

Determination of Stresses at Inclusions in Single Crystal Superalloy using HR-EBSD, Crystal Plasticity and Inverse Eigenstrain Analysis

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Abstract

The complementary techniques of high-resolution electron backscatter diffraction (HR-EBSD), crystal plasticity finite element modelling and the inverse method of eigenstrain are utilized for evaluating stresses resulting from the mismatch in thermal expansivities of a nickel single crystal containing a carbide particle. The EBSD method is employed to measure the complete residual elastic strain tensor on the free surface of the nickel matrix around a particular carbide particle. With these experimental results, the 3D inverse problem of eigenstrain is reconstructed to determine the complete residual stresses local to the particle at the *sub-surface* of the sample. A gradient-enhanced crystal plasticity finite element (CPFE) model has been developed for the same sample and loading conditions, and detailed comparisons of the eigenstrain and CPFE predicted stresses at the sub-surface of the sample are presented. In addition, free-surface residual elastic strains measured by HR-EBSD and predicted by the CPFE model are compared. Free-surface results show very good agreement, but some differences are apparent for sub-surface results. The eigenstrain technique relies on the assumption of uniform plastic strain in the direction normal to the free surface, and the CPFE approach provides an assessment of this assumption. The free-surface effects of 3D particle depth are also assessed.

Keywords: Residual stress; eigenstrain; HR-EBSD; crystal plasticity, nickel superalloys

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1. Introduction

Knowledge of the residual stresses which exist at key microstructural features in structural materials plays an increasingly pivotal role in component lifing methodologies. Fatigue crack nucleation, for example, is known to occur preferentially at second-phase particles (SPPs) in aluminium alloys [1], and at agglomerates in powder metallurgy processed nickel alloys [2] which act as stress raisers and the sites of highly localised slip. Modelling methods appropriate for the microstructural level such as crystal plasticity and discrete dislocation techniques are increasingly being employed in order to determine stress and plastic strain at key fatigue-inducing microstructural features (grain boundaries, triple junctions, twins, SPPs) [3]. Such models are growing in their sophistication providing additional microstructure-level information such as lattice rotations and curvatures and densities of geometrically necessary dislocations [3]. There is, therefore, great need for rigorously tested and validated models and a powerful experimental technique which can facilitate this is high resolution EBSD [4]. However, the technique is generally confined to free-surface measurement, albeit with the current resolution (tens of nanometers) enabling accurate elastic lattice strain (and hence stress) and rotation to be calculated [5]. Recently, highly successful 3D HR-EBSD has been reported [6], but this remains a destructive technique, and indeed the serial sectioning process leads to a change of stress state from what originally existed sub-surface to one which becomes approximately plane stress as material is machined away. Hence, the HR-EBSD-measured strains are those corresponding to free-surface, plane stress conditions rather than to those which existed before the serial sectioning process. Often, it is of more importance to gain knowledge not of free-surface microstructural stress state, but that which occurs sub-surface at the key microstructural features. For this reason, microstructural inverse eigenstrain techniques have been established [7] for a given class of problem which utilise the

measurement of free-surface elastic strain from HR-EBSD in order to establish full field 3D (sub-surface) stress distributions. The technique necessitates use of the commonly adopted assumption of invariant sub-surface eigenstrain distribution (often meaning plastic strain), and it is important that the validity of this simplification be tested. Previous work [12] also employed crystal plasticity modelling to assess the agreement between HR-EBSD measurements of free surface strains and crystal plasticity model predictions. However, this work did not address the eigenstrain technique nor assess the role of the particle depth on free-surface strain distribution. Hence, the aim of this work is multifold. Firstly, we aim to demonstrate an independent validation of the recent development [7] that extends the capability of the inverse problem of eigenstrain to determine residual stresses at sub-surface single-crystal nickel – carbide particle interfaces. Secondly, we investigate the accuracy of microstructure-based crystal plasticity modelling in capturing with fidelity the HR-EBSD-measured elastic lattice strains local to the single crystal – SPP interface, and thirdly, the effect of the sub-surface particle depth on the free-surface HR-EBSD-measured elastic strains is investigated. In the presentation, length-scale-enhanced crystal plasticity finite element modelling is introduced to simulate the single-crystal-particle material subjected to cooling from 820 °C to 20 °C from a nominally stress-free state thus causing the development of thermal mismatch strains as a result of the differing thermal expansivities of the Ni matrix and the carbide particle. Detailed comparisons between 3D crystal model predictions and HR-EBSD free-surface measurements are firstly presented. This is then followed by an analysis of the sub-surface stresses developed local to the particle obtained from the eigenstrain technique which are assessed against the crystal plasticity model predictions. An assessment is then presented of the key assumptions utilized in the eigenstrain technique. Finally, we examine the role of the particle out-of-plane length on influencing the free-surface strain and stress distributions measured using HR-EBSD.

2. Experimental Background

The material used in this study (supplied by Rolls-Royce plc) is a high tantalum and niobium-containing variant of the nickel superalloy MAR-M-002, which is widely used in the manufacture of industrial gas turbine engine components, as well as in aero gas turbine engines that operate at elevated temperature and at high stress. Changes in temperature for this type of superalloy resulting from heat treatment processing result in residual stresses around the carbide particles due to differences in thermal expansion coefficients between the nickel matrix and the carbide particles.

The resulting free-surface strains developed around carbide particles determined using HR-EBSD were reported in the work of Karamched and Wilkinson [8], and hence only a brief description is given here. The material was aged for 16 hours at 820 °C to precipitate large γ' (~20 μm) particles within a single crystal Mar-M-002 matrix, followed by cooling to 20°C. HR-EBSD was then carried out in order to determine, at a representative SPP, the bulk single-crystal crystallographic orientation together with the details of the local lattice strains and curvatures [8]. Figure 1 shows the geometry of the carbide particle-matrix combination over which HR-EBSD measurements were performed. The origin of the coordinate system is the centre of the free surface. EBSD patterns were obtained using a 1 mega pixels CCD camera at full resolution on a tungsten filament JEOL-6500 FEG SEM with TSL EDAX OIM DC v5.3 software, at 20 keV beam energy and using a probe current of approximately 10 nA. All maps were obtained using a 250 nm step size, which is well above the spatial resolution of the method. Lattice rotations and elastic strains in the sample result in small shifts in the positions of zone axes and these small pattern shifts are measured using image-processing based on cross-correlation analysis at four or more regions of interest [9,10] which enables the determination of eight of the nine degrees of freedom. Since EBSD measurements are

obtained within a few tens of nanometers of the free surface, with the help of the plane stress assumption, the last degree of freedom can be calculated. Hence, all six components in the strain tensor and all three terms in the lattice rotation tensor are fully determined. In addition, EBSD also enables determination of the crystallographic orientation and in the sample studied here, the orientation was found to be $\phi_1 = 83.5^\circ$, $\phi_2 = 91.5^\circ$ and $\phi_3 = 0^\circ$ in Bunge notation.

3. Crystal Plasticity Model

In this section, we introduce the crystal plasticity finite element modelling technique employed in this study to predict residual stresses/strains caused by mismatched thermal expansivities between the single-crystal nickel alloy and the carbide particle due to thermal cooling from 820 °C to 20 °C from a nominally stress-free state. The model takes account of the activity on each individual slip system and dislocation type (edge versus screw) for geometrically necessary dislocation (GND) accumulation.

Crystal plasticity kinematics depend upon deformation gradient $\mathbf{F}$, which can be multiplicatively decomposed into elastic and plastic parts as

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} = \mathbf{F}^e \mathbf{F}^p \text{ with } \dot{\mathbf{F}}^p = \mathbf{L}^p \mathbf{F}^p \text{ and } \mathbf{L}^p = \sum_{i=1}^n \dot{\gamma}^i \mathbf{s}^i \otimes \mathbf{n}^i, \quad (1)$$

in which the plastic part of the velocity gradient $\mathbf{L}^p$ consists of contributions from each active slip system, with normal vector $\mathbf{n}^i$ and slip direction vector $\mathbf{s}^i$ corresponding to the i^{th} slip system, and is computed according to a defined slip rule. The slip rule used here is that developed by Dunne et al [11]

$$\dot{\gamma}^i = \rho_s^m b^{i2} \exp\left(-\frac{\Delta F}{kT}\right) \sinh\left(\frac{(\tau^i - \tau_c^i) \gamma_0 \Delta V^i}{kT}\right) \quad (2)$$

with

$$\Delta V^i = l b^i{}^2 \text{ where } l = \frac{1}{\sqrt{\psi(\rho_s^s + \rho_G)}}, \quad (3)$$

in which ρ_s^m and ρ_s^s are the mobile and sessile statistically-stored dislocation (SSD) densities, and ρ_G is the overall GND density, b^i the Burger's vector magnitude for slip system i , ν the frequency of attempts (successful or otherwise) by dislocations to jump the energy barrier, ΔF the Helmholtz free energy, k the Boltzman constant, T the temperature in Kelvin (K), τ^i the resolved shear stress, τ_c^i the critical resolved shear stress, γ_0 the shear strain that is work conjugate to the resolved shear stress, ΔV the activation volume, l the pinning distance, and ψ is a coefficient that indicates that not all sessile dislocations (SSDs or GNDs) necessarily act as pinning points. Each slip system becomes active when the resolved shear stress is equal to or greater than the critical resolved shear stress ($\tau^i \geq \tau_c^i$). Eq. (3) provides a length scale-dependent slip rule because the density of GNDs is geometrically dependent [12]. The densities of both mobile and immobile SSDs are, for simplicity, assumed to be the same and we choose not to evolve the SSD density directly with deformation, but allow the dislocation density to change as a result of the GND evolution which is described below.

The density of GNDs is calculated by using the relation between inelastic strain gradients and dislocation densities [12] originally proposed by Nye [13] and subsequently developed by Busso et al. [14] and Acharya and Bassani [15] but here is given with respect to the deformed configuration as

$$\mathbf{G} = \sum_i (\mathbf{b}^i \otimes \boldsymbol{\rho}_G^i) = (\text{curl } \mathbf{F}^{e-1})^T \quad (4)$$

in which, $\mathbf{F}^e$ is the elastic deformation gradient, the summation is over all active slip systems, and $\mathbf{G}$ is introduced for convenience.

The solution of Eq. (4) potentially leads to a non-uniqueness problem (Arsenlis and Parks [16] and Dunne et. al. [12]) in which the number of distinct dislocation types may exceed the nine independent components of the Nye tensor, so that a unique solution for the dislocation density may not be possible. In such circumstances, the GND density may be obtained by solving Eq. (4) with an imposed constraint such as minimization of stored energy or dislocation line length. The material model is implemented in to the commercial finite element code ABAQUS using a user defined element (UEL), as described in [11].

The cross-sectional dimensions of the finite-element model were chosen to be the size of the EBSD map (36 μm height $\times$ 26 μm wide). The geometry of the carbide on the free surface, shown in Figure 1, is known from EBSD measurements. However, sub-surface, neither the cross-sectional area nor depth of the carbide particle are known with certainty. As a result, in the crystal modelling work, two depths of the carbide particle are considered to investigate its effect on residual stresses/strains. The particle depth in the first crystal model is taken to be 75 μm which is equal to the nickel matrix depth modelled. In the second model, the carbide particle depth is taken to be 5 μm embedded within the nickel matrix of depth 25 μm . In the simulations the nickel sample is modelled as face-centered cubic (FCC) single crystal. We assume that the carbide particle remains elastic. A three-dimensional finite element mesh comprising 20-node brick elements with reduced integration is used for the discretization of the samples as shown in Figure 2. The mesh is finer close to the nickel-carbide boundary to allow for good resolution of plastic deformation.

The values of the material properties for the nickel alloy studied are as follows: Young's modulus 207 GPa, Poisson's ratio 0.28, critical shear resolve stress 230 MPa, Burger's vector magnitude 3.5072 Å, Boltzman constant $k = 1.381 \times 10^{-23} \text{ JK}^{-1}$, the frequency of dislocation jumps $\nu = 1 \times 10^{11} \text{ s}^{-1}$, the activation energy $\Delta F = 3.45 \times 10^{-20} \text{ J}$, nominal strain $\gamma_0 = 8.33 \times 10^{-6}$,

and $\psi = 1.5 \times 10^{-4}$. The initial density of the mobile and sessile statistically stored dislocations are taken to be $\rho_s^m = \rho_s^s = 5 \times 10^{10} \text{ m}^{-2}$, and the initial density of GNDs is taken to be zero. The thermal expansion coefficient for the nickel alloy is taken to be $13.0 \times 10^{-6} \text{ K}^{-1}$ and that for the carbide particle is $9.5 \times 10^{-6} \text{ K}^{-1}$. The carbide material is assumed to be elastic and isotropic with a Young's modulus of 550 GPa and Poisson's ratio of 0.24. The initial temperature is taken to be 1093 K.

4. Inverse Problem of Eigenstrain

The principle of the inverse method used in this work is based on computing the eigenstrain from the measured residual elastic strains on the free surface of the nickel matrix zone and then the residual stresses are derived from the eigenstrain. The source of all residual stress is incompatible strain in a body which is the so-called eigenstrain. Many researchers have studied residual stresses in engineering components using eigenstrain [17-20]. The motivation for using eigenstrain to determine residual stresses is that it is independent of geometry. In other words, a change in geometry of a body alters the residual stress distribution but not the eigenstrain and hence if the eigenstrain variation in the body is known, the residual stress can be calculated for any configuration sectioned from this body.

The method used in this study aims to determine residual stresses at the sub-surface by means of the measured residual elastic strain tensor on the free surface (Figure 1). The one additional assumption for the inverse problem of eigenstrain in this work is that the distribution of eigenstrain in the principal (longitudinal) direction is constant. This is because of the fact that the experimental data set in this work is only available on the free surface (one plane), and the variations of the eigenstrain components in the direction normal to the measured surface may not be directly calculated without further information. Hence, the most

reasonable approximation is to assume that the eigenstrain distribution is uniform in that direction.

The computation of the eigenstrain from the measured residual elastic strains can be achieved by using the inverse solution technique which is widely used for residual stress measurement methods such as incremental hole drilling [21] and crack compliance [22]. In previous studies reported in the literature, only one non-zero eigenstrain component was assumed to exist [19] or multiple eigenstrain components have been studied but their spatial variations were usually limited to 1D [23] or 2D [18]. In the recent work of Kartal et al. [7], fully 3D inverse eigenstrain analysis has been carried out. In other words, Kartal et al. [7] have demonstrated the capability of the inverse problem of eigenstrain applicable for fully 3D sample geometry, 3D residual elastic strain components, 3D eigenstrain components, and 2D spatially varying eigenstrain. Data reduction has been explicitly explained in the previous work [7], and hence will only be briefly discussed in the next section.

It is assumed that the unknown eigenstrain variation (as a function of (x,y)) can be expressed as the sum of basis functions:

$$\epsilon_{xx}^*(x, y) = \sum_{i=1}^{(m+1)(n+1)} A_i^{xx} S_i(x, y) \quad (5)$$

where ϵ_{xx}^* is the unknown eigenstrain component (here, only one component of eigenstrain is denoted; other unknown eigenstrain components can be expressed in a similar manner), A_i^{xx} are unknown coefficients of the series for the unknown eigenstrain component ϵ_{xx}^* to be calculated, and S represent bivariate Legendre polynomials which are the dot product of two Legendre polynomials as basis functions such that

$$S_i(x, y) = P_k(x)P_l(y). \quad (6)$$

Legendre polynomials have been chosen as basis functions in this work. Here, the k index is the order of the polynomials in the x direction (ranges from 0 to m), the l index is the order of polynomials in the y direction (ranges 0 to n), and $i=l+(n+1)k+1$. Hence, the determination of each eigenstrain component is accomplished by solutions of a number $(m+1)(n+1)$ of unknown coefficients. Assuming elastic behaviour and using superposition, the measured residual elastic strains $e^{(m)}$ can be represented by a linear combination of unknown coefficients as [7]

$$\{e^{(m)}\} = \{A\}\{C\}, \quad (7)$$

in which C is the compliance matrix which constitutes the resulting residual elastic strain tensor induced by each eigenstrain component for each term in S_i and A is a coefficient matrix involving all unknown values for each eigenstrain components [7]. In this case, the measured residual elastic strain $e^{(m)}$ is a column matrix including all residual elastic strain data points on the measured surface for each component, and the compliance matrix C has i columns for each eigenstrain component, and rows for all data points for the predicted residual elastic strain tensor computed on the free surface.

The unknown coefficient matrix A is computed by means of a least squares fit between the measured and eigenstrain data [7]:

$$\{A\} = ([C]^T[C])^{-1}[C]^T\{e^{(m)}\}. \quad (8)$$

Once the unknown coefficients of the series expansion have been found using Eq. (8), the eigenstrain distribution can be calculated from Eq. (5).

Note that the residual elastic strains obtained from EBSD were measured only on the free surface of the nickel matrix region. In other words, the strain values in the carbide zone are

unknowns. However, because the carbide is a very hard material, it is fair to assume that it shows no plasticity. Then, it follows that a constant eigenstrain (from thermal cooling) exists in the carbide. This becomes one additional constant to fit along with the eigenstrains in the matrix. Hence it is sufficient to utilize eigenstrain distributions in the nickel matrix only. In other words, the zeroth order of the polynomial series (constant) used in the nickel matrix region accounts for the difference in thermal expansions between the two materials.

Mathematically, eigenstrain can be expressed as a dot product of the anisotropic thermal expansion tensor and unit temperature change. Therefore, any misfit strain can be imposed on to an FE model by using thermal analysis. For implementation of the inverse eigenstrain analysis, the residual elastic strain tensor obtained from EBSD on the free surface of the nickel material was initially evaluated at the locations of the nodes in the finite element model and they were assembled in the form of $\{e^{(m)}\}$. Each term in the bivariate Legendre basis function series was input into the nickel matrix region in the FE model as the known eigenstrain distribution of each component (ϵ_{xx}^* , ϵ_{yy}^* , ϵ_{zz}^* , ϵ_{xy}^* , ϵ_{xz}^* and ϵ_{yz}^*). Next the resulting predicted elastic strain components were 'measured' on the free surface of the nickel matrix region in the FE model. After the compliance matrix was assembled, the unknown coefficients were found by least squares solution of Eq. (8). Then, all the unknown eigenstrain components were computed. Finally, these determined eigenstrain distributions were then included in the FE model, resulting in elastic strains and the residual stresses. Note that the Legendre polynomial is valid in the interval $[-1:1]$ and therefore the co-ordinates were normalized with dimensions of the thickness and the width before computing the values for each eigenstrain component.

5. Results

Figure 3 (a-d) shows contour plots of residual elastic strain components (e_{xx} , e_{yy} , e_{zz} and e_{xy} respectively) obtained from high-resolution EBSD measurements local to the particle, on the material free surface together with the corresponding CPFE results for the 75 μm depth model in which both particle and matrix are assumed to have the same depth, as shown in Figure 2(a). Note that EBSD strain measurements are not available in the carbide particle. Hence, strains within the carbide obtained from CPFE have not been shown in the field plots. Dilatational strains resulting from temperature change are not captured by EBSD, so that in order to make direct comparisons of the calculated CPFE results, the purely dilatational strains and plastic strains have been subtracted from the total strains in order to generate the distortional elastic strains.

As can be seen from the figures there are some aspects of the strains which are in good agreement but others for which this is not the case. For example, the longitudinal strain component e_{zz} is found to be wrongly predicted in sign particularly very close to the particle. In contrast, shear strain components predicted by CPFE are in good comparison with EBSD measurement. Figure 4(a-d) shows the contour plots of residual stress components (σ_{xx} , σ_{yy} , σ_{zz} and σ_{xy}) calculated using the CPFE and eigenstrain models at the subsurface ($z=-37.5$ μm). While in-plane stress components obtained from the inverse method of eigenstrain show similarities in comparison with those determined from the CPFE model, there are significant differences for the longitudinal stress component. In order to illustrate the variation of the residual stress component σ_{zz} obtained from both methods, a line plot along $y=0$ is extracted from the contour plot and shown in Figure 5. It is apparent that comparison shows very poor agreement. A potential explanation for these differences is that the assumption made about the carbide particle depth is wrong, and in order to investigate this, a second model is analysed in which the particle depth is now taken to be considerably smaller than previously assumed. In the second model, the carbide depth is now taken to be 5 μm within a 25 μm

depth nickel matrix, as shown in Figure 2(b). Figure 6(a-d) shows the contour plots of residual elastic strain components (e_{xx} , e_{yy} , e_{zz} and e_{xy} respectively) obtained from high-resolution EBSD measurements local to the particle, on the material free surface together with the corresponding CPFE and the inverse eigenstrain technique. As can be seen, the eigenstrain technique provides a very good fit to the EBSD-measured strains for the 20-order polynomial series on the sample free surface. This is anticipated since the EBSD strains are used as part of the eigenstrain fitting process whereas the CPFE model is an independent validation of both the EBSD and eigenstrain results. In order to carry out a more quantitative assessment, all the elastic strain components along the horizontal line ($y = 0$) on the free surface have been extracted from the EBSD measurements, eigenstrain calculations and CPFE model predictions and are shown in Figures 7(a-d).

Maps of eigenstrain technique and CPFE predicted residual stress components (σ_{xx} , σ_{yy} , σ_{zz} and σ_{xy}) at the subsurface when $z = -2.5 \mu\text{m}$ are shown in Figures 8(a-d). There is close qualitative agreement between the two sets of stress contours. Once again, in order to illustrate the quantitative variation of the residual stress components obtained from both methods, a line plot along $y=0$ is extracted from the contour variations and shown in Figures 9 (a-d). Much better agreement between the two predictive methods is now achieved than in the previous (deep particle) analysis.

6. Discussion

In this work, we have aimed to provide detailed comparisons of the results of gradient enhanced crystal plasticity modelling with the inverse eigenstrain technique to calculate subsurface residual stresses from free-surface EBSD measurements. Because the combination of the EBSD method and the inverse technique of eigenstrain has been recently developed [7], this work presents the first numerical validation of the technique.

EBSD techniques (generally) give information only on free surfaces which can be quite different to stress states which can exist sub-surface. Experimental observations in nickel alloys show that fatigue crack nucleation is often associated with second phase particles embedded within the primary phase matrix. In addition, geometrically necessary dislocations can affect crack nucleation and growth [24], the initiation of recrystallization and the nucleation of twinning. Hence, predictive lifing techniques require correctly determining stress state and magnitude local to second phase particles at the appropriate length scale. Thus, we have aimed to test the ability of the eigenstrain technique to provide this information at a particular microstructural heterogeneity, and compared results with gradient-enhanced crystal plasticity.

The approach is currently limited to problems containing only prismatic micro-structural features. Consequently, some of the observed differences in the measured and predicted results are due to the modelling assumptions made. For instance, while free-surface modelling results have been compared properly with free-surface EBSD measurements, in the model, the particle is assumed prismatic sub-surface; this is unlikely to be strictly the case in reality. In addition, in the experimental Ni alloy material, while it is known that the carbide particles are well-dispersed such that inter-particle effects are likely to be small, they cannot be eliminated and the possibility always remains that additional local particles exist and interact.

Comparisons of the sub-surface residual stresses from the CPFE model with the inverse eigenstrain technique are promising, though there are some disagreements. Some of these differences are attributed to the assumption made for the eigenstrain technique for which the experimental data, on which the technique is based, is only available on the free surface (one plane), and the variation of the eigenstrain components in the direction normal to the measured surface therefore has to be assumed. In this work, the eigenstrains are assumed to

be invariant with particle depth. In order to investigate the validity of this assumption, the variation of effective plastic strain from the CPFE model in Figure 2 (b) along the z-direction from the free surface at the position $y=0$, $x=3.4\text{ }\mu\text{m}$ (that is, very close to the nickel - carbide boundary) is shown in Figure 10. As can be seen from the figure, crystal plasticity finite element results produce a non-uniform plastic strain in the longitudinal direction, and this therefore brings in to question the validity of the assumption being made for the eigenstrain analysis. In addition, the assumptions being made in the crystal analysis may also be reasonably questioned.

7. Conclusions

A 3D gradient-enhanced crystal plasticity finite element model has been developed for a single-crystal nickel alloy sample containing a carbide particle subjected to cooling. Comparisons of HR-EBSD-measured, CPFE and eigenstrain model-calculated elastic strains at the sample free-surface show good quantitative agreement.

The elastic strains local to the embedded particle but now sub-surface are also predicted from CPFE and eigenstrain modelling (in which the latter technique utilizes knowledge of the free surface strains). The quality of the agreement is found to depend on the depth assumed for the embedded particle, with better quantitative agreement being obtained for a shallow (as opposed to prismatic, full-length) particle.

As expected, the key simplifying assumption made in the eigenstrain model that the 2D inherent strain distribution remains uniform through the sample depth is found to lead to differences in the CPFE and eigenstrain sub-surface elastic strain (and hence stress) predictions. However, the results indicate that very good qualitative estimates of sub-surface stresses are achievable using the eigenstrain technique with knowledge of EBSD free-surface measurements.

8. References

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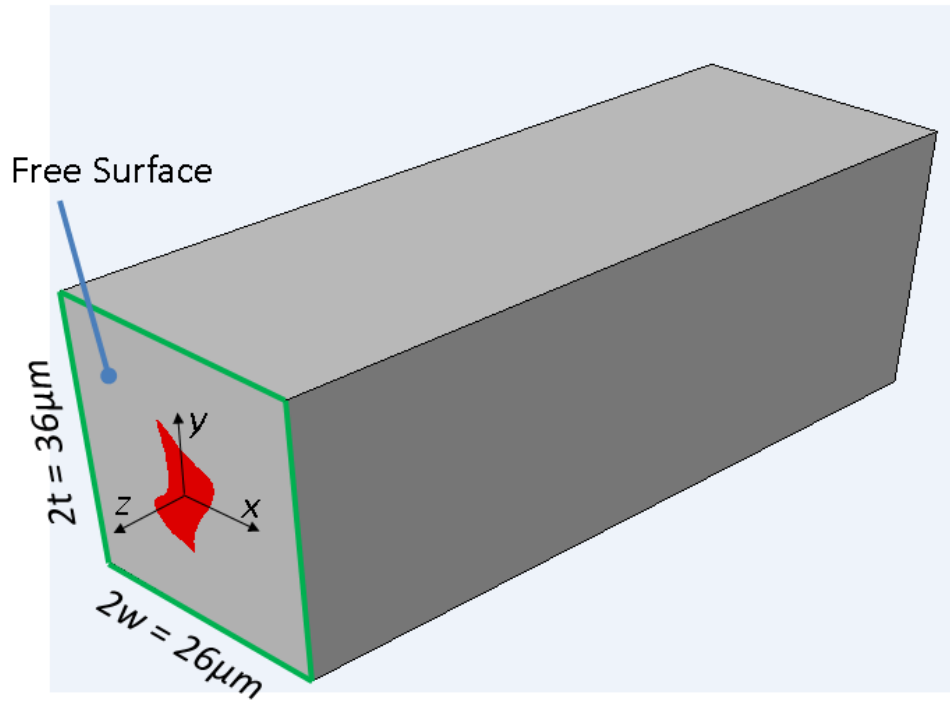


Figure 1 Geometry of the Nickel alloy matrix containing a large carbide particle

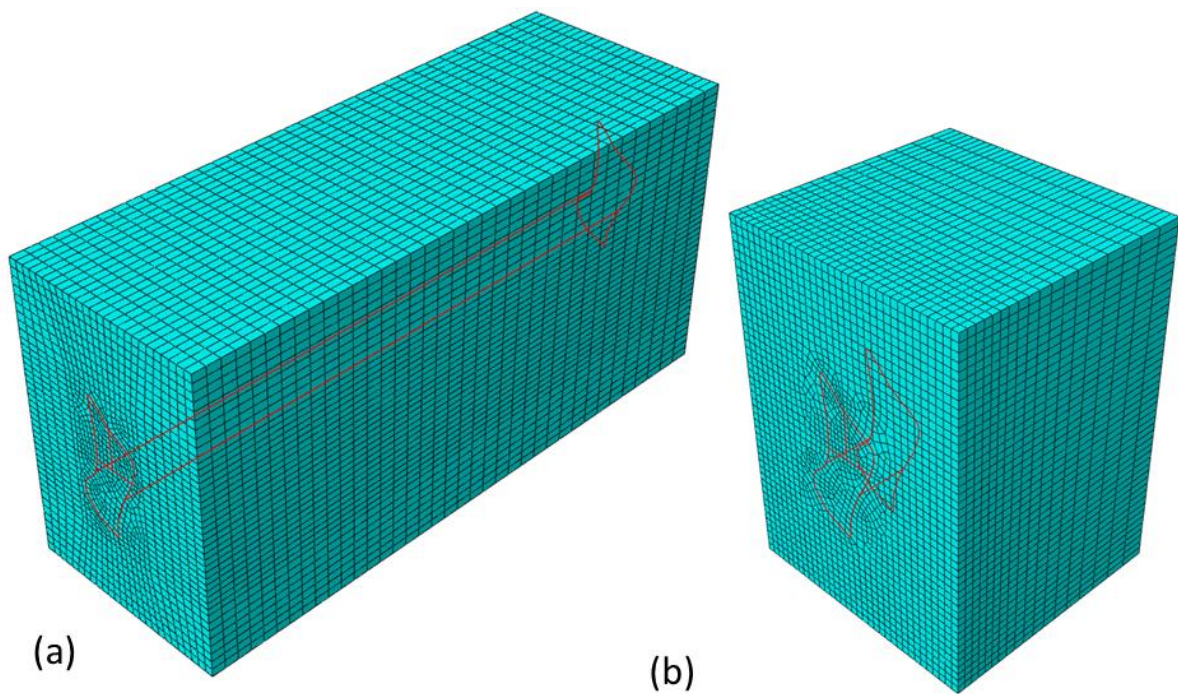
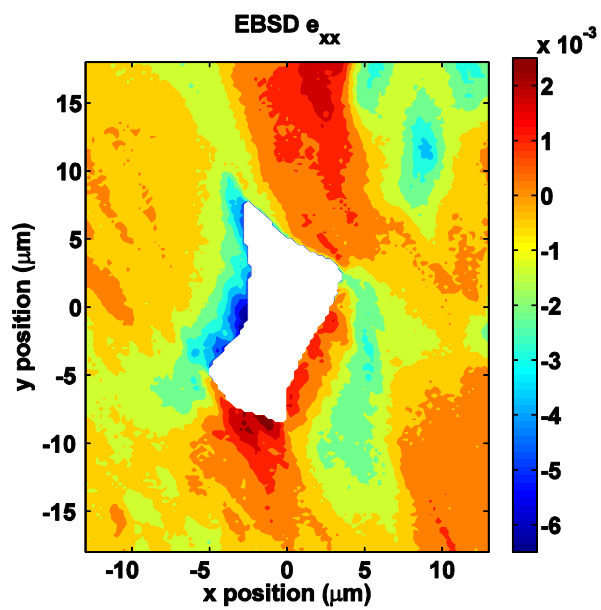
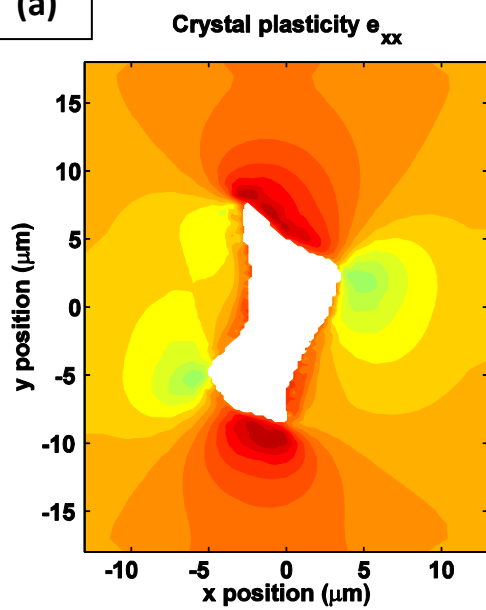
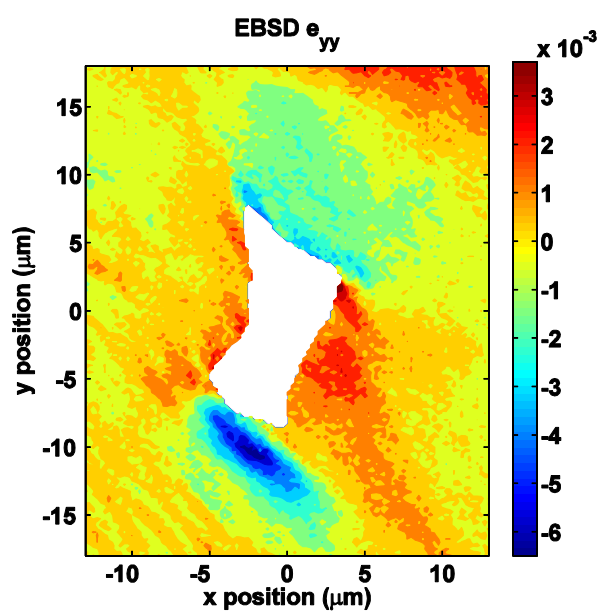
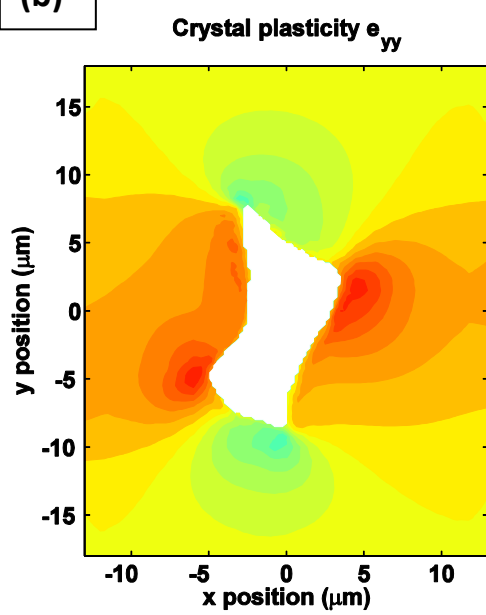


Figure 2 Finite element models of the Nickel alloy matrix containing a particle for a) $75\mu\text{m}$ b) $5\mu\text{m}$ uniform carbide length.

(a)



(b)



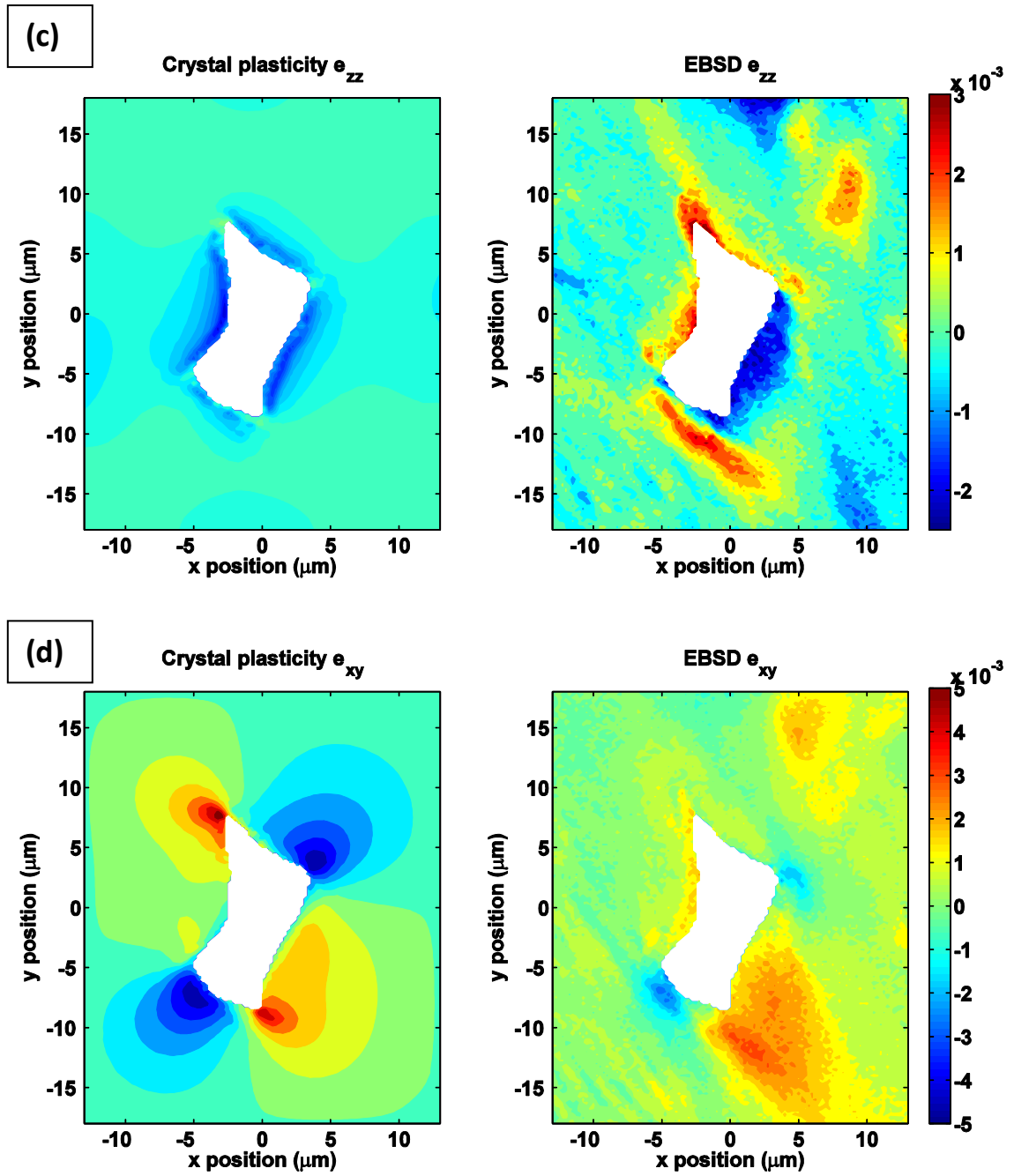
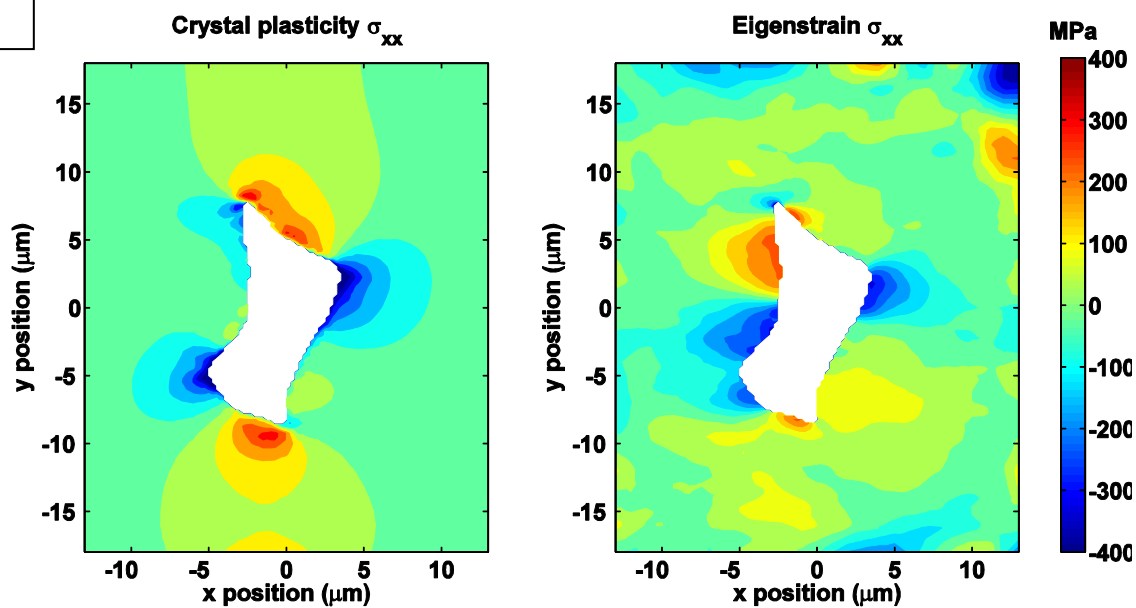
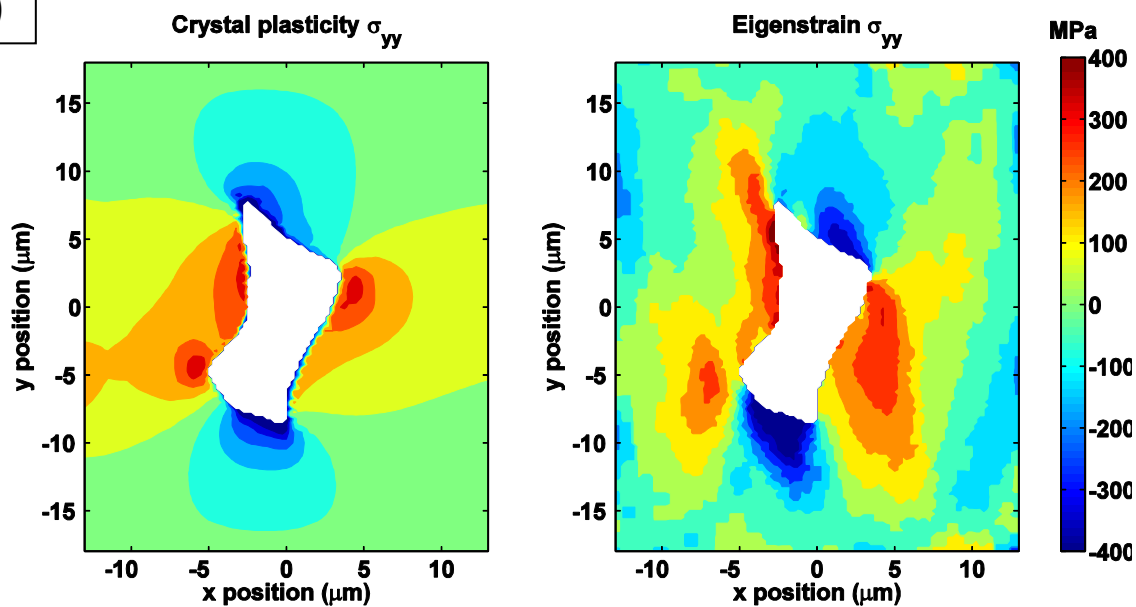


Figure 3 Comparison between 75 μm thick crystal plasticity finite element model and EBSD measurements on the free surface of the Nickel alloy matrix for the residual elastic strain components (a) e_{xx} , (b) e_{yy} , (c) e_{zz} and (d) e_{xy}

(a)



(b)



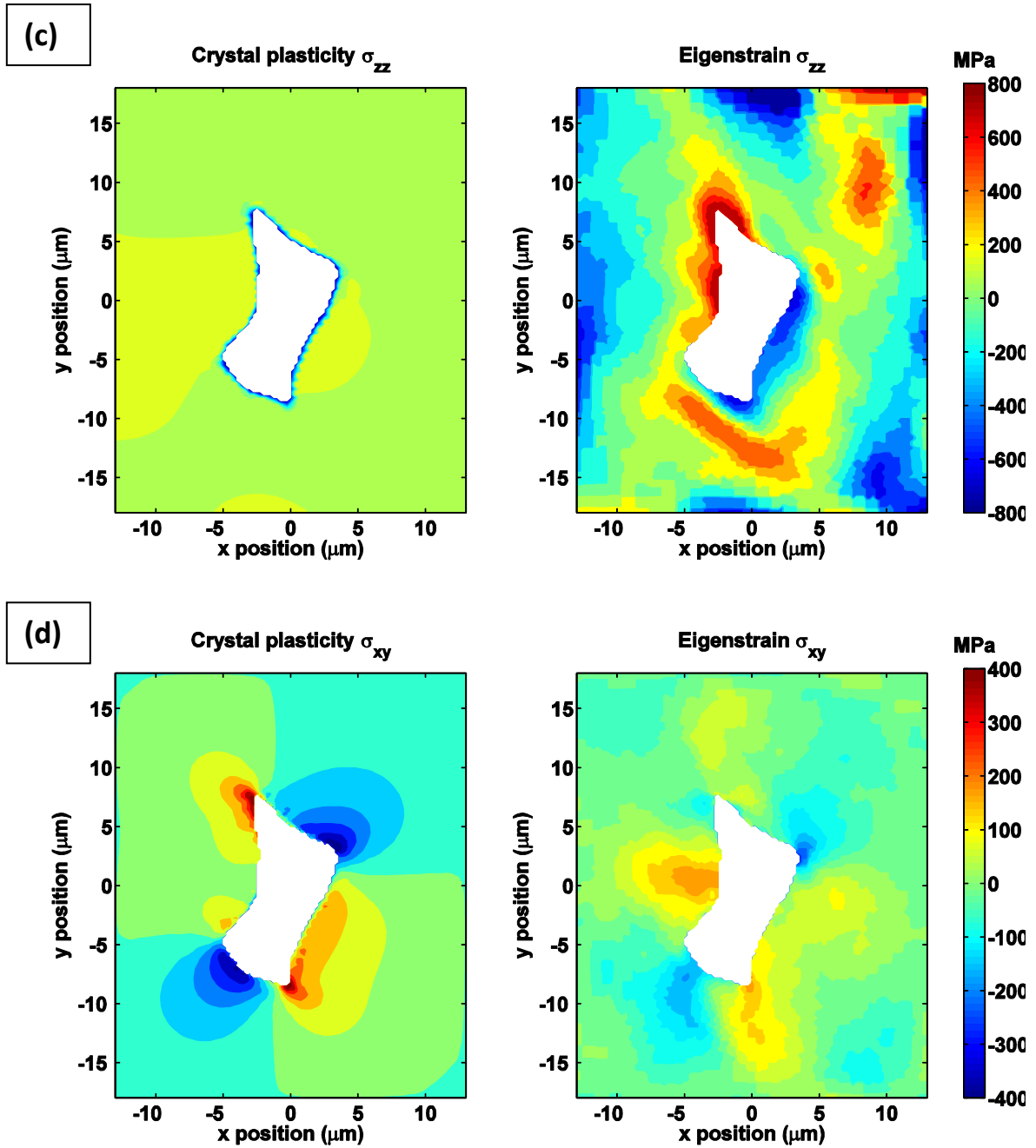


Figure 4 Comparison between 75 μm CPFE model and the inverse eigenstrain analysis at the sub-plane ($z = -37.5 \mu\text{m}$) for the residual stress components (a) σ_{xx} , (b) σ_{yy} , (c) σ_{zz} and (d)

σ_{xy}

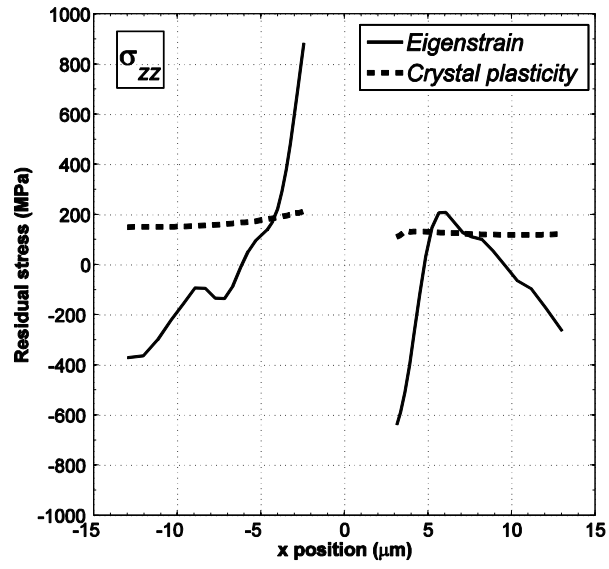
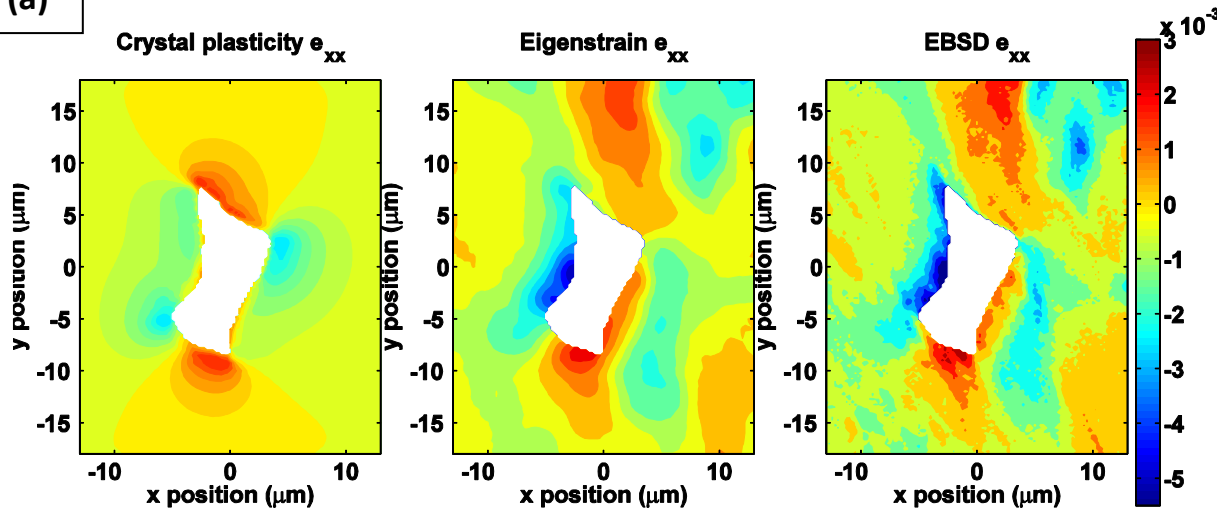
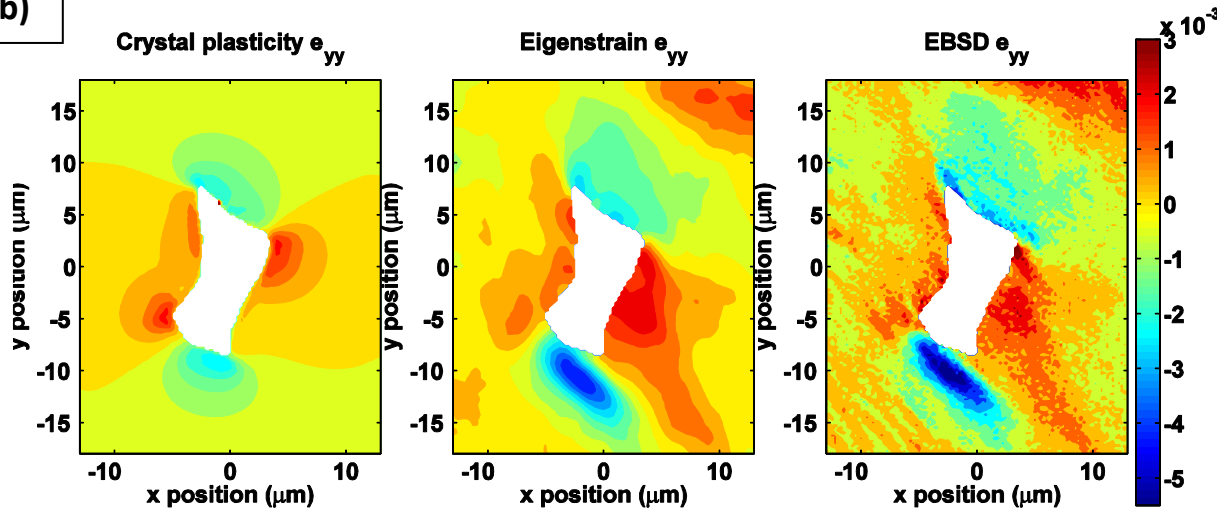


Figure 5 Residual stress component σ_{zz} along the line $y = 0 \mu\text{m}$ at the sub-plane ($z = -37.5 \mu\text{m}$).

(a)



(b)



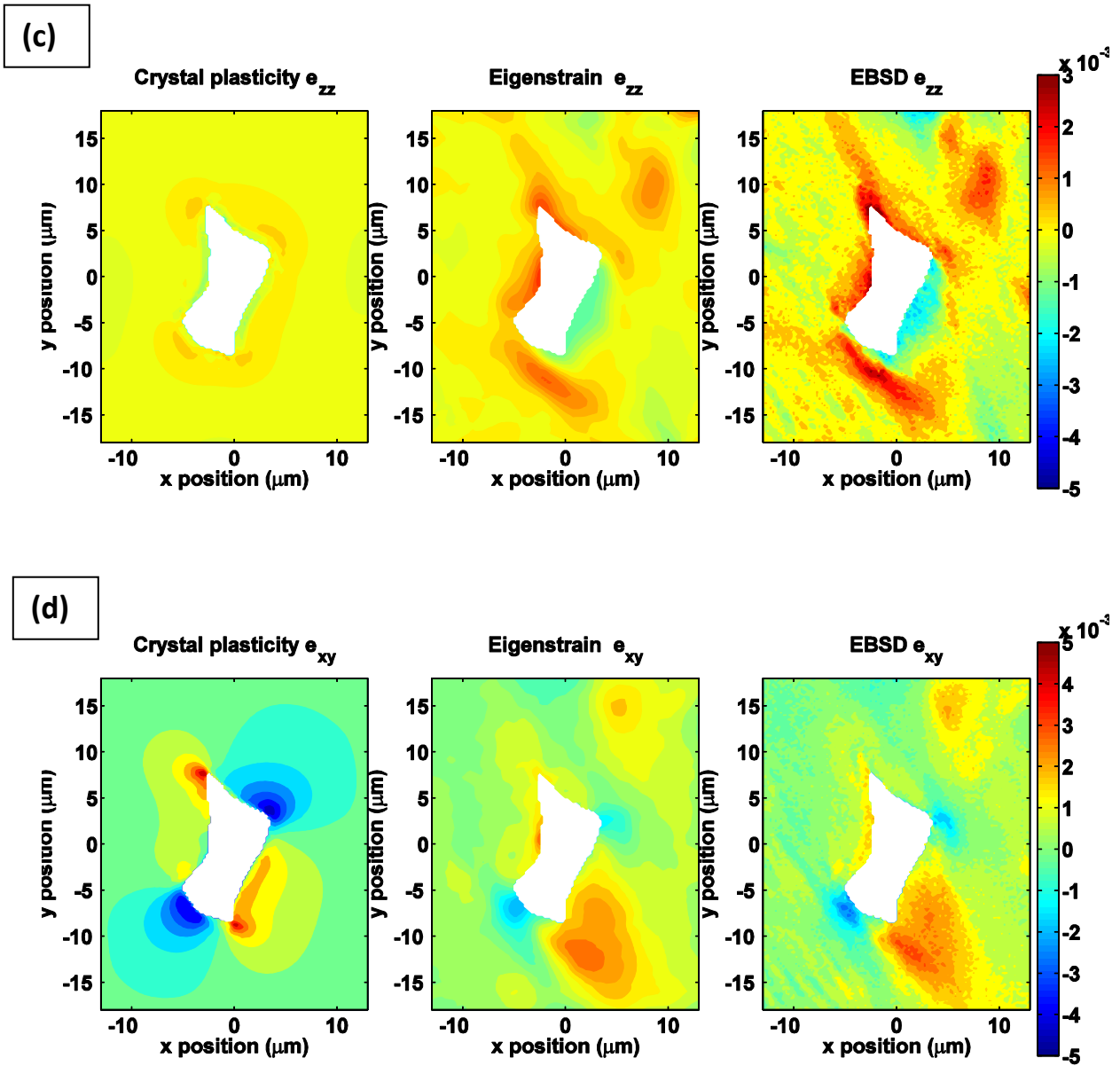
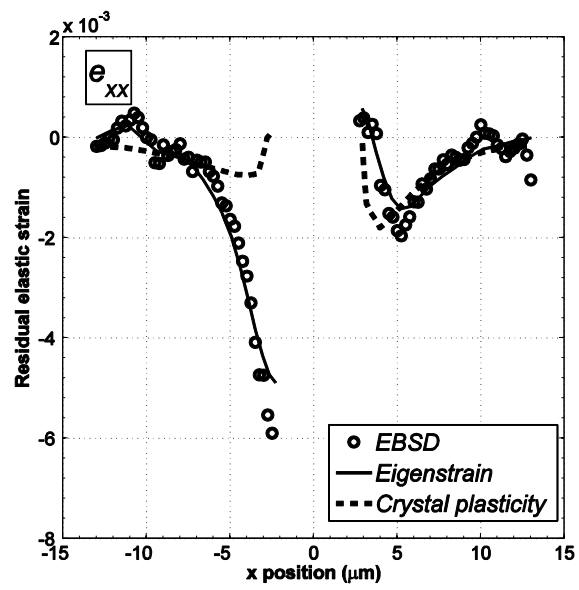
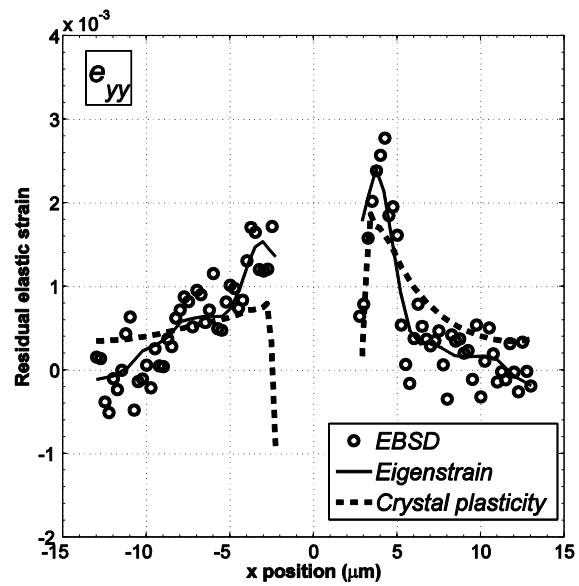


Figure 6 Contour plots obtained from EBSD measurements, CPFE model and the inverse problem of eigenstrain for 25 μm thick model on the free surface of the Nickel alloy matrix for residual elastic strain components (a) e_{xx} , (b) e_{yy} , (c) e_{zz} and (d) e_{xy}

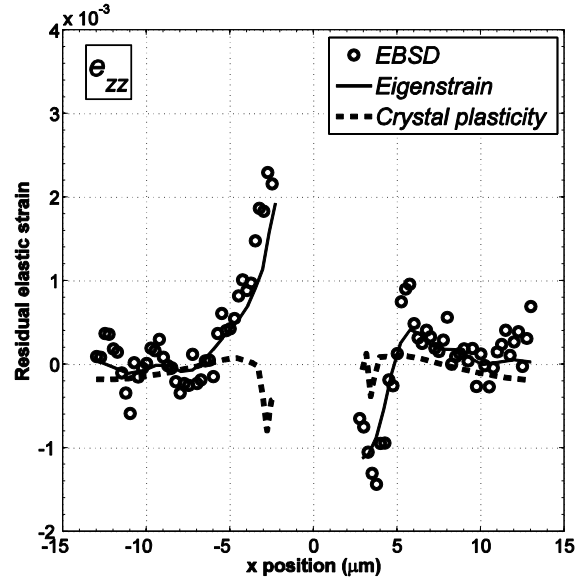
(a)



(b)



(c)



(d)

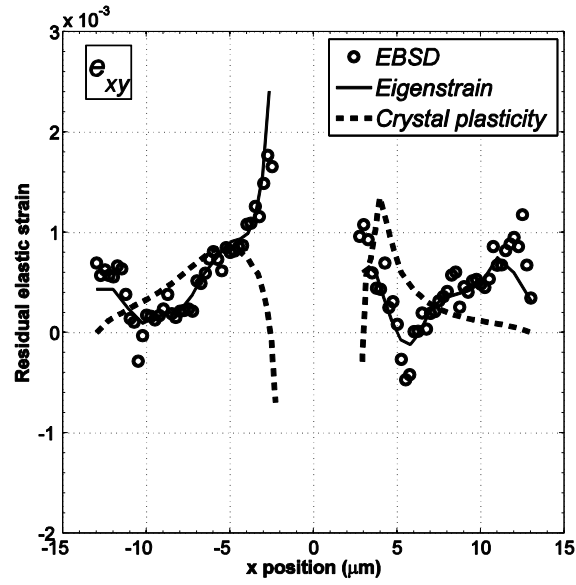
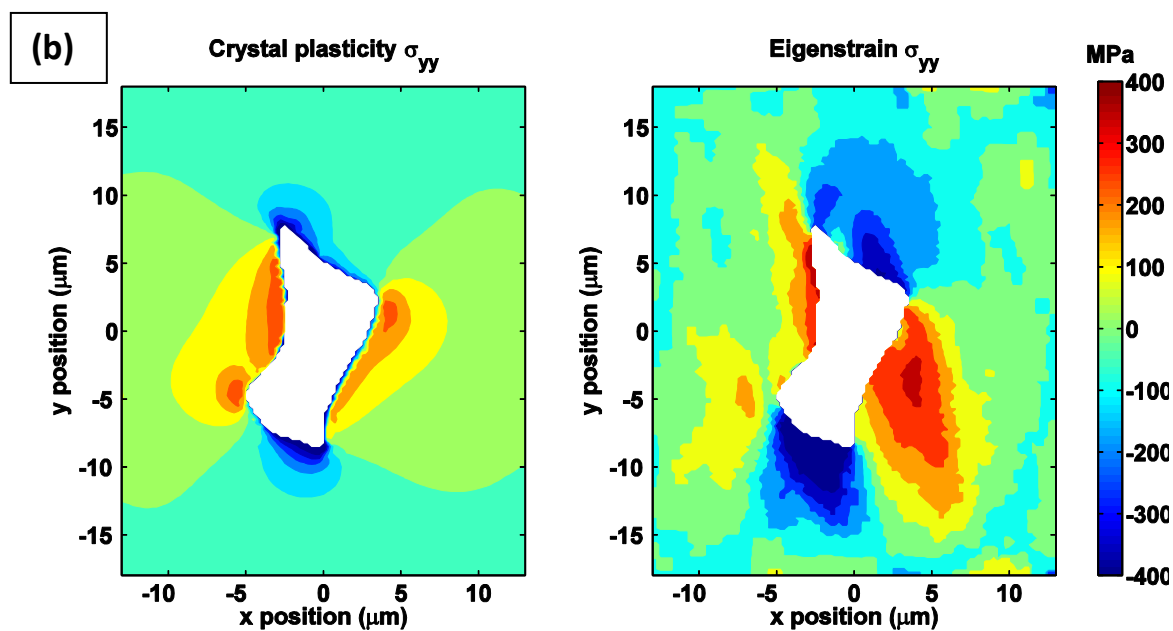
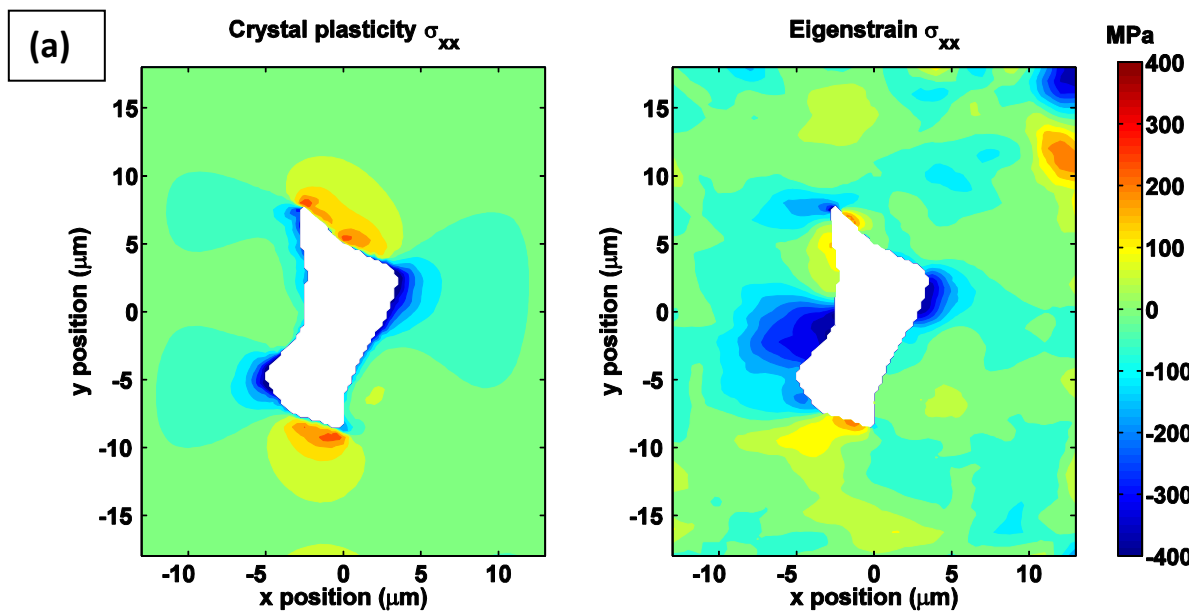


Figure 7 Line plots obtained from EBSD measurements, CPFE model and the inverse problem of eigenstrain for 25 μm thick model on the free surface of the Nickel alloy matrix along the line $y = 0 \mu\text{m}$ for residual elastic strain components (a) e_{xx} , (b) e_{yy} , (c) e_{zz} and (d) e_{xy}



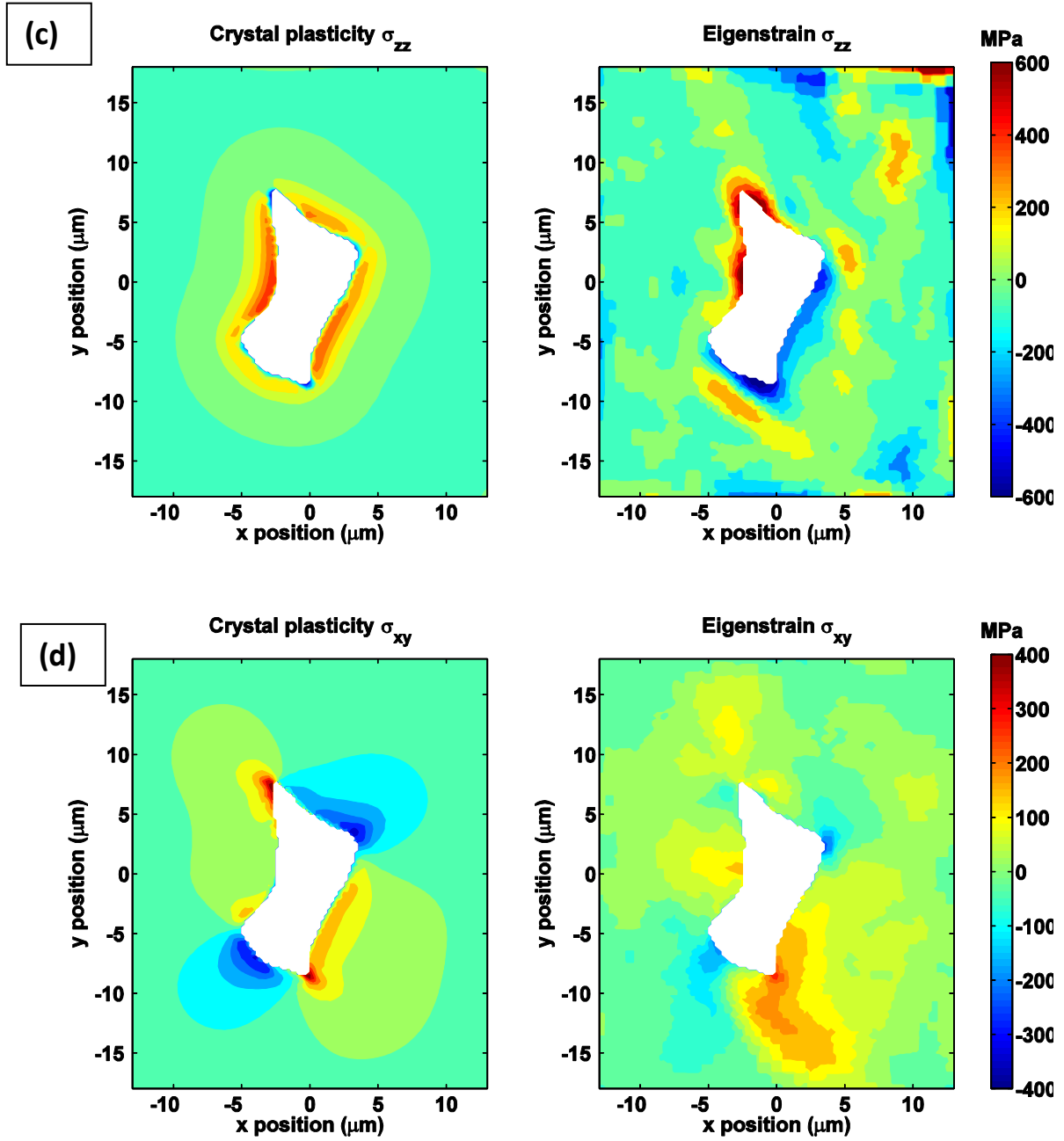
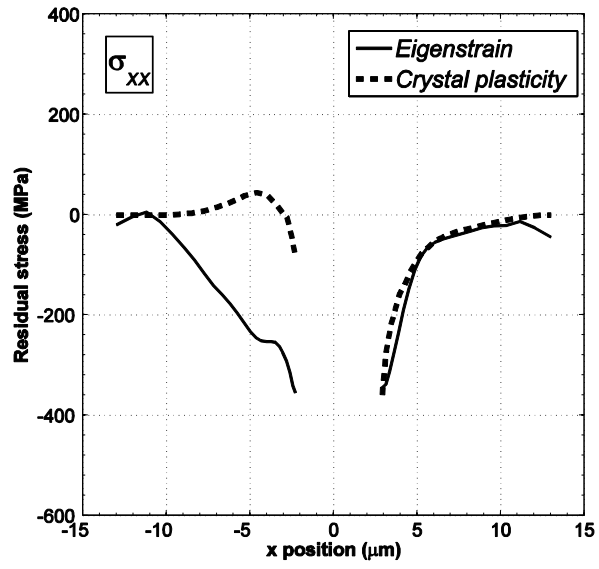
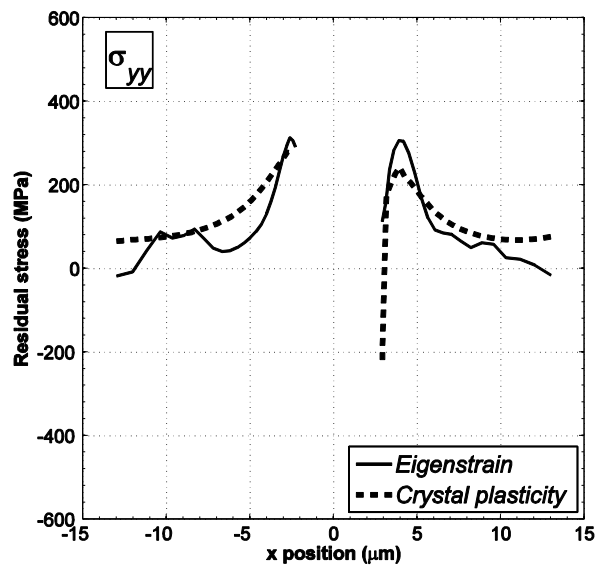


Figure 8 Comparison between 25 μm CPFE model and the inverse eigenstrain analysis at the sub-plane ($z = -2.5 \mu\text{m}$) for the residual stress components (a) σ_{xx} , (b) σ_{yy} , (c) σ_{zz} and (d) σ_{xy}

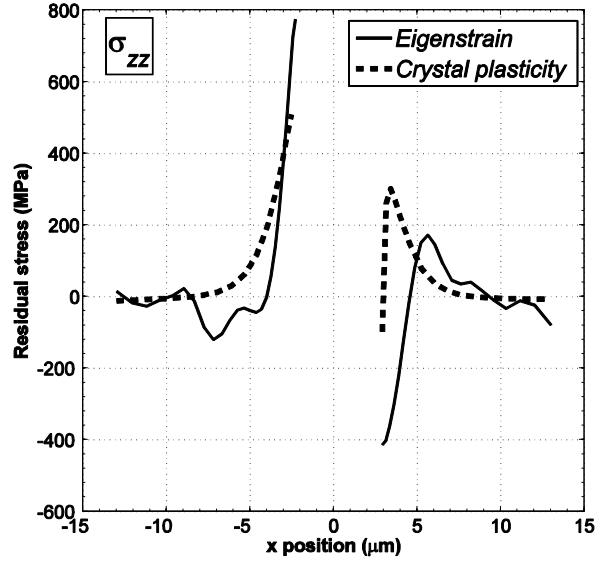
(a)



(b)



(c)



(d)

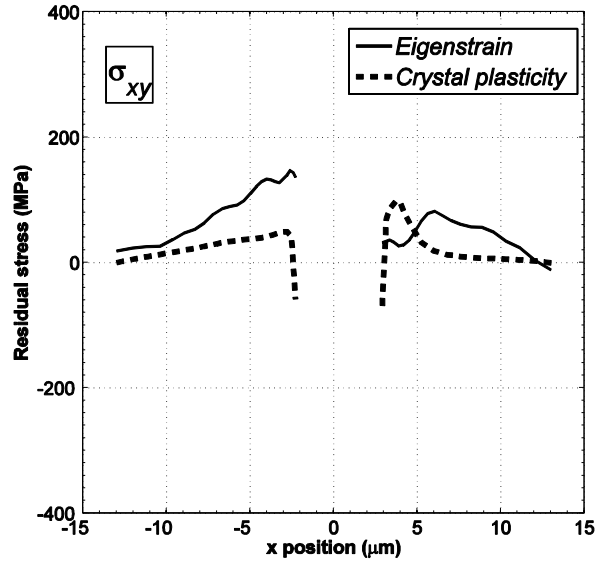


Figure 9 Line plots obtained from CPFE model and the inverse problem of eigenstrain for 25 μm thick model at the sub-plane ($z = -2.5 \mu\text{m}$) along the line $y = 0 \mu\text{m}$ for the residual stress components (a) σ_{xx} , (b) σ_{yy} , (c) σ_{zz} and (d) σ_{xy}

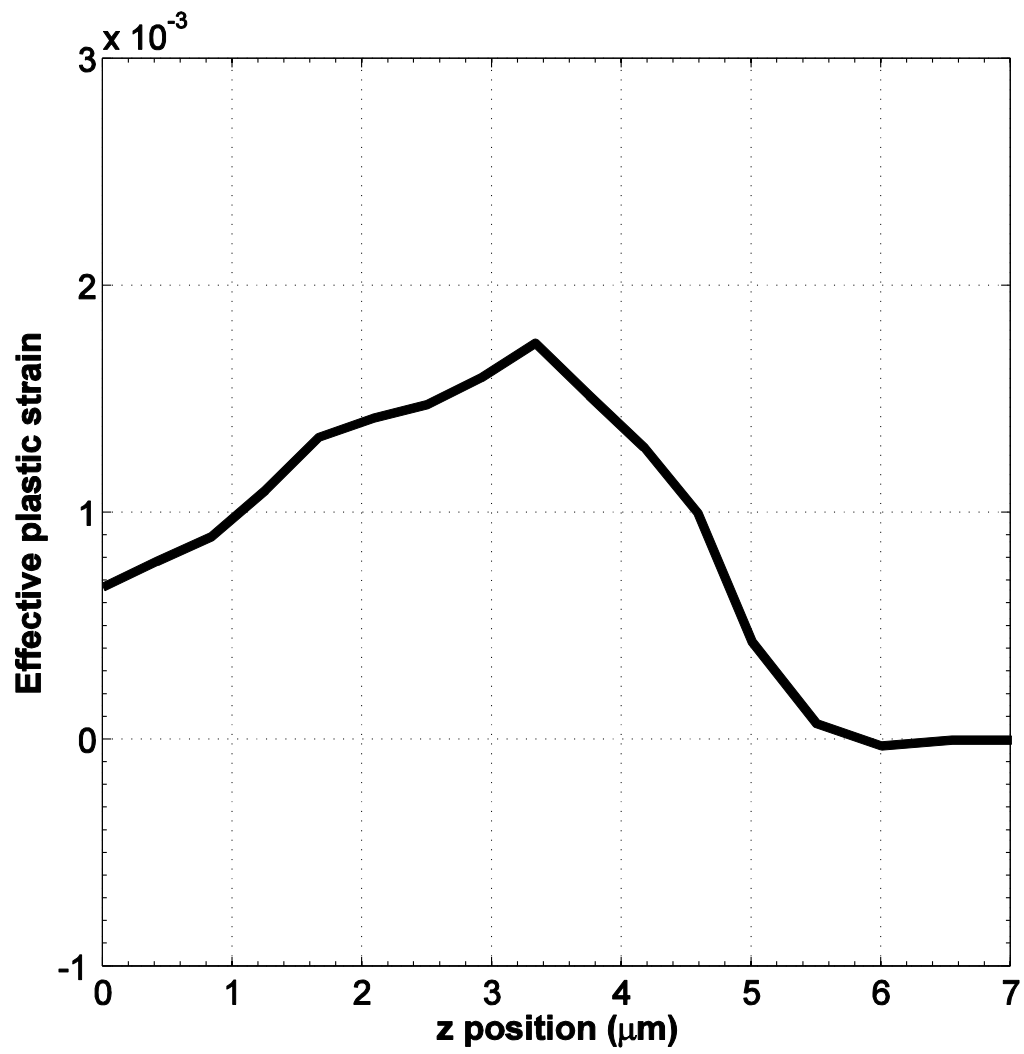


Figure 10 Line plots of effective plastic strain obtained from CPFE model in the z direction