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Degradation modelling in process control applications

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Declaration of originality

I confirm that all material in this dissertation which is not my own work has been properly acknowledged.

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Abstract

Degradation of industrial equipment is often influenced by how a system is operated, with certain operating points likely to accelerate degradation. The ability to mitigate degradation of an industrial system would result in improved performance and decreased costs of operation. The thesis aims to provide ways for managing degradation by adjusting the operating conditions of a system.

The thesis provides original insights and a new classification of models of degradation to facilitate the integration of degradation models into process control applications. The thesis also develops an adaptive algorithm for degradation detection and prediction in turbomachinery, which is able to predict the expected future values of a degradation indicator and to quantify the uncertainty of the prediction. The thesis then proposes two frameworks for load-sharing in a compressor station in which the compressors are subject to degradation. One framework considers management of degradation and the other one focuses on power consumption of the whole station. These examples show how modelling of degradation can have an impact on the operation of an industrial system.

The approaches have been evaluated with case studies developed in collaboration with industrial partners. As demonstrated in the case studies, the outcomes of the research presented in this thesis provide new ways to take account of degradation in process control applications. The thesis discusses steps and directions for future work to facilitate the technology transfer from academic to industrial implementation.

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We're supposed to meddle with things we don't understand. If we hung around waitin' till we understood things we'd never get anything done.

Terry Pratchett

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Chapter 1

Introduction

1.1 Motivation

During its lifetime, equipment in an industrial system is subject to varying operating conditions, including the operating mode or environmental conditions. As a result, the system changes and the operation of the system often becomes inefficient. In this context, *system* refers to “a combination of interacting elements organized to achieve one or more stated purposes” (BSI, 2015c), while a “detrimental change in physical condition, with time, use, or external cause” is called *degradation* (BSI, 2017).

Degradation is often influenced by how the system is operated, with certain operating points likely to accelerate degradation. Similarly, degradation of equipment will impact the ability of a system to perform its desired function. Hence it may be desirable to manipulate the operating set points to manage degradation and to reduce the risk of an unplanned stoppage in advance of a scheduled maintenance overhaul.

The analysis of equipment degradation has traditionally developed in two main directions. One approach, of great interest for control system design, has been to consider that degradation causes fundamental changes to the system. Another approach, used in optimal maintenance planning and production scheduling, considers degradation as a separate process that affects performance but does not necessarily change the way the system operates. This thesis considers primarily the first approach and focuses on how to improve the operation of a system taking degradation into account.

Current operating strategies rarely take degradation into account. In particular, the mutual relationships between degradation and the behaviour of a system are overlooked. As a result, the costs of operating a degrading system increase. The control and optimization of industrial systems would thus benefit from analysis of the impact of degradation. The thesis aims to provide ways for managing degradation by adjusting the operating conditions of a system.

1.2 Problem statement

The ability to mitigate the detrimental influence of degradation on the system would result in improved performance and decreased costs of operation. The research question addressed in the thesis is if and how the adjustment of the operating conditions leads to improved operation of an industrial system.

Answering the research question faces the following challenges related to mitigation of the detrimental influence of degradation on industrial systems:

- Challenge 1. Analysis of the interactions between degradation and the system,
- Challenge 2. Quantification of degradation of industrial systems,
- Challenge 3. Analysis of relationships within a system subject to degradation,
- Challenge 4. Design of control systems to manage degradation.

To tackle these challenges, existing models of degradation will be first classified anew with respect to the relationship between the degradation and the behaviour of the system. Furthermore, an algorithm for on-line degradation quantification will be developed. Finally, new strategies will follow that explore the relationships between the degradation and the behaviour of a system to improve the performance. One strategy will explore degradation management. The other strategy will investigate the influence of degradation in a constrained optimisation problem from the perspective of power minimisation. The results of the thesis will be applied in a case study from a compressor station and tested with industrial data.

1.3 Industrial case study

The case study analysed in the thesis focuses on degradation of compressors working in a compressor station. A compressor station is typically a part of a gas transport network and provides a boost for transporting the gas to the receivers (BSI, 2014a). Improving the operation of a compressor station would thus improve the performance of the whole network. The importance of the topic is reflected in the number of development projects submitted in the Ten-Year Network Development Plan prepared by European Network of Transmission System Operators for Gas (2018). Compressor stations were considered in 71% of all the proposed projects.

Table 1.1: Gas imported in the UK in 2014-2018 (millions cubic meters) (Department for Business, Energy & Industrial Strategy, 2019)

	Belgium	the Netherlands	Norway	LNG	in total
2014	365	6567	25561	11327	43821
2015	196	3327	28104	13925	45552
2016	1387	4402	31708	10746	48243
2017	2649	1869	35890	6749	47156
2018	3197	2709	34086	7169	47160

Figure 1.1 shows the gas transportation network in the North Sea, with emphasis on the pipelines connected to the UK (European Network of Transmission System Operators for Gas, 2019). Currently, the UK imports approximately 50% of its natural gas, with forecasts increased to 72% by 2035 (Oil & Gas Authority, 2019). As indicated by Department for Business, Energy & Industrial Strategy (2019), gas transport in the UK is primarily imported from Norway (72%), Belgium (7%), and the Netherlands (6%). The remaining 15% is shipped as liquefied natural gas. Table 1.1 presents the amount of gas imported into the UK in years 2014-2018 in millions cubic meters. The numbers emphasize the importance of gas transport network, in particular from Norway, as the gas delivered to the UK accounts for approximately one third of the gas produced in Norway (Norwegian Petroleum, 2019) in 2018. The thesis focuses on analysis of compressor stations as they are responsible for boosting the gas through the pipelines.

The case study was developed in collaboration with ABB AS in Norway as a part of PRONTO H2020 project. The details of the project can be found in Appendix A.

1.3.1 Gas transport

Rios-Mercado and Borraz-Sanchez (2015) provided an overview of the operation of gas transportation networks focused on natural gas processing. They distinguished two types of objectives for a natural gas transportation system with compressor stations:

- “Exploration, drilling, extraction, production and long-term storage of natural gas”,

- “Gathering, short-term storage, transportation and distribution of natural gas”.

A schematic of a gas transport network is presented in Fig. 1.2 (adapted from (BSI, 2014a)). There are two compressor stations (1), connected to a transmission line (5). A metering station (3), if available, measures the properties and the amount of the gas. The compressor stations deliver the gas to the distribution system (2) or to a storage facility (4).

The objectives for a gas transport network indicated by Rios-Mercado and Borraz-Sanchez (2015) concern both design and operation of the gas transport system. Whereas the design of the network influences the operation of the transportation system (Kurz et al., 2012), the operating performance depends on how the compressor stations in the network are operated (Rios-Mercado and Borraz-Sanchez, 2015).

Kashani and Molaei (2014) summarised the objectives that have been considered for the operation of a compressor station in a gas transport network. They included economic objectives, such as maximisation of profit or minimisation of operating costs, technical objectives, such as minimisation of power and fuel consumption, minimisation of the number of running compressors or maximisation of the throughput, and environmental objectives, such as minimisation of harmful emissions. This thesis adds to the list by investigating the influence of degradation in compressor stations, as will be presented in Section 1.3.4.

1.3.2 Compressor stations

Typically, a compressor station consists of two or more compressors working in series or in parallel. An example of a compressor station with two compressors in parallel is shown in Fig. 1.3. Each compressor A and B is usually characterized by a *compressor map*, which expresses the relationship between the suction and discharge pressures, compressor speed, mass flow, and compressor efficiency. The map is normally provided by the manufacturer, or it may be derived from first principles (Gravdahl, 1998). The compressor maps will be described in detail in Chapter 2.

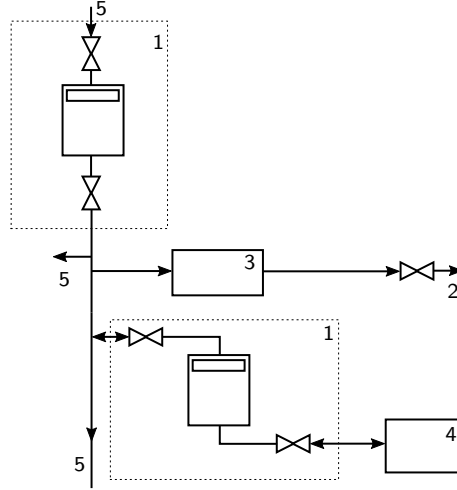


Figure 1.2: A schematic of a gas transport network adapted from BSI (2014a)

The load-sharing module allocates the loads to each compressor, so that the demand for the gas is satisfied. The term *load* refers to the amount of gas processed by a compressor. The demand might come from the final customers or from other compressor stations (Xenos et al., 2016b). The system from Fig. 1.3 includes flow controllers FC which receive their set-points from the load-sharing module. The flow controllers then provide set-points to the respective speed controllers SC (Cortinovis et al., 2016b).

The existing approaches to operating a compressor station, both as a part of the network and independently, will be presented in Section 3.2.

1.3.3 Compressor degradation

The main source of degradation of compressors is related to *fouling*. Fouling is caused by deposits forming on the surfaces inside the equipment. It affects the performance of a compressor and leads to increased costs of operation.

Figure 1.4 shows the efficiency of an offshore compressor with data provided by Equinor ASA (Zagorowska et al., 2020a). The data were collected over a period of approximately two years, with one sample per day. The black line represents the efficiency calculated from measurements. The black line is noisy and has short-term variations, but shows a clear long-term trend (dashed red line). Prediction of this trend is the objective of Chapter 5.

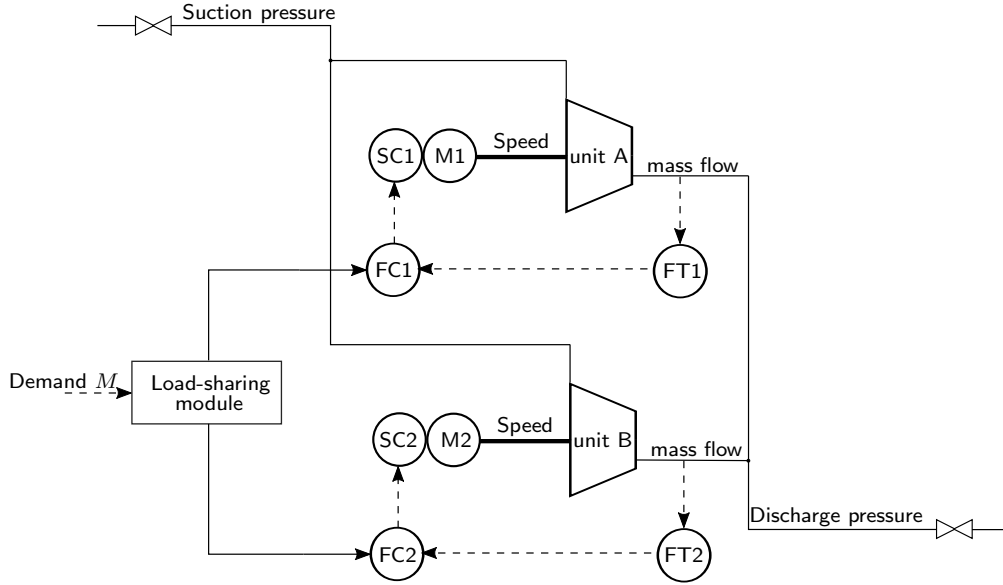


Figure 1.3: A compressor station (denoted with (1) in Fig. 1.2) with two compressors working in parallel

The overall degradation of efficiency consists of various types. Each can be (partly) reversed by specific maintenance actions (Aretakis et al., 2012; Meher-Homji et al., 2001). The red triangular area at the top of Fig. 1.4 denotes non-recoverable degradation, which can only be reversed by exchanging the internal parts of the compressor. The three maintenance types, *online washing*, *offline washing* and *compressor inspection*, reverse effects such as fouling. Online washing is the least expensive option and quick, but cannot remove all the deposits on the compressor blades (green). As the online washing cannot fully remove the deposits, a remainder builds up which requires offline washing. Offline washing is more thorough (yellow) and removes more deposits than online washing. During a compressor inspection, the compressor is shut down and the casing opened which enables better washing and a check of all mechanical components in the compressor system (orange). The trend data in Fig. 1.4 aligns well with the cleaning events at the end of the episodes C1, C2, and C3, and shows that the efficiency of the compressor was restored. After the compressor inspection was performed at the end of the episode CS, only the non-recoverable degradation remained (red).

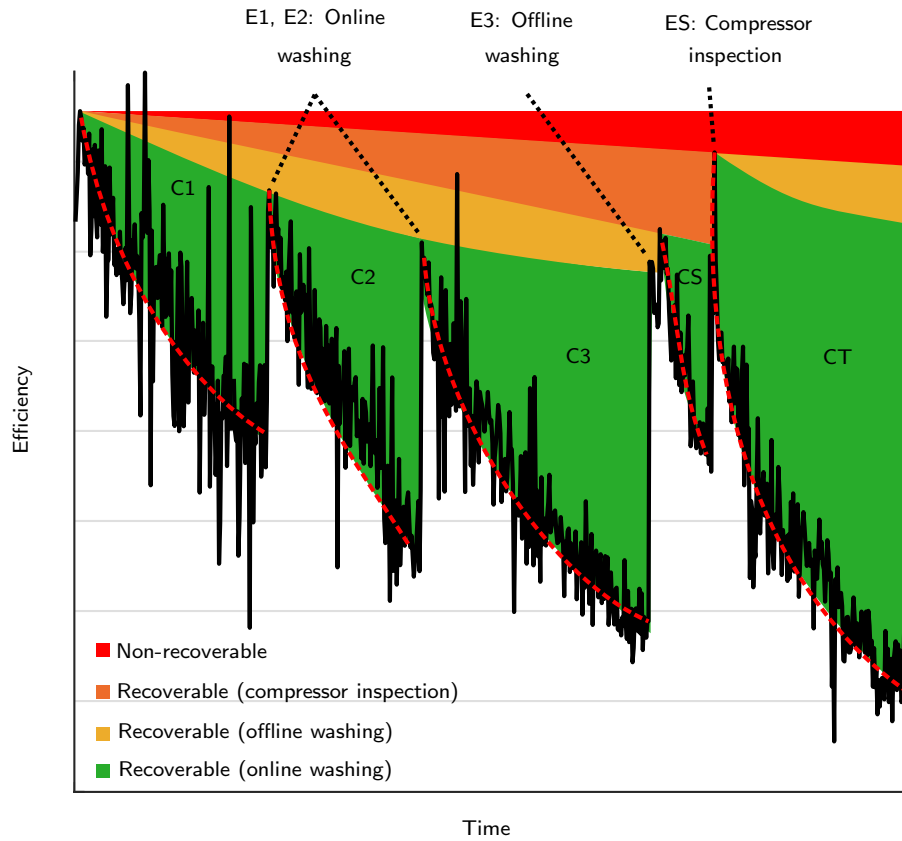


Figure 1.4: Efficiency degradation with maintenance types. The episodes C1, C2, C3, CS, and CT are used for developing and testing the algorithm for degradation prediction in Chapter 5. Each episode ends with a maintenance event (E1, E2, E3, ES) (Schulze Spüntrup, 2019)

1.3.4 Objective of the case study

The work reviewed in Section 1.3 shows the role of compressor stations in a gas transport network and introduces the challenges related to including degradation in operating a compressor station. The thesis will present a framework for operating a single compressor station with degrading compressors working in parallel. Degradation-aware load-sharing will be used to improve the performance of the compressor station and mitigate the influence of compressor degradation. The load-sharing algorithms based on the relationships between the compressors within a compressor station will be proposed in Chapters 6 and 7. The results from Chapters 6 and 7 will demonstrate that it is possible to manage degradation of industrial compressors in order to improve the operation of the whole station. The industrial data used for these chapters come from natural gas export compressors, and were retrieved from Xenos (2015) and Nørstebø (2008).

1.4 Contributions of the thesis

This section presents the contributions of the thesis that are aimed at tackling the challenges described in Section 1.2. Three auxiliary publications on compressor modelling are also listed.

1.4.1 Contribution to degradation analysis for process control applications

The first challenge in Section 1.2 indicated that there is need for understanding the effects of degradation on a process control system, and vice versa. The literature review in Chapter 3 will show that degradation of equipment has primarily been considered for maintenance purposes. Thus, the degradation is described from the perspective of diagnostics and prognostics, typically as a *mathematical model* that captures the relationship between the degradation and the operating environment. However, these models do not consider the changes in a system due to degradation, nor the operation-related factors that influence the degradation. Therefore degradation typically has not been considered in process control applications. To facilitate the integration of degradation models in process control applications, the thesis provides original insights and a new classification according to the relationships between degradation and behaviour of a system. The new classification has been published in a survey by Zagorowska et al. (2020b).

1.4.2 Contribution to modelling of turbomachinery degradation

Existing models of degradation of turbomachinery usually have a predefined functional form. Thus, they are unable to adapt to an unanticipated form of degradation. The approach proposed in the thesis tackles the second challenge from Section 1.2 and provides an adaptive algorithm for degradation detection and prediction. The adaptive algorithm can be used to predict the expected values of future degradation indicator and to quantify the uncertainty of the prediction. The influence of the parameters of the algorithm will be demonstrated and their impact on the accuracy of the prediction discussed.

The flexibility of the algorithm allows application for a range of turbomachinery units, as indicated in a conference publication (Zagorowska et al., 2019) and a journal publication with industrial partners (Zagorowska et al., 2020a).

1.4.3 Contribution to degradation management in a compressor station with degradation

The thesis addresses the third and fourth challenge from Section 1.2 first by exploiting the interactions between the compressors to allocate the load between them to manage the degradation. It expands the current state of the art in compressor load-sharing in the following ways:

- The degradation is explicitly taken into consideration. The new load-sharing algorithm is based on the degradation ratio between the compressors rather than on the degradation of each compressor separately;
- The algorithm can be used for degradation management, if the relationships between the degradation and the factors influencing degradation are known. It can be used for mitigation of the degradation of a compressor. Conversely, it can be used to accelerate degradation of a compressor.

The work on degradation management has been partly described in a conference paper in collaboration with industrial partners (Zagorowska et al., 2018b).

1.4.4 Contribution to degradation and power management in a compressor station with degradation

The integration of degradation and process control and optimisation expands the state-of-the art in compressor station operation. The thesis shows how degradation influences the power necessary to run a compressor station. An optimisation problem is solved that explicitly takes degradation into account and analyses the influence of degradation on the solution. The contributions address the third and fourth challenge from Section 1.2 and are summarised as:

- Giving conditions for when the problem of minimisation of compressor power is equivalent to maximisation of compressor efficiency,
- Performing sensitivity analysis of the optimisation problem to changes induced by degradation in the objective function and in the constraints,
- Formulating and solving a multi-period optimisation problem with degradation using the degradation modelling approach described in Section 1.4.2.

1.4.5 Other publications

The project included also works:

- On a new method of approximation of compressor maps based on Chebyshev polynomials. This work was aimed at capturing the operating range of a compressor (Zagorowska and Thornhill, 2017);
- On dynamic modelling of a compressor with Chebyshev polynomial approximation of a compressor map. This work aimed to provide a more accurate framework compared to those based on other polynomial approximations (Zagorowska et al., 2018a);
- On the differences between power and efficiency optimisation in a compressor station (Zagorowska and Thornhill, 2020b).

1.5 Structure of the thesis

The thesis is structured as follows. The main definitions regarding degradation and process control are introduced in Chapter 2. Chapter 2 describes also the background for the case study.

Chapter 3 presents the state-of-the art in process control applications taking degradation into account. Furthermore, it presents the existing approaches to operation of a compressor station, with and without degradation. Chapter 3 ends with identified gaps of knowledge and outlines how the thesis is going to address these gaps.

Subsequently, Chapter 4 presents a survey of models of degradation in process control applications. It introduces the new classification based on relationships between the degradation and the behaviour of the system, and classifies the existing control strategies that make use of the degradation models.

Chapter 5 presents an adaptive degradation modelling algorithm for turbomachinery degradation. It introduces the algorithm and shows its performance on industrial data sets obtained from Equinor ASA. This chapter also presents a comparison with existing models of degradation applied to turbomachinery.

Degradation management is analysed in Chapter 6. The approach is applied in two case studies: a compressor station with two compressors and a compressor station with five compressors.

Degradation and power management using optimisation is described in Chapter 7. The problem is formulated as a nonlinear programming problem for day-ahead planning. An industrial case study is analysed with degradation influencing the objective function.

Chapter 8 provides a critical evaluation of the methods and results. Chapter 8 also lays out a roadmap for future research in the area. Finally, Chapter 9 concludes the thesis, and is followed by references and appendices.

Part I

Background and context

Chapter 2

Background

The analysis of degradation is well established for maintenance planning (Chen and Patton, 1999) with the resulting models being used to aid decisions related to restoring the performance of a system. However, models of degradation for use in process control applications must also capture the relationship between the way in which a system is operated and how its component parts degrade. To tackle the challenges indicated in Chapter 1 and to devise process control strategies taking degradation into account, it is necessary to explain the underlying concepts for both topics, using standards as reference, among others. It is also necessary to put the definitions in the context of a compressor station for the case study.

The chapter has thus been divided in three parts. Section 2.1 presents the definitions for degradation with the terminology gathered in the Glossary of Terms in Appendix B. Process control is described in Section 2.2. Finally, the chapter introduces the characteristics of a compressor in Section 2.3.

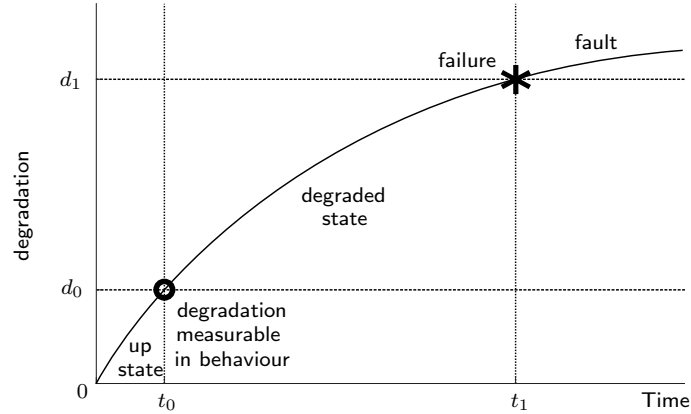


Figure 2.1: Evolution of degradation in relation to the behaviour of a system (based on BSI (2017) and Langeron et al. (2013))

2.1 Degradation

Figure 2.1 shows an illustration of how degradation may progress over time. Throughout the chapter, t refers to time and d refers to degradation. During the first stage, for time $t < t_0$, the value of degradation is too small to significantly influence the system. The system is therefore considered to be in *up state*. The black circle denotes the moment when the system enters a *degraded state*. When the degradation, d , is higher than a predefined threshold d_1 , a *failure* occurs, marked with an asterisk. The last stage, when time $t > t_1$ and degradation $d > d_1$, represents faulty operation where the system is no longer able to perform one or more required functions.

BSI (2017) introduces the term *failure mechanism*, describing “physical, chemical or other processes which may lead or have led to failure”. These failure mechanisms belong to a more general group of *influencing factors*, where an influencing factor is an “observable qualitative or measurable quantitative item that affects a system property”. BSI (2016a) categorises influencing factors according to their sources, such as the environment or operating personnel. The relationship between the influencing factors and the detrimental changes in the system properties is not explicitly described in the standard. For such relationships, the scientific literature suggests a variety of models depending on the type of influencing factors, the degradation processes, and also on the application.

2.1.1 Degradation path model

Meeker and Escobar (1998) described the concept of a *degradation path model*. They analysed degradation d as a function of time and discerned three forms of degradation path, characterized with initial value d_0 and constant parameter C (Meeker and Escobar, 1998; Hong et al., 2011):

- Linear: $d(t) = d_0 + Ct$,
- Convex: $d(t) = d_0 \exp(Ct)$,
- Concave: $d(t) = d_0(1 - \exp(-Ct))$.

In these models, the degradation starts at t_0 with a value $d(t_0) = d_0 \geq 0$. Taking $d_0 > 0$ allows situations where the system is initially in a degraded state to be modelled. Such situations may be caused by deficiencies in the manufacturing, storage or installation of an element in the system or may occur after maintenance activities where only partial performance has been restored.

Degradation path models have been widely used to represent various types of degradation. Reviews of these approaches were conducted by Haghighi et al. (2010) and Xu et al. (2016). Hong et al. (2011) summarised a number of methods of modelling degradation primarily for reliability assessment and maintenance planning purposes. Shahraki et al. (2017) indicated that degradation path models may be insufficient because of the rigidity of the functional form.

2.1.2 Degradation models for prognostics

The characterization from Meeker and Escobar (1998) assumes that degradation is a strictly increasing function of time. However, Gertsbakh and Kordonskiy (1969) suggested that it is not mandatory to model degradation as a monotone function of time. Shahraki et al. (2017) indicated that degradation path models might be oversimplified and therefore insufficient. Thus, more general approaches have been proposed. The standard BSI (2015b) divides models of degradation into five groups:

- Physics-based models,
- Statistical models,
- Heuristic models,

- Data-driven models,
- Hybrid models.

These are often aggregated into three groups:

- Physical models,
- Data-driven model,
- Knowledge-based models.

Physical, data-driven, and knowledge-based models of degradation are widely used for prognostic applications, including condition monitoring and maintenance planning as indicated by Jardine et al. (2006), Heng et al. (2009), Peng et al. (2010), Gorjian et al. (2010), Sikorska et al. (2011), Le (2015), Shahraki et al. (2017), and Lei et al. (2018). Similarly, van Noortwijk (2009), Si et al. (2011), Si et al. (2013), and Zhang et al. (2018) reviewed applications of selected types of models of degradation for prognostic purposes, whereas Nguyen et al. (2018) conducted a survey on selection of models of degradation using condition monitoring data. Zhang et al. (2015) presented a review of data-driven methods used for estimation of degradation for prognostics. A summary of existing reviews of models of degradation is in Table 2.1.

Modelling of degradation for prediction is well established (Chen and Patton, 1999) with the resulting models being used to aid decisions about restoring the performance of a system. However, degradation models for process control must also capture the relationship between the way in which a system is operated and how its component parts degrade. Previous work has included model-based fault-detection and diagnosis (Isermann, 2005), degradation modelling for reliability analysis (Gorjian et al., 2010) and prognostics, including factors that may influence degradation (Shahraki et al., 2017). Isermann (1984) provided a survey of methods for process fault detection based on comparisons between the up state and the degraded state of a system. The methods were classified according to the system variables which are affected by degradation. However, modelling of the degradation itself was not discussed. A new classification of degradation models and an analysis of the relationships between degradation and the control system will be presented in Chapter 4.

Table 2.1: Existing reviews of models of degradation

Authors	Classification	Applications
Isermann (1984)	Based on measurable signals Based on nonmeasurable state variables Based on nonmeasurable parameters Based on nonmeasurable characteristic quantities	Fault diagnosis
Jardine et al. (2006)	Time-domain Frequency domain Time-frequency domain Value type Data analysis combining event data and condition monitoring data	Diagnostics and prognostics
van Noortwijk (2009)	Based on Gamma process	Maintenance
Heng et al. (2009)	Physics-based Data-driven Integrated	Rotating machinery prognostics
Peng et al. (2010)	Physical Knowledge-based Data-driven Hybrid	Condition-based maintenance
Gorjian et al. (2010)	General Degradation Path Random process models Mixture model Time series Stress strength inference Cumulative damage Based on Markov models Based on Wiener process Based on Gamma process	Reliability analysis
Sikorska et al. (2011)	Knowledge-based Based on life expectancy Based on Artificial Neural Networks Physical	Diagnostics and prognostics
Si et al. (2011)	Statistical data driven	Prognostics
Qin (2012)	Physical Data-driven Based on deviations from the expected behaviour	Condition monitoring
Liao and Köttig (2014)	Experience-based Data-driven Physical Hybrid	Prognostics
Le (2015)	Experience-based Evolutionary and trending Physical Stochastic	Prognostics

Table 2.1 (Continued): Existing reviews of models of degradation

Authors	Classification	Applications
BSI (2015a)	Physical	Condition monitoring
	Data driven	
	Knowledge based	
Shahraki et al. (2017)	Data-driven	Reliability analysis, maintenance planning, prognostics
	Physics-based	
Alaswad and Xiang (2017)	Discrete	Maintenance
	Proportional hazard models	
	Continuous	
Zhang et al. (2018)	Based on Wiener process	Prognostics
Lei et al. (2018)	Depending on a single health indicator	Prognostics
	Depending on a health indicator and time	
	Depending on a health indicator and a stage sequence	
	Depending on multiple health indicators	
	Hybrid	

2.2 Process control

A *process control system* “responds to input signals from the process, its associated equipment, other programmable systems and/or operators and generates output signals causing the process and its associated equipment to operate in the desired manner” (BSI, 2016a). Stephanopoulos (1984) indicates that optimal operation of a system is a goal of a process control system in chemical plants. Typically, this goal is achieved by an automatic control system supported by human operators.

2.2.1 Control systems

An automatic control system is depicted in Fig. 2.2, and consists of a system controlled through adjustments of the manipulated variables by the actuators. The actuators adjust the manipulated variables according to the controller output, which is calculated using the information about the desired *set point* of the system and its current state measured by the sensors and fed back to the controller. The feedback controller drives the error between the set point and the feedback signal to zero in steady state (ANSI/ISA, 2010).

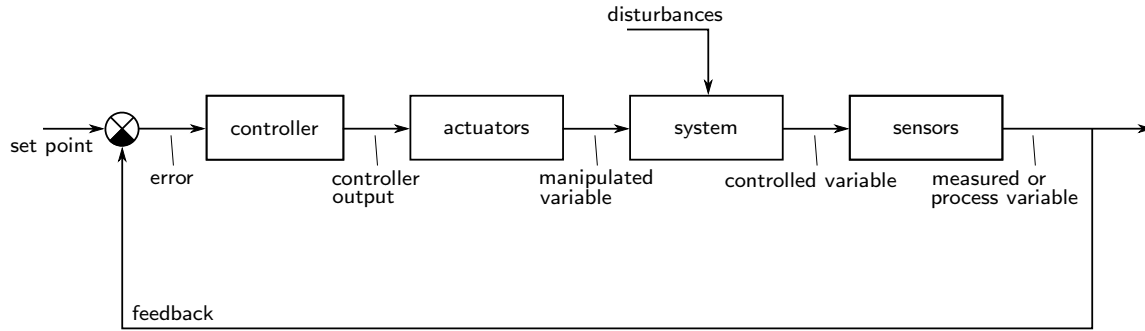


Figure 2.2: ANSI framework for feedback control including degradation processes (ANSI/ISA, 2010)

Figure 2.2 presents a *feedback control system* or *closed-loop system*. If there is no feedback, the system works in an *open loop*. Seborg et al. (2010) indicated that a closed-loop system reacts to deviations of the controlled variable, whereas an open-loop control system requires knowledge about expected deviations beforehand. Thus, an open-loop control system relies on accurate mathematical representation of the system affected by the disturbances. An accurate mathematical representation of a system might also help in design and simulation of a closed-loop control system, as it captures how a system reacts to its surroundings (Söderström and Stoica, 1989). These representations are called *mathematical models*. The performance of a system can be described by its *behaviour* introduced in Section 2.2.3.

2.2.2 Automation hierarchy

The tasks of automatic process control are typically grouped into layers distinguished with timescales. According to ANSI/ISA-95 standard (ANSI/ISA, 2010), industrial systems are structured in four layers:

- Regulatory control, maintaining selected variables at their set points,
- Supervisory control, providing set-points to the regulatory control taking limitations of the variables into account,
- Optimisation, recalculating the operating point of the system taking economic objectives into account,
- Scheduling, focusing on the operation on the highest level, e.g. on the entire process plant.

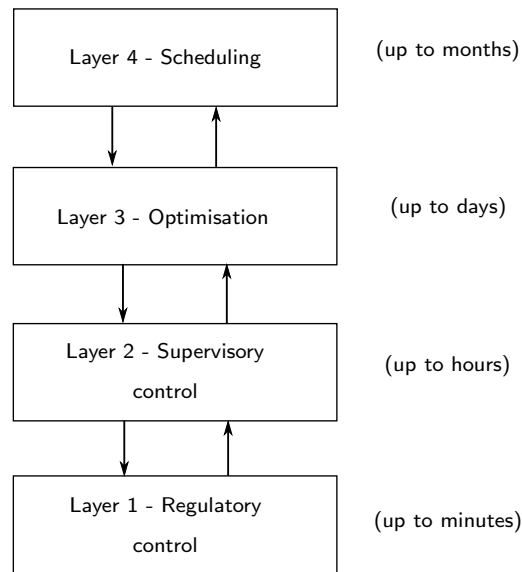


Figure 2.3: Control layers as described by BSI (2013); ANSI/ISA (2010), and adapted from Seborg et al. (2010), Skogestad and Postlethwaite (2007), Baldea and Harjunkski (2014). The lines between the levels represent flows of information

The four layers of industrial control systems are presented in Fig. 2.3. Regulatory control primarily protects the system against disturbances. Supervisory control also takes disturbances into account, but is able to handle additional constraints on process variables as well. The regulatory control and supervisory control layers typically have a closed-loop structure. The optimisation layer usually works in an open-loop structure (Skogestad and Postlethwaite, 2007) and takes into account varying operating conditions to improve the performance of the system. Finally, the scheduling layer considers both technical and economic aspects of a process, with the objective of maximising business profits (Seborg et al., 2010).

The lines between the levels represent flows of information, for instance controller set points are sent from the supervisory layer to the regulatory layer. In particular, Fig. 2.3 shows that changes in the regulatory control layer due to degradation will propagate to all layers.

2.2.3 Process behaviour

Sontag (1990) defines the effects of the inputs on the outputs of a system as its *input-output behaviour*. To analyse the behaviour in terms of the relationships between the inputs and the outputs, *mathematical models* are used, defined as “a set of equations that represents the system” (Ogata, 1997). Such equations include dynamic state-space equations and algebraic relationships. Other characteristic quantities such as duration of a batch reaction may also be included. Mathematical models that capture the behaviour of a system are called *behavioural models* (Willems, 2007).

The definitions related to behaviour of a system and mathematical models of the behaviour used in this paper are gathered below:

- *Input-output behaviour of a system* is a response of a system “to input signals from the process, its associated equipment, other programmable systems and/or an operator” (BSI, 2016a), i.e. “the effect that inputs have on the outputs” (Sontag, 1990);
- *Inputs* are “signals that can be manipulated” (Ljung, 1999);
- *Outputs* are “observable signals that are of interest” (Ljung, 1999);
- *Behavioural models* are mathematical models of the input-output behaviour of a system (Willems, 2007).

2.2.3.1 Dynamic behaviour

In up state, the dynamic behaviour of a system can be described as a set of nonlinear time-variant equations:

$$\dot{x}(t) = f(t, x(t), u(t)) \quad (2.1a)$$

$$y(t) = g(t, x(t), u(t)) \quad (2.1b)$$

with boundary conditions $x(t_0) = x_0$ where $t \in [t_0, \infty) \subset \mathbb{R}_+$ is time, $x \in \mathbb{R}^n$ is a vector of n state variables, $u \in \mathbb{R}^m$ is a vector of m input variables. The outputs of the system are denoted with $y \in \mathbb{R}^k$. The functions $f : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ and $g : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^k$ are nonlinear functions describing the dynamics of the system (Khalil, 2014).

Table 2.2: Variables for describing the behaviour of a system (following Isermann (1984))

Input and output variables u and y in Eq. (2.1)
State variables x in Eq. (2.1)
Parameters, such as matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ in Eq. (2.2)
Characteristic quantities that are functions of x and u

Equation (2.1) includes linear systems of form:

$$\dot{x}(t) = \mathbf{A}(t)x(t) + \mathbf{B}(t)u(t) \quad (2.2a)$$

$$y(t) = \mathbf{C}(t)x(t) + \mathbf{D}(t)u(t) \quad (2.2b)$$

where $\mathbf{A}(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times n}$, $\mathbf{B}(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times m}$, $\mathbf{C}(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^{p \times n}$, $\mathbf{D}(t) : \mathbb{R}_+ \rightarrow \mathbb{R}^{p \times m}$ are matrices with functions of time as coefficients (Khalil, 2014).

2.2.3.2 Characteristic quantities

Characteristic quantities are related to items of equipment within a system. For instance, a compressor map describes the relationships between pressure ratio, mass flow rate and speed of a compressor, while a valve characteristic relates valve position to the flow rate through the valve.

With appropriate assignment of u , x , and y , characteristic quantities are modelled by a time-invariant version of Eq. (2.1b):

$$y = g(x, u) \quad (2.3)$$

For instance, a compressor map can be modelled as $P_2 = P_1 \Psi(m, N)$, where P_2 and P_1 are the discharge and suction pressures, and Ψ is a non-linear function of mass flow rate m and speed N . Here, P_2 would be an output y , m would be a state variable x , and N an input u . The suction pressure P_1 might be a state variable or an input depending on how the system is operated.

Characteristic quantities do not have to be system variables and may instead be performance indicators. An example is compressor efficiency $\eta = \eta(m, N)$, where η is an output y , and is a nonlinear function of the state and input variables on the right hand side. Further examples discussed by Zagorowska et al. (2020b) included the length of a batch in a multi-product batch production.

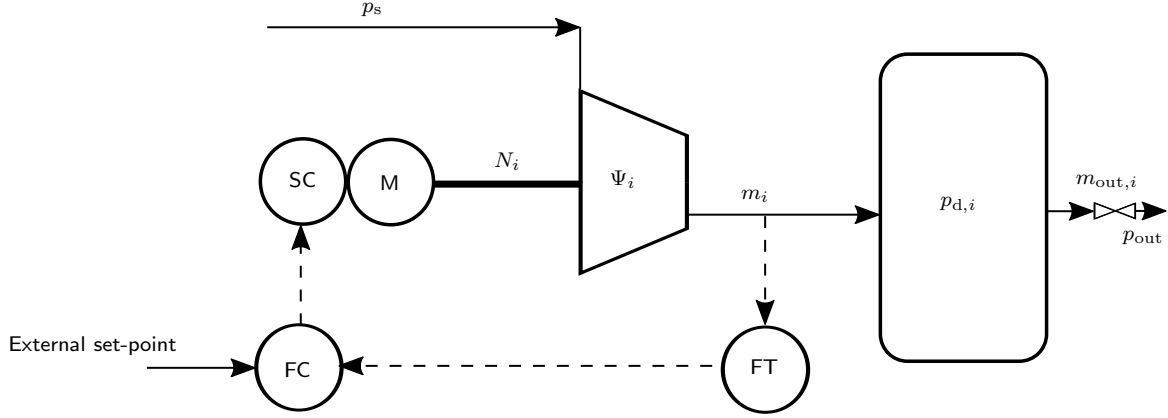


Figure 2.4: Single compressor train consisting of a compressor with a variable speed drive and a discharge tank

2.3 Compressor behaviour

A compressor unit is shown in Fig. 2.4 and typically consists of a compressor with a motor, a discharge tank and a piping system. In some cases, the compressor unit includes also a suction tank.

The system from Fig. 2.4 includes also a flow controller FC which receives its set-point from an external source, such as load-sharing if a compressor is a part of a compressor station, or a dispatch centre (Cortinovis et al., 2016b). The flow controller provides set-points to a speed controller SC.

The behaviour of a single compressor is captured by its dynamics and steady-state characteristics.

2.3.1 Dynamic behaviour of a compressor

The system in Fig. 2.4 can be described by a third order system of ordinary differential equations (Gravdahl, 1998) derived from mass balance in the tank (Eq. (2.4a)), momentum balance in the pipe (Eq. (2.4b)) and momentum balance on the compressor shaft (Eq. (2.4c)):

$$\dot{p}_{d,i} = \frac{a_{01}^2}{V_i} (m_i - m_{out,i}) \quad (2.4a)$$

$$\dot{m}_i = \frac{A_1}{L_c} (\Psi_i(m_i, N_i) p_s - p_{d,i}) \quad (2.4b)$$

$$\dot{\omega}_i = \frac{1}{J_i} (\tau_{t,i} - \tau_{c,i}(m_i, \omega_i)) \quad (2.4c)$$

where p_s is the suction pressure, $p_{d,i}$ is the pressure in the discharge tank, m_i denotes the mass flow rate through the compressor, ω_i is the compressor speed in rad s^{-1} and $\omega = \frac{2\pi N}{60}$ where N is the speed in rpm. The variable $\tau_{c,i}$ is the reaction torque of the compressor. The mass flow through the outlet $m_{\text{out},i}$ valve is a function of the valve opening $u_{\text{out},i}$ and external pressure p_{out} . The variable $\tau_{t,i}$ denotes the external torque delivered to the compressor by the motor. The constants V_i , A_1 , L_c , and a_{01} capture the geometry of the tank and the piping system, and J_i denotes the inertia of the shaft. The parameter Ψ_i shows the pressure ratio across a compressor as a function of compressor flow and speed. The model from Gravdahl (1998) can be extended to take a suction tank into account (Cortinovis et al., 2015a).

The model of dynamic behaviour of a compressor from Eq. (2.4) is used for control on the regulatory or supervisory layer (Gravdahl, 1998; Cortinovis et al., 2015a). The objective is typically to maintain a set point and ensure correct operation by keeping the compressor within its operating range described with a compressor map. In this thesis, the model will be used in Chapter 6 to show the influence of degradation on the system and develop a load-sharing algorithm based on interactions between the degradation levels of the compressors.

2.3.2 Compressor map

A compressor map expresses the relationship between pressure ratio, compressor speed, mass flow rate, and compressor efficiency. The map is normally provided by the manufacturer, or it may be derived from first principles (Gravdahl, 1998). Compressor maps are used in all layers of the automation hierarchy from Fig. 2.3.

2.3.2.1 Compressor head and efficiency

Compressor head captures the thermodynamics of a compression process. Lip-tak (2005) provides the formula for calculating compressor head at a given pressure ratio Ψ_i :

$$H = \frac{ZRT_1}{M_W\phi} \left(\Psi_i^\phi - 1 \right) \quad (2.5)$$

where $\phi = \frac{n-1}{n}$ with n as the polytropic coefficient and $\Psi_i = \frac{p_{d,i}}{p_s}$ is the pressure ratio across the compressor. M_W denotes the molecular weight of the gas, R is the gas constant, Z denotes the compressibility of the gas, and T_1 is the inlet temperature of the compressor.

The efficiency η can be obtained using experimental formulas for the isentropic exponent k and polytropic exponent n . Mokhatab et al. (2015) uses an experimental formula to calculate the unknown isentropic exponent k :

$$k = \left(1.6 - 0.44 + 0.097\rho^2\right) (1 + 0.0385\rho - 0.000286T) \quad (2.6)$$

where T is the temperature in K, and ρ is the gas relative density calculated as the ratio of gas molecular weight to air molecular weight.

The polytropic path coefficient n is estimated as:

$$n = \left(1 - \frac{\log \frac{T_2}{T_1}}{\log \frac{p_{d,i}}{p_s}}\right)^{-1} \quad (2.7)$$

where T_1 and T_2 denote the suction and discharge temperatures, respectively (Lüdtke, 2013).

Polytropic efficiency of a compressor is then calculated using Eq. (2.6) and (2.7):

$$\eta = \frac{\frac{k-1}{k}}{\frac{n-1}{n}} \quad (2.8)$$

Figure 2.5 shows an example of a compressor map, adapted from Nørstebø (2008). The vertical axis shows the head of a compressor, whereas the horizontal axis presents the mass flow rate through the compressor. The compressor head is shown as a function of the mass flow rate and the speed of a compressor. Thick solid black curves in Fig. 2.5 are called *speed lines* and show the compressor head as a function of the mass flow for a constant speed. The efficiency of a compressor is usually shown as a function of the compressor head and the mass flow rate and imposed on the compressor map as constant efficiency lines which form *efficiency islands* (ellipses in Fig. 2.5). The dash-dotted line in Fig. 2.5 shows where the efficiency is maximal for a given speed.

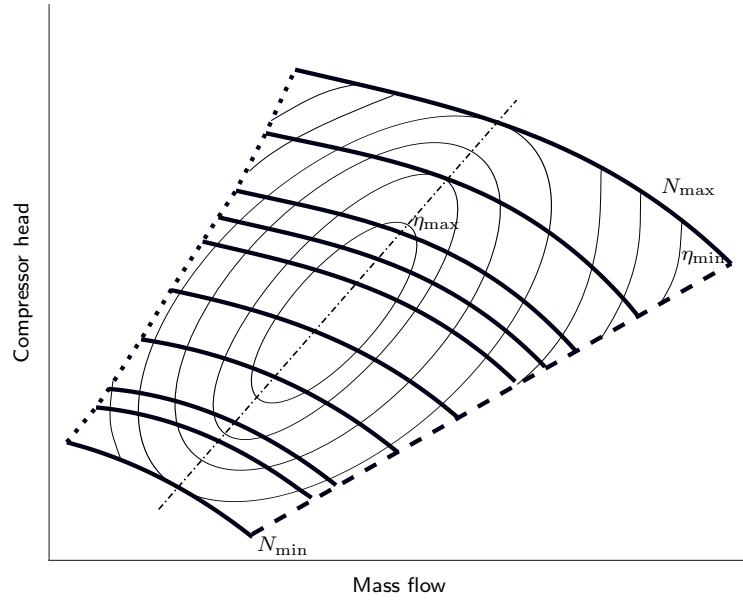


Figure 2.5: A compressor map with speed lines (—) and efficiency islands (---) with the maximal efficiency line (---). The minimal speed line $N_{\min}$, the maximal speed line $N_{\max}$, the surge line (···), and the choke line (--) show the operating range of a compressor

2.3.2.2 Compressor operating range

A compressor map captures the operating range for the compressor defined by a set of curves:

- Surge line, which corresponds to a dynamic instability consisting in reversing the flow inside the compressor (dotted line in Fig. 2.5);
- Choke line, which depends on the aerodynamics of a compressor (Budinis, 2015) (dashed black line in Fig. 2.5);
- Minimal speed (solid black curve corresponding to $N_{\min}$ in Fig. 2.5);
- Maximal speed (solid black curve corresponding to $N_{\max}$ in Fig. 2.5).

2.3.2.3 Compressor map approximations

Instead of using Eq. (2.5) or Eq. (2.6) for calculations of the head and efficiency of a compressor, polynomial approximations are considered for compressor control and optimisation, because they do not require a first principles model. The head of a compressor can be approximated with a polynomial function of the flow and the speed of a compressor (Wu et al., 2000b; Rasmussen and Jakobsen, 2000; Egeland

and Gravdahl, 2002):

$$H = a_1(N) + a_2(N)m + a_3(N)m^2 + a_4(N)m^3 \quad (2.9)$$

The efficiency can be then approximated as a function of the mass flow and the head as:

$$\eta = b_1(H) + b_2(H)m + b_3(H)m^2 + b_4(H)m^3 \quad (2.10)$$

The coefficients a_i and b_i are also polynomial functions with parameters estimated from operating data. Other approximations are also used, such as an approximation with Chebyshev polynomials that is able to capture the behaviour of a compressor outside the operating range (Zagorowska and Thornhill, 2017).

2.3.3 Compressor power

The behaviour of a compressor is also characterized by the power necessary to run the system at the desired operating conditions (Wu et al., 2000b).

2.3.3.1 Power calculation

The power necessary to run a compressor with efficiency η at the head H and the flow m is given as:

$$W = \frac{Hm}{\eta} \quad (2.11)$$

Figure 2.6 presents the power as a function of the flow for fixed values of head. The power curves are limited by the flow constraints defined in Section 2.3.2.2.

2.3.3.2 Power approximation

The power of a compressor can be approximated with a function of the mass flow and the head:

$$W = p_{00} + p_{10}m + p_{01}H + p_{20}m^2 + p_{11}Hm + p_{02}H^2 \quad (2.12)$$

where p_{ij} are constant parameters calibrated from plant data (Wu et al., 2000b). If $H = \text{const}$, Eq. (2.12) becomes a second order polynomial function of the mass flow m :

$$W = am^2 + bm + c \quad (2.13)$$

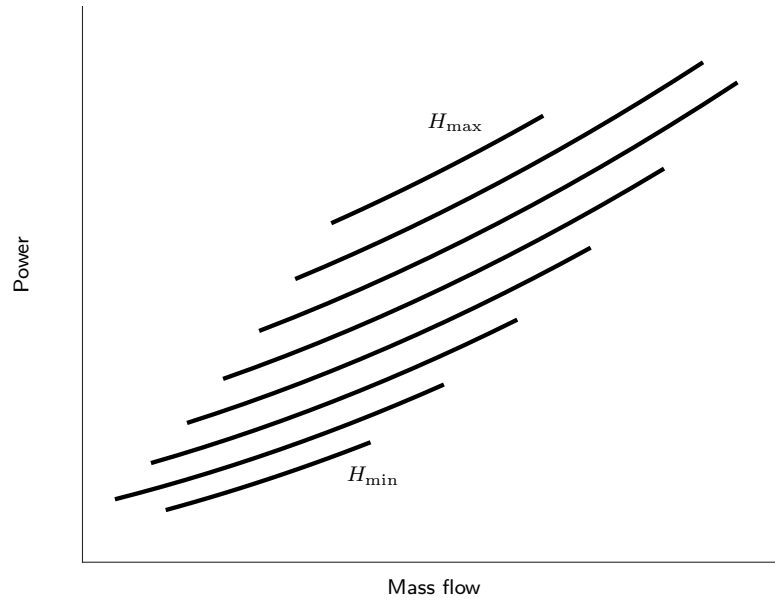


Figure 2.6: Compressor power from Eq. (2.11) as a function of the mass flow for various values of head

where $a = p_{20}$, $b = p_{10} + p_{11}H$, $c = p_{00} + p_{01}H + p_{02}H^2$. The approximation of power from Eq. (2.13) will be used in Chapter 7 to derive the optimal operating strategy for a compressor station with degrading compressor units.

2.4 Chapter summary

This chapter introduced the background for the thesis, divided into three main topics:

- Degradation in Section 2.1, which introduced definitions following the existing standards with the terminology gathered in Appendix B and indicated that degradation is primarily considered for prediction and maintenance planning;
- Process control and automation in Section 2.2, where the behaviour of control systems was introduced and characterized by dynamics of a system and characteristic quantities;
- Compressor behaviour in Section 3.2, which presented an overview of compressor behaviour for the case study introduced in Chapter 1.

The subsequent chapters build on the definitions introduced in this chapter. Models of degradation will be further investigated in Chapter 4, in particular in the context of process control applications. Control systems subject to degradation will be analysed in Chapter 3 and further investigated in Chapter 4 that gives a new classification of models of degradation for control applications. The classification offers a systematic and structured way to incorporate degradation modelling into control systems. The behaviour of the compressor will be used in Chapters 6 and 7 to devise load-sharing frameworks taking degradation into account.

Chapter 3

State-of-the art in process control with equipment degradation

The topic of degradation analysis spans across a wide range of industrial operations. Primarily arising in maintenance and condition monitoring applications, nowadays it becomes more prominent in process control as well. Nevertheless, the connection between degradation and control, and in particular the influence of operating conditions and control on degradation, has not been yet fully investigated. This chapter presents an overview of the topic of degradation in process control and indicates existing gaps of knowledge. In particular, the chapter reviews the literature relevant to the industrial case study introduced in Section 1.3.

The outline of Chapter 3 is as follows. The state of the art of process control approaches that take degradation into account is presented in Section 3.1. Section 3.2 then focuses on the operation of a compressor station, in particular with degrading compressors. Finally, the chapter closes with identified gaps of knowledge in Section 3.3.

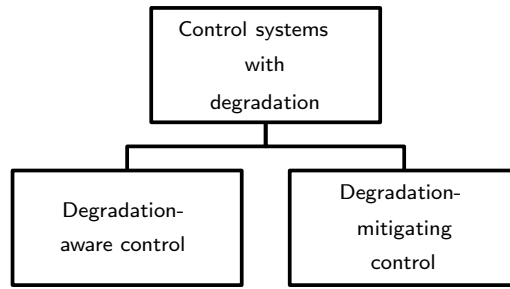


Figure 3.1: Classification of approaches to control of systems subject to degradation

3.1 State-of-the art regarding degradation in process control

A question arises how the degradation should be taken into account within a control system to preserve the overall performance. The relationship between the influencing factors such as control effort or operating temperature and degradation also raises a possibility of mitigating the detrimental changes by adjusting the operating conditions. Thus, there are two main groups of control approaches for degrading systems (Fig. 3.1):

- Control systems aware of degradation,
- Control system mitigating degradation.

Control systems aware of degradation compensate for degradation of a system. Control systems mitigating degradation take degradation into account and also attempt to mitigate degradation itself. This section discusses these two approaches emphasizing the relationships between the degradation and the behaviour of a system. A detailed review of degradation models used for control purposes is done in Chapter 4, where a new classification of models is developed.

3.1.1 Control systems aware of degradation

Control systems aware of degradation should compensate for the degraded behaviour of the controlled system. They therefore require knowledge about the degradation process and its influence on the system. For instance, Milosavljevic et al. (2016) and Cortinovis et al. (2016b) designed model-based optimising control for a compressor station, taking into account that the behaviour of a compressor might be different than expected. Both examples belong to the supervisory control layer in Fig. 2.3. Milosavljevic et al. (2016) simulated a load-sharing problem assuming that the characteristics of each compressor differed from the model used as a proxy for the real system. Cortinovis et al. (2016b), on the other hand, assumed that the characteristics of a compressor change with time. For optimisation, they approximated the characteristics with a polynomial and identified the parameters of the polynomial online. Furthermore, Ahmad et al. (2014) used a model of degradation in a control system of the temperature in steel-making process. They were able to improve the performance of a feed forward controller in the regulatory layer from Fig. 2.3 due to improved accuracy of the model used in the controller.

According to BSI (2017), a *fault* means that a system is unable to perform the required function. Ensuring the correct operation of a system in the presence of faults is a task of fault-tolerant control (Isermann, 2006). However, the literature on fault-tolerant control usually assumes a relaxed definition of a fault. Isermann and Ballé (1997) defined a fault as “an unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard condition”. Moreover, a fault “may develop abruptly (stepwise) or incipiently (drift-wise)” (Isermann, 2006). This relaxed definition of a fault is similar to the definition of a degraded state from BSI (2017) provided in Table B.1. Therefore, the term *fault-tolerant control* includes the control of systems in a degraded state as well as when the system is unable to function.

The field of fault-tolerant control has been extensively researched by Blanke et al. (1997), Isermann (2006), Zhang and Jiang (2008), Muenchhof et al. (2009), Hwang et al. (2010), and Jiang and Yu (2012). The text book by Mhaskar et al. (2012) presents methods and applications of fault-tolerant control.

These examples show that control systems aware of degradation have been studied in depth and there exist algorithms that ensure correct operation of a system with degraded behaviour. However, it would also be useful to study the effects of the operation of the system on degradation, as it would enable a better understanding of the system and help manage the degradation.

3.1.2 Control systems mitigating the degradation

Mitigation of degradation is usually considered a part of maintenance engineering. On the other hand, some control frameworks attempt to mitigate degradation. They make use of a model of degradation in the controller, or use the model to predict degradation for decision support in the optimisation layer of the automation hierarchy. Usually, a form of optimal control is used for degradation mitigation, typically a linear-quadratic controller or a model-predictive controller. These approaches allow straightforward inclusion of models of degradation, either as constraints or directly in the objective function.

Such control approaches were explicitly proposed by Ray et al. (1994b) and Ray et al. (1994c) as *life-extending control* or *damage mitigating control*. Using a detailed model of degradation, they designed a control algorithm to keep degradation below a threshold. Later on, a patent by Fuller (2005) gave a generic approach to life-extending control using model-predictive control with constraints on degradation, with estimates of degradation obtained from a model of degradation.

Degradation mitigation has also been applied in aircraft applications by Dai and Ray (1996), Ray and Caplin (2000), Ray et al. (2000), and Ray et al. (1994a). They all used models of crack propagation taking into account factors that influence degradation to design a model-based controller which would ensure correct operation of the degraded system. Tangirala et al. (1998) demonstrated a control system aimed at mitigation of crack development in aircraft, and demonstrated its effectiveness on a laboratory test structure. The influence of degradation on the behaviour of the system was not considered.

Other applications include Pereira et al. (2010), Vieira et al. (2015), Grosso et al. (2016) who considered factors causing degradation in a model-predictive controller, but did not model the effects of degradation on the controlled system. Salazar et al. (2017a), Sanchez et al. (2017), and Pour et al. (2019) also used model predictive control, and included models of degradation in the objective function.

Escobet et al. (2002) investigated a concept similar to life-extending control, but with the main focus shifted towards predictive health monitoring and decision making, and called it *health-aware control*. They used models of degradation to predict the health of the system. The predicted value was then used to choose between either reconfiguration of a controller, or performing maintenance. This example shows how degradation modelling contributes to decision support for the optimization and scheduling layers of the automation hierarchy.

Langeron et al. (2012) developed an extended fault-tolerant control framework for a DC motor including a model of degradation in the design of a PID controller. The developed controller was tested in simulation. Langeron et al. (2013) used a probabilistic description of health of a system and included degradation in the objective function of a linear-quadratic controller. This approach was applied by Langeron et al. (2016) to a generic control system, and by Langeron (2015) and Langeron et al. (2017) who designed a linear-quadratic control system for a drilling system. The application included a model of actuator degradation in a quadratic objective function.

Further applications presented by Pour et al. (2018) and Verheyleweghen et al. (2018) described hierarchical control systems that mitigated degradation. Both applications were based on model-predictive control. Pour et al. (2018) included a model of degradation directly in the objective function, whereas Verheyleweghen et al. (2018) added a constraint on the value obtained from the model of degradation.

The review in this section shows that it is possible to mitigate the loss of performance of a control system by including degradation as a constraint or in the objective function of an optimisation-based control system. It would also be useful to study how the degradation changes the behaviour of a system to further mitigate the degradation and improve the operation of the system.

3.1.3 Summarising comment

The work reported in this section shows the need for degradation modelling in control. The findings show that an approach based on Isermann and Ballé (1997) could be a starting point for addressing the first challenge stated in Chapter 1, namely to do an analysis of the interactions between the degradation and the system. The reported work has also shown gaps in the area of the interactions between the actuators subject to degradation, where there has not been much previous work. In particular, the question arises on how to use less degraded actuators to manage degradation of more degraded actuators. It will be important to make progress in this areas to achieve degradation management. The field of process control would also benefit from an in-depth analysis of degradation models that have already been used in degradation-aware and degradation-mitigating control systems. It would facilitate the degradation modelling in a wide range of applications, removing the need for application-specific models.

3.2 State-of-the art in compressor station operation

A compressor station can use up to 50% of the budget of the operator of gas transport network (Rios-Mercado and Borraz-Sanchez, 2015). As compressor degradation increases the power necessary to run the compressor, the costs of running the compressors in degraded state would also increase. To develop the case study introduced in Chapter 1, the existing approaches to operation of a compressor station will now be described.

The approaches to developing an operating strategy focus not only the compressor stations, but also include the pipelines and surrounding equipment in the gas transport network. However, due to the complexity of the problem, in some cases only the operation of the compressor station is considered. Thus, two groups of approaches to compressor operation can be distinguished:

- Operation of a compressor station as a part of a network,
- Operation of a compressor station as a separate process.

The main difference between considering the whole gas network and a single compressor station lies in the level of detail that is taken into account in modelling the compressor station. Considering a compressor station as a separate process reduces the size of the problem and takes into account more details relevant to the operation of the compressors. The additional details might for instance relate to the degradation of the compressor units. Thus, another possible classification arises, which will be employed in this thesis:

- Operation of compressor station in up-state,
- Operation of compressor station in degraded state.

Figure 3.2 presents a hierarchical tree with the classification of the approaches to operation of compressor stations. The first level categorizes the approaches according to the state of the compressors. Subsequent layers present internal classification adopted within each category. Thus, in up state, the approaches are grouped according to the extent of the system which treats the compressor stations either as a part of the network or a separate process. In degraded state, the approaches are classified according to the relationship between the operation and the degradation of the compressors into degradation-aware operation and degradation-mitigating operation.

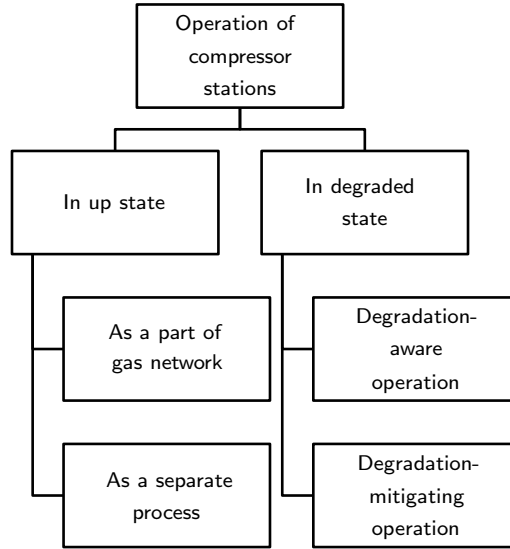


Figure 3.2: Classification of approaches to operation of compressor stations

3.2.1 Operation of compressor station in up state

If it can be assumed that the compressors operating in parallel are identical, optimal load-sharing in up-state can be done by assigning the same load to each unit (Liptak, 2005). However, it is almost impossible to have two units with the same characteristics due to age, wear, or design differences. Thus, several approaches have been proposed in the literature to allocate the load among the compressors. Liptak (2005) indicated that the most common approaches include assigning the loads in the order of decreasing efficiency of the units, or making all the compressor work at equal distance to respective surge lines. These approaches are used in the industry. Table 3.1 lists some patents related to industrial practices.

Usually, the strategies proposed by patents listed in Table 3.1 have two levels: surge control, and load-sharing control. Most of the inventions focus on ensuring the predefined distance to surge line for each compressor, either by dedicated individual controllers or by following the unit chosen as leader. The patents by Cortinovis et al. (2015b) and Itou and Takeda (2004) focus on improving the operation of the compressors using optimisation. A review of the approaches to the optimal operation of a gas network including compressor station was presented by Rios-Mercado and Borraz-Sanchez (2015).

Table 3.1: Patents for load-sharing controllers for compressor system

Author	Company		Year	Title
A. J. Fisher, T. S. Smoot, D. E. Kincer (Fisher et al., 1994)	Ingersoll-Rand company	Com-	1994	Method and apparatus for controlling a system of compressors to achieve load sharing
N. Staroselsky, S. Mirsky, P. A. Reinke, P. M. Negley, R. J. Sibthorp, (Staroselsky et al., 1994)	Compressor Corporation	Controls	1994	Load sharing method and apparatus for controlling a main gas parameter of a compressor station with multiple dynamic compressors
T. Itou, K. Takeda (Itou and Takeda, 2004)	Ingersoll-Rand company	Com-	2004	Control method of plural compressors and compressor system
D. Lau (Lau, 2004)	Dieter Lau		2004	Method for optimizing the operation of a plurality of compressor assemblies of a natural-gas compression station
A. Saxena, M. LeDizes (Saxena and LeDizes, 2003)	Dresser-Rand company		2003	Management and optimization of load sharing between multiple compressor trains for controlling a main process gas variable
S. Staroselsky, B. W. Batson, S. Mirsky, V. Shapiro (Staroselsky et al., 1998)	Compressor Corporation	Controls	1998	Method and apparatus for load balancing among multiple compressors
N. Staroselsky, S. Mirsky (Staroselsky and Mirsky, 1987)	Compressor Corporation	Controls	1987	Method for controlling a multicompressor station
E.J. Wichert, (Wichert, 2002)	Scales Air Compressor		2002	Method and control system for controlling multiple compressors
A. Cortinovis, M. Mercangoz, L. Dominguez, A. Zanarini, D. Pareschi (Cortinovis et al., 2015c)	ABB		2014	A method for calculating a load distribution among a plurality of rotary machines of a machine station
W. Grassens, B. Eldredge (Grassens and Eldredge, 2015)	Woodward		2015	Load sharing control for compressors in series

3.2.2 Compressor station as a part of the network

As the running of a compressor station is a major cost of the operation of the whole network, minimisation of the cost either by minimisation of the power necessary to run the compressor or by maximisation of the profit is typically considered. Three groups can be distinguished when considering the compressor station as a part of the network:

- Optimisation using steady-state and dynamic models,
- Multiobjective optimisation,
- Optimisation under uncertainty.

Optimisation using steady-state and dynamic models

Steady state analysis of a gas network with compressor stations models the parts of the network as time-invariant nonlinear functions (Al-Sobhi and Elkamel, 2015). The decision variables in steady state analysis are the flows in each segment of the network and the pressures in the pipelines, which define the operating pressures for the compressor stations (Rios-Mercado and Borraz-Sanchez, 2015; Koch et al., 2015). The compressors are modelled using steady state characteristic quantities, such as compressor maps or power characteristics. Examples of such maps can be found in Nørstebø et al. (2007b, 2006, 2008) who modelled the gas transport system in Norway for optimisation of maintenance.

Wu (1998) and Wu et al. (2000b) presented an analysis of compressor characteristics for steady-state optimisation. They considered a compressor station with compressors working in parallel and solved the problem of fuel minimisation as a mixed-integer linear program. A formulation of the optimisation problem as linear programming was also done by De Wolf and Smeers (2000), who approximated nonlinear characteristics of the network with piecewise linear functions. The model from Wu (1998) and Wu et al. (2000b) was also used by Kazda and Li (2018) who showed how to obtain the optimal linear approximation of compressor characteristics. Rios-Mercado et al. (2006) presented a heuristic approach to optimisation of a network in steady state.

An extension of the model from Wu et al. (2000b) was presented by Christo et al. (2008), who assumed dissimilar compressors and solved a mixed-integer quadratic programming problem. The fuel consumption problem was also solved by Tabkhi (2007) who added nonlinear constraints on the flows in the network, and solved the problem by mixed-integer nonlinear programming (MINLP). An MINLP formulation was also used by Geißler et al. (2018) and Chen et al. (2019) for optimisation of a network with dissimilar compressors, while Osiadacz (2011) considered a simplified problem, without integer variables, and formulated the problem as nonlinear programming.

Vostry et al. (1994) indicated that the results of a steady state optimisation might serve as an input to dynamic problems, for instance providing the final states for dynamic programming. Typically, in network optimisation problem, the dynamics of the flow in the pipelines is considered, but not the dynamics of the compressors. Marques and Morari (1986, 1988) formulated the analysis as a predictive control problem, minimising the overall cost of running the station over the whole operating horizon with varying demand. A dynamic problem with varying demand was also considered by Zlotnik et al. (2015), Mak et al. (2015), and Baumrucker and Biegler (2010) who optimised the dynamic behaviour of a network for day-ahead planning. Marques and Morari (1986), Zlotnik et al. (2015), and Baumrucker and Biegler (2010) included the dynamics of the network in the form of partial differential equations for the pipelines to enable more accurate modelling of the gas flow.

Conversely, Aalto (2005) approximated the dynamics of the gas transport network with ordinary differential equations. In particular, Aalto (2005) linearised the dynamics and obtained transfer functions for describing the overall dynamics of the network including compressor stations. A transfer function description allowed the author to reduce the complexity of the model and formulate the problem as quadratic programming with linear constraints.

Multiobjective optimisation

As indicated by Kashani and Molaei (2014), there are several objectives for optimal operation of a gas transport network. Some of the objectives are equivalent, for instance maximisation of a profit can be obtained by minimisation of the cost related to running the network. However, some of the objectives are contradictory and thus justify using multi-objective approaches. In particular, a trade-off between the total throughput in the network and the cost of the operation is sought, but some authors include environmental objective functions or storage costs.

For instance, Rodriguez et al. (2010) analysed the trade-off as a way of finding minimum and maximum emission of CO₂ if the throughput is maximised. The multi-objective approach to network optimisation was also considered by Kashani and Molaei (2014) who used genetic algorithms to analyse a tradeoff between the throughput, the cost of operation, and the amount of gas stored in the pipelines. Demissie et al. (2017) and Haddad and Behbahani (2013) also applied genetic algorithms to find a trade-off front between total throughput in the network and the cost of the operation. Arya and Honwad (2017) used ant colony optimisation for the same objectives. Both Demissie et al. (2017) and Kashani and Molaei (2014) presented a sensitivity analysis with respect to the number of compressors in the compressor station. In all the cases, a steady-state approach was used.

Optimisation under uncertainty

Zavala (2014) indicated that the demand for the gas required from the network might be uncertain and thus the problem of optimisation of the network becomes stochastic in nature. They proposed to solve the problem as a scenario-based optimisation with three profiles of demand and dynamic model of the pipelines. Wu et al. (2018) presented a review of non-deterministic algorithms applied to optimisation of a gas network, both in single and multi-objective cases. Furthermore, Aßmann et al. (2019) used a steady-state model to solve a robust optimisation problem with uncertainty in the demand and in the parameters of the pipelines. Cay et al. (2019) at the same time analysed load-sharing as a multi-period scheduling problem. The compressor station was required to meet the demand with predefined probabilities when the ramping constraints are used, i.e. the rate of change of the flows is limited between periods.

3.2.3 Compressor station as a separate process

There are two main approaches that tackle the operation of a single compressor station:

- Without considering compressor dynamics,
- Taking compressor dynamics into account.

Operation without considering compressor dynamics

Formulation of optimisation of a compressor station without considering compressor dynamics is obtained directly from optimisation of the whole gas transport network (Carter, 1996). The primary objective is to allocate the loads to compressors in a station to minimise the cost of operation. Thus, similarly to the results described in Section 3.2.2, the total energy or fuel consumption is minimised. Abbaspour et al. (2005) presented a comparison of optimisation results obtained for the whole network with the results of optimisation of a single compressor station. Optimisation of a single compressor station allowed a simplification of the problem by fixing the suction and the discharge pressures of the station and thus reducing the dimensionality of the optimisation problem.

Wu (1998) and Wu et al. (2000b) indicated that the cost of running a compressor is a nonlinear function of the flow. Thus, the cost minimisation can be formulated as a nonlinear programming problem. Nonlinear optimisation was previously analysed by Osiadacz (1980) and Osiadacz and Bell (1981), who solved a mixed-integer nonlinear programming problem to find the number of compressors in operation and then to assign an appropriate load to each of them. The non-linearity was due to a polynomial approximation of the fuel consumption as a function of power. Furthermore, Osiadacz and Bell (1981) used the nonlinear optimisation as a first step in a two-step control system to adjust the loads according to individual characteristics of the compressors. The individual characteristics of the compressors were also considered by Jenicek et al. (1995) who used nonlinear optimisation to allocate the loads assuming a fixed number of units in operation. They showed that depending on the chosen objective function, ‘cheaper’ compressors were run at capacity. The term ‘cheaper’ meant ‘more efficient’ if power was minimised, and ‘less expensive’ if the overall cost was taken into account.

Nonlinear programming was also applied to optimisation of a compressor station by Abbaspour et al. (2005), who focused in addition on the influence of gas temperature on the steady-state model of compressors. They calculated the power necessary to run the compressor from Eq. (2.11) using a second order polynomial approximation of compressor efficiency. A similar approach was employed by Zhang et al. (2014) who included an external demand requirement for the compressor station in the optimisation. Furthermore, Mahmoudimehr and Sanaye (2013) and Zhang and Wu (2015) solved the nonlinear optimisation problem for a compressor station using genetic algorithms. Recently, Rose et al. (2016) presented an overview of methods used for solving the problem of optimisation of the cost or power consumption in a compressor station with the focus on their computational efficiency.

Peyrl and Cortinovis (2015) also solved a nonlinear optimisation problem to assign the loads in a compressor station. They did not consider the dynamics of a compressor in the optimisation, but they included the recycling flow of each unit in the objective function. The objective of the recycling flow is to prevent the compressor from crossing the surge line (dotted line in Fig. 2.5). The aim of the work by Peyrl and Cortinovis (2015) was thus to minimise the recycle flow while maximising the efficiency of the compressors. By approximating the efficiency as a second order polynomial of compressor head and flow, they reformulated the nonlinear problem as quadratic programming.

Due to the complexity of the nonlinear programming, optimisation of a compressor station is sometimes formulated as linear programming. A linear formulation of optimisation of a compressor station was previously presented by Carter (1996), who compared it with nonlinear and heuristic approaches. Whereas the linear programming performed better than heuristics, the nonlinear approach was more accurate (Carter, 1996). However, a linear approach is the quickest (Deng, 2015) which makes it useful in multi-period optimisation of a compressor station. An example of multi-period optimisation of a compressor station was presented by Zuo et al. (2016), who used a linear steady-state model to minimise the power consumption of a compressor station.

Operation taking compressor dynamics into account

The behaviour of a single compressor is typically described with a compressor map which provides the operating range for each unit (Section 2.3.2). Another approach takes the dynamics of a compressor into account and ensures that the compressor stays within the operating range from Fig. 2.5 during transient periods. For this purpose, Smeulers et al. (1999) developed a control system for a compressor station based on model predictive control that manipulates the flow through the compressor and keeps the compressors at the required distance to surge. Jung et al. (2017) proposed an open loop control system that took into account both the load-sharing and the distance to surge.

Overall, model-predictive control outlined by Smeulers et al. (1999) is the most common approach taking compressor dynamics into account. A predictive controller for load-sharing was used by Øvervåg (2013), who maximised the efficiency of the compressors in parallel taking compressor dynamics into account. Mercangöz et al. (2016) used predictive control to control a compressor station with two compressors in series. Jones et al. (2017) used the weights of a predictive controller to analyse the relationships between the two units in parallel and in series. In all cases, the efficiency of a compressor was approximated as a second order polynomial and the dynamics were described with the model from Egeland and Gravdahl (2002). Efficiency maximisation with compressor dynamics was also considered by Cortinovis et al. (2016b) and Kumar and Cortinovis (2017), whereas Milosavljevic et al. (2016) minimised the power calculated using approximated efficiency.

3.2.4 Operation of a compressor station in degraded state

3.2.4.1 Degradation-aware operation

In most of the cases in Section 3.2.2, the behaviour of the compressors was fixed. The load-sharing problem with varying compressor characteristics was analysed by Paparella et al. (2013), Milosavljevic et al. (2016), and Kumar and Cortinovis (2017). They all approximated the characteristics of a compressor using polynomial functions and updated the parameters on-line to match the approximation to the real system. The updated characteristics of compressor efficiency and head were then used for optimal load-sharing, but the authors did not analyse the reason for changes in the compressor map. In practice, changes are often due to varying operating conditions, such as gas parameters, or fouling, which influence the behaviour of the compressor. Some evidence and examples will be presented in Chapters 6 and 7.

The papers by Milosavljevic et al. (2016) and Kumar and Cortinovis (2017) included the updated maps in the model of dynamic behaviour of a compressor from Eq. (2.4), but without considering the effect of the changes on the behaviour, and in consequence on the optimal load-sharing strategy. An analysis of the influence of degradation on the power necessary to run a compressor station was performed by Foss (2013) who simulated the operation of a compressor station with degradation. He indicated that equal load distribution is optimal if the compressors degraded at the same rate, but the operation of a compressor station can be improved by optimal load-sharing if the compressors degrade at different rates.

Tran et al. (2018) considered the influence of failure of a compressor on the performance of the whole gas transport network. Thus, they focused on the last stage in Fig. 2.1, when the compressors were no longer able to perform their required function. They modelled the failures as sources of uncertainty in the flows delivered by the compressors and maximised the throughput across the whole network using a steady-state optimisation.

However, none of the authors, who focused on degradation-aware operation of a compressor, took into account how the degradation affected the behaviour of the system. Analysing the mutual influences between the system and its degradation would enable new approaches that not only ensure the correct operation of a system with degradation, but that also adjust the operation of a system to mitigate degradation.

3.2.4.2 Degradation-mitigating operation

Hovland and Antoine (2004), Nørstebø (2008), Nguyen et al. (2008), Xenos (2015), and recently Schulze Spüntrup (2019) worked on improving the operation of compressor stations with degradation by optimal scheduling of maintenance activities. Hovland and Antoine (2004) used a dynamic model of a compressor, whereas Nguyen et al. (2008), Nørstebø (2008), Xenos (2015) and Schulze Spüntrup et al. (2019) did steady-state optimisation. Furthermore, Nørstebø (2008) formulated the optimisation problem as nonlinear programming. Nguyen et al. (2008), Xenos (2015), and Schulze Spüntrup et al. (2019) used mixed integer linear programming. In particular, Xenos (2015) used piecewise linear approximation of compressor maps to capture the inherent nonlinear nature of their operation. A review of optimisation of compressor stations for maintenance purposes was done by Xenos (2015) and by Schulze Spüntrup (2019).

The works by Hovland and Antoine (2004), Nørstebø (2008), Nguyen et al. (2008), Xenos (2015), and Schulze Spüntrup et al. (2018) included degradation of compressors as constraints in the optimisation problem. Thus, they developed degradation mitigating control systems by limiting the maximal value that degradation could reach before maintenance was due.

Verheyleweghen et al. (2018) and Verheyleweghen and Jäschke (2017) proposed a load-sharing approach based on mitigation of degradation by adjusting the operating conditions. They assumed that degradation depended on the flow through the compressor and its speed. However, they also included the degradation as constraints in the optimisation problem, without considering its influence on the compressors. They applied the proposed optimisation framework to find the maximal profit strategy for a subsea compressor station.

These examples show that the mitigation of degradation can be achieved by maintenance activities. However, it would also be useful to include the effect of degradation in the description of the behaviour of the system, because it would improve the accuracy of the description and enable better performance.

3.2.5 Summarising comment

Section 3.2 presented the existing approaches to devising an operating strategy for a compressor station. There are multiple approaches for optimising the operation of the compressor station in up state. Including degradation in the optimal load-sharing problem makes the optimisation more complex, because it affects the model used for calculations, for instance by adding new constraints or changing the behaviour. The existing work acknowledges that the degradation has a detrimental effect on the performance of the station and plans the maintenance activities to mitigate it. However, there is a gap in the area of analysing interactions between the degradation and the system. To fill this gap and address the first challenge from Chapter 1, both dynamic and steady-state models of a compressor station will be used to show the effects of degradation on the load-sharing problem. Having a model of how degradation affects the compressors would enable an analysis of the interactions within the compressor station with degrading units. This area has not yet been explored either in degradation-aware, or degradation-mitigating frameworks. Devising a load-sharing strategy using the interactions between the compressors addresses the third challenge from Chapter 1.

3.3 Gaps of knowledge and contributions of the thesis

3.3.1 Gap of knowledge related to degradation in process control

The literature review in Section 3.1 and 3.2 revealed that there is a gap in the area of considering the impact of degradation in process control applications. The existing approaches are typically application-specific. The existing classifications of degradation models described in Section 2.1 are mostly focused on modelling for prognostics and do not capture either the influence of degradation models on the behaviour of a control system, or the effect of adjusting the operating conditions on degradation.

To address this gap, this thesis will present a detailed review of existing degradation models in control applications, with particular focus on process control. A new classification of degradation models will be proposed that emphasizes the influence of degradation on control systems. The classification proved to be exhaustive and can be used for choosing degradation models depending on the objective of the control system, be it fault-tolerant control, degradation-aware control, or degradation mitigating control.

3.3.2 Gap of knowledge related to obtaining degradation models

In several industrial applications, the existing degradation models are insufficient. This is due to multiple reasons, including fixed functional forms of degradation models, or unavailability of degradation data to fit models such as the degradation path model. Although control systems exist to take degradation into account, their performance will not be adequate without an accurate degradation model.

A new algorithm for degradation modelling in turbomachinery will be proposed, to avoid the use of fixed functional forms. The new approach draws from existing algorithms to provide an adaptive algorithm for degradation detection and prediction. The algorithm is then tested on real industrial data for offshore turbomachinery.

3.3.3 Gap of knowledge related to degradation management in a compressor station

As indicated in Section 3.1, existing approaches to degradation-aware and degradation mitigating operation do not consider that degradation affects the behaviour of a system. In particular, it is not taken into account that pieces of equipment within one system will rarely have the same level of degradation. Therefore, exploring the relationships between the level of degradation of equipment within a system might provide a way of managing degradation and improving the performance of a system.

Thus, a load sharing strategy has been proposed in the thesis that takes into account how the degradation of compressors in a compressor station affects the behaviour of each unit. In particular, the new strategy assigns the loads to the compressors by comparing the values of degradation of the compressors.

3.3.4 Gap of knowledge related to optimal operation of a compressor station with degradation

The existing approaches to optimal operation of compressor stations subject to degradation focus on optimisation for maintenance purposes without considering the influence of degradation on the system. In particular, the effects of degradation on the optimal load sharing and the power consumption of a compressor station are not considered.

This thesis analyses the influence of degradation on the behaviour of the compressor, formulates and solves the load-sharing problem for degraded compressors as an optimisation problem to minimise the power consumption of a compressor station. An analysis will be performed to provide insight into how various types of degradation and the external demand affect the optimal load assignment in a compressor station with degrading compressors.

3.3.5 Industrial case study as a way of fulfilling the gaps of knowledge

The industrial problem with the gaps of knowledge described in Section 3.3 can be graphically summarised by Fig. 3.3 with bold connections emphasizing multiple compressors working in parallel. The compressor station must deliver the required demand. The problem solved in the thesis focuses on how to distribute the load among the compressors taking their degradation into account (Gaps 3.3.2 and 3.3.4). For this purpose, a degradation indicator is required that will be fed back to the load-sharing algorithm (Gap 3.3.2). The load-sharing algorithm will allocate the flows among the compressors taking into account both the value of the degradation indicator and the chosen performance metric (Gap 3.3.4). The flows delivered by the compressors contribute to the operation of the whole compressor station and thus influence its performance.

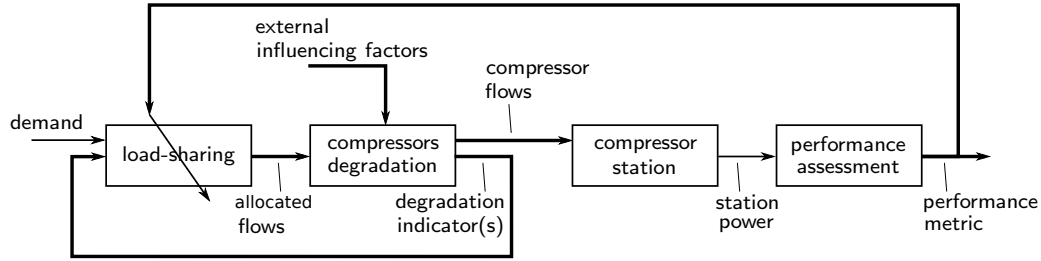


Figure 3.3: A graphic summary of the industrial load-sharing problem with degrading compressors

3.4 Chapter summary

The chapter introduced the state-of the art in degradation analysis in process control applications and identified the gaps of knowledge. Even though degradation is typically analysed for maintenance purposes, there are two control frameworks that include degradation: degradation-aware control and degradation-mitigating control. However, neither of them explicitly considers the mutual effects between the degradation and the behaviour of a system. This thesis aims to address this gap in Chapter 4.

Section 3.2 presented the literature relevant to the industrial case study introduced in Chapter 1. A review of the existing approaches to devising operating strategies for a compressor station revealed that most authors focused on operation in up state. Degradation-aware and degradation-mitigating operation of a compressor station is analysed from the perspective of maintenance planning, but there is a gap concerning the effect of degradation on the behaviour of the compressors in a compressor station. This gap will be addressed in Chapters 6 and 7 by proposing load-sharing strategies based on the levels of degradation of the compressors in a compressor station. To this end, a method for quantification of compressor degradation will be developed in Chapter 5.

Part II

Technical contributions of the thesis

Chapter 4

Degradation in process control applications

The first challenge indicated in Chapter 1 was related to analysis of mutual interactions between degradation and process control systems. This chapter aims to bridge the gap between degradation modelling and control systems, in particular in process industries. As indicated in Chapter 2, modelling of degradation for maintenance planning is well established (Chen and Patton, 1999) with the resulting models being used to aid decisions about restoring the performance of a system. However, degradation models for process control must also capture the relationship between the way in which a system is operated and how its component parts degrade.

Previous work presented in Chapter 2 has included model-based fault-detection and diagnosis, degradation modelling for reliability analysis and prognostics (Isermann, 2005; Gorjian et al., 2010; Shahraki et al., 2017). Isermann (1984) provided a survey of methods for process fault detection based on comparisons between the up state and the degraded state of a system. The methods were classified according to the system variables which are affected by degradation. However, modelling of the degradation itself was not discussed.

The chapter explains how degradation has been included in mathematical models of the behaviour of a system for control purposes. It builds on the approach from Isermann (1984) to capture the system variables describing the behaviour of control systems with degrading equipment. An extended version of this chapter has been published by Zagorowska et al. (2020b).

The chapter is divided into five parts. A new classification of degradation models based on behaviour of a system and the influencing factors is presented in Section 4.1. Section 4.2 reviews the existing classifications of degradation models. Section 4.3 presents the models of degradation that have been used in control applications and structures them according to the new classification. Section 4.4 shows the significance of degradation modelling in the context of control systems introduced in Chapter 3. The chapter ends with a summary in Section 4.5.

4.1 Classification of degradation models

The degradation models described in Section 2.1 predict the degradation of individual items of equipment. However, modelling of degradation within a process control system requires a description of how degradation of the equipment affects the ability of the system to perform its function. Also, the model has to capture how degradation is affected by the way the system is operated. A model for degradation therefore has to be embedded within a model for the behaviour of the system. The model may also need to take account of the factors influencing degradation.

4.1.1 Classification based on the behaviour of a system

The aim of this chapter is to model both the influence of degradation on the system to perform its function, and the influencing factors that affect the degradation. To achieve this, a new classification of behavioural models is proposed using the following criteria:

- Degradation-dependent models of behaviour if degradation affects the behaviour of the system;
- Degradation-independent models of behaviour if degradation does not affect the behaviour of a system.

The mathematical formulation of such models is presented in the upper section of Table 4.1. Degradation-independent systems (right column in Table 4.1) do not change and their behaviour is described with Eq. (2.1) introduced in Chapter 2,

Table 4.1: Mathematical models for systems with degradation

Behavioural models	
Degradation-dependent	Degradation-independent
$\dot{x}_D(t) = f(t, x_D(t), u_D(t), h(d))$	$\dot{x}(t) = f(t, x(t), u(t))$
$y_D(t) = g(t, x_D(t), u_D(t), h(d))$	$y(t) = g(t, x(t), u(t))$
Degradation models	
Factor-free	Factor-based
$h(d) = h(d(t))$	$h(d) = h(d(t, v, x(t), u(t), y(t)))$

recalled here for convenience:

$$\dot{x}(t) = f(t, x(t), u(t)) \quad (4.1a)$$

$$y(t) = g(t, x(t), u(t)) \quad (4.1b)$$

with unchanged boundary conditions. Degradation dependent models of behaviour (left column) are explicitly affected by degradation and either or both of Eq. (4.1a) or Eq. (4.1b) can change:

$$\dot{x}_D(t) = f(t, x_D(t), u_D(t), h(d)) \quad (4.2a)$$

$$y_D(t) = g(t, x_D(t), u_D(t), h(d)) \quad (4.2b)$$

with boundary conditions $x_D(t_0) = x_0$ where $t \in [t_0, \infty) \subset \mathbb{R}_+$ is time. The variable x_D denotes the states in degraded state, y_D corresponds to the outputs, and u_D represents the inputs to the system. The functions $f : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ and $g : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^k$ are nonlinear functions describing the dynamics of the system.

The new classification proposed in this thesis describes a model of behaviour as *degradation-dependent* when it is possible to monitor and quantify the degradation (usually by means of measurements of the input and output of the system). An example of degradation requiring a degradation-dependent model would be fouling of a centrifugal compressor, which is detectable from deviations in discharge pressure. A degradation-dependent model augments Eqs. (4.1a) and (4.1b) by including the degradation explicitly in a model for degraded system variables x_D in Eq. (4.2a) and y_D in Eq. (4.2b).

Conversely, a model of behaviour is considered to be *degradation-independent*, when the degradation does not influence the relationships between the model inputs and outputs. For example, the development of a leak in a heat exchanger may be modelled using a degradation independent behavioural model. This is because it is not possible to detect or quantify the severity of developing cracks leading up to the leak other than by specialist equipment, typically not considered part of a process control system. The formulation of a degradation-independent model is identical to Eq. (2.1). The behaviour of the system is indistinguishable from the system in up state, and the effect of degradation is not detectable in the system variables.

Degradation affecting the system, and which affects the inputs or the outputs of the system, requires a degradation-dependent model of behaviour of the form shown in the upper left side of Table 4.1. A degradation-independent model of behaviour is appropriate when the degradation does not affect the system.

4.1.2 Classification based on influencing factors

Factors that influence degradation may be considered as inputs of the degradation model:

- Factor-based models for degradation that depends on influencing factors

$$h(d) = h(d(t, v, x(t), u(t), y(t))) \quad (4.3)$$

- Factor-free models for degradation that is independent of any influencing factors

$$h(d) = h(d(t)) \quad (4.4)$$

Examples of influencing factors that may be included in factor-based models of degradation include the way in which the system is operated, and external factors v such as ambient temperature or humidity in Eq. (4.3). Factor-free degradation models typically depend only on time. These formulations are presented in the lower half of Table 4.1.

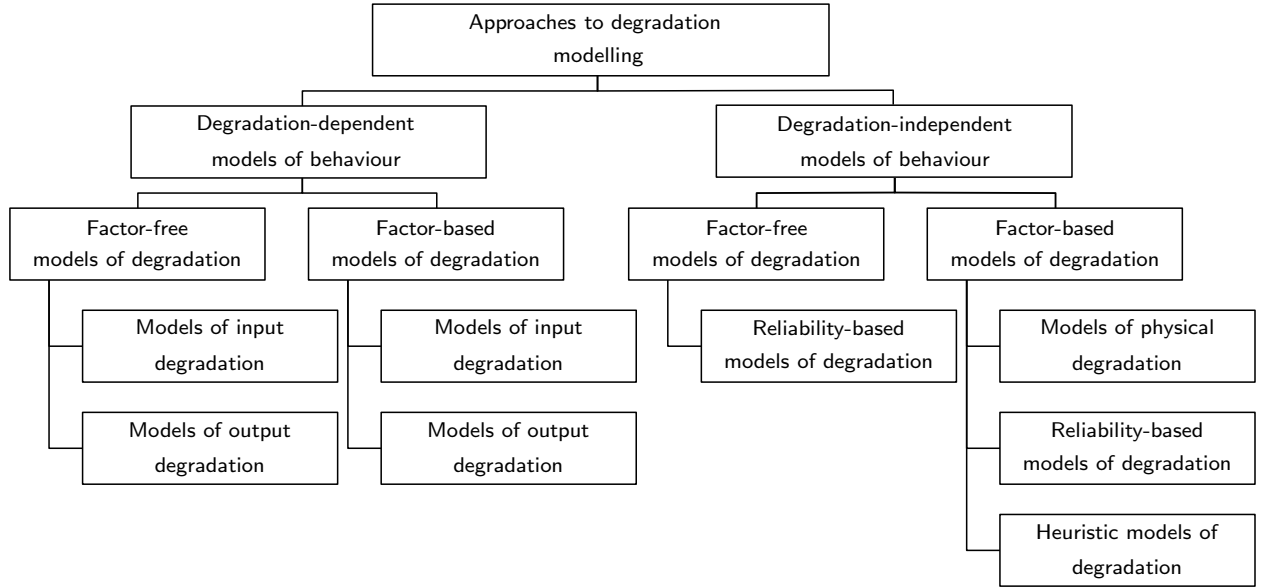


Figure 4.1: A hierarchical tree with classification of models of degradation relative to the control systems

4.1.3 Combinations of models based on detectability and influencing factors

Both the degradation-dependent and degradation-independent models of behaviour must be linked with a description of the degradation. This description can be factor-free or factor based, as shown in the lower row of Table 4.1. Either of the models of behaviour can be combined with either of the models for the influencing factors, thus there are four possible combinations of model structure. The four combinations form the top layer of the hierarchical tree in Fig. 4.1. In particular, a degradation-dependent model with factor-based degradation models has the potential to model the effect of degradation on the system, and also the effect of the system on the degradation.

4.2 Models of degradation

The classification of models of degradation based on the influencing factors and introduced in Eq. (4.3) and Eq. (4.4) allows an analysis of the influence of the surroundings on degradation.

Moreover, two subclassifications are described that divide the models of degradation according to:

- How the variables in a system can be influenced:
 - Additive models of degradation,
 - Multiplicative models of degradation;
- Which parts of a control system can be influenced:
 - Models of input degradation,
 - Models of output degradation.

4.2.1 Factor-free and factor-based models of degradation

A model of degradation gives an expression or a value of a degradation function $h(d)$ for use in Eqs. (4.1a) and (4.1b) as shown in Eq. (4.3) and Eq. (4.4). In the table, d is the degradation, and $h(d)$ is called a *degradation function* that reflects the effect of degradation on a system variable.

4.2.1.1 Factor-free models of degradation

In a factor-free model of degradation, the degradation d is assumed independent of any influencing factors. The degradation does not take into account either factors from outside the system, or the system variables x , u and y . Degradation can however evolve with time, i.e. $d = d(t)$, as time itself is not considered an influencing factor. McPherson (2013) presented an overview of general time-dependent models of degradation.

In a system described by a factor-free model of degradation with a degradation-dependent model of behaviour, the factor-free degradation can affect the parameters and variables, as shown in Table 4.1. However, the structure of the model for behaviour remains the same. For instance, the linear system described by Eq. (2.2) will remain linear with respect to x , u , and y . If d varies with time, the parameters of the system will also vary with time.

4.2.1.2 Factor-based models of degradation

Factor-based models of degradation provide a description of degradation in the form of Eq. (4.3), where $v(t)$ represents variables that influence the degradation but which are not included as system variables from Table 4.1, for instance environmental humidity or temperature.

Factor-based degradation might affect not only the values of the parameters and system variables, but also the structure of a degradation-dependent model of behaviour. For instance, it might introduce nonlinearity to a linear model or change the structure of Eq. (2.1).

4.2.2 Additive and multiplicative models of degradation

The effects of degradation on the system variables, listed in Table 4.1 can be considered as additive or multiplicative (Isermann, 2006).

4.2.2.1 Additive models of degradation

In an additive model of degradation, the degraded value of a generic system variable V from Table 4.1 is offset relative to the equivalent value in up state. If the degradation of V is additive, it is modelled as:

$$V_D = V + h(d) \quad (4.5)$$

where V_D denotes the degraded value, V is the value in up state without degradation, d is from the model of degradation and $h(d)$ is degradation function that reflects the effect of degradation on V . Therefore, the degradation function $h(d)$ would represent a deviation between the degraded value of a variable, V_D , and its counterpart in up state, $h(d) = V_D - V$.

A data-driven estimate of degradation would attempt to fit a model for degradation to measurements of V_D . Conversely, if V_D is not measurable, it would be possible to use a model of $h(d)$ to estimate V_D .

4.2.2.2 Multiplicative models of degradation

In a multiplicative model of degradation, the degraded value of a system variable is scaled according to a degradation function $h(d)$ as $V_D = h(d)V$. In particular, $h(d) = (1 - d)$ yields:

$$V_D = (1 - d)V \quad (4.6)$$

which can be rearranged to a form called the relative model of degradation:

$$d = \frac{V - V_D}{V} \quad (4.7)$$

Equation (4.7) is widely used for degradation monitoring (Loboda et al., 2007; Hameed et al., 2009). As with the additive case, Eq. (4.6) allows V_D to be estimated on the basis of a model for d , or conversely Eq. (4.7) allows estimation of d from measurements V_D if they are available.

Additive and multiplicative models can be also used together:

$$V_D = h_1(d)V + h_2(d) \quad (4.8)$$

with $h_1(d)$ representing a multiplicative degradation function, and $h_2(d)$ representing additive degradation function.

4.2.2.3 Uses of additive and multiplicative models

Additive and multiplicative models of degradation are used in fault-tolerant control systems that can accommodate failures in a component. In particular, they are used to describe the behaviour of a degraded actuator (Isermann, 2006; Noura et al., 2009). In that case, variable V is typically a characteristic quantity of a system, for instance, the flow through a valve calculated from valve position. Polycarpou and Helmicki (1995) indicated also that $h(d)$ might have various functional forms, such as polynomial or rational.

4.2.3 Input and output models of degradation

Figure 4.2 presents a block diagram representing a control system with degradation. The dashed blue box shows the parts of the system that would be described by a model of output degradation. The dotted green box represents a situation with input degradation.

Degradation that influences the outputs of the degradation-dependent model of behaviour will be considered separately from degradation influencing the inputs.

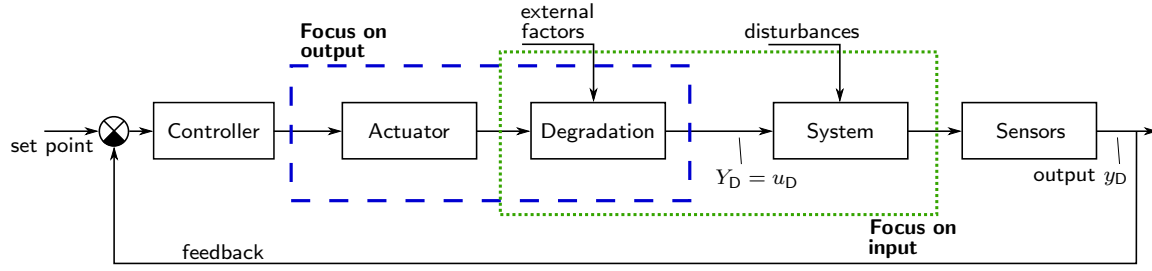


Figure 4.2: Degradation-dependent control system showing input and output degradation

4.2.3.1 Output degradation

Degradation may be modelled as a change in the output such that the output in degraded state y_D is a function of degradation:

$$\dot{x}(t) = f(t, x(t), u(t)) \quad (4.9a)$$

$$y_D(t) = g(t, x(t), u(t), h(d)) \quad (4.9b)$$

In the static case when $\dot{x} = 0$, the output Y is an algebraic function of the inputs U , where U and Y are static values of the input and output variables.

Models of output degradation are useful for subsystems that describe actuators. For instance, they can model compressor fouling that affects compressor efficiency or heat exchanger fouling that affects the heat transfer coefficient.

4.2.3.2 Input degradation

Degradation may alternatively be modelled as a change in the input. For multiplicative degradation the degraded input is $u_D = h(d)u(t)$, where $u(t)$ is the input in up state without significant degradation. Hence:

$$\dot{x}_D(t) = f(t, x_D(t), h(d)u(t)) \quad (4.10a)$$

$$y_D(t) = g(t, x_D(t), h(d)u(t)) \quad (4.10b)$$

where x_D and y_D are the degraded state and input variables.

Table 4.2: Interpretation of additive and multiplicative faults on behaviour of an actuator (Noura et al., 2009)

	No additive faults	Additive faults
No multiplicative faults $h(d) = 1$	Healthy actuator	Bias
Multiplicative faults $h(d) \in (0, 1)$	Partial loss of effectiveness	
Multiplicative faults $h(d) = 0$	Failure	Blockage

As an example, an input degradation approach is useful when considering the effect of a degraded actuator in a control loop. Noura et al. (2009) provided an interpretation of additive and multiplicative faults of an actuator, assuming

$$u_D = h(d)u \quad (4.11)$$

Their interpretation is summarised in Table 4.2.

4.2.3.3 Uses of models of input and output degradation

The input u_D to the controlled system is the output Y_D of an actuator such as a valve, compressor or heat exchanger, $u_D = Y_D$. The output of an actuator, Y , can be usually approximated by a linear combination of functions of the input U , $\nu_i(U)$:

$$Y = \sum_{i=0}^n k_i \nu_i(U) \quad (4.12)$$

where k_i are constant parameters. A model for the degraded output Y_D is thus given as:

$$Y_D = h_1(d) \sum_{i=0}^n k_i \nu_i(U) + h_2(d) \quad (4.13)$$

where h_1 is a multiplicative degradation function, and h_2 is an additive degradation function.

The behaviour of the actuator is usually described with the algebraic Eq. (4.9b) because according to Aldhaferi and Khalil (1996) the dynamics of the states x of the actuator are usually much faster than the dynamics of the states of the controlled system.

4.3 Models with degradation in control systems

This section of the chapter discusses the approaches introduced in Section 4.1 and highlights prior work in each area that is related to their application in control systems.

4.3.1 Degradation-dependent models of behaviour with factor-free models of degradation

As discussed by Chen and Patton (1999), factor-free models of degradation do not attempt to describe the mechanism of degradation. Rather, they assume the degradation is fixed, time-dependent or stochastic. Factor-free models of degradation have been combined with degradation-dependent models of behaviour for understanding the behaviour of a control system in the presence of degradation of components within it. The approaches can be classified as models of output degradation and models of input degradation, as defined in Section 4.2.3.

4.3.1.1 Factor-free output degradation

Degradation-dependent models of behaviour with factor-free models of output degradation describe how the outputs or characteristic quantities of a system or sub-system change with degradation (i.e. y in Eq. (4.1b) or Eq. (4.13)). These models are primarily used in complex processes where the output of a degradation-dependent system constitutes an input to another system. Table 4.3 presents selected applications of such models.

4.3.1.1.1 Fixed values of output degradation

Models with fixed values of degradation treat the degradation function $h(d)$ as constant with n possible values, i.e. $h(d) \in \{h_1, h_2, \dots, h_n\}$. Using models of output degradation with n fixed values effectively yields n outputs y_i , $i = 1, \dots, n$, corresponding to separate values of h_i . Each output can be considered separately, as the transitions from h_i to h_j , $i, j = 1 \dots, n$, $i \neq j$ are not defined. Examples can be found in works by Tsoutsanis et al. (2015), Meher-Homji et al. (2001), Aker and Saravanamuttoo (1989), and Kurz and Brun (2001) in which the performance of gas turbines was calculated for several values of fouling-induced degradation. They detected fouling from the behaviour by analysing changes of compressor characteristics when the degradation was constant. Furthermore, Kurz and Brun (2001) used the results to demonstrate the impact of an optimal cleaning schedule on the overall performance.

Behaviour and degradation cannot be analysed independently in a system described by a degradation-dependent model of behaviour. The controlled system has a different behaviour in the degraded state than in up state. As such, the control algorithm or the optimisation function must be adapted. Such an integrated analysis was done by Milosavljevic et al. (2016), who incorporated a factor-free model of degradation of a static compressor map with a degradation-dependent model of behaviour of a compressor to simulate optimal load-sharing among three compressors.

Table 4.3: Applications of factor-free models of output degradation

Model of degradation	Application
Fixed value model	Gas turbine (Tsoutsanis et al., 2015; Meher-Homji et al., 2001; Kurz and Brun, 2001; Goebel et al., 2008), compressor fouling (Aker and Saravanamuttoo, 1989), (Milosavljevic et al., 2016), pneumatic actuator (Graves et al., 2018; Beganovic and Söffker, 2017)
Time-dependent - deterministic	Compressor fouling (Tarabrin et al., 1996; Ciccioiti, 2015), CSTR (Lao et al., 2013), micro gas turbine (Zaccaria et al., 2019)
Time-dependent - stochastic	Gas turbine (Li and Nilkitsaranont, 2009), control valve (McGhee et al., 2014)

4.3.1.1.2 Time dependent and stochastic models of output degradation

Time-dependent models of degradation $h(d)$ take into account transitional behaviour as degradation moves between several values. Time-dependent models of $h(d)$ were used by Tarabrin et al. (1996) and Ciccioiti (2015) for analysis of compressor efficiency and pressure ratio in degraded state. The degradation function $h(d)$ was modelled as an exponential function of time with constant parameters. In particular, the time-dependent approach to modelling of fouling has been applied by Escher (1995), who modelled $h(d)$ as a nonlinear time trend to capture turbomachinery degradation. Li and Nilkitsaranont (2009), Loboda et al. (2007), and Loboda et al. (2009) modelled degradation in turbomachinery using time trends. Lao et al. (2013) used a logistic function to approximate a degraded behaviour of a heater in continuous stirred tank reactors (CSTR).

The degradation function $h(d)$ may also be modelled as stochastic, with degradation moving between random values at random times. McGhee et al. (2014) modelled degradation of a valve using a stochastic model to capture sticking valve behaviour:

$$y_D(t+1) = y(0) - \delta y(t) \quad (4.14)$$

where y_D denotes a decreasing area of the pipe inside the valve in relation to the initial area $y(0)$ and the random variable δ comes from a uniform distribution.

Time-dependent and stochastic output degradation implies that the output changes during the operation of the system. Therefore the control algorithm has to handle the transitions. Recent reviews of stochastic models that can be used for degradation modelling were presented by Si et al. (2011) and Zhang et al. (2015).

Table 4.4: Applications of factor-free models of input degradation

Model of degradation	Application examples
Fixed value model - two cases: up state or fully degraded	A fourth order dynamic system (Veillette, 1995), aircraft (Maki et al., 2004; Zhao and Jiang, 1998)
Fixed value model - several cases	A second order system (González-Contreras et al., 2007), three tank system (Theilliol et al., 2002, 2008), space vehicle (Gao et al., 2011), aircraft (Chamseddine et al., 2015; Chen et al., 2016; Li et al., 2017; Zhang and Jiang, 2001; Jiang and Zhang, 2006; Zhang et al., 2008; Yang et al., 2014; Ye and Yang, 2006; Yu et al., 2018; Shi et al., 2017), a third order system (Chen et al., 2013; Wang and Yao, 2010; Liu et al., 2015), heat exchanger (Ballé et al., 1998), TE benchmark (Yin et al., 2014), pulp mill (Zumoffen and Basualdo, 2008), CSTR (Prakash et al., 2005; Mhaskar et al., 2006), (Mhaskar et al., 2012), hydrothermal process (Boussaid et al., 2011), a wheeled robot (Ji et al., 2003)
Time-dependent model - fixed time and value	A third order system (Tao et al., 2001), aircraft (Wu et al., 2000a), tank level control (Zhang and Qin, 2009), hydrogen production (Du et al., 2014), Shell control problem (Kettunen et al., 2008)
Time-dependent model - fixed time	A third order system (Tao et al., 2001), ball-beam system (Zhang et al., 2010), valve leakages (Arici and Kara, 2018), electrical systems (Kaviarasan et al., 2016), aircraft (Jiang and Chowdhury, 2005; Liu and Crespo, 2012), CSTR (Wang et al., 2007; Yu et al., 2005), burner (Baldi et al., 2017), planetary lander (Boskovic et al., 2009), robotic arm (Jiang et al., 2006)
Time-dependent - stochastic time	Aircraft (Zhang and Jiang, 1999; Tian et al., 2010), second order system (Gu et al., 2012), DC motor (Langeron et al., 2013), control valve (Mo and Xie, 2016)
Stochastic - Markov process	A first order system (Mahmoud et al., 2002), inverted pendulum (Wei et al., 2017), vehicle suspension system (Shen et al., 2014), landing vehicle (Huang et al., 2014)

4.3.1.2 Factor-free input degradation

Degradation-dependent models of behaviour with factor-free models of input degradation assume that the degradation may be detected from the behaviour of a system. The system can be typically identified as being in up state, degraded state, or fault state as defined in Fig. 2.1. Table 4.4 presents selected applications of factor-free models of input degradation.

Input degradation means that the inputs to a system change due to degradation. In models of behaviour of form Eq. (2.1), input degradation may be detected from the value of u which becomes u_D . Typically, u_D captures the multiplicative nature of input degradation, where $u_D = h(d)u$. In particular, the input degradation represents how an output of a degraded actuator enters the system after leaving the dashed blue box in Fig. 4.2.

4.3.1.2.1 Fixed values of input degradation

As mentioned in Section 4.3.1.1.1, in the simplest models, the degradation function $h(d)$ takes fixed values, $h(d) \in \{h_1, h_2, \dots, h_n\}$. In consequence, there are n different input cases $u_D \in \{u_1, u_2, \dots, u_n\}$ where typically $u_i = h_i u$. Fan et al. (2017) observed that fixed values of $h(d)$ could emulate degradation processes which result in stuck actuators. Veillette (1995), Maki et al. (2004), and Zhao and Jiang (1998) considered an input to a system from an actuator that was either:

- In up state with $h(d) = 1$, and thus $u_D = u$,
- Failed with $h(d) = 0$ and $u_D = 0$.

González-Contreras et al. (2007) assumed that the performance of a degraded actuator changed from 100% to 90%, from $h(d) = 1$ to $h(d) = 0.9$. Ballé et al. (1998), Theilliol et al. (2002), Gao et al. (2011), Chamseddine et al. (2015), Graves et al. (2018), Chen et al. (2016), Li et al. (2017), and Theilliol et al. (2008) have modelled several values of degradation function, between 100% and 0% of performance. Liu et al. (2015) used a fixed value model of degradation to design a controller for a third-order dynamic system with uncertain parameters. In all cases, degradation-dependent models of behaviour with $u_D = h(d)u$ were considered.

Wang and Yao (2010) considered multi-actuator systems. Some actuators could only be in up state $h(d) = 1$ or faulty with $h(d) = 0$, whereas other actuators could operate with partial performance $0 < h(d) < 1$. Similarly, Ji et al. (2003) designed a controller for a wheeled robot with several degraded actuators. Yin et al. (2014) showed that fixed values of input degradation extend to a more general class of inputs than stuck actuators by considering two levels of feed ratio as degradation of the Tennessee Eastman benchmark process.

In linear models of the form of Eq. (2.2), modifying the input u is equivalent to modifying the matrix $\mathbf{B}$ (Mahmoud et al., 2002). Zhang and Jiang (2001), Jiang and Zhang (2006), Zhang et al. (2008), and Shi et al. (2017) defined values for the matrices $\mathbf{B}$ of the linear system (2.2a) in degraded states. Additive models of degradation with fixed matrices from Eq. (2.2a) were used by Prakash et al. (2005).

A degradation-dependent model of behaviour is required when the presence of degradation can be detected from the behaviour of the system. In some cases, degradation may be detectable and also quantifiable. Yang et al. (2014) and Ye and Yang (2006) have used an adaptive identification procedure to estimate the value of $h(d)$ in aircraft applications. Boussaid et al. (2011) and Yu et al. (2018) also assumed constant values of degradation, and estimated the values using online observers.

Similarly to the models of output degradation described in Section 4.3.1.1, if input degradation is modelled with fixed values h_i , it is possible to analyse the inputs separately for each i .

4.3.1.2.2 Time-dependent models of input degradation

Tao et al. (2001) proposed a model of an actuator subsystem that degrades after a predefined time t_0 . The degradation function $h(d)$ is therefore piecewise constant:

$$h(d(t)) = \begin{cases} h_0, t < t_0 \\ h_1, t \geq t_0 \end{cases} \quad (4.15)$$

In consequence, if multiplicative degradation is considered:

$$u_D = \begin{cases} h_0 u, t < t_0 \\ h_1 u, t \geq t_0 \end{cases} \quad (4.16)$$

Time-dependent models of the form of Eq. (4.15) are an extension of the models described in Section 4.3.1.2.1. If degradation of an actuator can be described with such a model of degradation, then the system must be able to handle a stepwise change in the input signal u_D . Wu et al. (2000a) used such an approach to model the degraded control surfaces of an aircraft system, whereas Zhang and Qin (2009) approximated degradation of pumps in a three tank system using this approach.

Other approaches proposed by Tao et al. (2001) modelled the degradation of the actuator as additive changes of the actuator output u :

$$u_D(t) = u(t) - h(d(t)) \quad (4.17a)$$

$$h(d(t)) = \tilde{u} + \tilde{d}(t) + \tilde{\delta}(t) \quad (4.17b)$$

where $\tilde{u}$ is an unknown constant, and $\tilde{\delta}(t)$ is bounded, but unknown. In Eq. (4.17), $\tilde{d}(t) = \sum_j \tilde{d}_j f_j(t)$, where $\tilde{d}_j$ are constant parameters and $f_j(t)$ are predefined functions. For instance, if $u(t)$ is the flow through a valve as in Arıcı and Kara (2018), then Eq. (4.17) provides a model such that the degraded flow rate u_D can vary according to $\tilde{d}(t)$ and can also be influenced by an unknown but bounded function $\tilde{\delta}(t)$. Additive time-dependent models of form Eq. (4.17) also represent continuous degradation that would influence the behaviour regardless of the input u .

The model (4.17) is not application-specific and can be applied for modelling degradation in various applications. Such models were used for degradation of hydraulic actuators (Tao et al., 2001; Zhang et al., 2010), continuous stirred tank reactor degradation (Wang et al., 2007), valve leakages (Arici and Kara, 2018), electrical systems degradation (Kaviarasan et al., 2016), degraded aircraft system (Jiang and Chowdhury, 2005; Liu and Crespo, 2012), and burner degradation (Baldi et al., 2017). Yu et al. (2005) simulated two kinds of degradation in a CSTR system: a fixed value model of degradation of a pump, and a time-dependent model of degradation representing a loss of inlet temperature. In all cases the degradation influenced the input signal $u_D = h(d)u$. Jiang et al. (2006) used additive models of degradation for design of a control system for a robotic arm.

4.3.1.2.3 Stochastic models of input degradation

Stochastic models typically assume that the value of the degradation function $h(d)$ changes only at certain time instants and is constant in between. The timings of the changes are not known in advance. Such an approach was applied by Zhang and Jiang (1999) and Tian et al. (2010) who assumed the constant value between the instants of change was:

$$h(d) = 1 - \theta \quad (4.18)$$

where θ had a normal distribution with positive mean and fixed variance. Gu et al. (2012) proposed a model that merged the random loss of performance of an actuator with its saturation, effectively combining the simple approach based on constant values with stochastic estimation and a nonlinear degradation function. A stochastic approach was also used by Mahmoud et al. (2002), Wei et al. (2017), and Shen et al. (2014) who defined degradation as a Markov process such that the degradation at time t depended on the value at $t - 1$. A Markovian model of degradation was also used by Huang et al. (2014) who included time-dependency in the stochastic model, i.e. the random changes of $h(d)$ depended on time, as well as on the previous value of $h(d)$. Mo and Xie (2016) modelled degradation d as a stochastic loss of effectiveness of a control valve and included it in the dynamic behaviour using $u_D = (1 - d)u$.

Stochastic models of degradation introduce both the transitional elements of time-dependent models of degradation, and the uncertainty. Therefore, despite their relative simplicity, if the degradation is described by such a model, Gu et al. (2012) indicated that the controller must take stochastic nature of degradation into account.

4.3.2 Degradation-dependent models of behaviour with factor-based models of degradation

Singpurwalla and Wilson (1998) indicated that the amount of use affect degradation of equipment. Factor-based models of degradation

$$h(d) = h(d(t, v(t), x(t), u(t), y(t))) \quad (4.19)$$

take into account that control actions may have a direct impact on the degradation of the system. Degradation-dependent models of behaviour with factor-based models of degradation consider further that such degradation is detectable from the behaviour of the system as shown in the Eq. (4.1), originally (2.1), for $\dot{x}_D(t)$ and $y_D(t)$, and hence that the behaviour is itself influenced by use and operation.

4.3.2.1 Factor-based models of output degradation

Factor-based models of output degradation are often analysed using a physics-of-failure approach (Modarres et al., 2017), and are case-specific. The main focus is on the degradation processes, such as

- Erosion (Li et al., 2018),
- Wear (Cao and Dai, 2015),
- Corrosion and pitting (Chookah et al., 2011),
- Fouling (Brahim et al., 2003).

Suri and Onori (2016) described battery ageing with a physical model relating influencing factors such as state-of-charge, battery temperature, and power level to degradation. They analysed the influence of degradation of the battery in a vehicle simulation. Ahmad et al. (2014) described degradation of a ladle in a steel making process as a function of repeated usage. They used the model of degradation for feedforward control of temperature during the process.

4.3.2.2 Factor-based models of input degradation

In the simplest case of a factor-based model of input degradation, the degradation function $h(d)$ depends directly on an input, u :

$$h(d) = h(d(u)) \quad (4.20)$$

For instance u might be the control effort. Such a model was proposed by Vieira et al. (2015) who described the degradation function as a first order discrete dynamic relationship:

$$h(d(k+1)) = h(d(k)) + \alpha|u(k)| \quad (4.21)$$

in which $\alpha > 0$. They observed that the relationship assumes the degradation will remain constant if there is no change in $u(t)$. This is not always the case, however. As an example, Meher-Homji et al. (2001) demonstrated that the degradation processes in turbomachinery would continue even after shut down of the piece of equipment.

4.3.2.2.1 Control effort as an influencing factor

Vieira et al. (2015) included the degradation function with influencing factors in the model of behaviour of the form of Eq. (2.2). They proposed to switch between the matrices of a model of a system with an actuator in up state and in degraded

state:

$$\mathbf{B}(h(d)) = \begin{cases} \mathbf{B}^{\text{nom}} & \text{if } h(d) < h^{\text{lim}} \\ h(d)\mathbf{B}^{\text{nom}} & \text{if } h(d) \geq h^{\text{lim}} \end{cases} \quad (4.22)$$

where $\mathbf{B}^{\text{nom}}$ is the nominal value in up state, $h(d)$ is the solution of Eq. (4.21), and h^{lim} is a predefined limit.

A similar approach to designing control systems taking account of degradation was considered by Langeron et al. (2013) who used stochastic processes to model the degradation function $h(d)$ and relate it with the behaviour of a system. They divided the behaviour into three phases, depending on the value of degradation d as in Fig. 2.1, i.e. $d \leq d_0$, $d \in (d_0, d_1)$, and $d \geq d_1$. Langeron et al. (2015, 2016, 2017) used shock models to find the values of the degradation function $h(d)$ related to degradation of a pump, and devised a predictive controller that took degradation into account. The shocks were defined as changes of the input u , i.e. the control action was considered a factor influencing degradation.

Both Vieira et al. (2015) and Langeron et al. (2016) used optimisation-based control to compensate for degradation. Therefore the feedback control problem became an optimisation problem with the degradation function included in the objective function (Langeron et al., 2016, 2015, 2017) or in the constraints (Vieira et al., 2015).

4.3.2.2.2 External influencing factors

Singpurwalla and Wilson (1998) considered both usage and external factors as influencing factors in a model of input degradation. Using this approach, Nguyen (2015) modelled $h(d)$ as:

$$h(d) = h_0 - W^{\text{nd}} - W^{\text{om}} \quad (4.23)$$

where W^{nd} and W^{om} denote a loss of effectiveness due to natural degradation and operating condition, called operating mode, respectively. W^{nd} and W^{om} are modelled as shock deterioration models:

$$W^{\text{nd}}(t) = \sum_{j=0}^{N(t)} \Delta W_j^{\text{nd}} \quad (4.24)$$

and:

$$W^{\text{om}}(t) = \sum_{j=0}^{M(t)} \Delta W_j^{\text{om}} \quad (4.25)$$

where here $N(t)$ is the number of shocks due to external factors and $M(t)$ refers to the number of changes of the control action that happened during a certain time period t . The values ΔW_j^{nd} and ΔW_j^{on} denote the change in degradation after j -th shock (change) and “are independently and identically distributed” (Nguyen et al., 2015). Nguyen et al. (2014a,b,c) applied the model from Eq. (4.23) to a tank system to describe degraded performance of a pump subject to random shocks. Every time a shock occurred, i.e. with every change of $h(d)$ according to Eq. (4.23), the performance of an actuator decreased by a quantity $w(t)$ following a uniform distribution on a fixed interval $[0, \delta]$. Nguyen et al. (2015) modelled the performance of an actuator as changes of the matrix $\mathbf{B}$ from Eq. (2.2a):

$$\mathbf{B}(t) = \mathbf{B}^{\text{nom}} - w(t) \quad (4.26)$$

Thus:

$$u_{\text{D}} = (\mathbf{B}^{\text{nom}} - w)u \quad (4.27)$$

In contrast to Langeron et al. (2016, 2015, 2017) and Vieira et al. (2015), Nguyen et al. (2014a,b,c) analysed the influence of degradation of a pump on a system with a PID controller, without explicitly including the degradation function in the controller design.

Table 4.5 presents selected applications with factor-based input degradation. An overview of models of degradation that can be used to approximate the degradation function depending on the influencing factors was provided by Singpurwalla (1995) and Bagdonavičius and Nikulin (2001a). A more recent review of such approaches was done by van Noortwijk (2009) and Bagdonavičius and Nikulin (2009).

4.3.3 Degradation-independent models of behaviour with factor-free models of degradation

Models of degradation that are independent from influencing factors and where degradation is not detectable in the behaviour have been of little use for control pur-

Table 4.5: Applications of factor-based models of input degradation

Model of degradation	Factors	Application
Performance loss - heuristic	Control effort $u(k)$	Second order system (Vieira et al., 2015)
Performance loss - Wiener process	Changes of control effort $u(k)$ - discrete	Tank level control (Nguyen et al., 2015, 2014a,b,c)
Performance loss - gamma process	Changes of control effort $u(k)$ - continuous	DC motor (Langeron et al., 2013), maintenance (Langeron et al., 2016), drilling unit (Langeron et al., 2015, 2017)
Performance loss - physical	Discharge current, state-of-charge (SOC), and operating temperature	Battery charging (Allam et al., 2015; Zou et al., 2018)

poses. However, they are widely used in reliability engineering for prognostics and maintenance planning (Mann et al., 1974; Rausand and Høyland, 2004). *Reliability* is defined as an “ability of an item to perform a required function under given conditions for a given time interval” (BSI, 2010a). It is described with a reliability function that captures the probability that the item will not fail in the time interval $[0, t]$ (Mann et al., 1974). The moment when an item fails at t_1 shows when degradation has reached a failure threshold in Fig. 2.1. The reliability function $R(t)$ measures the probability that the degradation will not reach a failure threshold in the given interval $[0, t]$. Alternatively, a failure (hazard) rate is used instead of $R(t)$:

$$z(t) = -\dot{R}(t) \quad (4.28)$$

which represents the probability that the item will fail in the interval $[t, t + \Delta t]$ if it has survived until the time t (Mann et al., 1974).

Factor-free reliability-based models of degradation can be found in industrial databases, such as ‘OREDA Offshore & Onshore Reliability Data’ (SINTEF, 2002) for oil and gas applications, and ‘Military Handbook: Reliability Prediction of Electronic Equipment: MIL-HDBK-217F’ (United States of America: Department of Defense, 1986) for electronics. As an example, Kopnov (1999) modelled crack growth as a set of fixed values, stochastically changing at random times. The probability of transitions was predefined (hence factor-free) and the cost of failure due to crack growth was included in the cost of operation. The behaviour of the system was not affected, because typically the effects of a crack are not observable in the behaviour until a component breaks.

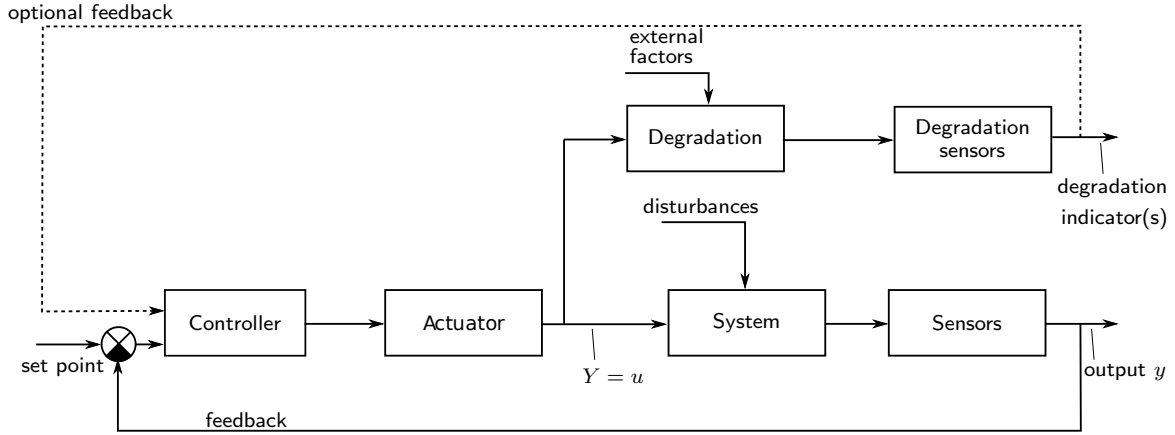


Figure 4.3: Feedback control loop with a degradation-independent system

Figure 4.3 shows an interpretation of a controller with a degradation independent system. Degradation is assumed to be measured separately from the output y , and the input u does not depend on the degradation. The optional feedback loop sends information about the degradation back to the controller. The controller adapts its output taking into account both the error between the setpoint and the feedback signal, and the current degradation. The adaptation might consist of changing the parameters of the controller as in a gain-scheduling controller, or might require a recalculation of the control output if model predictive control is used. The degradation and the system in Fig. 4.3 are considered separately from the controller point of view.

4.3.4 Degradation-independent models of behaviour with factor-based models of degradation

A factor-based model of degradation is useful when degradation is not detectable from the behaviour of the system. Degradation-independent models of behaviour with factor-based models of degradation can be divided into three groups:

- Physical models of degradation that is measurable before its influence is visible in the behaviour of the system. This could for example be during the early stages of Fig. 2.1 where $t < t_0$;
- Reliability-based models of degradation that are focused on the end of the degradation period i.e. when the system has failed ($t > t_1$ in Fig. 2.1);

Table 4.6: Physics-based relationships between degradation d and influencing factors as presented by Nelson (2009) and Escobar and Meeker (2006)

Name	Formula	Factors	Applications
Arrhenius life relationship	$d = A \exp(-BT)$ with constants A, B	Temperature T	Electrical insulation, semiconductor devices, batteries, plastics, lubricants
Inverse power relationship	$d = \frac{A}{V^\gamma}$ with constants A, γ	V - voltage, temperature range (Coffin-Manson r-ship), load (Palmgren), velocity (Taylor)	Electrical insulation, bearings, lamps, metal fatigue
Exponential relationship	$d = \exp(\gamma_0 - \gamma_1 V)$ with constants γ_0, γ_1	V	Dielectrics
Exponential-power relationship	$d = \exp(\gamma_0 - \gamma_1 x^{\gamma_2})$ with constants $\gamma_0, \gamma_1, \gamma_2$	x - voltage, temperature	Electrical components
Polynomial relationship	$d = \sum_{i=0}^K \gamma_i x^i$ with constants K, γ_i	x	Metal fatigue
Elastic-plastic relationship	$d = AN^{-a} + BN^{-b}$ with constants A, B, a, b	N - number of cycles	Metal fatigue
Eyring relationship	$d = \frac{A}{T} \exp(-BT)$ with constants A, B	T - temperature	Chemical degradation
Log-linear relationship	$\log d = \sum_{i=0}^K \gamma_i x_i$ with constants K, γ_i	x_i	Insulating tape, batteries
Generalised Eyring relationship	$d = \frac{A}{T} \exp(-BT) \exp(V(C - DT))$ with constants A, B, C, D	T - temperature, V - voltage	Capacitors

- Heuristic models of degradation that are based on knowledge and assumptions about the degradation and the system, applicable for $t > 0$.

4.3.4.1 Factor-based physical models of degradation

Factor-based physical models are used to describe degradation that either is not advanced enough to be detectable in the behaviour of the system, or else actually does not change the behaviour. The degradation may be measurable or diagnosable in itself, perhaps with specialist equipment. An example would be development of a crack in a turbine blade that does not change the behaviour of the turbine until the blade fails.

According to Modarres et al. (2017), a physics-of-failure approach uses knowledge of physical and chemical properties of materials, load profile, environmental conditions, failure mechanisms and accelerated test data to build models of degradation and time to failure. The physical nature of the models indicates that the models of degradation will be case-specific.

Some physical phenomena are common enough to justify the use of a set of predefined models. Xu et al. (2016) presented an overview of general degradation path models, which are application independent. Application-specific models were described by Martin et al. (1983), who focused on mechanical degradation, whereas Collins et al. (2015) presented material-specific models of metal degradation, with the underlying influencing factors explained by Fisher (2015). Comprehensive reviews of the physical models of degradation with influencing factors were given by Escobar and Meeker (2006) and Nelson (2009). A brief summary is presented in Table 4.6. The models given in Table 4.6 show how the degradation d depends on the influencing factors. The value of d represents the degradation itself, whereas $h(d)$ is used if degradation d is not measurable.

The degradation d in the physical relationships from Table 4.6 assumes the influencing factors are constant. The evolution of the influencing factors over time is also of importance and should be taken into account. For that purpose, cumulative damage indicators (or exposure functions) are considered (Xu et al., 2016). The degradation function has the form:

$$h(d(t)) = f(D(v(t), \beta), b) + \varepsilon \quad (4.29)$$

where $v(t)$ are influencing factors, β , b are constant parameters of the model, and ε represents measurement noise. The function $D(v(t), \beta)$ denotes the cumulative degradation up to time t (Nelson, 2009), sometimes called the *additive accumulation of damages* (Bagdonavičius and Nikulin, 2001b):

$$D(v(t), \beta) = \int_0^t d(v(\tau), \beta) d\tau \quad (4.30)$$

Palmgren-Miner's rule is an example of such an indicator, used for metal fatigue modelling and applied by Ray et al. (1994b). Palmgren-Miner's rule assumes that $D(v(t), \beta)$ is a linear function of influencing factors (Nelson, 2009). Table 4.7 summarises the physical models used for control purposes.

Table 4.7: Applications of degradation-independent models with factor-based physical models of degradation

Degradation type	Model	Applications
Fatigue (crack propagation)	Differential equations - Palmgren-Miner's relationship (Inverse power relationship (Nelson, 2009)), Difference equations (Patankar and Ray, 2000)	Aircraft (Ray and Caplin, 2000; Caplin et al., 2001), rocket engine (Ray et al., 1994b,c,a), power plant (Kallappa et al., 1997; Gallestey et al., 2002; Ferrari-Trecate et al., 2002; Kallappa and Ray, 2000), utility boiler (Li et al., 2005, 2006), gas turbine engine (Tangirala et al., 1998), mass-beam system (Tangirala et al., 1999; Zhang et al., 2000b,a), actuator cylinder (Chen et al., 2014), wind turbines (Beganovic and Söffker, 2016; Sanchez et al., 2017), bearing lifetime (Gökdere et al., 2006), pasteurisation plant (Pour et al., 2018, 2017)
Thinning of a wall	Geometric damage indicator	Rocket engine (Dai and Ray, 1996)
Thermal degradation	Arrhenius relationship	Winding insulation (Gökdere et al., 2006, 2005), electromechanic actuator (Brown et al., 2009), batteries (Hatzell et al., 2012)
Thermal degradation	Polynomial relationship	Synchronous motor (Samaranayake and Longo, 2018)
Wear model	Stochastic differential equations	General machine wear (Rishel, 1991), (Lefebvre and Gaspo, 1996), metallic structures in mechanical systems (Ray, 1999), race car tire degradation (Heilmeyer et al., 2018)
Surface film forming	Partial differential equations	Electric cars battery charging (Moura et al., 2013)
Wear model	Stochastic differential equations	Cutting tool degradation (Hao et al., 2017a)

A variant of model predictive control can be used with the objective function including the degradation function. Ray et al. (1994a,b,c) introduced the term *damage mitigating control* to emphasise that degradation is explicitly included in the objective function. This concept will be further described in Section 3.1.2. Another example is control of pasteurisation temperature (Pour et al., 2018, 2017). By including the degradation of the pump in the objective function, Pour et al. (2018, 2017) were able to mitigate the degradation, at the same time satisfying constraints on energy consumption.

4.3.4.2 Factor-based reliability models of degradation

The two main approaches to factor-based models of degradation based on reliability analysis are:

- Parametric models, where the reliability function is often based on one of the physical relationships gathered in Table 4.6,
- Non-parametric models or semi-parametric, usually in form of proportional hazard models (PHM).

Table 4.8: Applications of factor-based physical models of degradation

Model of degradation	Factors	Application
PHM with exponential distribution and failure rate $z_i = d_i(t, l_i)$	Actuator loads l_i	Reconfigurable control (Guenab et al., 2006) of a heating system (Guenab et al., 2011), aircraft applications ($i = 1$) (Khelassi et al., 2010; Weber et al., 2012)
PHM with exponential distribution and failure rate $z_i = d_i(t, u_i)$	Actuator loads u_i	Drinking water network (Salazar et al., 2017b; Pour et al., 2019)
PHM with exponential distribution and failure rate $z_i = d_i(f(u_i))$ with $f(u_i) = u_i $ or $f(u_i) = u_i^2$	Actuator loads u_i	Control allocation for Unmanned Aerial Vehicle (UAV) (Chamseddine et al., 2014; Theilliol et al., 2015), Drinking water network (Salazar et al., 2017a)
PHM with exponential distribution and failure rate $z_i = d_i(\beta_i, u_i^2)$ with constant β_i	Actuator loads u_i	Control allocation for F-16 aircraft (Khelassi et al., 2011b)
PHM with Weibull distribution and scale parameter $z_i = \frac{z_i^0}{\exp(U_i)}$ with U_i the root-mean-square of u_i and constant z_i^0	Actuator loads u_i	Control allocation for aircraft application (Khelassi et al., 2011a)
PHM with Weibull distribution and failure rate $z_i = d(y_i(t))$	Aggregated performance indicators y_i	Load allocation in an export compressor station (Nystad, 2008)

Escobet et al. (2002) suggested a generic approach to control taking degradation into account called *health-aware control* that has been described in more detail in Section 3.1.2. Since then, several authors used proportional hazard models in control applications, as listed in Table 4.8. In most cases, the control effort $u(t)$ was considered to be a factor influencing degradation. This approach was analysed in aircraft applications to allocate the control effort among several actuators with varying reliability functions (Khelassi et al., 2010; Weber et al., 2012; Theilliol et al., 2015). Similarly, Guenab et al. (2011) applied reliability-based control to a heating system with degrading pumps. Salazar et al. (2017b) and Pour et al. (2019) controlled a drinking water network based on the reliability of the pumps to improve the reliability of the whole system.

Table 4.8 summarises the applications of reliability-oriented models of degradation for control purposes. All entries in this table used a degradation-independent model of behaviour, as the reliability does not influence the behaviour of a system. The controllers are designed as model predictive control or adaptive optimal control (Chamseddine et al., 2014).

Table 4.9: Applications of factor-based heuristic models of degradation

Model of degradation	Factors	Application
$d(k+1) = d(k) + \Gamma u(k) + \Psi \Delta u(k) $ with constant parameters Γ, Ψ	Flow through a pump $u(k)$	Tank level control (Pereira et al., 2010), Drinking Water Network (Grosso et al., 2012)
$d(k+1) = d(k) + \psi u(k) + \eta_k$ with constant parameter γ , and η_k from a known normal distribution	Flow through an actuator: a pump or a valve $u(k)$	Drinking water network (Grosso et al., 2016)
$\Delta d = -(p_N N^3 + p_\Delta \Delta N ^3) \exp(1 - GV F)$ with p_N and p_Δ from a known normal distribution	Speed N , Gas Volume Fraction $GV F$	Subsea compression system (Verheyleweghen and Jäschke, 2017)
$\dot{d} = E \left(p_1 \phi_0 + \int_0^\infty (p_2 \phi(\tau) + p_3 \dot{\phi}(\tau)) d\tau \right)$ with constant weights p_1, p_2, p_3	Compressor mass flow ϕ dimensionless	Subsea compression system (Verheyleweghen et al., 2018)
$d = V_s - V_b$	Voltages corresponding to the electrode surface V_s and the inner part of the electrode V_b	Battery in electronic vehicles (Fang et al., 2017)

4.3.4.3 Factor-based heuristic models

Physical and reliability-based models of degradation are not always available. As indicated by Meeker and Escobar (1998), the unavailability of a model of degradation might be due to lack of information about the degradation itself. However, available knowledge about the process and the influencing factors can be applied to build heuristic models of degradation. Even though the heuristic models do not have any physical interpretation, they provide a knowledge-based description of degradation. Therefore the models are application-specific.

Heuristic approaches can be used for control purposes. Thus, optimisation-oriented approaches are of use as they combine objective functions and constraints related to the behaviour and the degradation. For instance, Verheyleweghen et al. (2018) presented a hierarchical framework for model-based control of a subsea compressor station including degradation as a constraint and ensuring stable operation of the station. Pereira et al. (2010) designed a predictive controller that took into account limits on the degradation of pumps. Table 4.9 presents further control applications of heuristic models of degradation that do not have any underlying physical or reliability-oriented interpretation.

4.4 Influence of degradation on process control

Process control at all layers of Fig. 2.3 has a role in reducing degradation and avoiding failures. For instance, regulatory control would consider fast changes due to degradation. Optimization and scheduling should take into account slower degradation processes (Baldea and Harjunkski, 2014), such as fouling of centrifugal compressors (Xenos, 2015).

The role of degradation modelling in process control applications is shown in Table 4.10, which presents a summary of the degradation models with respect to the corresponding process control layers of Fig. 2.3. The column headings reflect the models of behaviour introduced in Fig. 4.1, either a degradation-dependent model of behaviour, or degradation-independent. The rows indicate whether the model of degradation was factor-based or factor-free.

The examples listed in the table show that degradation-dependent models of behaviour dominate in regulatory control applications and to some extent in supervisory control applications. This is because the regulatory and supervisory control operate at similar timescales as the system and thus the knowledge about the behaviour is necessary to ensure correct operation of the system.

Degradation-independent models of behaviour find many uses in the scheduling and optimisation layers for improving the performance of a system given the correct operation is already ensured by regulatory and supervisor control layers. A degradation-independent model of behaviour can be sufficient at these higher levels even if the real system does have degradation-dependent behaviour. This is because scheduling and optimisation operate at different timescales than the behaviour of a system. From their perspective, the degraded state of a system is masked by regulatory and supervisory control.

As an example, the paper by Verheyleweghen et al. (2018) presented a model of compressor degradation applied for control of a subsea compression system. The proposed degradation model depended on the operating point of the compressor, as well as on the changes in the operating point, and thus is an example of a factor-based model of degradation. At the same time, the degradation model was not incorporated into the behavioural model of a compressor, hence the system from Verheyleweghen et al. (2018) is classified as degradation-independent model with factor-based degradation. By contrast, Theilliol et al. (2002) used a degradation-dependent model to control a three-tank system with a degrading pump. The behaviour of the tank system was influenced by the degradation of the pump, but the degradation of the pump was arbitrarily introduced and did not depend on either the system variables or external conditions. Thus, the pump degradation model used by Theilliol et al. (2002) is classified as a factor-free model of degradation.

Table 4.10 shows that factor-based models of degradation have tended to be used mainly with degradation-independent models of behaviour, and mainly at the higher levels of automation. Factor-based models of degradation are useful for these levels of automation because their timescales are close to the timescale of factor-based degradation. Moreover, if the factors influencing degradation are known, the scheduling and optimisation layers can improve the performance of the system by explicitly attempting to mitigate degradation. For instance, Wiebe et al. (2018) proposed a data-driven optimisation framework that mitigates degradation of equipment by planning maintenance activities in chemical processes.

Table 4.10: Uses of models of degradation and their influence on the models of behaviour, grouped according to the automation hierarchy from ANSI/ISA (2010)

	Degradation-dependent models of behaviour	Degradation-independent models of behaviour
Scheduling - factor-based models of degradation		Wind turbines (Beganovic and Söffker, 2016; Sanchez et al., 2017; Verheyleweghen et al., 2018; Verheyleweghen and Jäschke, 2017), compressor station (Nystad, 2008), benchmark problem with three units (Hao et al., 2017a,b), chemical plants (Wiebe et al., 2018)
Scheduling - factor-free models of degradation	Crack propagation (Beganovic and Söffker, 2017; Zhou et al., 2011)	Oil and gas (SINTEF, 2002), electronics (United States of America: Department of Defense, 1986)
Optimisation - factor-based models of degradation	Drilling unit (Langeron et al., 2015), (Langeron et al., 2017)	Turbomachinery (Dai and Ray, 1996; Ray et al., 1994b,c,a), windings, bearings and cylinders (Chen et al., 2014; Ray, 1999), mass-beam system (Zhang et al., 2000b,a), pasteurisation plant (Pour et al., 2018, 2017), tank level (Pereira et al., 2010), water network (Grosso et al., 2016, 2012; Salazar et al., 2017a,b), batteries (Fang et al., 2017)
Optimisation - factor-free models of degradation	Control valve (McGhee et al., 2014), turbomachinery (Aker and Saravanamuttoo, 1989; Tarabrin et al., 1996; Ciccotti, 2015; Li and Nilkitsaranont, 2009; Tsoutsanis et al., 2015; Meher-Homji et al., 2001; Kurz and Brun, 2001), TE benchmark (Yin et al., 2014), pulp mill (Zumoffen and Basualdo, 2008)	Power plant (Gallestey et al., 2002; Ferrari-Trecate et al., 2002; Kallappa et al., 1997; Kallappa and Ray, 2000)
Supervisory control - factor-based models of degradation	General dynamic system (Vieira et al., 2015), tank system (Nguyen et al., 2015, 2014a,b,c), battery charging (Suri and Onori, 2016; Allam et al., 2015)	Turbomachinery (Tangirala et al., 1999), batteries (Hatzell et al., 2012; Moura et al., 2013), mass-beam system (Tangirala et al., 1998), electromechanical system (Gökdere et al., 2006, 2005), aircraft (Ray and Caplin, 2000; Caplin et al., 2001; Brown et al., 2009), general machine wear (Rishel, 1991; Lefebvre and Gaspo, 1996), heating system (Guenab et al., 2006, 2011)
Supervisory control - factor-free models of degradation	CSTR (Prakash et al., 2005; Mhaskar, 2006; Mhaskar et al., 2012), vehicles (Boskovic et al., 2009), hydrothermal process (Boussaid et al., 2011), electrical systems (Langeron et al., 2013; Samaranayake and Longo, 2018, 2015), burner (Baldi et al., 2017), Shell control problem (Kettunen et al., 2008), hydrogen production (Du et al., 2014), heat exchanger (Ballé et al., 1998)	Utility boiler (Li et al., 2005, 2006; Baldi et al., 2017)
Regulatory control - factor-based models of degradation	Steel making process (Ahmad et al., 2014)	Aircraft (Khelassi et al., 2011a, 2010, 2011b; Weber et al., 2012)

Table 4.10 (Continued): Uses of models of degradation and their influence on the models of behaviour, grouped according to the automation hierarchy from ANSI/ISA (2010)

	Degradation-dependent models of behaviour	Degradation-independent models of behaviour
Regulatory control - factor-free models of degradation	General dynamic system (Tao et al., 2001; Veillette, 1995; Chen et al., 2013; Wang and Yao, 2010; González-Contreras et al., 2007; Gu et al., 2012; Mahmoud et al., 2002), aircraft (Graves et al., 2018; Zhang and Jiang, 1999; Tian et al., 2010; Jiang and Chowdhury, 2005; Wu et al., 2000a; Shi et al., 2017; Yang et al., 2014; Ye and Yang, 2006; Yu et al., 2018; Chamseddine et al., 2015; Chen et al., 2016; Li et al., 2017; Maki et al., 2004; Zhao and Jiang, 1998; Zhang and Jiang, 2001; Jiang and Zhang, 2006; Zhang et al., 2008), three tank system (Theilliol et al., 2002, 2008; Zhang and Qin, 2009), vehicles (Gao et al., 2011; Shen et al., 2014; Huang et al., 2014), ball-beam system (Zhang et al., 2010), control valve (Arici and Kara, 2018; Mo and Xie, 2016), electrical systems (Kaviarasan et al., 2016), inverted pendulum (Wei et al., 2017), CSTR (Wang et al., 2007)	

4.5 Chapter summary

This chapter has presented a perspective on degradation modelling for control applications with an emphasis on the links between degradation of components and the behaviour of a system. It addresses the first challenge from Chapter 1, which was related to analysis of interactions between degradation and a control system. Section 3.3.1 identified that there is gap of knowledge in the area of including degradation in process control applications. This chapter fulfils this gap by giving an in-depth discussion of models of behaviour of the system and models of degradation of the components of the system. The models of behaviour have been classified as degradation-dependent or degradation-independent, to reflect whether the degradation affects the behaviour. The models of degradation have been classified as factor-free or factor-based, to capture how influencing factors affect degradation (Table 4.10).

The survey examines control applications where degradation modelling has been used. Table 4.10 groups the findings according to the proposed classification. Section 4.4 of the chapter highlighted the following points:

- Degradation-dependent models of behaviour dominate in regulatory and supervisory control because such control needs accurate knowledge of the behaviour over short timescales. However, the models of degradation are often factor-free, for instance degradation is assumed constant;
- Factor-based models of degradation are used in scheduling and optimisation, because the scheduling and optimisation layers make adjustments to the factors to mitigate degradation;
- Degradation-independent models of behaviour are the most widely used in scheduling and optimisation. The assumption is that degradation does not change behaviour because any degradation that does appear is compensated for by the regulatory and supervisory control.

In particular, degradation modelling is of importance for degradation-aware control and degradation-mitigating control, which were introduced in Chapter 3. Degradation-aware control is focused on the behaviour of the system subject to degradation. With but a few examples, it considers degradation-dependent models of behaviour with factor-independent models of degradation. Conversely, the focus of degradation-mitigating control is shifted towards modelling the degradation itself. As such it uses the knowledge about factors influencing degradation to adjust the operating conditions and mitigate the effects of degradation on the behaviour of a system.

Chapter 5

Adaptive detection and prediction of performance degradation

Performance-based maintenance of industrial machinery relies on an assessment of the current condition of the machinery and on prognosis of future loss of performance (Jardine et al., 2006; Tahan et al., 2017). Loss of performance is commonly caused by degradation, introduced in Chapter 2. Hong et al. (2011) indicate that degradation could be characterized by a *degradation indicator* that calculates the loss of performance as the machine operates. However, the calculation can be difficult because the mathematical formula can be based on unknown physical relationships, and also because the calculation makes use of operational measurements. A degradation indicator calculated from operational measurements can be highly variable if the calculation does not account for disturbances. Short-term variability caused by disturbances can mask the overall trend of the indicator. Therefore, there is a need to provide a method for quantifying degradation using process data.

Section 3.3.2 identified a gap of knowledge in this area, namely inaccurate degradation models due to fixed functional forms of the model or lack of data for finding the parameters of the model. This chapter addresses the gap of knowledge 3.3.2. It describes a data-driven algorithm for improved estimation and prediction of a degradation indicator for turbomachinery. It combines a moving window approach with adaptive regression analysis to predict the expected value of degradation and quantify the uncertainty of the prediction. The algorithm is tested on industrial degradation data from offshore compressors and is compared with four other approaches from the literature. The parameters of the algorithm such as the prediction window and the threshold for model adaptation are discussed, and the chapter provides recommendations for optimal choices of these parameters. Paper Zagorowska et al. (2020a) is based on the work in this Chapter.

The chapter is structured as follows. Degradation modelling for turbomachinery is introduced in Section 5.1. Section 5.2 describes the industrial data used for tuning and testing the algorithm, which is presented in Section 5.3. The results and analysis of the parameters of the algorithm is conducted in Section 5.4, whereas Section 5.5 compares the performance of the algorithm with four other methods for degradation prediction. The chapter ends with a summary in Section 5.6.

5.1 Turbomachinery degradation

Degradation of turbomachinery is typically associated with fouling, i.e. deposits forming on the surfaces inside the equipment. However, measuring the level of fouling is only possible during manual inspections. To get an insight into the current degradation without performing an inspection, a *degradation indicator* is used.

5.1.1 Efficiency degradation

Typically, the degradation indicator d shows a deviation of the performance variables from the healthy values (Loboda et al., 2007), as indicated by Eq. (4.7) and recalled here:

$$d = \frac{V - V_D}{V} \quad (5.1)$$

where V is the characteristic quantity in up state and V_D is the characteristics quantity in degraded state. Usually, efficiency η is the characteristic quantity used for degradation assessment (Tarabrin et al., 1996). The expected value of η can be calculated based on data from the manufacturer of the turbomachinery or has to be estimated from thermodynamics (Cicciotti, 2015). The actual value η_D can be obtained from measurements using thermodynamic relationships based on experimental correlations to find unknown thermodynamic quantities (Mokhatab et al., 2018). For the purpose of the thesis, it is assumed that the degradation indicator d is available in real-time.

5.1.2 Degradation modelling for turbomachinery

This chapter describes a data-driven algorithm that models the degradation indicator d as a function of time. Tarabrin et al. (1996) described fouling of a compressor as having an exponential approach to an asymptotic value that was reached after 1000-2000 hours of operation. Cicciotti (2015) used such a model in a real-time framework. Assuming that the degradation started immediately after the compressor was turned on, he fitted the exponential model to degradation data as new data points arrived and the window used for approximation increased. The expanding window allowed him to adapt to the varying time of stabilisation of the degradation indicator. He also considered maintenance events that could restore partial performance, but the exponential model was applied between maintenance events and assumed strictly increasing. A similar approach was also assumed by Puggina and Venturini (2012) who approximated degradation of a simulated gas turbine as a linear function of time between maintenance events.

Li and Nilkitsaranont (2009) considered a simulated gas turbine engine in which the simulation included degradation and added random noise. The degradation was linear function of time initially and then started to change more rapidly. They estimated the underlying degradation by linear regression and then switched to fitting of a quadratic form to follow the more rapid degradation. They were able to give predictions with confidence intervals for the remaining useful life.

Tsoutsanis and Meskin (2017) also simulated a gas turbine and proposed a moving window approach with adaptation of the window size. They assumed degradation was monotonically increasing, and treated it as locally linear within each window. They showed a good fit between predictions of degradation from extrapolation from the previous window and the actual degradation.

The existing approaches discussed above cannot be applied to real data from offshore compressors and turbines. In practice, the underlying degradation indicator is not strictly increasing. Also, the degradation indicator calculated from operational measurements is significantly affected by disturbances, making it challenging to estimate and predict the true underlying degradation.

The chapter addresses the problem of detection and prediction of performance degradation in offshore turbomachines. It proposes an algorithm that uses both linear and exponential regression in an expanding moving window framework. The underlying degradation indicator is estimated between maintenance events. The proposed method enables predictions of future degradation, and gives confidence bounds for the predictions.

5.2 Industrial data for efficiency degradation

The studies on degradation modelling used a data set from an offshore compressor, courtesy of Equinor ASA, and a data set from an offshore turbine from Brekke et al. (2009). These data sets are discussed now, in order to introduce, motivate and explain the problems related to degradation modelling.

5.2.1 Determining the degradation of efficiency by smoothing of data

As indicated in Section 5.1, the value of expected efficiency is necessary to calculate the degradation indicator d according to Eq. (4.5) or Eq. (5.1). The expected efficiency in Eq. (4.5) and Eq. (5.1) can be taken to be the efficiency after a complete overhaul, which can be measured, $\tilde{\eta} = \eta_0$.

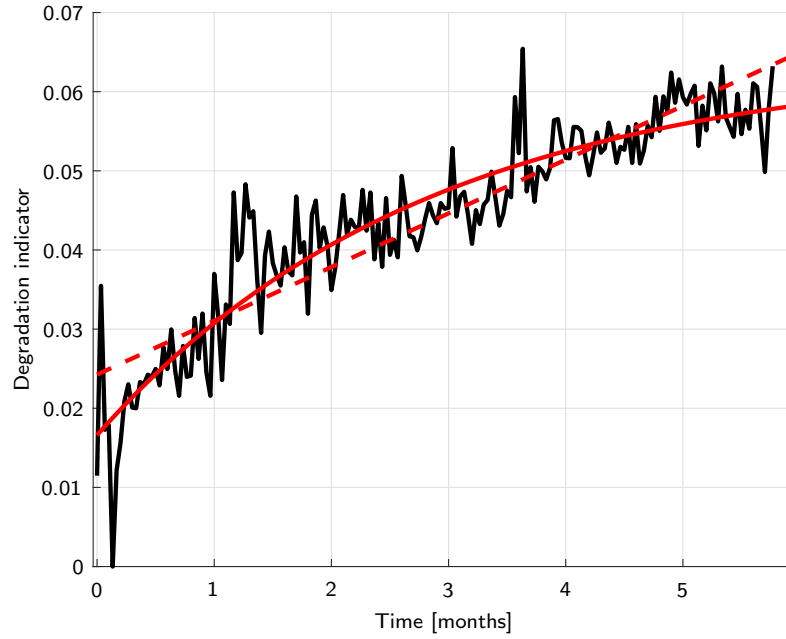
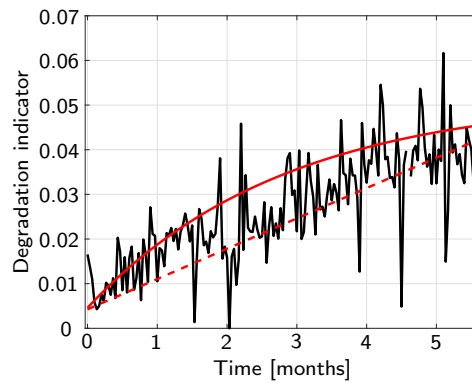


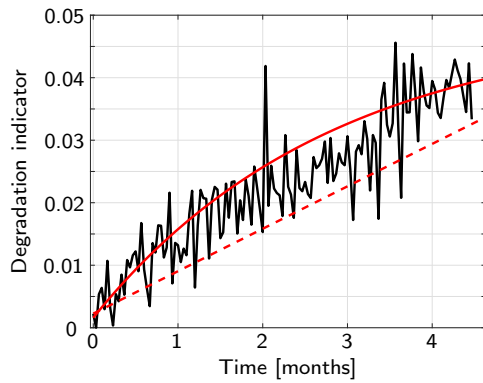
Figure 5.1: Degradation indicator (—) for the compressor from period CT with fitted linear trend (--) and an exponential trend (—)

Figure 5.1 shows the degradation indicator calculated from Eq. (4.5) for episode CT from Figure 1.4. The calculated degradation indicator needs to be smoothed in order to give a better estimate of the true underlying value. The exponential model fits more accurately than a linear trend. This is expected because according to Syverud (2007), the rate of increase of fouling slows down as the deposits build up.

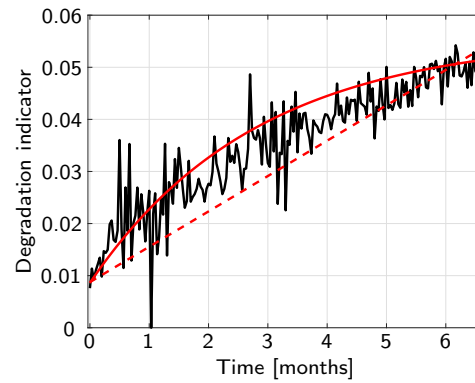
Figure 5.2 shows the three periods that were used for testing the algorithm. They are also noisy and indicate a need for a smoothing algorithm. Moreover, the red lines confirm that neither linear nor exponential approximations are good enough. The trends will be further analysed in Section 5.5.



(a) Degradation indicator in period C1



(b) Degradation indicator in period C2



(c) Degradation indicator in period C3

Figure 5.2: Degradation indicator (—) for the compressor divided in periods a linear trend (---) and an exponential trend (—)

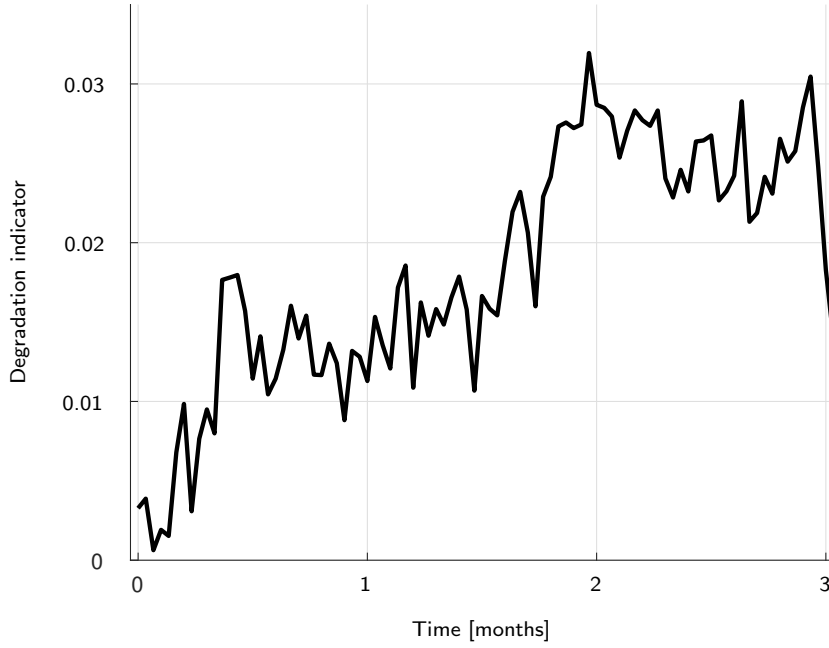


Figure 5.3: Degradation indicator for a turbine from Brekke et al. (2009) (T1)

5.2.2 Degradation of turbine efficiency

A further industrial data set comes from a GE LM2500 engine operating offshore in the Norwegian Sea and was obtained from Brekke et al. (2009) using software developed by Rohatgi (2018). The original data were collected during a period of operation of approximately three months without maintenance events that could mitigate the loss of performance. The samples were collected every minute.

The degradation indicator for the turbine is presented in Fig. 5.3 and cannot be approximated with either linear or exponential trend.

The degradation data presented in Figs. 5.1, 5.2, and 5.3 confirm that there is a need for improved approximation algorithms that would capture the changes in the degradation indicator, at the same time smoothing the noisy variations in the datasets. The fact that the approximations with a single trend are not sufficient have also been demonstrated by Li and Nilkitsaranont (2009) and Tsoutsanis and Meskin (2017), and is further confirmed in Section 5.5. As indicated by Zagorowska et al. (2019), a good candidate for the approximation of the degradation indicator would combine both linear and exponential functions. The algorithm described in this chapter presents a model that switches in a systematic way between these two functional forms.

5.3 Algorithm description

The aim of the algorithm is to estimate the value of the underlying degradation indicator $\hat{d}$ from the calculated degradation indicator d shown as the black lines in Figs. 5.1, 5.2, 5.3. Section 5.2 showed that a single linear or exponential approximation is insufficient for the estimation of $\hat{d}$. This section introduces a moving window algorithm with a model for the underlying degradation that has constant, linear, and exponential terms:

$$\hat{d}(t) = b_0^i + b_1^i t + b_2^i \exp(-b_3^i t) \quad (5.2)$$

where i denotes the current approximation window. It adapts by switching between a linear and exponential approximation as the window moves. The switching is done by adjusting the values of the model parameters b_0^i to b_3^i , as described in Section 5.3.2.1.

The resulting model of degradation $\hat{d}$ belongs to the group factor-free models of degradation, described in Chapter 4.

5.3.1 Moving windows

The algorithm makes use of three moving windows:

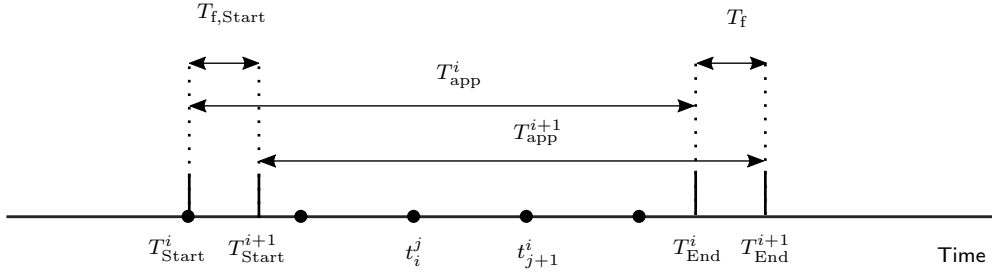


Figure 5.4: Explanation of moving approximation window, adapted from Zagorowska et al. (2019)

- An approximation window in which the measured degradation indicator (black lines in Figs. 5.1-5.3) is approximated with either linear or exponential function. The i -th approximation window is defined by an interval $(T_{\text{Start}}^i, T_{\text{End}}^i) = (T_{\text{Start}}^i, T_{\text{Start}}^i + T_{\text{app}}^i)$;
- A detection window needed for switching the functional form in the approximation window. The i -th detection window is defined by an interval $(\mathcal{T}_{\text{Start}}^i, \mathcal{T}_{\text{End}}^i) = (\mathcal{T}_{\text{Start}}^i, \mathcal{T}_{\text{Start}}^i + \mathcal{T}_{\text{app}}^i)$;
- A prediction window in which the model fitted to data in the approximation window is extrapolated. It is defined by an interval $(T_{\text{End}}^i, T_{\text{End}}^i + T_{\text{pred}})$ with constant T_{pred} .

Online smoothing of the calculated degradation indicator, and its forward prediction, require a moving window approach because of the varying nature of the degradation indicator. Figure 5.4 illustrates the idea of a moving approximation window. The i -th approximation window starts at T_{Start}^i , ends at T_{End}^i and lasts for T_{app}^i . The subsequent approximation window, $i+1$, is described with T_{Start}^{i+1} and T_{End}^{i+1} . The time difference between the ends of two consecutive windows, $T_f = T_{\text{End}}^i - T_{\text{End}}^{i+1}$, is constant, and characterises the update rate of the approximation. The dots on the time axis denote the measurement instants t_j^i in the i -th window.

The approximation window T_{app}^i and the difference between two consecutive starting points for the approximation $T_{f,\text{Start}} = T_{\text{Start}}^{i+1} - T_{\text{Start}}^i \in \{0, T_f\}$ are derived from the adaptation procedure described in subsequent sections.

5.3.2 Fitting of the model to data within an approximation window

The modelling approaches from Ciccioiti (2015), Li and Nilkitsaranont (2009), Puggina and Venturini (2012) and Tsoutsanis and Meskin (2017) have used regression methods to estimate the parameters of the model of the degradation indicator. As indicated by Cavarzere and Venturini (2011) who compared regression with more complex tools, such as Kalman filtering or Bayesian forecasting, regression is well suited to linear and nonlinear modelling and is used in this thesis.

5.3.2.1 Numerical considerations

Identification by regression of all four parameters b_0 to b_3 in Eq. (5.2) can give non-unique estimates. If the underlying degradation indicator does not change much within a window, as seen for instance from 0.5 to 1 months in Fig. 5.3, then the linear term dominates and there are many combinations of b_1 , b_2 , b_3 that would fit such data. In this case, the algorithm must select one combination. This is done by setting b_2 to zero when the underlying degradation indicator does not change significantly within a window. The model is then linear, $\hat{d} = b_0 + b_1 t$, and is fitted to the data in the approximation window using linear regression.

On the other hand, the underlying degradation indicator changes significantly at other times, as seen for instance between 1.5 and 2 months in Fig. 5.3. The literature reviewed in Section 5.1.2 shows that, for underlying physical reasons, an exponential model provides a good fit during such periods. If the underlying degradation indicator changes significantly within a detection window then the parameter b_1 is set to zero and the model is $\hat{d} = b_0 + b_2 \exp(-b_3 t)$. The model is fitted using non-linear regression.

The above arguments suggest that adaptation requires an assessment for each window of whether there has been a significant change in the underlying degradation indicator within the window. This is done with a parameter θ as described in Section 5.4.1.3.

An additional logical step is needed when starting to use the exponential model, because it is necessary to determine the starting point for exponential approximation. This is done by examining whether significant changes in the value of $\hat{d}$ began in the current (i -th) window, or in the previous window. If there was no significant change in the previous window, it means that the degradation indicator only started to increase or decrease significantly in the current window. The middle of the current detection window is taken as a starting point for the exponential approximation, and the exponential model is fitted to the data in the second half of the window. Taking the middle of the detection window as a starting point for the approximation results in shortening of the approximation window from the default value T_{app} to T_{app}^i . This is to ensure that the exponential model is fitted to the nonlinear part from the detection window.

5.3.2.2 Linear regression

It is assumed that the observed value of $d(t_j^i)$ at each time moment $t_j^i \in [T_{\text{Start}}^i, T_{\text{End}}^i]$, $j = 1, \dots, K^i$, with K^i denoting the number of samples in i -th approximation window, is a random variable (Montgomery and Runger, 2014), whose mean value is the true underlying value of the degradation indicator at time t_j^i . The expected value of the degradation indicator d at each t_j^i , $E(d(t_j^i))$, is a smoothed estimate of the true underlying value of the degradation indicator. Such an estimate removes variability that is present when the degradation is calculated from operating data. The expected value is calculated as:

$$E(d(t_j^i)) = b_0^i + b_1^i t_j^i \quad (5.3)$$

where b_0^i, b_1^i are unknown coefficients found using linear regression.

The estimated degradation indicator in i -th approximation window $[T_{\text{Start}}^i, T_{\text{End}}^i]$ is:

$$\hat{d} = b_0^i + b_1^i t \quad (5.4)$$

5.3.2.3 Nonlinear regression

When linear regression is not sufficient, an exponential model is used in i -th approximation window:

$$f(t, b^i) = b_0^i + b_2^i \exp(-b_3^i t) \quad (5.5)$$

The model (5.5) is obtained from Eq. (5.2) by setting $b_1 = 0$. Using nonlinear regression with constraints $b_0^i \in \mathbb{R}$, $b_2^i \in \mathbb{R}$ and $b_3^i > 0$ yields an estimate of the degradation indicator in i -th approximation window $[T_{\text{Start}}^i, T_{\text{End}}^i]$:

$$\hat{d} = f(t, b^i) \quad (5.6)$$

To find the parameters in both linear and non-linear cases, the function `fit` from the Curve Fitting Toolbox in Matlab is used.

5.3.3 Determining the change in degradation

The process for choosing the function form for approximation is conducted in i -th detection window by fitting a linear model of the form $y = mx + c$ to the data within the detection window, where x is time. Although this looks superficially similar to the linear model $\hat{d} = b_0 + b_1 t$ discussed in Section 5.3.2.2, y is not necessarily the same as $\hat{d}$. The quantity $\hat{d}$ is an estimate of the true underlying value of the degradation indicator within the approximation window. If it transpires from the analysis in the detection window that a linear model is appropriate for the given approximation window, then indeed $\hat{d} = y$ throughout the approximation window. However, the quantity y will be generated for all detection windows, even for those where it transpires that an exponential model is appropriate. If an exponential model is appropriate, then y is not the same as $\hat{d}$ throughout the approximation window.

An estimate of the change in $\hat{d}$ from the start to the end of the detection window is given by $\Delta y = m(\mathcal{T}_{\text{End}} - \mathcal{T}_{\text{Start}})$, where m is the gradient of the fitted straight line, and $\mathcal{T}_{\text{End}}^i$ and $\mathcal{T}_{\text{Start}}^i$ are the times at the end and start of the i -th detection window.

5.3.4 Choosing the model structure

The model structure is chosen by comparing Δy in i -th detection window with a value given by:

$$\tilde{\theta} = \frac{\theta \cdot \mathcal{T}_{\text{app}}^i}{T_{\text{app}}} \quad (5.7)$$

where θ is a constant parameter and $\mathcal{T}_{\text{app}}^i = \min\{T_{\text{app}}, \mathcal{T}_{\text{End}}^i - \mathcal{T}_{\text{Start}}^i\}$. The selection of an appropriate value for θ is discussed in Section 5.4.1.3. Equation (5.7) can be rewritten as:

$$\tilde{\theta} = \begin{cases} \theta, & \text{if } T_{\text{app}} \leq \mathcal{T}_{\text{End}}^i - \mathcal{T}_{\text{Start}}^i \\ \frac{\theta \cdot (\mathcal{T}_{\text{End}}^i - \mathcal{T}_{\text{Start}}^i)}{T_{\text{app}}}, & \text{if } T_{\text{app}} > \mathcal{T}_{\text{End}}^i - \mathcal{T}_{\text{Start}}^i \end{cases} \quad (5.8)$$

The first case is valid if the default window is used for detection, whereas the second case is used after shortening of the window, as indicated in Section 5.3.2.1.

If $|\Delta y| \leq \tilde{\theta}$, then the underlying degradation will be determined using linear regression by setting parameter b_2 to zero and fitting the model $\hat{d} = b_0 + b_1 t$ to the data in the approximation window. If $|\Delta y| > \tilde{\theta}$, the underlying degradation will be determined from nonlinear regression by fitting the model $\hat{d} = b_0 + b_2 \exp(-b_3 t)$. The test uses $|\Delta y|$ rather than Δy because the degradation indicator can decrease in some circumstances.

The logical steps in the algorithms are presented in the flow chart in Fig. 5.5. The left path represents fitting the linear model, the middle path represents the first window after detection that $|\Delta y| > \tilde{\theta}$, and the right hand path presents fitting the exponential model in subsequent windows. The three blocks are described in detail in Section 5.3.4.1.

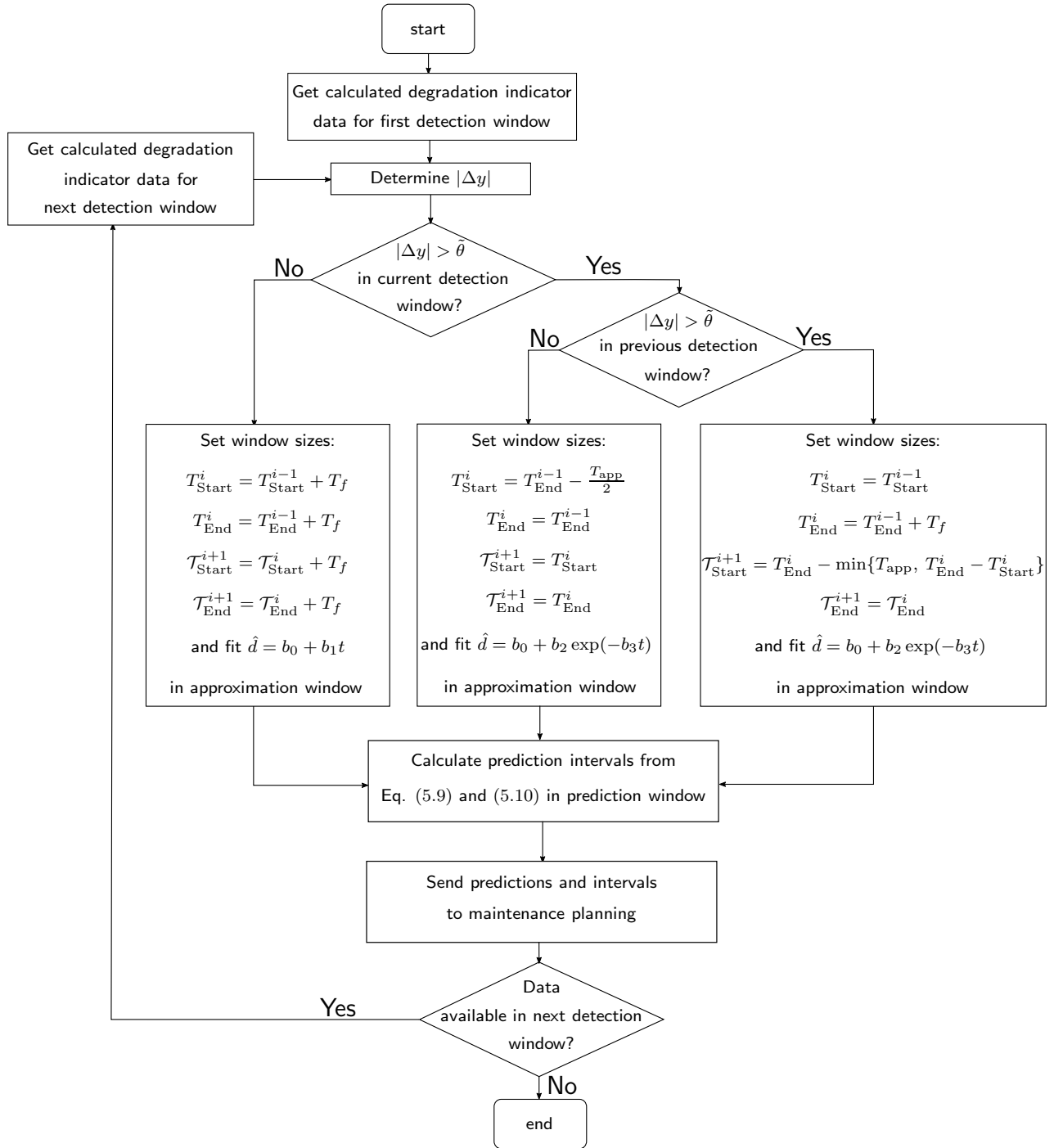


Figure 5.5: Flowchart of the Adaptive Degradation Prediction algorithm (ADP)

5.3.4.1 Variable detection and approximation window sizes

As indicated in Fig. 5.5 there are three possible paths for setting the sizes of the detection and the approximation window. By default, at the beginning the detection and prediction windows are the same with $T_{\text{app}}^i = T_{\text{app}}^{i+1}$ and $\mathcal{T}_{\text{Start}}^i = T_{\text{Start}}^i$, $\mathcal{T}_{\text{End}}^i = T_{\text{End}}^i$. This case represents the left path in Fig. 5.5, so $|\Delta y| \leq \tilde{\theta}$. The i -th approximation window starts at T_{Start}^i , ends at T_{End}^i and lasts for T_{app}^i . The subsequent approximation window, $i+1$, is described with T_{Start}^{i+1} and T_{End}^{i+1} . The time difference between the ends of two consecutive windows, $T_f = T_{\text{End}}^i - T_{\text{End}}^{i+1}$, is constant, and characterises the update rate of the approximation. The dots on the time axis denote the measurement instants t_j^i in i -th approximation and detection windows.

If $|\Delta y| > \tilde{\theta}$, the size of the approximation window and the detection window is derived as follows. The algorithm detects that the exponential model is suitable in i -th detection window, so $T_{\text{Start}}^h = \mathcal{T}_{\text{Start}}^h$ and $T_{\text{End}}^h = \mathcal{T}_{\text{End}}^h$, for $h \leq i$. The i -th approximation window is calculated as $T_{\text{app}}^h = T_{\text{End}}^h - T_{\text{Start}}^h$ and is equal to the detection window $\mathcal{T}_{\text{app}}^h$. Then the $i+1$ approximation window is shortened with $T_{\text{Start}}^{i+1} = T_{\text{Start}}^i$ that will remain fixed as long as the change is detected, and $T_{\text{End}}^{i+1} = T_{\text{End}}^i + T_f$. This corresponds to the middle path in Fig. 5.5.

Next, the algorithm moves to the right hand path in Fig. 5.5. If the shortened window is smaller than the default value T_{app} , the detection window will be the same as the approximation window, because $\min\{T_{\text{app}}, T_{\text{End}}^i - T_{\text{Start}}^i\} = T_{\text{End}}^i - T_{\text{Start}}^i$. The value of T_{Start}^i is then fixed as long as the change is detected $T_{\text{Start}}^{i+j} = T_{\text{Start}}^i$, whereas $T_{\text{End}}^{i+j} = T_{\text{End}}^i + jT_f$, where j is the number of detection windows where a change is detected. The fixed starting point and increasing endpoint of the approximation window indicate that the approximation window will expand as long as $|\Delta y| > \tilde{\theta}$ is detected. The detection window, $\mathcal{T}_{\text{app}}^{i+j}$ is calculated from the formula $\min\{T_{\text{app}}, T_{\text{End}}^{i+j} - T_{\text{Start}}^{i+j}\}$. The detection window is then set as $\mathcal{T}_{\text{End}}^{i+j} = T_{\text{End}}^{i+j}$ and $T_{\text{Start}}^{i+j} = T_{\text{End}}^{i+j} - \min\{T_{\text{app}}, T_{\text{End}}^{i+j} - T_{\text{Start}}^{i+j}\}$. If the change is not detected any more, the approximation window and the detection window become the same again, $T_{\text{Start}}^m = \mathcal{T}_{\text{Start}}^m$ and $T_{\text{End}}^m = \mathcal{T}_{\text{End}}^m$ with $\mathcal{T}_{\text{app}}^m = T_{\text{app}}^m = T_{\text{app}}$.

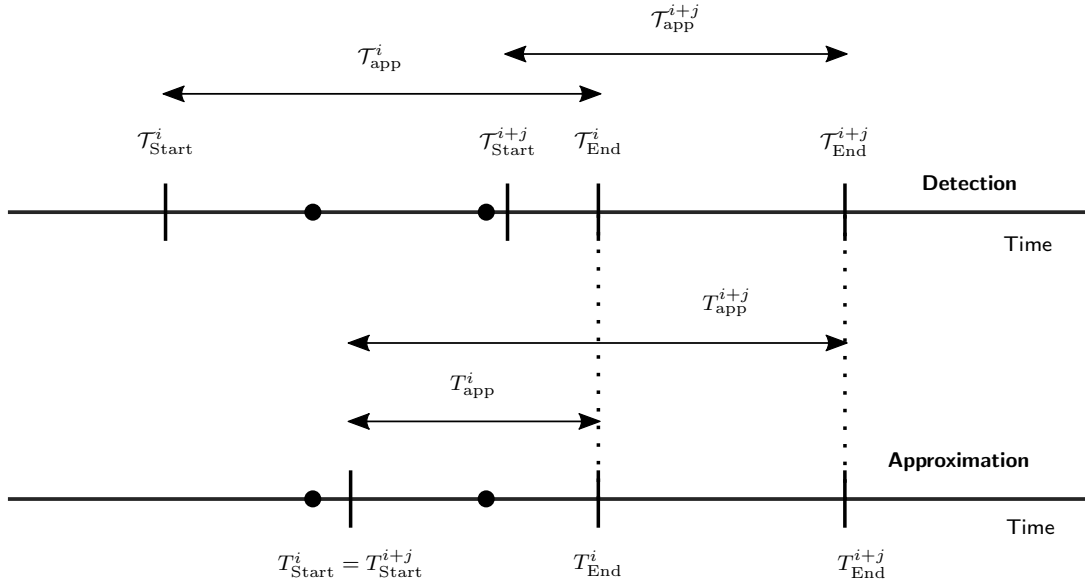


Figure 5.6: Variable detection and prediction windows for adaptation

The variable windows are depicted in Fig. 5.6. The top axis shows the detection window, whereas the bottom one presents the approximation window. The exponential model is chosen in i -th detection window and applied in approximation windows until $i + j$ iteration. The prediction window is omitted for clarity as T_{pred} is constant.

5.3.5 Prediction intervals

Prediction consists of extrapolation of the approximating linear or exponential function into the future, i.e. for $t \in [T_{\text{End}}^i, T_{\text{End}}^i + T_{\text{pred}}]$. The uncertainty of the prediction is captured by prediction intervals assuming that the errors $\varepsilon_i = d - \hat{d}$ are normally distributed (Montgomery and Runger, 2014).

5.3.5.1 Prediction intervals

The prediction intervals at a selected percentile $100(1 - \alpha) \%$ at time $t = t_0$ are given by:

$$\hat{d}(t_0) - \Delta \leq d(t_0) \leq \hat{d}(t_0) + \Delta \quad (5.9)$$

with:

$$\Delta = t_{\alpha/2, K^{i-2}} \sqrt{\hat{\sigma}^2 (1 + \hat{a}_0^\top (\mathbf{A}^\top \mathbf{A})^{-1} \hat{a}_0)} \quad (5.10)$$

Table 5.1: Parameters for calculating the prediction intervals for linear and exponential models

Linear model	Exponential model
$\hat{a}_0 = \begin{bmatrix} \frac{\partial f(t_0, b^i)}{\partial b_0^i} & \frac{\partial f(t_0, b^i)}{\partial b_1^i} \end{bmatrix}^T$ $= [1 \quad t_0]^T$	$\hat{a}_0 = \begin{bmatrix} \frac{\partial f(t_0, b^i)}{\partial b_0^i} & \frac{\partial f(t_0, b^i)}{\partial b_2^i} & \frac{\partial f(t_0, b^i)}{\partial b_3^i} \end{bmatrix}^T$ $= [1 \quad \exp(-b_3 t_0) \quad -t_0 b_2 \exp(-b_3 t_0)]^T$
$\mathbf{A} = \begin{bmatrix} \frac{\partial f(t_p, b^i)}{\partial b_0^i} & \frac{\partial f(t_p, b^i)}{\partial b_1^i} \end{bmatrix}_{p=1, \dots, K^i}$ $= \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_{K^i} \end{bmatrix}$	$\mathbf{A} = \begin{bmatrix} \frac{\partial f(t_p, b^i)}{\partial b_0^i} & \frac{\partial f(t_p, b^i)}{\partial b_2^i} & \frac{\partial f(t_p, b^i)}{\partial b_3^i} \end{bmatrix}_{p=1, \dots, K^i}$ $= \begin{bmatrix} 1 & \exp(-b_3 t_1) & -t_1 b_2 \exp(-b_3 t_1) \\ 1 & \exp(-b_3 t_2) & -t_2 b_2 \exp(-b_3 t_2) \\ \vdots & \vdots & \vdots \\ 1 & \exp(-b_3 t_{K^i}) & -t_{K^i} b_2 \exp(-b_3 t_{K^i}) \end{bmatrix}$

where $t_{\alpha/2, K^i-2}$ is the $100(1-\alpha)$ th percentile of Student's t-distribution with K^i-2 degrees of freedom and where $\hat{\sigma}^2$ is the estimated variance of d . The $\hat{a}_0$ is a vector of derivatives of $f(t, b^i)$ calculated at t_0 and $\mathbf{A}$ is a matrix of derivatives of $f(t, b^i)$ calculated at times $t_p \in [T_{\text{Start}}^i, T_{\text{End}}^i + T_{\text{pred}}]$ with $f(t, b^i)$ given by the appropriate form of the right hand side of Eq. (5.2). The formulae for $\hat{a}_0$ and $\mathbf{A}$ for the linear and exponential cases are given in Table 5.1. The numerical values for Δ were obtained in Matlab using the function `predint`.

The prediction intervals given by Eq. (5.9) show that $100(1-\alpha)\%$ of the data points in the prediction window will fall in the prediction interval. This thesis assumes $\alpha = 5$ which means that 95% of the data points will be within the prediction interval.

Figure 5.7 illustrates the prediction intervals in the online moving window algorithm for four windows, R and P1, P2, P3. Window R is enlarged in the bottom right and presents the approximation (thick solid red curve) of the degradation indicator (black) in the current window. The red dotted line shows the expected value $\hat{d}$ in the prediction window defined by T_{pred} (light green shading). The thin blue line shows the prediction intervals for data from the approximation window $[T_{\text{Start}}^i, T_{\text{End}}^i]$. The dotted lines show the prediction intervals for $t \in [T_{\text{End}}^i, T_{\text{End}}^i + T_{\text{pred}}]$.

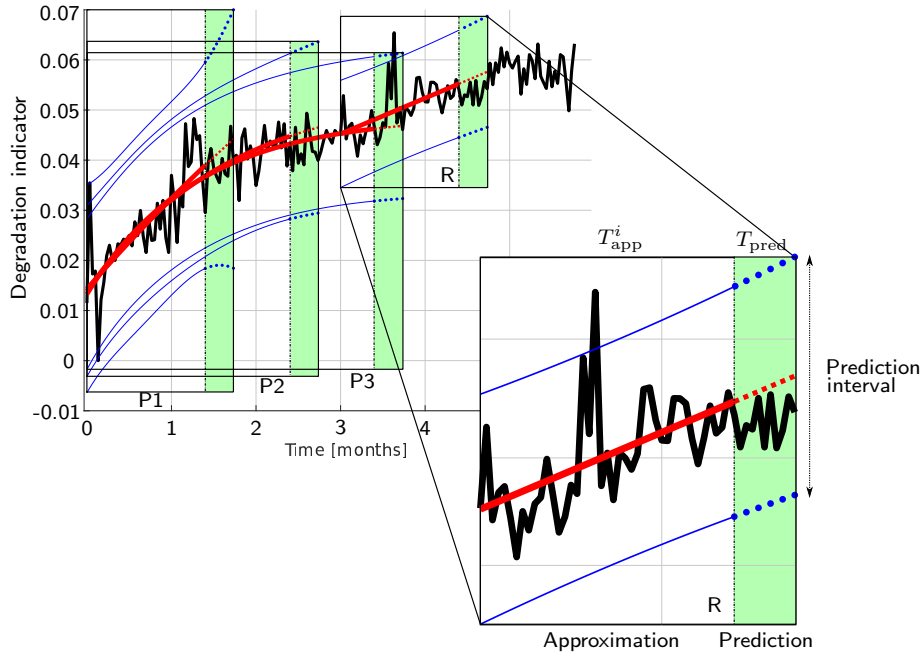


Figure 5.7: An online application of the algorithm ((—) for approximation and (—) for prediction bounds) on the tuning data set (—), with the structure of a time window in window R, and presenting the expanding approximation windows in time windows P1, P2, and P3

Windows P1, P2, and P3 present how the prediction intervals depend on the size of the approximation window. The shortest approximation window P1 has the largest prediction intervals. As indicated by Montgomery and Runger (2014), the prediction interval is a decreasing function of the number of samples K^i . Therefore, increasing number of samples K^i in i -th approximation window, for example by expanding the duration of the time window P1 to P2 and P3, results in narrowing of the prediction intervals. The same effect can be obtained by increasing the number of samples for fixed window size.

5.4 Industrial case study

The proposed adaptive algorithm was applied to real data sets coming from off-shore applications as described in Section 5.2. For the purpose of this chapter, the data from Fig. 1.4 were divided in two sets:

- Tuning data (CT) were used for tuning of the algorithm;

- Test data were used for comparison with other approaches (C1, C2 and C3 in Fig. 1.4).

The objective of testing the algorithm on data sets coming from the same compressor that was used for tuning is to evaluate the adaptation method and the quality of predictions. Furthermore, the algorithm is tested on the turbine data set T1 to evaluate whether the settings of the algorithm are portable.

5.4.1 Tuning

To apply the algorithm from Section 5.3 to industrial data, it is necessary to define the values of its settings: the default approximation window T_{app} , the prediction window T_{pred} , and the desired threshold θ . The values of the settings can be found by analysing historical datasets. The algorithm was tuned using the tuning data set CT. Then the same settings were applied to test data C1, C2, C3 and T1 to analyse the accuracy of prediction and the influence of the tuned parameters on the performance.

Moreover, the value of T_f was set to one day, as the new values of the degradation indicator were available once per day.

5.4.1.1 Approximation window

As indicated by Tarabrin et al. (1996), compressor degradation due to fouling typically increases over 1000-2000 hours of operation (42-84 days) and then stabilizes. Thus, all variations shorter than 42 days in the calculated degradation indicator (the black lines in Fig. 1.4) are considered to be due to disturbances. This suggests that the default window size should be in the range 42-84 days of operation to mitigate the influence of the disturbances. The approximation window for the compressor was chosen to be 42 days, $T_{\text{app}} = 42$, to capture the fastest degradation, while being longer than variations due to disturbances.

Syverud (2007) indicated that the duration of disturbances would depend on the type of the turbomachinery. Thus, a procedure for adapting the approximation window to a different piece of equipment is shown in Section 5.4.2.2.

5.4.1.2 Prediction window

The prediction window will have an impact on the accuracy of the prediction and the prediction interval. Two cases are considered:

- $T_{\text{pred}} = 10$ days - short-term prediction window,
- $T_{\text{pred}} = 30$ days - medium-term prediction window.

The choice of the prediction window depends on the application. In this thesis, short-term predictions are applicable to scheduling of online washings, whereas medium-term predictions relate to offline washings (Schulze Spüntrup et al., 2019).

Both prediction windows were explored in order to examine their influence on the algorithm and to give a recommendation that would be portable to other applications.

They were chosen to make sure that the approximation window and the prediction window together fit in 2000 hours, as this is the maximal expected duration of a single degradation period, from the start to stabilisation:

$$T_{\text{app}} + T_{\text{pred}} = 42 + 10 = 52 \text{ days} < 84 \text{ days} \quad (5.11)$$

for the short prediction $T_{\text{pred}} = 10$ days and:

$$T_{\text{app}} + T_{\text{pred}} = 42 + 30 = 72 \text{ days} < 84 \text{ days} \quad (5.12)$$

for medium-term prediction with $T_{\text{pred}} = 30$ days.

5.4.1.3 Detection threshold

The next step is to find θ , i.e. to define how large a change is considered significant. The procedure to find θ is based on historical data and has two steps:

- Finding the minimal and maximal value of $\frac{\Delta y}{T_{\text{app}}}$ over the whole range of the tuning data set to find the range for θ ,
- Running an optimization procedure that finds a value of θ such that the prediction error is minimised.

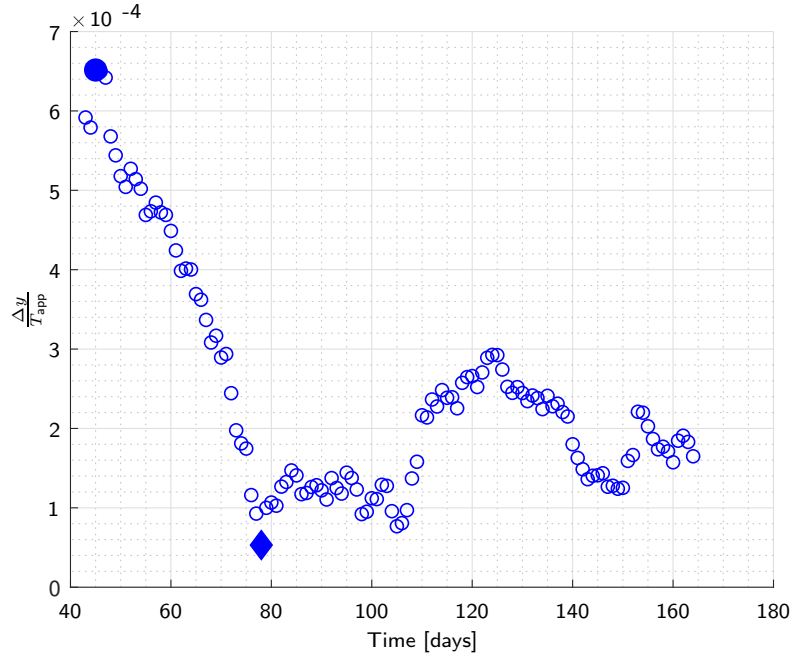


Figure 5.8: Minimal (◆) and maximal (●) value of $\frac{\Delta y}{T_{\text{app}}}$ for fixed $\mathcal{T}_{\text{app}}^i$ and T_{pred}

Figure 5.8 shows the values of $\frac{\Delta y}{T_{\text{app}}}$ over the whole period, for fixed $\mathcal{T}_{\text{app}}^i$ and T_{pred} . As $\mathcal{T}_{\text{app}}^i = T_{\text{app}}$ for all i , $\tilde{\theta} = \theta$ from Eq. (5.8). The minimal value of $\frac{\Delta y}{T_{\text{app}}}$ was denoted with a diamond, $\frac{\Delta y}{T_{\text{app}}} = 0.5 \times 10^{-4}$, and the maximal value with a filled circle, $\frac{\Delta y}{T_{\text{app}}} = 6.5 \times 10^{-4}$. This is equivalent to a change of between 0.02% and 2.7% of degradation indicator d during the approximation window T_{app} . As T_{app} was constant, $\Delta y = \theta$ and the value of θ is therefore sought in range $[0.002, 0.027]$.

To find θ , an indicator measuring the mean difference $\overline{P}$ between the value calculated from the measurements, d_k^i , and the expected degradation $\hat{d}(t_k^i)$ in the i -th prediction window is used:

$$\overline{P} = \frac{\sum_{i=1}^N \left(\sum_{k=1}^{K^i} (d(t_k^i) - \hat{d}(t_k^i))^2 \right)}{N} \quad (5.13)$$

where $t_k \in [T_{\text{End}}^i, T_{\text{End}}^i + T_{\text{pred}}]$ denotes the i -th prediction window with $k = 1, \dots, K^i$ and $i = 1, \dots, N$ with N denoting the number of windows for the whole tuning data set.

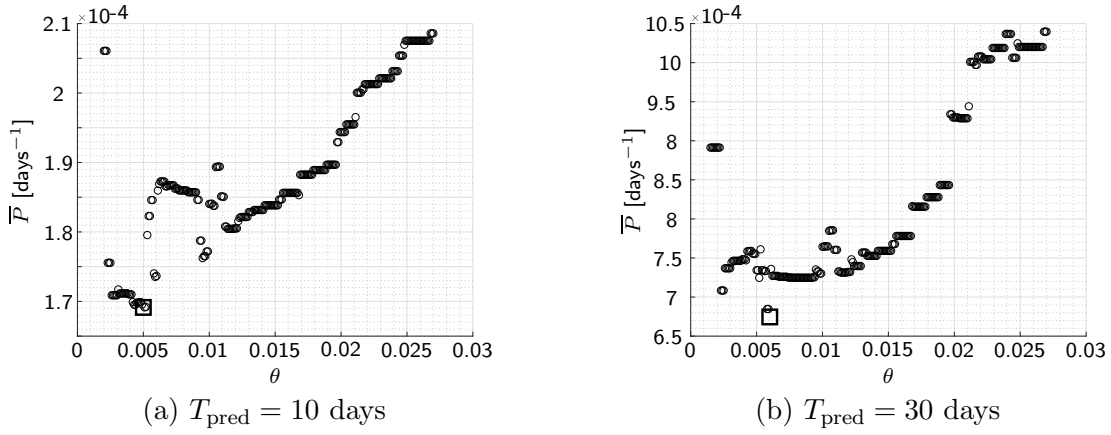


Figure 5.9: The indicator $\bar{P}$ for $\theta \in [0.002, 0.027]$ with minimal value ($\square$)

To show the influence of θ on prediction accuracy measured by the indicator (5.13), an exhaustive search was performed for $\theta \in [0.002, 0.027]$. As indicated with a square in Fig. 5.9, the smallest value of $\bar{P}$ was obtained for $\theta = 0.005$ for $T_{\text{pred}} = 10$ days, and $\theta = 0.006$ for $T_{\text{pred}} = 30$ days. These values were also confirmed with Global Optimisation Toolbox in Matlab.

Figure 5.9 also shows that overly small and overly large values of θ would result in inaccurate predictions. If θ is too small, for example if $\theta < 0.003$ for the tuning data CT $|\Delta y| > \tilde{\theta}$, where $\tilde{\theta}$ is calculated from Eq. (5.8), is detected more often. This forces the algorithm to use exponential approximation for longer periods which results in overestimating the value of the degradation indicator as indicated in Fig. 5.2. At the same time, if θ is too large, $\theta > 0.011$, a linear regression would be used for prediction, as the $|\Delta y| \leq \tilde{\theta}$. As presented in Fig. 5.2, linear approximation underestimates the degradation indicator.

5.4.1.4 Tuning results

The default T_{app} , θ , and T_{pred} , were chosen based on the knowledge about past degradation data. Following the discussions in Sections 5.4.1.1, 5.4.1.2 and 5.4.1.3, the default values were set to: $T_{\text{app}} = 42$, $\theta = 0.005$ for $T_{\text{pred}} = 10$ days (short term prediction), and $\theta = 0.006$ for $T_{\text{pred}} = 30$ days (medium term prediction). The objective of this section is to illustrate the results of the algorithm applied to period CT, before testing it on the data sets from C1, C2, C3, and T1.

The results of approximation and prediction on the data set CT are depicted in Fig. 5.10. Figures 5.10a and 5.10b show the degradation indicator d (black) with the approximating functions $\hat{d}$ (red) for the two prediction periods, $T_{\text{pred}} = 10$ days and $T_{\text{pred}} = 30$ days. In both cases, the red approximation $\hat{d}$ follows the underlying trend of the degradation indicator. However, the influence of the prediction window T_{pred} is noticeable in the third month, where the red curves are above the black indicator, i.e. the prediction diverges from the value calculated from the measurements if the prediction window is longer. However, when the approximation adapts to the significant changes of the degradation indicator, the predictions become accurate.

The moments when the algorithm switches from fitting of a linear model to fitting of an exponential model are visible in Figures 5.10c and 5.10d. Red filled circles show that $\tilde{\theta}$ was exceeded (black horizontal lines), i.e. where $|\Delta y| > \tilde{\theta}$. The first red circle means that a check is performed on the value of Δy in the preceding window. As it is the first one, the algorithm chooses the middle path in Fig. 5.5, and uses exponential approximation in the second half of the current window. The middle point of the window is then fixed, whereas T_{End} increases as new data arrive. As long as the change was detected, the algorithm approximated the degradation indicator with an exponential function over an expanding approximation window (right hand path in Fig. 5.5). When a change was no longer detected, $|\Delta y| \leq \tilde{\theta}$ (left path in Fig. 5.5), the algorithm switched to linear approximation (grey circles). Subsequently, the degradation indicator increased again, and the algorithm detected that $|\Delta y| > \tilde{\theta}$ and switched back to exponential approximation (month four for both $T_{\text{pred}} = 10$ days and $T_{\text{pred}} = 30$ days).

Table 5.2: Algorithm settings for the case studies

	Compressor (CT, C1, C2, C3)	Turbine (T1)
Medium-term prediction window	$T_{\text{pred}} = 30$ days, $\theta = 0.006$	$T_{\text{pred}} = 7$ days, $\theta = 0.002$
Short-term prediction window	$T_{\text{pred}} = 10$ days, $\theta = 0.005$	$T_{\text{pred}} = 3.5$ days, $\theta = 0.0017$
Default approximation window	$T_{\text{app}} = 42$ days	$T_{\text{app}} = 14$ days

5.4.1.5 Prediction intervals for tuning data set

As long as the algorithm is detecting $|\Delta y| > \tilde{\theta}$, the approximation window is extended from the value $\frac{T_{\text{app}}}{2}$. Therefore, the uncertainty of prediction is reduced because more data points are available for approximation. The reduction of the uncertainty is also visible in the prediction intervals, as both for $T_{\text{pred}} = 10$ days and $T_{\text{pred}} = 30$ days, the prediction intervals are wide at the beginning, but then they narrow down and follow the degradation indicator more closely. Figure 5.10a shows the prediction intervals for the whole data set and indicates resetting of the prediction intervals when $|\Delta y| > \tilde{\theta}$ was detected, as the prediction intervals widen in month four.

The longer approximation window resulted in detecting longer periods when $|\Delta y| \leq \tilde{\theta}$ (there are more grey circles between the threshold lines for longer T_{pred} in Fig. 5.10d than in Fig. 5.10c). A period with $|\Delta y| \leq \tilde{\theta}$ means that there are fewer switches between linear and exponential approximation. This tendency for less frequent changes had an impact on the prediction intervals. As the detection of $|\Delta y| > \tilde{\theta}$ is related to shortening of the approximation window, frequent changes result in frequent shortenings, and, in consequence, fewer datapoints are used for approximation. Hence shorter approximation windows yield more uncertainty and wider prediction intervals. Therefore, the more frequently $|\Delta y| > \tilde{\theta}$ is detected, the more uncertain the prediction.

5.4.2 Testing

The values of the parameters used for testing come from the tuning procedure, described in Section 5.4.1, and gathered in Table 5.2.

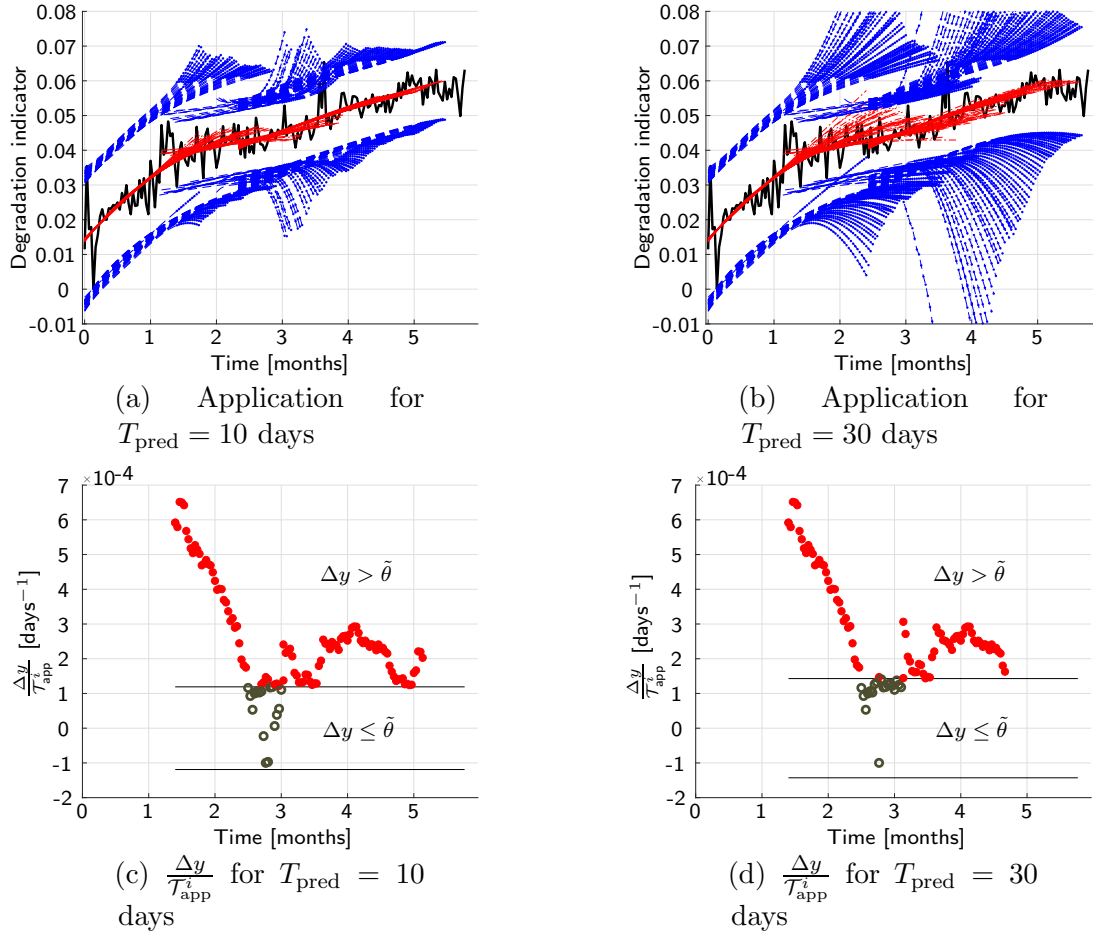


Figure 5.10: Application of the algorithm ((—) for approximation and (—) for prediction bounds) to the tuning data set CT (—) for $T_{\text{pred}} = 10$ days in Fig. 5.10a and $T_{\text{pred}} = 30$ days in Fig. 5.10b, together with respective $\frac{\Delta y}{T_{\text{app}}}$ (Fig. 5.10c and Fig. 5.10d)

5.4.2.1 Testing on compressor data sets

The results of application to the data sets C1, C2, and C3 are depicted in Fig. 5.11. The left column shows the approximation for $T_{\text{pred}} = 10$ days, and the right side shows the approximation for $T_{\text{pred}} = 30$. In both cases, the red curves follow the trend of the black degradation indicator. Nonetheless, Fig. 5.11 shows that medium term predictions are less accurate than short-term prediction. In particular, the inaccuracies are visible for C3 in the third month (Fig. 5.11f), where the algorithm underestimates future degradation values in the medium-term window compared to the shorter prediction window in Fig. 5.11f. This is due to the assumption that the underlying degradation indicator can be approximated by extrapolation of the same function for $t \in [T_{\text{End}}^i, T_{\text{End}}^i + T_{\text{pred}}]$.

The medium-term prediction window resulted also in more uncertainty in the prediction, characterized by the blue prediction intervals. The prediction intervals for $T_{\text{pred}} = 30$ days are approximately twice as wide as the prediction intervals for the shorter window. In particular after detection of $|\Delta y| > \tilde{\theta}$ resulting in a shortening of the approximation window, the prediction intervals increases for medium term window.

The results confirm that tuning of the algorithm enables application for multiple periods of degradation for the same compressor. The settings of the algorithm can be applied for real time approximation and prediction of the degradation indicator after tuning on a historical dataset.

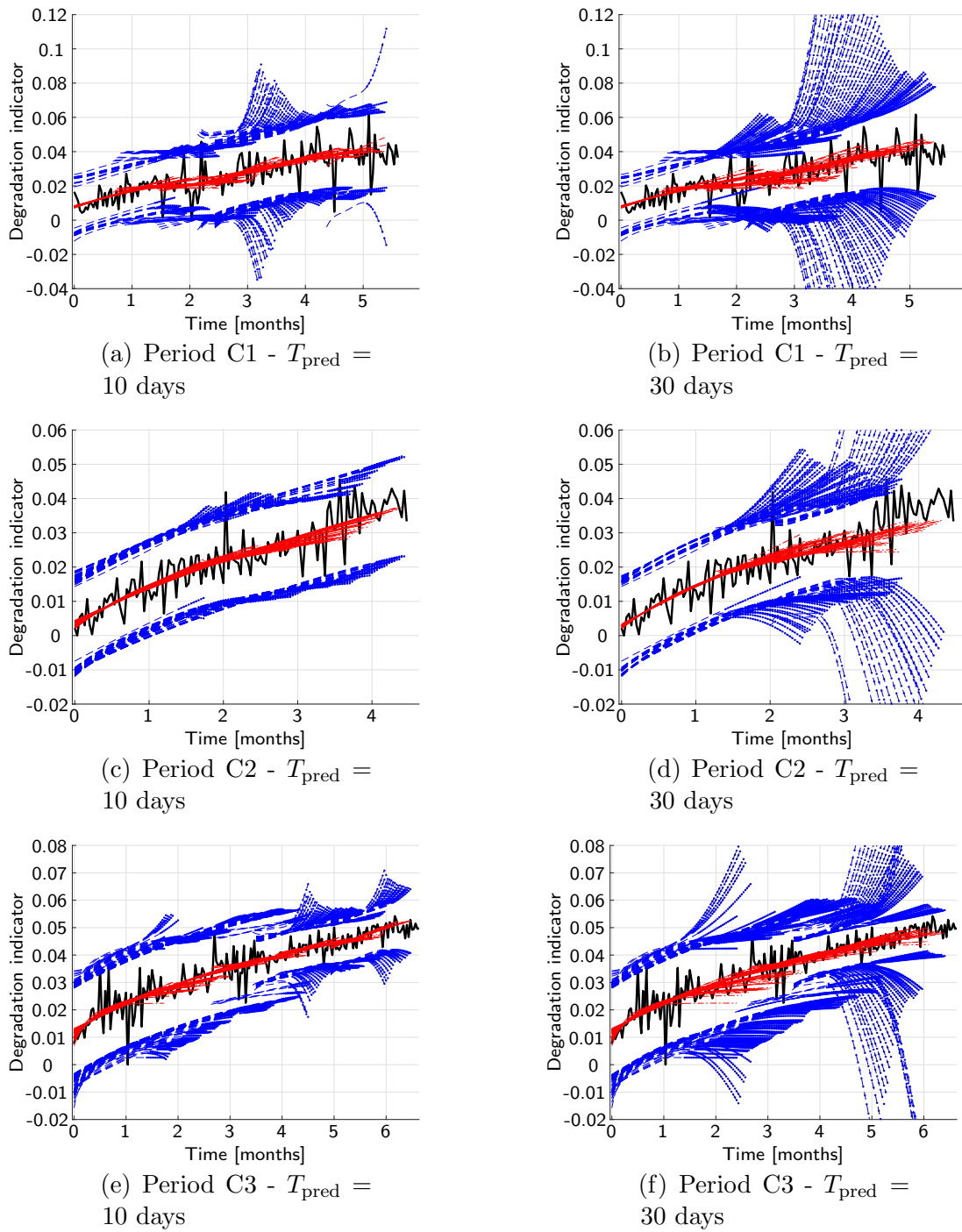


Figure 5.11: Results ((—) for approximation and (---) for prediction bounds) for test data sets (—), C1, C2, C3, for the offshore compressor

5.4.2.2 Testing for turbine data set

To further evaluate the performance of the algorithm and check the portability of the settings, the algorithm was applied to the data set T1 from an offshore turbine. Contrary to the previous data sets, the data set T1 is shorter (three months). The stabilisation periods are also of comparable duration to period with increasing degradation indicator (two weeks). For instance, in the first month in Fig. 5.12, the black degradation indicator stabilizes around 0.012 after two weeks and remains around this value for another two weeks. To follow the underlying degradation indicator, the default approximation window was fixed to two weeks, $T_{\text{app}} = 14$ days. It is also assumed that the degradation indicator d in a turbine is similar to the degradation indicator in a compressor. Therefore, the value of θ for a turbine has the same interpretation as discussed in Section 5.4.1, but with a shorter T_{app} . By proportions:

$$\theta_{\text{turb}} = \frac{\theta_{\text{comp}} T_{\text{app,turb}}}{T_{\text{app,comp}}} \quad (5.14)$$

where $T_{\text{app,turb}}$ denotes the default approximation window for the turbine, $T_{\text{app,comp}}$ denotes the default approximation window for the compressor.

The prediction windows of 3.5 and 7 days were chosen to fulfill $14 + 3.5 < 21$ days and $14 + 7 < 21$ days which was considered the length of a period when $\Delta y > \tilde{\theta}$ for the turbine dataset by Zagorowska et al. (2019). The corresponding values of θ are $\theta = 0.0017$ for 3.5 days and $\theta = 0.002$ for 7 days.

The results of the application of the algorithm to the data set T1 are depicted in Fig. 5.12. Again, the red curves follow the underlying degradation indicator $\hat{d}$ for both short and medium term prediction. However, due to more pronounced transitions between fitting of an exponential model and fitting of a linear model, the inaccuracies resulting from the longer prediction window are more visible (Fig. 5.12b). The algorithm overestimates the degradation in weeks three and 10, i.e. when the underlying degradation indicator stabilizes. When the degradation indicator starts to increase, the predictions underestimate the underlying value, which is visible in week eight. The inaccuracies suggest that the algorithm should be primarily applied for short-term prediction. However, the prediction intervals (dashed blue lines) capture the uncertainty related to medium term prediction and can be used for decision support in planning.

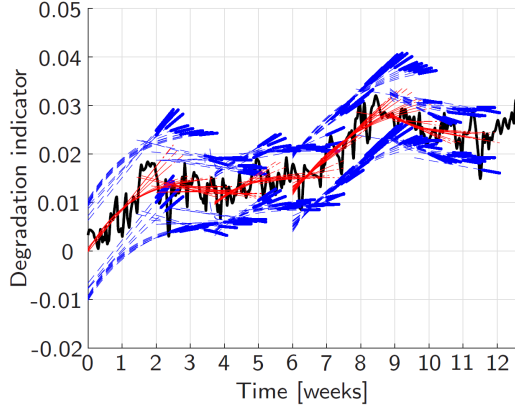
The T1 data set also shows how the algorithm successfully adapts to decreasing values of the underlying degradation indicator. In weeks 3-5 and 10-12, a decrease in the indicator was detected, and the algorithm switched from a linear to a decreasing exponential function. These transitions for both prediction windows are depicted in Fig. 5.12c and 5.12d. The red filled circles denoting $\Delta y > \tilde{\theta}$ are below the threshold, so the linear approximation used for detection is decreasing and $\Delta y < -\tilde{\theta}$.

Finally, the dashed blue lines in Fig. 5.12a and 5.12b show the influence of the number of samples on the prediction intervals. The prediction intervals are narrower for the turbine data set than for the compressor data set, despite shorter approximation windows. The narrow intervals were obtained thanks to frequent sampling, i.e. for the turbine data set the degradation data is available every minute, whereas the compressor had one sample per day.

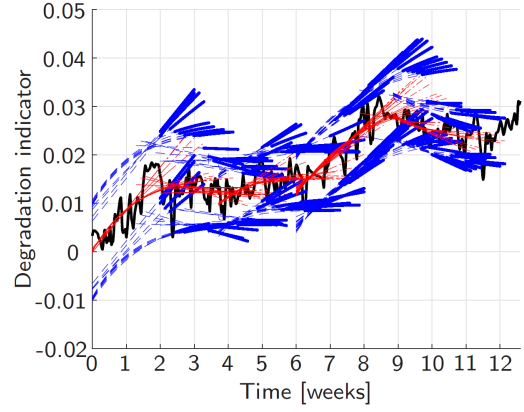
5.5 Comparison with prediction using other approaches

The algorithm was compared with four other approaches:

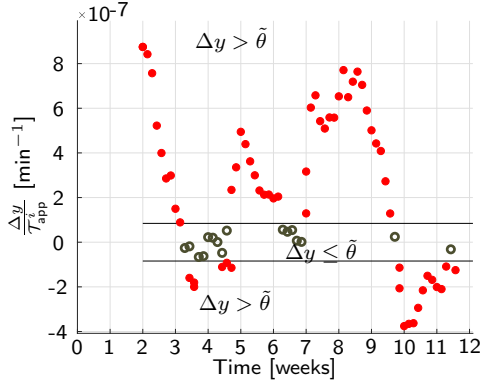
- Mean value over constant approximation window (Mean), 42 days for the compressor and 14 for the turbine,



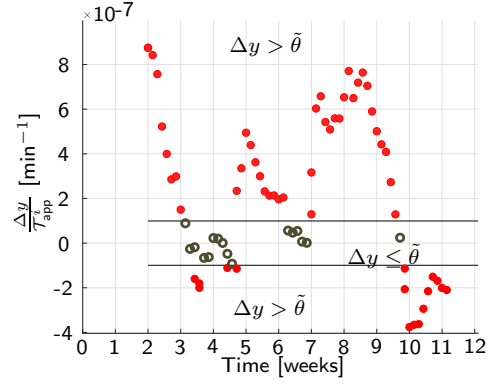
(a) Degradation indicator with approximating lines for $T_{\text{pred}} = 3.5$ days



(b) Degradation indicator with approximating lines for $T_{\text{pred}} = 7$ days



(c) $\frac{\Delta y}{T_i^{\text{app}}}$ for $T_{\text{pred}} = 3.5$ days



(d) $\frac{\Delta y}{T_i^{\text{app}}}$ for $T_{\text{pred}} = 7$ days

Figure 5.12: Application to the turbine data set T1 (—) from Brekke et al. (2009). The results of approximation are in Fig. 5.12a and Fig. 5.12b ((—) for approximation and (—) for prediction bounds) and the corresponding values of $\frac{\Delta y}{T_i^{\text{app}}}$ in Fig. 5.12c and 5.12d

- Linear regression over constant approximation window (LinearC), 42 days for the compressor and 14 for the turbine,
- Linear regression over varying approximation window as the algorithm (adaptation of the algorithm proposed by Tsoutsanis et al. (2016)) (Linear),
- Exponential approximation with fixed starting point and expanding approximation window (Cicciotti, 2015) (Fixed Exponential).

5.5.1 Performance metric for comparison

The performance of the algorithms was measured by a performance metric given by Eq. (5.15) as a sum over all the windows:

$$I = \sum_{i=1} \frac{P^i}{K^i} \quad (5.15)$$

where:

$$P^i = \sum_{k=1}^{K^i} (d(t_k^i) - \hat{d}(t_k^i))^2 \quad (5.16)$$

where i denotes the number of windows for each data set, K^i is the number of samples in i -th prediction window, $k = 1, \dots, K^i$ and $\hat{d}(t_k^i)$ denotes the estimated value of the degradation indicator at time t_k^i . The metric from Eq. (5.15) measures the error between the expected degradation indicator $\hat{d}$ and the measured values d . Thus, the smaller the value of I , the better the prediction. The metric from Eq. (5.15) is equivalent to integrating the mean value of a sum of squares of the differences in each prediction window $[T_{\text{End}}^i, T_{\text{End}}^i + T_{\text{pred}}]$ over the whole data set. Taking the mean value mitigates the influence of short spikes in the data set.

5.5.2 Compressor comparison

The performance of the algorithm was assessed against other approaches over the two prediction windows, 10 days and 30 days. The results of the comparison for the tuning data set from Fig. 5.1 are depicted in Fig. 5.13 which shows the value of P^i as a function of the window number i for both prediction windows $T_{\text{pred}} = 10$ days (Fig. 5.13a) and $T_{\text{pred}} = 30$ days (Fig. 5.13b). The red curve with dots depicts the performance of Adaptive Degradation Prediction, ADP, the light blue with crosses curve shows the results of the linear approximation over the same window as the algorithm, Linear, dotted magenta presents the linear approximation over the fixed window $T_{\text{app}} = 42$ days, LinearC, dark blue with circles curve shows exponential approximation with expanding approximation window, Fixed Exponential, and dashed green shows the results of averaging over constant $T_{\text{app}} = 42$ days, Mean.

The red curve with dots shows that the ADP algorithm presented in this thesis with moving window and switching ability gives the best results. This result is also confirmed in the first and sixth rows in Table 5.3. For both prediction windows, $T_{\text{pred}} = 10$ and $T_{\text{pred}} = 30$, the ADP in the first part ($i < 50$) behaves like the Fixed Exponential approximation (dark blue with circles) and is better than both linear approximations (light blue with crosses and dotted magenta). In particular for medium-term prediction window, i.e. longer T_{pred} , the linear approximations were inaccurate at the beginning. This is due to the fact that the linear approximations do not follow the underlying degradation indicator and assumes that it can be approximated by extrapolation of a linear function. As the linear approximations at the beginning use the data from the first 1.5 months from Fig. 5.1, they overestimate the degradation indicator. In the second part ($i > 50$), the ADP was able to follow the underlying degradation indicator, whereas the Fixed Exponential approximation was no longer good enough. This is because Fixed Exponential approximation extrapolates the approximation from the first three months into the future, whereas Fig. 5.1 shows that the underlying degradation indicator started to increase in month four.

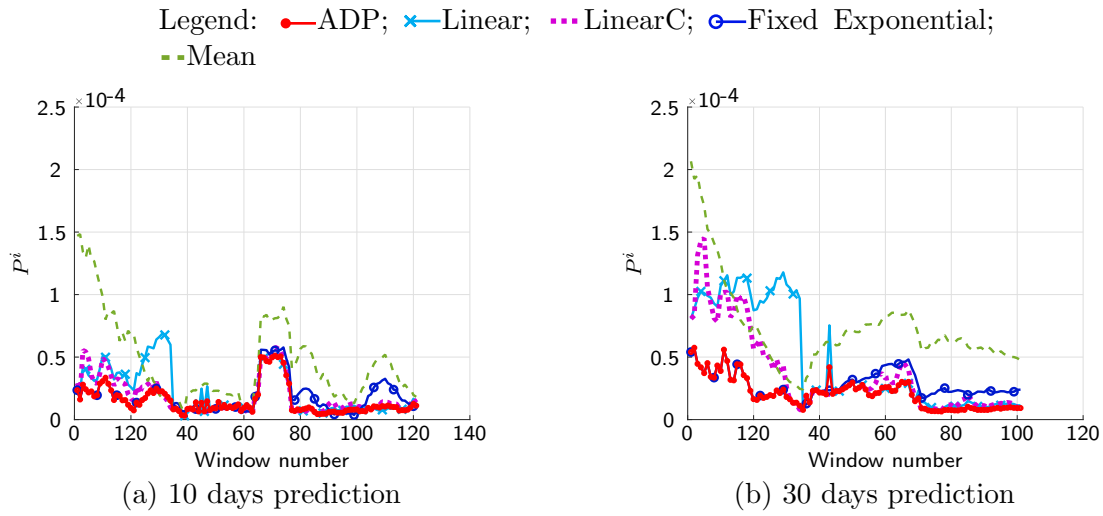


Figure 5.13: Comparison with other approaches for the tuning data set CT

Table 5.3 compares the three data sets C1, C2, and C3 (rows 2-4 and 7-9) using the metric I from Eq. (5.15). The values show the difference in percentage from the best obtained value for each data set, thus zero means that the method performed the best. For short term prediction linear approximations are enough, but their performance decreases for medium-term prediction. This is due to the fact that the linear approximation does not follow the underlying degradation indicator over the whole data set, as indicated in Fig. 5.2, and is not able to predict correctly. In all cases, the worst results were obtained for mean value approximation which predicts the degradation indicator to be constant.

5.5.3 Turbine comparison

Figure 5.14 shows the value of P^i as a function of the number of window i for both prediction windows $T_{\text{pred}} = 3.5$ days (Fig. 5.14a) and $T_{\text{pred}} = 7$ days (Fig. 5.14b) for the turbine data for five algorithms: ADP (red), Linear (light blue with crosses), LinearC (dotted magenta), Fixed Exponential (dark blue with circles) and Mean (dashed green).

Table 5.3: Value of performance indicator I for five algorithms, for two prediction windows. The values show the difference in percentage from the best obtained value for each data set: CT, C1, C2, C3 for the compressor, T1 for the turbine

	data set and prediction window	ADP	Linear	LinearC	Fixed Exponential	Mean
1.	CT - short	0	50	28	22	211
2.	C1 - short	2	6	2	0	27
3.	C2 - short	4	0	0	4	143
4.	C3 - short	5	5	0	5	141
5.	T1 - short	0	30	10	70	30
6.	CT - medium	0	124	81	38	252
7.	C1 - medium	0	8	4	1	55
8.	C2 - medium	0	25	15	15	205
9.	C3 - medium	18	5	0	14	241
10.	T1 - medium	0	38	23	54	23

All the algorithms give a high error at the beginning (window number smaller than 5). This is a result of choosing the default $T_{\text{app}} = \mathcal{T}_{\text{app}} = 2$ weeks. It can be seen in Fig. 5.12 that the underlying degradation indicator is increasing in this period. Thus, extrapolating the approximating model into the future overestimates the value of the underlying degradation indicator, which is confirmed in Figs. 5.12a and 5.12b where the red curves are above the black curve in week three. The best results in this period are obtained from the approximation with the Mean value (dashed green) because it averages the expected degradation indicator over two weeks and does not overestimate future values. Nonetheless, ADP was able to adapt to the underlying degradation indicator, and as a result gave the best predictions from window six.

Furthermore, all the approaches based on moving window framework, i.e. ADP (red), Linear (light blue with crosses), and LinearC (dotted magenta) yield smaller values of P^i in windows 40-60, compared with Fixed Exponential (dark blue with circles) and Mean (dashed green). This is because the Mean algorithm estimates the future degradation indicator based data until week seven, while it starts to increase in week eight. Therefore, the Mean algorithm underestimates the future degradation indicator. The Fixed Exponential algorithm tries to fit one exponential model to the whole data set available to that point. Therefore, it is not able to take into account the increasing period in week eight. Using a moving window approach results in removing older data from the approximation window, and in consequence, the ADP, Linear, and LinearC algorithms are able to better follow the underlying degradation indicator.

Contrary to the results for the tuning data set, depicted in Fig. 5.13, for the data set from Fig. 5.3, the linear approximation over a constant window, LinearC (pink dotted line), has a similar performance as the ADP (red line). This is due to the fact that the prediction window for the turbine data set is shorter than for the compressor. Therefore, it is more likely that the extrapolated values would follow the underlying degradation indicator more closely over a shorter time period. This is also confirmed by comparing the performance of LinearC method in Fig. 5.14a and 5.14b. In both cases, the ADP (red) and the LinearC (magenta) are close, but for $T_{\text{pred}} = 7$ days, the value of P^i for LinearC is higher than the value for ADP, which means that linear approximation is well suited for shorter prediction windows.

The confirmation of the best results for the turbine data set is in rows five and 10 in Table 5.3.

5.6 Chapter summary

This chapter tackles the second challenge from Chapter 1 related to quantification of degradation in industrial systems. It presents a method for quantification of degradation for industrial turbomachinery. It fulfils the gap of knowledge 3.3.2 by providing a data-driven algorithm for online prediction of turbomachinery degradation.

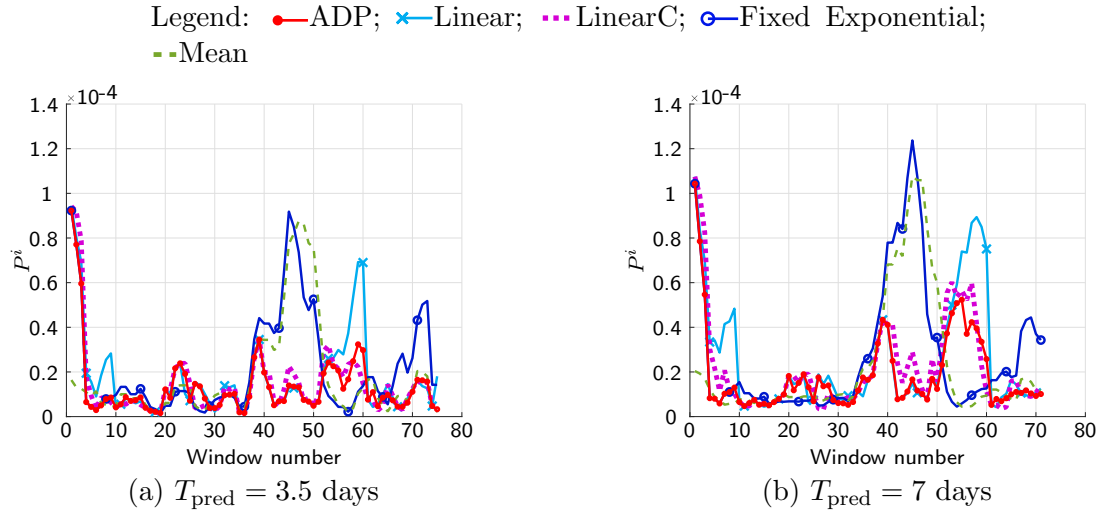


Figure 5.14: Comparison of different approaches for turbine from Brekke et al. (2009)

The algorithm is based on time-trend analysis and combines a moving window approach with adaptive regression analysis and predicts the expected value of degradation and quantifies the uncertainty of the prediction. The switching between the linear and exponential regression functions is also decided in a moving window fashion when the change in the value of the degradation indicator within a window exceeds a threshold.

The algorithm was used to predict the expected values of future degradation indicator and to quantify the uncertainty of the prediction by providing upper and lower prediction intervals in each window. An optimisation scheme was used for tuning of the parameters of the algorithm.

The results using real data from industrial case studies show that the algorithm predicts the degradation indicator accurately in the short and medium terms. The moving window framework combined with nonlinear regression allows adaptation of the algorithm to the variations of the trend of the degradation indicator. The adaptation properties were demonstrated in comparison with four window-based approaches.

The chapter shows that it is possible to combine the existing approaches in degradation modelling to improve the accuracy of the prediction, thus making the algorithm useful in industrial performance-based application, such as maintenance scheduling.

The model of degradation from this chapter is further expanded in Chapter 6 to take into account factors influencing degradation. The industrial data from the case study are used in Chapter 7 to demonstrate the performance of the new optimisation framework taking degradation into account.

Chapter 6

Load-sharing with degradation management

The concept of designing a control structure taking into account the degradation of the system has been introduced in Chapter 3. Degradation is induced both by ageing and varying operating conditions (BSI, 2010b). The mitigation of degradation is sought in order to retain the available performance of the system. In particular, a question arises on how to distribute the control effort among multiple compressors subject to degradation. A compressor station is an example of such a system where the distribution of the loads among the compressors is of importance.

One of the challenges indicated in Chapter 1 was related to an analysis of the relationships within the system subject to degradation. The objective of this chapter is to tackle this challenge by proposing a load-sharing method which takes the current level of degradation indicator into account and attempts to mitigate further degradation. For this purpose, the ratios between the levels of degradation indicators of compressors in a compressor station are investigated using the model of compressor introduced in Chapter 2. This chapter builds on previous approaches from the literature and expands the model of degradation from Chapter 5 to include influencing factors introduced in Chapter 4.

The chapter is structured as follows. Section 6.1 presents how degradation affects the behaviour of a compressor. Section 6.2 describes the model of degradation used in this chapter. The new load-sharing algorithm is introduced in Section 6.3 and the effects of its parameters are analysed in Section 6.4. The results for a compressor station with two compressors are presented in Section 6.5. An extension of the algorithm to five units is presented in Section 6.6. The chapter ends with a summary in Section 6.7.

6.1 Degradation-dependent models of compressor behaviour

The behaviour of the compressors introduced in Chapter 2 will be affected by degradation, for example due to fouling. The influence of degradation will appear both in the compressor maps and in the dynamics. As indicated in Chapter 2, the behaviour of a compressor is captured by a set of dynamic equation from Eq. (2.4) and recalled here:

$$\dot{p}_{d,i} = \frac{a_{01}^2}{V_i} (m_i - m_{\text{out},i}) \quad (6.1a)$$

$$\dot{m}_i = \frac{A_1}{L_c} (\Psi_i(m_i, N_i) p_s - p_{d,i}) \quad (6.1b)$$

$$\dot{\omega}_i = \frac{1}{J_i} (\tau_{t,i} - \tau_{c,i}(m_i, \omega_i)) \quad (6.1c)$$

where p_s is the suction pressure, $p_{d,i}$ is the pressure in the discharge tank, m_i denotes the mass flow rate through the compressor, ω_i is the compressor speed in rad s^{-1} and $\omega = \frac{2\pi N}{60}$ where here N is the speed in rpm. The variable $\tau_{c,i}$ is the reaction torque of the compressor. The mass flow through the outlet $m_{\text{out},i}$ valve is a function of the valve opening $u_{\text{out},i}$ and external pressure p_{out} . The variable $\tau_{t,i}$ denotes the external torque delivered to the compressor by the motor. The constants V_i , A_1 , L_c , and a_{01} capture the geometry of the tank and the piping system, whereas J_i denotes the inertia of the shaft. The parameter Ψ_i shows the pressure ratio across a compressor as a function of compressor flow and speed. The values of the parameters were adapted from Bakken et al. (2002) and Nørstebø (2008).

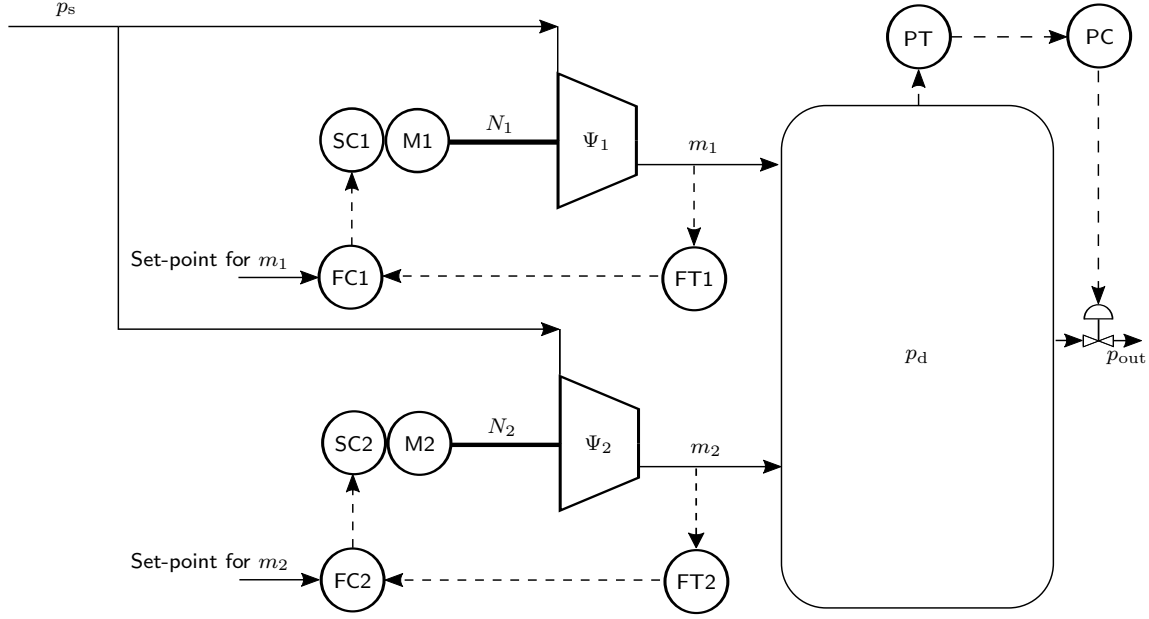


Figure 6.1: Compressor station with two compressors connected to the same discharge tank

A common scenario in compressors is that the mass flow rate is specified by the set point to a mass flow controller, the external and suction pressures are fixed by pipelines, and the speed has to adjust to be consistent with the mass flow point on the compressor map. This scenario is also used in this chapter and Fig. 6.1 shows a P&ID diagram of a single compressor station, where:

- The suction pressure p_s is fixed upstream and is the same for all the compressors;
- The discharge pressure p_d is controlled by a pressure controller, which adjusts the outlet valve opening. Both compressors are connected to the same discharge tank with volume V , so Eq. (6.1a) becomes:

$$\dot{p}_d = \frac{a_{01}^2}{V}(m_1 + m_2 - m_{out}) \quad (6.2)$$

- The external pressure p_{out} is fixed downstream;
- The mass flows m_i are controlled by flow controllers, each of which adjusts the speed of the compressor to reach the set-point.

The objective of the load-sharing algorithm is to provide set-points to the mass flow controllers.

6.1.1 Influence on pressure ratio

As indicated by Ciccioiti (2015), degradation due to fouling will affect the pressure ratio. The compressor map in degraded state, Ψ_D , can be obtained from the map in up state Ψ as:

$$\Psi_D = h(d_\Psi)\Psi \quad (6.3)$$

with a degradation function $h(d_\Psi)$, which captures how the pressure ratio changes with degradation indicator d_Ψ . The degradation function $h(\cdot)$ has been introduced in Chapter 4 and captures the influence of degradation on a selected variable or characteristic quantity. The function $h(\cdot)$ used in this chapter will be further described in Section 6.2.4.

The grey dashed lines in Fig. 6.2 show how a compressor map changes for a constant level of degradation indicator $d_\Psi = 3\%$ and $h(d_\Psi) = 1 - d_\Psi$. The grey lines correspond to the same speeds as the black lines in up state. The compressor map from Fig. 6.2 is an equivalent of the generic compressor map from Fig. 2.5. The data for the compressor maps were obtained from Nørstebø (2008) using the software from Rohatgi (2018). The data used by Nørstebø (2008) come from a real compressor station transporting natural gas and are used here as representative of a generic compressor in the oil and gas industry.

The red line represents a constant pressure ratio with the operating limits for the mass flow marked with a circle (lower limit $m^{\min}$, on the dashed surge line) and an asterisk (upper limit $m^{\max}$, on the dotted choke line). The operating point in up state is marked with a triangle, whereas the cross indicates the operating point in degraded state if there was no flow controller. If there is no flow controller, the speed and pressure do not change, because there was no change in the external operating conditions which would require adjustments of the speed. However, Fig. 6.1 indicates that the flow controller is a part of the system and to keep the compressor at the operating point marked by the red triangle, it is necessary to increase the speed of the compressor.

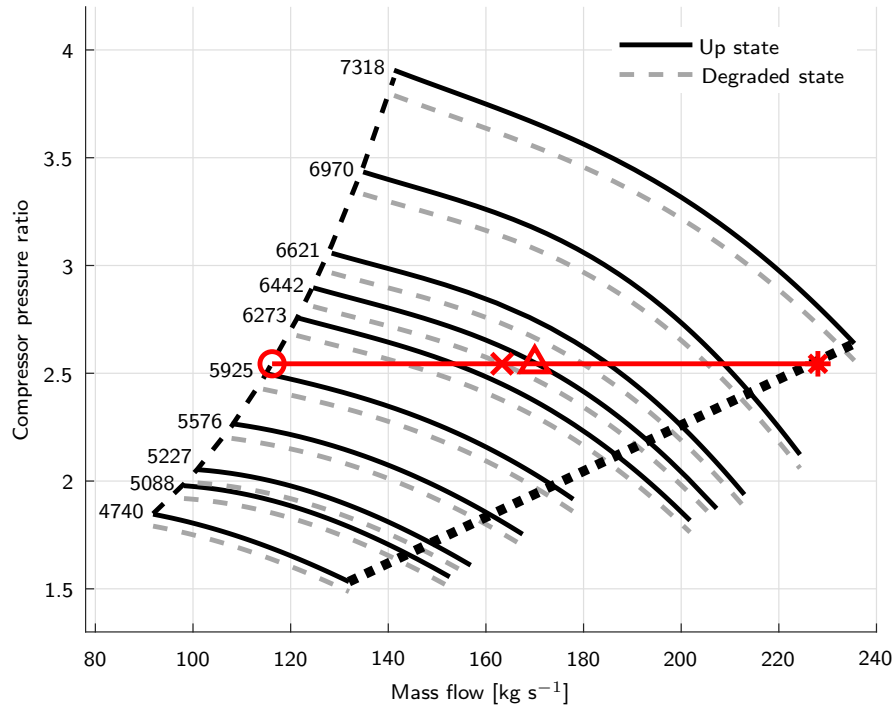


Figure 6.2: A compressor map with the pressure ratio as a function of the mass flow and compressor speed in up state and in degraded state. The labels indicate compressor speed in rpm. The red line (—) shows the constant pressure ratio with the operating limits for the mass flow marked with a circle (○, lower limit $m^{\min}$, on the dashed surge line) and an asterisk (*, upper limit $m^{\max}$, on the dotted choke line). The operating point in up state is marked with a triangle (Δ), whereas the cross (×) indicates the operating point in degraded state if there is no flow controller. The data for the compressor maps were obtained from Nørstebø (2008) using the software from Rohatgi (2018)

6.1.2 Influence on dynamics

Degradation of a compressor changes the dynamic behaviour from Eq. (6.1), because it affects the pressure ratio Ψ according to Eq. (6.3). Introducing the degradation of the form of Eq. (6.3) in Eq. (6.1) changes the dynamic behaviour of the system, as the pressure ratio will be smaller for the same mass flow and compressor speed.

6.1.3 Influence on efficiency

The need for increasing the speed for the same operating pressure ratio and mass flow results in a loss of efficiency, d_η , as described in Chapter 5:

$$\eta_D = (1 - d_\eta)\eta \quad (6.4)$$

where η is the efficiency in up state.

This thesis assumes that the value of the degradation indicator for pressure ratio is the same as the value of the degradation indicator for the efficiency, i.e.:

$$d_\eta = d_\Psi \quad (6.5)$$

Figure 6.3 shows the compressor map from Fig. 6.2 with efficiency contours (obtained from Nørstebø (2008) using the software from Rohatgi (2018)) in up state (Fig. 6.3a) and in degraded state (Fig. 6.3b) with $d_\eta = 3\%$ in Eq. (6.4). According to Eq. (6.4), the efficiency in degraded state is lower than the efficiency in up state, which is reflected in the absence of yellow colour in Fig. 6.3b denoting the highest value of efficiency.

6.1.4 Influence on power

For a given load, Eq. (2.11) states that the power W necessary to run the compressor in up state is inversely proportional to the efficiency:

$$W \sim \frac{1}{\eta} \quad (6.6)$$

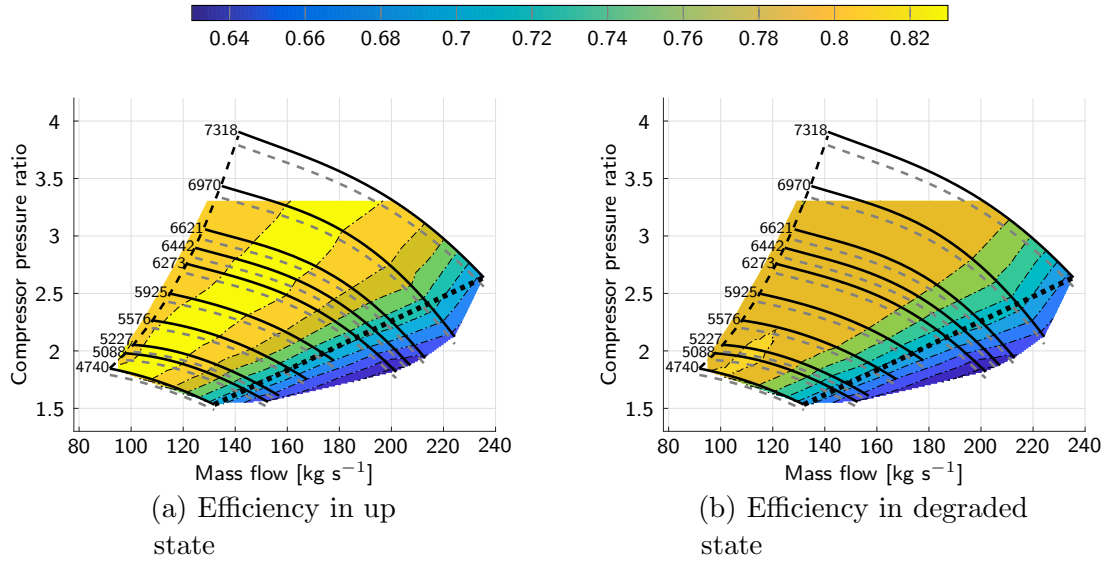


Figure 6.3: Compressor map with efficiency contours in up state (left) and in degraded state (right), with the surge line (---), and the choke line (···). The white area above the pressure ratio 3.3 is due to physical constraints imposed on the system used by Nørstebø (2008)

The power in degraded state W_D is inversely proportional to the efficiency in degraded state η_D :

$$W_D \sim \frac{1}{\eta_D} = \frac{1}{(1 - d_\eta)\eta} \quad (6.7)$$

Using the polynomial approximation of power from Eq. (2.13), the power in degraded state can be written as a function of the mass flow m :

$$W_D = h_W(d_\eta)(am^2 + bm + c) \quad (6.8)$$

where h_W represents a multiplicative degradation function introduced in Chapter 4:

$$h_W(d_\eta) = \frac{1}{(1 - d_\eta)} \quad (6.9)$$

Figure 6.4 illustrates how the power characteristics from Fig. 2.6 change with degradation for $h_W(d_\eta)$ from Eq. (6.9) with $d_\eta = 3\%$. For a given pressure ratio and flow rate, the power necessary to run a compressor in degraded state is larger than the power used if the compressor is in up state.

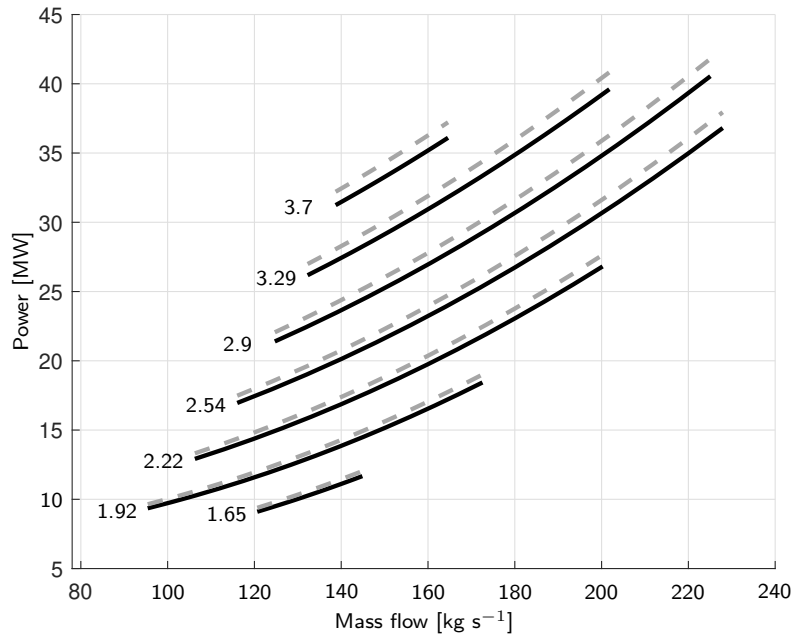


Figure 6.4: Compressor power in up state (—) and in degraded state (---), as a function of the mass flow for various pressure ratios indicated with labels

6.2 Factor-based model of compressor degradation

Chapter 5 introduced the approaches to modelling degradation in turbomachinery. A common way is to treat degradation as a function of time as suggested, among others, by Cicciotti (2015) and Tarabrin et al. (1996). Chapter 5 showed that it is possible to accurately capture compressor degradation as function of time.

At the same time, Chapter 4 indicated that it is necessary to include the influencing factors in the model of degradation to be able to mitigate degradation. Verheyleweghen and Jäschke (2017) suggested that the speed of the compressor is one of the main factors contributing to the degradation.

The model of degradation proposed in this chapter will have two elements:

- A factor-free component that depends on time,
- A factor-based multiplier that depends on the speed of the compressor.

6.2.1 Factor-free component of the model of degradation

The degradation models from Ciccioiti (2015) and Tarabrin et al. (1996) depend solely on time and can be linked with the fouling phenomenon. For the i -th compressor it is modelled as an exponential function of form:

$$d_{T,i}(t) = \alpha + \beta \exp(-\gamma_i t) \quad (6.10)$$

where $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}_-$, and $\gamma_i \in \mathbb{R}_+$ are constant parameters defining the maximal degradation level and the rate of change. The subscript T emphasises the time-dependence of the degradation function. The parameters in Eq. (6.10) can be identified in the way described in Chapter 5, because Eq. (6.10) is derived from Eq. (5.2) with $b_0 = \alpha$, $b_1 = 0$, $b_2 = \beta$, $b_3 = \gamma_i$.

6.2.2 Factor-based multiplier of the model of degradation

To include the influence of the speed on the degradation in Eq. (6.12), a factor-based multiplier $F_{N,i}(N)$ is introduced:

$$F_{N,i}(N) = a_i \left(\frac{N}{r_i} \right)^3 \quad (6.11)$$

where a_i and r_i are constant parameters. The subscript N emphasises the dependence on the speed. The values of a_i and r_i used in this work are given in Table 6.2. They were chosen so that $F_{N,i}(N) = 1$ for $N = 5848$ rpm. The form of Eq. (6.11) was derived from Verheyleweghen and Jäschke (2017) who indicated that degradation of a compressor depends on the third power of the compressor speed.

Both $d_{T,i}$ and $F_{N,i}$ are strictly increasing functions with respect to their arguments, i.e., time t and speed N . The required speed varies with time, because N is a combination of the load-sharing algorithm and the flow controller. Therefore, $F_{N,i}$ is non-monotonic with respect to time if $N(t)$ is a non-monotonic function of time.

6.2.3 Factor-based model of degradation

A factor-based model is now introduced to capture the degradation of efficiency with compressor speed as an influencing factor:

$$d_i(t) = d_{T,i}(t)F_{N,i}(N(t)) \quad (6.12)$$

where N denotes the speed of the i -th compressor and d_i is the degradation of efficiency. The function $d_{T,i}(t)$ represents a factor-free component, and $F_{N,i}(N)$ represents a factor-based multiplier of degradation. Focusing on the speed as the main factor influencing the degradation allows a more direct analysis of the influence of control on degradation as it bypasses the dynamic effect introduced by the inertia of the compressor and allows Eq. (6.1c) to be omitted in the model. If not specified, the term ‘degradation indicator’ in this chapter refers to d_i from Eq. (6.12).

The model from Eq. (6.12) ensures that the compressors are in up state at time $t = 0$. This is obtained because the exponential formula of the factor-free component from Eq. (6.10) is equal to unity for $t = 0$.

The exponential formula of Eq. (6.10) indicates also that for a fixed value of compressor speed N in Eq. (6.11), the model from Eq. (6.12) will saturate at a certain level. This is obtained because the exponential part of the factor-free component from Eq. (6.10) tends to zero for t increasing:

$$\lim_{t \rightarrow \infty} d_i(t) = a_i \left(\frac{N}{r_i} \right)^3 \alpha \quad (6.13)$$

6.2.4 Degradation function

The degradation function $h(d)$ captures how degradation affects a selected variable describing the behaviour of a unit. In this chapter, it is defined as:

$$h(d) = 1 - d \quad (6.14)$$

where d is a variable that describes the value of degradation. For instance, if $d = d_i(t)$, the degradation function $h(d)$ from Eq. (6.14) represents the degradation of pressure ratio. The degradation function $h(d)$ is maximal when $d = 0$, i.e. when there is no degradation. Conversely, when d reaches its upper limit, the value of $h(d)$ is minimal.

The degradation function $h(d)$ from Eq. (6.14) can be interpreted as *health* of a unit with degradation d . For instance, if it is assumed that Compressor 1 is more degraded than Compressor 2 i.e.:

$$d_1 \geq d_2 \quad (6.15)$$

where d_i is given by Eq. (6.12), Eq. (6.14) yields:

$$h(d_1) \leq h(d_2) \quad (6.16)$$

with an interpretation that Compressor 2 is *healthier* than Compressor 1.

The inequality (6.16) is a basis for the analysis of the relationships between the compressors in this chapter.

6.3 Load-sharing with degradation management

The new algorithm is now introduced to show how the load is shared in a compressor station with two compressors. Assuming that the compressors are similar, i.e., of the same type and configuration, and have the same characteristics, the main approach to load-sharing is based on keeping a selected parameter equal for both compressors, usually the distance to the surge line (Liptak, 2005). Based on the ratio $\frac{Q_2}{Q_1}$ where Q_i denotes the surge set-point parameters for the i -th compressor, the load is proportionally scaled according to formula (6.17):

$$m_2 = m_1 \frac{Q_2}{Q_1} \quad (6.17)$$

where m_i is the load assigned to i -th compressor. If the compressors are identical, then $Q_2 = Q_1$ and $m_1 = \frac{M}{2}$ where M is the desired overall flow from the station.

Typically, the value of Q_i is expressed as a *distance to surge*. For a given pressure ratio, the distance to surge denotes how far the operating point is from the surge line. Figure 6.2 shows that if there is no flow controller, the distance to surge will decrease in degraded state. Formula (6.17) does not take into account the possible changes in the system due to degradation. For instance, it does not consider that to keep the same distance to surge, it is necessary to increase the speed of a compressor.

6.3.1 Load-sharing algorithm for two compressors

The proposal in this thesis is to assign the loads m_1 and m_2 according to Eq. (6.18):

$$m_1 = \frac{M}{2} \exp(q(1 - f(d_1, d_2))) \quad (6.18a)$$

$$m_2 = M - m_1 \quad (6.18b)$$

where M is the required overall demand, d_i denotes the current degradation level of i -th compressor, and $q > 0$ is a *scaling factor*. The function $f(d_1, d_2)$ captures the relationships between the compressors, i.e. which compressor is more degraded as indicated by inequality (6.15). Depending on the value of $f(d_1, d_2)$, it is possible to manage the degradation of the compressors. Suitable forms for f are presented in Section 6.3.2.

6.3.2 Degradation management

It is now assumed that the two compressors are identical in up state and the default load assignment is done by dividing the demand in two. When the compressors degrade, each compressor is characterised by its own value of degradation, calculated according to Eq. (6.12). The management of degradation refers to changes in the value of the degradation indicator d_i from Eq. (6.12) relative to the value obtained in the equal load approach. The management of degradation of the i -th compressor has one of the objectives:

- To decrease degradation, if the degraded compressor gets less load than in the equal load approach. Load sharing strategy that decreases degradation d_i relative to an equal load strategy is called *mitigation of degradation*;
- To increase degradation, if the degraded compressor gets more load than in the equal load approach. Load sharing strategy that increases degradation d_i relative to an equal load strategy is called *intensification of degradation*.

The mitigation of degradation aims to preserve the degrading compressor and extend its lifetime. The counter-intuitive idea of intensifying degradation also has industrial applications. For instance, it might be useful to intensify degradation if a pre-scheduled maintenance event is coming up for the degraded unit. As the timing of the maintenance event has already been fixed, increasing the value of degradation helps to make the most of the maintenance. For instance, if a piece of machinery is to be replaced, it is better to remove it when it is no longer able to perform the required function, instead of replacing a still functioning machine.

Degradation management is done by choosing the functional form of $f(d_1, d_2)$. The functional forms of $f(d_1, d_2)$ will be analysed for:

- Mitigation of degradation:

$$f_M(d_1, d_2) = \frac{h(d_2)}{h(d_1)} \quad (6.19)$$

- Intensification of degradation:

$$f_I(d_1, d_2) = \frac{h(d_1)}{h(d_2)} \quad (6.20)$$

Table 6.1: Degradation management as influenced by the functional form of $f(d_1, d_2)$

	Compressor 1 degrades more quickly	Compressor 1 degrades more slowly
f_M	Mitigation of degradation of Compressor 1, intensification of degradation of Compressor 2	Intensification of degradation of Compressor 1, mitigation of degradation of Compressor 2
f_I	Intensification of degradation of Compressor 1, mitigation of degradation of Compressor 2	Mitigation of degradation of Compressor 1, intensification of degradation of Compressor 2

If the second compressor has a smaller value of degradation indicator and inequality (6.15) is fulfilled, the value of f_M from Eq. (6.19) is larger than unity, $f(d_1, d_2) > 1$ and the exponent in Eq. (6.18) is negative. As a result, the load assigned to the first compressor according to Eq. (6.18a) will be smaller than the load in equal load approach. Conversely, the value from Eq. (6.20) will be larger than unity, $f(d_1, d_2) < 1$ and the exponent in Eq. (6.18a) is positive. As a result, the load assigned to the first compressor according to Eq. (6.18a) will be larger than the load in equal load approach. If the inequality (6.15) is not fulfilled, the relationships in Eq. (6.19) and (6.20) will be inverted. The results of the degradation management based on the relationships between the degradation functions are gathered in Table 6.1 and will be shown in Section 6.5.

6.3.3 Relation to equal load assignment

The functional forms of the function $f(d_1, d_2)$ from Eq. (6.19) and (6.20) mean that the compressors will always get equal load when $d_1 = d_2$. Equation (6.18) becomes equivalent to equal load approach with:

$$m_1 = m_2 = \frac{M}{2} \quad (6.21)$$

It will be shown in Chapter 7 that the equal load approach is optimal if the compressors are identical.

6.3.4 Scaling factor

The scaling factor q influences what part of the load will be assigned to each compressor and therefore contributes to the degradation management described in

Section 6.3.2. From Eq. (6.18a):

$$\exp(q(1 - f(d_1, d_2))) = \frac{m_1}{\frac{M}{2}} \quad (6.22)$$

The choice of q and its influence on the system is analysed in Section 6.4 taking into account also the power necessary to run the compressors.

6.4 Scaling factor in degradation management

The scaling factor q indicates how much load the compressors will receive depending on the current degradation function $f(d_1, d_2)$. Assuming that the inequality (6.15) is fulfilled, a large q will allow a larger mitigation of the degradation of Compressor 1 using Eq. (6.19). Conversely, a large q will also intensify the degradation of Compressor 1, if Eq. (6.20) is used.

6.4.1 Influence on compressor operating range

As q affects the mass flow assigned to each compressor, the choice of q must be made with respect to the operating range of the compressors. For a fixed pressure ratio the mass flow assigned to each compressor has to satisfy the inequality:

$$m_i^{\min} \leq m_i \leq m_i^{\max} \quad (6.23)$$

The values of $m_i^{\min}$ and $m_i^{\max}$ in the inequalities (6.23) are obtained for a fixed value of pressure ratio from the compressor map bounded by the surge line m_i^s , minimal speed line $m_i^{N\min}$, maximal speed line $m_i^{N\max}$ and choke line m_i^c :

$$m_i^{\min} = \max\{m_i^s, m_i^{N\min}\} \quad (6.24a)$$

$$m_i^{\max} = \min\{m_i^c, m_i^{N\max}\} \quad (6.24b)$$

It is assumed that the compressors have the same operating ranges $m_1^{\min} = m_2^{\min} = m^{\min}$ and $m_1^{\max} = m_2^{\max} = m^{\max}$. For the pressure ratio $\Psi = 2.52$ marked in Fig. 6.2, the values are:

$$m^{\min} = 116 \text{ kg s}^{-1} \quad (6.25a)$$

$$m^{\max} = 228 \text{ kg s}^{-1} \quad (6.25b)$$

and are marked with a circle and an asterisk in Fig. 6.2.

The constraints from Eq. (6.23) will affect the possible values of q depending on the objective: the mitigation or intensification of degradation.

The following inequalities are obtained from Eq. (6.18) and Eq. (6.24):

$$\frac{M}{2} \exp(q(1 - f(d_1, d_2))) \geq m_1^{\min} \quad (6.26a)$$

$$\frac{M}{2} \exp(q(1 - f(d_1, d_2))) \leq m_1^{\max} \quad (6.26b)$$

$$M - \frac{M}{2} \exp(q(1 - f(d_1, d_2))) \geq m_2^{\min} \quad (6.26c)$$

$$M - \frac{M}{2} \exp(q(1 - f(d_1, d_2))) \leq m_2^{\max} \quad (6.26d)$$

Inequalities (6.26a) and (6.26d) will be considered to find the value of q for mitigation of degradation for Compressor 1, whereas inequalities (6.26b) and (6.26c) will be taken into account if the objective is to intensify the degradation of Compressor 1.

In some cases it is impossible to find a value of q such that the constraints from inequalities (6.26b) and (6.26c) are not violated. This happens when the demand is such that the compressors work close to the boundaries if the load is distributed equally. A possible solution proposed in Chapter 8 involves removing the upper constraint (6.26b) by assuming it is possible to obtain the excess flow from alternative sources.

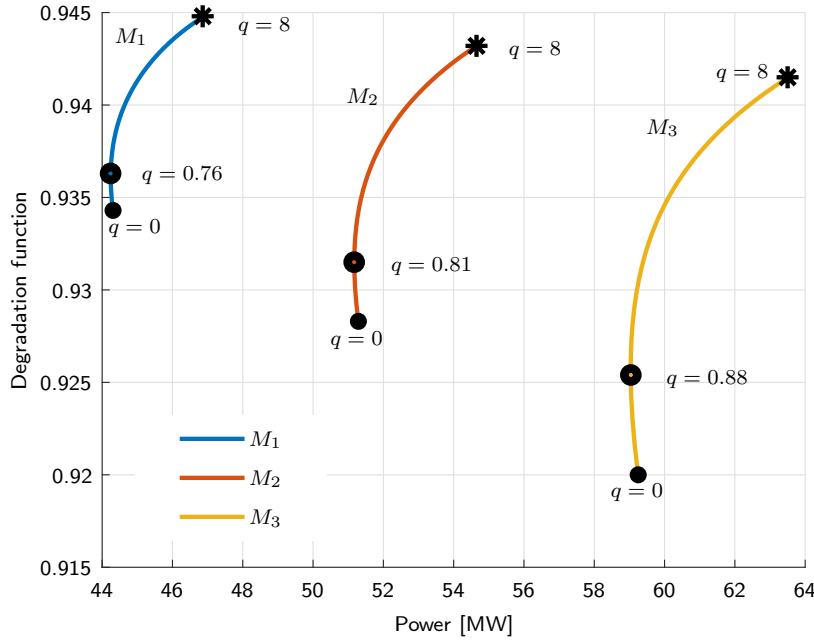


Figure 6.5: Trade-off between power (horizontal axis) and degradation (vertical axis) if only one compressor degrades, for three values of demand $M_1 < M_2 < M_3$. The value of q changes between zero (●) and eight (*), whereas the minimal power is denoted with a circle (●)

6.4.2 Influence on power consumption

Figure 6.5 shows the relationship between degradation of one compressor (vertical axis) and the power consumption of the station (horizontal axis) for the values of q between zero and eight for three values of demand, $M_1 = 300 \text{ kg s}^{-1}$, $M_2 = 340 \text{ kg s}^{-1}$, and $M_3 = 380 \text{ kg s}^{-1}$. Figure 6.5 shows that the maximal value of degradation function, which denotes the healthiest compressor, is obtained for the largest value of q (star) for all values of demand M_1 , M_2 , M_3 . In contrast, the value of q for which the smallest value of power is obtained depends on the demand (black circles). The equal load approach, equivalent to $q = 0$ is optimal neither from the perspective of power minimisation, nor health improvement. Thus, this chapter assumes $q = 2$, which keeps the compressors within the operating range and is not any of the special cases described above.

6.5 Analysis for two compressors

To evaluate the algorithm proposed in Section 6.3 the following scenarios are analysed:

- A scenario with identical compressors,
- A scenario with compressors degrading at different rates.

The scenarios are analysed both in terms of degradation mitigation using f_M from Eq. (6.19) and intensification of degradation with f_I from Eq. (6.20).

6.5.1 Identical compressors

The scenario with identical compressors occurs in two cases:

- If the compressors are identical in up state;
- If the value of the degradation functions are equal.

Identical compressors in up state do not have any degradation, $d_1 = d_2 = 0$, and in consequence $h(d_1) = h(d_2) = 1$. Then $f_M(d_1, d_2) = f_I(d_1, d_2) = 1$ in Eq. (6.19) and Eq. (6.20). Equation (6.18) yields:

$$m_1 = m_2 = \frac{M}{2} \quad (6.27)$$

From the point of view of the algorithm the compressors are also seen as identical if their degradation functions $h(d_i)$ have the same value. Then the load assignment will revert to the equal load approach.

6.5.2 Two compressors degrading

Section 6.5.1 indicates that two compressors degrading at the same rate are considered identical from the point of view of the algorithm. This section analyses the load-sharing algorithm from Eq. (6.18) if the compressors degrade at different rates for a fixed value of demand $M = 340 \text{ kg s}^{-1}$.

It is assumed that the degradation of Compressor 1 is larger than the degradation of Compressor 2, $d_1 > d_2$. This assumption preserves the mitigating and intensifying character of the function from Eq. (6.19) and (6.20), as indicated in Table 6.1. The values of the parameters of degradation were chosen to simulate an overall degradation level of 7% over a period of 90 days.

Table 6.2: Parameters of degradation model from Eq. (6.12) for two compressors

	a_i	r_i in rpm	α_i	β_i	γ_i
Compressor 1	4×10^{-5}	200	0.05	-0.05	2×10^{-6}
Compressor 2	4×10^{-5}	200	0.05	-0.05	6.25×10^{-7}

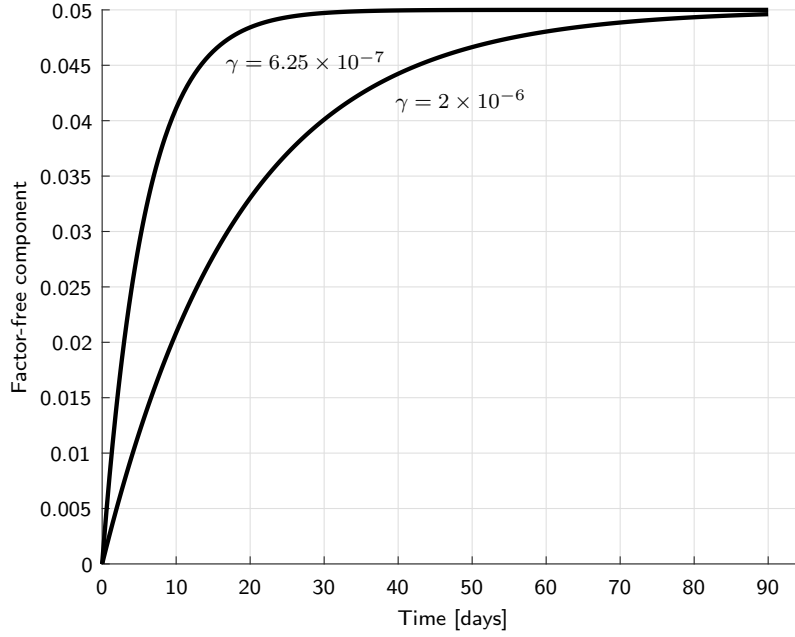


Figure 6.6: Factor-free component d_T of degradation d_i , as given by Eq. (6.10), for two values of γ

The speed affects both compressors in the same way and the parameters a_i and r_i in Eq. (6.11) are the same. The parameters of the factor-free degradation component $d_{T,i}$ from Eq. (4.17) influence how quickly a compressor is expected to degrade. The two functions of time are in Fig. 6.6.

The influence of the load-sharing on the compressors will be done by comparing the time at which the degradation function from Eq. (6.10) reaches 93%.

6.5.2.1 Two compressors degrading - load sharing

The results for the case when the Compressor 1 degrades more quickly are in Fig. 6.7. The left column in Fig. 6.7 shows the mass flows (Fig. 6.7a), the speed (Fig. 6.7c), and the power (Fig. 6.7e) if a mitigation strategy was used, with f_M from Eq. (6.19). The blue and the red lines show the operating range for the compressor, with blue denoting the maximal mass flow $m^{\max}$, and red denoting the minimal flow $m^{\min}$. The results from the new strategy (black lines in Fig. 6.7) are compared with the equal load approach (green lines in Fig. 6.7). The degraded compressor gets less load at the beginning (dashed line in Fig. 6.7a). This is due to the fact that Compressor 1 degrades more quickly and therefore the value of $f(d_1, d_2)$ increases.

After ten days, however, the ratio $\frac{d_2}{d_1}$ starts to decrease. This is due to the fact that the quickly degrading compressor reaches its lower bound of degradation after approximately 10 days, whereas the degradation function of the slowly degrading compressor still decreases. From day 60, the compressors become identical again and get assigned the same value of the load.

The fact that both compressors degrade is also noticeable in the maximal value of the flow and the speed of each compressor. The speed of both compressors increases to keep the desired flow, as indicated in Section 6.1.2. Figure 6.7a shows the maximal flow below 180 kg s^{-1} . This means that the new strategy keeps the compressors within their operating range (red and blue lines).

6.5.2.2 Two compressors degrading - power consumption

A low flow has a direct impact on the power necessary to run the compressors. As shown in Fig. 6.7e, the power necessary to run the compressors does not differ from equal load assignment. This is because now the strategy takes into account the degradation of the second compressor and does not assign too much load to it. Thus, as soon as the ratio is close to unity, the strategies reverts to equal load. Assigning a lower load from day 0 to day 60 to the more degraded compressor improved its health relative to the equal load approach, shown with a degradation function in Fig. 6.8a. The degradation function with load-sharing reached 93% ten days later compared with equal-load. At the same time, the compressor with a slower degradation rate reached 93% only two days earlier. The results show that it is possible to mitigate degradation by sharing the load using f_M .

Conversely, the left column of Fig. 6.7 indicates that it is also possible to intensify degradation of a compressor using the functional form f_I . The ratio $\frac{d_1}{d_2}$ is shown in Fig. 6.8f. Assigning a larger load to the more degraded compressor (dashed line in Fig. 6.7b) results in an increased value of compressor speed, and in consequence, in faster degradation. The degradation function of the degraded unit reaches 93% five days earlier than if the loads are assigned equally. The degradation function of the slowly degrading unit is further slowed down and the threshold is reached two days later.

The intensification of degradation affects the power necessary to run the compressors, shown in Fig. 6.7f. Until day 50, the power necessary to run the two compressors is larger than in the equal load case. This indicates that the degraded compressor requires more power than can be saved by slowing down the degradation of the slowly degrading unit. After the degradation ratio tends to unity, the power becomes equal to the equal load case.

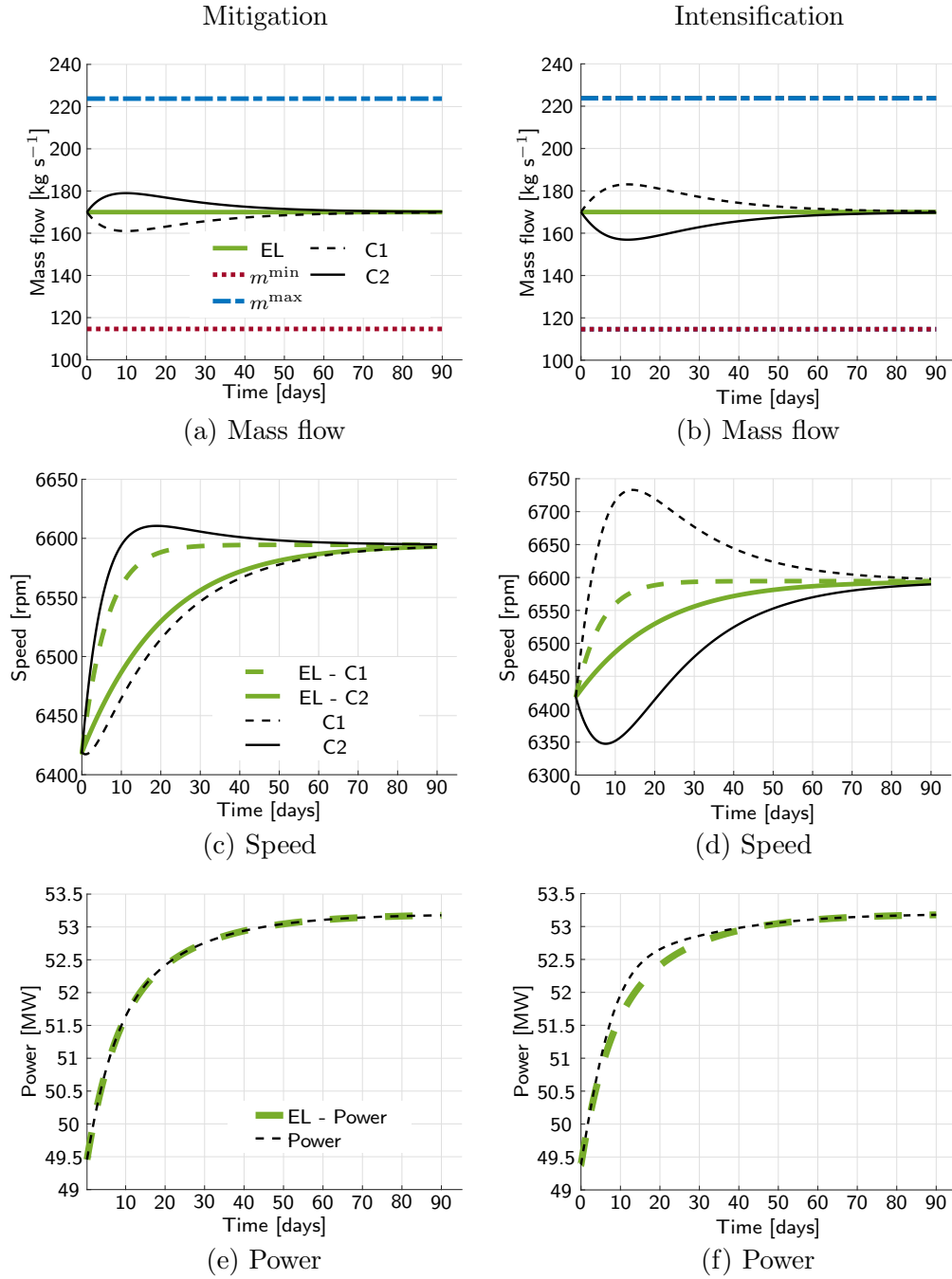


Figure 6.7: Process variables in the case of mitigating degradation using f_M (left column) and intensifying degradation with f_I (right column) if Compressor 1 (C1) degrades more quickly than Compressor 2 (C2), compared with equal load approach (EL)

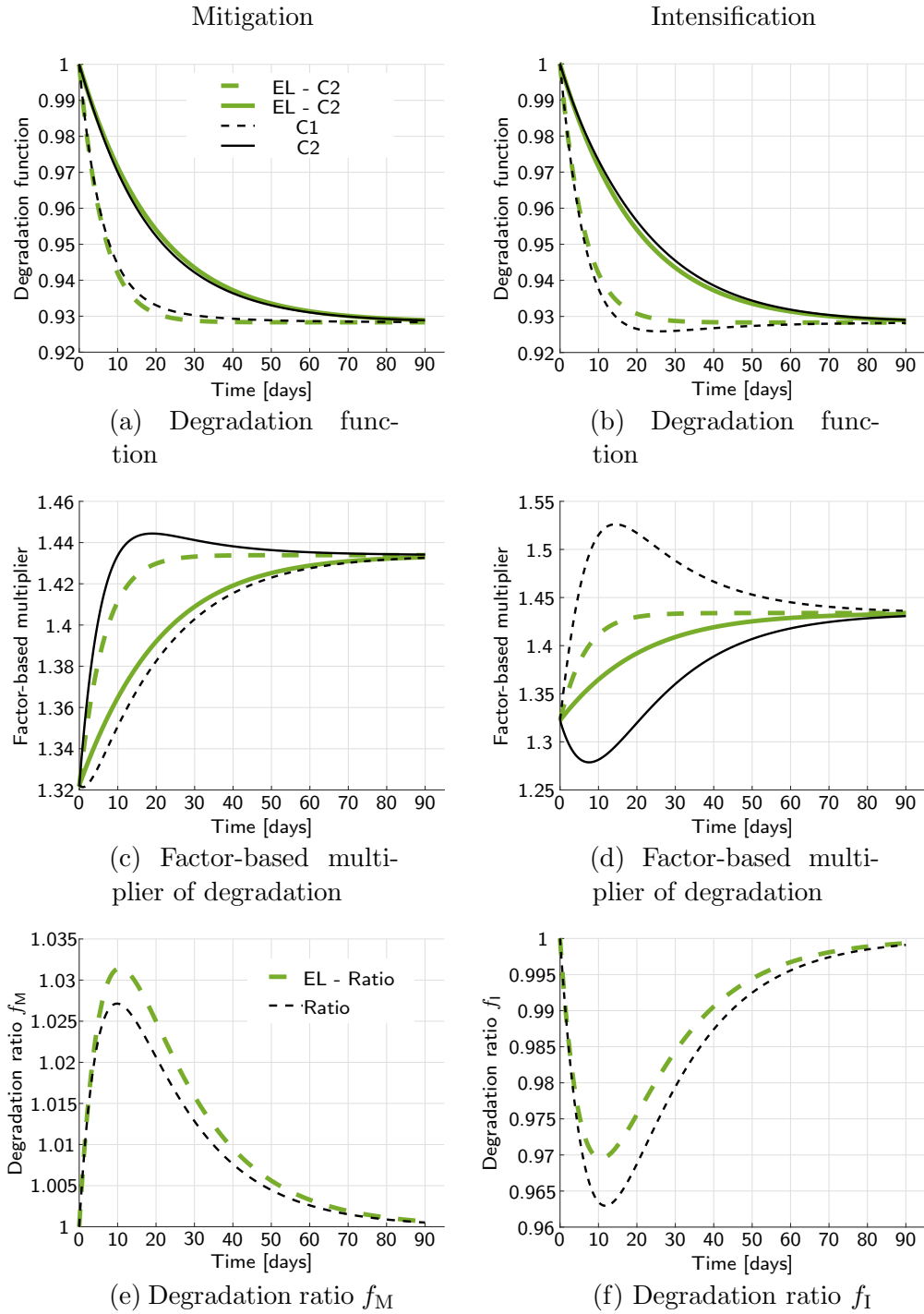


Figure 6.8: Degradation variables in the case of mitigating degradation using f_M and intensifying degradation with f_I if Compressor 1 (C1) degrades more quickly than Compressor 2 (C2), compared with equal load approach (EL)

6.6 Application to multiple compressors

An extension of the algorithm from Eq. (6.18) is presented to take into account compressor stations with more than two compressors and to further investigate the relationships between the extent of degradation of the compressors. It is now assumed that a compressor station has K compressors in parallel, and the compressors are identical in up state. Each compressor has a different degradation rate γ_i in Eq. (6.10) and the units are ordered from the most degraded to the least degraded:

$$d_1 \geq d_2 \geq \dots \geq d_K \quad (6.28)$$

The algorithm will focus on the compressors ordered according to Eq. (6.28).

The algorithm from Eq. (6.18) is expanded to:

$$m_i = \frac{M}{K} \exp \left(q \left(1 - \frac{h(d_{i+1})}{h(d_i)} \right) \right) \quad (6.29a)$$

$$m_K = M - \sum_{i=1}^{K-1} m_i \quad (6.29b)$$

where m_i is the mass flow assigned to the i -th compressor, $i = 1, \dots, K - 1$. The algorithm from Eq. (6.29) compares the degradation indicator of i -th compressor d_i with the compressor with the most similar, but smaller degradation d_{i+1} . The approach from Eq. (6.29) will be called a *collaboration strategy*, because the loads are assigned to compressors based on the ratio of degradation functions of each two compressors with most similar degradation.

6.6.1 Interpretation for multiple compressors

It is assumed that the compressors are ordered according to expression (6.28) for the whole time. If the ordering of the values of degradation indicators changes, the compressors can be reordered to keep expression (6.28) fulfilled.

Since the compressors are ordered according to expression (6.28), all the ratios are larger than or equal than unity:

$$\frac{h(d_{i+1})}{h(d_i)} \geq 1 \quad (6.30)$$

The case when the ratio from expression (6.30) becomes unity is a special case and the load-sharing algorithm from Eq. (6.29) reverts to equal load approach. From now on, it will be assumed that the strong inequality is fulfilled in expression (6.30). The expression (6.30) indicates that the i -th compressor, $i < K$, is assigned a lower load than in the equal load case. The least degraded compressor with degradation d_K gets the remaining load, which is larger than in the equal load case. How much less each compressor will get depends on the ratio $\frac{h(d_{i+1})}{h(d_i)}$. If the compressors i and $i + 1$ have degradation indicators close to each other, the ratio will be close to unity and the more degraded compressor does not get much less load. If the ratio $\frac{h(d_{i+2})}{h(d_{i+1})}$ is larger, compressor $i + 1$ will get less load than in the equal load approach.

In consequence, the algorithm does not necessarily assign the lowest load to the most degraded compressor. Instead, the algorithm makes the degradation of a compressor more similar to that of a compressor with the most similar degradation function, but in better health. If the degradation values are significantly different between the compressors, the algorithm is able to mitigate the degradation of the more degraded units. If the degradation values are similar, the algorithm still improves the more degraded unit.

Table 6.3: Parameters of factor-free component of degradation

	Compressor 1	Compressor 2	Compressor 3	Compressor 4	Compressor 5
γ_i	0.2×10^{-5}	0.067×10^{-5}	0.05×10^{-5}	0.04×10^{-5}	0.033×10^{-5}

The interpretation suggests an alternative to the collaboration strategy from Eq. (6.29). The alternative is:

$$m_i = \frac{M}{K} \exp \left(q \left(1 - \frac{h(d_K)}{h(d_i)} \right) \right) \quad (6.31a)$$

$$m_K = M - \sum_{i=1}^{K-1} m_i \quad (6.31b)$$

If the strategy from Eq. (6.31) is used, the compressor that is initially least degraded becomes significantly more degraded using this strategy, compared to the equal load strategy. Nevertheless, Compressor K remains the healthiest one of all the compressors, so that the assumption about the order of the compressors is fulfilled. The values of degradation of the remaining four compressors tend to the degradation of the least degraded unit, if the strategy from Eq. (6.31) is used. The approach from Eq. (6.31) will be called a *leader strategy*, because the degradation functions of the i -th compressor, $i = 1, \dots, K - 1$ are related to the degradation function of Compressor K , which can be considered the *leader*.

6.6.2 Results

The results from the collaboration strategy and the leader strategy will be now presented in simulation for a compressor station with five compressors, $K = 5$. The parameters of the degradation due to speed are the same for all compressors and are $a_i = 4 \times 10^{-5}$, $r_i = 200$ rpm. The parameters of the degradation due to time are $\alpha_i = 0.05$, $\beta_i = 0.05$, but the rates γ_i are different for each compressor and are presented in Table 6.3. This ensures that all the compressors reach the same value of degradation, but at different times. The simulation period was extended to 180 days to capture the varying degradation rates. The demand is $M = 850 \text{ kg s}^{-1}$.

6.6.2.1 Discussion of the collaboration strategy

The results are presented in Figs. 6.9, 6.10 and 6.11, ordered from the top according to the value of degradation. The top plot represents the values for the most degraded Compressor 1 with degradation d_1 and the bottom plot corresponds to the healthiest unit, Compressor 5, with degradation d_5 . The results of the leader strategy are the flows (Fig. 6.9a), the speeds (Fig. 6.10a), and the factor-based multiplier from Eq. (6.11) (Fig. 6.11a) for the five compressors.

The results show that, compared to the equal load assignment, the algorithm in the form of the leader strategy assigns lower loads to four degraded compressors and a higher load to the least degraded compressor. As a result, the speeds of Compressors 1-4 decrease compared to the equal load assignment of loads, and in consequence, the factor-based multiplier $F_{N,i}$ decreases as well (Fig. 6.11a).

Figure 6.12b presents the degradation function for all five compressors (black) if the collaboration strategy is used compared to the values obtained in the equal load case (green). It shows that the overall change of degradation function is primarily visible for Compressors 1 and 5.

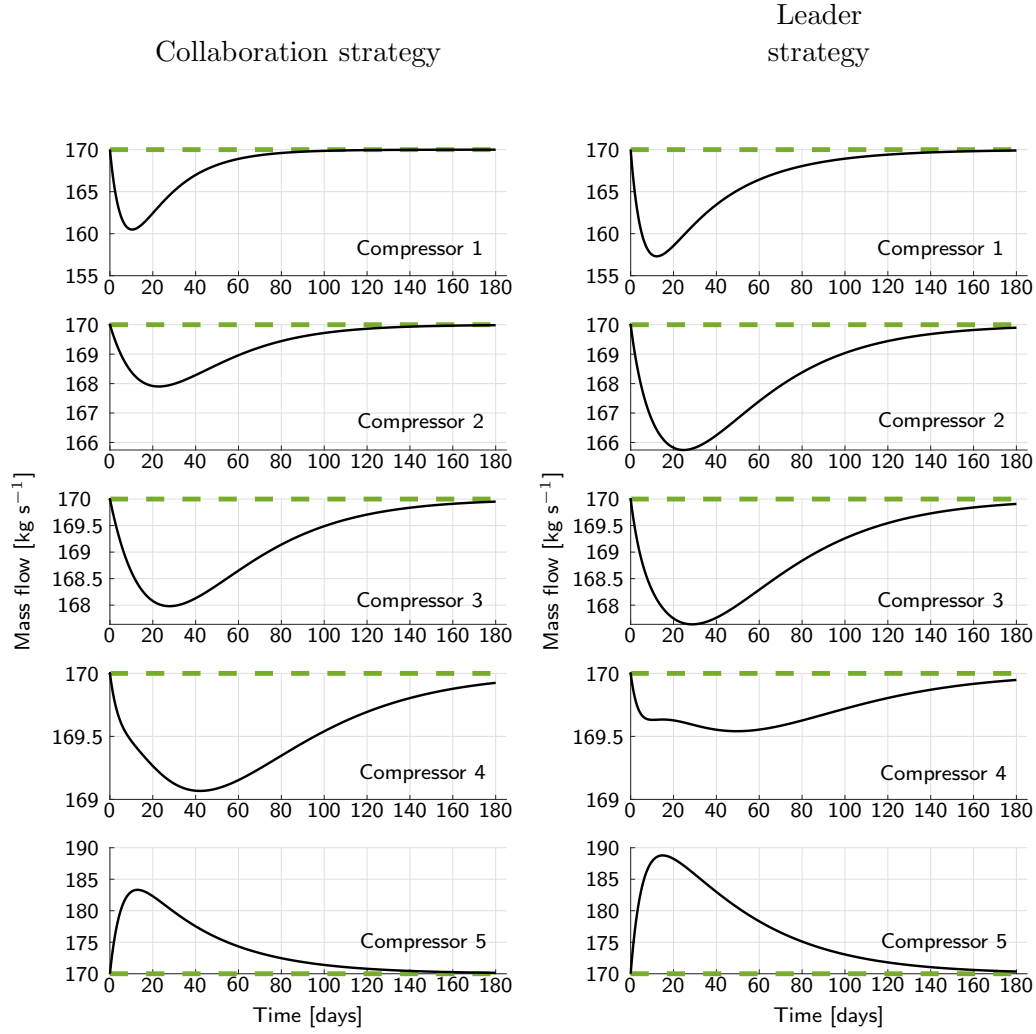
The ratios of degradation functions for the five compressors are in Fig. 6.13a. In this case, the maximal values of all the ratios can be ordered as:

$$\max \left(\frac{h(d_2)}{h(d_1)} \right) > \max \left(\frac{h(d_3)}{h(d_2)} \right) > \max \left(\frac{h(d_4)}{h(d_3)} \right) > \max \left(\frac{h(d_5)}{h(d_4)} \right) \quad (6.32)$$

Therefore, the Compressor 1 gets the smallest part of the equal load assignment. In consequence, it reaches the threshold of 93% 22 days later than in the equal load case (Table 6.4).

6.6.2.2 Discussion of the leader strategy

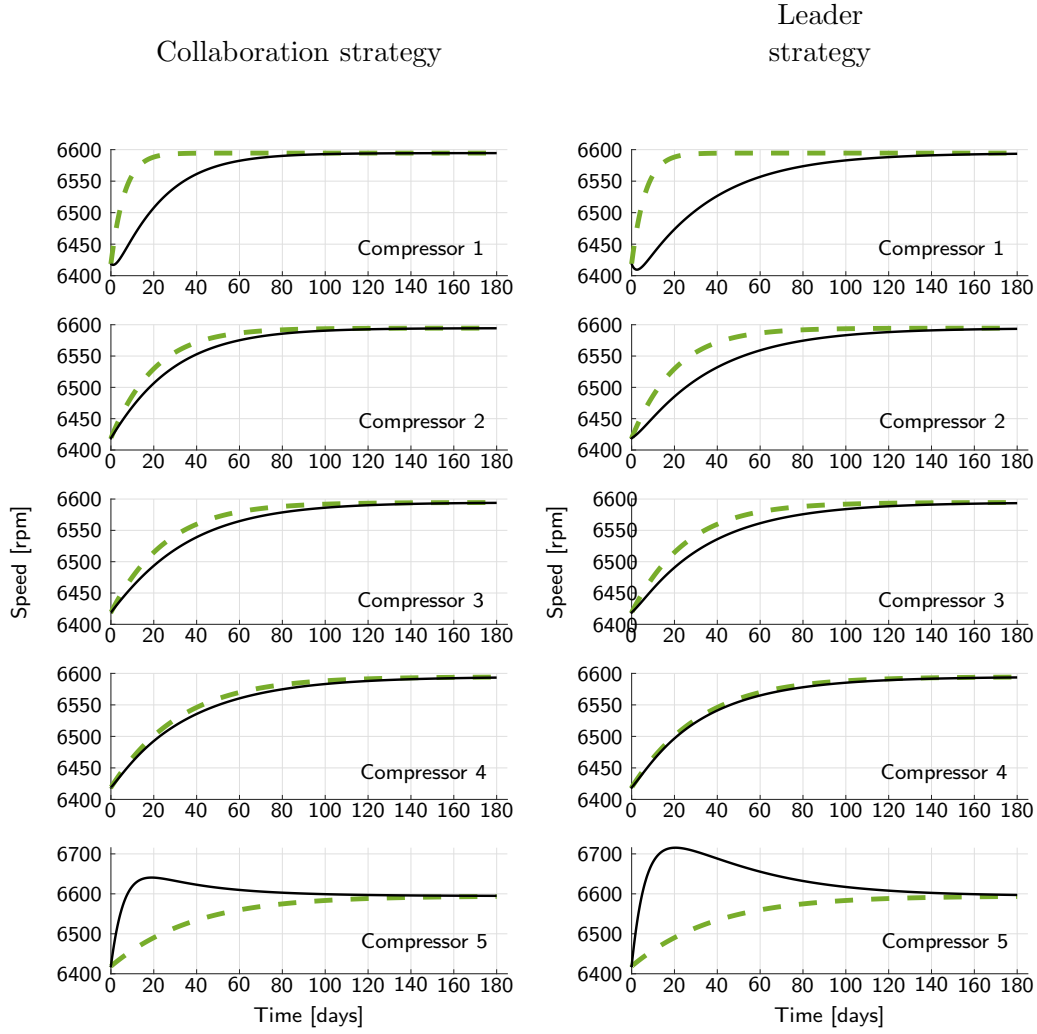
The results of the simulation from the leader strategy are: the flows in Fig. 6.9b, the speeds in Fig. 6.10b, and the degradation due to speed in Fig. 6.11b. Compressor 1 to 4 get less load, whereas Compressor 5 gets the largest load. In consequence, the speed of Compressor 1 to 4 decrease and the factor-based multiplier of degradation is also smaller than in equal load assignment.



(a) From Eq. (6.29)

(b) From Eq. (6.31)

Figure 6.9: Mass flows for five compressors, obtained from the collaboration strategy (—, left) and from the leader strategy (—, right) compared with equal load approach (---)



(a) From Eq. (6.29)

(b) From Eq. (6.31)

Figure 6.10: Speeds of five compressors, obtained from the collaboration strategy (—, left) and from the leader strategy (—, right) compared with equal load approach (---)

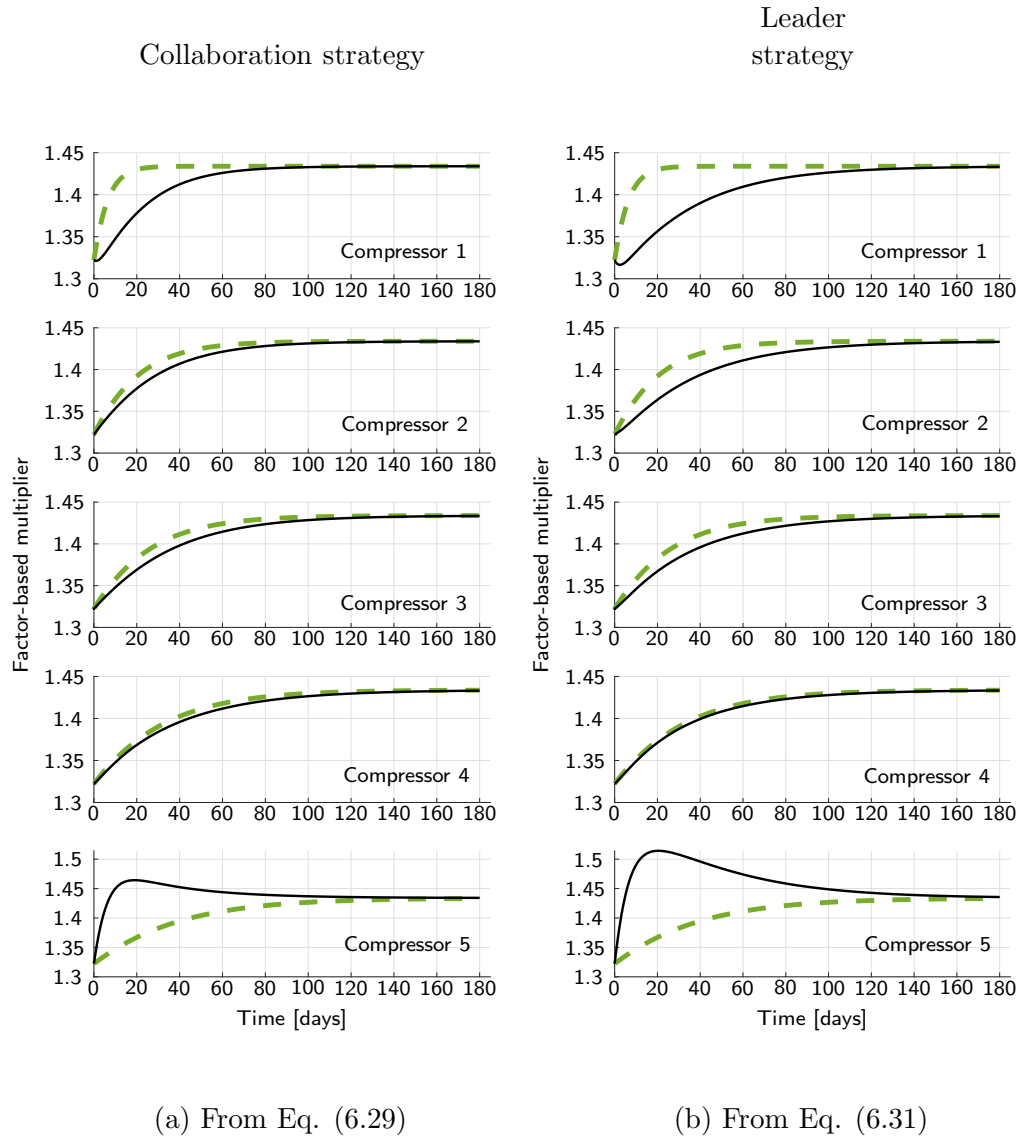
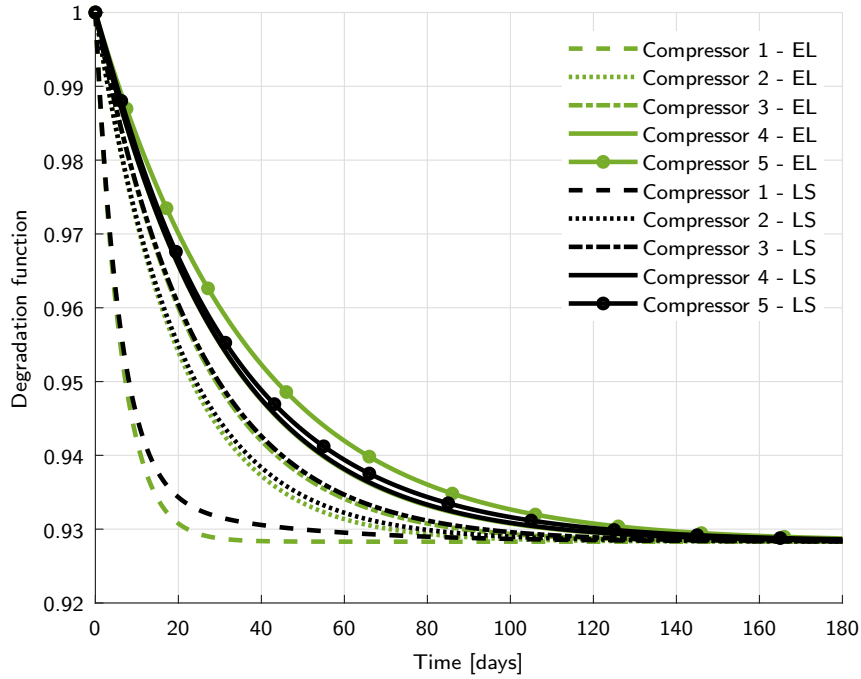
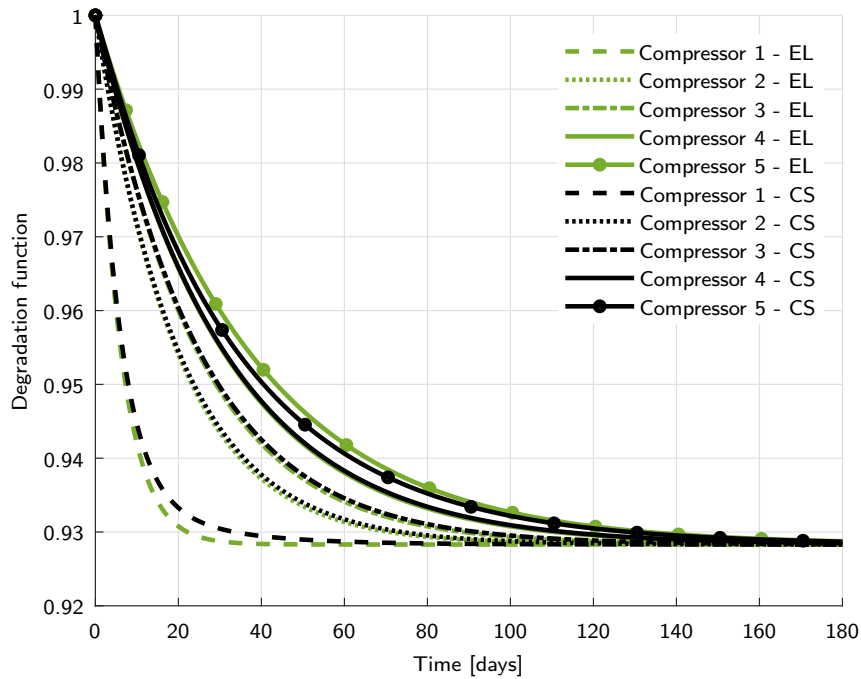


Figure 6.11: Factor-based multiplier of compressor degradation for five compressors, obtained from the collaboration strategy (—, left) and from the leader strategy (—, right) compared with equal load approach (---)



(a) Leader strategy from Eq. (6.31)



(b) Collaboration strategy from Eq. (6.29)

Figure 6.12: Degradation functions for five compressors, obtained from the collaboration strategy CS (bottom) and from the leader strategy LS (top) compared with equal load approach EL

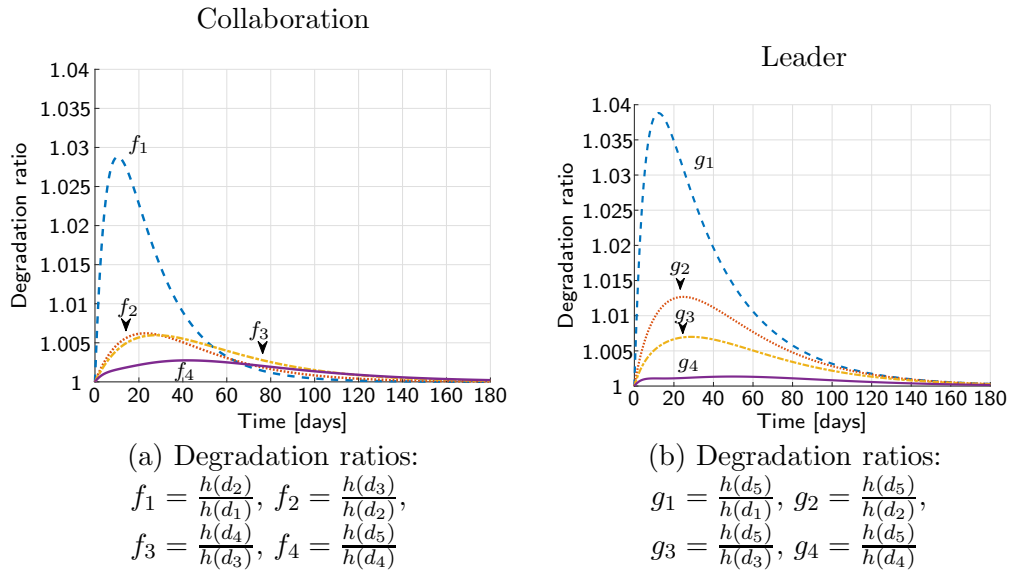


Figure 6.13: Degradation ratios for five compressors, obtained using the collaboration strategy (left) and the leader strategy (right)

The plot for Compressor 4 in Fig. 6.9b shows what happens when the ratios are close to each other. As indicated in Fig. 6.13b, the degradation ratio between Compressor 5 and Compressor 4 is the smallest. Moreover, it changes very slowly around day 20. As a result, the load assigned to Compressor 4 also changes very slowly around day 20. When the ratio increases after day 20, the load assigned to Compressor 4 decreases.

Figure 6.12a presents the degradation function for all five compressors (black) if the leader strategy is used compared to the values obtained in the equal load case (green). It shows that the overall change of degradation function is primarily visible for Compressors 1 and 5. The reasons for this behaviour will be analysed in Section 6.6.2.3 where the two approaches are compared.

6.6.2.3 Comparison between the collaboration and leader strategies

Figure 6.13 shows the degradation ratios in both the collaboration strategy (Fig. 6.13a) and the leader strategy (Fig. 6.13b). The dashed blue line shows the degradation ratio between Compressor 1 and Compressor 2 (Fig. 6.13a) between Compressor 1 and Compressor 5 (Fig. 6.13b). Since $d_2 > d_5$, the maximal ratio of the blue curve in Fig. 6.13a is smaller than in Fig. 6.13b. This means that using the leader strategy enables a lower load for the most degraded Compressor 1. This extended the time before reaching 93% threshold from 22 to 27 days for Compressor 1 (Table 6.4). The extension is also visible in Fig. 6.12. The dashed black line in Fig. 6.12b which corresponds to the collaboration strategy, reaches the threshold earlier than in Fig. 6.12a.

The dotted red line and the dash-dotted yellow line show how the degradation functions of Compressor 2 and 3 interact with each other and with the degradation function of Compressor 5, respectively. The ratio between the degradation functions of Compressor 3 and Compressor 2 (dotted red line in Fig. 6.13a) is smaller than the ratio between Compressor 5 and Compressor 2 (from the assumption in Eq. (6.28)) and shown with dotted red line in Fig. 6.13b. Therefore, the load-sharing strategy from the leader strategy helped Compressor 2 and extended the time before it reached the 93% threshold from three to seven days. The leader strategy also enabled a longer time for Compressor 3, but only two days longer. This is due to the fact that the ratio in Fig. 6.13b is only slightly higher than in Fig. 6.13a (dash-dotted yellow line).

Table 6.4: Number of days gained (+) or lost (-) for each compressor

	Compressor 1	Compressor 2	Compressor 3	Compressor 4	Compressor 5
Using the collaboration strategy	22	3	3	2	-5
Using the leader strategy	27	7	5	1.5	-10

Such behaviour is a result of increased load assigned to Compressor 5. Increasing the load intensifies the degradation of Compressor 5, which reached the threshold 10 days earlier than in equal load case (solid line with dots in Fig. 6.12a). The column for Compressor 4 in Table 6.4 also shows how the increased degradation of Compressor 5 affected Compressor 4. In addition to an increase of the load due to the decrease of load in Compressors 1 to 3, the leader strategy also resulted in an increase of the degradation function of Compressor 4 to make it closer to the degradation function of Compressor 5. This means that the ratio between the degradation functions of Compressor 5 and Compressor 4 decreased compared to the algorithm from the collaboration strategy (purple line in Fig. 6.13b and 6.13a). As a result, Compressor 4 improved only by 1.5 days instead of 2.

Fig 6.13a shows some of the ratio curves crossing over and intersecting with each other. This observation is of no great significance, so long as the ratios are above unity. The intersections indicate that in the collaboration strategy the compressors have similar degradation functions.

At the same time, Compressor 5 reached the threshold five days more quickly. It means that assigning the loads in order of increasing degradation (decreasing health) allows a mitigation of quickly degrading compressors. However, the degradation of the healthiest compressor was intensified and the compressor reached its 93% degradation threshold ten days earlier than in the collaboration strategy.

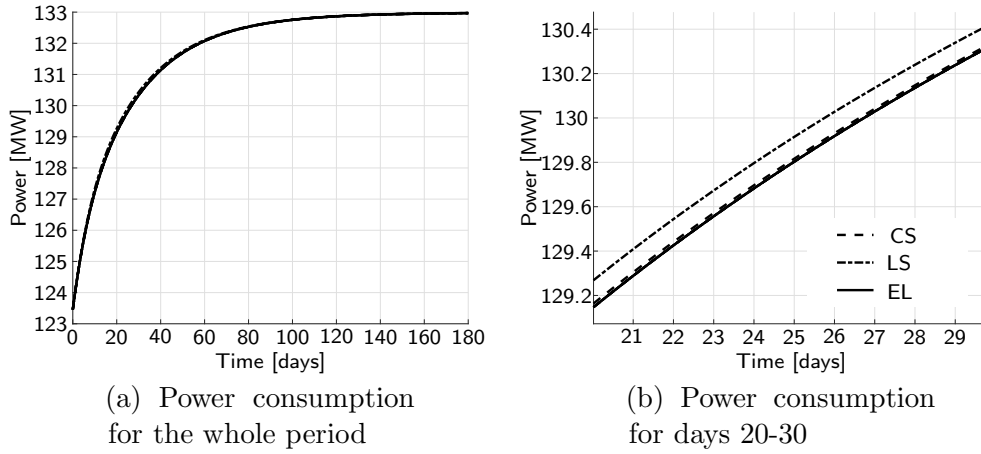


Figure 6.14: Power consumption of five compressors, obtained using the collaboration strategy (CS) and the leader strategy (LS), compared with the equal load approach (EL)

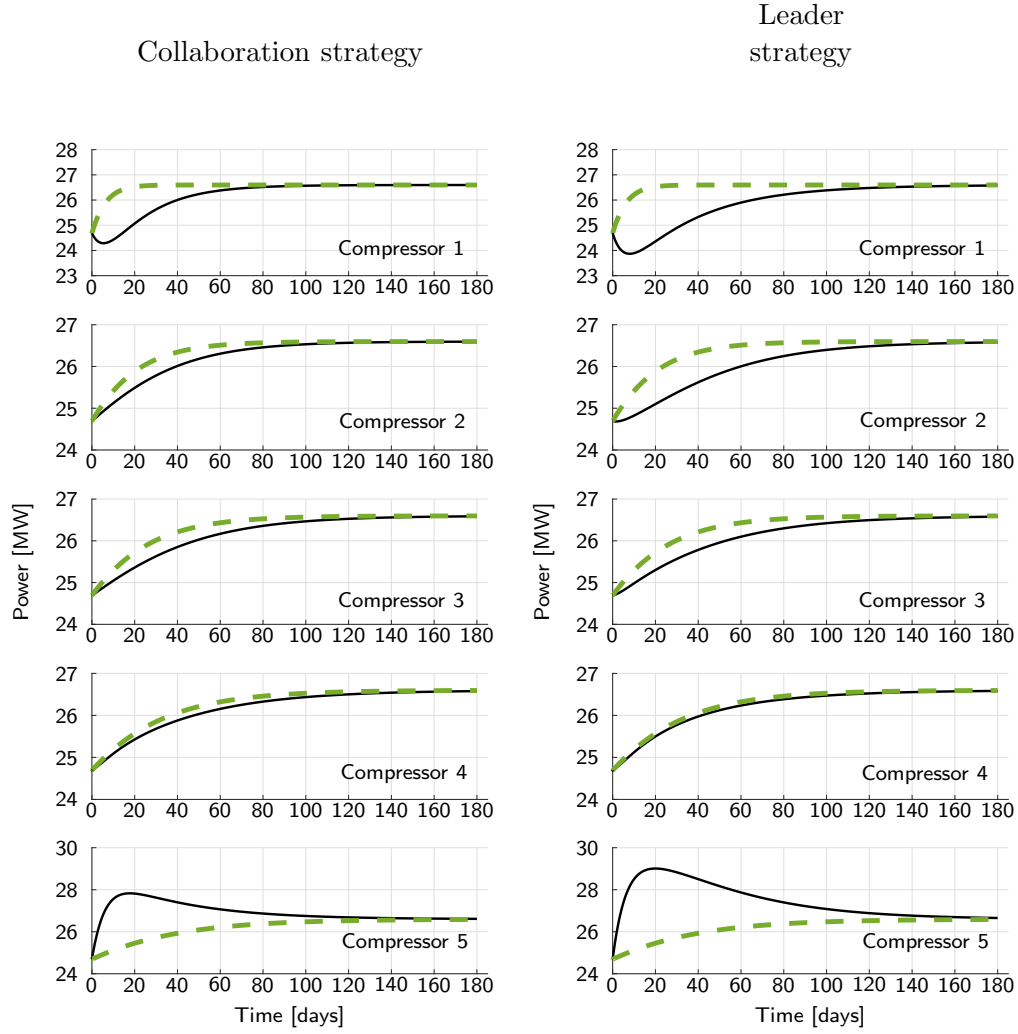
The results from the leader strategy confirm that the largest mitigation of degradation is achieved when the ratio of the degradation functions between two compressors is high. This is why using the leader strategy improved the most degraded compressor the most relative to the equal load strategy, and then the improvements were decreasing. Table 6.4 also shows that the larger improvements were done at the expense of the healthiest unit, which degrades more quickly than in the equal load strategy if the leader strategy is used.

6.6.2.4 Comparison between the collaboration and leader strategies - power consumption

Figure 6.14 shows the power consumption of the whole station in the whole period (Fig. 6.14a) and for days 20-30 (Fig. 6.14b). The power consumption increases throughout the whole period for the equal load approach and the two new strategies. However, Fig. 6.14a shows that the power consumption of the leader and collaboration strategies is similar to the power consumption obtained in the equal load approach. This means that by adjusting the loads, the new algorithms did not have a significant impact on the power consumption. At the same time, they mitigated the degradation of Compressor 1 to 4, as described in Section 6.6.1.

Figure 6.14b shows that even though both new strategies yield similar results, the power consumption in the collaboration strategy is more similar to the equal load approach. This is because the collaboration strategy assigns the loads using the ratios between two compressors with similar degradation functions, whereas the leader strategy always assigns the largest load to the least degraded compressor. Therefore, the power consumption of the least degraded compressor is larger in the leader strategy than in the collaboration strategy, which results in the overall increase of power consumption.

The power consumption of individual compressors is shown in Fig. 6.15. Both the collaboration and the leader strategy reduced the power consumption for Compressor 1 to 4 in relation to the equal load approach (the black lines, which correspond to the two strategies are below the green lines, which correspond to the equal load approach). In particular, the leader strategy reduced the power consumption of Compressor 1, but at the expense of the power consumption of Compressor 5. The difference between the power consumption of Compressor 1 in the leader and in the collaboration strategy is due to the different values of the ratios of degradation functions, as indicated in Fig. 6.13. A larger value of the ratio between the degradation functions of Compressor 1 and 5 was obtained in the leader strategy and resulted in a lower load assigned to the most degraded Compressor 1. A lower load enabled mitigation of degradation, as shown in Fig. 6.12. In turn, a smaller value of degradation means that less power is needed to compensate for the degraded characteristics. Conversely, the leader strategy assigned a larger load to Compressor 5, which intensified its degradation and increased the power consumption, compared to the collaboration strategy.



(a) From Eq. (6.29)

(b) From Eq. (6.31)

Figure 6.15: Power consumption for five compressors, obtained from the collaboration strategy (—, left) and from the leader strategy (—, right) compared with equal load approach (-.-)

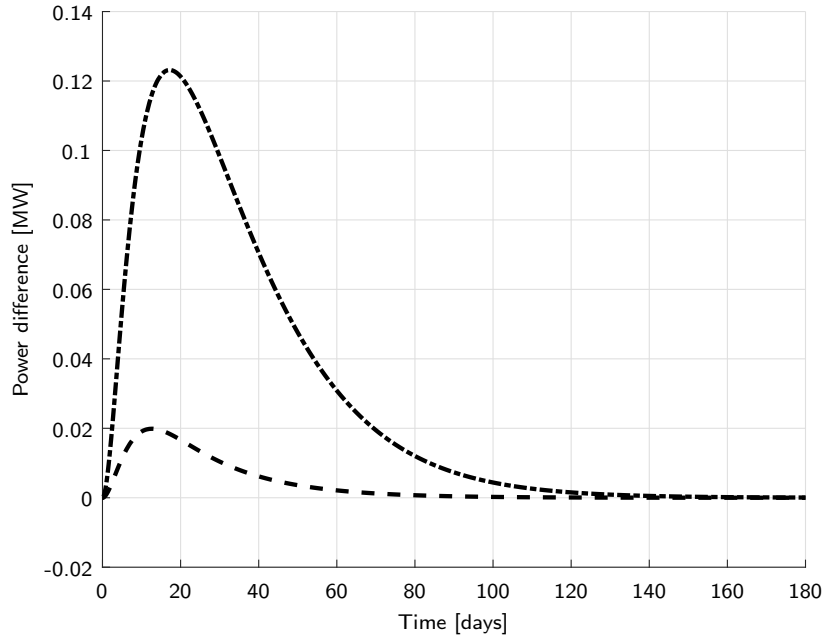


Figure 6.16: Difference of power consumption between the collaboration strategy and equal load approach (---), and between the leader strategy and equal load approach (-.-.-)

Figure 6.16 presents a comparison of power necessary to run the station obtained as a difference between the proposed strategies and the equal load approach for each day. The dashed line shows the difference between the power consumption needed for the collaboration and the equal load approaches. The dash-dotted line shows the difference between the power consumption in the leader strategy and the equal load approach. The collaboration strategy required almost the same amount of power throughout the whole period, whereas the leader strategy required more power until day 120.

In both cases the relative increase was calculated as:

$$\varepsilon = 100 \frac{W_X - W_{EL}}{W_{EL}} \quad (6.33)$$

where W_{EL} is the power in the equal load approach, W_X is the power in the proposed algorithms with X denoting either the leader or the collaboration strategy. The power increase ε was smaller than 0.1% for both strategies. Therefore, the approaches from this chapter allow a potential reduction in maintenance effort, because they are able degradation, especially of the most degraded compressor.

6.7 Chapter summary

The chapter tackles the third challenge from Chapter 1 concerning analysis of relationships within a system subject to degradation and fulfils the gap of knowledge 3.3.3, related to degradation management in a compressor station. The relationships in this chapter describe the ratios of the values of degradation of the compressors in a compressor station.

A new framework has been presented for load-sharing in a compressor station subject to degradation. The main objective of the framework was to use the relationships between the degradation indicators of the compressors to manage the degradation of the compressors. For this purpose, the model of degradation from Chapter 5 has been expanded to take influencing factors into account. The new model of degradation combines time-related degradation with a modifying factor including compressor speed as an influencing factor.

The framework for degradation management has been developed and analysed for the compressor station from Fig. 1.3. The parameters of the algorithm have been described and the influence of the parameters on the outcome of the load-sharing strategy has been evaluated for two objectives of degradation management:

- Mitigation of degradation,
- Intensification of degradation.

The performance of the new framework has also been analysed in a larger compressor station with five compressors. Two new strategies were proposed:

- A collaboration strategy, where degradation function of a compressor is compared with its less degraded neighbour in terms of the degradation functions;
- A leader strategy, where degradation function of a compressor is compared with the healthiest compressor in terms of the degradation functions.

It was shown that the new strategies allow the more degraded compressor to follow the less degraded compressor in terms of the degradation functions. As a result, the leader and the collaboration strategies do not significantly increase the power necessary to run the compressors, compared to the equal load approach.

Chapter 7

Load-sharing with power management

This chapter presents an alternative framework for optimal load-sharing in a compressor station with degrading compressors. In the alternative framework, the objective is to allocate the loads to compressors in a compressor station to minimise the power to run the station. The goal of this chapter is to model the influence of degradation on the power necessary to run a compressor station and analyse how the optimal load-sharing changes when the compressors are degrading. In this way, the chapter tackles the first and the third challenge indicated in Chapter 1.

Several approaches have been used to share the load among compressors working in parallel. If the compressors are identical, optimal load sharing is obtained by assigning the same load to each unit (Liptak, 2005). However, if the compressors are dissimilar, the equal load approach might not be optimal and should be adjusted to the nonlinear characteristics of the units (Carter, 1996; Wu, 1998; Wu et al., 2000b). A review of the approaches to the optimal operation of a compressor station was presented by Rios-Mercado and Borraz-Sanchez (2015) and summarised in Chapter 3. However, the existing approaches do not take degradation into account.

This chapter assumes that the individual characteristics in the form of the power curves of the compressors change with degradation. The load-sharing framework in this chapter minimizes the power necessary to run a compressor station when the compressors are subject to degradation. The degradation is explicitly included in the objective function. The chapter analyses the effects of the changes of the compressor characteristics on the solution of the optimal load-sharing. Another novel contribution is to show when compressors with dissimilar characteristics can be considered identical for optimisation purposes.

The chapter builds on the degradation modelling approaches from Chapter 4 and, similarly to Chapter 6, it uses a factor-based model of degradation to analyse the influence of degradation on the optimal load-sharing. The degradation data for the industrial case study is adapted from Chapter 5.

The rest of the chapter is structured as follows. Section 7.1 introduces the necessary tools for solving a nonlinear optimisation problem. Section 7.2 expands the factor-based degradation from Chapter 6 and presents its effects on the power to run the compressor. The solution of the optimisation is presented in Section 7.4 whereas Section 7.5 analyses the sensitivity of the solution. The effects of degradation on the solution are presented in Section 7.6. Finally, Section 7.7 applies the new framework to an industrial case study.

7.1 Optimisation background

The problem of optimal load-sharing in a compressor station is formulated as a nonlinear optimisation problem. The influence of degradation on the optimal load allocation is performed using sensitivity analysis.

7.1.1 Formulation of the optimisation problem

A nonlinear optimisation problem is to find the values of *decision variables* x such that minimise the *objective function* $f(x, p)$:

$$\min f(x, p) \tag{7.1}$$

and fulfil the constraints:

$$h(x, p) = \rho \quad (7.2a)$$

$$g(x, p) \leq r \quad (7.2b)$$

where p , ρ , r are vectors of constant parameters, $\rho = [\rho_i]$, $r = [r_j]$, and $p = [p_k]$. The function $h(x, p)$ represents *equality constraints* and $g(x, p)$ represents *inequality constraints*. The solution of the problem (7.1) will be denoted by x^* . If $g(x^*, p) = r$, then the constraints are called *active*.

7.1.2 Lagrangian function

The Lagrangian function for the problem with the objective function from Eq. (7.1) and constraints given by (7.2) is:

$$\mathcal{L}(x, \lambda, \mu, p, \rho, r) = f(x, p) + \lambda^\top (h(x, p) - \rho) + \mu^\top (g(x, p) - r) \quad (7.3)$$

where λ and μ are vectors of variables of the sizes corresponding to the number of equality constraint and inequality constraints, respectively, and $\mu \geq 0$. The variables λ and μ are called *dual variables* associated with equality and inequality constraints, respectively, as shown by the text book by Boyd and Vandenberghe (2009). The Lagrangian function from Eq. (7.3) will be used to solve the nonlinear optimisation problem from Section 7.1.1.

7.1.3 Optimality conditions

It is assumed that the problem with the objective function from Eq. (7.1) and the constraints from Eq. (7.2) is convex, i.e. the objective function is convex, the functions $g(x, p)$ are convex and the functions $h(x, p)$ are affine (Nocedal and Wright, 1999). If there exist x^* and a vector λ^* and a vector $\mu^* \geq 0$ such that the

conditions are fulfilled:

$$\nabla_x \mathcal{L}(x^*, \lambda^*, \mu^*, p, \rho, r) = 0 \quad (7.4a)$$

$$\mu^{*\top} (g(x^*, p) - r) = 0 \quad (7.4b)$$

$$h(x^*, p) - \rho = 0 \quad (7.4c)$$

$$g(x^*, p) - r \leq 0 \quad (7.4d)$$

$$\mu^* \leq 0 \quad (7.4e)$$

then the vectors x^* , λ^* and μ^* are optimal solution of the problem with the objective function from Eq. (7.1) and constraints from Eq. (7.2) (Boyd and Vandenberghe, 2009). The conditions given by Eq. (7.4) are called *Karush-Kuhn-Tucker conditions* (KKT conditions) (Nocedal and Wright, 1999).

7.1.4 Sensitivity analysis

Sensitivity describes the changes of the value of the objective function $f(x^*, p)$ if the parameters p , ρ , or r change. The sensitivity of the optimal objective function value $f(x^*, p)$ to the changes in the parameters ρ and r of the constraints is the negative value of the corresponding dual variables (Castillo et al., 2006), i.e.:

$$\frac{\partial f(x^*, p)}{\partial \rho_i} = -\lambda_i^* \quad (7.5)$$

and:

$$\frac{\partial f(x^*, p)}{\partial r_j} = -\mu_j^* \quad (7.6)$$

The interpretation of the sensitivity has been provided by Boyd and Vandenberghe (2009):

- If μ_j^* is large and r_j is replaced with a smaller value, then the optimal value $f(x^*, p)$ will increase significantly;
- If μ_j^* is small and r_j is replaced with a larger value, then the optimal value will not decrease too much;
- If λ_i^* is large and positive, and ρ_i is replaced with a smaller value, the optimal value $f(x^*, p)$ will increase;

- If λ_i^* is negative and $|\lambda_i^*|$ is large, and ρ_i is replaced with a larger value, the optimal value $f(x^*, p)$ will increase.

The sensitivity of the optimal objective function value $f(x^*, p)$ to the changes of parameter p correspond to a partial derivative of the Lagrangian with respect to the elements of vector p :

$$\frac{\partial \mathcal{L}(x^*, \lambda^*, \mu^*, p, \rho, r)}{\partial p_k} \quad (7.7)$$

evaluated at the optimal solution x^*, λ^*, μ^* (Castillo et al., 2006). The larger the value obtained from formula (7.7), the more sensitive the objective function in Eq. (7.1) is.

7.2 Compressor degradation

7.2.1 Influence of degradation on power and efficiency

Chapter 6 indicated that the power to run a compressor is inversely proportional to the efficiency of the compressor. The effect of efficiency degradation on power is captured by a multiplicative model of degradation from Eq. (6.8).

The model from Chapter 6 is now extended to take into account also additive degradation, which was considered among others by Xenos (2015). The power to run a compressor in degraded state is written as:

$$W^D = h_1(d)(am^2 + bm + c) + h_2(d) \quad (7.8)$$

where $h_1(d)$ and $h_2(d)$ represent a multiplicative and an additive degradation function, respectively. Degradation increases the power to run the compressors, so it is assumed that $h_1(d) > 1$ and $h_2(d) > 0$. The degradation indicator d is typically obtained from a condition monitoring system.

7.2.2 Factors influencing degradation

Chapter 4 described the significance of influencing factors in control systems and showed how they can be included in the models of the behaviour of a system. The factors influencing degradation of a compressor can be divided in two groups:

- Factors that are independent of the mass flow, for instance the ambient temperature;

- Factors that depend on the mass flow, such as compressor speed.

Chapter 6 considered the speed of the compressor as the main influencing factor. This chapter focuses on the power as a function of the flow, so it is necessary to express degradation as a function of the flow as well. The speed of a compressor can be expressed as a function of the flow. Figure 7.1 shows a plot of speed N as a function of mass flow m for constant head H . A single curve from Fig. 7.1 can be approximated as a polynomial function of order S of the flow (Egeland and Gravdahl, 2002):

$$N(m) = \sum_{i=1}^S a_i m^i \quad (7.9)$$

Therefore, if degradation is a polynomial function of the speed, it will be a polynomial function of the flow as well. The analysis of the influence of factor-dependent degradation on the optimisation will be conducted assuming degradation is a polynomial function of the mass flow:

$$d_m(m) = \sum_{i=1}^Z a_i m^i \quad (7.10)$$

where Z is the order of the polynomial.

7.3 Optimisation of power in a compressor station

7.3.1 Optimisation in up state

The objective of this chapter is to minimise the power necessary to run a compressor station with K compressors over a horizon T in the form suggested by Wu et al. (2000b). The objective function from Eq. (7.1) is:

$$\min_{m_{it}} \sum_{t=1}^T \sum_{i=1}^K W_{it} \quad (7.11)$$

where W_{it} is calculated according to Eq. (2.13) and recalled here:

$$W_{it} = a_{it} m_{it}^2 + b_{it} m_{it} + c_{it} \quad (7.12)$$

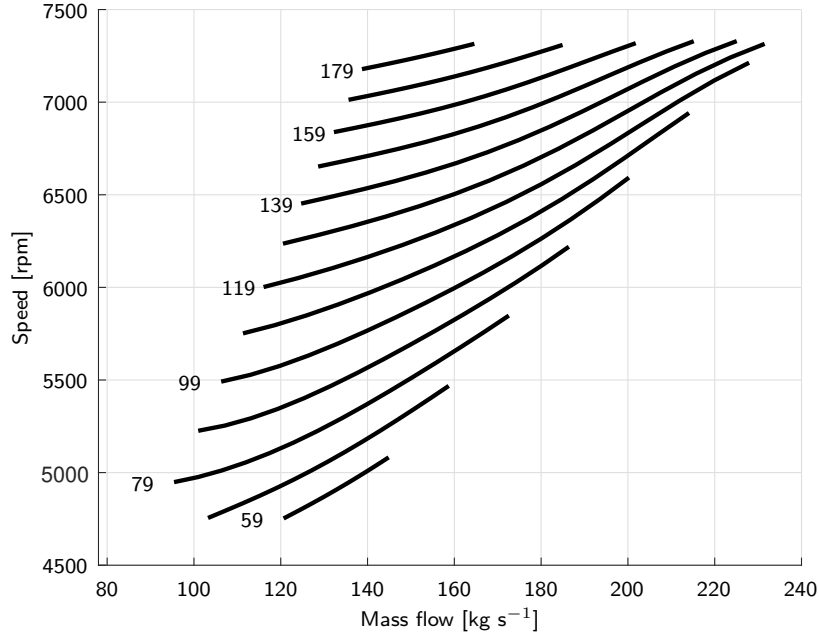


Figure 7.1: Compressor speed as a function of flow for various values of head in kJ kg^{-1} shown with labels

The power W_{it} calculated according to Eq. (7.12) captures the power curve from Fig. 2.6 in a given period t for the i -th compressor. The coefficients of the objective function might change between period t and period $t + 1$. The variable m_{it} indicates the flow set point for i -th compressor in period t .

The compressor station must satisfy the demand M_t in each period $t = 1, 2, \dots, T$:

$$\sum_{i=1}^K m_{it} = M_t \quad (7.13)$$

which provides an *equality constraint* corresponding to Eq. (7.2a). The load assigned to each compressor must be within the operating range defined by the compressor map in each period $t = 1, 2, \dots, T$:

$$m_{it}^{\min} \leq m_{it} \leq m_{it}^{\max} \quad (7.14)$$

which provides *inequality constraints* corresponding to Eq. (7.2b). The values $m_{it}^{\min}$ and $m_{it}^{\max}$ in the inequality (7.14) are obtained from a compressor map such as that shown in Fig. 2.5. For a fixed value of head, they are bounded by the surge line m_i^s ,

minimal speed line m_i^{Nmin} , maximal speed line m_i^{Nmax} and choke line m_i^c :

$$m_i^{\min} = \max\{m_i^s, m_i^{\text{Nmin}}\} \quad (7.15a)$$

$$m_i^{\max} = \min\{m_i^c, m_i^{\text{Nmax}}\} \quad (7.15b)$$

The values of minimal and maximal flows for each compressor $m_{it}^{\min}$, $m_{it}^{\max}$, as well as the demand M_t in the t -th period can change from period to period. A single-period optimisation is formulated by setting $T = 1$ and corresponds to day-ahead planning (Baumrucker and Biegler, 2010).

The objective function (7.11) is a convex function of the flows, because the power to run each compressor is assumed convex. The demand constraint from Eq. (7.13) and the mass constraints from Eq. (7.14) are linear. The objective function given by (7.11) and the constraints (7.13) and (7.14) is therefore a convex quadratic optimisation problem. Thus, the Karush-Kuhn-Tucker conditions from Eq. (7.4) can be used to find the optimal solution (Nocedal and Wright, 1999).

7.3.2 Optimisation in degraded state

The optimisation problem from Section 7.3.1 is now rewritten taking degradation into account. Using Eq. (7.8), the objective function from Eq. (7.11) becomes:

$$\min_{m_{i,j}} \sum_{t=1}^T \sum_{i=1}^K (h_1(d)W_{it} + h_2(d)) \quad (7.16)$$

where W_{it} is calculated from m_{it} according to Eq. (7.12) and d is a degradation indicator estimated over the horizon T . The constraints regarding the minimum and maximum flow are assumed independent of the degradation and the compressor operates in the same range as in up state. Similarly, the demand constraint is independent of degradation.

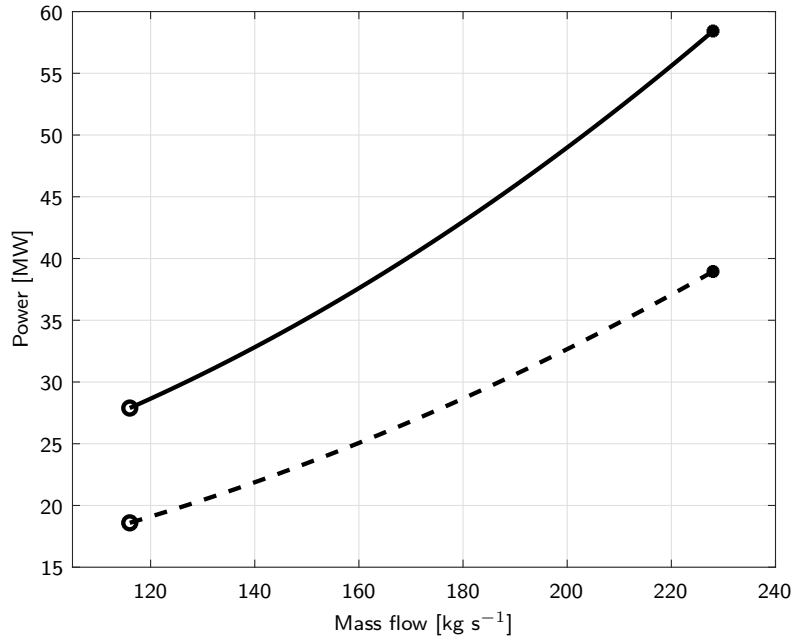


Figure 7.2: The power curves used for the interpretation of the optimisation problem for Compressor A (---) and for Compressor B (—) with the maximal flow $m_A^{\max} = m_B^{\max} = 228 \text{ kg s}^{-1}$ (●) and the minimal flow $m_A^{\min} = m_B^{\min} = 116 \text{ kg s}^{-1}$ (○)

Both $h_1(d)$ and $h_2(d)$ in Eq. (7.16) influence the objective function and can affect the convexity. A sensitivity analysis of the optimisation problem in up state provides insights into the influence of the degradation function $h_1(d)$ and $h_2(d)$ on the solution without the necessity of solving a potentially non-convex nonlinear optimisation with the objective function from Eq. (7.16).

7.3.3 Graphic interpretation of the optimisation problem

An interpretation of the optimisation problem from Section 7.3.1 is now described for a compressor station with two compressors, $K = 2$, and one period, $T = 1$. The two compressors have individual power versus flow characteristics shown in Fig. 7.2. The power curve W_B for Compressor B is steeper than the power curve for Compressor A W_A . Thus, the gradient with respect to the mass flow of the power curve W_B is larger than that for the power curve W_A .

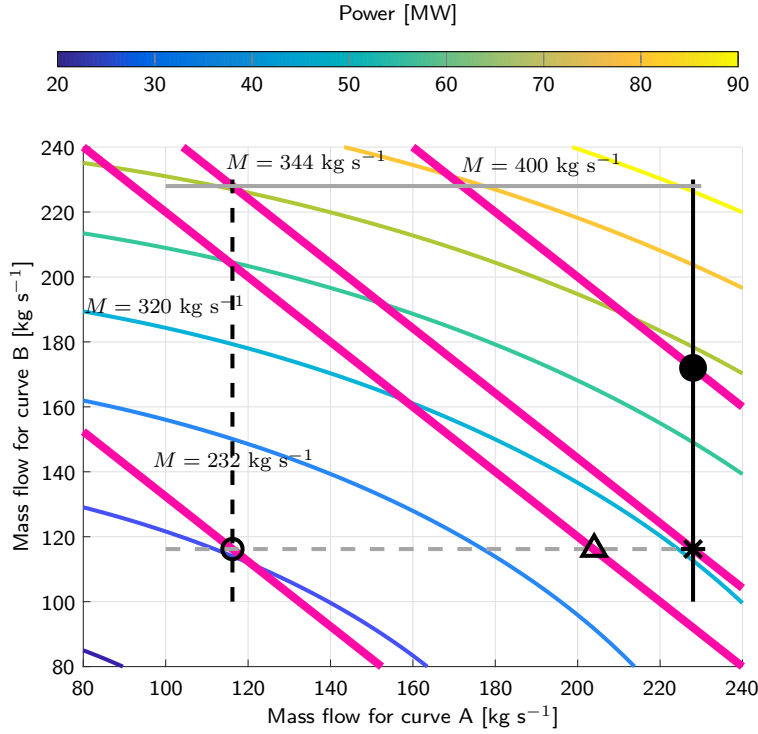


Figure 7.3: A graphical interpretation of the problem with the objective function (7.11), the demand constraint from Eq. (7.13) (—) and the mass constraints from Eq. (7.14): minimal (---) and maximal (—) value of the flow for Compressor A, minimal (---) and maximal (—) value of the flow for Compressor B. The black markers (○, △, *, ●) denote the solution for each value of demand

The problem from Section 7.4 consists in finding the flows for the compressors, such that the flows are within the operating ranges, the demand is satisfied, and the value of power to run the station is minimal. A graphic interpretation of the optimisation problem from Section 7.3.1 for a compressor station with Compressor A and B is depicted in Fig. 7.3 assuming $T = 1$ and $K = 2$. The colourful curves depict the objective function from Eq. (7.11) as a function of the mass flows through Compressors A (horizontal axis) and B (vertical axis). The black and grey lines correspond to the inequalities (7.14) and show the operating ranges for the compressors. The pink lines show the demand constraint from Eq. (7.13) for four values of the demand M_t (denoted with labels). Graphically, the problem from Section 7.4 consists in finding the flows on the pink lines, such that both are within the grey-black rectangle and the value of power is minimal.

For the compressor station with Compressor A and B with characteristics from Fig. 7.2, the assigned loads are:

- For $M = 232 \text{ kg s}^{-1}$ both compressors work at their lower limits ($\bullet$);
- For $M = 320 \text{ kg s}^{-1}$, Compressor B works at its lower limit ($\blacktriangle$);
- For $M = 344 \text{ kg s}^{-1}$, Compressor A works at its upper limit, whereas Compressor B works at its lower limit ($\ast$);
- For $M = 400 \text{ kg s}^{-1}$, Compressor A works at its upper limit ($\bullet$).

In particular, $M = 232 \text{ kg s}^{-1}$ is a special case that does not leave any flexibility in assigning the loads, because both compressors have to work at their lower limits to satisfy the demand. Such cases will not be considered in further analysis. The cases for $M = 320 \text{ kg s}^{-1}$, $M = 344 \text{ kg s}^{-1}$, and $M = 400 \text{ kg s}^{-1}$ indicate that the solution depends on the value of the demand M , as well as on the individual characteristics of the compressors.

7.4 Solving the optimisation problem

The Lagrangian function from Eq. (7.3) for the problem with the objective function (7.11) and constraints (7.13) and (7.14) with $T > 1$ is:

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^K \sum_{t=1}^T \left(a_{it} m_{it}^2 + b_{it} m_{it} + c_{it} \right) + \sum_{t=1}^T \lambda_t \left(\sum_{i=1}^K m_{it} - M_t \right) \\ & + \sum_{i=1}^K \left(\sum_{t=1}^T \mu_{it} \left(m_{it}^{\min} - m_{it} \right) \right) + \sum_{i=1}^K \left(\sum_{t=1}^T \gamma_{it} \left(m_{it} - m_{it}^{\max} \right) \right) \end{aligned} \quad (7.17)$$

with γ_{it} , μ_{it} , λ_t as *dual variables* (Nocedal and Wright, 1999). The variables μ_{it} correspond to the constraints related to the minimal flow of the compressors $m_{it}^{\min}$ and the variables γ_{it} correspond to the constraints related to the maximal flow of the compressors $m_{it}^{\max}$.

The KKT conditions from Eq. (7.4) are:

$$W'_i(m_{it}) + \lambda_t - \mu_{it} + \gamma_{it} = 0 \quad (7.18a)$$

$$\mu_{it}(m_{it}^{\min} - m_{it}) = 0 \quad (7.18b)$$

$$\gamma_{it}(m_{it} - m_{it}^{\max}) = 0 \quad (7.18c)$$

$$\sum_{i=1}^K m_{it} - M_t = 0 \quad (7.18d)$$

$$\mu_{it} \geq 0 \quad (7.18e)$$

$$\gamma_{it} \geq 0 \quad (7.18f)$$

where W'_i is the gradient of the power W_{it} with respect to mass flow m_{it} :

$$W'_{it}(m_{it}) = \frac{dW_{it}}{dm_{it}} \quad (7.19)$$

The power to run a compressor in this chapter is assumed to be approximated by a quadratic function of the flow from Eq. (7.12), and then:

$$W'_{it}(m_{it}) = 2a_{it}m_{it} + b_{it} \quad (7.20)$$

For a given period t , the conditions from Eq. (7.18) are equivalent to the conditions describing a single period optimisation problem, because Eq. (7.18) does not depend on two or more time instants. Thus, the problem of multi-period optimisation with constraints (7.13) and (7.14) is equivalent to solving a single-period optimisation in each period.

Equation (7.18) will be solved in two parts:

- If none of the compressors are at the limits;
- If some of the compressors are at the limits.

This section presents selected parts of the solution, whereas the Appendix C.2 presents the full solution to the problem with the objective function (7.11) and constraints (7.13) and (7.14).

7.4.1 None of the compressors are at the limits

If it assumed that none of the compressors are at their limits, $m_{it}^* \neq m_{it}^{\min}$ and $m_{it}^* \neq m_{it}^{\max}$ which means that the corresponding dual variables are zero, $\mu_{it}^* = 0$ and $\gamma_{it}^* = 0$, to fulfil Eq. (7.18b) and (7.18c). Then:

$$\lambda^* = - \frac{\sum_{i=1}^K \frac{W'_i \left(\frac{2M_t}{K} \right)}{a_{it}}}{\sum_{i=1}^K \frac{1}{a_{it}}} \quad (7.21)$$

and:

$$m_{it}^* = \frac{-\lambda^* - b_{it}}{2a_{it}} \quad (7.22)$$

The solution from Eq. (7.21) and (7.22) depends on the parameters of the quadratic function describing the power to run a compressor from Eq. (7.12). In particular, they will change if the derivative of the function changes. This aspect will be analysed in Section 7.6.

Equation (7.22) confirms also that equal load assignment is optimal if compressors are identical, as it yields:

$$m_{it}^* = \frac{M}{K} \quad (7.23)$$

An analysis of when the compressors can be considered identical will follow in Section 7.5.

7.4.2 Some compressors work at the limits

If some compressors work at their limits, the KKT conditions from Eq. (7.18) can be written:

$$W'_j(m_{jt}) + \lambda - \mu_{jt} + \gamma_{jt} = 0, \text{ for } j \in \{1, 2, \dots, k-1\} \quad (7.24a)$$

$$W'_k(m_{kt}) + \lambda - \mu_{kt} + \gamma_{kt} = 0 \quad (7.24b)$$

$$W'_v(m_{vt}) + \lambda = 0, \text{ for } v \in \{k+1, \dots, K\} \quad (7.24c)$$

$$\sum_{i=1}^K m_{it} - M_t = 0 \quad (7.24d)$$

$$\mu_{it}(m_{it}^{\min} - m_{it}) = 0 \quad (7.24e)$$

$$\gamma_{it}(m_{it} - m_{it}^{\max}) = 0 \quad (7.24f)$$

where the index j indicates the compressors working at the limits and the index v indicates unrestrained compressors, i.e. compressors which work not at their limits. The objective of this section is to derive conditions that cause compressor denoted with index k to work at its limits.

It is assumed that there exist at least two compressors that are not at their limits. The case with one compressor not at limits is a special case, because it does not leave any flexibility for the unrestrained compressor. This case will not be considered in this chapter, but a solution is presented in Appendix C.2.

The conditions when the compressor k works at its limit are:

$$\mu_{kt}^* = W'_k(m_{kt}^*) - W'_v(m_{vt}^*) > 0 \quad (7.25)$$

if the compressor k works at its minimal limit $m_{kt}^* = m_{kt}^{\min}$ and:

$$\gamma_{kt}^* = -W'_k(m_{kt}) + W'_v(m_{vt}^*) > 0 \quad (7.26)$$

if the compressor k works at its maximal limit $m_{kt}^* = m_{kt}^{\max}$. The values m_{vt}^* for the unrestrained compressors can be obtained from Eq. (7.24d) with Eq. (7.24c). Therefore, the compressor k will work at its boundary depending on the relationship between the gradients W'_k and W'_v at their respective flows. For instance, for Eq.

(7.25) to be a valid solution for μ_{kt}^* , the inequality must be fulfilled:

$$W'_k(m_{kt}^*) > W'_v(m_{vt}^*) \quad (7.27)$$

Inequality (7.27) guarantees that μ_{kt}^* is greater than zero. Therefore, the compressor k will work at its minimal bound if the gradient of its power curve with respect to mass flow at $m_k^{\min}$ is larger than the gradient of the power curve of an unrestrained compressor v at its flow.

The values obtained from Eq. (7.25) and (7.26) will be used to analyse the sensitivity of the optimisation in Section 7.7 according to the interpretation from Section 7.1.4.

7.5 Analysis of the solutions

The values of the dual variables derived in Section 7.4.2 have the form related to the gradient of the power curves with respect to the respective mass flows. To satisfy the requirement that the dual variables μ_{it} and γ_{it} are positive, the following condition must be fulfilled:

$$W'_i(x) \geq W'_j(y) \quad (7.28)$$

Equation (7.28) shows that an inequality between the gradients of the power curves with respect to the flow at two points x and y must hold. The inequality will be fulfilled if $W_j(y)$ is locally flatter around y than $W_i(x)$ around x , regardless of the values of x and y . This indicates that if the compressors are identical in up state and degradation affects the gradient, the solution will change. The change of the solution will depend on the kind of degradation and will be analysed in Section 7.6.

7.5.1 Sensitivity analysis

The sensitivity of the load-sharing problem with the objective function (7.11), the demand constraint from Eq. (7.13) and the mass constraints from Eq. (7.14) is now analysed. The changes due to degradation will affect both the value of the objective function and the optimal solution. As degradation influences the objective function, the sensitivity analysis provides a starting point for an analysis of the influence of degradation on the optimal load-sharing problem.

Table 7.1: Derivatives of the Lagrangian with respect to the parameters of the objective function from Eq. (7.11) at the optimal solution m_i^* , $i = 1, 2, \dots, K$

$$\text{Compressor } i \mid \frac{\partial \mathcal{L}}{\partial a_{it}} = (m_{it}^*)^2 \quad \frac{\partial \mathcal{L}}{\partial b_{it}} = m_{it}^* \quad \frac{\partial \mathcal{L}}{\partial c_{it}} = 1$$

7.5.1.1 Sensitivity of the objective function to changes in its parameters

The sensitivity to the changes of a parameter of the objective function can be obtained by analysis of a derivative of the Lagrangian function (Castillo et al., 2008). As W_{it} is a quadratic function of the respective flows, there are $3K$ parameters to consider: a_{it} , b_{it} , c_{it} . The derivatives of the Lagrangian calculated at the solution m_{it}^* are gathered in Table 7.1.

The analysis of Table 7.1 indicates that the value of the objective function is sensitive to changes in all the parameters, because all the derivatives are non-zero. As all the derivatives are positive, a larger value of one of the parameters would result in a larger value of the objective function (Castillo et al., 2008).

The analysis of Table 7.1 indicates that the value of the objective function will change if the parameters change. Graphically, if $T = 1$ and $K = 2$, the changes of the parameters of the objective function correspond to modifying the colourful curves in Fig. 7.3. Section 7.5.1.4 will analyse the influence of the changes of the parameters of the objective function on the solution, if the changes are due to degradation.

7.5.1.2 Sensitivity of the objective function to changes in the inequality constraints

The sensitivity to the changes in the parameters of the inequality constraints (7.2) is described by the value of non-zero μ_i and γ_i , if the inequalities are given in form of formula (7.15) (Boyd and Vandenberghe, 2009). The inequality for minimal flow is rewritten as:

$$-m_{it} \leq -m_{it}^{\min} \tag{7.29}$$

If any of $(-m_{it}^{\min})$ is decreased, i.e. if $m_{it}^{\min}$ is increased, the value of the objective function would increase (Boyd and Vandenberghe, 2009). The increment of the objective function will depend on the absolute value of μ_i^* . Similarly, decreasing $m_{it}^{\max}$ if γ_i^* is not zero will increase the value of the objective function.

Graphically, if $T = 1$ and $K = 2$, the changes in the inequality constraints are related to shrinking the grey-black operating range in Fig. 7.3. For instance, for $M = 320 \text{ kg s}^{-1}$, increasing the minimal flow for Compressor B would move the dashed grey curve upwards along the vertical axis. Therefore, the value of the objective function will increase, because the dashed grey curve will cross the pink line closer to the yellow curve denoting the maximal value of the objective function. At the same time, changes of the other grey or black lines in Fig. 7.3 would not affect the value of the objective function. This is because these constraints are not active and the corresponding dual variables are zero.

7.5.1.3 Sensitivity of the value of the objective function to demand

The sensitivity of an optimisation problem to the changes in the parameters of the equality constraint depends on the sign and the absolute value of the corresponding dual variable λ_t^* (Boyd and Vandenberghe, 2009). The values of λ_t^* for the problem (7.18) with the demand constraint from Eq. (7.13) are strictly negative. Therefore, increasing the demand M_t would increase the value of the objective function (Boyd and Vandenberghe, 2009).

Graphically, if $T = 1$ and $K = 2$, the changes of the demand correspond to shifting the pink lines vertically in Fig. 7.3. If the demand increases from $M = 320 \text{ kg s}^{-1}$ to $M = 400 \text{ kg s}^{-1}$, the value of the objective function increases. This is because shifting the pink lines vertically moves them closer to the maximal value of the objective function denoted with yellow.

7.5.1.4 Sensitivity of the solution to changes in parameters of the objective function

The solution given by Eq. (7.22) depends on the value of parameters a_i and b_i of the objective function. Thus, the value of the solution will change if either of the parameters a_i and b_i change, but will remain the same if c_i changes (Castillo et al., 2008).

Graphically, if $T = 1$ and $K = 2$, the changes of the parameters of the objective function correspond to modifying the colourful curves in Fig. 7.3. Changes of parameters a_i and b_i correspond to scaling of the colourful curves and therefore may affect the solution. Changes in parameter c_i corresponds to changing the colour of colourful curves in Fig. 7.3. For instance, the orange curve will be yellow if c_i is changed to a larger value, or conversely, it will be green if c_i is changed to a smaller value. Therefore, the solution will remain optimal.

From the perspective of optimisation, the power curves W_A and W_B given as:

$$W_i = am + bm + c_i \quad (7.30)$$

where $i \in \{A, B\}$ are considered identical. This means that the load allocation will not change if one power curve is shifted vertically in Fig. 7.2. In particular, the equal load allocation remains optimal if one compressor has a power curve shifted vertically with respect to the other, for example due to degradation.

7.6 Influence of degradation on the optimisation

As indicated in Section 7.2, degradation would influence the power necessary to run the compressor station. The additive and multiplicative degradation functions will influence the problem in different ways, depending on the influencing factors.

7.6.1 Factor-free degradation

Additive and multiplicative degradation functions $h_2(d)$ and $h_1(d)$ affect the power in two different ways and therefore will have a different influence on the optimisation problem with the objective function (7.11) and constraints (7.13) and (7.14).

7.6.1.1 Additive factor-free degradation

The additive factor-free degradation function $h_2(d)$ corresponds to a uniform degradation of k -th compressor across the whole operating range. It shifts the power curve W_{kt} up the vertical axis of Fig. 7.3:

$$W_{kt}^D = W_{kt} + h_2(d) \quad (7.31)$$

where W_{kt}^D is the power in degraded state. Additive degradation is equivalent to shifting the objective function vertically, so $h_2(d)$ must be expressed in Watts. It represents an overall increase of power if a compressor is subject to degradation. This model was used by Xenos (2015) for maintenance planning in a Norwegian compressor station.

The power of a compressor is a quadratic function of the flow and additive degradation will change the value of parameter c :

$$W_{kt}^D = a_{kt}m_{kt}^2 + b_{kt}m_{kt} + c_{kt} + h_2(d) \quad (7.32)$$

The optimal solution to the load-sharing problems is independent of the value of c_{kt} . Thus, additive factor-free degradation $h_2(d)$ will not change the solution of the load-sharing problem. It will, however, increase the power necessary to run the compressors, as indicated in the fourth column of Table 7.1.

7.6.1.2 Multiplicative factor-free degradation

A factor-free multiplicative degradation of form:

$$h_1(d) = \beta \quad (7.33)$$

corresponds to scaling the power to run the k -th compressor in up-state, W_{kt} by a certain value $\beta > 1$ to obtain the power in degraded state, W_{kt}^D :

$$W_{kt}^D = \beta a_{kt}m_{kt}^2 + \beta b_{kt}m_{kt} + \beta c_{kt} \quad (7.34)$$

The multiplicative degradation function $h_1(d)$ is dimensionless.

The solution from Eq. (7.21) and Eq. (7.22) changes if one the compressor is affected by the degradation function from Eq. (7.33). The loads assigned in the multiplicative case are different than the loads obtained in the additive case. The reason for this difference is that the multiplicative degradation affects parameters a_{kt} , b_{kt} of the power curve W_{kt} , and thus the gradient of the degraded power curve $(W_{kt}^D)'$ will change:

$$(W_{kt}^D)' = \beta W_{kt}' \quad (7.35)$$

Therefore, if multiplicative degradation takes place, it is necessary to consider the possibility of compressor k working at its limits.

7.6.1.3 Additive and multiplicative factor-free degradation

Degradation might also have a two-fold influence on the power and include both the additive and multiplicative degradation functions:

$$W_{kt}^D = h_1(d)W_{kt} + h_2(d) \quad (7.36)$$

A combination of the additive and multiplicative degradation can be seen in Fig. 7.2 as first scaling the power due to multiplicative degradation function $h_1(d)$, and then shifting it up the vertical axis as per the additive degradation $h_2(d)$. As indicated in Section 7.5.1.4, shifting the power curves up the vertical axis of Fig. 7.2 does not influence the solution. Thus, from the perspective of obtaining a solution, a compressor subject to both additive and multiplicative factor-free degradation can be treated as a compressor subject to multiplicative degradation.

7.6.2 Factor-based degradation

Additive factor-based degradation will shift the power curve up the vertical axis similarly to the additive factor-free degradation. If the factors influencing degradation are independent of the flow, the additive factor-based degradation corresponds to shifting the power curve up the vertical axis in the same way as with factor-free additive degradation. Only the value of the parameter c_{kt} will be changed and thus the solution remains the same. If the additive degradation depends on the flow through the compressor, it will influence the steepness of the power curve.

Multiplicative factor-based degradation changes the optimisation problem, regardless of the form of the degradation function $h_1(d)$.

7.6.2.1 Linear additive factor-based degradation

If the additive degradation function $h_2(d)$ is a linear function of the flow, then:

$$h_2(d) = \alpha_0 m_1 \quad (7.37)$$

with $\alpha_0 > 0$. The parameter α_0 has dimensions of $\text{m}^2 \text{s}^{-2}$. In this case, the power in degraded state becomes:

$$W_{kt}^D = a_{kt}m_{kt}^2 + (b_{kt} + \alpha_0)m_{kt} + c_{kt} \quad (7.38)$$

with derivative with respect to the mass flow m_{kt} :

$$(W_{kt}^D)' = 2a_{kt}m_{kt} + \alpha_0 + b_{kt} \quad (7.39)$$

Thus the solution in Eq. (7.21) and Eq. (7.22) will be influenced, because the coefficient of m_{kt} changed.

Moreover, as the additive factor-based degradation $h_2(d) = \alpha_0 m_1$ influences the steepness, it is necessary to check whether the k -th compressor may be at the limits.

7.6.2.2 Quadratic additive factor-based degradation

If degradation is a quadratic function of the flow, then:

$$h_2(d) = \alpha_1 m_{kt}^2 \quad (7.40)$$

with $\alpha_1 > 0$ in $\text{m}^2 \text{kg}^{-1} \text{s}^{-1}$. In this case, the power in the degraded state becomes:

$$W_{kt}^D = (a_{kt} + \alpha_1)m_{kt}^2 + b_{kt}m_{kt} + c_{kt} \quad (7.41)$$

Thus the solution in Eq. (7.21) and Eq. (7.22) will be influenced, because the coefficient of m_{kt} changes.

Moreover, the conditions for the k -th compressor to work at its limits may be active.

7.6.2.3 Other types of degradation

If degradation is a function of the flow such as a polynomial of order higher than two, or a non-polynomial function, it changes the optimisation problem. In particular, the minimised function from Eq. (7.11) might be no longer quadratic. Similarly to Section 7.6.2.1, if the influencing factors do not depend on the flow, multiplicative factor-based degradation is equivalent to multiplicative factor-free degradation. Any other kind of the degradation function $h_1(d)$, which depends on the flow, would necessitate a new solution to the optimisation problem. For instance, the degradation function is an affine function of the flow $h_1(d) = \beta m_{kt}$, the problem from Eq. (7.11) would not be quadratic any more.

If the objective function from Eq. (7.11) is not quadratic any more, it is still possible to use the KKT conditions from Eq. (7.18) to solve the problem. However, the new objective function might be non-convex and the solution might be non-unique. In particular, this means that there are several assignments of the flows to the compressors and the power consumption in all these assignments would be identical. An investigation of these matters is beyond the scope of the current thesis and is recommended for future work.

7.6.3 Synopsis of the influence of degradation

Table 7.2 summarizes the findings of this section classifying degradation with respect to its dependence on the flow.

Table 7.2: Cases considered for analysis of the influence of degradation on the objective function and the solution

	Independent from the flow		Additive and linear in the flow	Dependent on the flow		Multiplicative
	Additive	Multiplicative		Additive and quadratic in the flow	Additive and neither quadratic, nor linear in the flow	
Affects the value of the objective function	✓	✓	✓	✓	✓	✓
Affects the solution, but keeps the quadratic character of the optimisation problem		✓	✓	✓		
Affects the solution, changes the character of the optimisation problem					✓	✓

Regardless of the type, degradation will always change the value of the objective function, which means that the power to run a compressor station with compressors in degraded state increases in relation to the compressor station not affected by degradation. However, in some case the solution obtained for the compressor station without degradation remains optimal if the compressors degrade. The solution will remain optimal if the degradation is independent of the flow and affects the power in an additive manner. For instance, if degradation depends on the ambient temperature, the compressors will require more power to run, but the load assignment will remain the same regardless of the temperature. In most cases, however, the solution will change, because degradation will affect the gradient of the power with respect to the mass flow. This happens when additive degradation depends on the flow or when degradation affects the power in a multiplicative way.

A special case is considered if degradation is factor-free, but has a multiplicative influence on the power. The case of factor-free multiplicative degradation is common, because this type of degradation reflects the influence of the loss of efficiency due to fouling in a compressor. The effects of fouling on the optimal load sharing will be presented in Section 7.7.

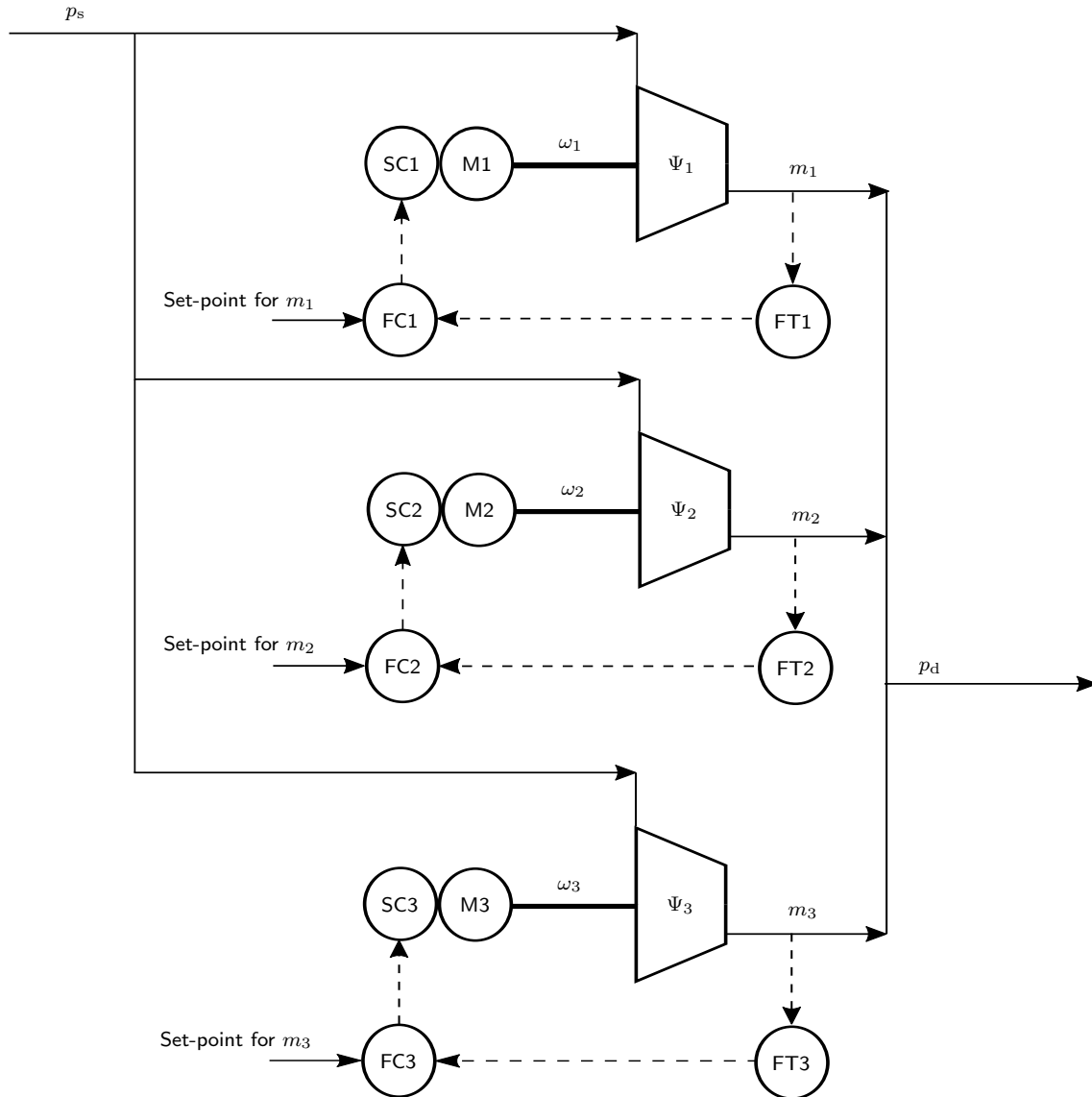


Figure 7.4: Compressor station with three compressors connected to the same pipeline

7.7 Industrial case study

The load-sharing problem is analysed in a compressor station with three compressors from Fig. 7.4, $K = 3$ over a period of 90 days, $T = 90$. The compressors are identical in up state. Each of the compressors has the characteristics in up state from Fig. 6.2 and power characteristic as a function of mass flow is given by Fig. 6.4.

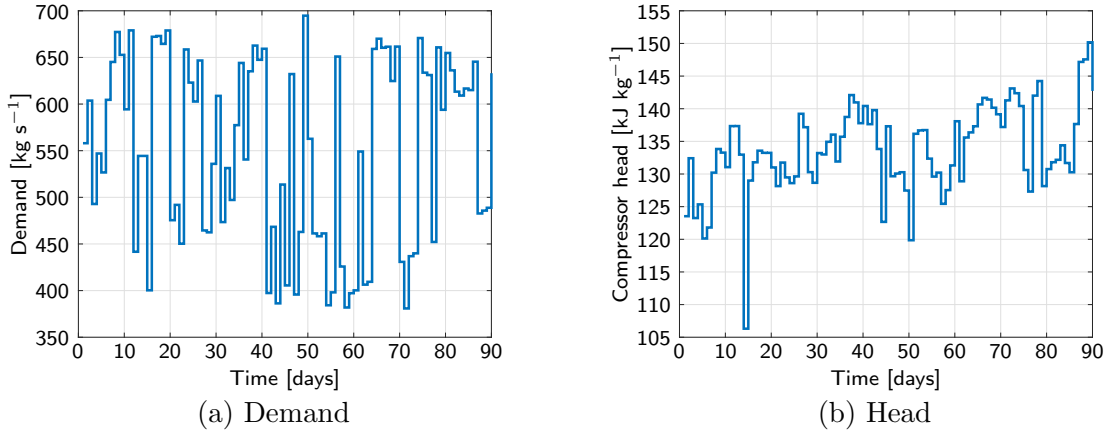


Figure 7.5: Compressor head and demand for the whole station over the period of 90 days. The values were obtained from Xenos (2015) using software from Rohatgi (2018)

It is assumed that all the compressors are connected to the same suction and discharge pipelines and thus work at the same pressures p_d and p_s . Therefore, all three compressors work at the same head in each day, because the gas properties are assumed constant throughout the whole period. Figure 7.5a shows the demand, whereas Fig. 7.5b shows the head over a period of 90 days. The gas properties and the data for the demand and the head were adapted from Xenos (2015) using software from Rohatgi (2018). The controllers FC1, FC2, and FC3 ensure that the flows are equal to their set-points. The calculation of these set-points taking compressor degradation into account is the objective of this section.

7.7.1 Degradation modelling

The degradation data for the current work were adapted from Chapter 5 and come from a period of 90 days. In the current chapter, both the predicted degradation and the uncertainty bounds are used to analyse how the solution of the load-sharing problem is affected by degradation.

The degradation d is quantified using a factor-free degradation indicator introduced in Chapter 5 and recalled here:

$$d = \frac{\eta - \eta_D}{\eta} \quad (7.42)$$

where η is the efficiency in up-state and η_D is the efficiency in degraded state. Equa-

tion (7.42) can be transformed as:

$$\eta_D = (1 - d)\eta \quad (7.43)$$

From Eq. (2.11), the power in degraded state W_D is inversely proportional to the efficiency η_D :

$$W_D = \frac{Hm}{\eta_D} = \frac{Hm}{(1 - d)\eta} = \frac{1}{(1 - d)} \cdot W \quad (7.44)$$

Thus, $h_1(d)$ was obtained as:

$$h_1(d) = \frac{1}{(1 - d)} \quad (7.45)$$

Equation (7.45) shows that $h_1(d)$ is a factor-free multiplicative degradation function, if d is a factor-free degradation indicator. Thus, it can be expected that, depending on the demand on a given day, one of the compressors is forced to work at its boundary, if the optimisation problem from Section 7.3.1 is affected by $h_1(d_i)$ from Eq. (7.45).

Figure 7.6 shows the course of degradation d calculated from Eq. (7.42) for 90 days. The results of approximation from Chapter 5 were used assuming that degradation is positive. To show the performance of the optimisation, it was assumed that each compressor has a different value of degradation. For that purpose, the upper bound d^{upper} , the lower bound d^{lower} , and the expected value d^{expected} were used. The upper bound d^{upper} (●) has been assigned to Compressor 1, the expected value d^{expected} (●) has been assigned to Compressor 2, and the lower bound d^{lower} (●) corresponds to Compressor 3.

It is assumed that the degradation function for additive degradation h_2 is zero. This is because a factor-based model of degradation was not available and a factor-free additive degradation would not change the solution, as shown earlier in this chapter.

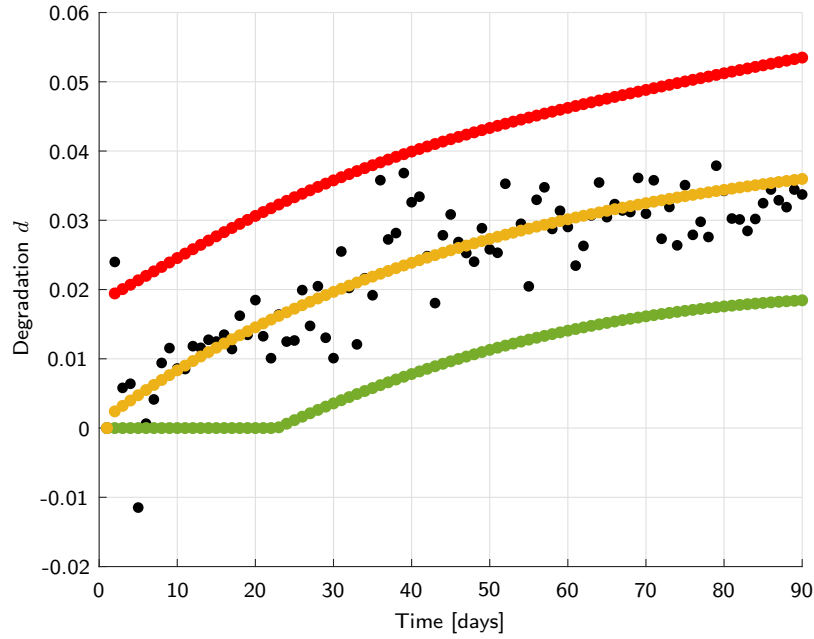


Figure 7.6: Degradation data adapted from Chapter 5 with the measured values of d ($\bullet$), the upper bound d^{upper} ($\bullet$), which has been assigned to Compressor 1, the expected value d^{expected} ($\bullet$), which has been assigned to Compressor 2, and the lower bound d^{lower} ($\bullet$), which corresponds to Compressor 3

7.7.2 Results of the case study

The optimisation problem with the objective function from Eq. (7.11) and the constraints from Eq. (7.13) and (7.14) was conducted with Pyomo 5.6.6. in Python 3.7.3 using CPLEX®Interactive Optimizer 12.9.0.0. The three values of degradation were used presented in Section 7.7.1.

7.7.2.1 Analysis of the load sharing

The optimal allocation of the load is affected by degradation of the compressors, as well as their constraints and the demand for the whole station. If factor-free degradation is considered and the compressors in up-state were identical, the results from Section 7.5.1.4 indicate that the most degraded compressor will get the lowest load, whereas the healthiest compressor will get the largest load.

The results of the load sharing algorithm are shown in Fig. 7.7 and compared with the results from the equal load approach. Figure 7.7a shows the loads assigned to Compressor 1, 7.7b shows the loads assigned to Compressor 2, and 7.7c shows the loads assigned to Compressor 3. The minimal and maximal flow for a given day are also indicated to show that the optimisation algorithm does not violate the constraints from Eq. (7.14).

The black line in Fig. 7.7 shows the flows assigned from both algorithms compared with the equal load approach (green line). The degradation from Fig. 7.6 is a factor-free degradation and affects the power to run Compressor 1 in a multiplicative way. Therefore, the value of the objective function, as well as the solution is different than the equal load assignment. However, the quadratic nature of the objective function (7.11) does not change.

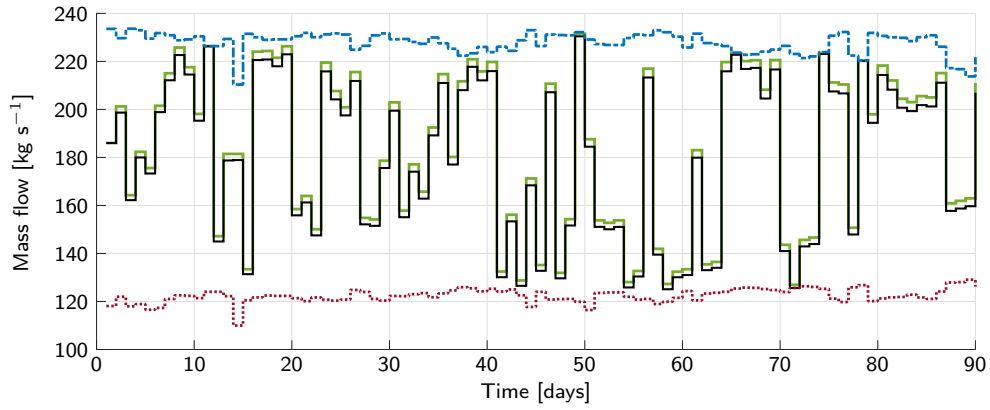
The assignment of degradation values described in Section 7.7.1 to Compressors 1, 2, and 3 indicates that Compressor 1 is the most degraded and Compressor 3 is the least degraded. The degradation from Section 7.7.1 affects the power to run a compressor in a multiplicative way. From Eq. (7.44) it is obtained that:

$$\frac{dW_{1t}d_1}{dm_{1t}} > \frac{dW_{2t}d_2}{dm_{2t}} > \frac{dW_{3t}d_3}{dm_{3t}} \quad (7.46)$$

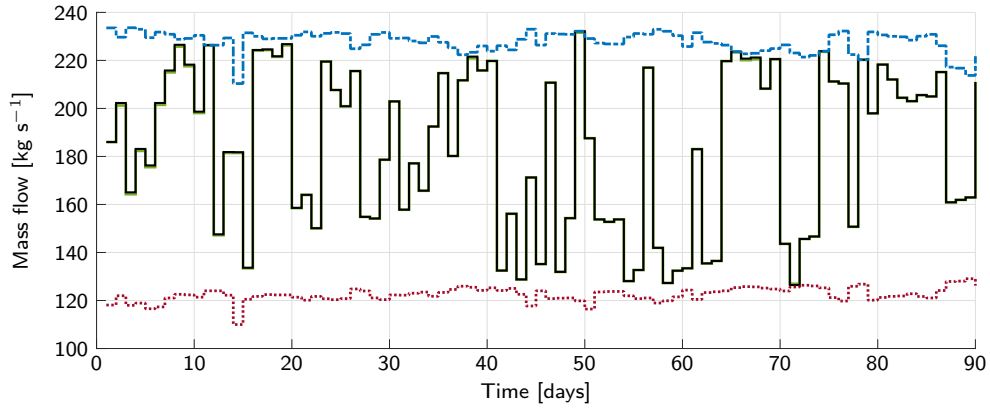
because it is assumed that the compressors are identical in up state, $W_{1t} = W_{2t} = W_{3t}$. Equation (7.46) suggests that Compressor 1 will get the smallest value of the load, because it has the largest gradient of power versus flow characteristics with respect to the flow (Fig. 7.7a). At the same time, Compressor 3 will get the largest value of the load, because it has the smallest value of the gradient (Fig. 7.7c).

Equation (7.24) provides conditions when the compressors will work at their limits. There are four possibilities:

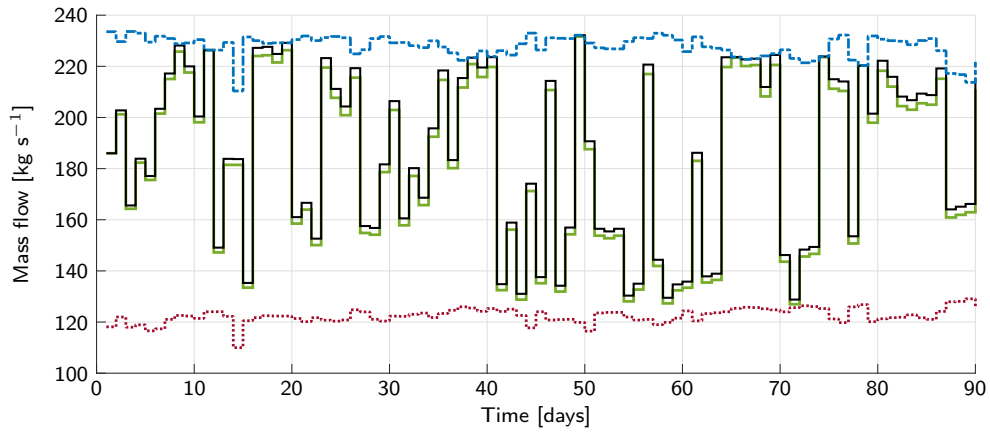
- Compressor 1 is at its lower limits (day 11 in Fig. 7.7a);
- Compressors 1 and 2 are at the lower limits;
- Compressors 2 and 3 are at the upper limits (days 11, 49, 65, 74, and 78 in Figs. 7.7b and 7.7c);
- Compressor 3 is at the upper limit (days 19, 38, 66, and 67 in Fig. 7.7c).



(a) Mass flow through Compressor 1



(b) Mass flow through Compressor 2



(c) Mass flow through Compressor 3

Figure 7.7: The mass flows through Compressors 1, 2, and 3, if all the compressors degrade, from two algorithms: power management (—) compared with the equal load approach (—). The maximal limit (---) and the minimal limit (---) for the flows in each day are also indicated

For instance, there are four days when Compressor 3 will be at the upper limit, but not Compressor 2. This is due to the degradation of Compressor 3, which increased the gradient of the power to run Compressor 3. When Compressor 3 is at the upper limit, the remaining load is shared between the two compressors in such a way that the overall power consumption is minimised. Because Compressor 1 is more degraded than Compressor 2, it gets less load, so the remainder must be assigned to Compressor 2. This is why Compressor 2 is above the equal load case in days 66 and 67, and reaches its upper bound in day 65.

In particular, the fact that Compressor 2 will work at the upper bound if Compressor 3 is already at its upper bound, confirms the results from Jenicek et al. (1995), that ‘cheaper’ compressors are run at full capacity. In contrast to their results, where ‘cheaper’ meant more efficient or less expensive in terms of the cost, here ‘cheaper’ denotes the compressor with the slowest change of power as a function of mass flow. This means that the compressors with flat power versus flow characteristics from Fig. 2.6 would get more load than the compressors with steeper characteristics. This allows power savings, but it could lead to degradation of the cheaper compressors, if degradation depends on the flow.

The conditions for when the compressors work at the limits will be further analysed in Section 7.7.2.2.

7.7.2.2 Analysis of sensitivity

The sensitivity analysis for the case study is performed using the solutions for dual variables from Eq. (7.25) and (7.26). Figure 7.8 presents days when the compressor will work at their upper and lower limits. The vertical axis corresponds to the value of the dual variables γ_2 and γ_3 , because Compressor 2 and Compressor 3 work on their upper boundaries, and μ_1 , because Compressor 1 works on its lower boundary. The sensitivity analysis from Section 7.5.1.2 provides insights into the

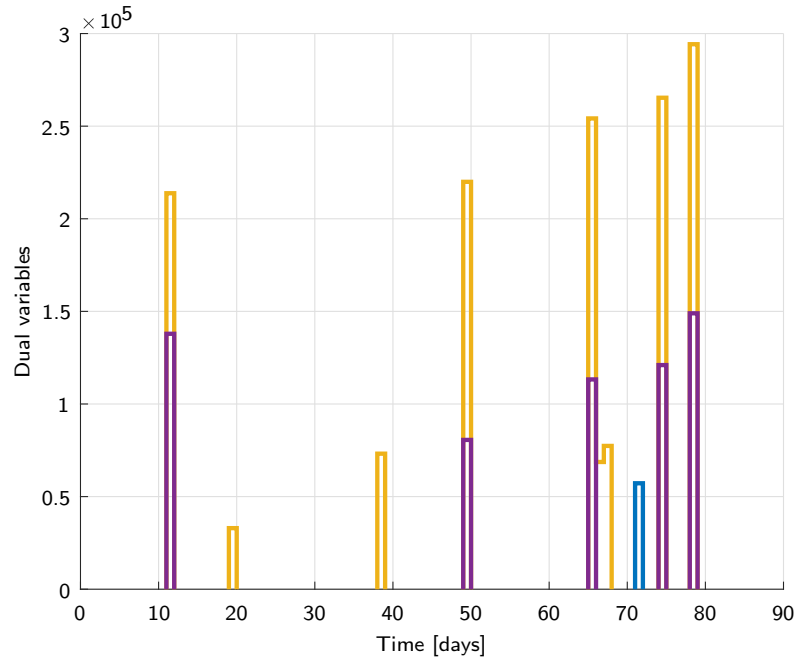


Figure 7.8: The days when compressors work at their limits: Compressor 1 can work at its lower limit (—), Compressors 2 and 3 are both at their upper limits (—), and Compressor 3 is at its upper limit (—)

significance of the values from Fig. 7.8. The sensitivity analysis indicates that if the active constraint changes, i.e. the upper limit for a compressor becomes smaller or the lower limit becomes larger, then the objective function will increase significantly. Therefore, if the flow constraints change on any of the days indicated in Fig. 7.8, the value of the objective function will increase. For instance, if the upper limit for Compressor 3 decreases on day 78, when the dual variable is the largest, the value of the objective function will increase. This is because Compressor 3 will only be able to take a smaller flow, which means that the more degraded Compressor 1 and Compressor 2 would have to take more load.

As indicated by Boyd and Vandenberghe (2009), the opposite change of the limits, i.e. increasing the upper limit or decreasing the lower limit, will have the opposite effect on the value of the objective function. The value of the objective function will decrease, and the large value of the dual variables will correspond to a large change of the objective function. Therefore, modifying the constraints by allowing a larger or smaller flow could improve the optimal value of the objective function. For instance, the constraints can be modified by improving the speed controllers SC from Fig. 7.4, as demonstrated by Cortinovis et al. (2015a).

7.7.2.3 Analysis of power consumption

Figure 7.9 presents selected power curves from the period of 90 days for the three compressors. In particular, the consecutive Figures 7.9b-7.9f show how the power curves evolve from being identical in Fig. 7.9a and how they compare to each other if degradation increases. The observations are:

- In Day 2, Compressor 1 needs more power, whereas Compressor 2 and 3 have a similar power consumption. This is because degradation of Compressor 2 is much closer to degradation of Compressor 3 in Day 2, whereas degradation of Compressor 1 is significantly larger (Fig. 7.6);
- From Day 20, the power characteristics do not visibly change in relation to each other, with Compressor 1 requiring the most power and Compressor 3 requiring the least power. This is because the degradation shown in Fig. 7.6 increases with time at similar rates;
- From Day 30, the power characteristics for all compressors move up the vertical axis. This is because the degradation of all the compressors (Fig. 7.6) increases with time, so compressors require more power.

Moreover, a comparison of Fig. 7.9 with Fig. 7.2 indicates that degradation does not result in a significant change of the power curves. Therefore, it can be expected that the power to run the system will be similar in degraded state and in up state. In particular, this fact suggests that the power to run a compressor in degraded state with optimal load sharing will be similar to the power consumption if equal load is used, because equal load is optimal in up state for identical compressors.

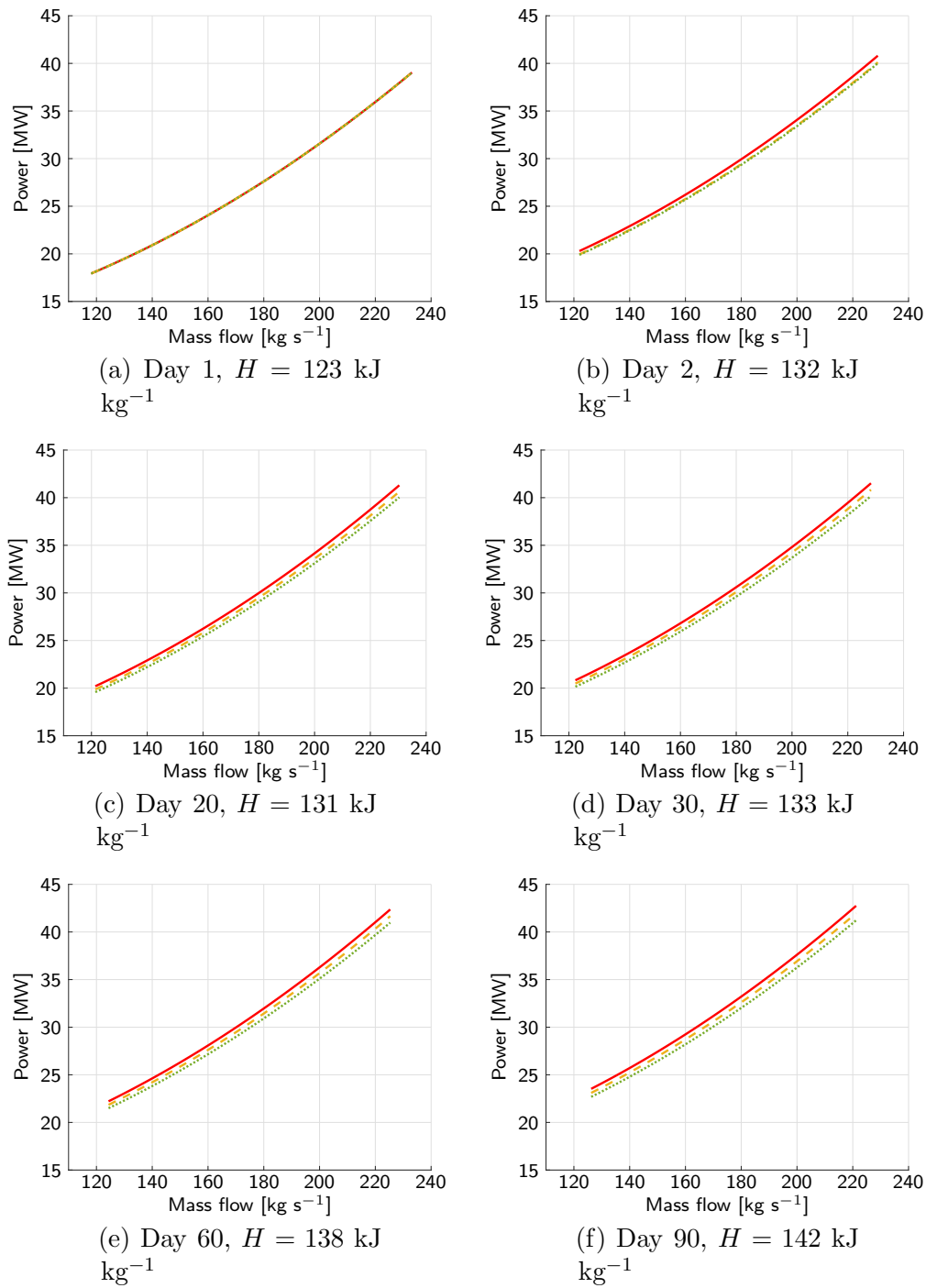


Figure 7.9: Power characteristics as a function of the mass flow for selected values of compressor head for Compressor 1 (—), Compressor 2 (---), Compressor 3 (···)

Figure 7.10 shows the power consumption of the three compressor in relation to the power if the equal load approach was used. The power consumption of each compressor follows the same pattern as the flows assigned to the compressors in Fig. 7.7:

- Compressor 1 has a lower power consumption, because it gets a lower load than in the equal load approach;
- Compressor 2 has a similar power consumption, because the assigned loads are similar in the optimal load sharing and in the equal load approach;
- Compressor 3 has a larger power consumption, because the assigned flows are larger than in the equal load approach.

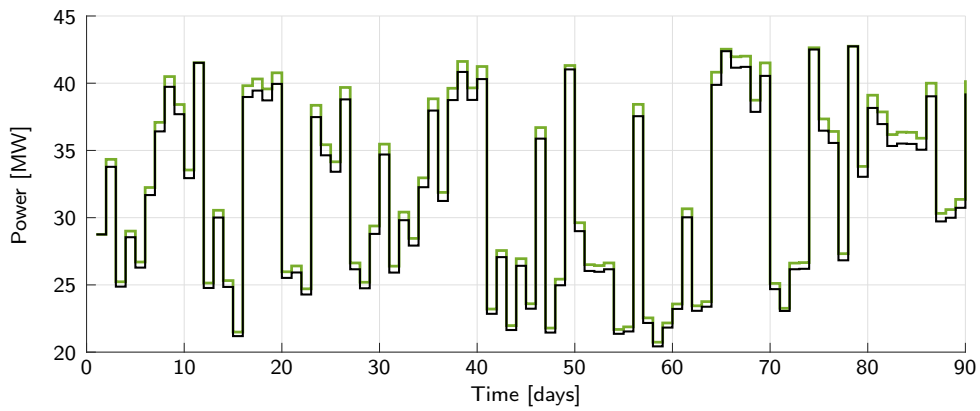
The overall power consumption in the two approaches is shown in Fig. 7.11a. The results of the optimisation are shown in black and the equal load approach is denoted with green. The overall power consumption is very similar in both approaches, which was expected from Fig. 7.9. Figure 7.11b indicates that the optimisation does reduce the power in relation to the equal load approach and the difference is primarily visible if degradation is large.

The relative power difference was calculated as:

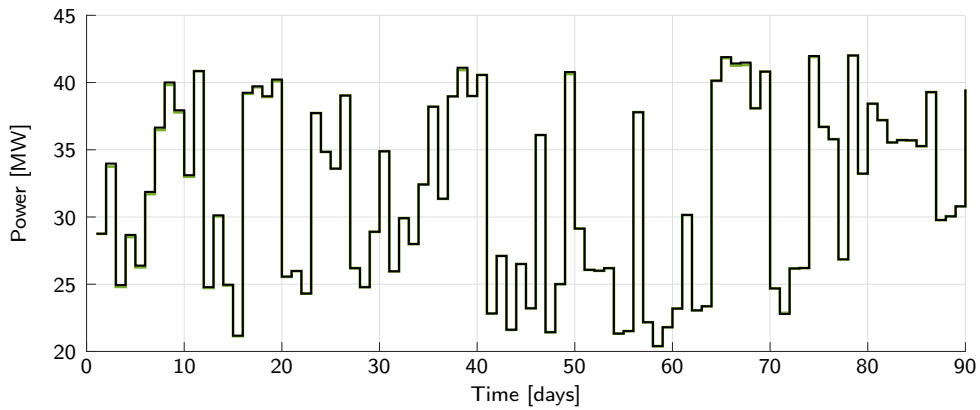
$$\varepsilon = 100 \frac{W_{OL} - W_{EL}}{W_{EL}} \quad (7.47)$$

where W_{EL} is the power in the equal load approach, W_{OL} is the power in the optimal load sharing. The values close to zero in Fig. 7.11 in days 11, 49, 65, 74, and 78 indicate that the power to run the compressors is close to the equal load assignment when a compressor is working at its limits. This result means that to save the power to run a compressor station with degradation, it is best to avoid the compressors working at the limits. The possibility of working at the limits is also affected by the demand required from the station.

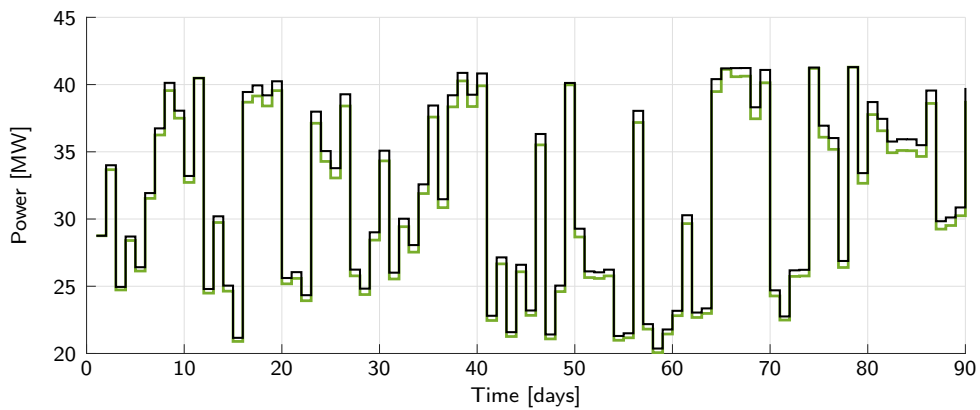
If the compressors are often run at their limits, this might prompt a decision to modify the structure of the compressor station. A new compressor might be added if the existing one is working on the upper limit. Conversely, one of the compressors might be removed, if the existing ones often work at their lower limit.



(a) Power consumption of Compressor 1

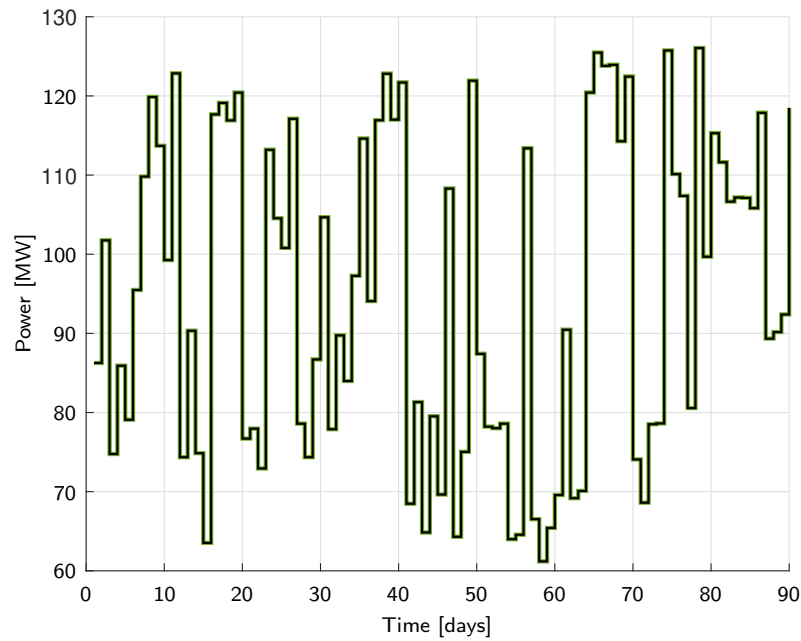


(b) Power consumption of Compressor 2

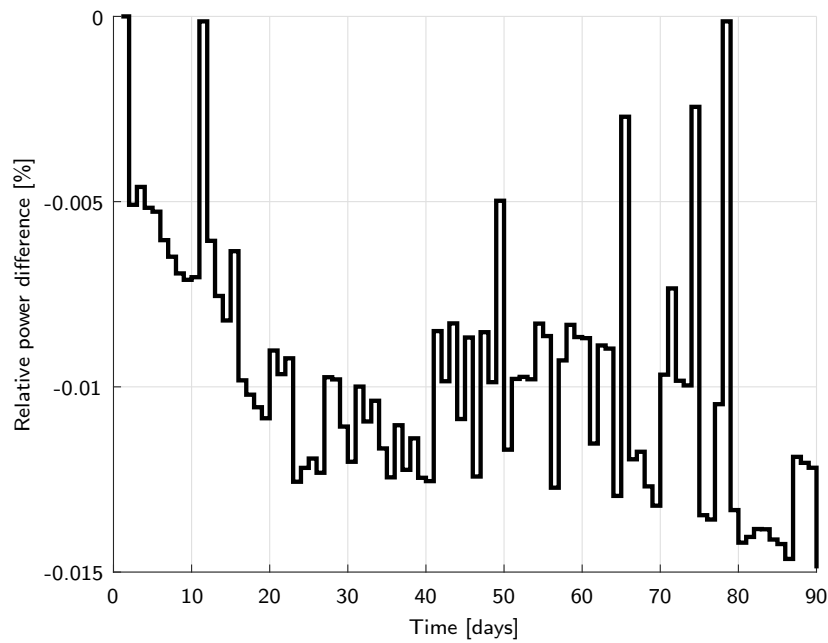


(c) Power consumption of Compressor 3

Figure 7.10: Power consumption of Compressors 1, 2, and 3, if all the compressors degrade, from two algorithms: power management (—) compared with the equal load approach (—)



(a) Power consumption of the whole station from two algorithms: power management (—) compared with the equal load approach (—)



(b) Relative differences in power consumption between the power management algorithm and the equal load approach

Figure 7.11: Analysis of power consumption of the whole station

7.8 Chapter summary

The chapter proposed an optimisation framework for load-sharing in a compressor station taking degradation of the compressors into account. The compressors are subject to degradation due to time, use and varying operating conditions, which results in an increase of power necessary to run the station. The optimisation model proposed in this work improves the performance of the compressor station by explicitly including degradation in the objective function and analysing its influence on the optimal solution. The chapter builds on the degradation modelling approaches from Chapter 4 and uses the factor-based model of degradation proposed in Chapter 6 to analyse the influence of degradation on the optimal load-sharing.

The findings of the chapter can be summarised as follows:

- Compressor degradation affects the power to run the compressors, but not necessarily the optimal load assignment;
- The load-assignment depends on how the gradient of power of each compressor changes with the mass flow and the current value of demand;
- The effect of degradation on the power consumption and the optimal load-sharing depends on the type of degradation;
- The sensitivity analysis indicates that allowing a larger flow for compressors working on the upper limits or smaller flow for compressors working on the lower limits could improve the power consumption.

In particular, degradation due to fouling, which is a typical phenomenon in industrial turbomachinery, will change the optimal load-assignment, and it will increase the power to run the station.

The optimisation framework was applied to an industrial case study with a three-compressor station and real data for degradation from Chapter 5. The overall demand and the operating pressures were adapted from real data available in the literature. The case study indicated that the framework for optimal load sharing taking degradation into account is able to reduce the power to run the station in relation to the equal load approach, but the largest power savings are obtained if the compressors work away from the limits. The industrial case study shows also that the compressors will be assigned the loads corresponding to the value of their degradation. This fact leads to an observation that the most degraded compressor must work at its lower limit, and only then the compressors with a smaller value of degradation will work at their lower limits. Conversely, the compressor with the lowest value of degradation must work at its upper limit, and only then the more degraded compressors will work at their upper limits.

The industrial case study led to the following practical recommendations:

- To save the power to run a compressor station with degradation, it is best to avoid the compressors working at the limits;
- If the compressors are often run at their limits, this might prompt a decision to modify the structure of the compressor station.

Part III

Discussion, future work, and conclusions

Chapter 8

Critical evaluation and future work

This chapter evaluates the results presented in Chapters 4-7 and discusses how they addressed the challenges indicated in Chapter 1.

The challenges from Chapter 1 and the gaps of knowledge identified in Chapter 3 have been confronted in the following ways. Chapter 4 introduced a classification of existing models of degradation from a process control perspective, which enabled an analysis of mutual interactions between degradation and the system. Chapter 5 presented an adaptive model of degradation in turbomachinery to quantify how the efficiency changes if a unit is degrading. Chapter 6 developed an algorithm based on comparison of the value of degradation within a compressor station to manage the degradation of the compressors. Chapter 7 designed an optimisation framework for power minimisation that takes degradation into account and demonstrated the performance of the proposed method in an industrial case study.

The current chapter discusses the advantages and limitations of the work in Chapters 4-7 and outlines a roadmap for future work in the wider context of control systems with degradation.

Table 8.1: A summary of the challenges introduced in Chapter 1 with gaps of knowledge identified from the literature in Chapter 3 and the corresponding chapters

Challenges	Gaps of knowledge	Contribution
Challenge 1: Analysis of the mutual interactions between degradation and the system	Gap 3.3.1: The existing models of degradation do not capture the relationships between degradation and the behaviour of process	Chapter 4, Chapter 7
Challenge 2: Quantification of degradation of industrial systems	Gap 3.3.2: The existing models of degradation are inaccurate due to fixed functional forms or insufficient data for identification of parameters	Chapter 5
Challenge 3: Analysis of relationships within the system subject to degradation	Gap 3.3.3: The existing approaches to degradation management do not take into account the interactions between the elements of a degraded system	Chapter 6, Chapter 7
Challenge 4: Design of control systems to mitigate degradation	Gap 3.3.4: The existing strategies for optimisation of compressor station do not take degradation into account	Chapter 6

8.1 Synopsis of challenges and contributions

The challenges are recalled in the first column of Table 8.1. The corresponding gaps of knowledge, identified in Chapter 3, are summarised in the second column. The third column in Table 8.1 assigns the work from Chapters 4-7 to the challenges and respective gaps of knowledge.

Figure 8.1 recalls the block diagram for the case study. The compressor station must deliver the required demand. The problems solved in the thesis focused on how to distribute the load among the compressors taking their degradation into account (Chapters 6 and 7). For this purpose, a degradation indicator was developed that can be fed back to the load-sharing algorithm (Chapter 5). The load-sharing algorithms are able to allocate the flows among the compressors taking into account both the value of the degradation indicator and the chosen performance metric (Chapters 6 and 7). The flows delivered by the compressors contributed to the operation of the whole compressor station and thus influence its performance measured by the power consumption of the station.

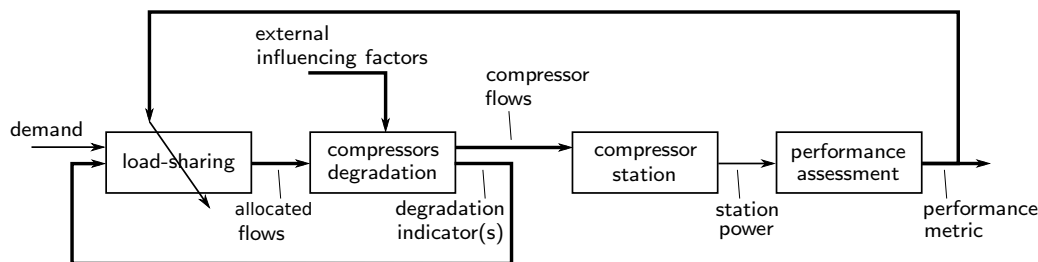


Figure 8.1: A graphic summary of the industrial load-sharing problem with degrading compressors

8.2 Contribution to degradation analysis in process control applications

8.2.1 Discussion of the contribution

Chapter 4 introduced a classification of models of degradation, which enabled an analysis of the mutual influence between degradation and the behaviour of a system. The models of behaviour have been classified as degradation-dependent or degradation-independent, to reflect whether the degradation affects the behaviour of a system. The models of degradation have been classified as factor-free or factor-based, to capture how influencing factors affect degradation. The models of behaviour can be then combined with the models of degradation, as recalled in Fig. 8.2.

The new classification was aimed at facilitating the integration of degradation modelling into process control applications. The chapter built on the definitions from standards, which make it useful for industrial applications. It used the nomenclature from the standards gathered in Appendix B and provided a direct link between industry and academia. By focusing on the behaviour of the system, the classification captured the effects that degradation has on the system. Therefore, degradation can be modelled from the perspective of the system as a change of the behaviour as described with dynamic or steady-state models. For this purpose, the chapter introduced the notion of a degradation function $h(d)$ that captures the influence of degradation d on selected characteristics of the system.

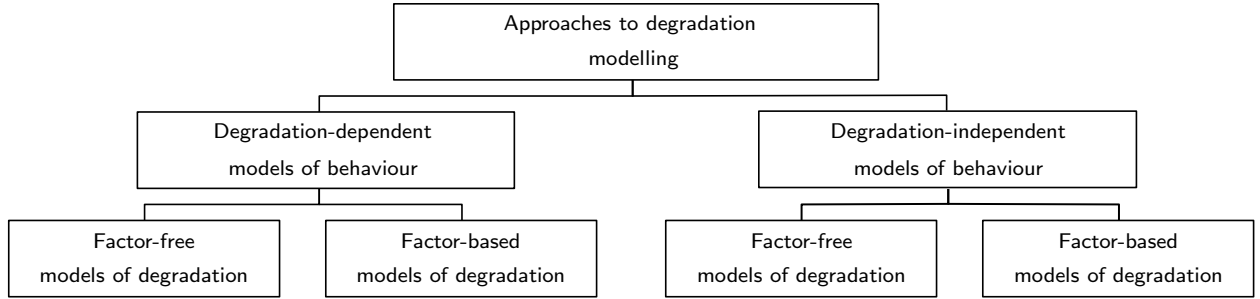


Figure 8.2: A hierarchical tree with classification of models of degradation relative to the control systems

The survey from Chapter 4 also presented how degradation models have already been integrated in control systems in a wide range of applications. The grouping proposed by the new classification emphasized the similarities in degradation modelling that span across multiple disciplines, from motor racing and aviation to process control. It showed the flexibility of the classification. Zagorowska et al. (2020b) have extended the impact of the survey by showing an example of application to multi-product batch scheduling.

Finally, Chapter 4 showed a perspective on the requirements for models of degradation for application in control systems taking degradation into account. In particular, the fields of degradation-aware control and degradation-mitigating control could benefit by including the models of degradation in the design of an optimal control structure.

8.2.2 Limitations

The classification proposed in Chapter 4 is based on models of degradation and models of behaviour (Fig. 8.2). Therefore the formulations of the models are of importance and is a source of limitations.

The classification does not distinguish between different kinds of degradation within a system. For instance, degradation of a compressor can be related to fouling, but also to leakages from the auxiliary piping system. Both kinds of degradation can be modelled either as factor-free or factor-based. It is a limitation in a sense that factor-free and factor-based models of degradation do not specify what part of equipment is degrading. Neither does the new classification indicate how the model of degradation was obtained, in contrast to more common classifications as data-driven models and models derived from first principles. The reason for this is that the new classification shifts the focus from degradation itself to the interactions between degradation and the behaviour of a system.

The survey did not provide ways of choosing the model of degradation, and in particular the form of the degradation function $h(d)$. Even though Chapter 4 emphasized the similarities of models of degradation across a range of applications, the choice of the degradation function was not discussed and is likely to be both system- and application-specific. Ideally a model of a degraded component would drop easily into the mathematical representation of a control system, much as Hammerstein or Wiener models can be used for representing generic nonlinearity in actuators.

The survey mentioned that certain forms of models of degradation change the functional form of the models of behaviour, by introducing nonlinearity or making the parameters of the system time-dependent. The chapter did not consider the consequences of these choices. An example of the consequences was highlighted in Chapter 7. Using a factor-based model of degradation with a degradation-dependent model of behaviour in an optimisation problem affected the quadratic objective function and required nonlinear solvers. This indicates that the integration of models of degradation in process control might require new solution methods, which either have not yet been applied in a specific system, or have to be tailored to the specific application.

8.2.3 Future work

A finding revealed in Chapter 4 is that there is a gap in the literature concerning the use of factor-based models of degradation at the regulatory control layer. A regulatory control system needs accurate knowledge of the behaviour of a system over short timescales, and this is why there is a preference for degradation-dependent models of behaviour. However, the level of degradation is mainly being estimated from factor-free models of degradation, whereas in practice degradation may depend on the way in which the system is operated, and also on external factors. There may be a promising research direction in specifying some generic functional forms for Eqns. (4.9) and (4.10) whose parameters can be identified from data.

Chapter 4 also indicated that a degradation-independent model of behaviour is typically considered sufficient in the scheduling and optimisation layers. It seems useful to verify this assumption, because degradation models in such schemes are often used as a proxy for the real system when testing and verifying the schemes. The work surveyed in Chapter 4 highlighted examples of scheduling and optimisation schemes being validated in a simulation that used a degradation-independent model of behaviour, even when the real system is known to have degradation-dependent behaviour. There seems to be scope for improved simulation environments that would be based on more realistic degradation-dependent models of behaviour. Using degradation-dependent models would therefore lead to an improved integration of control and scheduling, in a partial fulfilment of the gaps indicated by Baldea and Harjunkski (2014).

A control scheme in any layer of the automation hierarchy may incorporate models of degradation and models of behaviour. Also, simulations are often used by researchers to test and demonstrate their ideas. The simulation has to use an accurate representation of the real system. Validation of the models involves real data ideally recorded during field operation. However, there remains an open research question about a systematic way to validate models of degradation and degradation-dependent models of behaviour.

8.3 Contribution to modelling of turbomachinery degradation

8.3.1 Discussion of the contribution

Chapter 5 proposed an algorithm that is able to adapt to the evolution of degradation in turbomachinery, such as turbines and compressors. The work in this chapter corresponds modelling the degradation indicator from Fig. 8.1. The proposed algorithm was based on time-trend analysis and combines a moving window approach with adaptive regression analysis. It predicts the expected value of degradation and quantifies the uncertainty of the prediction. The results using real data from industrial case studies showed that the algorithm predicts the degradation indicator $h(d)$ accurately in the short and medium terms. The moving window framework combined with nonlinear regression allowed adaptation of the algorithm to the variations of the trend of the degradation indicator.

The algorithm gives a prediction of the value of the degradation indicator and also an estimate of the uncertainty of the prediction. Thus, additional information is provided about the degradation indicator that can be used for scheduling under uncertainty. The predictions can be used as an input to decision-support systems such as production and maintenance scheduling systems, which operate in short and medium timescales (Schulze Spüntrup, 2019). The benefit of using an online updated approximation function is that changes in the performance degradation are considered in the decision-making process of when to perform maintenance. As indicated in the turbine case study in Chapter 5, the uncertainty of the prediction depends on the number of samples in the data set. Thus, increasing the sampling frequency when collecting the data would further improve the predictions and narrow down the uncertainty.

The new algorithm may be used for decision support. In particular, the quantification of uncertainty can provide an estimate of expected power increase over the prediction window T_{pred} . Denoting the lower prediction bound as d^{lower} and the upper prediction bound as d^{upper} , Equations (6.8) and (6.9) yield:

$$h(d^{\text{lower}})W < W_D < h(d^{\text{upper}})W \quad (8.1)$$

where W is the power to run the system in up state and W_D is the power to run the system in degraded state. Some industrial sites, in particular in off-shore settings, are not equipped with the infrastructure for online maintenance. The estimates obtained in Eq. (8.1) might help creating the decision-support environment for equipment upgrades, such as investing in the system for online washings. Knowing the prediction bounds of the power consumption will make it possible to assess the uncertainties in the payback time for the investment.

8.3.2 Limitations

The algorithm proposed in Chapter 5 is data driven and therefore has limitations related to gathering and using the data. It was assumed that the datasets used in Chapter 5 are not affected by the operating conditions. In particular this means that the changes of efficiency are only due to degradation and are independent of the operating conditions. This is equivalent of assuming:

- The operating conditions do not change throughout the period considered;
- The efficiency in up state would be constant regardless of the operating conditions.

These assumptions were considered justified since the focus on the thesis was on using a model of degradation for load-sharing. Application of the method in practice would require a validation of the assumptions. If they are not fulfilled, the influence of the operating conditions would have to be accounted for. This could be done by using detailed first-principles models of thermodynamics of a compressor, such as the models developed by Ciccioiti (2015).

The new algorithm assumes that degradation data are available in real time. This indicates that both the efficiency in degraded state and the expected efficiency in up state have to be calculated online. Still, the new algorithm requires only one data set for tuning compared to more complex approaches based on machine learning and data science, such as neural networks. The new algorithm tackles in this way the challenges of insufficient data indicated by Fentaye et al. (2019), who indicated that one of the challenges for most algorithms is the unavailability of data due to lack of sensors.

It was also assumed that the data are available for the whole period and there are no missing periods. The datasets were obtained by reviewing the available degradation data from off-shore compressors and choosing the periods that fulfilled this assumption. In practice, data points can be missed due to sensor failures, planned maintenance events, or changes of operating regimes such as turning the compressors off. If the reasons for the missing periods are known, it is possible to take them into account and model the degradation combining the information from disparate sources, as proposed by Stief (2019).

The performance of the algorithm if there are missing periods has yet to be evaluated. The fact that a linear function is used to detect the change and possibly model the degradation indicates that it only requires two data points to provide a valid model. However, the algorithm is sensitive to the choice of parameters. Correct tuning of the parameters requires either historical degradation data or knowledge about similar systems, as indicated in Section 5.4.2.2.

The algorithm works best in short and medium term. This predicates it for the use in short and medium term applications, such as control on the regulatory or supervisory layer in Fig. 2.3, or scheduling for a short horizon. In some applications, for instance maintenance planning or fleet management (Schulze Spüntrup, 2019) operating in longer timescales, the prediction horizon would be too short. A possible solution would be to use alternative approaches designed for long-term prediction of time trends, such as the method proposed by Sorjamaa et al. (2007) who dynamically change the subsets of historical data to predict future values using pattern recognition.

There exist also other methods for detection of degradation, for instance based on Bayesian modelling (Adams and MacKay, 2007) or pattern recognition methods (Gao et al., 2015b). Trend analysis was chosen because:

- It has already been used for modelling of degradation in turbomachinery, for example by Li and Nilkitsaranont (2009) and Tsoutsanis et al. (2016);
- It was proven to be able to detect gradual degradation, not only to detect a step-like change.

The algorithms for the detection of the point of change can be used to take into account the maintenance events, which result in an abrupt change in the degradation indicator. A survey on such methods was done by Venkatasubramanian et al. (2003a,b,c) and Gao et al. (2015a).

At the same time, using trends enabled exploration of the known properties of degradation of turbomachinery, in particular its exponential character. Exploring this property of degradation required less data than other knowledge-based approaches.

8.3.3 Future work

The algorithm has proved to be portable and the results are promising for improving performance-based maintenance in short and medium term. One of the ways of improving the algorithm would be to analyse how much data is required to detect and predict the trend, as mentioned by Weatherhead et al. (1998). Collection of data in a systematic and consistent way can be a significant issue and an algorithm able to predict degradation using a minimal amount of data would have useful industrial implications.

The industrial case study in Section 5.4 suggested that there are similarities in the course of degradation in various kinds of turbomachinery. There seems to be a scope for analysing the available degradation data to provide a pool of parameters for the algorithm. Such a database could be used to tune the algorithm for monitoring new equipment without a history of degradation.

The expected value of degradation can be used for maintenance scheduling to provide the optimal schedule of washings and minimise the overall power consumption. Optimisation frameworks for maintenance scheduling with degradation modelling have been proposed by Hovland and Antoine (2006), Nørstebø et al. (2007a), Xenos et al. (2016a), Schulze Spüntrup et al. (2019), who all used linear models of degradation to minimise the power W_D in large oil and gas applications. Zulkaffi and Kopanos (2016) proposed a framework for maintenance scheduling for utilities, including compressor systems and modelled degradation as piecewise-linear between maintenance events. These frameworks would benefit from the more accurate model of degradation as provided by the new algorithm. In particular, the prediction would be more accurate and might extend the time between the maintenance activities such as compressor washings, or, conversely, bring forward an activity to prevent a failure.

A future step could then be to include the uncertainty estimates and to establish that the estimates provide useful information for scheduling of maintenance. Taking the uncertainty into account would enable development of multiple scenarios for maintenance scheduling from the perspective of economic objectives, such as used by Schulze Spüntrup (2019), or environmental constraints.

8.4 Contribution to degradation management

8.4.1 Discussion of the contribution

The framework proposed in Chapter 6 provided a method for sharing the load among multiple compressors in a compressor station. It served as a load-sharing module providing set-points to internal flow controllers (allocated flows in Fig. 8.1). The algorithm did not affect lower level control structures, such as speed control or surge protection. The loads were assigned to the degraded compressor by multiplying the value obtained from an equal load approach by an exponential function of the ratio of degradation functions $h(d)$. The algorithm is therefore independent from the functional form of degradation indicator and can handle both factor-free and factor-based models of degradation. The simplicity of the method makes it a promising solution for complex process industries.

The algorithm enables both mitigation and intensification of degradation, provided that the degradation is factor-based. Therefore, it can be used in maintenance planning frameworks, in particular if the timings of maintenance activities are fixed. The algorithm can extend the lifetime of a compressor by mitigating its degradation, or, conversely, can intensify the degradation to reach the maximum level in time for planned maintenance. The results from this chapter show also that the new algorithm is able to manage degradation from the perspective of power consumption of the whole station.

8.4.2 Limitations

The algorithm from Chapter 6 has the following limitations. In contrast to more advanced solutions, such as an optimisation framework from Chapter 7, assigning the loads based on the ratio of degradation functions does not explicitly take into account the operating range of the compressors. It is therefore necessary to tune the parameters so that the compressors stay within their operating ranges. However, the possible values of parameters depend on the demand required from the compressor station. Therefore, it is necessary to provide additional bounds of form (6.24) to make sure that the compressors stay within the operating ranges.

The performance of the algorithm has been demonstrated using a dynamic model of a compressor train from Fig. 2.4 with parameters from Bakken et al. (2002) and Nørstebø (2008). Moreover, the compressors were considered identical in every aspect, save the degradation. As indicated by Liptak (2005), this is rarely the case. To apply the algorithm in real life, it would be necessary to identify the compressor maps and power characteristics and other parameters such as the geometry of the passages inside the compressor.

The approach from Chapter 6 neglected the internal dynamics of the compressor by omitting the rotational dynamics of the compressor. It was assumed that the internal dynamics are much faster than the process behaviour and the degradation. However, as indicated by Besselmann et al. (2017) and Cortinovis et al. (2016a), the effects of the drive torque on the behaviour are of importance, and should be included in the behaviour. Even though these effects primarily happen in the regulatory control layer, in particular for surge protection, they should be considered on upper layers of control, because of their impact on the behaviour. The degradation-dependent model of behaviour of a compressor should therefore include the effect of the drive torque. This would enable a hierarchical control structure of a compressor station encompassing both regulatory and supervisory control layers, as suggested by Verheyleweghen et al. (2018).

The algorithm proposed in Chapter 6 assumed that it was possible to give a rank order to the compressors according to the value of their degradation indicators and that the ordering remained the same throughout the period when the load sharing strategy is in operation. In reality, it is possible that the ordering changes during the operation of the compressors, for instance due to differences between the compressors at the manufacturing level. The algorithm is not sensitive to the changes in the ordering, but Chapter 6 shows that reversing the order reverses the load assignment and mitigation of degradation becomes intensification of degradation. Therefore, the following steps are proposed to apply the algorithm, if the objective is to mitigate degradation:

1. Obtaining the values of the degradation indicators, for example following the approach from Chapter 5,
2. Ordering the values of the degradation indicators as required in Chapter 6,
3. Performing the load assignment according to the algorithm from Chapter 6.

Degradation due to fouling affects the compressor map. The need for increasing the speed for the same operating pressure ratio and mass flow results in a loss of efficiency. To show the performance of the algorithm, it was assumed that fouling

has the same impact on compressor pressure ratio and efficiency:

$$d_\eta = d_\Psi \quad (8.2)$$

In reality, it might not be true, as indicated by Ciccioiti (2015). However, the algorithm proposed in Chapter 6 can use either indicator, because it is not sensitive to the functional form of degradation functions. At the same time, it was assumed that the degradation affected only the pressure ratio and the efficiency. However, degradation can also modify the operating range of the compressors. Such changes would require retuning of the parameter q in Eq. (6.18) in an online fashion, to comply with the reduced operating range.

The factor-based model of degradation was based on a simplification of heuristic considerations from Verheyleweghen and Jäschke (2017). It was assumed that the degradation depends only on the speed of the compressors. Other factors that would affect degradation are related to the gas composition and include the amount of salts in the gas indicated by (Brekke et al., 2009) or gas volume fraction used by Verheyleweghen and Jäschke (2017). The possible dependence of the models of degradation on other factors does not affect the algorithm, because the algorithm does not distinguish between the functional forms. It will however have an effect on how much load is assigned to the compressors:

- If the other factors depend proportionally on the mass flow, changing the flow would result in a larger mitigation or intensification of degradation;
- If the other factors depend on the mass flow in an inversely proportional way, changing the flow would invert the mitigation and intensification of degradation;
- If the degradation is not factor-based, then the algorithm assigns a lower or larger load to a compressor, but the assignment does not affect the degradation any more.

This indicates that the performance of the load-sharing algorithm is affected by taking the influencing factors into account. This emphasises the need for developing factor-based degradation models. The parameters of both the factor-free and factor-based models of degradation used in Chapter 6 were arbitrarily chosen to simulate the degradation of up to 7%. In reality, the parameters of the models should be found in real time or come from past experiments.

8.4.3 Future work

Section 8.4.2 gave some suggestions for future work. In particular, the importance of factor-based models of degradation has been emphasised. Development and validation of factor-based models of degradation is therefore a potential area for future work. The work can be conducted experimentally, using laboratory set-ups for inducing fouling in compressors, as proposed by Igie et al. (2014), Fouflias et al. (2010), and Syverud (2007). Another approach would be to use system identification methods to identify the relationships between the mass flow and degradation (Ljung, 2010). This approach treats degradation as a separate system and models degradation as a black- or grey-box models, depending on how much information on degradation is already available. It can include data-driven methods such as neural networks if the physics of degradation are unknown.

The algorithm from Chapter 6 focused on degradation management in a compressor station with multiple units. It can be used as an initial step for analysis of the trade-offs between the value of degradation of the units. The parameter of the algorithm can be then chosen in such a way that the load-sharing aims to strike a balance between multiple compressors.

8.5 Contribution to power management

8.5.1 Discussion of the contribution

In contrast to Chapter 6, which focused on the management of degradation, Chapter 7 investigated how degradation due to fouling affects the power to run a compressor station (Fig. 8.1). For this purpose, an optimisation problem was solved with the purpose of minimising the power consumption. The compressor station had to satisfy the demand for the mass flow and deliver the required amount of gas. The effects of both factor-free and factor-based degradation were considered and their influence on the problem was investigated.

The chapter shows that including degradation in the problem of power minimisation will have a twofold effect:

- The power necessary to run the compressors will always increase due to degradation, regardless of the influencing factors and the demand;
- The optimal load assignment will not change if the degradation does not change the gradient with respect to the mass flow of the power curves.

Therefore, the framework provides insights into load-sharing with multiple types of degradation. For instance, factor-free additive degradation, which was considered by Xenos (2015) for maintenance planning, would not affect the solution of the load-sharing problem, but it will affect the overall power to run the station. This means that the assignment of the loads remains optimal even if the value of degradation changes. This result can compensate for the fact that the actual value of degradation might be unknown, in particular for a long optimisation horizon. Additive factor-based degradation can also preserve the solution, if the factors do not depend on the mass flow through the compressors.

In most cases, the influencing factors would affect both the value of the objective function and the solution. Nonetheless, the formulas derived in Chapter 7 can be used to find the optimal assignment of the loads if the factor-based degradation does not affect the quadratic character of the optimisation problem. Multiplicative factor-free degradation and additive factor-based degradation which is linear or quadratic functions of the flow belong to this group. Factor-based multiplicative degradation, as well as some kinds of additive degradation would require a new solution of the optimisation problem.

Finally, the sensitivity analysis enabled a study on how the solution of the power minimisation problem would change if the constraints describing the operating range, for instance due to degradation. If the degradation affects the operating range, there are two possibilities:

- The solution and the value of the objective function change. This case takes place when one of the compressors was forced to work at its limit in up state, i.e. if the constraint affected by degradation was active in up state;
- The solution does not change, if the constraint affected by degradation was not active in up state.

The case study from this chapter has been summarised in Fig. 8.1.

8.5.2 Limitations

The optimisation problem has the following limitations. The algorithm was formulated as a continuous optimisation problem, without considering discrete variables related to operating regimes, such as indicators if a compressor is on or off. There are three reasons for this assumption:

- It is assumed that the demand is feasible for all the compressors. If it were not feasible, then the optimisation problem would be reduced to a lower number of compressors;
- The evolution of degradation is not known in the periods when a compressor is off. Xenos (2015) indicated that some kinds of degradation might continue when a compressor is off;

- The maintenance activities are not considered. There would be no improvement in degradation if the compressor was turned off and as the evolution of degradation is not known, it would remain the same.

However, Xenos (2015), whose work was used as a source of the demand data in this thesis, took into account the possibility of turning the compressors off. At the same time, the compressor maps used in the thesis might be different than the ones used by Xenos (2015), because they were obtained from Nørstebø (2008) who provided numerical values for the flows and compressor head. This means that the demand required from the three compressors in the work of Xenos (2015) might not be feasible in the current thesis with the maps from Nørstebø (2008). The problem was solved by adjusting the demand on the days when the equal load approach would not be feasible. The adjustments were done by reducing or increasing the demand in such a way that the equal load approach fell within the range of 2 kg s^{-1} from the minimal and maximal limit on a given day. The value of 2 kg s^{-1} was considered large enough to leave some flexibility in assigning a lower or higher load to a compressor. Other operating conditions, i.e. suction and discharge pressures, were not changed. The adjustments of the demand do not affect the results from the case study, because the optimisation problem assumed that the demand was feasible and the adjustments enable fulfilling this assumption.

It has not been evaluated how the optimal load-sharing for power management will affect degradation. This was because of the unknown true value of the factor-based degradation, which was replaced by heuristic models dependent on the mass flow. As indicated in Section 8.4.2, finding a factor-based model of degradation remains a challenge. Therefore, it would only be possible to analyse the influence on a heuristic model. The results of such analysis are likely to be invalid for a different model of degradation.

The optimisation problem does not have any inter-period constraints, such as ramping constraints from Zuo et al. (2016) that would limit the change of mass flow in a compressor from day to day. Therefore, it was possible to analyse the multi-period optimisation problem as multiple single-period optimisation problems. This assumption was considered sufficient, because the station had to satisfy the demand for the mass flow in each day and by default there were no external sources of gas to supply the missing amount. The dynamic behaviour of the compressor was also neglected, as it was assumed that the dynamics of a compressor are much quicker than the degradation. However, since the case study considers varying demand, the dynamic behaviour should be taken into account in the transitions between the values of demand.

In terms of the operation of the compressors, the case study analysed only the load-sharing problem, and neither took low-level regulatory control such as surge protection into account, nor proposed a maintenance scheduling framework. However, these topics are of importance and their integration with the load-sharing framework would further improve the operation of a compressor station and would enable an economic analysis of the operation of the station. A hierarchical approach proposed by Verheyleweghen et al. (2018) can be used for this purpose.

The optimisation problem was assumed to be deterministic, even though Chapter 5 indicated that there can be uncertainty in degradation modelling. A deterministic model of degradation was considered sufficient for the analysis of the influence of degradation on the optimisation problem, because it allowed a direct interpretation of the influence of degradation on the power. Nevertheless, uncertainty is unavoidable in real life applications. The uncertainty in degradation modelling would primarily affect the power curves. Therefore, it would change the gradient with respect to mass flow and the load assignment would be different.

The case study in Chapter 7 has the following limitations. The value of degradation for the whole period was assumed to be known. However, predicting the value of degradation as well as the operating conditions for 90 days into the future remains a challenge. The limitation was partly removed by showing that a multi-period optimisation problem is equivalent to multiple single-period problems, if there are no inter-period constraints. Therefore, it was enough to know the value of the demand and the degradation only one day in advance.

The case study focused on degradation of compressors due to fouling. However, in complex systems, there is more than one kind of degradation that can be considered. A natural extension for a compressor station would be to take into account vibrations as an indicator of internal degradation (Qin et al., 2017) or to focus on degradation of mechanical or electrical components, such as bearings or insulators.

8.5.3 Future work

The fact that the assignment of the loads depends on the power curves emphasizes the importance of the power curves. The influence of the accuracy and uncertainty related to the power curves is an area for future work. The possibilities include stochastic programming, which would enable minimisation of the expected value of power consumption, as well as robust optimisation, which is focused on the worst-case scenario (Sahinidis, 2004).

8.5.3.1 Expanding the optimisation problem

One of the possibilities of expanding the work from Chapter 7 is to shift the focus from power management to joint management of power and degradation. The influence of the load-sharing on the degradation can be evaluated, provided a factor-based model of degradation is available.

The study of the influence of the optimal solution on degradation would also provide a method for multi-objective analysis of how the degradation of each compressor changes. Ultimately, this can lead to a system with degradation control with the objective of keeping the degradation of selected compressors at the desired level.

The framework of power minimisation in a compressor station with degrading units can be expanded to take into account additional constraints, such as ramping constraints or minimal shut-down and operating periods. In particular, including the interperiod constraints is of importance, as it will change the optimisation problem from Chapter 7. Such constraints can be derived from the area of power generation and unit commitment problem, described by Padhy (2004). An example for a compressor station was provided by Zuo et al. (2016), but without considering degradation and assuming that power to run a compressor was a linear function of the flow. The work in this thesis enables a formulation of the load-sharing problem in a compressor station with degrading compressors as a unit commitment problem with a convex quadratic objective function. The additional constraints for the compressors would result in a formulation of the problem as a mixed-integer quadratic programming.

8.5.3.2 Analysis of power and efficiency

More theoretical work is necessary to fully investigate the relationships between the power consumption and the efficiency of the compressors as suggested by the results from Appendix D. An initial step in this direction is provided in Appendix C where conditions for strict convexity of power are derived taking efficiency into account. The convexity of power can be further exploited. The fact that the power is a convex function of the mass flow would facilitate the formulation of the optimisation problem with integer variables or uncertainty.

8.5.3.3 Analysis of uncertainty

Power management in a compressor station is a topic of considerable industrial interest (Schulze Spüntrup et al., 2019; Schulze Spüntrup, 2019; Xenos, 2015; Kumar and Cortinovis, 2017; Cortinovis et al., 2016b; Nørstebø et al., 2007a; Nørstebø, 2008). The power management concepts presented here have potential for further improving the operation of a compressor station by directly considering the effects of degradation. However, further research is needed in the area of factor-based degradation modelling, as well as expanding the optimisation problem to take into account more realistic constraints. This would enable a formulation of the optimisation problem as non-linear optimisation problem with uncertainty which reflects the real operating conditions.

It would also be useful to take into account uncertainty inherent in real-life applications. Possible sources of uncertainty are:

- Degradation modelling, as indicated in Chapter 5. For instance using shock models discussed briefly in Chapter 4 would enable analysis of abrupt changes of the flow, for example due to varying demand;
- The value of demand. The chapter assumes that the demand is known, but it might be impossible to predict the demand.

This problem would be of particular importance if the overall costs of running the station are considered. The costs are proportional to the power, but the cost for a time period of work of a compressor can vary from period to period. Adjusting the load taking into account the variability of the costs, the demand, and the degradation would give a more accurate representation of the operation of a compressor station and the economic impact of degradation. Examples of solving optimisation problems for degrading units with uncertain demand were provided by Zhang et al. (2016).

8.6 Roadmap for the future

This section presents the major topics that have arisen from the thesis, presented in two groups:

- Modelling-oriented work in Section 8.6.1,
- Control-oriented work in Section 8.6.2.

Section 8.6.3 describes the steps necessary to transfer the work from the thesis from academia to real-life applications.

8.6.1 Modelling-oriented work

In recent years, the focus of modelling for monitoring and predictions has shifted towards data-driven approaches. In the context of maintenance and prognostics, Sikorska et al. (2011) indicated that contrarily to physical methods, which provide most accurate descriptions of the phenomena in a system, but require detailed knowledge about the system, data driven approaches are simpler to implement and can be built on incomplete knowledge about the process. One of the challenges related to data-driven modelling is the quality of the data for building the models (Tahan et al., 2017).

There is a challenge in obtaining representative data to get degradation models. Degradation typically happens slowly in relation to the dynamics of the system. Collection of data in a systematic and consistent way over many periods of operation can be a significant issue, especially for academic researchers as highlighted by Jardine et al. (2006). In particular, if the degradation data were available for process applications, the models of degradation could be derived using data-based methods, such as Bayesian modelling already used for electric motors by Stief et al. (2019). It would enable the modelling of degradation that is not easily captured with first principles.

The need for developing new models of degradation has already been indicated by Xenos (2015), but without specifying the requirements for the model. The current thesis emphasizes that the models of degradation should be developed in parallel to the models of the behaviour of the processes. A concurrent and holistic approach to modelling degradation in control systems would expand the applications of degradation-aware and degradation-mitigating control.

A step in this direction is provided by the modifier adaptation framework from Marchetti et al. (2009), who presented an optimisation framework able to take into account the mismatch between the plant and the model. Incorporating models of degradation could minimise the plant-model mismatch by providing a more accurate description of a system in degraded state.

Integration of degradation into more comprehensive frameworks from regulatory control to scheduling remains an open question. Ideally, degradation would be treated as an integral part of the system. In particular it would not be unexpected - and the control systems would take it into account from the start. Currently, control is separated from monitoring, and monitoring algorithms are separated from materials science and chemistry. To devise comprehensive frameworks that will work for the whole plant, it is necessary to combine knowledge from several fields.

8.6.2 Control-oriented work

An important topic in process control is related to integration of various layers of process automation. Baldea and Harjunoski (2014) and Du et al. (2015) indicated that integrating scheduling and control would improve both aspects. A further improvement in this area can be achieved by explicitly including degradation in all layers of process automation. In particular, the propagation of degradation from the physical system to regulatory and supervisory control layers should be analysed taking into account how the operating strategies affect the degradation. The propagation of degradation to upper layers of control systems can be considered as a part of existing degradation-aware control strategies.

Degradation management in the form of controlling the effects of the operating conditions on degradation remains a new concept, in particular in the area of the intensification of degradation. An example of how a comprehensive approach can improve the performance was given by Engell and Harjunoski (2012). An integrated approach encompassing factor-based degradation models, regulatory and supervisory control, and scheduling would enable optimisation of the operation and asset management.

An interesting expansion of the work in this thesis would be to combine the optimisation of power consumption in degrading systems with other objectives, such as mitigation of harmful emissions or other environmental and economic objectives indicated by Kashani and Molaei (2014). This would lead to solving a multi-objective optimisation problem. The resulting trade-offs could be a decision support tool for choosing the strategy for the development of the process.

The analysis of degradation in control systems is a part of larger topic on integration of condition monitoring in all layers of industrial automation. By putting degradation in context of changes in the behaviour of a system, the analysis conducted in this thesis enables an application of control methods to systems with degradation. At the same time, by focusing on the factors influencing degradation, it provides ways of mitigating the degradation, not only adjusting the system to take degradation into account.

A closer integration of all the layers of process automation in systems with degradation would lead to improved decision support tools. It would also enable further development of autonomous systems in process industries. Even though autonomous systems are more and more common in the automotive industry or aviation, there is little work on application of autonomous systems in process control. Gamer et al. (2019) indicated that there may be several steps necessary to achieve full autonomy in process plants, from no autonomy and relying on human actions (Step 0), to full independence from human actions (Step 5). Including the analysis of degradation in the system would enable automation of some activities, such as adjustments of the set points if degradation were detected, thus providing a step toward completely autonomous operation. Some initial works in the area of sub-sea compressor stations done by Eriksson (2011) and Verheyleweghen et al. (2018) indicate that this might be a promising future research direction. Due to a wide range of process control applications, it would be necessary to devise comprehensive frameworks, instead of application-specific approaches. Therefore, it is likely that this work will spark a closer collaboration between control theorists, condition monitoring specialists, and data scientists.

8.6.3 Technology transfer

This thesis is a result of work delivered within the European Industrial Doctorate framework, described in Appendix A. As such, it has a strong focus on technology transfer suitable for real-life industrial applications. A number of industrial case studies, namely degrading offshore turbomachinery and compressor stations, were analysed to provide insights into degradation modelling in real-life process control. The case studies were developed in collaboration with industrial partners of the project, ABB Energy Industries and Equinor ASA. ABB provided training in industrial research and development in a 20 month placement and gave guidance on the requirements for transferring academic results into industry. Equinor hosted a short visit to enable obtaining degradation data used in Chapters 5 and 7.

8.6.3.1 Theoretical perspective

The new classification of degradation models from Chapter 4 was developed in collaboration with ABB Corporate Research Center in Poland and BASF, who both provided an industrial perspective in the area of degradation modelling. In particular, their guidance enabled anchoring the survey in the intersection between control and condition monitoring. Thus, the survey provides a theoretical framework for including degradation modelling in process control applications that spans across multiple fields of engineering.

The new classification could be included in the industrial standards. For instance, the existing classification of degradation models from BSI (2015a), which is focused on diagnostics and prognostics could be expanded to take into account the mutual influences between degradation and industrial systems. At the same time, the classification from Chapter 4 could be incorporated in an update of BSI (2015b) to recognise that degradation is an inherent part of process control systems. The new way of thinking about degradation will therefore facilitate the integration of the automation layers, which was suggested as a challenge by Baldea and Harjankoski (2014) and Du et al. (2015).

8.6.3.2 Implementation perspective

The limitations of the approaches from the thesis affect the possibility of technology transfer. The limitations of the algorithm from Chapter 5 relate mostly to obtaining the data, whereas the limitations of the load-sharing algorithms from Chapters 6 and 7 pertain to the simplifying assumptions about the operation of a compressor station. This indicates that the approaches from the thesis are at different stages of suitability for technology transfer.

The algorithm from Chapter 5 is ready for implementation in a pilot environment. Parts of the degradation prediction algorithm have already been trialled with ABB Energy Industries. At this stage, there is a need to evaluate the performance on many more example data sets, in order to confirm that the suggested default parameter settings are performing well.

In terms of the load-sharing algorithms from Chapters 6 and 7, the next step would be to give a demonstration of the method on a high-fidelity industrial simulator that acts as a proxy for a real compressor station. Degradation data from the real compressors would also be needed, and realistic degradation models would have to be included in the simulator, to match the real system. The simulation is likely to emphasize the need for expanding the load-sharing algorithms for degrading equipment to take into account other operational constraints, such as turning the equipment off or on.

8.7 Chapter summary

This chapter discussed the contributions of the thesis described in Chapters 4-7. Each chapter has been evaluated in terms of:

- Discussion of the contribution,
- Limitations of the contribution,
- Future work related to the chapter.

The contributions of each chapter were discussed in Sections 8.2-8.5. Section 8.6 described a proposed roadmap for future research in the area of process control systems with degradation and proposed steps to make the contributions of the thesis suitable for technology transfer.

Chapter 9

Summary and conclusions

The objective of the thesis was to investigate the role of operating conditions in degradation management. The challenges identified in Chapter 1 were:

- Challenge 1. Analysis of the interactions between degradation and the system,
- Challenge 2. Quantification of degradation of industrial systems,
- Challenge 3. Analysis of relationships within the system subject to degradation,
- Challenge 4. Design of control systems to mitigate degradation.

By addressing these challenges, the work presented in this thesis provides an ability to mitigate the detrimental influence on the system due to degradation. The mitigation of the detrimental effects leads to improved performance and decreased costs of operation.

Section 9.1 explains how the thesis tackled the challenges, whereas the results of the work are summarised in Section 9.2. Section 9.3 provides closing remarks.

9.1 Meeting the objectives

The challenges from Chapter 1 and the gaps of knowledge identified in Chapter 3 have been tackled in the following ways. Challenge 1 has been dealt with in Chapter 4. Chapter 4 introduced a classification of existing models of degradation from process control perspective, which enabled an analysis of mutual interactions between degradation and the behaviour of the system. This work has been submitted as a journal article (Zagorowska et al., 2020b).

Challenge 2 was considered in Chapter 5. Chapter 5 presented an adaptive model of degradation in turbomachinery to anticipate future degradation on the basis of online measurements of efficiency. This work has been published as a conference paper (Zagorowska et al., 2019) and journal article (Zagorowska et al., 2020a).

Chapter 6 tackled Challenge 3 and 4 by developing an algorithm based on mutual relationships within a compressor station to manage the degradation of the units. The relationships were defined based on the values of degradation of the compressors. Part of this work has been published in a conference paper (Zagorowska et al., 2018b).

Chapter 7 addressed Challenge 1 and 3 by proposing an optimisation framework for minimisation of power consumption that takes degradation into account. The industrial case study in Chapter 7 shows how the optimisation framework can be used to tackle the challenges in real-life applications. The work from this chapter is in preparation for a journal publication (Zagorowska and Thornhill, 2020a).

The advantages and limitations of the algorithms presented in the thesis were discussed in Chapter 8 and possible extensions to the work in the thesis were presented. Chapter 8 outlined also a roadmap for future research on process control systems with degrading units.

In addition, the work in this thesis resulted in two conference papers on modelling of a compressor train (Zagorowska and Thornhill, 2017; Zagorowska et al., 2018a).

9.2 Summary of technical work

9.2.1 Summary of Chapter 4

Chapter 4 focused on the Challenge 1 related to the analysis of the interactions between degradation and control systems. The chapter presented a new classification of degradation models and divided them according to the interactions with both the system and its surroundings. The models of behaviour have been classified as degradation-dependent or degradation-independent, to reflect whether the degradation affects the behaviour of a system. The models of degradation have been classified as factor-free or factor-based, to capture how influencing factors affect degradation. The models of behaviour can then be combined with the models of degradation. The chapter examined also control application where degradation modelling has been used and structured them according to the process automation hierarchy.

Chapter 4 also discussed and gave examples of two main approaches to control of degrading systems introduced in Chapter 2:

- Control systems aware of degradation that compensate for the degraded behaviour of the controlled system. These systems include fault-tolerant control;
- Control systems mitigating the degradation that manage the degradation, also known as life-extending control.

9.2.2 Summary of Chapter 5

Chapter 5 presented a new data-driven algorithm for modelling degradation in turbomachinery and therefore tackled the Challenge 2 from Chapter 1 and recalled at the beginning of this chapter. The algorithm uses a factor-free model of degradation to give an on-line estimate of degradation, together with uncertainties in the estimate.

The parameters of the algorithm were discussed and a tuning procedure has been proposed. The results have been tested with industrial data from two off-shore applications. The performance of the algorithm has been compared window-based approaches: linear regression over a fixed and a varying approximation window, exponential regression, and analysis of mean value in a fixed window.

9.2.3 Summary of Chapter 6

Chapter 6 presented a load-sharing algorithm for management of degradation in the compressors of a compressor station. The new algorithm is based on the ratio of degradation functions between the compressors and therefore fulfils Gap 3.3.3, which indicated that there is a lack of approaches that take into account the interactions between the elements of a system. Thus, the new load-sharing algorithm contributes to Challenge 3 indicated at the beginning of this chapter. The loads are assigned to degrading units based on a function of the degradation ratio. For this purpose, a degradation model with compressor speed as influencing factor was developed to show that it was possible to manage degradation, provided a factor-based model of degradation is available.

The choice of the function and the parameters of the algorithm were discussed and their role in the algorithm analysed. In particular, the impact of the parameters on the degradation and the power necessary to run the compressors was investigated. The new approach was applied to simulated compressor stations with two and with five degrading compressors, respectively. The results show that it is possible to manage the degradation of the compressors in a compressor station, provided knowledge about factors influencing degradation is available.

9.2.4 Summary of Chapter 7

Chapter 7 presented an optimisation framework for power management in a compressor station with degrading compressors. The chapter fulfils Gap 3.3.4, which indicated that the existing strategies for compressors stations do not take degradation into account, and tackles Challenge 1 and Challenge 3 from the list at the start of the chapter.

The optimisation problem is formulated in two versions:

- Optimal load-sharing if there is no degradation;
- Optimal load-sharing if there is degradation.

The optimisation problem without degradation is formulated as a convex quadratic programming problem with linear constraints. The Karush-Kuhn-Tucker conditions give a unique optimal solution. A sensitivity analysis of the solutions is used to provide information on how the optimisation problem will change if the compressors degrade.

The chapter shows that the optimal load assignment depends on how the power curves from Fig. 6.4 change as functions of the mass flow. The solution to the optimization needs an examination of compressor-specific power characteristics that show power consumption as a function of the flow through the compressor for various pressure ratios. Chapter 7 gave an explanation of the criteria and showed how to conduct such analysis.

Further, Chapter 7 presented also an industrial case study for a compressor station with three compressors. The degradation data used for the evaluation come from Chapter 5. The factor-free models of degradation developed in Chapter 5 were used to analyse the algorithms for load sharing. The case study revealed that the flexibility of the algorithm is affected by how close the equal load approach is close to the limits of the operating ranges of the algorithm. It was demonstrated that the algorithm enables power savings compared to the equal load approach.

9.3 Final comment

The operational lifetime of industrial equipment, in particular centrifugal compressors used in gas processing and export facilities, is 30-50 years. Varying operating conditions, ageing and use lead to degradation of the equipment. The degradation should be taken into account for control purposes, such as for load-sharing. Taking degradation into account in the operating framework will lead to improved operation of the units, as demonstrated in this thesis.

The results in the thesis prove that by adjusting the operating conditions, it is possible to manage degradation in an industrial setting. The approaches proposed in the thesis can be a foundation for further research on industrial systems subject to degradation, including analysis of various types of degradation or uncertainty inherent in real-life applications.

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Part IV

Appendices

Appendix A

PRONTO project

PRONTO (PROcess NeTwork Optimization) was a European Industrial Doctorate program funded by the Marie Skłodowska-Curie Actions scheme under EU Horizon 2020 and led by prof Nina Thornhill from Imperial College London. The objective of the PRONTO project was to provide coherent academic-industrial PhD training for a cohort of Marie Skłodowska-Curie Early Stage Researchers (ESRs) to identify, analyse and solve operational challenges in the process industries of Europe.

The research topics of PRONTO were:

1. Data analytics for assessment of the condition and performance of networks of equipment used for production in the process industries,
2. Optimization of use of resources in process networks taking account of real-time information about the condition and performance of the process equipment,
3. New concepts for process operation identified as having high potential for impact by industrial partners.

The thesis contributes to the second and third research topics. The objective of the thesis was to devise a method for real-time adaptation of process control algorithms taking account of the condition of the equipment, to strike a balance between preserving equipment condition and control performance.

A detailed description of the project and the results can be found at PRONTO (2020).

Appendix B

Glossary of degradation terms

Table B.1: Glossary of terms related to industrial control systems and their degradation

Term	Explanation
Actuator	Functional unit that generates the manipulated variable, required to drive the final controlling element, from the output variable of the controlling element (BSI, 2016c)
Advanced process control (APC) and optimisation (APC-O)	APC: control strategy to cope with processes characterised by large time delays, non-minimum phase, non-linearity, loop instability and multi-variable coupling. APC enhances basic process control by addressing particular performance or economic opportunities in the process. Optimisation: decision-making strategy to meet the business objective under a weighted set of conditions and concerns. APC-O: Collection of advanced process control and optimisation strategies (BSI, 2015b)
Ageing failure	Failure whose probability of occurrence increases with the passage of calendar time (BSI, 2012b)
Basic Process Control System	System which responds to input signals from the process, its associated equipment, other programmable systems and or operators and generates output signals causing the process and its associated equipment to operate in the desired manner BSI (2016a)
Behaviour of a system	Response of a system to input signals from the process, its associated equipment, other programmable systems and/or operator (following BSI (2016a))
Behavioural model of a system	Mathematical models of the input/output behaviour of a system (Willems, 2007), including differential and algebraic equations, as well as other characteristic quantities (see: Characteristics)
Characteristics (of equipment)	Distinguishing attributes, qualities and properties of equipment and its subsystems which, by their presence and the relative magnitudes of their effects, define the configuration, performance, behaviour and capabilities of the equipment (BSI, 2016b)
Condition(s)	Characteristics and parameters of the actual state of an item (BSI, 2010a)
Condition monitoring	Activity, performed either manually or automatically, intended to measure at predetermined intervals the characteristics and parameters of the actual state of an item (BSI, 2010a)
Degradation	Detrimental change in physical condition, with time, use, or external cause (BSI, 2017)
Degradation (of performance)	Undesired departure in the operational performance of any device, equipment or system from its intended performance (BSI, 2012a)
Degraded state	State of reduced ability to perform as required, but with acceptable reduced performance BSI (2017)
Dependability	Ability to perform as and when required (BSI, 2012b)
Diagnostics	Examination of symptoms and syndromes
Environment	All external conditions influencing a system at any given moment (BSI, 2018)
Equipment	Machine or group of machines including all machine or process control components (BSI, 2012b)

Table B.1 (Continued): Glossary of terms related to industrial control systems and their degradation

Term	Explanation
Failure	Termination of the ability of an item to perform a required function (BSI, 2010a)
Failure criteria	Pre-defined conditions to be accepted as conclusive evidence of failure (BSI, 2010a)
Failure mechanism	Physical, chemical or other processes which may lead or have led to failure (BSI, 2010a)
Fault	State of an item characterised by inability to perform a required function, excluding the inability during preventive maintenance or other planned actions, or due to lack of external resources (BSI, 2010a)
Functionality	Extent to which the system provides functions to perform tasks required by the system mission (BSI, 2016b)
Induced environment	Conditions external to a system generated as a result of the operation of the system (BSI, 2018)
Influencing factor	Observable qualitative or measurable quantitative item that affects a system property BSI (2016a)
Item	Part, component, device, subsystem, functional unit, equipment or system that can be individually described and considered (BSI, 2010a)
Loss (of function or performance)	Temporary degradation (BSI, 2016b)
Maintenance	Combination of all technical, administrative and managerial actions during the life cycle of an item intended to retain it in, or restore it to, a state in which it can perform the required function (BSI, 2017)
Measurement	Process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity (BSI, 2016b)
Model	Mathematical or physical representation of a system or a process, based with sufficient precision upon known laws, identification or specified suppositions (BSI, 2016b)
Natural environment	Conditions generated by the forces of nature and the effects of which are experienced by a system when it is at rest as well as when it is in operation (BSI, 2018)
Operating conditions	physical loads and environmental conditions experienced by the item during a given period BSI (2017)
Parameter	Variable representing some significant measurable system characteristic (BSI, 2012b)
Performance	Precision and speed with which the system executes its tasks under defined conditions (BSI, 2016b); behaviour, characteristics and efficiency of a technological process, running in a machine derived by measurement and calculation of one or more parameters, for example, power, flow, efficiency or speed, which singly or together provide the necessary information (BSI, 2012b)
Physical properties	See Characteristics
Prognostic	Analysis of the symptoms of faults to predict future condition and residual life within design parameters (BSI, 2012b)
Reliability	Ability of an item to perform a required function under given conditions for a given time interval
State (actual)	A circumstance or condition of an ITEM at a time (BSI, 2014b), Characteristics of an item at a specific point in time (BSI, 2001) (obsolete)
System	In condition monitoring and diagnostics: set of interrelated elements that achieve a given objective through the performance of a specified function (BSI, 2012b); in process control: a subsystem of Basic Control System (BCS); Basic Process Control System (BPSC), system which responds to input signals from the process, its associated equipment, other programmable systems and (or) an operator and generates output signals causing the process and its associated equipment to operate in the desired manner (BSI, 2016b)
System property	Defined parameter suitable for the description and differentiation of BPCS(s) (BSI, 2016b)
Up state	State of an item characterised by the fact that it can perform a required function, assuming that the external resources, if required, are provided (BSI, 2017)
Wear-out-failure	Failure whose probability of occurrence increases with the operating time or the number of operations of the item and the associated applied stresses (BSI, 2017)

Appendix C

Theoretical appendices

C.1 Monotonicity and convexity of power as a function of mass flow for constant head

Figure 6.3 shows that for the data obtained from Nørstebø (2008), the efficiency is a concave function of the flow, but it is not monotonic. Conversely, the power to run the compressors is strictly increasing and convex. This section presents the conditions that the approximation of efficiency has to fulfil, so that the power is a strictly increasing and convex function of the flow.

The assumptions in the current section are:

- The efficiency η is a continuous and differentiable function of the mass flow, $\eta : [m^{\min}, m^{\max}] \rightarrow [\eta^{\min}, \eta^{\max}]$, and $\eta \in C^2$
- The efficiency is a strictly concave function of the mass flow and it attains the maximal value $\eta^{\max}$ at $m_{\eta}^{\max}$. In particular, it is assumed that $\frac{\partial \eta}{\partial m} > 0$ for $m \in [m^{\min}, m_{\eta}^{\max}]$, $\frac{\partial \eta}{\partial m} = 0$ if and only if $m = m_{\eta}^{\max}$, $\frac{\partial \eta}{\partial m} < 0$ for $m \in [m_{\eta}^{\max}, m^{\max}]$

where $m^{\min}$ and $m^{\max}$ are the minimal and maximal flow for each compressor, $m_{\eta}^{\max}$ is the mass flow for which the efficiency is maximal, $\eta = \eta^{\max}$, and $\eta^{\min}$ is the minimal efficiency.

C.1.1 Monotonicity of power

Theorem 1. *The power of a compressor is an increasing function of the flow for fixed head in the whole operating range if the efficiency is decreasing in the interval $m \in [m_{\eta}^{\max}, m^{\max}]$ and the tangent lines to the efficiency curve in the interval $m \in [m^{\min}, m_{\eta}^{\max}]$ have positive intercepts with the vertical axis.*

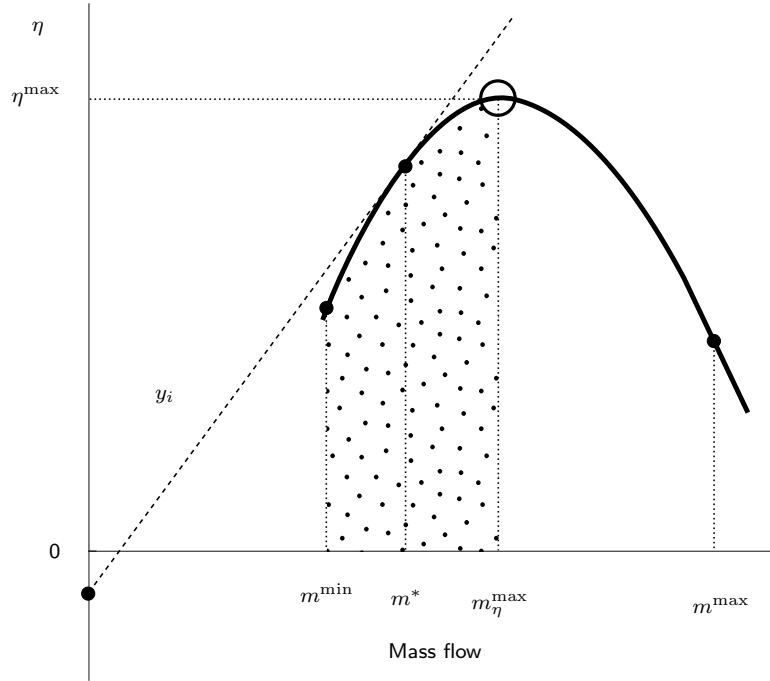


Figure C.1: Efficiency as a function of flow rate for fixed head

Proof. For a fixed head, $H = \text{const}$, the efficiency is a concave function of the flow, increasing for $m \in [m^{\min}, m_{\eta}^{\max}]$ and decreasing for $m \in [m_{\eta}^{\max}, m^{\max}]$ (Fig. C.1). These two cases for efficiency increasing for $m \in [m^{\min}, m_{\eta}^{\max}]$ and decreasing for $m \in [m_{\eta}^{\max}, m^{\max}]$ are considered separately:

- Case 1: $m \in [m_{\eta}^{\max}, m^{\max}]$. The power is given by Eq. (2.11). It is an increasing function of the flow if the first derivative with respect to mass flow is positive. Calculating the first derivative of the power W yields:

$$\frac{\partial W}{\partial m} = \frac{H\eta - Hm \cdot \frac{\partial \eta}{\partial m}}{\eta^2} \quad (\text{C.1})$$

As the efficiency in the interval $m \in [m_{\eta}^{\max}, m^{\max}]$ is decreasing, the derivative $\frac{\partial \eta}{\partial m}$ is negative. Thus, the expression $-Hm \cdot \frac{\partial \eta}{\partial m}$ is positive and, in consequence, $H\eta - Hm \cdot \frac{\partial \eta}{\partial m}$ is positive. Therefore, as the derivative from Eq. (C.1) is positive, the power in the interval $m \in [m_{\eta}^{\max}, m^{\max}]$ is an increasing function of the flow m .

- Case 2: $m \in [m^{\min}, m_{\eta}^{\max}]$. The power is also given by Eq. (2.11) with the derivative from Eq. (C.1), but as $\frac{\partial \eta}{\partial m} > 0$, the sign of $H\eta - Hm \cdot \frac{\partial \eta}{\partial m}$ is undefined.

Proof. As the efficiency is a concave function of the flow, the values of the extrapolated tangent at each point m_i^* are larger than the values of the efficiency. It is possible to choose a sequence $\{m_i^*\}$ for the calculation of the tangents.

The formula for the tangent y_i at m_i^* is given as

$$y_i = a_i m + b_i \quad (\text{C.2})$$

where $i = 2, 3, \dots$ and

$$a_i = \eta'(m_i^*) \quad (\text{C.3})$$

and

$$b_i = -\eta'(m_i^*)m_i^* + \eta(m_i^*) \quad (\text{C.4})$$

The tangent y_i is marked with a dashed line in Fig. C.1. Let now define an auxiliary function W_i as:

$$W_i = \frac{Hm}{y_i} \quad (\text{C.5})$$

As $y_i \geq \eta$ for all m , the power from Eq. (2.11) is bounded by W_i :

$$W = \frac{Hm}{\eta} \geq W_i \quad (\text{C.6})$$

The power W will be bounded for all m . Hence, if all W_i are increasing with the mass flow, the power W will also be an increasing function of the flow. Lemma 1 shows that the monotonicity of the functions W_i depends on the sign of the y -intercept, b_i . Thus, if $b_i \geq 0$ for all i , the power is bounded from below by an increasing function. The sequence for m_i^* can be chosen arbitrarily and thus Lemma 2 shows that the power must be increasing for all $m \in [m^{\min}, m_\eta^{\max}]$. $\square$

It can be noted that the requirement $b_i \geq 0$ with b_i given by Eq. (C.4) is equivalent to requiring the numerator in Eq. (C.1) to be positive. $\square$

Lemma 1 shows what conditions must be fulfilled for the tangents of power W_i from Eq. (C.5) to be increasing function of the flow m .

Lemma 1. *A function $W_i(m) = \frac{Hm}{a_i m + b_i}$ with $H = \text{const}$ is an increasing function of m if $b_i > 0$.*

Proof. The monotonicity of $W_1 = \frac{Hm}{am+b}$ can be calculated from the sign of the first derivative with respect to the flow:

$$\frac{\partial W_1}{\partial m} = \frac{Hb}{(am+b)^2} \quad (\text{C.7})$$

The sign of the derivative from Eq. (C.7) depends on the sign of the parameter b . If $b > 0$, W_1 is increasing. $\square$

Lemma 2 shows that if a function is bounded from below by a tangent curve and the tangent curve is increasing, then the function will also be increasing. In this case, the power W is bounded by the tangents W_i , so if W_i is an increasing function of the flow, then W is an increasing function of the flow as well.

Lemma 2. *Let $f(x) : [a, b] \rightarrow [c_f, d_f]$, $a > 0$, $c > 0$ be a continuous function. If $\forall x^* \in [a, b] \exists g^*(x) : [a, b] \rightarrow [c_g, d_g]$ such that $g^*(x^*) = f(x^*)$, $\forall x \in [a, b] \setminus x^*$ such that $g^*(x) < f(x)$, and $g^*(x)$ is increasing, then $f(x)$ is increasing.*

Proof. (By contradiction) Assume that $f(x)$ is decreasing locally in $[a_1, b_1] \subset [a, b]$. Then

$$\forall x_1, x_2 \in [a, b] : x_1 < x_2 \implies f(x_1) > f(x_2) \quad (\text{C.8})$$

But at x_1 there exists a function g^* such that $g^*(x_1) < g^*(x_2)$. As $g^*(x) < f(x)$, then in particular $g^*(x_2) < f(x_2)$. Thus $g^*(x_1) < f(x_2)$. But $g^*(x_1) = f(x_1)$, so $f(x_1) < f(x_2)$ which contradicts the assumption from (C.8). $\square$

C.1.1.1 Convexity of power

It is necessary to prove that the power of a compressor is a convex function of the flow in the given operating range.

Theorem 2. *The power W is a convex function of the flow in the current operating range for a fixed head if the equation*

$$2\eta'^2 m - 2\eta' \eta - m\eta'' \eta = 0 \quad (\text{C.9})$$

does not have a solution within the operating range.

Proof. Similarly to the proof for monotonicity of the power, the proof of convexity is divided in two parts: for efficiency increasing for $m \in [m_{\eta}^{\min}, m_{\eta}^{\max}]$ and decreasing for $m \in [m_{\eta}^{\max}, m^{\max}]$.

- Case 1: $m \in [m_{\eta}^{\max}, m^{\max}]$. A twice-differentiable function of one variable is convex if and only if its second derivative is non-negative. Calculating the second derivative of power yields:

$$\frac{\partial^2 W}{\partial m^2} = \frac{2\eta\eta'^2 H m - 2H\eta'\eta^2 - Hm\eta''\eta^2}{\eta^4} \quad (\text{C.10})$$

It is enough to check the sign of the numerator in Eq. (C.10). The efficiency is decreasing for $m \in [m_{\eta}^{\max}, m^{\max}]$, so $\eta' < 0$ in this interval. The efficiency is also a concave function of m for $m \in [m_{\eta}^{\max}, m^{\max}]$, so $\eta'' \leq 0$. Hence

$$\underbrace{2\eta\eta'^2 H m}_{>0} + \underbrace{(-2H\eta'\eta^2)}_{>0} + \underbrace{(-Hm\eta''\eta^2)}_{\geq 0} > 0 \quad (\text{C.11})$$

Dividing both sides of inequality (C.11) by $H\eta$ does not change the inequality because $H\eta$ is positive. Thus:

$$2\eta'^2 m - 2\eta'\eta - m\eta''\eta > 0 \quad (\text{C.12})$$

which means that Eq. (C.9) does not have a solution for the flows in the interval $[m_\eta^{\max}, m^{\max}]$. Thus, the power is a convex function of the flow for $m \in [m_\eta^{\max}, m^{\max}]$.

- Case 2: $m \in [m^{\min}, m_\eta^{\max}]$. It is not possible to check the sign of the numerator of Eq. (C.10), because $\eta' > 0$ for $m \in [m^{\min}, m_\eta^{\max}]$. Therefore, if there exists a solution such that Eq. (C.10) is zero, if there exists $m = \bar{m}$ such that

$$2\eta'^2 \bar{m} - 2\eta'\eta - \bar{m}\eta''\eta = 0 \quad (\text{C.13})$$

then the power will have an inflection point there. If $\bar{m} < m^{\min}$, then the power will be a convex function of the flow in the whole operating range, because it is a convex function for $m \in [m_\eta^{\max}, m^{\max}]$. Graphically, if $\bar{m}$ lies to the left of the dotted area in Fig. C.1, the power will be a convex function of the flow in the operating range.

□

C.2 Solution of the optimisation

The Lagrangian function for the problem with the objective function (7.11) and constraints (7.13) and (7.14) with $T > 1$ is:

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^N \sum_{t=1}^T \left(a_{it} m_{it}^2 + b_{it} m_{it} + c_{it} \right) + \sum_{t=1}^T \lambda_t \left(\sum_{i=1}^N m_{it} - M_t \right) \\ & + \sum_{i=1}^N \left(\sum_{t=1}^T \mu_{it} \left(m_{it}^{\min} - m_{it} \right) \right) + \sum_{i=1}^N \left(\sum_{t=1}^T \gamma_{it} \left(m_{it} - m_{it}^{\max} \right) \right) \end{aligned} \quad (\text{C.14})$$

The KKT conditions are:

$$W'_i(m_{it}) + \lambda_t - \mu_{it} + \gamma_{it} = 0 \quad (\text{C.15a})$$

$$\mu_{it}(m_{it}^{\min} - m_{it}) = 0 \quad (\text{C.15b})$$

$$\gamma_{it}(m_{it} - m_{it}^{\max}) = 0 \quad (\text{C.15c})$$

$$\sum_{i=1}^N m_{it} - M_t = 0 \quad (\text{C.15d})$$

$$\mu_{it} \geq 0 \quad (\text{C.15e})$$

$$\gamma_{it} \geq 0 \quad (\text{C.15f})$$

where

$$W'_i(m_{it}) = 2a_{it}m_{it} + b_{it} \quad (\text{C.16})$$

For a given period t , the conditions (C.15) are equivalent to the conditions describing a single period optimisation problem, because neither of them depend on two or more time instants. Thus, the problem of multi-period optimisation with constraints (7.13) and (7.14) is equivalent to solving a single-period optimisation in each period.

C.2.1 Optimisation without demand constraint

The first case analysed is the problem from Eq. (7.11) and constraints (7.14), but without the demand constraint. The Lagrangian from Eq. (C.14) is simplified to:

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^N \sum_{t=1}^T \left(a_{it}m_{it}^2 + b_{it}m_{it} + c_{it} \right) \\ & + \sum_{i=1}^N \left(\sum_{t=1}^T \mu_{it} (m_{it}^{\min} - m_{it}) \right) \\ & + \sum_{i=1}^N \left(\sum_{t=1}^T \gamma_{it} (m_{it} - m_{it}^{\max}) \right) \end{aligned} \quad (\text{C.17})$$

The KKT condition are:

$$W'_i(m_{it}) - \mu_{it} + \gamma_{it} = 0 \quad (\text{C.18a})$$

$$\mu_{it}(m_{it}^{\min} - m_{it}) = 0 \quad (\text{C.18b})$$

$$\gamma_{it}(m_{it} - m_{it}^{\max}) = 0 \quad (\text{C.18c})$$

$$\mu_{it} \geq 0 \quad (\text{C.18d})$$

$$\gamma_{it} \geq 0 \quad (\text{C.18e})$$

The conditions from Eq. (C.18b) and (C.18c) are satisfied if:

- $\mu_{it}^* = 0$ or $m_{it}^* = m_{it}^{\min}$
- $\gamma_{it}^* = 0$ or $m_{it}^* = m_{it}^{\max}$

Assuming $\mu_{it}^* = 0$, then the assigned flows are different than the minimal value, $m_{it}^* \neq m_{it}^{\min}$. Substituting $\mu_{it}^* = 0$ into Eq. (C.15a) yields:

$$\gamma_{it}^* = -W'_i(m_{it}^*) \quad (\text{C.19})$$

As $W(m_{it}^*)$ is strictly increasing, the derivative $W'_i(m_{it}^*)$ is positive. Thus, the inequality (C.18e) is violated. Thus, $\mu_{it}^* \neq 0$ and $m_{it}^* = m_{it}^{\min}$. As the problem from Eq. (7.11) with constraints (7.14) is a convex quadratic optimisation, the solution $m_{it}^* = m_{it}^{\min}$ is unique.

The result indicates that if there is no demand constraint of the form from Eq. (7.13), the compressors will work at their minimal allowed flows.

C.2.2 Optimisation with the demand constraint

It will be now shown that the assignment of loads in a multi-compressor station will depend on the relationships between the compressors if the demand M_t must be satisfied in each period. The KKT conditions are given in Eq. (C.15). Again, conditions (C.15b) and (C.15c) will be satisfied if:

- $\mu_{it} = 0$ or $m_{it}^* = m_{it}^{\min}$
- $\gamma_{it} = 0$ or $m_{it}^* = m_{it}^{\max}$

Two cases are considered:

- None of the compressors work at their limits which gives $\mu_{it}^* = 0$ and $\gamma_{it}^* = 0$

- A number k of compressors work at their limits which gives $\mu_{jt}^* \neq 0$ and $\gamma_{jt}^* \neq 0$ for $j \in \{1, 2, \dots, k\}$

It is also assumed that the demand does not require all the compressors to work at their minimal and maximal limits. If $M_t = \sum_{i=1}^N m_{it}^{\min}$, there is no flexibility in assigning the loads m_{it} . Similarly, there is no flexibility if $M_t = \sum_{i=1}^N m_{it}^{\max}$, so these cases are not considered.

C.2.2.1 None of the compressors are at the limits

As none of the compressors are assumed at their limits, $m_{it}^* \neq m_{it}^{\min}$ and $m_{it}^* \neq m_{it}^{\max}$ which means that the corresponding dual variables are zero, $\mu_{it}^* = 0$ and $\gamma_{it}^* = 0$, to fulfil Eq. (C.15b) and (C.15c). The KKT conditions from Eq. (C.15) become a system of $NT + 1$ equations with $NT + 1$ variables:

$$W_i'(m_{it}^*) + \lambda^* = 0 \quad (\text{C.20a})$$

$$\sum_{i=1}^N m_{it}^* - M_t = 0 \quad (\text{C.20b})$$

The system of equations (C.20) can be solved by expressing all m_{it}^* as functions of λ^* from Eq. (C.20a) and substituting them in (C.20b). Then:

$$\lambda^* = - \frac{\sum_{i=1}^N \frac{W_i'\left(\frac{2M_t}{N}\right)}{a_{it}}}{\sum_{i=1}^N \frac{1}{a_{it}}} \quad (\text{C.21})$$

and

$$m_{it}^* = \frac{-\lambda^* - b_{it}}{2a_{it}} \quad (\text{C.22})$$

In particular, for $N = 2$ it is obtained that:

$$m_{1t}^* = \frac{2a_{2t}M_t + b_{2t} - b_{1t}}{2a_{2t} + 2a_{1t}} \quad (\text{C.23})$$

and

$$m_{2t}^* = M_t - m_{1t}^* = \frac{2a_{1t}M_t + b_{1t} - b_{2t}}{2a_{2t} + 2a_{1t}} \quad (\text{C.24})$$

Equation (C.22) confirms also that equal load assignment is optimal if com-

pressors are identical, as it yields:

$$m_{it}^* = \frac{M}{N} \quad (\text{C.25})$$

C.2.2.2 Compressors work at the limits

If there is a number k of compressors working at their limits, the KKT conditions from Eq. (C.15) can be written:

$$W'_j(m_{jt}) + \lambda - \mu_{jt} + \gamma_{jt} = 0, \text{ for } j \in \{1, 2, \dots, k-1\} \quad (\text{C.26a})$$

$$W'_k(m_{kt}) + \lambda - \mu_{kt} + \gamma_{kt} = 0 \quad (\text{C.26b})$$

$$W'_v(m_{vt}) + \lambda = 0, \text{ for } v \in \{k+1, \dots, N\} \quad (\text{C.26c})$$

$$\sum_{i=1}^N m_{it} - M_t = 0 \quad (\text{C.26d})$$

$$\mu_{it}(m_{it}^{\min} - m_{it}) = 0 \quad (\text{C.26e})$$

$$\gamma_{it}(m_{it} - m_{it}^{\max}) = 0 \quad (\text{C.26f})$$

where the index j indicates the compressors working at the limits, the index k indicates the compressor for which the conditions will be derived, and the index v indicates unrestrained compressors, i.e. compressors which work not at their limits. It is assumed that $k \leq N-1$, so that there exist at least two compressors that are not at their limits. The case with one compressor not at limits is a special case and will be considered separately. The conditions when the compressor k works at its limit can be derived as follows. Since a compressor is able to work only at one limit at the time, the indicator 1_μ shows which limits is active for which compressor:

$$1_{\mu_i} = \begin{cases} 1 & \text{if } m_{it}^* = m_{it}^{\min} \\ 0 & \text{if } m_{it}^* = m_{it}^{\max} \end{cases} \quad (\text{C.27})$$

Introducing:

$$\xi_{it} = -1_{\mu_i}\mu_{it} + (1 - 1_{\mu_i})\gamma_{it} \quad (\text{C.28})$$

in Eq. (C.26b) yields:

$$W'_j(m_{jt}^*) + \lambda^* + \xi_{jt} = 0, \text{ for } j \in \{1, 2, \dots, k-1\} \quad (\text{C.29a})$$

$$W'_k(m_{kt}^*) + \lambda^* + \xi_{kt} = 0 \quad (\text{C.29b})$$

Equation (C.26d) can be rewritten as:

$$\sum_{j=1}^{N-k} m_{jt}^* + \sum_{v=1}^{N-k+1} m_{vt}^* - M_t = 0 \quad (\text{C.30})$$

which is an equation with $N - k + 1$ unknown variables m_{vt}^* . Combining Eq. (C.30) with Eq. (C.26c) gives $N - k + 1$ equations with $N - k + 1$ variables that can be used to find the values of m_{vt}^* and λ^* . In particular, Eq. (C.26c) yields:

$$\lambda^* = -W'_v(m_{vt}^*) \quad (\text{C.31})$$

Inserting Eq. (C.31) in (C.29b) and solving for ξ_{kt} yields:

$$\xi_{kt} = -W'_k(m_{kt}^*) + W'_v(m_{vt}^*) \quad (\text{C.32})$$

Equation (C.32) gives:

$$\mu_{kt}^* = W'_k(m_{kt}^*) - W'_v(m_{vt}^*) > 0 \quad (\text{C.33})$$

if the compressor k works at its minimal limit $m_{kt}^* = m_{kt}^{\min}$ and

$$\gamma_{kt}^* = -W'_k(m_{kt}) + W'_v(m_{vt}^*) > 0 \quad (\text{C.34})$$

if the compressor k works at its maximal limit $m_{kt}^* = m_{kt}^{\max}$. Therefore, the compressor k will work at its boundary depending on the relationship between the gradients. For instance, for Eq. (C.33) to be a valid solution for μ_{kt}^* , the inequality must be fulfilled:

$$W'_k(m_{kt}^*) \geq W'_v(m_{vt}^*) \quad (\text{C.35})$$

Therefore, the compressor k will work at its minimal bound if the gradient of its power curve at $m_k^{\min}$ is larger than the gradient of the power curve of an unrestrained compressor v at its value obtained from Eq. (C.30) with Eq. (C.26c).

From Eq. (C.26c), it is obtained that for any $v_1, v_2 \in \{k+1, \dots, N\}$

$$W'_{v_1}(m_{v_1 t}^*) = W'_{v_2}(m_{v_2 t}^*) \quad (\text{C.36})$$

Therefore, the condition from Eq. (C.35) holds for all $v \in \{k+1, \dots, N\}$.

The case with one compressor not at limits is a special case for certain values of demand, depending on Eq. (C.30):

$$m_{Nt}^* = M_t - \sum_{j=1}^{N-1} m_{jt}^* \quad (\text{C.37})$$

If $m_{Nt}^{\min} < m_{Nt}^* < m_{Nt}^{\max}$, then the corresponding μ_{Nt}^* and γ_{Nt}^* are zero and the problem can be solved to obtain Eq. (C.33) and (C.34). If the compressor N is forced to work at its limits for M_t , then the corresponding parameter μ_{Nt}^* or γ_{Nt}^* will be different than zero. Nocedal and Wright (1999) indicate that in that case there will be infinitely many values of λ^* that satisfy Eq. (C.26).

Equation (C.26) gives N equations with $N+1$ variables

$$W'_j(m_{jt}^*) + \lambda^* + \xi_{jt} = 0, \text{ for } j \in \{1, 2, \dots, N\} \setminus \{k\} \quad (\text{C.38a})$$

$$W'_k(m_{kt}^*) + \lambda^* + \xi_{kt} = 0 \quad (\text{C.38b})$$

Equation (C.38) has infinitely many solutions of form:

$$\lambda^* = -W'_k(m_{kt}^*) - \xi_{kt} \quad (\text{C.39})$$

and

$$\xi_{jt} = W'_k(m_{kt}^*) - W'_j(m_{jt}^*) + \xi_{kt} \quad (\text{C.40})$$

where ξ_{kt} is treated as a parameter. Taking into account that $\mu_{jt}^* > 0$, $\gamma_{jt}^* > 0$, $\mu_{kt}^* > 0$, $\gamma_{kt}^* > 0$ yields the conditions for μ_{kt}^* and γ_{kt}^* :

$$\mu_{kt}^* \in (W'_k(m_{kt}^*) - W'_u(m_{ut}^*), W'_k(m_{kt}^*) - W'_w(m_{wt}^*)) \quad (\text{C.41})$$

and

$$\gamma_{kt}^* \in (W'_w(m_{wt}^*) - W'_k(m_{kt}^*), W'_u(m_{ut}^*) - W'_k(m_{kt}^*)) \quad (\text{C.42})$$

where index w denotes compressors working at their maximal limits and index u denotes compressors working at their minimal limits.

Appendix D

Power and efficiency optimisation

This section shows that maximisation of efficiency is not equivalent to minimisation of the power to run the compressors due to differences in the properties of the functional forms of the power curves and efficiency curves. It is shown that the power is a convex and strictly increasing function of the flow, whereas the efficiency is only strictly concave. The results from this appendix have been submitted as a conference publication (Zagorowska and Thornhill, 2020b).

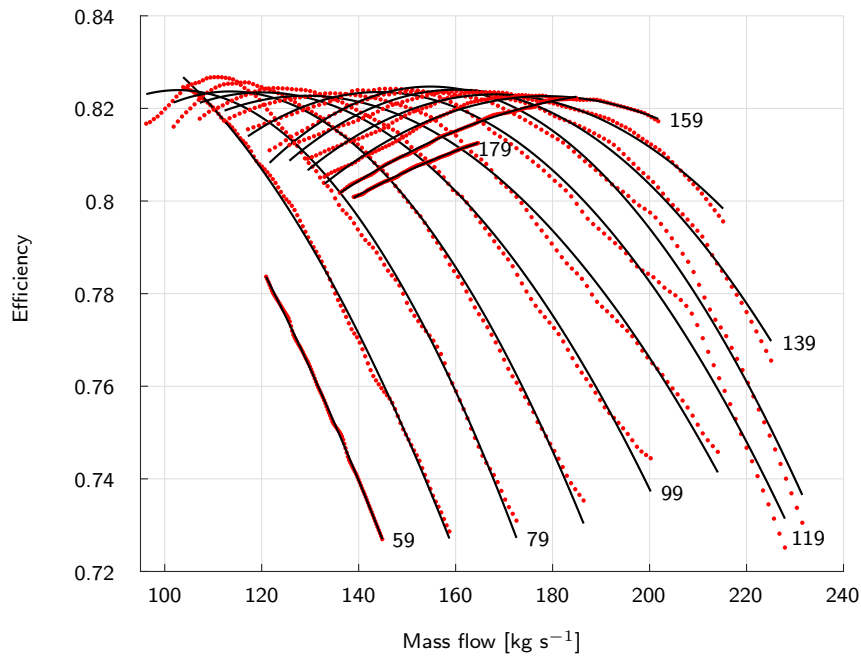
Figure D.1a shows the efficiency from Fig. D.2a as a function of the mass flow for a fixed head. The red dotted lines indicate the efficiency obtained from Nørstebø (2008), whereas the black solid lines denote the quadratic approximation, as suggested by Cortinovis et al. (2016b). For a given head, the efficiency of a compressor can be approximated with a quadratic polynomial of the mass flow m :

$$\eta = am^2 + bm + c \quad (\text{D.1})$$

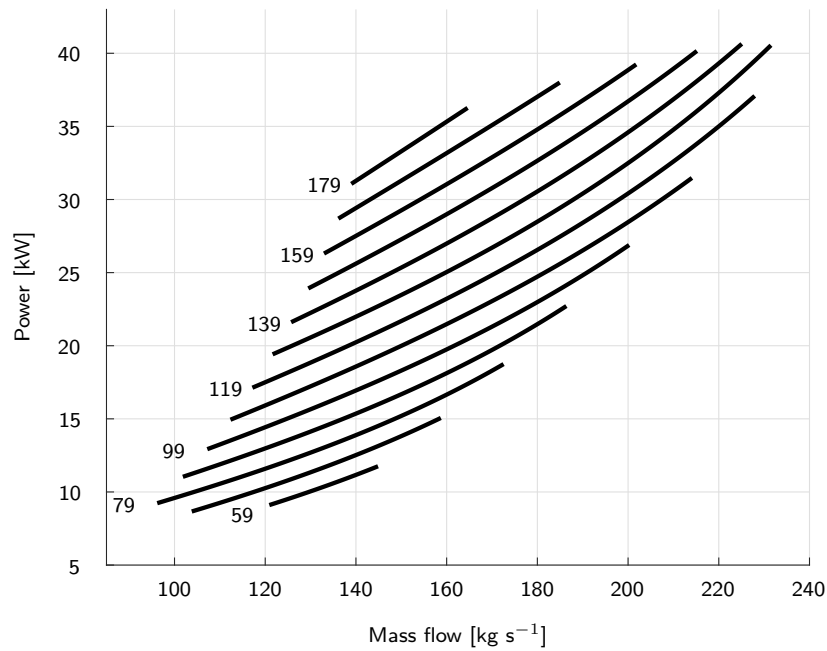
where a , b , and c are constant parameters. In particular, $a < 0$, which indicates that the efficiency from Eq. (D.1) is a concave function of the mass flow. This approach is also considered in this work.

The relative error of the approximation for the data from Fig. D.1a is calculated according to formula

$$\epsilon = \frac{|\eta_{\text{real}} - \eta_{\text{A}}|}{\eta_{\text{real}}} \quad (\text{D.2})$$

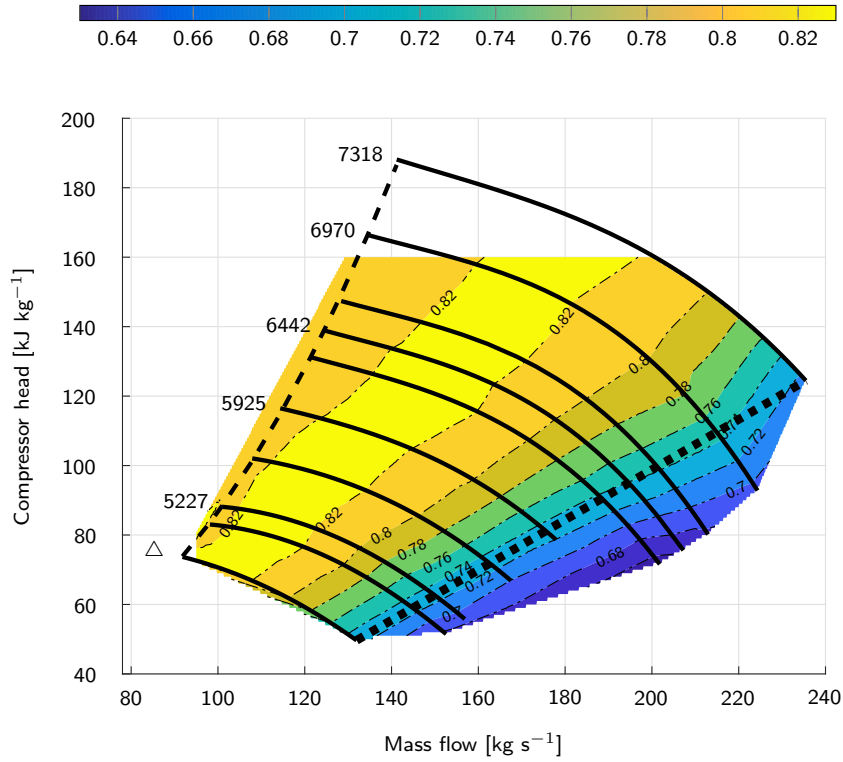


(a) Compressor efficiency from Nørstebø (2008) (---) with the quadratic approximation (—)

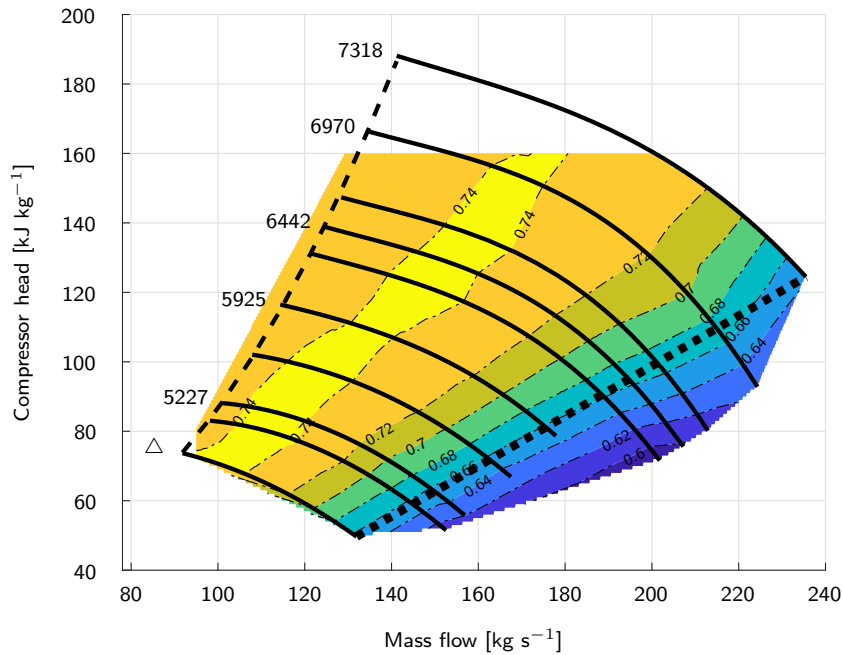


(b) Compressor power calculated for approximated efficiency

Figure D.1: Compressor efficiency and power as functions of the flow for various head in kJ kg^{-1}



(a) Compressor map of Compressor A



(b) Compressor map of Compressor B, with efficiency obtained as 90% of the efficiency of Compressor A

Figure D.2: Compressor map with speed lines (—), efficiency contours, the surge line (-- --), and the choke line (...). The line labelled with a triangle denotes the minimal speed 4740 rpm. The white area above 160 kJ kg^{-1} is due to physical constraints imposed on the system used by Nørstebø (2008)

where η_{real} is obtained from Nørstebø (2008) and η_A is calculated from Eq.(D.1). The error ϵ is smaller than 1%, so the quadratic approximation is used in further calculations in Section D.1.

The power necessary to run a compressor with efficiency η at the head H and the mass flow m is given as Wu et al. (2000b):

$$W = \frac{Hm}{\eta} \quad (\text{D.3})$$

Figure D.1b presents the power to run a compressor with efficiency approximated by Eq. (D.1) for the data obtained from Nørstebø (2008).

Analysis of Fig. 6.3 indicates that the maximal efficiency is obtained inside the operating range for a compressor (yellow area is inside the operating range). At the same time, Fig. D.1 shows that the minimal power to run is required for the minimal flow. It will be shown in Section D.1 that the mass flow for which the maximal efficiency is obtained is not the same as the flow for which the power to run a compressor is minimal.

D.1 Maximisation of efficiency and minimisation of power consumption

D.1.1 Optimisation formulation

The problem of optimal load-sharing in a compressor station is formulated as a nonlinear convex optimisation problem. If the objective is to minimise the power necessary to run a compressor station with N compressors Wu et al. (2000b), the optimisation problem becomes:

$$\min \sum_{i=1}^N W_i \quad (\text{D.4})$$

where W_i is calculated according to Eq. (D.3). Conversely, if the objective is to maximise the efficiency, the objective function is

$$\min \left(- \sum_{i=1}^N \eta_i \right) \quad (\text{D.5})$$

where η_i is calculated as a quadratic function according to Eq. (D.1).

The objective function from Eq. (D.4) is convex, as shown in Fig. D.1b and the fact that a sum of convex functions is convex. It is assumed that the efficiency is strictly concave with respect to the flow, so the objective function from Eq. (D.5) is therefore convex, because it has a negative sign. Therefore, the optimisation problem is convex and can be solved using the approaches described by Boyd and Vandenberghe (2009).

Regardless of the objective function, the compressor station must satisfy the demand:

$$\sum_{i=1}^N m_i = M \quad (\text{D.6})$$

The load assigned to each compressor must be within the operating range defined by the compressor map:

$$m_i^{\min} \leq m_i \leq m_i^{\max} \quad (\text{D.7})$$

The values $m_i^{\min}$ and $m_i^{\max}$ in the inequality (D.7) are obtained for a fixed value of head from the compressor map from Fig. D.1. The head is constant, because the suction and discharge pressure are constant.

D.1.2 Application

For this work, a compressor station with two compressors was chosen as an example, and hence $N = 2$. The characteristics of both compressors are shown in Fig. D.3 with the efficiency in Fig. D.3a and power in Fig. D.3b obtained for $H = 119 \text{ kJ kg}^{-1}$. The flows through each compressor must be in the range $116 \text{ kg s}^{-1} \leq m_i \leq 228 \text{ kg s}^{-1}$. The solid lines in Fig. D.3 come from Nørstebø (2008) and represent the characteristics of Compressor A. The dash-dotted lines correspond to Compressor B and were obtained as 90% of characteristics of Compressor A. The efficiency for Compressor A is given as:

$$\eta_A = -1.3126 \times 10^{-5} m_A^2 + 0.0038 m_A + 0.5508 \quad (\text{D.8})$$

with m_A denoting the mass flow through Compressor A. The efficiency for Compressor B is given as:

$$\eta_B = 0.9(-1.3126 \times 10^{-5} m_B^2 + 0.0038 m_B + 0.5508) \quad (\text{D.9})$$

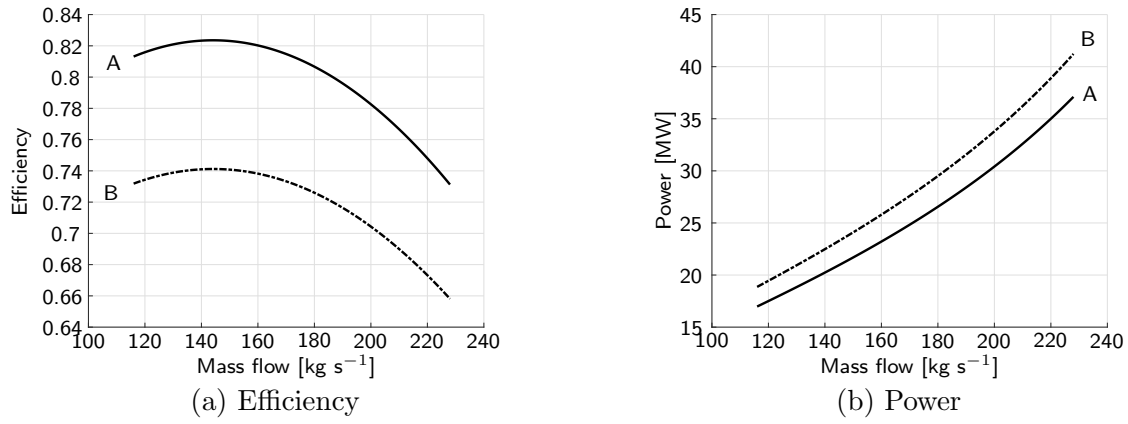


Figure D.3: Compressor characteristics as a function of the mass flow for fixed head $H = 119 \text{ kJ kg}^{-1}$, chosen for further analysis. The solid line corresponds to Compressor A directly from Nørstebø (2008), the dash-dotted line corresponds to Compressor B with characteristics obtained as 90% of characteristics of Compressor A

with m_B denoting the mass flow through Compressor B.

Figure D.4 shows the two objective functions from Eq. (D.4) (colour) and (D.5) (—) with the operating range for Compressor A (—) and for Compressor B (—). The largest value of efficiency corresponds to the smallest dark blue circle. Conversely, the coloured lines correspond to the contours of the objective function for minimisation of power, with the yellow line in the top right corner denoting the largest value and the blue line in the bottom left corner denoting the lowest value of power to run the two compressors. The dashed lines show the minimal flows for both compressors, with black corresponding to Compressor A and grey corresponding to Compressor B. Similarly, the solid black and grey lines show the maximal flows for both compressors. The pink lines correspond to two values of demand M from Eq. (D.6) and will be analysed in Section D.1.4.

D.1.3 No specified value for the flow

If there is no specified value for the flow, for instance if the compressors are only used to maintain the downstream pressure p_d , the minimisation of power would make the compressors work at their lower limit (thick black asterisk in Fig. D.4). In contrast, maximising the efficiency would place the compressors within the operating range at the point marked with a thick black circle.

This fact indicates that if the flow is not specified, there is no demand constraint from Eq. (D.6) and the minimisation of power is not equivalent to maximisation of efficiency.

D.1.4 With demand for the flow - efficiency maximisation

Figure D.4 shows also that the solutions obtained from minimisation of power (asterisk) would be different than obtained from maximisation of efficiency (circle) for a fixed value of demand. The dashed pink line shows the demand constraint for $M = 250 \text{ kg s}^{-1}$ and the dash-dotted pink line for $M = 400 \text{ kg s}^{-1}$. The values were chosen to put the equal load on both sides of the maximal efficiency $\eta^{\max}$ in Fig. D.3a. The grey-black rectangle shows the operating range for the station, with dashed lines showing the minimal and the solid lines the maximal flows for each compressor. Black lines show the limits for Compressor A and grey showing the limits for compressor B.

The flows assigned to each compressor if the efficiency is maximised are indicated by the open circle symbols in Fig. D.4. They are:

$$\begin{aligned} m_A &= \frac{1.8aM - 0.1b}{3.8a} \\ m_B &= \frac{2aM + 0.1b}{3.8a} \end{aligned} \tag{D.10}$$

Equation (D.10) was obtained by solving a quadratic optimisation problem with the objective function from Eq. (D.5). Analysing the solution from Eq. (D.10) and indicated by open circle symbols in Fig. D.4 reveals that for $M < 289.5$ the value assigned to Compressor A will be smaller than the value assigned to Compressor B. Conversely, for $M > 289.5$ the value assigned to Compressor A will be larger than the value assigned to Compressor B. If $M = 289.5$, then the compressors will have the same mass flow rate, $m_A = m_B$, because it corresponds to the maximum of efficiency.

D.1.5 With demand for the flow - power minimisation

To find the flows assigned to each compressor if the power is minimised, it is necessary to solve a nonlinear optimisation problem with the objective function

from Eq. (D.4) using Eq. (D.3). It was shown in Wu et al. (2000b) the relationship between power consumption and flow can also be approximated by a second order polynomial. it was shown in Wu et al. (2000b) that the power can be approximated by a second order polynomial as well

$$W_i = a_i m_i^2 + b_i m_i + c \quad (\text{D.11})$$

where a_i , b_i , and c_i are constant parameters, $i \in \{A, B\}$. Following the assumption that the efficiency of Compressor B is 90% of the efficiency of Compressor A and using Eq. (D.3) indicates that the power to run Compressor B is 111% of the power to run Compressor A. For the power curves from Fig. D.3b, it is obtained that:

$$W_A = 510m_A^2 + 6300m_A + 11 \times 10^6 \quad (\text{D.12})$$

with m_A denoting the mass flow through Compressor A. The power for Compressor B is given as:

$$W_B = 1.11(510m_A^2 + 6300m_A + 11 \times 10^6) \quad (\text{D.13})$$

with m_B denoting the mass flow through Compressor B. Solving a quadratic optimisation problem with the objective function from Eq. (D.4) yields:

$$\begin{aligned} m_A &= \frac{2.11a_A M + 0.11b_A}{4.22a_A} \\ m_B &= \frac{2a_A M - 0.11b_A}{4.22a_A} \end{aligned} \quad (\text{D.14})$$

It can be shown using Eq. (D.14) that Compressor B will always get a smaller load than Compressor A, because the gradient of its power curve with respect to the flow is larger than that of Compressor A. Because of the convexity of power, the gradient is larger in the whole operating range, so Compressor B will always get less load, regardless of the value of the demand. Physically, if compressor A is more efficient than compressor B, then the power consumption can be reduced by giving more flow to Compressor A than to Compressor B. This is true even if the extra flow means compressor A is not working at its own maximum efficiency.

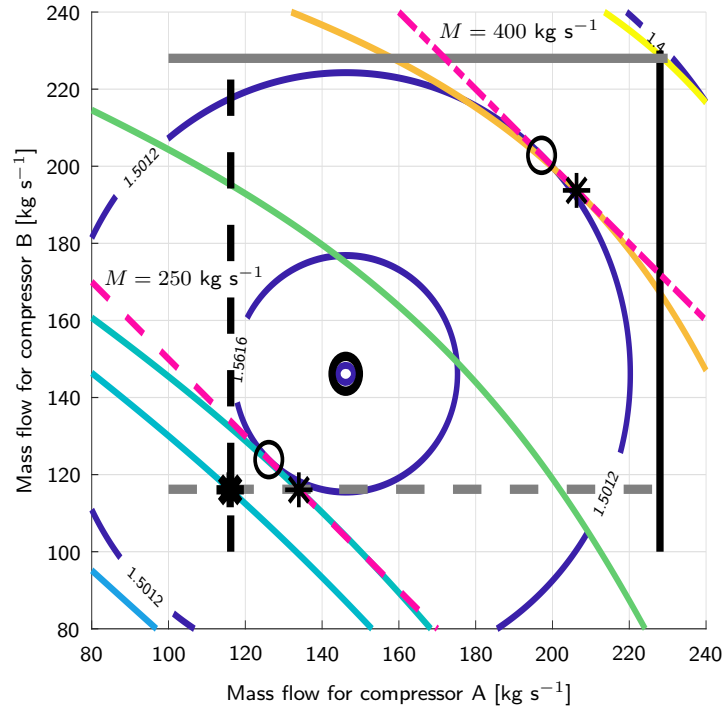


Figure D.4: Two objective functions from Eq. (D.4) (colour) and (D.5) (dark blue) with the operating range for Compressor A in black and for Compressor B in grey. Maximal efficiency corresponds to the smallest dark blue circle. Minimal power is shown with light blue curve in bottom left corner and yellow corresponds to the largest power. Optimal solutions are shown for two values of demand: $M = 250 \text{ kg s}^{-1}$ (---) and $M = 400 \text{ kg s}^{-1}$ (—) obtained for power minimisation (*) and efficiency maximisation (○). Thick black circle and thick asterisk show the optimal solution if no demand for the flow is required

D.1.6 With demand for the flow - comparison

Figure D.4 shows that the results obtained from maximisation of efficiency (circles) are different than the results obtained from minimisation of power (asterisks). These findings are further confirmed in Fig. D.5a and D.5b, which show enlarged areas around the solutions obtained for the demand $M = 400 \text{ kg s}^{-1}$ and $M = 250 \text{ kg s}^{-1}$. In both cases, the solution maximising the efficiency and denoted with a circle lies on the dark blue efficiency contour, but not on the power contour (yellow in Fig. D.5a and light blue in Fig. D.5b). Conversely, the solution obtained for minimisation of power consumption and denoted with the asterisk lies on the power contour, but not on the efficiency contour.

In particular, Figure D.5b shows that the solution obtained for minimisation of power (asterisk) lies on the boundary of the operating range of Compressor B. Conversely, the solution obtained for maximisation of efficiency (circle) lies inside the operating range. This fact confirms that a compressor attains its maximal efficiency inside the operating range, but it will require minimal power if the flow is minimal. Therefore, the minimisation of power consumption in a compressor station is not equivalent to maximisation of the efficiency of each compressor.

D.2 Discussion and conclusions

The work shows that optimal load allocation in a compressor station yields different results if efficiency is maximised or if the power to run the station is minimised. The two approaches are not equivalent both in the case when the overall mass flow is not specified and when it is.

If the demand is not specified, then the maximisation of efficiency will assign the loads inside the operating range which correspond to the maximal efficiency of each compressor. The minimisation of power will assign the lowest possible loads to each compressor and therefore the two approaches are not equivalent. It can be observed, however, that the two approach would assign the same flows if the maximal efficiency for each compressor is obtained for the lowest flow.

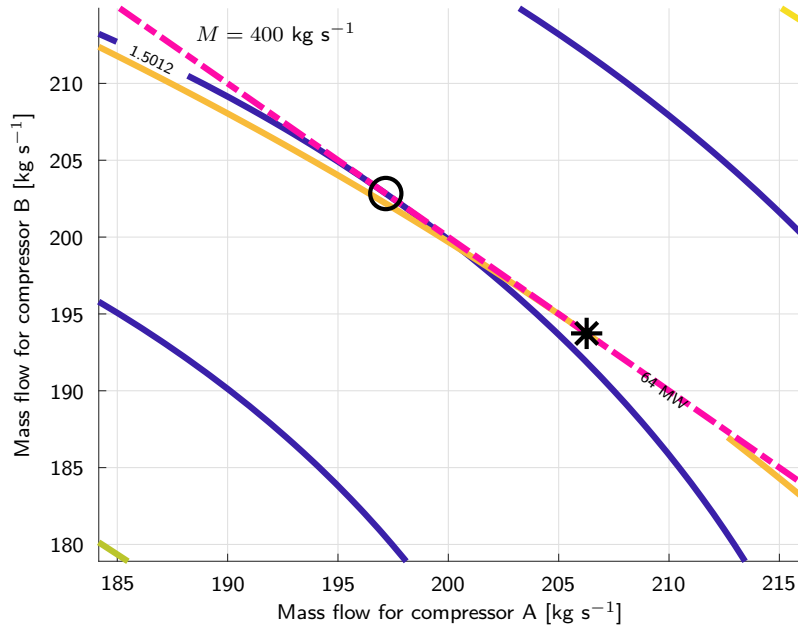
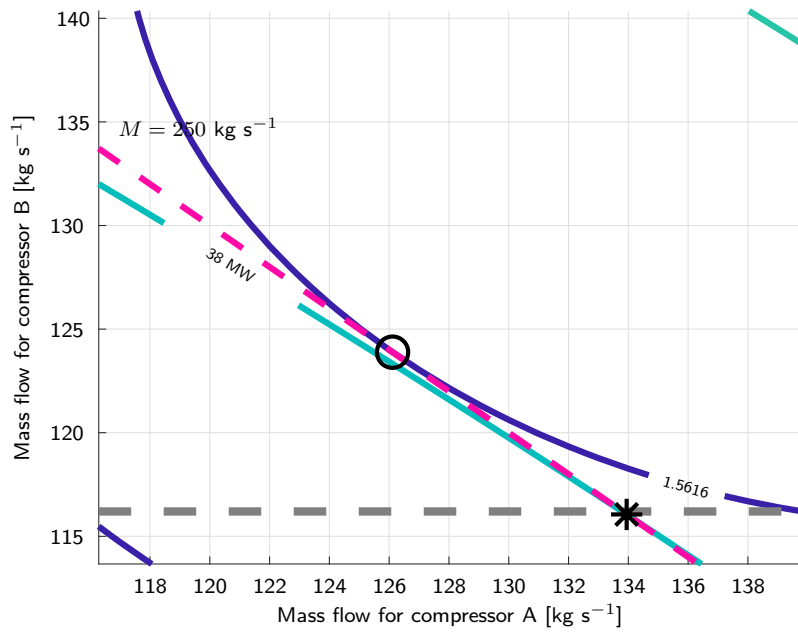
(a) Optimal solutions for $M = 400 \text{ kg s}^{-1}$ (b) Optimal solutions for $M = 250 \text{ kg s}^{-1}$

Figure D.5: Optimal solution for two values of the demand obtained for power minimisation (*) and efficiency maximisation (○)

If the mass flow is specified and the compressor station has to satisfy the demand, the values assigned to each compressor depend on the demand and on the gradient of the efficiency and the power. In the case of maximisation of efficiency, the less efficient compressor does not necessarily get a lower load, because the efficiency is a concave function of the mass flow. The fact that the efficiency is concave is illustrated in Fig. D.4 for the two values of demand on both sides of the maximum. Since Compressor A has a larger value of efficiency, the flow assigned to Compressor A will be closer to the flow for which the efficiency is maximal, here larger than the flow assigned to Compressor B. Conversely, for demand smaller than 289.5 kg s^{-1} , the efficiency of both compressor decreases. Again, since Compressor A will be assigned a flow closer to the flow for which maximal efficiency is obtained, in this case the flow is smaller than the flow assigned to Compressor B. The power, on the other hand, is a strictly increasing and convex function of the flow as shown in Fig. D.3b. Therefore, from Eq. (D.14) it is obtained that Compressor B will always get a lower flow.

Finally, both the maximisation of efficiency and the minimisation of power are formulated as quadratic optimisation problems. Moreover, the power and the efficiency of a compressor are strictly related to each other, which suggests that one can be obtained if the other is known. Therefore, even though the two approaches are not equivalent, they both enable optimisation of a compressor station taking compressor performance into account.