

Imperial College of Science, Technology and Medicine  
Department of Earth Science and Engineering

**Numerical modelling of wave-structure  
interaction with rubble-mound breakwaters  
and dynamic mesh optimisation**

Lluís Via-Estrem

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# **Abstract**

The coastal engineering community has historically relied on physical experiments for developing analytical methods and validating the design of coastal structures. However, the scale effects and costs associated with physical modelling have limited the progress in some key areas such as the design of rubble mounds. These structures are commonly used as breakwaters for harbours and coastal protection, the latter becoming more important in the recent years due to sea level rise. Computational modelling has become a realistic alternative over the past two decades with the increase of computer power and can already simulate many of the processes involved in wave-structure interaction at a low cost and without scale effects.

While numerical modelling of structural responses such as wave overtopping are already accepted methods by the coastal engineering community for designing and evaluating new and existing structures, the assumptions and approaches taken for obtaining such responses cast aside the potential capabilities of evaluating rubble mounds stability.

Over the past few years new developments in the Applied Modelling Computation Group (AMCG) at Imperial College on the FDEM model Solidity ([www.Solidityproject.com](http://www.Solidityproject.com)) have lead to the implementation of different tools for building and analysing numerical rubble mound structures. In the meantime, great progress has been made on Computational Fluid Dynamics (CFD) FEM AMCG codes, implementing new approaches for highly accurate interface capture and tracking and mesh optimisation with a reduced computational cost.

The purpose of this research is to further develop, optimise and validate AMCG FEM codes for simulations of Numerical Wave Tanks (NWT) and Wave-Structure Interaction (WSI) with rubble mounds, to develop numerical rock armour layer building capabilities, and finally to provide a few application cases such as measuring scale effects or the impact of sea level rise. Once ready and fully validated, such technologies have the potential to become a more accurate and cheaper alternative for rubble mound breakwaters design than physical modelling.



## **Declaration of Authorship and Copyright**

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*"The equation which I have formulated is a provisional truth in an advance toward the definite truth"*

*Ramon Iribarren Cavanilles, 1938*





# List of Abbreviations

$P_1DGP_1(CV)$  New spatial discretisation scheme with first order Discontinuous Galerkin for the velocity field and Control Volumes for the pressure field. See Section 4.2.2.

$P_{n_1}DGP_{n_2}$  Standard spatial discretisation scheme with order  $n_1$  and Discontinuous Galerkin for the velocity field, and order  $n_2$  and Continuous Galerkin for the pressure field. See Section 4.2.2.

**AMCG** Applied Modelling and Computation Group at Imperial College, London.

**BLc** Particle Blockiness.

**CFL** Courant–Friedrichs–Lewy condition.

**CG** Continuous Galerkin.

**CV** Control Volumes.

$D_n$  Particle nominal diameter for the  $n^{th}$  percentile of a sample of multiple particles.

**DEM** Discrete Element Method.

**DG** Discontinuous Galerkin.

**DMO** Dynamic Mesh Optimisation.

**FDEM** Finite and Discrete Element Methods combined.

**FE** Finite Element.

**FEM** Finite Element Method.

**IC-FERST** Imperial College - Finite Element Reservoir SimulaTor.

**$k_t$**  Layer thickness coefficient.

**LES** Large Eddy Simulation.

**LT** Particle Aspect Ratio.

**$M_n$**  Particle mass for the  $n^{th}$  percentile of a sample of multiple particles.

**NWT** Numerical Wave Tank.

**OWC** One Way Coupling, output from CFD modelling as input to solids modelling.

**TWC** Two Way Coupling, CFD modelling two-way coupled with solids modelling.

**VC-FDEM** Virtual Crane with FDEM.

**WSI** Wave-Structure Interaction.

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# Chapter 1

## Introduction

### 1.1 Background

Currently 8 of the top 10 largest cities in the world are located by the coast, and over 40 per cent of the World's population live in areas within 150km from the coast (UN - Oceans Atlas; Small and Nicholls, 2003; Nicholls and Cazenave, 2010). This concentration is significantly increasing in some areas in Asia and some regions in Africa expected to follow, adding a greater pressure on both land and sea.

In Europe, 43 per cent of the inhabitants of the EU member states live in coastal regions (Eurostat: Unit E1 and rural development, 2009), and over 70 per cent of the jobs are in the service sector, including leisure activities and tourism. Coastal defenses for protecting existing buildings and structures or stabilising the coastline and leisure areas now also have to face the impact of sea level rise and more frequent coastal flooding and other disasters (Neumann et al., 2015).

Moreover, due to globalisation and a continuous emphasis on shipping industry optimisation processes sea trade has risen to a blooming 90 per cent of the total world trade (International Chamber of Shipping, 2017), increasing the demand for new harbours and the expansion of existing ones.

The design of coastal structures has, since the very first pre-Hellenic harbours (Bruun, 2013), been following “trial and error” or “trial and failure” approach until few decades ago. New approaches, and the beginning of the Coastal Engineering discipline as we know it, pushed for a deeper analysis and understanding of coastal processes and wave-structure interaction. Since the beginning of the modern discipline, the coastal engineering community has relied on hydraulic physical modelling for formulating empirical response functions to feed analytical methods, and for validating design of new structures and solutions to coastal problems. Although physical modelling may have some disadvantages such as scale effects and cost, it is still nowadays the most accepted approach.

More recently, computational modelling has also positioned itself as a solution and a realistic alternative to physical modelling (Cavaleri et al., 2007; Tolman, 2010; Goda, 2010), thanks to the increase in computing power and the availability of new numerical modelling tools (Higuera et al., 2013; Jacobsen et al., 2012; Gomez-Gesteira et al., 2012; Vire et al., 2016). Some models are already able to give realistic answers for processes such as run-up and overtopping on coastal structures. However, the complexity of the processes involved in stability of coastal structures, and rubble-mounds in particular, makes this a very challenging problem for numerical modelling still nowadays.

Recent research on rubble mound breakwaters has started looking at the stability of rubble mound layers in depth with experiments and measurements of pressures and forces on individual units (Burcharth, 1985; Scott et al., 1989; Sakakiyama and Kajima, 1991; van der Meer and Heydra, 1991; Tørum, 1994; d’Angremond et al., 1995; Cornett, 1995; Hald, 1998; Moghim and Tørum, 2012; Jensen et al., 2014; Hofland et al., 2018), some of them with accelerometers to measure displacement.

In parallel, numerical models have also been used for modelling the rubble mound layers first approximating the outer layers with a porous layer (ideally for run-up and overtopping analysis), and later modelling a detailed section of the breakwater with a realistic representation of the units (Dentale et al., 2009, 2014; Altomare et al., 2012, 2014; Latham et al., 2015). The latter, however, remain a tentative approach with small sections of rubble mounds that do not

represent real breakwaters.

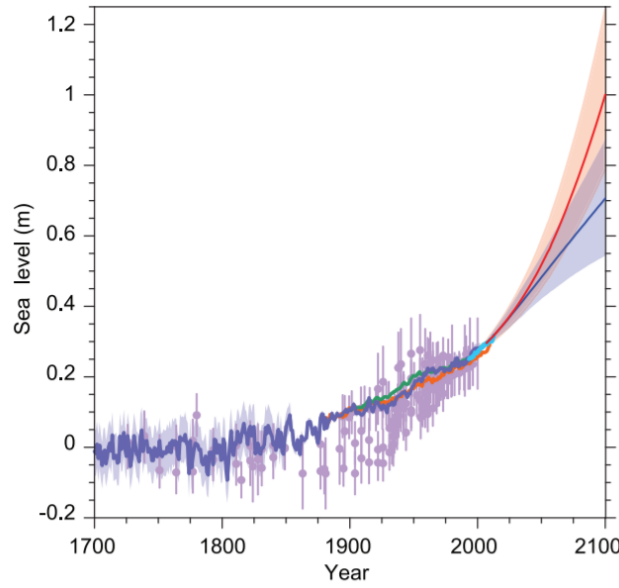


Figure 1.1: Global Sea Level Rise (SLR) past records and future projections. Projection with current trend in red (RCP8.5), and controlled emissions (RCP2.6) in blue. IPCC, Fifth Assessment Report by Working Group 1, 2013

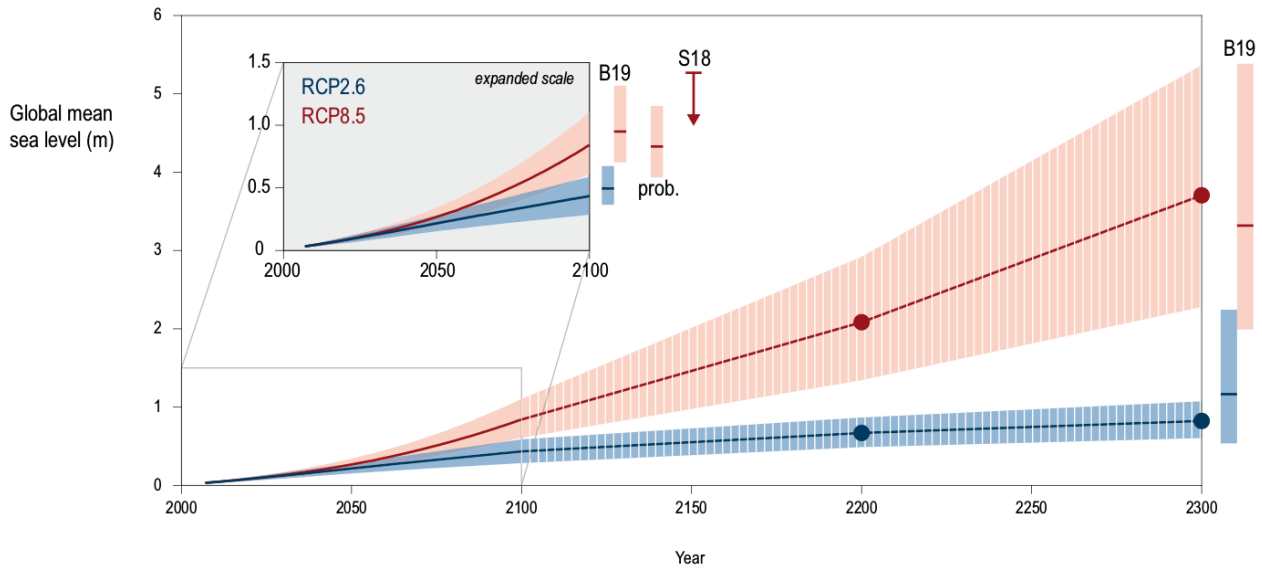


Figure 1.2: Projected sea level rise (SLR) until 2300 (Oppenheimer et al., 2019). Projection with current trend in red (RCP8.5), and controlled emissions (RCP2.6) in blue.

Progress on numerical modelling tools for rubble mounds stability analysis has been slow and no existing approach has been fully validated. This is believed to be critical before the coastal engineering community can make use of these new modelling tools for designing new structures and updating existing ones, in a very crucial moment with ever growing pressure on coastal

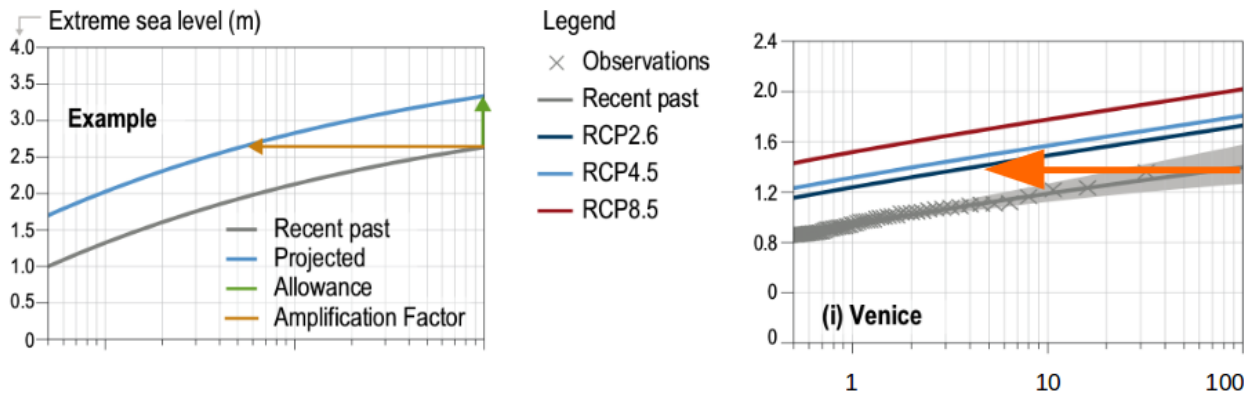


Figure 1.3: Projected change in extreme sea levels under different scenarios for the city of Venice. (Oppenheimer et al., 2019). Generic example on the left. Venice real case on the right, where it can be observed that a 100 years return period event with current scenario can become a 4-5 years return period under the RCP2.6 scenario (controlled emissions).

zones and the increasing threats from Climate Change. Although sea level rise (SLR) forecasts vary by region and emissions scenario, various reports (Stocker et al., 2013; Oppenheimer et al., 2019) estimated that water levels may rise globally by more than 1 m before the end of the century (see Figures 1.1 and 1.2). Moreover, extreme sea levels are expected to increase and return period values to decrease, with cases such as the city of Venice, where extreme sea levels of a 100 years return period at present may become 4-5 years return period events even in the most optimistic emissions scenario (Oppenheimer et al., 2019), as shown in Figure 1.3. New structures and existing ones needing adaptation to the new scenarios can benefit from new numerical tools for iterating and validating their design (van Gent, 2019).

Coastal cities around the globe are already increasing their budget for tackling sea level rise, and some countries even consider retreating neighbourhoods and entire cities to higher and safer locations. New tools are needed for better planning coastal protection structures and optimising their design and cost where retreating is not an option. This thesis is aiming to improve the knowledge of rubble mounds stability while delivering new and better tools for researchers and engineers for analysing and designing them.



## 1.2 Research context

Over the past few years new developments at Imperial College Applied Modelling and Computing Group (AMCG) with the combined Finite Element and Discrete Element Model (FDEM) Solidity ([www.Solidityproject.com](http://www.Solidityproject.com)) have opened new simulation capabilities offering applications of FDEM modelling in many fields of research. The most relevant contribution for the coastal engineering community has been the fluid-solid interaction. Work by Xiang, Latham, Vire and co-workers pursuing this direction lead to a two-way coupling approach based on the immersed body method. This was first implemented between Solidity and Fluidity (Finite Element Method -FEM- developed by AMCG) (Xiang et al., 2012; Vire et al., 2015) and later between the Imperial College Finite Element Reservoir Simulator (IC-FERST) and Solidity (Yang et al., 2016; Obeysekara, 2018; Joulin, 2019). This technology has however not been parallelised and remains unsuitable for cases with large number of particles or domains equivalent to NWTs.

Alternatively to the two-way coupling approach, a one-way coupling wave-structure interaction technology (“Wave Proxy”) was considered (Latham et al., 2015; Xiang J., 2019) with the Numerical Wave Tank (NWT) libraries developed by Higuera et al. (2013) for OpenFOAM (Weller et al., 1998). Computational Fluid Dynamic (CFD) simulation results are provided in order to impose the loading-induced forces from Wave-Structure Interactions (WSI) on the numerical breakwater modelled in Solidity, which results to a wave load equivalent displacement if initiation of movement is reached. This approach is considered valid for cases of initiation of movement and small displacements, since the variations of the location and rotation of the breakwater particles are not updated on the CFD simulation. The numerical breakwaters were built with a numerical rock shape library with realistic rock shapes (Latham and Munjiza, 2004) and the tools developed, in collaboration with Industry partners Baird & Associates and Artelia, for building realistic concrete unit rubble mound breakwaters numerically using the FDEM solver (Anastasaki, 2015).

Finite Element Methods (FEM) are typically associated with unstructured meshes and they are often considered computationally expensive, they can reproduce complex shapes accurately

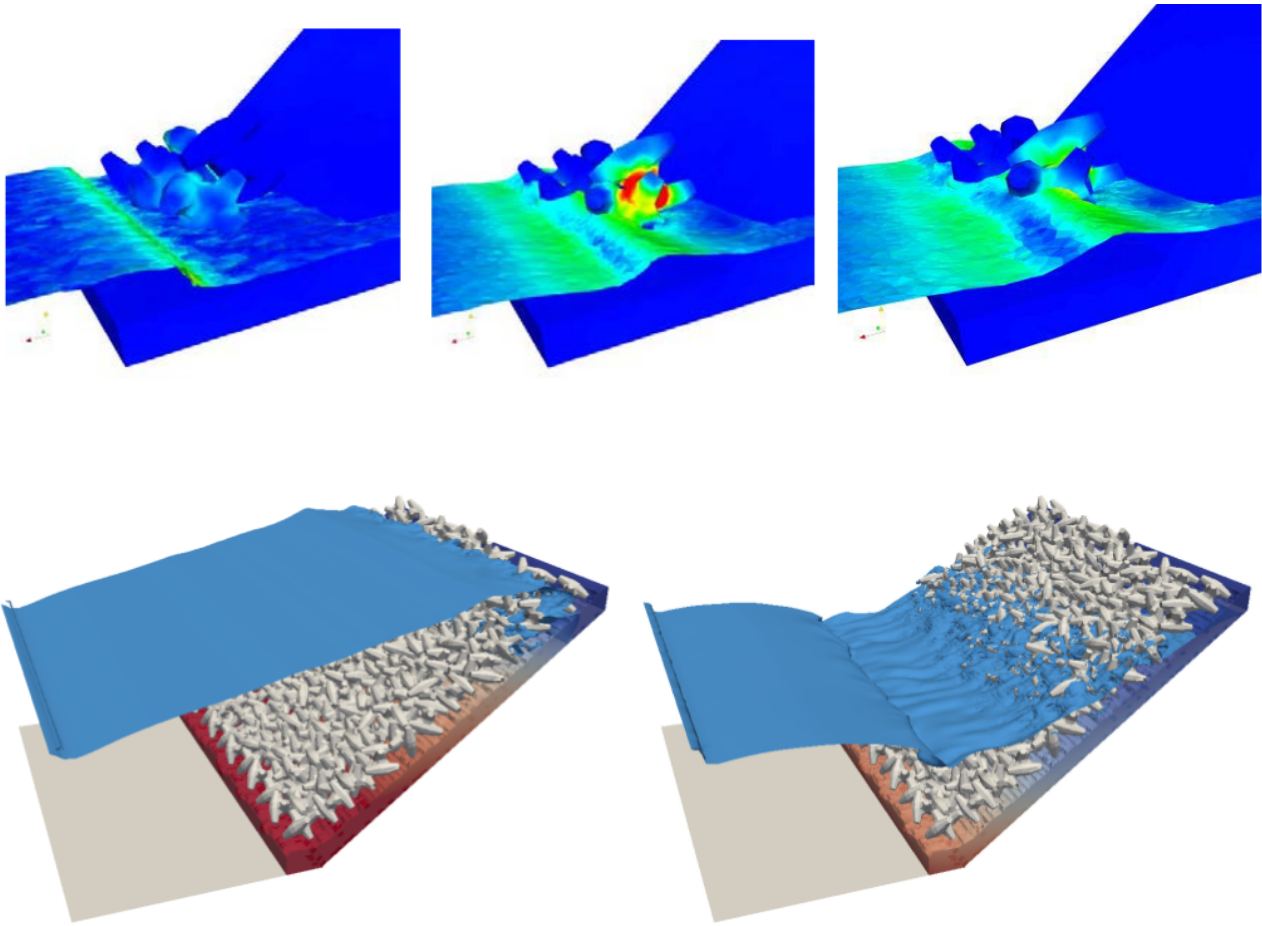


Figure 1.4: Example of two-way coupling with Solidity and Fluidity (top) at different time steps of the simulation with 3 Core-Loc units resting at the toe of a slope under the impact of a wave generated with a Collapsing Water Column (Xiang et al., 2012)), and one-way coupling with OlaFlow(OpenFOAM) and Solidity (bottom) at two different time steps of a simulation with a breaking wave on a Core-Loc layer (Latham et al., 2015; Xiang J., 2019)).

and present a wide range of solver and discretisation options (Donea and Huerta, 2003; Elman et al., 2014) making them very attractive and versatile for very different problems. AMCG FEM codes have already been tested for a wide range of CFD applications, such as interface or surface tracking and wave generation and propagation (Mindel, 2008; Spenneken et al., 2012; Vire et al., 2016; Xie et al., 2017b). Moreover, IC-FERST (Pain et al., 2001; Adam et al., 2016) latest and continuous developments make it a potential CFD code for coastal engineering, with dynamic mesh algorithms for higher accuracy where is needed with minimum computational cost, and various approaches for tackling interfaces such as compressive advection and surface tension Pavlidis et al. (2014, 2016); Xie et al. (2014, 2016, 2017a), and robustness for highly anisotropic meshes (Salinas et al., 2017).

The potential of AMCG codes for coastal engineering applications is however still limited to a very few simplified scenarios until new wider NWT capabilities are developed and validated. The readiness of such tools is considered a crucial milestone for the two-way coupling technology when it is parallelised, and for one-way coupling thanks to the increased accuracy AMCG FEM codes can provide. Moreover, an increased pressure on coastal defenses demands new faster and better solutions for their upgrades or design, making numerical modelling tools much needed and desired in the near future.

## 1.3 Approach

The aim of this PhD is to further develop and validate existing technology at AMCG for NWT simulations and make it ready for industry with a wide range of potential applications for rubble mound stability analysis, including one-way and two-way fluids-solids model coupling. This PhD and Thesis cover, to different extents, a very wide range of subjects affecting wave-structure interaction.

Two novel new approaches are explored for building numerical rock armour layers with the existing rock shape library (Latham and Munjiza, 2004), adding new potential capabilities to the already validated technology for building single concrete armour layers (Anastasaki et al., 2013).

IC-FERST has been optimised with different new and existing approaches (Pavlidis et al., 2016; Salinas et al., 2017) for simulating NWTs. Special attention has been given to interface capture and mesh optimisation for tracking the water free surface, critical for numerical wave modelling. New libraries have been developed for wave generation with a fixed boundary or inlet approach and sensitivity tests are performed for evaluating wave propagation and numerical energy dissipation. The NWT capabilities have also been validated for WSI with benchmark cases from literature and with new experiments.

A collaboration between Imperial College London, University of Ottawa, Baird and Associates, and the National Research Center (NRC-OCRE) in Canada has been set to produce a series

of physical modelling tests with single monitored units and monitored layers (Simpalean, 2019; Eden, 2019; Eden et al., 2019; Douglas et al., 2019). Measurements from these experiments have been used for validation of existing and newly developed numerical modelling tools within AMCG. Wave induced pressures and forces from single unit tests are compared to numerical model results for different unit locations and wave conditions. Sensitivity tests are run for mesh resolution and time-stepping.

Simulations with full concrete layers and rock armour layers are carried out for showing the full potential of the newly developed technology. Mesh limitations are discussed and solutions proposed and successfully tested. Finally, two applications of the different developed and validated technologies are proposed for real engineering problem, such as the impact of Climate Change and the increase of Sea Level Rise (SLR) on wave induced stresses on rubble mounds and the capturing and quantification of scale effects with NWT.

The modelling of WSI with rubble mounds layers and dynamic meshes while resolving the geometry of the particles accurately, even when considering motion-less particles, can be considered a significant contribution to the current state of the art. Although the modelling of the fluid flow through the filter layer (also under-layer) and the core of the structure (when permeable) could also contribute to higher accuracy of the physics of WSI overall, this is left for future investigation due to added complexity the modelling of these parts of the structure would add to the presented research.

## 1.4 Thesis Overview

### 1.4.1 Thesis Layout

This thesis is organised by chapters grouping the different contributions made to this research. Some of their content can also be found in papers published in Journals and Conference Proceedings. The chapters are organised as follows: Chapter 1 briefs on the current state of the art of rubble mound breakwaters stability analysis, and its potential future and motivation

of this research. It also outlines the contributions of this thesis and a description of the thesis layout.

Chapter 2 reviews the historical background and trends in coastal structures design, wave theories and understanding and how numerical methods have been used recently for different applications relevant to this research. Special attention is given to recent research modelling realistic rubble mound structures numerically, and to the contributions to this field by the AMCG at Imperial College London.

In Chapter 3 two different novel approaches for building numerical rock armour layers are proposed and tested.

In Chapter 4 a description of the CFD code and its implemented methods for modelling NWTs is given. Compressive advection, Dynamic Mesh Optimisation algorithms, spatial and time discretisation schemes are explained, discussed and validated with 2D and 3D benchmark cases from literature. Material in this Chapter has already been presented in Via-Estrem et al. (2020).

The implementation of wave generation schemes for the CFD code is detailed in Chapter 5. Results of 2D sensitivity tests for the different schemes and input conditions are shown. Energy dissipation and mesh optimisation for 3D simulations are discussed in depth and guidelines for future work are given. Chapter 5 also validates the NWT newly developed capabilities for WSI with experiments of single unit on slope tests.

Simulations of full sections of rubble mound layers are shown in Chapter 6. Simulations of simplistic examples of layers with spherical particles, CoreLoc single layers, and rock armour double layers are presented.

Potential applications of the numerical simulation of rubble mounds for stability analysis and structures design are also given in Chapter 6. A realistic case of the wave induced stresses changes due to climate change and potential analysis of scale effects using NWT are discussed.

An in-depth discussion of the meaning and impact of the research carried out during this PhD is presented in Chapter 7. Concluding remarks of this research and proposals for future work are also given in Chapter 7.

### 1.4.2 Contributions

The contributions of the author of this research are summarised below and linked to the different chapters of this Thesis:

- *Numerical rock layers:* Development of python based libraries wrapping FDEM Solidity inputs, running and outputs, and a novel algorithm (parallelised) for placing rocks on a slope, including the manipulation of rocks from the rock shape library. Testing and validating Blender as an alternative software for building rock layers. Analysis tools have been developed to validate these novel and realistic numerical rock slopes. This is explained in Chapter 3.
- *Free surface flows with IC-FERST:* Optimising, calibrating and validating IC-FERST for free surface flows in general, and wave modelling in particular. This includes testing new schemes such as  $P_1DGP_1(CV)$  and compressive-advection schemes, and the development of new test cases for the IC-FERST version-control system and regular contributions to the development and debugging of IC-FERST within AMCG. This is covered in Chapter 4.
- *Wave generation with IC-FERST:* Development of python libraries for a wave-generation scheme to be used as BC inputs for IC-FERST. Pre- and post-processing tools have also been developed. Chapter 5 describes the development and validation of wave modelling with IC-FERST.
- *Validation of NWT and fluid-solid hydrodynamics:* Calibration and validation of IC-FERST results for single unit tests. This includes 1 month visit at University of Ottawa and NRC, helping setting up single unit physical experiments. Chapters 5 and 6 cover single unit and multi-particle layer simulations, respectively.
- *Applications:* Generating and carrying out simulations illustrating the potential use of the novel technology with real engineering applications, as shown in Chapter 6.

### 1.4.3 Publications

Some of the work carried out for this PhD has been published in Journals and Conference Proceedings. During this PhD the author has also contributed to other relevant research projects. The full list of publications relevant to this Thesis is below.

Journal Papers:

- L. Via-Estrem, P. Salinas, Z. Xie, J. Xiang, J.P. Latham, S. Douglas, I. Nistor, C. C. Pain (2020). Robust control volume finite element methods for numerical wave tanks using extreme adaptive anisotropic meshes. *International Journal for Numerical Methods in Fluids*.
- A. Obeysekara, P. Salinas, C.E. Heaney, L. Kahouadji, L. Via-Estrem, J. Xiang, N.Srinil, A. Nicolle, O.K. Matar, C. Pain (2021). Prediction of multiphase flows with sharp interfaces using an integrated modelling approach with anisotropic mesh optimisation. *Advances in Engineering Software*.
- A. Kampitsis, L. Via-Estrem, K. Kapasakalis (2021). An Integrated FEA-CFD Simulation of Offshore Wind Turbines with Vibration Control Systems. Submitted to *Engineering Structures*.

Conference Papers:

- J. Xiang, J.P. Latham, P. Higuera, L. Via-Estrem, D. Eden, S. Douglas, A. Simplean, I. Nistor, A. Cornet (2019). A fast and effective wave proxy approach for wave-structure interaction in rubble mound structures. *Coastal Structures Conference, Hannover, 2019. Conference Paper*.
- J.P. Latham, J. Xiang, L. Via-Estrem, P. Higuera, S. Douglas, I. Nistor, A. Cornet, S. Logan, W.F. Baird (2017). A physics-based rubble mound stability simulator for armour layers. *ICE Coasts, Marine Structures and Breakwaters 2017: Realising the Potential. Liverpool, 2017. Conference paper*.

- D. Eden, S. Douglas, A. Simpalean, T. Kozlowski, I. Nistor, A. Cornett, D. Anglin, L. Via-Estrem, J.P. Latham, J. Xiang (2019). Quantifying forces and pressures on Core-Loc armour units via instrumented units and image processing. Coastal Structures Conference, Hannover, 2019. Conference Paper.
- S. Douglas, D. Eden, A. Simpalean, I. Nistor, A. Cornett, D. Anglin, L. Via-Estrem, J.P. Latham, J. Xiang (2019). Experimental study of wave induced loading on breakwater armour layers. Coastal Structures Conference, Hannover, 2019. Conference Paper.
- P. Salinas, C. Jacquemyn, A. Kampitsis, L. Via-Estrem, C. Heaney, C. Pain, and M. Jackson (2019). A Parallel Load-Balancing Reservoir Simulator with Dynamic Mesh Optimisation. SPE Reservoir Characterisation and Simulation Conference and Exhibition, Abu Dhabi, 2019. Conference Paper.



# Chapter 2

## Literature Review

### 2.1 An historical introduction to stability analysis of rubble mound breakwaters

The Coastal Engineering discipline as we know it is a very young discipline with just over 50 years of existence. The Coastal Engineering term was not used in print until 1950 (Shinji and Masahiko, 2015). However, first coastal structures and harbours date back to pre-Hellenic period (Bruun, 2013), the oldest harbour facilities known to date being at Wadi el-Jarf (2600 B.C.) in Egypt (Tallet, 2012). It is widely accepted that the design of coastal structures long before the creation of the Coastal Engineering term and discipline were following "trial and error" or "trial and failure" approach.

One of the first and most significant formulae, still relevant nowadays, for rubble mound breakwater design was by Iribarren Cavanilles (1938). He approached the problem of stability looking at the threshold of movement (Eq 2.1) considering wave height ( $H$ ), density ( $\rho_r$ ) and weight ( $W$ ) of the units, slope of the breakwater ( $\tan \alpha$ ) and a coefficient ( $K$ ) to be adjusted depending on the unit type and shape. Many have contributed to this topic after Iribarren Cavanilles (1938), but his approach has remained the most popular for most new formulae.

$$W = K \frac{H^3 \rho_r}{(\cos \alpha - \sin \alpha)^3 (\rho_r - 1)^3} \quad (2.1)$$

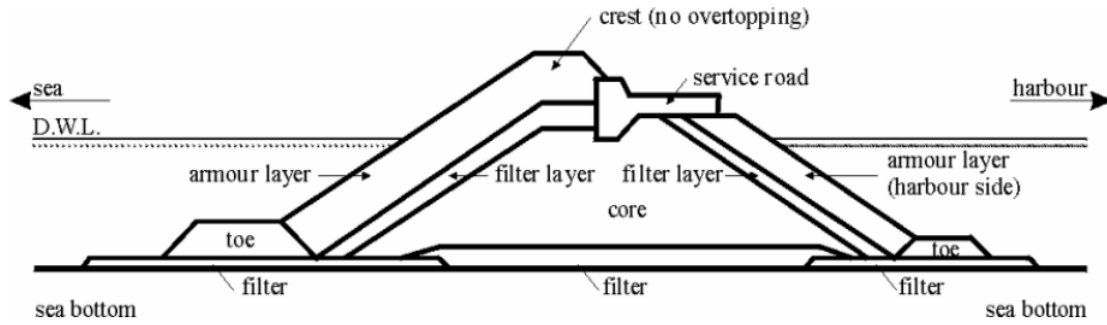


Figure 2.1: Typical cross section of a rubble mound breakwater (Troch, 2001) with its main components: the core, the filter and the main armour layers and the toe of the structure.

The next biggest and most known contribution was made by Hudson (1959), who adapted Iribarren's stability formula (Equation 2.1) into Equation 2.2 and validated it with experiments:

$$W_{50} = \frac{\rho_r g H^3}{K_D \Delta^3 \cot \alpha} \quad (2.2)$$

introducing  $g$  as gravity acceleration ( $m/s^2$ ),  $W_{50}$  as the median weight of the armourstone, the relative buoyant density  $\Delta = \rho_r / \rho_w - 1$  (-) and  $K_D$  as stability coefficient (-), suggesting  $K_D = 2.0$  and  $K_D = 4.0$  for breaking and non-breaking waves respectively.

By considering that  $H(m)$  is equivalent to  $H_{max}$  in real sea states, that  $H_{max} = 1.27 H_s$  generally applies, and by introducing Hudson's stability number  $N_s = H_s / (\Delta D_{n50})$ , Equation 2.2 could be simplified (CIRIA et al., 2007) to:

$$\frac{H_s}{\Delta D_{n50}} = \frac{(K_D \cot \alpha)^{1/3}}{1.27} \quad (2.3)$$

The stability number introduced by Hudson can also be used to describe the structure type regarding its response (CIRIA et al., 2007). For instance, caissons or seawalls have a  $N_s \leq 1$  with no damage allowance.  $N_s$  between 1 and 4 indicate a limited damage acceptance which corresponds to statically stable breakwaters (the subject of this research project). Values above 4 indicate certain reshaping allowance.

Although Equation 2.3 is still widely used in the coastal engineering community due to its simplicity, it lacks some important parameters and permeability  $P$  (Hedar, 1960, 1986), wave period  $T$  (Ahrens, 1975; Losada and Gimenez-Curto, 1979; Pilarczyk and Den Boer, 1983) and number of waves  $N$  (Thompson and Shuttler, 1975) were later introduced.

Covering the mentioned contributions, Van der Meer (1988a,b) produced an extensive adaptation of Hudson's formula for rock armour layers, moving also from a no damage approach by Hudson to a damage level approach with a dimensionless damage concept ( $S_d = A/D_{n50}^2$ ).  $S_d$  gives the eroded area of a breakwater section and indicates no damage with  $S_d \leq 2$  (for structure slopes with  $\cot \alpha \leq 3$ ) and failure from 8 (for  $\cot \alpha \leq 2$ ) or 15 (for  $\cot \alpha \geq 3$ ) onwards.

Van der Meer realised differences in results depending on wave breaking types and proposed two different formulae. The different breaking types were derived comparing a critical value  $\xi_{cr} = f(P, \tan \alpha)$  to Battjes (1975) surf similarity parameter  $\xi = \frac{\tan \alpha}{\sqrt{s_o}}$ , where  $s_o$  is the theoretical steepness of the wave in deep-waters ( $s_o = H/L_o$ ).

The surf similarity parameter is also known as Iribarren number as proposed by Battjes due to the importance of this concept and to honour "the man who introduced it and who has made many other valuable contributions to our knowledge of water waves" (Battjes, 1975).

Equation 2.4 is for plunging waves when  $\xi_m < \xi_{cr}$  and Equation 2.5 for surging waves when  $\xi_m \geq \xi_{cr}$  where  $\xi_m$  stands for Iribarren number using  $T_m$ .

$$N_S = c_{pl} P^{0.18} \left( \frac{S_d}{\sqrt{N}} \right)^{0.2} \xi_m^{-0.5} \quad (2.4)$$

$$N_S = c_s P^{-0.13} \left( \frac{S_d}{\sqrt{N}} \right)^{0.2} \sqrt{\cot \alpha} \xi_m^P \quad (2.5)$$

Equations 2.4 and 2.5 are applicable only to rock armour layers. Van der Meer (1988b) proposed stability formulae for designing concrete armour layers with cubes, tetrapod and Accropode(R) units, for which he introduced a new damage concept  $N_{od}$ , which corresponds to number of displaced units divided by the number of units in the tested section. The number of units in the tested section can be approximated as width of tested section divided by the  $D_n$  of the

units composing the layer.

Another significant contribution was made a decade later by the same author (Van der Meer, 1999) covering the design of double layer (cubes and tetrapods) and single layer (Accropodes and Core-locs) concrete units, covering some of the most used units in new designs. Formulae for double layers followed the same approach of damage levels as for rock armour layers whereas single layers had a very restrictive damage level only set with the stability number  $N_s$ .

Concrete units with inter-locking properties were originated in the 1950s with the Tetrapods with the aim to improve armour stability and porosity of the armour layers while reducing the concrete demand. The stability of each unit is increased by inter-locking with its neighbours, creating effectively a system of units working together. However, when units are extracted due to reduced inter-locking, the stability of the system is significantly decreased. Due to this rapid stability reduction, systems with inter-locking units are considered to fail as soon as very few units are extracted from the system. Several units shapes have emerged over the past decades, aiming to increase the inter-locking effect while reducing the concrete needed, such as the very popular Accropode and Core-loc units created by Concrete Layer Innovations (CLI).



Figure 2.2: Accropode (left), Accropode II (centre) and Core-Loc (right) concrete units (CLI) with inter-locking performance for single layer armour layers.

A different approach is to allow the breakwater to reshape within its lifetime. This in effect reduces the rock armour size needed for the main armour layer and allows the breakwater to reshape to a more stable section. Berm breakwaters provide a very attractive solution as much smaller armour stones can be used compared to conventional breakwaters (Sigurdarson and Viggosson, 1994). The authors Sigurdur Sigurdarson and Dr Jentsje van der Meer combine over 4 decades of expertise in designing these type of breakwaters, which have become very popular in some countries such as Iceland (Sigurdarson and Meer, 2013; Sigurdarson and van der Meer,

2014; Van der Meer and Sigurdarson, 2016). For more information on other units or types of rubble mound breakwaters see The Rock Manual (CIRIA et al., 2007) and the Coastal Engineering Manual (CERC, 2004).

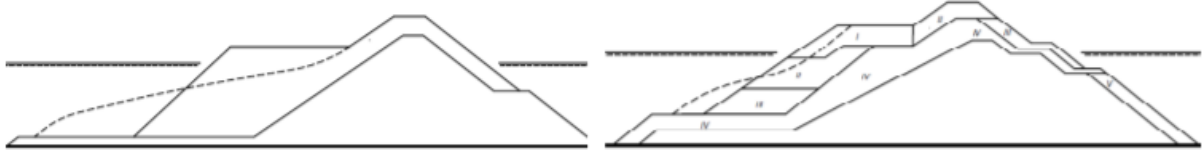


Figure 2.3: Examples of berm breakwater cross-sections: mass armoured berm breakwater (left) and Icelandic-type berm breakwater (right) with its different rock size and grading zones. Expected reshaping shown with the dotted line, (Sigurdarson and van der Meer, 2014).

Double and single layered breakwaters, however, have generally been looked at as systems of units with little knowledge of the physics of the interaction of waves with single units. In an effort to improve the understanding of the complex wave-structure interaction processes, tests have been performed looking at rubble mound layers with individual rock and concrete instrumented units, measuring wave loads in terms of pressure and forces on individual units (Burcharth, 1985; Scott et al., 1989; Sakakiyama and Kajima, 1991; van der Meer and Heydra, 1991; Tørum, 1994; d'Angremond et al., 1995; Cornett, 1995; Hald, 1998; Moghim and Tørum, 2012; Jensen et al., 2014; Hofland et al., 2018; Simpalean, 2019; Eden, 2019). Numerical models have also been used with this purpose considering different approaches for modelling the rubble mound layers. Several studies are available in literature and a summary of them is presented in Section 2.4. Most relevant ones for stability of rubble mounds are Dentale et al. (2009, 2014, 2018); Altomare et al. (2012, 2014); Latham et al. (2015); Xiang J. (2019).

Despite the continuous advances, rubble mounds have been and continue to be designed based on empirical experience in a direct way (i.e. structures tested in a scaled down hydraulic physical model) or indirect way (i.e. empirical formulae based on experiments). These experiments however have associated scale effects, as scaling cannot keep all parameter ratios constant (i.e., scaling for rubble mounds stability experiments follow the Froude similarity law, based on the Froude number  $Fr = U/\sqrt{gh}$ ) and also associated costs. Numerical model experiments have therefore become more relevant in the past decade, but they have yet to become the tools

for designing new structures without relying on experiments-based formulae. Moreover, most commonly used empirical formulae have been obtained with non-depth-limited wave breaking scenarios. Numerical models have the potential of becoming the tool to test a wider range of scenarios and overcome such limitations.

## 2.2 Water waves and Wave theory

Airy's work in first order theory of water waves (Airy, 1845), also known as the standard linear wave theory, is nowadays the most widely used wave theory, thanks to being the most simple analytical solution for water waves. It is considered valid and accurate for small amplitude waves and some may still use it when its accuracy remains within an acceptable range.

Small amplitude wave theory assumes that the fluid is incompressible (mass conservation), irrotational, that unsteady potential flow applies, waves are periodic in space and time, and wave amplitude is small compared to its length and water depth. With these assumptions one can obtain the free surface elevation (Eq 2.6), vertical (Eq 2.7) and horizontal (Eq 2.8) particle velocities of the wave:

$$\eta = a \cos(kx - \omega t + \phi) \quad (2.6)$$

$$u = \frac{a\omega \cosh(kz)}{\sinh(kh)} \sin(\omega t - kx + \phi) \quad (2.7)$$

$$w = \frac{a\omega \sinh(kz)}{\sinh(kh)} \cos(\omega t - kx + \phi) \quad (2.8)$$

where  $x$  and  $z$  are horizontal and vertical position,  $t$  is time,  $a$  the wave amplitude,  $k$  is the angular wave number ( $k = \frac{2\pi}{L}$ ),  $\omega$  the wave frequency ( $\omega = \frac{2\pi}{T}$ ),  $\phi$  is the phase,  $T$  is the wave period and  $L$  is the wave length. The wave length can be obtained as  $L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi h}{L}\right)$ , which can be simplified as  $L_O = \frac{gT^2}{2\pi}$  for  $h/L > 0.5$

Sir George Gabriel Stokes, who taught physics at the early days of the Government School of Mines (O'Connor and Robertson, 2003) -nowadays the Royal School of Mines at Imperial

College London-, continued Airy's work on first order -also known as Stokes I or first order wave theory- and added a second order term to it, known as Stokes II or second order wave theory (Dean and Dalrymple, 1991). The second order -see Equations 2.9, 2.10 and 2.11- term makes crests higher and steeper and troughs flatter (see Figure 2.4)

$$\eta = a \cos(\theta) + a^2 k \frac{3 - \sigma^2}{4\sigma^3} \cos(2\theta) \quad (2.9)$$

$$u = \frac{a\omega \cosh(kz)}{\sinh(kh)} \sin(\theta) + \frac{3a^2\omega k \cosh(2kz)}{4 \sinh^4(kh)} \cos(2\theta) \quad (2.10)$$

$$w = \frac{a\omega \sinh(kz)}{\sinh(kh)} \cos(\theta) + \frac{3a^2\omega k \sinh(2kz)}{4 \sinh^4(kh)} \sin(2\theta) \quad (2.11)$$

where  $\theta = kx - \omega t + \phi$  and  $\sigma = \tanh(kh)$ .

Stokes I and II are the most widely used wave theories in the coastal engineering community, both in physical and numerical experiments. Although they apply only to relatively small amplitude waves, they are often used out of their original range as they are easier to implement and reproduce waves fairly well in many cases.

In order to represent higher amplitude or crested waves one can also find higher order waves theories, such as 5th order by Skjelbreia and Hendrickson (1960) and Fenton (1985), for higher amplitude waves. To obtain free surface elevation and velocity components for Stokes V one first has to resolve a two equations system (Equations 2.12 and 2.13):

$$\frac{\pi a}{2h} = \frac{L}{h} [\lambda + \lambda^3 B_{33} + \lambda^5 (B_{35} + B_{55})] \quad (2.12)$$

$$L = L_0 \tanh\left(\frac{2\pi h}{L}\right) (1 + \lambda^2 C_1 + \lambda^4 C_2) \quad (2.13)$$

where  $B_{ij}$  and  $C_i$  are constant coefficients. Then one can resolve the amplitude ( $a_{vel_i}$ ) of each term, which depend on  $\lambda$  and  $A_{ij}$ , and finally obtain free surface elevation (Eq 2.14) and horizontal (Eq 2.15) and vertical (Eq 2.16) velocities. Coefficients  $A_{ij}$ ,  $B_{ij}$  and  $C_i$  are factors

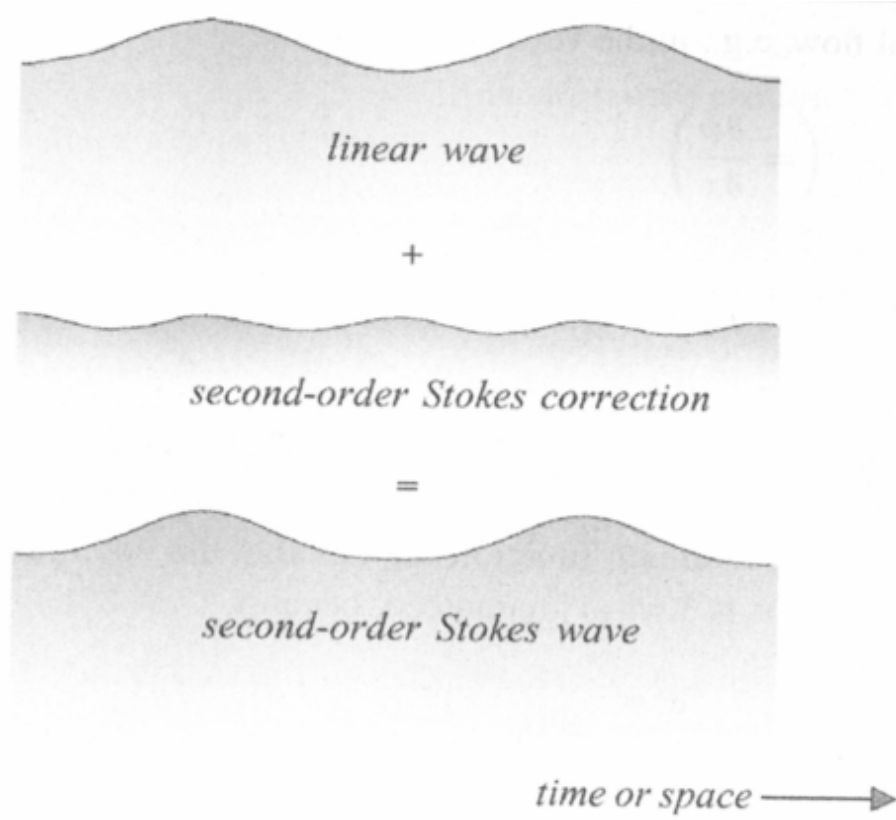


Figure 2.4: Second order wave theory (Holthuijsen, 2010) as a superposition of a linear wave and a second-order correction.

obtained from polynomial expressions that depend only on  $h$  and  $L$ .

$$\eta = \frac{1}{k} [\lambda \cos(\theta) + (\lambda^2 B_{22} + \lambda^4 B_{24}) \cos(2\theta) + (\lambda^3 B_{33} + \lambda^5 B_{35}) \cos(3\theta) + \lambda^4 B_{44} \cos(4\theta) + \lambda^5 B_{55} \cos(5\theta)] \quad (2.14)$$

$$u = \sum_{i=1}^{n=5} a_{vel_i} \cosh(ikz) \cos(i\theta) \quad (2.15)$$

$$w = \sum_{i=1}^{n=5} a_{vel_i} \sinh(ikz) \sin(i\theta) \quad (2.16)$$

Other waves theories can be found in literature, such as Cnoidal and Streamfunction (Fenton, 1988) for shallow steep waves and Solitary waves. Figure 2.5 shows most common water wave theories and their applicability range. Stream function theory presented by Fenton (1988) has some advantages over other theories as it adapts to different depths, giving a linear solution for small amplitude waves and/or waves in deep waters and higher order for higher amplitude



waves and shallow waters.

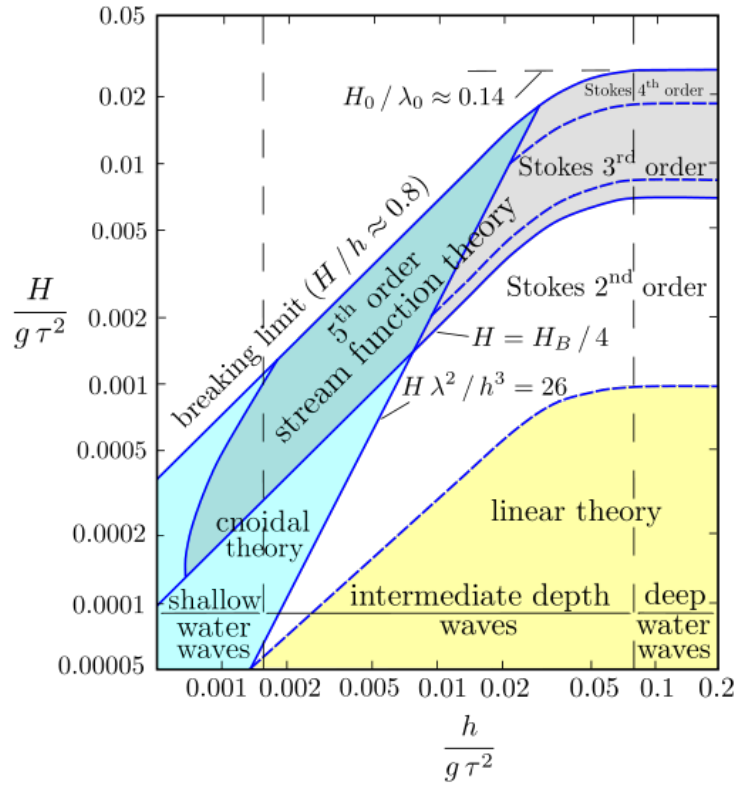


Figure 2.5: Water wave theories (Le Mehaute, 2013) and their domain of applicability. Non-dimensional axis obtained with  $H$  as the wave height (m),  $h$  the water depth (m),  $\tau$  for the wave period (s),  $g$  for the gravity ( $m/s^2$ ) and  $\lambda$  for the wave length (m).

Most physical modelling laboratories use still nowadays wave generation paddle movements based only on first order wave theory, and sometimes second order, expecting waves to transform over the bathymetry between the wave generation zone (normally deep or intermediate water conditions) and testing location (normally shallow water conditions). In numerical modelling, it is very common to reduce the NWT domain (for reducing computational cost) and have the wave generation zone closer to the test location, this results to shorter lengths for wave transformation and higher order wave generation schemes are often used.

### 2.2.1 Sea states

Mentioned wave theories in represent single frequency or monochromatic waves. In reality, however, sea states are a composition of different frequency waves also known as irregular

or random sea states (Komen et al., 1996; Goda, 2010). If the interaction between different frequencies is neglected, one can assume that the total sea states can be simply obtained by superposing each frequency component as a regular wave sequence.

Despite real structures being under irregular sea states, the main goal of this research project is to validate the hydrodynamics modelling of the fluid-particle interaction, and long simulations representing real sea states are not considered. For this reason only regular sea states (single frequency) are used here.

On the other hand, some understanding of concepts regarding realistic sea states might be required in the following sections (i.e. for applications chapter) and therefore some sea state concepts and parameters are described here.

Considering any random sea state, a very common way to analyse it is to decompose its free surface elevation over time in a series of monochromatic waves using a Fourier transform (see Figure 2.6. Following this process one can obtain the sea state's every frequency component, its amplitude and therefore its energy. This can be easily converted (Goda, 2010) and plotted as Spectral Density ( $m^2 s$ ) in the vertical axis and frequency ( $Hz$ ) in the horizontal axis. This is known as energy spectrum. Most wave measurement devices are nowadays using this procedure and then deriving a series of parameters from the obtained energy spectrum.

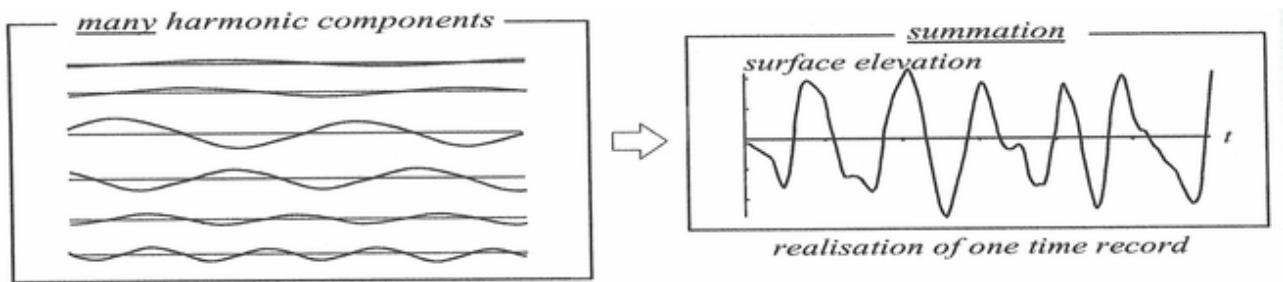


Figure 2.6: Free surface elevation of irregular waves (right) and its different regular wave components contributing to it (left) (Holthuijsen, 2010)

Most used parameters for describing a sea state are:

- significant wave height ( $H_s$ ) in meters. It is equivalent to the height average of the 1/3 higher waves. It can be obtained as  $H_s = 4\sqrt{m_o}$ , where  $m_o$  (0 moment of the energy density spectrum) is the total energy contained in the energy spectrum.

- mean ( $T_m$ ) or peak ( $T_p$ ) period in seconds. The peak period is equivalent to the maximum energy band of the energy spectrum. The mean period can be seen as  $T_{m02}$ , which is obtained as  $T_{m02} = \sqrt{\frac{m_0}{m_2}}$  and represents the mean period weighting the high frequency energy bands, or as  $T_z$ , which is the statistical zero crossing mean period. These two values can differ and they should be described accordingly.
- mean or peak direction in degrees from true North, where the direction reference is where waves are coming from.

Another parameter becoming more important in the recent years is  $T_{m-1,0} = \sqrt{\frac{m-1}{m_0}}$ , which is a weighted spectral period towards the low frequency bands, that is taking longer periods into account.

## 2.3 FDEM for building Numerical Breakwaters

In the fields of particle technology and granular matter, multi-particle systems had been historically approached with numerical models based on Discrete Element Methods (DEM) for its motion tracking and collision capabilities. However, this approach did not allow numerical models to represent internal stresses, deformations or fracture of the particles. In order to model these, Finite Element Methods (FEM) were required.

Munjiza (2004) pioneered the combined FEM-DEM model in the early 1990's capable of modelling particles stresses and deformations in a multi-particle interacting system. This was based on finite element tools modelling particles with separate meshes and discrete element techniques for the interaction between them. Algorithms to handle contact detection and interaction (based on a penalty term), transition from continua to discontinua or finite strain elasticity-plasticity made FDEM a very powerful code for many potential applications (Munjiza et al., 2010).

The FDEM model can solve the contact mechanics of complex shapes using a distributed contact force approach. The contact and small overlap volumes are considered for collisions and a penalty function delivers the contact forces (Munjiza, 2004). This method proved particularly

useful for handling several particles with complex shapes and their interactions (Latham and Munjiza, 2004), making it very attractive for coastal engineering applications such as rubble mounds (Latham et al., 2008b).

The collaboration between Latham, Xiang and Munjiza resulted in a 3D FDEM with a higher order and large strain formulation (Xiang et al., 2009a) which lead to the development of rubble mound breakwaters building capabilities (Xiang et al., 2011). A rigid version (no deformation or stresses modelling and therefore computationally less demanding) of this code together with new placing tools (POSITIT) pushed further its capabilities of building breakwaters with concrete units at a very reduced computational cost (Anastasaki et al., 2013). The units were layered on a rock layer representing a typical filter layer of rubble mounds, using rock shapes from a numerical rock shape library by Latham et al. (2008a).

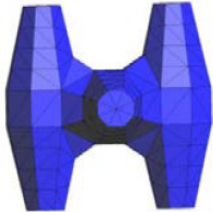
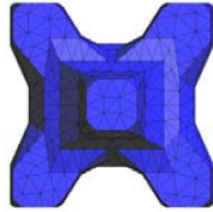
|                         |                                                                                    |                                                                                     |                                                                                      |
|-------------------------|------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| Type of unit            | Core-Loc                                                                           | Accropode II                                                                        | Accropode II, NS                                                                     |
| Meshed unit             |  |  |  |
| Number of unit elements | 767                                                                                | 11203                                                                               | 2930                                                                                 |

Figure 2.7: Numerical representation of a Core-Loc unit (left), and Accropode II unit (centre) and a simplified version (no studs) of an Accropode II unit (right) with their respective number of elements (Anastasaki, 2015).

Work carried out by Anastasaki (2015) tested such numerical breakwaters replicating the expected settlement of units under mild wave loads with the vibration of the layer. In the meantime, forces on the units by realistic wave loads were extracted from NWT (Latham et al., 2015) and applied to numerical layers, giving real unit displacement and overall breakwater damage levels (Latham et al., 2018), which agreed with experiments (Xiang J., 2019). The continuous development of the FDEM code and new pushes in other applications such as complex shape pellets, hydro-fracture and thermal fluid-solid coupling (Farsi, 2017; Obeysekara, 2018; Joulin, 2019) has been done under the Solidity project, leading to the FDEM model to be renamed Solidity.

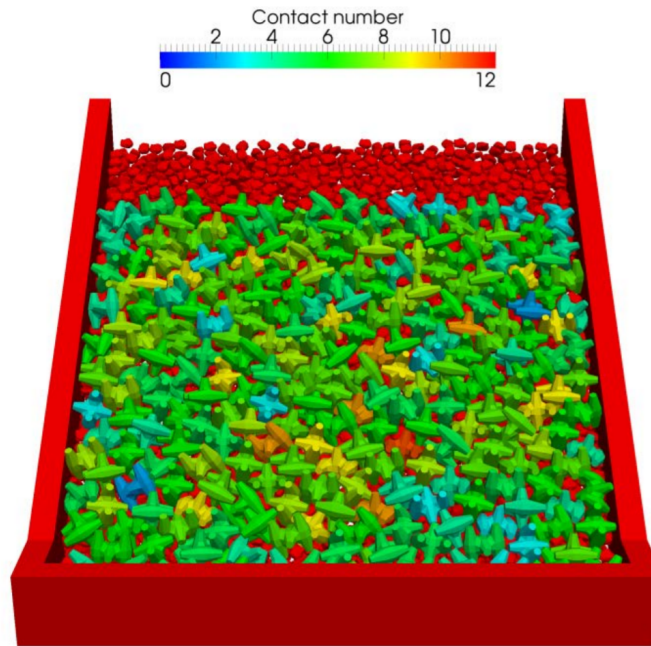


Figure 2.8: Numerical armour layer of Core-Loc units built with POSITIT (Anastasaki, 2015). It contains a base, an underlayer of fixed rock particles and the Core-Loc particles as the main layer.

## 2.4 Numerical Modelling of Wave-Structure Interaction for Rubble mounds

Numerical wave models have been used regularly by both research and industry for decades. First computer-aided wave models are believed to be wave forecasts in the 1950's (Tolman, 2010). Nowadays wave models are very diverse and numerous and generally approach only one or few aspects of the wave life. From its generation, propagation and large scale transformation using global or regional spectra-based models, to a smaller scale coastal models using shallow water equations, or to detailed wave-structure interactions modelling wave hydrodynamics.

Two main groups of wave models can be identified: i) phase-averaged models, without computing the wave profile which facilitate very large domain simulations ( $\gg 100$  wavelengths); and ii) phase-resolving models that resolve the wave profile, but have a limitation of small domain simulations (normally  $< 100$  wavelengths) (Goda, 2010).

In terms of equations they solve, there is a huge variety and it is not part of the scope of

this thesis to describe them in detail. Energy balance (phase-averaged), mild slope (can be phase-averaged or phase-resolving), boundary integral (phase-resolving), Boussinesq (phase-resolving), and shallow water (phase-resolving) equations, to mention a few, are widely used by the coastal engineering community (Goda, 2010; Holthuijsen, 2010; Panchang and Demirbilek, 1998). Some of them are simplifications of the Navier-Stokes equations, which became very popular for their low computational time cost compared to resolving their complete version. Also historically very popular (phase-resolving) for non-breaking waves there is the Boundary Element Method (BEM) based on potential flow (Grilli et al., 2001).

First attempts to model WSI with rubble-mounds in the 1990's (Norton and Holmes, 1993; Van Gent, 1995) used a simplistic fluid modelling depth-averaged velocity approach to resolve the nonlinear shallow-water equations, coupled with a drag force from Morison-type of equations to capture forces on the solid particles of berm breakwaters.

With the increasing computational power, however, complete Navier-Stokes equations models have been gaining users and now can be seen in both academia and industry. As phase-resolving models they can only be applied to small domains, but their capability of modelling wave-structure interaction has made them very popular. Main models used by the coastal community are OpenFOAM (Weller et al., 1998) based codes Wave2FOAM (Jacobsen et al., 2012) and OlaFlow (Higuera et al., 2013) using Eulerian methods (mesh discretisation), and the Smooth Particle Hydrodynamics (SPH) model SPHysics (Gomez-Gesteira et al., 2012) based on a Lagrangian (mesh free) method.

The first two models are based on a finite volume discretisation using the Volume Of Fluid (VOF) approach for a two phase (water and air) problem. In this way, free surface elevation of waves can be modelled accurately if an appropriate resolution is applied. For modelling wave hydrodynamics they use a Reynolds Average Navier-Stokes (RANS) approach with OpenFOAM coded turbulence models and they have the capability of modelling porous media (Volume Averaged, VA-RANS) to represent rubble mounds. A typical approach for domains discretised with rectangular grids, typically FVM such as OlaFlow and Wave2FOAM, is to have higher resolution near the free surface and around the structures, and lower resolution in the rest of

the domain, in order to reduce computational cost.

The main difference between OlaFlow (Higuera et al., 2013) and Wave2FOAM (Jacobsen et al., 2012) is the wave generation approach. While Higuera et al. (2013) uses an inlet Boundary Condition, which can apply moving paddle and static inlet methods, Jacobsen et al. (2012) applies a wave generation and absorption zone of 2 or more wave lengths. Although the wave generation and absorption zone can reduce the wave reflection upstream significantly, it also leads to larger domains when compared to Inlet BC. Dimakopoulos et al. (2016) presented resolution guidelines for wave generation and absorption zones in order to reduce computational cost induced by the generation and absorption zone approach.

The third model, with mesh-less domain, has been implemented for Central Processing Units (CPU) (Gomez-Gesteira et al., 2012), and also for Graphics Processing Units (GPU) under the name of DualSPHysics (Crespo et al., 2011) in order to reduce computational cost. Commercial codes for simulating NWT are also available. Commonly used are FLOW-3D by Flow Science, FLUENT and CFX by ANSYS, or STAR-CCM by CD-Adapco.

Most relevant studies on wave-structure interactions with rubble mounds, looking at the flow through and on the layer particles, are by Dentale et al. (2009, 2014, 2018) for Tetrapode and Accropode layers using Flow3D with Finite Differences Method (with a rectangular grid of different resolutions) and by Altomare et al. (2012, 2014); Zhang et al. (2018) for Antifer-cube and Cube layers using DualSPHysics (with SPH, single resolution and single water phase). With both cases a short flume was used and a narrow width (between 3 and 5 units wide) of the breakwater was simulated. Largest domain tests are however by Latham et al. (2015) and Xiang J. (2019), who modelled a full breakwater (between 10 and 12 Core-Loc units wide) with OpenFOAM wave generation library OlaFlow (Higuera et al., 2013) with the aim to apply wave load forces on a model (FDEM) capable of resolving unit displacements under a fluids-to-solids model coupling (Latham et al., 2015).

Potential alternatives to the models mentioned above can be found within Finite Element Methods (FEM), typically associated with unstructured meshes, and therefore capable of resolving complex geometries at higher accuracy than rectangular grids. Spenneken et al. (2012) showed

that single-phase wave generation and propagation with AMCG FEM code Fluidity (Pain et al., 2001) could maintain a very low level of numerical energy dissipation. The Imperial College Finite Element Reservoir Simulator (IC-FERST), also developed in the AMCG group and capable of modelling inertia dominated multi-phase flows (Pavlidis et al., 2014; Adam et al., 2016), has been able to model the generation and propagation of solitary waves with two (water and air) phases (Xie et al., 2017b).

Although FEM codes are often considered more computationally expensive than Finite Volume (FVM) or Finite Difference Methods (FDM), FEM can reproduce complex shapes accurately and present a wide range of solver and discretisation options (Donea and Huerta, 2003; Elman et al., 2014) making them very attractive and versatile for very different problems. For multi-phase problems a common approach is to use the Control Volume discretisation (CVFEM) for the phase field. Applications on NWT can be found in Mindel (2008); Vire et al. (2016); Xie et al. (2017b).

Both Fluidity and IC-FERST have been able to model fluid-solid interactions with a two-way fluid-solid coupling (Xiang et al., 2012; Vire et al., 2015; Yang et al., 2016), but these have yet to be parallelised and computational costs of these simulations remain very high, making them out of reach for rubble mound breakwater simulations.

The classical method in CVFEM is to discretise using finite elements for the pressure (continuous Galerkin, CG) and velocity (discontinuous Galerkin, DG) fields, typically with polynomials of order 0, 1 or 2 and with control volumes for the phase field. Higher order discretisations offer higher accuracy but also increase computational cost. Dynamic Mesh Optimisation (DMO) schemes can be used in order to significantly reduce computational cost without losing accuracy (Peraire et al., 1987; Pain et al., 2001; Piggott et al., 2009; Xie et al., 2014; Alauzet and Loseille, 2016). This is particularly attractive for multiphase problems where the interface needs special attention, such as NWT. Compressive advection schemes (Pavlidis et al., 2016) can simulate very compressive water-air interfaces and these can be resolved with a moving high resolution mesh while the rest of the domain can be modelled with lower resolution, resulting to a decrease of number of nodes overall. Moreover, anisotropic meshes (high ratio side-to-side elements) can



help reduce the number of nodes even further.

Furthermore, Salinas et al. (2017) has recently proposed using control volume discretisation for the pressure field to overcome mesh quality limitations in porous media flow problems, often found when using highly optimised anisotropic meshes that lead to elements with large angles, which can make the pressure solution take significantly longer or even fail to converge (Babuška and Aziz, 1976; Shewchuk, 2002).

# Chapter 3

## Construction of Numerical Rock Armour Layers

### 3.1 Introduction

Rubble mound structures such as rock revetments, rock groynes, rock reefs and breakwaters are all widely used to protect coastlines from erosion and flooding. Often the most attractive solution, breakwaters are more commonly constructed from loose concrete units than from rock armour for deep water applications. The advantage of using rubble-mound construction for breakwaters comes from their low cost and flexibility, and their ability to absorb efficiently the wave energy between the granular component rock pieces, or concrete units (CIRIA et al., 2007).

The granular nature of the rock rubble does make for challenges in design and construction of rock rubble mound structures. Their design used in codes of practice is based on empirical methods with design formulae for stability (e.g. Van der Meer (1988a)) and overtopping derived from extensive laboratory scale model tests, (Van der Meer et al., 2018). The physics governing wave-structure interactions for these granular, often adjusting, structures remains extremely challenging. This has held back coastal engineering structural design where stability criteria cannot easily be based on fundamental understanding of loading and resistance forces.

In recent years, computer modelling of WSI has advanced considerably. Structural responses such as overtopping have already been successfully modelled albeit with simplifications such as using porous layers (using Darcy or Forchheimer for a linear or quadratic loss, respectively) instead of particulate systems. However, rubble mound stability analysis with numerical modelling still remains in its infancy due to the need to resolve particle geometries accurately and the subsequent high computational cost.

The particulate variability of rubble mound material has become a realistic and worthwhile objective with a growing interest in numerical modelling as an alternative or supplement to traditional expensive scaled hydraulics testing methods. Although significant progress in constructing numerical models of armour layers consisting of identical shape and size packs of concrete units has been made (Anastasaki et al., 2013; Anastasaki, 2015), there is still no literature supporting any improved realism in the numerical representation of rock armour that would be needed to capture its behaviour in wave-structure interactions.

Concrete armour layers require a placing grid and varying degrees of control of the unit orientations depending on unit type, to ensure their interlocking response is maintained during wave attack. However, rock armour layer stability is based mainly on gravitational forces, contact interaction and frictional forces acting on each unit individually. Moreover, unit sizes and shapes can vary and therefore a grid and orientation strategy cannot be applied for building rock rubble mounds. In this way, a completely different numerical construction strategy is needed.

Crane operators follow both placing recommendations (CIRIA et al., 2007; Sigurdarson and van der Meer, 2018) and their own experience for placing individual rock units on the structure slope. This expertise and approach could be replicated numerically with machine learning algorithms given a good training data set of built layers (Mosser et al., 2017; Bergen et al., 2019), but the complexity of collecting such data from crane operators or existing structures mean that this is still an unrealistic approach.

Two much simpler approaches have been used in this chapter to tackle the complexity of building rock armour layers. First, a ‘brute-force’ approach is presented in Section 3.2.2, where rocks

are laid and then compacted with a concrete slab. This approach has been used for generating artificial rock armour layers in a fast and robust manner. Aiming to simulate crane operators with one-by-one methods, a second approach has also been developed and is described in Section 3.2.3. The numerical approach of the second method is inspired by the famous "tetris" game; an algorithm that aims to fit one rock at a time on account of the geometry built so far.

The realistic rock shapes needed to build realistic rock layers have been taken from the rock shape library created by Latham et al. (2008a) and available on the open source repository Sourceforge, under the name 'VGW'. Rock layers have been created using random samples of several rock shapes and have been scaled in order to achieve given  $D_n$  and gradation coefficient targets. This is explained in Section 3.2.1.

Both novel rock armour layer building approaches have been developed and tested during this PhD, and their preliminary results are shown in this Chapter. Although further development and sensitivity analysis are required, the novel approaches and their results are promising, and can be considered an important milestone towards the development of numerical methods to build rock armour layers. Moreover, simulations of wave-structure interaction using the generated rock armour layers in this section have already been carried out (see Chapter 6), which proves the contribution of the presented work with real applications and potential future research.

## 3.2 Building rock armour layers

Conventional rubble mound breakwaters, and in this case rock rubble mounds, typically consist of a core to avoid or reduce wave transmission through the structure, and a protection made of armour on the seaward slope. An underlayer is added between core and armour layer as a filter to avoid losing material from the core. Main dimensions of such structures can be reduced to crest freeboard and width, slope angle, armour layer thickness, underlayer thickness, and other parameters defining toe and leeward components of the structure (CIRIA et al., 2007).

The main role of the armour layer, also known as main layer, is to endure wave loadings and

dissipate part of their energy. The performance of conventional rock armour layers (and their stability) is highly dependent on the size of the rock units, their shape and interlocking capacity, together with the slope of the layer and its thickness.

National and international standards and guidance documents define and establish rock shape parameters, such as blockiness (BLc) and length-to-thickness ratio (LT), and rock size and mass distribution given by grading curves and the target design rock size (CIRIA et al., 2007).

### 3.2.1 Determining realistic rock shapes

Rock shapes from the rock shape library by Latham et al. (2008a) are used here as the closest available approximation of rocks used in prototype rock armour. However, the high resolution of rock surfaces from the rock shape library makes the simulation of motion and interaction in FEM models of rock surfaces computationally expensive as the number of elements can be extremely high. Therefore, rock surfaces need to be coarsened thereby reducing the number of elements used and lowering the computational cost. These changes to the surface resolution, as seen in Figure 3.1, will change the shape and the descriptors of the rocks, and need to be considered with caution.

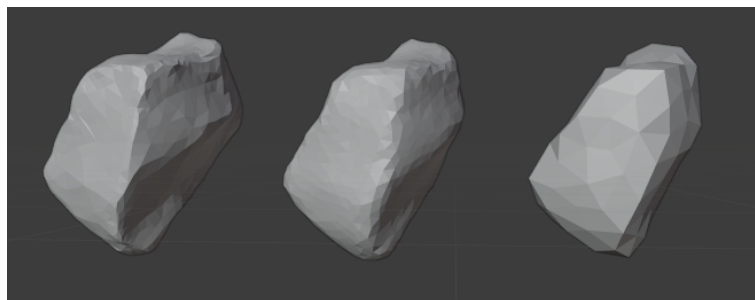


Figure 3.1: Effect of surface mesh resolution on shape representation. Shape library Latham et al. (2008a) (left) with 2710 surface elements, 0.1 m resolution (centre) with 2282 surface elements and 0.3 m resolution (right) with 260 surface elements.

It is worth bearing in mind that Latham derived the rock samples from a crushed aggregate stockpile due for use as track ballast. As such, the aggregate shapes are likely to be less blocky and of generally higher aspect ratio than would be expected in a typical armour layer built from oversize blocks of a metre or more in size as stockpiled in a quarry (CIRIA et al., 2007).

Rock samples for building numerical rock layers can be easily obtained by exporting rock shapes with the desired shape descriptor ranges, such as blockiness (BLc), sphericity or aspect ratio (LT). Once the shape groups are selected, rock armour size distribution can be easily replicated by giving a number of points on the gradation curve matching different sizes, i.e.  $D_{n15}$   $D_{n50}$ ,  $D_{n85}$ . A sample of rock armour can be randomly selected from the given pool of shapes and their numerical mesh can be easily scaled up or down to fit the required gradation curve points. As a result, a sample of rocks with different shapes and sizes is obtained.

Typical rock armour layers consist of 2 layers of rocks. Layer thickness is given as  $2k_t D_{n50}$ , where the  $k_t$  depends on rock shape and expected packing density, as given in Table 3.1. Blocky and Irregular rock shape categories are typically assigned to BLc values of around 0.65 and 0.50, respectively. Previously selected rock shape groups A, C and D present BLc values between 35 and 50, and therefore are considered under the Irregular category. The survey method used for measuring the top surface of the armour layer can also affect the resulting layer thickness and lead to different layer thickness coefficients. The expected range of porosities under different layer placements, rock shapes and survey methods is also given in Table 3.1.

| Layer placement type | Parameter | Blocky (1) - Irregular (1) | Blocky (2) - Irregular (2) |
|----------------------|-----------|----------------------------|----------------------------|
| Double standard      | $k_t$     | 0.91-0.87                  | 0.96-0.92                  |
| Double standard      | $n_v$     | 32-35                      | 36-40                      |
| Double dense         | $k_t$     | 0.91-0.87                  | 0.96-0.92                  |
| Double dense         | $n_v$     | 31-32                      | 35-36                      |

Table 3.1: Layer thickness coefficient and layer porosity for different layer placement types and rock shapes surveyed with the Reference (spherical foot staff) survey method (1) and the Highest point survey method (2). Extracted from The Rock Manual, Table 3.9 (CIRIA et al., 2007).

The slope angle of the structure and the armour layer is frequently limited by the building process and material costs. The number of armour units required and the construction time both depend on slope steepness. Therefore, bulky double layers are often placed on slopes of 1V:1.5H to 1V:2.5H to reduce material needs and building costs, however, slopes of 1V:3H are not uncommon for structures at shallow areas, such as small harbours (i.e. Lyme Regis), groynes and sea defenses (CIRIA et al., 2007). The advantage of the mild slopes (1V:3H) in shallow areas is that they absorb more energy from the waves, reducing wave reflection that

could otherwise affect neighbouring sites, structures or navigation channels.

A slope of 1V:3H has been chosen here for showcasing the two different approaches for building numerical rock armour layers. Both approaches use the same slope for purposes of comparison. The reasons to consider a mild slope (1V:3H) over steeper slopes are twofold. First, the rock shapes provided by the shape library present BLc values of 0.5 or below ('Irregular' rock shape category), suggesting that the interlocking of particles is low. A mild slope is therefore considered more appropriate for stability reasons. Second, the more dissipative effect of a mild slope on wave loadings and its effect on inducing wave breaking, make mild slopes an ideal scenario for testing and showcasing complex WSI with rock rubble mounds with CFD codes (see Chapter 6).

### 3.2.2 Simultaneous deposition and force compaction

This section discusses the application of a 'brute-force' approach for the generation of numerical rock armour layers. This very simplistic approach has been chosen as a first attempt to generate numerical rock armour layers and is based on laying rock armour units on a sloped base and compacting the layer with a heavy slab positioned on top.

For this approach a different software, Blender, has been used. Blender is an open source CAD and rendering tool that includes the physics engine Bullet library (Coumans et al., 2013) for multi-body simulations with contact detection and collisions (<https://github.com/bulletphysics/bullet3>). This 'physics engine' is widely used in the animation and gaming industry, but it has not been widely validated to assess the mechanical realism of the motion dynamics with benchmark cases by the scientific community and its results have to be used with caution. The reason for choosing Blender for this approach over Solidity, which has been already validated for armour layer building, is to test the CAD and format capabilities of this software.

Shape groups A, C and D have been chosen from the shape library (Latham et al., 2008a) for their shape parameters fitting better the rock armour layer recommendations (CIRIA et al., 2007). Elongated (very high LT) and rounded (very high sphericity) shapes were discarded.

Figure 3.2 and Table 3.2 show the shape groups chosen and their main shape descriptors.

| Shape group | Sphericity | BLc (%) | LT      |
|-------------|------------|---------|---------|
| A           | 0.82-0.83  | 41-47   | 1.6-1.7 |
| C           | 0.80       | 44      | 2.2     |
| D           | 0.79-0.85  | 35-50   | 1.4-2.6 |

Table 3.2: Chosen shape groups for realistic armour layers and their main shape descriptors range (Latham et al., 2008a). BLc courtesy of Ed Clark MSci thesis

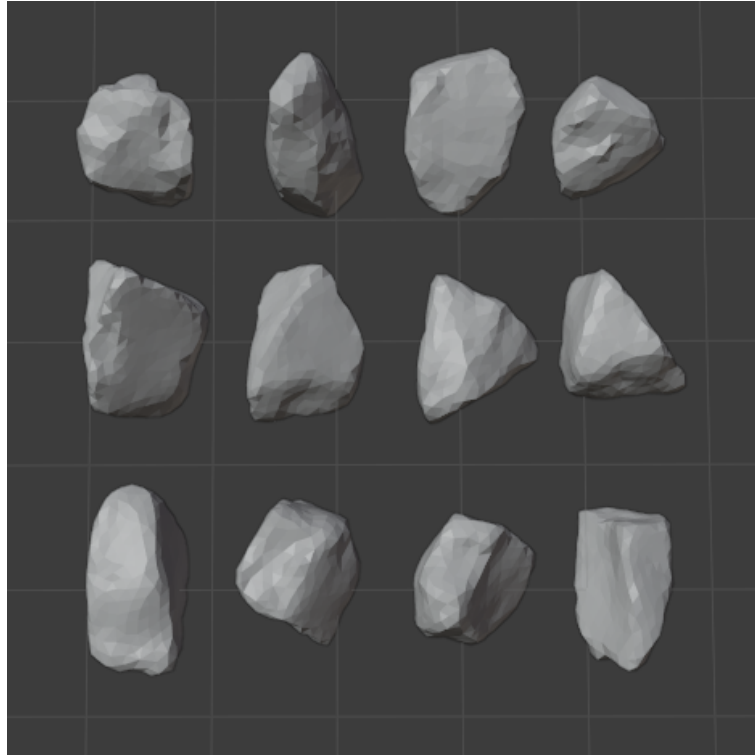


Figure 3.2: Chosen rock shapes from the Shape library Latham et al. (2008a) at 0.1 m resolution.

A sample of 200 randomly selected rocks (see Figure 3.3) has been generated with shapes A, C and D, which have been scaled to obtain a gradation curve with  $D_{n50}=1.39$  m,  $D_{n15}=1.3$  m,  $D_{n85}=1.49$  m, following  $D_{n85}/D_{n15}$  ratio recommendations of 1.2 for main layers ( $< 1.5$ ) by CIRIA et al. (2007). Rock surface resolution has been lowered to elements of 0.3 m.

A breakwater base with a slope of 3H:1V, 10 m width (y axis) and 10 m height (z axis) has been generated. Rocks have been laid in parallel alternating grids, as shown in Figure 3.4. A spacing between rock particles of approximately 3 and 2  $D_{n50}$  is manually given on the slope parallel (1V:3H) and the slope across (y) axis, respectively. The purpose of this layout and these grids is to allow particles of a given tier to fit in the spacing given by the tier below as it is being laid in the armour layer. This aims to increase the contact points of all particles and



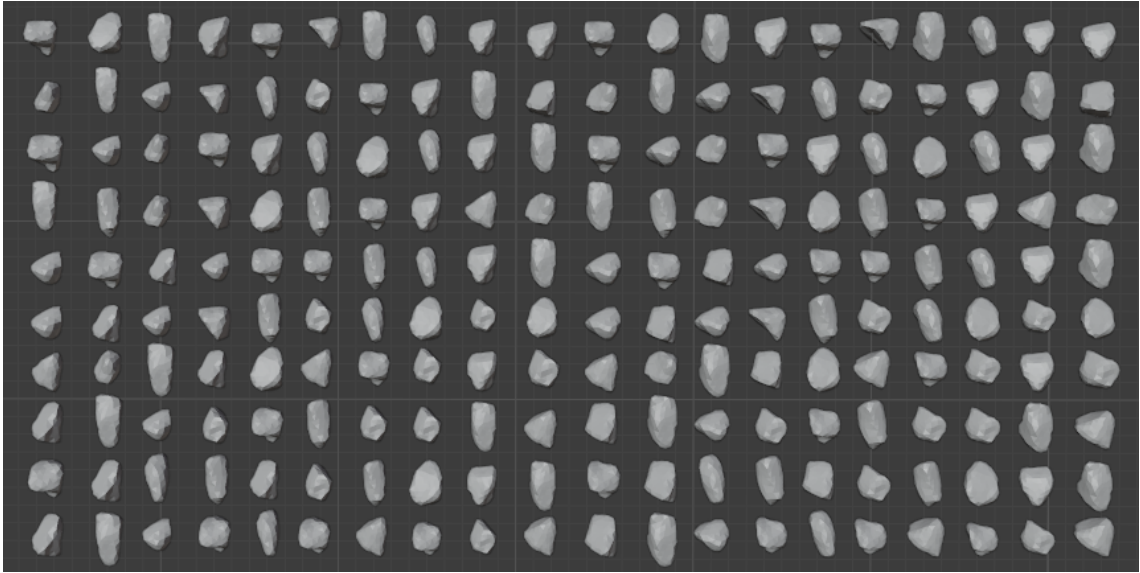


Figure 3.3: Sample of 200 rock units for building the numerical rock armour layer with the compaction slab approach. Shapes are at 3 different scales. Rocks have been manually placed on an horizontal plane for visualisation purposes.

the interlocking effect between them. Alternating grids of 3 and 2 rows of 10 particles are set for this purpose, which cover the full extent of the layer in the x axis.

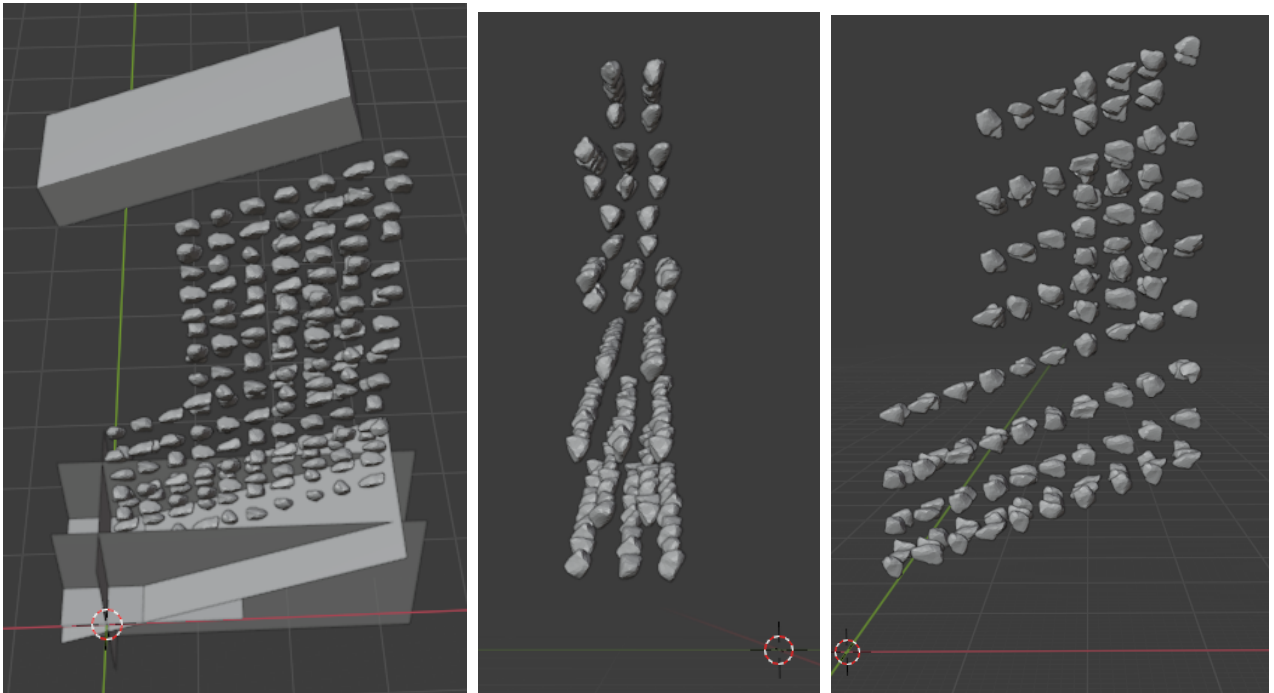


Figure 3.4: General view of the layout setup for the compaction slab approach (left), its front view from the bottom of the slope (centre), and side view (right), where the far left corresponds to the bottom of the slope.

Particles rolling and sliding down after making contact with the slope plane can lead to an

increase in particle density at the bottom of the slope. This difference in particle density can be addressed by reducing the number of particles being laid at the bottom half of the layer while increasing the number of particles being laid at the upper half. For this reason, four grids with rows of only 7 particles are added at the upper half of the slope on top of the previous laid grids. Grids with only 3 particles have also been added to increase the density at the upper end of the layer. A slab has been laid (parallel to the layer 3H:1V slope) on top of the rock layer with the aim to homogenise the layer thickness. The extra particle density at the top end of the layer is observed to help the homogenisation of the layer surface with the compacting slope approach. Other layouts have not been tested due to time constraints.

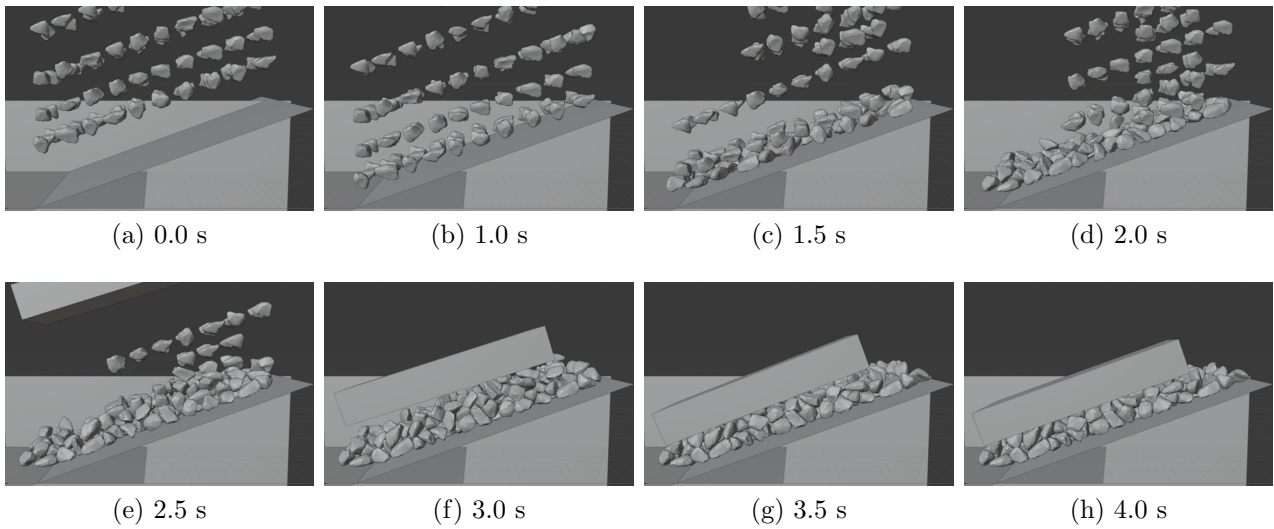


Figure 3.5: Side view of the simulation results of the rock layer compaction slab approach with Blender at different time steps, being 0.0s (a, top left) the starting time and 4.0s (k, bottom right) the last time step.

Figure 3.5 shows a side view of the simulation at different times. The grids and slab are laid with no initial velocity and accelerate with the force due to gravity. All the particle grids are laid in the first 3 s of the simulation. The slab touches down and compresses the layer immediately after the particles are laid, between 3 s and 3.5 s of the simulation. The total simulation time is 10 s, but no significant movement is observed after 4s.

Different views of a layer built using this compaction slab approach can be seen in Figure 3.6. A nearly flat top surface of the layer is observed with a constant layer thickness as the surface rocks are forced to rotate and slide to conform with an enveloping surface defined by the slab plane parallel to the slope.

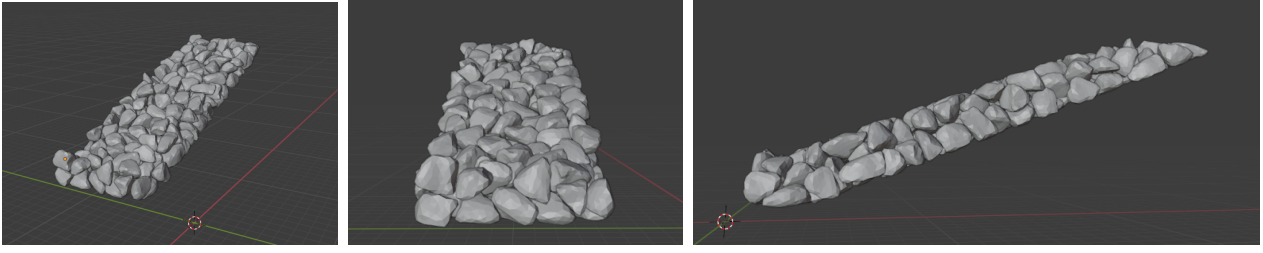


Figure 3.6: Simulation results of the layer-compacting approach with friction coefficient of 0.5. General view of the final stage of the rock layer particles (left), front view from the bottom of the slope (centre), and side view (right).

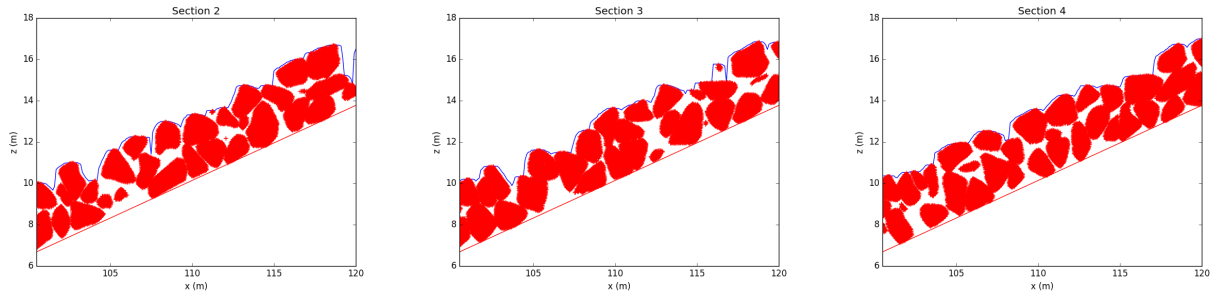


Figure 3.7: Example of extracted section and discretisation with top surface of the layer. Particles represented in red.

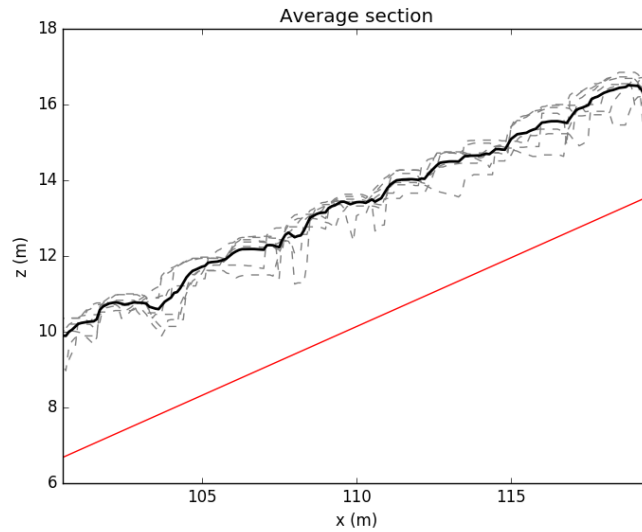


Figure 3.8: Results of averaging armour layer top surfaces from all layer sections (black) and the top surface of the layer at all considered sections (dashed grey).

In order to check tolerances of the layer profile heights in a manner similar to those used on site, a series of sections has been extracted from the armour layer. These have been extracted at 0.5 m intervals ( $y$  axis) and each rock has been spatially discretised in the  $x$ - $z$  plane with a resolution of  $D_{n50}/10$ . The maximum  $z$ -value of a vertical plane has been extracted at each

column in the discretised  $x$  axis to obtain the layer top surface. Porosity is also obtained from the section of the layer and the surface of solid from the sliced domain. Figure 3.7 shows a sample of different extracted sections and their discretisation where the top surface is obtained. Figure 3.8 shows averaged surface (black line) using all layer sections (in dashed grey lines).

The friction coefficient used in Blender physics library Bullet typically ranges between 0 (no friction) and 1 (no displacement allowed). It represents a multiplicative fraction applied to forces/velocities during collision/integration and is subject to time-dependency. The impact of the rock friction coefficient used in Blender for rock-on-rock and rock-on-slab or boundary on layer results has been tested with values of 0.5, 0.6 and 0.7. Results of all friction coefficients can be seen in Figure 3.9. Table 3.3 shows the obtained porosity and the averaged layer thickness (perpendicular to the slope). Although no significant differences between friction coefficients are observed, the friction coefficient of 0.5 is shown to achieve slightly lower values in layer porosity and standard deviation on layer thickness. This is expected as less energy is required to rearrange the particles into a tight pack defined by the slab constraint. The lowest thickness value for the 0.6 friction coefficient (instead of the expected 0.5 friction coefficient) can be explained with the higher standard deviation values (higher variability), which may suggest the presence of lower local thickness values for the 0.6 friction coefficient simulation that could lower the overall average value.

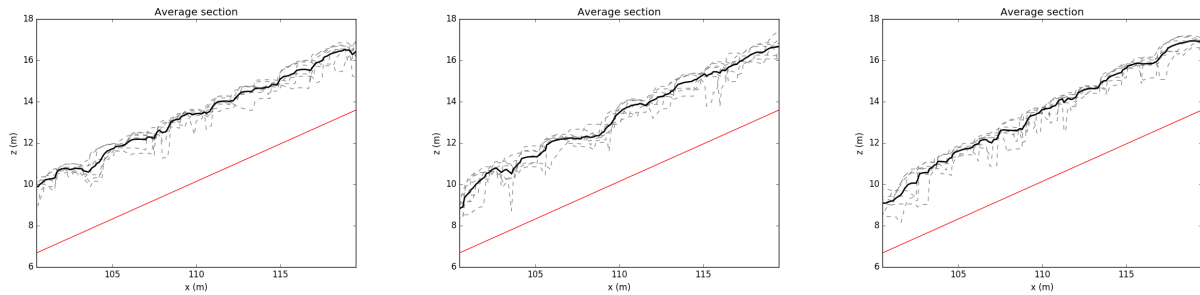


Figure 3.9: Results of averaged profiles from layer-compacting approach with friction coefficients of 0.5 (left), 0.6 (centre) and 0.7 (right).

Considering all shape groups and survey methods, layer thickness predictions by CIRIA et al. (2007) suggest a maximum thickness range between 2.42 m (i.e.  $k_t = 0.870$ ) and 2.67 m ( $kt = 0.960$ ) for rock layers with  $D_{n50}$  of 1.39 m, see Table 3.1, where ‘Blocky’ (2) and ‘Irregular’ (1) categories have been used for minimum and maximum values, respectively. Predictions of

| Friction Coef. | Av. Porosity | Av. layer thickness | St.Dev. on layer thickness | $k_t$ |
|----------------|--------------|---------------------|----------------------------|-------|
| 0.5            | 0.34         | 2.8                 | 0.47                       | 1.01  |
| 0.6            | 0.36         | 2.7                 | 0.50                       | 0.97  |
| 0.7            | 0.36         | 2.9                 | 0.56                       | 1.04  |

Table 3.3: Averaged porosity and layer thickness values for different friction coefficients using the slab compaction approach.  $k_t$  is the resulting layer thickness coefficient from the current values.

layer porosity present a wider range, between 0.31 (Double dense ‘Blocky’ (1)) and 0.4 (Double standard ‘Irregular’ (2)).

The results of the survey method used in the numerical extraction of the layer thickness do not replicate either of the given survey methods in Table 3.1, but they do present some similarities to both of them and the numerical layer thickness value should therefore remain in or near the provided range by the site survey methods. Porosity values from the numerical layer (0.34 - 0.36) remain within range and show agreement with predictions for two categories: the Double standard layer with ‘Irregular’ shapes and the ‘reference’ (spherical foot staff) survey method; and the Double dense layer with both ‘Irregular’ and ‘Blocky’ shapes with the ‘highest point’ survey method. However, the numerical thickness range of 2.7 m - 2.9 m (depending on friction coefficient) remains slightly higher than maximum thickness predicted values (2.42 m - 2.67 m). Moreover, the maximum thickness prediction from CIRIA/CUR data on real double layers applies a layer thickness coefficient leading to a thickness of 2.67 m from ‘Blocky’ rock shapes (BLc values around 0.65), which does not correspond to the ‘Irregular’ shape category (BLc values around 0.5) used for building this numerical layer.

While the overall good match with porosity predictions shows the potential use of the slab compaction approach, the slightly higher layer thickness results when compared to thickness predictions suggests that further research on other grid layouts for laying rocks should be carried out. Nevertheless, it is important to mention that this simple and robust approach has a very low computational cost due to the visualisation and manipulation tools provided by Blender’s GUI, together with a very fast solver speed. The building of the layout can take less than an hour by a trained Blender user, and the simulation time cost has remained below 1 min for all computers on which it has been tested. However, results need to be used with caution due to

the physics engine Bullet library (Coumans et al., 2013) not being fully validated.

In summary, Blender has been used here to rapidly assess the profile surface height variability, bulk packing density and layer thickness arising from the supply of one rock source sample. This digital rock sample was taken from a shape library of aggregate sized shapes, chosen to represent rock armour sized pieces of known shape and size distribution. This suggests that if enough is known about the shape characteristics of the source of rock armour, and if the shape library is populated with enough shapes typical of a wide range of armourstone supplies, the simulation approach might be useful for predicting the construction induced properties of the layer.

Alternatively, if the objective was to create a realistic digital armour layer for hydraulic simulation purposes, e.g. overtopping or stability studies, the resulting properties could be compared with best practice guidance for layer thickness and packing density as a function of mean Blockiness and Aspect ratio, as suggested in the Rock Manual (CIRIA et al., 2007). However, the rock coalescence behaviour and spreading towards fixed boundaries by sliding and rotating when using Blender’s approximate collision solver along with the need for calibration of the best friction coefficient assumption means that when using this approach, the limitations should be fully understood. Nonetheless, particle displacement and rotation naturally occurring in real structures during the initial months after construction could potentially be replicated by this layer building approach, for which further research should be encouraged.

### 3.2.3 Rock-by-rock placing algorithm

This section discusses the application of a virtual crane approach for building numerical rock armour layers. The algorithm, developed for this purpose, aims to replicate the flexibility of crane operators on site, in terms of freedom of rock movement and rotation, and is inspired by the famous ”tetris” game. The Virtual Crane FDEM (VC-FDEM) developed and shown here is using the Solidity (FDEM) solver for final placement of the rocks.

The ‘tetris’ game approach aims to place new shapes in a given cell-based-domain with the goal

of filling all the cells in all the rows, leaving no cells unoccupied. In the VC-FDEM rock placing algorithm the main goal is to find the best location and orientation that fulfils tolerance levels. The ‘no-space-left’ (no cells unoccupied) criterion has been substituted here by a weighted best-fit based on neighbour distance criteria described below. Given that this is a sequential placement process, only a small portion of the domain surrounding the location targeted for the next placement needs to be analysed for each placement. This approach is similar to the crane operator’s working practice and is computationally less demanding than analysing each rock placement possibilities for all potential locations in the whole layer domain.

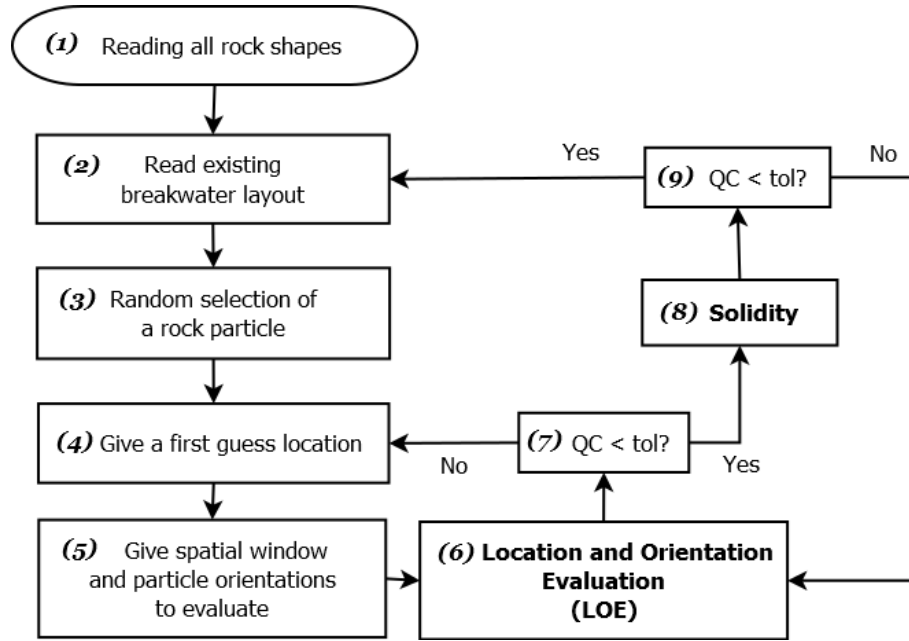


Figure 3.10: Simplified VC-FDEM rock placing algorithm structure with its different components and the different quality control (QC) checks and their possible outcomes. Parallel processes in bold.

Figure 3.10 shows the general structure of this newly developed algorithm. The required inputs for the algorithm are the rock shapes (step (1)) considered for building the layer or already in the layout (i.e. underlayer or toe of the structure), and the preexisting layout (step (2)). The pre-existing layout only requires a base with the given slope for the layer, but it can already include any particles of the main layer or even an underlayer if it is provided. Rock particles are fed into the algorithm for one-by-one placement. For each placement, a particle is randomly selected (step (3)) from the given rock particles sample (step (1)).

A first guess location needs to be provided to the VC-FDEM algorithm for initial placement

(step (4)). This first placement can be considered as the starting location for building the layer (i.e. on top of the toe of the structure) and is updated after each successful or unsuccessful placement. The rotations to be tested for each particle are also required, together with a spatial window around the given guess location (step (4)) for rock placement testing and evaluation. The spatial window (i.e.  $4D_n$  in x and y axis) and the rotations (i.e. every 15, 30 or 45 degrees on all or only certain axes) are given in step (5) and can depend on the size or the shape of the selected rock particle.

The Location and Orientation Evaluation routine (LOE) (step (6)), operating with cell-discretised domains, provides the best  $N$  available placement locations and orientations according to score-based criteria explained below. The lowest score achieved for a placement location and orientation that fulfils the given tolerances (step (7)) is then used for feeding the selected rock particle into Solidity (step (8)) for final placement in the layer domain. If no combination of orientation and location fulfils the given tolerances, a new location for the same particle is proposed (step (4)) by applying a given (x,y) shift (i.e. shift of  $D_{n50}$  (m) on the x or y axis).

Finally, the results from simulating the rock placement with Solidity (FDEM) and the particle resting location are validated (step (9)). It is important to bear in mind that the LOE routine (9) is based on a cell-based discretisation, while Solidity uses a more accurate FDEM spatial discretisation that ensures a mechanically viable resting place. This can sometimes lead to significant differences in placement and this is why the step (9) check is required. When the final particle location given by Solidity is as predicted or comes within a given tolerance, the algorithm moves to step (2) for reading the updated layout, which includes the recently added particle. Alternatively, if the particle is not in the expected location and is not fulfilling the given tolerance, this particular combination of location and orientation is discarded and the next best combination of these quantities from step (6) is fed into Solidity.

Figure 3.11 shows the LOE internal processes with its inputs (6.0) and final output (6.3). Cell-based discretisation (6.1) is used for the layer rock particles and all the combined orientations of the new particle separately. The discretisation is carried out using a slicing algorithm for obtaining the outer surface polygon, and a generic ‘point in polygon’ algorithm to detect



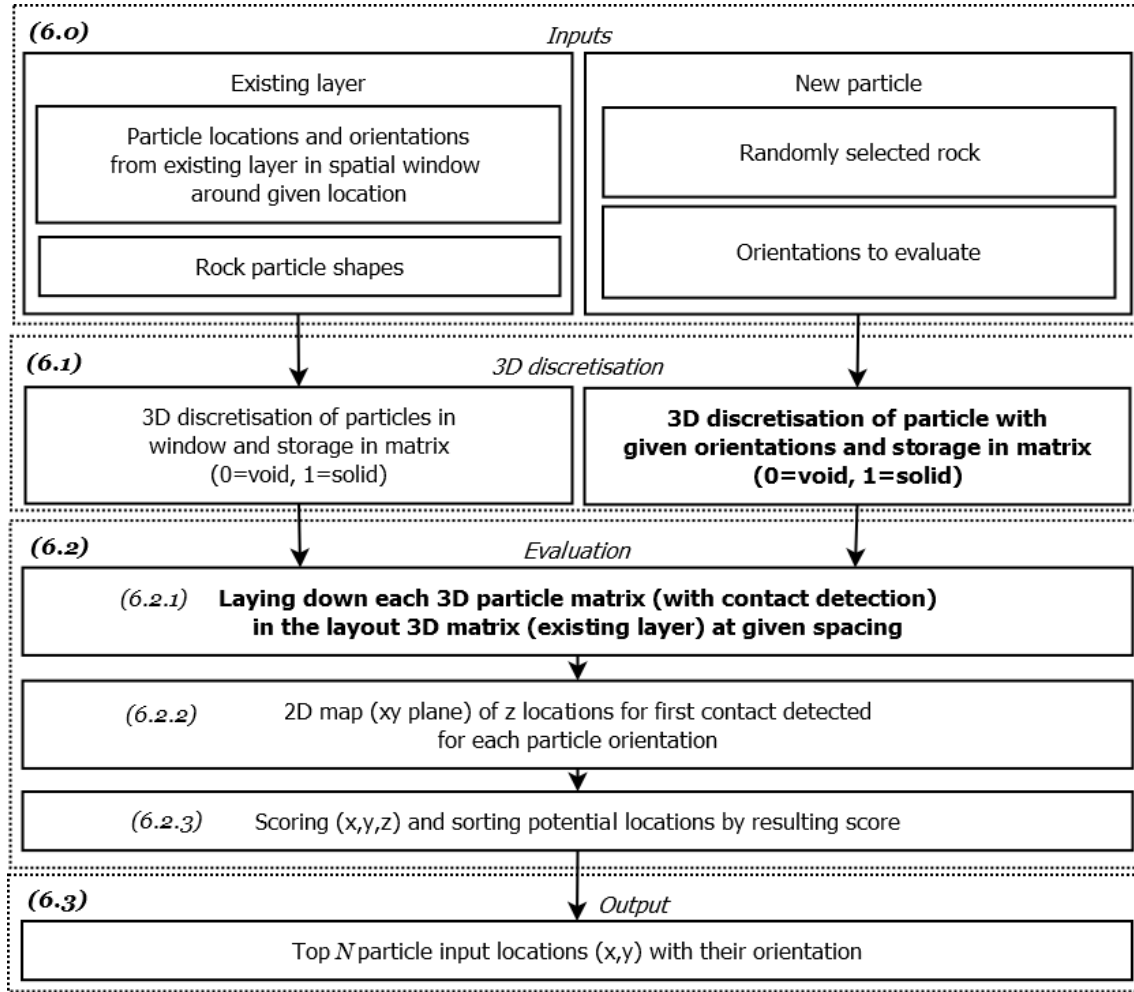


Figure 3.11: Location Orientation Evaluation (LOE) grouped input, routines and output components. Parallel (multiple CPU) components shown in Bold.

whether points lie inside or outside the polygon. Due to a potentially high number of orientations to be tested for the placement of a particle, the particle discretisation process has been parallelised (each core discretises one or more orientations, depending on the available cores and total number of orientations). All cell-based discretisations are stored in a 3D matrix format where each element defines a point or cell in the domain with 0 for void cell or 1 for solid cell. The number of rows and columns is defined by the resolution used for the particle discretisation.

The matrix system helps storing and optimising the process of laying down each rock orientation (the vectorisation of operations helps to increase the speed of Python codes), and has also facilitated the implementation of a very simple contact detection function (6.2.1). All particle orientations are placed at given columns and at sequential heights (z axis or matrix row) of the

layout domain, performing a sum operation at each step between domain matrix and particle matrix, until a contact between the added particle and any particle in the domain is detected. This detection is performed by a simple ‘max’ function on the resulting matrix from the sum operation. Any particle overlapping will also lead to their solid cells (represented by a value of 1) overlapping, which is detected by the ‘max’ function returning a value of 2. The last coordinates of the particle before contact are stored by the LOE routine. Due to the high computational cost of testing all rotations in all considered locations, the **(6.2.1)** routine has also been parallelised.

The returned last position before contact does not guarantee stability or resting state of the laid particle (only 1 contact point or point of support). The stable or resting position can be obtained from looking at neighbouring returned contact heights, which will provide local minimum values that can be considered as potential near-stable locations for the particle with the given orientation. The laying and contact detection routine **(6.2.1)** performed at all or several (x,y) locations in the spatial window results into a 2D map of contact heights **(6.2.2)**, as shown with different examples in Figure 3.12 for very few (top left) to several (bottom right) particles in the layer. The figure shows heights in a colour scheme from blue (zero height) to red (highest), a spatial window of  $2.5D_n$  by  $1.5D_n$  (black rectangle), the first guess location (black circle), and an example of a best placement location (red circle). The locations are shown using a circular symbol of diameter  $D$  as a simplified way of generalising the unknown shape and orientation of the particle being considered for placement.

The evaluation of all the locations in the resulting 2D map **(6.2.2)** can be achieved in many ways. Here a scoring scheme of multiplying the coordinates of each location by previously given ‘scoring factors’ is proposed. This effectively automates the scoring and sorting of potential locations depending on their spatial location and a given criteria represented by the (x,y,z) scoring factors. With this approach, lower scores are given to locations with lower coordinate values. For instance, lower x score is given to lower x coordinate, which indicates that the particle is closer to previous row particles. Consequently, a low score in y indicates that the particle is closer to the previous particle in the same row.

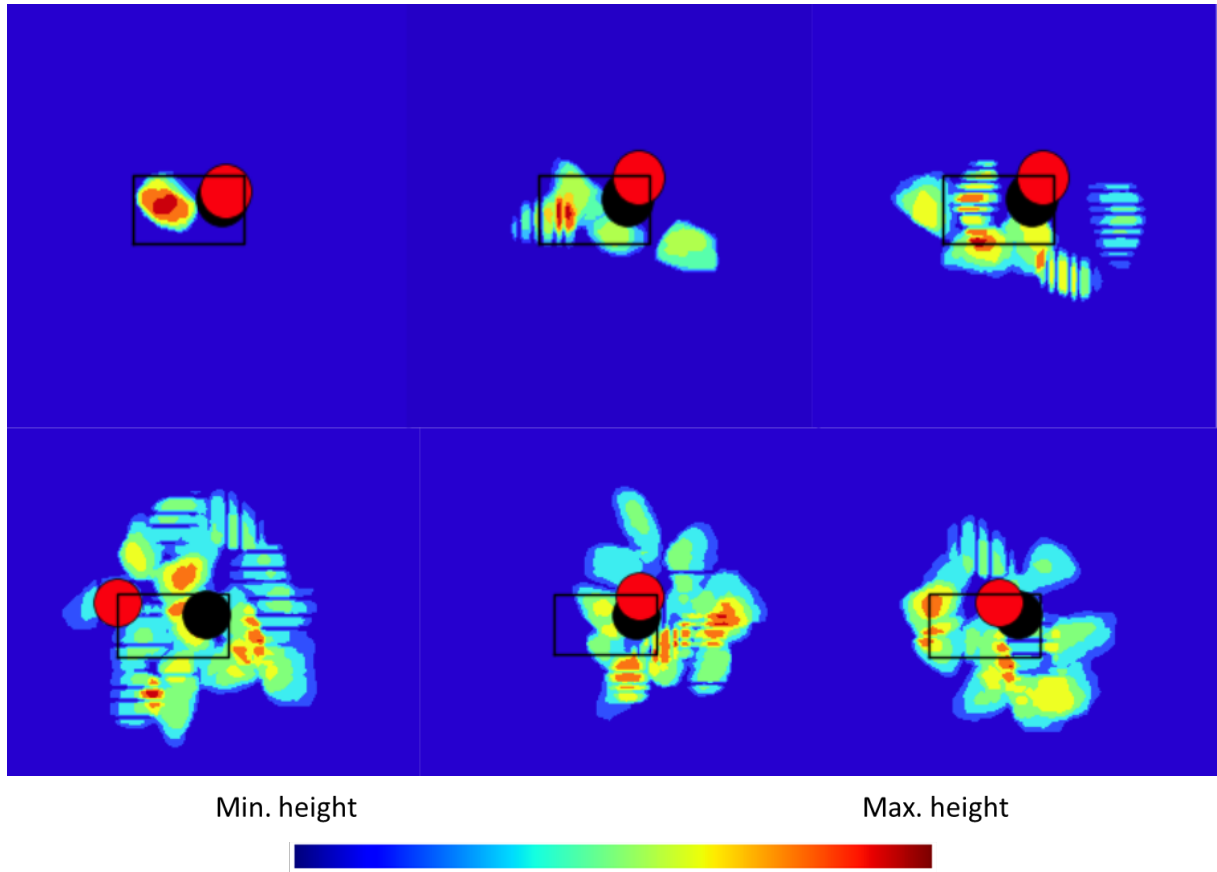


Figure 3.12: Examples of rock laying and scoring for scenarios with different number of particles in the domain. The colour scale represents the height of a potential placement before contact with existing particles. Blue is for min. height and red for max. height. Black and red dots represent the initial guess and final best location, respectively. Black rectangle represents the spatial window given to the VC-FDEM algorithm where to look for possible placements. Spatial discretisation used in this example is  $0.2D_n$ .

The total score is given by the sum of all coordinates multiplied by their factors, and the locations and rotations with lowest  $N$  scores are considered to be the best combinations for placement **(6.2.3)** (outputs of LOE routine). The factors or multipliers applied to the coordinates can be used to prioritise the locations closer to a previous rock in the same row (higher y factor over x factor) or closer to the previous row (higher x factor over y factor). The location outputs **(6.2.3)** are finally filtered by location (x,y,z) tolerance or threshold to fulfill row, slope and thickness dimensions of the layer (step **(9)** in Figure 3.10).

The complexity of the VC-FDEM algorithm and the LOE routines contrasts with the simplicity of its implementation and its running requirements in terms of file types and structure. Figure 3.13 shows the 3 files containing the main routine, the functions library and the setup or inputs

file (left), and all the file formats used (right).

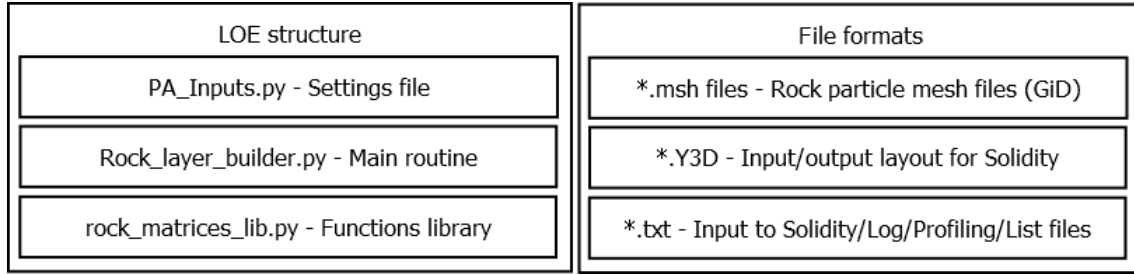


Figure 3.13: LOE algorithm file structure and formats. Available upon request (lluisve@gmail.com) and on [https://github.com/ImperialCollegeLondon/LVE\\_tools](https://github.com/ImperialCollegeLondon/LVE_tools) (permission required)

## Results

An example of building a rock armour layer with the VC-FDEM method is showcased here. A base for the main layer has been created with a 10 m width, 10 m height and a 3H:1V slope. Short flat 4 m (x) surfaces have been added to the bottom and top of the slope. For an initial test, rock shapes D1, D5, D6, D8, G5 (see Figure 3.14) have been chosen for their values of sphericity (0.8-0.9) and blockiness (40-60%), which should lead to a good fit. Shape descriptors of these rock shapes are shown in Table 3.4. A toe has been pre-built using 24 rocks of the D1 shape.

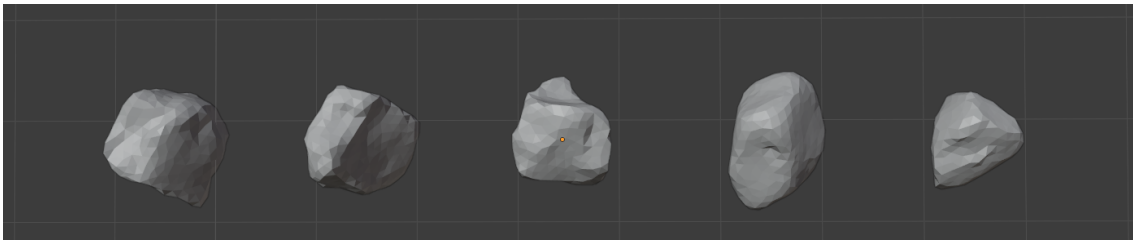


Figure 3.14: Chosen rock shapes Latham et al. (2008a) for testing the rock placing algorithm. From left to right: D1, D5, D6, D8, G5.

The location scores for the LOE routine  $[x,y,z]$  for this example have been weighted to  $[10,1,100]$ , which gives priority to locations with lower  $z$  coordinate values and aims to place rocks closer to neighbours in the same row (lower  $x$  coordinate values). Spatial window dimensions for placement are  $[0.5D_n, 0.5D_n]$  forward (with increasing  $x$ ) and  $[1D_n, 2D_n]$  backward (decreasing in  $x$  and  $y$  directions, respectively) from the guess location (total of  $1.5D_n$  and  $2.5D_n$  in the  $x$

| Shape | Sphericity | BLc (%) | LT  |
|-------|------------|---------|-----|
| D1    | 0.85       | 51      | 2.0 |
| D5    | 0.81       | 39      | 1.4 |
| D6    | 0.85       | 43      | 1.6 |
| D8    | 0.84       | 49      | 1.5 |
| G5    | 0.91       | 60      | 1.7 |

Table 3.4: Chosen shape groups for the rock placing algorithm and their main shape descriptors (Latham et al., 2008a).

and y axis, respectively). Spatial discretisation has been set to 0.2m ( $0.14 D_n$ ). Main [x,y,z] placement tolerances (target location against final location) are set to  $0.5D_n$ . Rotations are considered from the range -90 to +90 degrees with 30 degree intervals for y and z rotation axis. Only rotations on two axis are considered here for simplicity.

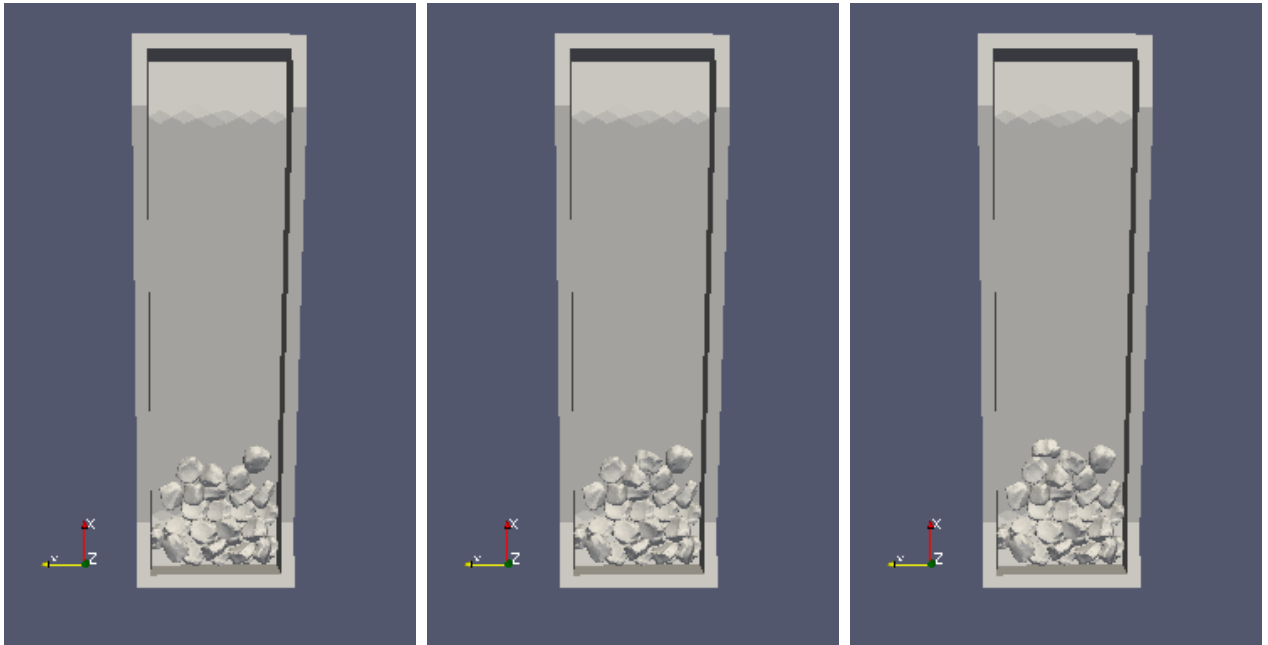


Figure 3.15: Example of rocks being placed on first rows at different time steps by the VC-FDEM algorithm. Section width of 10 m and  $D_{n50}$  of 1.39 m.

Each layer of the double layer has been built separately. Figure 3.15 shows the sequence of 3 new consecutive rocks being placed on the first layer. In this case the algorithm is shown to choose the next location successfully, placing each rock in contact with the previous row and the previously placed rock particle, which is the desired behaviour. Figure 3.16 shows different stages of the building of the first layer. Generally, a small gap can be observed by the wall at the end of some rows. This is induced by keeping a margin of  $D_{n50}$  between the particle centroid and the wall for placement, which leads to particles being placed in the next row if

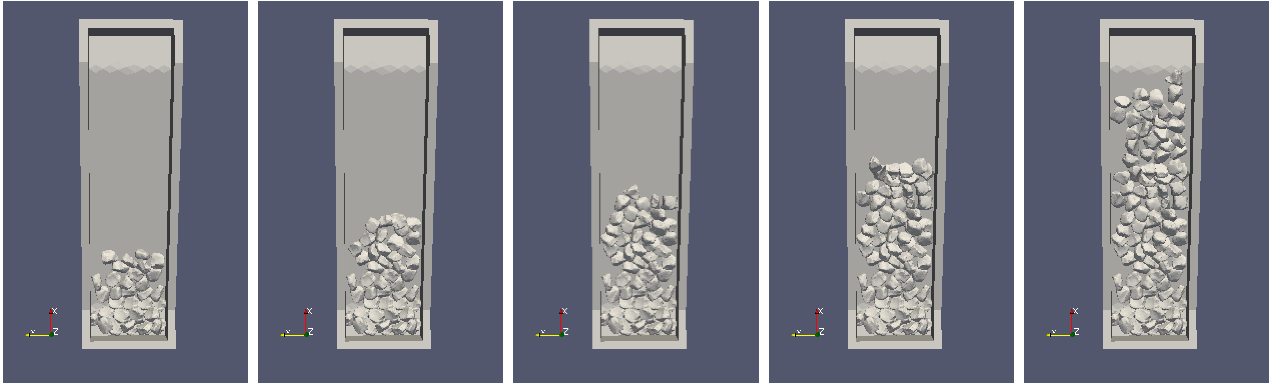


Figure 3.16: Example of a sequence of rocks being placed (accumulative placement) on first layer by the VC-FDEM algorithm. Section width of 10 m and  $D_{n50}$  of 1.39 m.

the next location falls within the  $D_{n50}$  safety margin, sometimes leaving a gap of up to  $D_{n50}$  at the end of the previous row. No significant gaps between rock particles are observed otherwise.

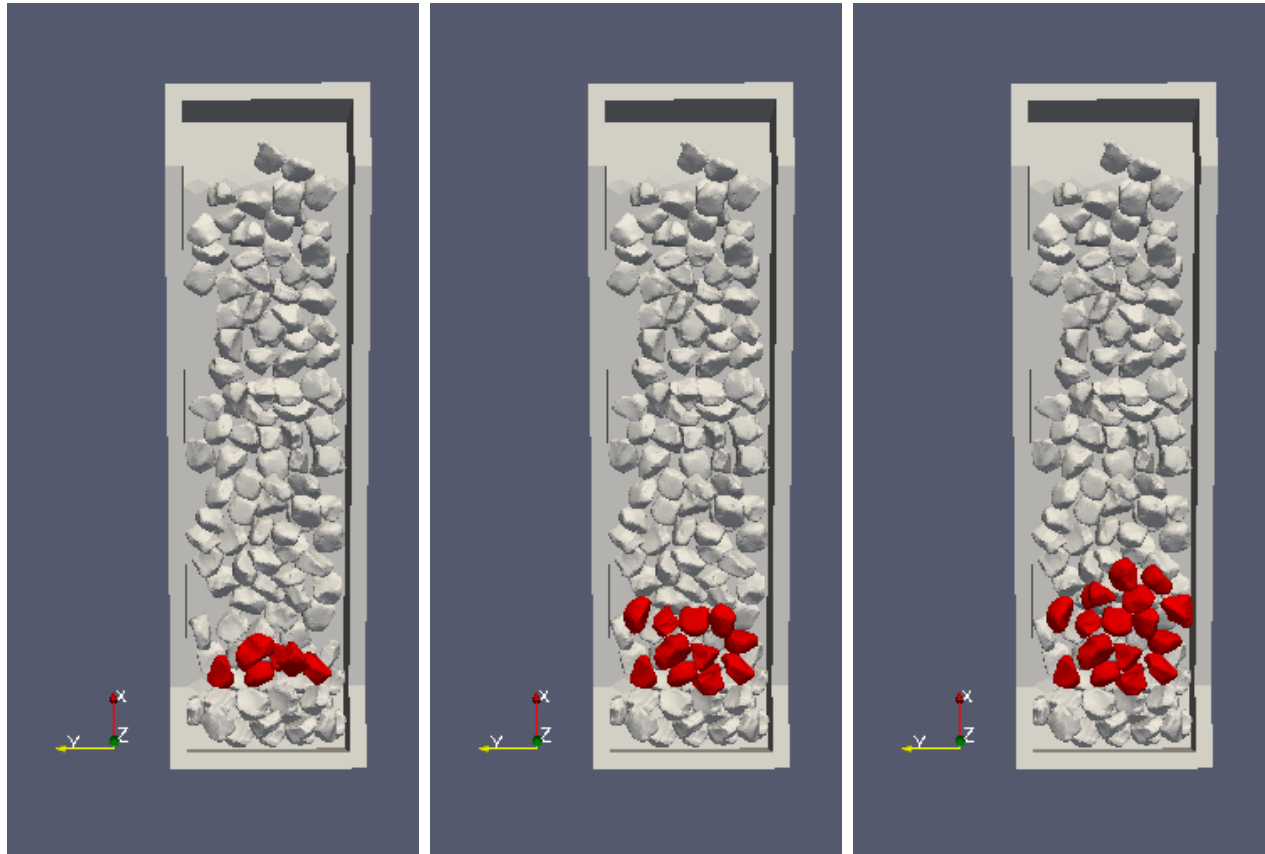


Figure 3.17: Example of rocks being placed on bottom rows of second layer, on top of existing first layer, by the VC-FDEM algorithm. Section width of 10 m and  $D_{n50}$  of 1.39 m.

The algorithm is seen to perform well when placing the second layer, as seen in Figure 3.17, however the spacing between units in the same row and in some cases between rows is seen to increase. Gaps in the second layer can also be induced by particles in the first layer protruding

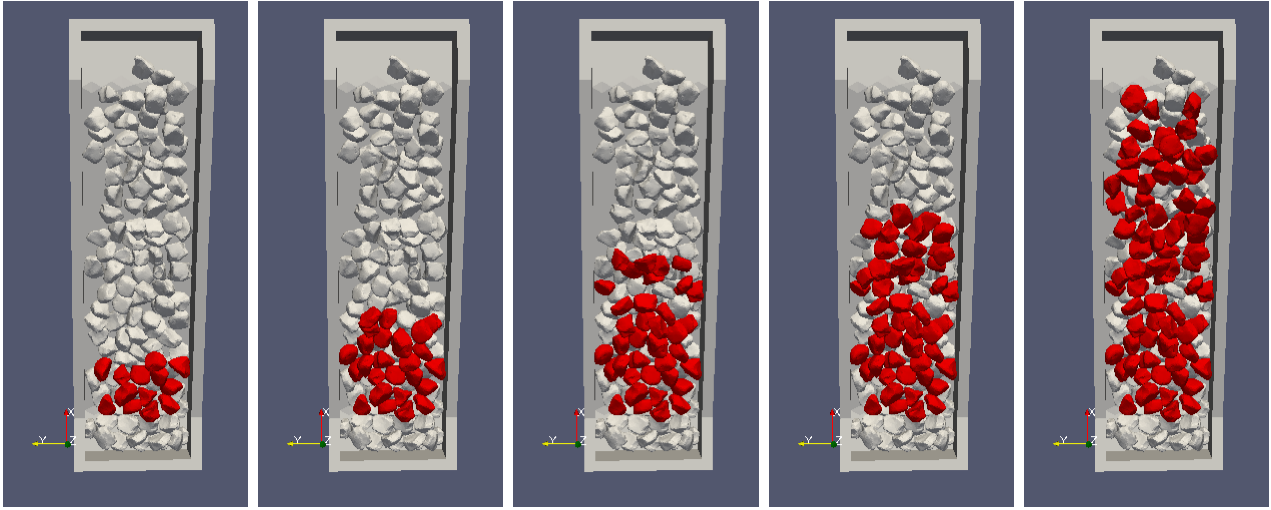


Figure 3.18: Example of a sequence of rocks being placed (accumulative placement) second layer by the VC-FDEM algorithm. Section width of 10 m and  $D_{n50}$  of 1.39 m.

into the second layer. Figure 3.18 shows the different stages of construction of the second layer with the VC-FDEM method. Gaps and units coming out of the layer can be observed in different locations of the second layer that are not seen in the layer compacting approach outlined in Section 3.2.2. This method uses upper compacting and parallel bounding planes for the lower slope.

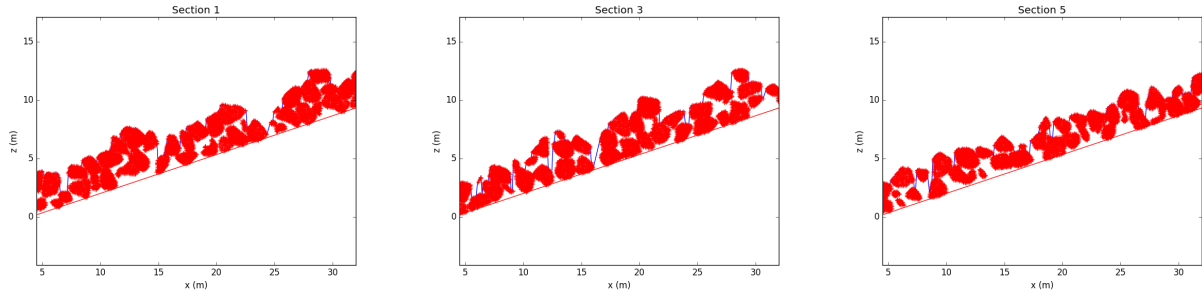


Figure 3.19: Example of extracted section and discretisation with top surface of the layer built by the VC-FDEM algorithm. Particles represented in red.

| Porosity | Av. layer thickness | St. Dev. on layer thickness | $k_t$ |
|----------|---------------------|-----------------------------|-------|
| 0.20     | 2.33                | 0.93                        | 0.84  |

Table 3.5: Porosity and layer thickness values of the layer built by the VC-FDEM algorithm.  $k_t$  is the resulting layer thickness coefficient from the current values.

Cross-sections of layers are shown in Figure 3.19 and the averaged surface of the layer is shown in Figure 3.20. An irregular surface of the layer is observed along the slope, induced by gaps on the second layer and rock particles coming out of the layer above a reference layer thickness

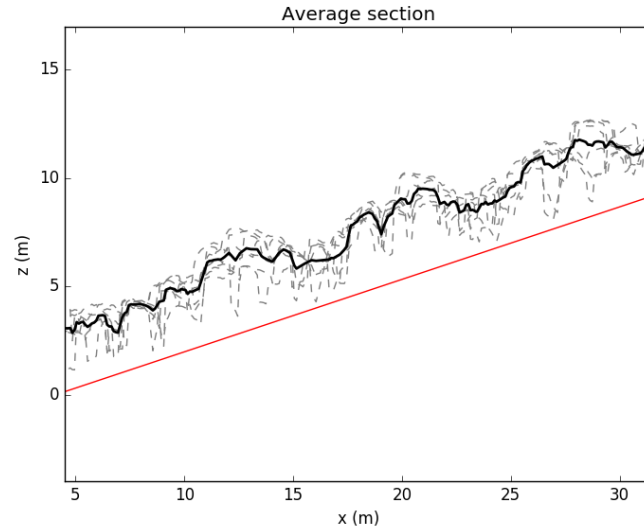


Figure 3.20: Results of averaging armour layer top surfaces from all layer sections (black) and the top surface of the layer at all considered sections (dashed grey), from the initial VC-FDEM results.

of  $2D_{n50}$  (higher thickness than design layer thickness). Table 3.5 shows porosity and layer thickness values of the layer. Considering an ‘Irregular’ shape sample (BLc of 0.5), the lowest predicted layer thickness based on CIRIA/CUR guidance is for  $k_t = 0.87$  leading to a predicted value of 2.42 m (using  $2k_tD_{n50}$ , with  $D_{n50} = 1.39$  m, and considering the ‘Reference’ (1) survey method), which is higher than the  $k_t = 0.84$  and 2.33 m thickness obtained with this example from a VC-FDEM simulation. The high variability in layer thickness seen in Figure 3.20 is confirmed by the 0.93 standard deviation value, which is considered significantly high for a thickness of 2.33.

The high variation in layer thickness can also explain why the obtained porosity of 0.20, is substantially lower than the CIRIA/CUR minimum predicted porosity of 0.31 for any survey method and blockiness. This has been obtained by considering the layer volume as the volume enclosed by the 3H:1V base of the layer and the top surface extracted from the surface of the rocks, which means that some of the void space in the layer above the ‘top’ surface is excluded from the porosity calculation. This difference between the surface definition obtained here and the practical ones used with field surveys could be adjusted for by using a moving average function on the top surface as a smoothing function. However, in order to compare results from this section and the previous section, Section 3.2.2, and in order to show raw results, no



smoothing function of the top surface has been applied.

Overall, this rock-by-rock approach shows promising results for placing the first layer away from the side walls, but results of the second layer could be improved. Further research could help tighten the current rules for setting tolerances and quality scoring for the LOE subroutine for different structural slope angles and rock friction coefficients. It will be important to ensure the VC-FDEM works well for the variety of quarry shapes and a modest grading in sizes. However new algorithmic principles may also be needed to improve success near domain boundary walls. Some potential improvements are summarised and briefly discussed in Section 3.2.4.

### 3.2.4 Future developments

The LOE generation and ranking algorithm has a simplistic trial and error approach to select likely initial locations and heights of each successive rock. The final resting position after modelling with Solidity can sometimes differ from the expected location by the LOE routine. In order to obtain information on the rest-state of the rock to be placed, an analysis of the principal axes of inertia could be carried out before placement. The information on principal axes of inertia is already calculated within the FDEM solver for purposes of rotation and could easily be implemented within the location evaluation process.

A more expensive alternative is feeding Solidity with several of the best scoring locations before fixing what is assumed the best Solidity placed outcome. This would facilitate the decision on the best choice to be based on final equilibrium locations instead of guesses that that could likely be improved upon if several Solidity rest states could be evaluated.

A much simpler improvement is expected by adding a new quality control step where the maximum height of the rock surface is checked. Quality control checks on the rock centroid exist already, but these do not include the rock surface topography and its maximum height. This new step could be easily introduced in the evaluation of potential new locations and be included in the location scoring and ranking in anticipation of obtaining a smoother top surface of the layer.

An independent two layer (one-by-one) construction approach has been developed for rock placement. This approach can be adapted to reproduce other more popular and well established methods such as the layer building approach by Sigurdarson and van der Meer (2018), where rocks are placed in both layers simultaneously, with the aim of increasing the internal interlocking behaviour of the layer.

### 3.3 Discussion

Building numerical rock armour layers is the next logical step to follow on from the successful work by Anastasaki et al. (2013) of building numerical concrete armour layers. The construction of rock armour layers has a random-like aspect when compared to the construction of concrete layers, which adopt units of identical size and shape, even though their topology is also very stochastic in nature. This, and the acceptance of a certain damage level (or slope re-shaping) during their service life, makes rock armour layers ideal candidates for numerical simulations as a tool with which to explore the impact of different setups, building methods or damage levels and to avoid the high computational costs of physical experiments. Structural failure risk analysis can also benefit from an increased knowledge of the structural performance of armour layers at different stages of their life, i.e. in reducing uncertainty and helping to plan when structural repairs may be required. The impact of rock armour erosion and fracturing are also a potential area of future research.

The work presented in this chapter shows two very different approaches both with promising results: a ‘volume filling’ approach where rocks are laid in descending arrays and sheets of rocks and compacted on the layer; and a ‘one-by-one’ rock placing approach that goes some way towards mimicking a crane placement with a discrete build up of separate layers. The contact forces for the placed topology of rocks with a reasonably set friction value are preserved for any further numerical investigation of rock armour stability. For example, if the packing topology can be shown to be realistic for the rock characteristics modelled, (e.g. as was achieved for concrete armour placements with a row by row method, (Latham et al., 2013)) the method of

numerical construction is available for both one-way and two-way coupling stability studies, or, after freezing in the rock topology, run-up and overtopping could be investigated with a numerical flume.

An individual rock placing approach can aim to realistically replicate crane operators placing rocks one-by-one on the slope. Numerical algorithms have the advantage of being very flexible and their decision making easily adjusted. General and numerical settings and scoring of new locations can be easily adjusted and results are expected to significantly change. Moreover, process improvements and other approaches such as (Sigurdarson and van der Meer, 2018) can be easily implemented within the same python library.

There is probably enough scope in the objective of designing a numerical equivalent to a crane or excavator grab with operator to warrant a full PhD. Provisional results are promising, but are still short of being ready for a validation study using experimental and field scale trial panels such as were used in the work of Stewart et al. (2003).

Although the volume filling approach is much cheaper to apply, test and validate, this is assumed too simplistic and does not replicate realistic building techniques for the initial layout of a recently built layer, however, this could be considered for a layer with natural particles settlement after  $>1000$  waves. The compacting rock layers approach does produce rock layers within the recommended range of porosity values and it is expected capable of meeting thickness thresholds with further research, which also makes it a potential future tool if interlocking effects added by the method are to be investigated and quantified.

To date, no other approaches for building numerical rock armour layers have been successfully applied. In this way, this chapter can be used as a first step for continuing the already promising approaches tested here or as an encouragement for developing new ones.

Two software packages have been used and proved successful in building numerical rock armour breakwaters, providing a wider range of possibilities for future work. While Blender has a significantly larger potential for contributions from its user community and versatility with the included CAD tools and formatting capabilities, the rigid version of Solidity provides a

validated and more accurate physics solver.

# Chapter 4

## Numerical modelling of free surface inertial flows

### 4.1 Introduction

*Key content from Chapters 4 and 5 has been published in Via-Estrem et al. (2020). A more detailed and more in-depth explanation of the methods and a wider range of sensitivity tests are presented in this thesis so that the content differs from the published paper. The content of this Chapter published in Via-Estrem et al. (2020) has been referenced accordingly with the given permission by John Wiley Sons, Ltd.*

The main challenges when using Numerical Wave Tanks (NWT) are to resolve the free surface elevation and the wave-structure interaction accurately. The balance between high accurate results and computational cost remains a key limiter, specially for simulations with domains of the size of a wave tank.

Dynamic Mesh Optimisation (DMO) algorithms (Pain et al., 2001) have been validated and extensively used for capturing the interface in multiphase simulations (Xie et al., 2016, 2017a,b) thanks to the capabilities of the coarsening and refinement of the mesh following the free surface motion, which helps reducing the number of elements required and their associated

computational cost. Mesh anisotropy can also help reducing the number of elements (Formaggia et al., 2004; Piggott et al., 2009; Xie et al., 2014; Alauzet and Loseille, 2016; Alauzet et al., 2018), and is of particular interest for simulations with dominant directions (in the equations to be solved by the model) where additional elements can be added on the main direction while reducing the elements in the non dominant directions.

A  $P_1DGP_1(CV)$  spatial discretisation scheme (Salinas et al., 2017), this is polynomials of order 1 for the velocity field (with discontinuous Galerkin, DG) and polynomials of order 1 for the pressure field (with control volumes, CV), has been applied as a novel approach for inertial flow simulations. The  $P_1DGP_1(CV)$  scheme has the potential to reduce number of elements by increasing the mesh anisotropy (element side to side ratio), as already demonstrated for Darcy flow simulations (Salinas et al., 2017), thanks to the faster and more robust convergence of the solution. Moreover,  $P_1DGP_1(CV)$  can help reducing the overall computational cost when compared to typical and more expensive first  $P_1DGP_1$  and second  $P_1DGP_2$  order discretisation schemes (with continuous Galerkin, CG) for the pressure field.

Different schemes can be considered for interface capture, such as a combined implicit backward Euler (robust) and explicit forward Euler scheme (compressive interface) time discretisation scheme, a compressive advection scheme (Pavlidis et al., 2016) to simulate very sharp water-air interfaces, and a continuous Galerkin projection (Geuzaine et al., 1999; Farrell and Maddison, 2011) for the component volume fraction mesh to mesh interpolation.

The aim of this chapter is to test and combine all the mentioned methods and validate them for free surface flow applications. This chapter is structured as follows: the numerical schemes proposed here are described in Section 4.2. 2D sensitivity analysis for the interface capture, the DMO and the spatial discretisation schemes are shown in Sections 4.3.1, 4.3.2 and 4.3.3, respectively. Section 4.3.4 shows the validation of the technology with a 2D collapsing water column benchmark case (Colagrossi and Landrini, 2003). A 3D collapsing water column benchmark case (Kleefsman et al., 2005) is used for a full 3D validation and results are shown in Section 4.3.5.

## 4.2 Governing equations and numerical methods

### 4.2.1 Governing equations

The Navier-Stokes equations for viscous incompressible flows are used here for multi-component models (Ewing and Heinemann, 1984; Pavlidis et al., 2014), where the component advection equations are formulated within the pressure and continuity equations, which results in local mass balance. A number of components ( $\mathcal{N}_C$ ) exist in one or more phases (one is assumed here), where  $\alpha_i$  is the volume fraction of component  $i$ ,  $i = 1, 2, \dots, \mathcal{N}_C$ , and the system is constrained to:

$$\sum_{i=1}^{\mathcal{N}_C} \alpha_i = 1. \quad (4.1)$$

For each component  $i$  the equation of conservation of mass is defined as:

$$\rho_i \left( \frac{\partial \alpha_i}{\partial t} + \nabla \cdot (\mathbf{u} \alpha_i) \right) = 0, \quad \mathbf{i} = 1, 2, \dots, \mathbf{N}_C \quad (4.2)$$

where  $t$  is time,  $\rho_i$  is the component density and  $\mathbf{u}$  is the velocity vector. The equation of motion of an incompressible viscous fluid is:

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot \mu (\nabla \mathbf{u} + \nabla^T \mathbf{u}) + \rho \mathbf{g} \quad (4.3)$$

where  $p$  is the pressure field,  $\mu = \sum_{i=1}^{\mathcal{N}_C} \alpha_i \mu_i$  is the bulk dynamic viscosity,  $\rho = \sum_{i=1}^{\mathcal{N}_C} \alpha_i \rho_i$  is the bulk density and  $\mathbf{g}$  is the gravity acceleration vector.

### 4.2.2 Computational grids and spatial discretisation with $P_1 DGP_1(CV)$

The classical approach for CVFEM is to solve equations 4.2 and 4.3 using a FE-based discretisation for both pressure and velocity fields with the continuous and discontinuous Galerkin

| Nomenclature   | Velocity field discretisation | Pressure field discretisation |
|----------------|-------------------------------|-------------------------------|
| $P_0DGP_1$     | constant discontinuous (DG)   | 1st order continuous (CG)     |
| $P_1DGP_1$     | 1st order DG                  | 1st order CG                  |
| $P_1DGP_1(CV)$ | 1st order DG                  | Control Volumes               |
| $P_1DGP_2$     | 1st order DG                  | 2nd order CG                  |

Table 4.1: Summary of nomenclature for different spatial discretisation schemes

method, respectively, typically with polynomials of order 0, 1 or 2, and with control volumes (CV) for the phase field. Higher order discretisations offer higher accuracy but also increase computational cost. Figures 4.1 and 4.2 shows second order or quadratic FE discretisation (left), linear or first order FE discretisation (centre), and constant CV discretisation (right) with their degrees of freedom for 1D and 2D representations, respectively.

A typical domain discretisation  $P_1DGP_2$  would be using a first order and discontinuous  $P_1DG$  discretisation for the velocity field, this is that the velocity value on the elements is represented with a linear polynomial function, and that the value at the node where two consecutive elements meet may differ between elements (see DG representation in Figure 4.1, left). The pressure field would be using a second order continuous  $P_2$  discretisation, which means that the value of the pressure field is represented with a quadratic polynomial function, which requires an extra discretisation point on the element, and that the value on the node where two consecutive elements meet has to be the same. The phases or components in the  $P_1DGP_2$  would typically be represented using CV, this is giving a constant value on the nodes where consecutive elements meet (see Figure 4.1, right).

Alternatively, a spatial discretisation using  $P_1DGP_1$  would be differing from  $P_1DGP_2$  with a first order continuous function (see Figure 4.1, centre) for the pressure field representation on the elements instead of a second order function, which would not need an extra discretisation point on the element and therefore is generally expected to be computationally less demanding. However, lower order can also lead to lower accuracy of the solution, and other schemes may be required to compensate this loss in accuracy. An example of compensating the accuracy loss in the pressure field with  $P_1DGP_1$  is presented by Maddison et al. (2011), where a pressure decomposition is proposed with the hydrostatic pressure being resolved separately.



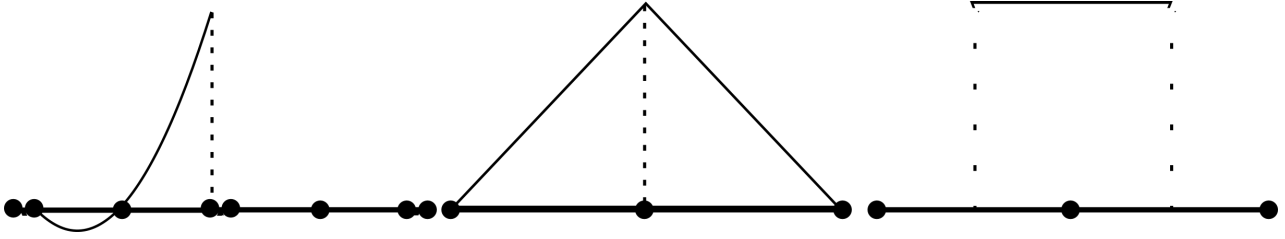


Figure 4.1: 1D representation of element discretisation using: polynomial function of order 2 ( $P_2$ ) with discontinuous Galerkin (DG), left; polynomial function of order 1 ( $P_1$ ) and continuous Galerkin (CG), centre; and Control Volumes on the element nodes (right). The black dots represent the nodes for element discretisation. Elements are represented with the thick black line. The representation of the variable discretised on each elements is presented as thin black line. For CV (right) a constant value is assigned to the node between elements; for  $P_1$  (centre) the value is assigned on the element with a first order polynomial function; and for  $P_2$  (left) the value is assigned on the element using a second order or quadratic polynomial function, which also has an extra discretisation point. The DG discretisation allows different values at the common node between two consecutive elements, which is represented with two black dots at the element ends (left), one for each element.

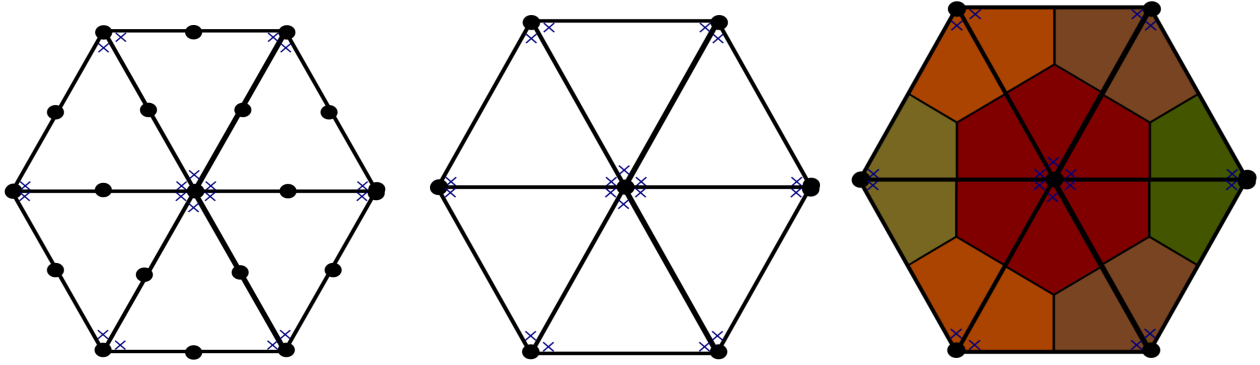


Figure 4.2: 2D representation of element discretisation using: polynomial function of order 2 ( $P_2$ ) with discontinuous Galerkin (CG), left; polynomial function of order 1 ( $P_1$ ) and continuous Galerkin (CG), centre; and Control Volumes on the element nodes (right). The elements are represented with a black thick lines. The black dots represent the points for element discretisation.  $P_2$  (left) is discretised using the element end nodes and an extra discretisation point for resolving a second order or quadratic polynomial function.  $P_1$  (centre) is discretised using the element end nodes for resolving a linear or first order polynomial function. CV (right) is discretised with a constant value (represented with different colours) on the element ends. The 'x' represent the Gauss or Integration points, where numerical integration is carried out.

### Novel CV discretisation for pressure

Using highly optimised anisotropic meshes for computational cost reduction can lead to elements with large angles, which can make the pressure solution take significantly longer or even fail to converge (Babuška and Aziz, 1976; Shewchuk, 2002). Salinas et al. (2017) has recently proposed using control volume discretisation for the pressure field to overcome mesh quality limitations

in porous media flow problems. His formulation is applied to the standard  $P_0DGP_1$  scheme, but the low polynomial order for discretising the velocity field makes it unsuitable for inertia dominated flows (allowing too much numerical dissipation of the momentum equation).

Here a  $P_1DGP_1(CV)$  is proposed based on the higher order  $P_1DGP_1$ . When compared to classical  $P_1DGP_1$ , the  $P_1DGP_1(CV)$  approach uses a control volume pressure spatial discretisation and this offers a more robust solution when applied to very anisotropic elements (see Salinas et al. (2017)). Compared to other classical discretisations schemes, such as  $P_1DGP_2$ , the  $P_1DGP_1(CV)$  is faster because of lower order discretisation for the pressure field. In order to keep higher accuracy for the pressure field, a balanced pressure decomposition method is used (Maddison et al., 2011), which solves the hydrostatic pressure separately with a quadratic element discretisation scheme. This method is found to lead to very significant improvements for the  $P_1DGP_1$  finite element pair, at a negligible increase in computational cost (see Maddison et al. (2011)). The pressure decomposition applies only to the momentum equation and will be omitted in the spatial discretisation section for simplicity.

Assuming finite element (FE) and control volume (CV) representations for  $\mathbf{u}$  and  $p$ , respectively, for the discretisation of the governing equations, these can be expressed in terms of their FE basis functions  $\mathbf{Q}_k$  and CV basis function  $P_j$ :

$$\mathbf{u} = \sum_{k=1}^{\mathcal{N}_u} \mathbf{Q}_k \mathbf{u}_k \quad \text{and} \quad p = \sum_{j=1}^{\mathcal{N}_p} P_j p_j \quad (4.4)$$

being  $\mathcal{N}_u$  and  $\mathcal{N}_p$  velocity and pressure number of degrees of freedom, respectively.

The mass conservation equation (Eq. 4.2) can be simplified by dividing it by  $\rho_i$  before summing all components. By multiplying Eq. 4.2 then with a linear Continuous Galerkin (CG) test function  $P_j$  and applying integration by parts and Green's theorem, as normally adopted, the discrete form of the continuity equation can be formulated as:

$$\begin{aligned}
& \sum_{i=1}^{\mathcal{N}_e} \left( \int_V P_j \left( \frac{\partial \alpha_i}{\partial t} + \nabla \cdot (\mathbf{u} \alpha_i) \right) dV \right) \\
& \approx \sum_{j=1}^{\mathcal{N}_p} \left( \int_V P_j \left( \frac{\alpha_i^{n+1} - \alpha_i^n}{\Delta t} \right) dV \right. \\
& \quad + \int_{\Gamma_{CV}} \theta^{n+1/2} (\mathbf{n} \cdot \mathbf{u}^{n+1} \alpha_i^{n+1}) d\Gamma \\
& \quad \left. + \int_{\Gamma_{CV}} (1 - \theta^{n+1/2}) (\mathbf{n} \cdot \mathbf{u}^n \alpha_i^n) d\Gamma \right) \quad (4.5)
\end{aligned}$$

where  $\mathbf{n}$  is the outward-pointing normal vector,  $\Gamma$  is the contour surface of volume  $V$  and  $n$  the current time level.

Multiplying the momentum equation (Eq. 4.3) with a linear Discontinuous Galerkin (DG) test function  $Q_k$ , applying integration by parts twice over each element  $e$ , with the  $\theta$  time discretisation method, and Green's theorem for the pressure term, the discrete form of the momentum equation can be formulated as:

$$\begin{aligned}
& \int_{V_E} \mathbf{Q}_k \left( \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) + \nabla p - \nabla \cdot \mu (\nabla \mathbf{u} + \nabla^T \mathbf{u}) - \rho \mathbf{g} \right) dV \\
& \approx \int_{V_E} \mathbf{Q}_k \left( \frac{\rho (\mathbf{u}^{n+1} - \mathbf{u}^n)}{\Delta t} - \rho \mathbf{g} \right) dV \\
& \quad + \theta (\mathbf{S}_{kADV}^{n+1} + \mathbf{S}_{kVIS}^{n+1}) + (1 - \theta) (\mathbf{S}_{kADV}^n + \mathbf{S}_{kVIS}^n) \\
& \quad + \int_{\Gamma_{CVj} - \Gamma_{\Omega_p}} \mathbf{Q}_k p_{CVj}^{n+1} \mathbf{n} d\Gamma + \int_{\Gamma_{\Omega_p}} \mathbf{Q}_k \mathbf{n} p_{bc}^{n+1} d\Gamma = 0, \quad (4.6)
\end{aligned}$$

where  $\mathbf{S}_{kADV}$  and  $\mathbf{S}_{kVIS}$  are the advection and viscous terms, respectively, and  $p_{CV}$  is pressure resolved with CV. Note that in this case, because of Pj, the gradient of the pressure field cannot be explicitly evaluated as in the standard FE approach, we therefore need to apply the Green theorem. It has been observed that this combination improves the quality of the pressure matrix, helping the convergence and solution of the pressure field (Salinas et al., 2017).

The advection term is expressed for time level  $n + 1$  as:

$$\mathbf{S}_{kADV}^{n+1} = \int_{V_E} \mathbf{Q}_k \rho \mathbf{u}_*^{n+1} \nabla \cdot \mathbf{u}^{n+1} dV - \int_{\Gamma_E} \mathbf{Q}_k \rho \mathbf{u}_*^{n+1} \mathbf{n} \cdot (\mathbf{u}^{n+1} - \mathbf{u}_{kn}^{n+1}) d\Gamma, \quad (4.7)$$

where  $\mathbf{u}_{kn}$  is the high order upwind velocity calculated from the weak form of the boundary conditions or from the neighbouring element and  $u_*$  is the latest value during the iteration in one time-step. The advection term for the time level  $n$  can be obtained in the same way. The viscosity term can be expressed as:

$$\mathbf{S}_{kVIS}^{n+1} = \int_{V_E} \nabla \mathbf{Q}_k \cdot \tau^{n+1} dV - \frac{1}{2} \int_{\Gamma_E} \mathbf{Q}_k \mathbf{n} \cdot (\tau^{n+1} + \tau_{nb}^{n+1}) d\Gamma, \quad (4.8)$$

where  $\tau = \mu (\nabla \mathbf{u} + \nabla^T \mathbf{u})$ ,  $\tau_{nb}$  is the value of  $\tau$  in the neighbouring element along the face. The viscous terms for time level  $n$  can be obtained in a similar way. More details on how to evaluate  $\tau$  can be found in Xie et al. (2016).

### 4.2.3 The transport equation and the compressive-advection scheme

The transport or flux of each component's volume fraction is modeled with the transport equation:

$$\frac{\partial \phi_i}{\partial t} + \nabla \cdot \mathbf{u} \phi_i - \nabla \cdot \kappa \nabla \phi_i = 0 \quad (4.9)$$

$\phi_i$  is the transported constituent, in this case  $\phi_i = \alpha_i \rho_i$ , and  $\kappa$  is the diffusion term. Here a Petrov-Galerkin method (Brooks and Hughes, 1982) is used to obtain  $\kappa$  as an anti-diffusion term, as described by Pavlidis et al. (2016). This method is equivalent to applying a negative diffusion on the interface in order to compensate for the model induced artificial diffusion and

make the interface more compressive.

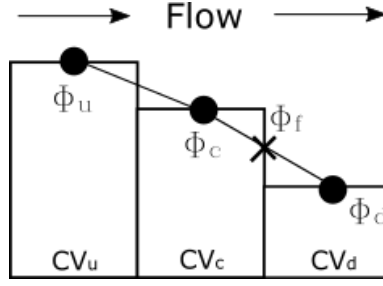


Figure 4.3: 1D representation of upwind ( $CV_u$ ), centre ( $CV_c$ ) and downwind ( $CV_d$ ) Control Volumes with the flux face limit value  $\phi_f$ .  $\phi_u$ ,  $\phi_c$  and  $\phi_d$  stand for the flux value at the  $CV_u$ ,  $CV_c$  and  $CV_d$ .

The discretised form of equation 4.9 can be obtained averaging the equations over each control volume, multiplying by the continuous Galerkin test function  $P_j$ , and applying integration by parts and Green's theorem, obtaining:

$$\begin{aligned} & \sum_{j=1}^{\mathcal{N}_c} \left( \int_{\Gamma_{CV}} \theta^{n+1/2} \left( \mathbf{n} \cdot \mathbf{u}^{n+1} \tilde{\phi}_i^{n+1} - \mathbf{n} \cdot \kappa^{n+1} \nabla \phi_i^{n+1} \right) d\Gamma \right. \\ & \left. + \int_{\Gamma_{CV}} (1 - \theta^{n+1/2}) \left( \mathbf{n} \cdot \mathbf{u}^n \tilde{\phi}_i^n - \mathbf{n} \cdot \kappa^n \nabla \phi_i^n \right) d\Gamma \right) = 0 \end{aligned} \quad (4.10)$$

where  $\tilde{\phi}$  is equal to the flux or transport limiter, which guarantees mass continuity and the method to be conservative (Pavlidis et al., 2016). This is based on the Normalised Variable Diagram (NVD) and given in Eq. 4.11 as non-dimensional high-order face value  $\tilde{\varphi}_f$ :

$$\tilde{\varphi}_f = \begin{cases} \max(\min(\varphi_f, \max(\frac{1}{\tilde{\gamma}C}, \gamma)\varphi_c, 1), \varphi_c) & , \quad \text{for } \varphi_c \in (0,1); \varphi_c > 1 \end{cases}$$

The flux-limited value  $\tilde{\gamma}$  controls the compressiveness of the scheme, here  $\tilde{\gamma} = 3$  is used.  $\gamma$  represents the slope, for which  $\gamma = 2$  is used.  $\varphi_f$  is the non-dimensional upwind face value and  $C$  is the Courant number.

Diffusion term  $\kappa$  can be obtained as:

$$\kappa = -\frac{r_{max}\lambda^*r_{max}}{\|\nabla\phi_{FEM}\|^2} \quad (4.12)$$

where

$$r_{max} = \frac{\|\mathbf{u}\|}{h_{cd}} \quad \text{and} \quad \lambda^* = \beta_\lambda \frac{h_{cd}}{\|\mathbf{u}^*\|} \quad \text{and} \quad \mathbf{u}^* = \frac{r\nabla\phi_{FEM}}{\|\nabla\phi_{FEM}\|^2} \quad (4.13)$$

$\phi_{FEM}$  is calculated using a FEM representation of the CV solution (Farrell and Maddison, 2011) and  $h_{cd}$  is the distance between centroids of local CV and downstream CV.  $\beta_\lambda$  is a constant that equals to 0.5 and 0.25 for linear and quadratic elements, respectively.

$$\Phi^* = \min\{4\phi_{cd}, 1\} \quad (4.14)$$

in which  $\phi_{cd}$  is the gradient of the constituent transported between the centroids of local CV ( $c$ ) and the downstream neighbour ( $d$ ). This effectively applies the negative diffusion term to higher  $\phi$  gradients and decreases linearly to null negative diffusion for zero  $\phi$  gradients. This is found to work very well in all tested cases and produce a well-defined free-surface.

#### 4.2.4 Temporal discretisation

Adaptive discretisation and adaptive time-step schemes described by Pavlidis et al. (2016) are proposed. Time-step size is adapted based on the Courant-Friedrichs-Levy (CFL) number. An adaptive discretisation scheme is used for solving the component volume fraction field based on a parameter  $\theta$ , which varies between 0 and 1 following equation by Pavlidis et al. (2016):

$$\theta_{m-1/2}^{n+1/2} = \max \left\{ \theta_{aim}, 1 - \frac{1}{2} \min \left\{ \left\| \frac{1}{\xi_{m-1/2}^{n+1/2}} \right\|, \left\| \frac{1}{\xi_{m+1/2}^{n+1/2}} \right\| \right\} \right\} \quad (4.15)$$

where:

$$\xi_{m-1/2}^{n+1/2} = \left( \frac{h_{m-1/2}^n - h_{m-1/2}^{n+1/2}}{\phi_m^{n+1/2} - \phi_m^n} \right) \frac{\Delta t}{\Delta x_m} \quad \text{and} \quad \xi_{m+1/2}^{n+1/2} = \left( \frac{h_{m+1/2}^n - h_{m+1/2}^{n+1/2}}{\phi_m^{n+1/2} - \phi_m^n} \right) \frac{\Delta t}{\Delta x_m} \quad (4.16)$$

being  $n$  is the time level,  $m$  the boundaries,  $h$  the flux limiting values and  $\Delta x_m$  is the width of the  $m$ th CV.  $\theta$  may vary between  $\theta = 0$ , when explicit forward Euler scheme is used (which results in a compressive interface) and  $\theta = 1$ , when a more robust implicit backward Euler scheme is used. The advantage of an adaptive discretisation scheme is that it is using implicit Euler, forward Euler whenever possible or a combination of the two where required, leading to a very robust and a more compressive interface. The velocity field is solved with an implicit backward Euler scheme in order to guarantee stability. Another advantage of using implicit or combined time discretisation schemes is that large CFL numbers can be imposed for the adaptive time-step scheme. Simulations with CFL numbers between 0.1 and 2 have been successfully tested and are shown in Section 4.3 and Chapter 5.

### 4.2.5 Solving the system of equations

A pressure projection method is used to solve the system of equations. The matrix form of the discretised continuity, momentum and flux equations for incompressible flows at time level  $n + 1$  are:

$$\mathbf{B}^T \underline{\mathbf{u}}^{n+1} = 0 \quad (4.17)$$

$$(\mathbf{M}_u + \mathbf{A}) \underline{\mathbf{u}}^{n+1} = \mathbf{C} \underline{\mathbf{p}}^{n+1} + \underline{\mathbf{s}}_u^{n+1} \quad (4.18)$$

$$\mathbf{D}(\mathbf{u}) \phi + \mathbf{K} \phi = \underline{\mathbf{s}}_\phi^{n+1} \quad (4.19)$$

where  $\mathbf{M}_u$  is the mass matrix (element-wise block diagonal),  $\mathbf{A}$  is a matrix containing advection and diffusion contributions,  $\mathbf{C}$  is the gradient matrix for pressure,  $\mathbf{B}$  and  $\mathbf{D}(\mathbf{u})$  divergence matrices.  $\mathbf{D}(\mathbf{u})$  and  $\mathbf{K}$  are the flux advection and diffusion matrices, respectively.  $\underline{\mathbf{s}}_p$ ,  $\underline{\mathbf{s}}_u$  and  $\underline{\mathbf{s}}_\phi$  are the source terms for the different equations. The projection method initiates by obtaining a guessed pressure  $\underline{\mathbf{p}}_*^{n+1}$  from initial or previous time-step and then solving for an intermediate velocity  $\underline{\mathbf{u}}_*^{n+1}$ . Eq. 4.18 becomes:

$$(\mathbf{M}_u + \mathbf{A}) \underline{\mathbf{u}}_*^{n+1} = \mathbf{C} \underline{\mathbf{p}}_*^{n+1} + \underline{\mathbf{s}}_u^{n+1} \quad (4.20)$$

Considering that the equation for velocity to be satisfied is:

$$\mathbf{M}_u \underline{\mathbf{u}}^{n+1} + \mathbf{A} \underline{\mathbf{u}}_*^{n+1} = \mathbf{C} \underline{\mathbf{p}}^{n+1} + \underline{\mathbf{s}}_u^{n+1} \quad (4.21)$$

and then subtracting Eq. 4.20 and 4.21, we obtain the velocity correction equation:

$$\mathbf{M}_u (\underline{\mathbf{u}}^{n+1} - \underline{\mathbf{u}}_*^{n+1}) = \mathbf{C} (\underline{\mathbf{p}}^{n+1} - \underline{\mathbf{p}}_*^{n+1}) \quad (4.22)$$



The pressure correction equation is obtained by multiplying Eq. 4.22 by  $\mathbf{B}^T \mathbf{M}_u^{-1}$  and using Eq. 4.17 to eliminate out  $\underline{\mathbf{u}}^{n+1}$ , resulting to:

$$\mathbf{B}^T \mathbf{M}_u^{-1} \mathbf{C} \left( \underline{\mathbf{p}}^{n+1} - \underline{\mathbf{p}}_*^{n+1} \right) = -\mathbf{B}^T \underline{\mathbf{u}}_*^{n+1} + \underline{\mathbf{s}}_p^{n+1} \quad (4.23)$$

The solution method solves first  $\underline{\mathbf{p}}^{n+1}$  with Eq. 4.23 and then corrects velocity with Eq. 4.22. Finally, transport Eq. 4.19 is solved with the corrected velocity. The solution of the system is evaluated and the algorithm moves to next time-step or iteration depending on whether convergence is achieved. A flow chart of the algorithm to solve the system of equations is shown in Figure 4.4. GMRES Krylov subspace solver (Saad and Schultz, 1986) from the PETSc framework (Balay et al., 1997) with the Boomer algebraic multi-grid preconditioner from the HYPRE library (Yang et al., 2002) is used to solve for pressure, velocity and transport.

#### 4.2.6 Dynamic mesh optimisation and mesh to mesh interpolation

Dynamic Mesh Optimisation (DMO) schemes can be used in order to significantly reduce computational cost without losing accuracy (Peraire et al., 1987; Pain et al., 2001; Piggott et al., 2009; Xie et al., 2014; Alauzet and Loeuille, 2016). This is particularly attractive for multiphase problems where the interface needs special attention, such as NWT.

The mesh adaptivity scheme used here is based on the anisotropic mesh optimization library of Pain et al. (2001), in which elements may be altered combining two different techniques: mesh refinement or ‘h’ method; and node relocation or ‘r’ method. The re-meshing algorithm adapts the mesh to the solution without sacrificing the integrity of external and internal boundaries of the domain. It starts by estimating the  $L_2$  norm of the local interpolation error introduced by the FE representation, using the weighted norm of the Hessian matrix of the specified fields  $\mathbf{H}$ . Variables defined into the CV space are projected into FE. The Hessian of a smooth field is then approximated by a Galerkin projection of the linear interpolation on the mesh (Alauzet

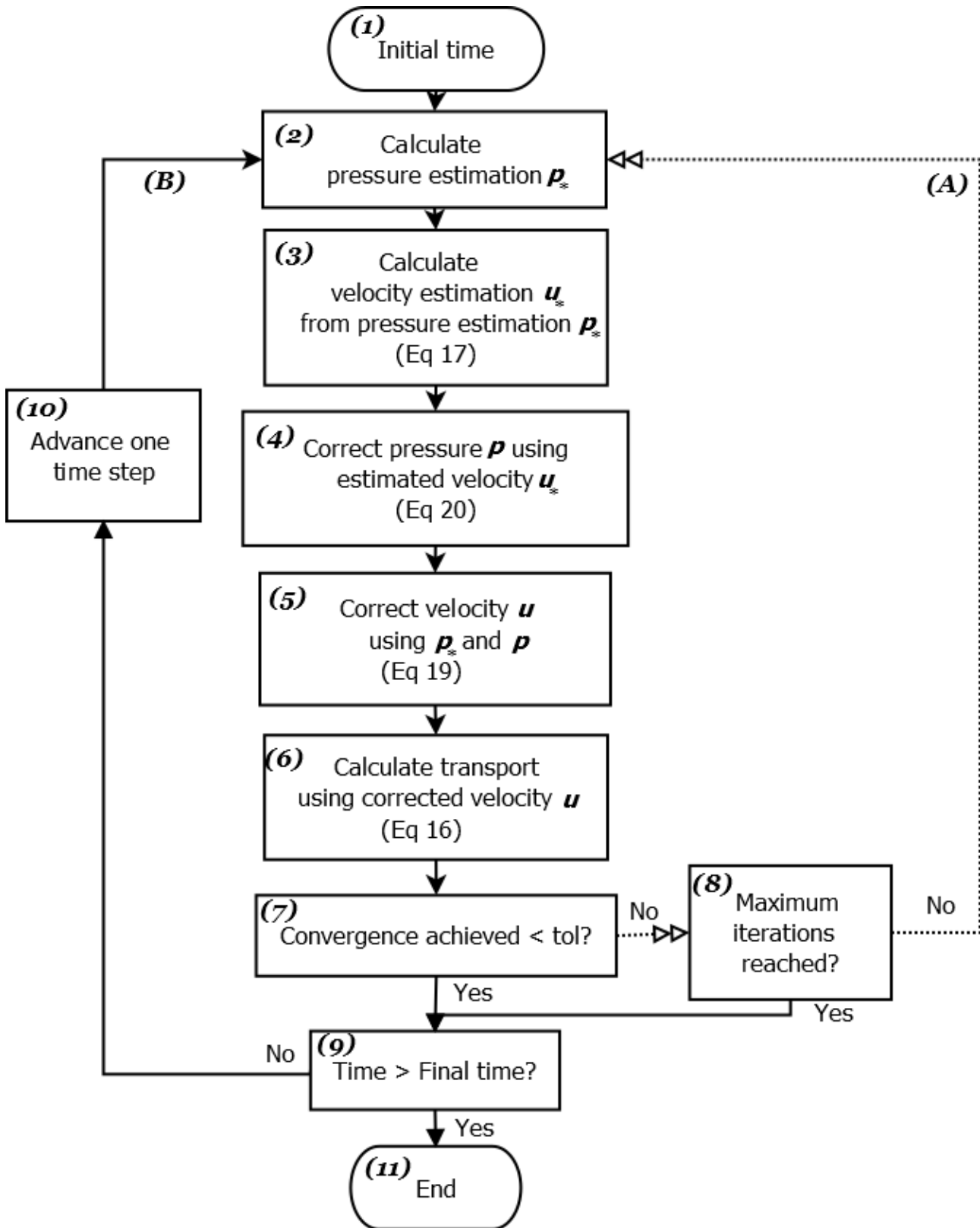


Figure 4.4: Pressure projection method used to solve the system of equations (Via-Estrem et al., 2020).

and Loseille (2016)). The estimation of the interpolation error is therefore based on information already available on the mesh. The quality of the mesh can be defined as:

$$\mathbf{I} = \sum_{i \in \text{edges}} (\mathbf{e}_i^t \mathbf{E} \mathbf{e}_i - 1)^2 \quad (4.24)$$

where  $e_i$  define the edges connecting the vertices of the mesh and

$$\mathbf{E} = \det(\mathbf{H})^{\frac{1}{2\gamma\delta}} \frac{|\mathbf{H}|}{\epsilon} \quad (4.25)$$

defines the estimation of the error metric along each edge.  $\gamma$  indicates the degree of the polynomial of the norm used and  $\delta$  the number of dimensions of the problem.  $\epsilon$  specifies the required tolerance for the field introduced by the user.

The objective is to obtain a mesh with uniform interpolation error in any direction. The water component volume fraction is used as solution field for the mesh adaptivity scheme, guaranteeing higher resolution on the interface. Mesh anisotropy, mesh gradation level and maximum number of elements are also criteria to be taken into account if given by the user. A metric advection method (Wilson, 2009) is used here in order to take into account the expected movement and direction of the high resolution area between mesh adapts.

Mesh to mesh interpolation is used once the new mesh has been generated (Adam et al., 2016). A standard consistent interpolation is proposed for the velocity field. For pressure and component fraction fields a continuous Galerkin projection (Geuzaine et al., 1999; Farrell and Maddison, 2011) is used because of its high order and mass conservation, which also leads to better interface capture as explained in section 4.2.7.

#### 4.2.7 Interface capture schemes

The different options proposed here for keeping a sharp interface without increasing the computational cost can be summarised:

- A more compressive interface can be obtained with the adaptive time discretisation scheme described in section 4.2.4.

- A continuous Galerkin projection (Geuzaine et al., 1999; Farrell and Maddison, 2011) is used for the component volume fraction (see Section 4.2.6). This guarantees a conservative, non-dissipative and non-bounded projection of the water volume fraction field.
- A high-order accurate Petrov-Galerkin-based compressive advection scheme Pavlidis et al. (2016) with a gradient-based scaling is applied. This method described in Section 4.2.3 is equivalent to adding a negative diffusion into the advection equation.
- A high-order FEM down-winding scheme (see Section 4.2.3) is used to obtain fluxes on the CV boundaries, which are subjected to a flux-limiting normalised variable diagram (NVD) approach (Leonard, 1991; Darwish, 1993; Darwish and Moukalled, 2003), to obtain compressive and bounded solutions.

### 4.3 Sensitivity tests and Validation

Collapsing water column, also called dam break, problems are very common for validating multiphase inertia dominated flows as they are seen as a good representation of inertial flows and free surface problems, typically with air and water phases. A few simple test cases are shown here as sensitivity analysis of the techniques described in Section 4.2. These were carried out using IC-FERST.

Two benchmark cases are also used for full validation of the methodology proposed here. A 2D dam break problem is compared to experiments by Colagrossi and Landrini (2003). Finally, a 3D dam break with a rectangular obstacle in the tank (Kleefsman et al., 2005)] is used for a full 3D validation and for showing full potential of the methods for other applications.

#### 4.3.1 2D sensitivity analysis of the compressive advection scheme

A high-order accurate Petrov-Galerkin-based compressive advection scheme Pavlidis et al. (2016) is considered for reducing the numerical interface dissipation typically associated to

multiphase fluid flow simulations. An assessment is presented here to evaluate the compressiveness of this scheme and the energy dissipation reduction for a collapsing water column case (case ‘A’).

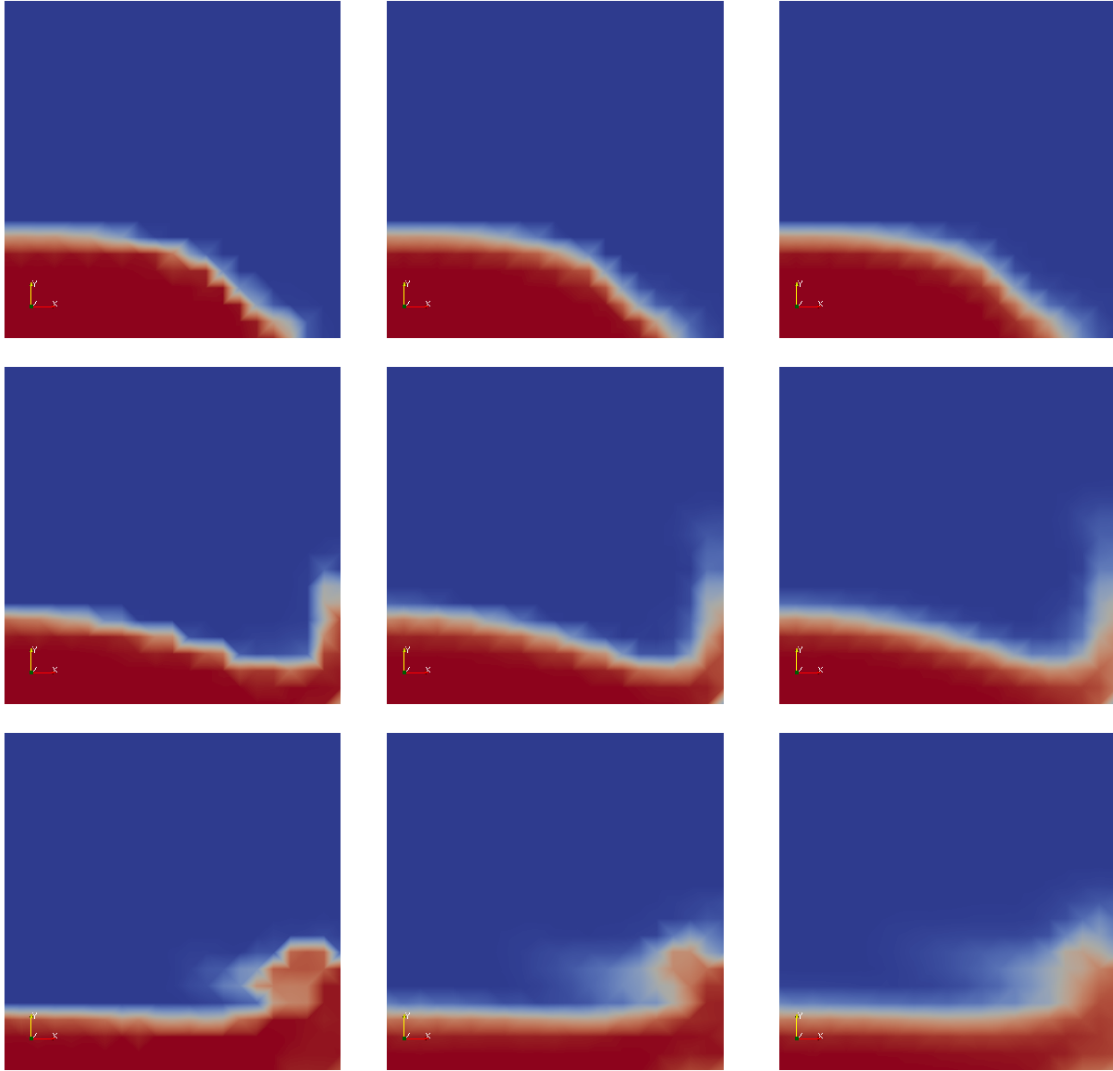


Figure 4.5: Compressive advection scheme results with negative diffusion (left), compressive advection scheme results without negative diffusion (centre) and Sweby scheme (right), at  $t=0.25$  s (top),  $t=0.5$  s (centre) and  $t=1$  s (bottom)

A compressive advection scheme with no negative diffusion is added to the comparison (case ‘B’), this is, using the same face value limiter approach as the standard compressive advection scheme, but in this case there is no negative diffusion term added to the phase or component value at the interface. The analysis includes a face value limiter by Sweby (1984) for comparison (case ‘C’).

The 2D domain of the water tank is  $2 \times 2$  m and the water column is 1.2 m wide and 0.6 m high.

The gridded mesh of 0.1 m resolution is fixed throughout the simulation so the compressiveness of the interface between the different schemes cannot be affected by mesh differences.

Figure 4.5 shows that the results from the compressive advection scheme (left) are more compressive than the non-negative diffusion and Sweby schemes at all time-steps. Moreover, the interface diffusion in (B) and (C) increases with time, while the interface compressiveness in (A) is preserved.

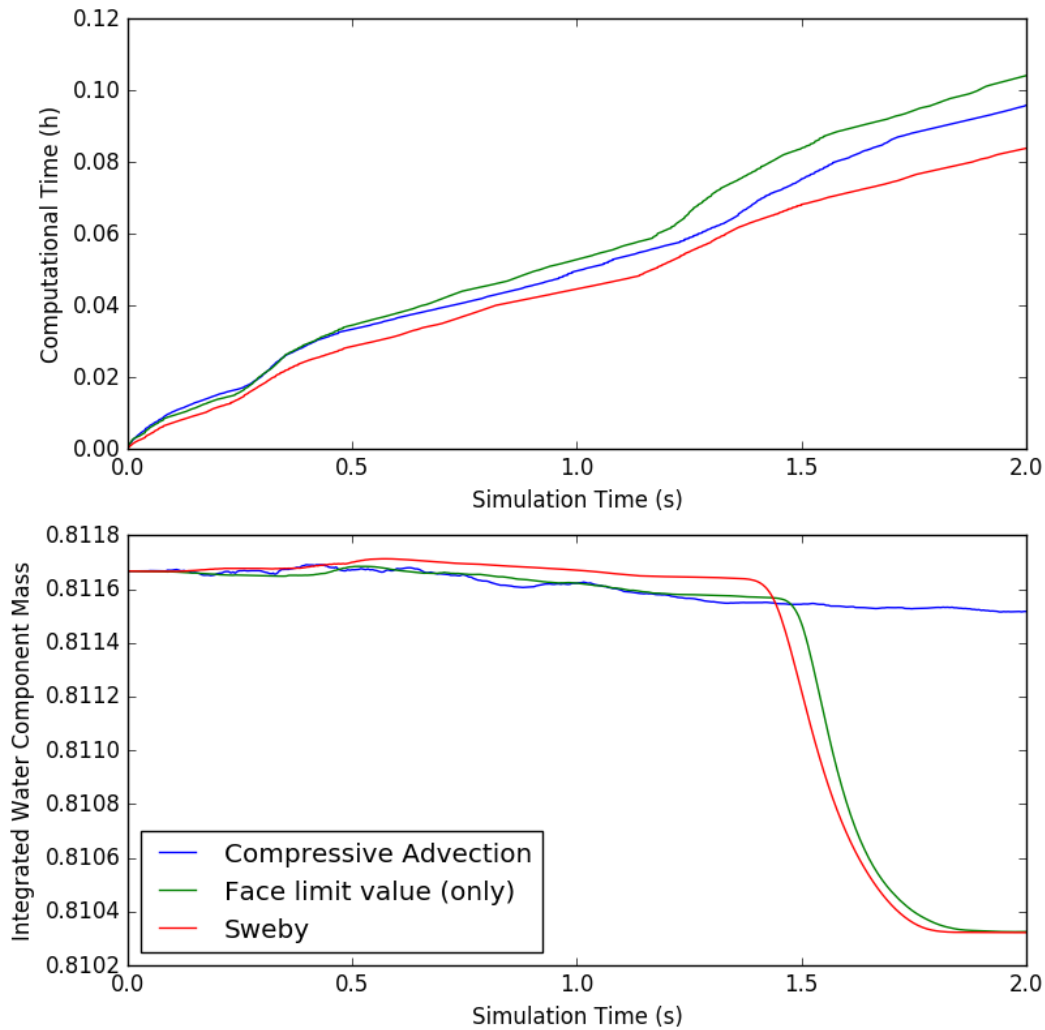


Figure 4.6: Simulation performance of Compressive Advection with limiter, the face value limiter (only), and Sweby schemes. Computational cost (top) and Integrated Water Component Mass in the domain (bottom) over the simulation time.

In terms of computational time, the compressive advection schemes with or without nega-

tive diffusion do not have a significant cost when compared to Sweby, as seen in Figure 4.6. Moreover, the Compressive Advection scheme with negative diffusion maintain the original water mass accurately, while the other schemes can lead to small mass loss towards the end of the simulation, when the water surface becomes flat across the domain and the flow speed is reduced.

### 4.3.2 2D sensitivity analysis of the Dynamic Mesh Algorithms and mesh to mesh interpolation

Dynamic mesh optimisation algorithms (DMO) can improve accuracy by increasing resolution in areas of interest, such as the interface for multiphase problems. The mesh can be adapted when a certain criteria is met, such as reaching a given number of time-steps or a given elapsed time. Once the mesh is adapted, the field information from the old mesh needs to be passed onto the new mesh. This interpolation between meshes can however affect the results (e.g. mass or momentum conservation)

The value of the phase or component field (discretised with CV) is very important for free surface flow problems since it resolves the phases or components and most importantly the interface. Farrell and Maddison (2011) method of a Galerkin based interpolation is considered a potential contribution for interface capture, since it can project sharp transitions between nodes.

A set of sensitivity cases with a simple collapsing water column case have been set for validating the Galerkin projection method (case ‘B’) against a fixed mesh (case ‘A’) simulation (no mesh-to-mesh interpolation errors), and a simple consistent interpolation (case ‘C’), which sets the values of a field at the nodes of the new mesh to the values at those physical positions on the old mesh.

A 2D domain of 2x2 m with a water column of 1.2 m wide and 0.6 m high is used. A fixed mesh (‘A’) is set with a grid of 0.1 m resolution. Two DMO simulations are set with 0.1 m and 0.5 m as minimum and maximum element edge length, respectively, one with a Galerkin

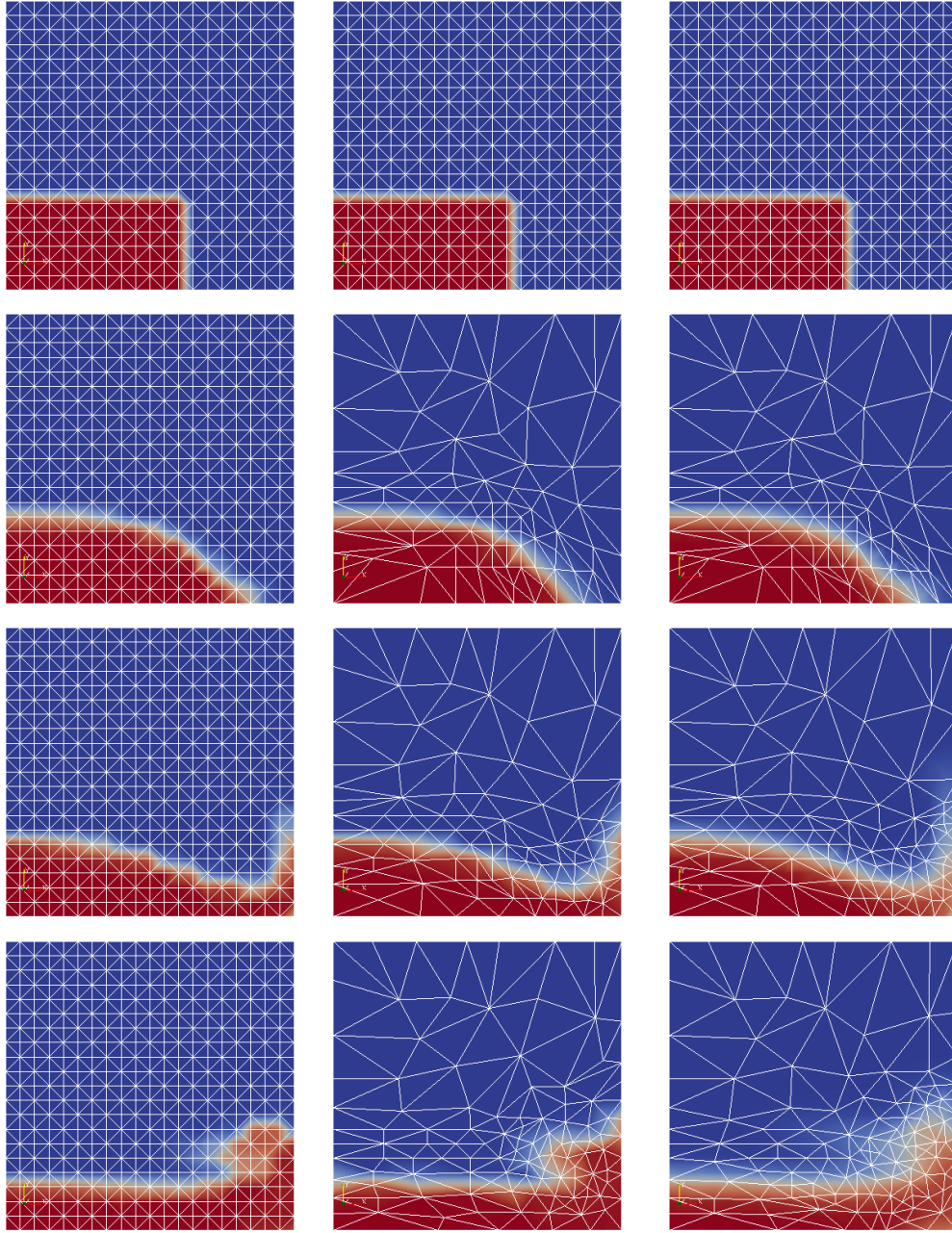


Figure 4.7: Simulation results using fixed mesh (left), dynamic mesh with Galerkin projection (centre) and consistent projection (right) for a dam break problem at  $t=0$  s (top row),  $t=0.25$  s (second row),  $t=0.5$  s (third row) and  $t=1$  s (bottom row).

projection method ('B') and another one with the consistent projection method ('C'), and mesh is adapted every 5 time-steps.

Figure 4.7 shows water column with a detailed mesh for cases 'A' (left), 'B' (centre) and 'C' (right) for instants  $t=0$  s (top row),  $t=0.25$  s (second row),  $t=0.5$  s (third row) and  $t=1$  s (bottom row). The simulation speed (top), with a computational time versus simulation time,



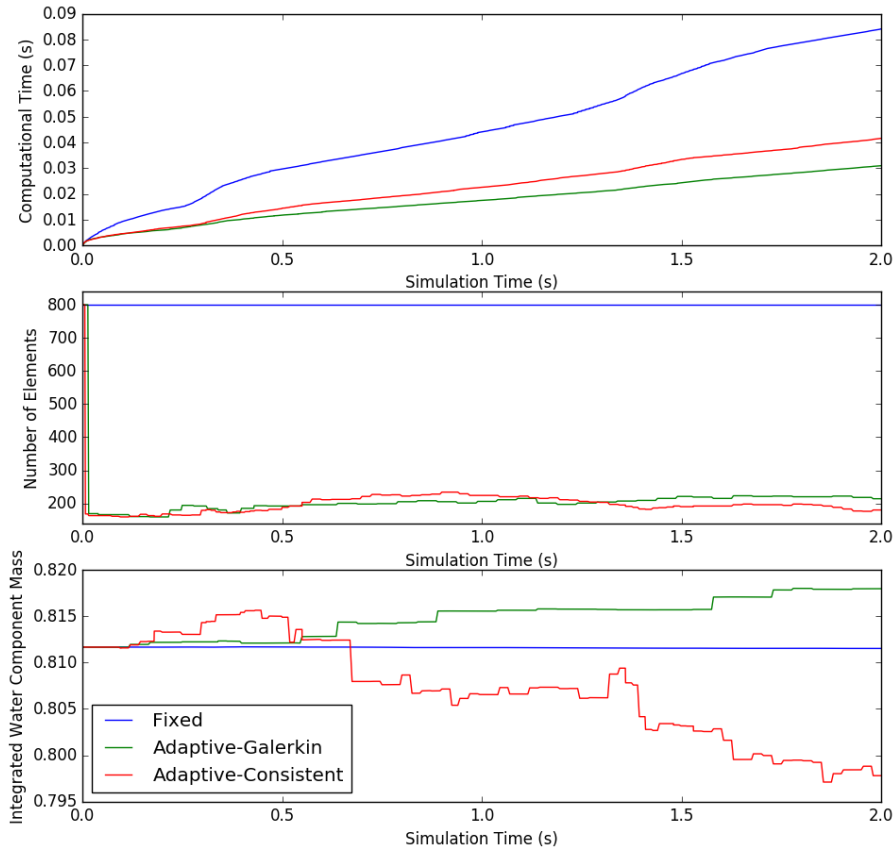


Figure 4.8: Simulation performance of fixed mesh and adaptive mesh with Galerkin and consistent projections. Computational cost (top), number of elements (centre) and Integrated water mass (bottom) evolution over simulation time.

and number of domain mesh elements through the simulation (centre) are shown in Figure 4.8, together with the integrated Water mass within the domain (bottom). Both DMO simulations ('B' and 'C') kept a high resolution on and around the interface and significantly reduced the computational cost to a half when compared to the fixed mesh simulation ('A'). The number of elements have decreased from 800 by the fixed mesh 'A' to around 200 elements by the DMO meshes 'B' and 'C'.

The Consistent projection simulation 'C' resulted to a more dissipative interface as seen in Figure 4.7 and a loss of water mass as seen in Figure 4.8. The projection Galerkin resulted to a more compressive interface (see Figure 4.7) and a very small water mass gain over the simulation (4.8), giving a more conservative result. The Consistent projection is believed to be

more expensive in this simulation due to lower convergence (more iterations needed in every time step) when compared to Galerkin projection.

### 4.3.3 2D sensitivity analysis of the $P_1DGP_1(CV)$ scheme

Compressive interface and adaptive mesh are a potential approach for reducing the computational cost of current simulations. This potential is however limited to the convergence and stability of the solution. With distorted meshes a lower convergence is achieved and computational cost can increase significantly.

The new  $P_1DGP_1(CV)$  (see Section 4.2.2) is tested here for tackling mesh related low convergence and compared to the standard  $P_1DGP_1$ . Two different fixed meshes are considered: an isotropic mesh with 1 in 1 element side ratio (mesh ‘A’), and a highly anisotropic mesh with a 1 in 25 aspect ratio (mesh ‘B’).

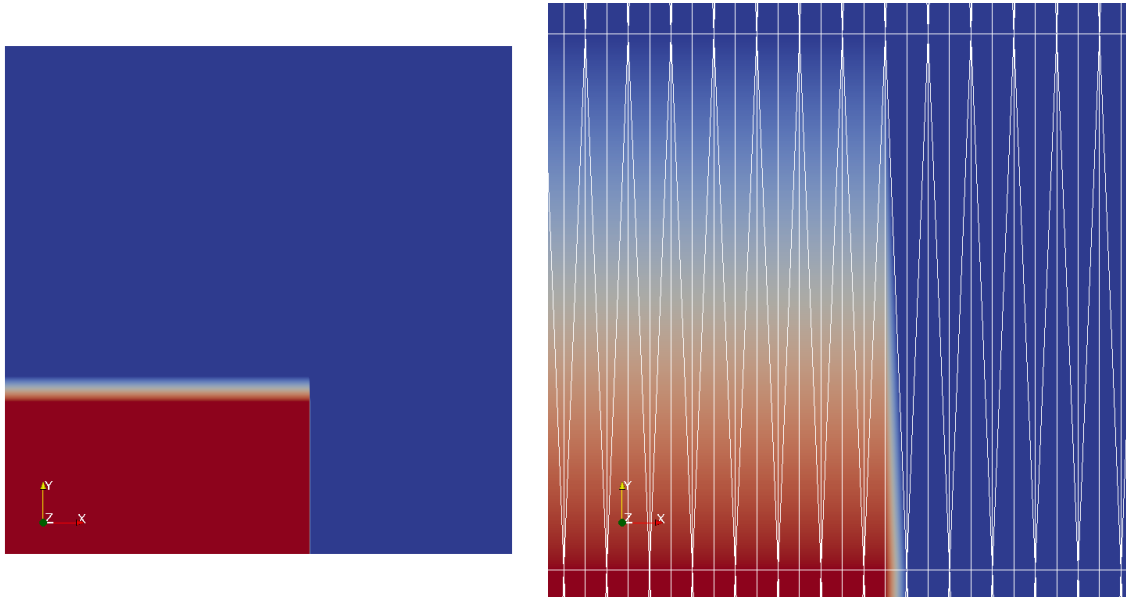


Figure 4.9: Water (red) and air (blue) in the domain with its initial imposed high anisotropic mesh on the interface (left), and a detail of the mesh imposed high anisotropy at the top right corner of the water column (right).

The domain is a 2 m by 2 m water tank with a water column of 1 m width and 0.6 m high. Mesh ‘A’ resolution is set to 0.02 m (x) and 0.02 m (y). Mesh ‘B’ resolution is set to 0.004 m (x) and 0.100 m (y). Figure 4.9 shows the resulting domain (left) and the detail of the top

corner of the water column (right), at  $t=0$ . Only 3 non linear iterations (for details on pressure projection method, see Section 4.2.5) are used and a top limit of 500 iterations for the Pressure field solver is set.

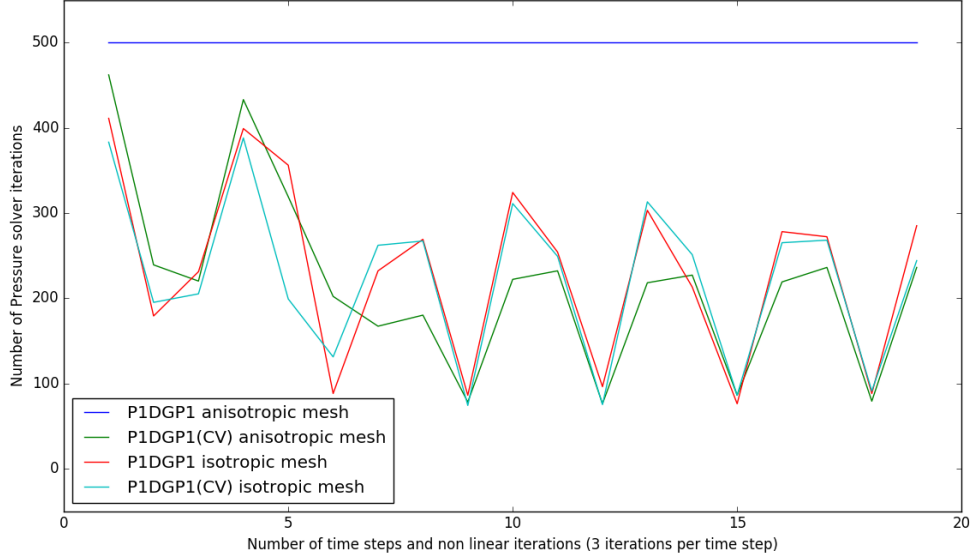


Figure 4.10: Convergence of the Pressure field solver for the new  $P_1DGP_1(CV)$  and standard  $P_1DGP_1$  schemes for isotropic and highly anisotropic meshes over time. The vertical axis shows the number of iterations required to achieve convergence at each time step.

Convergence performance of simulations with  $P_1DGP_1(CV)$  and  $P_1DGP_1$  schemes on meshes ‘A’ and ‘B’ are shown in Figure 4.10. Both schemes solve the pressure field with very similar performance for the isotropic mesh ‘A’. For the anisotropic mesh ‘B’ case the newly proposed  $P_1DGP_1(CV)$  can solve the pressure field for the mesh ‘B’ as well as for mesh ‘A’. However, the standard  $P_1DGP_1$  scheme reaches the maximum number of iterations for the pressure field solver with mesh ‘B’, proving this mesh to be too challenging for the standard scheme. This shows the potential of  $P_1DGP_1(CV)$  and its robustness against challenging meshes that the standard  $P_1DGP_1$  fails to solve.

The computational cost between  $P_1DGP_1(CV)$  and  $P_1DGP_1$  schemes remain similar in longer simulations with the isotropic mesh ‘A’, with a small cost reduction by the  $P_1DGP_1(CV)$  scheme, as seen in Figure 4.11. Maximum pressure is also shown with a slightly smaller peak by the  $P_1DGP_1(CV)$  scheme. This slightly lower pressure maximum affects the velocity field, although at a much smaller rate. The lower maximum pressure peak by the  $P_1DGP_1(CV)$

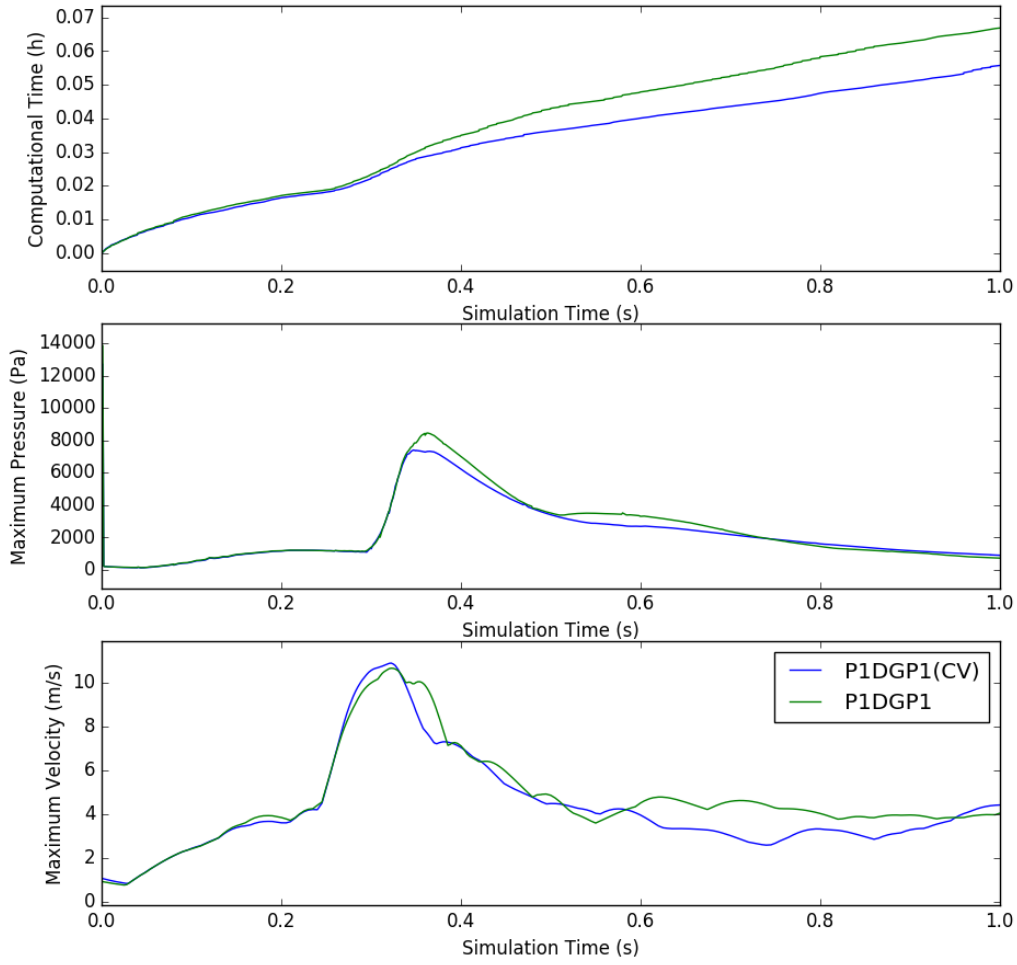


Figure 4.11: Comparison of computational performance with computational cost (top), Pressure (centre) and Velocity (bottom) field maximums, for  $P_1DGP_1(CV)$  (blue) and  $P_1DGP_1$  (green), over simulation time.

is assumed to be induced by the lower discretisation scheme when compared to the standard  $P_1DGP_1$ . The reduction of computational cost is however higher in comparison to the loss at the pressure peak, making  $P_1DGP_1(CV)$  very attractive when stability or computational cost are critical. Moreover, the slight to insignificant pressure accuracy reduction can be improved with a variation of mesh resolution or the time-step size, although this could increase the computational cost. The change of mesh resolution and time-step size are discussed in depth in Section 4.3.4.

Figure 4.12 shows the simulation result snapshots of  $P_1DGP_1(CV)$  and  $P_1DGP_1$  schemes,

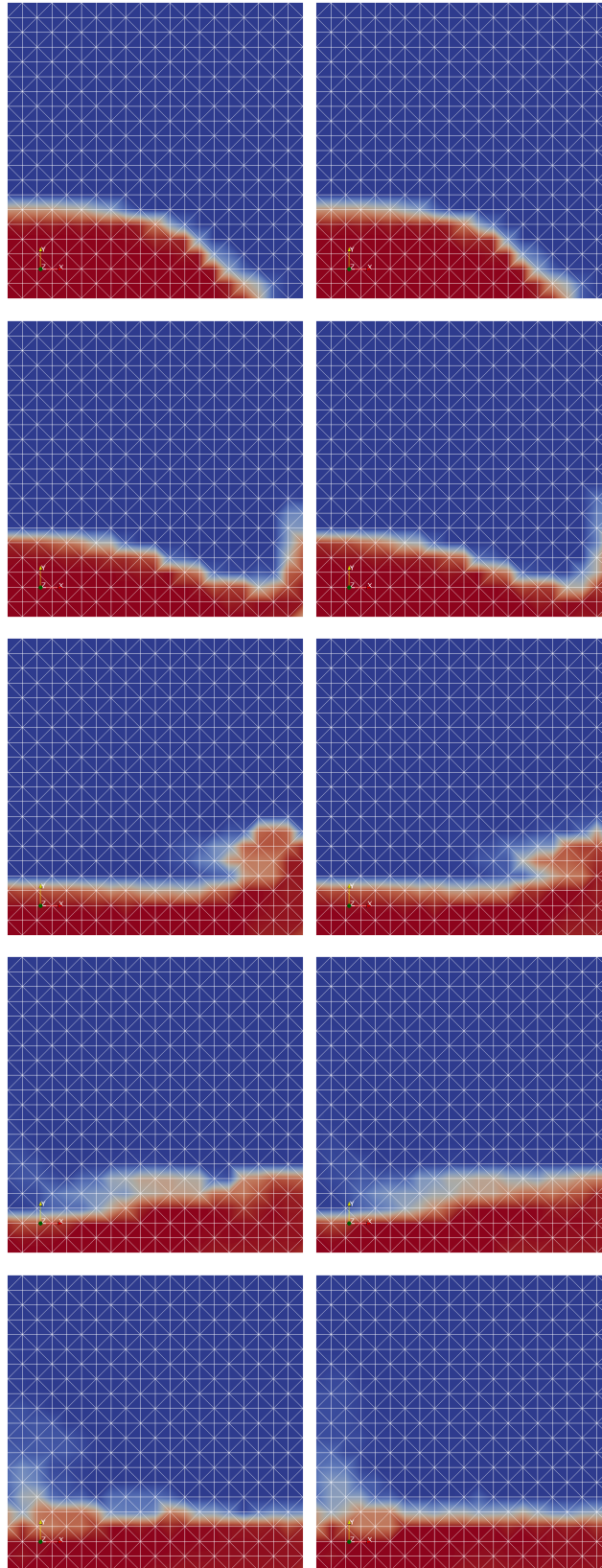


Figure 4.12: Visualisation of the collapsing water column simulation results using  $P_1DGP_1(CV)$  (left) and  $P_1DGP_1$  (right), with snapshots at  $t=0.25$  s (top),  $t=0.5$  s,  $t=1$  s,  $t=1.5$  s and  $t=2$  s (bottom)

showing small to no differences between them at the different stages of the simulation.

#### 4.3.4 2D benchmark case of a Collapsing Water Column

A 2D collapsing water column benchmark case by Colagrossi and Landrini (2003) is used here for mesh and time-step sensitivity analysis. The main purpose is to identify the numerical energy dissipation by different resolutions and time-steps, and compare the results to the experiments.

The water tank domain is 3.22 m (x) by 2 m (y). The water column is 1.2 m (x) by 0.6 m (y), located at the left end of the domain. Water measurements are available at 2.725 m (H1) and 2.220 m (H2) from the left boundary. Pressure measurements are taken at 0.16 m height at the right wall of the water tank (P2).

Simulations are set with different mesh resolutions with minimum element lengths of 0.05 m (case ‘A1’), 0.010 m (case ‘A2’) and 0.005 m (case ‘A3’). The minimum-maximum element length ratio is set to 20, this is, the maximum element length is set to 20 times the minimum element length for every case. Simulations for the adaptive time-step analysis use CFL=2 (case ‘B1’), CFL=0.5 (case ‘B2’) and CFL=0.1 (case ‘B3’). For the mesh sensitivity analysis an adaptive time-step for CFL=0.25 is chosen. For the CFL sensitivity analysis a resolution of 0.01 m is selected. A Large Eddy Simulation (LES) model is used here to tackle high turbulence flow in all cases.

Figure 4.13 shows results of water level at H1 and H2, and pressure at P2, for the different mesh resolutions against experiments. Differences can be observed in H1 and H2 at the peaks at 0.5 s and 0.7 s. The simulation ‘A1’ (lower resolution) shows a later arrival of the water front and a rapid increase of the water level at both H1 and H2 gauges when compared to ‘A2’ and ‘A3’. This is assumed to be due to the combination of the compressive interface with longer elements, making the water front higher (steep increase), and a higher energy dissipation by long elements (late arrival). Towards the end of the simulation a higher variation with smaller peaks is observed for the simulation ‘A3’. These peaks are the result of local turbulence and water drops or small waves appearing on the water surface, which cannot be resolved with lower

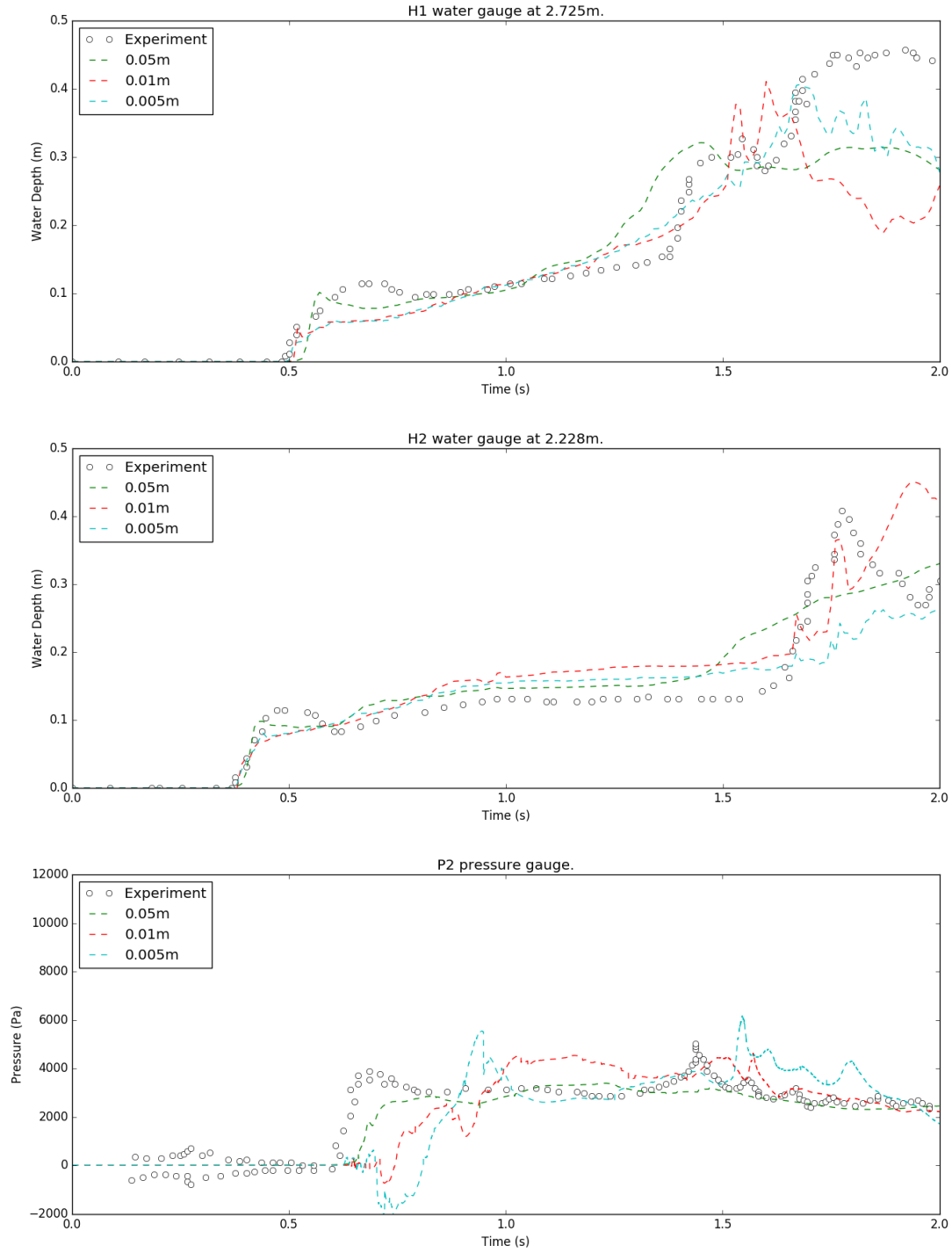


Figure 4.13: Water levels at H1 (top) and H2 (centre) and pressure measurements (P2, bottom) for the mesh sensitivity analysis, for mesh finest resolutions with 0.050 m (green), 0.010 m (red) and 0.005 m (blue), against experiments (black).

resolution (cases ‘A1’ and ‘A2’).

Mesh size has a bigger impact on the pressure results at P2 (See Figure 4.13c). Simulations ‘A2’ and ‘A3’ fail to resolve the first pressure peak at 0.6 s and show a pressure decline just

after the initial fluctuation with the water front reaching the wall. The simulation ‘A1’ shows the best fit to the experiments. Element lengths here and the water front height has therefore an impact on the pressure results at P2. Lower water front (‘A2’ and ‘A3’) impacting just below the pressure sensor reduces the pressure peak at P2, seen in Figure 4.13c, while ‘A1’ water front impacts higher and reaches directly P2, in this case at a later time due to energy dissipation. ‘A3’ case with higher resolution shows for P2 the same high variability as seen and discussed for the water level results.

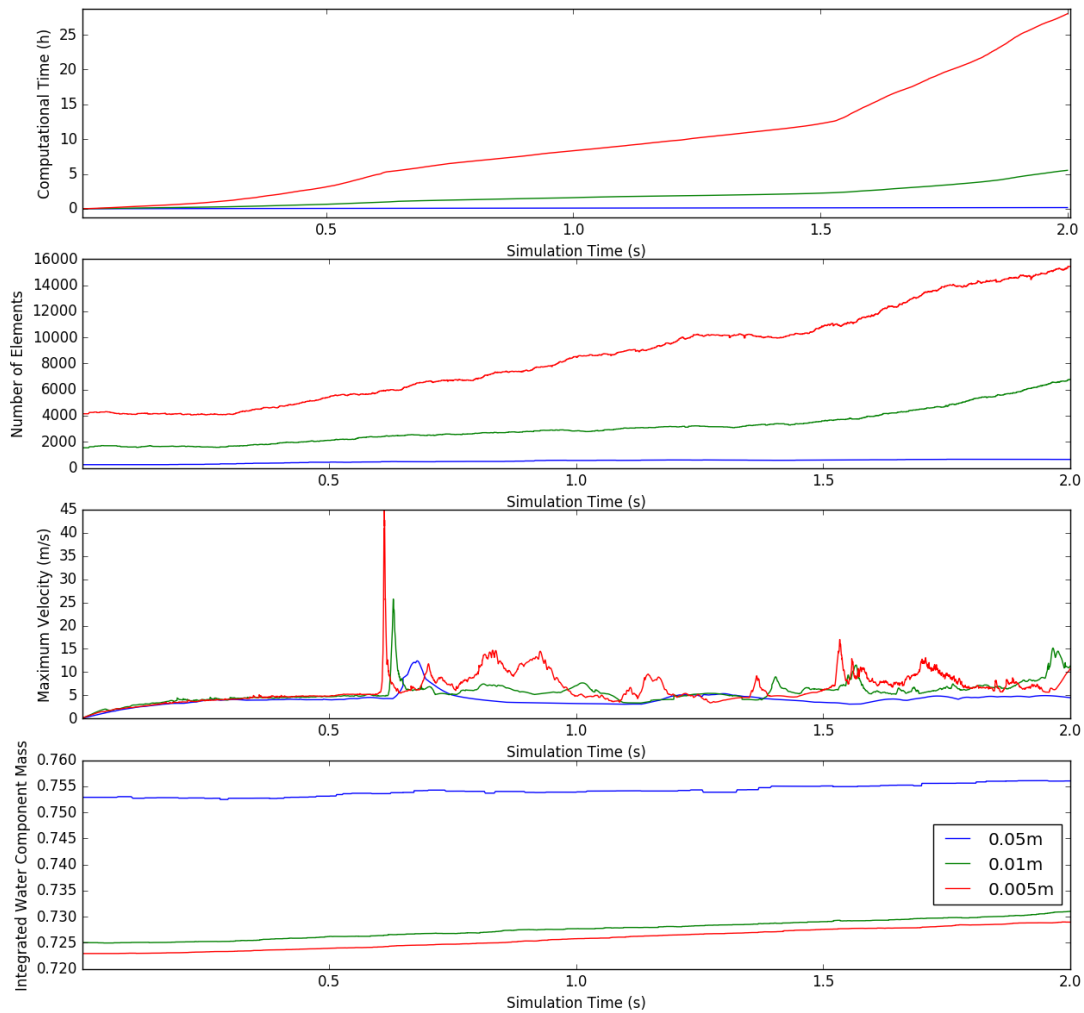


Figure 4.14: Performance of simulations with mesh resolution of 0.050 m (blue), 0.010 m (green) and 0.005 m (red). Evolution of the computational cost (top), the number of elements (second from top), the maximum velocity field value in the domain (second from bottom) and the integrated water mass (bottom) over the simulation time.



Air trapped in the simulation at the bottom right corner at the moment of impact can also affect significantly the pressure results at P2. This air trapped is likely to have lower impact in real 3D simulations, where air can escape easily.

Figure 4.14 shows the performance of simulations ‘A1’, ‘A2’ and ‘A3’ in terms of computational cost (top), number of elements, maximum velocity and water mass (bottom). While ‘A1’ show very low number of elements (1000) and computational cost (1 hour), it comes at an energy dissipation cost, with lower maximum velocity peak with around 0.1 s delay. Simulation ‘A2’ uses up to 7000 elements but remains at around 5 h computational time. Simulation ‘A3’ leads to a maximum of around 15000 elements (double of ‘A2’), over 25 hours of computational time (5 times longer than ‘A2’), but only a maximum velocity peak of 45 m/s (25 m/s for ‘A2’). Water mass shows differences at the initial time-step due to using initial meshes with different element length, but differences throughout the simulation remain insignificant for every mesh case. The computational performance analysis suggests that the improvement of global accuracy with higher resolution can lead to a significant increase in computational cost, and perhaps this is not translated to local accuracy improvement, as seen in Figure 4.13. Good judgement is therefore required when choosing the mesh resolution.

Figure 4.15 shows results of water level at H1 and H2, and pressure at P2 for adaptive time-step simulations with CFL=2 (‘B1’), CFL=0.5 (‘B2’) and CFL=0.1 (‘B3’) values against experiments. Very similar trends to the mesh sensitivity analysis can be observed, with higher dissipation for CFL=2 (‘B1’) as for ‘A1’ (lower resolution) and lower dissipation for ‘B3’ as for ‘A3’. ‘B2’ and ‘B3’ also show higher variability due to resolving with more time-steps the local physics and turbulence. For P2, however, ‘B1’ show worse agreement with experiments, which is assumed to be due to shorter elements when compared to ‘A1’. Similar time delay for the first peak is observed for ‘B1’ at H1, H2 and P2 when compared to experiments ‘B2’ and ‘B3’.

Performance of simulations ‘B1’, ‘B2’ and ‘B3’ are shown in Figure 4.16. Computational cost for ‘B1’ is around 1 h, for ‘B2’ between 3 and 4 h, and for ‘B3’ increases to 19 h. Velocity field shows however maximum peaks of the same order of magnitude in all three CFL cases. An increase of water mass is observed in all cases, having CFL=0.1 the largest increase, which is

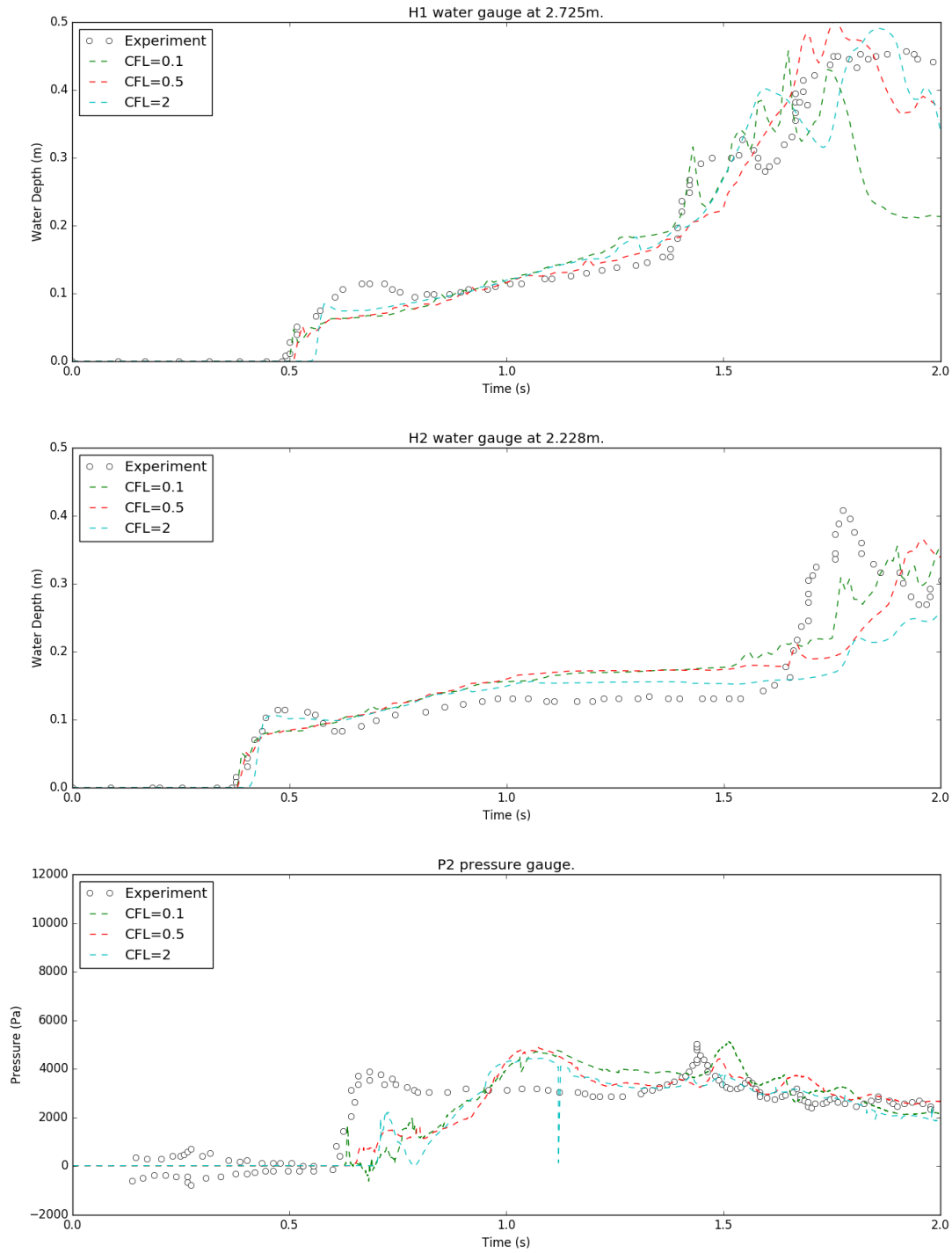


Figure 4.15: Water levels at H1 (top) and H2 (centre) and pressure measurements (P2, bottom) for the time-step sensitivity analysis, with adaptive time-step using CFL=2 (blue), CFL=0.5 (red) and CFL=0.1 (green), against experiments.

still below 1% at the end of the simulation.

Results of mesh and time-step sensitivity analysis show generally minor changes globally and more significant differences locally. Energy dissipation is larger with longer time-steps (higher

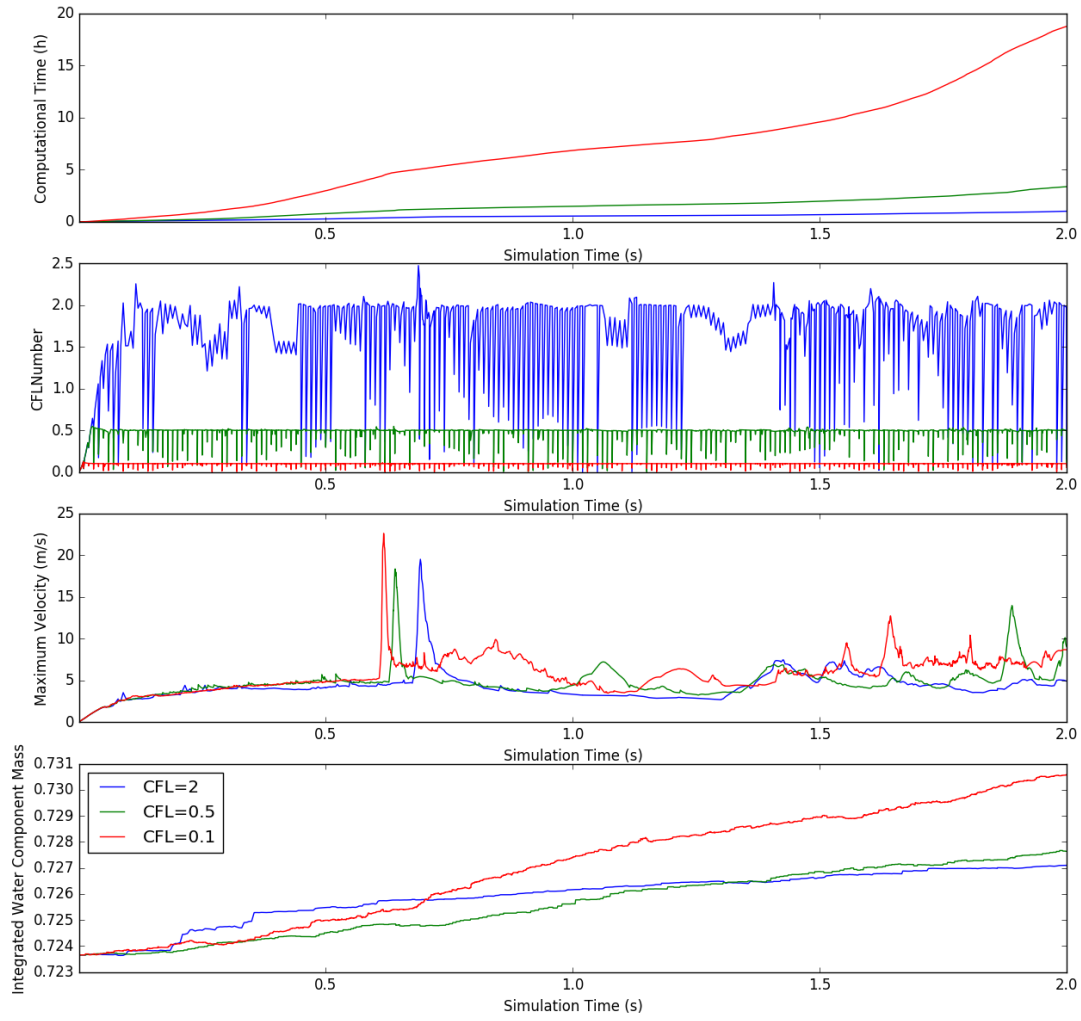


Figure 4.16: Performance of simulations with adaptive time-step using CFL=2 (blue), CFL=0.5 (green) and CFL=0.1 (red). Evolution of the computational cost (top), the CFL maximum value in the domain (second from top), the maximum velocity field value in the domain (second from bottom) and the integrated water mass (bottom) over the simulation time.

CFL) and with larger elements, but computational costs are reduced significantly when these more dissipative options are used. This highlights the need to choose time-step and mesh resolution wisely.

### 4.3.5 3D benchmark case of a Collapsing Water Column with an obstacle

The 3D experiments of Kleefsman et al. (2005) have been chosen for free surface inertia dominated flows validation due to the presence of a rectangular obstacle in the collapsing water column path and several available pressure measurements from sensors on the obstacle's front and top faces.

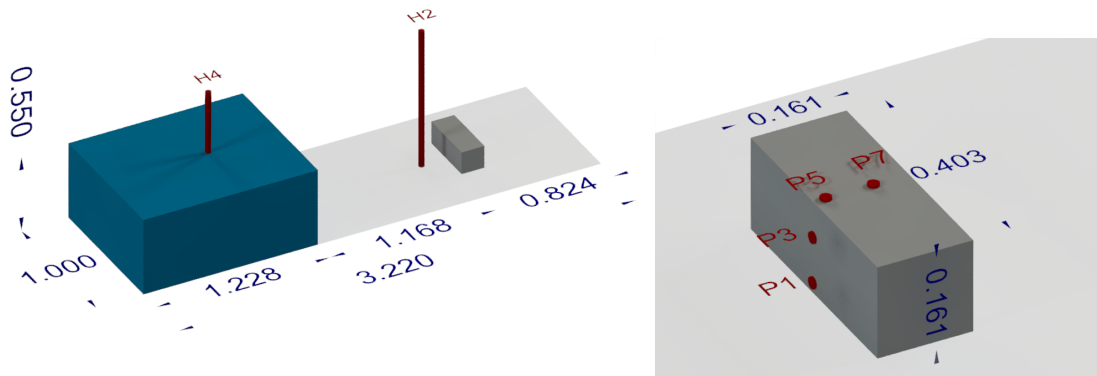


Figure 4.17: a) General setup of the experiments and numerical model domain, with two probes (H2 and H4) measuring water level time series; b) Pressure sensors on the front (P1, P3) and top (P5, P7) faces of the obstacle. (Kleefsman et al., 2005; Via-Estrem et al., 2020)

| Component | Density ( $kg/m^3$ ) | Viscosity ( $kg/(ms)$ ) |
|-----------|----------------------|-------------------------|
| Water     | 1000                 | $9.892 \times 10^{-4}$  |
| Air       | 1.125                | $1.81 \times 10^{-5}$   |

Table 4.2: Densities and viscosities for air and water used in the simulation of the experiments by Kleefsman et al. (2005).

The experiment layout and setup is shown in Figure 4.17. Pressure sensors at the front and top faces of the obstacle are shown in Figure 4.17b. Water level gauges are located at 0.72 m from the tank wall (on the water column), and 1.00 m from the front of the water column. Viscosities and densities of air and water are shown in Table 4.2.

A mesh sensitivity analysis is carried out for finding a sensible resolution that keeps a balance between computational cost and results accuracy. Three maximum resolutions of 0.01 m, 0.02 m and 0.05 m are tested with adaptive time-step for CFL=1. Figure 4.18 shows the performance of the simulations with the different resolutions. Figures 4.19 and 4.20 show the pressure

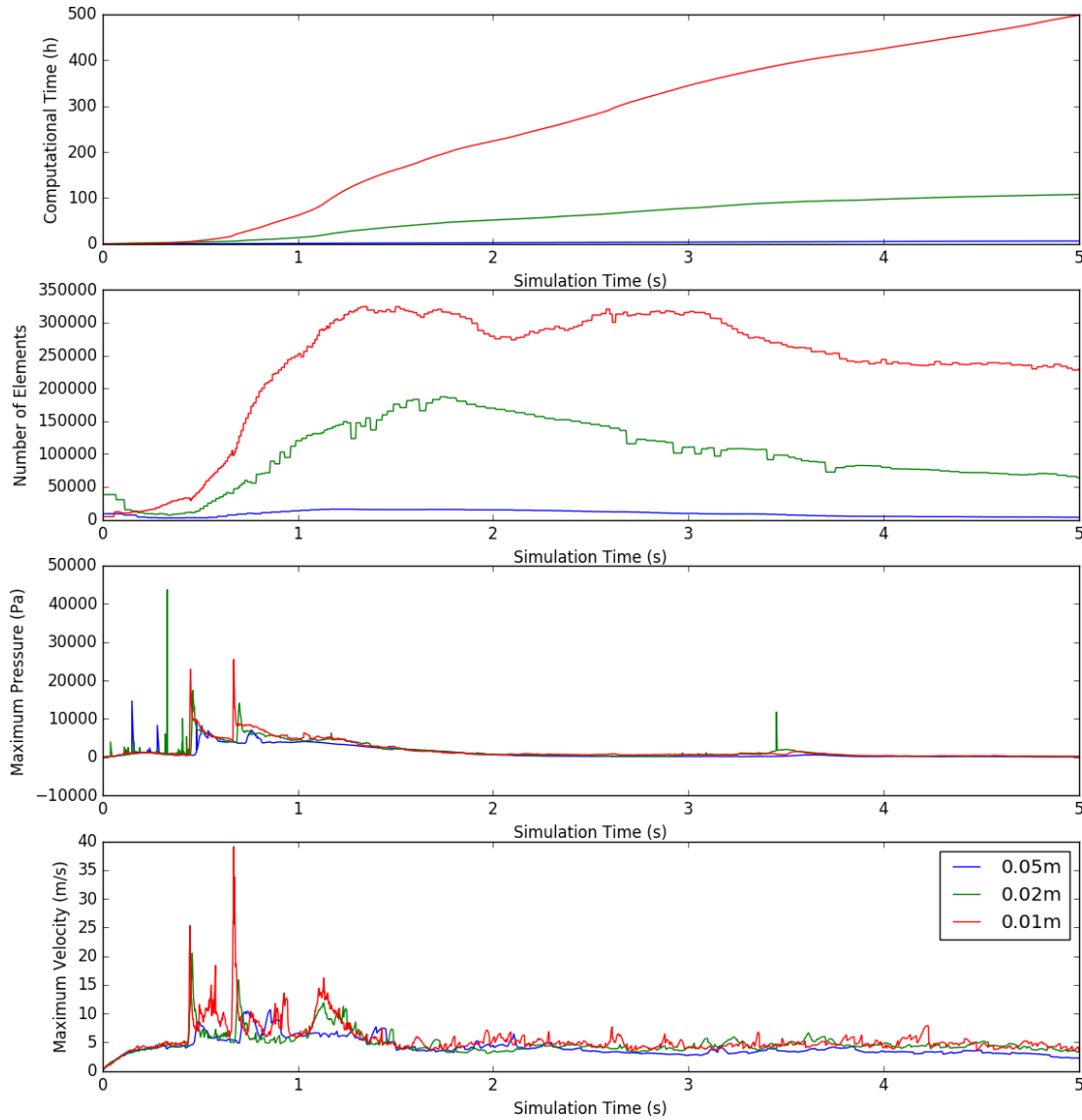


Figure 4.18: Computational performance comparison for mesh sensitivity analysis with meshes with finest resolutions of 0.01 m (red), 0.02 m (green) and 0.05 m (blue). Evolution of the computational cost (top), the number of elements (second from top), the maximum pressure (second from bottom) and maximum velocity (bottom) field values in the domain, over the simulation time.

measurements at the front face (P1 and P3) and water level measurements at H2 and H4, respectively. Results show good agreement between 0.02 m and 0.01 m for P1 sensor and a much lower peak for 0.05 m, indicating high dissipation and lower accuracy for the water front impact on the obstacle. Pressure readings at P3 show similar initial peak for 0.01 m and 0.02

m, but 0.02 m continues with a higher variability after the peak. This is also seen for H4. Water levels at H2 remain slightly higher for 0.01 m when compared 0.02 m.

Although running with a high resolution of 0.01 m is five times more expensive than with a resolution of 0.02 m (see Figure 4.18), the highest resolution (0.01 m) is considered here the most appropriate to proceed with due to overall better accuracy in all pressure sensors and water level gauges. Moreover, the simulation with 0.01 m is expected to be more capable to capture air trapping in the fluid and water drops in the air as seen later in this chapter.

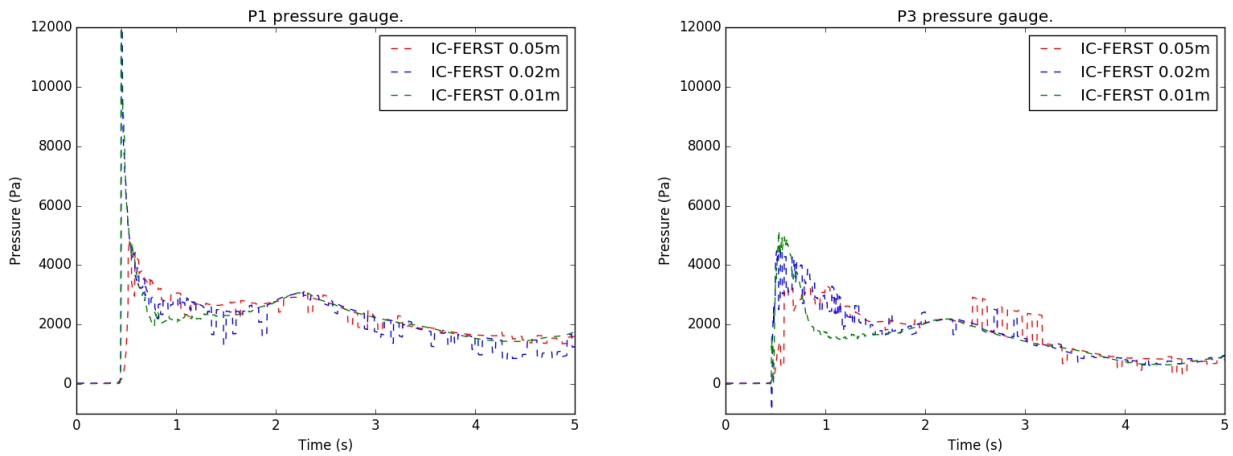


Figure 4.19: Comparison of simulation results for the mesh sensitivity analysis using meshes with finest resolutions of 0.01 m (green), 0.02 m (blue) and 0.05 m (red) at pressure sensors at the front face of the obstacle P1 (left) and P3 (right).

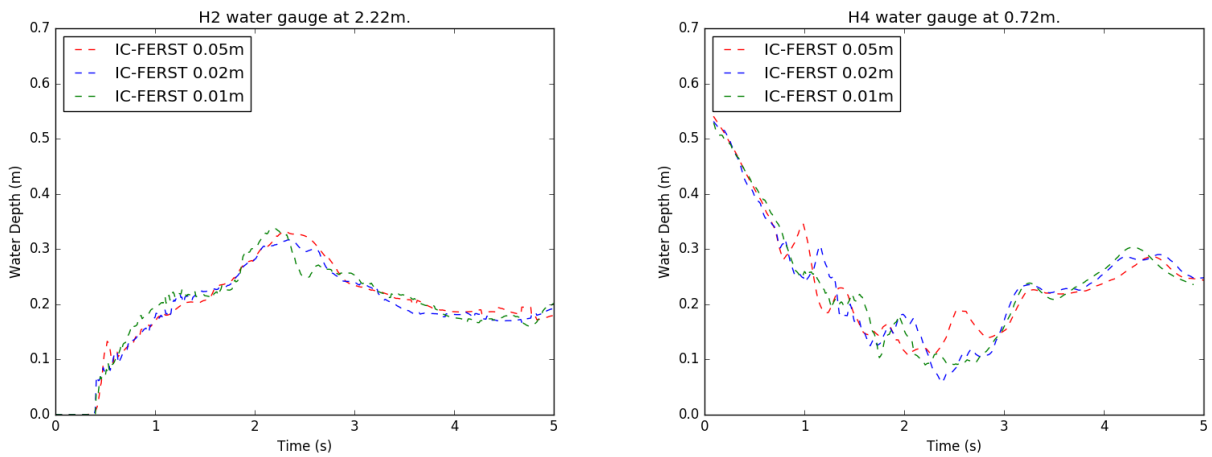


Figure 4.20: Comparison of simulation results for the mesh sensitivity analysis using meshes with finest resolutions of 0.01 m (green), 0.02 m (blue) and 0.05 m (red) at water level gauges H2 (left) and H4 (right).

The numerical mesh with a 0.01 m maximum resolution is chosen and adapted by the mesh

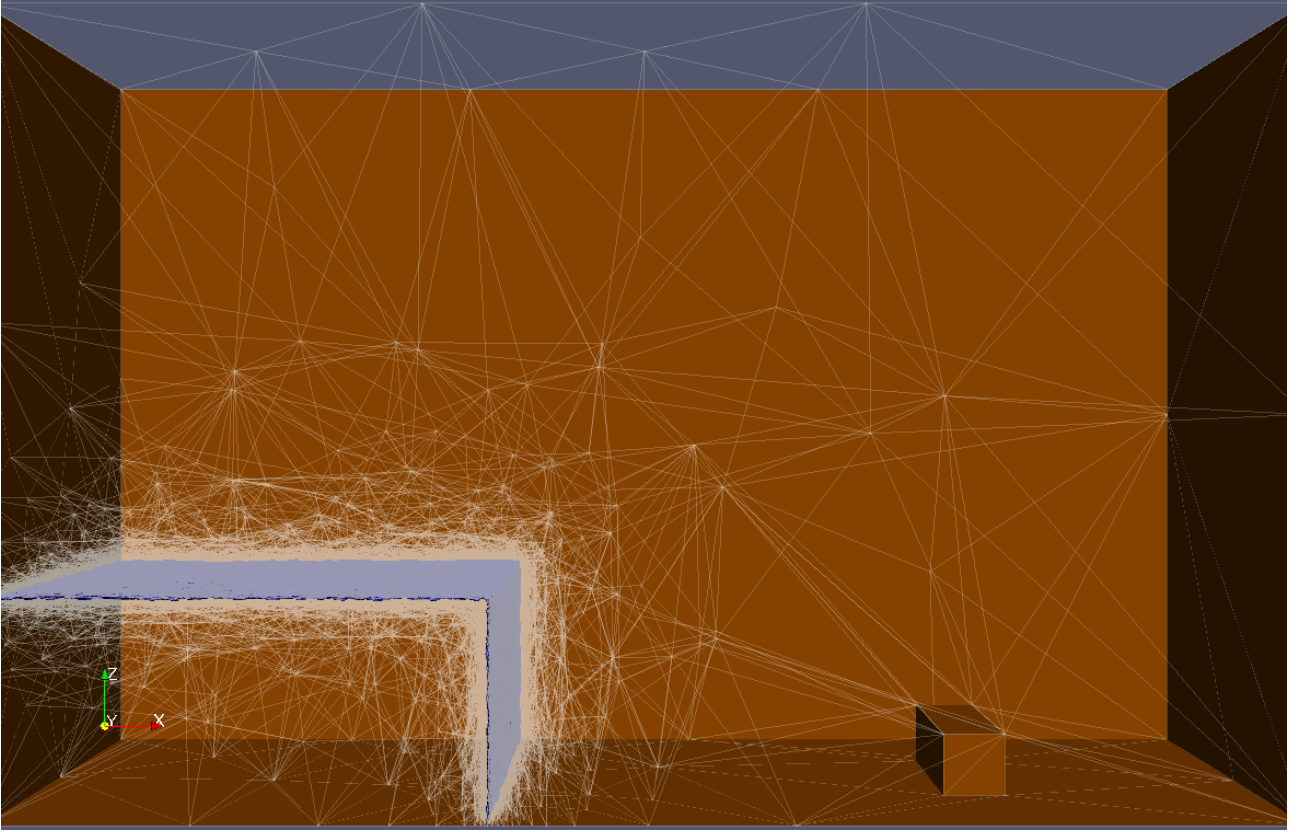


Figure 4.21: Numerical domain of the experiment with the initial mesh at the start of the simulation ( $t=0$  s). Mesh elements represented in grey. Element resolution increase is observed around the water surface (blue) of the collapsing water column (bottom left). (Via-Estrem et al., 2020).

adaptivity algorithm for water component volume fraction with maximum and minimum element sizes of 1.00 m and 0.01 m, respectively, at  $t=0$  s. Figure 4.21 shows the adapted mesh at time  $t=0$  s in preparation for highly resolved interface tracking with the previously described compressive interface scheme and a highly anisotropic mesh with a maximum aspect ratio of 100. Table 4.3 shows the mesh quality in terms of element angles at  $t=0$  s, where up to 8% of elements present a very large angle (usually associated with lower quality mesh but with no negative impacts in this approach). Simulations are run with an adaptive time-step scheme

| Angle  | N. elements | Percentage |
|--------|-------------|------------|
| 120.00 | 3377        | 72.59      |
| 150.00 | 2027        | 43.57      |
| 170.00 | 930         | 19.99      |
| 175.00 | 369         | 7.93       |

Table 4.3: Mesh quality quantitative analysis: number of elements within the given angle thresholds in the domain by maximum element angles.

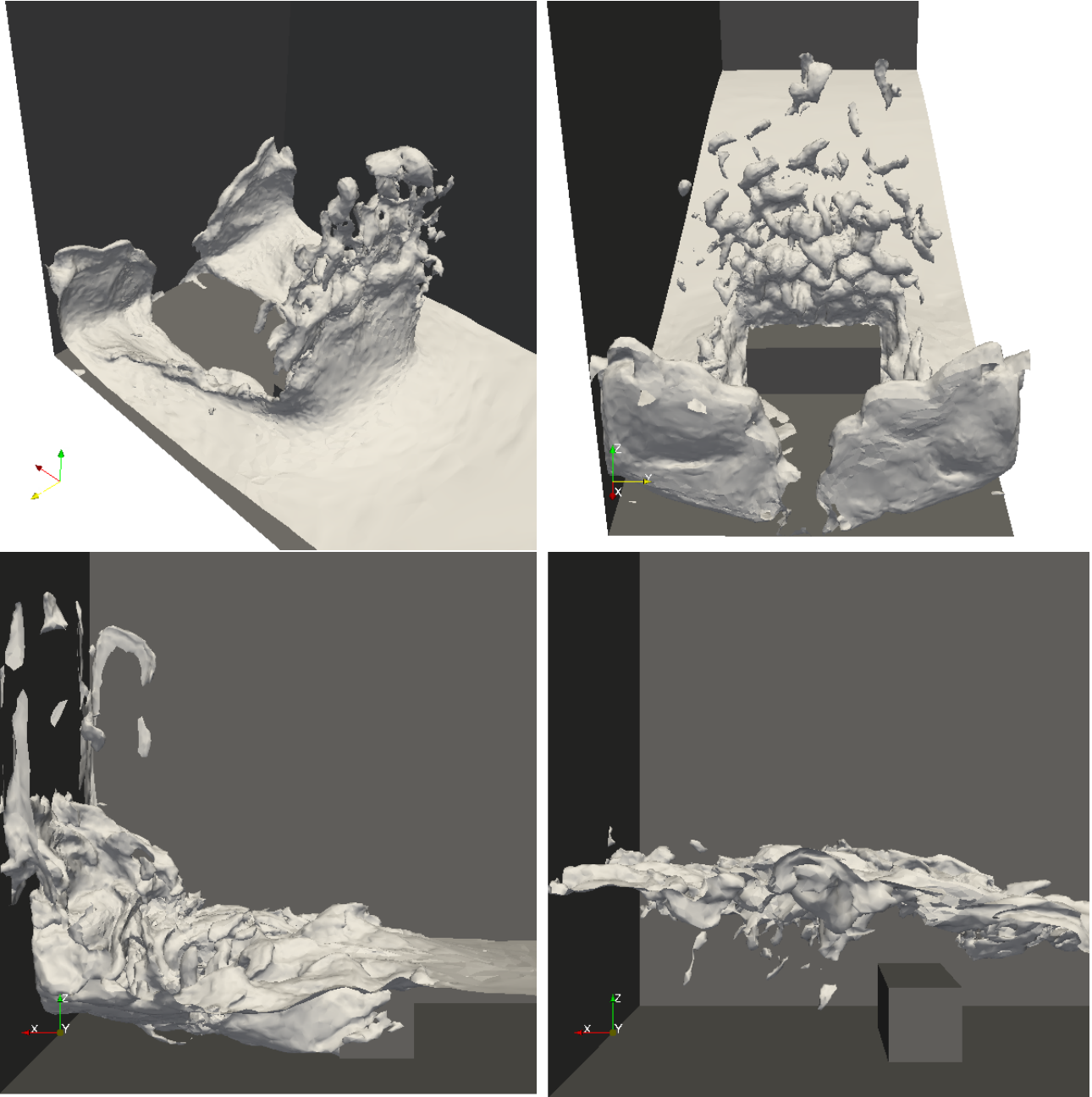


Figure 4.22: Detail of air-water trapping: water drops at 0.75 s (top left) and 0.80 s (top right) and air trapped in water at 1.21 s (bottom left) and 2.07 (bottom right) (Via-Estrem et al., 2020).

with CFL numbers of 0.1 and 1 for the velocity field in order to provide a full range of time-step sizes and their impact on results. The simulation is run for 5 s from the dam break.

Figure 4.22 shows snapshots with water drops at 0.75 s (top left) and 0.80 s (top right) and air trapped in water at 1.21 s (bottom left) and 2.07 (bottom right). These results show the



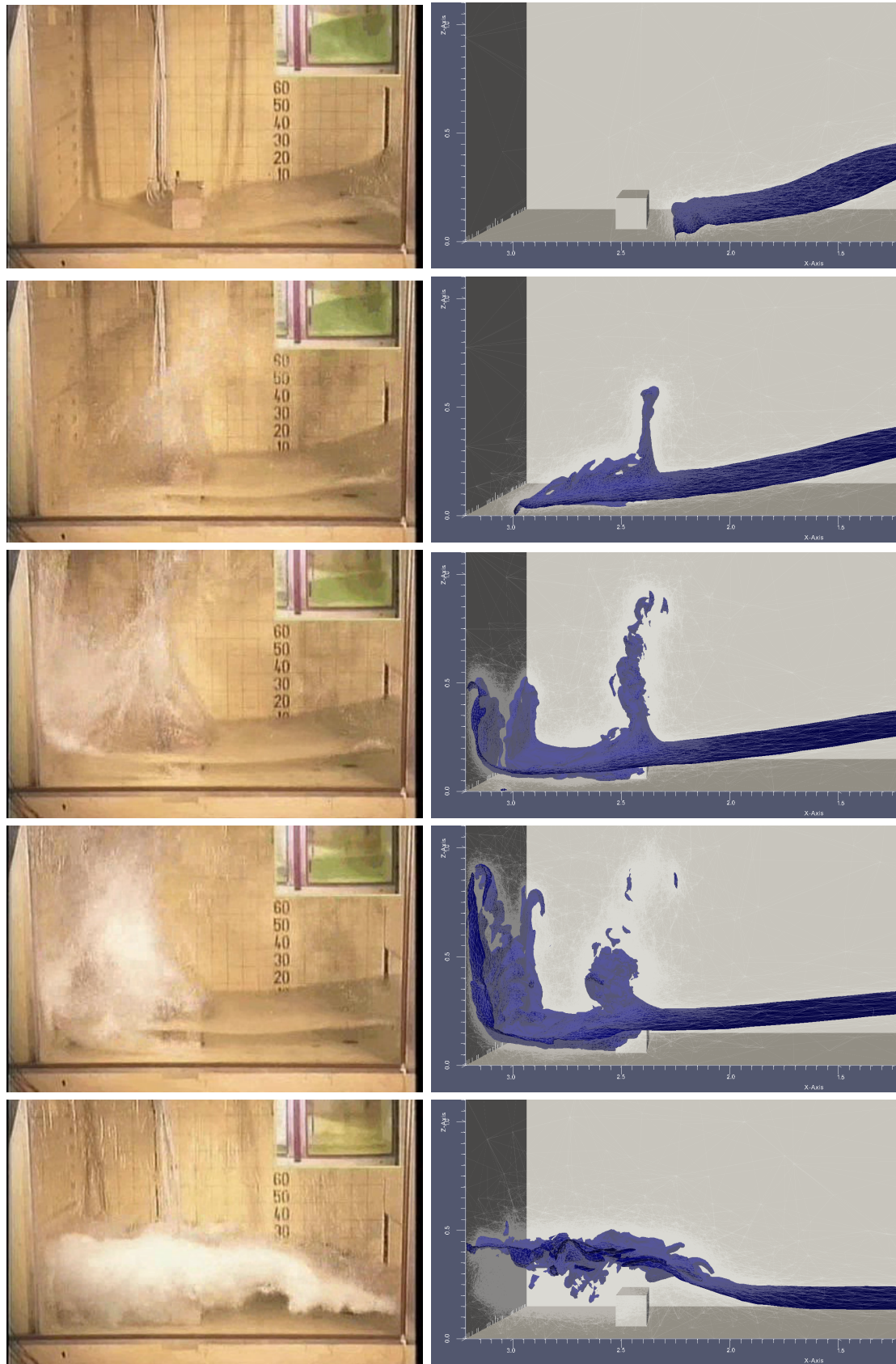


Figure 4.23: Water-air interface (0.5 water component isosurface) from numerical model results (right) compared with physical model experiments (left) (Kleefsman et al., 2005; Via-Estrem et al., 2020) at same times

potential of resolving complex two-phase flow with droplets and trapped air bubbles, which can be significantly improved by adding surface tension schemes (Xie et al., 2016, 2017a). A complete sequence of snapshots of experiments and numerical model results with CFL of 0.1 are presented in Figure 4.23.

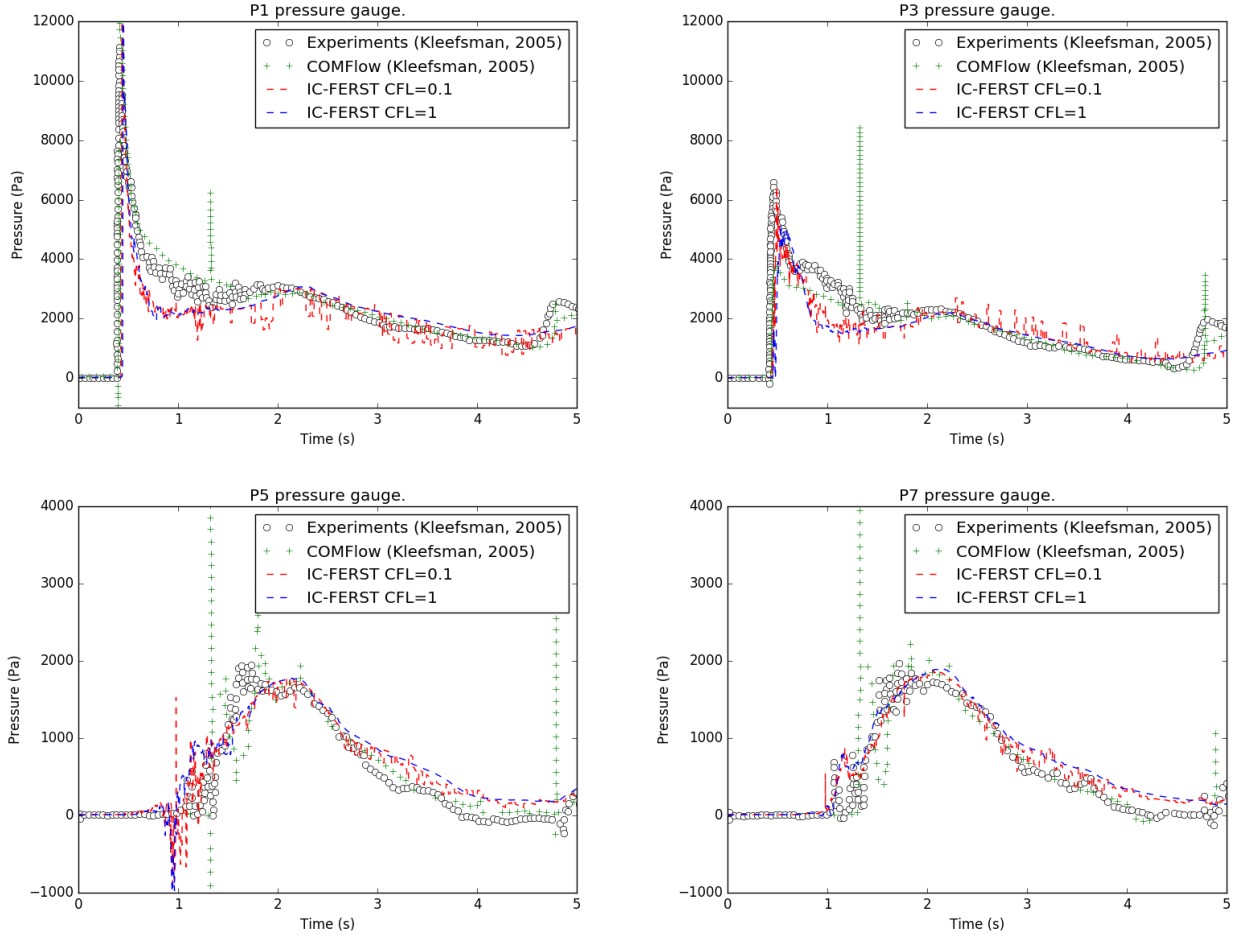


Figure 4.24: Pressure comparison at the obstacle front face sensors P1 (top left) and P3 (top right) and top face sensors P5 (bottom left) and P7 (bottom right). Numerical model results using CFL of 0.1 (red) and 1 (blue) against physical model experiments (black circles) and COMFlow (green) (Kleefsman et al., 2005; Via-Estrem et al., 2020).

Pressure measurements at the obstacle front and top faces are shown in Figure 4.24. A detail of the maximum pressure peak is shown in Figure 4.25. Figure 4.26 shows the water level measurements from experiments and numerical simulations. Experiments and COMFlow results, which is a numerical method based on Finite Volumes Method (FVM) presented in the original paper by Kleefsman et al. (2005), are also shown for comparison. Good agreement is observed between the  $P_1 DGP_1(CV)$  results and experiments on peak pressures at the bottom

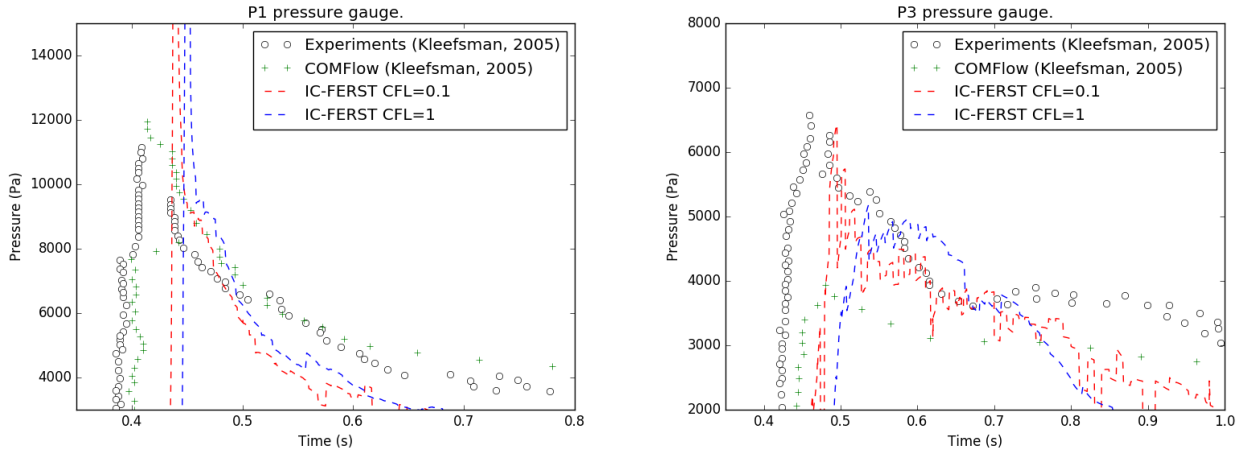


Figure 4.25: Peak pressure comparison at the obstacle front face sensors P1 (left) and P3 (right). Numerical model results using CFL of 0.1 (red) and 1 (blue) against physical model experiments (black circles) and COMFlow (green) (Kleefsman et al., 2005; Via-Estrem et al., 2020).

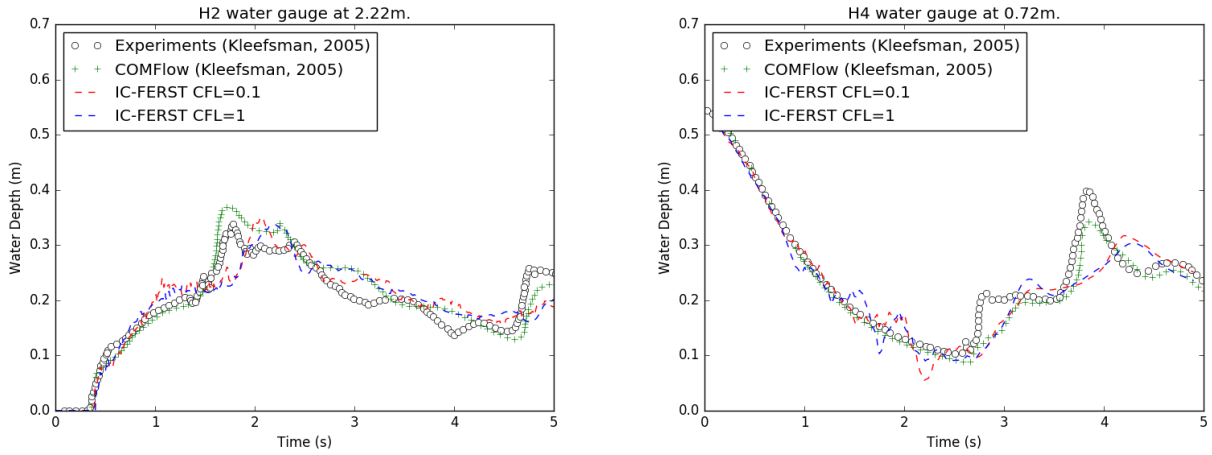


Figure 4.26: Water level comparison at water level gauges H2 (left) and H4 (right). Numerical model results using CFL of 0.1 (red) and 1 (blue) against physical model experiments (black circles) and COMFlow (green) (Kleefsman et al., 2005; Via-Estrem et al., 2020).

of the front face (P1) for both 0.1 and 1 CFL cases. The peak pressure from the model with the presented formulation is expected to be higher (25 kPa in this case) than in the experiment due to pressure sensor measuring at lower frequency than the simulation time-stepping, as seen in 4.25.

Results of pressure at the upper half of the front face (P3) show very good agreement for CFL=0.1 and a slightly smaller peak pressure for CFL=1, compared to the experiments. The author believe this is induced by a higher energy dissipation due to the longer time-step. In

both cases the presented method shows better agreement in the comparison of simulation results. Pressures measured at P5 and P7 (top face) of the obstacle show very good agreement for all cases using  $P_1DGP_1(CV)$ , with the different pressure spikes assumed to be related to water drops falling and turbulent flow on the surface of the obstacle before this is completely submerged. Good agreement with experiments is also found on water level measurements (Figure 4.26). Small magnitude spikes are observed in pressure plots (for  $CFL=0.1$ ), but these are very low level fluctuations when compared to other good models such as COMFlow.

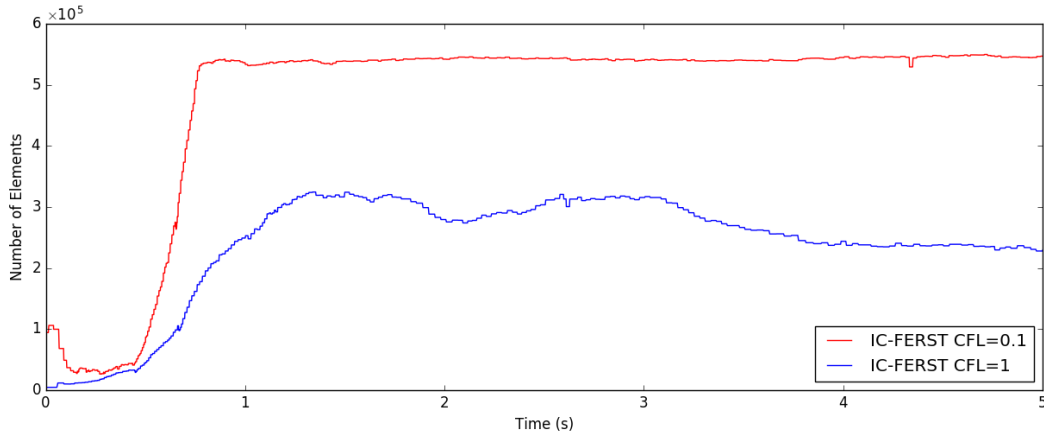


Figure 4.27: Evolution of the number of elements in the mesh for simulations using  $CFL=1$  (blue) and  $CFL=0.1$  (red) over the simulation time. The number of elements upper limit in the mesh set to  $5 \times 10^5$  is reached in the simulation using  $CFL=0.1$ .

Numerical dissipation over time is observed in Figure 4.26 as a delay on H2 (a) water level peak at 1.8 s, and as a smoothing effect on peaks at 4.8 s (H2), and 2.8 s and 3.8 s (H4). Numerical energy dissipation can be expected for  $P_1DGP_1(CV)$  due to solving the pressure field with CV (low order) when compared to higher order discretisations (FEM with polynomials of order 1 or higher), which should become more evident at the later stages of the simulation. However, the energy dissipation in this simulation does not affect the general good agreement with experiments.

The number of elements variation during the simulation induced by the dynamic mesh optimisation can be seen in Figure 4.27. For  $CFL=1$ , a maximum of  $2 \times 10^5$  elements is achieved at 1.3 s. For  $CFL=0.1$  a maximum threshold is set to  $5 \times 10^5$ . The elements threshold is reached at 0.8 s and therefore does not affect simulation results of earlier pressure peaks at sensors P1 and P3. Mesh resolution requirements, i.e. maximum (1.00 m) and minimum (0.01 m) edge

element lengths, are the same for all three cases. Time-step remains between  $10^{-3}$ s and  $5 \times 10^{-3}$ s for CFL=1 and between  $5 \times 10^{-4}$ s and  $3 \times 10^{-3}$ s for CFL=0.1.

Overall, the global behavior of the  $P_1DGP_1(CV)$  scheme results using CFL=0.1 and CFL=1 agree very well with the experiment. Impact peaks are in good agreement, especially for P3 with CFL=0.1, which was missed by COMFlow (Kleefsman et al., 2005) simulation.

## 4.4 Discussion

Traditionally more expensive unstructured grids can be combined with DMO helping optimise the mesh and reduce the computational cost of the simulations. However, the standard formulations would fail to solve the system with very obtuse elements.

The novel  $P_1DGP_1(CV)$  scheme for inertial flow is proven to be as accurate as  $P_1DGP_1$  standard CVFEM scheme, while being more robust and capable of resolving free surface flow problems with highly anisotropic meshes accurately. These developments can be applied to other multiphase inertial flow applications effectively and accelerate FEM approaches with their typical higher accuracy. The FEM speed up can be attributed to a new level of mesh optimisation with a significant reduction in the number of elements and computational cost, making  $P_1DGP_1(CV)$  more competitive and attractive. This model acceleration is also attributed to a faster convergence of the solution.

Sensitivity cases have shown that other schemes such as the Galerkin projection between meshes and the compressive advection scheme can help improving the interface capture of the simulations resulting in a sharp transition between water and air while the model remain robust.

Energy dissipation under the  $P_1DGP_1(CV)$  scheme is of the same order of magnitude as for the standard  $P_1DGP_1$  scheme. The numerical energy dissipation in the validation cases is mostly attributed to the interface capture. The interface capture schemes used here are proven to be conservative in terms of energy and water mass dissipation.

The presented formulation allows us to push the boundaries of DMO by allowing to increase the

aspect-ratio of elements and further reduce the computational costs while maintaining accuracy. Therefore, this is not only providing a new scheme to overcome a long standing problem for classical FEM models, but also has a great potential for a wide range of applications. This is of particular interest for 2D and 3D cases where one or more dimensions have less importance in solving the physics of the problem during part of the simulation or parts of the domain, such as with NWT applications. This is discussed in depth in Chapter 5.

# Chapter 5

## Optimisation and validation for Numerical Wave Tanks

### 5.1 Introduction

*Key content from Chapters 4 and 5 has been published in Via-Estrem et al. (2020). A more detailed and more in-depth explanation of the methods and a wider range of sensitivity tests are presented in this thesis so that the content differs from the published paper. The content of this Chapter published in Via-Estrem et al. (2020) has been referenced accordingly with the given permission by John Wiley Sons, Ltd.*

Numerical Wave Tank (NWT) simulations have the potential to replace physical experiments as computer power increases and numerical codes with new capabilities become available. Once validated, NWTs can simulate Wave-Structure Interaction (WSI) as accurately as physical experiments, and they also can provide information that would be very difficult or impossible to obtain from physical experiments due to instrumentation limitations.

Main challenges for numerical models are to achieve the desired sea state accurately for realistic simulations, which requires a numerical wave generation scheme, and to absorb wave energy efficiently when required. Jacobsen et al. (2012) and Higuera et al. (2013) developed wave

generation libraries for the widely used Finite Volume Method model OpenFOAM, using a wave generation zone and wave inlet methods, respectively. A piston wave paddle approach has been used by SPH models such as in DualPhysics (Altomare et al., 2014), replicating the piston wave paddle movement in experimental facilities. For wave absorption, most common approach is to use Darcy or Forchheimer formulae for imposing linear or quadratic energy loss, respectively (Dentale et al., 2009). The complexity of wave breaking both in deep and shallow waters also present a big challenge for NWT. Numerous researches have investigated the wave breaking on slope employing numerical modeling, the most significant of which is summarized by Xie (2015).

Current limitations of NWT are the computational cost and the accuracy that one can achieve under model stability and computational cost restrictions. Thanks to new FEM optimisation approaches and the dynamic mesh optimisation (DMO) described in Chapter 4, FEM can also become a very attractive alternative to previously mentioned models for NWT applications. Furthermore, FEM can typically achieve higher accuracy, and could be used with a two way coupling approach with solid models such as FDEM (Vire et al., 2015; Yang et al., 2016), allowing the displacement or movement and deformation of the structure under wave attack. Spinneken et al. (2012) and Vire et al. (2016) work with FEM codes showed already promising results on wave generation with DMO for one phase (water) simulations.

This chapter aims to validate and show the potential of the technology presented in Chapter 4 for NWT applications, including a newly written wave generation scheme for two-phase simulations. This chapter also covers the model optimisation for 3D simulations in order to significantly reduce computational cost. Experiments of a single CoreLoc unit on a slope under wave attack, run at the National Research Center (NRC) in Canada as part of a joint collaboration by the University of Ottawa, Imperial College London, Baird Associates and the NRC, are used for validating the WSI physics capture by the model.

This chapter is structured as follows: the implementation and validation of the wave generation and a description of the required libraries is explained in Section 5.2. Wave breaking is analysed with 2D simulations in Section 5.4. Sensitivity tests on the wave breaking and wave energy decay



are shown in subsection 5.3.1. Section 5.4 explains, using a 3D case of a Solitary wave breaking on a slope, an promising approach for optimising the numerical model for 3D simulations and reducing the computational cost. The full potential of 3D optimisation is shown with the numerical validation studies with experiments of WSI with single CoreLoc unit on a slope, in Section 5.5.

## 5.2 Wave Generation

Different approaches for wind waves generation are available in literature. Most commonly used are wave generation zone, wave inlet and wave paddle boundary conditions (see Section 2.2). A wave inlet, this is wave velocities profile and free surface elevation information provided at the inlet boundary, is chosen here due to reduced domain requirements and therefore lower computational cost. Moreover, imposing common linear wave theory at the inlet (replicating most physical wave tank or flume facilities) requires several wave lengths in the domain for waves to transform from linear waves to a higher order waves (when applicable). The formulation by Fenton (1988) is capable of generating waves of any order from linear to 5th order, depending on wave conditions and depth, reducing significantly the length needed for waves to transform from linear to non-linear. For this reason the wave generation using stream function by Fenton (1988) is implemented here.

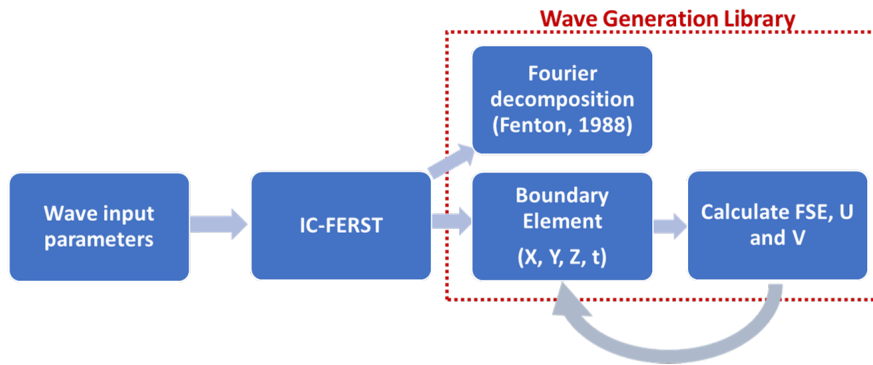


Figure 5.1: Components of the newly developed numerical wave generation routine. The wave parameters are read by IC-FERST, which calls the Fourier decomposition function (only on the first time step) and obtains the fluid free surface elevation (FSE), and fluid horizontal (U) and vertical (V) velocities on all elements at initial conditions and at the inlet boundary elements at every time step.

The calculations of the wave velocities and free surface elevation profiles by Fenton (1988) are captured and implemented within a newly written wave generation library (Python). Figure 5.1 shows the schema of the wave generation routine and subroutines implemented for IC-FERST. These include the original Fourier code by Fenton (1988) that provides the Fourier coefficients for solving wave velocity and surface profiles, which is only called once at the start of the process. IC-FERST initial and boundary conditions functions import the Fourier coefficients and calculate the free surface elevation and velocity vectors every time-step. Equations 5.1, 5.2 and 5.3 describe how free surface elevation, horizontal and vertical velocities are calculated for a given wave using Fenton (1988) formulation, respectively:

$$\eta = h \sum_{i=1}^N Y_i \cos i(kx - wt) \quad (5.1)$$

$$u = c - \bar{U} + \sqrt{gh^3k^3} \sum_{i=1}^N iB_i \frac{\cosh ikz}{\cosh ikh} + \cos ik(xk - wt) \quad (5.2)$$

$$v = \sqrt{gh^3k^3} \sum_{i=1}^N iB_i \frac{\sinh ikz}{\cosh ikh} + \sin ik(xk - wt) \quad (5.3)$$

where  $Y_i$  and  $B_i$  are the Fourier coefficients for free surface elevation and velocity, respectively,  $k$  is the wave number,  $x$  is the coordinate of the point of interest within the wave length,  $h$  is the depth and  $w$  is wave frequency. For the horizontal velocity solution the wave celerity  $c$  and the current  $\bar{U}$  are also taken into account.

The wave generation library includes a ramp linear function approach for transitioning from still water to a fully developed wave system (regular or irregular). The approach taken here for the wave generation library is to generate a given (by the user) number of waves in the NWT domain starting with a fully developed wave at the inlet location and progressively (linearly) decreasing the waves to reach still water level at the end of the specified wave lengths. This new approach (developed here) is very attractive due to having a shorter simulation time and giving a full wave scenario at the start of the simulation, which gives more momentum to the simulation (having higher velocities and time-step adjusted to these) than starting with still water. It is worth noting that the option of starting the simulation with still water and

generating waves progressively at the inlet is also implemented in the author's wave generation library.

The wave generation scheme can also be easily expanded if irregular waves are to be used. Irregular first order waves can be generated as a linear superposition of linear wave theory waves for a given number of components (N):

$$\eta = \sum_{i=1}^N a_i \cos(k_i x - \omega_i t + \phi) \quad (5.4)$$

$$u = \sum_{i=1}^N \frac{a_i \omega_i \cosh(k_i z)}{\sinh(k_i h)} \sin(\omega_i t - k_i x + \phi) \quad (5.5)$$

$$w = \sum_{i=1}^N \frac{a_i \omega_i \sinh(k_i z)}{\sinh(k_i h)} \cos(\omega_i t - k_i x + \phi) \quad (5.6)$$

However, in this Thesis only regular waves are used. This is due to the higher simplicity of validation and testing of regular wave simulations when compared to irregular waves.

### 5.2.1 Validation of inlet wave generation

Several wave conditions are used for validation of the newly implemented wave generation scheme for IC-FERST using DMO and the latest developments presented in Chapter 4. 2D simulations are used for validating the wave generation at the inlet boundary (weak form) and at different locations away from the generation boundary (validating wave transformation). The simulation results obtained at the boundary and different locations of each wave condition are compared against their analytical solution.

A set of wave conditions corresponding to wave theories of different order are considered. A range of depths, wave heights and wave periods are chosen from values typically seen in physical experiments (most common model scale factors are between 1:20 and 1:50). The decision of using common scaled down model values instead of full prototype scale values was taken in order to be representative of what physical modelling facilities commonly operate with and are

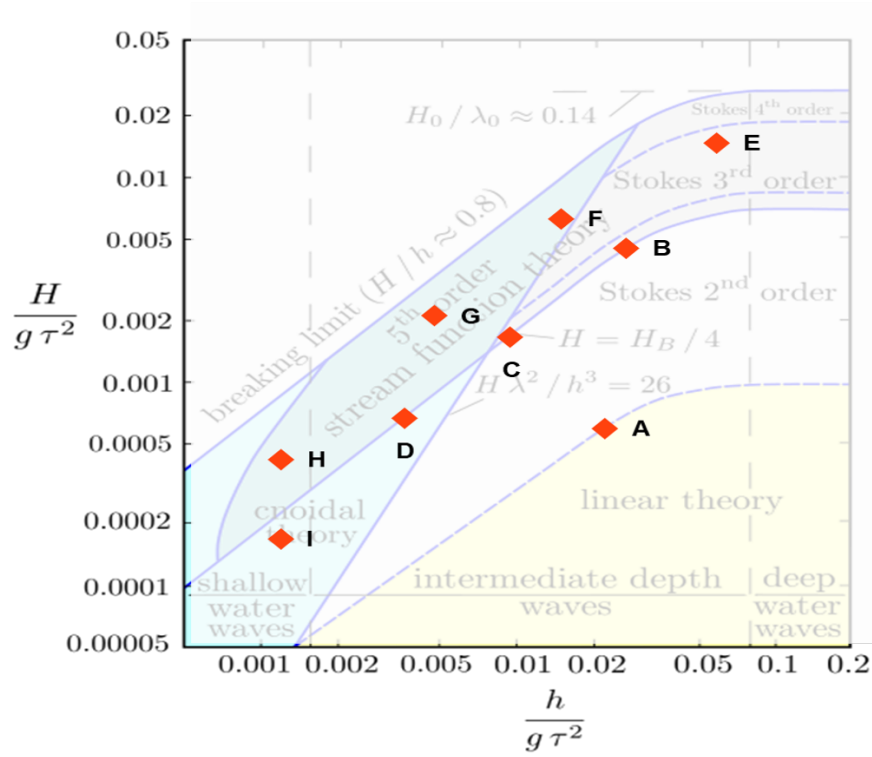


Figure 5.2: Generated wave conditions for the validation of the wave generation scheme and their corresponding wave theory range. See Table 5.1. Non-dimensional axis obtained with  $H$  as the wave height (m),  $h$  the water depth (m),  $\tau$  for the wave period (s),  $g$  for the gravity ( $m/s^2$ ) and  $\lambda$  for the wave length (m).

| Label | H    | h   | T   |
|-------|------|-----|-----|
| A     | 0.05 | 2   | 3   |
| B     | 0.1  | 0.6 | 1.5 |
| C     | 0.1  | 0.6 | 2.5 |
| D     | 0.1  | 0.6 | 4   |
| E     | 0.15 | 0.6 | 1   |
| F     | 0.25 | 0.6 | 2   |
| G     | 0.25 | 0.6 | 3.5 |
| H     | 0.25 | 0.8 | 8   |
| I     | 0.1  | 0.8 | 8   |

Table 5.1: Wave conditions used for the validation of the newly implemented wave generation scheme. H stands for wave height, T for wave period and h for water depth.

easier to validate against (with physical model results). Figure 5.2 and Table 5.1 describe all tested wave conditions. Full scale prototype wave generation can be found in Chapter 6.

Wave probes are placed at 0.01 m (WP0) in front of the inlet boundary for validating the generated waves at the boundary, and at 0.1 m (WP1) and 0.5 m (WP2) to validate their

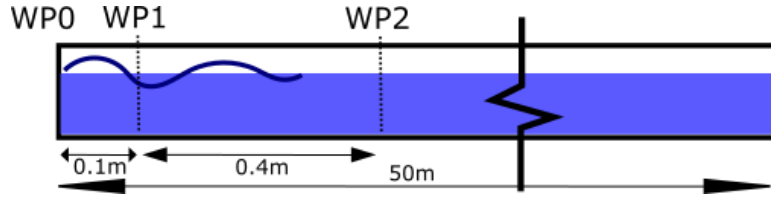


Figure 5.3: Representation of the 50 m length NWT, with its different wave probe (WP) locations, for the wave generation validation simulations. The wave generation inlet is located at the left boundary.

propagation, see Figure 5.3. The domain is set as a 50 m long flume with a flat bottom. Simulations are done using adaptive time-step with  $CFL=1$  and the mesh requirements are set to a minimum and maximum edge lengths of 0.005 m and 0.1 m, respectively. This single configuration of time-step and mesh resolution is set for evaluating the new wave generation scheme. Numerical energy dissipation of regular waves with mesh resolution and time-stepping sensitivity analysis is discussed and compared against physical modelling results in Sections 5.4 and 5.5.

Free surface elevation is extracted from the numerical outputs as an integration of the water column. This integration gives an accuracy up to the minimum edge length of the mesh. Numerical results are given at 0.1 s frequency, which is the output frequency of the numerical simulations. Results are compared for a minimum of either the first 3 wave lengths or 10 s of the simulation. This limitation is set in order to avoid any reflected waves at the end of the flume to reach back the wave probes (no wave generation zone was implemented at the downstream end of the flume).

Figure 5.4 shows wave generated at the inlet for wave conditions A (linear wave theory), B and C (Stokes 2nd order). All wave conditions show very good agreement between numerical simulations and analytical solution at the inlet. A good match is also observed as the wave propagates to 0.1 m and 0.5 m from the inlet. Small differences observed away from the inlet around the peak and trough of the regular waves correspond to the minimum edge length (0.005 m) established for the mesh (and accuracy of the water column integration method). This is visually more obvious for wave conditions A (top) as the mesh resolution corresponds to 10% of the wave height (0.05 m), whereas it only corresponds to a 5% of the wave height (0.1 m) for wave conditions B and C.

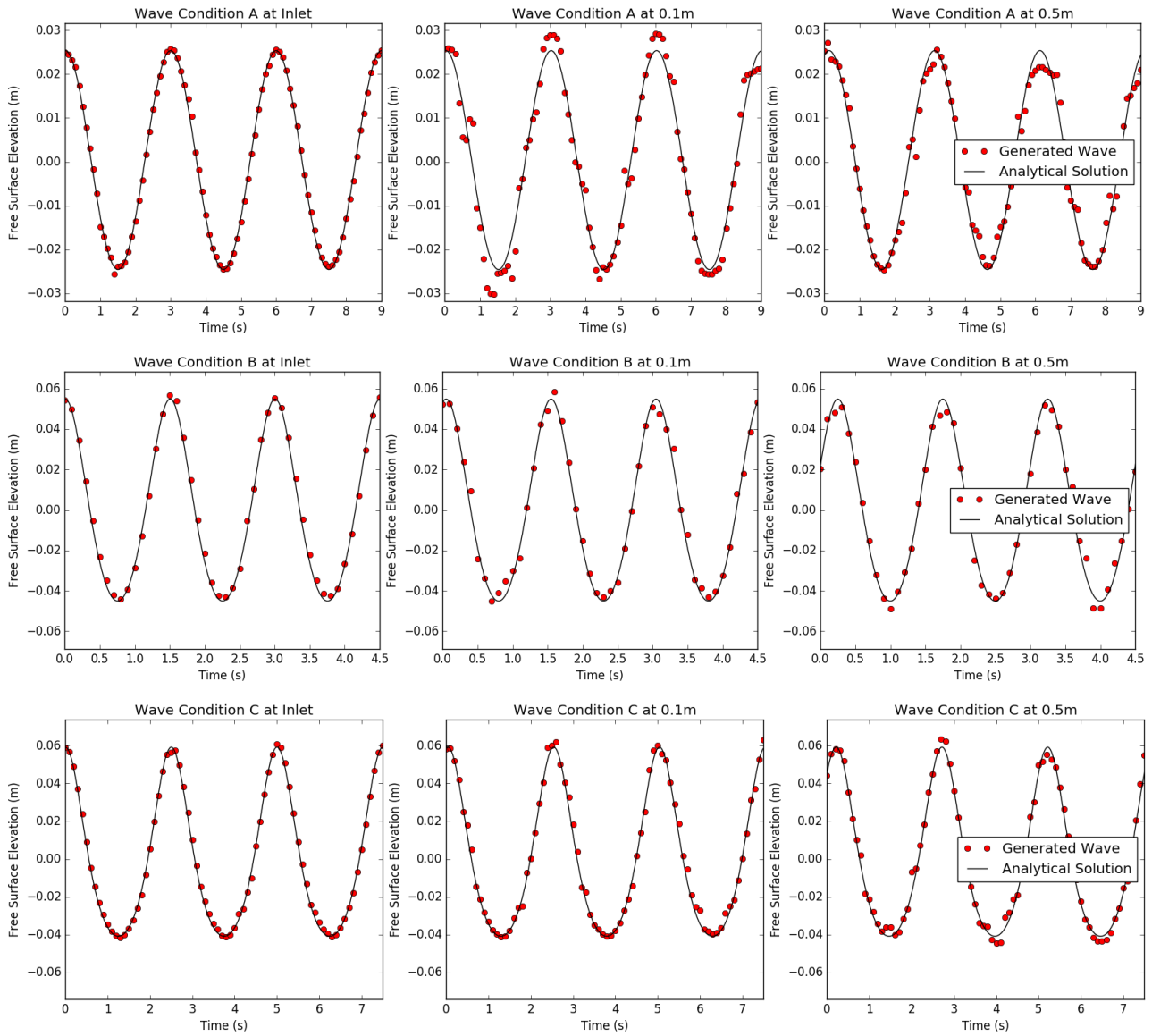


Figure 5.4: Comparison of results from the new wave generation scheme for wave conditions A (top), B (centre) and C (bottom), at the inlet (WP0, left), and at 0.1 m (WP1, centre) and 0.5 m (WP2, right) from the inlet, against the analytical solution for the free surface elevation.

As expected, higher peaks and shallower troughs are observed as waves increase their steepness and the order of the representation by Stokes theory (see Figure 5.4). Second order Stokes theory free surface (wave conditions B and C) shows a shift towards higher values (the maximum and minimum values are not at the same distance to the 0 -Mean Water Level- line) when compared to first order Stokes or linear wave theory (wave condition A).

This shift is increased in higher order wave theories. Figure 5.5 shows wave conditions E (Stokes 3rd order), F and G (Stokes 5th order). Good match is observed with wave conditions E, F

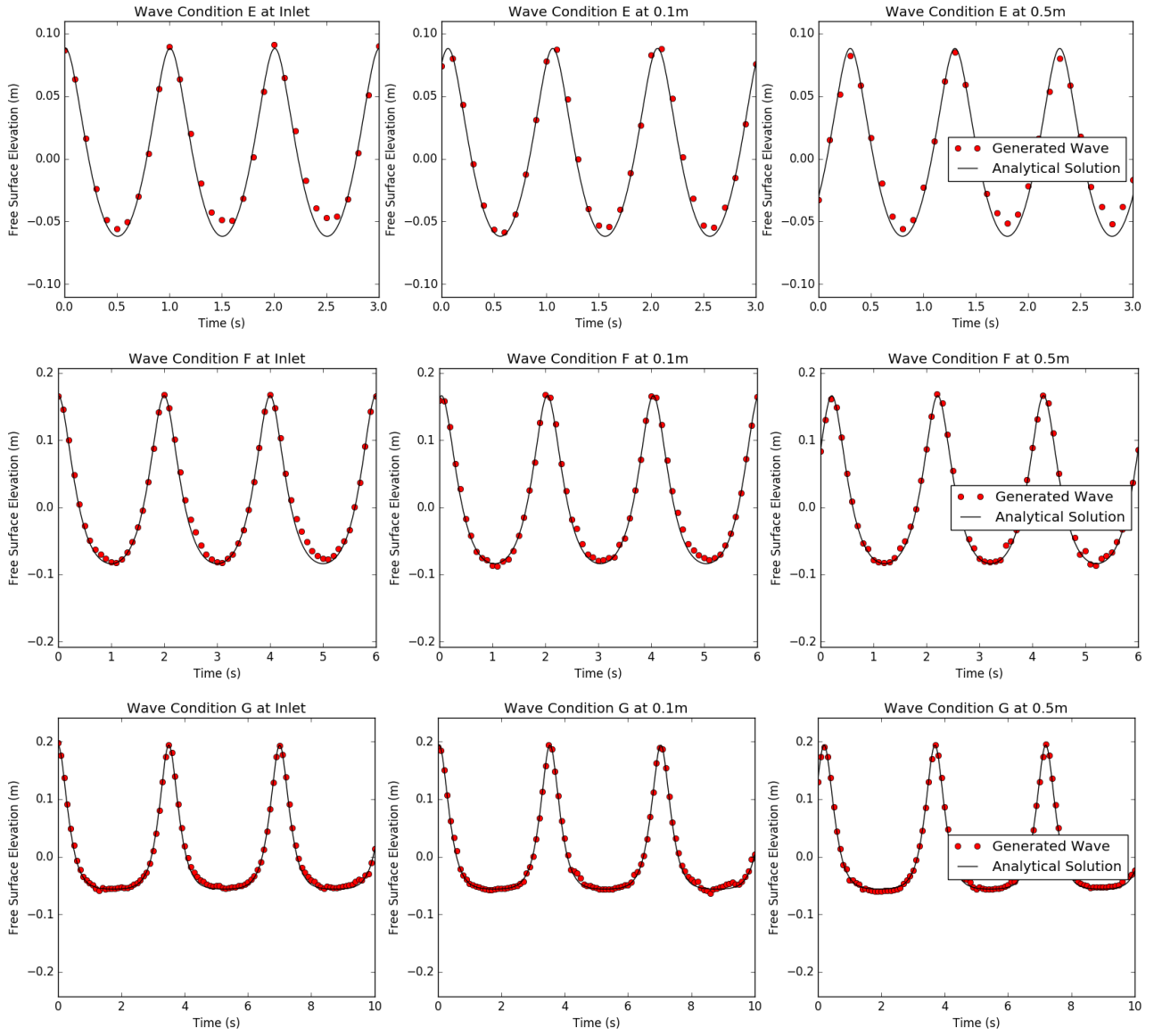


Figure 5.5: Comparison of results from the new wave generation scheme for wave conditions E (top), F (centre) and G (bottom), at the inlet (WP0, left), and at 0.1 m (WP1, centre) and 0.5 m (WP2, right) from the inlet, against the analytical solution for the free surface elevation.

and G at the inlet and at 0.1 and 0.5 m from the inlet. Small differences can be observed at the second and third troughs of wave condition E at the inlet. However, these are assumed to be artificially induced with the wave generation scheme and are significantly dissipated at 0.1 m from the inlet. Steeper waves and nearly flat troughs are observed with fifth order Stokes theory waves (bottom).

Figure 5.6 shows wave conditions D, H and I (within cnoidal theory range, see Figure 5.2). A very good match is observed between numerical results and analytical solution at and near the

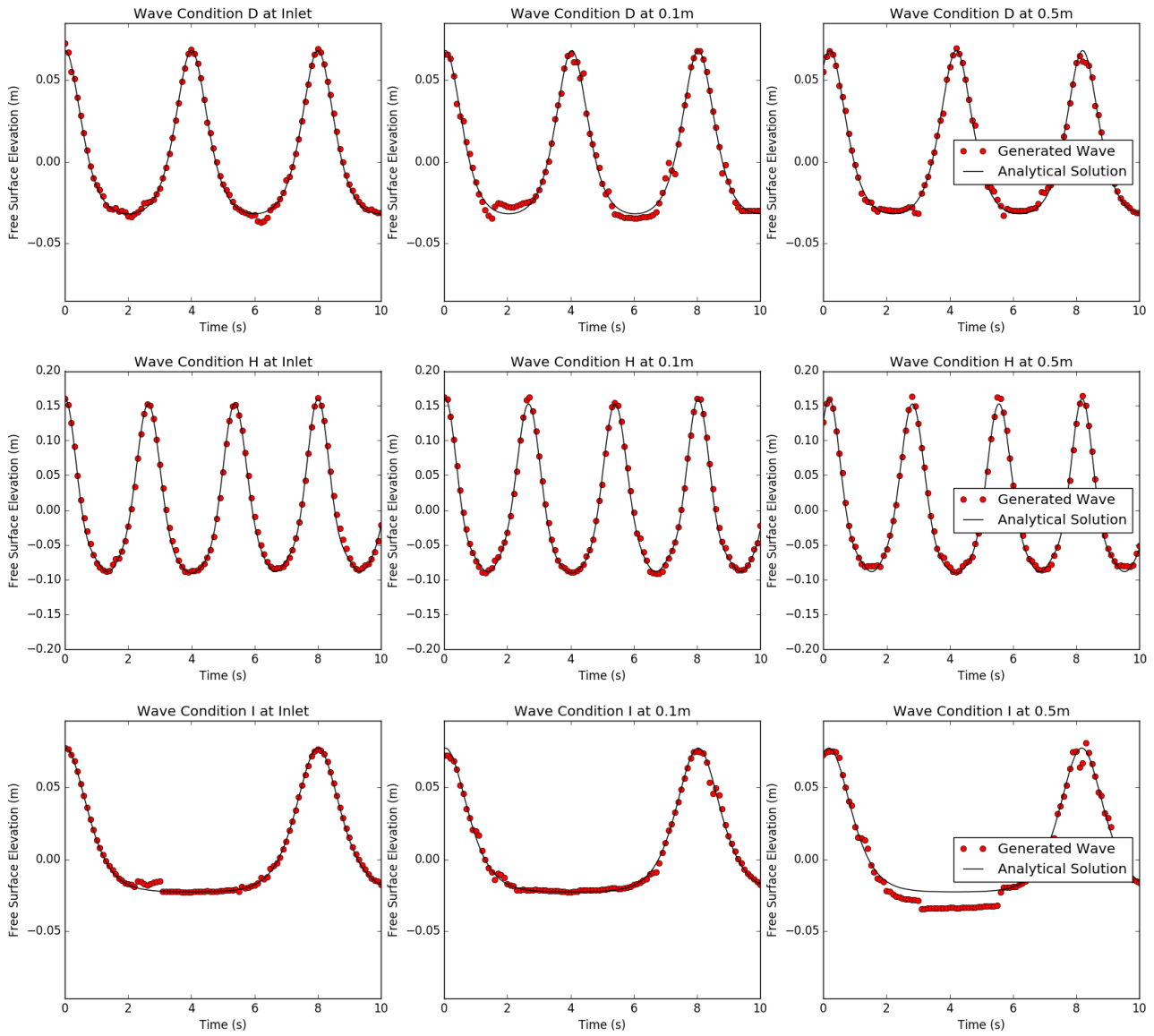


Figure 5.6: Comparison of results from the new wave generation scheme for wave conditions D (top), H (centre) and I (bottom), at the inlet (WP0, left), and at 0.1 m (WP1, centre) and 0.5 m (WP2, right) from the inlet, against the analytical solution for the free surface elevation.

inlet. A discontinuity is observed at the trough of the 0.5 m wave gauge for wave conditions I (bottom right). This discontinuity coincides with the order of magnitude of the mesh resolution (and accuracy of the water column integration method discussed above) and it is believed to be due to the low accuracy of the wave gauge numerical reading from the outputs. Overall, the results show that new wave generation scheme is capable to accurately generate waves of any order.



### 5.3 Capturing Wave Breaking

Coastal structures are often placed in shallow waters and therefore their design need to consider wave breaking scenarios. This section aims to analyse and validate the modelling of wave breaking using IC-FERST newest developments. The explanation of the different types of wave breaking can be found in literature (Dean and Dalrymple, 1991; Goda, 2010), which depend on sea bed slope and wave characteristics. In this section a plunging breaking wave type is considered due to their higher energy release at the breaking point and due to the availability of experimental results. This breaking wave type also presents an ideal scenario for analysing the potential of water-air interface capture and air trapping in the breaking zone. Different numerical approaches, in particular spatial discretisation and interface capture schemes, are also considered for comparison purposes and are discussed in this section. Sensitivity tests on wave breaking and wave energy decay are shown in subsection 5.3.1.

The benchmark case used here is a solitary wave breaking on a slope based on the experiments run by Li (2000). This case is run in 2D for sensitivity tests in this Section, and in 3D for full validation and the optimisation of a numerical wave tank in Section 5.4.

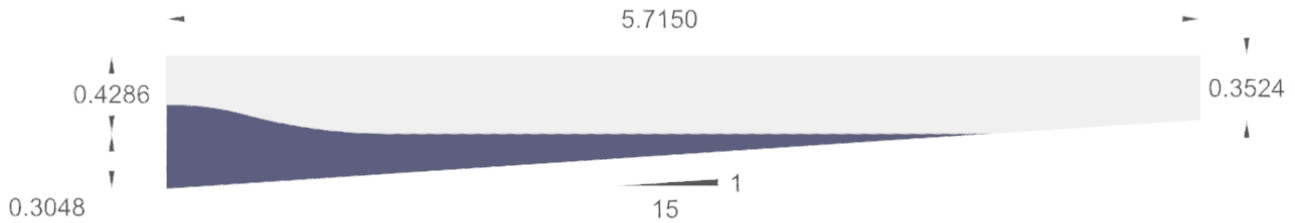


Figure 5.7: General setup of the sloped Numerical Wave Tank (NWT) (Li, 2000). The wave inlet and half of the profile of the solitary wave are located on the left and deepest point of the NWT.

The dimensions and slope of the numerical wave tank are shown in Figure 5.7. Water depth and wave height at the deepest point are  $h=0.3048$  m and  $H=0.45 \cdot h$ , respectively. A different wave generation approach from Section 5.2 is considered here for the solitary wave. Half of the solitary wave is added to the domain by imposing the free surface and horizontal and vertical velocities as initial conditions. The sloped (non-slip), right and side boundaries are imposing

velocity vectors of 0 magnitude. The top boundary is considered open with an imposed constant pressure of 0. The solitary wave is generated using the analytical solution by Lee et al. (1982), see Equations 5.7 to 5.11. Initial and inlet boundary conditions (left boundary) are imposed using free surface elevation  $\eta$ , horizontal  $u$  and vertical  $w$  velocities as follows:

$$\eta(x, t) = H \operatorname{sech}^2 \left[ \sqrt{\frac{3H}{4h^3}} X \right] \quad (5.7)$$

$$X = x - Ct - x_L \quad (5.8)$$

$$C = \sqrt{g(h + H)} \quad (5.9)$$

$$u(x, z, t) = C \left[ \left[ \frac{H}{h} + 3 \left( \frac{H}{h} \right)^2 \left( \frac{1}{6} - \frac{1}{2} \left( \frac{z+h}{h} \right)^2 \right) \right] \frac{\eta}{H} - \left( \frac{H}{h} \right)^2 \left( \frac{7}{4} - \frac{9}{4} \left( \frac{z+h}{h} \right)^2 \right) \left( \frac{\eta}{H} \right)^2 \right] \quad (5.10)$$

$$w(x, z, t) = C \sqrt{\frac{3H}{h}} \frac{z+h}{h} \frac{\eta}{h} \tanh \left( \sqrt{\frac{3H}{4h^3}} X \right) \left[ 1 + \frac{H}{2h} \left[ 1 - 7 \frac{\eta}{H} - \left( \frac{z+h}{h} \right)^2 \left( 1 - \frac{3\eta}{H} \right) \right] \right] \quad (5.11)$$

where  $x$  and  $z$  are horizontal and vertical coordinates,  $t$  time and  $g$  the gravitational accelera-

tion. Density and viscosity of the water and air components are the same as detailed in Table 4.2.

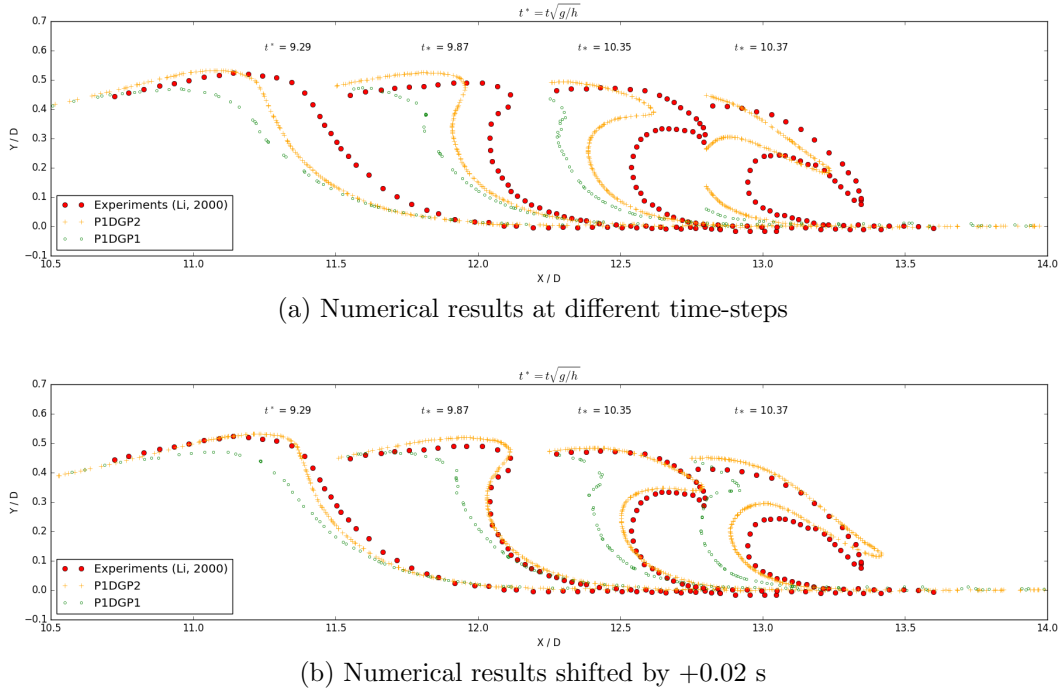


Figure 5.8: Free surface of the P1DGP1 and P1DGP2 2D simulations of a solitary wave breaking on a slope at different stages. Ratio of wave height to depth at the inlet is  $H/h=0.45$ . Numerical results shifted by 0.00 s (top) and +0.02 s (bottom). (Li, 2000)

A nearly regular mesh is initially generated with gmsh (Geuzaine and Remacle, 2009) and adapted at  $t=0$  by the DMO algorithm to give higher resolution to the interface, with maximum and minimum element sizes in  $(x, y)$  of  $(h, h)$  and  $(h/100, h/100)$ , respectively, being  $h=0.3048$  m. Different orders for spatial discretisation are considered in order to evaluate accuracy of the results against experiments (Li, 2000). Figure 5.8a shows the wave profile at different breaking stages for simulations using classical  $P_1DGP_2$  and  $P_1DGP_1$  spatial discretisations with no compressive schemes for the interface (see Chapter 4). Both  $P_1DGP_1$  and  $P_1DGP_2$  schemes show a delay of the wave breaking when compared to experiments due to numerical dissipation. This delay can be corrected for  $P_1DGP_2$  by shifting the numerical results by  $t=0.02$  s or  $t^*=0.11$  (see Figure 5.8b) for visualisation. Although the delay could be the result of both the numerical dissipation and some inaccuracies during physical experiment measurements, more dissipative spatial discretisation schemes (shown in the remainder of this section) are seen to leading to longer delays, suggesting that the numerical dissipation is likely to be the main cause of these

delays.

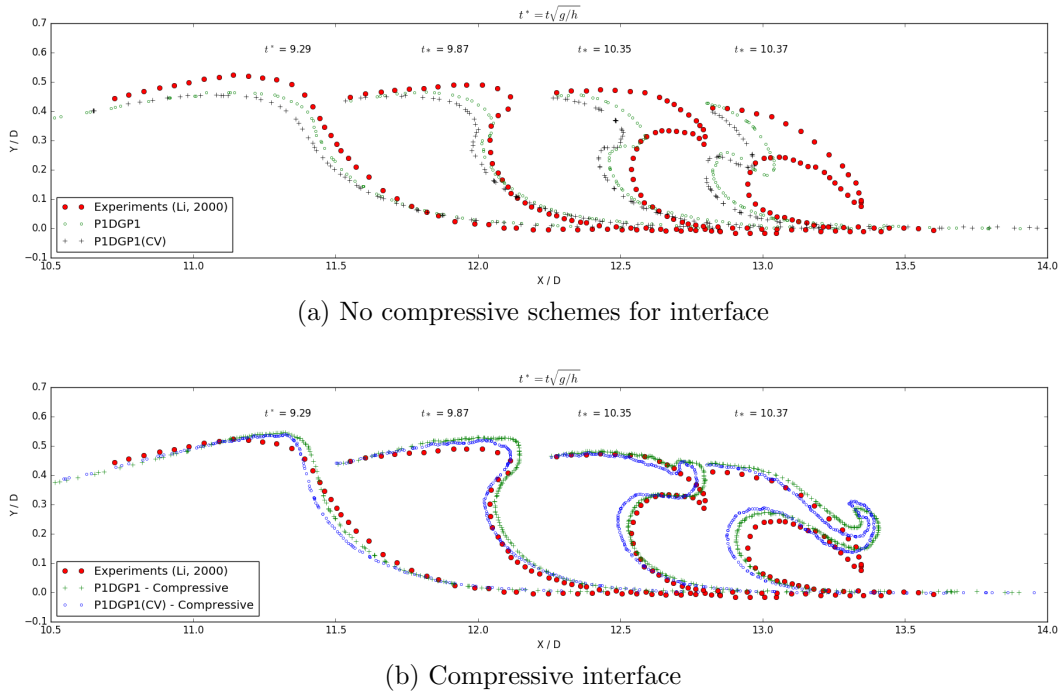


Figure 5.9: Free surface of the  $P_1DGP_1$  and  $P_1DGP_1(CV)$  2D simulations of a solitary wave breaking on a slope at different stages. Ratio of wave height to depth at the inlet is  $H/h=0.45$ . Numerical results shifted by  $+0.03$  s. (Li, 2000; Via-Estrem et al., 2020)

Once shifted, the  $P_1DGP_2$  discretisation scheme shows good agreement with experiments for all 4 breaking stages of the wave, as seen in Figure 5.8b. The  $P_1DGP_1$  scheme shows a significant delay and a lower maximum crest level before and during breaking. Moreover,  $P_1DGP_1(CV)$  is expected to add some extra numerical energy dissipation due to using a lower discretisation order for the pressure field. Expected dissipation for both  $P_1DGP_1$  and  $P_1DGP_1(CV)$  can be countered by using compressive interface schemes (described in Chapter 4), as seen in Figure 5.9a (no compressive interface) and 5.9b (compressive interface). The compressive interface scheme is proven to significantly reduce numerical dissipation at the interface, although it adds a tail shaped extension to the plunging jet of the wave before impacting the forward free surface, which is not observed in experiments. This effect not seen in experiments can be reduced by using a turbulence model such as LES, as seen in Figure 5.10.

Figures 5.11, 5.12 and 5.13 show the evolution of this wave breaking plunging jet for compressive interface (left) and non compressive interface simulations (right). Velocity and pressure field results are shown on top and bottom plots, respectively, with the free surface 0.5 contour line

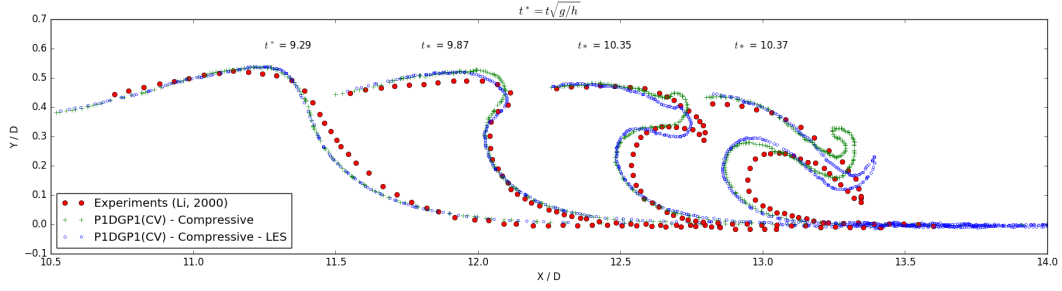


Figure 5.10: Free surface of the LES - P1DGP1(CV) 2D simulations of a solitary wave breaking on a slope at different stages. Ratio of wave height to depth at the inlet is  $H/h=0.45$ . Numerical results shifted by  $+0.03$  s. (Li, 2000; Via-Estrem et al., 2020)

marked in white and the 0.25 and 0.75 contour lines in pink, showing the compressiveness of the interface in each case.

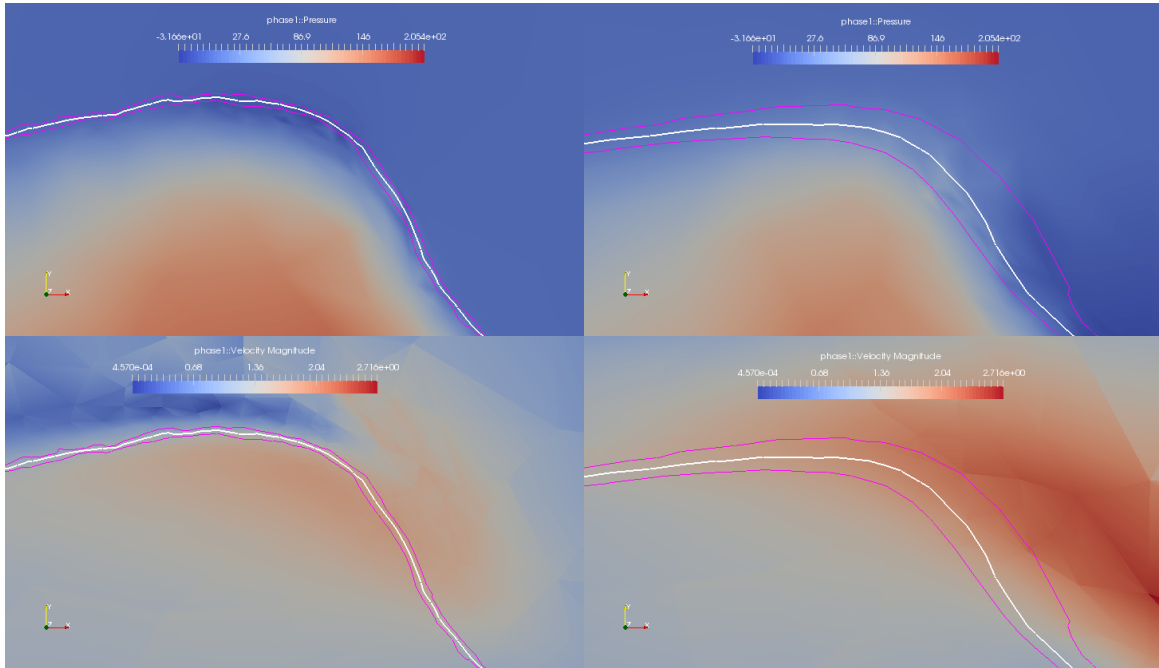


Figure 5.11: Detail at 1.68 s of the wave breaking jet with pressure (top) and velocity (bottom) fields for LES Compressive  $P_1DGP_1(CV)$  (left) and non-compressive  $P_1DGP_1(CV)$  (right). A low pressure area is observed off the surface of the water only for the compressive interface case (left).

Negative pressures (Hydrostatic Pressure component is not considered in this figure, see Chapter 4) are observed around the wave crest at the start of the breaking process of the compressive simulation. This is assumed to be artificially induced by the compressive interface scheme and affects the velocity field at later stages as observed in Figures 5.12 and 5.13. This leads to the generation of the hammer shape on the plunging jet not observed in experiments and non compressive interface schemes, where the pressure and velocity field show smooth transitions

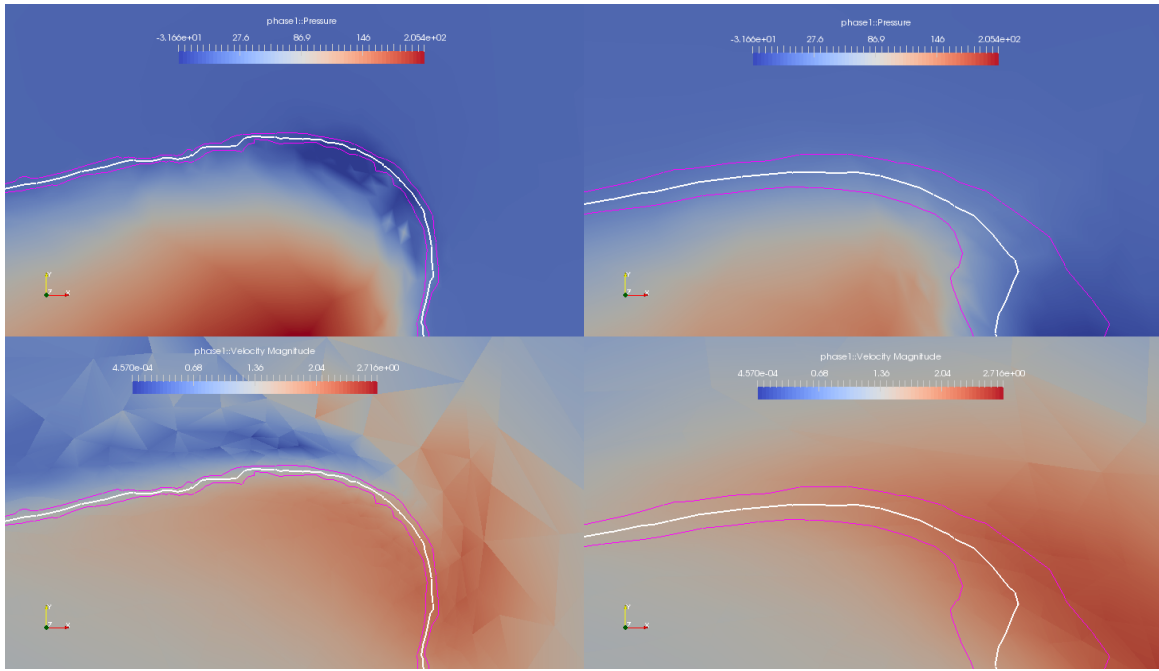


Figure 5.12: Detail at 1.76 s of the wave breaking jet with pressure (top) and velocity (bottom) fields for LES Compressive  $P_1DGP_1(CV)$  (left) and non-compressive  $P_1DGP_1(CV)$  (right). A low pressure area is observed off the surface of the water only for the compressive interface case (left).

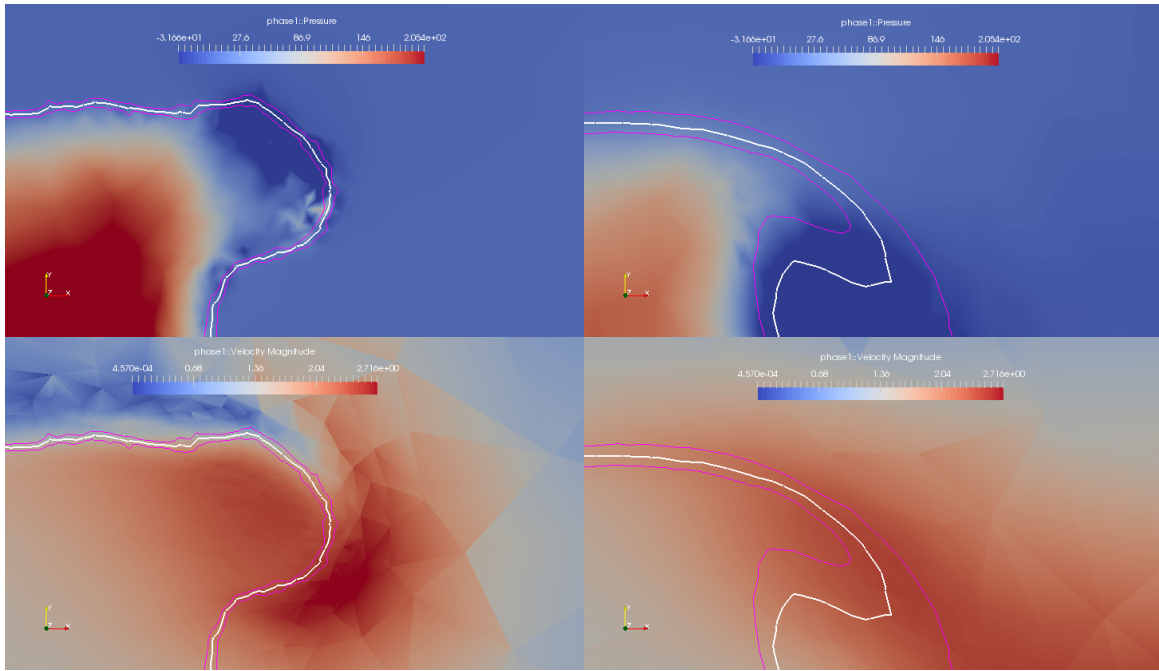


Figure 5.13: Detail at 1.85 s of the wave breaking jet with pressure (top) and velocity (bottom) fields for LES Compressive  $P_1DGP_1(CV)$  (left) and non-compressive  $P_1DGP_1(CV)$  (right). A low pressure area is observed off the surface of the water only for the compressive interface case (left).

between air and water components.

These differences in shape of the plunging jet in the wave breaking process could be reduced by applying different compressive interface schemes. This assumption will be investigated in future work. The compressive interface schemes used herein are considered appropriate for reducing the numerical dissipation associated with a dissipative interface and are used in the simulations shown in the following sections, unless stated otherwise.

### 5.3.1 Wave breaking and numerical wave energy dissipation

Wave height evolution plots are used here for analysing and quantifying numerical dissipation using different schemes. Figure 5.14 shows wave height evolution of the solitary wave transforming on its propagation up the slope, with a short transition of the generation inlet and a progressive increase as the wave propagates on shallower depths. This transition is induced by the difference of the incoming half wave from the inlet and the existing half wave imposed with the initial conditions (initial conditions assumed a constant depth in front of the inlet). This effect is observed in all numerical schemes discussed here, with similar magnitude for all cases, and is not expected to add any difference to the breaking of the wave.

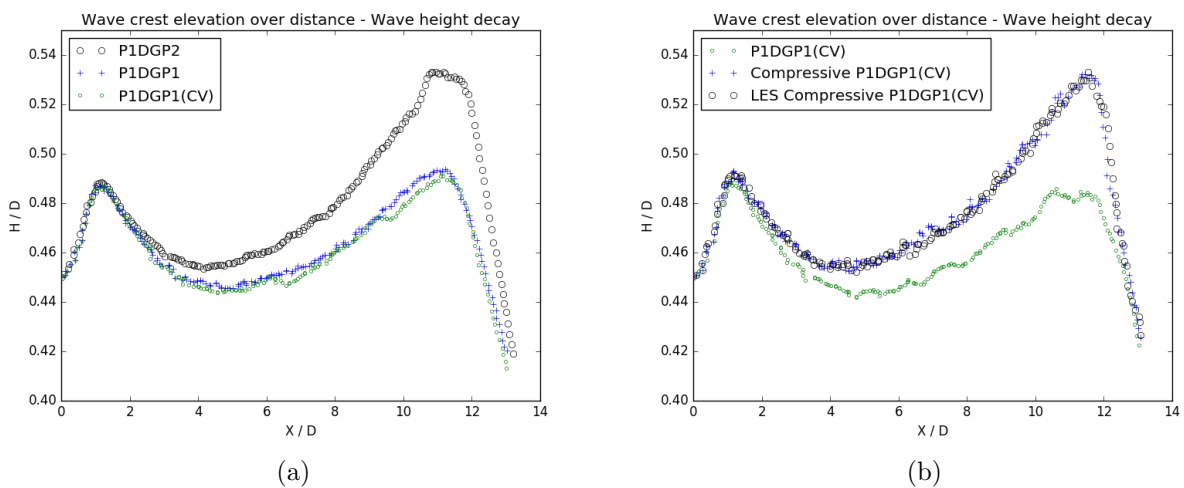


Figure 5.14: Wave height decay comparison for different spatial discretisation schemes (left) and compressive interface schemes (right), represented with the non-dimensional maximum wave height (vertical axis) along the non-dimensional longitude of the flume.

Wave height evolution using  $P_1DGP_2$ ,  $P_1DGP_1$  and  $P_1DGP_1(CV)$  is shown in Figure 5.14 (left) with significant differences between  $P_1DGP_2$  and  $P_1DGP_1$  wave heights before the wave starts breaking process (range of 0.05 -10%-). This is expected to be induced by higher numerical dissipation added by the lower spatial discretisation scheme ( $P_1DGP_1$ ). However, only minor numerical dissipation is added with  $P_1DGP_1(CV)$  when compared to the standard  $P_1DGP_1$ .

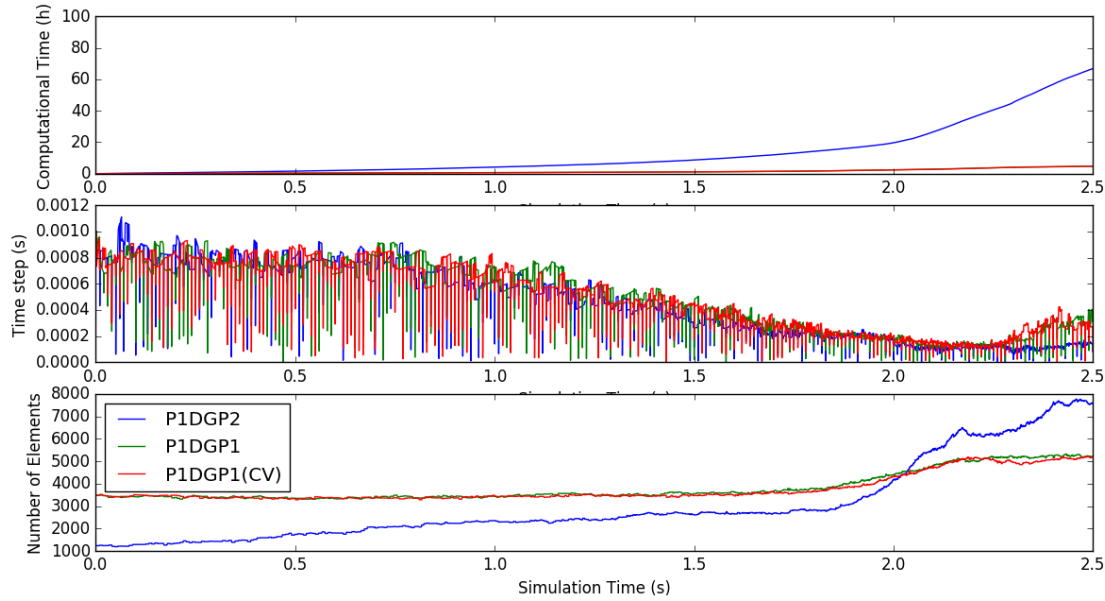


Figure 5.15: Computational performance comparison for  $P_1DGP_2$  (blue),  $P_1DGP_1$  (green) and  $P_1DGP_1(CV)$  (red) schemes. Evolution of the computational cost (top), the time step (centre), and number of elements in the domain (bottom), over the simulation time.

Numerical dissipation can be significantly reduced when using a compressive interface scheme (see Chapter 4) as it is observed in Figure 5.14 (right). It can also be observed that the impact of adding the LES turbulence model is negligible in terms of overall numerical energy dissipation.

In terms of simulation performance, a significant computational cost increase is observed for  $P_1DGP_2$  when compared to lower order discretisation schemes such as  $P_1DGP_1$  and  $P_1DGP_1(CV)$ , as seen in Figure 5.15. The Compressive interface scheme, see Figure 5.16, contributes to an increase to computational cost, but this remains significantly lower than the cost of a higher  $P_1DGP_2$  scheme, and it comes with the significant advantage of reducing the expected numerical dissipation of the interface in  $P_1DGP_1$  and  $P_1DGP_1(CV)$  schemes. The extra computational cost associated to using the LES turbulence scheme here is proved negligible.



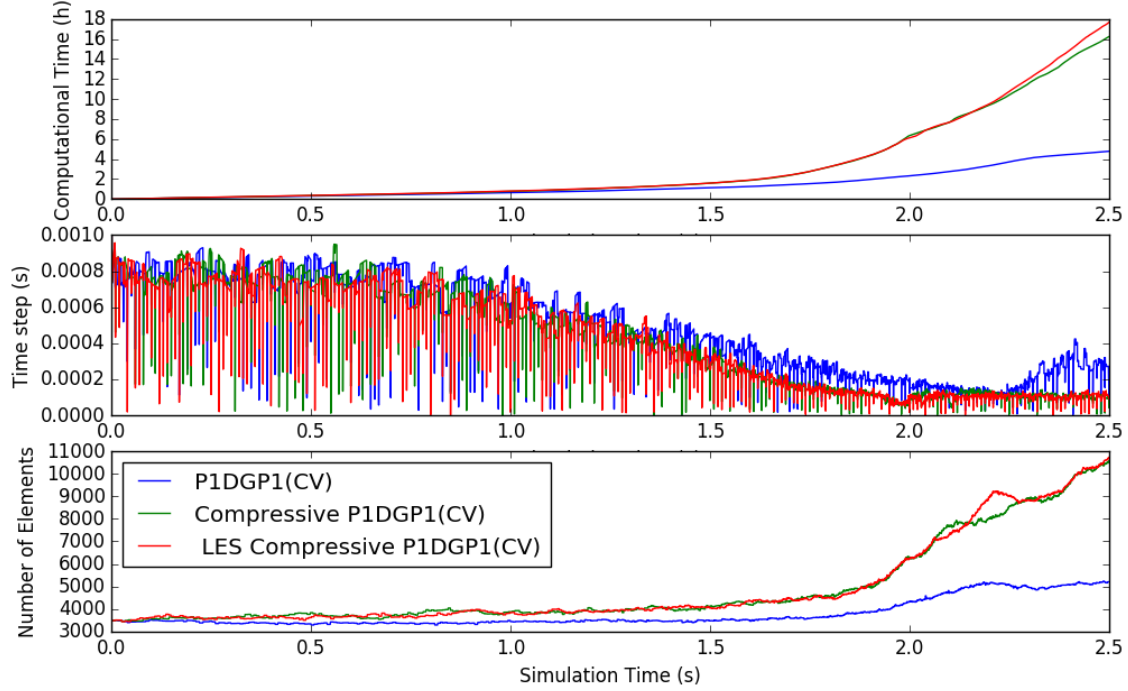


Figure 5.16: Computational performance comparison for  $P_1DGP_1(CV)$  (blue), Compressive  $P_1DGP_1(CV)$  (green) and LES Compressive  $P_1DGP_1(CV)$  (red) schemes. Evolution of the computational cost (top), the time step (centre), and number of elements in the domain (bottom), over the simulation time.

Time-stepping and mesh resolution was set for  $CFL=0.25$  and maximum and minimum element sizes in  $(x, y)$  of  $(h, h)$  and  $(h/100, h/100)$ , respectively, being  $h=0.3048$  m, for all the simulations shown above.

Sensitivity analysis of numerical energy dissipation and performance of wave breaking were performed with CFL values of 0.1, 0.25, 0.5 and 1 for the adaptive time-step and their results are shown in Figure 5.17. As expected, higher CFL value requirement leads to higher numerical dissipation and therefore to a delay on the wave breaking stage. However, some differences in shape at the final stages of the breaking wave are observed with  $CFL=0.1$ . The irregularities seen for  $CFL=0.1$  are not observed in experiments and are believed to be due to the model trying to solve the breaking jet anomaly previously discussed (using  $CFL=0.25$ ) with finer time resolution and lower energy dissipation, and therefore amplifying the otherwise dissipated negative pressure problem on the water jet. Differences between  $CFL=0.1$  and  $CFL=0.25$  remain otherwise small, same as between  $CFL=0.25$  and  $CFL=0.5$ . Simulation performance

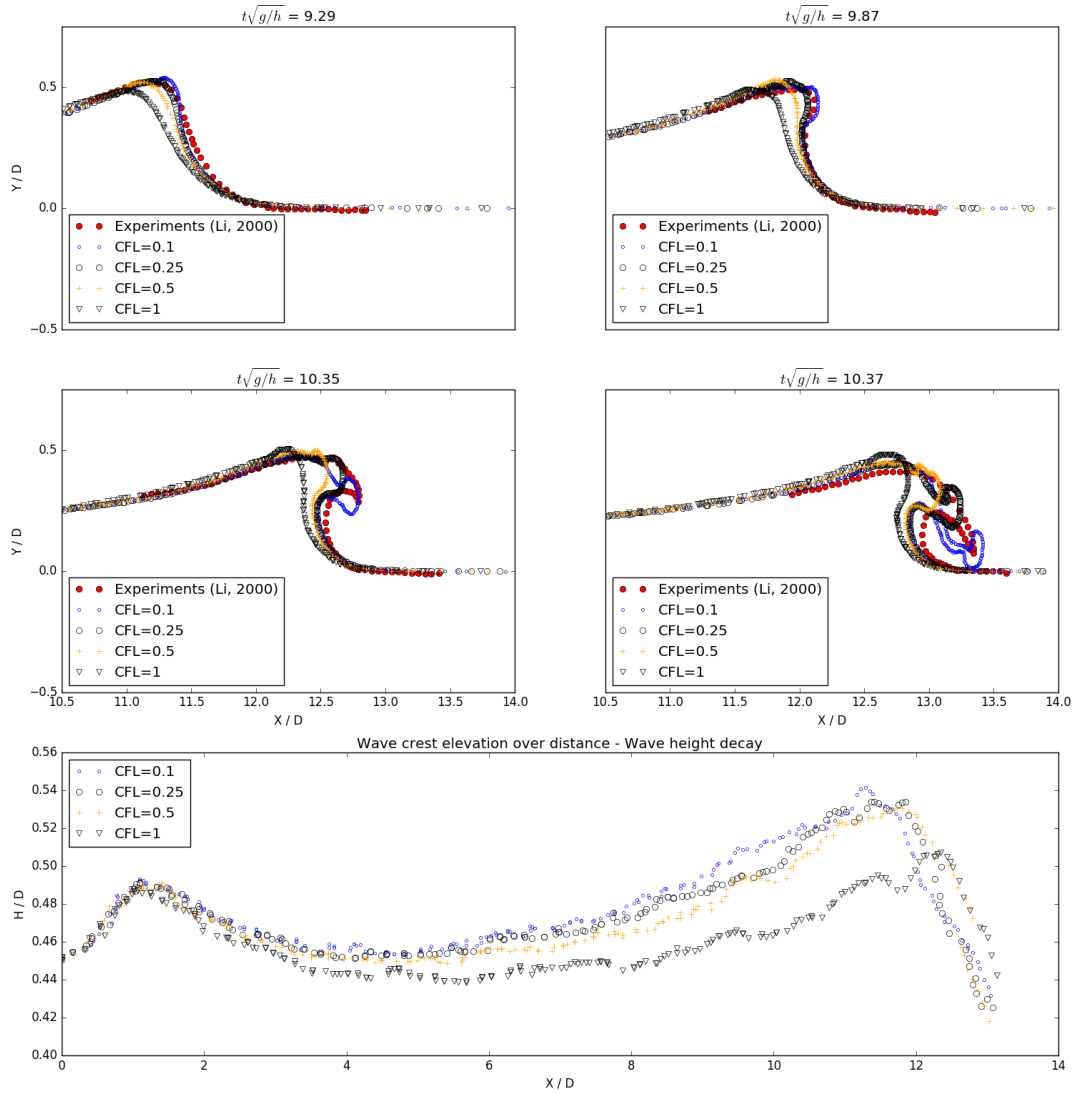


Figure 5.17: Comparison of the wave breaking shape (top and center) at different stages and Wave height decay representation using non-dimensional maximum wave height and NWT longitude (bottom) for the sensitivity analysis with different CFL values used for the adaptive time-step scheme.

analysis shows the expected increase in computational cost and shorter time-steps for lower CFL values, as seen in Figure 5.19. All CFL sensitivity simulations used maximum and minimum element sizes in  $(x, y)$  of  $(h, h)$  and  $(h/100, h/100)$ .

Results for mesh resolution sensitivity analysis are shown in Figure 5.18, with minimum and maximum edges ( $[min, max]$ ) of  $[3*h/100, 3*h]$ ,  $[2*h/100, 2*h]$ ,  $[h/100, h]$  and  $[h/200, h/2]$ . An

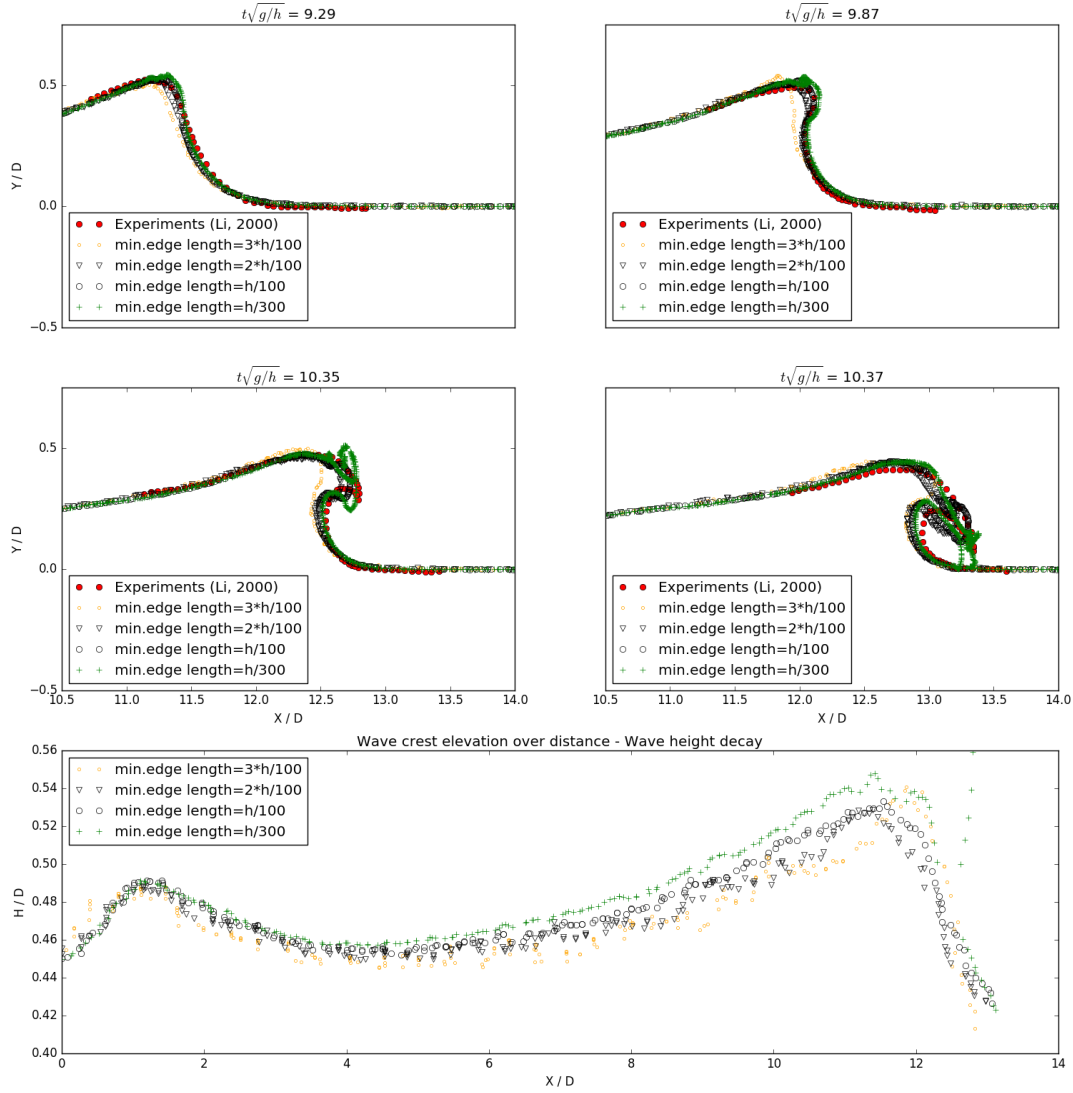


Figure 5.18: Comparison of the wave breaking shape (top and center) at different stages and Wave height decay representation using non-dimensional maximum wave height and NWT longitude (bottom) for the sensitivity analysis with different adaptive mesh resolution requirements.

increase of the breaking jet anomaly is also observed for the highest resolution case ( $[h/200, h/2]$ ), which is due to adding more elements and resolution to an artificially induced effect. With longer minimum and maximum edge lengths an increased dissipation is observed. It is important to note that the dissipation effect for changing mesh resolution remains smaller than the effect of changing the CFL value of the adaptive time-step scheme for the discussed values.

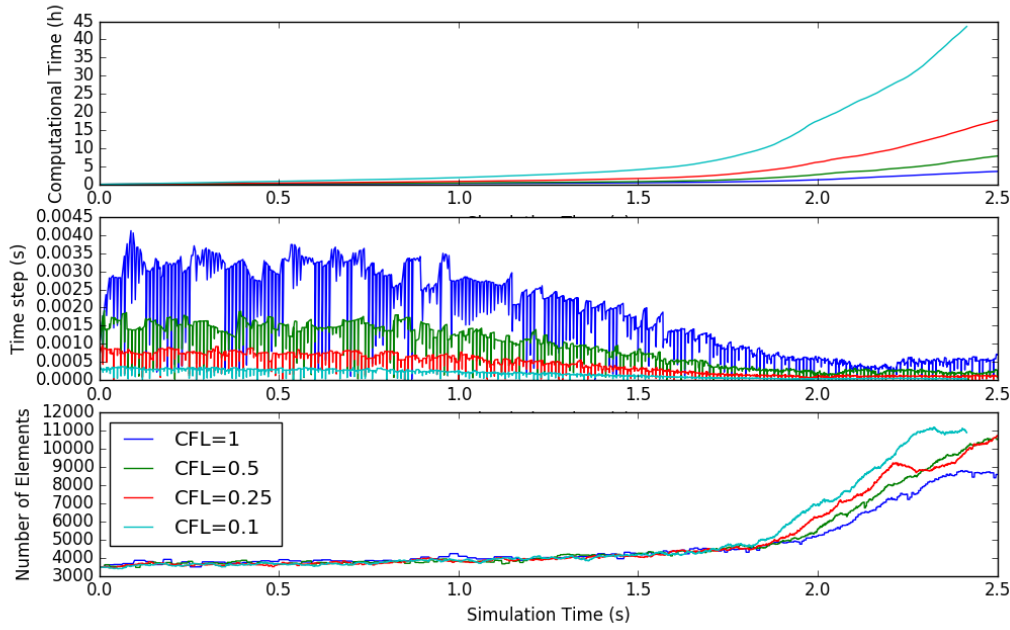


Figure 5.19: Computational performance comparison for different CFL values for the adaptive time-step scheme. Evolution of the computational cost (top), the time step (centre), and number of elements in the domain (bottom), over the simulation time.

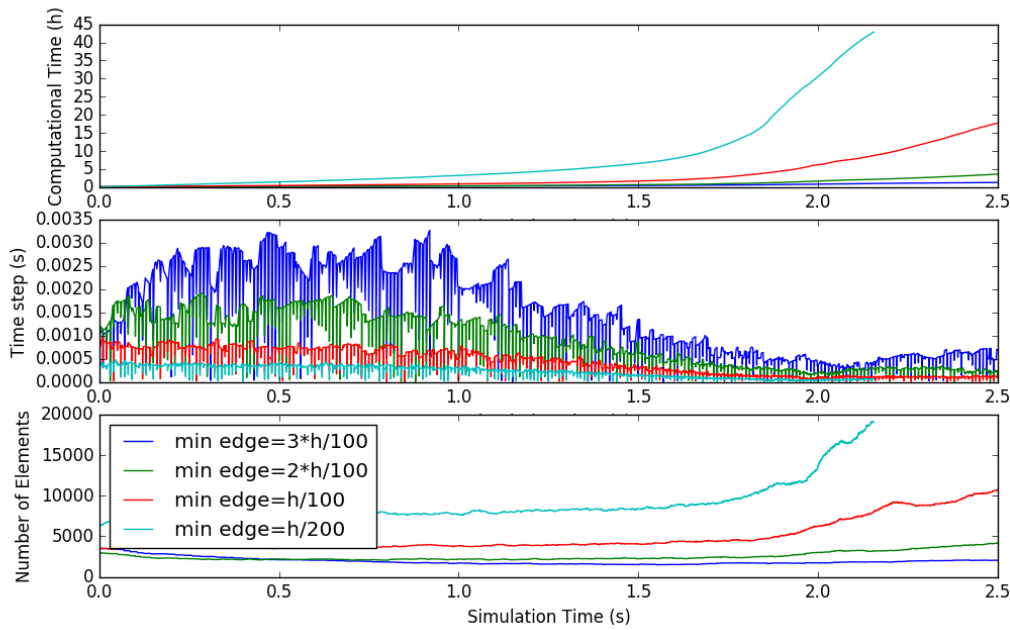


Figure 5.20: Computational performance comparison for different adaptive mesh resolution requirements. Evolution of the computational cost (top), the time step (centre), and number of elements in the domain (bottom), over the simulation time.

Figure 5.20 shows the simulation performance for the different adaptive mesh requirements used in the sensitivity analysis. As expected, the coarser mesh (longer elements leading to

overall lower number of elements) case has a significantly lower computational cost due to lower number of elements, but also due to leading to a higher time-stepping (maintaining the CFL requirement). All mesh sensitivity simulations used the same adaptive time-step scheme settings of CFL=0.25.

## 5.4 3D optimisation with DMO

Numerical approaches such as reducing spatial discretisation order, increasing the time-step or coarsen mesh resolution, and their impact on computational cost have been discussed in previous sections of this chapter and in Chapter 4 for 2D domains. Dynamic Mesh Optimisation (DMO) can also play a key role in lowering significantly the computational cost for 3D domains by lowering the number of elements needed without losing accuracy.

This section discusses and shows the potential towards 3D mesh optimisation and how to make the best possible use of the new  $P_1DGP_1(CV)$  spatial discretisation scheme with dynamic highly anisotropic meshes. This is particularly promising for 3D domains such as 3D numerical wave flumes, where one of the dimensions (width of the flume) play a much smaller role in the simulations outcome than the other two (length and depth of the flume) dimensions in most part of the domain, making anisotropic meshes more attractive.

A 3D simulation of the solitary wave breaking on a slope by (Li, 2000) is used here. The setup of this simulation and experiments are explained in previous Section for the simplified 2D simulation of this experiment. The dimensions (x and z axis) are described in Figure 5.7. The width (y axis) of the numerical flume is set to 0.5 m. A solitary wave with  $H/h=0.4$  is simulated for this Section and compared to experiments with the same wave characteristics published by Li (2000). Further sensitivity analysis of dissipation in 3D is provided in Section 5.5.

Three different resolution settings are used here in order to showcase the full potential of mesh optimisation: a standard isotropic mesh with minimum edge lengths in  $(x, y, z)$  of  $(h/100, h/100, h/100)$  in and maximum aspect ratio of 1 in 5; a highly anisotropic mesh with  $(h/100, h/10, h/100)$

minimum edge lengths and 1 in 20 aspect ratio; and an even higher aspect ratio mesh with  $(h/100, h/1, h/100)$  minimum element lengths (1 in 100 aspect ratio).

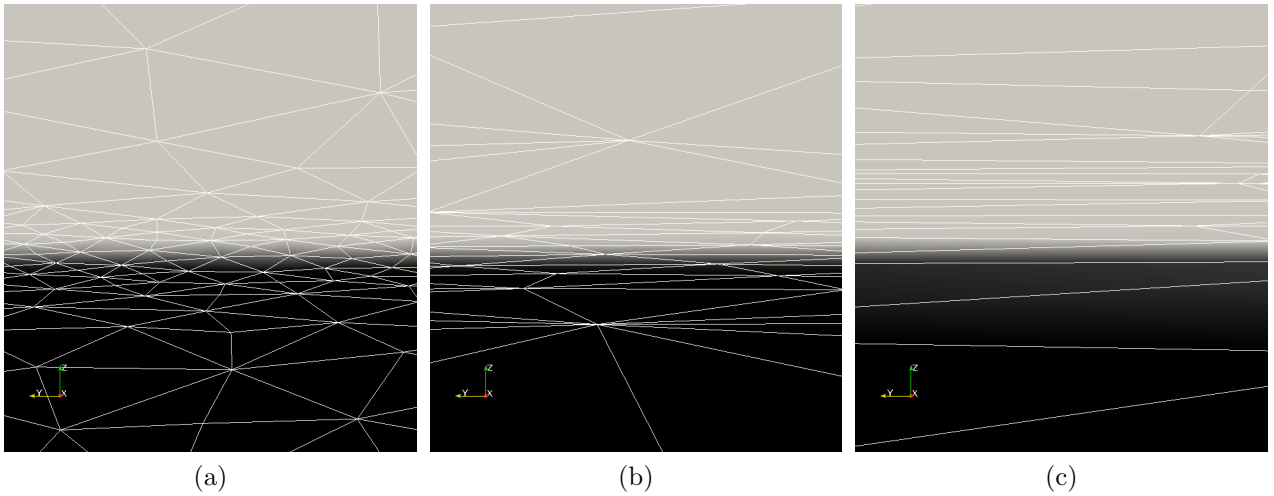
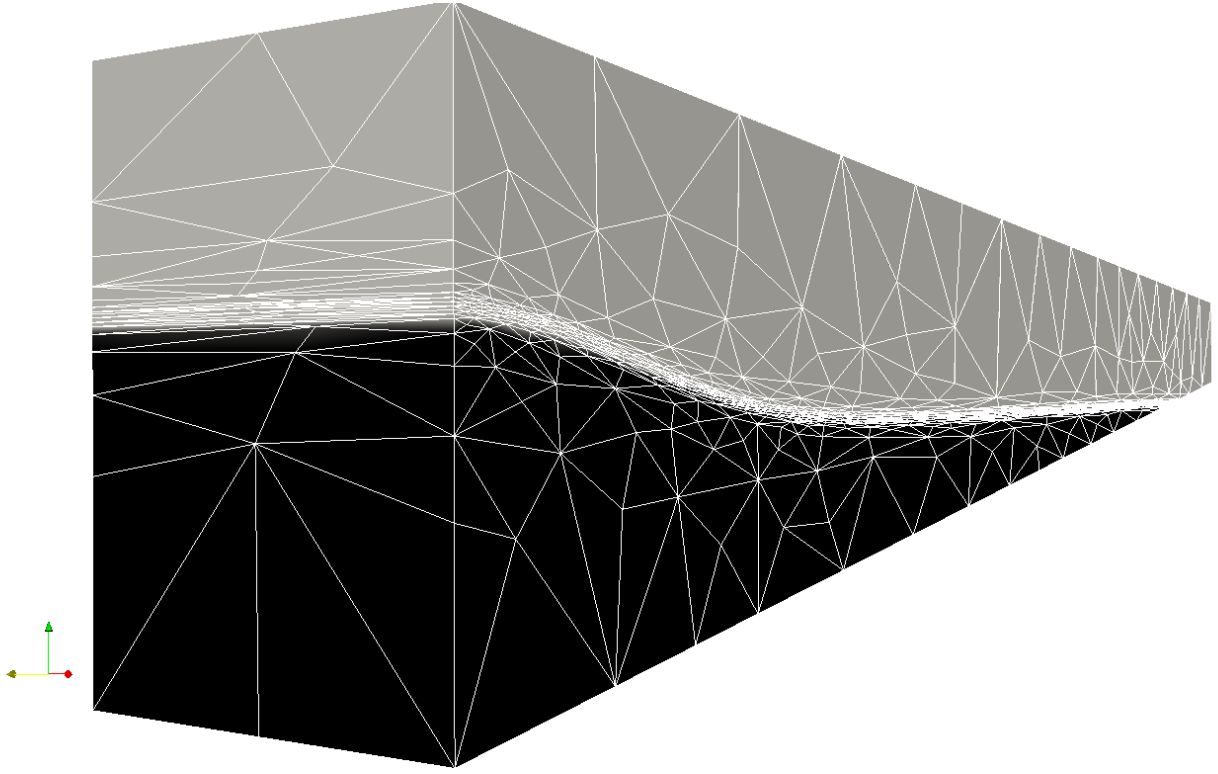


Figure 5.21: 3D multiphase model meshes: a standard isotropic mesh with a maximum aspect ratio of 1/5 (bottom left), a highly anisotropic mesh with maximum aspect ratio of 1/20 (bottom center) and very highly anisotropic mesh with maximum aspect ratio of 1/100 (bottom right). (Via-Estrem et al., 2020)

Figure 5.21 shows the 3D mesh refinement around the interface for a standard isotropic mesh (a, left), a highly anisotropic mesh (b, center) and the challenging anisotropic mesh (c, right). These three meshes have a starting number of elements of 472342 (a), 30915 (b) and 8742 (c),

| Angle  | No. elements (a) | % (a) | No. el. (b) | % (b) | No. el. (c) | % (c) |
|--------|------------------|-------|-------------|-------|-------------|-------|
| 120.00 | 297739           | 63.03 | 25415       | 82.21 | 7036        | 80.49 |
| 150.00 | 40571            | 8.59  | 12675       | 41.00 | 4835        | 55.31 |
| 170.00 | 6                | 0.00  | 1908        | 6.17  | 1603        | 18.34 |
| 175.00 | 0                | 0.00  | 111         | 0.36  | 553         | 6.33  |

Table 5.2: Mesh quality quantitative analysis: number and percentage of elements within the given angle thresholds in the domain by maximum element angles, for standard isotropic mesh with a maximum aspect ratio of 1/5 (left), a highly anisotropic mesh with maximum aspect ratio of 1/20 (centre) and very highly anisotropic mesh with maximum aspect ratio of 1/100 (right).

respectively. Table 5.2 shows element angle quality statistics for the three mesh cases considered here.

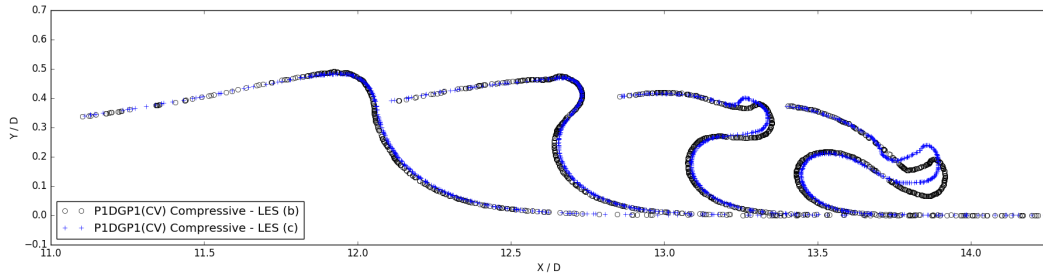


Figure 5.22: Free surface of the LES - P1DGP1(CV) 3D simulations of a solitary wave breaking at different stages with mesh settings for highly anisotropic (Figure 5.21b) and challenging anisotropic (Figure 5.21c). Ratio of wave height to depth at the inlet is  $H/h=0.4$ .

Results from mesh setups (b) and (c) can be seen 5.22, which shows a very close match up to the 4th breaking stage with the plunging jet before impact on the water surface. This indicates that the later stages of the wave breaking process are more sensitive to mesh changes. However, as seen in 2D simulations, this could be due to the high sensitivity from the compressive interface schemes used here.

|              | Isotropic<br>(a) | Highly<br>anisotropic<br>(b) | Challenging<br>anisotropic<br>(c) |
|--------------|------------------|------------------------------|-----------------------------------|
| No. elements | 472342           | 30915                        | 8742                              |

Table 5.3: Starting total number of elements in the domain for standard isotropic mesh with a maximum aspect ratio of 1/5 (left), a highly anisotropic mesh with maximum aspect ratio of 1/20 (centre) and very highly anisotropic mesh with maximum aspect ratio of 1/100 (right).

A qualitative comparison of the wave breaking (plunging) is shown in Figure 5.23 with the very challenging anisotropic mesh setup, which is in very good agreement with experiments

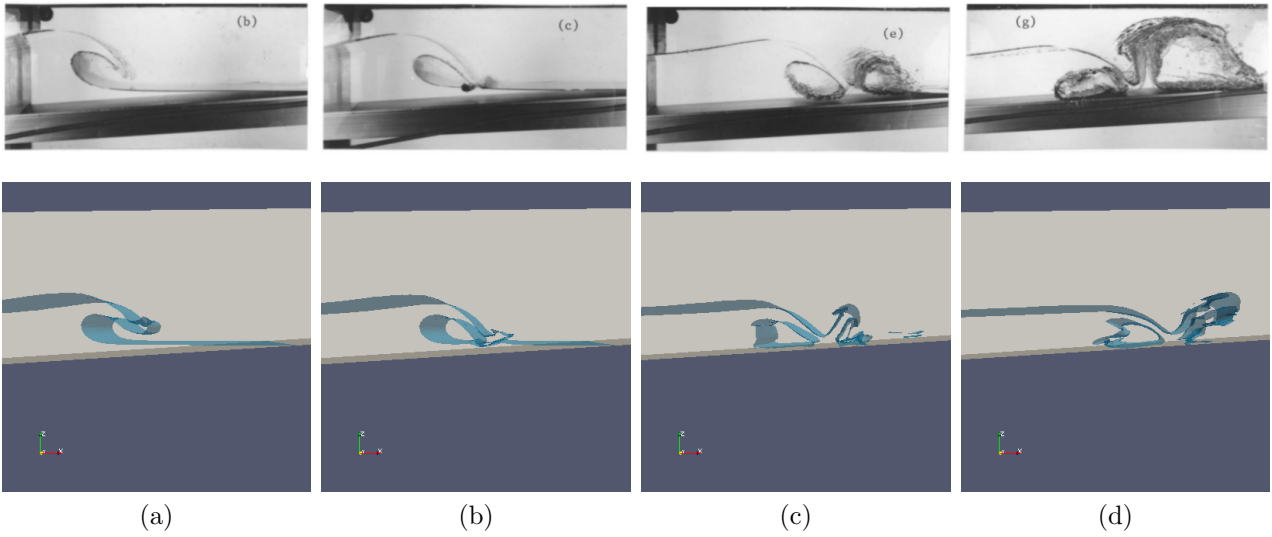


Figure 5.23: Comparison of the results of the 3D wave breaking simulation at different stages against physical modelling results for wave height to depth ratio of  $H/h=0.4$  at the inlet (Li, 2000; Via-Estrem et al., 2020).

(Li, 2000). Using the two anisotropic mesh settings as described above, the total number of nodes needed for achieving accurate results is reduced by two and one orders of magnitudes, respectively (see Table 5.3). It is important to note that no comparison between  $P_1DGP_1(CV)$  and the classical  $P_1DGP_1$  schemes could be shown here as the classical  $P_1DGP_1$  scheme failed to converge with mesh B and mesh C.

Figure 5.24 shows the performance with time-step, number of elements and computational cost for the first 0.1 s of the simulation for all A, B and C mesh setups. The simulation was manually stopped shortly after for cases A and B to avoid the higher computational costs.

## 5.5 Validation of Wave-Structure interaction with experiments of single CoreLoc on slope.

This section aims to validate IC-FERST and the methods described in Sections 4 and 5.2 for NWT and WSI applications. Simulation results are compared against results from physical experiments carried out in the National Research Center of Canada during this PhD (Simpalean, 2019; Eden, 2019; Douglas et al., 2019). The author of this thesis was involved in the exper-



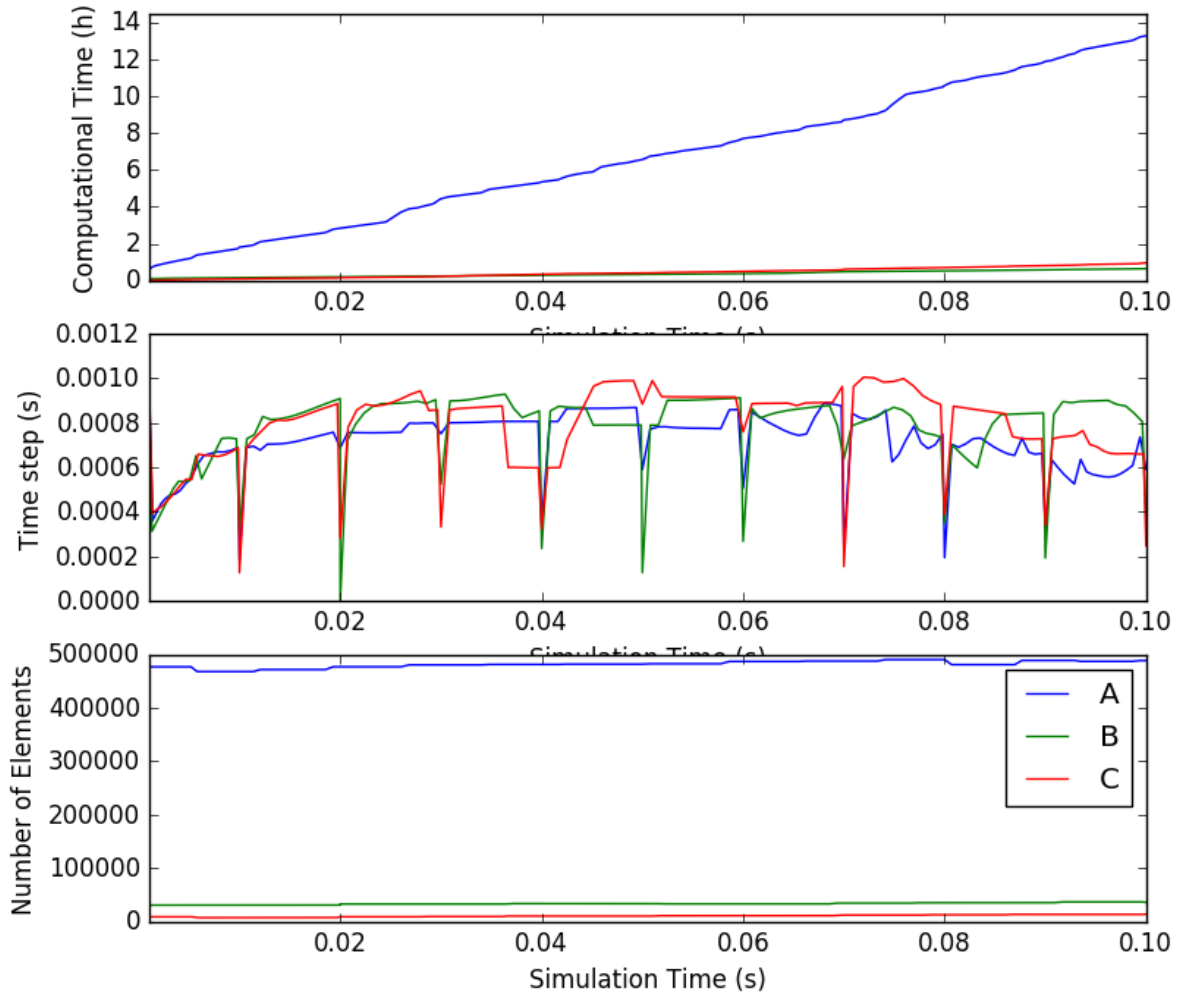


Figure 5.24: Computational performance comparison for different anisotropy levels for the DMO scheme. Evolution of the computational cost (top), the time step (centre), and number of elements in the domain (bottom), over the simulation time.

iments preparation and setup during 1 month placement as Visiting Researcher at University of Ottawa (November 2018).

The experiments were set up in a 50 m flume with a piston type wave paddle using First Order (also known as Linear) Wave Theory. The bathymetry and general setup of the flume is shown in Figure 5.25 and detailed in Table 5.4. A sloped (4H:3V) structure was located at 38.71 m, where an instrumented single CoreLoc unit (3D printed) was placed at different heights of the slope. Pressures were measured using pressure sensors placed at different locations on the surface of the unit. Global forces on the unit were captured by a load cell, which held the unit

in position with a welded rod.

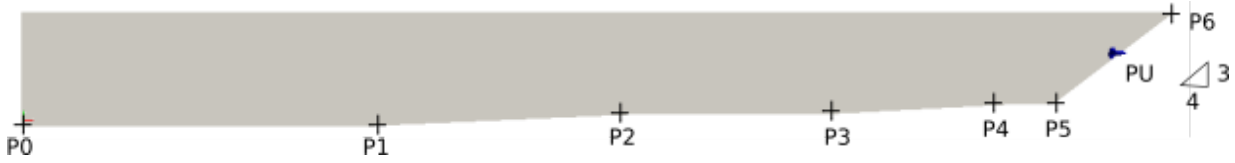


Figure 5.25: Numerical Wave Tank geometry for single CoreLoc unit on slope. Points coordinates shown in Table 5.4. Point PU shows the location of the instrumented Core-Loc unit.

| Point | X (m) [NRC] | X (m) [NWT] | Z (m) |
|-------|-------------|-------------|-------|
| P0    | 27.78       | 0.00        | 0.00  |
| P1    | 31.31       | 3.53        | 0.00  |
| P2    | 34.31       | 6.53        | 0.13  |
| P3    | 36.31       | 8.53        | 0.13  |
| P4    | 38.31       | 10.53       | 0.236 |
| P5    | 38.71       | 10.93       | 0.24  |
| P6    | 39.99       | 12.21       | 1.20  |
| PU    | 39.35       | 11.57       | 0.72  |

Table 5.4: Coordinates of Points describing the domain of the NRC physical modelling flume (left) and the NWT (right). The length of flume between the wave generation paddle (0 m) and P0 (27.78 m) of the physical model facility was not included in the NWT domain.

Wave paddle generation in the experiments using linear wave theory required a long section of the flume for waves to transform to higher order waves corresponding to their length and depth. Waves in the NWT have been numerically generated at the inlet with the streamfunction scheme (see Section 5.2) and therefore no transformation from linear wave theory to a higher order is needed. This allows the shortening of the numerical wave flume and a significantly lower computational cost thanks to reducing the number of elements. The 3D Numerical Wave Tank domain for the validation cases has been set to 1.2 m width and 12 m length (instead of 1.2 m and 40 m in experiments). The wave inlet is implemented at 27.78 m from the wave paddle in the physical model flume, which corresponds to the flat bathymetry and where a wave gauge (WG5) was located, so generated waves can be validated at the inlet. Free surface elevation in the NWT is validated again at the toe of the structure with measurements from experiments (WG7).

Numerical model resolution has been set to 0.01 m and 0.1 m for minimum and maximum element edge lengths (1 in 20 maximum edge length ratio), respectively, as the general setup of the wave flume. Minimum and maximum edge lengths have been set to 0.005 m and 0.05 m,

respectively, from 10 m from the inlet towards the slope and CoreLoc unit. A 1 in 1 maximum edge length ratio has been set around the unit. A linear transition between the flume coarser resolution and the unit finer resolution has been set between 8.5 m and 10 m.

The NRC experiments were carried out with regular waves and short irregular wave tests. In this section only the regular wave tests have been considered for numerical validation due to the shorter simulation length needed, and due to regular waves being easier to replicate from experiments than short irregular wave trains (higher variability and strong dependency on the seed).

| Wave Conditions | H (m) | T (s) |
|-----------------|-------|-------|
| A               | 0.12  | 1.8   |
| B               | 0.15  | 2.0   |
| C               | 0.15  | 2.4   |

Table 5.5: Wave conditions used for validation of numerical simulations against experiments. Depth is constant at  $h=0.72$  m for all conditions.

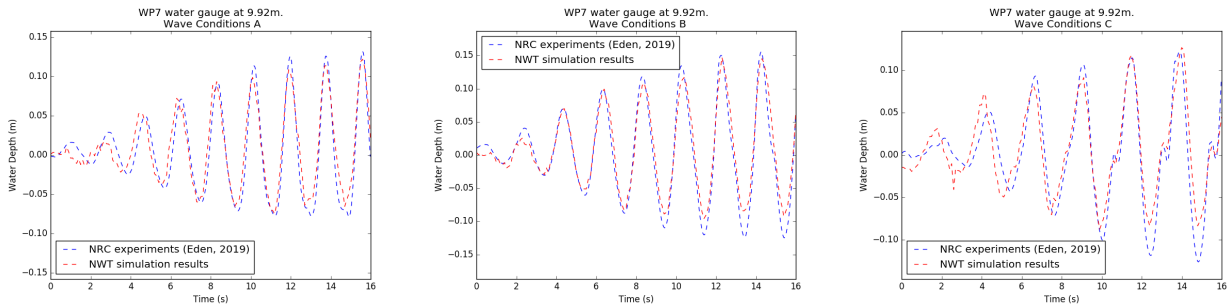


Figure 5.26: Free surface measurements at WP7 from the NWT simulation results (red) against NRC experiments (blue) for Wave Conditions A (left), B (centre) and C (right) (Eden, 2019).

A combination of 2 different wave heights and 3 different wave periods, shown in Table 5.5, have been considered for this validation. Figure 5.26 shows the comparison of free surface elevation from simulations with adaptive time-step scheme ( $CFL=2$ ) and the experiments observations at the toe of the structure. Overall, good agreement is observed between numerical simulations and experiments. Minor differences can be seen at the wave troughs in B for the last 3 wave lengths. These differences are slightly higher in results from C. A small ripple (0.02 m at its peak) in experiments measurements is observed for Wave Conditions C around the 0 m (SWL) line prior to the wave run up. This anomaly appears on the third and fourth waves

and increases in magnitude towards the end of the simulation, coinciding with the deepening of the trough of the waves and where differences between experiments and simulation are higher. This effect is only observed for Wave Conditions C and it is assumed to be induced by different wave reflection in the experiment from the slope for wave with longer periods (above 2 s), or by background waves reflected in the flume from previous experiments.

A qualitative comparison between experiments and numerical results is shown in Figure 5.27. Visual observations at 13.0 s and 13.1 s show good agreement between simulation and experiments and good capture of the air entrapment as the water surface reaches and rises towards and above the CoreLoc unit. Good match between experiments and numerical simulation is also observed during the run down of the wave on the slope, seen in frames from 13.8 s to 14.2 s. Small differences in maximum wave run up can be observed at 13.6 s, having slightly higher run up in the numerical simulation, suggesting that the wave reflection from the slope could also differ between experiments and numerical simulations.

Pressures and forces from the simulation results are compared to experiments with the readings from the pressure sensors and a load cell, respectively. The CoreLoc was attached by a threaded rod to a load cell measuring forces on the unit normal and parallel to the 4H:3V slope. The detail of the load-cell setup is shown in Figure 5.29. Pressure sensors were integrated at different locations on the Core-Loc surface, as shown in Figure 5.28. Pressure readings on the pressure sensor locations from numerical simulations have been obtained internally by IC-FERST and are given at every time-step. Pressure data have been extracted at 4 different points around each of the pressure sensors, taking the maximum value at each sensor for validation against experiments measurements.

Simulations have been run with the adaptive time-step scheme and different CFL values for sensitivity. Figure 5.30 shows the pressure results for all tested CFL values at the different sensor locations for Wave Conditions B and compares them to measurements from experiments. Experimental data show a different evolution or increase of the the pressure peaks, as expected due to having different wave generation ramping up set up (the wave paddle ramping up from experiments was not provided). Overall, simulations results show good agreement for all CFL

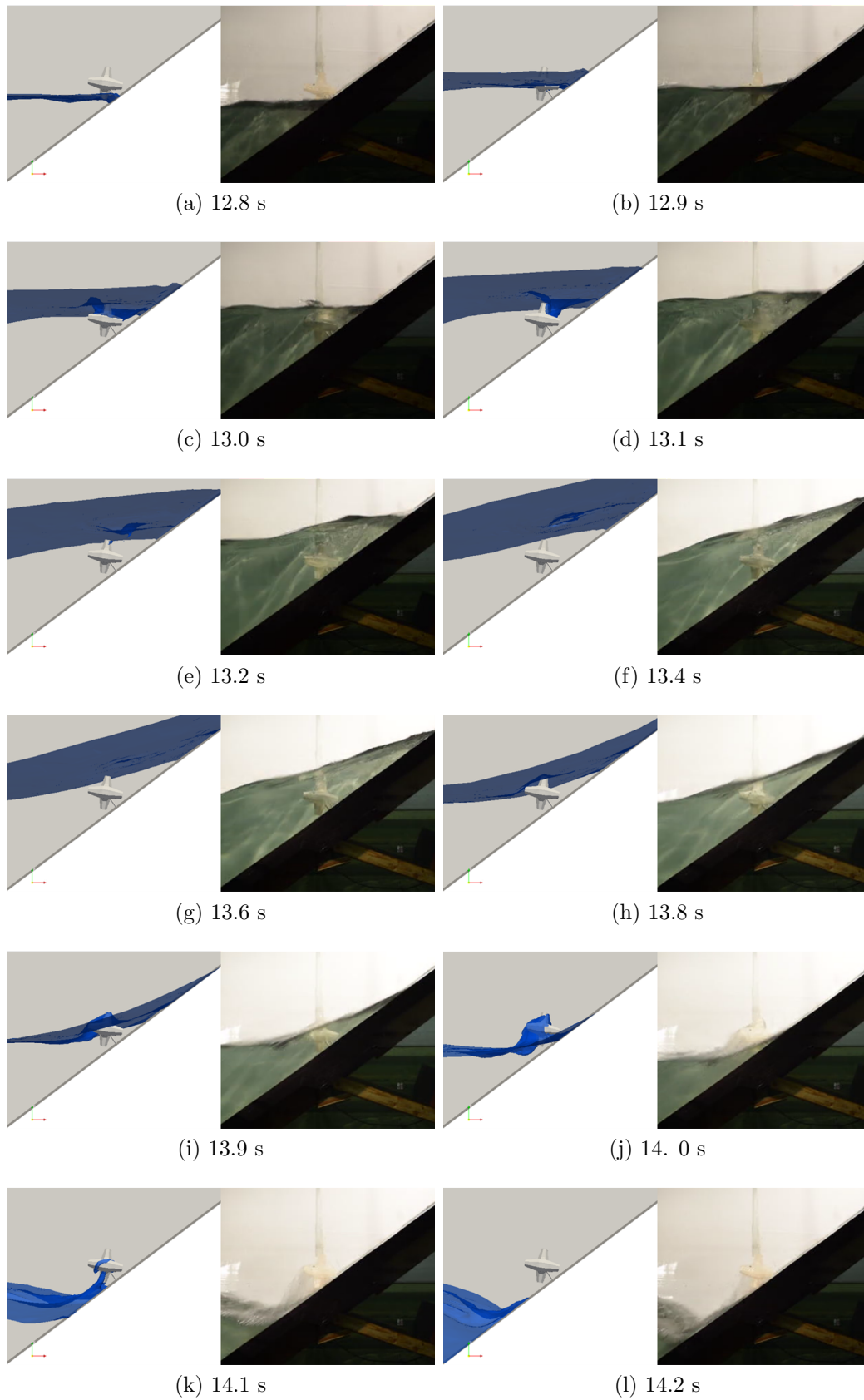


Figure 5.27: Qualitative comparison between experiments and numerical simulation (CFL=2) results of wave-structure interaction with a CoreLoc unit on a slope for Wave Conditions B. Side view.

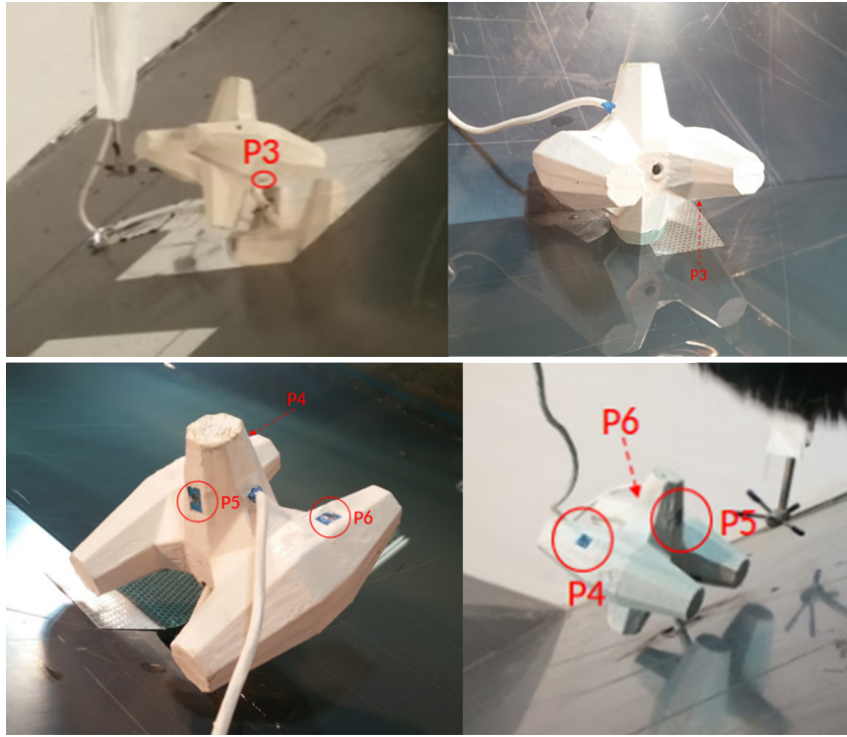


Figure 5.28: View of the instrumented CoreLoc on the sloped plane and location of pressure sensors on the CoreLoc surface (Eden, 2019).

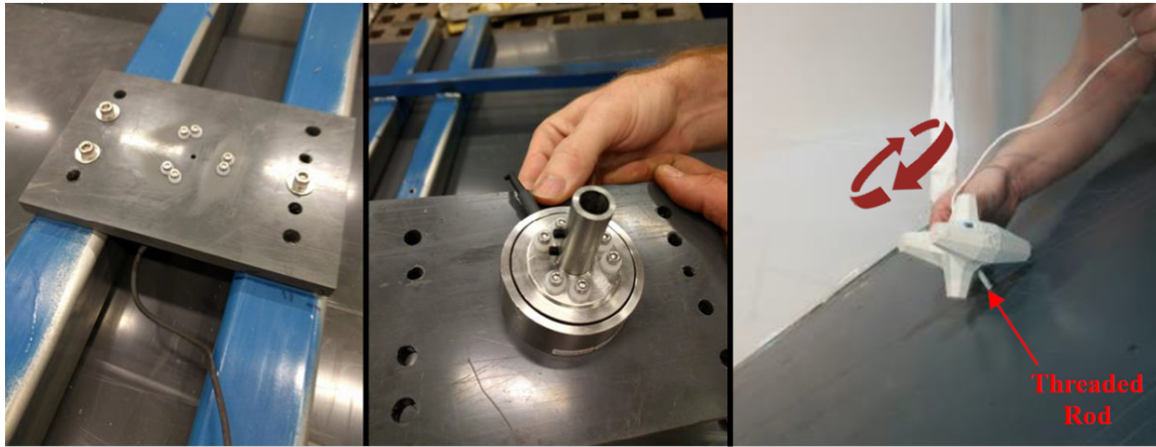


Figure 5.29: Load cell platform setup under the slope and installation of the CoreLoc on the sloped surface, The Core-Loc is held by the load cell via a threaded rod going through a drilled hole in in the platform (Eden, 2019).

values with experiments for the last 3 waves, in the second half of the simulation, coinciding with the fully generated waves observed at the WP7 (Figure 5.26). Simulation results using CFL of 0.5 show generally higher pressure values than higher CFL values (more energy dissipation) as expected. Higher differences between CFL values are observed for sensor P3, while very small to insignificant differences are observed for P4, P5 and P6. Pressure sensors show pressure peaks



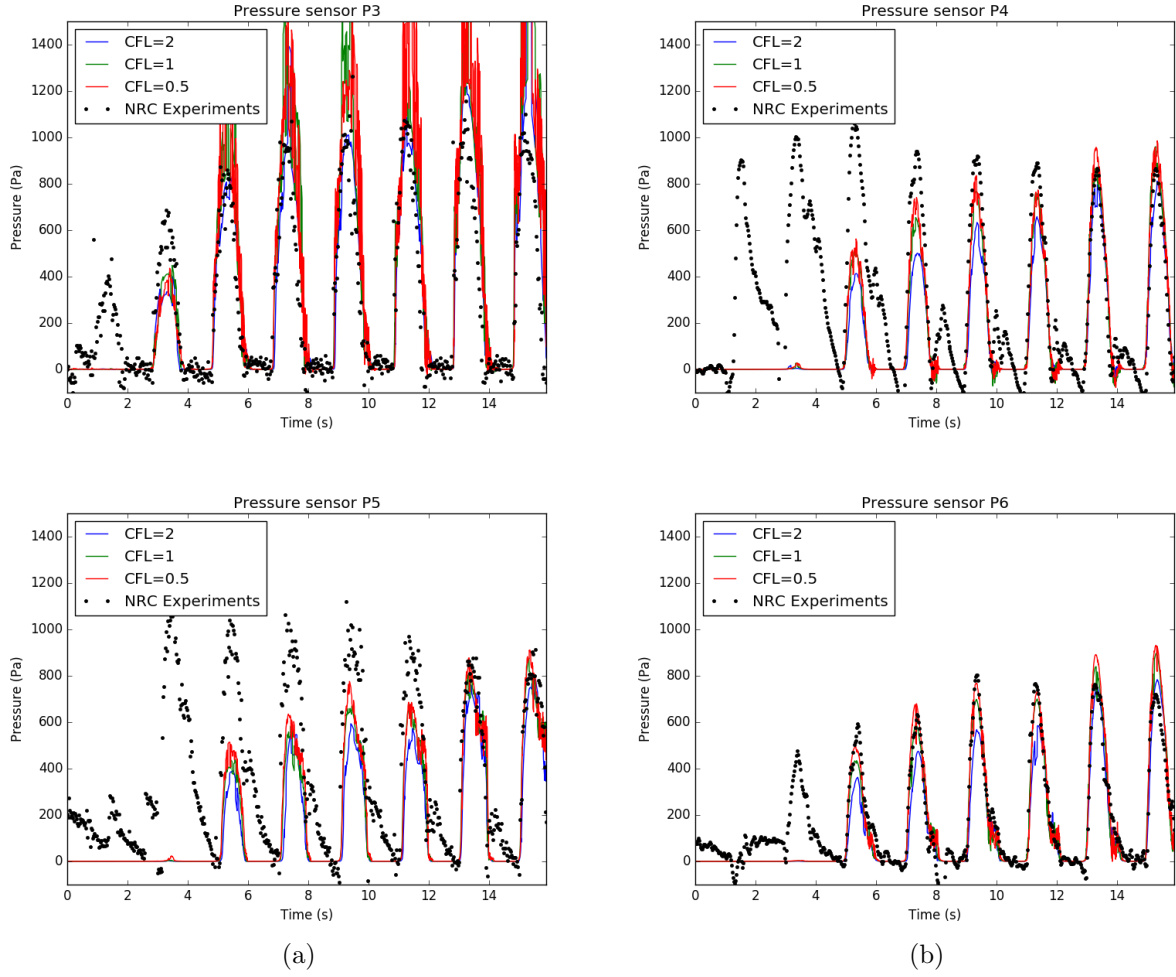


Figure 5.30: Pressure readings at pressure sensors P3 (top left), P4 (top right), P5 (bottom left) and P6 (bottom right), for Wave Conditions B from IC-FERST model results under adaptive time step for CFL=2 (blue), CFL=1 (green) and CFL=0.5 (red), against experiments (black).

of the same order of magnitude at all locations, suggesting no wave impact or wave breaking occurs around the unit during this test, as this would likely lead to much higher pressure values in the sensors directly impacted by the wave slamming force or breaking.

Individual pressure sensors on different locations can also give a good understanding of the WSI physics and how IC-FERST is resolving them. Sensor P3, which is located at the surface underneath one of the CoreLoc horizontal arms, shows good agreement with experiments and the pressure due to wave run up against the bottom surface of the CoreLoc, with even higher values than experiments for the simulation with lower CFL value.

Pressure sensors P4 and P6 present similar values as a result of both being located on upwards

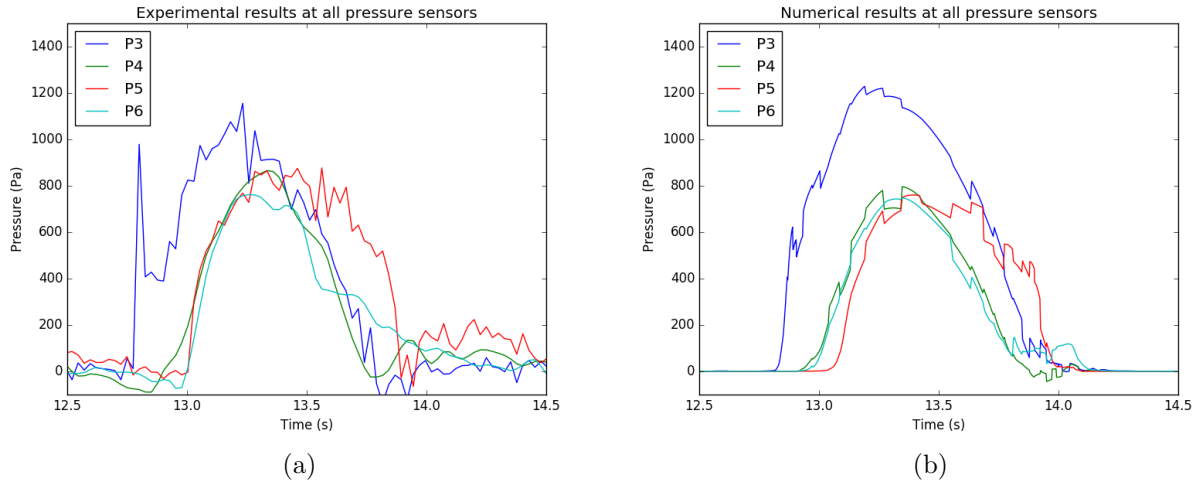


Figure 5.31: Wave induced pressures at pressure sensor locations P3 (dark blue), P4 (green), P5 (red) and P6 (light blue) from experiments (left) and numerical simulations (right). Wave Conditions B.

orientated surfaces of the horizontal arms of the CoreLoc. A good match is observed between experiments and simulations for CFL of 0.5 and 1 on all waves, and a gradually improving match towards the last waves of the simulation for CFL of 2. This is believed to be induced by higher dissipation during the first waves of the test for higher CFL values.

Pressure Sensor P5, located at the back of the vertical central arm of the CoreLoc, shows similar pressure built up towards the end of the simulation and a good match at the end of the simulation. This sensor shows the wave induced pressures of the wave run down on the slope. Differences between simulations with CFL=2, CFL=1 and CFL=0.5 are of the same order of magnitude as for P4 and P6 sensors.

Figure 5.31 shows the pressure peak evolution between 12.5 and 14.5 s for all four pressure sensors in the numerical simulation (left) and in experiments (right). Pressure peaks for the different pressure sensors are seen to be occurring at different times in both figures, as expected, due to the sensors different locations and orientations. The first pressure peak is at P1, which is located at the lower face of an horizontal arm of the CoreLoc and is the first sensor reached by the water surface, in both numerical and experimental data. A small early peak is observed in the experiments (left), but it is attributed to be noise or errors in the signal from the sensor. Small differences are observed at the tail, where the experimental data show a faster decay in



pressure than in the numerical data.

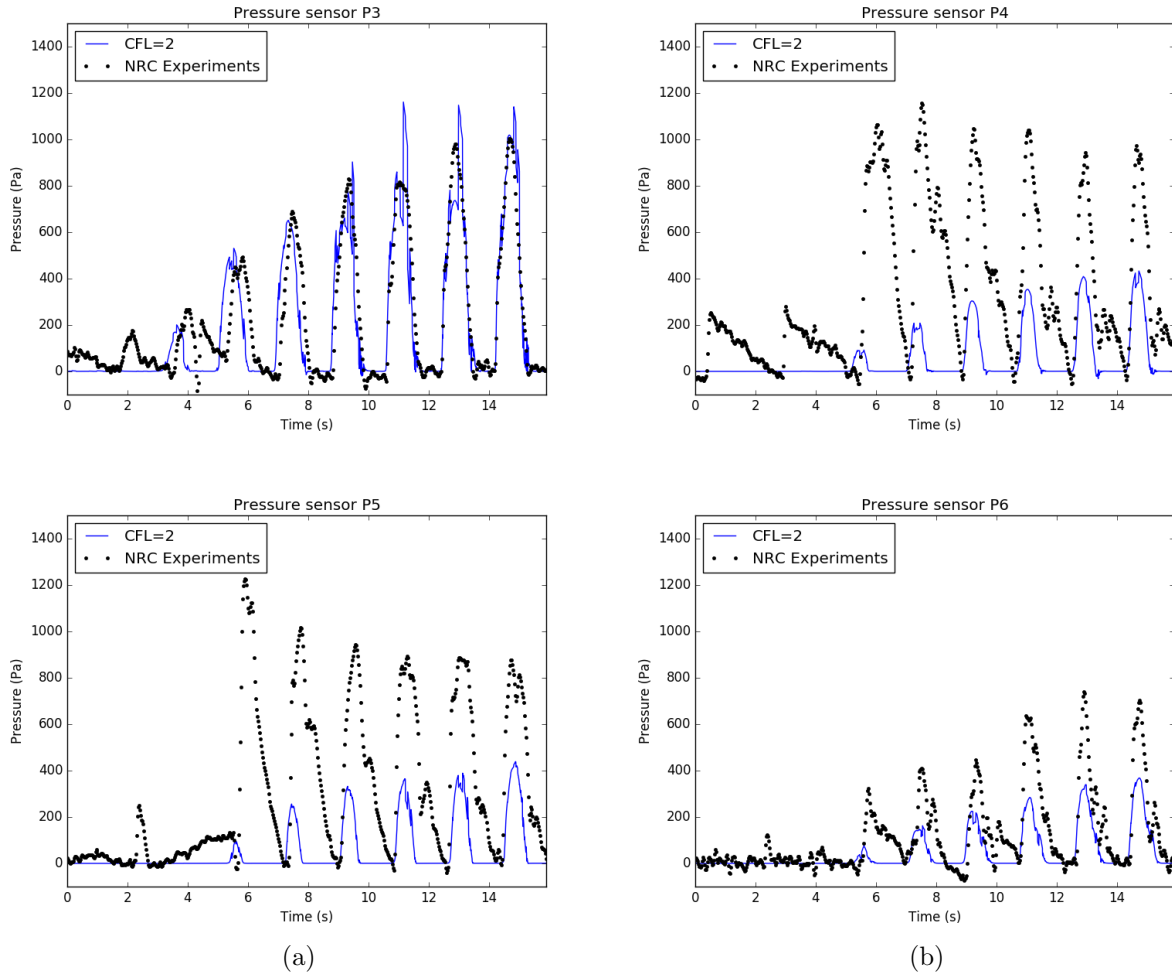


Figure 5.32: Pressure readings at pressure sensors P3 (top left), P4 (top right), P5 (bottom left) and P6 (bottom right), for Wave Conditions A from IC-FERST model results using adaptive time-step with CFL=2 (blue), against experiments (black).

Pressure sensors P4 and P6, on the upper surfaces of the horizontal CoreLoc arms, show pressure peaks just after P3 readings and no significant differences are observed between numerical simulation (right) and experiments (left). Pressure sensor P5 figures show some differences between simulation and experiments in terms of starting times of the pressure value increase. While the pressure in P5 rises at 0.1 s after P4 and P6 do in the numerical simulation, pressure rises in P5 at the same time as P4 and P6 in experiments, indicating that the water surface in the wave run down may differ between numerical simulation and experiments due to differences in wave reflection from the slope.

Figures 5.32 and 5.33 show simulation results for Wave Conditions A and C, respectively, with

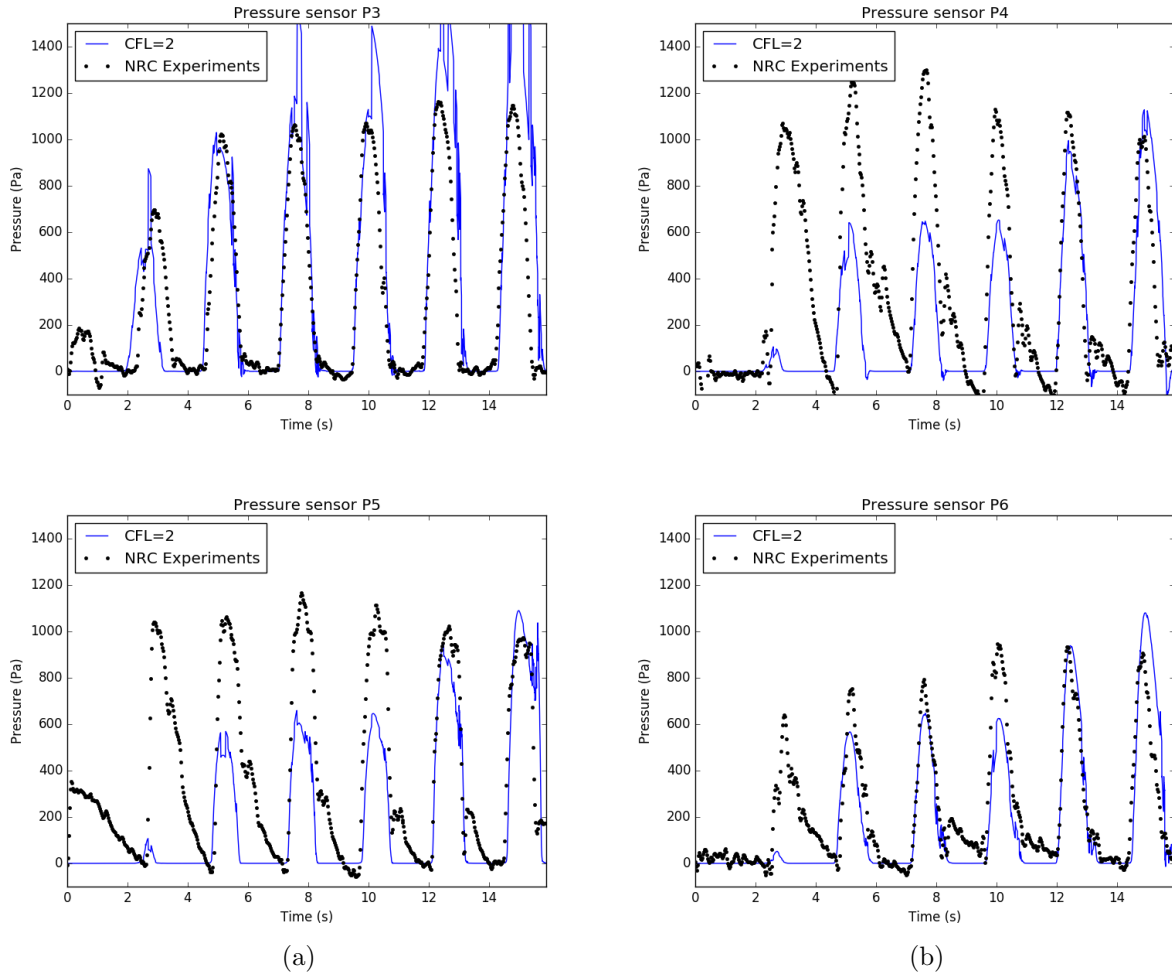


Figure 5.33: Pressure readings at pressure sensors P3 (top left), P4 (top right), P5 (bottom left) and P6 (bottom right), for Wave Conditions C from IC-FERST model results using adaptive time-step with CFL=2 (blue), against experiments (black).

an adaptive time-step scheme for CFL=2. While a good match is observed between experiments and simulation results for Wave Conditions C (Figure 5.33) at the end of the simulation, significant differences can be seen in peak pressure magnitudes for Wave Conditions A. Major differences for Wave Conditions A can be a consequence of higher dissipation in shorter wave periods when high CFL values are used for the adaptive time-step scheme. Moreover, an increase in pressure magnitude (when compared to experiments) is observed when moving from wave period of 2 s to wave period of 2.4 s, adding on the hypothesis of lower periods resulting into higher wave energy dissipation. However, this hypothesis needs further investigation since no significant differences are observed in wave height for the different wave periods at WP7 (Figure 5.26).

Force measurements from the experiments can also be used for numerical model validation. In this case, IC-FERST did not have an already implemented method for force extraction from the simulations and a new routine has been written. This preliminary work is based on the assumption that the total force on the CoreLoc unit could be obtained by integrating the pressure around the unit surface.

The newly developed pressure integrated force routine only requires the simulation outputs (\*vtu or \*pvtu formats) and a given geometry (ideally a surface stl format file), and can extract the total force on any surface or unit independently, including for simulations with domains with full breakwater sections (see Chapter 6 for full layer breakwater simulations). The pressure is extracted at every node of the triangulated surface and averaged at the centre of each triangle. The force is then obtained by calculating each triangle's area and using the surface normal for the force direction, resulting into a vectorised form of the force. This routine has been parallelised using the "multiprocessing" python library.

This integration of the pressure around the unit surface based on output files of the simulation implies that the force extraction frequency coincides with the simulation results output frequency (simulation results files were pre-set to an output frequency of 10 Hz for this validation). This differs from the pressure extraction from individual pressure sensor locations. This frequency difference is due to computational cost. Pressure sensors represent much fewer extraction points and can be extracted internally by IC-FERST without significantly increasing the simulation computational cost. The advantage of extracting pressure internally is the resulting normally higher frequency of data points when compared to the output files and that there is no need to read the often high memory demanding output files (above 1 GB each). Alternatively, the external based process of pressure integration reduces the computational cost since IC-FERST does not need to extract hundreds to thousands of points internally at every time-step. While the external routine based on output files gives lower frequency of data points and therefore lower accuracy in time evolution, it gives much higher versatility and allows choosing what units want to be processed at any time, whereas the pressure readings from individual points need to be set before the start of the simulation.

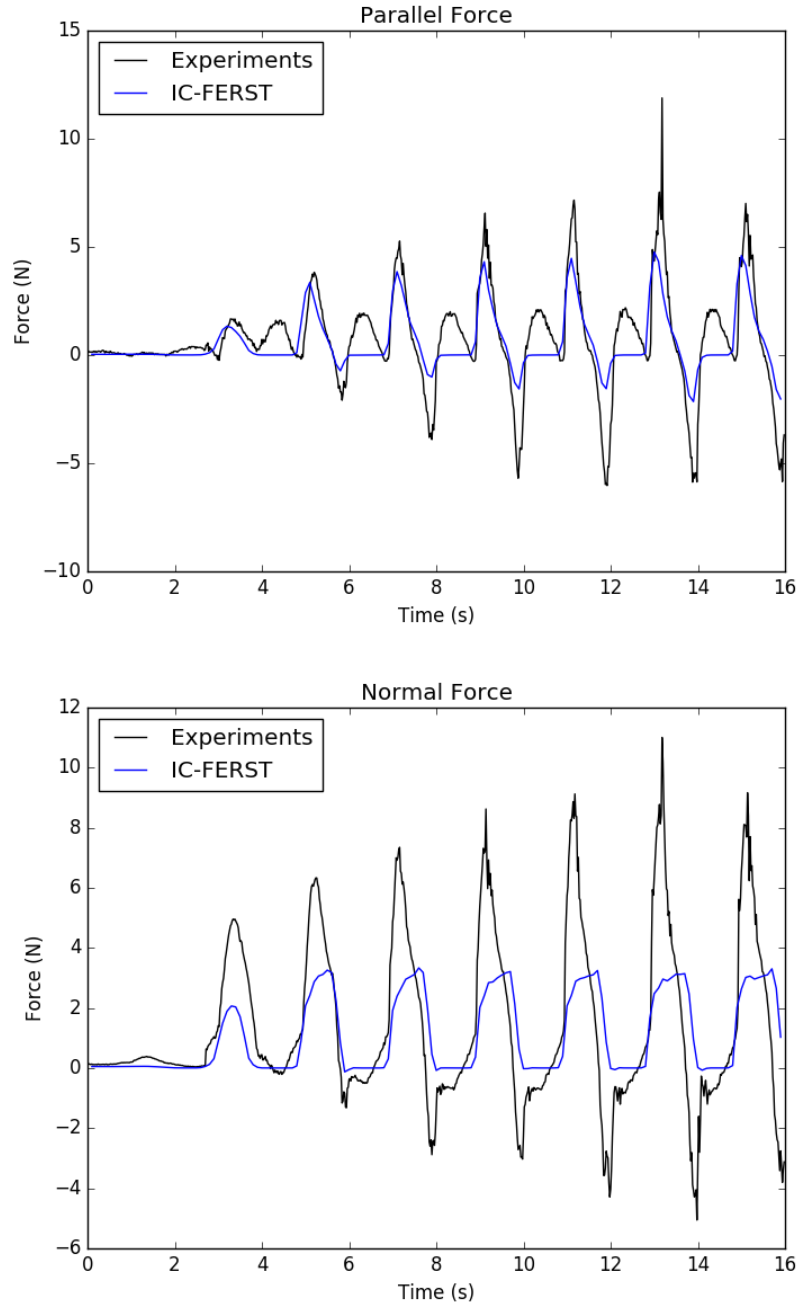


Figure 5.34: Comparison of hydrodynamic parallel (top) and normal (bottom) forces to the slope on the CoreLoc unit between experiments (black) and preliminary pressure integrated force results from numerical simulations with CFL=2 (blue). Wave Conditions B.

The preliminary results of pressure integrated forces are compared to experiments and shown in Figure 5.34 (using adaptive time-step with CFL=2), for parallel forces to the slope (top) and normal forces to the slope (bottom). Positive force indicates a force up the slope for parallel forces and off the slope for normal forces. A small force peak (nearly constant in magnitude

since its first appearance) is observed in experiments and not in numerical simulation results. This peak, located after the main and most significant force peak (occurring periodically), is only observed in the slope parallel force figure. Its only appearance in parallel force data, and its peak force remaining constant even when the main peak is increasing in time, suggests that this could be caused by unexpected movement of the rod (holding the CoreLoc) and the rod interacting with the sloped platform by the rod hole sides. A simple explanation is that a small bending spring bounce back effect (out of synchronisation with the main wave-induced swaying of the rod) could be generating this force on the load cell, but this could not be verified by the authors of the experiments. This hypothesis would also suggest that an inertia component of the total force in experiments could be missed due to the static condition of the CoreLoc in the simulation (Lipkowitz, 2020). Displacements of the CoreLoc due to the elastic deformation of the rod were not considered significant and were not monitored during the experiments, and therefore this hypothesis can not be investigated in depth (inertial component of the force could not be isolated). A sensible match is observed for positive parallel forces in the main force peaks otherwise, with simulation results showing up to 30% lower peak maximum values. Negative force values for parallel forces are significantly smaller in simulation results, which indicates that forces downwards are not fully captured by the method of pressure integrated force or by the model.

Normal forces to the slope show disagreement at the force peaks, with simulation results data down to 40% of experiments data. While normal force values in experiment measurements have a steep increase and decrease around a very short peak, simulation results show nearly flat long peaks after a short steep increase. This suggests that the force captured by the pressure integrated approach is highly dominated by the buoyancy of the unit and the uplift drag force generated is underestimated. Moreover, no negative values (force against the slope) are observed in simulation results for normal force. Missing the uplift drag force in numerical simulations could be explained by different hypotheses. A first hypothesis is that the free surface elevation during the run-up and run-down of the wave differs between experiments and numerical simulations (i.e. due to non captured physical model non-planned or non-observed phenomena). The small differences in maximum run-up observed in Figure 5.27 could support

this hypothesis, but free surface differences are considered low when compared to the very significant force differences. The second hypothesis is that the CFD model does not capture the wake effect of fluid flow on the unit, but this hypothesis would not be supported by the good match of pressure measurements in all pressure sensors, which should show significant differences to make this a valid hypothesis. A third hypothesis suggests that the drag force by the fluid-solid friction is significantly high, but that this is not captured by the model and by the force extraction approach.

Although the friction of a flat surface with fluid flow and the subsequent skin drag force has initially been assumed small for the integrated pressure approach applicability, as it would be expected from both model and prototype CoreLoc units, this is not the case of the unit used in these experiments. The unit was 3D printed and very small steps corresponding to the different layers laid out by the 3D printer could be observed at the unit surface. This non-flat outer surface of the unit is expected to have generated an unusually higher friction with the fluid flow and could explain the significant differences observed between experiments and numerical simulations. Furthermore, no breaking or wave impact is observed in these tests and therefore the force generated by the friction of the unit with the fluid flow is expected to have a larger impact on the total force when compared to slamming wave impact scenarios. The capture of potential skin drag forces induced by the solid surface friction or roughness, and alternative force extraction approaches, are discussed in Chapter 7 as part of Future Work.

Results from sensitivity analysis with different CFL values for the model simulations, shown in Figure 5.35, show only minor differences. Small positive increase and small negative increase are observed for CFL=0.5 at the parallel force positive and negative force peaks, respectively, when compared to CFL=1 and CFL=2. No significant differences are observed for the normal forces to the slope when using different CFL values.

The distance between the CoreLoc unit and the slope in experiments was not provided and a rod longitude ( $L_r$ ) of 0.063 m in the model has been assumed. Figure 5.36 shows the results of exploring the variation of rod length and the impact  $L_r$  can make on numerical simulation results. Significant differences are observed between  $L_r = 0.073$  m and  $L_r = 0.058$  m, with

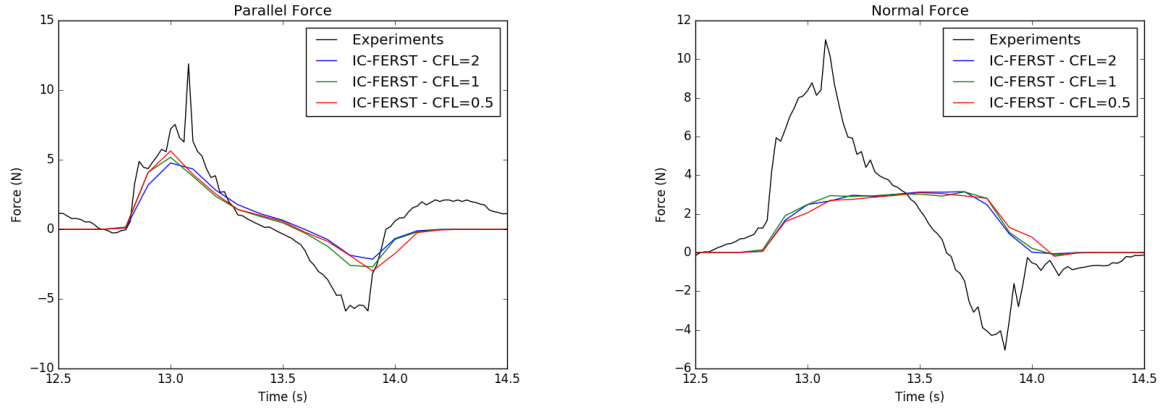


Figure 5.35: Comparison of hydrodynamic parallel (left) and normal (right) forces to the slope on the CoreLoc unit from numerical simulations using CFL=2 (blue), CFL=1 (green) and CFL=0.5 (red) for the adaptive time-step scheme, against load cell measurements from experiments (black). Wave Conditions B.

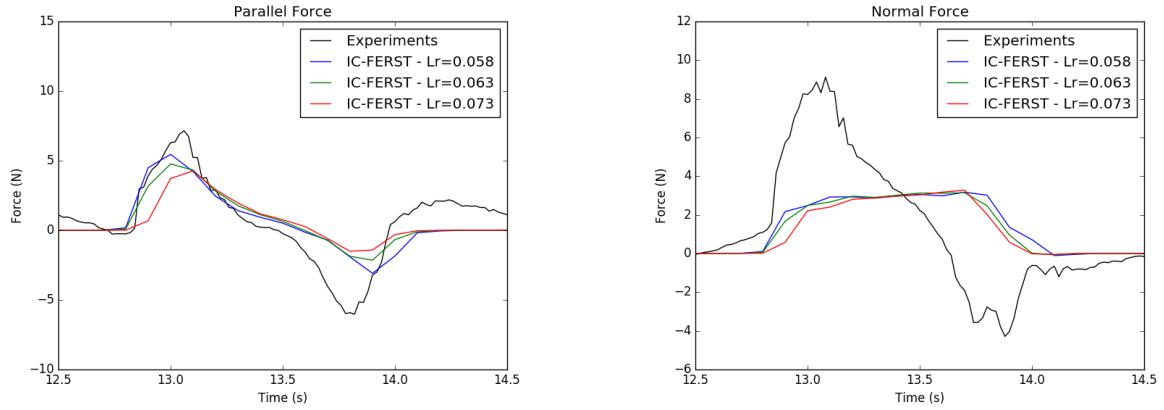


Figure 5.36: Comparison of hydrodynamic parallel (left) and normal (right) forces to the slope on the CoreLoc unit from numerical simulations using different CoreLoc rod lengths against experiments results. All simulations using CFL=2. Wave Conditions B.

larger horizontal force values both in the positive and negative force peaks. However, only very small differences in normal force peak magnitude are observed, which adds onto the assumption that the main component of the normal force from these results comes from the buoyancy force.

Figure 5.37 shows the parallel and normal forces on the unit for experiments and numerical simulations (CFL=2) under Wave Conditions A and C. The small peak prior to the main peak is also observed for parallel forces, as observed in Wave Conditions B. Normal forces for wave conditions A and C show same flat peak trend as seen in wave conditions B. However, a better match and slightly smaller differences between model and experiments are observed for longer

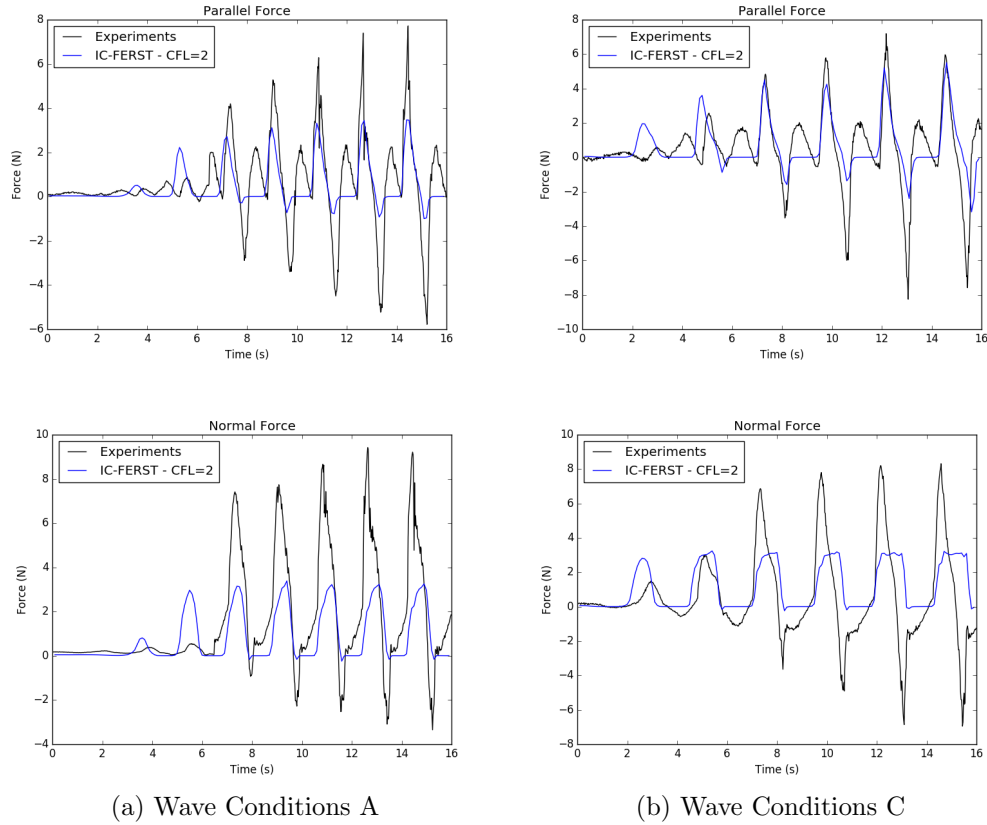


Figure 5.37: Comparison of hydrodynamic parallel (left) and normal (right) forces to the slope on the CoreLoc unit between experiments (black) and preliminary pressure integrated force results from numerical simulations using CFL=2 (blue). Wave Conditions A (left) and C (right).



period wave conditions (C), as previously seen when analysing the pressure measurements. This is assumed to be induced by using same mesh resolution for all wave heights and periods, which leads to higher energy dissipation for shorter wave periods and smaller wave heights.

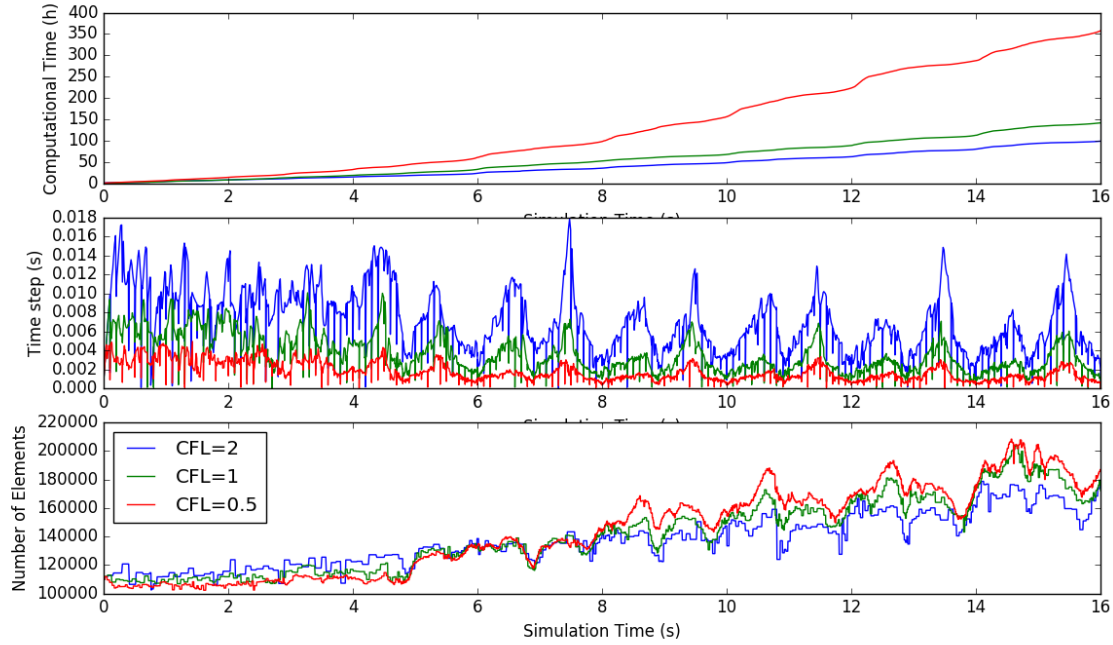


Figure 5.38: Numerical performance of the single CoreLoc unit simulation (Wave Conditions B) using CFL=2 (blue), CFL=1 (green) and CFL=0.5 (red) for the adaptive time-step scheme. Evolution of the computational cost (top), the time step (centre), and number of elements in the domain (bottom), over the simulation time.

Figure 5.38 shows the computational performance of the simulations with CFL=2, CFL=1 and CFL=0.5 for wave conditions B in terms of computational cost (time), time-step and number of elements. All simulations have been run using 12 cores in parallel processing.

## 5.6 Discussion

Very good match is observed between analytical solution and the numerical results using the IC-FERST setup described in Chapter 4 and the newly implemented wave generation library described in this chapter. The wide range of tested wave conditions, which include main wave theories observed in nature and in experiments, provide with enough evidence for the validation

of the wave generation scheme used here.

Although plunging (breaking) waves have presented a big challenge for the compressive interface scheme, the differences between experiments and numerical simulations have been smoothed out by using an LES turbulence model, with no computational cost increase.

FEM methods have been successfully used here with the latest IC-FERST developments and DMO for 3D domains, and now deployed for the first time in a NWT, which helped in reducing significantly the cost and the required number of elements used by the traditionally expensive FEM codes. Results of the validation studies of a single CoreLoc unit on a slope show that these cost reduction schemes and approaches lead to no reduction in accuracy.

Although pressure measurements are well matched by the simulation results in all wave conditions, pressure observations in the model are seen to improve when simulating longer periods as wave energy dissipation is expected to be reduced. This can be reduced by increasing mesh resolution when required.

Preliminary work on force extraction from numerical simulations using a surface pressure integration around the unit show promising results, although it seems to be underestimating the friction force, specially when modelling units with non-flat outer surface. For these cases an alternative approach or adding friction in the simulation should be considered. Displacements of the CoreLoc due to the elastic deformation of the rod were not monitored during the experiments, and therefore the inertial component of the force could not be isolated and included in the force analysis comparison.

Overall, this chapter shows the potential of the presented methods for NWT applications and WSI with a CoreLoc unit at model scale. This technology and its full potential is shown in the next chapter (Chapter 6) with simulations of full sections of breakwater layers at model and prototype scales.

# Chapter 6

## Numerical modelling of rubble mound breakwaters

### 6.1 Introduction

Numerical models capable of modelling wave-structure interaction with breakwaters have become available over the last decade. The most common approach for modelling rubble mounds is however to consider the armour layer as a porous layer. Although this simplification of the layer is widely accepted for overtopping analysis on breakwaters (Richardson et al., 2013), it does not resolve the rubble mound particles geometry and it is generally regarded as inadequate for stability analysis of the particles and the breakwater layer as a whole.

Most relevant studies that properly modelled wave-structure interactions with the real geometry of the layer particles are by Dentale et al. (2009, 2014) using Flow3D with Finite Differences Method (with a rectangular grid of different resolutions), by Altomare et al. (2012, 2014); Zhang et al. (2018) using DualSPHysics (with SPH, single resolution and single water phase) and by Latham et al. (2015); Xiang J. (2019) using OpenFOAM wave generation library OlaFlow (Higuera et al., 2013)(Finite Volume Method with rectangular grid of different resolutions).

Recent developments within the AMCG CFD codes (Pavlidis et al., 2014; Xie et al., 2014;

Adam et al., 2016; Pavlidis et al., 2016; Xie et al., 2016, 2017a,b) and the new contributions and optimisation techniques developed during this PhD on numerical schemes (see Section 4) and NWT applications (see Section 5) have made FEM AMCG CFD codes with dynamic mesh optimisation a realistic potential alternative to other CFD methods for modelling rubble mound layers with their real geometry. Moreover, newly developed and tested schemes making dynamic mesh optimisation robust with highly anisotropic meshes can lead to a better capturing of the geometry of fixed (or moving) particles in the domain while keeping high accuracy and low computational cost (Via-Estrem et al., 2020).

The reasons to consider this model and its potential as a promising technology when compared to other already existing models are twofold. First, AMCG CFD models have already been successfully coupled with solids FDEM models (Solidity) for coastal and maritime applications (see Latham et al. (2008b, 2009); Vire et al. (2015, 2016)), but these had not yet been capable of modelling two-phase WSI in NWT with a conventional wave generation scheme, which are now addressed by this thesis and showcased in this chapter. Second, the one-way or wave-proxy approach (Latham et al., 2015; Xiang J., 2019), which considers the modelling of the initiation of movement, can be loosely coupled with any CFD code and has the potential to benefit from higher numerical and geometrical accuracy demonstrated by DMO and FEM-DG methods.

The construction capabilities of realistic numerical armour layers are key for any modelling of wave-structure interactions. In this way, the development and validation of new approaches for building concrete armour layers by Anastasaki et al. (2013); Anastasaki (2015) was a key achievement for modelling realistic concrete armour layers. A new computationally rapid bulk layer placement approach and a sophisticated one-by-one virtual crane approach for building rock armour layers have also been explored and tested, and their results discussed in Section 3.

The aim of this chapter is to demonstrate the novel capabilities of modelling realistic armour layers, with a sufficient number of armour units to form a representative armour layer, using the CFD model and NWT tools presented in Chapters 4 and 5. This chapter also discusses meshing challenges for simulations with dense multi-particle systems. Special attention is given to elements restrained by the domain boundary, such as in intersections (i.e. unit-boundaries)

and complex topology of the void space between solid bodies, which can lead to very low quality elements and subsequent numerical instabilities.

The chapter is structured as follows. A simplistic version of a rubble mound layer using spheres is used in Section 6.2 for showing flow through multi-particle system, where intersections of unit-unit and unit-boundary surfaces are initially tested. This is followed by CoreLoc armour layer simulations in Section 6.3, where mesh problems linked to unit-boundary intersections and very close neighbouring units are exposed and successfully tackled. Finally, rock layers are successfully modelled in Section 6.4. Forces and velocities are extracted for the simulations of rock armour layers as explained in Section 5.5.

Two examples are shown to demonstrate the potential applicability of the work presented in this Chapter. First, the impact of different climate change scenarios on rubble mound stability is shown in Section 6.4.2. The second example considered here has looked at how this technology can be used to evaluate scale effects on results from physical modelling experiments, and is shown in Section 6.4.3.

## 6.2 Simplistic spherical particle layers

A very simplistic version of a rubble mound breakwater is considered here with a double layer of spherical units of 1.4 m in diameter. This simplification of the layer particles is used for analysis of the mesh quality and feasibility of modelling this layer with the CFD code presented in Sections 4 and 5. Three different configurations of sphere grids are considered on a 1H:3V slope (replicating 1H:3V slope of rock armour layers presented in Section 3). Figure 6.1 and Table 6.1 show grid spacing between spheres for each configuration, where  $r$  is the sphere radius (0.7 m).  $dy$  indicates the distance between centroids of neighbouring particles in the same row.  $dx'$  is the distance upslope between rows and  $dz$  is the vertical distance between layers.

Configuration A consists of a regular grid of non-intersecting (unsupported in space) spheres at a spacing of 2.2 radius between sphere centroids. This configuration has intersections only between spheres and domain boundaries at both ends of the slope. Configuration B consists

| Configuration | grid                  | porosity | dy<br>(width) | dx'<br>(slope)      | dz<br>(height) |
|---------------|-----------------------|----------|---------------|---------------------|----------------|
| A             | regular               | 0.48     | 2.2 r         | 2.2 r               | 2.2 r          |
| B             | regular               | 0.36     | 1.9 r         | 1.9 r               | 1.9 r          |
| C             | 45deg<br>shifted rows | 0.37     | 2.2 r         | 1.9 r<br>(at 45deg) | 2.2 r          |

Table 6.1: Layer configuration with spherical particles and their grid spacing with sphere radius  $r=0.7$  m. Configurations A and B represent particles in upper layer being placed on top of particles in bottom layer. Configuration C represents a double layer where particles in top layer are shifted at each row by  $dy/2$  on the y axis. Axis  $dx'$  represents an axis parallel to the slope.

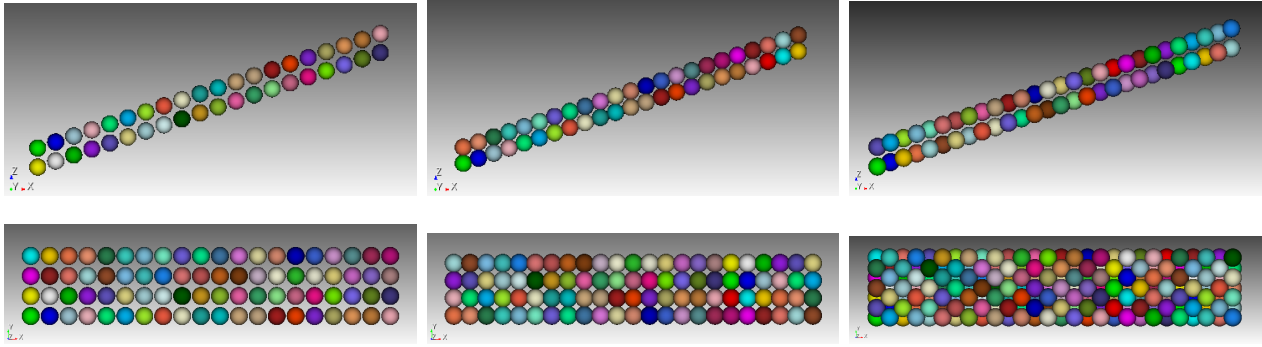


Figure 6.1: Visualisation of the layer configurations layouts with spherical particles: A (left), B (centre) and C (right). Side views (top figures) and views from the top (bottom figures). Layout descriptors given in Table 6.1.

of a regular grid of intersecting spheres at a spacing of 1.9 radii between sphere centroids, i.e. the overlap is 0.05 times the diameter. This second configuration therefore considers the intersection between spheres and the intersection between spheres and domain boundaries (the latter already existing in configuration A). These intersections are very important in terms of mesh quality, since a higher complexity on the resulting boundary surface is more likely to lead to lower quality elements generated at the computational domain boundary. A third configuration C is considered for comparison with a more complex sphere to domain boundary intersection. Configuration C consists of shifting rows at 45 degrees (shifting the centroid of consecutive rows at an 45 degrees angle), resulting in spheres of consecutive rows intersecting at 45 degrees from the row alignment, but no intersection between spheres in the same row.

The flume domain is shown in Figure 6.2, which consists of a vertical 20 m high inlet boundary, a 40 m flat bed in front of the inlet, a 600 m transition with a slope of 1 in 10 between the deep flat bed and the toe of the structure, and a 1V:3H slope for the rubble mound structure

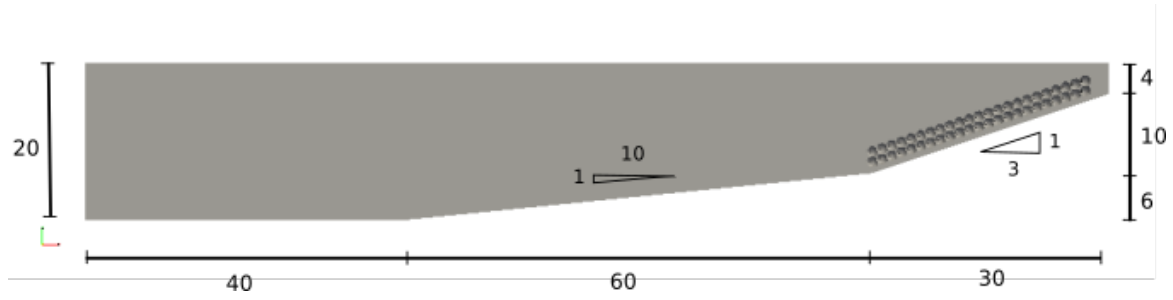


Figure 6.2: Numerical Wave Tank geometry for double sphere layer simulations (in metres). Computational domain in grey with a 40 m flat bed, a 60 m 1V:10H transition slope, and structure with a slope of 1V:3H with the sphere double layers placed on the slope of the structure.

(10 m high).

Regular waves have been generated at the inlet with  $H=3$  m and  $T=9$  s. Water level has been set to 12 m. The simulation length is 36 s and an adaptive time-step aiming at a CFL value of 2 has been chosen in order to improve simulation speed and reduce computational cost.

In order to replicate the dissipating effect of an underlayer, a porous layer has been considered. A dissipation term has been added to the velocity field for the region of the domain under the sphere double layer. This term can be applied to any element of the domain as a constant or as an equation with any inputs from the simulation scalar or vector fields. The velocity absorption term used here has been set to a constant value of  $1 \times 10^3 \frac{kg}{m^2s^2}$ , which is subtracted from the momentum term in the momentum equation. No sensitivity tests on the velocity absorption term have been carried out due to time limitation, and further research on the application of the absorption term is left for future work.

| Configuration | > 90   | > 120 | > 150 | > 170 | > 175 |
|---------------|--------|-------|-------|-------|-------|
| A             | 37.93% | 0.06% | 0.00% | 0.00% | 0.00% |
| B             | 39.25% | 0.07% | 0.00% | 0.00% | 0.00% |
| C             | 39.68% | 0.45% | 0.19% | 0.08% | 0.00% |

Table 6.2: Initial mesh quality in terms of number of elements with maximum angles as a % of the total angles above 90, 120, 150, 170 and 175 degrees.

The domain has been created and meshed with the CAD and meshing software ‘Trelis’ (Coreform). Sphere contours discretisation has been set to  $r/2$ . Table 6.2 shows mesh quality for every sphere layer grid configuration before DMO is applied. Configuration A and B present

similarly good mesh quality (i.e. a very small number of elements with angles above 120 degrees and none above 150 degrees). This indicates that sphere-sphere intersections in Configuration B do not result in bad quality elements at the boundary. Configuration C mesh quality is significantly lower with larger number of elements with angles above 120 and 150, and even some elements with elements with angles above 170 degrees. These elements can lead to non-convergence of the solution when using standard FEM spatial discretisation methods. As discussed in Chapter Section 4, using  $P_1DGP_1(CV)$  has enabled WSI simulations to run robustly, although an increase in computational cost can still be expected due the solver needing lower time-steps for solving the equations on lower quality elements.

The dynamic mesh algorithm has been set to minimum and maximum element lengths of 0.005 m and 0.3 m, respectively, for the layer and a region of 1.5 m around it. The minimum and maximum element lengths for the rest of the domain are set to 0.3 m and 2 m, respectively.

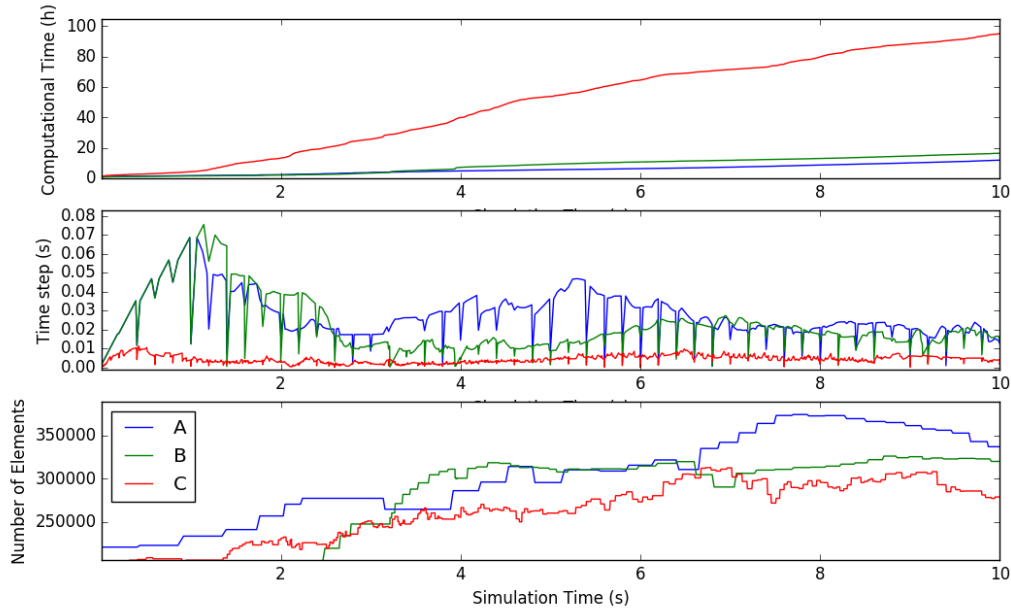


Figure 6.3: Numerical performance for 10 seconds of a regular wave interacting on a rubble-mound structure consisting of three different sphere configurations. Configurations A and B shown in blue and green, and Configuration C shown in red. Computational cost (top), time step (centre) and total number of elements (bottom) over simulation time for each Configuration.

All simulations have been run in parallel using 12 processors Intel Xeon(R) CPU E5-2650 v4 @ 2.20GHz. Simulation performance for all 3 configurations is shown in Figure 6.3. Configuration



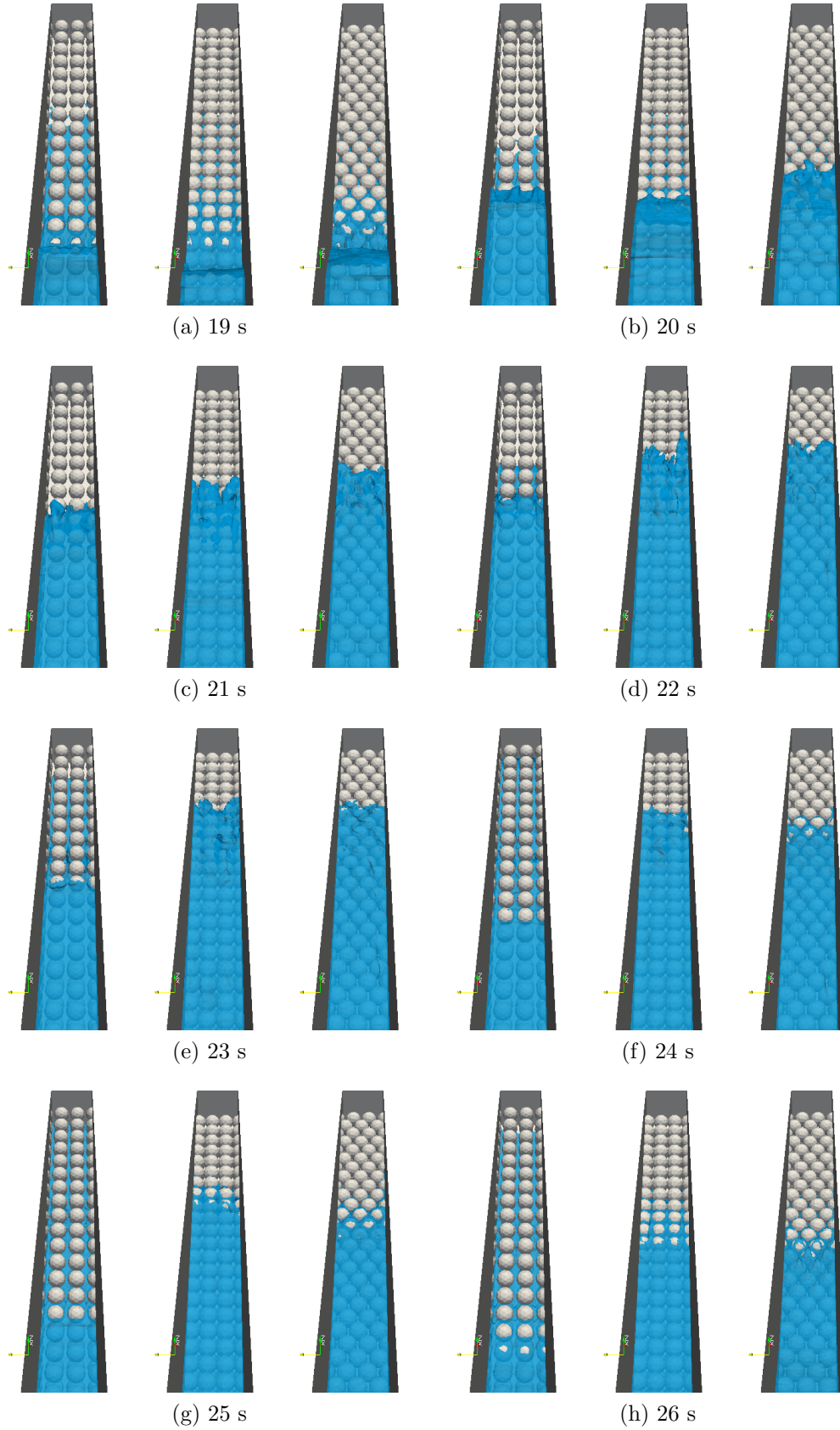


Figure 6.4: Visualisation of the double sphere layer simulation results at different simulation times for all three Configurations: A-left, B-centre, C-right. View angle of 30deg below the horizontal plane.

C (irregular grid of intersecting spheres) has significantly higher computational cost due to its lower convergence speed and smaller time-steps induced by lower quality elements. Configuration A (no intersection between spheres) has slightly higher time-stepping than Configuration B. Although configuration A has slightly higher number of elements than Configuration B, Configuration A has overall lower computational cost due to higher time-steps.

Both Configurations B and C have intersecting spheres and very similar porosity (0.36 and 0.37 for B and C, respectively). However, their pattern of the spheres layout and their numerical performance differs significantly. This suggests that domain boundary complexity (in this case as a result of sphere with sphere and sphere with boundary intersections) can have a large impact on numerical performance and overall computational cost. This boundary complexity needs therefore to be tackled when modelling full layers of any real geometry where units are expected to be in contact or close to their neighbours. This is shown for real geometries of armour units (CoreLoc and rock armour) in Sections 6.3 and 6.4.

Figure 6.4 shows simulation results at the same given times for configurations A, B and C. At 19 s a steep wave about to break is observed at the bottom of the slope. At 20 s the wave has broken for configurations B and C (lower porosity layers). The higher porosity of configuration A reduces the wave run up on top the slope, as seen at 21 s, 22 s and 23 s. The run up, however, is significantly higher under the double sphere layer in A than in configurations B and C. The run down can be seen for all configurations from 23 s to 26 s, where the fluid flow is captured around the spheres with a very high resolution. Thin and narrow film of water can be seen at different locations as it retrieves at 24, 25 and 26 s for B and C configurations.

### 6.3 CoreLoc Armour Layers

A validation study of numerical simulations with a single CoreLoc unit on a slope is shown in Section 5.5. In this section a full CoreLoc armour layer is tested with the CFD code presented in Chapters 4 and 5, and its results and numerical performance are discussed. As it has been introduced in previous Section 6.2, element quality at the numerical domain boundary is

expected to be critical due to the boundary elements being restricted by the boundary itself, leading to smaller angles and lower quality elements, and by the boundary elements being more likely to retain their low quality throughout the simulation. Other mesh limitations for CoreLoc layers, and concrete layers in general, are the elements quality between close and nearly in-contact surfaces of two neighbouring units.

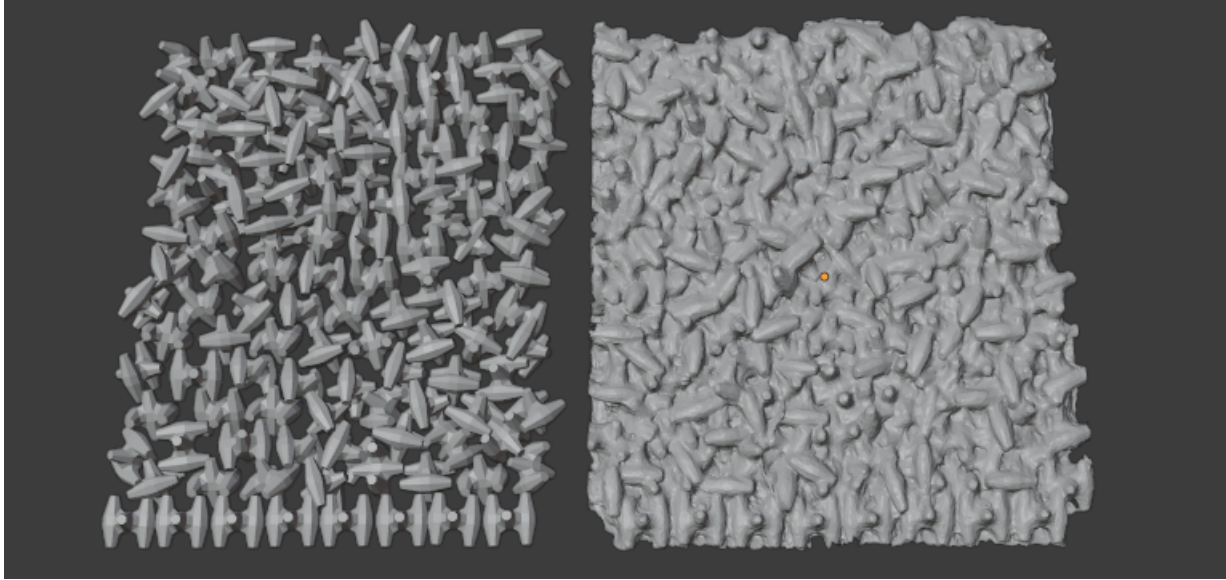


Figure 6.5: Example of (different) numerical CoreLoc layer (left) and Laser Scan output surface of a physical model layer (right).

A CoreLoc numerical armour layer is built here based on a layer layout obtained from physical modelling experiments at the NRC facilities in Ottawa, Canada. CoreLoc centroids and orientations were extracted from a physical model to provide the basis for an equivalent to a numerical representation of the layer. The layer was scanned with a Laser Scan and the top surface of the layer was extracted. Numerical manual placement of the CoreLoc units was carried out by students from Ottawa University using the layer upper surface obtained from the Laser scan as a reference (see Figure 6.5). CoreLoc units were moved and rotated one by one and row by row with Blender until agreement between CoreLoc and the top layer surface were reached, while avoiding a virtual and therefore an non-physical overlap intersection of neighbouring CoreLoc units. This pilot approach was reported to take several hours and was carried out only once.

Although experiments were run with instrumented CoreLoc units at different locations mea-

suring pressure and forces as part of the collaboration between ICL and the Canadian partners, the layer shown here was only built for conversion from physical to numerical layers purposes and did not include any instrumented unit.

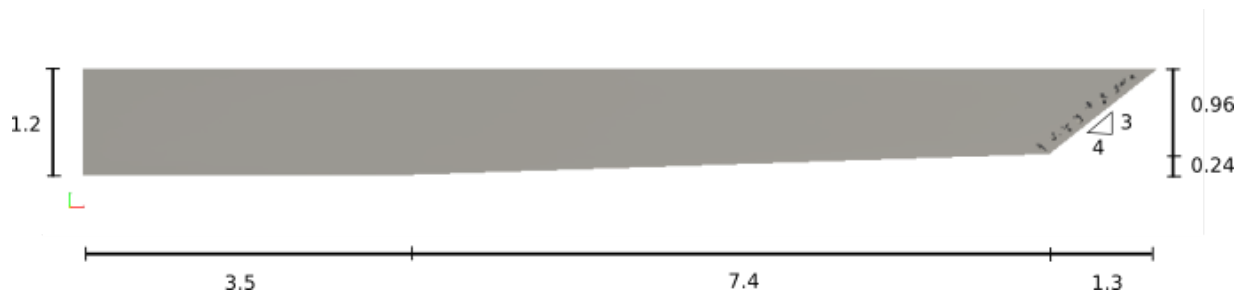


Figure 6.6: Numerical Wave Tank geometry for single CoreLoc layer simulations (in metres) with a flat bed in front of the wave inlet, a transition slope, and the structure with a 3V:4H slope and the CoreLoc layer sample placed on the slope of the structure.

Figure 6.6 shows the domain dimensions of the 1.2 m height and 0.5 m wide NWT used for the CoreLoc armour layer numerical simulations. The wave generation inlet is placed at the beginning of a 3.5 m flat bed. A bed slope of around 3% is placed between the flat bed and the structure slope. The slope of the structure and the CoreLoc armour layer are placed at the end of the 10.9 m flume. Dimensions are based on the wave flume facility where armour layers were tested in the NRC, Canada.

The CoreLoc units (0.12 m) have been imported (unit centroids and orientations) into the domain and meshed with Trelis. A 3-units wide section of the provided breakwater section has been considered for testing the simulating capabilities and performance. Although the unit layout is provided as a replica from a real physical model layer, units are already provided with reduced extra spacing between them to avoid any intersection between units during the numerical manual placement. This could be understood as a slight divergence from the physical model layer. However, it helps reduce the likelihood of bad elements appearing between very close units during the simulation (by increasing the space between them) and therefore CoreLoc data has been used as provided.

While CoreLoc units geometry is formed by outer flat surfaces, the resulting intersection of nearly random unit orientations and locations with the numerical domain boundaries could lead to computationally challenging elements. This could be fixed by manual small shifts on

unit locations and orientations as part of a quality check process, but it would require a trial and error approach. This process would add uncertainty to the simulations results and impact computational performance, as they would depend on the experience and decisions taken by the person carrying out this task at an unknown degree. Therefore, units intersecting the domain boundaries were not included in the simulation domain. An approach to reduce the higher fluid flow at the now unit-free side boundary regions of the flume is presented in Section 6.4.

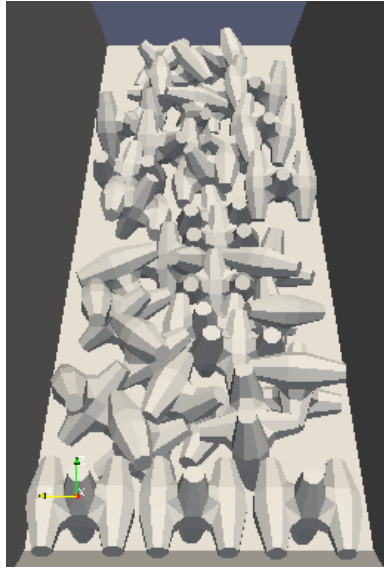


Figure 6.7: Visualisation of the numerical CoreLoc armour layer in the NWT domain. View of the layer from an horizontal plane from an upstream point.

Figure 6.7 shows the final layer in the NWT with 30 CoreLoc armour units. A porous underlayer between the flat slope and the CoreLoc layer has been set with a velocity field absorption term as described in Section 6.2.

The simulation has been run with regular waves of 0.15 m height and 2 s period. Water level has been set to 0.72 m from the flat bed in front of the wave inlet. Minimum and maximum element edge lengths for the DMO algorithm have been set to 0.004 m and 0.04 m, respectively, around the armour layer. For the rest of the domain minimum and maximum element edge lengths are set to 0.01 m and 0.1 m. The adaptive time-step scheme has been run with CFL=2.

Figure 6.8 shows different time frames of the free surface elevation from numerical simulations covering a full wave cycle. Wave run down can be observed in frames from 12 s to 12.4 s and starting again in 13.6 s and 13.8 s. Wave run up is observed between 12.6 s and 13.4 s. The non

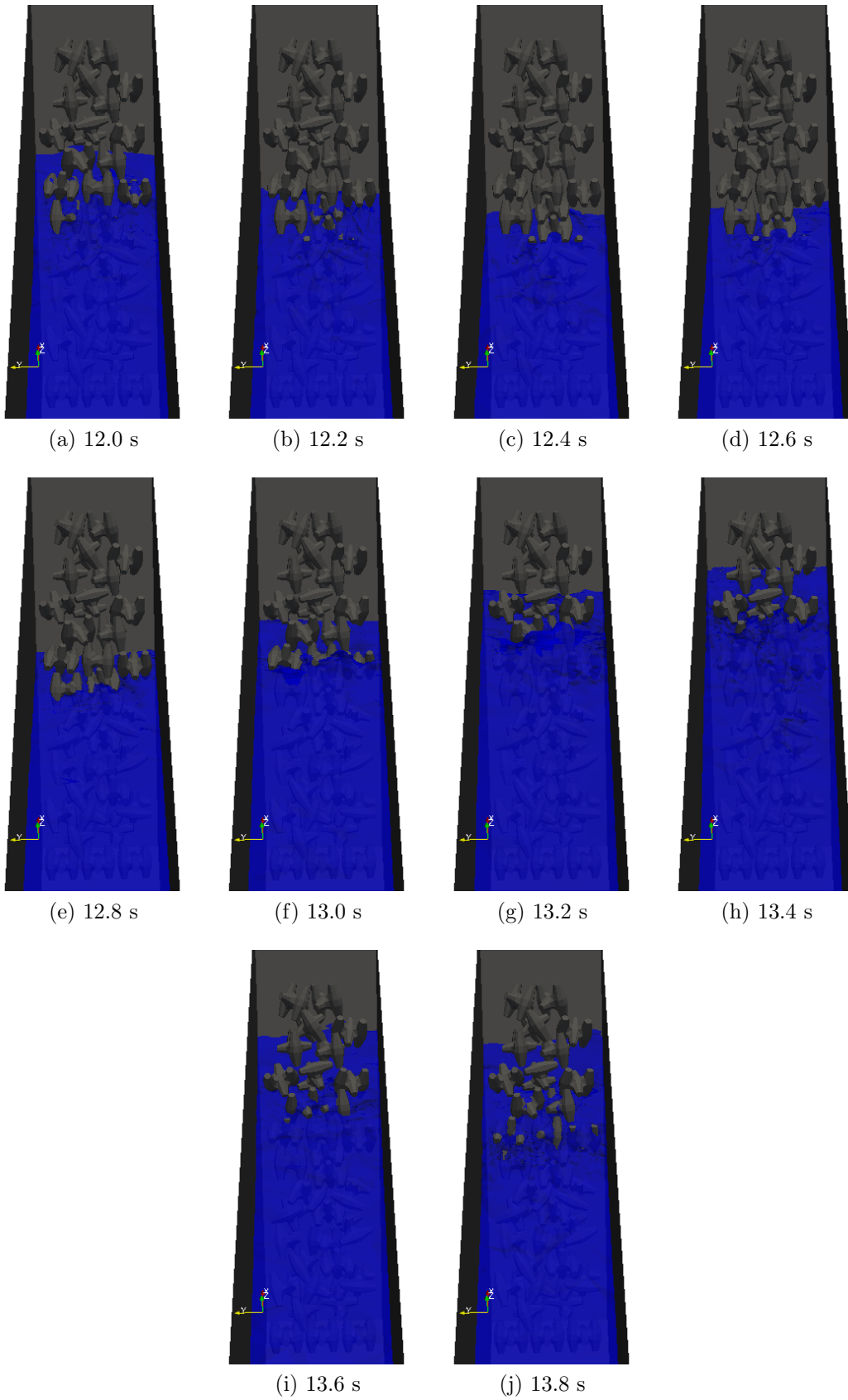


Figure 6.8: Visualisation of the CoreLoc armour layer simulation results at different simulation times. View angle of 30deg below the horizontal plane. Water free surface elevation in blue.



breaking wave is seen to cause no air entrapment during the run up. Overall, the simulation results show, at different scale and wave conditions, similar free surface patterns as observed in other simulations and published literature (Latham et al., 2015, 2018; Xiang J., 2019) on WSI with CoreLoc armour layers, suggesting good modelling of WSI physics by the technology presented in this Thesis.

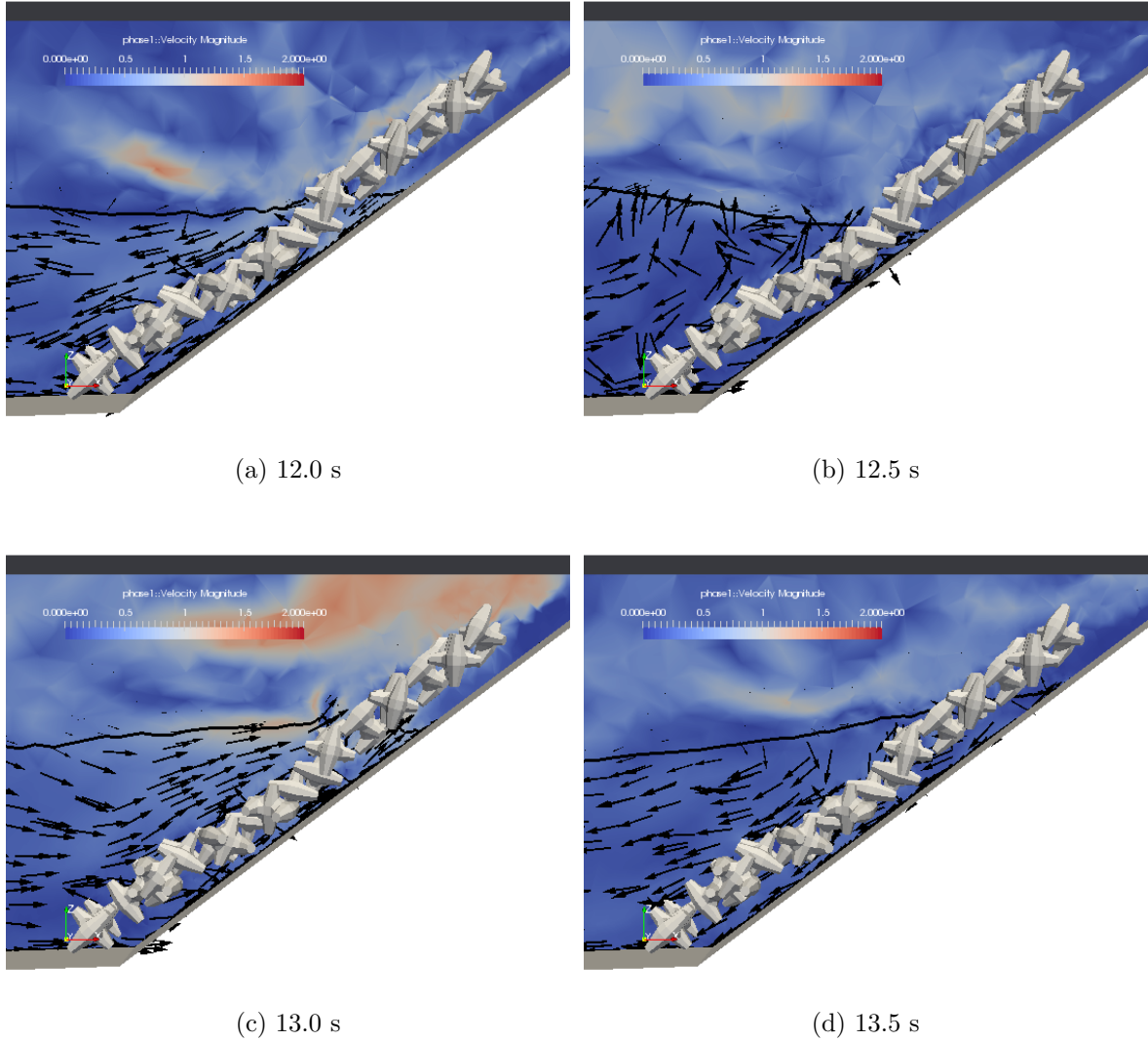


Figure 6.9: Velocity magnitude and velocity vectors on a layer cross section from CoreLoc armour layer simulation results, at different simulation times representing the run down of the wave on the structure (a) and the following run up stages (b, c and d).

Figure 6.9 shows velocity profiles (including water velocity vectors) of simulations results at 12 s, 12.5 s, 13 s and 13.5 s, on the flume cross section (centre) with the CoreLoc units overlapped on top for visualisation purposes. Velocity vectors show global trend downwards at time 12 s and 13.5 s, corresponding to the run down of the wave at different stages. A global upwards

trend is shown at 13 s with velocity vectors pointing towards the slope (left, in front of the slope) and up the slope (above, on and below the layer). A transition with vortices appearing on top of the submerged part of the layer is shown in time frame 12.5 s.

Although this simulation cannot be compared to experiments due to the lack of available data and therefore its results cannot be considered conclusive, its results are a demonstration of the potential of the technology presented in this Thesis. Figure 6.10 shows the numerical performance of the simulation and its computational cost. The 16 s simulation has run for 110 h (top) with 12 processors Intel Xeon(R) CPU E5-2650 v4 @ 2.20 GHz. Discontinuities (vertical lines in the figure) in computational time at 8 s and 12 s are induced by terminating and restarting the simulations due to non-simulation related computational resource limitations, which also lead to short term element number increase (bottom). Number of elements throughout the simulation span between 120,000 and 220,000 outside this interruption-induced peaks. Time-step (centre) ranges between 0.001 s and 0.006 s (using CFL=2 for the adaptive time-step scheme).

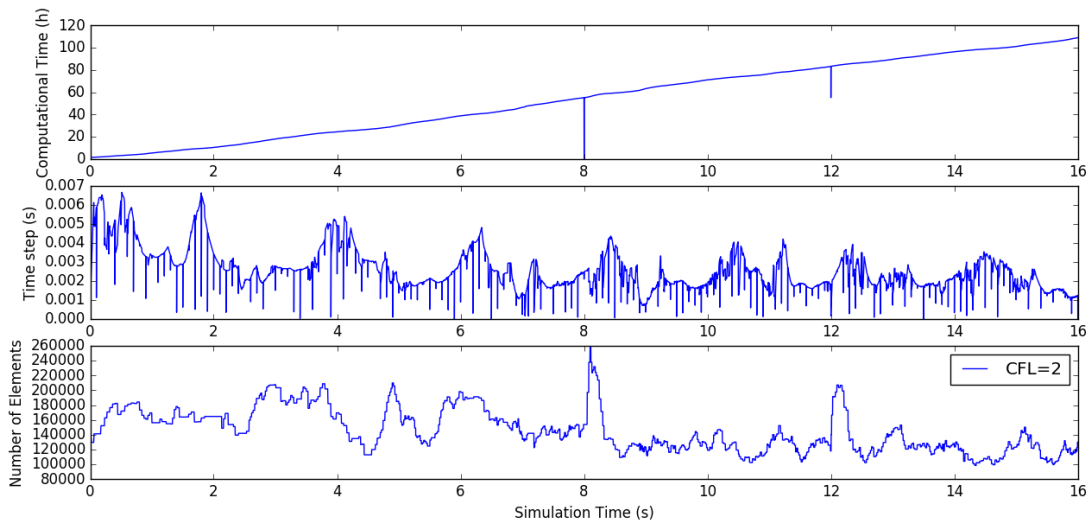


Figure 6.10: Numerical performance of a simulation using CFL=2 for the time step for 16 seconds of regular waves interacting with the CoreLoc armour layer. Computational cost (top), time step (centre) and total number of elements (bottom) over simulation time.



## 6.4 Rock Armour Layers

A double rock layer is modelled here using the CFD code presented in Chapters 4 and 5. The rock particle shapes are extracted from the rock shape library (Latham et al., 2008a) and scaled up and sampled with  $D_{n50}$  of 1.4 m and  $M_{50}$  of 7tonnes. The layer has been built on a 1V:3H slope following the compacting layer approach with Blender as presented in Section 3.2.2. Rock particles have been imported into the NWT using the centroids and orientations extracted from Blender.

Due to expected challenges when generating domains with very close or nearly parallel surfaces and initial meshing problems with third party meshing software (Trelis and Gmsh), a scaling down of the rock particles has been considered to create greater physical separation between rock surfaces and remove this cause of meshing difficulties. Two different scales have been applied to all rock particles (based on each particle centroid) for sensitivity. Rocks have been scaled down to 95% and 90% of their original size (x, y and z axis) in order to generate extra space between neighbouring rock particles, as seen in Figure 6.11, and to improve the mesh and the quality of its elements.

Although this is expected to improve numerical performance of the simulation, the porosity of the layer is increased and it distorts the representation of real voids between particles, which is expected to increase the flow through the layer. However, the author believes this can still be considered a sensible approach for structural performance analysis if results are interpreted carefully.

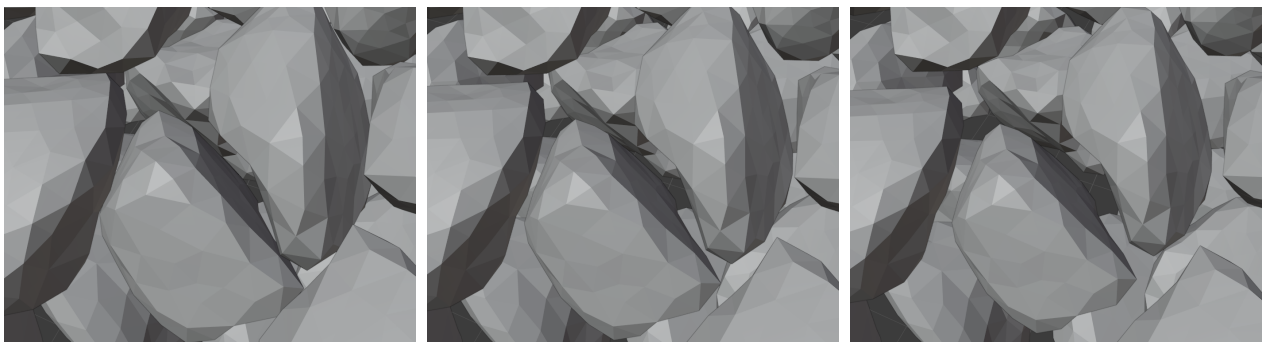


Figure 6.11: Detail of the size of neighbouring rocks and the spacing between them at full 100% scale (left), 95% (centre) and 90% (right).

Figure 6.12 and Table 6.3 show the impact of the scaling of the particles, leading to 35% and 15% porosity increases for the 90% and 95% scales, respectively. Porosity values are calculated using the aggregated rock volume in the layer, and the total volume encapsulated by the layer top surface (obtained from rock surface) and a simplified surface at the bottom of the rock layer (1V:3H slope). Decrease in layer thickness can be up to 11% for the 90% scale and goes down to 6% for the 95% scale. Resulting absolute values for porosity are 0.4 and 0.45 for a 95% and 90% scales, respectively. Layer thickness values are 2.78 m ( $k_t = 0.99$ ) for 95% scale, and 2.62 m ( $k_t = 0.94$ ) for the 90%.

| Particles Scale | Porosity | Averaged layer thickness | St.Dev. on layer thickness |
|-----------------|----------|--------------------------|----------------------------|
| 1.00            | 0.34     | 2.96                     | 0.47                       |
| 0.95            | 0.40     | 2.78                     | 0.51                       |
| 0.90            | 0.46     | 2.62                     | 0.58                       |

Table 6.3: Porosity and layer thickness values for particles scaling factors of 1 (100%), 0.95 (95%) and 0.9 (90%).

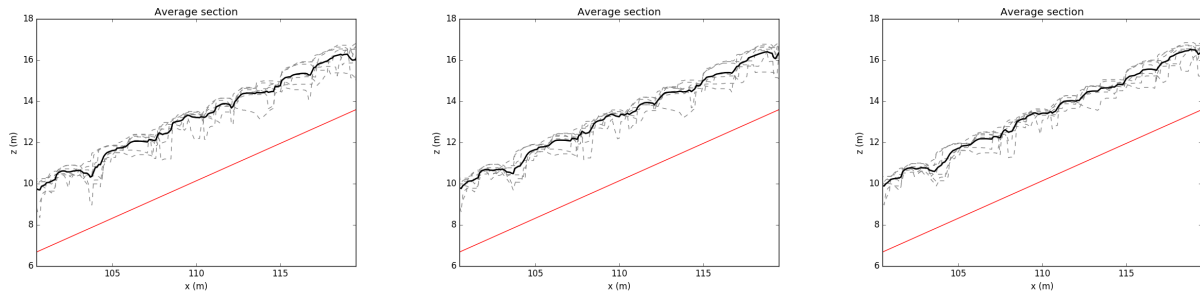


Figure 6.12: Averaged layer thickness profiles for layers with particles scaled to 90% (left), 95% (centre) and 100% (right) of the original layer.

To address unit intersections with side boundaries and mesh quality related problems described in previous sections of this Chapter two different approaches have been adopted. First, side boundaries have been set to cut the numerical layer spherical particles at both flume sides (Section 6.2). This has been done for several spherical particle configurations placed in grids where the author had full control of the grid patterns and subsequent control of the resulting intersections between unit and boundaries. The second approach has been to remove intersecting particles with the side boundary (Section 6.3) due to near-random (no given grid) intersection of units and boundaries leading to an increase in number of bad quality elements. This approach is however unrealistic when looking at the free flow artificially imposed at either side of

rubble mound layer representation after rocks are removed in those regions.

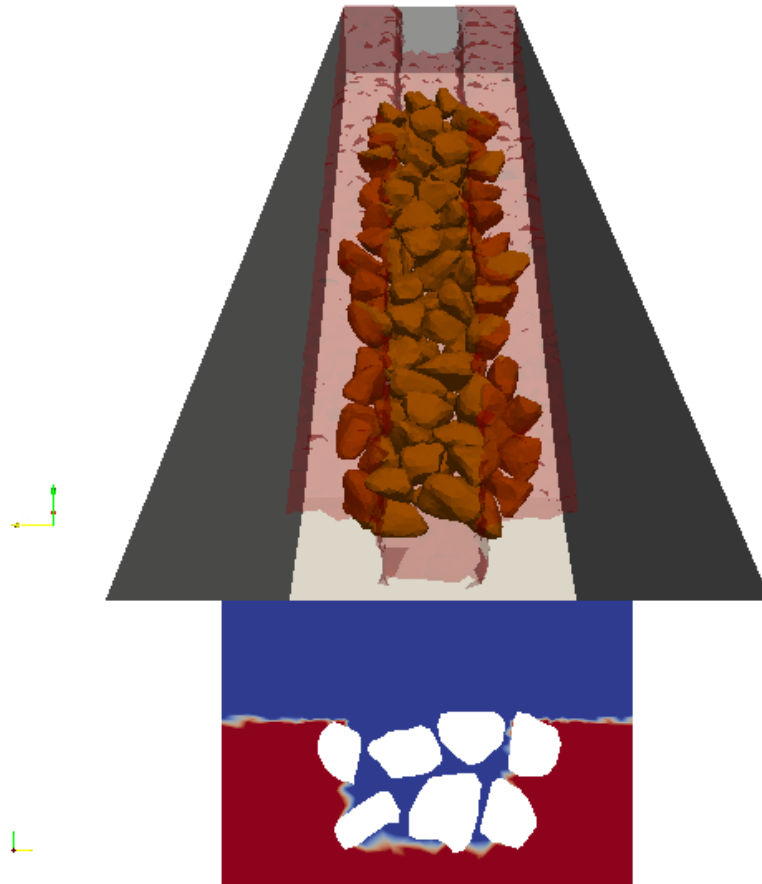


Figure 6.13: Full view (top) of the rock armour layer with the absorption zone for the velocity field in red (50% transparency applied for clarity) and a layer cross-section at the center of the slope (bottom), where red represents the absorption zone and blue the free flow zone.

A new approach has been used here, which avoids the problem of intersections between rock particles and flume boundaries and reduces the free flow at the boundaries. It consists of excluding from the computational domain all the rock particles to either side of the central cross-shore region of interest occupying a width of three or four rock diameters. The extra volumes of fluid that replace the rock volumes down each side have their flows retarded by applying an absorption term for the velocity field to the space left between particles and flume sides, as seen in Figure 6.13. This procedure replicates a porous media layer at the flume sides and allows us to model this complex domain.

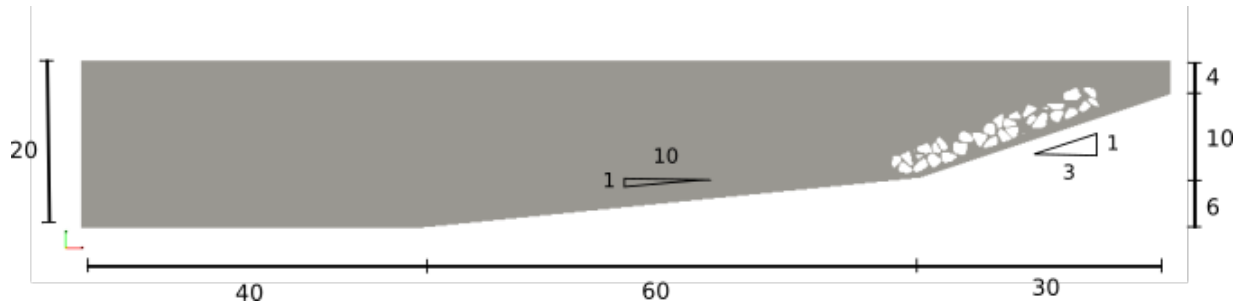


Figure 6.14: Numerical Wave Tank geometry for double rock layer simulations (in metres). Computational domain in grey with a 40 m flat bed, a 60 m 1V:10H transition slope, and structure with a slope of 1V:3H with the rock double layer placed on the slope of the structure.

### 6.4.1 Results

Regular waves with 3 m height and 9 s period have been generated at the inlet. The length of the simulation has been set to 36 s and an adaptive time-step aiming at a CFL value of 2 has been chosen in order to improve simulation speed and reduce computational cost. Figure 6.14 shows the 10 m wide NWT used for this simulation. A 40 m long flat bed in front of the inlet is followed by a 1 in 10 slope for 60 m. The toe of the 1V:3H rubble mound starts at the top of the 1V:10H slope and at 6 m height from the deep flat flume bottom. Water level has been set to 12 m, resulting to a 6 m depth at the toe of the structure. Two extra wave conditions covering potential climate change scenarios have been run and are discussed in Section 6.4.2.

Figure 6.15 shows free surface elevation of the simulation results with rock scales 95% (left) and 90% (right) for a whole wave period cycle (9 s) including a wave breaking and run up/down on the rock slope at 1 s steps (no simulations were run at 100% due to mesh quality problems described in Section 6.4). Good performance of the side boundary absorption zones (shown in grey) is observed with no cross flume reflections. Although differences in layer porosity are expected to lead to differences in fluid flow through the layer, no significant divergence is observed in free surface elevation between 90% and 95% scales. However, the small differences in free surface can be explained by layer porosity changes having smaller impact than expected on water flow above the layer surface with breaking waves. This is further investigated with profiles or sections of the velocity field.

Velocity profiles have been extracted at the centre of the flume (5 m from both flume sides)

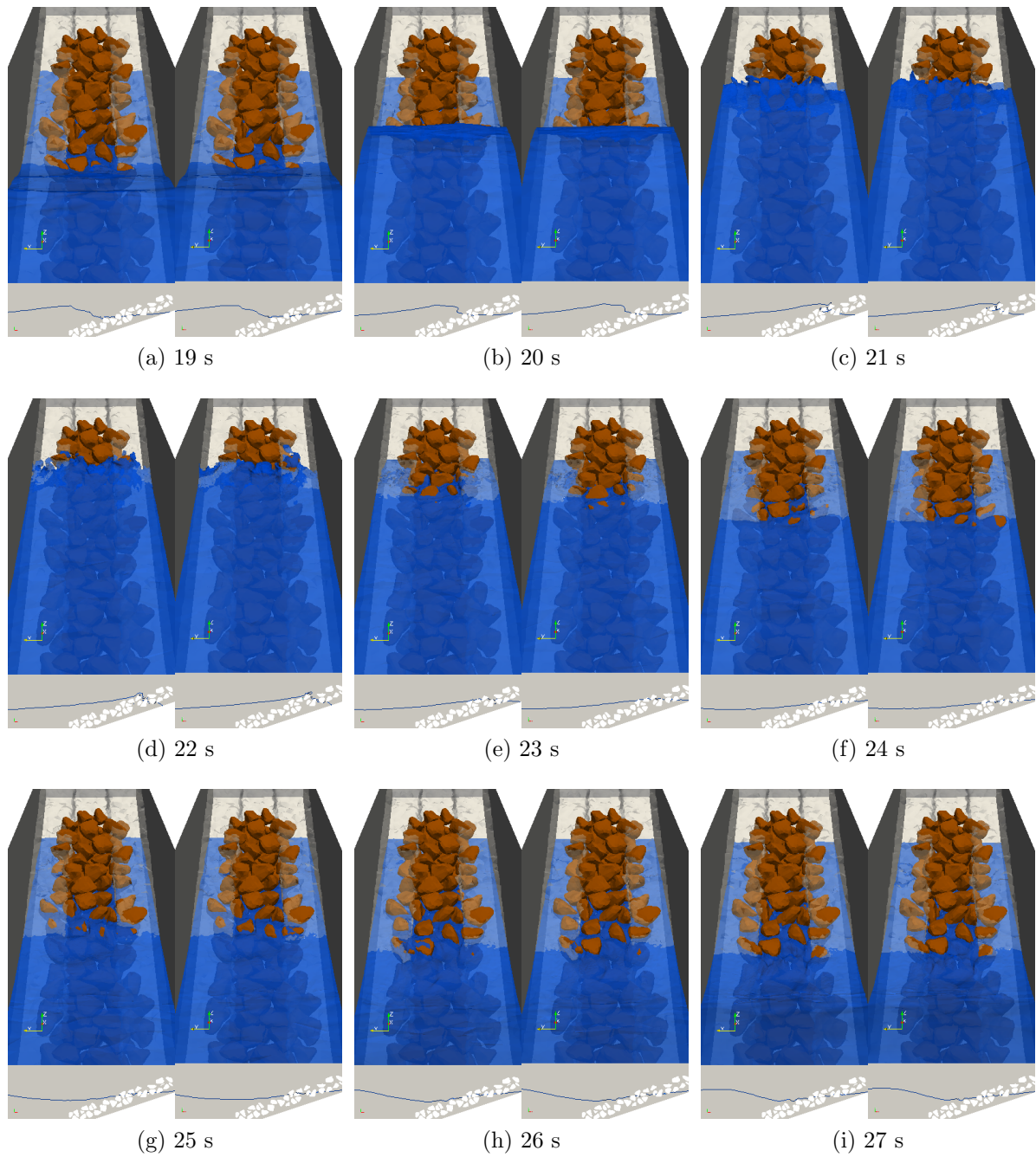


Figure 6.15: Double rock armour layer simulation results at different simulation times for 90% (right) and 95% (left) scaled down particles. Rock particles in brown, water surface in blue.



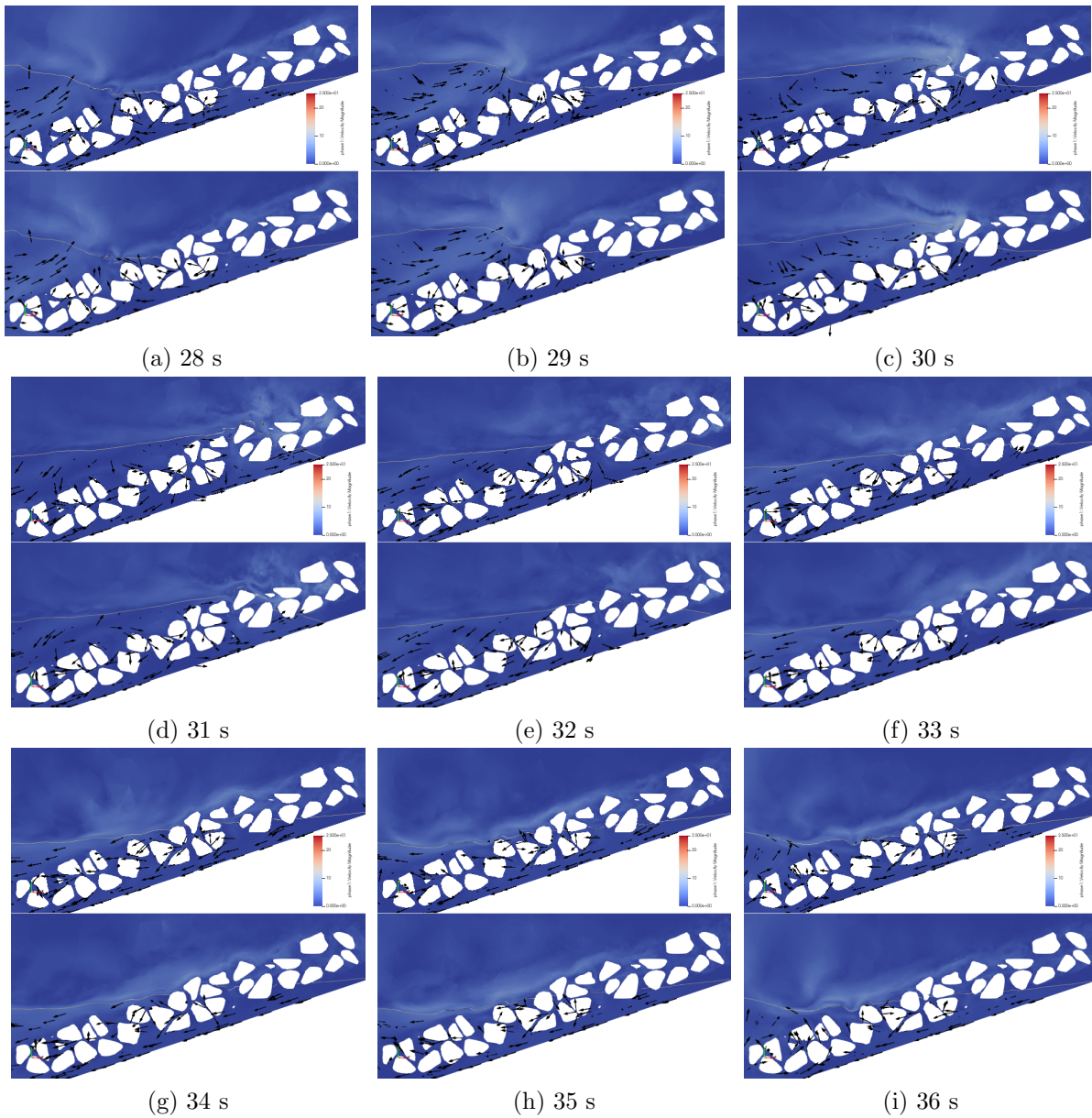


Figure 6.16: Velocity magnitude and velocity vectors on a layer cross section of the double rock armour layer simulation using 95% (top) and 90% (bottom) scaled down particles, at 1 s intervals for different simulation times showing a wave run-up and run-down cycle.

and are shown in Figure 6.16. While no overall significant differences are observed in terms of breaking and run up of the wave, slightly higher velocities can be observed on the top face of the rock layer for the 95% rock scale scenario during run up and run down of the shown wave cycle. This suggests that the lower porosity of the layer (for 95% scale) leads to a reduced flow through the layer, which causes the increase in flow velocities above the layer surface.

Moreover, water velocities on and inside the layers have been extracted at 4 different locations, see Table 6.4 and Figure 6.17, with a frequency of 0.1 s. Velocity measurements also show

higher velocity values on the surface of the layer for the rock layer with 95% scale particles, as seen in Figure 6.18, showing velocities rushing over the upper rocks reaching peak velocities of about 20-45% higher values for the 95% (lower porosity layer) than for the 90% case. Velocities measured inside the layer (VL2 and VH2) present higher variability between the two layers, however, and no significant differences are observed overall.

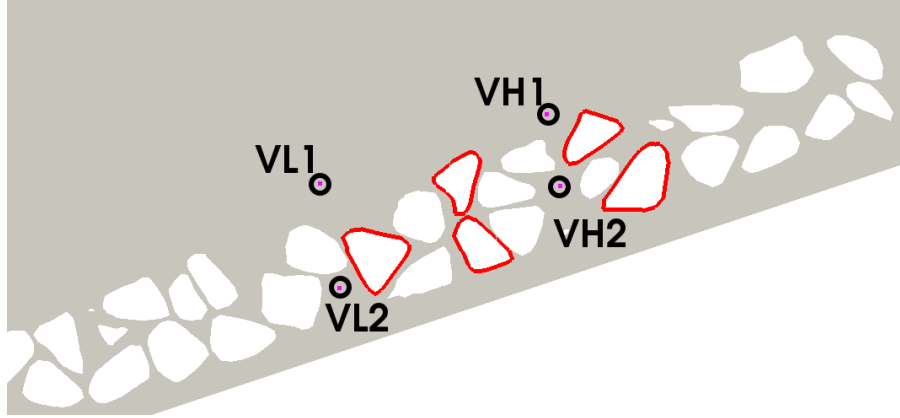


Figure 6.17: Location of points (red points highlighted with a black circle) on a cross section for velocity field measurements. Rock particles shown with red contour are discussed below for forces on units analysis. Points ending with 1 are located on top of the layer surface. Points ending with 2 are located in pores between particles inside the layer.

| Point | X (m)   | Y (m) | Z (m)  |
|-------|---------|-------|--------|
| VL1   | 105.684 | 5.000 | 12.708 |
| VL2   | 106.234 | 5.000 | 9.850  |
| VH1   | 111.932 | 5.000 | 14.622 |
| VH2   | 112.304 | 5.000 | 12.650 |

Table 6.4: Point coordinates in the NWT cross section (at 5 m in the y axis) for velocity field extraction.

The simulation results have also been used to extract other information of the wave-structure interaction. Pressure and forces acting on the units, as seen in Section 5.5 for CoreLoc units, can also be calculated in space and time. In this case, forces are measured with the extraction and integration of the pressure around rock unit surfaces, and therefore any rock on the slope can be analysed. This approach for obtaining the force on a unit is believed to underestimate the shear force on the unit for non-breaking waves (see Section 5.5). However, breaking waves impacting on a unit are expected to increase the pressure term of the total force and reduce the significance of the shear force term in the total force sum. Moreover, the reduced flow through the layer is also expected to reduce the shear force on the layer units when compared to single

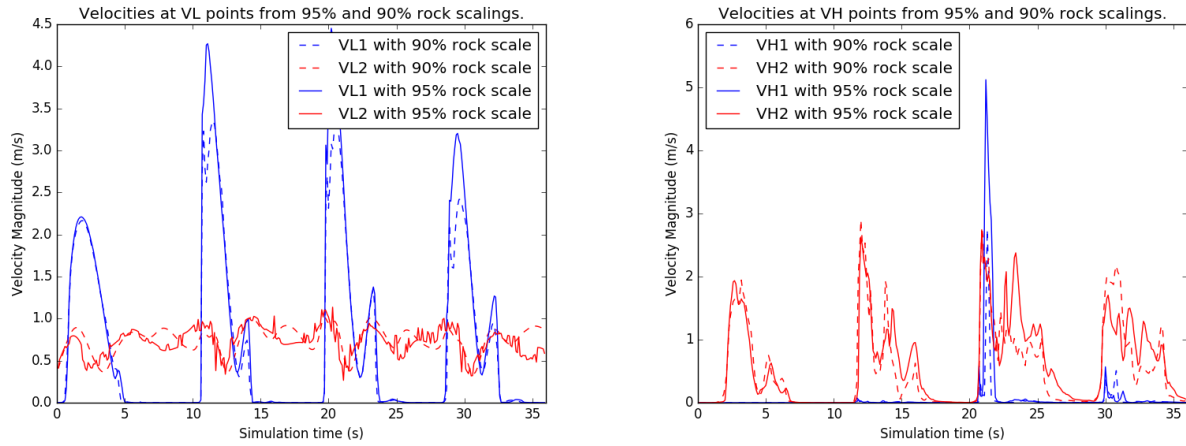


Figure 6.18: Velocity measurements from simulations (CFL=2) using 95% (continuous line) and 90% (dashed line) scaled down rock particles. Measurements at location names ending with 1 (on the surface) in blue and ending with 2 (in the layer) in red.

unit tests (Section 5.5).

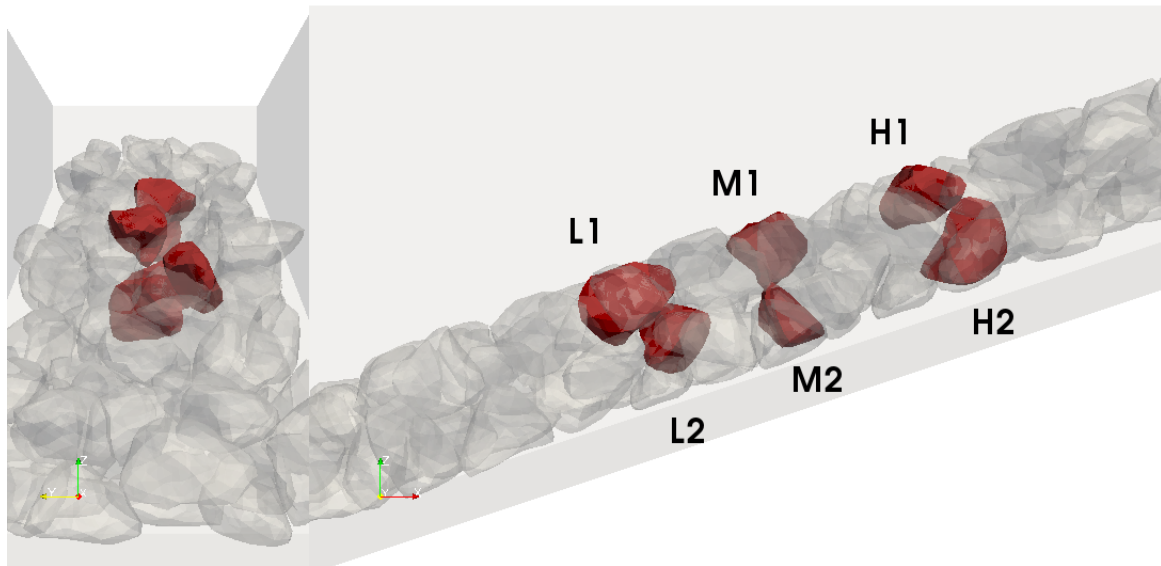
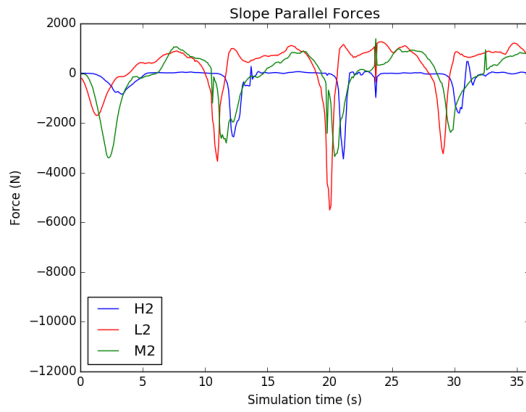


Figure 6.19: View of rocks selected for analysis of hydrodynamic forces on particles. Particle names ending with 1 indicate that particles are placed on the upper layer. Particle names ending with 2 indicate that particles are placed on the lower layer.

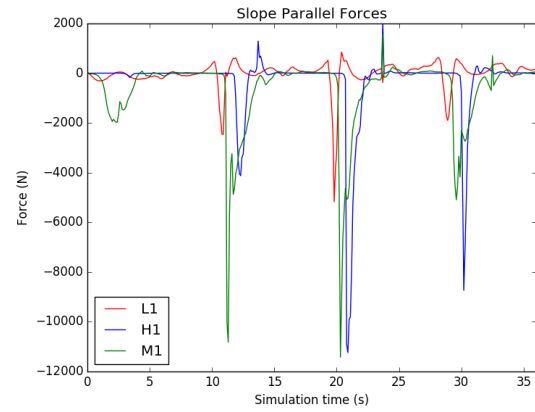
For the case of 95% scaling, 40% porosity layer, Figure 6.19 shows the rocks considered at the centre of the rock armour layer for the Force extraction and analysis. Three rocks (H1, M1 and L1) on the top face of the layer have been selected at different heights aiming to cover a wide range of the wave breaking and run up/down stages. Three other rocks (H2, M2, L2) placed approximately below the previously selected top surface rocks have been also considered in order to showcase potential differences between exposed rocks on the surface and rocks hidden



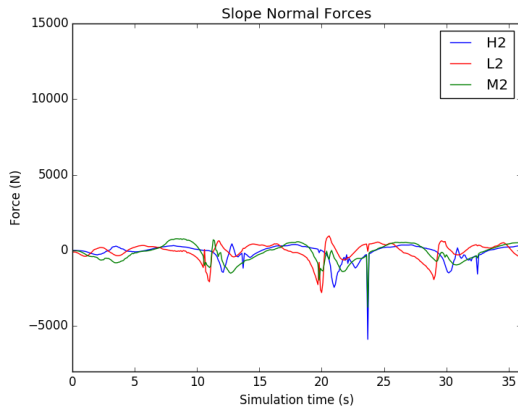
in the layer.



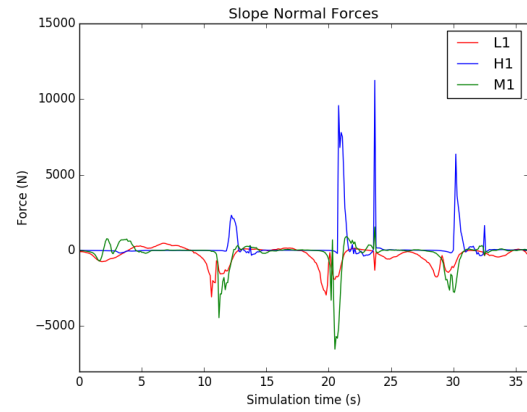
(a) Rocks placed on bottom layer



(b) Rocks placed on upper layer



(c) Rocks placed on bottom layer



(d) Rocks placed on upper layer

Figure 6.20: Parallel and normal hydrodynamic forces to the slope on rock particles H1, H2, M1, M2, L1 and L2 over simulation time, extracted from numerical simulations using CFL=2.

Parallel and normal forces to the slope are shown in Figure 6.20, where the positive force is equivalent to forces down the slope and against (i.e. up and outwards from) the slope for parallel and normal force plots, respectively. The frequency of the measurements is 10 Hz, which corresponds to the simulation output frequency.

Four waves reach the structure during the simulation time, being the first a small one generated by the ramping up wave generation library. Maximum force peaks are observed at around 20 s of the simulation, which coincides with the third wave reaching the structure. The detail of the FSE and the wave breaking on the selected rocks is shown in Figure 6.21 for 19 s, 20 s, 21 s and 22 s, where the wave is seen to break and impact on the rocks placed on the surface of the layer at different times. The detail of air entrapment with the wave breaking around

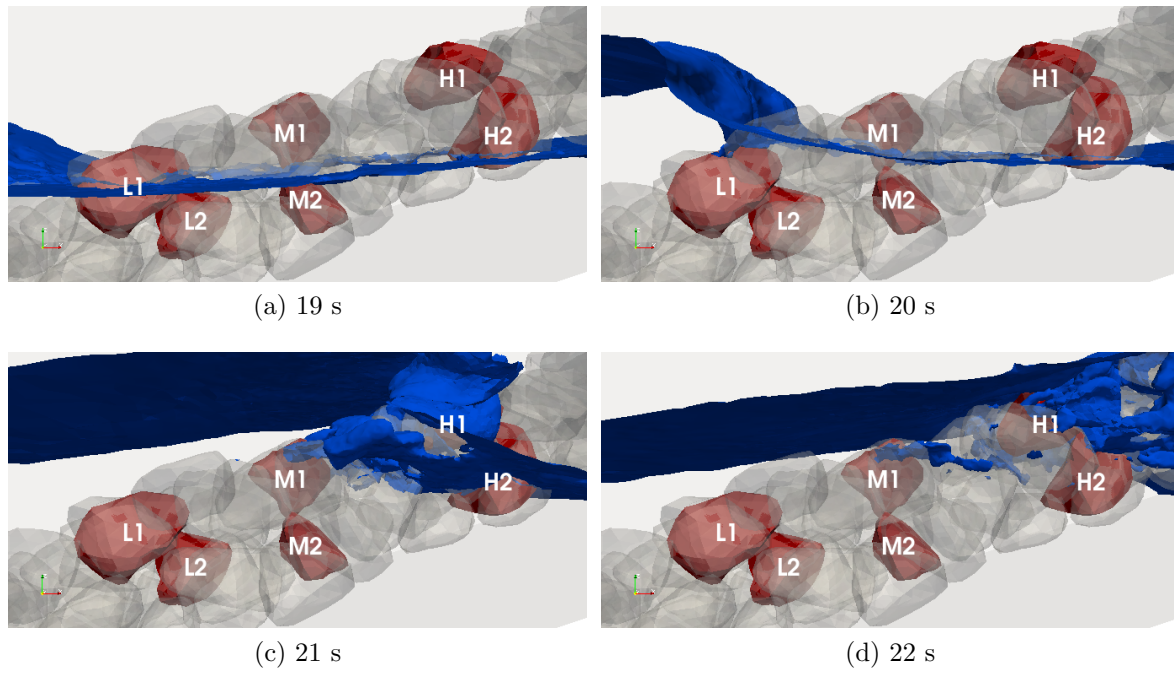


Figure 6.21: Simulation results at 19 s, 20 s, 21 s and 22 s. Detail of Free Surface Elevation (FSE), in blue, on the rock armour layer (grey with 50% transparency). Analysed rocks shown in red.

rock particles and how bubbles of air are formed and how they evolve with the turbulent flow through the layer is shown in Figure 6.21c and d.

A very sharp peak or anomaly is observed at around 24 s in all plots and all analysed rocks, which is assumed to be induced by either a numerical global instability of the simulation or a numerical error during the writing of the simulation outputs. This anomaly shows no impact on the overall simulation since all the force measurements return to their previous values immediately after the anomaly. For this reason the author believes this discontinuity is likely to be caused by the output writing and will consequently be ignored in the results analysis.

Expected force peaks can be observed at intervals of 9 s, which coincide with the wave period. Force peaks are observed with a short delay between L (lower rocks), M (middle rocks) and H (higher rocks), indicating that the wave breaking and run up/down stages are reaching the different rocks at different times. Maximum parallel forces are observed at both H1 and M1 rocks, with force peaks around 11 - 12 kN pushing up the slope. For normal forces against the slope a maximum force peak of 10 kN is observed at H1, while a maximum force off the

slope is observed at M1 with a peak of 6 - 7 kN. It is important to highlight the normal forces acting in opposite directions at H1 and M1. This is due to H1 receiving direct impact of the water jet of the breaking wave which is pushing rock H1 against the slope, while rock M1 is mainly affected by uplifting forces. This can be confirmed by the visualisation of the breaking wave stages in Figure 6.21(c), where M1 is shown below the direct wave impact region (H1). Particles placed inside the layer (L2, M2 and H2) show significantly lower force measurements, with around 6 kN parallel force pushing up the slope (H2) and around 1 kN and 3 kN of normal forces against and off the layer, respectively. Forces down the slope (positive parallel force), which are expected during the wave run down, are shown to be smaller than forces up the slope. This is assumed to be a particular case of the wave conditions used in this simulation. Further analysis with other wave conditions is expected to give a wider range of forces on units and general patterns. Moreover, using a porous core (here considered impermeable) is also expected to change the WSI patterns and increase this range of structural response.

Overall, destabilising forces (negative forces) are shown to be smaller when compared to rock unit weight and unit contact forces. Uplifting force peak remains under 10 kN, which is far below the 70 kN needed to lift 7 tonnes armour rock blocks off the slope. Moreover, rock contact forces with other units are expected to increase the restraining forces above 70 kN. However, observed forces could be sufficient for making rock particles roll if contact forces are locally small and this could lead to structural damage over time.

All simulations have been run in parallel using 12 processors Intel Xeon(R) CPU E5-2650 v4 @ 2.20 GHz. Simulations performance is shown in Figure 6.22. Although the 95% scale has generally slightly higher number of elements and slightly shorter time-step across the simulation, the computational cost of both rock scaling factors remain very similar, with 120 and 125 hours for the 90 and 95% scales, respectively. An increase in time-step and spatial resolution (increase in number of elements) is observed at around 21 s, coinciding with the force peaks observed in Figure 6.20. This is also seen in all other wave breaking events throughout the simulation, every 9 s, as expected for a 9 s wave period.

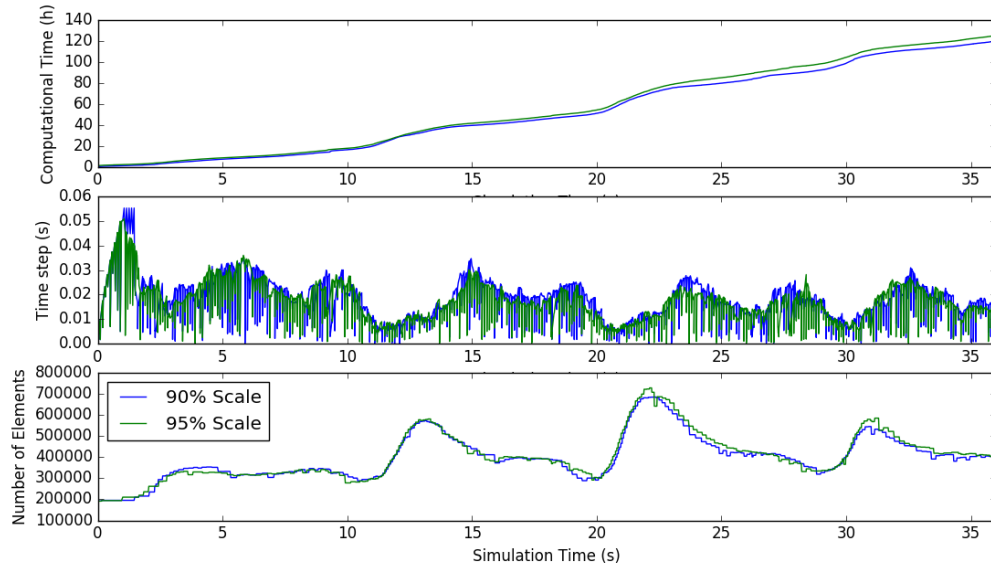


Figure 6.22: Numerical performance of rock layer simulations using  $CFL=2$ . Computational cost (top), time step (centre) and total number of elements (bottom) over simulation time for rock scaling factors of 90% (blue) and 95% (green). The run-up stages of wave breaking on the double layer are shown to increase the number of elements and reduce the time step.

#### 6.4.2 Existing structures against projected Climate Change scenarios

Climate change can increase the stresses on coastal structures with sea level rise and potentially an increase in wave energy. Although sea level rise forecasts vary by region and emissions scenario, the Fifth Assessment Report by the Working Group 1 of the International Panel for Climate Change (2013) estimated that water levels may rise globally by more than 1 m before the end of the century. Global average increase under the RCP2.6 scenario is estimated to be around 4 mm/year, and areas around the UK may have regional increase of 1 - 2 mm/year on top of the global average value. A 5 mm/year is considered here as an optimistic scenario for rubble mounds analysis. Less optimistic scenarios with higher sea level rise (as seen in later updates by Oppenheimer et al. (2019)) would lead to results with higher impact and higher stresses on the structure from the ones presented here, so this 5 mm/yr increase can be considered as a baseline scenario for future comparisons.

The Environment Agency (United Kingdom) has given in the guidelines “Flood risk assess-

ments: climate change allowances” the values to be considered for wave heights, with projected increases of 5% by 2055 and 10% from 2055 onwards. Storm surge increases (see UKCP18 for the United Kingdom), which are strongly influenced by the geography of each location, have not been included in the projected scenarios for simplicity.

| Scenario | H (m) | T (s) | Water Level (m) |
|----------|-------|-------|-----------------|
| Current  | 3.00  | 9     | 12.00           |
| +50yrs   | 3.15  | 9     | 12.25           |
| +100yrs  | 3.30  | 9     | 12.50           |

Table 6.5: Derived wave conditions for the comparison between current and two projected climate change scenarios (+50 year and +100 year projections).

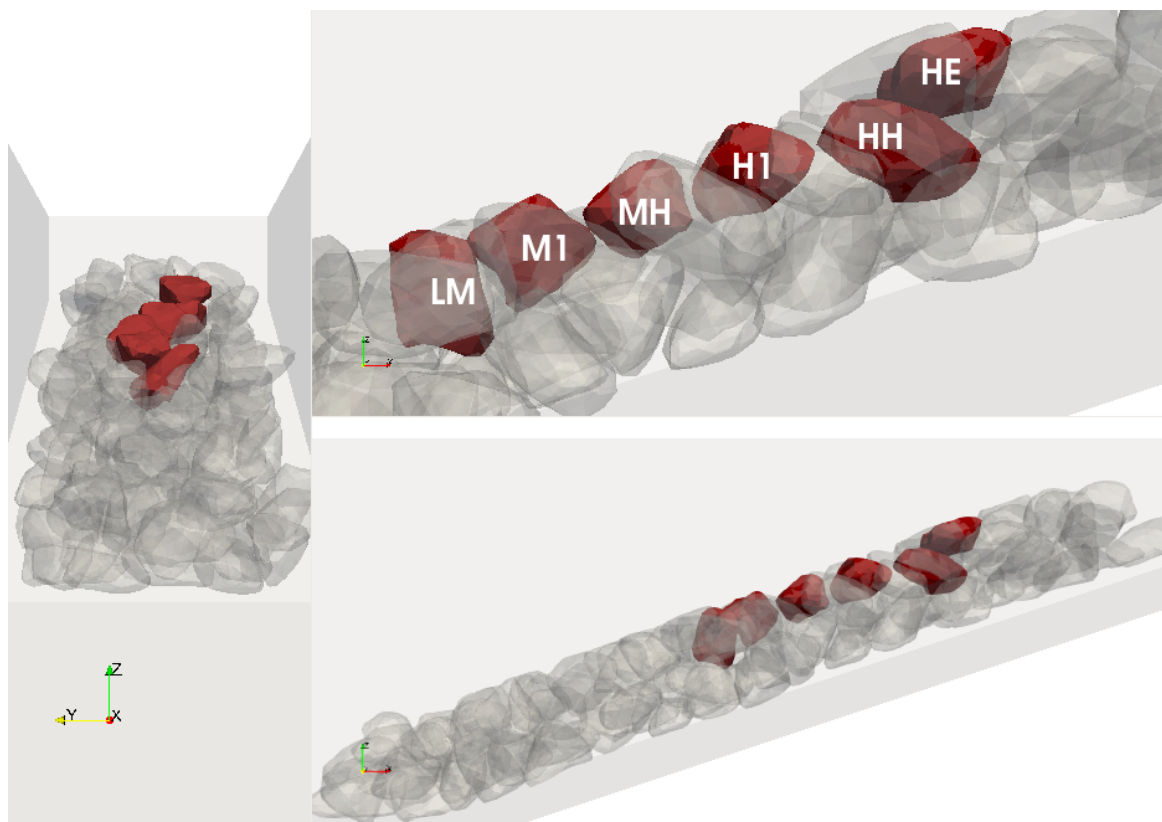


Figure 6.23: View of rocks selected (red) for the analysis and comparison of hydrodynamic forces on particles under current conditions and projected climate change scenarios. M1 and H1 correspond to particles analysed in Section 6.4.1 with the same name. Particles LM, MH, HH and HE are added to the selection to represent a line of consecutive rock particles on the upper layer. Front (left) and side (right) views of the layer and selected particles.

A structure designed and built in the 2000’s is considered here with design conditions of 3 m wave height and 9 s wave period and a planned life of 50 years. Two projections for 50 years and 100 years are considered for the end of the life of the structure and for potential future life extensions, respectively. Table 6.5 shows the summary of the current and projected scenarios

at 50 years and 100 years modelled for this analysis, with increases of 5 mm/year in water levels and increases of 5% and 10% in wave height.

The domain dimensions and layout are the same as in Section 6.4 and can be seen in Figure 6.14. Regular waves have been generated at the inlet and the simulation has been run for 36 s for each of the current and projected scenarios. Forces have been extracted from several rock particles in order to cover a wide range of locations affected with the impact of wave breaking (Figure 6.23). Rock particles M1 and H1 coincide with the ones extracted and analysed in Section 6.4, and rock particles ML, MH, HH, and H have been added to the analysis for this section.

Figure 6.24 shows the total force magnitude measured at all considered rock particles under all scenarios. Force magnitude is given at this stage instead of force vectors for simplicity when comparing different scenarios at all rocks. Force vectors are provided below for the most relevant rocks. Force peak anomalies or discontinuities observed at 13 s and 30 s for the 100 years projection results (see ML, M1, MH, H1 and HH) are assumed to be a numerical error from the output files and are not considered in the discussion.

As expected from a wave period of 9 s, force peaks are observed at 9 s frequency for all scenarios. Significant increases are observed in force peaks on rocks H1, HH and HE in the projected climate change scenarios simulations, with peaks doubling their values for 100 years projected scenario when compared with the original or current scenario. Force components parallel and normal to the slope are shown for particles H1, HH and HE in Figure 6.25. Higher increases are observed for forces parallel to the slope in the uprush direction for future scenarios, with the 100 years projected scenario reaching values that double or triple the force magnitude of the current climate scenario. Normal forces to the slope show increases of 30 to 40% for the 100 years projection when compared to the current scenario. The anomalous force peaks at 13 s and 23 s are again included in the plots in order to provide the reader with a transparent analysis of the raw data.

Figure 6.26 shows the FSE at a 5 m cross-section of the flume for all considered scenarios at different time-steps around the largest force peak associated with the second wave with the

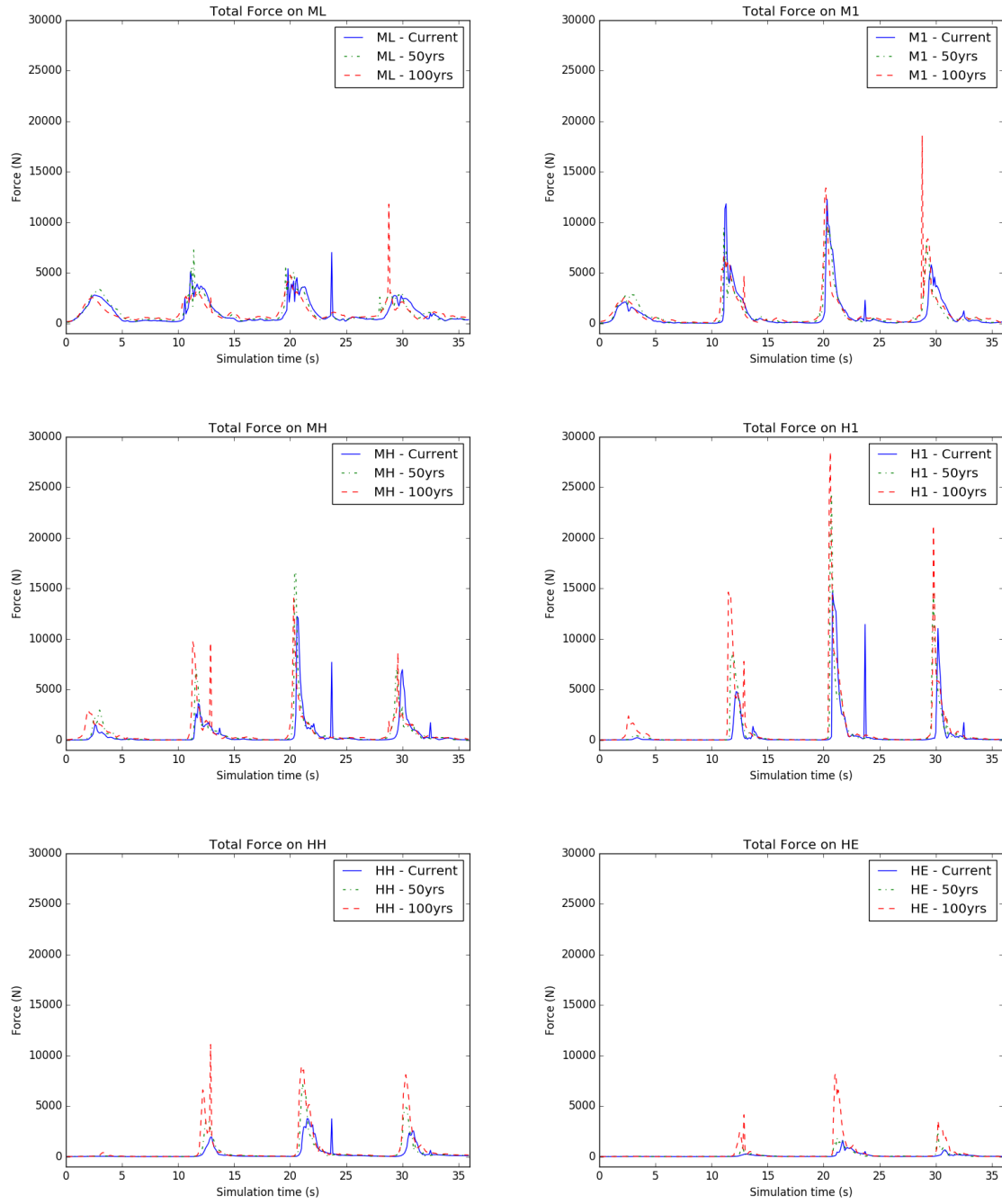


Figure 6.24: Total force magnitude at the different measured rocks under current (blue, continuous line) and climate change scenarios at +50 years (green, dashed line) and +100 years (red, dashed line).

rocks used for force analysis in the background. Wave breaking is seen to reach H1 in all scenarios and HH and HE only in the projected 50 and 100 years scenarios. Rock H1 has a shorter force peak duration but much larger (2x) uplift force normal to the layer compared with today. A simple calculation using Hudson formula (Equation 2.2) might expect the increase of force or weight of a block needed to compensate for that increase in wave height to be given

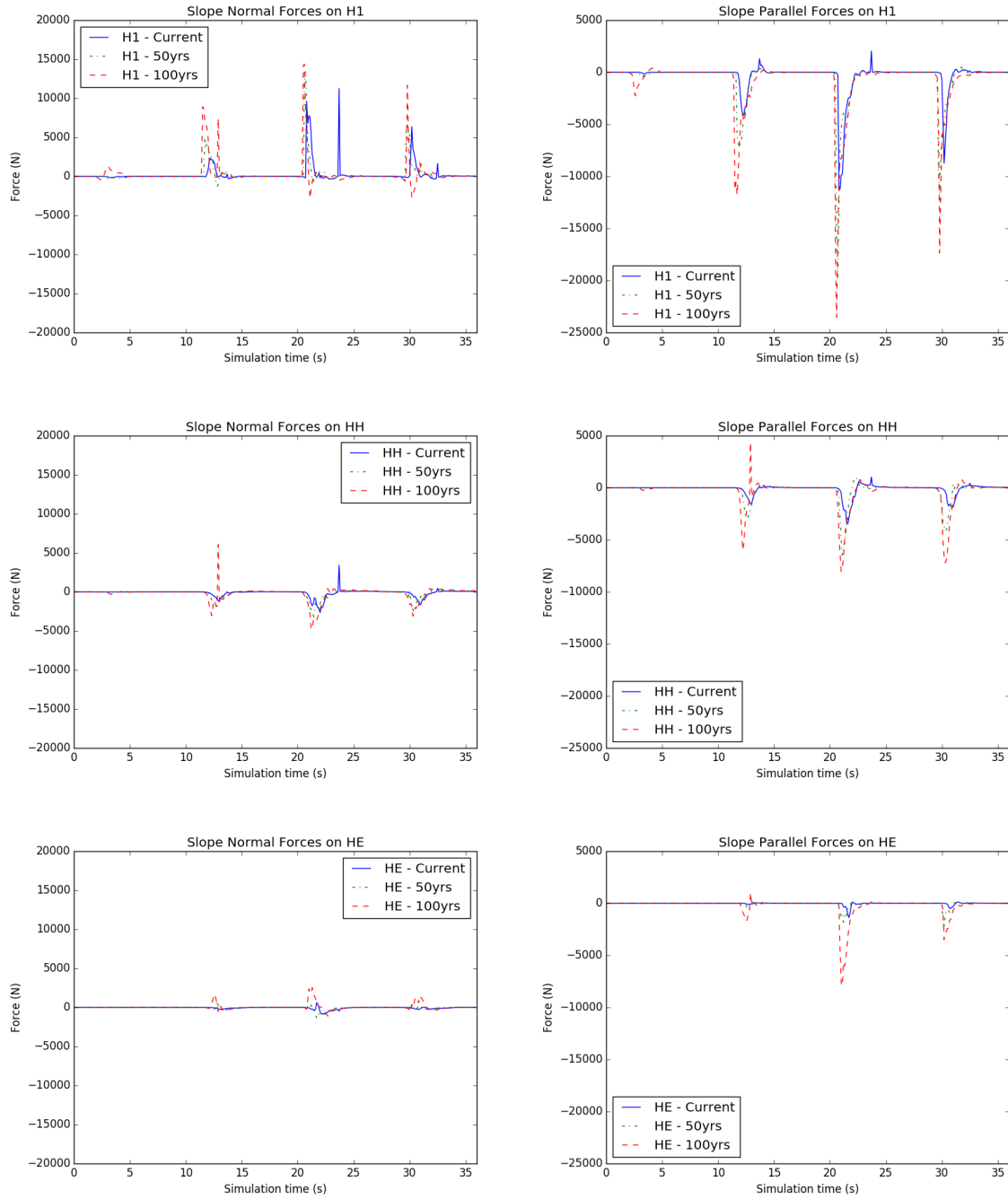


Figure 6.25: Measured parallel and normal forces to slope on particles H1 (top), HH (centre) and HE (bottom) under current (blue, continuous line) and climate change scenarios at +50 years (green, dashed line) and +100 years (red, dashed line).

by  $1.1^3$  or 33%. Here the uplift force seems to be 100% higher for this rock, but it neglects the role of contact friction and interlocking contact forces that would be activated in deriving the stability relation in Hudson's formula.

It is worth pointing out that results with this particular wave combination and impermeable core do not show downslope drag forces. This, as already introduced in Section 5.5, should be



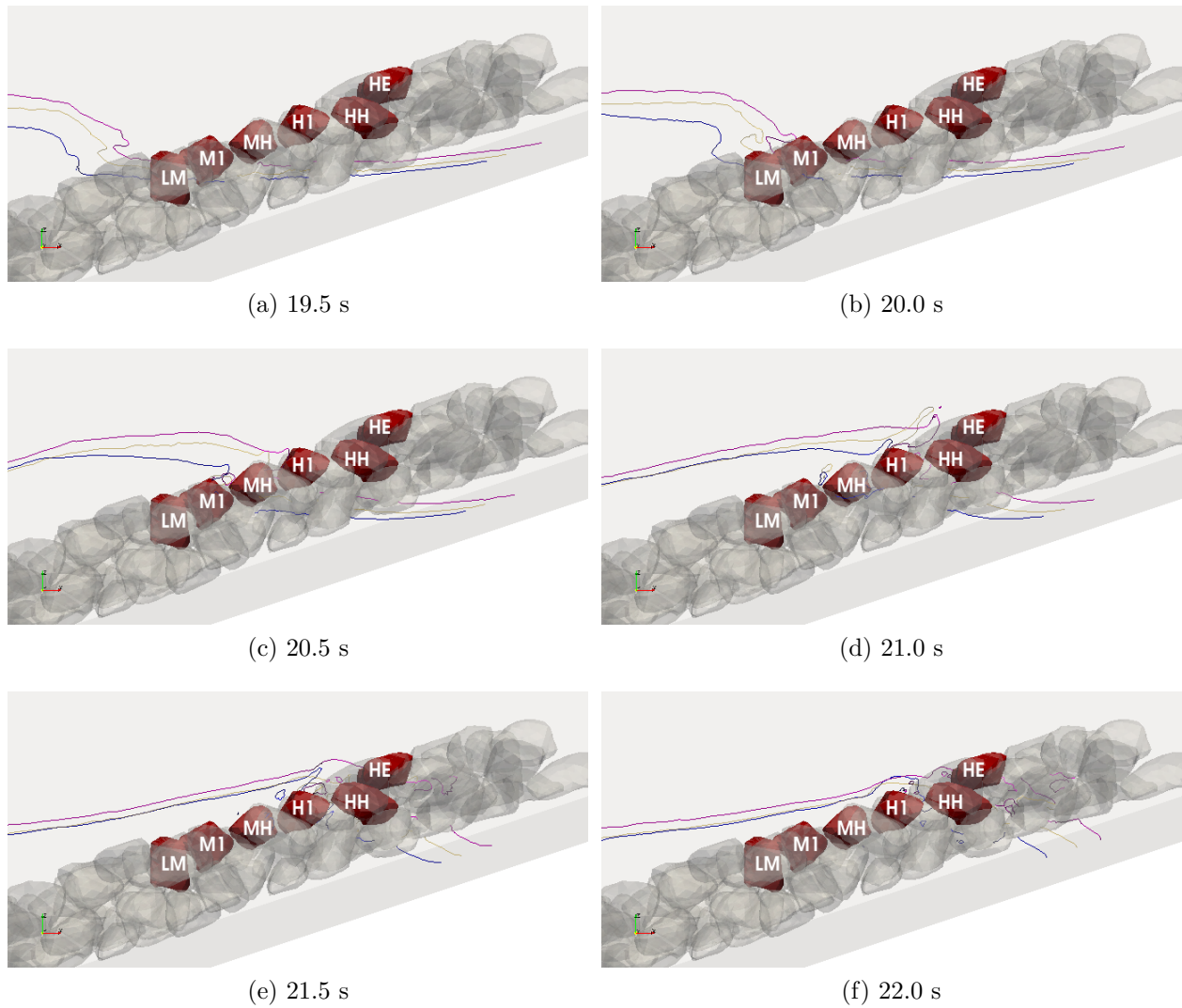


Figure 6.26: Detail of Free Surface Elevation at the centre of the flume for simulations with current (blue), 50 years (orange) and 100 years (pink) projected scenarios at 19.5 s, 20.0 s, 20.5 s, 21.0 s, 21.5 s and 22.0 s.

considered carefully and other approaches for force extraction could be considered. The capture of drag forces and the potential effect of the spurious "hammer head" tip to the breaking wave (seen in Figure 6.26) on the total force obtained in the model could be reviewed in depth as part of future validation of this technology.

Nevertheless, current results of the projected scenarios show significant increases in destabilising uplift forces and peaks in impulsive upslope forces from wave breaking. Existing structures designed in the early 2000's or earlier have not taken into account climate change impact and are therefore not designed to resist the increased loadings seen in Figures 6.24 and 6.25. These preliminary results are not taking into account storm surge (see UKCP18 for the United

Kingdom), which are expected to have an even larger impact on loadings. The work presented in this thesis can help review existing structures and update them when their risk of failure against new and more detailed climate scenarios during their life time is considered too high.

### 6.4.3 Analysis on potential scale effects from scaled down modelling

Scale effects from reducing the length scale of armour elements and structures in wave flumes or basins have been studied for several decades in order to understand potential errors when inferring structural and hydraulic performance (i.e. stability and or overtopping) at prototype scale from structures tested at smaller scales. Air entrainment and viscous forces can affect the wave energy dissipation through armour layers and structural response may differ at different scales. Examples of scale effects can be seen in overtopping analysis (Van der Meer et al., 2018) and wave impact pressures on vertical walls (Cuomo et al., 2010), however, no scale effects analysis can be found in the literature on wave breaking induced flow through rubble mound layers.

Simulations of a structure at prototype scale and at 1:25 scale have been run in order to show how the work presented in this PhD can be used for scale effects analysis. The wave flume for the prototype scale is shown in Figure 6.14. The Froude scaling law, which considers the gravity ( $g$ ) as the predominant factor in the fluid motion ( $u$ ) and states that the Froude number:

$$Fr = \frac{u}{(gD)^{0.5}} \quad (6.1)$$

should remain the same in model and prototype, has been applied for scaling down to a model scale.

Table 6.6 shows the scale factors applied to the different input parameters. Rock armour mass scaling has followed Equation 6.2 (CIRIA et al., 2007), where rock and water densities ( $\rho$ ) have been assumed of the same value for simplicity, and therefore the rock scaling can be assumed as volume scaling.

|              |                 |
|--------------|-----------------|
| Length       | $\lambda$       |
| Time         | $\lambda^{1/2}$ |
| Volume       | $\lambda^3$     |
| Velocity     | $\lambda^{1/2}$ |
| Force        | $\lambda^3$     |
| Acceleration | 1               |

Table 6.6: Scaling factors for common parameters using the Froude scaling law.

$$\text{Mass scale factor} = \lambda^3 * \frac{\rho_{a,m}}{\rho_{a,p}} * \frac{\Delta_m^3}{\Delta_p} \quad (6.2)$$

Same prototype input fluid viscosities and densities have been used for the model scale simulation. Prototype wave height of 3 m and period of 9 s have scaled down to wave height of 0.12 m and period of 1.8 s. Mesh resolution has been scaled down using the length scaling factor. CFL number for the adapting time-step scheme has remained 2.0 for both simulations.

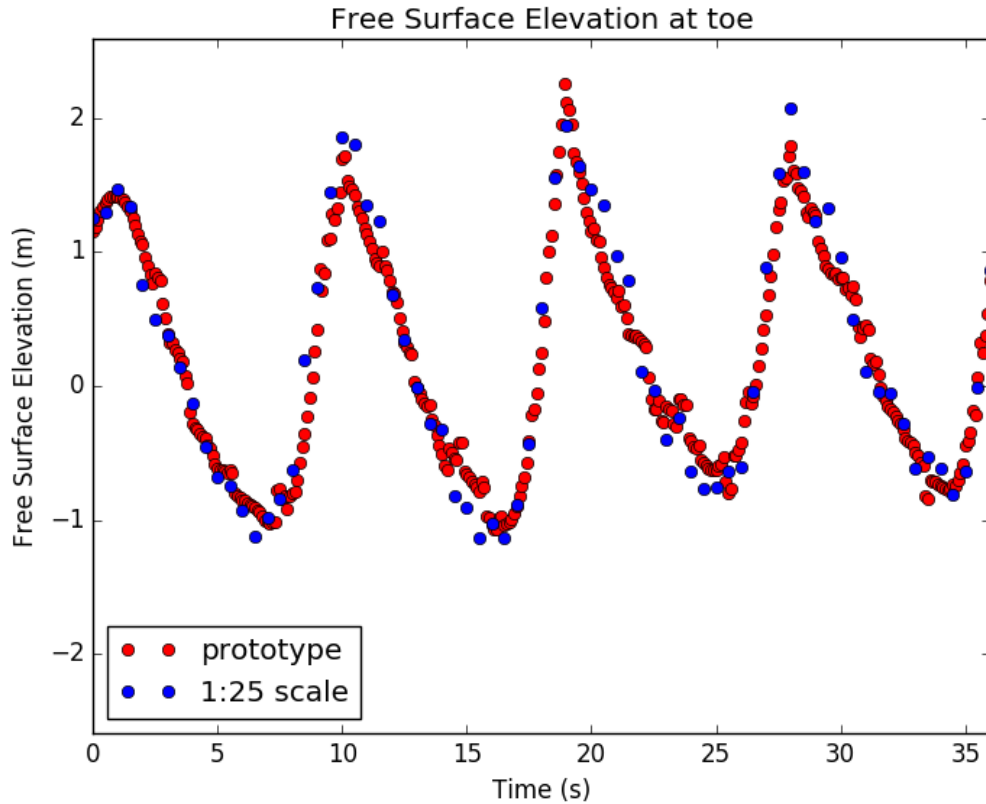


Figure 6.27: Free surface elevation at the toe of the structure for prototype (red) and model (blue) scale simulations. Results from 1:25 model scale simulations are scaled to prototype scale.

Figure 6.27 shows the free surface elevation evolution at the toe of the structure, where very good agreement is observed between scales. Measurements from the model scale simulation have been scaled up to prototype scale. Increase in wave height is due to the ramping up wave generation scheme and the higher reflected waves from the structure. The difference in frequency points is due to both simulations outputting at 0.1 s frequency, giving higher point spacing to model scale simulation when scaled to prototype scale.

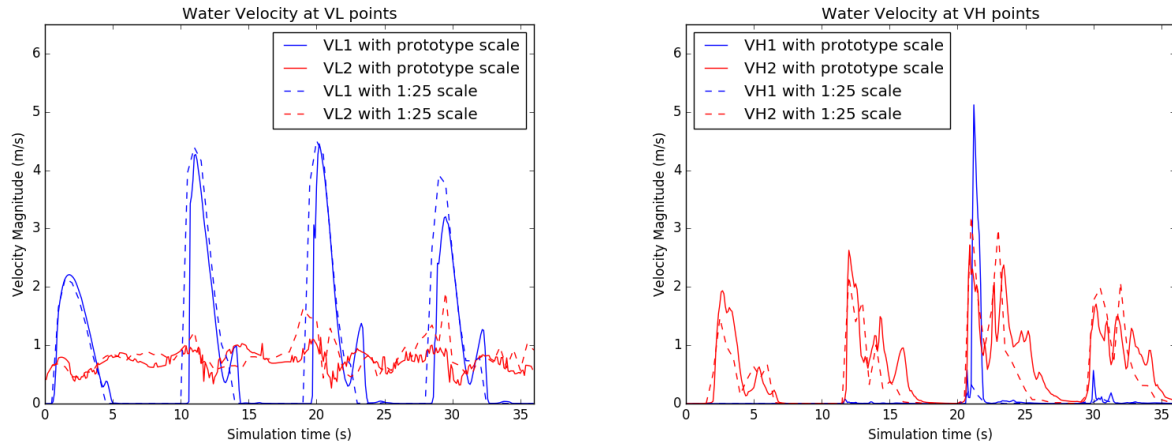


Figure 6.28: Velocities at VH (right) and VL (left) points for prototype and model scale simulations. Velocities and simulation time for the model are multiplied by 25 and root 25 respectively to compare directly with at prototype scale values.

Velocities have been extracted at points VH1, VH2, VL1 and VL2 (see Figure 6.17) and are compared in Figure 6.28 with prototype scale values. Good agreement is observed between simulations at both scales overall, however, velocity peak seen in VH1 for prototype scale is missed by the 1:25 scale simulation results. This is believed to be a consequence of the 1:25 model scale simulation being sampled at the lower output frequency. This suggests that an in depth output frequency sensitivity analysis should be a requirement of any future work on scale effects analysis.

Forces based on the pressure integration around the surface have been extracted for rocks L1-2, M1-2 and H1-2 and are shown in Figure 6.29. Model scale forces have been scaled up to prototype scale in this figure for comparison. No significant differences are observed in L1, L2, M1 and H2 rock particles. However, a general trend on higher force peaks is observed in M2 rock for 1:25 model scale simulation, which suggests a difference in the flow at different scales

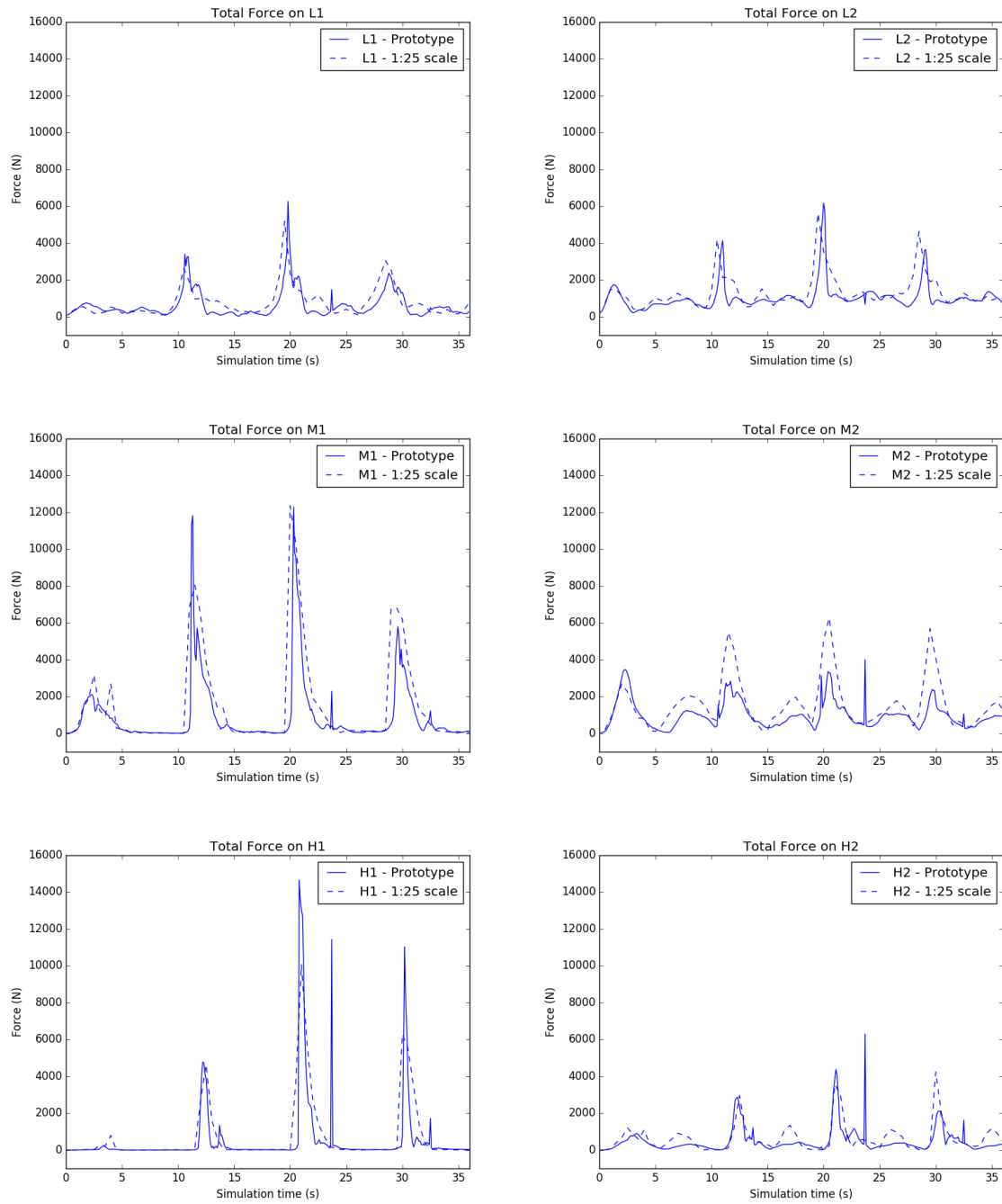


Figure 6.29: Total hydrodynamic force on particles L1 (top left), L2 (top right), M1 (centre left), M2 (centre right), H1 (bottom left) and H2 (bottom right) for simulations at prototype (blue continuous line) and 1:25 model (blue dashed line) scales.

could be expected in the interior of the rock layer at the breaking zone. Differences are also observed at H1 particle, in this case being the 1:25 model scale simulation results lower than the prototype scale results. The differences seen in H1 and M2 should encourage further research on scale effects analysis for rubble mounds, following the approach used by Cuomo et al. (2010) for physical experiments of vertical walls and where force scaling factors had already been

suggested.

The LES turbulence model and other numerical schemes, such as the adaptive time-step or the compressive advection scheme for interface compressiveness, may contribute to the differences observed in this section results. The preliminary results shown here should therefore be considered carefully. However, the observed differences already show the potential of the work presented in this Thesis for the analysis of different aspects of scaling structures. The ability to investigate model scale effects using numerical technologies becomes increasingly important when the fluid-solid coupled models are being investigated and the range of influencing forces being included in the simulation is more complex, such as dynamic stresses caused by motion and collision of armour units.

## 6.5 Discussion

Numerical modelling of rubble mounds has historically been approached with the simplification of representing the main rubble-mound layers as homogeneous porous layers, avoiding the modelling of the real geometry of the layer particles due to computational cost and meshing challenges. This approach has been very useful for obtaining accurate overtopping rates on rubble mound and composite breakwaters, but cannot be used for stability analysis of such structures. This chapter shows the combination of methods presented in Chapters 3, 4 and 5 and the newly obtained capabilities of modelling the real geometry of the particles and the physics of the flow through them in a full rubble mound layer domain. The presented approach code has demonstrated capability of modelling WSI with wave breaking on rubble mounds with real geometries at a very high level of detail, including the modelling and capturing of air entrapment around rock particles.

The main factors hindering the modelling of the rubble mound layers can be linked to the mesh of the domain, in particular the meshing of the intersections of the neighbouring particles surface, and the meshing of the often small spacing between particles. Here different approaches have been successfully introduced and used for modelling CoreLoc and rock armour layers.

CoreLoc layers have proven to be very challenging due to their mesh quality limitations on particle surface intersections with domain boundaries. This has led to consideration of a NWT domain with no units intersecting the sides of the flume. A physical model layer has been successfully imported into a NWT thanks to the manual placement of the CoreLoc units using a laser scan surface as reference. This pilot study for importing a replica of physical layers onto numerical domain is however considered time demanding and cannot be considered for more than one layer. Other approaches based on machine learning algorithms should be explored in order to reduce the time cost of the numerical manual placement. This is left for future discussion and as a potential future work due to its potentially significant contribution for physical and numerical layers validation projects.

CoreLoc and Rock armour layer simulations have performed with particularly fast run-times (120 h and 125 h for eight and four wave periods, respectively). A 5 metres wide section of the rock layer has been presented in this Chapter (excluding the absorption zones at the sides of the slope). This can be extended to larger (wider) domains and it is expected to scale up efficiently with an increased number of cores and harnessing more massively parallelised HPC resources.

The small sample of information provided in this chapter on flow velocities and forces on rock particles are already a novel contribution to the understanding of WSI with rock armour layers, demonstrating the potential of the new model and tools presented here. Applications for analysing the impact of climate change scenarios or the impact of scale effects on the fluid flow through the particle layers are shown here with promising preliminary results.

# Chapter 7

## Discussion

### 7.1 Overall Conclusions

This thesis presents the development and application of different numerical schemes for modelling Numerical Wave Tanks (NWT) with the Finite Element Methods (FEM) model IC-FERST. The combination of the FEM  $P_1DGP_1(CV)$  spatial discretisation, the Dynamic Mesh Optimisation (DMO) and the compressive advection schemes has been shown to achieve accurate results at a reduced computational cost. The compressive advection scheme for interface capture has been shown to reduce the interface thickness and significantly reduce numerical energy dissipation. The implementation of the  $P_1DGP_1(CV)$  spatial discretisation scheme for inertia dominated flow simulations has been shown to increase robustness of simulations with highly anisotropic meshes with no accuracy loss. Finally, DMO schemes have helped reduce the number of elements and the computational cost for NWT simulations while keeping a high resolution in the areas of interest (i.e. free surface elevation).

The combination of such methods with an LES turbulence model has been validated with benchmark cases with the focus on NWT applications. A numerical wave generation scheme has been subsequently implemented and validated against analytical solutions. Moreover, experiments with a single CoreLoc unit on a slope have been used to validate WSI in detail, which have shown high accuracy on free surface elevation and pressure measurements.



Main layers with spherical particles have been used for showcasing domain boundary meshing requirements for the IC-FERST model and the DMO algorithm. Realistic CoreLoc armour layers and rock armour layers with realistic rock shapes have been successfully and robustly modelled under wave breaking and non-breaking scenarios. This has been achieved by avoiding armour unit intersections with side boundaries and by applying an absorption term to the velocity in the regions where rocks are removed, which aims to replicate the general flow behaviour of a porous armour layer by reducing flow velocity and side reflections. Furthermore, the complexity of contacts and potential reduced void space between nearly parallel surfaces of neighbouring rock particles has been simplified by reducing the particles to 90 and 95% of their original size. The high quality mesh achieved in the main layer with this approach has significantly reduced computational cost, leading to simulations of 4 wave periods (36s) being run under 125h with 12 processors Intel Xeon(R) CPU E5-2650 v4 @ 2.20GHz.

In addition, two very different approaches for building rock armour layers have been developed: (i) a volume-filling approach where rocks are laid and compacted on the layer, and (ii) one-by-one rock placing approach with a newly developed algorithm (VC-FDEM) aiming to imitate crane operator placements. Initial results are considered promising, but they are yet to fulfill predictions given in standard engineering guidance documents on layer thickness and porosity and therefore need further investigation.

Finally, two numerical investigations have been undertaken to showcase the readiness of the CFD technology presented in this thesis and its potential applications. First, simulations with simplified different climate change scenarios have been used to assess wave induced force increase on different rock particles of a rubble mound. Simulation results showed nearly a 100% increase in destabilising force, while increases by the widely used empirical equation (Hudson formula, Equation 2.2) predicted only a 30 to 40% increase. Second, a numerical investigation of scale effects has been carried out with simulations of a rock armour layer at 1:25 and at prototype scales, which suggested the flow through the layer could differ between scales. This suggests that further investigation is needed to determine if the armour destabilising forces in small scales are correctly representing those at full scale when Froude scale modelling rules are adopted. Although results from these case studies are not conclusive, they successfully

showcase potential use of the presented methods and future improvements in these methods developed here.

## 7.2 Final Remarks: Limitations and Recommendations

AMCG CFD code IC-FERST and the schemes used have been proven capable of modelling WSI with wave breaking on full sections of rubble mounds at a very high level of detail, including the modelling and capturing of air entrapment around particles. This is seen in Chapter 6 with simplistic versions of rubble mound breakwater or coast protection revetment cover layers using spherical particles, using CoreLoc units, and using rock layers with realistic rock shapes (Latham et al., 2008a). The main limitations faced for the modelling of the rubble mound layers have been linked to the meshing of the domain, in particular around unit surface and rectangular domain boundaries, where bad elements are more likely to be formed due to the spatial restrictions by the boundary surfaces. In addition, neighbouring particles with very close surfaces can also lead to bad quality elements. General approaches have been suggested for avoiding such mesh quality limitations, however, these approaches can be replaced or complemented by manual location shifting or rotation of particles locally. This is considered particularly attractive for geometrically complex armour shapes such as the CoreLoc. The layer layout used in Chapter 6 has been made available to ICL after manual shifting and rotation of units aiming to replicate a real layer from experiments, while also aiming to avoid unit intersections. While applying additional manual shifts to CoreLoc units before the numerical simulation as a mesh quality assessment procedure is expected to increase spacing between units and improve mesh quality, they have not been used here due to the need for objective and reproducible methods.

Alternatively, double layer rubble mounds or layers with convex or nearly convex shapes can benefit from scaling particles to facilitate extra spacing between them. In this way, 90% or 95% scaling factors for rock armour layers (double layer) have shown good performance in terms of mesh quality and computational cost. Although the impact of such particles scaling is yet to be fully investigated, for which a validation case may be needed, it remains a very

attractive solution for highly populated layers, where manual shifting may not be a sensible option. The modelling of the underlayer with a velocity field dissipative region for CoreLoc and rock armour breakwaters has shown promising results and this could be investigated further, including testing this approach for the potential modelling of a permeable core.

Wave breaking validation simulations in 2D have shown that the water jet of a plunging breaking wave can differ slightly from what is observed in physical experiments (Chapter 5). This has also been observed at a smaller scale for 3D plunging waves on rock layers in Chapter 6. This difference in shape from physical observations is believed to be induced by the numerical compressive interface scheme, which is needed for interface capture and helps reducing numerical energy dissipation through the interface, and is reduced by using a turbulence model such as LES.

Simulations of wave interaction with single CoreLoc unit on slope have been used for an in depth sensitivity analysis of wave dissipation using different adaptive time-step CFL targets and mesh resolutions. These tests have shown very good agreement with pressures against experiments at the different pressure sensor locations. Initial analysis on wave induced forces on the unit using integrated pressure on the CoreLoc unit surface have been performed and compared to experiments, showing sensible agreement for forces parallel to the slope and significant underestimation of normal forces to the slope. This is believed to be induced by missing the shear force on the unit in the run up and run down of the wave on the slope, and therefore forces extracted using this approach should be taken with caution.

In terms of computational cost of the simulations presented here (using 12 processors Intel Xeon(R) CPU E5-2650 v4 @ 2.20 GHz), single CoreLoc unit simulations have been run under 100h and 150h for CFL=2 and CFL=1 (number of elements between 100,000 and 200,000 in both cases) for 8 wave periods simulations. The more demanding 3D CoreLoc (100,000-260,000 elements) and Rock (200,000-700,000 elements) armour layer simulations have performed with 120h and 125h for eight and four wave periods, respectively. This has been achieved by using an optimal setup with an adaptive time-step for CFL=2 and a minimum spatial resolution of around 1 order of magnitude lower than the wave height. While this correlation between

mesh resolution and wave height can be used for initial guess for any future simulation, further sensitivity analysis is encouraged when using other wave conditions as wave periods have also been seen to impact overall numerical wave energy dissipation (shorter periods lead to more dissipation). Only 3 to 4 units wide layers have been modelled for this thesis, but this can be extended to larger (wider) domains and it is expected to scale up efficiently with an increased number of cores and harnessing more massively parallelised HPC computational resources.

Two very different approaches for building rock armour layers have been approached in this thesis aiming to fill the gap in existing literature regarding numerical rock armour layers. While no numerical rock armour building approach tested during this PhD can be considered ready, the one-by-one approach (VC-FDEM) has benefited from significantly more time in development and at the same time offers a lot more flexibility on inputs and approaches (i.e. using the recommended placement sequencing as presented by Sigurdarson and van der Meer (2018)). It also has the advantage of using the sophisticated multi-body interaction mechanics of the FDEM code Solidity which has already been validated (Xiang et al., 2009b). This one-by-one virtual crane approach could not be fully investigated and fine-tuned during this PhD, but it has the potential to provide a quick route for future researchers to develop a fully validated and more realistic approach for building rock armour layers in the short-to-mid-term (say 2 to 3 further years of PhD researcher effort).

## 7.3 Applications

The CFD technology presented in this thesis is shown to be ready for potential WSI investigations such as the climate change impacts assessment or scale effects investigations discussed in Chapter 6. Improving the understanding of the flow through rubble mounds and the hydraulic forces generated by waves on these multi-particle systems and its individual units can be considered of real value to both academia and industry. This technology and the new knowledge acquired from using it can open many possibilities, such as refining existing design formulae or testing structural design or existing structures under new scenarios. This is of special interest

under a changing climate, which will lead to an increased loading on the ageing existing coastal defenses and the need of planning new ones. The optimisation and better performance of these will help reduce costs and risk of failure.

Although not considered during this PhD due to time limitations, adding accurate underlayer and permeable core modelling capabilities to the existing model can contribute to better understand permeable structures and their performance and stability. Many other research areas involving WSI can also benefit from the presented technology, such as vertical or composite breakwaters, offshore wind turbines or any other fixed or floating structure under wave or multiphase fluid loadings and induced stresses.

This technology effectively presents a new much cheaper alternative to traditional physical modelling, but it can also be used as a complement to it. The reduced overall cost of using this technology and the reduced computational cost (4 days with only 12 CPU cores) of simulating rubble mounds in NWT offers the opportunity to test a significantly higher iterative possibilities of any structure (i.e. different design sections, water levels and/or wave conditions) at a much reduced cost and effort than when using physical modelling (which would require rebuilding the structure after each test). Moreover, this technology presents the potential to extract flow velocities and wave induced pressures and forces in any location or particle in the NWT domain without the need to plan and install any instrumentation, increasing the data available for judgement at the decision and design stages.

Other research carried out within the AMCG group in recent years on the behaviour and motion of the solid particles of rubble mounds under wave attack using the solids FDEM code Solidity can also potentially benefit from the technology developed during this PhD. To address instability and motion modelling of armour units, AMCG proposes the coupling of Solidity and the fluids modelling with two different methods: a fluid-to-solid one-way coupling (OWC) where wave induced forces on stationary units are extracted from a CFD code and applied to the solids motion and interaction code (Latham et al., 2015; Xiang J., 2019); and a fluid-to-solid-to-fluid two-way coupling (TWC) based on the immersed body method (Xiang et al., 2012; Vire et al., 2015; Yang et al., 2016), where the fluid forces are passed to the solid model, and the resulting

displacements and rotations are passed back to the fluid in an iteratively and loose coupling framework.

The advantage of one way coupling is its much lower computational cost and that the codes are already parallelised and work independently, making any CFD code compatible with the solids code Solidity. The main problem, however, is that the displacement of units is not updated in the CFD code and therefore can only be used for initiation of movement or initiation of damage analysis.

The two way coupling technology has benefited from new developments, optimisation and validation in a wide variety of fluid-solid interaction problems by Yang et al. (2016); Obeysekara (2018); Joulin (2019) with small domains, but it has not yet been parallelised and it is therefore very expensive for larger domains such as rubble mound layers. This technology is already implemented for AMCG CFD code IC-FERST and once the fluid-solid two way coupling method is parallelised, it has the potential to become the first tool for modelling wave-structure interaction with particles motion for breakwater stability analysis with full armour layer sections. Moreover, the two way technology and its immersed body method meshing tools, which are capable of generating a new mesh around the units approximating their surface with the given mesh resolution, could be also used for generating the initial mesh of the rubble mound layers and avoid or reduce manual quality checks on the initial mesh.

## 7.4 Future Work

The research presented in this thesis covers a wide range of topics and, as such, a wide range of potential new research areas have been identified throughout the PhD. A summary of 4 of the potential next lines of investigation that could potentially improve the current results is presented in this Section.

### 7.4.1 Building Numerical Rock Armour Layers

Further investigations of the two different approaches for building armour layers are expected to improve the preliminary results presented in this Thesis. First, the volume-filling and compacting approach can benefit from further sensitivity analysis of the layout grids composition and spacing while aiming to improve layer thickness results. The volume-filling approach can also be implemented with the FDEM Solidity code, which would provide valuable (validated) data on rock stabilising and interlocking forces.

Alternatively, the VC-FDEM can also benefit significantly from further investigation on refining the already available setup parameters and from further development of the existing routines. In addition, the layer-by-layer and row-by-row combined approach for the double armour layer by the VC-FDEM algorithm could also be replaced by other building approaches, such as building both layers in parallel (Sigurdarson and van der Meer, 2018). Figure 7.1 shows the comparison of the layer-by-layer and row-by-row approach followed by the VC-FDEM (left) and the proposed tier-by-tier and outer-inner rock placing approach (Sigurdarson and van der Meer, 2018) (right). The VC-FDEM main routine is easily adaptable and would only require an update of the "first guess" function for allowing such new approach.

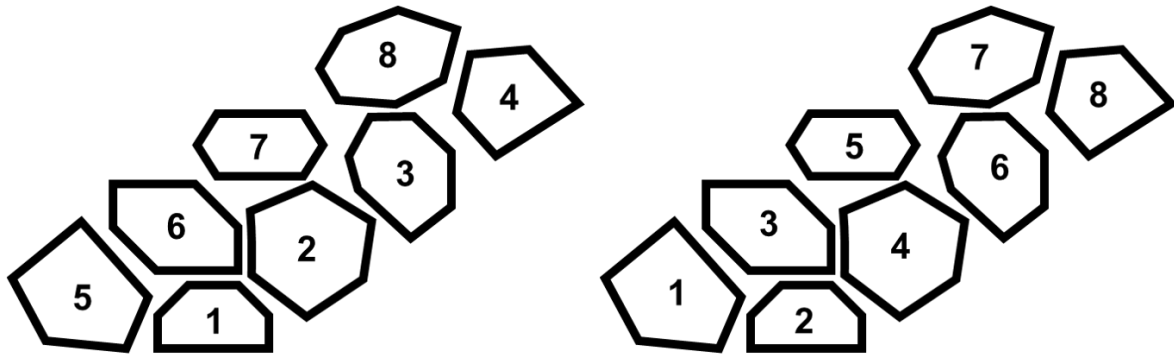


Figure 7.1: Rock placing order comparison between the current version of VC-FDEM (left) and Sigurdarson and van der Meer (2018) (right).

### 7.4.2 Solid Particles as Immersed Bodies within a CFD Domain

The immersed body method (Vire et al., 2015) for the already implemented two-way coupling (TWC) for Solidity and IC-FERST (Yang et al., 2016; Obeysekara, 2018; Joulin, 2019) has the initial mesh of the fluid domain generated by the DMO algorithm and represents the solid particles in the fluid model domain as zero velocity regions, instead of the particles representing an external boundary of the numerical domain as used for this thesis. Additionally, the DMO algorithm refines the mesh at the particles boundaries to resolve the shape geometry (at a given resolution) and the fluid-solid interaction accurately.

The approach of including rock particles in the CFD model domain as zero fluid velocity regions could be easily implemented for non-coupled fluid-only simulations. Although the particles geometry may change due to mesh resolution limitations, this can also present some advantages. Close neighbouring particles and intersections between particles and domain boundaries can be resolved with a given resolution by the user, leading to small void spaces by nearly parallel particle surfaces being considered as zero velocity regions and not imposing a fixed outer domain boundary that would potentially reduce the elements quality. The mesh resolution can be easily set by changing the minimum edge length requirements for the DMO algorithm. In this way, investigations on the impact of mesh resolution changes in particle shapes can be investigated, together with its impact on fluid flow velocities.

### 7.4.3 Capturing Shear Forces

The integration of the pressure around a particle surface is shown to be a very flexible approach for extracting wave induced forces on particles from numerical simulations. However, this is observed to miss the shear force generated by the fluid flow on the unit surface, also known as skin drag force. Skin drag forces are highly dependant on surface roughness, fluid viscosity, and boundary-layer scale physics, which have historically made them difficult and costly to estimate in large domains or for multiple particles. Simpler approaches for estimating forces on bodies, and rubble mound layer units in particular, under wave action have been used in



literature (Morison et al., 1950; Tørum, 1994; Moghim and Tørum, 2012; Jensen et al., 2014; Simpalean, 2019).

Although the total drag force can be conceptually considered as the sum of 'form' drag and 'skin friction' drag forces, the general trend in the Coastal Engineering community has been to calculate total drag forces using empirical formulae such as Morison et al. (1950). This empirical formulae (Morison et al., 1950) is still in use nowadays thanks to its simplicity, and represents the drag force as the sum of two terms: the mass term and the drag term, the latter including drag from wake effect and skin friction. This approach however needs empirical coefficients  $C_M$  and  $C_D$  to be known as these are not constant for every shape and in every wave condition. Research (Simpalean, 2019) carried out recently presented experimental data with CoreLoc units and used it for obtaining  $C_M$  and  $C_D$  parameters. Although these were obtained from submerged CoreLoc only tests, these values could be used as an initial guess for estimating total forces and compare them with experiments of single CoreLoc units, where units are only partially submerged. Similarly, coefficients used in recent work on layers and OWC validation (Xiang J., 2019) could be considered for the single unit case and compared with current results.

A more general approach that can be applied to any shape and makes use of the already existing pressure integration approach is to follow the drag force conceptual definition of the sum of 'form' and 'skin friction' drag forces. The 'form' drag can be represented by the pressure integration as it captures the normal forces on the particle, including both frontal pressure and wake resulting pressure at the rear, and the 'skin friction' drag force capturing shear forces can be estimated by extracting fluid velocity on each of the particle surface elements (with the same pressure extraction routine) and converting them into a friction drag force with  $F = \int_{surface} \frac{1}{2} \rho \mathbf{u}^2 C_f dA$ . While this approach would still require a coefficient  $C_f$  for obtaining such force from the velocity field, this single coefficient would be much easier to estimate than the two Morison coefficients due to this being dependant only on Reynolds number (fluid velocity and viscosity) and surface roughness, and not depending on shape and orientation of the particle. This has been implemented and tested in Kampitsis et al. (2021), but it requires further validation.

Alternatively, the Two Way Coupling (TWC) approach for modelling particle motion under hydraulic forces which uses the immersed body method (Vire et al., 2015) can be used to extract forces on solid particles and simulate the particle motion and particle-particle interaction in a FDEM model. The higher resolution applied to the particle geometry boundary buffer zone, known as the shell, has the potential to help resolve the boundary-layer physics of flows with high Reynolds numbers (Yang et al., 2016), and therefore obtain the ‘skin’ drag force or shear force on the unit surface. This method could also be further adapted and implemented as part of a CFD-only simulation if required, and benefit from the model performance optimisation techniques presented in this Thesis, which can significantly reduce the computational cost of adding a shell mesh around a high number of particles.

The available experimental data on wave induced forces on a CoreLoc unit can help investigate all suggested and other potential approaches for extracting forces from CFD simulations. In addition, experimental results on forces on rock particles available in literature (Tørum, 1994; Hald, 1998; Moghim and Tørum, 2012; Jensen et al., 2014) can be used for validation of IC-FERST WSI simulations with rock rubble mounds.

#### 7.4.4 Interface capture

The compressive advection scheme (Pavlidis et al., 2016) used here for interface compressiveness is shown to reduce the numerical energy dissipation by the interface significantly, however, wave breaking validation simulations have shown that the water jet of a plunging breaking wave can differ slightly from what is observed in physical experiments. Other more compressive approaches have recently become available (Obeysekara et al., 2021) as part of a combined force balancing approach (including surface tension) within AMCG and should be investigated for NWT applications with a focus on the wave breaking and potential energy dissipation reduction. In addition, further investigations could be carried out to find and reduce the artificial effect of the compressive advection scheme on pressure and velocity fields.

Alternatively, other interface tracking schemes available in literature could be investigated and potentially be implemented for IC-FERST. The HYPER-C face limit method implemented for

Fluidity by Wilson (2009) has shown good validation results with dam break benchmark cases, and could potentially become an alternative to current compressive advection type schemes if wave breaking results show good agreement with experiments.

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