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**A sensitive horizontal atom interferometer for
testing acceleration from an in-vacuum source mass**

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Abstract

Light-pulse atom interferometry has been powerful in testing fundamental physics. Because of its precision and sensitivity to inertial forces, the study of gravitational phenomena using an atom interferometer with various configurations has become a new trend in this community. This thesis presents my effort in characterising and improving the acceleration sensitivity of an ^{87}Rb atom interferometer that is sensitive to the horizontal acceleration of atomic motion, which can be used to constrain chameleon dark energy force from an in-vacuum source mass.

We achieved $\approx 10^7$ atoms for interferometry in 800 ms including loading ^{87}Rb atoms into a magneto-optical trap, vertically launching them upwards, preparing their internal state in $|F = 1, m_F = 0\rangle$ state and velocity selection. After minimising AC Stark shifts with a novel scheme using microwave spectroscopy, this centimeter-scale fountain configuration allowed us to interrogate our atom interferometer with three Raman pulses separated by $T = 33$ ms, which achieved an acceleration sensitivity of 30-40 $\mu\text{m}/\text{s}^2$ per shot. This acceleration sensitivity is more than a factor of two better than 86-156 $\mu\text{m}/\text{s}^2$ per shot reported for the last generation. We quantified the noise contribution to our acceleration sensitivity, which was found to be dominated by phase noise. With this improved interferometer, we measured an acceleration of $-2.597 \pm 0.186 \mu\text{m}/\text{s}^2$ in 92 hours. This negative result is not compatible with a chameleon dark energy induced fifth-force, which leads to a discussion of systematics.

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I declare that this thesis and the work presented here are my own. I have appropriately referenced all sources used in the thesis wherever they are required.

Guanchen Peng (2023)

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To the lasers that have become broken in my experiments.

Physics isn't the most important thing. Love is.

- Richard P. Feynman

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Chapter 1

Introduction

1.1 Motivation: dark energy in a nutshell

In the past few decades, one of the biggest discoveries is the accelerating expansion of our universe [1]. As shown in Fig. 1.1 (a), $\approx 70\%$ energy density in the universe is made of dark (unknown) energy. This is concluded from measurements of high red-shift supernovae explosion (SNe) [2] and baryon-acoustic oscillation (BAO) [3], combined with a flat cosmic geometry inferred from the temperature anisotropies in the cosmic microwave background (CMB) [4]. In standard cosmology, the dynamics of the universe are described by the Friedmann acceleration equation [5]

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3c^2}(\rho + 3p), \quad (1.1)$$

where $a(t)$ is a dimensionless scale factor characterising the size of the universe, G is the Newtonian constant of gravitation, ρ and p are the energy density and pressure in Einstein's stress-energy tensor. Since the energy densities of the matter (including dark matter) and dark energy are both measured as positive values in Fig. 1.1 (a), the accelerating expansion ($\ddot{a}(t) > 0$) implies the dark energy must have negative pressure ($p_{\text{DE}} < 0$). To identify this negative-pressure component, the ratio of its pressure to its energy density, $w_{\text{DE}} = p_{\text{DE}}/\rho_{\text{DE}}$, is measured to be ≈ -1 as shown in Fig. 1.1 (b). This makes dark energy distinguishable from the other four cosmic components [6], which are baryonic non-relativistic matter ($p_{\text{baryon}} \approx 0$), photon radiation ($p_{\text{photon}} = 1/3$), neutrinos ($p_{\text{neutrino}} = 1/3$) and cold dark matter ($p_{\text{CDM}} \approx 0$).

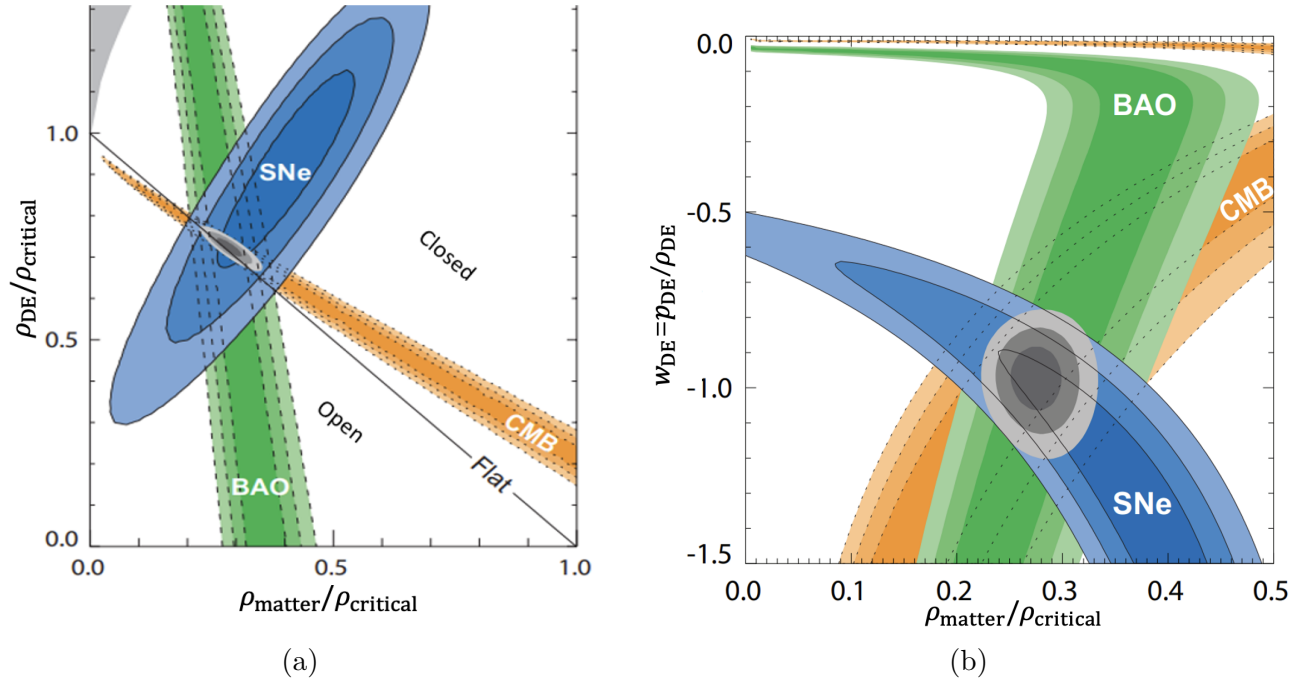


Figure 1.1: Cosmological probes of dark energy. The combinations of three cosmological probes indicate a positive missed dark energy density accounts for 70% of the total energy density in a flat universe. (a) Constraints of the dark energy density, ρ_{DE} , and the matter density, ρ_{matter} . (adapted from [1, 7]). The line $\rho_{\text{DE}} + \rho_{\text{matter}} = \rho_{\text{critical}}$, where $\rho_{\text{critical}} = \frac{3H_0^2}{8\pi G}$, describes the total energy density in a universe of a flat spatial geometry at an expanding rate of the Hubble parameter, H_0 . Total energy density above and below ρ_{critical} results in a sphere-like and saddle surface-like 3D space [5]. (b) Constraints of the equation of state for dark energy [7]. A cosmological constant, Λ , would have a pressure-to-energy-density ratio of $w = w_{\Lambda} = -1$. Current constraints allow a dynamic component with $w = w_{\text{DE}} \approx -1$ to give the accelerating expansion.

In the standard cosmology model, this negative-pressure component can be explained with Einstein's mathematical constant, Λ_{Ein} , in his famous field equation [8]. However, it has two controversial problems, known as the fine-tuning problem and the coincidence problem. Weinberg's comments on these two problems are summarised below [9].

Fine-tuning problem: The observed cosmology constant, Λ_{DE} , has at least two contributions, which are Einstein's mathematical term, Λ_{Ein} , describing the property of an empty timespace and the zero-point energy of the quantum field of particles. Since the observed energy density $\Lambda_{\text{DE}} \approx 10^{-47} \text{GeV}^4$ is some 120 orders of magnitude lower than the expectation of the zero-point energy of the quantum fields [1], this requires fine-tuning the value of Λ_{Ein} such that the zero-point energy contribution can be cancelled to match the observed Λ_{DE} .

Coincidence problem: If Λ_{DE} can be explained by the property of empty spacetime and zero-point energy of the quantum field, the anthropic principle seems to be the only one to

explain why the observed dark energy density is comparable to the matter density today.

These two problems have motivated people to look for alternative origins for the observed cosmological constant. One popular alternative is a cosmological dynamic scalar field known as quintessence [10] to be the fifth dynamic component contributing to the energy density of the universe. In this scenario, the pressure and the energy density of this scalar field ϕ are $\rho_\phi = \frac{1}{2}\partial_t^2\phi + V(\phi)$ and $p_\phi = \frac{1}{2}\partial_t^2\phi - V(\phi)$, where $V(\phi)$ is its self-interaction potential energy density. Thus the equation of state for a slow rolling scalar field ($\partial_t^2\phi \ll V(\phi)$) is $w_Q \approx -1$, which is also consistent with the measured w_{DE} in Fig. 1.1 (b).

In the scenario of the quintessence cosmic scalar field, the observed dark energy density is identified as the self-interaction potential energy density, $V(\phi)$, assuming the vacuum energy contributions from quantum fields are cancelled to zero by some unknown mechanism [10, 11]. The coincidence problem can be answered with a scalar field whose self-interaction potential energy density of a power-law form, $\frac{\Lambda_Q^{(n+4)}}{\phi^n}$ (n is the power-law index) [12], which leads to a tracker-behaviour of the quintessence energy density dynamically following the matter density as the universe evolves. Therefore, a slow-rolling scalar field is a plausible alternative to the cosmological constant to explain the accelerating expansion of our universe. Note that it does not solve the fine-tuning problem, instead, it explains the observed dark energy density differently and leaves the vacuum energy problem for other mechanisms beyond the standard model.

An interesting consequence of this cosmic scalar field is the consideration of its coupling to standard model particles [13]. Taking coupling to the baryonic sector as an example, this quintessence field can have spatial variation due to coupling to local baryonic clumps. The length scale of this spatial variation can be characterised by the Compton wavelength of the excitation of this quintessence field. Since its slow-rolling behaviour indicates a nearly-massless excitation, or equivalently a long Compton wavelength, coupling to standard model particles should mediate a long-range ‘fifth’ force between local baryonic clumps via this quintessence field. This opens up the possibility to detect the quintessence scalar field by probing a dark energy force.

1.2 Target: dark energy scalar field with chameleon screening

The cosmic scalar field as a dark energy candidate is expected to have a mass of the order of the Hubble parameter, H_0 , to evolve cosmologically today. This nearly massless scalar with coupling to matter predicts a long-range ‘fifth’ force that will not obey the equivalence principle (EP) [13]. However, the universality of free fall tested at 10^{-15} level [14] suggests the quintessence-matter coupling needs to be unnaturally small [13].

This situation is alleviated by a quintessence-matter coupling depending on the local matter energy density, ρ , which leads to a scalar field whose mass (force range) varies as ρ changes [15]. Therefore, in the high-density region where EP tests were performed, the scalar field can have a sufficiently large mass (sufficiently short force range) to satisfy the EP constraints. Meanwhile, it can become nearly massless on the cosmological scales, where the matter density is 10^{30} times smaller, to give the dark energy phenomenon.

The chameleon screening to give the ρ -dependent force range is explained in Section 1.2.1 and experimental constraints to parameters of a chameleon model are presented in Section 1.2.2.

1.2.1 Chameleon Screening: Environment-dependent mass

The equation of motion of a scalar field demonstrating the chameleon screening mechanism is given by [16]

$$\partial_\mu \partial^\mu \phi = \partial_\phi V_{\text{eff}}(\phi) = \underbrace{\partial_\phi \left(\Lambda^4 + \frac{\Lambda^{n+4}}{\phi^n} \right)}_{\text{self-interaction}} + \underbrace{\partial_\phi \left(\rho \frac{\phi}{M} \right)}_{\text{coupling to matter}}, \quad (1.2)$$

where the effective potential, $V_{\text{eff}}(\phi)$, includes a self-interaction potential $V_{\Lambda,n}(\phi) = \Lambda^4 + \frac{\Lambda^{n+4}}{\phi^n}$ with a self-interaction strength characterised by Λ , which mimics the tracker potential of a quintessence scalar field, and a field-matter interaction potential proportional to the local matter energy density, ρ , with a coupling strength characterised by M . From now on, we only focus on the power-law index $n = 1$ model as an example.

Inside a material of homogeneous matter density ρ (ignoring any boundary effect), the

solution is a constant field, $\phi_{\Lambda,M}^{\text{eq}}(\rho) = (\Lambda^5 M/\rho)^{1/2}$, reaching the equilibrium state of the chameleon field, which minimises its effective potential, i.e. $\partial_\phi V_{\text{eff}}(\phi)|_{\phi=\phi_{\Lambda,M}^{\text{eq}}} = 0$. If the field slightly deviates from its equilibrium value, its deviation $\delta\phi$ satisfies

$$\partial_\mu \partial^\mu \delta\phi = \partial_{\delta\phi} \left(V_{\text{eff}}(\phi_{\Lambda,M}^{\text{eq}}) + \partial_\phi V_{\text{eff}}(\phi)|_{\phi=\phi_{\Lambda,M}^{\text{eq}}} \delta\phi + \frac{1}{2} \partial_\phi^2 V_{\text{eff}}(\phi)|_{\phi_{\Lambda,M}^{\text{eq}}} \delta\phi^2 + O(\delta\phi^3) \right) \approx m_{\text{eff}}^2 \delta\phi, \quad (1.3)$$

where $V_{\text{eff}}(\phi)$ was Taylor expanded around $\phi_{\Lambda,M}^{\text{eq}}$ and changed the variable using $\phi = \phi_{\Lambda,M}^{\text{eq}} + \delta\phi$. Therefore, the spatial variation, captured by $\delta\phi$, satisfies the Klein-Gorden equation describing a particle of an effective mass $m_{\text{eff}}(\rho) = (\frac{4\rho^3}{\Lambda^5 M^3})^{1/4}$. On the one hand, the gradient of the chameleon field exponentially decays over a length scale of $m_{\text{eff}}^{-1}(\rho)$, which can be sufficiently short in a high-density region to evade EP violation tests. On the other hand, the matter density is sufficiently low on the cosmological scale such that $m_{\text{eff}}(\rho) \approx H_0$ to make the chameleon field a dark energy candidate with $V_{\text{eff}}(\phi) \approx \Lambda^4$. Thus $\Lambda = \Lambda_{\text{DE}}$ can make the chameleon field a dark energy candidate. However, cosmological probes gave little knowledge about the chameleon-matter coupling, M , which calls for constraints from experiments.

1.2.2 Constraints from dedicated experiments

Various constraints on the parameters $\{\Lambda - M\}$ of the prototypical (n=1) chameleon model are shown in Fig. 1.2 (a) [17]. The experimental efforts span from the measurements on the smallest scale, such as measuring electron magnetic moment (g/2) [18], atomic precise spectroscopy of hydrogen [19] and atom interferometry [20], to test with macroscopic objects, such as plates in Casimir force sensing [21] and torsion balance experiment [22], and finally even to the astrophysical objects such as Cepheids [23] and pulsars [24]. Among all the experimental efforts, atom interferometry stands out for tightly constraining the chameleon field parameter space because the size of nuclei is so small that atoms are unscreened [25]. Since the chameleon field only couples to the mass within a thin shell of a macroscopic object, the chameleon acceleration on a macroscopic object becomes negligible because the entire mass is the inertia to respond to the chameleon force on the thin shell. However, in the case of using an atom as a test mass, the entire mass of an atom plays both roles of coupling to the chameleon field and being the inertia

to respond to the chameleon force, which makes atom interferometry especially suited to sense chameleon-screened scalar fields.

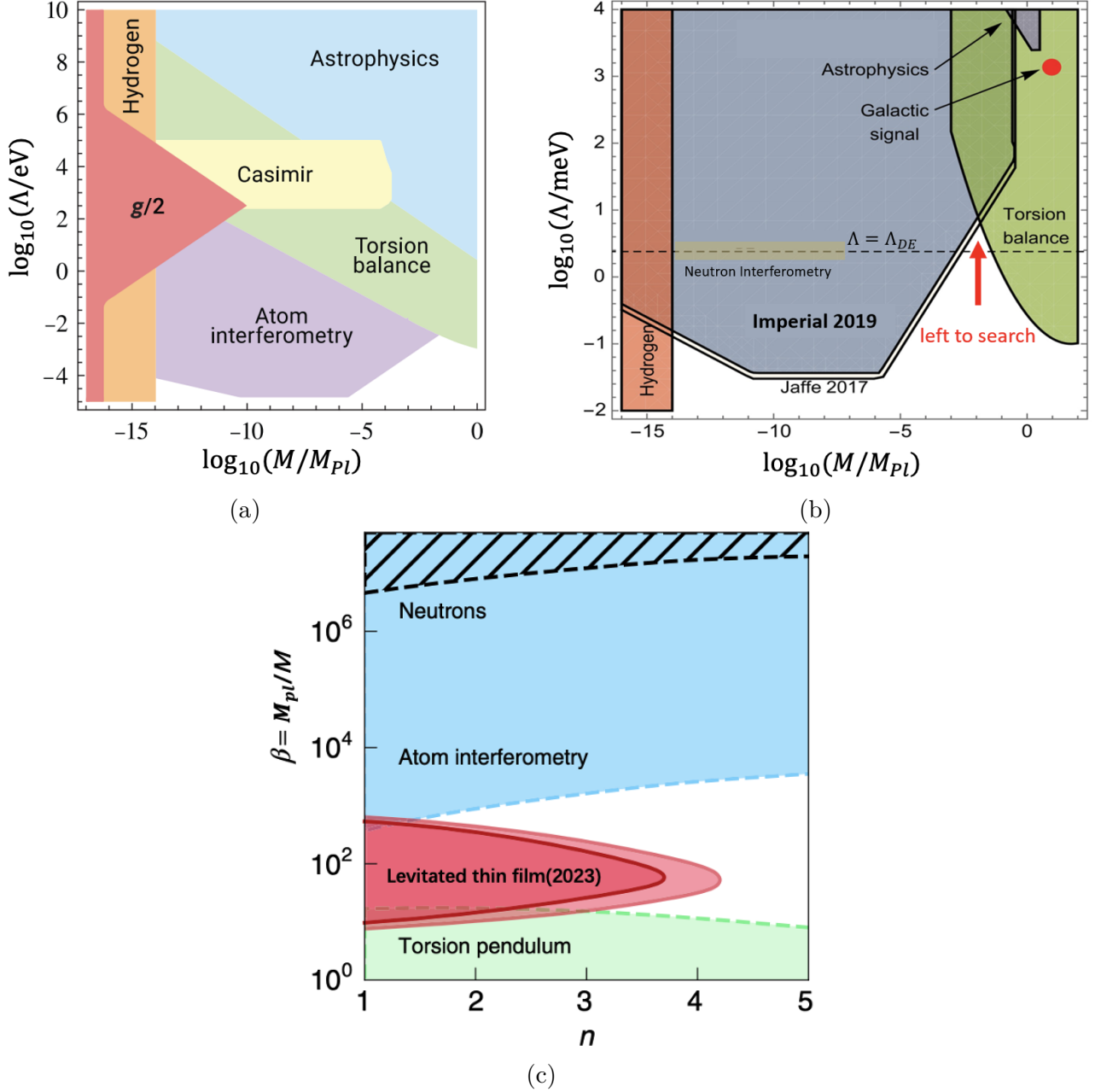


Figure 1.2: Parameter space for the chameleon model. Λ is the strength of chameleon self-interaction. $1/M$ is the strength of chameleon-matter coupling. M_{Pl} is the Planck mass and n is the power-law index for the chameleon model. (a) Constraints of the parameter space for $n=1$ Chameleon field [17] from various experimental efforts, where Λ is in the unit of eV displayed in a log scale. (b) The target parameter space to search [26] for $\Lambda \approx \Lambda_{DE} \approx 2.4 \text{ meV}$, where Λ is in the unit of meV displayed in a log scale. My work is the second generation following the result and setup from Imperial 2019. (c) Constrained chameleon-matter coupling strength, M , at $\Lambda = \Lambda_{DE}$ for various power-law index, n .

As shown in Fig. 1.2 (b), two independent atom interferometry experiments from Berkeley (Jaffe2017 [27]) and Imperial College London (Imperial 2019 [26]) have ruled out most of

the parameter space around $\Lambda \approx \Lambda_{\text{DE}} \approx 2.4 \text{ meV}$ for chameleon screened scalar field to be considered as a dark energy candidate. There is an unexplored parameter space for this dark energy candidate, where chameleon-matter coupling strength (in the unit of the Planck mass M_{Pl}), $2.8 \times 10^{-3} < 1/\beta = M/M_{\text{Pl}} < 7.4 \times 10^{-2}$, as indicated by a red arrow in Fig. 1.2 (b). This unexplored parameter space of the chameleon-matter coupling is the target during the development of our experiment, which is the second generation ungraded from the setup used in [26].

Very recently, a group probed our target area for chameleon model of $n = 1$ with a sensitive force sensor using a levitated thin film made of polyimide in August 2022 [20]. Our experiment using ^{87}Rb atoms can still be useful since the proposed scalar field may be sensitive to the nuclear composition of the source and test masses. Their source mass was outside and our source mass is in a vacuum. This difference made their result become less powerful on containing M at $\Lambda = \Lambda_{\text{DE}}$ for chameleon models of $n > 4$ as shown in Fig. 1.2 (c), which can be constrained with our setup using atom interferometry.

1.3 Tool: light-pulse atom interferometry

Interferometry experiments have the tradition of being precise tools for testing fundamental physics. The most famous experiment dates back to Michelson–Morley experiment in 1887 using light interferometry to deny the existence of aether [28]. The modern version of a Michelson–Morley light interferometer can sense a change in distance 10,000 times smaller than a proton to detect gravitational waves [29]. Since Louis de Broglie proposed the wave property of matter in 1924, matterwave interferometry with material gratings has been demonstrated with electrons [30], neutrons [31], atoms [32] and even organic molecules [33].

After replacing the material gratings with light pulses in 1992 [34], which overcomes inevitable defects in the material grating, light-pulse atom interferometry has been developed to test fundamental physics. Besides measuring fundamental constants such as the fine-structure constant [35, 36] and the Newtonian gravitational constant [37, 38], probing phase shifts induced by a source mass becomes a direction rapidly developed in the last decade. Taking Earth as the source mass, Earth’s gravity has been probed by a fountain atom interferometer of 10-meter free-fall distance [39] and a lattice atom interferometer of 20-second coherence time [40]. With an out-vacuum source mass, the gravitational Aharonov–Bohm effect has been detected [41]. With an in-vacuum source mass, the dark energy scalar field forces have been tested [42, 43] and blackbody radiation force has been illustrated [44].

Regarding to our target, we followed a horizontal atom interferometer to measure the acceleration from an in-vacuum source mass [43]. The expected acceleration of an atom induced by the chameleon-screened scalar field is given by

$$\mathbf{a}_\phi = -\lambda_a \frac{1}{M} \nabla \phi_{\Lambda, M} = -\lambda_a \frac{\beta}{M_{\text{planck}}} \nabla \phi_{\Lambda, M}, \quad (1.4)$$

where λ_a is the screening parameter for atoms, $\phi_{\Lambda, M}$ is the generic chameleon field profile sourced from the in-vacuum mass which can only be solved numerically, $\beta = M_{\text{planck}}/M$ is a dimensionless representation of the chameleon-matter coupling.¹ The parameters of interest is $2.8 \times 10^{-3} < M < 7.4 \times 10^{-2}$ at $\Lambda = \Lambda_{\text{DE}}$. As shown in [45], λ_a is equal to 1 for the

¹ β^2 means the chameleon-matter coupling strength relative to the gravitational coupling strength.

target parameters because atoms are too small and are effectively unscreened. Meanwhile, the chameleon field, $\phi_{\Lambda, M}$, is shown to be M -independent and limited by the geometry of the source mass relative to the chamber wall [46]. Since we inherited the system from [43] and did not change the geometry, improving the previous constraint on $|\mathbf{a}_\phi| < 183 \text{ nm/s}^2$ should proportionally improve the constrain on $M/M_{\text{planck}} > 8.9 \times 10^{-4}$ ($\beta < 358$) to approach the target parameter region.

To improve the previous constraint using the same setup, I have built a centimetre-height ^{87}Rb fountain atom interferometer, shown in Fig. 1.3, whose population in a particular internal state ($F=2$) is sensitive to the acceleration along the Raman beam axis experienced by the atoms. The interference fringe in atom interferometry is the population in $F=2$ as a function of acceleration,

$$P_{F=2}(a_{\text{mirror}} + \mathbf{n} \cdot \mathbf{g} + a_{\text{ball}}), \quad (1.5)$$

where a_{mirror} is the vibrational acceleration of the Raman mirror that generates the Raman-I beam, $\mathbf{n}$ is the unit vector of the normal direction of the mirror, $\mathbf{g}$ is the Earth's gravity and a_{ball} is the projection of the ball-induced acceleration on the Raman axis.

To distinguish the ball-induced acceleration from the Raman mirror vibration acceleration and the Earth's gravity, the source mass position was switched between two symmetric positions labelled as θ_+ and θ_- . This action makes the ball-induced acceleration become a differential signal, which manifested as the relative phase shift between two interferometer fringes at the corresponding ball position. The common-mode noise from the Raman mirror vibration and the Earth's gravity at two ball positions were recorded by a classical accelerometer, which also randomly sampled each fringe.

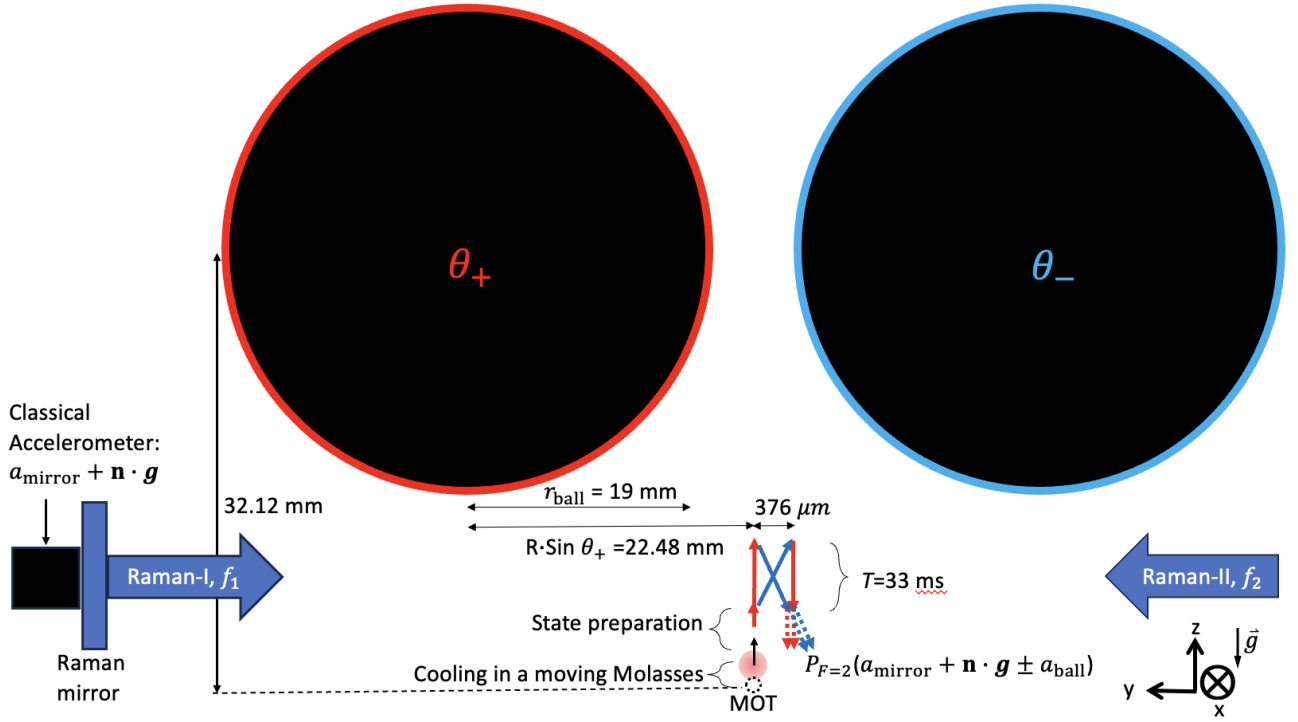


Figure 1.3: Overview of a fountain atom interferometer near an in-vacuum source mass. To build this fountain, we started with a magneto-optical trap (MOT) before vertically launching them in moving optical molasses. Before applying the first interferometry pulse, atoms were prepared in a particular internal state ($|F = 1, m_F = 0\rangle$). When the atoms were illuminated with the first pulse of Raman lasers, each atom wavepacket was in a superposition state of $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$. Because of the associated momentum kick for atoms transferred from $|F = 1, m_F = 0\rangle$ to $|F = 2, m_F = 0\rangle$, each atom wavepacket got split into two arms that reach their apex after $T=33 \text{ ms}$. At their trajectory apex, the second Raman pulse inverted their internal state and these two arms got recombined by the third Raman pulse after another $T=33 \text{ ms}$. The Raman lasers need two laser frequencies, f_1 and f_2 , with certain polarisation configurations. The incoming laser of f_1 and the retro-reflected f_2 are not shown in the graph for simplicity.

1.4 Thesis outline

My PhD work has two goals. The first is to improve the acceleration sensitivity of an atom interferometer and repeat the constraints on the chameleon acceleration of the last generation but in a significantly shorter time. The second is to characterise the noise budgets of the acceleration measurement for further improvement to achieve the scientific goal of probing the chameleon field. The efforts to achieve this goal are organised in this thesis as follows.

Part I reviews the theoretical framework of the Raman atom optics. In Chapter 2, we start by describing the evolution of a two-level atom, before proceeding to the velocity-sensitive Raman transitions between two clock states of ^{87}Rb that form an effective two-level system. Throughout Chapter 2, the AC Stark shifts to atomic energy levels are extensively discussed because they are the leading source of detuning error introduced by Raman transitions. Chapter 3 describes the acceleration sensitivity of a Mach-Zehnder atom interferometer using three Raman pulses and discusses its theoretical visibility, limited by the momentum distribution of velocity-selected atoms and the spontaneous emission in each Raman pulse.

Part II presents the experimental apparatus. Chapter 4 lists all the equipment for laser cooling ^{87}Rb and key microwave components to monitor the AC Stark shifts. Chapter 5 provides information about our Raman lasers and Raman optics as well as how to suppress the impact of mirror vibration on our atom interferometer with a classical accelerometer.

Part III is a collection of our results. Chapter 6 characterises the loading rate of our magneto-optical trap and the performance of our small fountain. Chapter 7 presents our novel scheme for measuring AC Stark shifts with microwave spectroscopy, modelling and optimisation of the damped velocity-sensitive Rabi oscillation, and finally, the discussion of our chameleon measurement with noise characterisation and systematics budgets.

In Chapter 8, we conclude that the acceleration sensitivity achieved by our $2T = 66\text{ ms}$ fountain interferometer, using velocity-selected 5×10^6 - 2×10^7 atoms (loaded in 0.7 seconds), has been improved by more than a factor of two compared to the acceleration sensitivity achieved with 2×10^6 atoms (loaded in 1.2 seconds) in the previous $2T = 32\text{ ms}$ free-drop interferometer. Currently, our measurement is limited by some unknown systematics linked

to the in-vacuum source mass. I also provide some prospects on how to potentially reduce this unknown systematics as well as an outlook on further improvement of the acceleration sensitivity.

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Part I

Theory

Chapter 2

Atom-light interaction

The state of an atom can be manipulated using light. Section 2.1 presents the technique of using propagators to describe the evolution of a two-level atom interacting with a laser field and introduces the AC Stark shift in the far-detuned limit. Section 2.2 extends the propagator technique to a three-level atom interacting with two laser fields, which can be reduced to an effective two-level system via Raman coupling. Finally, Raman couplings and AC Stark shifts are applied to the multi-level structure of ^{87}Rb in section 2.3.

2.1 Two-level atom model

A two-level atom model captures most of the physics of a real multi-level atom. This section focuses on using a propagator to describe the evolution of a two-level atom. Section 2.1.1 starts with describing an unperturbed atom to introduce how to obtain a propagator for a system described by a time-independent Hamiltonian. Section 2.1.2 proceeds to obtain the propagator for a system perturbed by a time-dependent light field by transforming the system into a rotating frame. Finally, section 2.1.3 discusses the concept of AC Stark shifts by examining the propagator in the far-detuned limit.

2.1.1 The propagator: the evolution of a two-level atom

Assuming a two-level system has two eigenstates $|1\rangle$ and $|2\rangle$ with corresponding atomic energies $\hbar\omega_1^A$ and $\hbar\omega_2^A$, the evolution of an arbitrary state $|\psi(t)\rangle = c_1(t)|1\rangle + c_2(t)|2\rangle$ with complex amplitudes $(c_1(t), c_2(t))^T$ is described by the time-dependent Schrödinger equation,

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle, \quad (2.1)$$

where $\hat{H} = \hat{H}_0 = \hbar\omega_1^A |1\rangle\langle 1| + \hbar\omega_2^A |2\rangle\langle 2|$ is the unperturbed Hamiltonian for our two-level atom. Solving the dynamics of this two-level system is to find the amplitudes $(c_1(t), c_2(t))^T$ at an arbitrary time $t = \tau$, while they satisfy a pair of non-coupled ordinary differential equations,

$$\begin{pmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \end{pmatrix} = -i \begin{pmatrix} \omega_1^A & 0 \\ 0 & \omega_2^A \end{pmatrix} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}, \quad (2.2)$$

whose solution is trivial. However, writing this trivial solution in the matrix form,

$$\begin{pmatrix} c_1(\tau) \\ c_2(\tau) \end{pmatrix} = \begin{pmatrix} e^{-i\omega_1^A\tau} & 0 \\ 0 & e^{-i\omega_2^A\tau} \end{pmatrix} \begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix} = \hat{U}_0(\tau; 0) \begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix}, \quad (2.3)$$

produces a notably useful matrix, $\hat{U}_0(\tau; 0)$, which is the propagator of the unperturbed system that evolves for a duration, $t = \tau$, from the start of the evolution, $t = 0$. Solving Eq. (2.2) is equivalent to obtaining this propagator, which can also be obtained with the exponential of a time-independent Hamiltonian, $\hat{H} = \hat{H}_0$,

$$\hat{U}_0(\tau; 0) = e^{-\frac{i}{\hbar}\hat{H}_0\tau} = e^{-i\omega_1^A\tau} |1\rangle\langle 1| + e^{-i\omega_2^A\tau} |2\rangle\langle 2|. \quad (2.4)$$

Therefore, if the dynamics of a system are governed by a known time-independent Hamiltonian, the final state at any time, t , is straightforward to obtain with the propagator calculated using Eq. (2.4) for a given initial state at time $t = 0$, where $|1\rangle$ and $|2\rangle$ are the eigenstates of $\hat{H}_0$ with corresponding eigenvalues ω_1^A and ω_2^A .

This technique of calculating propagators with eigenvalues and eigenvectors is also useful in

the presence of couplings between states, where $\dot{c}_1(t)$ depends not only on $c_1(t)$ but also $c_2(t)$. This is the topic of section 2.1.2. Using propagators to track the state evolution is crucial to study the interferometer discussed in chapter 3.

2.1.2 Rotating frame and rotating-wave approximation: the evolution involving time-dependent couplings

A non-trivial situation is how the state evolves in the presence of couplings, where our two-level atom is subject to an oscillating electric field $\mathbf{E}(t) = \mathbf{E}_0 \cos(\omega^L t + \phi_0)$ with an amplitude, $\mathbf{E}_0$, angular frequency, ω^L , and initial phase, ϕ_0 , at the start of the interaction at $t = 0$. In this case, because of the electric-dipole interaction, $\hat{V}(t) = -\mathbf{E}(t) \cdot \hat{\mathbf{d}}$, the Hamiltonian of the system becomes time-dependent $\hat{H}(t) = \hat{H}_0 + \hat{V}(t)$, whose matrix representation is

$$\frac{1}{\hbar} \hat{H}(t) = \begin{pmatrix} \omega_1^A & 0 \\ 0 & \omega_2^A \end{pmatrix} + \begin{pmatrix} 0 & \Omega_0 \cos(\omega^L t + \phi_0) \\ \Omega_0^* \cos(\omega^L t + \phi_0) & 0 \end{pmatrix}, \quad (2.5)$$

where $\Omega_0 = -\mathbf{E}_0 \cdot \langle 1 | \hat{\mathbf{d}} | 2 \rangle / \hbar$ is the Rabi frequency, which is a complex number. The diagonal terms of $\hat{V}(t)$ are zero because the dipole operator, $\hat{\mathbf{d}}$, has odd parity, while the eigenstates of $\hat{H}_0$, $|1\rangle$ and $|2\rangle$, have either odd or even parity. In the case where the transition electric dipole momentum $\langle 1 | \hat{\mathbf{d}} | 2 \rangle = 0$, e.g. $|1\rangle$ and $|2\rangle$ have the same parity, the magnetic-dipole interaction $\hat{V}(t) = -\mathbf{B}(t) \cdot \hat{\boldsymbol{\mu}}$ can be used to transfer population between $|1\rangle$ and $|2\rangle$. The same mathematical description is applicable, but the Rabi frequency needs to be replaced with $\Omega_0 = -\mathbf{B}_0 \cdot \langle 1 | \hat{\boldsymbol{\mu}} | 2 \rangle / \hbar$, where $\mathbf{B}_0$ is the amplitude of an oscillating magnetic field and $\hat{\boldsymbol{\mu}}$ is the magnetic dipole operator. The electric-dipole coupling is the building block of the Raman coupling for the acceleration-sensitive interferometry, while the magnetic-dipole coupling was used to transfer the population for preparing atoms in a magnetic-insensitive state.

To study the dynamics of the atomic state governed by a time-dependent Hamiltonian shown in Eq. (2.5), transforming into a cleverly chosen ‘rotating’ frame and applying the rotating-wave approximation produce a time-independent Hamiltonian, making its propagator trivial to calculate with Eq. (2.4).

Rotating frame transformation

The rotating frame transformation [1, p.13] refers to a mathematical transformation where $|\psi(t)\rangle = \hat{T}(t) |\psi_{rot}(t)\rangle$ with $\hat{T}(t) = e^{-\frac{i}{\hbar}\hat{H}_0 t}$, or more explicitly presented in its matrix form as

$$\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = \begin{pmatrix} e^{-i\omega_1^A t} & 0 \\ 0 & e^{-i\omega_2^A t} \end{pmatrix} \begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix}. \quad (2.6)$$

Substituting Eq. (2.6) into Eq. (2.1) and simplifying it with the unitary identity, $\hat{T}^\dagger \hat{T} = \hat{I}$, gives

$$i\hbar \frac{d}{dt} |\psi_{rot}(t)\rangle = \underbrace{(\hat{T}^\dagger \hat{H} \hat{T} - i\hbar \hat{T}^\dagger \frac{d}{dt} \hat{T})}_{\hat{H}_{rot}} |\psi_{rot}(t)\rangle, \quad (2.7)$$

which leads to an equation for $(b_1(t), b_2(t))^T$ satisfying

$$\begin{pmatrix} \dot{b}_1(t) \\ \dot{b}_2(t) \end{pmatrix} = -i \begin{pmatrix} 0 & \Omega_0 \cos(\omega^L t + \phi_0) e^{-i\omega_0 t} \\ \Omega_0^* \cos(\omega^L t + \phi_0) e^{i\omega_0 t} & 0 \end{pmatrix} \begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix}, \quad (2.8)$$

where $\omega_0 = \omega_2^A - \omega_1^A$ is the atomic transition frequency. So far everything is exact. This cleverly chosen transformation $\hat{T}(t)$ using the unperturbed Hamiltonian, H_0 , allows us to focus on the (transformed) non-diagonal terms, which can be further simplified by performing the rotating-wave approximation discussed below.

Rotating-wave approximation

Equation 2.8 is still a time-dependent differential equation that can't be solved easily. However, the time dependence has two contributions

$$\cos(\omega^L t + \phi_0) e^{-i\omega_0 t} = \underbrace{\frac{1}{2} e^{i(\omega^L - \omega_0)t} e^{i\phi_0}}_{\text{Co-rotating}} + \underbrace{\frac{1}{2} e^{-i(\omega^L + \omega_0)t} e^{-i\phi_0}}_{\text{Counter-rotating}}. \quad (2.9)$$

We are interested in the situation where the laser field frequency, ω^L , is near the resonance of the atomic transition frequency, ω_0 , and both are much larger than the Rabi frequency, Ω_0 . Thus the detuning parameter $\delta := \omega^L - \omega_0 \ll \omega^L + \omega_0 \approx 2\omega_0$ suggests that the second contribution

(counter-rotating term) in Eq. (2.9) varies much faster than the first term (co-rotating term). As the counter-rotating term quickly averages to zero at a time scale of $2\pi/\Omega_0$, the dynamics of $(b_1(t), b_2(t))^T$ can be approximated as

$$\begin{pmatrix} \dot{b}_1(t) \\ \dot{b}_2(t) \end{pmatrix} \approx -i \begin{pmatrix} 0 & \frac{\Omega_0}{2} e^{i\delta t} e^{i\phi_0} \\ \frac{\Omega_0^*}{2} e^{-i\delta t} e^{-i\phi_0} & 0 \end{pmatrix} \begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix}. \quad (2.10)$$

Neglecting the much faster coupling term is the so-called *rotating-wave approximation* [2, p.125]. This approximation simplifies the non-diagonal coupling term to an exponential, whose time-dependence can be transformed out with one more rotating-frame transformation discussed below.

Rotating-frame II

Note that the rotating basis transformation, Eq. (2.6), removes the constants in the diagonal terms at the expense of multiplying the off-diagonal term with a time-dependent exponential term. This section makes use of this property by performing another rotating-frame transformation

$$\begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix} = \begin{pmatrix} e^{-i(-\frac{\delta}{2})t} & 0 \\ 0 & e^{-i\frac{\delta}{2}t} \end{pmatrix} \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix}, \quad (2.11)$$

Substituting Eq. (2.11) into Eq. (2.10) gives the dynamics of $(a_1(t), a_2(t))^T$ following

$$\begin{pmatrix} \dot{a}_1(t) \\ \dot{a}_2(t) \end{pmatrix} = -i \underbrace{\begin{pmatrix} \frac{\delta}{2} & \frac{\Omega_0}{2} e^{i\phi_0} \\ \frac{\Omega_0^*}{2} e^{-i\phi_0} & -\frac{\delta}{2} \end{pmatrix}}_{\hat{H}_{\text{rot}}} \begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix}, \quad (2.12)$$

which now has no time dependence. Thus its solution can be obtained with Eq. (2.4) as

$$\begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix} = \underbrace{\begin{pmatrix} \Lambda_{\cos}(t) & -i\Lambda_{\sin}(t)e^{+i\phi_0} \\ -i\Lambda_{\sin}(t)e^{-i\phi_0} & \Lambda_{\cos}^*(t) \end{pmatrix}}_{\hat{U}(t;0)} \begin{pmatrix} a_1(0) \\ a_2(0) \end{pmatrix}, \quad (2.13)$$

using

$$\hat{U}(t; 0) = e^{-i\hat{H}_{\text{rot}}t} = e^{-i\lambda_+t} |+\rangle \langle +| + e^{-i\lambda_-t} |-\rangle \langle -|, \quad (2.14)$$

where $|+\rangle$ and $|-\rangle$ are eigenvectors of $\hat{H}_{\text{rot}}$ with corresponding eigenvalues of λ_+ and λ_- . The matrix entries of $\hat{U}(t; 0)$, $\Lambda_{\text{cos}}(t)$ and $\Lambda_{\text{sin}}(t)$, are defined as

$$\begin{aligned} \Lambda_{\text{cos}}(t) &= \cos\left(\sqrt{\delta^2 + |\Omega_0|^2} \frac{t}{2}\right) - i \frac{\delta}{\sqrt{\delta^2 + |\Omega_0|^2}} \sin\left(\sqrt{\delta^2 + |\Omega_0|^2} \frac{t}{2}\right), \\ \Lambda_{\text{sin}}(t) &= \frac{\Omega_0}{\sqrt{\delta^2 + |\Omega_0|^2}} \sin\left(\sqrt{\delta^2 + |\Omega_0|^2} \frac{t}{2}\right). \end{aligned} \quad (2.15)$$

Propagators for $b_n(t)$ and $c_n(t)$ can be obtained with the corresponding transformations.

2.1.3 AC Stark shift

It is the easiest to understand the AC Stark shift using the propagator for $c_n(t)$. At the limiting situation where $\delta \gg \Omega_0$, $\Lambda_{\text{cos}} \rightarrow e^{-i\sqrt{\delta^2 + |\Omega_0|^2}t/2}$ and $\Lambda_{\text{sin}} \rightarrow 0$. This leads to an evolution of $c_n(t)$ as

$$\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = \underbrace{\begin{pmatrix} e^{-i(\omega_1^A + \frac{|\Omega_0|^2}{4\delta})} & 0 \\ 0 & e^{-i(\omega_2^A - \frac{|\Omega_0|^2}{4\delta})t} \end{pmatrix}}_{\hat{U}(t;0)} \begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix}. \quad (2.16)$$

Compared with the propagator Eq. (2.3) for the trivial situation where no couplings are present, this propagator for atoms interacting with a far-detuned laser field results in a shift of the energy level of the atomic ground state by $\Omega^{\text{AC}} = \frac{|\Omega_0|^2}{4\delta}$. This is the AC Stark shift of the atomic energies as an atom interacts with an oscillating electric field [3, p.155].

2.2 Three-level atom model

A three-level system can support stimulated Raman transitions, which two-level systems cannot. Section 2.2.1 summarises the velocity-sensitive Raman transition between two clock states via an intermediate excited state, which is the building block of coherently splitting, redirecting, and recombining atomic wavepackets. The Raman transition introduces an inevitable systematic shift of the clock transition frequency due to the differential AC Stark shift, which is discussed in section 2.2.2.

2.2.1 Velocity-sensitive stimulated Raman transition

Stimulated Raman transitions can drive an electric-dipole forbidden transition between two states, $|1\rangle$ and $|2\rangle$, via electric-dipole couplings to a common intermediate state $|e\rangle$. To understand this, we start with a three-level atom interacting with two laser fields (oscillating electric fields), $\mathbf{E}_n \cos(\omega_n^L t - \mathbf{k}_n \cdot \mathbf{x} + \phi_0^n)$, where $\mathbf{E}_n$ are the amplitudes, ω_n^L and $\mathbf{k}_n$ are the angular frequencies and wavevectors for the corresponding oscillating electric fields ($\mathbf{k}_1$ and $\mathbf{k}_2$ are of opposite direction to make a velocity-sensitive transition), and ϕ_0^n are the initial phases at time $t = 0$ at a reference location $x = 0$ for individual laser fields ($n = 1, 2$). Assuming each laser field only contributes to one coupling for simplicity, the Hamiltonian of this three-level atom with interactions can be represented in its internal states basis $\{|1\rangle, |2\rangle, |e\rangle\}$ and centre-of-mass frame as

$$\begin{aligned} \frac{1}{\hbar} \hat{H}(t) = & \underbrace{\begin{pmatrix} \frac{1}{\hbar} \frac{\hat{\mathbf{p}}^2}{2m} + \omega_1^A & 0 & 0 \\ 0 & \frac{1}{\hbar} \frac{\hat{\mathbf{p}}^2}{2m} + \omega_2^A & 0 \\ 0 & 0 & \frac{1}{\hbar} \frac{\hat{\mathbf{p}}^2}{2m} + \omega_e^A \end{pmatrix}}_{\hat{H}_0} \\ & + \underbrace{\begin{pmatrix} 0 & 0 & \Omega_1 \cos((\omega_1^L - \mathbf{k}_1 \cdot \dot{\mathbf{x}}^c(t_j))(t - t_j) - \mathbf{k}_1 \cdot (\hat{\mathbf{x}} - \mathbf{x}^c(t_j)) + \phi_{1,j}) \\ 0 & 0 & \Omega_2 \cos((\omega_1^L - \mathbf{k}_2 \cdot \dot{\mathbf{x}}^c(t_j))(t - t_j) - \mathbf{k}_2 \cdot (\hat{\mathbf{x}} - \mathbf{x}^c(t_j)) + \phi_{2,j}) \\ c.c. & c.c. & 0 \end{pmatrix}}_{\hat{V}(t)}, \end{aligned} \quad (2.17)$$

where $\Omega_1 = -\mathbf{E}_1 \cdot \langle 1 | \hat{\mathbf{d}} | e \rangle / \hbar$, $\Omega_2 = -\mathbf{E}_2 \cdot \langle 2 | \hat{\mathbf{d}} | e \rangle / \hbar$ are single-photon Rabi frequencies, $\mathbf{x}^c(t_j)$ is the classical coordinate of the centre of mass of the atom at time t_j . Note that the dipole interaction has co-rotating terms proportional to $e^{i\omega_n^L t} e^{-i\mathbf{k}_n \cdot (\hat{\mathbf{x}} - \mathbf{x}^c)}$, which couples $|n, \mathbf{p}\rangle$ to $|e, \mathbf{p} + \hbar \mathbf{k}_n\rangle$ because $e^{-i\mathbf{k}_n \cdot (\hat{\mathbf{x}} - \mathbf{x}^c)} |\mathbf{p}\rangle = |\mathbf{p} + \hbar \mathbf{k}_n\rangle$ where $\mathbf{p}$ is the momentum in the centre-of-mass frame. This observation simplifies solving the time-dependent Schrodinger equation within a momentum family $\{|1, \mathbf{p}\rangle, |2, \mathbf{p} + \hbar \mathbf{k}_1 - \hbar \mathbf{k}_2\rangle, |e, \mathbf{p} + \hbar \mathbf{k}_1\rangle\}$ for this Hamiltonian, whose solution for each momentum family has the form of $|\psi_{\mathbf{p}}(t)\rangle = c_{1,\mathbf{p}}(t) |1, \mathbf{p}\rangle + c_{e,\mathbf{p}+\hbar \mathbf{k}_1}(t) |e, \mathbf{p} + \hbar \mathbf{k}_1\rangle + c_{2,\mathbf{p}+\hbar \mathbf{k}_1-\hbar \mathbf{k}_2}(t) |2, \mathbf{p} + \hbar \mathbf{k}_1 - \hbar \mathbf{k}_2\rangle$. Substituting a time-dependent transformation $c_{\nu,\mathbf{p}_{\nu}}(t) = e^{-i\omega_{\nu,\mathbf{p}_{\nu}}^A t} b_{\nu,\mathbf{p}_{\nu}}(t)$, where $\omega_{\nu,\mathbf{p}_{\nu}}^A = \omega_{\nu}^A + \frac{1}{\hbar} \frac{\mathbf{p}_{\nu}^2}{2m}$ ($\nu=1,2,e$), into Eq. (2.7) with rotating-wave approximation gives

$$\begin{pmatrix} \dot{b}_{1,\mathbf{p}}(t) \\ \dot{b}_{2,\mathbf{p}+\hbar \mathbf{k}_1-\hbar \mathbf{k}_2}(t) \\ \dot{b}_{e,\mathbf{p}+\hbar \mathbf{k}_1}(t) \end{pmatrix} \approx -i \begin{pmatrix} 0 & 0 & \frac{\Omega_1}{2} e^{i\Delta_1(\mathbf{p})t} e^{i\phi_{1,j}} \\ 0 & 0 & \frac{\Omega_2}{2} e^{i\Delta_2(\mathbf{p})t} e^{i\phi_{2,j}} \\ c.c. & c.c. & 0 \end{pmatrix} \begin{pmatrix} b_{1,\mathbf{p}}(t) \\ b_{2,\mathbf{p}+\hbar \mathbf{k}_1-\hbar \mathbf{k}_2}(t) \\ b_{e,\mathbf{p}+\hbar \mathbf{k}_1}(t) \end{pmatrix}. \quad (2.18)$$

Note that single photon detunings for a momentum family, $\Delta_1(\mathbf{p})$ and $\Delta_2(\mathbf{p})$, have three contributions, which are laser detunings Δ_n^L , single-photon Doppler shifts $\delta_n^D(\mathbf{p})$, and single-photon recoil shift δ_k^{recoil} , as

$$\begin{aligned} \Delta_1(\mathbf{p}) &= \omega_1^L - (\omega_{e,\mathbf{p}_e}^A - \omega_{1,\mathbf{p}_1}^A) = \underbrace{\omega_1^L - (\omega_e^A - \omega_1^A)}_{\Delta_1^L} - \underbrace{\frac{\mathbf{p}}{m} \cdot \hbar \mathbf{k} - \mathbf{k} \cdot \dot{\mathbf{x}}^c}_{\delta_1^{\text{Doppler}}(\mathbf{p})} - \underbrace{\frac{(\hbar \mathbf{k})^2}{2m}}_{\delta_k^{\text{recoil}}}, \\ \Delta_2(\mathbf{p}) &= \omega_2^L - (\omega_{e,\mathbf{p}_e}^A - \omega_{2,\mathbf{p}_2}^A) = \underbrace{\omega_2^L - (\omega_e^A - \omega_2^A)}_{\Delta_2^L} + \underbrace{\frac{\mathbf{p}}{m} \cdot \hbar \mathbf{k} + \mathbf{k} \cdot \dot{\mathbf{x}}^c}_{\delta_2^D(\mathbf{p})} - \underbrace{\frac{(\hbar \mathbf{k})^2}{2m}}_{\delta_k^{\text{recoil}}}, \end{aligned} \quad (2.19)$$

where the approximation $\mathbf{k}_1 \approx -\mathbf{k}_2 = \mathbf{k}$ has been applied.¹

In our special case where $\Delta_1(\mathbf{p}), \Delta_2(\mathbf{p}) \gg \Omega_1, \Omega_2$, Eq. (2.18) can be reduced to the dynamics of an effective two-level system [4] as

$$\begin{pmatrix} \dot{b}_{1,\mathbf{p}}(t) \\ \dot{b}_{2,\mathbf{p}+\hbar \mathbf{k}_1-\hbar \mathbf{k}_2}(t) \end{pmatrix} = -i \begin{pmatrix} \Omega_1^{\text{AC}} & \frac{\Omega_{12}^{\text{eff}}}{2} e^{i\phi_j^{\text{eff}}} e^{i\delta_{12}^{\text{eff}}(\mathbf{p})t} \\ c.c. & \Omega_2^{\text{AC}} \end{pmatrix} \begin{pmatrix} b_{1,\mathbf{p}}(t) \\ b_{2,\mathbf{p}+\hbar \mathbf{k}_1-\hbar \mathbf{k}_2}(t) \end{pmatrix}, \quad (2.20)$$

¹ $\omega_1^L - \omega_2^L$ is at the 7 GHz level which is much smaller than ω_1^L, ω_2^L at the 384 THz level.

where $\Omega_n^{\text{AC}} = \frac{\Omega_n \Omega_n^*}{4\Delta}$ is the AC Stark shift to the energies of the state $|n\rangle$ ($n = 1, 2$), $\Omega_{12}^{\text{eff}} = \frac{\Omega_1 \Omega_2^*}{2\Delta}$ is the effective Rabi frequency using the approximation $1/\Delta_2(\mathbf{p}) \approx 1/\Delta_1(\mathbf{p}) \approx 1/\Delta$,² the Raman phase, ϕ_j^{eff} , defined in Eq. (3.3) is discussed with more details in Section 3.1, and the two-photon Raman detuning for this momentum family, $\delta_{12}^{\text{eff}}(\mathbf{p})$, is given by

$$\begin{aligned} \delta_{12}^{\text{eff}}(\mathbf{p}) &= \Delta_1(\mathbf{p}) - \Delta_2(\mathbf{p}) = (\omega_1^{\text{L}} - \omega_2^{\text{L}}) - (\omega_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}^{\text{A}} - \omega_{1,\mathbf{p}}^{\text{A}}) \\ &= \underbrace{(\omega_1^{\text{L}} - \omega_2^{\text{L}})}_{\delta_{12}^{\text{L}}} - \underbrace{(\omega_2^{\text{A}} - \omega_1^{\text{A}})}_{\omega_{\text{HFS}}} + \underbrace{\hbar(\mathbf{k}_1 - \mathbf{k}_2) \cdot \left(\frac{\mathbf{p}}{m} + \dot{\mathbf{x}}^c\right)}_{\delta_{2\gamma}^{\text{Doppler}}(\mathbf{p})} + \underbrace{\frac{\hbar^2(\mathbf{k}_1 - \mathbf{k}_2)^2}{2m}}_{\delta_{2\gamma}^{\text{recoil}}}. \end{aligned} \quad (2.21)$$

In this expression, δ_{12}^{L} is the two-photon laser detuning, ω_{HFS} is the hyperfine splitting of the $^{87}\text{Rb } 5^2S_{1/2}$ orbital, $\delta_{2\gamma}^{\text{Doppler}}(\mathbf{p})$ is the two-photon Doppler shift for this momentum family and $\delta_{2\gamma}^{\text{recoil}}$ is the two-photon recoil shift. As $\mathbf{k}_1 \approx -\mathbf{k}_2 = \mathbf{k}$, different momentum families have different two-photon Doppler shifts, $\delta_{2\gamma}^{\text{Doppler}}(\mathbf{p})$, which makes this Raman transition velocity-sensitive.

Comparing to Eq. (2.5) where there was no direct coupling between $|1\rangle$ and $|2\rangle$, Eq. (2.20) shows an effective Raman coupling, Ω_{12}^{eff} , between clock states because of the intermediate state. This is achieved by substituting an approximate solution of $b_{3,\mathbf{p}+\hbar\mathbf{k}_1}(t)$ [5]

$$\begin{aligned} b_{3,\mathbf{p}+\hbar\mathbf{k}_1}(t) &= -i\frac{\Omega_1^*}{2}e^{-i\phi_{1,j}} \int b_{1,\mathbf{p}}(t)e^{-i\Delta_1(\mathbf{p})t}dt - i\frac{\Omega_2^*}{2}e^{-i\phi_{2,j}} \int b_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}(t)e^{-i\Delta_2(\mathbf{p})t}dt \\ &\approx -i\frac{\Omega_1^*}{2}e^{-i\phi_{1,j}}b_{1,\mathbf{p}}(t) \int e^{-i\Delta_1(\mathbf{p})t}dt - i\frac{\Omega_2^*}{2}e^{-i\phi_{2,j}}b_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}(t) \int e^{-i\Delta_2(\mathbf{p})t}dt \\ &\approx \frac{\Omega_1^*}{2\Delta}e^{-i\phi_{1,j}}b_{1,\mathbf{p}}(t)e^{-i\Delta_1(\mathbf{p})t} + \frac{\Omega_2^*}{2\Delta}e^{-i\phi_{2,j}}b_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}(t)e^{-i\Delta_2(\mathbf{p})t}, \end{aligned} \quad (2.22)$$

into Eq. (2.18). Treating $b_{1,\mathbf{p}}(t)$ and $b_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}(t)$ as constants in the integrals assumes the variation of $b_n(t)$ is much more slower than $e^{-i\Delta_n(\mathbf{p})t}$ ($n = 1, 2$). This technique of treating slowly varying terms as constants to eliminate the excited state from the problem is the so-called *adiabatic elimination*. It is self-consistent since the solution of Eq. (2.20) varies at the order of $\frac{\Omega_1^* \Omega_2}{\Delta}$ and $\delta_{12}^{\text{eff}}(\mathbf{p})$, which are indeed much slower than Δ .

Finally, it is worth noting that this Raman coupling introduces AC Stark shifts Ω_n^{AC} to the clock state $|n\rangle$, which is the topic of section 2.2.2. Both the AC Stark shifts, Ω_n^{AC} , and Raman

²This approximation is valid because the two-photon laser detuning, $\delta_{12}^{\text{L}} = \Delta_1^{\text{L}} - \Delta_2^{\text{L}} \leq 10$ MHz, single-photon Doppler shifts (≈ 100 kHz) and single-photon recoil shifts (≈ 3.8 kHz) are much smaller than $\Delta_1^{\text{L}} \approx 1$ GHz.

coupling, Ω_{12}^{eff} , are independent of the momentum state (to the leading order), even though the Raman transition is velocity-selective because of the two-photon Doppler shift in the Raman detuning.

2.2.2 AC Stark shift and the magic intensity ratio

Couplings to the intermediate state shown in Fig. 2.1 not only produce a Raman coupling, Ω_{12}^{eff} shown in Eq. (2.20), between the two clock states $|1\rangle$ and $|2\rangle$, but also generate energy shifts $\Omega_n^{\text{AC}} = \frac{|\Omega_n|^2}{4\Delta}$ to each clock state $|n\rangle$ ($n=1,2$) as discussed in section 2.1.3. These energy shifts change the resonance condition by the differential AC Stark shift, $\delta^{\text{AC}} = \Omega_2^{\text{AC}} - \Omega_1^{\text{AC}}$, of the two clock states.

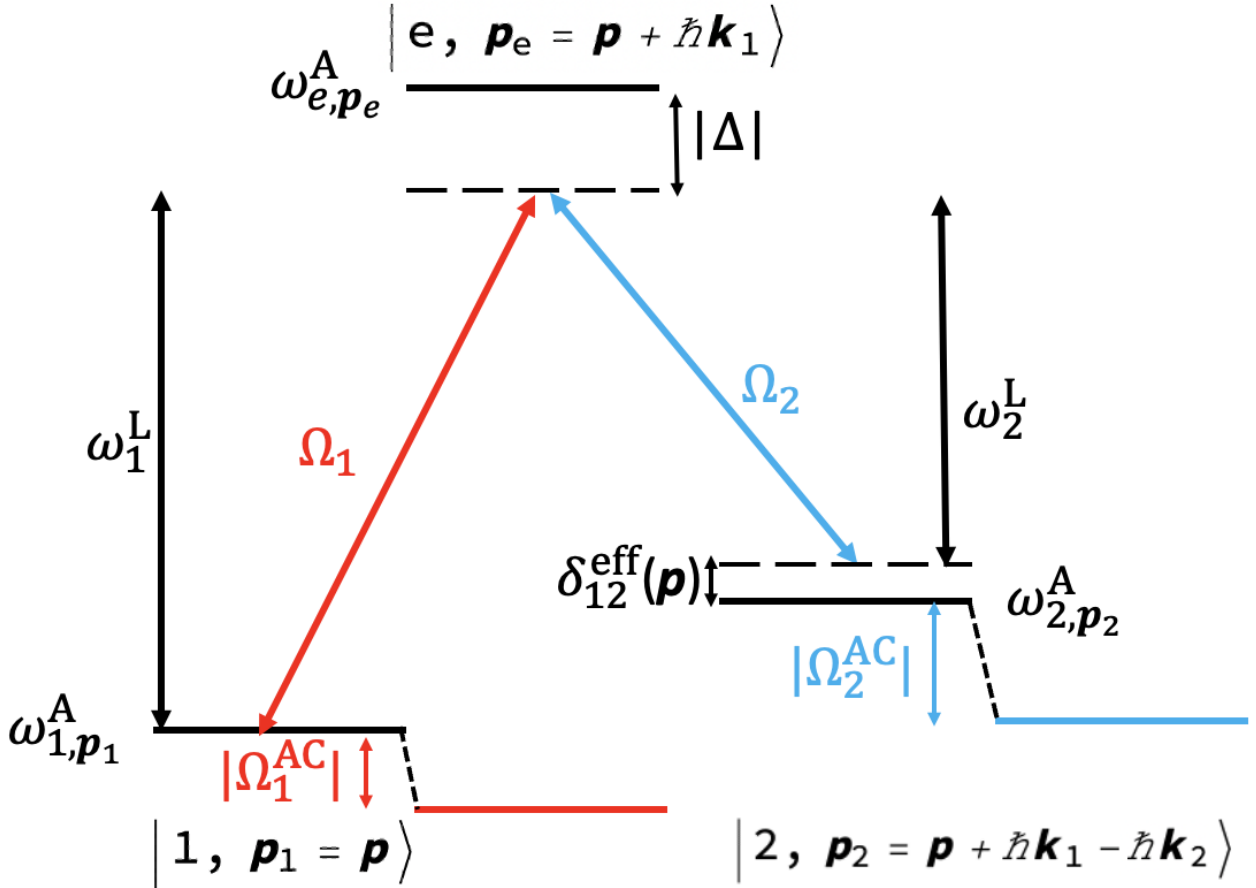


Figure 2.1: Velocity-sensitive stimulated-Raman transition and AC Stark shifts. An atom in state $|1\rangle$ can be transferred to $|2\rangle$ by absorbing a photon from an electric field oscillating at ω_1^L then emitting a photon into another electric field oscillating at ω_2^L . For the momentum family, $\{|1, \mathbf{p}_1\rangle, |2, \mathbf{p}_2\rangle, |e, \mathbf{p}_e\rangle\}$, each oscillating field has a single-photon detuning, $\Delta_n(\mathbf{p}) = \omega_n^L - (\omega_{e,p_e}^A - \omega_{n,p_n}^A) \approx \Delta$, and introduces an energy shift to the clock state $|n\rangle$ by $\hbar\Omega_n^{\text{AC}} = \hbar\frac{|\Omega_n|^2}{4\Delta}$ ($n = 1, 2$). The two-photon detuning, $\delta_{12}^{\text{eff}}(\mathbf{p})$, makes this Raman transition velocity-sensitive.

The change in the resonance condition due to the differential AC Stark shift can be shown by substituting a second rotating-frame transformation,

$$\begin{pmatrix} b_{1,\mathbf{p}}(t) \\ b_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}(t) \end{pmatrix} = \begin{pmatrix} e^{-i(\frac{\Omega_1^{\text{AC}}+\Omega_2^{\text{AC}}}{2}-\frac{\delta_{12}^{\text{eff}}(\mathbf{p})}{2})t} & 0 \\ 0 & e^{-i(\frac{\Omega_1^{\text{AC}}+\Omega_2^{\text{AC}}}{2}+\frac{\delta_{12}^{\text{eff}}(\mathbf{p})}{2})t} \end{pmatrix} \begin{pmatrix} a_{1,\mathbf{p}}(t) \\ a_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}(t) \end{pmatrix}, \quad (2.23)$$

into Eq. (2.20), which gives the dynamics of $(a_{1,\mathbf{p}}(t), a_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}(t))^T$ following

$$\begin{pmatrix} \dot{a}_{1,\mathbf{p}}(t) \\ \dot{a}_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}(t) \end{pmatrix} = -i \begin{pmatrix} \frac{\delta-\delta^{\text{AC}}}{2} & \frac{\Omega_{12}^{\text{eff}}}{2} e^{i\phi_j^{\text{eff}}} \\ c.c. & -\frac{\delta-\delta^{\text{AC}}}{2} \end{pmatrix} \begin{pmatrix} a_{1,\mathbf{p}}(t) \\ a_{2,\mathbf{p}+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2}(t) \end{pmatrix}, \quad (2.24)$$

where the differential AC Stark shift, δ^{AC} , effectively shifts the clock frequency, $\omega_0^{\text{A}} = \omega_2^{\text{A}} - \omega_1^{\text{A}} \rightarrow \omega_2^{\text{A}} - \omega_1^{\text{A}} + \delta^{\text{AC}}$, and c.c. is the complex conjugate. This differential AC Stark shift is proportional to the intensities of the individual oscillating fields shown below

$$\delta^{\text{AC}} = \Omega_2^{\text{AC}} - \Omega_1^{\text{AC}} = \frac{1}{4\Delta} \frac{2}{\epsilon_0 c \hbar^2} \left(I_1 |\langle 1 | \mathbf{e}_1 \cdot \hat{\mathbf{d}} | e \rangle|^2 - I_2 |\langle 2 | \mathbf{e}_2 \cdot \hat{\mathbf{d}} | e \rangle|^2 \right), \quad (2.25)$$

where ϵ_0 is the vacuum permittivity, c is the speed of light, and $\mathbf{e}_n$ are the unit vectors of the polarisation of the corresponding oscillating fields. Therefore, a carefully chosen intensity ratio, known as the magic ratio,

$$R_{\text{magic}} = \frac{I_1}{I_2} = \frac{|\langle 2 | \mathbf{e}_2 \cdot \hat{\mathbf{d}} | e \rangle|^2}{|\langle 1 | \mathbf{e}_1 \cdot \hat{\mathbf{d}} | e \rangle|^2}, \quad (2.26)$$

can zero this differential AC Stark shift to the clock frequency.

2.3 Application to ^{87}Rb

Applying the theory to ^{87}Rb atoms, which are used to build our atom interferometers, requires us to consider its multi-level structure and the Raman beam configurations. The Raman beams are delivered to the atoms with co-propagating Raman-I and Raman-II beams of $\sigma_I^- - \sigma_{II}^+$ polarisation configuration, while only the Raman-I beam gets retro-reflected with the Raman optics shown in section 5.2 to become the counter-propagating Raman-I' beam of $\sigma_{I'}^+$ polarisation (prime refers to the reflected beam). These considerations don't change the physics discussed but make calculations of the effective Raman couplings and AC Stark shifts more complicated. Section 2.3.1 evaluates the effective Rabi frequency due to Raman couplings specifically for ^{87}Rb atoms interacting with Raman-I' and Raman-II beams, followed by an evaluation of AC Stark shifts in section 2.3.2 and the scattering rate in section 2.3.3 due to interaction with these three Raman beams.

2.3.1 Raman coupling

As shown in Eq. (2.18) and 2.22, adiabatically eliminating one excited state gives an effective coupling, $\Omega_{12}^{\text{eff}} = \frac{\Omega_1 \Omega_2^*}{2\Delta}$, between the two clock states $|1\rangle$ and $|2\rangle$. Specific to our application of using ^{87}Rb , the two clock states are $|1\rangle = |F=1, m_F=0\rangle$ and $|2\rangle = |F=2, m_F=0\rangle$ of the $5^2S_{1/2}$ orbital. However, due to the multi-level structure of ^{87}Rb , Raman beams of a $\sigma^+ - \sigma^+$ polarisation configuration leads to an effective Raman coupling,

$$\Omega_{12}^{\text{eff}} = \sum_{|e\rangle} \frac{\Omega_{1,e} \Omega_{2,e}^*}{2\Delta_e}, \quad (2.27)$$

after adiabatically eliminating all of the possible excited states. Note that each individual contribution is inversely proportional to the corresponding single-photon detuning, Δ_e , relative to the excited state $|e\rangle$, the dominant contributions are from the D_2 lines, which are shown in Fig. 2.2.

Consequently,

$$\Omega_{12}^{\text{eff}} \approx \frac{\Omega_{1,2'}^{(1)} \Omega_{2,2'}^{(2)*}}{2(\Delta + |\Delta_{2',3'}|)} + \frac{\Omega_{1,1'}^{(1)} \Omega_{2,1'}^{(2)*}}{2(\Delta + |\Delta_{2',3'}| + |\Delta_{1',2'}|)} = \sqrt{\frac{I_1'}{\text{W/m}^2}} \sqrt{\frac{I_2}{\text{W/m}^2}} 2\pi \cdot 224.59 \text{ Hz}, \quad (2.28)$$

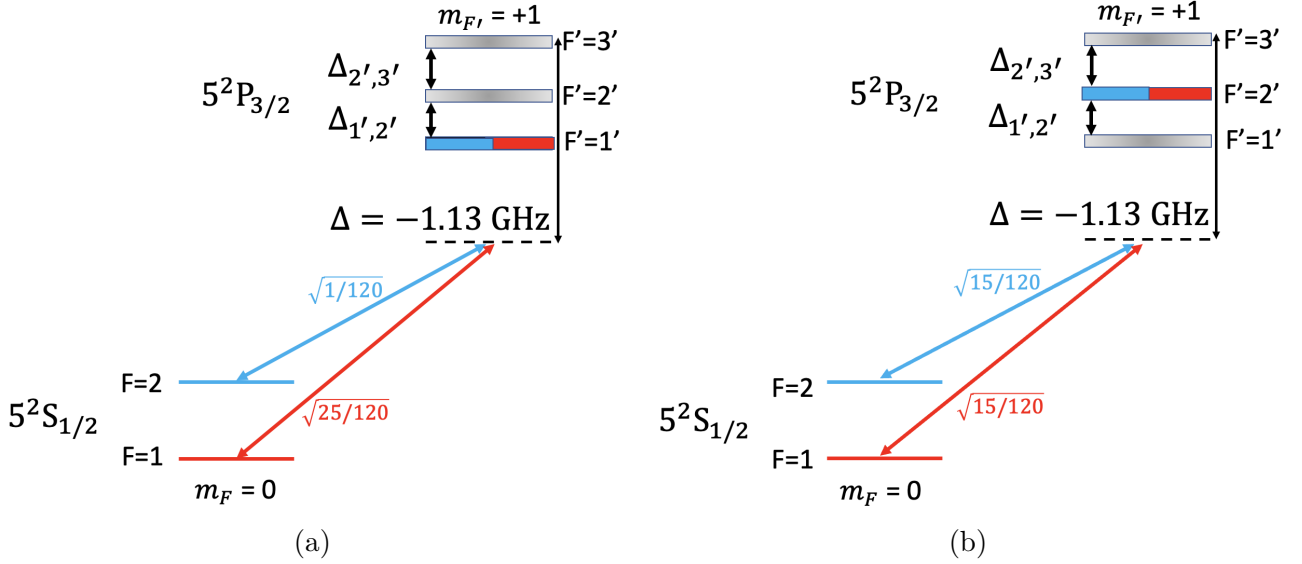


Figure 2.2: Raman routes in ^{87}Rb subject to Raman beams of σ^+ - σ^+ polarisation configuration. (a) Raman coupling between the clock states via $|F' = 1', m_{F'} = 1\rangle$. (b) Raman coupling between the clock states via $|F' = 2', m_{F'} = 1\rangle$. Couplings due to the Raman-I' beam and the Raman-II beam are shown as red and blue arrows respectively with transition dipole moments shown in units of $\langle J = 1/2 | e r | J' = 3/2 \rangle = 3.58 \times 10^{-29} \text{ C}\cdot\text{m}$. The single-photon detuning, Δ , relative to $F' = 3'$ (ignoring Zeeman shifts) is set to -1.13 GHz. Hyperfine splittings of the $5^2P_{3/2}$ states, $\Delta_{1',2'} = 156.95$ MHz and $\Delta_{2',3'} = 266.65$ MHz, are obtained from [6] ($F' = 0'$ has been omitted for the clarity).

where the single-photon detuning relative to the $|F' = 3'\rangle$ state, Δ , is set to -1.13 GHz (this is discussed in Section 2.3.2) and the hyperfine splitting in the excited states, $\Delta_{1',2'}$ and $\Delta_{2',3'}$, are indicated in Fig. 2.2, $\Omega_{n,j'}^{(n)} = \mathbf{E}_n \cdot \langle F = n, m_F = 0 | \hat{\mathbf{d}} | F' = j', m_F = 1 \rangle / \hbar$ are single-photon Rabi frequencies, and I_1' and I_2 are the intensities of the Raman-I' and Raman-II beams. Corrections from the D_1 lines change this result by 0.01%.

2.3.2 Differential light shift and the magic ratio

Including the multi-level structure of ^{87}Rb modifies the expression of the differential AC Stark shift shown in Eq. (2.25) to become

$$\delta^{\text{AC}} = \underbrace{\sum_{X=\text{I,II,I}'} \sum_{|e\rangle} \frac{|\Omega_{2,e}^{(X)}|^2}{4\Delta_e}}_{\Omega_2^{\text{AC}}} - \underbrace{\sum_{X=\text{I,II}} \sum_{|e\rangle} \frac{|\Omega_{1,e}^{(X)}|^2}{4\Delta_e}}_{\Omega_1^{\text{AC}}}, \quad (2.29)$$

where the contributions are not only summed over all possible excited states $|e\rangle$ that have single-photon couplings to the corresponding clock states $|1\rangle$ and $|2\rangle$, but also take into account

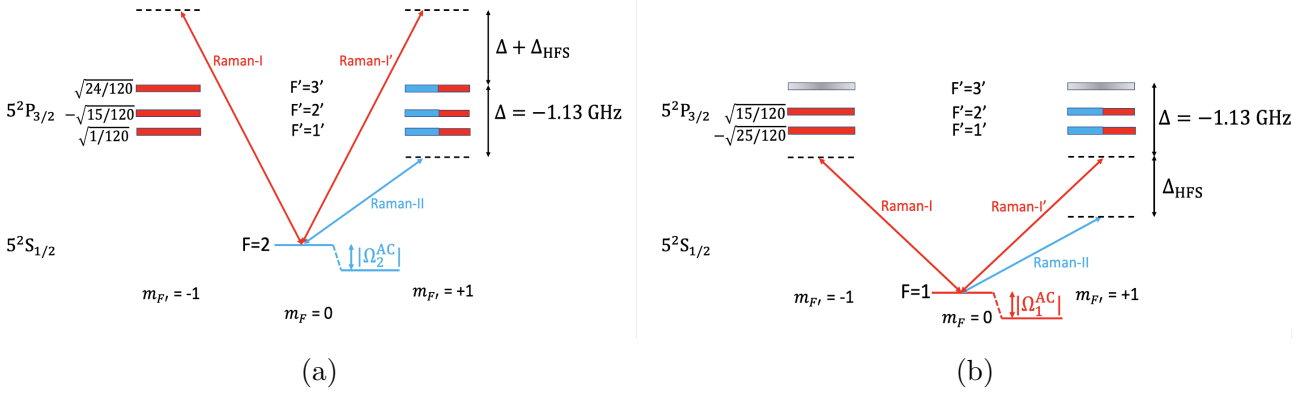


Figure 2.3: Couplings that lead to AC Stark shifts. Energy shifts to the corresponding clock states are Ω_2^{AC} and Ω_1^{AC} . Transition dipole moments are expressed as multiples of $\langle J = 1/2 || er || J' = 3/2 \rangle = 3.58 \times 10^{-29} \text{ C}\cdot\text{m}$ [6] (a) AC Stark shifts to the $|F = 2, m_F = 0\rangle$ state. The Raman-II beam (blue arrow) gives the dominant shift at the single-photon detuning (relative to $|F' = 3\rangle$) of -1.13 GHz, while the Raman-I and Raman-I' beam (red arrows) also contribute non-negligible shifts. (b) AC Stark shifts to the $|F = 1, m_F = 0\rangle$ state. Dominant contributions come from Raman-I and Raman-I' beams (red arrows). $\Delta_{\text{HFS}} = 6.835$ GHz is the hyperfine splitting of the $5^2S_{1/2}$ orbital, to which the frequency difference between the Raman-I (I') beam and the Raman-II beam was set.

all the oscillating electric fields in the system, i.e. the forward propagating Raman-I and Raman-II beams, and the retro-reflected Raman-I' beam. Although the forward propagating Raman-I beam doesn't contribute to the Raman coupling shown in Eq. (2.28), because there is no corresponding beam of the same polarisation to drive the two-photon transition, this beam still contributes to AC Stark shifts. Couplings to D₂ lines illustrated in Fig. 2.3 lead an differential AC Stark shift (evaluated at $\Delta = -1.13$ GHz) by

$$\delta^{\text{AC}} = (\alpha_1(\Delta)I_1 + \alpha_2(\Delta)I_2 + \alpha'_1(\Delta)I'_1) \Big|_{\Delta=-1.13\text{GHz}} = 2\pi \cdot \left(0.274 \frac{I_1 + I'_1}{\text{W/m}^2} - 0.160 \frac{I_2}{\text{W/m}^2} \right) \text{ kHz}, \quad (2.30)$$

where α s are the intensity-sensitivity coefficient of the differential AC Stark shift, δ^{AC} , to the corresponding beams.

Thus a magic ratio between the intensity of two Raman beams, assuming $I'_1 = I_1$, is given by

$$R_{\text{magic}} = \frac{I_1}{I_2} = \left| \frac{\alpha_2(\Delta)}{2\alpha_1(\Delta)} \right| \Big|_{\Delta=-1.13\text{GHz}} = 0.292, \quad (2.31)$$

which cancels out the differential AC Stark shift. The contribution of the D₁ lines changes the

magic ratio at the 10^{-7} level as both clock states are shifted by very similar amounts by the D_1 lines, whose single-photon detunings of approximately 7 THz are much larger than the hyperfine splitting of 6.8 GHz.

In a three-level system, the magic ratio calculated with Eq. (2.26) is independent of the single-photon detuning. However, the multi-level structure of ^{87}Rb makes it into a function of Δ as shown in Fig. 2.4. The single-photon detuning relative to the $|F' = 3\rangle$ state, $\Delta = -1.13$ GHz, used in this experiment minimises the sensitivity of the magic ratio to the single-photon detuning, i.e. $dR_{\text{magic}}/d\Delta|_{\Delta=-1.13 \text{ GHz}} = 0$.

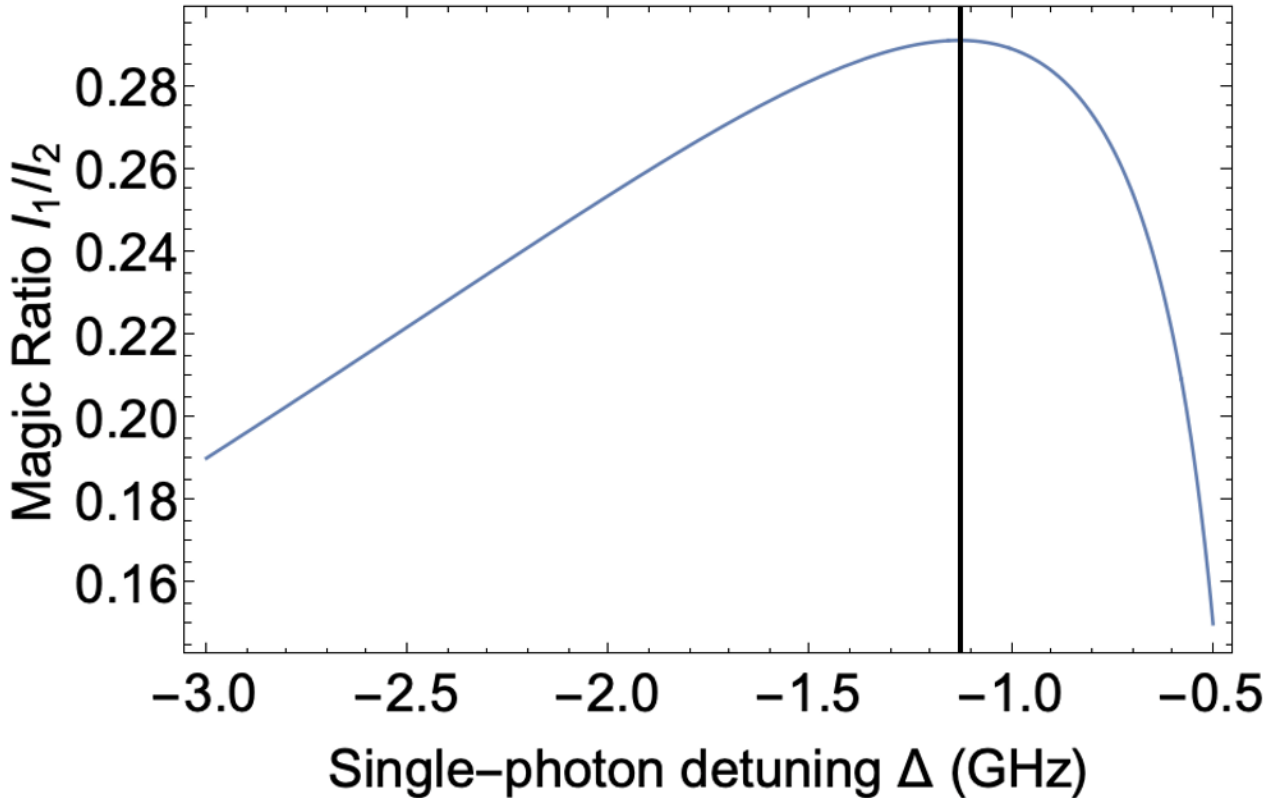


Figure 2.4: Magic ratio as a function of the single-photon detuning for ^{87}Rb (illuminated with a Raman-II rejected beam configuration) taking into account all the $5^2\text{P}_{3/2}$ states. The single-photon detuning, Δ , is defined relative to the cooling transition $|F = 2\rangle \rightarrow |F' = 3'\rangle$. The black line indicates $\Delta = -1.13$ GHz that minimises the sensitivity of the magic ratio to the laser frequency fluctuations.

2.3.3 Spontaneous emission

Each far-detuned single-photon coupling shown in Fig. 2.3 not only introduces an energy shift but also scatters photons. Considering the two states, $|n\rangle$ and $|e\rangle$, involved in each coupling, $\Omega_{n,e}^{(X)}$, the population transferred to the excited state, $|e\rangle$, can be approximated as $\rho_{ee} \approx \frac{|\Omega_{n,e}^{(X)}|^2}{4|\Delta_e|^2}$ in the far-detuned limit [6]. Therefore, the scattering rate can be calculated with $\rho_{ee}\Gamma$, where $\Gamma \approx 38 \times 10^6 \text{ sec}^{-1}$ ($2\pi \cdot 6 \text{ MHz}$) is the scattering rate of an atom in the excited state $|e\rangle$. Considering the D₂ lines of ^{87}Rb , atoms of $|1\rangle = |F=1, m_F=0\rangle$ and $|2\rangle = |F=2, m_F=0\rangle$ scatter photons from the Raman beams with the scattering rates, R_1^{sc} and R_2^{sc} , given by

$$R_1^{\text{sc}} = \sum_{X=\text{I,II,I'}} \sum_{|e\rangle} \frac{|\Omega_{1,e}^{(X)}|^2}{4|\Delta_e|^2} \Gamma = \left(12.32 \frac{I_1 + I_1'}{W/\text{m}^2} + 0.12 \frac{I_2}{W/\text{m}^2} \right) \text{sec}^{-1}, \quad (2.32)$$

$$R_2^{\text{sc}} = \sum_{X=\text{I,II,I'}} \sum_{|e\rangle} \frac{|\Omega_{2,e}^{(X)}|^2}{4|\Delta_e|^2} \Gamma = \left(0.21 \frac{I_1 + I_1'}{W/\text{m}^2} + 7.18 \frac{I_2}{W/\text{m}^2} \right) \text{sec}^{-1}, \quad (2.33)$$

assuming photons are independently scattered from the far-detuned Raman-I, Raman-II and Raman-I' beams of intensities I_1 , I_2 and I_1' . Including the D₁ lines produces relative changes of 10^{-6} to the above results.

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Chapter 3

Atom interferometry

A light-pulse atom interferometer uses the light-atom interaction to couple the internal degrees of freedom and the external degrees of freedom of an atom. As discussed in the Section 2.2.1, the Raman coupling can induce a Rabi oscillation between $|1\rangle = |g_1, \mathbf{p}\rangle$ and $|2\rangle = |g_2, \mathbf{p} + \hbar\mathbf{k}_{\text{eff}}\rangle$. Section 3.1 discusses the manipulation of the momentum state by controlling the interaction time of a velocity-sensitive Raman transition, which plays the role of atom optics. Specifically, an atom in an initial state $|1\rangle$ can evolve into an equal superposition of $|1\rangle$ and $|2\rangle$ with a beam splitter pulse, or completely transfer to $|2\rangle$ with a mirror pulse. With these atom optics, a Mach-Zehnder atom interferometer can be formed with three sequential pulses, which coherently split the wavefunction of each atom into two arms with a beam splitter pulse, reflect the two arms with a mirror pulse, and finally recombine them with another beam splitter pulse. The interference contrast and acceleration sensitivity are discussed in Section 3.2.

3.1 Atom optics

To obtain the final population in $|g_2\rangle$ after applying three sequential pulses to atoms initially prepared in $|g_1\rangle$, it is necessary to track the evolution of each momentum family $|\psi_{\mathbf{p}}(t)\rangle = c_{1,\mathbf{p}}(t)|g_1, \mathbf{p}\rangle + c_{2,\mathbf{p}+\hbar\mathbf{k}_{\text{eff}}}(t)|g_2, \mathbf{p} + \hbar\mathbf{k}_{\text{eff}}\rangle$. The presentation of this section closely follows the derivation in [1] to track the evolution of the state in the rotating frame, $|\psi_{\text{rot},\mathbf{p}}(t)\rangle = b_{1,\mathbf{p}}(t)|g_1, \mathbf{p}\rangle + b_{2,\mathbf{p}+\hbar\mathbf{k}_{\text{eff}}}(t)|g_2, \mathbf{p} + \hbar\mathbf{k}_{\text{eff}}\rangle$, with the notation defined in Section 2.2.1. Working with $|\psi_{\text{rot},\mathbf{p}}(t)\rangle$ reduces many phase dependencies to unity, while the final population of interest is simply given by $|c_{2,\mathbf{p}+\hbar\mathbf{k}_{\text{eff}}}(t)|^2 = |b_{2,\mathbf{p}+\hbar\mathbf{k}_{\text{eff}}}(t)|^2$ since $c_{j,\mathbf{p}_j}(t) = b_{j,\mathbf{p}_j}(t)e^{-i\omega_{j,\mathbf{p}_j}t}$.

With Eq. (2.23) and Eq. (2.12), one can obtain a propagator $\hat{U}(\tau; t_i)$ for this effective two-level system with the result for a true two-level system given in Eq. (2.13),

$$\begin{pmatrix} b_{1,\mathbf{p}}(t_j + \tau) \\ b_{2,\mathbf{p}+\hbar\mathbf{k}_{\text{eff}}}(t_j + \tau) \end{pmatrix} = \underbrace{\begin{pmatrix} \Lambda_{\cos}(\tau; \mathbf{p})e^{i\frac{\delta_{12}^{\text{eff}}(\mathbf{p})}{2}\tau} & -i\Lambda_{\sin}(\tau; \mathbf{p})e^{i\frac{\delta_{12}^{\text{eff}}(\mathbf{p})}{2}\tau}e^{+i\phi_j^{\text{eff}}} \\ -i\Lambda_{\sin}^*(\tau; \mathbf{p})e^{-i\frac{\delta_{12}^{\text{eff}}(\mathbf{p})}{2}\tau}e^{-i\phi_j^{\text{eff}}} & \Lambda_{\cos}^*(\tau; \mathbf{p})e^{-i\frac{\delta_{12}^{\text{eff}}(\mathbf{p})}{2}\tau} \end{pmatrix}}_{\hat{U}(\tau; t_i)} \begin{pmatrix} b_{1,\mathbf{p}}(t_i) \\ b_{2,\mathbf{p}+\hbar\mathbf{k}_{\text{eff}}}(t_i) \end{pmatrix}, \quad (3.1)$$

$$\begin{aligned} \Lambda_{\cos}(\tau; \mathbf{p}) &= \cos\left(\sqrt{(\delta_{12}^{\text{eff}}(\mathbf{p}))^2 + |\Omega_{12}^{\text{eff}}|^2}\frac{\tau}{2}\right) - i\frac{\delta_{12}^{\text{eff}}(\mathbf{p})}{\sqrt{(\delta_{12}^{\text{eff}}(\mathbf{p}))^2 + |\Omega_{12}^{\text{eff}}|^2}}\sin\left(\sqrt{(\delta_{12}^{\text{eff}}(\mathbf{p}))^2 + |\Omega_{12}^{\text{eff}}|^2}\frac{\tau}{2}\right), \\ \Lambda_{\sin}(\tau; \mathbf{p}) &= \frac{\Omega_{12}^{\text{eff}}}{\sqrt{(\delta_{12}^{\text{eff}}(\mathbf{p}))^2 + |\Omega_{12}^{\text{eff}}|^2}}\sin\left(\sqrt{(\delta_{12}^{\text{eff}}(\mathbf{p}))^2 + |\Omega_{12}^{\text{eff}}|^2}\frac{\tau}{2}\right), \end{aligned} \quad (3.2)$$

where the Raman phase is given by

$$\phi_j^{\text{eff}} = (\omega_1^{\text{L}} - \omega_2^{\text{L}})t_j - (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{x}_j + \phi_0^{\text{I}} - \phi_0^{\text{II}}, \quad (3.3)$$

is the local phase difference between two Raman lasers at the start of the interaction, t_j , at the interaction point $\mathbf{x}_j$. ϕ_0^{I} and ϕ_0^{II} are the initial phases ($t = 0$) of the individual lasers Raman-I and Raman-II at an arbitrary reference location $\mathbf{x} = 0$.

For a momentum family that matches the resonance condition, $\delta_{12}^{\text{eff}}(\mathbf{p}) = \delta_{12}(\mathbf{p}) - \delta^{\text{AC}} = 0$,

there are two special propagators with carefully controlled interaction durations, $\tau_{\pi/2} = \frac{\pi/2}{\Omega_{12}^{\text{eff}}}$ and $\tau_{\pi} = \frac{\pi}{\Omega_{12}^{\text{eff}}}$. They are

$$\hat{B}(\tau_{\pi/2}; t_j) = \hat{U}(\tau_{\pi/2}; t_j) = \begin{pmatrix} \frac{1}{\sqrt{2}} & -i\frac{1}{\sqrt{2}}e^{+i\phi_j^{\text{eff}}} \\ -i\frac{1}{\sqrt{2}}e^{-i\phi_j^{\text{eff}}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (3.4)$$

$$\hat{M}(t_j + \tau_{\pi}; t_j) = \hat{U}(t_j + \tau_{\pi}; t_j) = \begin{pmatrix} 0 & -ie^{+i\phi_j^{\text{eff}}} \\ -ie^{-i\phi_j^{\text{eff}}} & 0 \end{pmatrix}, \quad (3.5)$$

where $\hat{B}$ represents a beam splitter pulse ($\pi/2$ -pulse) because it maps an atom in state $|1\rangle$ (or $|2\rangle$) into an equal superposition state, while $\hat{M}$ represents a mirror pulse (π -pulse) because it transfers an atom in state $|1\rangle$ (or $|2\rangle$) into $|2\rangle$ (or $|1\rangle$).

3.2 Three-pulse Mach-Zehnder atom interferometer

A Mach-Zehnder atom interferometer can be formed with three sequential pulses, which coherently split the wavefunction of each atom into two arms with a beam splitter pulse, reflect the two arms with a mirror pulse, and finally recombine them with another beam splitter pulse. Section 3.2.1 discusses the acceleration sensitivity of the relative phase difference between the two arms within the short-pulse limit. Section 3.2.2 presents contrast estimation considering pulses of finite durations and Section 3.2.3 briefly discusses the impact of spontaneous emission for an atom interferometer.

3.2.1 Acceleration sensitivity in the short pulse limit

The population in $|2\rangle$ at the output of an atom interferometer, P_2 , after three sequential pulses shown in Fig. 3.1, depends on the interferometer phase, Φ , such that $P_2 = \sin^2(\Phi/2)$. The interferometer phase, Φ , is the phase difference accumulated between two arms, which has three contributions

$$\Phi = \Delta\phi_{\text{Laser}} + \Delta\phi_{\text{FE}} + \Delta\phi_{\text{Sep}}, \quad (3.6)$$

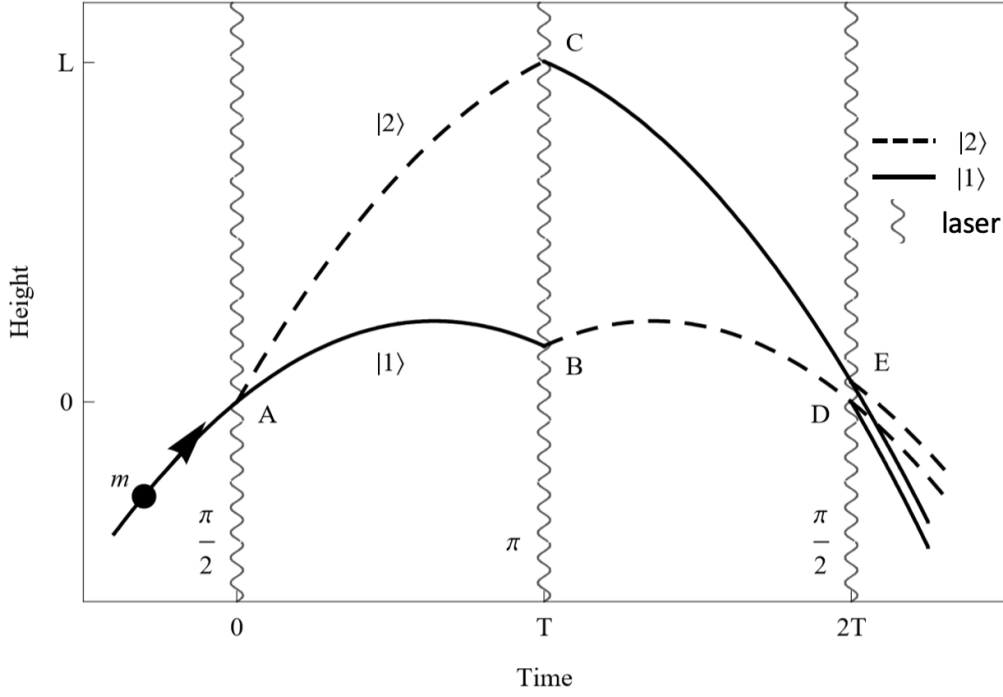


Figure 3.1: Schematics of a light-pulse atom interferometer in the three-pulse Mach-Zehnder configuration [1]. After interacting with a light pulse of duration $\tau_{\pi/2}$, an atom of mass, m , evolves from $|1\rangle$ into an equal superposition of $|1\rangle$ and $|2\rangle$. These two arms become spatially separated because of the momentum transfer at point A. After free-evolution for a time, T , a second pulse of duration τ_{π} flips its internal states at points B and C and redirects two arms towards each other. Then the final pulse of duration $\tau_{\pi/2}$ recombines the two arms and forms a closed Mach-Zehnder interferometer at point D(E).

where the laser phase difference $\Delta\phi_{\text{Laser}}$ is the only non-zero contribution in a closed three-pulse Mach-Zehnder atom interferometer (D=E in Fig. 3.1) subject to uniform acceleration. Detailed discussion about the free-evolution phase difference, $\Delta\phi_{\text{FE}}$, and separation phase difference, $\Delta\phi_{\text{Sep}}$, can be found in [1]. The discussion below focuses on the laser phase shift on each arm in the short-pulse limit.

The short pulse limit [2] assumes that the effective Rabi frequency is much larger than the effective detuning, $\Omega_{12}^{\text{eff}} \gg \delta_{12}^{\text{eff}}$, and the pulse area remains the same, i.e. $\Omega_{12}^{\text{eff}}\tau_{\pi/2} = \pi/2$ and $\Omega_{12}^{\text{eff}}\tau_{\pi} = \pi$. This limit reproduces the expression of a beam splitter pulse (Eq. (3.4)) and a mirror pulse (Eq. (3.5)) because all the momentum families effectively resonate with the light pulses. Noting that the Raman laser phase, defined in Eq. (3.3), has a negative sign when the wavepacket receives a photon (A and B) and a positive sign when the wavepacket is recoiled by releasing a photon (C and D), then the difference between the laser phase shifts of the two arms can be calculated with $\Delta\phi_{\text{Laser}} = (\phi_A^{\text{eff}} - \phi_B^{\text{eff}}) - (\phi_C^{\text{eff}} - \phi_D^{\text{eff}})$.

The acceleration sensitivity arises from the spatially dependent part in $\Delta\phi_{\text{Laser}}$ by rewriting it as

$$\begin{aligned}
 & \mathbf{k}_{\text{eff}} \cdot \left\{ \left(\mathbf{x}_c(t_1) - \frac{\mathbf{x}_c^{\text{upper}}(t_2) + \mathbf{x}_c^{\text{lower}}(t_2)}{2} \right) - \left(\frac{\mathbf{x}_c^{\text{upper}}(t_2) + \mathbf{x}_c^{\text{lower}}(t_2)}{2} - \mathbf{x}_c(t_3) \right) \right\} \\
 &= \mathbf{k}_{\text{eff}} \cdot \left\{ \left(-\mathbf{v}_1 T - \frac{1}{2} \mathbf{a} T^2 \right) - \left(-\mathbf{v}_2 T - \frac{1}{2} \mathbf{a} T^2 \right) \right\} \\
 &= \mathbf{k}_{\text{eff}} \cdot (-\mathbf{v}_1 + \mathbf{v}_2) T = \mathbf{k}_{\text{eff}} \cdot \mathbf{a} T^2,
 \end{aligned} \tag{3.7}$$

where v_j is the velocity of the centre of the mass of the superposed wavepackets after the j^{th} pulse. This can be interpreted as a classical acceleration estimation with three-position measurements using a ruler with graduations separated by $2\pi/k_{\text{eff}} \approx 390$ nm [3]. Notably, it is the quantum mechanical interaction (Raman transition) that equips this classical method with sub- μm resolution.

3.2.2 Contrast reduction with finite-duration pulses

In the short-pulse limit, the detuning effect has been completely ignored, where all momentum families are effectively resonant with the atom optics regardless of their Doppler shifts. However, experimentally, $\Omega_{12}^{\text{eff}} \approx 40$ kHz is limited by the power in our Raman beams. Therefore, a π -pulse cannot completely transfer atoms from $|1\rangle$ ($|2\rangle$) into $|2\rangle$ ($|1\rangle$) for the momentum families of Doppler shifts comparable or larger than Ω_{12}^{eff} . Meanwhile, these momentum families get extra momentum-dependent phase shifts in addition to the Raman laser phase shifts in Eq. (3.3), because $\Lambda_{\cos}(\mathbf{p})$ and $\Lambda_{\sin}(\mathbf{p})$ in Eqs. (3.1) and (3.2) become complex numbers. This section presents a numerical evaluation of $P_2(\mathbf{p})$ for each momentum family follows the discussion in [2] and assume $\delta^{AC} = 0$ for simplicity. This condition is experimentally approached by choosing the magic intensity ratio between two Raman beams.

Taking into account the projection of gravity on our horizontal Raman axis, the centre of mass velocity, $\dot{\mathbf{x}}^c$, can introduce different Doppler shifts at three pulses for each momentum family. This modifies the two-photon Doppler shift in Eq. (2.21), $\delta_{2\gamma}^D(\mathbf{p}) = \hbar(\mathbf{k}_1 - \mathbf{k}_2) \cdot (\mathbf{p}/m + \dot{\mathbf{x}}^c) \rightarrow \hbar(\mathbf{k}_1 - \mathbf{k}_2) \cdot (\mathbf{p}/m + \dot{\mathbf{x}}_j^c)$, where $\dot{\mathbf{x}}_j^c$ is the velocity of the centre of mass at the j -th pulse.

To calculate the final population in $|2\rangle$ after a sequence of three pulses $\frac{\pi}{2} - \pi - \frac{\pi}{2}$, it is

necessary to propagate the state during the free-evolution. The free-evolution propagator, $\hat{U}_F$, can be obtained by taking $\Omega_1^{\text{AC}} = 0$, $\Omega_2^{\text{AC}} = 0$, $\Omega_{12}^{\text{eff}} = 0$ for the propagator in Eq. (3.1) because no Raman light during the free-evolution, which gives

$$\hat{U}_F = \hat{U}(T; t_j) \Big|_{\Omega_1^{\text{AC}}, \Omega_2^{\text{AC}}, \Omega_{12}^{\text{eff}} \rightarrow 0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (3.8)$$

since $\Lambda_{\cos}(\tau; \mathbf{p}) \rightarrow e^{-i\frac{\delta_{12}^{\text{eff}}}{2}\tau}$ and $\Lambda_{\sin}(\tau; \mathbf{p}) \rightarrow 0$ in this limit.¹ Then the overall propagator for a Mach-Zehnder atom interferometer, $\hat{U}_{\text{MZ}}$, can be written as

$$\hat{U}_{\text{MZ}} = \hat{B}(\tau_{\pi/2}; t_0 + \tau_{\pi/2} + \tau_{\pi} + 2T) \hat{U}_F \hat{M}(\tau_{\pi}; t_0 + \tau_{\pi/2} + T) \hat{U}_F \hat{B}(\tau_{\pi/2}; t_0), \quad (3.9)$$

where the beam splitter propagator, $\hat{B}$, and mirror propagator, $\hat{M}$, are defined in Eq. (3.4) and Eq. (3.5) respectively. Using this Mach-Zehnder propagator, the final state of each momentum family can be calculated as $|\psi_{\mathbf{p}}(t_f)\rangle = \hat{U}_{\text{MZ}} |\psi_{\mathbf{p}}(t_0)\rangle$. It is shown as a function of the momentum-induced two-photon detuning in Fig. 3.2 for an interferometer of $T = 33$ ms and $\Omega_{12}^{\text{eff}} = 2\pi \cdot 40$ kHz, assuming $|\psi_{\mathbf{p}}(t_0)\rangle = (1, 0)^T$ for atoms initially prepared in $|1\rangle$ for all the momentum families.

The transition probability spectrum depends on the acceleration-induced phase difference between two arms, where the spectra of $\Phi = 0$ and $\Phi = \pi$ are shown in Fig. 3.2 (a) and Fig. 3.2 (b) respectively. This transition probability is highly-oscillatory as a function of the detuning (solid blue shade in Fig. 3.2 (a) and Fig. 3.2 (b)), which is illustrated around the $\delta_{12}^{\text{eff}} \approx 0$ for spectra of $\Phi = 0$ and $\Phi = \pi$ in Fig. 3.2 (c) and Fig. 3.2 (d). This oscillatory behaviour has a quasi-period of $1/2T$, which can be understood with atoms that are not perfectly resonant with the middle π -pulse forming a $\pi/2$ - π - $\pi/2$ Ramsey interferometer.

For momentum families satisfying the short-pulse limit condition, $\delta_{12}(\mathbf{p}) \ll \Omega_{12}^{\text{eff}}$, their final population in $|2\rangle$ matches $P_2 = \sin^2(\Phi/2)$ with the ideal visibility $V = (P_2^{\text{max}} - P_2^{\text{min}})/(P_2^{\text{max}} + P_2^{\text{min}}) = 1$. However, in the experiment, the momentum distribution is determined by a velocity selection pulse, shown as black curves in Fig. 3.2 (a) and Fig. 3.2 (b), the visibility can be calculated with a numerical integration over the momentum distribution (black curves),

¹This simple expression of $\hat{U}_F$ results from using the rotating frame for $(b_1(t), b_2(t))^T$.

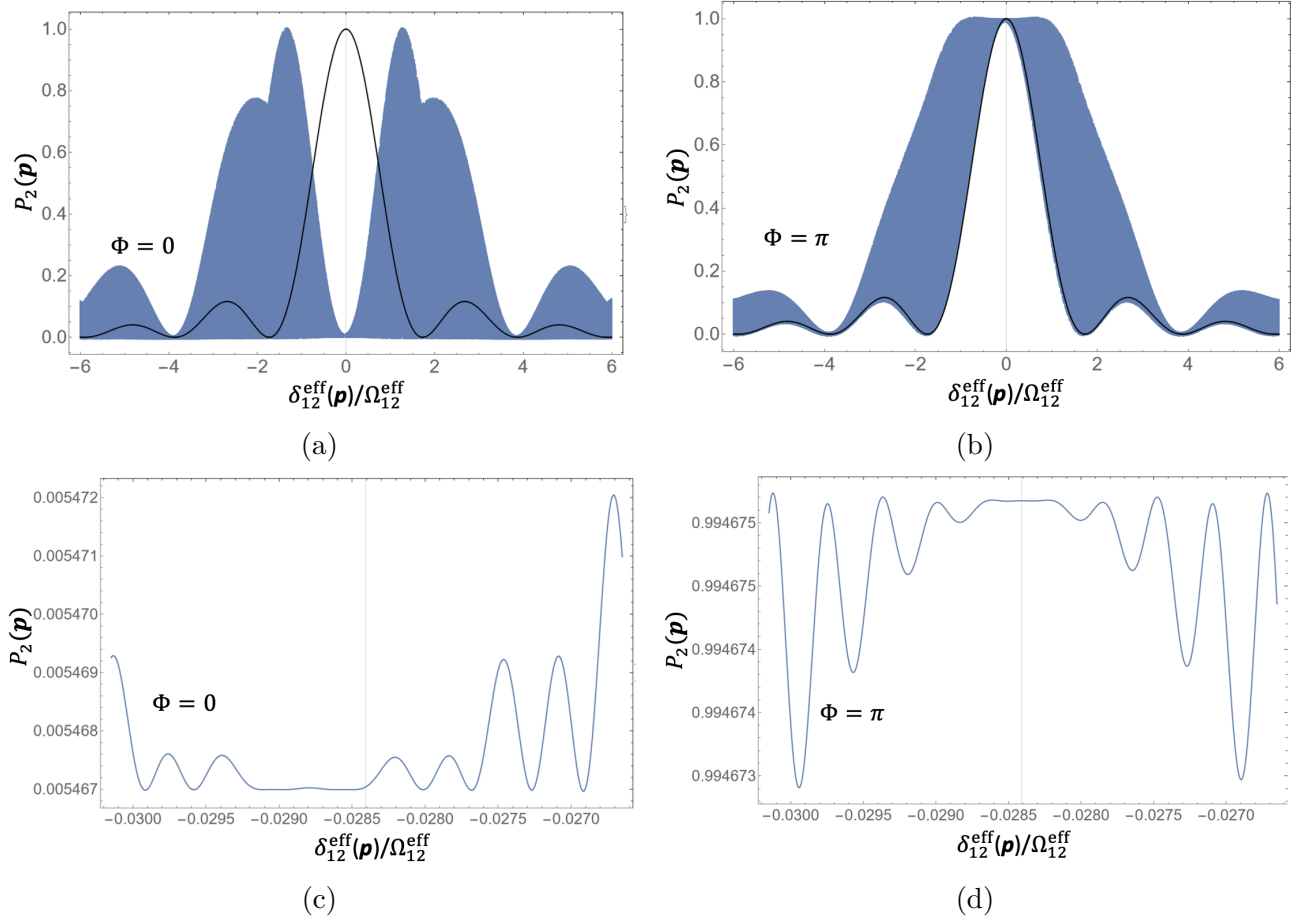


Figure 3.2: Transition probability as a function of the two-photon detuning for a $\pi/2$ - π - $\pi/2$ interferometer with $T=33$ ms, $\Omega_{12}^{\text{eff}}=2\pi\cdot 40$ kHz ($2T\cdot\Omega_{12}^{\text{eff}} = 5280\pi$). (a) Transition probability spectrum (blue) for the interferometer phase $\Phi = 0$. A velocity-selection pulse determines the momentum distribution (black), where the two-photon detuning of each momentum family, $\delta_{12}^{\text{eff}}(\mathbf{p})$, is given by its two-photon Doppler shift. (b) Transition probability spectrum (blue) for interferometer phase $\Phi = \pi$. These spectra have highly oscillatory behaviour shown as blue shades in (a) and (b), while its quasi-period is $2\pi/(5280\pi)$ as shown in (c) and (d). (c) Transition probability spectrum (blue) near $\delta_{12}^{\text{eff}} \approx 0$ for interferometer phase $\Phi = 0$. (d) Transition probability spectrum (blue) near $\delta_{12}^{\text{eff}} \approx 0$ for interferometer phase $\Phi = \pi$. The centre of the transition probability is non-zero because the mirror pulse is slightly detuned due to acceleration-induced two-photon Doppler shift for the centre-of-mass momentum p^c .

$G(\mathbf{p})$, weighted by the transition probability (blue curves), $P_2(\mathbf{p}, \Phi)$. For our experimental parameters, it is expected to be $V = 68.7\%$ with $P_2^{\text{max}} = \int d\mathbf{p} P_2(\mathbf{p}, \Phi = \pi) G(\mathbf{p}) = 0.695$ and $P_2^{\text{min}} = \int d\mathbf{p} P_2(\mathbf{p}, \Phi = 0) G(\mathbf{p}) = 0.129$.

3.2.3 Contrast reduction by spontaneous emission

Spontaneous emission can reduce the contrast for an atom interferometer. Substituting the magic ratio $I'_1 = I_1 = 0.292I_2$ into Eq. (2.32) and Eq. (2.33) reveals the scattering rates of Raman light for atoms in $|1\rangle$ and $|2\rangle$ are $R_1 = \frac{I_2}{W/m^2} 7.31 \text{ sec}^{-1}$ and $R_2 = \frac{I_2}{W/m^2} 7.30 \text{ sec}^{-1}$. Since these scattering rates are very similar, the scattering rate of Raman light for an atom in any superposition state of $|1\rangle$ and $|2\rangle$ can be approximately taken as $R_{\text{atom}} = \frac{I_2}{W/m^2} 7.3 \text{ sec}^{-1}$. Then the overall scattered Raman photons per atom during the $\pi/2$ - π - $\pi/2$ interferometer can be estimated with $R_{\text{atom}}\tau_{2\pi} = 0.06$, where the total interaction time with Raman beams is $\tau_{2\pi} = 2\pi/\Omega_{\text{eff}}$. This is independent of the intensities of the beams because the total pulse duration $\tau_{2\pi}$ is inversely proportional to I_2 while R_{atom} is proportional to I_2 .

This suggests that 6% of atoms will lose their coherence rather than contribute to the interference signal. Thus spontaneous emission in the Raman coupling reduces our interferometer contrast to 94% of the contrast without taking this effect into account.

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Part II

Experimental Apparatus

Chapter 4

Laser cooling and trapping

To make a sensitive atom interferometer, it is essential to minimise the expansion of cloud during interferometer sequences. The transverse expansion, combined with the finite extent of the Raman beams, limits the contrast of an interferometer by inhomogeneous Rabi frequencies, while the axial velocity distribution along the Raman axis limits the contrast due to inhomogeneous two-photon Doppler shifts. Therefore, it is critical to have atoms as cold as possible.

As such, the equipment required to cool the atoms to sub-Doppler temperatures is an important feature of this experiment. Section 4.1 presents an overview of the devices and the control software. Section 4.2 illustrates the vacuum system we used to conduct the laser cooling and trapping. To trap atoms and cool them to the sub-Doppler temperature, various magnetic bias and gradient fields used in this experiment are discussed in Section 4.3, while the laser systems are described in Section 4.4. Finally, a microwave system was found useful in characterising the magnetic field seen by atoms, which is presented in Section 4.5.

4.1 Overview of devices

To conduct a sophisticated experiment, various hardware devices need to be controlled to run in a specified pattern. The core software for generating the sequence patterns and data acquisition for this experiment were MOTMaster2 and AxelHub [1], which have been both developed using C# in the Navigation Lab at the Centre for Cold Matter of Imperial College London.¹

A schematic diagram of the software and hardware used in this experiment is shown in Fig. 4.1. The analogue outputs were generated from the PXI-6723 (13-Bit, 32 channels), which was upgraded to a PXI-6738 with lower noise. These analogue outputs controlled the position of the source ball (Ball position drivers) and the current to generate gradient and shim magnetic fields (coils drivers). They also ramped the frequency of the cooling laser to perform molasses cooling and set the frequencies of acoustic-optic modulators (AOMs drivers) and microwaves (Microwave drivers). Coils, AOMs and microwave were switched on/off with PXI-6541 using digital signals.

The computer sent serial commands to various lasers to control their frequencies and powers. A telecommunication client connected to the μ Quans lasers to control the Raman lasers for the interferometer and the lasers for creating a two-dimensional magneto-optical trap (2D-MOT), whose error signals were displayed with *gPlot* (μ Quans). The client also set the gain of the repump laser amplifier (*BoosTA*). We used *SOF8-409* to control the DDS frequencies (Novatech, 409B) sent to AOMs. We used *Windfreak-Synth* to control the microwave frequency for the state preparation. Finally, the computer configured the camera (Allied Vision, Pike F-505C) and took image data with *Vimba*.

The crucial data in this experiment are the acceleration of the Raman mirror and the fluorescence signal from atoms. The acceleration was measured with a MEMS accelerometer, while the fluorescence signal was collected by a photodiode. Both signals were digitised by PXI-4462 before being processed by Axelhub and saved to the computer. With Axelhub, the fluorescence signals were averaged over six samples (30 μ s) after trimming off the transients in each 100 μ s detections shown in Fig. 7.26. The acceleration and temperature signals of the

¹My contribution was adapting Telenet communication into MOTmaster2, such that serial commands can be sent to μ Quans lasers via an Ethernet cable to control the laser frequencies for producing cold atoms and the Raman light-pulse atom interferometry.

MEMS were recorded around the interferometer with an additional 1 ms data before the first Raman pulse and an extra 1 ms data after the last Raman pulse. These 2 ms extra data allowed us to study the potential synchronising issue between the atom interferometer and the MEMS interferometer with an adjustable delay discussed in Section 7.6.2.

The repetition rate of our experiment was roughly 0.4 Hz. The experiment sequence of loading atoms and performing interferometry took less than 800 ms, but the acquisition of the MEMS temperature and acceleration data took more than one second.

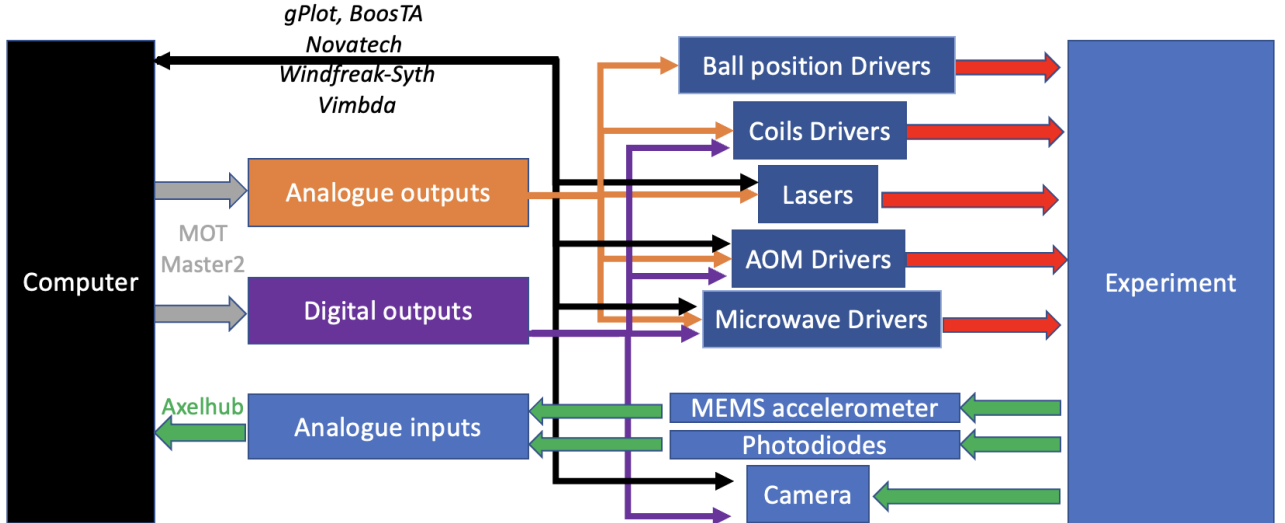


Figure 4.1: Schematics of the devices with their controls and data acquisition software (adapted from [2]). The pattern of the experimental sequence was generated with the MOTMaster2 (grey arrows). Then the programmed analogue outputs (orange arrows) were delivered to various hardware drivers, triggered by 3.3 V digital outputs (purple arrows). Before the start of each sequence, serial commands (black arrows) setting frequencies were sent to lasers, AOM drivers (DDS devices) and microwave drivers. During the experiment, the Raman mirror acceleration was measured with a MEMS accelerometer, while atom fluorescence signals were collected by a photodiode and a camera. Both acceleration data and fluorescence signals were processed with the Axelhub and saved on the computer. The data flow is shown with green arrows.

4.2 Vacuum system

The vacuum system was inherited from the previous-generation experiment, whose design details can be found in [3]. As shown in Fig. 4.2, it consisted of two vacuum parts: the glass cell for the two-dimensional magneto-optical trap (2D-MOT) and the science vacuum chamber for producing cold atoms and performing interferometry. In this section, I will summarise the key dimensions and a few changes I have made for this generation of experiment.

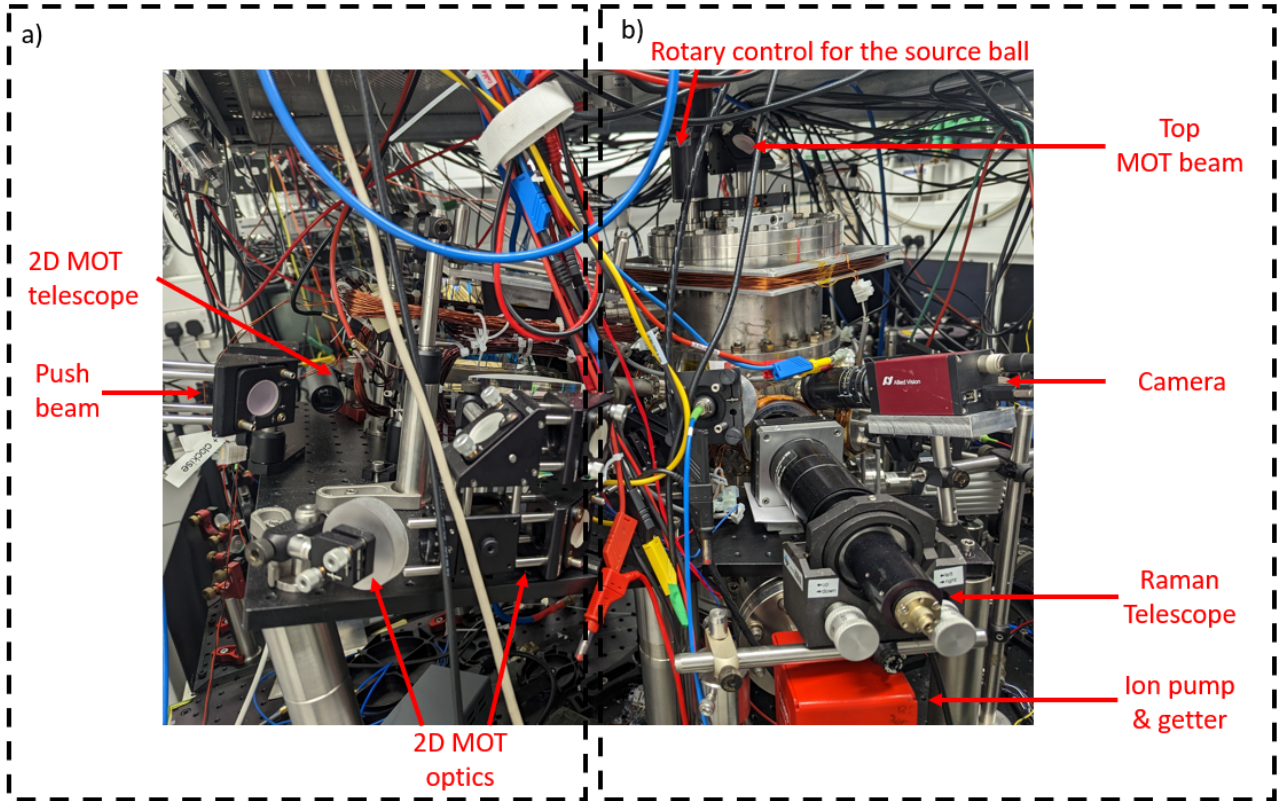


Figure 4.2: Overview of the vacuum system. (a) 2D MOT assembly. (b) The science chamber for the 3D MOT and atom interferometry.

4.2.1 Differential pumping tube

In this generation of experiment, the 2D MOT glass cell was connected to the 3D MOT chamber (MCF800-SphOct-G2C8) with a new differential pumping tube (adapted from [4, p.60]) as shown in Fig. 4.3 (a). This tube had an aperture of 2 mm diameter, d , with a pipe of diameter, $D = 6.95$ mm, and length, $l = 140$ mm. This differential pumping tube created a pressure difference between the 3D MOT chamber (low pressure $\approx 10^{-9}$ mbar) and the 2D MOT glass

cell (high pressure $\approx 10^{-6}$ mbar), which allowed a loading rate as fast as $\approx 2 \cdot 10^8$ /s by loading atoms into a three-dimensional magneto-optical trap (3D MOT) from the two-dimensional magneto-optical trap (2D MOT). This design provided an intense atomic beam from the 2D MOT because of the high vapour pressure in the glass cell, and a low-loss rate 3D MOT due to a low background-gas collision rate, which benefited from the low pressure in the science chamber.

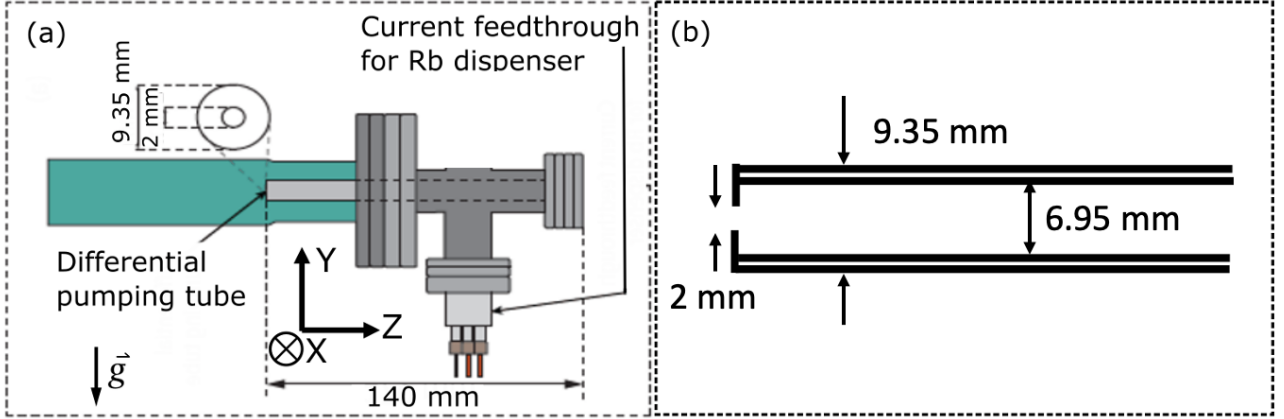


Figure 4.3: Schematics of the differential tube connecting the 2D MOT glass cell to the science chamber (adapted based on [3, 4]). (a) Side view from the X direction with indicated length and aperture of the differential pumping tube. (b) Schematics of the differential pumping tube with inner diameter illustrated.

The total conductance of this tube C_{tot} can be calculated with [5]

$$\frac{1}{C_{\text{tot}}} = \frac{1}{C_{\text{aperture}}} + \frac{1}{C_{\text{pipe}}}, \quad (4.1)$$

where the conductance of the aperture, C_{aperture} , and the conductance of the pipe, C_{pipe} , in the molecular flow regime, are given by

$$C_{\text{aperture}} = \frac{\pi(d/2)^2}{4} \bar{v}_{th}, \quad (4.2)$$

$$C_{\text{pipe}} = \frac{\pi D^3}{12l} \bar{v}_{th}, \quad (4.3)$$

where $\bar{v}_{th} = \sqrt{\frac{8k_B T}{\pi m}}$ is the mean thermal velocity, k_B is the Boltzmann constant, m is the mass of ^{87}Rb atom, and T is the temperature of particles. Assuming particles are at room temperature

$T = 300$ K, substituting the tube dimensions into Eqs. (4.1) to (4.3) gives a total conductance of $C_{\text{tot}} = 0.09$ litre/s. This total conductance is a factor of two smaller than the old design of a total conductance of 0.18 litre/s, which is obtained by evaluating Eqs. (4.1) to (4.3) with $d = 2$ mm, $D = 16$ mm and $l = 240$ mm provided in [3].² This is three orders of magnitude smaller than the 300 litre/s pumping speed of an ion pump (SAES NEX Torr D-300) in the science chamber. Therefore, a pressure difference of three-order magnitude was maintained between the 2D MOT glass cell and the science chamber.

More importantly, the new differential pumping tube is much closer to the 2D MOT, thus the collisions between the laser-cooled atoms and the hot background atoms were minimised. This is significantly different from the previous generation [3, p.21], where transversely cooled Rb atoms have to travel an additional distance of ≈ 5 cm through the hot background gas before entering the science chamber.

Another small modification is that the 2D MOT optics was put on a tilted table such that this new differential pumping tube was connected to a DN20 flange of the primary chamber. This action freed a DN40 flange. This free DN40 flange was replaced with a 2-inch window to give additional optical access for the experiment, which was used to estimate the scattered Raman light from the in-vacuum source mass discussed in Section 7.5.4.

4.2.2 In-vacuum source mass

The source mass for the chameleon measurement in Section 7.4 is an aluminium (alloy 6082) ball of a 19 mm radius placed inside the science chamber via a rotary feedthrough (LewVac, M-BRM16CF) as shown in Fig. 4.4a, which allowed us to adjust the position of the mass relative to the atoms shown in Fig. 4.4b. To minimize the undesired scattering of Raman beams, the ball was coated with high-absorption paint as same as the last generation [3, p.49]. Given the chameleon force is a short-range force, the distance from the centre of the atom cloud to the surface of the ball was chosen as short as ≈ 1 cm [6], which placed the atom cloud near the surface of the source mass before scattering too much Raman light. The shape of the source

²Noting in [3, p.23] there is a factor of π missed in its Eq.(3.2) compared to the Eq. (4.3) used here, which gives an incorrect estimation of $C_{\text{tot}} = 0.14$ litre/s. Correcting this error gives the total conductance of $C_{\text{tot}} = 0.18$ litre/s for the old design.

mass was chosen to be a 3D sphere for simplifying calculations. The radius of the source was chosen to be 19 mm because this radius can further simplify calculations of the chameleon force as from a 1D parallel-plane [7]. The material was chosen to be aluminium because its light density generates a very negligible Newtonian gravitational pull (details are discussed in Section 7.5.1), which doesn't change the chameleon force from the source mass of the same size because chameleon field only sources from the surface of bulk material.

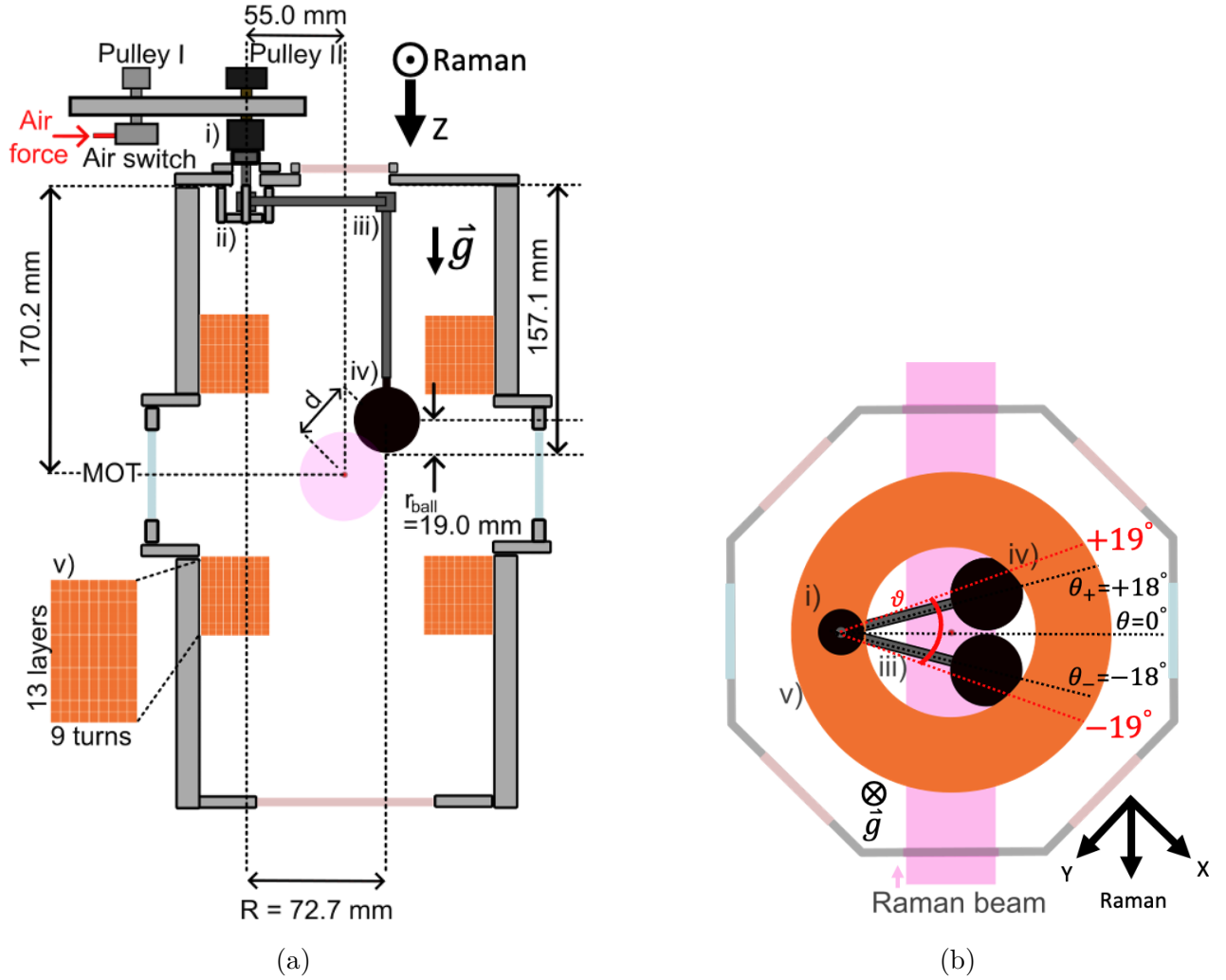


Figure 4.4: Schematics of the rotary vacuum feedthrough to control the ball position. (a) Side view with key geometric dimensions shown in the graph. i) An UHV-compatible rotary feedthrough and a pulley system to control its rotation angle. ii) The barrier system limits the rotation range of the feedthrough. iii) A right-angle extension arm formed by two rods to place the source mass near the atoms. iv) The aluminium source mass of a radius $r_{\text{ball}} = 19 \text{ mm}$. v) Zoom-in illustration of the in-vacuum MOT coils. (b) Top view. The ball is switched to be placed at $\theta_- = -18^\circ$ and $\theta_+ = +18^\circ$, while the rotation angle ranged from -19° to 19° give a total rotation angle $\vartheta = 38^\circ$.

The chameleon force that could be detected by this experiment is the projection of the

ball-induced acceleration onto the Raman axis, which is determined by the distance, d , shown in Fig. 4.4a, and θ shown in Fig. 4.4b. The chameleon force is smallest when the angular location $\theta = 0^\circ$ and largest when $\theta = \pm 19^\circ$. This means the magnitude of the chameleon acceleration induced by the ball is largest at $\theta = 0^\circ$ and smallest at $\theta = \pm 19^\circ$. However, the projection on the Raman axis is smallest at $\theta = 0^\circ$ and largest at $\theta = \pm 19^\circ$. This combination leads to an optimum angle, $\theta_{\text{opt}} = \pm 14^\circ$, that maximises the projected chameleon acceleration onto the Raman axis [3, p.50].

In the last generation of this experiment [3, p.49], a stepper motor was used to control the angular position θ . However, since stepper motors use solenoids to control the position of the motor, the magnetic field produced by the motor will vary with θ and may lead to systematic errors in the chameleon measurement. To suppress this problem, the stepper motor has been replaced with an air-driven pulley system. This pulley system on the top of the chamber has negligible changes in the magnetic field associated with its rotation, which minimised the possible systemic associated with a step motor system.

The actuators of the pulley system consisted of an air switch and a rotary control as shown in Fig. 4.5a. The air switch only has two states, which are 0° and 90° with an arbitrary reference angular position. However, the rotary feedthrough is desired to be varied from $\theta_{\text{opt}}^+ = 14^\circ$ to $\theta_{\text{opt}}^- = -14^\circ$. Therefore, a pulley system was designed as shown in Fig. 4.5b, where a 90° turn on the 24-teeth pulley-I (RS Pro, 183-928) caused a 36° turn on the 60-teeth pulley-II (RS Pro, 778-4742), and caused a change in θ of $\pm 18^\circ$. This operation was found to minimise the detection noise raised from scattered MOT light but lead to a $\approx 7\%$ reduction in the sensitivity from the optimal cases where $\theta_{\text{opt}}^\pm = \pm 14^\circ$. These two operational states, $\theta_\pm = \pm 18^\circ$, were switched by the direction of the airflow. The airflow direction controller (SY5120-6LOU-01F-Q) was placed far away from the chamber to minimise the systematic changes in the magnetic field seen by atoms.

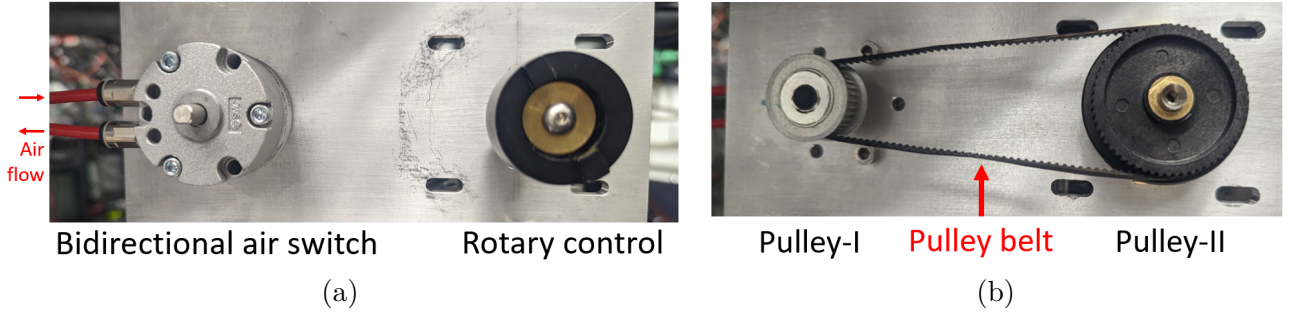


Figure 4.5: An air switch to control the rotary feedthrough. (a) Bottom view. The bidirectional air switch actuator rotated 90° subject to the direction of the airflow, which was determined by an airflow direction controller (not shown in the diagram). The actuator of the vacuum feedthrough can rotate up to $38^\circ (\pm 19^\circ)$ in total before being stopped by barriers shown as in Fig. 4.4 (a-ii). (b) Top view. Pulley-I co-rotated with the air switch actuator. Pulley-II controlled the actuator of the rotary vacuum feedthrough. They were connected with a belt of 2 mm pitch.

4.3 Magnetic field control

In this experiment, there were two systems generating the magnetic fields required for magneto-optical trapping in the 2D and 3D MOT. They consisted of coil pairs in an anti-Helmholtz configuration, which are referred as gradient coils (discussed in Section 4.3.1 and Section 4.3.3). In addition, there were another three sets of bias field coils. Two were used to cancel any stray magnetic fields in the 2D and 3D MOT and the third provided a magnetic bias field along the Raman axis. These bias field coils are discussed in Section 4.3.2 and Section 4.3.4.

4.3.1 3D MOT gradient coils

The 3D MOT gradient coil was a pair of anti-Helmholtz coils placed in the science chamber with details described in [3, p.23-p.26]. In summary, this MOT coil pair was designed to produce a magnetic field gradient of 1.3 G/cm/A along the axial direction. These two circular coils were separated by 11 cm. Each coil has a minimum radius of 55 mm and was made of 9 radial turns and 13 axial layers of $2.7 \text{ mm} \times 4 \text{ mm}$ rectangular wires as illustrated in Fig. 4.4 (a-v).

4.3.2 3D MOT shim field and Raman bias field

Four identical designs with operational amplifiers (OPA-549) were utilised to deliver bipolar currents into four pairs of Helmholtz coils. Three of them were shim coils that adjusted the

B-fields along the X, Y, and Z directions to cancel the stray magnetic fields at the centre of the science chamber. The fourth coil pair applied a biased magnetic field along the Raman axis, which is in the X-Y plane as illustrated in Fig. 4.4. As shown in Fig. 4.6 (a), the sensing resistor, $R_s=0.22\ \Omega$, converted its current to a voltage, V_{sense} , which is tightened to the control voltage, $V_{\text{sense}} = V_{\text{in}}$, via the negative feedback. Due to the high input impedance of an operational amplifier, the output current through the corresponding coil pair was the same as the current through the sensing resistor, $I_{\text{coil}} = I_{\text{sense}}$, regardless of the coil pair's resistance. Results of a calibration test are shown in Fig. 4.6 (b), where all four coils show approximately the same control responses that is consistent with the expected value of $1/R_s = 4.5\ \text{A/V}$.

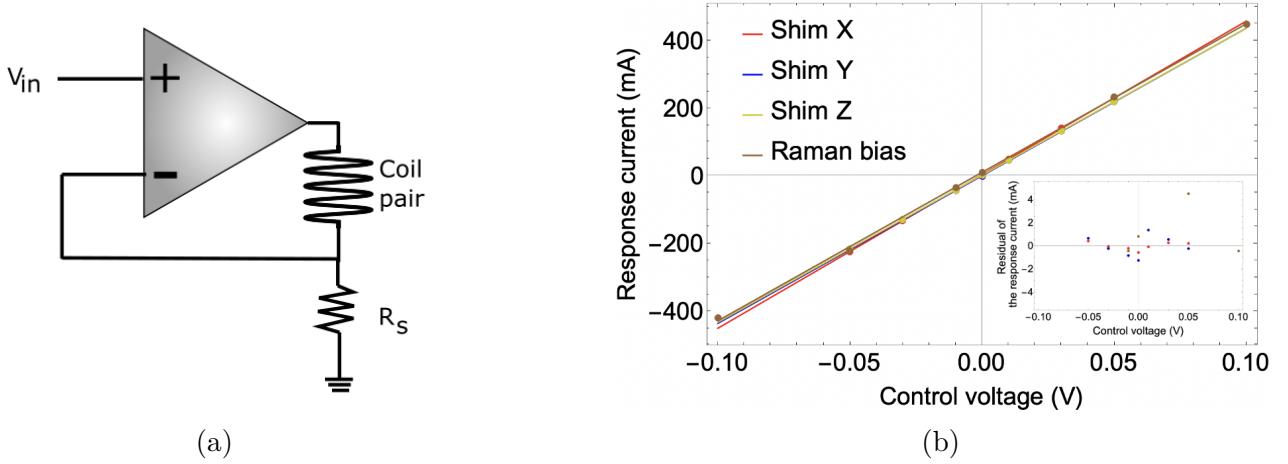


Figure 4.6: The current driver of the 3D MOT magnetic shim fields and the Raman magnetic bias field. (a) The current driver with an operational amplifier (OPA-549) for each coil pair. The sensing resistor, R_{sense} , is $0.22\ \Omega$. The maximum current has been set to 4 A with a $10\ \text{k}\Omega$ resistor (not shown in the graph). (b) Controlled-current response test for various bias coils. They all show the linear responses as the same as designed to be around $1/R_{\text{sense}}$, where fitting residuals were at mA-level. The continuous operation was below 1 A, beyond which power dissipation can result in significant heat to trigger its thermal protection.

The magnetic field generated by each coil pair is proportional to the current flowing through the coil. The stray magnetic field at the location of the atoms was compensated with shim X,Y,Z coil pairs by monitoring the Zeeman splitting in the microwave transitions. After compensation of the stray magnetic fields, the magnetic field generated by the Raman coil was tested on atoms using co-propagating Raman transitions shown in Fig. 4.7 (a). It displays an offset of 1 mV in the control voltage to produce a zero field crossing and was verified with a fine calibration shown in Fig. 4.7 (b). The fine calibration shows that a zero control voltage from

the computer produces a non-zero current. This is limited by the resolution and offset of the digital-to-analogue converter (DAC) in the analogue voltage generation device (PXI-6723).

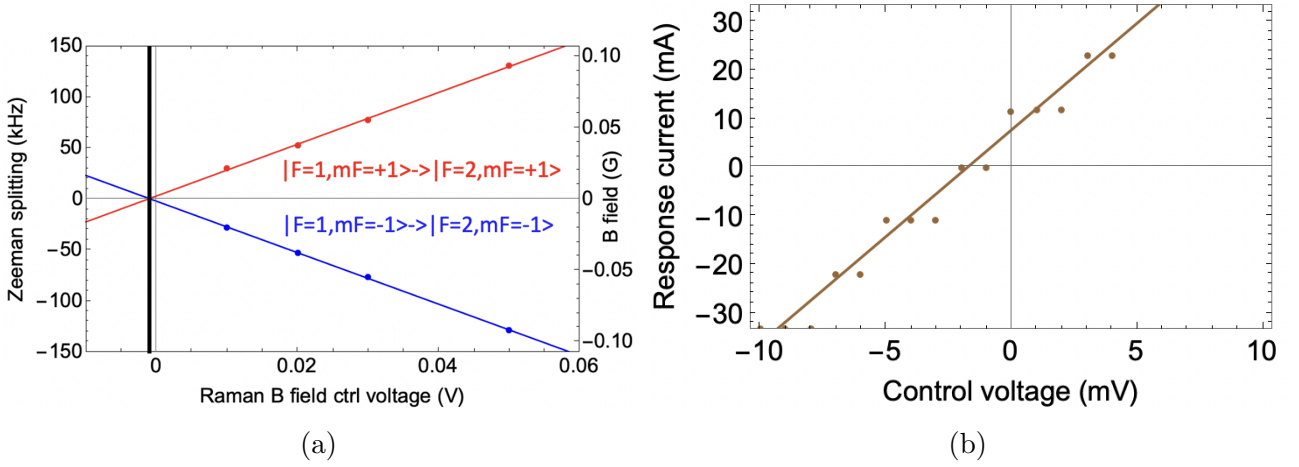


Figure 4.7: Calibrations of the Raman magnetic bias coil. Both calibrations show a 1 mV offset in the control voltage to produce a zero current generating a zero magnetic bias field along the Raman axis. (a) Calibration with the Zeeman splitting of ^{87}Rb atoms using co-propagating Raman transitions. The magnetic field was calculated with the Zeeman splitting sensitivity of $+0.7 \text{ MHz/G}$ per m_F for $F=2$ and -0.7 MHz/G per m_F for $F=1$ [8]. (b) Fine calibration of the response current.

4.3.3 2D MOT gradient field

The magnetic gradient field of the two-dimensional magneto-optical trap (2D MOT) was generated with two pairs of square coils arranged in the anti-Helmholtz configuration. The dimension of the square coil was (on average) $60 \text{ mm} \times 120 \text{ mm} \times 17 \text{ mm}$ as shown in Fig. 4.8 with wire of 1 mm diameter. To calibrate the number of turns, the y-component of the B field component is measured to be 7.1 G/A with a flux gate (Bartington, Mag-03MC100) at the centre of the upmost layer top coil. This is consistent with 63 turns using a numerical simulation of the Biot-Savart law. The simulation also gave a magnetic field gradient of 3.73 G/cm/A at the 2D MOT location. An optimal loading rate of 3D MOT from the 2D MOT was observed when the current is set to 4.3 A, which indicated the magnetic field gradient in the 2D MOT was 16 G/cm in both x and y directions for the optimal loading rate.

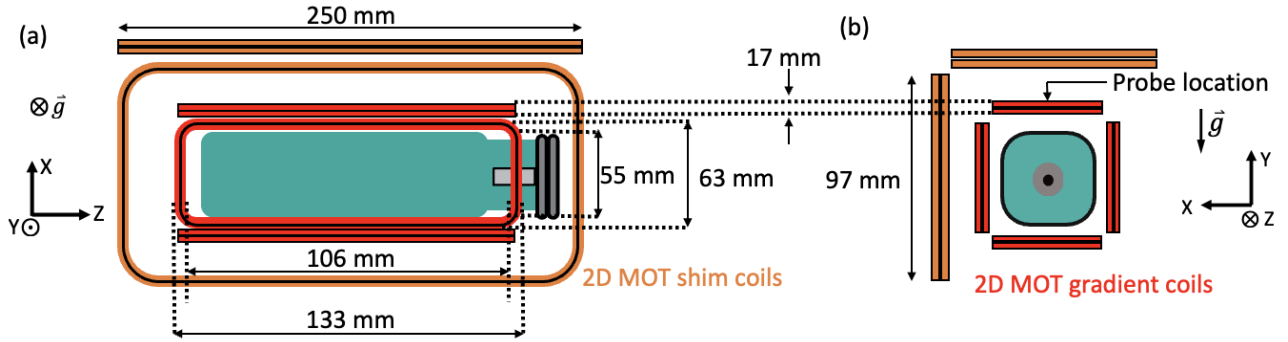


Figure 4.8: Geometry of the 2D MOT gradient coils (red) and 2D MOT shim coils (orange). (a) Top view. 2D MOT optics have been omitted to make the geometry of coils clear. (b) Side view. The location of the magnetic field probe to calibrate the number of turns has been indicated.

4.3.4 2D MOT shim field

The 2D MOT shim coils used a single square coil (rather than coil pair) of a dimension of about $97 \text{ mm} \times 250 \text{ mm}$, because of the limited space to add shim coils around the 2D MOT optics. These coils have not been carefully characterised. However, as shown in Fig. 4.9, sending $\approx 1 \text{ A}$ to the y-shim coil and $\approx 0.2 \text{ A}$ to the x-shim coil aligned the column of the trapped atoms to the centre of the differential pumping tube. This adjustment improved the loading rate by a factor of two as shown in the yellow curves of the inserted diagrams in Fig. 4.9.

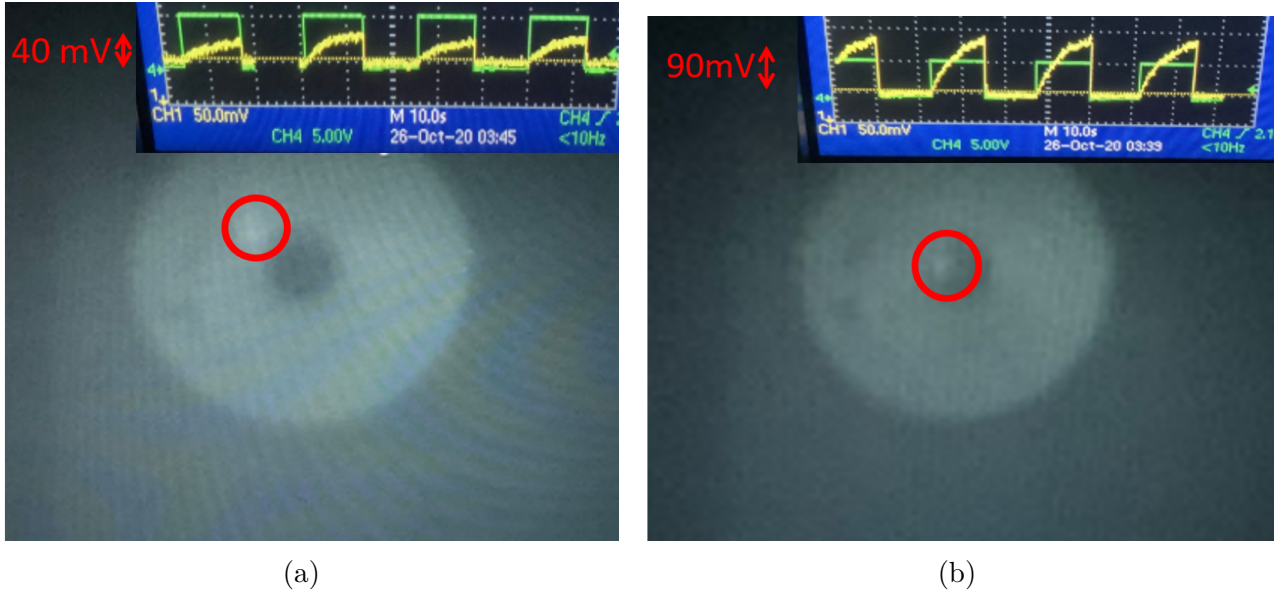


Figure 4.9: Fluorescence images of the 2D MOT with and without the shim coils. (a) 2D MOT relative to the differential pumping tube without shim coils. (b) 2D MOT relative to the differential pumping tube with shim coils. Using shim coils improved the 3D MOT loading rate by a factor of two, which is probed by a photodiode (yellow traces in the inset diagrams). The trigger to load a 3D MOT from the 2D MOT is shown as green TTL signals.

4.4 Laser system

We need various laser frequencies in this experiment to perform laser cooling, trapping, detection and interferometry as shown in Fig. 4.10. The laser frequencies used in this experiment are close to the transition frequency from $5^2S_{1/2}$ to $5^2P_{3/2}$. Their hyperfine states will be labelled as F for $5^2S_{1/2}$ states and F' for $5^2P_{3/2}$ states in this thesis. This section describes how the 2D MOT repump and cooling frequencies, f_r^{2D-MOT} and f_c^{2D-MOT} , were generated with a commercial laser system (μ Quans) in Section 4.4.1, and how a combination of home-built external-cavity diode lasers (ECDLs) and commercial laser systems (Toptica) was used to achieve the 3D MOT repump and cooling frequencies, f_r^{3D-MOT} and f_c^{3D-MOT} , in Section 4.4.2. The control of f_c^{3D-MOT} to achieve different detuning are detailed in Chapter 6 for various purposes, such as pushing a 2D MOT into 3D MOT, trapping atoms in a 3D MOT, sub-Doppler cooling in an optical molasses, state preparation with optical pumping, blow-away atom of undesired state and fluorescence detection. Details of Raman laser frequencies are discussed in Section 5.1.

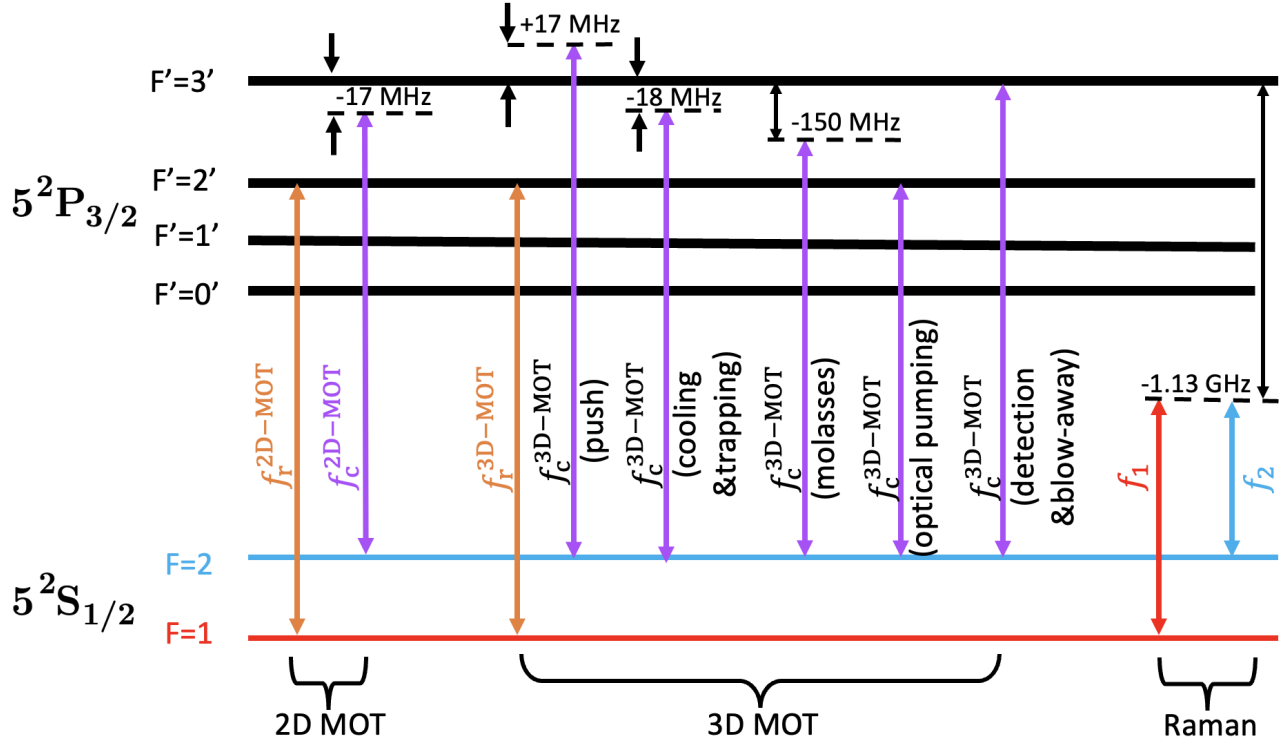


Figure 4.10: Overview of the laser system adapted from [2]. There are three laser systems to generate the frequencies used in this experiment. f_r^{2D-MOT} and f_c^{2D-MOT} are the repump and cooling frequencies to produce a 2D-MOT. f_r^{3D-MOT} and f_c^{3D-MOT} are the repump and cooling frequencies to produce a 3D-MOT. The 3D-MOT cooling frequency f_c^{3D-MOT} was varied during the sequence to achieve the frequencies needed for the push beam, Doppler-cooling and trapping in MOT, sub-Doppler cooling in optical molasses, optical pumping for state preparation, blowing away atoms in the undesired state and resonant detection. Two Raman frequencies, f_1 and f_2 , to drive the interferometer were detuned from $F' = 3'$ by -1.13 GHz.

4.4.1 2D MOT laser system

To produce a 2D MOT, two laser frequencies were used as shown in Fig. 4.10. One drove the cooling transition $F=2 \rightarrow F'=3'$, while the other drove the repump transition $F=1 \rightarrow F'=2'$. The laser system used to provide these two frequencies is a commercial system (μ Quans, ILS laser series). The system has four diode lasers, where the master and slave0 diode lasers were used for producing the 2D MOT light (cooling and repump) and the schematics are shown in Fig. 4.11. The other two diode lasers were used for Raman beams, which are discussed in Section 5.1 with more details.

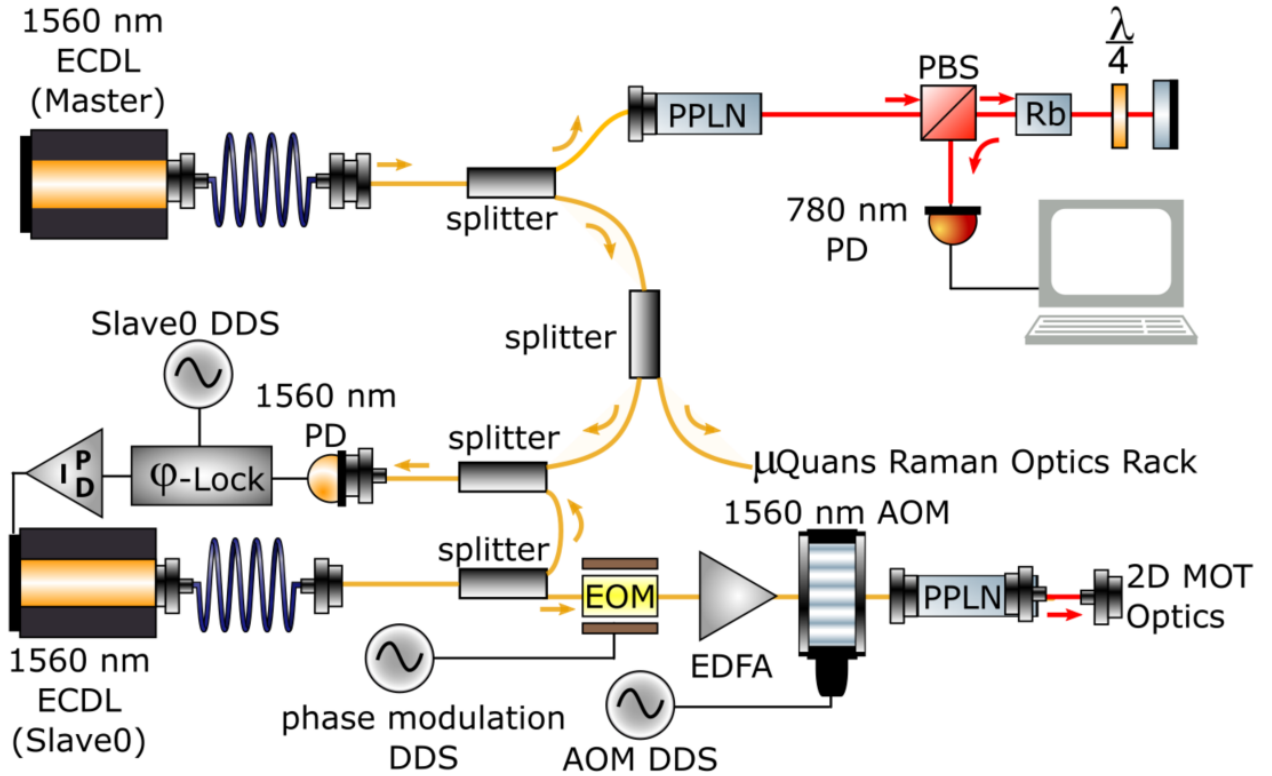


Figure 4.11: Schematics of the 2D MOT laser system. Master and slave0 diodes were operated at 1560 nm. The Master laser was frequency-doubled to 780 nm via a periodically-poled Lithium-Niobate waveguide (PPLN) and locked to a cross-over feature of ^{85}Rb ($F=3 \rightarrow F'=3',4'$) with saturation spectroscopy. Slave0 laser frequency was stabilised by phase locking the slave0-master beat note to a low-phase noise direct-digital synthesizer (slave0 DDS). An electro-optic modulator (EOM) generated sidebands to address the repump frequency.

Master diode laser

The μ Quans laser system has a 1560 nm diode laser that served as the master laser providing a frequency reference for the four μ Quans laser diodes (slave0 for 2D MOT cooling and repump light, slave1 and slave2 for Raman light). The diode temperature and current were manually tuned via the front panel of the μ Quans system, allowing coarse control of the laser frequencies near the lock points before locking them. Typically, the master diode was operated at 23.3 °C with the diode current set to 126 mA.

To operate at a useful frequency for Rb atoms, this master diode was frequency-doubled to 780 nm with a periodically-poled Lithium-Niobate (PPLN) waveguide. After this frequency doubling, the master diode was locked to saturated absorption spectroscopy of a rubidium vapour cell with modulation spectroscopy. The current was modulated and generated an error signal shown in Fig. 4.12 (b) with a lock-in detection.³ After a warming-up period of about 30 min, the saturation spectrum was observed in μ Quans software (*gPlot*) as shown in Fig. 4.12. Although this laser system was designed for laser cooling ^{87}Rb , the master diode locked to the ^{85}Rb cross-over resonance ($F=3 \rightarrow F'=3', 4'$) which has the largest signal-to-noise ratio. ^{85}Rb has a larger signal because of the greater natural abundance of the ^{85}Rb isotope. The cross-over resonance has a larger signal because two velocity groups are saturated rather than only one velocity group saturated in the transition resonance [10, p.160].

Cooling and repump lasers

The 2D MOT cooling and repump lasers were provided by the slave0 laser diode as shown in Fig. 4.11. Its output was split into two paths: the first path provided a seed for phase-locking the slave0 laser to the master diode at 1560 nm, and the second path supplied a seed for power amplification before coupling into the output fibre.

The phase-lock seed was combined with a pick-off from the master laser to generate a beat note on a fast photodiode. This beat note signal (at 1560 nm) was phase-locked to a tunable direct-digital synthesizer (slave0 DDS). The slave0 DDS frequency was set to 81.212 MHz to

³This is very similar to the modulation transfer spectroscopy in [9], where the difference was current was modulated for the master laser instead of using an EOM in [9].

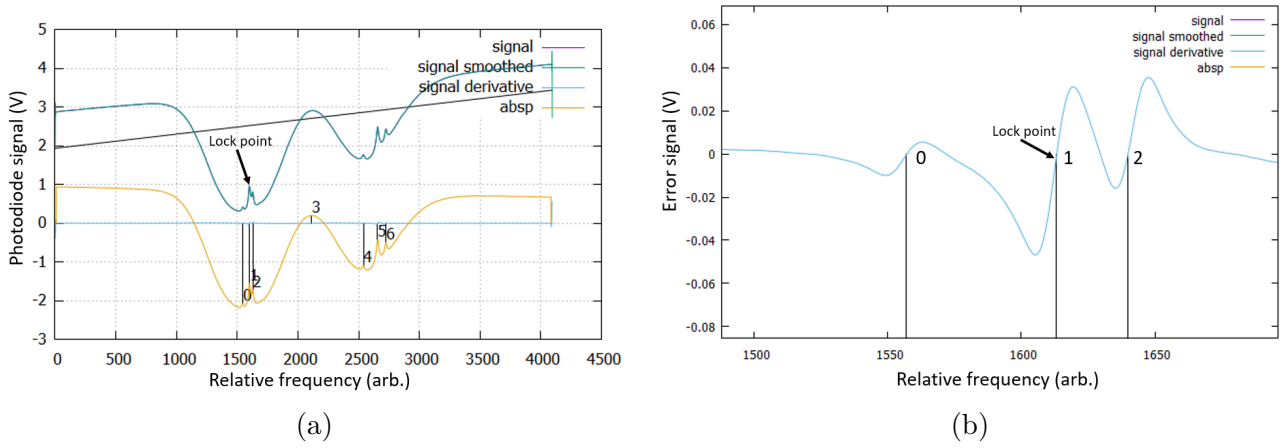


Figure 4.12: Absorption spectroscopy of an Rb vapour cell using the master diode laser. (a) The yellow curve is the raw saturation spectroscopy data for absorption measurement of the frequency-doubled master laser. The blue curve shows the error signal that is generated by frequency-modulation of the spectroscopy signal and is proportional to its derivative. The dark green curve is a saturation spectrum providing the same information as the yellow curve, but compensating for an estimated linear background that is shown in black. The diode was locked to the largest cross-over resonance of ^{85}Rb ($F=3 \rightarrow F'=3', 4'$). (b) Zoom-in view of the error signal near the lock point near the largest cross-over resonance of ^{85}Rb ($F=3 \rightarrow F'=3', 4'$).

lock the master-slave0 beat note (at 1560 nm) to 649.696 MHz (8×81.212 MHz).⁴ Since the AOM DDS frequency had been set to 110 MHz to get the maximum power, the lock-point of the cooling laser was changed by changing the slave0 DDS to optimise the loading rate in Section 6.1.1.

The seed for the power amplification went through an electro-optic modulator (EOM) to generate a sideband for repumping light by phase modulation. The phase modulation frequency was sourced from another internal direct-digital synthesizer (phase-modulation DDS). After passing through an Erbium-Doped Fiber Amplifier (EDFA), the cooling seed and repump sideband got an additional frequency shift by passing through an AOM at 1560 nm before frequency-doubling to 780 nm. The maximum power obtained from the fibre output was around 200 mW.

⁴The factor of 8 comes from the designed phase-lock loop.

4.4.2 3D MOT laser system

To produce a 3D MOT, frequency-stabilised cooling and repump laser light were distributed into six MOT beams. This section describes the diode lasers for addressing ^{87}Rb cooling and repump transitions as well as their frequency stabilisation strategies.

Frequency stabilisation of the reference laser via polarisation spectroscopy

For an optimal sub-Doppler cooling, ramping cooling laser over ≈ 140 MHz was reported to keep 90 % of the atoms trapped by MOT while further cooling below 10 μK [2, p.57,p.58]. A home-built external-cavity diode laser (ECDL) was constantly operated at the cooling transition frequency, to which the 3D MOT cooling laser was locked with a variable frequency offset (see more details in Section 4.4.2). This allowed us to detune the cooling laser frequency much further than the bandwidth of an AOM and also keep the power fixed.

The error signal to stabilise the reference laser frequency used the polarisation spectroscopy adapted from [11, 12].⁵ This adaption explored a retro-reflection geometry shown in Fig. 4.13 (a) to make it more compact. Scanning with only the piezo voltage, the reference ECDL showed a mode-hop-free range of more than 2 GHz by patiently tuning its temperature and current. This operation enabled us to observe the Rb D_2 lines of both isotopes as shown in Fig. 4.13 (b). A typical trace of the error signal near the ^{87}Rb cooling transition is shown in Fig. 4.13 (c).

The principle of the polarisation spectroscopy is to use a *circularly* polarised pump beam to pump the Rb atoms into the stretched magnetic state, which acts as a ‘Faraday rotator’ for another *linearly* polarised probe beam. Choosing the pump beam propagation vector ($\mathbf{k}$) as the angular momentum quantization axis, a pump beam of σ^+ polarisation optically pumped most of the atoms into $|F = 2, m_F = +2\rangle$. Interrogated by a *linearly* polarised probe beam, atoms in $|F = 2, m_F = +2\rangle$ display different refractive indices for the σ^+ and σ^- components of the probe beam. This circular birefringence leads to a rotation of the probe beam polarisation plane. The angle of rotation depends on the laser detuning from the $|F = 2\rangle \rightarrow |F' = 3'\rangle$ transition. This dependency gives an error signal for frequency stabilisation to the ^{87}Rb cooling transition.

⁵As a modulation-free technique to lock the laser to an atomic transition, its pros and cons compared against other methods are detailed in [13].

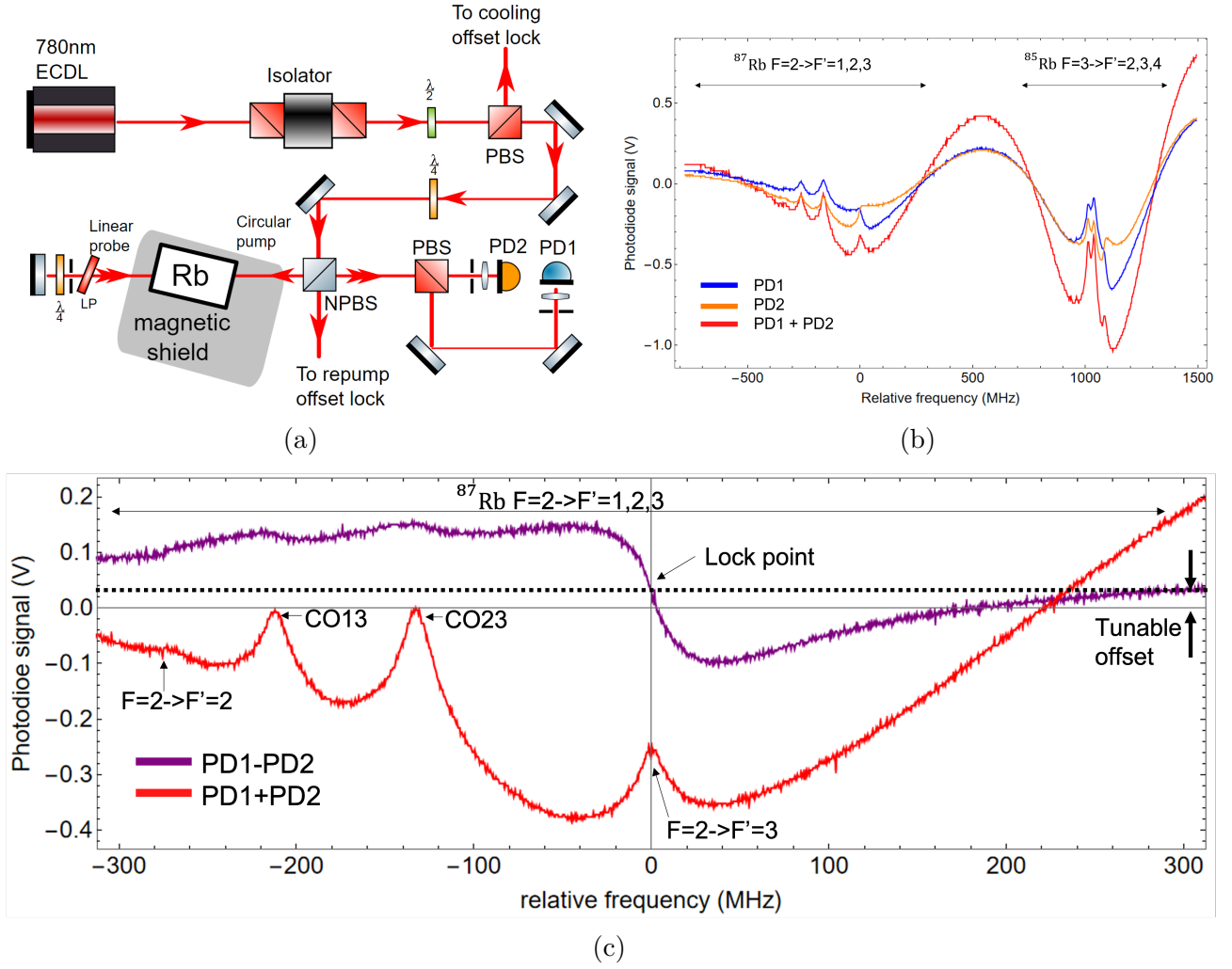


Figure 4.13: Polarisation spectroscopy of the reference laser. (a) Schematic of the optical set-up used to perform the polarisation spectroscopy of the reference laser. The reference laser was emitted from the ECDL. An optical isolator suppressed undesired external optical feedback to the reference ECDL. The half-waveplate ($\lambda/2$) and a polarising beam splitter (PBS) after the isolator controlled the power into the polarisation spectroscopy module. The quarter-waveplate ($\lambda/4$) before the 50:50 non-polarising beam cube (NPBS) produced a circularly polarised pump beam. Double passing a linear polariser (LP) and a quarter-waveplate after retro-reflection, the pump beam became a linearly polarised probe beam. It went into a balanced polarimeter made of a PBS and two photodiodes (PD). The iris before photodiodes reduced the room light background. The lenses focused down the beam to minimize the impact of beam path drift. The iris and tilted optical surfaces along the probe beam reduced the undesired reflection that goes into the balance polarimeter. (b) Polarisation spectroscopy of Rb D-II lines. The blue curve and orange curve illustrate traces detected by PD1 and PD2, the sum of which was the conventional saturation spectroscopy [14]. (c) Zoom-in polarisation spectroscopy of ^{87}Rb cooling transition. The purple curve is the polarisation spectroscopy signal, which was the difference between the two photodiode signals. Noting that this Doppler-free signal has a large capture range of more than 400 MHz, which makes this locking scheme robust to accidental bumps to the optical table. The linear part was used to lock the reference laser. The offset was tuned by changing the relative orientation between the LP of the probe beam and the PBS of the balanced polarimeter. Tuning the offset lead to a lockpoint at the ^{87}Rb cooling transition, which was calibrated by saturation spectroscopy (red curve).

Specifically, as shown in Fig. 4.13 (a), our circular pump laser was obtained from the reference ECDL passing through a polarising beam splitter (PBS) and a quarter waveplate (QWP). This circularly polarised pump beam ($\approx 24 \text{ mW/cm}^2$) went through a shielded Rb vapour cell to create the anisotropy in the atomic population of $F=2$ by optical pumping. After retro-reflection and double passing a linear polariser (LP) and another QWP, the circularly polarised pump beam became our linearly polarised probe beam ($\approx 0.1 \text{ mW/cm}^2$). The relative orientation between the QWP and the LP determined the probe beam intensity. This probe beam was then detected by a polarimeter made of a PBS and two photodiodes (PD). As illustrated in Fig. 4.13 (c), the difference between these two photodiode signals produced the polarisation spectroscopy signal, while the sum gave the conventional saturation spectroscopy signal [14] for frequency calibration.

To understand our polarisation spectroscopy signal, we can look at two laser detuning regimes. At far detuning from the cooling transition, the probe laser polarisation experienced a negligible rotation, thus, the two photodiode signals become very similar and their difference was independent of the frequency. Near the atomic transition, the probe beam obtained a strong frequency-dependent rotation due to the sharp change in the refractive index around the resonance. This lead to a significant difference between the two photodiode signals, thus, the polarisation spectroscopy had a dispersive shape around the cooling transition. The flat tail could have a non-zero offset, which was determined by the relative orientation between the probe beam LP and this polarimeter PBS. To lock our reference laser to the cooling transition frequency, it was necessary to tune the offset to make the zero-crossing coincide with the lock point that was calibrated by the saturation spectrum. The error signal was asymmetric about the cooling transition. This complexity arises from non-closed transitions and cross-over resonances, which are explained in [15].

Cooling laser

A commercial diode laser (Toptica TA pro) was used to address the ^{87}Rb cooling transition $|F = 2\rangle \rightarrow |F' = 3'\rangle$. As shown in Fig. 4.14, the laser light was split for spectroscopy (spectroscopy module), frequency stabilisation (offset-lock module) and the experiment (Master-Oscillator Power Amplification module), which are described individually below.

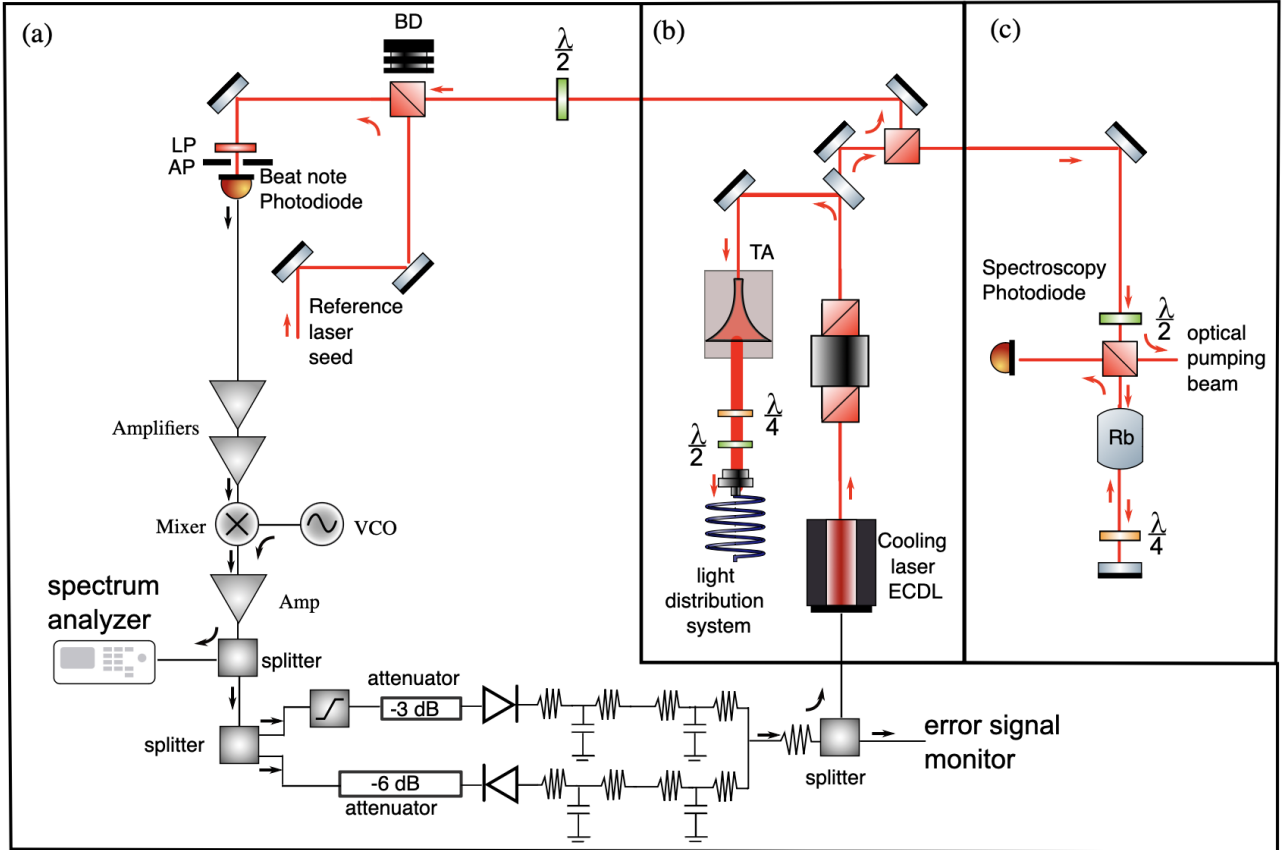


Figure 4.14: Schematics of the laser system to provide 3D MOT cooling light. (a) Offset-lock module. It consists of an optical part to produce a beat signal by overlapping pickoff of reference laser and cooling laser on a photodiode, and an electrical part that locks the beat signal frequency. (b) Master-oscillator power amplifier (MOPA) module. It consists of a tapered-amplifier (TA) to amplify the cooling laser up to 1.6 W, and a home-made polarisation adjuster (a quarter-waveplate and a half-waveplate on rotational mounts) for coupling the amplified cooling light into a polarisation-maintaining fibre. (c) A saturation spectrometer to monitor the cooling laser frequency, which is used to calibrate the error signal for locking the cooling laser.

The spectroscopy module is shown in Fig. 4.14 (c). The light used for the spectroscopy had a horizontally linear polarisation and double-passes through an Rb vapour cell. The retro-reflected component changed its linear polarisation from horizontal to vertical after double passing through a quarter-waveplate. This vertically polarised component served as a probe beam and

produced saturation spectroscopy [10, p.155] on a photodiode. This provided the sub-Doppler resolution to observe the hyperfine splittings and the cross-over resonances, which are shown in Fig. 4.15.

The offset-lock module shown in Fig. 4.14 (a) received 300 μ W from the MOPA module and was combined with a reference seed of 2 mW to generate a beat signal detected by a photodiode (Newport, 818-BB-21A). The beat signal power was optimised by changing the orientation of the linear polariser before the beat detection photodiode to have sharp error signals shown in Fig. 4.15. Since the reference laser frequency has been stabilised with the polarisation spectrum, stabilizing the cooling-reference beat frequency makes the cooling laser frequency stable. Considering the MOT cooling laser is switched on/off by a double-pass AOM (AA Opto-Electronics, MT110-A1-IR) with an optimal deflection efficiency at 110 MHz, we choose to stabilise the beat frequency to 240 MHz. Thus the optimal cooling power can be delivered at -20 MHz red-detuned ($-240 \text{ MHz} + 2 \times 110 \text{ MHz}$) from the cooling transition. Stabilizing the beat frequency $f_{\text{beat}} = |f_c^{3\text{D-MOT}} - f_{\text{ref}}|$ to 240 MHz was achieved by mixing the photodiode signal with a local oscillator (LO) before sending it through a frequency discrimination circuit (FDC) adapted from [16]. As shown in Fig. 4.15, a high-pass filter (Minicircuit, PHP-100+) of a 3dB attenuation at $f_{3\text{dB,HP}} = 82 \text{ MHz}$ and a voltage-controlled oscillator (Minicircuits, ZOS-300+) operating at $f_{\text{LO}} = 158 \text{ MHz}$ were used to produce sharp features at the condition $|f_{\text{beat}} - f_{\text{LO}}| = f_{3\text{dB,HP}}$, when the filtered branch and unfiltered branch had the same power. This resulted in four sharp features at $f_c^{3\text{D-MOT}} - f_{\text{ref}} = \pm f_{\text{LO}} \pm f_{3\text{dB,HP}}$ shown in Fig. 4.15. Other zero-crossings were due to harmonics, which can be suppressed with additional filters suggested in [17, p.75]. To distinguish all available zero-crossings and choose one of them to lock our laser, we display the FDC error signal together with the saturation spectroscopy feature as illustrated in Fig. 4.15. To lock to the correct zero-crossing, one has to decide if an inverter is needed and reduce the scan range within the capture range before feeding it back to the laser piezo controller.

The MOPA module shown in Fig. 4.14 (b) needs the seed from the cooling laser ECDL aligned with the tapered-amplified (TA) to produce the maximum power to the light distribution system in Fig. 6.1. The gain of amplification was determined by the current, which was

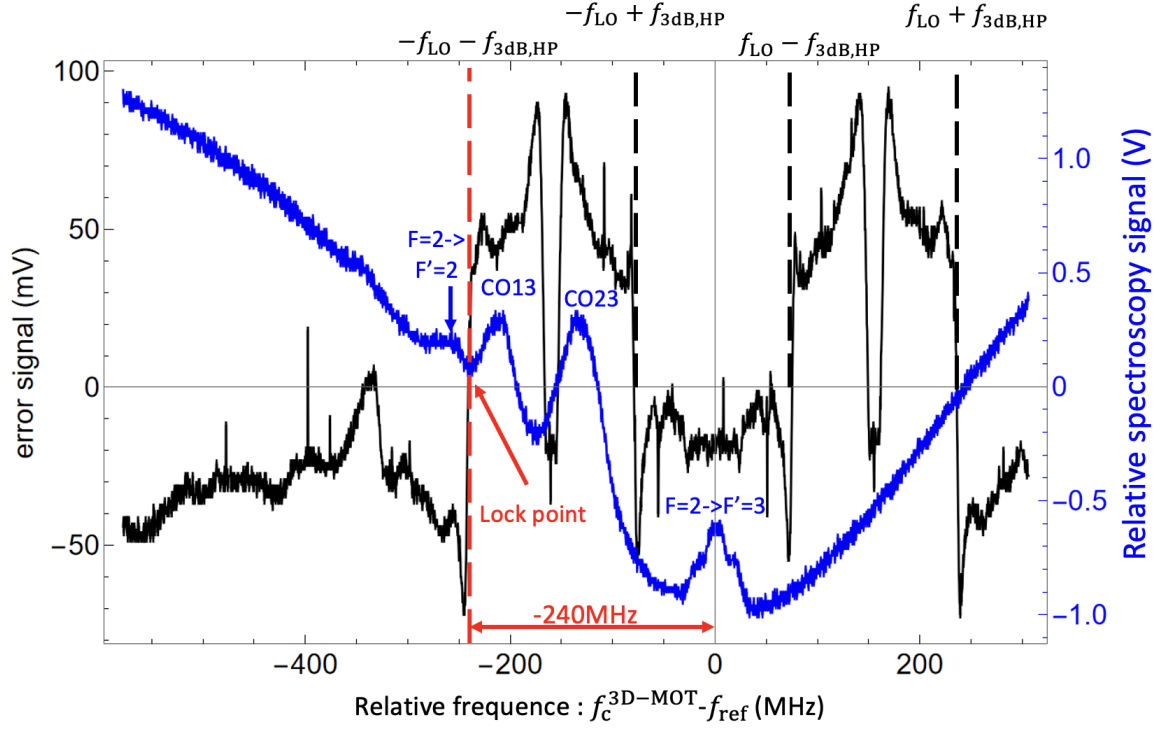


Figure 4.15: Error signal to lock cooling laser. The error signal (black) was generated by a frequency-discrimination circuit (FDC) adapted from [16]. The cooling laser lock point (indicated in red) was calibrated with its saturation absorption spectroscopy (blue), which was 240 MHz red-detuned from $F=2 \rightarrow F'=3'$ cooling transition.

controlled by the Toptica controller (Toptica, DCC-110: 5000 mA). It was operated at 2823 mA to produce 1.6 W at the input of the cooling laser fibre and delivered 350 mW at its output. This overall 20% coupling efficiency was optimized by walking the fibre dock alignment mounted on the TA Pro. It was limited by the transverse spatial mode matching between the diode output mode (non-Gaussian) and a single fibre coupling mode (Gaussian). We checked this assumption by sending some 780 nm light back propagated from the target fibre. The laser mode after TA and the back-propagated fibre mode were not very well overlapped. A single-mode fibre (780HP) was used to maintain the polarisation. The input polarisation was aligned with a homemade polarisation adjuster consisting of a quarter-waveplate and a half-waveplate on the rotational mounts. The quarter-waveplate compensated the ellipticity due to birefringence and the half-waveplate then aligned the compensated linear polarisation with respect to the polarisation-maintaining axis of the fibre. This operation optimised the polarisation stability monitored by a commercial polarimeter (Thorlab, PAX1000IR). More details on aligning polarisation-maintaining fibre are provided in Appendix B.

Repump laser

As shown in Fig. 4.16, the repump laser was seeded from a home-built ECDL before being split into two modules. The frequency stabilization locked the reference-repump beat signal to a VCO, while the amplification module produced approximated 100 mW repump (more than being adequate) to be distributed into MOT beams.

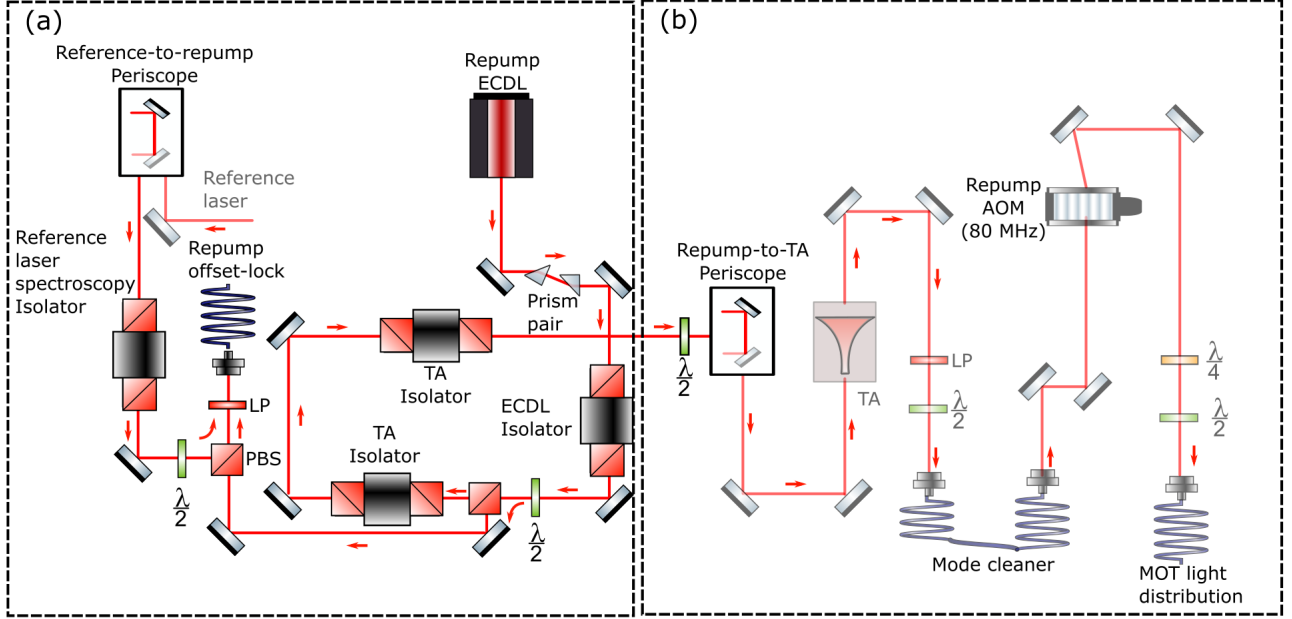


Figure 4.16: The optical layout of the repump laser. (a) Frequency stabilisation module. A pair of prisms was used to adjust the spatial mode of the repump laser. The ECDL isolator suppressed undesired optical feedback into the repump ECDL. After the ECDL isolator, a half-waveplate (HWP) and a polarising beam splitter (PBS) controlled the power that went into the repump offset-lock module and power amplification module. Because the repump laser optics had a slightly different height from that of the reference laser optics, the reference laser periscope lifted the reference seed laser to beat with the repump laser. (b) Power amplification with a tapered amplifier. The repump-to-TA periscope worked in the same way as the reference-to-repump periscope. It lowered the height of repump seed before sending it to a tapered amplifier (TA) for power amplification.

Its frequency stabilization scheme has the same principle as locking the cooling laser. As shown in Fig. 4.16 (a), 50 μ W reference laser was lifted to the same height as the repump laser with the reference-to-repump periscope. After combining with 380 μ W repump laser into the repump offset-lock fibre, they were delivered to an ultrafast photodiode (Thorlab, DX12CF) responding to 6.8 GHz. Then this beat signal was sent to a FDC similar to the one shown in Fig. 4.14 (a) used for the cooling laser. The major difference was that the frequency of this repump-reference beat signal was stabilised to 6.487 GHz. Then an 80 MHz AOM (Gooch &

Housego, M080-2B/F-GH2) in Fig. 4.16 (b) shifted up the repump laser frequency to address the $|F = 1\rangle \rightarrow |F' = 2'\rangle$ transition. The local oscillator was operated at 6.450 GHz (Minicircuits, ZX95-6640C-S+), while a high-pass filter (Minicircuits, PHP-50) was chosen to have a 3dB attenuation at 37 MHz. Other cautionary design details can be found in [3].

Scanning the repump laser piezo control voltage generated an error signal as shown in Fig. 4.17. The asymmetry in the error signal in a fast scan indicated some unknown relaxation process in the system of a time scale of 1 ms. This relaxation behaviour disappeared when the piezo was scanned at a slower rate. Ultimately, this 1 ms relaxation may limit the bandwidth of the feedback loop and the frequency stability of the repump laser.

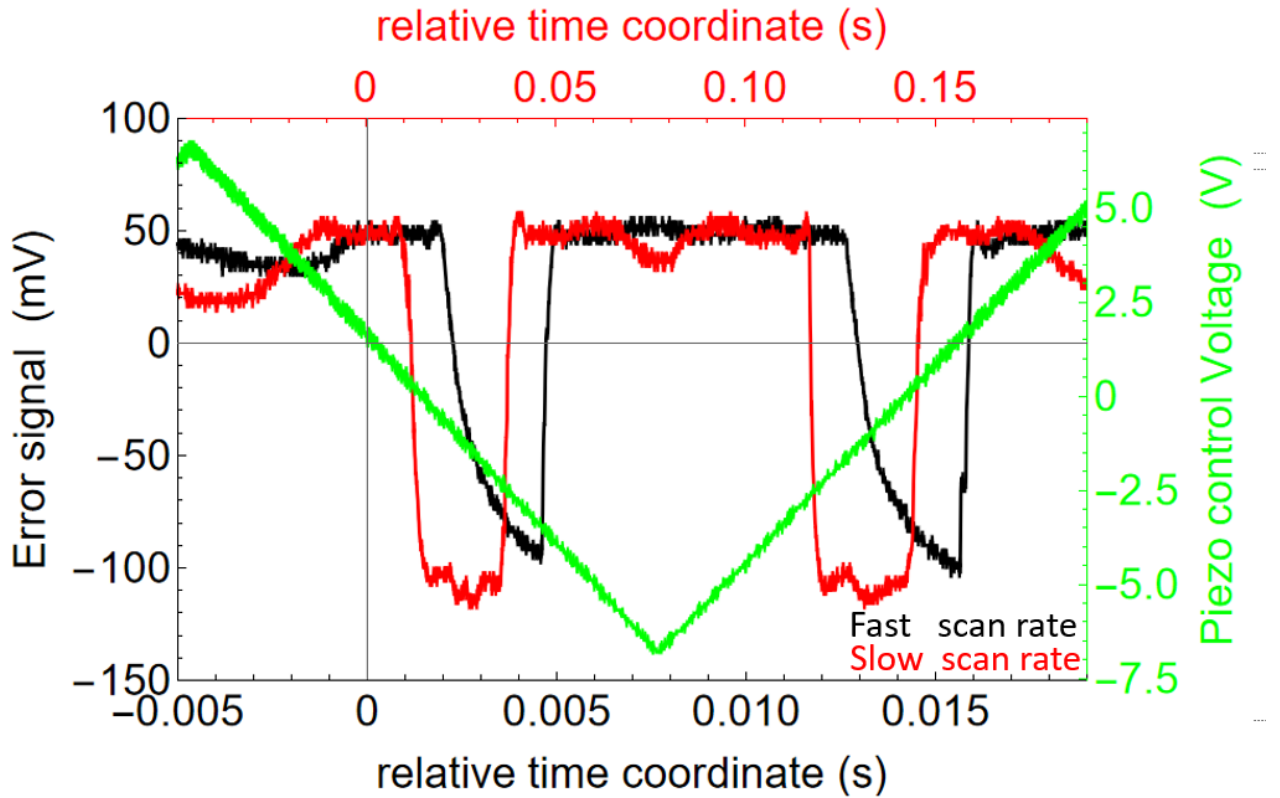


Figure 4.17: Repump Error signal as scanning the piezo. As the piezo voltage (green) was scanned between -7 V to +7 V at two scanning rates, the error signal displayed different symmetry. The lack of symmetry at a fast scan rate (black), compared to the symmetry at a slow scan (red), indicates a relaxation time scale of about 1 ms.

The power amplification module of the repump laser shown in Fig. 4.16 (b) had a seed power of 4 mW. To deliver enough power into the 3D MOT light distribution optics, a tapered amplifier (Toptica BoosTA) was used. The amplification gain was sensitive to the alignment and polarisation of the seed. The alignment was optimised with the steering mirrors between

the periscope and the TA, while the polarisation was optimised with the orientation of the half-waveplate before the TA periscope. With a TA current of 2100 mA, the repump seed of 4 mW was amplified to 470 mW, which was reduced to 400 mW at the input of a polarisation-maintaining (PM) fibre after cleaning its polarisation with the linear polariser (LP). This single-mode PM fibre cleaned the diode laser mode to be Gaussian mode with a typical coupling efficiency of 17% (70 mW at the output of this mode-cleaning fibre). Then 50 mW was deflected by a single-pass AOM of 80MHz that switched on/off the repump light. Coupling into the fibre, 15 mW repump light was delivered into the MOT light distribution system.

4.5 Microwave

Microwaves of around 6.8 GHz (to drive transitions between $|F = 1\rangle$ and $|F = 2\rangle$) were extensively used in our experiment. It was helpful for preparing the state of atoms with adiabatic passage Section 6.3.2 and monitoring the AC stark shift due to Raman light Section 7.1. This section presents the technical details.

There were two microwave (MW) sources in this experiment. One of them (Windfreak technologies, SynthHD) accepted a level-sensitive trigger (not an edge-sensitive trigger) to update its frequency within one shot of the sequence. This triggerable MW source was mainly used for the state preparation using the adiabatic passage, because the microwave frequency had to be toggled between two frequencies as shown in Fig. 6.22. The other microwave source (a built-in DDS from the μ Quans lasers system) had a significantly lower phase noise,⁶ which was mainly used to perform microwave Ramsey spectroscopy to study the noise of the detection system at the early stage (not presented in this thesis). Both microwave sources can be used to drive Rabi oscillations and perform Rabi spectroscopy to calibrate their frequencies.

As shown in Fig. 4.18 (a), the power of the WindFreak microwave source was controlled by a switch for 6.8 GHz (Hewlett-Packard, SPST 100 MHz-18 GHz 33142A) whose variable attenuation was up to 57 dB. The power control of the μ Quans microwave source was achieved by a VVA and switched on/off via a TTL switch (both are inside the μ Quans laser rack). The

⁶The frequency can only be changed from shot to shot via sending a command through an Ethernet cable. But it is also possible to become triggerable with extra development with μ Quans.

total maximum isolation with the VVA and the TTL switch was 70 dB. We selected one of them via another two-way switch (ZYSWA-2-50DR) that delivered the microwave into a 20 dB amplifier (Qorvo, QAP1019EVB, output power up to 10 W) before illuminating the atoms via a microwave horn shown in Fig. 4.18 (b).

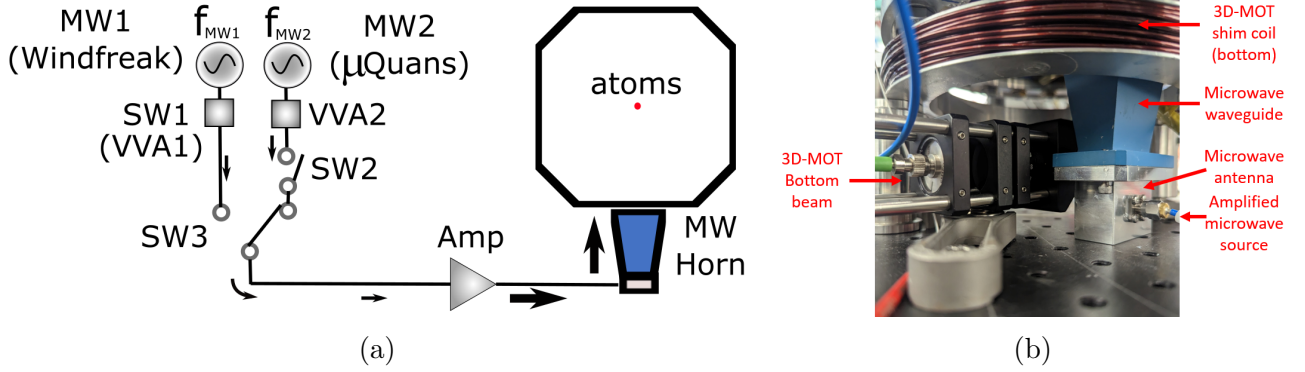


Figure 4.18: Schematics of the equipment for the microwave system. (a) Schematics of the electronic control system. The two-way switch (SW3) controlled the selection of microwave sources. Each microwave source had an additional separate switch. The switch for the Windfreak microwave source (SW1) was controlled by an analogue voltage, which effectively worked as a voltage-variable attenuator. A TTL signal controlled the switch for the μ Quans microwave source (SW2), while the voltage-controlled attenuator (VVA2) controlled its power with an analogue voltage. (b) A photo of the microwave horn. The horn was placed next to the 3D MOT upwards beam (at the bottom of the science chamber), which was offset from the centre. The microwave horn consisted of an antenna module (silver) and a waveguide (blue).

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Chapter 5

Interferometry system

Our interferometry system consisted of three parts: Raman laser system, Raman optics, and the vibration compensation system. I will describe how our Raman lasers were generated with a commercial laser system in Section 5.1 along with a summary of their frequency stabilisation strategy. Section 5.2 presents the optical layout used to drive velocity-sensitive transitions with counter-propagating Raman beams. As an acceleration-sensitive interferometer, it is inevitably sensitive to the vibration acceleration in our experiment. Section 5.3 describes how a crystal pendulum accelerometer was useful to suppress the impact of vibration on our chameleon measurement.

5.1 Raman lasers

As shown in Fig. 5.1 (a), our Raman beams were produced by two external-cavity diode lasers (ECDLs), slave1 and slave2, of the μ Quans laser system, which operated at 1560 nm. Both 1560 nm ECDLs were amplified and frequency doubled to 780 nm with periodically-poled Lithium Niobate (PPLN) crystals before being delivered to the atoms. The slave1 laser was frequency doubled to 780 nm (Raman-I) to produce one of the two Raman frequencies, f_1 , while the frequency-doubled slave2 laser produces the other frequency (Raman-II), f_2 . As shown in Fig. 5.1 (b), Raman-I coupled $|F = 1, m_F = 0\rangle$ to the P states and Raman-II coupled $|F = 2, m_F = 0\rangle$ to the same P states. The total output Raman power was controlled by the Raman AOM, while their individual power (and power ratio) was controlled by the corresponding Erbium-doped fibre

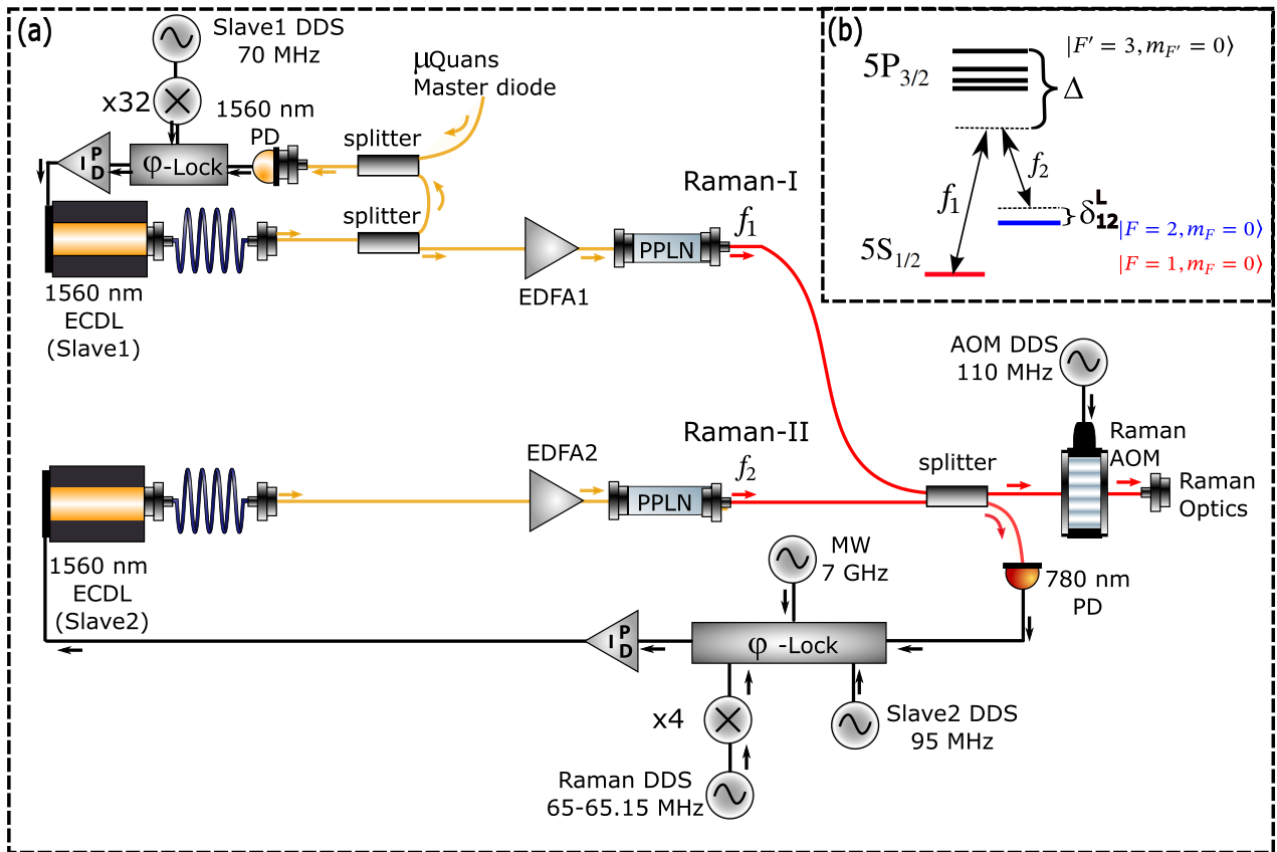


Figure 5.1: Schematic layout of the Raman lasers. (a) Raman frequencies generation. Two external-cavity diode lasers (ECDLs) at 1560 nm were amplified with Erbium-doped fibre amplifiers (EDFAs) and frequency-doubled using periodically-poled Lithium Niobate (PPLN) crystals to generate the two Raman frequencies, f_1 and f_2 , at 780 nm. Fibres for 1560 nm lasers are marked in orange, while fibres for 780 nm lasers are marked in red. (b) Raman frequencies relative to the ^{87}Rb atomic structure. The single-photon detuning, Δ , is relative to $|F' = 3', m_{F'} = 0\rangle$, and the two-photon Raman laser detuning, δ_{12}^L , is relative to the microwave transition frequency between $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$.

amplifiers (EDFAs). In our experiment, the power of the Raman-I laser was set to 109 mW (peak intensity of 283 W/m²) and the power of the Raman-II laser was set to 376 mW (peak intensity of 976 W/m²). This power setting approached the magic ratio for the 3-beam configuration where the retro-reflected Raman-II was rejected (more discussion in Section 7.1). The polarisations of the two Raman lasers were linearly polarised and orthogonal to each other. The output powers of the EDFAs were monitored and stabilised with an internal feedback loop.

Controlling these two Raman frequencies to have a proper single-photon detuning, $\Delta = -1.13$ GHz, and a tunable two-photon Raman detuning, δ_{12} , over a few MHz is important to operate an atom interferometer. The schematic of the frequency chain to lock the two Raman lasers is illustrated in Fig. 5.1, while the resultant Raman frequencies are summarised in Fig. 5.2.

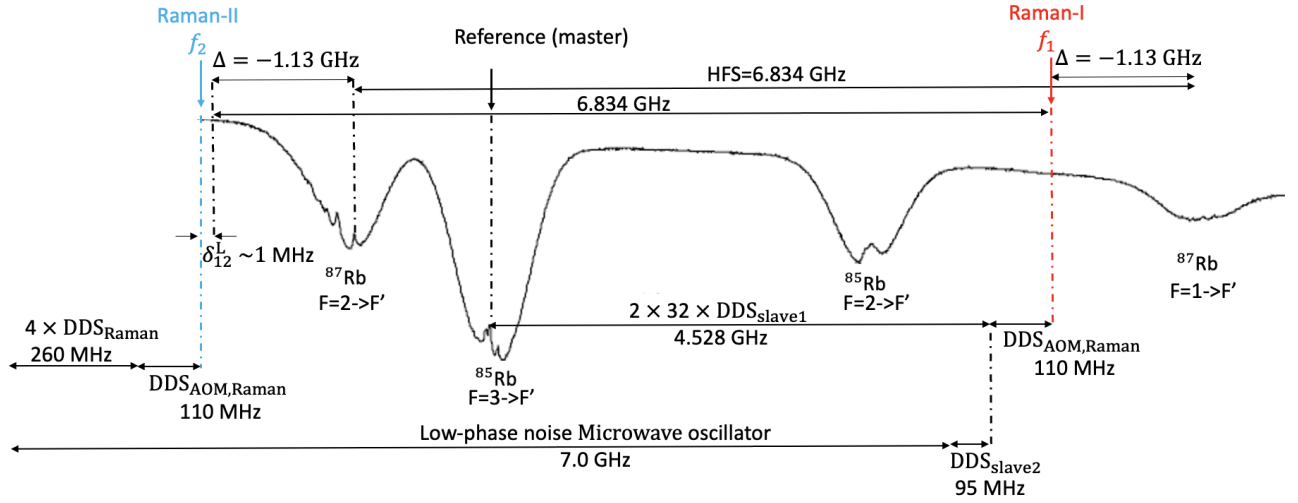


Figure 5.2: Lock points of the Raman lasers relative to Rb transition frequencies. Frequency chains used to generate relevant frequencies are shown at the bottom of the Rb saturation absorption spectroscopy adapted from [1], while the frequencies of the Raman lasers relative to the atomic frequencies are indicated at the top of the diagram. The hyperfine splitting (HFS) between the ground states of ^{87}Rb is 6.834 GHz. The single-photon detuning, Δ , was tuned by the slave1 DDS frequency, where the factor of 32 comes from the design of the internal phase-lock loop and the factor of 2 comes from frequency doubling. The two-photon Raman laser detuning, δ_{12}^L defined in Eq. (2.21), was tuned by the Raman DDS frequency.

The slave1 diode was locked to the master laser diode of the μQuans laser system (described in Section 4.4.1) by phase-locking their beat note at 1560 nm to a local oscillator (slave1 DDS). Adjusting the slave1 DDS frequency allows tuning of the single-photon detuning, Δ , of the Raman lasers. The slave2 diode was locked to the slave1 diode by phase-locking their beat note at 780 nm to another more complicated local oscillator (LO). This complicated LO was derived by mixing upwards a microwave oscillator at a fixed frequency of 7.0 GHz with the slave2 DDS which is constantly tuned to be 95 MHz. Then this 7.095 GHz LO was further mixed downwards with the Raman DDS, whose frequency is tuned around 65 MHz. Varying the Raman DDS frequency by 1 MHz changes the two-photon Raman detuning, δ_{12}^L defined in Eq. (2.21), by 4 MHz because of the internal multiplication shown in Fig. 5.1. Typical error signals for locking the slave1 and slave2 lasers can be found in the user manual.

5.2 Raman optics

Our experimental arrangement to drive $\sigma^+ - \sigma^+$ Raman transitions is shown in Fig. 5.3. The incoming two Raman lasers, labelled as Raman I and Raman II, have frequencies f_1 and f_2 with intensities I_1 and I_2 in the lin $\perp$ lin polarisation configuration. They were delivered to the atoms via the same fibre and expanded to a beam of 15.6 mm radius (e^{-2} intensity) with the same telescope (Schafter+Kirchhoff, 60FC-T-4-M200-37). After going through the first quarter waveplate, the polarisation of the Raman I laser becomes left-handed circularly polarised, while the polarisation of the Raman II laser became right-handed circularly polarised. This suppressed the co-propagating transition from $|F = 1, m_F = 0\rangle$ to $|F = 2, m_F = 0\rangle$, because the Raman transition between these two states needs Raman beams of the same circular polarisation. After doubly passing the second quarter waveplate and a high-extinction (30dB) Polarising Beam Splitter (PBS), the polarisation of the retro-reflected Raman I laser became the same as the polarisation of the incoming Raman II laser. This pair of counter-propagating laser fields drove the velocity-sensitive transition between $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$.

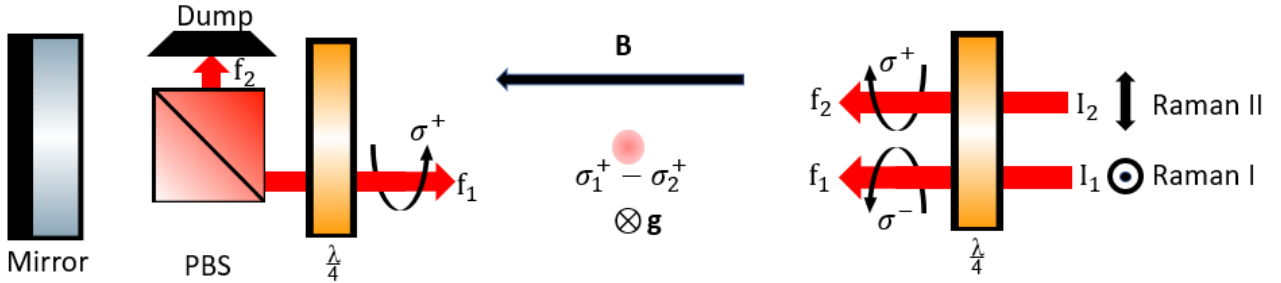


Figure 5.3: Top view of the experimental arrangement to drive the velocity-sensitive Raman transition. The incoming two Raman lasers, Raman I and Raman II, were overlapped. Here they are displayed separately for clarity. They had independent power controls to have different intensities, I_1 and I_2 . Their initial polarisations were both linear but orthogonal to each other. Passing through a quarter waveplate of a carefully chosen orientation made them into opposite circular polarisations. Passing the second quarter waveplate converted them back into orthogonal linear polarisations. The orientation of this second quarter waveplate was chosen to reject the Raman II laser with the polarising beam splitter (PBS). As a result, the polarisation of the reflected Raman I laser became the same as the incoming Raman II laser. This counter-propagating laser pair of the same circular polarisation drove the clock transition between $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ in the presence of a magnetic bias field, B , which lifted the degeneracy of the Zeeman substates. The direction of gravity, g , is also shown.

Meanwhile, the high-extinction PBS strongly suppressed the power of the retro-reflected Raman II laser delivered to the atoms. This operation reduced the chance of driving a dual-

interferometer of a complicated Rabi oscillation [2] because of the degenerate Raman transitions, which lead to a low-contrast interferometer [3, 4]. However, this rejection left the incoming Raman II laser beam unpaired, which modifies the magic ratio considering AC stark shift due to two laser beams ($I_1 : I_2 = 0.583$) to become $I_1 : I_2 = 0.292$, which consider AC stark shifts arising from three laser beams. This motivated us to probe the total differential light shift experimentally after setting the power ratio roughly equal to $I_1 : I_2 = 0.292$, which is discussed in Section 7.1.

5.3 Vibration suppression system

During each shot of the interferometer sequence, our acceleration-sensitive atom interferometer responded to not only the constant acceleration of the ball but also the dynamic vibration of the retro-reflection mirror. Therefore, a classical accelerometer (Innalab, AI-Q-2120) was used to track the motion of the retro-reflection mirror. This section describes how we used this information to isolate the vibration acceleration from our atom interferometer measurement.

5.3.1 Atom interferometer sensitivity to a time-dependent acceleration

The atom interferometer sensitivity to a constant acceleration has been shown as $\underline{k}_{\text{eff}} \cdot \underline{a}T^2$ in Eq. (3.7). This result is valid for the acceleration due to the in-vacuum ball, but not exact for the mirror vibration, because vibration acceleration was not constant during the total interferometer time $2T = 66$ ms. This section summarises the key results of the sensitivity to a time-dependent acceleration [5], and provides a straightforward interpretation inspired by [6].

The key result is the phase difference between the two arms of the atom interferometer, $\Delta\Phi_a$, due to the vibration of the non-inertial reference frame, $\underline{a}(t)$, during the interferometer is [5]

$$\Delta\Phi_a = \underline{k}_{\text{eff}} \cdot \int_{-\infty}^{+\infty} f(t)\underline{a}(t)dt, \quad (5.1)$$

where $\underline{k}_{\text{eff}} = \underline{k}_1 - \underline{k}_2$ is the effective wave vector for Raman lasers with individual wave vectors

k_1 and k_2 , while the sensitivity function, $f(t)$, is given by

$$f(t) = \begin{cases} \frac{1}{\Omega_{\text{eff}}^{(1)}} \left(1 - \cos(\Omega_{\text{eff}}^{(1)} t) \right) & 0 < t \leq \tau_{\pi/2}^{(1)}, \\ -\tau_{\pi/2}^{(1)} + \frac{1}{\Omega_{\text{eff}}^{(1)}} + t & \tau_{\pi/2}^{(1)} < t \leq \tau_{\pi/2}^{(1)} + T, \\ T + \frac{1}{\Omega_{\text{eff}}^{(1)}} + \frac{1}{\Omega_{\text{eff}}^{(2)}} \sin\left(\Omega_{\text{eff}}^{(2)}(t - T - \tau_{\pi/2}^{(1)})\right) & \tau_{\pi/2}^{(1)} + T < t \leq \tau_{\pi/2}^{(1)} + \tau_{\pi/2}^{(2)} + T, \\ -t + 2T + \tau_{\pi/2}^{(1)} + \tau_{\pi/2}^{(2)} + \frac{1}{\Omega_{\text{eff}}^{(1)}} & \tau_{\pi/2}^{(1)} + \tau_{\pi/2}^{(2)} + T < t \leq \tau_{\pi/2}^{(1)} + \tau_{\pi/2}^{(2)} + 2T, \\ \frac{1}{\Omega_{\text{eff}}^{(1)}} - \frac{1}{\Omega_{\text{eff}}^{(3)}} \sin\left(\Omega_{\text{eff}}^{(3)}(t - 2T - \tau_{\pi/2}^{(1)} - \tau_{\pi/2}^{(2)})\right) & \tau_{\pi/2}^{(1)} + \tau_{\pi/2}^{(2)} + 2T < t \leq \tau_{\pi/2}^{(1)} + \tau_{\pi/2}^{(2)} + \tau_{\pi/2}^{(3)} + 2T, \\ 0 & \text{otherwise.} \end{cases} \quad (5.2)$$

where $\tau_{\pi/2}^{(1)}, \tau_{\pi/2}^{(2)}, \tau_{\pi/2}^{(3)}$ are the duration of the first, second and third pulse with the corresponding pulse area equal to $\frac{\pi}{2}$, π and $\frac{\pi}{2}$. The effective Rabi frequency for the i^{th} pulse, $\Omega_{\text{eff}}^{(i)}$, relates to the corresponding pulse duration and pulse area. T is the separation between two pulses, where atoms move only under inertial forces without experiencing the light-atom interaction.

The Raman pulse durations were determined by the velocity-sensitive Rabi oscillations after optimisation (more details are discussed in Section 7.2) as shown in Fig. 5.4. The first and the third pulse should have a pulse area of $\pi/2$, thus $\tau_{\pi/2}^{(1)} = \tau_{\pi/2}^{(3)} = 2 \mu\text{s}$ (because $\tau_{\pi}^{(1)} = \tau_{\pi}^{(3)} = 4 \mu\text{s}$) are assigned to the beam splitter pulse (Fig. 5.4 (a)) and the recombiner pulse (Fig. 5.4 (c)). The second pulse should have a pulse area of π to perform the spin echo. Therefore, $\tau_{\pi}^{(2)} = 5 \mu\text{s}$ was assigned to the mirror pulse (Fig. 5.4 (b)). The damping behaviour was limited by the intensity inhomogeneity in the Raman beam sampled by the atom cloud, which is concluded in Section 7.2.3.

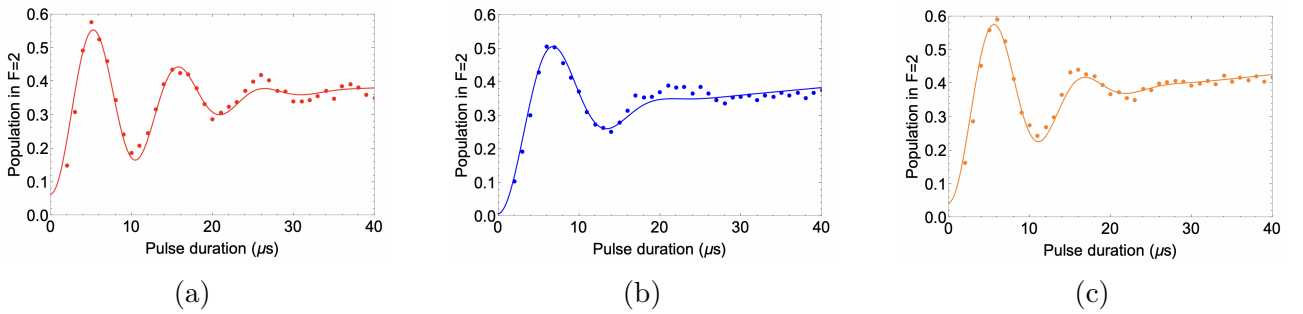


Figure 5.4: Velocity-sensitive Rabi oscillations on a velocity-selected ensemble as a function of pulse duration. (a) Rabi oscillation of the the beam splitter pulse. (b) Rabi oscillation of the mirror pulses. (c) Rabi oscillation of the recombiner pulses.

Evaluating $f(t)$ for a $\tau_{\pi/2}^{(1)}-\tau_{\pi}^{(2)}-\tau_{\pi/2}^{(3)}$ atom interferometer with Raman pulse durations $2\mu\text{s}-5\mu\text{s}-2\mu\text{s}$, where pulses are separated by $T=33\text{ ms}$, is shown as blue solid lines in Fig. 5.5. One could evaluate Eq. (5.2) in the short pulse limit, where pulse durations go to zero, the effective Rabi frequencies go to infinity and the pulse areas remain the same. In the short-pulse limit, evaluating Eq. (5.1) with a constant acceleration restores the result $\Delta\Phi_a = \underline{a} \cdot \underline{k}_{\text{eff}} T'^2$, where $2T'=2T+\tau_{\pi/2}^{(1)}+\tau_{\pi}^{(2)}+\tau_{\pi/2}^{(3)}$. The short-pulse limit is plotted as red dashed lines in Fig. 5.5. They capture most of the triangular shape of the original $f(t)$ except during the finite pulses shown in Fig. 5.5 (b). The difference between the finite-pulse calculation and the short-pulse limit is on the order of $(\tau_{\pi/2}^{(1)} + \tau_{\pi}^{(2)} + \tau_{\pi/2}^{(3)})/T \approx 0.03\%$.

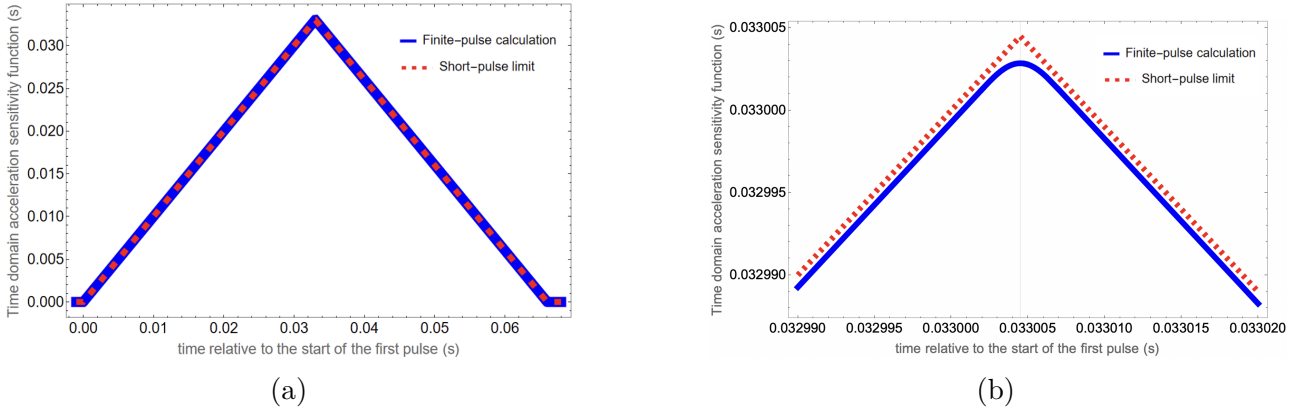


Figure 5.5: Time-domain acceleration-sensitivity function of a $\tau_{\pi/2}^{(1)}-\tau_{\pi}^{(2)}-\tau_{\pi/2}^{(3)}$ atom interferometer with Raman pulse durations $2\mu\text{s}-5\mu\text{s}-2\mu\text{s}$, where Raman pulses were separated by $T=33\text{ ms}$. The sensitivity function with short-pulse limit is calculated with $2T'=2T+\tau_{\pi/2}^{(1)}+\tau_{\pi}^{(2)}+\tau_{\pi/2}^{(3)}$ (a) Sensitivity function during the interferometry sequence. (b) Sensitivity function around the mirror pulse. The difference between the finite-pulse calculation and the short-pulse limit is on the order of $(\tau_{\pi/2}^{(1)} + \tau_{\pi}^{(2)} + \tau_{\pi/2}^{(3)})/T \approx 0.03\%$.

From the triangular shape of the acceleration sensitivity function, it is obvious to see that atoms have the largest acceleration sensitivity at the π -pulse. Below I provide an intuitive interpretation of this triangular shape with perturbation theory in the short pulse limit.

In the non-inertial frame defined by the retro-reflection mirror, atoms experience an additional homogeneous time-dependent fictitious force $\underline{F} = -m\underline{a}(t)$, or equivalently, a time-dependent perturbation potential $V(\underline{x}) = m\underline{a}(t) \cdot \underline{x}$. As shown in Fig. 5.6, this time-dependent perturbation changes the trajectories of the two arms of the interferometer. Consequently, it induces an additional phase shift, $\Delta\Phi_a$, between the two arms at the output of the interferometer. $\Delta\Phi_a$

can be evaluated by summing up the propagation phase difference, laser phase difference, and separation phase difference on the perturbed trajectories.

The same phase shift can also be evaluated by performing the same calculation on a conceptual unperturbed trajectory with the perturbation potential [6]. On the unperturbed paths shown as dashed lines in Fig. 5.6, the calculation of individual contributions gets dramatically simplified. Firstly, it is straightforward to see both contributions from laser phase difference and separation phase difference are zero. In addition, evaluating the propagation phase difference on these unperturbed paths simplifies to only considering the potential terms, that is

$$\frac{\Delta S}{\hbar} = \frac{1}{\hbar} \int (L_1 - L_2) dt = \frac{1}{\hbar} \int (V(x_1(t), t) - V(x_2(t), t)) dt = \frac{1}{\hbar} \int m \Delta x(t) a(t) dt, \quad (5.3)$$

where ΔS is the difference between actions along the upper arm of the unperturbed trajectory, $x_1(t)$, and the unperturbed lower arm, $x_2(t)$. Evaluating the difference between the two perturbed Lagrangians, L_1 and L_2 , on the unperturbed path cancels the kinetic energy terms because of the symmetry of the Mach-Zehnder configuration. Finally, the potential energy terms to calculate the action difference between the two arms is proportional to $\Delta x(t)$, the separation between two unperturbed arms, that is

$$\Delta x(t) = \begin{cases} \frac{\hbar k_{\text{eff}}}{m} t & 0 < t \leq T, \\ \frac{\hbar k_{\text{eff}}}{m} (2T - t) & T < t \leq 2T, \\ 0 & \text{otherwise.} \end{cases} \quad (5.4)$$

Substituting equation 5.4 into Eq. (5.3) reproduces the relation shown by Eq. (5.1) in the short-pulse limit. This interpretation can provide guidance to draw acceleration sensitivity functions for atom interferometers of different configurations.

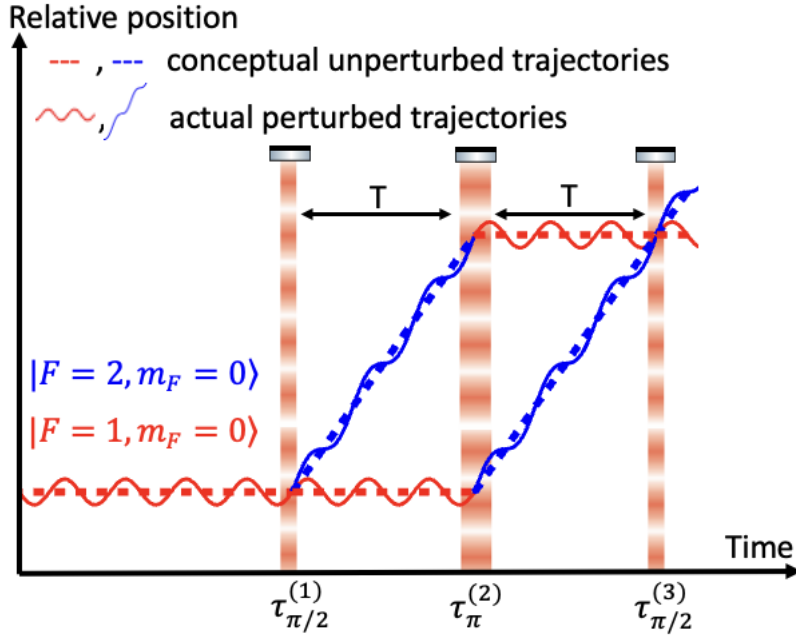


Figure 5.6: Trajectories of atomic wavepacket in the non-inertial frame of the vibrating mirror. In this non-inertial frame, mirrors are stationary, but actual atom trajectories (solid) are oscillating because of the fictitious force. Dashed lines are the unperturbed trajectories without fictitious forces in this non-inertial frame, which can be a useful conceptual tool for calculation. Three sequential pulses of durations $\tau_{\pi/2}^{(1)}$, $\tau_{\pi/2}^{(2)}$ and $\tau_{\pi/2}^{(3)}$ are separated by free-fall time, T .

5.3.2 MEMS

A micro-electro-mechanical system (MEMS) accelerometer was used to track the acceleration of the Raman retro-reflection mirror. This device (Innalab, AI-Q-2120) is a micro-fabricated quartz pendulous servo accelerometer [7]. Its three principal axes are shown in Fig. 5.7 (a). The orientation in our experiment is shown in Fig. 5.7 (b), where the acceleration signal, a_S , is the acceleration sensed along the input axis. The accelerations along the hinge axis (output axis), a_O and pendulous axis, a_P , are also indicated.

The MEMS accelerometer has two outputs. One is to measure the acceleration of the Raman mirror, while the other one is to measure the temperature of the MEMS accelerometer.

The acceleration output sensed the acceleration signal along the input axis (IA) as shown in Fig. 5.7 (a). The device used in this experiment has a specified scale factor $K_1 = 1.24996$ mA/ g_{Dublin} , where $g_{\text{Dublin}} = 9.81792$ m/s² is the local gravity at the test centre in Dublin. A precise load resistor, $R_L^{\text{acc}} = 6$ k Ω (Vishay Precision Group Foil Resistor, SMNZ 2k0 T 2k0), converted this current signal into a voltage signal, $V_m(t)$, before being digitised and recorded.

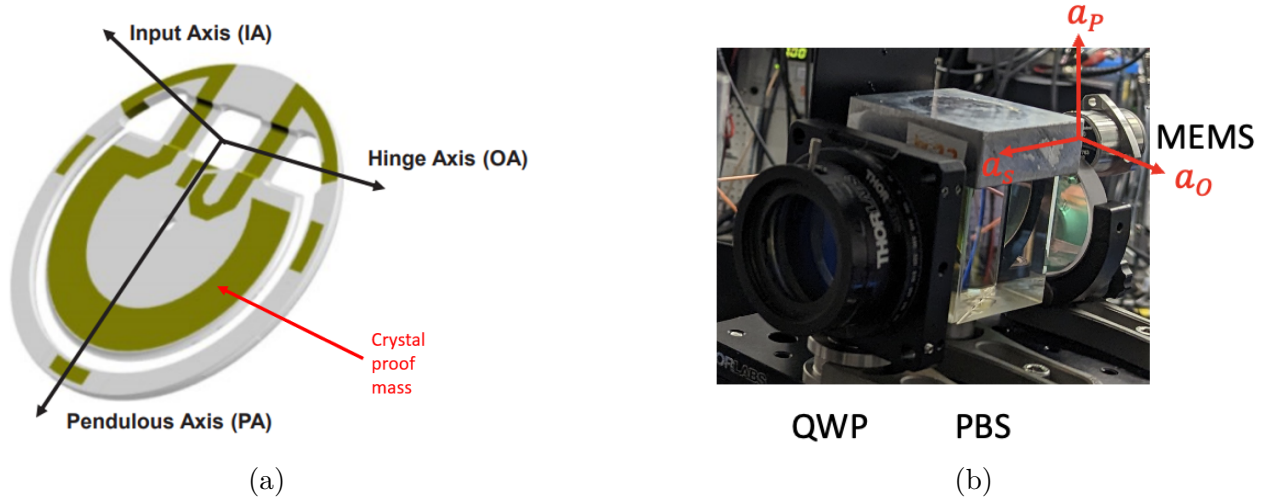


Figure 5.7: MEMS accelerometer on the retro-reflecting Raman mirror. (a) Conceptual diagram of the MEMS accelerometer. (b) Orientation of the accelerometer mounted on the Raman mirror.

Thus, our measured acceleration signal, $a_m(t)$, was given by¹

$$a_m(t) = \frac{V_m(t)}{R_L^{\text{acc}} K_1} = K_0 + a_s(t) + \delta_p a_o(t) + \delta_o a_p(t) + \epsilon(t). \quad (5.5)$$

The measured acceleration, $a_m(t)$, was slightly different from the actual acceleration signal, $a_s(t)$, which was the retro-reflection mirror acceleration along the MEMS input axis shown in Fig. 5.7b. Besides a bias offset $K_0 = -0.292 \text{ mg}_{\text{Dublin}}$, there were accelerations along the pendulum axis, $a_p(t)$ and acceleration along the output axis, $a_o(t)$. They contributed to this measured acceleration, $a_m(t)$, due to misalignment angles $\delta_p = -0.305 \text{ mrad}$ and $\delta_o = 1.810 \text{ mrad}$. The unmodelled error $\epsilon(t)$ accounts for the intrinsic electronic noise. More details on the impact of MEMS electronics noise and misalignment acceleration noise are discussed in Section 7.6.2.

The temperature output was used to monitor the MEMS temperature. Because both specified values of the bias and the scale factor are calibrated at 30°C , while they have temperature sensitivities of $dK_0/dT_{\text{MEMS}} = 14.49 \text{ } \mu\text{g}_{\text{Dublin}}/^\circ\text{C}$ and $dK_1/dT_{\text{MEMS}} = -14.83 \text{ ppm}/^\circ\text{C}$. These temperature dependents are important in our experiment as the chameleon measurement was carried out by comparing two blocks of measurements separated by one minute. Over this period,

¹I have dropped a quadratic non-linearity factor $K_2 a_s^2(t)$. Because our signal acceleration is horizontal with a typical small magnitude $a_s^{rms} \sim 1 \text{ mm/s}^2$, which is too small for this non-linearity to have any impact on our measurement.

the MEMS readout might drift due to the change in temperature as addressed in Section 7.6.1 with illustration in Fig. 7.22. Therefore, the temperature current output of the MEMs was passed through another load resistor $R_L^{\text{temp}} = 2 \text{ k}\Omega$ (Vishay Precision Group Foil Resistor, SMNZ 2k0 T 2k0), across which the voltage signal was simultaneously recorded over the interferometer sequence.²

²Temperature output pins are not shown in the diagram 5.7. Since the AI-Q-2120 only differs from the AI-Q-2020 by its dynamic range, pin connections are the same for these two devices and can be found in the AI-Q-20x0 user manual.

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Part III

Results

Chapter 6

Preparing atoms below 10 μK

Performing atom interferometry needs cold atoms of sub-Doppler temperature to minimise the contrast loss due to thermal expansion of the atom cloud. This chapter describes our sequence of producing ultra-cold atoms below 10 μK . Section 6.1 describes trapping cold atoms of Doppler temperature with a magneto-optical trap and its characteristic loading rate optimised with respect to laser intensities and detunings. After trapping more than 10^8 atoms, they were further cooled to the sub-Doppler temperature of 5 μK and get vertically launched with moving optical molasses, which are discussed in Section 6.2. At the end of the optical molasses stage, atoms are distributed in all the magnetic substates in $F=2$. Section 6.3 describes the state preparation with optical pumping beams and microwave pulses to concentrate more than 80% of atoms into interferometry clock state $|F = 2, m_F = 0\rangle$. The temperature of these state-prepared atoms was still below 10 μK . The calibration of the detection equipment was described in Section 6.4.

6.1 Magneto-Optical Trap (MOT)

Our experiment started with trapping more than 10^8 atoms in the science chamber with a magneto-optical trap (MOT). Since ^{87}Rb MOT has been extensively studied, whose principle can be found in [1, 2, 3], this section focuses on the implementation and optimisation of loading a 3D MOT from a 2D^+ MOT after distributing 350 mW cooling light and 15 mW repump light into six 3D MOT beams and a push beam shown in Fig. 6.1. Section 6.1.1 characterises the 2D^+ MOT assembly with the number of atoms loaded into the 3D MOT. Section 6.1.2 characterises the 3D MOT loading rate with respect to the intensity and frequency control of the 3D MOT light distribution system.

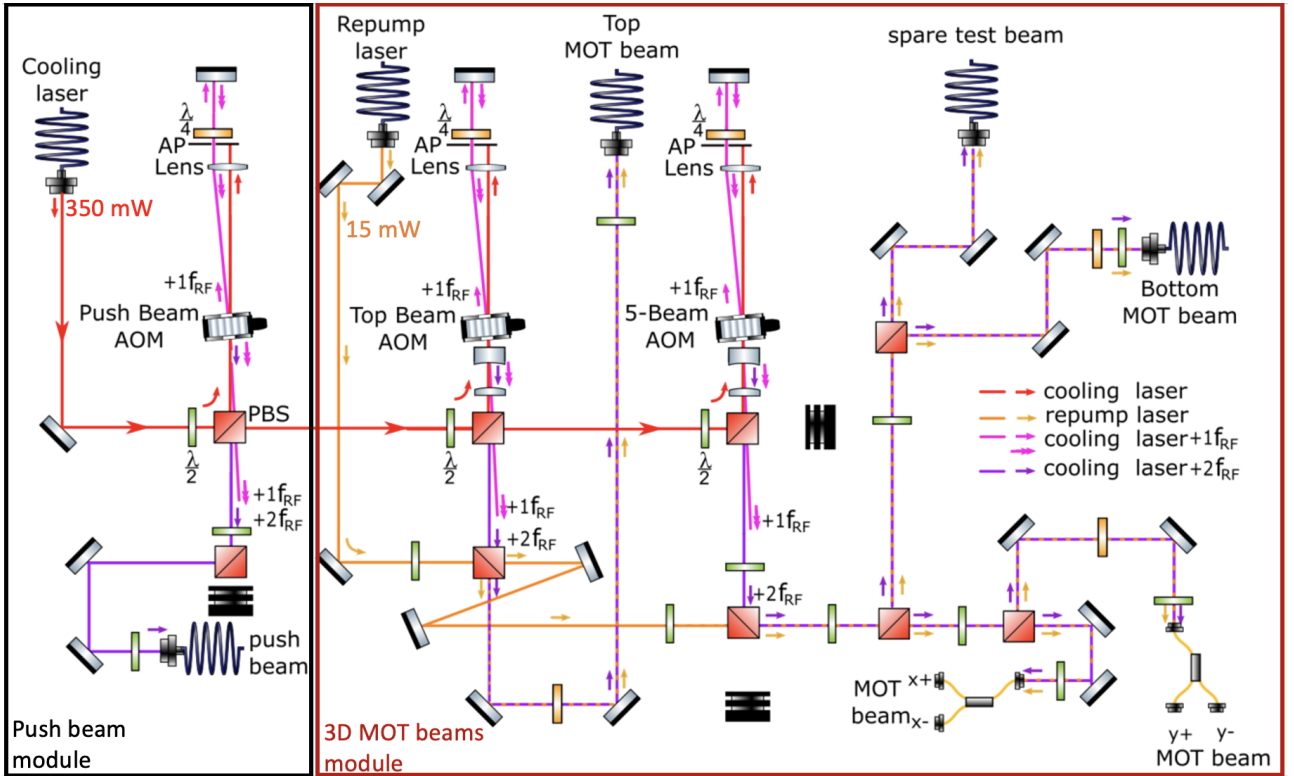


Figure 6.1: Overview of the light distribution system where control electronics for AOMs have been omitted. It consists of two distribution modules: the push beam module and the 3D-MOT beams module. Three double-pass AOMs were used to switch on/off the push beam, the 3D MOT top (downwards) beam and the other five 3D MOT beams.

6.1.1 2D^+ MOT characterisation

The 3D MOT was loaded from a 2D^+ MOT, which is a 2D MOT plus an additional push beam as shown in Fig. 6.2. After setting the current through the magnetic field gradient coil (see Section 4.3.3) to 4.3 A, the magnetic field gradient is expected to be 16 G/cm, which was found to give the optimum loading rate into 3D MOT. At this optimum magnetic field gradient, the frequency and intensity optimisation of the 2D MOT beam and the push beam are described as below.

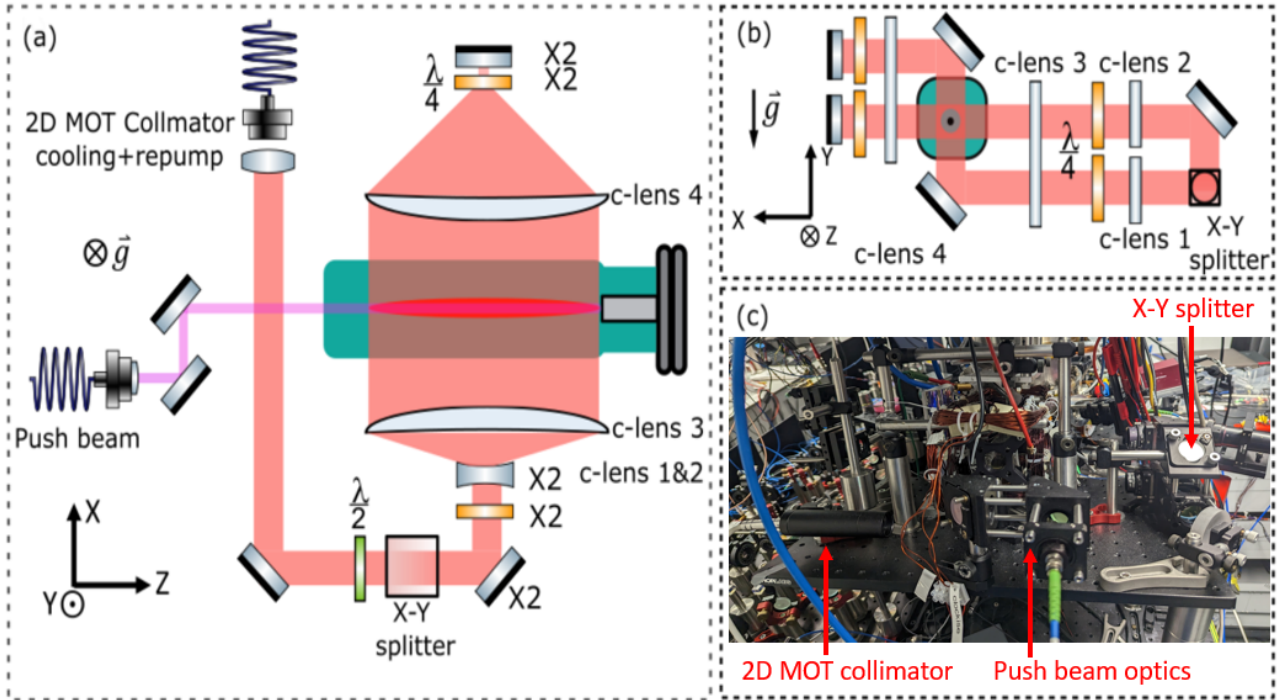


Figure 6.2: 2D MOT assembly (adapted based on [4, 2]). (a) Top view from the Y direction (along the gravity). (b) Side view from the Z direction (along the 2D MOT). The 2D MOT beam was split into two beams separated vertically by an X-Y splitter. To achieve a large capture volume, a pair of cylindrical lenses (c-lens 1 and 3) expanded the circular 2D MOT Y-beam along the z-direction to an elliptical shape of $1.6\text{ cm} \times 6.6\text{ cm}$. The other pair of cylindrical lenses (c-lens 1 and 2) expanded the 2D MOT X-beam. (c) The photo of the 2D MOT assembly.

2D MOT beam optimisation

A 2D MOT has been found to provide an intense cold atom beam to rapidly load a 3D MOT [5]. Our 2D MOT was formed with retro-reflected beams. The retro-reflection mirror position in Fig. 6.2 (a) was tuned to make the reflected light slightly converge, which compensated for power loss after passing through the glass cell. The power of the 2D MOT cooling and repump light distributed into the 2D MOT X/Y beams were then optimised by monitoring the number of atoms (estimated with the fluorescence detection explained in Section 6.4.2) in the 3D MOT as rotating the half-waveplate relative to the X-Y splitter shown in Fig. 6.2 (b). These efforts gave a 2D MOT that was visible on a camera placed at the push beam position viewing along the z-axis. After aligning the column of trapped atoms to the centre of the differential tube shown in Fig. 4.9 with shim coils, cooling laser intensity was optimised as shown in Fig. 6.3 (a). The maximum loading rate was limited to 2×10^8 atoms/s by the total power of the 2D MOT cooling beam. After setting the power to be the maximum power available, the detuning of the 2D MOT cooling was varied as shown in Fig. 6.3 (b). The optimum laser detuning was found to be -17 MHz.

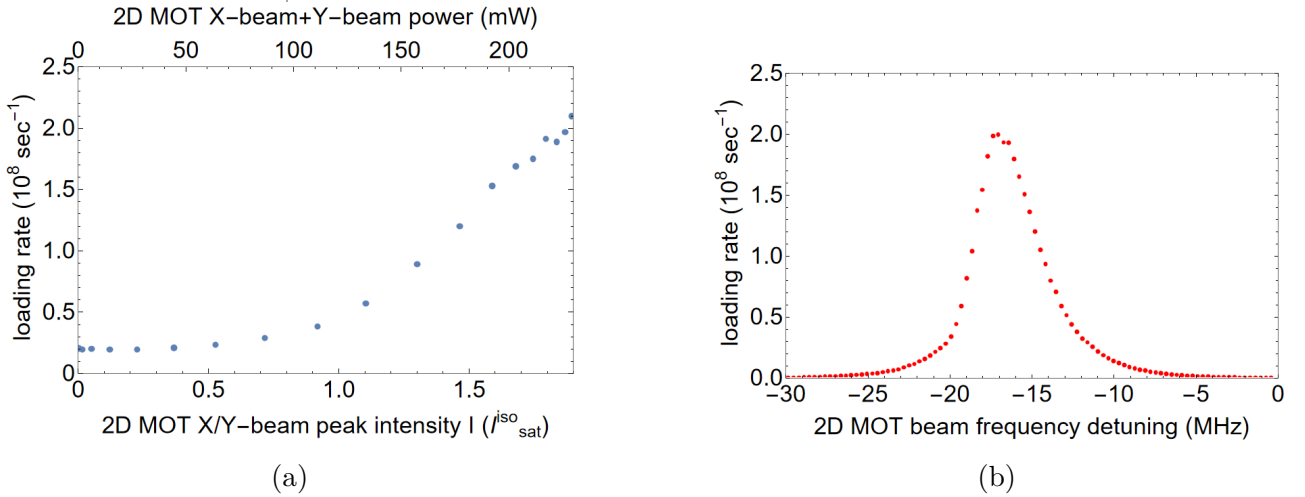


Figure 6.3: 3D MOT loading rate optimisation with respect to the 2D MOT. (a) Intensity optimisation. The total power of the cooling laser in the 2D-MOT beams was limited to 230 mW, which gave the maximum intensity $I/I_{\text{sat}}^{\text{iso}} = 1.8$ per elliptical beam of $1.6 \text{ cm} \times 6.6 \text{ cm}$. (b) Frequency optimisation. The frequency of the 2D MOT cooling laser was varied by changing its lock point controlled by the slave0 DDS.

Push beam optimisation

A push beam has been found useful [6, 7, 8, 9, 10, 11, 2, 12, 13] in significantly increasing the loading rate of a three-dimensional magneto-optical trap (3D MOT) from a two-dimensional magneto-optical trap (2D MOT). Interestingly, different setups do not agree on whether the optimal push beam frequency should be blue-detuned [7, 11, 2] or red-detuned [8, 13] from the cooling transition, though they all agree that a push beam is necessary to obtain a high loading rate as loading a 2D MOT into a 3D MOT.

A double-pass acousto-optic modulator (AOM) was used to control the frequency of our push beam with an optical layout following [14] as shown in Fig. 6.4, where the double-pass configuration minimises the fibre coupling sensitivity to the change of the deflection angle as f_{RF} is varied for optimising the 3D MOT loading rate. With a half-waveplate and a polarising beam splitter, $P_0 = 2 \text{ mW}$ of the incoming cooling beam of linear polarisation was delivered into the push beam acoustic-optical modulator (AOM). This AOM switched on/off the push beam by turning on/off the radio signal source from a voltage-controlled oscillator. It also controlled the cooling laser frequency of the push beam.

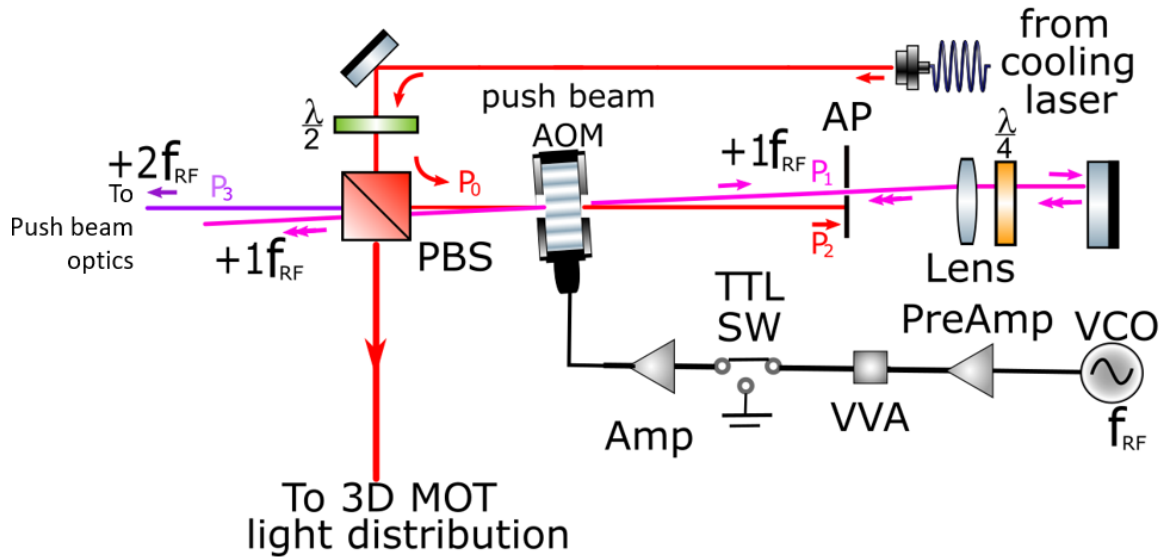


Figure 6.4: Optical layout of controlling a push beam with a double-pass acousto-optic modulator (AOM). P_0 is the incident power of the beam. $P_1 + P_2$ is the transmitted power, which was more than 90% of P_0 . The single-pass efficiency in the main text (diffraction efficiency) refers to $P_1/(P_1 + P_2)$, while the double-pass efficiency refers to $P_3/(P_1 + P_2)$.

After optimising the AOM alignment, $P_1/(P_1 + P_2) = 60\%$ (defined in Fig. 6.4) of the push

beam seed was diffracted into the +1 order of a frequency $f_{\text{cooling}} + f_{\text{RF}}$. An iris blocked other orders before the +1 order got retro-reflected. The retro-reflected beam passed the same AOM and was deflected again. The +1 order deflection of the retro-reflected beam had frequency $f_{\text{cooling}} + 2f_{\text{RF}}$, which delivered up to 160 μ W into the push beam. With a collimator (Thorlabs,F280APC-780) of 4 mm intensity e^{-2} diameter, this power leads to a peak intensity of 25.5 W/m² in our push beam seen by the 2D MOT.

We optimised the 3D MOT loading rate with respect to the intensity and detuning of the push beam. Fig. 6.5 (a) shows the loading rate as a function of the push beam power. The loading rate was strongly saturated with more than 100 μ W into the 2D MOT. After setting the push beam power to its optimum, the detuning of the push beam was optimised. As shown in Fig. 6.5(b), our system displayed an optimal loading rate of 2.3×10^8 atoms/s with a blue-detuned push beam (+17 MHz).

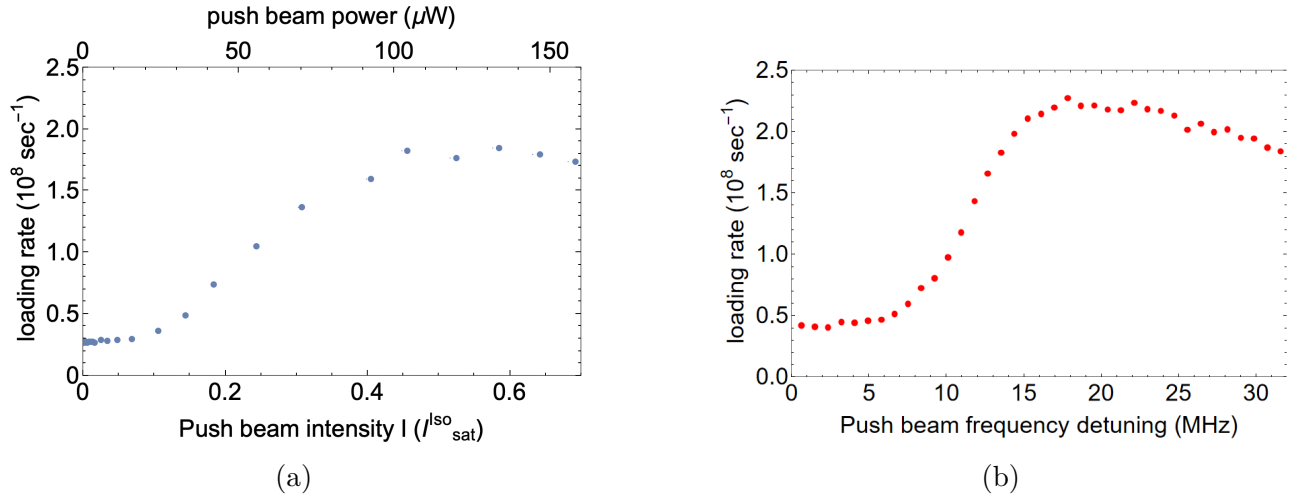


Figure 6.5: 3D MOT loading rate optimisation with respect to the push beam. (a) Power optimisation. The loading rate saturated after putting more than 100 μ W into the push beam (b) Frequency optimisation. The optimal loading rate was obtained with a blue-detuned push beam of +17 MHz detuning.

6.1.2 3D MOT characterisation

Vertically launching atoms needs at least two independent frequency controls for the upwards cooling beam and the downwards cooling beam. Therefore, one AOM controlled the top beam going downwards, and another AOM controlled the other five beams as shown in Fig. 6.6. This ‘1+5’ configuration allows us to have a frequency difference between the top and the bottom beam to launch atoms (more details in Section 6.2). For loading atoms into a 3D MOT, the frequencies of the top beam and the other five beams were set to be the same.

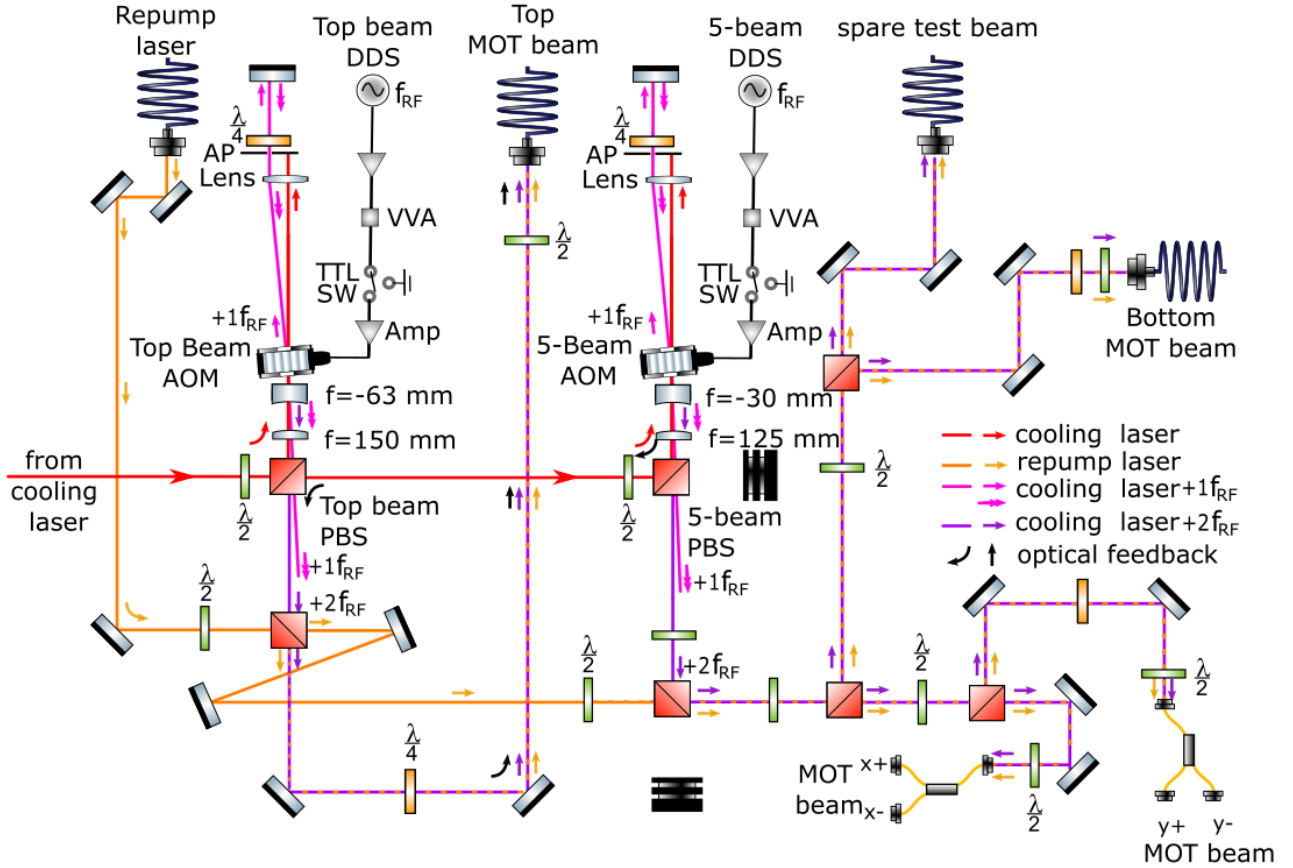


Figure 6.6: The optical layout of controlling 3D MOT beams in a ‘1+5’ configuration. There were two independent acoustic-opto modulators (AOMs) controlling the top MOT beam (downwards propagating along the gravity) and the other five beams. The undeflected beams are shown in red. The single-diffracted beam (light purple) acquired a frequency shift equal to the modulation frequency f_{RF} . The double-diffracted beam (dark purple) after double passing an AOM acquired a frequency shift as twice as the modulation frequency. An undesired feedback due to the finite extinction ratio of the 5-beam PBS was marked as black arrows.

To maximise the power of cooling light delivered into six MOT beams, telescopes were helpful to improve the deflection efficiency of AOMs up to 75% by carefully choosing lens pairs [14]. The deflection efficiency of an AOM depends on the relative beam size compared to the active

aperture of the crystal and the beam divergence through the crystal. Therefore, telescopes made of a plano-convex lens and a plano-concave lens were used to reshape the beams before sending them into the corresponding AOMs as shown in Fig. 6.6. For the top-beam AOM, a telescope of $f_1 = 125$ mm (LA1986-B) and $f_2 = -30$ mm (LC2879-B) improved the double pass efficiency from 43% to 70%. For the 5-beam AOM, a telescope of $f_1 = 150$ mm (LA1433-B) and $f_2 = -63$ mm (unknown model) improved the double pass efficiency from 35% to 58.5%. The improvements are also visible with a beam card as shown in Fig. 6.7.

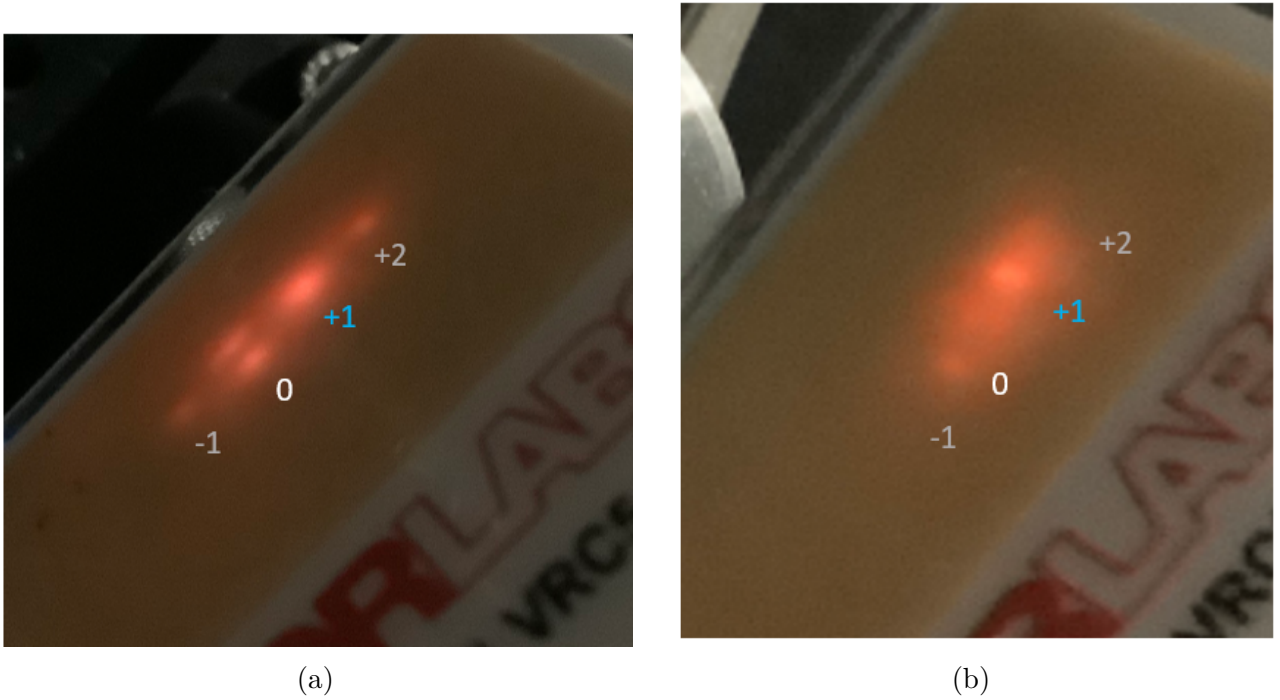


Figure 6.7: Performance of a single-pass AOM. (a) Without a telescope. A dark stripe is visible at the centre of the zero-order beam. The dark stripe corresponds to the active aperture of the crystal, which had a smaller size than the incident beam. (b) With a telescope. Most of the power was diffracted into the first-order of interest.

After optimising the performance of the 2D MOT, the number of atoms loaded in 700 ms was optimised by varying the beam power as shown in Fig. 6.8 (a), where each MOT beam was collimated with an $f = 50$ mm convex lens to have an intensity e^{-2} radius of 5 mm with maximum powers of 7 mW per horizontal beam and 10 mW per vertical beam. Although the loading rate started to show a saturation behaviour after the power gave more than the saturation intensity of isotropic polarisation, the loading rate still increased to 1.5×10^8 atoms/s as the power increased to the maximum power available. After setting the power to its maximum,

we varied the frequencies of 3D MOT cooling light. As shown in Fig. 6.8 (b), the optimum loading frequency of the 3D MOT cooling laser was -18 MHz ($\approx -3\Gamma$) at 15 G/cm in the science chamber by sending 12 A current through the 3D MOT gradient coil. This operation lead to an optimum loading rate of 2×10^8 atoms/s.

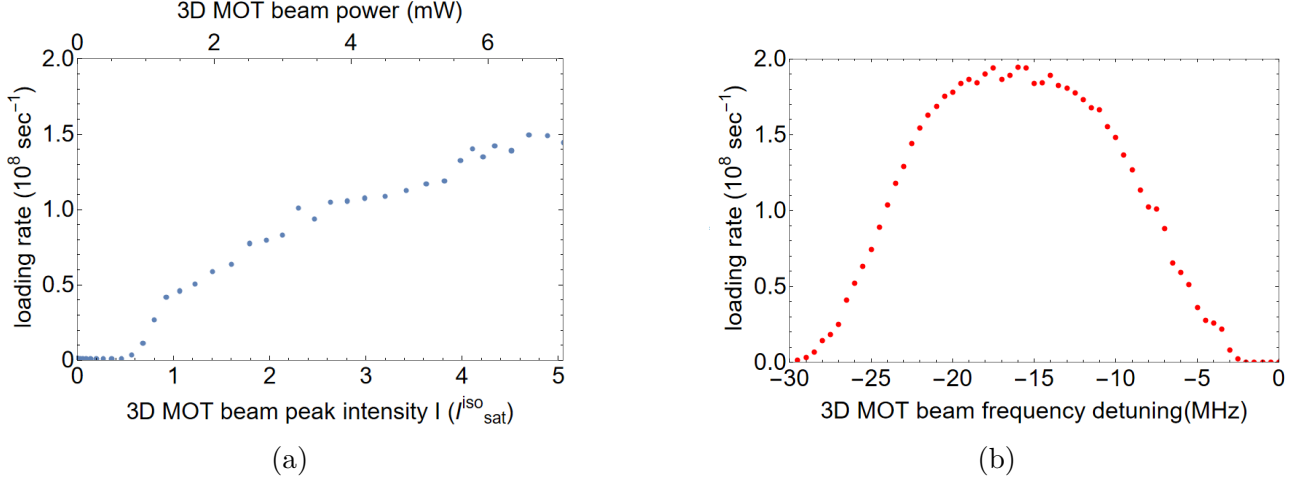


Figure 6.8: 3D MOT loading rate optimisation with respect to the 3D MOT beams. (a) Power optimisation. The loading rate increased as the beam power increased. The loading rate started showing a saturation behaviour after the intensity reached the saturation intensity $I_{\text{sat}}^{\text{iso}}$. (b) Frequency optimisation. The loading rate with the maximum power had an optimal loading frequency at -18 MHz.

Undesired optical feedback

Light distribution with sequential PBS-AOMs in our experiment had an intrinsic drawback of optical feedback, which is indicated as black arrows in Fig. 6.6. This undesired feedback may limit the launching efficiency discussed in Section 6.2.2.

The cooling light (dark purple with $+2 f_{\text{RF}}$) was designed to be delivered to the MOT beams fibres other than the top MOT beam after double-passing the 5-beam AOM. However, due to a finite extinction ratio ($T_p:T_s=240$) of this 5-beam PBS,¹ it can leak to the top-beam cooling-repump combination PBS. This created an optical beat note as shown in Fig. 6.9 between the desired cooling light and the undesired feedback light in the top beam. We found a better PBS (Thorlab, PBS25-780) of 22 times higher extinction ratio ($T_p:T_s=5300$) suppresses the beat note amplitude approximately by a factor of four (from 3.41% to 0.88% relative to its mean power).

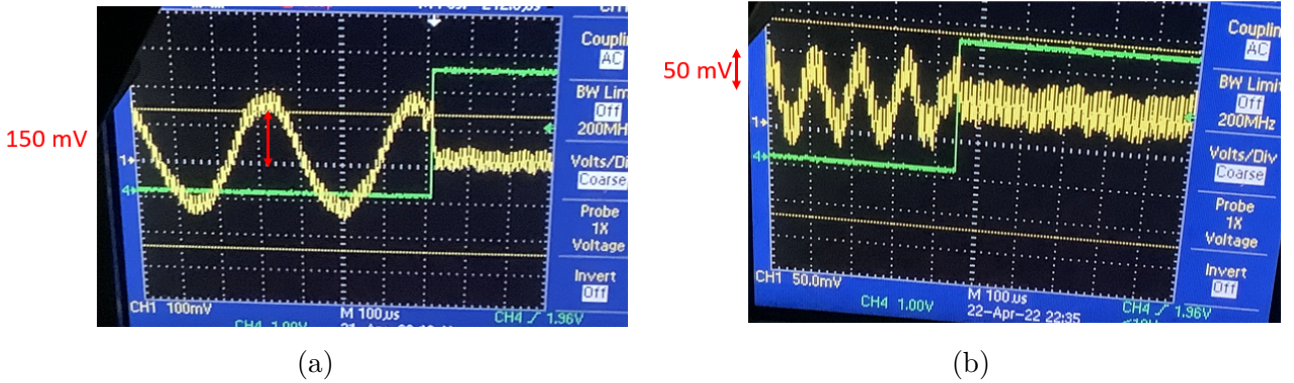


Figure 6.9: Optical beat note in the top MOT beam due to undesired optical feedback. Green signals were the TTL triggers, whose rising edge was the moment of switching off the 3D MOT 5-beam AOM, while yellow signals were the power in the 3D MOT top beam fibre measured with a photodiode. (a) The beat note amplitude was 150 mV. This was 3.41 % relative to its mean DC value of 4.4 V (not shown here), (b) Replacing the 5-beam PBS with another PBS of a 22 times higher extinction ratio suppressed the beat note amplitude (50 mV) to become 0.88% relative to the mean value (5.68 V, not shown in the graph), which was approximately 4 times smaller.

¹The transmitted power of the 5-beam PBS at the beam dump shown in Fig. 6.6 was monitored as a function of the orientation of the half-wave plate placed before this PBS. Then the extinction ratio was measured by taking the ratio between the minimum transmitted power and the maximum transmitted power.

6.2 Optical molasses

To minimize the cloud expansion during the interferometry time up to 66 ms, the temperature of the atom cloud needs to be below 10 μ K. However, Doppler cooling mechanism in the MOT only cools atoms to the Doppler limit of $0.5\hbar\Gamma/k_B = 144\mu\text{K}$, where $\Gamma \approx 2\pi \cdot 6\text{ MHz}$ is the natural line width of the excited state. This motivates us to load atoms into optical molasses by switching off the MOT quadruple magnetic field. Without magnetic fields, ^{87}Rb atoms have been reported to be cooled as cold as around 5 μ K [11, 10, 9, 6]. This sub-Doppler cooling mechanism originates from a multi-level atom seeing a changing polarisation as it diffuses in a light field [15]. Section 6.2.1 describes how 10^8 ^{87}Rb atoms were cooled to be around 5 μ K in 5 ms and Section 6.2.2 describes how to launch them upwards to obtain a small atomic fountain.

6.2.1 Temperature measurement and optimization

In a 1D situation, the polarisation gradient cooling can be understood in two distinct situations [15, 6]: Sisyphus cooling in the light field of a $\text{lin} \perp \text{lin}$ polarisation configuration, and motion-induced orientation cooling in the light field of a $\sigma^+ - \sigma^-$ configuration. In our 3D situation using three pairs of $\sigma^+ - \sigma^-$ beams, both cooling mechanisms contribute to the final temperature. For beams of intensity I_0 and laser detuning Δ , the final temperature is expected to be proportional to I_0/Δ .²

The sequence to achieve our sub 5 μ K cloud is shown in Fig. 6.10. More than 90% of atoms ($> 10^8$ atoms) were kept in the optical molasses after loading from the MOT by switching off the 3D MOT quadruple field and waiting for 30 ms to let the eddy current-induced B field decay away while the atoms diffused in the light field. This loading efficiency is significantly higher than the MOT-to-molasses loading efficiency of less than 10% reported in the previous generation. (7×10^6 atoms were kept after molasses cooling [4], while 10^8 atoms was loaded [16]).

Then the cooling laser frequency seen by atoms was ramped from $\Delta = -2\pi \cdot 18\text{ MHz}$ to $-2\pi \cdot 150\text{ MHz}$ by ramping the offset lock point of the cooling laser via the VCO control discussed

²This result is estimated with a semi-classical model, which breaks when the atom cloud temperature T is approaching the recoil temperature T_R , at which the atom DeBroglie wavelength becomes comparable with light wavelength. For ^{87}Rb , the recoil temperature is about 300 nK.

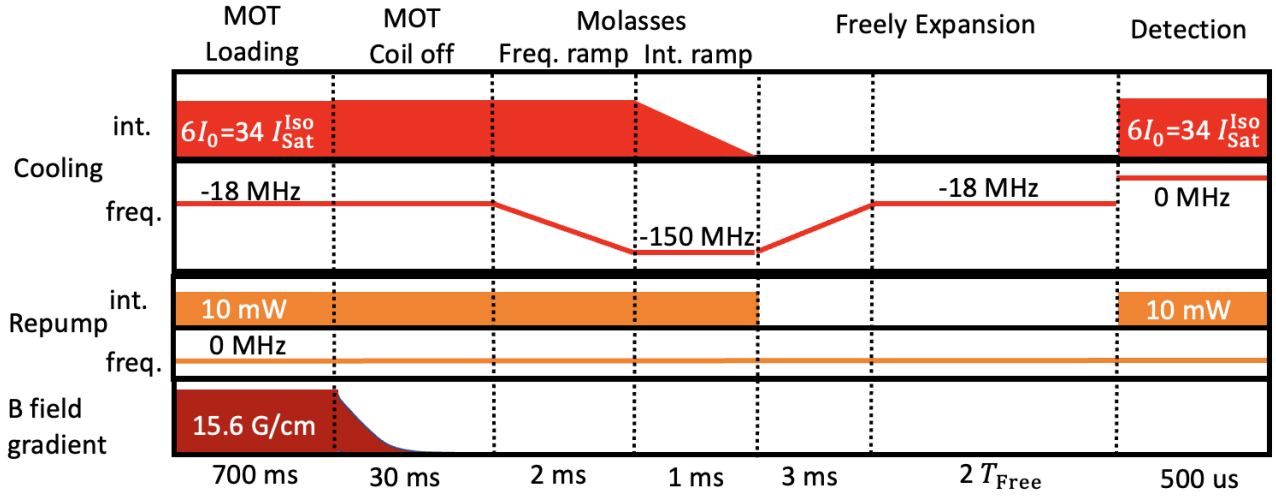


Figure 6.10: Sequence to produce atoms of 5 μK . It started with loading atoms with a magneto-optic trap. Then the magnetic field gradient was switched off to load the MOT into optical molasses for the sub-Doppler cooling. The freely expansion stage allows determining the cloud temperature by detecting its size at various free-fall time, $3 \text{ ms} + 2T_{\text{Free}}$, after releasing the cloud.

in the explanation of the offset-lock module in Section 4.4.2.³ As shown in Fig. 6.11 (a), this programmed frequency ramp was verified by monitoring the Doppler feature of the cooling laser saturation spectroscopy. The corresponding photodiode signal of the spectroscopy is calibrated to the cooling laser detuning as shown in Fig. 6.11 (b). Ramping the current of the cooling laser diode during the frequency ramps was found to be essential to achieve ramping laser frequencies over more than 100 MHz in a few ms.

Finally, we released our atom cloud by ramping down the cooling light intensity in 1 ms by ramping down the analogue control voltage of the voltage-varied attenuators (VVA) shown in Fig. 6.6, which reduced the RF power sent into the AOMs, before shutting off six MOT beams completely by switching off an RF switch (ZYSWA-2-50DR).

After releasing the cloud, we estimated the cloud temperature by taking fluorescent images of 500 μs resonance detection at variable wait time $2T_{\text{free}}$ ranging from 1 ms to 40 ms as shown in Fig. 6.12, where the distance indicator of 1 mm was calibrated with a ruler at the same focal plane as atoms (Section 6.4.1).

Assuming a Gaussian distribution for the cloud's spatial profile, its $e^{-1/2}$ radius, $\sigma(t)$, scales

³One may worry about the $F=2 \rightarrow F'=2$ transitions, relative to which our cooling laser is blue-detuned. This is bad considering Doppler heating (negligible at the large detuning), but good for Sub-Doppler cooling in 3D because of dark states [17].

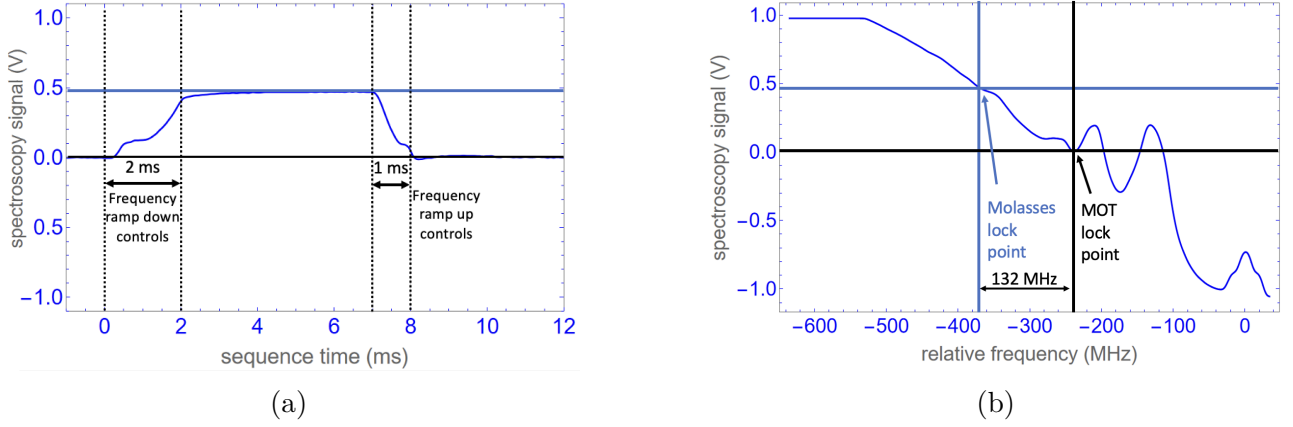


Figure 6.11: Frequency ramp of the 3D MOT cooling laser during the optical molasses. (a) Ramping the frequency of the offset-locked cooling laser. The frequency was monitored with the Doppler feature of the saturation spectroscopy. There was an additional 5 ms gap to show the laser took an extra 2 ms to settle down to the desired new lock-point after applying the frequency ramp sequence. (b) Calibration of the Doppler feature of the saturation spectroscopy for the cooling laser.

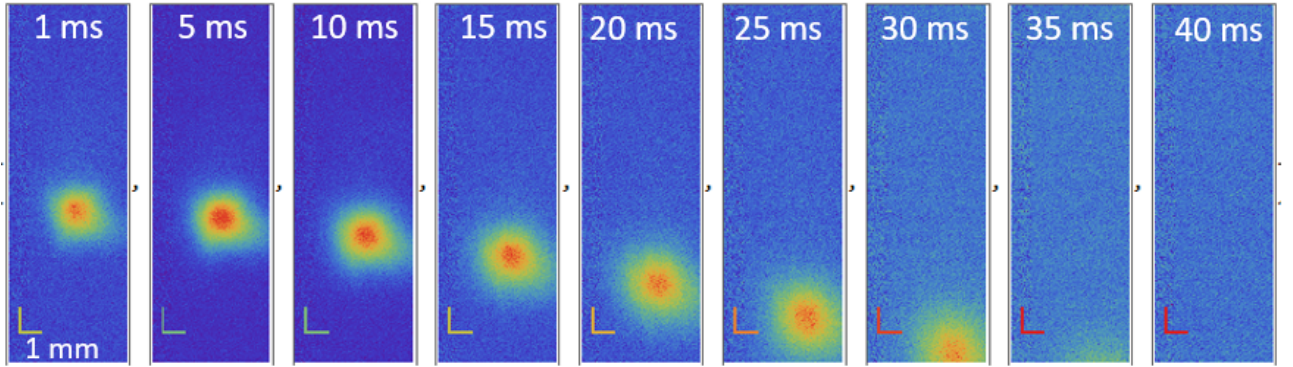


Figure 6.12: Atom cloud released from the optical molasses. It displayed a ballistic expansion as freely falling under gravity. The distances of 1 mm are indicated.

with the ballistic-expansion time $t = 3 \text{ ms} + 2T_{\text{Free}}$ due to a finite temperature T [9, p.75] as

$$\sigma(t)^2 = \sigma_0^2 + \frac{k_B T}{m} t^2, \quad (6.1)$$

where σ_0 is initial $e^{-1/2}$ radius of the initial cloud of a Gaussian distribution, k_B is the Boltzmann constant and m is the mass of ^{87}Rb atom. The width as a function of the ballistic expansion time is plotted in Fig. 6.13 (a), which gives temperature estimation of the vertical direction as $T_V = 8.8 \pm 0.3 \mu\text{K}$ and the temperature of the horizontal direction as $T_H = 5.7 \pm 0.7 \mu\text{K}$. An optimization of this geometric temperature with respect to the vertical shim B field is shown in Fig. 6.13 (b), which illustrates the magnetic field sensitivity of the sub-Doppler cooling mechanism. The final optimal temperature achieved 3-4 μK after minimising the magnetic field

seen by atoms, which was probed by the Zeeman splitting of atoms in their microwave spectrum.

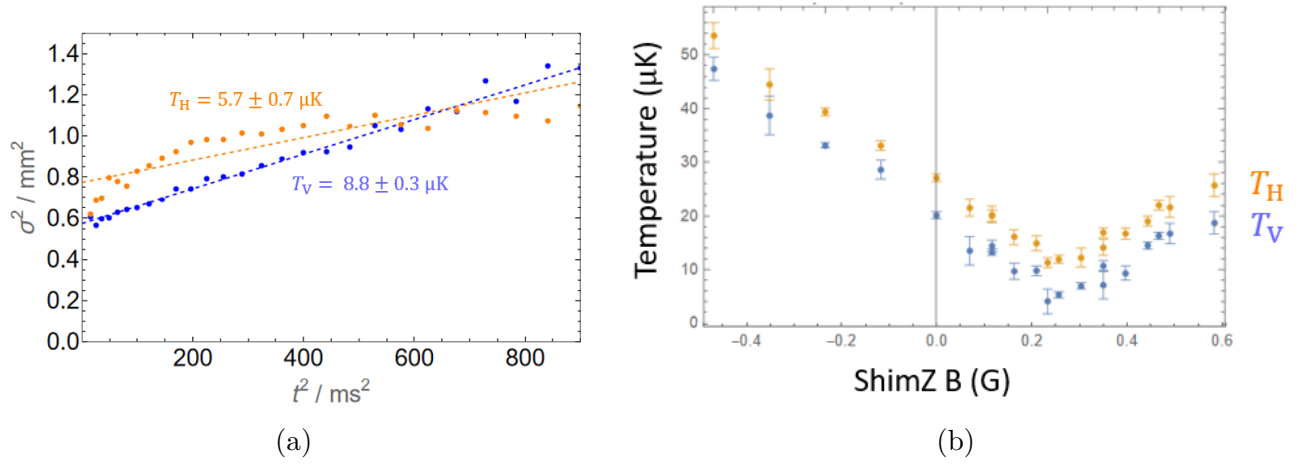


Figure 6.13: Temperature estimation and optimisation of atoms cooled by optical molasses. (a) Temperature estimation with the ballistic expansion. The horizontal temperature estimation shows some systematics due to the clip of the Gaussian tails in the image Fig. 6.12 (a). (b) Each temperature measurement (and its error bar) was obtained from the fitting shown in Fig. 6.13. The temperatures along both directions are sensitive to the stray magnetic field. Applying a shim magnetic field to cancel the stray field helped minimise the temperature.

6.2.2 Launching atoms with moving molasses

To increase the atom interferometer acceleration sensitivity, $k_{\text{eff}} T_{\text{Free}}^2$, we aim to increase the interferometer time $2T_{\text{Free}}$ by upwards launching atoms $\approx 1\text{ cm}$ to create a small atomic fountain. By this means, atoms fell back to the centre of our detection zone at the interaction of six MOT beams of an intensity e^{-2} radius of 5 mm. This allows us to increase $2T_{\text{Free}}$ from 32 ms to 66 ms in this second generation. In addition, this symmetric fountain configuration also suppresses systematic phase shifts raised from AC stark shift and Zeeman shift of the clock transition frequency, which are further discussed in Section 7.5.

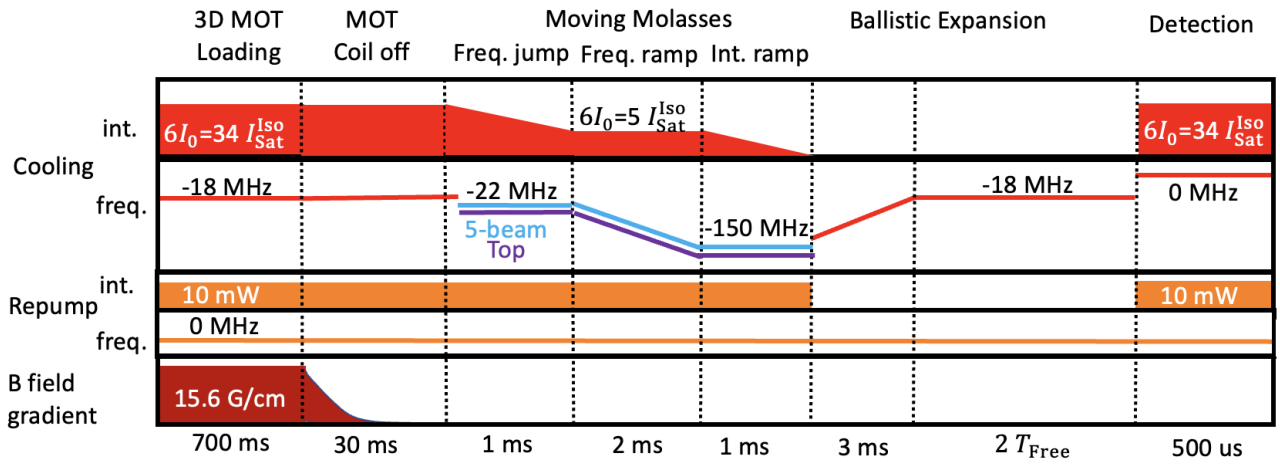


Figure 6.14: Sequence for vertically launching atoms with moving molasses. The top beam had a lower frequency than the other five beams to upwards launch the atoms. Their frequency difference determined the launching velocity, while a further 4 MHz red-detuning in their mean frequency from the loading frequency was found to launch the largest fraction of the atoms trapped by the MOT.

To achieve this small fountain, we implemented the moving molasses technique [18, 19] with a sequence shown in Fig. 6.14. Compared to the sequence used to make stationary molasses shown in Fig. 6.10, the key difference is an additional step that created a frequency difference of $\Delta f_{\text{light}} = 600\text{ kHz}$ between the top and bottom beam by applying a frequency difference of $\Delta f_{\text{AOM}} = 300\text{ kHz}$ to two phase-coherent DDS sources supplying to the corresponding double-pass AOMs.⁴ This leads to a moving molasses of velocity $v_{\text{eq}} = \Delta f_{\text{light}} \lambda / 2 = 0.234\text{ mm/ms}$, where $\lambda \approx 780\text{ nm}$ is the wavelength of the cooling laser. Atoms cooled in the moving molasses obtain an initial velocity that is equal to v_{eq} .

⁴We used to have a lot of trouble in launching atoms with two independent VCOs, whose relative phase noise was too large, checked with the beat note between the top and bottom beams on a photodiode, to make a moving molasses. On the contrary, two phase-coherent DDS frequency sources lead to a coherent beat signal and controlled moving molasses.

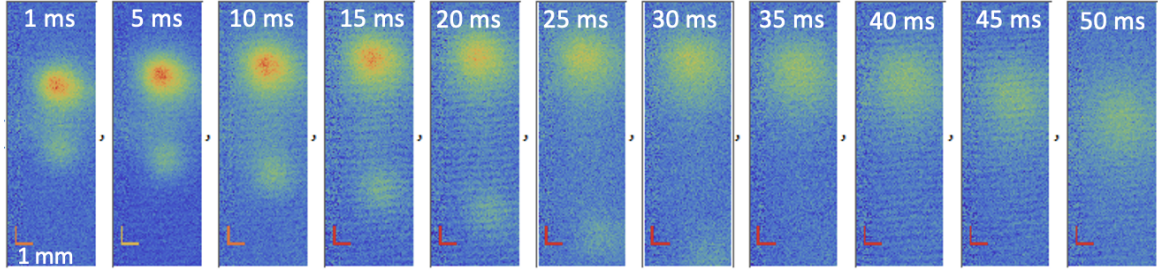


Figure 6.15: A launched cloud of atoms with the moving molasses detected by 500 μs fluorescence imaging. The launching velocity was expected to be $v_{\text{eq}} = 0.156 \text{ mm/ms}$ controlled by the frequency difference between the top and bottom beams. Here the frequency difference between the two beams was set to 400 kHz, which was achieved by setting 200 kHz frequency difference between the corresponding AOM DDS sources. The measured launching velocity was larger than the designed one may be due to the systematics associated with the detection duration discussed in Section 6.4.1.

A test launched cloud with the moving molasses of $\Delta f_{\text{AOM}} = 200 \text{ kHz}$ is shown in Fig. 6.15. The launching efficiency reached more than 70% after optimising the power balance between two beams and the detuning of the mean frequency of two beams equal to -22 MHz . Figure 6.16 (a) illustrates controlling the initial velocity of the launched atoms with the AOM DDS frequency difference $\Delta f_{\text{AOM}} = f_{\text{top,AOM}} - f_{\text{5-beam,AOM}}$. Figure 6.16 (b) shows that the temperature of the launched atoms with various Δf_{AOM} was not significantly different from the free-fall atoms ($\Delta f_{\text{AOM}}=0 \text{ kHz}$), because atoms were cooled in a moving frame with the same mechanism.

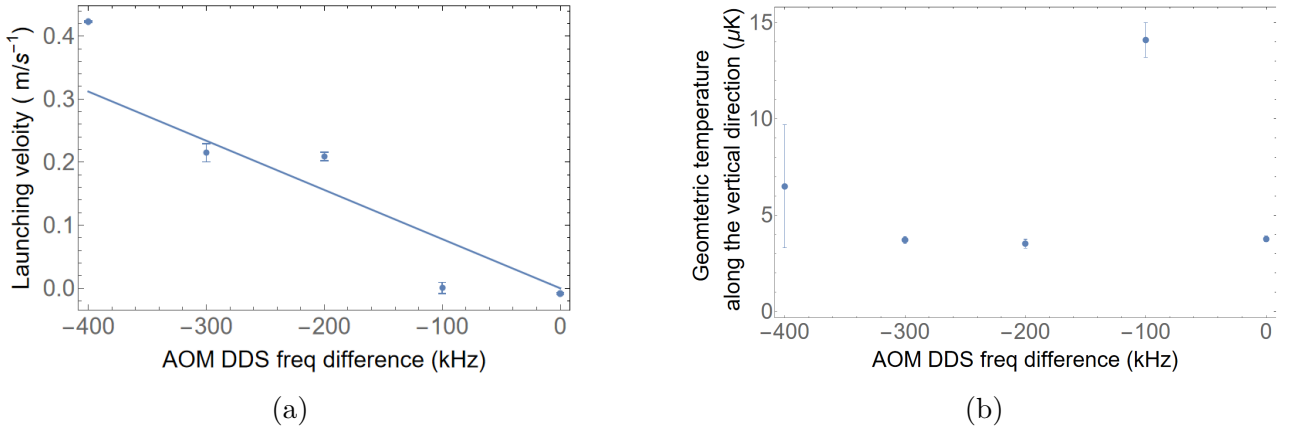


Figure 6.16: Characterisation of the launched atoms. (a) Launching velocity as a function of the frequency difference. The initial velocity of the launched cloud scales linearly with respect to the frequency difference between the top and bottom beam. The top beam had a lower frequency than the bottom beam to give an upwards launch. (b) Geometric temperature as a function of the frequency difference. The cloud temperature along the vertical direction was not significantly changed as the frequency difference increased. The temperature at AOM DDS frequency difference equal to -100 kHz appears as an outlier, because the launched and the unlaunched clouds only became separable after 9 ms at this small launching velocity, which lead to systematics in its temperature estimation.

6.3 State preparation

Atoms for interferometry need to be prepared in the magnetic insensitive state, which in our case is $|F, m_F\rangle = |1, 0\rangle$ before the first interferometer pulse. Atoms in $|F, m_F\rangle = |1, 0\rangle$ also need to have a momentum distribution smaller than the bandwidth of the interferometer π -pulse. In the previous generation of the experiment, only 20% atoms were prepared in the magnetic-insensitive state before performing the velocity selection [4, p.69]. In this generation, we developed two schemes to improve the efficiency of the state preparation. Scheme-I, inspired by [8, p.59-p.62], involved additional optical pulses, which lead to 33% atoms in the magnetic-insensitive state before applying two velocity selection pulses. Scheme-II applied optical pumping and adiabatic passage to prepare more than 80% atoms into $|F = 2, m_F = 0\rangle$ before velocity selection.

Scheme-I discussed in Section 6.3.1 was implemented in the interferometer measurement presented in Chapter 7, while scheme-II was not integrated with interferometer sequences yet but it was characterised in Section 6.3.2 for the next generation of experiment.

6.3.1 Scheme-I: double pulse velocity selection

The sequence of using the double-pulse velocity selection to prepare the launched atoms is shown in Fig. 6.17. The MOT loading and moving molasses steps were the same as discussed in Section 6.2.2. At the end of launching atoms upwards using moving molasses, atoms were almost distributed in the $F=2$ state while some were left in the $F=1$ state (population ratio depends on the scattering rate of the repump and cooling light). As illustrated in Fig. 6.18, these atoms of sub-Doppler temperature (red) were firstly prepared in $F=1$ state by the -150 MHz red-detuned cooling light (without repump light), which deposited 33% of the total atoms in the magnetically insensitive state $|1, 0\rangle$. Then a Raman velocity-sensitive π -pulse of $6\text{ }\mu\text{s}$ coherently transfer $\approx 27\%$ atoms in the $|1, 0\rangle$ state ($\approx 9\%$ of the total number of atoms) to $|2, 0\rangle$ state, whose momentum distribution was narrowed down and limited by the bandwidth of this Raman π -pulse. At this point, if atoms in $F=1$ could be blown away, then the left atoms in $|2, 0\rangle$ could be transferred to $|1, 0\rangle$ using another Raman pulse to be ready for interferometer pulses.

However, we did not have a laser to address the cycling transition $F=1 \rightarrow F'=0$ for blowing

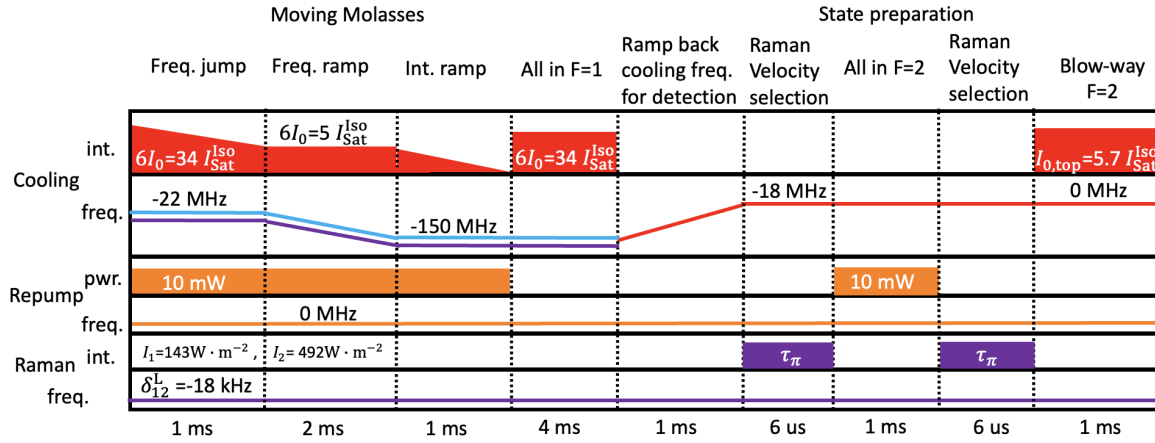


Figure 6.17: Sequence of preparing atoms with the double pulse velocity selection. $I_{\text{Sat}}^{\text{Iso}} = 3.58 \text{ mW/cm}^2$ is the saturation intensity for the $F=2 \rightarrow F'=3$ transition considering isotropic polarisation [20, p.17]. The frequencies of the cooling laser, repump laser and Raman lasers (δ_{12}^L) are relative to the cooling, repump and Raman transition frequencies shown in Fig. 4.10.

away atoms in $F=1$. This led to performing the second velocity selection after optically pumping all the atoms left in $F=1$ into $F=2$ with the repump light (no cooling light). The atoms coherently selected by the second velocity selection pulse were almost the same atoms selected by the first velocity selection pulse, which was about 9% of the total number of atoms. But now, the thermal atoms without velocity selection were left in $F=2$, which were blown away with a resonant cooling light from the top beam. This is the end of our state preparation implemented for interferometry measurement in Chapter 7.

Note that, the second velocity selection π -pulse also incoherently optically pumped $1/8 \times 3\% = 0.4\%$ of the total atoms into each magnetic substates of the $F=1$, where $1/8$ comes from the decay branching ratio from the P states and 3% comes from 6% of the total atoms were scattered by a 2π -pulse as discussed in Section 3.2.3.

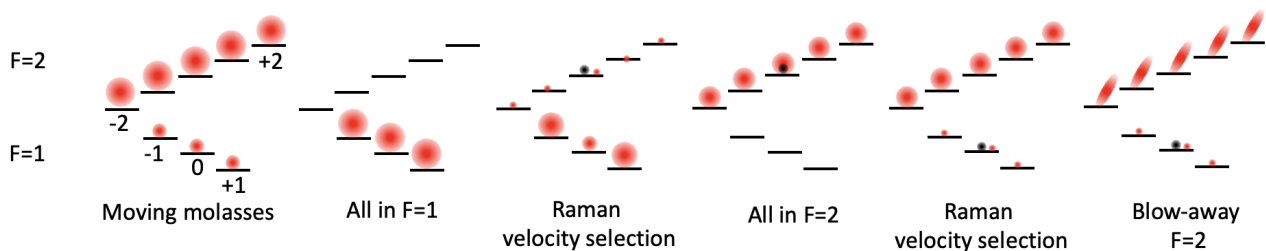


Figure 6.18: Schematics of preparing atoms with the double pulse velocity selection. Thermal atoms of sub-Doppler temperature are shown in red, while the velocity-selected atoms are shown in black.

6.3.2 Scheme-II: optical pumping and microwave adiabatic passage

Figure 6.19 illustrates the goal of our state preparation using microwave adiabatic passage. More than 80% of atoms, which were evenly distributed in $F=2$ magnetic states (Fig. 6.19 (a)), can be prepared into the magnetic insensitive state $|F = 2, m_F = 0\rangle$ shown in Fig. 6.19 (b) with optical pumping and two adiabatic microwave pulses discussed below.

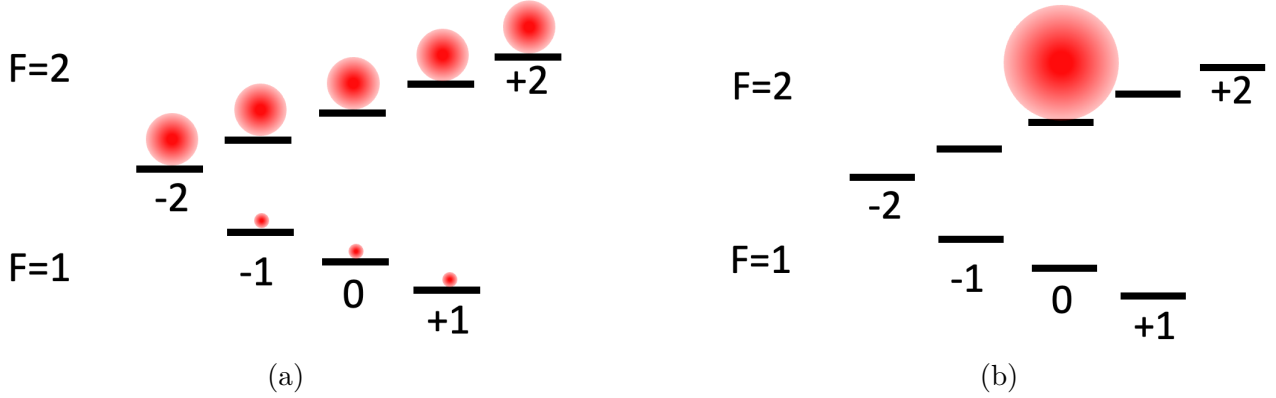


Figure 6.19: Overview of state preparation with microwave adiabatic passage. (a) Population distribution over the Zeeman states before state preparation. At the end of optical molasses, most of the atoms were evenly distributed in the $|F = 2\rangle$ Zeeman states. (b) Target population distribution after state preparation.

Zeeman State Concentration with Optical Pumping

At the end of the molasses cooling, atoms predominantly accommodated $|F = 2\rangle$ states, while some were left in the $|F = 1\rangle$ as illustrated in Fig. 6.19 (a). The cooling light in the 3D MOT beams was turned off while repump light was kept on for 1 ms to optically pump all atoms into $|F = 2\rangle$. Simultaneously, another optical pumping beam of circular polarisation was applied to drive the $|F = 2\rangle \rightarrow |F' = 2\rangle$ transition as shown in Fig. 6.20 (a). This transition minimises the heating effect during the optical pumping process, because the target state $|F = 2, m_F = -2\rangle$ doesn't scatter photons from the optical pump beam.⁵

As illustrated in Fig. 6.20 (b), this optical pumping beam was achieved by delivering a linearly polarised light through a polarisation-maintaining (PM) fibre and passing through a

⁵The optical pumping beam propagating axis was slightly tilted from the Raman magnetic field axis that stabilised the energy quantization axis. This axis misalignment allowed the left-handed circular polarisation to slightly drive the σ^+ and π^0 transitions, while the σ^- transition was the predominant one. Consequently, the equilibrium state was not 100% $|F = 2, m_F = -2\rangle$.

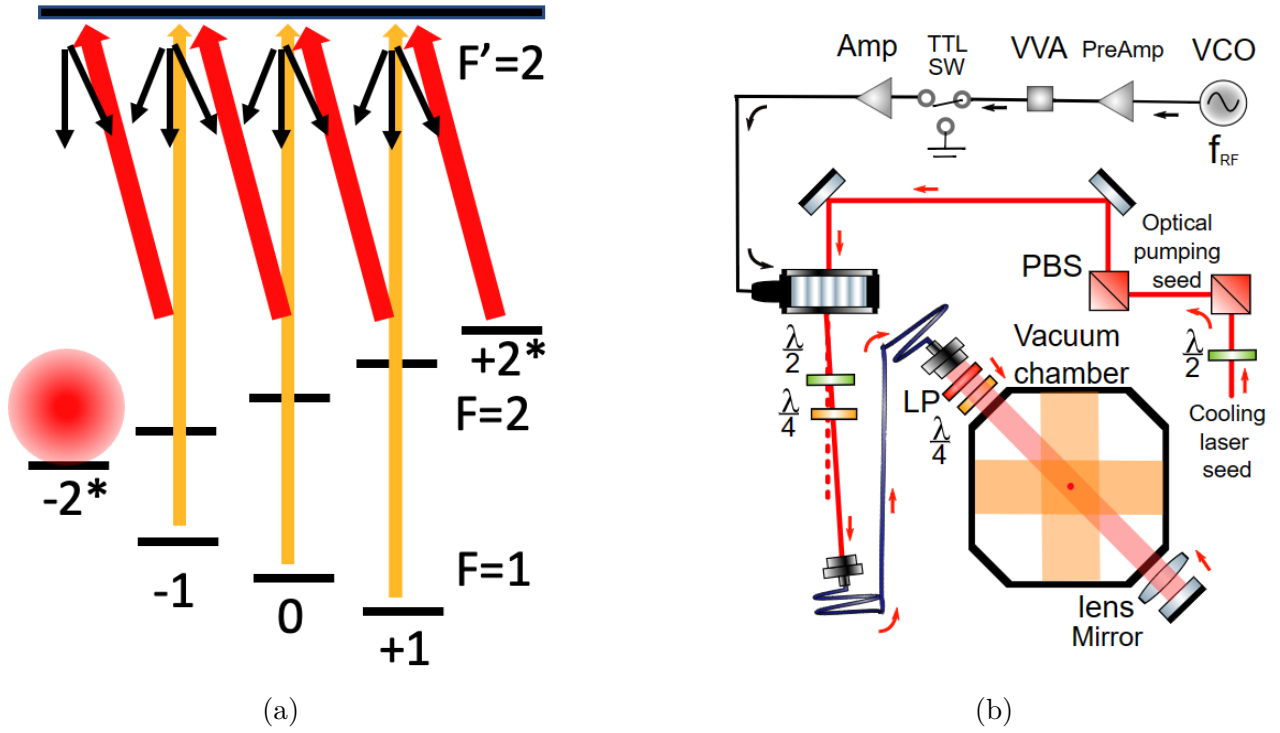


Figure 6.20: Schematics of the optical pumping to concentrate the Zeeman state. (a) Preparing atoms into a dark state with optical pumping. Repump (orange) light of isotropic polarisation pumped all of the atoms from $|F=1\rangle$ to $|F=2\rangle$. A pair of balanced optical pumping beams (red) drove σ^- transitions from $|F=2, m_F\rangle$ to $|F'=2, m'_F = m_F - 1\rangle$. (b) The optical layout for the optical pumping beam. It was a small fraction of the 3D MOT cooling laser. At the end of the molasses cooling, it locks to -370 MHz relative to the cooling transition $|F=2\rangle \rightarrow |F'=3\rangle$. The single pass AOM operates at 104 MHz, which shifts the cooling laser to address the optical pumping transition $|F=2\rangle \rightarrow |F'=2\rangle$ that is -266 MHz relative to the cooling transition.

quarter waveplate (QWP) before sending it into the vacuum chamber. To minimize pushing atoms during the optical pumping, a lens and a retro-reflection mirror were used to balance the radiation force.

The optical pumping efficiency was evaluated with an adiabatic-pulse spectrum (details provided in the discussion of adiabatic passage step B) shown in Fig. 6.21 (a). Applying the optical pumping beam, the spectrum displayed an enhanced $|F=2, m_F = -2\rangle \rightarrow |F=1, m_F = -1\rangle$ ($-2^* \rightarrow -1$) transition dip and reduced dips of other transitions. The depth of the $-2^* \rightarrow -1$ transition dip indicated more than 80% of atoms had been pumped into the $|F=2, m_F = -2\rangle$ state before being adiabatically transferred into $|F=1, m_F = -1\rangle$ that is dark to our fluorescence detection. The orientation of the QWP at the optical pumping beam output in Fig. 6.20 (b) set the beam to have a circular polarisation. Its orientation was then optimised to maximize

the transfer efficiency measured by the $-2^* \rightarrow -1$ transition dip depth in Fig. 6.21 (a). After this optimisation, the optical pumping efficiency combined with an adiabatic transfer efficiency approached 89% as shown in Fig. 6.21 (b).

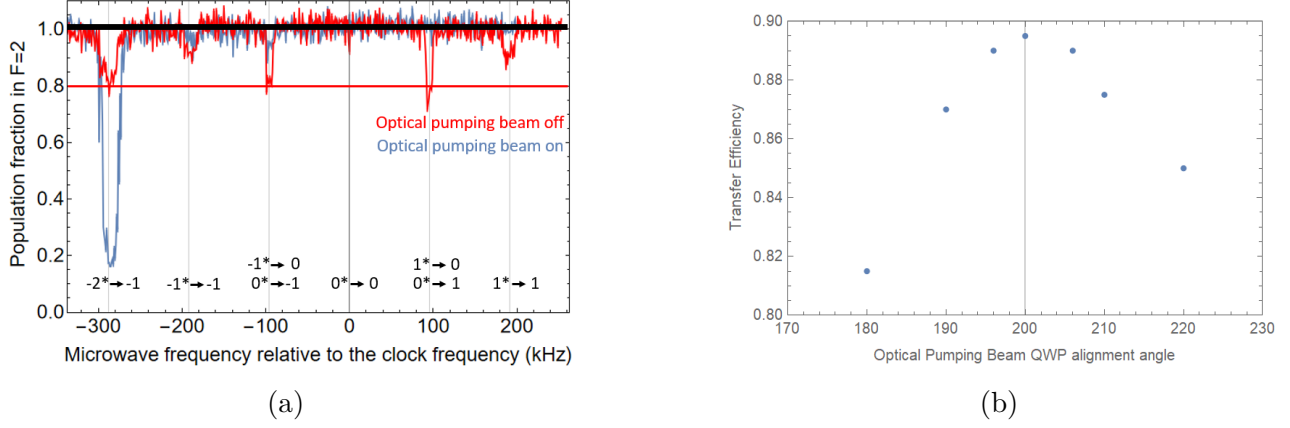


Figure 6.21: Optical pumping efficiency probed by an adiabatic microwave pulse. (a) Comparison between population before and after optical pumping. The horizontal red line shows the maximum depth of a non-degenerate transition with an even population distribution across the five sub-states. The horizontal black line indicates the background level centred around 1, remarking the estimator of population fraction in $F=2$ has been calibrated. The dip depth at $-2^* \rightarrow -1$ shows 85% atoms were transferred into the state $|F=1, m_F=-1\rangle$ from $|F=2, m_F=-2\rangle$. Small residuals at $-1^* \rightarrow -1$ and degenerate $-1^* \rightarrow 0$ dips indicate the polarisation of optical pumping light is not purely σ^- with respect to the Raman magnetic field. (b) Optimisation of the optical pumping efficiency with respect to the optical pumping beam polarization. The $-2^* \rightarrow -1$ dip depth is monitored as varying the orientation of the optical-pumping-beam quarter-waveplate.

Zeeman State Transfer with Adiabatic Passage

After optical pumping atoms into $|F=2, m_F=-2\rangle$, we describes how they were transferred to a magnetically insensitive state in two successive steps: $|F=2, m_F=-2\rangle \rightarrow |F=1, m_F=-1\rangle$ (step A) and $|F=1, m_F=-1\rangle \rightarrow |F=2, m_F=0\rangle$ (step B) using adiabatic microwave pulses. The oscillating magnetic field of a microwave pulse allows population transfer between $F=1$ and $F=2$ states via the magnetic-dipole interaction. We used adiabatic pulses rather than simple square pulses because a square pulse only transferred up to 80% in each step as displayed in Fig. 6.23 (b) and Fig. 6.24 (b). They displayed damping behaviours which limited the two-step efficiency using square pulses to 64%. The damped Rabi oscillations were due to an inhomogeneous Rabi frequency across the cloud, because the in-vacuum source mass and the chamber wall were highly reflective to the microwaves and created standing waves. In addition,

the transfer efficiency using square pulses drifted because the magnetic-sensitive transition frequencies changed over time due to the ambient magnetic noise sourced from 50 Hz mains power.⁶

To overcome these two limitations, an adiabatic pulse with a time-dependent power profile and a time-dependent magnetic bias field was implemented with the sequence shown in Fig. 6.22, which is in contrast to a square pulse with constant power in a constant magnetic bias field. The principle of the adiabatic passage with visualisation using the Bloch sphere has been discussed in

⁶We have triggered the experiment with a TTL that syncs with the mains power. This suppresses the drift but leaves a residual effect due to the fact that the mains-power TTL is not a stable 50 Hz oscillator.

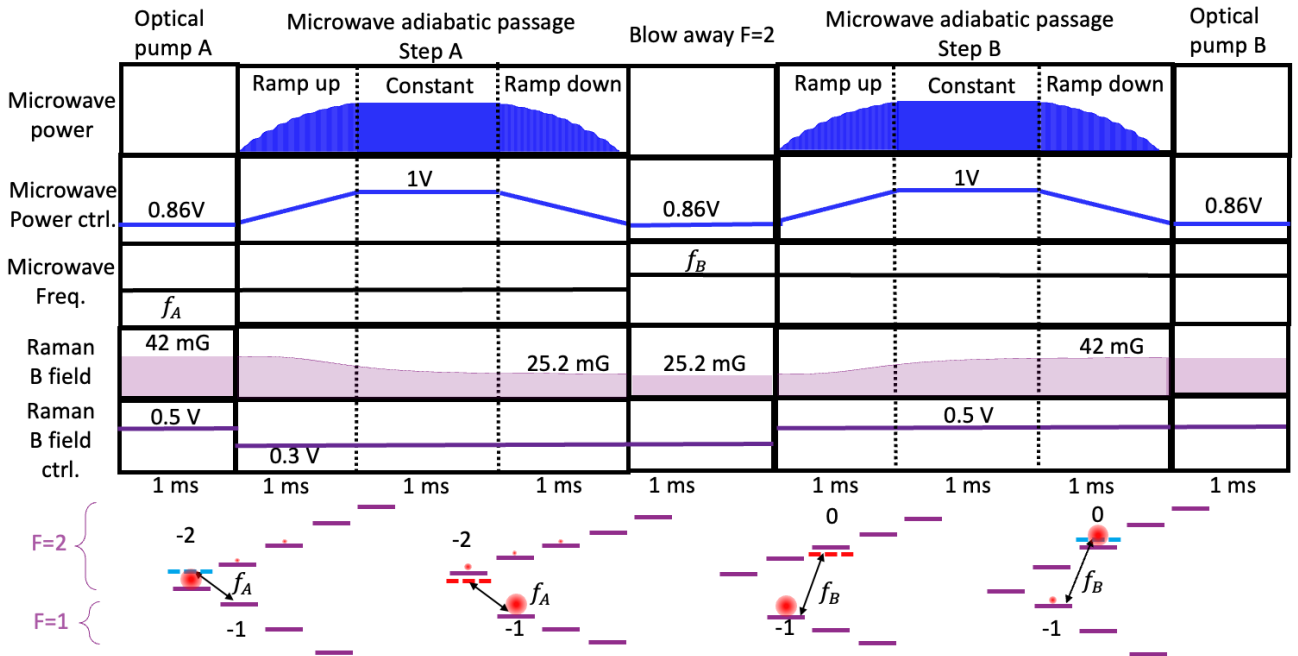


Figure 6.22: The sequence for the two-step Zeeman state transfer with the adiabatic passage. During step A, the microwave frequency was fixed to $f_A=6834.537$ MHz. At the start of step A, f_A was blue-detuned from $|F=2, m_F=-2\rangle \rightarrow |F=1, m_F=-1\rangle$ transition. At the end of step A, f_A became red-detuned. Because the $|F=2, m_F=-2\rangle \rightarrow |F=1, m_F=-1\rangle$ transition frequency was adiabatically ramped across f_A by ramping the Raman magnetic bias field B via its control voltage. After that, the microwave frequency was changed to $f_B=6834.637$ MHz and kept during the step B. At the start of step B, f_B was red-detuned from $|F=1, m_F=-1\rangle \rightarrow |F=2, m_F=0\rangle$ transition. At the end of step B, f_B became blue-detuned. Because the $|F=1, m_F=-1\rangle \rightarrow |F=2, m_F=0\rangle$ transition frequency was adiabatically ramped across f_B . The magnetic field profile was measured outside the vacuum chamber with a Hall probe (LakeShore Cryotronics, Gaussmeter), which was proportional to the actual Raman magnetic bias field profile at atoms. The absolute value of the Raman magnetic field at atoms for each voltage was calibrated with microwave spectra at various constant Raman magnetic fields. The microwave power profile was measured with a crystal detector (Hewlett-Packard, 420A). The power response to its control voltage is determined by the characteristics of the microwave switch (Hewlett-Packard, SPST 100 MHz-18 GHz 33142A).

[21, p.135]. In summary, there are two useful concepts, a Bloch vector and a drive vector. The Bloch vector represents the atomic state, which initially points to the north pole of the Bloch sphere. It precesses around the drive vector, $(\Omega(t) \cos \phi, \Omega(t) \sin \phi, \delta(t))$, at a precession angular frequency of $\sqrt{\Omega^2(t) + \delta^2(t)}$, where $\Omega^2(t)$ is the Rabi frequency determined by the instant power ($\propto |\Omega(t)|^2$) and $\delta(t)$ is the detuning of the microwave adiabatic pulse. During an adiabatic pulse, $\delta(t) = \omega_{\text{MW}} - \omega_0^B(t)$ changed its sign as the transition frequency of a magnetic sensitive transition, $\omega_0^B(t)$, was ramped across the constant microwave frequency ω_{mw} . Consequently, the drive vector gradually tipped from one pole of the Bloch sphere to the other pole. Importantly, the flip in the drive vector direction was achieved regardless of the Ω inhomogeneity and drift in ω_0^B , as long as $\delta(t)$ was swept over a sufficiently large range. The same is true for the Bloch vector if it can adiabatically follow the drive vector. In the adiabatic limit, flipping the Bloch vector from the North pole to the South pole is equivalent to transferring atoms from their initial state to the target state.

Adiabatic passage step A

As shown in Fig. 6.23 (a), applying an adiabatic pulse (square shape of power and ramped detuning) after optical pumping can transfer 89% atoms from $F=2$ into $F=1$ after optical pumping. Optimising power shaped to be ramped as shown in Fig. 6.22 increased this overall transfer efficiency to 92%, which is a combination of the optical pumping efficiency for concentrating atoms into $|F=2, m_F=-2\rangle$ and the adiabatic passage efficiency for transferring atoms from $|F=2, m_F=-2\rangle$ to $|F=1, m_F=-1\rangle$. The 92% efficiency to transfer atoms from $|F=1, m_F=-1\rangle$ to $|F=2, m_F=-2\rangle$ of this adiabatic pulse was higher than the efficiency of 78% with a square pulse as shown in Fig. 6.23 (b). More importantly, the adiabatic pulse is less sensitive to the drift in $|F=2, m_F=-2\rangle \rightarrow |F=1, m_F=-1\rangle$ transition frequency because this transition frequency was ramped over a range larger than the drift level during the adiabatic pulse.

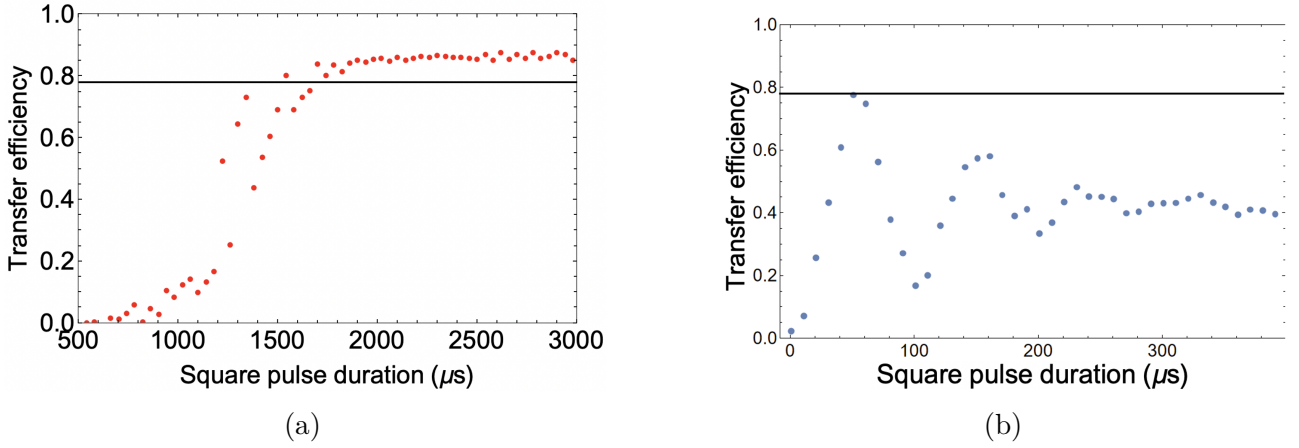


Figure 6.23: Comparison between Zeeman state transfer with adiabatic pulse and square pulse for step A. (a) Transferring atoms from $|F = 2, m_F = -2\rangle$ to $|F = 1, m_F = -1\rangle$ with an adiabatic pulse after optical pumping. This shows more than 78% atoms (black line) can be transferred into the $F=1$ state after applying optical pumping and an adiabatic pulse. (b) Transferring atoms from $|F = 2, m_F = -2\rangle$ to $|F = 1, m_F = -1\rangle$ with a square pulse. The damping behaviour fits with a model considering a Gaussian spread of the Rabi frequency, which limited the maximum transfer efficiency to 78% (black line).

Adiabatic passage step B

After transferring 92% atoms into $|F = 1, m_F = -1\rangle$ at the end of step A, cooling laser light resonant with the $|F = 2\rangle \rightarrow |F' = 3\rangle$ transition was applied to blow away the residual 8% atoms in $|F = 2\rangle$ with the top MOT beam. Then another adiabatic MW pulse was applied as shown in Fig. 6.22. The power control voltage linearly increased from 0.86V to 1 V in 1 ms before holding at 1V for 1 ms and linearly decreased to 0.86V in 1 ms, which smoothly varied $\Omega(t)$ from 0 kHz to 2.5 kHz and back to 0 kHz. Meanwhile, the transition frequency (Raman magnetic field) control voltage jumped from 0.3 V to 0.5 V, which resulted in an exponential profile of $\omega_B(t)$ changing from -36 kHz to -60 kHz relative to the clock transition frequency because the magnetic field relaxed from 25.2 mG to 42 mG.

An MW spectrum with an adiabatic pulse is shown in Fig. 6.24 (a) by varying the applied microwave frequency. This indicates about 90% atoms in $|F = 1, m_F = -1\rangle$ were transferred to $|F = 2, m_F = 0\rangle$ ($-1 \rightarrow 0^*$), which was higher than 83% efficiency with a square MW pulse shown in Fig. 6.24 (b). The spectrum in Fig. 6.24 (a) near the transition from $|F = 1, m_F = -1\rangle$ to $|F = 2, m_F = 2\rangle$ ($-1 \rightarrow 0^*$) showed 95% efficiency for this adiabatic pulse of parameters varied as shown in Fig. 6.22.

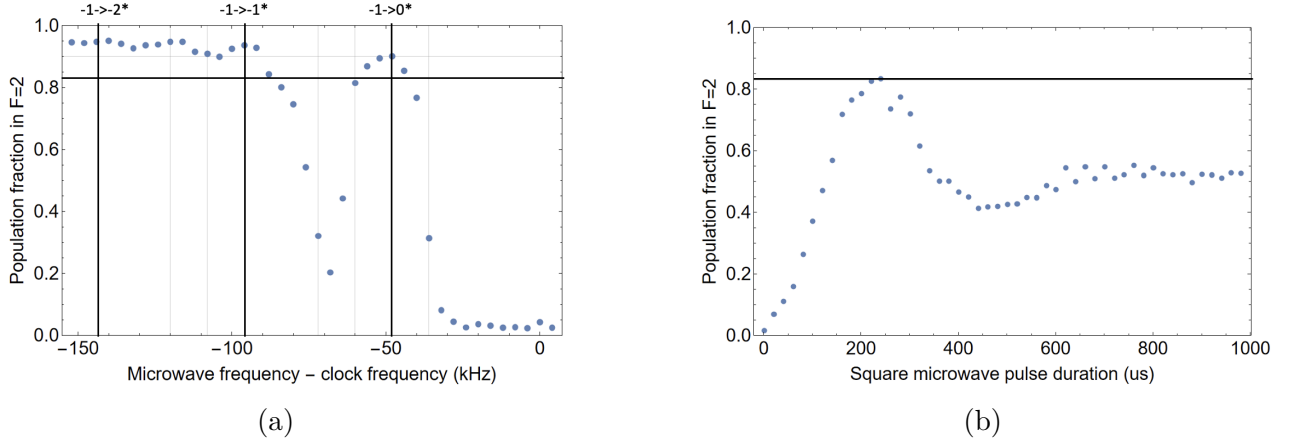


Figure 6.24: Comparison between Zeeman state transfer with an adiabatic pulse and a square pulse for step B. (a) Transferring atoms from $|F = 1, m_F = -1\rangle$ to $|F = 2, m_F = \{-2, -1, 0\}\rangle$ with an adiabatic pulse. The peak around -50 kHz shows about 90 % atoms in $|F = 1, m_F = -1\rangle$ was transferred to $|F = 2, m_F = 0\rangle$ with our adiabatic pulse. The plateau around -150 kHz shows the same adiabatic pulse transferred 95% atoms from $|F = 1, m_F = -1\rangle$ to $|F = 2, m_F = 0\rangle$. (b) Transferring atoms from $|F = 1, m_F = -1\rangle$ to $|F = 2, m_F = 0\rangle$ with a square pulse. The maximum transfer efficiency was 83% and was limited by the damping behaviour. The damping is well described with Rabi oscillation averaged over Gaussian-distributed Rabi frequencies.

Finally, we optically pumped the residual 10% atoms from $|F = 1\rangle$ to $|F = 2\rangle$ with the repump light. This action prepared atoms very close to the target state shown in Fig. 6.19 (b) and atoms were ready for velocity selection with Raman beams to perform atom interferometry.

Summary

In conclusion, this scheme prepared more than 80% atoms into the target state $|F = 2, m_F = 0\rangle$. As discussed in the adiabatic passage step A, the combined efficiency of the optical pumping and the adiabatic transfer approached 92%. From the spectrum in step B, the adiabatic transfer efficiency from $|F = 1, m_F = -1\rangle$ to $|F = 2, m_F = 0\rangle$ was 90%, while $|F = 1, m_F = -1\rangle$ to $|F = 2, m_F = -2\rangle$ was 95%. This indicates the overall state preparation efficiency was 83% (optical pumping 97%, step A adiabatic passage 95%, step B adiabatic passage 90%), which is significantly higher than the previous 20% state preparation scheme. [4, p.69]

After this state preparation scheme, the geometric temperature slightly increases from $2.9 \pm 1.1 \mu\text{K}$ to $4.4 \pm 1.3 \mu\text{K}$ for the vertical direction and $4.4 \pm 1.3 \mu\text{K}$ to $6.5 \pm 0.4 \mu\text{K}$ for the horizontal direction. These increments in temperature were negligible for our interferometer.

6.4 Detection system

In this experiment, the size of the atom cloud was useful to estimate its temperature. This spatial information was captured by a camera system (Section 6.4.1) to obtain fluorescence images. The other important information is the number of atoms in $F = 2$, which was estimated by measuring the fluorescence photons collected on a photodiode (Section 6.4.2).

6.4.1 Fluorescence imaging with a camera

A camera (Allied vision, Pike F-505-B) was used to capture the fluorescence images of the atom cloud using a lens system of a variable focal length. To calibrate this imaging system, the focal length was adjusted and fixed to make atom images as sharp as possible. Then this imaging system was used to take a calibration image of a ruler, whose position was chosen to make the ruler image as sharp as possible. This calibration gave a conversion relation from the pixel distance on the image to the actual distance on the focal plane as 0.207 mm/pixel.

We then used calibrated images like Fig. 6.12 to measure free-fall acceleration and found them to be 0.80 g, 0.85 g and 0.98 g for 1000 μ s, 500 μ s and 100 μ s resonance detection. This exposure-time dependence in the measured gravity implies some power imbalance between the six MOT beams, because the power imbalance could kick atoms in a particular direction during the detection. The 0.98 g measurement with the shortest exposure time to get enough signal-to-noise was consistent with the camera at a tilting angle of 12°. This verifies that the ruler calibration was reliable.

6.4.2 Photodiode characterisation

To estimate the number of atoms in an MOT or cooled after molasses, a photodiode (OSI Optoelectronics, PIN-RD100) collected fluorescence photons scattered from atoms in $F = 2$, which were illuminated by six MOT beams of strong cooling light that resonates with the $F = 2 \rightarrow F' = 3$ transition and weak repump light that kept atoms in $F = 2$ states.

The scattering rate of a single atom with resonant light is given by [20]

$$R_{sc} = \frac{\Gamma}{2} \frac{s_{\text{tot}}}{1 + s_{\text{tot}}}, \quad (6.2)$$

where $\Gamma \approx 36 \times 10^6 \text{ s}^{-1}$ ($2\pi \cdot 6 \text{ MHz}$) is the natural linewidth of the excited state and s_{tot} is the total intensity seen by atoms in the unit of saturation intensity I_{sat} . For ^{87}Rb illuminated by beams of isotropic polarisation, the saturation intensity is $I_{\text{sat}} = I_{\text{sat}}^{\text{iso}} = 3.58 \text{ mW/cm}^2$ [20]. Since each horizontal MOT beam of 5 mm (intensity e^{-2} radius) delivered 7 mW and each vertical MOT beam delivered 10 mW, the scattering rate can be safely approximated as $\Gamma/2 \approx 18$ photons per μs since $s_{\text{tot}} = 34$.

Subject to the dimension of the science chamber, our photon-collecting optics was placed at $r = 15 \text{ cm}$ away from the centre of the chamber where atoms were located. The lens occupied a solid angle $\Omega \approx 0.25d^2/r^2$, where the lens diameter was $d = 2 \text{ inch}$, which leads to a photon collection rate per atom as

$$R_{\text{collect}} = \frac{\Omega}{4\pi} R_{\text{sc}} = 0.71\% \times 18 \cdot \text{photons}/\mu\text{s} \approx 0.13 \text{ photons}/\mu\text{s}/\text{atom} \quad (6.3)$$

The fluorescence photons that arrived at the photodiode were then converted into a voltage signal with a load resistor. This voltage signal was then amplified before being recorded by the data-acquisition device (PXI-4462). The conversion relation between the photon rate and the amplified voltage was calibrated with a beam of a known power at 780 nm as shown in Fig. 6.25. This gives a linear relation as

$$\frac{V_{\text{pd}}}{V} = \underbrace{0.480}_{\kappa_{\text{power}}} \frac{P_{\text{collect}}}{\mu\text{W}} - 0.022 = \underbrace{1.22 \times 10^{-5}}_{\kappa_{\text{photon}}} \frac{R_{\text{collect}}}{\text{photons}/\mu\text{s}} - 0.022, \quad (6.4)$$

with a power responsivity $\kappa_{\text{power}} = 0.480 \text{ V}/\mu\text{W}$ or photon counting rate responsivity $\kappa_{\text{photon}} = 1.22 \times 10^{-5} \text{ V}/(\text{photons}/\mu\text{s})$. Combining the collecting rate per atom given by Eq. (6.3), this photodiode has an atom number (N_{atom}) responsivity

$$\frac{V_{\text{pd}}}{V} = \underbrace{1.6 \times 10^{-8}}_{\kappa_{\text{atom}}} \frac{N_{\text{atom}}}{\text{atoms}} - 0.022, \quad (6.5)$$

with $\kappa_{\text{atom}} = 1.6 \times 10^{-5} \text{ mV}/\text{atoms}$ and a minimal detectable atoms of 1.38×10^6 to give signal above 22 mV. Below this threshold, our photodiode behaved non-linear.

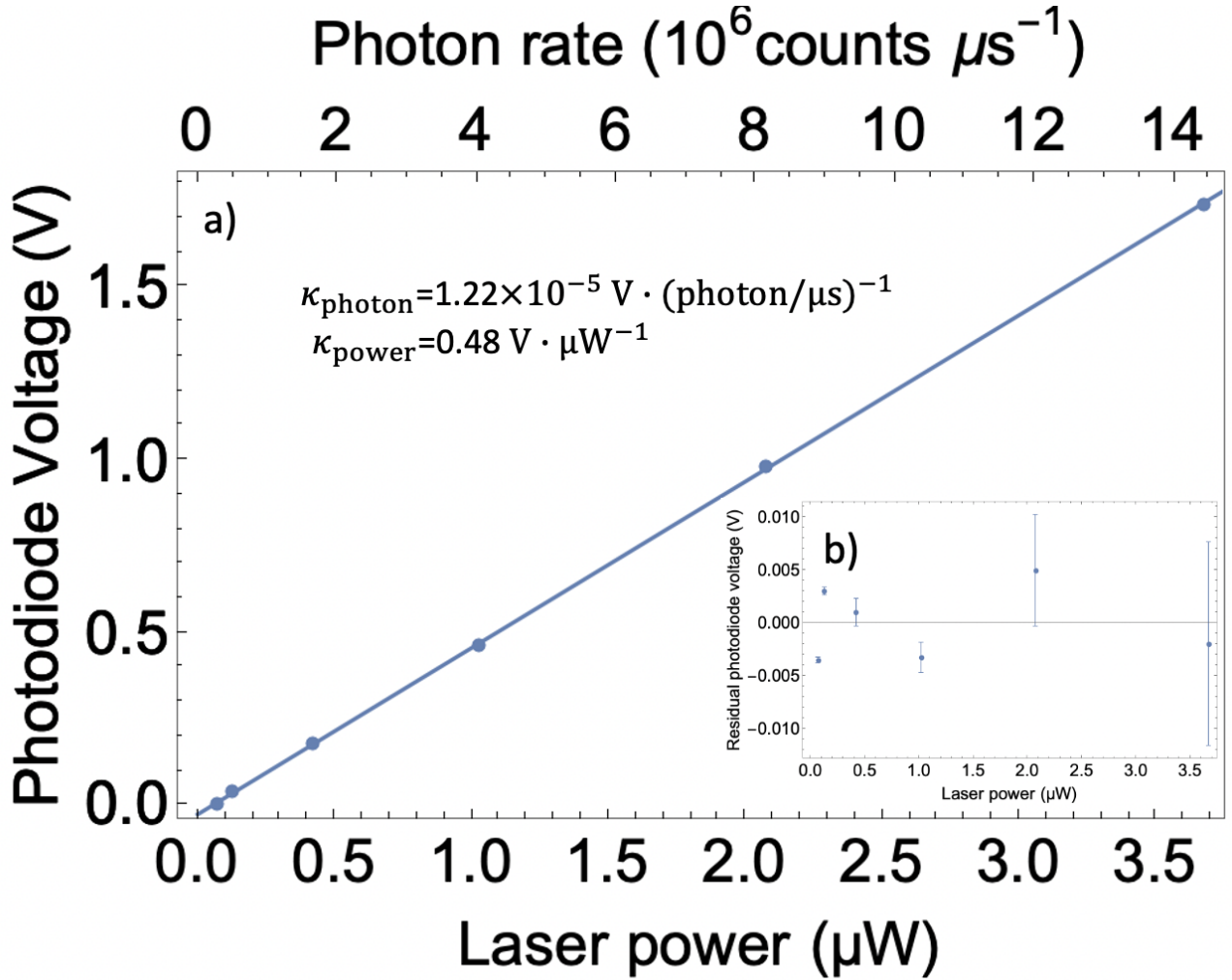


Figure 6.25: Calibration of the photodiode that was used to estimate the number of atoms in $F = 2$. a) The voltages were measured with the same data-acquisition device for atom detection. One of the MOT beams was used as the test beam, whose power was varied with the 5-beam AOM and measured with a power meter (Thorlabs, PM100D). The gradient of the linear fit shows that the photodiode had a power responsivity $\kappa_{\text{power}} = 0.480 \text{ V}/\mu\text{W}$ or a photon counting rate responsivity $\kappa_{\text{photon}} = 1.22 \times 10^{-5} \text{ V}/(\text{photons}/\mu\text{s})$. b) Fitting residuals.

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Chapter 7

Acceleration-sensitive interference

After preparing atoms of sub-10 μK , this chapter presents how a ball-induced acceleration was measured with an acceleration-sensitive atom interferometer.

Section 7.1 presents a novel method to minimise the AC Stark shifts to the clock transition of our interferometer. After that, Section 7.2 models the damping behaviour of velocity-sensitive Rabi oscillations and discusses the optimisation of this damping with respect to momentum distribution, Raman beam intensity ratio and retro-reflection configuration, and the intensity inhomogeneity. It concludes that the damping in our Rabi oscillations after velocity-selection were limited by the intensity inhomogeneity.

With this optimised velocity-sensitive Rabi oscillation, Section 7.3 presents measuring the cloud temperature by applying a single Raman pulse to drive counter-propagating transitions. It also discusses the minimisation of the inevitable co-propagating transitions. Then Section 7.4 presents measuring the ball-induced acceleration by applying three Raman pulses to build an atom Mach-Zehnder interferometer. The acceleration sensitivity of our atom interferometer in the fountain configuration is concluded to be at least a factor of two more sensitive than the last generation in the free-fall configuration. The statistical noise in our interferometer is analysed in Section 7.6, with systematics to explain our non-zero measurement discussed in Section 7.5.

7.1 Measuring the differential AC Stark shift

The differential AC Stark shift of the clock transition limited the efficiency of a velocity-sensitive Rabi oscillation driven by a pair of counter-propagating Raman beams, because different atoms could experience different local intensities of the Raman beam and acquire different two-photon resonance frequencies.

To minimise the impact of this detuning inhomogeneity, the magic ratio $I_1/I_2 = 0.583$ had been calculated for the two-beam configuration and used in the last generation [1, p.14]. In this generation, we noticed that this was incorrect for the three-beam configuration. With the microwave spectra in two-photon detuned Raman light fields discussed in this section,¹ the magic ratio was directly measured as $I_1/I_2 = 1.16 \pm 0.05$ ($\frac{I_1}{I_2+I'_2} = 0.58 \pm 0.03$) for the Raman-I rejection configuration shown in Fig. 7.1 (a) and $I_1/I_2 = 0.29 \pm 0.01$ ($\frac{I_1+I'_1}{I_2} = 0.58 \pm 0.02$) for the Raman-II rejection configuration shown in Fig. 7.1 (b).

Although the discussion throughout Section 7.1 implemented the *Raman-I* rejection configuration to illustrate our method of using microwave spectroscopy to obtain the magic ratio, the same method was applied to obtain the intensity ratio in our chameleon measurement using the *Raman-II* rejection configuration, whose better performance is discussed in Section 7.2.2.

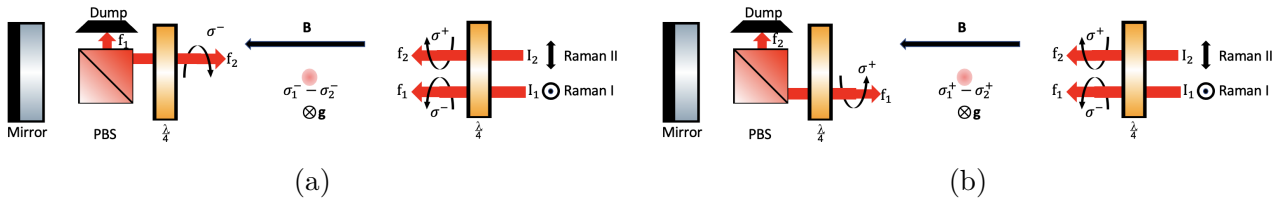


Figure 7.1: Schematics of rejecting different retro-reflected Raman beams. Switching between these two configurations was achieved by changing the relative orientation between the PBS and the quarter waveplate before the PBS. The polarisations of the incident Raman beams are linear and orthogonal to each other as indicated in the graphs. a) Rejecting the Raman-I laser after retro-reflection, which was used to illustrate the measurement of AC Stark shift using microwave spectroscopy. b) Rejecting the Raman-II laser after retro-reflection, which was used for the chameleon measurement.

¹The method was inspired by [2, p.108] and developed independently in November 2021. However, we then realised a very similar technique published in February 2021 [3]. Another direct measurement technique, within our best knowledge, was measuring displacement [4].

7.1.1 Sequence

The sequence of measuring the differential light shift is shown in Fig. 7.2. We first prepared our laser-cooled atoms into the non-fluorescent $F=1$ state (approximately uniformly distributed into three magnetic substrates) by turning off repump light at the end of the optical molasses stage. Applying a magnetic bias field, $B \approx 0.4\text{G}$, along the Raman axis lifts the degeneracy of magnetic substates belonging to the same hyperfine state. The Zeeman splitting between substates is expected to be about 280 kHz [5], however, the allowed Raman transitions are separated by $2 \times 280\text{ kHz}$, because the selection rule only allows $\Delta m = 0$ transitions driven by Raman beams of the same polarisation. To suppress a significant population transfer from $F=1$ to $F=2$ via Raman transition, we deliberately set the two-photon detuning to be 280 kHz away from the clock transition. Considering a 280 kHz two-photon detuning is much smaller than the single-photon detuning $\Delta \approx -1.1\text{ GHz}$, which was measured by a wavemeter (HighFinesse WS-6), the differential light shift of interest only gets a negligible systematic change of 0.028% .

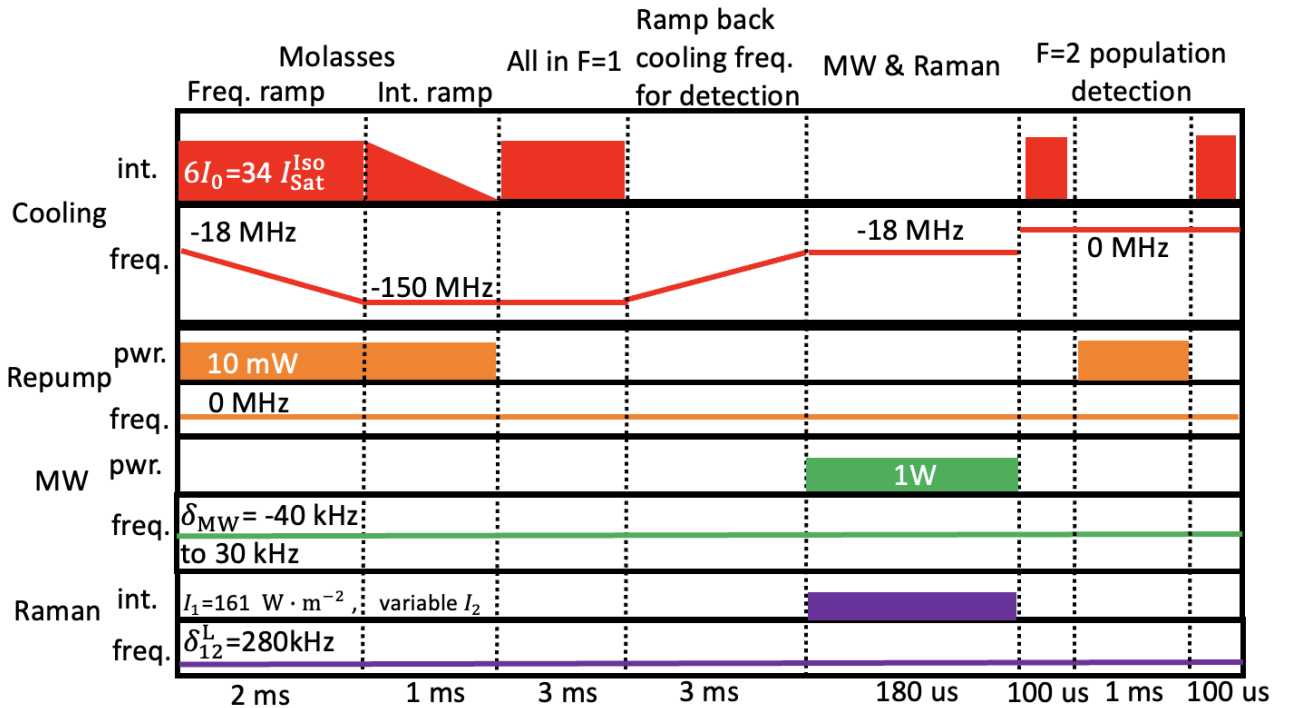


Figure 7.2: Sequence of measuring the differential AC Stark shifts to the clock transition using microwave spectroscopy. The steps of loading atoms into 3D MOT have been omitted for clarity. The two-photon Raman laser detuning was set to 280 kHz . Varying the microwave detuning from -40 kHz to 30 kHz produced the spectrum that can probe the differential AC Stark shifts to the clock transition frequency.

In the presence of two-photon detuned Raman laser fields, we simultaneously applied a microwave (MW) π -pulse of 180 μs , which transferred the population from the non-fluorescent $F=1$ state to the fluorescent state $F=2$. This population transfer was measured by normalized fluorescence detection [2] with six MOT beams, whose frequency was resonant with the cycling transition between $|F=2\rangle \rightarrow |F'=3\rangle$. By scanning the frequency of the microwave near the original clock transition frequency between $|F=1, m_F=0\rangle \rightarrow |F=2, m_F=0\rangle$, we obtained a microwave transfer spectrum that is perturbed by the total differential light shift. Comparing the perturbed MW spectra with an unperturbed spectrum reveals the information about the differential AC Stark shift, which is the theme of Section 7.1.2.

7.1.2 Measuring the magic ratio

The differential AC Stark shift on the clock transition frequency is seen in the microwave spectra shown in Fig. 7.3 (a). Data in black shows the unperturbed $|F=1, m_F=0\rangle$ to $|F=2, m_F=0\rangle$ transition driven by a microwave π - pulse of 180 μs . Data in other colours are the same transition, while Raman II laser intensity was varied by varying the EDFA2 power, but Raman I laser intensity was kept constant at 161 W/m². They clearly display an intensity-dependent frequency shift as summarised in Fig. 7.3 (b). These frequency shifts are our direct monitors of the mean differential AC Stark shifts averaged over the cloud of atoms. The zero-crossing gives us the correct intensity of Raman-II to minimise the differential AC Stark shifts to the clock transition. Thus the magic ratio was measured as $I_1/I_2 = 1.16 \pm 0.05$ for the Raman-I rejected three-beam configuration. Assuming the retro-reflected Raman-II' beam has the same intensity as the incident Raman-II beam, $\frac{I_1}{I_2+I_2'} = 0.58 \pm 0.03$, is consistent with the two-beam predication of 0.583.

Solid lines in Fig. 7.3 (a) are fittings with a Lorentzian model. In the absence of any inhomogeneity, the spectral lineshape of the population transferred into $F=2$ state, $P_{F=2}(\delta)$, is expected to be [6]

$$P_{F=2}(\delta) = \frac{\Omega^2}{\Omega^2 + \delta^2} \sin^2 \left(\sqrt{\Omega^2 + \delta^2} \tau / 2 \right), \quad (7.1)$$

where Ω is the magnetic dipole coupling strength between the clock states determined by

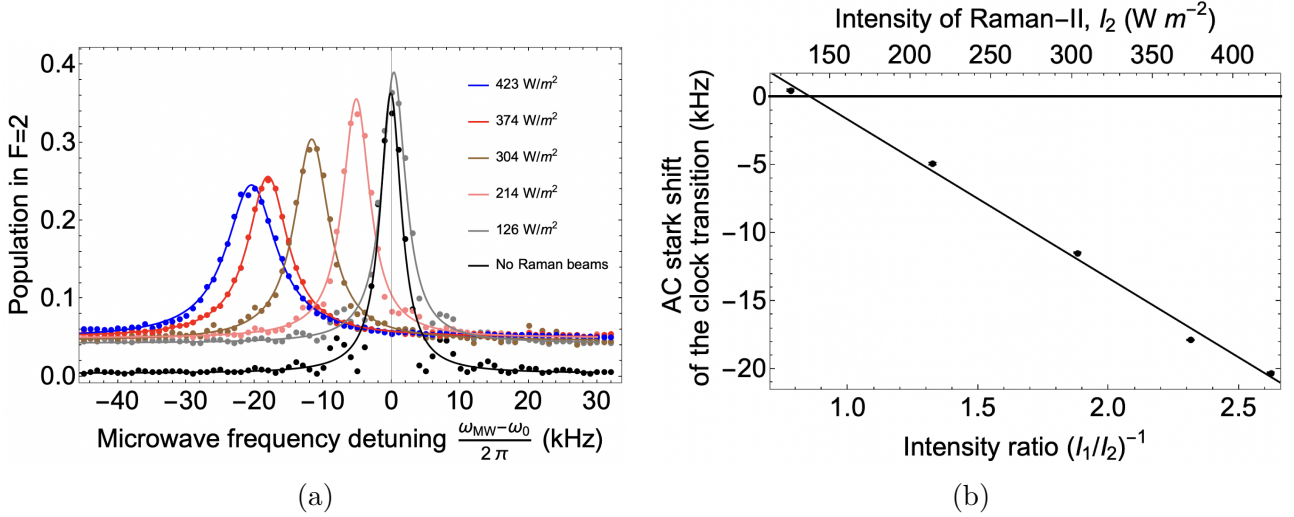


Figure 7.3: Microwave spectra in the presence of two-photon detuned Raman fields when the Raman-II intensity was varied as the Raman-I intensity was kept to be 161 W/m^2 . (a) The light shift modified microwave spectra fitted with Lorentzian models. The model better describes the lineshape for peaks of larger shifts. (b) The AC Stark shift as a function of the intensity of the Raman-II beam. It follows the expected linear relationship, which was discussed in Section 2.3.2 for Raman-II rejected beam configuration but is also valid for the Raman-I rejected beam configuration here.

the power of the microwave pulse, $\delta = \omega_{\text{MW}} - \omega_0(I_1, I_2)$ is the detuning between the tunable microwave frequency ω_{MW} and the transition frequency $\omega_0(I_1, I_2)$ that has been shifted by the Raman light and finally τ is the pulse length of the microwave that was set to $180 \mu\text{s}$. However, averaging over the distribution of detuning leads to a more complicated lineshape, which is a convolution between the detuning distribution and $P(\delta)$. In the limiting case, one can simplify the convolution result by replacing $\sin^2\left(\sqrt{\Omega^2 + \delta^2} \tau/2\right)$ in Eq. (7.1) with $1/2$, and expect the lineshape to approximately follow $P_{F=2}(\delta) \approx \frac{1}{2} \frac{\Omega^2}{\Omega^2 + \delta^2}$ [6]. This is a Lorentzian model.²

7.1.3 Measuring the distribution of the differential light shifts

In Fig. 7.3 (a), the peaks of the microwave spectra became broadened due to inhomogeneous differential AC Stark shifts. Using the unperturbed MW spectrum as a deconvolution kernel, we can obtain the distribution of the differential AC Stark shifts across the cloud of atoms as shown in Fig. 7.4 (a). All of the distribution of AC Stark shifts agree with a Gaussian model very well. The reference profile in Fig. 7.4 (a) was obtained by the deconvolution of the unperturbed spectrum with itself, which marks where the zero differential AC Stark shift

²It is worth noting that the Lorentzian lineshape here is not associated with intrinsic lifetime limited states, but merely an approximation because of the detuning inhomogeneity induced by the differential AC Stark shifts.

was and characterised the intrinsic width of 2 kHz for this method. When the intensity of the Raman-II beam increased, the widths of the distribution obtained from the perturbed spectra became larger as the centre of the Gaussian distribution shifted further away from the non-shift position. As shown in Fig. 7.4 (b), their full widths at half maximum (FWHM) are larger than the quadratic sum of the intrinsic width due to the sampling step and their lifetime limited FWHM calculated by the single-photon scattering rates with Eqs. (2.32) and (2.33) (modified for Raman-I rejected configuration). Therefore, the observed Gaussian distribution can be understood by a constant intensity gradient sampled by the atom cloud of a Gaussian profile.

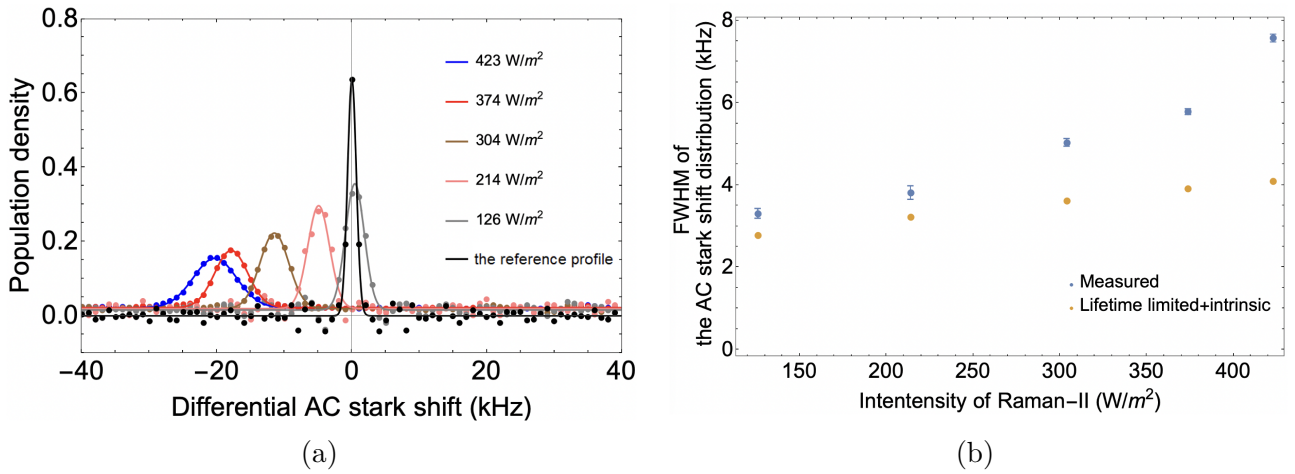


Figure 7.4: Distribution of the differential AC Stark shift across the atom cloud. (a) The population density as a function of the differential AC Stark shift. It was obtained by the deconvolution of the microwave spectra perturbed by the two-photon detuned Raman fields with the unperturbed microwave spectrum in Fig. 7.3 (a) as the deconvolution kernel. The coloured lines are Gaussian models fitted to data with various Raman-II intensities, while the Raman-I intensity was kept as 161 W/m². (b) The full-width at half maximum (FWHM) of the differential AC Stark shifts distributions. The measured FWHM from Fig. 7.4 (a) are all larger than the quadratic sum of the lifetime limited FWHM calculation and the intrinsic resolution. This suggests that the observed distribution of AC Stark shift was caused by an intensity gradient across the atom cloud.

After removing the mean AC Stark shifts to the clock transition frequency, $\delta_{AC} = \omega_0(I_1, I_2) - \omega_0$, the residual AC Stark shifts that were not perfectly cancelled can lead to a systematic phase shift, ϕ_{AC} , to our three-pulse interferometer signal as

$$P_{F=2} \propto \cos(\mathbf{k}_{\text{eff}} \cdot \mathbf{a}T^2 + \phi_{AC}). \quad (7.2)$$

To infer the phase shift induced by acceleration, $\mathbf{k}_{\text{eff}} \cdot \mathbf{a}T^2$, the most common technique is using k-reversal [7, 8, 9, 10, 11], which generates a pair of fringes

$$P_{F=2}^{(\pm k)} \propto \cos(\pm \mathbf{k}_{\text{eff}} \cdot \mathbf{a} T^2 + \phi_{\text{AC}}). \quad (7.3)$$

By comparing the relative phase shift between $P_{F=2}^{(+k)}$ and $P_{F=2}^{(-k)}$, the signal phase shift, $\mathbf{k}_{\text{eff}} \cdot \mathbf{a} T^2$, induced by the acceleration can be separated from the systematic phase shift, ϕ_{AC} , induced by δ_{AC} . This k-reversal technique needs two pairs of counter-propagating Raman beams that have distinguishable Doppler shifts, $\pm \mathbf{k}_{\text{eff}} \cdot \mathbf{v}_{\text{atom}}$, where $\mathbf{v}_{\text{atom}}$ is the velocity of the atom. However, $\pm \mathbf{k}_{\text{eff}} \cdot \mathbf{v}_{\text{atom}}$ in our setup was so small that two Raman transitions ($\pm \mathbf{k}_{\text{eff}}$) are indistinguishable. If these two nearly-degenerate Raman transitions were driven simultaneously, the interferometer contrast would be dramatically reduced [1, p.79]. Therefore, only one of the incoming Raman beams was retro-reflected as shown in Fig. 7.1 to avoid simultaneously driving the $\pm \mathbf{k}_{\text{eff}}$ transitions, which also made the k-reversal technique not applicable in our setup.

To suppress the systematics induced by the AC Stark shift, the position of the source mass was switched to two symmetric positions, $\theta_{\pm} = \pm 18^\circ$ in Fig. 7.10, to generate a pair of fringes

$$P_{F=2}^{(\pm)} \propto \cos(\mathbf{k}_{\text{eff}} \cdot \mathbf{a}_{\text{mirror}}^{\text{weighted}} T^2 \underbrace{\pm \mathbf{k}_{\text{eff}} \cdot \mathbf{a}_{\text{ball}} T^2}_{\phi_{\pm}} + \phi_{\text{AC}}), \quad (7.4)$$

where $\mathbf{a}_{\text{mirror}}^{\text{weighted}}$ was the weighted vibration-induced acceleration phase shift of the Raman mirror, which was measured with the MEMS accelerometer shown in Fig. 5.7 using a triangular weighting function described by Eq. (5.1) in the short-pulse limit, $\mathbf{k}_{\text{eff}} \cdot \mathbf{a}_{\text{ball}}$ was the projection of the ball-induced acceleration on the Raman axis, which was inverted by the switch of the ball position instead of the k-reversal technique. Comparing the phase shift between $P_{F=2}^{(+)}$ and $P_{F=2}^{(-)}$, $\phi_{\pm}$, gives the ball-induced acceleration phase shift, which is discussed with more details in Section 7.4.1.

7.2 Optimising the damped velocity-sensitive Rabi oscillation

After compensating for the differential AC Stark shifts on the clock transition $|F = 1, m_F = 0\rangle \rightarrow |F = 2, m_F = 0\rangle$, we could build an acceleration-sensitive interferometer with three velocity-sensitive Raman Rabi pulses. This section studies damping behaviours in a velocity-sensitive Rabi oscillation and optimises the Rabi oscillations with respect to the momentum distribution of the atom cloud by velocity selection (Section 7.2.1), the beam configuration and intensity ratio (Section 7.2.2) and finally the intensity homogeneity by enlarging the retro-reflection optics (Section 7.2.3).

7.2.1 Velocity selection

To drive a velocity-sensitive Rabi oscillation, the two-photon Raman detuning was fixed to be -50 kHz (Raman DDS frequency = 65.067 MHz) before applying a square pulse of Raman light to our sub- 10 μ K atoms. With a magnetic bias field of 0.38 G along the Raman axis to lift the Zeeman degeneracy,³ only atoms of $|F = 1, m_F = 0\rangle$ in the correct velocity group were transferred into $|F = 2\rangle$ and detected. The intensity ratio was set to minimise the AC Stark shift on the clock transition for the Raman-II rejection configuration, which is described in Section 7.2.2. As shown in Fig. 7.5 (a), the velocity-sensitive Rabi oscillation for a thermal ensemble of sub- 10 μ K was dominated by a linear background, which is modelled by

$$P_{F=2}(\tau) = \underbrace{B + R \cdot \tau}_{\text{incoherent transfer}} + \underbrace{\frac{A}{2} - \frac{A}{2} \exp\left(-\frac{\sigma \Omega_G^2 \cdot \tau^2}{2}\right) \cos(\mu \Omega_G \cdot \tau)}_{\text{coherent transfer}}, \quad (7.5)$$

where B is the constant background, R is the accumulation rate for the linear background due to an incoherent population transfer by the off-resonance single-photon scattering, and, finally, A is the coherent transfer amplitude. The coherent transfer from the correct velocity group was calculated by assuming that the generalised Rabi frequency of the Raman transition, $\Omega_G = \sqrt{\Omega^2 + \delta^2}$, followed a Gaussian distribution $G(\mu \Omega_G, \sigma \Omega_G)$ (due to the intensity inhomogeneity

³ $|F = 1, m_F = \pm 1\rangle \rightarrow |F = 2, m_F = \pm 1\rangle$ are ± 533 kHz away from the clock transition.

across the atom cloud suggested by Section 7.1.3), where $\mu\Omega_G$ is the mean generalised Rabi frequency and $\sigma\Omega_G$ is the width.

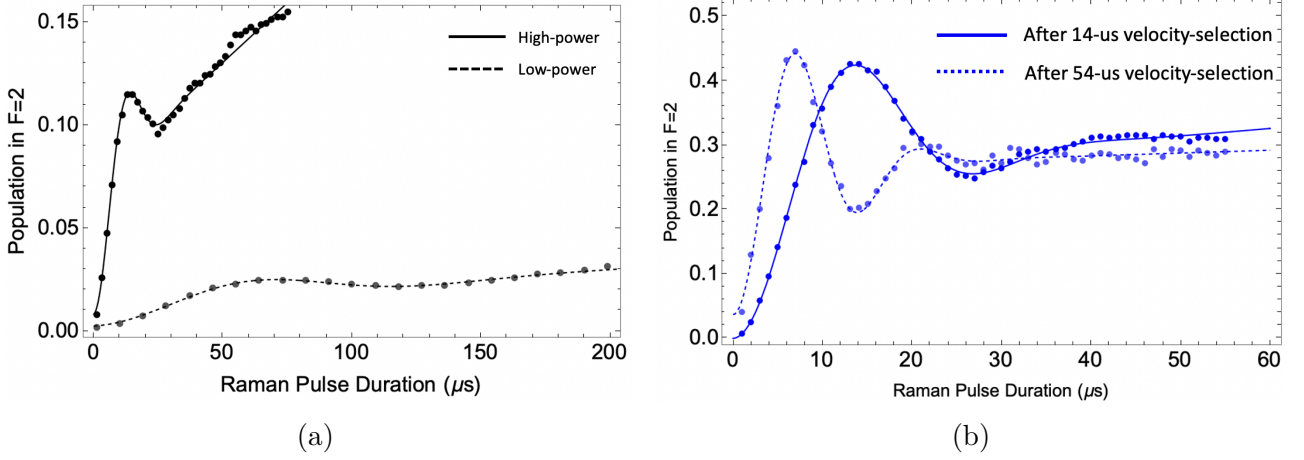


Figure 7.5: Optimising the velocity-sensitive Rabi oscillation with respect to the momentum distribution. Data were fitted to model Eq. (7.5). (a) Velocity-sensitive Rabi oscillation for a thermal atom cloud of sub-10 μK with Raman beams of different powers. (b) Velocity-sensitive Rabi oscillation for a velocity-selected ensemble produced by different selection strategies.

As shown in Fig. 7.5 (a), the fraction of the selected atoms by a high-power velocity selection pulse (coherently transferred 9.2%) was four times larger than that by a low-power velocity selection pulse (coherently transferred 1.9%), because their momentum spreads determined by their π -pulse durations, $\tau_\pi^{\text{HP}} = 14 \mu\text{s}$ and $\tau_\pi^{\text{LP}} = 54 \mu\text{s}$, approximately differed by a factor four.

To minimise the incoherent transfer from the wrong velocity classes, we applied double velocity selection pulses as discussed in Section 6.3.1. This velocity selection process significantly improved the velocity-sensitive Rabi oscillation as shown in Fig. 7.5 (b), where the coherent transfer amplitude increased to $\approx 40\%$ for the 14 μs selected atoms. To understand if the momentum spread was the limiting factor, the Raman pulse of the maximum power (whose π -pulse duration $\approx 7 \mu\text{s}$) was applied to the low-power selected atoms with $\tau_\pi = 54 \mu\text{s}$. It shows a very similar transfer amplitude.

As a conclusion, the coherent transfer fraction of a velocity-sensitive Rabi oscillation can be significantly improved by a velocity-selection Raman pulse if the atom ensemble has a momentum distribution much larger than the bandwidth of the probe Raman pulse. However, once the momentum spread was selected to be comparable (or smaller) than the bandwidth of the probe Raman pulse, the coherent transfer population was limited by other factors.

7.2.2 Intensity ratio of the Raman beams

Different intensity ratios and rejection choices of the retro-reflected beam can change the performance of the velocity-sensitive Rabi oscillation for velocity-selected atoms. As shown in Fig. 7.6, the worst Rabi oscillation, corresponding to $I_2 = 423 \text{ W/m}^2$ in Fig. 7.4 (a), had an intensity ratio $I_1/I_2 = 0.381 \pm 0.002$ in the Raman-I rejected configuration (blue). After setting to the measured magic ratio $I_1/I_2 = 1.16$ discussed in Section 7.1.2 for the same beam configuration, the damped Rabi oscillation (green) obtained an improved amplitude and dephasing time. Finally, the same method was applied to the Raman-II rejection beam configuration shown in Fig. 5.3, whose magic ratio was found to be $I_1/I_2 = 0.29$. Reducing the Raman-I power to achieve this ratio lead to the Rabi oscillation (black) of the largest transfer amplitude. Thus Raman-II rejection beam configuration was chosen to perform our interferometry experiment. In addition, our model described by Eq. (7.5) fits all of the damped Rabi oscillations fairly well, which implies that intensity gradient across the cloud is the major source of dephasing in our system.

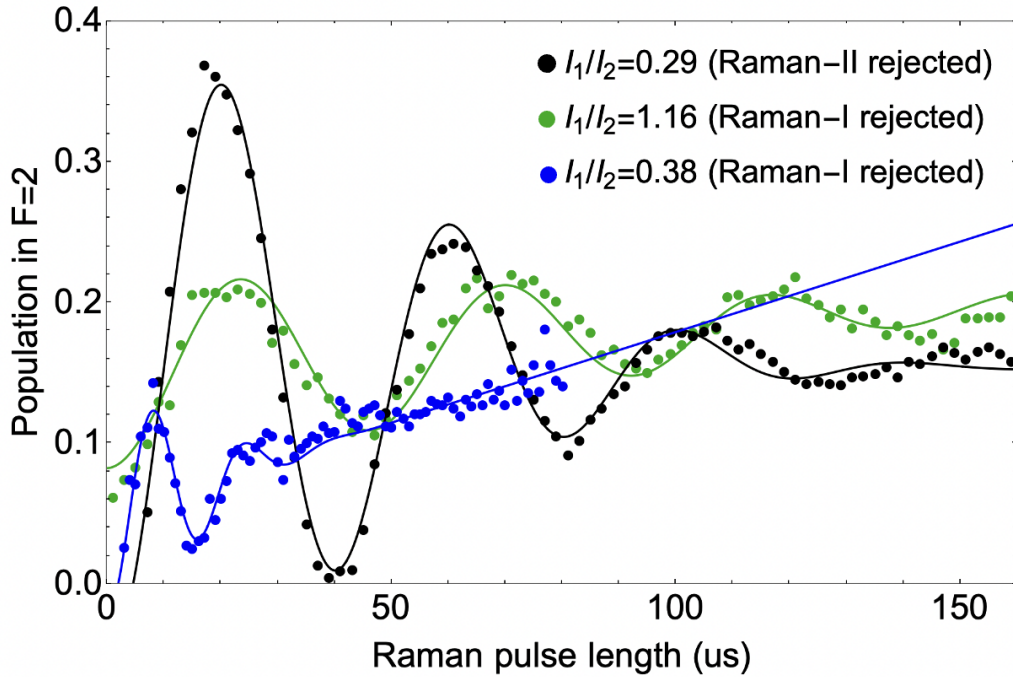


Figure 7.6: Optimising velocity-sensitive Rabi oscillations with respect to different intensity ratios and Raman beam configurations. The retro-rejected beam shown in Fig. 5.3 has two configurations. The Rabi oscillation with a non-magic intensity ratio (blue) had a smaller amplitude than that with a magic intensity ratio (green). Setting to the corresponding magic ratios, the Raman-II rejected configuration (black) had a larger amplitude than that of the Raman-I rejected configuration (green).

7.2.3 Raman PBS

In the last generation of the experiment, a 1-inch Raman PBS was used to realise the Raman-I rejection beam configuration to drive a free-fall atom interferometer. For the fountain atom interferometer, the performances of the Rabi oscillations for atom optics pulses can be significantly improved simply by replacing the 1-inch PBS with a 2-inch PBS, where the ratio of the peak intensities was set to be $I_1/I_2 = 0.29$ ($I_1 = 143 \text{ W/m}^2$, $I_2 = 492 \text{ W/m}^2$) for the Raman-II rejection configuration.

As shown in the dashed lines of Fig. 7.7 (a)-(c), all of the damped behaviours with the 1-inch Raman PBS fitted well with the model described by Eq. (7.5), suggesting the transfer efficiency was limited by the inhomogeneity of the Raman beam intensities. Notably, the mirror pulse showed the strongest damping, while the beam splitter pulse and the recombiner pulse illustrated similarly modest damping. This indicates that the atoms experienced more intensity inhomogeneities at their trajectory apex than at their initial and final position.

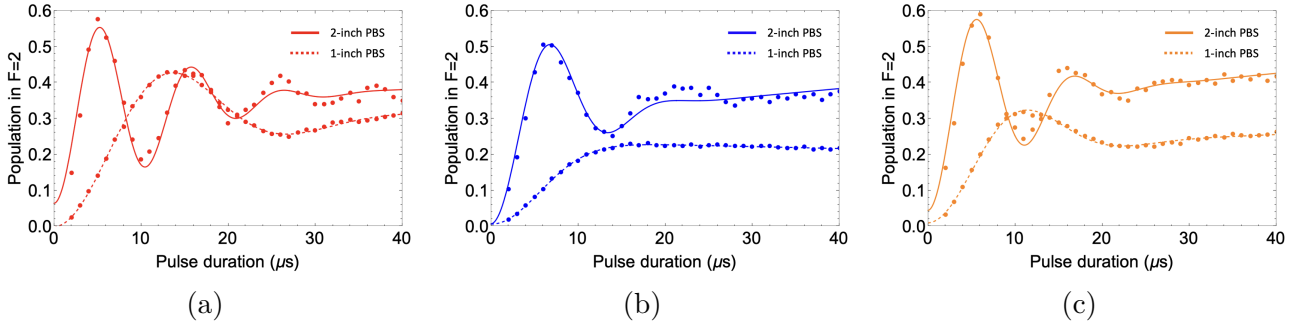


Figure 7.7: Improving the velocity-sensitive Rabi oscillations by enlarging the Raman PBS. The launched atoms were prepared in $|F = 1, m_F = 0\rangle$ with the double velocity-selection scheme discussed in Section 6.3.1, and interrogated with the same Raman pulses as used in the $2T=66$ ms interferometer. Data were fitted to the model where the Rabi oscillation was averaged over Gaussian-distributed Rabi frequencies described by Eq. (7.5). Data fitted to the dashed lines were obtained with the 1-inch PBS, and data fitted to the solid lines were obtained with the 2-inch PBS. (a) Beam splitter pulse. (b) Mirror pulse. (c) Recombiner pulse.

We performed a simulation taking into account the thermal expansion of a 5-10 μK cloud, the vertical travelling distance ≈ 1 cm for the launched atoms, and the e^{-2} intensity radius of the Raman beam $w = 15.6 \pm 0.6$ mm. The simulation could not explain the observed damping with the 1-inch PBS. Since the travelling distance ≈ 1 cm was comparable to the half size of the 1-inch Raman PBS, we replaced it with a 2-inch PBS. This replacement gave an improved Rabi

oscillation of the corresponding Raman pulses shown as solid lines in Fig. 7.7 (a)-(c).

This improvement concludes that the limiting factor of the transfer amplitude mentioned in Section 7.2.1 was the intensity inhomogeneity across the cloud.

7.3 Raman spectrum

Although the velocity-sensitive Raman pulse cannot transfer a significant population for a thermal ensemble without velocity selection, it can still map the Doppler shifts induced by the momentum distribution of an atom cloud. Section 7.3.1 presents the temperature measurement of the atom cloud with a velocity-sensitive Raman spectrum and its systematic error associated with the Raman pulse duration. The velocity-sensitive Raman spectrum also displayed a co-propagating Raman transition feature that is velocity-insensitive. Section 7.3.2 discusses the minimisation of the Rabi frequencies of the co-propagating Raman transitions.

7.3.1 Temperature measurement with counter-propagating Raman transitions

To measure the velocity distribution of the cloud, the Raman power was adjusted to drive a π -pulse of $\tau_\pi = 29 \mu\text{s}$. Then scanning the Raman DDS frequency gave the population in the $F=2$ as a function of two-photon Raman detuning, δ_{Raman} , displayed in Fig. 7.8 (a). The wings of this spectrum (red) were fitted to a Gaussian distribution,

$$P_2(\delta_{\text{Raman}}) = B + A \frac{1}{\sqrt{2\pi}\sigma_{\text{Raman}}} \exp\left(-\frac{(\delta_{\text{Raman}} - f_{\text{offset}})^2}{2\sigma_{\text{Raman}}^2}\right), \quad (7.6)$$

leads to an apparent σ -width of $\sigma_{\text{Raman}} = 90.2 \pm 1.8 \text{ kHz}$, while A is the coherent transfer amplitude and B is the constant background. The offset from the clock transition frequency, $f_{\text{offset}} = 12.8 \pm 0.1 \text{ kHz}$, was found to be different from the two-photon recoil shift of 15 kHz . This can be explained with a small horizontal velocity of 0.85 mm/s .

Since the Fourier width of the Raman pulse also contributed to the apparent width, σ_{Raman} , it is expected to be consistent with the actual width induced by the axial velocity distribution,

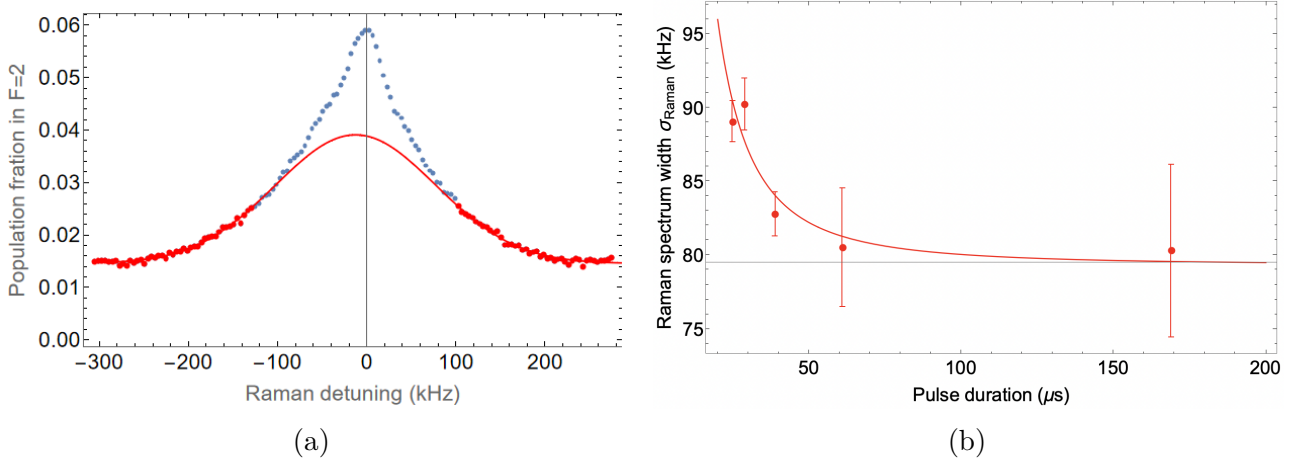


Figure 7.8: Temperature measurement with a velocity-sensitive Rabi oscillation. (a) Raman spectrum. The Gaussian feature is driven by the counter-propagating transitions, which was induced by the axial velocity distribution of the thermal cloud. The Gaussian wings (red) were used to measure the temperature. The middle peak was the co-propagating transition feature (more explanation in Section 7.3.2). (b) The width of the counter-propagating Raman spectrum as a function of π -pulse durations. At short pulse durations, the Fourier width of the Raman pulse had 10% contributions to the measured spectrum width.

Δf_{Raman} , when the probe pulse τ_π becomes infinitely long. For finite pulse durations, $\sigma_{\text{Raman}} = \sqrt{\Delta f_{\text{Raman}}^2 + (1/\tau_\pi)^2}$ [12, p.96] was verified by measuring the Raman spectrum with different π -pulse duration (using different Raman powers) as shown in Fig. 7.8 (b). This reveals $\Delta f_{\text{Raman}} = 79.4 \pm 1.4 \text{ kHz}$, which gives the axial temperature of our cloud, $T_{\text{axial}} = 10 \pm 0.35 \mu\text{K}$, by

$$T_{\text{axial}} = \frac{m \cdot (\lambda/2)^2 \cdot (\Delta f_{\text{Raman}})^2}{k_B}, \quad (7.7)$$

where m is the atomic mass of ^{87}Rb , λ is the wavelength of the Raman beam ($\approx 780 \text{ nm}$), the factor of the two comes from two-photon transition and k_B is the Boltzmann constant.

7.3.2 Minimise the coupling of co-propagating Raman transitions

Incoming Raman beams with an ideal $\sigma_1^- \sigma_2^+$ polarisation illustrated in Fig. 5.3 are not able to drive co-propagating Raman transitions. However, as seen in Fig. 7.8 (a), co-propagating features was inevitable. This comes from polarisation imperfection where Raman-I beam had σ^+ components. This section presents the technique to minimise this imperfection.

After blocking the retro-reflected Raman beams, we performed resonant Rabi oscillations with co-propagating Raman beams at various quarter-waveplate (QWP) orientations. To make

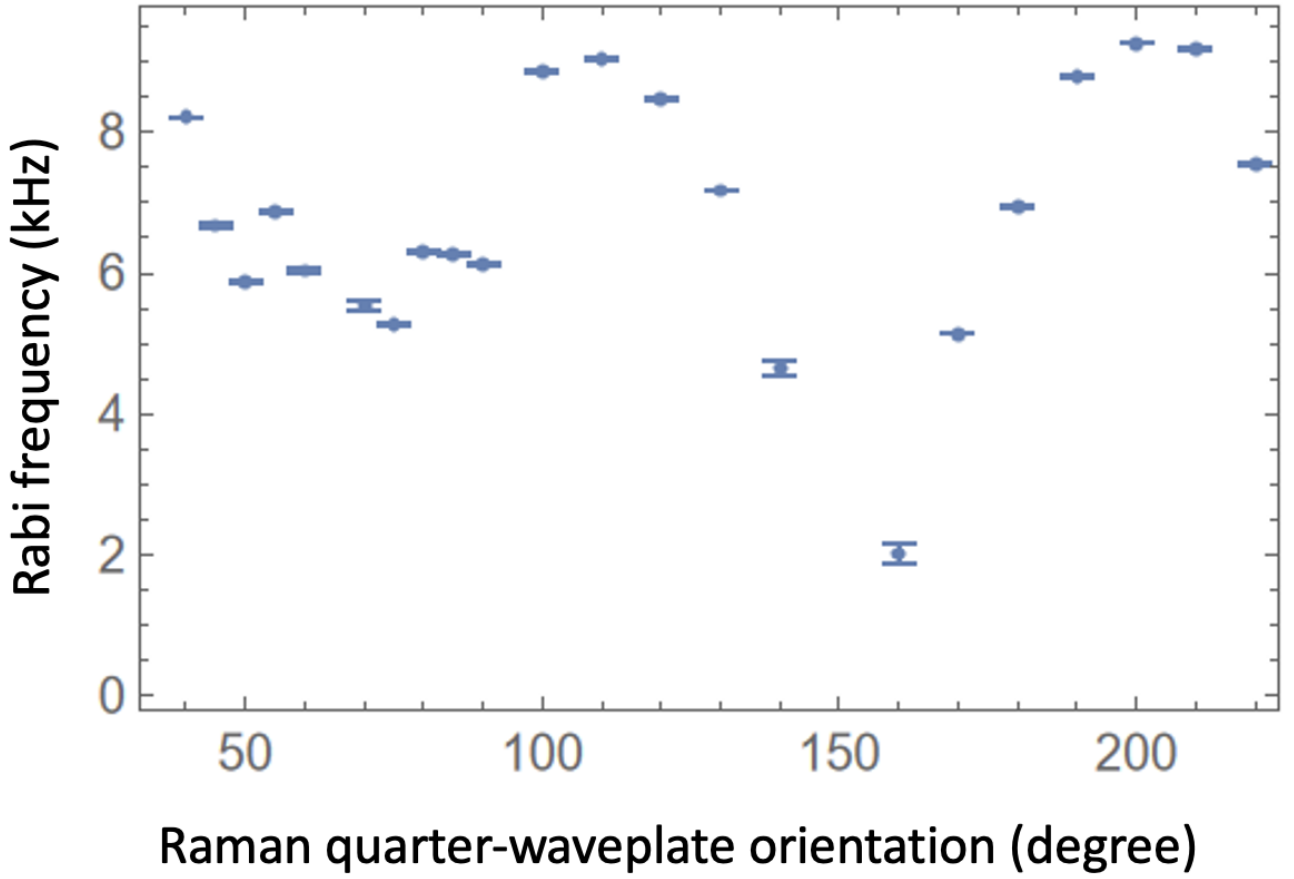


Figure 7.9: The Rabi frequencies of the co-propagating transition as a function of the Raman quarter-waveplate orientation. The minima at 70° and 160° did not have the same Rabi frequency, suggesting that the two Raman lights did not have perfect linear polarisation.

sure the co-propagating Raman pulse on resonance, its Raman spectra at various QWP angles were obtained by scanning the two-photon Raman detuning with a fixed pulse duration. The peak locations of the spectra changed as a function of the QWP orientation, presumably due to uncanceled differential AC Stark shift that varied with different polarisation configurations.

If the incident Raman-I and Raman-II beams had perfectly orthogonal linear polarisation, the Rabi frequencies should go down to zero at every 90° . As shown in Fig. 7.9, it displayed local maxima/minima every 90° , but the local minima separated at 90° did not agree with each other and were not equal to zero. This suggests polarisation imperfections in the incident Raman beams. Verified with a polarimeter, their polarisations were found not perfectly linear, though fairly orthogonal to each other. Setting the QWP orientation to 160° , we found the couplings ratio between counter-propagating transition and co-propagating transition $\Omega_{CT}/\Omega_{CO} \approx 6$.

7.4 The measurement of Chameleon acceleration

This section describes our two-stage chameleon measurement. The first stage is to compare the relative phase difference between two acceleration-sensitive interference fringes, where each fringe was taken at the ball position $\theta_{\pm} = \pm 18^\circ$ shown in Fig. 7.10. The second stage is to convert the relative phase shift to the ball-induced acceleration, $\mathbf{a}_{\text{ball}}$. This two-stage measurement is called a block and generates one chameleon acceleration measurement of $\mathbf{n} \cdot \mathbf{a}_{\text{ball}}$ ($\mathbf{n}$ is the unit vector along the Raman axis). This sequence is detailed in Section 7.4.1. Then 459 blocks of $\mathbf{n} \cdot \mathbf{a}_{\text{ball}}$ were averaged over 24 hours to give an adjusted weighted mean of $(\mathbf{n} \cdot \mathbf{a}_{\text{ball}})^{\text{AWM}} = -3.144 \pm 0.270 \mu\text{m/s}^2$ presented in Section 7.4.2. Section 7.4.3 concludes our acceleration sensitivity has been improved for at least a factor of two.

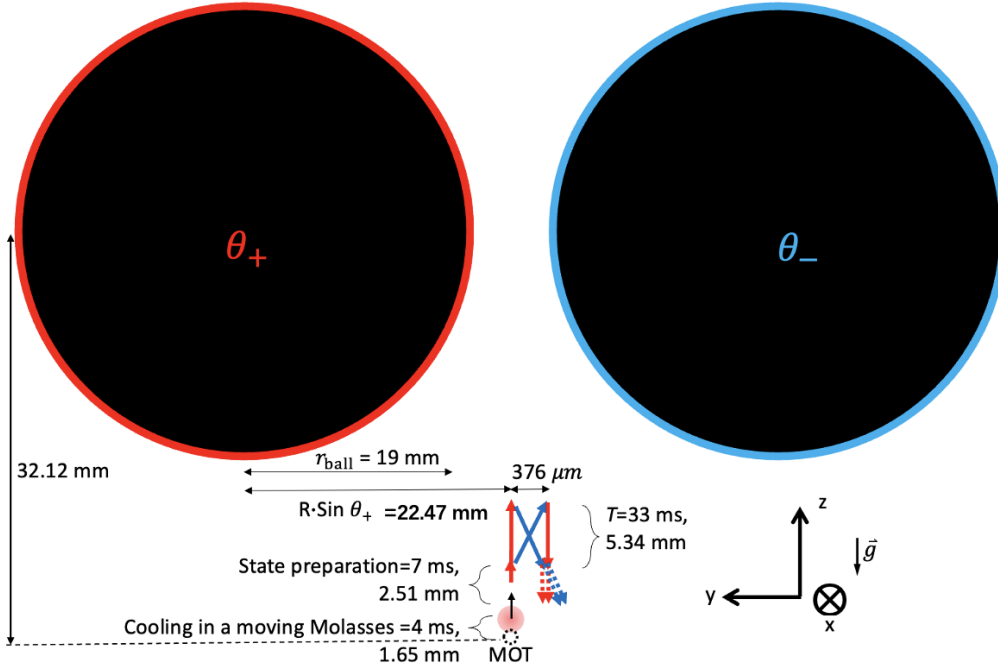


Figure 7.10: Schematics of the ball positions, $\theta_{\pm} = \pm 18^\circ$, relative to the atomic fountain. The coordinates of the ball centre are $\{-55 + R \cdot \cos \theta_{\pm}, R \cdot \cos \theta_{\pm}, 32.12\}$ in the unit of mm, where $R = 72.74 \text{ mm}$ is the length of the crossbar that moves the ball shown in Fig. 4.4b. The origin of the coordinates was chosen to be the centre of the chamber at which the MOT was formed. The x-y axes are perpendicular and parallel to the Raman axis, which is 45° off from the X-Y axes in Fig. 4.4b that were aligned with the X-Y MOT beams. The cloud of atoms was cooled in a moving molasses for 4 ms with the sequence shown in Fig. 6.14. The distance of 1.65 mm assumes they moved at a constant speed for $\approx 4 \text{ ms}$ after following the moving molasses rapidly. The state preparation involves Raman velocity selection and optical pumping between hyperfine states shown in Fig. 6.17, during which the distance the atoms moved was calculated by assuming they were decelerated by the Earth's gravity for most of the 7 ms after being released from the moving molasses. The state preparation took 6 ms and an extra 1 ms for completely turning off the cooling and repump light for the interferometry.

7.4.1 Sequence pattern of switching the ball position

It took successive blocks of measurements shown in Fig. 7.11 to sense the force induced by the intra-vacuum source mass. After switching the ball to $\theta_{\pm} = \pm 18^\circ$, 35 shots of $\{a_{\text{mirror}}^{\text{weighted}}, P_{F=2}\}$ were taken at each ball position. The chameleon-sensitive phase shifts of the n -th block, $\phi_{\pm}^{(n)}$, were estimated by fitting the last 25 shots of $\{a_{\text{mirror}}^{\text{weighted}}, P_{F=2}\}$ at each ball position⁴ to a constrained three-parameter model,

$$P_{F=2}(a_{\text{mirror}}^{\text{weighted}}; A, B, \phi_{\pm}^{(n)}) = B + A \cos(k_{\text{eff}} T^2 a_{\text{mirror}}^{\text{weighted}} + \phi_{\pm}^{(n)}), \quad (7.8)$$

where B is the mean value of the fringe and A is the fringe amplitude with parameter constraints $\{A > 0, B > 0, 0 < \phi_{\pm}^{(n)} \leq 2\pi\}$. The acceleration sensitivity factor, $k_{\text{eff}} T^2$, was assigned by substituting $k_{\text{eff}} = \frac{2\pi}{780.241209686/2}$ rad /nm and $T = 33$ ms using the vacuum wavelength of the $5^2S_{1/2} \rightarrow 5^2P_{3/2}$ transition. Each block took less than three minutes (2.78 mins) to generate one chameleon acceleration measurement, $a_{\text{ball}}^{(n)} = \mathbf{n} \cdot \mathbf{a}_{\text{ball}}^{(n)}$, by

$$\mathbf{n} \cdot \mathbf{a}_{\text{ball}}^{(n)} = \frac{1}{2} \frac{1}{k_{\text{eff}} T^2} (\phi_+^{(n)} - \phi_-^{(n)}) \quad (7.9)$$

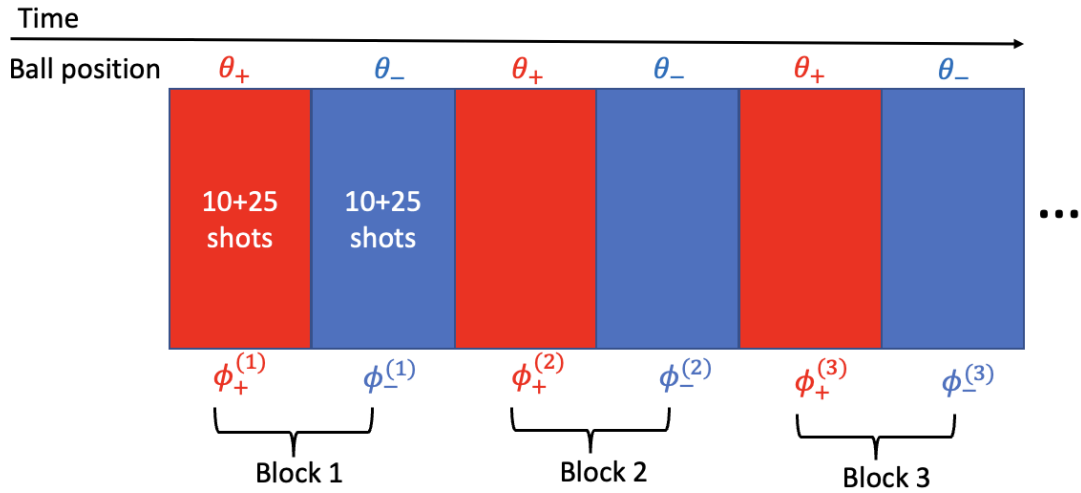


Figure 7.11: Experimental pattern of sensing the ball-induced acceleration. Each ball-induced acceleration measurement was generated by one block that took 35 shots of $\{a_{\text{mirror}}^{\text{weighted}}, P_{F=2}\}$ for each ball position $\theta_{\pm} = 18^\circ$. At each ball position, the first 10 shots were discarded for pressure relaxation.

⁴The first 10 shots at each ball position were discarded because the pressure of the science chamber increased from 5×10^{-9} mbar to 3×10^{-8} mbar as switching the ball position. It relaxed down below 8×10^{-9} mbar after 20 seconds (10 shots). This is due to the ageing of the rotary vacuum feed-through, which is under replacement.

7.4.2 Preliminary result of the Chameleon acceleration

Following the sequence pattern discussed in Section 7.4.1, 487 blocks of measurements were conducted in 24 hours on March 25, 2023. The measurement of the chameleon acceleration is displayed in Fig. 7.12 (a). Each data point was generated by a fringe comparison inside a block shown in Fig. 7.12 (b), where the mirror acceleration was randomly sampled by vibration noise and recorded by the MEMS accelerometer. The error bars of $a_{\text{ball}}^{(n)}$, $\sigma a_{\text{ball}}^{(n)}$, are obtained by propagating the standard error of $\phi_{\pm}^{(n)}$, $\sigma \phi_{\pm}^{(n)}$, through Eq. (7.9), where $\sigma \phi_{\pm}^{(n)}$ are given by the constrained non-linear model fit of Mathematica (default settings). The lowest and the highest 3% of $a_{\text{ball}}^{(n)}$ data (rounded to the floor), marked as red in Fig. 7.12 (a), were removed to give a more robust estimation of the statistics. The rest 94% of the data (459 blocks marked in black) gives a weighted mean (WM)

$$a_{\text{ball}}^{\text{WM}} = -3.144 \pm 0.141 \mu\text{m/s}^2 \quad (7.10)$$

using $a_{\text{ball}} = \frac{\sum_n (\sigma a_{\text{ball}}^{(n)})^{-2} a_{\text{ball}}^{(n)}}{\sum_n (\sigma a_{\text{ball}}^{(n)})^{-2}}$, and the standard error of the weighted mean of $0.141 \mu\text{m/s}^2$ was calculated with $((\sum_n (\sigma a_{\text{ball}}^{(n)})^{-2})^{-1})^{\frac{1}{2}}$.

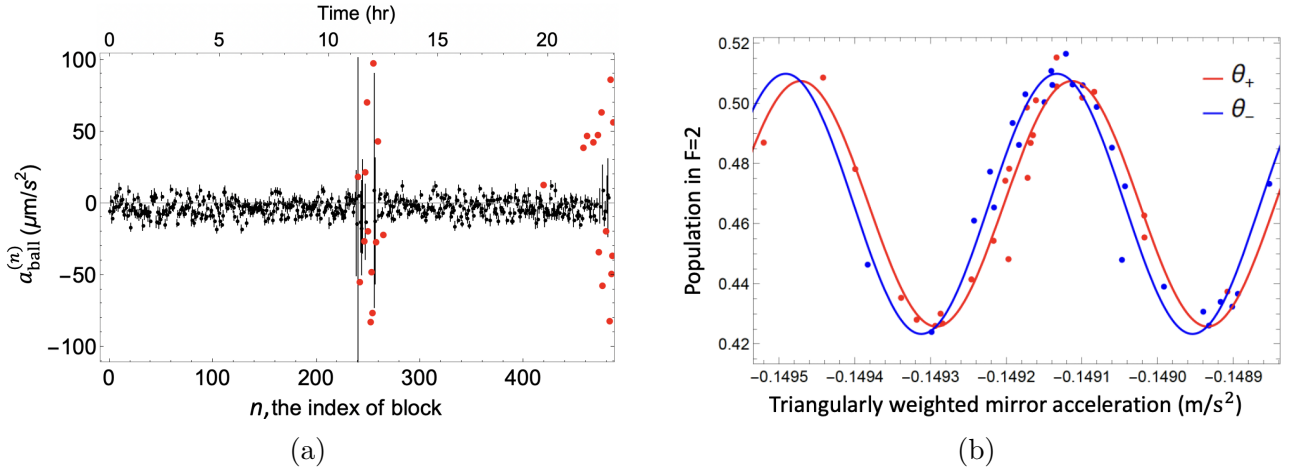


Figure 7.12: Measurement of Chameleon acceleration with a $2T=66$ ms atom interferometer of a fountain configuration. (a) 24-hour measurements of a_{ball} . Red data were trimmed off, around which the repump laser became unlocked. (b) Fringe fittings for each ball position within one block. The MEMS voltage signals were weighted with a triangular weighting function shown in Fig. 5.5 before converting into acceleration with specifications detailed in Section 5.3.2.

The error of the weighted mean is valid only if $\sigma a_{\text{ball}}^{(n)}$ had been correctly estimated. To check this, we calculated the reduced chi-square,

$$\chi_r^2 = \frac{1}{N_{\text{trim}} - 1} \sum_n (a_{\text{ball}}^{(n)} - a_{\text{ball}})^2 / (\sigma a_{\text{ball}}^{(n)})^2 = 3.67, \quad (7.11)$$

where $N_{\text{trim}} = 459$ is the number of blocks to give the trimmed weighted mean. The reduced chi-square is significantly larger than 1, indicating an underestimation of $\sigma a_{\text{ball}}^{(n)}$ in each measurement. Taking this underestimation (further discussed in Section 7.6) into account, the adjusted weighted mean (AWM) is

$$a_{\text{ball}}^{\text{AWM}} = -3.144 \pm 0.270 \mu\text{m/s}^2, \quad (7.12)$$

where the standard error of $a_{\text{ball}}^{\text{AWM}}$ is obtained with multiplying the standard error of $a_{\text{ball}}^{\text{WM}}$ by $\chi_r = \sqrt{3.67}$. This shows a non-zero negative result of more than $10\text{--}\sigma$ significance, which is confirmed in Section 7.4.3 with more discussion on this negative result in Section 7.4.4.

The stability of the fringe fitting process is summarised in Fig. 7.13. The root mean square of the population fitting residuals for fringes at both ball positions, shown in Fig. 7.13 (a), drifted over time but displayed a very similar level of noise. Therefore, the fitting quality changed over this dataset taken in more than 20 hours. However, the relative phase shifts between a pair of fringes shown in Fig. 7.12 (b) were not due to different fitting qualities, since the fitting qualities were very similar for both fringes all the time.

The phase estimation of $\phi_{\pm}$ changed over time as shown in Fig. 7.13 (b). However, both ϕ_+ and ϕ_- illustrated the same drifting pattern, which indicates that our switching strategy in Fig. 7.11 significantly suppressed this drift. Meanwhile, the ϕ_+ were overall smaller than ϕ_- , which was a different presentation of the same information shown in Fig. 7.12 (a).

The ball-induced accelerations showed quieter measurements whose index of block ranged from $n = 140$ to $n = 150$. However, neither of the fringe amplitudes, A , and fringe middle points, B , which were defined in Eq. (7.8), had the same quieter characteristics within the same range of the index of blocks shown in Fig. 7.13 (c) and Fig. 7.13 (d). This shows that our fitting quality was not the dominant source of noise in the estimation of the $a_{\text{ball}} = (\phi_+ - \phi_-)/(2k_{\text{eff}}T^2)$.

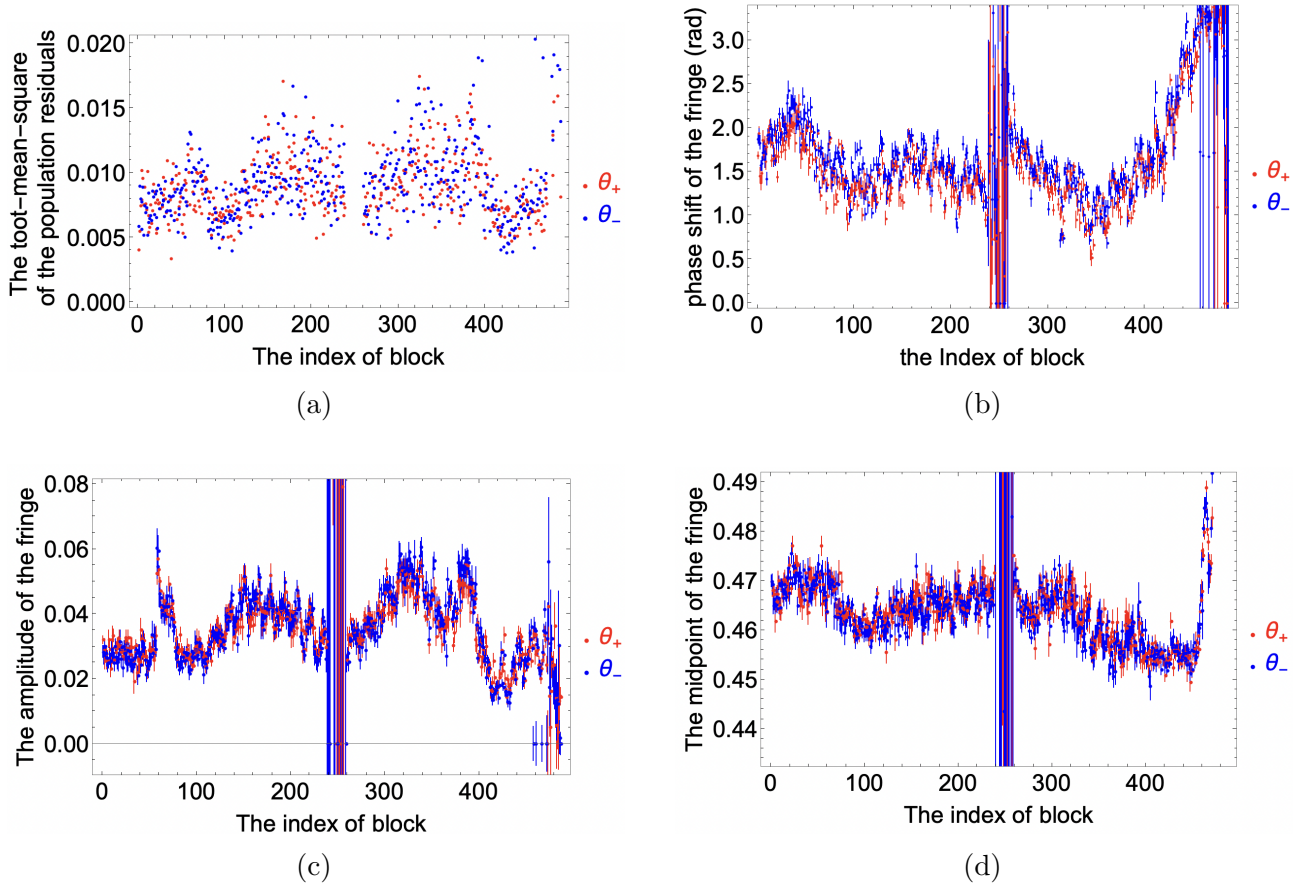


Figure 7.13: Stability of the fringe fittings for the chameleon measurement. (a) The root mean square of the fitting residuals as a function of the index of the block. (b) Fringe phases as a function of the index of the block. (c) Fringe amplitudes as a function of the index of the block. (d) Fringe mid-point as a function of the index of the block.

To check if the relative weights given by $(\sigma a_{\text{ball}}^{(n)})^{-2}$ make a difference, the unweighted sample distribution (Fig. 7.14 (a)) was evaluated against the weighted sample distribution (Fig. 7.14 (b)). The standard deviation of the weighted sample, $\sigma_{\text{weighted}} = 5.79 \mu\text{m/s}^2$, is slightly smaller than the unweighted sample, $\sigma_{\text{unweighted}} = 5.95 \mu\text{m/s}^2$. This is also observable as a slightly higher peak in Fig. 7.14 (b), whose area is the same as Fig. 7.14 (a) equal to the total number of blocks in the trimmed dataset. Finally, the standard error of the weighted mean, $\sigma a_{\text{ball}}^{\text{AWM}}$ agrees with $\sigma_{\text{weighted}}/\sqrt{N_{\text{trim}}}$, which justifies the adjustment of the weight $(\sigma a_{\text{ball}}^{(n)})^{-2} \rightarrow (\sigma a_{\text{ball}}^{(n)} \chi_r)^{-2}$ and validate the use of the adjusted weighted mean (Eq. (7.12)) over the weighted mean (Eq. (7.10)).

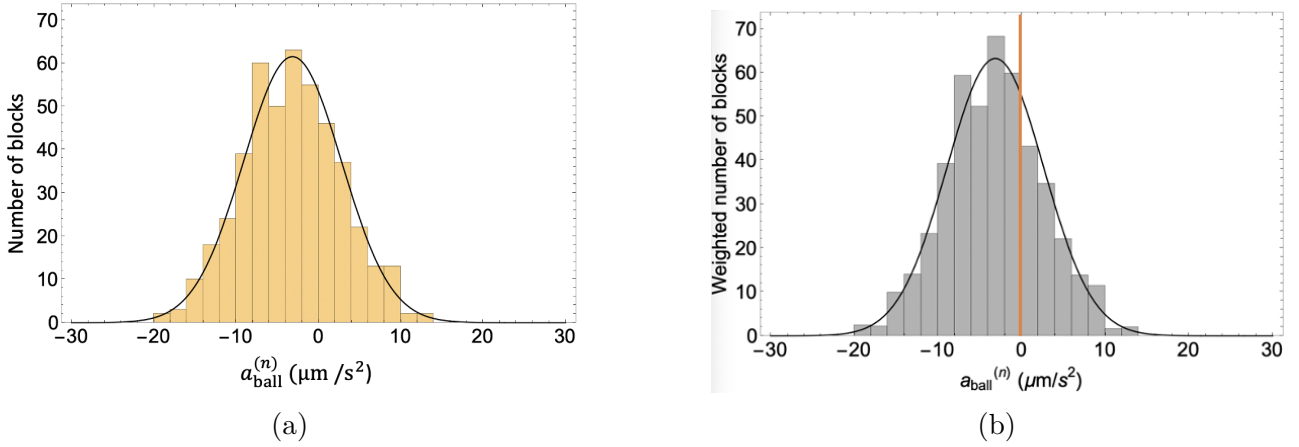


Figure 7.14: Comparison of the unweighted distribution of the chameleon acceleration a_{ball} and the weighted distribution. (a) The unweighted distribution of 459 blocks after trimming the lowest and the highest 3% of the original data set. (b) The weighted distribution. The weights were using the error propagation of $\sigma\phi_{\pm}$ estimated by the non-linear fitting solver. The mean of the distribution is statistically different from zero (indicated as an orange vertical line).

7.4.3 Sensitivity evaluation

Although a single-shot measurement cannot really give information about the acceleration sensed by atoms without prior knowledge of the fringe, it still provides a useful concept to estimate the number of shots for reaching a target acceleration and compare our sensitivity with the previous generation.

From the section above, we reached a statistical sensitivity of the acceleration as $5.79 \mu\text{m/s}^2$ per block using 2×25 shots of $\{a_{\text{mirror}}^{\text{weighted}}, P_{F=2}\}$. This gives an estimation of acceleration sensitivity of $41 \mu\text{m/s}^2$ per shot. In the last generation, it was reported $156 \mu\text{m/s}^2$ per shot in a quick test ($11 \mu\text{m/s}^2$ with 200 shots [1, p.108]) and $85.8 \mu\text{m/s}^2$ per shot for the chameleon force search. ($0.133 \mu\text{m/s}^2$ with $36 \times 2 \times 5774$ shots [13]). Therefore, our acceleration sensitivity per shot has been at least improved by a factor of two compared to the last generation of experiments.

Combining these three datasets with 46 hours of measurement listed in Table 7.1 produces a weighted mean $\mathbf{n} \cdot \mathbf{a}_{\text{ball}} = -2.597 \pm 0.186 \mu\text{m/s}^2$. This indicates that we have the potential to reach the acceleration uncertainty of $0.133 \mu\text{m/s}^2$ in 92 hours, which needed 432 hours in the previous generation. Further noise analysis is discussed in Section 7.6.

Table 7.1: Adjusted weighted mean (AWM) and equivalent acceleration sensitivity per shot.

Dataset label	Number of blocks after trimming	Duration (hrs)	AWM $\mathbf{n}_k \cdot \mathbf{a}_{\text{ball}}$ ($\mu\text{m/s}^2$)	Acc. sensitivity per shot ($\mu\text{m/s}^2$)
24Mar2023	163	8	-2.704 ± 0.349	30.5
25Mar2023	459	24	-3.114 ± 0.270	41
26Mar2023	194	14	-1.436 ± 0.382	37.6

7.4.4 Discussion of the non-zero result

After validating the statistics in Eq. (7.12), in this section I show that the statistically non-zero result is linked to the ball position.

I start by considering the accelerations measured when the ball is in the same position. The weighted statistics of the successive differences of $\phi_{\pm}^{(n)}$, $(\phi_{\pm}^{(n)} - \phi_{\pm}^{(n+1)})/2$, are shown in Fig. 7.15. The successive differences at both ball positions give adjusted weighted means

$$\frac{\phi_+^{(n)} - \phi_+^{(n+1)}}{2} = -0.114 \pm 0.289 \mu\text{m/s}^2, \quad \frac{\phi_-^{(n)} - \phi_-^{(n+1)}}{2} = 0.121 \pm 0.302 \mu\text{m/s}^2, \quad (7.13)$$

which are consistent with zero. This shows that our non-zero result is linked to the ball position.

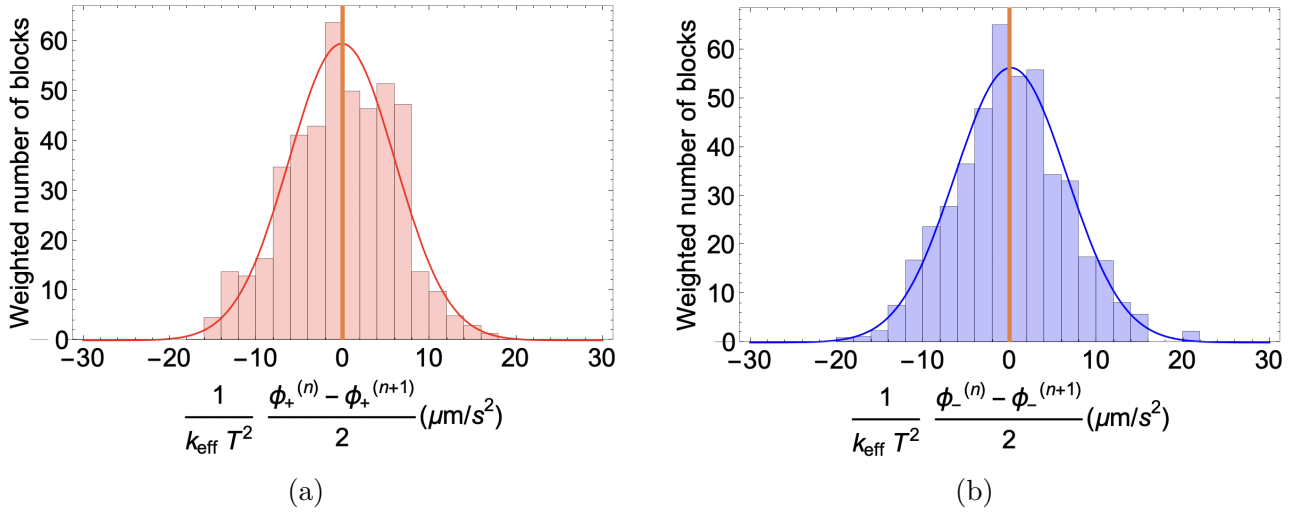


Figure 7.15: The successive difference of the phase estimation at each ball position. (a) Ball position was set to $\theta_+ = +18^\circ$. (b) Ball position was set to $\theta_- = -18^\circ$. Their mean values are both statistically consistent with zero.

This ball-induced acceleration had a negative sign as shown in Table 7.1, which indicated a repulsive force between the ball and the atoms [1, p.93]. However, the target chameleon-screened scalar field is expected to generate an attractive fifth force. This leads to an investigation of systematics in Section 7.5.

7.5 Systematics

Given that a non-zero result of $\mathbf{n} \cdot \mathbf{a}_{\text{ball}} = -2.597 \pm 0.186 \mu\text{m/s}^2$ (45.6 mrad) sourced from the ball was measured in this experiment, this section reviews the Newtonian gravity (Section 7.5.1), electrostatic (Section 7.5.2), and the second-order Zeeman shift (Section 7.5.3) in the context of our fountain configuration. These three systematics were found to be too small ($< 3 \text{ nm/s}^2$) to explain the observed negative (repulsive) acceleration from the ball, which leads to a qualitative discussion of the systematic phase shift from the scattered Raman light in Section 7.5.4. These known systematics are summarised in Section 7.5.6 and found not enough to explain the observation. It discusses other candidates for the unknown systematics.

7.5.1 Newtonian gravity

Using the trajectory shown in Fig. 7.10, the Newtonian acceleration along the Raman axis can be calculated as shown in Fig. 7.16 (a) taking the density $\rho_{\text{ball}} = 2.7 \text{ g/cm}^3$. Note this is a time-varying acceleration because the centre-to-centre distance changed during the interferometry shown in Fig. 7.16 (b), as well as the angle between the Newtonian force and the Raman axis. The weighted acceleration using Eq. (5.1) for a $2T=66 \text{ ms}$ interferometer gives $\mathbf{n} \cdot \mathbf{a}_{\text{ball}}^{\text{Newtonian}} = 2.61 \text{ nm/s}^2$ in the short pulse limit. This is 95% of the largest value (2.74 nm/s^2) when the atoms reached their trajectory apex and became closest to the ball.

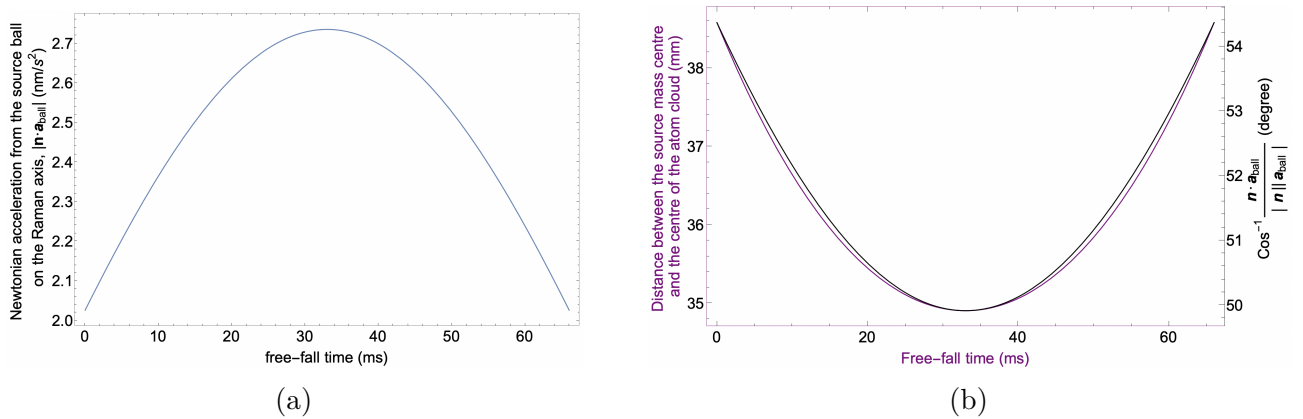


Figure 7.16: Newtonian gravity sourced from the ball as a function of time during the interferometry. The time coordinates are relative to the first interferometry Raman pulse. (a) The Newtonian gravity from the ball projected on the Raman axis. It reached the highest value of 2.74 nm/s^2 when the atom cloud approached the nearest of the ball. (b) The centre-to-centre distance as a function of time (purple) and projection angle on the Raman axis (Black).

7.5.2 Electrostatic

Although the source mass (ball) is grounded through the vacuum feedthrough and the chamber wall, it can still accommodate charges in its alumina layer. This alumina layer of ≈ 4 nm can support a potential difference of up to ≈ 1 V before breaking down [13]. This alumina layer can be regarded as a spherical capacitor (shell of electric dipoles), which can generate an electric field gradient at the atom cloud like a point charge at the centre of the ball. This electric field gradient can produce a systematic acceleration of $|\mathbf{a}_{\text{ball}}^{\text{E}}| < 2 \text{ nm/s}^2$ by shifting both clock states in the same way via their scalar polarisabilities. [1, p.96]. As analysed in Section 7.5.1, the atom is most sensitive at the middle of the interferometer sequence, where the wavepackets separation becomes the largest, it is safe to take the cos projection angle to be $\langle \mathbf{n}, \mathbf{a}_{\text{ball}}^{\text{E}} \rangle = 50^\circ$ as shown in Fig. 7.16(b). Thus, we can conclude $\mathbf{n} \cdot \mathbf{a}_{\text{ball}}^{\text{E}} < 1.3 \text{ nm/s}^2$.

7.5.3 Second-order Zeeman shift

Since the source ball was made of paramagnetic material (aluminium alloy 6082), it can be slightly magnetised by the Raman bias magnetic field during the interferometry. As the ball was switched between $\theta_{\pm}$ positions, the magnetic field sourced from the ball can induce different phase shifts, $\Phi_{\pm}^{\text{B}}$, at the output of the interferometer. This is due to the second-order Zeeman shifts $\Delta E_n = \hbar \alpha_n B^2$ to the energy levels of the clock states $|n\rangle = |n, 0\rangle$, where $-\alpha_1 = \alpha_2 = \alpha/2 = 2\pi \times 287.575 \text{ Hz/G}^2$ and B is the magnitude of the magnetic field. As these energy shifts are the same but have opposite signs, the wavepacket has no net acceleration. Thus, $\Phi_{\pm}^{\text{B}}$ only have contributions from the propagation phase difference instead of the laser phases,

$$\Phi_{\pm}^{\text{B}} = \left(\int_0^T \alpha_1 B_{\pm}^2(\mathbf{r}_{\text{AB}}(t)) dt + \int_T^{2T} \alpha_2 B_{\pm}^2(\mathbf{r}_{\text{BD}}(t)) dt \right) \quad (7.14)$$

$$- \left(\int_0^T \alpha_2 B_{\pm}^2(\mathbf{r}_{\text{AC}}(t)) dt + \int_T^{2T} \alpha_1 B_{\pm}^2(\mathbf{r}_{\text{CD}}(t)) dt \right), \quad (7.15)$$

where $B_{\pm}$ is the magnitude of the total magnetic field $\mathbf{B}_{\pm} = \mathbf{B}_{\text{Raman}} + \mathbf{B}_{\pm}^{\text{ball}}$, whose primary contribution is the Raman bias magnetic field along y-axis in Fig. 7.16 (a), $\mathbf{B}_{\text{Raman}}$, and a

secondary contribution from the paramagnetic source mass, $\mathbf{B}_{\pm}^{\text{ball}}$, at two positions, $\theta_{\pm}$. Since these two magnetic fields have significantly different magnitudes, $B_{\text{Raman}} \gg B_{\pm}^{\text{ball}}$, $\Phi_{\pm}^{\text{B}}$ can be simplified to be $-2B_{\text{Raman}}\alpha_1 \frac{\partial B_{y,\pm}^{\text{ball}}}{\partial y} 2v_{\text{recoil}}T^2$ after taking the below approximation [1, p.112]

$$B_{\pm}^2 \approx |\mathbf{B}_{\text{Raman}}|^2 + 2\mathbf{B}_{\text{Raman}} \cdot \mathbf{B}_{\pm}^{\text{ball}}(\mathbf{r}(t)) = B_{\text{Raman}}^2 + 2B_{\text{Raman}}(B_{y,\pm}^{\text{ball}}|_{r=0} + \nabla B_{y,\pm}^{\text{ball}}|_{r=0} \cdot \mathbf{r}(t)), \quad (7.16)$$

assuming Raman bias magnetic field was homogeneous over the interferometer region. The volume susceptibility of the same aluminium alloy sample was measured as $\chi = (2.17 \pm 0.03) \times 10^{-5}$ with a vibrating sample magnetometer, which gives $\frac{\partial B_{y,\pm}^{\text{ball}}}{\partial y} = \mp 0.41 \text{ } \mu\text{G}/\text{cm}$ at the centre of the chamber. This leads to systematic phase shift $\Phi_{\text{ball}}^{\text{B}} = (\Phi_+^{\text{B}} - \Phi_-^{\text{B}})/2 = 1.8 \text{ } \mu\text{rad}$ for 1 G Raman bias magnetic field, which is equivalent to an acceleration systematics of $0.1 \text{ nm}/\text{s}^2$ for a $2T = 66 \text{ ms}$ interferometer. This is 20 times smaller than the previous generation.⁵

7.5.4 Scattered Raman light

As discussed above, neither of the systematic acceleration of the Newtonian or the electrostatic interaction, nor the systematic phase shift from the magnetic field gradient induced by the ball can explain the observed ball-induced phase shift. This section discusses a non-trivial phase shift from the scattered Raman light. The qualitative analysis below is close to the observed phase shift of 45.6 mrad (equivalent to $2.579 \text{ } \mu\text{m}/\text{s}^2$).

The Raman beams that drive the atom interferometer (AI) have a coupling strength $\Omega_{\text{Raman}}^{\text{AI}} \propto E_1 e^{i\phi_1} E_2 e^{-i\phi_2}$. Ignoring the momentum states, the scattered Raman light can drive the same transition of coupling strengths $\Omega_{\text{Sc},1} \propto E_1^{\text{Sc}} e^{i(\phi_1 + \phi_1^{\text{Sc}})} E_2 e^{-i\phi_2}$, $\Omega_{\text{Sc},2} \propto E_1 e^{i\phi_1} E_2^{\text{Sc}} e^{-i(\phi_2 + \phi_2^{\text{Sc}})}$, where the scattered electric fields E_n^{Sc} can have additional different phase $e^{i\phi_n^{\text{Sc}}}$ ($n = 1, 2$). When they drive the same transition together, these coupling strength can interfere each other coherently because

$$\Omega_{\text{Raman}} = \Omega_{\text{Raman}}^{\text{AI}} + \Omega_{\text{Sc},1} + \Omega_{\text{Sc},2} \propto E_1 e^{i\phi_1} E_2 e^{-i\phi_2} \left(1 + \frac{E_1^{\text{Sc}}}{E_1} e^{i\phi_1^{\text{Sc}}} + \frac{E_2^{\text{Sc}}}{E_2} e^{-i\phi_2^{\text{Sc}}} \right). \quad (7.17)$$

⁵The leading contribution is from the arm segments AB and CD that are displaced $376 \text{ } \mu\text{m}$ in y-direction. Being in the same state gives a factor of two suppression because $\alpha_1 = -0.5\alpha$. The contribution from the z-direction motion of 5 mm is cancelled in this fountain configuration, which was the leading contribution in the last generation. The contribution from y-direction displacement of $376 \text{ } \mu\text{m}$ gives another suppression factor of 10.

Thus, the worst scenario $\phi_1^{\text{Sc}} = -\phi_2^{\text{Sc}} = \pi/2$ gives $\Omega_{\text{Raman}} \approx \Omega_{\text{Raman}}^{\text{AI}} e^{i\phi_{\text{Sc}}}$ with $\phi_{\text{Sc}} = \frac{E_1^{\text{Sc}}}{E_1} + \frac{E_2^{\text{Sc}}}{E_2}$. The incident Raman beams had 109 mW in the slave1 and 376 mW in the slave2 beams that had intensity e^{-2} radius of 15.6 mm. We estimated the scattered field strength by putting a photodiode at the spare 2-inch window shown in Fig. 7.17 (a) without loading atoms. As the ball was switched from θ_+ to θ_- (a→b→c) and switched back (d→e→f), the corresponding photodiode trace is shown in Fig. 7.17 (b).

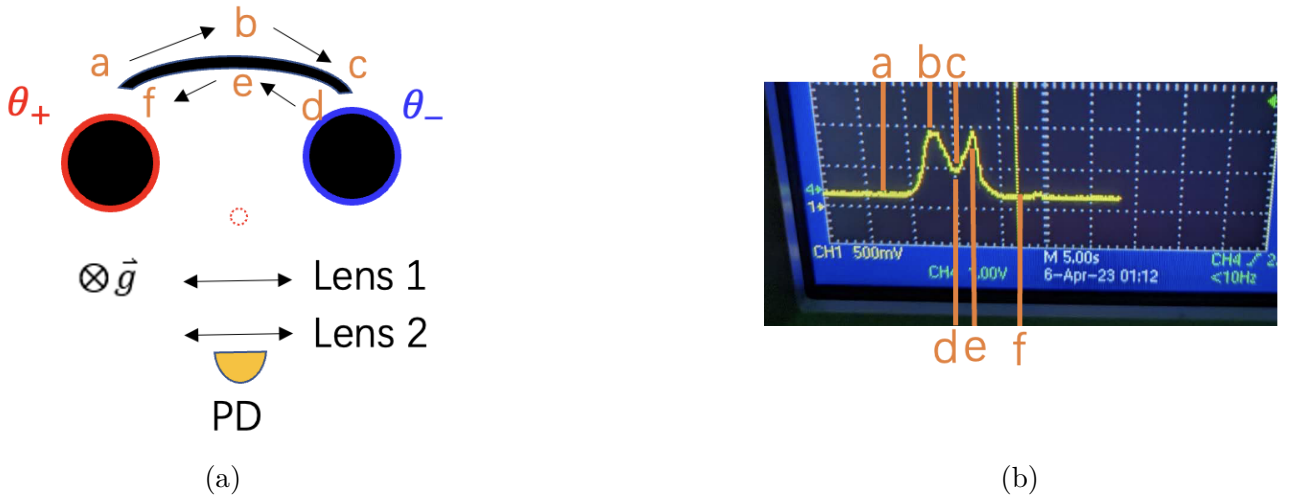


Figure 7.17: Estimating the power of the scattered Raman light from the ball. (a) The top view of the schematics for measuring the scattered Raman light with a photodiode (PD). The location of placing the PD and its lens system for scattering light test is at the window opposite to the location where the photodiode was used for detecting atoms in F=2. The dashed circle indicates where the atoms were in the interferometer. (b) The photodiode signal as the ball position was switched manually.

From the photodiode responsibility $\kappa_{\text{power}} = 0.480 \text{ V}/\mu\text{W}$ obtained in Eq. (6.4), the total Raman power scattered from the ball to the photodiode were at least $P_{\text{Sc}}^+ = 0.2 \mu\text{W}$ (a or f) and $P_{\text{Sc}}^- = 1 \mu\text{W}$ (c or d). Assuming the scattered Raman light from the ball illuminated the interferometer region uniformly across an area of 1 cm^2 , which gives $|\phi_+^{\text{Sc}}| = 1.3 \text{ mrad}$ and $|\phi_-^{\text{Sc}}| = 6 \text{ mrad}$ per Raman pulse. Considering a three-pulse interferometer, it can accumulate a systematic phase shift by the scattered light $\Phi_{\pm}^{\text{Sc}} = \phi_{1,\pm}^{\text{Sc}} - 2\phi_{2,\pm}^{\text{Sc}} + \phi_{3,\pm}^{\text{Sc}}$. In the worst scenario, this scattered Raman light from the ball can result in a systematic phase shift up to $\Phi_{\text{ball}}^{\text{Sc}} = (\Phi_+^{\text{Sc}} - \Phi_-^{\text{Sc}})/2 < 14.8 \text{ mrad}$, which is equivalent to $0.8 \mu\text{m/s}$. This upper bound could be underestimated, because the photodiode might not collect all the scattered Raman light from the ball due to its location shown in Fig. 7.17 (a). This may also explain why more scattered

photons were collected when ball was located at b and e positions shown in Fig. 7.17 (b).

7.5.5 Wavefront distortion

Wavefront distortion of the Raman beams can cause a systematic phase shift in the chameleon measurement. At θ_+ position, the corner edge of the incident Raman-II was clipped by the in-vacuum source mass shown in Fig. 7.18 (a). Aside from the impact of the scattered Raman discussed in Section 7.5.4, this clipping can also lead to wavefront distortions of the Raman-I' (prime indicates the retro-reflected Raman-I) beam shown in Fig. 7.18 (b) due to diffraction of the unscattered beam. As illustrated in Fig. 7.18 (c), when the ball was switched to the θ_- position, both Raman-I' and Raman-II beams would have this wavefront distortion. This ball-position-determined wavefront distortion can introduce a systematic phase shift, which needs further quantitative characterisation.

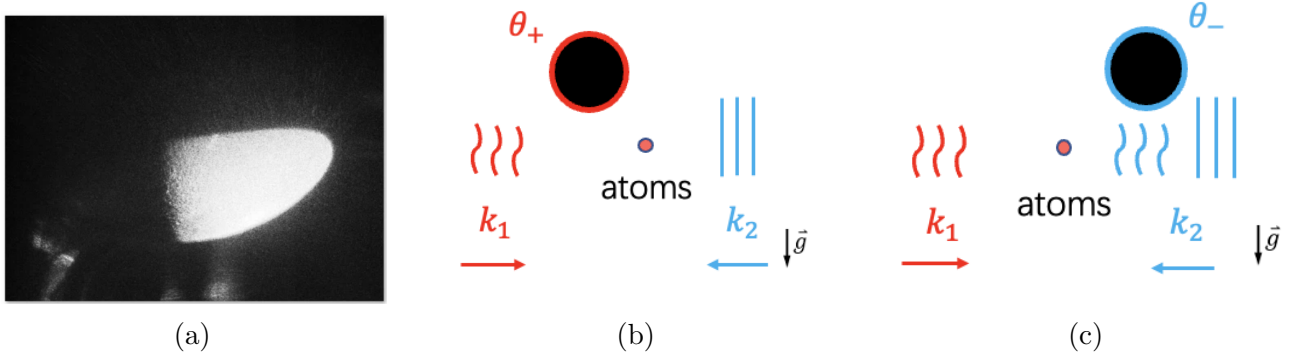


Figure 7.18: Wavefront distortion due to the clipping of the Raman beam by the ball. (a) A photo was taken at the side view to visualise the scattered Raman light from the ball. (b) Schematics of the wavefront distortions when the ball was at θ_+ position. The wiggling red curves represent the distorted wavefront of the retro-reflected Raman-I' beam, while flat blue lines represent the undistorted incoming Raman-II beam. They drive the velocity-sensitive Raman transitions to form our atom interferometer. (c) Wavefront distortions when the ball was at θ_- position.

7.5.6 Summary of the systematics

The known systematic corrections are summarised in Table 7.2, where the largest contribution is the scattered Raman light, which is expected to contribute one-third of the measured phase shift linked to the ball position. Thus, the wavefront distortion discussed in Section 7.5.5 could be responsible for the unknown systematic phase shift of 30 mrad. Meanwhile, the upper limit of the scattered Raman light phase could also be underestimated.

Table 7.2: Systematic corrections to $\mathbf{n} \cdot \mathbf{a}_{\text{ball}}$

Quantity	Symbols	Phase shift (mrad)	Acceleration ($\mu\text{m}/\text{s}^2$)
Measured phase shift	$\mathbf{n} \cdot \mathbf{a}_{\text{ball}}$	45.6	(2.597)
Newtonian gravity	$\mathbf{n} \cdot \mathbf{a}_{\text{ball}}^{\text{Newtonian}}$	(4.6×10^{-2})	2.61×10^{-3}
Electric field gradients	$\mathbf{n} \cdot \mathbf{a}_{\text{ball}}^{\text{E}}$	$(< 2.3 \times 10^{-2})$	$< 1.3 \times 10^{-3}$
Magnetic field gradients	$\Phi_{\text{ball}}^{\text{B}} = (\Phi_{+}^{\text{B}} - \Phi_{-}^{\text{B}})/2$	1.8×10^{-3}	(1.0×10^{-4})
Scattered Raman light	$\Phi_{\text{ball}}^{\text{Sc}} = (\Phi_{+}^{\text{Sc}} - \Phi_{-}^{\text{Sc}})/2$	< 14.8	(< 0.8)
Unknown systematics	Φ_{unknown}	≈ 30	≈ 1.7

Given that both of the largest systematics, scattered Raman light and unknown systematics potentially due to the wavefront distortion of the Raman beams, are associated with the source mass clipping the Raman beam, an aperture that clips the corner of the Raman beam can put the source mass in the shadow of the aperture. This can suppress the leading systematics and make our measurement become limited by statistical uncertainties.

7.6 Noise analysis for the phase estimation

The chameleon measurement presented in Section 7.4 involves two steps: estimating of the fringe phase at each ball position and comparing these two fringe phases. Each step can accumulate noise to the ball-induced acceleration estimation.

Section 7.6.1 separates the phase noise from the detection noise in a fringe phase estimation. It reveals that the interferometer phase noise was much larger than the detection noise in estimating a fringe phase. In addition, an unknown fringe phase instability is found to be the primary noise in a comparison between two successive fringe phases like our chameleon measurements.

As the secondary noise in chameleon measurement, the interferometer phase noise due Raman beatnote phase noise and the MEMS acceleration noise are discussed in Section 7.6.2. It concludes that there is an unknown interferometer phase noise source, which might be the air fluctuation in the retro-reflection optical path.

Being the smallest noise contribution to the chameleon measurement, detection noise was characterised in Section 7.6.3 in terms of the background noise and probe laser noise. It was found that our detection noise with $\approx 10^7$ atoms was limited by the probe laser noise.

7.6.1 The phase estimation uncertainty

To characterise the uncertainty in the phase estimation for a fringe of a given number of shots, 674 successive shots of $\{a_{\text{mirror}}^{\text{weighted}}, P_{F=2}\}$ were recorded in 25 minutes at one ball position, which is shown in Fig. 7.19 (a). The fringe was randomly sampled by the mirror vibration and slowly scanned from left to right by the uncontrolled table tilt, which was the same for the MEMS accelerometer ($a_{\text{mirror}}^{\text{weighted}}$) and the atom interferometer ($P_{F=2}$).

To analyse the statistics of the population fluctuation as a function of the interferometer phase shift Φ , I fitted all the data in Fig. 7.19(a) with a three-parameter model $P_{F=2}^{\text{Fit}}(a_{\text{mirror}}^{\text{weighted}}; A, B, \phi_0) = B + A \cos(k_{\text{eff}} T^2 a_{\text{mirror}}^{\text{weighted}} + \phi_0)$ to convert the horizontal axis from acceleration to phase. Then data from different fringe orders are folded into one period, displayed in Fig. 7.19 (b), by taking the remainder of $\Phi = k_{\text{eff}} T^2 a_{\text{mirror}}^{\text{weighted}} + \phi_0$ divided by 2π . After folding them into one period,

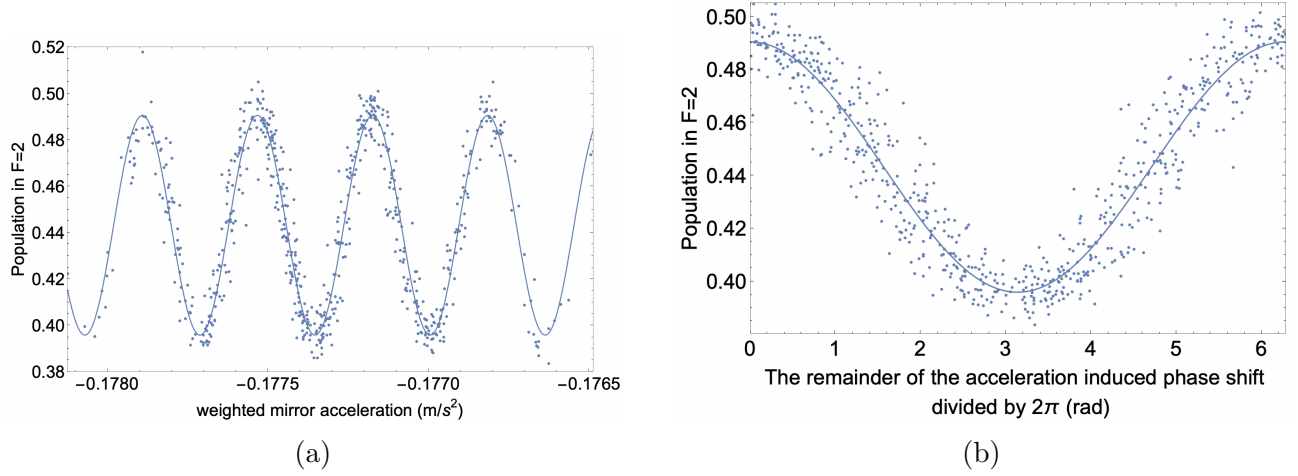


Figure 7.19: The test dataset for understanding the noise of the fringe phase estimation. (a) Population in $F=2$ as a function of the acceleration seen by the mirror. As the ambient vibration and the uncontrolled table tilt sampled the fringe from the left to the right, the population in $F=2$ shows a sinusoidal fringe over more than 4 fringe periods. (b) Folded fringe data after converting the horizontal unit from acceleration to interferometer phase. The data has been folded into one period by taking the remainder of the acceleration-induced interferometer phase divided by 2π .

the deviation of the measured population from the fitting model for all 674 shots is shown in Fig. 7.20. The population fluctuation shows the largest variance at $\Phi = \pi/2$ and $3\pi/2$ and the smallest variance at $\Phi = 0, \pi$ and 2π , which is a signature of phase noise dominated measurement. To quantify the phase noise per shot, $\sigma\Phi_{N=1}$, the residual data in Fig. 7.20 were partitioned into bins with a width of 100 mrad. The standard deviation of data grouped in each bin, $\sigma P_{F=2}(\Phi_i)$, was calculated and marked in red, where Φ_i is the bin centre of the i^{th} bin and bins containing less than six data were discarded. Fitting $\sigma P_{F=2}(\Phi_i)$ with a model [11]

$$\sigma P_{F=2}(\Phi) = \sqrt{\underbrace{\left(\frac{d\langle P_{F=2}(\Phi) \rangle}{d\Phi} \sigma\Phi_{N=1}\right)^2}_{\text{Phase noise contribution}} + \underbrace{\alpha + \beta \cdot \langle P_{F=2}(\Phi) \rangle^2}_{\text{Detection noise contribution}}}, \quad (7.18)$$

separates the phase noise and the detection noise with the knowledge of the expectation of the population in $F=2$, $\langle P_{F=2}(\Phi) \rangle$, which can be approximated as $\langle P_{F=2}(\Phi) \rangle \approx P_{F=2}^{\text{Fit}}(\Phi)$. The smallest phase estimation uncertainty per shot can be obtained at the side of the fringe, $\Phi = \pi/2$, which is given by $\sigma P_{F=2}(\Phi = \pi/2)/A = 289 \text{ mrad}$, where the phase noise per shot $\sigma\Phi_{N=1} = 273 \text{ mrad}$ is the dominant contribution,⁶ while the detection noise contributes to 95 mrad per shot at the side of the fringe of a fringe amplitude $A = 0.047$.

⁶The bin width of 100 mrad can contribute 35 mrad systematics in the $\sigma\Phi_{N=1}$, which is negligible.

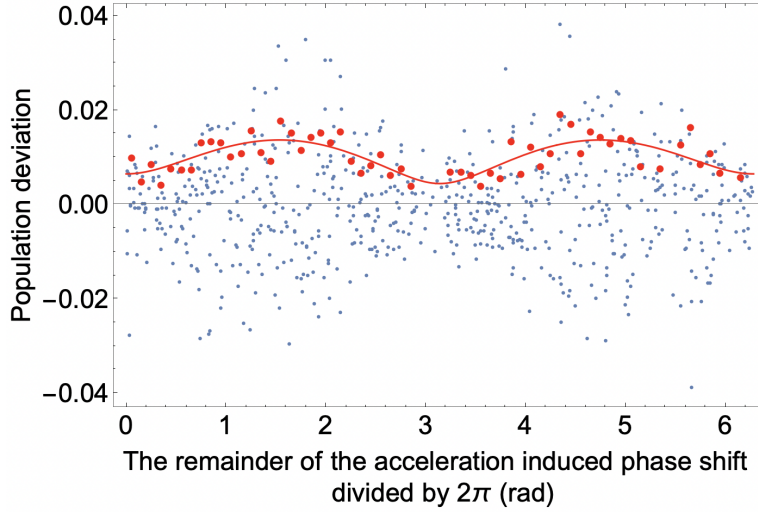


Figure 7.20: Population deviation as a function of the interferometer phase. Red dots are the standard deviation of the fitting residuals (blue dots) partitioned in the bins of 100 mrad width, which can be described by a model Eq. (7.18) (solid red line), which separates the phase noise and detection noise.

In our chameleon measurement, the phase was always estimated by fitting to Eq. (7.8) over more than 2π (Fig. 7.12 (b)) rather than being estimated at the side of the fringe ($\Phi = \pi/2$). To check whether the phase uncertainty estimated by fitting a fringe agrees with our analysis at the side of the fringe, this test dataset was partitioned into M groups of N points ($M = \lfloor 674/N \rfloor$). For the m^{th} group of N points, we obtained the uncertainty of the phase estimation, $\sigma\phi_{\text{fit}}^{(m)}(N)$, from the non-linear fit solver. Averaging them across M groups produces the mean phase uncertainty estimation using N shots, $\langle\sigma\phi_{\text{fit}}^{(m)}(N)\rangle$, shown as red dots in Fig. 7.21. Our phase estimation uncertainty from fitting a fringe (red dots) shows a $1/\sqrt{N}$ behaviour, which is consistent with the prediction from the phase estimation uncertainty at the side of the fringe (red solid and red dashed lines). This consistency agrees with a phase-noise-dominated estimation.⁷ Thus, we conclude that the phase estimation uncertainty by fitting is limited by the phase noise of the measurements, which will be discussed more in Section 7.6.2.

Meanwhile, we also care about the uncertainty in the phase comparison between a pair of phase estimations like the procedure in a block for the chameleon measurement. This is equivalent to checking the stability of the best-fit phase estimation, $\phi_{\text{fit}}^m(N)$, for a fringe fitted by

⁷For a fringe only having phase noise, the phase estimation uncertainty is the same everywhere across the fringe, because the population residual at any point of the fringe is proportional to the slope of the fringe. In this case, every point has the equally lowest acceleration uncertainty to sense acceleration. An example is a projection-noise-limited measurement.

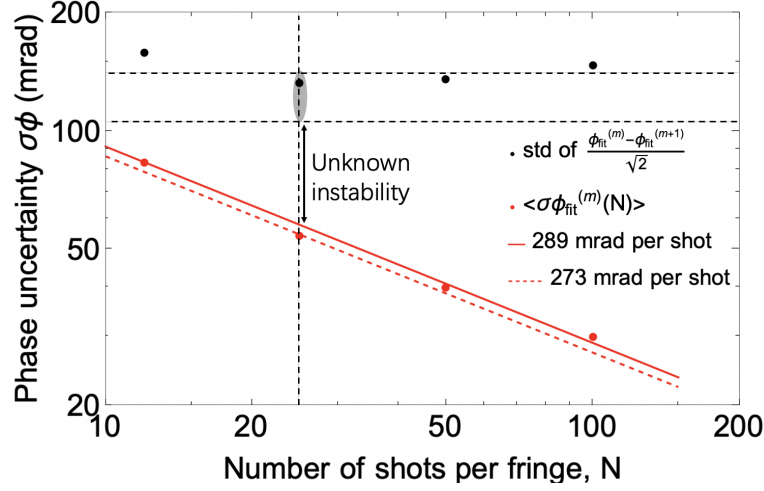


Figure 7.21: Understanding the noise of the fringe phase estimation using the test dataset shown in Fig. 7.19 and Fig. 7.20. Red dots are the mean phase uncertainties estimated by the non-linear fitting of N -shot fringes, $\langle \sigma \phi_{\text{fit}}^{(m)}(N) \rangle$. They are consistent with the phase uncertainty estimated at the side of a fringe, which is $N^{-1/2} \cdot \sigma P_{F=2}(\Phi = \pi/2)/A = 289 \text{ mrad}/\sqrt{N}$ (red solid lines). The phase noise floor $N^{-1/2} \sigma \Phi_{N=1} = 273 \text{ mrad}/\sqrt{N}$ is indicated as the red dashed line. The standard deviation of $(\phi_{\text{fit}}^{(m)}(N) - \phi_{\text{fit}}^{(m+1)}(N))/\sqrt{2}$ estimated the stability of $\phi_{\text{fit}}^{(m)}(N)$. It agrees with the standard deviation of $(\phi_+ - \phi_-)/\sqrt{2}$ for three chameleon datasets (light dark region), but significantly larger than the corresponding mean phase uncertainty $\langle \sigma \phi_{\text{fit}}^{(m)}(N = 25) \rangle$.

N shots of measurements in this test dataset. If $\phi_{\text{fit}}^{(m)}(N)$ is stable, then its rescaled successive difference, $(\phi_{\text{fit}}^{(m)}(N) - \phi_{\text{fit}}^{(m+1)}(N))/\sqrt{2}$, should have a standard deviation equal to the uncertainty in the phase estimation, $\langle \sigma \phi_{\text{fit}}^{(m)}(N) \rangle$. However, as the black data shown in Fig. 7.21, the standard deviation of $(\phi_{\text{fit}}^{(m)}(N) - \phi_{\text{fit}}^{(m+1)}(N))/\sqrt{2}$ over the M fringes of N shots displays a relatively flat behaviour instead of showing a $1/\sqrt{N}$ behaviour. $N=25$, marked as the vertical dashed line, is of particular interest because 25 data points per fringe were used for the chameleon measurement. The standard deviation of $(\phi_{\text{fit}}^{(m)}(N = 25) - \phi_{\text{fit}}^{(m+1)}(N = 25))/\sqrt{2}$ is 133 mrad, which agrees with the observed standard deviation of $(\phi_+ - \phi_-)/\sqrt{2}$ in the three datasets of chameleon measurements (between 105 mrad and 140 mrad). However, they are much larger than the corresponding phase estimation uncertainty of $\langle \sigma \phi_{\text{fit}}^{(m)}(N = 25) \rangle = 54 \text{ mrad}$. This suggests that an unknown instability contributes to an additional noise of 122 mrad between the successive phase estimations for fringes of 25 shots.

One candidate for the instability was the bias drift of the MEMS accelerometer, which was induced by the temperature drift. As shown in Fig. 7.22 (a), the temperature of the MEMS accelerometer was measured by its built-in thermal sensor. It illustrated $\approx 0.08^\circ\text{C}$ variation

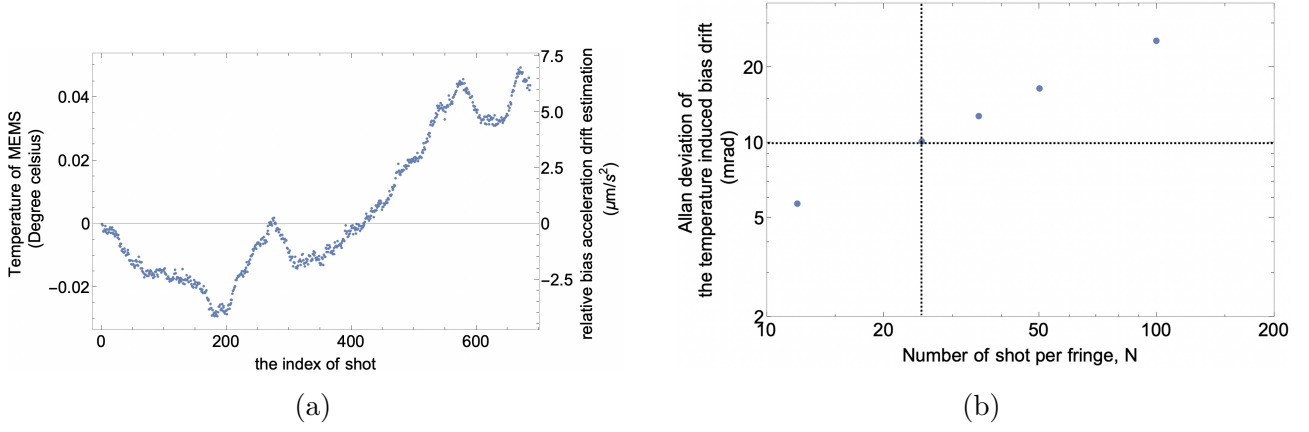


Figure 7.22: The temperature drift of the MEMS accelerometer and its impact in phase estimation. (a) Temperature drift of the MEMS accelerometer. (b) The instability of the interferometer phase induced by the bias drift of the MEMS.

over the 674 shots of this test dataset, which corresponded to $\approx 10 \mu\text{m/s}^2$ variation. To calculate the impact of this variation on the time scale we were interested in, these 674 temperature measurements were also partitioned into M groups of N points ($M = \lfloor 674/N \rfloor$). For the m^{th} group of N points, we firstly calculated their mean $\bar{T}_{\text{MEMS}}^{(m)}(N)$ before calculating their stability by evaluating their Allan deviation [14], $\sigma T_{\text{MEMS}}(N)$,

$$\sigma T_{\text{MEMS}}(N) = \sqrt{\frac{1}{M-1} \sum_{m=1}^M (\bar{T}_{\text{MEMS}}^{(m)}(N) - \bar{T}_{\text{MEMS}}^{(m+1)}(N))^2}, \quad (7.19)$$

which is plotted in Fig. 7.22 (b). As we were using 25 shots of measurement to estimate the phase of a fringe, $\sigma T_{\text{MEMS}}(N = 25)$ is of particular interest, which was equivalent to 10 mrad after converting this temperature instability into interferometer phase with the bias thermal sensitivity, $dK_0/dT_{\text{MEMS}} = 14.49 \mu\text{g}_{\text{Dublin}}/\text{°C}$, and the acceleration sensitivity of our interferometer, $k_{\text{eff}}T^2 = 17.5 \text{ mrad}/(\mu\text{m/s}^2)$. This 10 mrad level of instability was not enough to explain the instability observed in Fig. 7.21.

Therefore, we conclude there was some unknown instability in the phase estimation by fitting a fringe. Sorting out the instability between phase comparisons can reduce the noise in our chameleon measurement, $\phi_{\text{ball}} = (\phi_+ - \phi_-)/2$, by a factor of two. If this can be achieved, the uncertainty of ϕ_{ball} will be limited by the phase-noise dominated estimation of $\phi_{\pm}$. The characterisation of the phase noise is discussed in Section 7.6.2.

7.6.2 Interferometer phase noise

As discussed at the end of Section 7.6.1, $\phi_{\text{ball}} = (\phi_+ - \phi_-)/2$ potentially will be limited by the phase noise, $\sigma\Phi_{N=1} = 266 \text{ mrad}$, if the extra noise introduced by the unknown instability between ϕ_+ and ϕ_- can be controlled. This section discusses the possible sources that contribute to the observed interferometer phase noise per shot $\sigma\Phi_{N=1}$.

Our interferometer phase, Φ , is contributed from the Raman laser phases at three pulses shown in Fig. 3.1, $\Phi = \mathbf{k}_{\text{eff}} \cdot \mathbf{a}T^2 + \phi_1^{\text{Beat}} - 2\phi_2^{\text{Beat}} + \phi_3^{\text{Beat}}$. The acceleration-dependent part comes from the round-trip optical path between the atoms and the Raman mirror as discussed in Section 3.2.1. The laser phases between the co-propagating beams,⁸ ϕ_j^{Beat} , at the j^{th} pulse was omitted from the discussion, because they are assumed to be the same for a perfect phase lock. Thus, the interferometer phase noise has two contributions, the acceleration noise and the beatnote phase noise, which are estimated below. Another possibility of the interferometer phase uncertainty due to unsynchronised MEMS measurements relative to the atom interferometer was ruled out with the discussion below.

Synchronising the MEMS accelerometer with the interferometer

A chosen delay between the unweighted MEMS acceleration signal and its triangular weight function has been reported to be able to reduce the fringe phase uncertainty by a significant amount of 35% [15, p.109] in the last generation. However, I only found $\approx 3\%$ improvement for the phase uncertainty in my system as shown in Fig. 7.23. This difference was attributed to the fact that we used a resistor to read our MEMS output current without any low-pass filter, but the previous generation used an extra low-pass filter [15, p.47] that introduced phase lag for the MEMS signal. Thus, the phase noise floor was not due to synchronisation between the MEMS accelerometer and the atom interferometer.

⁸It is actually the phase difference between two co-propagating Raman lasers relative to the phase of the local reference oscillator that phase-locks the Raman beatnote.

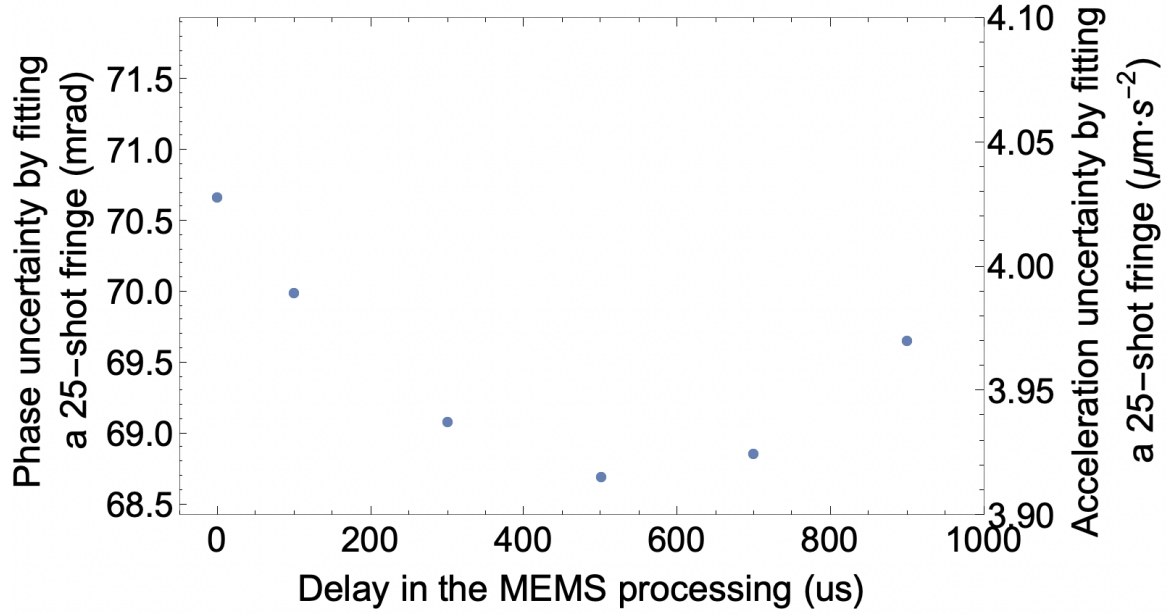


Figure 7.23: Synchronising the atom interferometer and the MEMS accelerometer. We added a variable delay time between the MEMS data and its triangular weight function. The fringe phase uncertainty by fitting 25 shots of $\{a_{\text{mirror}}^{\text{weighted}}, P_{F=2}\}$ only showed a small amount of improvement of $\approx 3\%$ at 500 μs compared to no delays.

Raman beatnote phase noise

On the one hand, an atom interferometer behaves as a second-order high-pass filter of $f_H = 1/(2T)$ because the slow drift of the optical phase with characteristic time larger than T is strongly suppressed by the second-order difference $\phi_1^{\text{Beat}} - 2\phi_2^{\text{Beat}} + \phi_3^{\text{Beat}}$. On the other hand, an atom interferometer behaves as a first-order low-pass filter of $f_L = 1/\tau_\pi$, because the fast fluctuation of the optical phase with characteristic time smaller than τ_π is quickly averaged out during each pulse. Therefore, atom interferometer acts as a bandpass filter for the noise in the Raman beatnote phase ϕ^{Beat} .

The μQuans datasheet presents a noise power spectrum of ϕ^{Beat} , which gives an interferometer phase noise per shot of $\sigma\Phi_{\mu\text{Quans}}^{\text{Beat}} = 19.75 \text{ mrad}$ for a $2T=50 \text{ ms}$ interferometer using π -pulse of $\tau_\pi = 20 \mu\text{s}$. In the last generation, a free-fall interferometer of $2T = 32 \text{ ms}$ using Raman pulses of $\tau_\pi = 5 \mu\text{s}$ estimated $\sigma\Phi_{\text{Free fall}}^{\text{Beat}} = 36.17 \text{ mrad}$ per shot, where the increment of interferometer phase noise compared to the $\sigma\Phi_{\mu\text{Quans}}^{\text{Beat}}$ came from a shorter π -pulse. In this generation of $2T = 66 \text{ ms}$, $\tau_\pi = 6 \mu\text{s}$, I conservatively estimated $\sigma\Phi_{\text{Fountain}}^{\text{Beat}} < 50 \text{ mrad}$ per shot.⁹

⁹Since the performance of the phase lock is related to the powers of individual lasers that are different from the last generation, it will be better to directly measure this with a low-phase-noise local oscillator [12, p.75].

MEMS intrinsic noise

The other interferometer phase noise source is the MEMS noise in measuring the mirror acceleration. We measured this noise by comparing two MEMS. One was the Innalab-2110 for the chameleon measurement of this generation, while the other was Honeywell Q-750 used in the last generation [13]. To make sure both MEMS are recording the same acceleration, they were placed face down on the same platform that was attached to a floating table for vibration reduction. Their acceleration axes were both aligned along the gravity to minimise the impact of the uncontrolled tilt of the floating table. Five seconds of data taken at 200 kHz sampling frequency were post-processed with the triangular weighting function of different interferometer time, $2T$. An example of $T = 33$ ms of the accelerations measured with both devices is shown in Fig. 7.24 (a). The standard deviation of the acceleration difference as a function of T is shown in Fig. 7.24 (b). It displays a good $1/\sqrt{T}$ behaviour between 10 ms and 50 ms (purple data and purple line), which indicates the noise spectrum of their differences was dominated by white noise of $2\text{ }\mu\text{m/s}^2/\sqrt{\text{Hz}}$ between 20 Hz and 100 Hz. This noise level is within the specified largest intrinsic noise level [16, 17], which is $700\text{ }\mu\text{m/s}^2/\sqrt{\text{Hz}}$ from 10 Hz to 500 Hz and $20\text{ }\mu\text{m/s}^2/\sqrt{\text{Hz}}$ below 10 Hz. The deviation from $1/\sqrt{T}$ above $T = 50$ ms possibly due to the slow drift below 20 Hz between two MEMS were intrinsically different and it was larger than the white noise contribution below 20 Hz.

We are particularly interested in the MEMS noise after averaging with the triangular weighting function of $T = 33$ ms, indicated as the orange vertical line in Fig. 7.24 (b), which gives $(\Delta a)^{\text{rms}} = 12\text{ }\mu\text{m/s}^2$ for their differences. Assuming both MEMS intrinsic noise contributed equally to the observed $(\Delta a)^{\text{rms}}$, we can estimate individual MEMS have an intrinsic acceleration noise of $a_{\text{MEMS}}^{\text{rms}}(T = 33\text{ ms}) = 8.5\text{ }\mu\text{m/s}^2$, which can contribute to an interferometer phase noise of $\sigma\Phi^{\text{MEMS}} = 150\text{ mrad}$ per shot for a $2T = 66$ ms interferometer.

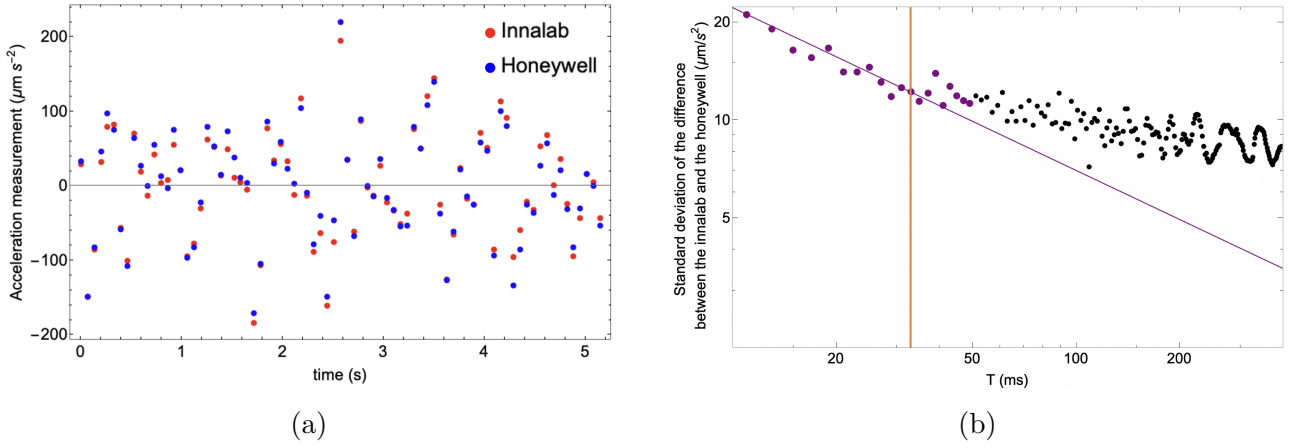


Figure 7.24: MEMS noise characterisation. (a) Comparison between acceleration simultaneously measured by Innalab and Honeywell devices. They were sampled at 200 kHz before averaging over $2T = 66$ ms with a triangular weight function shown in Fig. 5.5 (b). (b) Standard deviation of the acceleration difference measured with two devices as a function of T . It shows a $1/\sqrt{T}$ behaviour between $T = 10$ ms and $T = 50$ ms (purple). $T = 33$ ms marked as an orange vertical line is where the chameleon measurement was operated.

Unknown phase noise

As shown in the summary table Table 7.3 below, there is still 220 mrad unknown phase noise dominating the observed phase noise per shot.

Table 7.3: Budget of the interferometer phase noise per shot

Quantity	Symbols	Phase noise (mrad)	Acceleration noise ($\mu\text{m/s}^2$)
Measured interferometer phase noise per shot	$\sigma\Phi_{N=1}$	273	(15.6)
Raman beatnote phase noise per shot	$\sigma\Phi^{\text{Beat}}$	< 50	(< 2.85)
MEMS acceleration noise per shot	$a_{\text{MEMS}}^{\text{rms}}(T = 33 \text{ ms})$	(150)	8.5
Unknown noise per shot	$\Phi_{\text{unknown}}, a_{\text{unknown}}$	≈ 220	≈ 12.5

From the laser phase source, the finite speed of light can introduce extra noise, $\sigma\Phi \approx \pi^2(2L/c)\sqrt{S_{\nu_1}^0/\tau_\pi}$ [18], to the Raman phase seen by atoms compared to the Raman phase that was locked to the local reference oscillator, where $2L \approx 80$ cm is the extra round trip distance of the retro-reflected beam, $\tau_\pi = 6 \mu\text{s}$ is the π -pulse duration. This needs that the master laser had a white-noise-equivalent frequency noise power of $\sqrt{S_{\nu_1}^0} = 3 \text{ kHz} \cdot \text{Hz}^{1/2}$ over $1/\tau_\pi \approx 160 \text{ kHz}$ to explain the unknown phase noise of 220 mrad, which is unlikely large. Alternatively, if the

refractive index of the air fluctuates and can change the round-trip optical path length by 10^{-6} over the interferometer time $T = 33$ ms. This can explain this unknown phase noise of 220 mrad, which is quite likely.

From the MEMS perspective, a misalignment angle, θ_{align} , between the mirror normal axis and the MEMS sensing axis can introduce extra phase noise in the fringe. As shown in Fig. 7.25, even there is no horizontal acceleration along the Raman mirror, which an atom interferometer cares about, a misalignment angle of MEMS, θ_{align} , can measure the projection of the vertical acceleration, $a_{\text{MEMS,align}}^{\text{rms}} = \theta_{\text{align}} \cdot a_{\text{vert}}^{\text{rms}}(T = 33 \text{ ms})$, where the vertical acceleration fluctuation is measured as $a_{\text{vert}}^{\text{rms}}(T = 33 \text{ ms}) = 70 \mu\text{m/s}^2$ in Fig. 7.24(b). Therefore, it needs $\theta_{\text{align}} = 10^\circ$ to explain the observed unknown phase noise of 220 mrad, which is also very unlikely.

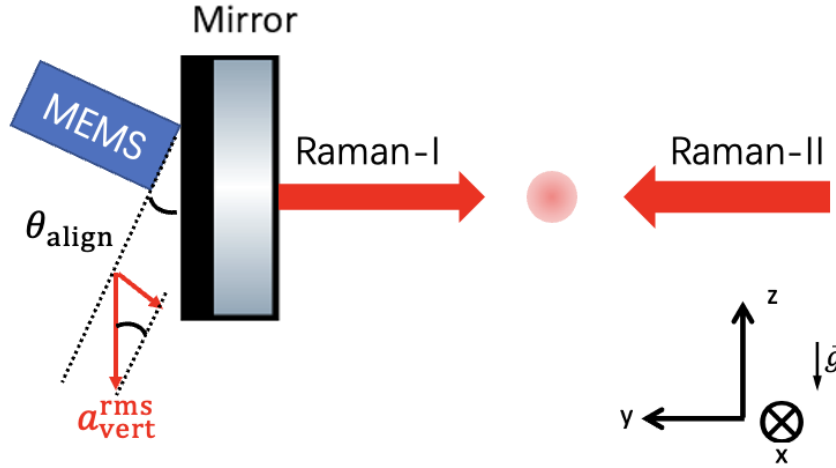


Figure 7.25: Noise from MEMS misalignment. If the mirror normal and Raman beams were perfectly horizontal, atoms won't see any acceleration fluctuation in the vertical direction, $a_{\text{vert}}^{\text{rms}}$. But a misalignment angle, θ_{align} , can lead the MEMS to sense the projection of $a_{\text{vert}}^{\text{rms}}$ on its sensing axis.

Therefore, the fluctuation of the air is the most likely candidate for the unknown phase noise of 220 mrad, which can be reduced by enclosing the retro-reflected beam path.

7.6.3 Detection noise

Although the phase estimation uncertainty was dominated by the phase noise, characterising the sources of the detection noise, $\sigma P_{F=2}^D = \sqrt{\alpha + \beta \cdot P_{F=2}^2}$, is helpful for further improvement. The population estimator, $\tilde{P}_{F=2}$, is a combination of four sequential voltage measurements,

$$\tilde{P}_2 = \frac{\tilde{V}_{N_2} - \tilde{V}_{B_2}}{\tilde{V}_{N_{\text{tot}}} - \tilde{V}_{B_{\text{tot}}}}, \quad (7.20)$$

where $\tilde{V}_X$ is the photodiode voltage averaged over the middle 30 μs in step $X = \{N_2, N_{\text{tot}}, B_2, B_{\text{tot}}\}$ of the sequence shown in Fig. 7.26 to minimise the impact of transient behaviours.

Thus, the noises in $\tilde{V}_X$ are the sources to the detection noise $\sigma P_{F=2}^D$, which have two physical catalogues. One is the fluctuation of the scattering rate of atoms, $\sigma R^{\text{Sc}}(t)$, in the steps of $\{N_2, N_{\text{tot}}\}$, while the other is the fluctuation of the detection background ground, $\sigma B_g(t)$, in all four steps. It is more convenient to define the averaged relative fluctuation of the scattering rate, $\sigma_L^D = \langle \sigma R^{\text{Sc}}(t) \rangle / R^{\text{Sc}}$, and the averaged background fluctuation in the unit of atom numbers, $\sigma_{Bg}^D = \langle \sigma B_g(t) \rangle / \kappa_{\text{atom}}$, with the conversion factor, $\kappa_{\text{atom}} = 1.6 \times 10^{-5} \text{ mV/atoms}$, calibrated in Eq. (6.5). The time average $\langle \cdot \rangle$ was performed on the trimmed detection of 30 μs .

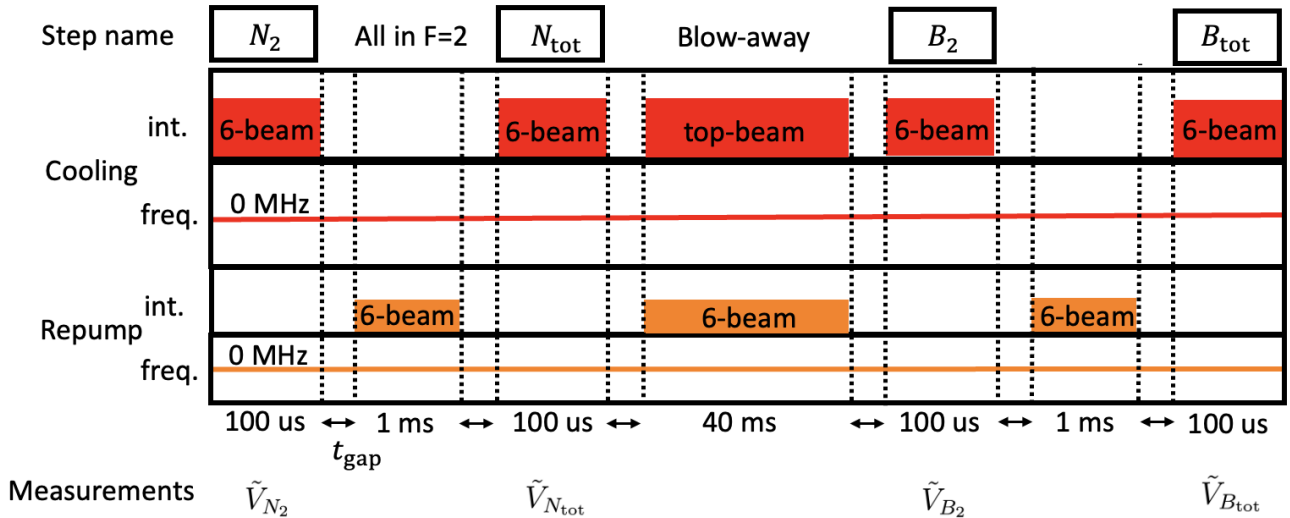


Figure 7.26: Detection sequence to estimate the population in $F=2$. $\tilde{V}_{N_2}$ is the photodiode signal averaged over the middle 30 μs in the step N_2 of 100 μs , where six MOT beams of cooling light (no repump) induced the fluorescence of atoms via the $F = 2$ to $F' = 3$ transition. Although the beam configuration for $\tilde{V}_{N_{\text{tot}}}$ was the same, it effectively detected the total number of atoms due to the step of all in $F=2$. After blowing away atoms with the cooling light from the top MOT beam, the scattered light background of the six MOT beams of cooling light, $\tilde{V}_{B_2}$ and $\tilde{V}_{B_{\text{tot}}}$, were collected. $t_{\text{gap}} = 200 \mu\text{s}$ was found useful to minimise the contamination of repump light in the detection.

Assuming that four voltage measurements were independent, error propagation leads to

$$\sigma P_{F=2}^D = \sqrt{\underbrace{\frac{2(\sigma_{Bg}^D)^2}{N_{\text{tot}}^2}}_{\alpha} + \underbrace{\left(2(\sigma_L^D)^2 + \frac{2(\sigma_{Bg}^D)^2}{N_{\text{tot}}^2}\right)}_{\beta} P_{F=2}^2}, \quad (7.21)$$

which shows the two parameters of the detection noise, α and β , are determined by σ_L^D and σ_{Bg}^D . To characterise σ_L^D and σ_{Bg}^D , atoms were prepared in $F = 2$ before performing 36 measurements of $\tilde{P}_{F=2}$ to estimate its standard deviation at a fixed N_{tot} with a constant loading time. As shown in Fig. 7.27 (a), the mean value was found to be $\langle \tilde{P}_{F=2} \rangle = 1.097$ instead of 1. We attributed this to a 10% loss of atoms in the N_2 detection step due to heating and rescaled all observed population estimations by this value. In this case, $\sigma P_{F=2}^D = \sigma \tilde{P}_{F=2} / \langle \tilde{P}_{F=2} \rangle$ is expected to be $\sqrt{2(\sigma_L^D)^2 + 4(\sigma_{Bg}^D)^2 / N_{\text{tot}}^2}$ after substituting $P_{F=2} = 1$ into Eq. (7.21).¹⁰ Then $\sigma_L^D = (2.2 \pm 0.4) \times 10^{-3}$ and $\sigma_{Bg}^D = (3.44 \pm 0.06) \times 10^3$ were determined by measuring $\sigma P_{F=2}^D (P_{F=2} = 1)$ as a function of N_{tot} (by varying the loading time) as shown in Fig. 7.27 (b). During the chameleon measurements, our atom numbers drifted between 5×10^6 and 2×10^7 (light black region), which indicates the detection noise was dominated by the probe laser noise.

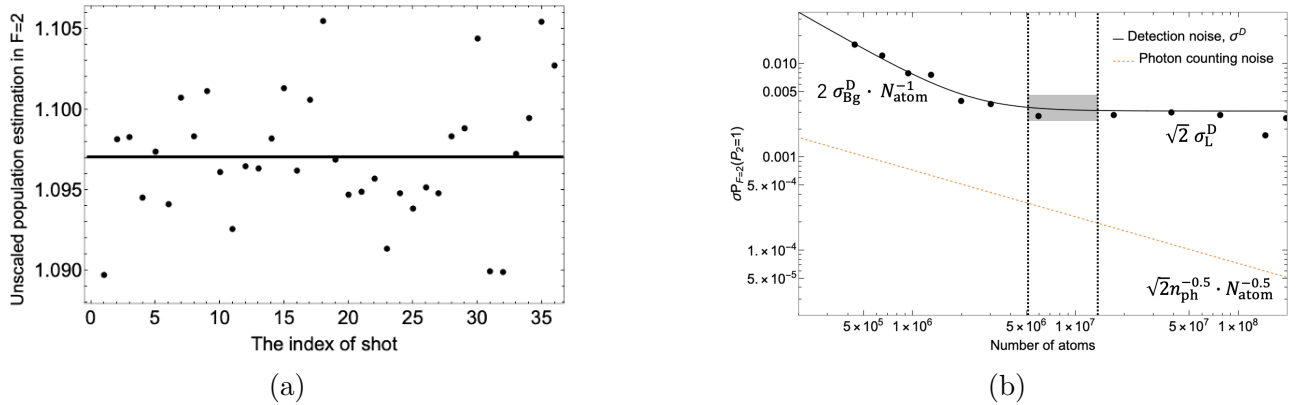


Figure 7.27: Detection noise characterisation. (a) Measuring the statistics of the unscaled population estimator $\tilde{P}_{F=2}$. The unscaled population estimator was slightly larger than 1 presumably due to the heating of atoms in the resonant detection during the N_2 step. The MOT was loaded for 700 ms to get $\approx 10^7$ atoms after velocity selection. (b) The statistics of the rescaled population estimation, $P_{F=2} = \tilde{P}_{F=2} / \langle \tilde{P}_{F=2} \rangle$, as a function of the number of atoms. Its standard deviation, $\sigma P_{F=2}$, was proportional to N_{atom}^{-1} at low number of atoms and became independent of N_{atom} at high number of atoms. The fundamental noise level (dashed yellow line) due to photon counting was ≈ 10 times below the detection noise during the chameleon measurement (light-dark region).

¹⁰The factor of two for $(\sigma_L^D)^2$ came from two measurements, $\tilde{V}_{N_2}$ and $\tilde{V}_{N_{\text{tot}}}$, where probe lasers were on. The factor of four for $\frac{(\sigma_{Bg}^D)^2}{N_{\text{tot}}^2}$ came from the detection background noise contributing to all four voltage measurements.

Probe laser noise

The probe laser noise sourced from the fluctuation of the relative scattering rate, $\sigma_L^D = \langle \sigma R^{\text{Sc}}(t) \rangle / R^{\text{Sc}}$, has two contributions of the probe laser (3D MOT cooling laser), which are intensity fluctuations and frequency fluctuations. The discussion below focuses on the scattering rate noise due to laser frequency fluctuation and leaves the impact of intensity fluctuations to the discussion of background noise.

We measured the laser frequency fluctuations with two methods. Firstly, we converted the error signal of the locked laser with a frequency sensitivity factor of $\kappa_{\text{error}} = -12.7 \text{ mV/MHz}$, which was calibrated in Fig. 4.15, to the frequency fluctuation shown in Fig. 7.28 (a). The frequency fluctuation measurement was noisy because the electronic noise is comparable to the $\approx 10 \text{ mV}$ -level variation of the error signal. To determine the frequency fluctuation more precisely, we used a cavity (Thorlabs, SA-200-5B). After calibrating the frequency sensitivity of the cavity feature with a spectroscopy feature, we adjusted the cavity length such that the transmission of the locked probe laser was parked at the cavity's positive slope, whose frequency sensitivity was $\kappa_{\text{cavity}} = 129.6 \text{ mV/MHz}$. Then the fluctuation of the cavity transmission was simultaneously recorded when the error signal in Fig. 7.28 (a) was recorded. Converting the cavity transmission into the probe laser frequency fluctuation gives results shown in Fig. 7.28 (b), which agrees with the frequency fluctuation measured in Fig. 7.28 (a) of a much better signal-to-noise ratio.

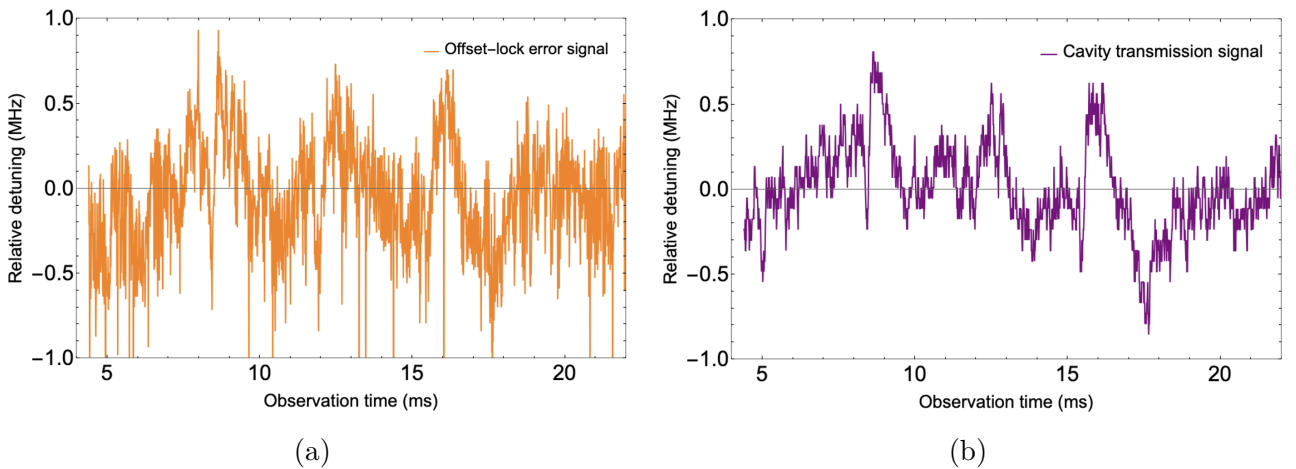


Figure 7.28: Characterisation of the frequency fluctuation of the probe laser. (a) Measuring frequency fluctuation with the variation in the offset-lock error signal. (b) Measuring frequency fluctuation with the variation of the cavity transmission. Both were simultaneously recorded.

To estimate the contribution to $\sigma_L^D = \langle \sigma R^{\text{Sc}}(t) \rangle / R^{\text{Sc}}$ from the laser frequency fluctuation measured in Fig. 7.28, the relative scattering rate was measured as a function of the probe laser detuning as shown in Fig. 7.29. The frequency dependence of the relative scattering rate was estimated by the frequency dependence of $\tilde{V}_{N_2}$, which was detected with various detection frequencies at fixed loading time and loading frequency. To reduce the impact of frequency fluctuation on this frequency dependence measurement, we extended the trimmed detection duration to 150 μs . Fitting a quadratic model to the data shown in Fig. 7.29 gives

$$R(\delta) \propto -0.189\delta^2 + 0.0129\delta + 2.0867. \quad (7.22)$$

Substituting $\delta(t)$ measured in Fig. 7.28 (b) into $\sigma_{L,\text{freq}}^D = \sigma \left(\langle R^{\text{Sc}}(\delta(t)) \rangle / R^{\text{Sc}}(\delta = 0) \right)$ gives $\sigma_{L,\text{freq}}^D = 1.7 \times 10^{-3}$, which agreed with the probe laser detection noise $\sigma_L^D = (2.2 \pm 0.4) \times 10^{-3}$.

We conclude that the primary contribution to the probe laser detection noise, $\sigma_L^D = \langle \sigma R^{\text{Sc}}(t) \rangle / R^{\text{Sc}}$, was the frequency fluctuation induced scattering rate fluctuation. The uncontrolled probe laser intensity fluctuation had a negligible contribution to the scattering rate fluctuation.

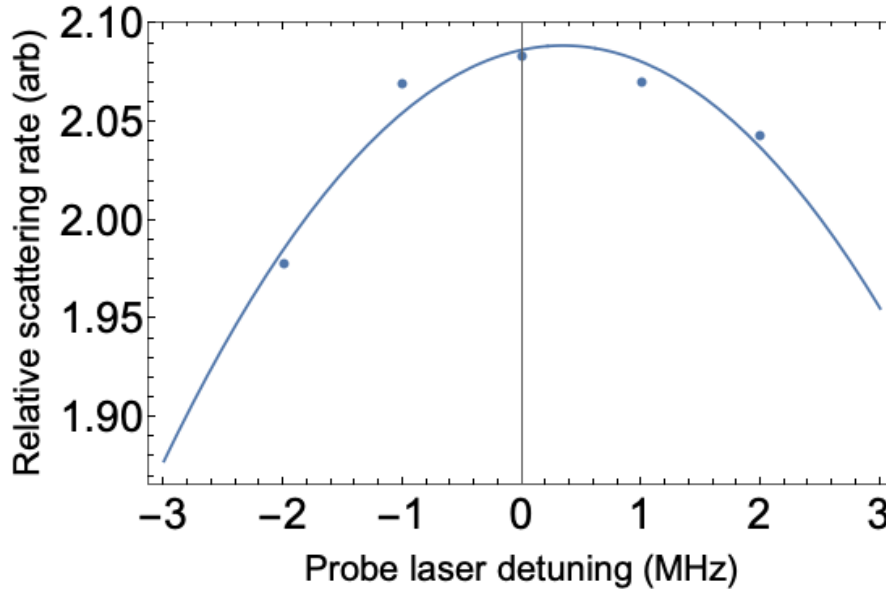


Figure 7.29: Relative scattering rate as a function of laser detuning. The frequency dependence of the relative scattering rate was estimated by the frequency dependence of $\tilde{V}_{N_2}$ at various detection frequencies with a fixed loading time at a fixed loading frequency. Then a quadratic model was fitted to the data.

background noise

An independent characterisation of $\sigma_{\text{Bg}}^{\text{D}}$ is to look at the standard deviation of $(\tilde{V}_{B_2} - \tilde{V}_{B_{\text{tot}}})/\sqrt{2}$, σV_{B} , during the detection sequence shown in Fig. 7.26, which gives $\sigma V_{\text{B}} = 50 \mu\text{V}$ and is consistent with $\sigma_{\text{bg}}^{\text{D}}/\kappa_{\text{atom}} = (55 \pm 10) \mu\text{V}$ measured in Fig. 7.27. This fluctuation in the light background has three sources, which are intensity fluctuation, photon counting noise and electronic noises from the photodiode.

To individually characterise these three sources, instead of collecting fluorescence photons from atoms, we used the same detection sequence in Fig. 7.26 to collect photons directly from one of the 3D MOT beams. In this case, $\tilde{V}_X$ yields four successive measurements of the beam power in the four steps $X=\{N_2, N_{\text{tot}}, B_2, B_{\text{tot}}\}$ within one shot. Then we repeated 36 shots and obtained 36 power ratios between the steps of B_2 and B_{tot} , $R_{\text{power}} = \langle \tilde{V}_{B_2} \rangle / \langle \tilde{V}_{B_{\text{tot}}} \rangle$. Their standard deviation, σR_{power} , was plotted as a function of the mean power in the B_2 step, $V_B = \langle \tilde{V}_{B_2} \rangle$, shown in Fig. 7.30. They agree with a noise model

$$\sigma R_{\text{power}} = \sqrt{\underbrace{\frac{2(\sigma_{\text{Bg,PD}}^{\text{D}})^2}{V_B^2}}_{\text{electronic noise}} + \underbrace{2(\sigma_{\text{Bg,Inten}}^{\text{D}})^2}_{\text{intensity fluctuation}}}, \quad (7.23)$$

where $\sigma_{\text{Bg,PD}}^{\text{D}} = 293 \pm 3 \mu\text{V}$ is the electronic noise in each step and $\sigma_{\text{Bg,Inten}}^{\text{D}} = (2.2 \pm 0.2) \times 10^{-3}$ is the relative power variation in each step. The prediction of the photon counting noise (orange) was more than a factor of two smaller than these two sources, thus it was ignored in Eq. (7.23).

It is surprising that the electronic noise from the photodiode, $\sigma_{\text{Bg,PD}}^{\text{D}} = (293 \pm 3) \mu\text{V}$, in this beam power measurement was measured to be much larger than the fluctuation of the scatter light background, $\sigma_{\text{Bg}}^{\text{D}} = 50 \mu\text{V}$, in the noise characterisation with atoms in Fig. 7.27. One possibility is the undesired optical feedback found in Section 6.1.2 contaminated this power measurement by adding an additional light whose power fluctuated at $\sigma_{\text{Bg,PD}}^{\text{D}}/\kappa_{\text{power}} = 0.6 \text{ nW}$ using the power sensitivity calibrated in Eq. (6.4).

Nevertheless, the relative power fluctuation, $\sigma_{\text{Bg,Inten}}^{\text{D}} = (2.2 \pm 0.2) \times 10^{-3}$, should be the same for this power measurement and the noise characterisation with atoms. It suggests that the power of each beam was varied at 0.2% level after averaging over the detection duration of 30 μs .

Since the mean scattered light background was measured as $\langle \tilde{V}_{B_2} \rangle \approx 40 \text{ mV}$, the beam power fluctuation can contribute a background noise as $\langle \tilde{V}_{B_2} \rangle \sigma_{\text{Bg,Inten}}^D \approx 88 \pm 8 \mu\text{V}$, which roughly agreed with $\sigma_{\text{Bg}}^D = 50 \mu\text{V}$. Note that this relative power fluctuation also induced noise in the atom scattering rate using Eq. (6.2),

$$\sigma_{\text{L,Inten}}^D = \frac{\langle \sigma R^{\text{Sc}}(s_{\text{tot}}(t)) \rangle}{R^{\text{Sc}}} \approx \frac{1}{R^{\text{Sc}}} \frac{dR^{\text{Sc}}}{ds_{\text{tot}}} \langle s_{\text{tot}} \rangle \sigma_{\text{Bg,Inten}}^D = \frac{\sigma_{\text{Bg,Inten}}^D}{1 + \langle s_{\text{tot}} \rangle}, \quad (7.24)$$

However, the atoms were strongly saturated because $s_{\text{tot}} = 34$ was estimated in Section 6.4.2. This gave $\sigma_{\text{L,Inten}}^D = (6.3 \pm 0.6) \times 10^{-5}$, which was a negligible contribution to the observed relative scattering rate fluctuation $\sigma_{\text{L}}^D = (2.2 \pm 0.4) \times 10^{-3}$.

We conclude that the relative power fluctuation of $\sigma_{\text{Bg,Inten}}^D = (2.2 \pm 0.2) \times 10^{-3}$ during the $30 \mu\text{s}$ detection was a candidate to explain the background fluctuation of $\sigma_{\text{Bg}}^D = 50 \mu\text{V}$ for the measurement background, $\langle \tilde{V}_{B_2} \rangle \approx 40 \text{ mV}$, due to the scattered detection light.

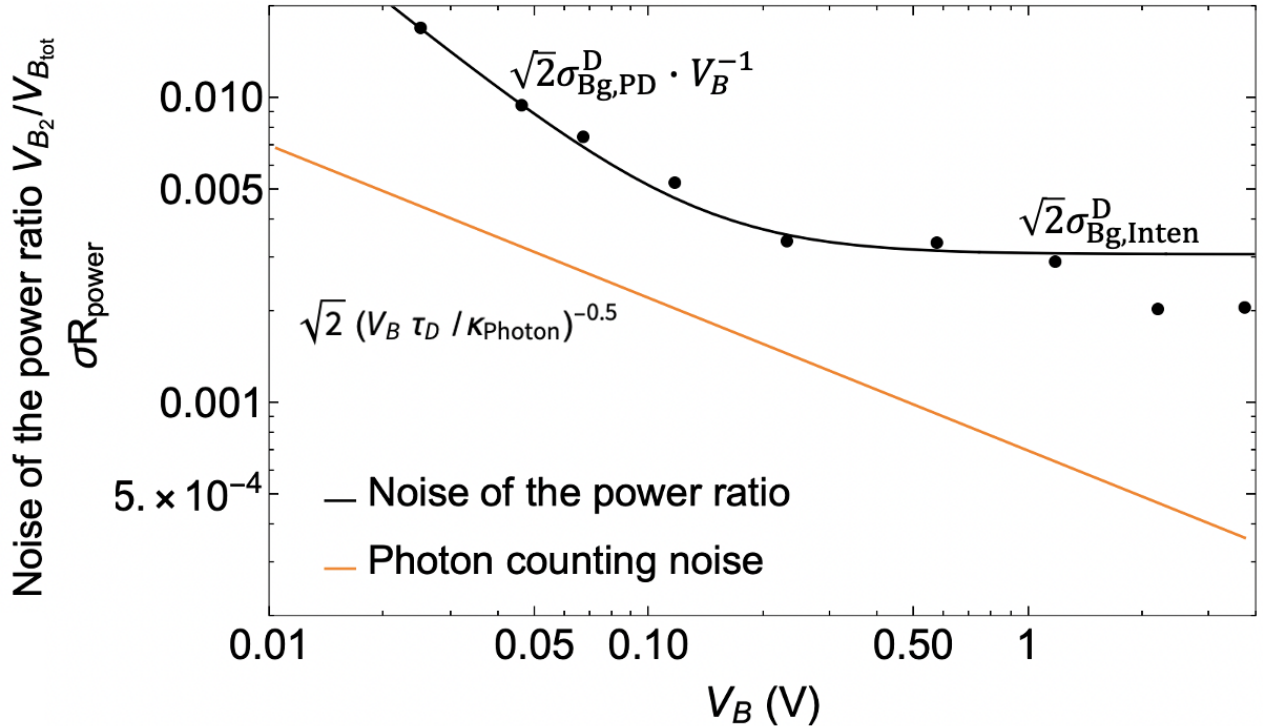


Figure 7.30: Characterisation of the background noise. The ratio of $\langle \tilde{V}_{B_2} \rangle$ to $\langle \tilde{V}_{B_{\text{tot}}} \rangle$ is effectively the ratio of two successive beam power measurements. Its standard deviation (black) was plotted as a function of $V_B = \langle \tilde{V}_{B_2} \rangle$. At small V_B , the photodiode noise, $\sigma_{\text{Bg,PD}}^D$, dominated the noise of the power ratio. At large V_B , the relative intensity noise, $\sigma_{\text{Bg,Inten}}^D$, became dominant. The photon counting noise during the detection time $\tau_D = 30 \mu\text{s}$ was calculated with the photodiode photon counting rate responsivity $\kappa_{\text{photon}} = 1.22 \times 10^{-5} \text{ V}/\mu\text{W}$ calibrated in Eq. (6.4).

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Chapter 8

Conclusion and prospect

8.1 Summary of achievements

The major achievement is that I have improved the statistic sensitivity by more than a factor of two. My interferometer shows an acceleration sensitivity smaller than $41 \mu\text{m/s}^2$ compared to the last generation which illustrated an average acceleration sensitivity of $85 \mu\text{m/s}^2$. This improvement was achieved by upgrading the free-drop atom interferometer to our fountain interferometer, which was realised by implementing the moving molasses. This increased the interferometer time from $2T_{\text{free-drop}} = 32 \text{ ms}$ to $2T_{\text{fountain}} = 66 \text{ ms}$. More atoms in the interferometer also contributed to my improvement of the statistic sensitivity. I increased the number of atoms after velocity selection from $< 2 \times 10^6$ to $\approx 10^7$. The number of atoms was increased by redesigning a differential pumping tube between the 2D-MOT cell and the science chamber and rapidly ramping the cooling laser frequency by controlling the laser diode current. I implemented a state preparation using double velocity selection pulses on atoms in $F=1$ where 33% of them were in the magnetic insensitive state before velocity selections. This is better than 20% of atoms in the magnetic insensitive state before velocity selection used in the last generation. I also demonstrated a ready-to-implement scheme using microwave adiabatic passage that prepared more than 80% of atoms into the magnetic insensitive state before velocity selection.

The second achievement is I have characterised the noise budget for our system, which was not

provided by the last generation. It reveals that our phase estimation of a fringe was limited by a phase noise of 273 mrad per shot ($15.6 \mu\text{m/s}^2$). This is more than a factor of two smaller than the chameleon acceleration sensitivity of $41 \mu\text{m/s}^2$ by fitting two fringes and comparing their phases. This is attributed to some unknown phase instability between two phase comparisons. The detection noise of $F=2$ at the steepest slope was found to give the phase estimation noise floor of $\sigma P_{F=2}^D(P_{F=2} = 0.44)/A = 37 \text{ mrad}$ ($2.1 \mu\text{m/s}^2$), where $\sigma P_{F=2}^D(P_{F=2} = 0.44) = (1.7 \pm 0.2) \times 10^{-3}$ is the population detection noise at the middle of a fringe and $A = 0.047$ is the fringe amplitude. $\sigma P_{F=2}^D(P_{F=2} = 0.44)$ has a primary contribution from the probe laser frequency fluctuation of $(1.37 \pm 0.25) \times 10^{-3}$ and a secondary contribution from the fluctuation of the scattered light background $(9.7 \pm 1.7) \times 10^{-4}$ because of the beam power fluctuation at 0.2% level.

The third achievement is we have improved the damped velocity-sensitive Rabi oscillation. I have developed a model that well describes the damping behaviour, which was found to be limited by the intensity inhomogeneity of the Raman beam. As a by-product, a novel method of directly measuring the differential AC Stark shifts to the clock transition using microwave spectrum was developed, which experimentally confirmed the magic ratio for the Raman-II rejection beam configuration.

The last achievement is that we measured a non-zero result linked to the ball, $\mathbf{n} \cdot \mathbf{a}_{\text{ball}} k_{\text{eff}} T^2 = -46 \pm 3 \text{ mrad}$ ($\mathbf{n} \cdot \mathbf{a}_{\text{ball}} = -2.597 \pm 0.186 \mu\text{m/s}^2$), which indicated a repulsive force between atom and the source mass that was not the target attractive chameleon force and was not reported in the previous generation of experiment. This repulsive result motivated us to revise the study of systematic error in the context of our fountain interferometer. We evaluated the Newtonian gravity acceleration taking into account its time dependence, which was treated as a constant in the last generation. The phase shift induced by the magnetic field gradient was suppressed by a factor of 20 because the trajectory during the first half interferometry time $0 < t < T$ is identical to the second half $T < t < 2T$. The theory of estimating the phase shift due to scattered Raman light was developed, but found not enough to explain the observed systematics. We concluded the wavefront distortion due to clipping of the Raman beams by the source mass could cause a systematic phase shift that behaved like the negative phase shift.

8.2 Prospect

Our negative non-zero result implied a repulsive force between atoms and the in-vacuum source mass. However, an attractive force is expected in the context of chameleon gravity. In the near future, we are going to use an aperture to clip the incident beam such that the in-vacuum source mass is in the shadow, which should suppress the systematics from the Raman light scattered from the source mass.

Once the systematic error gets suppressed, we need to figure out the source of the instability between phase comparisons. Note that our current repetition rate is 2.2 s per shot, which is limited by the software configuration time and data acquisition time (1.5 s) rather than the time for loading atoms (0.7 s). An optimisation of the software can potentially improve our repetition rate by a factor of two. Taking data quicker may help suppress the instability issue. Solving the instability between two phase comparisons is expected to improve the acceleration sensitivity from $\mathbf{n} \cdot \mathbf{a}_{\text{ball}} = 41 \mu\text{m/s}^2$ per shot to $\mathbf{n} \cdot \mathbf{a}_{\text{ball}} = 15.6 \mu\text{m/s}^2$, which is limited by the phase noise of 273 mrad per shot from an unknown source. A candidate for the unknown phase noise might be the air turbulence in the retro-reflection beam path. If the unknown phase noise can be solved by putting an enclosure around the retro-reflection Raman beam to minimise the air turbulence, our measurement will be limited by the MEMS intrinsic acceleration noise floor measured to be $8.5 \mu\text{m/s}^2$ (150 mrad) per shot for a $2T=66$ ms interferometer. Further improvement can be done by using two MEMS devices together that may give another factor of $\sqrt{2}$ improvement to approach a statistical sensitivity of $6 \mu\text{m/s}^2$ (106 mrad) per shot. Or, we need a better device to track the horizontal motion of the mirror due to vibration.

If the phase noise can be suppressed below the current population detection noise of $\sigma P_{F=2}^D(P_{F=2} = 0.44)/A = 37$ mrad ($2.1 \mu\text{m/s}^2$) per shot, further effort on stabilising the probe laser frequency is needed. Alternatively, a different detection scheme of using a low-noise camera to read out the two interferometer outputs simultaneously can suppress the impact of probe laser frequency noise [1, supplementary material]. In that scenario, the detection noise might be limited by the detection background noise, whose impact on population estimation can be reduced by increasing the number of atoms in the interferometer.

References

- [1] Victoria Xu, Matt Jaffe, Cristian D. Panda, Sofus L. Kristensen, Logan W. Clark, and Holger Müller. Probing gravity by holding atoms for 20 seconds. *Science*, 366(6466):745–749, 2019.

Part IV

Appendix

Appendix A

Damped Rabi Oscillation due to intensity inhomogeneity

As discussed in Chapter 2, the clock transition frequency can be shifted due to differential AC Stark shifts. Given the spatial profile of the Raman beam, there are spatial-dependent clock frequency shifts and Raman Rabi frequencies across a cloud of atoms, whose radius was about 1 mm. Consequently, the Raman Rabi oscillation will show a damping behaviour. To understand the damping behaviour, we started with a single atom of the correct velocity class in $F=1$ located at $\{x,y\}$, whose transition probability to $F=2$ (ignoring the spontaneous emission process) is given by

$$P_{F=2}^{\text{atom}}(x, y) = \frac{\Omega_{\text{Raman}}^2(x, y)}{\Omega_{\text{Raman}}^2(x, y) + \delta_{\text{AC}}^2(x, y)} \sin^2(\sqrt{\Omega_{\text{Raman}}^2(x, y) + \delta_{\text{AC}}^2(x, y)} \tau/2), \quad (\text{A.1})$$

where $\Omega_{\text{Raman}} = \Omega_1 \Omega_2 / \Delta$ is the effective Raman coupling and δ^{AC} is the differential AC Stark shift to the clock transition frequencies. The observed P2 estimator in the experiment is a weighted average over the atom cloud given by $P_{F=2} = \int dx dy \rho(x, y) P_{F=2}^{\text{atom}}(x, y)$. If the atom cloud can be assumed to be a two-dimensional Gaussian centred at $\{x_0, y_0\}$, the weight function can be described with $\rho(x, y) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp(-\frac{(x-x_0)^2}{2\sigma_x^2}) \frac{1}{\sigma_y \sqrt{2\pi}} \exp(-\frac{(y-y_0)^2}{2\sigma_y^2})$.

Considering an intensity profile of the Raman beam described by a dimensionless profile function $B(x, y)$, then the Raman Rabi frequency and AC Stark shift are both proportional to $B(x, y)$ as shown below

$$\Omega_{\text{Raman}}(x, y) = \sum_i \frac{\Omega_{1,i}(x, y)\Omega_{i,2}(x, y)}{\Delta_i} = \underbrace{\sum_i \frac{d_{1,i}d_{i,2}}{\Delta \epsilon_0 c^2} \sqrt{I_1 I_2}}_{\kappa_\Omega} B(x, y) \quad (\text{A.2})$$

$$\delta_{\text{AC}}(x, y) = \sum_i \frac{\Omega_{1,i}(x, y)^2}{\Delta_i} - \frac{\Omega_{i,2}(x, y)^2}{\Delta_i} = \underbrace{\sum_i \left(\frac{d_{1,i}d_{i,1}}{\Delta_i} I_1 - \frac{d_{2,i}d_{i,2}}{\Delta_i} I_2 \right)}_{\kappa_\delta} B(x, y) \quad (\text{A.3})$$

where I_1, I_2 are the peak intensities of the Raman-I and Raman-II beams, Δ_i is the single-photon detuning to the excited state $|i\rangle$, $d_{a,b} = \langle a | \hat{d} | b \rangle$ is the transition dipole moment, and finally, $|i\rangle$ was summed over all possible excited states.

For simplicity, we could Taylor expand the beam profile $B(x, y)$ around the centre of the cloud $\{x_0, y_0\}$, which gives $B(x, y) \approx B_0 + \partial_x B|_{x_0, y_0} \cdot (x - x_0) + \partial_y B|_{x_0, y_0} \cdot (y - y_0)$, where $\partial_x B|_{x_0, y_0}$ and $\partial_y B|_{x_0, y_0}$ describes the intensity gradient along the x direction and the y direction at the centre of the cloud. Assuming $\partial_y B|_{x_0, y_0} \ll \partial_x B|_{x_0, y_0}$, the x-dependence in the beam profile is going to be ignored from now on. Then the population in $F=2$ after applying a square Raman pulse of duration τ is expected to be

$$P_{F=2}(\tau) = \frac{\kappa_\Omega^2}{\kappa_\Omega^2 + \kappa_\delta^2} \int dy \frac{\exp\left(-(y - y_0)^2 / (2\sigma_y^2)\right)}{\sigma_y \sqrt{2\pi}} \sin^2\left(\sqrt{\kappa_\Omega^2 + \kappa_\delta^2} (B_0 + \partial_y B|_{x_0, y_0} \cdot (y - y_0)) \tau / 2\right) \quad (\text{A.4})$$

$$= \frac{\kappa_\Omega^2}{\kappa_\Omega^2 + \kappa_\delta^2} \int d\Omega_G \frac{\exp\left(-(\Omega_G - \mu\Omega_G)^2 / (2\sigma\Omega_G^2)\right)}{\sigma\Omega_G \sqrt{2\pi}} \sin^2\left(\Omega_G \frac{\tau}{2}\right) \quad (\text{A.5})$$

where $\mu\Omega_G = \sqrt{\Omega_{\text{Raman}}^2(x_0, y_0) + \delta_{\text{AC}}^2(x_0, y_0)}$ is the generalised Rabi frequency at the centre of the atom cloud, $\sigma\Omega_G$ characterised the inhomogeneity of the generalised Rabi frequency Ω_G across the cloud. Evaluating Eq. (A.5) leads

$$\frac{\kappa_\Omega^2}{\kappa_\Omega^2 + \kappa_\delta^2} \frac{1}{2} \left(1 - \exp(-\sigma\Omega_G^2 \cdot \tau^2 / 2) \cos(\mu\Omega_G \cdot \tau) \right), \quad (\text{A.6})$$

which is the model I used for the coherent transferred population in Eq. (7.5).

Appendix B

Aligning polarisation-maintaining fibers

Polarisation-maintaining (PM) fibres are extensively used in our experiment. Its working principle is quite similar to a waveplate, which has two principle axes of significantly different refractive indices. As shown in Fig. B.1 (a), the cross-section of a PM fibre has two ‘panda eyes’, which are stress rods. Therefore, the reflection index for incident light of a linear polarisation aligned with the panda eyes, n_e , is significantly different from that of a linear polarisation perpendicular to the panda eyes, n_o . Consequently, incident light with vacuum wavelength, λ_0 , of an arbitrary polarisation repeats its polarisation after propagating an integer number of the beat length, $L_{\text{beat}} = \lambda_0 / (n_e - n_o)$.

Ideally, a PM fibre reproduces the incident polarisation at the fibre output if its length, L_{fibre} , satisfies $L_{\text{fibre}} = m \cdot L_{\text{beat}}$ where m is an integer. In practice, any change in the refraction indices due to the ambient temperature drift can change the beat length, causing the drift of the polarisation at the PM fibre output. This section summarises how I characterise the stability of the output polarisation of a PM fibre and maximise its stability with a half waveplate and a quarter waveplate.

To make the polarisation of an incident beam as linear as possible, a polarising beam splitter shown in Fig. B.1(a) was used to clean its polarisation. After that two mirrors were used to steer the beam such that most of the power was coupled into the target PM fibre, whose output polarisation was monitored with a polarimeter (Thorlabs, PAX1000IR1/M). The polarisation state of the light was mapped to a point on the Poincare sphere. Initially, the polarisation state

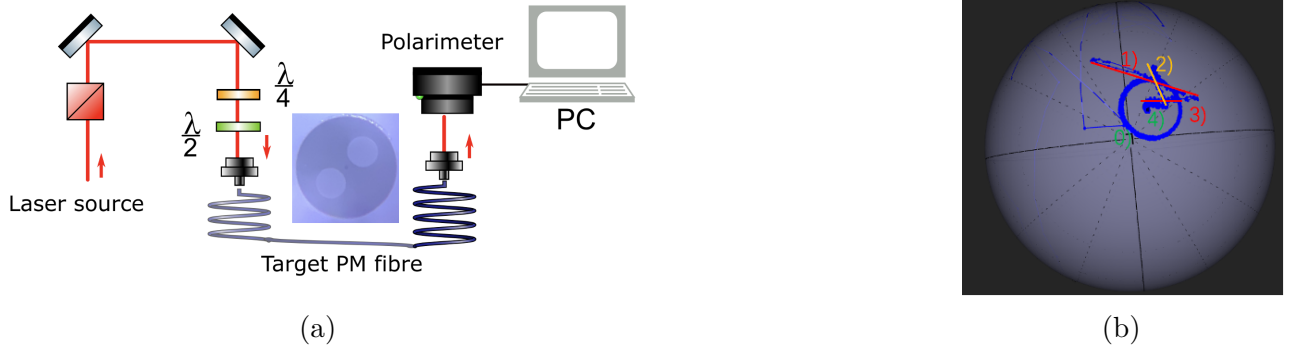


Figure B.1: Aligning a polarisation maintaining fibre. a) The schematics of the experimental setup. The quarter waveplate combined with the half waveplate was found useful to align the polarisation to the principle axis of a polarisation-maintaining (PM) fibre. b) The polarisation is visualised with a polarimeter. The trace of the polarisation as the half waveplate was rotated is marked in red lines (1 and 3). The polarisation trace as the quarter waveplate was rotated is marked in orange lines (2). The polarisation trace formed a circle as the fibre was warmed up by hand (0 and 4).

draws a circle on the Poincare sphere (0,green) as shown in Fig. B.1 (b) when I warmed the fibre with my hand. This circular trace on the Poincare sphere indicates the output polarisation was sensitive to the change in the ambient temperature. To achieve better stability, which is equivalent to a circular trace of a smaller radius when the fibre was warmed, I used the conventional method of rotating the half-waveplate. However, it only lead to a line (1,red) that was almost tangent to the initial circle and cannot significantly get to the centre. Then I added a quarter-waveplate and tweak its orientation, which moved the polarisation state closer to the centre of the initial circle. The corresponding trajectory is shown as the orange line labelled as 2. Finally, a rotation of the half-waveplate can move the polarisation state to the centre of the initial circle (3,red). Now, warming the fibre only lead to a circular trace of a much smaller radius than the radius of the initial circular trace. This finishes the alignment of the input polarisation to one of the principle axes of the PM fibre.

Note that the quarter-wave plate in my alignment process was necessary. An explanation is the incident beam had some birefringence because of the two steering mirrors. This was initially compensated by only using the half-waveplate with a $\approx 45^\circ$ tilt around the axis perpendicular to the optical table. Then we found adding an additional quarter waveplate is more efficient to compensate for the extra birefringence. Our two-waveplate method achieved the target stability in much fewer steps than the conventional method of only using a half-waveplate that needed extra adjustment in the tilt.