

Effectiveness and heat-transfer downstream
of three-dimensional film-cooling slots.

by

Ashok Kumar Rastogi

B.Tech., M.Tech.

Thesis submitted for the diploma of
Membership of Imperial College of Science
and Technology, and the degree of Doctor
of Philosophy in the Faculty of Engineering,
University of London.

October, 1972.

ABSTRACT

An experimental investigation of the influence of slot geometry parameters on the effectiveness and heat transfer coefficient downstream of three-dimensional discrete hole film-cooling slots is presented for uniform density flows and in the presence of density gradients. The parameters investigated are the longitudinal distance, hole diameter, lip thickness and lip length, height of the hole above the wall, pitch of the holes, open area and slot to mainstream velocity ratio.

A procedure to predict the influence of the above parameters on the effectiveness and heat transfer coefficient downstream of discrete hole slots is described. The procedure is based on a finite difference solution of the time-averaged three-dimensional boundary layer form of the Navier Stokes equations. A modified form of the Prandtl mixing length hypothesis is used. The predictions are compared with a wide range of experimental data, including the present data, and the validity of the solution procedure is assessed. The relevance of the present prediction procedure to a practical film-cooled combustion chamber is examined.

A procedure to predict the effect of longitudinal curvature on the effectiveness and aerodynamic properties of two-dimensional wall jets is presented.

A three-equation turbulence model has been used to predict the effectiveness downstream of thin-lip two-dimensional film-cooling slots in the presence of density

gradients. The present predictions are compared with the previous predictions effected by the Prandtl mixing length hypothesis and the merits of using the present model are assessed.

ACKNOWLEDGEMENTS

I would like to acknowledge my sincere gratitude to Dr. J. H. Whitelaw for his excellent supervision. His deep personal interest and encouragement helped me immensely to achieve the objectives of the present research. His constructive suggestions and polite criticism on my writing have been very useful in presenting this thesis.

I wish to thank Dr. S.V. Patankar for introducing the three-dimensional prediction procedure to me and for the patient advice he gave me during its development in the present work. Thanks are also due to Dr. S.J. Peerless and Dr. M.M. Gibson, who were on my thesis committee and helped me in keeping up the speed of the work. I would like to express my thanks to all my colleagues and the technical staff of the heat-transfer section - in particular to Mr. King and Mr. Robinson who helped in the construction of the experimental rig.

Finally the financial support of the Ministry of Defence is gratefully acknowledged.

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CHAPTER 1

1. Introduction

1.1 Statement of the problem and its significance

Many items of engineering equipment are subjected to high-temperature gas streams and require cooling to ensure that the surface temperature does not exceed some predetermined metallurgical limit. Gas turbine combustion chambers, reheat nozzles of aircraft engines, gas turbine blades and rocket nozzles are a few examples of such equipment. One way of cooling the surfaces exposed to hot gases, is to inject a cool fluid along the surface in the form of a film. This process of protecting the surface by a cool fluid is known as film-cooling. In certain applications, such as de-icing of aircraft wings and de-misting of windscreens, a film of hot fluid is used and the process is known as film-heating. Film-heating and film-cooling are similar processes and the study of one leads to understanding of both.

The present thesis is particularly related to film-cooling in gas turbine combustion chambers. The cooling demands of the modern combustion chambers have increased considerably with the increase in gas temperatures and pressures. Film-cooling performance depends upon the device through which the film is injected and the amount of coolant used. The cooling slot experiences severe stresses as a result of large temperature gradients and must, therefore, be mechanically strong. At the same time it should be easily and accurately machined. After long experience with different types of slots, it has been

recognised that machined rings possess these qualities.

A machined ring consists of discrete holes drilled in one piece of material, e.g. see Fig. 3.2.8⁹. Fluid flow downstream of such a device is evidently three-dimensional and is significantly more complicated than the flow downstream of a two-dimensional device: nevertheless the latter must be well understood as a basis for understanding the former.

Extensive experimental work (1,2,3,4,5) has been done on two-dimensional slots and the effects of various flow and geometrical parameters have been investigated. Procedures have been developed (6,7) to predict the performance of such slots. These procedures require the specification of the laws for turbulent exchange of momentum, mass and enthalpy and the accuracy of the predictions essentially depends upon the validity of the exchange hypotheses. A form of Prandtl's mixing length hypothesis has been used (6) to give reliable predictions of film-cooling effectiveness downstream of two-dimensional slots within a limited range of conditions. Although it proved adequate for predicting the wall-effectiveness, it did not represent the local flow properties exactly and the inaccuracy may be attributed to the assumptions on which it is based. A complex model of turbulence, which has been shown to represent the details of asymmetric flows correctly, has been employed in the present work in order to estimate its benefit over the mixing length hypothesis, for predicting wall effectiveness and local flow properties.

The temperature assumed by the combustion-chamber

wall is such that the net heat received by it through radiation and convection from the flame is equal to the heat lost through its outer surface by radiation and convection. Film-cooling influences the convective mode of heat transfer within the combustion-chamber. It is convenient to represent the convective heat transfer, in the presence of film-cooling, through two quantities: the adiabatic-wall effectiveness and the heat-transfer coefficient. In the absence of radiation and chemical reaction between the film and the hot gas stream, the adiabatic-wall effectiveness, η , is a measure of the cooling performance of the film. η is defined by the expression

$$\eta = \left[\frac{T_{a,w} - T_G}{T_C - T_G} \right]_{cp = \text{const.}} \quad (1.1.1)$$

and depends upon the mixing characteristics of the two streams. It can be considered as a measure of the preservation of the identity of the cooling film. It does not provide information of the resistance of the film to the heat transfer through it. A heat-transfer coefficient based on the adiabatic-wall temperature, provides the necessary information and is defined as:

$$hf = \frac{\dot{q}_w''}{(T_w - T_{a,w})} \quad (1.1.2)$$

It is evident from the definitions of η and hf that a cooling device should be designed to result in high values of η and low values of hf . The values of these two quantities depend upon the several factors discussed below.

In combustion-chamber practice, there are density gradients between the film and the hot-gas stream and the amount of coolant available is restricted. It may be

expected that η will increase with increasing mass flow of the coolant; on the other hand, large quantities of cooling air cannot be used because of the adverse effect on combustion efficiency and reduction in the amount of air available for dilution purposes. Therefore the mass of the coolant required to give the maximum cooling effectiveness should be minimised. Other important parameters affecting η and h_f include the geometry of the cooling device, the distance from the slot exit, pressure gradients in the flow field, turbulence intensities in the hot gas stream and the film and property variations because of the high temperature gradients. The present investigation deals mainly with the identification and the influence of important parameters that have significant effect on η and h_f downstream of three-dimensional cooling slots.

The influence of the above parameters can, of course, be found experimentally though the number of experiments required would be excessively large. In addition, it is expensive in time and effort to isolate some effects from each other experimentally, e.g., turbulence intensity from shape of the velocity profile or Reynolds number from boundary-layer thickness. It would definitely be more convenient to use a mathematical model to predict the influence of the relevant parameters. A previous practice has been to take measurements and then to correlate them by guessed functional relationships in order to characterise the findings. Such relationships are valid only for the particular cases upon which they are based and usually need more information than they provide. The approach in the present work has been to develop a general

prediction procedure based on the fundamental equations of fluid motion and to make use of a limited number of measurements to check the validity of the procedure.

This approach reduces the number of experiments required and provides a general and inexpensive method to predict the influence of each parameter separately.

1.2 Objectives and Scope of the present study

The principal objectives of the research were as follows:

1. to identify experimentally the important parameters that influence effectiveness and heat transfer coefficient downstream of three-dimensional discrete hole film-cooling slots and to assess their influence. This experimental investigation covered the parameters relevant to gas turbine combustion-chamber design within the following limits:

slot to main stream velocity ratio;	(.1 to ∞)
slot to main stream density ratio;	(1.0 to 4)
hole diameter	(2mm to 12.5mm)
lip thickness to diameter ratio;	(.2 to .8)
lip length to diameter ratio;	(0 to 3.0)
pitch to diameter ratio;	(1.4 to ∞)
height of the hole centre from the wall;	(.5D to 1.5D)

2. to develop a prediction procedure which can accurately predict effectiveness and heat transfer coefficient downstream of three-dimensional slots with various initial and boundary conditions.

The validity of the procedure was tested against the above measurements.

3. to examine the relevance of the above prediction procedure to the practical application of film-cooled combustors.

The present research included work on two-dimensional film-cooling with the following subsidiary objectives:

4. to provide a procedure for predicting the influence of longitudinal curvature on the effectiveness downstream of two-dimensional slots.

The procedure was tested against wall-jet data on curved surfaces; this is relevant to the film-cooling of blades with high curvature.

5. to assess the merits of using a three equation model of turbulence (equations for turbulent kinetic energy, dissipation of turbulent kinetic energy and turbulent shear stress) over Prandtl's mixing length hypothesis, in predicting effectiveness downstream of two-dimensional slots.

Effectiveness predictions downstream of tangential slots with thin lip and density gradients were compared.

6. to provide a general prediction procedure for predicting effectiveness downstream of two-dimensional slots which have recirculating flow in the upstream and boundary layer flow in the downstream region.

The procedure was tested against measurements with tangential slots having a thick lip and slots having a backward facing step in the plane of the slot exit.

1.3 Outline of the thesis content

The present thesis has been sub-divided into seven chapters including this introduction. The following

chapter presents a critical review of previous work on two-dimensional and three-dimensional film-cooling. It places the present work in perspective.

Chapter 3 describes the experimental investigation. The design and construction of the apparatus and the instrumentation is described. This is followed by a brief statement of the preliminary tests, carried out with a fully turbulent boundary layer on a flat plate, designed to assess the performance of the equipment. The experimental method and results for all the cooling slots used are presented and discussed. The chapter concludes with a summary of results.

Chapters 4 and 5 are concerned with the prediction of two-dimensional and three-dimensional film-cooling performance respectively. The general format of both the chapters is similar, i.e. a statement of the mathematical problem in the form of appropriate equations to be solved; a description of the physical hypothesis; a description of the procedure used; and lastly, comparison of the predicted results with the test data.

Chapter 6 discusses the practical relevance of the results achieved in the present study. It demonstrates how well the wall temperature of a combustion chamber can be predicted with the procedure developed in the present work. The last chapter summarises all the conclusions made in the previous chapters and gives suggestions for future work.

CHAPTER 2

2. Brief literature survey.

The systematic study of film-cooling started about 30 years ago and, until very recently, most of the work was confined to two-dimensional film-cooling. Two-dimensional film-cooling is simpler to understand, more amenable to available techniques in fluid mechanics and heat transfer and its knowledge is very necessary to the understanding of the more complex and practical problem of three-dimensional film-cooling. The present chapter briefly outlines the previous investigations of film-cooling with two-dimensional and three-dimensional slots.

The performance of a film-cooling slot strongly depends upon the initial and boundary conditions of the flow domain. The initial conditions pertain to the slot exit, i.e. slot height, lip thickness and length, open area ratio, boundary layer thicknesses, fluid properties in the plane of the slot exit etc., whereas the boundary conditions pertain to the rest of the three surfaces enclosing the flow such as pressure gradients in the free stream, zero or constant heat-flux at the wall etc. The purpose of this review is to identify the conditions which have previously been investigated; to indicate their significance; and to indicate the influences that remain to be studied. As demonstrated in sections 2.1 and 2.3 influences of various initial and boundary conditions on the performance of two-dimensional slots are well understood. Detailed measurements indicating the important influences are available and

prediction procedures have been developed that can reliably predict the important influences. This state of knowledge has not been reached for three-dimensional slots. It is also an objective of this review to apply the information obtained with two-dimensional slots, to three-dimensional slots and thus to identify the important parameters for practical film cooling. The available experimental information for three-dimensional slots is critically examined and the direction for further experiments provided. Various prediction procedures available for three-dimensional flows are listed and a choice is made for the present work.

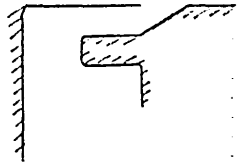
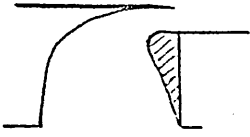
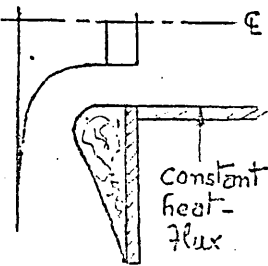
Reviews of film-cooling investigations up to 1969 may be found in references (11), (12), (13) and (6) for two-dimensional film cooling and in ref. (14) for three-dimensional film cooling. Some of these investigations are mentioned here for the sake of completeness but more attention is paid towards the work done between 1969 and 1972.

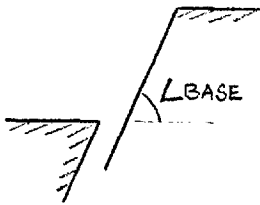
2.1 Experimental studies of two-dimensional film-cooling.

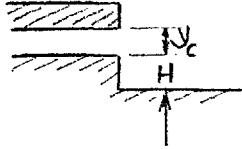
Table (2.1) gives details of the two-dimensional slot geometries studied with the range of parameters investigated and a brief account of the results. This section critically examines these investigations and identifies the important parameters that influence effectiveness and heat transfer coefficient.

For slots with tangential injection ρ_C/ρ_G , U_C/U_G and t/y_c are the most important initial conditions. It is advantageous to blow a heavier gas through the slot for a particular velocity ratio. Effectiveness always decreases

Table (2.1). Summary of experimental studies of two-dimensional film-cooling.

AUTHOR	YEAR	INITIAL CONDITIONS INVESTIGATED							BOUNDARY CONDITIONS INVESTIGATED		SLOT-GEOMETRY	REMARKS
		ρ_C / ρ_G	U_C / U_G	t / y_c	δ / y_c	H / y_c	$\angle \text{sec.}$	$\angle \text{Base}$	Kp	$\sqrt{u_G^2} / U_G$		
WIEGHARDT (15)	1944	-	.22 to ∞	-	-	-	-	-	-	-		Maximum η around $U_C / U_G = 1.0$. At a particular x / y_c and U_C / U_G , η was same for for two values of y_c .
SEBAN (16)	1960	-	.27 to .76	-	1 to 10	-	-	-	-	-		Influence of δ on η was small. η decreased with increasing δ .
HARTNETT et al. (3), (17)	1961	-	.28 to 1.10	-	-	-	-	-	Moderate Favorable & adverse pr. gradients	-	Similar WIEGHARDT. Constant heat flux at the wall.	For $x / y_c > 30$, hf reached value corresponding to flat plate value. Small influence of pressure gradients on η - η reduced.
PAI & WHITELAW (18), (19)	1968 1969	.069, 1.0, 1.38, 4.17	.30 to 3.10	-	-	-	-	-	1, 1.82, 3.30, -1.0 $\times 10^6$	-		No maxima in η for $\rho_C / \rho_G \neq 1.0$. η was higher for $\rho_C / \rho_G > 1.0$ at a particular value of U_C / U_G . η decreased in presence of dP/dx .
PAI (6)	1969	-	.39 to 3.55	.35 and 1.0	-	-	-	-	-	-		hf increased with U_C / U_G . For $U_C / U_G > 1.2$ hf tends to a value higher than fully developed pipe flow and for $U_C / U_G < 1.2$ to a value lower. No effect of t / y_c on hf for $10 < x / y_c < 50$.

AUTHOR	YEAR	INITIAL CONDITIONS INVESTIGATED							BOUNDARY CONDITIONS INVESTIGATED		SLOT-GEOMETRY	REMARKS
		ρ_C/ρ_G	U_C/U_G	t/yc	δ/yc	H/yc	$\angle \text{ sec.}$	$\angle \text{ Base}$	K_p	$\sqrt{u_G^2}/U_G$		
CARLSON & TALMOR (20)	1968	2.76	.19 to .90	-	-	-	-	30°, 45°, 60°	-	-		η decreased as $\angle \text{ Base}$ increased, as a result of favorable pressure gradient experienced by flow.
MATTHEWS (21)	1972	1.20	.7 to 2.0	-	-	-	-	30°, 45°	-	-		Same as above. For the same value of $\rho_C U_C / \rho_G U_G$, η was higher for $\rho_C / \rho_G > 1.0$
WHITELAW (22)	1967	-	.29 to 2.7	-	-	-	-	-	-	-		η decreased as t/yc increased. For large t/yc ($>.4$), maxima in η for $\rho_C / \rho_G = 1$ and $U_C / U_G \approx 1$ disappeared.
SIVASEGARAM & WHITELAW (23)	1969	-	.4 to 5.0	.05 to 1.6	-	-	-	-	-	-		
KACKER & WHITELAW (1)	1969	-	.3 to 2.7	.126 to 1.90	-	-	-	-	-	-		Effect of Freestream turbulence (FST) on η was small, decreased with FST. Similar influence was found of slot turbulence in the range .055 to .095.
KACKER (7)	1969	-	.3 to 2.7	-	-	-	-	-	-	.005, .035, .07		
		-	"	-	2.42 to 10.2	-	-	-	-	-		Similar to the influence of ref. (16).

AUTHOR	YEAR	INITIAL CONDITIONS INVESTIGATED							BOUNDARY CONDITIONS INVESTIGATED		SLOT-GEOMETRY	REMARKS
		ρ_C / ρ_G	U_C/U_G	t/yc	δ/yc	H/yc	$\angle$ sec.	$\angle$ Base	K_p	$\sqrt{u_G^2}/U_G$		
MATTHEWS & WHITELAW (24)	1971	1.0 and 2.0	.7 to 2	.125, 1.0	-	0 to 2.0	-	-	-	-		η decreased as H increased. This influence was very small for thick lip case. For the same value of $\rho_C U_C / \rho_G U_G$, η for $\rho_C / \rho_G = 2.0$ was smaller than $\rho_C = \rho_G$
METZGER et al. (25)	1968	-	-	-	-	-	20°, 60°					Heat transfer coefficient increased with increase in injection angle.
MATTHEWS (5)	1972	1.0 and 2.0	.7 to 2.0	-	-	-	30°, 45°	-	-	-		η decreased as the angle of injection increased.
CARLSON & TALMOR (20)	1968	-	.17 to .67	-	-	-	20°	-	-	.032 .12 .22		Considerable reduction in η was observed with increase in Free stream turbulence.

with increasing lip thickness as a result of turbulent mixing generated by the lip. The data of ref. (1) shows that for the velocity ratio of 0.86 and the density ratio of unity, at 25 slot heights downstream the slot exit, the wall effectiveness decreases from .85 to .45 as t/y_c is increased from .126 to 1.14. Burns and Stollery (26) found the adverse effect of lip thickness for the density ratio of 4.17. The measurements of Pai (6) indicate that the heat transfer coefficient, based on the difference of the wall temperature and the adiabatic wall temperature, is unaffected by t/y_c for $x/y_c > 10$. However it should be noted that the heat transfer at the wall will increase as t/y_c increases. The measurements of detailed profiles of the aerodynamic quantities such as turbulence intensities, kinetic energy of turbulence, velocity, u' -spectra and turbulent shear stress downstream of tangential slots have been reported in refs. (2), (27), (28). The main purpose of these measurements has been to assess a hypothesis of turbulent momentum transport for predicting flow properties of two-dimensional film-cooling.

For slots in which the two streams meet at an angle, such as in the geometries of ref. (5), (20), (21) and (25), the angle between the two streams is an important parameter. There is a significant decrease in effectiveness and increase in heat transfer coefficient with increasing angle. Although there are quantitative differences in the effect measured by different workers listed in Table (2.1), all the measurements indicate the importance of this parameter.

The influence of free stream pressure gradient and free stream turbulence has been found to be small on the

effectiveness of two-dimensional slots. According to ref. (19) the reduction in effectiveness due to mainstream pressure gradients at any downstream distance was of the order of 5% of unity except for one case where the favorable pressure gradient was very severe ($\kappa_p = 3.30 \times 10^{-6}$) and the reduction was 20% of unity. The influence of freestream turbulence measured by Carlson and Talmor (20) should be treated with caution. In this case, the turbulence levels were not measured but were calculated for each grid used to generate turbulence. In addition, the influence of parameters like velocity profile shape and free stream velocity were not separated.

The above description demonstrates that, for two-dimensional film-cooling slots, conditions at the slot exit have more influence on the cooling performance than the conditions in the free stream. The parameters investigated for two-dimensional slots are present in three-dimensional slots but, in this case, there are additional important parameters. Experimental investigations of some of these parameters are described in the following section.

2.2 Experimental studies of three-dimensional film-cooling

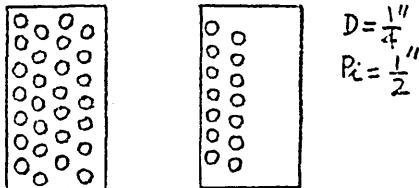
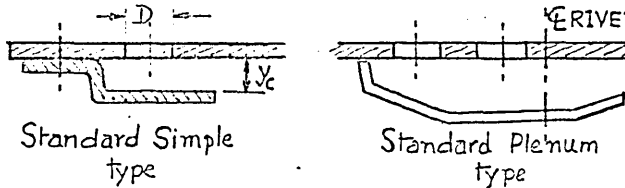
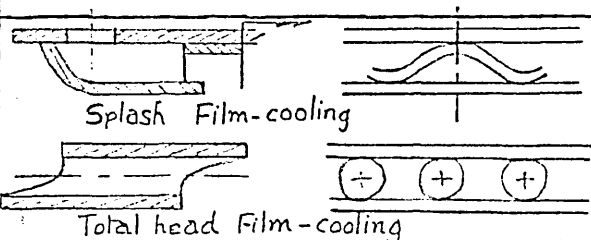
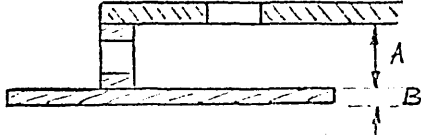
Unlike two-dimensional slots, three-dimensional slots have not been studied systematically and it is difficult to categorise the available measurements in terms of different parameters. Until recently most of the experimental work, e.g. refs. (29), (30), (31), (32) and (33), was concerned with the testing of cooling performance of specific cooling devices and subsequent correlation of the experimental results. It

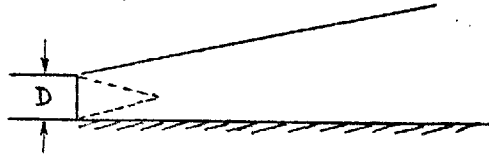
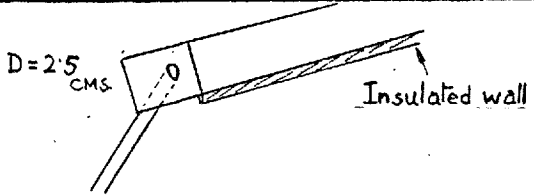
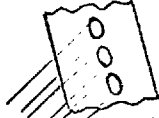
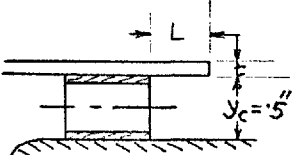
is convenient to describe each investigation separately; to summarise the list of parameters investigated; to indicate the available knowledge; and to indicate what remains to be attained.

Papell (34), during his experiments on angled two-dimensional slots, measured adiabatic wall-effectiveness downstream of two three-dimensional slots injecting coolant through normal holes. The slots consisted of a plate, flush with the wall to be cooled, containing rows of $1/4$ " holes at $1/2$ " pitch in a staggered pattern. One slot had two rows and the other had four rows as shown in Table (2.2). The main stream was at 1500°R and was subsonic whereas the secondary stream temperature could be varied between 550°R and 1100°R . He measured adiabatic wall temperatures for a range of velocity ratios (U_0/U_G) from .035 to 1. His measurements indicate that effectiveness, for slots with normal holes, was less than the two-dimensional slot injecting normally under similar flow conditions even when the open area of the two-dimensional slot was less than the other.

Palmer (29) investigated the performance of two types of splash cooling devices which he named as standard simple type and standard plenum type and are shown in Table (2.2). He kept the slot Reynolds no. constant at a value of 1.6×10^5 and changed the hot stream Reynolds no. from 2.92×10^5 to 8.41×10^5 . He concluded from his experiments that the simple type was more efficient than the plenum type and both the slots gave maximum effectiveness at a velocity ratio of about unity which was found for two-dimensional slots as well. Systematic comparison of the two slots tested by him was difficult as the open areas were different.

Table (2.2) Summary of experimental studies of three-dimensional film-cooling.

Author	Year	U_C/U_G	x/D OR x/y_c	Measured quantities	Slot geometry										
PAPELL (34)	1960	.035 to 1.0	0 to 140	Adiabatic wall effectiveness											
PALMER (29)	1964	.42 to .85	0 to 37	Adiabatic wall effectiveness											
PALMER (30)	1965	.58 to 1.16	0 to 48	Adiabatic wall effectiveness											
NOBBS and RICE (35)	1968	.385 to 3.31	0 to 25	Adiabatic wall effectiveness. Temperature and velocity profiles.	 <table border="1" data-bbox="2051 1320 2197 1514"><thead><tr><th>A''</th><th>B''</th></tr></thead><tbody><tr><td>.5</td><td>.25</td></tr><tr><td>.438</td><td>.188</td></tr><tr><td>.375</td><td>.125</td></tr><tr><td>.312</td><td>.063</td></tr></tbody></table>	A''	B''	.5	.25	.438	.188	.375	.125	.312	.063
A''	B''														
.5	.25														
.438	.188														
.375	.125														
.312	.063														

Author	Year	U_C/U_G	x/D OR x/y_c	Measured quantities	Slot geometry
NAIB (37)	1968	∞	0 to 60	Velocity profiles, growth rates	$D = 2.5 \text{ cms.}$ 
GOLDSTEIN ET AL. (38), (39)	1968	0.10 to 2.0	0 to 45	Adiabatic wall effectiveness, rates of growth	$D = 2.5 \text{ cms.}$ 
	1970	0.10 to 2.0	0 to 90	Adiabatic wall effectiveness	 $D = 1.18 \text{ cms}$ $Re = 3D$
NINA (14)	1969	0.3 to 2.5	0 to 40	Adiabatic wall effectiveness	 $t/y_c = 0.125, 0.50$ $L/y_c = 0, 2.36, 4.0$ $D = 0.29", 0.41"$ $Re/D = 1.22, 1.72, 2.44, 3.45$

He found that increasing the hot stream turbulence reduced the film cooling effectiveness. Palmer (30), after the above investigation, did performance tests with 9 splash cooling slots and one total head type slot. The generalised form of the slots is shown in Table (2.2). He changed different geometrical parameters from one slot to another to be able to suggest a better design and concluded that a splash device with a deflector was better than the total head device up to 25 slot heights and in the region downstream both were equally effective. Again the effective area of the two slots was not same and the comparison was effected at the same pressure drop through the device which implied different coolant mass through each slot. Although Palmer's work contained some useful data, the effects were found to be complex and it was difficult to say why the geometry selected by him gave the best performance.

Nobbs and Rice (35) have investigated the effect of geometrical parameters on effectiveness for splash-impingement cooling devices. They have found the influence of slot-gap, lip thickness and velocity ratio on the film cooling effectiveness while keeping the hole diameter, pitch and lip length constant. Results were limited to 25 slot heights which is the region of interest in chamber practice. They concluded that for a given coolant mass flow, to obtain maximum effectiveness, the slot gap should be maximum to give a blowing rate of approximately unity; lip thickness should be minimum and holes should be arranged in a staggered way in case of multirow slots. These trends are qualitatively very similar to those found for two-dimensional slots. A long lip

was recommended without any experimental evidence.

Most of the above work was done in industrial research organisations where, due to the need for rapid development, the results of research programs are more important than the reasons leading to those results. In attempts to understand the flow, systematic studies have been carried out in Universities and are described in the rest of this section. Experimental work began with the study of a single jet injected tangential to a wall and followed through to discrete hole film-cooling slots.

Naib (37) studied a round jet injected tangential to a wall in stagnant surroundings. He measured the rates of growth of the jet, decay of maximum velocity and velocity profiles. He observed that the flow could be divided into four regions: potential core up to two diameters, transition region up to 8 diameters, fully established flow up to 30 diameters and lastly the region of terminal flow which consisted of large scale eddies and could be regarded as still. He found that the lateral growth of the jet was about 6 times larger than the normal growth and both the boundaries of the jet grew linearly with distance. The profile shape could be defined by a power law near the wall while in the outer layer the shape resembled the free jet profiles. Decay of the maximum velocity was found to be inversely proportional to the longitudinal distance. Although this flow situation is far from the practical film cooling situation, the above information is useful for understanding the flow downstream of a three-dimensional film cooling slot.

Goldstein et al. (38) studied the rates of growth

and flow patterns of a heated round air jet injected at an angle of 35° and 90° to an adiabatic wall in the presence of a fully developed turbulent free stream boundary layer. They found that vertical penetration and lateral spread increased with increasing jet velocity. They investigated the influence of mass velocity ratio on effectiveness and concluded that, for inclined jets, the maximum effectiveness was achieved for a mass velocity ratio of about 0.5. They have measured the variation of adiabatic wall temperature in the spanwise direction. At a particular spanwise location, effectiveness increases with longitudinal distance in the upstream region and then gradually decreases. Jet Reynolds no. did not show a significant influence on the cooling performance. The work of ref. (38), on single inclined jets, was extended by Goldstein et al. (39) to the study of the cooling effectiveness as a result of injection through a row of inclined jets spaced laterally at a pitch of 3 diameters. For an inclination of 35° and blowing rates of less than unity, the row of jets resulted in same effectiveness at the centre line, as obtained by a single jet. At locations away from the centre line the row of holes resulted in slightly better effectiveness. For higher blowing rates the row of jets gave better cooling performance at all locations. At blowing rates of 1.5 and 2, the row of jets resulted in effectiveness values that remained constant with increasing longitudinal distance. Their measurements indicated that, for the row of jets, effectiveness reached a maximum value at a particular blowing rate. This blowing rate which gave maximum effectiveness was a function of the

downstream distance. This investigation gives information relevant to film-cooling of blades and should be extended to determine the influence of other parameters such as pitch to diameter ratio, angle of injection, density and pressure gradients etc. on the effectiveness and heat transfer.

Nina (14) measured adiabatic-wall effectiveness downstream of 11 total head discrete hole slots and 5 slots that had splash holes and holes tangential to the adiabatic wall. He used a heated secondary stream and measured effectiveness for a wide range of blowing rates (.3 to 2.5) and uniform density. The slots were designed to identify the influence of specific geometrical parameters. He concluded that an increase in lip thickness decreases the effectiveness and the effect decays as the blowing rate increases. Increasing the lip length increased the effectiveness as was suggested by Nobbs and Rice (35). Nina found that increasing the open area ratio increased the effectiveness for all blowing rates and all downstream distances. Similar results were found for the splash cooling slots which Nina tested. The measured effect of open area ratio should be considered only as an indication because, in changing the open area ratio, other geometrical parameters were not kept constant, e.g. open area. A comparison of measurements obtained for splash slots with those of tangential slots showed that both were equally effective except for blowing rates higher than 2.5; at these high blowing rates, splash slots were better. He also found that it did not matter whether the splash holes were in line with the tangential holes or in between the tangential holes so

far as effectiveness was concerned. The above work gives useful trends as a result of some geometrical changes and indicates that parameters should be changed with great care so that their effects can be isolated.

On the basis of the above experiments, the importance of lip length, lip thickness and velocity ratio, on three-dimensional film-cooling, has been confirmed. Lip thickness and velocity ratio were also important for two-dimensional film-cooling slots and showed similar influences on the effectiveness. This suggests that it is possible, within limits, to apply the results of two-dimensional investigations to the three-dimensional slots. This will help in identifying the parameters which may have significant influence on the performance of three-dimensional slots. The following implications can be drawn from the results of two-dimensional film-cooling.

The influence of free stream pressure gradients and intensity of turbulence is expected to be small on the performance of three-dimensional slots. In addition to lip length and lip thickness, the identifiable parameters at the slot exit are: density ratio, pitch to diameter ratio, height of the holes from the wall, open area and open area ratio. The influence of density ratio is of practical relevance and is expected to be significant. The height of the holes from the wall will have an influence similar to the height of the backward facing step in a two-dimensional situation; this influence is expected to be significant only for large values of the height as compared to the hole diameter. There is no two-dimensional result directly

applicable to imply the influence of parameters like pitch to diameter ratio, open area and open area ratio.

2.3 Prediction procedures for two-dimensional film-cooling

Different procedures have been used to predict the film-cooling effectiveness and heat-transfer coefficient downstream of two-dimensional film-cooling slots. The objective of this section is to consider merits and demerits of the different methods and to make a choice of the best procedure. All the available procedures may be classified into the following categories:

Correlations.

Integral methods.

Differential methods.

Correlations. Dimensional analysis and some observed trends in the experimental data form the basis of correlations. These are not necessarily based on physical or transport hypotheses. The work done by Weighardt (15), Seban (40), Samuel and Joubert (41) and Spalding et al. (42) are a few of the numerous examples in this category. Each correlation employed a different set of dimensionless groups and was restricted to a particular experimental investigation. As a result of uncertainty and non-generality of different correlations, the attention of film-cooling investigators, e.g., ref. (3), (43), was diverted to the methods that were based on conservation laws of heat and mass transfer.

Integral methods. Integral methods solve the integrated forms of the conservation equations for momentum, species concentration or enthalpy. The integral equations are normally ordinary differential equations and employ

assumptions regarding the shape of the velocity and temperature profiles. These methods require auxiliary relations to link quantities appearing in the equations with the dependent variables. These auxiliary relations may either be explicit functions derived from experiments or may be derived from a general hypothesis for eddy transport. Weighardt (15), Hartnett et al. (3), Klein and Tribus (43) and Stollery and El-Ehwany (13) have used integral procedures for film-cooling studies. Weighardt (15) assumed a power law for the velocity profile, an exponential law for the temperature profile and empirical flat-plate boundary-layer formulae, to obtain the following relation for effectiveness:

$$\eta = 5.44 (x/m y_c)^{-0.8} R_c^{0.2} \quad (2.3.1)$$

Similar expressions have been obtained in refs. (3), (43), (13) and differ in the values of the constant and the exponents. These explicit integral methods are more general than the correlations but still need empirical information in the form of auxiliary relations.

Another group of integral methods that have been used in the past are known as parametric integral methods. In methods of this type, velocity and temperature profiles are described by simple algebraic expressions, e.g., polynomials in distance from the wall, containing free parameters, e.g., the coefficients of the polynomials. The partial differential equations are multiplied by some functions of dependent or independent variables, usually named as weighting functions, and are integrated in the direction normal to the flow. Integration results in ordinary differential equations with the free parameters as the dependent variables and the downstream distance as the

independent variable. Matrix inversion is used to express the first order differential coefficient of each parameter explicitly and the differential equations are integrated simultaneously to give the desired expressions for the free parameters. Accuracy of this procedure can be increased by increasing the number of free parameters but, at the same time, complexity and computing time increase. In these solution procedures, matrix singularity is occasionally encountered and can be a serious drawback of this method. The limitations of the integral methods can be overcome by differential procedures which are described next.

Differential methods. Differential methods solve numerically the partial differential equations for conservation of mass, momentum and enthalpy. In recent years, finite-difference methods have been extensively used because of their simplicity, economy and speed. Attention here is devoted to differential methods using finite difference techniques. For two-dimensional boundary layer flows, two types of differential methods have been used, cross-stream integration type and marching integration. In the former the partial differential equations are transformed, using a finite-difference approximation, into a series of ordinary differential equations, valid for a succession of sections through the boundary layer, normal to the main direction of flow. Since the boundary conditions lie at the end of each region, trial-and-error iteration is necessary for solving the ordinary differential equations. This implies considerable increase of computing time and storage over methods which do not need iteration.

The marching integration methods require no iteration and calculation proceeds downstream by means of forward steps. Starting with known initial conditions, unknowns are calculated at a short distance downstream. The computed values then become the upstream values and the front of computation moves in the downstream direction to cover the whole domain. Methods developed by Glushko (44), Paskonov (45), Bradshaw, Ferriss and Atwell (46) and Patankar and Spalding (9) are of this type. These methods are much faster and more economical than the cross-stream integration methods and are preferred for boundary layer calculations. The method of ref. (9) has been extensively used for film-cooling predictions (6, 47). Pai (6) has predicted the influence of velocity ratio, density ratio, pressure gradients, lip thickness and downstream distance on the effectiveness and heat-transfer coefficient. Metzger et al. (47) have predicted the flow development and wall heat-transfer coefficient downstream of angled slots.

For two-dimensional recirculating flows, forward marching integration procedures cannot be used because the upstream flow is influenced by the downstream flow, e.g., flow in the near-slot region. The relevant differential equations are solved by iterative procedures. The procedure developed by Gosman et al. (10) solves the governing equations for two-dimensional recirculating flows by finite difference technique. Kacker and Whitelaw (28) and Matthews (48) have used the method of ref. (10) to predict effectiveness and flow development downstream of slots having a thick lip and a backward facing step respectively.

Differential methods need hypotheses for turbulent transport of momentum, heat and mass and the accuracy of the predictions largely depends upon these hypotheses. The algorithms of these methods do not depend upon the hypothesis used and hence can be used to evaluate the validity of a particular hypothesis. Pai (6) and Metzger et al. (47) used Prandtl's mixing length hypothesis for their predictions. Pai found that the mixing length distribution had to be modified in order to predict the influence of t/τ_c correctly. Gibson and Spalding (49) have predicted effectiveness using a more complex hypothesis with the procedure of ref. (9). They have used a physical hypothesis where they define effective viscosity in terms of kinetic energy of turbulence and frequency of turbulence. Kacker and Whitelaw (28) have predicted film-cooling effectiveness with the Prandtl-Kolmogorov hypothesis using an empirical length scale of turbulence. Matthews (48) improved the physical hypothesis by solving an equation for the turbulent length scale in terms of dissipation of turbulent kinetic energy and found that empirical information could be generalised to a greater extent.

The above review demonstrates that prediction procedures are available for two-dimensional film cooling and have been used to predict the measured influences of various initial and boundary conditions. Physical hypotheses have been developed that can adequately represent the flow although to cover a wide range of conditions they require slight changes in the empirical constants.

2.4 Prediction procedures for three-dimensional film-cooling.

Unlike two-dimensional film cooling, all the methods used so far for three-dimensional film-cooling predictions fall into the category of correlations. Because of the three-dimensionality of the flow and complexity of the physical hypotheses, no attempt has previously been made to solve the relevant equations of motion. It is only recently that general solution procedures have become available for three-dimensional flows. The suitability of these procedures for film-cooling will be discussed in this section and a recommendation will be made. Some of the correlations which are presently in use for designing practical slots are described first.

Correlations. Sturgess (31, 50) has systematically developed correlations for effectiveness data on practical film-cooling slots. He began with Spalding's correlation (51) for two-dimensional slots, which was based on a combined boundary layer and jet flow model. Spalding defined the two-dimensional effectiveness in the following way:

$$\begin{aligned} \eta &= 1 && \text{for } X < 7 \\ \eta &= 7/X && \text{for } X > 7 \end{aligned}$$

$$\text{where } X = .91 \left(\frac{U_{Gx}}{U_{CYC}} \right)^{.8} \left(\frac{U_{CYC}}{v} \right)^{-.2} + 1.41 \left(1 - \frac{U_G}{U_C} \frac{x}{y_C} \right)^{.5} \quad (2.4.1)$$

Marked discrepancies were observed between the measurements, on two practical slots, and predictions using eq. (2.4.1). The bad agreement was attributed to small potential-core length and larger transition length in practical slots as compared with ideal two-dimensional slots. Sturgess (50) then defined the following semi-empirical correlation and

obtained better predictions:

$$\eta = C_1 S_1^{-.65}$$

where $C_1 = f$ (slot geom., vel. ratio)

$$\text{and } S_1 = \{(\rho_G U_G / \rho_C U_C) (x/y_C)\} (U_C y_C / \nu)^{.25} \quad (2.4.2)$$

The constant C_1 was prescribed for each geometry and so this law could not be extrapolated beyond the range of available experimental information. Equation (2.4.2) was based on the empirical relations of flat plate turbulent boundary layer. Sturgess (31) further modified the analysis to result in the following relation:

$$\eta = k_2 [1 + k_3 (S_1)^{.65}]$$

$$\text{where } S_1 = \{(\rho_G U_G / \rho_C U_C) (\frac{x - x_P}{y_C})\} (\frac{U_C y_C}{\nu} \frac{\mu_C}{\mu_G})^{-.18} \quad (2.4.3)$$

and k_2 and k_3 are constants.

The above relation is only valid beyond the potential core and has the same limitation of having unique values of k_2 and k_3 for each cooling slot. The only advantage is that it is more flexible than eq. (2.4.2). Sturgess (31) also attempted to find a correlation based on a jet flow model which assumed that the flow downstream of a practical slot behaved like a free jet. This approach was found to be much worse than the boundary layer approach and was dropped as unsatisfactory. Sturgess (33) applied the above theory to the cooling of aircraft gas turbine chambers, taking into account the radiative and convective heat fluxes at the chamber walls. In his own words, "it is a crude analysis and no proper account of the disordered mixing region, which is the main region of interest, has been taken."

The above description of the work done by Sturgess suggests that it is impossible to find a unique law for

three-dimensional film-cooling slots. Variations of the correlation given by equation (2.4.3) have been found by other authors, e.g., ref. (32), for their own particular devices. It is evident from the form and the basis of these correlations that they cannot predict the influence of different initial and boundary conditions on the performance of three-dimensional slots. It is necessary to solve the governing partial differential equations to be able to predict the various influences correctly, on the lines parallel to the solution procedures of ref. (9) and (10) for two-dimensional film-cooling. The rest of this section examines the available differential procedures for three-dimensional flows and makes a choice of a procedure, suitable for predicting three-dimensional film-cooling.

Differential methods. Flow downstream of practical film-cooling slots has a predominant direction and gradients of dependent variables are larger in the directions normal to the main direction of flow. Therefore, we should consider particularly those calculation procedures that solve the boundary layer form of the general Navier Stokes equations. Patankar (52) has given a review of the available calculation procedures for steady three-dimensional boundary layers and has developed a procedure (8) of his own. In the remainder of this section we shall attempt to make a choice, from the available procedures, for the present work.

Table (2.4) lists the available calculation procedures for three-dimensional boundary layers. All of these procedures, except that of ref. (8), assume that $\frac{\partial P}{\partial y} = 0$ and P as a function of x and z is described by the

Table (2.4) Available differential methods for three-dimensional boundary layers.

Author	Year	Laminar/Turbulent	Finite-difference scheme
Raetz (53)	1957	Laminar	Explicit
Hall (54)	1967	Laminar	Implicit
Dwyer (55)	1968	Laminar	Implicit
Fannelop (56)	1968	Laminar	Implicit
Nash (57)	1969	Turbulent	Explicit
Krause et al. (58)	1970	Laminar	Implicit
Wang (59)	1970	Laminar	Implicit
Patankar & Spalding(8)	1972	Laminar & Turbulent	Implicit

free stream conditions. In addition they assume that diffusion is important only in the y-direction and solve equations of conservation for U and W only and calculate V from continuity. These assumptions are not valid for flow downstream of a three-dimensional slot. Jets injected parallel to the wall spread in lateral and normal directions and hence diffusion is important in both y and z directions. Also the zero pressure assumption gradient is inadequate. It is the pressure variation in yz plane which is responsible for cross-stream velocities in a film-cooling situation, so the pressure variation inside the flow should be calculated separately rather than assuming it to be known from the free stream. In addition to these common shortcomings, the method of Raetz (53) uses an explicit finite-difference scheme which limits the size of the forward step and Fannelop's method (56) is restricted to small velocities in the z-direction.

The procedure developed by Patankar and Spalding (8) does not have above limitations. It takes account of diffusion both in y and z directions and does not neglect pressure variation in the cross-stream plane. It is a general procedure and is not limited to laminar flows. These features together with easy availability made the author choose the procedure of ref. (8) for the present work.

The following are the main conclusions drawn from the present literature survey:

1. Exhaustive experimental results are available for two-dimensional slots and the important influences have been identified. Prediction procedures have been developed for two-dimensional flows and can accurately predict the significant

influences.

2. Experimental work should be continued on three-dimensional slots and the influence of density ratio, pitch to diameter ratio, open area, open area ratio, height of the holes from the wall, lip thickness and lip length, should be measured.

3. Differential methods should be used to predict the flow downstream of three-dimensional film-cooling slots. The procedure of ref. (8) is more suitable for film-cooling than the other differential procedures available.

CHAPTER 3

3. The experimental investigation.

The purpose of the present experimental program was to investigate the influence of mass velocity ratio ($\rho_C \bar{U}_C / \rho_G U_G$) and density ratio (ρ_C / ρ_G) on the effectiveness and heat-transfer rates downstream of discrete hole three-dimensional film-cooling slots with injection of the secondary fluid tangential to the wall. In addition to the above flow parameters, it was required to determine the influence of geometrical parameters such as diameter of the holes (D), pitch to diameter ratio (P_i/D), lip thickness to diameter ratio (t/D) etc. The influence of the above parameters was investigated in order to identify those that have significant influence on the performance of three-dimensional slots. The experimental program was designed to provide measurements suitable for testing a prediction procedure for three-dimensional film-cooling.

For these measurements, a low speed wind tunnel was designed and constructed. It could accommodate various slot geometries and allowed the systematic examination of the above-mentioned parameters. Section 3.1 gives details of the construction and operation of the apparatus, section 3.2 includes the presentation and discussion of results and section 3.3 gives the summary of results.

3.1 Description of apparatus.

The apparatus consisted of a subsonic, open-circuit wind tunnel with provision for secondary injection through

different interchangeable slots: it is shown schematically in Fig. 3.1.1. The components of the main flow system are described in the following subsection.

3.1.1 Design and construction of the wind-tunnel for effectiveness and heat transfer measurements.

Main flow circuit.

The test-section. The tunnel was designed to allow a uniform steady, two-dimensional and low turbulence main flow. The test-section was rectangular in cross-section (200 mm high x 300 mm wide) and was 900 mm long. This cross-section was chosen as large enough to ensure that the influence of the side-walls was negligible and yet small enough to allow economical foreign-gas injection. The side-walls were made of 6.35 mm thick perspex strengthened at the edges by duralumin angle-sections. One side wall had a number of static pressure holes along its length. The roof of the test section was made up of perspex strips of various sizes covering the whole length. At the top edges two silver steel rods of 19.05 mm diameter were fixed to guide the traverse mechanism in the longitudinal direction.

The base of the test-section was designed in three parts - splitter plate, secondary box exit and the base plate. As is clear from Fig. 3.1.1, the leading edge of the splitter plate divides the main stream and provides an origin for the wall boundary layer. A wire trip (1 mm dia.) was used at a distance of 60 mm from the leading edge to promote early transition to turbulent flow. The splitter plate was followed by the secondary box which housed the film cooling slots. Downstream of the slot position, the secondary box

was connected to a 6.35 mm thick and 950 mm long duralumin base plate which could be moved up or down between the side walls by 19 mm from its nominal position. This arrangement is shown in Fig. 3.1.1. Five rows of static pressure holes were drilled in the base plate at the same streamwise locations as the static pressure holes in the side wall. Fig. 3.1.2 shows the lay out of these static pressure holes which had the diameter of 0.5 mm and depth of 0.25 mm.

For heat-transfer measurements, the duralumin plate described above was replaced by a tufnol plate. On the top face of the tufnol plate a stainless-steel sheet, 600 mm long and 0.125 mm thick, was glued using a 1 mm thick hot melt film. One brass strip 3 mm thick and 25 mm wide was fixed on each end of the s.s. sheet. The s.s. sheet was electrically heated and the brass strips acted as buss bars to pass current uniformly over the complete length. 40 swg copper-constantan thermocouples were spot welded to the bottom surface of the s.s. sheet through holes in the tufnol plate. A lay out of the thermocouple junctions is shown in Fig. 3.1.3.

Primary fan and the contraction unit. A Gyro-flow centrifugal fan (type-20-A-1) driven by a 10 h.p. A.C. synchronous motor was chosen to supply the main flow. The fan was run by belt and pulley drives and velocities up to 45 m/s could be obtained in the test-section by changing the pulley ratio. For all the present measurements, the fan was operated at a speed which resulted in a free-stream velocity of 20 m/s.

Connecting the fan to the test-section were a diverging section, a settling chamber and a contraction. The diverging section and the settling chamber were

constructed with 8 mm thick plywood and the contraction with 3.2 mm thick mild steel. The inlet flange of the diverging section was joined to the outlet flange of the fan only with cloth tape and minimised the transmission of vibrations from the fan to the contraction unit. Three screens of 20 mesh and 28 swg and two sets of honeycomb, each of 9.6 mm cell size and 76 mm length, were used in the diverging section and the settling chamber. The locations of these screens and honeycombs are shown in Fig. 3.1.1. A contraction ratio of 12:1 was used and the profile was chosen as a scaled version of that used in a tunnel and known to be satisfactory. The exit flange of the contraction was also joined with cloth tape to the inlet flange of the test-section. The contraction unit and the test-section were designed as free standing components mounted on channel iron frames which were supported on the floor by anti-vibration rubber pads. These precautions resulted in satisfactory isolation of the test-section from the fan vibrations.

Secondary flow circuit. The secondary flow system was designed to allow air, Arcton-12 and mixtures of Arcton-12 and air to be injected through the slot. The general arrangement is shown in Fig. 3.1.4. Air was supplied by a small blower, driven directly by a 1 h.p. motor running at 2850 rpm. The amount of air was adjusted with a sliding throttle at the exit flange of the blower. A 3 KW heater was provided after the fan so that air could be heated to a desired temperature for heat-transfer measurements. A separate manifold was used for Arcton-12 so that six cylinders could be used in parallel. This was done because

of the relatively low pressures at which Arcton-12 is stored in the cylinders. The desired flow rate of Arcton-12 was achieved by manual adjustment of the cylinder valves. The secondary blower and the Arcton-12 manifold were connected in parallel leading to a section of Durapipe, 50 mm diameter and 3050 mm long. Two throttle valves were used for final adjustments of air and Arcton-12 or to disconnect one of them. The pipe carrying the Arcton-12 was fitted with a 12.7 mm orifice meter and the main pipe was fitted with a 25.4 mm orifice meter; these allowed the secondary mass flow rate to be measured. On both the orifice meters 'D and D/2' tappings were used in accordance with B.S. 1042 (1966).

The secondary pipe was connected to the slot through a 90° elbow joint, a diverging transition section, a settling chamber and the secondary box which opened between the splitter plate and the base plate of the test section. The settling chamber contained two 20 mesh and 28 swg screens and a honeycomb of 9.6 mm cell size and 76 mm long. Wooden pieces were fitted in the secondary box (Fig. 3.1.4) to form a contraction before the flow passed through the slot.

3.1.2 Instrumentation.

Traverse mechanism and Pitot-probes. The traverse mechanism, which was designed to give motion in all the three directions, is shown on Fig. 3.1.5. DISA traversing mechanism, type 55H01, was used for movement of the probes in the y-direction. This mechanism could be driven manually or with a two-speed motor. In the present investigation, manual drive was used and had a resolution of 0.01 mm. The DISA block was fixed

on a second block which could be moved in the x-direction with the help of a spring loaded 25 mm micrometer. For larger movements in the x-direction, the complete mechanism was moved on the steel rods fixed on the roof of the tunnel. The two end supports that moved on the steel rods were connected with a screw of 25.4 diameter and 1.27 mm pitch, running all along the width of the tunnel. The DISA block carrying the probe could be moved on this screw in the z-direction to cover the whole width of the tunnel. This screw was rotated manually and the minimum movement obtainable in the z-direction was 0.025 mm.

The pitot probes were constructed from stainless steel hypodermic tubing. A flat ended pitot probe with external dimensions of 0.4 mm x 0.25 mm was used to measure mean total-head pressures near the wall and a round pitot static tube of inside diameter 0.75 mm was used to measure mean total-head pressures away from the wall. The flat ended probe was also used to obtain isokinetic samples for concentration measurements.

For turbulence measurements, a DISA (type 55A01) constant temperature anemometer was used. One straight wire probe (55A25) and single inclined wire probe (55A29) were used to measure $\sqrt{u^2}$, $\sqrt{v^2}$, $\sqrt{w^2}$, and $\overline{uv}$.

Pressure measuring devices. A Hilger-Watt electronic micro-manometer with a variable capacitance pressure transducer (range 0 to $\pm$ 50 mm of water), connected to a digital voltmeter, was used to measure total head pressures from the impact probes. A Betz manometer (0 to 300 mm of water) measured the pressure drop across the orifice meters.

Pressure drops in excess of 300 mm of water were measured with a U-tube manometer containing water, paraffin or mercury. To measure streamwise distribution of static pressure, a bank of inclined-tube manometers, containing paraffin, was used.

Concentration measuring device. A device for concentration measurement was developed because, in part of the present experiments, impervious-wall effectiveness was measured; this is defined by eq. 1.1.1 with concentration replacing the temperatures. The impervious-wall effectiveness is analogous to adiabatic-wall effectiveness if the turbulent Lewis number is unity. The particular advantage of this analogy is that an impervious-wall condition can be realised more closely than an adiabatic-wall. A small quantity of helium was mixed with the secondary stream to permit a density ratio of unity and for higher density ratios between the secondary and the primary stream Arcton-12 or Arcton-12 and air mixtures were used. The use of the mass transfer analogy implied that mass fractions of the secondary injected gas were measured in the flow field rather than temperatures.

Concentration measurements were effected with a Servomex microkathorometer (MK. 158) of the flow through type. This kathorometer consists of two chambers of 2.6 μ l volume, containing platinum filaments, and was placed in a Shandon oven kept at a constant temperature of 50°C. Atmospheric air was drawn through one chamber and the gas sample through the other by means of a vacuum pump connected to the outlet of the kathorometer. The flow through each chamber was regulated with the help of rotameters. The platinum wire filaments of the kathorometer were a part of the bridge circuit

contained in the power unit which was used to supply a constant current to the filaments. The heat transfer rate from each filament was different because of different conductivity of the gas flowing in each chamber and results in different temperatures and different resistances of the filaments. This difference is a direct measure of the mass concentration of helium or Arcton-12 in the gas sample and was measured by displaying the output of the bridge circuit on a digital voltmeter. The response time of the kathorometer was of the order of milliseconds and more time was taken in flushing through the tubes leading the gas sample to the kathorometer. In order to reduce the flushing time, hypodermic stainless steel tubing of 1 mm outside diameter was used for all gas connections. 20 seconds was a typical time taken for one concentration measurement. An air selector valve (Norgren) with ten ports allowed the kathorometer to be connected to a new sample point, in a time which was negligible compared to the flushing time.

The kathorometer was calibrated for small concentrations of helium in air and Arcton-12-air mixtures ranging from 100% air to 100% Arcton-12. A small amount of helium was injected by a 1 ml Hamilton gas syringe into an air tight 1 litre flask which was stirred to make a homogeneous mixture of helium and air. The calibration curve for small amounts of helium in air was linear as shown in Fig. 3.1.6. Arcton-12 and air mixtures were made over water in a graduated jar. Dalton's law of partial pressures and the perfect gas law were used to calculate mass fraction from the measured volumes. The amount of water vapor in the mixtures

prepared in the above manner was assumed to be negligible. The calibration curve of Arcton-12-air mixtures for a bridge current of 100 mA is shown in Fig. 3.1.7.

Equipment used for heat-transfer measurements. A high amperage (0 to 1000 amperes), variac transformer was used to supply the current for heating the s.s. sheet. A 5 ampere Crompton ammeter was used, in conjunction with a portable current transformer for stepping down the current in the ratio 60:1, to measure the current through the heater. A Solartron a.c. digital voltmeter was used to measure the a.c. potential drop and another Solartron digital voltmeter was used for recording the thermo-e.m.f.-s. The resolution of both the voltmeters was 10 μ v.

3.1.3 Testing and operation of the apparatus.

It was decided to test the performance of the tunnel for two simplified flows for which established results are available, before commencing the measurements with the discrete hole cooling slots. The measurements of turbulent boundary layer development in zero pressure gradient and effectiveness downstream of a two-dimensional slot were done to test the tunnel performance. Results of these measurements together with the operation of the apparatus are described in this section.

The boundary-layer measurements were made to check the two-dimensionality, momentum balance and the nature of turbulence of the flow in the test section. The secondary box exit was covered with a perspex plate to level with the splitter plate and then the base plate was adjusted to

minimise the longitudinal pressure gradients due to the growth of the boundary layer.

Momentum balance. The following procedure was used to check two-dimensionality and momentum balance in the boundary layer flow. The flow was fully turbulent at a distance of 75 mm from the trip. Between this point and a point 1000 mm downstream of the leading edge, twelve velocity profiles were measured on the centre line of the base plate. At a few stations, velocity profiles were also measured at a lateral distance of 100 mm on the sides of the centre line. The flat ended pitot probe was used for the velocity profiles and the wall was located by obtaining the minimum total head reading. A displacement correction of 15% of the pitot probe height was allowed as suggested by MacMillan (60). The perfect gas law was used to calculate density from the room temperature and pressure and Sutherland's formula was used to calculate the absolute viscosity.

The boundary layer thickness, δ , was defined as the distance of the point, from the wall, where the velocity was 99% of the free stream velocity. Each measured velocity profile was numerically integrated, using the trapezoidal rule, to calculate displacement thickness, δ_1 , and momentum thickness δ_2 . The shape factor, H_{12} , and δ_2 had the same values on the off-centre line points as on the centre line to within 4%, and this demonstrated satisfactory two-dimensionality of the flow.

The local skin friction coefficient was evaluated using (i) Momentum Integral Equation (ii) Clauser chart with Patel's values of constants, K_1 (.4187) and E (9.793).

The Momentum Integral Equation for no pressure gradients has the following form:

$$cf/2 = \frac{d\delta_2}{dx} \quad (3.1.1)$$

The right hand side of the above equation was obtained by fitting a power law, between δ_2 and x , of the following form:

$$R\delta_2 = a Rx^b \quad (3.1.2)$$

where Rx is based on the longitudinal distance from the virtual origin and a and b are constants. The distance of the virtual origin from the trip was found by trial and error and was chosen to be the one which gave minimum standard deviation between $R\delta_2$ values calculated by eq. 3.1.2 and the experimental values. It was calculated to be 142 mm upstream of the trip. The values of a and b in eq. 3.1.2 were found by a least square fit. Knowing the power law in eq. 3.1.2 calculation of cf as a function of x is straight forward.

cf was not evaluated from the Clauser chart graphically, instead a numerical scheme was developed to calculate cf from the law of the wall,

$$U+ = \frac{1}{K_1} \ln(Ey+)$$

or $\frac{U}{U_G} \frac{2}{cf} = \frac{1}{K_1} \ln \left[E \frac{yU_G}{v} cf/2 \right] \quad (3.1.3)$

cf appears on both sides of eq. 3.1.3 and was found by iteration for each point in a velocity profile. The next step was to make groups of cf values that lay within the bands of cf limits corresponding to lines of constant cf on a Clauser chart. The group consisting of the maximum number of values corresponded to the log-law region of the boundary

layer. The value of c_f which had the minimum standard deviation in the largest group was found to give the desired skin friction coefficient. Ref. (61) gives the computing details of this numerical scheme.

The values of c_f obtained from the law of the wall were within 4% of those found by Momentum Integral Equation. c_f obtained by both the methods compared well with the Spalding-Chi (62) correlation, as can be seen in Fig. 3.1.8. c_f values obtained at the off-centre points were within 3% of those at the centre-line.

Turbulence measurements. The free stream-turbulence was measured at the contraction exit with a straight hot-wire, calibrated in the potential core of a round jet using King's law. The turbulence intensity was found to be 0.3% at a free stream velocity of 20 m/s and low frequency fluctuations were not observed.

At a distance of 635 mm downstream of the trip, three components of the fluctuating velocity and the correlation $\overline{uv}$ were measured in the boundary layer using a straight hot-wire, a 45° inclined hot-wire and the constant temperature hot wire anemometer. Kacker's (7) procedure was used for data reduction and is valid only for flows with low turbulence levels. These measurements compared satisfactorily with the measurements of Klebanoff (63) and are shown in Fig. 3.1.9.

From the above measurements of mean flow and turbulent flow properties on a flat plate, it was concluded that the tunnel was satisfactory for present purposes.

Measurement of Impervious-wall effectiveness. A mild steel

plate, 2 mm thick, was screwed to the splitter plate and the base plate was lowered to make a two-dimensional slot of height 6.7 mm. The slot height was uniform to within ± 0.25 mm and the base plate was adjusted to eliminate the longitudinal pressure gradient. The tunnel was set to operate at the desired velocity ratio by operating the throttle on the secondary fan. A small amount of helium was introduced into the secondary stream just after the blower and acted as a measurable tracer gas for effectiveness determination. The static pressure holes on the base plate were connected to the selector valve by 1 mm diameter stainless steel tubing. The common port of the valve was connected to the kathorometer sample chamber and flow rates in the two chambers were adjusted to be the same. A current of 140 mA was passed through the kathorometer and the bridge was balanced for air passing through both the chambers. After this preliminary setting of the kathorometer, helium-air samples were passed through from the secondary box and different locations on the base plate. The concentration reading at any position downstream of the slot was normalised with the box value to give effectiveness at that position.

Effectiveness was measured on the centre line for the velocity ratios of 1.6, 1.1 and .63 and also on the off centre lines for the velocity ratio of 1.1. There was no significant difference between the centre line and the off centre line effectiveness values. These measurements are shown in Fig. 3.1.10 together with the measurements of ref. (7). Agreement between the present measurements and the measurements of ref. (7) is excellent and confirms the

reliability of the operation of the present set-up.

Measurement of heat-transfer coefficient and adiabatic-wall effectiveness.

The test section was subjected to a heat loss test to determine the heat transfer from the heated s.s. sheet to the insulation and the surroundings. A constant current was passed through the s.s. sheet and the a.c. potential difference between the bus bars was measured. Under the steady state conditions without any flow in the test section, the electrical power input into the s.s. sheet was equal to the heat transferred to the surrounding air on the top surface by free convection and the heat lost through the insulation. The average heat-transfer coefficient, because of free convection, was calculated by the following formula

$$Nu_{\text{free}} = 0.54 (Gr Pr)^{1/4}. \quad (3.1.4)$$

The s.s sheet surface temperatures were measured and the heat-transfer through the insulation was determined by subtracting the heat transferred by free convection from the power input. The heat loss coefficient (h_L) was based on the temperature difference between the wall and the surroundings and was found to be independent of this temperature difference in the range between 15 and 25°C. Conduction along the s.s. sheet was evaluated and was found to be negligibly small.

The procedure for obtaining the heat-transfer coefficient and the adiabatic-wall effectiveness in the presence of film-cooling was as follows. The desired velocity ratio was set by adjusting the secondary fan throttle and the secondary air was heated by some 20°C above

room temperature. When the conditions were steady (in about an hour) the temperature distributions in the s.s. sheet and the slot were recorded by noting the thermo-e.m.f.-s developed by the thermocouples. Next, a current of approximately 170 A was passed through the sheet and the velocities and slot temperature were maintained at their previous values. Under steady conditions (in about half an hour), the thermo-e.m.f.-s, the a.c. potential difference between the bus bars and the current through the s.s. sheet were measured. The rate of heat generation was calculated from the product of the current through the s.s. sheet and the voltage gradient across it. Measurement of local a.c. voltage gradient across the s.s. sheet showed that the heat was uniformly generated and it was sufficient to measure the voltage drop between the bus bars to determine the net heat generated. A heat balance for an element of the s.s. sheet leads to the relation

$$\dot{q}''_{\text{gen}} = h_f(T_w - T_{\text{aw}}) + hL(T_w - T_{\text{a}}) \quad (3.1.5)$$

Since for each velocity ratio, the experiments were carried out for two values of $\dot{q}''_{\text{gen}}$ (zero and one non-zero value), the two unknowns in eq. (3.1.5), h_f and T_{aw} could be readily computed. To confirm that h_f was independent of $\dot{q}''_{\text{gen}}$, it was determined for two non-zero values of $\dot{q}''_{\text{gen}}$ and was found to remain unchanged.

3.2 Presentation and discussion of experimental results.

The measurements made in the apparatus described in the previous section are reported in this section. Hydrodynamic properties and impervious-wall effectiveness were

measured downstream of slots having a single injection hole tangential to the wall and impervious-wall effectiveness, adiabatic-wall effectiveness and heat transfer coefficients were measured downstream of slots having multiple injection holes. As explained in ref. (64) the former geometry was chosen as it is the simplest three-dimensional tangential wall jet situation and could aid the understanding and design of more complicated multi-hole geometries. Experimental results on single hole and multiple hole geometries are reported separately.

3.2.1 Film-cooling downstream a single injection hole.

Fig. 3.2.1 shows the general slot configuration. The influence of all the parameters shown in the figure, except t on the impervious-wall effectiveness was investigated. The range of values used for each parameter is shown in Table 3.2.1. Measurements were done in stagnant surroundings and with a free stream velocity of 20 m/s. Air and Arcton-12 were used as slot fluids and resulted in slot to mainstream density ratios of 1 and 4.17 respectively.

Hydrodynamic and species properties. Rates of growth of a round jet of 12.5 mm diameter, injected tangential to a wall in stagnant surroundings, were measured. Fig. 3.2.2(a) shows the centre-line values of U_{max} and $y_{1/2}$ for two jet velocities. Normal growth is slightly more for the jet injecting at the lower velocity. The growth rate parameters shown on Fig. 3.2.2(a) agree reasonably well with those measured by Naib (37). He used a jet of 25 mm diameter and his injection system was different. Agreement between

his and the present measurements indicates that, in the downstream region, the growth rate parameters are not strongly geometry dependent. This is further demonstrated by the similar measurements of Newman et al. (65), who used a jet of 3 mm diameter at the injection velocity of 50 m/s. He measured rates of growth in the far downstream region beyond the present range of longitudinal distances; however, the extrapolated values from his measurements agree satisfactorily with the present measurements.

Fig. 3.2.3 shows the lateral spread of the jet in the presence and in the absence of the free stream. These measurements were obtained with temperature sensitive Cholesteric liquid crystals. A crystal mixture which turned red at 26°C , green at 27°C and blue at 28°C was sprayed uniformly on a plane surface. A false tunnel base was used for this purpose and a warm jet was injected tangential to this base. The jet exit temperature was 31°C , which was 7°C above the room temperature. The boundaries of the blue region of the crystal spray are shown in Fig. 3.2.3 and correspond to lines of constant effectiveness equal to approximately 0.6. Beyond a distance of 12 diameters downstream of the jet exit, it was difficult to separate the region of blue colour from that of green colour and so lines in that region correspond to effectiveness values between 0.6 and 0.43. From the contours shown on Fig. 3.2.3(a) it is concluded that, in absence of a free stream, lateral spread of the faster jet is greater. Doubling the jet velocity, and hence the mass flow rate, does not double the surface area maintained within the constant effectiveness value. From

the depth of the blue colour it was seen that higher the jet velocity, the greater was the downstream penetration of the jet. The two contours shown in Fig. 3.2.3 (b) correspond to jets of different diameters and show that for the same velocity ratio the smaller jet spreads more and hence, at least up to 18 diameters downstream, the smaller jet makes more efficient use of coolant at the velocity ratio of 1.25. These results are more relevant to film cooling than those of previous investigations (37, 65), as they show the influence, on the growth rates, of the presence of the free stream. Fig. 3.2.3 indicates that for jet velocities higher than the free stream, the jet does not grow laterally with longitudinal distance as happens in the absence of the free-stream.

Fig. 3.2.4 presents velocity and concentration profiles for air injection at velocity ratios of .9 and 1.30. Concentration profiles are shown on off-centre line locations as well and demonstrate the three dimensionality of the flow. In the upstream region, concentration profiles at the centre line contain maxima where as, after 10 diameters, maxima appear in the off-centre line profiles and disappear from the centre line profiles.

Impervious-wall effectiveness. Fig. 3.2.2(b) shows effectiveness measured on the hole centre line plotted against x/D for the two jet velocities of Fig. 3.2.2(a) in the absence of the free stream. In the upstream region, effectiveness values for the higher jet velocity are higher but beyond 35 diameters downstream the trend is reversed. The discrepancy in the two curves of Fig. 3.2.2(b) would be substantially removed if the normalising parameter were $(xy_{1/2}/D^2)$, $(xw_{1/2}/D^2)$

or any alternative which tends to relate the surface cooled to the available flow rate. The practical difficulty is that $y_{\frac{1}{2}}$ or $w_{\frac{1}{2}}$ are not known a priori. The cross over in the effectiveness curves of Fig. 3.2.2(b), for the two jet velocities, is also likely to be removed by using one of the above parameters. Indications, from the present experiments, were that lateral growth increases significantly after 30 diameters and is more for higher jet velocities which explain lower effectiveness for the faster jet in the downstream region.

As in the two-dimensional case, the single hole slot geometry possesses a number of geometrical parameters that influence the mixing of the two streams and hence the effectiveness. The parameters investigated in the present work were the hole diameter, height of the hole centre above the wall, length of the lip and the way of manufacturing the hole.

Fig. 3.2.5 presents the impervious-wall effectiveness measured on the centre line, for air injection, downstream of four slots having different hole diameters. Each hole was located tangential to the wall, had a lip thickness (t) of 1.25 mm and had no lip length. In this figure, like many of the following figures in the present chapter, effectiveness is plotted against the velocity ratio for different values of x/D . The points represent the data and the lines are the smooth curves through them. These curves exhibit maxima similar to those previously found for two-dimensional slots. As a result of jet growth in two directions, maxima appear at a velocity ratio which increases with increasing downstream distance. Therefore, selection of

the hole size that will result in the maximum effectiveness for a given mass flow rate of the coolant depends upon the velocity ratio and the downstream location. For a particular hole size, the velocity ratio should be chosen to correspond to the maximum in the effectiveness curve at a particular downstream location. Within the present range of parameters, the 8 mm hole gave best results, for distances up to 100 mm. The measurements shown on Fig. 3.2.5 indicate the influence of an equivalent lip thickness; consequently as the hole size diminishes, a decrease in effectiveness may be anticipated, other parameters remaining unchanged. The same effect may be anticipated due to the influence of velocity profile shape at the hole exit.

Fig. 3.2.6 shows the measured values of effectiveness on and off the centre line of the slot with 12.5 mm hole, for air injection and three velocity ratios. These measurements indicate a significant reduction in the effectiveness values along the off centre lines when compared with the centre line values. The difference is more noticeable at upstream locations and at higher velocity ratios. For a velocity ratio of 1.3, effectiveness measurements on the centre line are presented for a slot which had a hole of 7° half angle taper, with the exit diameter of 12.5 mm. As shown in Fig. 3.2.6 the effect of diverging taper is to diminish the effectiveness significantly. This is expected because, as a result of divergence, the flow inside the hole has a tendency of separating and consequently at the exit grows more in the normal as well as the lateral direction. Larger growth rates result in more mixing and hence lower

effectiveness.

Also for the injection of air into air and for the particular case of a hole diameter of 10 mm, the influence of lip length (L) on the centre line effectiveness was investigated. The lip thickness used for these measurements was 1.25 mm and, in terms of hole diameter, may be considered thin. Use of lip lengths in the range of L/D from 0 to 2.2 indicated that the longer lip leads to a decrease in effectiveness in the downstream region ($40 \leq x/D \leq 80$) of approximately 10%. In the upstream region, i.e. up to 20 diameters from the slot exit, the results indicate that the lip length has little influence at velocity ratios in excess of 1.5 but has a detrimental effect for lower velocity ratios. The maximum decrease in effectiveness at $x/D = 20$ was 20% for $L/D = 2.2$. The detrimental effect of lip length is contrary to the findings of ref. (14) for the slots with multiple injection holes. The single jet is isolated from the free stream in the presence of a lip and grows much more in the lateral direction than it would have done in the presence of a free stream. For this reason the maximum decrease was found for $L/D = 2.2$ which was the longest lip used.

The influence of the height of the hole centre above the wall is shown in Fig. 3.2.7. These results show that an increase in the distance between the hole centre and the wall causes a decrease in effectiveness at all downstream locations. The effect is more significant at higher velocity ratios.

Fig. 3.2.8 presents effectiveness measurements on

the centre line of three slots for the injection of Arcton-12 into air. These curves show little trace of maxima as was observed in the case of two-dimensional slots. At a particular downstream location, effectiveness remains unchanged with increase in velocity ratio beyond a particular value of the velocity ratio. For a given mass flow rate the largest hole diameter compatible with this flat region of the effectiveness curve is desirable. Within the range of parameters investigated in the present measurements, the intermediate hole diameter of 8 mm was found to be the best for the injection of Arcton-12.

3.2.2 Film-cooling downstream of multiple hole slots.

Measurements were made downstream of the five multihole slots shown in Fig. 3.2.9 and described in Table 3.2.2. These configurations were chosen so that the influence of lip thickness to diameter ratio (t/D), hole pitch to diameter ratio (P_i/D), open area and open area ratio could be determined separately, on the impervious wall effectiveness. The dimensions of slot 1 were chosen to approximate a scaled up version of a practical combustor-cooling device. Slots 2 and 3 were selected so that influence of t/D and P_i/D could be determined by comparing their results with those of slot 1. Slot 4 was designed to provide version of slot 2 but with practical dimensions. Slot 5 represents an alternative means of injecting the coolant and its dimensions were chosen to provide an open area similar to that of slots 1, 2 and 3 and open area ratio same as slot 1: ^{it consisted of three rows of holes as indicated in Table 3.2.2.} Impervious-wall effectiveness measurements

were made for air into air and for the injection of a mixture of air and Arcton-12 into air which resulted in a slot to mainstream density ratio of 2.10. Heat transfer coefficient and adiabatic-wall effectiveness were also measured downstream of some of these slots.

Impervious-wall effectiveness. Fig. 3.2.10 to Fig. 3.2.14 show the effectiveness values measured on the centre line of the five slots for uniform density and density ratio of 2.1. Measurements of effectiveness between the hole centres for slot 1 indicated that wall effectiveness did not attain two-dimensionality as rapidly as had been suggested by the measurements of Nina (14). The rapid reversion to two-dimensionality was observed at the wall in Nina's measurements because of the conduction along the wall where as in the present mass-concentration technique no such error is introduced. For velocity ratios greater than unity, the region of three-dimensionality at the wall extended beyond 30 diameters. The results show trends which are, in general, similar to those observed for two dimensional slots. For the uniform density case, effectiveness curves reveal a maximum where as for non-uniform density this maximum is generally absent. The effectiveness at a particular value of x/D is always greater for the higher density ratio, provided the comparison is made at the same velocity ratio, this is not so if the comparison is made at equal mass velocity ratios.

Comparison of Fig. 3.2.10 with Fig. 3.2.11 shows that an increase in t leads to a decrease in effectiveness. The effect is more in the upstream region and reduces with

increasing distance downstream. This is in accord with the findings of Nina (14) where a slot lip extended beyond the slot exit. It is also in accord with the findings of Kacker and Whitelaw (1) for the two-dimensional geometry although the magnitude of the influence is much less if the comparison is made on the basis of t/y_c with y_c based on the open area.

From Fig. 3.2.12 and Fig. 3.2.10 it is deduced that effectiveness increases as the ratio of pitch to diameter decreases. This statement is strictly true only when the other parameters, in particular the open area, remain unchanged as was the case between slot 1 and slot 3. This implies that in the limit when P_i/D is zero, the effectiveness should be maximum, and indeed as previously shown in Ref. (4), effectiveness of a two dimensional slot is correspondingly higher than those reported here.

From Fig. 3.2.11 and Fig. 3.2.13, it can be seen that effectiveness values downstream of slots 2 and 4 are sensibly identical in the upstream region. The two slots were geometrically similar, except for the upper lip boundary layer thickness (approximately 12 mm in both cases) which resulted in slight decrease in effectiveness for the slot 4, at values of x/D greater than 30. The maximum difference was noticed at the velocity ratio of 1.12 and was 8% of unity at 50 diameters downstream of the slot exit.

Slot 5, at first sight, appears to provide a simple means of obtaining two-dimensional effectiveness downstream of a three-dimensional slot of practical structure. In practice the jets emerging from the various holes mix rapidly and create a region of relatively high turbulence. Compari-

son of the effectiveness values shown in Fig. 3.2.14 with those of a two-dimensional slot with a thin lip and the same open area as slot 5, shows that the two are similar for velocity ratios above 1.4 and uniform density. For lower velocity ratios the two-dimensional slot is more effective. This is due to the fact that thin lipped two dimensional slots result in effectiveness curves that have maxima around the velocity ratio of unity. For the density ratio of 2.1, the measurements of Ref. (6) suggest that slot 5 and two dimensional thin lipped slots result in similar performance over a much wider range of velocity ratios provided the open area is same. Fig. 3.2.14, when compared with Fig. 3.2.10, shows that the multirow slot 5 is always better than slot 1 although both have same open area and open area ratio. Slot 3 with its greater open area ratio is to be preferred to slot 5 for values of x/D greater than 30.

The influence of the height of the holes from the wall has been omitted in the present investigation. This parameter is analogous to a backward facing step in a two-dimensional situation and has been investigated for slots with single holes. The effect of having a backward facing step in a two-dimensional slot and moving the hole away from the wall in single hole slots, has been shown (24, 6⁴~~7~~) to reduce the effectiveness; however the effect was relatively small, compared to the effect of lip thickness. A smaller effect can be expected for multihole slots, as it is an intermediate geometry between a single hole slot and a two-dimensional slot.

Heat-transfer coefficient. The measured values of heat-

transfer coefficient, expressed as a Nusselt number based on hole diameter and conductivity at slot temperature, are shown in Fig. 3.2.15 to Fig. 3.2.17 along the hole centre lines for the slots 1, 3 and 5 respectively. Measurements are shown by open circles for three velocity ratios for each slot. The lines in these figures, except in Fig. 3.2.17, are the predictions and will be discussed in Chapter 5.

Measurements were not made in the immediate vicinity of the slot ($x/D < 5$), but in the downstream region ($10 < x/D < 50$) trends are clearly indicated. Nusselt number increases with increase in the velocity ratio and reaches the flat-plate value, within $\pm 10\%$, at the downstream location of 45 diameters. The maximum difference between the present measurements and the flat plate value is 14% and was observed at $\frac{x}{D}$ of 40 with the smallest velocity ratio of .495 in case of multirow slots. The flat plate values shown by short chained lines are based on the longitudinal distance from the virtual origin of the main flow turbulent boundary layer. The influence of velocity ratio is similar to that shown by the measurements of Pai (6) for a two-dimensional slot, although the magnitudes are higher in the present measurements.

Comparison of Fig. 3.2.15 and Fig. 3.2.16 reveals that, in the upstream region, the heat-transfer coefficient ~~along the hole centre line~~ increases with increase in pitch to diameter ratio, provided the open area and other parameters are kept unchanged. ^{This results from} the increase in secondary mass flow through an individual hole of the slot with larger P/D , ~~is expected as the resistance of the film to heat transfer and hence a larger diameter, although the net mass flow through each slot is same. will reduce if the holes are placed at a greater pitch.~~

Comparison of Fig. 3.2.15 and Fig. 3.2.17 shows that heat-transfer coefficient downstream of the multirow slots is

lower than the slot 1 which has the same open area and open area ratio as the multirow slot. The heat-transfer coefficient downstream of slot 5 compares reasonably with the measurements of ref. (6); this was also the case for effectiveness results. Thus, the overall performance of the multirow slot is comparable to two-dimensional slots.

Adiabatic-wall effectiveness. Figs. 3.2.15 to 3.2.17 show measured values of adiabatic-wall effectiveness along the hole centre lines for the three slots and velocity ratios for which heat transfer coefficients were measured and reported above. These measurements, represented by open circles, are plotted together with the measured values of impervious-wall effectiveness which are shown by solid circles. Agreement between the measured values of adiabatic and impervious-wall effectiveness is closer than 10% of unity for all cases except for one case in Fig. 3.2.17 where the values are small and small errors in temperature measurements could lead to significant differences in effectiveness. The agreement between the adiabatic and impervious-wall effectiveness provides further evidence for the unity-value of the turbulent Lewis number. The trends shown by these measurements are very similar to those shown by the impervious-wall effectiveness measurements already discussed in this chapter.

3.2.3 Summary of above results.

To conclude the present chapter, the main results are enumerated below:

1. The present measurements have identified U_C/U_G , ρ_C/ρ_G , t/D , P_i/D and H/D as the parameters that have significant influence on the effectiveness downstream of three-dimensional

discrete hole slots.

2. The effectiveness is significantly three-dimensional downstream of all the slots except for the multirow slot. This confirms the need for three-dimensional prediction procedures to correctly predict the film-cooling performance of discrete hole slots.

3. For the single hole case, the velocity ratio at which maximum effectiveness is obtained, depends upon the downstream distance. For the multihole slots, maxima in the effectiveness curves occur approximately at the same velocity ratio for all the downstream distances and the heat-transfer coefficient increases with the increase in velocity ratio.

4. The influence of blowing a heavier gas through the slot is to increase the effectiveness at all the downstream distances, provided the velocity ratio is kept the same. If the comparison is based on constant mass velocity ratio, then the injection of heavier gas results in higher effectiveness only in the upstream region (≈ 20 diameters) and for higher values of mass velocity ratio ($\rho_C U_C / \rho_G U_G \approx 2$).

5. To obtain maximum cooling effectiveness, holes should be located as close to the cooled surface as possible and the lip thickness should be as small as possible.

6. For a particular value of slot open area, a smaller value of pitch to diameter ratio results in an improved effectiveness and lower heat-transfer coefficient.

7. The understanding of the scaling parameters obtained from the present measurements has been tested by effectiveness measurements with two multihole slots scaled by a factor of 6.25. The results are identical in the near slot region and show that the important parameters have been

correctly identified in the region of practical interest.

8. Multi-row slots are preferable to slots with a single row on an open area and open area ratio basis. On the same basis they also compare well with two-dimensional slots except for uniform density flows where the velocity ratio is less than approximately unity.

9. From the complexity of the flow process, it appears that the correct approach for optimisation of various parameters is to develop a three-dimensional prediction procedure. The present measurements offer a means of checking the validity of such a procedure.

CHAPTER 4

4. Prediction of two-dimensional film cooling.

Two problems of two-dimensional film cooling are considered in this chapter. The first is the development of a procedure to predict the influence of longitudinal curvature on boundary layer flow: the main motivation for developing this procedure stems from the need to predict flow over modern highly loaded turbine blades with small radii of curvature. The procedure has been applied to predict the available measurements, using a form of Prandtl's mixing length hypothesis and comparison between the predictions and the experimental data has been reported. The second problem considered in this chapter is the prediction of effectiveness of two-dimensional slots using a three-equation model of turbulence which requires the solution of an equation for the turbulent shear stress. The purpose is to evaluate the merits and demerits of this model as compared to the mixing length hypothesis which has been used in the past (6, 7).

4.1 Effect of longitudinal curvature.

Table 4.1.1 gives details of the experimental information available for flows on curved surfaces. Most of the experimental work has been done on wall-jet flows only and it is against these data that the procedure has been tested. Although, for the present purposes, this restricted application is sufficient, the method is a general one and can be used to predict any boundary layer flow along a curved

surface. Fig. 4.1.1 shows the flow geometry and the coordinate system used. The free stream velocity need not be zero as shown in this figure.

4.1.1 Relevant equations of motion.

The Navier-Stokes and continuity equations for a two-dimensional, steady, incompressible flow in orthogonal curvilinear coordinates may be written (66) in the following form:

$$\begin{aligned} \frac{1}{1+Ky} U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + \frac{K}{1+Ky} UV &= - \frac{1}{\rho} \frac{1}{(1+Ky)} \frac{\partial P}{\partial x} + v \left[\frac{1}{(1+Ky)^2} \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right. \\ &- \frac{y}{(1+Ky)^3} \frac{\partial K}{\partial x} \frac{\partial U}{\partial x} + \frac{K}{1+Ky} \frac{\partial U}{\partial y} - \frac{K^2}{(1+Ky)^2} U + \frac{1}{(1+Ky)^3} \frac{\partial K}{\partial x} V \\ &\left. + \frac{2K}{(1+Ky)^2} \frac{\partial V}{\partial x} \right] \end{aligned} \quad (4.1.1)$$

$$\begin{aligned} \frac{1}{1+Ky} U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} - \frac{K}{(1+Ky)} U^2 &= \frac{1}{\rho} \frac{\partial P}{\partial y} + v \left[\frac{1}{(1+Ky)^2} \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right. \\ &- \frac{y}{(1+Ky)^3} \frac{\partial K}{\partial x} \frac{\partial V}{\partial x} + \frac{K}{1+Ky} \frac{\partial V}{\partial y} - \frac{K^2}{(1+Ky)^2} V - \frac{1}{(1+Ky)^3} \frac{\partial K}{\partial x} U \\ &\left. - \frac{2K}{(1+Ky)^2} \frac{\partial U}{\partial x} \right] \end{aligned} \quad (4.1.2)$$

$$\frac{\partial U}{\partial x} + \frac{\partial}{\partial y} [(1+Ky)V] = 0 \quad (4.1.3)$$

In the present work it has been assumed that the radius of curvature of the surface is of the same order of magnitude as the boundary-layer thickness, i.e. $1/K \equiv R = O(\delta)$.

Using this assumption together with boundary-layer approximations and retaining all terms of the order of unity and higher, eq. (4.1.1) and eq. (4.1.2) reduce to the following form:

$$U \frac{\partial U}{\partial x} + (1 + Ky) V \frac{\partial U}{\partial y} + KUV = - \frac{1}{\rho} \frac{\partial P}{\partial x} + v \left[(1+Ky) \frac{\partial^2 U}{\partial y^2} + K \frac{\partial U}{\partial y} \right]$$

$$- \frac{K^2}{(1+Ky)} U] \quad (4.1.4)$$

$$KU^2 = \frac{(1+Ky)}{\rho} \frac{\partial P}{\partial y} \quad (4.1.5)$$

If $K = 0$, these equations further reduce to the familiar Prandtl boundary-layer equations.

The shear stress in laminar flow along a curved path is given by the equation:

$$\tau = \mu \left[\frac{\partial U}{\partial y} - \frac{U}{R+y} \right] \quad (4.1.6)$$

Use of eq. (4.1.6) simplifies eq. (4.1.4) to give

$$U \frac{\partial U}{\partial x} + (1+Ky) V \frac{\partial U}{\partial y} + KUV = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \left[(1+Ky) \frac{\partial \tau}{\partial y} + 2K\tau \right] \quad (4.1.7)$$

Equations (4.1.5), (4.1.6) and (4.1.7) have been solved simultaneously with eq. (4.1.3) to predict properties of laminar boundary layer flows on curved surfaces. From this set of equations it is readily seen that momentum transfer in the stream-wise direction is coupled with the momentum transfer in the normal direction through the pressure. The momentum equation in the normal direction i.e. eq. (4.1.5) represents the equilibrium between the centrifugal forces due to the curvature and the pressure forces in the normal direction.

A similar set of equations, with effective shear stress replacing the laminar shear stress, has been solved for the prediction of turbulent flow on curved surfaces. The turbulent form of eq. (4.1.7) may be written as

$$U \frac{\partial U}{\partial x} + (1+Ky) V \frac{\partial U}{\partial y} + KUV = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \left[(1+Ky) \frac{\partial \tau_{\text{eff}}}{\partial y} + 2K \tau_{\text{eff}} \right]$$

where $\tau_{\text{eff}} = \tau - \rho \overline{uv}$ (4.1.8)

The right-hand side of eq. (4.1.8) reveals that the diffusion of longitudinal momentum is affected by curvature. Positive curvature, i.e., convex surface, augments diffusional flux whereas negative curvature, i.e., concave surface, suppresses it.

With the object of predicting the influence of longitudinal curvature on the impervious wall-effectiveness, following equation for conservation of species was solved as well:

$$U \frac{\partial c}{\partial x} + (1+Ky)V \frac{\partial c}{\partial y} + K_C V = \frac{1}{\rho} \left[(1+Ky) \frac{\partial}{\partial y} J_{\text{eff}} + 2K J_{\text{eff}} \right]$$

where $J_{\text{eff}} = J - \rho \overline{c'v}$ (4.1.9)

The boundary conditions used for the solution of the momentum and species conservation equations were as follows:

$$\begin{aligned} U &= 0; \quad \frac{\partial c}{\partial y} = 0 \quad \text{at } y = 0 \\ U &= U_G(x); \quad c = c_G \quad \text{as } y \rightarrow \infty \end{aligned}$$

4.1.2 Physical inputs.

For laminar flow, the equations presented in the previous section were solved with no additional information as the laminar transport properties are well known and are unaffected by curvature. For turbulent flow, a physical hypothesis was necessary, such as used in ref. (67), to calculate the turbulent diffusional fluxes in terms of mean flow properties.

Sawyer (68), in his investigation on curved wall jets, employed the following hypothesis for the effective shear stress:

$$\tau_{\text{eff}} = \mu_{\text{eff}} \left\{ \frac{\partial U}{\partial y} - A_c \frac{U}{R+y} \right\} \quad (4.1.10)$$

where μ_{eff} is given by the usual Prandtl's mixing length hypothesis i.e. $\mu_{eff} = \rho l^2 \left| \frac{\partial U}{\partial y} \right|$

Equation (4.1.10) has been used in the present predictions. This equation has the theoretical basis of Rayleigh's stability criterion (69) and was first suggested by Kassner (70). The experiments of Eskinazi and Yeh (71), in a curved channel, provide justification for its use. It is possible to use more complex physical hypotheses in place of eq. (4.1.10) however, since there is only a small amount of experimental data that include turbulence measurements, more sophisticated hypotheses cannot be justified at the present time.

In the present predictions the following length distribution (71) was used:

$$l = .435 y \quad 0 < y \leq \frac{.09}{.435} y_G$$

$$l = .09 y_G \quad y > \frac{.09}{.435} y_G$$

This distribution of length scale has been shown (9) to be representative of a large number of boundary layer flows. A value of 9 was used for the constant A_c appearing in equation (4.1.10). Various values of A_c have been used by others, e.g., refs. (68), (73), for different flow situations. The value of 9 has been found (72) to allow good predictions for the present wall jet flows over a wide range of radius of curvature and boundary layer thickness. Computational experiments showed that the influence of A_c was insignificant in the range from 7 to 11.

The diffusional flux of species was calculated using the following relation:

$$J_{eff} = \Gamma_{eff} \frac{\partial c}{\partial y} \quad (4.1.11)$$

$$\text{where } \Gamma_{eff} = \frac{\mu_{eff}}{\sigma_{eff}}$$

A value of .86 was used for σ_{eff} and was taken from ref. (4).

4.1.3 Solution Procedure.

Since the partial differential equations to be solved for laminar as well as turbulent flow are parabolic in form, the procedure of Spalding and Patankar (9) was used to obtain their solution. The method of ref. (9) is a forward-marching finite-difference scheme of the implicit type and is relatively free from restrictions on the step length. This method has been extensively used for different boundary-layer flows and has been found to give accurate solutions at low cost. Another advantage of this procedure is that it is a very general procedure and any physical hypothesis can be used without changing the framework of the method.

The conservation equations (4.1.3, 4.1.8 and 4.1.9) are cast into the following general form using the von Mises transformation:

$$\frac{\partial \phi}{\partial x} \bigg|_w + (a_1 + b_1 w) \frac{\partial \phi}{\partial w} \bigg|_x = \frac{\partial}{\partial w} \left(c_1 \frac{\partial \phi}{\partial w} \right) \bigg|_x + d_1 \quad (4.1.12)$$

where ϕ represents U or c . This differential equation is transformed to a set of finite difference linear algebraic equations which are solved to obtain the solution at a downstream location. Initial conditions are either chosen from

experimental data or are approximated. Special procedures are used near the wall to obviate the need of having a large number of grid lines to cover the region of steep gradients.

The following specific changes were made to the original computer program of ref. (9) to obtain the solution of the present problem:

Normal Pressure Gradient. The solution of equation (4.1.5) for conservation of normal momentum in conjunction with that for longitudinal momentum results in a value of $\partial P / \partial x$ which varies in the normal direction. $\frac{\partial P}{\partial x}$ was calculated as a function of y from eq. (4.1.5) according to the equation:

$$\left. \frac{\partial P}{\partial x} \right|_y = \left. \frac{\partial P}{\partial x} \right|_G + \frac{\partial}{\partial x} \int_G^y \frac{\rho K}{(1+Ky)} U^2 dy \quad (4.1.13)$$

The integral term appearing in the above equation was evaluated from the previous two velocity profiles and was used for the forward step. The first step was computed with $\partial P / \partial x$ uniform across the boundary layer and equal to the value at the outer edge of the layer. In order to evaluate the integral appearing in eq. (4.1.13), $U/(R+y)$ was assumed to vary linearly with y ; this practice is consistent with the assumptions made within the numerical scheme of ref. (9).

Forward step length. As a result of the coordinate system, lines normal to the surface diverge or converge according to whether the surface is convex or concave. Therefore the interval between any two normal lines to the surface is a function of normal distance, y . To take account of this variation, a one-dimensional array was used for Δx i.e. a separate value of Δx was used for each grid point. One of the implications of this variation can be seen through the

momentum integral equation (72) which reveals that wall shear stress decreases for a convex surface and increases for a concave surface.

The procedure for calculating the pressure gradient from the upstream stations required careful adjustment of the forward step. For the first 50 steps Δx at the wall was chosen to be 0.1% of the boundary layer thickness and this was gradually increased to a maximum of 1% of the boundary layer thickness.

Wall Functions. The first grid line of the finite-difference mesh was located within the viscous sub layer and the shear stress was evaluated on the basis of a viscous wall law. This was found necessary as near the wall $\frac{\partial P}{\partial y}$ can be considered negligibly small e.g. see eq. (4.1.5) and hence $\frac{\partial P}{\partial x}$ can be assumed constant. In the sublayer, convection of longitudinal momentum is very small and the following equation was assumed to hold:

$$\frac{\partial \tau}{\partial y} = \frac{dP}{dx}$$

$$\text{or } \frac{\partial}{\partial y} \mu \left[\frac{\partial U}{\partial y} - \frac{U}{R+y} \right] = \frac{dP}{dx} \quad (4.1.14)$$

Equation (4.1.14) was integrated to yield the shear stress coefficient

$$S = \frac{1}{R_E} - \frac{F}{2} - \frac{\overline{\Psi_I}}{R_R} \quad (4.1.15)$$

where $R_R = (\rho U)_{2.5} R / \mu$

It is clear from eq. (4.1.15) that the effect of positive R i.e. convex surface is to reduce wall shear stress and the effect of negative R i.e. concave surface is to increase the wall shear stress.

4.1.4 Comparison of predictions with the available data.

The results of turbulent wall-jet calculations on surfaces of constant radius of curvature and stagnant surroundings are shown in Figs. 4.1.2 and 4.1.3 together with the experimental data of Guitton (74) and Newman (75). Fig. 4.1.2 compares predicted surface pressure, decay of the maximum velocity and growth rate of a wall jet growing along the inside of a cylinder of radius 20.5 in with the measurements of ref. (74). A similar comparison is shown in Fig. 4.1.3 for a wall jet growing along the outside of a cylinder of radius 2.55 in (75). There is reasonable agreement between the predictions and the experiments except for the pressure coefficient on the concave surface, where the experiments show large scatter. Fig. 4.1.4 shows the results of laminar wall-jet calculations corresponding to the geometry of Newman (75), i.e., the radius of curvature used for the convex surface was 2.55 in and negative of this value was used to obtain the results for concave surface. The slot height was taken as $0.282 R$ and calculations presented are for the slot Reynolds no. of 2000. There are no laminar flow data with which the results shown in Fig. 4.1.4 can be compared. However, the figure demonstrates that the effect of longitudinal curvature is much less in laminar flow than in turbulent flow.

Fig. 4.1.5 shows computed values of decay of maximum velocity and surface pressure of a turbulent wall jet on logarithmic spiral surfaces and are compared with the data of ref. (73). Agreement is excellent both for convex and concave surfaces. A comparison of growth rates and velocity

profiles of wall jets on logarithmic spiral convex and concave surfaces with those on a plane surface is shown on Fig. 4.1.6. As expected, these results show that the jet on a convex surface grows more than that on a flat surface and the opposite is true for a concave surface. The velocity profiles possess maxima very close to the wall as a result of the mixing length hypothesis; this was previously observed by Kacker et al. (4) . The velocity profile for the concave surface is fuller near the wall than that for the convex surface. Fig. 4.1.7 shows the wall values of shear stress and effectiveness computed for wall jets on the logarithmic spiral surfaces and on a flat surface. The convex surface results in higher effectiveness than the flat surface and, in contrast, the concave surface results in decrease in effectiveness. These results can be related to the shear stress variations on the two surfaces. There are no measurements available to derive the influence of curvature on species or enthalpy flux law and hence, in obtaining Fig. 4.1.7, a simple flux law has been used for the species equation i.e., eq. (4.1.11). Nevertheless, it is expected that the general trends of Fig. 4.1.7 are correct. Trends shown for the wall-effectiveness are of particular relevance to modern turbine blades where the leading edge has a small radius of curvature. The results shown on Fig. 4.1.7, ^{rather surprisingly} imply that the surface curvature at the leading edge and on the suction surface will aid the performance of film-cooling where as on the pressure surface there will be an adverse effect of the surface curvature on the cooling performance. *Experimental confirmation of this result is required.*

All the predictions presented so far, have been for

wall jets in zero free stream. The measurements of Kind (76) were obtained downstream of a wall jet, in the presence of a moving free stream. As can be seen from Fig. 4.1.8 the free stream was subjected to a favorable, and then to an adverse pressure gradient. With this free stream pressure gradient as an input to the solution procedure, surface pressure, growth rate, velocity maximum decay and the wall shear stress have been predicted and are shown in Fig. 4.1.8. These quantities compare very well with the measurements, although the radius of curvature is particularly small, i.e., 0.563 in. Agreement between the calculated and the measured velocity profiles is also satisfactory as seen in Fig. 4.1.9. Adiabatic-wall effectiveness shows a similar trend, in this case, to the wall jet in a stagnant free stream, although the magnitude of effect is reduced as shown in Fig. 4.1.9.

The results shown in this section demonstrate the reliability of the present solution procedure and the physical hypothesis only for the wall jet flows. In an effort to assess the viability of the various assumptions for a non-wall jet type flow, an attempt was made to predict the data of Schubauer and Klebanoff (77). These data were obtained on a surface of 31 ft. radius of curvature and the flow was subjected to adverse pressure gradients. The use of present curvature corrections resulted in significant improvements in the predicted values of δ_2 , h_{12} and cf but the predictions were not yet in line with the measurements as can be seen in Table 4.1.2. The data of ref. 77 must be regarded with great caution as some three dimensionality is revealed in the data. Some further checks are required before the present

model can be called reliable for non-wall jet flows.

4.2 Effectiveness downstream of two-dimensional slots.

This section presents the predictions of impervious wall-effectiveness downstream of two-dimensional thin-lip slots, obtained with a three-equation model of turbulence. This turbulence model requires the solution of partial differential equations for conservations of turbulent shear stress, kinetic energy of turbulence and dissipation of the turbulent kinetic energy. The present section also describes a general solution procedure which solves the appropriate differential equations for flow downstream of the two-dimensional slots which have regions of recirculating flow as well as boundary layer flow. The reasons leading to the work presented in this section are as follows:

It is a known experimental fact that in a wall-jet flow, the turbulent shear stress has a finite value at the point where the mean velocity gradient is zero and vice versa. This implies that, to represent this flow correctly, the physical hypothesis should allow the turbulent viscosity to change sign and even go to infinity. This is not realised in the eddy viscosity variety of hypotheses where the turbulent shear stress is made proportional to the mean velocity gradient. The Prandtl's mixing length hypothesis is of this nature and, in addition, makes the turbulent viscosity proportional to the mean velocity gradient. These shortcomings of the mixing length hypothesis can be overcome by solving a differential equation for the turbulent shear stress, simultaneously with the longitudinal momentum equation.

This has been realised in the present model and, to permit the required closure, additional equations for the turbulent kinetic energy and dissipation of the turbulent kinetic energy have been solved. This turbulence model, indeed, will represent the details of a wall-jet flow more precisely than the mixing length hypothesis. However, the objective here is to assess the merits of this model, in predicting the wall-effectiveness, as compared with the Prandtl's mixing length hypothesis.

The general flow configuration of a two-dimensional film-cooling slot is shown on Fig. 4.2.1 where all the dimensions shown may be regarded as variables. There are two particular features of the flow downstream of the slot exit, regions of separated flow exist behind the lip of the slot and the backward-facing step. These separated flow regions are different in character, but both imply that diffusion is equally important in x and y directions. Thus in this region the two-dimensional form of the complete Navier-Stokes equations must be solved. These equations are elliptic in nature. Downstream of the near-slot region, recirculations and normal pressure gradients as a result of separation at the slot exit, are absent and the equations representing the flow can be simplified further to the two-dimensional boundary-layer form. Therefore in the presence of parameters like thicklip and backward facing step, the solution procedure should solve the appropriate conservation equations in the two regions.

4.2.1 Relevant equations in the two regions.

In the upstream region, equations were solved for stream function, vorticity, kinetic energy of turbulence and species concentration. These equations, with the turbulent viscosity hypothesis for diffusion terms may be written as indicated below:

$$\frac{\partial}{\partial x} \left(\frac{1}{\rho} \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{1}{\rho} \frac{\partial \psi}{\partial y} \right) + \bar{\omega} = 0 \quad (4.2.1)$$

$$\begin{aligned} \frac{\partial}{\partial x} \left(\bar{\omega} \frac{\partial \psi}{\partial y} \right) - \frac{\partial}{\partial y} \left(\bar{\omega} \frac{\partial \psi}{\partial x} \right) - \frac{\partial^2}{\partial x^2} (\mu_t \bar{\omega}) \\ - \frac{\partial^2}{\partial^2 y} (\mu_t \bar{\omega}) - \frac{\partial}{\partial x} \left(\frac{U^2 + V^2}{2} \right) \frac{\partial \rho}{\partial y} + \frac{\partial}{\partial y} \left(\frac{U^2 + V^2}{2} \right) \frac{\partial \rho}{\partial x} = 0 \end{aligned} \quad (4.2.2)$$

$$\begin{aligned} \frac{\partial}{\partial x} \left(k \frac{\partial \psi}{\partial y} \right) - \frac{\partial}{\partial y} \left(k \frac{\partial \psi}{\partial x} \right) - \frac{\partial}{\partial x} \left(\Gamma_{k,t} \frac{\partial k}{\partial x} \right) - \frac{\partial}{\partial y} \left(\Gamma_{k,t} \frac{\partial k}{\partial y} \right) \\ - 2 \mu_t \left\{ \left(\frac{\partial U}{\partial x} \right)^2 + \left(\frac{\partial V}{\partial y} \right)^2 \right\} - \mu_t \left[\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right]^2 + \frac{\rho k^{3/2}}{l} C_D = 0 \end{aligned} \quad (4.2.3)$$

$$\begin{aligned} \frac{\partial}{\partial x} \left(c \frac{\partial \psi}{\partial y} \right) - \frac{\partial}{\partial y} \left(c \frac{\partial \psi}{\partial x} \right) - \frac{\partial}{\partial x} \left(\Gamma_{c,t} \frac{\partial c}{\partial x} \right) \\ - \frac{\partial}{\partial y} \left(\Gamma_{c,t} \frac{\partial c}{\partial y} \right) = 0 \end{aligned} \quad (4.2.4)$$

and the appropriate boundary conditions as:

$$\frac{\partial \psi}{\partial y} = U_G \quad \text{as } y \rightarrow \infty$$

$$\bar{\omega} = c = 0 \quad \text{as } y \rightarrow \infty$$

$$k = \text{Free stream turbulence as } y \rightarrow \infty$$

$$\psi = \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = 0$$

$$\frac{\partial c}{\partial y} = 0 \quad \text{at } y = 0$$

$$k = \text{a prescribed false value at } y = 0$$

The downstream boundary condition was assumed to be

$$\frac{\partial \psi}{\partial x} = \frac{\partial \bar{\omega}}{\partial x} = \frac{\partial c}{\partial x} = \frac{\partial k}{\partial x} = 0$$

Equation (4.2.1) for stream function satisfies mass continuity and eq. (4.2.2) for vorticity has been obtained from the two momentum equations by eliminating the pressure

terms. The conservation equation for turbulent kinetic energy allowed the predictions to be influenced by the free stream and the slot turbulence intensity. The species equation (4.2.4) was solved because comparisons were to be made with mass concentration measurements.

In the downstream region, equations were solved for conservation of longitudinal momentum, turbulent shear stress, kinetic energy of turbulence, dissipation of turbulent-kinetic energy and species concentration. These equations are written in the following way:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} (-\overline{uv}) - \frac{1}{\rho} \frac{\partial \bar{P}}{\partial x} \quad (4.2.5)$$

$$U \frac{\partial \overline{uv}}{\partial x} + V \frac{\partial \overline{uv}}{\partial y} = \frac{\partial}{\partial y} (C_s \frac{k^2}{\epsilon \sigma_s} \frac{\partial \overline{uv}}{\partial y}) - C_{s2} (C_s k \frac{\partial U}{\partial y} + \frac{\overline{uv}}{k} \epsilon) \quad (4.2.6)$$

$$U \frac{\partial k}{\partial x} + V \frac{\partial k}{\partial y} = \frac{\partial}{\partial y} (C_s \frac{k^2}{\epsilon \sigma_k} \frac{\partial k}{\partial y}) - \overline{uv} \frac{\partial U}{\partial y} - \epsilon \quad (4.2.7)$$

$$U \frac{\partial \epsilon}{\partial x} + V \frac{\partial \epsilon}{\partial y} = \frac{\partial}{\partial y} (C_s \frac{k^3}{\epsilon \sigma_\epsilon} \frac{\partial \epsilon}{\partial y}) - C_{\epsilon 1} \frac{\epsilon}{k} \overline{uv} \frac{\partial U}{\partial y} - C_{\epsilon 2} \frac{\epsilon^2}{k} \quad (4.2.8)$$

$$U \frac{\partial c}{\partial x} + V \frac{\partial c}{\partial y} = \frac{\partial}{\partial y} (C_s \frac{k^2}{\epsilon \sigma_c} \frac{\partial c}{\partial y}) \quad (4.2.9)$$

Boundary conditions to the above equations are:

$$U_G \frac{dU_G}{dx} = - \frac{1}{\rho} \frac{d\bar{P}}{dx}$$

$$\overline{uv} = 0$$

$$U_G \frac{dk}{dx} = - \epsilon_G$$

$$U_G \frac{d\epsilon_G}{dx} = - C_{\epsilon 2} \frac{\epsilon_G^2}{k_G} \text{ as } y \rightarrow \infty$$

For the wall boundary when $y = 0$

$$U = 0$$

The mean momentum equation combined with a logarithmic velocity profile served as a basis for the specification of shear stress.

For the dissipation degenerate form of kinetic energy equation, in which convection and diffusion terms were neglected, was used.

Two additional equations, namely the equations for Reynolds stress and dissipation of kinetic energy, have been solved in the downstream region of boundary layer flow. The reason for this addition is clearly indicated in the following sub-section.

4.2.2 Physical Inputs.

The two sets of equations given in the previous section contain assumptions in the form of diffusion and dissipation terms. Diffusion of all the turbulent quantities and vorticity was assumed to have a gradient form and the laminar component was neglected. Dissipation of turbulent kinetic energy was assumed to take place at high wave numbers and has a form appropriate to isotropic turbulence, e.g. see eq. (4.2.3). In the downstream region this assumption was removed by solving a conservation equation for dissipation.

Turbulent viscosity used in the gradient type assumption of diffusion terms was defined by the Prandtl-Kolmogorov hypothesis, which states:

$$-\overline{uv} = \mu_t \frac{\partial U}{\partial y} \quad (4.2.10)$$

$$= \rho k^{\frac{1}{2}} l \frac{\partial U}{\partial y} \quad \text{as used in the upstream region}$$

$$= \rho C_s \frac{k^2}{\epsilon} \frac{\partial U}{\partial y} \quad \text{as used in the downstream region}$$

Diffusive fluxes of all quantities other than momentum were represented by way of equations like eq. (4.1.11) and different Prandtl or Schmidt numbers were used for each.

The set of equations (4.2.1) to (4.2.4) has been solved previously by Wolfshtein (78) and Runchal (79) and so the values of the constants appearing in these equations have been taken from these references. Similarly the constants appearing in the set of equations (4.2.5) to (4.2.9) have been taken from Hanjalic (80). Details about these constants can be found in ref. (81).

4.2.3 Solution procedure.

The set of elliptic equations, i.e., eq. (4.2.1) to (4.2.4) was solved by the procedure of ref. (10) which transforms the differential equations into a set of finite-difference equations that are solved from an initial guess to the converged solution. For this procedure, a control volume embodying a 26 x 26 mesh was defined in the region from the slot exit to 50 slot heights downstream. The upstream boundary conditions were taken from experiments and the downstream boundary conditions corresponded to zero longitudinal gradients for all variables.

The set of parabolic equations, i.e., eq. (4.2.5) to (4.2.9), was solved by the forward-marching finite-difference procedure of ref. (9) which is briefly described in section 4.1.3. The program solving the parabolic equations was joined with that solving the elliptic equations and the initial conditions for the parabolic grid were taken from the converged solution of the elliptic grid. The two

grids were matched at a downstream distance of 10 slot heights from the slot exit for the cases where slot lip was thin and 20 slot heights for the cases where slot lip was thick or height of backward facing step was non zero.

The elliptic equations were solved using a turbulent viscosity hypothesis and hence it was expected that in the upstream region profiles would not be predicted exactly. Since these profiles were used as the initial conditions for the downstream region, errors in these profiles could result in errors in the solution of parabolic equations. With the purpose of finding the sensitivity of the parabolic solution on the initial conditions, effect of changes in initial profiles on the predicted effectiveness was investigated. The remainder of this section presents the results of this investigation and indicates the precision with which the initial profiles should be provided to the parabolic program. The influence of initial profiles of mass concentration, turbulent kinetic energy, turbulent shear stress and dissipation of kinetic energy was investigated for three velocity ratios for density ratio of unity and two velocity ratios for density ratio of 4.17, spanning the practical range. This investigation was done for a geometry having a thin lip and no backward facing step.

Effect of mass concentration profiles. The mass concentration profile was changed in shape, e.g., see Fig. 4.2.2, so as to conserve the species flux. The effect of this change on the predicted values of effectiveness is presented in Table 4.2.1 which demonstrates that a shift in the concentration profile to provide more species away from the

wall results in a drop in effectiveness. Far downstream, the predicted values of effectiveness are similar for both initial conditions but in the range up to 50 slot heights, the imposed differences in the initial concentration profile caused practically significant differences in effectiveness.

Effect of turbulent kinetic energy profiles. The initial profiles generated by the elliptic procedure at $x/y_G = 10$ are shown on Fig. 4.2.3 for three velocity ratios with air injection and two velocity ratios with Arcton-12 injection. The augmentation of these profiles by 20 per cent, causes differences in the predicted effectiveness which were, in all cases, less than 1 per cent at 100 slot heights and less than 2% anywhere else upstream.

Effect of turbulent shear stress profiles. Turbulent shear stress profiles were calculated using eq. (4.2.10), with the kinetic energy and the velocity profiles as given by the elliptic program. These profiles are shown in Fig. 4.2.4 and were also augmented by 20% across the boundary layer thickness. The maximum resulting difference in effectiveness was 2.5% which diminished with downstream distance.

Effect of kinetic energy-dissipation profiles. Initial profiles for dissipation were obtained using the following equation:

$$\epsilon = C_D k^{3/2}/l \quad \text{with} \quad l = A_K y; \quad 0 < y \leq \frac{y_G \lambda}{A_K}$$

$$= \lambda y_G; \quad y > \frac{y_G \lambda}{A_K} \quad (4.2.11)$$

and $\lambda = .09$, $A_K = .42$

These profiles are shown in Fig. 4.2.5 and were augmented by 20% in a similar way to the shear stress profiles. No significant influence on effectiveness could be detected for

any of the cases.

The effect of initial profiles for longitudinal velocity was not investigated because it was not possible to change the velocity profile without disturbing the conservation of mass, species and momentum.

4.2.4 Comparison of predictions with the available data.

Fig. 4.2.6 compares present predictions with the measurements of Pai and Whitelaw (18) for the particular case of zero density gradient, thin lip and no backward facing step. The maximum deviation between the predicted and measured values is 10% of unity and this for the lowest velocity ratio. Comparison with the zero density gradient, thick lip measurements of Kackar and Whitelaw (1) is shown in Fig. 4.2.7. The agreement between the data and the predictions is excellent and the maximum discrepancy is 6% of unity in this case. Fig. 4.2.8 shows the comparison of the measurements of Matthews and Whitelaw (24) for zero density gradients and the present predictions. Here the slot geometry has been complicated by the presence of a backward facing step. Predictions are shown for a large value of the step height with a thin as well as a thick lip. This was done because the measurements of ref. (24) indicated that the step had a significant influence only for large step heights as compared to the slot gap y_c . As indicated by Fig. 4.2.8, the predictions are in very good agreement with experiments. Fig. 4.2.9 presents predictions made for the density ratio of 4.17 corresponding to the measurements of ref. (18). The agreement indicated on this figure is less satisfactory and

this was attributed largely to the initial conditions used for the parabolic equations and provided by the solution of the elliptic equations.

The above results show clearly that the present solution procedure can adequately predict the effectiveness downstream of a range of two-dimensional slot geometries. Further improvements can be made by the removal of the algebraic form of the length scale used in the solution of the elliptic equations. Indeed the best way may be to remove the Prandtl-Kolmogorov hypothesis itself, and solve the equations for the shear stresses and dissipation in the upstream region in a manner parallel to the downstream region. This would certainly improve the velocity and concentration profiles provided by the elliptic solution procedure and the larger discrepancies in the predictions of non-uniform density cases are likely to be removed.

4.2.5 Comparison with the previous predictions.

Data from ref. (18) has been previously predicted by Pai (6) using the mixing length theory. As mentioned earlier the turbulent viscosity type of hypotheses are not entirely suitable for wall-jet flows as they lead to coincidence of zero shear stress points with the points of maximum and minimum velocity. For this reason mainly, in the present predictions an equation was solved for the turbulent shear stress. The present predictions are compared with the predictions of ref. (6) in this section, so as to evaluate the advantages and disadvantages of using the present model over the simple mixing length hypothesis.

Pai (6), in his predictions, also used the solution procedure of Spalding and Patankar (9) and had taken initial profiles of velocity and concentrations from his own measurements. To compare his effectiveness predictions quantitatively with measurements, he calculated maximum deviation $D_{\max}$ and a statistical quantity nebula defined by:

$$\kappa_{L_1}^2 = \frac{1}{L_1} \int_0^{L_1} (\eta_{\text{prd}} - \eta_{\text{exp}})^2 dx \quad (4.2.12)$$

$D_{\max}$ and κ_{100} of the present predictions are compared with the corresponding values of Pai (6) in Table (4.2.2). The results shown in Table (4.2.2) reveal that the present solution procedure and the turbulence model allows predictions of impervious-wall effectiveness which are similar in precision to those previously obtained with the mixing length hypothesis.

It is not surprising that the three-equation turbulence model has not shown significant improvements over the mixing length hypothesis, in predicting the wall-effectiveness. Although the solution of the turbulent shear stress equation takes account of the non coincidence of locations of zero shear-stress and zero velocity gradient, the accuracy of the predictions will depend upon the empirical constants being used and the accuracy of the initial conditions. The present calculations have demonstrated the precision with which the initial conditions should be supplied to the parabolic program to allow accurate predictions in the downstream region. It is not likely that the one-equation turbulence model used for the elliptic equations in the present work, will give sufficiently accurate

initial conditions to the parabolic program. Therefore the physical hypothesis should be improved in the upstream region to result in better initial conditions for the solution procedure in the downstream region and then it is hoped that the present three-equation model will result in more precise predictions of wall effectiveness than the Prandtl's mixing length hypothesis.

4.3 Conclusions.

1. A modified version of the Spalding and Patankar solution procedure (9) for two-dimensional boundary layers, has been used to predict wall-jet properties along curved surfaces. The procedure incorporates a simple mixing length assumption. The predicted aerodynamic properties, such as rates of growth, maximum velocity decay and surface pressure agree satisfactorily with the available measurements.
2. A three-equation model of turbulence resulted in predictions of impervious wall-effectiveness in the downstream region of two-dimensional thin-lip slots, which were similar in precision to those obtained with the simple mixing length hypothesis. The author believes that the three-equation model is capable of more precise predictions only if the initial conditions can be supplied with greater accuracy than in the present computations.
3. The influence of longitudinal curvature on impervious-wall effectiveness is shown to be small for concave and large for convex surfaces, especially in the absence of a free stream. Effectiveness is shown to increase on a convex surface and decrease on a concave surface as compared to a

flat surface.

4. The combination of the solution procedure for the elliptic equations and the procedure for the parabolic equations has provided a single solution procedure for the upstream and downstream regions of two-dimensional film-cooling slots. The procedure is capable of accurately predicting the influence of lip thickness and the backward facing step height on the effectiveness over a wide range of values of these parameters.

CHAPTER 5

Prediction of three-dimensional film-cooling.

The present chapter is concerned with the prediction of effectiveness and heat-transfer coefficient downstream of three-dimensional film-cooling slots. The purposes are to present the values of wall effectiveness and heat-transfer coefficient, predicted with the aid of the computational method of Patankar and Spalding (8); to compare these predictions with existing measurements and thereby to assess the validity of the procedure; and to present the predicted values of effectiveness and heat-transfer coefficient outside the range of existing measurements for a range of practically relevant parameters.

The range of slot configurations considered is particularly relevant to gas-turbine practice. They consist of discrete holes, located parallel to the surface to be cooled (Fig. 5.2). The present chapter considers the influence of velocity ratio, density ratio and geometrical parameters such as pitch to diameter ratio, lip length and lip thickness to diameter ratio, and open area on effectiveness and heat-transfer coefficient. Exhaustive comparisons of these predictions are made with the measurements described in section 3.2; these were devised partly to allow prediction procedures, such as that of ref. (8), and the associated physical hypothesis to be evaluated. Comparison is also reported with the previous measurements of ref. (14).

As discussed in section (2.4), the general calculation procedure of ref. (8) is more suited than other

available methods to solving three-dimensional boundary layer equations relevant to practical film-cooling. Major modifications were introduced in the basic procedure of ref. (8) to allow the present predictions. These modifications and the salient features of the procedure are reported in section (5.1) together with a statement of conservation equations and boundary conditions. Section (5.1) also describes the physical hypothesis used and indicates reasons for its selection. The predictions and their comparison with the available data are reported in the first two sub-sections of section (5.2). The last sub-section presents predicted values of effectiveness and heat-transfer coefficient for a range of parameters of interest to designers and for which measurements are not presently available. Section (5.3) states the main conclusions.

5.1 Mathematical problem and the solution procedure.

5.1.1 Relevant equations of motion.

Differential equations of motion for three-dimensional boundary layer flows can be written in the following manner in cartesian coordinates:

$$\frac{\partial}{\partial x} (\rho U) + \frac{\partial}{\partial y} (\rho V) + \frac{\partial}{\partial z} (\rho W) = 0 \quad (5.1.1)$$

$$\frac{\partial}{\partial x} (\rho U^2) + \frac{\partial}{\partial y} (\rho UV) + \frac{\partial}{\partial z} (\rho UW) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y} (\tau_{u,xz}) + \frac{\partial}{\partial z} (\tau_{u,xy}) \quad (5.1.2)$$

$$\frac{\partial}{\partial x} (\rho UV) + \frac{\partial}{\partial y} (\rho V^2) + \frac{\partial}{\partial z} (\rho VW) = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial y} (\tau_{v,xz}) + \frac{\partial}{\partial z} (\tau_{v,xy}) \quad (5.1.3)$$

$$\frac{\partial}{\partial x}(\rho UW) + \frac{\partial}{\partial y}(\rho VW) + \frac{\partial}{\partial x}(\rho W^2) = -\frac{\partial P}{\partial z} + \frac{\partial}{\partial y}(\tau_{w,xz}) + \frac{\partial}{\partial z}(\tau_{w,xy}) \quad (5.1.4)$$

$$\frac{\partial}{\partial x}(\rho U\Phi) + \frac{\partial}{\partial y}(\rho V\Phi) + \frac{\partial}{\partial z}(\rho W\Phi) = S\Phi - \frac{\partial}{\partial y}(J_{\Phi,xz}) - \frac{\partial}{\partial z}(J_{\Phi,xy}) \quad (5.1.5)$$

where Φ stands for species concentration or stagnation enthalpy and the double suffix in the shear stress represents the plane on which the shear stress acts. In the above equations shear stresses and diffusion fluxes acting on yz plane have been omitted. This omission ensures that no influence from downstream can penetrate upstream in accord with the boundary layer approximations. Body forces have been neglected in the above equations. It should be noted that $\bar{P}$, in the equation for longitudinal momentum, is an average pressure over a cross-section and is given by the free-stream conditions. For the present flow situation $\frac{\partial \bar{P}}{\partial x}$ is zero. P appearing in equations for cross-stream momentum is calculated differently as will be explained in the next section.

Auxiliary Equations. The differential equations do not alone specify the problem; additional information is required in the form of boundary conditions and auxiliary equations that allow the calculation of shear stresses and diffusion fluxes in terms of dependent and independent variables. The boundary layer approximations lead to the following expressions for the shear stresses:

$$\tau_{u,xz} = \mu_{\text{eff}} \frac{\partial U}{\partial y}$$

$$\tau_{u,xy} = \mu_{\text{eff}} \partial U / \partial z$$

$$\begin{aligned}
\tau_{v,xz} &= \mu_{\text{eff}} (2\partial V/\partial y) \\
\tau_{v,xy} &= \mu_{\text{eff}} \left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right) \\
\tau_{w,xz} &= \mu_{\text{eff}} \left(\frac{\partial W}{\partial y} + \frac{\partial V}{\partial z} \right) \\
\tau_{w,xy} &= \mu_{\text{eff}} (2\partial W/\partial z).
\end{aligned} \tag{5.1.6}$$

Diffusion fluxes are approximated by

$$\begin{aligned}
J_{\bar{\phi},xz} &= - \frac{\mu_{\text{eff}}}{\sigma_{\text{eff}}} \partial \bar{\phi} / \partial y \\
J_{\bar{\phi},xy} &= - \frac{\mu_{\text{eff}}}{\sigma_{\text{eff}}} \partial \bar{\phi} / \partial z
\end{aligned} \tag{5.1.7}$$

The set of equations (5.1.2) to (5.1.5) takes the following general form after substitution of equations (5.1.6) and (5.1.7):

$$\begin{aligned}
\frac{\partial}{\partial x}(\rho U \bar{\phi}_1) + \frac{\partial}{\partial y}(\rho V \bar{\phi}_1) + \frac{\partial}{\partial z}(\rho W \bar{\phi}_1) &= \frac{\partial}{\partial y}(J_{\bar{\phi}_1,xz} \frac{\partial \bar{\phi}_1}{\partial y}) \\
+ \frac{\partial}{\partial z}(J_{\bar{\phi}_1,xy} \frac{\partial \bar{\phi}_1}{\partial z}) + P \bar{\phi}_1 + S \bar{\phi}_1
\end{aligned} \tag{5.1.8}$$

where functions $P \bar{\phi}_1$ and $S \bar{\phi}_1$ are given in the table below:

$\bar{\phi}_1$	$P \bar{\phi}_1$	$S \bar{\phi}_1$
U	$-\partial \bar{P} / \partial x$	0
V	$-\partial P / \partial y$	$\frac{\partial}{\partial y}(\mu_{\text{eff}} \frac{\partial V}{\partial y}) + \frac{\partial}{\partial z}(\mu_{\text{eff}} \partial W / \partial y)$
W	$-\partial P / \partial z$	$\frac{\partial}{\partial y}(\mu_{\text{eff}} \partial V / \partial z) + \frac{\partial}{\partial z}(\mu_{\text{eff}} \partial W / \partial z)$
$\bar{\phi}$	0	0

Appropriate boundary conditions for equations represented by eq. (5.1.8) are:

$$\begin{aligned}
U, V, W &= 0 & \text{at } y = 0 \\
\partial\phi/\partial y &= 0, \text{ constant at } y = 0 & \text{(Wall boundary)} \\
W, \phi &= 0 & \text{as } y \rightarrow \infty \\
U &= U_G & \text{as } y \rightarrow \infty \text{ (Free boundary)} \\
\frac{\partial U}{\partial z}, \frac{\partial V}{\partial z}, W, \frac{\partial \phi}{\partial z} &= 0 & \text{at } z = 0 \text{ (Symmetry line)} \\
\frac{\partial U}{\partial z}, \frac{\partial V}{\partial z}, W, \frac{\partial \phi}{\partial z} &= 0 & \text{at } z = B/2 \text{ (Symmetry line)} \\
V, \phi &= 0 & \text{as } z \rightarrow \infty \\
U &= U_G & \text{as } z \rightarrow \infty \text{ (Free boundary)}
\end{aligned}$$

The particular form of effective viscosity used was:

$$\mu_{\text{eff}} = \rho l^2 \sqrt{\left(\frac{\partial U}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial z}\right)^2} + [0.05 (t/D)^2 \rho_w \delta_w U_w] \quad (5.1.9)$$

where

$$l = 0.42y \quad 0 < y \leq \frac{0.09}{0.42} y_G$$

$$l = 0.09y_G \quad y > \frac{0.09}{0.42} y_G$$

The solution procedure of ref. (8) does not require this particular assumption or even the effective-viscosity hypothesis. Effective viscosity hypothesis based on Prandtl's mixing length assumption was used because past experience had shown that it was simple in concept and use and gave satisfactory degree of accuracy. The second term on the right hand side of eq. (5.1.9) was added to take account of the additional mixing due to the wake behind the lip of thickness t in the slot. This practice is similar to the one used by Pai (6) for two-dimensional film cooling. The value of effective Prandtl number was assumed to be unity in the present work.

5.1.2 Description of the Solution procedure.

The equations represented by eq. (5.1.8) are parabolic and the numerical procedure solves these equations by marching forward in the x-direction, with the solution in the yz plane at each x-station. A staggered grid, shown in Fig. 5.1.1, is used in the yz plane at which the variables U, V, W, P and Φ are stored. This arrangement has the convenient feature that the cross-stream velocities V and W are stored at just the points at which they are needed for the calculation of the convective contribution to the conservation of U and Φ . The pressures are stored at the node points so as to make it easy to calculate the pressure gradients which affect V and W. The differential equations are integrated over the micro-control volume shown in Fig. 5.1.2 for each grid point and the following finite-difference equations are obtained:

$$C^V\{(\rho V)_N - (\rho V)_P\} + C^W\{(\rho W)_E - (\rho W)_P\} = C^U\{(\rho U)_{P,U} - (\rho U)_P\} \quad (5.1.10)$$

$$U_P = A_N^U U_N + A_S^U U_S + A_E^U U_E + A_W^U U_W + B^U + D^U(\partial \bar{P} / \partial x) \quad (5.1.11)$$

$$V_P = A_N^V V_N + A_S^V V_S + A_E^V V_E + A_W^V V_W + B^V + D^V(P_P - P_S) \quad (5.1.12)$$

$$W_P = A_N^W W_N + A_S^W W_S + A_E^W W_E + A_W^W W_W + B^W + D^W(P_P - P_W) \quad (5.1.13)$$

$$\Phi_P = A_N^\Phi \Phi_N + A_S^\Phi \Phi_S + A_E^\Phi \Phi_E + A_W^\Phi \Phi_W + B^\Phi \quad (5.1.14)$$

Here the A coefficients contain mass fluxes, viscosities, diffusion coefficients etc.; B coefficients express the effects of convection from the upstream station and source terms; C's are areas and D's involve areas, mass flow rates etc. Subscript U stands for upstream values and

unsubscripted variables have downstream values. For the integration over the micro control volumes it has been assumed that the dependent variables have a stepwise variation in the x-direction and linear variation in the y and z directions. The problem is to solve equations (5.1.10) to (5.1.14) simultaneously for all the U's, V's, W's, P's and Φ 's at the downstream x station. The coefficients A's, B's, C's and D's can be taken as known because they are evaluated at the upstream station.

Equation (5.1.11), for longitudinal momentum, can be solved as $(\partial \bar{P} / \partial x)$ is known from the free stream conditions. Equations for cross-stream velocities are coupled through pressure and so cannot be solved individually as the pressures are not known in advance. A preliminary set of V and W values are calculated based on an estimated pressure field P, usually the upstream value of P, from:

$$V_P^* = A_N^V V_N^* + A_S^V V_S^* + A_E^V V_E^* + A_W^V V_W^* + B^V + D^V (P_P^* - P_S^*) \quad (5.1.15)$$

$$W_P^* = A_N^W W_N^* + A_S^W W_S^* + A_E^W W_E^* + A_W^W W_W^* + B^W + D^W (P_P^* - P_W^*) \quad (5.1.16)$$

The starred values of V and W, given by equations (5.1.15) and (5.1.16), will in general not satisfy the continuity equation (5.1.10) and will produce a mass source m_p for the point P, defined by

$$m_p \equiv C^V \{ (\rho V^*)_N - (\rho V^*)_P \} + C^W \{ (\rho W^*)_E - (\rho W^*)_P \} + C^U \{ (\rho U)_P - (\rho U)_{P,U} \} \quad (5.1.17)$$

Now the starred pressure and velocity fields are corrected so as to annihilate this mass source. A pressure correction P' is defined in the following manner:

$$P = P^* + P' \quad (5.1.18)$$

Using eq. (5.1.18) with equations (5.1.15), (5.1.16), (5.1.12) and (5.1.13) we can write

$$V_P = V_P^* + D^V (P_P' - P_S') \quad (5.1.19)$$

$$W_P = W_P^* + D^W (P_P' - P_W') \quad (5.1.20)$$

The substitution of equations (5.1.19) and (5.1.20) in the continuity equation (5.1.10) gives

$$P_P' = A_N^P P_N' + A_S^P P_S' + A_E^P P_E' + A_W^P P_W' + B^P \quad (5.1.21)$$

where A's involve C's and D's and mp has been incorporated in B^P . This equation gives pressure corrections and then V's and W's are computed from equations (5.1.19) and (5.1.20). After solving for U, V and W at the downstream station, these values are used to solve for ϕ (concentration or enthalpy) using equation (5.1.14).

To solve the finite-difference equations two sweeps of the standard tri-diagonal matrix algorithm (TDMA) are employed, one in the y-direction and one in the z-direction. Thus, for equation (5.1.14), ϕ_E and ϕ_W are taken as known and equal to the upstream values when the sweep is in y direction; similarly ϕ_N and ϕ_S are held constant for the sweep in the z-direction.

Some specific modifications had to be introduced, in the above procedure, to allow the present predictions. These specific features are described in the rest of the present section.

1. The grid line distribution in the yz plane was chosen so that the mass flow from the circular boundary of the jet at the starting station could be accurately represented by the rectangular grid. For the single hole geometry, a cross stream grid of 9 x 12 node points (9 pts. in the y-direction) was used and for the multihole slots a grid of 8 x 6 node points was used. The number of node points in the y-direction was increased by 2 when the lip thickness t was more than .2D. The mass flow emerging from this numerical, non circular slot exit was ascertained to be within 2% of the specified value. It was assumed that the mixing of the jets was uninfluenced by the small deviations from roundness.

2. Initial profiles of the dependent variables U and ϕ at the slot exit were taken from experiments wherever available; otherwise appropriate profiles were assumed. Initial values of V and W were assumed to be zero. The initial profile of U was very non uniform and resulted in oscillations and, in some cases, divergence of the solution. This was overcome by iteration and under-relaxation of the U velocity, and by adding a fictitious source term to the U equation. With this modification equation (5.1.11) has the form:

$$U_P + A_I U_P = A_N^U U_N + A_S^U U_S + A_E^U U_E + A_W^U U_W + B^U + A_I U_{P,U}$$

$$\text{or } (1+A_I) U_P = U_P (\text{ordinary}) + A_I U_{P,U} \quad (5.1.22)$$

Now depending upon the value of A_I , the downstream value of U_P will be under relaxed e.g. if $A_I = 1$

$$U_P = \frac{1}{2} U_P (\text{ordinary}) + \frac{1}{2} U_{P,U}$$

A_I was given a high value to start with and was gradually reduced to zero so that no relaxation was done. Similarly, for the first few steps, 5 iterations were required but the number gradually decreased until, at a downstream distance of 20 diameters, no iterations were required.

3. As has been indicated in the boundary conditions, the left face of the control volume was always taken to be the centre line of the jet. For calculations of the flow downstream of multihole slots, the control volume was $\pi/2$ wide and the right face also conformed to a plane of symmetry. For calculations downstream of single hole slots, the jet was allowed to spread freely in the y and z directions.

4. For free boundaries, the boundary values of V and W were obtained by performing a mass balance for the forward step. It should be noted that this specification is not a unique one and alternative procedures could be used to treat free boundaries.

5. A single sweep of TDMA in each of y and z directions was found adequate for the calculation of U, V and W but 3 sweeps for pressure and five sweeps for ϕ were found necessary to obtain accurate solutions.

5.2 Comparison of the predictions with the available data.

Most of the comparisons presented in this section are with the present measurements which represent the most complete set of data for present purposes. Predictions were made for a sample of conditions within the wide range of experimental parameters and indicate the range of applicability of the prediction procedure. In a way similar

to section (3.2), predictions for the single hole geometries are presented first, followed by the predictions for the multihole geometries.

5.2.1 Single hole geometries.

Predictions were made for two slots with the hole (diameters = 8 mm and 12.5 mm) located tangential to the wall ($H = 0.5\text{mm}$), lip thickness of 1.25 mm (see Fig. 3.2.1) and no lip overhang ($L = 0$). Fig. 5.2.1 compares the predicted values of effectiveness along the centre line of 8 mm hole slot, with the present measurements for density ratios of unity and 4.17. Measurements are represented by points and lines show the predictions. Fig. 5.2.2 shows a similar comparison for the hole diameter of 12.5 mm. Measurements with constant density are predicted with good accuracy and the predictions clearly show the correct influence of velocity ratio and hole diameter. The predictions are less satisfactory for the non-uniform density case in the upstream region. For the single hole geometry, the forward recirculation in the upstream region is significant, as a result of flow over the backward facing step through which the hole is drilled. Consequently in the upstream region the boundary layer form of equations do not correctly represent the flow and the discrepancy between the predictions and measurements is not very surprising. Larger discrepancies for the density ratio of 4.17 are also implied from the mixing length hypothesis used in the present predictions. This hypothesis defines effective-viscosity as proportional to the density which results in very high mixing for the higher density ratio of

4.17 as compared to the uniform density situation, especially in the upstream region.

Figs. 5.2.1 and 5.2.2 show the accuracy to which the present procedure can predict impervious wall-effectiveness downstream of slots having a single hole. These predictions show the influence of a few parameters within the range investigated experimentally. Due to the relatively small significance of the single hole geometry in the present investigation, the influence of the other parameters was not predicted; instead effort was devoted to the prediction of flow downstream of multihole slots.

5.2.2 Multihole geometries.

Fig. 5.2.3 compares predictions with measurements obtained with multihole slots 1 and 2 shown in Fig. 3.2.89 and described in Table (3.2.2) for the density ratio of unity. It indicates the ability of the present procedure to predict the influence of velocity ratio, downstream distance and lip thickness to diameter ratio upon the impervious-wall effectiveness. The maximum discrepancy in magnitude is 8% of unity for the thick-lip case and the trends are faithfully followed.

Slot 1 was used to permit the assessment of the influence of density ratio. Measurements and predictions which demonstrate this influence are shown on Fig. 5.2.4. Fig. 5.2.4 together with Fig. 5.2.3(a) demonstrate that the present procedure predicts the influence of density ratio correctly. The maximum difference in magnitude between measurement and prediction is 16% of unity but corresponds to

a significantly smaller velocity ratio and higher density ratio than those used in practice.

Fig. 5.2.5 compares predictions and measurements based on slot 3 which was designed along with slot 1 to see the influence of pitch to diameter ratio keeping the open area unchanged. The influence is precisely predicted for the two density ratios, at which measurements were done. Slot 4 of Table (3.2.2) is geometrically similar to slot 2 but has dimensions which more clearly relate to practice. The predictions shown in Fig. 5.2.6 compare very well with the measurements of slot 4 and demonstrate that the procedure provides satisfactory answers for flow and geometrical parameters similar to those found in gas-turbine practice.

The present measurements were confined to slots with no lip over hang at the slot-exit. To assess the capability, of the present procedure, to predict the influence of a finite lip length, a sample of the measurements of ref. (14) were predicted. Predictions and measurements of adiabatic-wall effectiveness are shown in Fig. 5.2.7. In these computations, the flow was treated as confined between two walls from the exit of the slot up to the end of the lip and then the free stream was included with a free boundary. The predictions are in close agreement with the measurements and indicate that the influence of lip length can also be predicted. Fig. 5.2.7 indicates that, in the presence of a finite lip length, effectiveness increases. The influence penetrates up to 25 slot heights for velocity ratios smaller than 1 and beyond 40 slot heights for velocity ratios greater than unity. Predictions were made for two velocity

ratios representative of the data of ref. (14).

Predictions of heat-transfer coefficient downstream of slot 1 and slot 3 are shown in Fig. 3.2.1⁵ and Fig. 3.2.1⁶ along with the measured values. As discussed in Chapter 3, this heat-transfer coefficient is based on the difference of wall temperature and adiabatic-wall temperature. The adiabatic-wall temperatures were computed first, with zero heat flux at the wall and then wall temperatures were computed with a constant heat flux at the wall. The heat-transfer coefficient was calculated from the values of the constant heat flux, the wall temperature and the adiabatic-wall temperature. The predicted values are in excellent agreement with the measured values with the exception of one case of slot 1 where the velocity ratio is much smaller than used in practice. The influence of velocity ratio is faithfully predicted and the predicted values tend to the flat plate results in the downstream region for all the velocity ratios. Comparison of Fig. 3.2.1⁵ and Fig. 3.2.1⁶ shows that the present procedure is capable of predicting the influence of pitch to diameter ratio on the wall heat-transfer coefficient. Predicted values of adiabatic-wall effectiveness are also shown in these figures and are the same as the predicted impervious-wall effectiveness values. This is so because the governing equations for species and enthalpy are same and a value of unity for the effective Prandtl no. was used for both.

The comparisons shown in this section provide confidence in the ability of the present prediction method and the physical hypothesis to predict impervious-or-

adiabatic wall effectiveness and heat transfer coefficient downstream of discrete hole slots. Short computing times (55 seconds from $x/D = 0$ to $x/D = 80$) and economical memory requirements (34K) are additional attractions of this procedure. The following section presents some predictions outside the range of available measurements and provides results which will guide engineer in the design of film cooling slots.

5.2.3 Prediction of useful trends.

The predictions presented in section (5.2.2) showed the values of impervious-wall effectiveness along the centre line of a slot which provided maximum values of effectiveness at any downstream station. To assess the overall cooling affected by a slot, spanwise variation of wall effectiveness must be known. Fig. 5.2.8 has been prepared to demonstrate the three-dimensionality of wall-effectiveness downstream of single-hole and multihole slots. Contours shown in Fig. 5.2.8(b) were computed for the multihole slot 1 ($D = 12.5$ mm, $Pi/D = 2$, $t/D = .18$), density ratio of unity and velocity ratio of 1.3 and indicate that wall-effectiveness does not vary in the spanwise direction after 20 diameters downstream the slot exit.

Figs. 5.2.9 to 5.2.12 demonstrate the influence of four geometrical parameters on impervious-wall effectiveness, for which systematic measurements are not available. Wherever measurements are available, they are shown on the figures. The influence of lip thickness is shown on Fig. 5.2.9 for a velocity ratio of 1.12 and for two density ratios. As has been suggested by the previous investigations of two-

dimensional slots and present investigation of three-dimensional slots, an increase in t causes a decrease in effectiveness and the influence is greater for a density ratio of unity.

The influence of the thickness of the boundary layer on the upper surface of the slot, δ , is shown on Fig. 5.2.10 for a small value of t/D , i.e. 0.18, and for two density ratios. The influence is small but could be important in a combustion chamber where the value of δ is small at the upstream end of the chamber and, due to the build up from several films, large at the downstream end.

Fig. 5.2.11 is concerned with the influence of hole pitch to diameter ratio and shows this influence for uniform density and for a density ratio of 2.1. The figure shows the effect of increasing P_i/D ratio by increasing pitch for a constant mass flow rate through the slot corresponding to a velocity ratio of 1.12. It was found that the manner in which P_i/D is varied is unimportant but, of course, if P_i/D is increased by diminishing the value of D then the effectiveness at a particular x/D corresponds to a smaller x than would be the case if P_i was increased. In practice P_i/D should be as small as is possible with D chosen to provide a velocity ratio which corresponds to maximum in the effectiveness Vs velocity ratio curve.

Fig. 5.2.12 demonstrates the influence of lip length with x measured from the hole exit rather than from the end of the lip. Clearly the effectiveness is improved by a finite lip and, in the practical range of L/D the influence is important close to the slot exit.

The influence of velocity ratio and density ratio on the wall heat-transfer coefficient in presence of film cooling is shown on Fig. 5.2.13. Fig. 5.2.13(a) shows the predictions already presented in section (5.2.2) along with the present measurements downstream of slot 1. This representation clearly indicates the increase in the heat-transfer coefficient with increasing velocity ratio. Similar influence is indicated for the density ratio of 2.02 on Fig. 5.2.13(b). The computations for the higher density ratio were effected by assuming that the slot fluid was at a temperature lower than the main stream by about 300°K . Comparison of Fig. 5.2.13(a) and (b) reveals that for a particular velocity ratio heat-transfer coefficient decreases with the increase in density ratio and the decrease is more at downstream locations.

Figs. 5.2.14 to 5.2.17 demonstrate the influence of lip thickness, thickness of the upper boundary layer, hole pitch and the lip length respectively on the heat-transfer coefficient for the density ratios of .94 and 2.02 and the velocity ratio of 1.12. Fig. 5.2.14 indicates that an increase in t causes a decrease in the heat-transfer coefficient which is based on the difference of adiabatic-wall temperature and the wall temperature for constant wall heat flux boundary condition. The influence is equally important for the two density ratios investigated. The decrease in heat-transfer coefficient with the increase in lip thickness is related to the decrease in adiabatic wall effectiveness with the increase in lip thickness. A similar influence was experimentally observed by Pai (6) in the

upstream region of a two-dimensional axisymmetric slot.

Fig. 5.2.15 shows the influence of thickness of the upper boundary layer on the heat-transfer coefficient downstream of slot 1 (i.e. $t/D = .18$). It indicates that in the presence of thin lip and δ/D ratios of up to 3.5, the influence of δ is small. Heat-transfer coefficient decreases with increasing δ .

The influence of pitch to diameter ratio is shown to be small on Fig. 5.2.16. This effect was computed by changing pitch for a particular value of hole diameter. Heat-transfer coefficient decreases in the upstream region for P_i/D ratios of greater than 4 and remains unchanged in the downstream region. Fig. 5.2.17 demonstrates the influence of lip length with x measured from the hole exit. Increasing the lip length does not effect the heat-transfer coefficient very much.

Trends shown on Figs. 5.2.14 to 5.2.17 for influence of geometrical parameters on the heat-transfer coefficient, are similar to the trends shown on Figs. 5.2.9 to 5.2.12 for the adiabatic or impervious wall effectiveness. The reason is that in the presence of constant non zero wall heat flux condition, the wall temperature is not as much influenced by the geometrical parameters of the slot, as the adiabatic wall temperature. This also results in smaller influence on the heat-transfer coefficient. It is clear from Figs. 5.2.14 to 5.2.17 that lip thickness has a more significant influence on the heat-transfer coefficient than δ/D , P_i/D , and L/D .

5.3 Conclusions.

The following main conclusions may be extracted from the results presented in section 5.2.

1. The numerical procedure, which solves the three-dimensional boundary-layer equations and incorporates a form of mixing-length hypothesis, predicts values of impervious-wall effectiveness and heat-transfer coefficient which are in close agreement with experimental data. The comparisons cover a range of initial conditions which satisfactorily validate the procedure for flow and geometrical parameters similar to those found in the film-cooling of gas-turbine combustors.

2. In general terms the procedure predicts that the downstream effectiveness increases with velocity ratio and density ratio while the heat-transfer coefficient increases with velocity ratio and decreases with density ratio.

Effectiveness and heat-transfer coefficient increase with D/t , D/Pi , L/D and D/δ . The influence of the geometrical parameters, except t , is more significant on effectiveness than on heat transfer coefficient. The influence of each parameter and their interactions are, however, complex and the availability of the procedure greatly diminishes the need for further experiments.

3. The procedure is economical of computer time and storage, approximately 55 seconds and 34k locations of CDC 6600 time and storage being required to predict the wall effectiveness for a single set of initial and boundary conditions to a downstream distance corresponding to x/D of 80.

CHAPTER 6

6. Practical relevance of the present results.

The discussion in previous chapters was mainly concerned with the performance of discrete-hole film-cooling slots in low-turbulence wind tunnels and in the absence of combustion and radiation. Radiation forms an important part of the conditions under which film-cooling slots operate in gas turbines and it is, therefore, important to incorporate it in the present prediction procedure and to obtain predictions of relevance to gas-turbine practice. This is the main purpose of the present chapter.

The important characteristics of the flow inside the flame tube of a combustion chamber and normally not present in the wind tunnels are as follows:

1. large radiative heat-fluxes from the hot gases to the tube wall and from the tube wall to the outer annulus,
2. assymetry and irregularities in the main flow which results in three-dimensionality in the whole region,
3. multiple slots in series resulting in overlapping films,
4. periodicity in the flow caused by instabilities in the recirculating flow pattern,
5. large temperature, velocity, density and concentration gradients due to combustion and chemical reaction,
6. large values of turbulence intensity and scale.

Thus the flow is of a complex nature and, particularly in the presence of periodicity, indicates that the time averaged quantities should be regarded with caution. Time dependent three-dimensional studies are not feasible at the present time because of the large computing time requirements and

attention will be restricted to time-averaged properties in the present chapter.

Previous attempts to effect predictions relevant to gas-turbine film cooling have been made, for example, by Pai (6), Whittaker (82) and Colladay (83). Pai predicted wall temperatures some 300°C lower than the thermal paint measurements in a combustion chamber. This discrepancy stemmed largely from the inability, at that time, to solve the equations appropriate to the flow downstream of three-dimensional film-cooling slots. Whittaker made use of various correlations in his analysis and these are known to be unreliable outside the range of data upon which they are based. Colladay investigated different cooling arrangements but neglected radiative heat fluxes and confined himself to two-dimensional cooling configurations. The present research has resulted in a general method which can predict the flow downstream of three-dimensional slots and it is the purpose here to show the extent to which the present advance can be employed to obtain realistic predictions to the wall temperature of a gas-turbine flame tube. The procedure to predict the wall temperature is outlined in the next section. Section 6.2 presents the predicted values of wall temperature and compares them with the available data. Predictions are also presented in section 6.2 for the conditions outside the range of available data. The last section states the conclusions.

6.1 Procedure to predict Flame-tube wall temperature.

The temperature assumed by the flame tube is such

that it represents the balance between the heat received from the hot gases and the heat lost to the surroundings. Heat is received and lost by the flame tube through radiation and convection; the various fluxes can be expressed in the following manner.

Internal Radiation, R_I .

The radiant heat flux from the hot gases to the wall may be calculated from the relation derived by Reeves (84) and expressed in the following way:

$$R_I = \sigma_B \left(\frac{1+\epsilon_w}{2} \right) (\epsilon_G T_G^4 - \alpha_w T_w^4) \quad (6.1.1)$$

In eq. (6.1.1) the effects of reflection and re-radiation at the wall are allowed for by the term $\left(\frac{1+\epsilon_w}{2} \right)$. The flame absorptivity, α_w , at wall temperature is calculated from the relation

$$\frac{\alpha_w}{\epsilon_G} = \left(\frac{T_G}{T_w} \right)^{1.5} \quad (6.1.2)$$

The derivation of eq. (6.1.1) involves major empiricism because of the uncertainties in various quantities, in particular ϵ_G and T_G ; these have significant influence on the wall temperature. The gas emissivity is a function of the pressure, temperature, effective fuel-air ratio and burning characteristics of the fuel. Current practice is to calculate it from empirical formulæ. In the present calculations the relation given by Reeves (84) has been used. The gas temperature is estimated from the effective fuel air ratio and combustion efficiency. This is a crude way of estimating T_G and does not take proper account of severe temperature gradients in the axial direction. In the present calculations, the gas temperature variation is assumed to be

known and has been taken from the available industrial calculations.

Internal Convection, C_I .

The heat received by the flame tube via convection depends upon whether a film-cooling strip is encountered or not. Before a film-cooling slot is encountered, hot gases transfer heat through the wall boundary layer; the flux can be adequately estimated by two-dimensional boundary layer theory, e.g., ref. (9). In the presence of a film-cooling strip, the convective heat flux is expressed by

$$C_I = hf (T_{a,w} - T_w) \quad (6.1.3)$$

The adiabatic-wall temperature is used in eq. (6.1.3) since it is independent of the heat transferred to or from the wall. Calculation of hf and $T_{a,w}$ in the past has also been made using flat plate theory and correlations which cannot give successful results for the flow downstream of three-dimensional practical slots. The results discussed in previous chapters have provided confidence in the ability of the present procedure to predict precisely the influences of various parameters of three-dimensional film cooling slots on hf and $T_{a,w}$. The procedure was tested for discrete hole slots only but is also suitable for other practical slots which do not have significant regions of forward recirculation. Given proper initial conditions and boundary conditions the solution procedure can predict hf and $T_{a,w}$ or η downstream of a particular slot configuration. Hence the cumulative effect of multiple slots can also be taken into account very conveniently in the present procedure. hf and η have been shown (6) to have important effects on the prediction of wall

temperature and therefore have been computed using the procedure presented in chapter 5, instead of previous two-dimensional procedures and correlations.

External Radiation, R_0 .

Heat lost from the flame tube by radiation to the outer casing can be expressed by the equation,

$$R_0 = \sigma_B \epsilon'_w (T_w^4 - T_C^4), \quad (6.1.4)$$

which assumes that the effective emissivity of the tube wall and the outer casing is the same. ϵ'_w is expressed in terms of ϵ_w and ϵ_c and the diameter of the flame tube and the casing, e.g., see ref. (82). T_C , in eq. (6.1.4) is taken as the coolant temperature instead of the wall temperature of the casing. As can be expected and has been shown in ref. (6), T_C in the above equation has an equally significant influence on T_w as T_G has in equation (6.1.1).

External Convection, C_0 .

Heat lost by convection can be expressed as

$$C_0 = h_2 (T_w - T_C) \quad (6.1.5)$$

h_2 in eq. (6.1.5) has been computed using two-dimensional turbulent boundary layer theory. This component in the heat balance depends upon the coolant mass which is utilised for convection cooling at a particular location. The net mass available for convection reduces, with increasing distance, as the coolant is bled off through film-cooling slots and dilution holes. The influence of the external convection on the wall temperature also depends upon the direction of flow in the annulus. Colladay (83) has shown, for a two-dimensional model, that a combination of convection and film-

cooling is particularly advantageous when the flow in the annulus is in a direction opposite to the gas flow.

Using equations (6.1.1) to (6.1.5), the wall temperature can be calculated so that

$$R_I + C_I = R_O + C_O \quad (6.1.6)$$

The next section presents sample calculations of the wall temperature under the operating conditions of a modern high-compression-ratio aero-engine, in order to show the extent to which the results of the present research can be used in practice. The influence of three geometrical parameters, namely, P_i/D , L/D and t/D is also demonstrated on the wall temperature under the same operating conditions. These parameters have been chosen on the basis of results presented in chapter 5.

6.2 Wall-temperature predictions.

Fig. 6.2.1 shows the predicted values of wall temperature along with the thermal paint measurements for the outer wall of an annular flame tube. The wall temperatures shown on this figure correspond to the primary zone of a combustion chamber and have been measured and computed downstream of the first cooling ring. The cooling ring consisted of discrete holes with coolant injecting tangential to the wall; it is shown on Fig. 6.2.1 along with the flow parameters of relevance to the prediction procedure. Wall temperature predictions, labelled by T_{wall} , represent the temperatures along the centre line of the holes and were obtained using the procedure outlined in section 6.1. The temperature shown by the dotted line represents the maximum wall temperature

measured downstream the first cooling ring as a result of a thermal paint test carried out for the whole flame tube. The thermal paint used for these measurements changed colour only at about 800°K and this temperature was obtained at 25 diameters downstream of the slot exit. Thus no measurements are available upstream of 25 diameters. The measurements shown on Fig. 6.2.1 should be treated with caution as at some circumferential locations and at the same streamwise distance, lower temperatures by some 100°C were recorded. The present predictions are 125° to 150°C below the maximum values of measured wall temperature in the region downstream of 25 diameters. The main reasons for this discrepancy are as follows:

- i) the measurements of wall-temperatures are imprecise and are influenced by the lack of symmetry of the flow,
- ii) eq. (6.1.1), to calculate the radiation flux from the hot gases to the wall, is based on oversimplified assumptions,
- iii) the values of gas temperature (Fig. 6.2.1) used in the present predictions are inaccurate.

The first two of the above reasons are outside the scope of this thesis: the third is discussed below.

The temperature of hot gases in a gas turbine flame tube cannot be measured accurately. Those shown on Fig. 6.2.1 are computed from combustion efficiency and air-fuel ratio and are, therefore, approximate. At a downstream distance of 35 diameters the predicted wall temperature would conform to the maximum measured wall temperature if the gas temperature was increased by 30%, i.e. a gas temperature

of about 1500°K ; this is a plausible value for modern combustion chambers. This indicates the significance of gas temperature on the prediction of wall temperature.

In the present calculations, the external convective heat-transfer coefficient, h_2 , was computed from existing correlations for simplified flows, e.g., fully turbulent flat plate boundary layer or a fully developed channel flow. The influence of calculating h_2 by different ways, on the prediction of wall temperature is discussed in the following paragraph.

Fig. 6.2.1 shows predicted values of wall temperature for two cases: a) parallel flow convection i.e. the flow in the outer annulus is in the same direction as the film; b) counter flow convection i.e. the flow in the outer annulus is in opposite direction to the direction of film inside the flame tube. For parallel flow convection, these predictions were obtained by computing h_2 from the correlation (85) for a fully turbulent boundary layer on a flat plate. The origin of the boundary layer was assumed 50 mm upstream of the slot exit and corresponds to the beginning of the flame tube. A calculation, assuming the origin of the boundary layer to be at the slot exit, resulted in wall temperatures about 7°C less than those shown in Fig. 6.2.1. This reveals that choice of the origin for external boundary layer does not significantly influence the wall temperature predictions. For the counter flow convection case h_2 was calculated using the same flat plate correlation and assuming the origin to be at 50 mm downstream of the last point calculated which was 35 diameters downstream. The coolant temperature and the mass

flow rate was assumed the same as the parallel flow case and the heating of the coolant before passing through the slot was neglected. As shown on Fig. 6.2.1 and also found by Colladay (83) counter flow convection results in lower wall temperatures (by some 25°C at 35 diameters downstream) than obtained in parallel flow convection. This is due to the complementary interaction of the film on inside and convection cooling on outside in counter flow case and means that, to maintain the wall temperature within a particular limit, for a given temperature of the hot gas and the coolant, counter flow convection would require a smaller amount of cooling air than parallel flow convection. The predictions based on h_2 calculated from correlation for a fully developed channel flow (85) resulted in wall temperature 20°C above the value shown on Fig. 6.2.1 at 35 diameters downstream for parallel flow convection. The reason was that for the present annulus gap fully developed channel flow relation gave smaller value of h_2 than that given by the flat plate boundary layer relation. It should be noted that use of the fully developed channel flow relation to compute h_2 would give the same wall temperatures for parallel flow and counter flow convection.

Due to the uncertainties in the wall temperature measurements and gas temperature estimates, the accuracy of absolute values of the predicted wall temperatures cannot be assessed. However the present calculation procedure can provide practically valuable indications of the influences of various flow and geometrical parameters on the wall temperature. Predictions showing these influences are presented and discussed in the following paragraph.

Fig. 6.2.2 shows the predicted influence of three geometrical parameters, P_i/D , L/D and t/D , on the wall temperature of flame tube, maintaining other parameters the same as shown on Fig. 6.2.1. By increasing the pitch to diameter ratio, the wall temperature increases; this is consistent with the previously indicated influence of P_i/D on η and h_f . Increasing the lip length decreases the wall temperature and increasing the lip thickness increases the wall temperature slightly. The influence of lip length and lip thickness is less significant than the influence of pitch on wall temperature. The results of chapter 5 have indicated that lip length does not influence η and h_f appreciably within the present range of lip length and, therefore, the small influence of lip length on the wall temperature in Fig. 6.2.2 is not surprising. Although η and h_f are significantly influenced by variation in lip thickness, the wall temperature is not influenced very much because the decrease in η and decrease in h_f with increase in lip thickness have opposite effects on the wall temperature. Calculations to predict the relative influence of P_i/D , L/D and t/D on wall temperature were also performed for a velocity ratio of 1.12 and density ratio of 2.0 at the slot exit and in the absence of pressure gradient in the main flow. The gas and coolant temperatures were maintained at the same values as shown on Fig. 6.2.1. For these predictions η and h_f values were taken from the results presented in chapter 5. These predictions revealed similar influences of P_i/D and L/D on wall temperature but the influence of t/D was more than shown on Fig. 6.2.2, e.g., increasing t/D from .2 to .8 resulted in

an increase in wall temperature of 22°C at 30 diameters downstream. This indicates that the influence of lip thickness is more significant for velocity ratios of approximately unity than for velocity ratios higher than unity at the slot exit.

6.3 Conclusions.

The following conclusions may be drawn from the results presented in section 6.2.

1. Gas temperature has a very significant influence on the prediction of flame tube wall temperature. Although the present procedure predicts wall temperature within the measured range, quantitative assessment is precluded by the unavailability of accurate values of gas temperature and wall temperature measurements.
2. To maintain the flame tube within a particular temperature limit, for a given coolant and gas temperature, film cooling with counter flow convection requires a smaller amount of coolant as compared to film cooling with parallel flow convection.
3. The procedure predicts that increasing pitch to diameter ratio increases the wall temperature and lip length and lip thickness have less significant influence on the wall temperature as compared to pitch of the holes.
4. The calculations presented in section 6.2 indicate that the wall temperature downstream of the practical slot (Fig. 6.2.1) can be reduced by about 5% at a distance of 68 mm from the slot exit, by judicious choice of parameters. For example a) by decreasing pitch to diameter ratio from

1.47 to 1.2 and maintaining the same coolant mass flow rate, a decrease of 25°C (3.7%) in the wall temperature would be obtained, b) using counter flow convection in the outer annulus a maximum decrease of 10°C (1.47%) would be obtained in the wall temperature. For the flow conditions shown on Fig. 6.2.1, increasing the lip length and decreasing the lip thickness within the practical range would make little difference to the wall temperature.

CHAPTER 7

7.1 General conclusions.

Conclusions specific to the work described in each of chapters 3, 4, 5 and 6 are listed in the last section of the relevant chapter. The present section lists the conclusions in a more general way in that it explicitly brings together the conclusions of individual chapters and indicates their inter-relationships. The conclusions presented in this section indicate the extent to which the aims of the present study, as listed in section 1.2 have been achieved.

1. The effectiveness and heat transfer coefficient downstream of three-dimensional discrete hole film-cooling slots are significantly influenced by slot geometrical parameters and the flow parameters at the slot exit. The present measurements imply that the effects are complex and a general prediction procedure, based on relevant differential equations, is necessary to understand the influences clearly. Such a prediction procedure must always take into account the values of P_i/D , t/D , L/D , H/D , U_C/U_G and ρ_C/ρ_G and should be able to quantify their influence on η and hf .
2. A prediction procedure based on the steady three-dimensional boundary layer equations of conservation of mass, momentum and species or enthalpy and the numerical algorithm developed by Patankar and Spalding is shown to yield satisfactory predictions of η and hf downstream of discrete hole film-cooling slots. The procedure incorporates a form of mixing length hypothesis and permits, the influence of

important parameters on effectiveness and heat-transfer coefficient to be satisfactorily predicted.

3. The balance of convective and radiative heat fluxes on the two sides of a combustion chamber flame tube wall has led to predicted wall temperatures which are within the possible error margin of the measured wall temperatures. The error margins in wall temperature measurements and computed values of gas temperature used for wall temperature predictions are large and make it impossible to assess the accuracy of wall temperature predictions. Although the precision of absolute magnitudes of predicted wall temperature cannot be assessed, the present procedure indicates the relative significance of different geometrical parameters on the wall temperature. These relative trends are of direct relevance to the design of practical film-cooling slots.

4. The solution procedure of Spalding and Patankar (9) for two-dimensional boundary layers has been modified to include relevant conservation equations with an appropriate version of Prandtl's mixing length hypothesis and thereby, to predict the influence of longitudinal curvature on two-dimensional wall-jets. The procedure has been shown to satisfactorily predict the effect of longitudinal curvature on the impervious wall effectiveness and growth rate parameters of wall-jets.

5. The merits of replacing Prandtl's mixing length hypothesis by a three equation turbulence model (equations for turbulent kinetic energy, dissipation of turbulent kinetic energy and turbulent shear stress), for the prediction of impervious wall effectiveness downstream of two-dimensional film-cooling slots have been assessed. The

accuracy of predictions obtained with the three equation model depends upon the precision of the initial conditions provided. For the initial conditions provided in the present calculations, the three-equation model results in predictions that are similar in precision to those obtained with the mixing length hypothesis.

6. The procedure of Gosman et al. (10) for solving two-dimensional elliptic equations has been combined with the procedure of Spalding and Patankar (9) for solving two-dimensional boundary layer equations and provides a general solution procedure to predict effectiveness downstream of two-dimensional slots. The procedure is shown to predict effectiveness satisfactorily in the presence of thick lip and backward facing step.

7.2 Recommendations for future work.

Although the aims formulated in chapter 1 of the present thesis have been achieved, the present work should be considered as a beginning for a systematic investigation of practical film-cooling. The gas conditions existing inside a combustion chamber and the performance of practical film-cooling slots, are the two main fields in which further research should be done before the mean wall temperature of a flame tube can be predicted with confidence. The research reported in the present thesis has contributed to the understanding of cooling performance of practical slots and the purpose of this section is to provide suggestions for future research in that direction.

1. In the immediate neighbourhood of the slot exit, forward

recirculation is present and in that region the complete three-dimensional time averaged Navier-Stokes equations are required to represent the flow. In the downstream region there are no forward recirculations and the three-dimensional boundary layer equations represent the flow correctly.

Therefore a calculation procedure should be developed which solves the relevant equations in the upstream and downstream regions, in a way similar to the method reported in section 4.2 for two-dimensional film-cooling. Such a procedure should also be used to predict the flow downstream of film-cooling slots that have normal injection holes along with the tangential holes, usually known as splash cooling slots and are used in practice.

2. The experimental investigation should be extended to the measurement of effectiveness, heat-transfer coefficient and detailed profiles downstream of splash cooling slots. The influence of various parameters, relevant to these slots, on the cooling performance should be investigated and the important parameters identified. Measurements should be made in a manner so that the validity of the solution procedure suggested in the above paragraph can be assessed.

3. The present work has reported some measurements downstream of a slot having three rows of holes. A more extensive experimental study should be carried out on multi-row slots to provide the optimum balance between the number of rows and machining costs. A prediction procedure should also be developed for such slots. Since the flow downstream of the multirow slots becomes two-dimensional at short distances from the slot exit, use of the present three-

dimensional procedure should not be preferred to use of two-dimensional solution procedures such as ref. (9). However in the vicinity of the slot exit, flow is complex and prediction in that region would require solution of complete Navier-Stokes equations with a complex turbulence model.

4. Although the present calculation procedure with the simple physical hypothesis has faithfully predicted a wide range of data on wall properties for discrete hole slots, it will not predict the details of the flow with the same precision as it does not take account of noncoincidence of zero shear stress and zero velocity gradient in wall jet flows. To be able to predict the details of the flow correctly, the present physical hypothesis should be replaced by a turbulence model which solves for each turbulent shear stress appearing in the governing equations. In addition to shear stress equations such a model will require the solution of turbulent kinetic energy and will be able to predict the influence of turbulence levels in the flow field. However for predicting the wall properties, in particular the wall temperature of a flame tube, an advanced model need not be used since reliable estimates of gas temperature and wall-temperature measurements are not available.

REFERENCES.

1. KACKER, S.C. and WHITELAW, J.H.: An experimental investigation of the influence of slot-lip-thickness on the impervious wall effectiveness of the uniform density, two-dimensional wall jet. Int. J. Heat and Mass Transfer, 12, 1196 (1969).
2. KACKER, S.C. and WHITELAW, J.H.: The turbulence characteristics of two-dimensional wall jet and wall wake flows. Imperial College, Mech. Eng. Dept., Rep. BL/TN/6 (1969).
3. HARTNETT, J.P., BIRKEBACK, R.C. and ECKERT, E.R.G.: Velocity distributions, temperature distributions, effectiveness and heat transfer for air injection through a tangential slot into a turbulent boundary layer. A.S.M.E., J. of heat transfer, 83, 293 (1961).
4. KACKER, S.C., PAI, B.R., and WHITELAW, J.H.: The prediction of wall-jet flows with particular reference to film cooling. Prog. in Heat and Mass Transfer, Vol. 2, Eckert presentation vol. Pergamon Press (1969).
5. MATTHEWS, L. and WHITELAW, J.H.: Private communications.
6. PAI, B.R.: The effectiveness of film cooling. Ph. D. Thesis, Univ. of London (1969).
7. KACKER, S.C.: The influence of slot geometry on the two-dimensional turbulent wall-jet. Ph. D. Thesis, Univ. of London, (1969).
8. PATANKAR, S.V. and SPALDING, D.B.: A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows. Imperial College Mech. Eng. Dept., Rep. BL/TN/A/5 (1971).
9. SPALDING, D.B. and PATANKAR, S.V.: Heat and mass transfer in boundary layers. Morgan-Grampian presses, London (1967).
10. GOSMAN, A.D., PUN, W.M., RUNCHAL, A.K., SPALDING, D.B. and WOLFSHTEIN, M.: Heat and mass transfer in recirculating flows. Academic press, London (1969).
11. WHITELAW, J.H.: Review of heat transfer literature pertaining to wall jets. Imperial College, Mech. Eng. Dept., Rep. IC/HRJ/25 (1965).
12. WHITELAW, J.H.: The effect of the geometry of the injection region on wall cooling processes. A.R.C. 27373, HMT. 78 (1965).
13. STOLLERY, J.L. and EL-EHWANY, A.A.: On the use of a boundary-layer model for correlating film-cooling data. Int. J. of heat and mass transfer, 10, 101 (1967).

14. NINA, M.N.R.: The influence of three-dimensional slot geometries on film-cooling. M.Sc. Thesis, Univ. of London (1969).
15. WIEGHARDT, K.: On the blowing of warm air for de-icing devices. German ref.: FB NO. 1900 (1944) - Zentrale f. wiss. Bericht (ZWB).
16. SEBAN, R.A.: Effects of initial boundary layer thickness on a tangential injection system. J. of heat transfer, 82, 392 (1960).
17. HARTNETT, J.P., BIRKEBACK, R.C. and ECKERT, E.R.G.: Velocity distributions, temperature distributions, effectiveness and heat transfer in cooling a surface with a pressure gradient. A.S.M.E., Int. Dev. in Heat Transfer, Part IV, 682 (1961).
18. PAI, B.R. and WHITELOW, J.H.: The influence of density gradients on the effectiveness of film cooling. A.R.C. C.P. 1013 (1968).
19. PAI, B.R. and WHITELOW, J.H.: The influence of strong pressure gradients on film cooling. Imperial College, Mech. Eng. Dept., Rep. EHT/TN/A15 (1969).
20. CARLSON, L.W. and TALMOR, E.: Gaseous film cooling at various degrees of hot gas acceleration and turbulence levels. Int. J. of Heat and Mass Transfer, 11, 1695 (1968).
21. MATTHEWS, L. and WHITELOW, J.H.: Private communications.
22. WHITELOW, J.H.: The effect of slot height on the effectiveness of the uniform density, two-dimensional wall-jet. Imperial College, Mech. Eng. Dept., Rep. EHT/TN/4 (1967).
23. SIVASEGARAM, S. and WHITELOW, J.H.: Film cooling slots; the importance of lip thickness and injection angle. J. of Mech. Eng. Sci., 11, 1, 22 (1969).
24. MATTHEWS, L. and WHITELOW, J.H.: Film cooling effectiveness in the presence of a backward-facing step. Imperial College, Mech. Eng. Dept., Rep. EHT/TN/A/34 (1971).
25. METZGER, D.E., CARPER, H.J. and SWANK, L.R.: Heat transfer with film cooling near nontangential injection slots. A.S.M.E., J. of Engineering for Power, 90A, 157 (1968).
26. BURNS, W.K. and STOLLERY, J.L.: The influence of lip thickness on the impervious wall effectiveness of a wall jet with foreign gas injection. Imperial College, Dept. of Aero., Rep. EHT/TN/19 (1969).

27. MATTHEWS, L. and WHITELOW, J.H.: Some characteristics of the flow downstream of a blunt trailing edge in the presence of a neighbouring wall. IUTAM Symposium on Unsteady Boundary Layers, Quebec, Canada (1971).
28. KACKER, S.C. and WHITELOW, J.H.: Some properties of the two-dimensional turbulent wall-jet in moving stream. A.S.M.E., J. of Applied Mechanics, Trans., 35, 4 (1968).
29. PALMER, J.F.: Interim report on performance investigations of two-dimensional models of Splash type film cooling. Bristol Siddeley Report No. 1248 (1964).
30. PALMER, J.F.: Second interim report on performance investigations into two dimensional models of film cooling. Bristol Siddeley Report No. P.D. 1396 (1965).
31. STURGESS, G.J.: Correlation of film cooling data from combustion chamber experimental devices. Bristol Siddeley Engines Ltd., combustion dept., G.N. 9327 (1966).
32. ODGERS, J. and WINTER, J.: Wall cooling processes in gas turbine combustion chambers. Lucas gas turbine equipment limited, Research laboratories, Burnley. Rep. No. B.48,314 (1963).
33. STURGESS, G.J.: Application of film cooling theory to the cooling of aircraft gas turbine chambers. J. of the Royal Aero. Soc., 71, 430 (1967).
34. PAPELL, S.S.: Effect on gaseous film cooling of coolant injection through angled slots and normal holes. NASA TND-299 (1960).
35. NOBBS, K.W. and RICE, P.: The performance of splash impingement cooling devices. G.T.C.C. Combustion sub-committee, Paper No. 255, The college of Aeronautics, Cranfield (1968).
36. STURGESS, G.J.: Review of film cooling research. A summary. G.T.C.C. combustion sub-committee, Paper No. 256, Loughborough Univ. of technology (1968).
37. NAIB, S.K.A.: Spreading and development of the parallel wall jet. Air-craft Engineering, 30, Dec. (1968).
38. GOLDSTEIN, R.J., ECKERT, E.R.G. and RAMSEY, J.W.: Film cooling with injection through holes: Adiabatic wall temperatures downstream of a circular hole. A.S.M.E., Paper No. 68-GT-19 (1968).

39. GOLDSTEIN, R.J., ECKERT, E.R.G., ERIKSEN, V.L. and RAMSEY, J.W.: Film cooling following injection through inclined circular tubes. Israel J. of Technology, 8, No. 1-2, 145 (1970).
40. SEBAN, R.A.: Heat transfer and effectiveness for a turbulent boundary layer with tangential fluid injection. A.S.M.E., J. of Heat Transfer, 82c, 303 (1960).
41. SAMUEL, A.E. and JOUBERT, P.N.: Film cooling of an adiabatic flat plate in zero pressure gradient in the presence of hot main stream and cold tangential secondary injection. J. of heat transfer, Paper No. 64 WA/HT - 48 (1965).
42. SPALDING, D.B., JAIN, V.K. and NICOLL, W.B.: Film cooling in incompressible turbulent flow: examination of experimental data for the adiabatic wall temperature. A.R.C., Rep. 25311 (1963).
43. KLEIN, J. and TRIBUS, M.: Forced convection from non-isothermal surfaces. Heat transfer symposium. Univ. of Michigan, Ann Arbor, Michigan (1953).
44. GLUSHKO, G.S.: Turbulent boundary layer on plane plates incompressible fluid. Izv. Akad. Nauk. Mech. No. 4, 13 (1965).
45. PASKONOV, V.M.: A standard programme for the solution of boundary-layer problems. Numerical methods in gas dynamics - edited by G.E. Raslyakov and L.A. Chudov. NASA Washington, TTF - 300, TT65 - 50138, 74 (1966).
46. BRADSHAW, P., FERRISS, D.H. and ATWELL N.P.: Calculation of boundary layer development using the turbulent energy equation. J. Fluid Mech., 28, pt. 3, 593 (1967).
47. METZGER, D.E., CARPER JR., H.J. and WARREN, J.M.: Predicted film cooling near flush slots - comparison with experiment. AIAA 7th thermodynamic conference, Paper No. 72 - 291 (1972).
48. MATTHEWS, L.: Prediction of film cooling effectiveness, in the presence of density and pressure gradients, by a recirculating flows procedure. Imperial College, Mech. Eng. Dept., Rep. EHT/TN/A/27 (1970).
49. GIBSON, M.M. and SPALDING, D.B.: The use of an improved model of turbulence for the calculation of film-cooling processes. Imperial College, Mech. Eng. Dept., Rep. EHT/TN/C/21 (1970).
50. STURGESS, G.J.: Comment on 'Prediction of adiabatic wall temperatures in film cooling systems'. A.I.A.A. J., 4, 763 (1966).

51. SPALDING, D.B.: Prediction of adiabatic wall temperatures in Film-cooling systems. A.I.A.A., Vol. 3, 965 (1965).
52. PATANKAR, S.V.: On available calculation procedures for steady three-dimensional boundary layers. Imperial College, Mech. Eng. Dept., Rep. BL/TN/A/44 (1971).
53. RAETZ, G.S.: A method of calculating three-dimensional laminar boundary layers of steady compressible flows. Northrop Aircraft Inc., Rep. No. NAI-58-73 (BLC-144), (1957).
54. HALL, M.G.: A numerical method for calculating steady three-dimensional laminar boundary layers. RAE Tech. Rep. 67145 (1967).
55. DWYER, H.A.: Solution of a three-dimensional boundary layer flow with separation. A.I.A.A. J., 6, 1336 (1968).
56. FANNELOP, T.K.: A method of solving the three-dimensional laminar boundary-layer equations with application to a lifting re-entry body. A.I.A.A. J., 6, 1075 (1968).
57. NASH, J.F.: The calculation of three-dimensional turbulent boundary layers in compressible flow. J. Fluid Mech., 37, 625 (1969).
58. KRAUSE, E. and HIRSCHEL, E.H.: Exact numerical solutions for three-dimensional boundary layers. Second International Conference on Numerical Methods in Fluid Dynamics, Univ. of California, Berkeley (1970).
59. WANG, K.C.: Three-dimensional boundary layer near the plane of symmetry of a spheroid at incidence. J. Fluid Mech., 43, 187 (1970).
60. MacMILLAN, F.A.: Experiments on pitot tubes in shear flow. A.R.C., R. & M. No. 3028 (1957).
61. RASTOGI, A.K.: A computer programme to replace the Clauser Chart. Imperial College, Mech. Eng. Dept. Rep. ET/TN/A/4 (1971).
62. SPALDING, D.B. and CHI, S.W.: The drag of a compressible turbulent boundary layer on a smooth flat plate with and without heat transfer. J. of Fluid Mechanics, 18, Pt. 1, 117 (1964).
63. KLEBANOFF, P.S.: Characteristics of turbulence in a boundary layer with zero pressure gradient. NACA TN 3178 (1953).

64. RASTOGI, A.K. and WHITELOW, J.H.: Film-cooling downstream of single, circular tangential jets. Imperial College, Mech. Eng. Dept., Rep. EHT/TN/A/33 (1971).
65. NEWMAN, B.G., PATEL, R.P., SAVAGE, S.B. and TJIO, H.K.: Three-dimensional wall jet originating from a circular orifice. McGill University report. (1971).
66. GOLDSTEIN, S.: Modern developments in Fluid-Mechanics, Part I, Oxford (1938).
67. BRADSHAW, P.: An analogy between stream line curvature and buoyancy in turbulent shear flows. J. of Fluid Mechanics, 36, 177 (1969).
68. SAWYER, R.A.: Two-dimensional turbulent jets with adjacent boundaries. Ph. D. Thesis, University of Cambridge (1962).
69. LORD RAYLEIGH: On the dynamics of revolving fluids. Scientific Papers, 6, (1916).
70. KASSNER, R.: Friction laws and energy transfer in circular flow. Wright-Patterson air force base, Report No. F-TR-2198-ND.
71. ESKINAZI, S. and YEH, H.: An investigation of fully developed turbulent flows in a curved channel. J. of Aero. Science, 23, 23 (1956).
72. RASTOGI, A.K. and WHITELOW, J.H.: Procedure for predicting the influence of longitudinal curvature on boundary layer flows. A.S.M.E., Paper No. 71-WA/FE-37 (1971).
73. GILES, J.A., HAYS, A.P. and SAWYER, R.A.: Turbulent wall-jets on logarithmic spiral surfaces. Aero. Quarterly, 17, 201 (1966).
74. GUITTON, D.E.: Two-dimensional turbulent wall jets on curved surfaces. McGill University, Mech. Eng. Dept. Report No. 64-7 (1964).
75. NEWMAN, B.G.: The deflection of plane jets by adjacent boundaries - Coanda effect. Boundary layer control principles and applications, by Lachmann, G.V., ed., Pergamon, 232 (1961).
76. KIND, R.J.: A calculation method for circulation control by tangential blowing around a bluff trailing edge. Aero. Quarterly, 19, 205 (1968).
77. SCHUBAUER, G.B. and KLEBANOFF, P.S.: Investigation of separation of the turbulent boundary layer. NACA TN 2133 (1950).

78. WOLFSHTEIN, M.: Convective processes in turbulent impinging jets. Ph. D. Thesis, University of London (1967).
79. RUNCHAL, A.K.: Transfer processes in steady two-dimensional separated flow. Ph. D. thesis, University of London (1969).
80. HANJALIC, K.: Two-dimensional asymmetric turbulent flow in ducts. Ph. D. thesis, University of London (1970).
81. MATTHEWS, L., RASTOGI, A.K. and WHITELOW, J.H.: The application of numerical methods to film cooling and associated problems. First Indian national heat and mass transfer conference, Paper No. HMT-3-71 (1971).
82. WHITTAKER, M.: Theoretical assessment of flame tube temperature in a combustor operating at typical S.S.T. conditions. Combustion and heat transfer in gas-turbine systems. Proceedings of an International Propulsion Symposium, Cranfield. ed. E.R. Norster (1969).
83. COLLADAY, R.S.: Analysis and comparison of wall cooling schemes for advanced gas turbine applications. NASA TN D-6638 (1972).
84. REEVES, D.: Flame Radiation in an Industrial Gas Turbine Combustion Chamber. N.G.T.E. Memo No. M285.
85. ECKERT, E.R.G. and DRAKE, R.M.: Heat and mass transfer. McGraw-Hill book company, INC. (1959).

NOMENCLATURE

<u>Symbol</u>	<u>Meaning</u>
a	constant in eq. (3.1.2)
a_1	coefficient in eq. (4.1.12)
A	coefficients in difference equations of section (5.1.2)
A_C	constant in eq. (4.1.10)
A_I	fictitious source term in eq. (5.1.22)
A_K	constant in mixing length
A_O	open area of the slot
A_t	total area of the slot
b	constant in eq. (3.1.2)
b_1	coefficient in eq. (4.1.12)
B	coefficients in difference equations of section (5.1.2)
c	concentration
c_1	coefficient in eq. (4.1.12)
c'	concentration fluctuation
C_1	constant in eq. (2.4.2)
C_D	constant in eq. (2.4.3)
C_f	shear stress coefficient
C_I	internal convection eq. (6.1.3)
C_O	external convection eq. (6.1.5)
C_P	specific heat at constant pressure
C_{P_1}	$(P_W - P_G)/\frac{1}{2}\rho_G U_G^2$
C_S	constants in eq. (4.2.6)
C_{S_2}	
C_{ϵ_1}	constants in eq. (4.2.8)
C_{ϵ_2}	

<u>Symbol</u>	<u>Meaning</u>
d_1	coefficient in eq. (4.1.12)
D	diameter of the hole OR coefficients in section (5.1.2) when appearing with a superscript
D_m	mean diameter of slot 5; equivalent to a single row of holes to give the same open area
E	constant in the law of the wall
F	pressure gradient parameter $\left[\frac{dp}{dx} \frac{y}{\rho U^2} \right]^{2.5}$
Gr	Grashof number
h_f	heat-transfer coefficient in presence of film-cooling
h_L	heat-transfer coefficient of losses
h_2	heat-transfer coefficient of external convection
H	defined in Fig. 3.2.1
H_L	defined in Fig. 3.2.1
H_{12}	shape factor
J	diffusional flux
k	kinetic energy of turbulence
K	reciprocal of radius of curvature
K_1	constant in the law of the wall
K_2	
K_3]	constants in eq. (2.4.3)
K_p	pressure gradient parameter $[v/U_G^2 \cdot dU_G/dx]$
ℓ	Prandtl's mixing length
ℓ_p	length of the hole
L	lip length
m	slot to main stream mass velocity ratio

<u>Symbol</u>	<u>Meaning</u>
m_p	mass source. eq. (5.1.17)
N	number of rows of holes
Nu_D	Nusselt's number based on the hole diameter
Nu_{free}	Nusselt's number of free convection
$\bar{P}$	average pressure over a cross-section
P	pressure in the y-z plane
P_i	pitch of the hole
Pr	Prandtl number
$\dot{q}''_{gen}$	generated heat-flux
$\dot{q}''_w$	heat-flux at the wall
R	radius of curvature
R_C	Reynolds number based on slot conditions
R_E	Reynolds number for the first grid interval near the wall
R_I	internal radiation. eq. (6.1.1)
R_O	external radiation. eq. (6.1.4)
R_R	Reynolds number based on the radius of curvature
R_X	Reynolds number based on longitudinal distance
$Re_{\frac{\delta}{2}}$	Reynolds number based on momentum thickness
S	shear stress coefficient. eq. (4.1.15)
S_1	correlation parameter of Sturgess. eq. (2.4.2) and eq. (2.4.3)
t	thickness of lip
T	temperature
u	fluctuating component of velocity in x-direction
u'	$\sqrt{u'^2}$ (fig. 3.1.9)
$\overline{uv}$	turbulent shear stress

<u>Symbol</u>	<u>Meaning</u>
U	mean velocity in x-direction
$U_C, \overline{U_C}$	mean velocity at the slot exit
U_W	velocity deficit in the wake $\left[\frac{U_{Cm} + U_G}{2} - U_{min} \right]$
U_+	dimensionless velocity $\left[\frac{U}{U_G} \frac{2}{cf} \right]$
v	fluctuating component of velocity in y-direction
v'	$\sqrt{\overline{v^2}}$
V	mean velocity in y-direction
w	fluctuating component of velocity in z-direction
w'	$\sqrt{\overline{w^2}}$
W	mean velocity in z-direction
x	longitudinal distance
x_p	length of the potential core
X	correlation parameter. eq. (2.4.1)
y	distance normal to the wall
yc	slot height
y_+	dimensionless distance $\left[\frac{y U_G}{v} \frac{cf}{2} \right]$
y_H	defined in fig. 3.2.9
z	lateral distance
α_w	flame absorptivity at the wall
Γ	diffusivity
Δx	length of the forward step
δ	upper lip boundary layer thickness
δ_1	displacement thickness
δ_2	momentum thickness
δ_W	width of the wake
ϵ	dissipation of kinetic energy of turbulence

<u>Symbol</u>	<u>Meaning</u>
ϵ_c	emissivity of the combustion chamber casing
ϵ_G	emissivity of the flame
ϵ_w	emissivity of the flame-tube wall
ϵ'_w	effective emissivity of the flame-tube wall and the casing
η	film-cooling effectiveness
Λ_{L_1}	defined in eq. (4.2.12)
λ	mixing length constant
μ	laminar viscosity
ν	kinematic viscosity
ρ	density
ρ_w	average density in the wake
τ	shear-stress
Φ_1	a conserved property
Φ	concentration or enthalpy
Ψ_I	an integral $\left[\int_0^{2.5} \rho U \, dy / (\rho U)_{2.5} \int_0^{2.5} dy \right]$
Ψ	stream function
σ	Prandtl or Schmidt number
σ_s	Schmidt number for turbulent shear stress
σ_B	Stefan-Boltzman constant
ω	non-dimensional stream function
$\overline{\omega}$	vorticity

<u>Subscript</u>	<u>Meaning</u>
a, w or ad	adiabatic wall
c	concentration
c, j	secondary flow
Cm	secondary maximum

<u>Subscript</u>	<u>Meaning</u>
c,t	concentration equation, turbulent component
E	east point
eff	effective
exp	experimental
G or ∞	main flow
imp	impervious wall
k	kinetic energy of turbulence
k,t	k equation, turbulent component
max	maximum
min	minimum
N	north point
P	grid point
prd	predicted
S	south point
t	turbulent
w	wall
W	west point
y	direction y
ϵ	dissipation of turbulent kinetic energy
$\frac{1}{2}$	length corresponding to half velocity difference
2.5	mid point of cell nearest to wall
<u>Superscript</u>	
P	pressure
U	mean longitudinal velocity
V	mean velocity in y-direction
W	mean velocity in z-direction
*	corresponding to guessed pressure
/	correction to guessed quantities

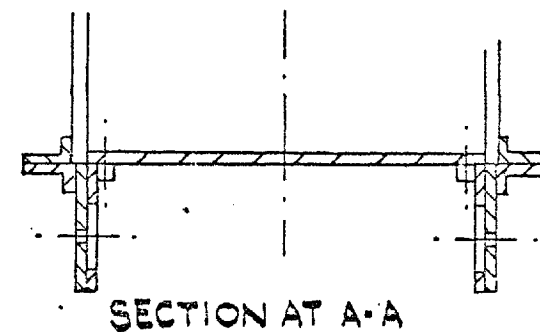
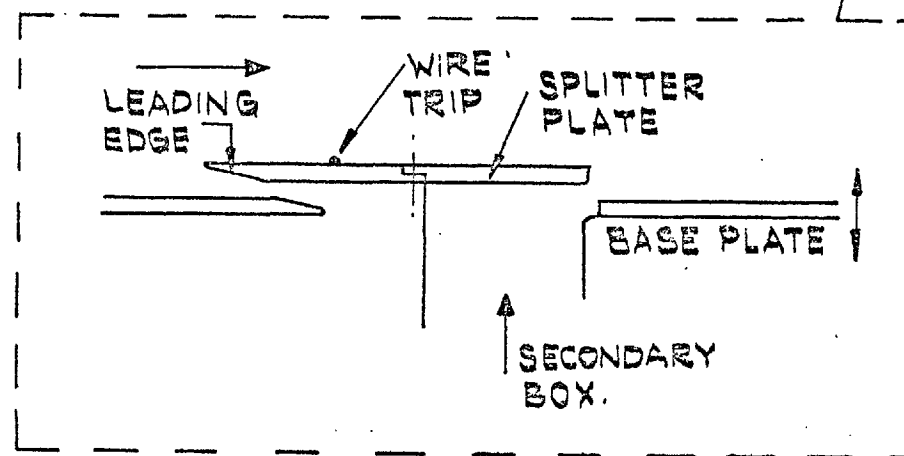
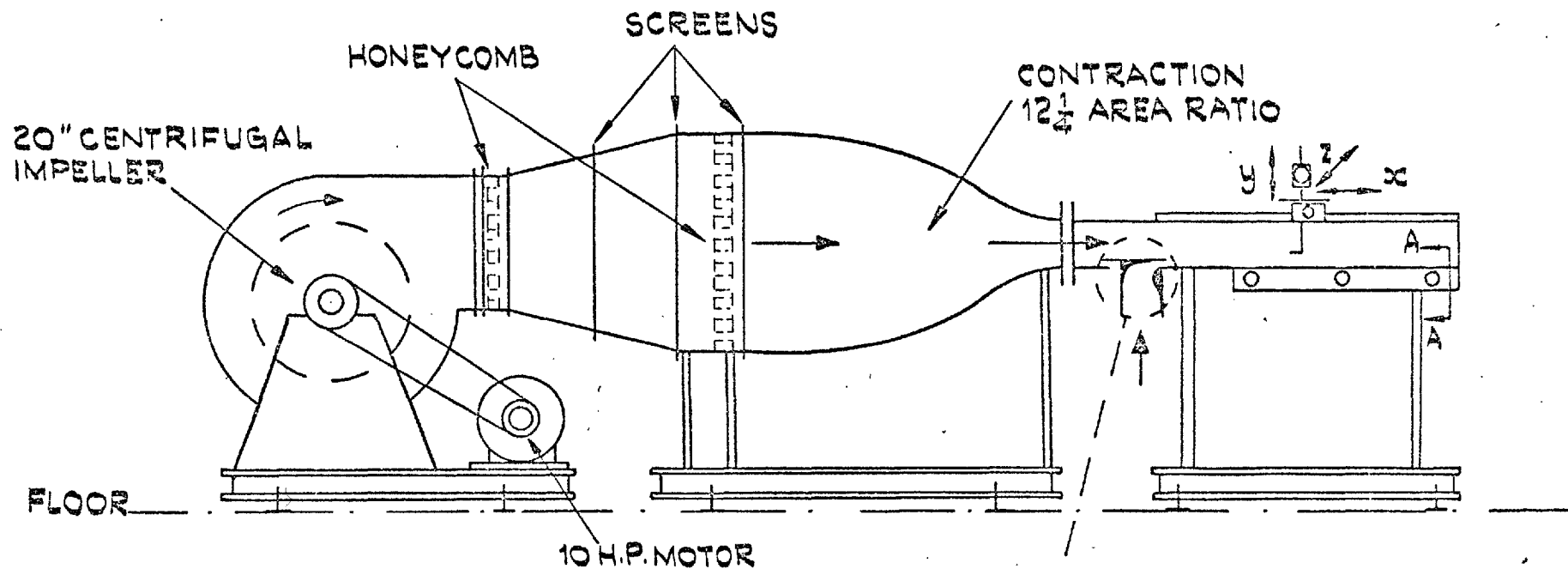
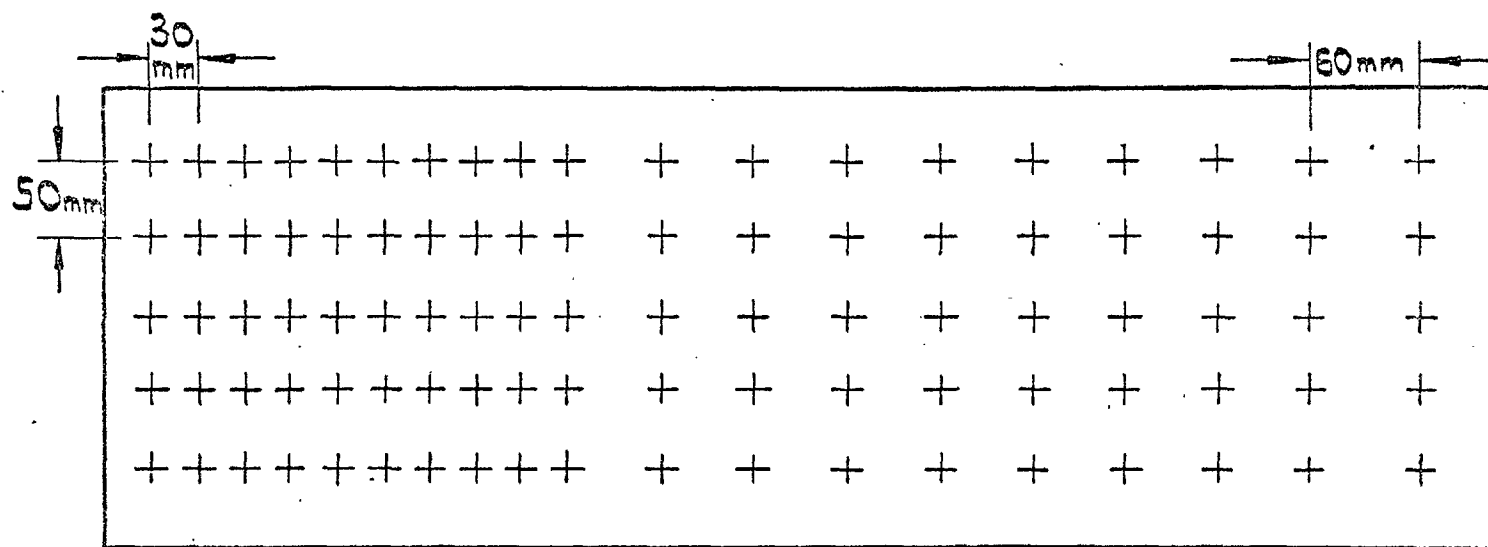


Fig. 3-1-1. Schematic diagram of the main flow circuit.



SCALE - 5mm:1mm

Fig. 3.1.2. Lay out of the static pressure holes in the base plate.

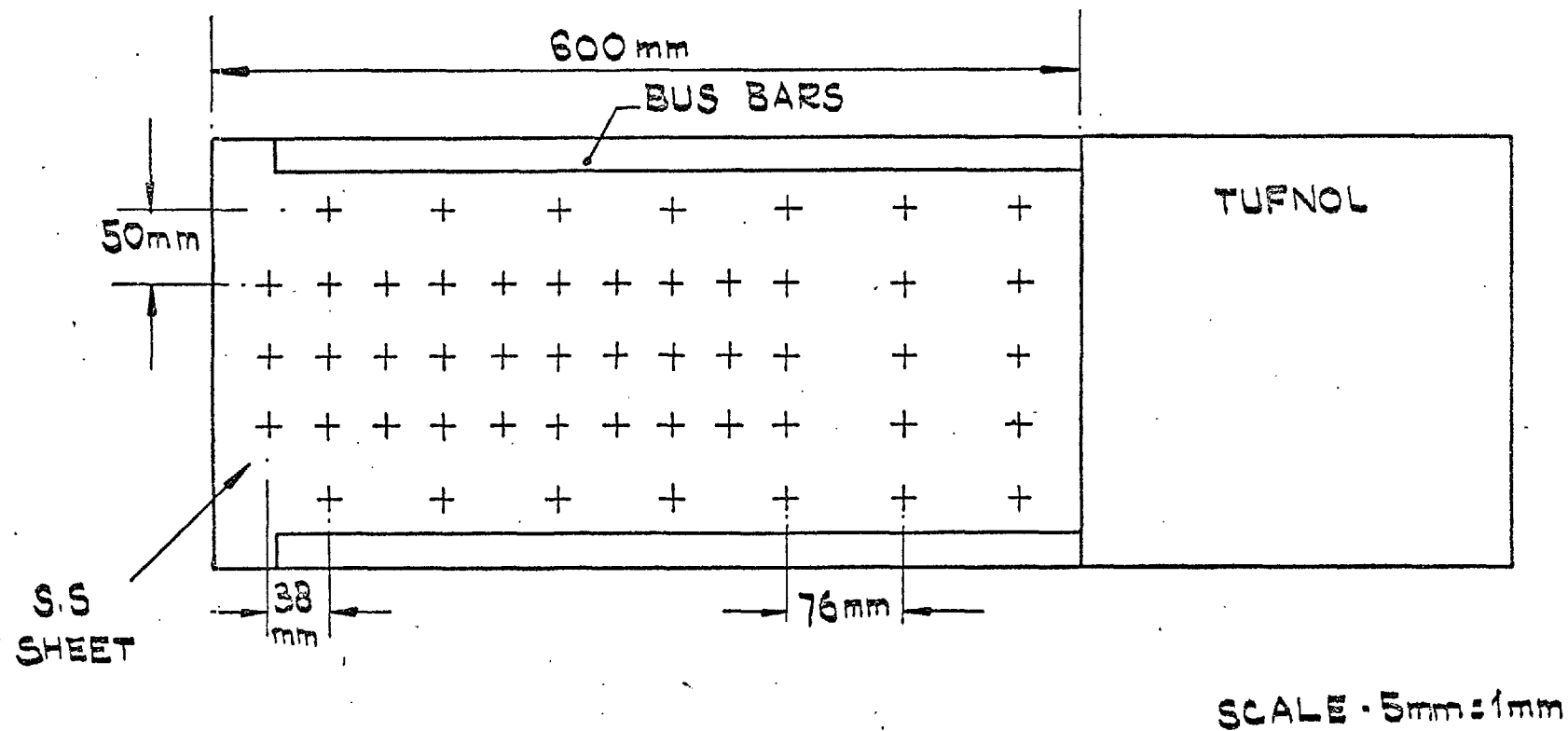


Fig. 3:1.3. Lay out of the thermocouple junctions on the heat-transfer plate.

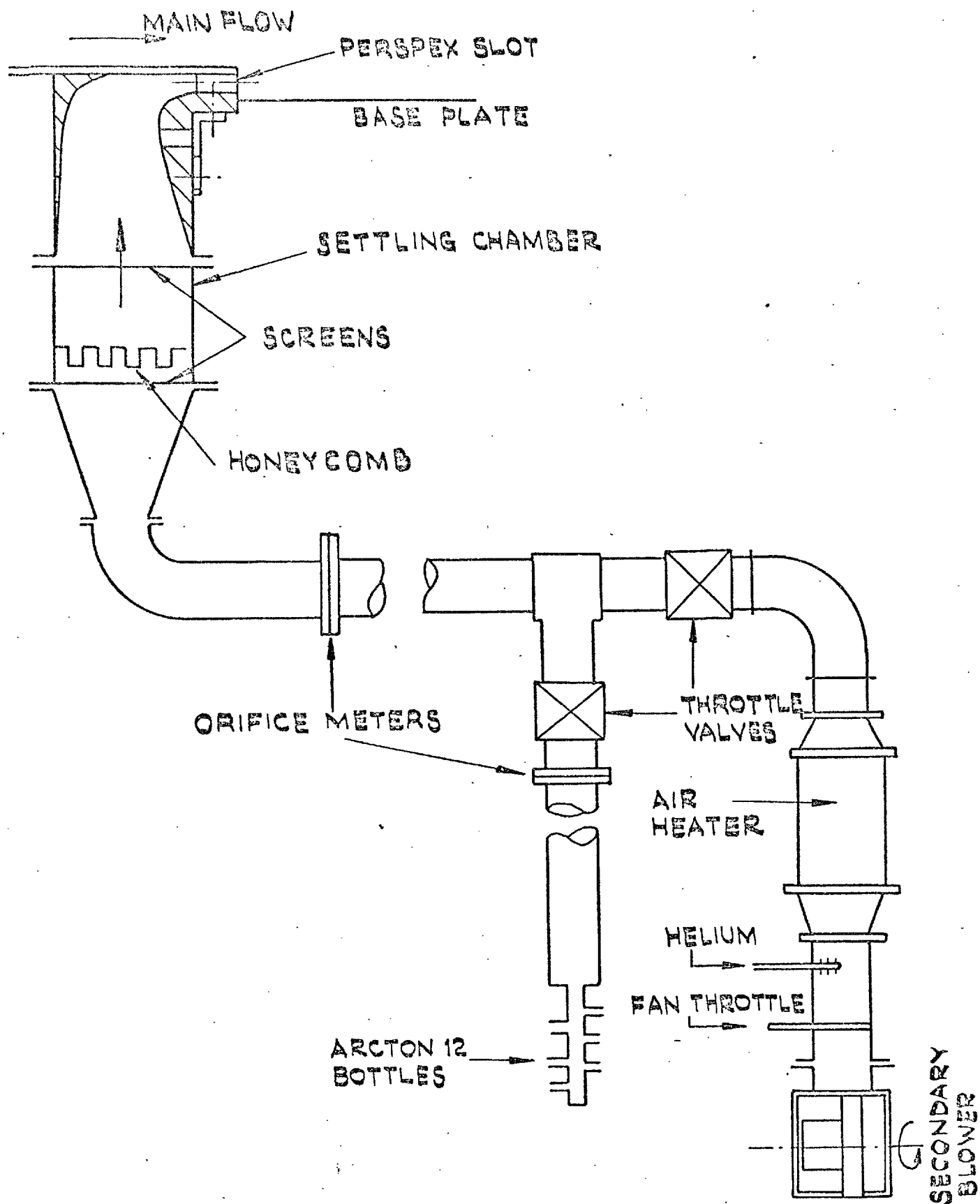


Fig. 3.1.4. Schematic diagram of the secondary flow circuit.

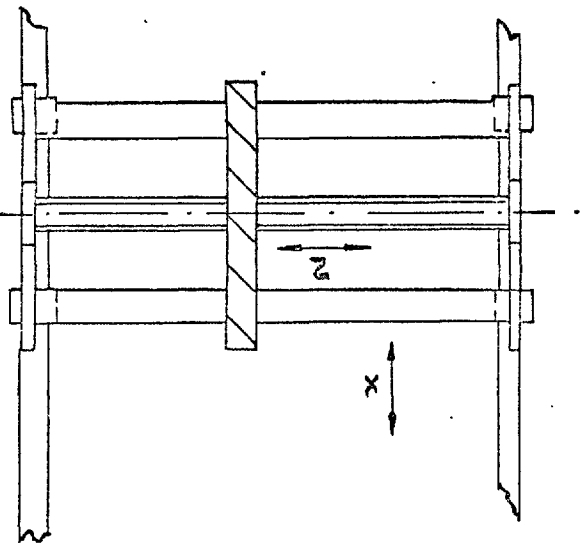
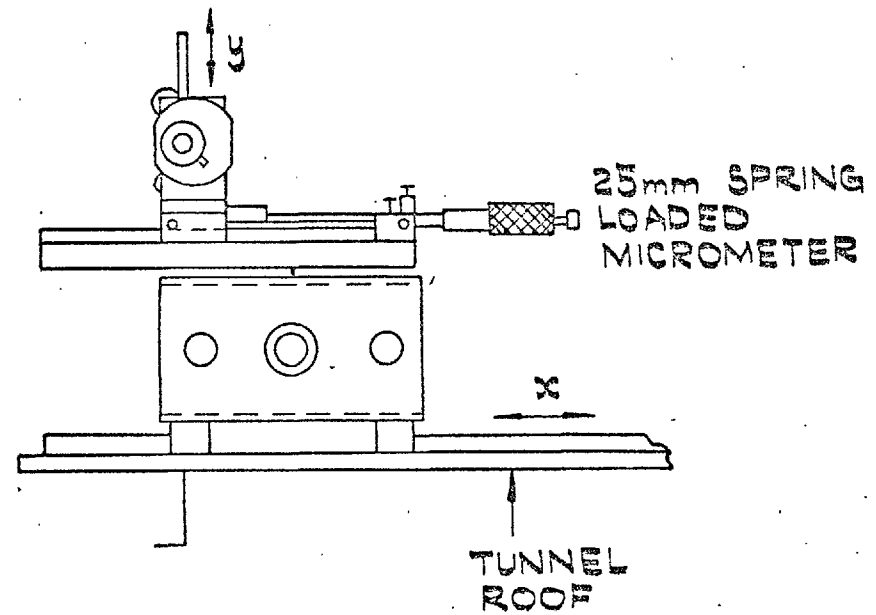
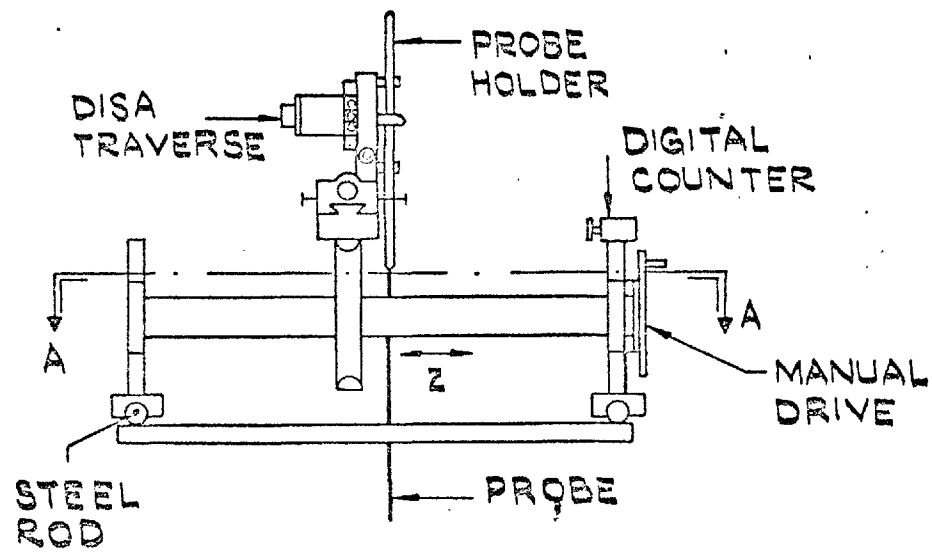


Fig. 3.1.5 The traverse mechanism.

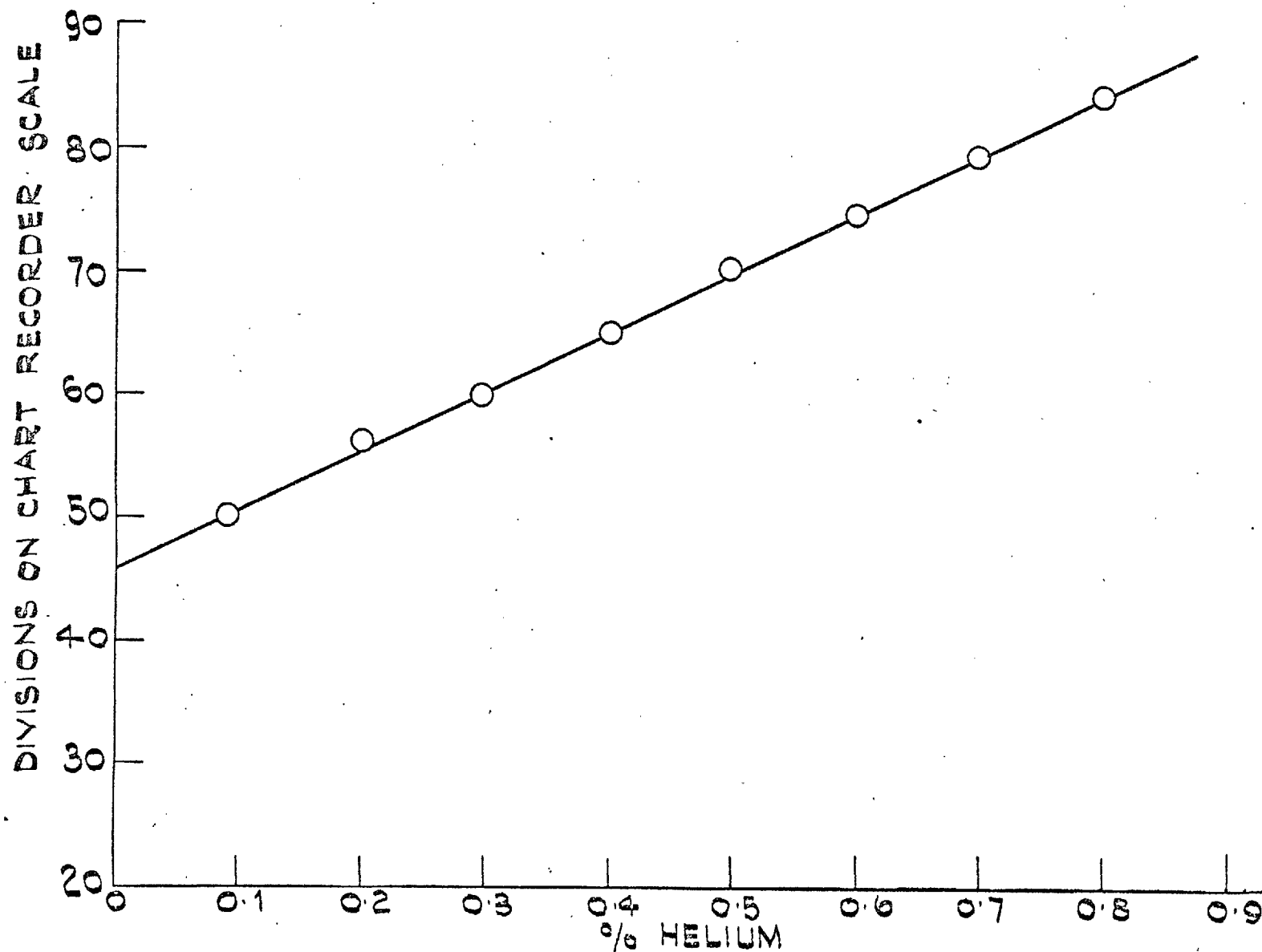


Fig. 3.1.6. The calibration curve of the kathorometer for small amounts of helium in air.

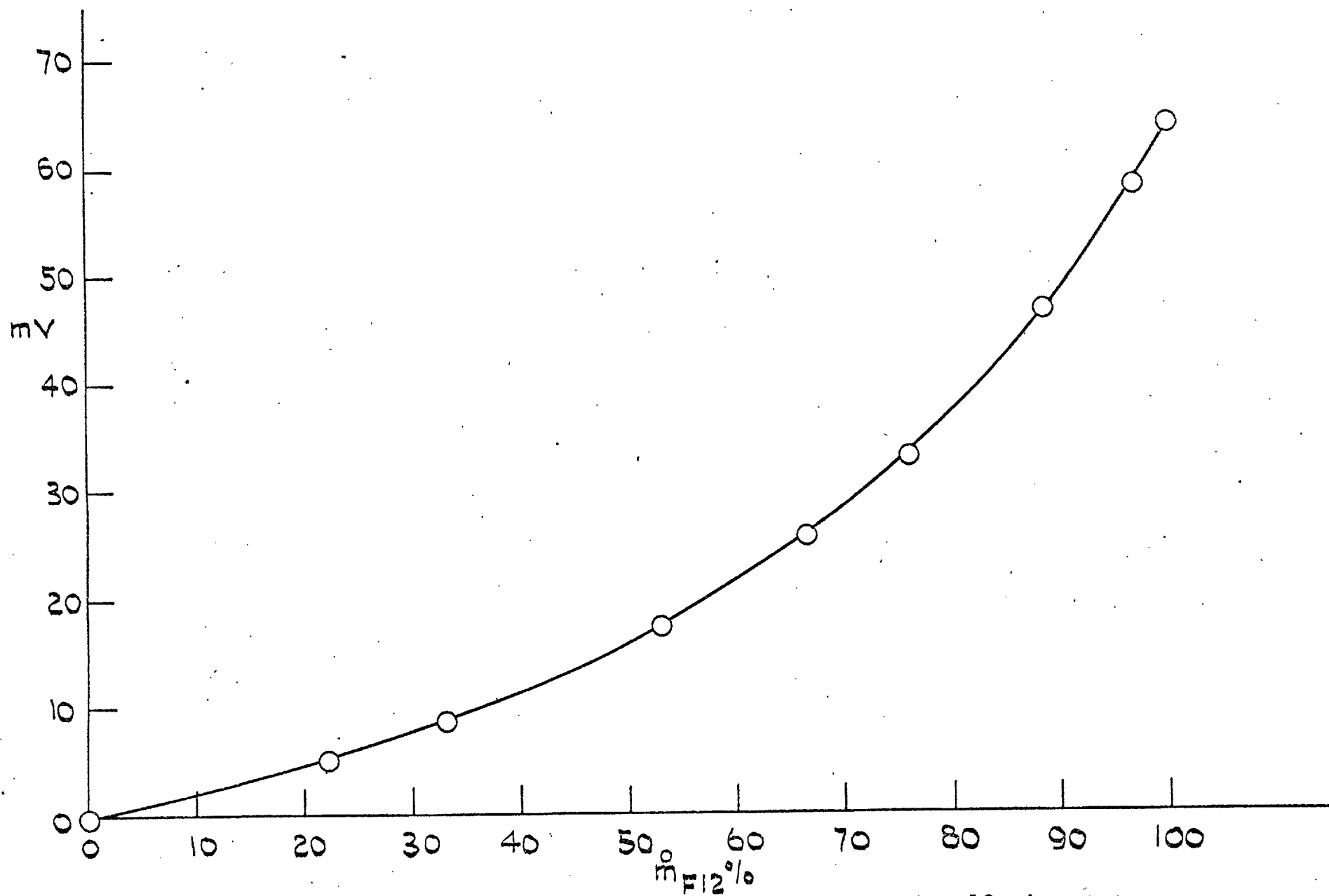


Fig. 3.1.7. The calibration curve of the kathorometer for Arcton-12-air mixtures.

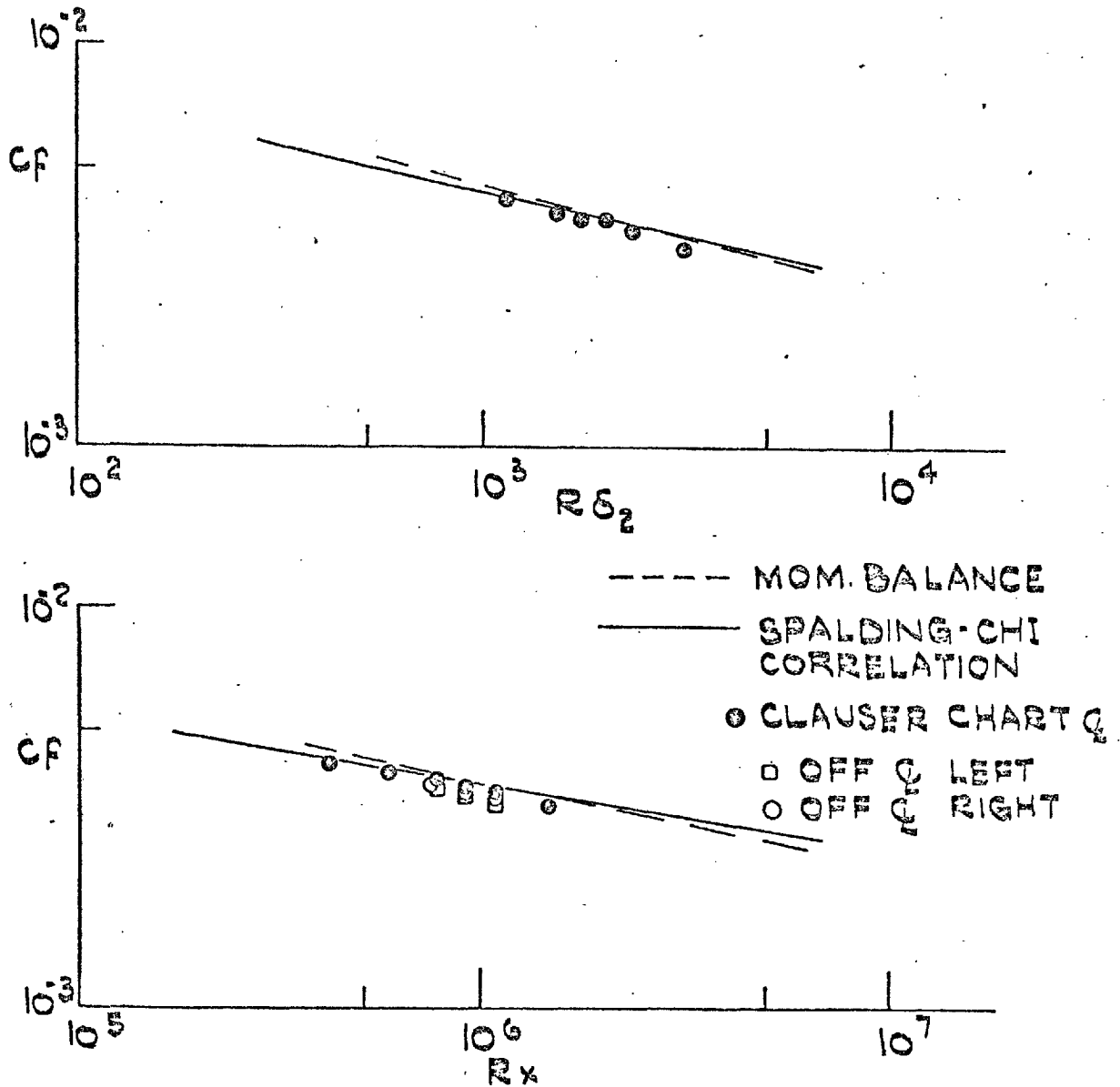


Fig. 3.1.8. Wall shear stress coefficient in a fully turbulent boundary layer on a flat plate.

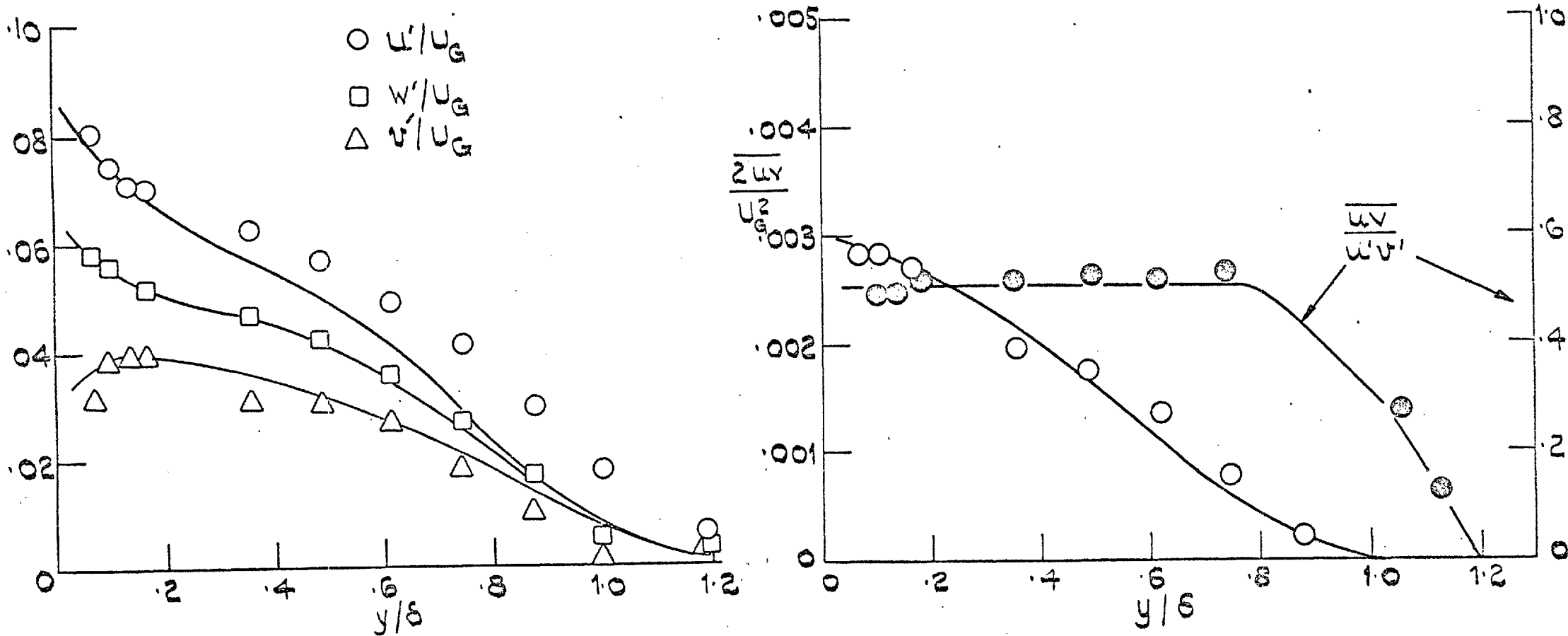


Fig. 3.1.9. Profiles of turbulence intensities and turbulent shear stress at $x = 635$ mm.

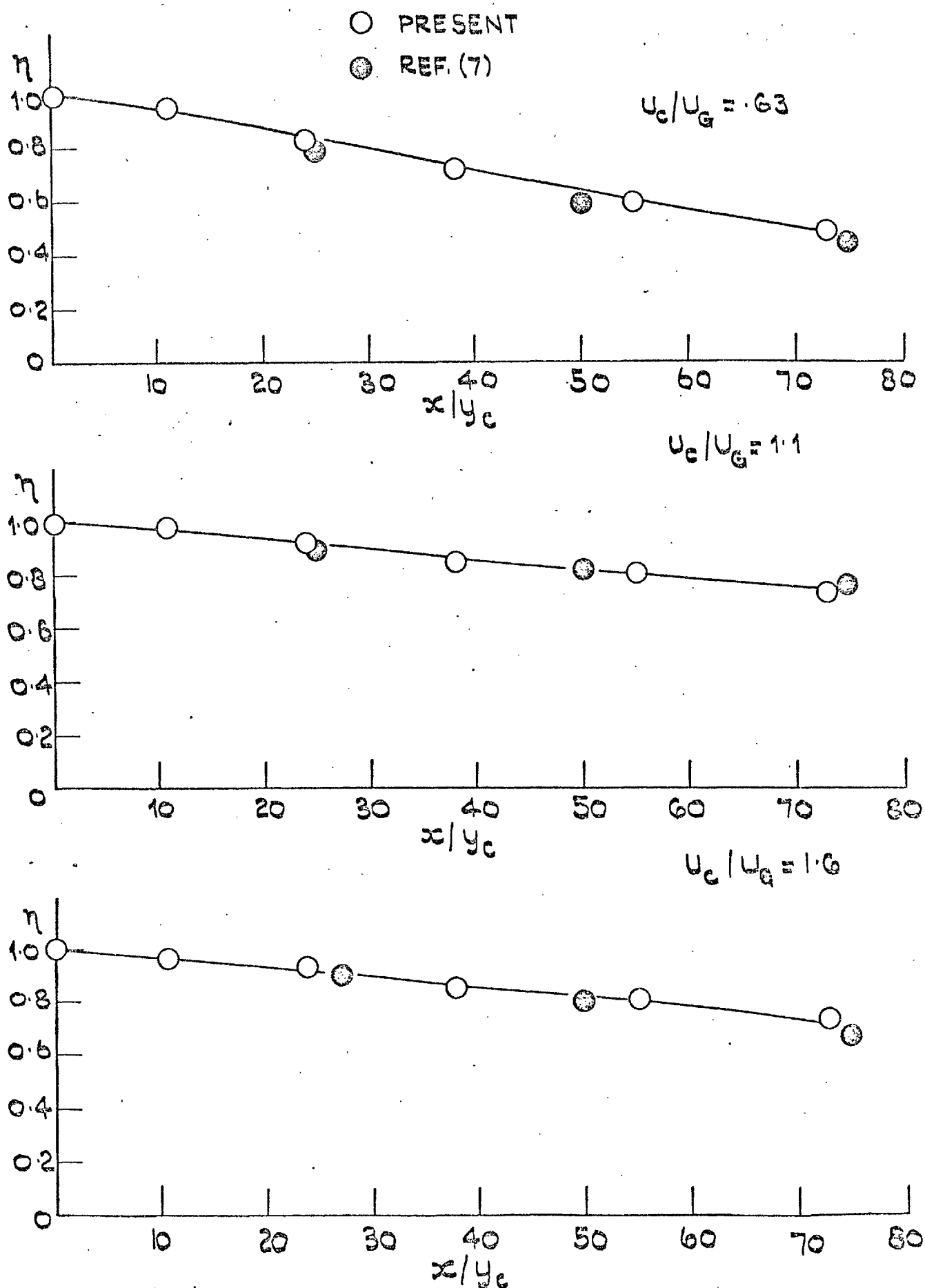


Fig. 3.1.10. Measured values of impervious wall effectiveness downstream of the two-dimensional slot.

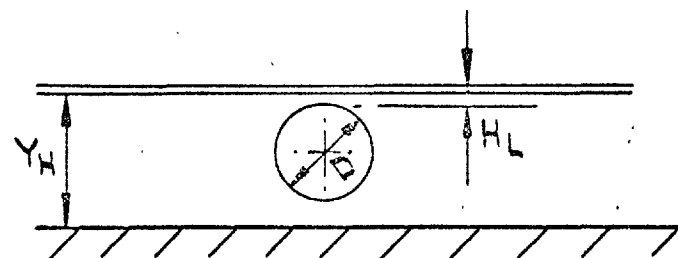
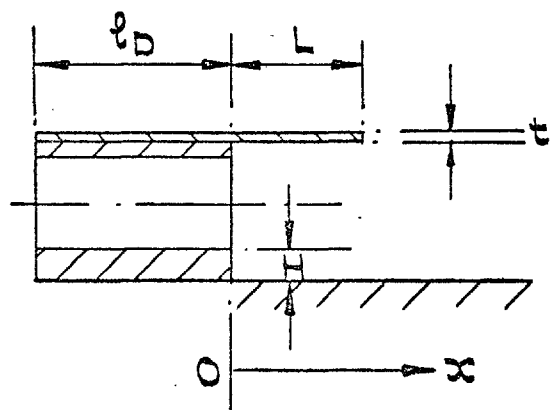


Fig. 3.2.1, Single hole slot geometry.

Table 3.2.1

X	D	L	H	H _L	ℓ _D	t	U _C /U _G	P _C /P _G
0 to 880	5, 8, 10, 12.5	0	.5	1.	25	1.25	.4 to 2.4	1.0
0 to 880	5, 8, 12.5 ~~5~~	0	.5	1.0	25	1.25	.2 to 1.0	4.17
0 to 880	12.5	0	.5	1.0	25	1.25	∞	1.0
0 to 640	8	0	.5 to 8.0	1.0	25	1.25	.85 to 1.8	1.0
0 to 880	10	0 to 2.2	.5	1.0	25	1.25	1.0 to 1.8	1.0

(all dimensions in mm).

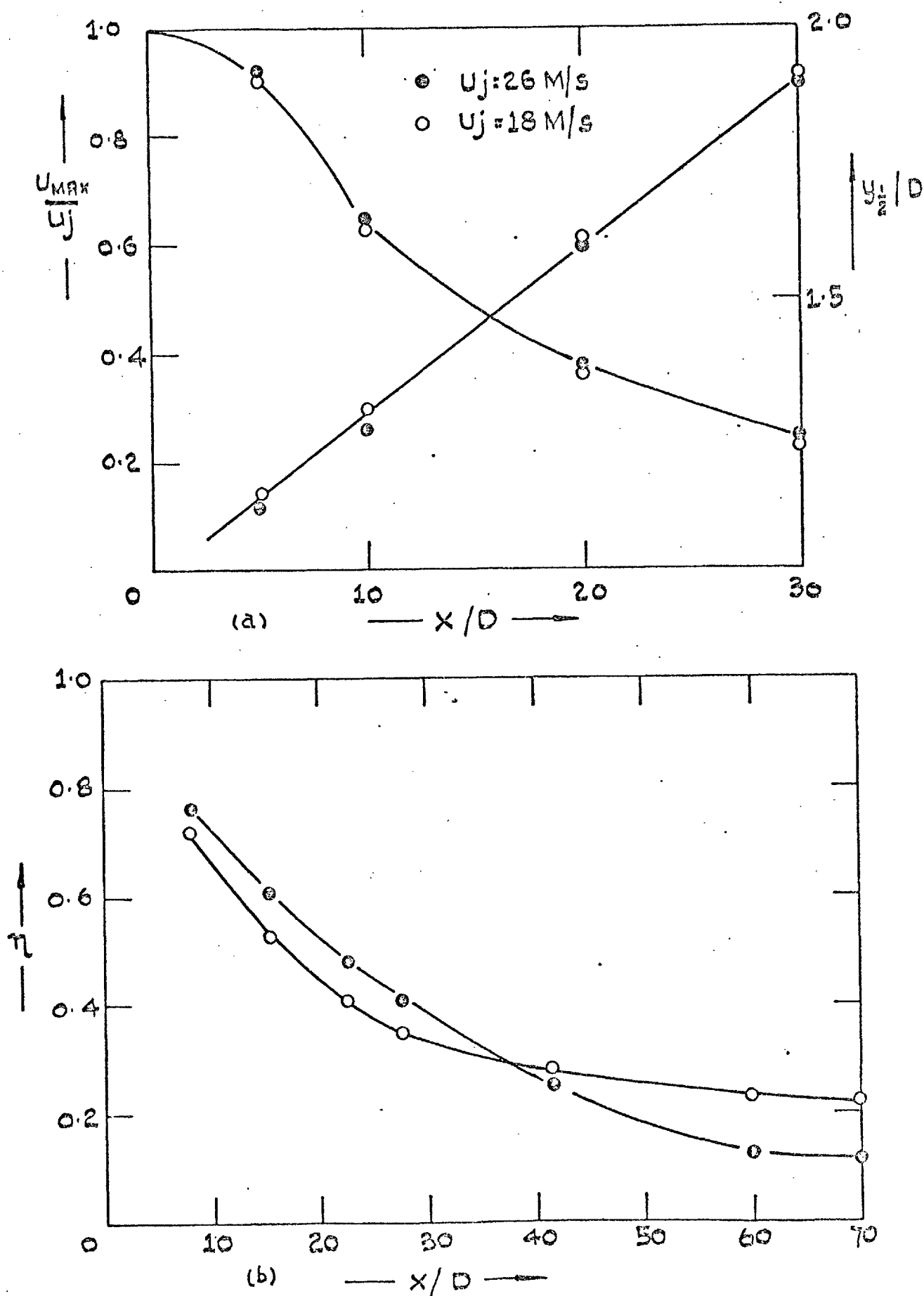


Fig. 3.2.2. Measured wall jet development and impervious wall effectiveness along the centre-line for $D = 12.5 \text{ mm}$; $U_G = 0$

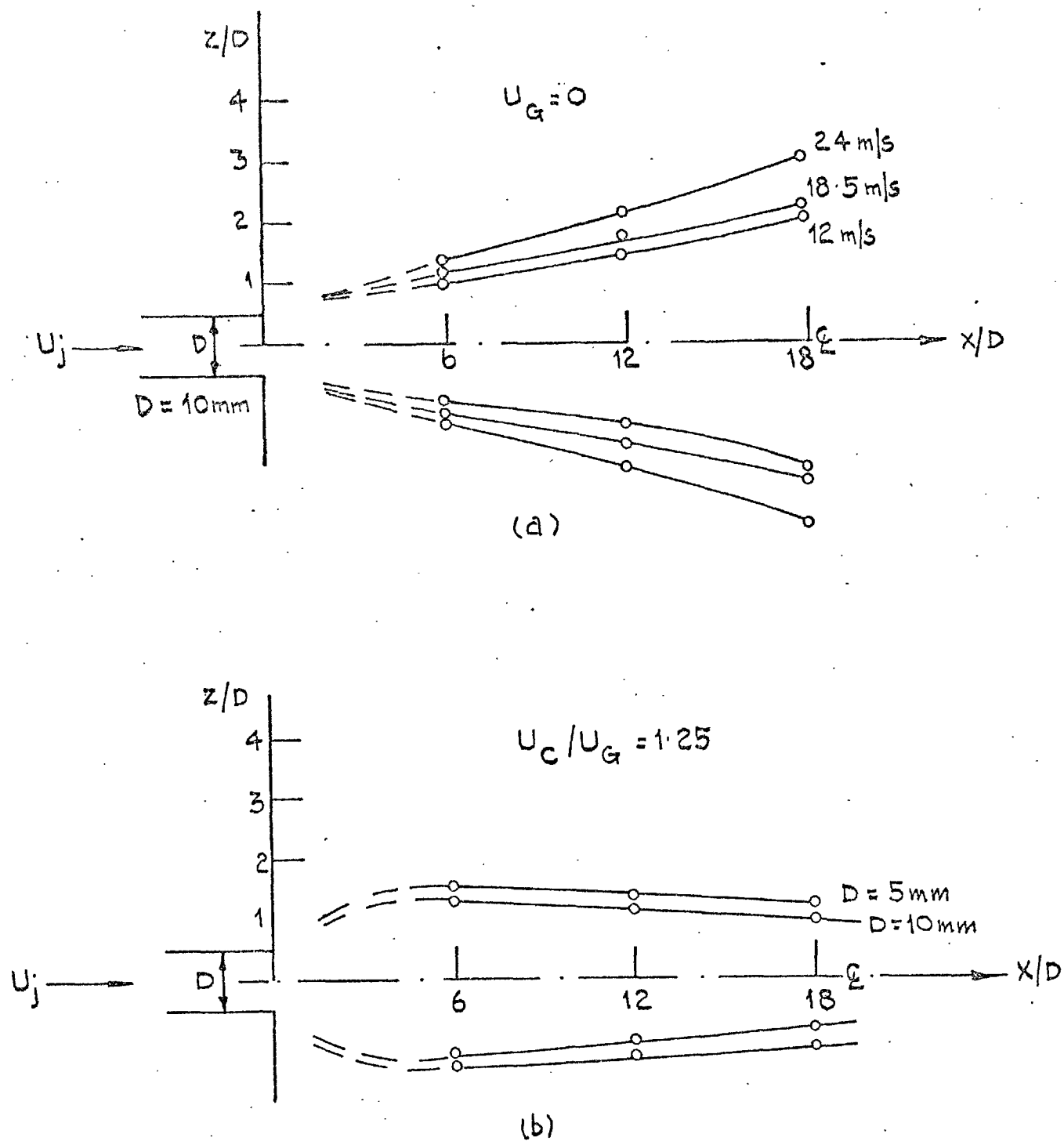


Fig. 3.2.3. Measured values of the lateral spread of round wall jets.

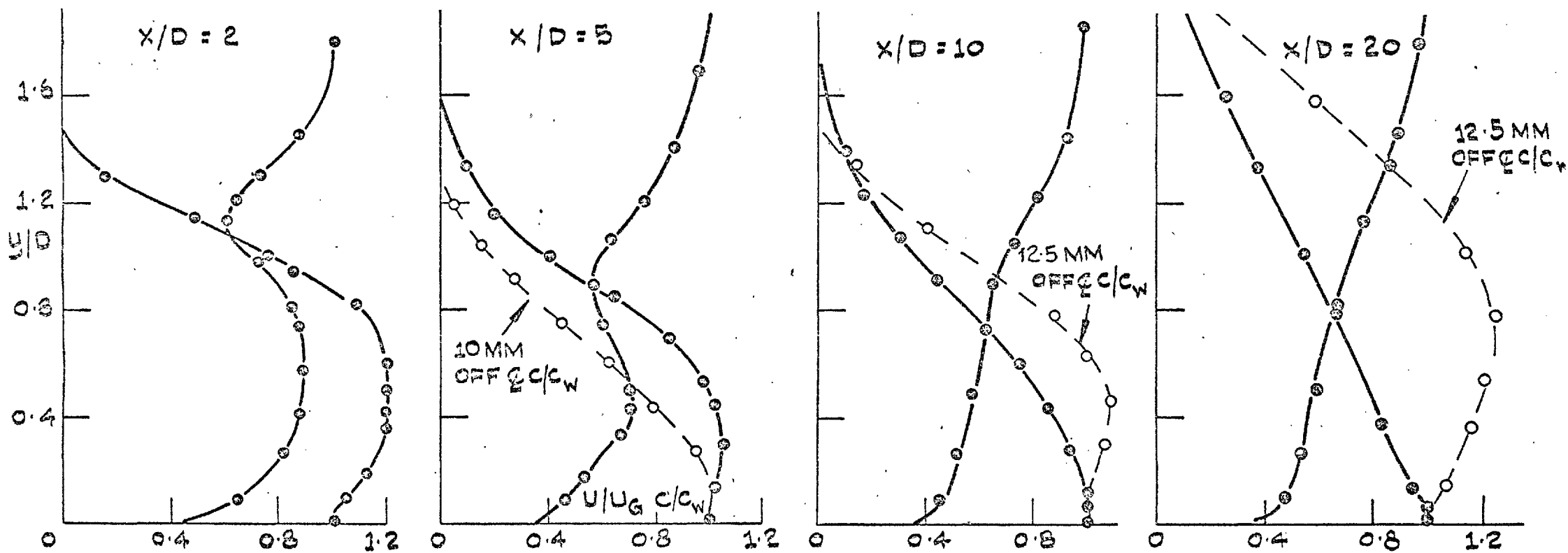


Fig. 3.2.4.(a) Measured profiles of longitudinal velocity and concentration on the centre line downstream of a single hole slot for $U_C/U_G = 0.9$

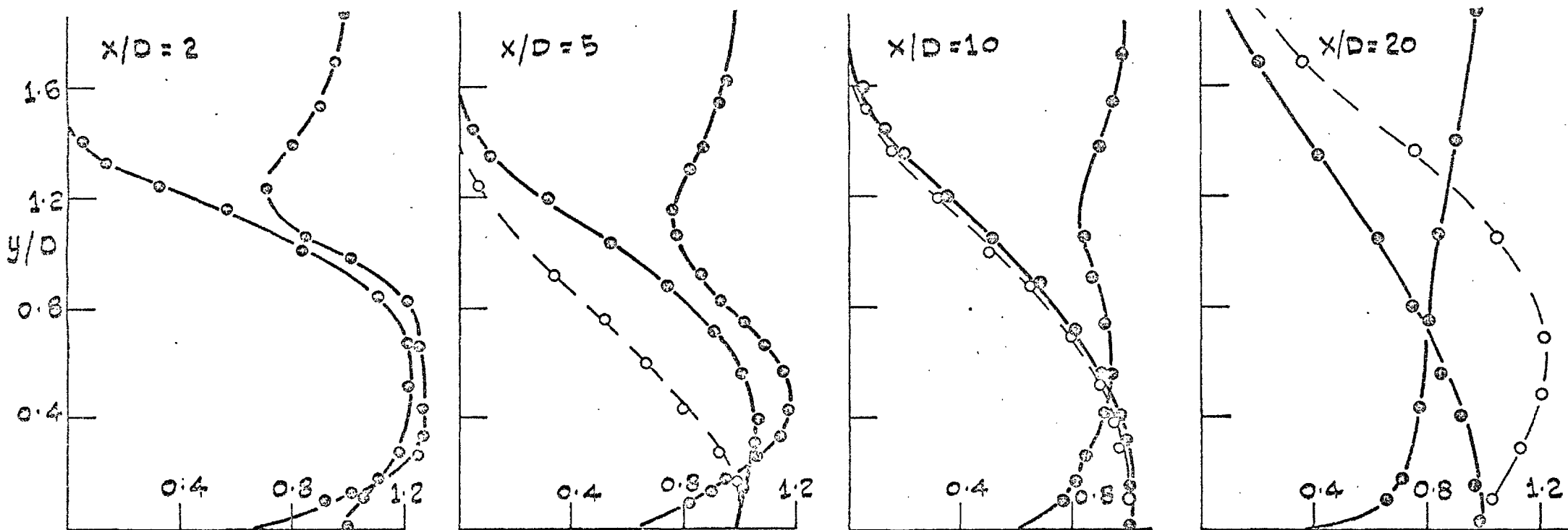


Fig. 3.2.4.(b) Measured profiles of longitudinal velocity and concentration on the centre line downstream of a single hole slot for $U_C/U_G = 1.3$

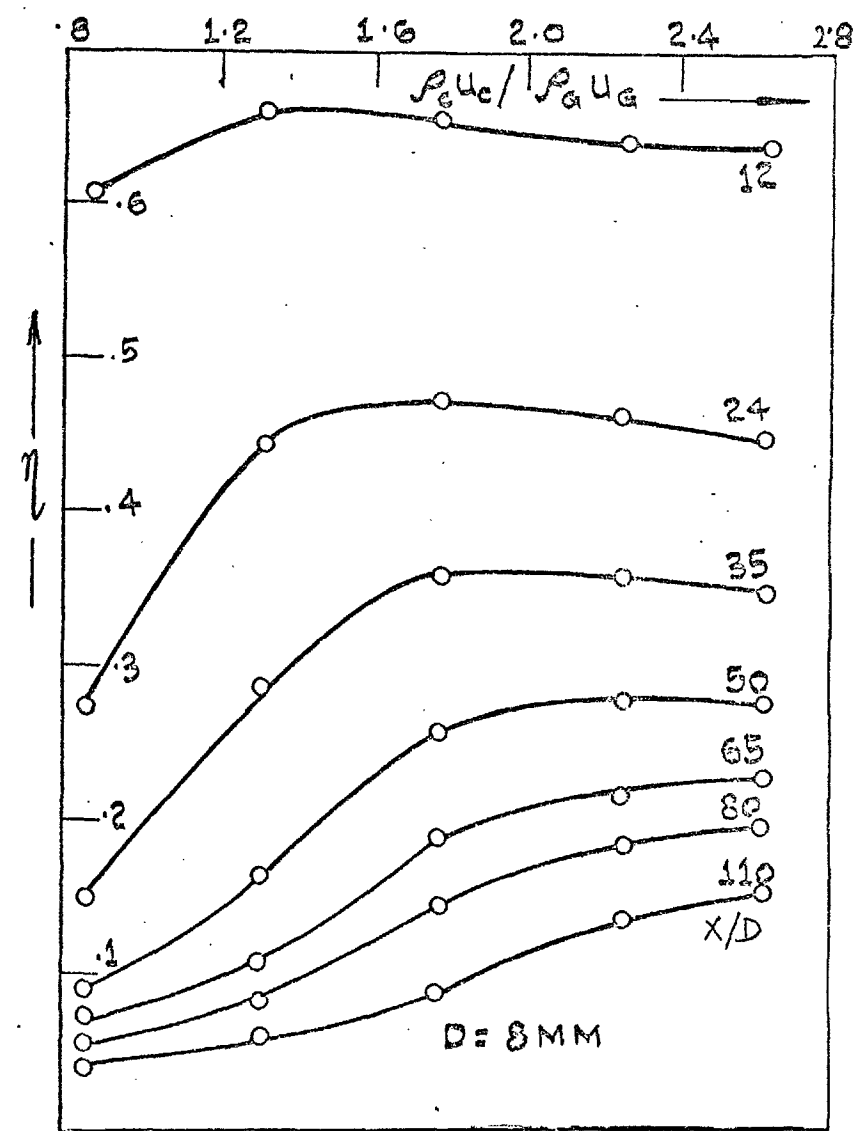
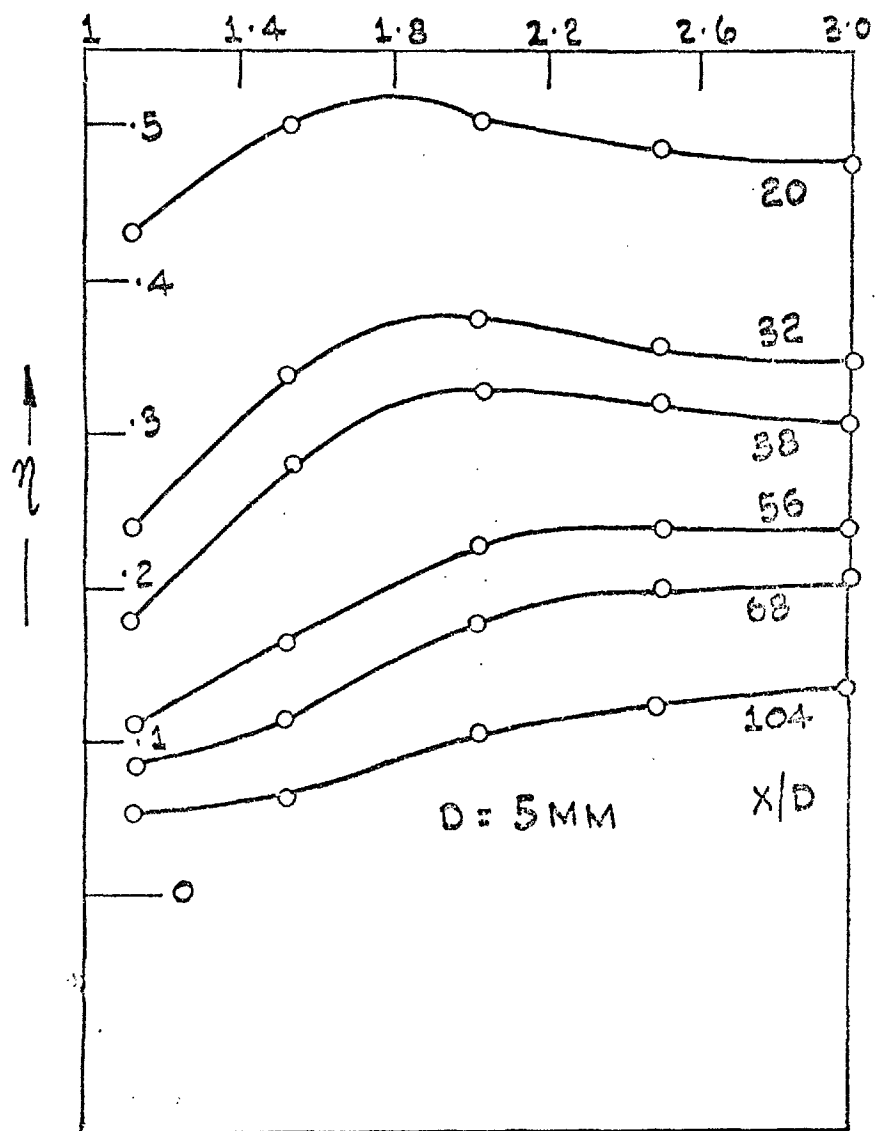


Fig. 3.2.5. Measured impervious wall effectiveness along the centre line of single hole slots for $P_c/P_g = 1.0$

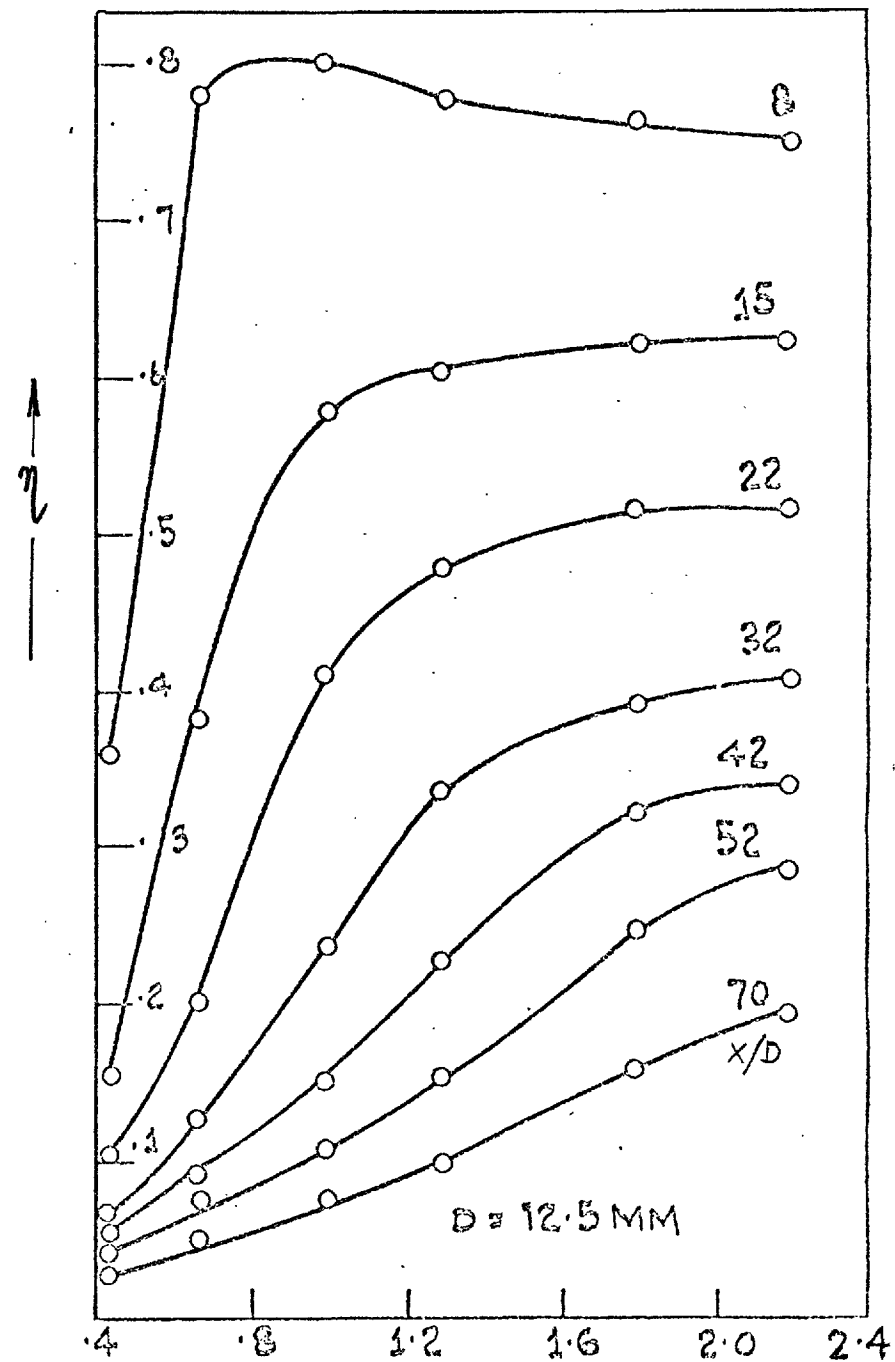
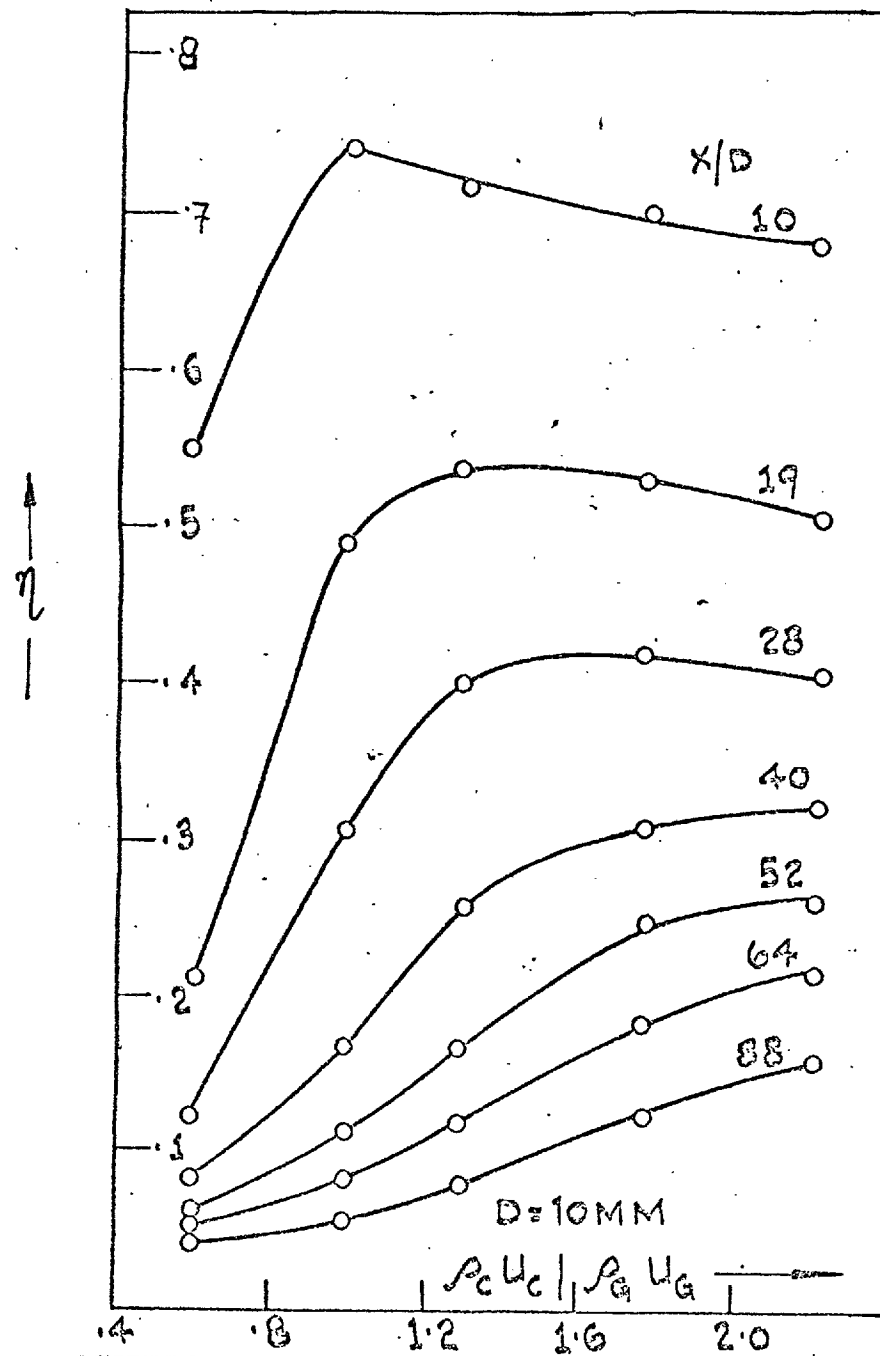
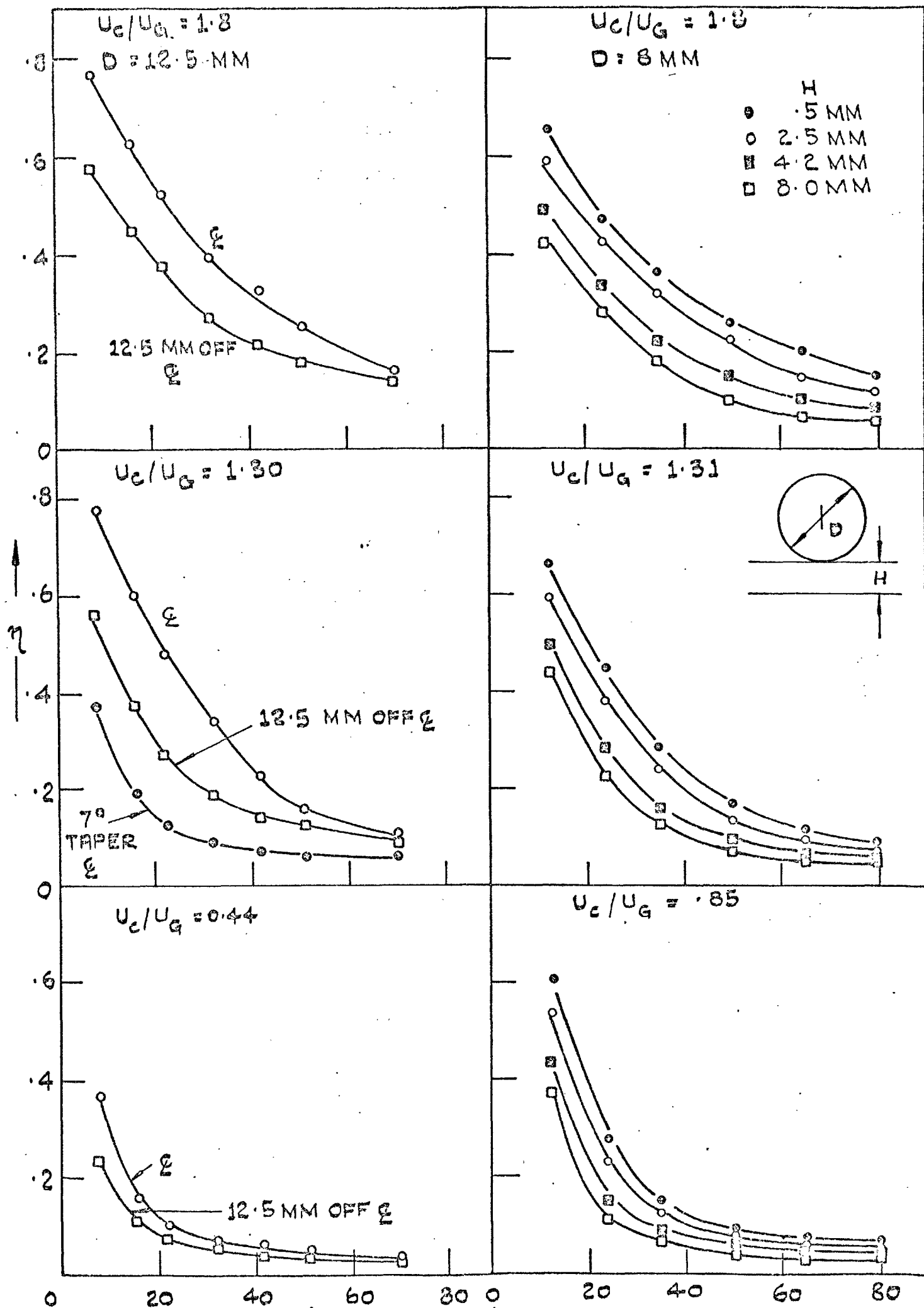


Fig. 3.2.5.(contd.) Measured impervious wall effectiveness along the centre line of single hole slots for $\rho_c / \rho_g = 10$



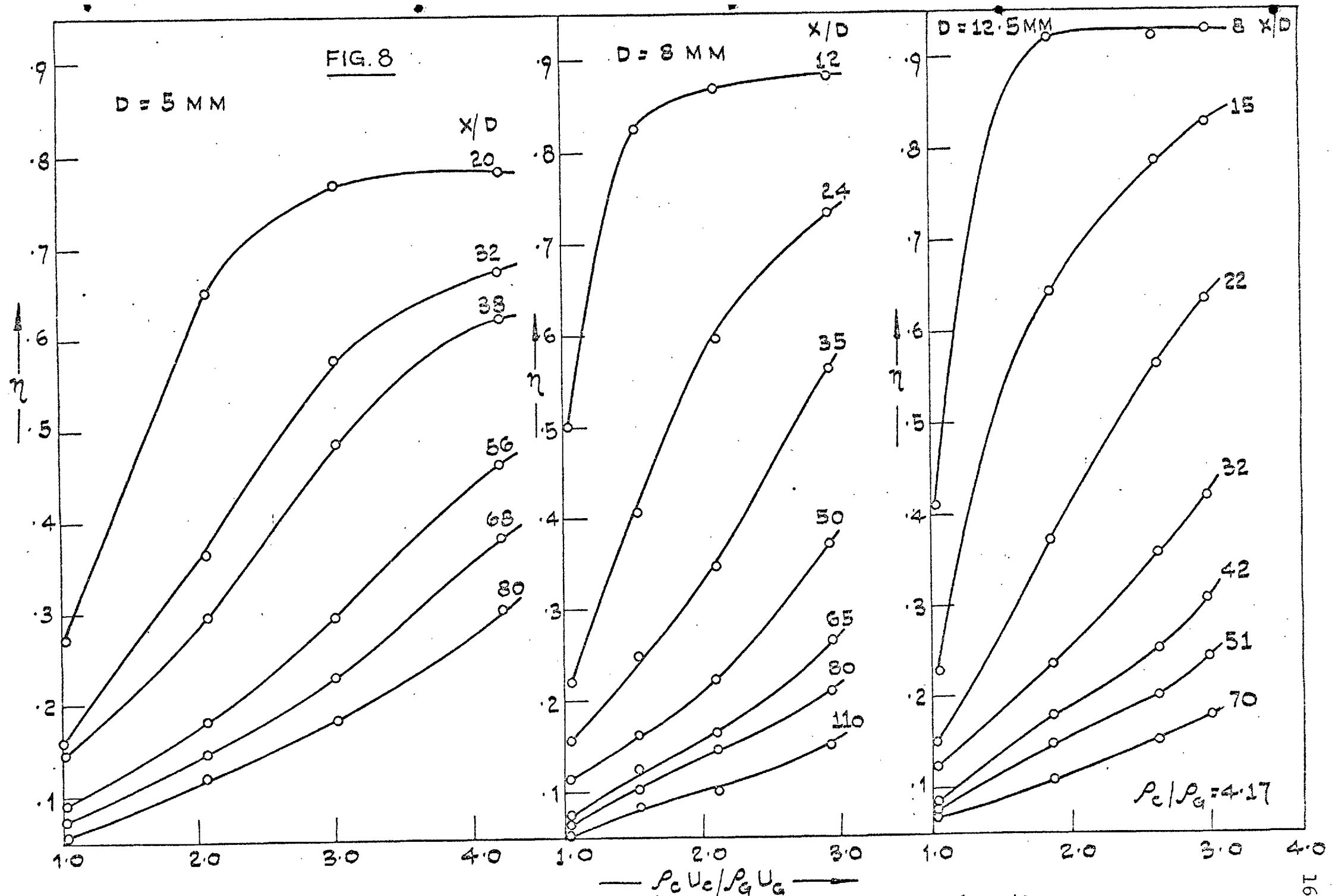


Fig. 3.2.8. Measured η_{imp} along the centre lines of three single hole slots for $\rho_c/\rho_g = 4.17$.

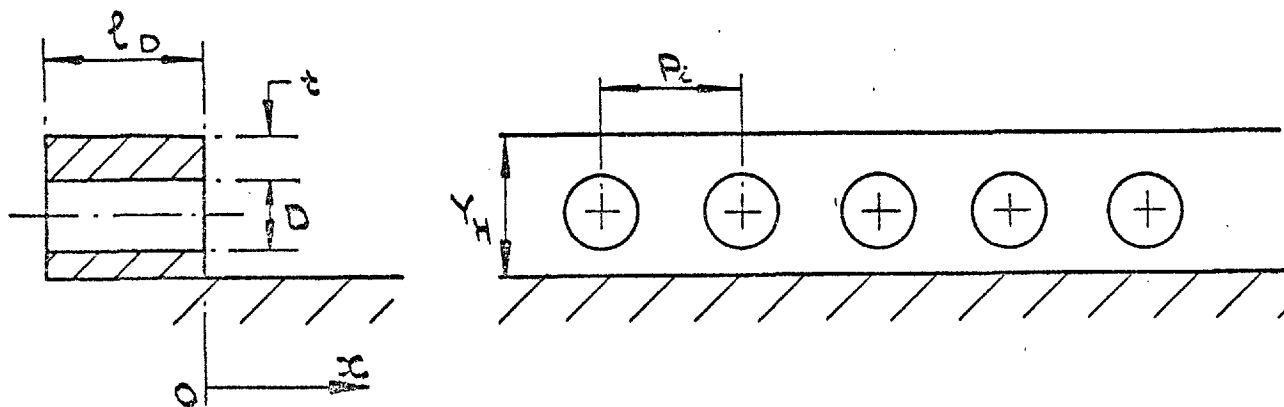


Fig. 3.2.9. Multi hole slot geometry.

TABLE 3.2.2

	Y_H	D	P_i	t	l_o	N	A_o	A_o/A_t
Slot 1	15.25	12.5	25	2.25	25	1	15.3×10^2	.328
Slot 2	23.25	12.5	25	10.25	25	1	15.3×10^2	.215
Slot 3	10.95	9	12.5	1.60	18	1	15.3×10^2	.457
Slot 4	3.72	2	4	1.64	4	1	2.45×10^2	.215
Slot 5	15.25	3.18	4.77	2.25	6.36	3	15.3×10^2	.328

(all length dimensions in mm)

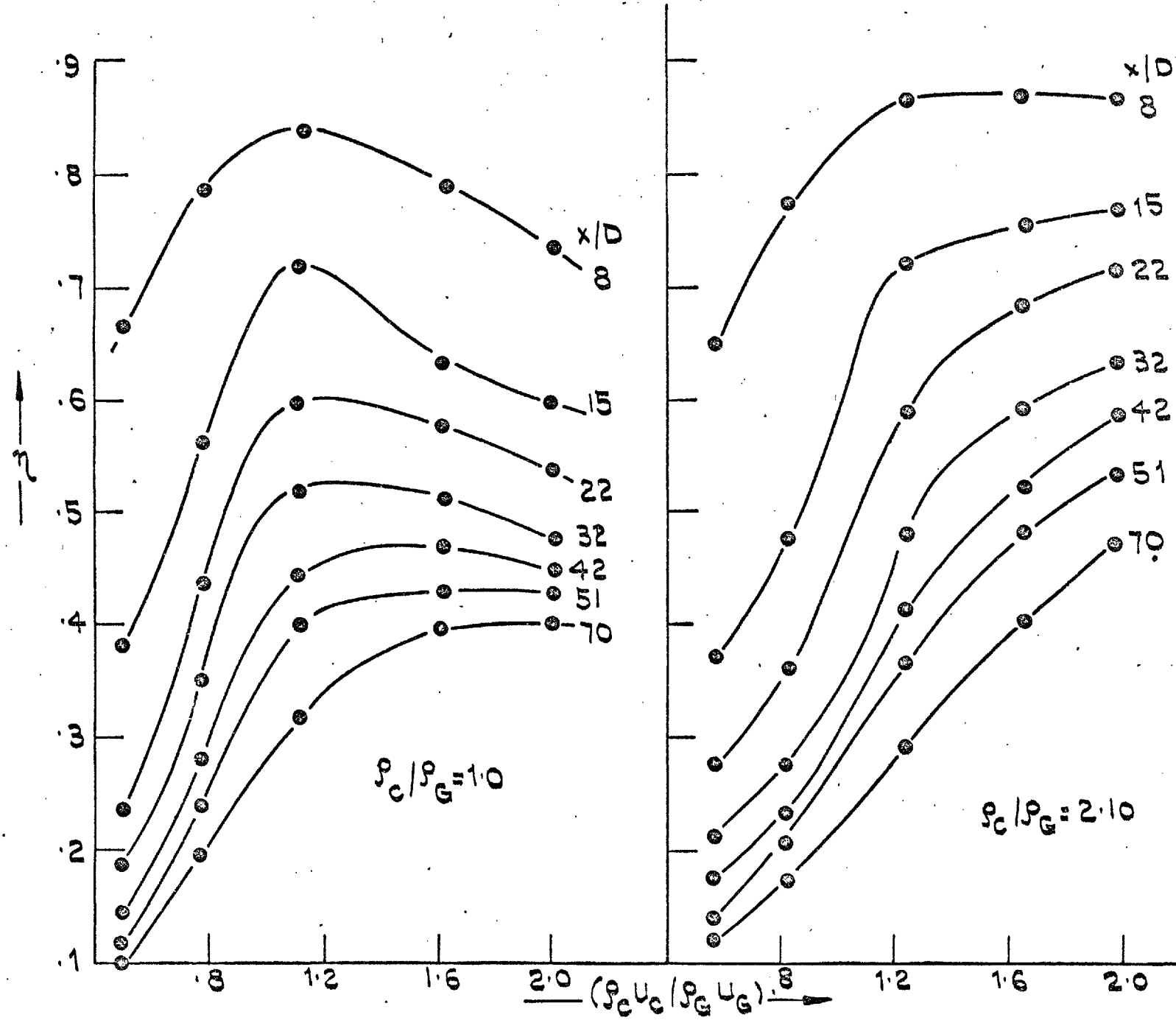


Fig. 3.2.10. Measured impervious wall effectiveness downstream of slot-1.

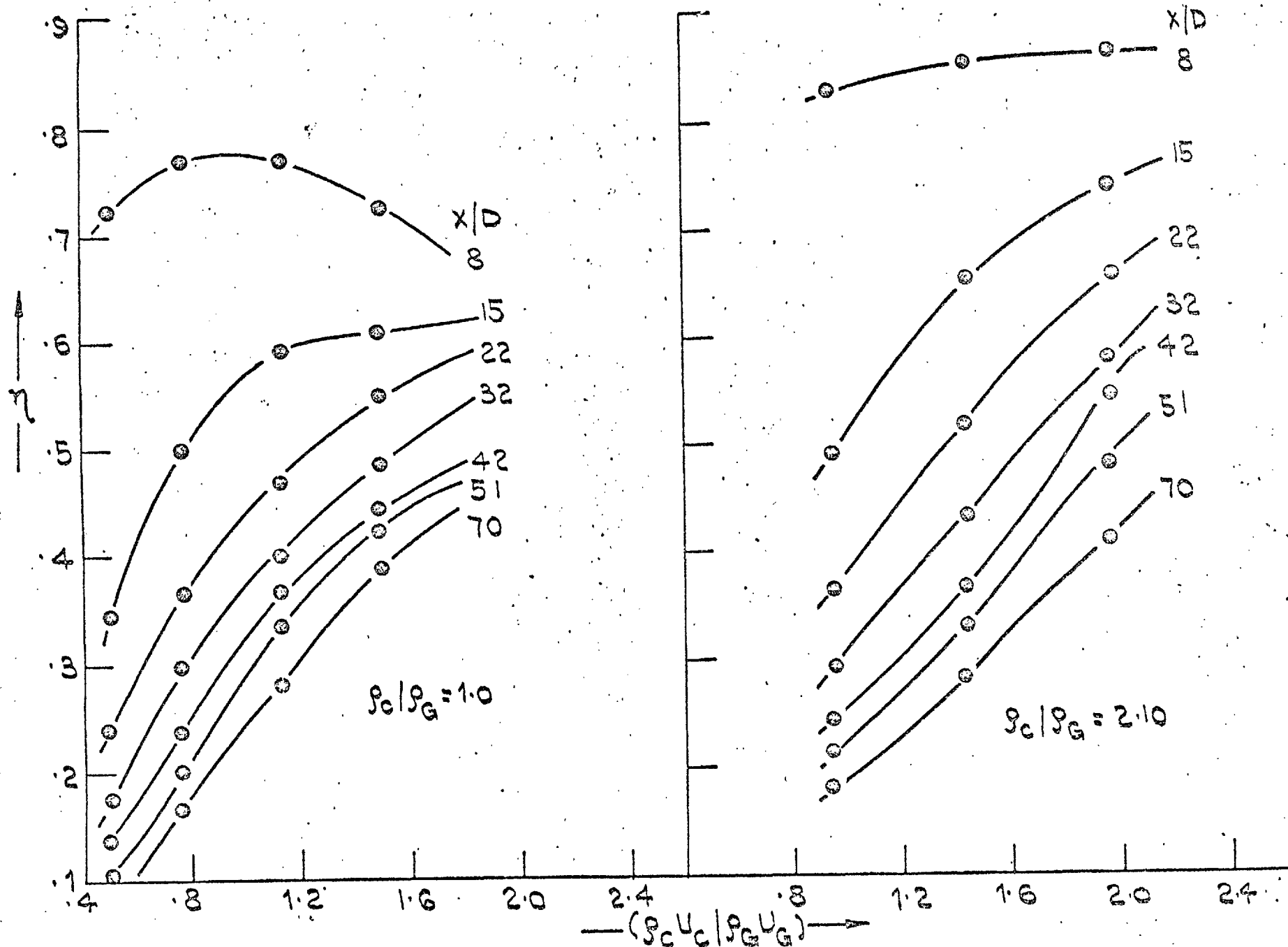


Fig. 3.2.11. Measured impervious wall effectiveness downstream of Slot 2.

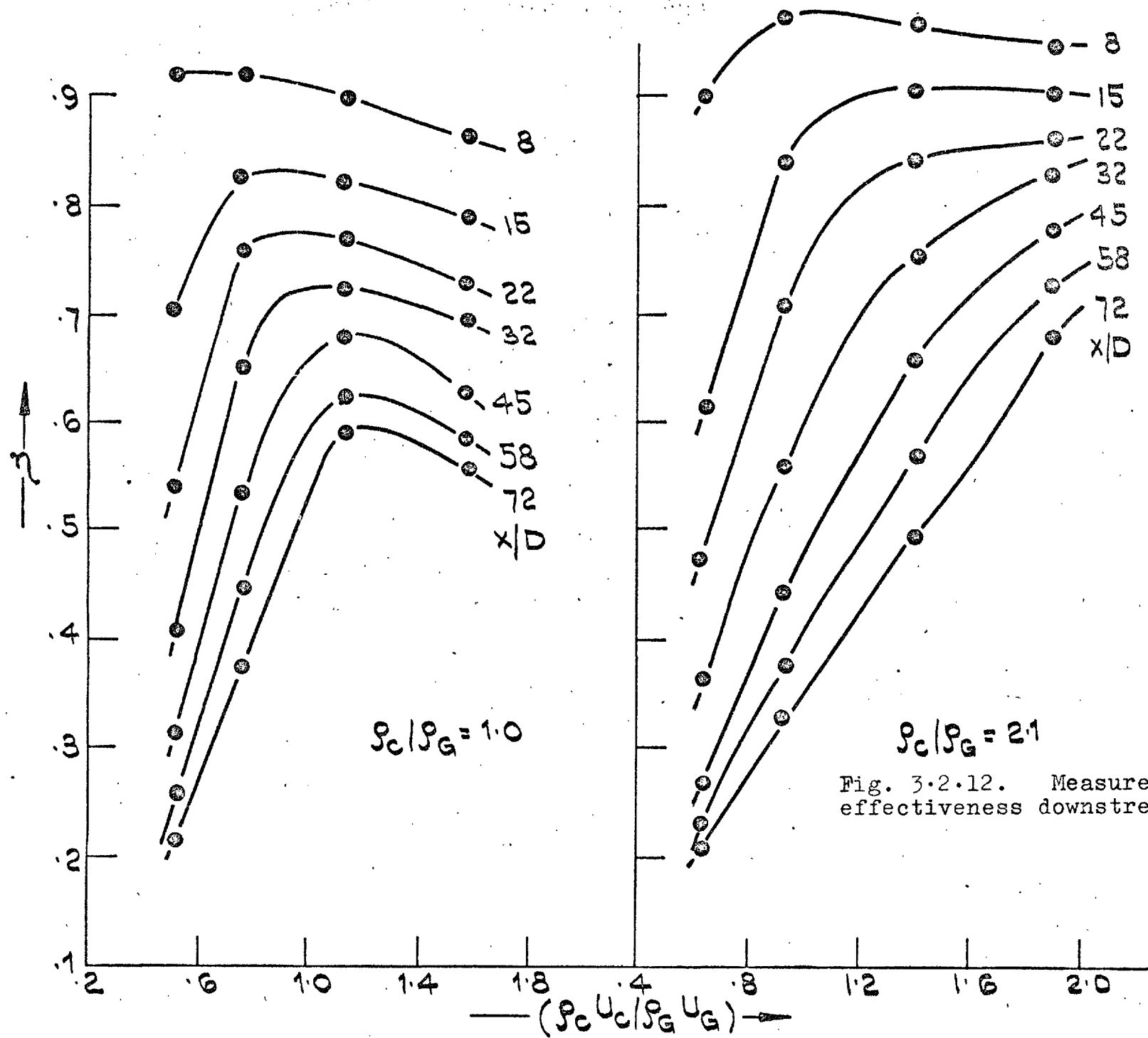
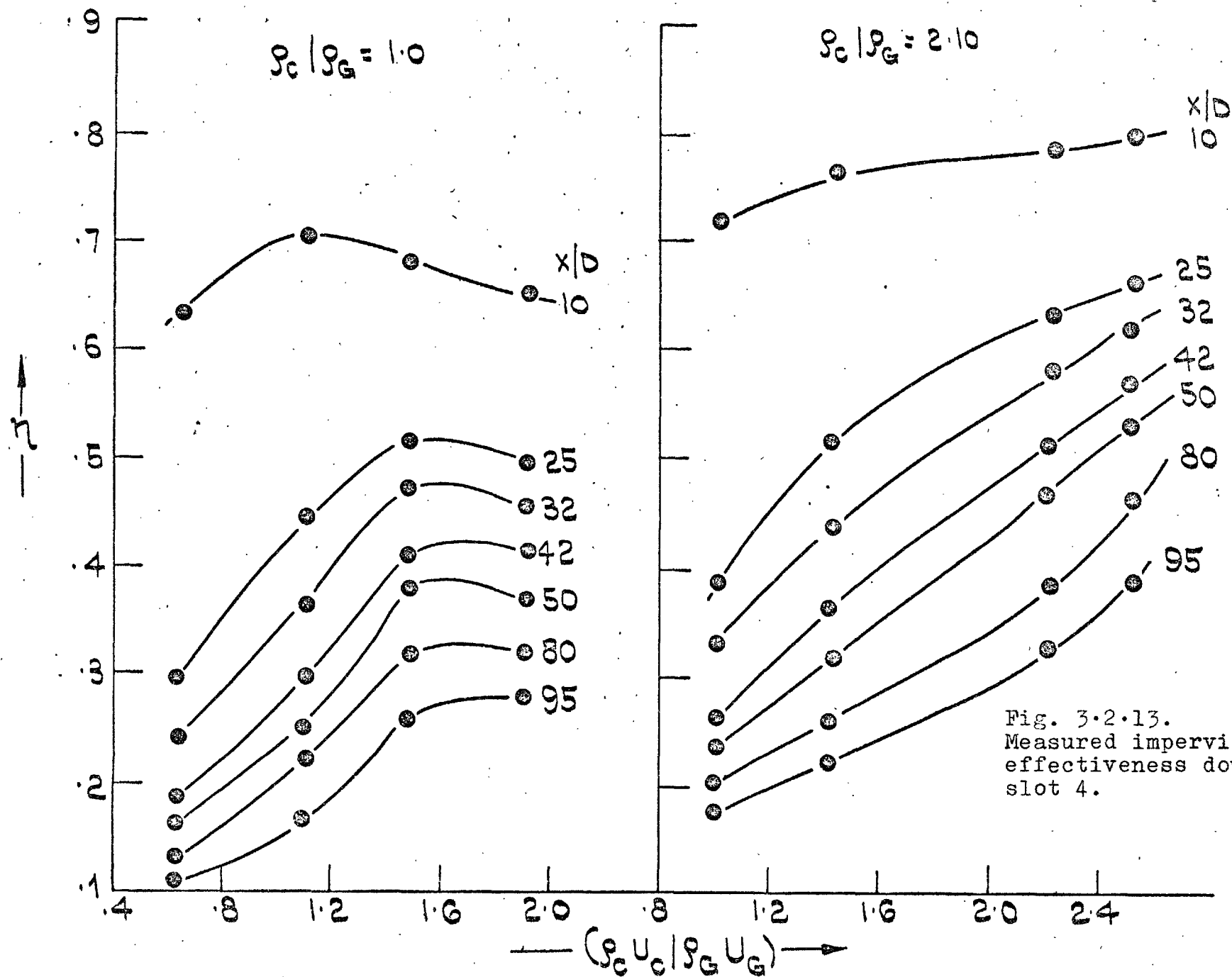
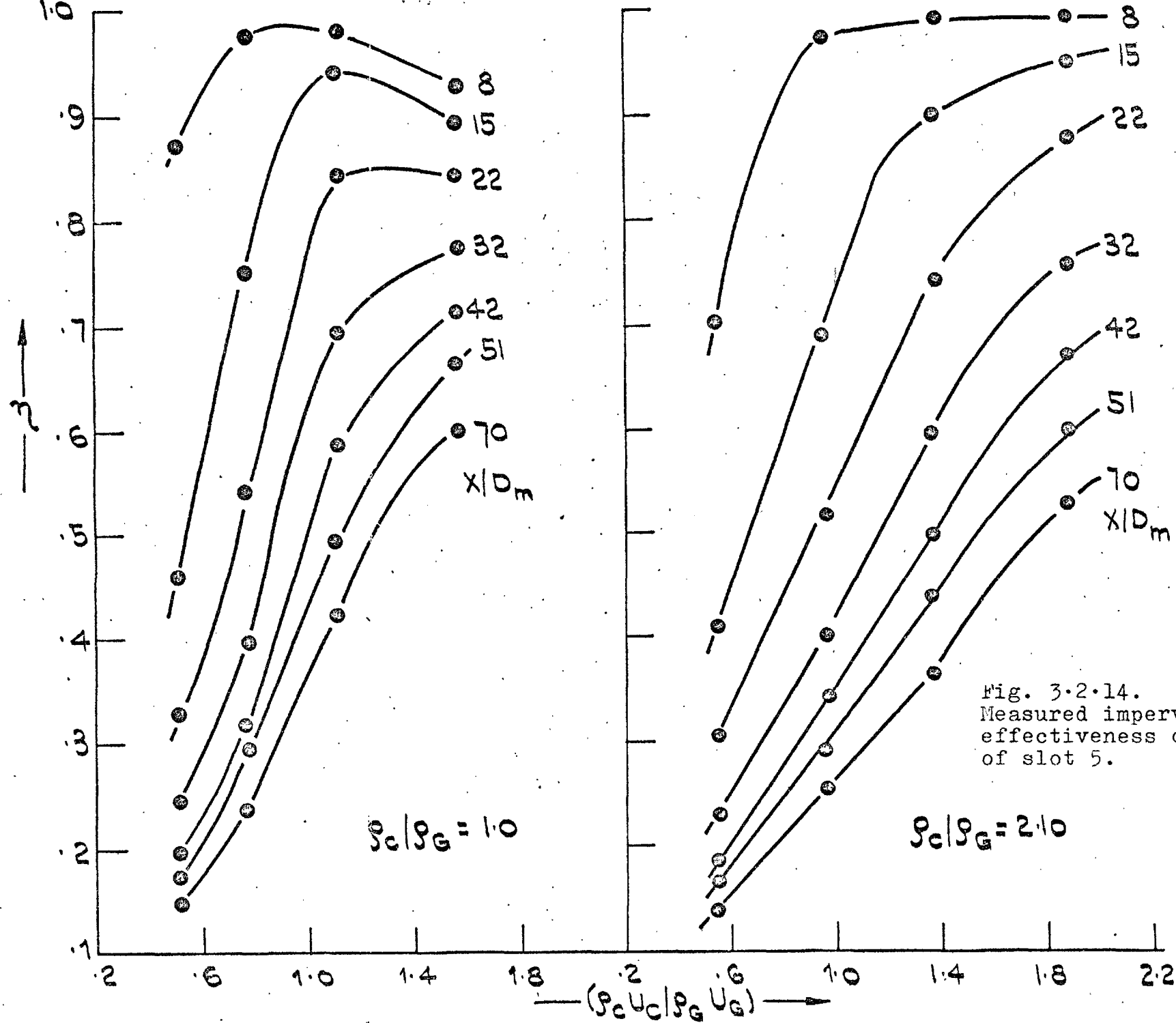


Fig. 3.2.12. Measured impervious wall effectiveness downstream of slot 3.





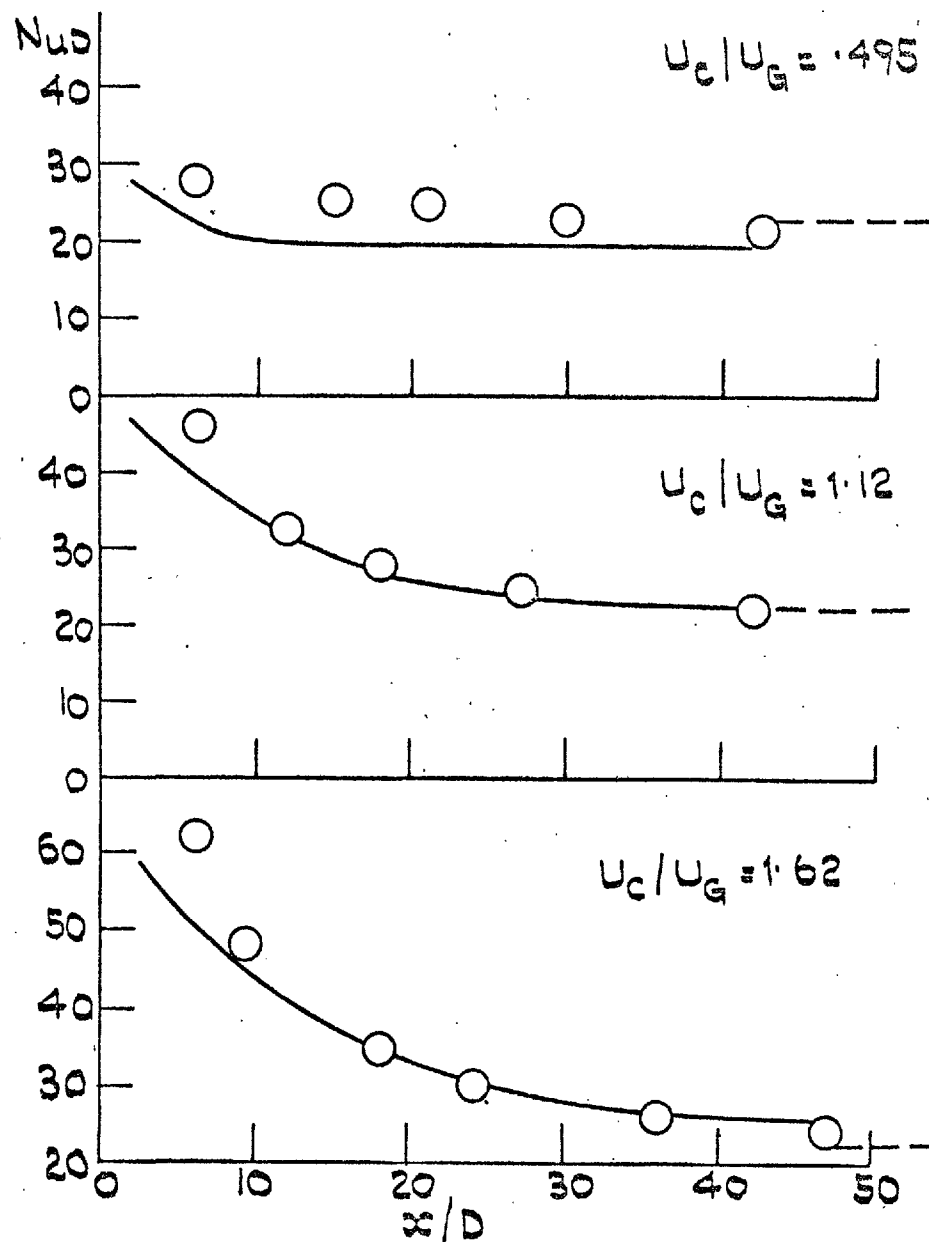
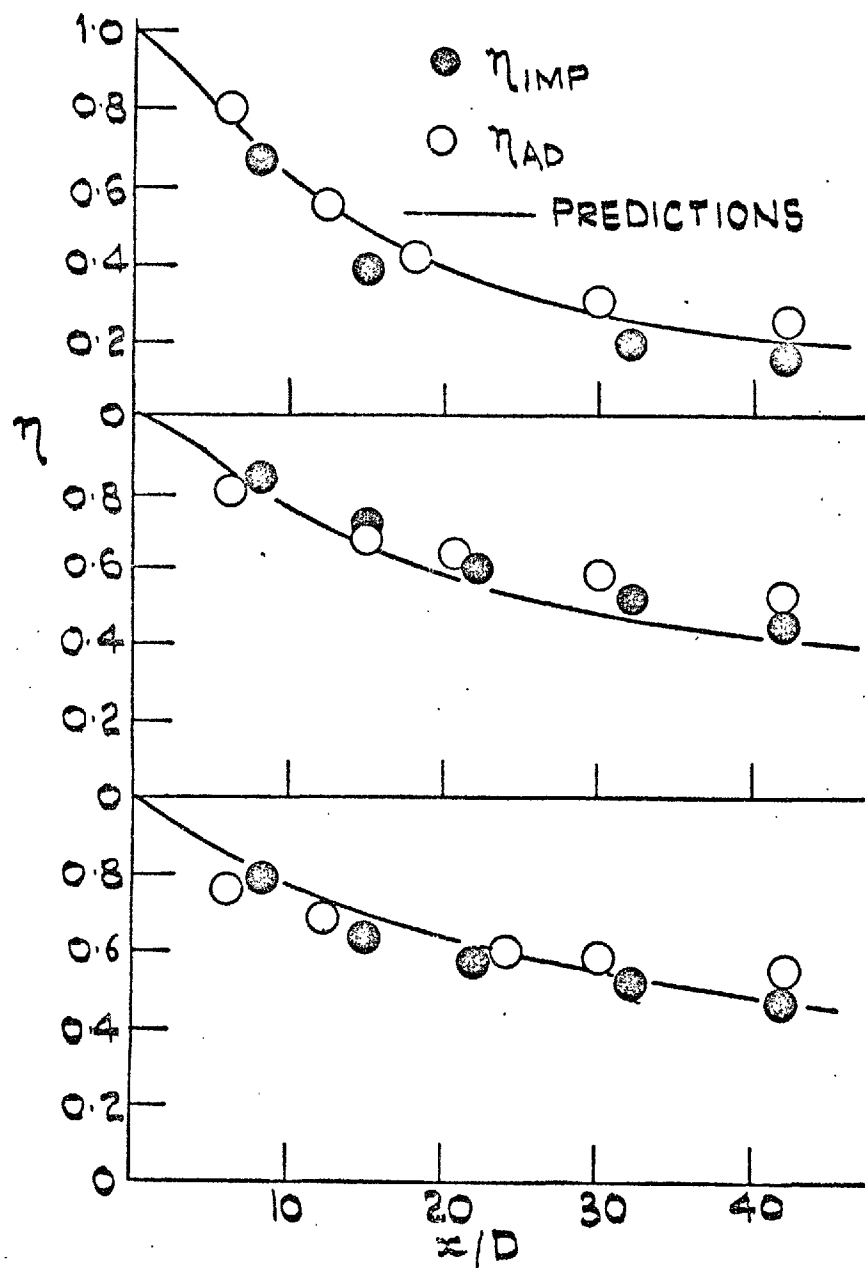


Fig. 3.2.15. Measured and predicted adiabatic wall effectiveness and heat-transfer coefficient downstream of slot-1 for $P_c/P_g = 0.94$

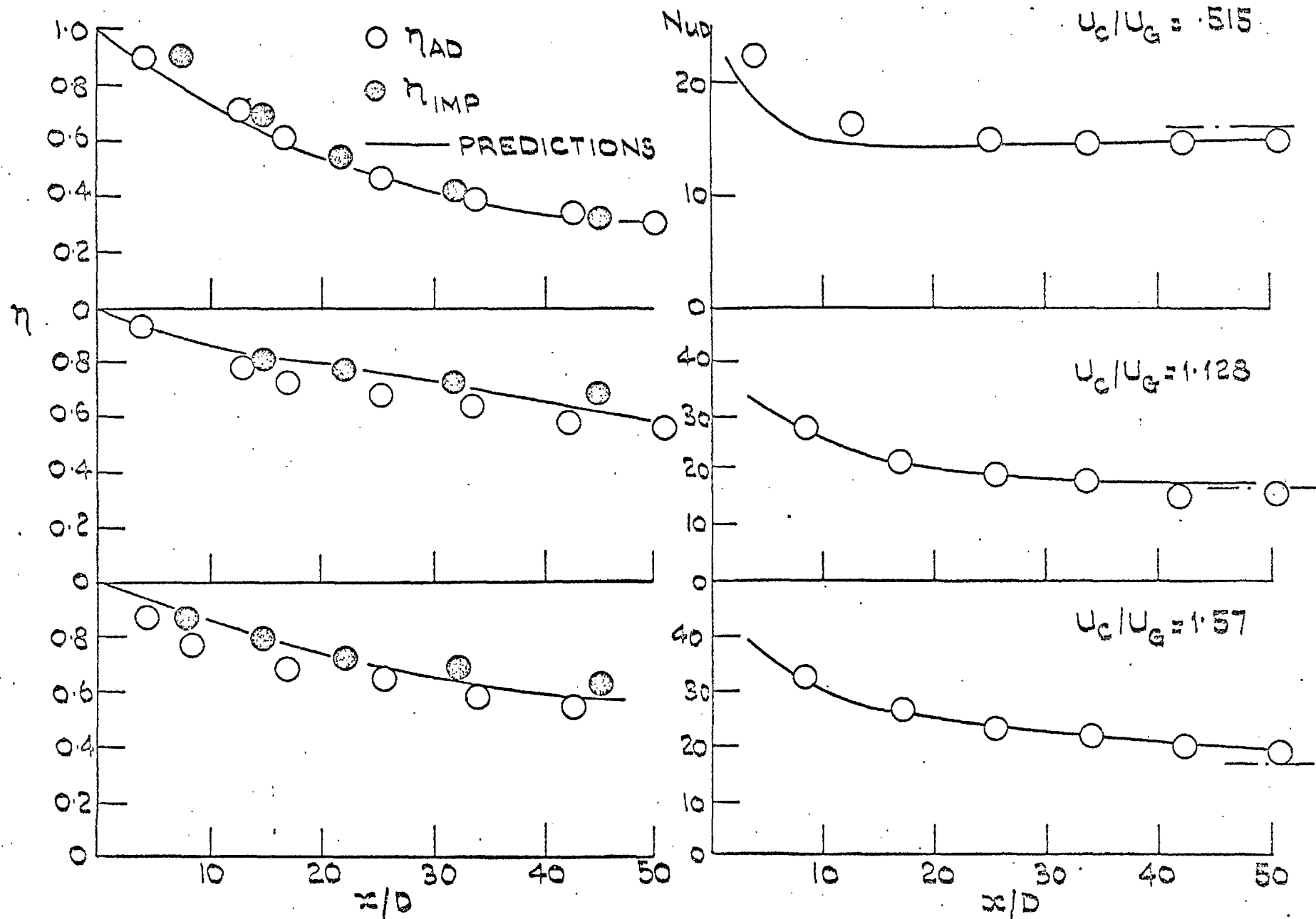


Fig. 3.2-16. Measured and predicted adiabatic wall effectiveness and heat-transfer coefficient downstream of slot 3 for $P_c/P_g = 0.94$

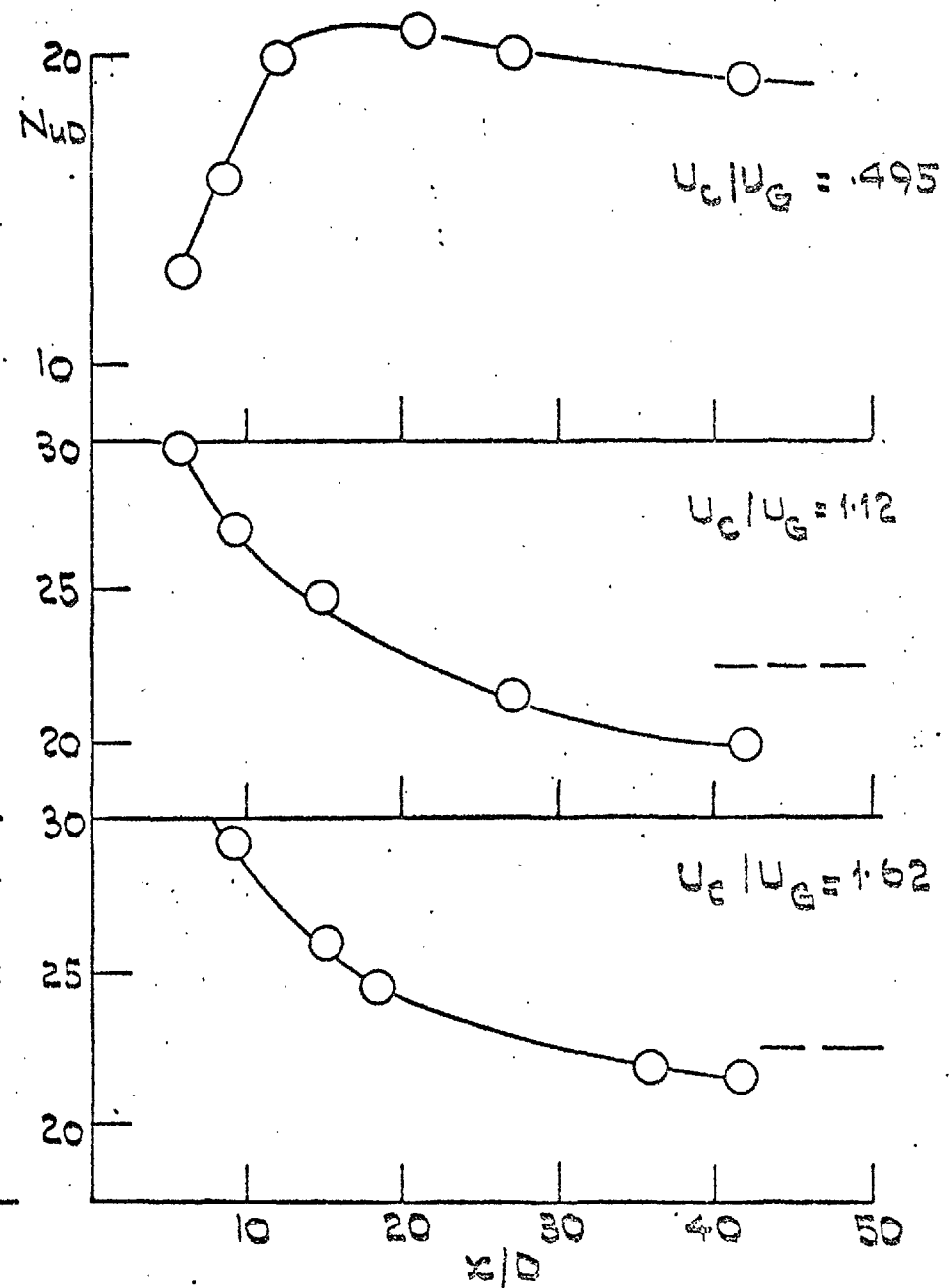
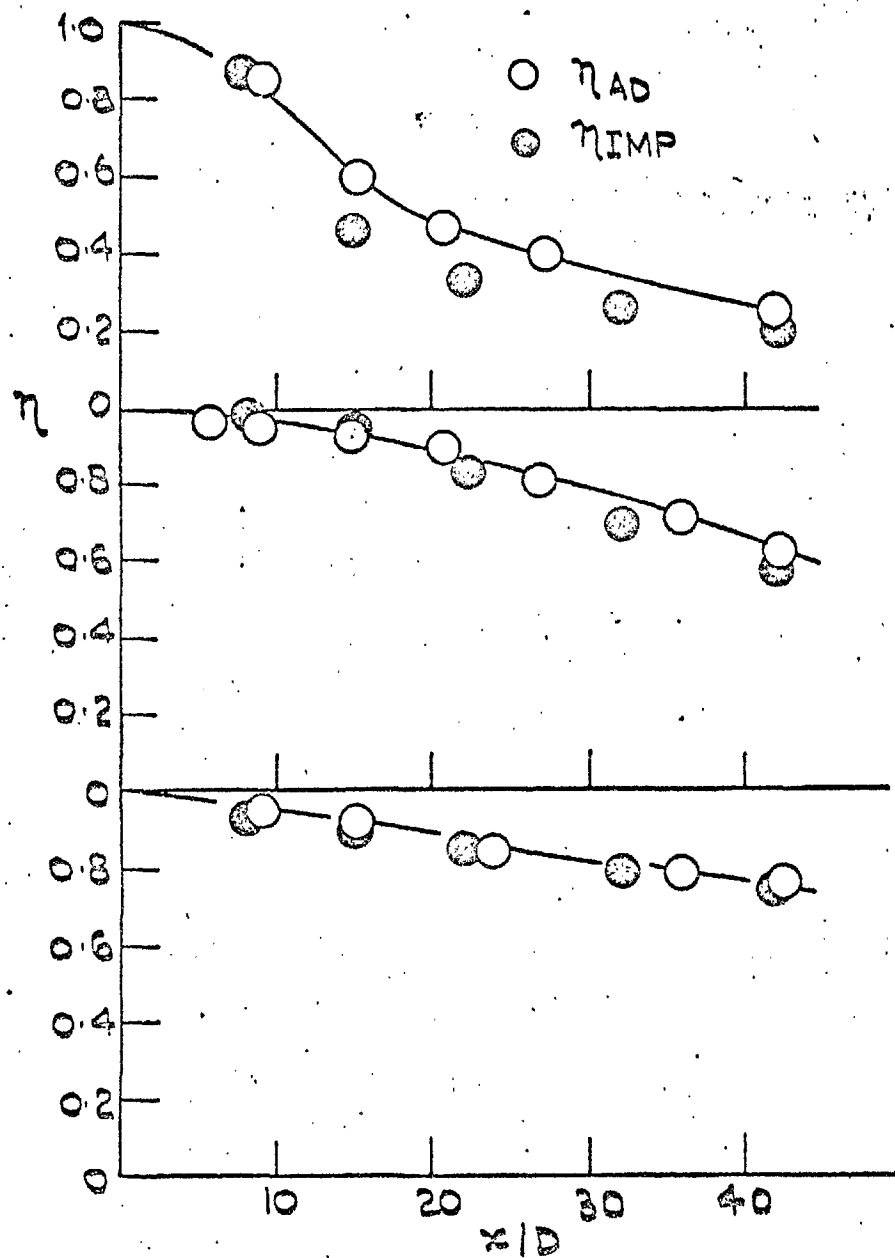


Fig. 3.2.17. Measured adiabatic wall effectiveness and heat-transfer coefficient downstream of slot 5 for $P_c/P_g = 0.94$

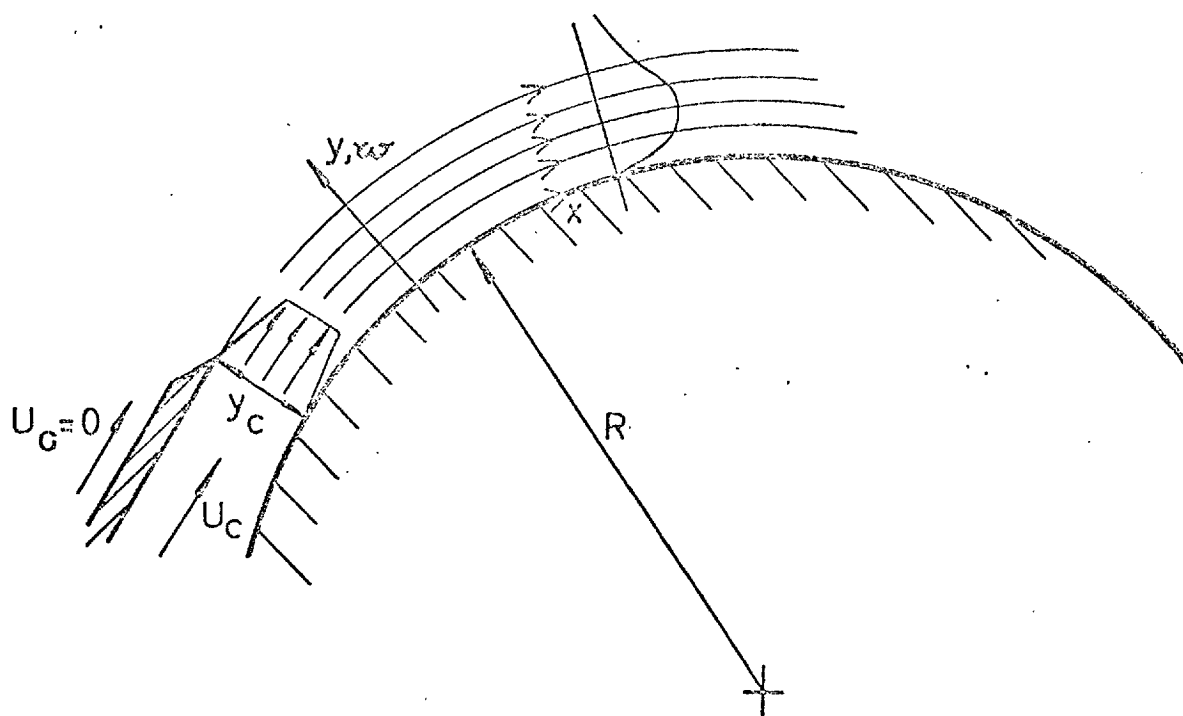


Fig. 4.1.1. Coordinate system for a wall jet over a curved surface.

T A B L E 4.1.1

Authors	Ref. No.	Measurements
R.A. Sawyer	68	Surface pressure, decay of maximum velocity, rate of growth and $\overline{u^2}$, $\overline{v^2}$, $\overline{uv}$ of turbulent wall jets on logarithmic spiral surfaces in absence of free stream.
S. Eskinazi & H. Yeh	71	Turbulence intensity, turbulent shear, and energy spectrum for flow in a curved channel. Mean flow properties are not measured.
B.G. Newman	75	Surface pressure, decay of maximum velocity, rate of growth of turbulent wall jet outside a circular cylinder in absence of free stream.
D.E. Guitton	74	Surface pressure, decay of maximum velocity, rate of growth of turbulent wall jet inside a circular cylinder in absence of free stream.
R.J. Kind	76	Surface pressure, decay of maximum velocity, rate of growth of turbulent wall jet outside a circular cylinder in presence of a free stream: lift coefficients for the aerofoil also measured.
G.B. Schubauer & P.S. Klebanoff	77	Displacement and momentum thickness, wall shear stress coefficient on a convex surface in presence of severe main-stream pressure gradients.

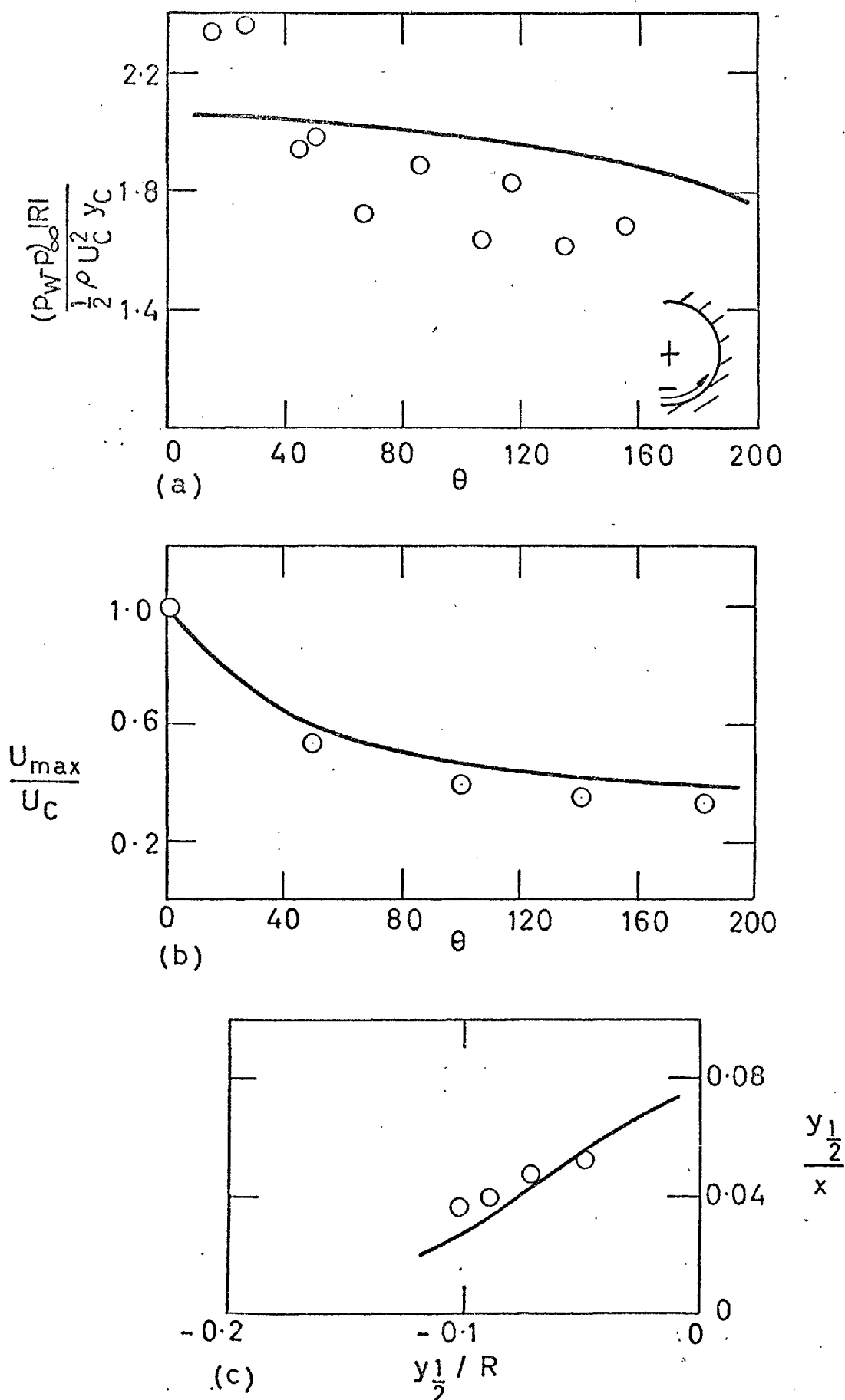


Fig. 4.1.2. Surface pressure, maximum velocity decay and growth rate of a turbulent wall jet on a concave surface. Data of Guitton (74)

$$R = 20.5''; \quad y_c = 0.0122R; \quad \frac{U_c \sqrt{y_c R}}{\nu} = 23.4 \times 10^4$$

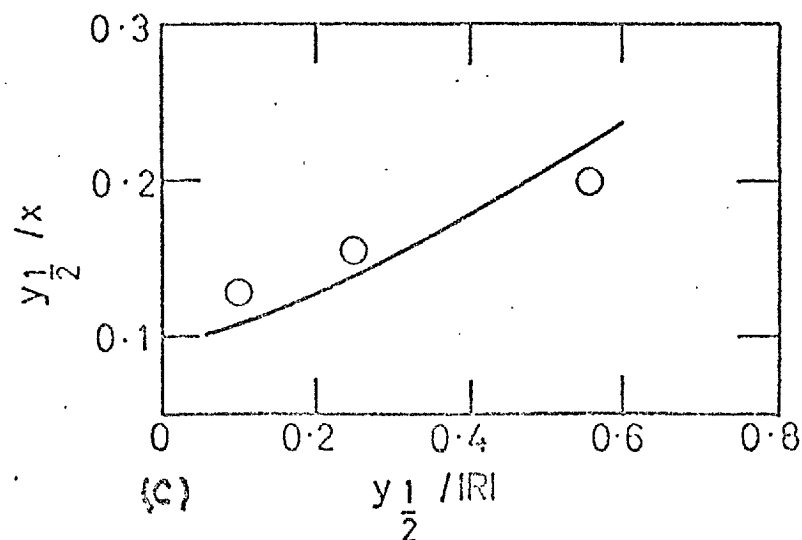
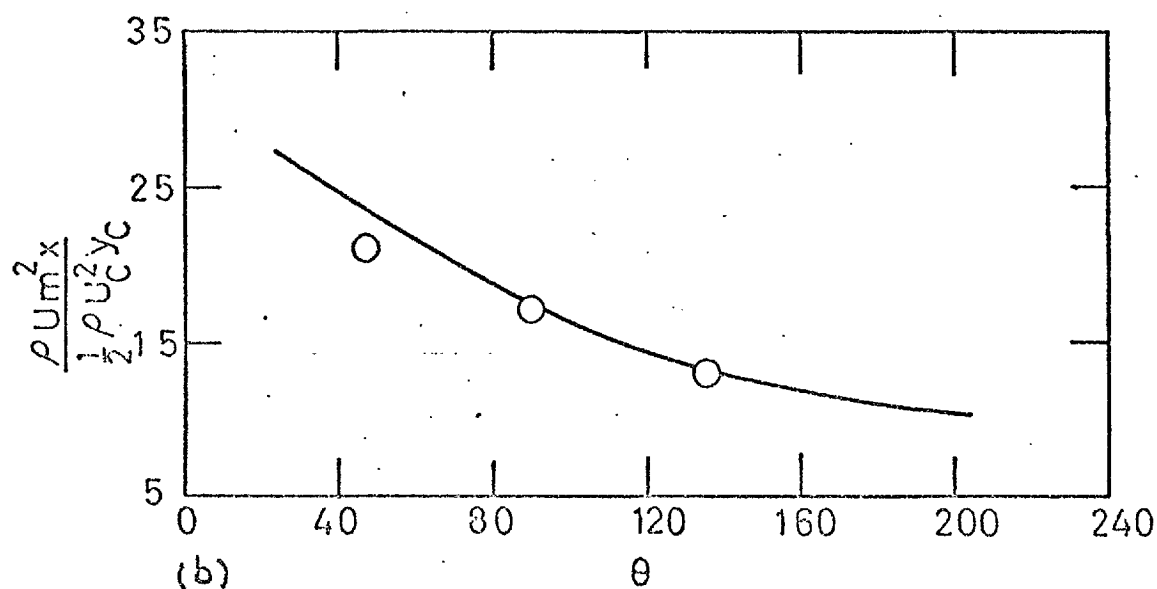
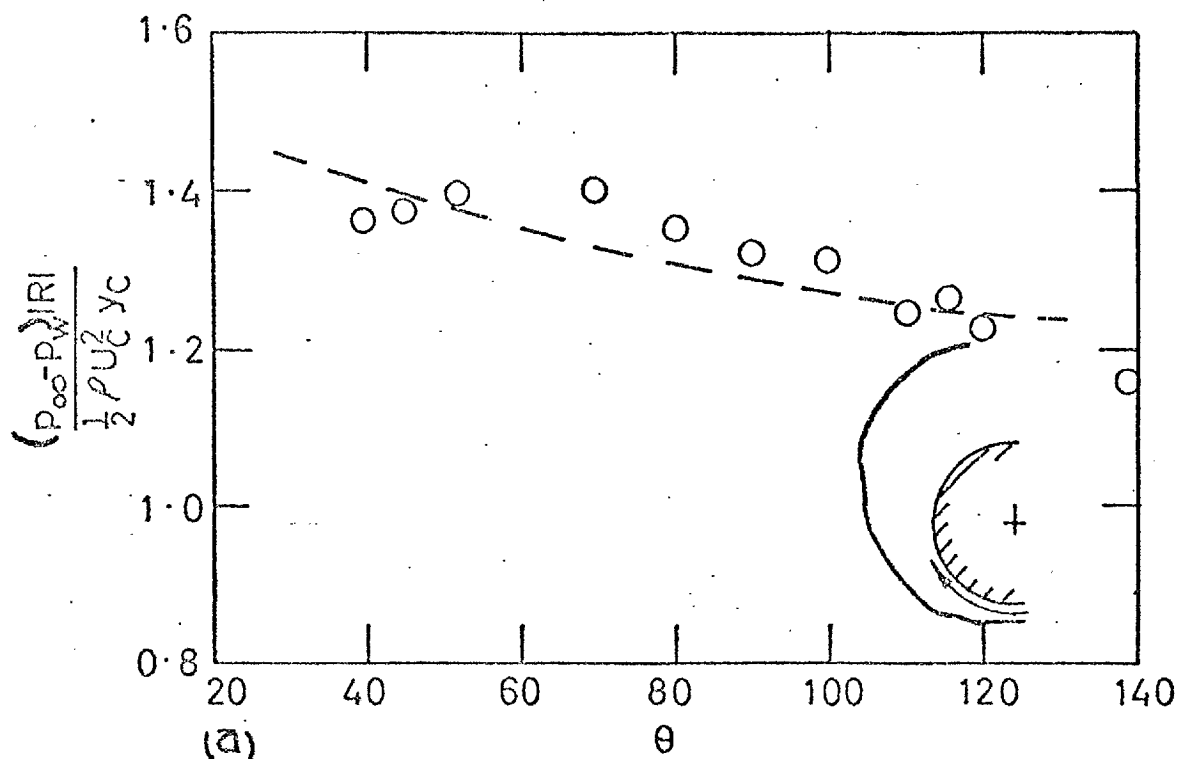


Fig. 4.1.3. Surface pressure, decay of maximum velocity and growth rate of a turbulent wall jet on a convex surface. Data of Newman (75).

$$R = 2.55 \text{ in}; y_c = 0.282 R; \frac{U_c \sqrt{y_c R}}{\nu} = 2 \times 10^3 \rightarrow 4.4 \times 10^4$$

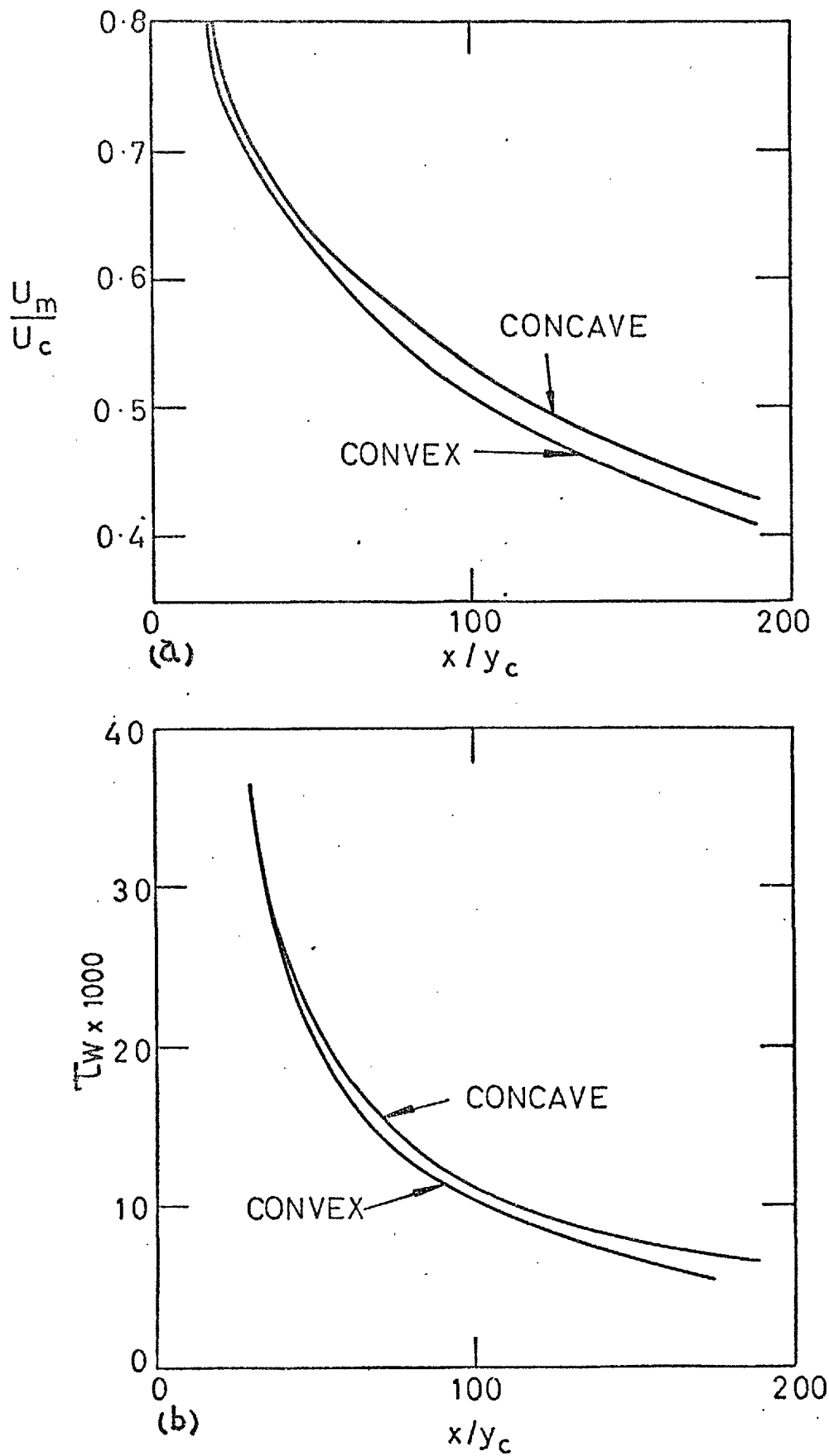


Fig. 4.1.4. Decay of maximum velocity and wall shear stress for laminar wall jet on curved surfaces. $R = 2.55''$; $y_c = 0.282R$; $\frac{U_c \sqrt{xR}}{\nu} = 2000$

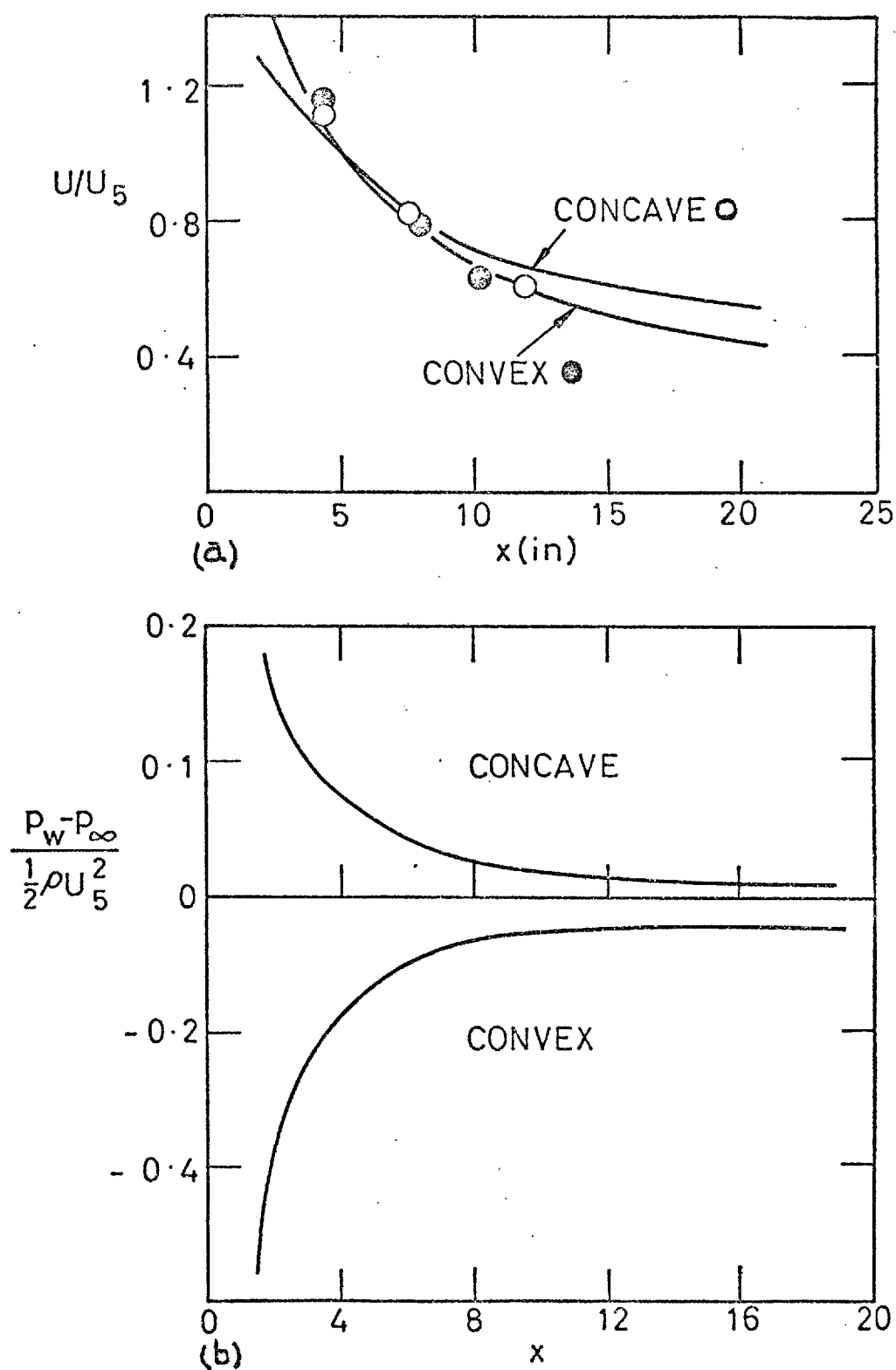


Fig. 4.1.5. Surface pressure and decay of maximum velocity of a turbulent wall-jet on logarithmic-spiral surfaces. Data of Sawyer (73), $U_{cyc} = 0.2$ in; $\frac{U_{cyc}}{\nu} = 6 \times 10^4$

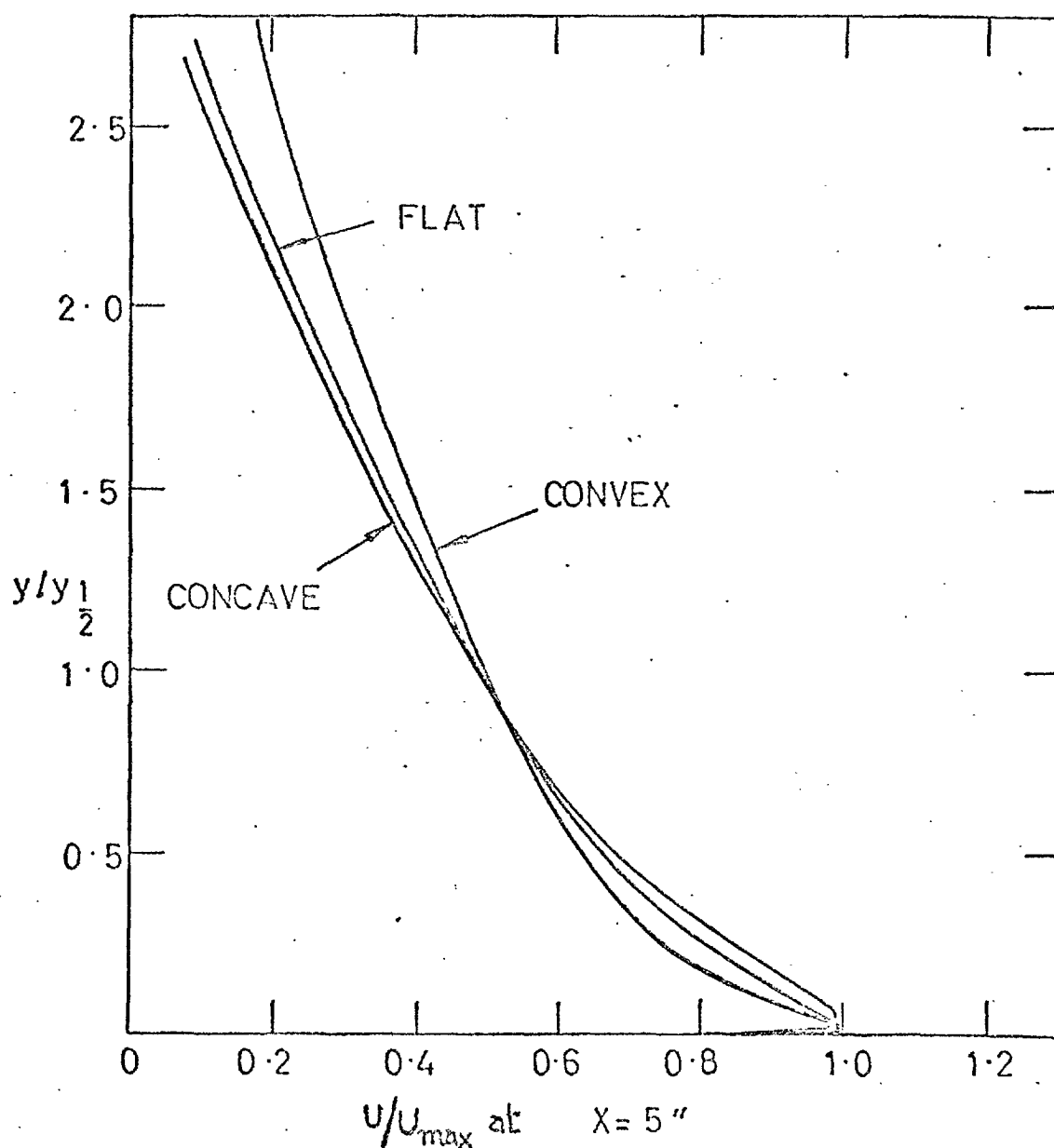
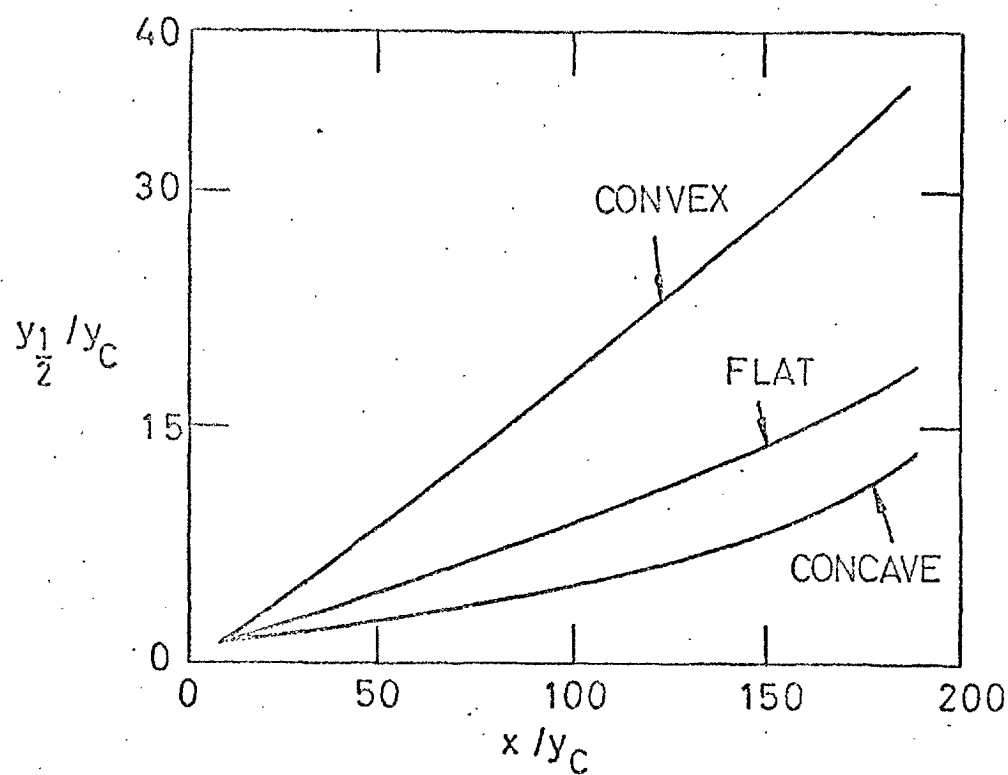


Fig. 4.1.6. Non dimensional growth rates and velocity profiles corresponding to Fig. 4.1.5.

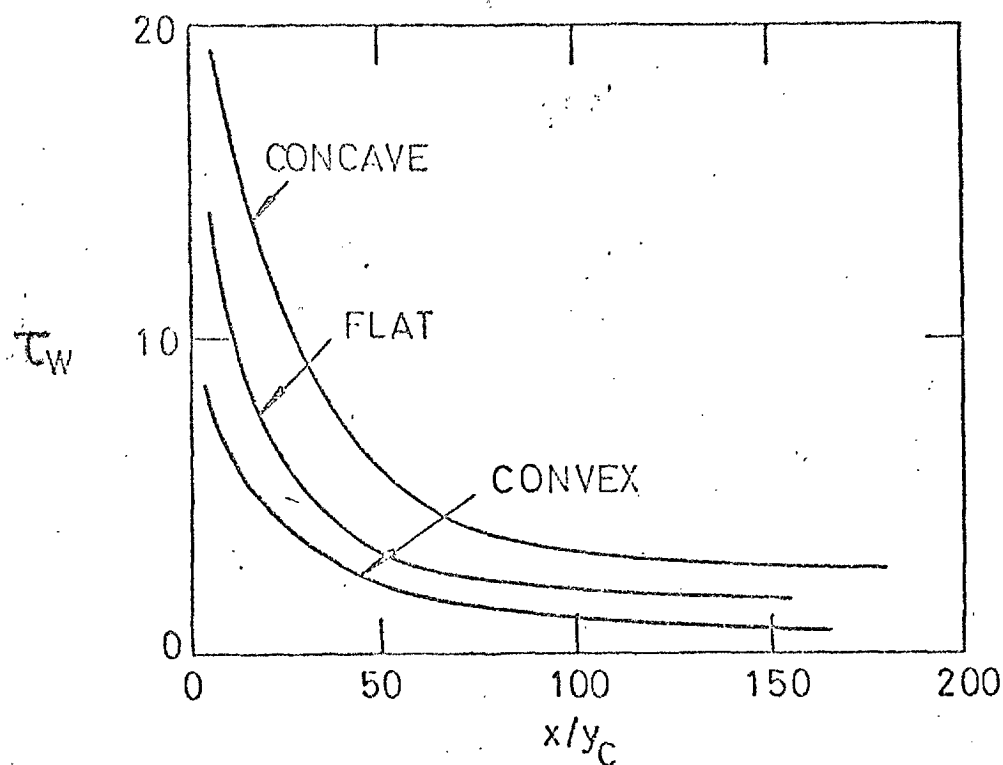


Fig. 4.1.7.(a) Wall shear stress corresponding to Fig. 4.1.6.

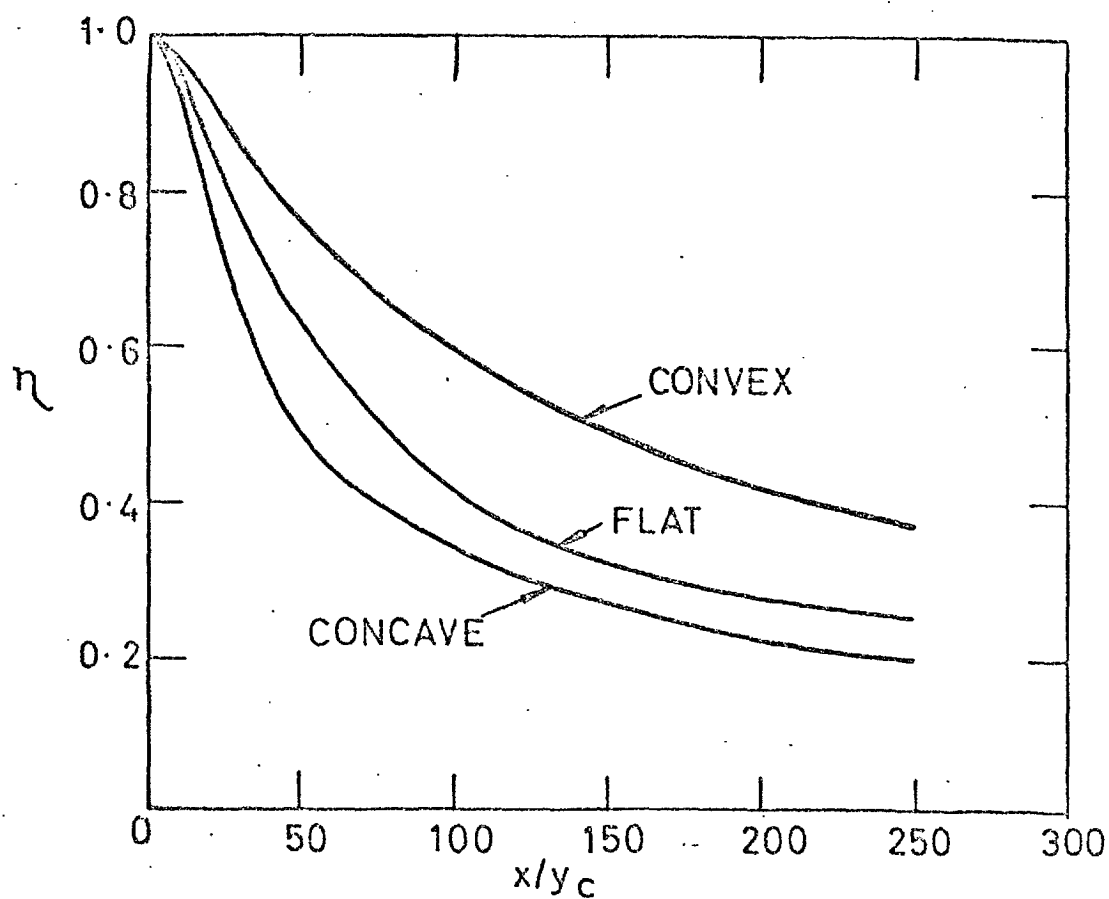


Fig. 4.1.7.(b) Impervious wall effectiveness corresponding to Fig. 4.1.6.

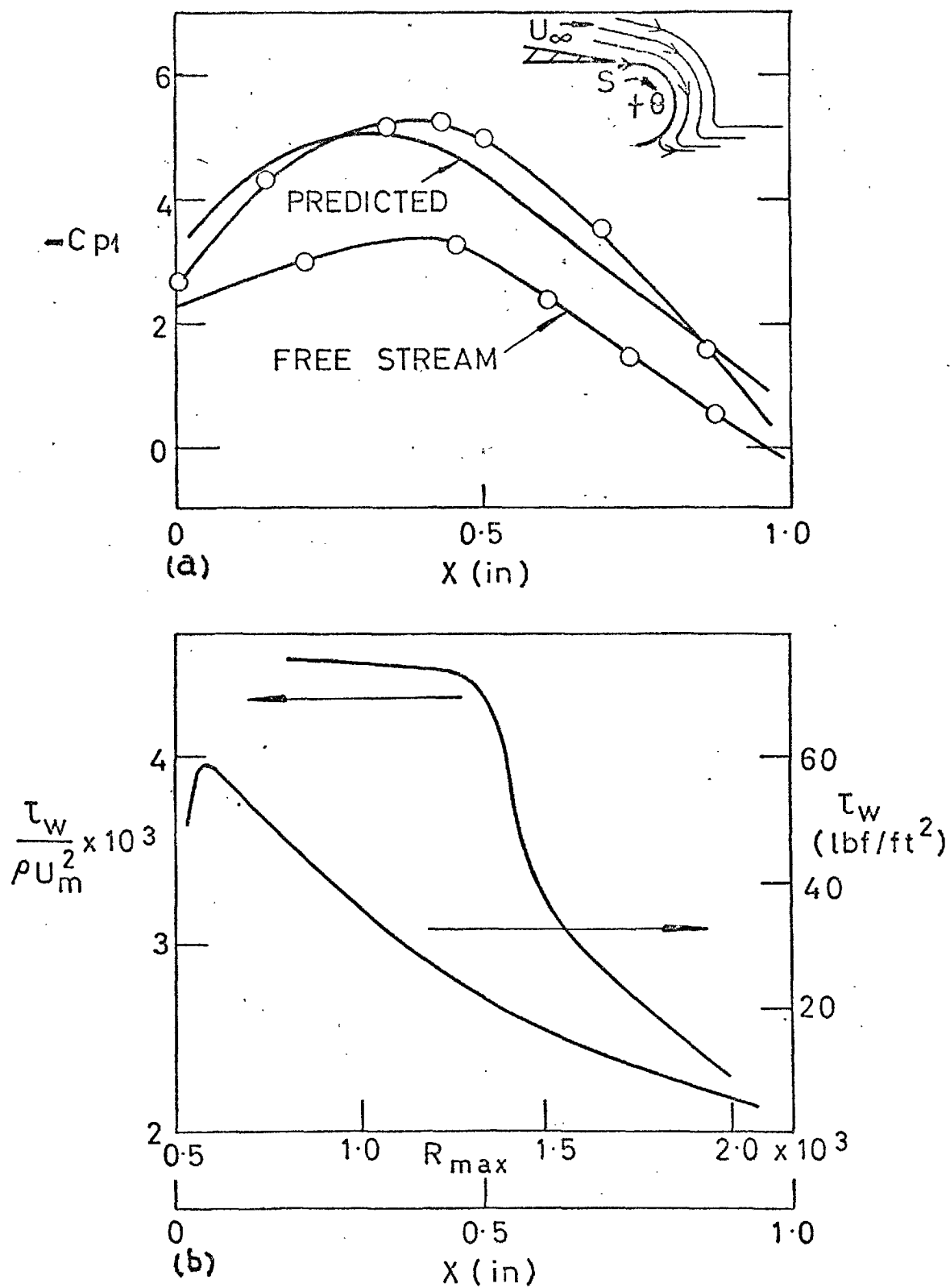


Fig. 4.1.8. Surface pressure and the wall shear stress for a turbulent wall jet on a convex surface in the presence of a free stream. Data of Kind (76).

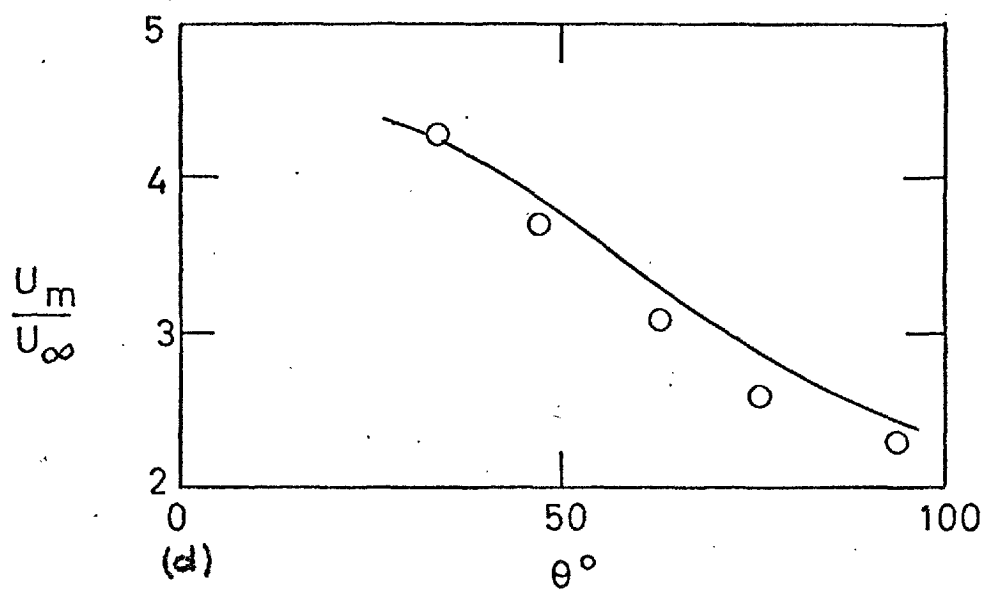
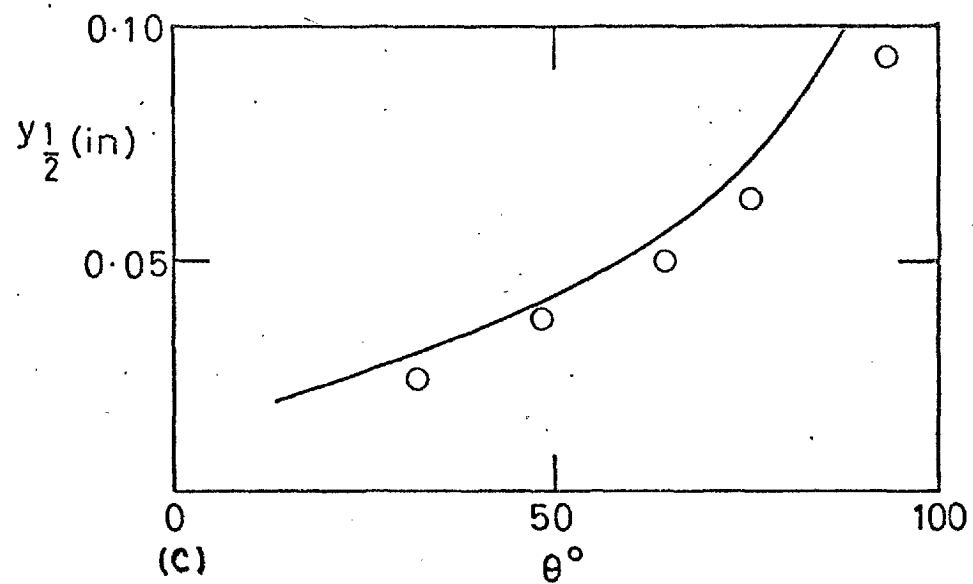


Fig. 4.1.8. (contd.) Growth rate and decay of maximum velocity for flow of ref. (76).

$$R = 0.563 \text{ in}; \quad Y_c = 0.0302 R; \quad U_g Y_c/2 = 900; \quad U_c/U_g = 4.5.$$

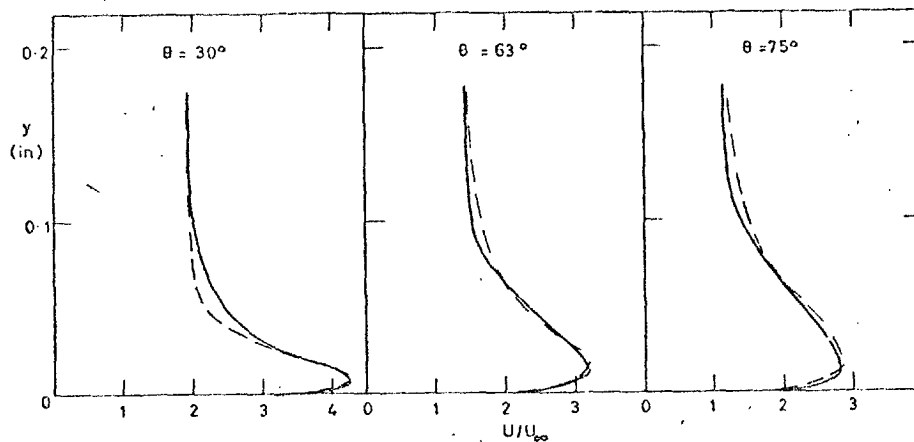


Fig. 4.1.9.(a) Velocity profiles corresponding to Fig. 4.1.8.

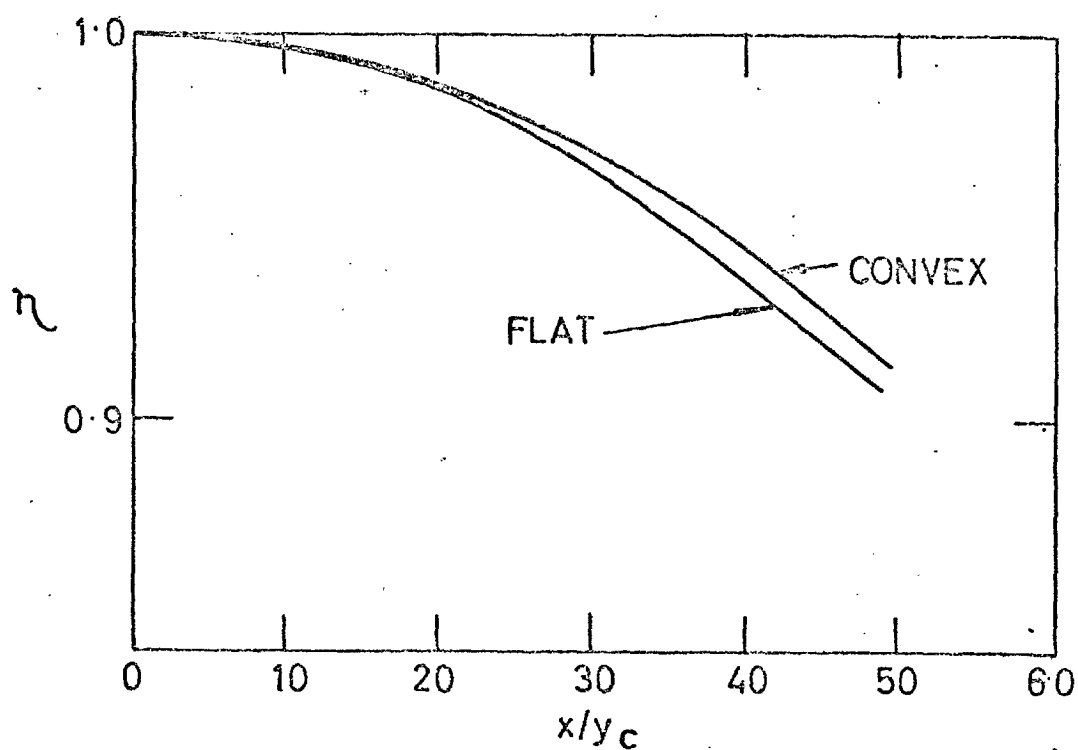


Fig. 4.1.9.(b) Impervious wall effectiveness corresponding to Fig. 4.1.8.

T A B L E 4.1.2

Comparison between the experimental result of reference 77
and computations with and without curvature modifications

x ft.	Experimental			With Curvature			Without Curvature		
	Cf	H ₁₂	δ ₂ in	Cf	H ₁₂	δ ₂ in	Cf	H ₁₂	δ ₂ in
16.5	.00238	1.310	.225	.00246	1.3143	.218	.00246	1.314	.216
18.0	.00236	1.298	.232	.00236	1.3151	.232	.00240	1.313	.240
20.0	.00194	1.359	.324	.00186	1.3728	.350	.00208	1.349	.312
23.0	.00102	1.613	.671	.00714	1.4797	.695	.00136	1.470	.60
25.0	.00025	2.324	1.136	.00088	1.5571	.996	.00116	1.530	.830

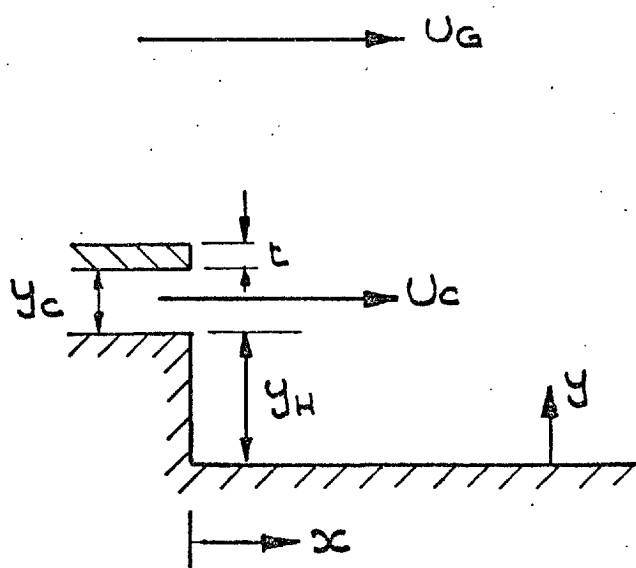


Fig. 4.2.1. The general flow configuration of a two-dimensional film-cooling slot.

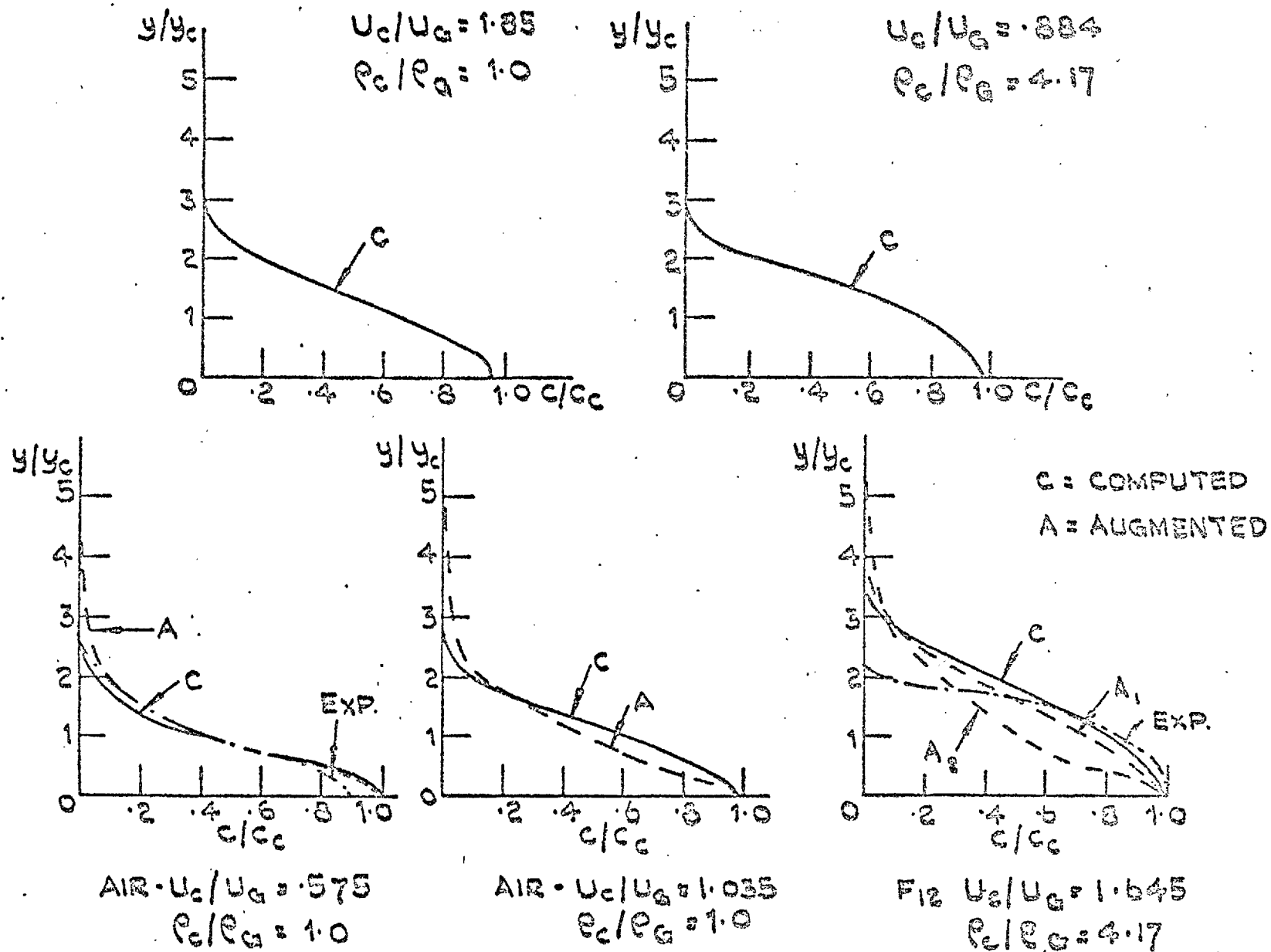


Fig. 4.2.2. Non dimensional initial profiles of concentration.

x/y_C	12.5	32.5	52.5	72.5	92.5	112.5	132.5	172.5	212.5	U_C/U_G
Exp	.930	.655	.530	.405	.318	.288	.244	.210	.170	.575
C	.984	.598	.401	.331	.288	.252	.224	.187	.162	Air
A	.935	.532	.362	.302	.267	.235	.211	.178	.156	
Exp	.955	.800	.661	.635	.565	.540	.511	.461	.394	1.035
C	.977	.862	.772	.706	.650	.601	.556	.472	.403	Air
A	.915	.751	.667	.611	.564	.525	.488	.420	.364	
Exp	.954	.807	.705	.608	.594	.520	.505	.461	.425	1.85
C	.957	.777	.674	.616	.575	.544	.518	.478	.446	Air
A	.90	.710	.605	.520	.480	.420	.400	.381	.345	
Exp	1.00	.975	.936	.920	.885	.870	.840	.785	.752	.884
C	.989	.976	.946	.912	.876	.839	.806	.747	.696	F_{12}
A1	.971	.945	.905	.868	.833	.798	.767	.716	.674	
A2	.932	.888	.829	.774	.725	.682	.645	.586	.540	
Exp	.996	.990	.960	.940	.922	.905	.890	.870	.845	1.645
C	.984	.935	.881	.841	.797	.765	.736	.692	.658	F_{12}
A1	.979	.931	.866	.810	.774	.740	.716	.675	.643	
A2	.938	.797	.713	.664	.617	.581	.555	.513	.481	

TABLE-4.2.1. Predicted values of impervious wall effectiveness with
initial profiles of concentration shown on Fig.4.2.2

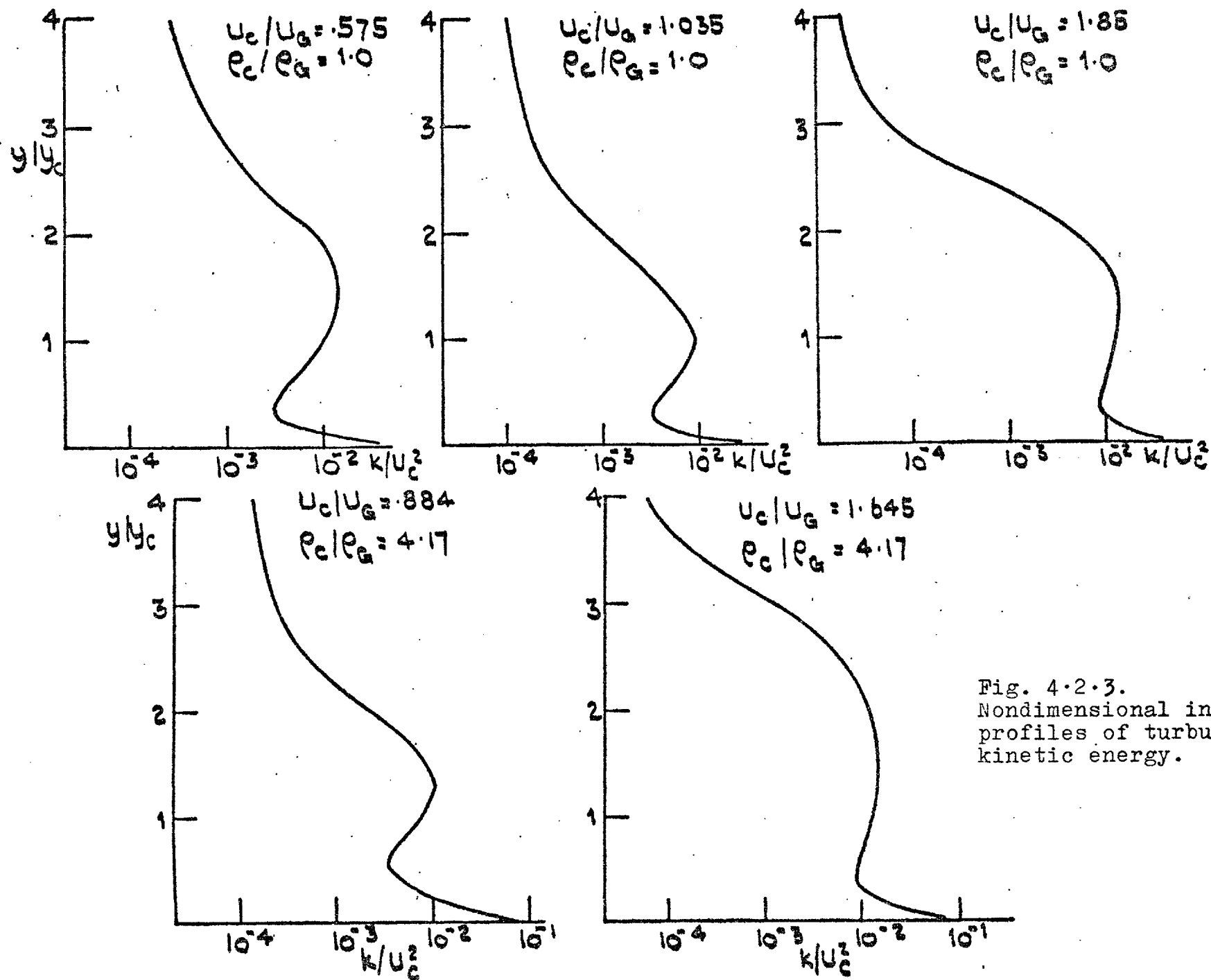


Fig. 4.2.3.
Nondimensional initial
profiles of turbulent
kinetic energy.

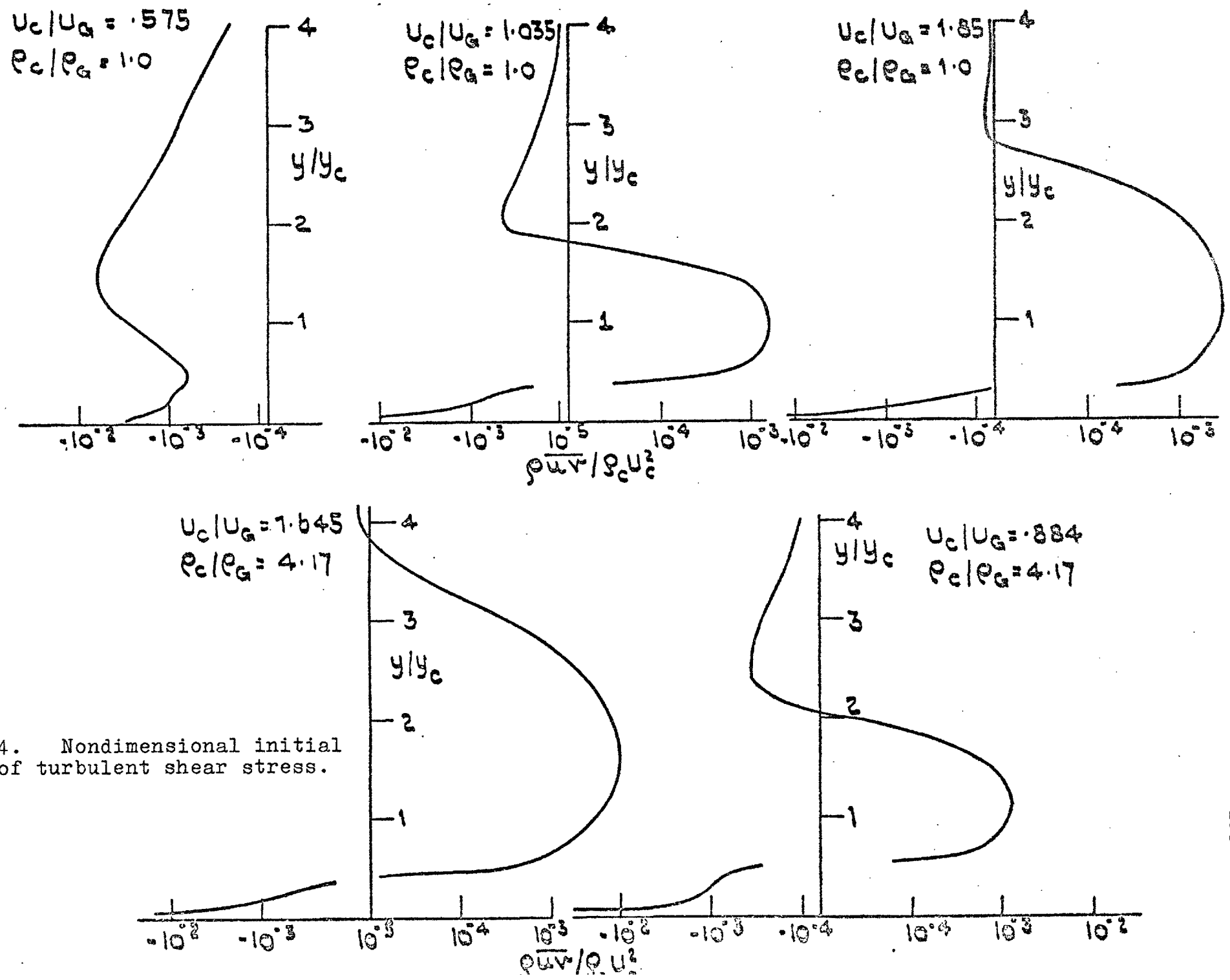


Fig. 4.2.4. Nondimensional initial profiles of turbulent shear stress.

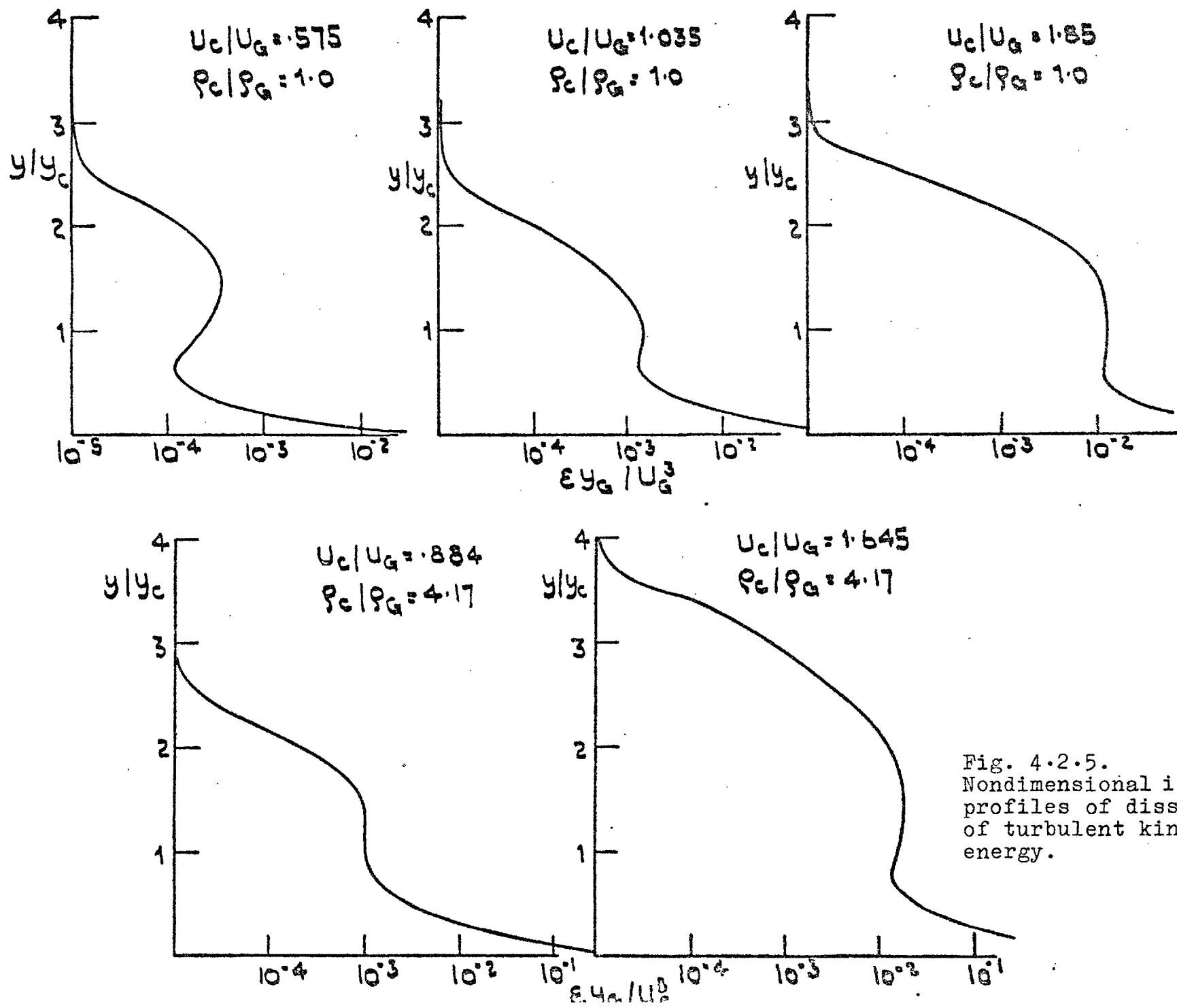


Fig. 4.2.5.
Nondimensional initial
profiles of dissipation
of turbulent kinetic
energy.

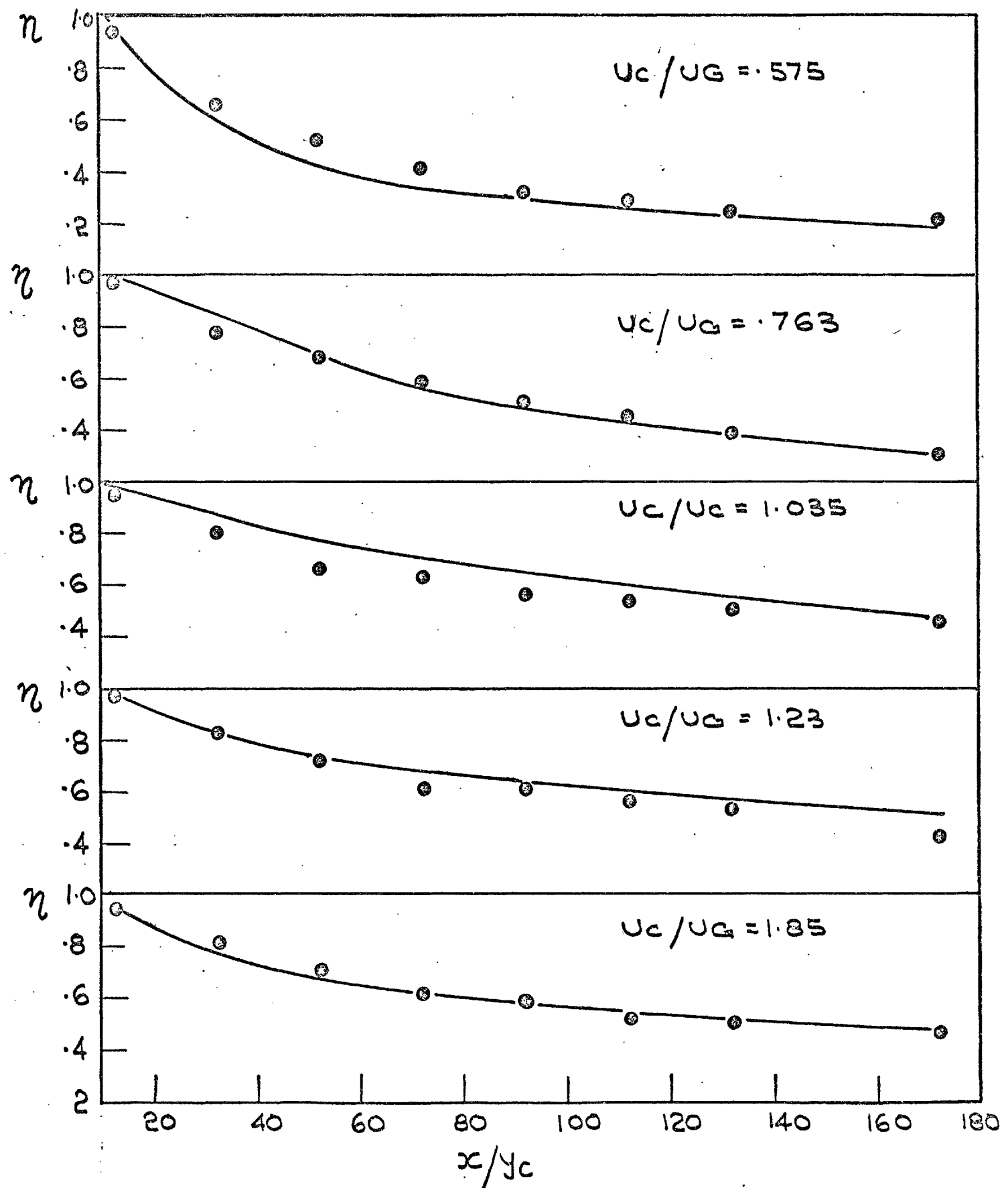


Fig. 4.2.6. Present predictions and the measurements of Pai and Whitelaw (18); $\rho_c/\rho_g = 1.0$

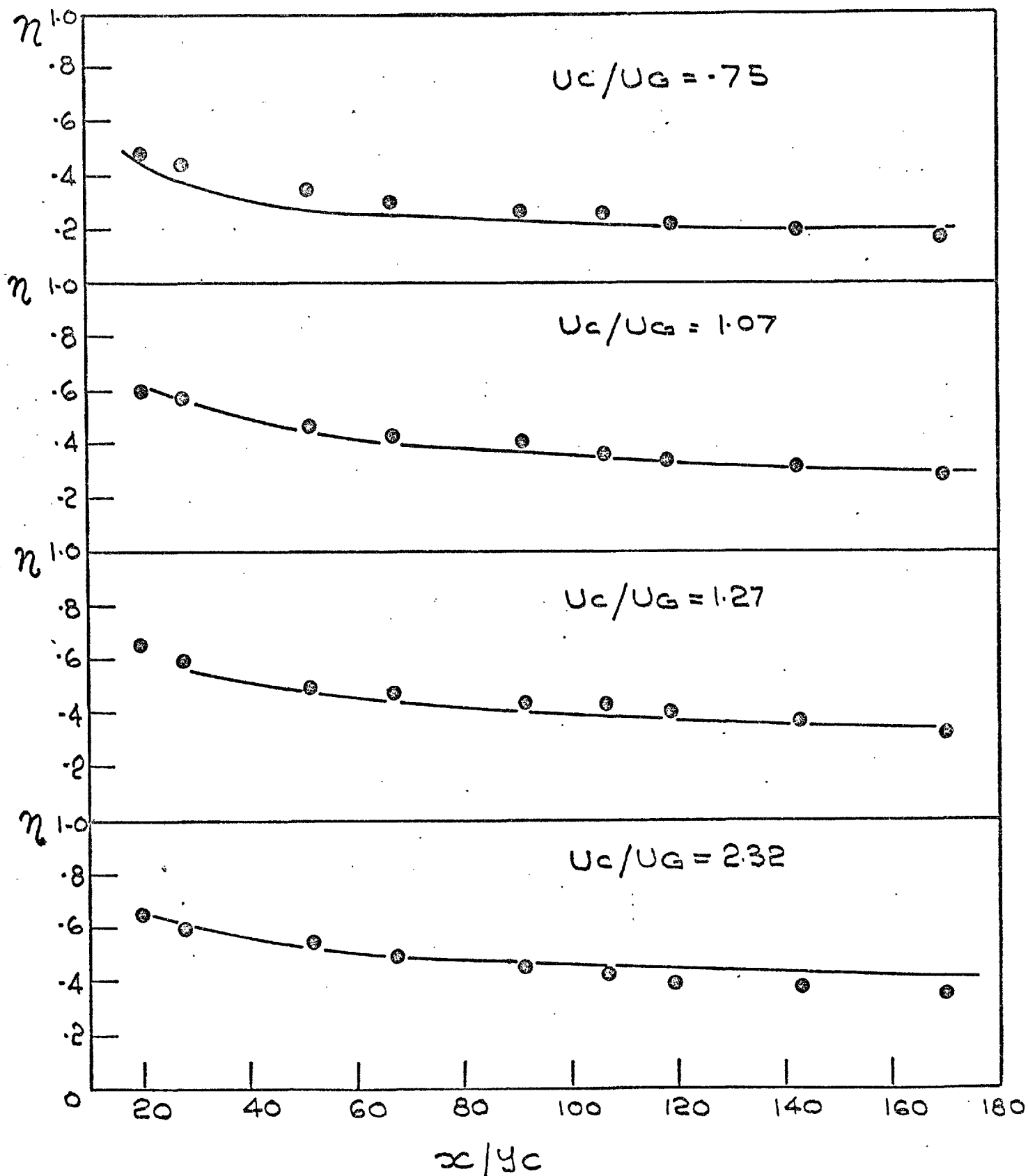


Fig. 4.2.7. Present predictions and the measurements of Kacker and Whitelaw (1); $\rho_c/\rho_G = 1.0$; $t/y_c = 1.14$

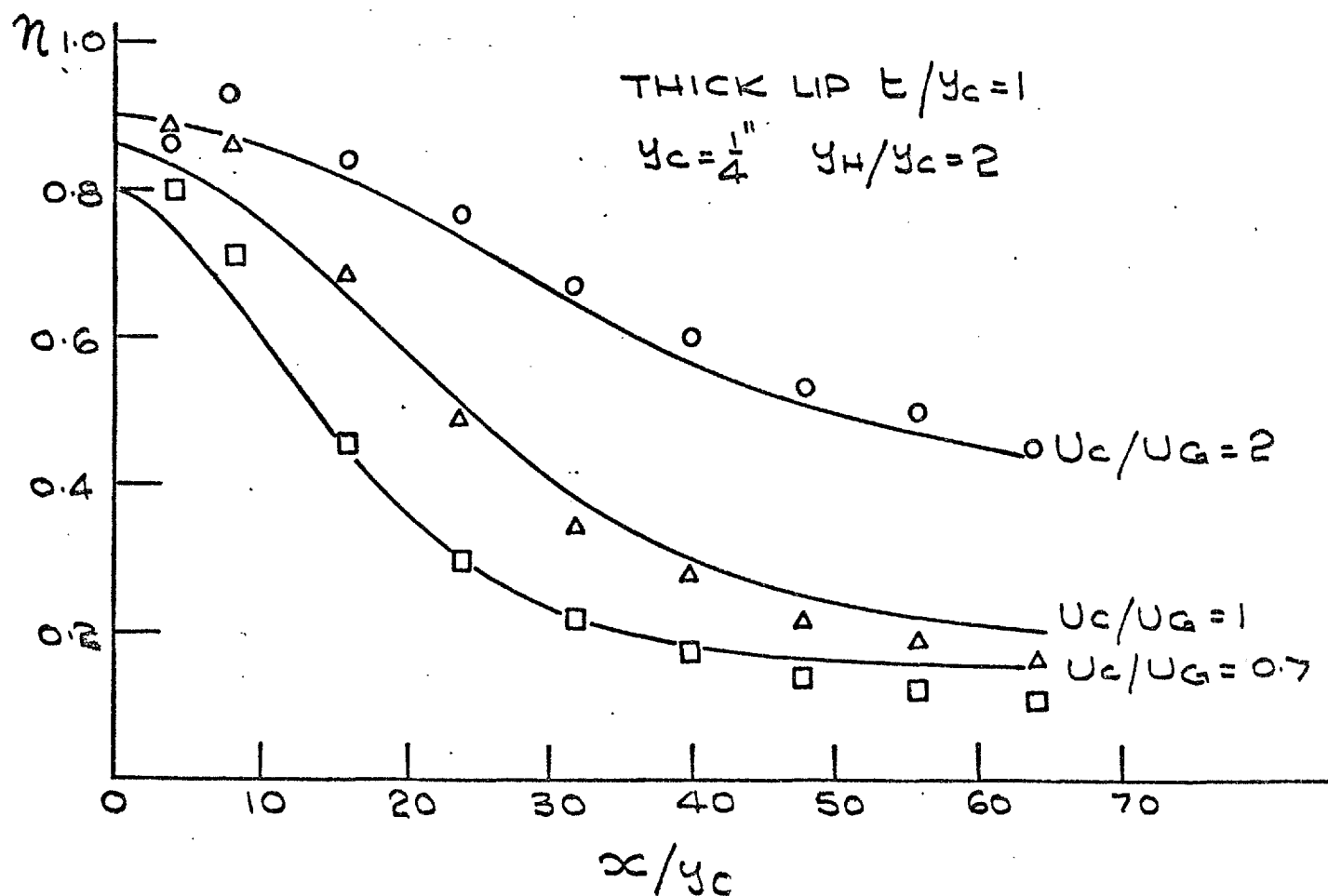
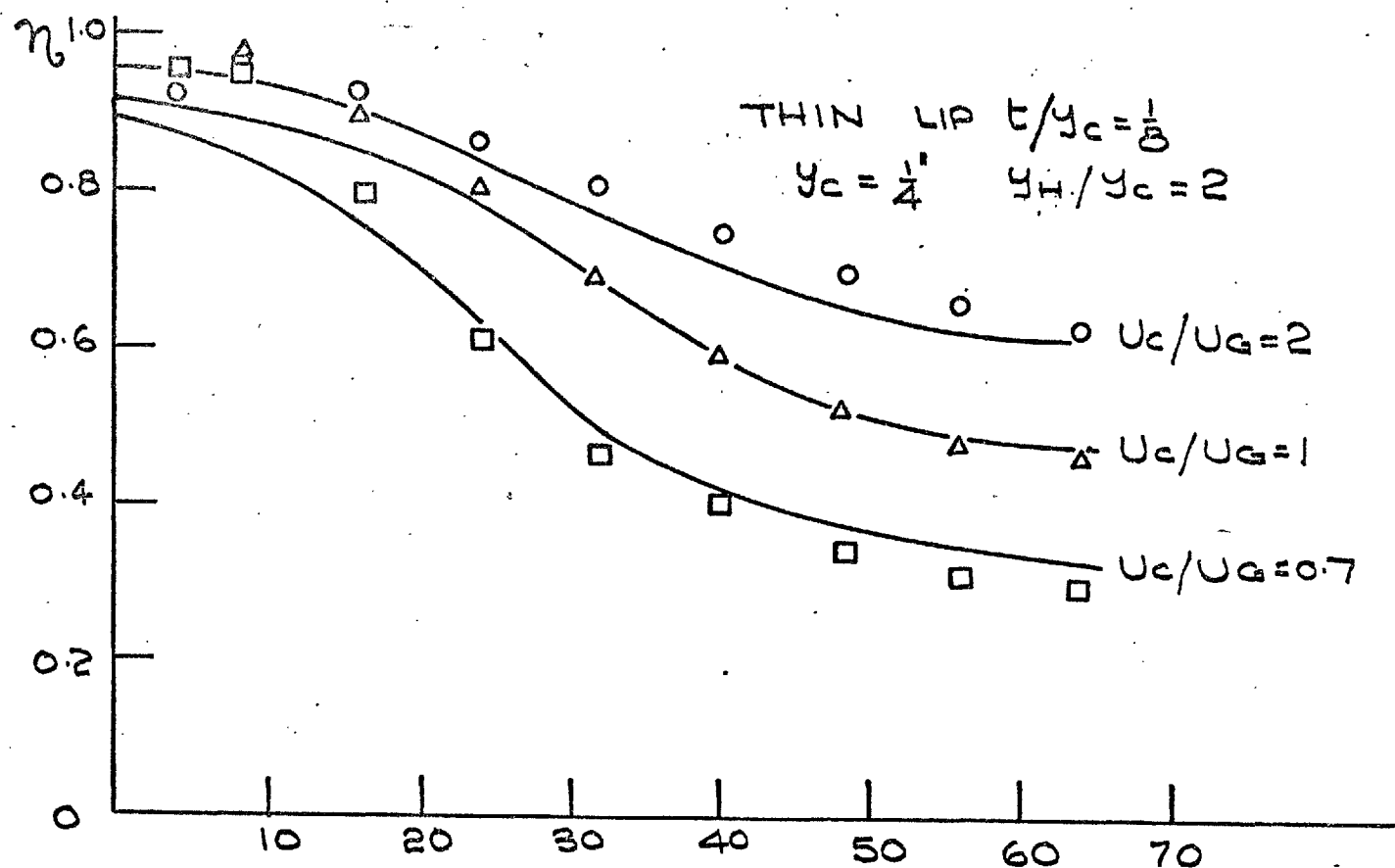


Fig. 4.2.8. Present predictions and the measurements of Matthews and Whitelaw (24); $\rho_c/\rho_g = 10$

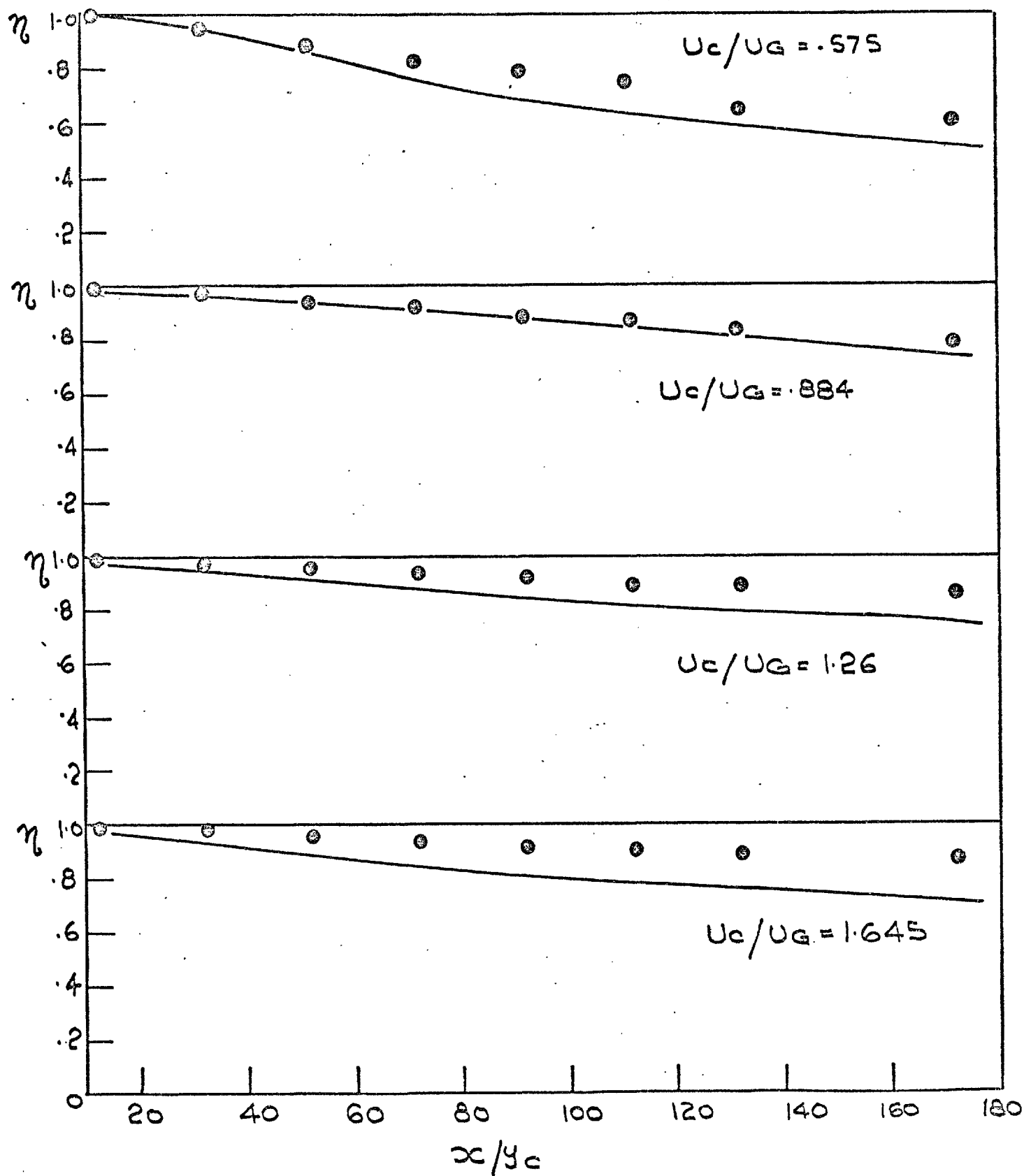


Fig. 4.2.9. Present predictions and the measurements of Pai and Whitelaw (18); $\rho_c/\rho_g = 4.17$

U_C/U_G	R_C	ρ_C/ρ_G	Mixing Length		Present Model	
			λ_{100}	$D_{\max}$	λ_{100}	$D_{\max}$
.55	1970	1.0	.030	± 0.03 good	.07	- .074 fair
.76	2620	1.0	.034	$\pm .03$ good	.050	$\pm .046$ good
1.23	4170	1.0	.024	+ .02 good	.034	+ .06 good
1.85	6330	1.0	.063	+ .07 fair	.016	- .031 good
.575	5150	4.17	.012	$\pm .02$ good	.064	- .120 fair
1.645	14250	4.17	.018	- .07 good	.085	- .110 fair

Ratings:

$\lambda < 0.05$ Good
 $0.05 < \lambda < 0.11$ Fair
 $0.11 < \lambda$ Poor

TABLE - 4.2.2.

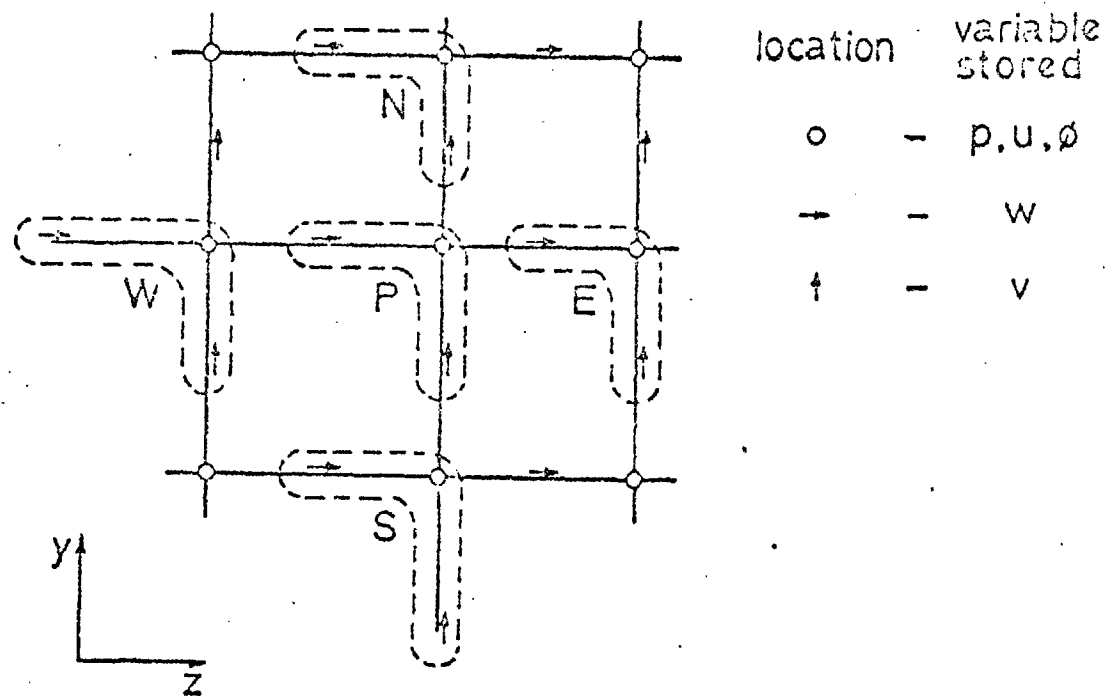


Fig. 5.1.1. The staggered grid

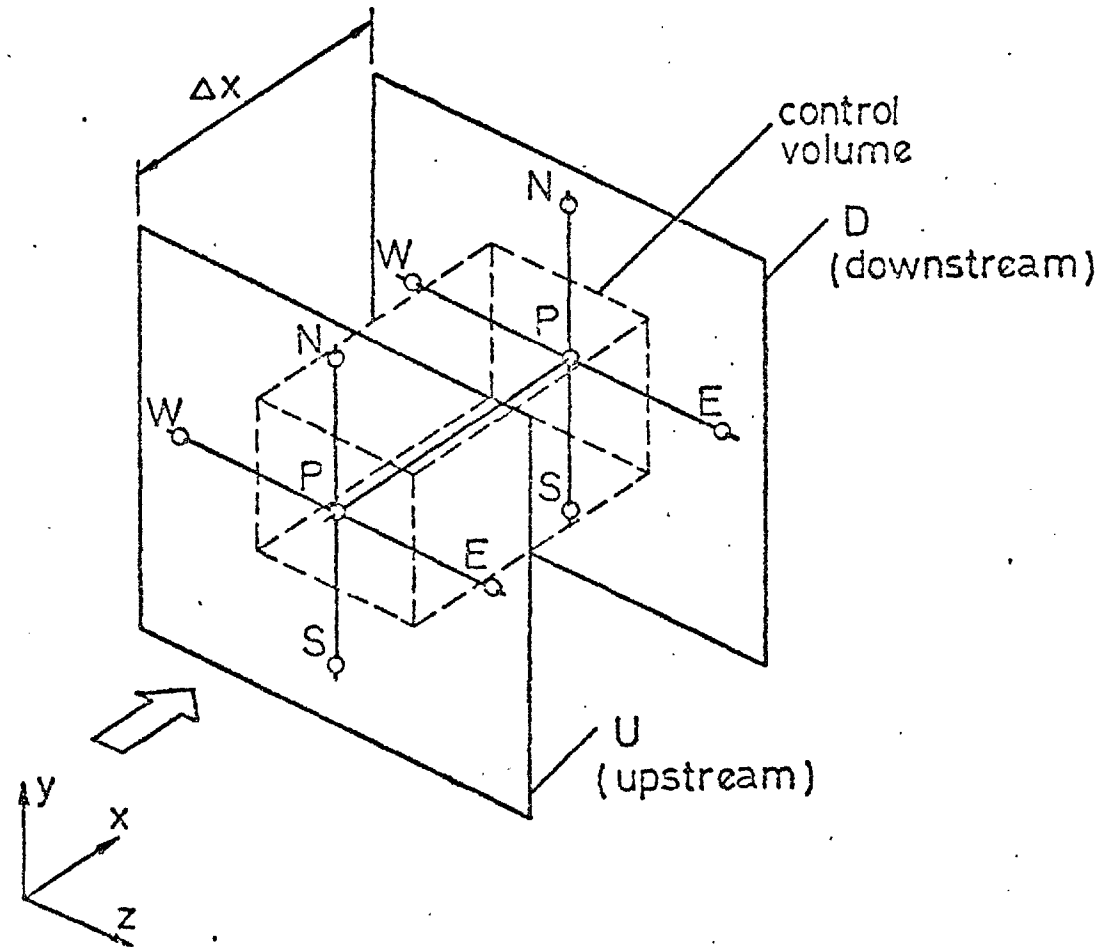


Fig. 5.1.2. The control volume used to obtain the finite-difference equation

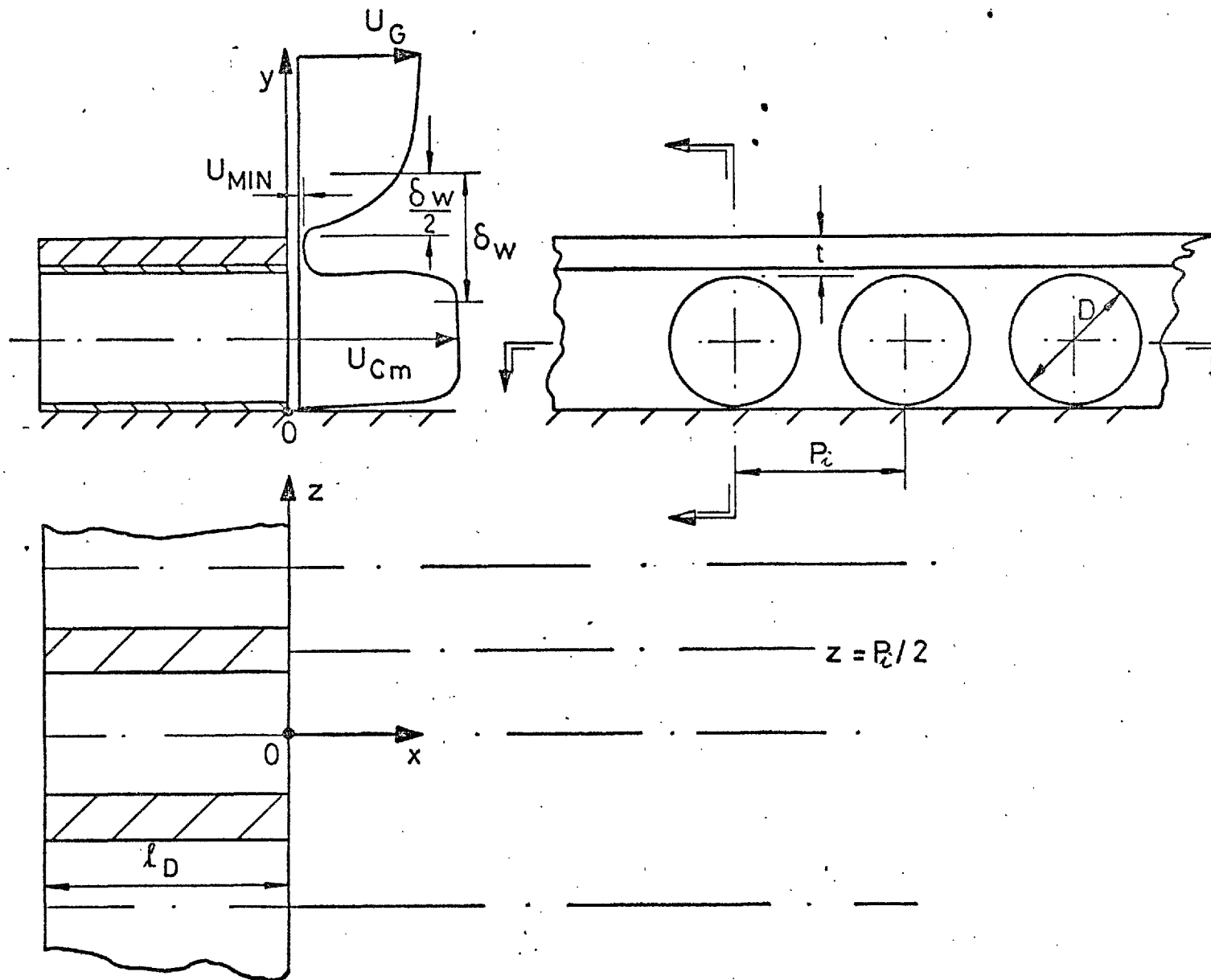


Fig. 5.2. Slot configuration and the coordinate system.

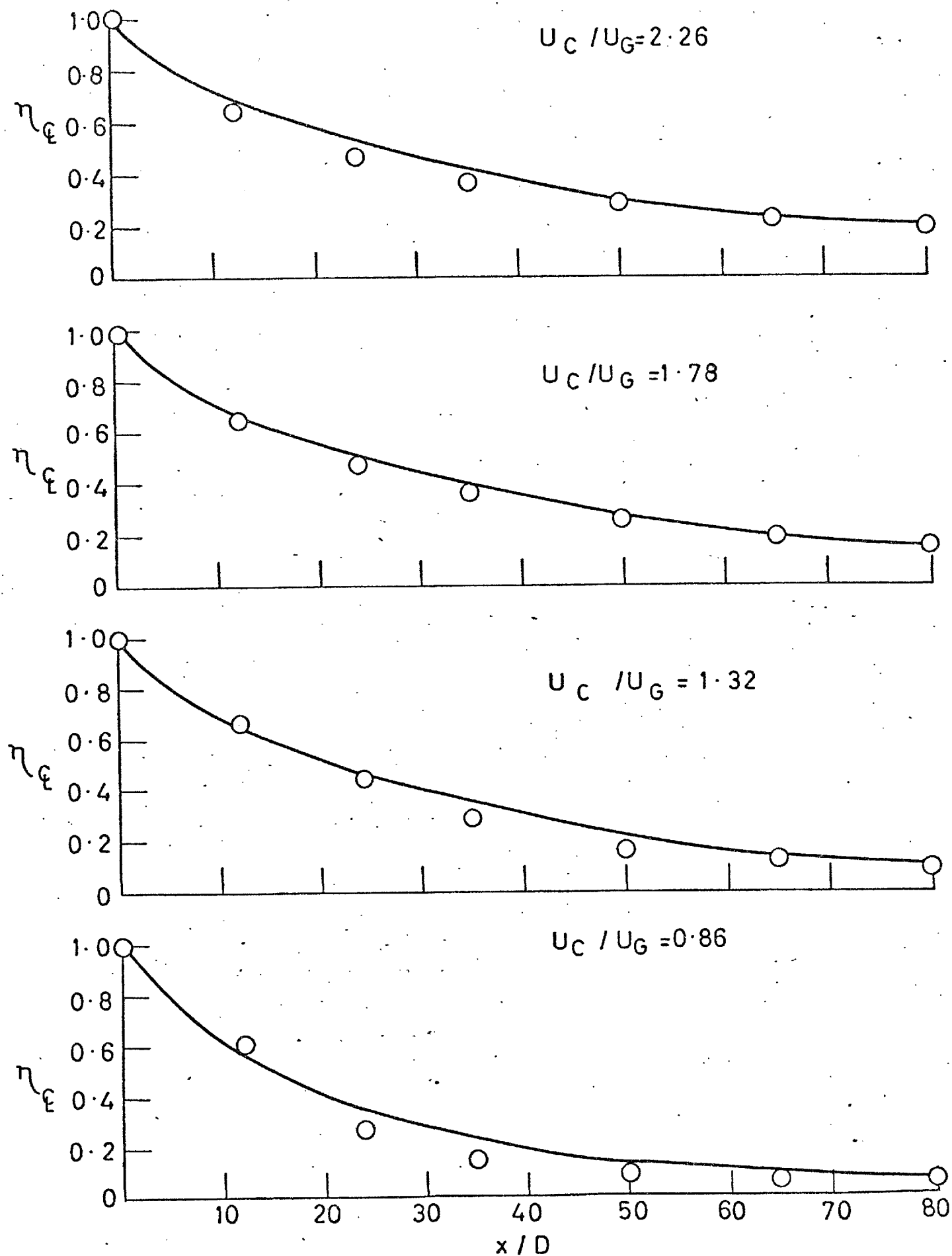


Fig. 5.2.1.(a) Comparison between predicted and measured values of impervious wall effectiveness, $D=8\text{ mm}$; $P_c/P_e \approx 10$

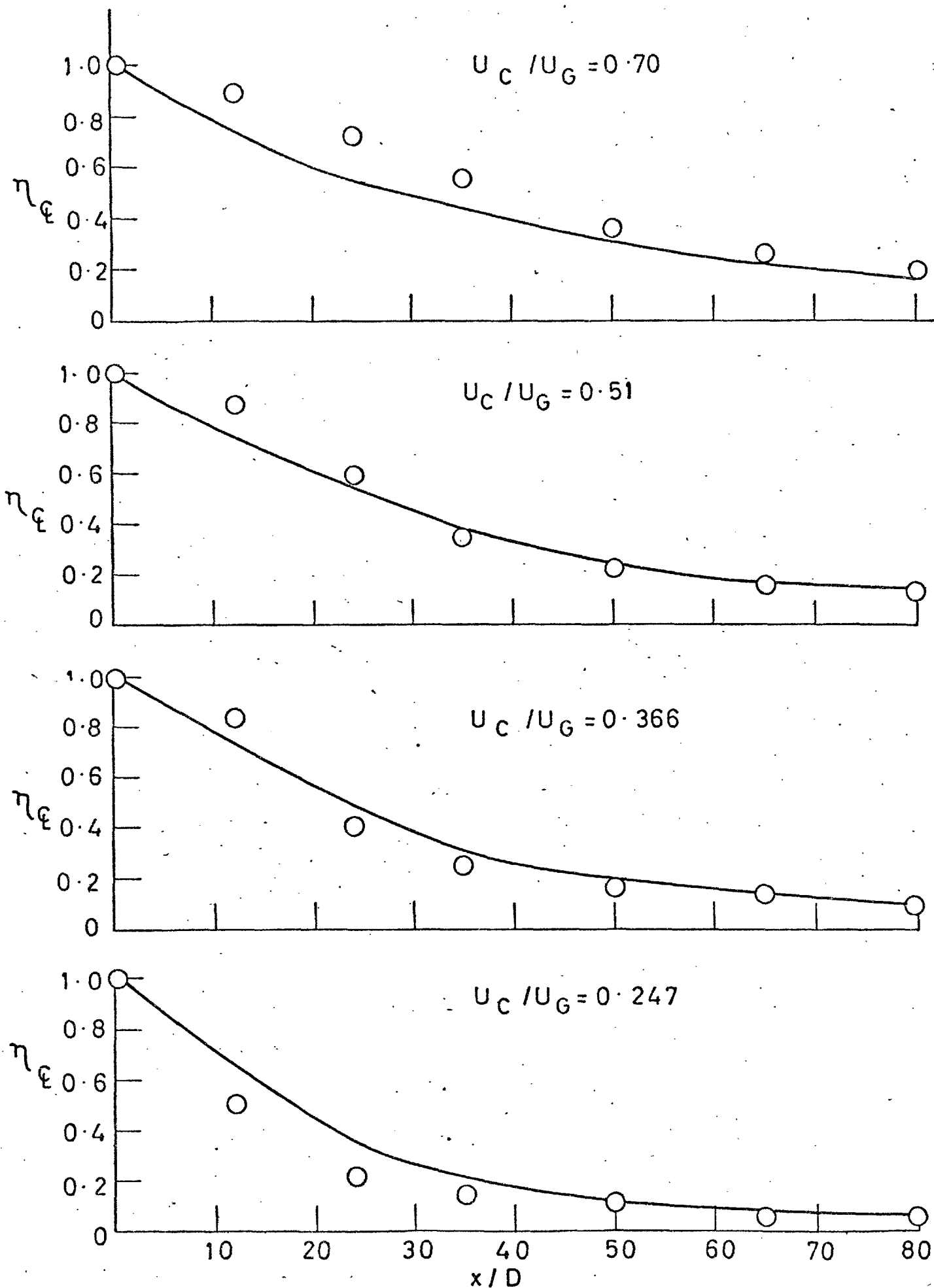


Fig. 5.2.1(b) Comparison between predicted and measured values of impervious wall effectiveness, $D=8\text{mm}$, $\rho_c/\rho_g = 4.17$

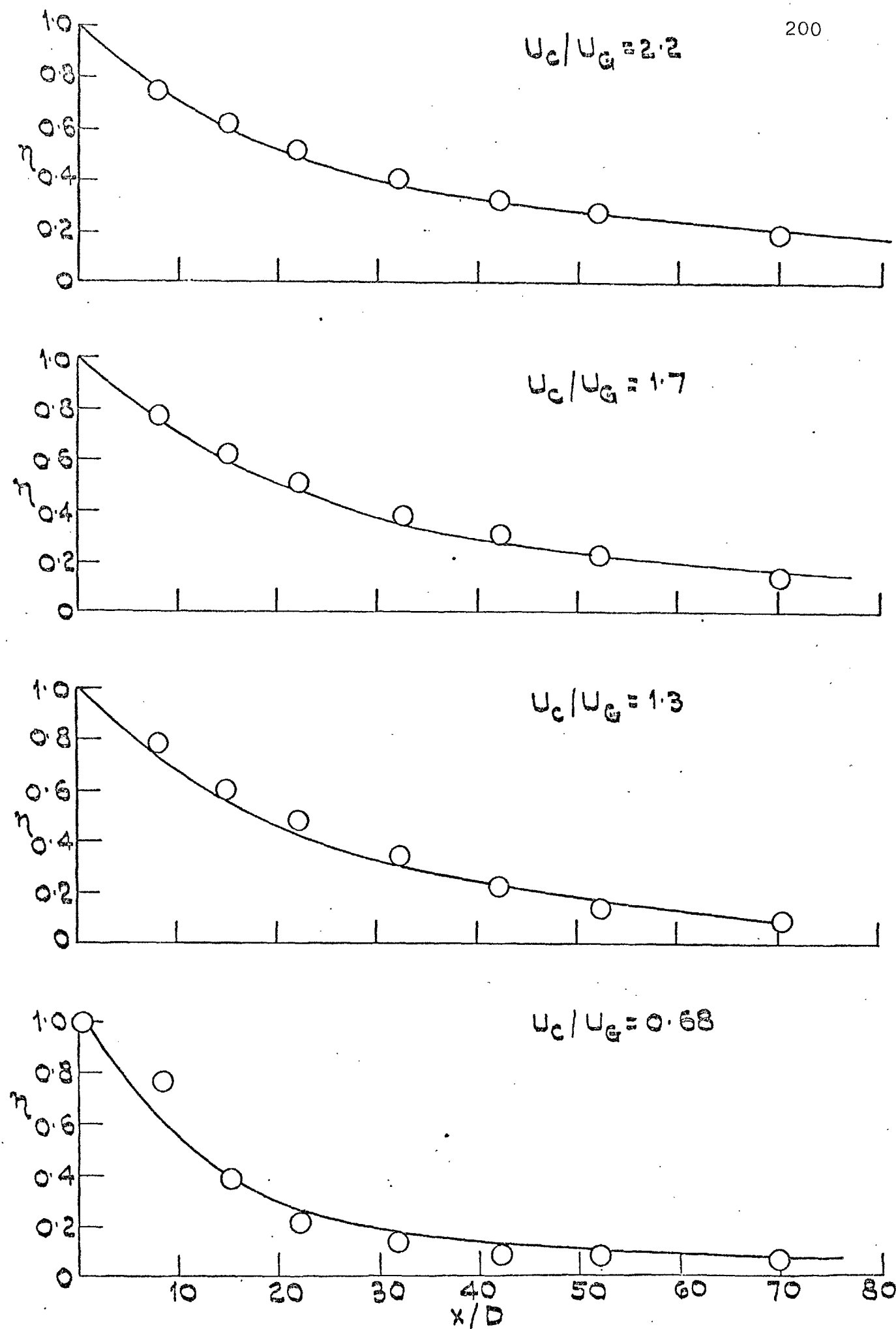


Fig. 5.2.2.(a) Comparison between predicted and measured values of impervious wall effectiveness, $D = 12.5 \text{ mm}$, $\rho_c/\rho_g = 1.0$

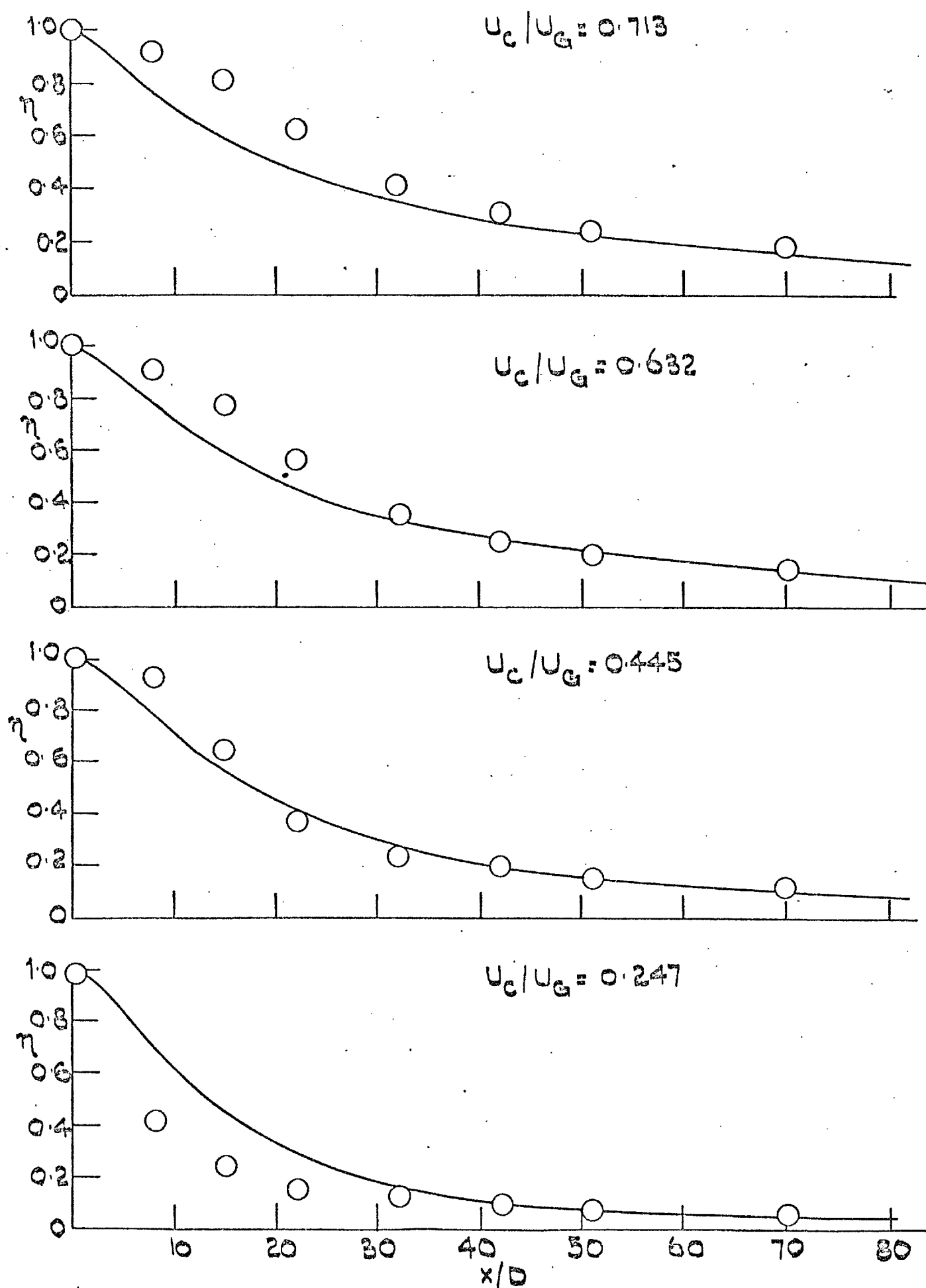


Fig. 5.2.2.(b) Comparison between predicted and measured values of impervious wall effectiveness, $D=12.5\text{mm}$, $P_c/P_G = 4.17$

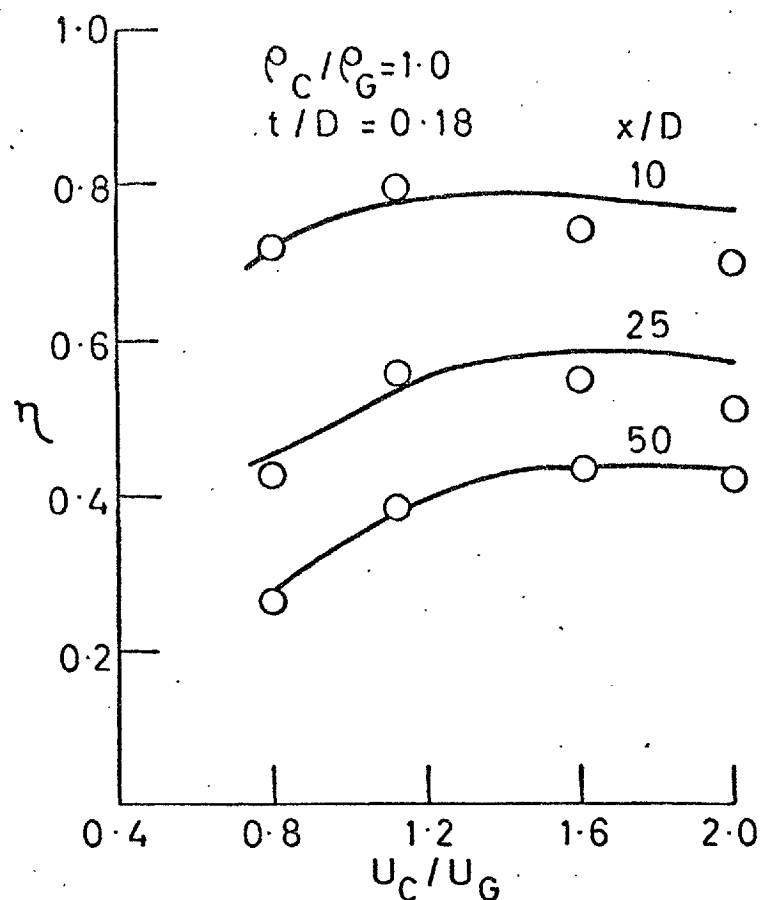


Fig. 5.2.3.(a) Predicted and measured values of $\eta_{imp.}$ downstream of slot 1.

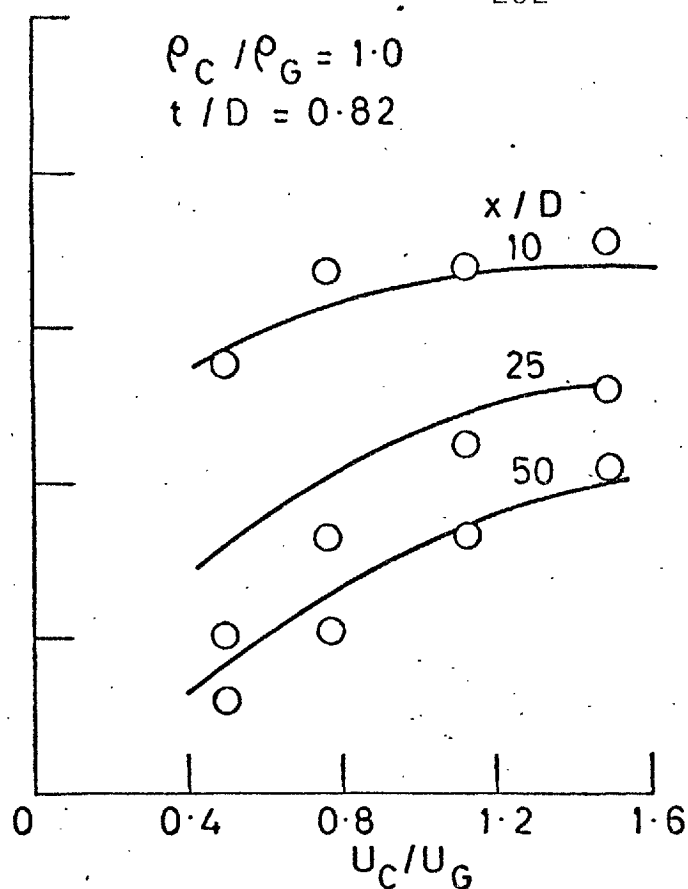


Fig. 5.2.3.(b) Predicted and measured values of $\eta_{imp.}$ downstream of slot 2.

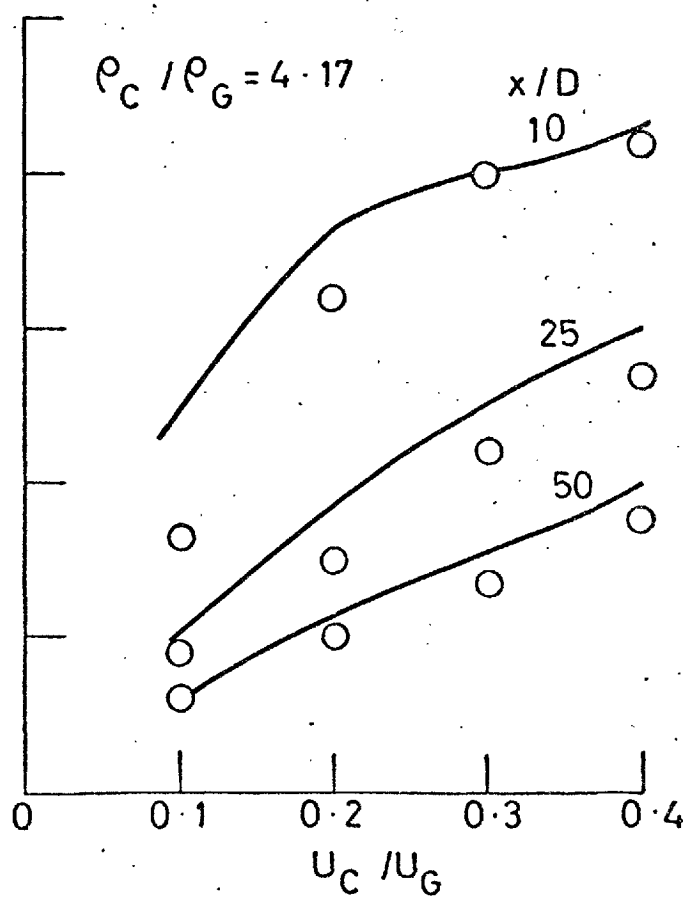
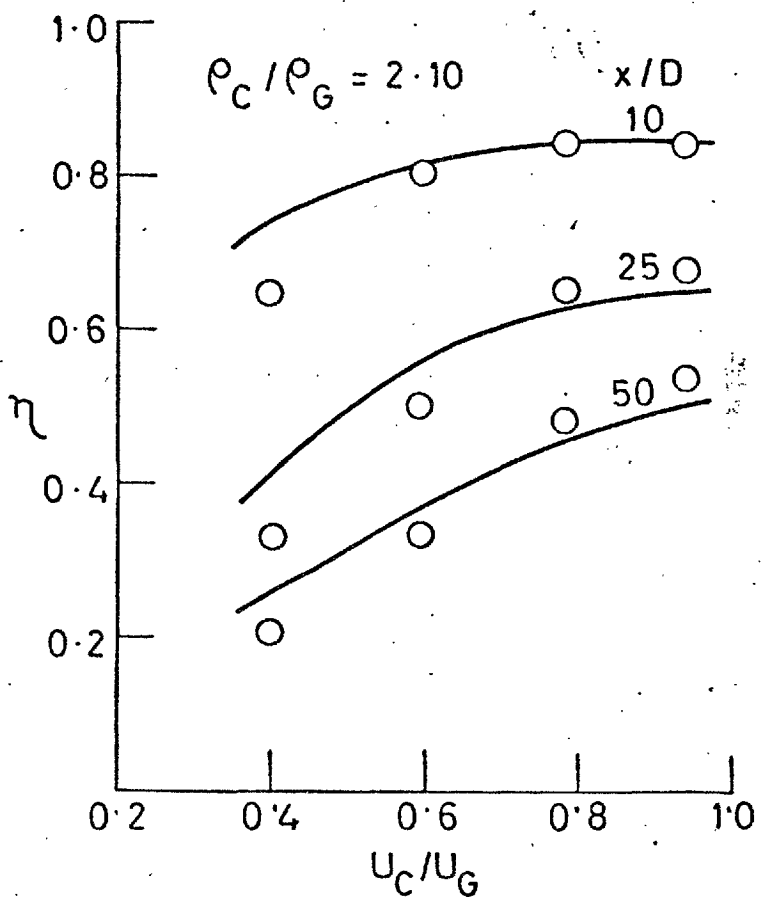
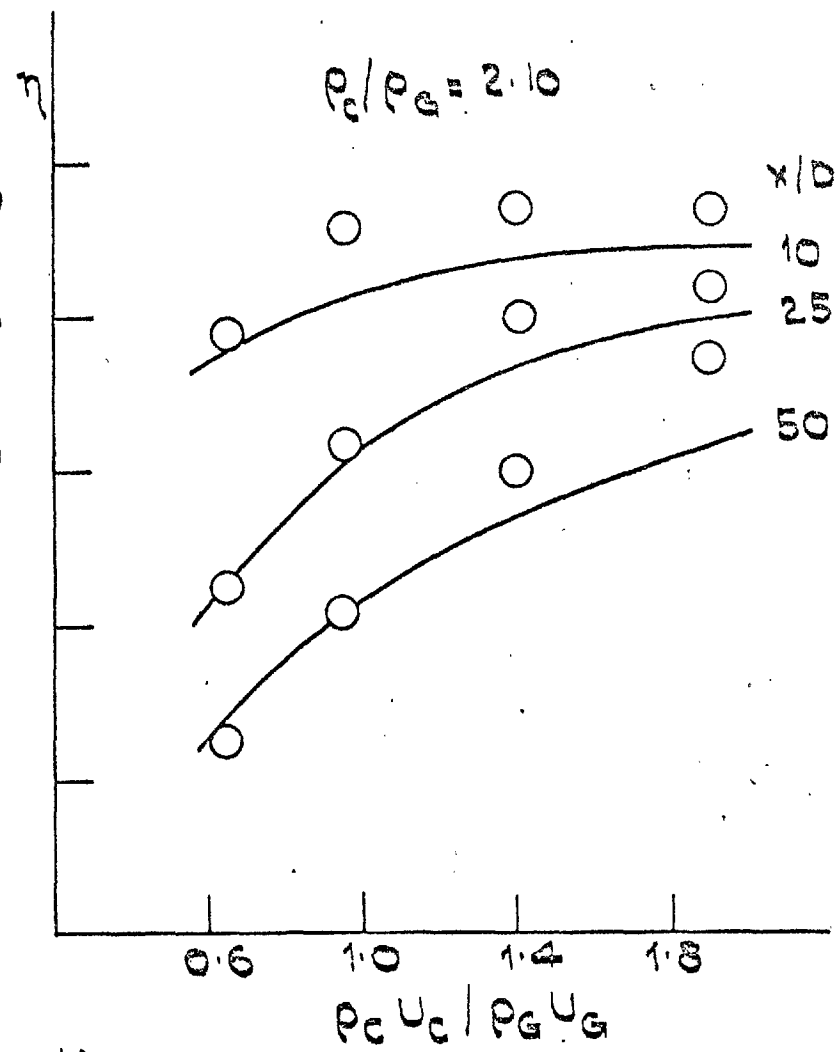
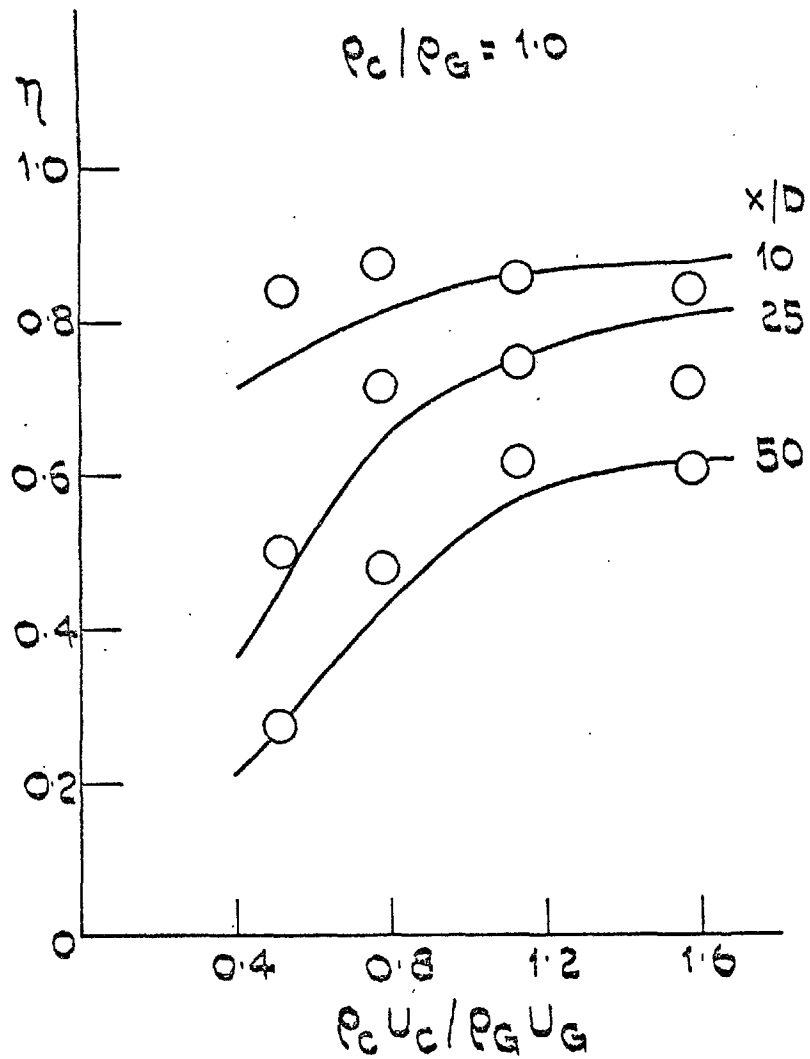


Fig. 5.2.4. Predicted and measured values of $\eta_{imp.}$ downstream of slot 1 in presence of density gradients.



a).
 Fig. 5.2.5. Comparison between predicted and measured values of impervious wall effectiveness downstream of slot 3.

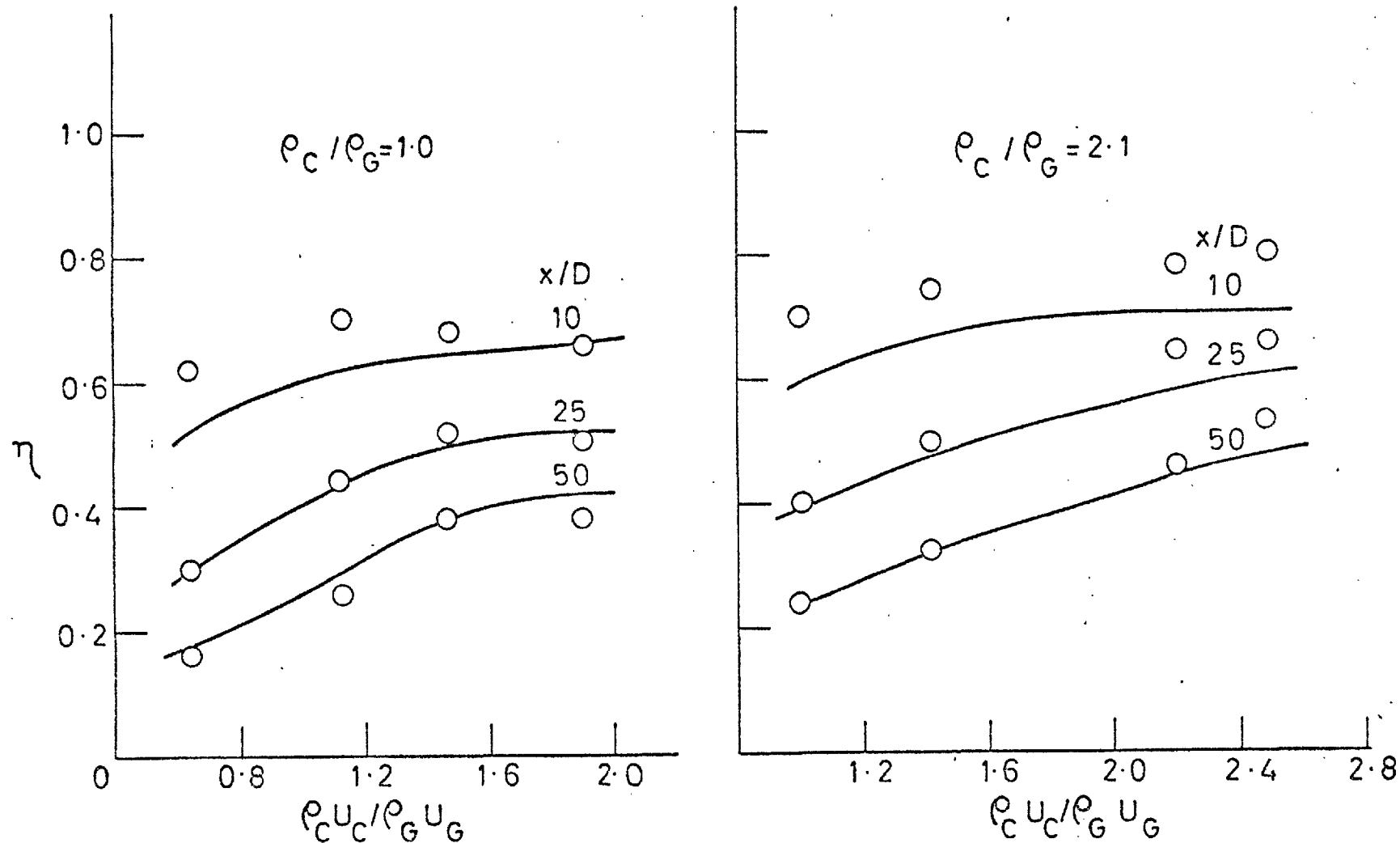


Fig. 5.2.6. Comparison between predicted and measured values of impervious wall effectiveness downstream of slot 4.

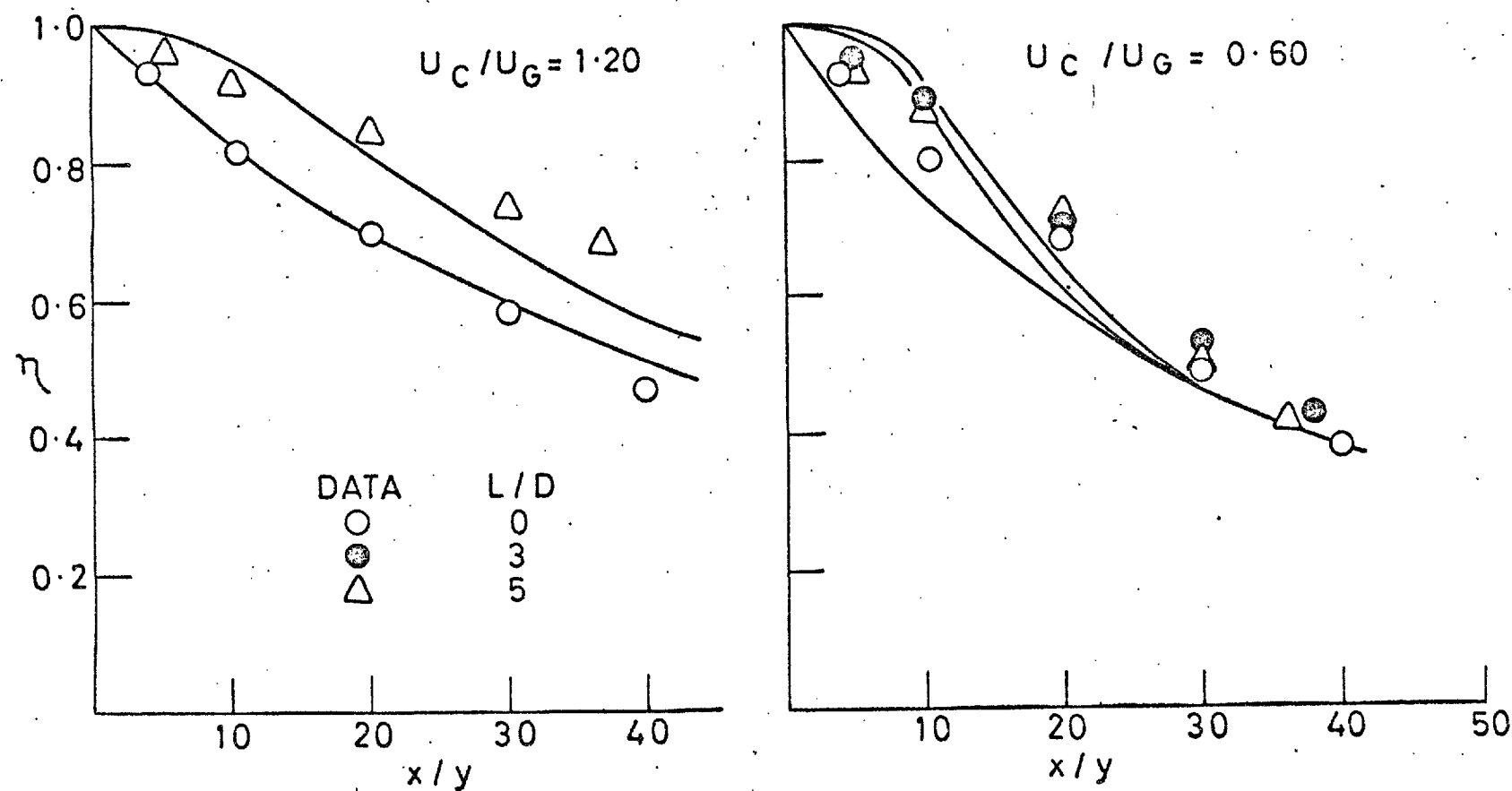


Fig. 5.2.7. Comparison between predicted values of adiabatic-wall effectiveness and the measurements of ref. (14).

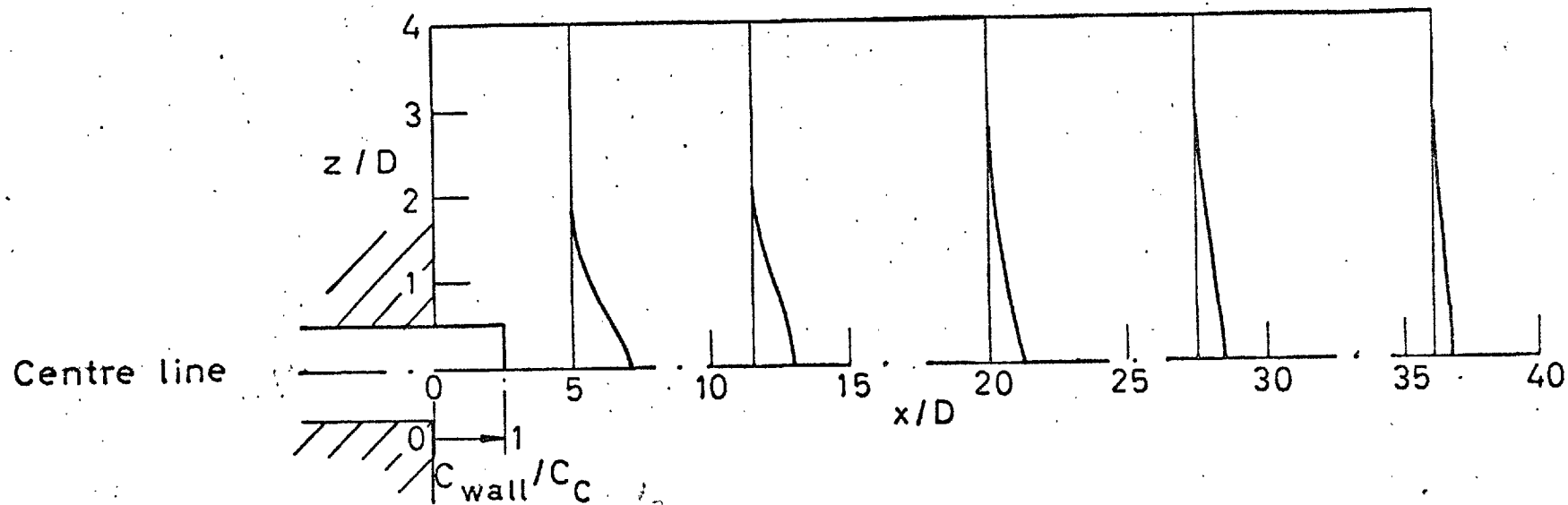


Fig. 5.2.8.(a) Spanwise variation of impervious wall-effectiveness for a single hole slot.

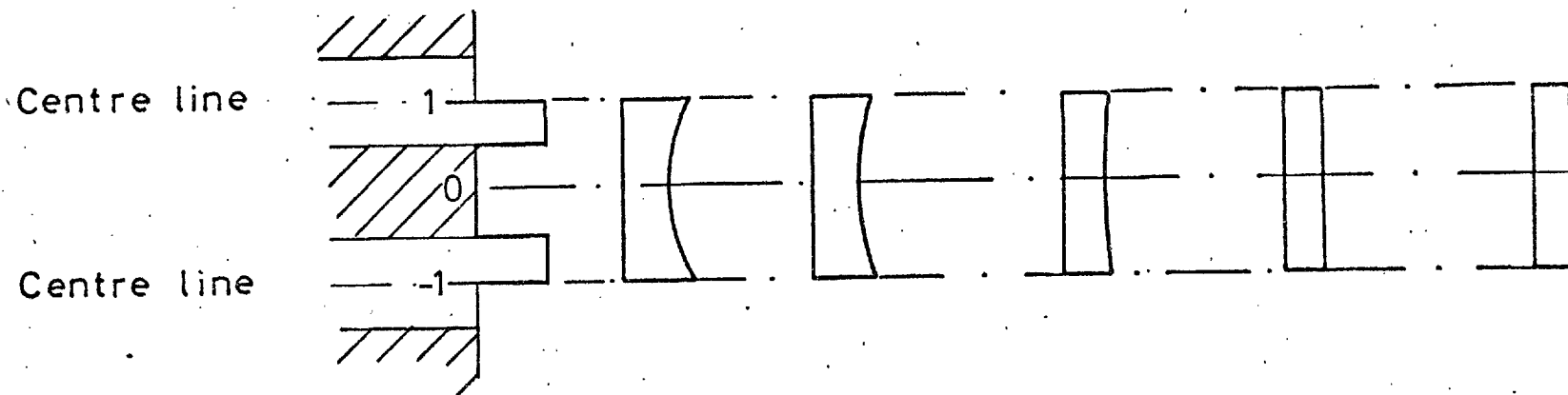


Fig. 5.2.8.(b) Spanwise variation of impervious wall-effectiveness for a multihole slot.

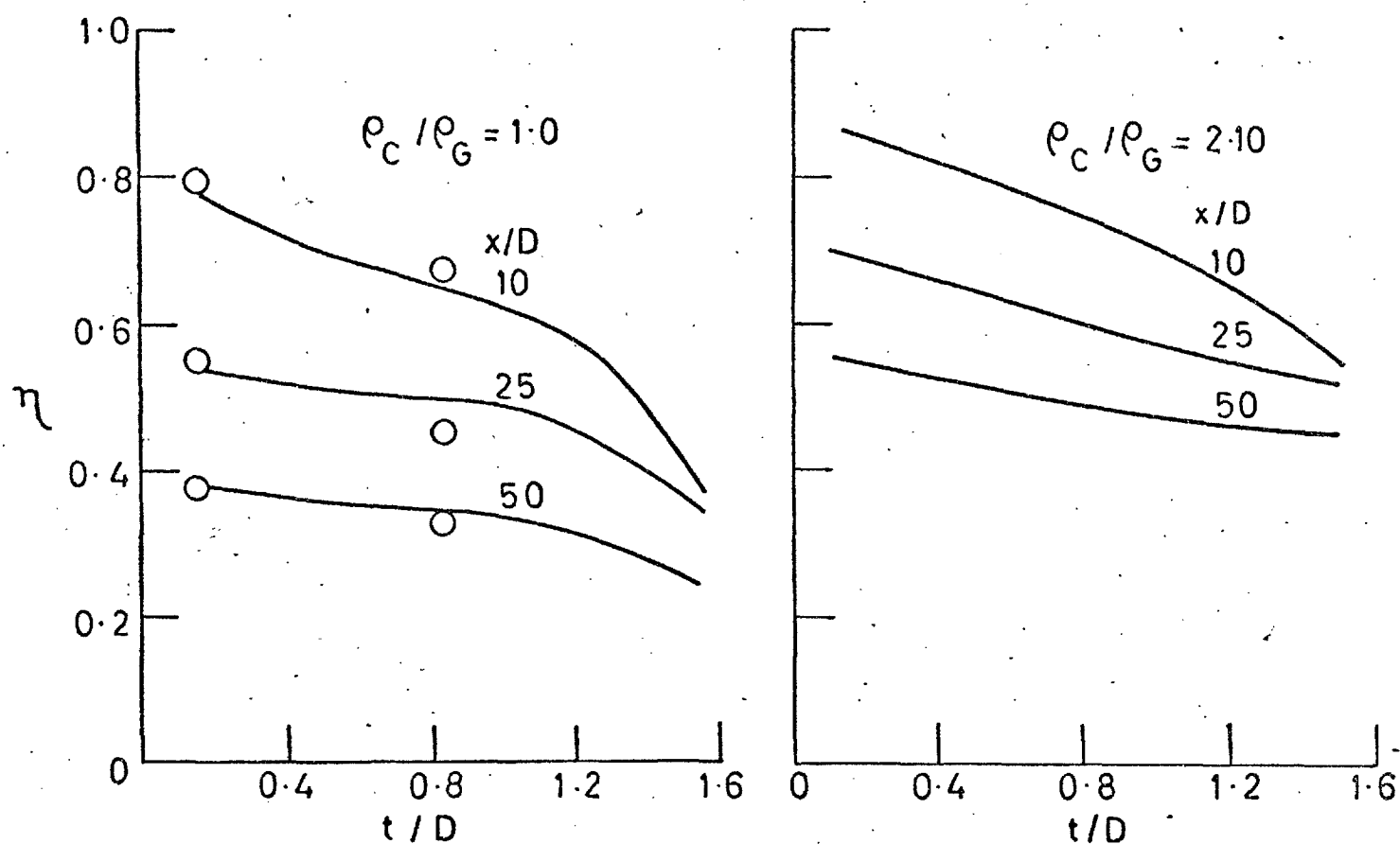


Fig.5.2.9. Influence of lip thickness on impervious wall effectiveness.

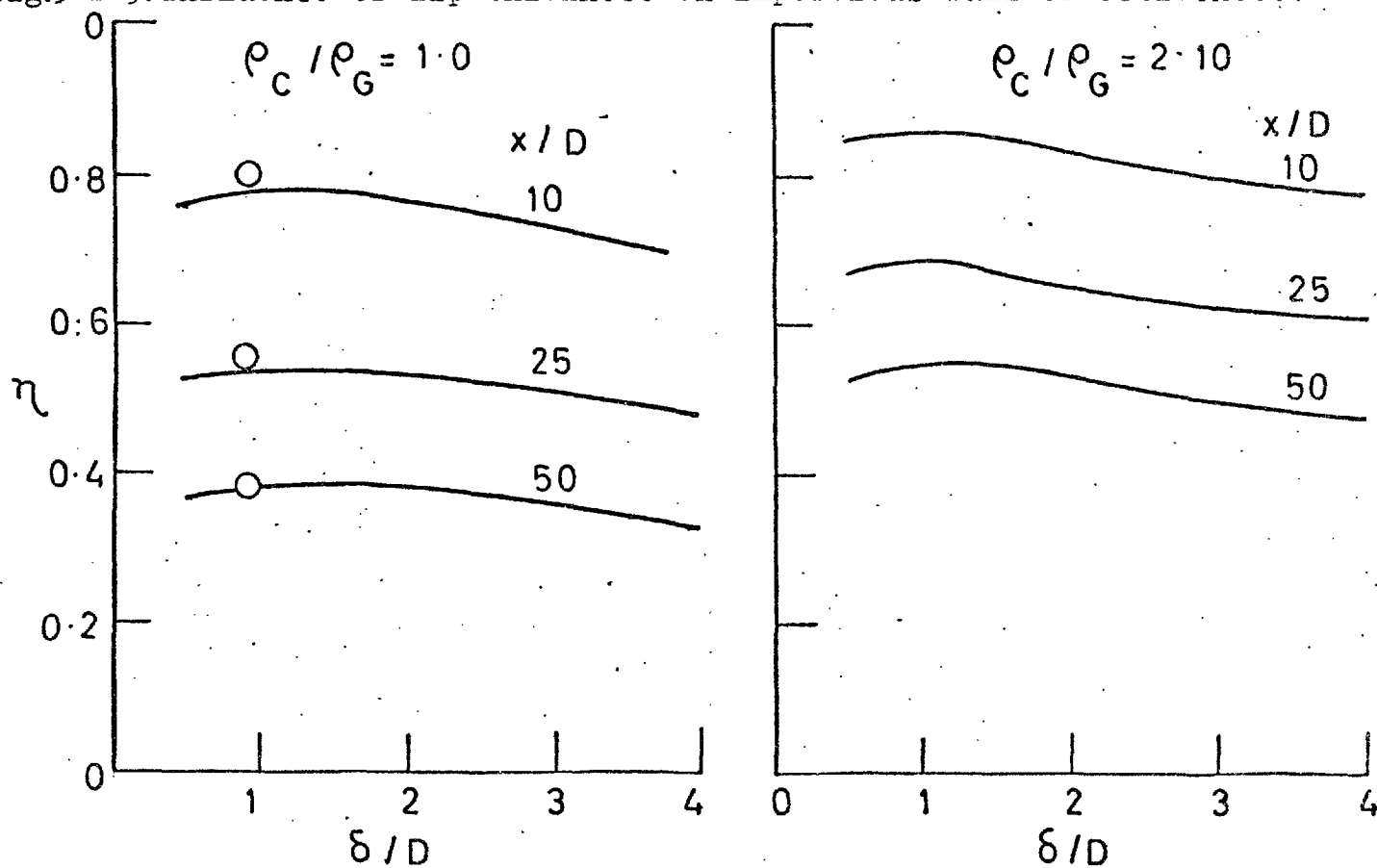


Fig.5.2.10. Influence of the thickness of the boundary layer on the upper surface of the slot, on η_{imp} .

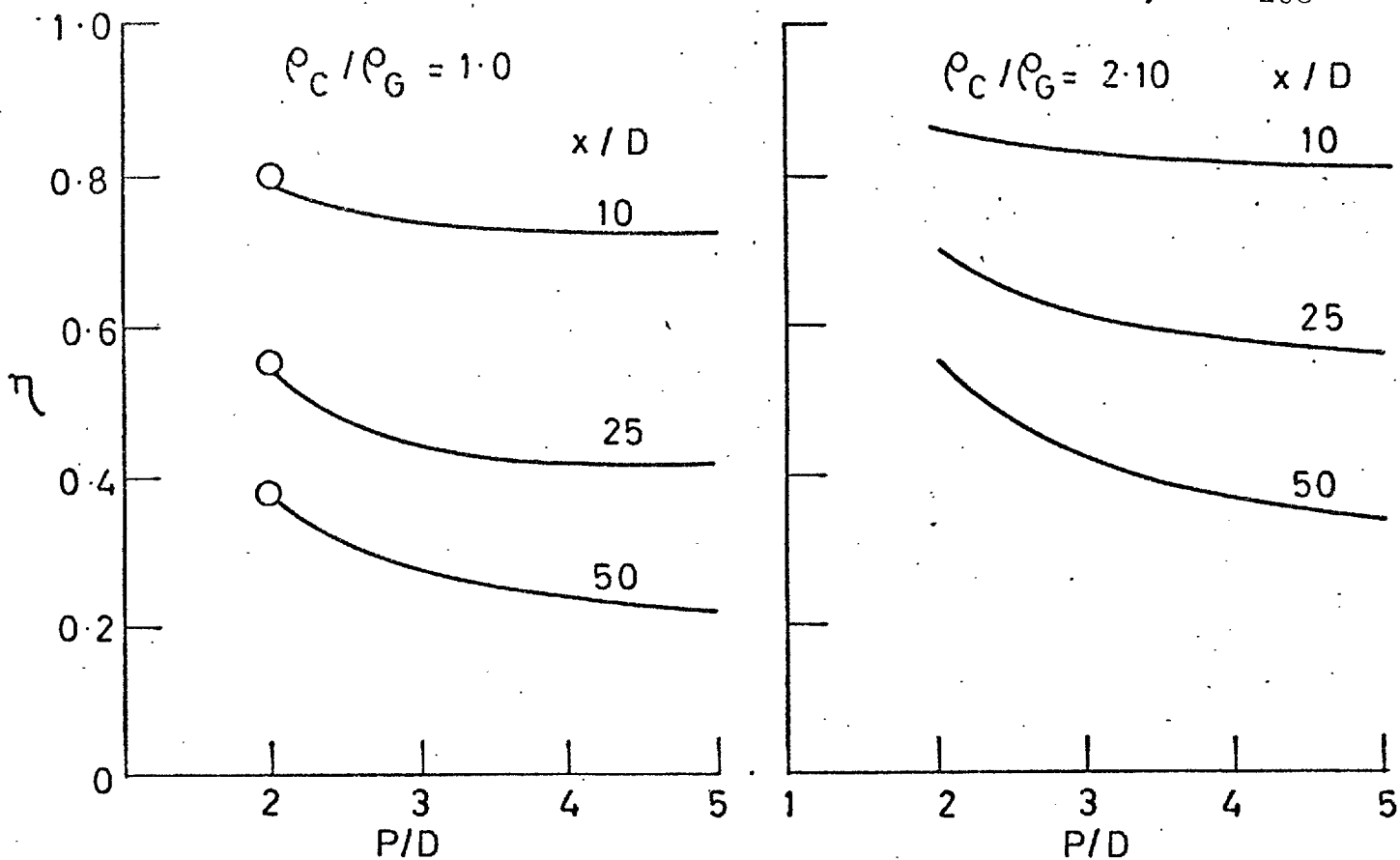


Fig. 5.2.11. Influence of hole pitch to diameter ratio on impervious wall-effectiveness.

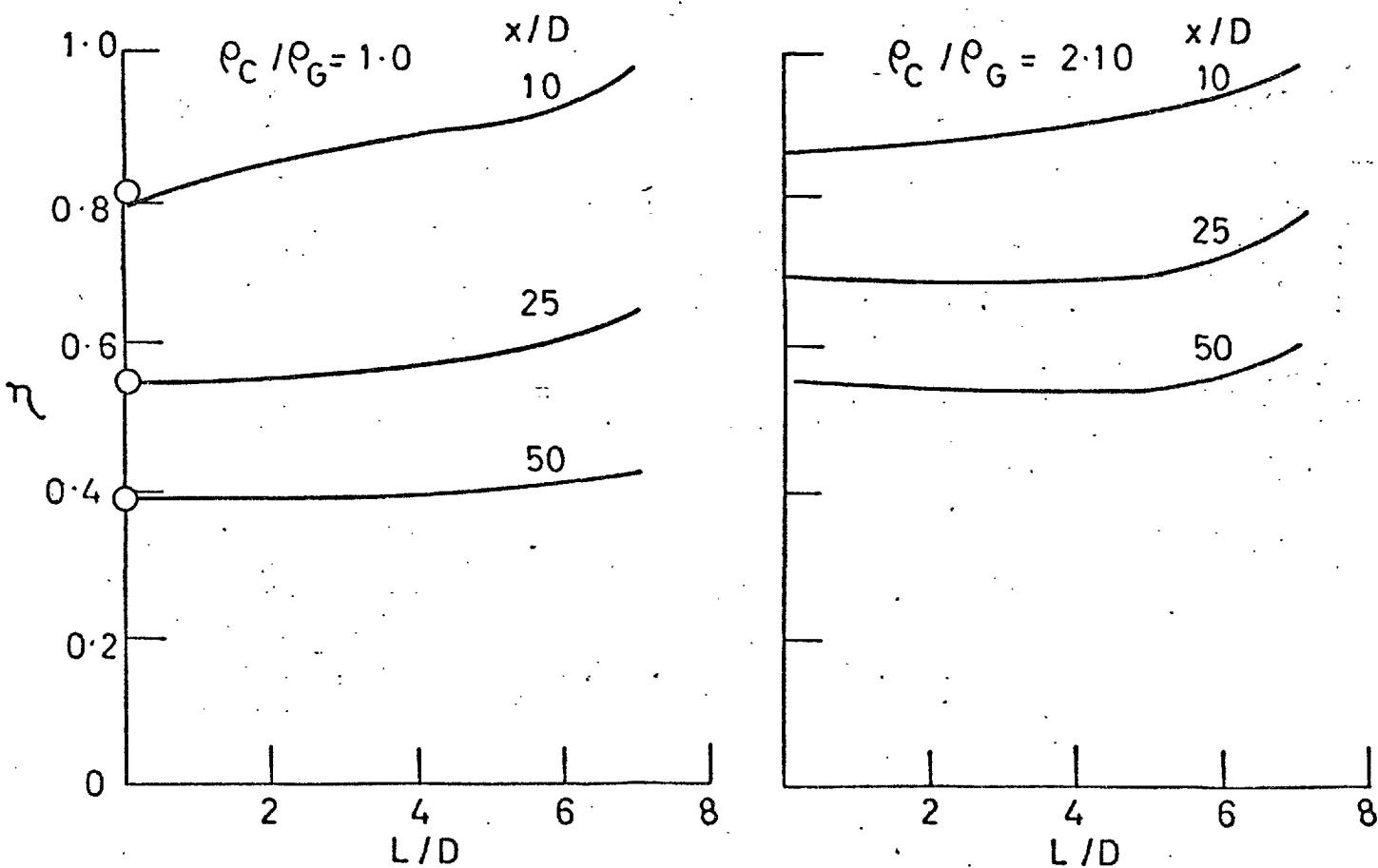


Fig. 5.2.12. Influence of lip length on impervious wall-effectiveness.

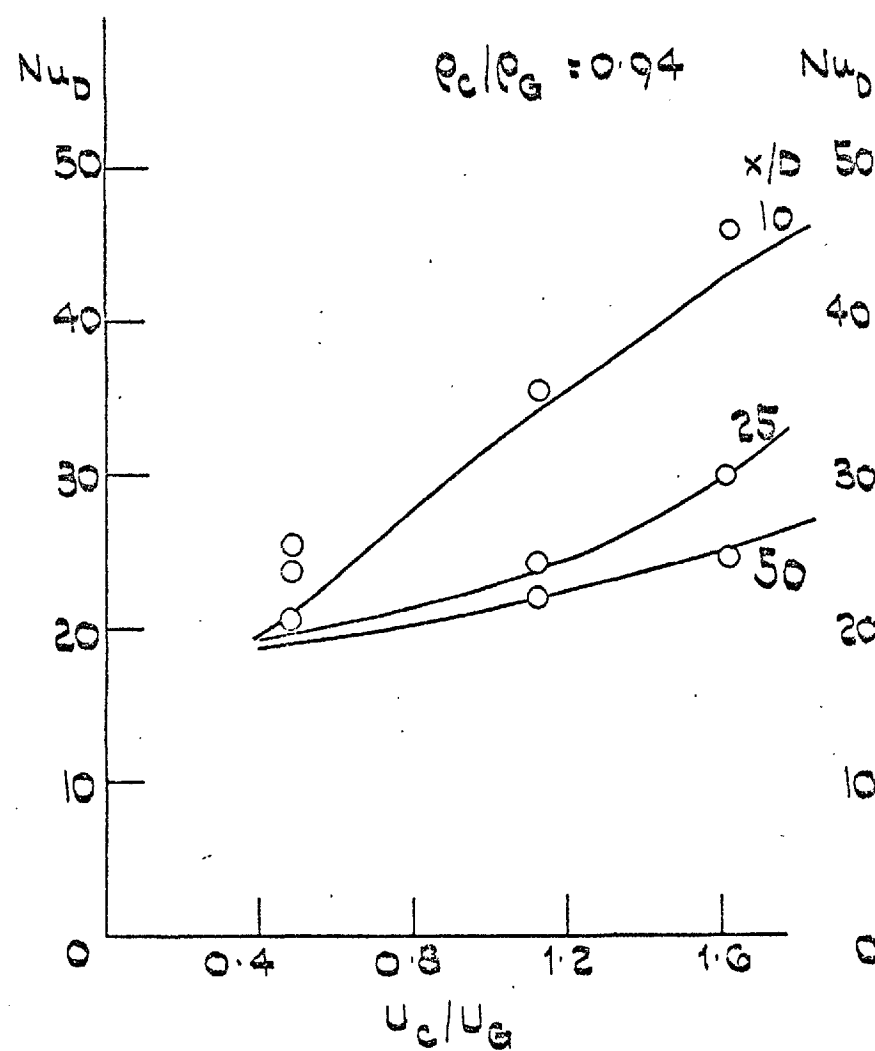


Fig. 5.2.13.(a) Comparison between measured and predicted values of heat-transfer coefficient downstream of slot 1.

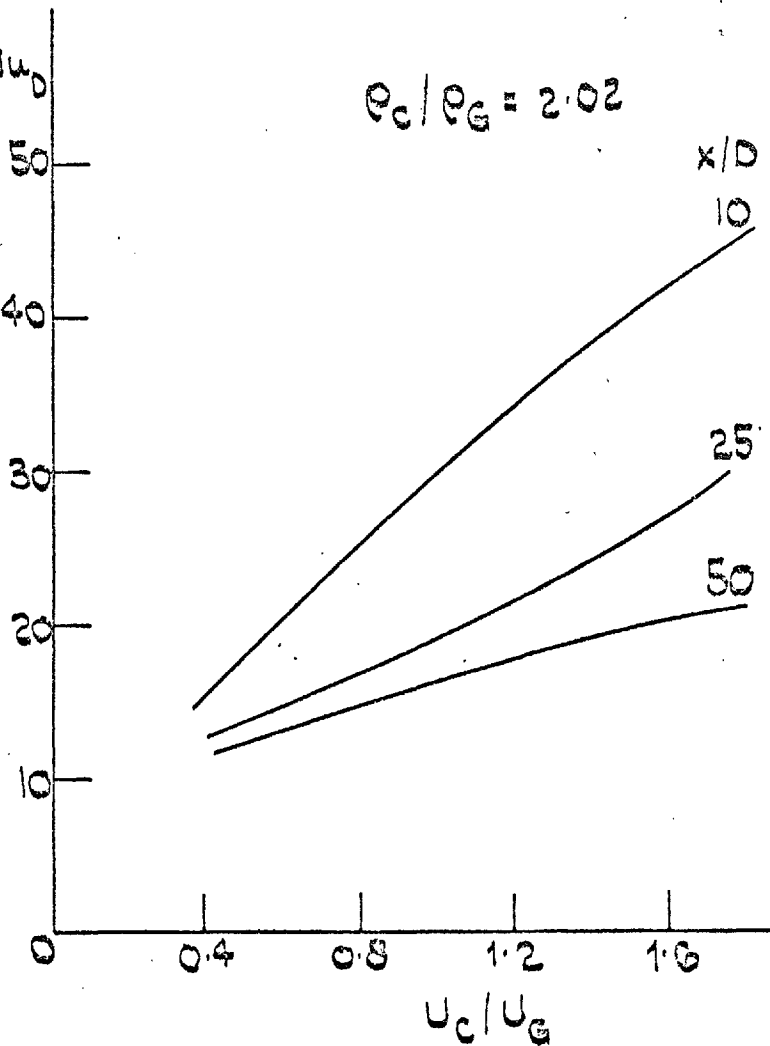


Fig. 5.2.13.(b) Predicted values of heat-transfer coefficient in presence of density gradients.

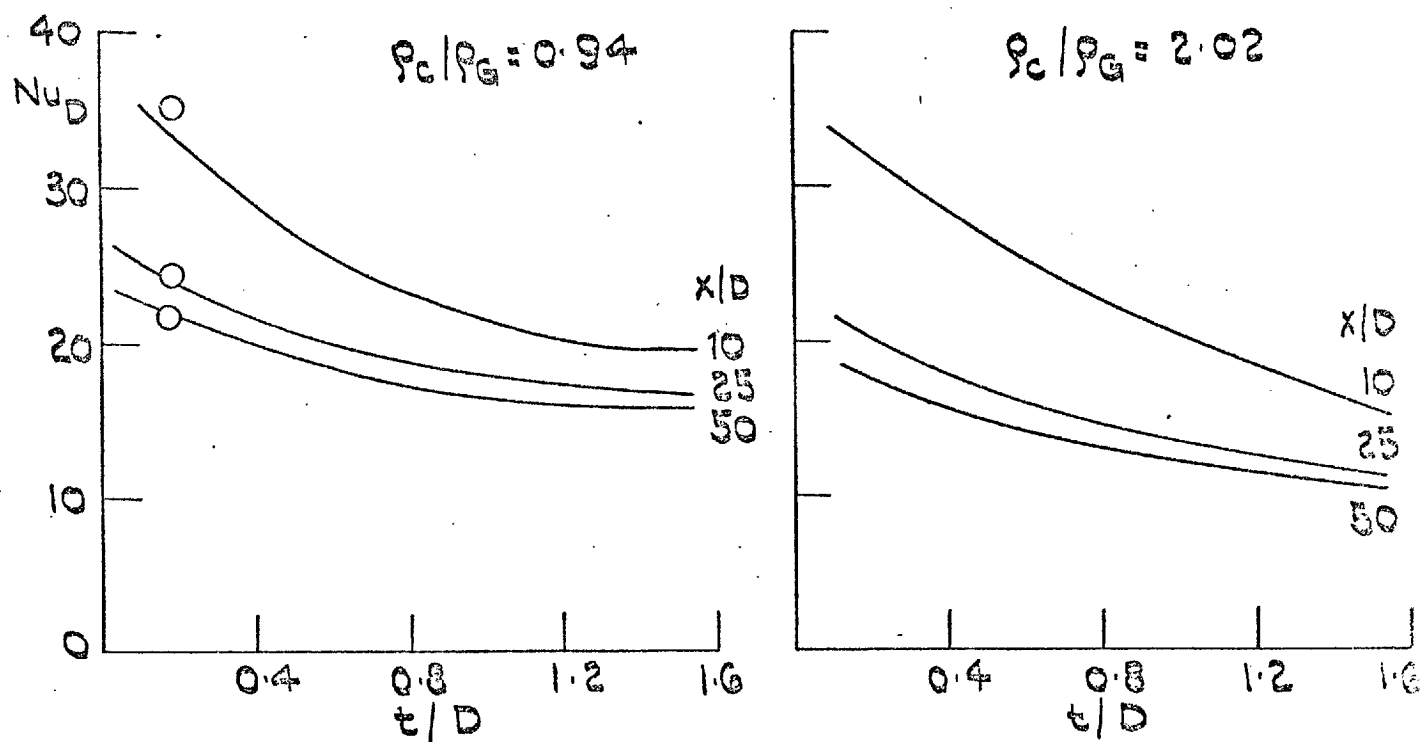


Fig. 5.2.14. Influence of lip thickness on heat-transfer coefficient.

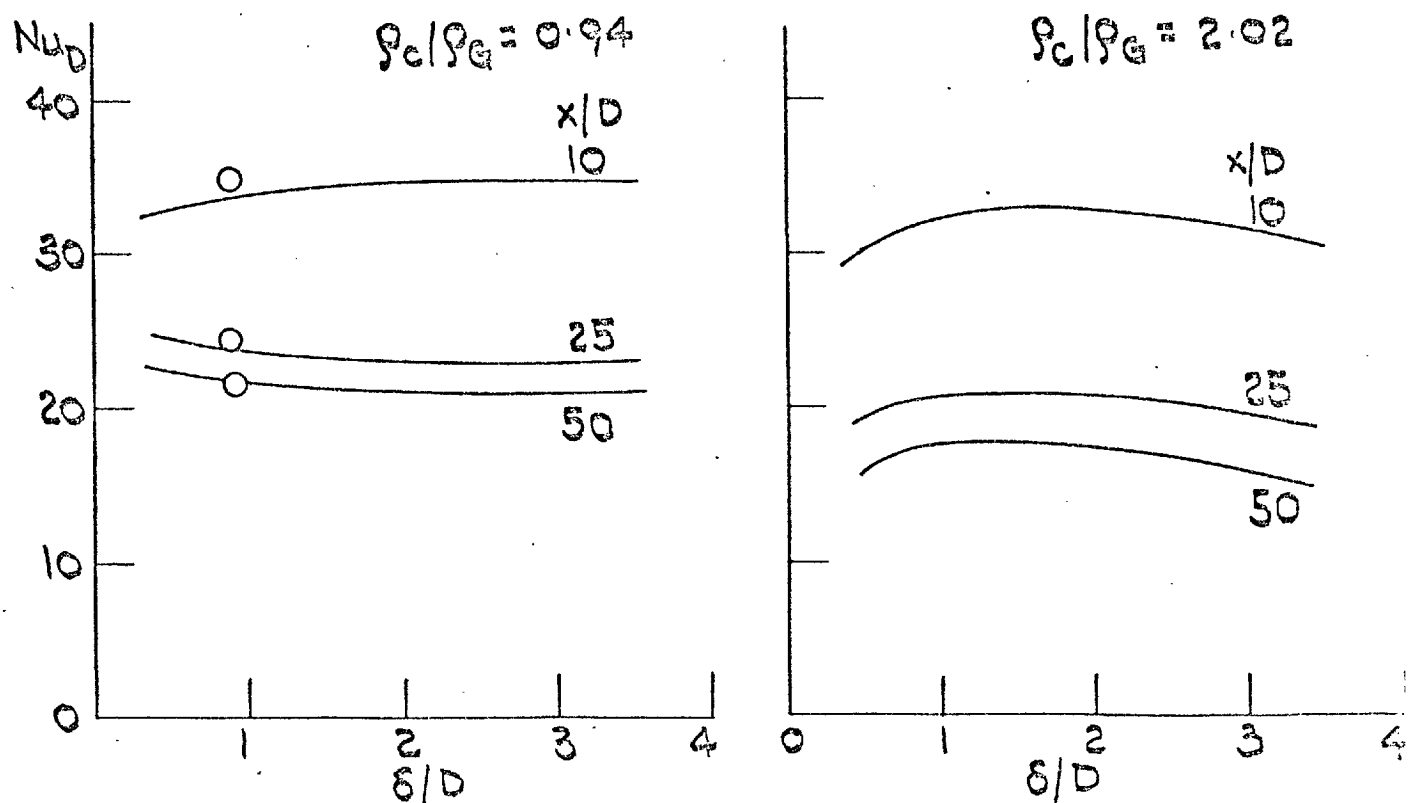


Fig. 5.2.15. Influence of thickness of the upper boundary layer on heat-transfer coefficient.

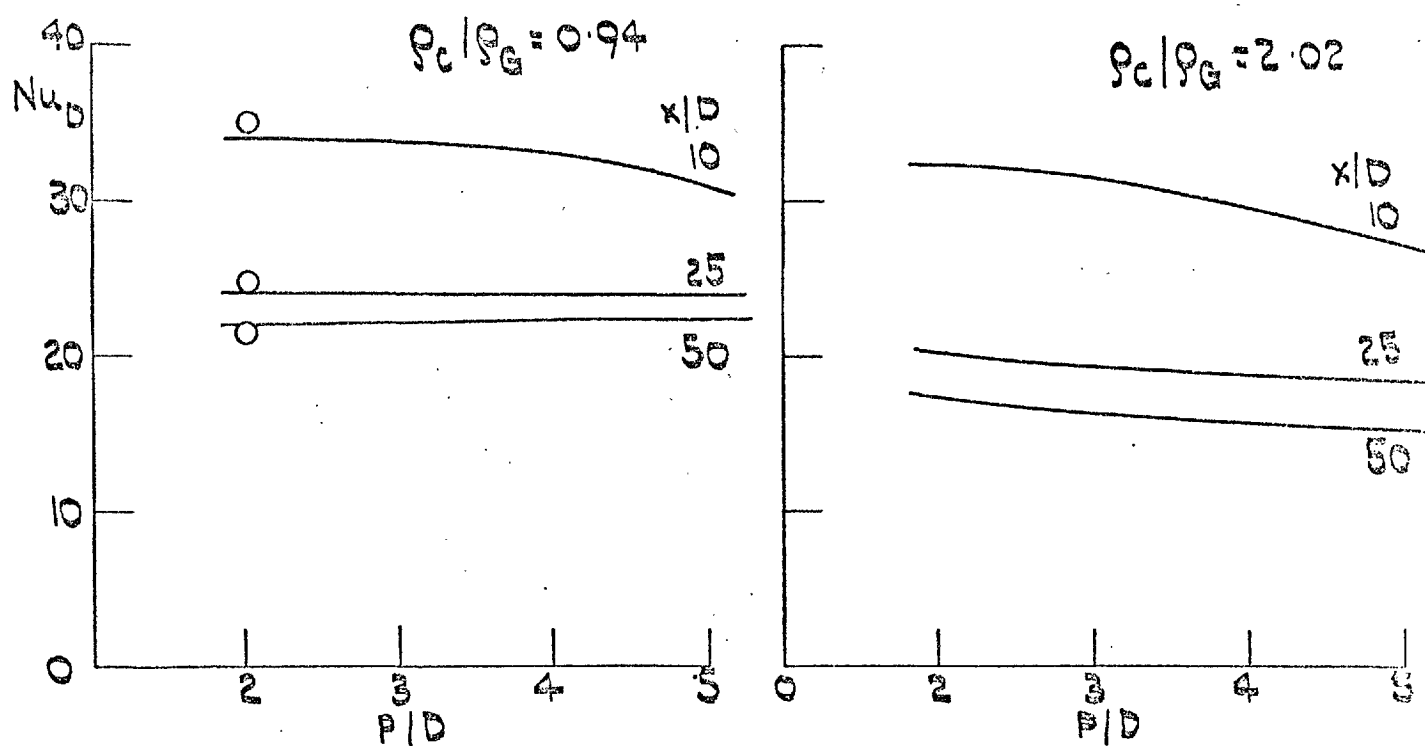


Fig. 5.2.16. Influence of hole pitch to diameter ratio on heat-transfer coefficient.

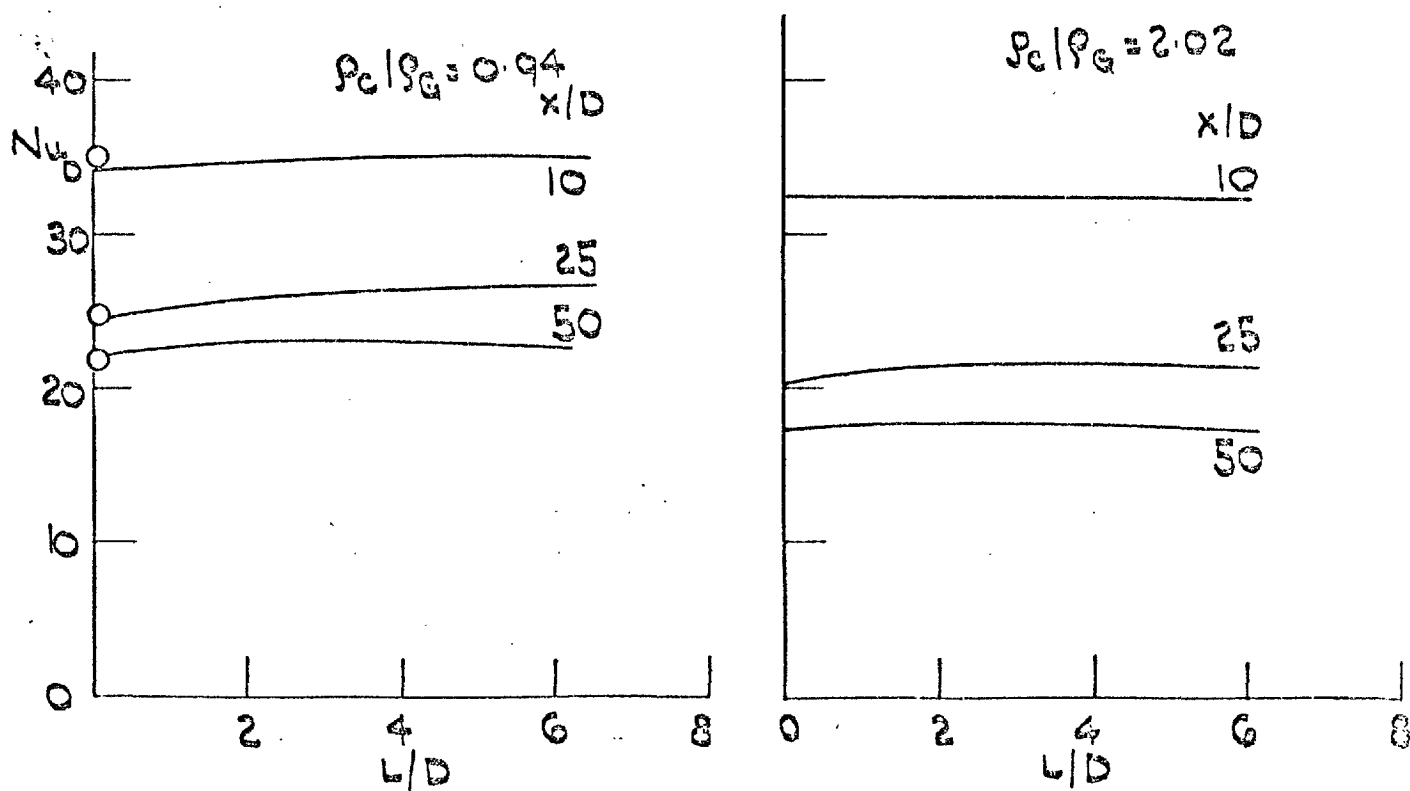


Fig. 5.2.17. Influence of lip length on heat-transfer coefficient.

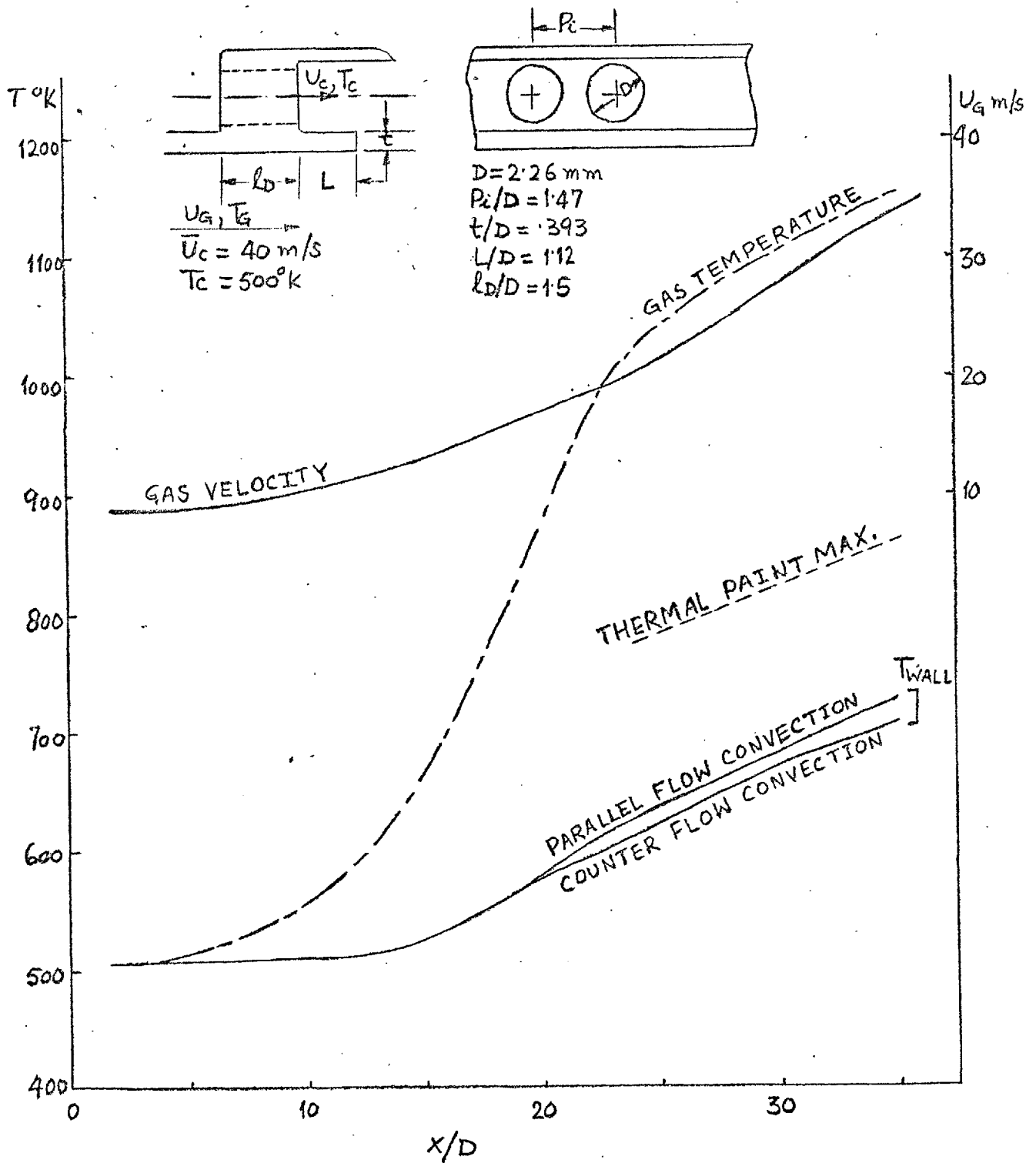


Fig. 6.2.1. Practical slot geometry, gas velocity and temperature variations and measured and predicted values of wall temperature.

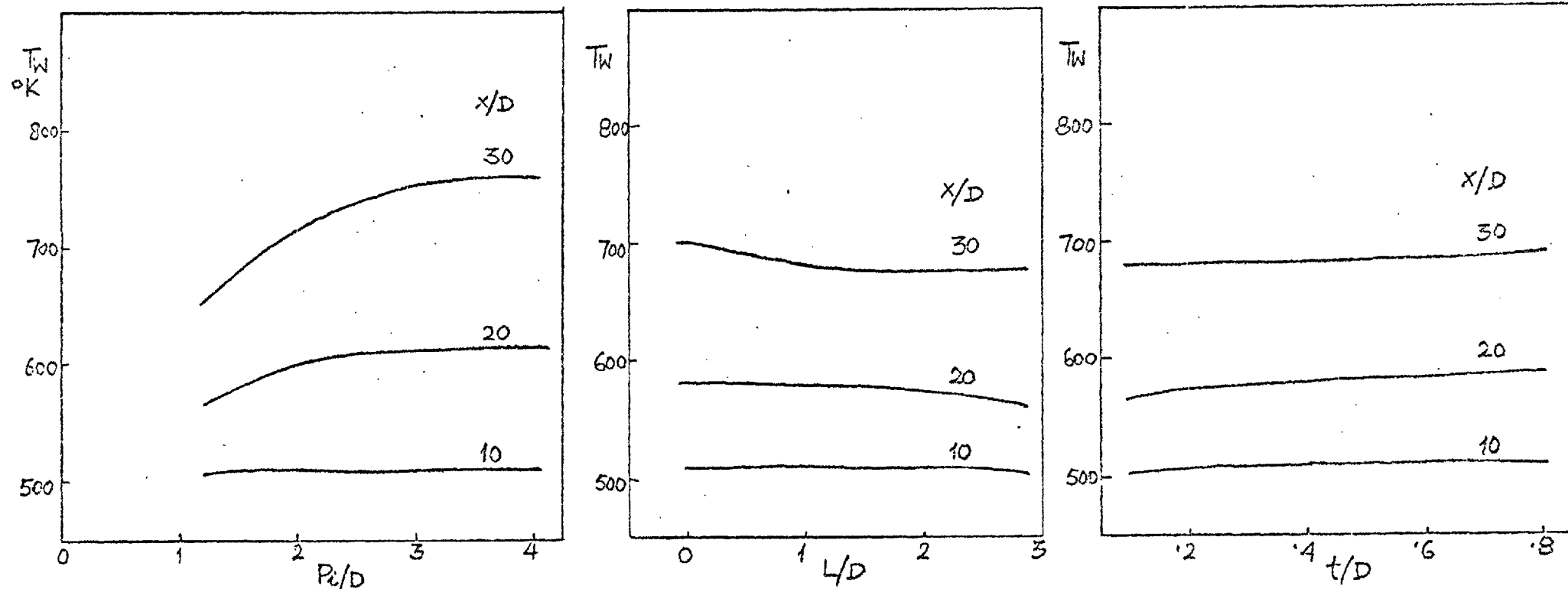


Fig. 6.2.2. Predicted influence of Pe/D , L/D , and t/D on the wall temperature of flame tube.