

U N I V E R S I T Y O F L O N D O N

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INTERFEROMETRIC MEASUREMENT OF
OPTICAL TRANSFER FUNCTIONS

by

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A B S T R A C T

Measurement of the optical transfer function for assessing the performance of an optical system is now well known, and techniques are well established. The number of theoretical results confirmed experimentally is, however small; and coma, in particular, has not yet been studied. There is, in this case the difficulty of accurate phase measurement and the problem of producing pure coma.

The measuring techniques used may be classified into two main types, namely scanning methods and the interferometric method. The latter is employed in the present work to measure both the modulus and phase of the transfer function of an optical system. The apparatus constructed has advantages ~~over previous devised apparatus~~, in its self-compensation for tilt, and in having the facility of automatic recording.

The optical transfer function of a defocused system has been measured for various values of the coefficient δW_{20} up to one wavelength. This was to check the performance of the apparatus; good agreement between the experimental results and theoretical results showed that the apparatus was functioning properly. A test lens having coma has been designed for the present work; the amount of coma produced can be varied from 0 up to 7 wavelengths. The optical transfer function of an optical system suffering from pure primary coma was measured

for values of the coma coefficient, W_{31} , from 0.63 up to 3.5 wavelengths, and these have confirmed theoretical results. The accuracy of measurement is about 0.02 of the maximum value for the modulus of the transfer function, and the error is less than 0.016 of a period for the phase of the transfer function.

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CHAPTER I

OPTICAL IMAGE FORMATION

1.1 Introduction

The ideal optical system is one that forms an image identical to object, and giving one-to-one correspondence between points in the object and those in the image plane. In fact the image of a point source formed by even a perfectly corrected optical system, and at the best focus, is not a point but a diffraction disc of finite size determined by the finite aperture of the optical system. This was first described by Airy (1834) and the fundamental theory developed subsequently by Kirchhoff (1882). The second factor affecting the correspondence between object and image depends on the optical system itself, that is its aberration. Rays traced through a lens do not always meet at one point in the image except for a particular shape of the lens; and even then, this will only be true for a certain object and image plane and over a small region of image plane. Geometrical optics is used to study the aberration of an optical system, and this may be expressed either in terms of ray aberration or asphericity of the wave-front. Near to the region of focus geometrical optics is valid only when the aberration of an optical system is very much greater than the wave-length of light concerned; for smaller

aberration the diffraction of light must be taken into account. This leads, incidentally, to an increase of aberration tolerances compared with those based on pure geometrical theory.

Another factor which has to be considered is the effect of the image detector. The human eye, for example, will resolve 10 line/mm. at the least distance of distinct vision (20^5 cm.); different photographic plates will resolve between 10 and 100 line/mm.; a television camera is usually limited to 8 line/mm.

Despite the use of resolving power, it has never proved possible to find a single number which constitutes an adequate criterion of the image quality of an optical system. Although aberration measurements, resolution tests, and intensity distributions in the images of certain objects have given a great deal of useful information, none of these is complete in itself and the correlation between result obtained and the performance of the lens is not simple. A better understanding of the process of image formation by an optical system was obtained when the methods of Fourier analysis were applied to optical systems, and a means of specifying the quality of image formation by an optical system was formulated. This introduced the concept of the optical transfer function of an optical system, which is the subject matter

of the present work.

1.2 Diffraction theory of a point source image.

In the eighteenth century the astronomers noticed that the image of a star, which is effectively a point source, formed by the best corrected telescope objective, at the best focus and under the best seeing condition, was still not a point but a finite disc of light surrounded by alternate dark and bright rings. It was found later by Fraunhofer in 1823 that this effect was dependent on the particular lens used, and that the diameter of the diffraction disc was inversely proportional to the diameter of aperture of the telescope objective. The intensity distribution in a perfect star image (when the objective is free from aberration) was calculated by Airy in 1834 as a function of aperture diameter and the wavelength of light used. Airy's formula is

$$I(Z) = \left(\frac{2J_1(Z)}{Z} \right)^2$$

where $I(Z)$ is the intensity distribution in the diffraction disc, as a function of the radial coordinate Z : $J_1(Z)$ is first order Bessel function, and $Z = k.n \sin \alpha \mathcal{F}$. Here $k = 2\pi/\lambda$ and $n.\sin \alpha$ is the numerical aperture of the objective and $\mathcal{F}$ the radius in the diffraction pattern. Hopkins (1943) showed that this formula was accurate to

a few per cent for systems of angular semi-aperture up to 30 degrees with a suitable interpretation of Z .

Since the perfect image is rarely formed in practice, many theoretical and experimental studies have been carried out to investigate the influence of aberration on the intensity distribution in the image of a point source. It is well recognized that in the presence of aberration, the image of a point source has a complicated structure, so that the problem is mathematically difficult, and the results obtained are difficult to interpret. It is rarely possible to interpret successfully any given criterion of performance in terms of it.

The investigation of the effect of defocusing on the intensity distribution in the image of a point source was made by Lommel (1884), extending considerably Airy's results. Conrady (1919) gave some numerical results for the influence of spherical aberration at different focal planes. These results were confirmed experimentally by Martin (1921-22) who also did work on the case of coma (1922-23). The case of astigmatism was tackled by Buxton (1921, 1923). Steward (1925) gave a more extension mathematical treatment, and Richter (1925) also studied the balancing of aberration terms needed to obtain maximum at the centre of the diffraction pattern. Cases of particular aberrations and effects of

small third order aberrations at the geometrical image plane were also treated by Steward (1925,1928) and Born (1932). Zernike (1934) using Strehl's criterion expressed tolerances for aberration coefficients in the case of small spherical aberration. These formulae were generalized by Nijboer (1943) using the circle polynomials, first introduced by Zernike in his diffraction theory of phase contrast microscope. The formulae obtained were comparatively simpler than those of Steward. Maréchal (1944, 1947) gave some results of practical use to lens designers on the optimum balancing of aberrations. Nienhuis (1949) extended the studies of Nijboer to include cases of larger amounts of aberration, and gave some experimental confirmation of diffraction theory. The problem of aberration large compared with the wave-length of light was also tackled with some success by Van Kampen (1949). The effect of radially non-uniform amplitude wave on the diffraction pattern was studied by Hopkins (1949), who showed that this had little effect as far as the ordinary lens was concerned. What has been described so far has concerned only the effect of aberration on the intensity distribution in the image of a point source, but no generally satisfactory performance criterion for the optical system was developed.

1.3 Image formation in a microscope

In the telescope the object is normally self-luminous, and therefore incoherent; with the microscope, on the other hand, the object is coherent or partially coherent when illuminated by a condenser system, there being then a non-zero phase correlation between different points in the object. Abbé (1873) was the first to realize the importance of wave-length and numerical aperture of the objective on the resolving power of an optical system. He made a very different approach to the problem of image formation in the microscope by taking into account the interference effects which he had noticed when using microscope. He suggested that the process of image formation was best described in term of diffraction produced by the object, which gave rise to diffraction spectra in the pupil of the microscope objective. Only spectra that lie within the lens aperture were transmitted by the objective and subsequently interfered and gave rise to the image in the image plane. The fidelity of image depended on the number of spectra transmitted; if all spectra were transmitted the image would be, apart from a constant factor, identical to the object. The image plane is uniformly illuminated if only one spectrum is accepted by the aperture of the objective. At least two spectra must be transmitted by the objective to show any structure at all. If the object is

a sine-wave grating in amplitude it will be imaged perfectly, so that the elementary object structure is a sine-wave grating and not a point source as was believed before. In contrast a point source assumes a most complicated structure; it includes an infinity of components ~~in its~~ Fourier frequency spectrum. Abbé established the classical expression determining the resolution limit of a microscope and noted further that the condition of illumination was also important in the imaging process. Rayleigh (1896) also dealt with image formation in the microscope; he considered light from each point in the object to give rise to an Airy disc in the image plane, and the superposition of these Airy discs gave image. Zernike(1942) using Fourier methods pointed out the equivalence of Abbé and Rayleigh's treatments. In Abbé's treatment, light is first diffracted by the object and then by the objective, thus in calculations a double integration must be made. Rayleigh considered the diffraction by the objective; each point in the object gives rise to an Airy disc, this is one integration and the second integration takes account of the structure of object. So there is no fundamental difference between these two treatments, only the order of a double integration that is inverted. These two treatments, although not recognized at that time, were the first application of Fourier methods to optical image

formation.

1.4 Point source image criteria

In the middle of nineteenth century astronomers became interested in the resolution of the components of double stars, which relates to the resolution limit as defined by Airy's formula. Experimental investigations of the condition determining the resolution of double stars was carried out by Dawes (1865). Rayleigh in 1879 introduced the Rayleigh criterion which states that the ~~images~~ of two close point sources are considered to be resolved when the intensity at the region of overlap of two images dropped to 20% of the maximum intensity; or, in other words, when the first dark ring of one Airy pattern fell on the central maximum of the other. Several modifications of this criterion have been suggested; in spectroscopy the resolution limit may be defined by the width of the intensity profile when the intensity falls to 50% of the peak intensity. A similar criterion was given by Sparrow (1916). According to this the resolution limit is reached when the second derivative of the intensity distribution is zero at the mid-point of the line joining the two images.

1.5 Tolerance criterion

The perfect optical system rarely exists: what the

optical designer attempts to do is to minimise all aberrations down to some limit so that their effects on the quality of image can be neglected. Rayleigh (1879) established the so-called $\lambda/4$ limit. According to this the aberration has negligible effect on the quality of image, provided it is small enough for the image-forming wavefront to be contained within two concentric spheres differing in radius by a quarter of a wavelength. This criterion is rather strict and can only be applied to a very good corrected optical system, such as a microscope objective. When the image forming wavefront satisfies the Rayleigh $\lambda/4$ criterion the general effect of aberration is to cause a loss of light from the centre of the pattern and the appearance of more light in the outer parts of the diffraction image. Resting on this fact, Strehl (1895) introduced the term "Definitionshelligkeit", or Strehl intensity ratio, which is the ratio of the intensity at the principal maximum in the diffraction image of a point source to the intensity obtained there in the absence of aberration and defocusing, i.e. at the centre of Airy disc pattern. The quality of the image remains very good if this intensity ratio does not drop below 0.8. This criterion is valid for the case of small aberration only; when the aberration is large compare with wavelength of light, the light distribution in the region of

focus no longer consists of a marked principal maximum surrounded by weaker light. For example, the case of large primary spherical aberration shows two equal peaks of intensity along the axis, which would suggest, according to this criterion, there being two image planes of equal image quality, whereas actually the images of extended object formed in these planes are significantly different.

1.6 Quality criteria

Generally speaking the finest detail in the image is an obvious criterion of quality. It would be useful if the index of performance of the optical system forming an image at a given point in the image plane could be correlated to the state of correction of the optical system or more particularly the wave-front aberration of the optical system. This correlation is not always simple with the index of performance to be described.

(a) Resolution test. The practical test to determine the image performance of an optical system was first introduced by Foucault (1858) who used a simple grating consisted of black and white strips of different line-spacing as test object. The performance of an optical system is then measured by the reciprocal of the spacing of the finest grating which can be distinguished as resolved in the image. Foucault also tried to relate this

result to the design data of the optical system but the wide range of contrast sensitivity of the eye makes the result unreproducible. The use of photography offers one solution; this method was used by Gardner (1937) and also Selwyn and Tearle (1946). The latter worked on aerial photography and suggested the use of low-contrast test objects. Since the results obtained depend on the test object and the detector used, these must be similar to those used in practice. Cobb (1915) introduced the "Cobb chart", having bar test patterns with spacing progressively decreasing from group to group. The resolution limit is determined by the closest spacing of the object in which its image is still be detected to be resolved. A fan-shaped sector test object was used by Ross (1924). Sandvik (1927) noticed that the photographic resolution is also a function of contrast of the test object; he examined the relation between resolution and the logarithm of the line-to-space ratio of Foucault type test-objects. Cobb and Moss (1928) investigated the factors influencing resolution in the eye and the test object used. They suggested the test object consisted of two black bars separated by a white one, all of the same width and the ratio length to width being 3:1. This type of test object was used by Fabry (1936), Washer (1939) and Sayce (1940). Howlett (1946) used a low contrast annular test object. Gardner

in 1950 introduced a test chart having the variations of line frequency and contrast in directions perpendicular to each other, to measure both the influences of resolving power and contrast. Hotchkiss and co-workers (1951) showed that in the presence of aberration there could be more than one resolution limit, and found that the plane giving best resolution was not necessarily the plane of best contrast. This was also shown by Higgins and Jones (1952). The sharpness of a picture judged by the subjective impression of an observer was studied by Wolfe and Eisen (1953). An attempt has been made to correlate these results with the performance of the system for extended objects. In using the photographic method it should be noted that the photograph is assessed on the reproduction of both coarse and fine details, whereas the measurement of resolution limit only provides information about the finest detail resolved.

All the tests carried out by this basis gave no general satisfactory means of specifying lens performance. It is obvious when the lenses are used in television camera or similar system that the resolution limit alone is not sufficient to specify the lens performance. Therefore, it may be concluded that only one number is inadequate to specify the quality of the lens.

(b) Other tests. As early as 1859 the knife-edge test of Foucault was recognized as a useful test. Even

nowadays the knife-edge test still proves to be useful and is widely used in the field of astronomical telescope objective manufacturing. The knife-edge test, Hartmann test (1904) or Ronchi test (1925) all give useful information about the aberrations of optical systems. The Michelson type interferometer was modified by Twyman and Green (1920-21) to measure directly the wave-front aberration of an optical system. Even this only measures the aberration of the wave-front forming the image, and does not specify directly the quality of image formed by the optical system, since the performance of an optical system cannot be predicted directly from the aberration coefficient. Coleman and Clark (1947) introduced the term "Interferometer quality" to assess the performance of an optical system from its interferogram. This measures the diameter of the largest circle which can be inscribed in a single interference area of the pattern obtained. It was found that this measurement agreed well with the resolving power tests. Huber (1943) measured the performance of an optical system by the diameter of the central core of the image containing 25% of the total light. R. E. Hopkins in 1954 and co-workers measured the radial energy distribution in the image of a point source taken with the lens under test at different focal setting and aperture. This test offered some useful information in certain cases.

1.7 Coherence theory in optics

The dependence of the image forming process on the condition of object illumination had been realized by Abbé and Rayleigh but only two extreme cases were considered at that time namely complete coherence and incoherence. The case of partial coherence was studied experimentally by Verdet (1860) and more detailed research was made by Michelson (1890-92) who found the relation between the source size and the coherence of illuminating wave in his measurement of stellar diameter. Berek (1924) first introduced the term "degree of consonance" while concerned with the image formation in the microscope. The degree of consonance is zero, or disconsonance, when there is no interference in the illuminating field; it is equal to unity for coherence and in-between are degrees of partial coherence. The relation of slit width and the degree of coherence on a plane illuminated by the slit was calculated by Van Cittert (1934), and simplified by Zernike (1938) using a complex degree of coherence which was defined by the visibilities of interference fringes. Hopkins (1951) developed this treatment to give a general solution for the case of partial coherence and the degree of coherence over a plane illuminated by any source. He also discussed a chromatic coherence factor of a light source (1956). The same question has been discussed by Duffieux (1953), Wolf (1955)

and Steel (1957). The latter was concerned with the effect of small aberrations on the images of partially coherence objects. The precise definition of partially coherence conditions was discussed by Slansky (1955), Blanc-Lapierre and Dumontet (1945, 1955). Some experimental work was carried out by Arnulf and co-workers (1952,1953) to confirm the theoretical work of Hopkins and Barham (1950). They also studied the relationship between the limit of resolution and the coherence of light. The effect of source size on coherence condition was studied experimentally by Baker (1955), Markovic (1956) and also by Wolf and Thompson (1957).

1.8 Coherent, partial coherent and incoherent elementary structure.

Formerly only two extreme cases of treatment were considered namely coherence and incoherence, until Hopkins in 1951,1953 extended Duffieux's treatment to include the case of partially coherence. He also introduced the idea of an effective source which could replace the actual source and give the same degree of coherence in the object plane. If the object is coherently illuminated the effective source will be a point; and it will be infinite in extent in the case of incoherent illumination. For partially coherent illumination the effective source will

have a finite size.

According to Abbe's theory, when the structural frequency of the object increases the diffraction spectra it produces will move toward the edge of the pupil. For simplicity let the object be a sinusoidal grating of the spatial frequency such that only two spectra are accepted by the lens pupil. Consider in general a state as shown in fig. 1.1a where two spectra are centred at points $-m, 0$ and $p, 0$. If the illuminated object is partially coherent the spectra will be two finite patches instead of two points as in the case of perfect coherent illumination. Part of light will be cut off by the lens pupil, and all of the light that is transmitted is not in a condition to interfere; only those parts common to two spectra can interfere, the rest is non-interfering light unavoidably transmitted to the image plane and causes reduction of contrast. The interfering light may be found by making two spectra coincide (Fig. 1.1b.); the interfering light contains in the shaded area common to two displaced pupils centred at point $-m, 0$, and $p, 0$ and the effective source centred at point $0, 0$. For incoherently illuminated object (the effective source is infinite in extent) the interfering light contains in the area common to two sheared pupils. The contrast in the image is proportional to the interfering light transmitted by the lens pupil; for the

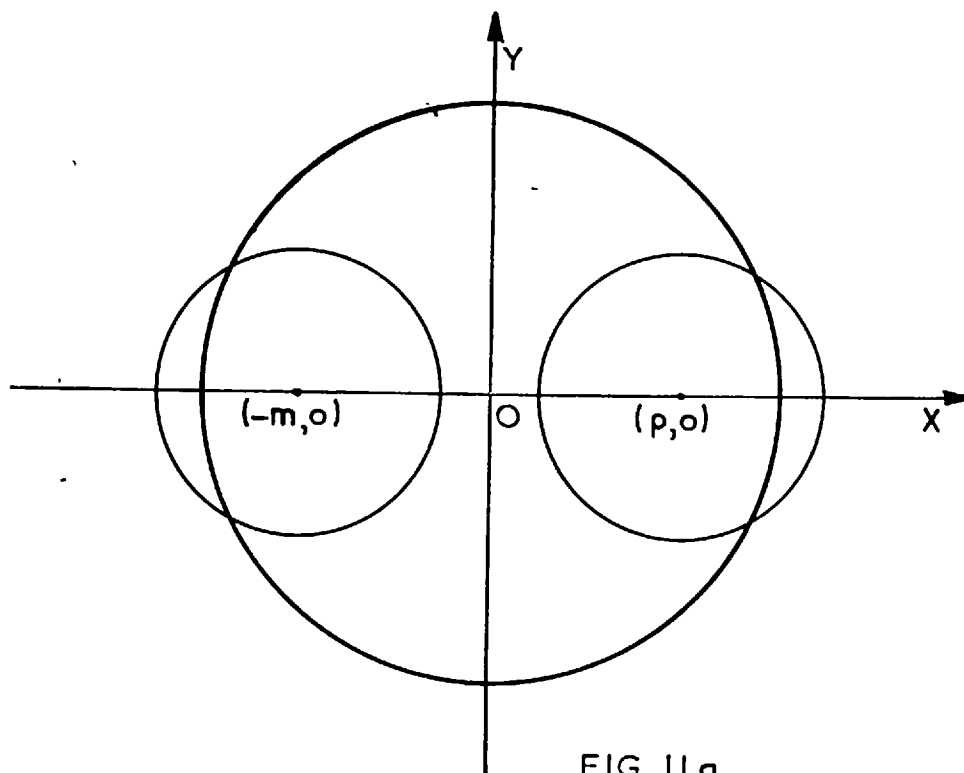


FIG. 1.1 a

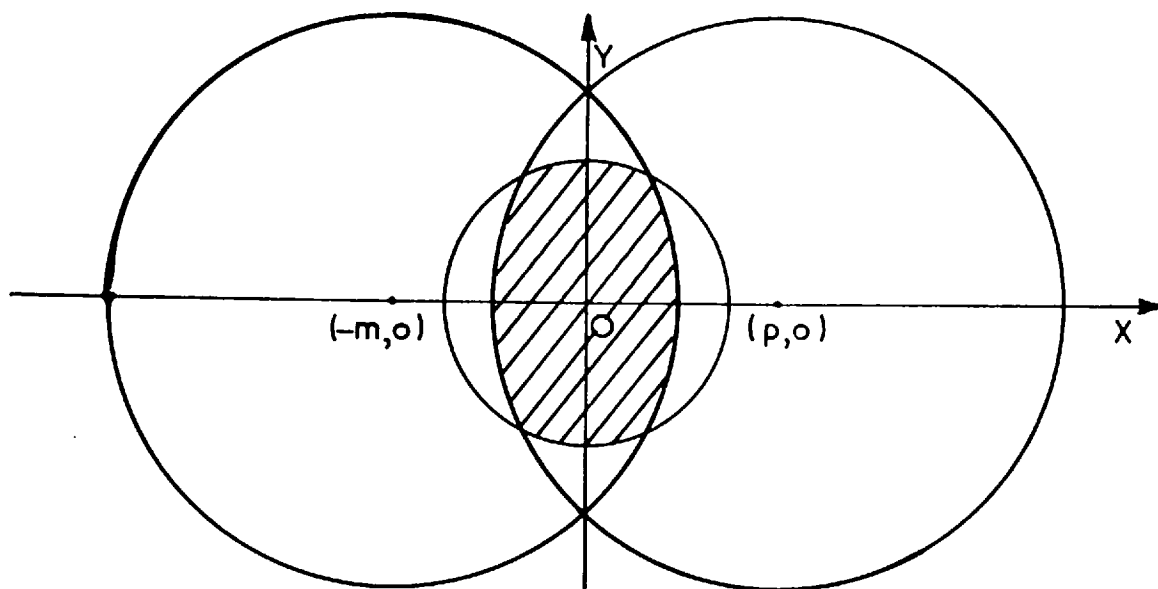


FIG. 1.1 b

case of perfect coherent illumination the contrast of the image is constant, when the frequency of the object increases, and drops abruptly to zero when the point spectra fall just outside the lens pupil. If the object is partially coherent illuminated the contrast in the image will be constant and gradually drops off finally to zero when the centres of spectra fall outside the lens pupil; whereas in the case of incoherent illumination the image contrast drops gradually from the beginning and reaches zero at the limit of resolution (Fig. 1.2). If aberration is present in the optical system, the fringes in the image plane will overlap and causes further reduction in contrast; the fringe position will be shifted laterally relative to its geometrical position if asymmetrical aberration such as coma is present. It will be seen that the information in the image is a function of both the property of an optical system and the condition of illumination.

This forms the basic idea of shearing interferometer for measuring the optical transfer function of an optical system which was suggested by Hopkins (1955).

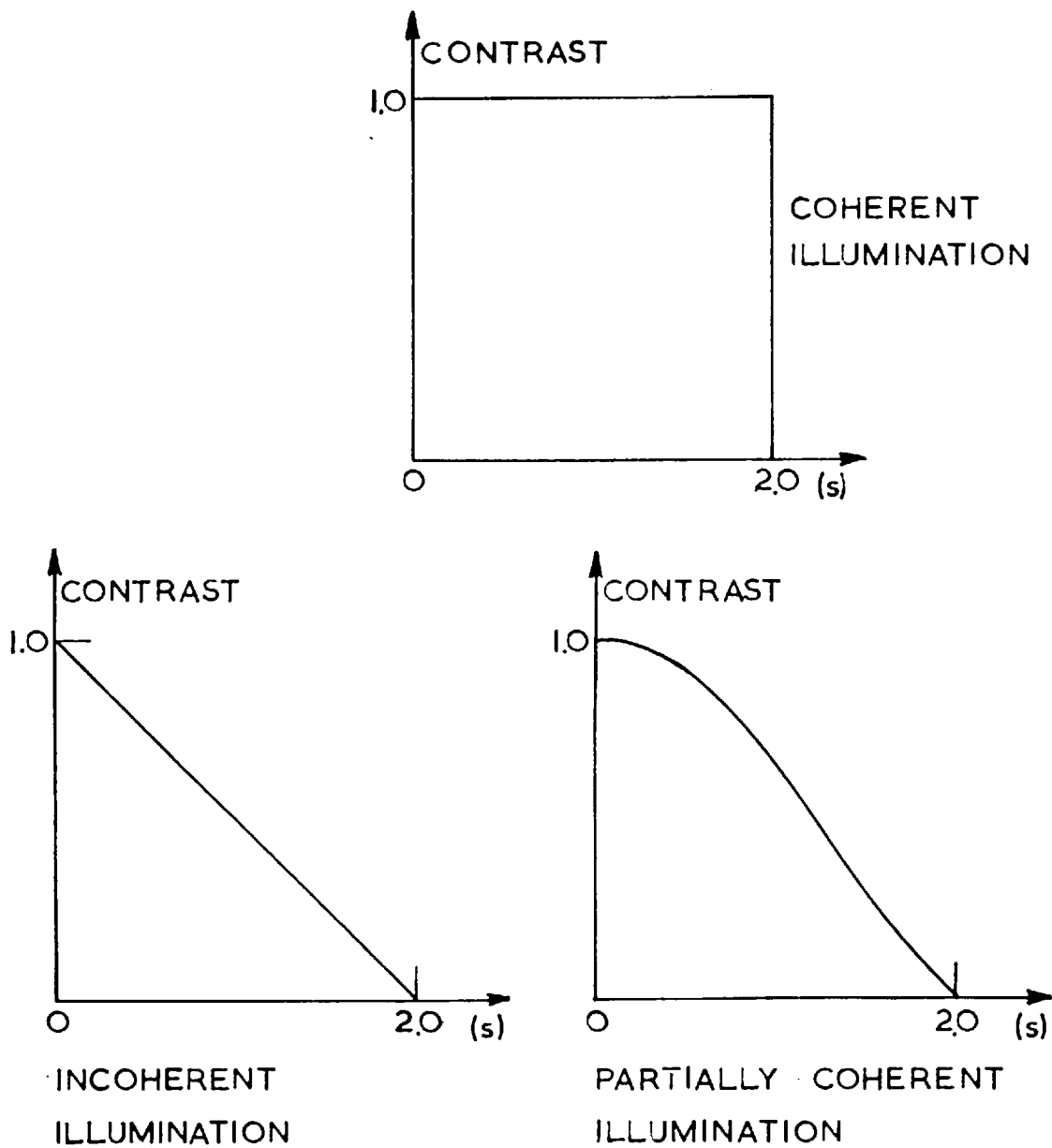


FIG. 1.2

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CHAPTER II

IMAGE ASSESSMENT BASED ON THE OPTICAL TRANSFER FUNCTION

2.1 Application of Fourier analytical method.

The recent development of the Fourier analytical treatment of optical image formation has shown the way to the more general solution of image quality evaluation. Apart from the earlier work of Abbé and Rayleigh, Mertz and Grey (1934) seem to have been the first to apply Fourier methods to optics. In dealing with the scanning of an optical image, as in a television system, they came to the conclusion that the intensity distribution in the object might be Fourier analysed into sets of sine-wave components, whose amplitudes of modulation would be modified by the process, there being a "Transfer Admittance" of the finite scanning aperture, which behaved like a low-pass filter attenuating the higher frequency components to a greater degree. The use of sine-wave signals or impulses for testing electrical networks suggests that a useful type of test object structure would be of sine-wave form or possibly square-wave.

Similar techniques were introduced by Duffieux (1940, 1942) to describe the process of image formation by a lens. For a one-dimensional object, the slit-image intensity profile (the line spread function) is used in the convolution theorem for Fourier transforms, and shows that the product

of the Fourier transform of the function denoting the image intensity distribution is given by the product of the transforms of the spread function and of the function expressing the distribution of intensity in the object. In symbols:

$$b'(s) = b(s) \cdot g(s)$$

where s denotes spatial frequency. The optical transfer function is thus given by the inverse Fourier transform of the intensity distribution in the image of a point or line source.

2.2 The optical transfer function.

The measurement of resolving power, aberration or edge gradient give incomplete or indirect information for describing the image quality of an optical system. For example the system that gives better edge gradient does not necessarily give good resolving power. On the other hand, if the optical transfer function of an optical system is known all the above-mentioned criteria can be determined. This is why the measurement of optical transfer function has become increasingly accepted as the best assessment criterion, and research in advancing the technique of practical measurement is still required.

Frieser (1938) had foreshadowed the measurement of optical transfer function as a means to assess the image performance of an optical system. Mathematical analysis of

the images of point, line or Faucault bar type test objects are difficult; it becomes simpler when a sine-wave test object is used because the image distribution is then, apart from magnification and reduction of contrast, identical to object. This property was noticed by Frieser (1935) while investigating the resolution of photographic film and he recommended this sine-wave object as a test object. Cox and Martin (1945) discussed the image intensity of an incoherent line source and stated that all informations and the assessment of lens performance were possible from this by applying the Fourier analysis method. Instead of applying quality criterion to this spread function directly, however, it is simpler to take the Fourier transform of the line spread function formed in different azimuths and obtain optical transfer functions in those azimuths. Selwyn (1946, 1948) studied the theoretical aspect of the resolving power of lens-emulsion combinations. He also introduced the function characterising the modulation of contrast of sine-wave test object for both the objective and emulsion and pointed out that these functions could be calculated from the intensity distribution in the image of a point or line source which made the theoretical argument simpler. Schade (1948) discussed the correlation and evaluation of the electro-optical characteristic of the imaging system. In his investiga-

tion on television system, he introduced "The Aperture Response" or optical transfer function of an optical system which could be obtained by calculation if the distribution of intensity in the point or line image was known.

The sine-wave contrast transfer function may be evaluated directly from the wave-front aberration of an optical system without first calculate the line or point spread function. Régnier (1950) calculated the contrast in the image of sine-wave test object for the presence of few cases of spherical aberration. Hopkins (1955) gave the first analytical solution and treatment of the effect of defocusing on optical transfer function. He showed that the band-width given by the optical transfer curve decreased rapidly when the defocusing term exceeded the Rayleigh tolerance and that Spurious resolution (reversal of contrast) appeared after the optical transfer function had dropped to zero. This effect was noticed practically earlier by Washer (1939) and Hochkiss (1951). The optical transfer function of a defocused system was also discussed by Steel (1956). De (1955) studied the influence of astigmatism on the optical transfer function, covering the range of astigmatism up to 20 wave-lengths. The case of spherical aberration was studied theoretically by Parrent and Drane (1956), who calculated a number of optical transfer curves, some of which were compared with experimental re-

sults. Black and Linfoot (1957) also treated the case of spherical aberration, in different focal planes, and for amounts of aberration of one and two wave-lengths only.

Such calculations involve double integrations, and the highly oscillatory nature of the integrand had presented a serious difficulty, until Hopkins (1957) proposed a simple method of numerical integration. This new method was used by Goodbody (1958, 1960) for studying the effects of spherical aberration and coma on the optical transfer function. The condition for optimum balancing of residual aberrations and the permissible amount of different kinds of aberration in an optical system were also treated by Hopkins (1957).

2.3 Experimental measurement of optical transfer function

The measurement of the optical transfer function as a means for optical image assessment is now well-known, techniques are well established but the number of theoretical results precisely ~~confi~~^rmed experimentally is relatively small. The experimental methods employed can be classified into two main types namely scanning methods and the interferometric method. The scanning method can further be sub-divided into those employing the measurement of the spread function followed by calculation and the measurement of optical transfer function directly.

(a) Direct scanning of spread function. An incoherently illuminated (geometrical) line source can be regarded as composed of an infinite set of equal amplitude sine-wave components, and may be represented by a delta function whose Fourier transform is unity. The complex ^{amplitude} of each Fourier component, after transmission through the optical system, is given by the inverse Fourier transform of the image intensity distribution. That is:

$$g(s) = \int_{-\infty}^{+\infty} G(u) \exp.(-i2\pi us) du$$

where $G(u)$ is the line spread function.

Hopkins (1956) stressed the fact that uni-dimensional objects (line sources) used in different azimuths could be employed for any system without loss of generality. Thus slit-images may always be scanned by slits, with considerable photometric gain (as high as 10^4) over the scanning of point image with apertures (Naish, 1954; R. E. Hopkins et al., 1954, 56).

Herriott (1947) scanned a line source image with a vibrating slit attached to the cone of an energized loud-speaker. By coupling the position of the scanning slit synchronously with the photo-electric signal, the intensity profile of a slit image was obtained from the oscilloscope trace. Ingelstam and Lindberg (1951) scanned a slit image

with a fine slit fixed before a photocell. The intensity profile was obtained by synchronising the movement of the whole scanning unit to that of chart-recorder. By this method he examined the out-of-focus image of a line source. A similar method was used by Sproson (1953) to test television camera lenses. Jones and Wolfe (1945) photographed a slit image through a linear density colloidal carbon wedge and obtained the logarithmic intensity profile of the slit image. Kuwabara (1955) used a test object consisted of slit and edge and examined it with a double slit system; one scanned the slit image in a usual manner and the other provided a comparison intensity.

(b) Direct measurement of optical transfer function.

With the last method only the spread function is measured and the optical transfer function is obtained by numerical calculation. To obtain directly the optical transfer function, which is the inverse Fourier transform of spread function, the incoherent point or line image is scanned with some form of periodic grating, or alternatively the image of a periodic aperture is allowed to move across a fixed narrow slit aperture. Ideally the periodic aperture should be a cosinusoidal transparency, but because of the difficulty in production, a square wave grating is often used and the fundamental Fourier component is filtered electronically.

Schade (1948) was the first to use this method in determining, what he called the "Transfer Admittance" of the lens aperture of a television camera. The sine-wave test object was obtained by recording a constant amplitude electrical a.c. signal on a variable-density sound track of a cine film. By scanning the point spread function with test patterns of different spatial frequency, he obtained the optical transfer function. Lindberg (1954) scanned a point source image with a constant speed fan-shaped sector disc. The spatial frequency was varied by moving the image along the radius of sector disc, and the fundamental frequency was picked out by electrical filtering. The optical phase-shift was measured by comparing the phase of the out put signal with that given by a magnetic pick-up which indicated the phase at the centre of each sector. Ingelstam (1956) also scanned a slit image with sinusoidal templates of different spatial frequencies; each had phase marks in the form of slits at each end. The phase term was found by comparing the separation of the phase marks on the template with those of the actual peaks occurring on the chart recorder. Suzuki (1957) employed a similar test pattern but with spatial frequency varying continuously from 10-110 line/mm. He obtained optical transfer functions for symmetrical systems only since there was no phase measurement device.

This method has been repeated by many observers using square-wave screens. The screen may be constructed in the form of a drum, and used either as a test object, Parker (1953) and Hutto (1955), or as a scanning screen as used by Rosenhauer and Rosenbruch (1957).

Since a series of sinusoidal screens of different spatial frequencies are difficult to make, Born, Fürth and Pringle (1945) used a screen of single frequency whose apparent frequency may be varied by mechanical optical means in his analysis of mathematical function. This method was adapted for the measurement of the optical transfer function by Hacking (1958). Birch (1961) scanned a slit image by a square-wave scanning drum of fixed spatial frequency and examined the output for its harmonic content to determine the optical transfer function. If the slit source is of negligible width its Fourier transform is a delta function and all harmonics present have the same amplitude. Each harmonic transmitted is sorted out by varying the speed of scanning drum to bring each harmonic to the frequency that is accepted by the selective amplifier. The normalised output from the chart-recorder gives the transfer factor for each spatial frequency. It should be noted that with multiple slit scanning the function is repeated periodically and the Fourier transform of this series of pulses are of amplitude equal to that of the Fourier trans-

form of each individual pulse at equal spaced positions on the spatial frequency axis, this spacing being determined by the period of repetition of the pulse. If the pulses are repeated periodically with an interval between their maxima of A mms. (say), the Fourier transform of the series will be finite at the spatial frequency corresponding to $1/A, 2/A, \dots$. At other spatial frequencies the Fourier transform is zero. With this method Birch confirmed some theoretical results for the case of a defocused system. Birch (1962) also used the image of a narrow horizontal section of the image of a vertical slit formed on a ruled glass drum; rotating this slit-image relative to the rulings the effective spatial frequency was changed. Lohmann (1957) used Moiré fringes of various spatial frequency as a test object and examined the output by a narrow slit. The image contrast was compared with that of the object to obtain the modulation transfer function; the phase term was also obtained from the comparison. Kubota and Ohzu (1957) used a technique involving the cross-correlation function between a random test chart and its image formed by the optical system under test. The measurement of the auto-correlation function gave the optical transfer function. Kapany and Keim (1958) used a shadowgraph procedure to measure the square-wave response of a lens system from its computed spot diagrams. This method has advantage

in assessing lenses before construction.

(c) Interferometric method.

In this method the optical transfer function is determined by measuring the total light flux in an interference pattern and the spatial frequency scale is determined by the fractional shear between two pupils. For an aberration-free optical system, with incoherent slit object, the optical transfer function is proportional to the area common to two sheared pupils. This technique was first suggested by Hopkins (1955) and the first instrument was built by Baker (1955). The instrument was a modified Michelson type interferometer with the two end-mirrors replaced by roof mirrors in order to separate the emergent and reflected beams. The shear was produced by the sideways movement of one of the roof mirrors, and a longitudinal movement of the other roof mirror, produced by magnetostriction, produced an optical path change accurate to $\frac{\lambda}{100}$. The output was detected visually by null setting of a two-beam flicker photometer. The optical transfer function of the lens under test was compared with that of a perfect lens to obtain the relative optical transfer function. The phase term was also obtained by a comparison method. Baker obtained a number of optical transfer curves for a defocused system which agreed within ± 0.05 with theoretical results.

Kelsall (1959) developed this method to make possible

for auto-matic recording of the optical transfer function. He used an interferometer similar to that of Baker, but the roof mirrors were replaced by the corner-cubes, giving a more compact and stable instrument. Shear of the wave-fronts was produced by rotating two plane parallel glass plates, one in each arm of the interferometer, and a continuous variation in optical path was obtained by the sideways movement of a small angle wedge prism arranged to move in one arm of the interferometer. The total light flux from the instrument was detected by a photomultiplier, a chart-recorder trace then giving the modulation transfer function of the optical system under test. The phase term was measured by using a comparison beam. A number of theoretical results for the case of defect of focus and spherical aberration were confirmed.

Montgomery (1961) employed a new type of interferometer which was similar to that of Brown (1954). This type of interferometer has an advantage over the previous type used, in that it is self compensating against tilt: it is also compact and uses a symmetrical arrangement of the optical components. Shear is produced by the rotation of two plane-parallel glass plates, one in each arm of the interferometer, and the optical path difference is produced by the movement of a small angle prism placed in one arm of the

interferometer. Montgomery was able to measure the modulation transfer function of an optical system using automatic recording. By using a polarization technique, he claimed that the phase term could be measured. Some theoretical values of the optical transfer function for the cases of spherical aberration, astigmatism and defocusing were confirmed.

Lohmann (1957) also proposed an interferometric method for determining the optical transfer function. His apparatus was a special form of the Baker type shearing interferometer. A Wallaston prism was inserted between two crossed polarisers to produce a small shear between the two emergent beams. The optical transfer function was then obtained by the measurement of the total light flux in the interferogram.

Viñot (1958) and also Wilczynski (1961) used an interferometric method to determine the optical transfer function of the lens-emulsion combination. A sine-wave pattern to illuminate a photographic transparency was produced by means of a two-beam interferometer. The measurement of the maxima and minima of the transmitted light fluxes, with lateral shift of the fringe system, served to measure the frequency density of the transparency function of the image.

Moss (1963) was also concerned with the measurement of optical transfer function of lens-emulsion combination.

He produced the sine-wave pattern by means of a Wallaston prism and other polarising devices.

The scanning method has usually employed a narrow slit or point object, and the image is scanned by a further narrow slit so that the amount of light transmitted to give any information is very low. If a slit image is scanned with a periodic aperture, the photometric efficiency is about the same as that of the interferometric method. The continuous determination of the optical transfer function for different spatial frequencies is often not possible with scanning methods. Although the use of a single screen of effectively variable frequency is one solution, this sort of screen is difficult to construct with high accuracy. The new method of Birch (rotating the horizontal section of a vertical slit image relative to the ruling on the glass drum) may produce a continuous varying of frequency but is photometrically worse than the direct scanning method.

The obvious advantage of the scanning method is that it can be used with white light, which is the usual conditions under which the optical system will be used in practice. Although different wave-lengths of light may be used in the interferometric method to obtain the optical transfer functions for various wave-lengths but it is not convenient. Nevertheless, to confirm theoretical results monochromatic light has to be used in any case.

The technique of direct image scanning is difficult to standardise and reproduce with an accuracy of (say) 5%. The interferometric method, on the other hand, uses the lens under test itself as a collimator lens for the instrument, whereas the scanning method needs an auxiliary collimator. Since the measurement of the optical transfer function is independent of the sign of the wave-front aberration, the effect of residual aberration of the collimator or of any auxiliary component cannot be subtracted from that of the aberration of the system itself. One way to overcome this problem is to choose a well corrected auxiliary component or at least the residual aberration should not be greater than $\frac{\lambda}{10}$. When white light is used in the scanning method, it is also necessary to correct for the spectral sensitivity of the detector used by inserting an appropriate filter before it. In measuring the distribution of intensity in the image of a point or line object by scanning it with a slit or pinhole, the intensity at any point cannot be measured but an average of the light over a small area. This integrating area should not be too small; the tolerance may be set such that the scanning slit width is 1/10th of the period of structure. A fine slit of the appropriate size can be made by depositing an opaque aluminium layer over a quartz fibre resting on glass.

The position of the scanning slit at each point of the

is also important, scan_x and should often be positioned with an accuracy of $1/2$ micron, but even good mechanical precision is only of the order of 2 microns, and makes for difficulty.

Attempts have been made to derive a single number from the optical transfer function to serve as an index of performance. For example "The equivalent bandwidth", which defines the width of the rectangular bandpass of area equal to that under the actual optical transfer curve has been used, but this averages out too strongly the form of the optical transfer curve. Thus two lenses having the same equivalent bandwidth may produce significantly different image qualities. This is illustrated in Fig. 2.1, where the two curves have the same equivalent bandwidth, but curve (a) gives significantly better image quality at lower spatial frequency. The optical system corresponding to curve (a) is suitable for television work where a restricted range of spatial frequency is of interest; while that corresponding to curve (b) is suitable for photography where resolution at higher frequencies is also required. It is, therefore, difficult to judge from the optical transfer curve alone whether one optical system is better than another, unless the application for which the optical system is going to be used in practice is known.

2.4 Conclusion

It may be concluded that the optical transfer function

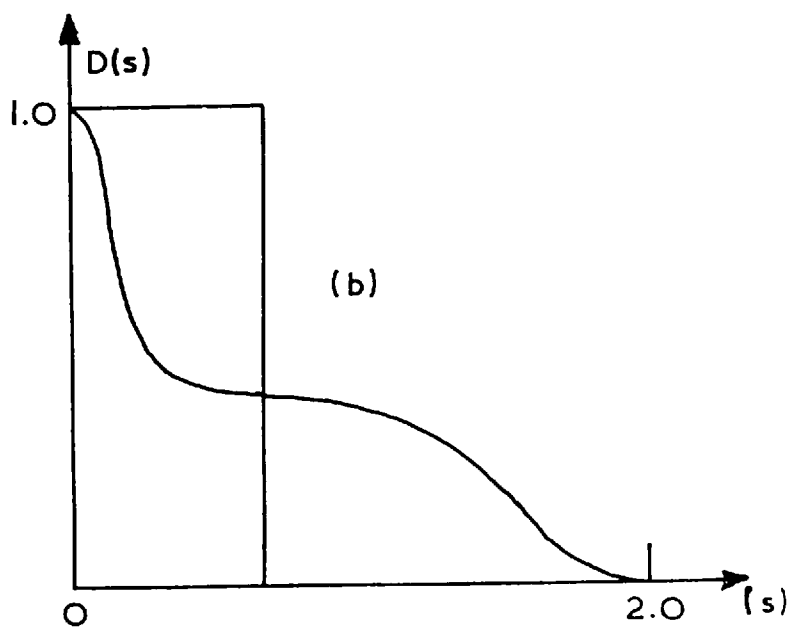
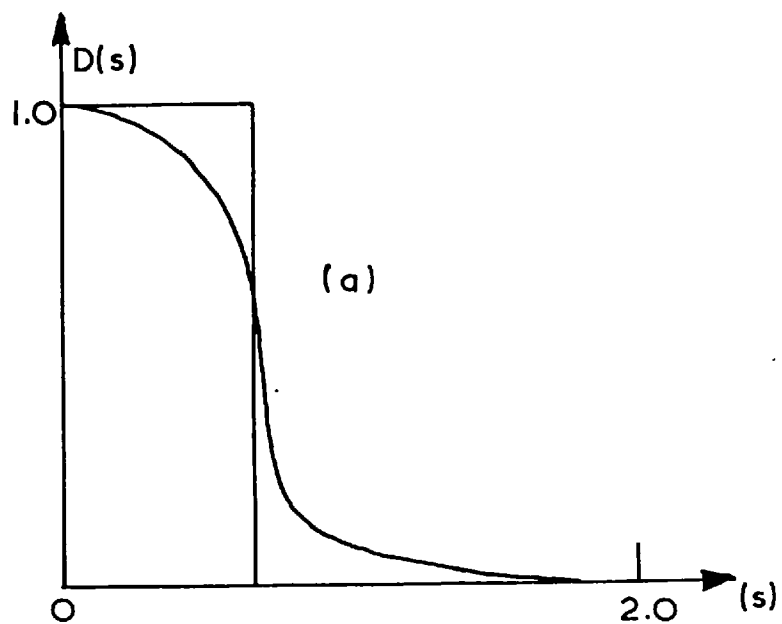


FIG. 2.1

is sufficient for describing analytically all the imaging properties of an optical system. However, the character of the object and the detector must be taken into consideration. Other disturbing influences, such as graininess of detector, are of not less importance.

The extent to which the image resembles
~~To have an image identical with~~ the object may be defined to be the "fidelity" of the image, and this may in principle be determined from the transmission characteristic of the optical system. Alternatively to have an image containing the greatest possible information, has to be assessed by information theory (Linfoot, 1955). Maréchal and Croce (1953) and others studied an "Equalisation process" with the aim of obtaining a uniform transmission throughout the spatial frequency range. They photographed the primary image where the higher spatial frequencies were of low contrast, this was then re-image using coherent light, with an aperture mask which attenuated the lower frequencies. The technique to control the shape of an Airy disc by coating a thin film in the pupil of the lens was introduced earlier by Boughon, Dossier and Jacquinet (1946) and also Wilkins (1950).

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CHAPTER III

THEORY OF TRANSFER FUNCTIONS AS APPLIED TO OPTICS

3.1 Introduction

It was mentioned in the previous chapter that the image of a cosinusoidal intensity distribution in the object imaged by an optical system is again a cosinusoidal intensity distribution whatever the state of correction of the optical system. The image is of course reduced in contrast, compared with that of the object, due to the effects of diffraction at the finite aperture of the optical system. If there is aberration present in the optical system the image will be further reduced in contrast and possibly shifted laterally from the geometrical image position (phase-shift); this occurs when an asymmetrical aberration such as coma presents (Fig. 3.1). The reduction of contrast and the phase-shift term may be combined into one complex number termed "The optical transfer function" of the optical system.

If the optical transfer function of an optical system for a given region in the image plane is known as a function of spatial frequency R , and for different azimuths ψ , the image forming properties of the optical system for that region of the image is specified. This is so because any spatial distribution of intensity may be Fourier analysed into sets of cosinusoidal spatial distributions of intensity. Each of these components is then transmitted to the

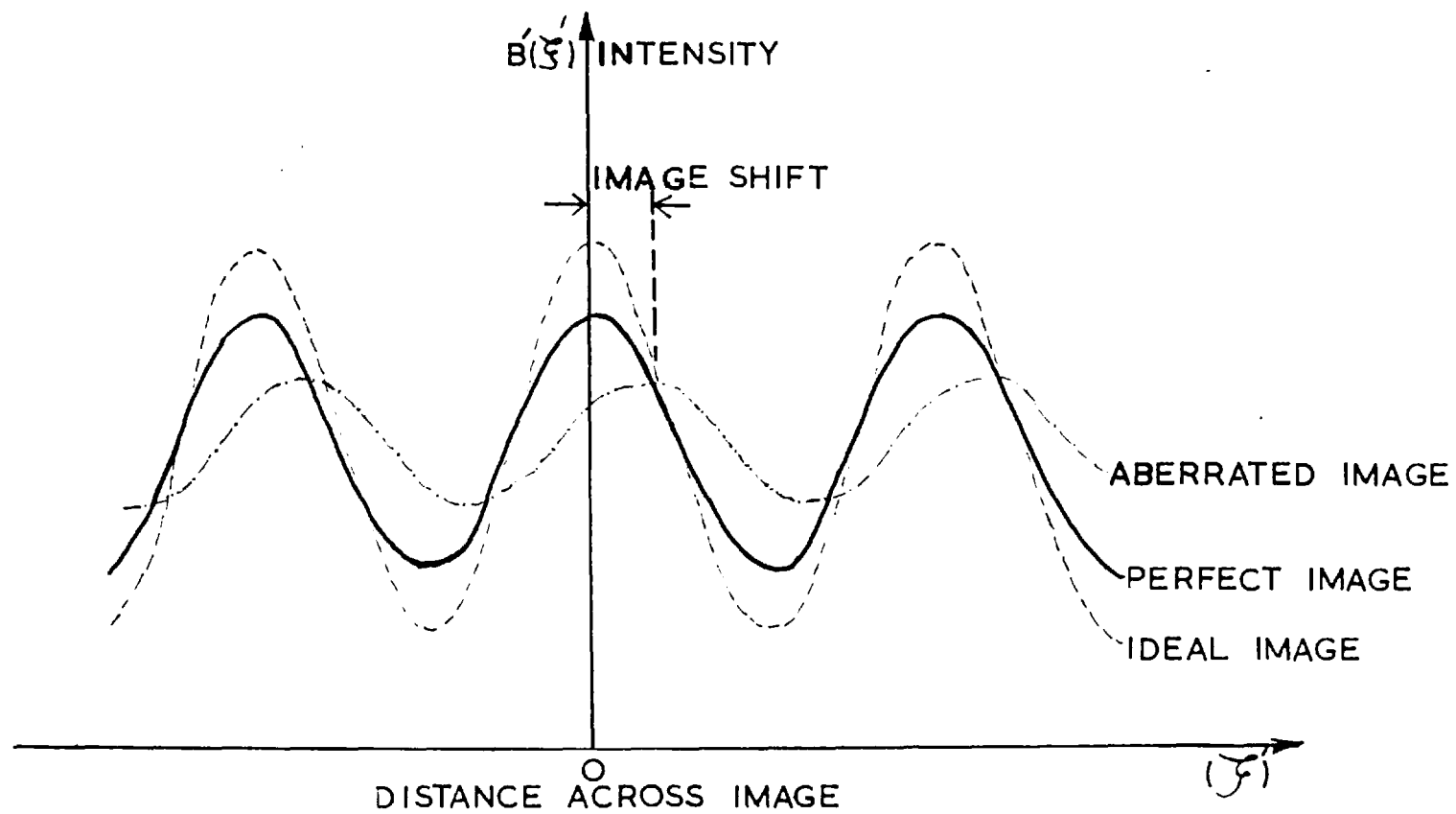


FIG. 3.1

image plane with its amplitude attenuated by a factor $T(R, \psi)$, and possibly laterally shifted by the amount $\theta(R, \psi)/2\pi R$. The intensity distribution in the image is found by the superposition of these components. The quantities used above are the modulus and argument of the complex transfer function of the system.

The optical transfer function may be derived from the image of an incoherent point or line source formed by an optical system (the so-called spread function) since these objects have a Fourier spectrum containing all spatial frequencies. The transfer function is also given by the suitably normalised, Fourier transform of the spread function, and may be expressed as an integral of the distribution of complex amplitude over the exit pupil of an optical system, that is the "pupil function". This pupil function is dependent on the aberration of the optical system so that if the aberration of the optical system is known, the optical transfer function of the optical system may be calculated.

3.2 Analysis of an object into a series of cosinusoidal gratings.

Any function of two variable representation may be represented in the form:

$$B(u,v) = \iint_{-\infty}^{+\infty} b(s,t) \exp. \left\{ i2\pi(us + vt) \right\} dsdt \dots (1)$$

where $B(u,v)$ and $b(s,t)$ are Fourier transform pair. If $B(u,v)$ represents the intensity distribution in the object, it must be real so that:-

$$b(-s, -t) = b^* (s,t)$$

Now $b(s,t)$ may be written in term of its modulus and argument,

$$b(s,t) = \beta (s,t) \exp. i \phi (s,t)$$

where $\beta (s,t)$ and $\phi(s,t)$ are all real, and substituting this back into (1) gives:-

$$B(u,v) = 2 \iint_0^\infty \beta(s,t) \cos \left[2\pi(us + vt) + \phi(s,t) \right] ds dt \quad \dots(2)$$

(u,v) are coordinates in the object plane and (s,t) are spatial frequency variables. If the axes are rotated through an angle ψ , the coordinates (u, v) referred to the new axes will be :-

$$u = \bar{u} \cos \psi - \bar{v} \sin \psi$$

$$v = \bar{v} \cos \psi + \bar{u} \sin \psi$$

and thus, $us + vt = \bar{u} (s \cos \psi + t \sin \psi) + \bar{v} (t \cos \psi - s \sin \psi)$

If the value of ψ is now chosen to make the coefficient of $\bar{v}$ zero, that is:

$$t \cos \psi - s \sin \psi = 0$$

and thus; $\psi = \tan^{-1} \frac{t}{s}$

The intensity distribution in (2) is now seen to vary

only along the direction $\bar{u}$ with a spatial frequency $\sqrt{s^2 + t^2}$; it is constant along the direction of $\bar{v}$. Hence any object having a two-dimensional intensity distribution can be represented by the superposition of unidimensional intensity distributions in different azimuths. This has the advantage that the imaging properties of any optical system may be determined by using a slit or grating in different azimuths as test object. Moreover, the imaging characteristics of a system may be adequately described by means of single variable curves for different azimuths, three such curves are normally adequate.

3.3 Image formation as a passive linear process.

Image formation by an optical system may be considered as a passive linear process. For this only two assumptions have to be made; namely, the conditions of superposition and isoplanatism. The first condition requires that the intensity distribution in the image be found by simple addition of the intensities produced by each elementary area of the object. If the intensity distribution in the object $I_1(u)$ gives rise to the intensity distribution in the image $O_1(u')$ and a second object intensity $I_2(u)$ gives an image intensity $O_2(u')$, the condition of superposition requires that when $I_1(u)$ and $I_2(u)$ are presented simultaneously the resulting object intensity should be $O_1(u') + O_2(u')$ or

symbolically:-

$$\begin{array}{rcl} I_1(u) & \longrightarrow & O_1(u) \\ I_2(u) & \longrightarrow & O_2(u) \\ I_1(u) + I_2(u) & \longrightarrow & O_1(u) + O_2(u) \end{array}$$

To satisfy this condition, light waves from different parts of the object must be mutually incoherent so that self-luminous and most naturally illuminated objects are included. The cases of coherent and partially coherent illuminated object has been treated by Hopkins (1953).

The second condition requires that the aberration change sufficiently slowly for the spread function to be regarded as constant over a small region of image plane, the spread function being defined to be the distribution of intensity in the image of an incoherent slit of limitingly small width. In practice the aberration of an optical system is not constant over the whole image plane, but in most cases the above condition is sufficiently well satisfied. This is because the extent of the spread function is very small in comparison with the total image size.

The condition of isoplanatism is more exactly satisfied the better the state of correction of an optical system, since both the spread function is then smaller in extent, and the aberration changes more slowly over the image plane.

This second requirement may be stated formally using the same notation as used above; if the input $I(u)$ is

displaced by an amount u_0 the output $O(u')$ has to be similarly displaced by the same quantity or in symbols:-

$$I(u - u_0) \longrightarrow O(u' - u_0)$$

In optics it means that a lateral displacement of the object only results in a corresponding lateral shift of image without changing its intensity distribution.

If a "unit impulse" (an incoherently illuminated infinitesimally narrow slit) at $u = 0$ gives rise to the 'output' $G(u')$, the spread function as defined above, the intensity distribution in the object $I(u)$ will have an intensity distribution in the image given by the integral:

$$O(u') = \int_{-\infty}^{+\infty} I(u) G(u' - u) du$$

Applying the convolution theorem this may be written as:-

$$O(s) = i(s) \cdot g(s)$$

This may be interpreted to mean that each Fourier component $i(s)$ of the object appears in the image, but is modulated by the complex factor $g(s)$, which is the inverse Fourier transform of the spread function.

3.4 The image of a cosinusoidal grating.

Supposing that the intensity distribution of an incoherently illuminated cosinusoidal grating takes the form:-

$$B(\mathcal{F}) = \alpha + \beta \cos 2\pi R \mathcal{F} \quad \dots (1)$$

where $\mathcal{F}$ is a rectangular geometrical coordinate in the

object plane, α is the mean intensity and β is the peak value. If ξ_0 is the period of the grating; $R = 1/\xi_0$ is the spatial frequency measured in lines per unit length (Fig. 3.2). The symbols with suffixes have the same interpretation but apply to the image plane. To avoid the complication of magnification through the system, the unit of length in the object and image planes are chosen to be in the same ratio as the magnification, so that $\xi = \xi'$ gives the geometrical image of the object point at ξ .

The Michelson visibility, or contrast, in the object is defined by:-

$$\text{contrast} = \frac{B_{\text{max.}} - B_{\text{min.}}}{B_{\text{max.}} + B_{\text{min.}}} = \frac{\beta}{\alpha}$$

The maxima of intensity in the object occur when $2\pi R \xi = 2\pi m$, where m is an integer or zero, so that

$$\xi_{\text{max}} = m/R$$

gives the positions of the maxima.

If the intensity distribution in the image of a line source of negligible width, placed at $\xi = 0$ is $G(\xi')$, the corresponding spread function of a line source placed at $\xi = \xi$ will be $G(\xi' - \xi)$. The intensity at the point ξ' produced by the whole object $B(\xi)$ is then given by

$$B(\xi') = \int_{-\infty}^{+\infty} B(\xi) G(\xi' - \xi) d\xi \quad \dots\dots(2)$$

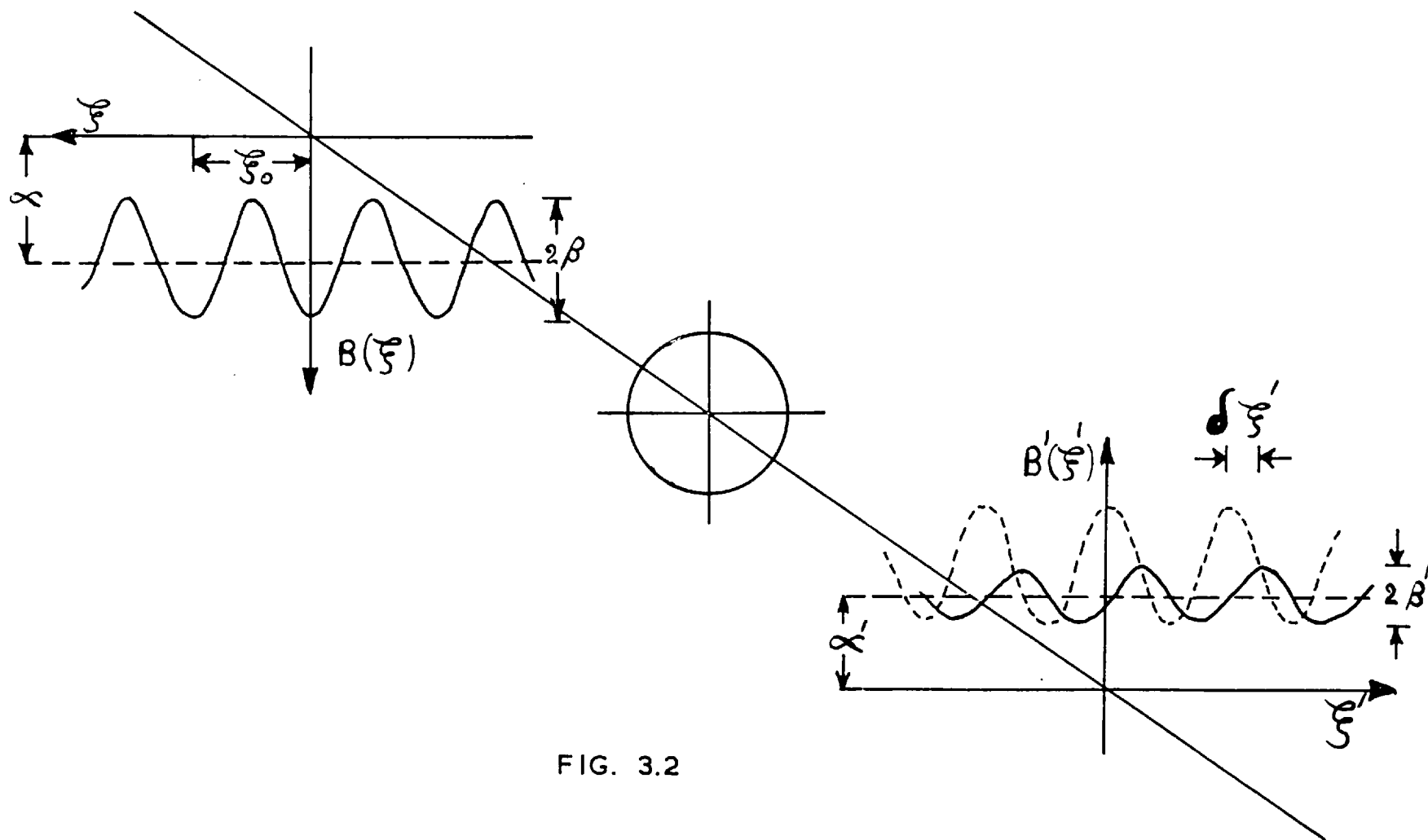


FIG. 3.2

The formal limits of integration may be set at $\pm \infty$ since the value of $G(\xi')$ is zero outside some finite range of ξ . Substituting from (1) in (2) gives

$$B(\xi') = \iint_{-\infty}^{+\infty} (\alpha + \beta \cos 2\pi R \xi) G(\xi' - \xi) d\xi \dots (3)$$

and replacing $(\xi' - \xi)$ by ξ , and writing:-

$$\begin{aligned} C(R) &= \int_{-\infty}^{+\infty} G(\xi) \cos 2\pi R \xi d\xi, & C(0) &= \int_{-\infty}^{+\infty} G(\xi) d\xi \\ S(R) &= \int_{-\infty}^{+\infty} G(\xi) \sin 2\pi R \xi d\xi, & S(0) &= 0 \end{aligned}$$

(3) is easily shown to give for the intensity distribution

$$B(\xi') = C(0) \cdot \alpha + C(R) \cdot \beta \cos 2\pi R \xi' + S(R) \beta \sin 2\pi R \xi' \dots (4)$$

If the constant factor $C(0)$ is omitted, implying a new choice for the unit of intensity for the image, equation (4) becomes:-

$$B(\xi') = \alpha + T(R) \beta \cos \left\{ 2\pi R \xi' + \theta(R) \right\} \dots (5)$$

$$\text{where } T(R) = \frac{\sqrt{C^2(R) + S^2(R)}}{C(0)}$$

$$\text{and } \theta(R) = \tan^{-1} - \frac{S(R)}{C(R)}$$

It is evident from (5) that the image of a cosinusoidal intensity distribution object is again a cosinusoidal inten-

sity distribution. The contrast of the image now being $T(R) \cdot \beta / \alpha$ so that:

$$\frac{\text{contrast of image}}{\text{contrast of object}} = \frac{T(R) \cdot \beta / \alpha}{\beta / \alpha} = T(R)$$

$T(R)$ is the contrast transfer factor or contrast attenuation of the optical system and $\theta(R)$ is the spatial phase shift.

From equation (5) the intensity of the image has maxima when

$$2\pi R \mathcal{F}' + \theta(R) = 2\pi m \quad (\text{where } m \text{ is again an integer})$$

that is when

$$\mathcal{F}' = \frac{m}{R} - \frac{\theta(R)}{2\pi R}$$

so that the image is shifted laterally by the amount:-

$$\delta(\mathcal{F}') = - \frac{\theta(R)}{2\pi R}$$

The contrast transfer factor $T(R)$ and the spatial phase shift $\theta(R)$ are conveniently combined into a single complex number given by:-

$$\begin{aligned} D(R) &= T(R) \exp. i \theta(R) \\ &= \frac{C(R) - i S(R)}{C(0)} \quad \dots\dots\dots(6) \end{aligned}$$

which is the optical transfer function of the optical system.

Substituting the values of $C(R)$ and $S(R)$ into equation (6) gives:-

$$\begin{aligned}
 D(R) &= \frac{1}{C(0)} \left(\int_{-\infty}^{+\infty} G(\xi) \cos 2\pi R \xi \cdot d\xi - i \int_{-\infty}^{+\infty} G(\xi) \sin 2\pi R \xi d\xi \right) \\
 &= \frac{1}{C(0)} \int_{-\infty}^{+\infty} G(\xi) \exp. \{ -i2\pi R \xi \} d\xi
 \end{aligned}$$

since $g(R) = \int_{-\infty}^{+\infty} G(\xi) \exp. \{ -i2\pi R \xi \} d\xi$ is the inverse Fourier transform of $G(\xi)$, and $C(0) = \int_{-\infty}^{+\infty} G(\xi) d\xi = g(0)$, this may be written:

$$D(R) = \frac{g(R)}{g(0)}$$

The presence of $g(0)$ in the denominator is merely to ensure that $D(R) = 1$, when $R = 0$. Hence it is again seen that the optical transfer function is the normalized inverse Fourier transform of the spread function.

If all asymmetrical aberrations such as coma are absent, and the optical system has a symmetrically shaped pupil, the spread function will be symmetrical, that is:

$$G(-\xi) = G(\xi)$$

Its inverse Fourier transform $g(R)$, and hence the optical transfer function, is then wholly real so that $\theta(R)$ is either 0 or π . In the latter case the image is shifted laterally by half a period, giving the effect of reversal of contrast or spurious resolution.

Applying Schwarz's inequality to the expression for $g(R)$ gives:-

$$|g(R)| \leq \int_{-\infty}^{+\infty} |G(\xi) \exp. -i2\pi R \xi| d\xi$$

$$\leq \int_{-\infty}^{+\infty} G(\xi) d\xi = g(0)$$

or $T(R) = \frac{1}{|D(R)|} = \left| \frac{g(R)}{g(0)} \right| \leq 1$

This indicates that the contrast in the image is always less than that in the object. They are equal when $R = 0$; that is, when the object has no structure at all, the image will be similarly uniformly illuminated.

The optical transfer function may also be related to the image of a point source. Thus if $G(\xi, \eta)$ is the point spread function, the line spread function $G(\xi)$ is found by integration:-

$$G(\xi) = \int_{-\infty}^{+\infty} G(\xi, \eta) d\eta$$

If now in the double transform:

$$g(s, t) = \iint_{-\infty}^{+\infty} G(\xi, \eta) \exp. \{-i2\pi(s\xi + t\eta)\} d\xi d\eta$$

we put, $s = R$ and $t = 0$

$$\begin{aligned} g(R, 0) &= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} G(\xi, \eta) d\eta \right] \exp. \{-i2\pi R\xi\} d\xi \\ &= \int_{-\infty}^{+\infty} G(\xi) \exp. \{-i2\pi R\xi\} d\xi \end{aligned}$$

$$g(R, 0) = g(R)$$

Hence the optical transfer function can also be determined directly from knowledge of the intensity distribution in

the image of a point source. Since the optical transfer function derived from the line spread function is different for different orientation of the spread function, it is necessary to include Ψ , the angle between the direction of line spread function and the meridian plane of an optical system, to indicate the dependence of $D(R)$ and $\Theta(R)$ on the azimuth. $D(R)$ and $\Theta(R)$ are then replaced by $D(R, \Psi)$ and $\Theta(R, \Psi)$ respectively.

3.5 Diffraction theory of optical transfer function.

For simplicity when using the diffraction theory, reduced coordinates are frequently used. The reduced coordinates for the entrance pupil are defined by:-

$$x = \frac{X}{h} ; \quad y = \frac{Y}{h}$$

where X, Y are rectangular geometrical coordinates and h is the entrance pupil radius. $\dot{x}, \dot{y}$, are similarly reduced coordinates in the exit pupil plane, and are defined as above but using $\dot{X}, \dot{Y}$ and $\dot{h}$. The reduced coordinates (u, v) for the object plane are related to the geometrical coordinates (ξ, η) by the expressions:

$$u = \frac{n \sin \alpha}{\lambda} \xi ; \quad v = \frac{n \sin \alpha}{\lambda} \eta$$

and similarly for the image plane:-

$$\dot{u} = \frac{\dot{n} \sin \alpha'}{\lambda} \dot{\xi} ; \quad \dot{v} = \frac{\dot{n} \sin \alpha'}{\lambda} \dot{\eta}$$

where $n \sin \alpha$ and $n' \sin \alpha'$ are the numerical apertures of the system in the object and image spaces respectively, λ is the vacuum wave-length of light used. (Fig. 3.3.)

Since both $n \sin \alpha$ and $n' \sin \alpha'$ are invariants, $u' = u$, $v' = v$ give the geometrical image of the point (u, v) .

Corresponding to the reduced coordinates (u, v) , there are reduced spatial frequencies (s, t) which are related to the geometrical spatial frequency by the relation.

$$s = \frac{\lambda}{n \sin \alpha} R$$

and similarly for t .

The pupil function $f(x, y)$ of an optical system represents the distribution of complex amplitude over the reference sphere. It is thus defined by:-

$$f(x, y) = \mathcal{T}(x, y) \exp. ikW(x, y)$$

where $W(x, y)$ is the wave-front aberration of the system, $k = 2\pi/\lambda$ and $\mathcal{T}(x, y)$ is the distribution of real amplitude, which is finite inside the pupil region A and zero outside the pupil. Hopkins (1949) showed that a small non-uniformity of amplitude over the wave-front has little effect on the quality of image so that the factor $\mathcal{T}(x, y)$ may usually be neglected in the above expression.

Apart from a constant factor, the distribution of complex amplitude in the image plane $F(u', v')$, according to Fraunhofer diffraction formula, is given by:-

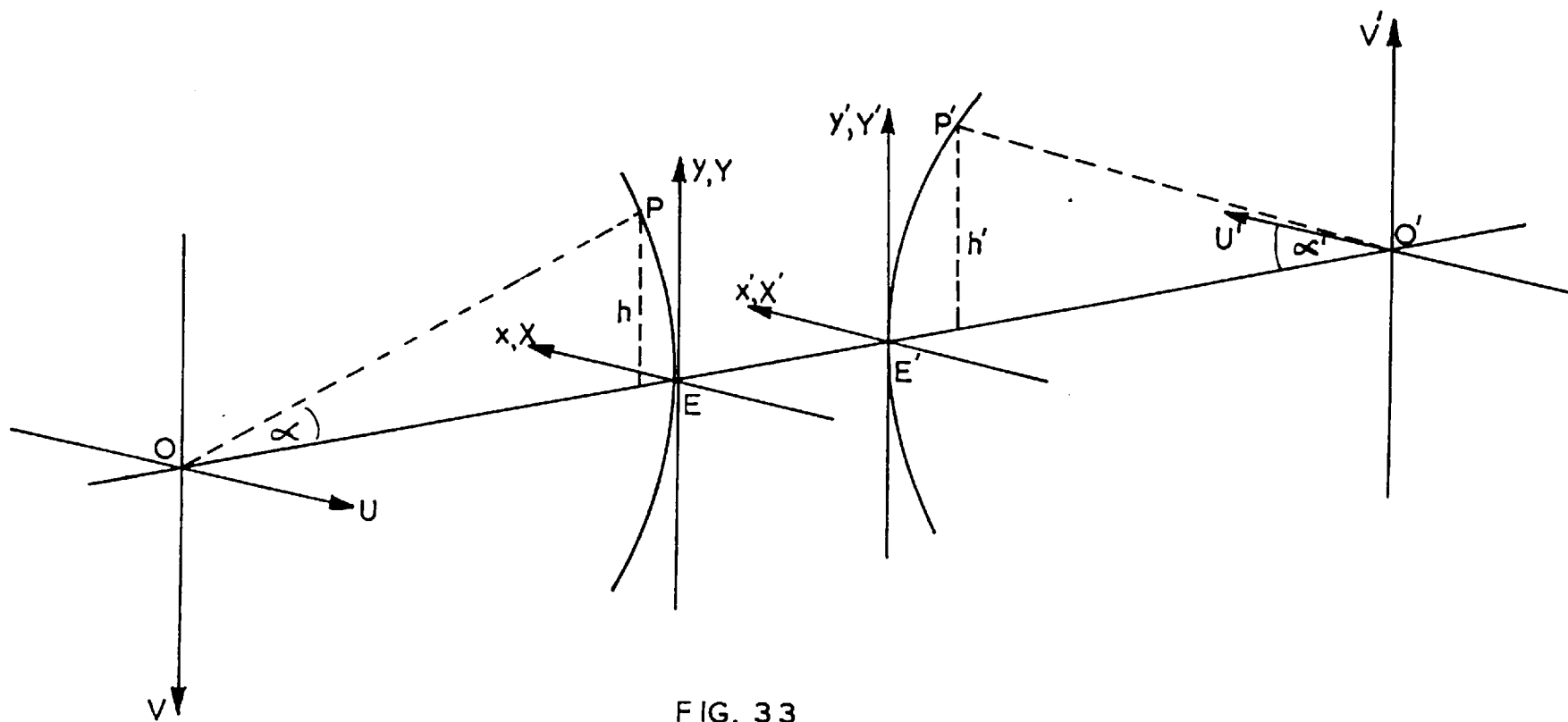


FIG. 3.3

$$F(\dot{u}, \dot{v}) = \iint_{-\infty}^{+\infty} f(x,y) \exp. \left\{ i2\pi(\dot{u}x + \dot{v}y) \right\} dx dy$$

which is exactly the Fourier transform of the pupil function. The formal limits of integration may extend from $-\infty$ to $+\infty$ because $f(x,y)$ is zero outside pupil area A.

The intensity distribution in the image plane is the squared modulus of the complex amplitude in the same plane, that is:

$$G(\dot{u}, \dot{v}) = |F(\dot{u}, \dot{v})|^2$$

and, since

$$g(s,t) = \iint_{-\infty}^{+\infty} G(\dot{u}, \dot{v}) \exp. \left\{ -i2\pi(\dot{u}s + \dot{v}t) \right\} d\dot{u} d\dot{v}$$

it follows that:

$$\begin{aligned} g(s,t) &= \iint_{-\infty}^{+\infty} F(\dot{u}, \dot{v}) \cdot F^*(\dot{u}, \dot{v}) \exp. \left\{ -i2\pi(\dot{u}s + \dot{v}t) \right\} d\dot{u} d\dot{v} \\ &= \iint_{-\infty}^{+\infty} f(x,y) \cdot f^*(x-s, y-t) dx dy \end{aligned}$$

By shifting the origin to make each pupil function symmetrical about the origin, this expression for $g(s,t)$ becomes:-

$$g(s,t) = \iint_{-\infty}^{+\infty} f(x + s/2, y + t/2) \cdot f^*(x - s/2, y - t/2) dx dy$$

(Fig. 3.4a)

If the coordinate axes are rotated through an angle $\psi = \tan^{-1} t/s$, the spatial frequency variable along the new x-axis is $\dot{s} = \sqrt{s^2 + t^2}$, and is correspondingly zero

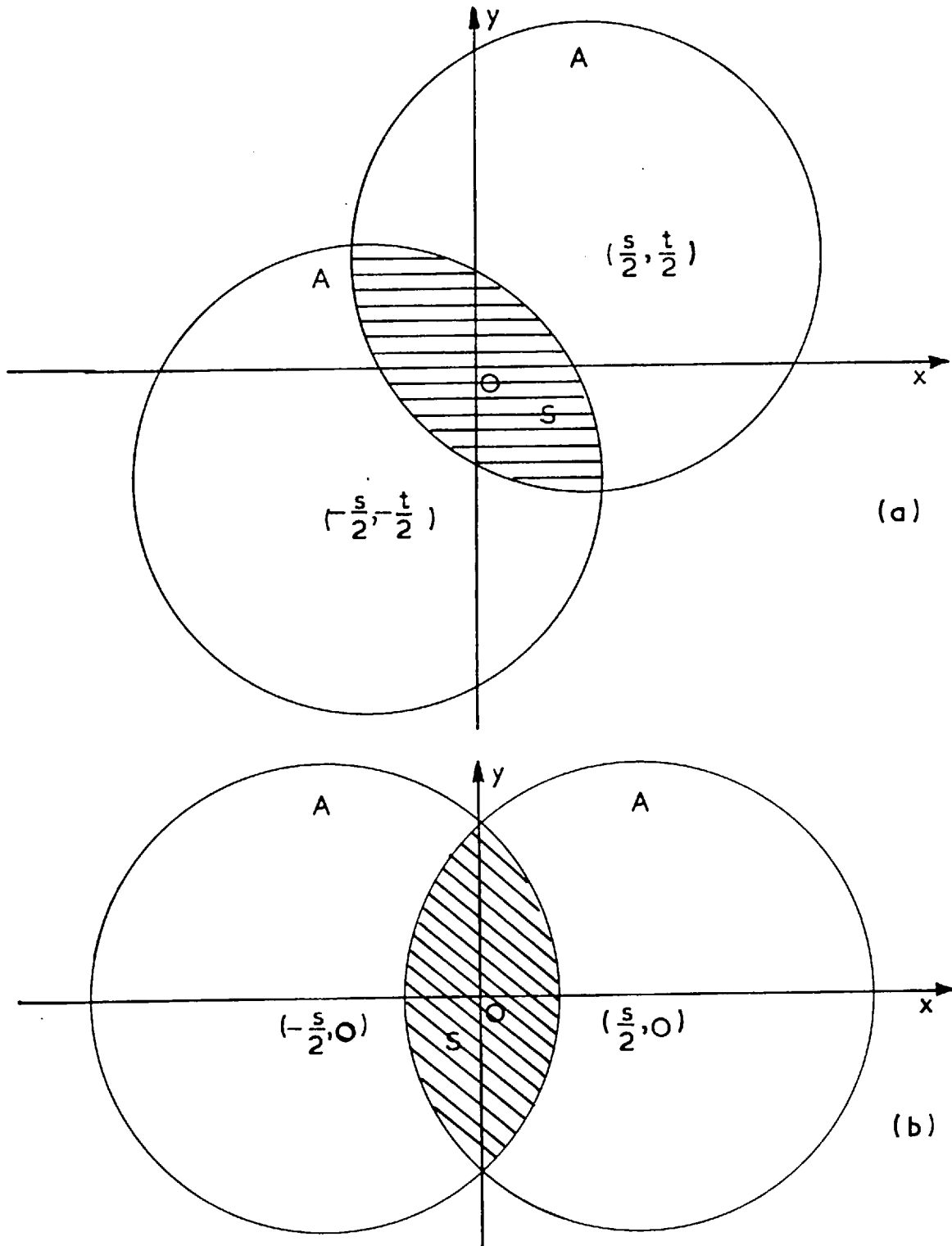


FIG. 3.4

along y-axis as described in section 3.2. The superscript on s will be omitted in what follows since the component frequencies (s,t) are not needed explicitly, so that the expression for g(s,t) becomes:-

$$\begin{aligned}
 g(s,0) &= \iint_{-\infty}^{+\infty} f(x + s/2, y) \cdot f^*(x - s/2, y) \, dx dy \\
 g(0,0) &= \iint_{-\infty}^{+\infty} |f(x,y)|^2 \, dx dy \\
 &= A, \text{ the area of either pupil.}
 \end{aligned}$$

where now the pupil function, $f(x,y)$, is referred to axes which are respectively perpendicular to and in the azimuth Ψ of the line structure in question. The formula for the transfer function is now:

$$D(s, \Psi) = 1/A \iint_{-\infty}^{+\infty} f(x + s/2, y) \cdot f^*(x - s/2, y) \, dx dy.$$

In the above expression, the factors $f(x + s/2, y)$, $f^*(x - s/2, y)$ are respectively zero outside pupils centred on the points $(-s/2, 0)$ and $(+s/2, 0)$. The optical transfer function is thus determined by an integral over the shaded area S common to two pupils centred on points $(-s/2, 0)$ and $(s/2, 0)$, with the frequency scale corresponding to the amount of shear s between two pupil (Fig. 3.4b). When the optical system is free from aberration, the optical transfer

function is then given simply by the ratio of the area common to two sheared pupils and the area of either pupil, that is:

$$D(s, \psi) = S/A$$

when $s = 2$ the two pupils are just touching each other and the common area S is zero. Since $s = (\lambda/n \sin \alpha)R$, the smallest spacing of object structure that can be just resolved will be given by $\xi_0 = 1/R = \lambda/(n \sin \alpha \cdot s) = 0.5 \lambda/n \sin \alpha$ which is the classical limit of resolution.

The above formula is also the basis of shearing interferometer used in determining the optical transfer function of an optical system.

Substituting the above form for $f(x,y)$ in the expression for $D(s, \psi)$ gives:-

$$D(s, \psi) = 1/A \iint_S \exp. ik \left\{ W(x + s/2, y) - W(x - s/2, y) \right\} dx dy$$

The region of integration is now over S , because outside this region the integrand is zero. By expanding each of the two wave-front aberration functions in a Taylor series this formula takes the form:

$$D(s, \psi) = 1/A \iint_S \exp. \left\{ iks \cdot V(x, y; s) \right\} dx dy$$

where $V(x, y; s) = \dot{W}_x(x, y) + 1/3! (s/2)^2 \dot{W}_x''(x, y) + \dots$

so that, from the knowledge of the wave-front aberration $W(x, y)$, the optical transfer function may be calculated for

any azimuth ψ .

3.6 Theory of the wave-front shearing interferometer.

The wave-front shearing interferometer uses the optical system itself as the input to an analogue device to solve the double integral in the transfer function formula:

$$D(s, \psi) = 1/A \iint_{-\infty}^{+\infty} f(x + s/2, y) \cdot f^*(x - s/2, y) dx dy$$

The combination of test-lens and interferometer may be considered as an analogue computer. This method was first suggested by Hopkins (1955), and used successfully by Baker (1955), Kelsall (1959) and Montgomery (1961). The main feature of the interferometer is to cause two sheared wave-fronts to interfere, and the total output light flux measured for different path differences gives then a measure of optical transfer function.

The interferometer unit used in the present work takes the form shown diagrammatically in Fig. 3.5 where the dielectric coated beam-splitter with a square hole at the centre is situated between two front-aluminised mirrors.

The collimated beam from the lens under test is split at one end of the beam-splitter, and after reflections the two beams cross at the central hole: they then recombine at the other end of the beam-splitter. Let the two recom-

bined wave-fronts be sheared by some means. Then the total light flux may be found as follow.

As indicated in figure 3.5, there will be two interference patterns produced by the pairs of beams labelled I and II. Each of the beams forming interference pattern I will have had one transmission and one reflection at the beam-splitter. On the other hand one beam in pattern II has two transmissions, and the other two reflections, at the beam-splitter. In both cases each beam is reflected once at each of the outer mirrors. The reflection losses at these mirrors is neglected.

It was at first intended to use both the interference pattern I and the complementary pattern II in the way described below. Although this was abandoned in favour of a simpler procedure, the theory will be outlined.

The total light flux in fringe plane I is given by:

$$F_I = \iint_{-\infty}^{+\infty} \left| tnf(x + s/2, y) + trf(x - s/2, y) e^{i\Delta_I} \right|^2 dx dy$$

where $f(x, y)$ is the pupil function, t and r are amplitude transmission and reflection coefficients for the beam-splitter, and Δ_I is the phase difference between two sheared wave-fronts. The square may be expanded to give

$$\begin{aligned}
F_I &= t^2 r^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |f(x + s/2, y)|^2 dx dy + t^2 r^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |f(x - s/2, y) e^{i\Delta_I}|^2 dx dy \\
&\quad + 2 \operatorname{Re} \left\{ t^2 r^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x + s/2, y) \cdot f^*(x - s/2, y) e^{i\Delta_I} dx dy \right\} \\
&= TR \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |f(x, y)|^2 dx dy + TR \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |f(x, y)|^2 dx dy \\
&\quad + 2TR \operatorname{Re} \left\{ e^{i\Delta_I} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x + s/2, y) \cdot f^*(x - s/2, y) dx dy \right\}
\end{aligned}$$

with $T = t^2$ and $R = r^2$. Writing:

$$A = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |f(x, y)|^2 dx dy$$

and:

$$T(s, \psi) \exp. i \theta(s, \psi) = \frac{1}{A} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x + s/2, y) f^*(x - s/2, y) dx dy$$

this becomes:

$$F_I = 2A TR + 2ATR \cdot T(s, \psi) \cos \left[\theta(s, \psi) + \Delta_I \right] \dots\dots(1)$$

Similarly the total light flux in fringe plane II is given by:-

$$F_{II} = A(T^2 + R^2) + 2ATR \cdot T(s, \psi) \cos \left[\theta(s, \psi) + \Delta_{II} \right]$$

or, since for the complementary pattern the phase difference is:

$$\begin{aligned}
\Delta_{II} &= \Delta_I \pm \pi, \\
F_{II} &= A(T^2 - R^2) - 2ATR \cdot T(s, \psi) \cos \left[\theta(s, \psi) + \Delta_I \right] \dots(2)
\end{aligned}$$

Subtraction of (2) from (1) gives then:

$$F_I - F_{II} = -A(T - R)^2 + 4ATR \cdot T(s, \psi) \cos \left[\theta(s, \psi) + \Delta_I \right] \dots(3)$$

Thus, if the phase difference Δ_I is varied in steps of $\pi/2$ the real part and imaginary part of the optical transfer

function can be obtained.

From (3), putting successively $\Delta_I = 0, \pi/2, \pi$ and $3\pi/2$ gives

$$(F_I - F_{II})_{\Delta_I = 0} = -A(T - R)^2 + 4ATR.T(s, \psi) \cos \theta(s, \psi) \dots (4)$$

$$(F_I - F_{II})_{\Delta_I = \frac{\pi}{2}} = -A(T - R) - 4ATR.T(s, \psi) \sin \theta(s, \psi) \dots (5)$$

$$(F_I - F_{II})_{\Delta_I = \pi} = -A(T - R) - 4ATR.T(s, \psi) \cos \theta(s, \psi) \dots (6)$$

$$(F_I - F_{II})_{\Delta_I = \frac{3\pi}{2}} = -A(T - R) + 4ATR.T(s, \psi) \sin \theta(s, \psi) \dots (7)$$

The difference (4) - (5) gives the real part of the transfer function

$$F_0 - F_{\pi} = 8ATR.T(s, \psi) \cos \theta(s, \psi) \dots (8)$$

while (7) - (5) gives the imaginary part

$$F_{\frac{3\pi}{2}} - F_{\frac{\pi}{2}} = 8ATR.T(s, \psi) \sin \theta(s, \psi) \dots (9)$$

From $\sqrt{(8)^2 + (9)^2}$, normalised to make $T(s) = 1$ when $s = 0$, the modulus and the phase shift $\arg. \left[(8) + i(9) \right]$ of the transfer function for the optical system may be obtained.

Because it proved difficult to set the slit image patterns in beams I and II on a scanning grating with sufficient precision a single-beam method was found to be more useful in practice. This is described later.

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CHAPTER IV

DESCRIPTION OF THE APPARATUS

4.1 Design of the interferometer

An interferometer for the present work should be stable, compact, insensitive to vibration, and easy to adjust; the two beams must also be completely separated so that a path-changing prism and suitable shear-plates may be inserted,

None of the standard types of interferometer has proved to be particularly suitable for the present work. Twyman and Green's (1920-21) adaptation of the standard Michelson (1890) type two-beam interferometer was used by Baker (1955), and subsequently by Kelsall (1959), in determinations of the optical transfer function in the presence of certain aberrations. The former replaced the two plane mirrors by roof mirrors in order to separate the outgoing and returning beams, while the latter used cornercube reflectors which also made the interferometer more stable against vibration. An objection to this type of interferometer is its size and the complicated adjustments which were found to be necessary. For example, the small residual errors in angle in the cornercubes cause the reflected beams not to be parallel to the incident beams and sets of tilt-compensating wedge prisms were required.

The Mach-Zehnder (1891) type interferometer has completely separated beams, but it is difficult to adjust and

easily disturbed because of the use of two separate beam-splitters. This type of interferometer was adapted by Bates (1947) for use as a shearing interferometer. Relative shear of the two wave-fronts may be introduced by rotating the second beam-splitter in the Mach-Zehnder type interferometer. Since the interferometer can be placed near the focus of the wavefront under test it needs not have a wide aperture. Simplified and more compact shearing interferometer was developed by Drew (1951) and Brown (1954).

Another type of interferometer employs two Kösters double-image prisms which was described among others by Saunders (1955, 1957). This type of interferometer can be made very compact and the most distinct advantage is that only one shear plate need be used, which simplifies the construction of the shear-plate mechanism and eliminates the differential path-difference which is introduced when two shear-plates are used. The drawback of this type of interferometer lies in its sensitivity of relative tilt between the wave-fronts which imposes very severe tolerances on the Köster prisms. The introduction of tilt-compensating plates could be a solution but this means more complication in both construction and adjustment.

A type of interferometer that seems to satisfy the stringent requirements of the present work is the type used by Montgomery (1961) for the determination of optical

transfer functions. This system was adopted in the present work: it is described in what follows.

4.2 The interferometer unit

The present type of interferometer is related to the Mach and Zehnder type, and more particularly the system used by Brown. The two beam-splitters are incorporated as one plate with a one inch square hole at the centre. In order to achieve the property of self compensation for tilt, each beam is made to undergo reflection at each of the interferometer mirrors. The resulting interferometer is compact, rigid and has been found to be extremely stable and insensitive to vibration; the overall size measures approximately 5 x 4 x 3 inches. A schematic diagram of the interferometer unit, approximately to scale, is shown in fig. 3.5.

The light beam from the slit source is collimated by the lens under test and is incident at an angle of 35° near to one end of the semi-reflecting side of the beam-splitter (Fig. 4.1). This angle is chosen to make the interferometer as compact as possible subject to there being no vignetting introduced when the shear plates and path changing prism are operated. The reflected and transmitted beams, leaving the beam-splitter, pass through a path-changing prism and a compensating prism respectively, and are then reflected from the side-mirrors. They pass again through

GENERAL LAY-OUT OF THE APPARATUS

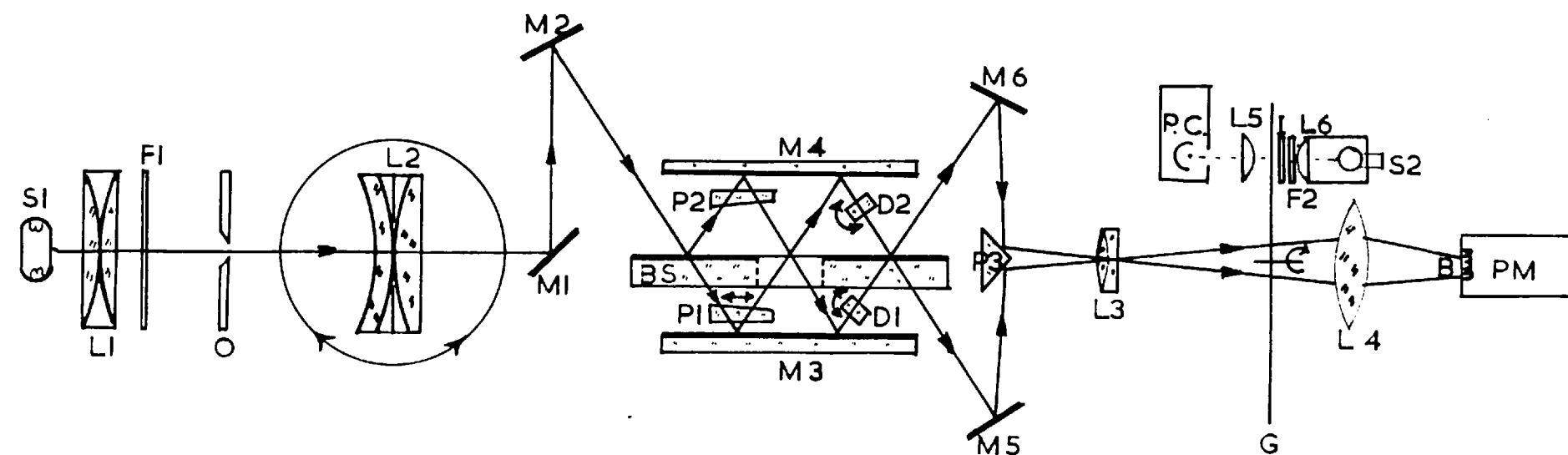


FIG. 4.1

the compensating prism and path changing prism after reflection. Then after crossing at the central square hole, both beams are again reflected at the side-mirrors, pass through the shear plates, and finally recombine at the other end of the beam-splitter. In fringe plane I both beams undergo one transmission and one reflection at the beam-splitter so that both have the same light intensity even if the beam-splitter does not give exactly equal transmission and reflection. This is true only for a dielectric coated beam-splitter, since for an aluminium beam-splitter the reflection coefficient for air-aluminium is not normally equal to that for a glass-aluminium interface. In fringe plane II one beam undergoes two transmissions and the other two reflections at the beam-splitter, so that the intensities in the two beams may be significantly different when the beam-splitter does not give equal transmission and reflection. This leads to loss of contrast in the fringes.

Supposing one beam has an intensity I_1 and the other has I_2 . When the two beams interfere, the resulting intensity is given by the formula:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

where δ is the phase difference between two beams. The Michelson visibility, or contrast, of the fringes is given by:

$$V = \frac{I_{\text{max.}} - I_{\text{min.}}}{I_{\text{max.}} + I_{\text{min.}}}$$

or

$$V = \frac{2\sqrt{I_1/I_2}}{(1 + I_1/I_2)}$$

If, for example, $I_1 = \frac{1}{2} I_2$, the visibility is given by

$$V = \frac{2\sqrt{0.5}}{(1 + 0.5)} \approx 0.9$$

It will be seen that inequality in intensity of the interfering beams has little effect on the visibility of fringes.

The frame of the interferometer unit is machined from a solid block of Meehanite closed-grain cast iron, which is found to be very rigid and stable with time. The two faces on which the mirrors are located are ground accurately parallel and perpendicular to the base piece; the separation between the two sides is two and a half inches. The beam-splitter is held in a strain-free Duralumin mount by means of four pairs of phosphor-bronze spring clips, acting directly against each other in each pair and the mount is in turn fixed firmly to the base piece of the interferometer unit between the two side-mirrors. The orientation of the beam-splitter is set to be parallel to the two side-mirrors and perpendicular to the base. In order to have the two beams crossing at the centre of the square hole

in the beam-splitter, the latter is not exactly half way between the two mirrors. The position of the beam-splitter giving no vignetting is found by simple ray tracing through the system. The two side-mirrors are not placed directly against the metal frame, but they are supported by three thin rubber pads and held in place by three screws acting directly opposite to each rubber pad to allow very slight adjustment of the mirrors with respect to the beam-splitter.

4.3 The beam-splitter and side-mirrors

The beam-splitter measures 12 x 5 cm., and is $\frac{1}{2}$ inch thick. The square hole at the centre of the beam-splitter was made by means of an ultrasonic drill; the width of glass left on either side of the hole is $\frac{1}{2}$ inch to prevent any distortion that might occur during manufacturing. The parallelism between its surfaces is not very critical, since any deviation caused by this will be the same and in the same sense for each of the two beams. Flatness of each surface is very important, and a tolerance of $\lambda/10$ over the whole surface area was imposed. The tolerance on the curvature of the beam-splitter is set so that the relative tilt between two emergent beams is not greater than 2 seconds of arc; in this case the sag at the centre of the beam-splitter of 12 cm. length should be less than $\lambda/4$.

The semi-reflecting side of the beam-splitter is coated

with a dielectric semi-reflecting coating (Balzers Type TF-TSIT-55) and the rear surface is coated with an anti-reflection coating (455/139) which reduces the percentage reflection to 0.125% for the mercury green line. A dielectric-coated beam-splitter is chosen because of its low absorption, which is less than 2% of the incident beam whereas as large as 30% absorption may be found with an aluminium beam-splitter; since there are two transmissions and two reflections at the beam-splitter, this is a factor of some importance. The beam-splitter was chosen to have nearly equal reflection and transmission coefficients for the wavelength of the mercury green line, $5461 \text{ \AA}$.

The two side-mirrors are front-aluminised and of interferometer quality. The overall flatness of each mirror was given a tolerance of $\lambda/10$. The thickness of each mirror is $3/8$ inch, sufficient to prevent any distortion.

It has been shown by Montgomery (1961), using vector form of the law of reflection, that this type of interferometer has self-compensation for relative tilt between the two wave-fronts. For example, when one of the outer mirrors is tilted both beams will be tilted by the same amount and in the same sense. Although the direction of two emergent beams may change slightly, they will remain parallel: only a small relative shear and path difference are introduced. If the tolerance on tilt is placed at 2 seconds of arc the

two mirrors and the beam-splitter are only required to be parallel with an accuracy of approximately ten minutes of arc. In fact the parallelism between the two mirrors and the beam-splitter can easily be set to better than this accuracy.

Kelsall (1959) showed that relative tilt between the two emergent wave-fronts in the direction of shear is more serious than that in the direction perpendicular to it. If the tilt angle is two seconds of arc, the error in the modulus of the transfer function, $T(s)$, for $s = 0.6$ is approximately 2%, which is of the same order as other errors in the measurement. The resulting error in a phase-measurement is more serious, the error is around 5%.

Montgomery (1961) found that the path-difference between two emergent beams resulting from an error of parallelism is given by the formula:

$$\frac{12 \theta^2}{\sin 55^\circ} \times \frac{10^5}{5} \quad \text{wavelengths.}$$

where θ is the angle of deviation of either mirror relative to the beam-splitter. If the tolerance on tilt of either mirror is set at 10 minutes of arc this path-difference will be 2 wave-lengths. Also the lateral separation between the two emergent beams, measured along the surface of the beam-splitter, is given by 12θ cms., where θ is again measured in radians. For θ equal to ten minutes of arc, this separation will be 1/3 mm. The tolerance on relative tilt of the wave-

fronts is thus satisfied if errors of parallelism between the outer mirrors and the beam-splitter are kept less than ten minutes of arc.

Ghost-images produced at the second surface of the beam-splitter, are avoided in the present case, the beam-splitter being $\frac{1}{2}$ inch thick. The ghost-images are then approximately 1 cm. away from the real image, and can easily be blocked out by applying a matt black paint on the unwanted part of the beam-splitter.

4.4 Path changing prism and compensating plate

The function of the path-changing prism is to change the path difference between two arms of the interferometer in steps of $\lambda/4$, so that the real and imaginary parts of the optical transfer function may be obtained. The prism has an angle of 9 minutes of arc, and measures $1\frac{5}{8} \times 1 \times \frac{1}{4}$ inches. Each polished surface is flat to $\lambda/10$ overall the surface, and all surfaces are coated with an anti-reflection coating. The path-changing prism is mounted on a universal adjustable kinematic mount, based on the principle of point, line and plane. The prism may be rotated about an axis perpendicular to its plane and also tilted about the line of slope relative to the mirror and beam-splitter. The kinematic mount is in turn fixed to a $1/8$ inch diameter silver steel rod which projects down into the interferometer beam from

a kinematic slide on a bridge position overhead. The steel rod is coupled to the kinematic slide through a plane brass bearing fixed to the slide, and having coarse and fine adjustments to enable the prism to be rotated about axes parallel to its plane and perpendicular to the line of slope. The translation of the path-changing prism along the direction of the line of slope is affected by means of a micrometer. This can be set accurate to 0.01 mm., and it contacts the kinematic slide through a steel ball at one end. The kinematic slide is spring-loaded against the steel ball and micrometer head by means of two long helical springs.

The bridge is made from a piece of 7/8 inch thick cast iron plate and resting on three legs. Each leg carries an adjustable screw tip at the end to allow height-adjustment and levelling of the bridge.

The compensating plate is exactly similar to the path-changing prism and is mounted in the opposite arm of the interferometer. It has the same adjustments as the path-changing prism. It has no translation, and is hung from the bridge by means of a simple screw clamp. The sole purpose of the compensating plate is to compensate the deviations of the light beams as they pass through the path-changing prism, thus avoiding relative tilt between the wave-fronts being introduced by the path-changing prism.

It also serves approximately to equalise the path length in two arms.

The relation between the translatory movement of the path-changing prism and the optical path difference introduced may be found as follow. Suppose that δt is the change in thickness of glass when the prism is displaced a distance δl , then, for a prism of angle θ , $\delta t = \delta l \tan \theta$, so that the optical path difference introduced when the light beam travels through the prism twice is given by $2(n - 1) \times \tan \theta \cdot \delta l$, the angle θ being small. To introduce an optical path-difference of $\lambda/4$, the prism has thus to be moved through a distance:

$$\delta l = \frac{\lambda}{4 \times 2(n - 1) \tan \theta}$$

Putting $\lambda = 0.0005 \text{ mm.}$, $n = 1.5$ and $\theta = 1.0 \text{ minute}$, gives

$$\delta l \approx 0.43 \text{ mm.}$$

for the displacement needed to introduce a path-change of $\lambda/4$.

Due to the double passage of light through the path-changing prism and compensating plate there will be compensation for any shear introduced by them. If the prism is rotated about an axis parallel to its face and perpendicular to the line of slope, so that the angle of incidence is increased, the shear produced on transmission is also increased, but the angle of incidence of the light beam

at the prism after reflection at the mirror will be decreased. The shear produced on this transmission is decreased, so that to the first approximation the total shear produced will be independent of the tilt and position of these two components.

The angular deviations produced by the path-changing prism and compensating plate do not cancel out completely; but, in fact, these deviations change slightly when either component is rotated about an axis perpendicular to its face or about an axis parallel to its face and perpendicular to the line of slope. Use is made of these deviations to eliminate any residual tilt between the finally emergent wave-fronts resulting from slight errors in the beam-splitter or mirrors.

The deviating power of a small angle prism is proportional to the angle between its two faces. If the median plane of the prism lies in the YZ plane (fig. 4.2), the prism having its line of greatest slope in the Z-direction, the deviation will be maximum in planes parallel to XZ, and is then zero in the Y-direction. If the prism is now rotated about the X-axis through an angle θ , the deviation parallel to the XZ plane becomes $\delta \cdot \cos\theta$, and along Y-direction will be $\delta \cdot \sin\theta$, where δ is the maximum deviation, namely when $\theta = 0$. Any vertical tilt of the wave-front may be compensated by counter-rotation of the second prism, which merely needs a rotation through an angle $-\theta$. The

deviation of the second prism parallel to the XZ-plane is again $\delta \cos \theta$, but along the Y-direction it will be $-\delta \sin \theta$. Thus the net deviation produced by two prisms along the Y-direction is zero; while it is equal to $2\delta \cos \theta$ parallel to the XY-plane (Jenkins and White, 1951).

Rotation of the prism about the Y-axis produces horizontal tilt in a manner explained by De (1955) (Fig. 4.3). Montgomery (1961) demonstrated this effect by inserting a prism of ten minutes angle, in one arm of a Twyman-Green interferometer, with its axis vertical. As in the present instrument, the light passed through the prism, suffered reflection at an angle of about 35° at a mirror, and then passed again through the prism. On rotating the prism through a known angle about a vertical axis, the magnitude of tilt in the horizontal plane was determined, and found to be minimum for an angle of incidence at the prism of 35° . Above or below this value the deviation tended to increase. The prisms are accordingly set at this angle in the interferometer being described. Rotation about this vertical axis was used as a fine adjustment to eliminate relative tilt between the two interfering wave-fronts. Coarse adjustment of this was made using counter-rotation of the two prisms about a horizontal axis as described above. Rotation of the prism about the Z-axis gives negligible effect on the tilt of wave-front.

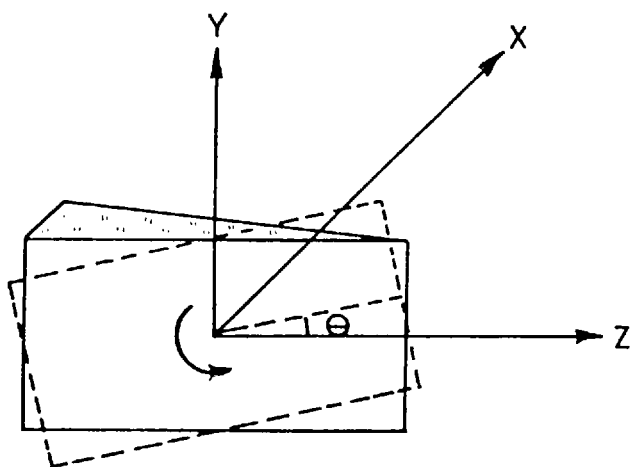


FIG. 4.2

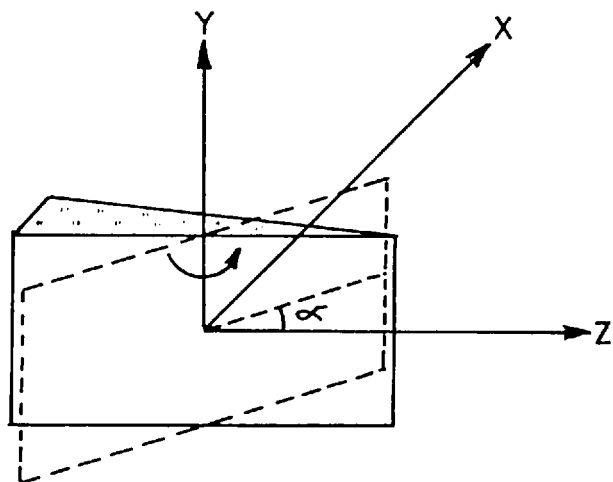


FIG. 4.3

4.5 Shear-plates and their operation

The shear between two wave-fronts is produced by the rotation about vertical axes of two plane-parallel glass plates, one in each arm of the interferometer. The shear plates are made from the glass of the same quality as that of the path-changing prism or compensating plate with each polished surface flat to $\lambda/10$; the parallelism between two surfaces is made to have an error less than 2 seconds of arc. All surfaces are coated with anti-reflection coatings. The two shear plates must be of exactly equal thickness, otherwise a large differential path-difference may be introduced when they are rotated to produce shear. To ensure this, the two shear plates are cut from the same piece of glass, after being optically worked. The size of each shear plate measures 1.3 x 1.5 x 1.25 cm. When one shear plate is put into the beam with its normal making an angle i with the direction of the incident light beam, the amount of lateral shear introduced may be found as follows.

Referring to Fig. 4.4. let t be the thickness of the shear plate of refractive index μ ; i, i' , are the angles of incidence and refraction respectively. The lateral displacement δ is seen to be given by

$$\delta = (AB) \sin (i - i')$$

or, putting $(AB) = t/\cos i'$, and expanding the sine.

$$\delta = t \sin i \left(1 - \frac{\sin i' \cos i}{\sin i \cos i'} \right)$$

By the law of refraction, $\sin i' / \sin i = \mu$, and writing $(\mu \cos i')^2 = \mu^2 - \sin^2 i$, one has finally

$$\delta = t \sin i \left\{ 1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right\}$$

If the angle i is small this reduces to

$$\delta = \frac{t i (\mu - 1)}{\mu}$$

The error in this approximation is only 0.5% even for $i = 10^\circ$.

Since the two shear plates act in opposite directions the total shear in our case is 2δ . The fractional shear s is defined to be (actual shear)/(radius of pupil), so that

$$s = \frac{2t \sin i}{a} \left\{ 1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right\}$$

where a = radius of the pupil. The value of s given by this formula is thus the reduced spatial frequency used in the diffraction theory of the optical transfer function described earlier.

If the axis of rotation of either shear plate is not parallel to the outer faces of the shear plate, a vertical shear is introduced which will vary with rotation. The angle determining the amount of vertical shear for any position of the shear plate is the angle between the projection of the normal to the shear plate and the direction of the beam in the plane containing the beam and the axis of rotation. If

the axis of rotation has an error of one degree, when the shear plate rotates through an angle of 10 degrees the change in vertical shear is approximately 1½% of its initial value.

If the two shear plates do not rotate through exactly the same angle, or if the angles of incidence at the shear plates are not exactly equal, a differential optical path-difference may be introduced. The effect of such a differential path-difference on the measurement of $T(s, \psi)$ is of little importance, provided it is never more than a few wave-lengths in magnitude. It merely affects slightly the output electrical frequency. But, to measure the phase angle accurate to $\pm 5\%$, the differential path-difference must either be less than $\lambda/50$, or suitable corrections made to the measurements.

Referring again to Fig. 4.4, the path-difference introduced when a shear plate of thickness t and refractive index μ is put into the beam, with an angle of incidence i , is given by

$$\mu_{AB} - AC = \frac{\mu t}{\cos i'} - \frac{t \cos(i - i')}{\cos i'}$$

For zero angle of incidence this reduces to $(\mu t - t)$, so that the path-difference introduced when the shear plate rotates from $i = 0$ to i is given as

$$\delta P = \frac{\mu t}{\cos i'} - \frac{t \cos(i - i')}{\cos i'} - (\mu t - t)$$

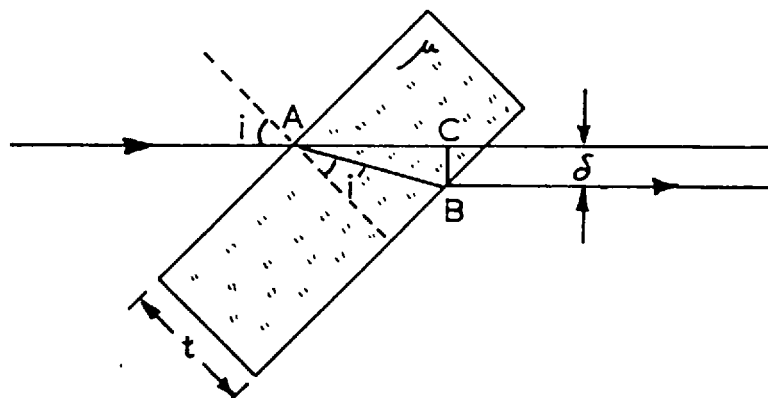


FIG. 4.4

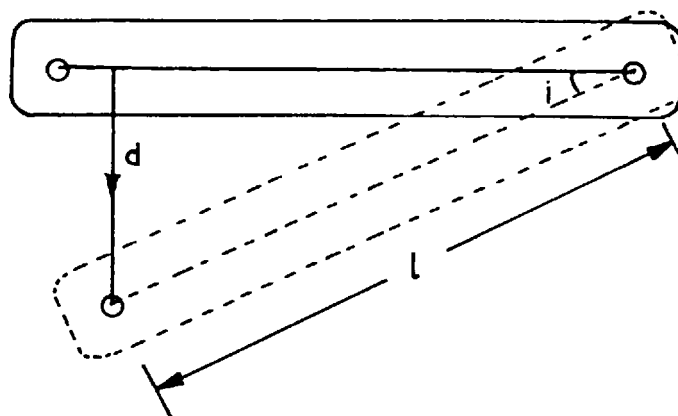


FIG. 4.5

which, expanding $\cos(i - i)$, easily reduces to

$$\delta p = t \left\{ (1 - \cos i) - (\mu - \sqrt{\mu^2 - \sin^2 i}) \right\}$$

If, now, the angle of incidence at one shear plate differs from that at the other by δi , the differential path difference introduced will be given by a differentiation of the above formula. After a slight rearrangement of terms, one finds,

$$\Delta(\delta p) = t \sin i \left\{ 1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right\} \delta i$$

where i is the mean angle of incidence.

The fractional shear has been found to be given by the formula

$$s = \frac{2 t \sin i}{a} \left\{ 1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right\}$$

so that the differential path-difference introduced for any shear s , is

$$\Delta(\delta p) = \frac{as}{2} \cdot \delta i$$

We thus have the important result that the differential path difference introduced is directly proportional to the fractional shear s . For δi less than $\frac{1}{2}$ degree, the differential path difference is less than one wave-length for $i = 0$, but when $i = 9^\circ$ it becomes sixteen wave-lengths.

If the tolerance on differential path-difference, namely

$\lambda/50$ is applied, the maximum permissible value of δi is approximately two seconds of arc, which is an extremely severe tolerance. Fortunately, as will be seen later, a phase-shift in the transfer function proportional to s

merely corresponds to a shift of origin in the spread-function, and in no way invalidates the measurements.

In order to cover the range of shear from $s = 0$ to $s = 1.2$ the shear plates must rotate together in the same sense through an angle of approximately twenty degrees. Each shear plate is mounted in an adjustable mount so that it can be tilted about the axis parallel to its face and perpendicular to its mechanical axis of rotation. Each mount is attached to a $3/16$ inch diameter silver steel rod, which projects down from the bridge to the interferometer unit. The steel rods rotate in a bearings consisting of two separated $\frac{1}{2}$ inch diameter angular contact ball races, which are fixed to the bridge. The separation between the two steel rods is not adjustable, being fixed initially to give the required positions of the shear plates. The rotations of the two shear plates are coupled by using a pair of identical linkage arms made from two 5 inches long Duralumin strips. There are coarse and fine adjustments provided so that each shear plate can be adjusted separately relative to the linkage arms. The parallel linkages carry a small ball race at the end which is kept in contact with a surface perpendicular to the motion of a kinematic slide by a weight acting through a pulley and string. The kinematic slide is in turn driven by an Achimedian heart-shaped cam, and using a ball-race cam-follower of the size for which the

cam was designed. The cam itself is mounted on a $\frac{1}{4}$ inch diameter ground steel rod, which passes through the bridge and rotates in a bearing made from two separated angular contact ball races. The drive shaft is connected by flexible couplings through two 100:1 reduction gears to an Evershed and Vignoles 1,500 r.p.m. synchronous motor. The motor and gear boxes are mounted on a heavy cast iron base plate which is isolated from the bench by means of a thick latex foam rubber cushion. The speed of the cam is thus one revolution per $6\frac{2}{3}$ minutes.

The rotation of the shear plates is not exactly linear with the rotation of the cam; in fact, the rotation of the cam is proportional to the sine of the angle of rotation of the shear plates (Fig. 4.5).

The effective length of each of the linkage arms is $4\frac{1}{2}$ inches and the stroke of the slide is $1\frac{1}{2}$ inches, so that the angle of maximum rotation of each of the shear plates is: $i = \sin^{-1} \frac{1.5}{4.5} = 19.45$ degrees.

If l is the length of the linkage arm and it has an angle of rotation i when the cam moves one end through a distance d (Fig. 4.5), then

$$\sin i = \frac{d}{l}$$

and the effect of an error δl in the length of one of the linkage arms is then given by

$$\delta l = - \frac{1}{\tan i} \cdot \delta i$$

where δi is the error in incidence angle produced at the corresponding shear plate.

It has been proved earlier that the differential path difference is given by

$$\Delta(\delta p) = \frac{as}{2} \cdot \delta i$$

Again applying the tolerance of $\lambda/50$ for $\Delta(\delta p)$, for a shear $s = 1$ and radius of pupil $a = 2.5$ mm., which gives a permissible error $\delta i = 8.7 \times 10^{-6}$ radians, the tolerance on the length of the linkage arm is

$$\delta l \approx \pm 0.004 \text{ mm.}$$

which assumes that $i = 14^\circ$ for $s = 1$. The appropriate calibration curve being given later. This is again a severe tolerance, but ~~one~~ whose effects may be compensated for in interpreting the measurements made with the interferometer.

4.6 Light source

The light source used to illuminate the slit object is an 80 watts, 240 volts type MB/U medium pressure mercury vapour lamp with the outer diffusing bulb cut off. This lamp gives an average light output through its life of about 2,700 lumens. Although other light sources, such as a

sodium lamp might be used, the mercury vapour lamp is chosen because of its high intensity with sufficiently narrow spectrum band width. The 5461 Å⁰ mercury green line is isolated by means of a Wratten No. 77 A green filter which transmits about 70% of the incident light at this wavelength. This particular lamp with its internal pressure and all glass shell, is capable of fracture. Because of the danger of explosion, it is important that the lamp should never be operated with the cover off. Some ultra-violet radiation is also emitted by the lamp, so it should never be looked at directly without the filter in place. The lamp is operated from a 200 volts, 320 watts Westat constant voltage power supply (Fig. 4.6a), which gives a d.c. voltage constant to better than 3%, with r.m.s. ripple less than 0.2% of the mean d.c. voltage. A suitable choke and ballast resistance (Fig. 4.6b) is put in series with the lamp to control the current through it, and also to prevent the power supply from over-loading when the lamp is started. A suitable current through the lamp is found to be about 0.8 amp. The fluctuation of the output light flux when this type of lamp was operated from the a.c. mains was studied by Vienot (1958) who found that a variation of 10% in mains supply voltage gave a variation of 20% in lamp current and 30% in the output light flux.

The lamp is housed in a light-tight housing with light

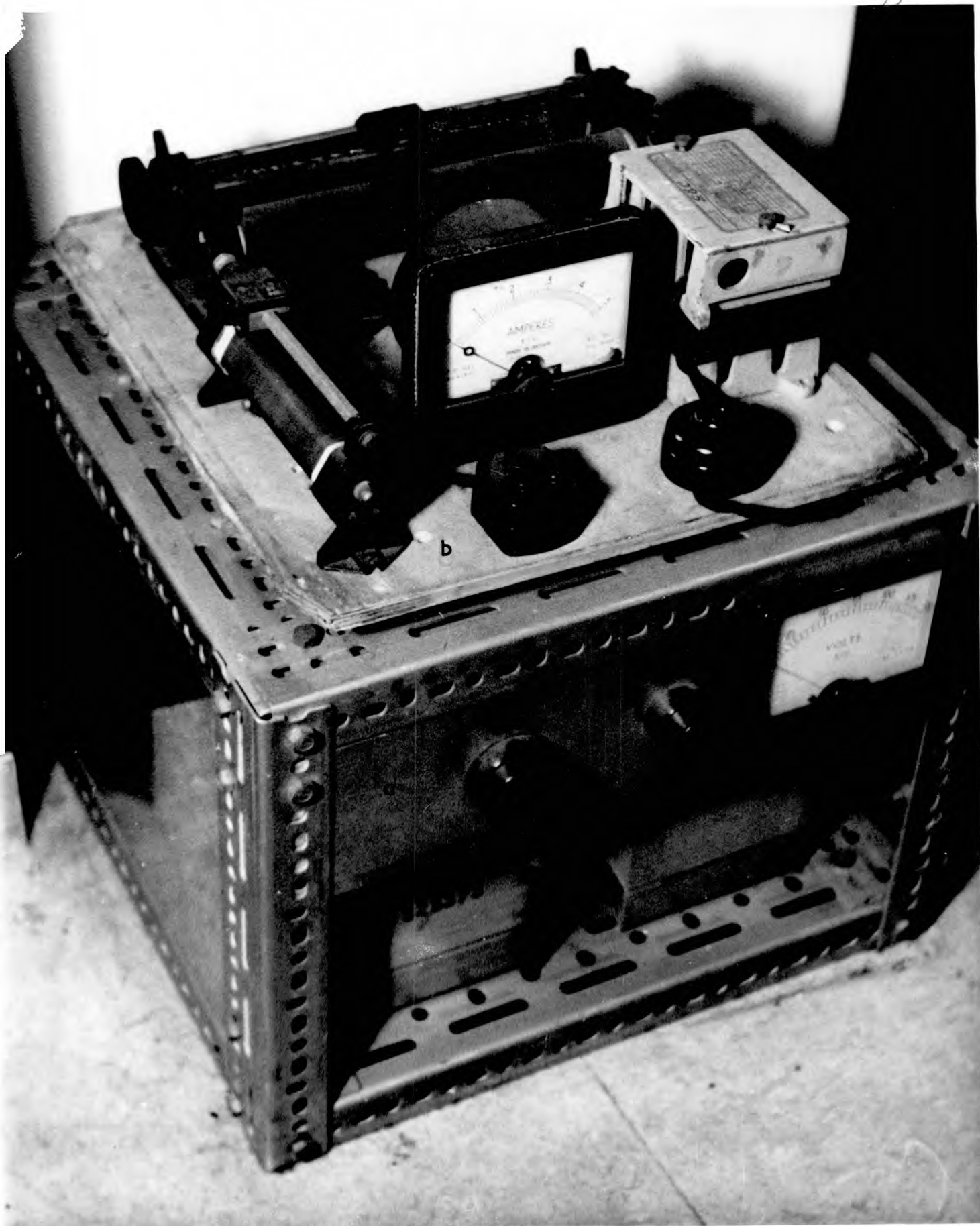


FIG. 4.6

traps, (Fig. 4.7a). It has ventilation passages at both top and bottom to prevent the lamp from over heating. Both the internal and external walls of the lamp housing are painted in matt black to keep the stray light down to a minimum. There is only one rectangular opening to let the light through. The whole system has the necessary adjustments to bring the image of light source parallel to, and on the centre of, the object slit. Since the light source has only to illuminate the slit object there is no need for more accurate location of its position: manual adjustments of a thumb screw on the vertical stand are all that is needed.

An ideal monochromatic source will give a wave train of infinite length, so that the spectral line profile would be a delta function. But practical sources give wave trains of finite length and the spectral line profile is of finite width. When two wave trains having a path-difference between them interfere the visibility of the fringes produced will not be 100%, but this effect is small if the path-difference between them is small compared with the length of wave train. Hopkins (1957) and Wolf (1955) have applied the theory of partial coherence to the case of coherence with a finite spectral range. Hopkins discussed the form of the chromatic coherence factor; assuming that the line profile was of rectangular form the visibility function may be written:-

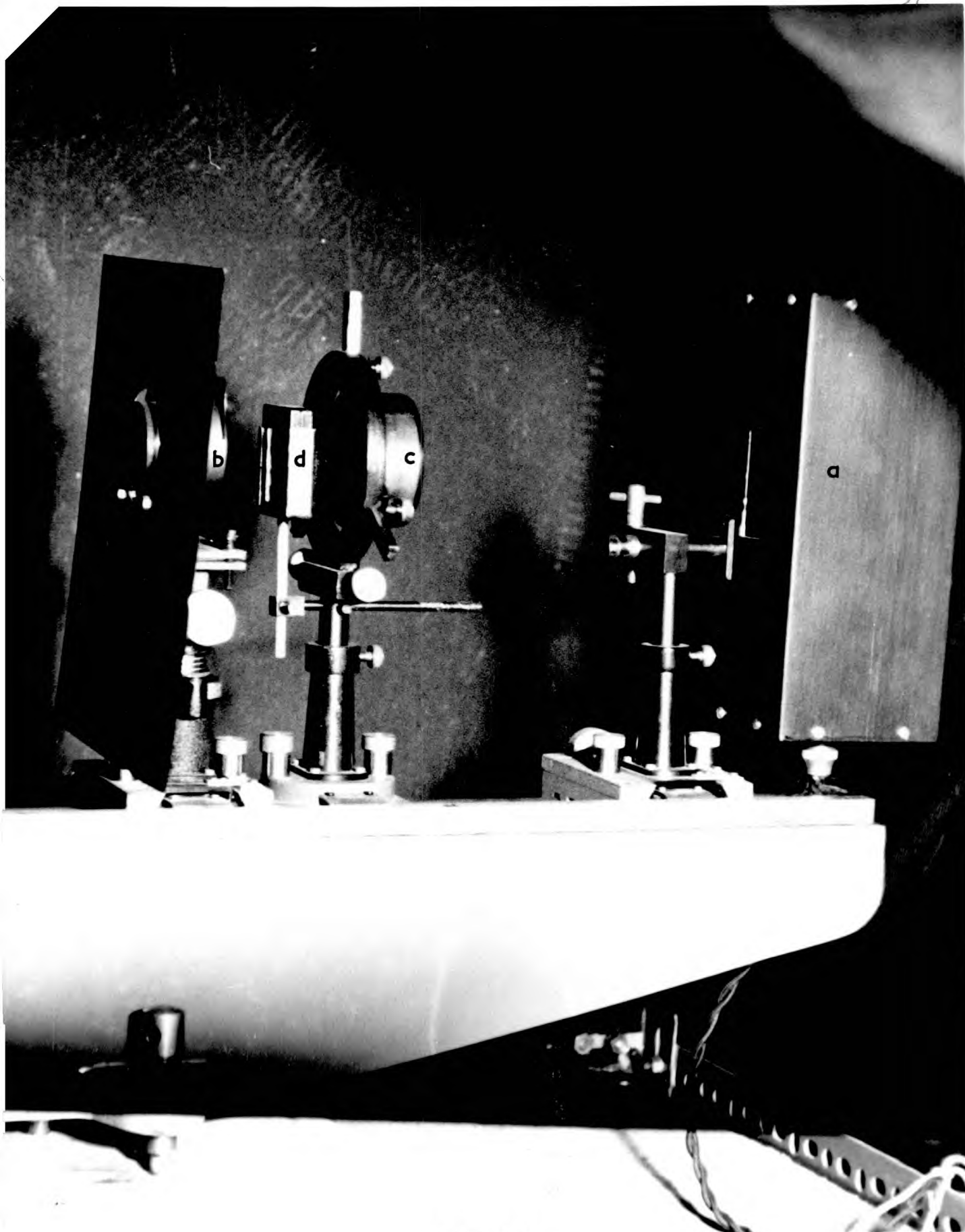


FIG. 4.7

$$(V_{21})_c = |\tau_{21}|_c = \left| \frac{\sin \pi(l_1 - l_2) \frac{\delta\lambda}{\lambda^2}}{\pi(l_1 - l_2) \frac{\delta\lambda}{\lambda^2}} \right| = \left| \frac{\sin \Psi}{\Psi} \right|$$

Where l_1, l_2 , are the optical path length along each beam, $(\tau_{21})_c$ is the chromatic coherence factor and $(V_{21})_c$ is the visibility which is in fact the modulus of $(\tau_{21})_c$.

If the tolerance for the visibility is set so that

$$\begin{aligned} (V_{21})_c &\geq 0.99 \\ \text{Then } 1 - \frac{\Psi^2}{6} &\geq 0.99 \\ \text{or } l_1 - l_2 &\leq 0.078 \frac{\lambda}{(\delta\lambda/\lambda)} \end{aligned}$$

The spectral line profile of the mercury green line 5461 Å produced by a medium pressure mercury vapour lamp was measured by Kelsall (1959) using a recording spectrograph. The half width (the width of the spectral line profile where the intensity falls to 50% of the maximum intensity) of the mercury green line 5461 Å was found to be 2.2 Å and the line profile was quite symmetrical in shape so that the phase term $(\beta_{21})_c$ in the expression $(\tau_{21})_c = (V_{21})_c \exp. i (\beta_{21})_c$ is zero.

Substituting this value in the expression for $|l_1 - l_2|$ gives

$$\begin{aligned} |l_1 - l_2| &\leq \frac{0.078 \times 5461}{2.2} \lambda's \\ &\approx 200 \text{ wavelengths} \end{aligned}$$

This means that the maximum path-difference for $(V_{21})_c \geq 0.99$

is about 200 wavelengths. In the present apparatus the path-difference is well within this tolerance.

If the visibility factor $(V_{21})_c$ is taken into account, the expression for evaluating the optical transfer function derived previously becomes

$$F_I - F_{II} = -A(T - R)^2 + (V_{21})_c \cdot 4A \cdot TR \cdot T(s, \psi) \left[\cos \theta(s, \psi) + A_1 \right]$$

It thus simply acts as a scaling factor, and will not cause any inaccuracy in the measurement: it merely reduces slightly the signal-to-noise ratio when the path-difference exceeds this tolerance.

4.7 The slit object

The slit object used is a two-speed Hilger spectrometer slit with symmetrical opening (Fig. 4.7b). Each division on the engraved drum corresponds to an increase in slit width of 0.001 mm. in the range from zero up to 0.1mm; and from 0.1 mm. onwards each division corresponds to 0.005 mm. This object slit is mounted so that it can be made perpendicular to the shearing axis defined in the instrument.

The slit is critically illuminated by means of a good quality condenser lens, of $2\frac{1}{2}$ inches focal length (Fig. 4.7c). This images the light source on to the slit, and therefore need not be of very high quality. If, however, the aberration is too large it may cause an uneven illumination on the slit. At first it was intended to use Köhler illumina-

tion, but then only a small fraction of the total area of the source is used. The intensity in the central region of the source is fairly uniform, and so it would have been an unnecessary complication to use ["]Köhler illumination when critical illumination can serve as well. The filter is mounted behind the condenser lens such that it can be swung in an out when desired, (Fig. 4.7d).

Theoretically the slit object should be of infinitesimally ~~width~~ width and be incoherently illuminated. This condition of illumination strictly calls for an illuminating cone of 180 degree, or in another words the so-called effective source needs to be infinite in extent. This is not possible to achieve in practice. Birch (1959) showed that this condition of illumination may be assumed when the illuminating cone is greater than four times of that of the lens under test. The slit object must also be of sufficient width to give enough light for accurate measurements to be made. The effect of this is to reduce slightly the coherence of the wave-front over the pupil below the theoretically desired condition of complete coherence.

The theory of partial coherence and its application to image-formation and interferometers has been described by Hopkins (1951), and later (1953, 1957). The phase-coherence factor, or complex degree of coherence, for rectangular slit aperture is given by:

$$\tau_{21}(u,v) = \frac{\sin(2\pi u)}{2\pi u} \cdot \frac{\sin(2\pi Kv)}{2\pi Kv}$$

where $K = b/a$ is the ratio of half length to half width of the slit, u, v , are reduced coordinates as defined previously. The visibility factor V_{21} which is the modulus of $\tau_{21}(u,v)$ will appear in the expression for determining the optical transfer function derived previously. This now has the form:

$$F_I - F_{II} = -A(T - R)^2 + (V_{21}) \cdot 4A \cdot TR \cdot T(s, \psi) \cos(\theta(s, \psi) + \Delta_I)$$

If the visibility does not change when the two pupil are sheared apart V_{21} will merely appear as a scaling factor, and will not affect the results of measurements.

If the tolerance on V_{21} is placed such that $V_{21} \geq 0.99$, and assuming that the vertical shear is zero, that is $v = 0$, u is given by

$$u = \frac{n \sin \alpha}{\lambda} \leq 0.04$$

For the pupil diameter 5 mm., the shear $s = 0.6$ will be corresponding to the actual shear 1.5 mm. The largest permissible value of α is then found to be

$$\alpha \leq \frac{0.04 \times 5461 \times 10^{-7}}{1.5} = 1.45 \times 10^{-5} \text{ radians}$$

If the focal length of the lens under test is 500 mm., the corresponding maximum permissible slit width will be 14.5 microns. In practice a slit width of 20 microns was used and no serious effects on the results was detected.

However, more light is available and hence a better accuracy in measurement is obtained. If the vertical shear is not zero, applying the same tolerance $V_{2\lambda} \gg 0.99$, the slit length is given by

$$\text{slit length} \leq \frac{2.2 \times 10^{-2}}{S_v} \text{ mm.}$$

where S_v is an actual vertical shear in mm. If the tolerance for the vertical shear is set at 0.01 mm., the permissible slit length will be about 2 mm. This tolerance needs not be applied if the vertical shear remains constant for all values of shear; its effect on the visibility is constant and merely acts as a scaling factor. A longer slit may be used to get more light; for a slit length of 6 mm. the visibility drops to about 90%. In the present work the slit length is kept at 2 mm. since this gives sufficient light for accurate measurement.

4.8 The mirror systems

It was intended initially to construct all optical components of the apparatus on the same optical bench and make it as symmetrical as possible. The interferometer unit is set so that the beam splitter is parallel to the optical axis along the length of the optical bench. Since the collimated beam from the lens under test has to be incident on the beam splitter at an angle of 35 degrees, there must be

some optical device to guide the beam to this direction. The original proposal made for this device consisted of a large prism of the shape shown in Fig. 4.8. Perfectly homogeneous glass of this size is not easy to obtain and the other disadvantage is that any error of figure of a reflecting surface will result in an error in optical path length of three times this. This is because the light passes through the defective region twice and the refractive index of glass is approximately 1.5, so that the resulting error in optical path length is three times as great as the surface error.

If mirrors are used, an error of figure of a reflecting surface will cause an error in optical path length only $2/3$ of those caused in the first case. The disadvantages of using mirrors are the difficulty of keeping them clean and the liability to tarnish with age. Nevertheless, the later were adopted to serve this purpose because of cost, accuracy in manufacturing and simplicity in construction.

The actual system consists of two good quality $1 \times 1\frac{1}{2}$ inch front-aluminised mirrors with their surfaces flat to

$\lambda/10$. Each mirror is mounted in a universal adjustable mount, which can be tilted in both horizontal and vertical directions. The first mirror is located on the optical axis to reflect the incident collimated beam to lie in the horizontal plane but perpendicular to the optical axis. The second mirror reflects this beam at the appropriate inclina-

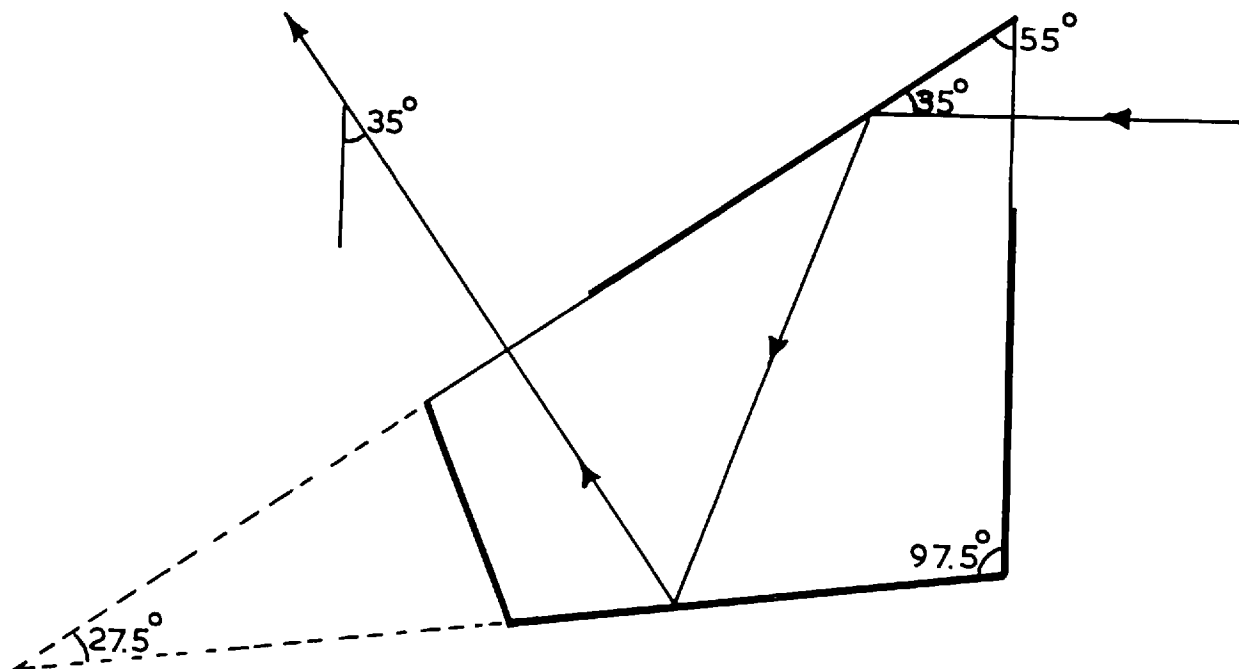


FIG. 4.8

SCALE 1:1

tion into the interferometer: it is seated on a kinematic slide, which is movable along the direction perpendicular to the optical axis by means of a micrometer screw. This movement serves as a fine adjustment for locating the beam on the beam splitter, the coarse adjustment being achieved by moving the whole mirror system along the optical axis.

A second mirror system is situated after the interferometer unit, and serves to bring the interfering pairs of beams, I and II, to be along the direction of the optical axis. This system consists of two good quality, $1 \times 1\frac{1}{2}$ inch, front-aluminised mirrors and one good quality, 1×1 inch 45 degree prism with aluminised surfaces used to provide two reflecting surfaces at right angle; all surfaces are flat to $\lambda/5$. The mirrors are mounted in universal adjustable mounts and reflect the appropriate beams on to the reflecting surfaces of the prism. The prism itself is seated on the turn table fitted with three levelling screws. After reflection each beam is brought approximately parallel to the optical axis, and is then focused by a well corrected 4 inches focal length telescope objective. In this way images of the slit object are formed on the scanning system.

4.9 The scanning system

It has been shown earlier that the optical transfer function may be measured in principle from the difference

of light fluxes in the two interference patterns I and II. The original intention was to use a scanning system based on a polarisation technique, and such a system was, in fact, ~~been~~ constructed. The beams I and II were perpendicularly polarised by means of two thin sheets of polaroid inserted one in each beam. The total pattern was scanned by a 4 inches diameter thin polaroid disc rotated at constant angular velocity about the axis perpendicular to its face. In this way each beam is alternately presented to the photomultiplier, and the output a.c. signal will be the measure of the difference of the light fluxes in the two beams. A thin Duralumin ring was glued to the edge of the polaroid disc to prevent distortion when rotated at high speed. The polaroid disc was driven by a 1,500 r.p.m. Evershed and Vignoles synchronous motor through a 5:14 gear train. It should be noted that the electrical output frequency is twice the frequency of rotation of the polaroid disc, and hence this frequency is equal to 140 c.p.s. This value was chosen to avoid interference from the mains frequency (50 c.p.s.).

The disadvantages of this type of scanning system are:-

- (1) Low light is transmitted, at least 50% of incident light is lost in transmission.

- (2) Uniform transmission polaroid disc of such dimensions is difficult to obtain; and any imperfection of this kind can give rise to serious error in the output signal.

If the polaroid sheet is mounted on a rotating tube so that the light beam is always incident on the same portion of the polaroid sheet (Baker, 1955), this error may be avoided.

(3) The response of the photomultiplier may not be the same for different directions of polarisation of the incident light.

(4) The axes of polarisation of the two beams must be set accurately perpendicular to each other, otherwise an error is introduced.

(5) Since the light beam is incident obliquely on the beam splitter some degree of polarisation may be produced (Longhurst, 1957), and this was in fact detected (see later), so that any apparatus employing a beam-splitter should avoid the use of polarisation technique.

Because of these difficulties a scanning drum similar to that used by Birch (1959) in his determination of optical transfer functions was considered. The drum consists of a good quality glass drum with a photographically made coarse grating wound around it. The drum is rotated at constant angular velocity about an axis through its centre. The images of the object slit are then formed on the periphery of the scanning drum, one falling on an opaque line while the other is formed on the adjacent clear part of the grating. The light flux from each beam is then alternately presented to the photomultiplier as the drum rotates, and again the output

a.c. signal is a measure of the difference of light fluxes in the two beams.

The optimum line-spacing of the grating drum is found from the width of the slit object, the focal length of the lens under test, the focal length of the collimator lens and the electrical frequency accepted by the detecting system. The focal length of lens under test for the present case is about 20 inches, the focal length of the collimator lens is 4 inches and the object slit width is 0.02 mm. so that the geometrical image width is 0.004 mm. The diameter of the diffraction pattern is $\frac{\lambda}{n \sin \alpha} \approx 0.022$ mm. Hence overall width of the image is 0.026 mm. In order to collect all the diffracted light, a factor of ten must be added, so that the width of the line or space of the grating should be $\gg 0.22$ mm.

In the present apparatus the slit object is in the vertical direction so that the scanning drum must be rotated about a vertical axis. In this manner the mounting and driving mechanism of the drum is rather complicated, although an image rotating prism (Birch, 1959) may be used: but this means an additional optical component in the system which is undesirable. Another point to be considered is the difficulty in manufacturing this kind of drum to a high standard.

The actual scanning system eventually used is a radial

grating having 500 lines, of 1:1 line-to-space ratio, on a 4.5 inches diameter glass disc. The accuracy in angular position of any line or space is better than 5 seconds of arc. The grating is produced as the contact copy of a master radial grating calibrated by the National Physical Laboratory. The emulsion used is Kodak maximum resolution type on a base of high quality twin ground glass plate of 1.5 mm. thick.

The radial grating is mounted on the output shaft of a 12.5:1 reduction gear box with its input shaft connected to the 1,500 r.p.m. Evershed and Vignoles synchronous motor through a flexible coupling so that the net output frequency of the scanning system is 1000 cycles per second.

The whole scanning system is seated on a $\frac{3}{4}$ inch thick cast iron bridge supported by three adjustable legs, and is isolated from the bench by two pieces of thick sponge rubber. The light beam leaving the scanning system is focused on the photocathode of a photomultiplier tube by means of a short focal length plano-convex lens. Strickly speaking the exit pupil of the first collimator lens, which is before the scanning system, should be imaged on the photocathode; but when the two pupils are sheared apart the two light patches in the pupil of the first collimator lens are not stationary; this means that different parts of photocathode are used, and the non-uniform sensitivity over the area of the photocathode may lead to error in the final result. To avoid this diffi-

-culty the slit images on the scanning disc are themselves imaged on the photocathode.

Suppose that the transparency of the grating is a square-wave function of period p . It may then be represented by the Fourier series

$$Z(u) = \sum_{n=-\infty}^{+\infty} Z_n e^{-i2\pi(n/p)u}$$

with

$$Z_n = \frac{1}{p} \int_p Z(u) e^{i2\pi(n/p)u} du$$

Here u is the coordinate across the grating and $Z(u)$ is the transparency function. If now, for the square-wave grating, in one period,

$$Z(u) = 1 \text{ when } |u| \leq \frac{kp}{2}$$

where k is any positive number (Fig. 4.9), the coefficient is given by

$$Z_n = \frac{\sin \pi nk}{\pi n}$$

Thus, if only the fundamental frequency is used, then with $n = 1$, Z_n is maximum when $k = \frac{1}{2}$, that is when the grating has equal line-to-space ratio.

4.10 The detector system

The output light fluxes from the interferometer unit, or more particularly from the scanning system, are detected with an E.M.I. 6094 B, 11 stage photomultiplier tube having Venetian Blind type dynodes coated with CsSb secondary emitting material, and the semi-transparent photocathode is of

CsSbO (S 11 type). This type of photomultiplier tube is chosen because of its high gain (20 amperes per lumen at 1,000 volts supply), very low dark current and high spectral response in the region of wavelength used ($5461 \text{ \AA}^0$); the large area dynodes of the Venetian Blind structure are likely to give more efficient and uniform electron collection. Since the electrons emitted by the photocathode take time in their transit through the tube to reach the anode, the input signal frequency is limited. For the Venetian Blind type dynode tube, the upper frequency limit is about 50 Mc/s. At the lower end of the frequency, the photomultiplier tube responds with-out loss, since the secondary-emission multiplier is, in effect, a linear d.c. amplifier for small input currents.

To ensure a good linear response, the resistor chain voltage divider to supply the voltage for each dynode is chosen so that the current through the resistor chain is high compared with the signal current to be drawn by the photomultiplier tube. The power supply for the photomultiplier tube comes from an A.E.R.E. stabilised power unit, type 1007 c (Fig. 4.11a), which gives a maximum output of 2,000 volts, 10 mA., constant to within 0.1% for up to 10% of input voltage variation. This high voltage is fed to the potentiometer unit, type 1007 B (Fig. 4.11b), which has a continuously variable output via two controls, one changing the output

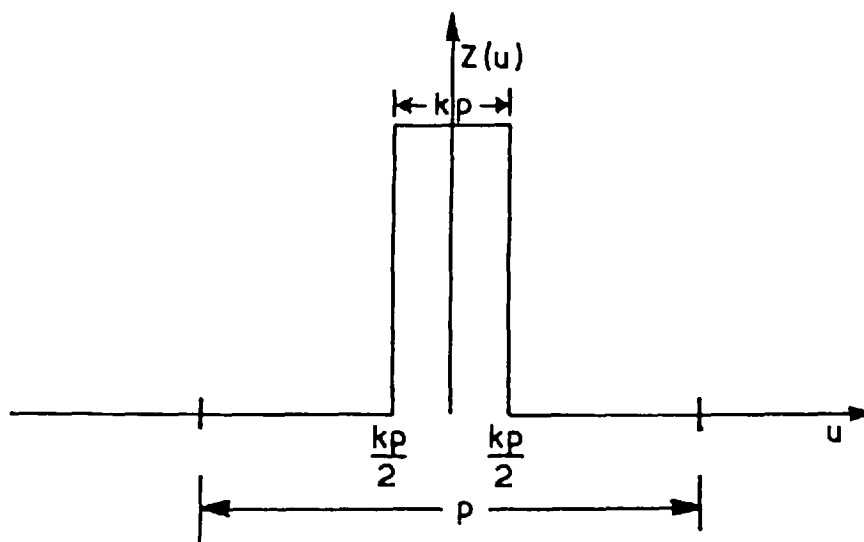


FIG. 4.9

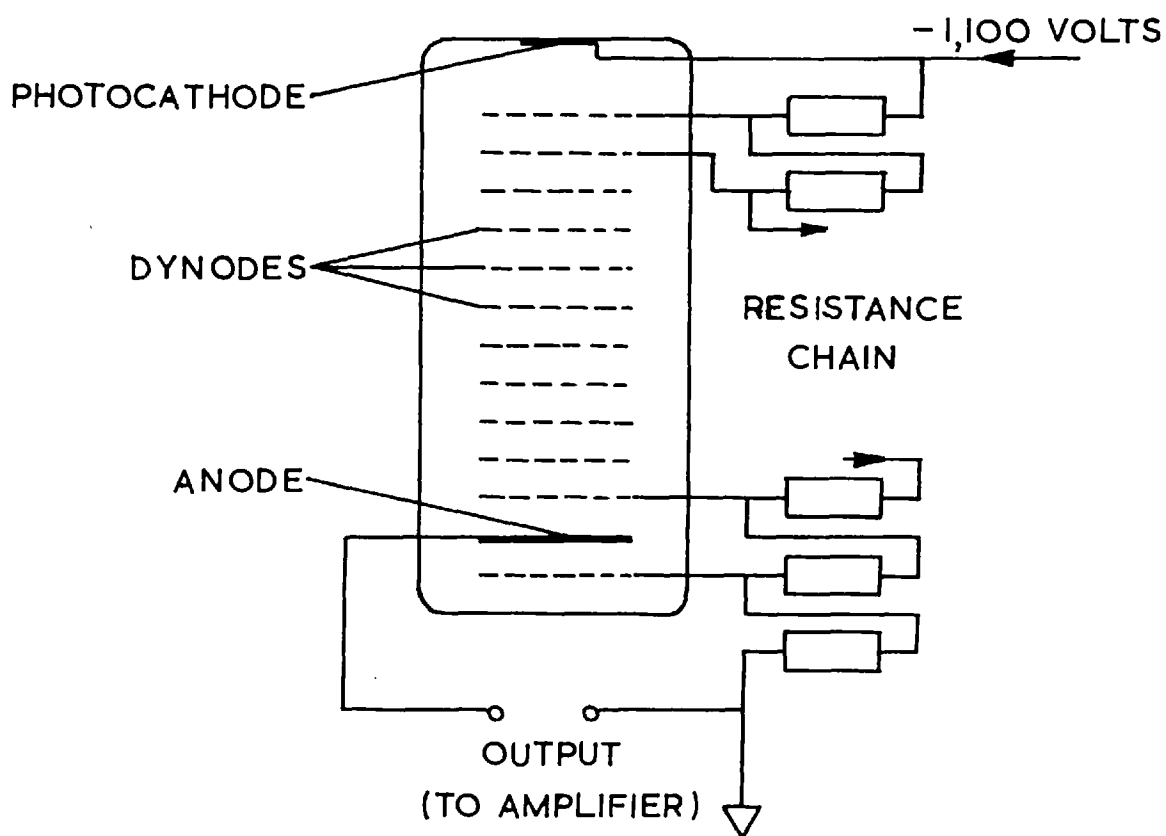


FIG. 4.10

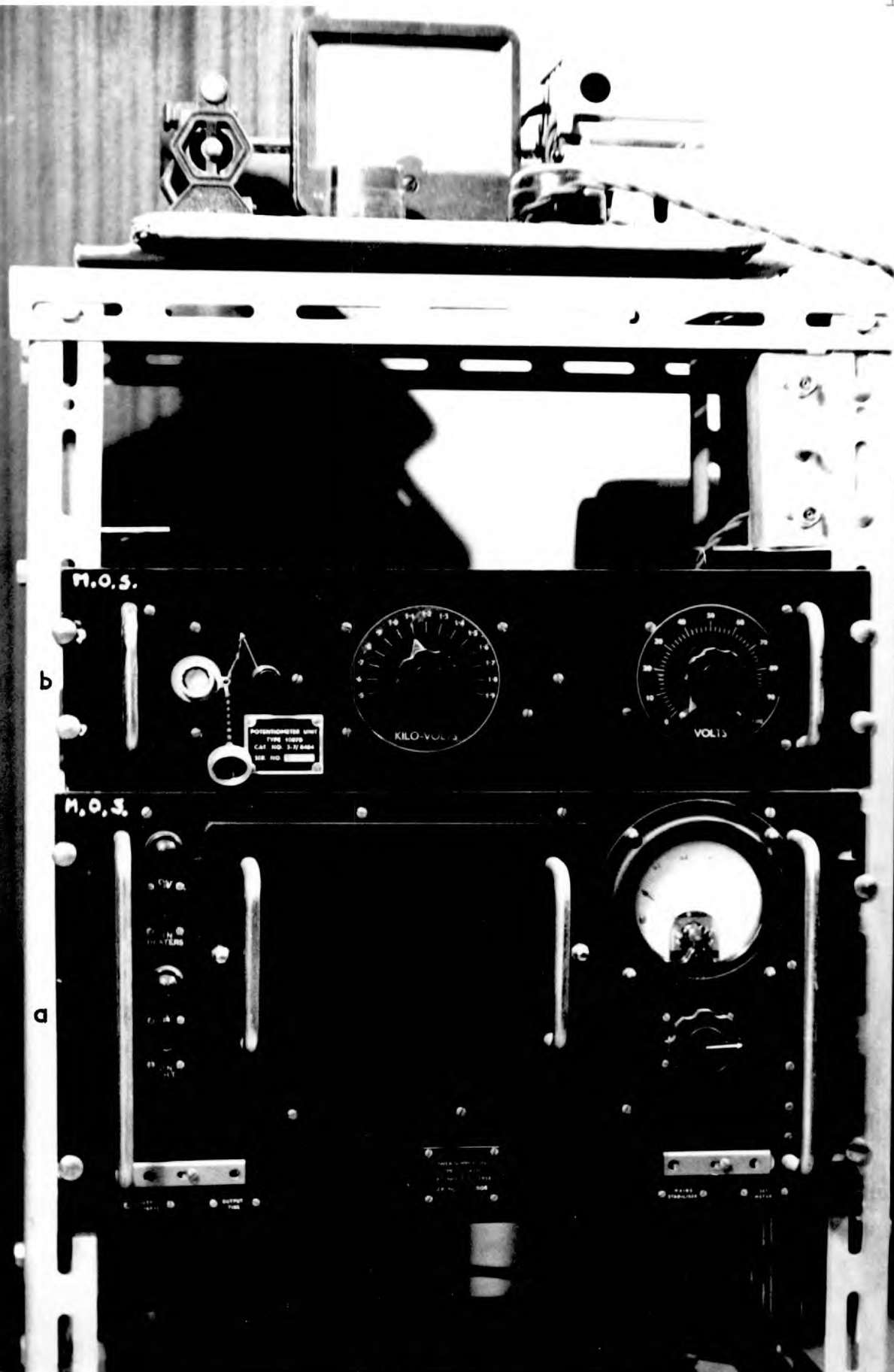


FIG. 4.11

in steps of 100 volts from 500 to 1,900 volts, and the other continuously variable from 0 to 100 volts. The output from this unit is supplied to the resistor chain with tappings for each dynode as shown in Fig. 4.10. In the present case the anode current is always less than $50\mu\text{A}$. so that the current of 1mA . through the resistor chain seems to be high enough. With the power supply of 1,200 volt across the resistor chain of 12 resistors, each resistor will have a value equal to 100K ohms.

The resistor chain is made very compact, using twelve $\frac{1}{4}$ watt high quality resistors criss-crossed over the 15 pin sockets of the photomultiplier tube. The photomultiplier tube together with the resistor chain are mounted in a light-tight brass cylinder with a one cm. diameter circular opening at one end, to expose the photocathode. The inner surface of the cylinder is painted in matt black to reduce stray light and the outer surface is left shiny to keep the photomultiplier tube cool over long running periods. The power supply and the output signal from the photomultiplier tube are conveyed by means of co-axial cable. To avoid any locally non-uniform sensitivity or fatigue of the photocathode, a (lenticular) scalloped grating diffusing screen (Baker, 1955) is inserted immediately before the photocathode. This spreads out the light uniformly over the photocathode. This type of diffusing screen has high light transmission compared

with a ground glass screen, which latter scatters light over a wide angle whereas the lenticular screen spreads light in a small forward direction of angle only. In the first design, the frequency selective amplifier (Fig. 4.12a) was intended to be used after the photomultiplier tube to filter out **undesired** frequencies and noise. This type of amplifier is based on the basic idea described by Fleisher (1948) using identical twin T network, stagger tuned to the required frequency. Because of its low gain, another high fidelity audio-frequency amplifier is needed. The main disadvantage of this selective amplifier is that the top of the amplification curve is not flat so that any change in signal frequency will give rise to a change in amplification.

An Elliott type B.801 synchronous amplifier (Fig. 4.12b) was finally used in the present work, because it has a high gain and low noise. The output is coupled to a phase-sensitive rectifier which rectifies those components of the amplifier output which are in synchronism with a reference signal derived externally, as explained later. The reference signal is automatically controlled to a constant level and can be continuously varied in phase before it is fed to the phase sensitive rectifier. The output is read from a 0 - 100 μ A. d.c. ammeter provided. The gain of the amplifier is controlled by coarse and fine attenuators in the following steps: of 20 dB. from 0 - 80; of 2 dB. from 0 - 20; and

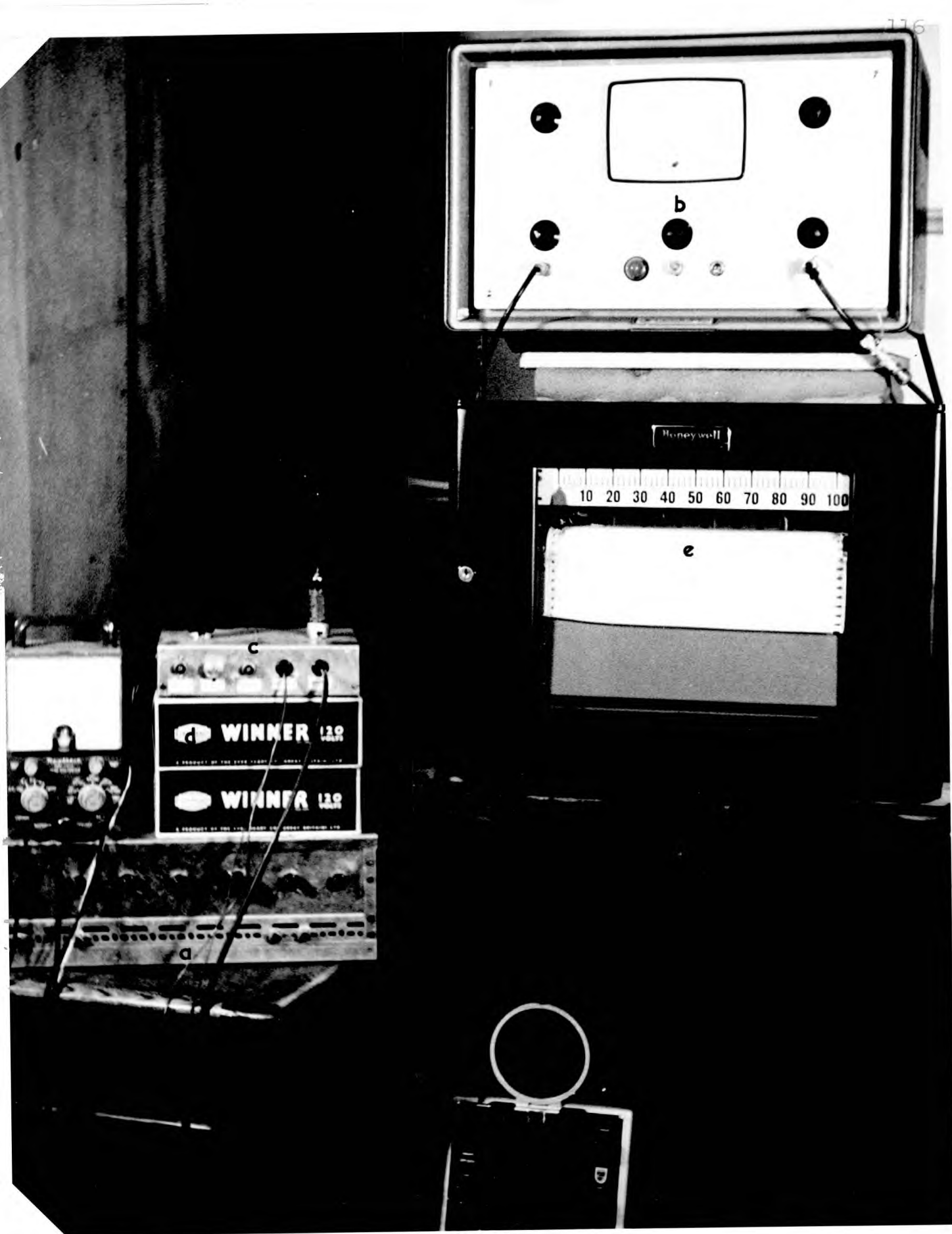


FIG. 4.12

a fine continuous variation from 0 - 6 dB. The amplifier works satisfactorily in the range of frequency from 400 c/s to 4,000 c/s with an input impedance adjustable from 10K to 100K ohms to match the generator impedance of the signal source. The input impedance of the reference channel is approximately 10K to 20K ohms and the reference signal should preferably lie between 2V. and 20V.r.m.s. Since the signal circuit is sensitive to a small fraction of a microvolt, it is most important that the connection from the signal source to the amplifier should be by means of a totally screened coaxial connector having a close mesh low resistance outer braid. The block diagram of the synchronous amplifier is shown in Fig. 4.13.

The reference signal of the synchronous amplifier is derived from the same scanning system used for the main signal, since it is essential that the reference signal be phase-locked to the signal being measured. The system consists of two parts; the light source illuminates one side of the grating, and the light transmitted is detected with a photocell situated directly opposite to the light source on the other side of the grating. The light source is a 6.5 volts, 0.3 amp. tungsten filament lamp mounted in a light-tight brass cylinder with a built-in condenser lens to image the source filament onto the photocathode of the photocell. A red Wratten filter is inserted in the beam to isolate the

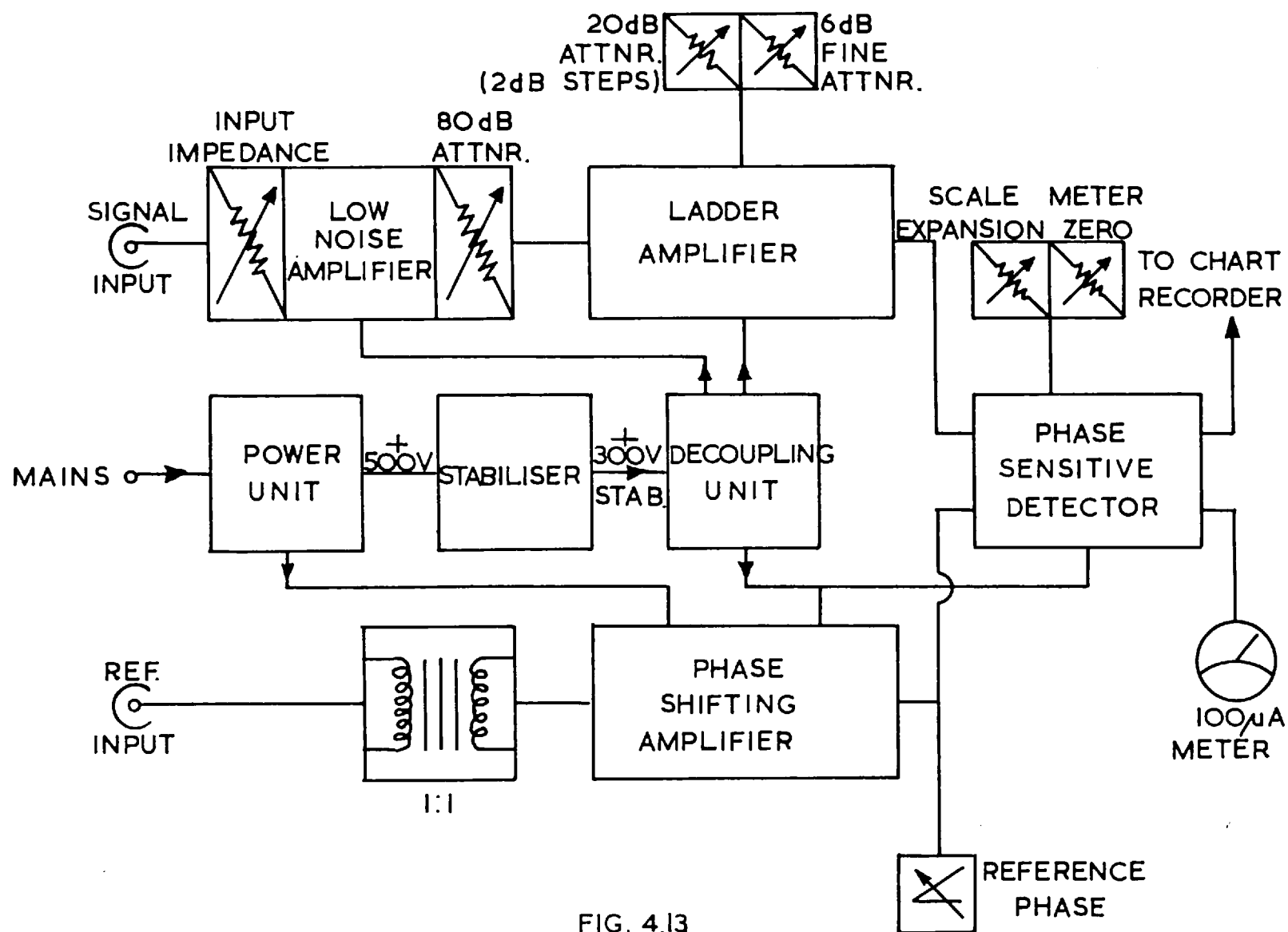


FIG. 4.13

light from that giving the main signal, and behind this is an index piece, which is a negative of the main grating disc, thus forming a multiple-slit source to give more light transmitted. The photocell used is a Mullard gas filled type 90 CG; this type is chosen because of its linearity of response, relatively low operating potential required, with a time lag smaller than 10^{-5} seconds and greater sensitivity in the red region of the spectrum. The frequency used should not be greater than 1,000 c/s, since after this limit the sensitivity tends to decrease. The photocell is also mounted in a light-tight brass cylinder with a small opening at one side directly before the photocathode. The output from the photocell is amplified by a simple two-valve class A resistance-capacitance coupled power amplifier (Fig. 4.12c and circuit diagram shown in Fig. 4.14) before feeding into the reference input channel of the synchronous amplifier. As there is a complete earth loop between the reference and signal channels, it is most important that the reference voltage should not be developed across a high resistance path in the earth circuit of the reference channel. This channel should be kept as short as possible and the cable used should have a closed woven braid of very low resistance and particular care should be taken with the plug connections at each end.

The power supply for the light source in the reference signal generator and for the power amplifier comes from a

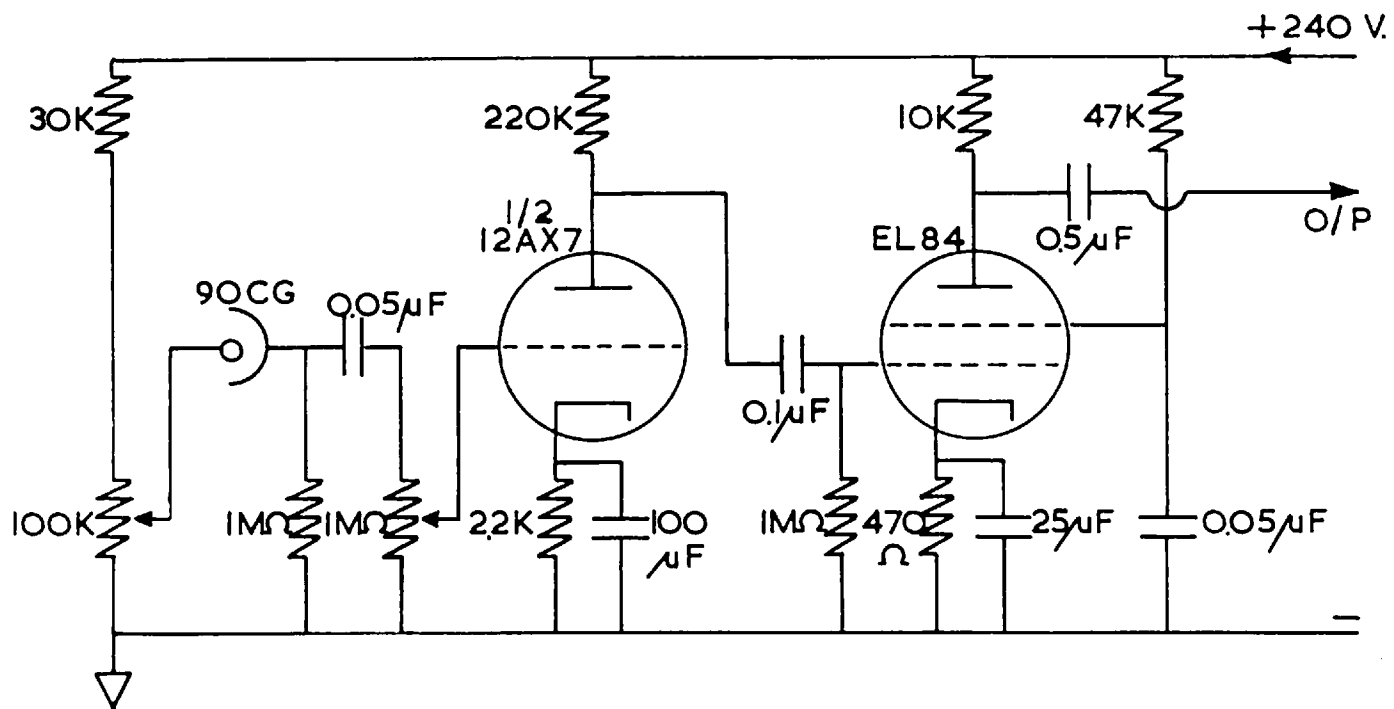


FIG. 4.14

Westat constant potential unit, which gives an output of 6.45 volts, 11.5 amps. The high tension supply for the power amplifier is supplied from two 120 volts standard power packs (Fig. 4.12d.).

Because the light beam passing from the index piece to the grating is not collimated, the square wave signal from the reference source is rounded to almost a perfect sine-wave. This is fortunate since the synchronous amplifier operates most satisfactory with a sinusoidal reference signal, although satisfactory results can be achieved with a triangular wave form.

The output signal from the synchronous amplifier is normally read off from the micro-ammeter, but this does not give continuous reading. Moss (1963) modified the system in order to feed the output signal to a Brown potentiometric chart recorder (Fig. 4.12e). This is done by tapping the output signal from the output circuit of the synchronous amplifier by means of a resistor net work; since the chart recorder needs as little as 10 MV. for its full scale deflection, this does not affect the linearity of the amplifier.

An automatic cut-off switch is attached to the driving shaft of the cam driving the shear plates, so that the cam stops automatically at the end of each revolution and always at the position where $s = 0$. The cam is started by a push-button control, and takes $6\frac{2}{3}$ minutes to complete one revolu-

-tion. The speed of the chart recorder is 2 inches per minute, so that one revolution of the cam is corresponding to $13\frac{1}{3}$ inches on the chart recorder. Each inch on the chart recorder is divided into three divisions, so that $13\frac{1}{3}$ has 40 divisions. The spatial frequency scale s for the recorded curves is determined from this relation.

The synchronous amplifier may be pictured as a gate which is open and closed periodically every cycle. Only a signal of the same frequency as the frequency of opening of the gate and of the same phase can get through; other frequencies or its harmonics will be integrated out and give zero output. This can be shown as follows.

Let $\dot{V}(t)$ be the output signal, varying as a function of time t ,

$V(t)$ be the input signal,

and $s(t)$ be the switching signal.

Then $\dot{V}(t) = V(t) \cdot s(t)$

Since $s(t)$ is a periodic function it may be written as a Fourier series:

$$s(t) = \sum_{n=-\infty}^{+\infty} S_n \cdot \exp. \left\{ -i2\pi(n \nu_0) t \right\}$$

Where ν_0 is the frequency of switching signal.

$V(t)$ may be written in term of its transform:-

$$V(t) = \int_{-\infty}^{+\infty} v(\nu) \exp. \left\{ i2\pi \nu t \right\} d\nu$$

If T is the integrating time of the circuit, the average output $\overline{V(t)}$ is:-

$$\begin{aligned}
 \overline{V(t)} &= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} V(t) dt \\
 &= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} V(t) \cdot s(t) dt \\
 &= \frac{1}{T} \sum_{n=-\infty}^{+\infty} S_n \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{-\infty}^{+\infty} v(\gamma) \cdot e^{i2\pi\gamma t} \cdot e^{-i2\pi(n\gamma_0)t} \cdot dt d\gamma \\
 &= \sum_{n=-\infty}^{+\infty} S_n \int_{-\infty}^{+\infty} v(\gamma) d\gamma \cdot \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \exp. \left\{ i2\pi(\gamma - n\gamma_0)t \right\} dt
 \end{aligned}$$

or finally:

$$\overline{V(t)} = \sum_{n=-\infty}^{+\infty} S_n \int_{-\infty}^{+\infty} v(\gamma) \cdot \frac{\sin \pi(\gamma - n\gamma_0)T}{\pi(\gamma - n\gamma_0)T} \cdot d\gamma$$

When $n = 1$, $\frac{\sin \pi(\gamma - n\gamma_0)T}{\pi(\gamma - n\gamma_0)T}$ is maximum when $\gamma = \gamma_0$ and is

zero when $\gamma - \gamma_0 = 1/T$ so that the spectrum $v(\gamma)$ of the input $V(t)$ is multiplied by zero except nearer to $\gamma = \gamma_0$; the band width of the system is $1/T$.

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CHAPTER V

ADJUSTMENT AND TESTING OF THE APPARATUS

5.1 General testing

Each part of the apparatus was tested separately before assembly. Most of the optical components were tested using the Twyman and Green interferometer, and were found to be of good interferometer quality, except in the case of the beam splitter which showed about one fringe at the region one centimetre from the end. This distortion probably arose from the heat treatment (about 400°C) during the coating process. It has been tried, but without success, to avoid this defective region by altering the angle of incidence of the incoming beam. A new beam splitter was then made 2 cm. longer than the old one to allow for any "turned edge", and it was found that no fringe could be detected either when tested separately or after assembly in the interferometer unit.

The beam splitter and mirrors of the interferometer unit were set parallel by means of an auto-collimator (Angle Dekkor). A small right angle prism was placed between the beam splitter and one of the mirrors; and, with the auto-collimator directed to its surface, several images of the graticule could be seen. The mirror on the far side of the beam-splitter was then adjusted to bring all these images to coincide. One graticule division of the auto-collimator corresponds

to an angle of two minutes of arc, and the mirror could be adjusted accurate to one-tenth of a division. The same procedure was carried out for the other mirror and for different parts of the beam-splitter. It was found that when the parallelism had been achieved for one position of the prism it was so for all other positions, showing that there is no curvature of the beam-splitter or mirrors to be detected as far as the accuracy of this procedure is concerned.

The width of the object slit was checked by a precision travelling microscope in the range to be used, and the readings were found to be shifted by one division relative to the measured width. This was in itself of no consequence, the slit opening and closing both smoothly and linearly with the rotation of the micrometer drum.

The scanning radial grating was centred precisely on the shaft of the driving mechanism by means of a travelling microscope having a setting accuracy of 0.01 mm.

The orientation of the index grating of the reference signal generator was also set accurately relative to the main grating, so that when the scanning grating was rotated slowly by hand an alternately completely dark and bright field could be observed.

The linearity of the synchronous amplifier and the chart recorder were checked initially by feeding into them an input signal of known magnitude, and of the frequency to

be used, from a signal generator, and noting the output for various magnitudes of the input signal in the range to be used. Within the accuracy of measurement ($< 1\%$) the system was found to give linear response.

5.2 The setting up of the apparatus

The whole apparatus, except the recording system, is enclosed in a light-tight cover (Fig. 5.1, 5.2) with the inner sides painted in matt black in order to reduce stray light to minimum. The illuminating system and the object slit are isolated in a separate part, having a circular opening to pass the light beam to the interferometer (Fig. 4.7). The optical bench is a standard two metre Beck bench. It is supported at both ends on an anti-vibration mounting, consisting of two 4 inches thick Latex foam rubber cushions with 1 inch thick wooden boards placed on top of each cushion to distribute the load over the whole area. The optical bench is levelled, using a spirit level. The necessary height of the optical axis from the optical bench is determined by the height of the bridge and the height of the stirrup-fittings from it. The object slit is then set to this height, with its face perpendicular to the optical axis. This setting is in fact not very critical since the jaws of the slit object are very thin and the length of the slit is only 2 mm. The object slit is set in a vertical direction by means of

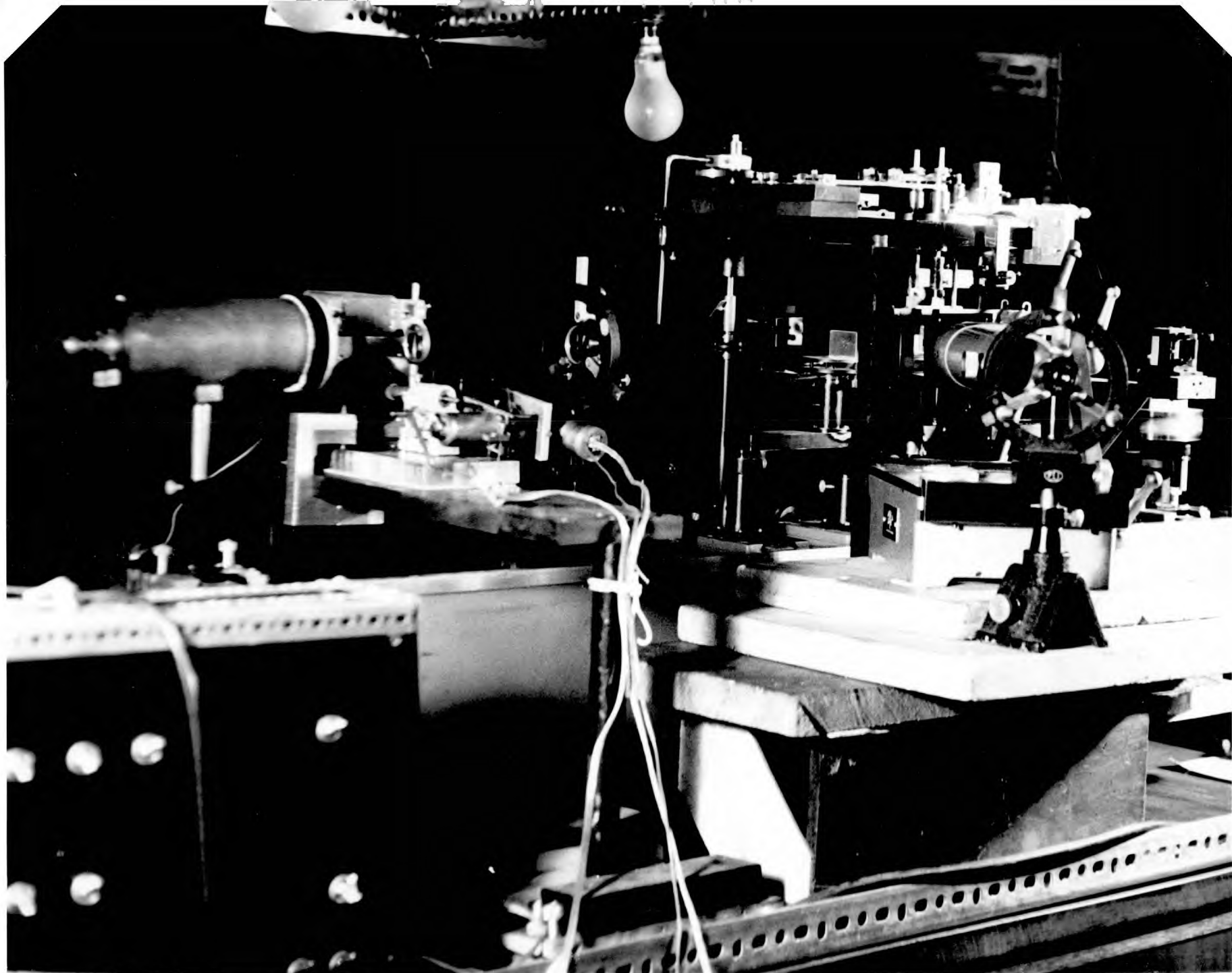


FIG. 5.1

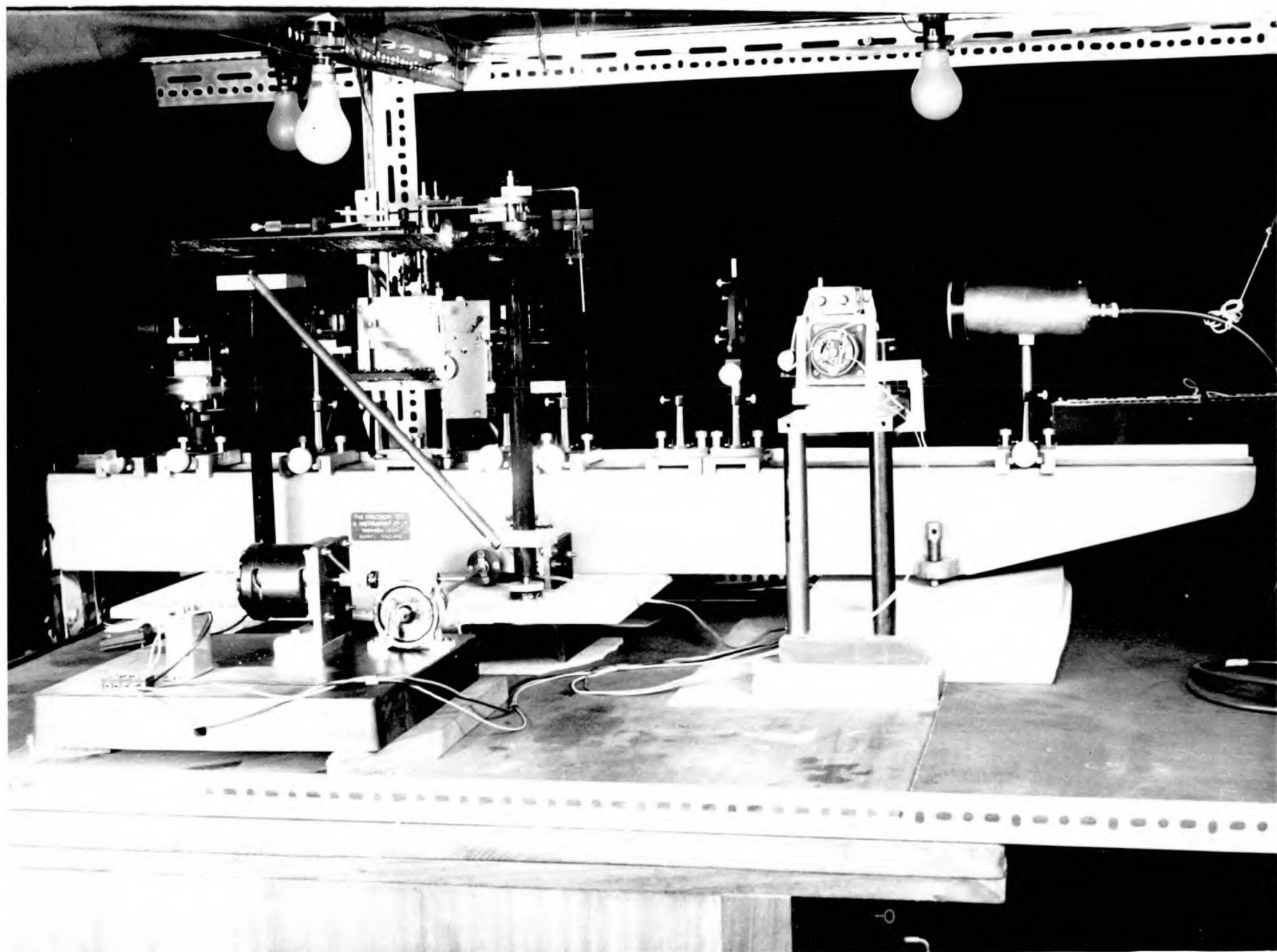


FIG. 5.2

a plumb line observed with a low power microscope. For this setting, the full length of the slit is used in order to give good precision of alignment. The slit is then closed down to a pinhole size and used as a reference for setting the other optical components. The mercury vapour lamp and the condenser lens are set so that a real reduced image of the source is focused on the slit object, with the centre of the discharge column parallel to and covering the slit. The Wratten green filter is then swung into position behind the condenser lens. A "three-pinhole method" for determining the optical axis, as described by Birch (1959) and Montgomery (1961), is also used. The setting up of the lens under test will be described in the next chapter.

The mirror system for directing the light beam into the interferometer was set by means of a Zeiss automatic Level, Ni2. With the test lens in position and the second mirror removed, the first mirror was adjusted against the level; the second mirror was then put in position and adjusted similarly.

The interferometer unit was positioned and levelled so that the beam-splitter was on the axis of the optical bench, with the beam-splitter and mirrors are parallel to the optical axis and in a vertical plane. The position of the incident beam on the beam-splitter is chosen so that the beams, after splitting and reflection at each side mirror cross at

the centre of the square-hole. The bridge, carrying the path-changing prism, compensating plate and shear plates, was then brought into position. To keep the bridge and the elements supported from it fixed in relation to the rest of the interferometer, a 1" thick wooden board, reinforced with two $\frac{1}{2}$ inch thick steel strips, was attached to the optical bench, the bridge rested on this platform. The positions of the bridge, prism pair, and shear plates were then adjusted to avoid any vignetting when they are all in full operation. With the interferometer unit temporarily out of the way, the prism pair and the shear plates were readjusted by means of the auto-collimator. This was to ensure that the normal to each surface of the prism is perpendicular to the optical axis and in the horizontal plane, and also that each shear plate rotates about a true vertical axis with its faces parallel to the axis of rotation. With the interferometer back into position, the position of zero shear was set by directing the auto-collimator to the emergent beams of the interferometer unit, so that the image of the object slit is seen at the centre of the graticule. Each shear plate was then adjusted to bring the back-reflected image of the graticule to coincide with the image of the object slit.

With a telescope focused on the exit pupil of the lens under test, a few fringes may be seen. These fringes can be eliminated by rotating the path-changing prism and(or) the

compensating plate as described in the last chapter. Rotation of either shear plate slightly by the fine adjustment device in either sense when there are a few horizontal fringes in the field of view, causes the latter to move up or down due to the change in optical path. At the turning point of this fringe movement, the optical path length is a minimum, and one knows that the light beam is incident normal to its face. Use is made of this to set the position of zero shear. The prism pair are then adjusted by fine adjustment devices until the field of view is free from fringes and uniformly illuminated. On moving the path-changing prism the field of view is alternately uniformly dark and bright, corresponding to the change of optical path length by $\lambda/2$. With the green filter removed, a coloured field of view is seen. The path-changing prism is then moved slowly until a colourless uniformly dark field is obtained which corresponds to there being zero optical path difference over a range of wavelengths. This position of the path-changing prism is noted and used as the zero position throughout the work.

Rotation slowly by hand of the cam which drives the shear plates through one complete revolution, a single fringe is observed in the region of overlap of the two sheared pupils. This fringe can be eliminated by small adjustments of the second mirror of the beam-deflecting mirror system.

This adjustment is very sensitive, since it changes the angles of incidence on the shear plates in opposite sense.

The second mirror system, at the exit end of the interferometer unit, is now brought into position, and set by means of the auto-collimator and Zeiss level. These mirrors are adjusted so that the light beams from the two sides of the interferometer are brought almost parallel to the optical axis. When the final collimator lens is put into position, two images of the slit object, one from each pair of beams, are formed side by side on the scanning grating, one on an opaque line and the other on the adjacent clear space. The slit images are set parallel to the lines of the scanning grating and exactly half a period apart as seen by a low power microscope. The images of the slit object on the scanning grating are now relayed by means of a second lens onto the lenticular screen, which is placed immediately before the photocathode of the photomultiplier tube. When all the electrical and electronics parts are now connected, the apparatus is ready to be operated.

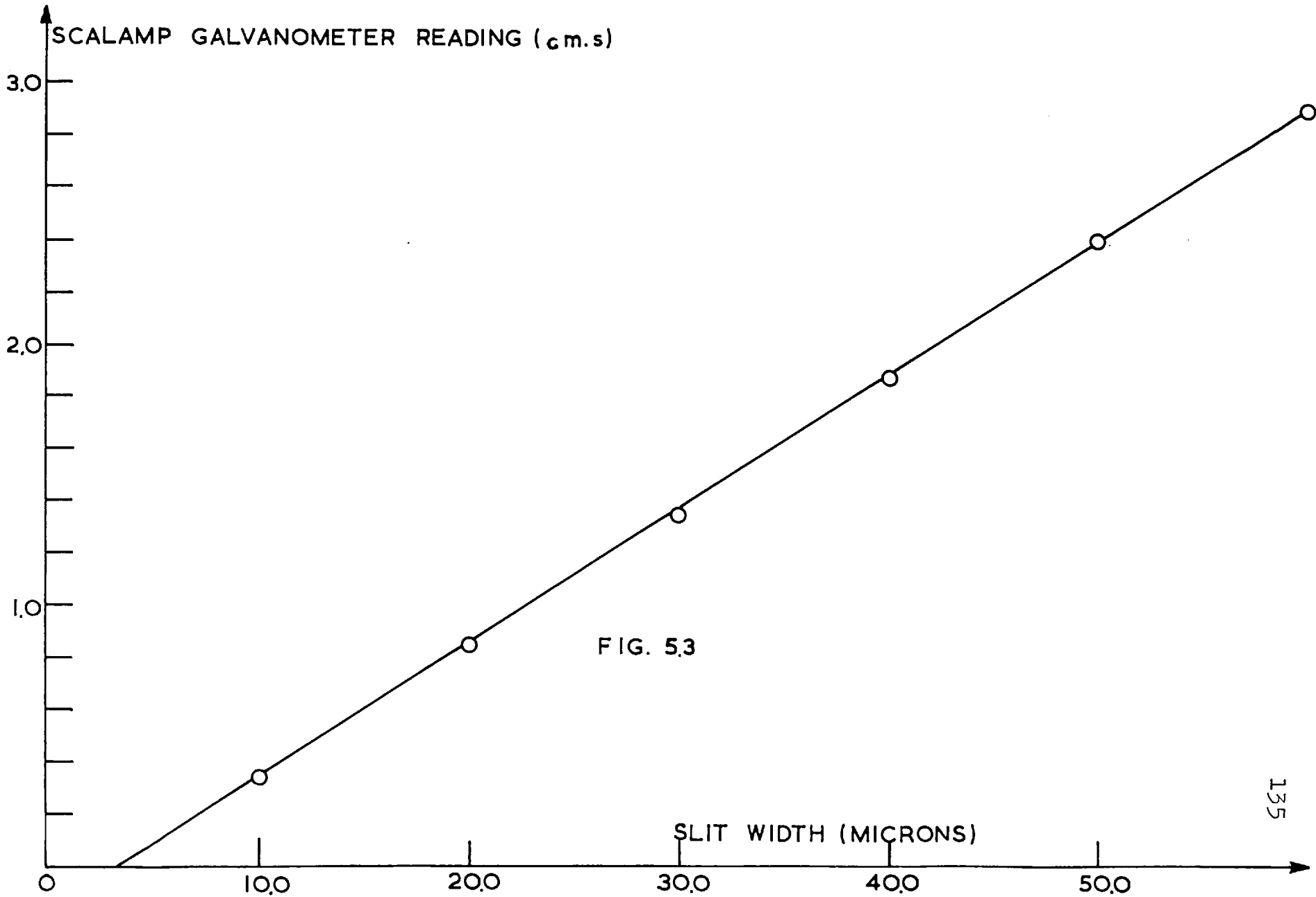
5.3 Test for linearity of the recording system

When all the electrical and electronics parts are switched on the apparatus is allowed to warm up for at least half an hour, or until the output signal remains constant for a period of approximately seven minutes, which is the time required

for one complete operation. It was found that the apparatus is affected by thermal change and the best operating condition is obtained when the temperature in the enclosure is around summer heat (76° F) so that switching on the lighting lamps in the enclosure for 1-2 hours before operation tends to shorten the warming up time.

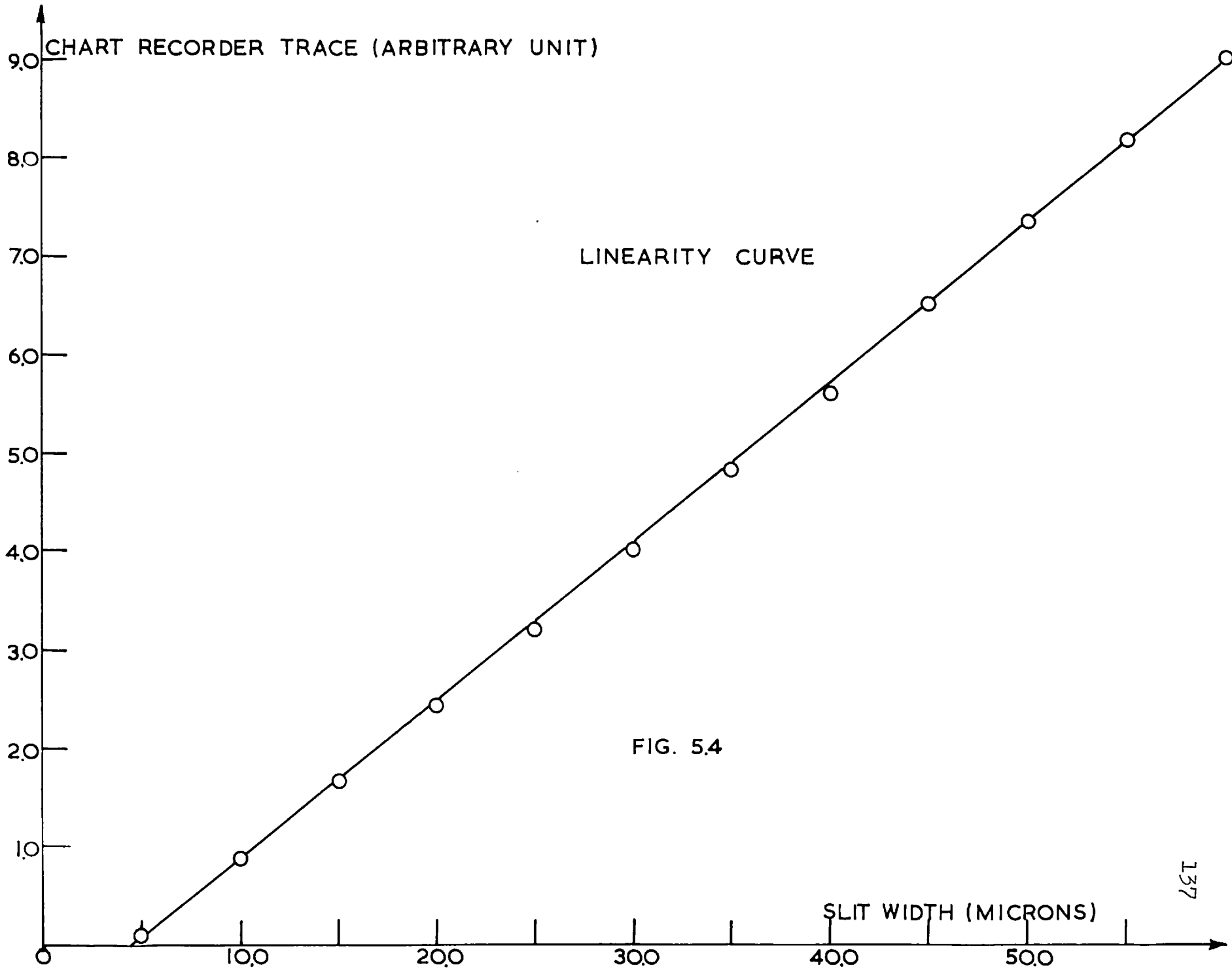
The photomultiplier tube was first tested by connecting the output to a PYE scalamp galvanometer, whose sensitivity is $0.0057 \mu\text{A/mm}$. The power supply for the photomultiplier tube was set at 1,100 volts, this being sufficient to give an output large enough for accurate measurement. With larger voltages, the dark current can become significant. To avoid the complication of interference effects, only the transmitted beam from one side of the interferometer was used. The scanning grating was fixed at the position giving maximum transmission. The length of slit was kept fixed at one millimetre for this test, the width of slit being varied in steps of ten microns. When the slit is small in extent and uniformly illuminated, the light flux transmitted may be assumed to be proportional to the slit width. The deflections of the scalamp galvanometers were plotted against the slit width and found to be perfectly linear within the accuracy of measurement, (Fig. 5.3). The output from the photomultiplier tube was then switched to the synchronous amplifier, whose output is fed into the chart recorder. With the scanning grating

LINEARITY CURVE



in rotation and the reference signal set at 7 volts r.m.s., the reference phase-control knob of the synchronous amplifier was adjusted to give maximum reading on the microammeter, and the scale-expansion control knob was adjusted to make the chart recorder and microammeter read the same value. The same procedure was then carried out, namely varying the slit width in steps of 10 microns. The output from the chart recorder was then plotted against the slit widths, and it was found that the maximum deviation from a straight line was about 0.02 which can be tolerated (Fig. 5.4).

A second method for testing the linearity of the recording system employs a polarising device, in the manner described by Kelsall (1959) and Vienot (1958). A polaroid sheet is placed in the collimated beam and followed by an analyser, which is driven slowly by a clockwork motor (Fig. 5.5a). The peak values of the sinusoidal output signals were found to be different for different regions over the analyser, suggesting that the extinction or transmission of the analyser is not uniform over all the disc. For this reason the analyser was arranged to be a polaroid sheet fixed to a hollow tube, and again driven by a clockwork motor through a worm gear (Fig. 5.5b). In this way the same part of polaroid is always used, and the non-uniform transmission of the analyser gives no difficulty. Without the polariser the output should be uniform, since it has been shown experimentally by Moss (1963)



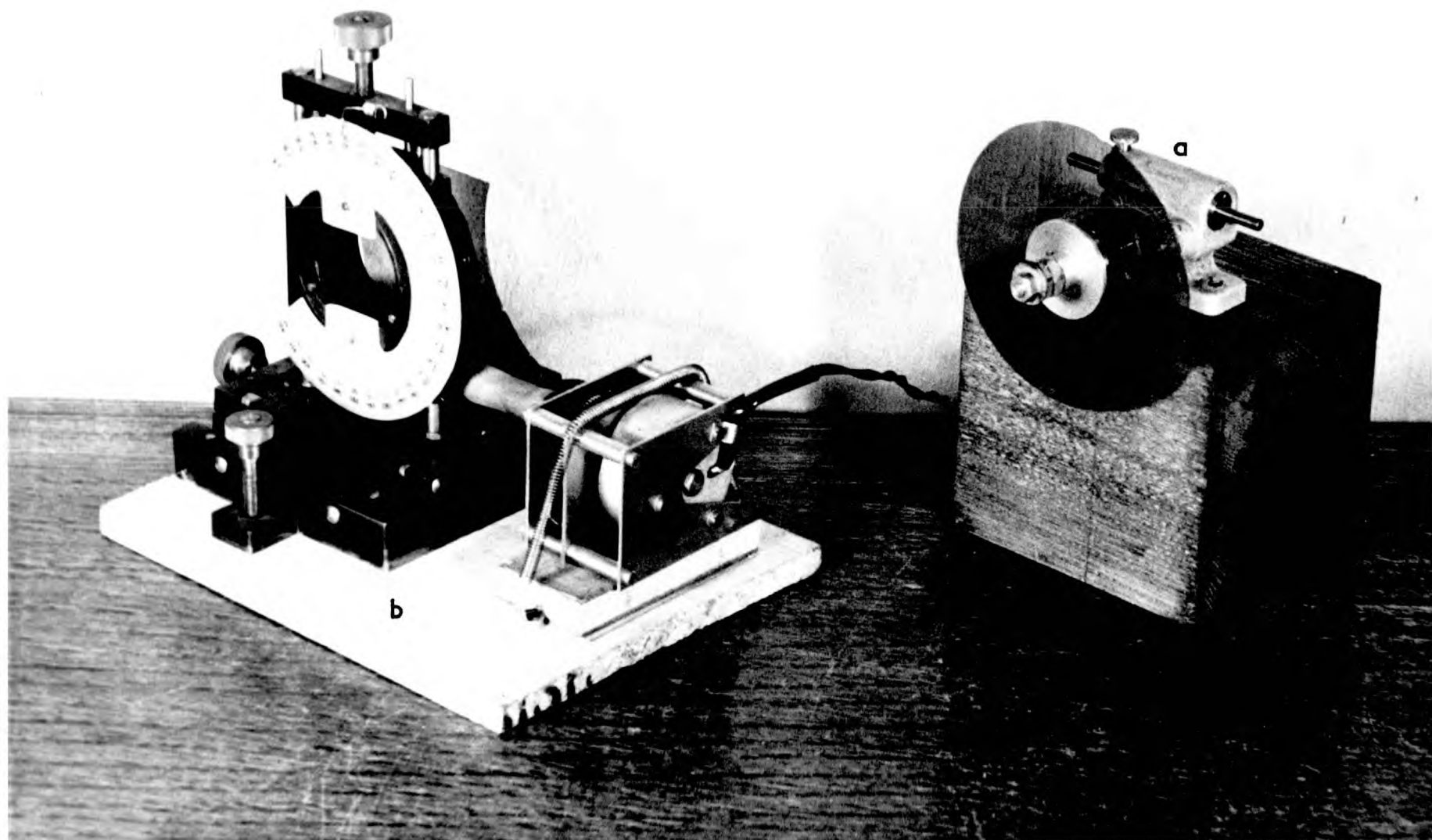


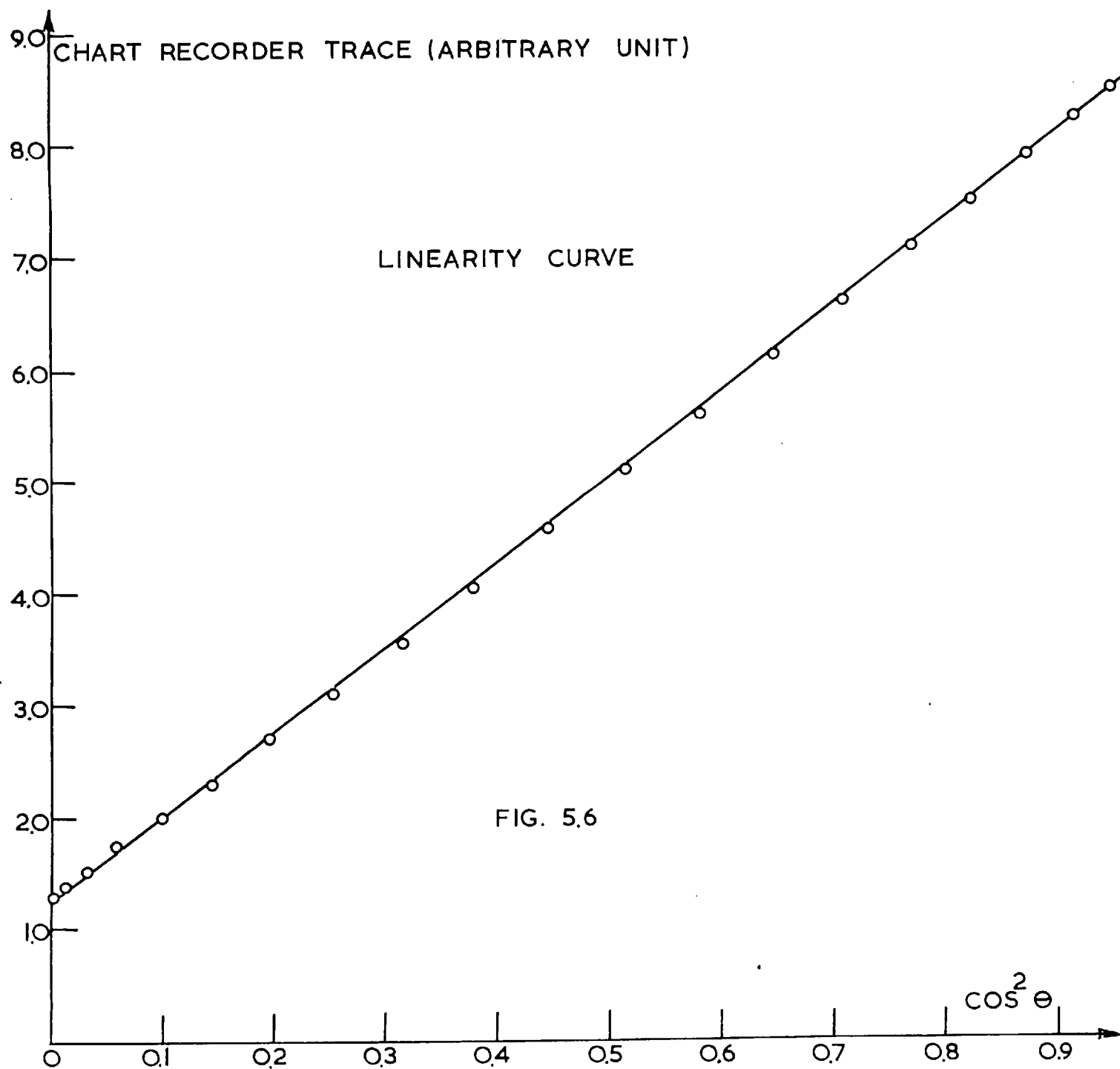
FIG. 5.5

the photomultiplier tube response is independent of the polarisation axis of the incident light. It was found that the light emerging from the interferometer unit is about 17% polarised. It is for this reason, as noted earlier, that any system employing a beam splitter is unsuitable for use in a photometric system using polarised light. With the polariser in position the output from the chart recorder was plotted against $\cos^2\theta$, where θ is the angle between the directions of polarisation of the analyser and polariser. The maximum deviation from a straight line was found to be about 0.01 (Fig. 5.6), which verifies that the recording system has good linear response.

5.4 Calibration of the shear axis

Since the fractional shear s between two beams is not quite linear with time, as the cam is rotated, it was necessary to calibrate the instrument for each position. If i is the angle of incidence at the shear plates and l is the length of the linkage arm (Fig. 4.5), then $\sin i = d/l$. Theoretically d is linear with time from the design of the cam. The expression for the fractional shear has already been derived to be:

$$s = \frac{2t \sin i}{a} \left\{ 1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right\}$$



In fact, the value of s may be calculated from this expression but it is more straight forward, and preferable in practice, to measure it directly. A fine vertical wire is placed at the exit pupil of the lens under test and a low power microscope, fitted with a micrometer eyepiece which can be read accurate to 0.01 of a millimetre, is focused on the sheared images of the vertical wire as formed by a good telescope objective. Setting the cam by hand at intervals of $1/50$ of a revolution, the separation of the two vertical line images was measured, and each value divided by the apparent radius of the pupil, thus giving the fractional shear for each setting. The resulting calibration curve is shown in Fig. 5.7.

5.5 Measurement of the transfer function for a defocused system.

The overall performance of the apparatus was tested by measuring a known optical transfer function; the most suitable one is that of a defocused optical system. The best focus of the optical system must first be found. The lens under test is set roughly by means of a telescope focused for infinity, and then the optical transfer function is measured. It has been claimed by some authors that, with the apparatus set at a fixed value of the shear s , moving the lens under test along the optical axis, will indicate the

CALIBRATION CURVE FOR (s)

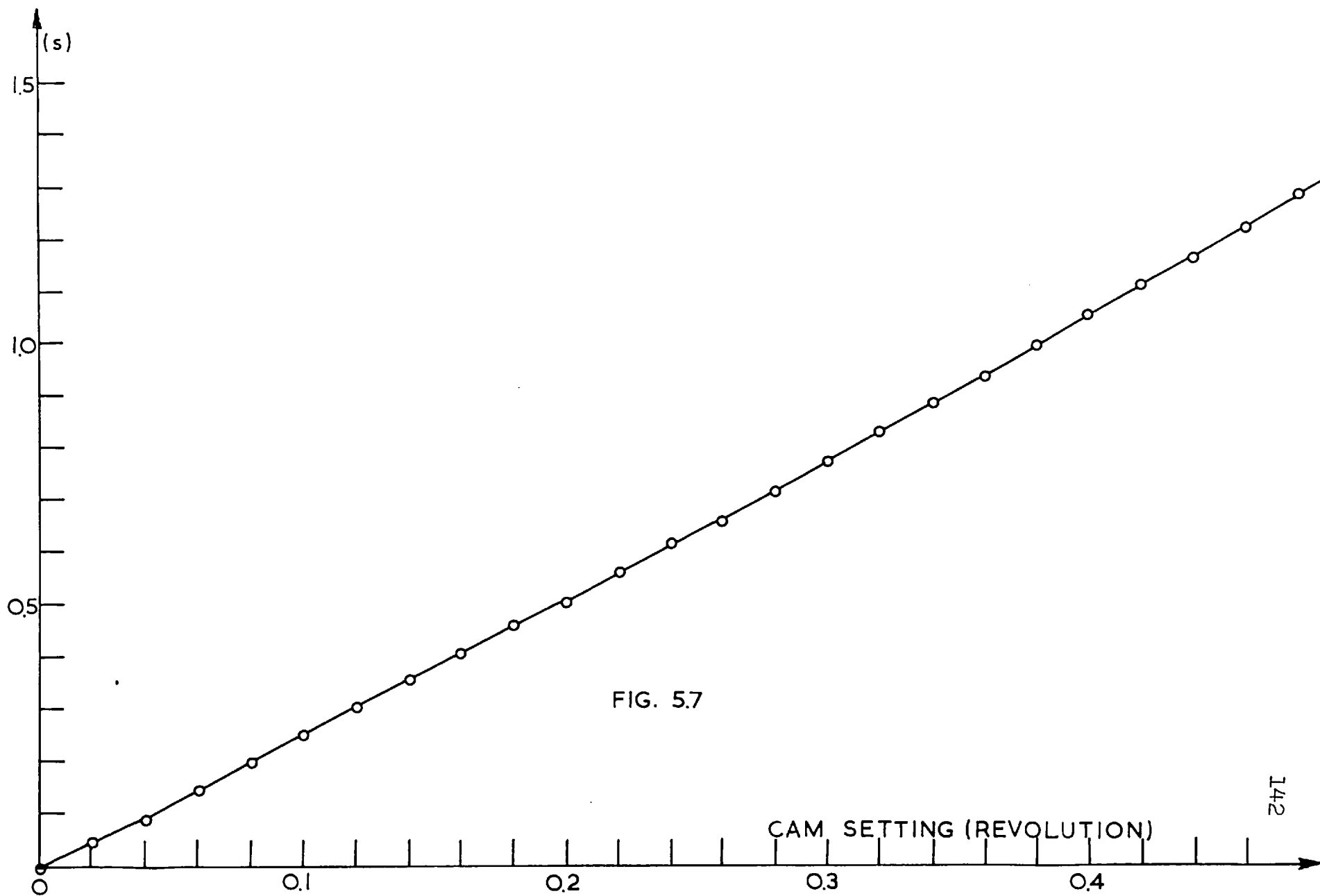


FIG. 5.7

position of best focus when the output signal is a maximum. This method was tried, but it was found that the output signal increased indefinitely when the lens under test was moved towards the object slit, and no turning point was detected. This happens because, in doing this, the numerical aperture of the lens under test increases and consequently the total light flux transmitted by the lens increases. A successful method used to determine the best focus of the lens under test, was found to be to measure the optical transfer function for various focal settings. The results obtained were then compared with the theoretical results given by Hopkins (1955). Since $T(s, \psi)$ decreases to either side of the best focus for any value of s the best focus may be interpolated.

Difficulty was found in setting the reference phase of the synchronous amplifier, when using the slit images formed by the two beams denoted previously by I and II. If the reference phase is set to be in phase with one of these slit images the other is in antiphase and vice versa. For this reason it was thought better to sacrifice one image, and to measure the light flux in one image only. The image in beam I was used for this.

From the formula given above, when only beam I is used, the total light flux in the image is given by:

$$F_I = 2 A \cdot TR. + 2 A \cdot TR. T(s, \psi) \cos \left[\theta(s, \psi) + \Delta_I \right]$$

If now the path difference Δ_I is changed from 0 in successive

steps of $\pi/2$, the light flux is given by:

$$F_{\Delta I=0} = 2A \cdot TR + 2A \cdot TR \cdot T(s, \psi) \cos \theta(s, \psi) \quad \dots\dots\dots(1)$$

$$F_{\Delta I=\pi/2} = 2A \cdot TR - 2A \cdot TR \cdot T(s, \psi) \sin \theta(s, \psi) \quad \dots\dots\dots(2)$$

$$F_{\Delta I=\pi} = 2A \cdot TR - 2A \cdot TR \cdot T(s, \psi) \cos \theta(s, \psi) \quad \dots\dots\dots(3)$$

$$F_{\Delta I=3\pi/2} = 2A \cdot TR + 2A \cdot TR \cdot T(s, \psi) \sin \theta(s, \psi) \quad \dots\dots\dots(4)$$

The difference (1) - (3) gives the real part of the optical transfer function, namely:

$$F_0 - F_\pi = 4A \cdot TR \cdot T(s, \psi) \cos \theta(s, \psi) \quad \dots\dots\dots(5)$$

while the difference (4) - (2) gives the imaginary part, that is

$$F_{3\pi/2} - F_{\pi/2} = 4A \cdot TR \cdot T(s, \psi) \sin \theta(s, \psi) \quad \dots\dots\dots(6)$$

The values of (5) and (6) are normalised to make $T(s, \psi) = 1$ when $s = 0$. For any other value of s , the modulus of the transfer function is given by

$$T(s, \psi) = \sqrt{\{(F_0 - F_\pi)^2 + (F_{3\pi/2} - F_{\pi/2})^2\}}$$

and the phase is given by

$$\theta(s, \psi) = \arg. \left\{ (F_0 - F_\pi) + i (F_{3\pi/2} - F_{\pi/2}) \right\}$$

so that complete information is known.

The defocusing coefficient, introduced when the object

slit is moved, may be written:

$$\delta W_{20} = -\frac{1}{2} \sin^2 \alpha_0 \cdot \delta Z \left\{ \frac{1}{1 - \frac{\delta Z}{F^2} (F - \Gamma')} \right\} \quad (\text{Hopkins } 1950)$$

where δW_{20} is the optical path difference between the marginal and axial rays, α_0 is the aperture angle of the lens, δZ is the actual longitudinal shift of focus, F is the focal length of the lens under test and Γ' is the distance of the aperture stop from the second principal plane of the test lens. If h_E is the radius of the pupil, $\sin \alpha_0 = h_E/F$; and putting $\delta W_{20} = n\lambda$, one then writes

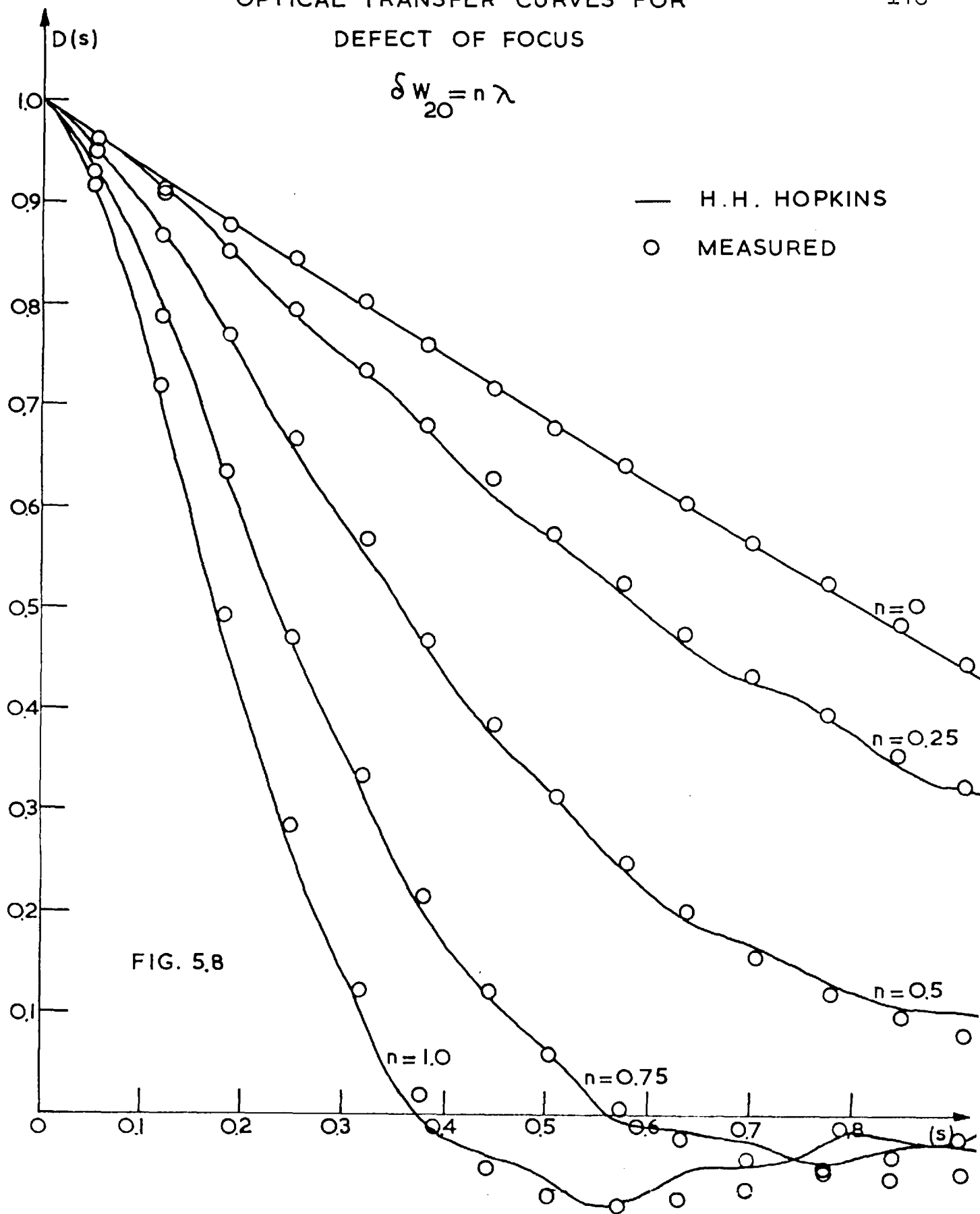
$$\delta Z = \frac{F^2}{(F - \Gamma') - \frac{h_E}{2n\lambda}}$$

for the shift of the object slit required to introduce a defocusing of amount n wavelengths, the values of δZ were calculated for $n = 0, 0.25, 0.5, 0.75$ and 1.0 . The optical transfer function curves corresponding to these values of defocusing are shown in Fig. 5.8. This measurement assumes that the lens under test is free from all aberrations other than defect of focus. This was ensured by using the lens on axis with a very low aperture ratio, equal to $F/100$. At this aperture even a simple lens produces no aberration of any consequence. The curve for $\delta W_{20} = 0$ was repeated at

OPTICAL TRANSFER CURVES FOR DEFECT OF FOCUS

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$$\delta W_{20} = n \lambda$$



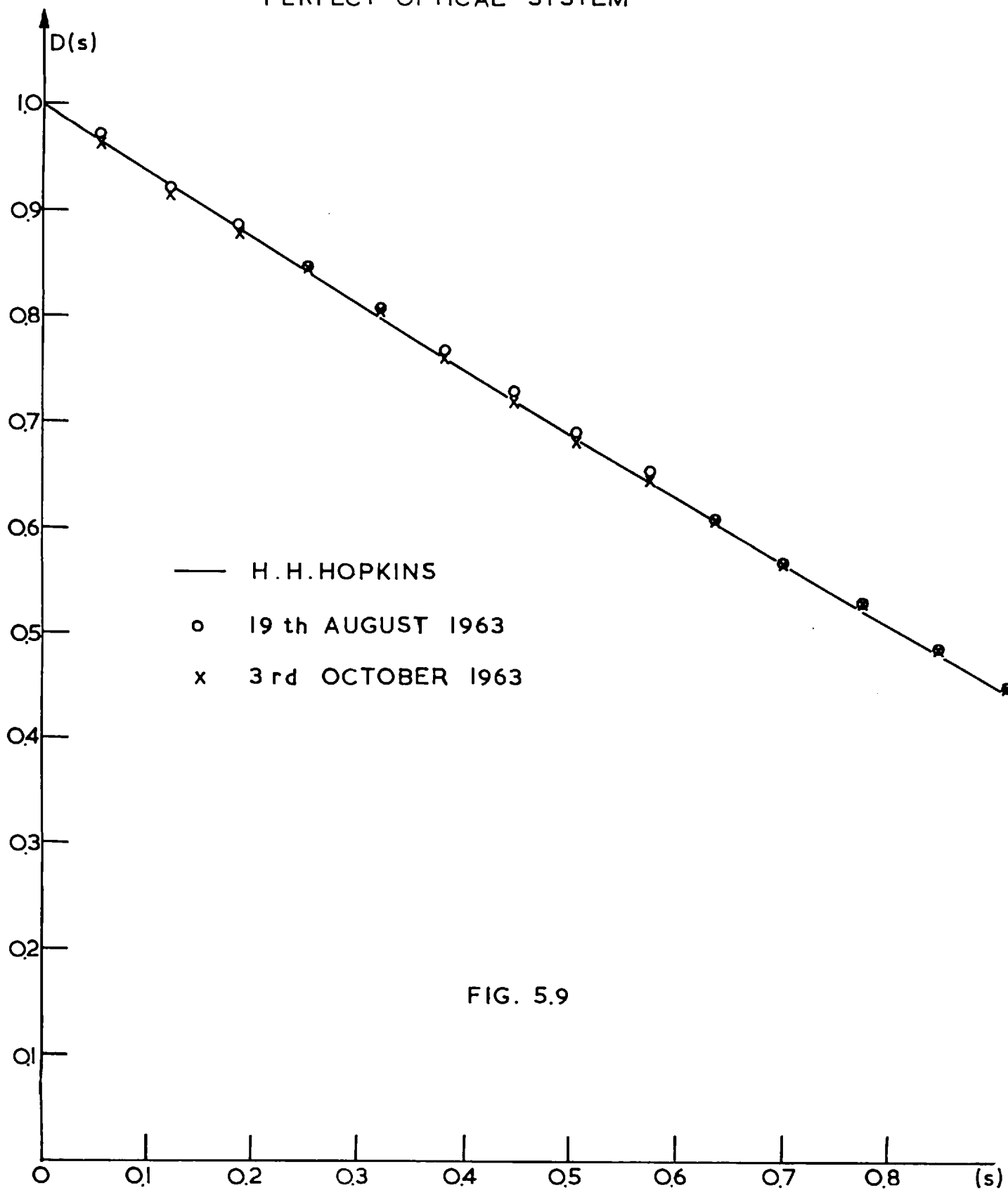
OPTICAL TRANSFER CURVE FOR
PERFECT OPTICAL SYSTEM

FIG. 5.9

OPTICAL TRANSFER CURVES FOR DEFECT OF FOCUS

$$\delta W_{20} = 0.25 \lambda$$

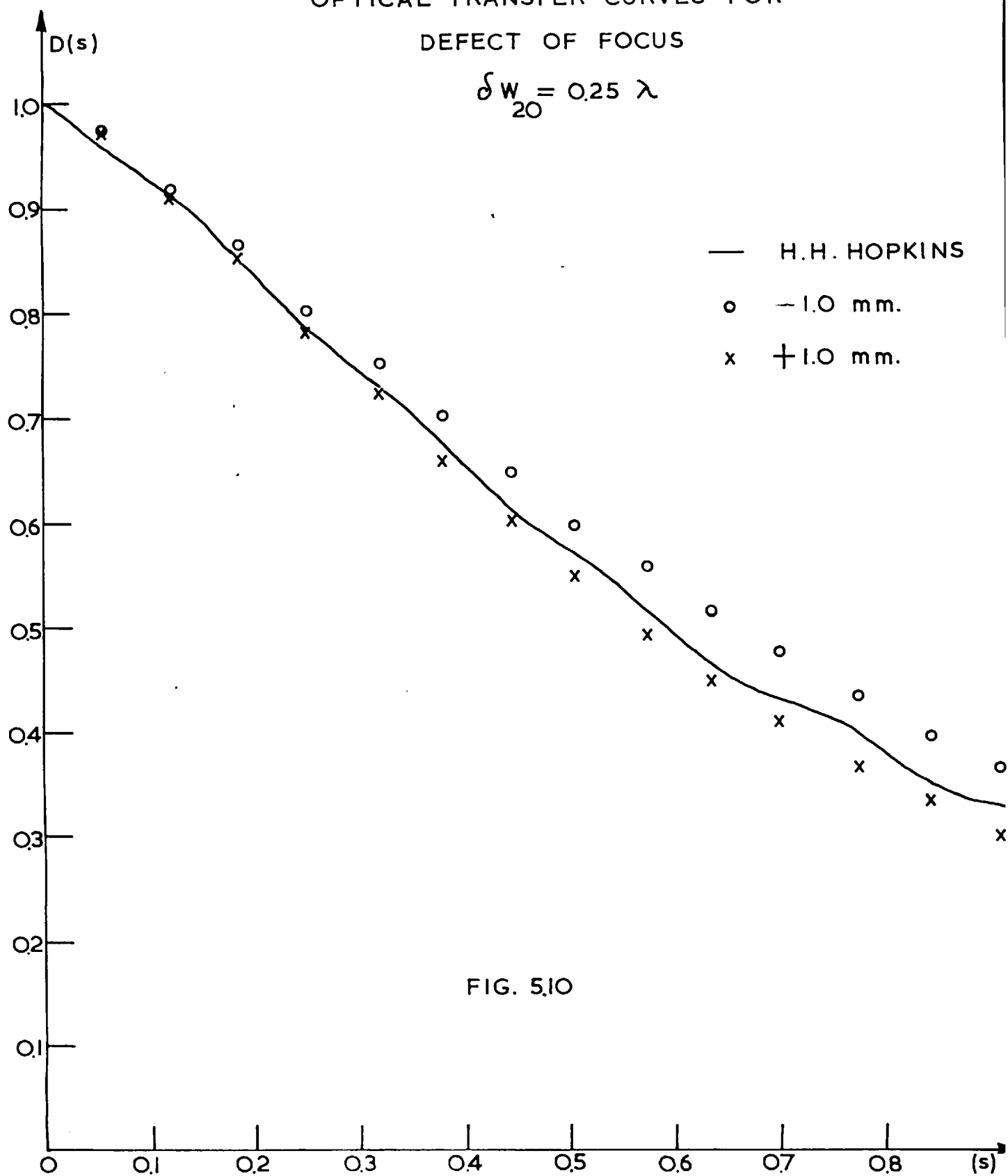


FIG. 5.10

$$\delta W_{20} = 0$$

No	(1) $\Delta_I=0$	(2) $\Delta_I=\frac{\pi}{2}$	(3) $\Delta_I=\pi$	(4) $\Delta_I=\frac{3\pi}{2}$	(1)-(3) (5) Real	(4)-(2) (6) Imagin.	(5) ² +(6) ²	$\sqrt{(5)^2+(6)^2}$	$r(s)$	$\frac{(6)}{(5)}$	$\theta(s)$
0	90	46.7	1.2	45.5	88.8	0	7885.44	88.80	1	—	—
1	87.2	37.5	2.4	53.8	84.8	16.3	7456.73	86.35	.972	—	—
2	81.2	28.5	7.0	63.0	74.2	34.5	6695.89	81.83	.921	—	—
3	75.0	21.7	12.3	69.3	62.7	47.6	6197.05	78.72	.886	—	—
4	68.5	17.6	17.9	73.1	50.6	55.5	5640.61	75.10	.846	—	—
5	63.1	15.3	22.8	74.7	40.3	59.4	5152.45	71.78	.808	—	—
6	58.8	15.1	26.2	74.9	32.6	59.8	4638.80	68.11	.767	—	—
7	55.7	15.5	28.7	74.2	27.0	58.7	4174.69	64.61	.728	—	—
8	53.7	16.3	30.4	73.0	23.3	56.7	3757.78	61.30	.690	—	—
9	52.8	17.6	31.2	71.4	21.6	53.8	3361.00	57.97	.653	—	—
10	52.5	19.5	31.7	69.2	20.8	49.7	2902.73	53.88	.607	—	—
11	52.7	21.5	31.8	67.1	20.9	45.6	2516.17	50.16	.565	—	—
12	53.2	23.3	31.2	64.6	22.0	41.3	2189.69	46.79	.527	—	—
13	54.0	26.1	30.0	61.8	24.0	35.7	1850.49	43.02	.484	—	—
14	55.3	29.1	28.5	58.4	26.8	29.3	1576.73	39.71	.447	—	—
15	56.2	32.2	27.8	54.6	28.4	22.4	1308.32	36.17	.407	—	—

FIG. 5.11

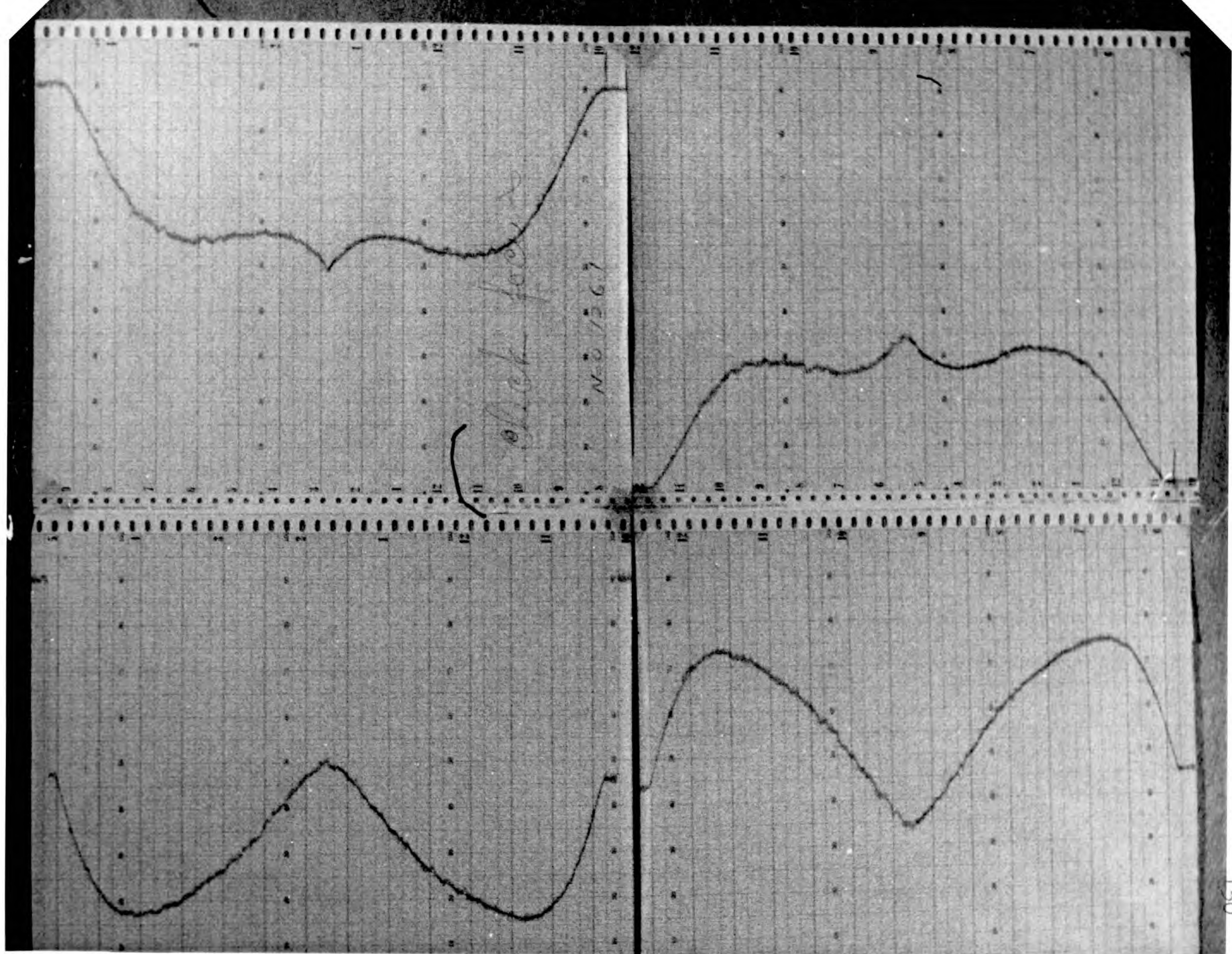


FIG. 5.12

an interval of time of two weeks, to show the stability and reproducibility of results obtained by the apparatus (Fig. 5.9). The curves for the positions 1 mm. on either side of focus, corresponding to $\delta W_{20} = 0.25\lambda$ are included, ^{there} to show the sensitivity to focusing (Fig. 5.10). Accuracy of this order would be difficult to achieve by any other methods.

The agreement of measurement with theory was found to be about 0.01 for smaller values of δW_{20} . For larger values of δW_{20} , especially when the optical transfer curves pass through zero, the error increases to about 0.04. The scheme of calculation and typical chart recorder traces are shown respectively in Fig. 5.11 and Fig. 5.12. The phase of the transfer function was not calculated from these measurements.

The apparatus has thus proved to be capable of measuring the optical transfer function of an optical system with good accuracy and repeatability.

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CHAPTER VI
TEST-LENS HAVING COMA

6.1 Introduction

The purpose of the present work was to develop a method capable of measuring accurately both the modulus and phase of the optical transfer function of an optical system; and in particular, to confirm theoretical results for an optical system suffering from pure primary coma. For this reason a system giving different amounts of primary coma had to be designed. This system should thus satisfy the following requirements:-

- (a) The system should give as near as possible pure primary coma.
- (b) The amounts of coma introduced should be of the magnitudes needed to confirm theoretical results.
- (c) The system should be simple and such that the amount of coma can be varied in a simple manner.

These requirements are less difficult to meet in the cases of chromatic aberration, spherical aberration, field curvature, astigmatism or distortion, than for coma. This is because of the inter-relationship between various aberrations. It is difficult, for example, for a simple lens or group of lenses to have coma without also having appreciable astigmatism. Thus, to obtain an appreciable quantity of coma without the presence of other aberration requires essentially

a flat-field lens having zero spherical aberration and negligible astigmatism.

6.2 Survey of aberration synthesizers

Coleman and Clark (1947) used a cemented doublet objective two inches in diameter, and rotatable about a transverse axis approximately mid-way between the objective and eyepiece, to produce different amounts of coma. The doublet used has a considerable amount of spherical aberration at full aperture, but this can be reduced by reducing the effective aperture by means of a stop placed near the lens. Off-axis aberrations other than coma are then minimised by choosing a suitable position for the stop relative to this lens. The objective is rotated about an axis through the stop, and not about nodal point of the lens, so that the image point will not be stationary. In order to observe the image the eyepiece must be moved laterally in the direction of image shift.

Fridge (1960) has described a system for generating six aberrations in continuous and almost independent amounts. Each aberration may be varied within a range of a few wavelengths in either sense. The system consists of a simple telescope doublet with four additional zero-power elements, two placed in front and two behind the doublet. The first and second are cylindrical elements which can introduce astigmatism by counter rotation about the optical axis. The third element is itself

a telescope doublet, but mounted flint leading, which will give rise to undercorrection for both spherical aberration and longitudinal chromatic aberration. The stop for the system is at the first surface of the doublet. The fourth element is a positive colour corrector, which compensates the chromatic aberration left in the objective. Moving it toward the objective produces overcorrection and vice versa. Lateral movement in either sense produces transverse chromatic aberration. The last element is a doublet; its longitudinal movement either overcorrects or undercorrects spherical aberration according to its position, and lateral movement produces coma. The transverse chromatic aberration is not completely independent of longitudinal chromatic aberration and also in the case of coma and astigmatism, a small amount of astigmatism is produced when there is a large amount of coma. Defocusing of the whole system introduces field curvature.

Scidmore and Yoder (1961) used one half of a planar type erecting system, as used in a low power wide-field telescope, to introduce coma. The system consists of two elements: the first is a meniscus-shaped cemented doublet, and the second is a single lens with an external diaphragm located a short distance in front of the first element, and acting as an aperture stop of the whole system. Astigmatism can be eliminated by choosing the correct position of stop. This system has zero field curvature so that the image can be observed at

smaller field angles without refocusing. The amount of coma can be varied by rotating the system about its second nodal point, and no lateral adjustment of the viewing system is then required, for the image remains stationary. This system has no colour correction so that monochromatic light has to be used.

Montgomery (1961) proposed a simple system to introduce coma similar to that used earlier by Nienhuis (1948). The system is a split doublet with each element mounted separately, so that they can be moved laterally perpendicular to the optical axis. The amount of coma is in first approximation proportional to the lateral shift and to the third power of aperture diameter. With this system Nienhuis obtained excellent coma patterns, which agreed very closely with those calculated by Nijboer (1943). Nienhuis varied the amount of coma by varying the diameter of aperture stop while the lateral shear between two components was kept fixed. Since he used the system at finite conjugates, the system was reversed by placing the negative component toward the object in order to eliminate the spherical aberration of the system. In contrast to this Montgomery kept the aperture stop fixed and varied the amount of coma by varying the lateral shear between the two components.

The coma produced in this manner arises from the spherical aberration produced by the positive component of an uncorrected doublet which can be written in the form

$$W_{40} = ah^4$$

where a is a constant and h is the radius of the aperture stop. This spherical aberration is in turn corrected by the negative component. If the two components are sheared laterally by an amount Δh , the aberration terms present, to a first approximation will be:-

$$\begin{aligned} W &= a(h + \Delta h)^4 \\ &= ah^4 + 4ah^3\Delta h + 6ah^2\Delta h^2 + 4ah\Delta h^3 \end{aligned}$$

The first term is spherical aberration which will be eliminated by the negative component, the second term is coma, the third term is astigmatism and the fourth term is distortion. Further terms, not shown in the expression, correspond to the change in direction of the collimated beam on leaving the system, which varies ^{with} lateral shift of the components. The principal part of this tilt arises from the paraxial focusing term. Thus, if

$$\Delta = bh^2$$

a change of h by Δh will give a new value

$$\Delta' = bh^2 + 2bh\Delta h + b\Delta h^2$$

in which the term $2bh\Delta h$ represents a tilt of the wavefront, which is rather large in practice.

From the relation $W_{31} = 4 \frac{\Delta h}{h} W_{40}$, and for a lateral displacement of 2 mms., giving two wavelengths of coma, the diameter of aperture stop must be 2.6 cms. In this case the amount of astigmatism introduced at the same time is also about two

wavelengths. The ratio between the amounts of astigmatism and coma introduced is equal to

$$\frac{W_{22}}{W_{31}} = \frac{3 \Delta h}{h}$$

Thus, for less than one wavelength of coma the amount of astigmatism may be ignored, but beyond this either the diameter of aperture stop has to be increased and the lateral displacement decreased or a very low power cylindrical lens has to be inserted to compensate for the astigmatism.

6.3 Design of a comatic lens

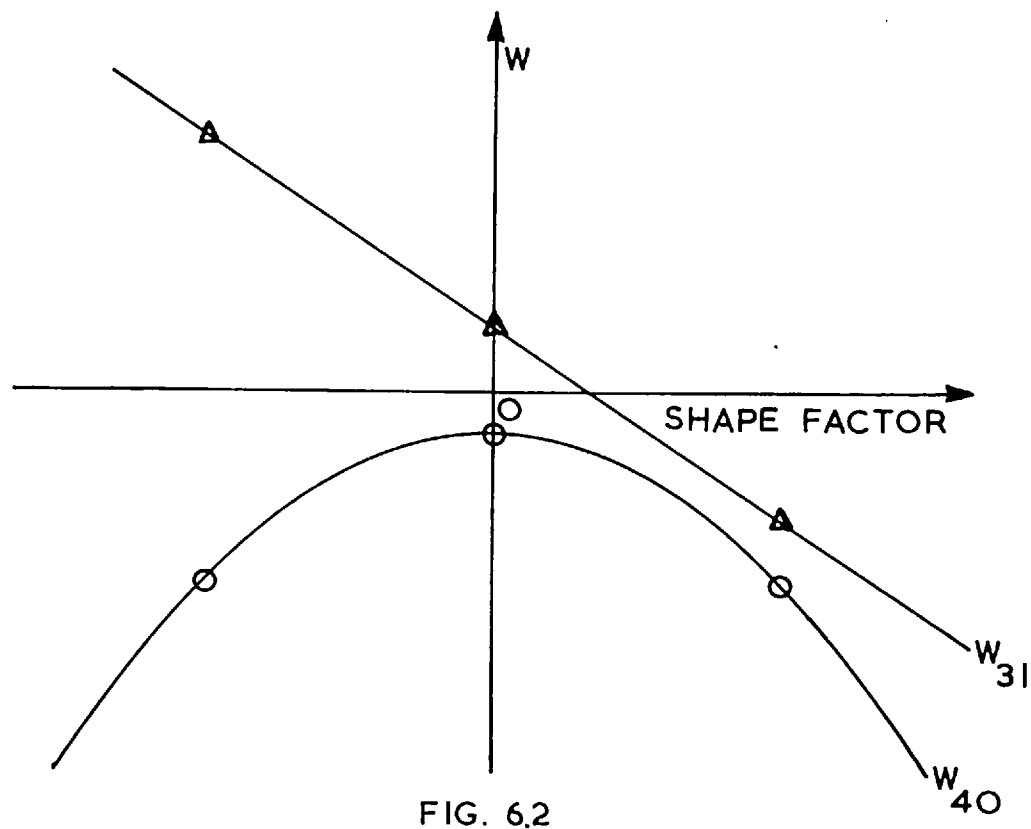
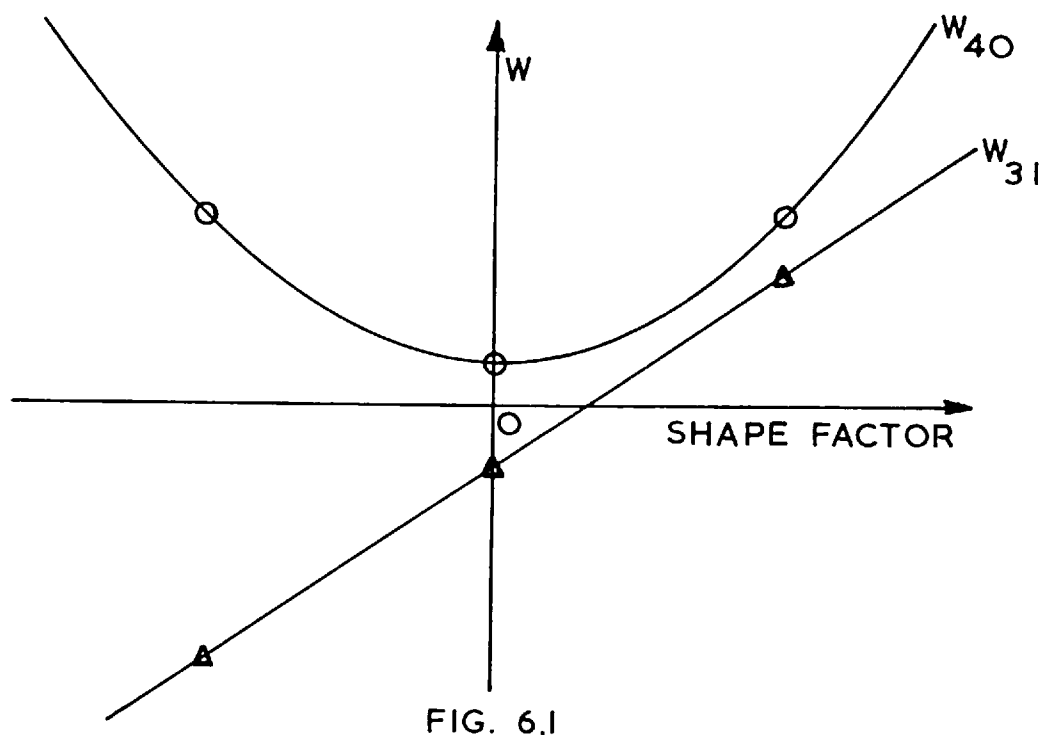
None of systems mentioned above seemed well suited for the present work. The system of Coleman and Clark does not give a stationary image. The one introduced by Fridge is complicated and does not give pure coma. The system of Scidmore and Yoder is promising, but the image produced by this system does not correspond exactly to the theoretical comatic star image intensity distribution pattern, and also this system is intended to be used at finite conjugates. Changing of the conjugates would disturb the balance of aberrations in the system. The system suggested by Montgomery has been tried in the present work, but it was found that the system was very sensitive to relative tilt between the two components, and this made it impossible to set for zero coma unless the

mechanism driving the two components was made to a very high accuracy. The diameter of the beam needed by this system to introduce any appreciable amount of coma is also too large (2.6 cms.) for immediate use in the present apparatus, which is limited to about 5 mms. in diameter.

For these reasons it was decided to design a new system for introducing coma. The first system studied was, in principle, a zero-power^{element} intended for use in conjunction with a perfect lens. Since on bending a positive lens the amount of coma and astigmatism vary with shape as shown in Fig. 6.1, and for a negative lens as in Fig. 6.2, it is possible to choose for the positive and negative lenses shapes such that their spherical aberrations cancel and their comas add. The initial system was given the following construction:

$$\begin{array}{ll}
 r_1 & = \infty \\
 d_{12} & = 6 \text{ mm.} \quad \mu_{12} = 1.51899 \\
 r_2 & = \overline{200} \text{ mm.} \\
 d_{23} & = 0.05 \text{ mm.} \quad \mu_{23} = 1.0 \\
 r_3 & = \overline{200} \text{ mm.} \\
 d_{34} & = 4 \text{ mm.} \quad \mu_{34} = 1.51899 \\
 r_4 & \approx \infty
 \end{array}$$

The aperture stop was assumed to be in contact with the first surface. Both components are of the same type of glass, namely Hard Crown. Since it is intended to use with monochromatic light there is no need for colour correction. The



last radius of curvature was found by ray tracing to make the system of zero power. Each component was then bent separately, and the spherical aberration and coma for each component calculated using the primary aberration formulae given by Hopkins (1950). The spherical aberration and coma values were plotted against shape for each component, and from these curves shapes were chosen for the positive and negative components such that spherical aberration cancelled, but each having coma of the same sign. The radii of curvatures were found to be:-

$$\begin{aligned} r_1 &= 250 \text{ mm.} \\ r_2 &= 112.022 \text{ mm.} \\ r_3 &= 129.229 \text{ mm.} \\ r_4 &= 77.385 \text{ mm.} \end{aligned}$$

All the primary aberration coefficients were then calculated, and it was found that the field curvature and astigmatism were too large, namely of the order of $\lambda/2$ and λ respectively. A study of the lens as it stood indicated that to reduce these aberration the separation between the two components had to be increased and the negative component given a high refractive index. For this reason Dense Barium Crown ($\mu = 1.613000$) was used for the negative component and Hard Crown ($\mu = 1.51791$) for the positive component. The same procedure was repeated and the field curvature was reduced to $\lambda/10$ which is negligible. The astigmatism was still too large, but this could be eliminated by shifting the stop

position. The parameters of the system then became:-

$$\begin{aligned} r_1 &= 62.500 \text{ mm.} \\ r_2 &= 87.9314 \text{ mm.} \\ r_3 &= \overline{75.0272} \text{ mm.} \\ r_4 &= \overline{114.631} \text{ mm.} \end{aligned}$$

The aperture stop had now to be placed 7.15328 mm. from the first surface. This was felt to be inconvenient mechanically, and a slightly different approach was therefore adopted, which led to a system having itself a convenient equivalent focal length. With the aperture stop midway between two components, it was found that astigmatism was always too large and negative in sign. To eliminate this without altering the stop position meant reducing the power of the negative component while keeping its shape constant.

By doing this the astigmatism was decreased, but the spherical aberration increased, so that bending of the components was needed to eliminate this spherical aberration. The procedure of reducing the negative power and bending was repeated until both spherical aberration and astigmatism were negligible. At this state the field curvature was also very small. At this point the system was no longer of zero power, but had a back focal length of 436.5 mms. This proved fortunate, because the present apparatus is designed for use with an optical system of this order of focal length. After making small adjustment in order to use as far as possible existing tools, the data of the

system were as follows:

$$\begin{aligned}
 r_1 &= \infty \\
 d_{12} &= 4 \text{ mm.} & \mu_{12} &= 1.51791 \text{ (H.C.)} \\
 r_2 &= \overline{35.046} \text{ mm.} \\
 d_{23} &= 0.5 \text{ mm} & \text{stop} & \mu_{23} = 1.0 \\
 r_3 &= 34.257 \text{ mm.} \\
 d_{34} &= 2 \text{ mm.} & \mu_{34} &= 1.51791 \text{ (H.C.)} \\
 r_4 &= 17.960 \text{ mm.}
 \end{aligned}$$

This system then used in a computer programme to work out the aberration coefficients for different angles off-axis up to 20° . The results showed that all other aberrations were zero within close tolerances, and that the maximum amount of coma produced is about 7 wavelengths.

6.4 Testing of comatic lens

After the system had been manufactured, it was set up for the star-test. On the axis, a perfect Airy pattern was observed, which, when defocused, showed alternately a dark and bright centre in the diffraction pattern. When the system is moved off-axis an excellent coma flare is observed. To do a quantitative test, the system was set on the universal camera lens interferometer at the National Physical Laboratory. It was found that the system had only coma, and that the amount produced at different field angles agreed very closely with the computed values.

CALIBRATION CURVE FOR COMA

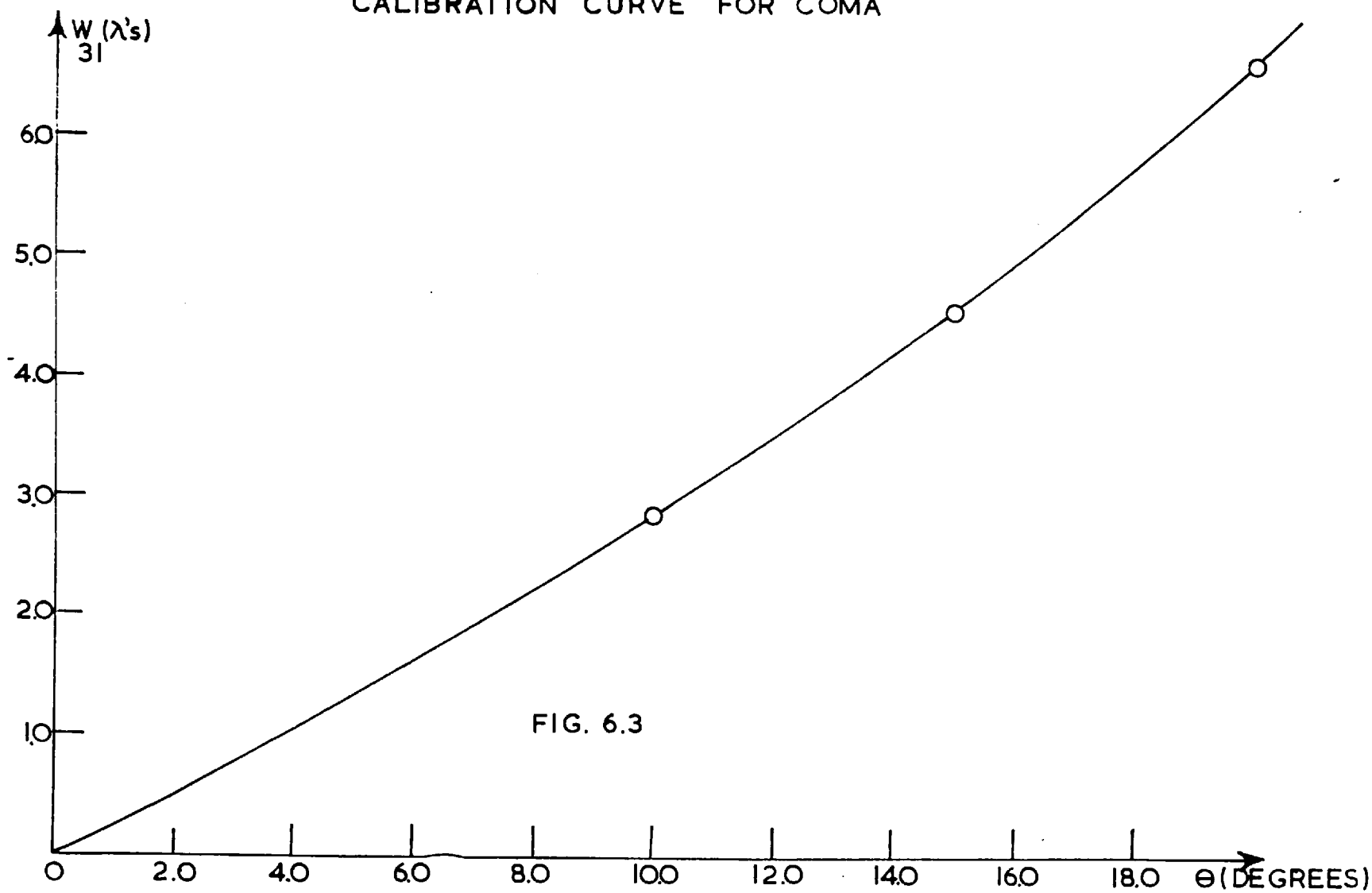


FIG. 6.3

CALIBRATION CURVE FOR COMA

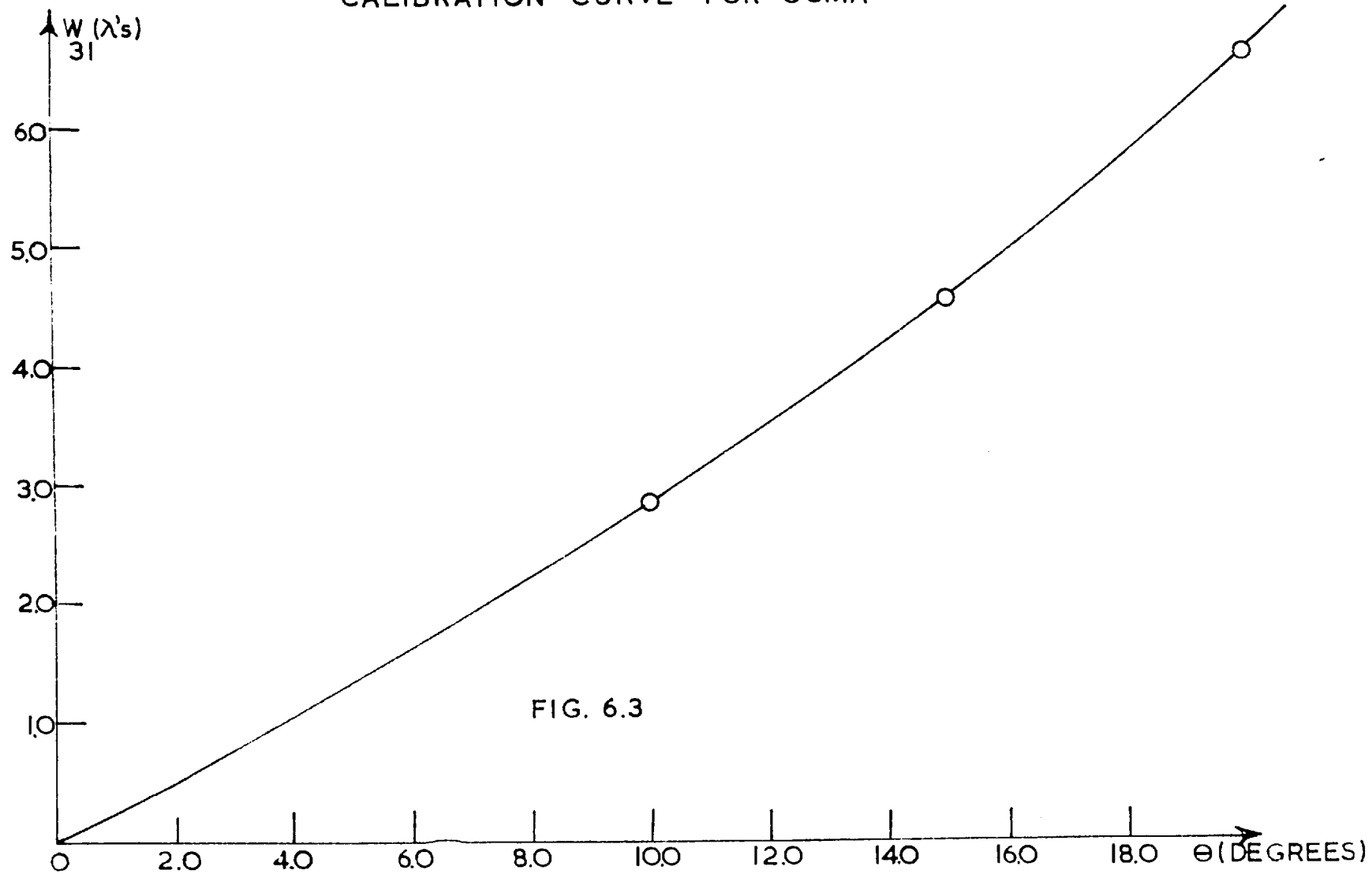


FIG. 6.3

The interferometer does not give the magnitude of coma directly. In practice the system was set at the best focus, and at a known angle off-axis with no tilt of the reference wave in the sagittal section. A small tilt was introduced in the tangential section in order to have a reasonable number of fringes (say 10). Then photographs of the fringes were taken, and the distance of each fringe from the centre of the interferogram measured by means of a medium power microscope fitted with a micrometer eyepiece. The curve of path difference against distance from the centre of the interferogram was plotted, and shows the contour of the wavefront forming the image. The wavefront aberration of the system at full aperture was then measured by the perpendicular distance from the tangent to the wavefront at the centre of the interferogram to the wavefront at the edge of the pupil. The tangent to the wavefront is not easy to draw so that a 'least square' method was used to calculate the wavefront aberration.

The coma term depends on y^3 in the meridian section, while a tilt in the wavefront gives a term depending on y , there being no tilt in x direction. A constant term represents a wrong choice for the centre of interferogram. Thus the expression for the measured wavefront aberration may be written

$$W_j = f(y_j) = C_0 + C_1 y_j + C_3 y_j^3$$

where $j = 1, 2, 3, \dots$ correspond to the different point

measured.

Small errors of measurement will mean that the values of W_j , y_j leave small residuals ϵ_j , defined by

$$W_j - \{C_0 + C_1 y_j + C_3 y_j^3\} = \epsilon_j$$

no matter what values are given to C_0 , C_1 or C_3 . The least square method will give the values of C_0 , C_1 , C_3 which minimise the sum of the squared residuals.

$$E = \epsilon_1^2 + \epsilon_2^2 + \dots \dots \dots \epsilon_n^2$$

writing the residuals ϵ_j in the form

$$\epsilon_j = W_j - (C_0 + C_1 y_j + C_3 y_j^3)$$

the squared errors sum to give

$$E = \sum_{j=1}^{j=n} \epsilon_j^2 = \sum_{j=1}^{j=n} \{W_j - (C_0 + C_1 y_j + C_3 y_j^3)\}^2$$

where E is now a function of C_0 , C_1 and C_3 , since the values W_j and y_j are known. To minimise E with respect to C_0 , C_1 and C_3 requires:-

$$\frac{\partial E}{\partial C_0} = \sum_{j=1}^{j=n} 2 \{W_j - (C_0 + C_1 y_j + C_3 y_j^3)\} (-1) = 0$$

$$\frac{\partial E}{\partial C_1} = \sum_{j=1}^{j=n} 2 \{W_j - (C_0 + C_1 y_j + C_3 y_j^3)\} (-y_j) = 0$$

$$\frac{\partial E}{\partial C_3} = \sum_{j=1}^{j=n} 2 \{W_j - (C_0 + C_1 y_j + C_3 y_j^3)\} (-y_j^3) = 0$$

Thus putting

$$\sum_p = \sum_{j=1}^{j=n} y_j^p \quad \text{then}$$

the equations

$$\begin{aligned} (n) \ C_0 + (\Sigma_1)C_1 + (\Sigma_3)C_3 - \sum_{j=1}^{j=n} W_j &= 0 \\ (\Sigma_1)C_0 + (\Sigma_2)C_1 + (\Sigma_4)C_3 - \sum_{j=1}^{j=n} W_j y_j &= 0 \\ (\Sigma_3)C_0 + (\Sigma_4)C_1 + (\Sigma_6)C_3 - \sum_{j=1}^{j=n} W_j y_j^3 &= 0 \end{aligned}$$

will give the best values for the coefficients C_0 , C_1 , C_3 .

The coma of the system at full aperture is then given directly by the value of C_3 .

The amount of coma for 5 degrees and 10 degrees off-axis were calculated in this way, and the results found to agree very closely with the computed values. For example, at 10 degrees off-axis the measured value is 2.83 wavelengths compared with 2.84 wavelengths by theory. The difference of 0.3% could easily be due to errors in measurement, because the interferometer used is intended to test lenses of much larger aperture, of the order of 5 inches in diameter. The overall magnification of the interferometer including the camera lens is 1/5; so that the present system being only 5 mms. in diameter, the interferogram recorded on the film was only 1 mm. in diameter. Although a fine grain Ilford F.p.3 film was used, the boundary of the pattern seen under the microscope was still not perfectly clear. If the wavefront aberration is

$$W(0,y) = W_{31} y^3$$

the error in W resulting from an error δy , will be

$$\delta W(0,y) = 3W_{31} \cdot y^2 \cdot \delta y$$

At the edge of pupil $y = 1$, this gives

$$\delta W(0,y) = 3W_{31} \cdot \delta y$$

Thus, for $W_{31} = 3$ wavelengths, the error of 0.3% found in $W(0,y)$ corresponds to an error of measurement in y given $\delta y = 0.001$; or, with a radius equal to 0.5 mm, a positional error of 0.0005 mm. This is clearly impossible to attain in practice.

6.5 Setting the comatic lens

The lens under test is mounted on a nodal slide which is in turn mounted on a graduated turn-table, which latter can be set accurate to 2 seconds of arc. The turntable must rotate about the true vertical axis parallel to the work surface of the lens mount. This may be checked by directing an auto-collimator (Angle Dekkor) to a microscope slide attached to the lens mount. If the work surface is parallel to the axis of rotation, turning the table through an angle of 180 degrees the back reflected image seen in the auto-collimator should be stationary. The lens mount has then to be levelled with respect to the axis of rotation of the turn-table until this condition is achieved. With the comatic lens in position, the whole system is set for height and lateral

position by forming a real image of a pinhole(the object slit reduced to a pinhole size) onto the focal plane of a low power microscope already set to the pinhole object. A good collimator is now set, by the self-conjugate method, to send a collimated beam along the optical axis. The comatic lens is put in, and a real image is formed. The second nodal point of the comatic lens arranged to be located on the axis of rotation by watching this image through a low power microscope as the comatic lens is rotated a few degrees off-axis. The image will be stationary when the right position is located. The position for zero coma is judged by the symmetry of the Airy disc as seen through the observing microscope.

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CHAPTER VII

DETERMINATION OF OPTICAL TRANSFER FUNCTION FOR PURE PRIMARY COMA

7.1 Introduction

Using the notation of Hopkins (1950), the primary coma term is given by an expression of the form ${}_1W_{31}\tau r \cos \emptyset$. Here the coefficient ${}_1W_{31}$ measures the optical path difference between the marginal ray in the meridian section and the axial ray. The variables τ and r denote the fractional field and polar radius in the aperture respectively. $\emptyset$ is the azimuth angle measured with respect to the meridian plane. Since this coma term varies as the first power of field, it is the first aberration to appear when moving from the axis. The wavefront error for coma is zero in the sagittal section, and a ^amaximum in the tangential plane: it decreases with the factor $\cos \emptyset$ in intermediate sections. Because of this asymmetrical nature, coma is very objectionable when present in an optical system. The permissible amount in an optical system is less than for any other aberrations.

Besides the Seidel coma there will frequently be terms having the same dependence on pupil variables, but dependent on the cube and higher powers of the field size τ . These are conveniently grouped for any given value of τ , by writing.

$${}_1W_{31}\tau r^3 \cos \emptyset + {}_3W_{31}\tau^3 r^3 \cos \emptyset = W_{31}r^3 \cos \emptyset$$

where

$$W_{31} = {}_1W_{31}\tau + {}_3W_{31}\tau^3$$

denotes the total coma of this type for any given value of τ . The presence of the term W_{31} is the reason why the calibration curve shown in Fig. 6.3 is slightly curved instead of straight line, which would be the case for the primary coma term in the Seidel aberration terms.

7.2 The measurement of modulation transfer function

The lens under test is set off-axis to give the required amount of coma according to the calibration curve given in chapter VI (Fig. 6.3). This setting may be made to give exactly the required amount of coma. Since the lens under test has a flat field, the object slit has only to be moved by an amount $F(\sec \theta - 1)$ to retain the correct focus when the lens is rotated through an angle θ . The second nodal point of the lens is a little behind the exit pupil, so that the mirror system has to be adjusted slightly for each setting of the off-axis angle in order that the beam is always incident at the same position on the beam splitter. To make this setting, a microscope is focused on the image of a vertical line placed in the exit pupil of the lens while the lens is on-axis. At any position of the lens off-axis the mirror system is then adjusted to bring the image of the vertical line back to its original position. The setting of the mirror system is then noted for each angle off-axis and used as an appropriate correction when required.

The experimental procedure then carried out, is that described in chapter V when dealing with a defocused system. The modulation transfer functions were measured for the values of pure primary coma equal, in wavelengths, to 0.63, 1.26, 1.89, 2.0; and then by a steps of 0.25 up to 3.5 wavelengths (Fig. 7.2 - 7.11). According to the transfer function integral,

$$D(s, \Psi) = \frac{1}{A} \iint_S \exp. \left\{ i k s \cdot V(x, y; s) \right\} dx dy$$

$V(x, y; s)$ is larger, when the aberration is larger, and the integral will then have the same value for the smaller value of s . For example, when the aberration is equal to 3.5 wavelengths the spatial frequency range of interest is that below 0.3.

Thus it is an advantage to have different shear plates for different ranges of s , suited to different values of W_{31} . In fact, two pairs were used having respectively thicknesses of 12.5 mm. and 4 mm. The curves obtained agree for the most part very closely with the theoretical results provided by Goodbody (1960) and Hindley (1964). For the higher spatial frequencies, or for larger amount of aberration, the errors are larger, but even then the maximum difference never exceeds 0.04. A curve for primary coma plus defocusing term (Fig. 7.12) is added to show the effect of focusing for an optical system suffering from coma. This curve also illustrates the fact that the performance of an optical system suffering from an odd aberration cannot be improved by introducing an even aberration to it, a result proved theoretically by Hopkins (1960-61).

ARGAND DIAGRAM OF OPTICAL TRANSFER FUNCTION

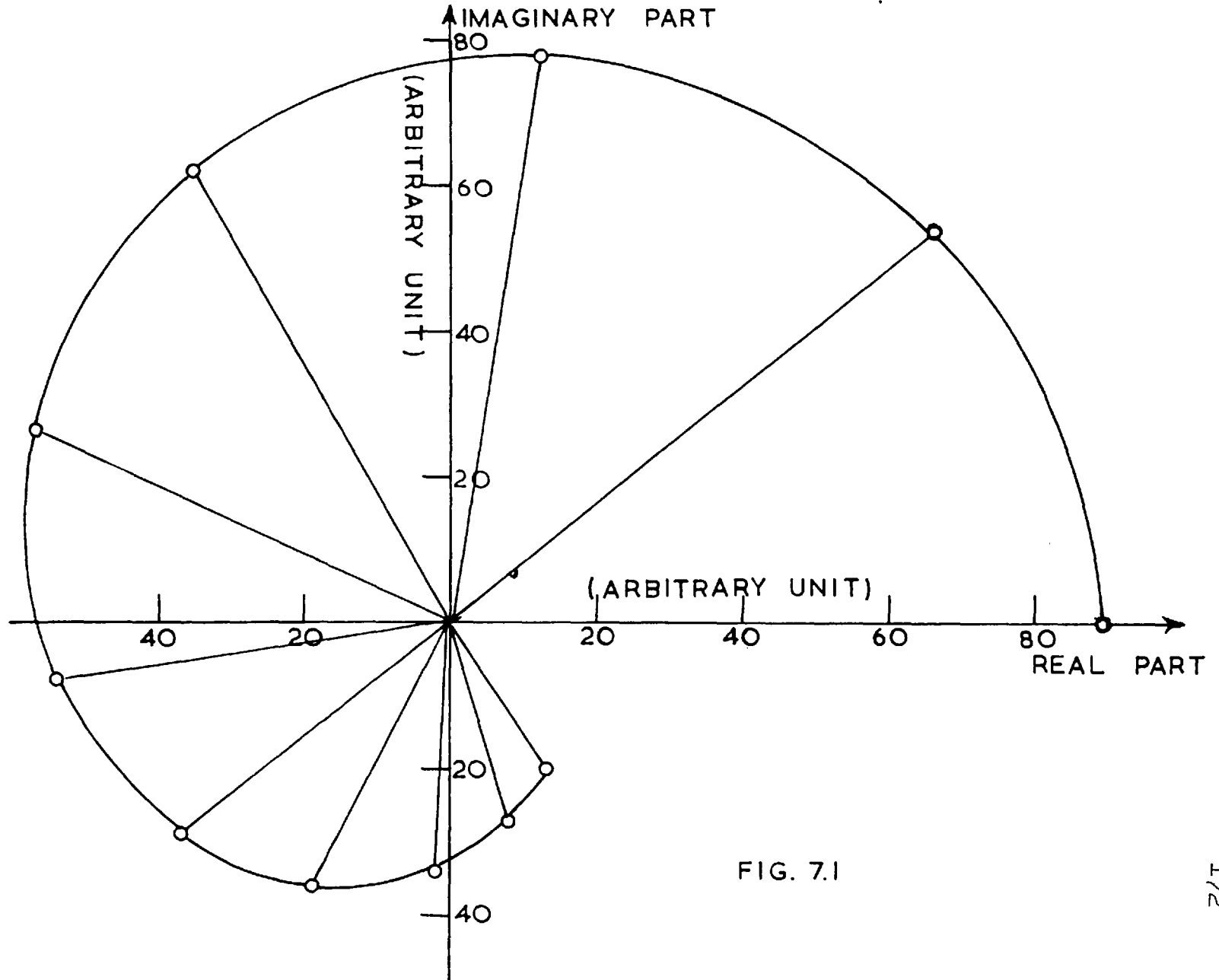
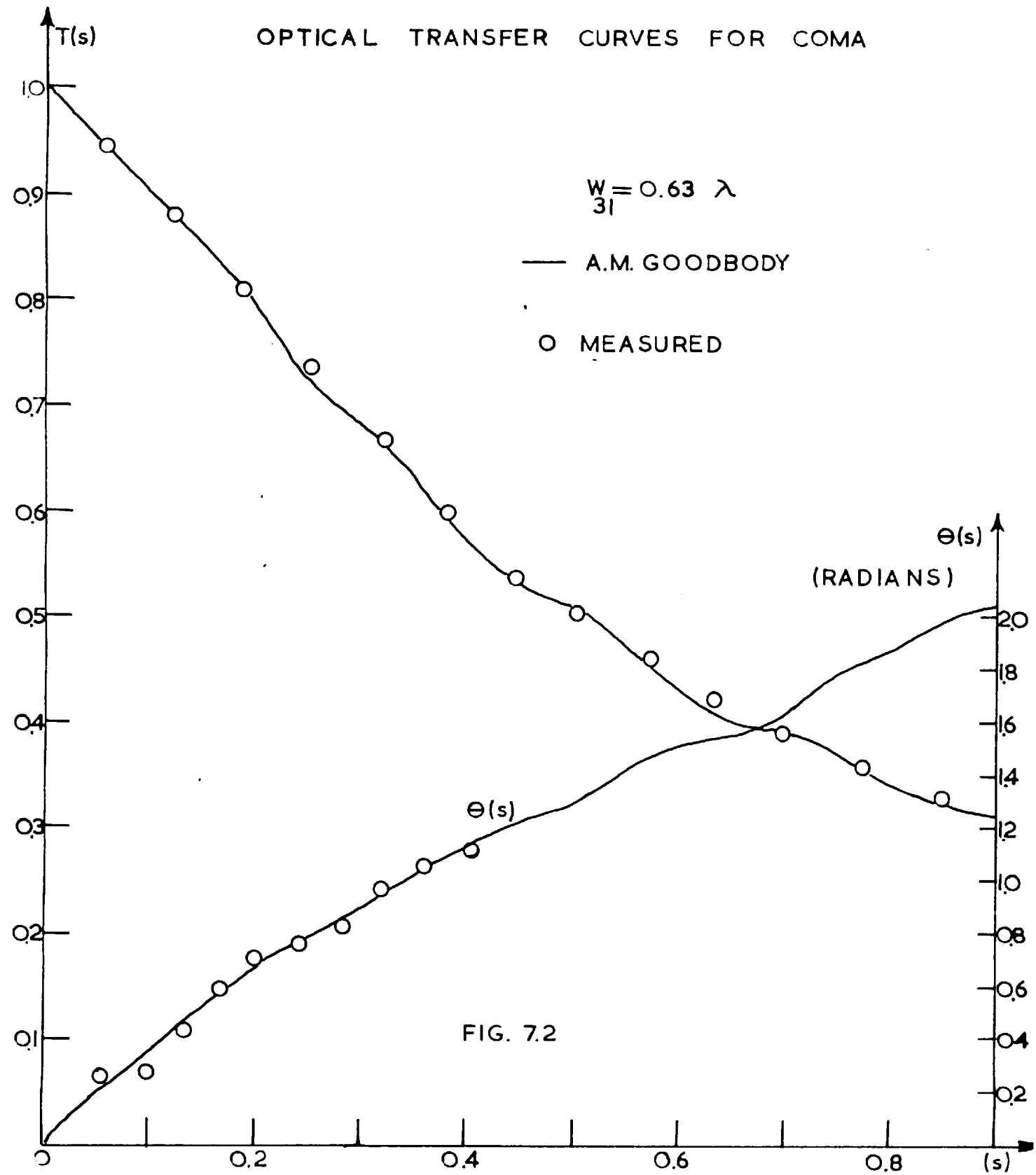


FIG. 7.1

OPTICAL TRANSFER CURVES FOR COMA



OPTICAL TRANSFER CURVES FOR COMA

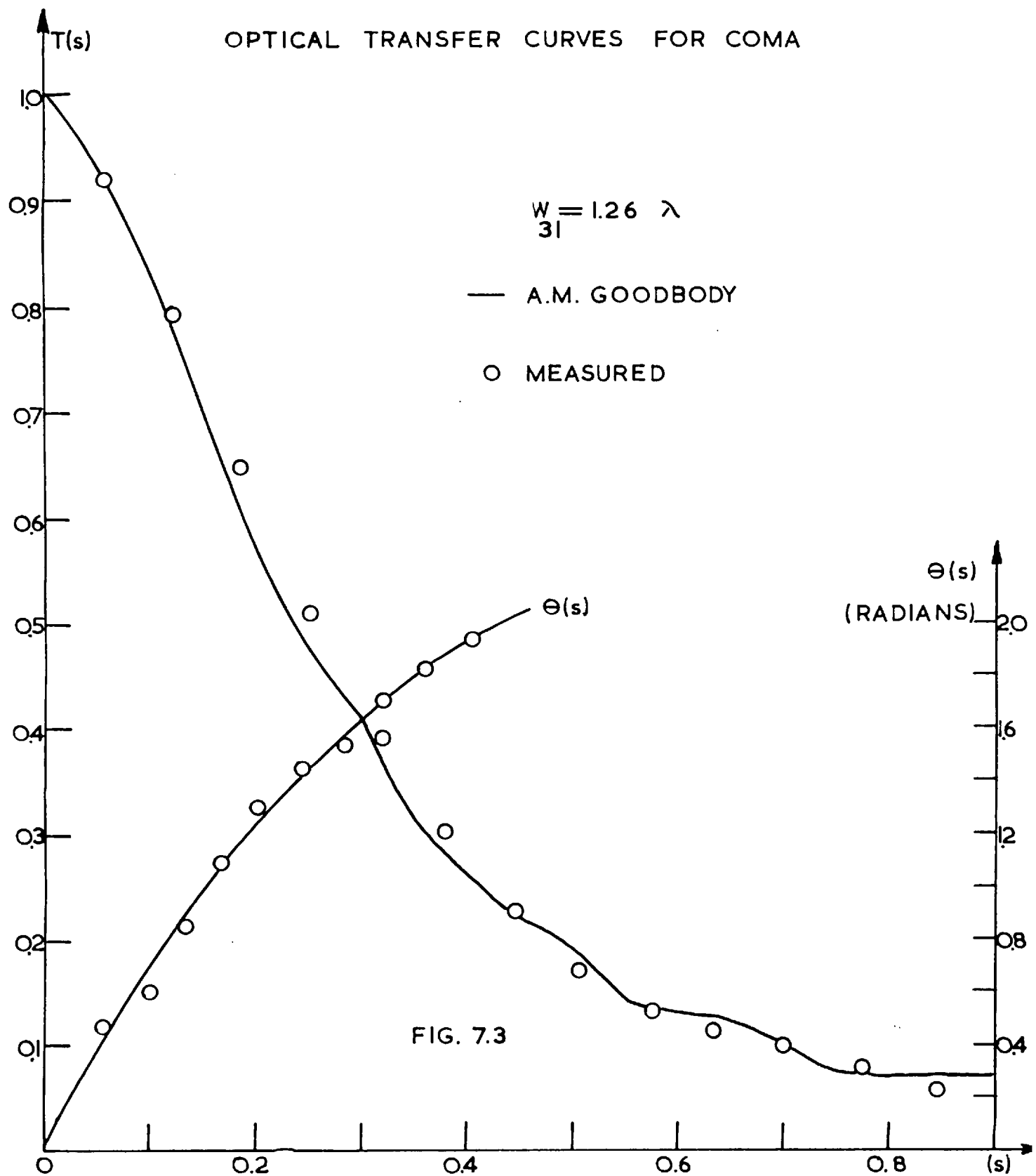
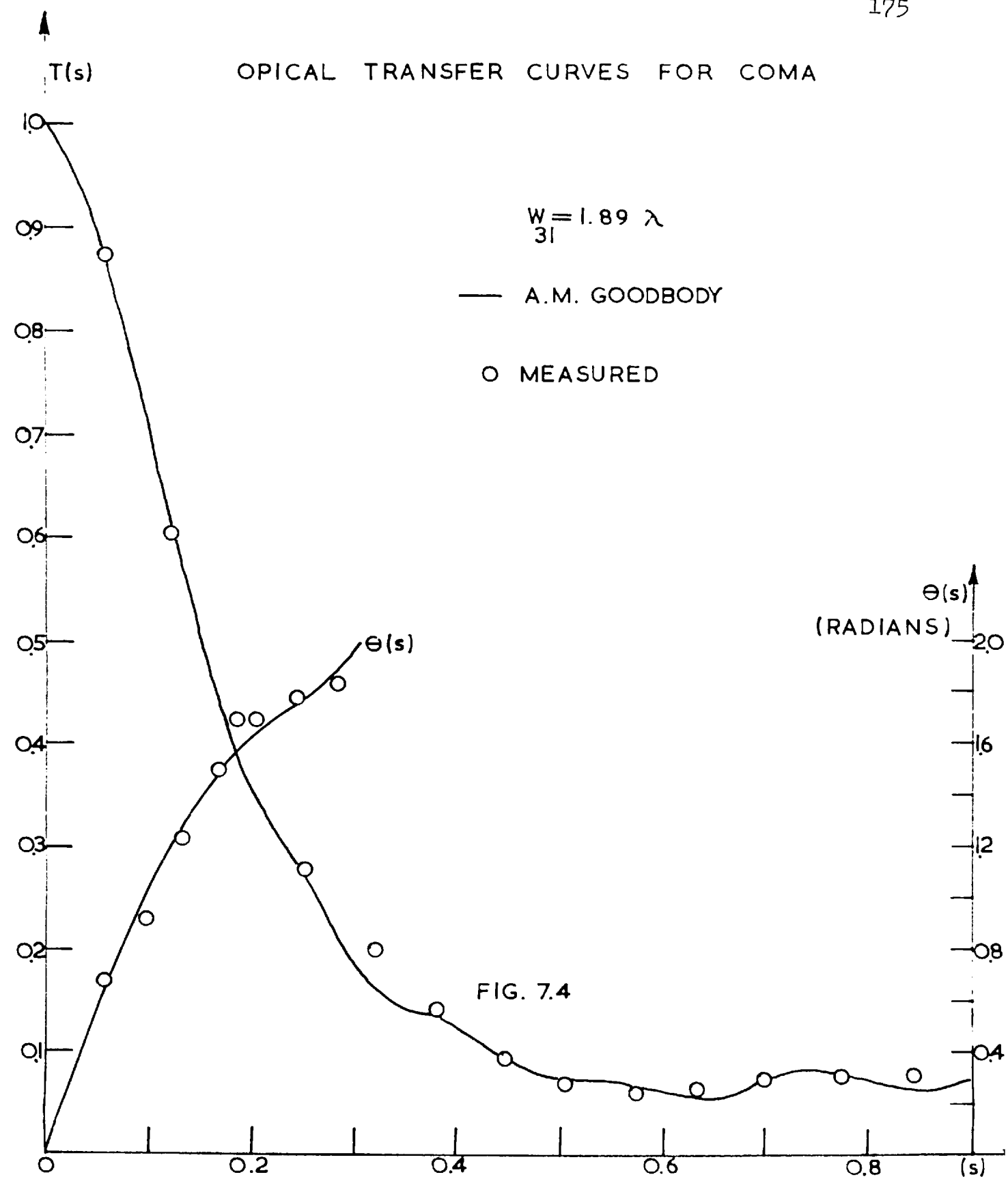


FIG. 7.3

OPICAL TRANSFER CURVES FOR COMA



OPTICAL TRANSFER CURVES FOR COMA

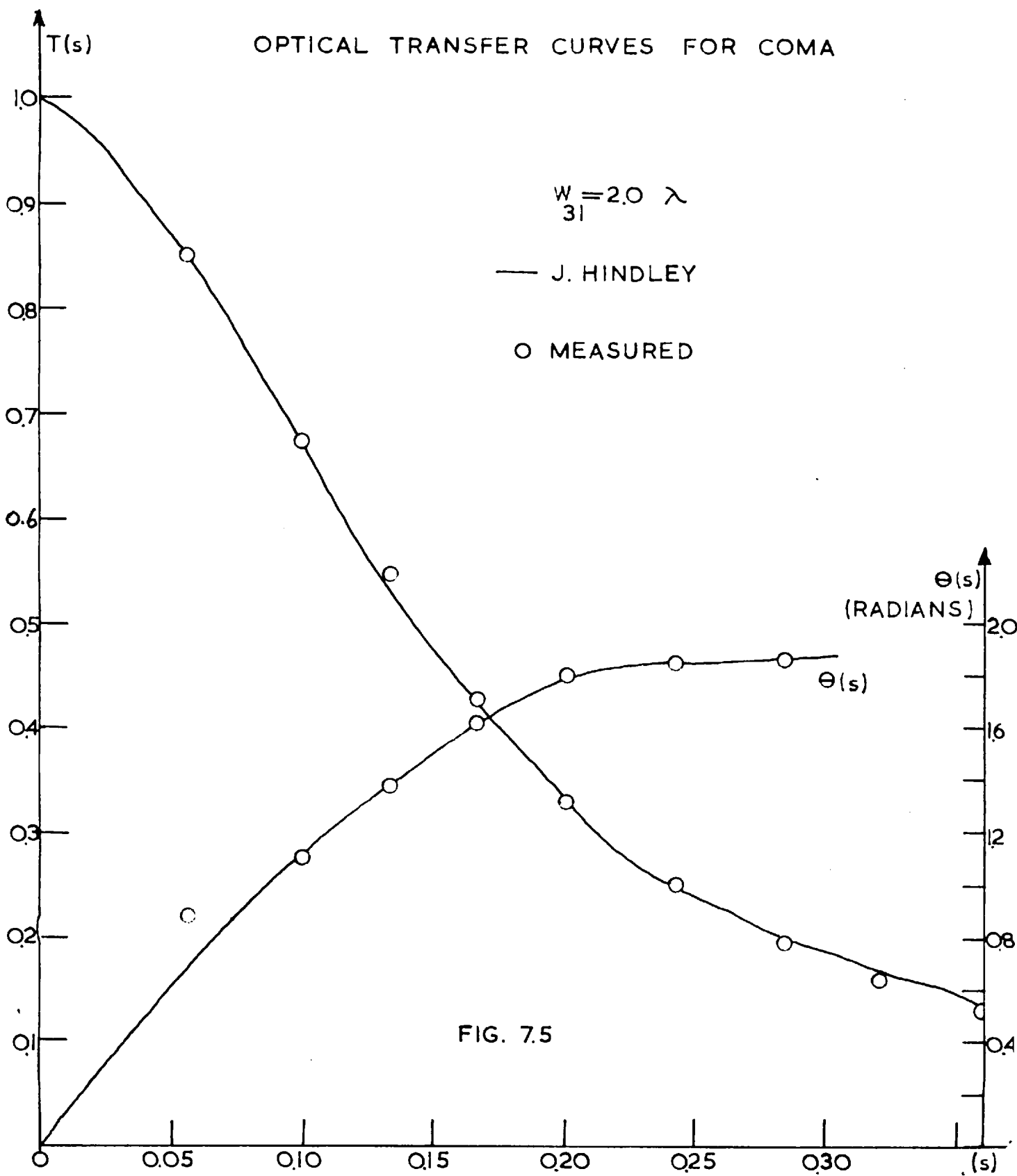


FIG. 7.5

OPTICAL TRANSFER CURVES FOR COMA

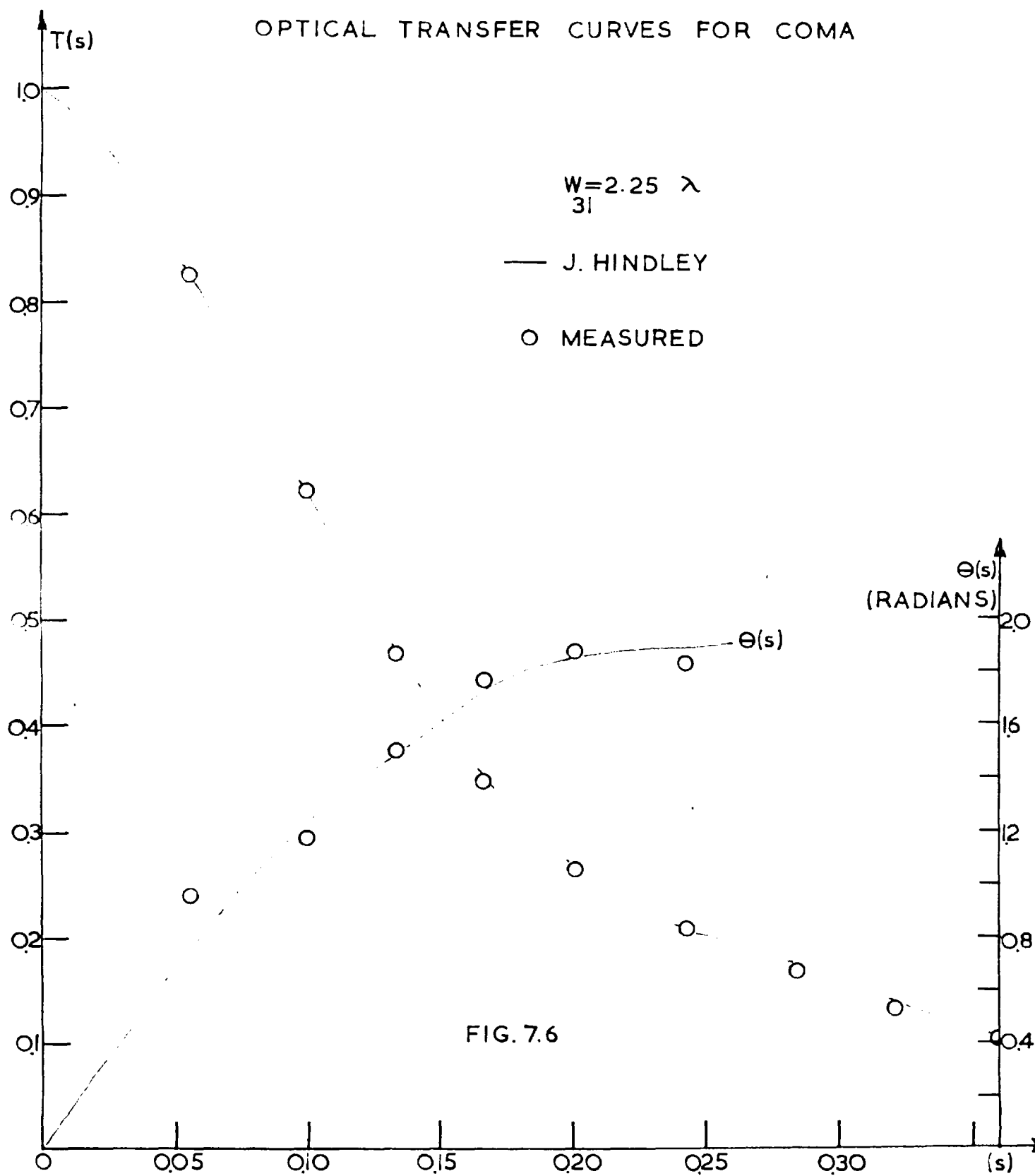
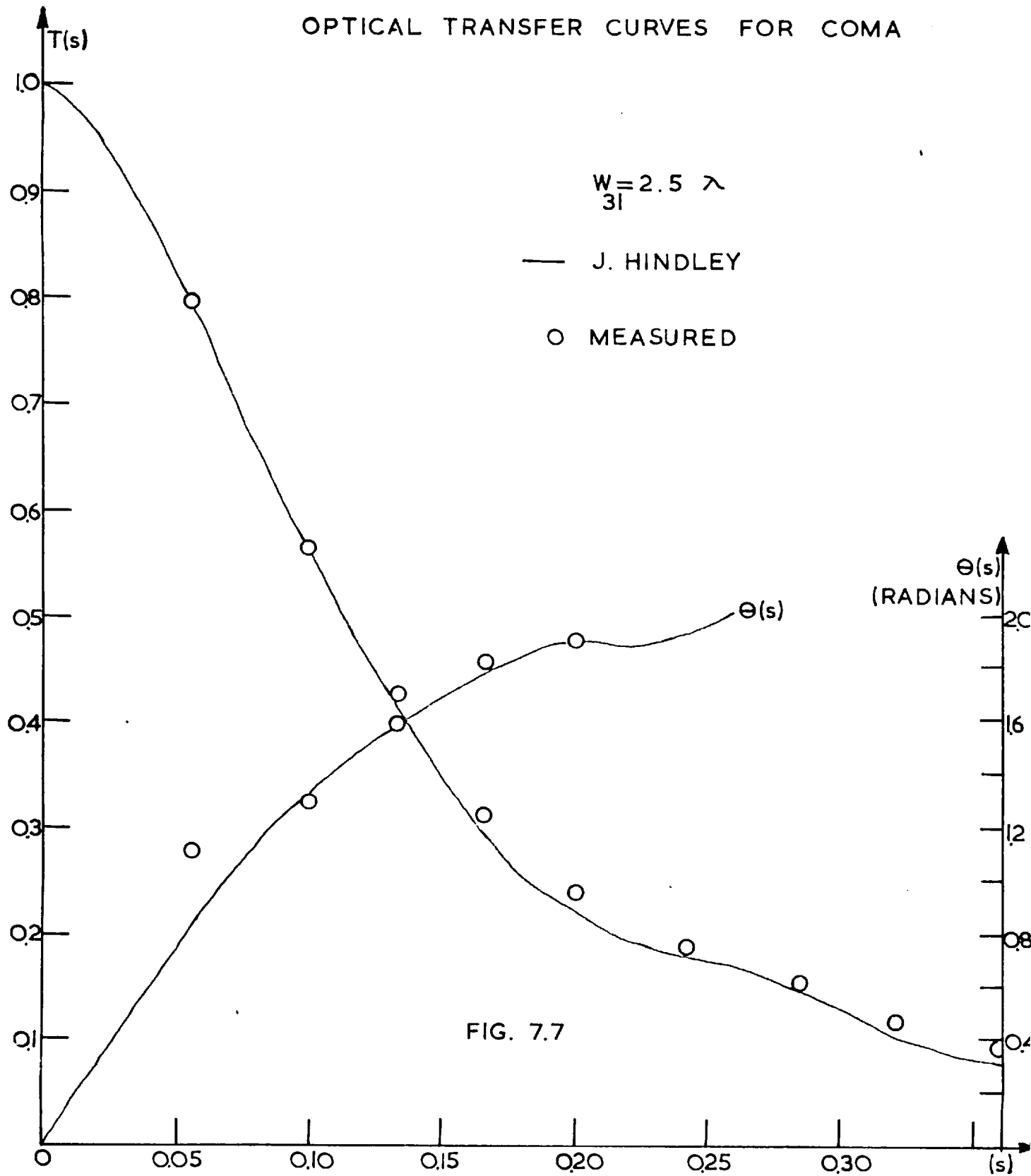
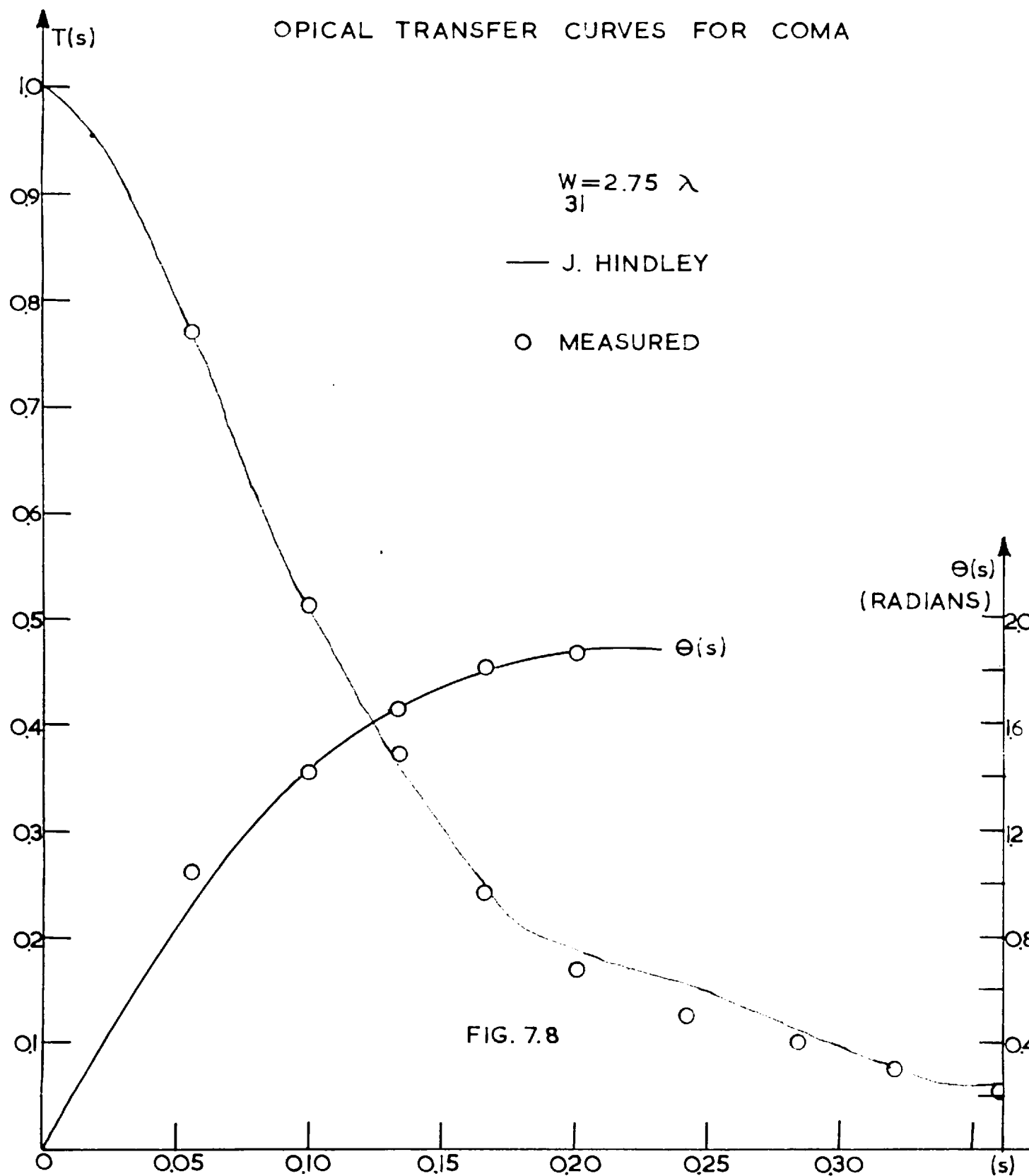


FIG. 7.6

OPTICAL TRANSFER CURVES FOR COMA



OPTICAL TRANSFER CURVES FOR COMA



OPTICAL TRANSFER CURVES FOR COMA

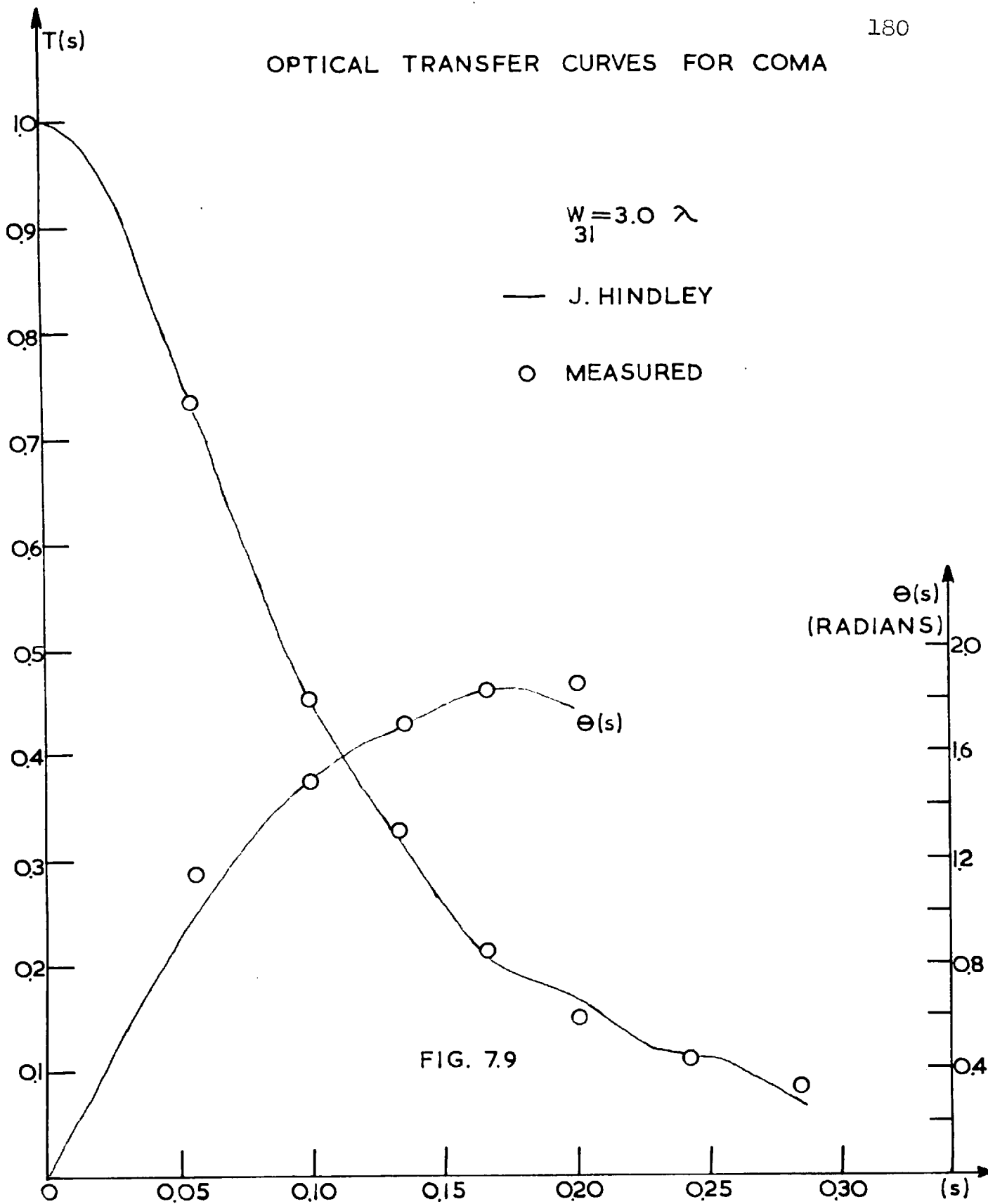
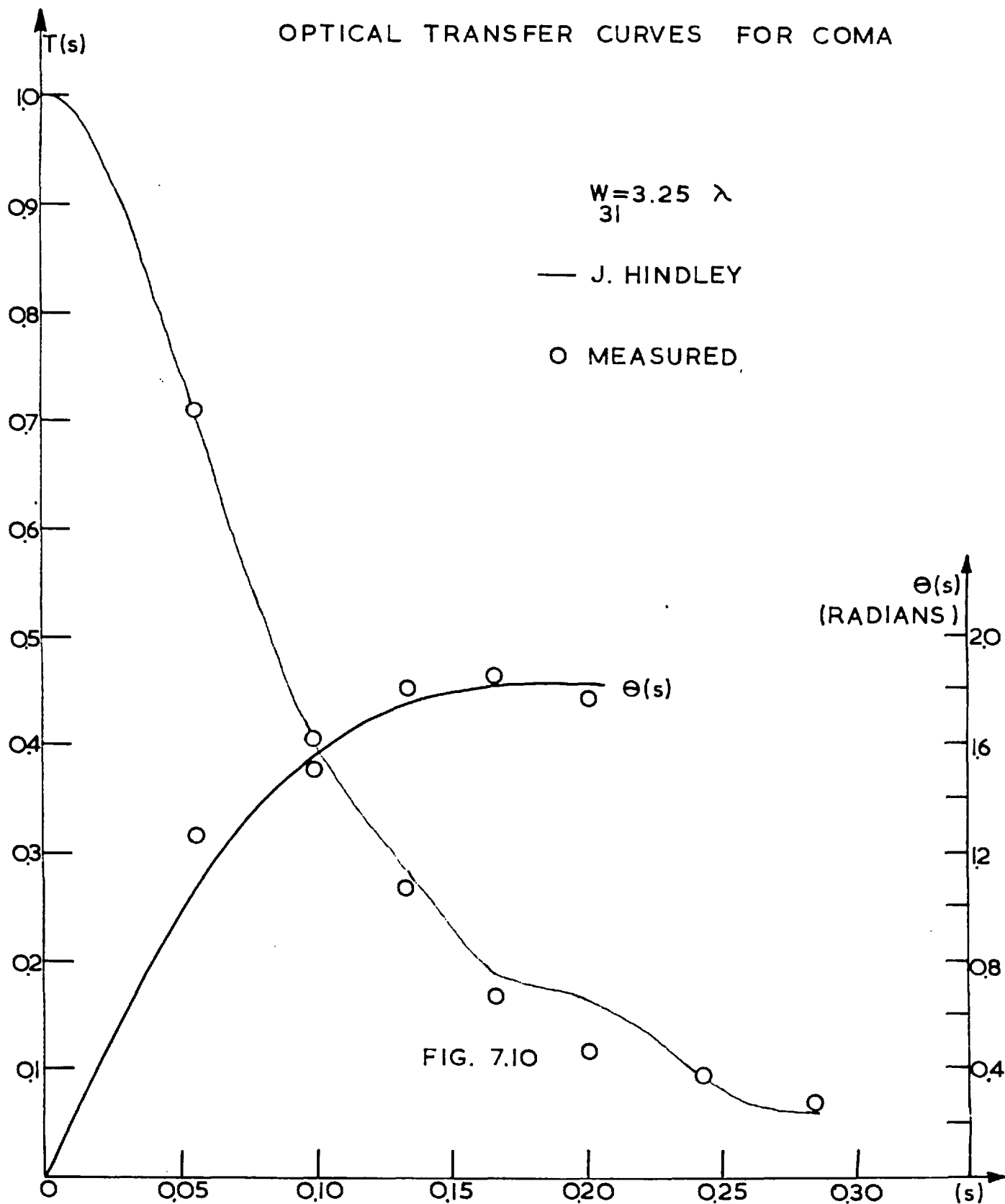
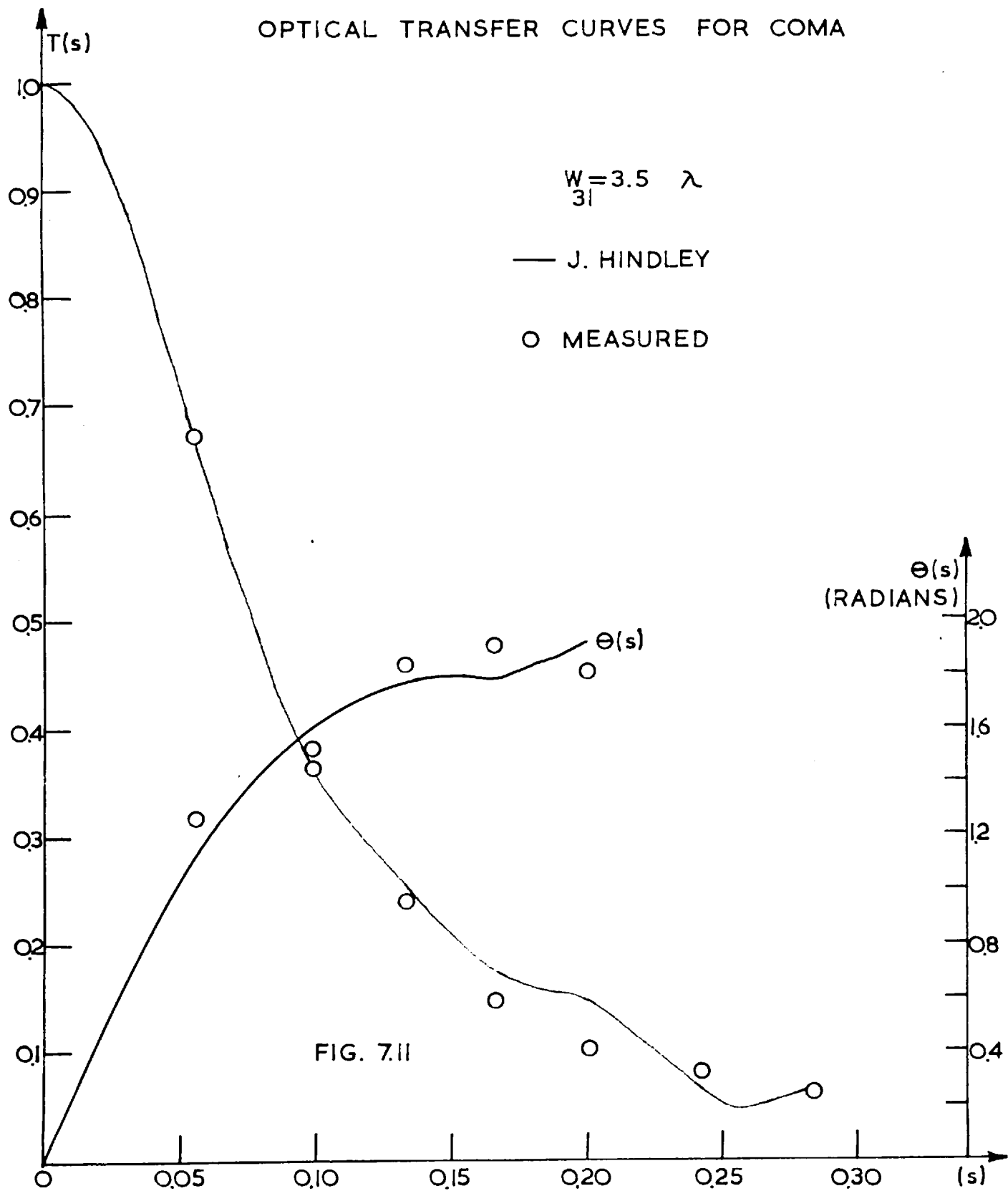


FIG. 7.9

OPTICAL TRANSFER CURVES FOR COMA



OPTICAL TRANSFER CURVES FOR COMA



MODULATION TRANSFER CURVES FOR COMA

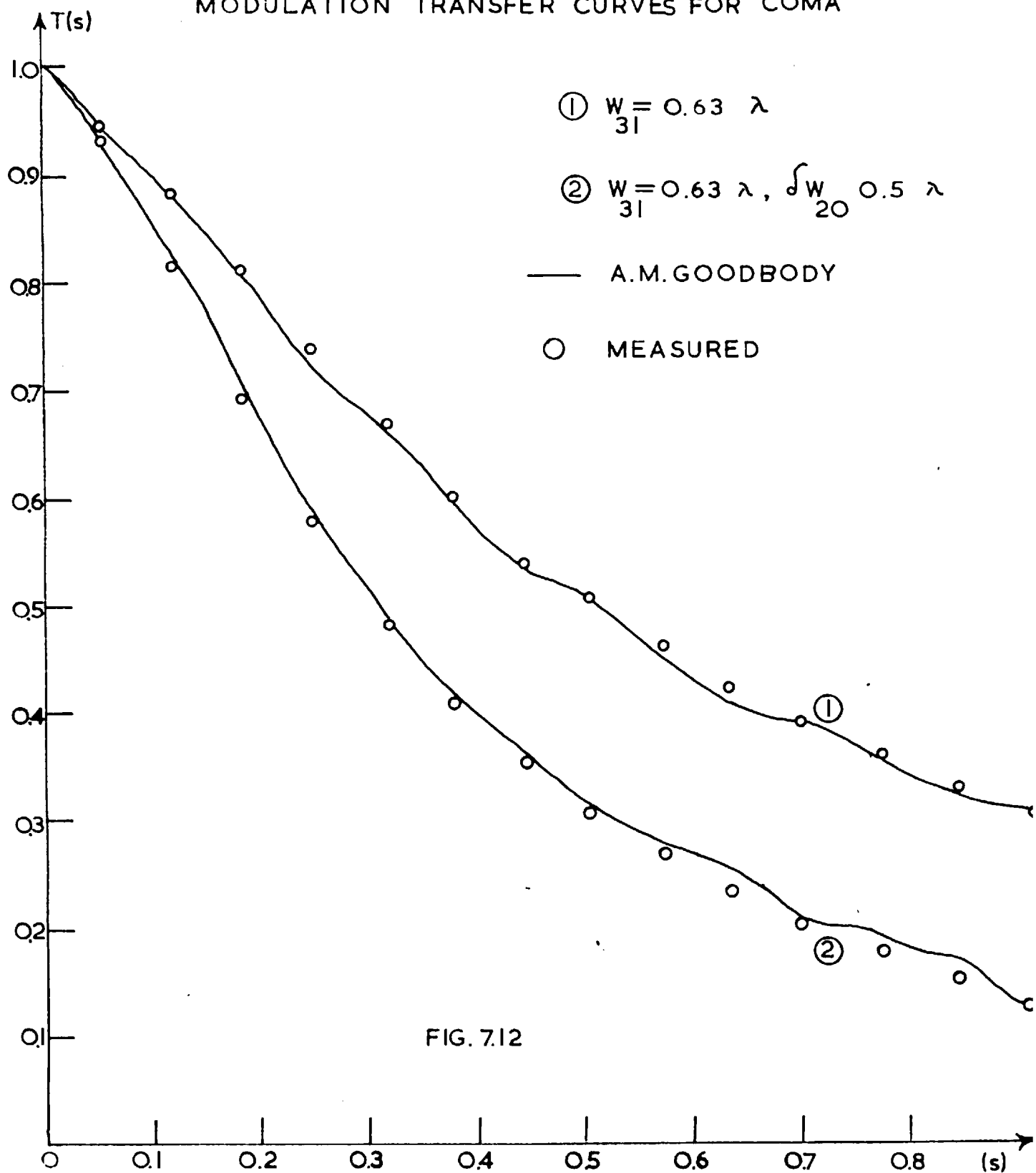


FIG. 7.12

7.3 The measurement of phase transfer function

The phase term $\Theta(s)$ that appears in the expression for the optical transfer function $D(s) = T(s) \exp.\{i \Theta(s)\}$ corresponds to a lateral shift of image position from the Gaussian image point for each spatial frequency component. At the present time there seems to have been no comparison of accurate measurements of this quantity with the theoretically expected values. It is not so necessary for the case of symmetrical aberrations, but it is important for this to be done in the case of asymmetrical aberrations such as coma. The phase term $\Theta(s)$ is an odd function of s . That is

$$\Theta(s) = as + cs^3 + ds^5 + \dots$$

If $c, d, \dots$, are all zero, $\Theta(s)$ is proportional to s , so that $\frac{\Theta(s)}{s} = -\delta u'$ is constant for all values of s . In this case the whole image is shifted bodily, and has no effect on the image quality. If $c, d, \dots$, are not zero the different spatial frequency in the image will be laterally shifted by different amounts and a deterioration of image quality will appear.

The main practical difficulty in the measurement of the phase lies in the differential path difference introduced by the shear plates. It seems to be impossible in practice to make two shear plates to rotate and produce exactly the same change in optical path. The tolerance for this differential path difference is very severe. Thus, to measure the phase $\Theta(s)$ accurate to ± 0.01 of a radian, the differential path difference must be less than 0.01 of a wavelength, and this means that the

angles of incidence at each of the two shear plates must be equal to within one second of arc at any position of shear.

Since there is, in fact, some differential path difference introduced by the shear plates as a function of s , we let this amount be denoted $f(s)$, so that the basic equation for the light flux given above becomes:-

$$F_I = 2A.TR. + 2 A.TR.T(s).\cos\left\{\theta(s) + f(s) + \Delta_I\right\}$$

The measured phase term will thus be the sum of $\theta(s)$ and $f(s)$, and the error term $f(s)$ has to be removed. For the case $W_{31} = 0.63\lambda$, a least square fit between the measured and theoretical phase curves showed a significant departure from the form $as + cs^3$. At this stage it was found that a term of the form bs^2 would be introduced by any inequality, δl , in the lengths of the linkage arms which impart the rotations to the shear plates. On the other hand the coefficient of the linear term, a , depends on δi , the difference in angle of incidence at the two shear plates. Any constant term in the measured values of phase as a function of s may be omitted, since this denotes a residual path difference in the interferometer when $s = 0$. Likewise any linear part, as , in the measured phase may also be subtracted out, since the effect of this is that the image suffers only from a bodily shift.

The real and imaginary parts of the optical transfer function were measured in each case using the procedure already described in chapter V. These were then plotted on the argand.

diagram, as shown in Fig. 7.1. The measured value of the phase angle is then given by $\arg. \{ (\text{real part}) + i (\text{imaginary part}) \}$. For the first case, namely for the value of $W_{31} = 0.63$ wavelengths, the coefficient of the square term, bs^2 , was determined by means of a least squares fit. This same value of b was then used for all other values of W_{31} . The coefficient of the linear term was found in each case by least squares fitting. In this way, the corrected phase of the transfer functions were obtained for the values $W_{31} = 0.63, 1.26, 1.89, 2.0$, and in steps of 0.25 up to 3.5 wavelengths (Fig. 7.2-7.11). In each case the modulus and phase of the transfer function have been plotted on the same graph for comparison with the theoretical values. The measured phase of the transfer functions agree very closely with the theoretical results, the agreement being in general better than 0.1 of a radian or 0.016 of a period. Small errors can result from any error in setting the path difference for the values $\Delta_1 = \pi/2$ and $3\pi/2$, since, for smaller values of s , the variation of light flux is rather rapidly. This has little effect to the measurement of modulation transfer function but can have a marked effect to the phase measurement.

The experimental errors can be estimated as follows. If I denotes the imaginary part and R the real part of the optical transfer function, then.

$$|\theta(s)| = \tan^{-1} \frac{I}{R}$$

and so; $\delta \{ \theta(s) \} = \frac{1}{1 + (I/R)^2} \delta \left(\frac{I}{R} \right)$

or, since $T(s) = \sqrt{R^2 + I^2}$

$$\delta\{\theta(s)\} = \frac{R\delta I - I\delta R}{T(s)^2}$$

Thus, when $T(s)$ is small, a small error in δR or δI will give rise to a large change in $\delta\{\theta(s)\}$ so that phase measurements corresponding to small values of $T(s)$ are not reliable. In the present work the phase measurement was restricted to cases when $T(s) \gg 0.2$. For the case where $W_{31} = 0.63 \lambda$, the phase term could thus be measured up to $s = 1.0$, but the particular pair of shear plates used in this measurement produced a maximum shear of only $s = 0.4$, so no measurement was made beyond this point.

The calculation of the phase uses the differences $(F_1 - F_3)$, $(F_4 - F_2)$ in the measured light fluxes. If K is the normalising constant needed to make $T(0) = 1$, the real and imaginary parts are given by

$$R = \frac{F_1 - F_3}{K}$$

$$I = \frac{F_4 - F_2}{K}$$

Here F_1 , F_2 , F_3 and F_4 are the total light fluxes measured when the path differences are 0 , $\pi/2$, π and $3\pi/2$ respectively. Thus, the error in $\theta(s)$ is given by

$$\delta\{\theta(s)\} = \frac{(F_1 - F_3)(\delta F_4 - \delta F_2) - (F_4 - F_2)(\delta F_1 - \delta F_3)}{(F_1 - F_3)^2 + (F_4 - F_2)^2}$$

where δF_1 , δF_2 , δF_3 , δF_4 are the numerical errors in the measured light fluxes. If these are all equal to δF , the

maximum error in $\theta(s)$ is given by

$$\delta \{ \theta(s) \} = \frac{(F_1 - F_3) + (F_4 - F_2)}{(F_1 - F_3)^2 + (F_4 - F_2)^2} \delta F$$

Substituting each value of F from each case into the above equation, and putting δF equal to 1% of the maximum value of F , it is found that this maximum error in $\theta(s)$ is of the order of 0.1 radian. This is the order of difference actually observed between the measured and theoretical values. An accuracy of phase measurement better than this order can thus not be expected.

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CONCLUSION

The apparatus constructed is compact, easy to adjust and capable of measuring the optical transfer function of any optical system; and furthermore, visual observation of the interferogram is possible at the same time. The results obtained for the case of a defocused optical system and for pure primary coma agree very closely with the theoretical results, which represents an important further step in confirming the correctness of the theoretical approach to image formation by Fourier methods. Although the phase measurement is not a direct method it is capable of accurate measurements to confirm theoretical results.

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