

PRACTICAL SOLUTIONS FOR SEISMIC FREE-SURFACE AND INTERNAL MULTIPLE ATTENUATION BASED ON INVERSION

By

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Declaration of Originality

I confirm that the work in this thesis is my own. I appropriately acknowledge the work of others which are referenced in this thesis.

Abstract

Multiple prediction through inversion (MPI) is an effective method for seismic multiple attenuation. The research in this thesis aims to make the MPI method more practical for both free-surface and internal multiple attenuation.

For free-surface multiple attenuation, the MPI scheme requires the input data to be dense and regularly sampled, and with one shot at each receiver position. In order to meet these requirements, I use a multilevel B-spline method for seismic data reconstruction. This method can perform regularisation and interpolation on seismic data without any prior-knowledge of models.

For free-surface multiple attenuation on marine data, MPI can generate superior results compared to SRME (surface-related multiple attenuation). However, MPI is more computationally expensive due to the large amount of matrix operations involved. The conventional implementation addresses this by approximating the multiple model prediction operator as a pentadiagonal or a tridiagonal matrix. Tackle this problem by solving the full prediction operator using a Graphic Processing Unit (GPU), this accelerates the processing and improve the multiple attenuation results, especially for far-offset traces.

As extensions of SRME for internal multiple attenuation, both the CFP (common-focus-point) technique and correlation method have problems. The results can be improved using the MPI method with GPU acceleration. The correlation method is preferred as the initial step for MPI because it can be implemented as a fully data-driven pre-stack domain approach in either forward data space or inverse data space. In all cases, the MPI scheme generates internal multiple models with improved kinematic and dynamic accuracy.

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Abbreviations

AVO Amplitude versus offset
CDP Common depth point
CFP Common focus point
CMP Common middle point
CPU Central processor unit
DTS Different time shift
EMCM Expanded multi-channel matching (Page 41)
EPSI Estimation of primaries by sparse inversion (Page 34)
GPU Graphics processor unit
GRT General Radon transform
IDS Inverse data space
MPI Multiple prediction through inversion
NMO Normal move-out
S/N Signal-to-noise
PEF Prediction error filter
SRME Surface-related multiple elimination

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Chapter 1 INTRODUCTION

In seismic exploration, the recorded data are used for the interpretation of subsurface geological structures or physical properties. However, seismic data are usually contaminated by a large number of different types of noise. Among them, multiples are the most complex and hardest to remove, and increase uncertainties and risks in any subsequent production and development. It is therefore important to remove seismic multiples effectively from the recorded data without affecting primary reflections. This thesis will tackle problems of both free-surface and internal multiple attenuations, in particular practical aspects of these methods.

1.1 MULTIPLE AND MULTIPLE ATTENUATION

In theory, recorded seismic traces are the convolution of the reflection coefficients and the source wavelet. Most imaging algorithms make the assumption that the recorded energy has been upward reflected only once in the subsurface, as shown in Figure 1.1 by the yellow lines. This kind of reflection is termed “primary”. In practice however, the reflected waves may also be bounced back downwards. This generates “Multiples”, which are defined as seismic events (energy) that have been reflected or scattered more than once in the subsurface (Sheriff, 1991). In Figure 1.1, the blue and green lines indicate multiple reflections.

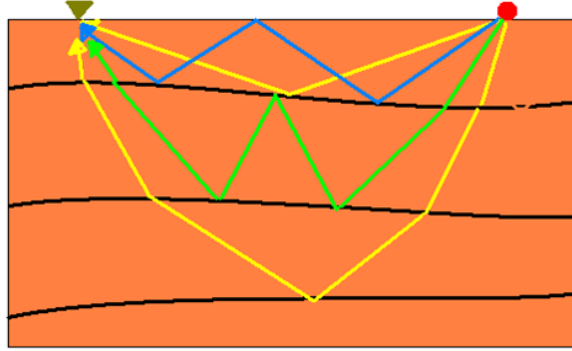
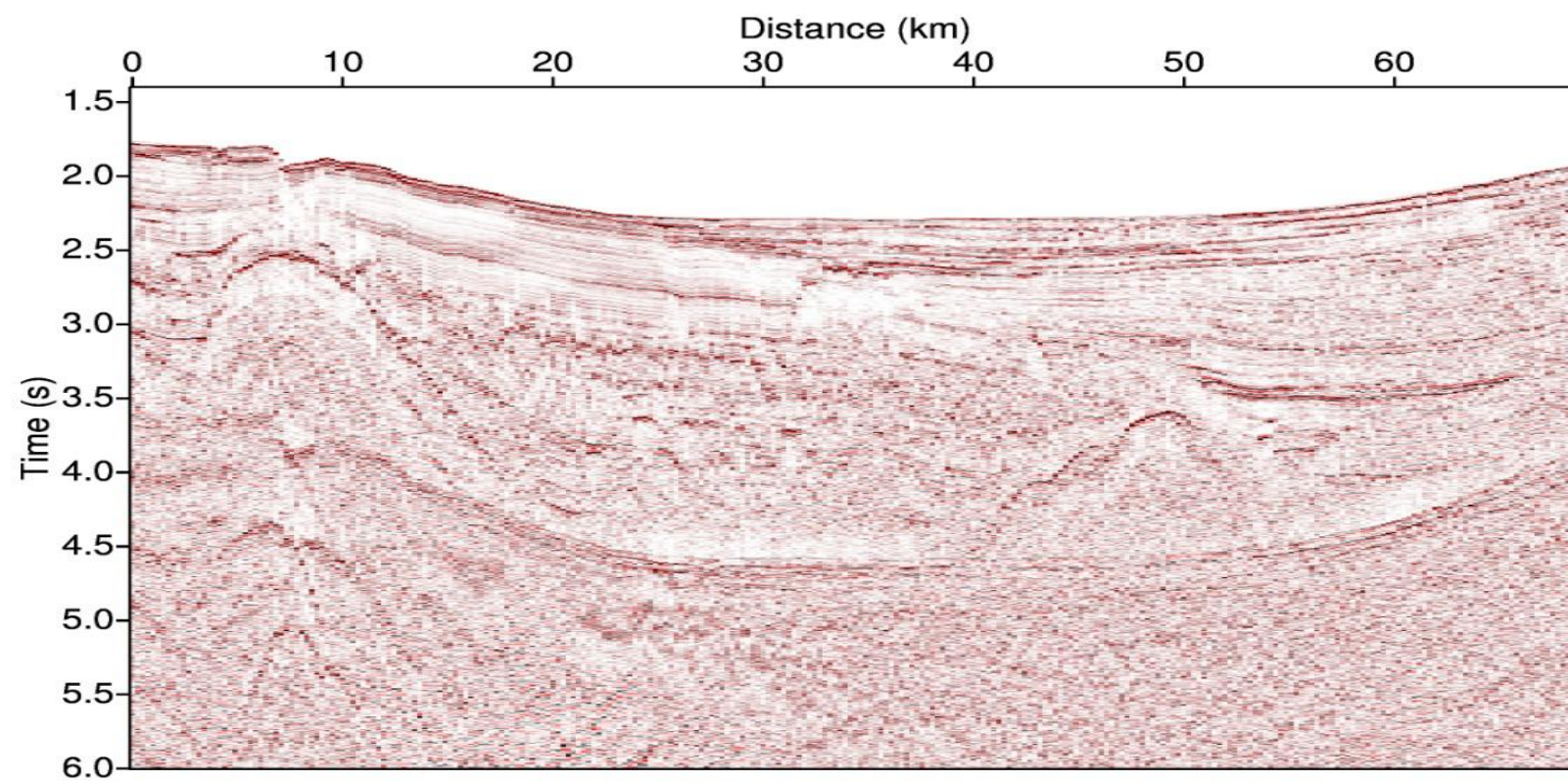


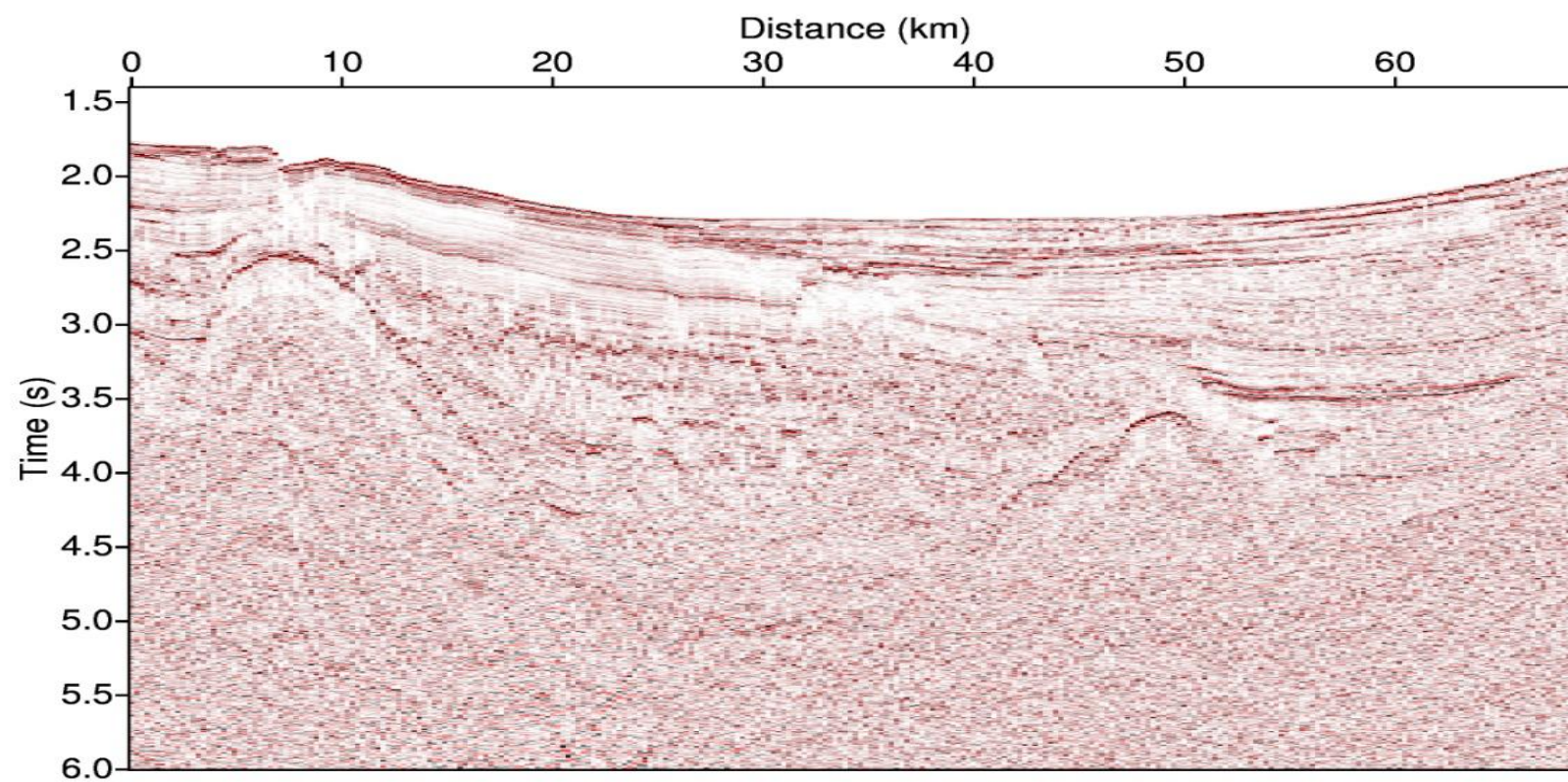
Figure 1.1 Primary reflections have only one upward reflection in the subsurface (yellow lines) and multiple reflections have at least one downward reflection (blue and green lines).

In seismic data, the energy generated by multiples contaminates primary reflections, and reduces both the spatial resolution and the signal-to-noise (S/N) ratio of the data. Furthermore, multiples can also obscure the accuracy and utility of many procedures, e.g. depth-migration, velocity-model building or AVO analysis, and can often lead to increased uncertainties and risks when using seismic data to guide production and development. It is therefore important to remove the multiples effectively from the recorded data without affecting the amplitudes of the primary reflections. Figure 1.2 shows two seismic sections from a 2D marine data set (Appendix A.1). The first (Figure 1.2a) is the original stack section without multiple attenuation while the second (Figure 1.2b) is the corresponding result after multiple attenuation. Before multiple attenuation, the seismic section displays a confused mixture of multiple and primary reflections which make the geology difficult to discern, while after multiple attenuation, the primary reflection response is clearly visible.



(a)

Figure 1.2 Stacked section of a marine seismic dataset: (a) before multiple attenuation.



(b)

Figure 1.2 Stacked section of a marine seismic dataset: (b) after multiple attenuation.

The attenuation of multiple reflections in seismic data has been of high priority in industry for a long time. There are many partial solutions which can tackle the problem under certain circumstances. However, no single method, or group of methods, can work in all cases. That is why multiple contaminations are still a dominating problem that reduces S/N ratio and resolution. This is particularly true for surface-related multiples on marine data and internal multiples on all data.

In this thesis, the core strategy for multiple attenuation is based on the multiple prediction through inversion (MPI) scheme. I will tackle the related problems of both free-surface and internal multiple attenuation and propose a few new approaches especially focusing on the practical solution aspects.

1.2 OUTLINE OF THIS THESIS

Chapter 2 Literature review. Many strategies have been developed to tackle multiple problems under different circumstances. These approaches are based on different physical principles and proved to be successful to some extent. All the work in this thesis is based on the previous contributions. I will first give a brief review of those methods, so as to clarify where my work has started.

Chapter 3 Practical implementation of MPI scheme for free-surface multiple attenuation on 2D marine data. For free-surface multiple attenuation, wave-equation based methods such as surface-related multiple elimination (SRME) and multiple prediction through inversion (MPI) are more effective than the conventional f - k and Radon transform domain filtering methods. In order to practically implement wave-equation based methods for real data, some careful pre-processing steps are requested. Among them, seismic data interpolation is essential.

In seismic acquisition, the original recorded data may be sparse and irregular. However, in both SRME and MPI methods for multiple attenuation, the input data set must be dense and regularly sampled with one shot at each receiver position. In order

to meet this requirement, I propose to use a multilevel B-spline method for seismic data regularisation and interpolation.

In this chapter, I will show the effectiveness of applying the multilevel B-spline method for seismic interpolation as a pre-processing step for multiple attenuation by MPI. Then I will implement the MPI scheme for free-surface multiple attenuation on a 2D real marine data set. The MPI method generates better results on this data set than SRME.

Chapter 4 Improving the effectiveness of MPI with a full prediction operator. MPI is an effective method for free-surface multiple attenuation but is computationally expensive due to the large amount of matrix operations it requires. In the past, this has been avoided by assuming the operator for multiple model prediction is a pentadiagonal or a tridiagonal matrix. In this chapter, this prediction operator is solved in full by inversion, with the aid of Graphic Processing Unit (GPU). Since the GPU has a highly paralleled structure, it is able to improve efficiency by two orders of magnitude for marine seismic multiple prediction.

Since it is affordable to calculate the full prediction operator in multiple modelling by MPI, one is able to improve the 3D multiple attenuation result especially for far-offset traces. The procedure involves two steps: SRME with a time-domain trajectory-stacking generalised Radon transform (GRT) to generate the initial multiple prediction; the MPI scheme is then used to refine the multiple model.

Chapter 5 The MPI scheme for the internal multiple attenuation. Multiple prediction through inversion (MPI) generates the multiple model iteratively by inversion. In this chapter, I extend this scheme from free-surface multiple attenuation to internal multiple attenuation. For the initial step of this iterative procedure, I test different methods including common-focus-point (CFP) method and correlation method.

As extensions of SRME for internal multiple attenuation, both the CFP and correlation methods have similar problems, for example, the edge effect that appears

in conventional SRME. As an improvement, these problems can be tackled by the extended MPI scheme. The performance of MPI with different methods as the initial step shows that the internal multiple models predicted by MPI are accurate not only in time but also in phase and amplitude; this could in turn reduce the non-linearity of the problem in the subsequent subtraction stage. I demonstrate the effectiveness of MPI for internal multiple attenuation using both synthetic and field data examples.

Chapter 6 An inverse data space method for the internal multiple attenuation. Conventional seismic processing can be considered as an implementation in the forward data space, with discrete seismic measurements arranged in a data matrix $\mathbf{P}$, each column of the data matrix representing a common shot gather and each row of the matrix representing a common receiver gather. For seismic multiple attenuation, this data matrix $\mathbf{P}$ can be replaced with its inverse $\mathbf{P}^{-1}$, the so-called inverse data space (IDS). In the inverse data space, multiples are mapped onto the zero time axis and can be eliminated simply by removing data at zero time.

In this chapter, I implement a fully data-driven inverse data space internal multiple attenuation method in CMP gathers, which is a 1.5D approach. This method can be used as an alternative approach to start MPI which then improves the internal multiple attenuation results iteratively. The effectiveness of the whole procedure is demonstrated with a synthetic example.

Chapter 7 Conclusions and perspectives. At the end of the thesis, I will come to some conclusions regarding my research work and provide suggestions for future research on seismic free-surface and internal multiple attenuation.

Chapter 2 LITERATURE REVIEW

The early publications on seismic multiple problems started in 1948 (Dix, 1948; Gutenberg and Fu, 1948; Hansen and Johnson, 1948; Waterman, 1948; Burg *et al.*, 1951). Since then, this issue has been a hot and difficult topic in seismic research both in academia and industry. Many different multiple-removal methods have been developed based on different characteristics of multiples. Predictive deconvolution is proved to be efficient in attenuating energy of short-period multiples. (Robinson, 1957; Kunetz and Fourmann, 1968; Silverman and Sparks, 1965; Peacock and Treitel, 1969; Sinton *et al.*, 1978; Taylor *et al.*, 1979; Gibson and Lerner, 1984; Taner *et al.*, 1995). Attenuation methods for long-period multiples attenuation can be briefly classified into two categories:

1. Methods based on the different spatial behaviours of primaries and multiples;
2. Methods based on wave-equation theories which predict a model of multiples and then subtract it from original data.

2.1 SEISMIC FREE-SURFACE MULTIPLE ATTENUATION METHODS

2.1.1 MULTIPLE ATTENUATION METHODS BASED ON FK AND RADON TRANSFORMS

For the first category of multiple attenuation methods, the characteristic most commonly adopted for discrimination is based on the different stacking velocities of primary and multiple events. If we consider two seismic reflection events that arrive at the receiver with roughly the same arrival time, the multiple reflection has been travelling into the shallower part of the earth, while the primary reflection has travelled the deeper part of the earth (Figure 2.1). If we assume that propagation velocities increase with depth then the primary reflection encounters higher velocities

compared to the multiple reflection. This difference can be enhanced by applying NMO. Figure 2.2 shows the separation between primary and multiple velocities in a CMP gather. In Figure 2.2a, the gather is NMO corrected with the primary velocity so that the multiples are under-corrected because of their low velocities. On the contrary, if the gather is corrected by the multiple velocity, the multiples are flat and all the primaries are over-corrected (Figure 2.2b). As a result, primaries and multiples can be separated according to their different apparent moveout velocities. A group of multiple-removal methods are based on this fact, including the f - k transform, Radon transform and their variations.

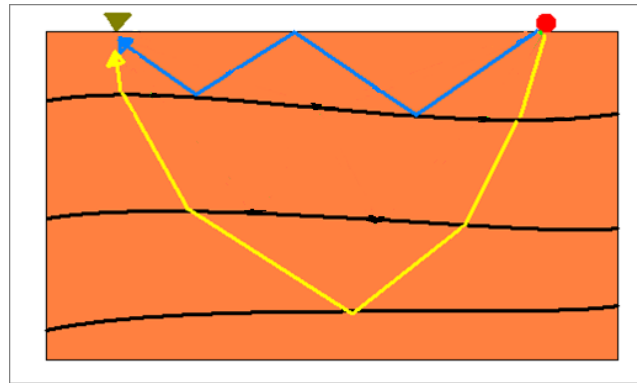


Figure 2.1 A primary reflection travels deeper in the subsurface (the yellow line) and encounters higher velocities compared to a multiple reflection that arrives at roughly the same time (the blue line).

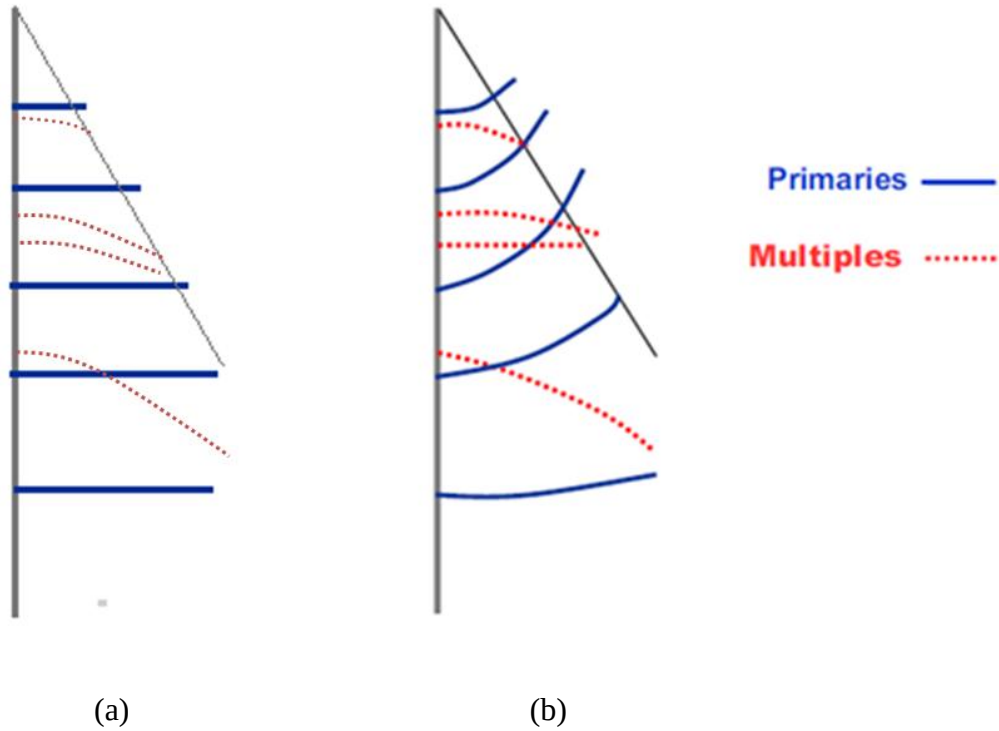


Figure 2.2 Application of NMO to a CMP gather (a) with the primary velocity, (b) with the multiple velocity.

The two-dimensional Fourier transform, or f - k transform, is a useful tool for attenuating dipping noise (e.g. the direct arrival and the ground roll). Ryu (1982) first described using the f - k transform to eliminate the multiples. In this approach, the de-multiple step is carried out after NMO correction with a velocity function that is between the primary and multiple velocities in CMP gathers. The primaries have negative wave-numbers when the NMO corrected gather (Figure 2.2b) is transformed into the f - k domain. This is because they are overcorrected and have negative dip, whereas the multiples have positive wave-numbers. Consequently, the multiples will be attenuated if the energy with positive wave-numbers is eliminated. The gather can then be transformed back to the t - x domain to produce the multiple-free data. The f - k de-multiple method performs poorly at near offsets because at small offsets primaries and multiples have similar dips. Another problem for the far offsets is spatial aliasing when the residual moveout of the multiples goes beyond the Nyquist Frequency and

the aliased energy enters the range of primary events (Yilmaz, 1987). The f - k transform scheme also requires regular increments of offset within the input gathers.

An alternative to f - k filtering is to use a Radon transform which stacks the data on common parabolic or hyperbolic slopes. In the Radon transform, parabolic functions are used to model the multiple events in the NMO corrected CMP gather (Hampson, 1986). This provides a better separation of the primary and multiple events because parabolic functions more closely match the moveout. The Radon transform is used to decompose the input data into zero-offset time and zero-offset curvature spectra. Events with different curvature are then more easily identified and removed in this domain than in the conventional t - x or f - k domains. The data can then be reconstructed from the transform with a limited move-out range to give a model of the data containing only the corresponding curvatures.

Compared with the conventional Radon transform implemented on 2D CMP, the generalized Radon transform (GRT) has an additional degree of freedom along the lateral location. Following Thorson and Claerbout (1985), the GRT can be defined as an inverse problem in the time domain. Hokstad and Sollie (2003) suggested a frequency-domain solution to this inverse problem using the scheme of Sacchi *et al.* (1998) to improve the efficiency.

All these filtering methods assume that the stacking velocities increase with depth, but this is not always valid. For example, shallow high velocity carbonates may cause an interval velocity inversion which then decreases the differences in stacking velocities between primary and multiple reflections with depth. This can result in no velocity difference between multiples and overlying primary events. Methods based on velocity discrimination fail when differences between primary and multiple velocities are not big enough (Kneib and Bardan, 1997). Moreover, the velocities of the primary events need to be accurate. Given the ambiguity of picking velocities on multiple affected data, seismic processors risk either removing primary energy or

incompletely removing the multiples. These techniques are therefore often unable to preserve the amplitudes of the primary reflections.

Figure 2.3 shows an example of applying the Radon transform method for the free-surface multiple attenuation to a CMP gather in a 2D marine data set (Appendix A.2). Figure 2.3a is the input gather, where strong free-surface multiple can be seen around 4.8 s and 7.1 s. Figure 2.3b shows the Radon transform de-multiple result. It can be seen that most of the free-surface multiples are attenuated. However, due the similarity of the dips, there are still residual multiples in the near-offset areas. Figure 2.3c shows the difference of the gather before and after Radon transform de-multiple processing, this shows that the amplitudes of the primary reflections are not preserved.

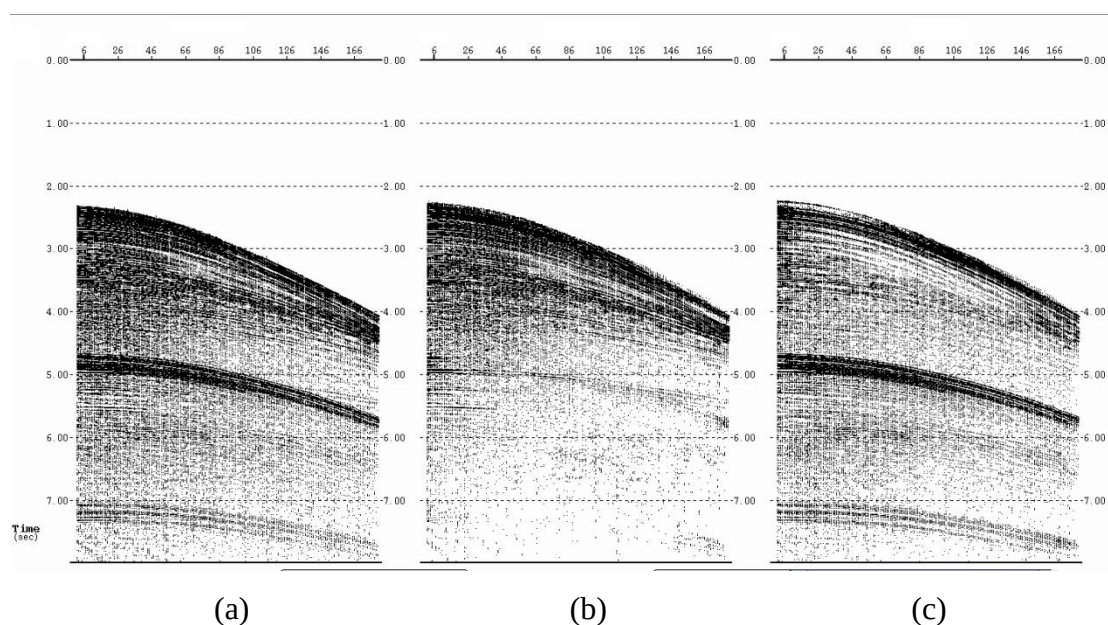


Figure 2.3 The effectiveness of Radon transform method for free-surface multiple attenuation: (a) The input gather, (b) the de-multiple result with Radon transform method. (c) the difference between (a) and (b).

2.1.2 SURFACE-RELATED MULTIPLE ELIMINATION (SRME)

Another group of successful algorithms are based on the fact that the multiples are predictable through modelling with the acoustic wave equation. These techniques,

known as surface-related multiple attenuation (SRME), have proved to be effective and widely implemented in seismic data processing. SRME is a two-step procedure, which includes multiples modelling and adaptive subtraction.

Anstey and Newman (1966) first found that convolving a trace with itself will generate the multiples within the trace. SRME method is also based on this principle. Adopting convolution techniques, it only requires data to predict multiples. This technique assumes that a seismic response is a convolution of the Earth's impulse response with a seismic source wavelet, and the foundation of the technique is spatial auto-convolution.

In spatial auto-convolution, the bounce of a wavefront at the free surface can be thought of as coming from a delayed source. If the shot and receiver spacing are equal and the data are recorded using a split spread geometry, the two bounces of the wavefront will be recorded as two primaries in the dataset. As showed in Figure 2.4, Primary *AB* is in the trace whose shot is located at point *A* and receiver is located at point *B*, respectively, primary *BC* is in the trace whose shot is located at point *B* and receiver is located at point *C*. These two traces are then convolved together to generate the trace *AC* whose shot is located at point *A* and receiver is located at point *C* and which corresponds to the predicated multiples. The predicted multiples are kinematically correct because the arrival time of multiple *AC* is simply the sum of the arrival times of the two primaries. This is the simple but powerful principle on which SRME is based. Every surface-predictable multiple, no matter how complicated its ray path is, consists of segments which, from a surface perspective, are primary events (Dragoset and Jericevic, 1998).

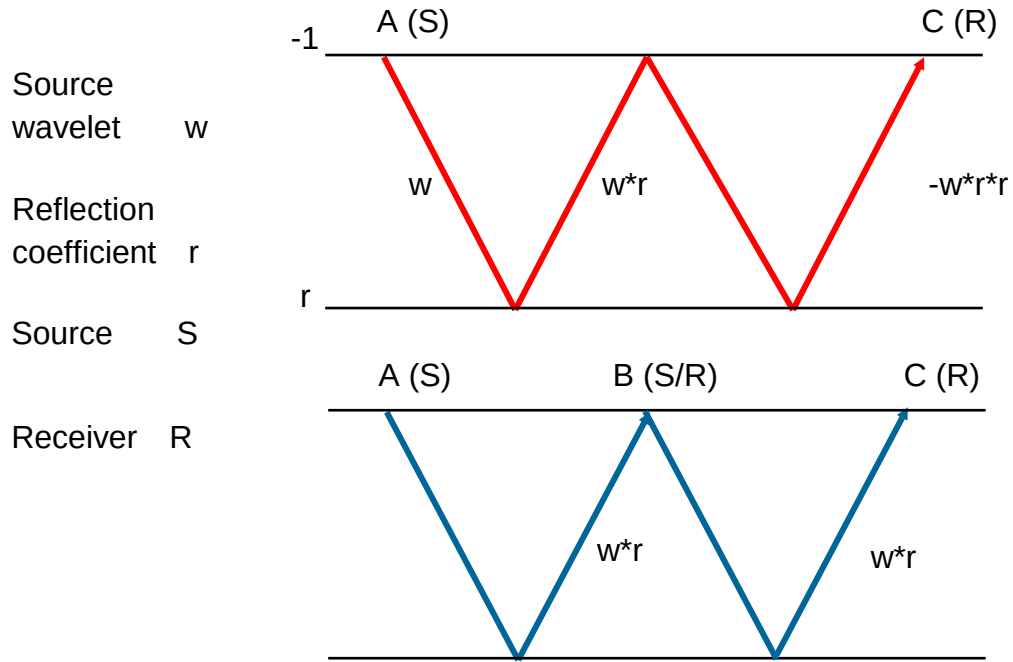


Figure 2.4 The principle of spatial convolution.

In practice to predict the multiples in a trace, several different groups of traces are needed. In fact, it is impossible to select which traces are convolved together to predict a particular individual trace. This problem is solved by convolving all traces in common shot gathers with traces in related common receiver gathers, and then combining them using the Kirchhoff integral to produce the ultimate prediction trace. This is the wave equation consistent solution used by both Verschuur (1991) and Dragoset and Jericevic (1998). Figure 2.5 illustrates the spatial convolution procedure.

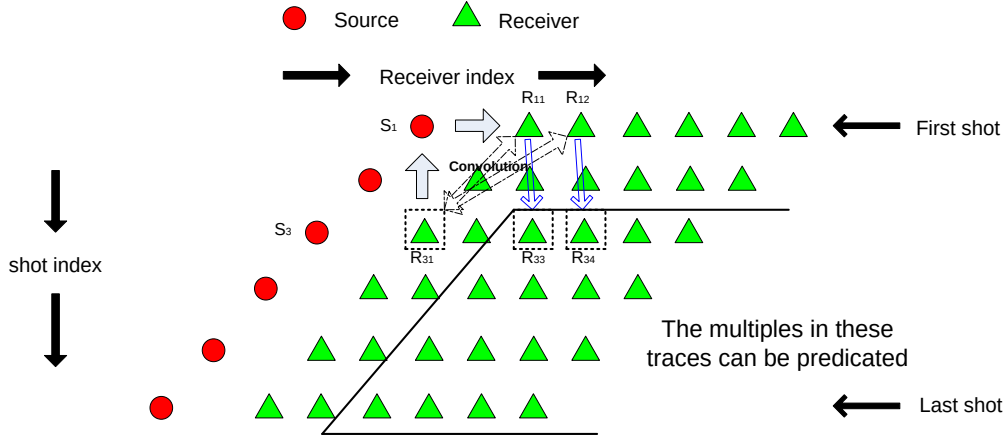


Figure 2.5 The procedure of spatial convolution.

This is a simple example consisting of 6 shot gathers with 6 receivers for each shot (there are 2 missing near-offset traces including zero-offset). The shot spacing is equal to the receiver spacing. The common shot records are placed along the rows. As a result, the common receiver records are the columns. In the third shot gather, the first receiver is defined as R_{31} . This receiver records the wavefield originally coming from source S_3 . According to the principle of spatial convolution, convolving the trace at R_{31} with the trace at R_{31} will predict the model of multiples coming from source S_3 which are downward-reflected at R_{31} . The traces whose shot is at R_{31} can be found in the shot gather whose source is S_1 as shown in Figure 2.5. In this shot gather, the first receiver R_{11} is in the same common receiver gather as receiver R_{33} , so convolving the trace at R_{31} with R_{11} will predict the multiple model at R_{33} . With the same approach, convolving the trace at R_{31} with R_{12} will predict the multiple model at R_{34} . As a result, the multiple models for all the traces in Figure 2.5 can be predicted. However, this also shows that conventional SRME cannot properly predict the multiples for near-offset traces.

Verschuur *et al.* (1992) derived the SRME scheme for single-component data assuming that only longitudinal (P) waves are measured. In this method, a 2D seismic line with fixed spread of N receivers is considered. The first shot is located at the first receiver position and moved along the receivers with the same spatial interval as the receivers. N shot records are produced at the end. The data are transformed to the f - x domain, and separated to N monochromatic common-shot gathers with N complex samples for each shot. Following Berkhout (1982), the data matrix is defined by storing monochromatic shot records into the columns of the matrix for this particular frequency. Such a data matrix describes the total 2D seismic data for one frequency component and can be constructed for every frequency component (Figure 2.6).

With the notation taken from Berkhout (1982), matrices are indicated with bold capitals, **P**, and a column of the matrix contains a shot record (or a wavefield) and a row contains the seismic response from all source positions to the same receiver. In this way, we get matrices of N dimensions with the zero offset data on the main diagonal and the common midpoint data on the anti-diagonals for 2D seismic data. Using this discrete notation, spatial convolutions in the t - x domain can be described by f - x domain matrix multiplications. In practice, the defined square matrices might be only partly filled with data (i.e., a band matrix) due to acquisition limitations.

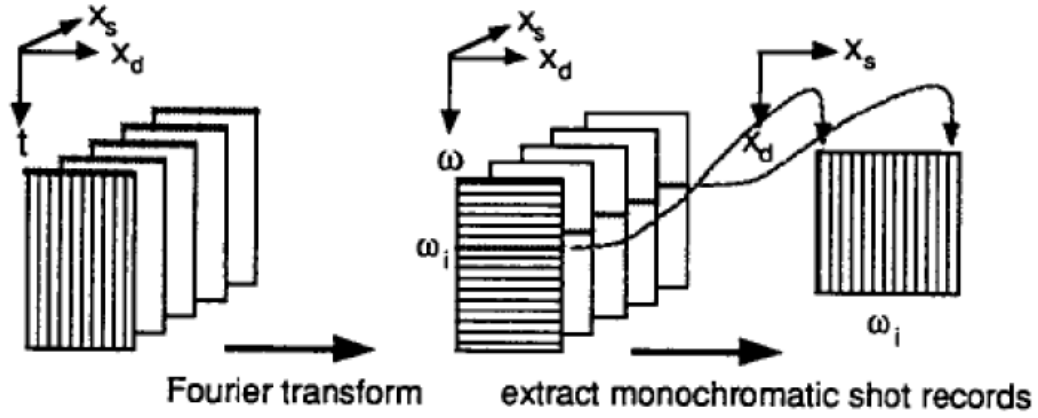


Figure 2.6 The indices of the data matrix used in SRME. The shot records are Fourier transformed to the frequency domain, and then sorted to monochromatic shot records with each frequency component stored in a column of the monochromatic data matrix. (Verschuur *et al.*, 1992).

With the notation extracted from Berkhout (1982), the upgoing seismic wavefield $\mathbf{P}_0$, at the surface can be formulated as:

$$\mathbf{P}_0 = \mathbf{X}_0 \mathbf{S}, \quad (2.1)$$

where $\mathbf{S}$ is the matrix containing the downgoing source signature and $\mathbf{X}_0$ is the subsurface response matrix for a non-reflecting surface. The response matrix includes all primary reflections and internal multiples.

With a free surface, however, all upgoing waves will be bounced back from the surface and changed into downgoing waves. Therefore, the total downgoing wavefield consists of not only the illuminating source $\mathbf{S}$, but also the downward reflected upgoing wavefield (including multiples), $\mathbf{R}\mathbf{P}$. As a result, Equation (2.1) should be modified to

$$\mathbf{P} = \mathbf{X}_0 [\mathbf{S} + \mathbf{R}\mathbf{P}], \quad (2.2)$$

where $\mathbf{P}$ is the total upgoing wavefield at the surface, and $\mathbf{R}$ stands for the reflectivity matrix of the free surface. Using Equation (2.2),

$$\mathbf{P} = [\mathbf{I} - \mathbf{X}_0 \mathbf{R}]^{-1} \mathbf{X}_0 \mathbf{S}, \quad (2.3)$$

which is the explicit expression for the total upgoing wavefield including surface-related multiples. If we expand the matrix inversion to a series, we get

$$\mathbf{P} = [\mathbf{I} + \{\mathbf{X}_0 \mathbf{R}\} + \{\mathbf{X}_0 \mathbf{R}\}^2 + \{\mathbf{X}_0 \mathbf{R}\}^3 + \dots] \mathbf{X}_0 \mathbf{S}. \quad (2.4)$$

Comparing with Equation (2.1), we find that the extra terms in Equation (2.4) are all surface-related multiples.

If the surface is pressure-free, as in the marine case, the reflectivity matrix $\mathbf{R}$ can be simplified to:

$$\mathbf{R} = r_0 \mathbf{I}. \quad (2.5)$$

Ideally, the scalar $r_0 = -1$. The simplification treats the reflection matrix as a unit matrix, which requires that both sources and receivers have a constant spacing.

Using Equation (2.5), Equation (2.3) simplifies to:

$$\mathbf{P} = [\mathbf{I} - r_0 \mathbf{X}_0]^{-1} \mathbf{X}_0 \mathbf{S}, \quad (2.6)$$

and by further expanding the inverse matrix into a series, we get:

$$\mathbf{P} = [\mathbf{I} + r_0 \mathbf{X}_0 + r_0^2 \mathbf{X}_0^2 + r_0^3 \mathbf{X}_0^3 \dots] \mathbf{X}_0 \mathbf{S}. \quad (2.7)$$

Note that the matrix $\mathbf{X}$ consists of every reflection that happens in the subsurface, and even includes elastic and anisotropic effects, as well as absorption (Verschuur *et al.*, 1992). We still need to assume that only P-waves are recorded at the surface and that the reflection coefficient r_0 can represent the surface reflectivity. Both assumptions hold well for marine data.

To remove the multiples from the data $\mathbf{P}$, Equation (2.2) can be inverted to get an explicit expression for $\mathbf{X}_0$:

$$\mathbf{X}_0 = \mathbf{P}[\mathbf{S} + \mathbf{R}\mathbf{P}]^{-1}. \quad (2.8)$$

If we define $\mathbf{X}$ as the response of the subsurface with surface-related multiples, then Equation (2.3) becomes $\mathbf{P} = \mathbf{XS}$, and thus the following formula can be derived from Equation (2.8):

$$\mathbf{X}_0 = \mathbf{X}[\mathbf{I} + \mathbf{RX}]^{-1}. \quad (2.9)$$

For Equation (2.9), a straightforward inversion may be unstable when strong multiples exist. This can be seen by expanding the inverse matrix in Equation (2.9) to the following series:

$$\mathbf{X}_0 = \mathbf{X}[\mathbf{I} - \{\mathbf{RX}\} + \{\mathbf{RX}\}^2 - \{\mathbf{RX}\}^3 + \dots]. \quad (2.10)$$

The inverse term in Equation (2.9) results in an infinite expansion in Equation (2.10). With strong multiple reflections, the series expansion converges rather slowly, and therefore direct inversion of Equation (2.9) is unstable. Verschuur *et al.* (1992) indicated that using a limited number of terms in equation (2.10) could stabilise the inversion. The number of terms depends on the highest-order surface-related multiples present in the data, because each additional term taken into account in Equation (2.10) results in one extra order of surface-related multiples.

Using Equation (2.5), Equation (2.10) becomes:

$$\mathbf{X}_0 = \mathbf{X} - r_0 \mathbf{X}^2 + r_0^2 \mathbf{X}^3 - r_0^3 \mathbf{X}^4 + \dots. \quad (2.11)$$

As can be seen from Equations (2.10) and (2.11), no model of the subsurface is required in this scheme. Only the seismic data after deconvolution for the source signature, i.e. $\mathbf{X}$, the free surface reflectivity properties, and the scalar r_0 , are used. In fact, the data contain all the necessary information for predicting the multiples and are used as the multiple prediction operators

Due to the structure of the complex matrices used in SRME, a group of shot records are needed to eliminate the multiples in one shot record and the matrix multiplication describe spatial and temporal 2D convolutions of the data with itself.

Based on the above theory, Berkhout and Verschuur (1997) developed the most popular SRME scheme currently in use. In particular, they rewrote the SRME scheme of Verschuur *et al.* (1992) as an iterative procedure:

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \mathbf{P}_0^{(n-1)} \mathbf{A} \mathbf{P} \quad (2.12)$$

In this equation the primary output is updated with a multiple removal step. The series expansion is retrieved when the iterative procedure is started with $\mathbf{P}_0^{(n=0)} = \mathbf{P}$ and several iterations are carried out. The matrix $\mathbf{P}$ is the raw input data, and $\mathbf{P}_0$ stands for the upgoing wavefield for all shot records without free-surface multiples. The surface operator $\mathbf{A}$ is defined as:

$$\mathbf{A} = [\mathbf{S}]^{-1} \mathbf{R} [\mathbf{D}]^{-1}, \quad (2.13)$$

where $\mathbf{S}$ and $\mathbf{D}$ describe the source and receiver properties, and $\mathbf{R}$ contains the surface reflectivity.

The source signature plays an important role in the procedure. Verschuur and Berkhout (1997) suggested the adoption of long global matching filters to take care of the source signature. Ziolkowski *et al.* (1999) proposed to measure the marine seismic wavefield for a precise source signature.

The estimation of primaries by sparse inversion (EPSI) method proposed by van Groenestijn and Verschuur (2009a) is based on the same primary-multiple model as SRME. But in contrast, it estimates the primaries as unknowns in a multidimensional inversion process rather than a subtraction process. The applications of this method are discussed in van Groenestijn and Verschuur (2009b, 2010 and 2011).

2.1.3 THE MULTIPLE PREDICTION THROUGH INVERSION (MPI) METHOD

Multiple prediction through inversion (MPI) proposed by Wang (2004) is a fully data-driven approach to multiple-model building for surface-related multiple attenuation. It generates the multiple model for a new iteration by updating the multiple wavefield attenuated in the previous iteration. It can be formulated as:

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{T}\mathbf{M}^{(n-1)}, \quad (2.14)$$

where $\mathbf{M}^{(n-1)}$ is the multiple wavefield attenuated in the $(n-1)$ th iteration, $\tilde{\mathbf{M}}^{(n)}$ is the multiple model for the n th iteration, and $\mathbf{T}$ is an updating operator as for any other iterative linear inverse problem. For the first iteration, the multiple model can be obtained by spatial convolution in the same way as for the SRME method, or using any other multiple prediction scheme.

The multiple wavefield $\mathbf{M}^{(n-1)}$ can be expressed as

$$\mathbf{M}^{(n-1)} = \mathbf{P} - \mathbf{P}_0^{(n-1)}, \quad (2.15)$$

where $\mathbf{P}$ is the original input data and $\mathbf{P}_0^{(n-1)}$ is the de-multiple result from the $(n-1)$ th iteration. Assuming that $\mathbf{P}_0^{(n-1)}$ needs to be updated further, the updating step length and direction from $\mathbf{P}_0^{(n-1)}$ to $\mathbf{P}_0^{(n)}$ can be approximated using $\mathbf{P}_0^{(n-2)}$ and $\mathbf{P}_0^{(n-1)}$. The updating operator $\mathbf{T}$ is therefore expressed as

$$\mathbf{T} = \mathbf{P}_0^{(n-1)}[\mathbf{P}_0^{(n-2)}]^{-1}. \quad (2.16)$$

Combining Equations (2.14), (2.15) and (2.16), the following MPI formula can be obtained (Wang, 2004):

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)}[\mathbf{P}_0^{(n-2)}]^{-1}(\mathbf{P} - \mathbf{P}_0^{(n-1)}), \quad (2.17)$$

where $n \geq 2$. Note that $\tilde{\mathbf{M}}^{(n)}$ with “ $\sim$ ” indicates the multiple model $\tilde{\mathbf{M}}$ is an approximation, and is distinguished from $\mathbf{M}$ (without “ $\sim$ ”), which is the attenuated multiple wave-field. The multiple model is then subtracted from the original data matrix as:

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \Lambda\tilde{\mathbf{M}}^{(n)}, \quad (2.18)$$

where Λ is a shaping operator and $\mathbf{M}^{(n)} = \Lambda\tilde{\mathbf{M}}^{(n)}$ is the attenuated multiple wavefield after the n th iteration. Multiple subtraction as described by Equation (2.18) may be performed adaptively in the time–space domain, in which the spatial-variant

operator Λ is implemented by localised adaptation (for example, trace by trace) in the subtraction phase.

The MPI formula given by Equation (2.17) can also be derived from the equation used in conventional SRME as shown below; the surface operator $\mathbf{A}$ is then estimated approximately and implicitly using the original data set $\mathbf{P}$ and the de-multiple results from two iterations. Its dependency on $\mathbf{P}$ can be understood physically in that the original data set $\mathbf{P}$ contains not only information about the subsurface structure but also all of the information that constitutes the surface operator. Such a relationship between $\mathbf{A}$ and $\mathbf{P}$ may well justify our attempt to take account of the nonstationarity of the surface operator in the SRME scheme.

Conventional SRME (Berkhout, 1982; Berkhout and Verschuur, 1997) can be expressed as:

$$\mathbf{P} = [\mathbf{I} - \mathbf{P}_0 \mathbf{A}]^{-1} \mathbf{P}_0, \quad (2.19)$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{P}_0$ is the primary response. If both $\mathbf{P}_0$ and the surface operator $\mathbf{A}$ are known, Equation (2.19) can be used to model the seismic record $\mathbf{P}$ consisting of primary and multiple reflections recorded at the surface. In this forward modeling procedure, the primary response $\mathbf{P}_0$ implicitly acts as a subsurface model whose components are the structure of the medium, reflectivity, raypath etc., and $[\mathbf{I} - \mathbf{P}_0 \mathbf{A}]^{-1}$ is the operator that predicts the seismic response, including all surface-related multiples.

Multiple attenuation means retrieving the primary response $\mathbf{P}_0$ from Equation (2.19). However, because we do not have any knowledge about the surface operator $\mathbf{A}$, the fundamental problem faced in conventional SRME is that two unknown variables, $\mathbf{A}$ and $\mathbf{P}_0$, are contained within one basic equation. This is the motivation for the derivation of the MPI scheme.

Equation (2.19) can be rewritten as

$$\mathbf{P}_0 = \mathbf{P} - \mathbf{P}_0 \mathbf{A} \mathbf{P}, \quad (2.20)$$

which indicates that the exact multiple wave-field is given by

$$\mathbf{M} \equiv \mathbf{P}_0 \mathbf{A} \mathbf{P}. \quad (2.21)$$

An iterative procedure is adopted by rewriting Equation (2.20) as

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \Lambda \mathbf{P}_0^{(n-1)} \mathbf{A}^{(n-1)} \mathbf{P}, \quad (2.22)$$

where

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)} \mathbf{A}^{(n-1)} \mathbf{P} \quad (2.23)$$

is the multiple-model prediction in the n th iteration and Λ is the undetermined shaping operator in Equation (2.18).

The surface operator $\mathbf{A}^{(n-1)}$ in Equation (2.22) is derived as follows. First, following Berkhout and Verschuur (1997), Equation (2.20) is rewritten as

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \mathbf{P}_0^{(n-1)} \mathbf{A}^{(n)} \mathbf{P}, \quad (2.24)$$

This can be used to improve the accuracy of $\mathbf{P}_0$ through iteration, if the surface operator $\mathbf{A}$ is supplied. Then $\mathbf{A}^{(n)}$ is derived explicitly as

$$\mathbf{A}^{(n)} = [\mathbf{P}_0^{(n-1)}]^{-1} (\mathbf{P} - \mathbf{P}_0^{(n)}) \mathbf{P}^{-1}. \quad (2.25)$$

Substituting $\mathbf{A}^{(n-1)}$ for the $(n-1)$ th iteration in Equation (2.22), we finally obtain the following expression:

$$\mathbf{P}_0^{(n)} = \mathbf{P} - \Lambda \mathbf{P}_0^{(n-1)} [\mathbf{P}_0^{(n-2)}]^{-1} (\mathbf{P} - \mathbf{P}_0^{(n-1)}). \quad (2.26)$$

This is the key equation in the MPI scheme; it indicates that, in the two-step MPI procedure, the multiple model $\tilde{\mathbf{M}}^{(n)}$ is predicted by the multiple wavefield after the previous iteration $\mathbf{M}^{(n-1)} = \mathbf{P} - \mathbf{P}_0^{(n-1)}$ and is adjusted by the operator $\mathbf{T} = \mathbf{P}_0^{(n-1)} [\mathbf{P}_0^{(n-2)}]^{-1}$.

Comparing MPI with conventional SRME, we can observe two differences. First, the new multiple model is not predicted by spatial convolution but is constructed by

updating an attenuated multiple wave-field. Secondly, the new multiple prediction approach eliminates the requirement for an explicit surface operator **A**.

2.1.4 3D SURFACE-RELATED MULTIPLE PREDICTION

One of the assumptions of 2D SRME is that the geological structures in the exploration area are 2D, i.e. no dips exist in the crossline direction. In general, this assumption does not hold, resulting in the so-called ‘3D effect’. In fact, multiple contributions also lie along crossline direction as shown in Figure 2.7. Multiple contributions in both the inline and crossline directions need to be accounted for the prediction to produce an accurate multiple model.

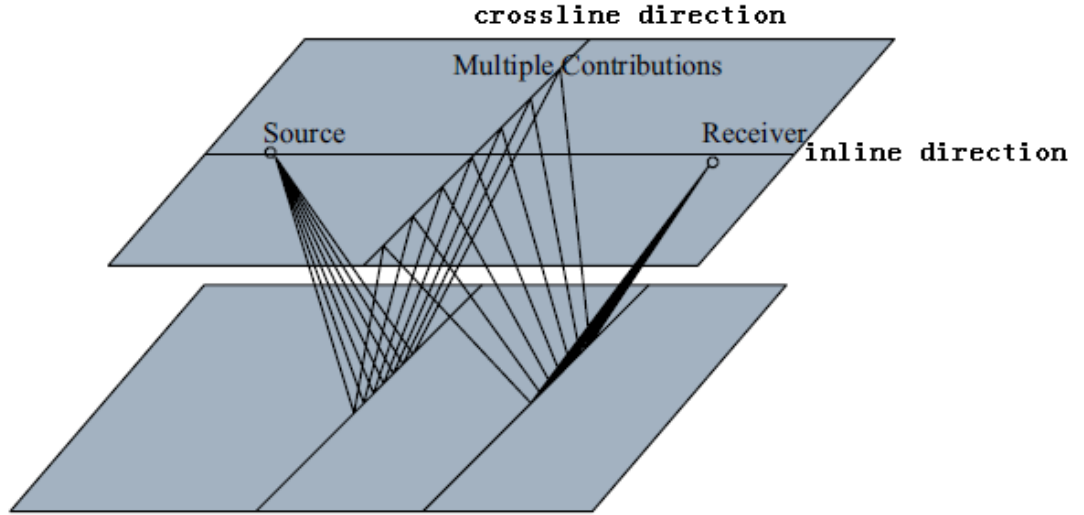


Figure 2.7 A description of 3D multiple contributions.

3D surface-related multiple prediction can be considered as an extension of the 2D case. In the frequency domain, it can be described with the formula (Van Dedem and Verschuur, 2005) for each frequency component ω as:

$$\begin{aligned}
 M(x_r, y_r, x_s, y_s, \omega) &= \sum_{y_k} \sum_{x_k} M_{xy}(x_k, y_k, \omega), \\
 &= \sum_{y_k} \sum_{x_k} P(x_r, y_r, x_k, y_k, \omega) P(x_k, y_k, x_s, y_s, \omega)
 \end{aligned} \tag{2.27}$$

where the predicted multiple trace is $M(x_r, y_r, x_s, y_s, \omega)$, and (x_s, y_s) and (x_r, y_r) are respectively the shot and receiver position of an input trace.

The summation in Equation (2.27) can be divided into two steps. The first step is the summation in the inline direction, just like the 2D SRME approach:

$$M_y(y_k, \omega) = \sum_{x_k} M_{xy}(x_k, y_k, \omega), \quad (2.28)$$

and the second step is the summation in the cross-line direction:

$$M(x_r, y_r, x_s, y_s, \omega) = \sum_{y_k} M_y(y_k, \omega). \quad (2.29)$$

As mentioned above, wave-equation based multiple attenuation techniques require one shot location for each receiver position. This is not realistic for most 3D acquisition geometries because 3D datasets normally have inadequacies in crossline sampling, limited aperture of the recorded data and irregularities in the acquisition geometry. Multiple contributions can only be calculated for the few crossline coordinates, at which source lines (approximately) coincide with receiver lines, because of the sparse crossline sampling of sources. Not only are the crossline multiple contributions sparsely sampled, but the crossline aperture is also limited.

Currently mainly two kinds of methodologies are proposed to tackle the problems. One solution is to apply interpolation on the input data so as to create the missing streamers and shot lines for the required convolution process (Duijndam *et al.*, 1999; Hindriks and Duijndam, 2000; Baumstein and Hadidi, 2004; Xu *et al.*, 2005). Interpolation, although a simple solution for the inadequately sampled crosslines, increases the volume of data and therefore adds considerable computational expense to the prediction process.

Another solution to the problem of inadequate data is to implement sparse inversion on the available multiple contributions (Van Dedem and Verschuur, 2005). Sparse inversion is used in this approach as a constraint of the Radon transform in the crossline direction which is used to predict 3D multiples, thereby avoiding direct interpolation of the input data. However, this approach has to solve large groups of

linear equations and produces good results only after a large number amount of iterations. This makes the method numerically very expensive. Another sparse inversion approach, which is numerically more efficient, is based on a frequency domain parabolic Radon transform (Hokstad and Sollie 2006). As opposed to the time domain scheme, this method does not have the capability to impose sparseness in the time direction, as demonstrated by Schonewille *et al.* (2004). Hence some accuracy is compromised to achieve numerical efficiency.

2.1.5 ESTIMATION OF THE WAVELET

The multiple models produced by the techniques discussed above are normally not very precise. This is due to the following factors: partly modelled 3D effects, inaccuracy in synthesising near offset traces, inaccuracy in the amplitude of wavelet, ambiguity in the phase of the wavelet and so on. Among these factors, the wavelet is essential. SRME and MPI include an adaptive subtraction step that essentially estimates the wavelet. Ziolkowski *et al.* (1999) proposed to measure the marine seismic wavefield for a precise source signature and then the wavelet can be replaced by an impulse, which will make the subtraction stage much easier. In practice, if the wavelet is not known, a robust adaptive multiple subtraction procedure is then essential for multiple attenuation.

2.1.6 MULTIPLE ADAPTIVE SUBTRACTION

The concept of adaptive subtraction was first proposed by Julien and Raoult (1989) who implemented the $L2$ norm to minimise the energy of seismic data after multiple attenuation. Currently, least-squares adaptive subtraction is widely adopted in two-step multiple attenuation strategies. It is commonly implemented as a multi-channel matching filter to employ the lateral coherency of adjacent traces as a constraint to suppress the effect of random noises and preserve the primaries.

Monk (1993) proposed to extend the multiple model traces to a group of their dependant traces using a constant phase rotation of the wavelet and constant time shift.

The dependant traces include their first derivative, their Hilbert transform and the derivative of the Hilbert transform. This concept was adopted by Wang (2003) to expand the conventional multi-channel matching filter to the so-called expanded multi-channel matching (EMCM) filter. The EMCM filter is given by:

$$p(t) = y(t) - \sum_{j=1}^N [f_{1,j}(t) * m_j(t) + f_{2,j}(t) * \dot{m}_j(t) + f_{3,j}(t) * m_j^H(t) + f_{4,j}(t) * \dot{m}_j^H(t)] \quad , \quad (2.30)$$

where $m(t)$ is multiple model trace, N is the number of channels involved in matching, $f(t)$ is the matching filter. $m^H(t)$ is the Hilbert transform of $m(t)$, and $\dot{m}$ and $\dot{m}^H(t)$ are the derivatives of $m(t)$ and $m^H(t)$, respectively. The raw input trace, $y(t)$, is matched in a $N \times N_c$ segment in the conventional multi-channel matching approach, where N_c is the number of segmented samples in the time direction. Three out of four input sections in the EMCM scheme are adjoined sections from the raw data section, and therefore the trace $y(t)$ is matched by a data cube of $4 \times N \times N_c$. This approach expands conventional multi-channel matching filter in new physical dimensions.

2.1.7 THE INVERSE DATA SPACE METHOD FOR FREE-SURFACE MULTIPLE ATTENUATION

Consider the full data matrix $\mathbf{P}$ expressed in the feedback model as (Berkhout, 1982)

$$\mathbf{P} = \mathbf{P}_0 + (\mathbf{P}_0 \mathbf{A}) \mathbf{P}_0 + (\mathbf{P}_0 \mathbf{A})^2 \mathbf{P}_0 + \dots, \quad (2.31)$$

where $\mathbf{P}_0$ is the surface-free response (including primaries and internal multiples) while $\mathbf{A}$ is the surface operator.

In Equation (2.31), multiplication with $\mathbf{P}_0\mathbf{A}$ mathematically means a spatial convolution; physically multiplication with $\mathbf{P}_0\mathbf{A}$ means adding to one round trip in the subsurface. Equation (2.31) can also be written as:

$$\mathbf{P} = [\mathbf{I} - \mathbf{P}_0\mathbf{A}]^{-1} \mathbf{P}_0. \quad (2.32)$$

From Equation (2.32), the formulation for free-surface multiple attenuation in the inverse data space can be derived as (Berkhout and Verschuur 2005b; Berkhout, 2006):

$$\mathbf{P}^{-1} = \mathbf{P}_0^{-1} - \mathbf{A}. \quad (2.33)$$

Equation (2.33) shows that primaries and multiples have a simple relationship in the inverse data space. The inverse surface-multiple free response should primarily be located at negative times. This is because the inversion process transforms the poles in the reverberant forward data to zeros in the non-reverberant inverse data (Berkhout, 2006). The surface operator $\mathbf{A}$ can be found at and around zero time in the inverse data space because it does not contain traveltimes according to Equation (2.13) (Berkhout, 2006).. As a result, the demultiple process in the inverse data space becomes a simple muting process which removes the surface operator $\mathbf{A}$ around time zero (Berkhout, 2006).

2.2 INTERNAL MULTIPLE ATTENUATION METHODS

For multiple attenuation methods based on velocity difference, both free-surface and internal multiples can be removed by one process. However, internal multiples generally have a higher velocity compared to the surface multiples, especially in deeper parts of seismic data. Thus for internal multiples generated in deeper structures, discrimination of velocities between primaries and internal multiples becomes problematic.

2.2.1 EXTENDED SRME METHOD FOR INTERNAL MULTIPLE ATTENUATION

A first-order surface-related multiple can be seen as the combination of two primaries that are connected to each other at the reflection. Thus primary reflections that are already available in the data can be used to construct first-order multiples. Similarly, internal multiples can be described as combinations of primary wave field components. In this case the downward reflections take place below the surface. In Figure 2.8, z_k represents the internal multiple generating interface. After downward extrapolating the shot record to the internal interface z_k , all multiples related to this interface can be removed by the surface-related algorithm. Note that only the downward scattering part of the internal multiple should be positioned at depth, whereas the sources and receivers could reside at the surface. Based on this concept, Berkhout (1982), suggested the extension of conventional free-surface multiple attenuation (SRME) method to internal-multiple attenuation by including inverse wavefield extrapolation. This idea was then reformulated in Berkhout and Verschuur (1997) by replacing common-shot gathers with common-focus-point (CFP) gathers. In this case, an inverse propagation operator and a muting process are required to extrapolate seismic data toward the internal-multiple-generating interface. This operator can be determined without explicit subsurface information with CFP technique, in which the muting becomes a straightforward process (Berkhout, 1997a and 1997b).

In Berkhout (1999), this concept was further also generalised by considering internal multiples generated by a complete layer instead of a single interface at a time. This means that the inverse wave field propagation is carried out towards to a depth level in-between interfaces but not towards an exact interface. In Figure 2.9, three types of events are combined, the first is the input data with the source at surface and the receiver at the defined level, the second is the reflection response from below the multiple generating boundaries with source and receiver at the defined level, the last

is the response with the source at the defined level and receiver at the surface. The half-redatumed data can be obtained in a similar way to the interface-related prediction method. The fully-redatumed data are obtained by applying focussing at both the source and receiver side, so that a new dataset is generated with virtual sources and virtual receivers at the chosen level. Note that the reflections related to the interface above the chosen level should be muted from this virtual source and virtual receiver gathers. Finally, internal multiples are predicted by two spatial convolutions along the chosen level of all these redatuming data. The implementation of this method was discussed in Berkhout and Verschuur (2005a) and Verschuur and Berkhout (2005) in details.

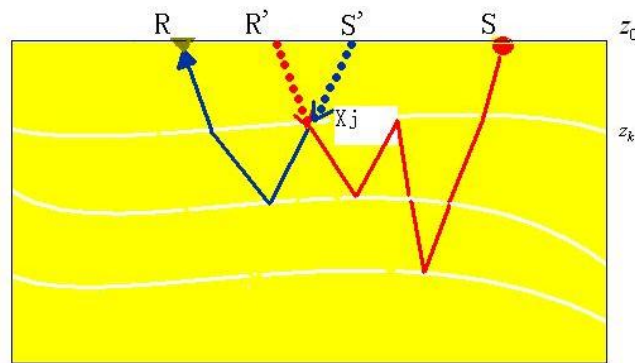


Figure 2.8 An internal multiple can be predicted using half-redatumed data is used. One dataset should have the source at the surface and the virtual receiver at the multiple generating boundary and the other dataset needs a virtual source at the boundary together with the receiver at the surface (Verschuur, 2006).

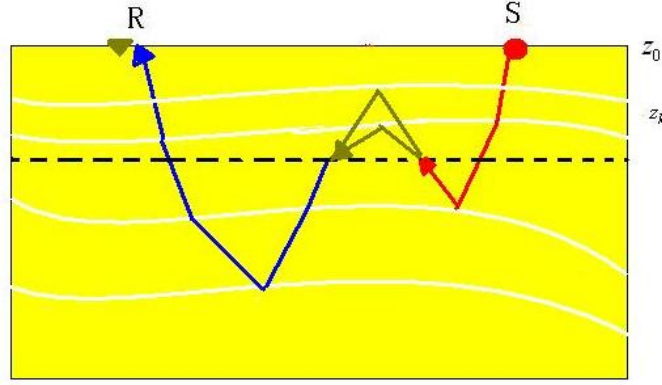


Figure 2.9 The prediction of internal multiples in the layer-related approach (Verschuur, 2006).

Jakubowicz (1998) realised that the two inverse wave field extrapolation operators needed to redatum the sources and receivers can be constructed from the time-inverse primary reflections from that interface. In Figure 2.8 this means the two dashed arrows together constitute a primary reflection which is present in the data. This consideration is also similar to the ray-based work of Keydar *et al.* (1997). As illustrated in Figure 2.10, the internal multiple is then decomposed into three components, two primaries SR' and $S'R$ minus a third primary $S'R'$. By extracting the required primary reflection from the data, the internal multiple prediction process can be written as a fully data-driven approach:

$$\mathbf{M}_k = \bar{\mathbf{P}}_{k-1} [\Delta \mathbf{P}_k]^H \bar{\mathbf{P}}_{k-1}, \quad (2.34)$$

where z_k is the internal-multiple generating interface, $\mathbf{M}_k$ is the model of the internal multiples generated by interface z_k . $\mathbf{P}_{k-1}$ is the wavefield in which multiples above level k have been removed, and $\bar{\mathbf{P}}_{k-1}$ with a top bar indicates that all primaries related to the k th interface z_k and above have been muted from $\mathbf{P}_{k-1}$. $\Delta \mathbf{P}_k$ means the isolated primary reflections only related to interface z_k and the superscript H means transposition and complex conjugation.

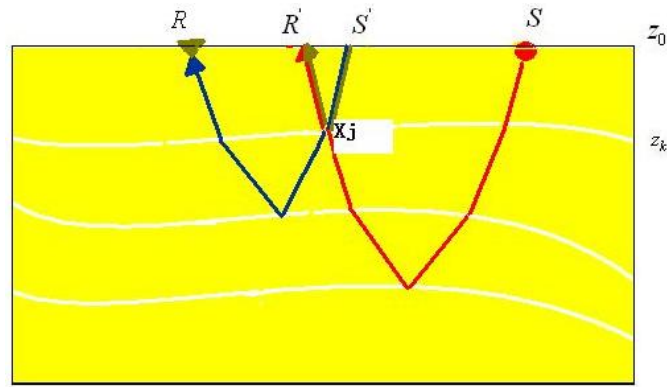


Figure 2.10 The prediction of an internal multiple can be done using three events that have both sources and receivers at the surface (Jakubowicz, 1998, as shown in Verschuur, 2006).

2.2.2 THE INVERSE SCATTERING SERIES METHOD FOR THE INTERNAL MULTIPLE ATTENUATION

For the approach based on the inverse scattering, as described by Araujo *et al.* (1994), Coates and Weglein (1996) and Weglein *et al.* (1997), one procedure predicts all possible internal multiples which can then be subtracted from the input data. This procedure is derived from an inverse scattering sub-series and can be interpreted as a sum over all possible combinations of three imaged data points that have a constrained lower-higher-lower relationship. The construction of the internal multiples is illustrated in Figure 2.11. These imaged data are the original pre-stack data migrated using a constant velocity. The multiple attenuation sub-series is a first order approximation to an internal multiple removal series. This sub-series properly predicts the travel time of all internal multiples. For typical seismic events, the amplitudes of the predicted multiples are 80-95% of the actual values. In practice, the main strength of this procedure is that all internal multiples are handled at the same time, without the need for any priori subsurface information according to Weglein *et al.* (1997). The weakness of this approach is that it is computationally very intensive.

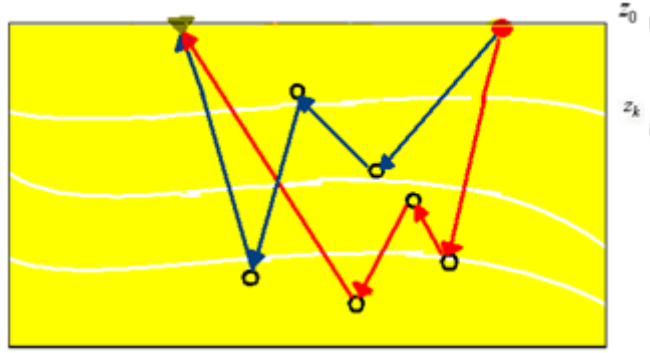


Figure 2.11 For the prediction of internal multiples using the inverse scattering series each point in the subsurface is considered as a potential scatter (Weglein, 1997, as shown in Verschuur, 2006).

2.2.3 THE INVERSE DATA SPACE METHOD FOR INTERNAL MULTIPLE ATTENUATION

Based on the concept of multiple attenuation in the inverse data space (Berkhout, 2006), Berkhout and Verschuur (2007) removed internal multiples in the inverse data space. As in the case of the extended SRME procedure for internal multiple attenuation, if we also downward extrapolate shot records from surface to the reflector and remove the response of that reflector, then the internal multiples generated from this reflector can be removed in the inverse data space by removing the pseudo surface factor. Luo *et al.* (2007) proposed a method based on correlation principles (Jakubowicz, 1998) to suppress internal multiples in the inverse data space in post-stack domain, this is a fully data-driven 1D approach.

2.2.4 OTHER METHODS FOR INTERNAL MULTIPLE ATTENUATION

Ikelle (2004) and Erez and Ikelle (2006) proposed the concept of virtual seismic events to attenuate internal multiples from surface seismic data. Virtual events are not directly recorded in standard seismic data acquisition but can be used to construct internal multiples with scattering points at the sea surface. Essenreiter *et al.* (1998 and 2001) trained a back propagation neural network to recognise and remove all

multiples. The training data consist of model data containing all multiples and the corresponding data containing only primaries. The models are based on the data from a real well log. The advantages of this method are that the neural network does not depend on restricted assumptions as in the case of conventional methods and also makes extensive use of a priori knowledge about the geology.

Wang (2007) showed that the extension of the MPI concept from surface-related multiples attenuation to internal multiples attenuation is much less problematic than the corresponding extension of conventional SRME approaches. This is because the MPI method does not require an explicit surface operator hence no explicit internal operator is needed for internal multiple attenuation. However the spatial variation of the internal operator still needs to be considered in the multiples prediction.

2.3 PRE-PROCESSING FOR MULTIPLE ATTENUATION

The seismic multiple attenuation approaches based on SRME and MPI need high quality data for the multiple modelling. The traces should be regularly and densely distributed with the same shot and receiver intervals and also have a high signal and noise ratio (S/N). This requires careful pre-processing.

2.3.1 NOISE ATTENUATION

There are several types of noise in seismic data and these are generally different for land and marine datasets. The quality of marine data is normally fairly high, but still has various types of coherent and random noises. For example, noises can come from the ships engines at near offsets, from side scattering from other objects (e.g. rigs, shorelines etc) and swell. Linear noise due to side scattering may be removed by dip filtering. Similarly, swell noise is generally random low frequency noise caused by movement of the cable and can be removed by low-cut filtering. As for land seismic data, this typically contains ground roll, refractor noise and side scattering, as

well as high levels of random noises. Ground roll usually has low frequency and slow velocity on near offsets, and it may be attenuated by either a low-cut filter applied to a cone around the sources or careful multichannel filtering.

FX-deconvolution is an effective tool for random noise attenuation (Canales, 1984). This is achieved by prediction error filtering (PEF) in the spatial direction as opposed to the time direction for conventional deconvolution approaches. It is deconvolution in the frequency space (f - x) domain, where linear events are predictable in the space direction and anything unpredictable can be considered to be noise. Curved events are also unpredictable, so filters must be short enough to approximate curved events as straight line segments. 3D FXY-deconvolution, in which data is predictable as a plane in x and y , can better preserve cylindrical events than the 2D approach (Chase, 1992). Compared with the 2D scheme, filters for 3D are two dimensional and may be shorter than in 2D, but remain stable because they possess the same number of points. In addition, linear dipping noise which is coherent in only one direction will be better attenuated in the 3D approach.

FXY-deconvolution may be applied in shot, receiver or common-offset domains. However, in practice it is rarely applied pre-stack except on common offsets due to the limitation of linear events. Wang (1999) proposed a real symmetric 3D FXY-deconvolution strategy. It improves the result by replacing the conventional average of the predictions in two directions with simultaneous prediction in both directions.

2.3.2 DATA INTERPOLATION

In marine acquisition, the shot interval is often twice the group interval (e.g. group interval of 12.5 m and shot interval of 25 m). The SRME and MPI methods require the values of the shot and group interval to be the same. In order to make a full use of all the data and avoid aliasing, an interpolation method is needed to produce data dense enough for the multiple modelling. A good interpolation method is the one which will not only honour the dip, frequency and amplitude of the data,

but also recover the aliased energy. A series of FX-interpolation methods have been proposed to tackle this problem.

Spitz (1991) introduced a seismic trace interpolation method that uses a full-step prediction-filter scheme in the frequency-space domain. With modifications to Spitz's method, Porsani (1999) proposed a half-step prediction-filter scheme that makes the interpolation process more efficient. In Wang (2002), this interpolation method is extended from 2D to 3D. Later, Naghizadeh and Sacchi (2007) proposed a multistep autoregressive interpolation method that can reconstruct data with gaps, while Naghizadeh and Sacchi (2009) proposed an adaptive interpolation method that avoids the need for spatial windows.

As in the case of FX-deconvolution, the FX-interpolation method operates on a window in both time and space. It first performs a Fourier transform to generate temporal frequencies of the windowed data. Then it designs a symmetric interpolation filter (Wang, 2002) for each potentially aliased frequency (i.e. a coherency filter from data at half the target frequency and applies the filter to the data to interpolate traces, leaving the input data unchanged.). Finally, it performs an inverse Fourier transform to reconstruct data in the space-time domain. The approach iterates over overlapping windows. FX-interpolation can be applied on sublines and crosslines, as well as CDP or shot gathers. When offset is used as one of the keys, it should be regularised so as to ensure correct grouping in that direction.

2.3.3 INCIDENT FIELD MUTING

For marine seismic data, the incident field is the direct wave from the source and its sea-surface reflection. It has to be removed before SRME because the spatial convolution only applies to the scattered field (Ziolkowski *et al.*, 1999). In practice, this can be done by muting (Verschuur, 1991) and can be helped using information on the water-bottom and water velocity.

In my examples, I have muted all the incident fields before the multiple prediction steps. The first arrivals are first picked, and then the energies arriving

earlier are muted. However, this muting process is problematical if first arrivals cross with direct waves. For example, for marine data in shallow water in far-offset sections, direct waves may arrive after first arrivals, which could not be muted effectively.

In the adaptive subtraction step, a wavelet can be estimated, which allows to calculate the incident field and then compare it with the muted-out component of the measured data. (Ziolkowski *et al.*, 1999).

2.4 SUMMARY

The problems with existing multiple attenuation methods can be summarised as follows:

(1) As the SRME and MPI methods require the seismic data to be dense and regular with one shot at each receiver position, it is often necessary to use interpolation before implementing these methods in practice.

(2) MPI is computationally expensive due to the large amount of matrix operations involved. Conventional implementations make the computation tractable by assuming that the operator $\mathbf{T}$ for multiple model prediction is a pentadiagonal or a tridiagonal matrix (Wang, 2004).

(3) As extensions of SRME for the internal multiple attenuation, both the CFP method and correlation method have similar problems, for example, the edge effect that appears in conventional SRME.

(4) The fully data-driven inverse data space method based on the correlation principle for the internal multiple attenuation has previously been implemented in the post-stack domain, which cannot produce as accurate results as in pre-stack domain.

In this thesis, I will tackle each of these problems as follows:

(1) I propose to use a multilevel B-spline method for seismic data interpolation and regularisation as the pre-processing step for multiple attenuation. This can do both 2D and 3D regularisation and interpolation on seismic data without any prior knowledge of models.

(2) I solve the full prediction operator $\mathbf{T}$ of MPI by inversion with the aid of Graphic Processing Unit (GPU) for efficient matrix computation. This accelerates the processing and improves the multiple attenuation result especially in far-offset traces.

(3) I extend the MPI scheme from free-surface to internal multiple attenuation, where the predicted internal multiple models have no edge effect and are accurate not only in time but also in phase and amplitude.

(4) I implement the fully data-driven inverse data space method based on the correlation principle in the pre-stack domain using CMP gathers (a 1.5D approach) and also integrate it into the MPI scheme.

Chapter 3 PRACTICAL IMPLEMENTATION OF MPI SCHEME FOR FREE-SURFACE MULTIPLE ATTENUATION ON 2D MARINE DATA

3.1 INTRODUCTION

Wang (2004) proposed multiple prediction through inversion (MPI) and demonstrated its effectiveness for free-surface multiple attenuation. The MPI scheme builds the multiple model by updating the multiple wavefield that was attenuated in the previous iteration to generate the multiple model for a new iteration, as is usually the case in an iterative inverse problem. Using this scheme for multiple prediction may minimise the edge effect in multiple modelling in the conventional SRME method and eliminate the need to synthesise near-offset traces which is also required by SRME. The model predicted by MPI is not only accurate in time but also in phase and amplitude, which in turn reduces the non-linearity of the problem in the subsequent subtraction stage. Wang (2007) documented two theoretical justifications of the MPI scheme. The first is that MPI considers seismic multiple attenuation as an optimisation problem, in which the cost function is optimised from one level to the next by updating the current model for each iteration. A desired updating direction should be normal to the next level of the cost function. The second is that MPI takes into account the surface operator implicitly. In MPI, the surface operator can be expressed as a derivative of the seismic data only and the surface operator from the previous iteration is included in the new iteration. This improves the accuracy of the multiple modelling.

In this chapter, I will test the MPI scheme for free-surface multiple attenuation on 2D marine data and focus on the practical aspects. As pre-processing steps, random noise attenuation and seismic data interpolation will be discussed.

3.2 MEDIAN FILTERING

The median operation has been used in signal processing for a long time. The median filter performs a nonlinear filtering operation where a window moves over a signal and at each point the median value of the data within the window is taken as the output. Median filtering has some desirable properties that cannot be achieved with linear algorithms. Its impulse response is zero, which makes it attractive in suppressing impulsive noise. Wang (2000) separates signal from noise using median correlative filtering. Zhang and Ulrych (2003) use a hyperbolic median filter to suppress multiples, although the median filter may destroy detailed structures that are smaller than the filter window used. Median filtering is also an effective tool for random noise attenuation. In Liu *et al.* (2007), the multistage median filter (MLM) was used for seismic random noise attenuation. In this method, 1D median filters are applied in several directions and a value derived from them is chosen to output at the centre of the 2D window. This method may attenuate the signal while reducing noise, the balance of which depends on the window size. It was also proposed to apply iterative filters with variable-sized length instead of a single filter with a length of constant (Liu *et al.*, 2007).

The median of N scalars $x_i, i = 1, \dots, N$ can be defined as the value x_{med} such that for all $j = 1, \dots, N$,

$$\sum_{i=1}^N |x_{med} - x_i| \leq \sum_{i=1}^N |x_j - x_i|. \quad (3.1)$$

It is known that x_{med} can always be chosen as one of the x_i . If N is odd, x_{med} is unique, and if N is even there can be an infinite number of possibilities for x_{med} . The 1D median filter, denoted by $median[]$ can then be defined as

$$median[x_i, i = 1, \dots, N] = x_{med}, \quad (3.2)$$

where N is chosen to be odd.

Let $a(x, y)$ be a 2D signal. A 2D multistage median filter (MLM) is defined as a $(2N+1) \times (2N+1)$ filter window with a center point located at (n_1, n_2) . Within the filter window, four sub-windows are defined as (Wang, 1992) (Figure 3.1):

$$\begin{cases} W_1[n_1, n_2] = \{(n_1 + k, n_2) : -N \leq k \leq N\} \\ W_2[n_1, n_2] = \{(n_1, n_2 + k) : -N \leq k \leq N\} \\ W_3[n_1, n_2] = \{(n_1 + k, n_2 + k) : -N \leq k \leq N\} \\ W_4[n_1, n_2] = \{(n_1 + k, n_2 - k) : -N \leq k \leq N\} \end{cases} \quad (3.3)$$

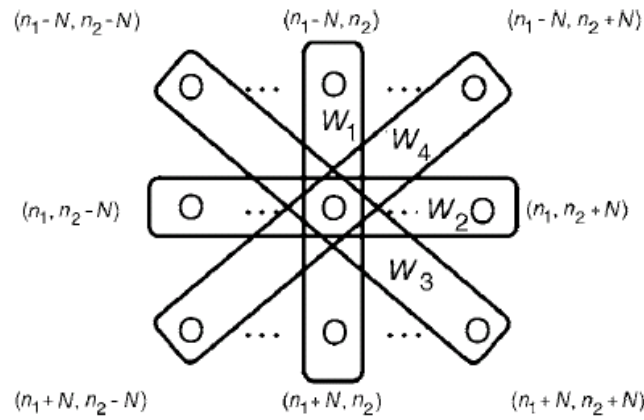


Figure 3.1 Illustration of the four sub-windows in MLM (Liu *et al.* , 2007)

Then the output of this MLM with the center point located at (n_1, n_2) is defined as:

$$Y_{MLM}(n_1, n_2) = median[Y_{\max}(n_1, n_2), Y_{\min}(n_1, n_2), a(n_1, n_2)], \quad (3.4)$$

where

$$Y_{\max}(n_1, n_2) = \max_{1 \leq i \leq 4} [Z_i(n_1, n_2)], \quad (3.5)$$

$$Y_{\min}(n_1, n_2) = \min_{1 \leq i \leq 4} [Z_i(n_1, n_2)], \quad (3.6)$$

$$Z_i(n_1, n_2) = \text{median}[(x, y) \in W_i[n_1, n_2]], i = 1, 2, 3, 4. \quad (3.7)$$

A synthetic test of applying this 2D multistage median filter to reduce random noise on seismic data is shown in Figure 3.2. Figure 3.2a is a synthetic shot record containing surface waves. Random noise was added to the shot record as shown in Figure 3.2b. Then I applied the 2D multistage median filter with a size of 13 points to this shot gather giving the result shown in Figure 3.2c. It can be seen that the random noise is significantly reduced, although there is still some residual noise. I then applied the 2D multistage median filter iteratively with different filter sizes. The result of three iterations with filter size of 13, 11 and 9 points respectively is shown in Figure 3.2d. In comparison with Figure 3.2c, the residual noise is further attenuated. Figure 3.2e shows the attenuated events, which are the difference between Figure 3.2b and d. It can be seen that the random noise is significantly attenuated while the useful reflected waves are well preserved.

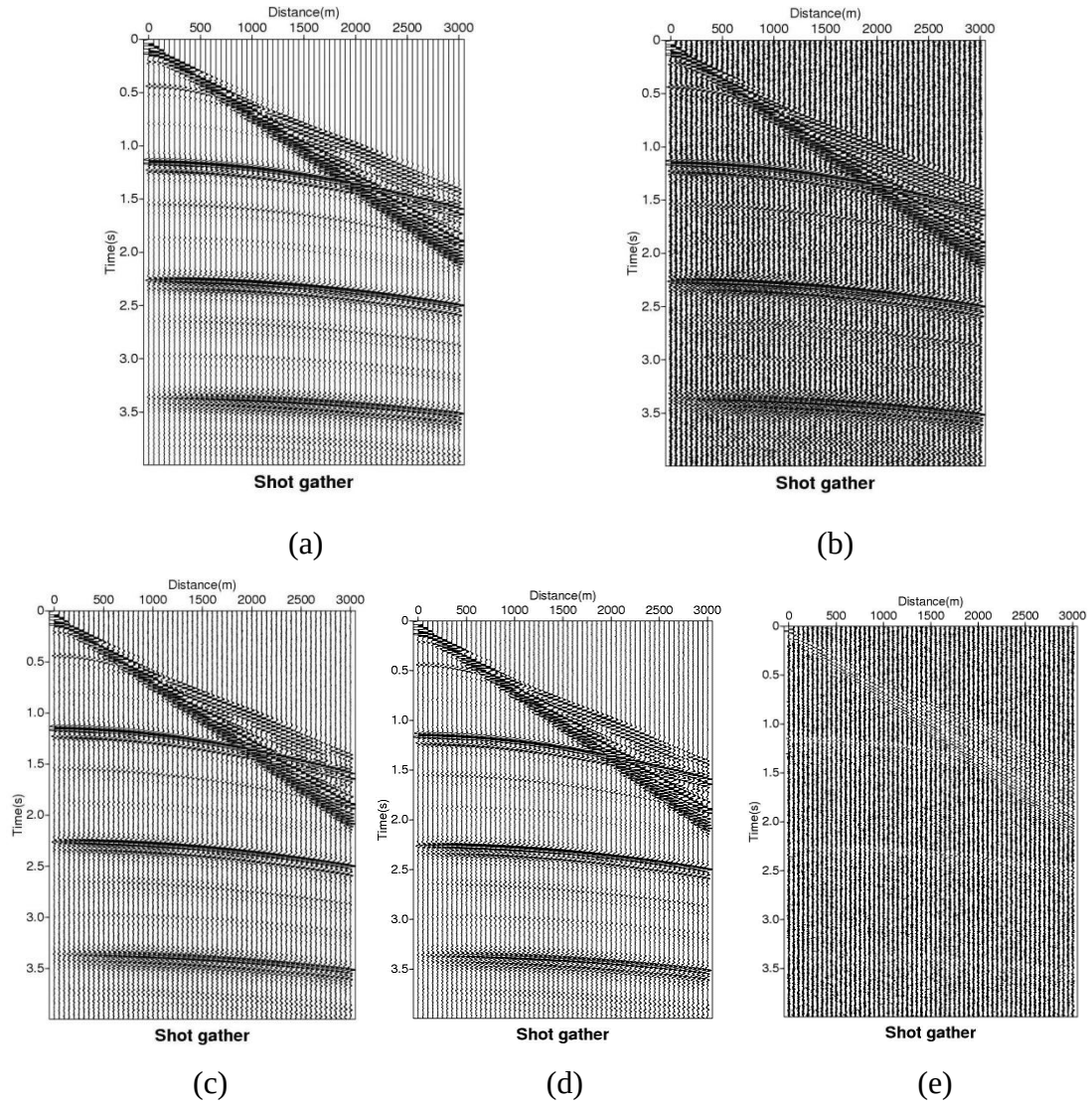


Figure 3.2 (a) A shot gather containing surface waves. (b) The shot gather with random noise. (c) The random noise elimination result after applying a multistage median filter with a size of 13 points. (d) The random noise elimination result after applying multistage median filter iteratively with filter sizes of 13, 11, and 9 points respectively. (e) The difference between (b) and (d).

3.3 SEISMIC DATA INTERPOLATION WITH MULTILEVEL B-SPLINE METHOD

3.3.1 INTERPOLATION FOR SEISMIC DATA PROCESSING

In seismic data processing, dense and regular data records are normally required. For example, SRME and MPI require the shot and group intervals to be the same. However, in the most common type of processing geometry, the shot interval is twice the group interval (e.g. the group interval is 12.5 m while the shot interval is 25 m). Another case is that, in seismic migration, too large a spatial sample interval leads to an aliasing problem which adversely affects migration and results in poor lateral resolution of the subsurface image. In addition, there are practical issues such as irregular sampling caused by feathering of streamers in marine seismic acquisition and dead or missing traces due to the restricted acquisition conditions on land. This leads to a need for methods to produce dense and regular seismic data are needed for all of the above situations

In this section, I implement an interpolation method for seismic data reconstruction which can do interpolation, gap filling, or regularisation in a same scheme. This method applies an effective B-spline approximation technique to a hierarchy of control lattices to generate a sequence of functions whose sum approaches the desired interpolation function. I will show the effectiveness of this method by using synthetic data examples.

3.3.2 THE MULTILEVEL B-SPLINE METHOD

Data interpolation with multilevel B-splines applies a B-spline function to a control lattice hierarchy to calculate intensity values on a regular 3D grid (Lee *et al.*, 1997). The use of a cubic B-spline allows both high accuracy and computational efficiency. In this section, a 2D example is used to demonstrate the method.

In the 2D space $\Omega = \{(x, y) | (0 \leq x \leq m, 0 \leq y \leq n)\}$, it is assumed that a set of scattered points $P = \{(x_c, y_c, I_c)\}$ are recorded, where (x_c, y_c) are the coordinates of the point and I_c is the corresponding intensity value. The purpose is to interpolate requested values on to a dense and regular grid using the input points. The basic concept which is used is that, if a function $f(x, y)$ can be set up to predict the value of these recorded points accurately, then this function $f(x, y)$ can also be used to estimate values at unmeasured locations. In order to find the function $f(x, y)$, the cubic B-spline basis functions are applied on a control lattice Φ overlaying the area Ω . If the value of the control point located at (i, j) is defined as $\phi_{i,j}$, then the function $f(x, y)$ used to estimate the recorded values $P = \{(x_c, y_c, I_c)\}$ can be defined as:

$$f(x, y) = \sum_{u=0}^3 \sum_{v=0}^3 B_u(r) B_v(s) \phi_{(i+u), (j+v)}, \quad (3.8)$$

where $i = \lfloor x \rfloor - 1, j = \lfloor y \rfloor - 1, r = x - \lfloor x \rfloor, s = y - \lfloor y \rfloor$ and $\lfloor x \rfloor$ means that the value of x is rounded to the nearest integer which is less than x , and B_u and B_v are uniform cubic B-spline basis functions defined as: (Lee *et al.*, 1997).

$$\begin{aligned} B_0(\tau) &= (1 - \tau^3) / 6 \\ B_1(\tau) &= (3\tau^3 - 6\tau^2 + 4) / 6 \\ B_2(\tau) &= (-3\tau^3 + 3\tau^2 + 3\tau + 1) / 6 \\ B_3(\tau) &= \tau^3 / 6, \end{aligned} \quad (3.9)$$

where $0 \leq \tau \leq 1$. Here τ refers to the distance between the estimated point and the nearest control point. The cubic B-spline basis functions are used to weight the contribution of each control point to the estimation function based on the distance. By minimising the errors between the estimated function $f(x, y)$ and the values at the

recorded points $P = \{(x_c, y_c, I_c)\}$, we can calculate values of the control points $\phi_{i,j}$ in Φ which can then be used to estimate the required values on a regular grid.

In Equation 3.8, changing the value of a control point $\phi_{i,j}$ affects the estimation only in the local 4×4 neighborhoods of $\phi_{i,j}$, thus the B-splines are locally controlled, which makes them computationally efficient even for a large number of control points. The density of the control lattice Φ directly affects the shape of the function $f(x, y)$. For a sparse Φ , a large number of points in $P = \{(x_c, y_c, I_c)\}$ will be affected by the same control point. This may cause these data points to be blended together yielding a smooth shape for $f(x, y)$ and compromise the accuracy of the estimation. In contrast, if Φ is dense, the influence of a control point is limited within a small range of neighborhoods and the points in $P = \{(x_c, y_c, I_c)\}$ can be estimated more accurately. In that case, $f(x, y)$ will contain local peak values near the data points.

To deal with the tradeoff between smoothness and accuracy, we use the multilevel B-spline algorithm with a hierarchy of control lattices, $\Phi^1 \dots \Phi^L$ each of which has a different control point spacing. The estimation starts with the sparsest control lattice Φ^1 . The resulting function f_1 provides a smooth initial estimation that possibly leaves large discrepancies at the data points in $P = \{(x_c, y_c, I_c)\}$. In particular, f_1 leaves an error of:

$$P_1 = \{(x_c, y_c, \Delta^1 I_c)\}, \quad (3.10)$$

where

$$\Delta^1 I_c = I_c - f_1(x_c, y_c). \quad (3.11)$$

For each point in $P = \{(x_c, y_c, I_c)\}$, a denser control point lattice Φ^2 , whose control point space is twice that of Φ^1 , is then used to obtain a function f_2 that estimates the difference $\Delta^1 I_c$ in the previous step. The sum $f_1 + f_2$ yields a smaller error:

$$\Delta^2 I_c = I_c - f_1(x_c, y_c) - f_2(x_c, y_c). \quad (3.12)$$

In general, for a level k in the hierarchy, we derive the function by using control lattice Φ^k to estimate the data

$$P_k = \{(x_c, y_c, \Delta^k I_c)\}. \quad (3.13)$$

The final estimation function f is then defined as the sum of the functions f_k :

$$f = \sum_{k=0}^L f_k. \quad (3.14)$$

The density of the finest lattice Φ^L controls the precision of the estimation. When Φ^L is sufficiently dense compared to the data distribution, f can interpolate the data without an estimation error by exactly fitting splines to every data point (Lee *et al.*, 2007). In my work, consistent with tests in Jiang *et al.* (2007), half of the target resolution will be used as the spacing of the densest lattice Φ^L , while the control point spacing of the coarsest lattice Φ^1 is chosen to be just larger than the maximum scattered point separation in the work area.

In the examples shown below, the interpolation with multilevel B-Spline is implemented for seismic data reconstruction on NMO corrected shot gathers.

3.3.3 SYNTHETIC DATA APPLICATION EXAMPLES

As a demonstration, I applied the multilevel B-spline interpolation to seismic data reconstruction on a synthetic data set. First I applied the interpolation to a regularly sampled data set. In Figure 3.3, a completed synthetic shot gather with 101 traces is shown. In this shot gather, the receiver spacing is 10 m. A resampled version of this shot gather is shown in Figure 3.4a, where only 51 traces are kept with a receiver interval of 20 m. Then I applied an approximate NMO correction followed by

multilevel B-spline interpolation and finally inverse NMO correction. The interpolation result is shown in Figure 3.4b. It can be seen that it is quite acceptable. The difference between the interpolated shot gather (Figure 3.4b) and the original shot gather (Figure 3.4a) is shown in Figure 3.4c.

The multilevel B-spline interpolation is then applied to fill gaps on a seismic data set. The shot gather with gaps is shown in Figure 3.5a and is the same as in Figure 3.3 but with 2 traces of every 15 traces were deleted. After interpolation with the multilevel B-spline method, the result is shown in Figure 3.5b where the reflection events are recovered smoothly, although there is a little energy leakage in the far offset steep events which can be found in Figure 3.5c.

In the last example, I applied multilevel B-spline interpolation to a simulated irregularly sampled data set. I randomly deleted 23 traces from Figure 3.3. One of the resulting shot gathers is shown in Figure 3.6a. The result of regularising the shot gather to a receiver interval of 10 m is shown in Figure 3.6b. The difference between the reconstructed data set and the original data set are shown in Figure 3.6c. It can be seen that there is a little energy leakage in a far offset large dip event where two neighbouring traces are both missing.

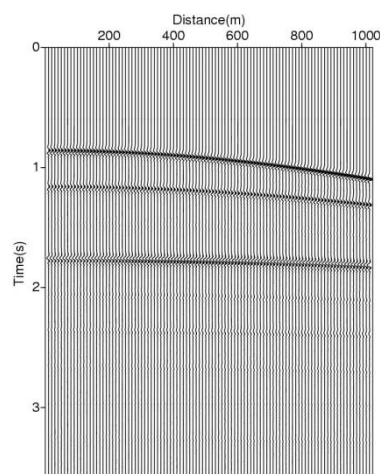


Figure 3.3 A synthetic shot gather with 101 traces, and a trace spacing of 10 m.

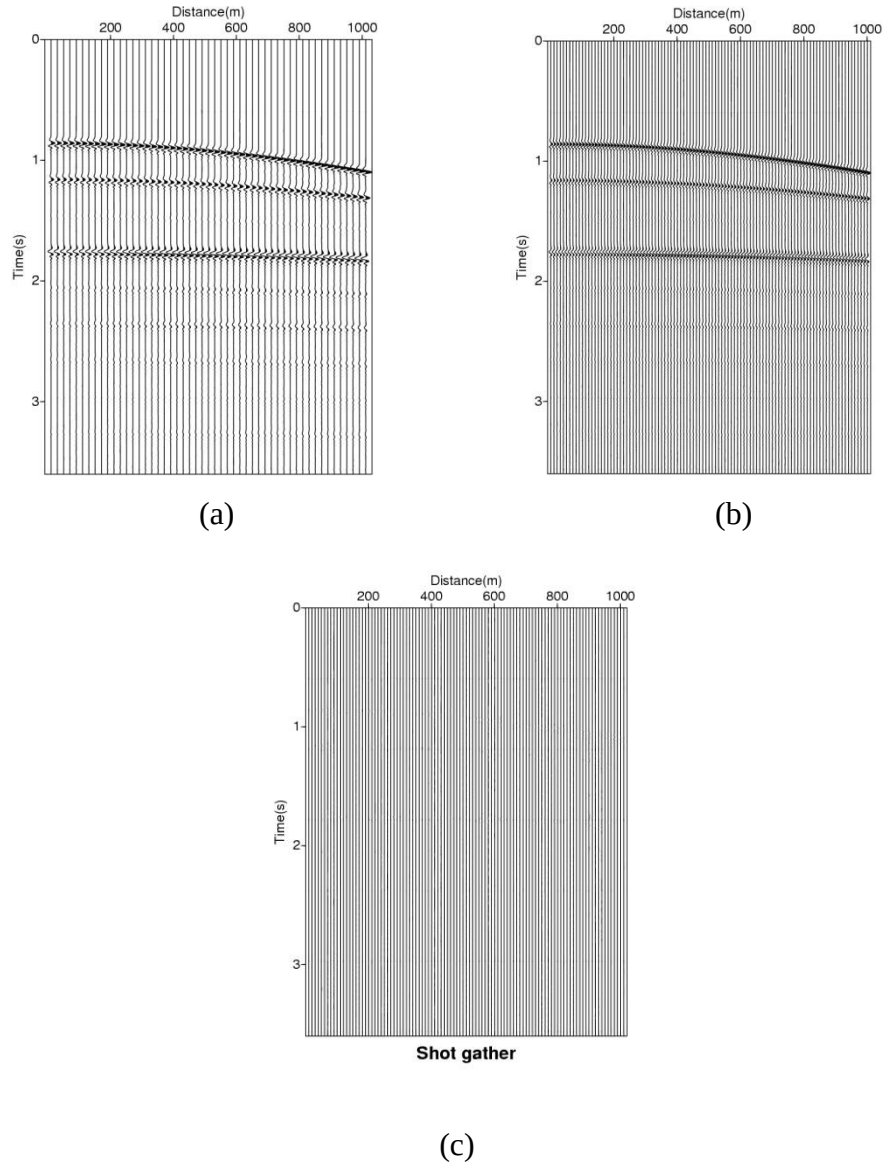


Figure 3.4 The result of applying multilevel B-spline to interpolate seismic traces on regularly sampled data set: (a) the sparse shot gather with 51 traces, where the trace spacing is 20 m; (b) the interpolation result with multilevel B-spline interpolation. (c) the difference between the interpolation result in b and the original shot gather in Figure 3.3.

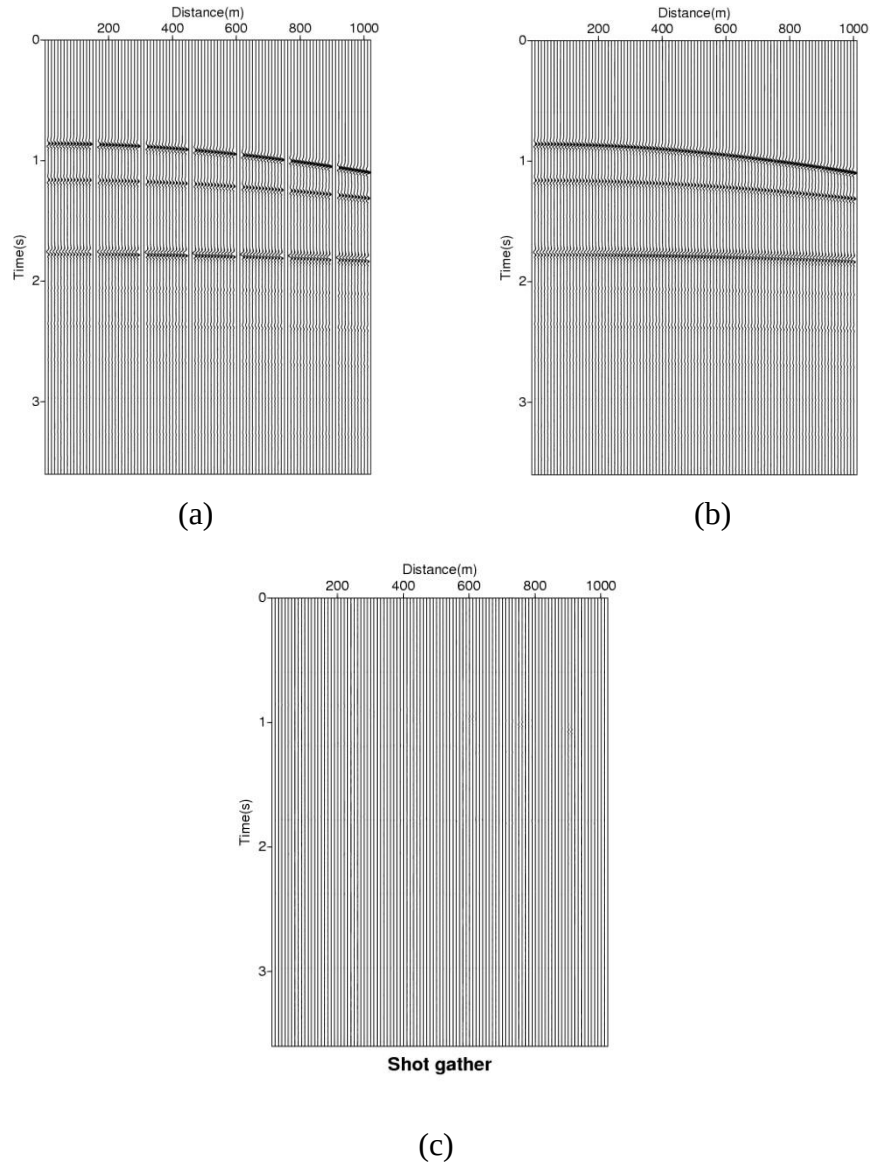


Figure 3.5. The result of applying multilevel B-spline interpolation to fill gaps on seismic data set: (a) the shot gather with gaps introduced by removing 2 traces of every 15 traces (12 traces in total) from Figure 3.3; (b) the result of filling the gaps with multilevel B-spline interpolation. (c) the difference between the gap filling result in Figure b and the original shot gather in Figure 3.3 .

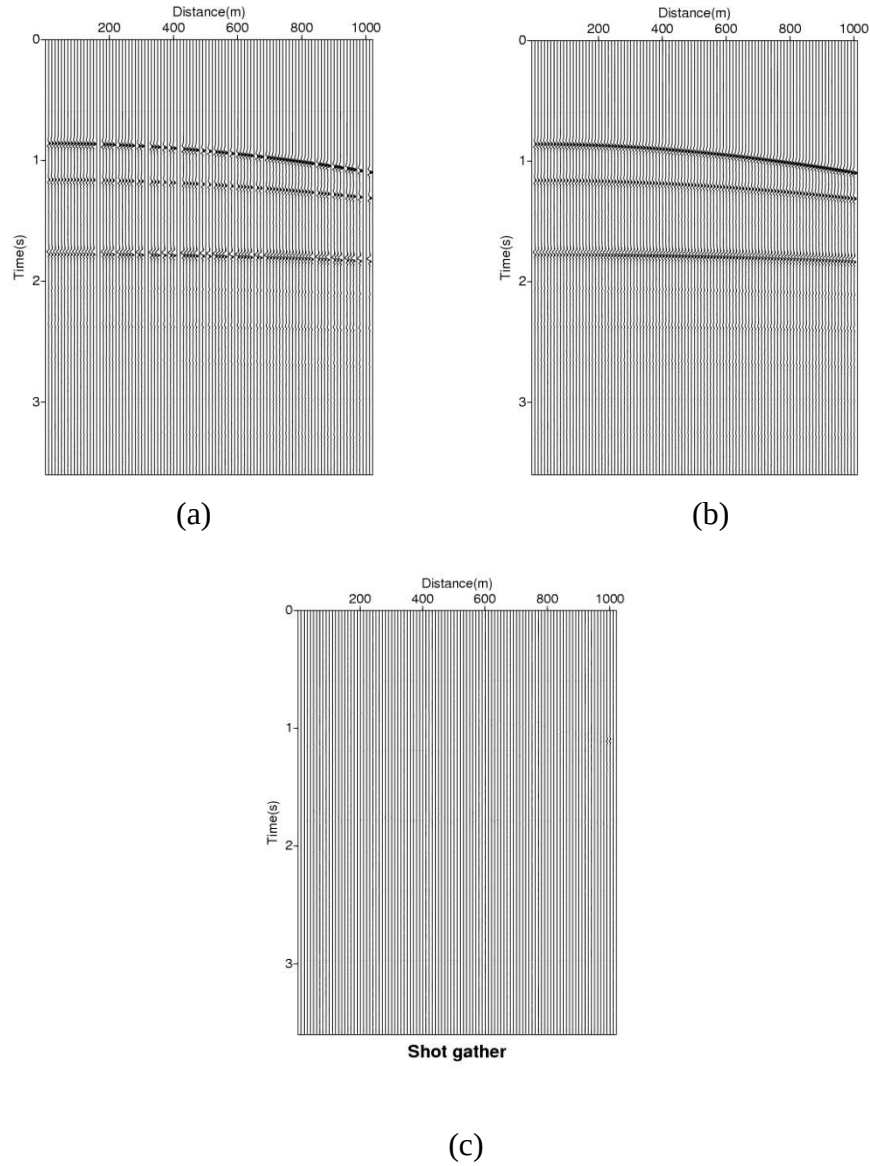


Figure 3.6 The result of applying multilevel B-spline to interpolate seismic traces on an irregularly sampled data set: (a) the shot gather with 23 traces randomly deleted from Figure 3.3; (b) the regularisation result obtained with the multilevel B-spline interpolation method. (c) the difference between the regularisation result in b and the original shot gather in Figure 3.3.

3.4 IMPLEMENTATION OF MPI WITH MULTILEVEL B-SPLINE INTERPOLATION

Wang (2004) proposed the MPI scheme for free-surface multiple attenuation. The theoretical discussion given here is in the frequency domain, but for simplicity the dependence on frequency is omitted.

The MPI scheme generates a multiple model by modifying the multiple wavefield attenuated in the previous iteration, as

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{T}^{(n)} \mathbf{M}^{(n-1)} \quad (3.15)$$

where $\mathbf{M}^{(n-1)}$ is the multiple attenuated in the previous iteration, $\tilde{\mathbf{M}}^{(n)}$ (with tilde) is the model in the current iteration, and $\mathbf{T}^{(n)}$ is the updating operator. The updating operator for the n th iteration $\mathbf{T}^{(n)}$ is borrowed from a previous iteration as $\mathbf{T}^{(n-1)}$ where the multiple attenuation results $\mathbf{P}_0^{(n-1)}$ and $\mathbf{P}_0^{(n-2)}$ in the $(n-1)$ th and the $(n-2)$ th iterations are known (Wang, 2004, 2007). The operator $\mathbf{T}^{(n-1)}$ accumulates all the information including modelling and subtraction in the $(n-1)$ th iteration, as

$$\mathbf{P}_0^{(n-1)} = \mathbf{T}^{(n-1)} \mathbf{P}_0^{(n-2)}, \quad (3.16)$$

for $n \geq 2$. So the updating operator can be expressed as

$$\mathbf{T}^{(n)} \approx \mathbf{T}^{(n-1)} = \mathbf{P}_0^{(n-1)} (\mathbf{P}_0^{(n-2)})^H [\mathbf{P}_0^{(n-2)} (\mathbf{P}_0^{(n-2)})^H]^{-1}, \quad (3.17)$$

where the superscript H denotes complex conjugate transposition.

In practice, it is assumed $\mathbf{P}_0^{(0)} = \mathbf{P}$, the original data matrix, and the SRME method can be used to estimate the wavefield $\mathbf{P}_0^{(1)}$. In SRME, the shot interval should be equal to the receiver interval and one shot record is required at each receiver position. The MPI scheme also requires the same dense and regular data geometry. However, in marine data acquisition, the shot interval is often larger than the group

interval. In this case, interpolation of the missing shot gathers is required for both SRME and MPI.

To meet this requirement, I use multilevel B-spline interpolation to reconstruct the missing seismic data. The effectiveness of this method is demonstrated on the synthetic examples. The traveltimes of the interpolated events are correct, although some energy leakages are observed. As the missing shot gathers need to be interpolated, the B-spline interpolation is implemented in common-offset gathers.

First, the original shot gathers are sorted into common-offset gathers. In the common-offset gather, when the shot interval is twice the receiver interval, two adjacent traces are from two consecutive shot gathers with the same source-receiver offset. The multilevel B-spline method is then used to interpolate one trace between every two traces. The interpolated traces reconstruct the data of the missing shot gathers at the same source-receiver offset. Figure 3.7a is a arbitrary selected original common-offset gather from a 2D marine data (Appendix A.2). Figure 3.7b shows the corresponding interpolation results, where it can be seen that the continuity of the structure is well preserved. Finally, after implementing the interpolation in each common-offset gathers, all the traces are sorted back to the shot domain, where the interpolated traces can form the originally missing shot records at each receiver position.

Following reconstruction, the multiple models can then be predicted by the MPI scheme. Figure 3.7c and d are respectively an arbitrarily selected original shot gather and the corresponding multiple models predicted by MPI. It can be seen that the predicted free-surface multiple model is accurate not only in time but also in phase and amplitude, although there are missing near-offset traces.

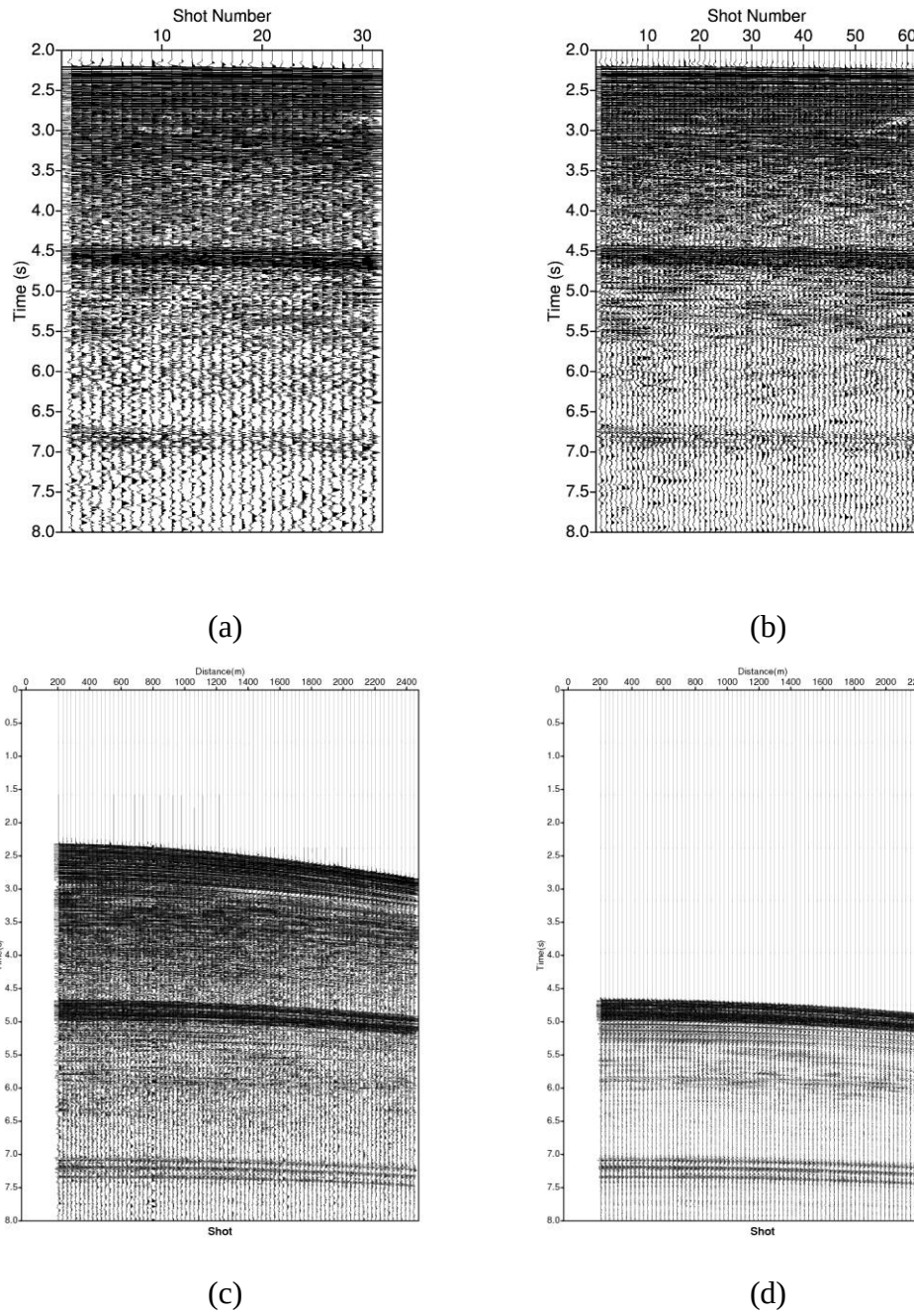


Figure 3.7 (a) An original common-offset gather (b) The common-offset gather after multilevel B-spline interpolation to reconstruct traces for missing shot gathers. (c) An original shot gather (d) The multiple model predicted by MPI.

3.5 FREE-SURFACE MULTIPLE ATTENUATION ON 2D MARINE DATA WITH MPI SCHEME

The MPI scheme is implemented for free-surface multiple attenuation on the 2D marine data (Appendix A.2). On this data set, the source interval of each shot is 26.67 m while the receiver interval in each shot is 13.33 m. The minimum source-receiver offset is 206 m. As the shot interval is not equal to the receiver interval, I used the multilevel B-spline interpolation method to interpolate the missing data. The expanded multiple channel matching (EMCM) filter (Wang, 2003) was used in the adaptive subtraction stage.

The flowchart of the whole procedure is shown in Figure 3.8.

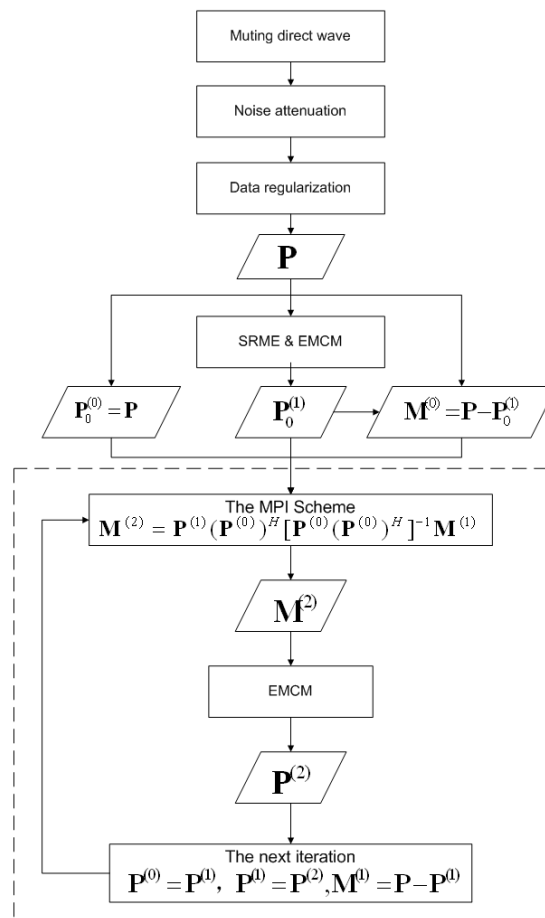
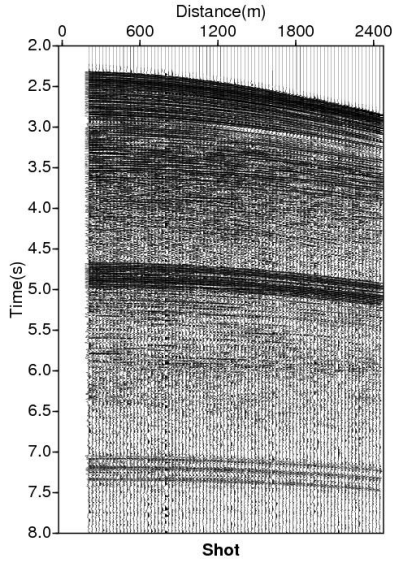


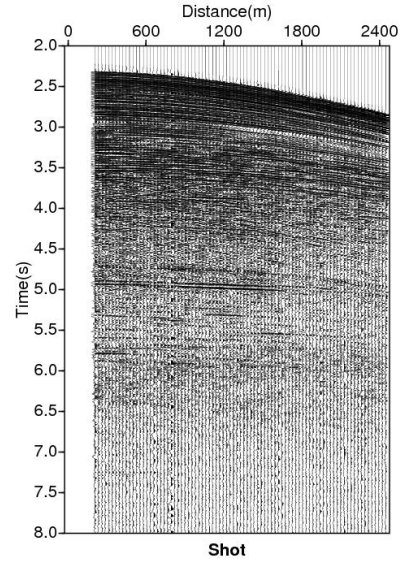
Figure 3.8 Flowchart for implementing MPI for free-surface multiple attenuation.

Figure 3.9 shows a comparison of the effectiveness of MPI and SRME. Figure 3.9a shows a shot gather where strong free-surface multiples are overlapping primaries. Figure 3.9b is the demultiple result with SRME to predict the multiple models and EMCM for subtraction (filter length = 5). It can be seen that there are still some residuals of the multiples along the offsets. Based on the same multiple model predicated by SRME method, if more aggressive filters are used in the subtraction stage, for example filter length = 10, the multiples will be over-subtracted and it leaves blank areas as shown in Figure 3.9c. In comparison, when we use MPI to improve the multiple model prediction for three iterations and apply EMCM with a moderate filter (filter length = 4) for subtraction, the free-surface multiples are better suppressed, as shown in Figure 3.9d. After MPI there are much less residual multiples and no blank areas.

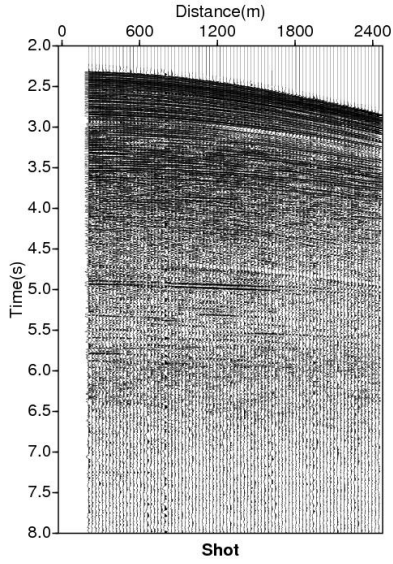
Figure 3.10 is a part of the stack section of this 2D line. Figure 3.10a is the stack section before multiple attenuation while 3.10b and c are respectively de-multiple result and attenuated multiples after implementing 3 iterations of the MPI scheme. It can be seen that, the primaries are well preserved after multiple attenuation.



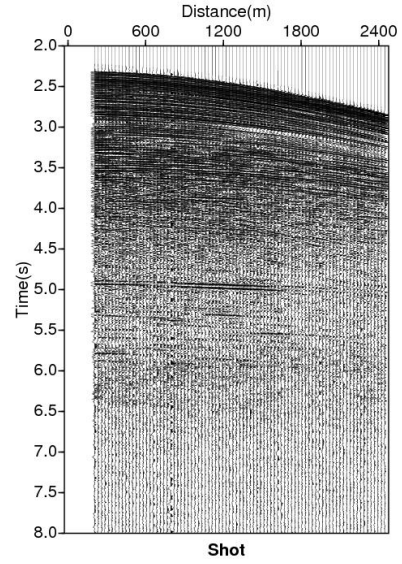
(a)



(b)

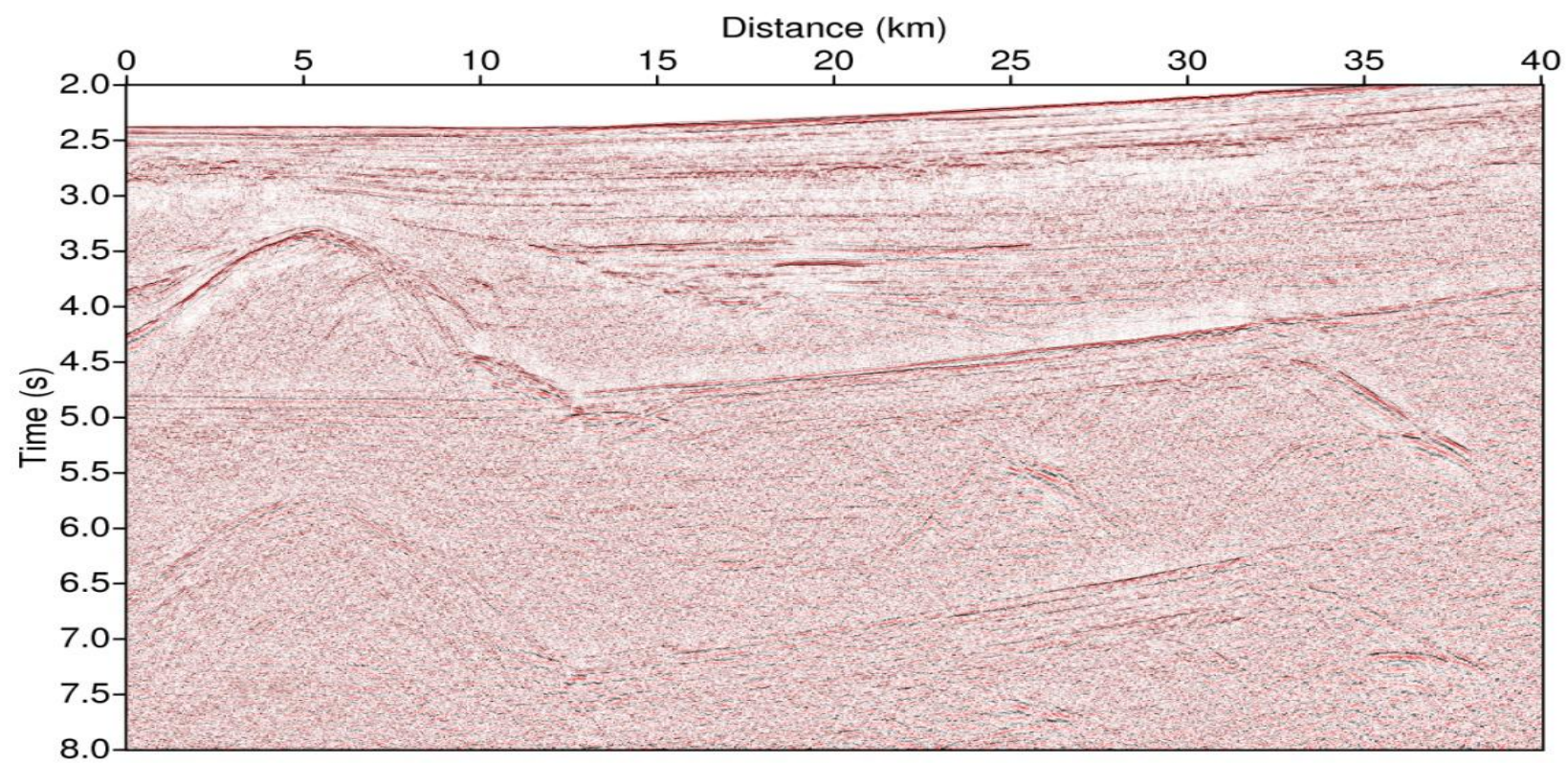


(c)



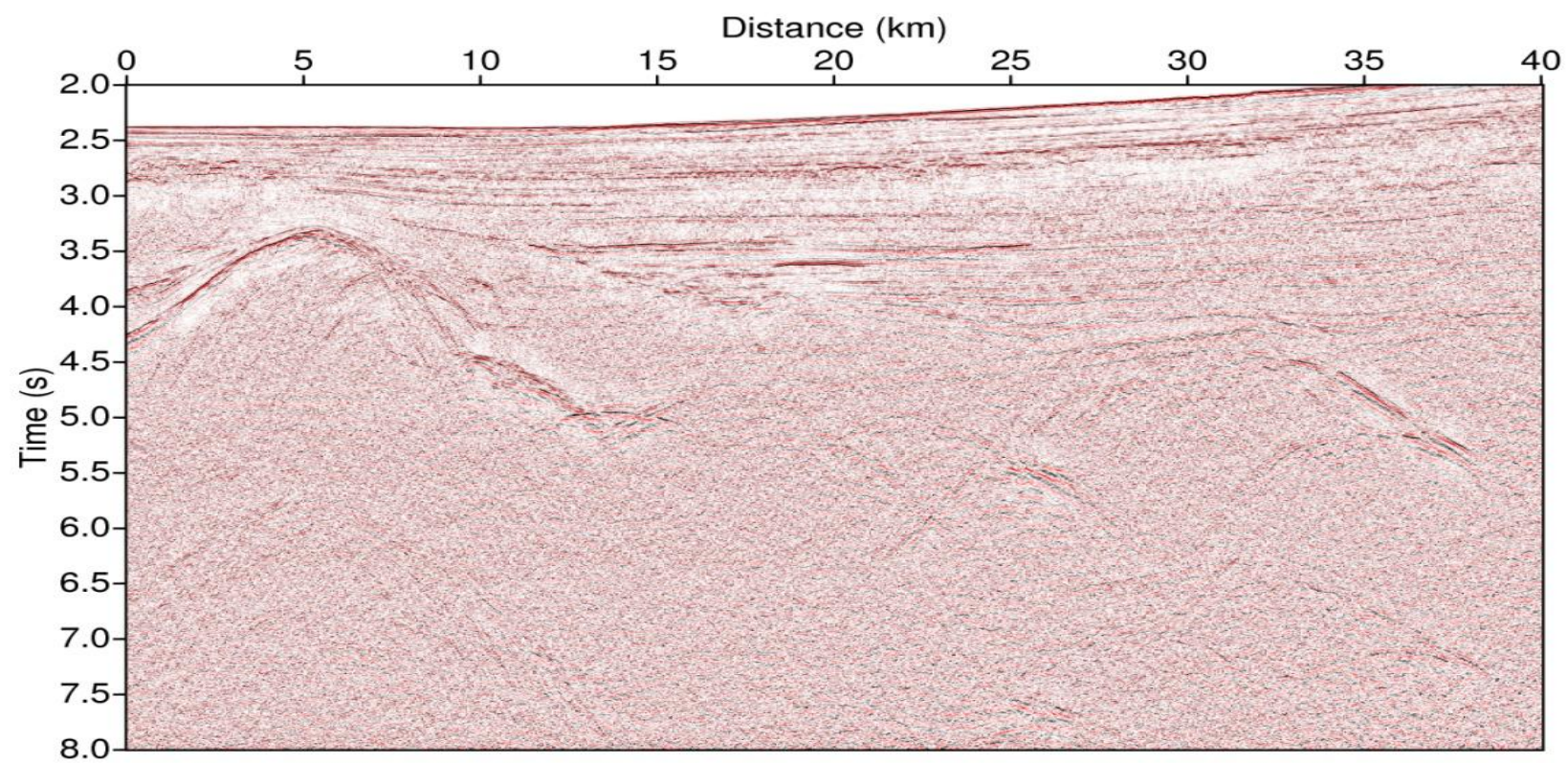
(d)

Figure 3.9 (a) A shot gather containing strong free-surface multiples (b) The demultiple results with SRME and EMCM (filter length = 5). (c) SRME and EMCM (filter length = 10) (d) MPI and EMCM (filter length = 4) after three iterations.



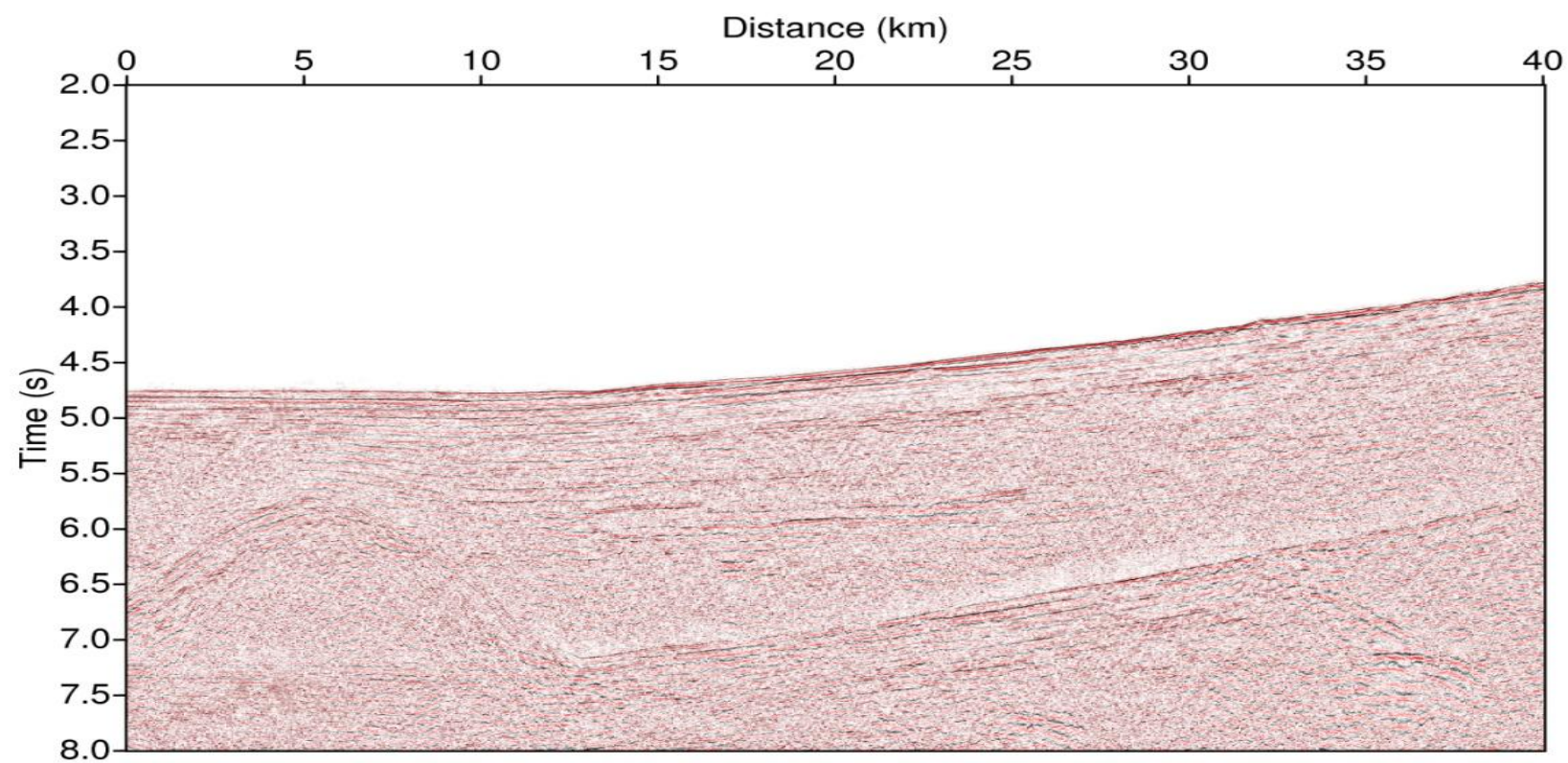
(a)

Figure 3.10 (a) A stack section before multiple attenuation.



(b)

Figure 3.10 (b) The stack section after applying 3 iterations of the MPI scheme for multiple attenuation.



(c)

Figure 3.10 (c) The attenuated multiple energy.

3.6 CONCLUSIONS

Regularisation and interpolation are essential pre-processing steps for seismic multiple attenuation with MPI, I have described an optimised scattered data interpolation with multilevel B-spline method for seismic data reconstruction. The effectiveness of this method has been demonstrated on synthetic and real data examples. This is a novel method that can be used to reconstruct a sparse and irregular seismic data on a desired dense and regular grid without using models.

With proper pre-processing, the MPI scheme can then be applied for free-surface multiple attenuation on real marine data. These examples demonstrate that the MPI scheme can generate good results when compared with SRME method.

Chapter 4 IMPROVING THE EFFECTIVENESS OF MPI WITH A FULL PREDICTION OPERATOR

4.1 INTRODUCTION

The effectiveness of the MPI scheme was demonstrated by Wang (2004, 2007), but the computational cost is considerably more expensive than SRME. In MPI, a key step is to generate an operator, $\mathbf{T}$, for multiple prediction solving an inverse problem. To improve the computational efficiency Wang (2004) suggested assuming the $\mathbf{T}$ operator as a diagonal band matrix such as a pentadiagonal or a tridiagonal matrix. In this chapter, I solve this operator in full by inversion, with the aid of efficient matrix computation using a Graphic Processing Unit (GPU).

GPU's are specialised circuits originally designed to offload graphics tasks from the CPU with the intention of performing them faster than the CPU (Sanders and Kandrot, 2011). They have excellent floating point performance with hundreds of cores working in parallel to draw triangles and polygons on the screen. Because of their highly parallel structure, GPU's have been considered for scientific computing where huge amounts of parallel computations are required. They have been applied to various fields including bio-informatics (Schatz *et al.*, 2007), fluid dynamics (Corrigan *et al.*, 2010), molecular dynamics (Maintz *et al.*, 2011), computer vision (Cornelis and van Gool, 2008), and so on. They have also been used in geophysics for speeding-up the computation involved in seismic attribute volume generation (Lin and Wei, 2007), seismic modelling (Mendoza *et al.*, 2009; Michea and Komatitsch, 2010) and migration (Foltinek *et al.*, 2009; Li *et al.*, 2009; Zhang *et al.*, 2009).

With GPU technology, I can now demonstrate the full effectiveness of MPI, as I am able to remove the assumption in the key step used to generate the $\mathbf{T}$ operator.

4.2 USING GPU FOR MPI

MPI generates a multiple model by modifying the multiple wavefield attenuated in the previous iteration, as

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{T}^{(n)} \mathbf{M}^{(n-1)}, \quad (4.1)$$

where $\mathbf{M}^{(n-1)}$ is the multiple attenuated in the previous iteration, $\tilde{\mathbf{M}}^{(n)}$ (with tilde) is the model in the current iteration, and $\mathbf{T}^{(n)}$ is the updating operator. The updating operator for the n th iteration $\mathbf{T}^{(n)}$ is taken from an earlier iteration $\mathbf{T}^{(n-1)}$ where the multiple attenuation results $\mathbf{P}_0^{(n-1)}$ and $\mathbf{P}_0^{(n-2)}$ respectively corresponding to the $(n-1)$ th and the $(n-2)$ th iterations are known (Wang, 2004, 2007). The operator $\mathbf{T}^{(n-1)}$ accumulates all the effort including modelling and subtraction in the $(n-1)$ th iteration, as

$$\mathbf{P}_0^{(n-1)} = \mathbf{T}^{(n-1)} \mathbf{P}_0^{(n-2)}, \quad (4.2)$$

where $n \geq 2$. The updating operator can then be expressed as

$$\mathbf{T}^{(n)} \approx \mathbf{T}^{(n-1)} = \mathbf{P}_0^{(n-1)} (\mathbf{P}_0^{(n-2)})^H [\mathbf{P}_0^{(n-2)} (\mathbf{P}_0^{(n-2)})^H]^{-1}, \quad (4.3)$$

where the superscript H denotes the complex conjugate transpose. In practice, we often assume $\mathbf{P}_0^{(0)} = \mathbf{P}$, the original data matrix, and use either SRME or f - k and Radon multiple suppression filtering to estimate the wavefield $\mathbf{P}_0^{(1)}$. The $\mathbf{T}$ operator converges to an identity matrix and three iterations are usually sufficient for real data processing (Wang, 2007). In this chapter, I use SRME for the initial estimate; this also involves a series of matrix multiplications and thus can be speeded up by a GPU (Appendix 4.A).

Equation 4.3 is a linear system of $\mathbf{AX} = \mathbf{B}$, as

$$\mathbf{P}_0^{(n-2)} (\mathbf{P}_0^{(n-2)})^H \mathbf{T}^H = \mathbf{P}_0^{(n-2)} \mathbf{P}_0^{(n-1)H}. \quad (4.4)$$

We can list the total computation costs of each MPI iteration explicitly as follows.

- (1) Each matrix multiplication $\mathbf{A} = \mathbf{P}_0^{(n-2)}(\mathbf{P}_0^{(n-2)})^H$ and $\mathbf{B} = \mathbf{P}_0^{(n-2)}\mathbf{P}_0^{(n-1)H}$ cost the same as a single iteration SRME.
- (2) When $\mathbf{A}$ and $\mathbf{B}$ are computed, we solve $\mathbf{AX} = \mathbf{B}$ to obtain the operator $\mathbf{T} = \mathbf{X}^H$.
- (3) Once the operator $\mathbf{T}$ is obtained, the computation $\mathbf{T}^{(n)}\mathbf{M}^{(n-1)}$ (equation 4.1) is equivalent to one more iteration of SRME.

The matrix multiplication of each MPI iteration is therefore equivalent to three times that needed for a SRME plus the cost for solving the system $\mathbf{AX} = \mathbf{B}$. A GPU can accelerate these three matrix multiplications and also speed up the solution of the linear system. The speedup provided by a GPU for MPI is tested in the next section.

Figure 4.1 outlines the workflow of CPU/GPU co-operation for the MPI method, where the matrix multiplications and the solution of linear system are calculated by the GPU and other steps are dealt with by the CPU.

The interface between CPU and GPU programming is called Compute Unified Device Architecture (CUDA) (Sanders and Kandrot, 2011). With CUDA, high-level languages such as C, C++ and FORTRAN can be used for GPU-related programming. The GPU used is a NVIDIA Tesla C2050 GPU with 448 cores and 3G dedicated memories. For the comparison, the CPU used is one core of a Intel Xeon Quad-Core X5647 (2.93 GHz).

Figures 4.2a and b are the multiple models predicted by MPI on CPU and GPU with the same input (from data in Appendix A.2), respectively, for an arbitrarily selected shot position. In the results to within single floating-point precision, the two predicted models are the same. The difference between the two models is zero as shown in Figure 4.2c. This demonstrates that the GPU application can keep the precision of calculation for predicting multiple models as CPU.

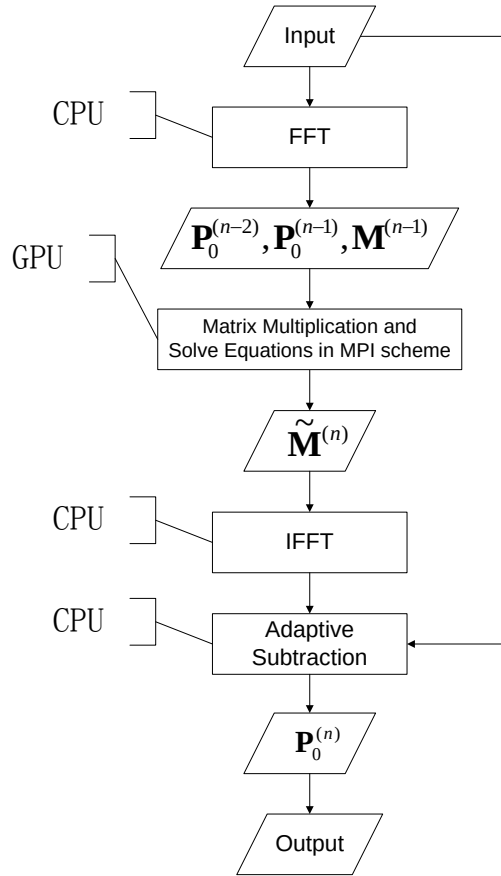


Figure 4.1 Workflow of CPU/GPU co-operation for seismic multiple attenuation based on the MPI method.

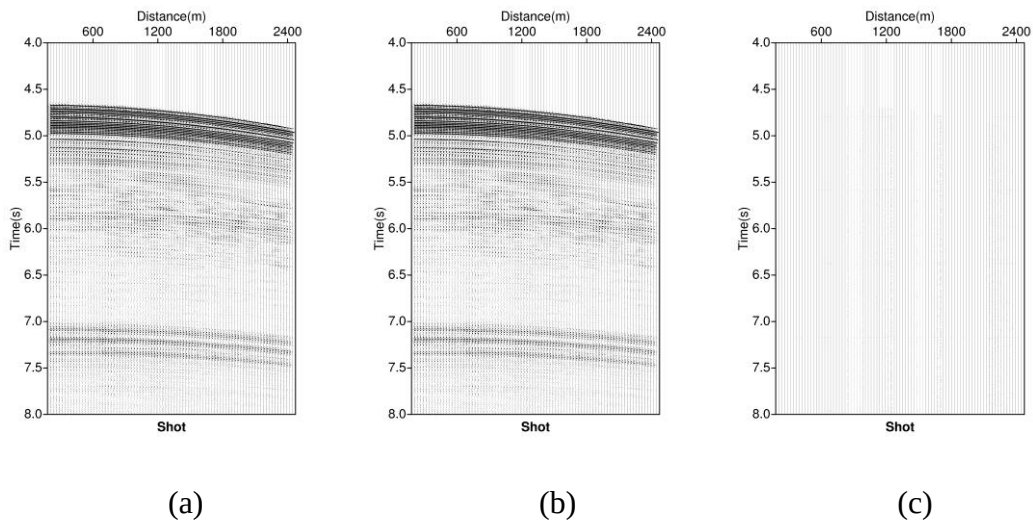


Figure 4.2. (a) The multiple model predicted by MPI on CPU. (b) The multiple model predicted by MPI on GPU. (c) The difference between (a) and (b).

4.3 PERFORMANCE

I demonstrate the efficiency of applying GPU in multiple model prediction with MPI on a 2D data set (Appendix A.3). I only compare the cost of the CPU and GPU for the matrix multiplication and solving the linear system, because the other steps are all dealt with by CPU.

First I test the computational efficiency of solving the linear system $\mathbf{AX} = \mathbf{B}$. I compare the computation cost in three scenarios. First, using the CPU to calculate a full $\mathbf{T}$ operator (i.e. $\mathbf{X}^H$). Then assuming the solution matrix $\mathbf{T}$ to be a diagonal band matrix such as a tridiagonal matrix, as suggested by Wang (2004, 2007). Thirdly, using the GPU to calculate the full $\mathbf{T}$ operator. The test was carried out on data matrices with different sizes. For 400×400 size matrix, the time costs of the three scenarios are 4600: 244: 25 s. When the matrix size is 1000×1000 , the costs are 760: 24: 5 minutes, respectively (Table 4.1). It can be seen that, compared with calculating the simplified $\mathbf{T}$ operator on CPU, the GPU can speed up processing even using the full operator.

	Matrix 400 x 400	Matrix 1000 x 1000
CPU(full $\mathbf{T}$)	4600 s	760 min
CPU(tri-diagonal $\mathbf{T}$)	244 s	24 min
GPU(full $\mathbf{T}$)	25 s	5 min

Table 4.1 Time for solving the linear system.

Next I test the computational efficiency of a GPU for an individual iteration of MPI with all matrix operations (and with the full $\mathbf{T}$ operator). For 400×400 matrices, processing on the CPU and GPU respectively cost about 204 minutes and 2 minutes. When the matrix size is set to 1000×1000 , the costs on CPU and GPU are 2838 minutes and 12 minutes. In summary, GPU can accelerate MPI up to 102 and 236 times for 400×400 and 1000×1000 matrices, respectively (Table 4.2).

	Matrix 400 x 400	Matrix 1000 x 1000
CPU	204 min	2838 min
GPU	2 min	12 min

Table 4.2 Time cost of one iteration of MPI based on full **T** operators.

Figure 4.3 shows the relationship between the time cost (in seconds) of an individual iteration of MPI for matrix sizes between 400 x 400 and 1000 x 1000. It can be seen that computation time is approximately linear. For an individual data set, a smaller size matrix means more blocks while bigger matrix means less blocks (Appendix 4.A). After tests with a GPU, for this 2D marine data line (Appendix A.3) with 4200 shots, the matrix size is chosen to be 500 for optimum efficiency. To process the whole line with one iteration of MPI took about 150 minutes.

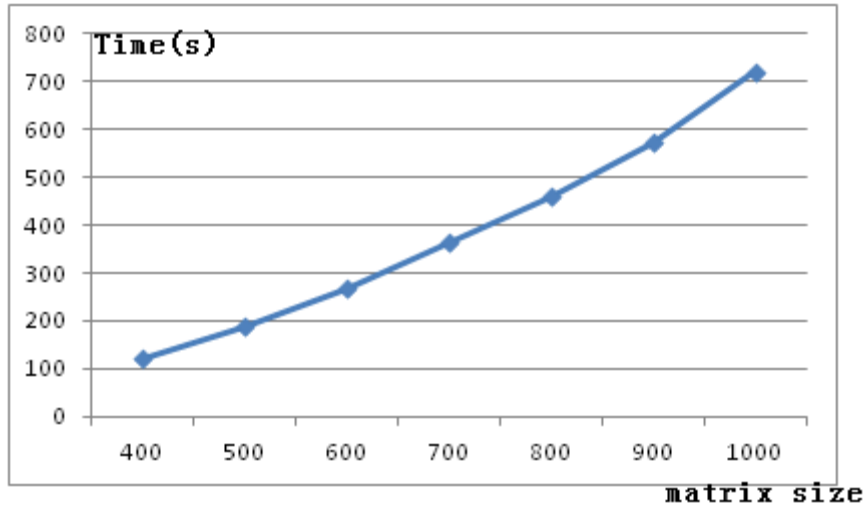
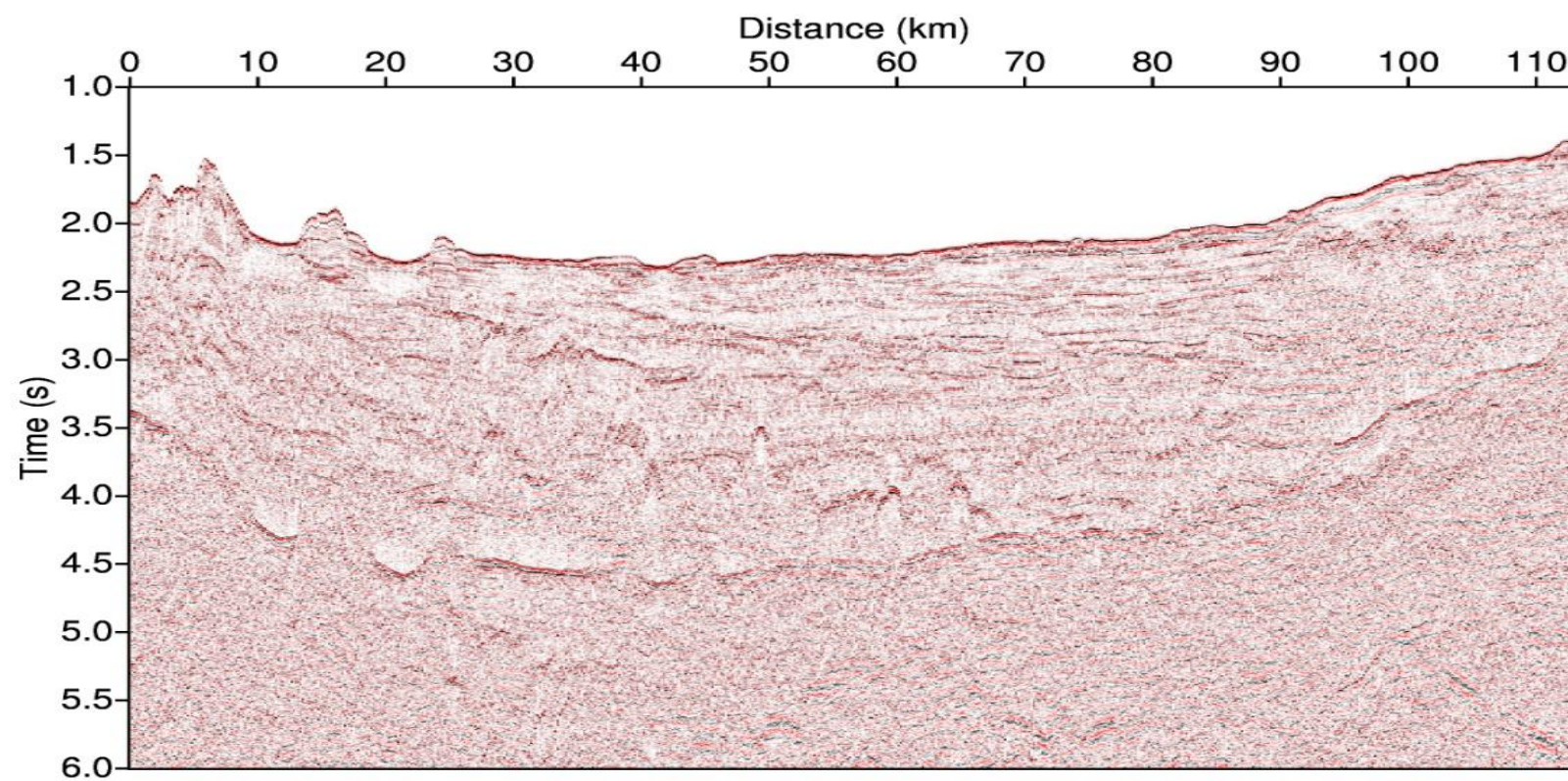


Figure 4.3 The relationship between the time cost (in second) of an individual iteration of MPI and matrix sizes between 400 x 400 and 1000 x 1000.

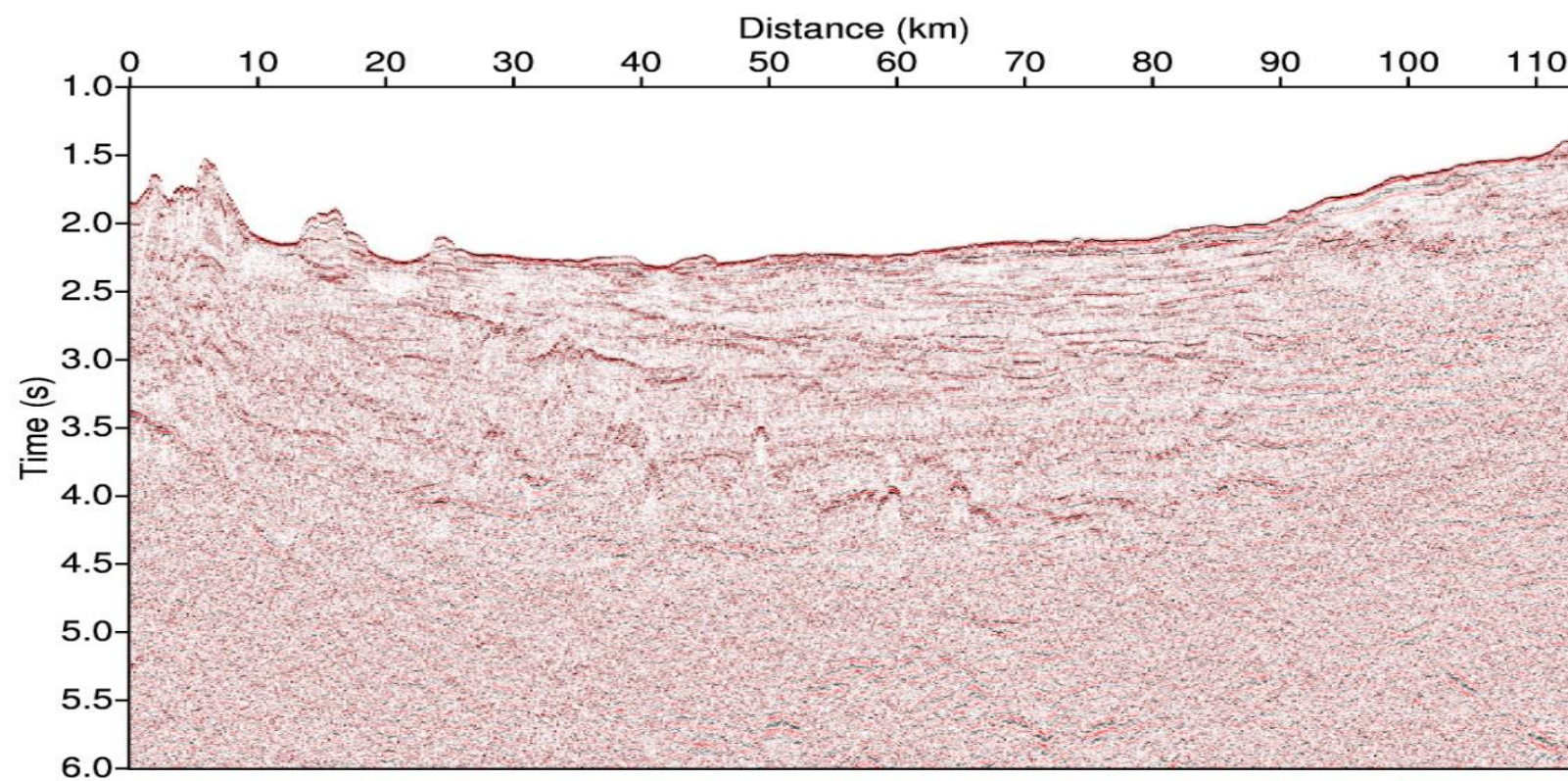
Finally, I will show the result of applying the GPU-accelerated MPI scheme for free-surface multiple attenuation on this 2D marine data (Appendix A.3). A data interpolation process based on the B-spline method (Chapter 3) is implemented before the multiple attenuation. Figure 4.4 displays the result in the stack domain. Figure

4.4a is a part of the stack section of the original data showing free-surface multiples along the whole the figure. Figures 4.4b and 4.4c are respectively the multiple-attenuation result after three iterations of MPI, and the attenuated multiple energy. It can be seen that the primary reflections are well preserved while the multiples are suppressed.



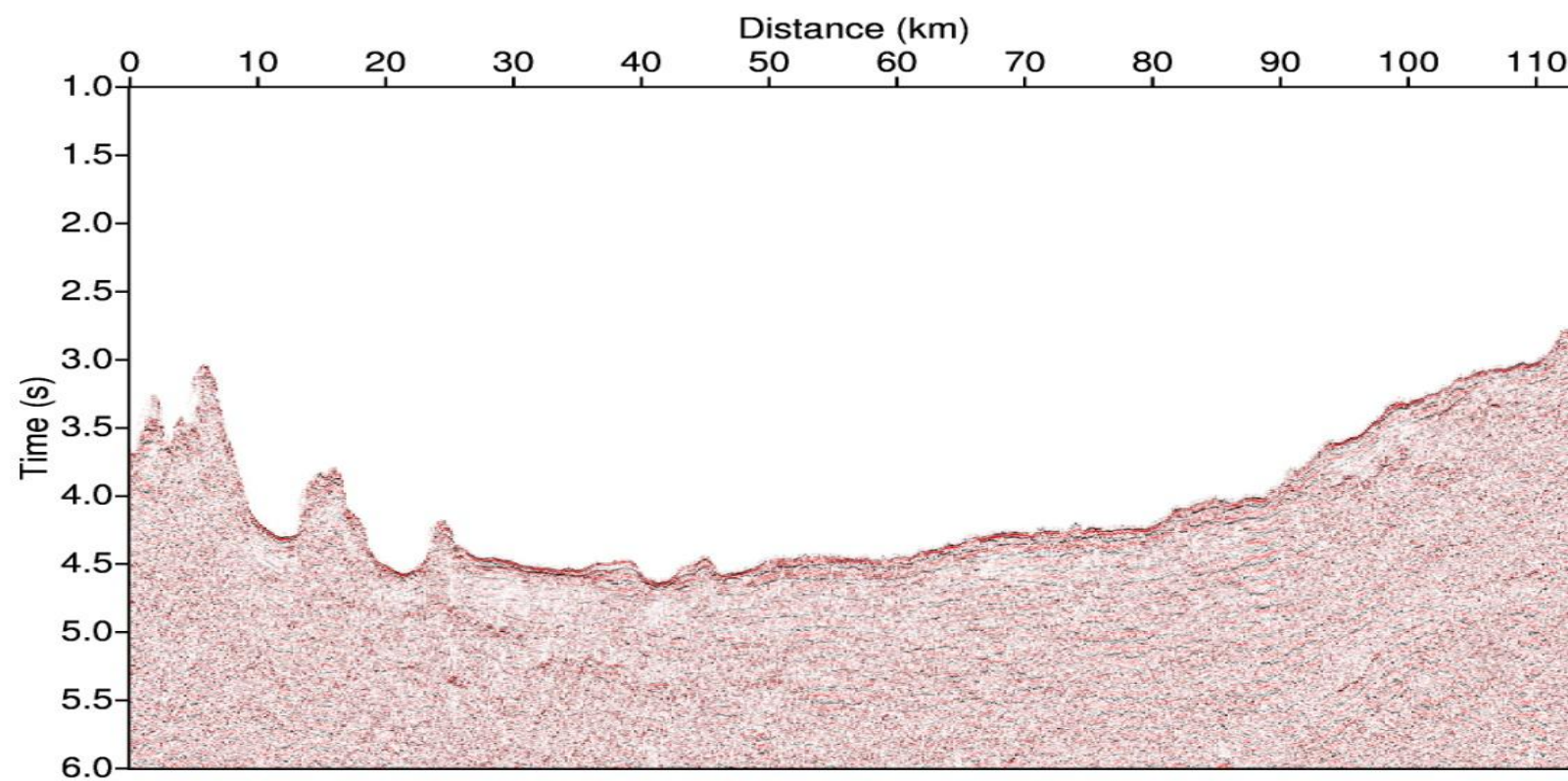
(a)

Figure 4.4 (a) Stack section of the original 2D seismic data.



(b)

Figure 4.4 (b) Stack section after free-surface multiple attenuation using MPI.



(c)

Figure 4.4 (c) The attenuated multiple energy.

4.4 APPLICATIONS TO 3D SEISMIC DATA

3D seismic data often have acquisition-related problems, such as inadequacies in crossline sampling, aperture limitation in the crossline direction and irregularities in the acquisition geometry. As a result, it is difficult to implement a multiple prediction method on 3D datasets without proper preprocessing. Van Dedem and Verschuur (2005) proposed to extract the required 3D information from the available coarsely sampled multiple contributions using a high-resolution Radon transform in the crossline direction. Hokstad and Sollie (2006) introduced a similar sparse inversion approach based on a parabolic Radon transform in the f - x domain. Huo (2009) used a semblance-threshold trajectory-stacking generalised radon transform (GRT) in the crossline direction, and then introduced an energy-spectrum approach to locate the reflection points and extract information for the 3D multiple prediction (Appendix 4.B). In this section, I apply the MPI scheme along the inline direction to update the multiple model predicted by the trajectory-stacking GRT method. The refined model is expected to be more accurate in phase and amplitude, which could in turn reduce the non-linearity of the problem in the subsequent subtraction stage. For subtraction, I use the Expanded Multichannel Matching filter (Wang, 2003).

I apply the proposed procedure to a deep-water 3D marine data set to show the effectiveness and efficiency of MPI with GPU acceleration. In this data set (Appendix A.4), the sail-line consists of 6 streamers with 100 m offset along the crossline direction. In the inline direction, each streamer records 1200 shots with 120 receivers in each shot. Both the shot interval and receiver interval are 25 m, the minimum source-receiver offset is 200 m, the trace length is 8 s, with sampling interval is 4 ms. The MPI method is implemented in frequency domain.

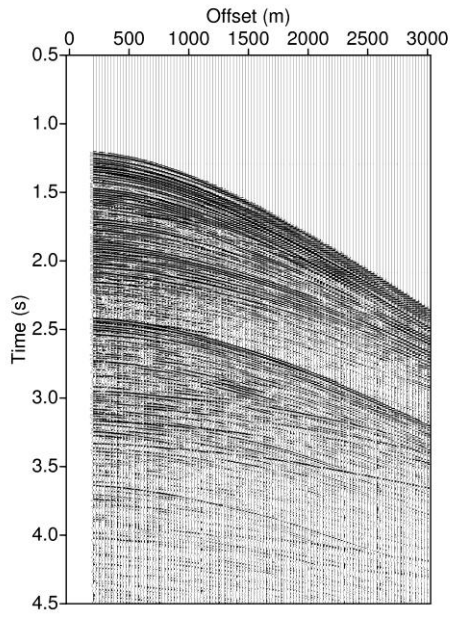
Figure 4.5a shows a full shot gather from the data where there are 8 missing near-offset traces. Figure 4.5b shows the free-surface multiple model predicted using the full $\mathbf{T}$ operator by MPI after 3 iterations. It can be seen that both the amplitude and

the phase of the model are accurate. This reduces the non-linearity for the subtraction. After adaptive subtraction with EMCM, the multiple-attenuation result is shown in Figure 4.5c. It can be seen that the free-surface multiples are well attenuated.

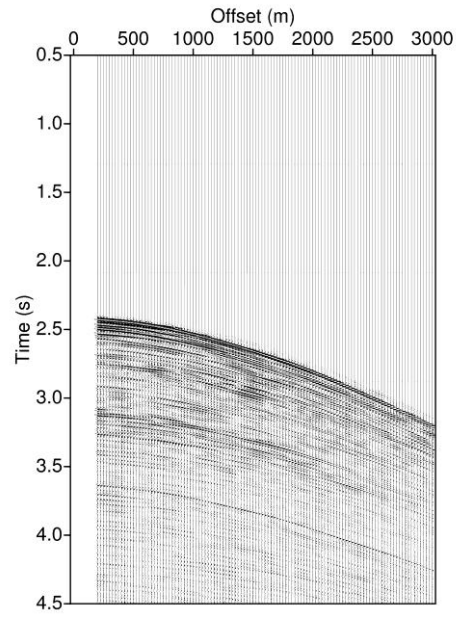
Figure 4.6a shows zoomed far-offset sections of the same shot gather as shown in Figure 4.5a. The reflections include two multiples starting at 2.8 s and 3.9 s. Figure 4.6b is the multiple-attenuation result when the $\mathbf{T}$ operator is assumed to be tridiagonal. It contains residual multiple energy indicated by the arrows. By contrast, Figure 4.6c shows the result of using the GPU to solve the full $\mathbf{T}$ operator; this provides much better attenuation of the multiples.

Figure 4.7a shows a stack section extracted from the inline direction of the 3D data set. Without multiple attenuation, strong multiples can be seen to cross primary reflections. Figure 4.6b shows the result of applying three iterations of the proposed procedure. It can be seen that the free-surface multiples are now well attenuated. The difference between Figures 4.7a and 4.7b is shown in Figure 4.7c. It can be seen that the primaries are well preserved after free-surface multiple attenuation.

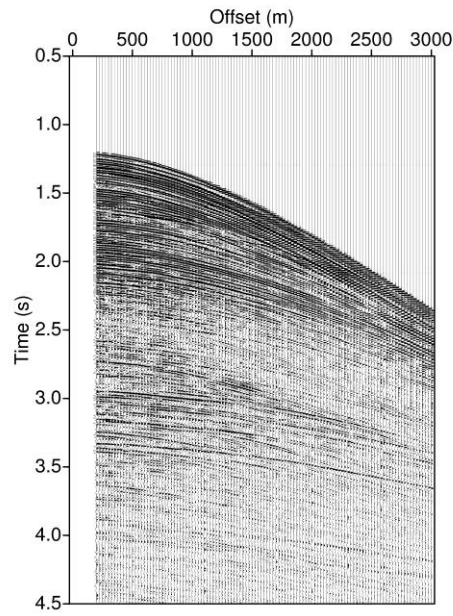
As a comparison of the computational efficiency of implementing MPI on this 3D data (Appendix A.4), free-surface multiple modelling with three iterations of MPI took about 10 hours for an individual sail-line using on GPU, whereas, the original test of processing the same scale of data on CPU took more than 1 week. This shows that MPI can be accelerated significantly using GPU technology making it much more practical for real data processing.



(a)

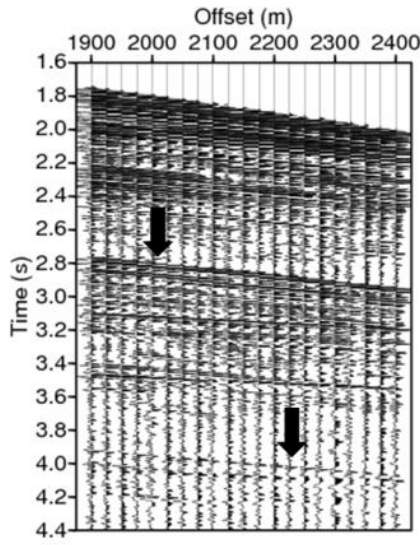


(b)

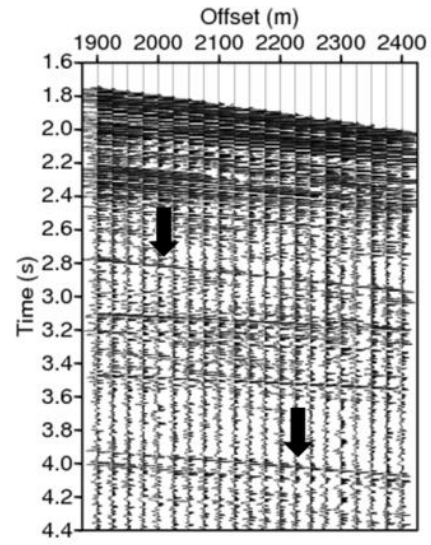


(c)

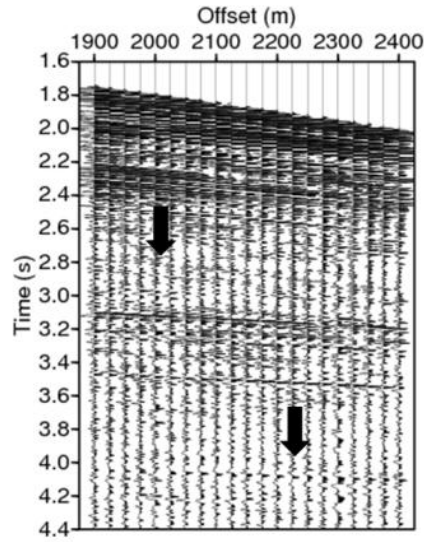
Figure 4.5 (a) Shot gather containing strong free-surface multiples. (b) Multiple model predicted by MPI. (c) Multiple-attenuation result after three iterations using the complete $\mathbf{T}$ operator.



(a)

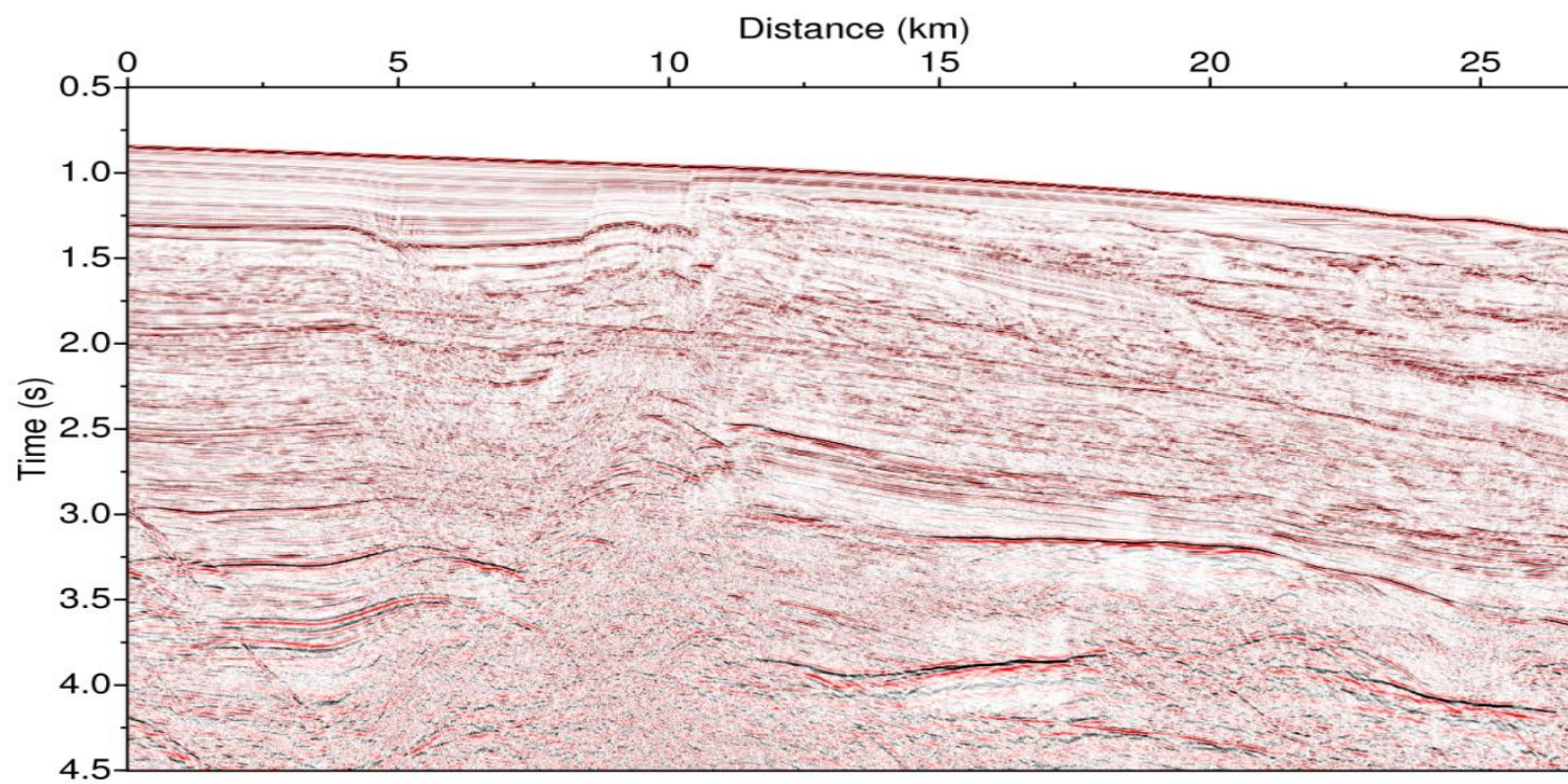


(b)



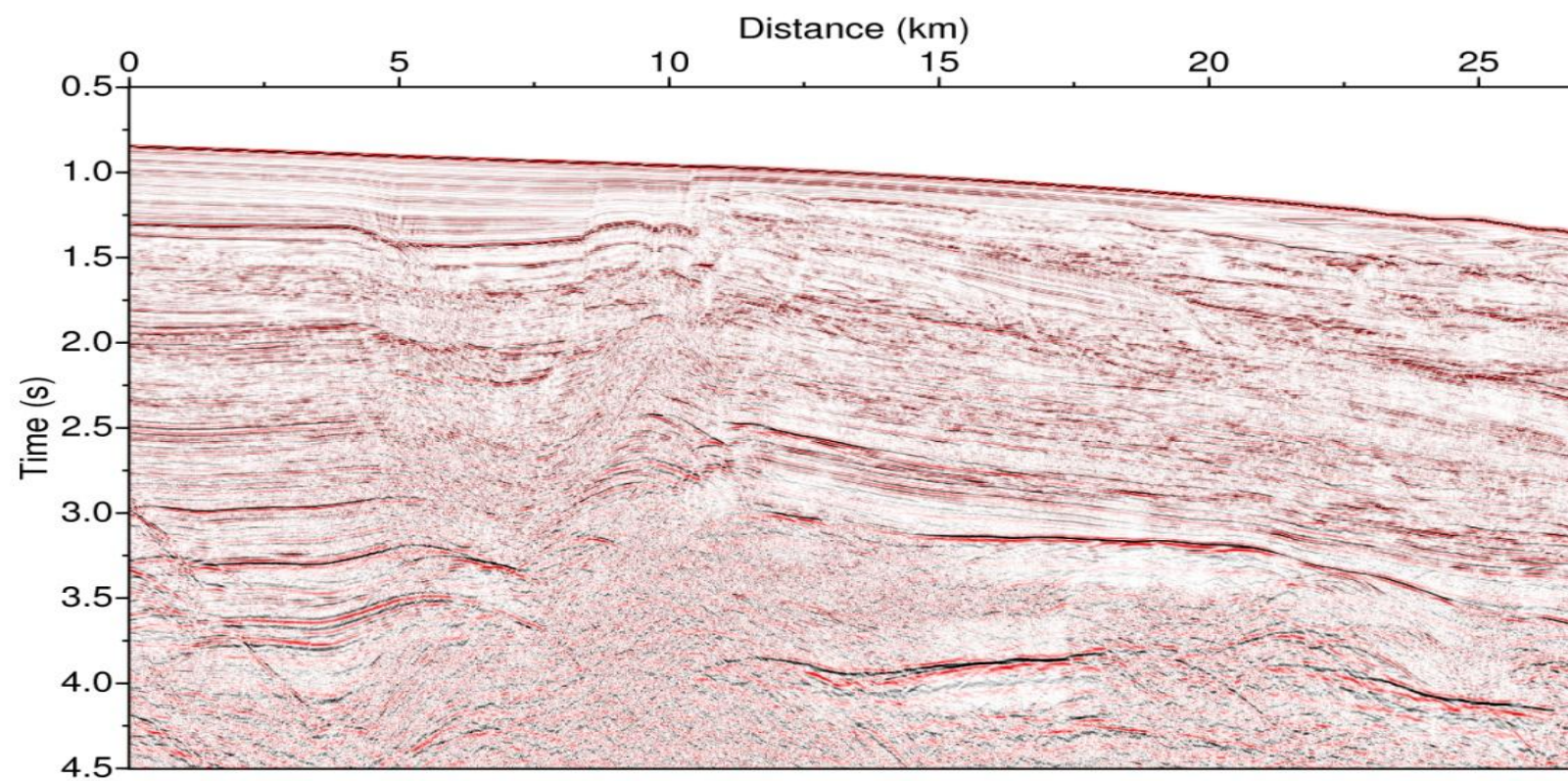
(c)

Figure 4.6 (a) The zoom-in section, focusing on the far-offset area, of a shot gather. (b) The multiple-attenuation result with the approximated tri-diagonal $\mathbf{T}$ operator. (c) The multiple-attenuation result with the full $\mathbf{T}$ operator.



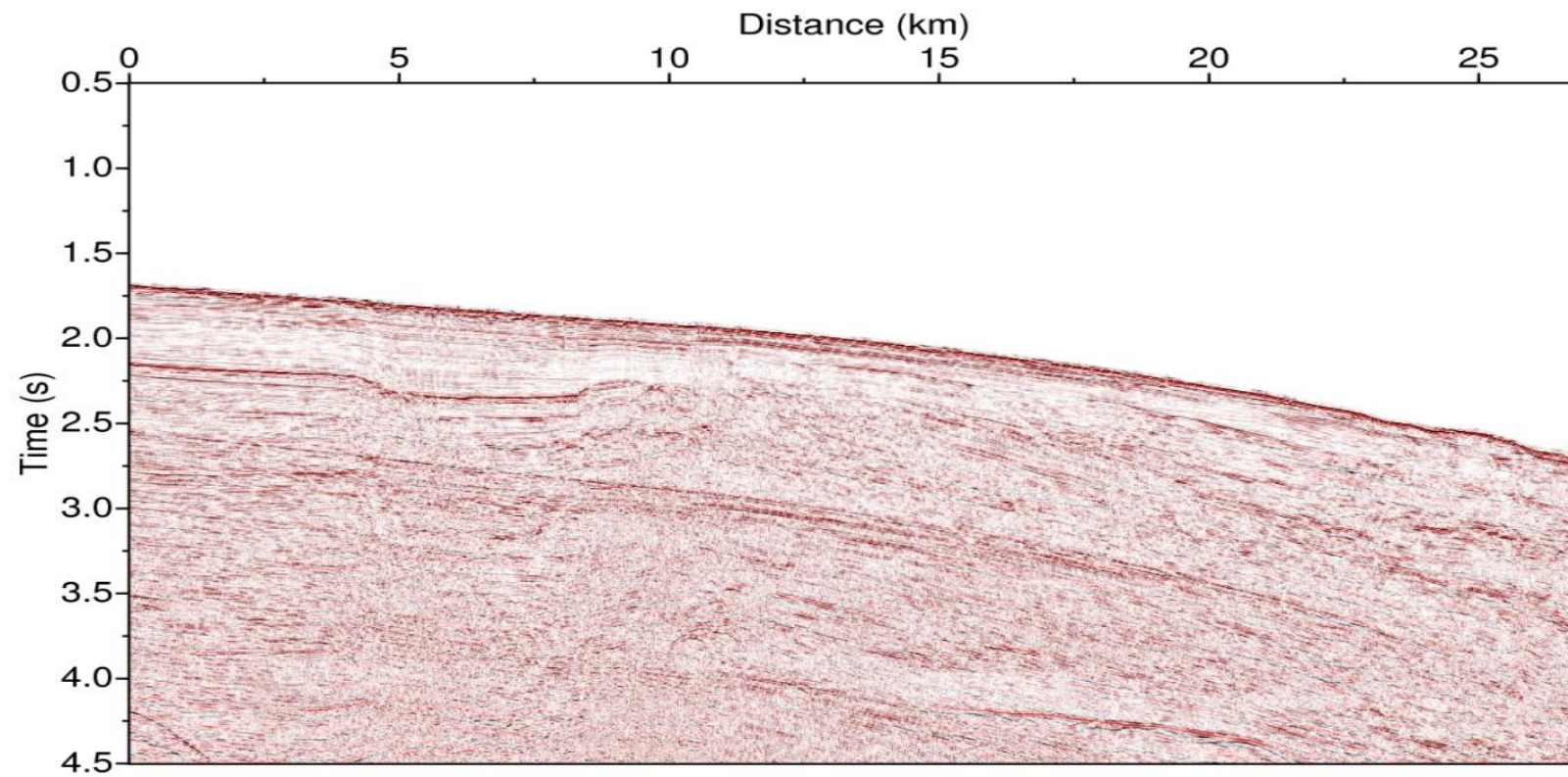
(a)

Figure 4.7 Stack section (a) before multiple attenuation.



(b)

Figure 4.7 Stack section (b) after multiple attenuation.



(c)

Figure 4.7 Stack section (c) The attenuated multiple energy.

4.5 CONCLUSIONS

In the MPI scheme for seismic multiple attenuation, the prediction operator $\mathbf{T}$ is calculated in multiple modelling. This operator is an inverse matrix, which previously was approximated to be a band matrix so that the matrix inverse computation was affordable. GPU technology can speed up the computation efficiency of MPI by two orders of magnitude making it realistic to calculate a full operator $\mathbf{T}$. With the full $\mathbf{T}$ operator, the multiple attenuation result can be improved, especially in far-offset traces.

Appendix 4.A GPU FOR SRME

In the MPI scheme for multiple model prediction, iteration starts from $n = 2$, and we need to know at least $\mathbf{P}_0^{(0)}$ and $\mathbf{P}_0^{(1)}$. In practice, we assume $\mathbf{P}_0^{(0)} = \mathbf{P}$. To estimate the initial wavefield $\mathbf{P}_0^{(1)}$, conventional SRME used to generate an improved initial multiple model.

In order to implement SRME, the seismic data are organized as a matrix $\mathbf{P}$ in frequency domain as shown in Figure 4.A1a, where each column is a common shot gather and each row is a common receiver gather (Berkhout, 1982). To predict the model of multiples at the position of i th row and j th column, the common receiver gather at the i th row is convolved with the common shot gather at the j th column, as a general matrix multiplication calculation (Figure 4.A1b). After the matrix multiplication, the free-surface multiple models are generated at each trace position.

SRME requires several adjacent shots to predict the multiple model of one shot. In particular, for marine data, if the number of receivers in each shot gather is N , the previous $N - 1$ shots as well as the following shots are needed to predict the model at each offset. For example, if the whole data matrix is processed at once, all the required information is not available for the first $N - 1$ shots. However, due to

computational limitations, the data matrix is always separated into small overlapping blocks for computational efficiency (Figure 4.A2). The size of the overlap depends on the number of receivers in each shot gather and a minimum of $N - 1$ overlapping shots are needed to ensure accuracy. SRME can be described as a series of matrix multiplications, which is a highly parallel computational process.

In my work, I use GPU technology to improve the computational efficiency in the modelling stage for the free-surface multiple attenuation. This requires a CPU/GPU co-operation work flow, where the matrix multiplications are calculated by GPU while the other steps are still dealt with by CPU. Figure 4.A3 shows the workflows for SRME. In this work flow, one matrix multiplication is calculated for each single frequency matrix.

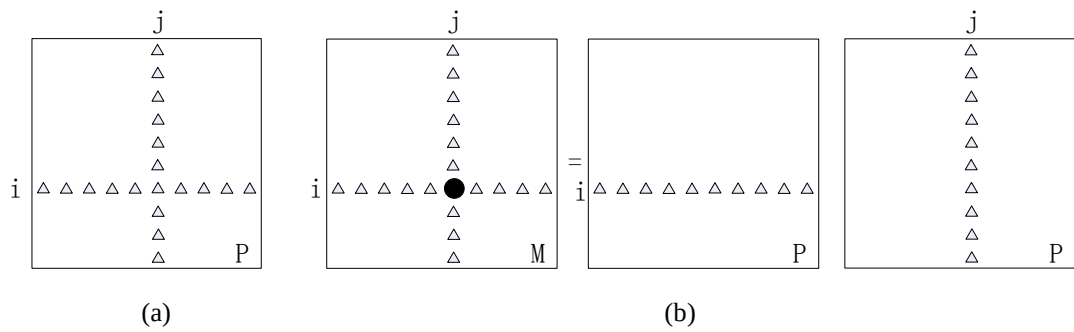


Figure 4.A1 Matrix multiplication in SRME (a) The data matrix where each column is a common shot gather and each row is a common receiver gather. (b) The matrix multiplication process to predict the multiple model at the position of i th row and j th column.

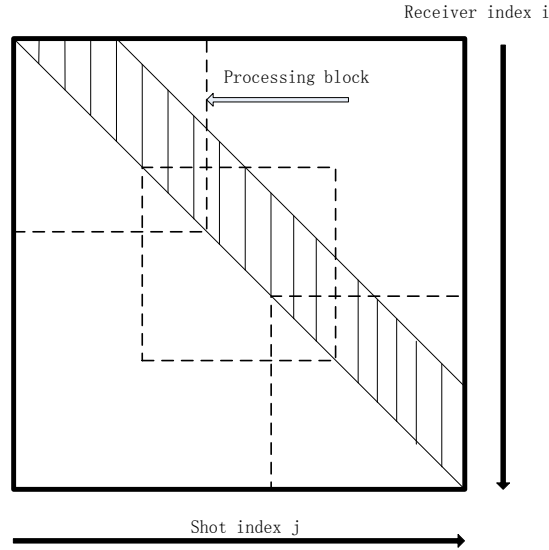


Figure 4.A2 The whole data matrix is separated into small overlapping blocks for calculation.

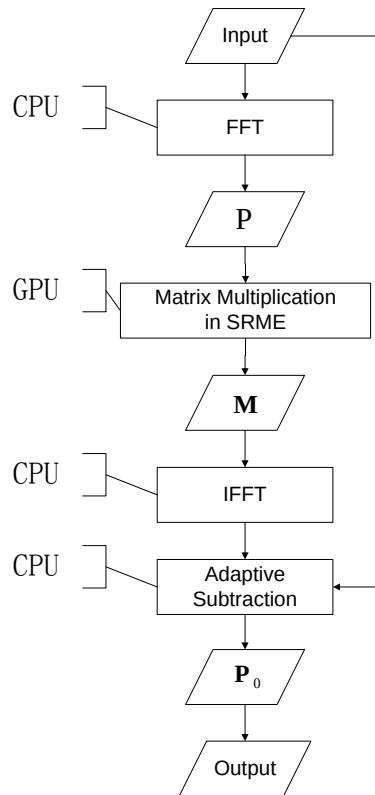


Figure 4.A3 CPU/GPU workflow for SRME.

I tested the computational efficiency of matrix multiplication for both a CPU and GPU. When the matrix size is 400×400 , the cost to calculate 2185 matrix multiplications for CPU and GPU respectively are 2487 and 33 s, i.e. the GPU is

about 75 times faster than the CPU. For a larger matrix size of 1000×1000 , the costs are 41,548 s and 133 s. CPU needs more than 11 hours, but GPU speeds up the calculation more than 300 times (Table 4.A1).

	Matrix 400 x 400	Matrix 1000 x 1000
CPU	2487 s	41548 s
GPU	33 s	133 s

Table 4.A1 Time cost of matrix multiplication

Appendix 4.B TRAJECTORY-STACKING GRT AND ENERGY SPECTRUM

This appendix closely follows Huo (2009), but is included here for the completeness.

In the frequency domain, the spatial convolution in 3D SRME can be divided into two summations: summation in the inline direction as in 2D SRME and summation in the cross-line direction. In the crossline direction the multiple contribution gathers are sparsely sampled which makes the summation unsatisfactory. Huo (2009) used a generalised Radon transform (GRT) in the crossline direction to locate the apexes and extract the curvature of the multiple contributions. Given the curvature of the multiple contribution gathers, trajectory stacking an almost identical result to the summation of dense sampling gathers.

The trajectory is parameterised with three parameters: the intercept time τ for the time location of the apex, the location of the apex y_0 , and the slowness p . With this parameterisation, the transformation from the data space $d(y_k, t)$ to the model space $m(\tau, p, y_0)$ can be achieved with the following linear operation:

$$m(\tau, p, y_0) = \sum_{y_k=y_{k, \min}}^{y_k=y_{k, \max}} d(y_k, t = \sqrt{\tau^2 + p^2 (y_0 - y_k)^2}) \quad (4.B1)$$

A weighting function based on semblance is applied to the trajectory-stacking Radon transform, so that the amplitude of a more coherent trajectory can be better preserved. A threshold is set to eliminate the effect of non-coherent trajectories and semblance values less than the threshold are set to zero.

Using the GRT model space, Huo (2009) developed an energy spectrum method to simplify the identification of the apexes corresponding to multiple reflection points. Each point in the energy spectrum is the summation of the energy of a cube centered at the point in the GRT model space, which can be formulated as

$$E = \sum_{W1} \sum_{W2} \sum_{k=1}^N z_k^2 \quad (4. B2)$$

where z_k are samples in the cube defined by $W1$, $W2$ and N in the GRT model space. In the energy spectrum, the energy blocks stand for the multiple contribution events. With the assumption that the trajectory closest to the real event yields the largest stacking energy, the largest value inside a block stands for the apex of a time-domain event. Then the multiple contributions in the crossline direction with these apex locations can be reconstructed.

Chapter 5 THE MPI SCHEME FOR INTERNAL MULTIPLE ATTENUATION

5.1 INTRODUCTION

Wang (2007) showed that the extension of MPI from free-surface multiple attenuation to internal-multiple attenuation is much less problematic than the similar extension of conventional SRME. This is because MPI does not require an explicit surface operator so no explicit internal operator for internal-multiple attenuation. However, the spatial variation of the internal operator still needs to be considered in the multiple prediction.

In this chapter, I extend the MPI scheme from the free-surface multiple attenuation to internal multiple attenuation. For the initial step of this iterative procedure, I use two different methods including the common-focus-point (CFP) method (Berkhout, 1982; Berkhout and Verschuur, 1997 and 2005a; Verschuur and Berkhout, 2005) and the correlation method (Jakubowicz, 1998). I then use the MPI scheme to improve the multiple models through further iteration. I will demonstrate the effectiveness of the MPI scheme for the internal multiple attenuation using both synthetic and field data examples.

5.2 MPI FOR INTERNAL MULTIPLE ATTENUATION

In MPI scheme, the multiple model for the current iteration is built by updating the multiple wavefield attenuated in the previous iteration as:

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{T}^{(n-1)}\mathbf{M}^{(n-1)}, \quad (5.1)$$

where $\tilde{\mathbf{M}}^{(n)}$ is the multiple model in the n th iteration and $\mathbf{M}^{(n-1)}$ is the multiple wavefield attenuated in the $(n-1)$ th iteration. $\mathbf{T}^{(n-1)}$ is the updating operator given by:

$$\begin{aligned}\mathbf{T}^{(n-1)} &= \mathbf{P}_0^{(n-1)} [\mathbf{P}_0^{(n-2)}]^{-1} \\ &= \mathbf{P}_0^{(n-1)} (\mathbf{P}_0^{(n-2)})^H [\mathbf{P}_0^{(n-2)} (\mathbf{P}_0^{(n-2)})^H]^{-1},\end{aligned}\quad (5.2)$$

where the superscript H stands for complex conjugate transposition. The MPI scheme can minimise the edge effect that appears in conventional SRME as well as handle data sets with missing near-offset traces without the need to synthesise near-offset traces (Wang, 2004).

Wang (2004) showed that MPI can be derived from the same equation used in conventional SRME. In SRME, the free-surface multiples can be predicted as:

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)} \mathbf{A} \mathbf{P}. \quad (5.3)$$

The MPI scheme can be understood as:

$$\tilde{\mathbf{M}}^{(n)} = \mathbf{P}_0^{(n-1)} \mathbf{A}^{(n-1)} \mathbf{P} \quad (5.4)$$

and

$$\mathbf{A}^{(n-1)} = [\mathbf{P}_0^{(n-2)}]^{-1} (\mathbf{P} - \mathbf{P}_0^{(n-1)}) \mathbf{P}^{-1}, \quad (5.5)$$

A detailed derivation can be found in Wang (2004). The surface operator $\mathbf{A}$ is implicitly taken into account using the seismic data only.

Internal multiple attenuation methods based on the extension of SRME treat internal multiples interface by interface, with the free-surface as a special case. The target internal-multiple-generating interface z_k acts as a pseudo “free-surface” and a corresponding pseudo “surface operator”, or internal operator $\mathbf{A}_k$, is involved in the multiple modelling process (Berkhout and Verschuur, 1997). Following the same concept, the MPI scheme can also be extended to predict the model of internal multiples interface by interface by taking into the internal operator implicitly as:

$$\mathbf{A}_k^{(n-1)} = [\mathbf{P}_k^{(n-2)}]^{-1} (\mathbf{P}_{k-1} - \mathbf{P}_k^{(n-1)}) \mathbf{P}_{k-1}^{-1} \quad (5.6)$$

and

$$\tilde{\mathbf{M}}_k^{(n)} = \mathbf{P}_k^{(n-1)} \mathbf{A}_k^{(n-1)} \mathbf{P}_{k-1}, \quad (5.7)$$

where index k means the internal-multiple generating interface at z_k , $\tilde{\mathbf{M}}_k^{(n)}$ is the internal multiple model generated by interface z_k in the n th iteration. Because the internal multiples should be attenuated interface by interface, $\mathbf{P}_{k-1}$ is the de-multiple result where multiples generated by interface z_{k-1} and all the interfaces above it have been removed, $\mathbf{P}_k^{(n-1)}$ and $\mathbf{P}_k^{(n-2)}$ are the wavefields in which multiples generated by interface z_k and all interfaces above have been removed for the $(n-1)$ th and $(n-2)$ th iteration, respectively, and $n \geq 2$.

Substituting Equation 5.7 into Equation 5.6, the extended MPI scheme for predicting the internal multiples generated by interface z_k can be expressed as:

$$\begin{aligned} \tilde{\mathbf{M}}_k^{(n)} &= \mathbf{P}_k^{(n-1)} [\mathbf{P}_k^{(n-2)}]^{-1} (\mathbf{P}_{k-1} - \mathbf{P}_k^{(n-1)}) \\ &= \mathbf{P}_k^{(n-1)} (\mathbf{P}_k^{(n-2)})^H [\mathbf{P}_k^{(n-2)} (\mathbf{P}_k^{(n-2)})^H]^{-1} \mathbf{M}_k^{(n-1)}, \end{aligned} \quad (5.8)$$

where $\mathbf{M}_k^{(n-1)}$ is the actual internal-multiple wavefield which has been attenuated in the $(n-1)$ th iteration.

Figure 5.1 is the flowchart for internal-multiple attenuation using the MPI scheme. In practice, we set $\mathbf{P}_k^{(0)} = \mathbf{P}_{k-1}$. To estimate the initial wavefield $\mathbf{P}_k^{(1)}$, we can use any other multiple-attenuation algorithms to obtain the attenuated internal-multiple wavefield. In the following section, I will compare the results when using different methods as the initial step in the MPI scheme.

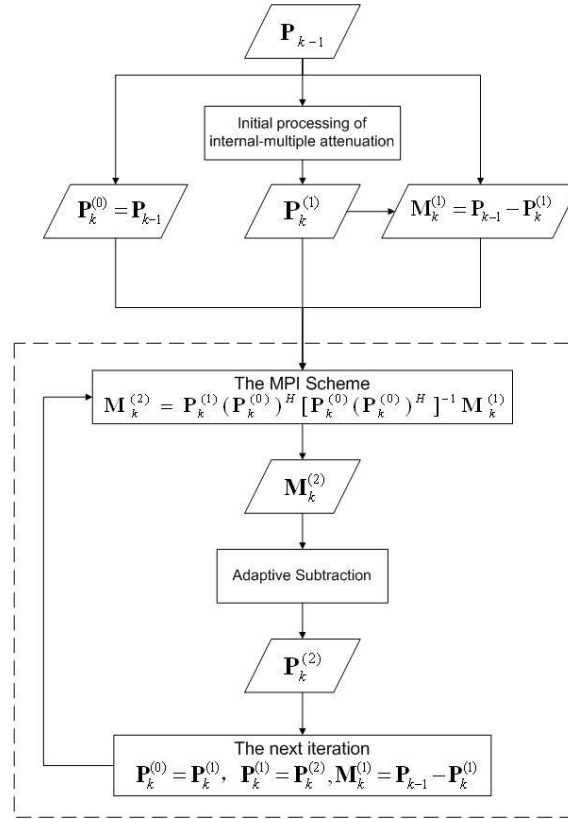


Figure 5.1 Flowchart for internal-multiple attenuation using the MPI scheme.

5.3 USING DIFFERENT METHODS FOR THE INITIAL STEP IN THE MPI SCHEME

I compare the use of CFP and correlation methods as initial steps for the MPI scheme. Using a synthetic subsurface model, which contains three flat interfaces at the depth of 600, 900 and 1800 m the velocities of each layer are 1500, 2000 and 3000 m/s respectively.

5.3.1 USING THE CFP METHOD AS THE INITIAL STEP IN THE MPI SCHEME

The common-focus-point (CFP) method (Berkhout, 1982; Berkhout and Verschuur, 1997 and 2005a; Verschuur and Berkhout, 2005) is extended from SRME for the internal multiple attenuation.

Given the seismic data $\mathbf{P}(z_0, z_0)$, with both source and receiver points being on the surface z_0 , we may redatum the data to either $\mathbf{P}(z_0, z_k)$ or $\mathbf{P}(z_k, z_0)$, where the first matrix corresponds to a redatumed data set with source points at interface z_k and receiver points at the surface z_0 , and the second matrix has source and receiver points at depth level z_0 and z_k , respectively. The internal multiples related to interface z_k may be written as:

$$\mathbf{M}_k(z_0, z_0) = \bar{\mathbf{P}}_{k-1}(z_0, z_k) \bar{\mathbf{P}}_{k-1}(z_k, z_0), \quad (5.9)$$

where z_k is the internal-multiple generating interface, $\mathbf{P}_{k-1}$ is the wavefield for which multiples above level k have been removed, and $\bar{\mathbf{P}}_{k-1}$ indicates that all primaries related to the k th interface z_k and above have been muted from $\mathbf{P}_{k-1}$. The redatumed data set $\mathbf{P}(z_0, z_k)$ in Equation 5.9 can be obtained as:

$$\mathbf{P}(z_0, z_k) = \mathbf{P}(z_0, z_0) \mathbf{F}(z_0, z_k), \quad (5.10)$$

where $\mathbf{F}(z_0, z_k)$ represent redatuming operators describing inverse wavefield extrapolations from the surface z_0 to the interface z_k . The redatuming operator is referred to as a focusing operator in the CFP technique (Berkhout, 1997a), and the redatumed data $\mathbf{P}(z_0, z_k)$ are CFP gathers which focus virtual sources at subsurface depth level z_k . In a similar way, using focusing operators $\mathbf{F}(z_k, z_0)$, we may perform re-datuming as

$$\mathbf{P}(z_k, z_0) = \mathbf{F}(z_k, z_0) \mathbf{P}(z_0, z_0) \quad (5.11)$$

to generate CFP gathers $\mathbf{P}(z_k, z_0)$ focusing in virtual receivers at subsurface depth level z_k . The CFP methods involve depths as specific parameters, therefore it is a model-driven approach (Verschuur *et al.*, 1999).

The focusing operator $\mathbf{F}(z_0, z_k)$ or $\mathbf{F}(z_k, z_0)$, can be obtained by a data-driven updating procedure which consists of the following steps (Berkhout, 1997b):

- 1) An initial focusing operator is constructed based on the NMO stacking velocity.
- 2) Creating a CFP gather with the current focusing operator.
- 3) Subtracting the traveltimes of the focusing operator from the CFP gather to generate a different time shift (DTS) panel.
- 4) Picking the time difference between the event of interest and the line $t = 0$ in the DTS panel.
- 5) Updating focusing operator by adding half of the picked time difference to the current focusing operator.

Repeating steps 2-5 iteratively until the event of interest in the DTS panel is close to $t = 0$.

Actually a focusing operator is a Green's function which represents traveltimes between any point in the surface and a focusing point in the subsurface. The principle assumption used in the procedure above is that traveltimes in a focusing operator are equal to the traveltimes of the reflection event in the corresponding CFP gather. This reciprocity is called the principle of equal traveltimes in the CFP technique (Berkhout, 1997b).

Figure 5.2a is a common-shot gather with the source point at $x_j = 500$ m at surface z_0 . Figure 5.2b shows the ray paths of a focusing operator related to one focus point at the lateral position $x_j = 500$ m and depth level $z_k = 600$ m. The correct focusing operator should be the Green's function from the focus point to the surface z_0 . We construct an initial focusing operator based on the approximate macro model in time-reversed version as shown in Figure 5.2c. We then correlate each trace in the shot gather (Figure 5.2a) with the trace at the same lateral location in the initial time-reversed focusing operator (Figure 5.2c), and sum up the resulting

traces (Figure 5.2d) to generate a single trace (Figure 5.2e) illuminated from surface z_0 and measured at level $z_k = 600$ m with the same lateral location $x_j = 500$ m. In this step, only the contribution around the stationary point contributes to the summation result. If we apply this focusing operator to all common-shot gathers, we obtain the initial CFP gather corresponding to a virtual receiver in the subsurface (Figure 5.2f).

To update the focusing operator, we obtain the so-called differential time shift (DTS) panel (Figure 5.3a) by subtracting the traveltimes of the focusing operator (Figure 5.2c) from the CFP gather (Figure 5.2f). From Figure 5.3a we can see that the event of interest is located above $t = 0$. We can then pick half of the time difference between the event and the line $t = 0$ to update the initial focusing operator. After five iterations, the updated DTS panel (Figure 5.3b) is correctly aligned at time zero. It can be seen that the updated focusing operator (Figure 5.3c) has the same traveltimes as the corresponding response in the CFP gather (Figure 5.3d). Thus, we obtain a CFP gather with a virtual receiver at $x_j = 500$ m and at the first interface.

Equation (5.9) also shows that, for internal-multiple prediction, muting must be applied to the redatumed data to remove all primary reflections that have their downward reflections above and including depth level z_k . In the CFP domain, muting becomes a straightforward process. If the focusing operator is correct, the time-reversed focusing operator will coincide in time with the corresponding reflection event in the CFP gather. Therefore, the focusing operator defines the mute curve for removing events from above the depth level z_k .

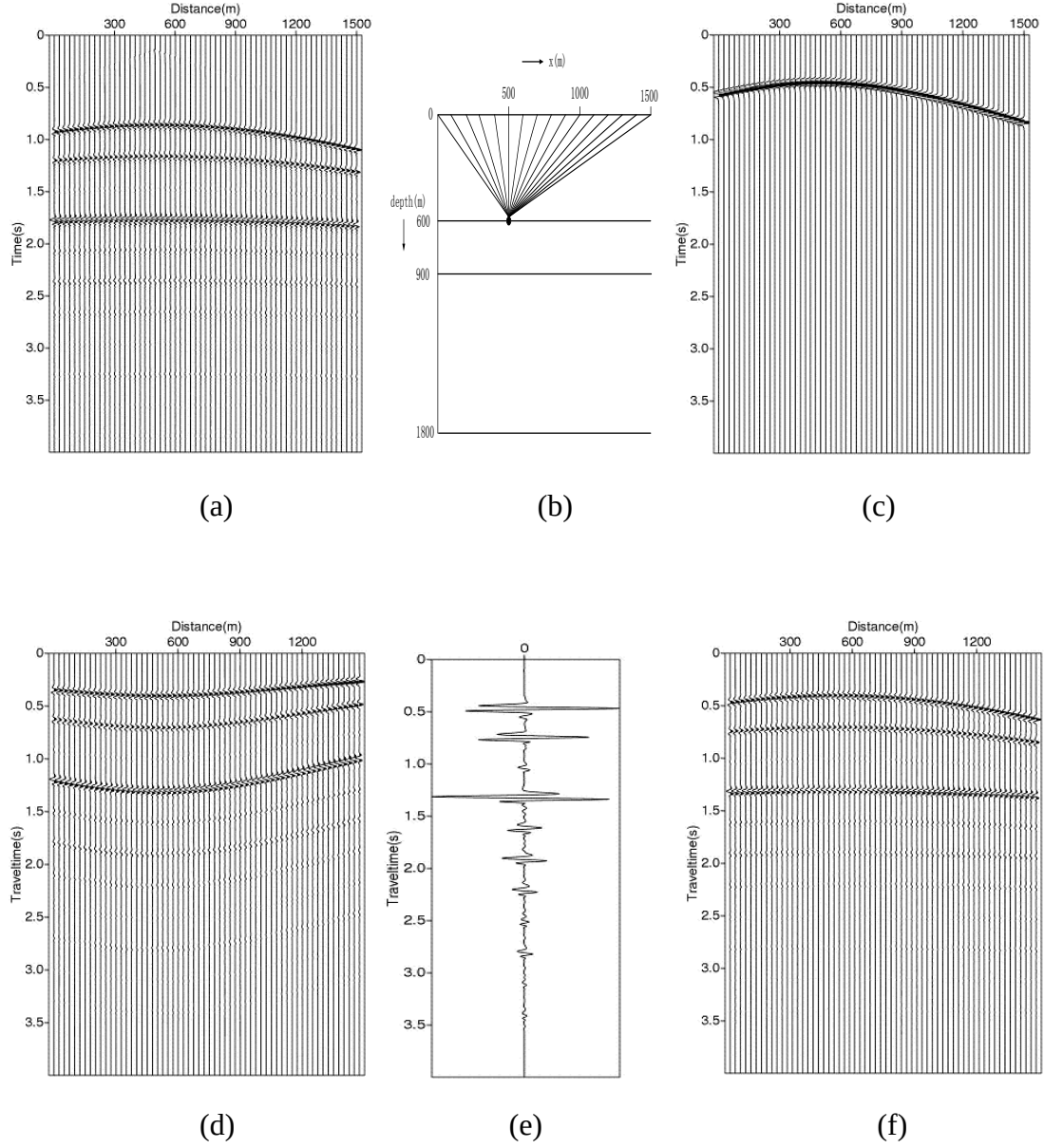


Figure 5.2 Generating a CFP gather: (a) A common-shot gather with the source point at the surface z_0 (a column of $\mathbf{P}(z_0, z_0)$). (b) Ray path for the focusing operator. (c) The initial focusing operator. (d) Trace-by-trace correlations of (a) and (c). (e) Summation result of all traces in (d). (f) The resulting CFP gather (a row of $\mathbf{P}(z_k, z_0)$).

Comparing internal-multiple attenuation using MPI (Equation 5.8) with the CFP method (Equations 5.9 and 5.10), the focusing operator $\mathbf{F}(z_0, z_k)$ or $\mathbf{F}(z_k, z_0)$ is

not needed for predicting the multiple model. These operators are spatially variable and are unlikely to be accurate estimates. However the effect of these operators can be implicitly considered in the iterative modelling process in MPI. In addition, MPI avoids the need to mute the primaries reflected from the internal-multiple generators during the iteration process, reducing the error caused by the muting process. As the CFP method is extended from conventional SRME, the need to synthesise near-offset traces still exists, which can be resolved by MPI.

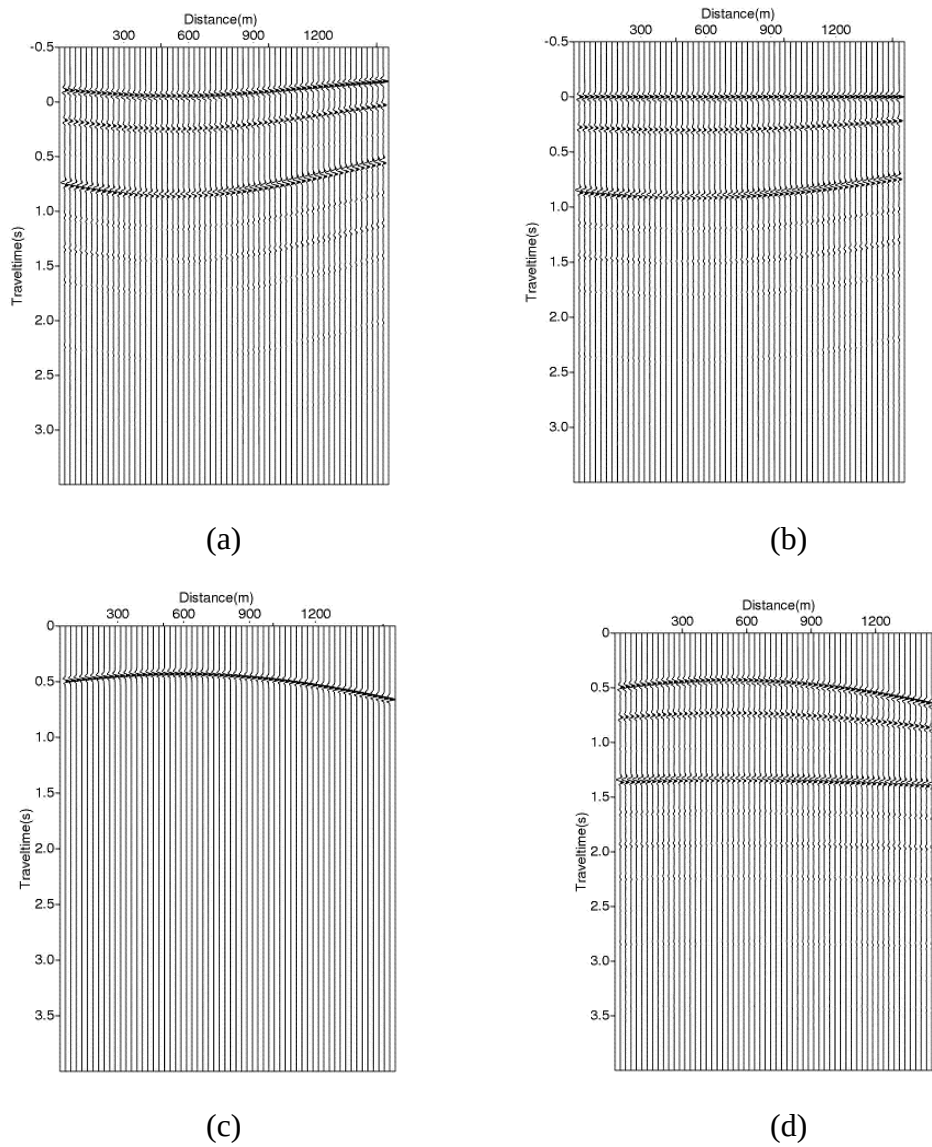
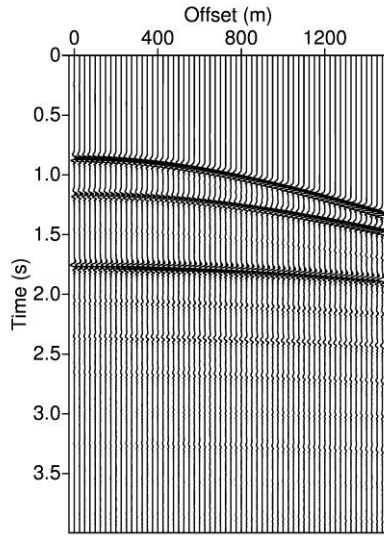


Figure 5.3 Focusing operator updating: (a) The initial DTS panel. (b) updated DTS panel after five iterations. (c) updated focusing operator. (d) updated CFP gather.

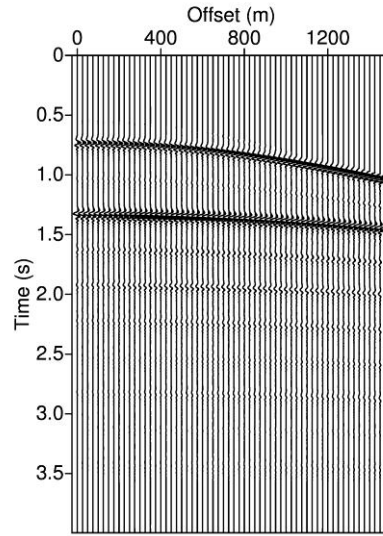
Figure 5.4a shows a synthetic shot gather containing primary reflections and all internal multiples (free-surface multiples are omitted). The second order first-interface-generating internal multiple should arrive at the same time (1.7 s) at zero offset as the third primary. However, this internal multiple could be omitted as its amplitude is very weak in comparison with the third primary. Figure 5.4b is a muted CFP gather related to the first interface. As the subsurface model consists of horizontal layers, the muted CFP gathers $\bar{\mathbf{P}}_0(z_0, z_1)$ and $\bar{\mathbf{P}}_0(z_1, z_0)$ are identical. Figure 5.4c is the prediction of the internal multiple model generated by the first interface. On the prediction, the travel times of the multiples are accurate, but the amplitudes and phases differ from the true internal multiples. In addition, the edge effect is visible.

Although imperfect, using the initial result $\mathbf{P}_1^{(1)}$ (Figure 5.4d) obtained from CFP method, (and $\mathbf{P}_1^{(0)} = \mathbf{P}_0$), we can now apply the MPI scheme. Figure 5.5a is the model of the first-interface-generated internal multiples, predicted by MPI. This multiple prediction is more accurate in both amplitude and phase than the one with the CFP method. There is also no near-offset effect. Figure 5.5b displays the corresponding multiple-attenuation result $\mathbf{P}_1^{(2)}$. We can see that the internal multiple generated by the first interface has been attenuated effectively.

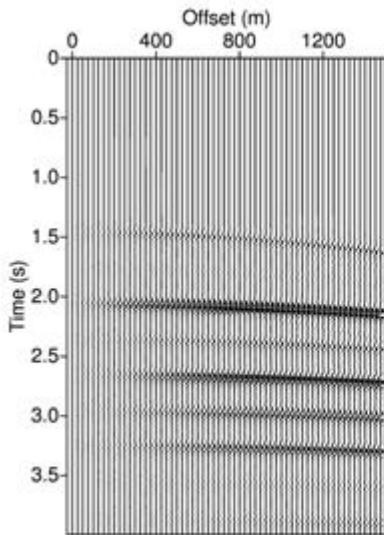
To remove the multiples (between 2.0 and 3.0 s in Figure 5.5b) generated by the second interface, we assume $\mathbf{P}_2^{(0)} = \mathbf{P}_1^{(2)}$ and then use the CFP method to predict the internal-multiple model (Figure 5.6a) and then to generate $\mathbf{P}_2^{(1)}$ (Figure 5.6b). Finally, we use the MPI scheme as the second iteration. Figure 5.6c is the internal-multiple models predicted by the MPI scheme. Figure 5.6d is the multiple-attenuation result $\mathbf{P}_2^{(2)}$, in which most of the internal multiples have been removed.



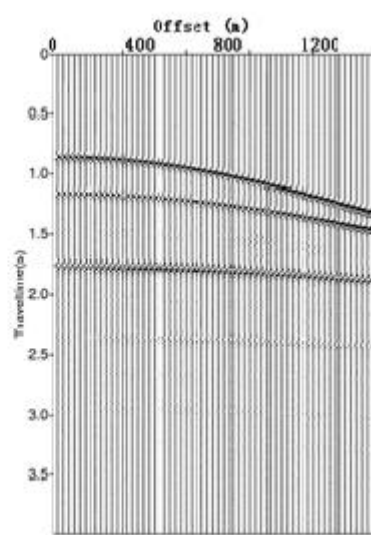
(a)



(b)

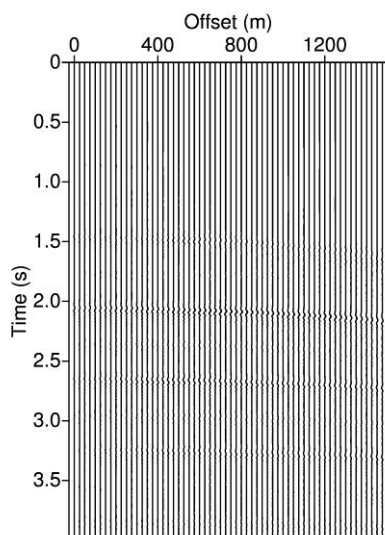


(c)

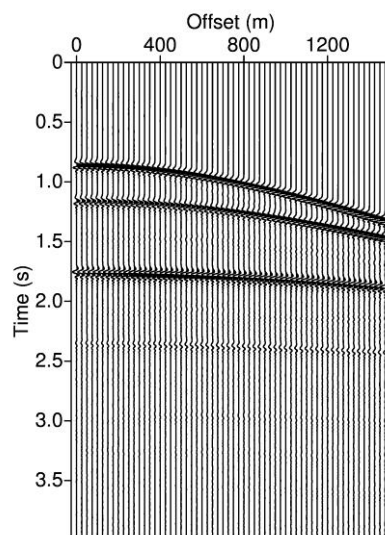


(d)

Figure 5.4. CFP method: (a) A synthetic shot gather with all internal multiples, but no free-surface multiples. (b) The muted CFP gather without primary reflections from the first interface. (c) The internal multiple model predicted by CFP method. (d) The multiple-attenuation result from CFP.



(a)



(b)

Figure 5.5. MPI method: (a) The internal multiple model predicted by MPI scheme.
(b) The multiple-attenuation result from MPI.

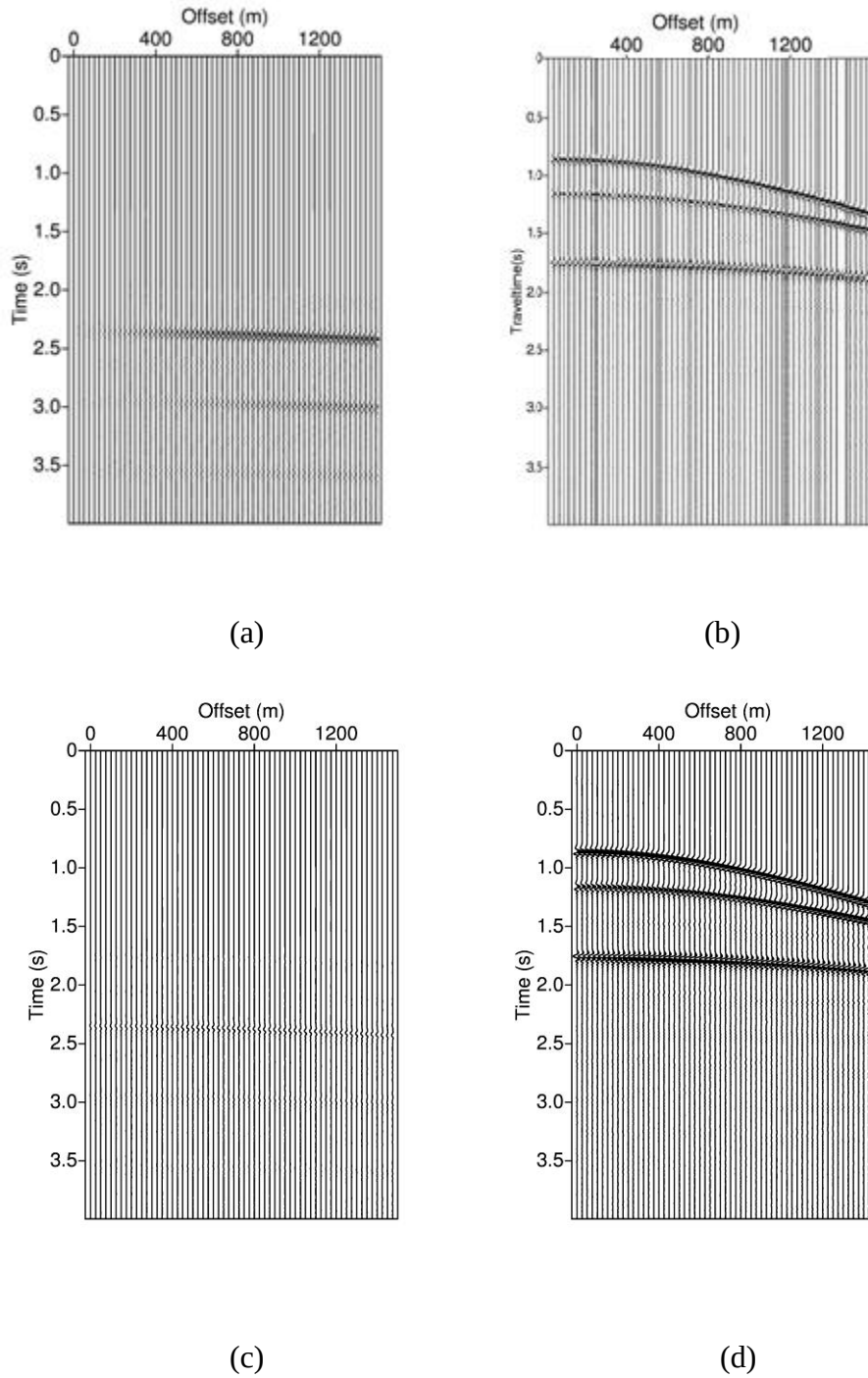


Figure 5.6. The internal multiple-attenuation related to the second interface. (a) The model of the second-interface-generated internal multiples predicted by CFP method. (b) The multiple-attenuation result based on the multiple model in (a). (c) The model of the second-interface-generated internal multiples predicted by MPI scheme. (d) The multiple-attenuation result based on the multiple model in (c).

5.3.2 USING THE CORRELATION METHOD AS THE INITIAL STEP IN THE MPI SCHEME

The inverse propagation operator in the CFP method can also be constructed from time-reversed primary reflections from the internal-multiple-generating interface (Jakubowicz, 1998). By extracting the required primary reflections from the data, the internal-multiple prediction process can be written as a correlation approach, where the internal multiples generated by the interface z_k can be predicted by correlation as:

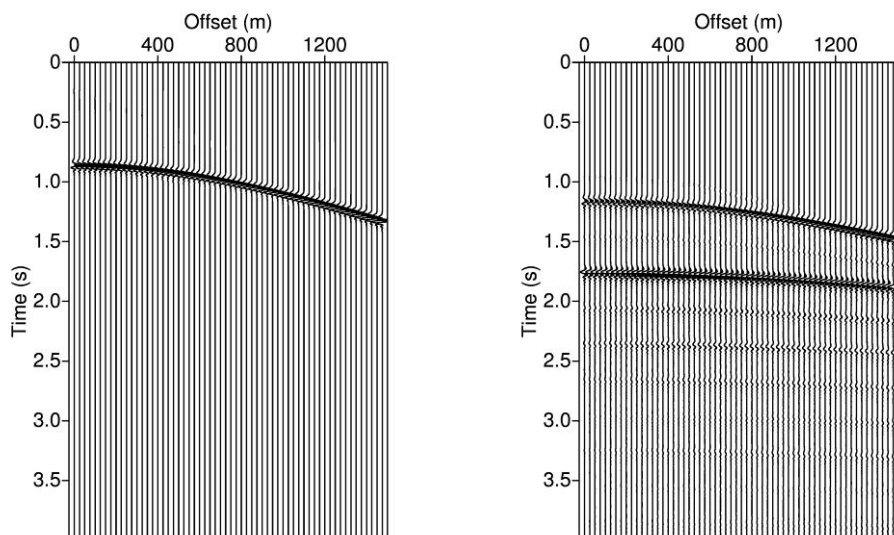
$$\mathbf{M}_k(z_0, z_0) = \overline{\mathbf{P}}_{k-1}(z_0, z_0) [\Delta \mathbf{P}_k(z_0, z_0)]^H \overline{\mathbf{P}}_{k-1}(z_0, z_0), \quad (5.12)$$

where $\Delta \mathbf{P}_k(z_0, z_0)$ means the isolated primary reflections only related to interface z_k and the superscript H means transposition and complex conjugation. In this method, the downward extrapolation operators in the CFP method are not needed. However, the primary reflections from the defined interface should be isolated from the full seismic response. Resorting the data into CMP gatherers and applying an NMO correction may facilitate this process (Verschuur, 2006).

Figure 5.7a shows the primary reflection from the first interface (extracted from the original shot gather shown in Figure 5.4a), which needs to be time-reversed. Figure 5.7b is the corresponding muted shot gather. Figure 5.7c is the model of the internal multiples generated by the first interface obtained using the correlation method. This involves interpreting the data to select the desired primaries from the multiple generating interfaces. The internal multiples predicted by the correlation method also have a near-offset effect as well as incorrect amplitude and phase.

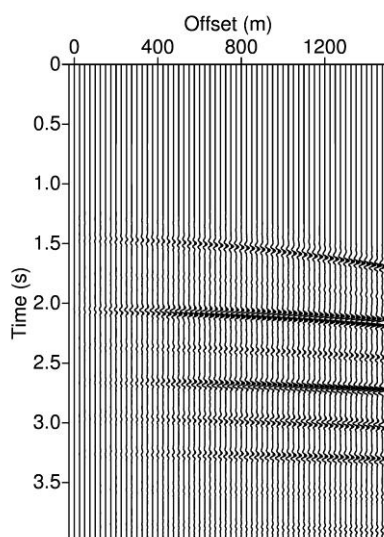
Figures 5.8a and 5.8b are the updated model of internal multiples and multiple-attenuation result obtained through the MPI scheme.

After attenuating the first-interface-generated internal multiples, the same procedure could be used to attenuate the internal multiples generated by the second interface.



(a)

(b)



(c)

Figure 5.7. Correlation method: (a) The isolated primary reflections. (b) The muted shot gathers. (c) The internal multiple model predicted by the correlation method.

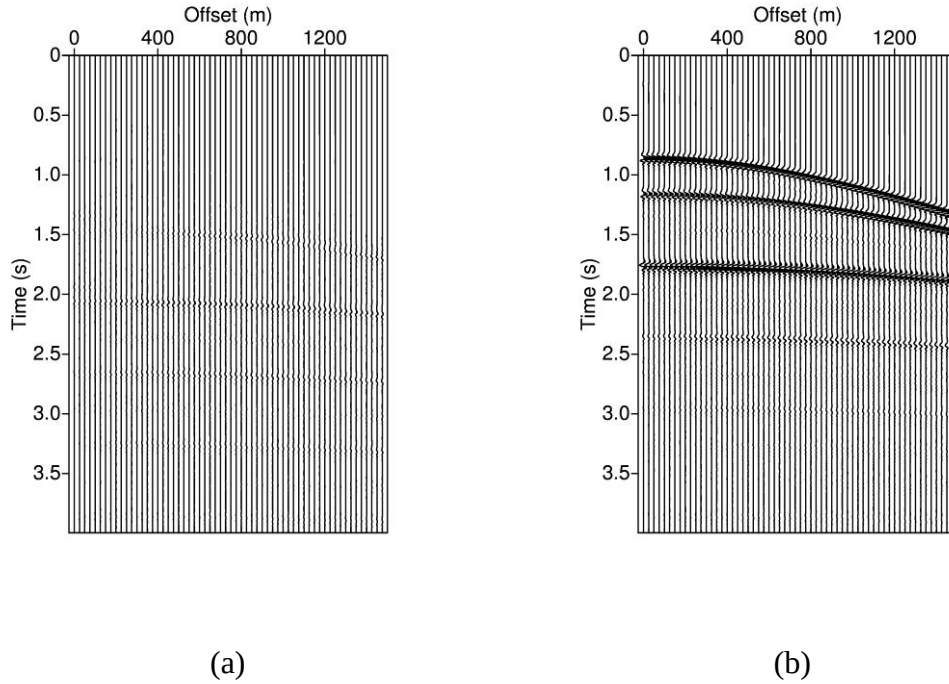


Figure 5.8. MPI method: (a) The internal multiple model predicted by MPI scheme. (b) The demultiple result from MPI.

5.3.3 DISCUSSION

In the CFP method, the inverse propagation operators are spatially variable and difficult to estimate accurately. In addition muting is a straightforward process in the CFP gather, but heavily depends on the accuracy of the estimated focusing operators. The operators could be determined by a data-driven updating procedure, but the focus point of an updated operator might be shifted with respect to the focus point of the initial operator, and this may influence the operator estimation (Bolte and Verschuur 1998; Bolte *et al.*, 1999). By comparison, the correlation method is more practical because it only involves seismic data with sources and receivers located at the surface and no propagation operators are needed; it does however need interpretation to select the desired primaries from the multiple generating interfaces. In real data applications, it is not easy to pick the desired primaries to generate the multiple models, especially in the early stages of processing. For areas with complicated dipping structures, starting only with a rough multiple-attenuation result, the MPI scheme can iteratively

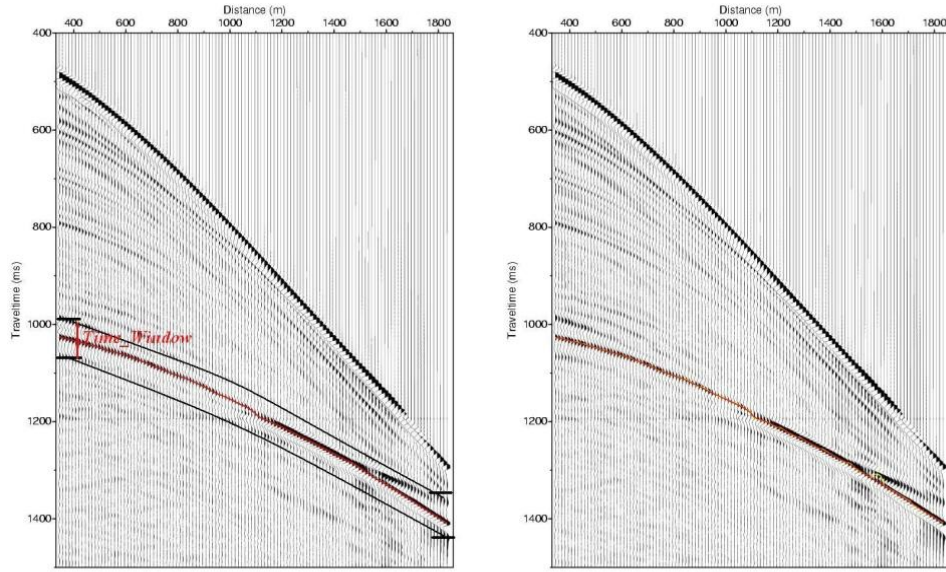
improve the model predictions. The above results have shown that MPI can attenuate internal multiples effectively when either the CFP method or the correlation method is used for the initial model.

5.4 APPLICATION EXAMPLES

5.4.1 TRAVEL TIME PICKING

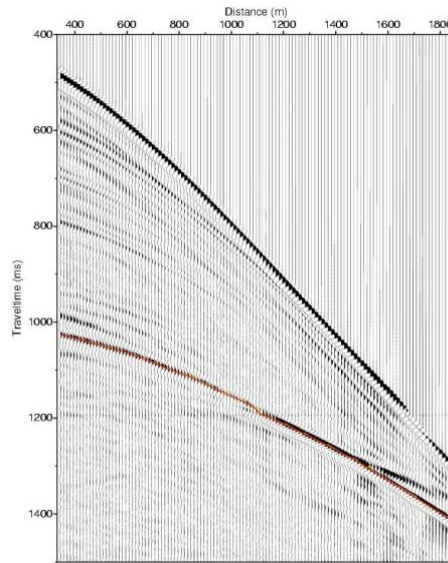
An essential step in both the CFP method and correlation method for the internal-multiple prediction, is picking traveltimes related to the internal-multiple-generating interface. In the examples shown here, I use a simple algorithm which takes the result from the previous shot gather as a reference to automatically generate the picking curve in the new shot gather.

In Figure 5.9a, a variable time window is defined which is an integer multiple of the sampling interval. Each point of the picking result from the previous shot gather lies at the midpoint of the time window. The traveltimes of the reflection events in the current shot gather are obtained by searching for the peak amplitude within the time window. Here the result of picking a shot gather is shown in Figures 5.9b and c. In Figure 5.9b, the size of time window was chosen to be four samples. The picking result is shown as the yellow line. The automated picks compare well with the red line which indicates the result from manually picking. However, around 1600 m, where there are crossing events, the automatic picking tracked wrong reflections on several traces. This is because the time window was too wide so that the previous peak was picked. The size of time window was then changed to twice sampling interval, producing the improved result shown in Figure 5.9c.



(a)

(b)



(c)

Figure 5.9 (a) The defined time window (b) Picking result with a window size of four samples. (c) Picking result with a window size of two samples.

5.4.2 GPU ACCELERATION FOR INTERNAL MULTIPLE ATTENUATION

In comparison with the wave-equation-based free-surface multiple prediction, internal multiple prediction based on the wave equation requires more computation.

For example, the correlation method for the internal multiple modelling includes two matrix multiplication processes for each frequency. In an individual iteration of the MPI scheme, there are three matrix multiplications and one solution of the linear system. All the matrix-related calculations are time-consuming. In Chapter 4, GPU technology was used to accelerate the MPI scheme for free-surface multiple attenuation. Based on the same approach, the extended MPI scheme for the internal multiple attenuation can also be accelerated by GPU technology. All the application examples below use a GPU to speed up the calculations.

5.4.3 A SYNTHETIC EXAMPLE

Here I show a synthetic example of the MPI scheme for internal-multiple attenuation using the correlation method as the initial step. The second data set is the Sigsbee2B model released by SMAART (Paffenholz *et al*, 2002). It is a 2D model generated using the acoustic wave equation and finite differencing with constant density (Appendix A.5). Figure 5.10 is the true interval velocity model of this dataset. There is a fast layer with the velocity of 2750 m/s at the seafloor, and the velocity of the salt is 4500 m/s. Both these interfaces are strong reflectors, and should therefore generate significant internal multiples. Figure 5.11a is a zoomed stack section between 7 km and 18 km along the line, where both free-surface and internal multiples can be observed. Figure 5.11b is the stack section without free-surface multiples. The B-spline method was implemented to interpolate the missing shot and then the MPI scheme with the correlation methods as the initial step was applied to attenuate the internal multiples generated by the seafloor. After two iterations, the internal multiple attenuation results and the attenuated internal multiple energies are as shown in Figure 5.11c and d respectively. It can be seen that internal multiples between the seafloor and the salt are significantly attenuated.

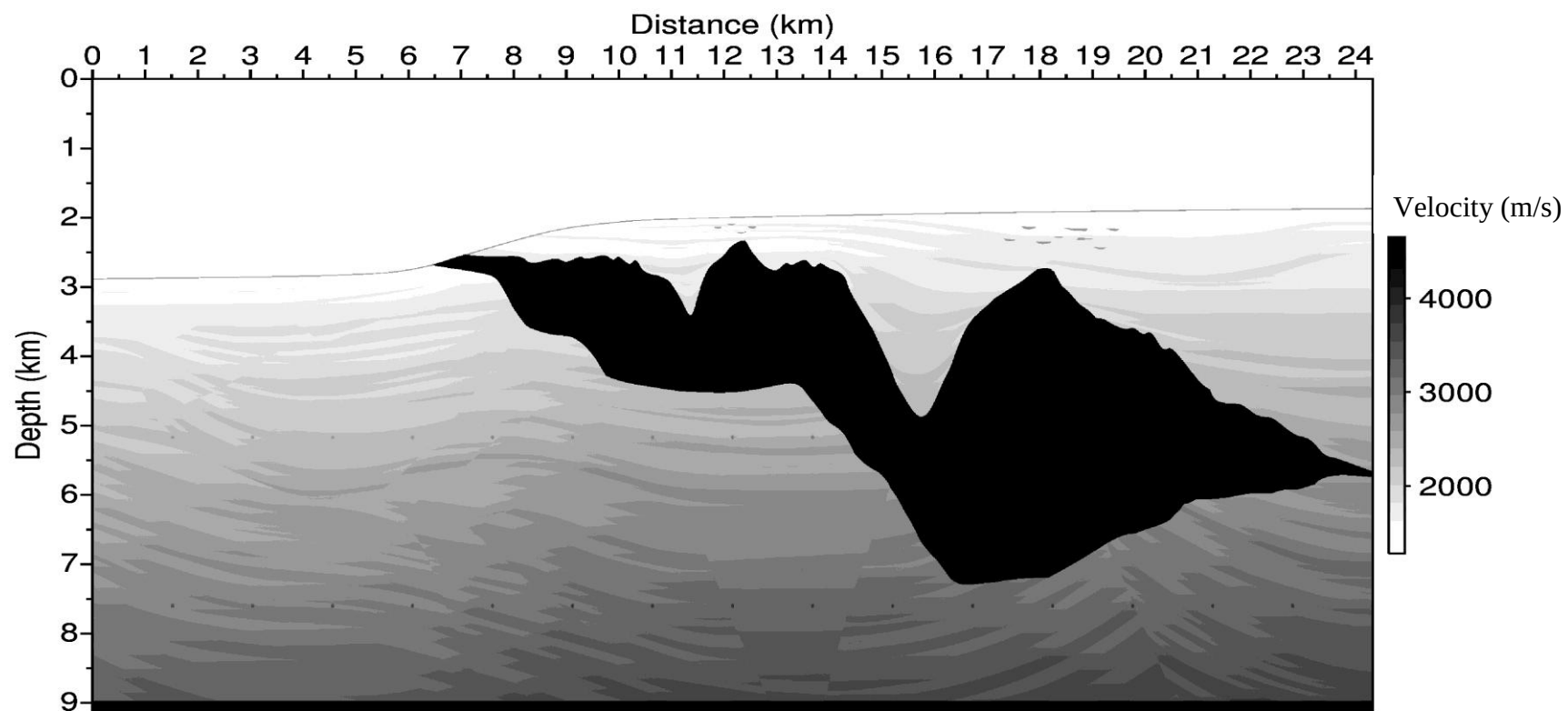
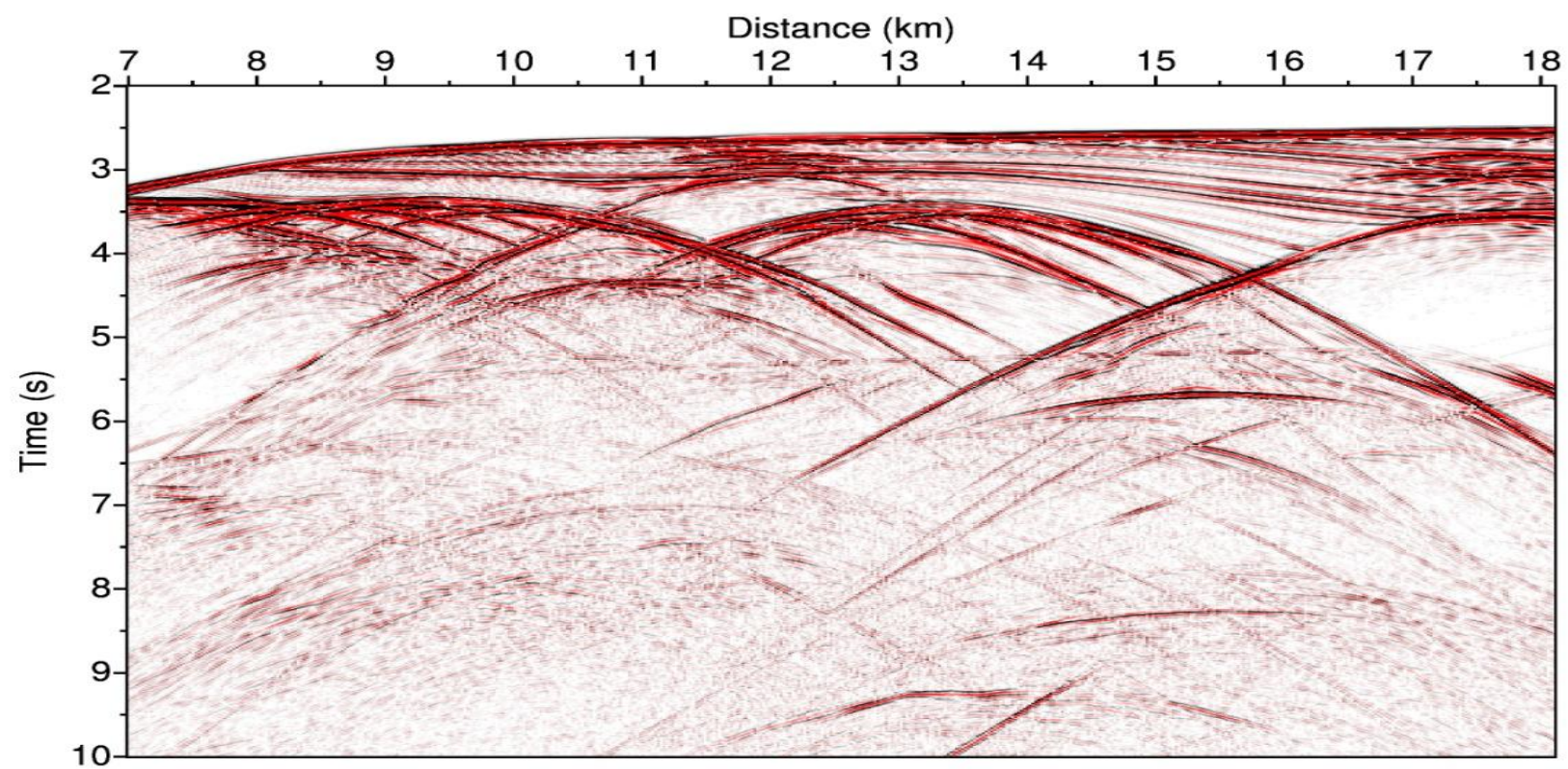
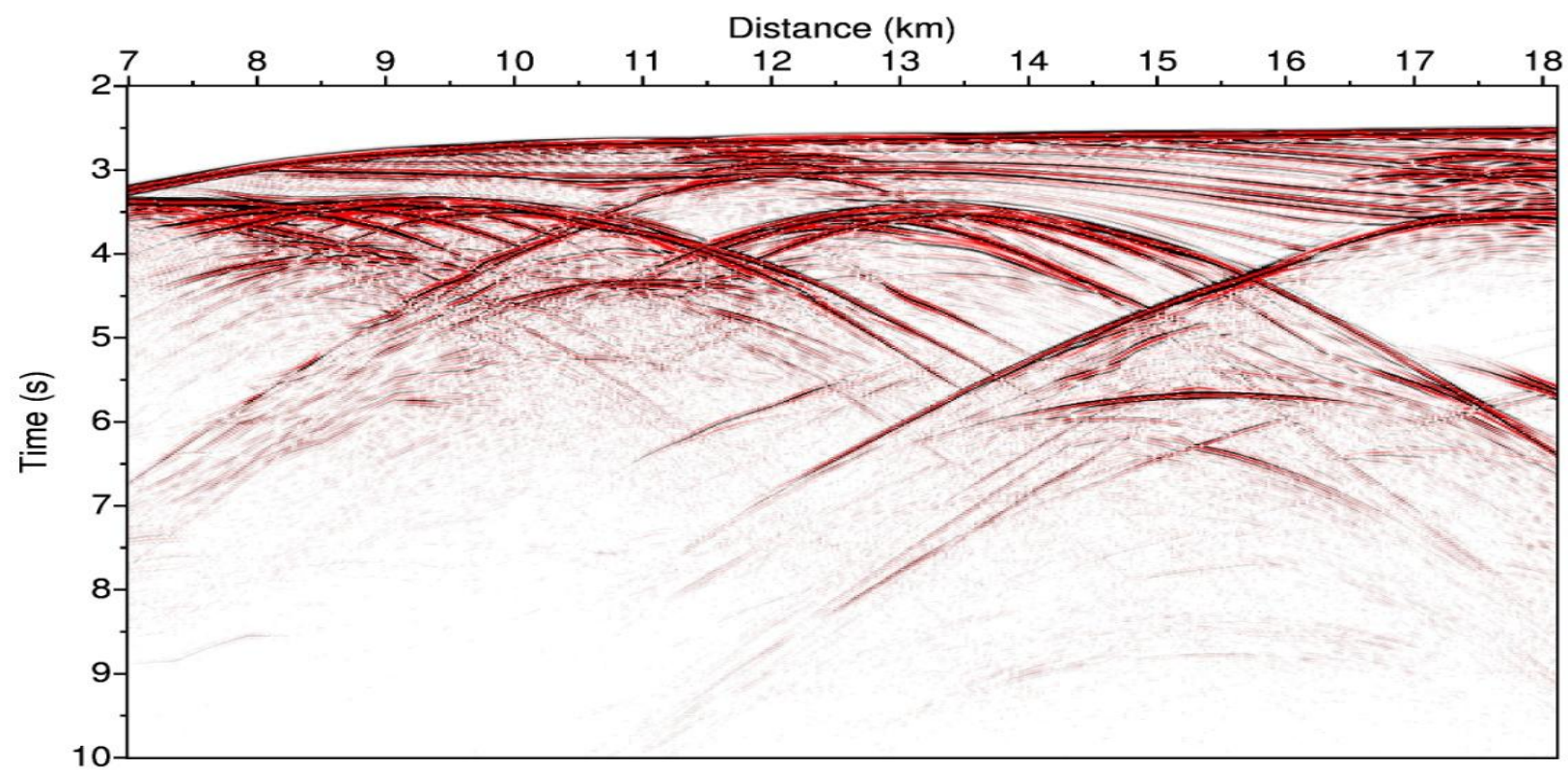


Figure 5.10 Interval velocity model of the Sigsbee2B dataset.



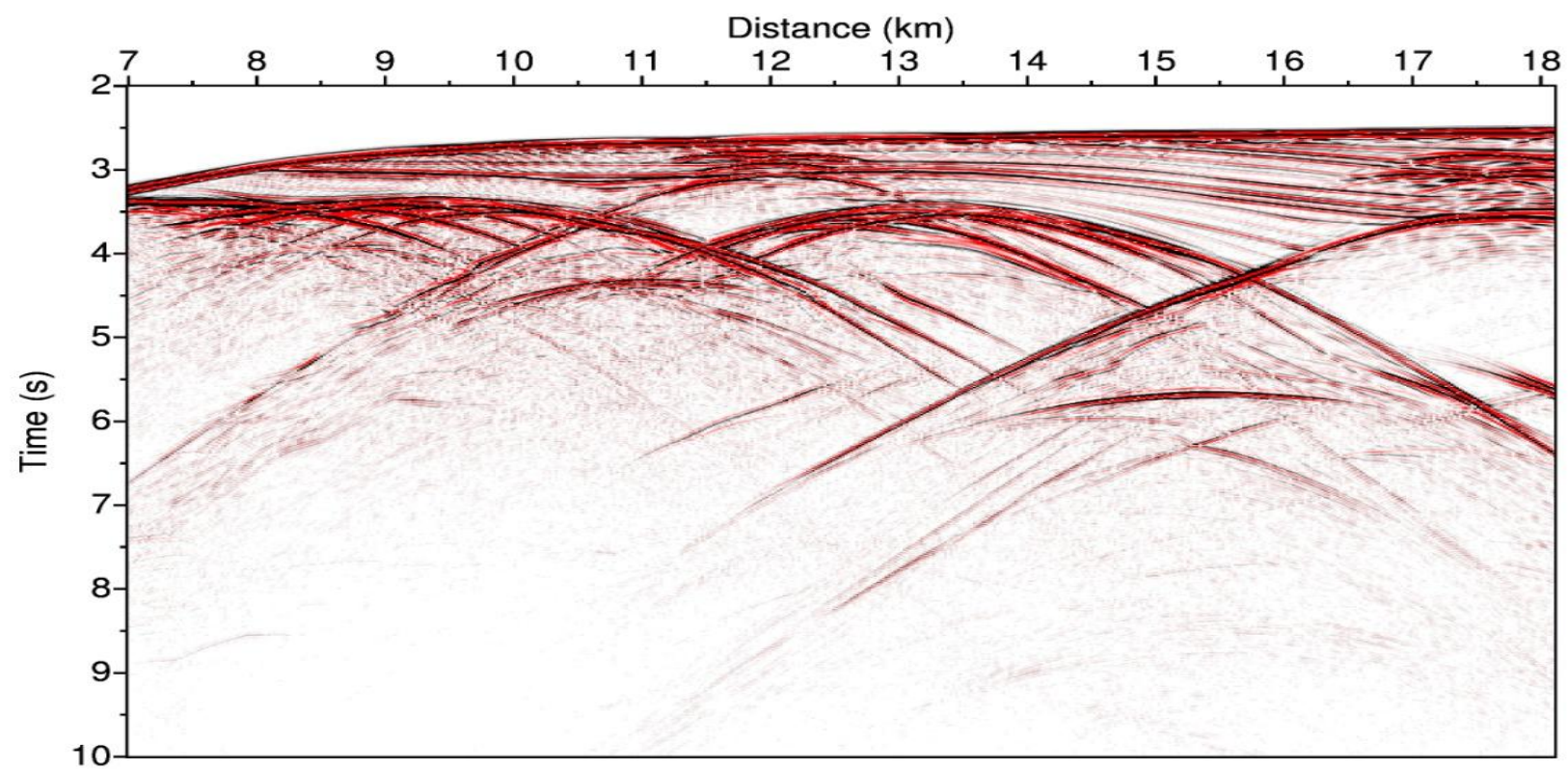
(a)

Figure 5.11 (a) The stack profile (The velocities for stacking are picked through velocity analyst).



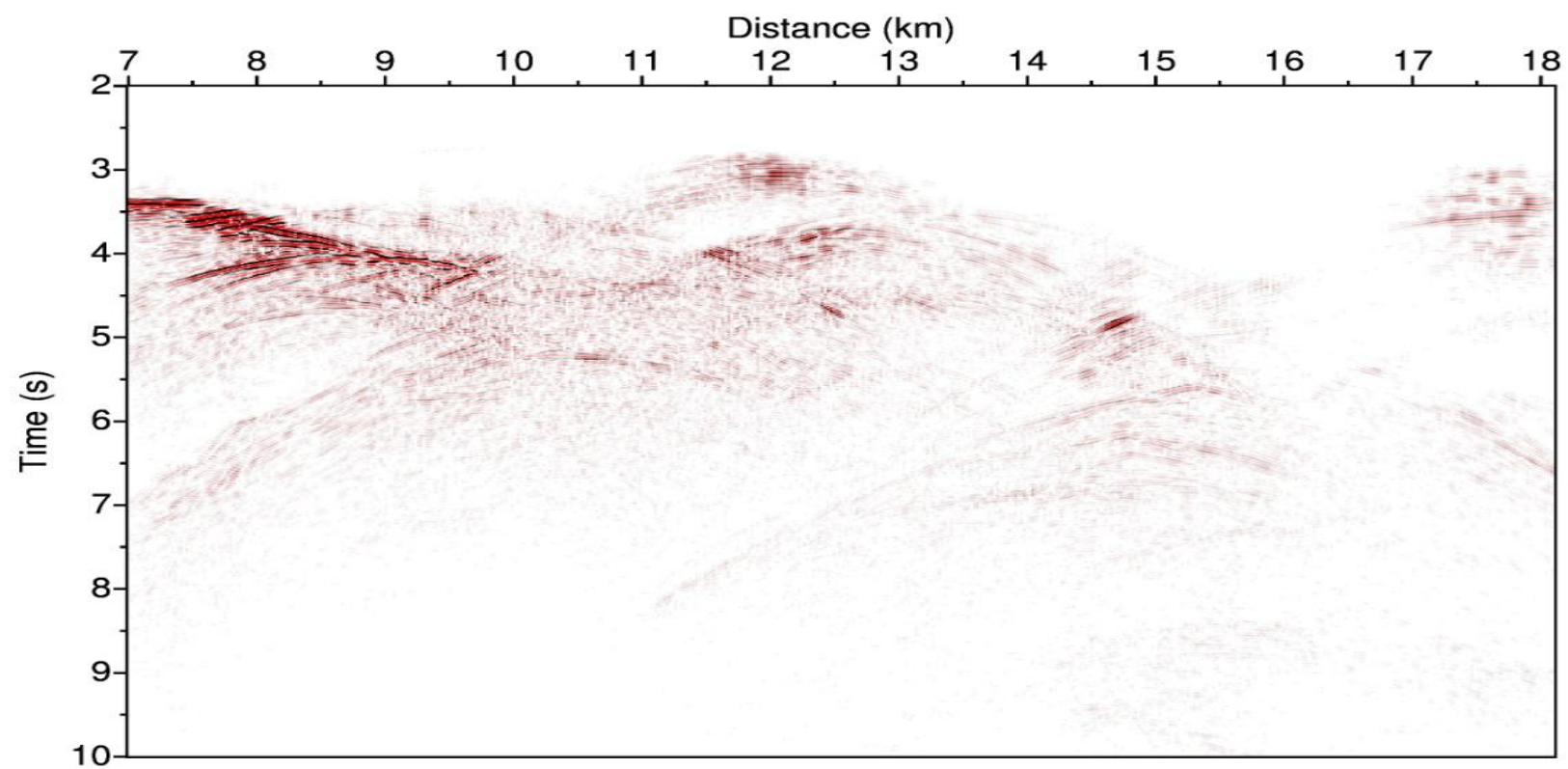
(b)

Figure 5.11 (b) The stack without free-surface multiples.



(c)

Figure 5.11 (c) The internal multiple attenuation result.



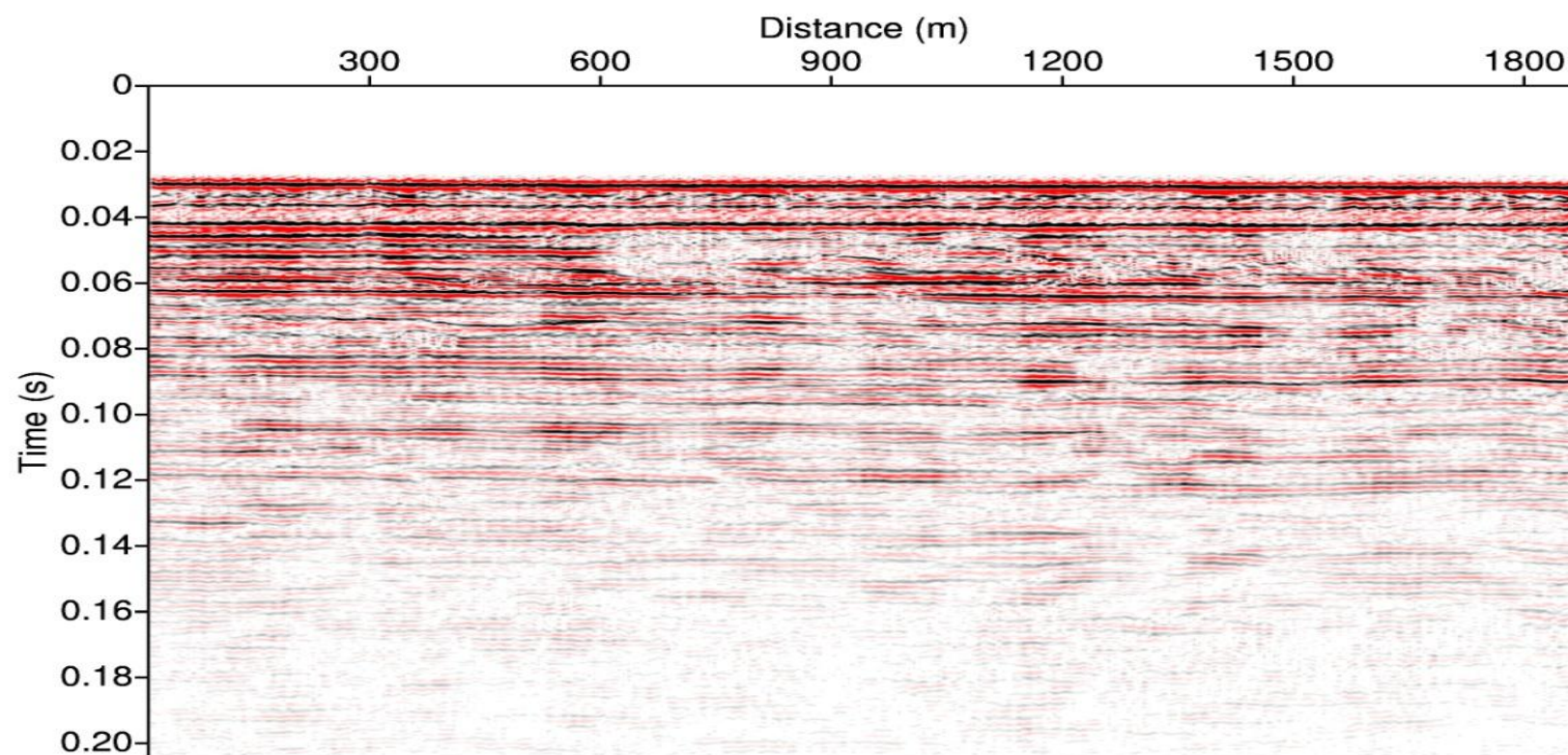
(d)

Figure 5.11 (d) Internal multiples attenuated from the data set.

5.4.4 FIELD DATA EXAMPLE

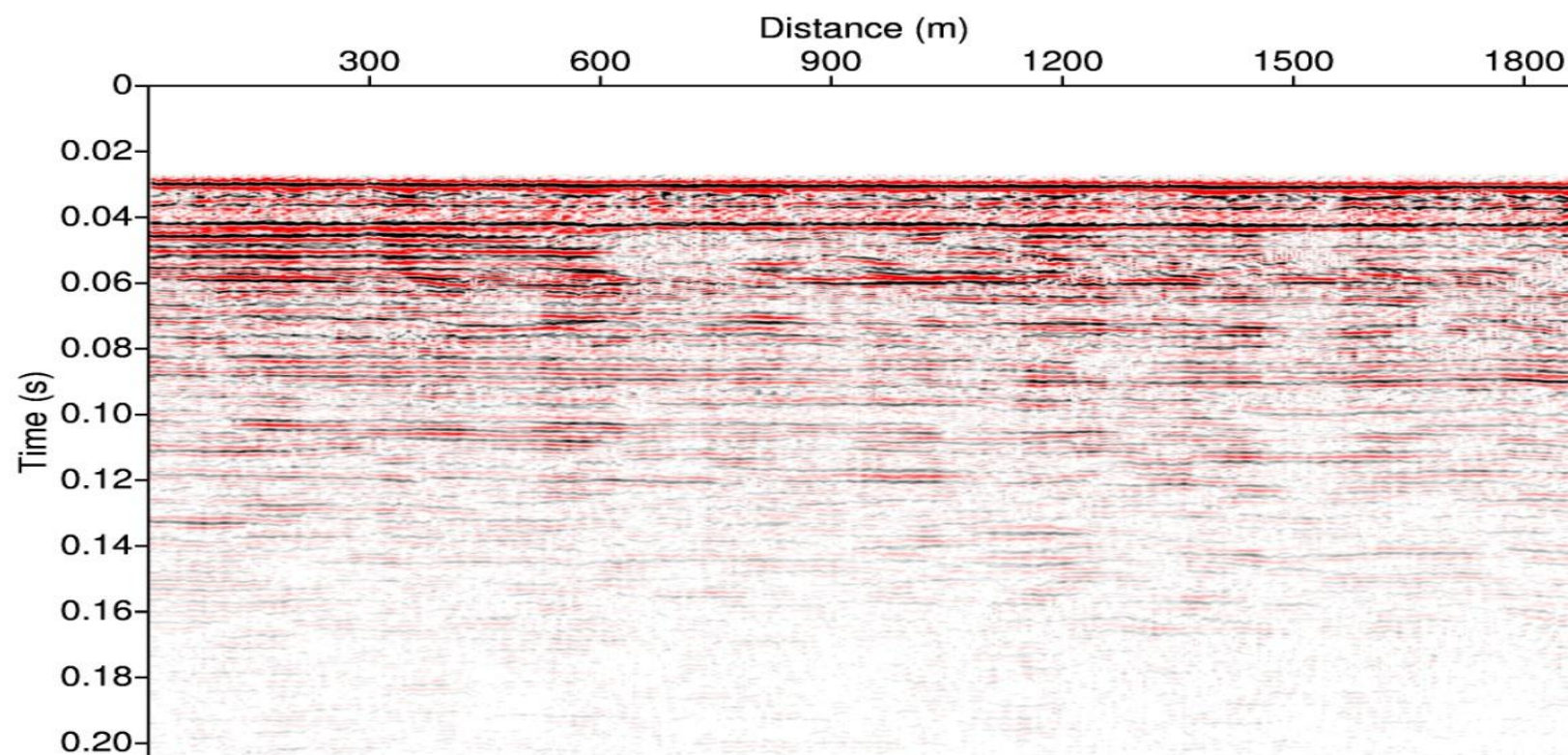
In this section I apply MPI to the attenuation of internal multiples in a real 2D marine data set acquired for an engineering project (Appendix A.6). These data consist of 270 shot gathers with 24 receivers in each gather. The source interval of each shot is 9.75 m and the receiver interval in each shot is 3.25 m. The minimum source-receiver offset is 9.75 m. Figure 5.13 shows a brute stack section (Figure 5.13a) and the result of after free-surface multiple attenuation (Figure 5.13b) using the moveout method of Wang (2003).

Next I consider the internal multiples generated by the sea floor using the correlation method as the initial step in MPI. As the shot interval is not equal to the receiver interval, we need to perform an interpolation before multiple modelling. I used the multilevel B-spline interpolation method (Chapter 3) to interpolate the data. Figure 5.13 c shows the stack section after using two iterations of MPI to attenuate sea-floor-related internal multiples, and Figure 5.13d shows the energy actually attenuated from the data. After the internal multiples have been attenuated, we are able to identify the weak primary reflection events in the deeper portion of the data.



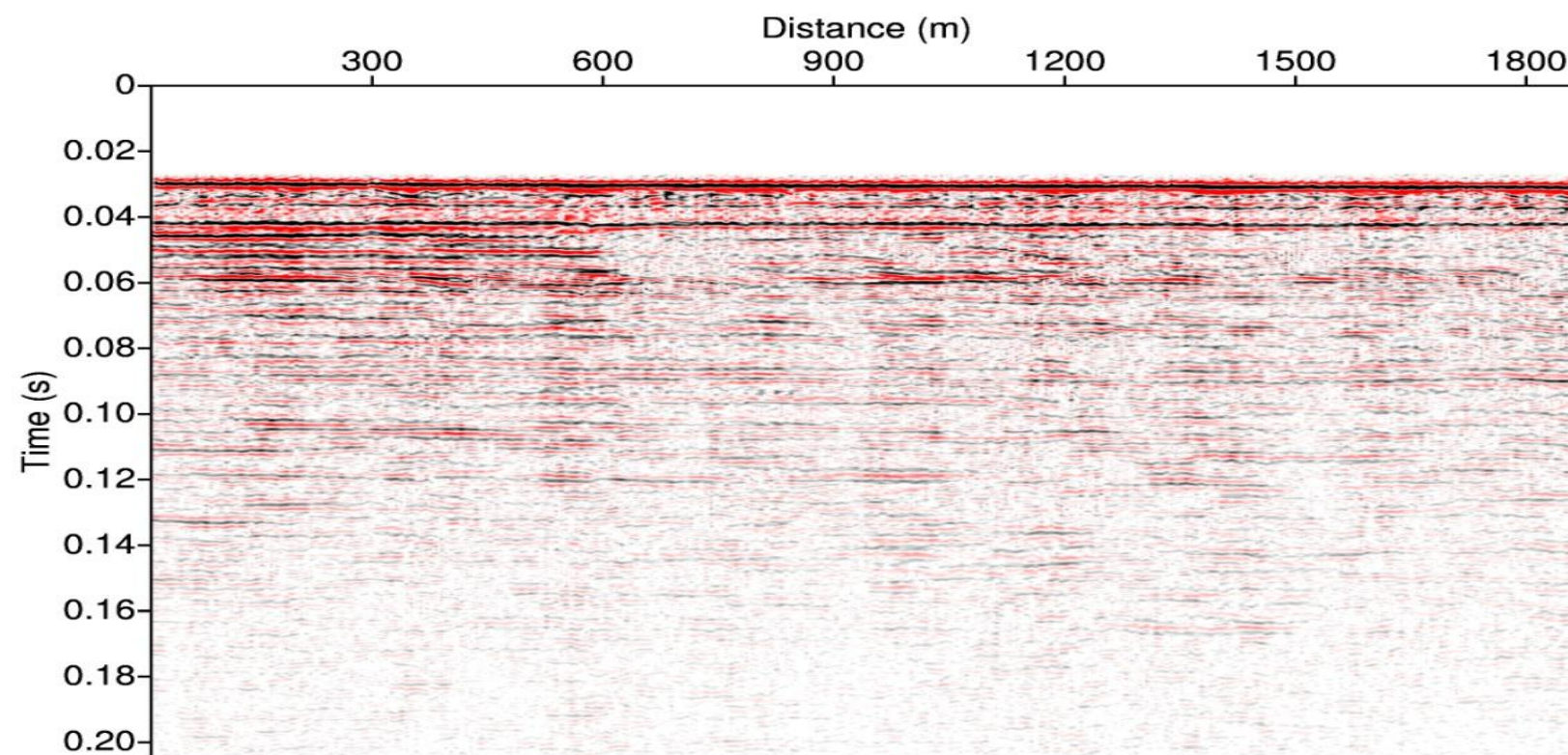
(a)

Figure 5.13 (a) Marine data set shown in the stack domain.



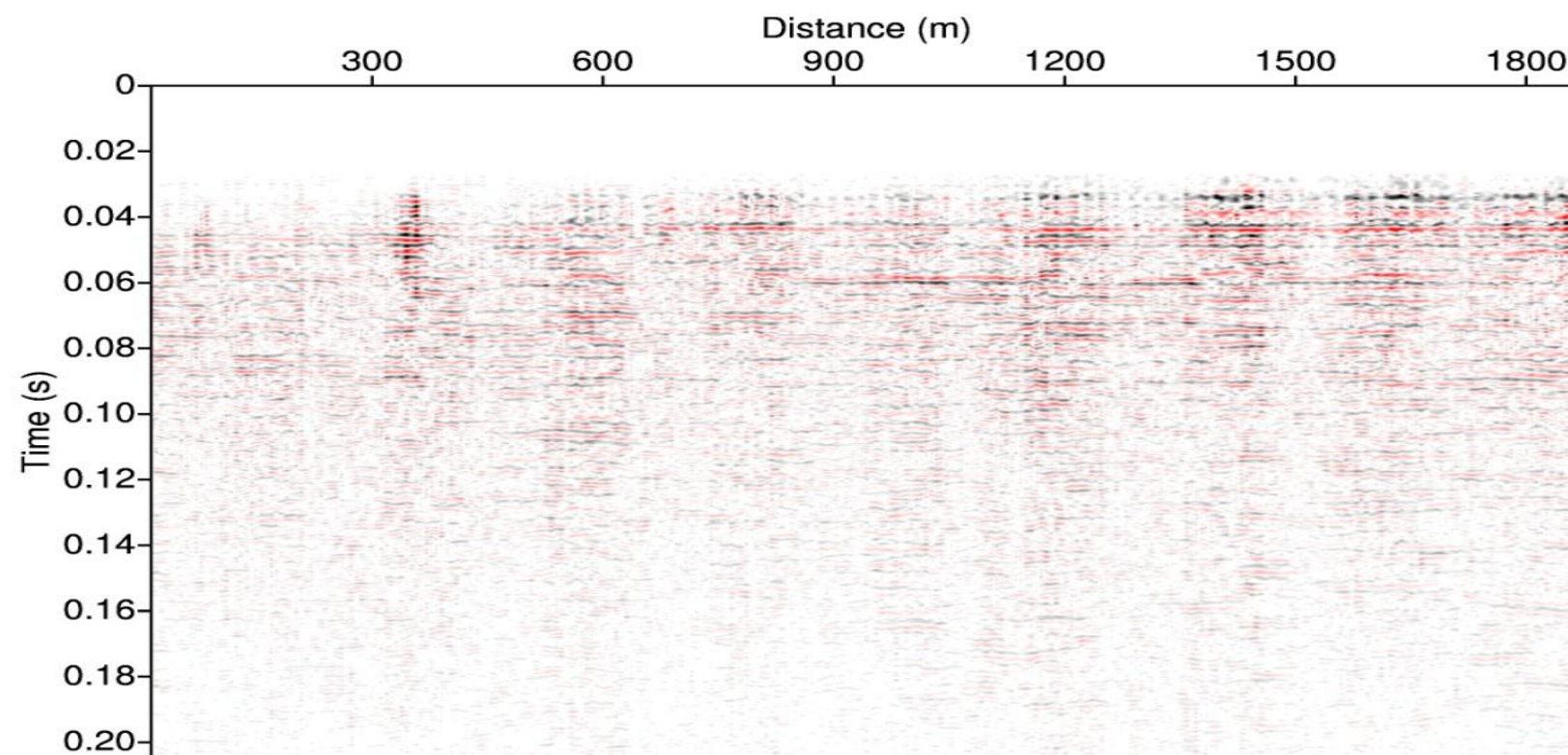
(b)

Figure 5.13 (b) The free-surface multiple attenuation result using the moveout method of Wang (2003).



(c)

Figure 5.13 (c) The internal multiple attenuation result obtained using MPI.



(d)

Figure 5.13 (d) Difference plot showing the multiples which have been removed.

5.5 CONCLUSIONS

I have successfully extended the MPI scheme from free-surface multiple attenuation to internal multiple attenuation. In comparison to other internal multiple attenuation methods that are extensions of SRME such as CFP method and correlation method, the MPI scheme can eliminate requirement for near offsets in the same way as for free-surface multiple prediction. Furthermore, there is no explicit internal operator or muting process needed in MPI for the multiple modelling. The models predicted by MPI are also accurate both kinematically and dynamically relative to the SRME extension methods.

After obtaining an initial attenuation result, we have used MPI to improve the result iteratively. Comparing the different methods used as the initial processing step in MPI, the correlation method is more practical for 2D datasets because it only involves seismic data with sources and receivers located at the surface.

Chapter 6 AN INVERSE DATA SPACE METHOD FOR INTERNAL MULTIPLE ATTENUATION

6.1 INTRODUCTION

The internal multiple attenuation methods described in Chapter 5 are based on the so-called data matrix $\mathbf{P}$, with each column of the data matrix representing a common-shot gather and each row of the matrix representing a common-receiver gather. In comparison, Berkhout (2006) introduced an inverse data space method for attenuating multiples by replacing the data matrix $\mathbf{P}$ by its inverse $\mathbf{P}^{-1}$. In the inverse data space, primaries and multiples have a much simpler relationship than in the forward space. In fact, multiples are concentrated in a very small area in the inverse data space so that they could be easily removed by a simple muting.

Luo (2007) introduced a fully data-driven inverse data space method based on the correlation principle (Jakubowicz, 1998) for internal multiple attenuation in post-stack domain. In this chapter, I implement this correlation based method in the pre-stack domain on CMP gathers as a 1.5D approach and integrate it into the MPI scheme.

6.2 MULTIPLE ATTENUATION IN THE INVERSE DATA SPACE

The conventional SRME method (Berkhout, 1982; Berkhout and Verschuur, 1997) can be expressed as:

$$\mathbf{P} = [\mathbf{I} - \mathbf{P}_0 \mathbf{A}]^{-1} \mathbf{P}_0, \quad (6.1)$$

where the surface operator $\mathbf{A}$ is defined as:

$$\mathbf{A} = [\mathbf{S}]^{-1} \mathbf{R} [\mathbf{D}]^{-1}, \quad (6.2)$$

where $\mathbf{S}$ and $\mathbf{D}$ describe the source and receiver properties, and $\mathbf{R}$ contains the surface reflectivity.

From Equation (6.1), the formulation for free-surface multiple attenuation in the inverse data space can be derived as (Berkhout, 2006):

$$\mathbf{P}^{-1} = \mathbf{P}_0^{-1} - \mathbf{A}. \quad (6.3)$$

Equation (6.3) shows that primaries and multiples have a simple relationship in the inverse data space. The inverse surface-multiple-free response $\mathbf{P}_0^{-1}$ should be primarily located at negative times, as the inversion process transforms the poles in the reverberant forward data to zeros in the non-reverberant inverse data (Berkhout, 2006). The surface operator $\mathbf{A}$ could be found at and around zero time in the inverse data space (Berkhout, 2006). As a result, the demultiple process in the inverse data space becomes a simple muting process by removing the surface operator $\mathbf{A}$ around time zero (Berkhout, 2006).

In practice, the calculation of the inverse matrix is difficult to accomplish in a stable manner if the main diagonal elements of the matrix become very small. In addition, there are two inverse matrix calculations needed, i.e. from $\mathbf{P}$ to $\mathbf{P}^{-1}$ and from $\mathbf{P}_0^{-1}$ to $\mathbf{P}_0$, which makes the implementation of this approach computationally very expensive. Verschuur (2009) proposed to redefine the inverse process as a parameter estimation problem so that the inverse data are calculated via an iterative process that only involves forward convolutions to resolve the instability. Ma *et al.* (2009) proposed to implement the inverse data space method for the free-surface multiple attenuation in the plane-wave domain, where the data can be compressed to reduce computation costs as well as stabilise the inversion process.

The effectiveness of free-surface multiple attenuation in the inverse data space can be simply demonstrated with a single trace in a 1D model. In this case, the matrices in Equation 6.3 become complex numbers. Figure 6.1a shows the primary

reflections $\mathbf{P}_0$ without any multiples while Figure 6.1b shows the data $\mathbf{P}$ with both primaries and free-surface multiples in the forward data space. The data $\mathbf{P}$ are then transformed from the forward data space into the inverse data space. The resulting data $\mathbf{P}^{-1}$ are shown in Figure 6.1c. It can be seen that in the inverse data space the surface operator A is located at zero time, which is enlarged in Figure 6.1d. After removing this surface operator by a simple muting process, the demultiple result in the inverse data space $\mathbf{P}_0^{-1}$ was obtained (Figure 6.1e). After another inverse process the demultiple result P_0 in the forward data space is shown in Figure 6.1f, where the free-surface multiples are successfully eliminated.

6.3 THE INVERSE DATA SPACE METHOD FOR INTERNAL MULTIPLE ATTENUATION

Berkhout and Verschuur (2007) applied the inverse data space method for the internal multiple attenuation using a downward extrapolation to move the data to the internal-multiple-generating interface. Luo *et al.* (2007) applied a fully 1-D data-driven method based on the correlation principle (Jakubowicz, 1998) to attenuate internal multiples in the inverse data space in the post-stack domain. As an improvement, in this chapter, I implement this principle in pre-stack domain using CMP gathers and a 1.5D approach.

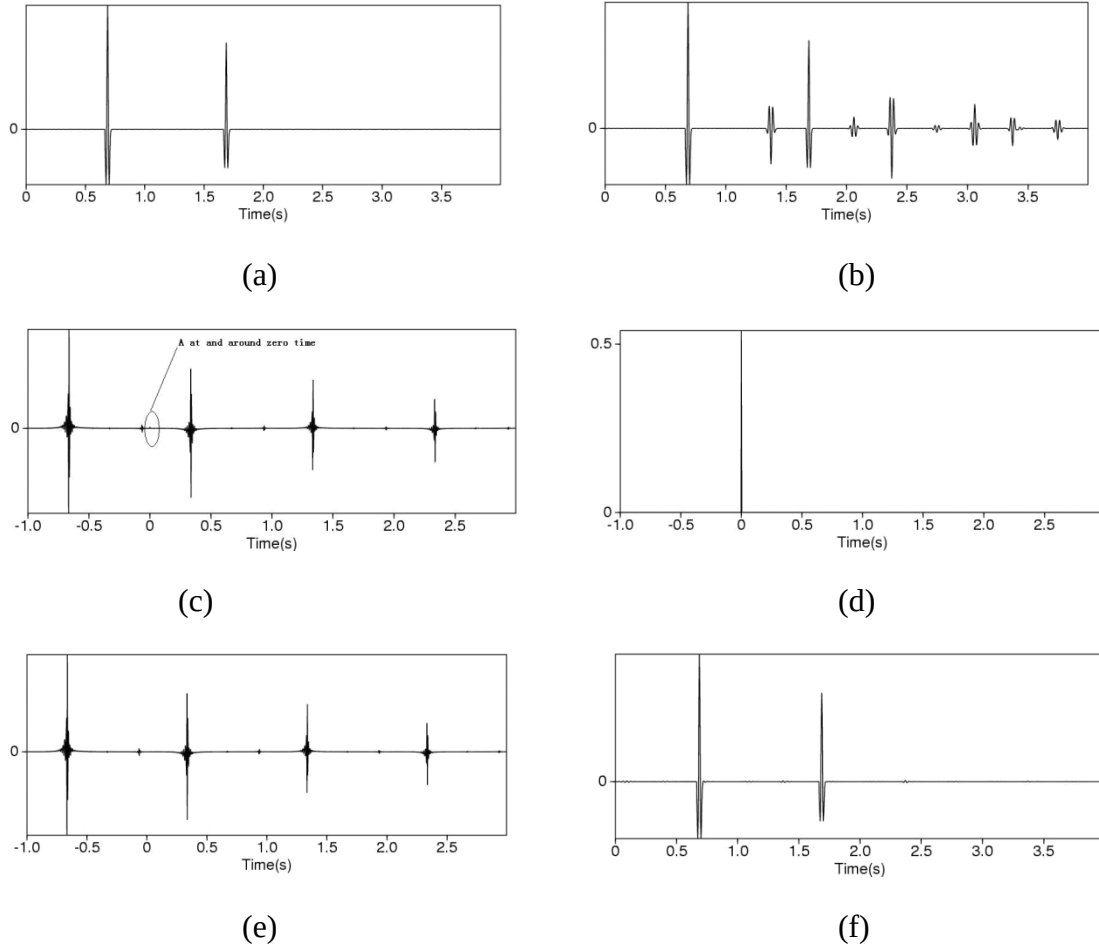


Figure 6.1 (a) Primary reflections P_0 in a 1D model with two reflectors. (b) Primary reflections and free-surface multiples (P) in the forward data space. (c) The data record in the inverse data space P^{-1} (d) The enlarged surface operator A in the inverse data space. (e) The demultiple result P_0^{-1} in the inverse data space. (f) The demultiple result P_0 in the forward data space.

For the multiple modeling based on the data matrix $\mathbf{P}$, assuming a low-relief earth and that the earth is laterally homogeneous, all shot records can be considered identical so that the spatial convolution between the different shots can be replaced by single complex multiplication of the data from one shot in the wavenumber-frequency domain. This method is best applied in CMP gathers because this relaxes the horizontally homogeneous assumption (Kelamis and Verschuur, 2000). In this case, applying the inverse data space method in CMP gathers means the calculation of the inverse data matrix in the frequency-space domain is reduced to that of obtaining the inverse of single complex numbers in the frequency-wavenumber domain.

The data-driven method of Jakubowicz (1998) for the internal-multiple prediction can be expressed as:

$$\bar{\mathbf{P}}_k = \bar{\mathbf{P}}_{k-1} + \bar{\mathbf{P}}_{k-1}[\Delta\mathbf{P}_k]^H \mathbf{A}\bar{\mathbf{P}}_k, \quad (6.4)$$

where z_k is the internal-multiple generating interface, $\mathbf{P}_k$ and $\mathbf{P}_{k-1}$ are the wavefields, in which multiples generated by interface z_k and all above interfaces have been removed, the top bar over a matrix indicates that all primaries related to the current interface and above have been muted, $\Delta\mathbf{P}_k$ is the primary reflections related to interface z_k and the superscript H means transposition and complex conjugation. When implementing Equation 6.8 in CMP gathers, the matrix is reduced to single complex numbers in the frequency-wavenumber domain, where the muted wavefield $\bar{p}_k$ can be expressed as:

$$\bar{p}_k = \bar{p}_{k-1} + \bar{p}_{k-1}\Delta p_k^* A\bar{p}_k, \quad (6.5)$$

where $\bar{p}_k$, $\bar{p}_{k-1}$ and Δp_k^* are all single complex numbers and $*$ denotes complex conjugation of Δp_k . Dividing both sides of Equation 6.5 by $\bar{p}_{k-1}\Delta p_k^* \bar{p}_k$, we obtain

$$(\bar{p}_{k-1}\Delta p_k^*)^{-1} = (\bar{p}_k\Delta p_k^*)^{-1} + A. \quad (6.6)$$

Equation 6.6 describes the relationship between primaries and internal multiples in the inverse data space. $(\bar{p}_{k-1}\Delta p_k^*)^{-1}$ is the wavefield before attenuating the internal multiples generated by interface k while $(\bar{p}_k\Delta p_k^*)^{-1}$ is the wavefield after attenuating the related internal multiples. The difference between $(\bar{p}_{k-1}\Delta p_k^*)^{-1}$ and $(\bar{p}_k\Delta p_k^*)^{-1}$ is the operator A and corresponds to the k th interface generating internal multiples in the inverse data space. This operator A still includes the information of source signature. Based on Equation 6.6, internal multiples can be attenuated in the inverse data space in the CMP domain, where the calculation of the inverse data matrix in the frequency-space domain is reduced to calculate the inverse of single complex numbers. However, it should be noted that a stabilisation is still needed to calculate the inverse of the single complex numbers.

6.4 SYNTHETIC EXAMPLE

As an illustration, I use a synthetic subsurface model consisting of three flat interfaces at the depth of 600, 900 and 1800 m with velocities of 1500, 2000 and 3000 m/s, respectively.

Figure 6.2a is a CMP gather containing three primary reflections and all internal multiples. To attenuate the internal multiples generated by the first interface, the primary reflections generated from the first interface (Δp_1) were isolated from the CMP gather, as shown in Figure 6.2b. Figure 6.2c is the resulting muted CMP gather ($\bar{p}_0$). Using Equation 6.6, $(\bar{p}_0\Delta p_1^*)^{-1}$ can be calculated in the inverse data space (Figure 6.2d). A zoomed section of this temporary data in the inverse data space is shown in Figure 6.2e. It can be seen that the energy located around zero time is the operator A which is related to internal multiples generated by the first interface in the inverse data space. After muting the energy around the zero time, the resulting $(\bar{p}_1\Delta p_1^*)^{-1}$ was obtained (Figure 6.2f), and following multiplication with Δp_1^* and

another inverse process, the demultiple result $\bar{p}_1$ can be obtained (Figure 6.2g). It can be seen that the internal multiples generated by the first interface were attenuated effectively.

In Figure 6.2g, some residual internal multiples generated by the first interface can still be seen. Therefore, as we already have the initial result, as suggested in Chapter 5, we can apply the MPI scheme to improve it. Figure 6.3a shows the internal multiple model predicted by MPI and Figure 6.3b is the corresponding demultiple result after subtraction from the original CMP gather. It can be seen that the internal multiple generated by the first interface is better attenuated after applying MPI. The remaining internal multiples generated by the second interface can be suppressed by the same procedure.

6.5 CONCLUSIONS

The synthetic example demonstrates the effectiveness of the fully data-driven inverse data space method for internal-multiple attenuation implemented in CMP domain. This method can be applied to a data set where the approximation of a low-relief earth is valid and the earth is laterally homogeneous.

As an alternative approach to start MPI, this method can also be as the initial step in MPI. With the initial result, the MPI scheme can then be applied to improve the result iteratively.

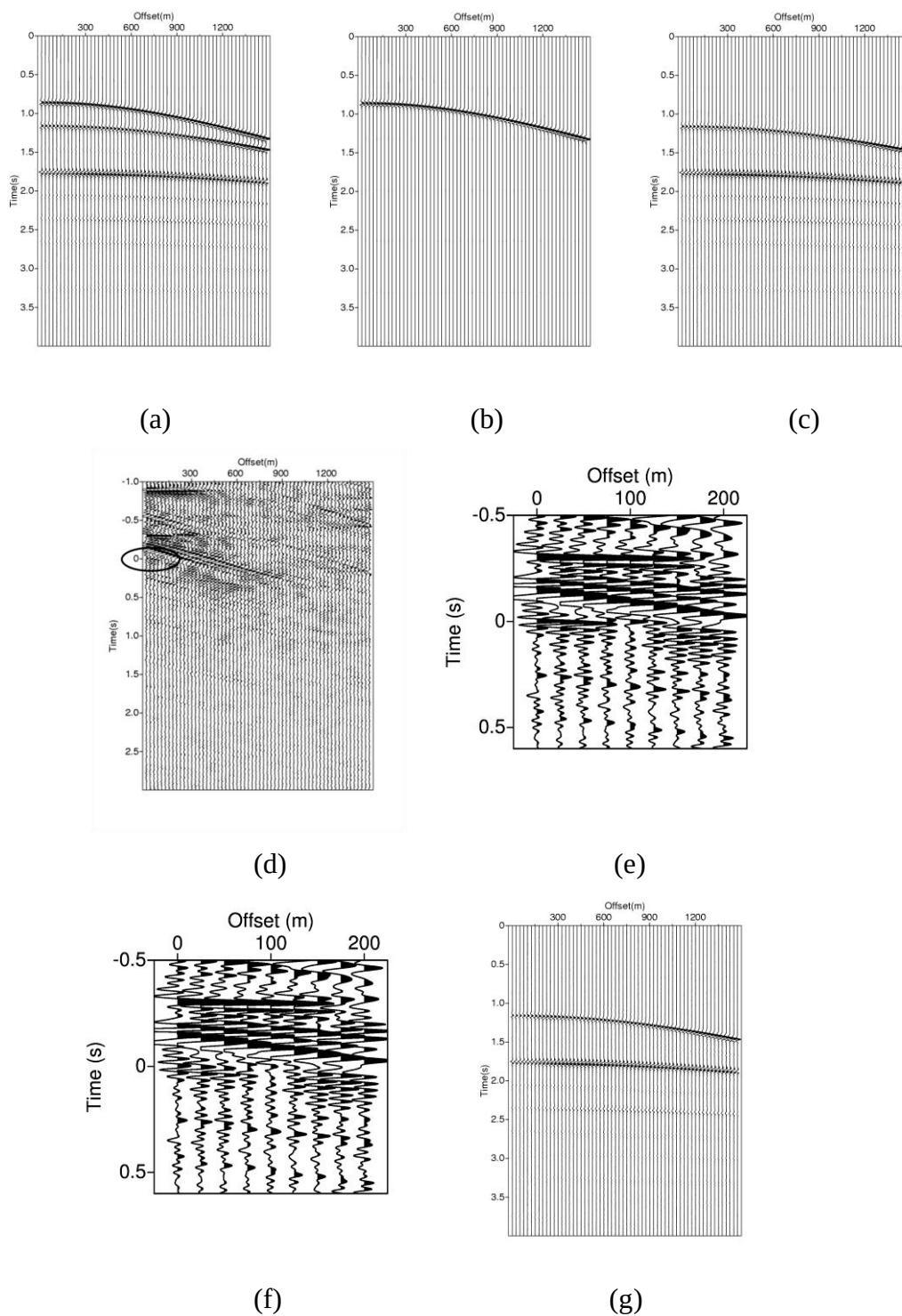


Figure 6.2 The inverse data space method: (a) CMP gather containing primaries and all internal multiples in the forward data space. (b) The isolated primaries from the first interface. (c) The muted CMP gather. (d) The temporary data in the inverse data space. (e) The temporary data in the inverse data space after muting the operator A around the zero time. (f) The muted CMP gather after attenuating internal multiples generated by the first interface.

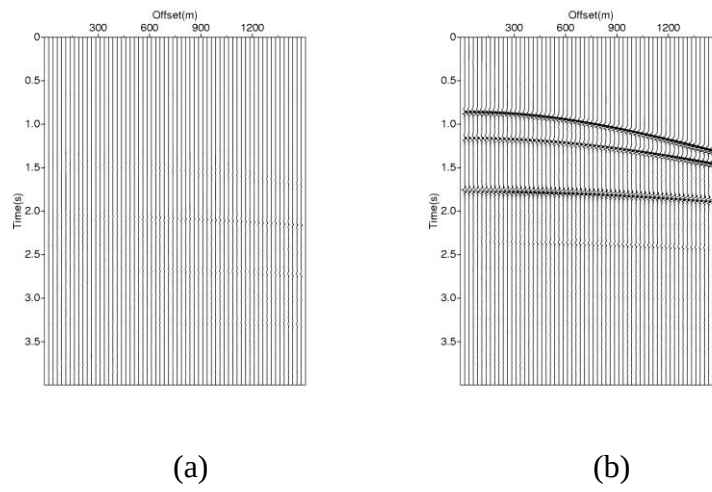


Figure 6.3 MPI method: (a) The model of internal multiples generated by the MPI scheme. (b) The multiple-attenuation result.

Chapter 7 CONCLUSIONS

7.1 SUMMARY

The strategy of my research for seismic multiple attenuation is based on the “multiple prediction through inversion”. The innovative point of MPI is to apply the concept of inversion into seismic processing. Based on this concept, this thesis treats MPI as a general workflow for seismic multiple attenuation, which involves both free-surface multiple attenuation methods and internal multiple attenuation methods implemented in forward data space and inverse data space respectively. The same basic idea is to treat seismic multiple attenuation as an optimisation problem, in which the cost function is optimised from one level to the next by updating the current results in each iteration, as in a general inverse problem.

To apply MPI in practice, I first develop a multilevel B-spline method for seismic data regularisation and interpolation. Then MPI is applied for free-surface multiple attenuation on real marine data, where superior results are obtained. However, the main limitation of applying MPI in real data is the high computation cost. This can be addressed using the highly parallel structure of a GPU. With the aid of a GPU, not only can I accelerate the processing of MPI, but can also improve the multiple attenuation results in far-offset traces by solving the prediction operator $\mathbf{T}$ in full. Finally, the improved MPI scheme with GPU acceleration is extended to deal with the problem of internal multiple attenuation. Several methods are compared as the initial step to start MPI. In the conventional forward data space, the correlation method is preferred as it involves surface data only. As an alternative, the correlation method is also implemented in the inverse data space as a 1.5D approach.

In all cases, the MPI scheme generates multiple models with both kinematically and dynamically improved accuracy. The free-surface and internal multiple attenuation results based on MPI demonstrate that the idea of applying the concept of

inversion into seismic processing is realistic, and encourages the possibility of using inversion methodology in other seismic processing steps.

7.2 PERSPECTIVES

7.2.1 IMPROVING SEISMIC MULTIPLE ATTENUATION RESULTS WITH MPI

Following this work, I suggest more effort is directed at the adaptive subtraction stage of the routine two-step wave-equation seismic multiple attenuation methods. The performance of MPI depends on the initial de-multiple results. If multiples strongly interfere with primaries and some primary reflections have been subtracted in the initial processing, they would be brought to the next iteration as multiples. Although the MPI scheme with EMCM could reduce damage to primaries to some extent, an improved adaptive subtraction method which can better preserve primaries is still desired. The pattern-based subtraction approach is a possible route for improvement, but needs to more study on real data applications.

In 3D, due to the sparse sampling in the cross-line direction, the MPI is currently implemented along the in-line direction only. In future, using new acquisition techniques, or proper interpolation methods or inversion based methods to estimate missing records in the cross-line direction, MPI should be implemented as a full 3D approach.

For internal multiple attenuation, the interpretation of primaries generated from internal-multiple-generating interfaces needs more attention, because accurate isolation of primary reflections is essential for both the CFP and correlation methods as a starting point for MPI. It is also difficult to identify internal multiples in real data sets. We still need proper datasets with prior knowledge of subsurface structure and tests to prove the internal multiple attenuation procedure.

As an alternative, the proposed inverse data space method could be a starting point for MPI and should be tested on real data sets where the approximation of a

low-relief earth is valid and the earth can be considered as laterally homogeneous. As there are inversion processes in the inverse data space method, it is important to use stabilisation for this to be successful.

7.2.2 ESTIMATION OF PRIMARIES BY SPARSE INVERSION

The estimation of primaries by sparse inversion (EPSI) method estimates the primaries as unknowns in a multidimensional inversion process rather than by a subtraction process. It is also an inversion approach and worth further investigation in comparison with MPI.

An interesting aspect of the EPSI method is that it produces a residual, which is the part of the input data not explained by primaries and multiples. This residual can be analysed and may provide useful information on the primary estimation process. Furthermore, EPSI does not depend on interpolated missing near-offset data because it can reconstruct missing data simultaneously.

7.2.3 USING SEISMIC MULTIPLES

Seismic multiples always have complicated ray-paths. At the same time, this means that multiples also contain more information on the subsurface structures and properties. It is therefore worthwhile considering whether free-surface and internal multiples can be used for other processing steps or interpretation. For example, internal multiples might be useful for imaging the structure below salt.

In the inverse data space, multiple energies are represented as the surface or internal operator $\mathbf{A}$, and this can be isolated more easily than in the forward data space. The operator $\mathbf{A}$ should be of interest, as it contains much information, such as source and receiver properties and surface or interface reflectivity.

7.2.4 HIGH PERFORMANCE COMPUTING IN GEOPHYSICS

Multiple modelling dominates in the computational cost of multiple attenuation processing, but currently only matrix multiplication and solving linear system in the

multiple modelling for MPI have been implemented on a GPU. In future, other seismic processing steps, e.g. FFT and EMCM, could also be sped up using a GPU.

MPI is based on the concept of inversion, which is implemented through solving linear equations. Therefore, other geophysical algorithms based on inversion, such as full waveform inversion, could also be implemented on a GPU.

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Appendix A: DATA ACQUISITION PARAMETERS

A.1 2D MARINE DATASET 1

Location	South China Sea
No. of receiver	600
Device	SYNTRAK 960
Receiver interval	12.5 m
Source interval	37.5 m
No. of fold	100
Sample interval	4 ms
Record length	12288 ms
Depth of receiver	6 m
Depth of source	5 m
Low frequency filter	3 HZ/6 db
High frequency filter	206 HZ/ 276 db
No. of shots	2708
Minimum offset	210 m
Distance	100 km

Table A.1 Acquisition parameters for 2D marine Dataset 1.

A.2 2D MARINE DATASET 2

Location	South China Sea
No. of receiver	180
Device	LRS-16A
Receiver interval	13.3 m
Source interval	26.7 m
No. of fold	45
Sample interval	4 ms
Record length	8192 ms
Depth of receiver	9 m
Depth of source	7 m
Low frequency filter	6 HZ/18 db
High frequency filter	188 HZ/156 db
No. of shots	3766
Minimum offset	206 m
Distance	100 km

Table A.2 Acquisition parameters for 2D marine Dataset 2.

A.3 2D MARINE DATASET 3

Location	South China Sea
No. of receiver	180
Device	LRS-16A
Receiver interval	13.33 m
Source interval	26.7 m
No. of fold	45
Sample interval	4 ms
Record length	8192 ms
Depth of receiver	9 m
Depth of source	7 m
Low frequency filter	6 HZ/18 db
High frequency filter	188 HZ/156 db
No. of shots	4200
Minimum offset	206 m
Distance	110 km

Table A.3 Acquisition parameters for 2D marine Dataset 3.

A.4 3D MARINE DATASET

Location	South China Sea
No. of receiver/streamer	120
No. of Streamer	6
Crossline offset	100 m
Receiver interval	25 m
Source interval	25 m
No. of fold	60
Sample interval	4 ms
Record length	8192 ms
No. of shots	1200
Minimum offset	200 m
Distance	30 km

Table A.4 Acquisition parameters for 3D marine dataset.

A.5 SIGSBEE2B DATASET

No. of receiver	348
Receiver interval	22.5 m
Source interval	45 m
No. of fold	87
Sample interval	8 ms
Record length	12 s
Depth of receiver	8 m
Depth of source	8 m
No. of shots	500
Minimum offset	0 m
Distance	23 km

Table A.5 Acquisition parameters for Sigsbee2B.

A.6 2D MARINE DATASET 4

No. of receiver	24
Receiver interval	3.25 m
Source interval	9.75 m
Sample interval	0.2 ms
Record length	0.2 s
No. of shots	270
Minimum offset	9.75 m
Distance	2600 m

Table A.6 Acquisition parameters for 2D marine Dataset 4

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