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TESTING THE EFFECTS OF EXPANSION ON SOLAR WIND TURBULENCE

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ABSTRACT

We present a multi-spacecraft approach to test the predictions of recent studies on the effect of solar wind expansion on the radial spectral, variance, and local 3D anisotropies of the turbulence. We found that on small scales (5000–10,000 km) the power levels of the B-trace structure functions do not depend on the sampling direction with respect to the radial suggesting that on this scale the effect of expansion is small possibly due to fast turbulent timescales. On larger scales (110–135 R_E), the fluctuations of the radial magnetic field component are reduced by ~20% compared to the transverse (perpendicular to radial) ones, which could be due to expansion confining the fluctuations into the plane perpendicular to radial. For the local 3D spectral anisotropy, the B-trace structure functions showed dependence on the sampling direction with respect to radial. The anisotropy in the perpendicular plane is reduced when the increments are taken perpendicular with respect to radial, which could be an effect of expansion.

Key words: magnetohydrodynamics (MHD) – plasmas – solar wind – turbulence

1. INTRODUCTION

The solar wind is a highly conductive quasi-neutral flow in which the lack of significant viscosity leads to turbulence (e.g., Coleman 1968; Siscoe et al. 1968). Turbulence in the solar wind is anisotropic, which arises because of the magnetic field breaking the isotropic symmetry (see review by Horbury et al. 2012). Understanding this anisotropy is particularly important to reveal how the turbulent cascade operates. Previous studies made progress toward understanding the anisotropy of the turbulent eddies, particularly their three-dimensional (3D) properties (e.g., Boldyrev 2006; Chen et al. 2012). It has recently been proposed that additional mechanisms can affect the 3D nature of the anisotropy and that besides the magnetic field, the expansion of the solar wind can also give a preferred direction to the turbulence. In this letter, we present a multi-spacecraft approach to test these predictions and discuss the implications for our understanding of solar wind turbulence.

Belcher & Davis (1971) observed that on large scales most of the energy in the solar wind is in Alfvénic fluctuations. Horbury et al. (1995) found that the minimum variance directions are approximately aligned with the mean magnetic field and the perpendicular fluctuations are 30 times larger than the field aligned ones, which is consistent with the highly Alfvénic nature of the turbulence. Horbury & Balogh (2001) studied the large-scale variance anisotropy of the fluctuations and concluded that the radial component has lower power than the two perpendicular ones. This is consistent with the constraint imposed by the solenoidality of the magnetic field (Turner et al. 2011). When a single spacecraft is used, the measurements can be understood as a 1D cut along the radial direction (k_x); thus, the observed power of δB_x is reduced as compared to the two perpendicular ones.

Early studies on MHD turbulence by Iroshnikov (1963) and Kraichnan (1965) assumed isotropy and predicted the spectral index of the power spectrum to be $-3/2$. Later, Montgomery & Turner (1981) investigated MHD turbulence in the presence of a strong DC magnetic field and concluded that it might be reasonable to assume rotational isotropy with respect to the mean magnetic field, but not for the other two directions. Goldreich & Sridhar (1995) showed that the Iroshnikov-Kraichnan model is not valid and suggested that MHD turbulence can be best described with $-5/3$ spectral index. The hypothesis of Goldreich & Sridhar (1995) is known as critical balance and implies that the perpendicular wavenumber (with respect to the magnetic field) of the eddies increases at a larger rate than the parallel one, and the scaling between them is $k_\parallel \sim k_\perp^{2/3}$. The results of several solar wind turbulence studies are in agreement with this scaling (e.g., Leamon et al. 1998; Horbury et al. 2008; Podesta et al. 2009; Wicks et al. 2010, 2011; Chen et al. 2011). Boldyrev (2006) extended the critical balance model to 3D and proposed different scaling along the two perpendicular directions with respect to the magnetic field resulting in 3D anisotropic eddies. The observations of Chen et al. (2012) in the fast solar wind are consistent with the 3D shapes of Boldyrev (2006), although the 3D scaling did not match the predictions in all directions.

The recent studies of Dong et al. (2014) and Verdini & Grappin (2015) utilized an expanding box model to study the large-scale evolution of the solar wind turbulence. During the simulation, the radial dimension of the box remained constant while the other two dimensions increased gradually, corresponding to a stretching of the plasma volume. Dong et al. (2014) studied the competing effects of the mean magnetic field and the expansion on the anisotropy of the turbulence. When only a magnetic field was added to the simulation, the eddies became elongated along the field lines as predicted by the critical balance theory. In contrast, the sole effect of expansion caused stretching of the eddies along the perpendicular direction with respect to the radial. When both expansion and mean radial magnetic field were included, weak spectral anisotropy was observed along the perpendicular to radial direction. The analysis of the root mean square (rms) amplitudes of the magnetic field fluctuations showed selective damping of the components: in the expanding run, the fluctuations of the radial component decayed quicker than the two perpendicular ones as a function of the radial distance from the Sun. Dong et al. (2014) interpreted these observations as a consequence of expansion, which, unlike in the non-expanding model, causes anisotropic energy distribution among the magnetic field components.

Verdini & Grappin (2015) studied the local 3D spectral anisotropy in terms of B-trace structure functions, which are defined as $\langle \sum_i (B_{i,1} - B_{i,2})^2 \rangle$, where i is the component of the magnetic field and $B_{1,2}$ denotes the magnetic field measurements at two different locations. The structure functions were binned in a local coordinate system, in which the two special orthogonal directions are defined by the local mean magnetic field ($\boldsymbol{B}_0 = (\boldsymbol{B}_1 + \boldsymbol{B}_2)/2$) and the perpendicular fluctuations ($\delta\boldsymbol{B}_\perp = \boldsymbol{B}_0 \times [(\boldsymbol{B}_1 - \boldsymbol{B}_2) \times \boldsymbol{B}_0]$). When the anisotropy was studied in the expanding model along the radial sampling direction, the power levels of the B-trace structure functions were in good agreement with the study of Chen et al. (2012) and the small-scale anisotropy was compatible with Boldyrev (2006) as well. When the samples were taken along the perpendicular direction with respect to the radial, the large-scale anisotropy disappeared but the small-scale anisotropy remained consistent with Boldyrev (2006). Verdini & Grappin (2015) suggested that this effect can be the result of expansion, which causes anisotropic energy distribution of the magnetic field components and confines the fluctuations in the perpendicular plane with respect to the radial direction.

In this Letter, a multi-spacecraft approach is presented to test the recent predictions about the effect of expansion on the anisotropy of the turbulence. The multi-spacecraft approach makes it possible to study the turbulence both along the radial and perpendicular to radial directions, which would not be possible with single-spacecraft data. We compare the observed features for the two different sampling directions to understand the possible effect of expansion on the radial spectral, variance, and local 3D anisotropy.

2. RESULTS

2.1. Radial Spectral Anisotropy

As discussed above, the expansion of the solar wind may counteract the effect of the mean magnetic field by causing the stretching of the eddies along the perpendicular direction with respect to the radial. This tendency may result in dependence of the B-trace structure functions on the sampling direction with respect to the radial.

To test this prediction, we used 4 s measurements of the magnetic field from the *Cluster* spacecraft (Balogh et al. 1997). The anisotropy was studied with a method based on that of Chen et al. (2010, 2011), in which the anisotropy of the B-trace structure functions was measured as a function of the parallel and perpendicular separation distances with respect to the local mean magnetic field using a multi-spacecraft technique. Here, we used the same technique but the B-trace structure functions were binned in 10° bins according to θ_R (angle between the spacecraft separation vector and the radial direction defined by the Earth–Sun line) and θ_B (angle between the spacecraft separation and the local mean magnetic field). The magnetic field data were carefully inspected by eye, and the intervals containing possible signatures of the terrestrial foreshock such as high energetic particles and Langmuir waves were not considered in this study. It was also ensured that the data do not contain interplanetary shocks, interplanetary coronal mass ejections (ICMEs), and corotating interaction regions (CIRs). Overall, 40 solar wind intervals were selected (43 hr of data in total) between 2001 September and 2015 June. The time-lagged spacecraft separation distance was kept in the range of 5000–10,000 km to ensure that the anisotropy is studied on approximately equal scales along all directions.

The results of the radial spectral anisotropy measurements can be seen in Figures 1(a) and (b). In both panels, the averaged B-trace structure functions are shown. In panel (a), one of the 40 investigated intervals is shown (51 minute interval), while panel (b) depicts the average of all the intervals (the B-trace structure functions for all the 40 intervals were binned together and the average of each bin was taken). The power levels of the structure functions showed some variation among the different intervals, which introduced additional scattering of the data in panel (b). Both panels confirm the results of earlier studies (e.g., Chen et al. 2011) that the power increases as a function of θ_B, which is consistent with anisotropic turbulence ($k_\perp > k_\parallel$). In contrast, no significant systematic trend can be seen in the structure function power levels as a function of θ_R. Expansion and turbulence are competing effects and both of them have associated timescales at which they operate. At small scales, the effect of expansion is small compared to the turnover of the eddies since expansion has larger timescales; therefore, this could explain why no anisotropy is measured with respect to the radial direction. We note that when the global magnetic field (average magnetic field over the whole interval) was used to define θ_B the results were similar to those shown in Figure 1 (although a bit noisier).

2.2. Variance Anisotropy

In the expanding solar wind model of Dong et al. (2014), it is predicted that expansion confines the fluctuations in the perpendicular plane with respect to the radial leading to anisotropic energy distribution among the field components. Previous studies of the variance anisotropy used single-spacecraft observations and concluded that the radial component has reduced power compared to the perpendicular ones. This is likely to be the combined effect of expansion and the solenoidality constraint. To separate these two effects, a multi-spacecraft approach is needed: the fluctuations of the magnetic field are studied along the radial (k_x) and transverse (k_y) sampling directions. It is expected that in both cases the field component along the sampling direction ($\delta B_x(k_x)$ and $\delta B_y(k_y)$, respectively) has the lowest power due to $\boldsymbol{k} \cdot \delta\boldsymbol{B}_k = 0$. Therefore, by comparing $\delta B_x(k_x)$ and $\delta B_y(k_y)$, the effect of expansion and solenoidality can be separated.

For testing this prediction, we studied the fluctuations of each magnetic field component separately ($\delta B_{i,2} = (B_{i,1} - B_{i,2})^2$, where i denotes the component of the magnetic field). For this study the *Advanced Composition Explorer* (*ACE*) and *Wind* spacecraft were chosen, which make it possible to study the solar wind turbulence on a 110–135 R_E scale. It is expected that more anisotropy is measured at large scales because the expansion time is more comparable to the turbulent time. The magnetic field data in the Geocentric Solar Ecliptic (GSE) coordinate system were used in which X is along the Earth–Sun line, Z points toward the ecliptic North, and Y completes the orthogonal system. The 3 s *Wind* magnetic field data (Lepping et al. 1995) was averaged to match to the 15 s *ACE* data (Stone et al. 1998, p. 1).

When *ACE*–*Wind* are separated ±10° to the $Y_{\rm GSE}$ axis, the distance between them is 110–135 R_E, while for the radial separation (±10° to the $X_{\rm GSE}$) it is only 30–40 R_E. In order to

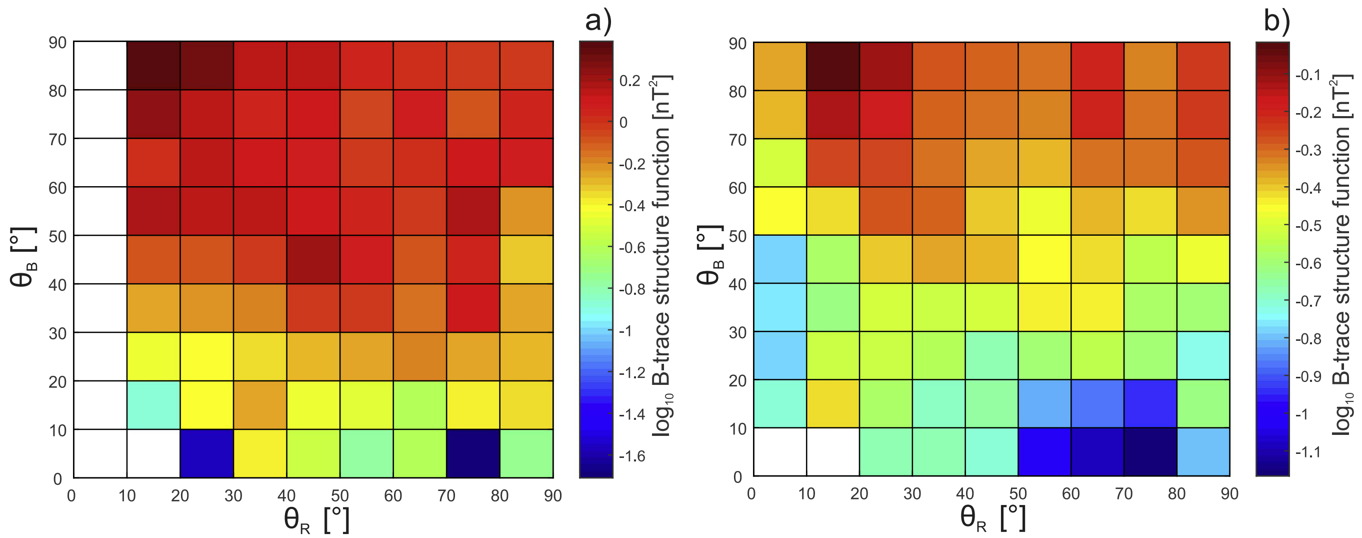

Figure 1. B-trace structure functions for one of the 40 investigated intervals (a) and averaged over all the investigated intervals (b). θ_B denotes the angle between the spacecraft separation vector and the local mean magnetic field, and θ_R is the angle between the spacecraft separation vector and the radial direction.

study the turbulence on equal scales along both directions, the Taylor hypothesis was used (Taylor 1938). The Taylor hypothesis is based on the fact that turbulent evolution is significantly slower than the solar wind speed, which is well satisfied at 1 au and allows us to measure the anisotropy at any radial separation distances. A time lag was added to the single-spacecraft measurements, and the δB_i^2 components were calculated based on the shifted data corresponding the 110–135 R_E separation distance. The accuracy of the Taylor hypothesis was tested: the δB_i^2 components were measured with the *ACE*–*Wind* double-spacecraft data along the radial direction (30–40 R_E) for 20 days of data, and then using the same intervals, the δB_i^2 components were derived again based on the time-shifted *ACE* and *Wind* data separately. It was found that the differences in the single- and double-spacecraft values are within 10%; therefore, the Taylor hypothesis is considered to be a good approximation.

The data from 2004 August to 2015 August were investigated, and the selection criteria were that the spacecraft should be separated $\pm 10°$ along the Y_{GSE} axis and the solar wind should not be disturbed by ICME/CIR events. Further down, selection was done to the data: the intervals were chosen when the local mean magnetic field was $\pm 15°$ along the Z_{GSE} axis. In this way, the variance anisotropy from the mean magnetic field is eliminated and the fluctuations in the X–Y_{GSE} plane can be compared. For the radial separation, the time-shifted data was produced (both with *ACE* and *Wind*), and the intervals with the approximately Z_{GSE} aligned local mean field were identified as well. 32 hr of data met the above requirements for the transverse and radial sampling directions, respectively.

The results are summarized in Table 1. The average fluctuations measured along the sampling directions are $\delta B_x^2(k_x) = 5.81 \pm 0.07$ and $\delta B_y^2(k_y) = 7.04 \pm 0.12$, which is a factor of 1.21 ± 0.025 difference. The components that are perpendicular both with respect to the sampling direction and the local mean magnetic field are $\delta B_x^2(k_y) = 9.29 \pm 0.21$ and $\delta B_y^2(k_x) = 11.53 \pm 0.18$, meaning a factor of 1.24 ± 0.034 difference. In the radial case, δB_x^2 and δB_y^2 were measured both with *ACE* and *Wind* separately, then the two data sets were merged, and the mean values were calculated. In both the radial and transverse cases, the fluctuations of the radial (x) component are smaller than the transverse ones (y). These results are qualitatively consistent with the prediction of Dong et al. (2014) and could be a possible effect of the solar wind expansion.

Table 1
Mean Values of δB_y^2 and δB_x^2 for the Radial and Transverse Sampling Directions (110–135 R_E), Respectively

	Radial Separation	Transverse Separation
δB_x^2	5.81 ± 0.07	9.29 ± 0.21
δB_y^2	11.53 ± 0.18	7.04 ± 0.12

2.3. Local 3D Anisotropy

Verdini & Grappin (2015) suggested that at large scales the local 3D anisotropy shows significant dependence on the sampling direction and the measured anisotropy in the perpendicular plane (with respect to the local mean magnetic field) is reduced when the samples are taken along the transverse direction with respect to the solar wind. A multi-spacecraft approach is needed to test this prediction: with a single spacecraft the B-trace structure functions can be measured only along the radial direction and two spacecraft are needed to sample along the transverse direction. We followed the approach of Chen et al. (2012) and studied the binned B-trace structure functions in the local coordinate system.

For this study the *ACE*–*Wind* pair of spacecraft were selected, which make it possible to study the anisotropy on 110–135 R_E scales, and due to the longevity of these missions, a significant amount of data was available to carry out a statistical study. From 2004 August to 2015 August, overall, 122 days of data were selected when the two spacecraft were separated $\pm 10°$ from the Y_{GSE} axis. Due to the orbit restrictions discussed in Section 2.2, the Taylor hypothesis was used to reconstruct the radial separation distance in the range of 110–135 R_E. The B-trace structure functions were binned in 10° bins in the local frame of reference according to θ_B (angle between the local mean magnetic field and the spacecraft

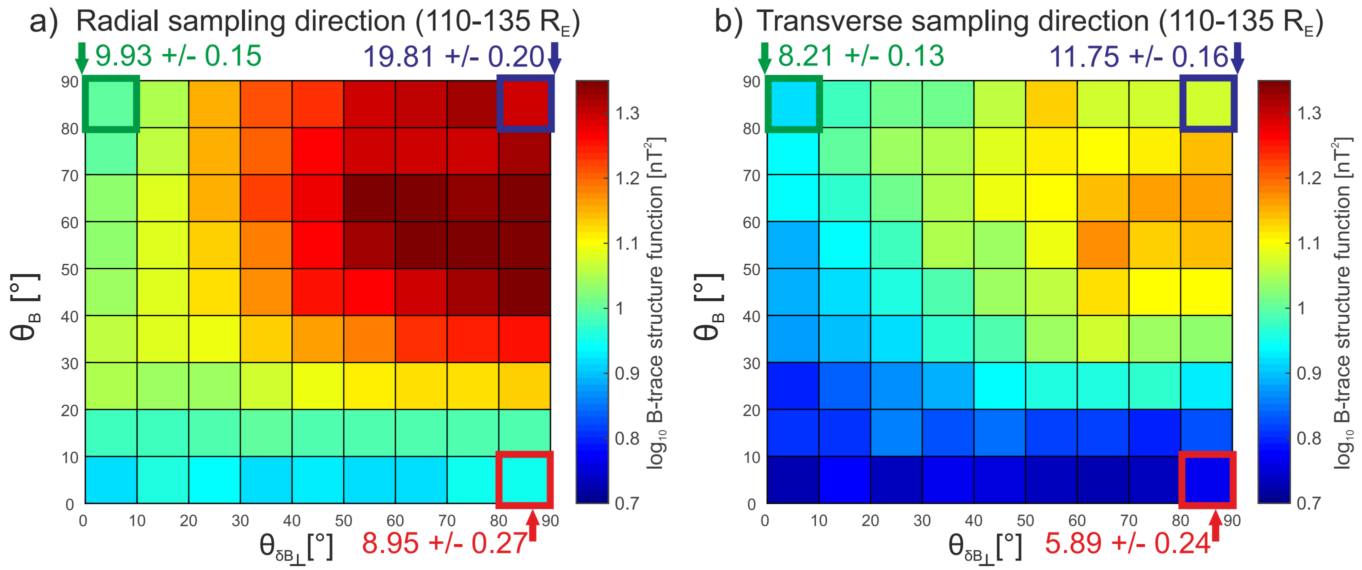

Figure 2. B-trace structure functions for the radial (a) and transverse (b) sampling directions. θ_B is angle between the local mean magnetic field and the spacecraft separation vector. $\theta_{\delta B_\perp}$ denotes angle between the local perpendicular fluctuation and the component of the spacecraft separation vector perpendicular to $\boldsymbol{B}_0$.

separation vector) and $\theta_{\delta B_\perp}$ (angle between the local perpendicular fluctuation and the component of the spacecraft separation vector perpendicular to B_0; see Chen et al. 2012). The angles larger than 90° were reflected below 90°, which has been found to be a good approximation (Podesta 2009).

Figures 2(a) and (b) show the B-trace structure functions for the radial and transverse sampling directions, respectively. Three bins are highlighted with green, blue, and red boxes: the green and blue bins (and the rest of the bins in the top row of the binned data) are in the perpendicular plane with respect to the local mean magnetic field, which is highlighted with the red box. The ratio of the power levels in the blue and green boxes can be used as a simple proxy to quantify the level of anisotropy in the plane perpendicular with respect to the local mean field. These ratios are 1.99 ± 0.036 and 1.43 ± 0.029 for the radial and transverse sampling directions, respectively. The measured anisotropy is significantly smaller for the transverse direction and the power levels are reduced by nearly 50%, which are in agreement with the simulations of Verdini & Grappin (2015).

3. DISCUSSION

In this Letter, a multi-spacecraft approach was used to study the signatures of expansion on the radial spectral, variance, and local 3D anisotropies.

The radial spectral anisotropy was studied with the *Cluster* data. The results showed that the power levels vary as a function of θ_B as expected from previous measurements (Chen et al. 2011). In contrast, no significant systematic variation was found in the power levels along θ_R. Although the expansion may cause stretching of the eddies at small scales, its timescale is slow compared to the turnover of the eddies, and as a result, the signature of expansion is not present. These findings imply that on small scales (5000–10,000 km) the anisotropy is primarily with respect to mean magnetic field and the role of expansion is negligible.

The variance anisotropy of the magnetic field components was studied with the *ACE* and *Wind* data. The limitations of the single-point observations were resolved with the use of the two spacecraft separated along the radial and transverse directions, respectively, and the effect of expansion, solenoidality, and the mean magnetic field were separated. The analysis of the fluctuations of the x and y components showed that $\delta B_x^2(k_x) < \delta B_y^2(k_y)$ and also $\delta B_x^2(k_y) < \delta B_y^2(k_x)$. These relations indicate anisotropic energy distribution, which is consistent with expansion. Horbury & Balogh (2001) found that the fluctuations of the transverse magnetic field components are approximately two times larger than the fluctuations of the radial one. The results of this Letter are consistent with the previous findings since $\delta B_y^2(k_x)/\delta B_x^2(k_x) \approx 1.98 \pm 0.039$; however, due to the multi-spacecraft approach, the sole effect of expansion was studied as well and the ratios of $\delta B_y^2(k_y)/\delta B_x^2(k_x)$ and $\delta B_y^2(k_x)/\delta B_x^2(k_y)$ were measured as 1.21 ± 0.025 and 1.24 ± 0.034. This implies that the solenoidality constraint causes the majority of the variance anisotropy observed with single spacecraft and expansion itself causes only weak anisotropy even on large scales.

The investigation of the local 3D spectral anisotropy revealed that the anisotropy of the B-trace structure functions depends on the sampling direction and the power levels were significantly reduced when the anisotropy was measured along the transverse direction. Both of these results are consistent with the predictions of Verdini & Grappin (2015) based on expansion. The defined proxy for quantifying the level of anisotropy was 1.99 ± 0.036 and 1.43 ± 0.029 for the radial and transverse sampling directions, respectively. These values suggest that some of the anisotropy is caused by expansion for the radial measurements. It is noted, however, that some anisotropy was found when the measurement was made along the transverse (perpendicular with respect to the radial) direction, which implies that additional mechanisms also contribute to the 3D anisotropy and the findings could be compatible with the model of Boldyrev (2006) as well.

This Letter presented a new approach to study the 3D nature of the turbulence. The results showed that under some circumstances the effect of solar wind expansion may be important. At large scales, expansion causes anisotropic energy distribution among the magnetic field components. As a further consequence, the turbulence measurements are affected in the local 3D coordinate system as well. It was found that anisotropy of the B-trace structure functions depend on the sampling direction with respect to the radial and the power levels are lowest when the anisotropy is measured along the transverse direction. Finally, at small scales, the effect of expansions is negligible, which is likely to be caused by the fast turbulent timescales, which are faster than the timescale of expansion, and as a result, at small scales the anisotropy is primarily with respect to mean magnetic field.

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