonant cavity demands the optical path length as an integer number of wavelength for each mode, which is a challenge to achieve in practice. We will discuss more about how to physically achieve triple resonance in Chapter 4. The JTI is also impacted by the cavity. Cavity enhanced JTA is calculated by the convolution of the Fourier transforms from JSA of signal resonance, pump resonance, and phase balancing factor. In the ideal case when the mode number is 1, both the JSA and JTA are separable.

In this chapter we investigated squeezed states generation from parametric down conversion inside a triply resonant cavity. We began with the classic theory of three-wave mixing and progressed to a quantum description of parametric down conversion. Next, we explored the relationship between the squeezing operator and the PDC Hamiltonian. Then we presented basic optical resonator theory with cavity modes, stability, and characteristics. Finally, we discussed the impact on SPDC photon pair generation from the cavity triple resonance and defined the conditions to generate a separable state.

After establishing the theoretical foundations, we move on to introduce the experimental design of the cavity in the next chapter. We will discuss both the optical and mechanical perspectives, as well as electrical controls for cavity tuning.

Chapter 3

Cavity design

Our exploration of three-wave nonlinear interaction, parametric down conversion, and cavity-enhanced squeezed state source forms the theoretical basis for this project's approach to identify the ideal spectral properties for the isolation of squeezing on a single pair of frequency modes. In this chapter, we discuss the practical design of an optical cavity, including its specifications for optics and custom optomechanics. Additionally, we describe electronic control of the temperature and length of the cavity, which can be used to fine-tune the operating parameters.

3.1 Optical design

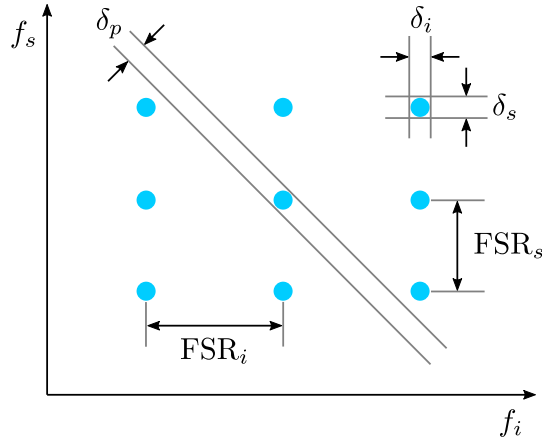


Figure 3.1: Single mode isolation using triple resonance. The blue region is a schematic representation of the cavity resonance joint spectrum. The distance between adjacent modes is the free spectral range (FSR). δ is the width of the mode. The subscript p, s, and i represent pump, signal, and idler field correspondingly. The FSR_s and FSR_i are different due to the birefringence of the crystal. By engineering the reflectivities and the pump bandwidth, the central mode can be isolated.

As mentioned in [section 2.2](#), we expect the cavity design to help isolate a single mode in the joint spectral intensity. The triply resonant cavity with a birefringent crystal provides possibilities to single mode isolation as shown in Fig. 3.1. The crystal birefringence contributes to different free spectral range for signal and idler fields. The pump resonance needs to be wide enough to isolate a single

mode and also narrow enough to not overlap with adjacent modes. With a shorter cavity optical path length, the free spectral range of the cavity is longer. It provides more flexibility for the required pump finesse to achieve single mode isolation. We chose to design the cavity optical path to be close to 10 mm in length in order to match the smallest radius of curvature given the current range of commercially available curved mirrors.

In this section, we start from an overview of the cavity. Then we introduce the cavity optical design with respect to the stability, higher-order transverse modes, finesse and reflectivity, the nonlinear crystal, and beam size matching.

3.1.1 Cavity overview

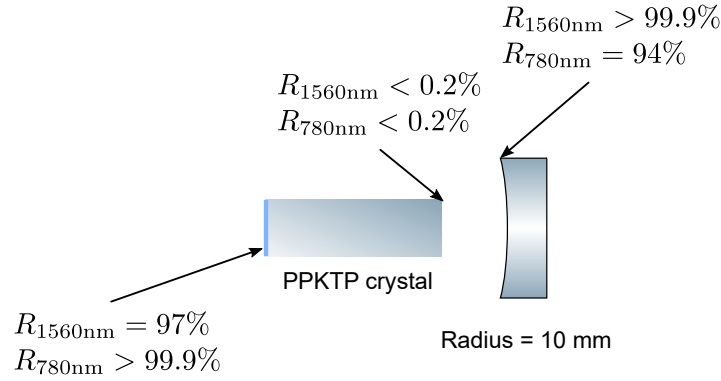


Figure 3.2: Cavity structure. Formed by a coated PPKTP crystal and a curved mirror, the cavity has an optical path length shorter than 10 mm. The type II phase-matched PPKTP has a length of 4 mm. The coating on the crystal has 97 % reflectance for 1560 nm and larger than 99.9 % reflectance for 780 nm. The inner side of the crystal surface is AR coated for both 780 nm and 1560 nm. The curved mirror has larger than 99.9 % reflectance for 1560 nm and 94 % reflectance for 780 nm.

As shown in Fig. 3.2, the cavity consists of a HR-coated curved mirror and a periodically-poled KTP (PPKTP) crystal whose one side is HR-coated and the other side is AR-coated. The curved mirror from Layertec has a 10 mm radius of curvature. Its coating has more than 99.9 % reflectivity at 1560 nm wavelength and 94 % reflectivity at 780 nm wavelength. The mirror substrate is made of fused silica with a diameter of 7.75 (- 0.1) mm and edge thickness of 4.0 ($\pm$ 0.1) mm. The crystal from Raicol is a type II crystal poled along x-axis with a period of 46.15 μm . It has a dimension of $1 \times 1 \times 4$ mm. The type II interaction produces the signal and idler photons with orthogonal polarisation. The refractive index of the PPKTP depends on the wavelength and polarisation. The refractive index data are calculated based on the Sellmeier equation of KTP from [116]. For wavelength of 780 nm, $n_y = 1.7578$ and $n_z = 1.8464$. For wavelength of 1560 nm, $n_y = 1.7347$ and $n_z = 1.8155$. The crystal end facing the curved mirror has an anti-reflective coating for both 780 nm and 1560 nm. The coating on the opposite side of the crystal has reflectivities of more than 99.9 % at 780 nm and 97 % at 1560 nm. The mirror is attached to a piezo actuator, which is glued on a homemade mirror

base. The crystal sits inside a holder, to which a heater and a sensor are attached in order to stabilise the temperature of the cavity. Temperature stability and piezo voltage stability play a vital role in controlling the optical path length to maintain fixed resonance frequencies.

3.1.2 Cavity stability

A stable mode output requires the cavity design to satisfy the requirement that the radiation field incident on a cavity mirror is essentially identical to that striking the cavity mirror after one round trip. A Gaussian mode can be represented with a complex parameter $q(z)$ which can be described as $\frac{1}{q(z)} = \frac{1}{r} - j\frac{\lambda}{\pi w^2}$ [115, 117], where r is the wavefront radius of curvature, λ is the wavelength of the mode, and w is the beam width of the mode at position along the propagation direction. In a paraxial approximated optical system, a Gaussian mode maintains its Gaussian shape after propagation. We can use the ABCD matrix [117] to calculate the status of the state after one round-trip with the input state. The corresponding matrix is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 - \frac{2}{R}(d_1 n_1 + d_2) & 2(d_1 n_1 + d_2) - \frac{2}{R}(d_1 n_1 + d_2)^2 \\ -\frac{2}{R} & 1 - \frac{2}{R}(d_1 n_1 + d_2) \end{pmatrix} \quad (3.1)$$

where d_1 is the crystal length along the propagation axis, d_2 is the free space distance between the crystal and the end mirror, n_1 is the refractive index of the crystal, R is the radius of curvature of the end mirror.

For a stable cavity, the ABCD matrix needs to satisfy $-1 < \frac{1}{2}(A + D) < 1$. The stability region is

$$0 < (d_1 n_1 + d_2) < R. \quad (3.2)$$

This means the optical path length needs to be smaller than 10 mm for the cavity to be stable. At the reflection mirror, the Gaussian mode has a wavefront with radius of curvature as

$$r = (d_1 n_1 + d_2) + \frac{(\pi w_0^2)^2}{\lambda^2(d_1 n_1 + d_2)} \quad (3.3)$$

where w_0 is the beam waist. This suggests that when the curvature of the reflection mirror matches the wavefront radius of curvature, the cavity is in its stability region.

3.1.3 Higher-order transverse modes and their loss

Different transverse modes exist in the cavity. In order to generate factorisable squeezed states, higher order transverse modes frequencies need to be isolated from the PDC process. Assuming the cavity has a length of L and the Rayleigh length of the desired mode is z_R , The free spectral range is determined

by $\text{FSR} = c/2L$. The frequency between transverse modes is [115]

$$\Delta f = \frac{\text{FSR}}{\pi} \times \cos^{-1} \left(\pm \sqrt{\frac{z_R^2}{L^2 + z_R^2}} \right). \quad (3.4)$$

With different cavity length L , three cases are discussed. When cavity length is much smaller than the Rayleigh length, i.e. $L \ll z_R$, the frequency difference between the first order mode and the fundamental mode Δf is

$$\Delta f = \frac{L}{\pi z_R} \text{FSR}. \quad (3.5)$$

When cavity length is around the Rayleigh length $L = z_R$, Δf is changed to

$$\Delta f = \frac{\text{FSR}}{4}. \quad (3.6)$$

When cavity length is much larger than the Rayleigh length, i.e. $L \gg z_R$,

$$\Delta f = \left(\frac{1}{2} - \frac{z_R}{\pi L} \right) \text{FSR}. \quad (3.7)$$

As the Rayleigh length is very short, our cavity length would match the third case that cavity length is much larger. The second order transverse mode would be close to the adjacent longitudinal mode. Because the phase-matching bandwidth applies to both fundamental mode and higher order modes, it is important to find out the loss for higher order modes in the cavity to avoid multi-mode output. We could design the cavity in a way to achieve high loss rates for higher order modes to prevent multi-mode emission.

Cavity loss originates from different aspects. One of the loss is from the physical restriction of aperture size. Because Gaussian modes have a Gaussian transverse spatial distribution, they will have a spatial loss when passing through an aperture. Hermite-Gaussian modes have an intensity distribution of [115]

$$I_{l,m}(x, y, z) = |A_{lm}|^2 \left(\frac{w_0}{w(z)} \right)^2 \mathbb{G}_l^2 \left(\frac{\sqrt{2}x}{w(z)} \right) \mathbb{G}_m^2 \left(\frac{\sqrt{2}y}{w(z)} \right) \quad (3.8)$$

where w_0 is the beam waist, $w(z)$ is the beam width of the mode at axial position z , $\mathbb{G}_l(u)$ is the Hermite-Gaussian function of order l , and $\mathbb{G}_l(u) = \mathbb{H}_l(u) \exp \left(-\frac{u^2}{2} \right)$. The loss from aperture can be calculated by the integrated intensity ratios. As shown in Fig. 3.3, with a fixed aperture size, losses for different order of modes are different. Higher order modes suffer from more loss than the fundamental mode with the same aperture size. The aperture size design is important to achieve a single mode output by suppressing higher order modes. An aperture size similar to the beam waist is ideal for restricting the cavity mode to the fundamental mode. However, any loss introduced inside the cavity

would reduce the squeezing. In our design, the aperture is 18 times larger than the fundamental mode beam waist to avoid intra-cavity loss.

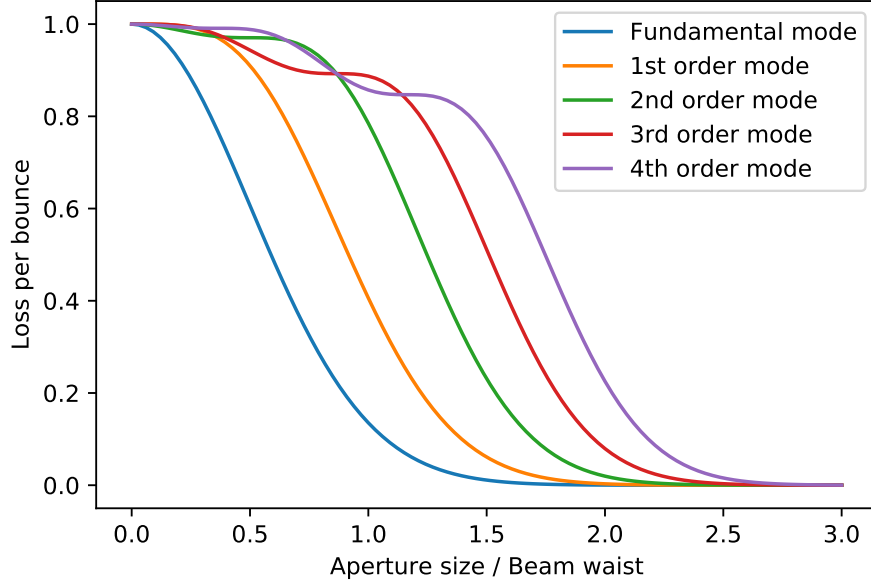


Figure 3.3: Loss per bounce due to finite aperture size for different modes in the cavity. When we restrict the aperture size to be the same as the beam waist, the fundamental mode has a much lower loss per bounce comparing with other higher order modes.

3.1.4 Finesse and escape efficiency

In the previous chapter we discussed how the reflectivities of cavity mirrors affect the spectral width of resonance frequency, which in turn impacts the output's joint spectral intensity. It is necessary to choose the appropriate reflectivities so that the triple resonance can isolate a single mode while preventing mode shape distortion.

As shown in Fig. 3.4, different reflectivities for the signal and idler fields on the end mirror affect the spectral width of each mode. With higher reflectivity, the spectral width is narrower, corresponding to a smaller size on the joint spectral intensity. We choose a type II SPDC to generate signal and idler photons that have different polarisation to aid mode isolation. The signal and idler fields have different refractive indices due to crystal birefringence. The modes are arranged in a rectangular shape in the joint spectral intensity as the free spectral range (FSR) are different for signal and idler mode, which allows the diagonal pump resonance mode to isolate one joint mode. We believe that in order to isolate the target mode without including adjacent diagonal modes or changing the target mode as shown in Fig. 3.1, the following conditions need to be satisfied:

$$\delta_p \geq c \max(\delta_s, \delta_i) \quad (3.9)$$

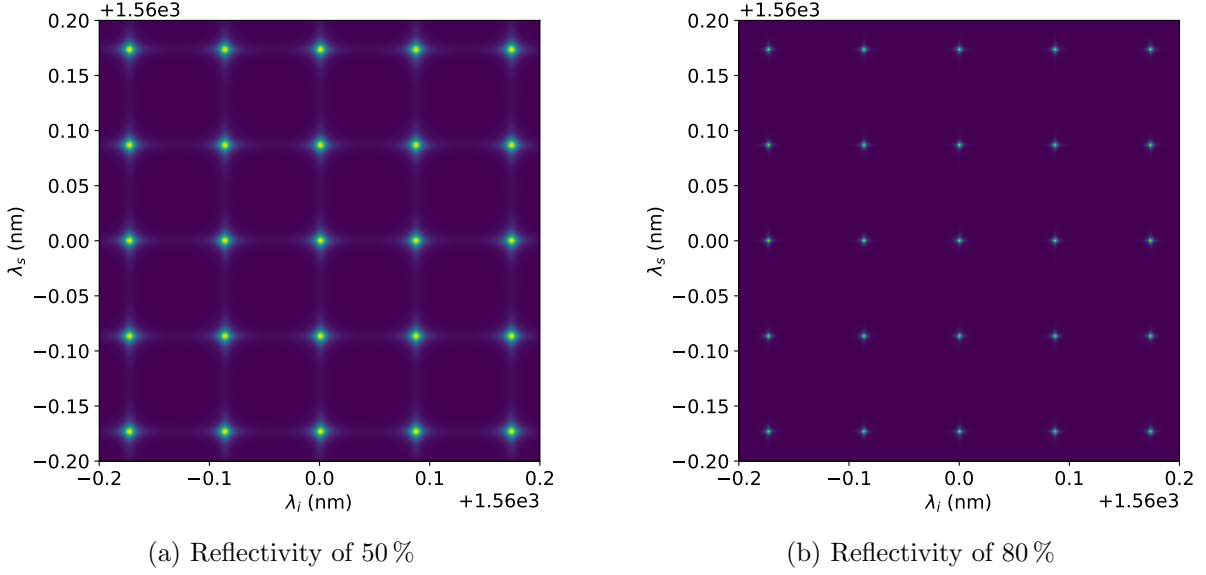


Figure 3.4: JSI comparison for cavity with different reflectivities. The bright region is a schematic representation of the joint spectrum for cavity resonant modes. When reflectivity is higher, the JSI becomes narrower.

and

$$\max(\text{FSR}_s, \text{FSR}_i) - \min(\text{FSR}_s, \text{FSR}_i) \geq \sqrt{2} \max(\delta_s, \delta_i) \quad (3.10)$$

where δ_μ is the spectral width of resonance for mode μ ($\mu = s, i, p$) and c is a constant that depends on the allowable overlap, $\text{FSR}_\mu = \frac{c}{2n_\mu L}$ is the free spectral range for mode μ ($\mu = s, i$). We need the refractive indices between the signal and idler modes to have enough difference to achieve the mode separation.

The specified reflectivities of the coatings have a manufacturing uncertainty of $\pm 1\%$. Thus, the finesse of the cavity in theory should be between 200 and 296 for 1560 nm wavelength and between 85.4 and 120 for 780 nm wavelength.

The reflectivities also impact the escape efficiency of the cavity. The escape efficiency is the probability for a photon to exit the cavity through the out-coupling mirror once generated. It is defined as [118]

$$\eta = \frac{1 - R_{\text{OC}}}{1 - g_{\text{rt}}^2} \quad (3.11)$$

where R_{OC} is the reflectivity of the out-coupling mirror, and g_{rt} is the round-trip gain of the cavity which is defined with the absorption coefficient α and the crystal length l as

$$g_{\text{rt}} = \sqrt{R_1 R_2} e^{-\alpha l}. \quad (3.12)$$

The escape efficiency of the photon pair is the product of each photon's escape efficiency, written as

$$\eta_{\text{pair}} = \eta_s \eta_i = \frac{1 - R_{1s}}{1 - R_{1s} R_{2s} e^{-2\alpha l}} \frac{1 - R_{1i}}{1 - R_{1i} R_{2i} e^{-2\alpha l}}. \quad (3.13)$$

In our design, if there is no intra-cavity loss, the escape efficiency of the photon pair η_{pair} is between 82.6 % and 90.9 %. It is very sensitive to loss inside the cavity.

3.1.5 Nonlinear crystal

The cavity loss comes from two sources. One is the imperfect reflectivity of mirrors. The other is the intra-cavity loss including the absorption and scattering loss inside the cavity. The reflectivities are designed for coatings on both sides of the cavity. The scattering loss usually comes from dust on the surfaces. We expect the crystal material to have low absorption at the photon pair wavelengths to reduce the intra-cavity loss.

High birefringence is another anticipated feature of the crystal. In order to isolate a single mode as in Fig. 3.1, it is important that the difference between the FSRs of signal and idler field is as large as possible. Otherwise, the pump field resonance would cover part or the whole adjacent modes, which results in a low factorisable output. With a higher birefringence, the separation between two FSRs would be larger.

Based on the two requirements stated above, we choose to use potassium titanyl phosphate (KTP) crystal to be part of the cavity. KTP’s absorption coefficient has different values for different propagation axis. The absorption is largely dependent on the manufacture processes. On Raicol’s product catalogue, the KTP crystal has an absorption coefficient smaller than $5 * 10^{-5} \text{ cm}^{-1}$ at 1064 nm.

The refractive indices of KTP at 1560 nm for three axes are: $n_x = 1.728$, $n_y = 1.7347$, and $n_z = 1.8155$ [116]. The crystal grows along the x-axis, that means we are using the birefringence between y and z axes. In terms of phase matching we discussed in the former chapter, the quasi phase matching is achieved by periodic poling, which is a fabrication of layers with alternating orientation of the birefringent crystal. The periodically poled KTP (PPKTP) is frequently employed in SPDC experiments due to its customised phase matching condition and widely available pump wavelength range.

The nonlinear crystal in our design is a type II phase matched PPKTP with a poling period of $46.5 \text{ }\mu\text{m}$. The size of the crystal is $1 \times 1 \times 4 \text{ mm}$. The hemispherical cavity design requires that one side of the crystal has a highly-reflective coating and the other side has an anti-reflective coating. In our setup, the y-axis of the crystal corresponds to vertical polarisation and the z-axis corresponds to horizontal polarisation. We use “spdcalc.org” maintained by NIST for joint spectral distribution and phase-matching curve information with customised parameters.

3.1.6 Cavity beam size matching

The cavity supports different modes for operation. A single mode SPDC output requires single mode coupling. To optimise the cavity mode coupling, we need to choose the beam size that matches the fundamental mode inside the cavity.

The transverse mode that matches with the cavity reproduces its electromagnetic field distribution after a round-trip inside the cavity. In a hemispherical cavity, the mode has its beam waist at the flat mirror and its wavefront's radius of curvature at the curved mirror matches the curved mirror radius of curvature. Based on our cavity design, we calculate the matching mode at 780 nm vertical polarisation has a beam waist of 35 μm . The matching modes at 1560 nm has a beam waist of 49.3 μm for vertical polarisation and 49.2 μm for horizontal polarisation.

The cavity sits between two aspherical lenses, which couple the beam into and out from the cavity. Both aspherical lenses have a focal length of 100 mm. The lens on the pump side and the lens on the signal side have an AR coating at 780 nm and 1560 nm, respectively. The lens for the signal and idler fields at 1560 nm (on the left of the cavity) should be 99.8 mm away from the flat mirror and provides a collimated beam waist of 1.0 mm. The lens for the pump field at 780 nm (on the right of the cavity) should be 94.1 mm away from the curved mirror back end and collimates the beam waist calculated as 0.9 mm.

The fibres at 780 nm and 1560 nm are polarisation maintaining single mode patch cables from Thorlabs. In order to match the fibre modes with the cavity modes, the pump field requires a fibre collimator with a focal length of 9.1 mm. The specific focal length can be achieved using the zoom fibre collimator ZC618FC from Thorlabs. The zoom fibre collimator is a variable lens set to launch the light field from fibre to free space into a collimated beam. The lens set provides a range of focus from 6 to 18 mm, corresponding to a collimated beam waist of 1.95 to 0.62 mm. We are able to adjust its focal length to 9.1 mm to match the pump beam size with the cavity. In practice, we can adjust the pump launching collimator's focal length and the aspherical lenses location to optimise the alignment.

3.2 Mechanical design

The cavity structure is supported and maintained by custom-designed mounts. In order to avoid further alignment within the cavity after construction, we design the mechanical mounts to obtain the required optical alignment during their assembly. The cavity mechanical design and the steps to align and construct the cavity are covered in this section. My colleague Rob Gardner specified the cavity mechanical designs. This section is based on his work and plays an essential role in presenting the cavity design. All the mounts and construction supporting parts introduced in this section were machined in the mechanical workshop of Department of Physics, Imperial College London.

3.2.1 Cavity parts

Fig. 3.5 shows the cavity construction parts: the crystal hold fills, crystal hold, PPKTP crystal, curved mirror, cavity spacers, piezo actuator, and mirror base. In this section, we will introduce the design of the machined parts and the usage of them.

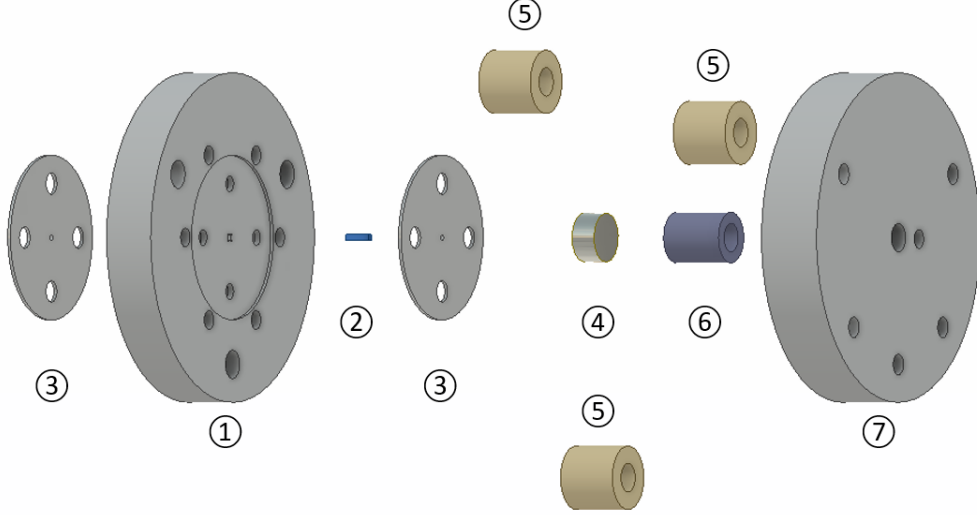


Figure 3.5: Exploded view of the cavity parts. Parts from left to right: ① crystal hold; ② PPKTP crystal ($1 \times 1 \times 4$ mm); ③ crystal hold fill; ④ curved mirror; ⑤ cavity spacers; ⑥ piezo actuator; ⑦ mirror base.

Crystal hold

The crystal hold helps to secure the crystal and control the temperature of the crystal. There is a cut through square hole designed with 1.06 ± 0.01 mm in width and corner relief radius of close to 0.15 mm. The three outer ring M4 clearance holes are connecting to cavity spacers which provide a fixed distance between crystal hold and mirror base. Among the six middle ring M3 holes, two of them are used to accommodate the temperature sensor and heater for the crystal. Made in aluminium, the crystal hold provides a sensitive and stable temperature control. The inner circle tapped M3 holes are made to connect the crystal hold fill, which keeps the crystal from falling out of the crystal hold.

Crystal hold fill

The crystal hold fill is a thin disk attached to the crystal hold to protect the crystal from the side. The central hole with 0.9 mm diameter allows the light field to enter the crystal. Meanwhile, the outer ring holes are aligned with the crystal hold for attaching them with M3 screws.

Mirror base

The mirror base supports the piezo and curved mirror. The central 4.5 mm through hole allows the light to go through. The 3 mm hole next to it accommodates the wiring of piezo actuator control so that the light would not be blocked accidentally by the wires. On the outer ring, the three 120° separated M4 tapped holes are connected to the cavity spacers. The two M4 holes symmetric around the central axis at the bottom are used to connect the mirror base to a U-shaped base to align the

height of the cavity.

Cavity spacers

A cavity configuration matches a light mode with an overlap for both spatial intensity and phase profile. With a certain configuration, a cavity matches the fundamental Gaussian mode the best and has a low coupling of higher order modes. There are three cavity spacers attached between the crystal hold and the mirror base to make the cavity length close to what is needed in order to match the fundamental mode.

Due to cavity stability requirements and tolerance of misalignment, we design the cavity spacer with a length of 9.40 mm to provide a cavity optical path length around 9.1 mm. With the fixed distance between crystal hold and the mirror base, the cavity length is finely adjusted with a 14 μm travel range piezo actuator by external voltage supply.

3.2.2 Cavity assembly

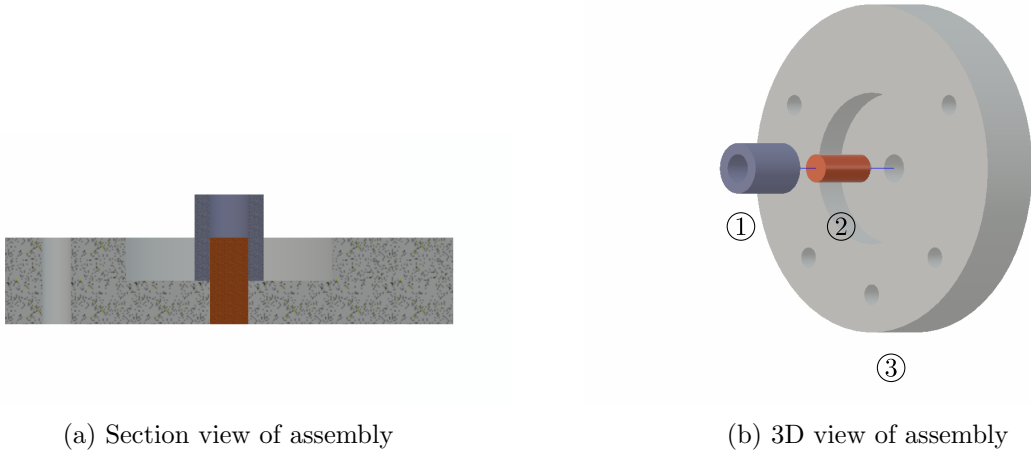
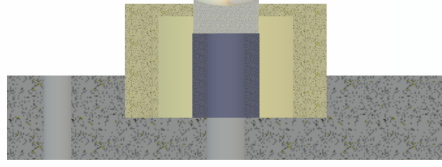


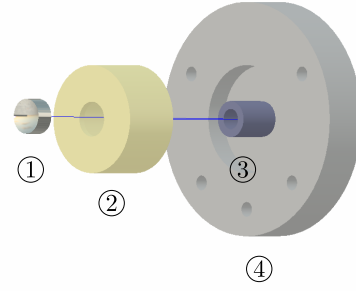
Figure 3.6: Piezo attachment with the dowel. The dowel ② fits in the central hole of the mirror base ③. Because the outer diameter fits both the mirror base hole and the piezo inner diameter, the piezo ① can be glued at the centre of the mirror base. The dowel will be removed after the glue is settled.

Three parts are designed for supporting the cavity alignment: dowel, mirror jig, and alignment tube. The dowel is a small cylinder that fits in the central hole of the mirror base. Its width matches with the piezo inner diameter within 50 μm . Shown in Fig. 3.6, with the help of the dowel, we glue the piezo on the centre of the mirror base.

The next step is to use the mirror jig to help with centring the mirror alignment. The mirror jig fits in the mirror base's inner pocket within 50 μm . Fig. 3.7 shows that the mirror would fit in the smaller hole of the mirror jig. Once we glue the mirror on the piezo and remove the mirror jig, the mirror base is aligned with the piezo and the curved mirror.



(a) Section view of assembly



(b) 3D view of assembly

Figure 3.7: Mirror assembly with the mirror jig. The outer diameter of the mirror jig ② matches the inner ring of the mirror base ④. After sliding the mirror jig on the mirror base, the mirror ① fits the inner diameter of the mirror jig. Once the mirror is glued with the piezo ③, the mirror jig would be removed.

After the crystal fits in the centre of the crystal hold, we loosely attach the mirror base with the cavity spacers and crystal hold in the Fig. 3.5 order. With the help of a dowel and mirror jig, the piezo and mirror can be attached along the centre of the mirror mount. Both the crystal hold and the mirror base are designed with the same diameter and both are close fit ($10\text{ }\mu\text{m}$) into an alignment tube as in Fig. 3.8. We use the alignment tube to align the cavity parts on the same axis as in Fig. 3.9. Once we tighten the fasteners and remove the alignment tube, the cavity is fully constructed and aligned.

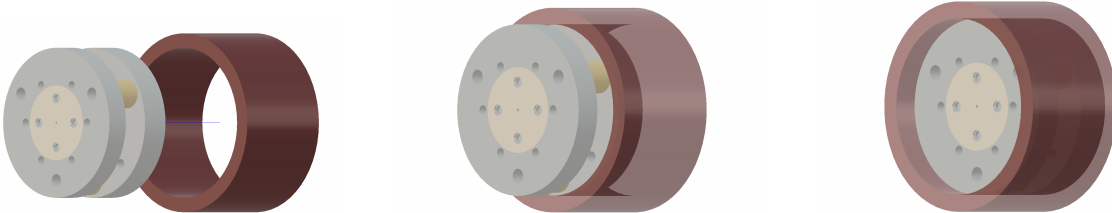


Figure 3.8: Cavity axis alignment with the alignment tube in 3D view. The loosely attached cavity is sent to the alignment tube, whose inner diameter matches the outer diameter of both the mirror base and the crystal hold. After the cavity is fitted in the alignment tube, the cavity fasteners will be tightened.

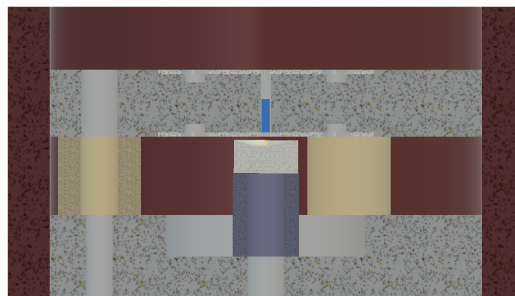


Figure 3.9: Section view of axis alignment with the alignment tube.

Assembly with collimating lenses

After assembling the cavity mounts, we attach the cavity to a U-shape base. Fig. 3.10 illustrates the parts that hold the cavity and couple light fields with the cavity. There are two spacers at each side of the U-base to connect to two 3-axis micro-positioning stages (Thorlabs nanoMax). On each stage there is a 100 mm focal length lens. These spacers provide a fixed distance between the stages and the cavity so that the light field is well coupled with the cavity structure.

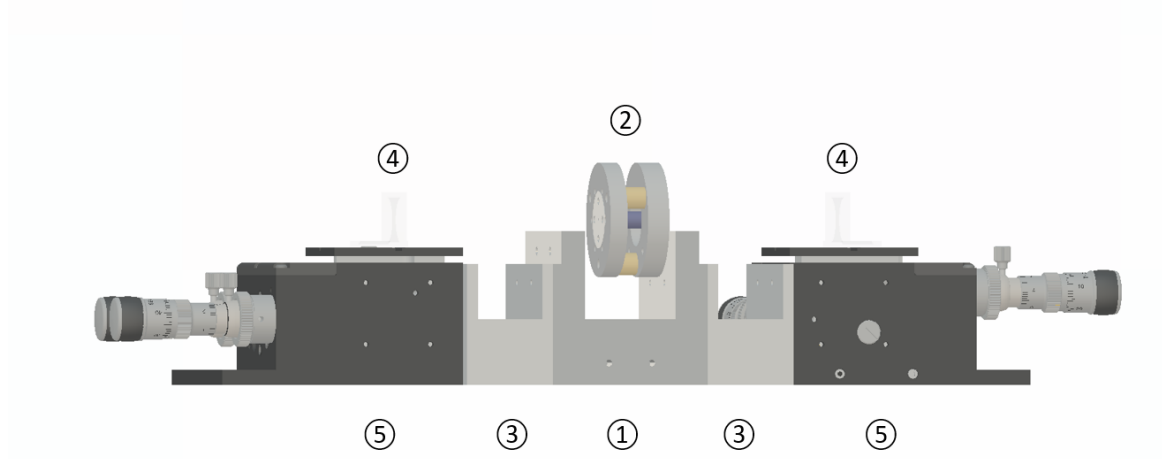


Figure 3.10: Cavity assembly with collimating lenses. Part names: ① U-shape base; ② Cavity; ③ Spacer; ④ 100 mm Focusing lens; ⑤ Micro-positioning stage.

3.3 Electrical control

In order to maintain the cavity length, we equip the cavity with a temperature control system and a piezo control unit. The temperature control stabilises the temperature of the crystal so that the refractive index remains stable for the input fields. The piezo attached at the back of the mirror can be controlled by input voltage to adjust the free space distance. In this section, we will introduce the details in the temperature control system and the piezo control system.

3.3.1 Temperature control

The temperature of the cavity is stabilised by a Lakeshore model 325 temperature controller through setting up a feedback loop to control a Thorlabs HT15W resistive cartridge heater and read from a Thorlabs TH100PT 100 Ω resistance temperature detector.

The temperature controller Lakeshore model 325 is capable of providing 2 PID control loops for temperature monitor and control. The stability of the temperature for the loop is 0.005° C from the controller.

The cartridge heater has a heater power of 15 W with 24 V voltage supply, and a maximum temperature of 260° C. The resistance temperature detector provides a temperature coefficient of $3.85 \times 10^{-3}/\text{K}$ and an repeatability of $\pm 0.1^\circ \text{C}$.

In order to minimise the disturbance from the environment, we also add a 3D printed cover on top of the cavity U-base to reduce the thermal conduction through air. The cover is shown in Fig. 3.11. The holes on both sides allow the pump beam and output modes pass through. The legs are matched with the U-base so that the cover would have a fixed position when assembled.

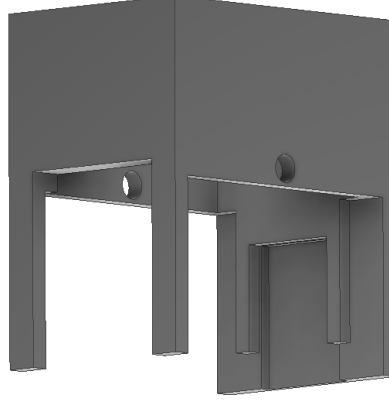


Figure 3.11: Custom designed cover made in 3D printed PLA material by Ultimaker. The cover is matched with the U-base and placed around the cavity to reduce thermal conduction.

3.3.2 Piezo control

When a voltage is applied to the piezo actuator, the piezoelectric effect induces a high-precision displacement. This property allows us to apply a voltage to adjust the cavity length, which affects the resonance frequency. It is employed when scanning the cavity to find and tune the resonance frequency. The piezo used in this cavity is the SR080410 ring piezo from PiezoDrive. It has a length of 10 mm and a diameter of 8 mm to allow the light field pass through. The voltage range for the piezo is -30 V to +150 V. This piezo has a travel range of $14 \mu\text{m} \pm 10\%$.

The piezo is controlled by a data acquisition system Labjack. Labjack provides automation of the piezo setting with analogue inputs and outputs connected to photodiode power readings and piezo voltage control. After reading the photodiode level, the Labjack is programmed to adjust the piezo voltage so that the cavity stays on resonance. The Labjack device used in this project is Labjack T4. It provides 4 analogue inputs with a range of $(\pm 10 \text{ V})$ and 2 analogue outputs. In next chapter, we discuss the use of analogue inputs to read the resonance monitor photodiodes and analogue outputs to supply the piezo voltage in a GUI program.

In this chapter, we introduced our design of the 10 mm optical-path-length cavity with a type II phase-matched PPKTP crystal and highly reflective mirrors. First, we discussed optical design

elements including cavity stability, higher order modes, finesse, nonlinear crystal, and beam size matching. Then, we presented the mechanical design of cavity parts and assembly steps. Lastly, we described the use of the temperature control and the piezo control to set and stabilise cavity parameters. In the next chapter, with a fully assembled cavity, we will measure the cavity's linear response and develop a tuning protocol.

Chapter 4

Linear response of the cavity

In the previous sections, we derived cavity SPDC source theory and designed the optical and mechanical aspects of the cavity. After assembling the cavity, the optical behaviour plays an important role in understanding the cavity performance. In this chapter, we measure the optical specifications of the fully-constructed cavity. We then observe the linear response of the cavity resonance from controlled parameters and test the cavity stability. In combination with a discussion of tuning theory, we form a tuning protocol to achieve triple resonance via the temperature of the crystal and laser wavelengths.

4.1 Optical characterisation

With the fully-constructed cavity, we'd like to explore the optical characteristics of the cavity for the pump, signal, and idler fields. Free spectral range (FSR) is the difference between adjacent resonance frequencies of a cavity. The linewidth of the field is defined by the full-width half maximum (FWHM) of the field's cavity resonance. We start the section by introducing the experimental apparatus and optical alignment. Then we characterise the cavity with its optical features for the three fields. By scanning the laser frequency for each field, we are able to measure the FSR and the linewidth of the three fields.

4.1.1 Optical alignment

The setup for optical alignment is shown in Fig. 4.1. The pump laser on the top is a Toptica DFB Pro laser that is collimated to free space with a lens. The Toptica laser has a wavelength tuning range of 778.5 - 781.0 nm. We use mirrors M1 and BS1 to steer the pump laser into the cavity. Each mirror mount has two knobs: one controls horizontal tilt, the other controls vertical tilt. By adjusting the four controls from two mirrors, we are able to scan the laser beam across all directions to match the cavity optical axis. During the alignment, we scan the piezo voltage with a range of 10 V to include at least one resonance feature.

We set up the mirrors at a position that can direct the beam along the optical table holes to

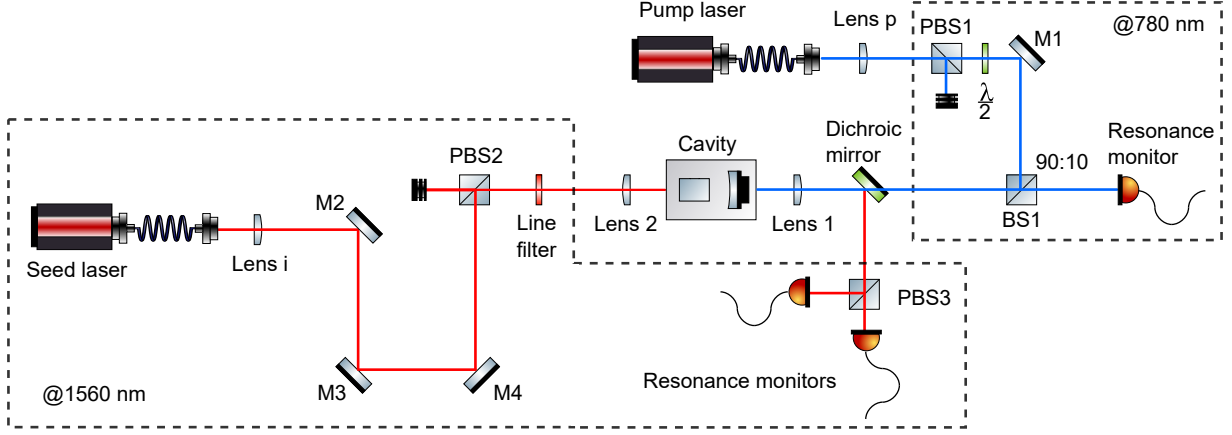


Figure 4.1: Optical alignment setup. The pump laser is collimated by a lens to free space. Then it is steered into the cavity using M1 and BS1. On the left of the cavity, the seed laser gets reflected from PBS2 to the cavity. The line filter protects single photon detectors in modified setups from pump field induced noise. Pump: Toptica DFB Pro laser; Seed: EXFO TS100S laser; M: mirror; PBS: polarising beam splitter; BS: non-polarising beam splitter; $\lambda/2$: half waveplate.

facilitate optical alignment. Every time, we turn a pair of knobs with the identical control function to steer the beam into the cavity. Applying back-and-forth adjustment of the horizontal and vertical directions, we are able to achieve the optimal coupling, in which case the reflection resonance monitor records a high visibility dip during the piezo voltage scan. The resonance monitor is a Thorlabs FPD310-FS-VIS Si switchable gain free-space amplified photodetector. We use a PicoScope 6824E to read electrical signals from the setup.

Similarly on the 1560 nm side, we back-propagate an EXFO T100S-HP tunable laser as the seed laser to align the beam into the cavity with the transmission side resonance monitor. The optimal coupling with seed laser provides high photon collection efficiency as we perform SPDC measurements. The resonance monitors are Thorlabs FPD310-FS-NIR InGaAs switchable gain free-space amplified photodetectors, which take the transmission feature of the cavity to check the cavity alignment for each polarisation. When the seed field couples in the cavity, the photodetector provides resonance peaks with the piezo voltage scan. The optimal coupling is achieved when the detector displays the highest peak.

4.1.2 FSR of the cavity fields

As introduced earlier in Section 2.2, the cavity FSR is linked with the optical path length. Considering a cavity formed with a crystal and air gap, the optical path length consists of both components:

$$L_{\text{OPL}} = L_{\text{air}} + n(\omega) \cdot L_{\text{crystal}} \quad (4.1)$$

where $n(\omega)$ is the frequency dependent refractive index, and L_{air} and L_{crystal} represent the air-gap length and crystal length correspondingly. With the constructed cavity from the last chapter, the air-gap length can be adjusted with the piezo voltage. The crystal length is fixed but we can change the refractive index by changing temperature or frequency.

The pump has different refractive index with signal and idler due to different frequency. The signal and idler fields generated from type II SPDC have different polarisation, resulting in different refractive indices due to the crystal birefringence. The optical path length for the three fields are different, which are reflected by different FSRs.

To measure FSR, we measure the power from cavity transmission and (or) reflection to check if a certain frequency is on resonance with the cavity. While scanning the piezo voltage to change the cavity optical path length, the transmission power presents peaks when the laser frequency is on resonance with the cavity length. Similarly, scanning the laser frequency for a fixed length cavity would show transmission peaks at resonance frequencies. If monitoring on the reflection side, the resonance presents optical power dips in a scan. In this subsection, we use the monitor photodiodes power with respect to scanning laser frequency to find the cavity resonances for all three fields. We control the current and temperature of the laser diode to adjust the pump laser frequency and power. For the seed laser we are able to use Python to adjust its frequency through its RS232 connection.

To measure the FSR for the pump field, we set a fixed piezo voltage, and measure the reflection power from the cavity while changing the laser diode temperature. When a minimum power is observed, which means the laser frequency is in resonance with the cavity, we record the laser frequency measurement from the wavemeter. Increasing the laser diode temperature until the next resonance frequency arrives the cavity, the reflection power reaches minimum again. By subtracting the adjacent resonance frequency, we have the FSR of the pump field. The same measurements are performed for the signal and idler fields with the transmission power to find power peaks. The seed laser frequency scan is achieved by a EXFO frequency scan Python script. The vertical polarisation FSR is 16.69 ± 0.02 GHz for wavelength of 780 nm. For wavelength of 1560 nm, the horizontal polarisation FSR is 16.22 ± 0.01 GHz, and the vertical polarisation FSR is 16.96 ± 0.01 GHz.

4.1.3 Linewidth of the cavity fields

Using the same setup, we can also measure the linewidth of the three fields. To measure the pump field linewidth, we use the reflection monitor at 780 nm to capture the reflection power of the cavity when scanning the pump frequency. The transmission peak of the cavity follows a Lorentzian curve [115]. The reflection dip is also Lorentzian as it equals to the total input power subtracted by the transmission power. The fine scan across one resonance feature is fitted with a Lorentzian function, whose full-width half maximum is the linewidth of the cavity pump field. Fig. 4.2 shows the fitting for one of the resonance curve measurement. Taking the average of measurements at different times, the

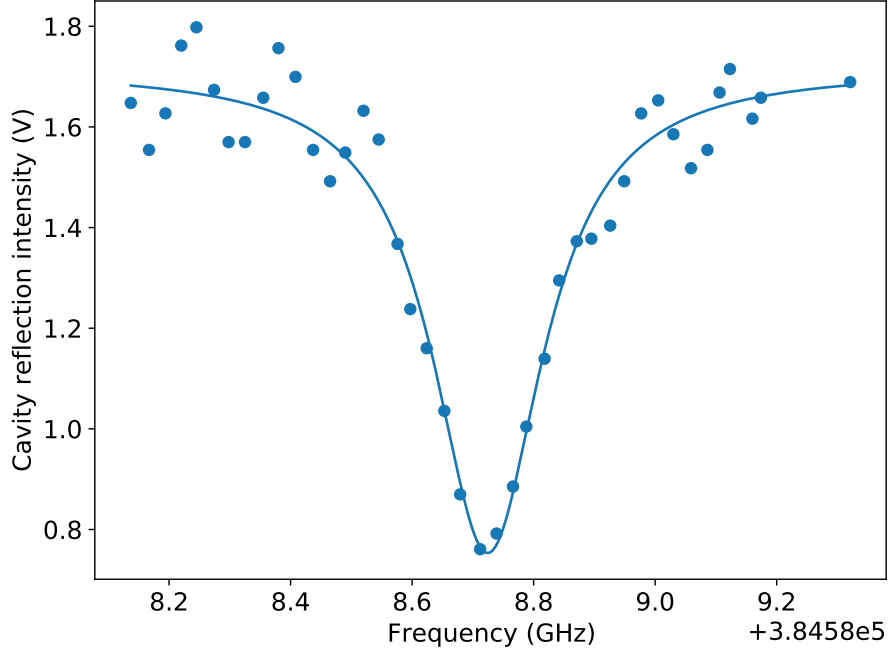


Figure 4.2: Pump resonance feature fitting with a Lorentzian function. The Lorentzian fitting shows the resonance dip is centred at 384588.72 GHz and has a linewidth of 0.220 GHz.

linewidth for the pump field is estimated to be 220 ± 4.96 MHz. The finesse of the cavity is 75.8 for the pump field, smaller than the theoretical range 85.4 to 120 calculated in Section 3.1.4. It's possible that the mode mismatch between the cavity and the pump width introduces extra loss from coupling higher order modes into the cavity. Using the measured finesse to calculate the cavity loss, we estimate there is at most 2.85 % intra-cavity loss, potentially caused by scattering and misalignment.

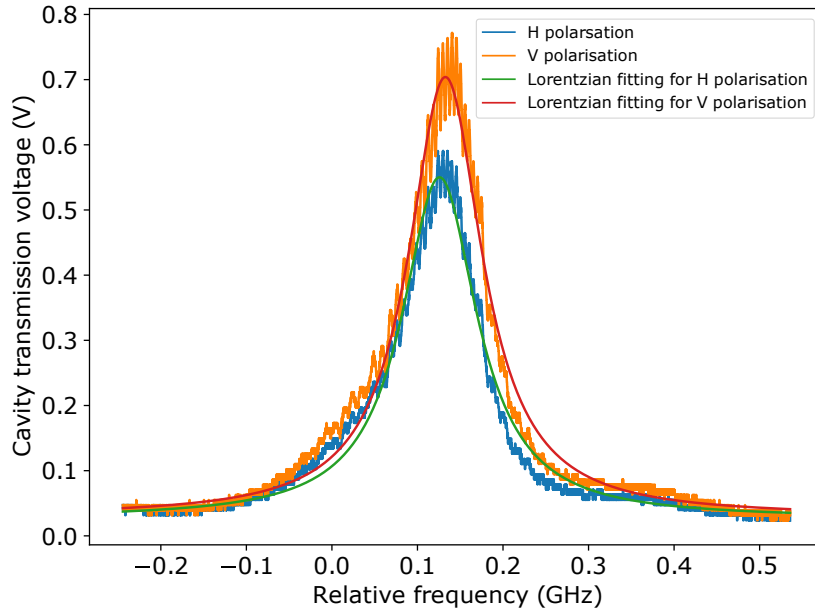


Figure 4.3: Signal and idler resonance peak fitting with a Lorentzian function. The Lorentzian fittings show these resonance peaks have a linewidth of 0.107 GHz for H polarisation and 0.105 GHz for V polarisation.

Similarly, we can measure the signal and idler linewidth. By using the dichroic mirror and the polarising beam splitter, we are able to record the cavity transmission power for both horizontal and vertical polarisation. We scan the seed laser frequency to capture the resonance feature and fit the transmission peak with a Lorentzian function in Fig. 4.3. The seed laser frequency fluctuates around its central frequency with a high speed. The wavelength meter takes around 1 second to read and save the frequency data. In this case, a slow frequency scan is not able to accurately reflect the resonance feature. We need to scan the laser faster than its fluctuation to capture its cavity resonance for linewidth measurements. The laser is scanned with a triangle shaped signal connected to the FSC port on the rear side of the EXFO laser. We use the PicoScope to record the photodiode voltages of all three fields simultaneously with the scanning triangle signal. We find the ratio of pump resonance FWHM to signal resonance FWHM. Using this ratio, we can solve for the signal linewidth. The idler linewidth is measured using the same approach.

The horizontal polarisation linewidth for 1560 nm is measured as 101.70 ± 10.45 MHz, which corresponds to a finesse of 159. And the vertical polarisation linewidth for 1560 nm is measured as 100.68 ± 10.85 MHz, which indicates a finesse of 168. Compared with the theoretical finesse between 200 and 296, the measurement indicates the extra loss are at most 1.8% on horizontal polarisation and 1.6% on vertical polarisation.

4.2 Resonance tuning and stability

In order to generate cavity-amplified difference signals or two-mode squeezed vacuum, we expect the cavity to be triply resonant, which means that it should resonate with the pump, signal, and idler fields. The signal and idler fields have different optical path lengths due to crystal birefringence. Finding the right settings to tune the cavity to triple resonance can be very challenging as any change of parameter affects all three fields at the same time. In this section, we will present the response of the cavity with respect to controlled parameters and some approaches to stabilise the cavity. We also introduce the theoretical approach and experimental protocol for triple-resonance tuning.

4.2.1 Cavity response

Before tuning the cavity to triple resonance, we need to investigate how cavity length responds to different parameters. Crystal temperature and piezo voltage are examples of parameters that we can control to change the cavity length. Other factors like room temperature may drift over time, contributing to cavity length change. Changes in cavity length reflect on different cavity resonance frequencies of each field. By tracking the resonance frequencies for different cavity configurations, we study the cavity response for controlled parameters.

The crystal temperature changes the optical path length by changing the refractive index of the

crystal. The piezo voltage affects the location of the mirror to change the air gap. When we have a fixed setting for temperature control and piezo voltage, over time we observe a drift for resonance frequencies. To find out how these two factors impact the resonance frequency, we investigate the cavity response to changes in crystal temperature and piezo voltage.

Temperature response

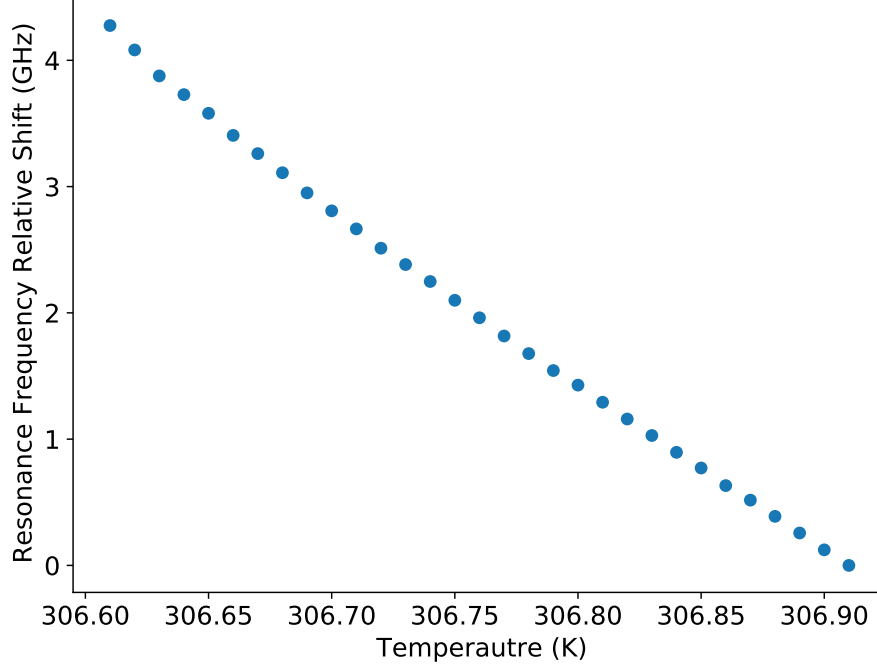


Figure 4.4: Cavity resonance frequency response of crystal temperature. Resonance frequencies are recorded and plotted at different crystal temperature. The response is linear with a coefficient of -13.982 GHz/K .

The temperature of the crystal affects the optical path length. With a fixed piezo voltage setting and a stable temperature on the mirror base, we explore the effect of the resonance wavelength in respect of the crystal temperature to check the response of the cavity. To make sure the piezo voltage is not affecting the tests, we short-circuited the electrodes of the piezo. Fig. 4.4 shows within a small range of temperature change, the resonance frequency decreases by 4.27 GHz. The response curve is approximately linear over this range with a coefficient of -13.982 GHz/K .

Piezo actuator voltage response

The piezo actuator changes its length with different voltage applied. In our cavity structure, changing the piezo length means the cavity air gap length changes, which would result in resonant frequency change. We set the temperature of the crystal to 313.5K one day before the piezo response measurement to avoid the temperature fluctuation. Then we change the piezo voltage with a step of 0.05 V and scan the laser frequencies to measure the cavity resonance frequency with each piezo setting. The

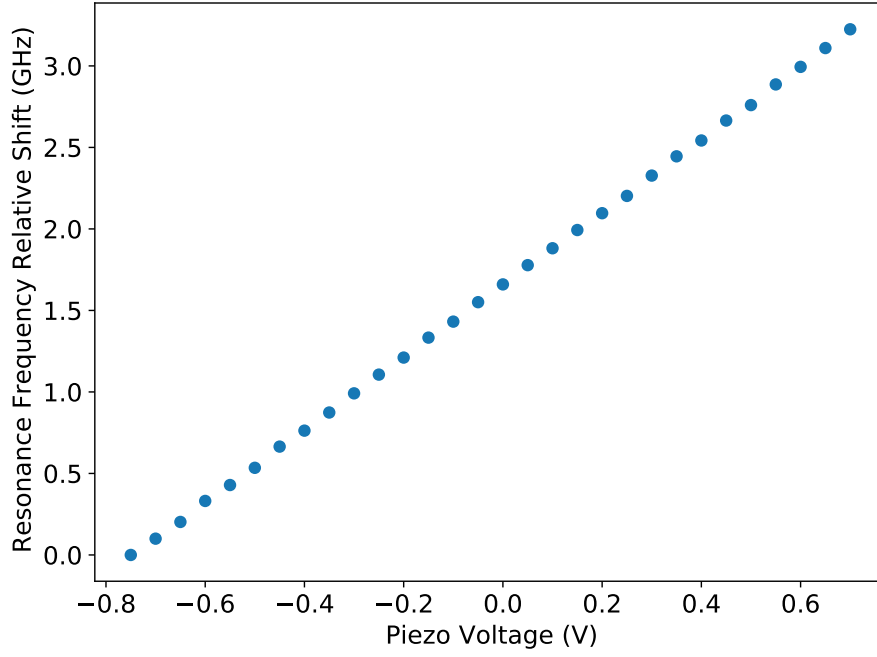


Figure 4.5: Cavity resonance frequency response of piezo voltage. Resonance frequencies are recorded and plotted at different piezo voltage with a step of 0.05 V. The response is linear with a coefficient of 2.23 GHz/V.

response is recorded in Fig. 4.5. Across a voltage change of 1.45 V, the resonance frequency shifted by 3.22 GHz. The response is linear with a coefficient of 2.23 GHz/V.

4.2.2 Cavity stabilisation

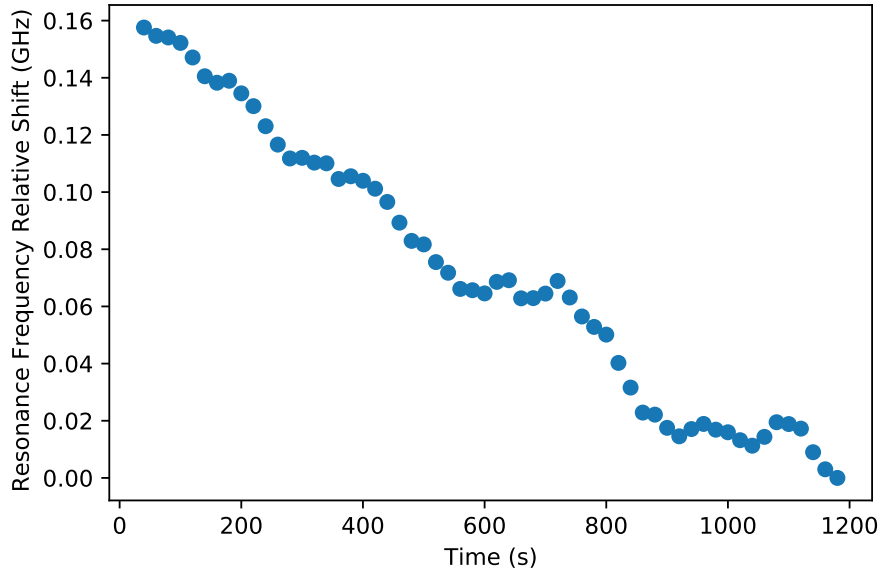


Figure 4.6: Cavity drifting in frequency over 20 minutes. Scanning laser frequency to capture the cavity transmission resonances, we record the resonance peak position repeatedly over 20 minutes. The cavity drifts over 20 minutes is 158 MHz.

The cavity is aligned separately for the fundamental mode of three fields. Given the linewidth

measurement from the last section, the cavity needs to be stabilised within 100 MHz for the pump field and 50 MHz in the signal and idler field. We expected the cavity would be steady enough to maintain the overlapping of resonance features. However, we record the drifting of pump resonance frequency over 20 minutes when the temperature of the crystal is steady and the piezo wires are grounded. From Fig. 4.6, we can see the resonance frequency is decreasing over time. After 20 minutes, the cavity resonance drifts 158 MHz.

To discover the possible causes, we look into the temperature of the cavity. When we heat up the cavity mirror base, the resonance frequency increased more significantly. Thus, we add a temperature control on the mirror base to stabilise the resonance frequency.

A stable squeezed-state output requires the stability of laser wavelength and the stability of cavity length. A stable laser wavelength helps to tune the cavity towards triple resonance. A stable cavity length helps to maintain the triple resonance condition. The cavity length needs to be stabilised with both temperature and the piezo voltage. The temperature of the crystal, controlled by Lakeshore temperature controller, provides a precision of 0.005° K . There is another temperature control loop at the mirror base to stabilise the temperature within a precision of 0.005° K . The piezo attached to the mirror is controlled with Thorlabs K-Cube piezo controller which has a voltage control with 0.05 V variance.

After adding the temperature control loop for the mirror base, the cavity resonance frequency noise is reduced to 15 MHz as shown in Fig. 4.7. This change improves the cavity's stability significantly but does not eliminate the fluctuation. There are other sources of instability in the cavity, one of which could be laser frequency.

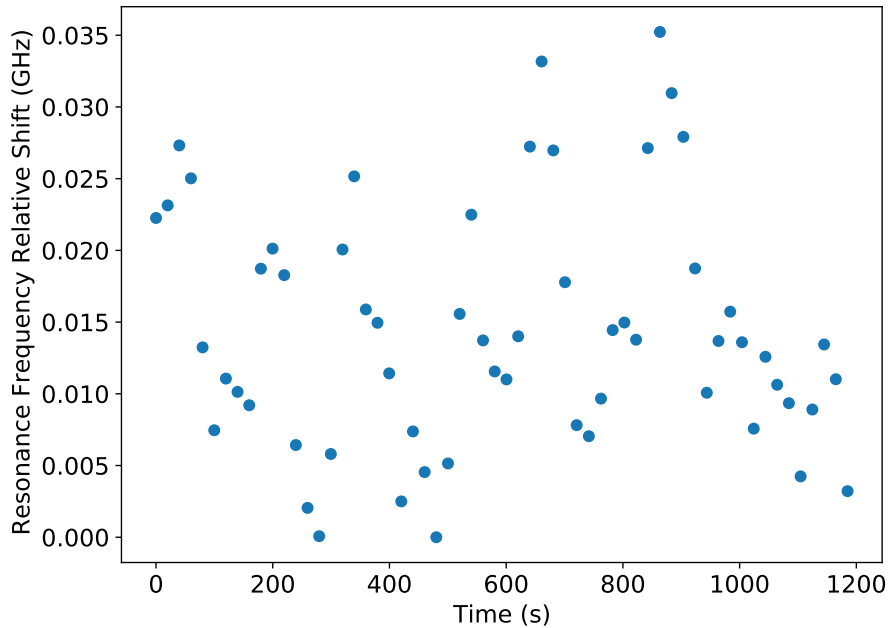


Figure 4.7: Cavity drifting in frequency over 20 minutes after adding the second temperature control loop. Scanning laser frequency to capture the cavity transmission resonances, we record the resonance peak position repeatedly over 20 minutes. The cavity fluctuation over 20 minutes is 15 MHz.

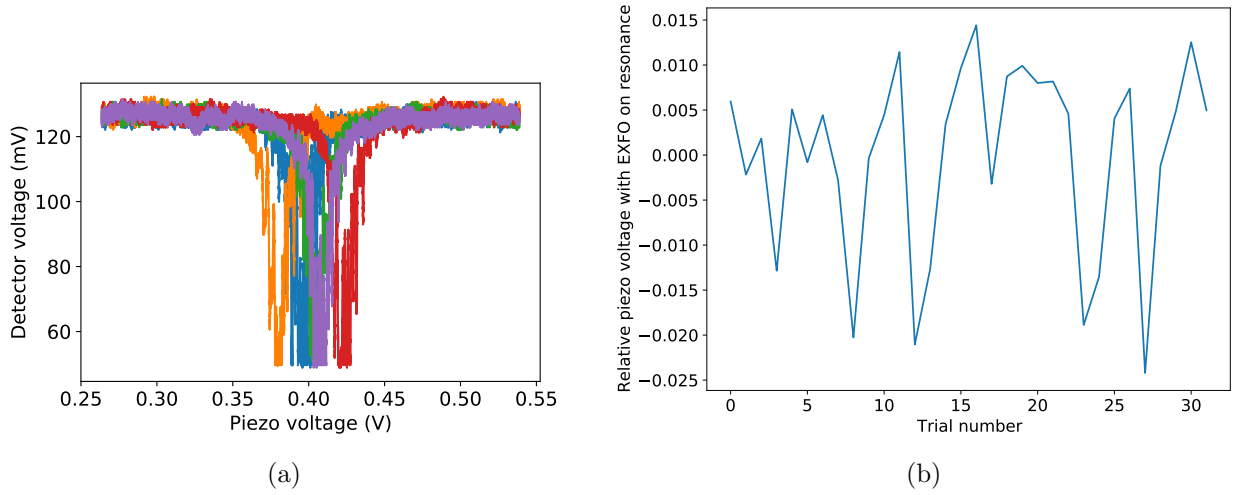


Figure 4.8: The cavity resonance using frequency-doubled EXFO as the pump laser by scanning the piezo. (a) Shows a few frames of the detector voltage with respect to the piezo voltage scan. Different colours represents different measurements. The resonance frequency is not stable, which causes the resonance feature to appear at different piezo voltages from different scans. (b) Shows the resonance frequency fluctuation within a short period of time. The piezo voltage corresponding to the cavity resonance fluctuates multiple times within this 0.28 s.

The EXFO seed laser fluctuates around a central frequency. The fluctuation is particularly large when the coherence control is switched on. In the experiment, we choose to use the constant current setting and keep the coherence control off to have the lowest fluctuation from the EXFO laser. We are able to observe the fluctuation by monitoring the cavity resonance. The EXFO laser gets frequency doubled to be 780 nm and then sent to the cavity from the pump side, i.e. input to the cavity from right to left. Consequently, the cavity resonance scan captures the reflection of the EXFO laser. Fig. 4.8 presents the resonance frequency fluctuation for EXFO laser with a piezo scan. The piezo voltage scans through a range of cavity length, which supports a continuous set of resonant frequencies. If the frequency of the laser is stable, the response should stay on resonance at the same piezo voltage. However, in Fig. 4.8a, different frames of measurements in different colours suggest that the frequency is not stable. Fig. 4.8b records 32 measurements within 0.28 s. We fit the Lorentzian data for each measurement and use the fitting to find resonance frequency dip corresponding piezo voltage. Within a short period of time, the frequency fluctuates multiple times. Fig. 4.9 shows the spectrum of the frequency fluctuation. The dominant fluctuation happens at around 210 Hz and 420 Hz. The intensity difference reaches more than 30 dB for these frequencies.

To reduce resonance fluctuation on measurement results, future experiments could use a Hansch-Couillaud method [119] to lock the EXFO laser frequency on resonance. In terms of data acquisition, we could also post select measurements to record data that only comes from a triple resonant cavity. We can also improve the cavity mechanical design. The metal base that supports the cavity is directly connected with the optical table. This introduces a large thermal conduction that could affect the cavity stability. We could change the base with a thermal insulated material to reduce the possible

sources of fluctuation.

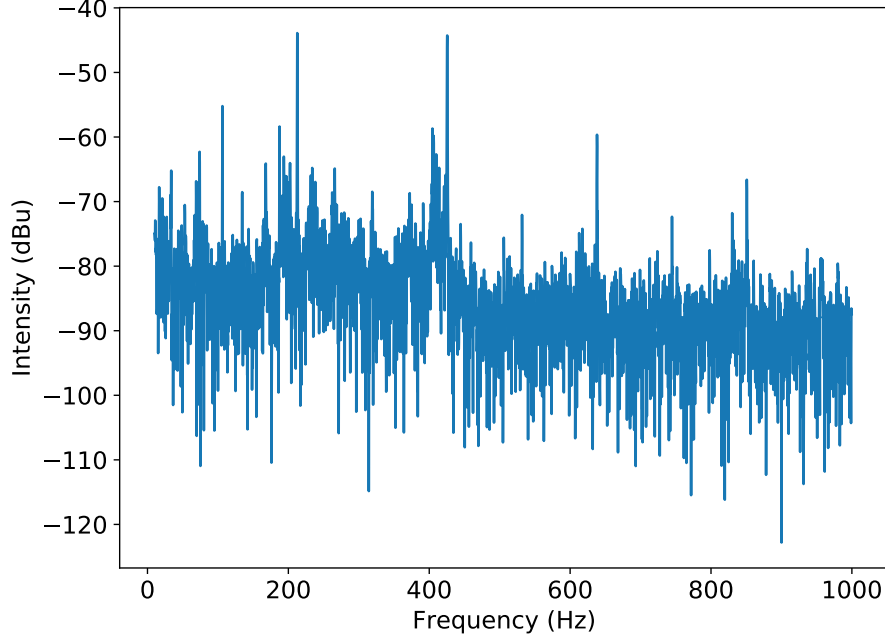


Figure 4.9: The frequency spectrum of the EXFO cavity resonance measured with an oscilloscope using an FFT. The measurement is taken when the triple resonance condition is set for the cavity and the monitor resonance fluctuates on- and off-resonance. The highest fluctuation happens at 210 Hz and 420 Hz with more than 30 dB intensity differences.

4.2.3 Triply resonant cavity tuning

A triply resonant cavity helps to enhance the SPDC interaction and output a factorisable state. It is not easy to tune a cavity to resonance in all three fields. In this section, we start from the tuning theories to find out what is needed. Then we introduce the tuning setup and the tuning protocol to help achieve cavity resonances for all three fields.

Tuning theory

Due to the birefringence and wavelength-dependent refractive index of the crystal, the tuning for triple resonance remains a complicated problem. In a general two-mirror cavity model shown in Fig. 4.10, the cavity with length L contains a crystal with length d . The crystal is operated with temperature T and tilted by an angle of θ . The resonance for each field can be tuned individually by the shown parameters. In this section we will discuss how to use these parameters to tune the cavity towards triple resonance.

Assuming the phase shifts inside the cavity are neglected, the cavity resonance frequencies are given by

$$f_{\epsilon,m} = m \times \text{FSR} = \frac{mc}{2g} \quad \text{where} \quad g = g(L, T, d, \theta) = \frac{d(T)}{\cos(\theta)}(n_{\epsilon}(T, \theta) - 1) + L \quad (4.2)$$

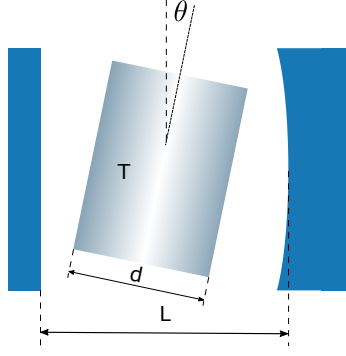


Figure 4.10: A simple model of cavity with tuning parameters. The cavity of length L contains a crystal of length d , operated under temperature T , and rotated by an angle θ .

where g is the optical path length, ϵ is the polarisation of the field, m is the longitudinal mode number, and n is the refractive index. For small changes of any listed parameter, we can write the frequency change in terms of its FSR as

$$\frac{\Delta f_{\epsilon,m}}{\text{FSR}} = \Delta m - \frac{2}{\lambda} \sum_i \frac{\partial g(\mathbf{x})}{\partial x_i} \Delta x_i \quad (4.3)$$

where x_i are the tuning degrees of freedom. For the parameters in the figure, the tuning responses are

$$\frac{\partial g}{\partial L} = 1 \quad \text{for total cavity length} \quad (4.4)$$

$$\frac{\partial g}{\partial T} = n \frac{\partial d}{\partial T} + d \frac{\partial n}{\partial T} \quad \text{for operating temperature} \quad (4.5)$$

$$\frac{\partial g}{\partial d} = n - 1 \quad \text{for crystal length} \quad (4.6)$$

$$\frac{\partial g}{\partial \theta} = d \theta (n - 1 - 2 \Delta n) \quad \text{for crystal tilt.} \quad (4.7)$$

In our setup, we consider a practical tuning scheme where we employ two parameters to adjust relative resonant frequencies instead of absolute resonant frequencies. Because it is challenging to control the crystal tilt and the crystal length accurately, here we mainly discuss the temperature and cavity length parameter to tune the cavity. The temperature tuning can be achieved with a crystal oven. Cavity length can be changed using a piezo actuator with high precision.

We consider two variables for the relative resonant frequencies tuning, the energy conservation condition f_{psi} and the degeneracy condition f_{si} , which are defined as

$$f_{\text{psi}} = f_{\text{p}} - f_{\text{s}} - f_{\text{i}} \quad (4.8)$$

$$f_{\text{si}} = f_{\text{s}} - f_{\text{i}} \quad (4.9)$$

where f_{μ} is the resonance frequency of field μ .

The tuning is targeted to reach a point where all three fields are resonant with the cavity. The

three fields are bounded with the energy conservation relation where $f_p = f_s + f_i$. Considering the practical tuning procedure, we need to check all three fields' resonant condition at the same time. If we choose to have degenerate signal and idler fields, we are able to use the same laser to input both fields with rotated polarisation. And when the cavity parameter changes, we don't need to adjust the laser frequency separately. This means $f_s = f_i$.

The above conditions correspond to $f_{\text{psi}} = 0$ and $f_{\text{si}} = 0$. With two parameters, cavity length L and operating temperature T , there should be a matrix A to linearly convert the parameter adjustments to the relative frequency change as [120]

$$\begin{bmatrix} \Delta f_{\text{psi}} \\ \Delta f_{\text{si}} \end{bmatrix} = A \begin{bmatrix} \Delta T \\ \Delta L \end{bmatrix} \quad (4.10)$$

where Δf is the relative resonant frequency change, A is formed as a 2×2 matrix, ΔT and ΔL are the temperature and cavity length change. Using the cavity parameters $d = 4$ mm, $L = 6.61$ mm, and KTP's thermo-optic dispersion formulas [116], we are able to write the resonance frequency change for each field with ΔT and ΔL . This allows us to estimate the elements in matrix A :

$$A = \begin{bmatrix} 22.9 \text{ MHz/K} & -273 \text{ MHz}/\mu\text{m} \\ 327 \text{ MHz/K} & -660 \text{ MHz}/\mu\text{m} \end{bmatrix}. \quad (4.11)$$

This means for a temperature change of 1 K, the f_{psi} changes for 22.9 MHz and the f_{si} changes for 327 MHz. With the cavity length change of 1 μm , the f_{psi} changes for -273 MHz and the f_{si} changes for -660 MHz.

For any arbitrary starting point in the cavity, we would like to find the amount of tuning needed for ΔT and ΔL . Because the relationship between resonance shift and tuning parameters change is linear, we can invert the above equation to find the relationship between tuning parameter and relative resonance frequency change. The tuning parameter change can be expressed as

$$\begin{bmatrix} \Delta T \\ \Delta L \end{bmatrix} = A^{-1} \begin{bmatrix} \Delta f_{\text{psi}} \\ \Delta f_{\text{si}} \end{bmatrix}. \quad (4.12)$$

The inverse matrix A^{-1} is calculated as

$$A^{-1} = \begin{bmatrix} -8.9 \text{ K/GHz} & 3.7 \text{ K/GHz} \\ -4.4 \mu\text{m/GHz} & 0.3 \mu\text{m/GHz} \end{bmatrix}. \quad (4.13)$$

The inverse matrix means if we want 1 GHz change in f_{psi} , we need to change the temperature by -8.9 K and the cavity length by -4.4 μm . When we want to change 1 GHz in f_{si} , it requires a temperature change of 3.7 K and a cavity length change of 0.3 μm .

The inverse matrix A^{-1} suggests that there is a combination of tuning parameter change that can achieve any desired relative resonance shifts. To achieve the triple resonance in the cavity, the specified frequencies need to be shifted at most by one FSR, which corresponds to the change of tuning parameters in Table 4.1. The cautious estimate considers the case when we want to shift f_{psi} for 1 FSR without affecting f_{si} , the temperature and the distance are required to change -138 K and -68.5 μm . Similarly, to change the f_{si} for 1 FSR without shifting the f_{psi} requires a temperature change of 57 K and a distance change of 4.8 μm . Any specific tuning can be optimised by using different longitudinal modes. In practice, we can find two closest longitudinal modes as a starting point to reduce the amount of parameter changes.

	ΔT (K)	ΔL (μm)
$\Delta f_{\text{psi}} = \text{FSR}, \Delta f_{\text{si}} = 0$	-138	-68.5
$\Delta f_{\text{psi}} = 0, \Delta f_{\text{si}} = \text{FSR}$	57	4.8

Table 4.1: Tuning parameters for relative resonance frequency change by one FSR in a cautious estimation. The amount of change in each row represents a change for one relative frequency without affecting the other.

According to the above discussions, the triple resonance tuning can be achieved with available tuning parameters. The more tuning parameters we are able to manipulate, the more easily triple resonance can be achieved. The tuning parameters also depend on the types of SPDC for photon-pair generation. In the type II phase-matched SPDC case, all three fields have different optical path lengths, which leads to different resonant frequencies. If there are three degrees of freedom, we will be able to tune the absolute frequencies to resonance. While if there are two degrees of freedom, the relative frequencies can be tuned to triple resonance. In the type I case, the signal and idler have the same polarisation. So we only need to consider two different fields to achieve resonance. Two degrees of freedom are needed to tune the absolute resonance. And one degree of freedom is needed for tuning relative resonant frequency.

Tuning protocol

When designing a triply resonant cavity, we have three degrees of freedom: temperature of the crystal, cavity length controlled by the piezo voltage, and laser wavelengths. With two parameters, we are able to tune the relative resonances. We choose to use the crystal temperature and laser wavelengths as our control parameters. It is worth noting that crystal temperature affects phase-matching wavelengths. However, given the practical tuning range in this project, there is a negligible impact on the phase-matching wavelength. Fig. 4.11 illustrates the experimental diagram for the tuning. The pump resonance monitor part on the right side are the same as the pump resonance tests setup. The seed laser EXFO provides 1560 nm wavelength field input from the crystal side of the cavity. We send the

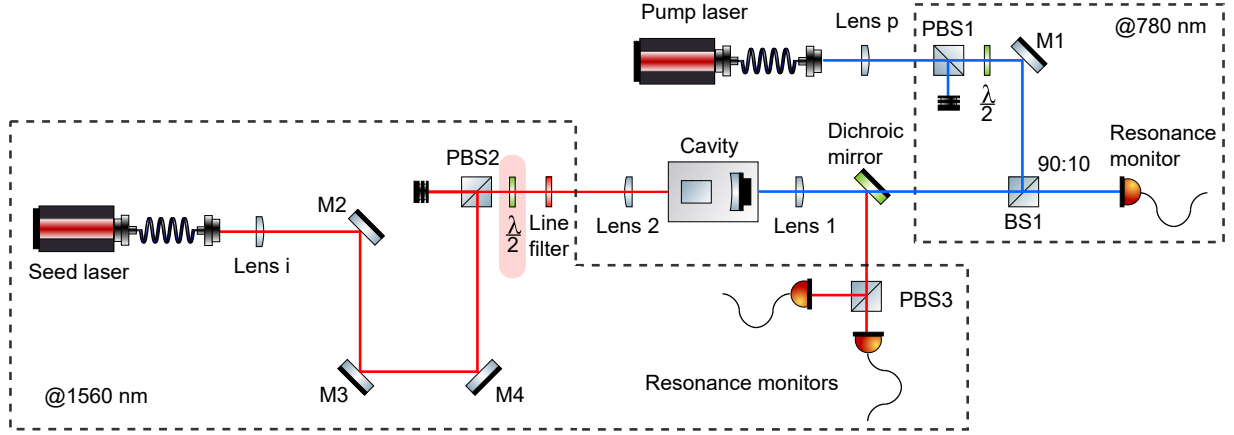


Figure 4.11: Experimental diagram for triply resonant cavity tuning. All three fields are present in the cavity. The pump resonance is monitored with reflection. The half waveplate is added to rotate the seed laser for 45° to input both horizontal and vertical polarisation in the cavity. Resonances of both polarisations of seed are monitored with transmission. Pump: Toptica DFB Pro laser; Seed: EXFO T100S-HP tunable laser; M: mirror; PBS: polarising beam splitter; BS: non-polarising beam splitter; $\lambda/2$: half waveplate.

collimated seed laser from the crystal side of the cavity through a PBS. The half waveplate between PBS and the cavity is rotated to have a 22.5° between the light polarisation and the waveplate's fast axis. This angle provides a diagonal polarisation input so that resonances of both horizontal and vertical polarisation are coupled in the cavity.

When we scan the piezo voltage with a ramp-up signal, the oscilloscope captures resonances signal for all three fields. Fig. 4.12 shows an example of the oscilloscope data. In the example, the pump resonance is measured with the reflection of the cavity. The signal and idler field resonances are presented using the transmission of the cavity from the two orthogonal polarisation from the seed laser. The pump resonance feature is between the signal and idler resonance.

A triply resonant cavity requires all three resonances overlap at the same time stamp on the oscilloscope. Firstly, we select the pump wavelength and corresponding degenerate signal/idler wavelength where the pump resonance is in between the seed and signal field. Secondly, changing the temperature makes the two polarisation resonances overlap. By now, there might be a small mismatch between pump resonance and the other two resonances. The piezo offset voltage is helpful for moving the resonances to the centre of the scope screen. And we repeat the steps above to move the pump resonance in between signal and idler wavelength and use the temperature change to overlap the signal and idler resonances again. Comparing the differences between pump resonance and signal-idler resonance as the tuning steps are repeated, we are able to determine whether the tuning is on the right direction, which leads to smaller differences and eventually triple resonance. An example of triple resonance on the oscilloscope is shown in Fig. 4.13. All three fields resonance features line up at a specific piezo voltage setting.

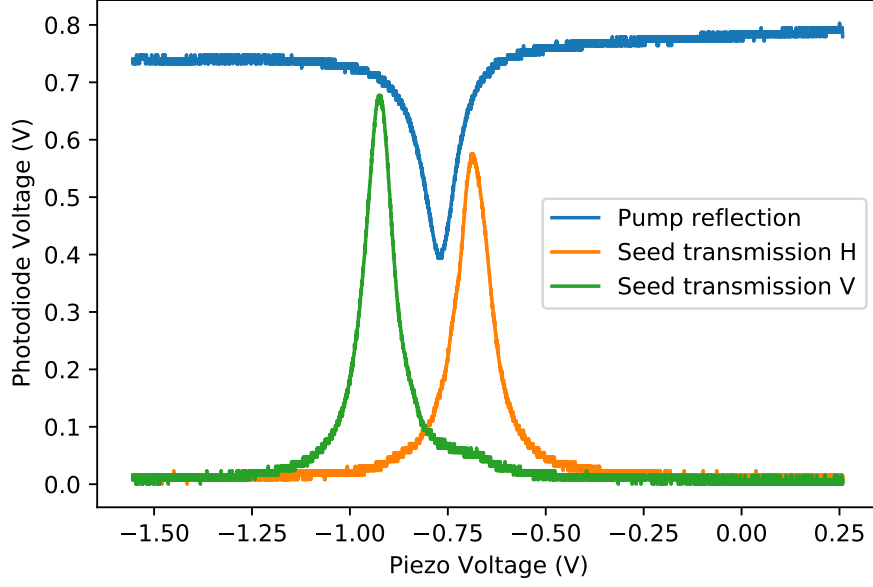


Figure 4.12: An example of three field resonances on oscilloscope by scanning piezo voltage. The pump resonance is in between the signal and idler resonances. The pump field is observed with cavity reflection signal. The signal and idler fields are monitored with cavity transmission signals.

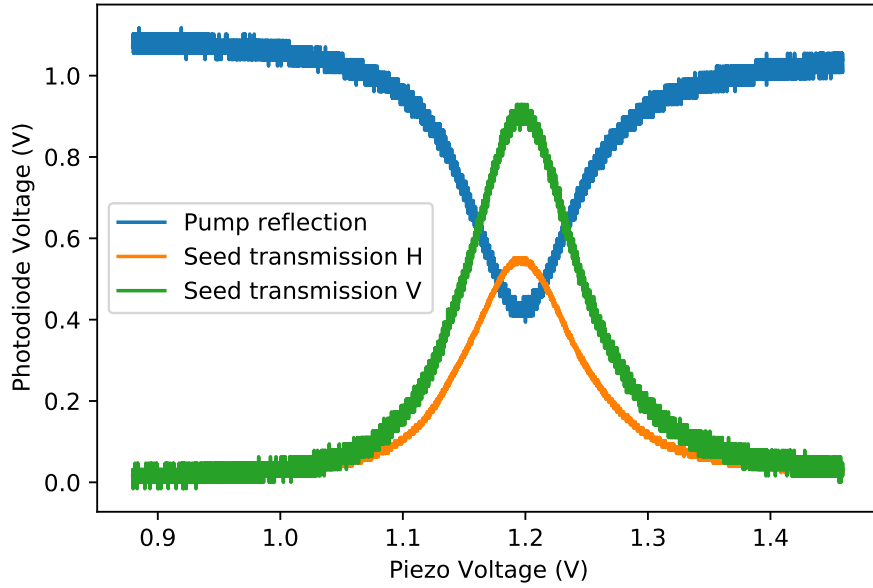


Figure 4.13: An example of triple resonance condition on oscilloscope by scanning piezo voltage. The three fields observed on resonance monitor detectors overlap with each other. The pump field is monitored from cavity reflection and both signal and idler fields are monitored from cavity transmission.

We develop a graphic user interface (GUI) in Python to interface multiple hardware equipment at the same time. The GUI allows users to set the frequencies of Toptica and EXFO laser, the temperature control loop set points, and to read the frequencies from the wavelength meter. It also enables a customised piezo voltage scan with Labjack, where we can set the start point, the end point, and the step of the scan. The GUI provides an option to fit the resonance feature with a Lorentzian curve. Users can choose to save one or more of the following options with a user-defined file name: the data, the plot, and the fitting curve. Fig. 4.14 is an example of the detectors' voltage plot when the cavity

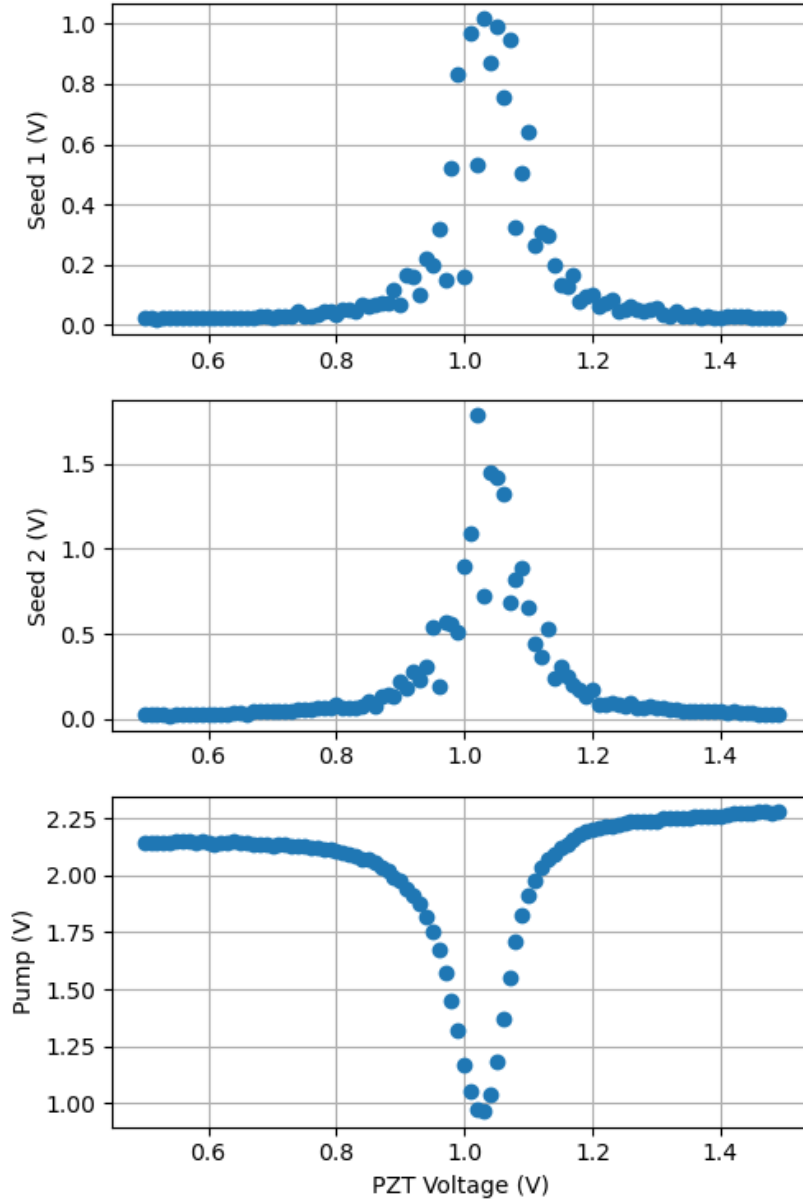


Figure 4.14: Triple resonance condition shown on GUI by scanning piezo voltage. The seed 1 and 2 correspond to the horizontal and vertical polarisation of the seed laser. Seed resonances are monitored with transmission power. The pump resonance is monitored with reflection power. On triple resonance condition, the three fields' resonances overlap at the same piezo point. The measurement of pump fits better to a Lorentzian curve than the two seed fields due to frequency stability of the lasers. The pump laser Toptica DLC is more stable in frequency than the seed laser EXFO T100S.

satisfies the triple resonance condition. The GUI scans the piezo in a slower speed so the EXFO laser fluctuation is more noticeable on the plot. It is an easy way to check the cavity resonance and set our piezo voltage to the triple resonance point.

This chapter focused on the cavity’s linear response. Following the optical alignment procedures, we measured the FSR and linewidth of the cavity fields. The FSR and linewidth of the three fields are shown in Table 4.2.

Wavelength (nm)	Polarisation	FSR (GHz)	Linewidth (MHz)
780	Vertical	16.69 ± 0.02	220 ± 4.96
1560	Horizontal	16.22 ± 0.01	101.70 ± 10.45
1560	Vertical	16.96 ± 0.01	100.68 ± 10.85

Table 4.2: The FSR and linewidth measurements of the three fields.

Since the resonance frequency is related to optical path length, we investigated the response with variations of the crystal temperature and the piezo voltage. We found that the cavity parameters affect the resonance frequency linearly. This investigation helped us develop the theory and procedure to tune the cavity towards triple resonance. We also developed a GUI to aid data acquisition and cavity resonance visualisation. In the next chapter, we will explore the nonlinear response of the triply resonant cavity. First, we observe the effects of second harmonic generation and difference frequency generation from the triply resonant cavity. Then, we perform spontaneous parametric down conversion with a continuous-wave pump and measure its photon statistics, collection efficiency, indistinguishability, and two-photon timing correlation.

Chapter 5

Nonlinear response of the cavity with a continuous-wave pump

In the prior sections, we built and calibrated the cavity to achieve a triply resonant state. In this chapter, we discuss the nonlinear response of the cavity. This chapter starts with introducing superconducting nanowire single-photon detectors used in the experiment, with their working principle and specifications. We investigate the second harmonic generation (SHG) effect from the seed laser inside a triply resonant cavity, verified with the SHG intensity relations and polarisation relations. By observing the difference frequency generation (DFG) from the triply resonant cavity with both pump and seed laser, we can estimate the intensity relations between DFG and SPDC. Then, we test the SPDC squeezed state generation with a continuous-wave pump and characterise the SPDC squeezed state source with arrival time difference and interference measurements. Using the photon counts, we are able to characterise the collection efficiency. By interfering the signal photon and idler photon, we observe a Hong-Ou-Mandel effect using the squeezed state source. Lastly, we discuss the two-photon timing measurements that forms a double exponential probability distribution and measure the timing correlation between the signal and idler photons.

5.1 Superconducting nanowire single-photon detectors - SNSPDs

To characterise the photon source, it is essential to determine the number of individual photons over a given period of time. This requires a detector with the ability to detect single photons. There are multiple approaches to detect single photons [121], such as single-photon avalanche photodiodes (SPAD), frequency up-conversion single-photon detector, superconducting transition-edge sensors (TESs), superconducting nanowire single-photon detectors (SNSPDs), etc. Some of them can provide the number of photons during each detection named as photon number resolving detectors, like TESs. Others are click detectors, which provide binary information between zero photons and one or more photons. SPAD, up-conversion detectors, and SNSPDs are examples of click detectors. In this work, we use the

SNSPDs as our single photon detector to characterise the squeezed state source.

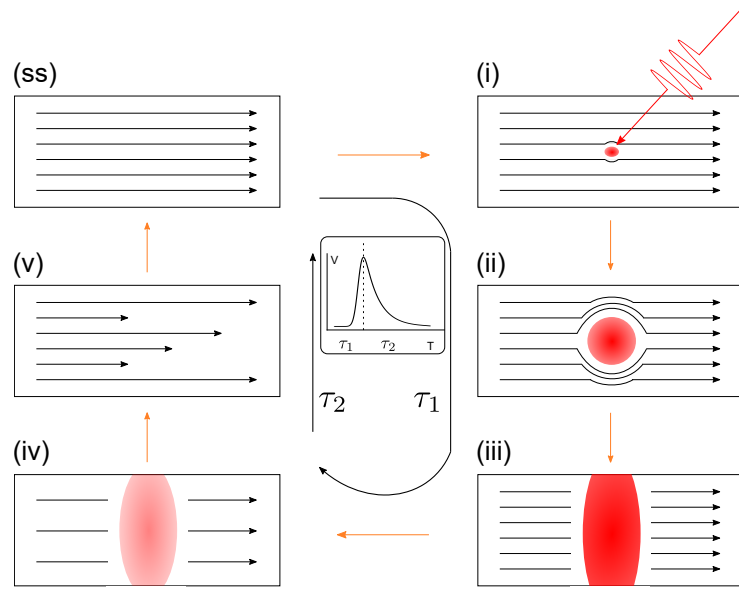


Figure 5.1: Single photon detection mechanism using a SNSPD [122, 123]. The superconducting nanowire on a thin film at steady state (ss) has a bias current close to the critical current. At stage (i), a single photon at near-infrared wavelength presents at the active area, triggering the photon absorption. The photon excited electrons lead to a creation and expansion of quasi-particles cloud and phonons at stage (ii). This induces a region of a local resistive region in the active area at stage (iii), where there is a hot-spot in the current distribution. The current flows towards the readout electronics at stage (iv) and the nanowire restores its superconductivity at stage (v). The voltage of the readout signal at each stage of the detection.

SNSPDs are single photon detectors based on superconducting materials with quick photo-responsive properties to provide fast photon detection [122, 123]. They are well-suited for near-infrared wavelengths. Fig. 5.1 shows the single photon detection process with different stages. SNSPDs operate at a temperature range of 1.5 - 4 K to maintain the superconductivity at steady state (ss). The nanowires are biased under the critical current density so that, when a photon is present in the superconducting region at stage (i), the photon absorption creates a cloud of quasi-particles and phonons at stage (ii). This changes the current distribution and leads to a local resistive region, so that the superconducting is broken at stage (iii). A voltage pulse is induced as the current flows toward the readout electronics at stage (iv). The detector is insensitive to more photons arriving at the nanowire within around 50 ps, which is the recovery time of the nanowire. Afterwards the nanowire cools down, and the superconductivity is restored to allow further photon detection at stage (v) and back to stage (ss).

Dark counts and jitter time are important performance parameters of SNSPD systems. The dark counts of an SNSPD are the accidental triggers of voltage pulses when there is no photon present. The dark counts grows exponentially when the bias current reaches the critical current I_c . It's necessary to detect photons with a bias current that provides high detection efficiency and low dark count rate. The jitter of an SNSPD is the timing uncertainty between an ideal single photon response and the

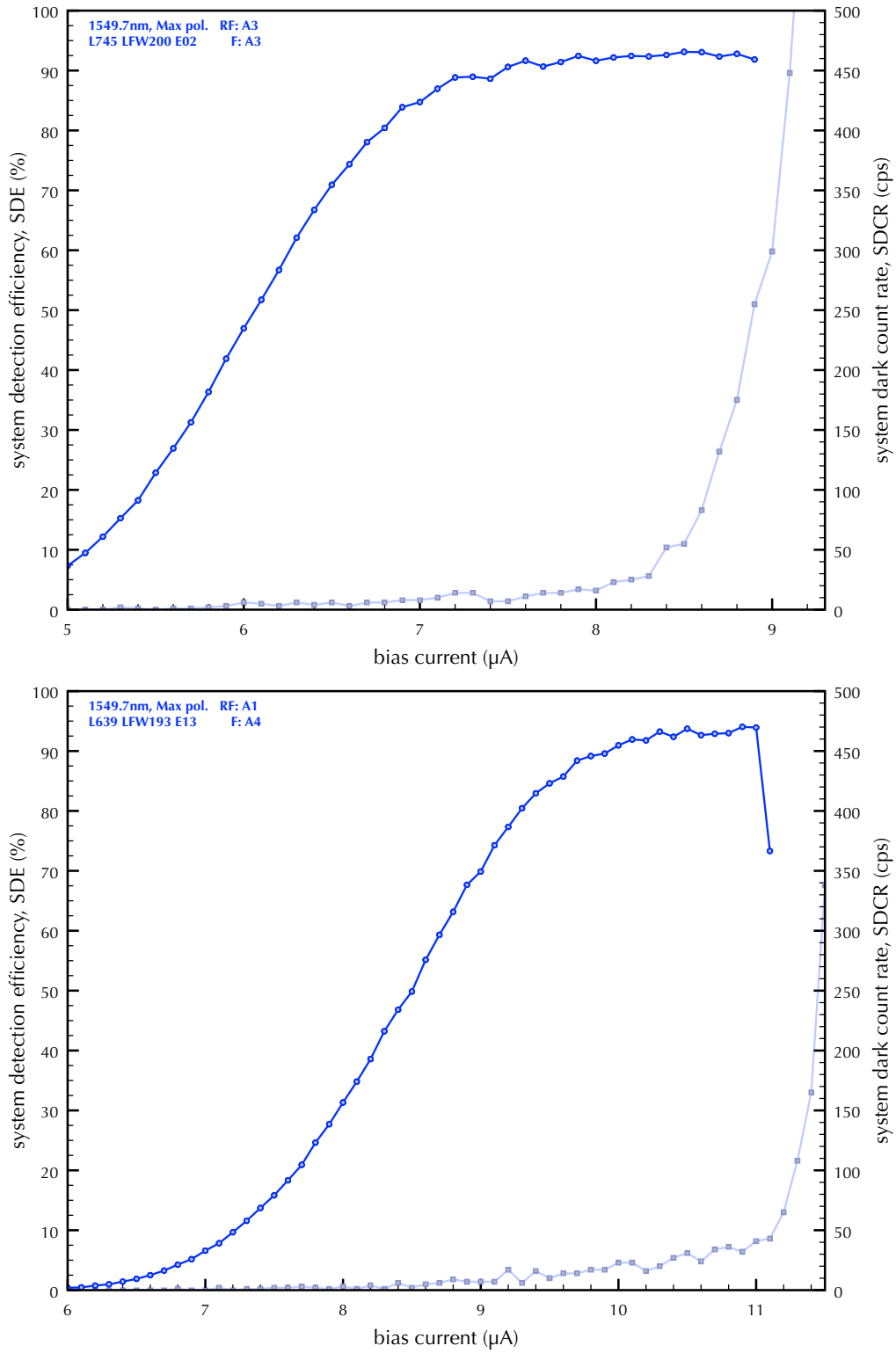


Figure 5.2: Characterisation for two channels of SNSPDs used in the measurements with various bias current provided by PhotonSpot Inc. The dark line is the system detection efficiency of detection (SDE). The light line is the system dark count rate (SDCR). With higher bias current, the SDE increases first and then saturate near 90 %. The SDCR grows exponentially after the bias current reaches a certain point. There is a bias current region for each channel that provides a high SDE ($> 85\%$) and a low SDCR (< 50 counts per second).

detection output voltage pulse. The full width at half maximum (FWHM) of the distribution can be used to quantify the jitter for a typical Gaussian distribution.

SNSPDs have the advantage of low dark counts, low jitter time, and short recovery time. The system used in this work is an array of 4 SNSPDs from PhotonSpot Inc. Its detection efficiency and dark counts in respect of the bias current is shown in Fig. 5.2. The detection efficiency grows gradually with the bias current and reaches saturation when bias current is high. The dark count rate stays low with a low bias current. After the bias current reaches a limit, the dark count rate grows exponentially with respect to the bias current. When using the SNSPD, we choose the bias current region with high detection efficiency and low dark counts. With bias current at 8 and 10 μA , the two channels could achieve $\sim 90\%$ detection efficiency and low dark count rates (< 50 counts per second).

5.2 SHG effect from a triply resonant cavity

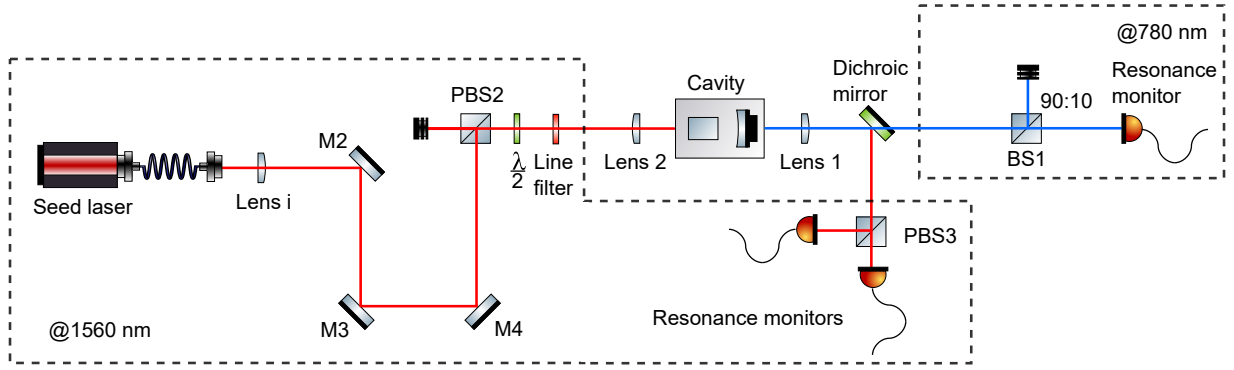


Figure 5.3: Setup to detect SHG signal from cavity triple resonance. The seed laser is sent to the cavity with diagonal polarisation using the half-wave plate. The triple resonance condition allows the horizontal and vertical polarisation components to generate 780 nm field. The pump laser is blocked to prevent false detection. Seed: EXFO T100S-HP tunable laser; M: mirror; PBS: polarising beam splitter; BS: non-polarising beam splitter; $\lambda/2$: half waveplate.

When the cavity reaches triple resonance, with a high power seed laser, we can perform second harmonic generation in the cavity. The pump beam is blocked from interrupting the measurements, and only the seed laser is sent to the triply resonant cavity from the left side as shown in Fig. 5.3. The rotated half-wave plate before the cavity enables both horizontal and vertical polarisation fields inside the cavity at the same time. We adjust the half-wave plate angle to 22.5° to balance the two fields with the same frequency. When the fields are strong enough, they can interact with the nonlinear crystal to perform second harmonic generation. We observe the transmission peak from the SHG using the detector for 780 nm as shown in Fig. 5.4. The noise level in the measurements is the same as the EXFO laser frequency noise. Because the resonance detector on the pump side is a silicon photodiode, it only detects the range of visible light and has no response for the 1560 nm field.

In order to confirm the measured signal is from the SHG effect inside the cavity, we fit the measurements with Lorentzian distribution to find the width and perform measurements with different

seed power and different half waveplate angles. With the same cavity piezo voltage scan, a frequency-doubled field should have a width that is half of the width from the fundamental frequency field. The detected signal has a width of 59 MHz, which is around half of the seed laser width fitting 101 MHz. This fitting is one of the supporting pieces of evidence for SHG effect detection.

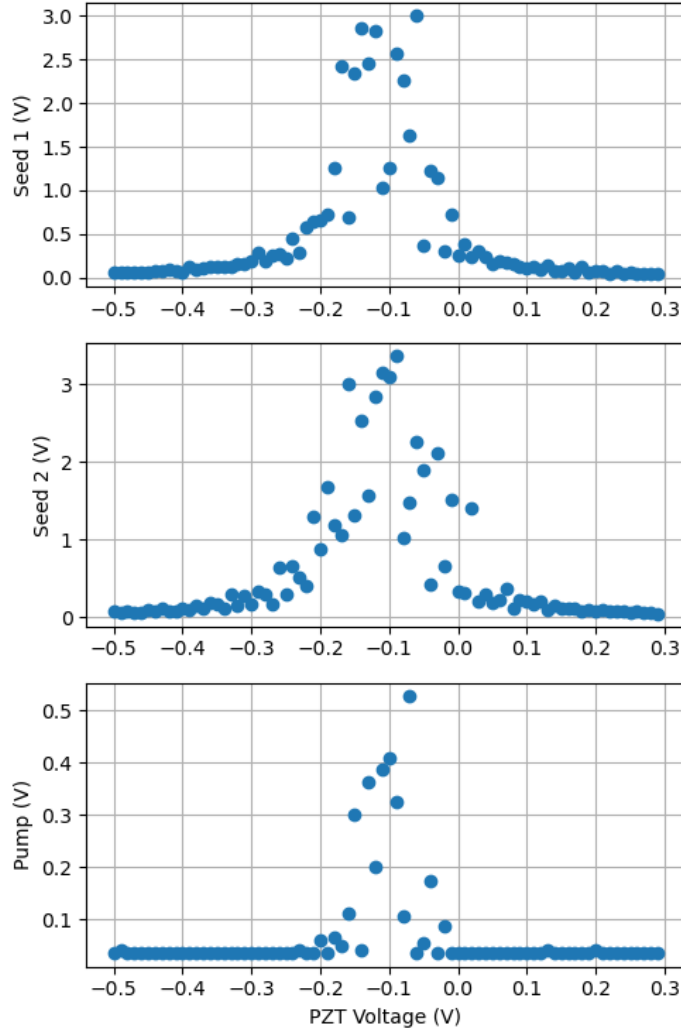


Figure 5.4: The detection of SHG effect from triply resonant cavity recorded with the Python GUI. The silicon detector presents a transmission peak of the SHG at 780 nm (bottom figure) from the seed laser with horizontal and vertical polarisation 1560 nm (top and middle figure).

When we increase the seed laser power, the field intensity detected with silicon photodiode increases quadratically. Fig. 5.5 shows a quadratic relation between the generated 780 nm signal and the seed laser power. The SHG effect makes the frequency-doubled field intensity grow with the square of the fundamental frequency field intensity. The output intensity measurement with different seed power provides further evidence for observing the SHG effect.

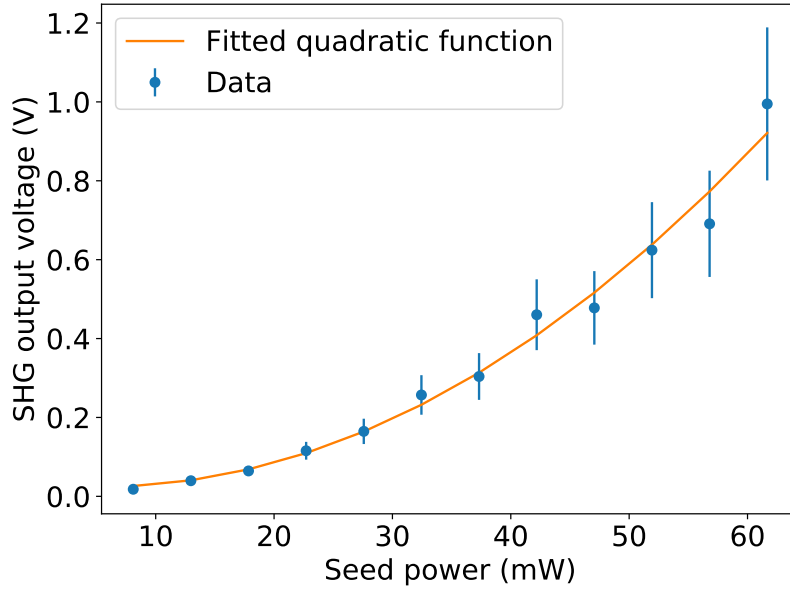


Figure 5.5: Relationship between detected signal on the silicon photodiode and the seed laser power. The silicon photodiode is transparent for 1560 nm wavelength, which means the detected signal are 780 nm from the SHG process. The quadratic relation between the detected signal and the seed laser power follows the SHG theory. The errors come from the laser frequency fluctuation.

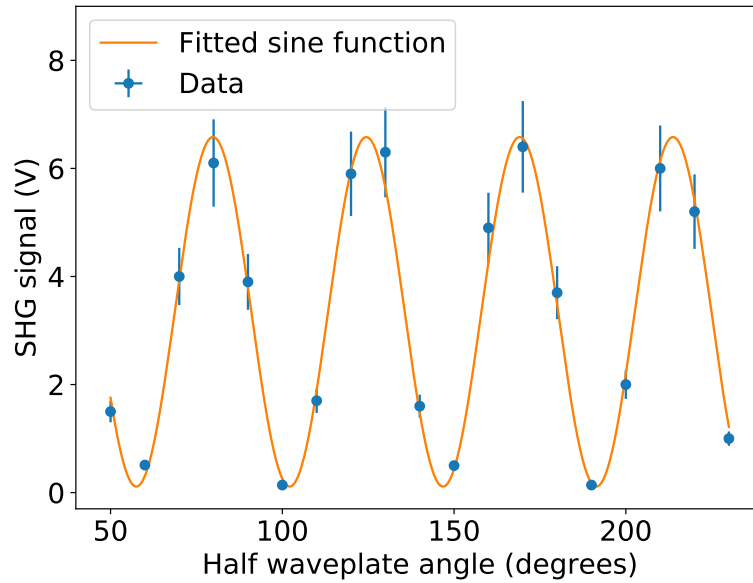


Figure 5.6: The second harmonic generation intensity varies with the polarisation of the input seed signal. The intensity reaches maximum when the input seed is rotated to diagonal polarisation with the waveplate. It has its minimum when the polarisation is rotated to either horizontal or vertical direction. The voltage errors come from laser frequency fluctuation. The half waveplate uncertainties come from manual control of the rotational mount.

We also take measurements with different half waveplate angles. The intensity of the SHG signal changes in a sinusoidal relation with the half waveplate angles, as shown in Fig. 5.6. Rotating the half waveplate changes the intensity proportion of the horizontal and the vertical polarisation fields. When the half waveplate provides equal contribution for the horizontal and the vertical polarisation fields,

the generated SHG field has the highest power. Similarly, when there's only horizontal polarisation or vertical polarisation field present in the cavity, the SHG has the lowest power. This is another supporting evidence for SHG behaviour. Above all, we can confirm that a diagonally polarised seed laser produces the SHG effect in the triply resonant cavity. The conversion efficiency of SHG is 17 %/(W · cm).

5.3 Difference frequency generation (DFG)

Difference frequency generation (DFG) experiment requires sending both the pump field and the seed field into the cavity, where a new field with an energy difference between the pump and seed fields is generated. The setup is the same as the resonance tuning setup in Fig. 4.11 apart from the transmission of the PBS is coupled to an SNSPD detector for photon counting.

We start with measuring the output with only the pump field in the triply resonant cavity. The input pump power is kept at 11.5 mW. The cavity output measurements are marked as SPDC counts in Fig. 5.7. We block the seed field input while maintaining the same pump laser power during the measurements. The SPDC counts data serves as a benchmark for comparison with the DFG effect.

Then we measure the output with only the seed field inside the cavity. If the PBS separates the orthogonal polarisation perfectly, there should be no photon arriving at the detectors. However, in reality, the PBS has a limited distinguishability of 0.1 % and that allows photons with vertical polarisation leaking through the transmission side of PBS with a low probability. The counts from the cavity with only seed laser is marked as the leak through counts. The leak through counts grows linearly against the seed power with a slope of 84 counts/second/nW.

Finally, we take measurements with both the pump and the seed present in the cavity. The results is marked as combined counts in Fig. 5.7. The combined counts data includes the impact from SPDC, leak through, and possibly DFG. With a growing seed power, the combined counts data has a much larger slope comparing with the slope of the leak through counts, indicating there are other impacts that contribute to the measurement, likely to be the DFG counts from three-wave mixing process. When we subtract the leak through counts from the combined counts, the remaining counts come from the sum of SPDC and DFG process. We fit the SPDC + DFG with a linear function and get a slope of 1756 counts/second/nW.

The difference between SPDC and DFG process is that DFG has a seeding component that enhances the three-wave mixing. For SPDC generation, the photons are generated from vacuum:

$$\hat{a}^\dagger |0\rangle = |1\rangle \quad (5.1)$$

where the $\hat{a}^\dagger$ is the creation operator for Fock basis.

For a very simple model of DFG, we can consider an initial state to contain n photons. In this

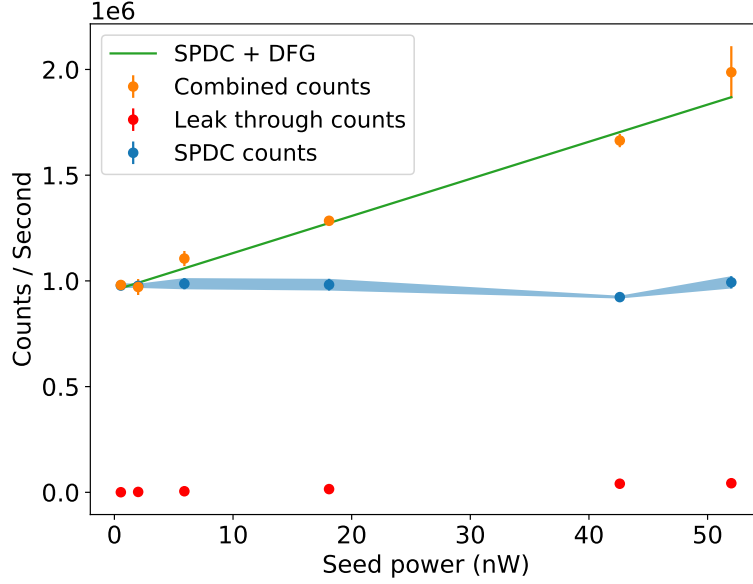


Figure 5.7: Measurements of count rates for different laser settings with different seed power. All the measurements are taken when the cavity is on triple resonance. The leak through are counted with only seed field. The SPDC counts are measured with only the pump field. The combined counts are taken when both pump and seed fields are in the setup. The SPDC + DFG curve shows the counts from combining SPDC and DFG generation.

case, the generated state of $n + 1$ photons is enhanced by the amplitude factor $\sqrt{n + 1}$:

$$\hat{a}^\dagger |n\rangle = \sqrt{n + 1} |n + 1\rangle. \quad (5.2)$$

In the DFG process, the seed photon is already inside the cavity. When there is 1 photon as state $|1\rangle$, the creation operator $\hat{a}^\dagger$ acts on the state provides an amplitude factor $\sqrt{2}$ on the generated state:

$$\hat{a}^\dagger |1\rangle = \sqrt{2} |2\rangle. \quad (5.3)$$

The intensity of the field from DFG is $|\sqrt{2}|^2 = 2$ comparing with a SPDC field intensity 1. The ideal DFG generated double the number of SPDC counts. If the cavity generates mixed states, the DFG would provide lower number than double the SPDC counts. In Fig. 5.7, the data point at 52 nW presents around double the number as the SPDC counts.

5.4 SPDC photon statistics

Using a triply resonant cavity, we generate SPDC photon pairs with a continuous-wave (CW) pump. The setup is shown in Fig. 5.8 with two SNSPDs as the signal and idler detectors. The CW pump is sent to the cavity to generate SPDC photon pairs. The resonance monitor at the pump wavelength is fed back to the temperature and piezo controls to keep the cavity in triple resonance state. The generated photon pairs are separated by a polarisation beam splitter. The photon statistics are measured using

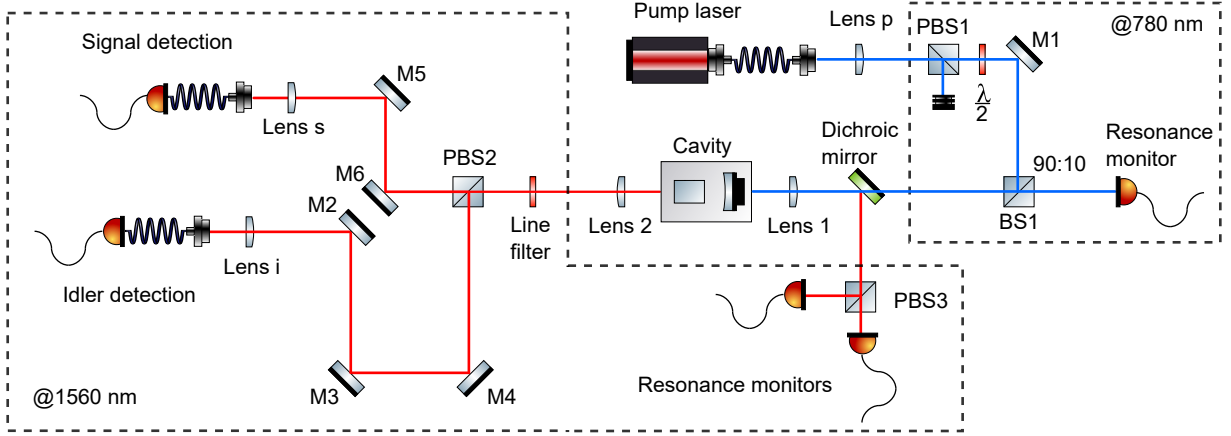


Figure 5.8: Experimental apparatus of SPDC squeezed state generation. The pump laser inputs into the cavity for SPDC generation. The generated photons have orthogonal polarisation. They leave the cavity from the left side and separate after PBS2 on two paths towards single photon detectors. Pump: Toptica DFB Pro laser; M: mirror; PBS: polarising beam splitter; BS: non-polarising beam splitter; $\lambda/2$: half waveplate.

a Hanbury Brown-Twiss setup.

5.4.1 Hanbury Brown-Twiss (HBT) setup

The state generated from PDC contains different number of photon pairs. The photon statistics can be measured with Hanbury Brown-Twiss interferometer [124]. The setup shown in Fig. 5.9 consists of a beam splitter, two detectors, and a timing electronics equipped with a start-stop circuit. It illustrates that the light field is sent to a beam splitter and detected with two detectors D1 and D2. The timer starts when one of the detectors registers a photon and stops when the other detector detects a photon after time τ . A time tagger is able to record all photons' arrival time and post-process the correlation data. It can present a histogram for $g^{(2)}(\tau)$ with high accuracy. The histogram of the start-stop delay times τ is proportional to $g^{(2)}(\tau)$.

In reality, the detectors to perform the measurements are not perfect. We denote the efficiency of the two detectors as η_1 and η_2 . When the pulsed light field generates one photon pair, the detectors have a probability of η_1 and η_2 to provide a click.

As discussed in Section 1.4, the pulsed light field can be written in the density matrix form as

$$\rho = P(0) |0\rangle \langle 0| + P(1) |1\rangle \langle 1| + P(2) |2\rangle \langle 2| + \dots \quad (5.4)$$

where $P(i)$ represents the probability of generating i photons from the light source. We assume that, in the low gain regime, the probability of generating more than 2 photons are very small so that

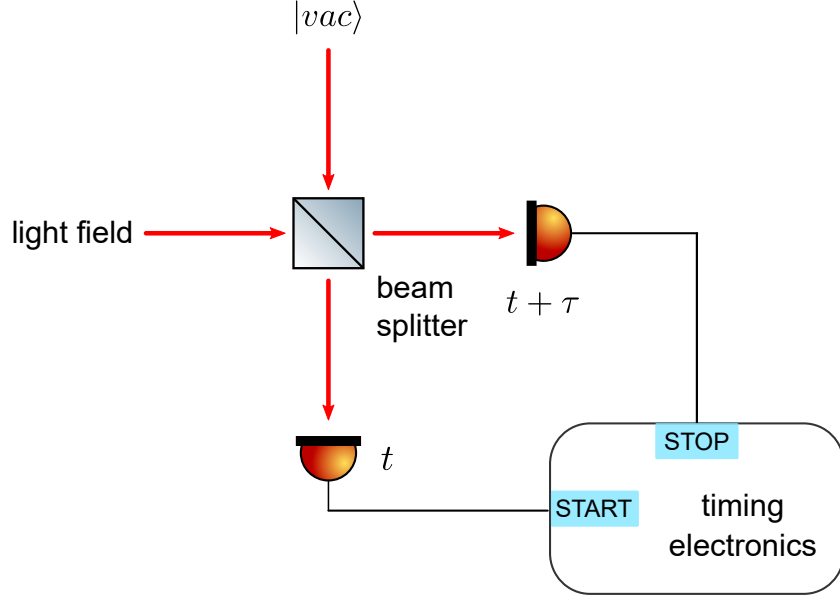


Figure 5.9: Hanbury Brown-Twiss interferometer setup. The light field propagates through the beam splitter. Two detectors from transmission and reflection side are connected to a timing electronics with a start-stop circuit. The timing electronics registers the arrival time of two detection events. The histogram for photon correlations measured by the two detectors can be used to characterise the input field.

$P(1) \gg P(2) \gg P(n > 2)$, and we can write the $g^{(2)}$ as

$$g^{(2)}(0) = \frac{2P(2)}{(P(1))^2} \quad (5.5)$$

as we derived in Eq. (1.41).

On the detection side, there are four possibilities: both detectors click, detector 1 clicks, detector 2 clicks, and neither detector clicks. Each scenario can be described with a probability expression as follows:

$$\mathcal{P}_{11} = P(2) * 2R(1 - R)\eta_1\eta_2 \quad (5.6)$$

$$\begin{aligned} \mathcal{P}_{10} &= P(1)R\eta_1 + P(2)R^2(1 - (1 - \eta_1)(1 - \eta_2)) + P(2)2R(1 - R)\eta_1(1 - \eta_2) \\ &\approx P(1)R\eta_1 \end{aligned} \quad (5.7)$$

$$\begin{aligned} \mathcal{P}_{01} &= P(1)(1 - R)\eta_2 + P(2)(1 - R)^2(1 - (1 - \eta_1)(1 - \eta_2)) + P(2)2R(1 - R)\eta_2(1 - \eta_1) \\ &\approx P(1)(1 - R)\eta_2 \end{aligned} \quad (5.8)$$

$$\mathcal{P}_{00} = 1 - \mathcal{P}_{11} - \mathcal{P}_{10} - \mathcal{P}_{01}. \quad (5.9)$$

Rearranging Eqs. (5.6) to (5.8), we can use the detection probability to represent the photon

probability in the light field:

$$(P(1))^2 = \frac{\mathcal{P}_{10}\mathcal{P}_{01}}{\eta_1\eta_2R(1-R)} \quad (5.10)$$

$$P(2) = \frac{\mathcal{P}_{11}}{\eta_1\eta_22R(1-R)}. \quad (5.11)$$

Thus, we can write Eq. (5.5) as

$$g^{(2)}(0) = \frac{\mathcal{P}_{11}}{\mathcal{P}_{10}\mathcal{P}_{01}} = \frac{C_{11}N_{\text{trial}}}{C_{10}C_{01}} = \frac{C_{11}T_{\text{total}}R_{\text{repetition}}}{C_{10}C_{01}} \quad (5.12)$$

where the $C_{a,b}$ is the count registered on the detectors for detecting a and b photons on the two detectors. T_{total} is the total time of the measurement, $R_{\text{repetition}}$ is the repetition rate with a unit of Hz, and $N_{\text{trial}} = T_{\text{total}}R_{\text{repetition}}$ is the number of trials during the measurement.

The HBT has an advantage in terms of $g^{(2)}(0)$ measurements that this scheme is not sensitive to the loss in the setup of Fig. 5.9. One of the sources of loss comes from the imperfection of the beam splitter. Another source of loss origins from the detection efficiencies of photon detectors. The HBT approach doesn't require the beam splitter to have a perfect 50:50 splitting ratio. It doesn't require the detection efficiencies of both detectors to be 100 % or even matched to each other. This provides a high flexibility in experimental options. The $g^{(2)}(0)$ would not be affected by the loss in the setup as long as it satisfy the low gain regime assumption of $P(1) \gg P(2) \gg P(n > 2)$.

5.4.2 Count rates

From the derivations in chapter 2, we have the expression for the SPDC output state with the pump amplitude:

$$|\Psi\rangle \propto |0\rangle + CE_pL \int \int d\omega_s d\omega_i f(\omega_s, \omega_i) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) |0\rangle \quad (5.13)$$

where C is a constant, E_p is the amplitude of the pump field, and L is the crystal length. The output state intensity is proportional to $|E_pL|^2 = I_pL^2$, where $I_p = |E_p|^2$ is the intensity of the pump field. The output intensity is linearly dependent on the pump intensity.

Using the HBT setup to measure the signal beam, we recorded the count rates for the output of the beam splitter. Fig. 5.10 shows the relationship between the single counts on each channel and the pump power. The single counts grow linearly with pump power. The measurement data matches the derivations from the SPDC theory.

To discuss the coincidence counts relation with the pump power, we use the two-mode squeezed state theory to explore the cases for registering a coincidence count. The two-mode squeezed state

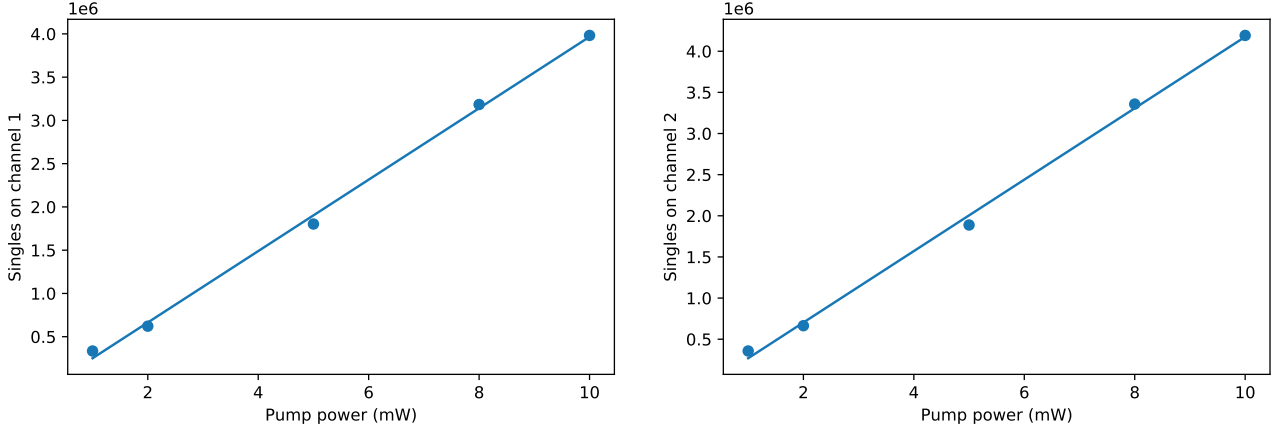


Figure 5.10: Single counts with different pump power. On both channels the singles grow linearly with respect to pump power. The error bar is negligible on the plot and the line is the linear fitting.

can be expressed in the form of

$$\hat{S}_2 |00\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (\tanh r)^n |nn\rangle = \frac{1}{\sqrt{1-\lambda^2}} \sum_{n=0}^{\infty} \lambda^n |nn\rangle \quad (5.14)$$

where we define the coefficient $\lambda = \tanh r$. The probability of generating n photons in signal and idler beam is

$$P(n) \propto \lambda^{2n}. \quad (5.15)$$

When there is one photon exiting in the signal beam, it would randomly chooses the path after the beam splitter and arrives at one detector. This means the count is recorded as a singles count instead of a coincidence count because no other photon is present to trigger the other detector. As we discussed and measured, the singles count is linearly related to the pump power. Such case has a probability of

$$P(1) \propto \lambda^2 \propto I_p. \quad (5.16)$$

The coincidence count in the HBT setup corresponds to the case when the signal arm has two photon emission and the two photons exit the beam splitter in different paths, assuming the SPDC is still operating in low gain regime. The probability of having 2 photons in the signal beam is

$$P(2) \propto (\lambda^2)^2 \propto I_p^2 \quad (5.17)$$

which means the coincidence count is proportional to the pump power squared.

The HBT start-stop measurement records the coincidence counts for the SPDC source. Fig. 5.11 shows the coincidence count measurements with a coincidence window of 80 ns. The coincidence count grows quadratically with the pump power, matching the theory of two photon probability in SPDC.

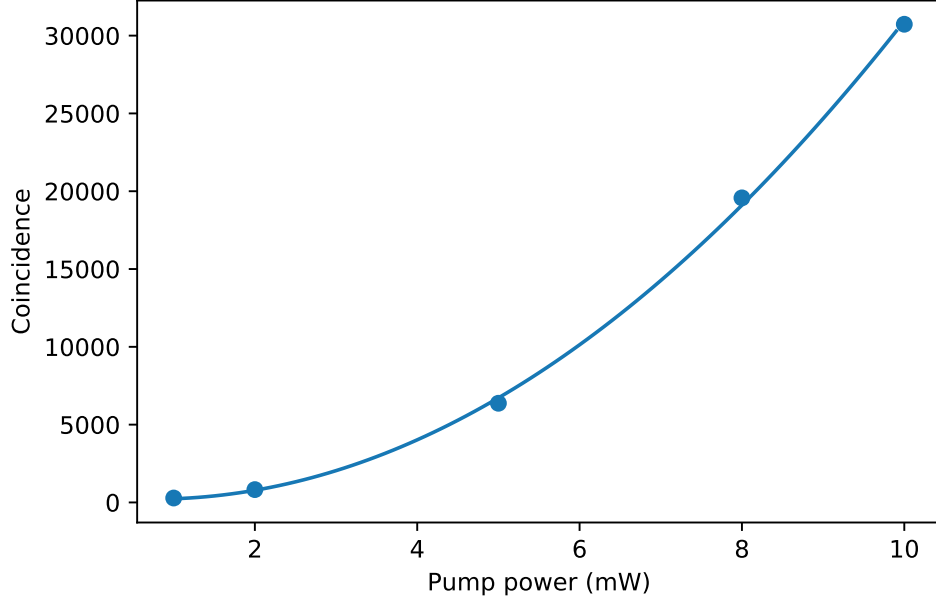


Figure 5.11: Coincidence counts with different pump power. The coincidence counts grow quadratically with the pump power. The error bar is negligible on the plot and the line is the quadratic fitting.

5.4.3 Heralded $g^{(2)}(0)$

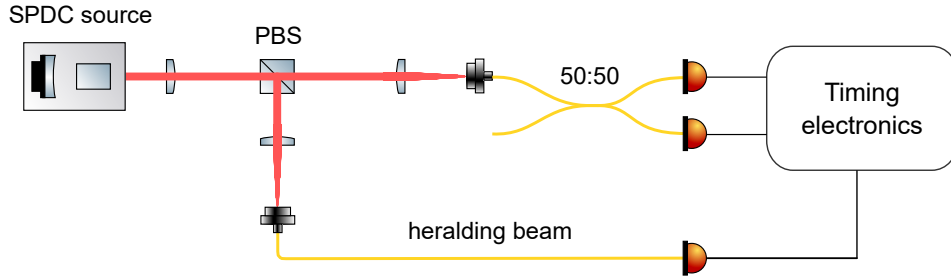


Figure 5.12: Heralded $g^{(2)}(0)$ measurement setup. The SPDC source generates orthogonally-polarised photon pairs, one of which is sent to a 50:50 beam splitter and detected by SNSPDs. The heralding photon is directly detected with an SNSPD as the condition for signal photon measurements. All of SNSPDs detection signals are recorded with a time tagger.

As we introduced in the first chapter, heralded $g^{(2)}(0)$ measures the second order correlation conditioned on heralding beam detection. A heralded $g^{(2)}(0)$ measurement employs a setup in Fig. 5.12. The photon pair generated from SPDC are separated by a polarising beam splitter. Photons in the signal arm pass through a 50:50 beam splitter to reach two detectors. Photons in the heralding arm are sent to another detector that is used to condition the timing electronics. We denote the detectors in the signal arm as detector 1 and 2, and the detector for the heralding arm as detector h. We can explore the heralded $g^{(2)}(0)$ relation with different pump powers by measuring the photon statistics. The detection signals are connected with a time-to-digital converter, which records the time of arrival for the photons and provides coincidence counts.

The photon counts registered on the time tagger can be used to calculate the heralded $g_{\text{Her}}^{(2)}(0)$. In Chapter 1, we have derived the expression for $g_{\text{Her}}^{(2)}(0)$ with conditional probabilities as

$$g_{\text{Her}}^{(2)}(0) = \frac{2P(2|H)}{(P(1|H))^2}. \quad (5.18)$$

Since the measurements are conditioned on the heralding channel detection, the number of trials becomes the number of heralding channel detection C_{H} . The heralded second order correlation using Eq. (5.12) can be written as [125]

$$g_{\text{Her}}^{(2)}(0) = \frac{\mathcal{P}_{11|H}}{\mathcal{P}_{10|H}\mathcal{P}_{01|H}} = \frac{N_{\text{trial}}C_{\text{H},1,1}}{C_{\text{H},1,0}C_{\text{H},0,1}} = \frac{C_{\text{H}}C_{\text{H},1,1}}{C_{\text{H},1,0}C_{\text{H},0,1}} \quad (5.19)$$

where the C_{H} is the count on the heralding channel, and $C_{\text{H},a,b}$ is the count for detecting non-zero photons on the heralding channel and a and b number of photons on detector 1 and 2 within the coincidence window.

To explore the relationship between $g_{\text{Her}}^{(2)}(0)$ and the pump power, we can write the counts in the above calculation as

$$C_{\text{H},1,0} = P(1)\eta_{\text{H}}R\eta_1 \quad (5.20)$$

$$C_{\text{H},0,1} = P(1)\eta_{\text{H}}(1-R)\eta_2 \quad (5.21)$$

$$C_{\text{H}} = P(1)\eta_{\text{H}} \quad (5.22)$$

$$C_{\text{H},1,1} = P(2)\eta_{\text{H}}R\eta_1(1-R)\eta_2 \quad (5.23)$$

where $P(n)$ is the probability of SPDC generating n photon pairs, η_i is the detector efficiency of detector i , R is the reflectance of the beam splitter which is 50 % in a perfect setting. By using these expressions, we find the heralded $g_{\text{Her}}^{(2)}(0)$ as

$$g_{\text{Her}}^{(2)}(0) = \frac{P(1)\eta_{\text{H}}P(2)\eta_{\text{H}}R\eta_1(1-R)\eta_2}{P(1)\eta_{\text{H}}R\eta_1P(1)\eta_{\text{H}}(1-R)\eta_2} = \frac{P(2)}{P(1)} \propto I_{\text{p}}. \quad (5.24)$$

The measurements are taken using time tagger to register singles and coincidence counts with a coincidence window of 80 ns. In the measurements, we are operating in the low gain regime where low probability of double photon-pair emission is expected. The heralded $g^{(2)}(0)$ should be very low. And the heralded $g^{(2)}(0)$ grows linearly with the pump power.

The results of $g_{\text{Her}}^{(2)}(0)$ are shown in Fig. 5.13, which matches the theory discussed above. The $g_{\text{Her}}^{(2)}(0)$ has a linear relation with the pump power with a negligible error bar. When we employ a higher pump power, double photon-pair emission is more probable, where the approximation under low gain regime no longer applies. With high gain operation, homodyne detection is more relevant to characterise the squeezing parameter of the SPDC source.

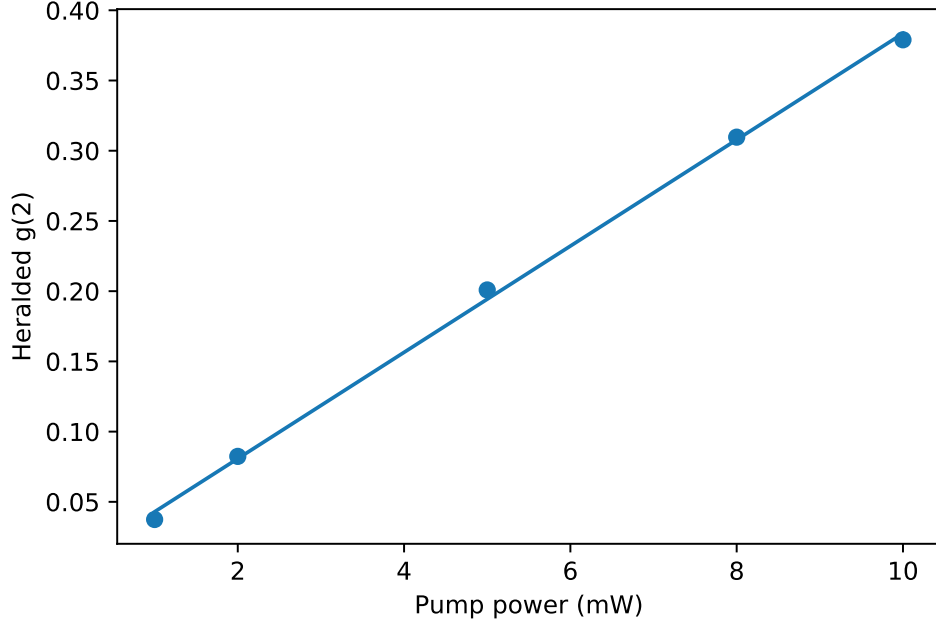


Figure 5.13: Heralded $g^{(2)}(0)$ with pump power. The heralded $g^{(2)}(0)$ grows linearly with pump power. It stays at near 0 level within the low-gain regime. The error bar is negligible on the plot and the line is the linear fitting.

5.4.4 Marginal $g^{(2)}(0)$

The marginal $g^{(2)}(0)$ is useful to characterise the state mode number. We are able to measure the second order correlation $g^{(2)}(0)$ using the setup in Fig. 5.14. The photon pairs are generated by SPDC and split at the PBS. The photons in the signal arm go through a 50:50 beam splitter. The outputs of the beam splitter are detected with SNSPDs. The detection signals are connected with a time tagger, which provides singles and coincidence counts during the measurement.

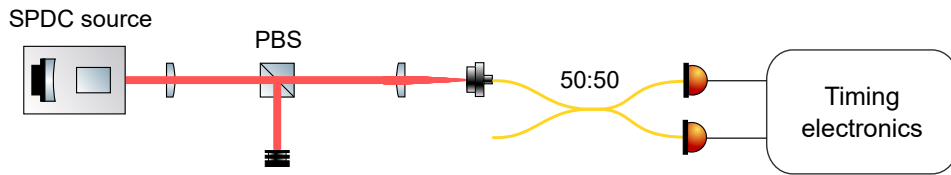


Figure 5.14: Marginal $g^{(2)}(0)$ measurement setup. The SPDC source generates orthogonally-polarised photon pairs, one of which is sent to a 50:50 beam splitter. The outputs of the beam splitter are detected with SNSPDs whose detection signals are recorded with a time tagger.

The SPDC output's mode number is represented by Schmidt number K of the state, which is linked to the marginal $g^{(2)}(0)$ as [107]

$$g^{(2)}(0) = 1 + \frac{1}{K}. \quad (5.25)$$

With a larger Schmidt number K , the second order correlation is closer to 1. Using a CW pump, we

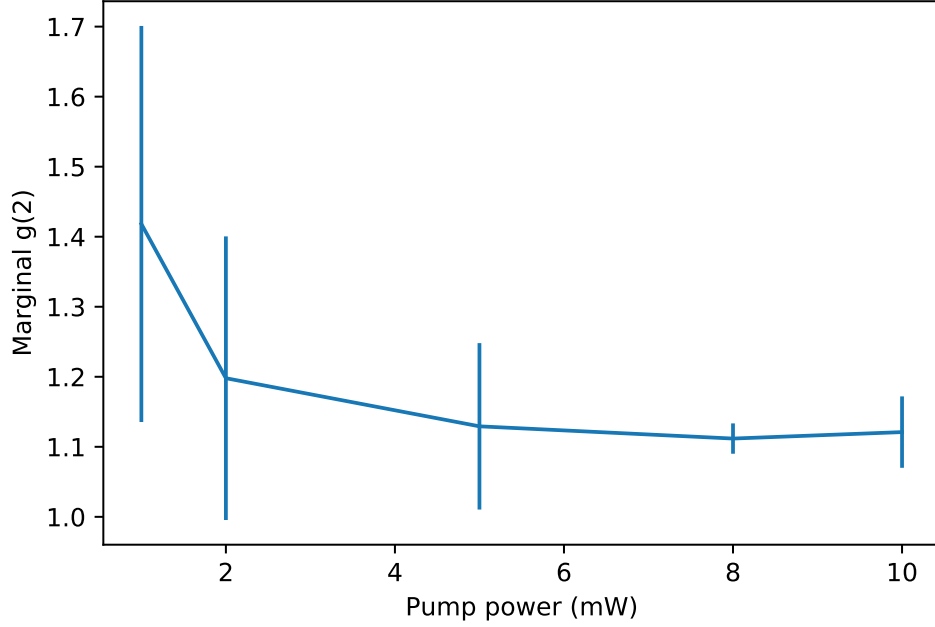


Figure 5.15: Marginal $g^{(2)}(0)$ with pump power. For the continuous-wave SPDC, the marginal $g^{(2)}(0)$ is close to 1 due to multi-mode output. The coincidence window is 80 ns. Each measurement takes 2 minutes.

are generating spectrally correlated photon pairs. Thus, the marginal $g^{(2)}(0)$ is expected to be 1 as the mode number should be large. We use the two-photon timing measurement to get the temporal mode length as 1.7 ns. The detailed discussion about the temporal mode length will be shown in Section 5.7. In our measurement, the coincidence window is 80 ns. In theory we should measure the marginal $g^{(2)}(0)$ to be approximately

$$g^{(2)}(0) = 1 + \frac{1}{80/1.7} = 1.02 \quad (5.26)$$

which is close to 1 in the ideal case. Fig. 5.15 shows the measurement for marginal $g^{(2)}(0)$ at different pump power levels. Each measurement takes 2 minutes. The measurement has large error bars due to the small number of counts with low pump power. The marginal $g^{(2)}(0)$ value is around 1.15, higher than expected.

One possible explanation for higher marginal $g^{(2)}(0)$ is the cavity stability. When the cavity is on resonance, the measured counts provides an accurate calculation. If the cavity fluctuates around the resonance condition, the photon pairs are not generated at all times. Applying the same calculation results in a higher $g^{(2)}(0)$ because of the empty time bins when the cavity is off resonance. Take an example of a simple model where half of the time the cavity is on resonance, the other half time it is off resonance. To calculate an accurate $g^{(2)}(0)$, the total time should be half of the measured time. However, when we use the equation derived earlier $g^{(2)}(0) = \frac{C_{11}T_{\text{total}}R_{\text{repetition}}}{C_{10}C_{01}}$, the calculation provides double the value of the correct $g^{(2)}(0)$. Depending on the actual on-resonance time and level, the calculated $g^{(2)}(0)$ from measurements can be larger than the expected value $g^{(2)}(0) = 1$.

Another possible reason is that the output polarisation of the SPDC source is rotated. Rather than perfectly splitting the signal and idler through the PBS, a small portion of single-mode squeezed states contribute to the marginal $g^{(2)}(0)$ measurements. The measured $g^{(2)}(0)$ is larger than expected due to the large $g^{(2)}(0)$ of a single-mode squeezed state. In our setup, we put a half waveplate in front of the PBS to obtain optimal polarisation, whose precision is limited by the manual control of the rotational mount. This can be improved with a motorised rotational mount in future measurements.

As mentioned, the coincidence window is 80 ns in our measurements. Based on Eq. (5.26), the Schmidt number K is related to the ratio between the coincidence window and the temporal mode length. If we reduce the coincidence window to take measurements, we should be able to observe a growing $g^{(2)}(0)$. When we set the coincidence window to be the same as the temporal mode length, we should measure $g^{(2)}(0) = 2$. We tried to reduce the coincidence window to prove this theory. The photon counts reduced because the coincidence window is shorter, which means that each experiment takes longer time to record data. The cavity resonance fluctuates too quickly compared with the experiment data recording. Thus, in our system the measurements have huge uncertainties that do not support us to validate the theory.

5.5 SPDC collection efficiency

The collection efficiency of SPDC is the probability of detecting a photon pair when there is one pair of photons generated during SPDC. After the photons are generated, they first exit the cavity. Then they propagate in free space and get coupled into fibres. The fibres are connected to SNSPDs for photon detection. Loss during any step would impact the collection efficiency.

In order to measure the collection efficiency, we introduce the Klyshko model [126], an approach to use the two-photon arrival time properties to calibrate the photon detectors. In the heralded $g^{(2)}(0)$ setup, we use three SNSPDs to detect photons. Two of them are after the beam splitter in the signal arm. One is used for detecting the heralding arm.

If the photons left the cavity and propagate in free space without loss, we can write the probability for the cases when each detector registers one count:

$$P_1 = P(1)R\eta_1 \quad (5.27)$$

$$P_2 = P(1)(1 - R)\eta_2 \quad (5.28)$$

$$P_h = P(1)\eta_h \quad (5.29)$$

where P_i is the single count probability, i is the notation for detectors, $P(1)$ is the probability of one pair of photons generated from SPDC, η_i is the efficiency of the specified detector, and R is the beam splitter reflectance.

With the help of timing electronics, we are able to measure the probability of coincidence counts between two channels:

$$P_{1,h} = P(1)R\eta_1\eta_h \quad (5.30)$$

$$P_{2,h} = P(1)(1 - R)\eta_2\eta_h \quad (5.31)$$

where $P_{i,j}$ is the detection probability of coincidence between detector i and j .

With the above expressions, we can find the calculation for detector efficiency using the ratios of coincidence probability and singles probability:

$$\eta_h = \frac{P_{1,h}}{P_1} = \frac{P_{2,h}}{P_2} \quad (5.32)$$

$$\eta_1 = \frac{P_{1,h}}{P_h R} \quad (5.33)$$

$$\eta_2 = \frac{P_{2,h}}{P_h(1 - R)} \quad (5.34)$$

where η_i is the Klyshko coefficient of detector i .

We take measurements for different pump powers with a coincidence window of 80 ns. The Klyshko coefficients of the three channels increases with pump power as shown in Fig. 5.16. The Klyshko coefficients represent the detection efficiency, which should be independent of the light source. In practice, when the pump power increases, the possibility of multi-photon emission increases, resulting in a higher coincidence count increase compared with the counts on single channels. Fig. 5.16 shows that the value of the Klyshko coefficients are quite low, which indicates that there are losses in the system.

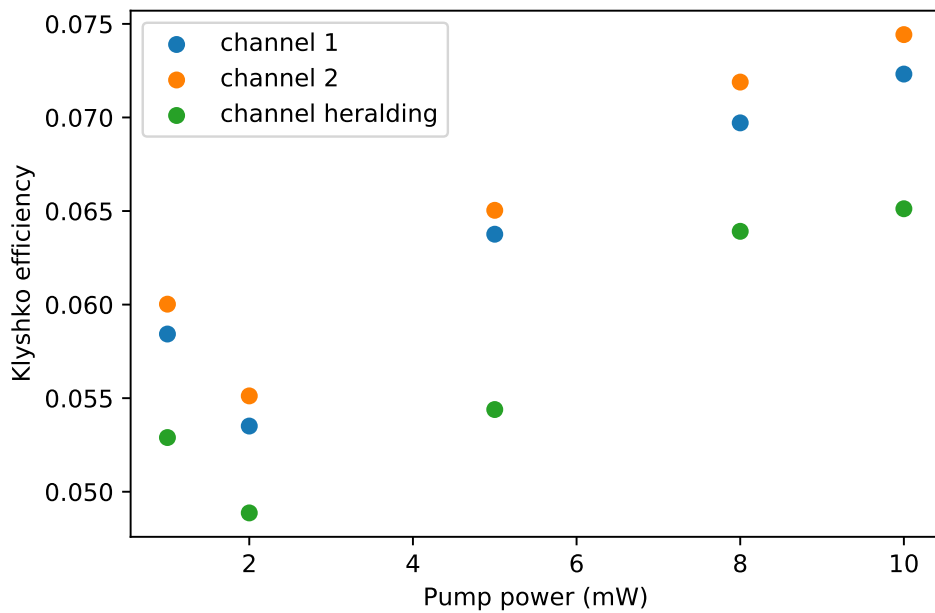


Figure 5.16: Klyshko coefficient with pump power. The Klyshko efficiency increases with pump power. The error bar is negligible on the plot.

Firstly, we consider the intra-cavity loss inside the cavity using the escape efficiency. The mirror coated on the crystal is the exit mirror for 1560 nm photons. It has a reflectance of $97\% \pm 1\%$. The lowest escape efficiency comes from the highest intra-cavity loss. To find the lower bound of escape efficiency, we consider the mirror reflectance as 98% , corresponding to a high intra-cavity loss. Based on the finesse measurement in Chapter 4, the round-trip loss is at most 1.8% for the horizontal polarisation and 1.6% for vertical polarisation at 1560 nm. The escape efficiency of the photon pair is 30% . This is the main source of loss for photon detection.

One possible explanation for the intra-cavity loss is from the crystal hold plate aperture in the middle of the cavity. The cavity-supported fundamental mode has a beam waist of $49.2\ \mu\text{m}$, which corresponds to a beam diameter of $156\ \mu\text{m}$ at the aperture from the crystal hold plate. The size of the aperture is designed to have a diameter of $900 \pm 50\ \mu\text{m}$. At least $350\ \mu\text{m}$ of misalignment is needed for a substantial loss from the aperture, which seems unlikely as the alignment is optimised with the photon counts. Nevertheless, the intra-cavity loss is not enough to explain the low Klyshko coefficient.

Then we consider the mode coupling between the output modes from the cavity and the fibre modes. For a normalised electric field, we can express it as

$$E(r) = \sqrt{\frac{2}{\pi w(z)^2}} \exp\left(-\frac{r^2}{w(z)^2}\right) \quad (5.35)$$

where r is the radial distance to the beam centre, and $w(z)$ is the beam size at z on its optical axis. The mode overlap between two fields can be calculated by

$$\int_0^\infty dr \int_0^{2\pi} d\theta r E_1(r) E_2(r) \quad (5.36)$$

where $E_1(r)$ and $E_2(r)$ are the two normalised fields.

In the experiment, the photons left the cavity are collimated by a 100 mm focal-length lens. The collimating beam in free space has a beam radius of 1.006 mm. The fibre mode collimated in free space by a 10 mm lens has a beam radius of 0.95 mm. Using the above equation to calculate the mode overlap between them, we get a mode overlap of 99.86% . It's very unlikely the mode matching contributes to a huge loss towards the collection coefficient.

The fibre coupling efficiency is around 85% for both detectors. The escape efficiency times fibre coupling efficiency would predict a collection efficiency of 22% . However, the measurement is smaller than this prediction, meaning there are other sources of loss we have yet to account for. One other potential reason for a low Klyshko coefficient might be relatively high noise.

5.6 SPDC Hong-Ou-Mandel experiment

To characterise the indistinguishability of the SPDC source, two-photon interference is the most recognised approach. We can use the Hong-Ou-Mandel experiment setup to perform the two-photon interference experiment. In our case the type II SPDC generates orthogonal polarisation states. A polarisation beam splitter (PBS) separates the horizontal and vertical polarisation spatially into its transmission and reflection ports. By taking advantage of a PBS, we are able to perform a polarisation HOM experiment to measure the interference outcomes.

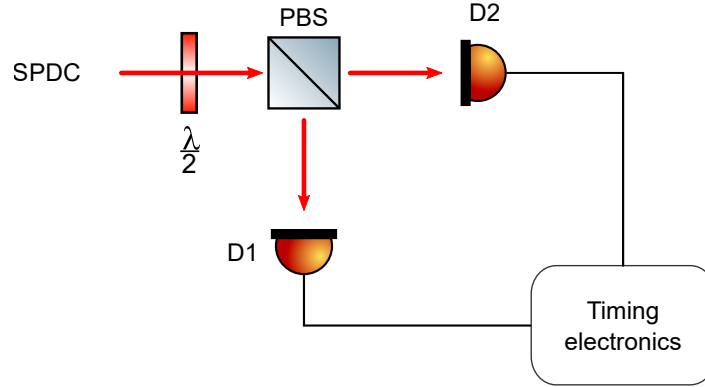


Figure 5.17: Diagram for polarisation Hong-Ou-Mandel setup. The photon pair generated from SPDC is rotated by a half waveplate. The rotated state is sent to the PBS to be measured in the $|H\rangle$ and $|V\rangle$ polarisation basis. The detection signal is recorded by timing electronics to provide information on the HOM interference. $\lambda/2$: half-wave plate; PBS: polarisation beam splitter; D1 and D2: detectors.

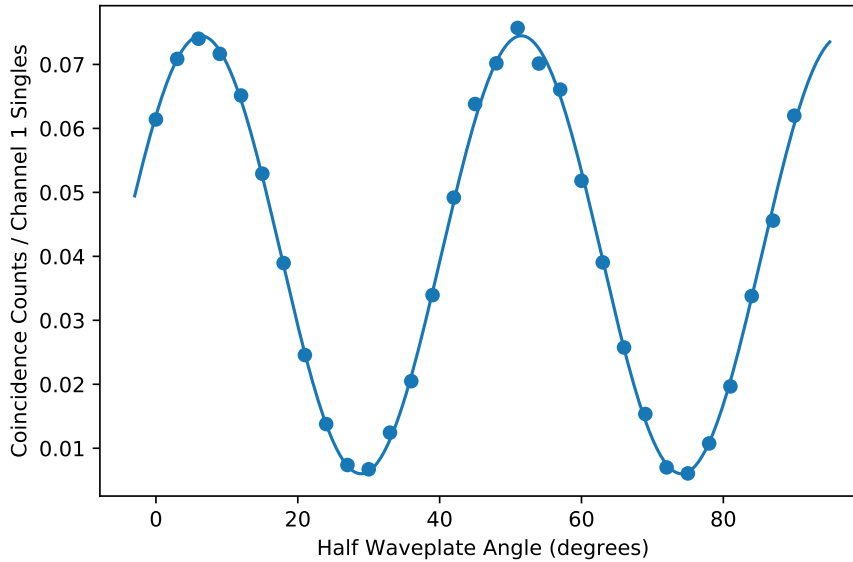


Figure 5.18: Polarisation Hong-Ou-Mandel measurements. The coincidence counts are measured with a time tagger. The coincidence rate changes periodically with the angle of half wave plate. The dip in the figure happens when the two photons interfere at the PBS. The visibility of the HOM dip is 92 %.

The measurements are taken with the setup shown in Fig. 5.17 described in Section 1.4.3. The

photon pair is sent through a half wave plate which makes the 45° rotation. When the rotated state arrives at the PBS, the two-photon interference happens and the outputs of the interference are presented in $|H\rangle$ and $|V\rangle$ basis. The detectors are placed at the outputs of the PBS, allowing us to measure the output states. The detection signals are sent to timing electronics to record the coincidences counts between the two output. Fig. 5.18 illustrates the coincidence rates in respect of the angle of the half wave plate. The figure presents a periodic pattern where every 45° rotation we can capture a minimum in the coincidence.

The coherence of the two photons can be characterised using the visibility from the interference pattern. The visibility of a HOM interference is defined as

$$V = \frac{I_{\max} - I_{\min}}{I_{\max}} \quad (5.37)$$

where $I_{\max}$ and $I_{\min}$ are the maximum intensity and minimum intensity of the measured signal. We calculate the visibility of the HOM dip to be 92 %, slightly less than the ideal visibility 100 %. The discrepancy might come from the frequency symmetry of the output. If the joint spectrum is not symmetric about the diagonal line, the visibility would be reduced. This might happen when the cavity fluctuates around the triple resonance condition. We can compare the results of the HOM dip with one that includes a frequency filter in the signal beam - if the visibility is improved, it suggests that there are contributions from other frequency modes.

5.7 SPDC two-photon timing measurements

The objective of the squeezed state source is to generate a factorisable highly-squeezed state. The factorisability can be quantified with the joint spectral relations between the signal and idler photons. As the time domain is related to the frequency domain with the Fourier transform, we can measure the timing relations between the two to help understand the frequency relations.

After two photons are created simultaneously through the SPDC process inside a cavity, they travel inside the cavity with a certain probability of exiting the cavity from one of the mirrors. We place two detectors on one side of the cavity that can capture the photons after they leave the cavity from this side. The difference in the detection time between the two photons are measured with SNSPDs and digitised using a time tagger. This section discusses the time difference measurements between recording a signal photon and an idler photon.

To simplify the discussion, we begin with a straightforward model where the cavity is homogeneous. The model is shown in Fig. 5.19a. The two photons are generated at the same time and travelling with the same speed inside the cavity. We define the time $t = 0$ as the time when both photons hit the exit mirror M_2 for the first time and time T as the round-trip travel time. The possibility of the signal photon leaving the cavity at $t = 0$ can be written as $P_0 = 1 - R_2$. If the signal photon doesn't

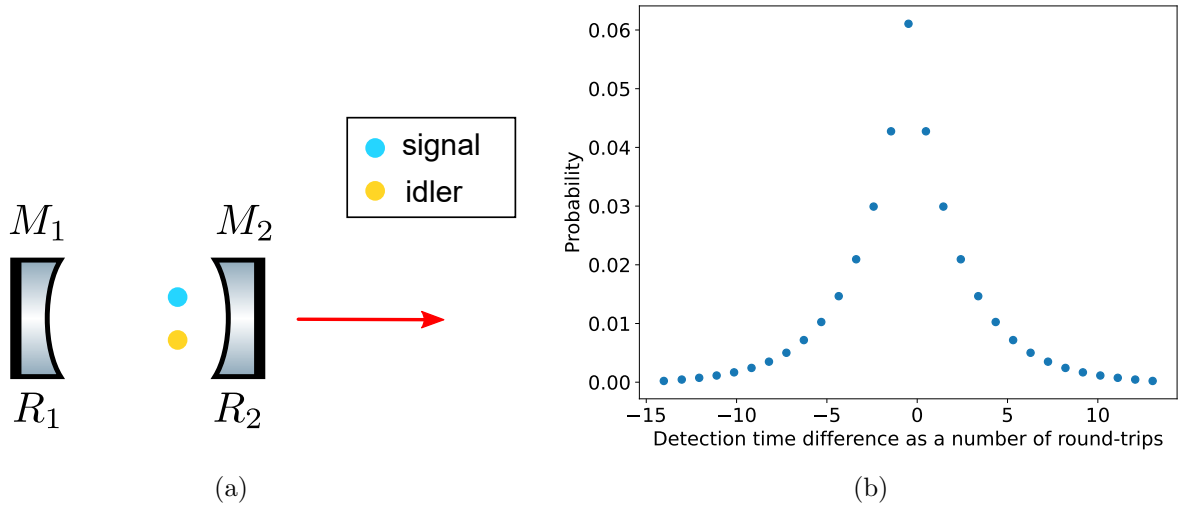


Figure 5.19: (a) Diagram of a simple photon picture model for the time correlation between signal and idler fields from cavity SPDC. The cavity with mirrors reflectances of R_1 and R_2 has the same optical path length for signal and idler photons. The time of signal and idler reaching M_2 for the first time is defined as $t = 0$. (b) Diagram of the probability of detecting two photons with a time difference as a number of round-trips. The relative detection only happens when two photon exiting the cavity at times with a integer number of round-trips. The discrete probability distribution has an exponential decay shape that suggests the photons are more likely to exit the cavity at the same time rather than separated for a few round-trips.

exit at $t = 0$, it would bounce inside the cavity for n round-trips and leave from M_2 with a probability

$$P_n = (1 - R_2)(R_1 R_2)^n \quad (5.38)$$

where n is the number of round-trips the signal photon has travelled before exit from M_2 . The idler photon shares the same exit cavity probability distribution since this model provides the same specifications for both photons.

When the detection time difference τ is zero, it means the signal and the idler photons leave the cavity at the same time. The probability of $\tau = 0$ is a sum of all such cases where the two photons exit the cavity after the same number of round-trips [127]:

$$P(\tau = 0) = P_0 P_0 + P_1 P_1 + \dots = \sum_0^\infty P_i^2 = \sum_0^\infty (1 - R_2)^2 (R_1 R_2)^{2i} = \frac{(1 - R_2)^2}{1 - (R_1 R_2)^2}. \quad (5.39)$$

When there is one round-trip time difference between the signal and idler, the probability is

$$P(\tau = T) = P_1 P_0 + P_2 P_1 + \dots = \sum_0^\infty P_{i+1} P_i = \sum_0^\infty (1 - R_2)^2 (R_1 R_2)^{2i+1} = \frac{(1 - R_2)^2 (R_1 R_2)}{1 - (R_1 R_2)^2}. \quad (5.40)$$

Similarly, we can write the probability for $\tau = kT$ as

$$P(\tau = kT) = \sum_0^\infty P_{i+k} P_i = \sum_0^\infty (1 - R_2)^2 (R_1 R_2)^{2i+1} = \frac{(1 - R_2)^2 (R_1 R_2)^k}{1 - (R_1 R_2)^2}. \quad (5.41)$$

The probability distribution for the signal and idler time difference is shown in Fig. 5.19b as a discrete distribution. The time difference τ in this case is always an integer number of round-trip time.

Now we consider a more complicated situation where there is birefringence inside the cavity. The round-trip time for the signal and idler are different. We call the round-trip time T for the signal and $T + \Delta$ for the idler as shown in Fig. 5.20a. And we define the time $t = 0$ as the time of the signal photon hitting the exit mirror for the first time. The birefringence changes the exit time relations along the time axis in Fig. 5.20b, because the exit time difference is shifted by extra time Δ per round-trip.

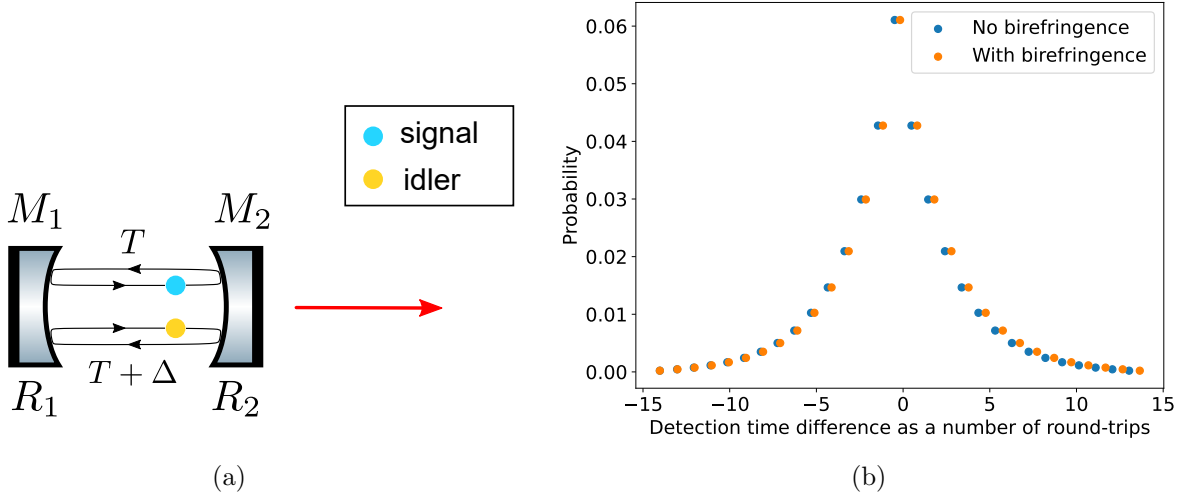


Figure 5.20: (a) Diagram of a type II cavity SPDC photon model for the time correlation between the signal and idler. The round-trip time for the signal and idler photons are different because of the birefringence inside the cavity. We define the signal photon round-trip as T and the idler photon round-trip as $T + \Delta$. (b) Impact of birefringence on the probability distribution for detection time difference between the signal and idler photons as a number of round-trips. The birefringence effect shifts the probability distribution along the time axis.

The detectors are not perfect in terms of response time. The SNSPDs used in the experiment have a time jitter of 50 ps. We use a Time Tagger 20 from Swabian Instruments with a RMS jitter of 34 ps. There is a 78.5 ps FWHM jitter in the detection system. The time jitter introduces a difference between the detection time reading and the actual detection time. The probability of detection with a time difference τ as a number of round-trips can be written as

$$P(\tau) = \sum_{\{t_i\}} P(t_i) P(\tau|t_i) \quad (5.42)$$

where $P(\tau|t_i)$ is a normal distribution with

$$P(\tau|t_i) = (\tau - t) \sim N(0, \sigma) \quad (5.43)$$

where σ is related to the time jitter of the detection system.

Using the above equations to plot the probability distribution with the jitter effect in Fig. 5.21, we discover that the distribution is a double-exponential curve with the detection time difference. The probability distribution becomes a continuous distribution, of which the peak is less sharp at $\tau = 0$. The probability distribution can be approximated as a continuous distribution.

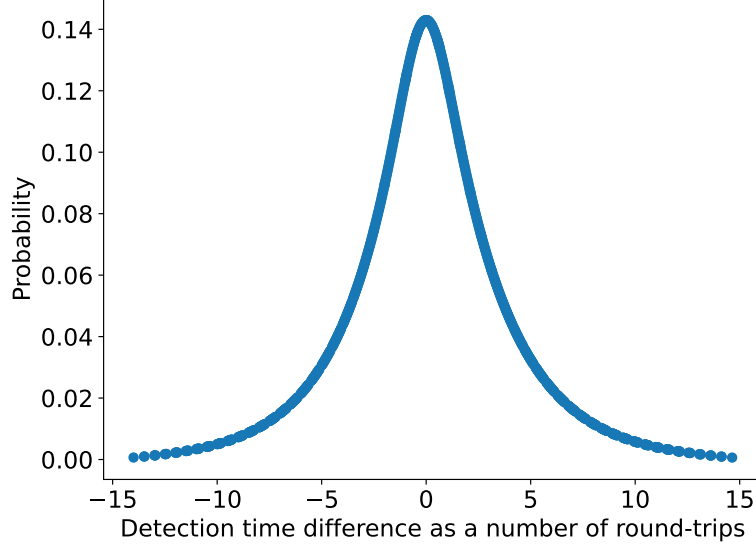


Figure 5.21: Probability distribution under time jitter effect with different detection time difference. It is a double exponential distribution with a less sharp peak at $\tau = 0$.

With a continuous distribution, we can rewrite the expression in Eq. (5.41) as an exponential form:

$$\begin{aligned}
 P(\tau = kT_{\text{rt}}) &= \frac{(1 - R_2)^2 (R_1 R_2)^k}{1 - (R_1 R_2)^2} \propto (R_1 R_2)^k = e^{\ln(R_1 R_2)^k} = e^{k \ln R_1 R_2} \\
 &= e^{k \ln(1 - T_1)(1 - T_2)} = e^{k(\ln(1 - T_1) + \ln(1 - T_2))} \\
 &\approx e^{k(-T_1 - T_2)} = e^{-\frac{(T_1 + T_2)\tau}{T_{\text{rt}}}} = e^{-\gamma\tau}
 \end{aligned} \tag{5.44}$$

where the approximation $\ln(1 - x) \approx -x$ applies with small x , and $\gamma = \frac{(T_1 + T_2)}{T_{\text{rt}}}$ is the inverse of the cavity lifetime.

With the two photon timing difference measurements, we can also explore what the timing relation means in the frequency domain. For cavities with high reflectance, R_1 and R_2 are close to 1. Their finesse can be approximated as [115]

$$\mathcal{F} = \frac{\pi(R_1 R_2)^{1/4}}{1 - (R_1 R_2)^{1/2}} = \frac{\pi(1 - \frac{T_1}{4})(1 - \frac{T_2}{4})}{1 - (1 - \frac{T_1}{2})(1 - \frac{T_2}{2})} \approx \frac{2\pi}{T_1 + T_2} \tag{5.45}$$

where $T_1 = 1 - R_1$ and $T_2 = 1 - R_2$ are the transmittance of the cavity mirrors. The free spectral range of the cavity is

$$\text{FSR} = \frac{1}{T_{\text{rt}}} \tag{5.46}$$

where T_{rt} is the round-trip time of the photon inside the cavity. We can rewrite the cavity linewidth

as a function of cavity transmittance and round-trip time:

$$\delta = \frac{\text{FSR}}{\mathcal{F}} = \frac{T_1 + T_2}{2\pi T_{\text{rt}}}. \quad (5.47)$$

As above equations suggest, the inverse of the cavity lifetime γ is 2π times of the cavity linewidth δ in frequency:

$$\frac{1}{\tau_{\text{lifetime}}} = \gamma = 2\pi\delta. \quad (5.48)$$

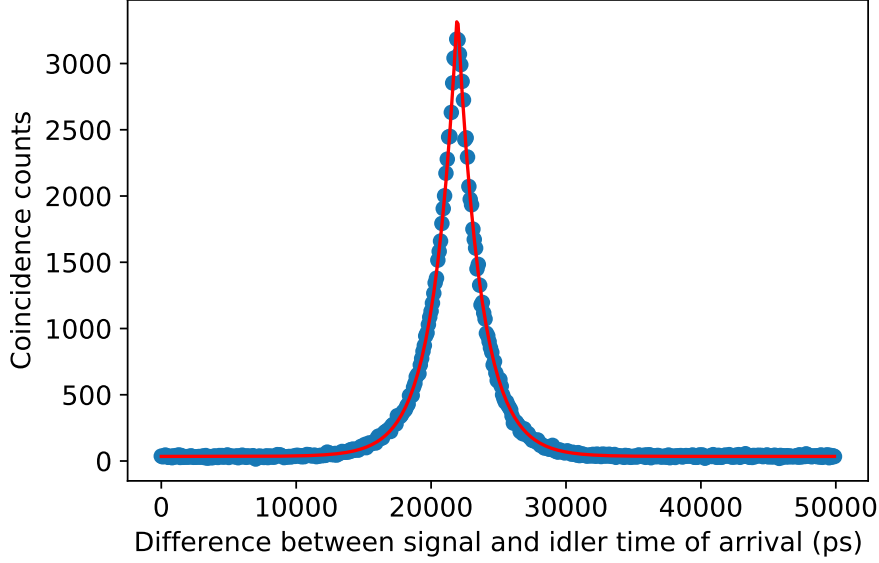


Figure 5.22: Coincidence counts from SPDC squeezed state generation. The time electronics records the arrival time difference between the signal and idler to produce this histogram. The histogram is fitted with Laplace distribution, which is the Fourier transform of a Lorentzian distribution from the cavity resonance spectrum.

The measured two-photon timing is shown in Fig. 5.22. It is a start-stop measurement that takes the arrival time difference between one pair of signal and idler photon. The coincidence counts with time difference presents a double-exponential shape with a fitting of 1.7 ns cavity lifetime. The measurements corresponds to a Lorentzian correlation in frequency with a linewidth of 94 MHz. The linewidth is on the same order as the cavity linewidth for both polarisations.

This chapter presented the nonlinear response of the cavity SPDC source. First, we introduced the superconducting nanowire single photon detectors, used for photon counting in the SPDC low gain regime. Then we observed SHG and DFG signals from the triply resonant cavity configuration. Next, using a continuous-wave pump, we characterised the SPDC photon statistics. The SPDC single counts and heralded $g^{(2)}(0)$ grow linearly with the pump power, and the coincidence counts increase quadratically with the pump power. The marginal $g^{(2)}(0)$ is around 1.15 with a large uncertainty, possibly from cavity instability and the mixing of signal and idler. The collection efficiency is calculated as 6.5 % for both detectors, largely due to the low escape efficiency of the cavity, but we haven't

identified other main sources of loss. We measured the SPDC photon indistinguishability at 92 % in the photon interference experiment. We also performed the two-photon timing measurement between the signal and idler and measured 1.7 ns for the temporal mode of the cavity source. The results in this chapter help us to understand the indistinguishability and the temporal mode length of the signal and idler fields generated in cavity SPDC. The experiments conducted to explore cavity SPDC behaviour with a CW pump serve as a stepping stone for further experimentation employing a pulsed pump. In the next chapter, we will introduce the pulsed pump system and investigate the cavity performance with a pulsed pump.

Chapter 6

Cavity SPDC with a pulsed pump

In the last chapter, we observed the SHG and DFG inside the triply resonant cavity and measured photon statistics of continuous-wave SPDC. The output of SPDC displayed a high photon indistinguishability. After characterising the continuous-wave SPDC source, the next step is to employ a pulsed pump system for factorisable squeezed state generation. In this chapter, we start with introducing the pulsed pump system and describe the behaviour of its components. Then we characterise the pulsed pump SPDC source by testing the photon statistics and its joint temporal intensity.

6.1 Pulsed pump system

Factorisable squeezed state generation requires a specially engineered pulsed pump which can isolate a single mode in the joint spectral intensity. The pulse duration that defines the spectral bandwidth of the pump is the key parameter for factorisable state generation. In this section, we introduce our design for a pulsed pump system.

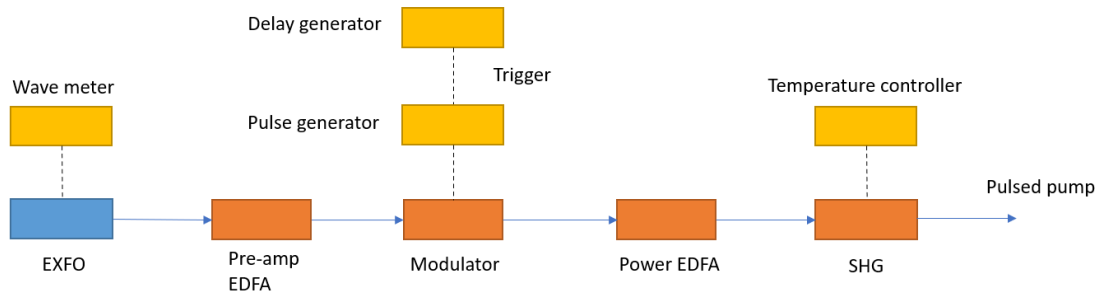


Figure 6.1: Pulsed pump system design. The 1560 nm EXFO laser is amplified with an EDFA. The modulator modifies the pulse shape for the laser with pulse generator. After another amplifier the pulsed laser interacts with a SHG waveguide to generate 780 nm pulsed pump.

In the pulsed pump system, the EXFO T100S laser is employed to generate 780 nm pump laser using second harmonic generation (SHG). The design is structured as shown in Fig. 6.1. The frequency-

locked EXFO laser is amplified with a pre-amp EDFA (Erbium-doped fibre amplifier) EDFA-CW-C0-PM-PR-21-21-FCA from NuPhoton Technologies. The amplified laser is modulated by an iXblue intensity modulator MBC-DG-LAB-A1 which is controlled by an electrical pulse. The shape and frequency of the electrical pulse is defined by a PPG512 PicoQuant programmable pulse generator and a SRS DG645 digital delay generator. After the laser pulse is shaped by the modulator, it gets further amplified with EDFA-CW-C0-PM-PR-33-33-FCA from NuPhoton Technologies. The high power pulsed laser doubles its frequency with a SHG waveguide with part number 1WM1560001111X1XCP021 from HC Photonics. A Koheron TEC100L temperature controller controls the operation of the waveguide. The temperature controller provides a stable environment with less than 0.1°C variance, which is necessary to optimise the SHG efficiency. The optical pulses are measured with high speed fibre-coupled detectors from Thorlabs. The 1560 nm pulses are recorded with DET08CFC/M Si detectors and the 780 nm pulses are recorded with DET025AFC/M InGaAs detectors. Using this system, we expect to generate tunable pulse shapes with a duration on the order of nanoseconds.

6.1.1 Pulse control

We use a programmable pulse generator from PicoQuant to write our electrical pulse. The PicoQuant pulse generator allows pulse writing with a resolution of 200 ps per byte and 512 bytes in total. The delay generator can generate delays between 0 s and 2000 s with 5 ps resolution and a typical RMS jitter of 12 ps. The electrical pulse is written with the pulse generator, which gets triggered by the delay generator with a desired repetition rate. This allows the repetition rate of the pulse matching with the cavity length, so that the pulses on the train enters the cavity at the same time as a former pulse bounces inside the cavity with an integer number of round-trips.

We programme the electrical pulse using a Gaussian temporal shape with various pulse durations. With longer pulses (e.g 10 ns - 20 ns), the pulse presents a well-defined Gaussian shape. However, with shorter pulses (1 - 2 ns), the pulse shape can be distorted with an unexpected spike after the defined pulse possibly due to the fast rising and falling time. The resolution for the pulse generator is limited to 200 ps, which leaves the short pulses defined only by a few points.

To generate a pulsed optical signal, we use the programmed electrical pulse to drive the iXBlue Lithium Niobate intensity modulator, whose electro-optic bandwidth is larger than 10 GHz. The modulator is based on an interferometer structure. Applying a voltage on the modulator would change the optical path length of the material inside the interferometer with electro-optic effect. The effect affects one of the optical paths inside the interferometer to provide continuous adjustment between maximum output (constructive interference) and minimum output (destructive interference). The half-wave voltage, the voltage required to modulate the output from maximum transmission to maximum extinction, is 6 V. The iXBlue modulator used in this experiment has an extinction ratio of 48.6 dB. With the iXBlue software control, the modulator can be auto-calibrated with a DC voltage

to provide minimum output. When the pulsed shape RF voltage applies to the modulator, the optical output has a similar shape as the electric signal.

Fig. 6.2 shows the comparison between the electrical pulse and the optical pulse. We measure the electrical pulses directly using oscilloscope. The optical pulses are measured with a Thorlabs DET08CFC/M detector. Comparing with the electrical signal, the optical signal is slightly shorter in time. It's possible that the time duration change comes from the modulation process. The modulation response has a sinusoidal dependence on the voltage. We need to set the modulator at the minimum output DC voltage for optical pulse modulation. The output intensity response from an electrical voltage change is not linear. The optical pulse intensity grows slower at the start of the electrical pulse. Furthermore, passing the peak of the electrical pulse, the optical pulse intensity reduces more slowly. As a result, the optical pulse has a higher peak. After normalisation, the optical pulse looks shorter in time compared with the electrical signal.

6.1.2 Erbium-Doped Fibre Amplifier (EDFA) characterisation

Erbium-doped fibre amplifiers (EDFAs) employ the concept of stimulated emission to amplify optical signals. Inside an EDFA, a pump laser excites the Erbium ions in the fibre, resulting in a population inversion between the ground state and excited state. The stimulated emission happens when an incoming photon passes through the fibre and interacts with an excited ion, causing the ion to decay to a lower energy level and emit an identical photon as the incoming photon. The amplification process in an EDFA takes place at around 1550 nm. Both the pre-amp EDFA and the power EDFA used in the pulsed laser system are designed for continuous-wave amplification from NuPhoton Technologies.

The output of the EDFA can be adjusted with its driving current for its pump laser. The pre-amp EDFA has one stage of amplification. The power EDFA has two stages of amplification, known as the pre-amp stage and the booster stage, that can be adjusted separately. For the power EDFA, we keep the pre-amp current to be constant as 100 % and characterise the output power with different booster currents. Fig. 6.3 shows the relationship between output power and the current settings on the EDFA. Both present a linear trend between the output power and the current settings. It is worth noting that the output power is independent of the input power for both EDFAs. With the characterisation for the two EDFAs, we are able to estimate the output power after amplification.

6.1.3 Temperature controller for SHG

The Koheron TEC100L temperature controller has a high precision control and adjustable PID gains to provide stable SHG temperature control using a Peltier element and a thermistor. The unit is built on a compact printed circuit board (PCB) with its configuration shown in Fig. 6.4a. It is able to drive the Peltier element with ± 1.15 A current. The controller is powered through the VCC and GND pins with 5 V using a bench-top power supply. After connecting the thermistor with TH+ and TH-

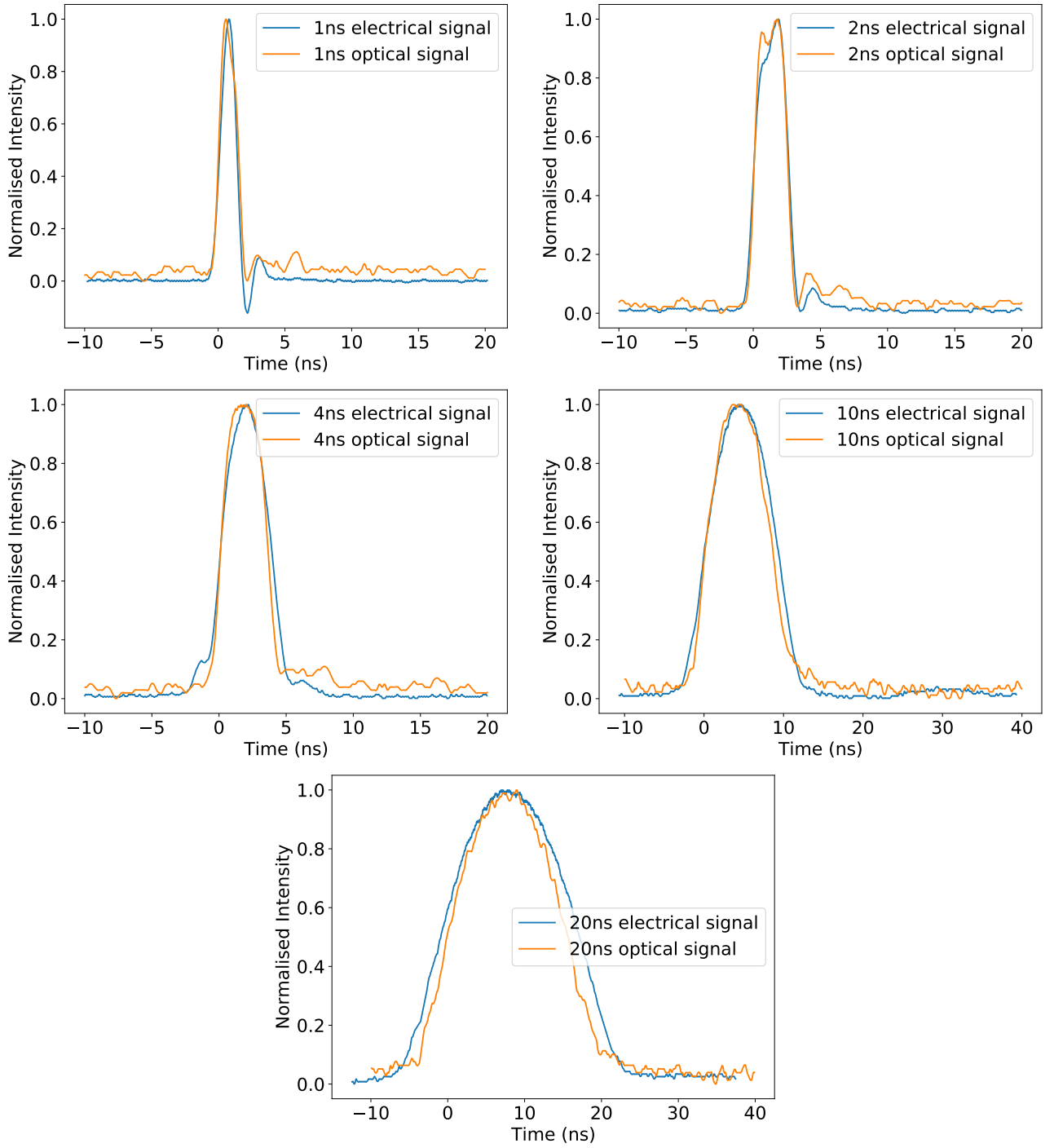


Figure 6.2: Comparison between electrical and optical pulses. The electrical pulses with Gaussian shape are edited using a PicoQuant programmable pulse generator. The optical pulses with Gaussian shape are modulated by iXBlue optical modulator controlled with the electrical signal. The optical pulses are slightly shorter than the electrical pulses due to the sinusoidal response of the modulator.

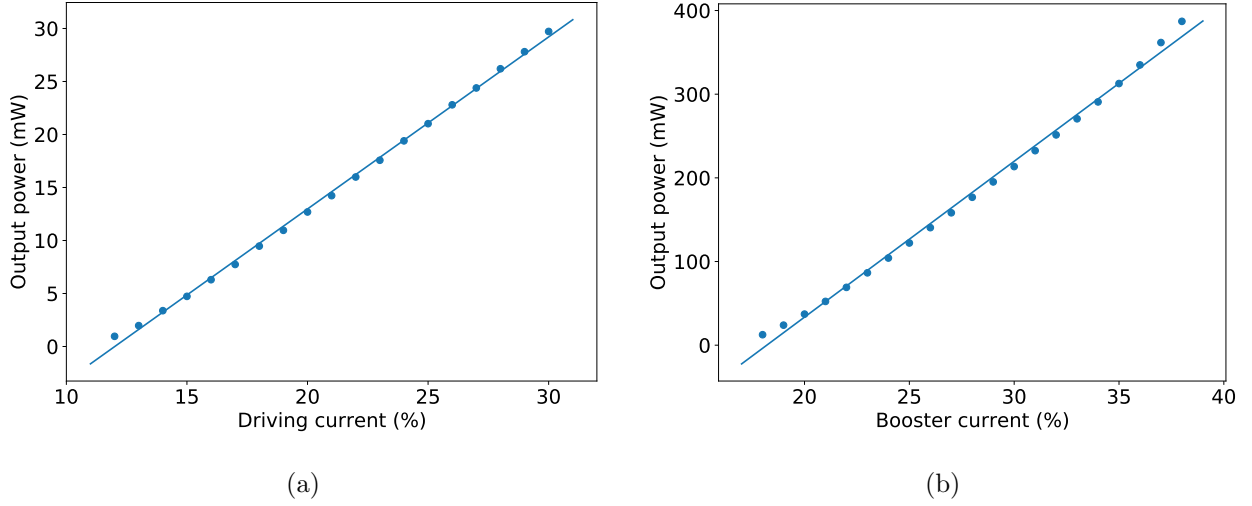


Figure 6.3: Output power characterisation of the EDFAs used in the experiment. (a) Shows the pre-amp EDFA characterisation for its output power versus the driving current setting. (b) Shows how the power-amp EDFA output power depends on the booster current setting. Both present a linear relationship between output power and the current setting.

pins, and the Peltier element with TEC + and TEC - pins, we are able to adjust the set point using the potentiometer at bottom left in Fig. 6.4a. The relationship between the potentiometer and the voltage to drive the heater is shown in Fig. 6.4b with the relation

$$R_{th} = 10k\Omega * (4.096V - (V_{TACT} - 1.65V)) / (4.096V + (V_{TACT} - 1.65V)). \quad (6.1)$$

The temperature controller drives the Peltier element with PID controls to match the V_{TACT} to V_{TSET} . We are able to monitor the voltage settings in real time and calculate the temperature with a Python interface “Koheron.py” which includes Eq. (6.1).

6.1.4 SHG waveguide

We have discussed the second harmonic generation in Chapter 2 as a demonstration of three-wave nonlinear interaction. In this pulsed pump system, we use a commercially available waveguide mixer to convert a 1560 nm laser into a 780 nm laser. The waveguide mixer is from HC Photonics with a part number 1WM1560001111X1XCP021. It includes a PPLN crystal to perform second harmonic generation. The phase-matching of the PPLN is typically 45 °C, with a range from 20 ° to 70 °C. The input and output of the SHG waveguide is coupled with PM850 fibres. After the pulsed-shape signal gets amplified in power, we send it to the temperature-stabilised SHG waveguide and generate a 780 nm pulsed output. As we discussed in Chapter 2, the SHG output intensity is proportional to the square of the input field intensity. To test the performance of SHG, we characterise the SHG waveguide with the amplified CW 1560 nm signal and the results are shown in Fig. 6.5.

The SHG waveguide is temperature dependent and wavelength dependent due to the phase match-

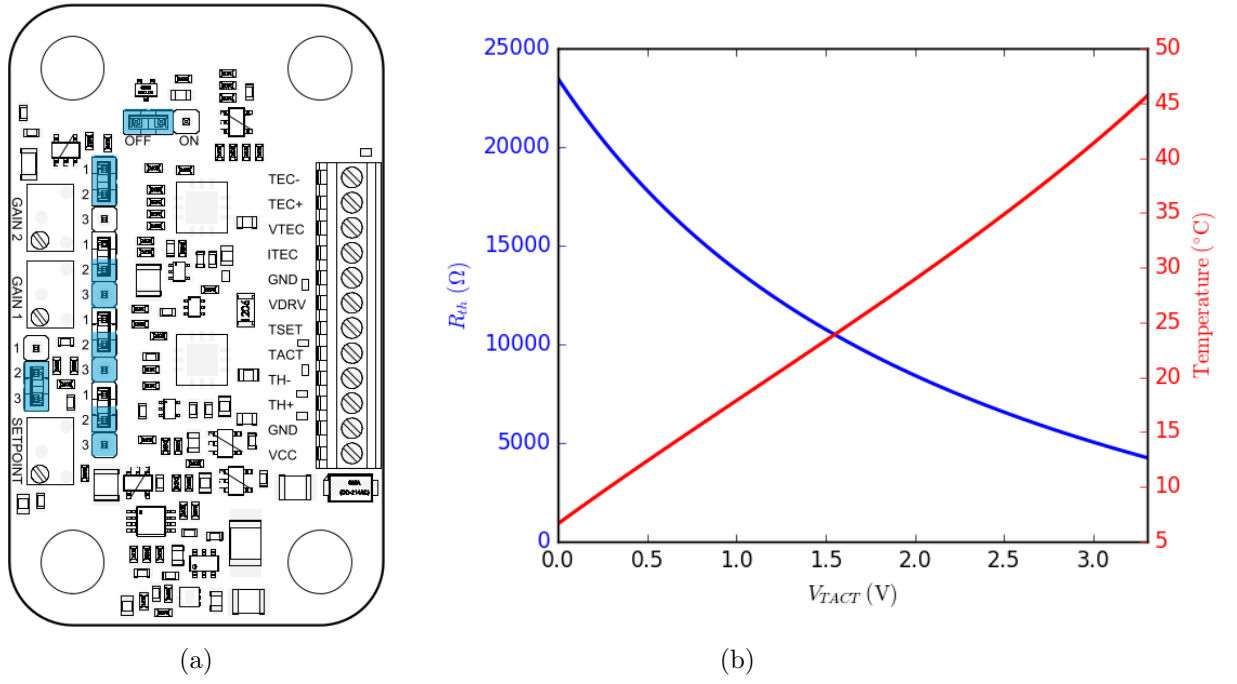


Figure 6.4: TEC100L temperature controller design and thermistor response provided by Koheron. (a) PCB pin diagram of the TEC100L temperature controller. The blue areas are the default jumper configurations for operation. The potentiometer on the left controls PID gains and temperature set point. The pins on the right are connected to power supply (VCC and GND), the thermistor (TH+ and TH-), and the Peltier element (TEC+ and TEC-). (b) The temperature detected from thermistor. The blue curve shows the relation between the thermistor resistance and the voltage on V_{TACT} pin, which corresponds to a temperature reading with the red curve.

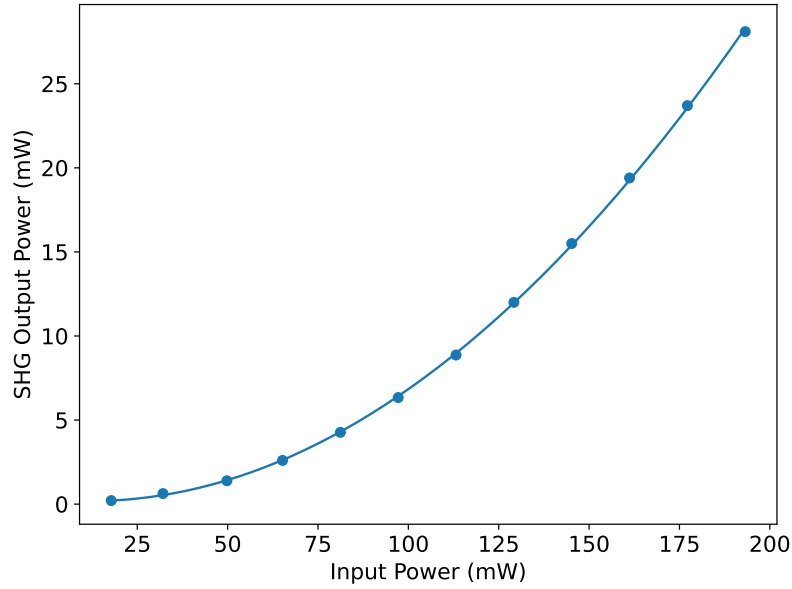


Figure 6.5: The SHG waveguide output for different input power. The relationship between the SHG output and input powers is quadratic.

ing of the crystal. The normalised output response with respect to the wavelength is measured in Fig. 6.6 with the temperature optimised for a central wavelength of 1560 nm. The phase matching covers a bandwidth of 2 nm. With a fixed temperature at 52.5 $^{\circ}\text{C}$, the SHG effect is negligible outside the range 1559 to 1561 nm. In the phase matching range, the SHG response is symmetric along the

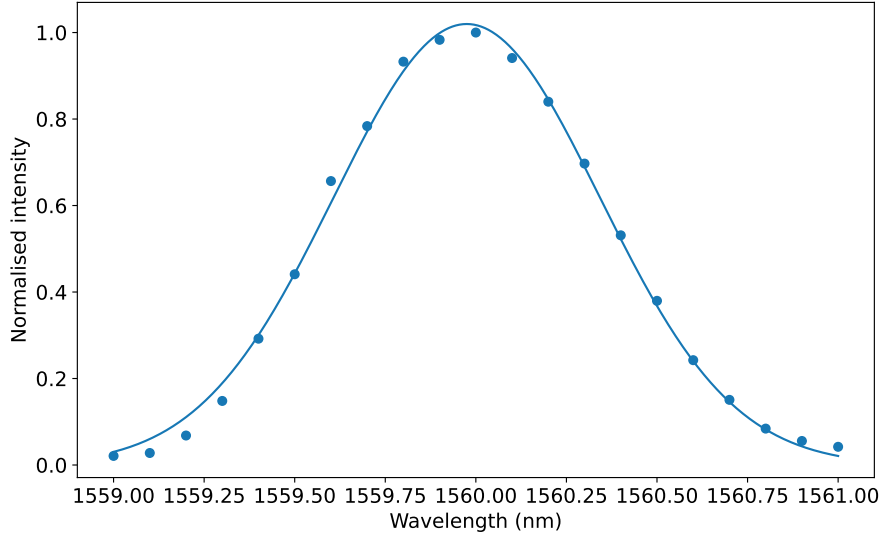


Figure 6.6: The SHG waveguide response for different wavelengths. The temperature is optimised for 1560 nm SHG interaction. The SHG intensity is symmetric around 1560 nm and drops significantly beyond 1 nm away from the central wavelength.

central wavelength.

6.1.5 Pulsed response from SHG waveguide

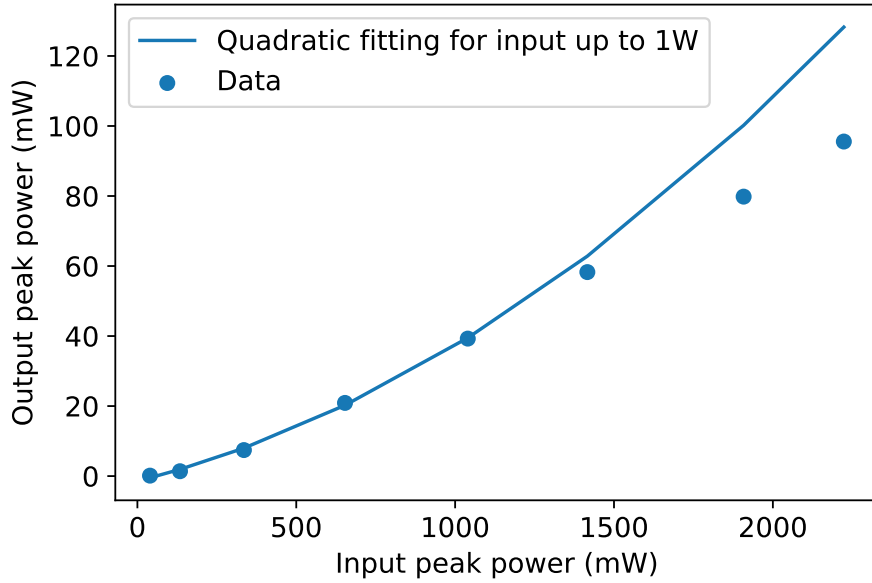


Figure 6.7: Power response for SHG waveguide with a 10 ns pulse. The peak power of pulse output increases with the growth of the input pulse peak power. When the input peak power is lower than 1 W, the output peak power grows quadratically. The SHG saturates when the input peak power is higher than 1 W, causing a slower growth of the output peak power.

We input a pulsed optical signal to the SHG waveguide to generate a frequency-doubled pulsed optical output. The pulse repetition rate is 5 MHz. With a fixed pulse duration 10 ns, we characterise the SHG input and output with their peak powers. The output pulse peak power increases with the input pulse peak power as shown in Fig. 6.7. When the pulse's peak power is lower than 1 W, the

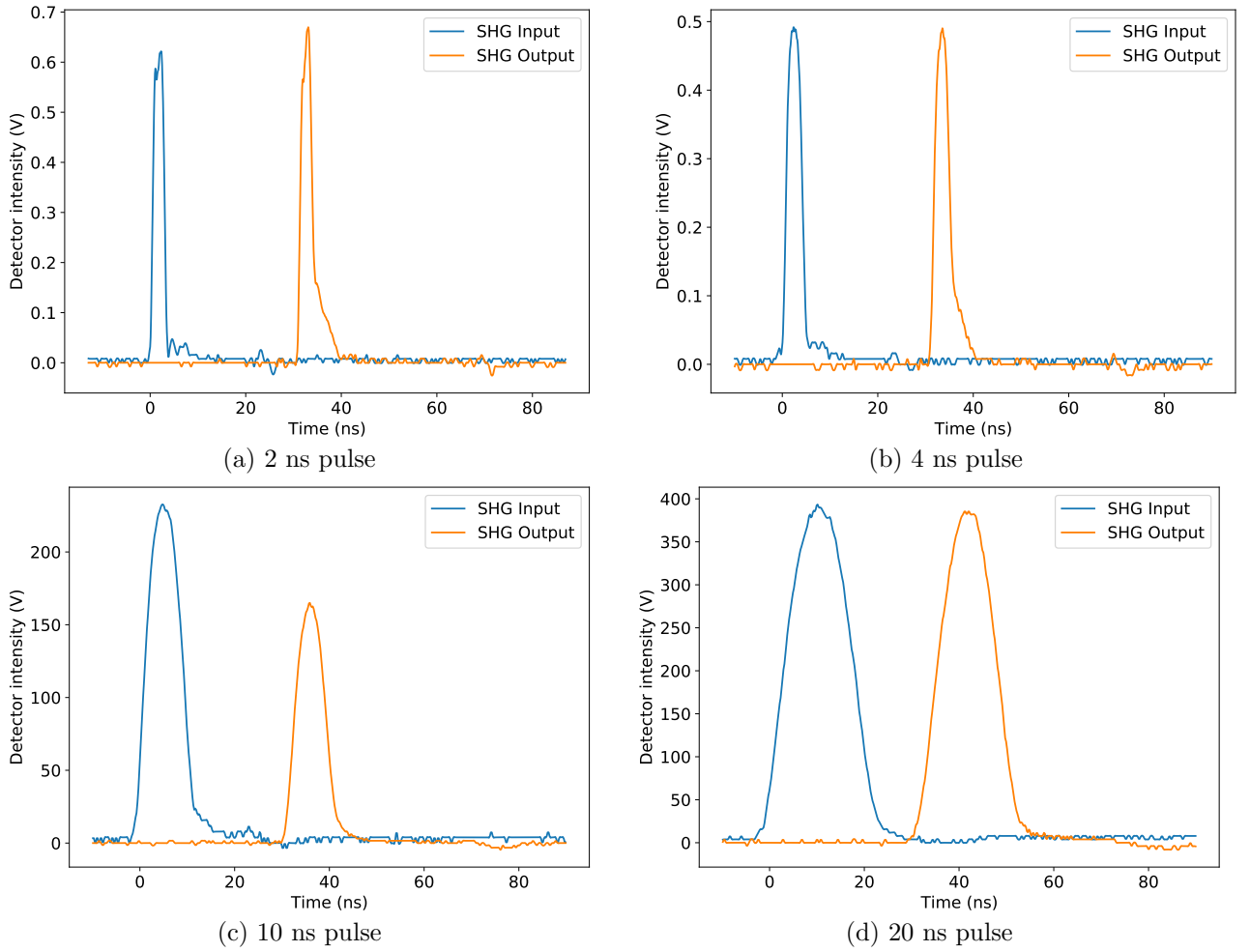


Figure 6.8: SHG waveguide response with different pulse duration. With shorter pulses like 2 ns in (a) and 4 ns in (b), the output pulses have a residual component at the end of the pulse comparing with the input pulse. The longer pulses like 10 ns in (c) and 20 ns in (d) maintain the shape as the input pulse.

output peak power grows quadratically. With a higher than 1 W input peak power, the SHG in PPLN reaches saturation and grows slower.

In Fig. 6.8, we compare the input pulse shape and the output pulse shape with different pulse duration. For shorter pulses like 2 ns and 4 ns, the output pulse has a tail at the end compared with the input pulse shape. Long pulses like 10 ns and 20 ns maintain the pulse shape as the input. The duration of the output pulse is also shorter than the input pulse, due to the SHG effect.

6.2 Output state characterisation

The pulsed pump enters the cavity using a similar experimental setup as the CW pump SPDC setup, with one change that the pump enters from the other side of the PBS. The setup is shown in Fig. 6.9. The polarisation of the pulsed pump is vertical, which reflects from the PBS1. The half-wave plate after PBS1 is rotated by 45° to provide a horizontal polarisation for cavity coupling. Each pulsed pump generates photon pairs, which exit the cavity with a certain time delay with respect to the

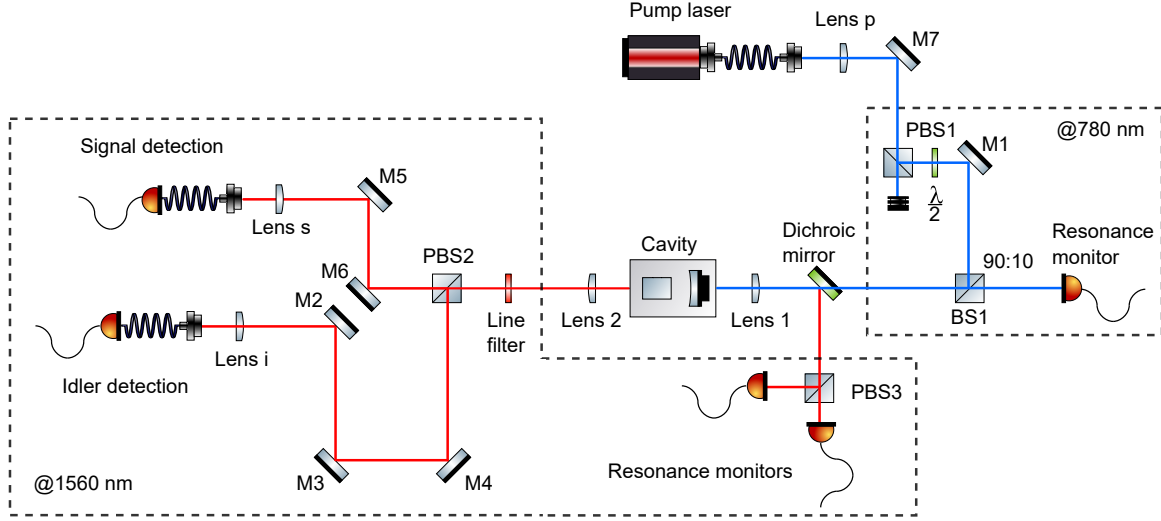


Figure 6.9: The pulsed pump experimental setup. Similar to the continuous-wave setup, apart from the pump beam before PBS1. The pulsed pump has a vertical polarisation that is reflected from the PBS. The angle of the half-wave plate is 45° different from the CW setup to guarantee the pump cavity coupling with the correct polarisation.

pump pulse. The time delays provide a time correlation among the signal, idler, and pump fields. We can measure this time correlation as joint temporal intensity of the signal and idler triggered by the pulsed pump photons. In this experiment, the pulse's repetition rate is set as 5 MHz. The pulsed pump is programmed as Gaussian shaped and the temporal length can vary from a few ns to 100 ns.

6.2.1 Photon statistics

This section presents the photon statistics measurements for cavity SPDC with a pulsed pump. We employ the HBT model used in the former chapter to measure with a pulsed pump cavity SPDC. Because the 4 ns Gaussian-shaped pulse present a better factorisability, which we will discuss in the next section, and a reasonable number of counts on the SNSPDs, we use it for photon statistics measurements. It takes 2 minutes to record the counts for each data point. The coincidence window in the measurements is 80 ns.

The singles counts for the signal beam after the beam splitter are shown in Fig. 6.10. The singles counts grow linearly with the pulsed pump peak power as the theory describes. The coincidence counts in Fig. 6.11 have the same quadratic behaviour with pulsed pump peak power as the continuous pump results.

Second order correlation for the SPDC source carries information of the purity of the source. We tried to measure both the heralded $g^{(2)}(0)$ and the marginal $g^{(2)}(0)$. The heralded $g^{(2)}(0)$ is shown in Fig. 6.12 with a linear relation with pump peak power. The marginal $g^{(2)}(0)$ measurement appears to be larger than 2, which does not have physical meaning as $g^{(2)}(0) = 1 + \frac{1}{K}$, where K is the Schmidt number representing number of modes. The reason for incorrect marginal $g^{(2)}(0)$ measurements is likely to be the laser stability and imperfect splitting of signal and idler photons under low pump

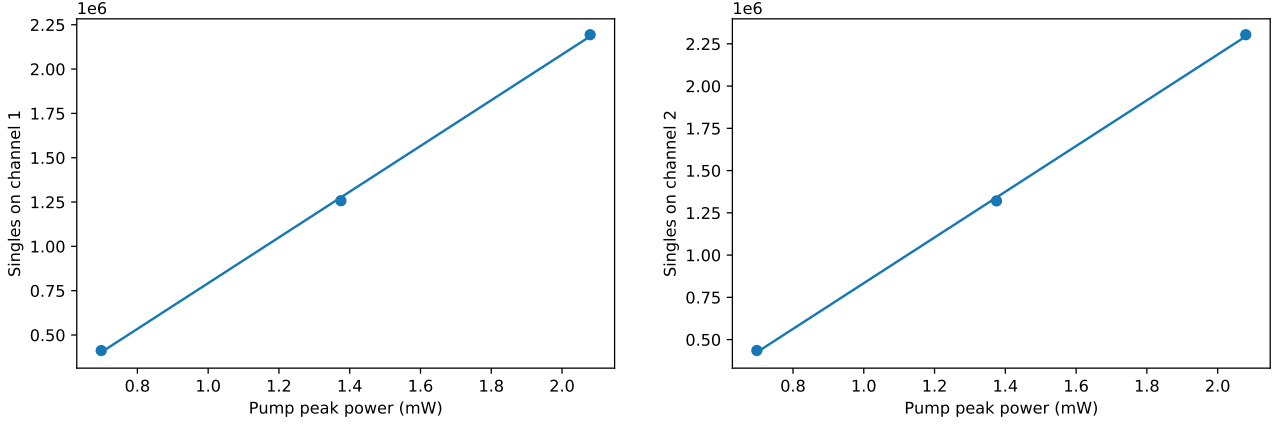


Figure 6.10: Singles counts from 4 ns pulsed pump cavity SPDC using HBT model with different pump pulse peak power. On both channels the singles grow linearly with respect to pump peak power. The error bar is negligible on the plot and the line is the linear fitting.

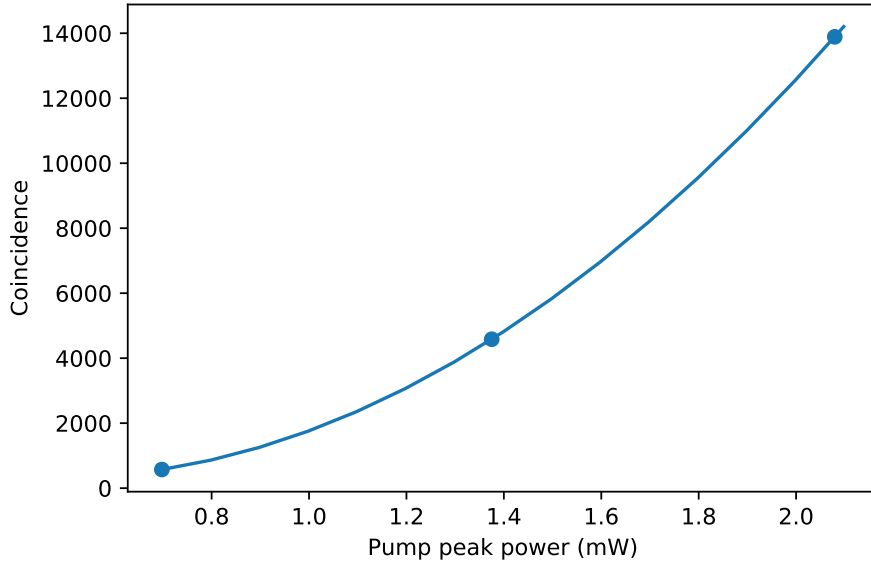


Figure 6.11: Coincidence counts from 4 ns pulsed pump cavity SPDC using HBT model with different pump peak power. The coincidence counts grow quadratically with the pump power. The error bar is negligible on the plot and the line is the quadratic fitting.

power. The measurement requires the triple resonance condition fulfilled at all times, whereas the pump laser frequency fluctuates with a few hundred Hertz, resulting in the cavity partially on triple resonance. If most of the time the cavity is off resonance, the marginal $g^{(2)}(0)$ is much larger than 2. In order to get a more reliable characterisation, we try to utilise the relationship between the heralded $g^{(2)}(0)$ and marginal $g^{(2)}(0)$ under the assumption of small mean photon number [107]:

$$g_{\text{her}}^{(2)}(0) = 2\langle n \rangle \left(1 + \frac{1}{K}\right) = 2\langle n \rangle g_{\text{mar}}^{(2)}(0) \quad (6.2)$$

where $\langle n \rangle$ is the mean photon number and the assumption is that $\langle n \rangle \ll K$.

In order to find the mean photon number, we make use of the Klyshko coefficient. The Klyshko coefficient measurement in Fig. 6.13 presents similar results as the measurement with a CW pump

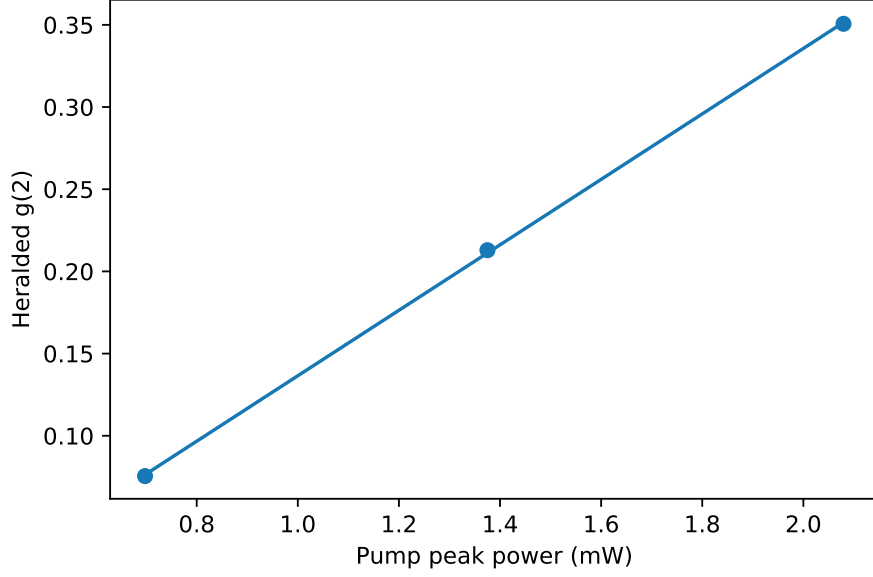


Figure 6.12: Heralded $g^{(2)}$ using 4ns pulsed pump with different pump peak power. The error bar is negligible on the plot and the line is the linear fitting.

in the last chapter. The low Klyshko coefficient mainly comes from the low escape efficiency of the cavity, aggregated with the fibre coupling efficiency. Using the heralding channel photon counts and Klyshko coefficient, we can estimate the total number of photons generated during the measurement period. Each measurement takes 120 s, which includes $120 \times 5 \times 10^6 = 6 \times 10^9$ number of trials. The mean photon number is calculated by the recorded singles per experiment trial divided by the Klyshko coefficient. We calculate $\langle n \rangle$ to be around 0.015. Then we use Eq. (6.2) to estimate marginal $g^{(2)}(0)$. The results are still larger than 2. This phenomenon requires further investigation.

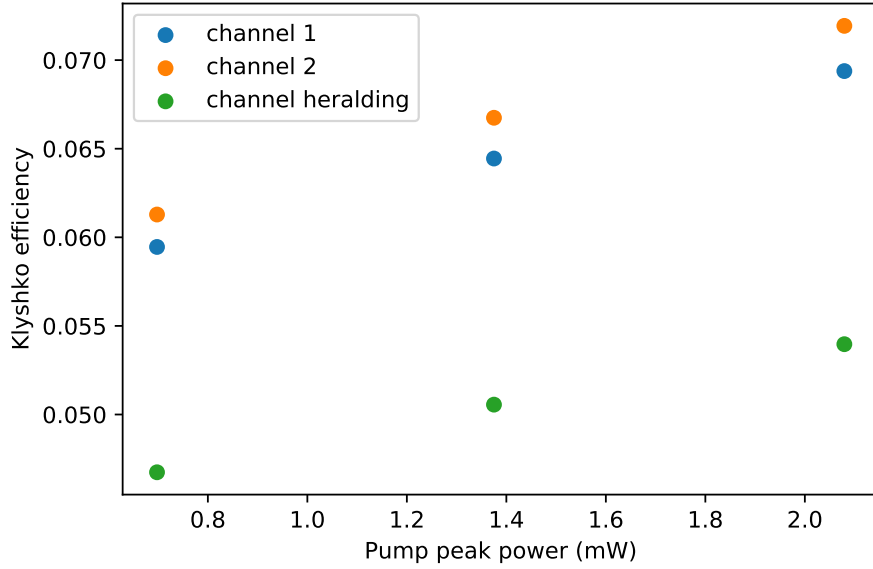


Figure 6.13: Klyshko coefficient for three channels using 4 ns pulsed pump with different pump peak power. The error bar is negligible on the plot.

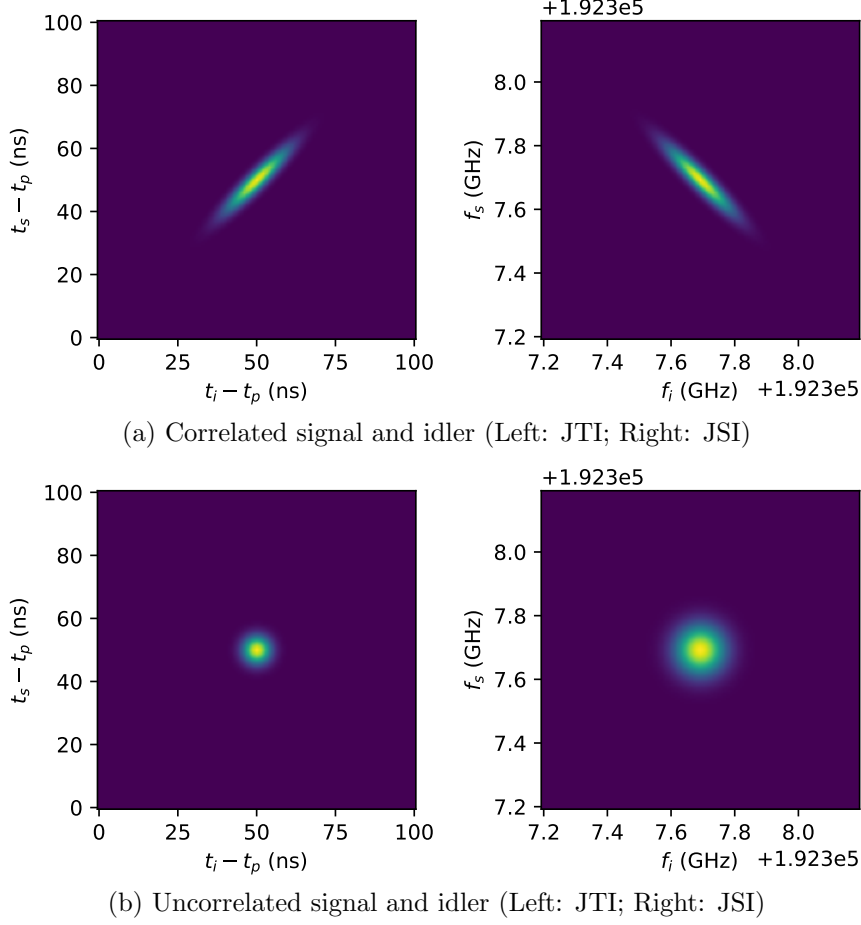


Figure 6.14: Joint temporal intensity (JTI) simulations and corresponding joint spectral intensity (JSI). The left figures are JTI figures with axes as signal and idler arrival time with respect to the pulsed pump arrival. The figures on the right are the corresponding JSIs. The correlated signal and idler presents 2.74 modes using Schmidt decomposition. The uncorrelated spectrum shows a single mode.

6.2.2 Joint temporal intensity

Joint spectral intensity (JSI) is an illustrative property to present the factorisability of a squeezed state. As we employ a cavity structure to generate squeezed states, the frequency spectrum is narrow. Its JSI measurement requires a resolution of at least 1 MHz, which is challenging to achieve in the frequency domain. As an alternative way to describe separability, the joint temporal intensity (JTI) is easier to measure with the help of a time-to-digital converter. It requires to record the arrival time information for photons generated from the cavity SPDC.

In Fig. 6.14 we simulate two types of JTI measurements and their corresponding JSI. The first example is a correlated state which can be written as a sum of a few product states. The Schmidt decomposition suggests there are 2.74 modes in this state. The second example shows a uncorrelated state with the signal and idler. It can be written as 1 product state. The Schmidt decomposition shows a single mode state. Our target of a pulsed pump cavity SPDC is to generate a factorisable state.

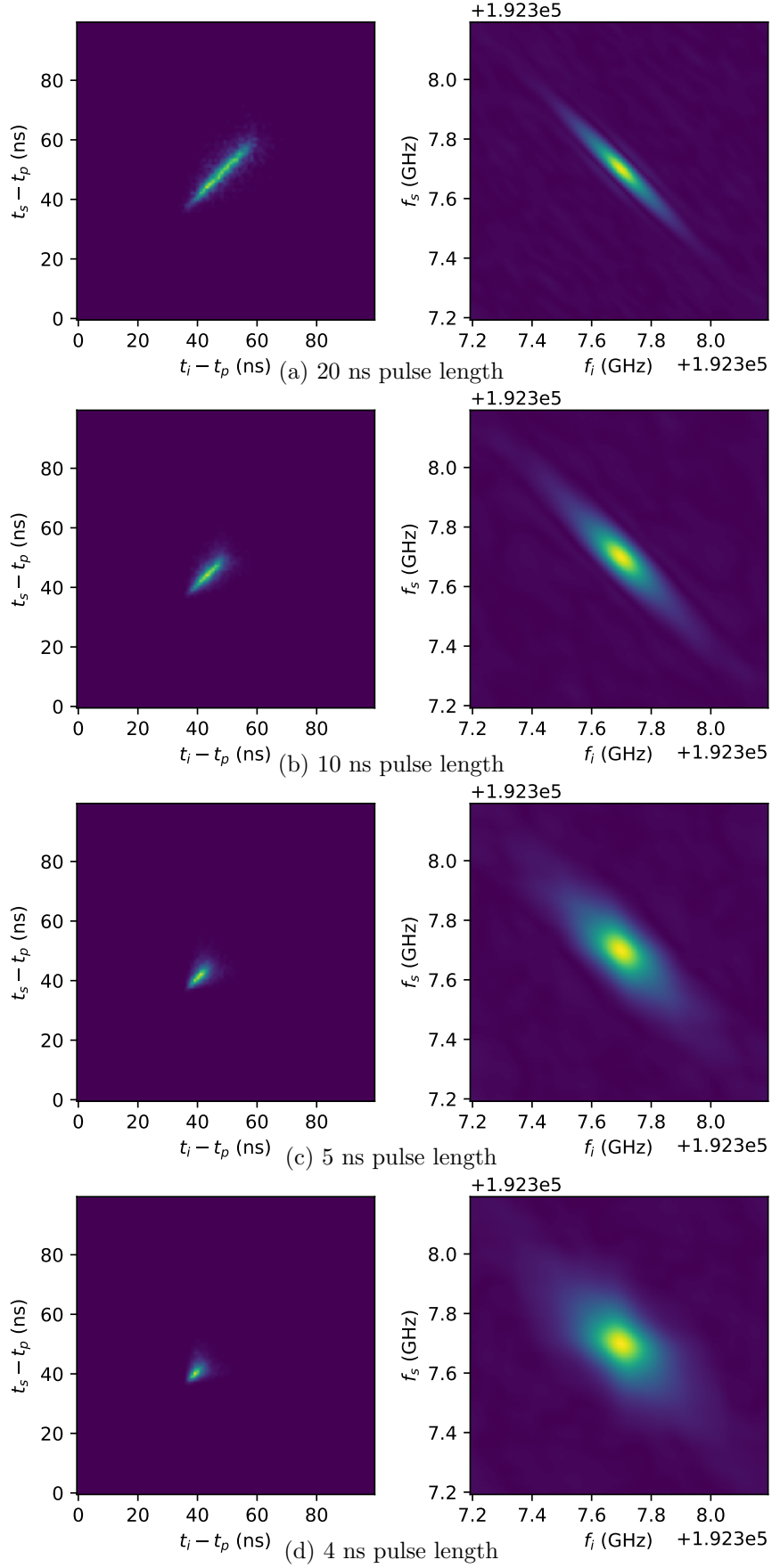


Figure 6.15: Joint temporal intensity (JTI) measurement (left) and its corresponding JSI (right) from the pulsed pump SPDC with the triply resonant cavity. The left figures are JTI figures with axes as signal and idler arrival time with respect to the pulsed pump arrival. The figures on the right are the joint spectral intensity (JSI). The pulse duration mentioned in the captions are the electrical pulse setting. With shorter pulse length, the arrival time correlation between signal and idler becomes more factorisable, which suggests the spectral correlation between signal and idler is more factorisable.

The JTI of the signal and idler can be presented with a 2D histogram, which shows the arrival time difference between the pump and signal photons, and the arrival time difference between the pump and idler photons. The histograms are measured with a bin-width of 1 ns and 100 bins for a duration of 20 minutes. Fig. 6.15 presents the JTI measurement on the left and its corresponding JSI on the right with different pulse lengths. The pulse duration mentioned in the captions are the electrical pulse setting. The centre of the pump pulse gets amplified more than the wings of the pulse during the SHG. After normalisation, the pump pulse looks shorter than the electrical pulse. With different pulse duration, the JTI and JSI present different degrees of correlation between the photon pairs. The signal and idler are more correlated using a 20 ns pulsed pump. When the pulse changes to a shorter length (4 ns), its joint spectral intensity becomes more factorisable.

Using Schmidt decomposition, we are able to calculate the mode number K of the output state. We decompose the square root of transformed JSI and calculate the mode number in the spectrum picture. Fig. 6.16 shows the relationship between the mode number and the pulse duration from the data and simulation of ideal cases. In the measurements, the pulse duration corresponds to the electrical pulse settings. The pump pulse duration arriving at the cavity is calibrated with the SHG effect. The 4 ns setting corresponds to 3.4 ns pulsed pump. It has 1.14 modes, the least mode number on the plot. The 20 ns setting corresponds to 13.2 ns pulsed pump. It generates 2.23 modes, indicating a higher degree of correlation.

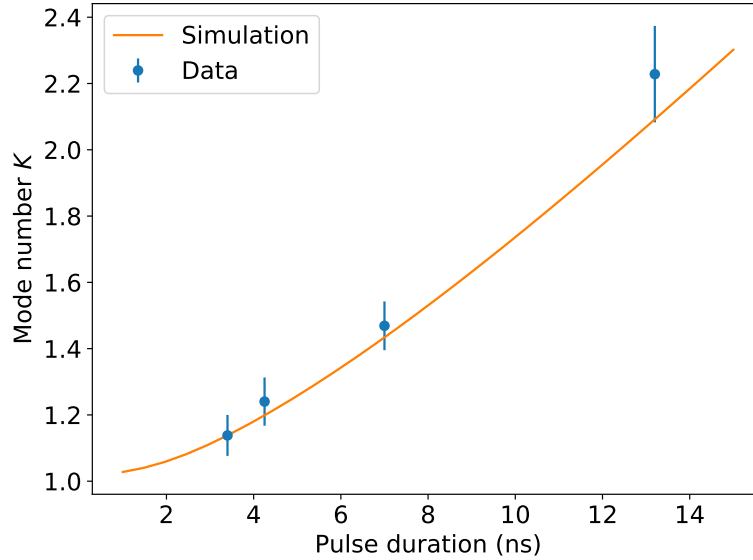


Figure 6.16: The calculated mode number from data and simulation with different pulse duration. With longer pulse duration, the output mode number is higher as it includes higher degree of correlation. The pulse duration in measurement data is the electrical pulse duration, which is re-calibrated for the SHG output. Assuming the photon number measurement is subject to Poissonian noise, our simulation agrees with the data. There are also pulse duration uncertainties subject to the Gaussian pulse shape deviation.

The simulations are shown as a line in Fig. 6.16. At a specified pulse duration, we use the Gaussian pulse time-bandwidth product to calculate the bandwidth of the pulsed pump field. We can use the

pump bandwidth in combination with the linewidth characteristics of signal and idler fields to calculate the JSA of the cavity output. The Schmidt decomposition of the JSA provides Schmidt coefficients, which can be used to calculate the mode number. We find the data approximately matches the simulations.

For a longer pulse or continuous wave pump which is considered as an infinitely long pulse, the pump bandwidth is narrow compared with the signal and idler resonances. In the JSI picture, the narrow pump resonance introduces correlation between signal and idler frequencies. This results in a large mode number. In contrast, a shorter pulse provides a wider pump bandwidth that overlaps well with the signal and idler resonances in JSI picture. The mode number decreases with a shorter pulse duration. In the simulation, a 1 ns pulse presents a mode number of 1.03, close to a single mode. As the mode number approaches 1, the output becomes more separable. The relation between JTI and JSI suggests that when the JTI is more factorisable, the JSI is also more separable. This signifies that with a suitable Gaussian pulse, type II SPDC in a triply resonant cavity can generate highly separable states.

Using the photon statistics and the mode number from a 4 ns pulsed pump, we can estimate the squeezing parameter. We use the Klyshko coefficient to calculate the average photon number as 0.015, and the Schmidt decomposition to find the mode number as 1.14. On average, each mode has 0.013 photons. We can use Eq. (1.22) to estimate the squeezing parameter as 0.114 (1.0 dB). If we increase the pulse power, we are able to achieve a higher squeezing output.

This chapter introduced the pulsed pump system and SPDC generation with the pulsed pump. First, we presented the pulsed pump system which includes two stages of optical amplification with EDFAs, electrical pulse control from a programmable pulse generator, electro-optic modulation to translate electrical pulses into optical pulses, and laser frequency doubling with an SHG component. Then, we tested the SPDC generation with a pulsed pump and measured its photon statistics and joint temporal intensity. We found that a 4 ns pulse duration provides better factorisability compared with a longer pulse duration. And using Schmidt decomposition of the joint spectral intensity, we calculated 1.14 modes for the 4 ns pulsed pump. As a result, we inferred the squeezing of the SPDC output is 1.0 dB. By analysing the relationship between pulse duration and mode number, we can identify the requirement of approximately 1 ns pulse to generate separable states.

Chapter 7

Conclusion

This thesis reports progress on generating an isolated optical squeezed vacuum state using pulsed SPDC in a cavity. In particular, we've presented the design and construction of an optical cavity that achieves simultaneous resonances of the three SPDC fields. The cavity has been characterised by its linear and nonlinear optical response. Finally, pulsed operation has been shown, providing evidence that control of the pulse pump duration can be used to achieve the desired mode isolation.

7.1 Summary of the thesis

Chapter 1 explains the motivations of the project to contribute to the continuous-variable quantum teleportation and photonic quantum computing. It introduces the two-mode squeezed vacuum concept and its spectral characteristics. This chapter also covers some characterisation methods for squeezed state sources.

Chapter 2 provides a detailed discussion of the SPDC theory inside a triply resonant cavity. It starts from the three-wave mixing theory with coupled wave equation and Maxwell equations. Then it presents the quantum description of the parametric down conversion, and the relationship between squeezed state operator and PDC expression. It also discusses the triply resonant cavity enhancement for the SPDC squeezed state generation, and the condition to generate a factorisable output.

Chapter 3 is about the design of the cavity SPDC source. It consists of the optical design, the mechanical design, and the electrical controls. The optical design specifies the important optical settings to achieve stability and high factorisability output. We apply the birefringence property of the PPKTP crystal to isolate a single state when the cavity is on triple resonance. The mechanical design includes the custom-made mounts and assembly parts of the cavity. As the cavity size is not standard with commercially available mounts, we designed the mounts to hold the crystal and the mirror to form the cavity. We also introduce the electrical controls for cavity optical path length. The temperature controller controls the crystal temperature to change its phase-matching and thermal expansion. The piezo voltage controller adjusts the air gap inside the cavity.

Chapter 4 presents the cavity linear response, including the optical characteristics and tuning response of the cavity fields. We tested the cavity with a FSR of 16.69 ± 0.02 GHz for vertically polarised pump field, and for the signal and idler fields we measured 16.22 ± 0.01 GHz for horizontal polarisation and 16.96 ± 0.01 GHz for vertical polarisation. The linewidth of the pump field is measured to be 220 ± 4.96 MHz. The linewidth for signal and idler are 100.68 ± 10.85 MHz, and 101.70 ± 10.45 MHz correspondingly. In order to tune the cavity resonant fields, we can adjust the temperature of the crystal, the piezo behind the curved mirror, or the laser frequencies. By tuning two parameters, we are able to achieve triple resonance for relative frequencies that satisfies the energy conservation and degeneracy condition.

Chapter 5 focuses on the nonlinear response of the cavity SPDC source with a CW pump. Using a triply resonant cavity, we observe the SHG effect and a DFG signal. The SHG signal matches the quadratic relation of input power with a strong seed laser input. It depends on the polarisation of the seed laser which matches the phase-matching condition of the nonlinear crystal. Then we perform SPDC generation with a continuous-wave pump and measure the photon statistics of the photon source, the collection efficiency, the HOM dip, and two-photon arrival timing. The photon statistics agree with the theory from SPDC source. We record a low collection efficiency, largely from cavity's low escape efficiency. The HOM dip has a visibility of 92 %. And the two-photon arrival time presents a double-exponential distribution.

Chapter 6 discusses the pulsed pump SPDC generation. The pulsed pump system is introduced with its equipment and controls for a pulsed pump. Then we apply the pulsed pump to the cavity source to measure the photon statistics and the joint temporal intensity. The resonance is not maintained during $g^{(2)}(0)$ measurements because the laser frequency fluctuates too quickly. The joint temporal intensity is transformed into spectral picture for a characterisation of the light source's factorisability. With a shorter pulse duration, the output state becomes more separable. The output state of a short pulse modulated by a 4 ns electrical pulse generates 1.14 modes, from which a 1.0 dB squeezing can be estimated.

7.2 Outlook

The characterisation of the pulsed quantum light in the thesis demonstrates the progress we have made so far. The next step is to measure the second order correlation for a more stable pulsed pump SPDC source. Ideally, the marginal $g^{(2)}(0)$ would be 2 for factorisable states. Depending on the measurements, we will be able to improve the squeezed states source in the short term and long term. This section is about the improvements we can make to optimise the generation rate and factorisability of the light source.

Firstly, we can improve the factorisability of the output state by adjusting the programmable

pulses. The pulse shape could be refined to counterbalance the electrical pulse shape defect and the distortion that occurs during electrical-to-optical signal conversion. If we are able to generate a better-defined pulse with a shorter pulse duration, the cavity will be able to generate a more factorisable state with more symmetric joint temporal intensity measurements.

Secondly, we can implement a system that monitors the cavity status closely and provides feedback to the parameters so that the triple resonance condition can be maintained. We noticed during the measurements that the cavity has a tendency to slowly drift away from the triple resonance condition if the cavity stability is not precisely controlled. The EXFO laser frequency as well, has a fast fluctuation that affects the statistics measurement. By recording both the output counts and the degree of overlap among the three fields, while also scanning the cavity parameters, we can proactively adjust to counterbalance the drift.

Finally, we can minimise the unexpected loss inside the cavity. There are multiple possible causes that could lower the Klyshko coefficient in our measurements. One cause comes from the intra-cavity loss. The high reflective mirrors makes the cavity source sensitive to the intra-cavity loss. To find the cause of intra-cavity loss and improve the performance helps to improve the Klyshko coefficient. Other causes could potentially include dust and impurities on the optical surface that can be handled with proper cleaning. Further investigation is required on the source of loss. We can also consider adjusting the mirror reflectivities to improve the escape efficiency.

Once the light source's performance has been optimised with the above improvements (adjusting the programmable pulses, implementing a system for cavity and laser stabilisation, and minimising its intra-cavity loss), the next step in our research is to explore the application of our apparatus in the high gain regime. In the high gain regime, we can no longer assume a negligible double photon-pair emission. As a result, the single photon detectors we have employed so far will not be able to accurately measure the second order correlations. In response, we will build a homodyne detection system capable of calculating the squeezing of the squeezed state source by measuring its quadrature variances. By applying our system to the high gain regime, we can generate highly separable photon pairs at a higher rate that can enable various applications in photonic quantum information.

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