

Numerical modelling of wave
evolution over variable coastal
bathymetry

Umniya Khalil Ahmed Al Khalili

IMPERIAL

Department of Civil and Environmental Engineering
Imperial College London

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To those who inspired the first step and sustained every step after.

Declaration of originality

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Abstract

Accurate modelling of coastal wave transformation is critical for safe, efficient design of coastal and offshore infrastructure, yet remains challenging due to nonlinear amplification, wave-wave interactions, bathymetric effects, and wave breaking. This thesis presents a numerical investigation of unidirectional random waves over planar bathymetry, focusing on how bed slope controls nearshore wave statistics and kinematics.

An extensive dataset is generated using the non-hydrostatic model SWASH, comprising over two million waves simulated across a wide range of offshore conditions and bed slopes. Model predictions are validated against laboratory experiments, with normalised wave and crest heights reproduced within 2% for moderate sea states.

Analysis shows that bed slope governs shoaling, breaking, and nonlinear energy transfer. Steeper slopes lead to larger wave and crest heights and higher near-surface velocities, while milder slopes enhance nonlinear evolution, broaden spectra, and promote stronger energy redistribution. These processes produce systematic spectral shifts and differences in wave asymmetry and higher-order statistics.

Existing engineering formulations for sea-state parameter evolution have limitations, including underestimation of mean energy period by up to 50%. New data-driven parameterisations are developed, incorporating sea-state steepness and bed slope. These improve wave height predictions by up to 21% and predict mean energy period, mean period, and mean zero-crossing period with accuracies of 13.8%, 6.9%, and 5.6%, respectively.

Phase-resolved analysis of wave kinematics shows that bed slope can alter shallow-water crest velocities by up to 48%. Regular wave methods perform well in intermediate depths but degrade in shallow water, with errors in the non-dimensional depth-averaged velocity profile of order $O(1)$. Random wave methods achieve improved accuracy ($\pm 5\%$) in intermediate conditions but underestimate velocities by up to 40% in shallow water, highlighting the complexities of the processes in this region.

Overall, this work delivers new insight and robust, slope-dependent tools for predicting coastal wave behaviour.

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الْبَحْرُ لَا يَبْخُ بِأَسْرَارِهِ إِلَّا لِمَنْ يَغُوصُ فِي أَعْمَاقِهِ.
مُصْطَفَى صَادِقِ الرَّافِعِيِّ

‘The sea whispers its secrets only to those who dare to dive deep into its heart.’

Mustafa Sadiq Al-Rafi'i

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DFSBC	Dynamic Free Surface Boundary Condition
DFT	Discrete Fourier Transform
FDM	Finite Difference Method
FEM	Finite Element Method
FFT	Fast Fourier Transform
FNPF	Fully Nonlinear Potential Flow
FVM	Finite Volume Method
JONSWAP	JOint North Sea WAve Programme
KFSBC	Kinematic Free Surface Boundary Condition
LRWT	Linear Random Wave Theory
MWL	Mean Water Level
NLSWE	Nonlinear Shallow Water Equation
NSE	Navier-Stokes Equation
RANS	Reynolds-Averaged Navier Stokes
RMSE	Root-Mean-Squared Error
SORWT	Second-Order Random Wave Theory
SPH	Smoothed-Particle Hydrodynamics
SWASH	Simulating WAves till SHore
SWL	Still Water Level

TMA	Texel–Marsden–Arsloe
VOF	Volume-of-Fluid
WKB	Wentzel–Kramers–Brillouin

Chapter 1

Introduction

1.1 Overview

The ocean has underpinned human civilisation for millennia, providing food, transport routes, and natural resources essential for societal development. In the modern era, it continues to support major economic sectors, including fisheries, aquaculture, international shipping, tourism, and coastal development. Coastal zones, where land–sea interactions are most dynamic, are particularly vital. Nearly 80% of global trade by volume is transported by sea (United Nations, 2018), underscoring the sustained dependence of international economies on maritime routes and coastal infrastructure.

These advantages have driven rapid coastal urbanisation. Approximately 40% of the world’s population resides within 100 km of the coast (Center for International Earth Science Information Network, 2012), and coastal megacities continue to expand. In light of this, the significance of coastal hazards is amplified. Accelerating climate change, sea-level rise, shifts in storm patterns, and an observed increase in extreme wave events threaten both natural and built coastal environments (Morim et al., 2021). Flooding, shoreline retreat, and intensified storm surges increasingly impact communities, transportation systems, and industrial facilities. This growing exposure heightens the need for accurate prediction of coastal wave conditions to inform hazard assessment and the design of resilient coastal infrastructure.

Coastal and offshore regions also hold strategic importance for global energy security. Offshore extraction remains substantial, with 28% of global crude oil production occurring offshore and offshore natural gas extraction increasing by roughly

50% since 2000 (International Energy Agency, 2018). Concurrently, the marine renewable-energy sector has expanded rapidly, driven by international decarbonisation commitments such as the Paris Agreement (United Nations, 2015). Offshore wind, wave-energy converters, and tidal-stream turbines are now integral to national energy strategies. For instance, offshore wind contributes more than 13% of electricity generation in the United Kingdom (Mawhood et al., 2020). As this sector grows, the demand for accurate prediction of nearshore wave transformation intensifies, as these processes directly influence design loads, fatigue cycles, and operational safety.

1.2 Research motivation

Accurate prediction of wave conditions is essential across coastal and offshore engineering applications, including breakwater design, harbour layout optimisation, sediment-transport analysis, and marine renewable-energy development. While wave propagation in deep water is well understood, wave transformation in intermediate and shallow water involves complex nonlinear processes. As waves approach shallower depths, they undergo shoaling, refraction, reflection, nonlinear interactions, and ultimately breaking. Variations in bathymetry further modify wave spectra and statistics in ways that cannot be represented by simple analytical descriptions.

Understanding how waves evolve over variable bathymetry is particularly important because even modest changes in seabed slope can significantly modify the nearshore wave field. These changes influence coastal hydrodynamics, sediment-transport pathways, and structural loading. Improved understanding and modelling of nearshore wave evolution is therefore essential for both hazard prediction and the safe, efficient, and sustainable development of coastal zones.

This thesis is motivated by three persistent gaps in coastal engineering research and practice. These motivations are expanded in the following subsections:

1. Experimentally-validating numerical modelling frameworks.
2. Advancing understanding of the nonlinear evolution of waves in the coastal zone, including both surface elevations and kinematics.
3. Converting fundamental insight into practical engineering guidance.

1.2.1 Reliable numerical modelling

Numerical modelling is indispensable in coastal engineering because it enables the investigation of hydrodynamic processes too complex for analytical solutions or laboratory replication alone (Dean & Dalrymple, 1991). Many governing equations for wave transformation, breaking, turbulence, sediment transport, and morphodynamics are highly nonlinear and coupled (Holthuijsen, 2010). Laboratory experiments, although essential, are constrained by scale effects, spatial resolution, and cost (Van Rijn et al., 2011). Moreover, isolation of individual wave processes is often difficult, with configurations being intrinsically three-dimensional in nature. Numerical models complement experimental and theoretical approaches by allowing controlled simulation of diverse sea-states and bathymetric configurations.

A key advantage of numerical modelling is the ability to isolate individual physical processes. In natural coastal systems, the interplay between different physical processes occurs simultaneously. Numerical models enable these mechanisms to be examined independently or in combination, enhancing both theoretical understanding and the interpretation of experimental or field data. Additionally, models can generate extensive datasets spanning thousands of scenarios, supporting the development of robust parametric tools for engineering design (Goda, 2000).

From a practical standpoint, numerical models are central to coastal-risk assessment and infrastructure planning. They underpin predictions of wave transformation, overtopping, flooding, and sediment dynamics under present and future climate conditions (Vousdoukas et al., 2018). Advances in computational power and numerical schemes have transformed these models into operational tools for forecasting and coastal design (Lesser et al., 2004). Their continued development and validation are therefore critical.

Recent advances in numerical modelling have focused on improving the accuracy and efficiency with which multi-scale coastal processes are resolved. In particular, the development of adaptive modelling techniques utilising unstructured mesh refinement enable the resolution of both small and large scale ocean processes. Much effort is made to allow these methods to conform to complex bathymetry and coast-line (Pain et al., 2005). By concentrating computational effort in regions of strong gradients or intricate bathymetry (Piggott et al., 2008), these approaches enhance the simulation of nonlinear processes while maintaining computational tractability. Such

developments have proven especially valuable in modelling spatially heterogeneous systems, including marine mixing (Robinson et al., 2016), tidal flows (Abolghasemi et al., 2016) and coupled ocean–ice interactions (Jordan et al., 2014), and reflect the broader trend towards flexible and scalable numerical frameworks in coastal engineering.

Among smaller-scale coastal models, Simulating WAves till SHore (SWASH; Zijlema et al. 2011) offers a promising balance between computational efficiency and physical fidelity. However, despite its widespread application, a comprehensive assessment of its performance for phase-resolved statistics across diverse wave and bathymetric conditions remains lacking. Establishing the validity and limitations of SWASH for predicting nearshore wave statistics forms a key motivation for this research.

1.2.2 Understanding of wave evolution

Understanding wave evolution in coastal waters is fundamental to predicting overtopping, inundation, structural loading, and erosion (Kobayashi & Wurjanto, 1989). As waves shoal from deep to shallow water, nonlinear processes intensify, altering wave heights, shapes, and kinematics (Mei, 1989). Considerable research has examined the effects of offshore wave steepness and reduced depth on these transformations (e.g. Tayfun, 2006, Katsardi et al., 2013, Karpadakis et al., 2019a). However, the influence of local bed slope on these processes remains comparatively under-explored. Numerical analyses by Bellotti (2007) and Bellotti et al. (2012) have highlighted complexities in nearshore wave transformation in the context of semi-enclosed basins, emphasising the significance of long waves in this region which can induce significant departures from simple analytical solutions.

Bed slope controls the rate of shoaling, the distance available for nonlinear energy transfers, and the onset and intensity of breaking (Lawrence et al., 2021b). Despite these clear mechanisms, limited systematic quantification exists for the effect of bed slope on surface-elevation statistics in highly nonlinear, irregular sea-states. This is a critical gap, because coastal regions with similar offshore conditions can experience vastly different nearshore climates depending on local bed slope. This is an effect commonly under-represented in present design tools (SPM, 1984, CEM, 2002, EurOtop, 2018).

Bed slope effects are also essential for understanding kinematics, which govern hydrodynamic loads on structures (Lighthill, 1986). As waves steepen over sloping beds, crest velocities and accelerations increase, amplifying loads on coastal and offshore structures, especially under breaking conditions (Cuomo et al., 2010). Yet relatively few studies have examined crest kinematics in irregular, strongly nonlinear seas (Lutio, 2017). Addressing this gap is vital for improving design-wave methods and kinematic models in engineering practice.

1.2.3 Practical applications

The design of coastal and offshore structures typically follows a multi-stage framework intended to capture both the long-term variability of sea-states and the short-term characteristics of waves that govern structural loading. This framework integrates historical data analysis, probabilistic methods, and detailed wave modelling to ensure that structures are designed to withstand extreme but plausible environmental conditions. At its core, the approach consists of three interconnected components:

Long-term analysis For a given geographical location of interest, the long-term characteristics of the environment are defined based on a collection of met-ocean data. These describe the behaviour of the sea and wind, through characteristic spectral wave parameters, over long time periods. This information is obtained through *in situ* measurements or most commonly, hindcast data. The latter refers to a numerical model that infers surface wave parameters from past meteorological observations, such as wind speed. Through this long-term analysis, representative *sea-state* parameters are desired. In this context, sea-state refers to a duration in time (usually 3 hours) over which statistical stationarity can be assumed (Ochi, 2005).

Typically, the sea-state is described by an associated pair of significant wave height and spectral peak period (H_s, T_p), a mean direction of wave propagation ($\bar{\theta}$), the directional spreading about the mean (σ_θ) and the spectral bandwidth (ν).

Once a dataset of sea-state parameters is established, the long-term distribution is calculated. This defines the probability of extreme events, such as the 1-in-100-year or 1-in-1000-year sea-states traditionally used in coastal design. According to Det Norske Veritas (2010), this may be achieved in two ways by either considering all sea-states (initial distribution method) or utilising a Peak-over-Threshold approach

(storm analysis method). In the former method, the entire dataset is used directly to construct the long-term distribution of key sea-state parameters. Because it does not rely on the identification of individual storm events, this approach can be applied even where the available time series is relatively short. A drawback, however, is that consecutive sea-states recorded during the same storm inevitably exhibit statistical dependence. If independence between observations is assumed without caution, the resulting estimates of extreme conditions can become slightly conservative. Additionally, careful attention must be given to how the tail of the distribution is modelled, since most measurements typically reflect moderate rather than extreme conditions. Despite these limitations, the all-seas method is widely considered suitable in regions where severe storms occur frequently, providing sufficient coverage of high-energy states over the observational period (Haver, 2017)

An alternative is to focus only on sea-states that exceed a chosen threshold, thereby isolating storm events. For each storm, the most energetic sea-state, usually the one with the largest significant wave height, is used to characterise that event. The extracted peaks are then fitted to a statistical model grounded in extreme-value theory (Coles et al., 2001). This approach benefits from a clearer justification of statistical independence, since individual storms can be treated as distinct events. However, its effectiveness depends heavily on the availability of sufficiently long observational records to ensure a representative number of storms; otherwise, the resulting estimates may be dominated by sampling uncertainty. In environments with infrequent extreme events but extensive long-term metocean datasets, this method is generally preferred. A variant of this approach uses annual maxima to construct the long-term distribution, although this also requires lengthy records to produce stable estimates.

In the nearshore region, transformation of the wave field is typically modelled using spectral wave propagation approaches that account for key physical processes such as refraction, shoaling, depth-induced breaking, bottom friction, and wave–current interaction. As waves propagate from deep to shallow water, spatial variations in bathymetry and currents redistribute wave energy across frequencies and directions, altering both the spectral shape and directional spreading. This process can be conceptually represented using a wave transformation matrix, which links offshore and nearshore spectral conditions through an operator that accounts for energy

propagation, redistribution, and dissipation (Dean & Dalrymple, 1991). In practice, however, this matrix form is rarely constructed explicitly, as wave evolution is more commonly obtained by numerically solving the governing spectral energy or action balance equation.

A widely adopted tool for this purpose is the third-generation spectral wave model SWAN (Simulating WAVes Nearshore), which solves the wave action balance equation in two horizontal dimensions while including source and sink terms for wind input, whitecapping, nonlinear wave-wave interactions, depth-induced breaking, and bottom friction (Booij et al., 1985). SWAN is particularly suited to coastal applications due to its ability to resolve complex nearshore bathymetry and transform offshore hindcast or measured sea-states into local design conditions. In engineering workflows, several methods such as SWAN amongst others are utilised to propagate offshore long-term sea-state statistics to nearshore locations. These methods will be evaluated in Chapter 5 of this thesis and new improved parameterisations will be proposed.

The long-term analyses derived from either method are often visualised as environmental contours (Winterstein et al., 1993), which define combinations of significant wave height and wave period corresponding to specified return periods (e.g., 1, 100, or 10 000 years). Points associated with large H_s values denote the most severe conditions for a given probability level, while steeper seas are typically associated with lower T_p values. These extreme or near-extreme sea-states form the basis for subsequent short-term analyses.

Short-term analysis For a given return period, the design sea-state is defined based on the long-term analysis of the wave climate by selecting a combination of H_s , T_p , $\bar{\theta}$, σ_θ and ν . Once a design sea-state has been identified, its wave-by-wave variability must be explored. Short-term analysis focuses on the statistical distribution of individual waves, such as crest heights or wave heights, within a fixed-duration sea-state. Traditional probabilistic models (Forristall, 1978, 2000) have commonly been applied for this purpose, but several studies have shown that their predictions can deviate notably from measured data, particularly in intermediate or shallow water depths where nonlinear processes become more prominent (Latheef & Swan, 2013). A major component of contemporary research therefore concerns improving these short-term models and identifying the conditions under which exist-

ing formulations fail (Karmpadakis et al., 2019b, 2020, Karmpadakis & Swan, 2022, Karmpadakis et al., 2022). Although these studies are extensive, they are limited to flat-bed conditions. The influence of bed slope, however, remains to be investigated.

Design wave With the relevant short-term characteristics identified, engineers must then select a representative deterministic wave for structural loading calculations. The standard practice is to match the design crest or wave height using a regular wave theory, commonly a high-order Stokes solution (Fenton, 1985) or Stream Function theory (Dean, 1965). This produces a single wave profile that approximates the extreme event expected within the design sea-state. Although this deterministic design-wave method is straightforward and computationally inexpensive, limitations exist when the effects of unsteadiness, nonlinearity, breaking and directional spreading become important.

An alternative is the implementation of linear random wave modelling (Airy, 1845) or empirical linear random wave alternatives (Wheeler, 1970, Molin, 2002, Chakrabarti, 1971, Donelan et al., 1992). Although these methods are regularly adopted in practice, few studies have systematically quantified the accuracy of these methods for instantaneous kinematics under extreme irregular waves especially in shallow water and with sloping beds.

1.3 Research aims and objectives

The aim of this research is to understand the dominant mechanisms that govern the evolution of uni-directional random waves in coastal regions and provide practical guidance for their prediction. Following the research gaps outlined above, the main objectives to address this aim are summarised below:

- (i) Numerically-simulate long random wave records in SWASH for a range of sea-states and bathymetries.
- (ii) Evaluate the validity of SWASH by comparing numerical simulations with equivalent laboratory measurements.
- (iii) Gain physical insight into coastal wave evolution, with particular emphasis on the influence of bed slope.

- (iv) Develop new formulations describing the evolution of sea-state parameters in coastal regions.
- (v) Provide practical guidance for the application of kinematic theory in engineering design.

1.4 Thesis layout

These objectives are addressed across four results chapters (Chapters 3–6), structured as follows:

Chapter 2 outlines the key technical background established in the literature. It first presents the fundamental theories describing regular and irregular waves, followed by an examination of the principal mechanisms governing wave evolution in coastal waters. The short-term statistical measures used to characterise sea-states are then introduced through both time- and frequency-domain representations. Finally, relevant numerical modelling approaches for capturing wave transformation are reviewed. Together, these components provide the theoretical foundation for the analyses developed in the remainder of the thesis.

Chapter 3 evaluates the capability of the SWASH non-hydrostatic model to reproduce regular and irregular wave transformation from intermediate depths to the shoreline. By benchmarking simulations against controlled laboratory measurements, the chapter establishes the model’s range of validity for nearshore wave and crest-height statistics, forming the basis of the modelling framework used throughout the thesis and directly contributing to Objectives (i) and (ii).

Chapter 4 investigates the influence of bed slope on nonlinear energy transfers and statistical properties of irregular waves. Using an extensive numerical dataset, the chapter provides a detailed spectral and statistical analysis of both surface elevation and kinematic quantities, quantifying how slope controls wave statistics, integral parameters and spectral properties. This process-based understanding addresses a key research gap concerning slope-dependent nonlinear evolution and therefore advances Objective (iii).

Chapter 5 applies the validated modelling framework to develop data-driven parameterisations for representative wave heights and periods over variable bathymetry. These formulations bridge the gap between computationally expensive process-based

models and simplified empirical methods, offering efficient, physically consistent tools for preliminary coastal engineering design and thereby fulfilling Objective (iv).

Chapter 6 presents the first systematic analysis of horizontal crest velocities in irregular, highly nonlinear sea-states across different bed slopes. After validating SWASH for phase-resolved kinematics, the chapter quantifies how sea-state steepness and bathymetric slope shape crest-velocity distributions and vertical velocity structure. The results are used to evaluate commonly applied kinematic theories and to translate these findings into practical guidance for engineering applications, completing Objective (v).

Chapter 7 summarises the key findings of the present work and discusses their practical implications. The chapter also outlines several avenues for future research.

Publications The work presented in the results chapters has either been published or under review for publication:

1. Chapter 3 - Al Khalili, U., Bellos, V., Christou, M., & Karpadakis, I. (2025). Regular and random wave modelling over mild uniform bathymetry using SWASH. *Coastal Engineering*, 199, 10475. <https://doi.org/10.1016/j.coastaleng.2025.104745>
2. Chapter 4 - Al Khalili, U., Christou, M., & Karpadakis, I. (2026b). The effect of bed slope angle on the statistics and evolution of wave fields over coastal bathymetry. *Journal of Fluid Mechanics*.
3. Chapter 5 - Al Khalili, U., Christou, M., & Karpadakis, I. (2026a). Data-driven parameterisation of representative wave heights and periods over coastal bathymetry. *Maritime engineering*.
4. Chapter 6 - Al Khalili, U., Christou, M., & Karpadakis, I. (2026c). On the prediction of wave particle velocities in irregular seas. *Applied Ocean Research*, 170, 105039. <https://doi.org/10.1016/j.apor.2026.105039>.

Together, these studies deliver a comprehensive investigation of coastal wave evolution over variable bathymetry. The thesis progresses from model validation (Chapter 3), through physical understanding (Chapter 4), to development of practical design-oriented tools (Chapters 5–6). Overall, the thesis advances both scientific understanding and engineering prediction of nonlinear coastal wave evolution,

highlighting the central role of bathymetry in shaping nearshore wave statistics and kinematics and establishing validated approaches for modern coastal engineering practice.

Chapter 2

Technical Background

This chapter presents the theoretical foundations required to understand wave propagation and evolution over variable coastal bathymetries. The material is presented concisely, while more specialised topics and discussions appear in subsequent chapters. The aim is to provide a clear overview of the governing equations, wave theories, statistical representations of sea-states, coastal transformation processes, and numerical modelling frameworks relevant to coastal wave evolution.

2.1 Governing equation

The fundamental description of water waves is based on the assumption that the flow is *incompressible*, *inviscid*, and *irrotational*. The inviscid and irrotationality assumptions are justified in the absence of wave breaking or near boundary layers. Under these assumptions, the velocity field can be expressed as the gradient of a scalar velocity potential, $\phi(x, y, z, t)$, and the governing equation is the Laplace equation:

$$\nabla^2 \phi = 0, \tag{2.1}$$

where $\phi(x, y, z, t)$ is the velocity potential, $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$ is the gradient operator in the (x, y, z) directions and t denotes time. $z = 0$ is defined at still water level with positive direction upward. The underlying velocity field $\mathbf{u}$ is

$$\mathbf{u} = \nabla \phi. \tag{2.2}$$

The Laplace equation must be solved in concert with boundary conditions that

define the physical domain, which in surface-wave problems include:

Kinematic Free Surface Boundary Condition (KFSBC)

The free surface is a material surface, so water particles on the surface must remain on it. This condition is expressed as:

$$\frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \eta}{\partial y} - \frac{\partial \phi}{\partial z} = 0, \quad \text{at } z = \eta, \quad (2.3)$$

where $\eta(x, y, t)$ is the surface elevation relative to the SWL.

Dynamic Free Surface Boundary Condition (DFSBC)

Assuming no overlying wind stress, the pressure at the free surface must equal the atmospheric pressure. Using Bernoulli's equation, this yields:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right] + g\eta = 0, \quad \text{at } z = \eta, \quad (2.4)$$

where g is gravitational acceleration.

Bottom Boundary Condition Assuming the seabed is impermeable:

$$\frac{\partial \phi}{\partial z} = 0, \quad \text{at } z = -d, \quad (2.5)$$

where d is the still-water depth.

Solving the Laplace equation with these boundary conditions is challenging due to the intrinsic nonlinearity of the free-surface conditions (Equations 2.3 and 2.4). These conditions are imposed on the unknown, time-varying surface $\eta(x, y, t)$, making the problem inherently *nonlinear*. To gain practical insight, a variety of wave theories have been developed, ranging from simple linear solutions to more advanced nonlinear formulations, which are discussed in the following section.

2.2 Wave theories

Several analytical and numerical approaches have been proposed to solve the water-wave problem. These can broadly be categorised into *regular* (Section 2.2.1) and

irregular (Section 2.2.2) wave theories. Regular wave theories describe monochromatic (single-frequency) waves, while irregular wave theories account for realistic sea-states composed of many interacting frequency components.

2.2.1 Regular wave theory

Airy (1845) first derived an analytical solution by applying a perturbation expansion based on the wave steepness ak , where a is the amplitude, $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength. Assuming small wave steepness ($a/\lambda \ll 1$ and $a \ll d$), the free-surface boundary conditions may be linearised, leading to solutions for monochromatic waves:

$$\eta(x, t) = a \cos(kx - \omega t), \quad (2.6)$$

$$\phi(x, z, t) = \frac{a\omega}{k} \frac{\cosh(k(z + d))}{\sinh(kd)} \sin(kx - \omega t), \quad (2.7)$$

where $\omega = 2\pi f = 2\pi/T$ is the angular frequency and T is the wave period. Substitution into the DFSBC yields the linear dispersion relation:

$$\omega^2 = gk \tanh(kd), \quad (2.8)$$

from which the phase speed $c = \omega/k$ follows.

Although linear theory provides important insight, the linearisation of the free surface boundary conditions (Equations 2.3 and 2.4) presents important limitations to the description of realistic waves which are inherently non-linear. As the wave steepness increases, higher order contributions must be included to accurately describe the properties of the wave. This was considered in the solution of Stokes (1847) by incorporating terms up to third-order of wave steepness (ak^3). Later, Fenton (1985) extended this to include terms up to fifth-order (ak^5). In addition to the *free* wave described by linear regular wave theory, these solutions provide additional *bound* wave components. These are wave components travelling at the same phase velocity as the *free* wave, despite being a different frequency. As such, they do not satisfy the linear dispersion relation but rather are *bound* to the linear component. These act to modulate the wave profile, evident as sharper and higher crests, and shallower and flatter troughs, manifesting stronger crest-trough asymmetry.

In shallow water, dispersive and nonlinear effects become significant. The classical KdV equation (Korteweg & De Vries, 1895) describes weakly nonlinear, weakly dispersive long waves and forms the basis for Cnoidal wave theory (Fenton, 1998) and solitary-wave theory (Wiegel, 1960). These theories remain limited to single-frequency waves, motivating the need for irregular wave descriptions.

While regular wave theory provides valuable analytical solutions for single-frequency waves and insight into nonlinear bound-wave components, real ocean conditions are far more complex. Irregular wave theory extends these concepts to a superposition of multiple interacting wave components, capturing the statistical nature of real sea-states.

2.2.2 Irregular wave theory

Real ocean waves consist of many interacting frequencies. Linear Random Wave Theory (LRWT) models the sea surface as a superposition of linear monochromatic waves. The linear surface elevation, $\eta^{(1)}$, and velocity potential, $\phi^{(1)}$, are given by:

$$\eta^{(1)} = \sum_{i=1}^{\infty} a_i \cos(\mathbf{k}_i \cdot \mathbf{x} - \omega_i t + \psi_i), \quad (2.9)$$

and

$$\phi^{(1)} = \sum_{i=1}^{\infty} \frac{a_i \omega_i}{k_i} \frac{\cosh(k_i(z+d))}{\sinh(k_i d)} \sin(\mathbf{k}_i \cdot \mathbf{x} - \omega_i t + \psi_i). \quad (2.10)$$

where subscript i denotes an individual wave harmonic with wave number vector $\mathbf{k}_i = (k_i \cos \theta_i, k_i \sin \theta_i)$ and initial phase ψ_i , and $\mathbf{x}$ defines the position vector in the (x, y) plane.

Longuet-Higgins & Stewart (1960) extended the description of random waves by considering the nonlinear interaction between two *freely-propagating* wave trains. By considering nonlinear boundary conditions and retaining terms up to second-order of wave steepness, additional *bound wave* contributions arise. More specifically, a *sum*- and *difference*- frequency bound wave are present. This foundational theory was extended by Sharma & Dean (1981) to consider an arbitrary number of two-wave interactions and finite depth. This theory is commonly referred to as Second-order Random Wave Theory (SORWT). The second-order contributions to the free surface, $\eta^{(2)}$, and the velocity potential, $\phi^{(2)}$ consist of sum and difference contributions, denoted by superscripts $+$ and $-$, respectively:

$$\begin{aligned} \eta^{(2)} = \frac{1}{4} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_i a_j \left\{ \left[\frac{D_{ij}^- - (\mathbf{k}_i \cdot \mathbf{k}_j + R_i R_j)}{\sqrt{R_i R_j}} + (R_i + R_j) \right] \cos(\Psi_i - \Psi_j) \right. \\ \left. + \left[\frac{D_{ij}^+ - (\mathbf{k}_i \cdot \mathbf{k}_j - R_i R_j)}{\sqrt{R_i R_j}} + (R_i + R_j) \right] \cos(\Psi_i + \Psi_j), \right\} \end{aligned} \quad (2.11)$$

and

$$\begin{aligned} \phi^{(2)} = \frac{1}{4} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} b_i b_j \frac{\cosh k_{ij}^-(d+z)}{\cosh k_{ij}^- d} \frac{D_{ij}^-}{\omega_i - \omega_j} \sin(\Psi_i - \Psi_j) \\ + \frac{1}{4} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} b_i b_j \frac{\cosh k_{ij}^+(d+z)}{\cosh k_{ij}^+ d} \frac{D_{ij}^+}{\omega_i + \omega_j} \sin(\Psi_i + \Psi_j) \end{aligned} \quad (2.12)$$

with

$$\begin{aligned} \Psi_i = \mathbf{k}_i \cdot \mathbf{x} - \omega_i t + \psi_i, \quad \Psi_j = \mathbf{k}_j \cdot \mathbf{x} - \omega_j t + \psi_j, \\ k_{ij}^- = |\mathbf{k}_i - \mathbf{k}_j|, \quad k_{ij}^+ = |\mathbf{k}_i + \mathbf{k}_j|, \end{aligned} \quad (2.13)$$

$$\begin{aligned} D_{ij}^+ = \frac{(\sqrt{R_i} + \sqrt{R_j}) [\sqrt{R_i}(k_j^2 - R_j^2) + \sqrt{R_j}(k_i^2 - R_i^2)]}{(\sqrt{R_i} + \sqrt{R_j})^2 - k_{ij}^+ \tanh(k_{ij}^+ d)} \\ + \frac{2(\sqrt{R_i} + \sqrt{R_j})^2 (\mathbf{k}_i \cdot \mathbf{k}_j - R_i R_j)}{(\sqrt{R_i} - \sqrt{R_j})^2 - k_{ij}^+ \tanh(k_{ij}^+ d)}, \end{aligned} \quad (2.14)$$

$$\begin{aligned} D_{ij}^- = \frac{(\sqrt{R_i} - \sqrt{R_j}) [\sqrt{R_j}(k_i^2 - R_i^2) - \sqrt{R_i}(k_j^2 - R_j^2)]}{(\sqrt{R_i} - \sqrt{R_j})^2 - k_{ij}^- \tanh(k_{ij}^- d)} \\ + \frac{2(\sqrt{R_i} - \sqrt{R_j})^2 (\mathbf{k}_i \cdot \mathbf{k}_j + R_i R_j)}{(\sqrt{R_i} - \sqrt{R_j})^2 - k_{ij}^- \tanh(k_{ij}^- d)}, \end{aligned} \quad (2.15)$$

$$R_i = k_i \tanh(k_i d), \quad (2.16)$$

and

$$b_i = \frac{a_i g}{\omega_i}. \quad (2.17)$$

Utilising these formations, the total surface elevation, η , and velocity potential, ϕ , are expressed as the sum of the linear and second-order (sum- and difference-) constituents:

$$\eta = \eta^{(1)} + \eta^{(2)} + O(a^3 k^3), \quad (2.18)$$

and

$$\phi = \phi^{(1)} + \phi^{(2)} + O(a^3 k^3). \quad (2.19)$$

Understanding both linear and nonlinear interactions in irregular waves provides a foundation for predicting how wave energy transforms as it propagates towards the shore. The following section explores these nearshore processes, including wave shoaling and breaking, which directly govern coastal evolution.

2.2.3 Wave Transformation Models

The wave theories described in the previous section provide analytical descriptions of wave motion under simplified conditions, typically assuming uniform water depth and idealised wave forms. In coastal environments, however, waves propagate over spatially varying bathymetry as they approach the shoreline. As a result, wave properties such as height, wavelength, and direction evolve through processes including shoaling, refraction, diffraction, and nonlinear interactions.

To describe these transformations, several models have been derived from the governing potential flow equations by introducing additional assumptions. Two commonly used formulations are the *depth-averaged shallow water equations*, which describe long-wave motion in very shallow water, and the *mild-slope equation*, which describes the transformation of linear waves over gently varying bathymetry while retaining dispersive effects.

Depth-Averaged Shallow Water Equations In very shallow water, where the wavelength is much larger than the water depth ($kd \ll 1$), vertical accelerations become small and the pressure distribution may be approximated as hydrostatic:

$$p = \rho g(\eta - z). \quad (2.20)$$

Under this assumption the *incompressible, inviscid Euler equations* can be integrated over the water column. By considering flow in one spatial dimension (x), the total water depth is defined as as

$$h(x, t) = d(x) + \eta(x, t), \quad (2.21)$$

and introducing the depth-averaged velocity

$$u(x, t) = \frac{1}{h} \int_{-d}^{\eta} u(x, z, t) dz, \quad (2.22)$$

integration of the continuity equation over the water column gives

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} = 0. \quad (2.23)$$

Similarly, integrating the horizontal momentum equation over the depth and applying the hydrostatic pressure assumption yields

$$\frac{\partial(hu)}{\partial t} + \frac{\partial}{\partial x} \left(hu^2 + \frac{1}{2}gh^2 \right) = gh(S_0 - S_f), \quad (2.24)$$

where $S_0 = -\partial d/\partial x$ represents the bed slope and S_f represents frictional resistance at the bed. These equations are known as the *Saint-Venant equations* (De St Venant, 1871).

The shallow water equations neglect frequency dispersion but retain nonlinear advection terms. They are therefore particularly suited to modelling long-wave processes such as tides, storm surges, and tsunami propagation, where the wavelength is large compared with the water depth.

Mild-Slope Equation For shorter waves propagating over gently varying bathymetry, dispersive effects become important and the shallow water approximation is no longer sufficient. In such cases, the transformation of linear waves may be described using the *mild-slope equation*.

For slowly varying bathymetry, we assume that the horizontal variation of wave amplitude is much slower than the vertical variation, allowing the vertical structure to be approximated by the local linear wave solution presented in Section 2.2.1:

$$\Phi(x, y, z) = A(x, y) \frac{\cosh k(z + d)}{\cosh kd}, \quad (2.25)$$

where $A(x, y)$ is a slowly varying complex amplitude. Substituting this ansatz into the Laplace equation (Equation 2.1) and integrating over depth yields the mild-slope equation for the slowly varying complex amplitude.

$$\nabla \cdot (c_g c \nabla A) + k^2 c_g c A = 0, \quad (2.26)$$

where the $c = \omega/k$ is the phase velocity, and c_g is the group velocity. The surface elevation is then defined as $\eta(x, y) = \Re \{A(x, y)e^{-i\omega t}\}$. This equation describes the combined effects of shoaling, refraction, and diffraction of waves propagating over gently varying bathymetry.

In addition to the classical mild-slope formulation, more general modal approaches have been developed to improve the representation of wave propagation over variable bathymetry. A notable example is the consistent coupled-mode theory proposed by Athanassoulis & Belibassakis (1999), which represents the velocity potential as a series expansion in vertical eigenfunctions whose amplitudes vary horizontally. Unlike the mild-slope equation, which assumes a single dominant vertical mode, the coupled-mode formulation retains multiple propagating and evanescent modes, allowing the governing potential flow equations to be satisfied more consistently over rapidly varying bathymetry. This approach provides improved accuracy in situations where depth variations are significant or where higher-order vertical structure becomes important, while remaining computationally tractable for coastal wave transformation problems.

Nonlinear Mild-Slope Models Extensions of the mild-slope framework have been developed to incorporate weak nonlinear interactions between wave components. These formulations are commonly referred to as *nonlinear mild-slope models* or *quadratic models*, and are typically expressed in the frequency domain. In this class of models, the nonlinear evolution of the wave field is represented through quadratic interactions between harmonic components.

The governing equation for the spatial evolution of the complex wave amplitudes may be written as

$$\partial_x a_n - ik_n a_n = -i \sum_r V_{r,n-r} a_r a_{n-r}, \quad (2.27)$$

where a_n and k_n denote the complex amplitude and wavenumber of the n th harmonic component, respectively, and $V_{r,n-r}$ represents the quadratic interaction coefficients describing the coupling between wave components. The free-surface elevation η is expressed through a Fourier representation,

$$\eta(x, t) = \sum_n a_n \exp(-i\omega_n t), \quad (2.28)$$

where ω_n is the angular frequency of the n th wave component. In this formulation, nonlinear wave transformation arises from pairwise interactions between spectral components, while linear dispersion is retained through the wavenumber k_n .

Quadratic formulations were first introduced by Freilich & Guza (1984), building on the Boussinesq-type equations originally developed by Peregrine (1967). These early models captured weakly nonlinear interactions while maintaining computational efficiency, making them attractive for coastal wave transformation problems. However, the underlying Boussinesq framework assumes weak dispersion, which limits the accuracy of these models in intermediate water depths (Agnon, 1999).

Subsequent developments sought to improve the dispersive properties of quadratic models. Weakly nonlinear Boussinesq formulations were proposed by Madsen & Sørensen (1993), while later work introduced fully dispersive, weakly nonlinear amplitude models (Agnon, 1999, Kaihatu & Kirby, 1995, Bredmose et al., 2005). These developments improved the linear dispersion characteristics of the models and extended their applicability to a broader range of coastal wave conditions.

Despite these improvements, limitations remain in the representation of nonlinear wave evolution. In quadratic models, nonlinearity is represented solely through a single quadratic mode-coupling term, which can lead to inaccuracies in the predicted evolution of wave spectra when nonlinear effects become significant. In particular, spurious amplitude dispersion and artificial growth associated with modulational instability have been reported in certain formulations.

Recent work by Akrish et al. (2024a) highlights these limitations and demonstrates that conventional quadratic formulations may produce unrealistic spectral evolution due to the restricted nonlinear structure of the governing equations. To address these issues, Akrish et al. (2024b) proposed an optimized quadratic formulation based on parameterisation against a large dataset of experimental and numerical results. The resulting model, QuadWave1D, was shown to provide improved predictions of both sea-swell components and infragravity wave responses in coastal environments.

2.3 Coastal evolution

Coastal evolution is driven by the transformation of incident wave energy as waves propagate from deep to shallow water and interact with the seabed, shoreline and one another. Two fundamental processes governing nearshore morphodynamics are wave shoaling (Section 2.3.1), which modifies wave height and wavelength as a function of depth and bathymetric gradients, and wave breaking (Section 2.3.2), which dissipates wave energy and produces turbulence, rollers, and mean flows that drive sediment transport and morphological change. This section summarises the theoretical foundations for these processes.

2.3.1 Wave shoaling

Wave shoaling describes how the amplitude of waves changes as they move into shallower water. The principal theoretical basis is the conservation of wave energy flux in the cross-shore direction in the absence of dissipation. For slowly varying bathymetry, the wave field may be treated under the Wentzel–Kramers–Brillouin (WKB) approximation (Mei, 1989) such that:

$$\frac{dP}{dx} = 0, \quad (2.29)$$

where $P = Ec_g$, E is the wave energy and c_g is the group velocity. For linear regular waves, the wave energy density (per unit horizontal area) is:

$$E = \frac{1}{2}\rho ga^2 = \frac{1}{8}\rho gH^2, \quad (2.30)$$

where under the linear assumption $H = 2a$ is the wave height and ρ is the fluid density. The group velocity, c_g , denotes the velocity of energy transport and is defined from the linear dispersion relation (Equation 2.8) as:

$$c_g = \frac{d\omega}{dk} = \frac{1}{2}c \left(1 + \frac{2kh}{\sinh(2kh)} \right). \quad (2.31)$$

Utilising Equation 2.30, the ratio of the local wave amplitude, a to the deep water wave amplitude, a_0 , is defined by the shoaling coefficient as:

$$K_s(d) \equiv \frac{H}{H_0} = \sqrt{\frac{c_{g0}}{c_g}}, \quad (2.32)$$

where c_g and $c_{g,0}$ are the local and deep water group velocity, respectively.

In the non-dispersive shallow-water limit $kd \ll 1$, the phase and group velocities coincide,

$$c_g \approx c \approx \sqrt{gd}, \quad (2.33)$$

hence from Equation 2.32

$$H \propto (gd)^{-1/4} \Rightarrow H \propto d^{-1/4}. \quad (2.34)$$

This relation is commonly referred to as Green's law (Green et al., 1838) and describes the approximate shoaling behaviour of long (relative to depth) waves. The shoaling expression (Equation 2.32) is based on linear regular wave theory and neglects wave dissipation due to bottom friction and breaking, as well as nonlinear energy transfer between wave components. In reality, as waves approach the shore, steepening, harmonic generation, and wave-wave interactions can significantly modify wave heights and shapes. These limitations motivate the need for nonlinear analytical solutions and numerical models that can capture the complex dynamics of nearshore wave transformation.

For random seas, a linear shoaling approximation is obtained by modifying Equation 2.32 to account for the energy distribution in the frequency domain. This is achieved by simply computing the wave shoaling coefficient at various frequency intervals in the spectrum and summing the results. This linear approximation has been utilised by Collins (1970), Battjes (1972) and Kuo (1974). For a nonlinear description, the finite amplitude of individual waves is considered by adopting finite amplitude theories such as those proposed by Shuto (1974). As such, according to Goda (1975a), the shoaling coefficient may be rewritten as:

$$K_s = \begin{cases} K_{si} & : d \geq d_{30} \\ (K_{si})_{30} \left(\frac{d_{30}}{d} \right)^{2/7} & : d_{50} \leq d < d_{30} \\ \sqrt{K_s - B} - C = 0 & : d < d_{50} \end{cases} \quad (2.35)$$

where K_{si} is the shoaling coefficient corresponding to a small amplitude wave as given by Equation 2.32, d_{30} and $(K_{si})_{30}$ are the water depth and shoaling coefficient of that water depth satisfying Equation 2.36, d_{50} and $(K_{si})_{50}$ are the water depth and shoaling coefficient of that water depth satisfying Equation 2.37. The constants B and C are given by Equations 2.38 and 2.39, respectively.

$$\left(\frac{d_{30}}{\lambda_0} \right)^2 = \frac{2\pi}{30} \frac{H_0}{\lambda_0} (K_{si})_{30}, \quad (2.36)$$

$$\left(\frac{d_{50}}{\lambda_0} \right)^2 = \frac{2\pi}{50} \frac{H_0}{\lambda_0} (K_{si})_{50}, \quad (2.37)$$

$$B = \frac{2\sqrt{3}}{\sqrt{2\pi H_0/\lambda_0}} \frac{d}{\lambda_0}, \quad (2.38)$$

$$C = \frac{C_{50}}{\sqrt{2\pi H_0/\lambda_0}} \left(\frac{\lambda_0}{d} \right)^{3/2}, \quad (2.39)$$

where λ_0 is the deep water wavelength and C_{50} is a constant given by

$$C_{50} = (K_s)_{50} \left(\frac{d_{50}}{\lambda_0} \right)^{3/2} \left[\sqrt{2\pi \frac{H_0}{\lambda_0} (K_s)_{50}} - 2\sqrt{3} \frac{d_{50}}{\lambda_0} \right]. \quad (2.40)$$

The coefficients B and C are obtained from nonlinear wave theory by combining energy flux conservation in shallow water with the nonlinear wave height–depth relationship. B represents the local depth-controlled shoaling effect and C arises as an integration constant determined from matching the solution to known wave conditions at a reference depth. For water depths d_{30} and d_{50} , Equations 2.36 and 2.37 are solved iteratively. However, water depths shallower than d_{50} require numerical methods to solve Equation 2.35.

2.3.2 Wave breaking

Wave breaking is the dominant mechanism for energy dissipation in the surf zone and strongly influences nearshore circulation, sediment transport and morphological evolution. Breaking is inherently a highly nonlinear and often turbulent process (Banner & Peregrine, 1993). Wave breaking is commonly classified by the breaker type on plane beaches through the Iribarren number defined as:

$$Ir = \frac{\tan \alpha}{\sqrt{H_0/\lambda_0}}, \quad (2.41)$$

where $\tan \alpha$ is the bed slope, and H_0 and λ_0 are the deep water wave height and wavelength, respectively. Utilising this definition, four breaker types are defined:

Spilling breaker ($Ir < 0.45$): Spilling breakers occur on gentle beach slopes and relatively low waves. The crest gradually becomes unstable and spills down the front face, dissipating energy over a wide area. These breakers produce relatively mild turbulence and are typical of sandy, low-gradient beaches.

Plunging breaker ($0.45 < Ir < 2.3$): Plunging breakers are common on moderately steep slopes. The wave crest curls over and plunges downward, forming a “barrel” or jet at the front face. This results in localized high turbulence and energy dissipation, which can cause significant sediment transport and erosion in the surf zone.

Collapsing breaker ($2.3 < Ir < 3.2$): Collapsing breakers occur on steeper slopes with intermediate waves. The wave front steepens and the crest loses its sharp overturning shape, collapsing down the slope. Energy is dissipated rapidly but over a shorter distance compared to spilling and plunging breakers, producing strong instantaneous forces on the shoreline.

Surging breaker ($Ir > 3.2$): Surging breakers develop on very steep beaches or with long, low waves. The wave front slides up the slope without significant crest overturning. Energy dissipation occurs mainly through friction along the slope rather than turbulent breaking, often resulting in very short surf zones with high instantaneous forces.

A widely used necessary condition for incipient breaking of a periodic wave was derived by Miche (1944) requiring that the particle orbital velocity at the crest does not exceed the phase speed. Practically, this criterion can be expressed as:

$$H_b k_b \leq 0.88 \tanh(k_b h_b), \quad (2.42)$$

where subscript b denotes values at breaking inception. This relation yields two limiting forms:

- Deep-water limit ($kd \gg 1$): $H_b/\lambda_b \lesssim 0.142$,
- Shallow-water limit ($kd \ll 1$): $H_b/d_b \lesssim 0.88$.

McCowan’s classical solitary-wave analysis (McCowan, 1894) arrived at a limiting shallow-water ratio $H/d \approx 0.78$ for an inviscid solitary wave. This number is often quoted as a guideline (the so-called “0.78 breaker index”), but experimental and field results show systematic dependence on slope, spectral shape and wave grouping so the constant is not universal (Goda, 2000, Thornton & Guza, 1983).

In coastal regions, the breaker height is described as a function of the Iribarren number such that:

$$H_b = \gamma_b d_b, \quad (2.43)$$

where the breaker index $\gamma_b = 1.16Ir^{0.22}$.

In contrast to regular wave breaking, random wave breaking occurs over a much wider spatial scale. Given the complexities of the mechanisms occurring in the surf zone, the description of breaking in random seas is often described by considering the macroscopic changes to the wave height distribution, as done in the theory of Goda (1975b). Prior to breaking, wave heights are assumed to have a Rayleigh distribution (see Section 2.4.1). From these waves, those exceeding the breaking limit (x_1) will break. To account for the variability of breaking in random seas, this is extended to consider waves smaller than the breaking limit up to x_2 . The probability of occurrence of breaking between x_1 and x_2 follows a linear variation. The broken waves lose a fraction of their energy and as such, redistribute to higher exceedance probabilities in the distribution of wave heights. This is formally described as:

$$\frac{H_b}{\lambda_0} = A \left(1 - \exp \left[-1.5 \frac{\pi d}{\lambda_0} (1 + 15 \tan^{4/3} \alpha) \right] \right), \quad (2.44)$$

where the coefficient A is set at 0.18 and 0.12 in the upper ($x = x_1$) and lower ($x = x_2$) breaking limit.

Alternative descriptions of random wave breaking were presented in the notable works of Battjes & Janssen (1978) and Thornton & Guza (1983). Unlike the breaking model of Goda (1975b), which only considers the local water depth, these models are based on integrating the wave energy flux balance equation from deep water. In general form, the cross-shore evolution of wave energy can be written as

$$\frac{d}{dx}(Ec_g) = -D_b, \quad (2.45)$$

where E is the wave energy density, c_g is the group velocity and D_b represents the rate of energy dissipation due to breaking.

The formulation of Battjes & Janssen (1978) introduces a probabilistic description of wave breaking based on the fraction of waves that are actively breaking. The dissipation is assumed to be related to the energy loss of a hydraulic bore (Le Méhauté, 1976) and can be expressed as

$$D_b = \frac{1}{4} \rho g f_p Q_b H_b^2, \quad (2.46)$$

where ρ is the water density, g is the gravitational acceleration, f_p is the peak wave frequency, Q_b is the fraction of breaking waves, and H_b is the representative breaking wave height. The fraction of breaking waves is determined implicitly from the truncated Rayleigh distribution of wave heights,

$$\frac{1 - Q_b}{\ln Q_b} = - \left(\frac{H_{\text{rms}}}{H_b} \right)^2, \quad (2.47)$$

where H_{rms} is the root-mean-square wave height and the breaker height is given by Equation 2.43. This approach links the probability of breaking to the local wave statistics and water depth through the saturation of the wave height distribution.

In contrast, the model of Thornton & Guza (1983) retains the energy flux balance framework but assumes that wave heights in the surf zone continue to follow a Rayleigh-type distribution that is not truncated at the breaking limit. Instead, the

probability of breaking is incorporated directly into the dissipation term through a weighting of wave heights that exceed the breaking threshold. In this formulation, the breaker index γ is treated as a constant and the fraction of breaking waves is implicitly represented within the statistical distribution rather than solved through an explicit breaking probability. Both approaches have been widely applied in nearshore wave transformation models due to their ability to represent depth-limited breaking in irregular wave fields while remaining computationally efficient.

Wave breaking redistributes momentum in the water column, generating depth-averaged forces known as *radiation stresses*. Conceptually, radiation stress represents the excess momentum flux due to the presence of waves (Longuet-Higgins & Stewart, 1964) and drives mean water level variations, including set-up (increase) in the surf zone and set-down (decrease) at breaking onset.

Through integration of the excess pressure force across the water column, the alongshore radiation stress component S_{xx} is expressed as:

$$S_{xx} = \frac{1}{16}\rho g H^2(1 + 2G), \quad G = \frac{2kd}{\sinh(2kd)}. \quad (2.48)$$

This results in mean water level (MWL) changes defined as:

$$\frac{d\bar{\eta}}{dx} = -\frac{1}{\rho g d} \frac{dS_{xx}}{dx}, \quad (2.49)$$

The resulting maximum set-down at breaking, $\bar{\eta}_b$, and surf-zone set-up, $\bar{\eta}_s$, can then be expressed as:

$$\bar{\eta}_b = -\frac{1}{8} \frac{H^2 k}{\sinh(2kd)}, \quad \bar{\eta}_s = \bar{\eta}_b + \frac{\frac{3}{8}\gamma_b^2(d_b - d)}{1 + \frac{3}{8}\gamma_b^2}, \quad (2.50)$$

where γ_b is the breaker index and d_b is the water depth at breaking.

Wave breaking is a critical process to include in modelling because it governs energy dissipation, which in turn drives surf-zone dynamics and controls sediment transport and coastal morphology.

2.3.3 Nonlinear wave-wave interactions

Nonlinear wave-wave interactions play a critical role in the transformation of ocean waves as they propagate into shallow coastal environments. Phillips (1960) first ex-

plained the effects of nonlinear interactions in ocean waves. Among these nonlinear processes are *triad interactions*, responsible for energy exchanges among three wave components. These interactions become increasingly important in shallow water where depth-induced nonlinearity becomes strong. Triad interactions reshape the wave spectrum, modify wave shape, and generate low-frequency motions that are fundamental to nearshore hydrodynamics (Herbers et al., 2000). This section introduces the physics of triad interactions, presents the governing equations, and explains their importance in coastal waters.

As wind-generated waves shoal, their steepness increases and nonlinear effects intensify. In shallow water, the dominant nonlinear mechanism is the *triad interaction*, which involves either *sum*- or *difference*- frequency coupling:

$$f_3 = f_1 + f_2, \quad f_3 = |f_1 - f_2|. \quad (2.51)$$

The mechanism for triad interaction is illustrated in Figure 2.1. Consider two wave components corresponding to wave number k_1 (shown in blue) and k_2 (shown in red), respectively. This pair of waves will interact with a third freely-propagating wave of wave number k_3 (shown in green) provided the proper conditions are met:

$$\mathbf{k}_3 = \mathbf{k}_1 \pm \mathbf{k}_2. \quad (2.52)$$

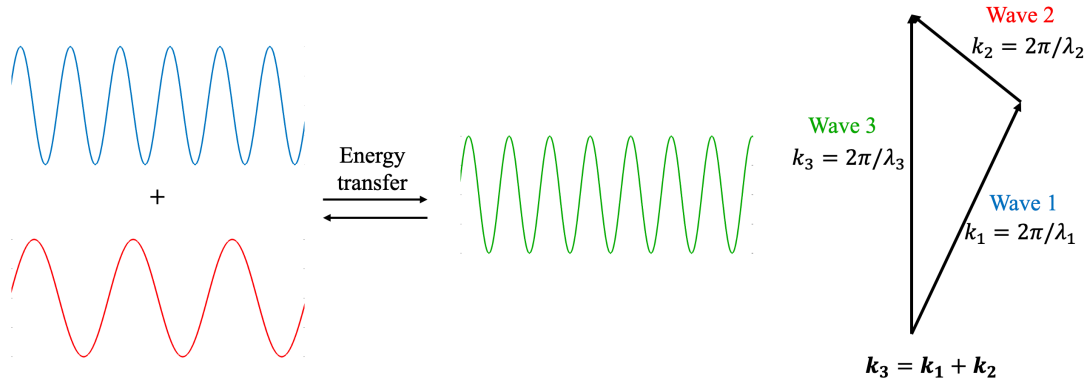


FIGURE 2.1: Schematic of the mechanism of triad wave-wave interactions. Interactions involve three wave components of wave numbers satisfying the resonance condition $\mathbf{k}_3 = \mathbf{k}_1 \pm \mathbf{k}_2$.

Such interaction between freely propagating wave components is referred to as *resonance*. More specifically, resonance occurs when the sum/difference of wave vectors still satisfies the dispersion relation for a freely propagating wave. Through

this interaction, energy is transferred between wave components. In shallow water, the dispersion relation (Equation 2.8) approaches a linear form, $\omega \approx k\sqrt{gd}$, which greatly increases the likelihood of satisfying resonance. Consequently, triad interactions intensify as the water depth decreases.

In the context of second-order random wave theory (SORWT), it is important to distinguish between triad interactions and bound waves, as they arise from fundamentally different mechanisms. Triads represent resonant, free-wave interactions that occur predominantly in shallow water, where the nonlinear coupling between wave components can transfer energy between different frequencies and directions. In contrast, second-order bound waves are non-resonant and arise in finite water depths as a direct consequence of the nonlinearities in the wave surface; they are bound to the primary waves and do not exist independently. While triads can redistribute energy among free waves, bound waves primarily modify the wave shape and contribute to phenomena such as wave asymmetry and set-up, which are essential for accurate modelling of wave forces and coastal processes.

Triad interactions produce several key transformations in coastal wave fields. Sum interactions transfer energy to higher frequencies by amplifying second and third harmonics, resulting in sharper crests and broader troughs. This enhances wave asymmetry and skewness, with important implications for sediment suspension and bed shear stresses (Longuet-Higgins & Stewart, 1962). At the same time, difference-frequency interactions transfer energy to low frequencies, generating long-period infragravity waves, typically between 0.003 and 0.05 Hz, which play a crucial role in surf-zone circulation, wave setup, runup, and resonant responses in harbours and lagoons (Herbers et al., 1994, 1995). More broadly, triad processes drive internal spectral redistribution during shoaling, in contrast to linear shoaling which predicts energy growth solely through depth reduction; this nonlinear redistribution can fundamentally reshape the nearshore wave spectrum.

Another form of nonlinear interaction are *quartet* (four-wave) interactions which satisfy the resonance condition (Hasselmann, 1962):

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4. \quad (2.53)$$

Quartet (four-wave) interactions dominate nonlinear energy transfers in deep water (Janssen, 2003), but they become negligible in shallow coastal environments

for several reasons. First, the dispersion relation becomes weakly dispersive as $\tanh(kd) \rightarrow kd$, causing the dispersion relation to approach linearity ($\omega = k\sqrt{gd}$) and collapsing the resonance manifold required for efficient quartet coupling. Second, triad interactions dominate the nonlinear energy transfer during shoaling because quadratic nonlinearities strengthen rapidly in shallow water and overwhelm the slower quartic processes. Third, quartet interactions operate on comparatively long timescales and require substantial propagation distances to accumulate, whereas triad interactions act much more rapidly over the short shoaling distances typical of coastal regions. For these reasons, although quartet interactions play a crucial role in deep-water wave growth and spectral evolution, they can typically be neglected in coastal wave modelling frameworks.

The physical processes governing wave transformation and breaking ultimately shape the statistical properties of the coastal wave field. Given the stochastic nature of ocean waves, a statistical description over short time intervals provides a practical framework for analysing and predicting wave characteristics.

2.4 Short-term statistics

Coastal waves are inherently stochastic, with characteristics that vary over time scales much longer than individual wave periods. As such, the statistical properties of a sea-state are often assumed stationary over a given time interval. This is commonly assumed to be 3 hours at field scale. Hence, within a given sea-state where the sea-state parameters remain constant, the short-term statistics of the waves may be defined.

Within the short-term statistics framework, wave properties can be obtained either by a *wave-by-wave analysis* (Section 2.4.1) or through *spectral analysis* (Section 2.4.2), as discussed below. Both approaches are complementary: wave-by-wave analysis is particularly useful for design parameters and understanding extreme wave behaviour, while spectral methods offer insight into energy distribution and nonlinear interactions.

2.4.1 Wave-by-wave analysis

For a given time series of the water surface elevation, $\eta(t)$, a zero-crossing analysis may be performed to infer individual wave properties. This may be achieved through either a down-crossing or up-crossing approach, as shown in Figure 2.2 by red crosses and circles, respectively. Utilising the former approach, considering changes in the surface elevation from positive to negative, the down-crossing wave height (H_d), wave period (T_d) and crest height (η_c) are defined between two successive zero-crossing points. Alternatively, the later approach considering changes in the surface elevation from negative to positive, may be adopted. Through this, the up-crossing wave height (H_u) and wave period (T_u) are extracted. Both definitions are equally valid, as such, unless otherwise stated, the present work will adopt a down-crossing definition. An alternative wave period definition is the trough-to-trough wave (T_{tt}), often regarded as a more stable measure compared to down-crossing or up-crossing alternatives. Ranking the individual wave heights, crest heights and wave periods yields empirical short-term statistical distributions of wave properties appropriate to engineering design.

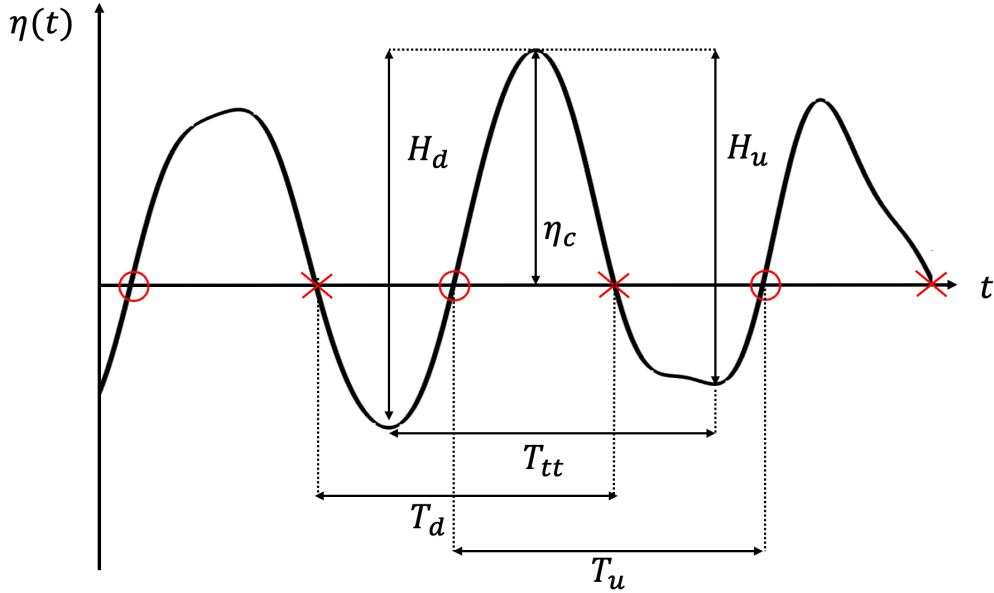


FIGURE 2.2: Schematic illustrating the zero-crossing analysis technique. Down-crossing [×] and up-crossing [○] parameters are denoted by d and u subscripts, respectively.

In practical applications, long-duration wave records are often unavailable, limiting the direct inference of short-term wave statistics. Given the importance of

accurately modelling short-term wave statistics for engineering design, a wide range of wave and crest height distributions have been proposed in the literature.

Wave height distributions By modelling the water surface elevation at a fixed spatial location as a Gaussian process (Rice, 1944, 1945), and assuming a linear, narrow-banded sea state (Longuet-Higgins, 1952), the wave height distribution can be expressed as

$$Q(h > H) = \exp \left[-\frac{1}{\alpha} \left(\frac{H}{\sqrt{m_0}} \right)^\beta \right], \quad (2.54)$$

where Q denotes the probability of exceedance of the wave height H by the random variable h , $\alpha = 8$, $\beta = 2$, and m_0 is the zeroth-order spectral moment. The n th-order spectral moment is defined as

$$m_n = \int_0^\infty \omega^n S_{\eta\eta}(\omega) d\omega, \quad (2.55)$$

where $S_{\eta\eta}(\omega)$ is the surface elevation power spectral density.

The original Rayleigh model is normalised by the root-mean-square wave height (H_{rms}), which for a linear, narrow-banded process in deep water is given by

$$H_{\text{rms}} = \sqrt{8m_0}. \quad (2.56)$$

In finite water depths, the narrow-banded assumption underlying Equation (2.56) becomes invalid. Under such conditions, spectral broadening is observed, leading to a de-correlation between wave crests and troughs and, consequently, a reduction in wave height magnitudes. Owing to the inherently broad-banded nature of wind seas, it is well established that the Rayleigh distribution tends to overestimate measured wave heights (Forristall, 1978, Haring et al., 1977, Goda, 1979).

Given these limitations, several alternative wave height distributions have been proposed. Among these, the two-parameter Weibull distribution introduced by Forristall (1978) is the most commonly adopted in practice. This distribution was calibrated using field measurements collected during a storm in the Gulf of Mexico and retains the functional form of Equation (2.54), but with parameters $\alpha = 2.126$ and $\beta = 8.42$. Unlike the Rayleigh distribution, which is derived under the assumptions of linear random wave theory, the Forristall distribution accounts empirically for the

effects of nonlinearity and wave breaking, which become increasingly important in reduced water depths. The simplicity of the Forristall formulation has contributed to its widespread use in engineering applications. However, as the model is calibrated to a specific dataset and depends on a single input parameter (m_0), its applicability beyond environmental conditions similar to those used in its derivation (very broad-banded seas) should be treated with caution.

Crest height distributions In the context of crest heights, the Rayleigh crest height distribution (Longuet-Higgins, 1952) was the first to be proposed, again under the assumption of a linear, narrow-banded sea state. By applying a change of variable such that $\eta_c = H/2$ and defining the significant wave height as

$$H_s = 4\sqrt{m_0}, \quad (2.57)$$

the exceedance probability of the crest height can be written as

$$Q(\eta > \eta_c) = \exp \left[-\frac{1}{\alpha} \left(\frac{\eta_c}{H_s} \right)^\beta \right], \quad (2.58)$$

where η_c denotes the crest height, $\alpha = 1/8$ is the scale parameter, and $\beta = 2$ is the shape parameter.

The Rayleigh crest height distribution is known to significantly underestimate crest heights at low probabilities of exceedance due to its underlying linear assumption (Tayfun, 1981, Prevosto et al., 2000). To address this shortcoming, Tayfun (1980) derived an analytical model incorporating second-order wave steepness effects for narrow-banded spectra. Second-order effects were also considered by Forristall (2000), who proposed a crest height distribution based on an empirical fit to second-order random wave simulations using realistic ocean wave spectra. The Forristall crest height model is frequently adopted in engineering design practice (Det Norske Veritas, 2010) and has historically formed the basis for the design of offshore and coastal structures. This formulation accounts for the effects of wave steepness and reduced effective water depth.

The functional form of the Forristall model follows Equation (2.58); however, unlike the Rayleigh formulation, the parameters α and β are no longer constants. Instead, they depend on the wave steepness parameter S_1 , which characterises the

degree of wave nonlinearity, and the Ursell number U_r , which quantifies the influence of effective water depth on nonlinear wave behaviour. For short-crested seas, Forristall's Weibull fit yields

$$\begin{aligned}\alpha &= \frac{1}{2\sqrt{2}} + 0.2568 S_1 + 0.0800 U_r, \\ \beta &= 2 - 1.7912 S_1 - 0.5302 U_r + 0.2840 U_r^2,\end{aligned}\tag{2.59}$$

where $S_1 = \frac{2\pi H_s}{gT_1^2}$, $U_r = \frac{H_s}{k_1^2 d^3}$, k_1 is the wavenumber associated with the mean wave period T_1 , and d is the water depth.

Several studies (Socquet-Juglard et al., 2005, Toffoli et al., 2008, Onorato et al., 2006, Waseda, 2006, Gibson & Swan, 2007) have demonstrated the importance of accounting for nonlinear amplification mechanisms beyond second order when modelling crest height statistics. Building on this, Tayfun & Fedele (2007) proposed an alternative formulation incorporating third-order wave steepness effects through a Gram–Charlier series expansion. When evaluating such models, particularly in the context of extreme waves, a key consideration is their ability to capture the competing influences of higher-order nonlinearities and wave breaking. The importance of these competing mechanisms has been highlighted by Latheef & Swan (2013) and Karpadakis et al. (2019a), especially for intermediate water depths.

A notable limitation of many existing crest height distributions is the neglect of wave breaking effects. This gap was addressed by Karpadakis & Swan (2022), who proposed a new crest height model based on an extensive database of experimental and field measurements. Although this model incorporates data from intermediate water depths, it is restricted to waves propagating over flat beds. Consequently, the added complexity associated with wave transformation over sloping bathymetry continues to limit the development of unified crest height distributions for such conditions.

2.4.2 Spectral analysis

While wave-by-wave analysis provides direct insight into individual waves, it does not easily reveal the distribution of energy across frequencies or the interactions between waves. Spectral analysis addresses these limitations, offering a complementary perspective by translating the time series of water surface elevation into energy

distributions across the frequency range.

Fourier transform Spectral analysis is a fundamental tool in coastal engineering for characterizing the frequency content of water surface elevation time series. The Fast Fourier Transform (FFT) provides an efficient algorithm to compute the Discrete Fourier Transform (DFT), converting a signal from the time domain to the frequency domain (Welch, 1967).

Consider the time series of the water surface elevation $\eta(t)$ sampled at a frequency $F_s = 1/\Delta t$ with length N samples. The DFT is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-i2\pi kn/N}, \quad k = 0, 1, \dots, N-1, \quad (2.60)$$

where $x[n]$ is the discrete time series and $X[k]$ are the complex Fourier coefficients. To prevent aliasing artefacts, the frequency content of the signal must be limited to $[0, F_s/2]$, with the upper limit defining the Nyquist frequency (Shannon, 2006). The sampling frequency is defined as $\Delta f = F_s/N$. The frequency corresponding to index k is:

$$f_k = \frac{kF_s}{N}, \quad \text{with } f_k \in [0, F_s/2]. \quad (2.61)$$

Utilising these definitions, the amplitude spectrum is:

$$|X(f)| = \frac{2}{N} |X[k]|. \quad (2.62)$$

and the power spectrum is:

$$P(f) = |X(f)|^2. \quad (2.63)$$

Thus, the variance density spectrum is defined as:

$$S_{\eta\eta}(f) = \frac{1}{2} \frac{P(f)}{\Delta f} = \frac{1}{2} \frac{|X(f)|^2}{\Delta f}. \quad (2.64)$$

Spectral moments derived from $S_{\eta\eta}(f)$ allow estimation of spectral parameters such as the significant wave height (H_s) and mean wave period (T_1). This provides important inputs to numerical wave models. Moreover, this approach is widely used in wave decomposition and energy partitioning between swell and wind-sea components.

Bispectral analysis While the power spectrum provides second-order statistics, it does not capture phase relationships between frequency components. Bispectral analysis, based on third-order statistics, captures phase coupling between triads of frequencies and is therefore fundamental for detecting nonlinear interactions such as depth-limited triad transfers (Hasselmann et al., 1963).

The bispectrum $B(f_1, f_2)$ is defined as:

$$B(f_1, f_2) = E [X(f_1)X(f_2)X^*(f_1 + f_2)], \quad (2.65)$$

where E denotes the expectation, $X(f)$ are Fourier coefficients as defined by Equation 2.60 and $*$ denotes complex conjugation. A non-zero bispectrum indicates quadratic phase coupling among f_1 , f_2 , and $f_1 + f_2$.

The amplitude of the bispectrum is conditional on the amplitude of the signal. If only the degree of phase coupling is of interest, the bicoherence is calculated as:

$$b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{E[|X(f_1)X(f_2)|^2] E[|X(f_1 + f_2)|^2]}, \quad (2.66)$$

with $0 \leq b^2 \leq 1$. A high bicoherence indicates persistent phase relationships, characteristic of deterministic nonlinear processes.

The utilisation of spectral and bispectral techniques in the analysis of coastal waves has been used in detecting energy transfer from primary waves to harmonics during shoaling. Moreover, it allows for the identification of triad interactions responsible for wave asymmetry and skewness (Elgar & Guza, 1985).

2.4.3 Realistic spectra

In the absence of long time series of the water surface elevation, design calculations adopt parametrised forms of the wave spectrum. These provide information relating to the amplitudes, frequencies and initial phases of the individual freely propagating wave components. The distribution of wave energy (or amplitudes) across the frequency range is described by the energy spectrum $S_{\eta\eta}(\omega)$, with the initial phases defined stochastically by sampling from a uniform distribution in $[0, 2\pi)$. The most commonly applied formulations are outlined below:

The Pierson-Moscowitz spectrum (Pierson Jr. & Moskowitz, 1964)

The PM spectrum is developed based on measurements in the North Atlantic, assuming a fully-developed wind sea. It incorporates the theory of Phillips (1958) which described the high frequency tail of the spectrum using a power decay law (f^{-5}).

The JOint North Sea WAVE Programme spectrum (Hasselmann et al., 1973)

The JONSWAP spectrum generalises the PM spectrum to account for fetch-limited seas by the inclusion of a peak enhancement factor, γ . The development of this spectrum involved a more extensive measurement campaign by Hasselmann et al. (1973). As such, this is the most commonly used spectrum in the offshore industry. This formulation is defined as:

$$S_{\text{JONSWAP}} = \frac{\alpha g^2}{\omega^5} \exp \left[-\frac{5}{4} \left(\frac{\omega_p}{\omega} \right)^4 \right] \gamma^{\exp \left(\frac{-(\omega - \omega_p)^2}{2\omega_p^2 \sigma^2} \right)}, \quad (2.67)$$

where ω_p is the angular frequency corresponding to the spectral peak, T_p , α is the Phillips' parameter and $\sigma = 0.07$ for $\omega \leq \omega_p$ and 0.09 for $\omega > \omega_p$. If $\gamma = 1$, this formulation is equivalent to that of the PM spectrum. The Phillips' parameter, α , is adjusted to control the significant wave height, H_s , as given by Equation 2.57.

The Texel–Marsden–Arsloe spectrum (Bouws et al., 1985)

In shallower water, Kitaigorskii et al. (1975) utilised the self-similarity concept to extend the Phillips' high frequency saturation concept to finite water depths. Later, Bouws et al. (1985) combined this concept with buoy measurements, to develop the Texel–Marsden–Arsloe (TMA) spectrum. This consists of a frequency dependent attenuation factor, $\Phi(\omega, d)$, which is applied to modify the JONSWAP spectrum such that:

$$S_{\text{TMA}} = S_{\text{JONSWAP}} \cdot \Phi(\omega, d), \quad (2.68)$$

where

$$\Phi(\omega, d) = \frac{k^{-3}(\omega, d) \frac{\partial k(\omega, d)}{\partial \omega}}{k^{-3}(\omega, \infty) \frac{\partial k(\omega, \infty)}{\partial \omega}}, \quad (2.69)$$

and k is defined based on the dispersion relation (Equation 2.8).

2.5 Numerical Models

Numerical models have become indispensable in coastal engineering for simulating wave transformation, breaking, and nearshore hydrodynamics. These models vary in complexity, from phase-averaged models that predict bulk wave properties to fully three-dimensional Navier–Stokes models that resolve turbulent flow and detailed wave-structure interactions. The choice of model depends on the spatial and temporal scales of interest, bathymetry complexity, and computational resources (Holthuijsen, 2010).

2.5.1 Phase-Averaged Models

Phase-averaged or spectral wave models do not resolve individual waves but compute the evolution of wave energy across frequencies and directions. They solve the spectral action balance equation, denoted as:

$$\frac{\partial N}{\partial t} + \nabla \cdot (\mathbf{c}_g N) = S_{in} + S_{ds} + S_{nl}, \quad (2.70)$$

where N is the wave action density, $\mathbf{c}_g$ is the group velocity vector, and S_{in} , S_{ds} , and S_{nl} represent energy input, dissipation (including breaking), and nonlinear wave-wave interactions, respectively. These models (e.g. SWAN; Booij et al. 1999) are efficient for large domains but cannot resolve wave-by-wave processes such as run-up or localized breaking. They are often used for coastal planning and wave climate assessment.

2.5.2 Phase-Resolving Models

Phase-resolving models calculate the instantaneous wave field, including amplitude and phase, and are capable of simulating wave transformation over variable bathymetry. They can be categorised into three major classes (in increasing computational cost): Nonlinear Shallow Water Equation (NLSWE) models, Boussinesq-type models and Fully Nonlinear Potential Flow (FNPF) models.

Nonlinear Shallow Water Equation Models NLSWE models neglect dispersive effects but fully resolve nonlinear advection, making them computationally efficient for surf-zone applications dominated by breaking waves. They solve:

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = 0, \quad \frac{\partial(h\mathbf{u})}{\partial t} + \nabla \cdot (h\mathbf{u} \otimes \mathbf{u}) + gh\nabla\eta = 0, \quad (2.71)$$

where $h = \eta + d$ is the local water depth and $\mathbf{u}$ the depth-averaged velocity. NLSWE models are widely used for coastal flooding, run-up, and nearshore hydrodynamics.

Boussinesq-type Models These models extend the shallow water equations to include weakly dispersive and nonlinear terms, suitable for intermediate and shallow water waves (Kirby, 2016). The governing equations are typically written as:

$$\frac{\partial \eta}{\partial t} + \nabla \cdot \mathbf{q} = 0, \quad \frac{\partial \mathbf{q}}{\partial t} + gh\nabla\eta + \mathbf{N}(\mathbf{q}) + \mathbf{D}(\mathbf{q}) = 0, \quad (2.72)$$

where $\mathbf{q}$ is the depth-integrated momentum, $\mathbf{N}$ is the nonlinear advection terms, and $\mathbf{D}$ is the dispersive corrections. Boussinesq models can reproduce wave shoaling, refraction, diffraction, and onset of breaking but are limited for fully turbulent breaking or strongly nonlinear waves. Variants include higher-order Boussinesq formulations that improve dispersion accuracy (Nwogu, 1993, Madsen & Schäffer, 1998) and fully nonlinear Serre-Green-Naghdi-type models (e.g. Wei et al., 1995, Cienfuegos et al., 2006, Bonneton et al., 2011).

Fully Nonlinear Potential Flow Models FNPF models solve the irrotational, incompressible Euler equations under the assumption that the flow is potential ($\mathbf{u} = \nabla\phi$). They retain *full* nonlinearity and *exact* linear dispersion, and therefore provide highly accurate phase-resolving predictions from deep to shallow water depths (Dold & Peregrine, 1984). The governing equations consist of Laplace’s equation (as given by Equation 2.1) in the fluid domain with fully nonlinear free-surface boundary conditions (Equations 2.3 and 2.4).

Numerical implementations typically rely on boundary element methods (Grilli et al., 1989, Hague & Swan, 2009, Christou, 2009, Archibald, 2011) or high-order spectral (HOS) approaches (Dommermuth & Yue, 1987, West et al., 1987, Bateman et al., 2001, Ducroz et al., 2016) to advance the free-surface potential and elevation. Because FNPF models resolve the vertical flow structure exactly, they capture strong nonlinear interactions associated with steep wave groups. They are often used in applications requiring precise wave kinematics, such as wave–structure interaction, wave loading on offshore platforms, and detailed wave transformation. However,

such models can become computationally expensive for large domains. Moreover, they are unable to model fully turbulent breaking.

2.5.3 Fully Nonlinear Navier Stokes Models

Fully three-dimensional models solve the Navier–Stokes equations and directly resolve turbulence, breaking, and free-surface deformation. Examples include computational fluid dynamics frameworks such as OPENFOAM (Jasak, 2009), BASILISK (Popinet, 2026), and FLUIDITY (Piggott et al., 2008), which commonly employ Volume-of-Fluid (VOF) methods (Hirt & Nichols, 1981), as well as particle-based approaches such as SPHYSICS based on Smoothed Particle Hydrodynamics (SPH) (Gingold & Monaghan, 1977). While these models provide highly detailed simulations of wave–structure interaction and complex turbulence, they are extremely computationally expensive, making them impractical for large-scale coastal evolution studies.

The choice of an appropriate numerical model for simulating coastal wave evolution depends on the required level of physical detail and computational resources. Coastal wave models can be broadly categorized into a hierarchy based on their ability to resolve wave processes:

$$\text{Phase-Averaged} \ll \text{Phase-Resolving} \ll \text{Fully Nonlinear Navier–Stokes}$$

Phase-averaged models, such as spectral wave models, provide statistical outputs of wave parameters at relatively low computational cost but neglect detailed phase information and higher-order nonlinear processes. Phase-resolving models, including Boussinesq-type and mild-slope formulations, offer a more accurate representation of wave shoaling, nonlinear transformations, and the onset of breaking, at moderate computational expense. Models such as SWASH successfully adopt this modelling approach for a range of coastal applications (see Chapter 3). At the highest level of fidelity, three-dimensional Navier–Stokes solvers resolve turbulence and detailed flow structures, capturing fully nonlinear wave dynamics; however, their computational cost restricts application to limited spatial and temporal domains. Consequently, model selection should be guided by the scope of the problem, balancing physical accuracy against computational feasibility.

2.5.4 Numerical Schemes

The governing equations for coastal wave evolution are generally too complex to solve analytically in realistic coastal environments, especially when considering nonlinear transformations, variable bathymetry, and wave breaking. Consequently, numerical solutions are employed. The choice of numerical scheme depends on the type of model (phase-averaged, phase-resolving, or fully nonlinear) and the characteristics of the flow being simulated.

Finite Difference Methods (FDM)

FDM approximate spatial derivatives with differences on structured grids. They are straightforward and efficient but can struggle with complex geometries (Smith, 1985). This makes them suitable for phase-resolving models where waves remain relatively smooth and the domain geometry is simple. However, FDM can struggle to accurately represent complex bathymetries or sharp discontinuities, such as wave breaking fronts.

Finite Volume Methods (FVM)

FVM conserve fluxes across control volumes, making them particularly robust for handling shocks, steep gradients, or highly nonlinear phenomena (Versteeg, 2007). They are commonly used in phase-resolving and fully nonlinear Navier–Stokes models to simulate breaking waves, turbulent flows, and interactions with coastal structures, where accurate representation of momentum and energy fluxes is critical.

Finite Element Methods (FEM)

FEM employ variational formulations with unstructured meshes, allowing flexible representation of irregular coastlines, varying bathymetry, and localized features (Cook et al., 2007). They are widely used in high-fidelity Boussinesq-type and non-hydrostatic models, where accurate spatial resolution of complex geometries is essential. FEM can handle both linear and nonlinear wave processes, and their adaptive meshing capabilities make them suitable for regions with steep gradients or strong wave-current interactions.

The choice of numerical scheme should align with the physical processes captured by the selected model. Phase-averaged models, which evolve wave energy spectra rather than individual waves, can typically employ simpler, structured discretizations (FDM

or low-order FEM), since fine-scale resolution of wave shapes is not required. In contrast, phase-resolving models must use schemes capable of accurately representing nonlinear advection, dispersive effects, and wave transformations; FVM and higher-order FEM are commonly applied in this context. These models are often combined with unstructured meshes to capture highly complex geometric effects (Piggott et al., 2008). Fully nonlinear Navier–Stokes models, which resolve turbulence, breaking, and complex free-surface dynamics, require robust schemes such as high-order FVM or specialized FEM formulations.

Chapter 3

Numerical set-up and validation

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3.1 Introduction

This chapter establishes and validates the numerical modelling framework used throughout the remainder of the thesis. As discussed in Chapter 1, the design of marine renewables and coastal structures, such as breakwaters, seawalls and jetties, requires knowledge of the statistical distribution of the wave parameters used to determine wave loads. Although waves propagating in deep water are well-understood and well-modelled, the behaviour of waves in intermediate and shallow water is more complex (Tayfun & Alkhalidi, 2020). This complex behaviour is driven by a range of physical effects such as nonlinear amplification, wave breaking and the influence of bathymetry (Martins et al., 2021). Given these complexities, analytic solutions describing wave propagation in shallow regions is highly limited. Moreover, conducting physical model testing to quantify these effects may be time-consuming and costly. In addition, there may be limitations as a consequence of the measuring equipment or physical limitations in the experimental set-up. These may include potential scale effects, unwanted reflections caused by insufficient energy dissipation at the downstream boundary and unwanted side wall effects.

In light of these constraints, alongside the demand to accurately understand wave

properties in the vicinity of the coastline, a plethora of numerical models have been developed. Liu et al. (2020) have shown the success of Boussinesq-type models at modelling the evolution of coastal waves from intermediate water up to the swash zone. This category of numerical model is based on an assumed variation of the horizontal flow velocity in the vertical direction, and thus, is only valid in the modelling of weakly nonlinear and weakly dispersive long waves. More recently, Benoit et al. (2024) have demonstrated the success of the Fully Nonlinear Potential Flow (FNPF) wave model, Whispers3D, at capturing the surface elevation statistics and particle kinematics to high accuracy. This is achieved by utilising a basis of orthogonal Chebyshev polynomials of the first kind to project the vertical structure of the velocity potential. Other formulations adopt an efficient pseudo-spectral approach (Ducroz et al., 2016), utilising a nonlinear potential flow description. Although this method is efficient in solving large spatial domains, it is unable to capture breaking or overturning waves due to the periodicity of the Fourier transform it employs. Other popular models utilise smoothed particle hydrodynamics (SPH) which is Lagrangian and meshless in nature (Violeau, 2012). Gomez-Gesteira et al. (2012) outlines that this method exhibits wave decay when modelling large spatial domains. In addition, it has a comparatively higher computational cost than the former methods outlined. However, it can be highly effective at modelling smaller spatial regions surrounding breaking and overturning waves (Altomare et al., 2015).

The present study is concerned with modelling the propagation of waves from intermediate water up to the shoreline. According to Reeuwijk (2002), problems in coastal regions involve nearly horizontal flow such that the length scales associated with the flow far exceed the water depth. Various models utilise this concept to simplify the complex Navier–Stokes equations (NSE) and provide a less computationally expensive method of solution. These models implement a hydrostatic pressure assumption such that the vertical accelerations of the flow are neglected. In doing so, the vertical momentum equation simplifies to the hydrostatic pressure relation. However, the limitation of this assumption is that only long time and length scales are accurately modelled. In other words, only waves where the wavelength significantly exceeds the water depth are well-described. As such, this category of models are referred to as ‘shallow water’ models.

The accuracy of numerical modelling techniques is often benchmarked against

equivalent experimental measurements to establish their range of validity. Previous work by Karpadakis et al. (2024) have coupled numerical simulations using SWASH (Zijlema et al., 2011) with experimental measurements to investigate wave propagation over complex bathymetries. Notably, their work outlines the applicability of relevant numerical models to simulate wave overtopping in the context of coastal flood defence, extending the findings of Suzuki et al. (2011). Such reliability depends on resolving the key physical phenomena occurring in the coastal zone, including the effects of shoaling, wave reflection and breaking. Several experimental campaigns have investigated these effects on planar beach slopes. Katsardi et al. (2013) considered the evolution of realistic ocean spectra over mild bed slopes ($\leq 1:100$). Their work investigates the implications of reduced effective water depth in altering the statistical distribution of wave and crest heights. Further experimental work conducted by Michallet et al. (2014) considered the implications of a slope configuration of 1:80 in the development of local nonlinear processes. Such nonlinear processes were also investigated by Beji & Battjes (1993) in the context of submerged bars. Boers (1996) provided further insight for waves in the surf zone under the influence of a barred beach. Moreover, the notable work of Ting & Kirby (1994) investigated the behaviour of monochromatic breaking waves in the surf zone, with a particular emphasis on turbulence generated through the wave breaking process. Efficient treatment of these effects has been achieved in deeper water by Huo et al. (2023) using hybrid approaches to combine deterministic and random simulations. However, further complexities manifest in shallower water as a consequence of depth-limited wave breaking, resulting in a significant modification in the description of wave and crest height statistics. Al Khalili & Karpadakis (2024) propose a modelling suite to probabilistically incorporate depth-limited wave breaking effects into existing crest height distributions for short-crested seas in intermediate water depths.

This work investigates the performance of the readily applied time-domain shallow-water model SWASH (Simulating WAVes till SHore). Although the non-hydrostatic model SWASH is widely used in coastal engineering, comprehensive validation across a broad range of regular and irregular wave conditions remains limited. This chapter focuses on numerically modelling regular and random nonlinear waves propagating from intermediate water depths to the shoreline over a mild slope. The accuracy of

the model is assessed by comparing the statistical distributions of crest heights and wave heights with equivalent sea-states measured experimentally. It is noted that, due to the limitations of the new formulation proposed by Vasarmidis et al. (2024), this version of SWASH is not implemented here (see Section 3.3.2 for further details).

This chapter outlines the numerical configuration adopted in this thesis, including the model formulation, boundary conditions, grid resolution, and wave generation, thereby establishing a transparent foundation for all subsequent simulations. The performance of SWASH is evaluated through direct comparison with controlled laboratory measurements. These comparisons quantify the model's range of applicability and limitations, directly addressing Objectives (i) and (ii) defined in Chapter 1.

3.2 Numerical Model

The SWASH model is a non-hydrostatic wave-flow model based on the solution of the nonlinear shallow water equations (NLSWE), which describe the mass and momentum balance for an incompressible fluid with a free-surface (Zijlema et al., 2011, Smit et al., 2013). SWASH overcomes the limitations associated with the assumptions of the NLSWE by appending a non-hydrostatic equation of vertical motion. As such, the solution of these equations is equivalent to solving the incompressible NSE (Zijlema & Stelling, 2008). For a one-dimensional flow, the conservation of mass is given by

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} (hu) = 0, \quad (3.1)$$

where $\eta(x, t)$ is the surface elevation measured from still water level (SWL), $d(x)$ is the water depth corresponding to SWL, $h = \eta + d$ is the total water depth and $u(x, t)$ is the depth-averaged flow velocity in the x direction. The conservation of momentum in the x coordinate is described by

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \eta}{\partial x} + \frac{1}{2} \frac{\partial q_b}{\partial x} + \frac{1}{2} \frac{q_b}{h} \frac{\partial (\eta - d)}{\partial x} + c_f \frac{u|u|}{h} = \frac{1}{h} \frac{\partial}{\partial x} \left(h \nu_t \frac{\partial u}{\partial x} \right), \quad (3.2)$$

where g is the gravitational acceleration, $q_b(x, t)$ is the non-hydrostatic pressure at the bed, c_f is the dimensionless bottom friction coefficient, ν_t is the horizontal eddy viscosity due to wave breaking and subgrid turbulence. In addition, the momentum associated with the velocities in the z -direction at the free surface (w_s) and at the bed

(w_b) must also be considered. The momentum equation for these vertical components is described by

$$\frac{\partial w_s}{\partial t} = \frac{2q_b}{h} - \frac{\partial w_b}{\partial t}. \quad (3.3)$$

A kinematic condition must also be imposed for vertical velocities at the bed and is given by

$$w_b = -u \frac{\partial d}{\partial x}. \quad (3.4)$$

The final governing equation imposes a condition of conservation of local mass such that

$$\frac{\partial u}{\partial x} + \frac{w_s - w_b}{h} = 0. \quad (3.5)$$

Moreover, unlike many other numerical models which solve the governing equations using depth-averaged properties, SWASH splits the vertical domain into a number of vertical grid points or layers. In doing so, the frequency dispersion is improved without requiring an increase in the order of the dispersion term (Zijlema & Stelling, 2008). This significantly improves the computational efficiency of the model.

Given that SWASH is a non-hydrostatic model, it is unable to describe the details of breaking waves due to the exclusion of physical effects such as air-entrainment, wave-generated turbulence and wave overturning (Smit et al., 2013). However, if only the macro-scale effects of wave breaking on the wave heights are of interest, it is not necessary to include the details of the breaking process. Le Mehaute (1962) explains that spilling and plunging breakers both evolve into a quasi-steady bore such that the front face of the wave is turbulent. Hence, a breaking wave may be described equivalently to a hydraulic jump. Therefore, integral properties such as the height of the hydraulic jump and the energy it dissipates can be determined by modelling the breaking wave as a discontinuity in the free surface and related velocities. Thus, if both mass and momentum conservation are ensured, the magnitude of energy dissipation may be calculated and consequently, the evolution of the wave height determined. However, Smit et al. (2013) explain that although vertical resolutions of 1–3 layers are sufficient to describe behaviour such as shoaling, refraction, non-linear interactions and diffraction outside the surf zone, significantly higher resolutions of 10–20 layers are required to describe dissipation due to wave breaking.

SWASH treats the dissipation of wave energy in a similar way to that proposed by Tonelli et al. (2010). At points where the vertical speed of the free surface exceeds

a fraction of the wave phase speed, the breaking process is initiated and hydrostatic pressure is assumed and remains so at the front face of the breaking wave. This ensures that a vertical face is developed at the wave front. This is referred to as the hydrostatic front approach. Hence, the condition for the initiation of breaking is given by

$$\frac{\partial \eta}{\partial t} > \alpha \sqrt{gh}, \quad (3.6)$$

where α is a threshold parameter to initiate wave breaking. It is important to note that these equations employ a shallow water approximation of the wave phase speed, $c = \sqrt{gh}$. Hence, this formulation is expected to work well in most shallow water studies where the celerity of the waves is equivalent to the shallow water wave celerity. However, caution should be taken when modelling waves that are breaking offshore. A similar condition is enforced subsequently to maintain breaking described as

$$\frac{\partial \eta}{\partial t} > \beta \sqrt{gh}, \quad (3.7)$$

where β is a threshold parameter at which to stop breaking dissipation. In other words, once breaking is initiated numerically, the breaking process continues to take place until Equation 3.7 no longer holds. According to Zijlema & Stelling (2008), this method correctly predicts energy dissipation on the front face of the breaking wave without the need for a dissipation model. Moreover, it preserves nonlinear wave properties such as asymmetry and skewness well. The default breaking parameters proposed by The SWASH Team (2024) are $\alpha = 0.6$ and $\beta = 0.3$. Although the default parameters are shown to work well for vertical resolutions greater than or equal to 10 layers, Smit et al. (2013) observe a delay in the onset of wave breaking and an underestimation of energy dissipation at low resolutions (<5 layers) when these default parameters are utilised. Thus, SWASH has the ability to calibrate these parameters in such cases to better capture experimental measurements.

SWASH employs a second-order finite difference method (FDM) such that mass and momentum are strictly conserved at the discrete level (Zijlema & Stelling, 2008). The time integration of the mass and momentum equations is done using the second-order Hansen scheme (Hansen, 1956). Conversely, the time integration of the non-hydrostatic pressure gradient is implemented using the θ -method, which is a linear combination of the explicit and implicit Euler method (Zijlema & Stelling, 2008).

The non-hydrostatic pressure is obtained through a second-order accurate pressure correction technique (Zijlema & Stelling, 2008). The time integration of the advective terms is conducted using the MacCormack predictor–corrector scheme, a second-order FDM. These are the methods most commonly adopted, but other options are available. A full description of SWASH alongside detailed explanation of the numerical methods adopted and boundary conditions can be found in Zijlema et al. (2011).

Several studies have validated the applicability of SWASH at modelling a variety of coastal phenomena. The work of Smit et al. (2013) investigated the nonlinear evolution and breaking of sea-swell waves, with a validation against the regular wave experiments of Ting & Kirby (1994) and random wave experiments of Battjes & Janssen (1978), Dingemans et al. (1987), Boers (1996) and Van Gent & Doorn (2000), over planar and barred topography, respectively. The findings of these works confirm the applicability of adopting the aforementioned hydrostatic front approach to model wave breaking. However, the regular waves investigated by Ting & Kirby (1994) consider a limited scope of only two wave conditions. Moreover, the aforementioned random wave experiments consider offshore wave steepnesses between $0.006 \leq S_p \leq 0.036$, where $S_p = \frac{2\pi H_{s,0}}{gT_p^2}$ and $H_{s,0}$ is the offshore significant wave height. This encompasses very mild to moderate offshore conditions. In the work of Smit et al. (2014), good correspondence is found between the results of SWASH and the experiments of Smith (2004) for random waves over a 1:30 planar bed slope for both single- and double-peaked spectral shapes. Although this work investigated a broad range of offshore wave conditions ($0.009 \leq S_p \leq 0.058$), the comparisons made were confined to the surf zone. Hence, minimal analysis was conducted in the shoaling zone to determine the pre-breaking wave behaviour.

The work of Rijnsdorp et al. (2014) have confirmed the ability of SWASH at modelling infragravity wave dynamics. This was also validated against field scale measurements (Rijnsdorp et al., 2015). Based on this, Rijnsdorp et al. (2022b) utilised SWASH simulations to provide physical insight into the surfzone energy balance in the context of infragravity wave dynamics. Independent comparisons between SWASH and experiments were also performed by De Bakker et al. (2016) investigating nonlinear energy exchanges for a range of planar bed slope configurations. In another context, Suzuki et al. (2017) utilised SWASH to model random wave overtopping in shallow foreshores. Their work displayed good accuracy in predicting mean overtop-

ping discharge compared to experiments. Conversely, Rijnsdorp et al. (2017) placed emphasis on the analysis of wave-induced current. Their work demonstrated the applicability and efficiency of the subgrid approach implemented in SWASH to capture the flow behaviour. However, they outline discrepancies of an order of magnitude in the turbulent flow field between the SWASH results and equivalent experimental measurements. Nevertheless, these works are all limited in terms of the offshore conditions they investigate, with only up to moderate offshore steepness explored ($S_p = 0.03$).

The literature presented above encompasses an investigation into a broad range of physical effects over variable coastal bathymetries. Moreover, the results have been validated across an extensive base of experimental measurements. However, the key limitations in the above works is the offshore steepness of the waves considered. This work seeks to bridge the gap in this field by establishing the validity of SWASH for a broader range of wave steepnesses that extend to more extreme offshore conditions. This is achieved by simulating a large range of regular and irregular waves both numerically and experimentally. Moreover, the comparisons between the numerical and experimental datasets are not limited to the surf zone only but extend to the shoaling region as well. This allows for a more detailed analysis into the ability of the numerical model to capture nonlinearities prior to the onset of wave breaking. Finally, unlike many of the works above that investigate the evolution of representative wave parameters, this work considers the distribution of crest heights and wave heights to provide a more complete description for the evolution of the modelled wave field.

3.3 Model set-up

3.3.1 Test cases

The present study concerns the propagation of unidirectional regular and random waves over a mild bed slope with gradient 1:50. As such, the test cases encompass waves propagating from intermediate to shallow water and include a range of steepness cases from near-linear sea-states to highly nonlinear cases. The offshore water depth for all simulations is $d = 0.5$ m. To allow for a valid direct comparison between numerical and experimental results, the same set of test cases are modelled

across the two data sets, adopting a 1:100 Froude scaling. As such, a length scale of $l_s = 1:100$ is implemented with a congruent time scale $t_s = \sqrt{l_s} = 1:10$. Given that direct comparisons will be made between the numerical predictions and the experimental measurements, all values presented in this chapter will correspond to those in laboratory scale. In the case of regular waves, a frequency range of 0.4 Hz - 1.4 Hz and amplitude range of 0.01 m - 0.10 m, with a step of 0.01 m, is adopted. As such, the selected test cases examine a range of effective water depths, kd , between 0.33 onshore to 3.95 at the offshore location. However, the sea-states that may be simulated experimentally are confined by two additional limitations. For the low frequency of 0.4 Hz, the amplitude of regular waves that may be modelled are limited by the paddle stroke such that $\frac{a}{\lambda} \leq 0.025$, where a is the wave amplitude and λ is the wavelength. Similarly, in the case of higher frequencies, 0.9 Hz–1.4 Hz, the amplitude of waves is limited by physical steepness limitations of $\frac{a}{\lambda} = 0.58$. Similar constraints apply for the case of random waves. The random experimental cases are given in Table 3.1.

TABLE 3.1: Random wave cases

Sea-state	$T_p[s]$	$H_{s,0}[m]$	$S_p = \frac{2\pi H_{s,0}}{gT_p^2} [-]$	$\frac{H_{s,0}k_p}{2} [-]$	$k_p d [-]$
A1	1.2	0.023	0.01	0.035	1.53-0.62
A2		0.046	0.02	0.070	
A3		0.068	0.03	0.105	
A4		0.091	0.04	0.140	
A5		0.111	0.05	0.170	
A6		0.124	0.06	0.190	
B1	1.4	0.029	0.01	0.035	1.22-0.52
B2		0.057	0.02	0.070	
B3		0.086	0.03	0.105	
B4		0.115	0.04	0.140	
B5		0.139	0.05	0.170	
B6		0.156	0.05	0.190	
C1	1.6	0.034	0.01	0.035	1.02-0.45
C2		0.069	0.02	0.070	
C3		0.103	0.03	0.105	
C4		0.137	0.04	0.140	
C5		0.167	0.05	0.170	
D1	1.8	0.040	0.01	0.035	0.88-0.40
D2		0.080	0.02	0.070	
D3		0.119	0.03	0.105	
D4		0.159	0.04	0.140	

The experimental data presented, for both the regular and random waves, is conducted in the Coastal Flume at the Hydrodynamics Laboratory of Imperial College London by Bellos et al. (2023, 2025). The facility is 23 m long, 0.6 m wide and has a working water depth of 0.5 m. The flume consists of a smooth glass bed and side walls to minimise viscous dissipation. Wave generation is performed using a piston-type wave paddle with active absorption which is operated in force control. Further details specific to this wave flume are provided in Aknin & Spinneken (2017). In all test cases, a high-resolution array of 80 gauges are positioned in the Coastal Flume. Each wave gauge records the free water surface elevation, $\eta(t)$, at a sampling frequency of 128 Hz and is calibrated daily. This ensures an accuracy of ± 0.5 mm as described by Karpadakis et al. (2019b).

In the case of random waves, the experimental data is generated using a JONSWAP spectrum (Hasselmann et al., 1973) with a peak enhancement factor of $\gamma = 2.5$. This is chosen to reflect realistic storm conditions occurring in intermediate waters (Jonathan & Taylor, 1997). For a given peak wave period, T_p , the offshore wave steepness, S_p , is incrementally increased to the test cases outlined in Table 3.1 with a step of $\Delta S_p \approx 0.01$ in sea-state steepness. Moreover, the phase-alignment approach of Karpadakis & Swan (2020) is utilised to allow for a direct comparison of individual waves in sea-states of the same peak period, T_p , but different offshore wave steepness, S_p . Both in the numerical and experimental simulation, for each test case, 20 random realisations (seeds) of the sea-state were conducted, with the initial phase of each seed defined stochastically from a uniform distribution in $[0, 2\pi)$. Each seed corresponds to approximately 3 h in field scale, containing approximately 18,000 waves. This is a considerably larger number than most coastal studies involving $O(1000)$ waves (Katsardi, 2007). Although this may be appropriate to define wave spectra and integral statistics, it limits the ability to define the tail of wave distributions (Latheef & Swan, 2013). Hence, the long length of the simulations in this work allow for a significant reduction in statistical uncertainties.

Figure 3.1 illustrates the bathymetry explored in this study, consisting of an initially flat bed region of width 2 m followed by a uniform bed slope of inclination 1:50 further downstream. The location of the wavemaker is illustrated with a vertical red line. The location of the gauges utilised in the experimental set-up are labelled $G1 - G80$. The first 12 gauges are spaced in 0.5 m intervals, whereas a higher

resolution of 0.2m is used for gauges $G13 - G80$ to better capture the effects of shoaling and breaking occurring in this region.

3.3.2 Numerical simulation

Equivalent numerical simulations to the experimental measurements were conducted using SWASH, which seek to numerically capture the physical effects observed in the laboratory investigation. Figure 3.1 explains the set-up utilised in the numerical model. Furthermore, the parameters adopted in the numerical simulation are summarised by Table 3.2.

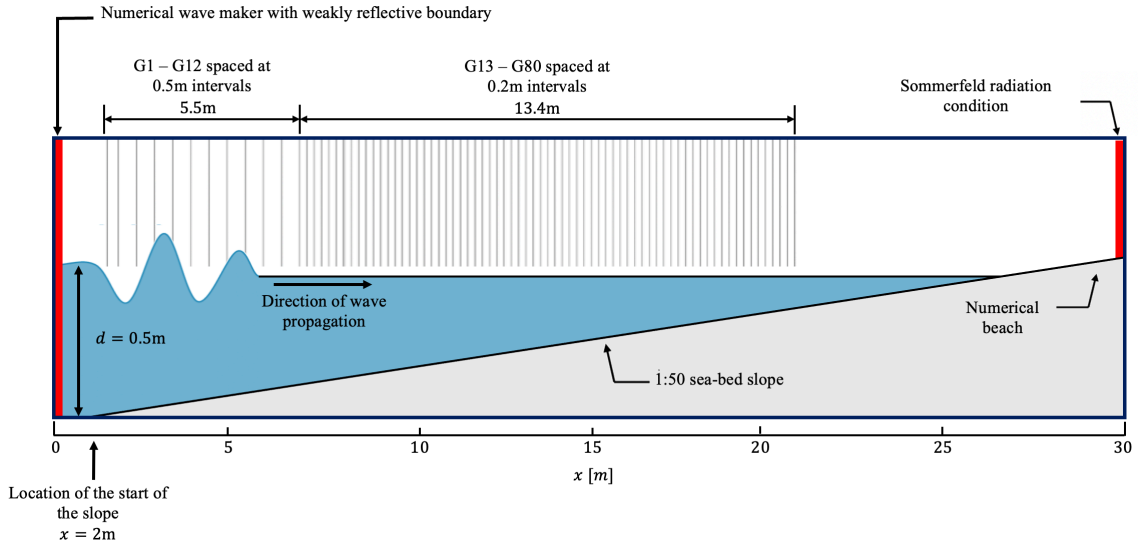


FIGURE 3.1: Schematic of the numerical set-up in SWASH and experiments.

The length of the horizontal domain is chosen to ensure the effects occurring in the swash zone are fully captured. This includes modelling a numerical beach that is of sufficient length to capture effects in this region. This is combined with a Sommerfeld radiation condition at the end of the numerical domain to minimise reflections at the outflow boundary. This differs from the experimental set-up which includes a perforated beach at the end of the spatial domain acting as a passive absorber. Unlike radiation conditions which are more effective at removing low frequencies, passive absorbers are more efficient at removing high frequencies (Hague, 2006). As such, it is expected that this will result in different characteristics of the observed reflections and is a point to acknowledge henceforth. In addition, a weakly reflective boundary condition is imposed at the generating boundary to ensure that the reflected waves

from the beach are not re-reflected at the wavemaker boundary.

TABLE 3.2: Properties of numerical simulations

Property	Value
Mode	One-Dimensional
Coordinate system	Cartesian
Length of domain (x)	40m
Mesh size (x)	60 grid points per (peak) wavelength
Mesh size (z)	10 non-equidistant layers of thickness comprising (from top to bottom) 1, 2, 3, 4, 5, 5, 10, 15, 25 and 30 % of the water depth, respectively
Inflow boundary condition	Weakly reflective
Friction	Manning coefficient = $0.019 \text{ m}^{-1/3} \text{ s}$,
Viscosity mode	Vertical mixing only, modelled using $k - \epsilon$ model
Ramping time	3 (peak) wave periods
Breaking parameters	$\alpha = 0.6$ and $\beta = 0.3$ (though varied in Section 3.4.2)
Courant number limits	0.1 - 0.5

The choice of the horizontal mesh size is chosen to ensure a high spatial resolution to resolve the physical effects of interest whilst avoiding numerical instabilities which may arise from an overly fine mesh. Previous work by Lashley et al. (2020) has shown that a maximum of 60 grid points per wavelength was required to ensure numerical stability. For a finer mesh resolution, Lashley et al. (2020) have observed strong oscillations in the water surface elevations surrounding the region of wave breaking. The instabilities associated with a high grid resolution have been thoroughly explored by Kazolea & Ricchiuto (2018). They explain that the cause of these instabilities is SWASH’s treatment of wave breaking using a hybrid approach. This approach suppresses the dispersive terms following the initiation of wave breaking based on the recommendations of Tonelli & Petti (2009). Following a mesh sensitivity study, the present work adopts a mesh size of 60 grid points per wavelength for regular waves or peak wavelength (λ_p) for random waves. This is found to provide the highest spatial resolution whilst maintaining stability across all sea-states investigated.

According to Zijlema et al. (2011), SWASH accurately describes wave dispersion of linear progressive waves up to $kd \approx 16$ with three non-equidistant vertical layers. However, in shallower water with $kd \leq 3$, two layers are sufficient to model progressive waves. Lashley et al. (2020) explains that utilising ten or more vertical layers allows SWASH to be depth-resolving. This refers to the ability of the numerical model to capture variations in the vertical flow structure. Given that SWASH imposes

depth-averaging within a given layer, the use of a greater vertical resolution allows for changes in the flow field in the vertical direction to be adequately resolved. Although the increased number of layers significantly increases the computational cost of a simulation, a fine vertical mesh will better capture effects associated with frequency dispersion. Moreover, a greater vertical resolution is required to adequately resolve highly nonlinear effects such as wave breaking. Hence, a vertical grid of 10 non-equidistant layers is adopted, which has a finer resolution closer to the water surface.

In this study, the bottom friction is modelled using

$$c_f = \frac{n^2 g}{h^{1/3}}, \quad (3.8)$$

based on Manning’s roughness coefficient, n , in combination with Equation 3.2. Zijlema et al. (2011) and Suzuki et al. (2017) outline that this formulation better captures the wave dynamics within the surf zone in comparison to other formulations, such as the Chezy coefficient and the Colebrook–White equation, both of which are options within SWASH. In this study, the default Manning coefficient value of $n = 0.019 \text{ m}^{-1/3} \text{ s}$, was found to be consistent with experimental measurements and will be used subsequently. Although a lower value of the Manning coefficient is recommended for glass (Suzuki et al., 2018), experimentally, additional frictional dissipation is present from the side walls. This is not present numerically where a one-dimensional configuration is considered. Hence, the higher chosen frictional coefficient acts to account for these effects and is shown to perform well for preliminary flat bed simulations conducted. However, it is important to note that the side wall dissipation is not uniform across the spatial domain in the context of this work. Instead, it plays a more dominant role in deeper water. Given that the implementation of a spatially varying frictional dissipation is beyond the scope of this work, this effect should be acknowledged in the comparisons that follow.

As outlined in Section 3.2, the initiation and termination of wave breaking is controlled by the α and β parameters in Equations 3.6 and 3.7. Unless otherwise stated, the default values of $\alpha = 0.6$ and $\beta = 0.3$ are utilised. In order to accurately capture the turbulent structure of the flow and dissipate wave energy within the breaking process, an appropriate viscosity model is required. According to The SWASH Team (2024), the horizontal eddy viscosity has a negligible role in coastal seas, which are

governed by bottom friction. This was further validated in the present study by comparing two simulations which were identical except from the viscosity model adopted. The former is modelled considering the full viscous structure (both horizontal and vertical components) whereas the later is modelled by considering the vertical viscosity alone. The results of the two simulations demonstrate minimal variation ($\leq 1\%$). Thus, given the negligible role of the horizontal viscosity, only vertical mixing is considered numerically. This is done by adopting the $k - \varepsilon$ model (Spalding et al., 1974), with k denoting the turbulent kinetic energy per unit mass and ε denoting the rate of turbulent kinetic energy dissipation per unit mass.

The initial time step of the numerical simulation is chosen to be equivalent to the sampling frequency of the experimental data, $F_s = 128$ Hz. Throughout the duration of the simulation, the time step is adjusted to be within the Courant number limits of 0.1–0.5, to ensure stability.

The simulations utilise 8 cores and 32 GB of memory, with each simulation requiring approximately 24 hours to complete. The outputs include water surface elevations and kinematics, which will be presented in Chapters 4 and 6.

The selection of the input signal to the numerical wavemaker is chosen to best correspond with the laboratory set-up. In the case of regular waves, the total time series of the water surface elevation measured in the first gauge in the laboratory for each sea-state is used as input to the numerical wavemaker. This signal is calibrated to ensure the same crest height is measured numerically and experimentally at the location of the first gauge. This process ensures that any diffusive and/or dissipative effects that may be present in the flat bed region are corrected. Figure 3.2 illustrates the measured time history of the water surface elevation at the first gauge, $G1$, for numerical and experimental measurements, shown in black markers and a red line, respectively. The results pertain to a wave of relatively high offshore steepness, $ak_0 = 0.33$, in intermediate offshore effective water depth, $kd_0 = 2.08$, where a is the wave amplitude and k is the wave number as defined by the linear dispersion relation (Equation 2.8). The zero subscript refers to quantities calculated offshore. Figure 3.2 demonstrates good correspondence between the input signal and the measured signal. Some very minor discrepancies are observed at the crest. However, given that this is a relatively large amplitude wave, this variation is deemed inconsequential. This further validates the methodology adopted in terms of the correct implementation

of the inflow boundary condition.

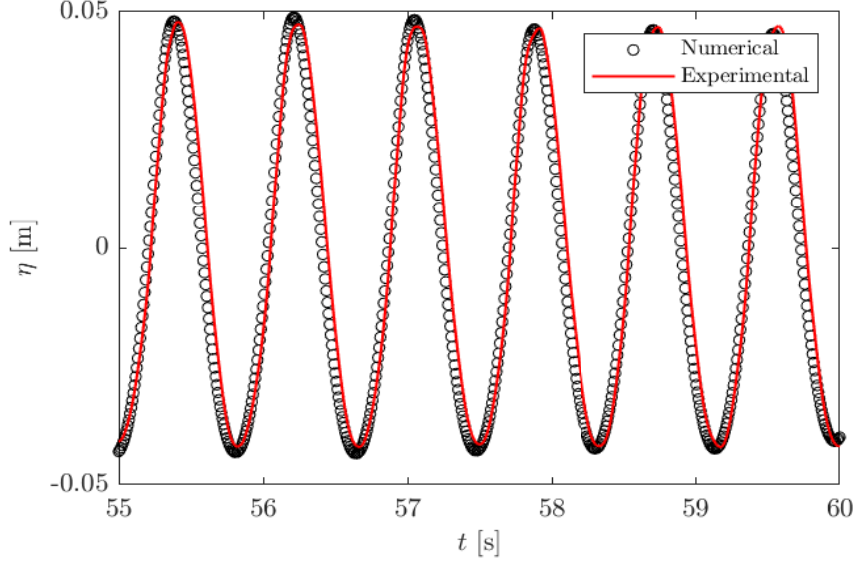


FIGURE 3.2: Time history of the water surface elevation at the location of the first gauge, $G1$, for $kd_0 = 2.08$ and $ak_0 = 0.33$. Numerical results are shown in circular markers and experimental measurements are shown in a red solid line.

In the case of random waves, the same JONSWAP input spectrum provided to the experimental wavemaker is utilised to generate a linear time series of the surface elevation. For each sea-state, the spectral input to the numerical wavemaker has been calibrated such that the significant wave height, H_s , (Equation 2.57) predicted numerically corresponds to that measured experimentally at the first gauge location within a 5% threshold. The validity of the input signal is established by considering the omni-directional energy spectrum, $S_{\eta\eta}$. This parameter is nondimensionalised by multiplying by the peak frequency, f_p , where $f_p = 1/T_p$ and normalised by the zero-th order spectral moment, m_0 , where the n th order spectral moment is given by Equation 2.55.

The variation of $S_{\eta\eta}f_p/m_0$ across the normalised frequency range, f/f_p , is shown in Figure 3.3(a)–(b) at location $G1$, corresponding to a mild and extreme sea-state, respectively. Numerical and experimental measurements are shown in blue and red, respectively. In order to remove the noise in the energy spectrum, a moving average filter is enforced with a 14 sub-series filtering window. The energy spectra numerically at the first gauge location demonstrate excellent correspondence with experimental measurements for up to extreme seas. The validity of the input sig-

nal is further investigated by considering the normalised crest height distribution, η_c/H_s , at $G1$. This is illustrated in Figure 3.3(c)–(d) for a mild and extreme sea-state, respectively. The results demonstrate good correspondence in capturing the crest statistics even for low probabilities of exceedance, thus validating the numerical wave generation methodology adopted.

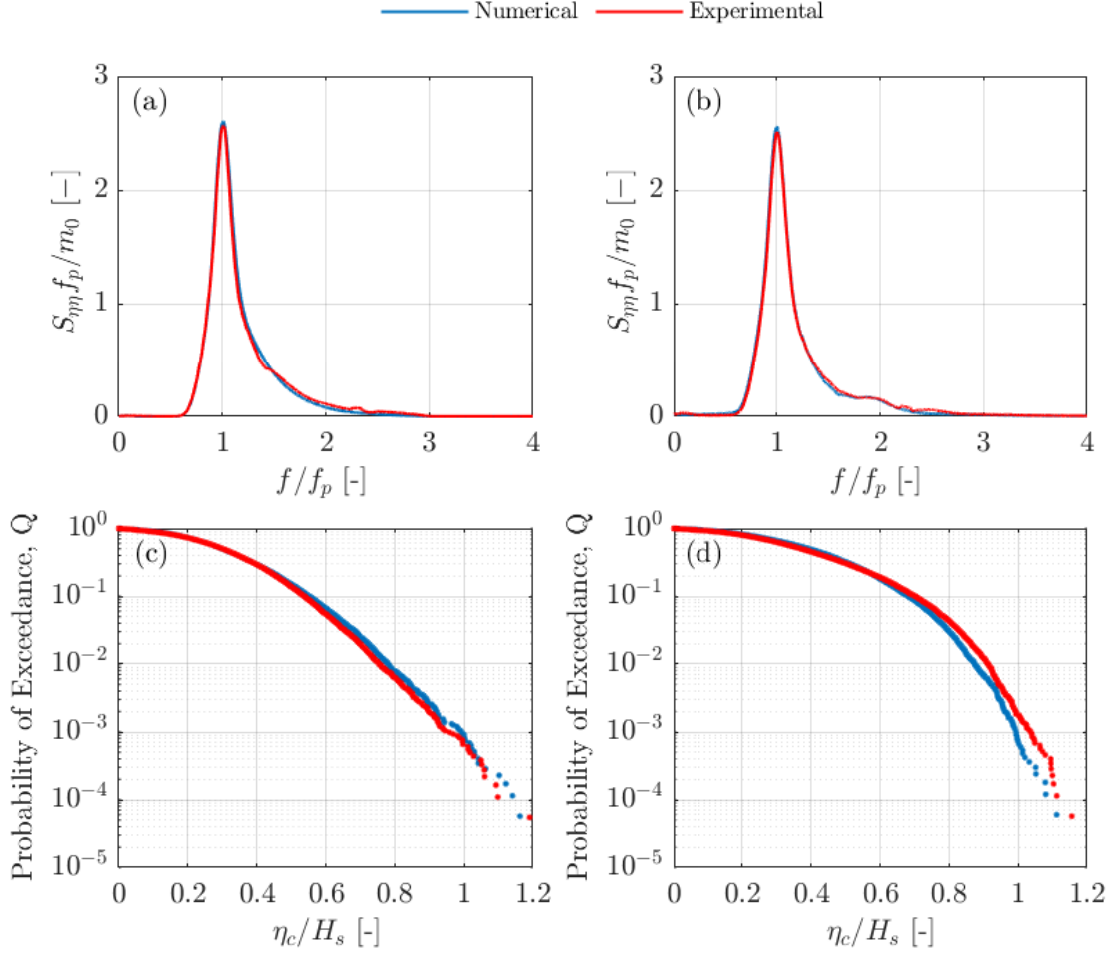


FIGURE 3.3: (a)–(b) Normalised energy spectrum, $S_{\eta\eta}f_p/m_0$ and (c)–(d) normalised crest height distribution, η_c/H_s , at the location of the first gauge, $G1$. Results in the left and right column correspond to sea-states B1 and B6, respectively. Numerical and experimental measurements are shown in blue and red, respectively.

Orszaghova et al. (2014) explain that a well-established issue of first-order wave generation is the formation of spurious free waves. Given that these wave components travel at a different speed than the second-order wave, a non-homogeneous surface profile arises. Recently, Vasarmidis et al. (2024) have conducted analysis into the non-linear properties of SWASH. Through this, they develop higher-order

hydraulic boundary conditions allowing for second-order wave generation in SWASH. This eliminates the generation of free spurious waves. In this study, comparisons between linear and second-order wave generation has demonstrated minor discrepancies in surface elevations measurements; however, these do not manifest in significant distinctions when considering surface elevation statistics ($<4\%$) or integral wave parameters ($\ll 1\%$). Moreover, the formulation of Vasarmidis et al. (2024) is limited to a maximum of four equidistant vertical layers. A sensitivity analysis on the number of vertical layers concluded that using 4 layers instead of 10 layers leads to a further increase of 1.2% in the dispersive error from the onshore to inshore location ($kd = 2.08$ to $kd = 0.77$). A higher vertical resolution is also required to accurately model wave breaking (The SWASH Team, 2024). For these reasons, the formulation of Vasarmidis et al. (2024) has not been further considered in the present work.

As outlined earlier, the differing outflow boundary condition numerically compared to experiments is expected to alter the characteristics of the reflections observed. To understand these variations, a reflection analysis is conducted based on the method of Lykke Andersen et al. (2016). This method extends the work of Lin & Huang (2004) to allow for the decomposition of the measured surface elevation into incident and reflected components for sloping bathymetries. Figure 3.4 shows the variation in the reflection coefficient, r , as a function of wave frequency, f , where r is defined as the ratio of the reflected to incident wave amplitude. The results shown in Figure 3.4, both numerically and experimentally, demonstrate the same overall trend. The reflection coefficient generally does not exceed 5% for moderate frequencies (0.4 Hz–1.0 Hz). In low frequency wave cases, experimentally, a greater magnitude of reflections is observed. This is a consequence of the use of a passive absorber experimentally which is less efficient at removing low frequency components. For higher frequencies, an increase in the reflection coefficient is observed as frequency is increased further for both the numerical and experimental measurements. For a given wave amplitude, higher frequency waves are associated with greater steepness, presenting additional nonlinear effects that influence the magnitude of reflections. Moreover, given the gauge spacing utilised in this work, there may be limitations with resolving frequencies $f \geq 0.85$ Hz (Goda & Suzuki, 1976).

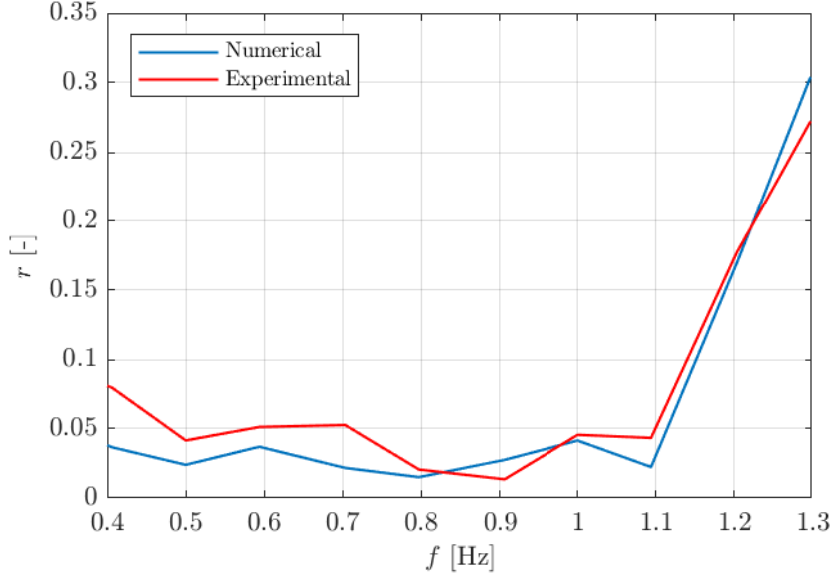


FIGURE 3.4: Variation in the reflection coefficient, r , as a function of wave frequency, f , for waves of amplitude $a = 0.03$ m. Numerical and experimental measurements are shown in blue and red, respectively.

3.3.3 Dispersion

Following the completion of the numerical simulations, the following analysis methodology is adopted to allow for a direct comparison between the numerical predictions and experimental measurements. The time series of the water surface elevation is extracted from the numerical simulation output at the location of the gauges in the experimental set-up. This is illustrated in Figure 3.5, which shows the time history of the water surface elevation at selected gauge locations, for the numerical (blue) and experimental (red) measurements. Subsequently, the first crest in the time series is tracked as it propagates from the first gauge ($G1$) to the gauges downstream. The location of this crest is marked using a solid and dashed black line in the experimental and numerical time series, respectively.

It can be seen that at the location of the first gauge, $G1$, the phasing of the inputs to the numerical model are equal to those of the experimental measurements. Some minor discrepancies are observed in terms of the nonlinearities of the waves. However, the calibration of the inflow boundary condition has ensured equivalent crest heights at this location. Thus, the discrepancies shown here are due to variation in the evolution of the waves within the flat bed region of the bathymetry. With regards to the phasing, it is expected that the two data sets will continue to be in-phase,

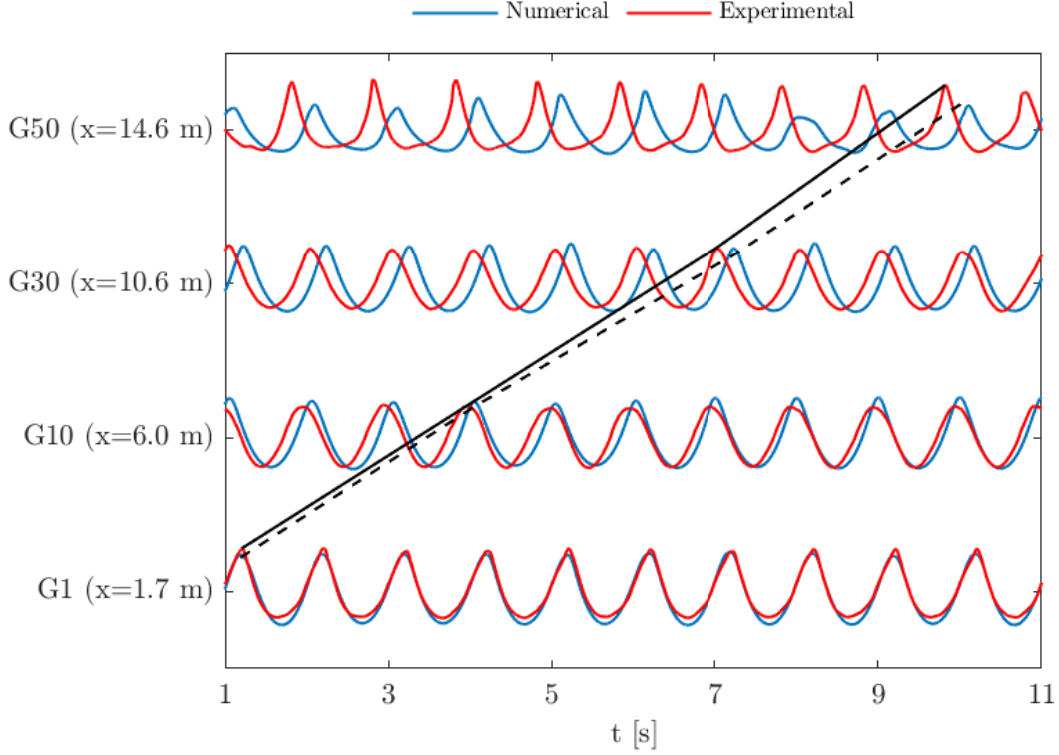


FIGURE 3.5: Time history of the water surface elevation demonstrating dispersion effects for $kd_0 = 2.08$ and $ak_0 = 0.33$ in the numerical (blue) compared to the experimental (red) measurements. The black solid and dashed lines track the location of a crest across gauges in the experimental and numerical set-up, respectively.

both spatially and temporally, at distances downstream of the first gauge. However, it is observed that there is a phase difference between the numerical simulations and the experimental measurements at distances downstream of the first gauge location, signifying the presence of dispersion. As evident in Figure 3.5, the slope of the line associated with the location of the experimental crest is steeper than that of the numerical crests. This signifies a lower dispersive behaviour of numerically-simulated waves compared to the dispersive effects observed in experiments, resulting in later crest arrivals at a downstream spatial location. The extent of this discrepancy is approximately a 3.2% reduction in the wave celerity observed numerically compared to experimental measurements. Numerical dispersion could act as a contributing factor. This occurs in even discretisation methods which have an odd leading truncation error. Given the discrete nature of the finite difference method used, the truncation of the Fourier series representing the wave propagation results in a marginal variation

in the wave celerity of the various components (Cavaleri et al., 2007). Hence, the Fourier components are described to “disperse”. In the case of the present numerical implementation, the non-hydrostatic pressure is obtained by a second-order accurate pressure correction technique, which is inherently dispersive. In comparison to existing literature, Zijlema et al. (2011) have shown a much lower dispersive error compared to experiments. However, the cases considered in their work are linear progressive waves ($a_0/d = 0.001$) which have a far lower steepness than the case shown in Figure 3.5 ($a_0/d = 0.16$). The findings of the present study are in line with previous literature in the context of low wave steepness conditions such that the dispersive behaviour predicted numerically corresponds well to experimental measurements for up to significant distances downstream of the inlet ($>10\lambda$). This means that the reported value of 3.2% in Figure 3.5 is representative of extreme wave cases.

3.3.4 Sensitivity analysis on number of vertical layers in SWASH

In order to assess the influence of the number of vertical layers used in the numerical model on the results of the numerical simulation, a sensitivity analysis is conducted such that all other input parameters to the model are unchanged, except the number of vertical layers. In the context of this sensitivity analysis, only equidistant layers are investigated. This analysis, pertaining to Section 3.3.4 only, is conducted over a flat bed region to ensure effects associated with wave propagation over a slope are not introduced. Hence, an additional 10 m flat bed region is appended at the start of the spatial domain such that the numerical wavemaker is situated at $x = -10$ m. As a metric for this sensitivity analysis, the mean crest height, $\overline{\eta}_c$, of regular waves is investigated. An explanation of how this parameter is obtained is provided in Section 3.4. Figure 3.6(a) demonstrates the spatial variation in the mean crest height, $\overline{\eta}_c$, normalised by the value at the ‘inlet’, $\overline{\eta}_c^{inlet}$ with each colour corresponding to 1, 3, 5, 8 and 10 vertical layers used in the SWASH model. In theory, the ‘inlet’ location should be taken at the location of the wavemaker, $x = -10$ m. However, given the local evanescent modes that occur in the vicinity of the wavemaker, the location of the ‘inlet’ for the purpose of this analysis is offset to $x = -6$ m. The black vertical dashed line denotes the position of the start of the slope such that the domain prior to this location is a flat bed.

In the case of regular waves propagating over a flat bed, a uniform mean crest

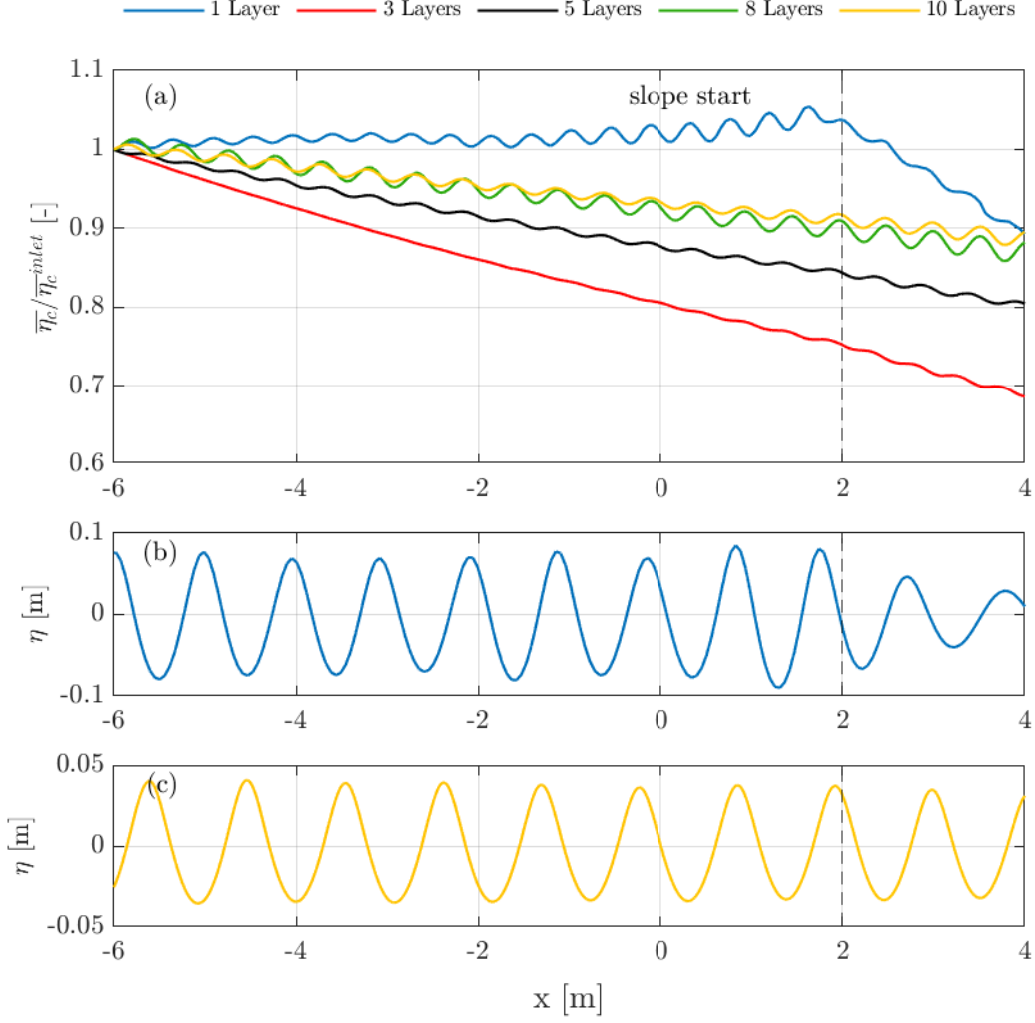


FIGURE 3.6: Spatial evolution of (a) the mean normalised crest height compared to the inlet, $\bar{\eta}_c / \bar{\eta}_c^{inlet}$. Evolution of the surface elevation in space for a (b) single-layer and (c) 10 layer model. Results pertain to a regular wave with $kd_0 = 2.91$ and $ak_0 = 0.29$. The colour of the line corresponds to a different number of equidistant vertical layers used in the SWASH model.

height or gradual reduction compared to the inlet is expected as a result of frictional dissipation losses. However, for the case of a single layer, it is evident that the mean crest height does not follow this expected trend. Instead, an increase in the mean crest height is observed. To understand these effects further, the surface elevation variation in space at one instance in time is shown in Figure 3.6(b) for a single-layer model. Prior to the start of the slope, modulations in the surface elevation are observed. This highlights the inability of SWASH to accurately model relatively steep high-frequency waves using a single vertical layer. For the cases modelled using 3–10 layers, it is shown that the increase in the number of layers results in a larger

predicted mean crest height across the spatial domain. This physically relates to more vertically asymmetric waves which are more “peaked” in nature, a property of waves with increased nonlinear effects. Moreover, within the flat bed region $x < 2$ m, Figure 3.6(a) demonstrates that there is a gradual reduction in crest heights within this region such that approximately a 10% reduction in the mean crest height is observed prior to the start of the slope. This is further highlighted in the spatial evolution of the time series shown in Figure 3.6(c) for the case of a SWASH model with 10 layers. A clear reduction in the magnitude of the wave crest is shown with distance away from the inlet. The magnitude of this reduction is comparable to the dissipative effects for similar experimental simulations of regular waves on a flat bed configuration. This further validates the frictional dissipation model implemented in this work.

Another important observation in Figure 3.6(a) is that the mean crest heights appear to fluctuate spatially, such that a modulation pattern is seen. It is hypothesised that this may be a consequence of the presence of high frequency wave reflections at approximately double the fundamental frequency. Although the use of higher-order wave generation, such as that proposed by Vasarmidis et al. (2024), may reduce these effects, this formulation is not adopted in this work considering its limitation to a maximum of four vertical layers. Given the nature of the sea-states of interest in this work, it is deemed that such a low vertical mesh resolution is unable to capture the full extent of the nonlinearities and wave breaking present, as demonstrated in Figure 3.6(a).

To obtain an understanding on the energy components present in the numerical simulation, a two-dimensional Fast Fourier Transform (FFT) analysis is conducted in both space and time for the surface elevation within the flat bed region of the domain in the case of 10 vertical layers. The results of this analysis are plotted in normalised wavenumber, k/k_f , and normalised frequency, ω/ω_f , space in Figure 3.7, where k_f and ω_f are the wavenumber and angular frequency, respectively, associated with the fundamental frequency of regular waves. The contours correspond to the log of the amplitude spectrum, $a(\omega, k)$. Clear peaks are observed at $k/k_f = \omega/\omega_f$, indicating the presence of bound terms up to fifth-order. This is to be expected given the nonlinear nature of the modelled sea-state. It also signifies the ability of SWASH to capture nonlinear bound effects; however, the accuracy of the magnitude of

these effects is yet to be established. Furthermore, some energy is located along the linear dispersion line, $\omega^2 = gk \tanh(kd)$, indicating the presence of freely-propagating components, which support earlier findings in Figure 3.4 made regarding the presence of wave reflections. Furthermore, there is some unwanted energy that is seen neither on the linear dispersion line nor on the $k/k_p = \omega/\omega_f$ line, which is unexplained physically. This is unwanted energy that is artificially introduced in the numerical domain. The implications of these discrepancies in the context of wave statistics will be further addressed in Section 3.4.1 and Section 3.4.3.

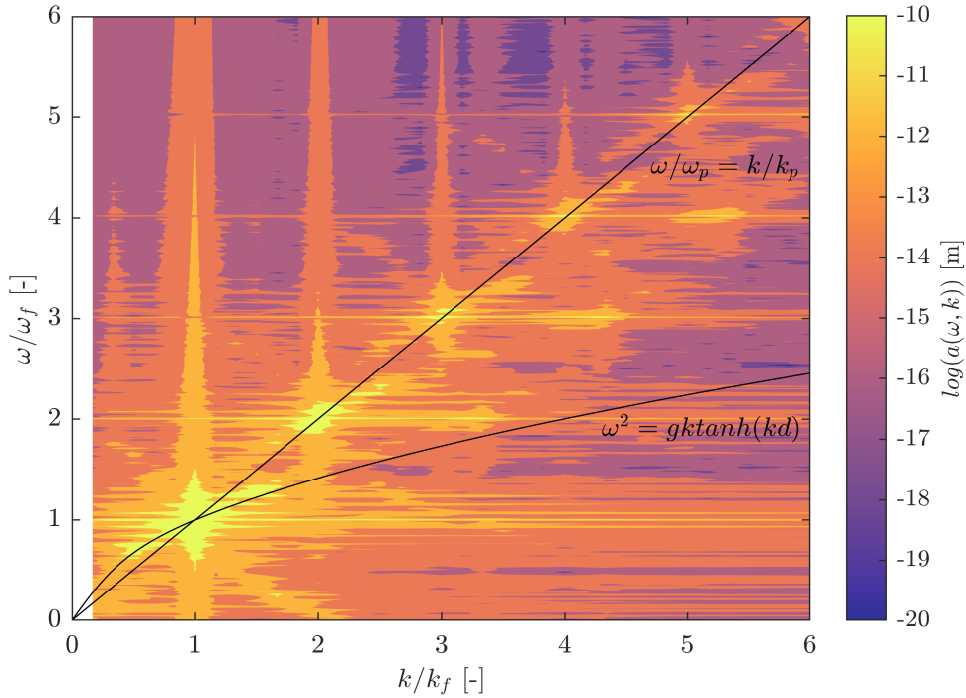


FIGURE 3.7: Spatial and temporal Fast Fourier Transform analysis showing contours of the logarithm of the two-dimensional amplitude spectrum, $\log(a(\omega, k))$, in normalised wave number, k/k_f , and normalised wave frequency, ω/ω_f , space for $kd_0 = 2.91$ and $ak_0 = 0.29$.

3.4 Regular waves

3.4.1 Crest height evolution

This study is primarily focused on analysing the numerical predictions of crest and wave height evolution and comparing them against experimental measurements.

Hence, a zero-crossing analysis is performed for each sea-state, as described in Section 2.4.1. This consists of firstly extracting the time series of the surface elevation at the location of the 80 wave gauge locations defined in Section 3.3.2 and Figure 3.1. Each time series is then truncated to remove the initial ramping period and subsequently, a zero-crossing analysis is performed to extract the properties of individual waves.

This section will investigate the evolution of the mean down-crossing crest height, $\overline{\eta}_c$. This parameter is calculated as the mean of the crest heights obtained from the zero-crossing analysis of the time series at each spatial location. Essentially, it is a spatially varying, temporally-averaged crest height. These values are then normalised by the offshore mean crest height measured at the first gauge location, $\overline{\eta}_{c,0}$. Figure 3.8 shows the variation of the normalised mean crest along the spatial domain for selected wave cases. The numerical predictions are shown in blue and the experimental measurements are illustrated in red and error bars corresponding to a 95% confidence interval in the mean normalised crest height, $\overline{\eta}_c/\overline{\eta}_{c,0}$, are indicated. The confidence interval is computed in the spatial domain based on 10 000 samples of the data generated using the bootstrap method (Efron & Tibshirani, 1994). In analysing Figure 3.8, there are two main points to consider. Firstly, the performance of SWASH in modelling seas of ranging steepness and secondly, its performance for seas of ranging frequency. The latter will serve as an indication for the relative performance of SWASH in different effective water depths.

To assess the performance across effective water depths, we compare Figure 3.8(a), (c) and (e), which all reflect the spatial variation of the mean normalised crest height, $\overline{\eta}_c/\overline{\eta}_{c,0}$ for waves of similar offshore steepness (ak_0) but for offshore effective water depths of $kd_0 = 1.19$, $kd_0 = 1.44$ and $kd_0 = 2.47$, respectively. In the case of low frequency sea-states as shown in Figure 3.8(a), the numerical model exhibits a stronger shoaling behaviour such that the mean normalised crest heights are generally overestimated relative to experimental measurements up to the breaking location ($kd \approx 0.7$). In this case, waves in the numerical model are shown to break earlier compared to waves in the experiments. That being said, the spatial maximum crest height observed across the two data sets is comparable. In more moderate frequency cases, such as that shown in Figure 3.8(c), the numerical model does well at capturing the observed mean crest height in the experiments up to the breaking location.

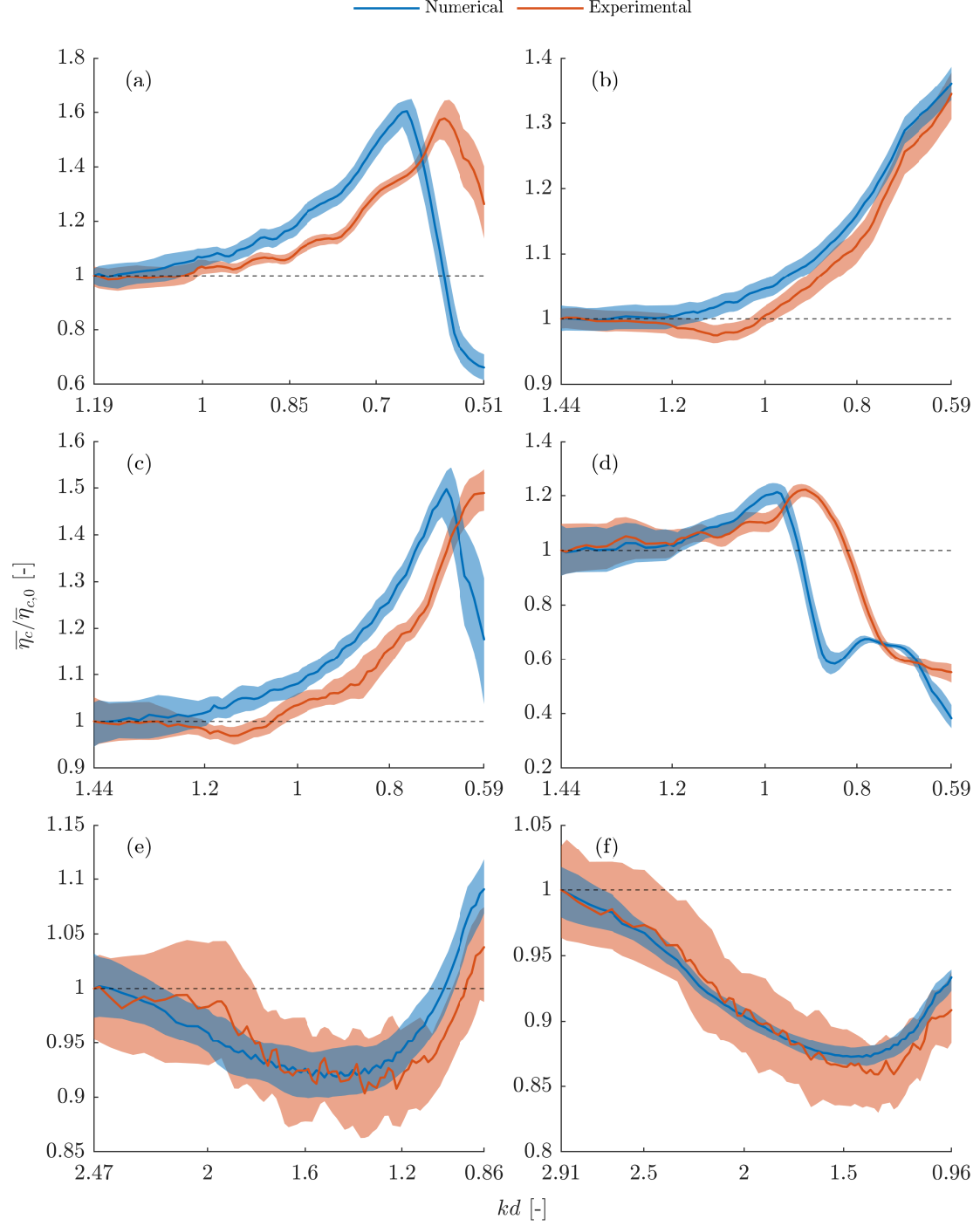


FIGURE 3.8: Spatial variation of the mean normalised crest height for numerical (blue) and experimental (red) measurements for regular waves with (a) $kd_0 = 1.19, ak_0 = 0.12$, (b) $kd_0 = 1.44, ak_0 = 0.09$, (c) $kd_0 = 1.44, ak_0 = 0.12$, (d) $kd_0 = 1.44, ak_0 = 0.23$. (e) $kd_0 = 2.47, ak_0 = 0.14$ and (f) $kd_0 = 2.92, ak_0 = 0.18$. Error bars correspond to a 95% confidence interval.

Similar to the case in Figure 3.8(a), the numerical waves begin to break towards the end of the spatial domain whereas breaking has not yet commenced in experiments

at the location of the last gauge. In other words, breaking is initiated at distances further downstream of the location of the last gauge experimentally. In deeper effective water depths, as shown in Figure 3.8(e) and (f), the evolution of the mean crest height is driven by shoaling effects. In Figure 3.8(e), the mean crest height both numerically and experimentally experiences a gradual reduction in space up to the centre of the slope ($kd \approx 1.3$), with the spatial variation of this reduction being similar in the two data sets. Moreover, for later spatial locations, the amplification in the wave crest experimentally is also accurately captured by the numerical model.

To address the performance of SWASH with increasing steepness, but equal effective water depth, we compare Figure 3.8(b)–(d). These figures pertain to offshore effective water depths of $kd_0 = 1.44$ and steepnesses $ak_0 = 0.09$, $ak_0 = 0.12$ and $ak_0 = 0.23$, respectively. In less steep seas, as in Figure 3.8(b), the numerical model does well to capture the mean crest height variation observed in the experiments across the spatial domain. This is understandable given that less steep seas are associated with less nonlinear interactions and as such, less complexity required by the numerical model to match experimental measurements. Furthermore, in such mild seas, the extent of wave breaking is minor and as such, the inaccuracies associated with determining the location of breaking instigation and the dissipation of wave energy following wave breaking are not reflected in the results. For seas of moderate steepness such as Figure 3.8(c), SWASH continues to do well to capture the shoaling behaviour observed in experimental measurements. However, discrepancies in the breaking location begin to be observed at the final gauge location such that earlier breaking initiation occurs numerically. In the steepest case, shown in Figure 3.8(d), a similar trend is observed in the pre-breaking performance of SWASH. However, unlike in Figure 3.8(a) where breaking is initiated earlier numerically, the numerical model more accurately predicts the location for the start of breaking in Figure 3.8(d). Nevertheless, in this context, the gradient of the reduction in mean crest height is steeper in SWASH compared to the experiments. This indicates excessive energy dissipation numerically for this test case compared to experimental findings. Additionally, at the end of the spatial domain, the mean crest height in the experiments remains relatively uniform spatially, whereas numerically the mean crest height exhibits more fluctuations. These fluctuations are driven by more pronounced wave reflections occurring in this region in comparison to experimental measurements.

It is also important to highlight the presence of reflections occurring in the laboratory measurements which are manifested as modulations in the mean normalised crest height. As explained in Section 3.3, given that the outflow boundary is different in the two set-ups, it is expected that the nature of the reflections observed will vary. A prominent example of the discrepancy in the reflection pattern is observed in Figure 3.8(d). In the experimental set-up, we observe stronger low frequency (large wavelength) wave reflections compared to what is observed numerically. This can be explained by the fact that a radiation condition is adopted numerically whereas a passive absorber is utilised experimentally. Although high frequency reflections are expected in the swash zone, low-frequency reflections indicate the experimental limitations of the dissipative beach.

In comparing the magnitude of the confidence intervals between the numerical and experimental data, it can be seen that the uncertainties associated with the numerical simulations are generally relatively constant. The confidence intervals associated with experimental measurements are similar to the numerical results for longer waves, (Figure 3.8(a)–(d)). However, shorter waves (Figure 3.8(e)–(f)) show increased uncertainties in the experiments. This variability arises due to wave reflections which are more pronounced in these wavelengths. In turn, these can lead to long- and cross- tank resonances and combine with other experimental sources of variability.

3.4.2 Wave breaking

In assessing the overall performance of SWASH in simulating regular waves on a 1:50 slope, following the discussion in Section 3.4.1, it is able to accurately model wave propagation across a range of effective water depths up to offshore steepnesses of $ak_0 = 0.2$. The primary discrepancies between numerical simulations and the experimental observations are centred around the behaviour associated with wave breaking. Within this region, even for moderate steepness waves, the performance of the numerical model is heavily dependent on the simulated regular wave. This is driven by two notable effects in the treatment of breaking in the numerical model. The first is the location for the initiation of wave breaking denoted by Equation 3.6. Although the recommended value of $\alpha = 0.6$ is used in all simulations, this parameter may be calibrated to either delay or advance the initiation of wave breaking numeri-

cally. However, such calibration is not undertaken in this study as no global α value was shown to perform well across all the modelled regular waves. For example, as outlined previously, in Figure 3.8(a), the numerical model initiates earlier breaking compared to the experiments so an increase in the α parameter would be required to match the experimental measurements. Conversely, in Figure 3.8(d), the breaking location is matched well. Thus, calibration of the breaking parameters would have to be conducted relative to a specific regular wave.

In order to further highlight the implications of the choice of breaking parameters, a sensitivity analysis is conducted on the α and β parameter, individually. Figure 3.9 depicts the spatial variation of the mean normalised crest height, $\bar{\eta}_c/\bar{\eta}_{c,0}$ measured at each gauge. The colour of the lines represents the results of the numerical simulation for the same input specifications but different α and β parameters. The grey shaded region represents the surf zone according to experimental measurements, such that waves in this region are observed to break, experimentally. The start of the surf zone is defined as the location at which a notable reduction in the mean normalised crest height is observed experimentally, as illustrated by the blue dotted line.

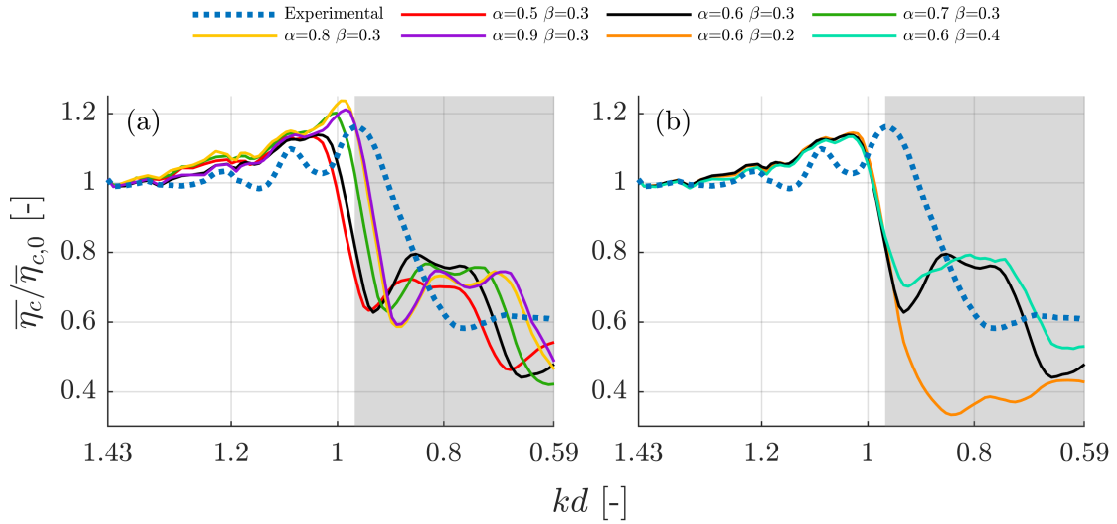


FIGURE 3.9: Spatial variation of mean normalised crest height showing sensitivity analysis for $kd_0 = 1.44$ and $ak_0 = 0.29$ on (a) α parameter and (b) β parameter. The surf zone, as determined from experiments, is indicated by the grey shaded region.

Figure 3.9(a) indicates the variation in the α parameter with a constant $\beta = 0.3$, whereas each line in Figure 3.9(b) corresponds to a different β parameter with a constant $\alpha = 0.6$. The results shown correspond to an effective water depth offshore

of $kd_0 = 1.44$ and offshore wave steepness $ak_0 = 0.29$. As expected, an increase in the α value results in a delay in the onset of wave breaking in the numerical model shown by a downstream shift in the maximum mean crest height, as observed in Figure 3.9(a). For the wave case shown in Figure 3.9(a), an α value larger than the default of 0.6 may better capture the location of breaking observed in the experiments. However, this is not so for all wave cases considered as illustrated in Figure 3.8(d), where the default breaking parameters perform well.

In examining the influence of the β parameter shown in Figure 3.9(b), it is seen that varying the value of this parameter alters the duration of wave breaking such that larger β values are associated with an earlier termination of the breaking process. Smit et al. (2013) have empirically determined the value of $\alpha = 0.6$ based on the experiments of Ting & Kirby (1994). However, recently Barranco & Liu (2021) have outlined that the calibration of the breaking parameters is also dependent on the grid size utilised in the numerical mesh. In their work modelling a dam-break bore, $\alpha = 0.8$ matches well to experimental measurements. Hence, given the several dependencies of such calibration for a wide range of sea-states, in all subsequent analysis, the default α and β parameters are utilised.

Another important observation shown by Figure 3.9 is the trend in the mean crest height prior to breaking. It is expected that varying the breaking parameters will only modify the results of the numerical simulation following the break point. However, from Figure 3.9, a minor fluctuation in the mean crest height is observed for varying breaking parameters prior to breaking. Moreover, the choice of breaking parameters has implications on the modulation pattern in the mean crest height observed within the surf zone. This supports the previous findings discussed which inferred the presence of wave reflections within the numerical simulation.

3.4.3 SWASH performance across range of regular waves

The ability of SWASH to capture experimental measurements may be summarised by the parameter R_1 , defined as

$$R_1 = \overline{\eta_{cNUM}} / \overline{\eta_{cLAB}}, \quad (3.9)$$

where $\overline{\eta_{cNUM}}$ is the mean numerical crest height and $\overline{\eta_{cLAB}}$ is the mean experimental crest height. Utilising this parameter, Figure 3.10(a) plots the contours of R_1 , measured at each gauge location for all regular wave cases as a function of the local effective water depth, kd , and wave steepness, ak . To better interpret the results of these contours, three slices through the plot are made, shown by dashed black lines in Figure 3.10(a). Figure 3.10(b) shows the variation of R_1 with effective water depth for a horizontal cut at $ak = 0.3$. Likewise, Figure 3.10(c) shows the variation of R_1 with wave steepness for vertical cuts at $kd = 0.75$ and $kd = 2.25$, shown in blue and red, respectively.

In considering Figure 3.10(a), it is observed that for up to moderate wave steepnesses, $ak = 0.25$, SWASH captures the magnitude of experimentally-measured crest heights within 5% accuracy. However, discrepancies become prevalent between numerical and experimental measurements for higher wave steepnesses, $ak > 0.25$. The characteristics of these discrepancies are dependent on the local effective water depth, kd . The variation in the model accuracy with effective water depth is investigated in Figure 3.10(b) for $ak = 0.3$. The results demonstrate an underestimation by the model in intermediate water depths, $kd \geq 0.8$, such that SWASH under predicts crest heights by the order of 10%. Referring back to Figure 3.10(a), these under predictions in intermediate water depths are greater as wave steepness increases. This can be explained by the challenges associated with modelling very steep waves with increased nonlinear effects numerically. Also note that the blue band observed for $kd < 1$ does not correspond to data sampled at the same spatial location. Rather, it is an effect observed in shallower water depths manifested due to the treatment of nonlinearities and wave breaking at these locations. In shallower water, $kd \leq 0.8$, a clear over estimation of up to 38% of the experimentally-measured crest height is observed for steeper waves ($ak \approx 0.33$). This is consistent with the observations outlined earlier relating to the complexities of capturing the breaking location and post-breaking behaviour numerically.

Similarly, Figure 3.10(c) shows the variation of R_1 with wave steepness for vertical cuts at $kd = 0.75$ (blue) and $kd = 2.25$ (red). As expected, less steep waves, $ak < 0.25$, are well modelled numerically across the spatial range such that the value of R_1 is within 5% of the experimental crest height. However, more significant deviations are observed for higher steepnesses. In shallow water, $kd = 0.75$, the

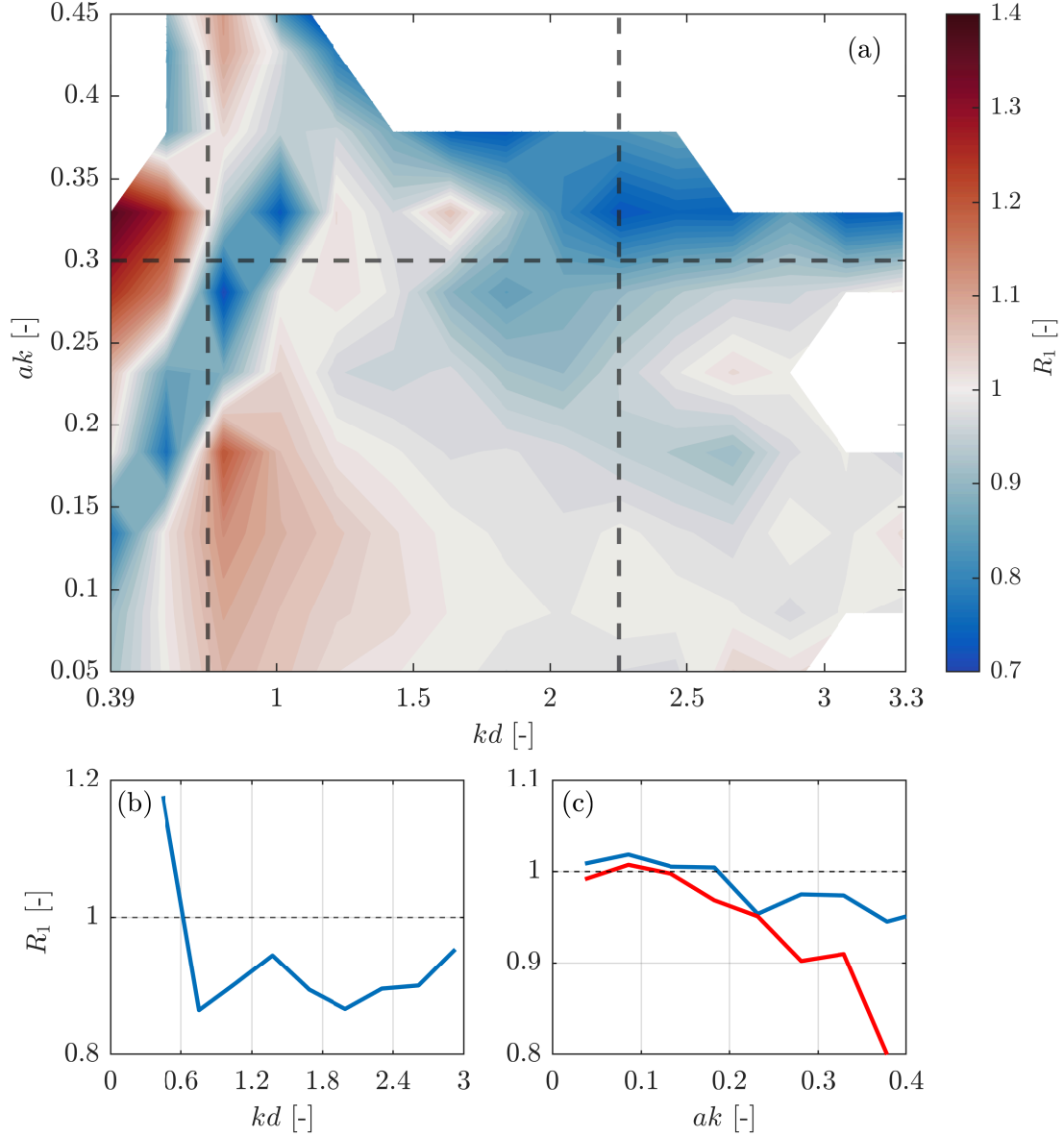


FIGURE 3.10: (a) Contour of the ratio of mean numerical to experimental crest heights, R_1 (Equation 3.9), for all regular sea-states investigated and measured at all gauge locations. (b) Variation of R_1 with effective water depth, kd , for steepness $ak = 0.3$. (c) Variation of R_1 with steepness, ak , for effective water depth $kd = 0.75$ (blue) and $kd = 2.25$ (red).

mean crest height is underestimated for $ak > 0.25$, due to an inability to capture the full nonlinearities of these steep waves numerically coupled with an overestimation of breaking energy dissipation. This has previously been shown in Figure 3.8(a) and (d). In intermediate water depths, $kd = 2.25$ for steep waves, crest heights are more greatly underestimated. These discrepancies are more notable with increased steepness such that errors of up to 20% of the experimental crest height are observed.

The discrepancies between SWASH and the experiments have inferred that SWASH is able to model the mean crest height of regular waves for up to moderate steepnesses ($ak = 0.25$) with an accuracy of 5%. More notable discrepancies are shown for steep waves ($ak > 0.25$) in deep to intermediate water depths ($kd > 0.9$) with a trend of underestimation of numerical crests by up to 20%, which indicates a limitation in SWASH's ability to capture the full extent of nonlinearity observed experimentally alongside an overestimation of breaking energy dissipation. In shallower water, limitations in the numerical treatment of wave breaking become more pronounced, both in terms of the location of breaking initiation and the subsequent energy dissipation. As a consequence, errors in the order of 38% of experimental mean crest heights are observed.

3.5 Random waves

To this point, the study has explored the evolution of regular waves, which are idealised in nature. However, realistic seas exhibit irregular wave behaviour. As such, this section will investigate the evolution of random waves for the sea-states outlined in Table 3.1. Hence, a similar zero-crossing methodology to that outlined in Section 3.4.1 for regular waves is conducted at the location of each experimental gauge for each realisation of the random sea-states explored. The results for each sea-state across the 20 seeds are then concatenated. In doing so, the wave properties may be reordered and their probability of exceedance determined. Given the length of the concatenated data set, with each seed being approximately three hours in field scale, probabilities of exceedance up to $Q = 10^{-4}$ are derived. As such, extreme waves associated with small probabilities of exceedance may be investigated.

3.5.1 Spectra

A key parameter to understand the behaviour of the numerical model compared to experimental measurements is the omni-directional spectral density function, $S_{\eta\eta}$. The variation of $S_{\eta\eta}f_p/m_0$ across the normalised frequency range, f/f_p , is shown in Figure 3.11, for $T_p = 1.4$ s at gauge 60 ($x = 16.6$ m) and $k_p d = 0.69$ for a range of steepness cases. For seas with low and moderate steepness, such as those shown in Figure 3.11(a) and (b), the numerical spectra correspond well with that of the exper-

iments, particularly considering that the results shown relate to gauge measurements towards the end of the slope, where significant nonlinear effects and wave breaking take place. Furthermore, the consistency of the results is encouraging in light of the increasing dissipation of waves with increased steepness. Some deviations are observed in steeper seas, such as those in Figure 3.11(c) and (d), where the peak in the numerical spectrum is lower compared to that of the experiments. However, the numerical model does well to accurately capture the magnitude of the low frequency ($f/f_p \approx 0.2$) and the high frequency ($f/f_p \approx 1.75$) peaks. These are associated with wave-wave interactions which shift energy from the spectral peak to the sub- and super-harmonic region (Sorensen, 2006). The key distinction in the shape of the spectra numerically and experimentally is that high frequency peaks numerically are sharper, indicating interactions associated with a narrower range of frequency components. Conversely, the infragravity peak numerically is marginally broader compared to experimental measurements. That being said, overall, the shape of the numerical spectra is in line with that of the experiments for up to steep seas (sea-state B6) and even at spatial locations in the surf zone.

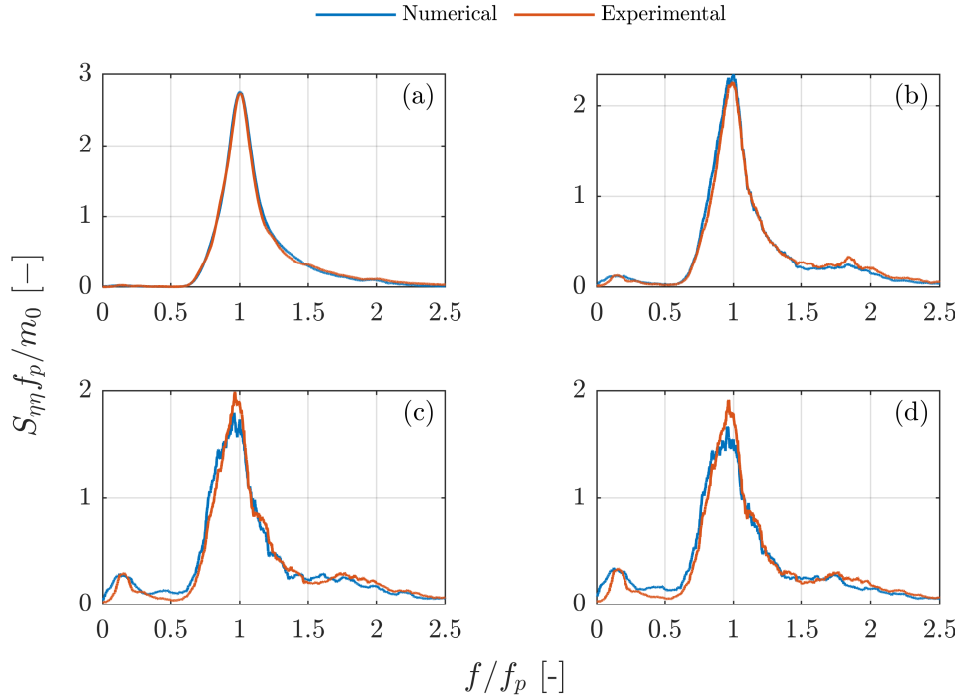


FIGURE 3.11: Normalised energy spectrum, $S_{\eta\eta}f_p/m_0$, for sea-states (a) B1, (b) B3, (c) B5 and (d) B6 for gauge 60 ($x = 16.6$ m) and $k_p d = 0.69$. Numerical and experimental measurements are shown in blue and red, respectively.

3.5.2 Statistical parameters

The energy spectrum may be used to derive met-ocean parameters characterising the sea-state. One such parameter is the significant wave height, H_s , which characterises the severity of the sea-state as defined by Equation 2.57. Other parameters of interest are the skewness, λ_3 , and (excess) kurtosis, λ_4 , of the surface elevation defined by Equation 3.10 and 3.11, respectively:

$$\lambda_3 = \frac{\overline{\eta(t)^3}}{\sigma_\eta^3}, \quad (3.10)$$

$$\lambda_4 = \frac{\overline{\eta(t)^4}}{\sigma_\eta^4} - 3, \quad (3.11)$$

where σ_η refers to the standard deviation of the surface elevation and the over bars reflect statistical averaging. The skewness defines the extent of the asymmetry of the surface elevation, observed in real seas as a result of second-order bound nonlinearities (Fedele & Tayfun, 2009). The excess kurtosis is a measure of the peakedness of the distribution. Fedele et al. (2016) explains that this encapsulates two effects: a dynamic component due to third-order quasi-resonant wave-wave interactions and a bound contribution induced by second- and third-order bound nonlinearities. Another important parameter to analyse is the mean water level. In the surf zone, Longuet-Higgins & Stewart (1964) outlines that the effect of wave shoaling as waves propagate on the slope results in an increase in wave radiation stresses which should manifest in a negative surface slope (set-down). Towards the end of the slope, within the surf zone, the reduction in wave radiation stresses following the onset of wave breaking should result in a positive surface slope referred to as a set-up.

The spatial evolution of each of these parameters obtained through numerical simulation will be examined subsequently and the accuracy of the numerical predictions will be established by comparison to equivalent experimental data. The spatial evolution of H_s as a function of $k_p d$ is shown in Figure 3.12 corresponding to a peak period of $T_p = 1.4$ s, with each line colour corresponding to a different offshore steepness, S_p . The solid line corresponds to experimental measurements whereas the dashed line corresponds to the results of the numerical simulation. It is observed that in the case of non-breaking waves, the significant wave height predicted numerically is larger than that observed in the experiments. This is attributed to a proportionally

greater dissipation of small waves experimentally by the side walls. In this context, non-breaking waves refer to the parts of Figure 3.12 with shallow gradients prior to the onset of wave breaking (for example $k_p d < 0.75$ for $H_{s,0} = 0.115$). Following the initiation of wave breaking, the reduction of the significant wave height is greater numerically compared to the experiments. This is observed by a steeper gradient of the curve in Figure 3.12. Although only sea-states corresponding to a single T_p case are shown, the aforementioned observations extend across all sea-states examined in Table 3.1.

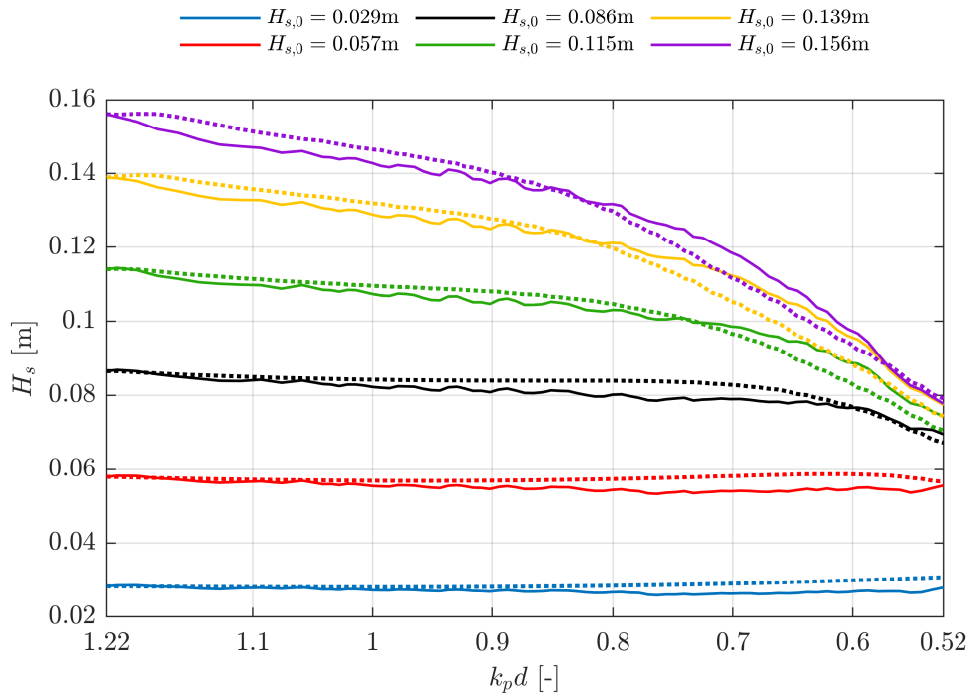


FIGURE 3.12: Spatial evolution of the significant wave height, H_s , for $T_p = 1.4$ s (sea-states B1-B6 in Table 3.1). Experimental and numerical measurements are shown with a solid and dashed line, respectively.

Figure 3.13 shows the spatial evolution of λ_3 and λ_4 in panels (a) and (b), respectively, corresponding to sea-states B1, B3 and B5. The spatial variation of the skewness of the numerically-simulated surface elevation is comparable to that of the experiments, indicating a good ability of SWASH to capture effects associated with second-order bound nonlinearities. The difference in skewness of the two data sets is only noticeable within the central portion of the slope, $0.6 \leq k_p d \leq 1$, where the numerically-simulated surface elevation is shown to have a larger skewness; this corresponds to the region in which H_s is also larger in SWASH (see Figure 3.12) and

therefore the waves are locally steeper. This variation is more evident for less steep waves. As such, it can be stated that for less steep waves which are fairly linear in nature, SWASH tends to overestimate the magnitude attributed by the second-order bound nonlinearities. This is a consequence of the earlier point made regarding a greater dissipation through sidewall friction of smaller waves experimentally. However, for more steep seas, such as $S_p = 0.05$ (black) in Figure 3.13(a), this contribution is estimated well. In steeper seas, the main discrepancy lies in shallower water, $k_p d < 0.7$, where numerically the skewness is under-predicted relative to experiments as a consequence of the earlier breaking numerically demonstrated in Figure 3.12.

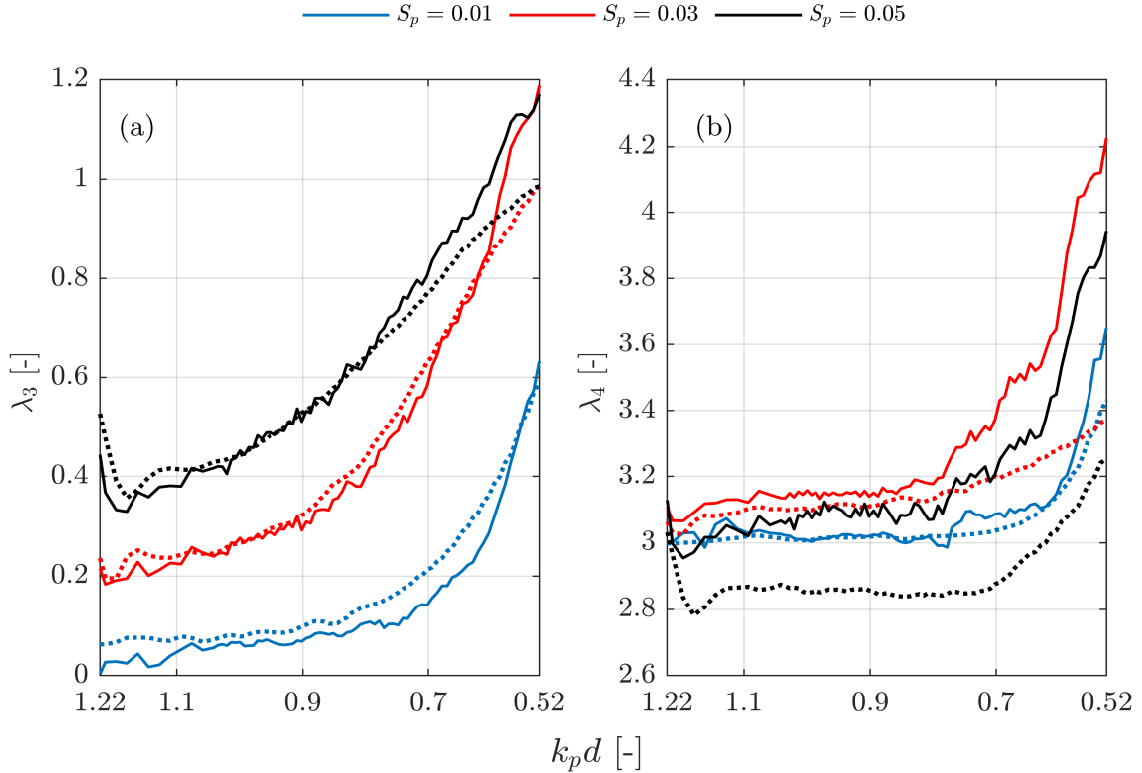


FIGURE 3.13: Spatial evolution of the (a) skewness, λ_3 , and (b) (excess) kurtosis, λ_4 , of the surface elevation for $T_p = 1.4$ s (sea-states B1, B3 and B5 in Table 3.1). Experimental and numerical measurements are shown with a solid and dashed line, respectively.

With regards to the excess kurtosis, it is observed that globally, the kurtosis of experimentally-simulated waves is higher compared to equivalent waves modelled numerically. This indicates that the magnitude of third-order effects, notably the quasi-resonant wave-wave interactions, and the second- and third-order bound contribution, is larger in the experiments. Although Figure 3.7 suggests that higher

order effects in regular waves are captured by SWASH, the lower kurtosis of the numerical simulations suggests that the full extent of higher order effects in random waves may not be fully captured. As expected, this discrepancy is most obvious for waves with a larger offshore significant wave height, and also waves located further up the slope which manifest as steeper waves as a result of shoaling effects.

The spatial evolution of the mean water level, MWL, is shown in Figure 3.14. Given the limited importance of wave breaking effects for near-linear seas, the numerical model is shown to accurately predict the mean water level for sea-states with steepness $S_p = 0.01$. However, discrepancies become evident for higher steepness waves. For sea-states with higher frequencies, $T_p = 1.2$ s, shown in Figure 3.14(a), the set-down predicted numerically is greater than that observed experimentally. Furthermore, the set-up observed numerically following breaking onset is greater than in the experiments, particularly for steep seas ($S_p < 0.05$). These observations are also evident for all other frequency cases, as shown in Figure 3.14(b). For these cases, the numerically-simulated mean water level is always higher than the observed mean water level in the experiments. Another important aspect to note in lower frequency wave cases ($T_p \geq 1.4$ s) is that the numerical model predicts a positive mean water level within the initial portion of the slope ($k_p d > 0.6$) for $S_p \geq 0.05$, where a set-down is expected and is observed in the experimental measurements. This is because for such steep sea-states, waves are observed to break at a short distance following their generation, both numerically and experimentally. Thus, the inaccuracies associated with numerically-predicting wave set-up become evident as early as the offshore location. At the end of the spatial domain, the set-up numerically can be as large as 3 times that observed in the experiments, albeit that this is in the case of very steep waves. An analysis into the water surface elevation in this region has also shown the presence of physically unrealistic spurious sub-harmonic waves resulting in group length scale mean water level changes. This finding is reinforced by Figure 3.11(d), showing that the energy associated with low frequency components ($f/f_p < 0.5$) is greater numerically compared to experimental measurements. Such discrepancies in the mean water level may also have implications for the magnitude of other wave parameters such as crest and wave height statistics, and should be considered in subsequent analysis.

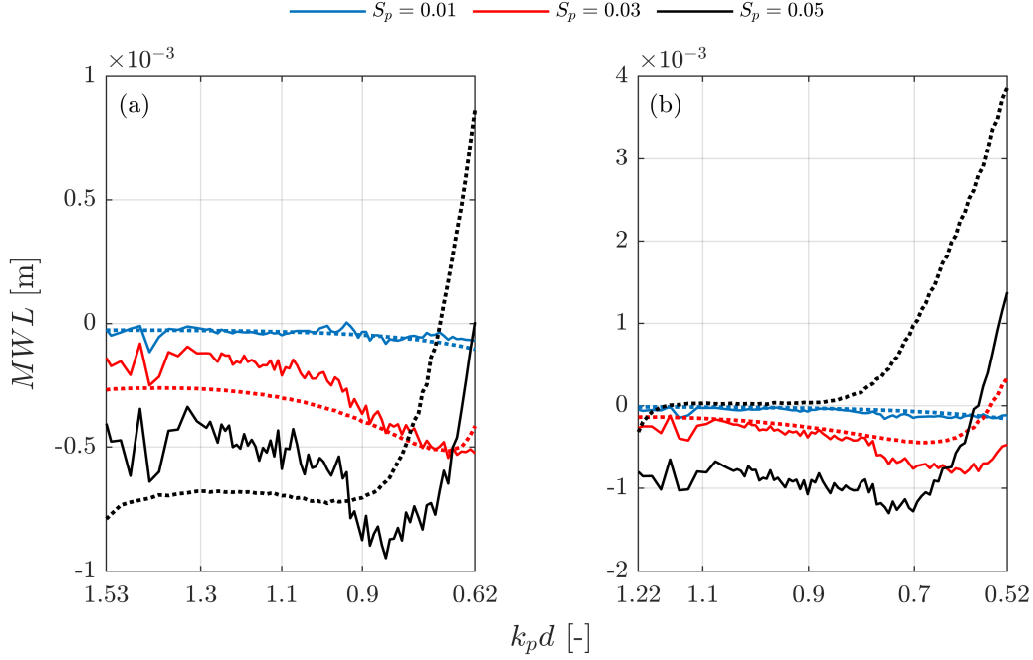


FIGURE 3.14: Spatial evolution of the mean water level, MWL, for (a) $T_p = 1.2$ s (sea-states A1, A3 and A5) and (b) $T_p = 1.4$ s (sea-states B1, B3 and B5). Experimental and numerical measurements are shown with a solid and dashed line, respectively.

3.5.3 Crest height statistics

This section will assess the accuracy of SWASH in predicting the evolution of crest heights across the spatial domain. A summary of these findings is shown in Figure 3.15, which demonstrates the variation of the crest height normalised by the significant wave height, η_c/H_s , with distance along the slope. The results shown correspond to $T_p = 1.4$ s; however, the findings that will be discussed subsequently are extended to all other frequency cases examined. Each subplot corresponds to a wave steepness case, S_p , with each line colour corresponding to a probability of exceedance. The numerical and experimental measurements are shown with a transparent and solid line, respectively. Note that the results are shown in effective water depth space, $k_p d$. Hence, the start of the x -axis corresponds to the start of the slope, not the start of the spatial domain. Within the flat bed region, nonlinear interactions and wave breaking may manifest resulting in discrepancies across the two datasets, even at the offshore boundary. Furthermore, it is important to state that the significant scatter in the lines associated with small exceedance probabilities is purely due to sampling variability in the tail of a distribution.

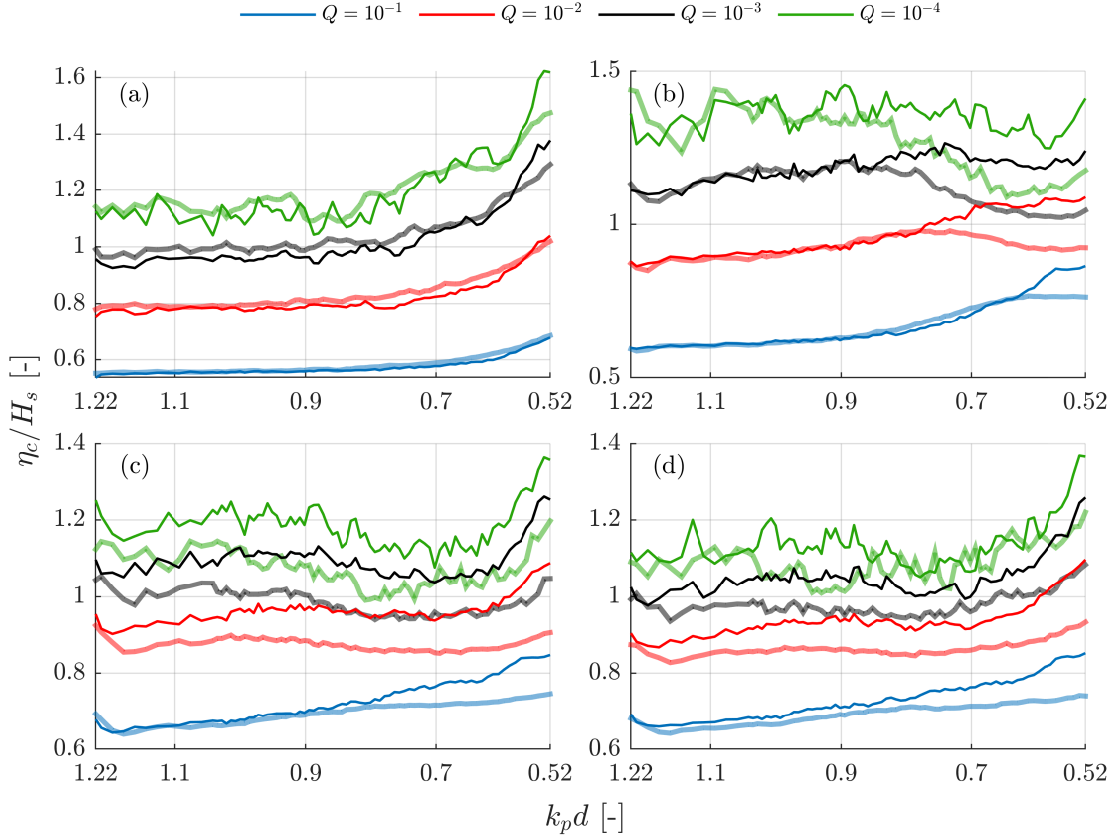


FIGURE 3.15: Spatial evolution of the normalised crest height, η_c/H_s , for sea-state (a) B1, (b) B3, (c) B5 and (d) B6. Experimental and numerical measurements are shown with a solid and transparent line, respectively. Each colour corresponds to a probability of exceedance given in the legend.

Considering Figure 3.15(a) corresponding to steepness $S_p = 0.01$, the numerical model does well to predict crest heights with small steepnesses even for small probabilities of exceedance, $Q = 10^{-4}$. Nevertheless, discrepancies become evident at moderate steepnesses such as those in Figure 3.15(b) corresponding to steepness $S_p = 0.03$. Although the normalised crest height evolution is well predicted within the initial deeper region of the slope prior to the onset of wave breaking, waves numerically break at an earlier spatial location compared to that observed in the experiments. This earlier breaking numerically is coupled with a stronger rate of reduction in crest height post-breaking across all probabilities of exceedance. The characteristics observed in Figure 3.15(b) following the onset of wave breaking continue to hold for larger steepness cases shown in Figure 3.15(c) corresponding to steepnesses $S_p = 0.05$ such that the numerical model underestimates crest heights across the spatial domain for most probabilities of exceedance. This reinforces the

uncertainties outlined earlier regarding the treatment of wave breaking in the numerical model which becomes more apparent with increased wave steepness. For the steepest seas such as that shown in Figure 3.15(d), the numerical model is shown to generally perform better compared to the results shown in Figure 3.15(c). Given the earlier findings shown in Figure 3.15(a)–(c), it would be expected that the discrepancies in the prediction of the normalised crest height would be more pronounced in Figure 3.15(d) as a result of the underestimation of higher order effects coupled with an overestimation of energy dissipation as a result of wave breaking. However, as discussed in Section 3.5.2, the mean water level estimated numerically is significantly higher than experimental measurements, particularly for the steepest sea-states $S_p \geq 0.05$, as a consequence of spurious sub-harmonic waves. As such, the influence of these wave components is also reflected in the crest height statistics for the steepest seas. In the context of Figure 3.15(d), the presence of these wave components seeks to artificially correct the underestimations caused by the lack of higher-order effects numerically and the overestimation of the energy dissipated following breaking onset.

In considering the normalised crest height corresponding to a probability of exceedance of $Q = 10^{-3}$ pertaining to the results shown in Figure 3.15 in black, the following comments can be made. Up to moderate steepness seas, $S_p \leq 0.03$, the numerical model estimates crest heights well within the initial region of the slope, $k_p d \geq 0.8$, with only minor discrepancies compared to the experimental measurements. The extent of these discrepancies are dependent on the relative steepness of the waves. For mild steepnesses, as shown by Figure 3.15(a) and (b), the numerical simulations marginally over predict the normalised crest height at a probability of exceedance of $Q = 10^{-3}$, indicating an accentuation of nonlinear effects in these cases, which are not observed experimentally. In steeper but non-breaking waves (within the initial deeper region of the slope), as shown by Figure 3.15(c), the opposite is observed. This indicates that for highly nonlinear waves, the full extent of the nonlinearity observed experimentally is not captured in the numerical model. This is consistent with the earlier findings shown in Figure 3.13. In such seas, the numerically predicted normalised crest height is smaller than that observed in the experiments. This underestimation is also observed for all sea-states where breaking is prevalent, as illustrated by trends observed in the end region of the slope associated

with small effective water depths. These observations reinforce the notion of stronger breaking observed numerically associated with a greater rate of energy dissipation compared to experimental waves. This effect, in conjunction with the inability to capture the full nonlinearity of very steep waves, gives rise to these underestimations observed.

The accuracy of SWASH in capturing experimental measurements may be summarised by determining the ratio of the numerical to experimental normalised crest height corresponding to a probability of exceedance of $Q = 10^{-3}$, R_2 , as defined by

$$R_2 = \frac{\eta_{c_{NUM}}^{Q=10^{-3}} / H_{s_{NUM}}}{\eta_{c_{LAB}}^{Q=10^{-3}} / H_{s_{LAB}}}. \quad (3.12)$$

The contours of R_2 measured at all gauges for all sea-states, is plotted in effective water depth, $k_p d$, and wave steepness, H_s/d , space in Figure 3.16(a). In general, for up to waves of moderate steepness with $H_s/d \leq 0.3$, the numerical model does well at capturing crest heights observed in the experiments within a 2% margin. For the waves of small steepness, $H_s/d \leq 0.25$, the numerical model tends to overestimate crest heights for $k_p d \leq 1.25$. An example where this is the case is labelled ‘P1’, corresponding to measurements at the second gauge ($x = 2.0$ m) in $k_p d = 1.21$ for $T_p = 1.4$ s and $S_p = 0.03$. To better understand the cause of this trend, the normalised crest height distribution for ‘P1’ is shown in Figure 3.16(b) and is compared to two short-term crest height distributions. The first and more simplistic model is the Rayleigh distribution (Longuet-Higgins, 1952), shown in blue, which adopts a linear and narrow-banded formulation. The second is the uni-directional Forristall distribution (Forristall, 2000), shown in red, based on an empirical fit to second-order numerical simulations.

In comparing the experimental and numerical crest heights to those predicted by the aforementioned crest height distributions, the following observations are made. Both the numerical and experimental normalised crest heights are larger than that of the Forristall crest height distribution (Forristall, 2000). Given that this is a second-order crest height distribution, these observations indicate that both numerically and experimentally, effects beyond second-order are present (Karpadakis & Swan, 2022). This is to be expected given the relatively large steepness of the sea-state chosen ($S_p = 0.03$). However, the numerical crest heights are shown to be greater

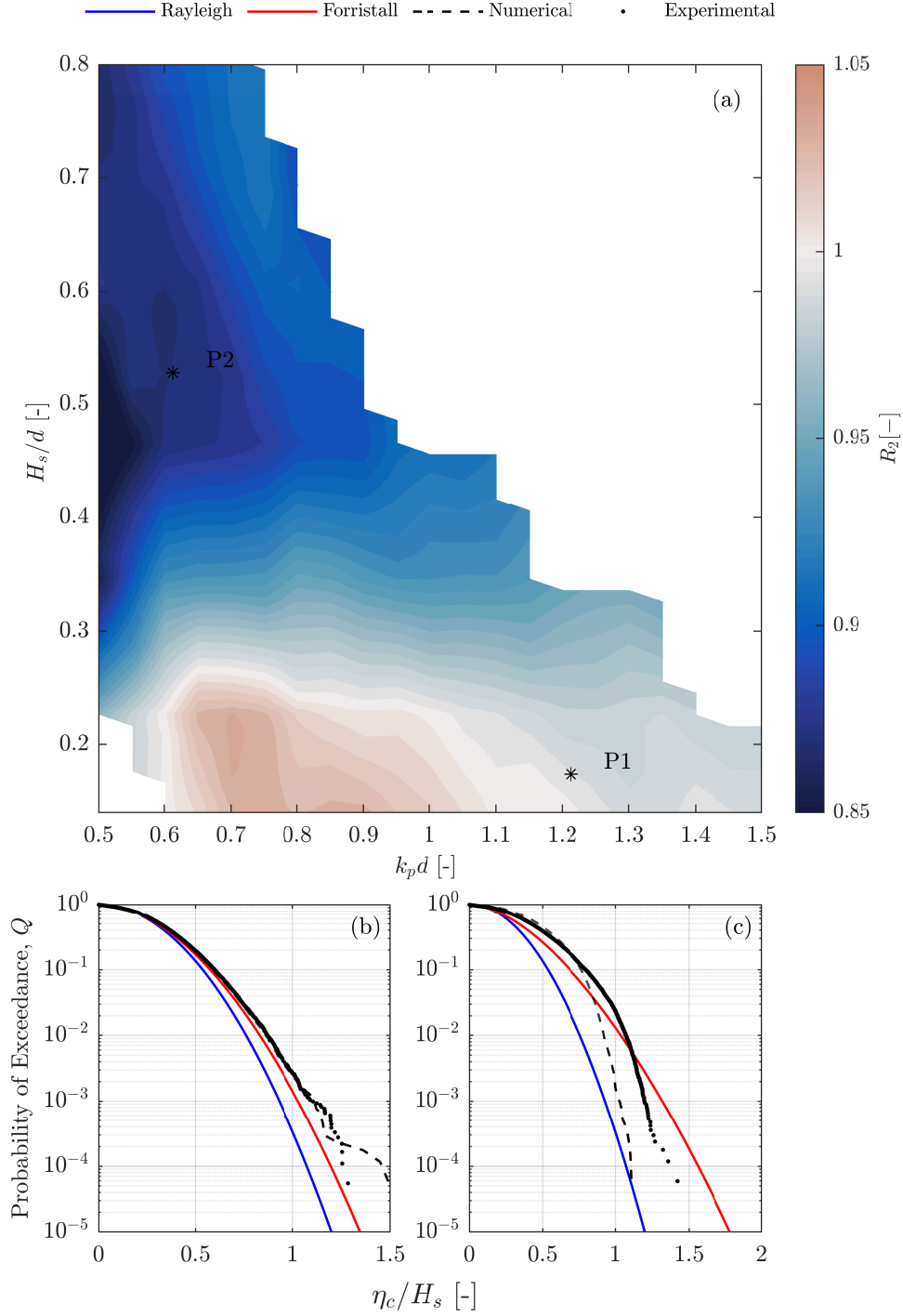


FIGURE 3.16: (a) Contour of the ratio of numerical to experimental crest heights, R_2 , corresponding to a probability of exceedance of $Q = 10^{-3}$. Normalised crest height distribution, η_c/H_s , corresponding to $T_p = 1.4$ s and $S_p = 0.03$ for (b) gauge 2 (labelled P1 in panel (a)) and (c) gauge 70 (labelled P2 in panel (a)). Blue line corresponds to the Rayleigh crest height distribution (Longuet-Higgins, 1952) and the red line corresponds to the Forristall crest height distribution (Forristall, 2000).

than that of the experiments, further validating the earlier point regarding SWASH's overestimation of nonlinear properties for small to moderate wave steepnesses. This is because smaller waves experience proportionally greater dissipation by sidewall friction experimentally compared to larger waves. As a consequence, the magnitude of numerically-measured waves are often comparatively larger, given that this effect does not occur numerically. Moreover, it should be noted that given the small magnitude of these waves, any minor discrepancy in the crest height will be proportionally magnified.

Referring back to Figure 3.16(a), for moderate to steep waves ($H_s/d \geq 0.3$), underestimations relative to experimental measurements are observed. In deeper water depths, $k_p d > 1$, the magnitude of these underestimations is marginal. This can be explained by the fact that nonlinear effects are less dominant in deeper waters (Karmadakis et al., 2019a,b). In shallower water, $k_p d < 1$, effects relating to wave breaking become significant. As previously highlighted, SWASH tends to predict an earlier and stronger breaking behaviour. As such, notable underestimations by the numerical model are observed. An example of where this is the case is illustrated by 'P2' in Figure 3.16(c), which shows the normalised crest height distribution corresponding to the same sea-state as that of 'P1' but measured at gauge 70 ($x = 17.6$ m) in $k_p d = 0.61$. Once again, the Rayleigh and Forristall crest height distributions are shown in blue and red, respectively. At this location, the influence of wave breaking is evident both numerically and experimentally shown by a strong reduction in the tail of normalised crest heights distribution. This figure also highlights the comparative extent of the breaking influence on the normalised crest height in the two data sets. Although the numerical crest heights are larger at the start of the spatial domain shown in Figure 3.16(b), they are smaller in Figure 3.16(c). This reinforces the previous comment regarding the 'stronger' influence of breaking in the numerical model such that crest heights reduce at a faster rate following breaking initiation compared to the experiments. Alluding back to Figure 3.16(a), the underestimation of crest heights corresponding to a probability of exceedance of $Q = 10^{-3}$ located within this region can be as notable as 15% of the experimental measured crest height.

3.5.4 Wave height statistics

This section will explore the evolution of wave heights in SWASH with a comparison to experimentally measured wave heights to establish accuracy. As outlined in the analysis of crest height statistics, there are two major aspects of interest. The first is the ability of SWASH to capture the full extent of the nonlinear behaviour of waves observed experimentally for very steep, non-breaking waves. Secondly, is the ability of SWASH to accurately model the breaking process, both in terms of accurately predicting the breaking location but also correctly dissipating energy following breaking onset (Karpadakis et al., 2020, 2022). These effects are prominent within the central region of the slope where it is observed experimentally that less steep waves have increased in nonlinearity given the effects of shoaling as they propagate up the slope. Similarly, for steeper sea-states, the effects of breaking become evident in this region. As such, the exceedance probability of normalised wave heights, H/H_s , is shown in Figure 3.17 at gauge 40 ($x = 12.6$ m) in $k_p d = 0.84$ for $T_p = 1.4$ s, with each subplot corresponding to a different sea-state steepness case, S_p . Numerical and experimental wave heights are shown in dashed and solid markers, respectively. In addition, the short-term wave height distribution corresponding to the Rayleigh wave height distribution (Longuet-Higgins, 1952) and Forristall wave height distribution (Forristall, 1978) are shown in blue and red, respectively. The Forristall wave height distribution is based on a two-parameter Weibull distribution of field measurements of storms in the Gulf of Mexico. As such, it captures to some extent the effects of nonlinearity and bandwidth which have been neglected in the Rayleigh wave height distribution, which assumes a linear, narrow-banded sea-state. However, given that it is based on an empirical fit, it corresponds well to sea-states with similar characteristics as that used in the model development, but may be unable to capture the full extent of real seas that are beyond this scope.

In mild seas, as shown in Figure 3.17(a), the numerical and experimental measurements correspond well, with the extent of the nonlinearity present in these cases being well-modelled by the semi-empirical Forristall model. The validity of the SWASH model in predicting wave heights continues to hold for steeper wave cases as shown in Figure 3.17(b). However, in very steep seas such as those shown by Figure 3.17(c) and (d), the wave height distribution is significantly influenced by the effects of wave breaking and hence, diverges from the predictions of the Rayleigh and Forristall

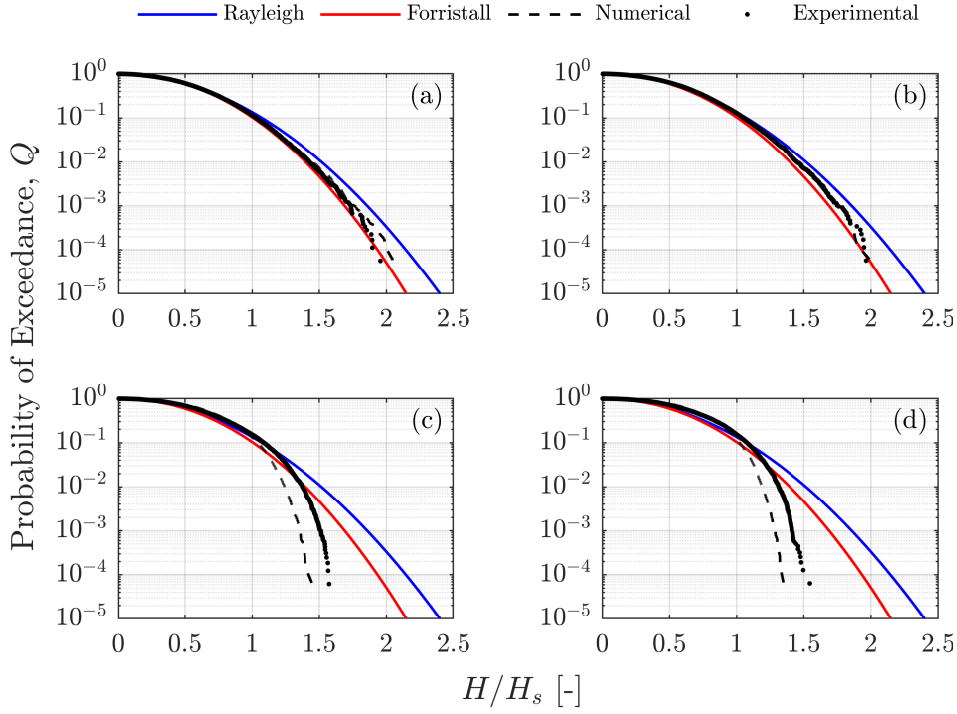


FIGURE 3.17: Normalised wave height distribution, H/H_s , corresponding to gauge 40 ($x = 12.6$ m) and $k_p d = 0.84$ for sea-states (a) B1, (b) B3, (c) B5 and (d) B6. Blue and red lines correspond to the Rayleigh wave height distribution (Longuet-Higgins, 1952) and Forristall wave height distribution (Forristall, 1978), respectively.

wave height models, which are both developed with deep-water conditions in mind. In comparing the results of the numerical to experimental wave heights, the overestimation of wave breaking in SWASH continues to be evident when considering wave height statistics. This results in an underestimation of wave heights, particularly for very steep waves.

A comparison of the wave height statistics in SWASH and the laboratory may be summarised by Figure 3.18, which demonstrates the variation of the wave height normalised by the significant wave height, H/H_s , with $k_p d$ for $T_p = 1.4$ s. Similar to Figure 3.15, each subplot corresponds to a wave steepness case, S_p , with each line colour corresponding to a probability of exceedance. Once again, the numerical and experimental measurements are shown with a transparent and solid line, respectively. From this plot, there are three notable points to be made. Firstly, in the case of less steep waves as shown in Figure 3.18(a), the numerical model accurately captures wave heights across the full range of effective water depths investigated experimentally ($0.52 \leq k_p d \leq 1.22$), across all probabilities of exceedance. The second point to

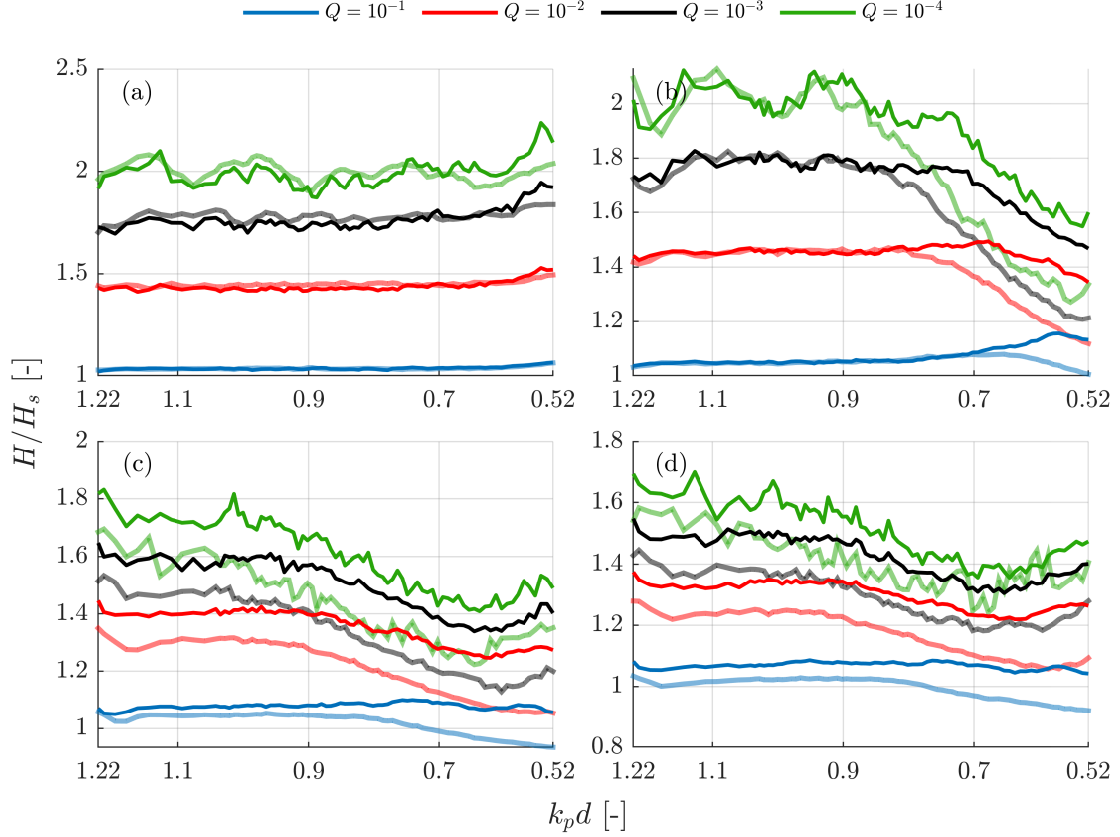


FIGURE 3.18: Spatial evolution of the normalised wave height, H/H_s , for sea-states (a) B1, (b) B3, (c) B5 and (d) B6. Experimental and numerical measurements are shown with a solid and transparent line, respectively. Each colour corresponds to a probability of exceedance given in the legend.

observe relates to sea-states of moderate steepness, as illustrated in Figure 3.18(b). The numerically predicted wave heights compare well to experimental measurements for $k_p d \geq 0.75$, reinforcing the ability of the numerical model to capture higher order nonlinear effects occurring in sea-states of moderate steepness. This is true across all probabilities of exceedance.

The final point to make relates to the numerical modelling of wave heights in regions dominated by breaking, such as the final region of the slope ($k_p d > 0.7$) in Figure 3.18(b) and in very steep seas which are dominated by wave breaking, as shown in Figure 3.18(c) and (d). In such cases, across the spatial domain, the more intense numerical breaking results in a global underestimation of wave heights across the spatial domain and across all probabilities of exceedance. As expected, these deviations become more notable for smaller exceedance probabilities and can be as significant as 18% of experimental measured wave heights.

3.6 Conclusion

The performance of the shallow water numerical model, SWASH, is assessed through conducting a series of numerical simulations of both regular and random, fully nonlinear waves propagating from intermediate water to the shoreline over a mild bathymetry with a 1:50 slope. The accuracy of the model is benchmarked through comparisons to equivalent sea-states generated experimentally, with a focus on the crest height and wave height statistics.

The numerical modelling of regular waves has demonstrated the ability of SWASH to accurately model the mean crest height of regular waves for low and moderate steepnesses ($ak \leq 0.25$) propagating from deep to shallow water within 5% of experimental measurements. In steeper test cases ($ak > 0.25$), in deep to intermediate water depths ($kd \geq 0.9$), underestimations of up to 20% in the mean crest height are observed numerically. This is a consequence of an inaccurate treatment of breaking waves or a lack of higher order nonlinear effects numerically which however, are observed experimentally. In shallower water, the limitations in the numerical breaking process are more notable with differences of up to 38% of experimental mean crest heights.

With regards to the modelling of random sea-states, the findings of the present study have highlighted the efficiency and accuracy of SWASH to capture the properties of waves propagating on mild slopes up to moderate steepness $H_s/d < 0.3$ within 2% of experimental observations. This includes an accurate description of normalised crest height and wave height evolution. Conversely, in steeper seas, the full extent of the nonlinearity observed experimentally is not captured numerically. Further limitations are observed in the model's ability to accurately capture the breaking process. Numerically, the extent of energy dissipation following wave breaking is larger than that of the experiments, resulting in underestimations of up to 15% and 18% of the experimental measured normalised crest height, η_c/H_s , and wave height, H/H_s , respectively. Overall, the extent of these discrepancies is more pronounced in extreme waves (small probabilities of exceedance) occurring in seas of greater steepness, located in shallower effective water depth.

Thus, it may be concluded that SWASH is able to act as a high fidelity phase-resolving model that is able to accurately capture effects associated with wave propagation in mild bathymetries in intermediate and shallow waters. However, care

should be taken for extreme wave cases, where breaking numerically is incompatible with experimental measurements. This validated numerical framework forms the basis for the subsequent analyses in this thesis.

Chapter 4

Effect of bed slope angle on wave fields

Author’s Note: The contents of this chapter are under review in Al Khalili, Christou, & Karpadakis (2026b). The effect of bed slope angle on the statistics and evolution of wave fields over coastal bathymetry. *Journal of Fluid Mechanics*.

4.1 Introduction

The preceding chapters have established the motivation for understanding nearshore wave transformation (Chapter 1), outlined the theoretical framework governing coastal wave processes (Chapter 2), and validated the numerical modelling approach used throughout this thesis (Chapter 3). The present chapter utilises this framework to examine the evolution of irregular waves as they propagate over variable sloping bathymetry, with a particular focus on how the bed slope influences the properties of wave surface elevations and kinematics. This represents a key step toward understanding the physical mechanisms that govern coastal wave fields.

Ocean waves propagating from deep to shallow water undergo pronounced nonlinear transformation, producing highly complex wave fields in coastal regions (Lamb, 1932). This evolution reflects the combined influence of wave shoaling, nonlinear energy transfers, and wave breaking, which together control the statistics, energy distribution, and shape of nearshore waves. Understanding how these three aspects evolve is fundamental for predicting coastal hydrodynamics, wave loading, sediment transport, and extreme events.

The evolution of coastal wave fields is governed primarily by triad wave-wave interactions, in which three wave components exchange energy when resonance conditions are satisfied (Phillips, 1960, Hasselmann, 1962, Longuet-Higgins & Phillips, 1962, Hasselmann et al., 1976). In deep water, these interactions are generally non-resonant and lead to negligible net energy transfer. As water depth decreases, however, resonance conditions are progressively fulfilled, enabling efficient redistribution of energy across the spectrum. Energy is transferred both to higher (super-harmonic) and lower (sub-harmonic) frequencies, fundamentally altering the spectral composition of the wave field. Transfers to the super-harmonic region drive the progressive distortion of wave shape from the nearly symmetric profiles of deep water to the strongly skewed forms observed in shallow water, with sharper crests and flatter troughs (Guza & Thornton, 1982, Elgar & Guza, 1985). The evolution of wave asymmetry is governed by the phase relationships between the primary component and its higher harmonics (Elgar & Guza, 1986, Doering & Bowen, 1986, 1995). While higher-order regular wave theories reproduce vertical asymmetry through bound harmonics (Stokes, 1847), horizontal asymmetry and forward pitching arise from progressive phase shifts between harmonics and the carrier frequency during shoaling.

Conversely, energy transfer to the sub-harmonic region produces long-period infragravity motions. The generation of these waves is largely attributed to two mechanisms: the shoaling of bound long waves and the variable breakpoint effect. The former arises from the growing phase lag between the incident long wave and the envelope of the short-wave group as the waves shoal, leading to energy transfer from the primary components to the bound long waves (List, 1992, Van Dongeren & Svendsen, 1997, Janssen et al., 2003, Battjes, 2004). The latter mechanism, the variable breakpoint, occurs due to the oscillatory motion of the breaking point in response to the wave group envelope, producing low-frequency oscillations in the mean water level, commonly referred to as surf beat (Baldock, 2012). Both mechanisms contribute to the amplification of infragravity energy in the nearshore.

In the surf zone, wave breaking becomes the dominant process dissipating short-wave energy (Battjes & Janssen, 1978), causing a rapid decay in wave heights across the breaking region (Ting & Kirby, 1994, 1995). In the absence of significant wind forcing, the equilibrium between the energy input from shoaling and the dissipation through breaking was classically described by Thornton & Guza (1983, 1986).

Breaking also redistributes energy within the short-wave spectrum, often flattening the high-frequency tail and altering the spectral balance.

Spectral evolution in the coastal zone has frequently been interpreted through the concept of self-similarity, where the transformation of the spectrum is governed by a small number of characteristic length scales. Building upon the local equilibrium formulation of Phillips (1958), Kitaigorodskii et al. (1975) established a theoretical framework for wind-wave growth based on energy balance principles. This underpins empirical spectral models such as the JONSWAP spectrum (Hasselmann et al., 1973), which introduces a peak enhancement factor to describe fetch-limited conditions. Later, extensions to finite water depths were made through the TMA spectrum (Bouws et al., 1985), incorporating depth-dependent modifications. Although these formulations account for water depth, they do not explicitly include the influence of bed slope, a factor known to affect nonlinear energy transfers and wave transformation in coastal environments.

In the context of surface elevation statistics, early studies demonstrated that waves propagating over sloping beds deviate strongly from Gaussian behaviour (Bitner, 1980), with steeper and more nonlinear sea-states exhibiting enhanced skewness, kurtosis, and extreme-event probability (Klahn et al., 2021a,b). Herbers et al. (1994), Norheim et al. (1998) and De Bakker et al. (2015) attributed this behaviour to stronger nonlinear interactions in more energetic sea-states. The skewness reflects wave asymmetry, which is critical for understanding breaking onset and loading on coastal infrastructure (Janssen, 2004), while the kurtosis is associated with the probability of extreme or rogue wave events. The pioneering field observations of Elgar & Guza (1985) revealed increasing skewness and kurtosis with decreasing water depth, followed by a reduction in very shallow regions due to breaking dissipation. Subsequent experiments (Trulsen et al., 2012, Kashima et al., 2014, Bolles et al., 2019, Moore et al., 2020) and numerical studies (Majda et al., 2019, Zhang et al., 2019) confirmed the existence of local maxima in both skewness and kurtosis under intermediate-depth conditions.

Although the statistics of surface elevation have been widely studied (Cherneva et al., 2005, Tayfun, 2006, Toffoli et al., 2007, 2009, Cherneva et al., 2009, Latheef & Swan, 2013, Gibson et al., 2014, Karpadakis et al., 2019a,b), comparatively little attention has been given to wave kinematics. Recent investigations (Trulsen et al.,

2020, Lawrence et al., 2022, Zhang et al., 2024) have shown that velocity skewness and kurtosis may differ markedly from those of surface elevation, reflecting second-order set-down effects and sub-harmonic modulations (Alberello et al., 2016). These findings highlight that the magnitude and extremes of near-surface velocities are governed by nonlinear processes that are sensitive to bathymetric transformation, and importantly may be different to those associated with surface elevation quantities. Nevertheless, a unified description linking bed slope effects on wave statistics, wave energy, and wave shape, particularly for wave kinematics, is still lacking.

The present study addresses this gap using an extensive numerical dataset of irregular waves propagating over planar bed slopes, as outlined in Section 4.2. The objective is to provide a systematic description of how the bed slope gradient governs the evolution of wave statistics, wave energy, and wave shape in the nearshore zone. Wave statistics are characterised through a zero-crossing analysis of surface elevation and kinematic parameters (Section 4.3), wave energy and dissipation processes are examined using spectral and bispectral measures (Section 4.4), and wave shape and asymmetry are quantified through phase-coupling diagnostics and higher-order statistical moments (Section 4.5).

4.2 Numerical set-up

Following the numerical framework described in Chapter 3, the dataset is extended to consider five uniform slope configurations are investigated: 1:10, 1:15, 1:30, 1:50 and 1:80. The numerical configuration is illustrated in Figure 4.1. The bathymetry consists of a flat offshore region of 200 m length followed by a planar sloping bed extending to the shoreline. The boundary conditions, mesh sizing and numerical parameters all follow the configuration described in Chapter 3.

Simulations are carried out under a 1:100 Froude-scaled model, and all results presented henceforth are converted to field scale for interpretation. To capture the full evolution of wave transformation, data are sampled at 80 spatial locations ($G1$ – $G80$), comprising five offshore gauges ($G1$ – $G5$) and 75 gauges ($G6$ – $G80$) distributed uniformly between $x = 200$ m and the shoreline (see Figure 4.1).

The sea-states described by Table 3.1 are all modelled for each of the five bed slopes. However, for clarity, the results of this chapter will focus on Group B of

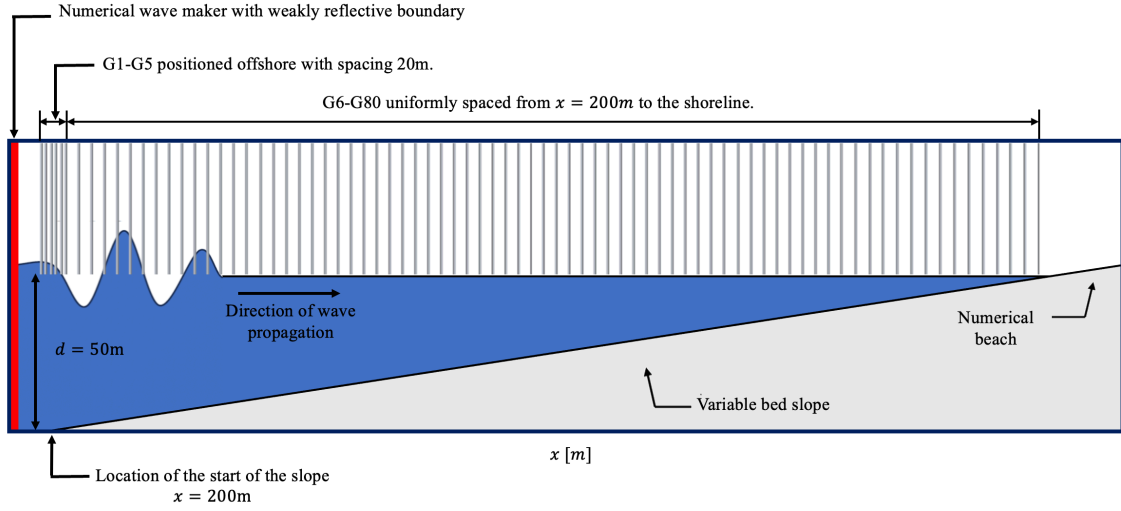


FIGURE 4.1: Schematic of the numerical set-up, where x is defined as a function of the bed slope.

the sea-states, sharing a common spectral peak period, T_p , but different offshore steepness, S_p . The selected range spans from near-linear to strongly nonlinear (storm-like) conditions.

4.3 Surface elevation and velocity statistics

This section examines how variations in bed slope influence the statistics of surface elevations and associated kinematic parameters during wave shoaling and breaking. The analysis is conducted through zero-crossing statistics, which provide a framework for characterising both the dynamical and statistical properties of irregular waves as they evolve across variable bathymetry. By tracking the spatial evolution of wave heights, crest elevations, and crest velocities, zero-crossing analysis enables direct quantification of exceedance probabilities and, in turn, the likelihood and intensity of extreme events. The focus of this section is therefore twofold: first, to describe how wave statistics evolve during shoaling and breaking for different sea-state steepnesses, and second, to establish how this evolution is modified by changes in the bed slope angle.

Accordingly, a zero-crossing analysis is applied independently to the time series of the surface elevation, η , horizontal velocity, u , and vertical velocity, w . Four parameters are defined: the wave height H , and the crest magnitudes η_c , u_c , and w_c in surface elevation, horizontal velocity, and vertical velocity, respectively. The

present method ensures that the local maxima in the kinematic quantities are properly identified, which is of particular interest in practical applications.

4.3.1 Distribution of waves

Figure 4.2 presents the probabilities of exceedance for (a) wave height H , (b) crest height η_c , (c) horizontal crest velocity u_c , and (d) vertical crest velocity w_c at different effective water depths. For wave heights (Figure 4.2(a)), reducing the effective water depth from $k_p d = 1.2$ to $k_p d = 0.9$ results in a largely unchanged distribution, with amplification confined to the tail ($Q < 10^{-2}$) due to wave shoaling. In contrast, the crest height distribution (Figure 4.2(b)) is more sensitive to wave shoaling, exhibiting increased exceedance probabilities across the entire distribution as the effective water depth is reduced from $k_p d = 1.2$ to $k_p d = 0.9$. This effect becomes more pronounced for horizontal crest velocities (Figure 4.2(c)). The distribution of vertical crest velocities (Figure 4.2(d)) is comparatively less affected by wave shoaling, with velocity amplification largely confined to the tail of the distribution.

A further reduction in effective water depth from $k_p d = 0.9$ to $k_p d = 0.6$ highlights the influence of wave breaking. For wave heights (Figure 4.2(a)), clear reductions are observed across the entire distribution. Similar behaviour is evident for crest elevations (Figure 4.2(b)), horizontal (Figure 4.2(c)) and vertical crest velocities (Figure 4.2(d)). These distributions exhibit characteristic features of breaking wave fields, namely a pronounced reduction in the tail combined with a hump at larger probabilities of exceedance. The latter is associated with the redistribution of broken waves from the tail towards higher exceedance probabilities. This effect has previously been identified in surface elevation statistics by Karmiadakis & Swan (2020) and Al Khalili & Karmiadakis (2025), and more recently in wave kinematics by Al Khalili et al. (2026c). It is noted that the influence of wave breaking on zero-crossing velocity parameters is more pronounced than for zero-crossing surface elevation parameters.

Figure 4.3 extends the findings of Figure 4.2 to show the full extent of the effective water depth, $k_p d$, domain by employing contours. The columns (from left to right) correspond to a mild, moderate and extreme sea-state, respectively. A dashed line is added to denote the onset of breaking saturation, defined as the point of maximum curvature in the distribution of wave heights. For the wave height H , the magnitude

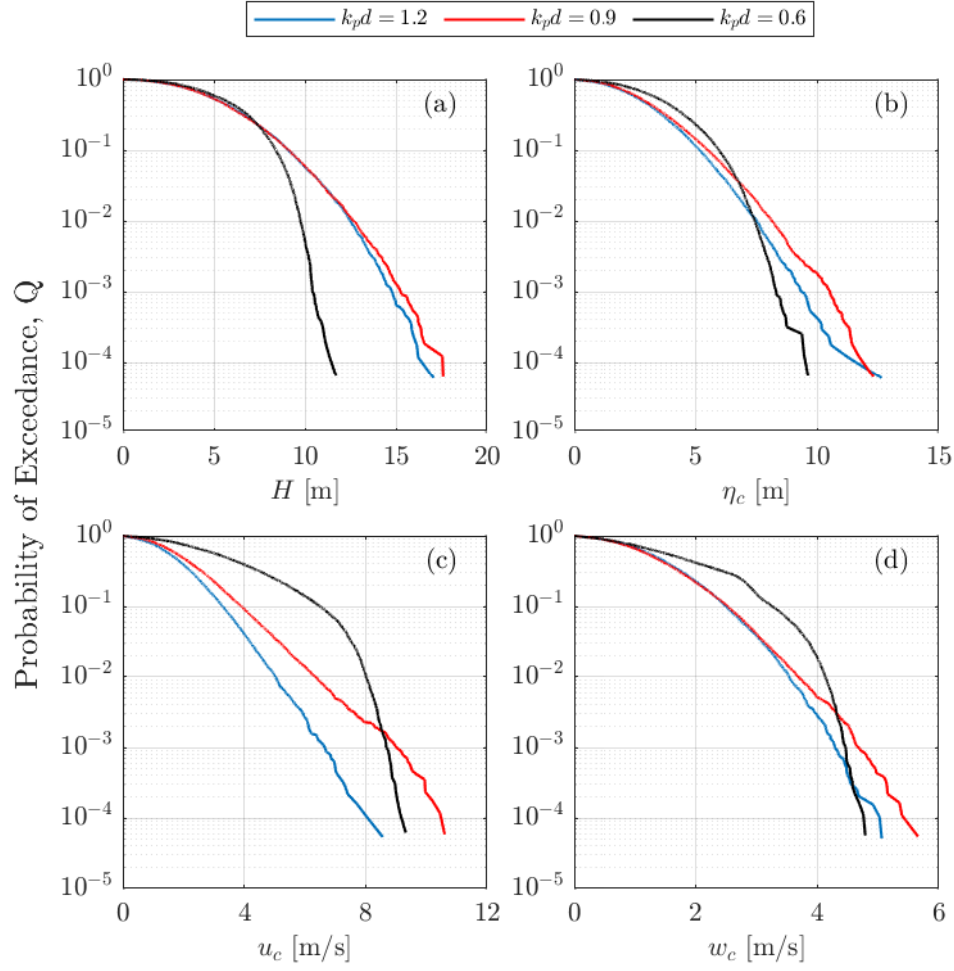


FIGURE 4.2: Exceedance probabilities for (a) H , (b) η_c , (c) u_c , and (d) w_c for sea-state B3 propagating over bed slope 1:30. The colour of the line corresponds to the local effective water depth, $k_p d$.

remains approximately constant for a given exceedance probability during shoaling, with only mild amplification observed in low-steepness sea-states (Figure 4.3(a)). In general, the competing effects of crest steepening and trough shallowing maintain an approximately stable wave height until a rapid reduction occurs upon breaking. As the sea-state steepness increases, breaking occurs progressively earlier for a given exceedance probability, as indicated by the leftward shift of the breaking saturation line. Moreover, breaking saturation is initiated in deeper effective water depths for larger waves associated with smaller exceedance probabilities, which explains the inclined shape of the breaking line. This inclination is more pronounced for steeper sea-states (Figure 4.3(c)) than in milder ones (Figure 4.3(a)), indicating a wider cross-shore region over which breaking occurs for steeper incident seas.

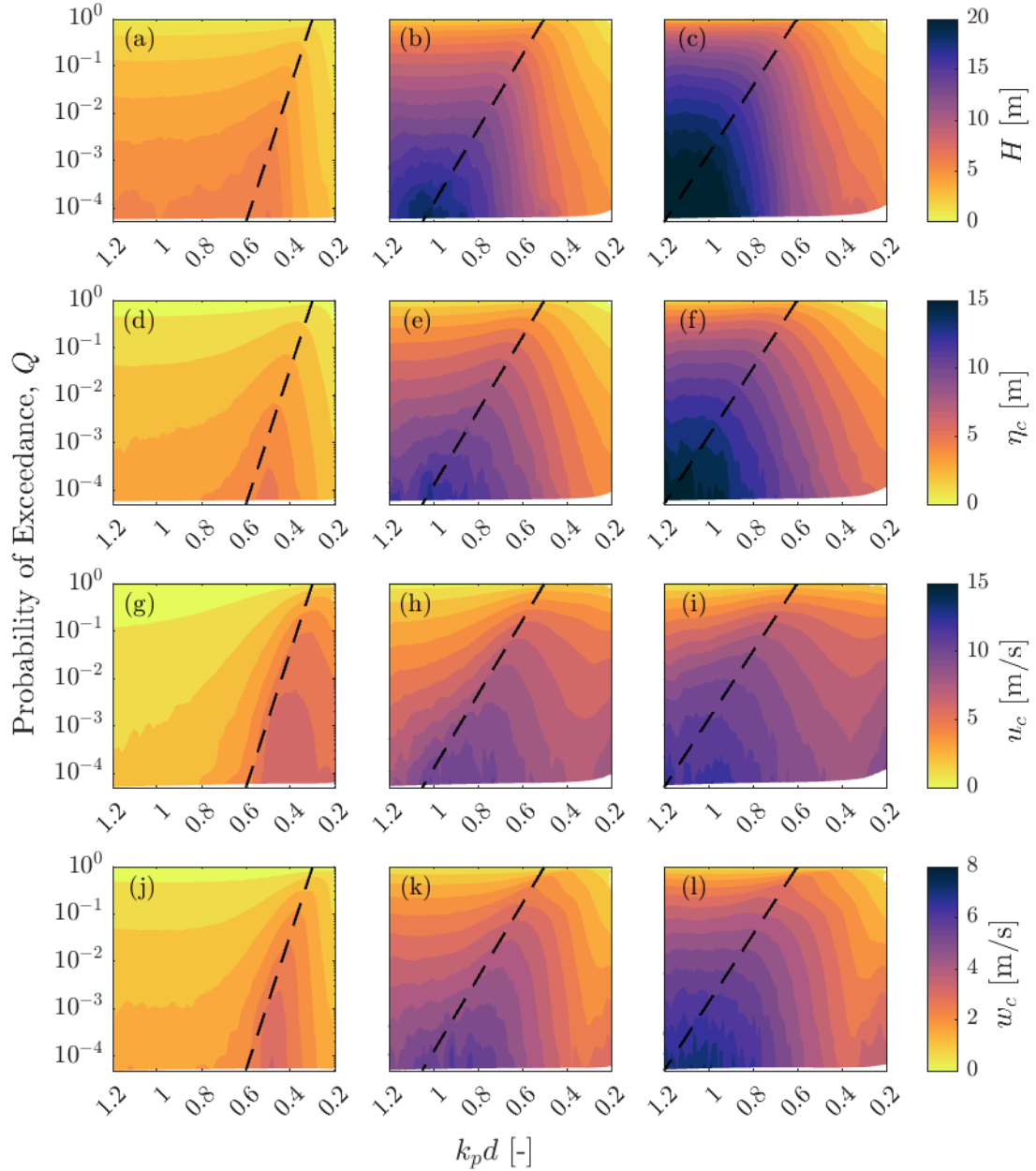


FIGURE 4.3: Spatial evolution of exceedance probabilities for (a–c) H , (d–f) η_c , (g–i) u_c , and (j–l) w_c over bed slope 1:30. Columns from left to right correspond to sea-states B1, B3, and B5, respectively. The black dashed line indicates the breaking saturation limit.

The evolution of crest elevation η_c (Figures 4.3(d)–(f)) follows a broadly similar trend, although crest heights exhibit stronger amplification during shoaling due to their greater sensitivity to nonlinear effects. Comparable behaviour is observed for the horizontal crest velocity u_c . In addition, increases in u_c are detected in very shallow water following breaking saturation for moderate (Figure 4.3(h)) and steep

(Figure 4.3(i)) sea-states. Svendsen (1984) describes the development of a strong shoreward-directed pressure gradient near the free surface induced by radiation stress gradients associated with wave set-up in the surf zone. This near-surface pressure-driven flow is balanced by a compensating seaward-directed return flow (undertow) in the lower part of the water column. Within the inner surf zone, the collapse of the breaking crest and the subsequent bore-like propagation generates strong, depth-varying horizontal velocities concentrated near the surface. Superimposed on the background pressure-driven circulation, these bore-induced motions can locally enhance the phase-averaged shoreward velocities, leading to an increase in u_c before the flow ultimately decays towards the shoreline.

Vertical crest velocities w_c (Figures 4.3(j)–(l)) exhibit trends similar to those of u_c during shoaling, but with more localised amplification near the breaking location. Following breaking onset, w_c decreases sharply, reflecting the elongation of particle trajectories in shallower water. Physically, the presence of the seabed constrains vertical particle motion and forces the water column to oscillate more uniformly, so that the restoring force becomes dominated by gravity acting on the free surface rather than by dispersive effects. This promotes coherent propagation and facilitates the development of nonlinear features such as wave asymmetry and steepening, which are commonly observed in shallow coastal regions (Svendsen, 2006).

It is further noted that the maxima in the velocity parameters (u_c and w_c) occur slightly shoreward of the breaking saturation line inferred from surface elevation statistics (H and η_c). Similar spatial lags between wave energy dissipation and momentum transfer have been reported by Lippmann (1995), Reniers & Battjes (1997), and Kuriyama & Nakatsukasa (2000). This behaviour reflects the finite adjustment length scale over which dissipated wave energy is converted into mean and oscillatory momentum, such that the maximum kinematic response may occur downstream of the location of maximum wave height decay.

4.3.2 Effects of bed slope on wave statistics

Having established the evolution of zero-crossing parameters as a function of sea-state steepness, the analysis now turns to the central objective of this section: quantifying the influence of bed slope on the magnitude of extreme waves and velocities. Figure 4.4 illustrates this effect by considering the 100 largest offshore waves. These

are taken as representative of design waves associated with the extreme tail of the distribution. Each of these waves are tracked inshore and their associated statistics are determined.

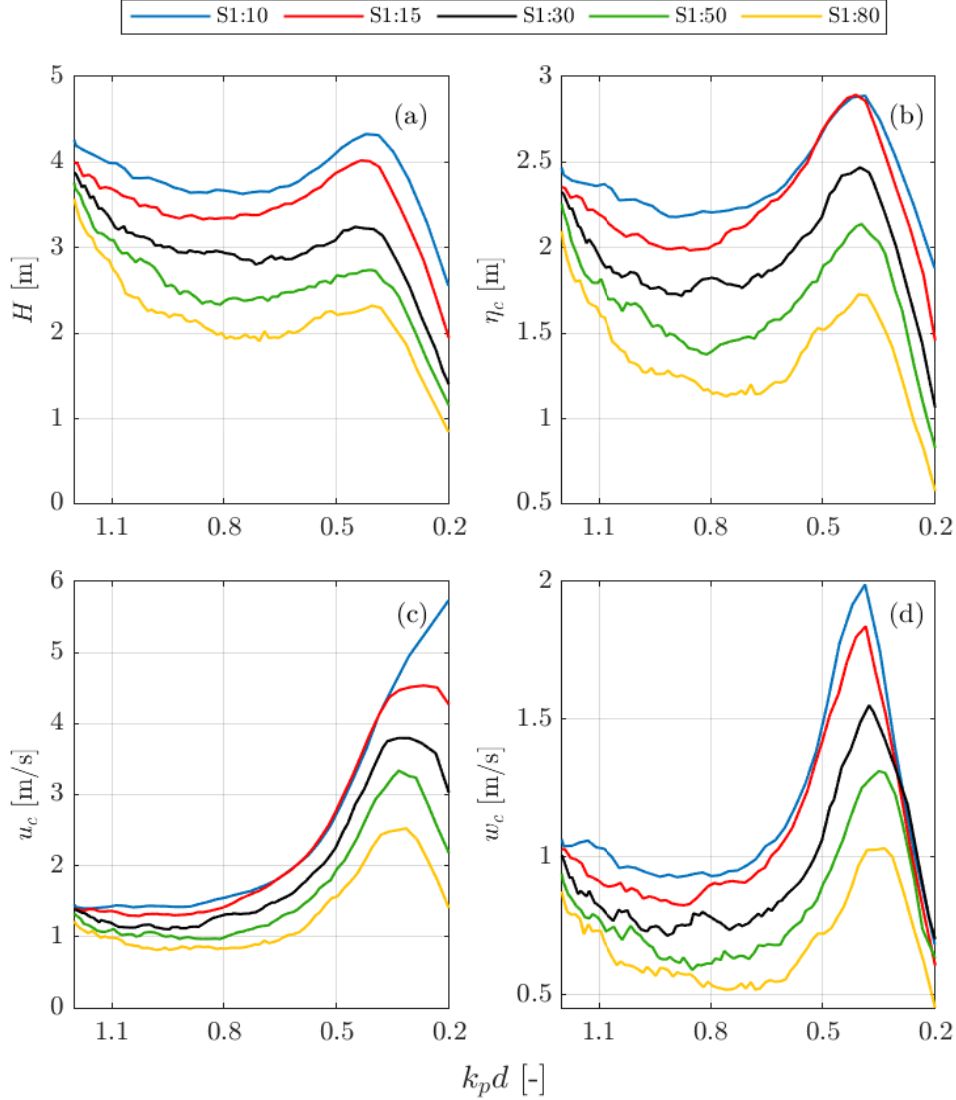


FIGURE 4.4: Evolution of (a) H , (b) η_c , (c) u_c , and (d) w_c for the 100 largest offshore waves (sea-state B1). Line colour indicates bed slope gradient.

The evolution of wave heights (Figure 4.4(a)) and crest heights (Figure 4.4(b)) show an initial reduction from the offshore region to $k_p d \approx 0.6$. This behaviour is associated with a decrease in the shoaling coefficient in intermediate water depths, resulting from local increases in group velocity (Dean & Dalrymple, 1991). In addition, since the largest offshore waves are considered, the onset of wave breaking in some waves further contributes to this reduction. Subsequently, an increase in

the shoaling coefficient leads to a growth in both wave and crest heights up to the breaking saturation point ($k_p d \approx 0.3$), beyond which further reductions occur due to enhanced wave energy dissipation.

Examining the wave height in Figure 4.4(a), larger maximum heights are observed over steeper bed slopes. This behaviour is in accordance with the breaker height formulation of Battjes (1974):

$$H_b = \gamma d, \quad (4.1)$$

where H_b is the breaker height, $\gamma = 1.16 N_I^{0.22}$, and $N_I = \tan \theta / \sqrt{H_0 / \lambda_0}$ is the Iribarren number. For identical offshore conditions ($H_{s,0} / \lambda_{p,0}$), steeper slopes (larger $\tan \theta$) therefore produce larger breaker heights. A similar dependence is observed for crest elevations (Figure 4.4(b)). In contrast, milder slopes are associated with wider surf zones and more progressive breaking, leading to greater cumulative reductions in η_c and smaller crest heights throughout the nearshore region. Previous studies (Janssen & Battjes, 2007, Mostert & Deike, 2020) have shown that the dissipation rate per unit length of breaking wave increases with bed slope, with plunging breakers dominating on steep slopes, whereas spilling breakers and more spatially distributed dissipation prevail on mild slopes (Ting & Kirby, 1995, 1996). Consequently, although the individual waves on steep slopes experience more intense, localised breaking, the total integrated dissipation is larger over mild slopes.

The evolution of the horizontal crest velocity u_c (Figure 4.4(c)) and vertical crest velocity w_c (Figure 4.4(d)) exhibits trends similar to those of H and η_c , attaining larger values on steeper slopes both during shoaling and after breaking onset. This behaviour is in line with the enhanced depth-induced nonlinearity and rapid wave compression associated with steep bathymetries, which amplify vertical accelerations in the crest region.

To interpret this mathematically, the one-dimensional Saint–Venant equations governing unsteady open-channel flow over variable topography are considered (De St Venant, 1871). Given that the flow is in relatively shallow water, assuming depth-uniform hydrostatic flow is deemed reasonable. The continuity and momentum equations are:

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} = 0, \quad (4.2)$$

$$\frac{\partial(hu)}{\partial t} + \frac{\partial}{\partial x} \left(hu^2 + \frac{1}{2} gh^2 \right) = gh(S_0 - S_f), \quad (4.3)$$

where $S_0 = -\partial z_b / \partial x$ is the bed slope and S_f is the friction slope. In the context of the present study, the friction slope is constant over different bathymetry as defined by the Manning coefficient. Thus, neglecting friction and unsteady terms for gradually varying shoaling flow, the momentum equation reduces to:

$$u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = g S_0. \quad (4.4)$$

This expression shows that the horizontal acceleration is directly proportional to the bed slope S_0 and the free-surface gradient $\partial h / \partial x$. For a steeper slope, both S_0 and the magnitude of $\partial h / \partial x$ increase, producing a stronger pressure-gradient force and gravitational driving term, and therefore a larger positive velocity gradient $\partial u / \partial x$.

From the continuity equation, the depth-integrated discharge per unit width $q = hu$ remains approximately conserved in the absence of strong dissipation. As the flow propagates upslope and the local depth decreases, the horizontal velocity must increase according to:

$$u = \frac{q}{h}. \quad (4.5)$$

Since a steeper slope induces a more rapid reduction in depth with distance, the associated increase in velocity is stronger than for a mild slope, leading to larger horizontal velocities on the steeper profile under otherwise identical offshore conditions, in agreement with the numerical results.

In summary, bed slope exerts a fundamental control on the statistics of wave and kinematic extremes in the coastal zone. Steeper slopes promote rapid shoaling, stronger nonlinearity, and more localised breaking, leading to larger crest elevations and enhanced horizontal and vertical crest velocities. In contrast, milder slopes produce broader surf zones and more progressive dissipation, resulting in reduced post-breaking amplitudes across all zero-crossing parameters. These findings demonstrate that extreme surface elevations and flow kinematics depend not only on offshore wave conditions but also critically on the local bathymetric gradient.

4.4 Wave energy

This section examines the transformation of waves over coastal bathymetry from an energetic perspective, with particular emphasis on how bed slope governs the

partition, redistribution, and nonlinear transfer of wave energy during shoaling and breaking. The analysis combines bulk energy diagnostics, spectral descriptions, and bispectral techniques to provide an interpretation of how energy is redistributed across frequency bands and between dynamical components. The section is organised as follows. First, the relative contributions of kinetic and potential energy are quantified to establish how energy partitioning evolves with depth and slope. Second, the evolution of the wave energy spectrum and its bandwidth is analysed to identify slope-dependent trends in spectral broadening and downshifting. Finally, bispectral diagnostics are used to interpret these trends in terms of the dominant nonlinear wave–wave interaction mechanisms.

4.4.1 Energy composition

The depth-integrated, time-averaged wave energy is decomposed into kinetic and potential contributions at each cross-shore location. The kinetic energy per unit horizontal area is computed as:

$$KE(x) = \frac{1}{2}\rho \int_{-h(x)}^{\eta(x,t)} \langle u(x, z, t)^2 + w(x, z, t)^2 \rangle dz, \quad (4.6)$$

where ρ is the water density and $\langle \cdot \rangle$ denotes time averaging. The gravitational potential energy associated with the free surface is computed as:

$$PE(x) = \frac{1}{2}\rho g \langle \eta(x, t)^2 \rangle. \quad (4.7)$$

Figure 4.5 shows the evolution of the relative proportion of kinetic and potential energy as a function of reduced depth, $k_p d$. Offshore, the energy is approximately equipartitioned, in agreement with classical linear wave theory (Dean & Dalrymple, 1991). During shoaling, the proportion of potential energy increases as wave crests steepen and free-surface variance grows. In contrast, within the shallow-water and breaking-dominated region, the kinetic contribution increases markedly, reflecting the conversion of wave potential energy into forward-directed momentum and intense near-surface velocities. This transition occurs much more abruptly over steep slopes (dashed line) than over mild slopes (solid line). Steep bathymetries therefore promote a rapid shift towards kinetic-energy dominance, whereas mild slopes exhibit a smoother and more progressive redistribution of energy.

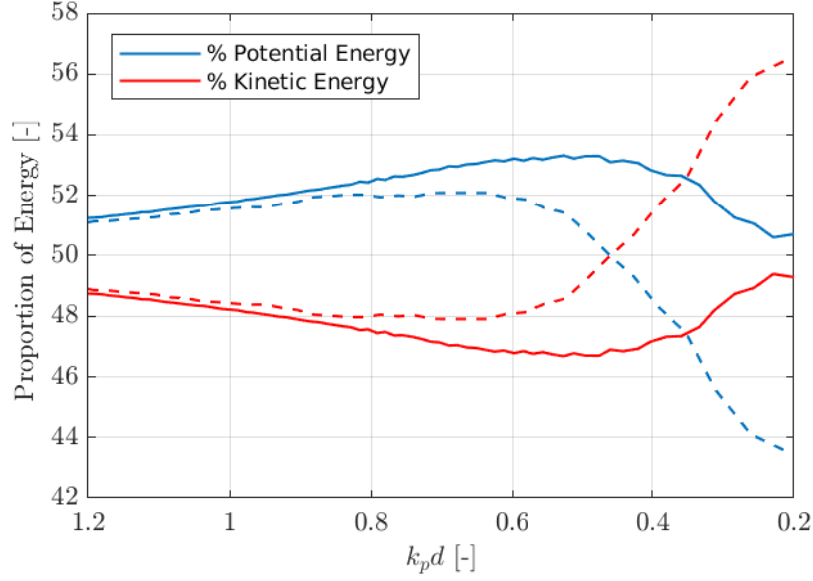


FIGURE 4.5: Evolution of the proportion of wave energy with reduced $k_p d$ for sea-state B3. The solid and dashed lines correspond to a mild and a steep slope, respectively.

Overall, the trends in Figure 4.5 confirm that steep slopes favour stronger momentum fluxes and larger crest velocities, consistent with the enhanced horizontal velocities in shallow water reported in Section 4.3. In contrast, the broader surf zones of mild slopes distribute dissipation over longer distances, limiting the growth of kinetic energy and producing weaker post-breaking velocities. This establishes bed slope as a primary control on the energetic balance between wave-induced motion and free-surface variability, and the invalidity of the energy equipartition assumption in coastal waters.

4.4.2 Distribution of wave energy

Energy spectra of surface elevation $S_{\eta\eta}$, horizontal velocity S_{uu} , and vertical velocity S_{ww} , all measured at the free surface, are analysed. Spectra are non-dimensionalised by the peak frequency $f_p = 1/T_p$ and the zeroth spectral moment m_0 . The n th spectral moment is defined as

$$m_n(x) = \int_0^\infty \omega^n S_{xx}(\omega) d\omega, \quad (4.8)$$

where x denotes the variable of interest (η , u , or w) and S_{xx} is the corresponding omni-directional spectral density measured at the free surface.

Figure 4.6 illustrates the evolution of the normalised spectra with decreasing $k_p d$ for a 1:30 slope. The spectra are shown in logarithmic scale to facilitate clearer depictions of the spectral evolution. As depth decreases, the surface elevation spectrum (Figure 4.6(a)) exhibits a progressive transfer of energy from the spectral peak to both sub- and super-harmonics due to triad interactions (Madsen & Sørensen, 1993). For $k_p d < 0.4$, wave breaking strongly dissipates high-frequency energy around $f/f_p \approx 2$, while energy in the subharmonic band increases. The horizontal velocity spectrum (Figure 4.6(b)) mirrors these trends, reflecting the tight dynamical coupling between surface elevation and horizontal motion. In contrast, the vertical velocity spectrum (Figure 4.6(c)) flattens with decreasing depth, congruent with the reduced dispersion and increasingly elliptical particle orbits characteristic of shallow water (Holthuijsen, 2010). Notably, infragravity energy in S_{ww} remains negligible, in agreement with second-order random wave theory (Sharma & Dean, 1981) and with the predominantly horizontal nature of infragravity motions (Herbers et al., 1994).

To isolate the influence of bed slope, Figure 4.7 compares spectra over steep (1:15) and mild (1:80) slopes at a fixed depth ($k_p d = 0.6$). Given that comparisons are conducted across bathymetries at a single $k_p d$ location, this is associated with a greater spatial propagation distance from the inlet in milder slopes compared to steeper slopes. For all variables, milder slopes exhibit broader spectra, with less energy concentrated at the peak and more distributed across sub- and super-harmonics. This is a consequence of the wider surf zone in such beds compared to steeper slope configurations, resulting in a greater extent of energy transfers.

In addition, both the primary and secondary spectral peaks are shifted towards lower frequencies over mild slopes, indicating enhanced spectral downshifting. While downshifting is well documented for deep-water and mildly sloping conditions (Huang et al., 1996, Bendykowska & Werner, 1988, Hashimoto et al., 2014, Saprykina et al., 2020), the present results demonstrate that the bed-slope angle exerts a previously underexplored control on this phenomenon. Milder bed slopes are associated with a slower rate at which second-order interaction kernels form. Thus, the mechanism for spectral energy transfer to generate higher harmonics across different bathymetries is physically different, resulting in variations in the evolution of the spectral tail. Further insight is provided by the gradual shift of spectral energy across frequency bands, which reflects a slower adjustment of the dispersion properties over milder

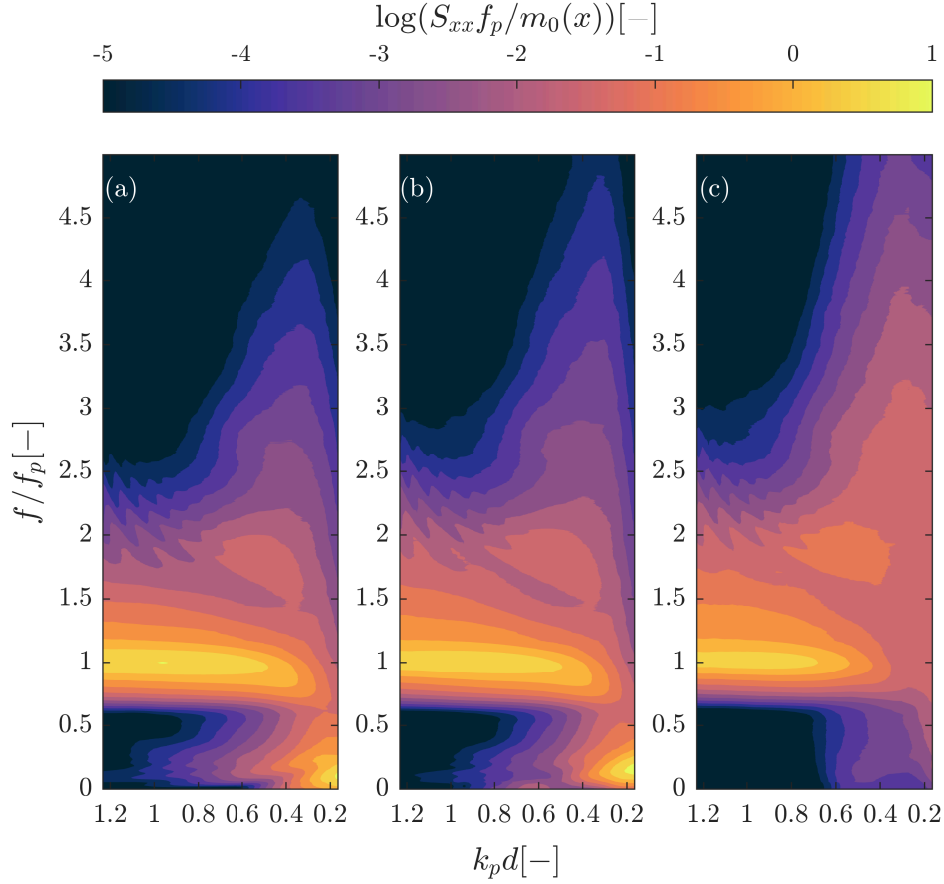


FIGURE 4.6: Normalised energy spectra for sea-state B3 over 1:30 slope: (a) $S_{\eta\eta}f_p/m_0(\eta)$, (b) $S_{uu}f_p/m_0(u)$, (c) $S_{ww}f_p/m_0(w)$.

slopes. As a result, the nature of the interactions, in terms of the wave numbers involved, is altered. This results in a greater downshift of the peak frequencies over milder bathymetries. This hypothesis will be further supported in Section 4.4.3 when considering the nature of the wave interactions involved.

In the infragravity band, milder slopes display stronger and lower-frequency sub-harmonic peaks in both $S_{\eta\eta}$ and S_{uu} , indicating longer and more organised wave groups and larger mean flows at the free surface. The enhanced infragravity energy promotes stronger interactions between long and short waves and facilitates spectral downshifting (Saprykina & Kuznetsov, 2022). By contrast, infragravity energy in S_{ww} remains weak and is dominated on steep slopes by breaking-induced vertical motions rather than by classical bound long waves.

Comparisons with standard spectral models further highlight the importance of slope effects. The JONSWAP spectrum (Hasselmann et al., 1973) shown in a black

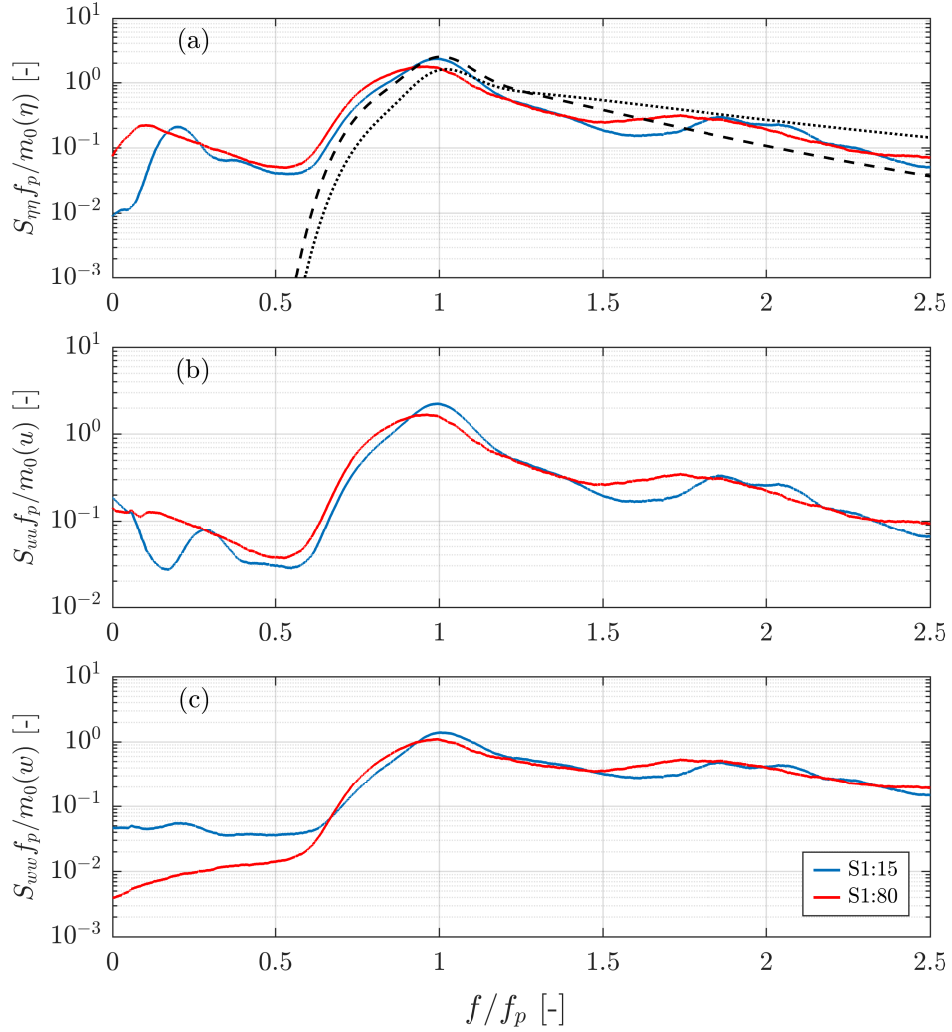


FIGURE 4.7: Normalised spectra at $k_p d = 0.6$ for sea-state B3 over slopes 1:15 (blue) and 1:80 (red). JONSWAP (dashed) and TMA (dotted) spectra shown for reference.

dashed line, which is the offshore shape of the spectrum in this study, is unable to capture the additional sub- and super-harmonics formed in shallower water ($k_p d = 0.6$). The TMA spectrum (Buows, 1979) shown in a black dotted line, which includes finite water depth effects, provides a closer match to the super harmonic peak. However, it underestimates and overestimates the low and high frequency tail, respectively, as well as underpredicting the spectral peak. Moreover, although this formulation embeds a dependence on water depth, it is formulated for mild slopes based on field data. As such, effects associated with steeper bed configurations are not captured. These discrepancies reveal a key limitation of existing spectral models and underscores the need to incorporate bathymetric-gradient effects into future formulations

of nearshore wave transformation.

The influence of slope on spectral broadening is summarised by the bandwidth parameter:

$$\nu = \sqrt{\frac{m_0(\eta)m_2(\eta)}{m_1(\eta)^2} - 1}. \quad (4.9)$$

Figure 4.8 shows that ν increases monotonically with decreasing $k_p d$, with a much faster growth rate over mild slopes, particularly within the surf zone. Gentle bathymetries therefore promote stronger spectral spreading by allowing nonlinear interactions to act over longer distances. Conversely, steeper slopes retain narrower-banded spectra, favouring more coherent superposition and larger wave and crest extremes, in accordance with the results of Section 4.3. This behaviour is consistent with the broader spectral shape observed over gentle bathymetries in Figure 4.7.

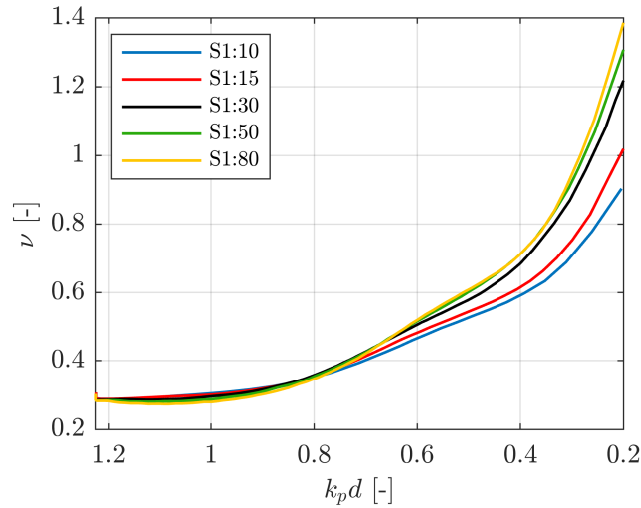


FIGURE 4.8: Spectral bandwidth, ν , versus $k_p d$ for sea-state B3. Line colour corresponds to bed slope gradient.

4.4.3 Nonlinear wave-wave interactions

Fourier bispectral analysis of the surface elevation time series is employed to identify the dominant triad coupling mechanisms responsible for energy transfers between frequencies, following the method of Hasselmann et al. (1963). This is achieved by adopting 1024 Hann-windowed records, which are overlapping by 50%. This is conducted independently per realisation of the sea-state and results are averaged across realisations for statistical stability. As such, the bispectrum B_{f_1, f_2} detects

the phase-coupling between frequency components f_1 and f_2 in a triad. Given the dominance of triad wave-wave interactions over quadruplet wave-wave interactions in coastal waters (Smit & Janssen, 2016), the utilisation of bispectral methods over higher order alternatives is deemed appropriate.

To establish the strength of nonlinear interactions, we consider the nonlinear energy exchange term, S_{nl} , determined through the integration of the imaginary part of the bispectrum as defined by Herbers et al. (2000):

$$S_{nl,f} = 3\pi \frac{\rho g f}{d} \Im \left(\sum_{f'=0}^f B(f', f - f') - 2 \sum_{f'=0}^f B(f', f) \right), \quad (4.10)$$

where $\Im(\cdot)$ denotes the imaginary part. Values of $S_{nl,f} < 0$ and $S_{nl,f} > 0$ indicate a loss and gain of energy, respectively, by frequency f .

Figure 4.9 compares the evolution of $S_{nl,f}$ with $k_p d$ for a (a) 1:15 and (b) 1:80 bed slope. Firstly, considering Figure 4.9, the nature of the energy exchanges can be discussed by considering three regions: the subharmonic ($f < 0.7f_p$), peak ($0.7f_p \leq f \leq 1.5f_p$) and superharmonic ($f > 1.5f_p$). At frequencies surrounding the spectral peak, energy exchanges result in a loss of energy, with the extent of these losses being greater with reducing effective water depth within the shoaling region ($k_p d > 0.6$). Conversely, nonlinear exchanges result in a growth of energy within the sub and superharmonic region, with these transfers being stronger as waves undergo shoaling. These observations reinforce the earlier findings regarding the transfer of energy from the peak to lower and higher frequencies through triad wave-wave interactions presented in Section 4.4.2.

Following wave breaking saturation ($k_p d < 0.6$), notable energy around the spectral peak ($0.7f_p \leq f \leq 1.5f_p$) is lost. In shallow water ($k_p d < 0.3$), the dominance of wave breaking also results in losses at higher wave frequencies ($0.5f_p \leq f \leq 3f_p$). Conversely, although energy gain around $f = 2f_p$ is reduced or diminished post-breaking saturation, energy continues to be gained at higher harmonics $f > 2.5f_p$ within the surf zone, provided the water depth is not too small ($k_p d \geq 0.3$).

If the observations made in Figure 4.9(a) in relation to steep bed slopes is compared to those in mild slopes, as shown in Figure 4.9(b), several distinctions can be made. Although overall the pattern of energy exchange is similar, the strength of interactions, both positive and negative, are significantly weaker on milder bed slopes. Moreover, the gain of energy in the superharmonic region is more restricted into

specific ‘bands’ surrounding the higher harmonics ($2f_p$, $3f_p$ etc.) in steeper slopes, whereas milder slopes exhibit more uniform gains and losses across the frequency domain. Furthermore, in line with the findings of Rijnsdorp et al. (2022a), energy is continuously being received within the infragravity band in steep slopes. Conversely, in mild slopes, although a gain in energy is observed initially, losses in this region are observed for $k_p d < 0.3$.

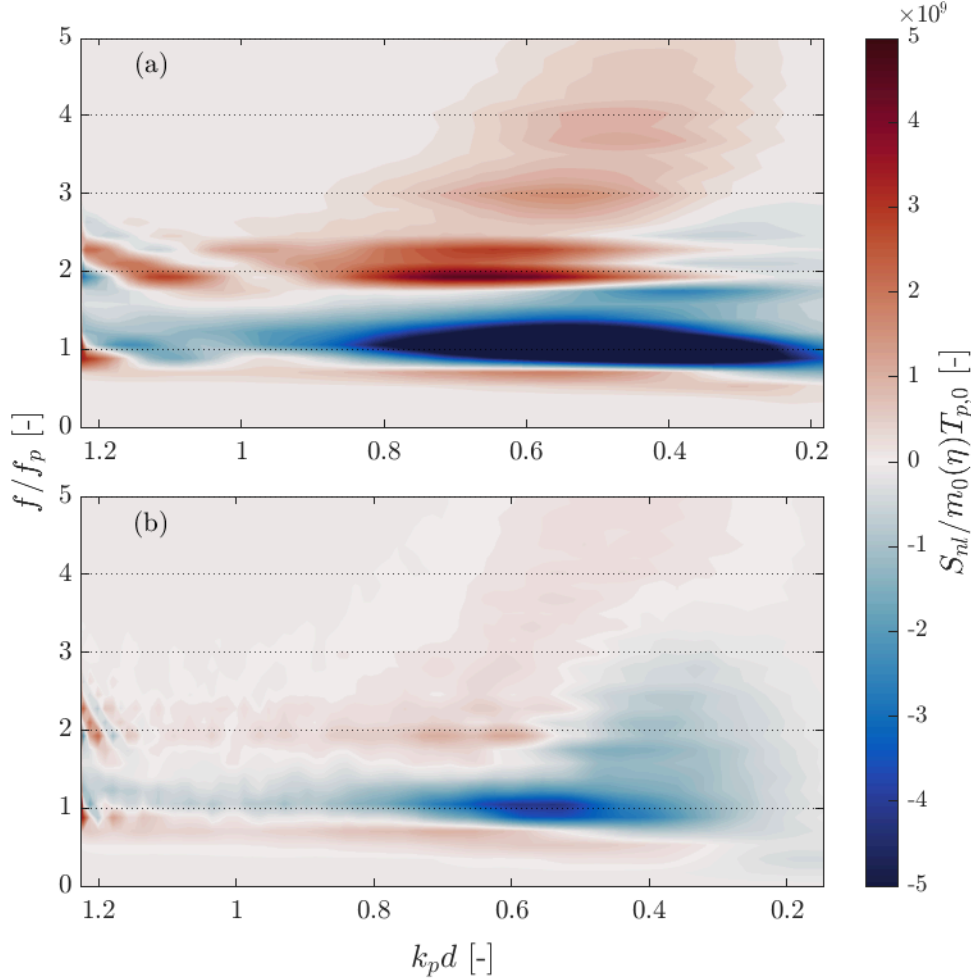


FIGURE 4.9: Evolution of normalised nonlinear energy exchange term $S_{nl}/m_0 T_{p,0}$ with $k_p d$ for sea-state B3 propagating over bed slope (a) 1:15 and (b) 1:80.

To elucidate these differences, Figure 4.10 presents the evolution of the imaginary part of the bispectrum, which characterises the phasing of triad interactions. Positive values (red) of B_{f_1, f_2} indicate energy transfer from f_1 and f_2 to the sum frequency $f_3 = f_1 + f_2$, while negative values (blue) indicate energy transfer from f_3 to f_1 and f_2 (with $f_2 = f_3 - f_1$). The imaginary bispectrum therefore captures both sum and difference interactions, with colour intensity representing the magnitude of energy

transfer.

At the offshore location (Figure 4.10(a)), a pronounced positive peak at $f_1 = f_2 = f_p$ indicates self-self interactions at the primary frequency, transferring energy to $f_3 = 2f_p$. This process leads to the formation of a secondary superharmonic peak, as seen in Figures 4.6–4.7. A strong negative peak at $f_1 = 0.875f_p$ and $f_2 = 0.175f_p$ is also observed, coupled to the primary harmonic. This reflects energy transfer from the spectral peak to the subharmonic region, driving the growth of bound infragravity waves and a downshift of energy around the peak, consistent with De Bakker et al. (2015) on mild beds.

Further into the shoaling region (Figure 4.10(b)), this negative coupling is amplified, highlighting enhanced infragravity generation through short-wave group envelope modulation. Physically, the increased phase lag of the bound long wave relative to the wave group during shoaling results in energy transfer from the primary frequency to low-frequency bound waves, in agreement with previous studies (List, 1992, Janssen et al., 2003, Battjes, 2004, Van Dongeren et al., 2007). Concurrently, the strength of the self-self interaction at the spectral peak diminishes, indicating that energy redistribution away from the peak is no longer dominated solely by self-self interactions.

Deeper into the shoaling region (Figure 4.10(c)), additional interactions emerge. A positive peak at $f_1 = 1.75f_p$ and $f_2 = 0.875f_p$ indicates interactions that broaden the high-frequency spectral tail ($f > 2.5f_p$), while a negative interaction at $f_1 = 1.75f_p$ and $f_2 = 0.175f_p$ transfers energy away from the secondary peak at $2f_p$. These interactions intensify towards the end of the shoaling region (Figure 4.10(d)), where higher-order couplings are also evident, with positive interactions at $f_1 = f_2 = 1.75f_p$ and negative interactions at $f_1 = 2.625f_p$ and $f_2 = 0.175f_p$. Collectively, these processes shift energy towards higher harmonics.

Within the surf zone (Figure 4.10(e)), overall phase coupling weakens, particularly at high frequencies, due to wave energy dissipation. However, a more distinct positive interaction at $f_1 = f_p$ and $f_2 = 0.175f_p$ indicates energy transfer from the infragravity to the short-wave regime, corresponding with observations by De Bakker et al. (2016) on steep slopes. Close to the shoreline (Figure 4.10(f)), interaction strengths are substantially reduced following extensive wave breaking.

Building on these observations, Figure 4.11 compares bispectral evolution over

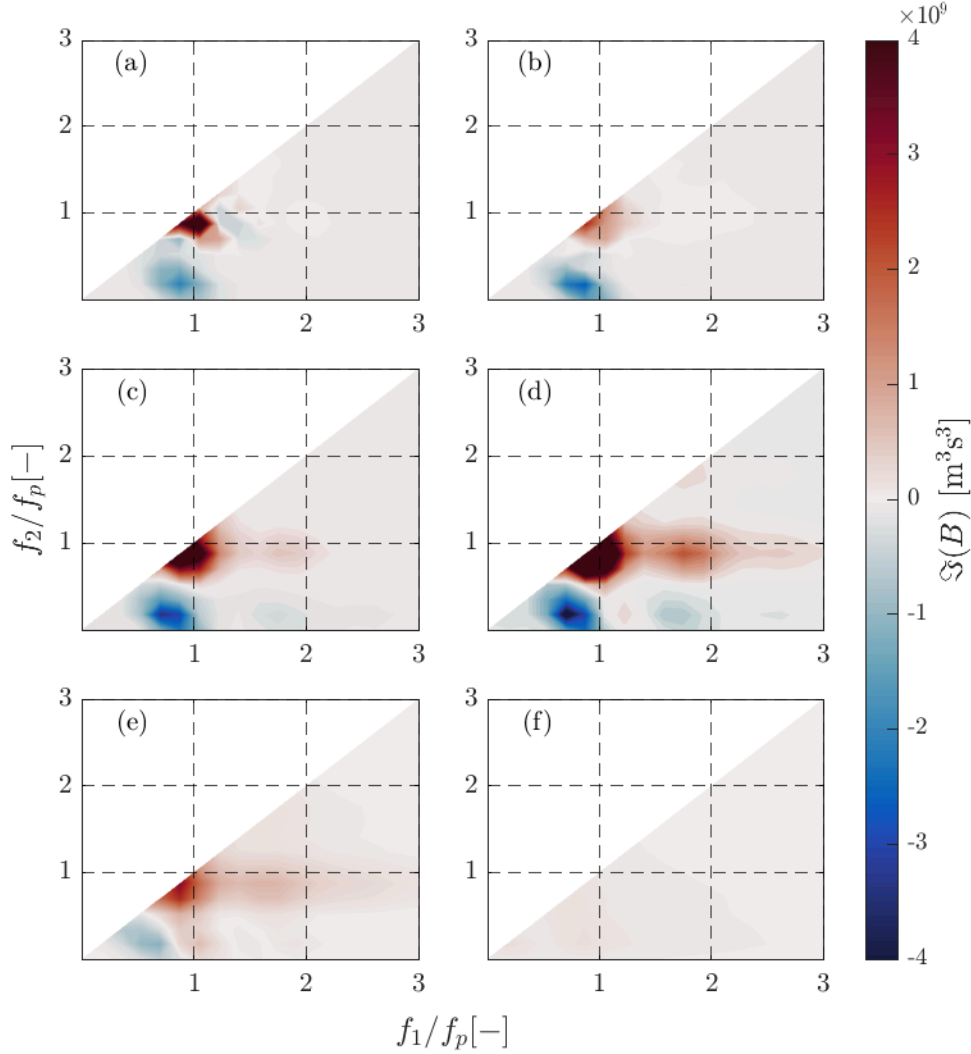


FIGURE 4.10: Imaginary part of the bispectrum for sea-state B3 propagating over bed slope 1:30 in (a) $k_p d = 1.2$, (b) $k_p d = 1.0$, (c) $k_p d = 0.8$, (d) $k_p d = 0.6$, (e) $k_p d = 0.4$ and (f) $k_p d = 0.2$.

steep (left column) and mild (right column) slopes. At the onset of shoaling (Figure 4.11(a–b)), two dominant couplings are evident: a positive self–self interaction at the peak frequency and a negative interaction between primary and long-wave components associated with bound-wave shoaling. These couplings are stronger for steeper slopes, as indicated by differing colour scales.

Towards the end of the shoaling region (Figure 4.11(c–d)), higher-order couplings similar to those in Figure 4.10(c) appear, but with substantially reduced strength over mild slopes. Instead, mild bathymetries exhibit additional interactions which only appear later in steeper cases (Figure 4.11(e)), notably a positive interaction at $f_1 = 1.225f_p$ and $f_2 = 0.175f_p$. This coupling transfers energy to higher harmonics

below $2f_p$, providing evidence for the downshift of the secondary peak observed in Figure 4.7 for mild slopes, and supporting earlier hypotheses regarding bathymetric control on the formation rate of second-order interaction kernels.

Within the surf zone (Figure 4.11(e–f)), slope-dependent differences become pronounced. Steep slopes exhibit more localised couplings, whereas mild slopes display a broader range of nonlinear interactions with less distinct peaks. Similar behaviour is evident within the shoaling zone (Figure 4.11(d)), where mild slopes show enhanced higher-order couplings involving infragravity energy. In steep slopes, negative interactions between higher harmonics ($f_1 > 1.5f_p$) and infragravity components ($f_2 = 0.175f_p$) indicate energy transfer from infragravity to higher frequencies, promoting infragravity steepening and enhanced infragravity breaking. Conversely, these interactions remain predominantly positive over mild slopes, implying weaker transfers to higher frequencies and more gradual infragravity dissipation within the surf zone.

Overall, the findings of this section have highlighted notable discrepancies in terms of the nature of interactions across different bed slopes. Although steeper bed slopes are associated with more intense interactions, these are confined to fewer nonlinear coupling modes. Conversely, milder slopes are associated with weaker interactions but involve a greater number of coupling modes. When combined with the wider surf zone in mild bathymetries, this leads to a greater rate of spectral evolution with respect to reductions in effective water depth. As such, wave fields propagating over milder slopes are broader in nature. In addition, the varying nature of the coupling modes results in a greater downshift in the spectral peak and secondary peaks in such bathymetries.

4.5 Wave shape

This section examines the evolution of wave shape during shoaling and breaking, with particular emphasis on the roles of wave steepness and bed slope in controlling vertical and horizontal asymmetry. Wave shape is characterised using complementary phase-coupling diagnostics (biphase), integral statistical moments (skewness and kurtosis), and infragravity-band steepness.

To examine wave shape characteristics, we focus on the 100 largest waves within

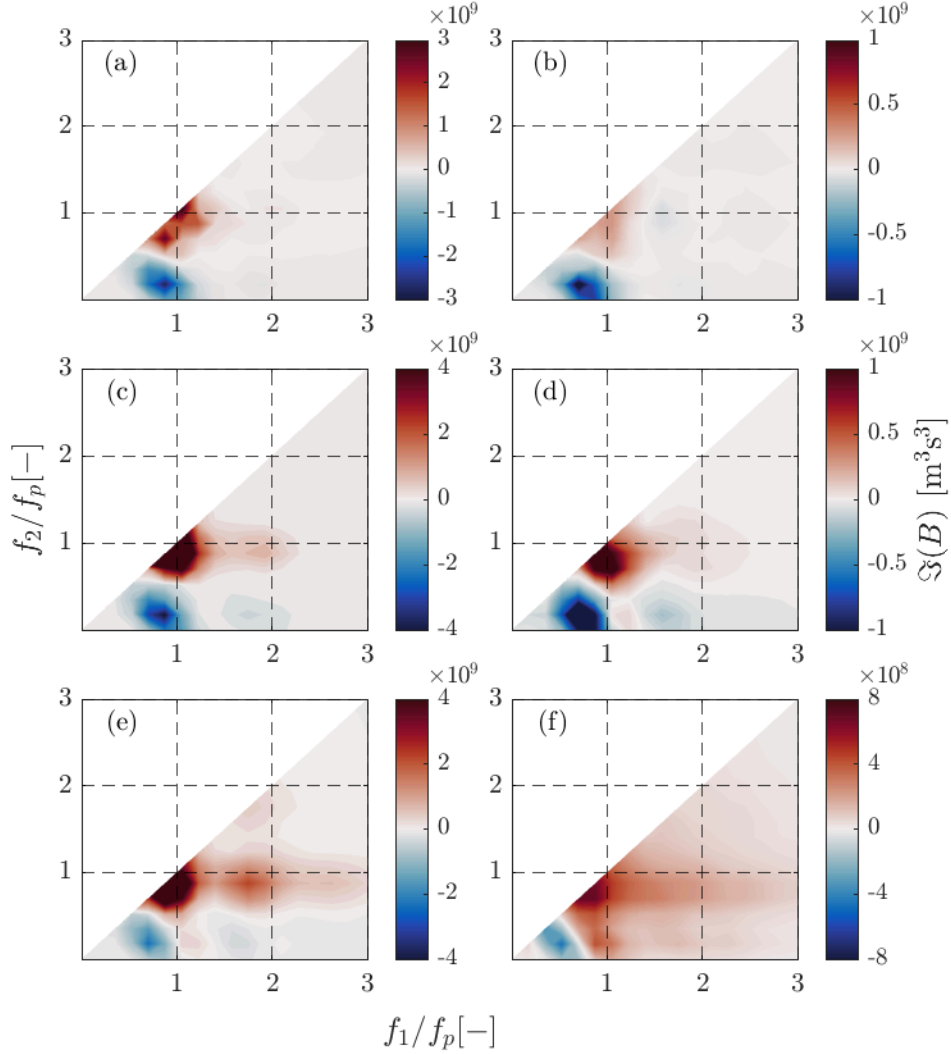


FIGURE 4.11: Imaginary part of the bispectrum for sea-state B3 in (a-b) $k_p d = 1.2$, (c-d) $k_p d = 0.8$ and (e-f) $k_p d = 0.4$. The left and right column correspond to bed slope 1:15 and 1:80, respectively.

each sea-state as representative of extreme events. Figure 4.12 shows a portion of the water-surface elevation time series for sea-state B3 corresponding to these extreme waves. The waves have been phase-aligned such that all crests occur at the same time. The results shown correspond to $k_p d = 0.6$, highlighting the effects of nonlinear wave evolution on wave shape. The left and right columns represent steep and mild bed slopes, respectively.

In terms of vertical asymmetry, waves propagating over steeper slopes exhibit a smaller crest-to-trough ratio compared to those over milder slopes, indicating enhanced vertical asymmetry over mild bed configurations. In contrast, waves over mild slopes display crests that are more symmetrically distributed about $t - t_c = 0$ s,

suggesting reduced horizontal asymmetry relative to waves over steeper beds. These initial observations will be quantified and examined in further detail in Sections 4.5.1 – 4.5.2.

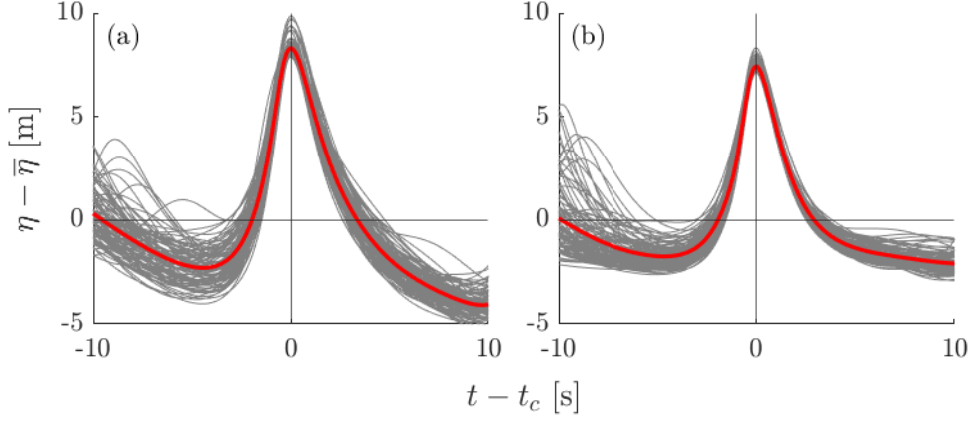


FIGURE 4.12: Shape of the 100 largest waves in sea-state B3 in $k_p d = 0.6$ propagating over bed slope (a) 1:10 and (b) 1:80. Individual waves are shown in grey and the mean shape in red.

4.5.1 Horizontal asymmetry

The extent of horizontal asymmetry is quantified by considering the biphase, defined as

$$\beta_{f_1, f_2} = \arctan \left(\frac{\Im(B_{f_1, f_2})}{\Re(B_{f_1, f_2})} \right), \quad (4.11)$$

where the real part $\Re(B_{f_1, f_2})$ and imaginary part $\Im(B_{f_1, f_2})$ of the bispectrum quantify vertical skewness and horizontal asymmetry, respectively (Elgar & Guza, 1985). Vertical skewness reflects sharper crests and shallower troughs, whereas horizontal asymmetry reflects a steeper front face relative to the rear face. According to Masuda & Kuo (1981), a zero biphase between the primary component and its first harmonic corresponds to vertically asymmetric but horizontally symmetric waves, while a biphase of $-\frac{1}{2}\pi$ denotes forward-leaning, horizontally asymmetric waves with negligible skewness.

Figure 4.13 shows the evolution of $\beta_{f_p, 2f_p}$ with decreasing effective water depth $k_p d$. To avoid contamination by small shifts in the spectral peak, biphase values are averaged over a frequency window $\pm f_p/5$ surrounding each interacting component. In agreement with the observations of Elgar & Guza (1985), the progressive reduction in water depth drives a transition from vertically peaked wave profiles offshore ($\beta_{f_p, 2f_p} \approx$

0) to strongly forward-leaning, sawtooth-like shapes in shallow water ($\beta_{f_p, 2f_p} \rightarrow -\frac{1}{2}\pi$). Comparisons across bed slopes further reveal a more pronounced reduction in $\beta_{f_p, 2f_p}$ over steeper bathymetries, leading to stronger horizontal asymmetry near the shoreline, in agreement with the observations in Figure 4.12.

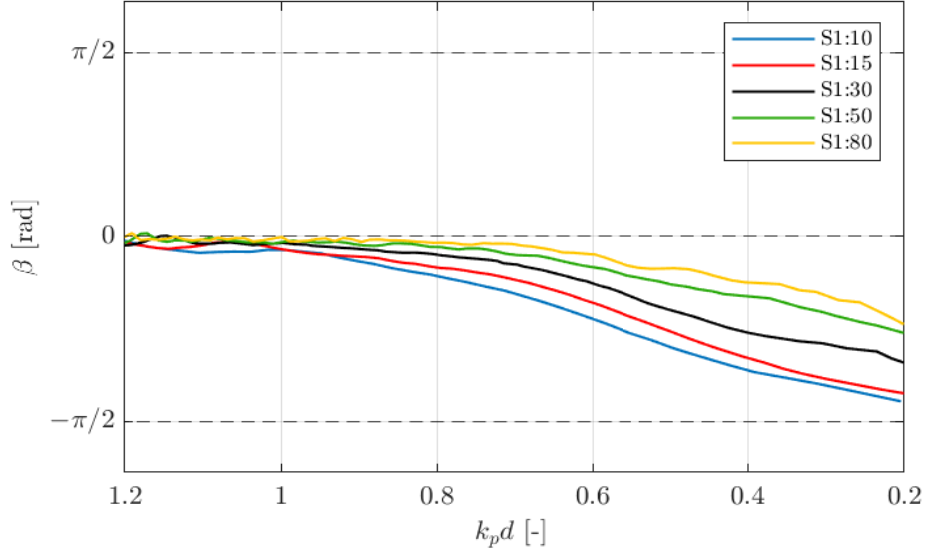


FIGURE 4.13: Evolution of the biphase $\beta_{f_p, 2f_p}$ with $k_p d$ for sea-state B3. The colour of the line corresponds to the bed slope gradient.

These findings are further supported by examining the mean ratio of the preceding trough depth, η_{pt} , to the following trough depth, η_{ft} , which is adopted here as a proxy for horizontal wave asymmetry:

$$\kappa = \frac{\eta_{pt}}{\eta_{ft}} \quad (4.12)$$

The resulting values of κ are shown in Figure 4.14. Values of κ close to unity indicate a horizontally symmetric wave profile, whereas departures from unity reflect increasing horizontal asymmetry. The trends in Figure 4.14 are consistent with those observed previously in Figure 4.13, with steeper bed slopes corresponding to more horizontally asymmetric wave shapes. Furthermore, $\kappa > 1$ indicates that preceding troughs are deeper than following troughs, a feature commonly associated with breaking waves (Karpadakis & Swan, 2020). This behaviour is expected, as the B3 sea state has reached breaking saturation at this spatial location ($k_p d = 0.6$).

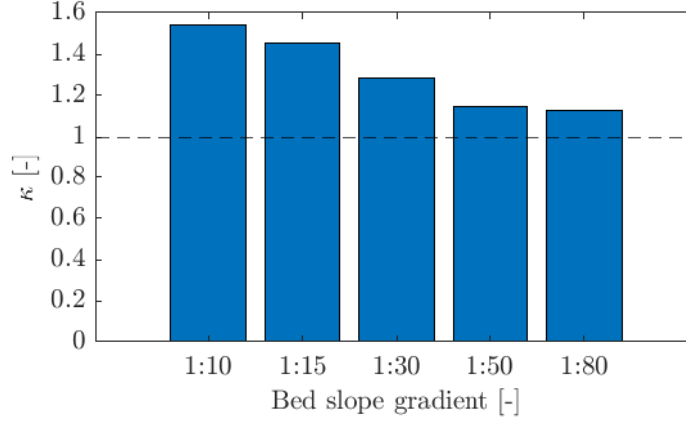


FIGURE 4.14: Variation in κ with bed slope gradient for sea-state B3 in $k_p d = 0.6$.

4.5.2 Skewness and kurtosis

Further details of the characteristics of wave shape are obtained by examining the skewness and kurtosis, with emphasis on the respective roles of incident wave steepness (Figure 4.15) and bed slope (Figure 4.16).

Figures 4.15(a,c) present the skewness,

$$\lambda_3(X) = \frac{\langle (X(t) - \overline{X(t)})^3 \rangle}{\sigma_X^3}, \quad (4.13)$$

where σ_X is the standard deviation of X . Skewness quantifies vertical wave asymmetry induced by second-order nonlinearities. For the surface elevation (Figure 4.15(a)), the skewness $\lambda_3(\eta)$ increases during shoaling as bound harmonics intensify and energy is transferred to higher frequencies, resulting in sharper crests and shallower troughs. Following the onset of wave breaking, a clear reduction in skewness is observed. Energy dissipation during breaking promotes a transition towards more linear wave forms, which are associated with lower skewness values. In addition, turbulence generation and phase randomisation redistribute spectral energy and weaken coherent bound-harmonic contributions, further contributing to the observed decline in skewness. Since wave breaking occurs in deeper water for steeper incident sea-states, the reduction in skewness begins earlier for larger values of S_p .

Horizontal velocity skewness (Figure 4.15(c)) exhibits analogous behaviour. Positive $\lambda_3(u)$ reflects stronger onshore velocities beneath crests and weaker offshore velocities beneath troughs (Stokes, 1847), consistent with the surface-elevation asymmetry. Maximum skewness occurs near breaking saturation for both η and u , followed

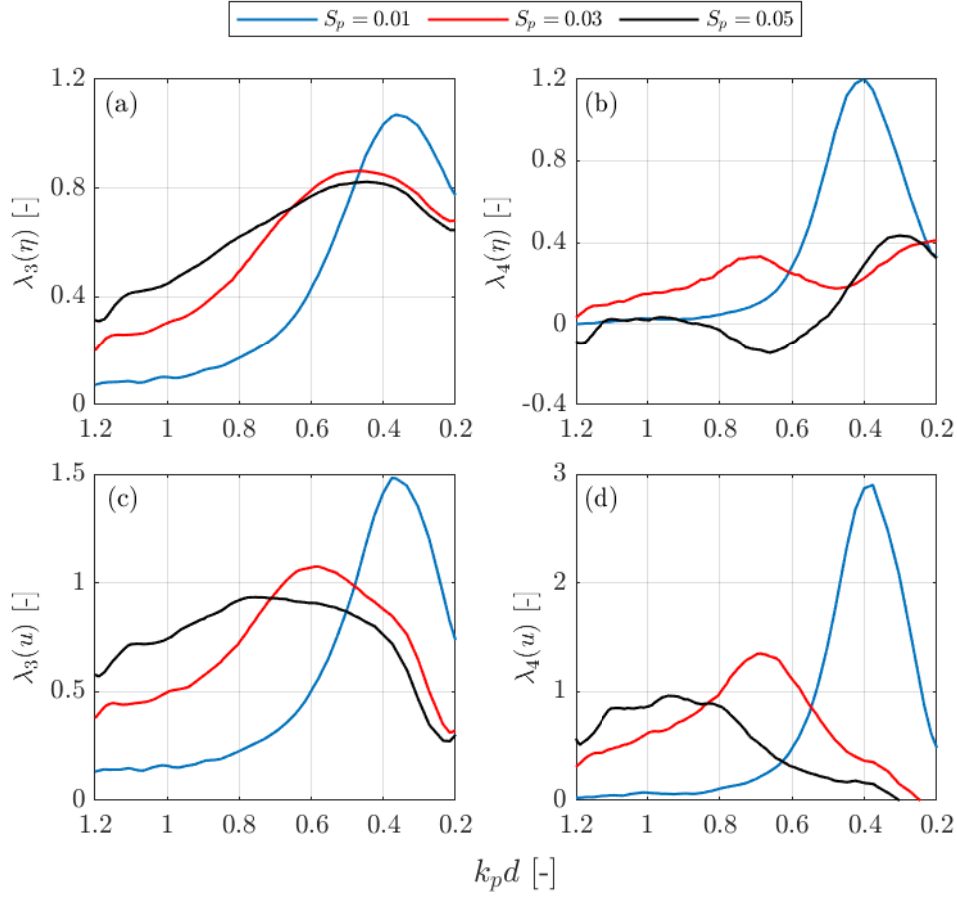


FIGURE 4.15: Evolution of integral wave parameters with effective water depth, $k_p d$, for sea-states B over bed slope 1:30. Line colour corresponds to sea-state steepness, S_p .

by a decline in the surf zone. Although steeper sea-states exhibit larger offshore skewness, the earlier onset of breaking yields higher shallow-water skewness for milder seas. The reduction of skewness in the inner surf zone indicates a redistribution of velocity fluctuations over the wave cycle: extreme crest velocities become less dominant relative to trough velocities, yielding a more symmetric distribution. This interpretation is supported by the corresponding probability density functions (not shown).

Figures 4.15(b,d) show the excess kurtosis,

$$\lambda_4(X) = \frac{\langle (X(t) - \overline{X(t)})^4 \rangle}{\sigma_X^4} - 3, \quad (4.14)$$

which measures tail heaviness and the propensity for extreme events. In the finite-depth regime considered here, kurtosis is governed primarily by bound nonlinear

wave components rather than by classical deep-water quasi-resonant interactions (Fedele et al., 2016). Both $\lambda_4(\eta)$ and $\lambda_4(u)$ attain local maxima near breaking. In the inner surf zone, however, a secondary maximum appears only in $\lambda_4(\eta)$: surface elevation retains intermittent extreme crests associated with individual breaking events, whereas horizontal velocities are dominated by bore-like motions and depth-dependent return flows that suppress extreme tails.

For the steepest incident sea ($S_p = 0.05$), negative values of $\lambda_4(\eta)$ are observed after breaking onset, indicating a suppression of extremes characteristic of heavily broken waves. In contrast, milder seas display larger shallow-water kurtosis, reflecting the persistence of rare extreme events once the most energetic waves in steeper seas have already broken. Vertical velocity skewness and kurtosis follow analogous trends and are omitted for brevity. As a whole, the skewness and kurtosis of velocities have larger magnitudes than their surface-elevation counterparts, underscoring the stronger sensitivity of kinematics to nonlinear distortion.

Figure 4.16 isolates the effect of bed slope. Both skewness and kurtosis of η (Figures 4.16(a–b)) and u (Figures 4.16(c–d)) are larger over milder slopes. The extended shoaling distance over such bathymetries promotes the gradual formation of additional bound wave contributions before breaking occurs. Moreover, the greater number of nonlinear coupling modes in milder bed slopes presented in Section 4.4.3 further support these observations. This leads to stronger vertical asymmetry in mild-slope environments.

4.5.3 Vertical structure of asymmetry

Figure 4.17 illustrates the vertical structure of $\lambda_3(u)$ and $\lambda_4(u)$ for sea-state B5 at selected spatial locations. For the sea-state shown (B5), breaking saturation occurs at $k_p d \approx 0.8$. Prior to breaking, surface skewness and kurtosis increase as $k_p d$ decreases from 1.2 to 0.9, followed by a reduction within the surf zone ($k_p d = 0.6$). Both moments decrease with depth, approaching zero or slightly negative values near the bed. This implies stronger orbital motions associated with large forward velocities under the wave crest, leading to more positively skewed horizontal velocities at the surface. Lower in the water column, velocity fluctuations become more symmetric (or negatively skewed) which reduce both the skewness and kurtosis. More specifically, in Figure 4.17(a), values of $\lambda_3(u)$ at the bed are more negative at the offshore location,

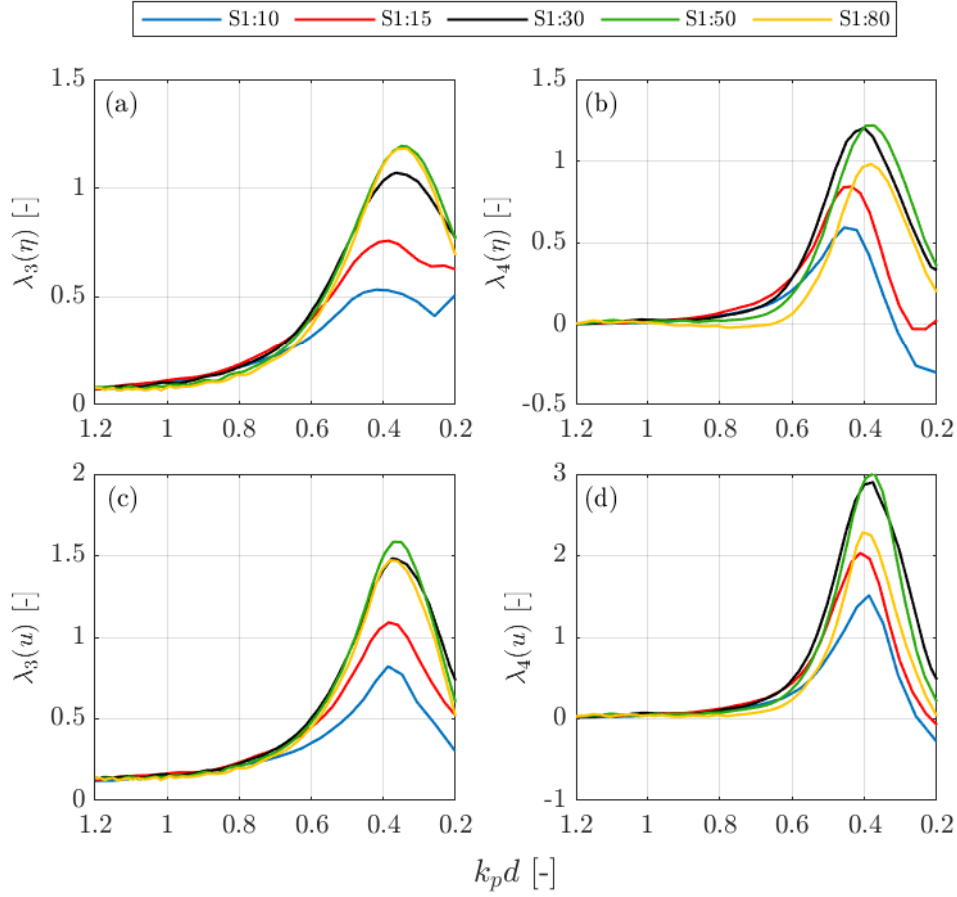


FIGURE 4.16: Evolution of integral wave parameters with $k_p d$ for sea-state B1. Line colour corresponds to bed slope gradient.

indicating a greater prominence of offshore-directed horizontal velocity components, associated with the return flow occurring at this location. In shallow water, the extent of offshore-directed horizontal flow components reduces, resulting in positive values of $\lambda_3(u)$ near the bed.

Considering the profile of $\lambda_4(u)$ shown in Figure 4.17(b), converse trends in the influence of reduced $k_p d$ are observed for values of $\lambda_4(u)$ lower in the water column. $\lambda_4(u)$ near the bed becomes more negative in shallow water (prior to breaking onset). Together with the findings of Figure 4.17(a), this indicates a flow in shallow water dominated by intermittent bursts of high momentum fluid sweeping down to the bed but without extreme turbulence spikes. After breaking onset, both skewness and kurtosis profiles become nearly depth-uniform, reflecting the homogenising influence of breaking-induced turbulence.

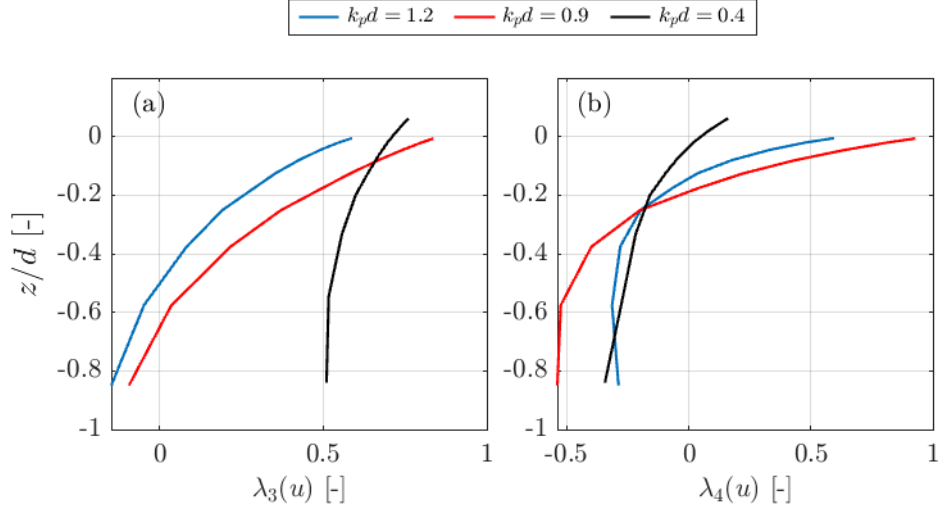


FIGURE 4.17: Depth profile of (a) skewness and (b) excess kurtosis of horizontal velocity, u , for sea-state B5 over bed slope 1:30. Line colour corresponds to $k_p d$.

4.5.4 Infragravityband wave shape

This section considers the infragravity-band specifically, and assesses the shape of the waves in this region. This is quantified using the root-mean-square horizontal surface slope,

$$S_{IG}(x) = \sqrt{\left\langle \left(\frac{\partial \eta_{IG}}{\partial x} \right)^2 \right\rangle}, \quad (4.15)$$

where η_{IG} denotes the band-pass-filtered infragravity surface elevation over $f = [0.01, 0.04]$ Hz (Okiihiro & Guza, 1995). This metric provides a direct measure of long-wave nonlinearity and crest steepening (Phillips, 1966).

Figure 4.18 shows that infragravity steepness increases as waves propagate into shallow water for all bathymetries. Over mild slopes, S_{IG} approaches a plateau, whereas over steep slopes it continues to increase, indicating sustained nonlinear growth and a greater susceptibility to infragravity breaking. This is inline with the observations made in Figure 4.11 where nonlinear energy transfers were found to promote the steepening of infragravity waves. These trends demonstrate that bed slope exerts a strong control on long-wave shape and steepening in the nearshore zone.

In summary, wave shape evolves systematically under the combined influence of shoaling, breaking, and bathymetric control. Progressive depth reduction promotes vertical and horizontal asymmetry through bound harmonic growth and phase

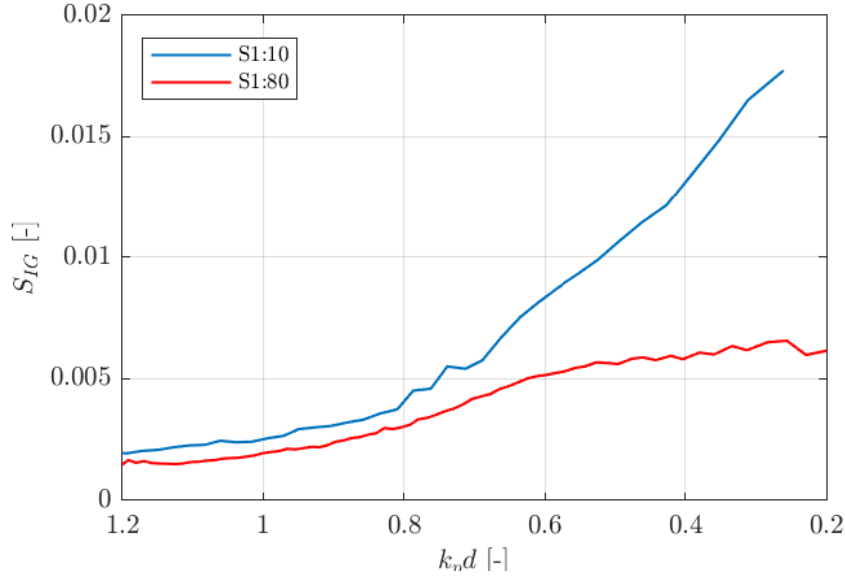


FIGURE 4.18: Steepness of infragravity waves, S_{IG} , versus $k_p d$ for sea-state B3. Line colour corresponds to bed slope gradient.

coupling, while breaking suppresses organised nonlinearity and redistributes energy. Mild slopes favour stronger nonlinear bound wave formation, manifesting in more vertically asymmetric profiles compared to steeper beds. Conversely, steep slopes promote rapid horizontal asymmetry development and enhanced infragravity steepening. Together, these results highlight the central role of bed slope in shaping the geometry of nearshore waves.

4.6 Conclusion

This chapter employs the non-hydrostatic wave–flow model SWASH to simulate unidirectional irregular waves propagating over a range of uniform bed slopes. The broad set of offshore sea-states considered enables a systematic investigation of how bathymetric slope governs the nearshore evolution of wave statistics, wave energy, and wave shape, using complementary statistical and spectral analyses of surface elevation and kinematic fields.

Analysis of wave statistics based on zero-crossing analysis demonstrates that bed slope exerts a primary control on shoaling and breaking behaviour. Steeper slopes promote stronger shoaling, yielding larger wave heights, crest elevations, and near-surface velocities throughout the coastal zone, together with earlier breaking

and reduced spatially integrated dissipation. In contrast, milder slopes produce wider surf zones and more gradual transformations, resulting in smaller post-breaking amplitudes. These findings highlight the sensitivity of extreme surface and kinematic responses to bathymetric gradient and confirm the role of slope in regulating both the location and intensity of breaking-limited wave growth.

The evolution of wave energy further reflects these slope-dependent dynamics. Examination of the relative contributions of kinetic and potential energy demonstrates the breakdown of the energy equipartition assumption in coastal waters. This energy transition is more abrupt over steeper beds. Moreover, spectral and bispectral analyses reveal that, while steeper slopes are characterised by more intense individual nonlinear interactions, milder slopes support a greater number of coupling modes and broader surf zones. This facilitates enhanced energy transfers to both sub- and super-harmonic frequency bands, producing broader inshore spectra. The more pronounced downshift of primary and secondary spectral peaks over mild slopes further illustrates the cumulative effect of extended nonlinear transformation.

Wave shape is likewise strongly governed by bed slope. Analysis of biphasic evolution and relative trough depth indicate more horizontally asymmetric wave profiles over steeper slopes. Conversely, the skewness and kurtosis of both surface elevation and velocity attain larger maxima over milder beds, indicating greater vertical asymmetry. These enhanced values arise from longer transformation distances and increased cumulative nonlinear evolution, whereas steeper slopes retain more narrow-banded spectra and support larger individual wave and crest magnitudes which are more horizontally skewed. Across all cases, velocity fields exhibit greater skewness and kurtosis than surface elevations, underscoring their stronger nonlinear response.

Taken together, these results provide a coherent picture of how bathymetric slope controls the coupled evolution of wave statistics, energy redistribution, and wave shape in coastal environments. The findings emphasise the importance of explicitly accounting for bed slope effects when modelling nearshore wave statistics and kinematics. This chapter provides the foundation understanding of the mechanisms that govern coastal wave evolution, directly addressing Objective (iii). These insights form the necessary basis for the practical applications to be presented in Chapters 5 and 6.

Chapter 5

Parameterisation of representative wave heights and periods

Author’s Note: The contents of this chapter are in press in Al Khalili, Christou, & Karmpadakis (2026a). Data-driven parameterisation of representative wave heights and periods over coastal bathymetry. *Maritime engineering*.

5.1 Introduction

Building on the analysis presented in the preceding chapters, this chapter shifts from process-based numerical modelling toward the development of practical tools for coastal design. The findings of Chapter 4 demonstrated the sensitivity of key wave statistics to bathymetric conditions. These findings reinforce the broader motivation outlined in Chapter 1: the need for reliable yet computationally efficient methods for estimating representative nearshore wave parameters. The present chapter examines how these insights can be translated into data-driven parameterisations suitable for early-stage coastal engineering applications.

In this design context, the selection of appropriate wave conditions becomes critical. The design of coastal infrastructure requires the selection of appropriate design conditions that ensure structural performance and safety under a wide range of hydrodynamic processes. For coastal structures exposed to waves, reliability is typically assessed based on wave-induced loading, often characterised by extreme wave events. In the case of vertical structures subject to breaking waves, additional consideration of wave impact loading is necessary, as this process can produce highly localized

pressure peaks and significant dynamic load contributions (Hu et al., 2017). Similarly, the assessment of wave run-up is essential when determining the crest height of coastal structures, particularly for revetments, seawalls and dikes designed to protect against coastal flooding (Van der Meer & Stam, 1992). For such infrastructure, achieving a tolerable overtopping discharge forms a central part of the design process. Run-up calculations are performed using formulae that we revisit in the present work.

Design conditions are typically defined in terms of representative or design wave parameters: height, period, direction, and frequency of occurrence. In practice, these parameters are often derived from offshore wave datasets obtained through hindcast modelling or in situ measurements (e.g. wave buoys). Once the offshore design conditions are established, they are transformed toward the project site through standard nearshore transformation procedures. These transformations account for physical processes such as shoaling, refraction, diffraction, and depth-limited wave breaking. In quantifying these effects, Chen et al. (2018) emphasized the significant influence of coastal bathymetry on wave transformation. This study focuses on quantifying this transformation for long-crested waves in coastal waters.

Given the complexity of wave transformation in coastal waters, empirical formulations are frequently employed to supplement theoretical descriptions, drawing upon laboratory and field observations (Vanem & Fazerer-Ferradosa, 2022). Although individual studies often target specific ranges of sea-states or bathymetric conditions, collectively they provide the foundation for many widely used empirical models. However, Koosheh et al. (2021) highlighted discrepancies between experimental observations and existing empirical formulations for wave overtopping, while similar limitations have been identified in the prediction of wave and crest height distributions (Katsardi et al., 2013, Karmpadakis et al., 2020, Karmpadakis & Swan, 2022). These findings suggest that current formulations may not fully capture the nonlinear transformation of wave statistics across varying coastal bathymetries. Moreover, advances in laboratory technologies, particularly in active wave generation and absorption, now enable the production of higher-fidelity datasets than those underlying many classical empirical models. This provides further motivation for the present work.

To overcome these limitations, numerical models that explicitly resolve nonlinear

hydrodynamic processes are often employed. The SWASH model, as described in Chapters 3 and 4, is extensively used in a wide range of practical design applications due to its ability to resolve many of the coastal processes of interest. Despite its accuracy, however, the computational expense of running high-resolution numerical simulations remains a major limitation for preliminary design applications. In spite of significant advances in both empirical and numerical modelling, there remains a need for efficient yet physically consistent parameterisations capable of reproducing key wave characteristics across diverse coastal bathymetries, particularly during early design stages. Data-driven approaches offer a promising means of bridging this gap by leveraging extensive numerical datasets to extract relationships between offshore conditions, bathymetric features, and nearshore wave statistics.

Accordingly, this study uses a large database of SWASH simulations to develop data-driven parameterisations of representative wave heights and periods over variable coastal bathymetries, addressing Objective (iv) defined in Chapter 1. The proposed formulations aim to retain the physical fidelity of process-based models while offering the computational efficiency required for practical design applications. In doing so, this work provides new insights into the evolution of wave statistics across the coastal zone and presents a framework for integrating data-driven techniques into the design of coastal infrastructure.

5.2 Background

5.2.1 Wave height parameters

In coastal and offshore design, the accurate representation of wave height statistics is fundamental to determining design loads, sediment transport rates, and run-up conditions. Because real seas are irregular and random, engineers require a representative wave height measure that captures the integral characteristics of a sea-state. The selection of this parameter has important implications for safety and cost in design applications.

The concept of a representative wave height was first introduced by Sverdrup & Munk (1947), who defined the significant wave height as the mean of the highest one-third of zero-crossing waves, $H_{1/3}$. This empirically derived definition was chosen to reflect the average wave height perceived visually by sailors. It is important to

distinguish this from the spectral definition of the significant wave height, H_s , as defined by Equation 2.57.

EurOtop (2018) recommends the use of H_s when estimating wave run-up and overtopping on coastal structures, noting that differences between H_s and $H_{1/3}$ may be up to 10-15% in shallow water (Longuet-Higgins, 1980). CEM (2002) also highlight the appropriateness of using H_s when computing sediment transport.

For offshore design, such as for wind turbines, the most probable maximum wave height, H_{max} , is often adopted. Goda (2012) defines this as the 1000th largest wave, which in deep water can be approximated as $H_{max} = 1.86H_{1/3}$. This approximation is consistent with guidance from EurOtop (2018) and has been widely employed in design formulae for composite breakwaters (Goda, 1975c, 2000). These relationships ensure conservative design against extreme wave events, though they depend on the statistical distribution and sampling of waves within a storm. The use of such extreme wave height statistics in the design of offshore structures is also suggested by CEM (2002) through the use of parameters such as $H_{1/10}$, $H_{2\%}$ or $H_{1\%}$.

The choice of wave height definition has a direct impact on predicted loads. Cuomo et al. (2010) demonstrated that using $H_{1/3}$ underestimates quasi-static forces on vertical seawalls when breaking is intense ($H_s/d > 0.6$), whereas using H_{max} yields more conservative predictions. A similar trend arises in impact load estimations using the classical approaches of Hiroi (1920), Minikin (1963), and Allsop & Vicinanza (1996). Cuomo et al. (2010) therefore proposed a new impact load parameterisation based on H_s , improving agreement with laboratory data. This highlights the importance of selecting appropriate representative wave height parameters in design.

Extreme wave heights within a sea-state are typically described statistically using a short-term distribution. The Rayleigh distribution (Longuet-Higgins, 1952), derived under the narrow-banded linear assumption, has long been the standard. However, this distribution tends to overestimate large waves in shallow regions (SPM, 1984), since it neglects wave breaking and spectral bandwidth effects (Karpadakis et al., 2020). The more recent distribution proposed by Karpadakis et al. (2022) includes both effects but remains limited to flat bed configurations.

In sloping bed environments, Goda (2000) introduced an empirical approach to describe wave height evolution accounting for shoaling and breaking effects, providing parametric maps for H_s and H_{max} as functions of depth and slope (1:10–1:100).

However, this approach omits infragravity wave contributions, which limits its accuracy in very shallow water (EurOtop, 2018).

An alternative is the distribution of Battjes & Groenendijk (2000) (hereafter BG00), which explicitly includes wave breaking by combining two Weibull distributions for pre- and post-breaking wave regimes:

$$Q(H) = \begin{cases} \exp \left[- \left(\frac{H}{H_1} \right)^{K_1} \right] & \text{for } H \leq H_{tr} \\ \exp \left[- \left(\frac{H}{H_2} \right)^{K_2} \right] & \text{for } H > H_{tr} \end{cases} \quad (5.1)$$

where

$$K_1 = 2, K_2 = 3.6, \quad (5.2)$$

and the transitional height is

$$H_{tr} = (0.35 + 5.8 \tan \alpha) d, \quad (5.3)$$

with α the bed slope angle in radians in the direction of dominant wave propagation. H_1 and H_2 are tabulated in BG00 based on the ratio of H_{tr} to the root-mean-squared (RMS) wave height, H_{rms} .

Laboratory and field studies (Katsardi & Swan, 2011, Caires & Van Gent, 2012) have validated BG00 in finite-depth and mildly sloping environments, though some underprediction occurs in shallow zones (Wu et al., 2016). Application of BG00 requires an estimate of H_{rms} , which is approximated as:

$$H_{rms} = \left(2.69 + 3.24 \frac{\sigma_\eta}{d} \right) \sigma_\eta, \quad (5.4)$$

where σ_η is the standard deviation of the surface elevation and can be related to the significant wave height as $\sigma_\eta = \sqrt{m_0} = \sqrt{\frac{H_s^2}{16}}$. We note the absence of a spectral bandwidth dependence on this formula (Karnpadakis et al., 2020). Regardless, the accurate estimation of H_{rms} is critical for the successful evaluation of the short-term wave height distribution. CEM (2002) also highlights the importance of describing H_{rms} to determine the set-up associated with irregular sea-state by adopting the formula of Dally et al. (1985).

When calculating H_{rms} , it is important to distinguish between the definition based on the spectral moment, H_{rms} , as given by Equation 5.5 and that based on

the zero-crossing wave height, $\hat{H}_{rms}$, as given by Equation 5.6.

$$H_{rms} = \sqrt{8m_0} \quad (5.5)$$

$$\hat{H}_{rms} = \sqrt{\frac{1}{N} \sum_i H_i^2} \quad (5.6)$$

where H_i are individual wave heights and N is the number of waves. The definitions given by Equation 5.5 and 5.6 are only equivalent in the narrow-banded limit of linear theory (Karmadakis, 2019). In the present work, the spectral definition, H_{rms} , will be adopted. It is also important to highlight the relationship between the spectral root-mean squared wave height, H_{rms} , and the spectral significant wave height, H_s . Given the definitions in Equations 2.57 and 5.5, the ratio H_{rms}/H_s remains a constant equal to $\frac{1}{\sqrt{2}}$.

To predict the spatial evolution of wave heights, many models adopt a representative wave approach, balancing energy flux with dissipation due to breaking:

$$\frac{\rho g}{8} \frac{\partial(H_{rms}^2 c_g \cos \theta)}{\partial x} = -D_B, \quad (5.7)$$

where ρ is the water density, c_g is the group velocity and $\cos \theta$ incorporates wave refraction effects. The accuracy of this method is largely dependent on obtaining an appropriate energy dissipation model, D_B , and breaker height H_b . Most existing formulations adopt empirical or semi-empirical models; however, the experimental conditions on which the models are calibrated are generally limited in scope. An extensive range of literature has been dedicated to obtaining accurate wave height evolution models. Table 5.1 summarises several widely used formulations.

Building on the findings of Le Mehaute (1962), Battjes & Janssen (1978) (BJ78) first estimated the dissipation rate of a breaking wave using a bore analogy. Thornton & Guza (1983) (TG83) subsequently adopted a modified version of the bore model to describe the distribution of breaking wave heights as a weighted Rayleigh distribution. Later, Battjes & Stive (1985) (BS85) retained the BJ78 dissipation model but modified the breaker height. Similarly, Southgate & Nairn (1993) (SN93) combined BJ78 with the breaker height formulation of Nairn (1990). In contrast, Baldock et al. (1998) (BHV98) proposed a new formulation integrating the dissipation of a single broken wave with the Rayleigh distribution. Rattanapitikon & Shibayama (1998)

(RS98) modified this model using the stable energy concept, and Ruessink et al. (2003) (RWS03) recalibrated BHV98 based on experimental measurements. Alsina & Baldock (2007) (AB07) further refined BHV98 to obtain a more complete dissipation formulation, which was subsequently adopted by Janssen & Battjes (2007) (JB07) alongside the breaker height of Nairn (1990).

Given the large number of available formulations, in coastal engineering design practice, the evolution of the root-mean-squared wave height, H_{rms} , is most commonly represented using the BJ78 formulation or its variants. These models are widely recommended in design manuals such as EurOtop (2018), CEM (2002), SPM (1984), and Det Norske Veritas (2010), primarily due to their simplicity, robustness, and ability to provide conservative estimates for design purposes. The BJ78 framework, often combined with the BG00 distribution for shallow water, is particularly recommended when wave breaking and limited water depth effects are significant. While more recent recalibrated formulations can improve accuracy for specific morphodynamic conditions, they are less commonly adopted in routine design practice and are typically reserved for site-specific studies or research applications. Section 5.4.1 provides comparisons between these models and evaluates their accuracy.

5.2.2 Wave period parameters

While wave height governs structural loading and energy transport, wave period plays a central role in defining design conditions for coastal infrastructure (Baarholm et al., 2010), controlling changes in coastal morphology (Aagaard & Greenwood, 2008), and predicting coastal hazards (Bowers, 1977). The probability density of the wave period developed by Bretschneider (1969) assumes the wave period squared follows a Rayleigh distribution, with mean period

$$T_{m1} = \frac{m_0}{m_1}. \quad (5.8)$$

EurOtop (2018) suggest that the mean period is related to the peak spectral period, T_p , as $T_p = 1.1 - 1.25T_{m1}$. Alternatively, Det Norske Veritas (2010) account for spectral bandwidth effects such that

$$\frac{T_{m1}}{T_p} = 0.7303 + 0.04936\gamma - 0.006556\gamma^2 + 0.0003610\gamma^3, \quad (5.9)$$

TABLE 5.1: H_{rms} evolution models

Model	Dissipation Formulation, D_b	Breaker height, H_b
BJ78	$Q_b \frac{\rho g H_b^2}{4T_p}$	$0.14L \tanh(0.91kd)$
TG83	$\frac{3\sqrt{\pi}}{4} \left(\frac{H_{rms}}{H_b} \right)^2 \left(1 - \frac{1}{(1+[H_{rms}/H_b]^2)^{2.5}} \right) \frac{\rho g H_{rms}^3}{4T_p d}$	$0.42d$
BS85	$Q_b \frac{\rho g H_b^2}{4T_p}$	$0.14L \tanh \left\{ \left[0.57 + 0.45 \tanh \left(33 \frac{H_{rms,0}}{L_0} \right) \right] kd \right\}$
SN93	$Q_b \frac{\rho g H_b^3}{4T_p d}$	$d \left[0.39 + 0.56 \tanh \left(33 \frac{H_{rms,0}}{L_0} \right) \right]$
BHV98	$\begin{cases} \exp \left[- \left(\frac{H_b}{H_{rms}} \right)^2 \right] \frac{\rho g (H_b^2 + H_{rms}^2)}{4T_p} & \text{for } H_{rms} < H_b \\ \exp[-1] \frac{\rho g 2H_b^2}{4T_p} & \text{for } H_{rms} \geq H_b \end{cases}$	$d \left[0.39 + 0.56 \tanh \left(33 \frac{H_{rms,0}}{L_0} \right) \right]$
RS98	$0.1Q_b \frac{c_g \rho g}{8h} \left[H_{rms}^2 - \left(d \exp(-0.58 - 2 \frac{d}{\sqrt{L H_{rms}}}) \right)^2 \right]$	$0.1L_0 \left\{ 1 - \exp \left[-1.5 \frac{\pi d}{L_0} (1 + 15m^{4/3}) \right] \right\}$
RWS03	$\begin{cases} \exp \left[- \left(\frac{H_b}{H_{rms}} \right)^2 \right] \frac{\rho g (H_b^2 + H_{rms}^2)}{4T_p} & \text{for } H_{rms} < H_b \\ \exp[-1] \frac{\rho g 2H_b^2}{4T_p} & \text{for } H_{rms} \geq H_b \end{cases}$	$0.14L \tanh [(0.86kd + 0.33)kd]$
AB07	$\frac{\rho g H_{rms}^3}{4T_p d} \left\{ \left[\left(\frac{H_b}{H_{rms}} \right)^3 + \frac{3H_b}{2H_{rms}} \right] \exp \left[- \left(\frac{H_b}{H_{rms}} \right)^2 \right] + \frac{3\sqrt{\pi}}{4} \left[1 - \operatorname{erf} \left(\frac{H_b}{H_{rms}} \right) \right] \right\}$	$0.14L \tanh \left\{ \left[0.57 + 0.45 \tanh \left(33 \frac{H_{rms,0}}{L_0} \right) \right] kd \right\}$
JB07	$\frac{\rho g H_{rms}^3}{4T_p d} \left\{ \left[\left(\frac{H_b}{H_{rms}} \right)^3 + \frac{3H_b}{2H_{rms}} \right] \exp \left[- \left(\frac{H_b}{H_{rms}} \right)^2 \right] + \frac{3\sqrt{\pi}}{4} \left[1 - \operatorname{erf} \left(\frac{H_b}{H_{rms}} \right) \right] \right\}$	$d \left[0.39 + 0.56 \tanh \left(33 \frac{H_{rms,0}}{L_0} \right) \right]$

where γ is the peak enhancement factor of the JONSWAP spectrum. It should be noted that Equation 5.9 relies on the local value of the peak period, T_p . As such, it cannot be applied if only the offshore magnitude of wave period parameters are known.

For coastal structure design, the mean zero-crossing period, $\overline{T_z}$, may also be adopted. This parameter requires long time histories of the surface elevation, which are often unavailable. In such cases, Cartwright & Longuet-Higgins (1956) suggest that it is proportional to

$$T_{m2} = \sqrt{\frac{m_0}{m_2}}, \quad (5.10)$$

which is further related to the peak period as $T_{m2} = 0.78T_p$ (Boccotti, 2000). The mean energy period, T_{m-1} , defined as

$$T_{m-1} = \frac{m_{-1}}{m_0}, \quad (5.11)$$

places additional weight on low-frequency waves. These low-frequency, infragravity waves play a critical role in sediment transport and beach morphology (Aagaard & Greenwood, 2008, Baldock et al., 2011, De Bakker et al., 2016), and their prominence in the shoaling and surf zone was first noted by Munk (1949) and Tucker (1950).

Infragravity waves are generated via two primary mechanisms. The first is the shoaling of bound low-frequency waves, where triad interactions between short-wave components and bound long waves amplify low-frequency energy during shoaling (Longuet-Higgins & Stewart, 1962, Freilich & Guza, 1984, Baldock & Huntley, 2002, Battjes, 2004, Janssen et al., 2003). Bound waves may also be progressively released as free long waves in shallow water. The second mechanism arises from the time-varying breakpoint (surf-beat), where the location of wave breaking varies temporally, generating free low-frequency waves (Symonds et al., 1982). The dominant mechanism depends on the bed slope and initial wave steepness (Van Dongeren et al., 2007), with bound-wave shoaling prevailing for mild slopes. This is quantified by the surf-similarity-like parameter

$$\beta_b = \frac{\alpha}{\omega} \sqrt{\frac{g}{d_b}}, \quad (5.12)$$

where α is the bed slope angle, ω is the angular frequency of the long waves, and d_b

is the mean breaker depth of the primary short waves.

The correlation of T_{m-1} has been established for a range of coastal processes, including wave overtopping (Altomare et al., 2016, Van Gent et al., 2007, Van Gent & Smith, 1999), reflection (Dekker et al., 2009, Zanuttigh & van der Meer, 2008), armour layer stability (Van Gent et al., 2003), and wave impact (Chen et al., 2016). In design, the empirical formulation of Hoffland et al. (2017) is often applied for T_{m-1} in shallow ($1 < d/H_{s,0} < 4$) to extremely shallow ($d/H_{s,0} < 0.3$) foreshores with mild slopes. For long-crested waves,

$$\frac{T_{m-1}}{T_{m-1}^o} = 1 + 6 \exp(-4\tilde{d}) + \exp(-\tilde{d}), \quad (5.13)$$

where $\tilde{d} = d/H_s^o (m/100)^{0.2}$, $m = \cot \alpha$, and H_s^o is the offshore spectral significant wave height. For short-crested waves with an offshore directional spread of 25° ,

$$\frac{T_{m-1}}{T_{m-1}^o} = 1 + 6 \exp(-6\tilde{d}) + 0.25 \exp(-0.75\tilde{d}). \quad (5.14)$$

In this chapter, existing formulations are assessed. Subsequently, new data-driven formulations for the evolution of representative wave periods are developed, explicitly accounting for slope and sea-state parameters. These include updated relationships for the mean energy period T_{m-1} , mean spectral period T_{m1} , and mean zero-crossing period $\overline{T_z}$. This provides improved predictive representation of wave transformation processes for engineering design while balancing physical insight and computational efficiency.

5.3 Numerical set-up

Following the numerical set-up explained in Chapter 3, the 21 sea-states outlined in Table 3.1 are numerically modelled in SWASH across five uniform bed slope configurations: 1:10, 1:15, 1:30, 1:50 and 1:80. This follows the bathymetric configuration outlined in Chapter 4. Hence, a total of 105 sea-state conditions are considered, each simulated using 20 realisations (seeds) as described in Chapter 3 to produce long time records. These form an extensive numerical dataset consisting of more than 2 million waves each simulated for approximately 3 hours in field scale.

To ensure that the proposed data-driven formulations generalise beyond the

dataset used for their development, the complete numerical dataset was divided into separate training and testing subsets. Simulations corresponding to the bed slope of 1:50 were retained as an independent testing set to evaluate predictive performance. This subset represents approximately 20% of the available dataset. The remaining 80% of simulations were used for training the regression models and determining the model coefficients. This partitioning approach ensures that the derived relationships are not over-fitted to a specific subset of the numerical data and remain representative of unseen conditions. The robustness of this approach will be discussed in Section 5.6.

Given that this section of work aims to provide practical guidance, all results presented in this chapter are converted to field scale. Core results will be presented using the test cases in Group B for brevity, since the findings in the present study are consistent across the whole parameter space.

5.4 Wave height parameters

First, we consider the evolution of wave height parameters, as defined in Section 5.2.1. Figure 5.1 compares the evolution of commonly used wave height parameters with decreasing effective water depth, $k_p d$, for waves propagating over a 1:30 slope in sea-state B3. As expected, as the exceedance probability associated with a given wave height parameter decreases (e.g. from $H_{1/3}$ to $H_{1/10}$), the corresponding wave heights increase across the spatial domain, as shown in Figure 5.1.

The most notable change is observed due to wave breaking. This acts to reduce wave height parameters with decreasing $k_p d$. However, there are variations in breaking behaviour, both in terms of the onset of breaking saturation and subsequent energy dissipation. Extreme statistics, representing larger waves, reach breaking saturation earlier (at larger $k_p d$), while parameters describing smaller waves saturate later. However, the more extreme wave height metrics also experience stronger energy dissipation after breaking, leading to a smaller difference between the various wave height parameters closer to the shore. Moreover, smaller exceedance probabilities (e.g. $H_{1\%}$, $H_{2\%}$, and $H_{1/10}$) exhibit a slight initial increase, which is not observed for the other parameters. This increase arises from the transition between the flat and sloping portions of the bathymetry, and its magnitude is more pronounced for

steeper slopes.

A subtle but important distinction evident in Figure 5.1 is between H_s and $H_{1/3}$. The former is based on spectral estimates, while the latter is obtained from time-series measurements. While they agree well in deeper water ($k_p d < 0.6$), notable discrepancies appear in shallow water ($k_p d > 0.6$). A similar observation applies to the spectral H_{rms} compared with the time-series-based $\hat{H}_{rms}$, which are often used interchangeably. These discrepancies arise due to spectral bandwidth effects, which are neglected under the narrow-banded assumption (Longuet-Higgins, 1980). Although these are established offshore, the implications of coastal wave evolution on these differences are not known.

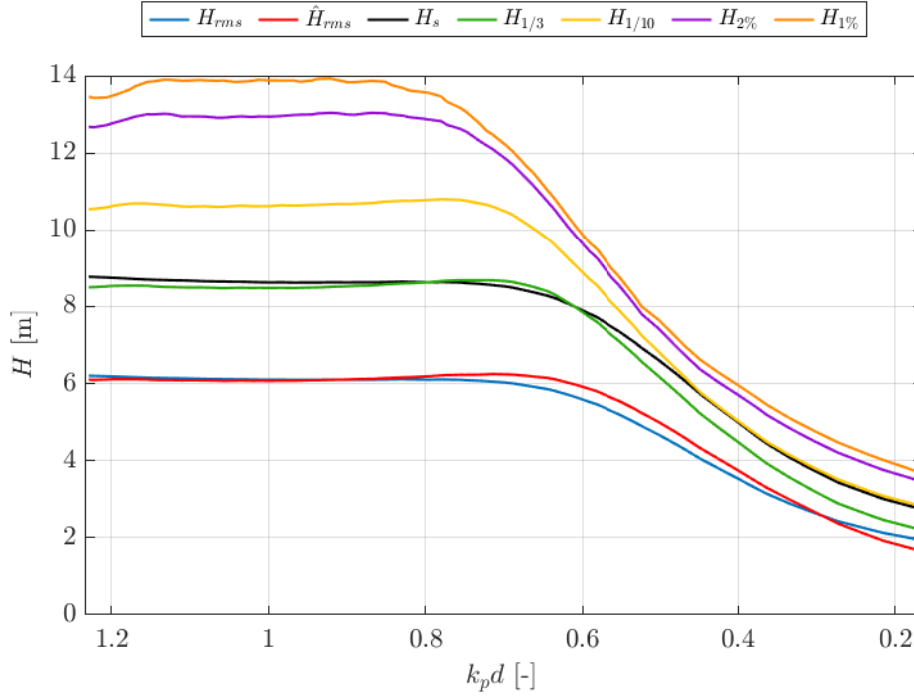


FIGURE 5.1: Evolution of wave height parameters with reduced effective water depth, $k_p d$, over a bed slope of 1:30 for sea-state B3. The colour of the line corresponds to the different integral wave height parameters.

5.4.1 Existing H_{rms} models

The BG00 distribution is assumed to provide a reasonable representation of the distribution of coastal waves propagating up uniform sea-bed slopes; this assumption is explored further in Section 5.4.3. Given its dependence on H_{rms} , this section

evaluates the existing parameterisations summarised in Table 5.1. Using SWASH simulations for mild (sea-state B2) and steep (sea-state B5) conditions over a 1:30 slope, the evolution of H_{rms} is compared in Figure 5.2.

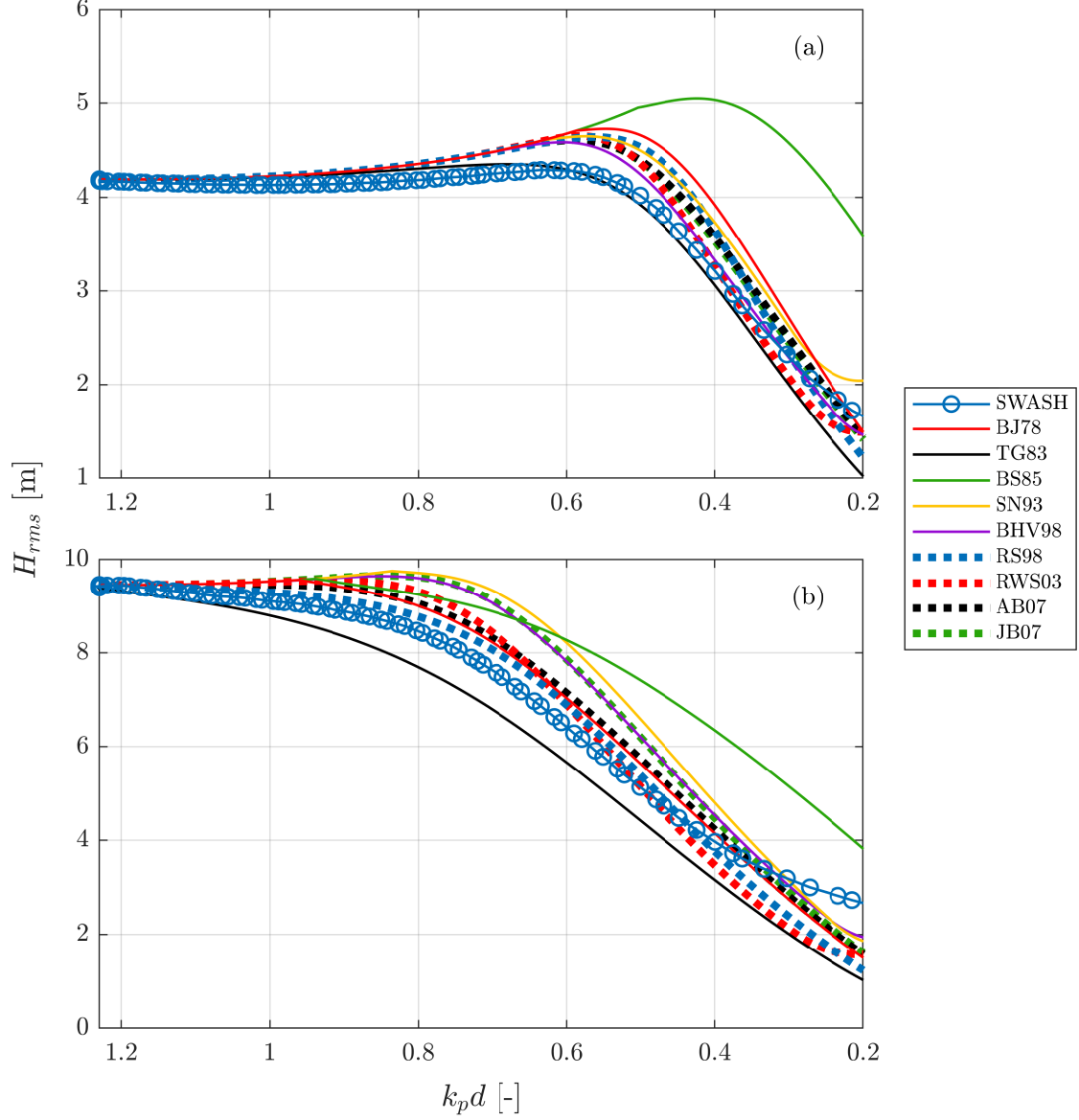


FIGURE 5.2: Evolution of root-mean-squared wave height, H_{rms} , with reduced effective water depth, $k_p d$, over a bed slope of 1:30 for sea-state (a) B2 and (b) B5. Each model is shown in a different line, with SWASH results distinguished by circular markers.

In mild sea conditions (Figure 5.2(a)), all models perform well for $k_p d \geq 0.8$, where nonlinear and breaking effects are limited. However, discrepancies become evident in shallower water, where most models tend to over-predict shoaling. Despite this, excluding BS85, the models converge toward the numerical results in shallow

water ($k_p d < 0.3$). The TG83 model performs best overall in this regime, maintaining good agreement for $k_p d \geq 0.2$.

In contrast, under extreme sea conditions (Figure 5.2(b)), TG83 overestimates the effect of breaking, leading to consistent underestimation of H_{rms} . The RS98 model provides the most accurate predictions in this context. Other models exhibit overly strong shoaling followed by an abrupt onset of breaking, whereas the SWASH results show more gradual breaking over an extended region. This is followed by a plateau in shallow water due to wave field saturation. Although recalibration may improve performance under specific conditions, none of the examined formulations perform consistently across all sea-states and bathymetries. Earlier work by Ratnapitikon (2007) recalibrated these models against experimental data; however, the improvements were minor. This highlights the motivation for the present study, to develop alternative data-driven parameterisations that achieve numerical-level accuracy without the computational cost.

5.4.2 H_{rms} evolution

The analysis of existing dissipation models reveals a clear need for a consistent and accurate formulation for H_{rms} evolution in coastal waters. This section develops a practical, data-driven model that captures H_{rms} behaviour across a range of bathymetries and sea-state conditions. Figure 5.3 presents the evolution of normalised H_{rms} , defined as $H_{rms}/H_{rms,0}$, with $k_p d$. The zero subscript refers to deep water quantities. Since the waves are generated in intermediate water, second-order random wave simulations (Sharma & Dean, 1981) are used to deshoal the field to deep-water conditions. These results suggest that the deep-water RMS wave height is about 5% larger than the deepest measurement in the simulations of sea-state B ($k_p d = 1.22$). Accordingly, $H_{rms}/H_{rms,0} = 1/1.05 \approx 0.95$ at this depth. This normalisation facilitates generalisation of the results, avoiding dependence on arbitrary offshore reference values.

The first factor to address is the physical effect of increased offshore steepness. In mild seas, as shown in Figure 5.3(a), the influence of wave shoaling is evident such that $H_{rms}/H_{rms,0}$ initially increases with reducing water depth. However, in moderate and steep seas as illustrated in Figure 5.3(b) and (c), respectively, the increased prevalence of wave breaking, as early as the start of the slope in the latter

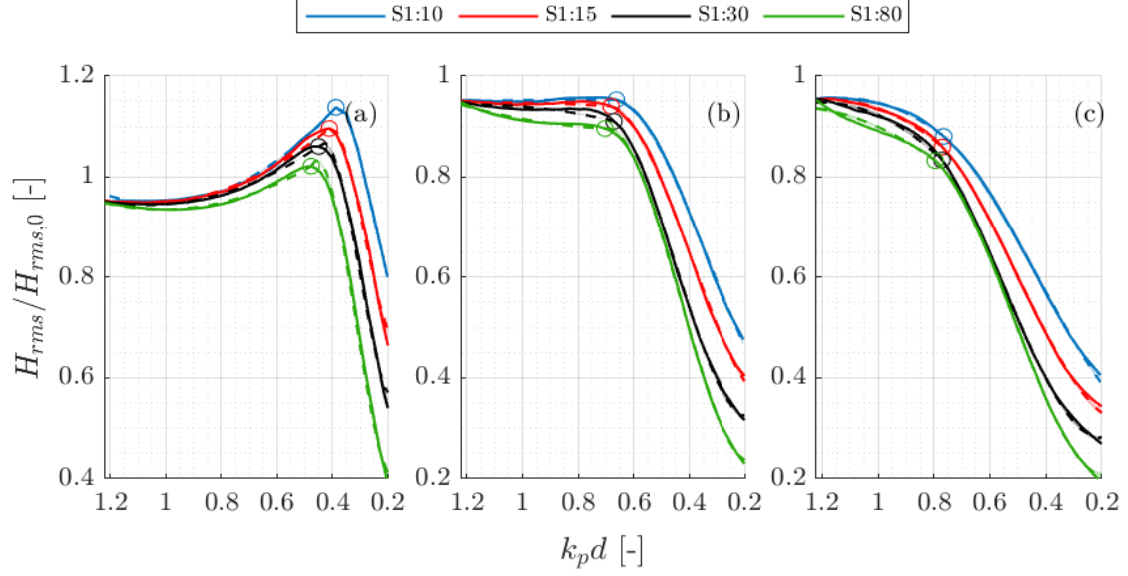


FIGURE 5.3: Evolution of normalised root-mean-squared wave height, $H_{rms}/H_{rms,0}$, with reduced effective water depth, $k_p d$, for sea-state (a) B1, (b) B3 and (c) B5. Line colour indicates bed slope gradient. The fitted distribution is shown by a dashed line. Circular markers indicate the transition between pre- and post-breaking saturation.

case, results in no notable shoaling behaviour. In fact, the earlier breaking saturation in steeper seas is associated with an earlier point at which the wave height begins to reduce. Moreover, the magnitude of energy dissipation following breaking initiation is related to the steepness of the sea-state such that steep seas are associated with a greater reduction in wave height relative to the offshore value.

The second factor to address is the influence of the bed slope gradient. In analysing Figure 5.3, it is evident that the bed slope gradient plays a fundamental role in the evolution of H_{rms} . In mild sea-states, as shown in Figure 5.3(a), the initial shoaling behaviour is observed to be more prominent in steeper bed slopes. As such, the normalised wave height, $H_{rms}/H_{rms,0}$, is globally always larger in steeper slopes. This is inline with the work of Battjes (1972) that defined the breaking height to be proportional to the Irribarren number, $Ir = \tan \alpha / \sqrt{H_0/L_0}$, for regular waves. Moreover, breaking saturation is observed to occur later in steeper slopes such that the maximum normalised wave height is observed in shallower effective water depths, compared to equivalent waves propagating over milder slopes. In steeper sea-states where breaking effects dominate (Figure 5.3(b–c)), the discrepancy between bed slopes is observed to converge in milder configurations such that there is minor

variation for bed slopes smaller than 1:30.

Utilising these observations, a new formulation to describe the evolution of $H_{rms}/H_{rms,0}$ is derived. This utilises a split structure to explicitly define the pre- and post-breaking saturation behaviour using a quadratic and cubic, respectively:

$$\frac{H_{rms}}{H_{rms,0}} = \begin{cases} a_1(k_p d)^2 + b_1 k_p d + c_1, & \text{for } k_p d \geq (k_p d)_{tr}, \\ a_2(k_p d)^3 + b_2(k_p d)^2 + c_2 k_p d + d_2, & \text{for } k_p d < (k_p d)_{tr}. \end{cases} \quad (5.15)$$

where $(k_p d)_{tr}$ denotes the transition depth corresponding to breaking saturation, and $[a_1, b_1, c_1]$ and $[a_2, b_2, c_2, d_2]$ are the pre- and post-breaking coefficients, respectively, expressed as functions of offshore steepness, S_p , and bed slope. The transition location is determined as the inflection point in the $H_{rms}/H_{rms,0}$ curve, defined as a point where a notable change in the slope of the line is observed, caused by the occurrence of strong wave breaking. The algorithm utilised to define this point is supplemented with the aforementioned physical findings. This ensures that the location of breaking saturation occurs further offshore in the case of steeper sea-states and milder slopes. Similarly, the model coefficients are determined utilising a constrained nonlinear regression model such that the magnitude of the coefficients is in line with the physical observations outlined earlier. This ensures that the coefficients $[a_1, b_1, c_1]$ and $[a_2, b_2, c_2, d_2]$ have a monotonic relationship with the model input parameters: sea-state steepness, S_p , and bed slope. This is particularly important to ensure the robustness of the model and allows for the model coefficients to be interpolated across sea-states and bed slope configuration whilst maintaining the model integrity. Moreover, it highlights the novelty of the present work compared to other data-driven alternatives, such that the physical insights observed are used to constrain the regression model. In addition, the wide scope of the sea-states and bathymetries considered in the present work provide a wide range of applicability for the proposed formulations. For brevity the numerical values of the proposed coefficients are provided in Appendix .1 alongside code for the model implementation. This new formulation is illustrated in Figure 5.3 using dashed lines with the location of the transitional effective water depth, $(k_p d)_{tr}$, denoted with circular markers. The proposed model reproduces the numerical data with high fidelity, substantially improving on the formulations shown in Figure 5.2. Since $H_s = \sqrt{2}H_{rms}$, this formulation can also describe H_s evolution.

5.4.3 Wave height percentiles

This section evaluates the accuracy of the BG00 wave height distribution (Equation 5.1) in predicting wave height percentiles, and examines the approximation of local H_{rms} to the local significant wave height, H_s (Equation 5.4).

Figure 5.4 shows the evolution of selected percentiles ($H_{1/3}$, $H_{1/10}$, $H_{2\%}$, $H_{1\%}$) with decreasing $k_p d$. Solid and transparent lines correspond to SWASH data and BG00 predictions, respectively. The BG00 distribution captures the general evolution of extreme wave height percentiles well for mild (Figure 5.4(a)) to extreme (Figure 5.4(c)) sea-states. As expected, the performance of the BG00 distribution is better for statistics associated with a larger probability of exceedance ($H_{1/3}$) compared to relatively smaller exceedance probabilities ($H_{1\%}$). This is particularly noticeable in highly nonlinear seas such as those in Figure 5.4(c), where the full extent of the nonlinearities present are not captured by the model of BG00. Moreover, the application of the BG00 model relies on the local value of H_s , a parameter that is not readily available in practice.

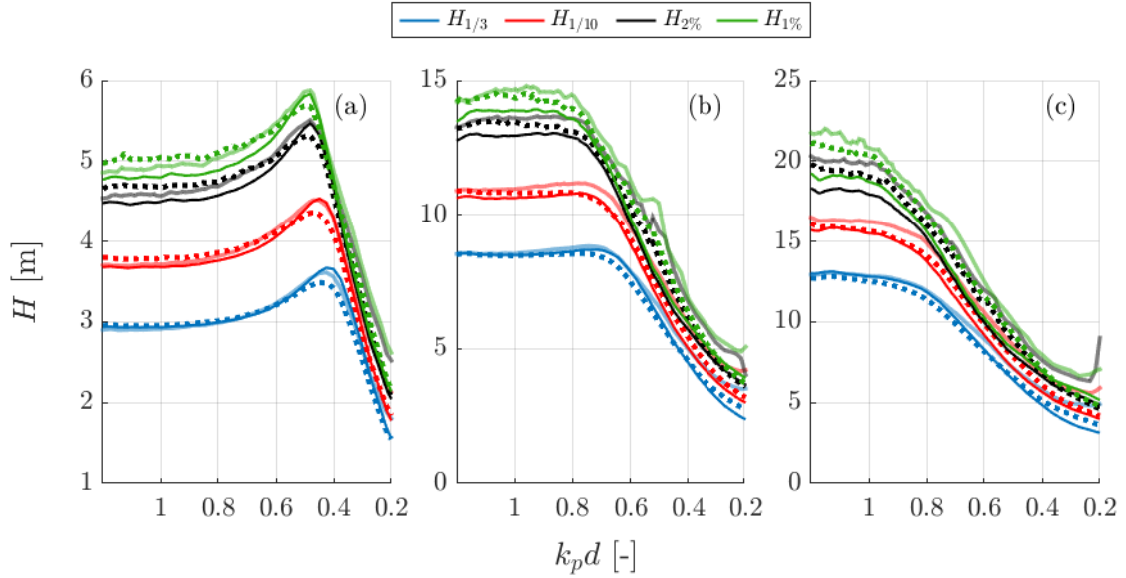


FIGURE 5.4: Evolution of wave height percentiles with reduced effective water depth, $k_p d$, over a 1:30 slope for sea-state (a) B1, (b) B3 and (c) B5. Colours correspond to the different percentiles. Solid, transparent, and dotted lines denote SWASH simulations, BG00, and the proposed formulation, respectively.

A method to improve the accuracy of the model of BG00 is to enhance the description of H_{rms} evolution, used as input to the model of BG00. Thus, instead

of adopting the definition in Equation 5.4, the new formulation proposed in Equation 5.15 is utilised. This is shown by a dotted line on Figure 5.4. This modification is advantageous as the new model proposed in this work for H_{rms} evolution relies solely on parameters measured in deep water, without requiring additional methods to transform the wave properties from deep water to the location of interest. The performance of this new approach is comparable to the model of BG00 for less extreme statistics ($H_{1/3}$ and $H_{1/10}$). In mild seas around the location of breaking saturation, as shown in Figure 5.4(a), the performance of this new approach is marginally compromised. However, enhanced correspondence to numerical simulations is observed for extreme statistics ($H_{2\%}$ and $H_{1\%}$), particularly in steep sea-states (Figure 5.4(b–c)). For very small exceedance probabilities ($H_{1\%}$), discrepancies still exist between the model and the data. Since the proposed H_{rms} evolution model corresponds well to the dataset, as shown in Figure 5.3, the discrepancies observed are due to limitations in the tail of the BG00 model. This observation has also been made by Karpadakis et al. (2022) for waves in intermediate water. Importantly, while the BG00 model requires the local value of H_{rms} , the new approach has comparable or improved accuracy utilising only the offshore properties as input, which is generally what is available in practice.

Quantitatively, model performance is evaluated using $\Delta H = (H_{model} - H_{SWASH}) / H_{SWASH}$, where H_{model} and H_{SWASH} are the wave height percentiles predicted by the model of BG00 and the present SWASH dataset, respectively. Figure 5.5 illustrates the variation of ΔH for selected $k_p d$ values. The first and second row correspond to modelling using the original BG00 and the proposed formulation, respectively. The colour scheme adopted is consistent with those in Figure 5.4. We observe that the discrepancy between the modelled and measured wave height percentile values increases with growing sea-state steepness, for both modelling methods. Moreover, these discrepancies are generally manifested as overestimations by the BG00 model. However, the utilisation of the new H_{rms} formulation in Equation 5.15 is shown to provide a mean reduction in the magnitude of ΔH by 5% across the wave statistics investigated. This improvement can be as large as 21% in the case of extreme wave statistics.

The integration of the new H_{rms} formulation therefore enhances predictive accuracy while maintaining practical usability, requiring only offshore wave parameters,

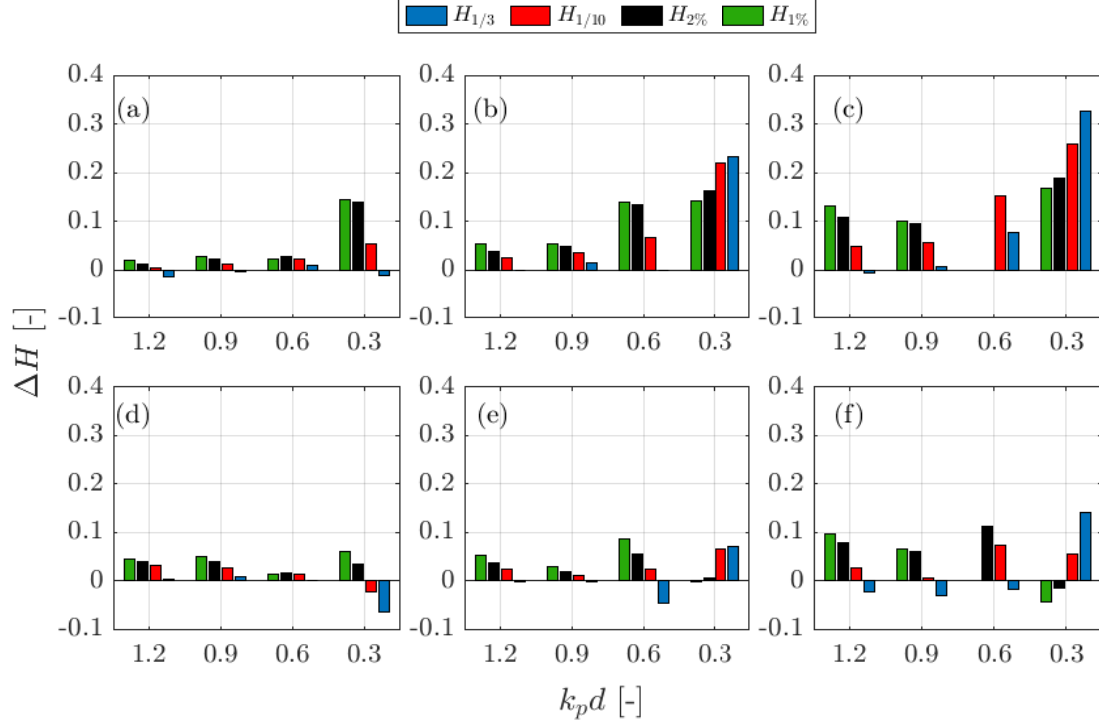


FIGURE 5.5: Error in predicted wave height percentiles, ΔH , for a 1:30 slope and sea-states (a)&(d) B1, (b)&(e) B3 and (c)&(f) B5. The first and second rows correspond to the BG00 and proposed formulation, respectively.

consistent with the information typically available in engineering design applications.

5.5 Wave period parameters

Understanding the characteristics of different wave period parameters is fundamental to selecting an appropriate critical measure for engineering applications. Figure 5.6 shows the evolution of wave period parameters normalised by their deep-water value, T/T_0 , with reduced $k_p d$ for a bed slope of 1:30 for sea-states (a) B1, (b) B3 and (c) B5. The colour of each line corresponds to the various wave period parameters considered. Following the methodology described in Section 5.4.2, second-order random wave simulations are employed to deshoal the wave field to deep water. This analysis indicates a growth of approximately 0%, 3%, 1%, 1% and 2% in T_p , $\overline{T_z}$, T_{m1} , T_{m2} and T_{m-1} , respectively, from deep water to the deepest gauge in the simulations (G1 in Figure 4.1).

Considering first the peak period, T_p , minimal changes are observed initially;

however, significant increases occur in shallow water. In this region, the spectral peak shifts into the infragravity regime, as illustrated by the dashed line in Figure 5.6. When the definition of T_p is restricted to periods near the input T_p as defined in Table 3.1, the corresponding evolution is represented by the solid blue line.

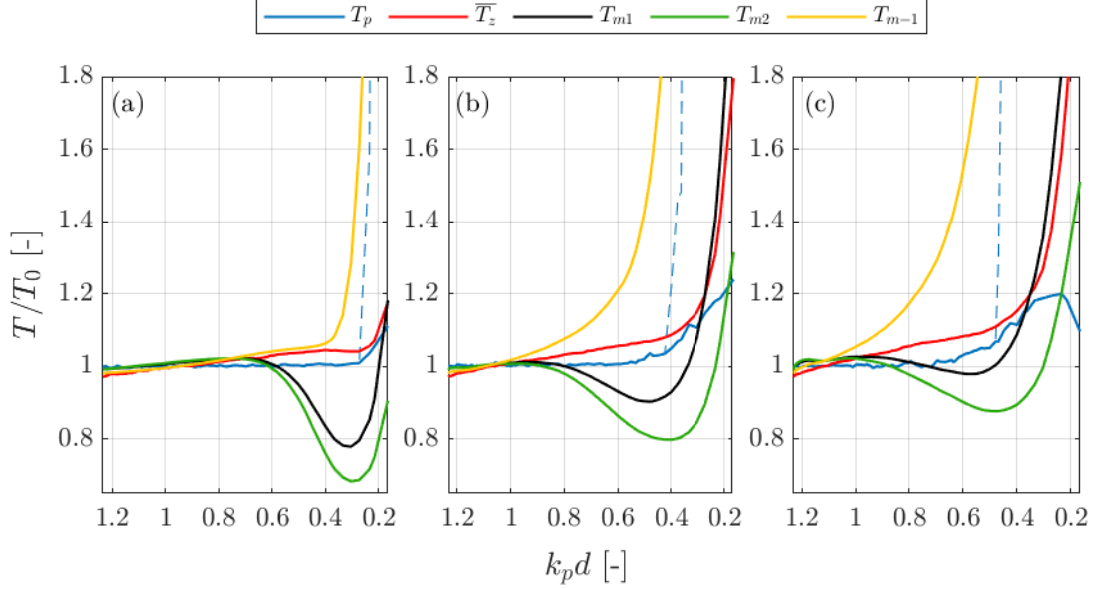


FIGURE 5.6: Evolution of normalised wave period parameters with reducing effective water depth, $k_p d$, for a bed slope of 1:30 for sea-states (a) B1, (b) B3 and (c) B5. The colour of each line corresponds to a different integral wave period parameter.

The evolution of the mean zero down-crossing wave period, $\overline{T_z}$, shows a gradual increase with reducing $k_p d$ from the offshore region. This increase indicates a progressive lengthening of the average wave period, partly due to energy transfer to infragravity waves. As with T_p , a pronounced increase in $\overline{T_z}$ is observed in shallow water, where sub-harmonic energy growth dominates.

The parameters T_{m1} and T_{m2} favour the weighting of higher frequency components given that they are defined based on the first and second spectral moment, respectively. Consequently, both parameters initially remain relatively stable followed by a notable decrease with reducing $k_p d$, as nonlinearities transfer energy away from the primary spectral peak. This energy redistribution forms a secondary peak in the superharmonic region. Since higher-order moments emphasise the high-frequency range, the relative reductions in m_1 and m_2 are smaller than in m_0 , resulting in a corresponding decrease in T_{m1} and T_{m2} . The global minima in these parameters mark the spatial locations where breaking saturation first occurs, which in turn oc-

curs in deeper water for steeper seas. Beyond this point, wave breaking removes high-frequency energy, while infragravity energy grows substantially, leading to increases in both T_{m1} and T_{m2} in shallow water.

In practice, T_{m2} is often used interchangeably with $\overline{T_z}$ in engineering design (Det Norske Veritas, 2010) and as an output parameter in phase-averaged models (Booij et al., 1999). However, the results in Figure 5.6 highlight a clear distinction between these parameters. Although their magnitudes are comparable in deeper water, T_{m2} becomes significantly smaller than $\overline{T_z}$ in intermediate and shallow depths. This behaviour aligns with the expected deviation between spectral and zero-crossing periods in non-Rayleigh seas, where dissipation of high-frequency energy during breaking reduces T_{m2} more strongly than $\overline{T_z}$ (Holthuijsen, 2010). Caution should therefore be exercised when substituting one parameter for the other in practical applications.

The parameter T_{m-1} instead places greater weighting on the infragravity region. Hence, the initial superharmonic growth during shoaling is not evident, while the subsequent infragravity amplification dominates. This manifests as an increase in T_{m-1} with decreasing effective water depth, most notably for steeper sea-states. Compared to other parameters, the enhanced weighting towards sub-harmonics results in substantially larger shallow-water values of T_{m-1} . This is attributed to the increasing relative contribution of infragravity wave components to the total spectrum as the short-wave energy is progressively dissipated by breaking. Given that breaking effects are more pronounced in shallower water for steeper sea-states, these cases correspond to the regions in which significant increases in T_{m-1} are observed. These findings are consistent with those of EurOtop (2018), where values in extremely shallow foreshores ($d/H_{s,0} < 0.3$) can reach up to eight times their offshore equivalents in steep seas.

The parameter T_{m-1} , defined using spectral moments over the full frequency range (Equation 5.11), is highly sensitive to changes in spectral shape associated with significant infragravity-wave growth and concurrent short-wave energy dissipation (Bertin et al., 2018). Consequently, increases in T_{m-1} do not necessarily indicate more severe hydrodynamic forcing but may instead reflect a redistribution of wave energy across the spectrum. In such cases, Altomare et al. (2016) demonstrated that the use of this wave period measure can lead to overestimations when applying existing wave overtopping formulae. This raises the question of whether short-

wave and low-frequency components should be treated separately when interpreting spectral wave period measures. Supporting this view, Dermentzoglou et al. (2025) showed that an alternative definition of T_{m-1} , computed using only the primary wave-frequency band, remains relatively constant over salt marshes, in contrast to the substantial increases obtained when the full frequency range is considered. These findings highlight the need for modified design approaches that explicitly account for frequency-dependent wave attenuation. Nevertheless, to maintain consistency with the existing literature, this chapter adopts the definition of T_{m-1} given in Equation 5.11. Accordingly, caution should be exercised when applying this wave period measure directly within existing overtopping design formulations (EurOtop, 2018, Van Gent & Smith, 1999).

5.5.1 Existing T_{m-1} models

Given the importance of T_{m-1} in coastal design, existing predictive formula are now assessed. The formulation of Hofland et al. (2017) is widely adopted to estimate T_{m-1} in coastal waters and will be the focus of this section. Figure 5.7 shows T_{m-1}/T_{m-1}^o plotted against the non-dimensional parameter $d/H_s^o(m/100)^{0.2}$. Here, T_{m-1}^o and H_s^o denote offshore values, consistent with Hofland et al. (2017). Since the effective water depths at the offshore limit of this work are comparable to Hofland et al. (2017) ($k_p d = 1.18$), this comparison is appropriate.

Numerical data are shown as coloured markers corresponding to bed slope. The long-crested and short-crested formulations of Hofland et al. (2017) are shown in blue and green, respectively. The long-crested formulation agrees well with the numerical results for a 1:15 slope, while the short-crested model corresponds better to the steeper 1:10 slope, which experiences weaker nonlinear interactions and less infragravity growth. However, for milder slopes ($\leq 1:30$), the Hofland et al. (2017) model significantly underpredicts T_{m-1} . This is noteworthy because the original formulation assumes mild slopes characterised by low surf-similarity parameters (β_b), where bound long-wave shoaling dominates infragravity generation. The observed underprediction implies that even when these conditions are met, infragravity energy magnitudes are considerably higher than predicted, leading to non-conservative design estimates of nearshore wave loading and overtopping. Additionally, the experimental dataset of Hofland et al. (2017) only considered $d/H_s^o > 0.15$, whereas

the present dataset extends to $d/H_s^o = 0.05$, allowing for the examination of stronger infragravity contributions in very shallow water.

An alternative formulation was proposed by Mahmoudof et al. (2023) based on field data from the Southern Caspian Sea. However, comparison with the present dataset showed minimal improvement relative to Hofland et al. (2017). Moreover, since the Mahmoudof et al. (2023) model was developed for mild seas, it is not considered further.

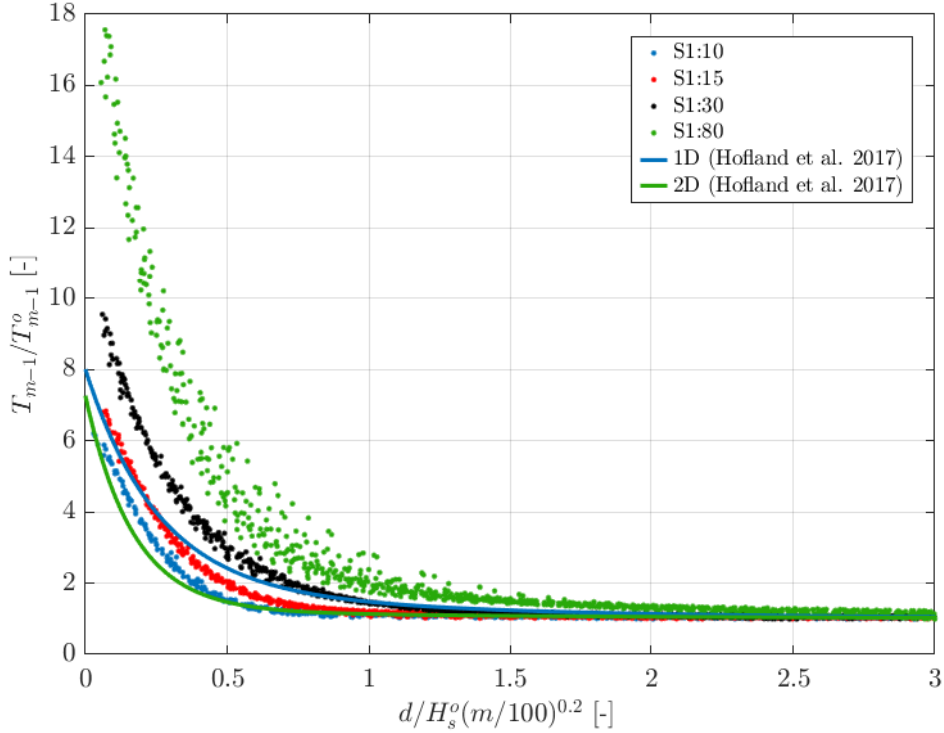


FIGURE 5.7: Assessment of the T_{m-1} formulation of Hofland et al. (2017), shown in blue and green lines for the long-crested and short-crested cases, respectively. Numerical results are shown as markers, with colour corresponding to bed slope gradient.

5.5.2 T_{m-1} evolution

Based on the lack of correspondence between existing formulations and the current dataset, it is evident that a new formulation is desired. This work seeks to provide a new empirical formulation with a wide scope such that it can be applied to any uniform bed slope configuration, without assumptions of the mildness of the

configuration a priori. The general expression for this new formulation is given by

$$\frac{T_{m-1}}{T_{m-1,0}} = 1 + a_4 \exp(-b_4 \check{d}), \quad (5.16)$$

where the explanatory variable $\check{d}$ is based on deep-water measurements:

$$\check{d} = d/H_{s,0}(m/100)^{0.2}. \quad (5.17)$$

This formulation offers improved generality for design applications. Unlike Hofland et al. (2017), the coefficients a_4 and b_4 are defined as explicit functions of bed slope, m . Figure 5.8(a) presents $T_{m-1}/T_{m-1,0}$ against $\check{d}$, with numerical data shown as coloured markers by slope. A constrained fitting algorithm was employed to determine a_4 and b_4 for each slope, yielding excellent agreement across all bathymetries.

Figures 5.8(b–c) show the variation of a_4 and b_4 with m , respectively. The coefficient a_4 increases linearly with milder slopes, following

$$a_4 = 0.18m + 5.03. \quad (5.18)$$

Conversely, b_4 decreases exponentially with milder slopes, as

$$b_4 = 2.97 + 5.16 \exp(-0.13m). \quad (5.19)$$

These results indicate a previously unaccounted slope dependence of the coefficients, absent from Hofland et al. (2017). Physically, as m increases (milder slopes), the relative enhancement of T_{m-1} becomes larger (greater a_4), while the depth-dependent rate of transformation becomes more gradual (smaller b_4). This reflects the enhanced persistence and gradual release of infragravity energy on gentler slopes.

5.5.3 T_{m1} and $\overline{T_z}$ evolution

Few existing formulations describe the evolution of the mean period, T_{m1} , and mean zero-crossing period, $\overline{T_z}$. Current design guidance typically relates these parameters to locally defined T_p , as discussed in Section 5.2.2. Here, the approach used in Section 5.5.2 is extended to develop empirical relationships for these parameters in the coastal zone utilising only offshore defined values.

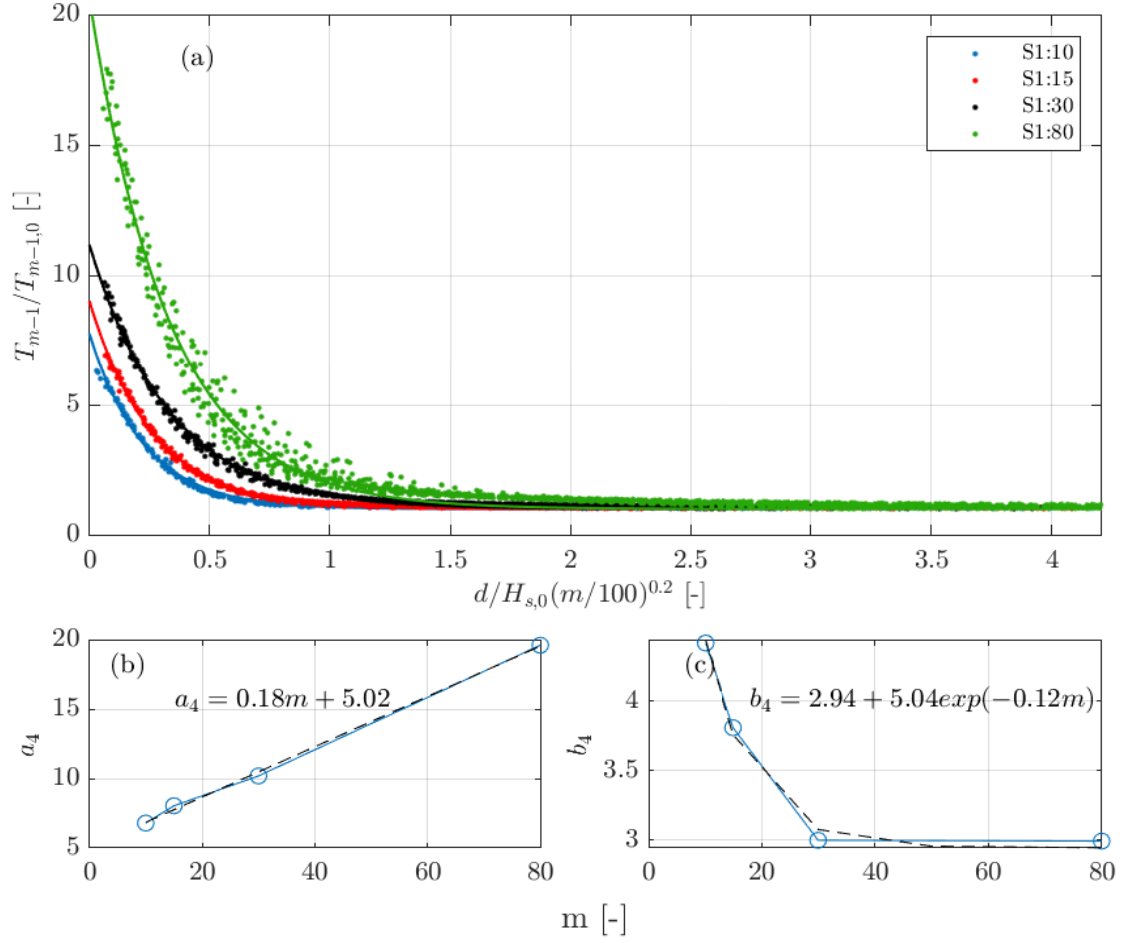


FIGURE 5.8: New T_{m-1} formulation. (a) Data are shown as solid markers, coloured by bed slope. Fitted lines for each slope are shown in corresponding colours. (b) Variation of model parameter a_4 and (c) b_4 with bed slope, m .

Given their distinct dependence on bed slope, the following explanatory variable is introduced:

$$\hat{d} = d/H_{s,0} (m/100)^{-0.4}. \quad (5.20)$$

The exponent was obtained through a sensitivity analysis to maximise correlation between each wave period parameter and $\hat{d}$.

Figure 5.9 illustrates $T_{m1}/T_{m1,0}$ and $\overline{T_z}/\overline{T_{z,0}}$ plotted against $\hat{d}$ in subplots (a) and (b), respectively. The variable $\hat{d}$ effectively collapses results across all bathymetries. Similar to T_{m-1} , both T_{m1} and $\overline{T_z}$ show limited change in deep water, followed by pronounced exponential growth in shallow regions due to infragravity enhancement.

The following empirical relationships describe this behaviour:

$$\frac{T_{m1}}{T_{m1,0}} = 3.89 \exp(-5.00\hat{d}) + 1.01, \quad (5.21)$$

and

$$\frac{\overline{T_z}}{\overline{T_{z,0}}} = 2.57 \exp(-4.23\hat{d}) + 1.10. \quad (5.22)$$

The fitted curves (red lines in Figure 5.9) capture the data well across sea-states and bathymetries. Some scatter is observed for $0.3 < \hat{d} < 1$, reflecting bathymetric variability, though the magnitude remains within the bounds discussed in Section 5.7.

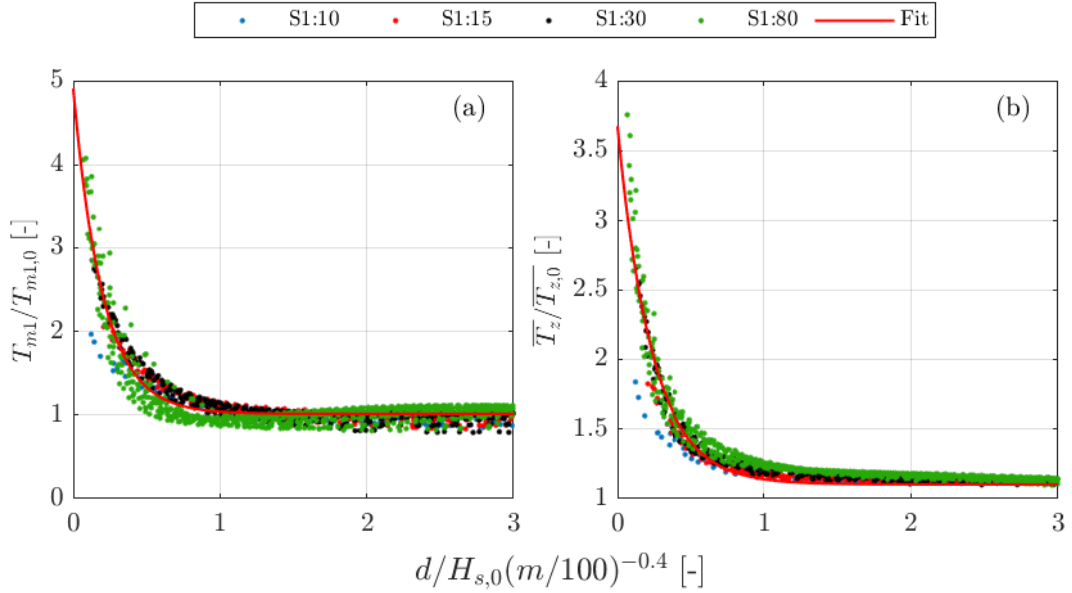


FIGURE 5.9: New (a) T_{m1} and (b) $\overline{T_z}$ formulations. Data are shown as solid markers, coloured by bed slope gradient. The fitted model is shown by a red line.

Overall, the analyses presented in this section demonstrate that while existing formulations such as Hofland et al. (2017) capture general trends, they substantially underpredict infragravity growth on milder slopes. The new formulations introduced here incorporate explicit slope dependence and accurately reproduce observed trends across all bathymetries, providing a more reliable framework for estimating representative wave periods in coastal engineering design.

5.6 Testing dataset

As outlined in Section 5.3, the complete numerical dataset was divided into separate training and testing subsets to evaluate the generalisability of the proposed data-driven formulations. The testing dataset corresponds to the bed slope of 1:50, which was not included during model training and represents approximately 20% of the available simulations. This section evaluates the robustness of the proposed formulations against this unseen dataset.

Figure 5.10 presents the performance of the proposed H_{rms} evolution model (Section 5.4.2) when applied to the independent testing data. Results are shown for sea-state group B, with marker colour indicating varying sea-state steepness. The comparison demonstrates very good agreement between the numerical simulations (circular markers) and the proposed model predictions (solid lines) across the full range of sea-state steepness conditions. The consistency between the model and unseen data highlights the predictive capability and physical robustness of the derived formulation.

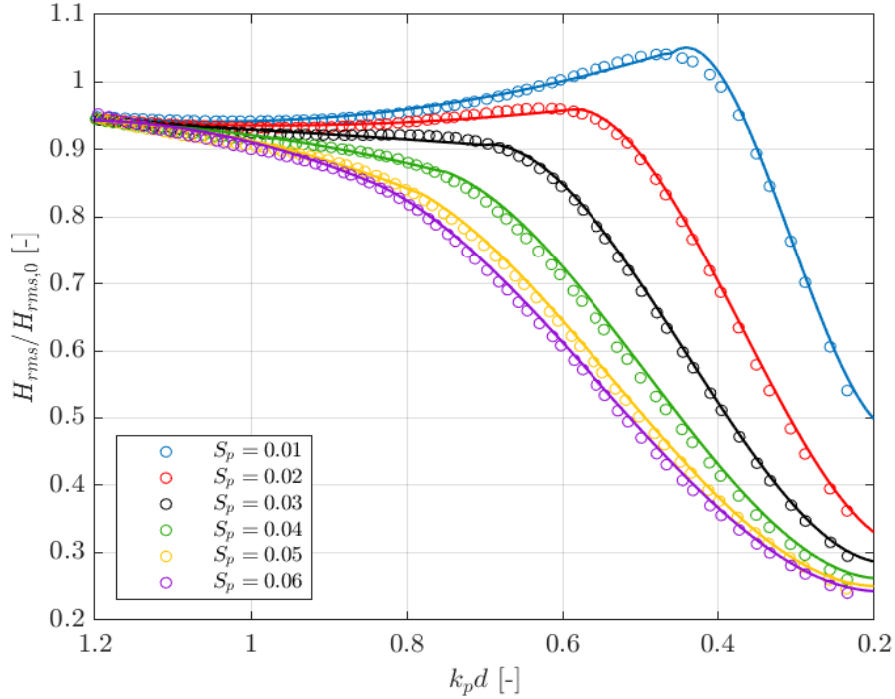


FIGURE 5.10: Evolution of H_{rms} for sea-state group B propagating over a bed slope of 1:50. Circular markers and solid lines correspond to the numerical data and the proposed formulation, respectively.

Similarly, the proposed wave period formulations presented in Section 5.5 are evaluated against the same testing dataset in Figure 5.11. The T_{m-1} formulation, shown in Figure 5.11(a), reproduces the numerical data well across the entire spatial domain, confirming its ability to capture the depth-dependent evolution of the energy period. Despite the moderate scatter observed in the T_{m1} data (Figure 5.11(b)), the proposed formulation successfully captures the overall trend and magnitude of variation. For the mean zero-crossing period, $\overline{T}_z$, shown in Figure 5.11(c), minor deviations are observed around $d/H_{s,0}(m/100)^{-0.4} \approx 1$. These discrepancies are primarily attributed to the weaker data collapse in milder slopes, as previously discussed in Figure 5.9(b). Nevertheless, the proposed model retains strong agreement with the numerical data overall.

The results presented in this section confirm that the proposed formulations are not over-fitted to the training data and demonstrate reliable predictive performance for unseen conditions. This further supports the applicability of the models across a broad range of sea-state and bathymetric configurations.

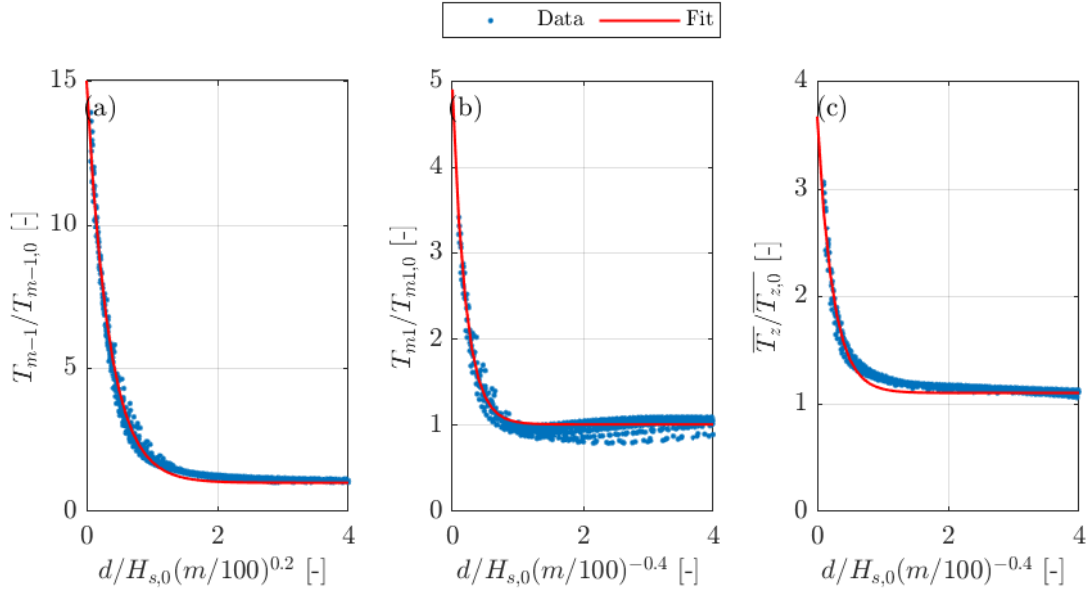


FIGURE 5.11: Evolution of (a) T_{m-1} , (b) T_{m1} and (c) $\overline{T}_z$ for waves propagating over a bed slope of 1:50. Data are shown as solid markers, and the proposed formulations are shown by red lines.

5.7 Sources of uncertainty

The formulations presented in this work are based on numerical simulations conducted using SWASH. The fidelity of this model has been established in numerous studies in the context of coastal wave transformation, swash dynamics, and infra-gravity wave generation (Zijlema, 2012, Smit et al., 2014, De Bakker et al., 2016, Ruju et al., 2014, Filipot, 2016, Al Khalili et al., 2025). These works demonstrate that SWASH accurately reproduces wave shoaling, breaking-induced dissipation, and run-up processes across a wide range of bed slopes (1:10–1:100) and spectral conditions.

Despite its robustness, uncertainties relative to experimental or field measurements may still arise. Key sources include grid resolution, boundary forcing, and the treatment of wave breaking and roller parameterisations (De Bakker et al., 2016, Ruju et al., 2014, Al Khalili et al., 2025). Reported discrepancies in the literature are typically of the order of 2–18% for wave height predictions, depending on wave nonlinearity and water depth.

Consequently, the formulations presented herein serve as practical and computationally efficient alternatives to fully nonlinear shallow-water simulations, providing reliable first-order estimates for coastal engineering design. Complete accuracy, particularly under highly nonlinear or site-specific conditions, may only be achieved through detailed laboratory or field measurements. Future work may further reduce uncertainty by integrating these numerical datasets with experimental (Trulsen et al., 2012) and field observations (Ruessink et al., 2000).

In presenting the wave height and period formulations above, it is important to outline the upper and lower bounds associated with these formulae. These bounds seek to encapsulate two components. Each of these are presented in Figure 5.12 in the context of the normalised H_{rms} evolution. Similar arguments hold for all formulations presented in this work.

The first type of uncertainty, E1, is shown in Figure 5.12(a). Given the random nature of the sea-states investigated, the evolution for each realisation of the sea-state (shown by the red dotted line) will be different. This is an aleatoric uncertainty as it derives from natural variability of the physical process. If only the ensemble mean of the evolution of all seeds per sea-state is considered (shown by the red dashed line), then upper and lower bounds to the mean may be defined based on the variation of

the evolution across seeds (shown in the blue shaded region).

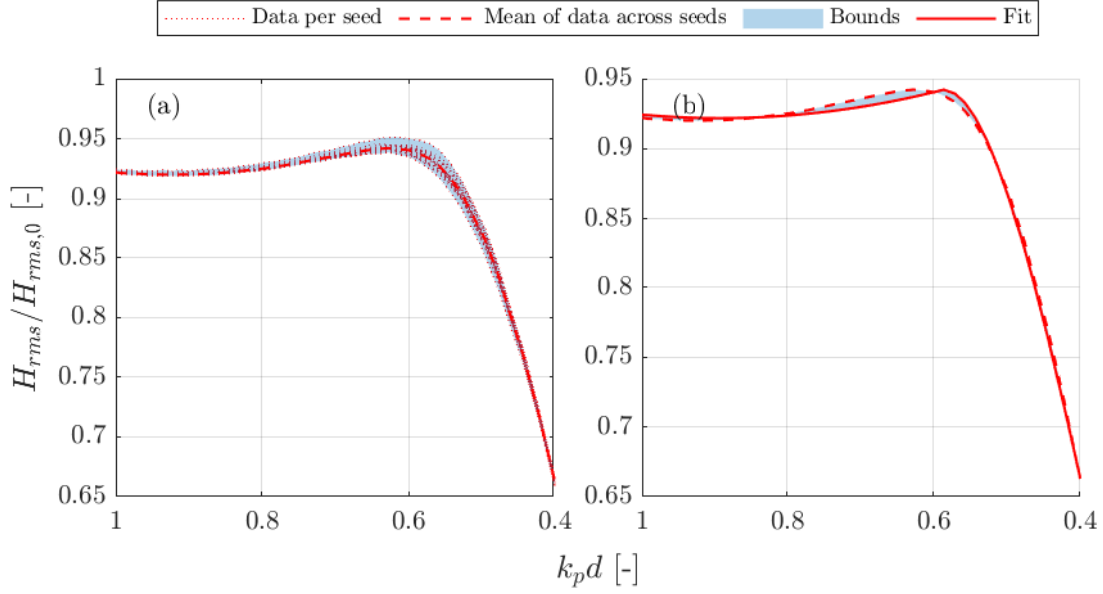


FIGURE 5.12: Evolution of $H_{rms}/H_{rms,0}$ with reduced effective water depth, $k_p d$, for a bed slope of 1:80 for sea-state B2. Bounds indicate (a) the variation in the evolution across different seeds (red dotted line) compared to the mean (red dashed line) and (b) the discrepancy between the fitted distribution (red solid line) compared to the mean of the data (red dashed line).

Secondly, there may be discrepancies associated with the fit of the model to the dataset, particularly in regions associated with high levels of data scatter. This is an epistemic uncertainty. Figure 5.12(b) shows the fitted distribution by a solid red line. Hence, the discrepancy between the data and the fit is illustrated by the blue shaded region. This defines the second source of uncertainty, E2. In the context of H_{rms} evolution, the predominant discrepancy is surrounding the location of breaking saturation.

E1 and E2 are calculated based on the root-mean-squared error across the sea-states and bathymetries investigated. For simplicity, these are chosen to be spatially-uniform and defined as a percentage of the modelled variable. The magnitude of these uncertainties are tabulated in Table 5.2. Assuming E1 and E2 are independent and random, we utilise the law of propagation of uncertainty as described by Taylor (1997) and International Towing Tank Conference (ITTC) (2021) to determine the total uncertainty associated with the formulations presented. The uncertainty for the formulation T_{m-1} is relatively high, given its dependence on the magnitude of infragravity wave energy, which is inherently difficult to capture numerically. This

TABLE 5.2: Bounds to formulations

	Uncertainty[$\pm\%$]		
	E1	E2	Total
H_{rms}/H_{rms}	1.1	0.6	1.2
$T_{m-1}/T_{m-1,0}$	2.5	13.6	13.8
$T_{m1}/T_{m1,0}$	0.8	6.8	6.9
$T_z/\overline{T}_{z,0}$	1.8	5.3	5.6

has been highlighted earlier in terms of the limited literature employing SWASH to investigate infragravity energy in very shallow water when breaking effects are dominant. The complexities of infragravity evolution are illustrated by the extent of scatter observed in Figure 5.8(a), particularly for mild slopes.

The present study considers unidirectional wave propagation over smooth beds, representing an idealised nearshore configuration that enables preliminary design quantification of wave transformation processes. While certain simplifying assumptions are adopted, these are consistent with many established nearshore modelling studies. Barred beach morphologies and longshore bathymetric variability are not explicitly represented, and the analysis therefore focuses on alongshore-uniform conditions. Similarly, the assumption of unidirectional wave propagation is most applicable to coastlines with limited curvature, where alongshore gradients and multidirectional effects are secondary. Processes such as directional spreading, spatially varying bottom roughness, wave-current interaction, and fully three-dimensional breaking dynamics are also not considered. Within these constraints, the formulations presented provide a robust and efficient initial design and analysis framework for modelling wave evolution over idealised nearshore environments, and form a sound basis for extension to more complex coastal settings.

5.8 Conclusion

Coastal engineering design requires simple and broadly applicable tools, particularly for initial calculations and scoping. During the early stages of design, when resources and data availability are limited, such formulations provide an essential means of estimating representative wave heights and periods before more intensive numerical or physical modelling is undertaken. This work seeks to expedite this phase by formulating transformations that are notoriously difficult to capture. This is achieved

by simulating 2 million waves in total over 21 sea-states and 5 bed slopes. This extensive dataset is utilised to achieve two key aims. Firstly, it is utilised to assess existing models used in practice for the evolution of representative wave heights and periods. Secondly, based on the discrepancies outlined between the existing literature and the present dataset, we propose new data-driven parameterisations.

Specifically in the context of wave heights, the root-mean-squared wave height, H_{rms} , is a key parameter employed in several practical applications. However, the results of this work have highlighted differences between the various modelling formulations. More specifically, no formulation is shown to accurately capture the evolution of H_{rms} observed for all cases investigated. Thus, novel practical guidance for the evolution of H_{rms} is provided, which is conditional upon the sea-state steepness and the underlying bed slope gradient.

In the context of wave periods, the parameter T_{m-1} is often adopted in coastal design due to its favourable weighting to low frequency components, which become increasingly important in shallow water. Assessment of the commonly adopted T_{m-1} formulation of Hoffland et al. (2017) has demonstrated non-conservative findings. As such, a new parametric relation is developed to describe the evolution of T_{m-1} . This new formulation is unique in that it includes additional bed slope effects which are not considered in existing formulae. This formulation is further extended to predict the evolution of the mean spectral period, T_{m1} , and the mean zero-crossing period, $\overline{T_z}$, thereby providing a comprehensive set of tools for practical coastal design.

Finally, all proposed models are accompanied by uncertainty quantification, allowing users to assess the reliability of predictions and identify scenarios where model performance may vary. To facilitate practical implementation, the computational tools developed in this work are made available as supplementary material. Together, these contributions offer a significant step towards more reliable, yet practical, design methods for coastal wave transformation, addressing Objective (iv) of the thesis.

Chapter 6

Numerical prediction of wave particle kinematics

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6.1 Introduction

The preceding chapters have demonstrated how nonlinear wave transformation over variable bathymetries governs the evolution of nearshore wave statistics, and how these changes can be captured using both process-based numerical modelling and data-driven parameterisations. While such statistics underpin many aspects of coastal design, the detailed kinematics of the wave field—particularly the instantaneous velocities beneath the crest—play an equally critical role in determining hydrodynamic loading, sediment mobilisation, and breaking behaviour. As highlighted in Chapter 1, the accurate characterisation of wave-induced velocities remains one of the key challenges in coastal engineering, especially in shallow and highly nonlinear conditions. Building on the physical understanding and validated modelling framework established in Chapters 3–5, this chapter now turns to the phase-resolved description of horizontal crest velocities in irregular waves propagating over sloping bathymetries, highlighting the importance of wave kinematics for practical applications.

Several works have highlighted the dependence of wave-induced forces on the

kinematics of waves, both in the context of ships (Salvesen et al., 1970, Faltinsen, 1993) and offshore structures (Morison et al., 1950, MacCamy & Fuchs, 1954, Kriebel, 1998). Given that both drag (Morison et al., 1950) and slamming loads (Faltinsen, 1993) are proportional to the square of the velocity, minor uncertainties in the calculation of wave kinematics are amplified in loading calculations. In another context, the understanding of wave motion plays a role in wave-induced sediment transport and sandbar migration in coastal regions. Such transfers of sediment govern the morphology of coastal areas (Fredsoe & Deigaard, 1992, Elfrink & Baldock, 2002), having notable socioeconomic implications through shoreline retreat and erosion. In combination with currents, this wave motion, drives the transport of salt, heat and natural or man-made tracers. This is an area of continual research, particularly in light of the environmental concerns associated with micro-plastic pollution (Zhang, 2017, Li et al., 2020).

More specifically, insight into particle velocities at the crest of a wave is of critical significance. The crest represents the most energetic region of a wave, where nonlinear effects dominate and dynamic processes such as wave breaking are initiated (Gemmrich & Garrett, 2010). Additionally, commonly adopted kinematic breaking criteria often hinge upon the ratio of horizontal fluid velocity at the crest to the wave's propagation speed (Perlin et al., 2013, Barthelémy et al., 2018). Accurately characterising crest velocities enables precise prediction of breaking onset, which in turn governs energy dissipation and turbulent dynamics in the nearshore zone. This understanding is essential for effective design and assessment of coastal protection measures under variable wave conditions.

Much work on the kinematics of waves in deep water exists in literature (Baldock et al., 1996, Johannessen & Swan, 2001, Stansberg et al., 2008), with well-established formulations for both regular (Airy, 1845, Stokes, 1847, Dean, 1965, Fenton, 1985) and irregular (Longuet-Higgins & Stewart, 1960, Sharma & Dean, 1981) waves. In an effort to overcome some of the classical difficulties of irregular wave theories, a plethora of empirically-stretched wave models have been proposed in the literature (Wheeler, 1970, Chakrabarti, 1971, Gudmestad & Connor, 1986, Rodenbusch & Forristall, 1986, Donelan et al., 1992, Molin, 2002). Several works have assessed the validity of these various methods in the context of deep water waves (Niedzwecki & Whatley, 1991, Gudmestad, 1993, Stansberg & Gudmestad, 1996). However, the

propagation of waves in coastal areas is highly complex due to the various physical processes at play, including wave refraction, reflection, shoaling and dissipation from bottom friction. Moreover, the reduction in the water depth increases the nonlinearity of the wave field (Freilich & Guza, 1984) and introduces depth-limited breaking effects (Barthelemy et al., 2018). This presents complexities in modelling wave kinematics in finite water depths (De Serio & Mossa, 2016, Smith & Swan, 2002, Isobe, 2013, Lutio, 2017). These studies have demonstrated concerns regarding the accuracy of existing methods, particularly in the context of highly nonlinear wave fields in coastal areas (Lawrence et al., 2021a).

Although wave kinematics in coastal regions have been extensively investigated, including irregular sea-state velocities measured in both laboratory and field settings, most existing studies have focused on their role in sediment transport and mean flow processes. However, the detailed statistical characterisation of horizontal crest velocities in irregular, highly nonlinear sea-states, particularly across different bathymetric slopes, remains limited. In addition, few studies have systematically validated numerical predictions of instantaneous kinematics under extreme irregular waves against laboratory measurements.

The present work addresses this gap through a comprehensive series of SWASH (Zijlema et al., 2011) simulations. The objective is to develop a quantitative understanding of how sea-state steepness and bed slope affect the distribution and vertical structure of horizontal crest velocities in coastal waters. The results are further used to assess the applicability of commonly used kinematic wave theories and to provide practical recommendations for their use in engineering design.

Although depth-integrated models such as SWASH were developed more than a decade ago, they remain widely used for resolving nearshore hydrodynamics and wave kinematics under both regular and irregular wave conditions. Their computational efficiency and ability to capture key nonlinear processes make them valuable for practical engineering applications where fully three-dimensional turbulence-resolving models are not feasible. The relatively fast computational time also allows for long-duration simulations at reasonable cost, enabling the prediction of waves with small exceedance probabilities, or in other words extreme events, that are of particular design relevance. The validity of SWASH for modelling wave-induced longshore currents has been demonstrated by Zhang et al. (2018), based on comparisons with the

experimental data of Visser (1991), Hulsbergen & ter Horst (1973), Reniers & Battjes (1997) and Hamilton & Ebersole (2001) across a range of regular and irregular waves and bed slopes (1:10 to 1:35). Similarly, Rijnsdorp et al. (2017) showed good correspondence with the experiments of Visser (1991), Ting & Kirby (1994) and Boers (1996) for the prediction of time-averaged cross-shore velocity and longshore current.

However, the validation of SWASH in the context of phase-resolved horizontal velocities under highly nonlinear sea-states has yet to be established. This work therefore begins by verifying the model against laboratory data and theoretical expectations in Section 6.2, before examining the drivers of crest-velocity distributions in Section 6.3. This addresses the first aim of the present chapter: to understand the mechanisms governing phase-resolved crest velocities in the coastal zone. Finally, Section 6.4 evaluates the performance of existing kinematic theories against the numerical results and provides quantitative guidance for engineering application, addressing the second aim of this chapter.

6.2 Model set-up and validation

The numerical simulations utilised in this chapter are equivalent to those outlined in Chapters 4-5. Hence, description of the numerical set-up and test-cases are omitted here for conciseness.

6.2.1 Validation against experiments

Several works have validated SWASH in a range of coastal contexts in terms of its ability to capture surface elevation statistics (Zijlema, 2012, Smit et al., 2014, De Bakker et al., 2016, Al Khalili et al., 2025). In the context of kinematics, time-averaged velocities have been considered. However, to the authors' knowledge, no work has examined its validity for phase-resolved wave crest velocities in irregular, highly nonlinear sea-states. As such, the numerical simulations are supplemented with experiments conducted in the Coastal Flume in the Imperial College Hydrodynamics Laboratory by Millar et al. (2025), for both regular and irregular waves, propagating over a bed slope of 1:30. The facility is 23 m long, 0.6 m wide and has a working water depth of 0.5 m. It utilises a piston-type wave paddle with active

absorption, operated under force control. Further detail of the facility are provided in Akinin & Spinneken (2017). Particle kinematics are recorded using down-looking Nortek Vectrino+ ADVs. Nortek (2018) have demonstrated a scale-factor bias of less than 1% of the measured velocity for these instruments. The Vectrinos are positioned at six locations along the flume, spanning water depths of 0.5 m to 0.16 m. Due to the relative importance of flow disturbance by the Vectrinos in shallow water, kinematics are not measured closer to the shoreline. At each spatial location, measurements are taken at 100 mm and 50 mm vertical spacing for $z \leq -0.3$ m and $z > -0.3$ m, respectively. Further details are provided in Millar et al. (2025).

The regular wave cases considered are summarised in Table 6.1. The dataset comprises four wave frequencies, f ; for each frequency, three to five wave amplitude, a , cases are considered. As such, a range of offshore effective water depths, $0.77 \leq kd_0 \leq 4.53$, and offshore steepness, $0.016 \leq ak_0 \leq 0.362$, are investigated. Figure 6.1 considers the velocity profile below the crest for selected sea-states at various spatial locations. To facilitate comparisons with theoretical expectations, 5th-order Stokes theory (Stokes, 1847) and the Stream function (Dean, 1965) are included in black and green, respectively. These are computed based on the measured numerical wave parameters. According to Fenton (1985), the former is based on a perturbation expansion, assuming periodic, irrotational, and inviscid flow with relatively small to moderate wave steepness. It is expressed as a Fourier series expansion given by:

$$u(x, z, t) = C_0 \sqrt{\frac{g}{k^3}} \sum_{i=1}^5 \left(\frac{Hk}{2} \right)^i \sum_{j=1}^5 A_{ij} \cosh(jk(z+d)) \cos(j(kx - \omega t)), \quad (6.1)$$

The coefficients C_0 and A_{ij} are given in Table 1 of Fenton (1985). In contrast, the Stream Function formulation (Dean, 1965) assumes steady, periodic, and incompressible flow, but without linearisation of the free-surface boundary conditions. It solves the governing equations in a fully nonlinear manner by directly satisfying kinematic and dynamic boundary conditions at the actual free surface. Consequently, it can accurately represent strongly nonlinear wave profiles and asymmetric velocity distributions near the crest. The corresponding expression for the horizontal velocity component is given by:

$$\begin{aligned}
u(x, z, t) = & (c - \bar{u}) \\
& + \sum_{n=4,6,8,\dots}^{N-1} \frac{(n-2)\pi}{\lambda} \cosh\left(\frac{(n-2)\pi}{\lambda}(z+d)\right) \\
& \left[X(n) \cos\left(\frac{(n-2)\pi}{\lambda}x\right) + X(n+1) \sin\left(\frac{(n-2)\pi}{\lambda}x\right) \right].
\end{aligned} \tag{6.2}$$

where c is the phase velocity, $\bar{u}$ is the mean velocity and $X(i)$ is determined by a least squares fit to the full nonlinear free surface boundary conditions.

TABLE 6.1: Regular wave cases

Sea-state	f [Hz]	a [m]	ak_0 [-]	kd_0 [-]
E1	0.05	1.0	0.016	0.77
E2		5.0	0.078	
E3		10.0	0.155	
F1	0.08	2.0	0.055	1.44
F2		4.0	0.115	
F3		7.0	0.202	
F4		10.0	0.288	
G1	0.11	1.0	0.049	2.47
G2		2.0	0.099	
G3		4.0	0.120	
G4		5.0	0.247	
G5		7.0	0.346	
H1	0.15	1.0	0.091	4.53
H2		2.0	0.181	
H3		3.0	0.272	
H4		4.0	0.362	

Steep waves generated numerically in intermediate water compare well to experimental measurements, as shown in Figure 6.1(a–b). Moreover, in both these cases, higher order Stokes theory is most appropriate according to the validity diagram of Le Méhauté (1976). We observe that the numerical solutions are in line with Stokes theory for these cases, with excellent agreement near the crest. This continues to be so in near-breaking wave cases located near the limit of applicability of Stokes theory, as shown in Figure 6.1(c). In shallower water depth, shown in Figure 6.1(d), although Stokes theory provides good predictions lower in the water column, the accuracy begins to break down near the crest, with the Stream function theory being appropriate. This is due to its ability to capture highly nonlinear effects associated with steep waves in shallower water. More importantly, excellent agreement between

the results of the numerical simulation and experimental measurements is observed at this water depth.

The value of root-mean-squared error (RMSE) in $u/\sqrt{gd}$ for each wave case is included in the caption of Figure 6.1. As expected, the accuracy of the model is higher further offshore (Figure 6.1(a)) compared to shallower water depths (Figure 6.1(d)). Nevertheless, the magnitude of these errors does not exceed 0.010, highlighting the accuracy of SWASH at modelling regular wave fields propagating in coastal environments.

Additionally, the 99% confidence interval in the numerical and experimental velocity profile are indicated by the blue and red shaded region, respectively. These are calculated using the bootstrap method of Efron & Tibshirani (1994). Overall, the extent of the variability in the numerical velocity profile is minimal, with the blue shaded area being extremely thin and therefore difficult to discern visually. This emphasises the strength of the adopted modelling approach such that effects of spurious waves or wave reflections, which may cause local variability in the wave field, are minimised. In comparison, the confidence intervals of the experimental wave profiles are generally wider in shallow water, as shown in Figure 6.1(d). This can be attributed to difficulties associated with performing wave measurements in smaller water depths (Mueller et al., 2007). This is both due to a more notable effect of flow disturbance by the instrument, as well as more difficulties associated with measuring highly nonlinear wave fields. However, despite these challenges, the experimental confidence intervals are still reasonable, providing confidence into the experimental methodology.

In addition to regular wave simulations, the random sea-states outlined in Table 3.1 are also considered. Further validation of the numerical model is conducted by evaluation against experimental measurements of sea-state B3 propagating over a bed slope of 1:30. Comparisons between numerical simulations and experimental measurements of the velocity profile under wave crests associated with a probability of exceedance $Q = 10^{-3}$ at each spatial location is shown in Figure 6.2. Utilising the Gaussian law of error (Bevington & Robinson, 2003), the profiles shown are an average across 30 wave crests distributed around $Q = 10^{-3}$. At the offshore location, shown in Figure 6.2(a), strong agreement is observed across the water column. This correspondence across datasets continues to hold down to effective water depths of

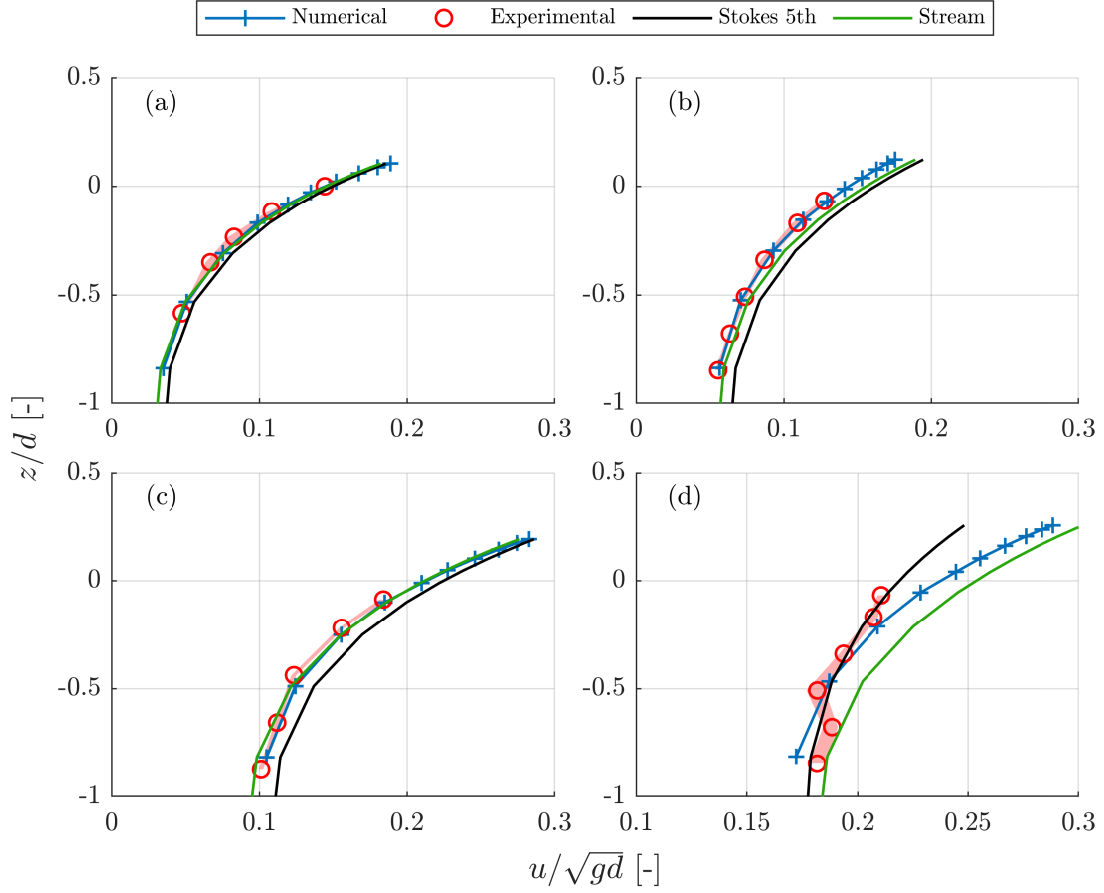


FIGURE 6.1: Velocity profile for waves propagating over bed slope 1:30 for (a) $kd = 2.12$, $ak = 0.22$ (sea-state G4, RMSE=0.002) (b) $kd = 1.55$, $ak = 0.19$ (sea-state G3, RMSE=0.002) (c) $kd = 1.28$, $ak = 0.22$ (sea-state G4, RMSE=0.004) and (d) $kd = 0.57$, $ak = 0.11$ (sea-state E2, RMSE=0.010). The 99% confidence interval in the numerical and experimental velocity profile are indicated by the blue and red shaded region, respectively.

$k_p d = 0.74$ as shown in Figures 6.2(b–d), reinforcing the applicability of the SWASH model to capture wave kinematics within the shoaling region. In the shallowest region considered experimentally, shown in Figure 6.2(f), the effects of wave breaking become important. We observe marginally higher velocities experimentally compared to numerical measurements. Moreover, it may be stated that the numerical model exhibits some systematic bias observed as over-predictions near the crest and under-predictions near the bed, relative to experimental measurements.

Overall, the numerical predictions agree well with the experimental measurements, with values of the non-dimensional root-mean-squared error in $u/\sqrt{gd}$ of approximately 0.05. This demonstrates the validity of SWASH to capture the profile of wave velocity for a range of sea-state conditions across a broad range of effective

water depths. This is particularly encouraging considering that the results of Figure 6.2 correspond to extreme waves in the sea-state. Moreover, as evident in Figures 6.1 and 6.2, significantly higher vertical resolution in the vicinity of the wave crest may be obtained numerically. Given that kinematics are larger at this location, their accurate description is critical to practical applications. This complements experimental measurements which are typically confined to point measurements. Based on the validation analysis conducted, the SWASH model is employed as the reference model for the kinematic field.

As a whole, this section has provided validation of the SWASH model across a broad range of conditions for both regular ($0.41 < kd < 4.53$ and $0.02 < ak < 0.30$) and irregular ($0.61 < k_p d < 1.22$ and $0.18 < H_s/d < 0.49$) waves. Good correspondence is observed across this range, except for long regular waves propagating in relatively shallow water ($kd < 0.5$) where discrepancies between numerical simulations and experiments may be observed. The earlier work of Al Khalili et al. (2025) assessed the SWASH model in the context of surface elevation statistics. Their results confirm the accuracy of SWASH at modelling crest height and wave height statistics for up to moderate steepness waves, $H_s/d < 0.3$, within 2% accuracy of experimental measurements. However, limitations were observed in steeper waves where the full extent of wave nonlinearity and breaking effects were not captured. As such, these limitations in the numerical model should be acknowledged when considering the findings of this work.

6.3 Horizontal crest velocity

Knowledge of particle velocities at the wave crest is essential, as this region controls the initiation of wave breaking and governs the maximum hydrodynamic loads exerted on marine structures. Recent laboratory investigations of wave forces on vertical caissons have highlighted the central role of near-crest kinematics in this context (Romano & Bellotti, 2023). Furthermore, accurate characterisation of crest velocities provides insight into nonlinear wave behaviour and is crucial for predicting extreme coastal hazards. In this section, we investigate the key drivers governing the distribution of the horizontal crest velocity, u_c . Here, u_c is defined as the horizontal particle velocity evaluated at the instantaneous location of a wave crest, whose

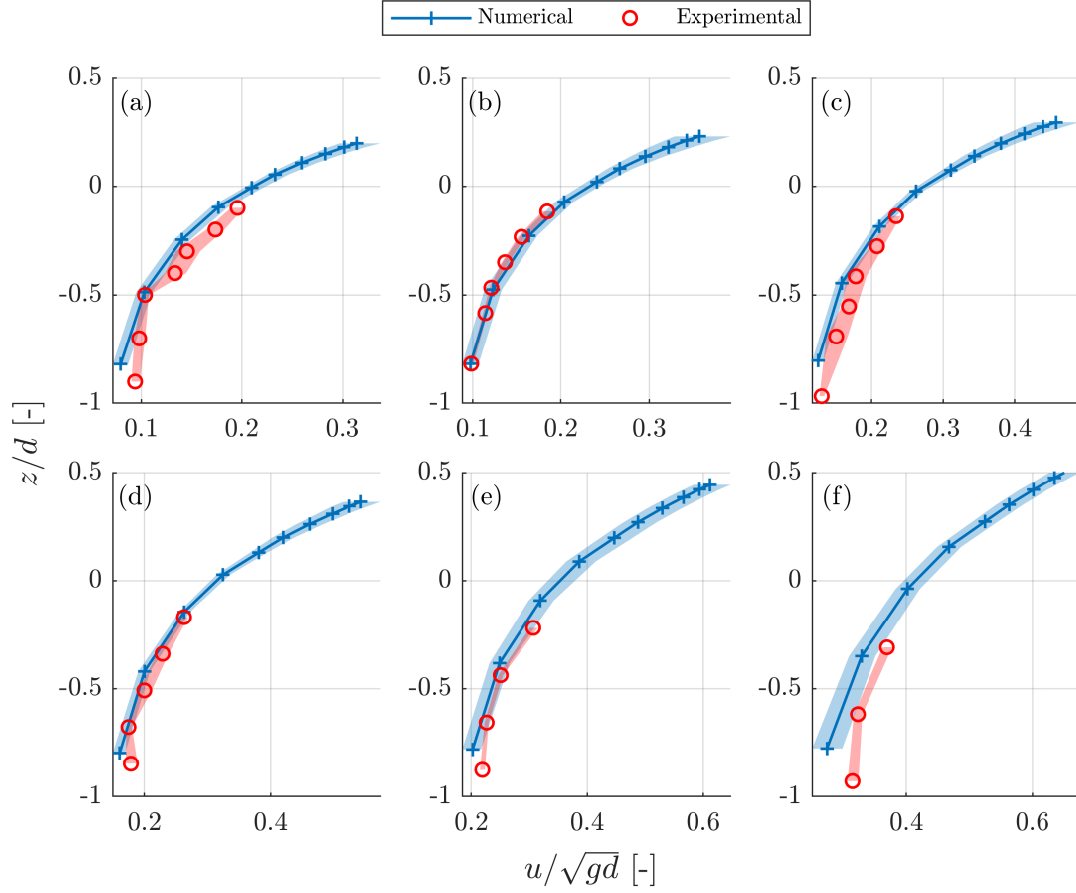


FIGURE 6.2: Velocity profile for sea-state B3 under crests associated with $Q = 10^{-3}$ propagating over bed slope 1:30 in (a) $k_p d = 1.22$ (RMSE=0.05), (b) $k_p d = 1.10$ (RMSE=0.04), (c) $k_p d = 0.98$ (RMSE=0.05), (d) $k_p d = 0.87$ (RMSE=0.05), (e) $k_p d = 0.74$ (RMSE=0.05) and (f) $k_p d = 0.61$ (RMSE=0.05). The 99% confidence intervals are indicated by the shaded region.

elevation is denoted by η_c . Crest elevations, η_c , are identified from the time series of the water surface elevation using a zero-crossing analysis, and the corresponding horizontal velocities are extracted at the same spatial and temporal locations. Where appropriate in later sections, both u_c and η_c are presented in non-dimensional form using local wave and depth scales.

To characterise waves associated with small probabilities of exceedance (extreme events), crest events are identified within each realisation and concatenated across all 20 realisations at each spatial location. The resulting ensemble of crest velocities is then reordered to form the exceedance distributions presented in this section. The analysis focuses on the influence of sea-state steepness and bed slope gradient on the distribution of u_c , conditional on the local effective water depth, $k_p d$. Following the validation analysis presented in Section 6.2, all results discussed herein correspond

to numerical predictions.

6.3.1 Influence of sea-state steepness

Figure 6.3 presents the exceedance probability of the crest velocity, u_c , for sea states of increasing steepness, S_p , using Weibull probability paper form (Weibull, 1951). The Weibull distribution is commonly used to describe extreme values in wave-related processes, as it provides a flexible representation of the relative frequency of large events through two parameters: a scale parameter, which controls the magnitude of typical extremes, and a shape parameter, which controls the decay rate of the tail of the distribution. When data are plotted on Weibull probability paper, a straight line indicates that the exceedance behaviour can be described by a Weibull distribution, with the slope and intercept corresponding to the shape and scale parameters, respectively.

In intermediate water depths (Figure 6.3(a)), the exceedance curves are approximately linear, indicating that the crest velocity statistics converge to a Weibull distribution. The slopes of the curves are similar across different sea-state steepnesses, whereas the intercepts vary systematically. This suggests that increasing steepness primarily affects the scale of crest velocities, i.e. the overall magnitude of extreme velocities, while the shape of the distribution remains largely unchanged. Physically, this implies that steeper waves increase crest velocities in a proportional manner without fundamentally altering the underlying kinematic variability.

In contrast, in shallow water (Figure 6.3(b)), a clear change in slope is observed near the upper tail of the distribution ($u_c \approx 1$), indicating a departure from a single Weibull distribution. Inspection of the corresponding wave fields shows that this transition coincides with the occurrence of depth-induced wave breaking, as indicated by enhanced local dissipation in the magnitude of wave crests at this location and the emergence of strongly asymmetric crest shapes. In this regime, the balance between nonlinear wave growth and dissipation is modified, leading to a distinct change in the exceedance behaviour of u_c . Consequently, the distribution is more accurately represented by two Weibull regimes separated by a transition point associated with the onset of breaking. A similar two-regime interpretation has previously been proposed for wave heights by Battjes & Groenendijk (2000); the present results suggest that an analogous signature can also be identified in wave kinematics when breaking

is explicitly accounted for.

Moreover, in shallow water the limiting effect of wave breaking constrains the maximum attainable crest velocities, causing the exceedance distributions for different sea states to converge. This convergence suggests that, once breaking dominates, both the shape and scale parameters of the Weibull distribution become largely independent of sea-state steepness.

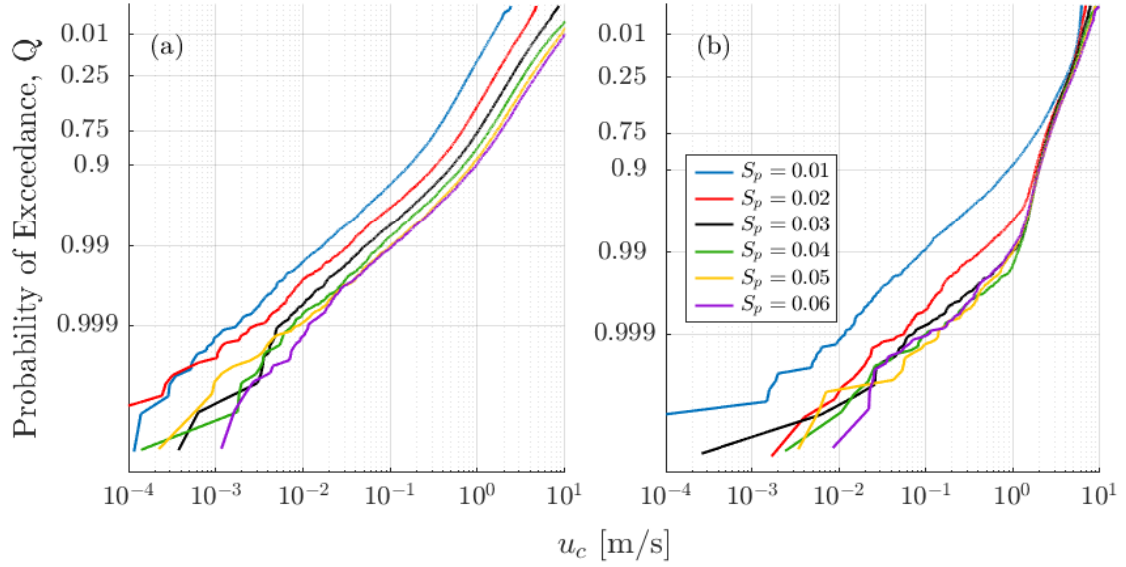


FIGURE 6.3: Probability of exceedance of u_c plotted in Weibull form in (a) $k_p d = 1.2$ and (b) $k_p d = 0.3$ propagating over bed slope 1:30. The colour of the line corresponds to the sea-state steepness, S_p .

To investigate the role of the local effective water depth further, Figure 6.4 considers the probability of exceedance of the normalised crest velocity, u_c/u_s , where u_s is the significant horizontal velocity. We define this parameter in a similar manner to the spectral definition of the significant wave height (Equation 2.57) such that:

$$u_s = 4\sqrt{m_0(u)}, \quad (6.3)$$

where $m_0(u)$ is the zero-th order spectral moment as defined previously by Equation 4.8. Considering Figure 6.4(a), initially, reducing the effective water depth from $k_p d = 1.2$ to $k_p d = 0.9$ acts to increase the magnitude of u_c/u_s . This is driven by nonlinear steepening of the wave field which manifests as higher and sharper crests and associated with greater horizontal particle velocities at these higher elevations (Stokes, 1847). Moreover, the reduced effective water depth acts to flatten particle

orbits. In order to maintain mass conservation, the horizontal velocity must increase to compensate for reductions in vertical motion, as explained in the early works of Kenyon (1996). In shallower water, $k_p d = 0.6$, greater nonlinear amplifications are observed over deeper distributions. However, investigation into the crest height distribution at this location has highlighted the dominant occurrence of wave breaking acting to reduce the magnitude of crest heights compared to what is observed further offshore. This effect also dominates the shape of the normalised velocity distribution. This is observed as a near vertical tail for probabilities of exceedance, $Q < 5 \times 10^{-2}$. Given that the largest waves in a sea-state break first, the waves in the tail of the distribution on Figure 6.4(a) will break and dissipate energy. As such, they will be associated with larger probabilities of exceedance further downstream. This redistribution of broken waves has been explained by Karmpadakis & Swan (2020) and Al Khalili & Karmpadakis (2025) in the context of crest heights. Here, we observe similar effects for u_c evident by the hump in the distribution at high probabilities of exceedance. It should be noted that the extent of the influence of wave breaking effects on the distribution of u_c is more notable compared to the distribution of η_c , as presented in the work of Al Khalili et al. (2025). This results in a prominent change in the shape of the distributions shown in shallower water, $k_p d = 0.6$, compared to the offshore location, $k_p d = 1.2$. This is even more pronounced in shallower water depths, $k_p d = 0.3$, as shown in Figure 6.4(a). Considering more extreme seas, as shown in Figure 6.4(b), this behaviour is evident as early at $k_p d = 0.9$. In other words, the steepening of the sea-state results in breaking saturation occurring further offshore and as such, the importance of considering wave breaking effects on the shape of the distribution becomes important in deeper effective water depths.

The 95% confidence intervals are shown in Figure 6.4 at the offshore location ($k_p d = 1.2$) by the blue shaded region, calculated using the bootstrap approach (Efron & Tibshirani, 1994, Huo et al., 2023, 2025). The uncertainty increases towards the tail of the distribution, reflecting the progressively smaller number of crest events contributing to these levels. At $Q = 10^{-3}$, the statistics are supported by a limited but still adequate number of samples, whereas at $Q = 10^{-4}$ the confidence intervals widen substantially, indicating increased sampling uncertainty given that only a few waves are present at this level. This behaviour is consistent with extreme-value statistics and highlights that quantitative statements at the lowest exceedance levels

should be interpreted with caution.

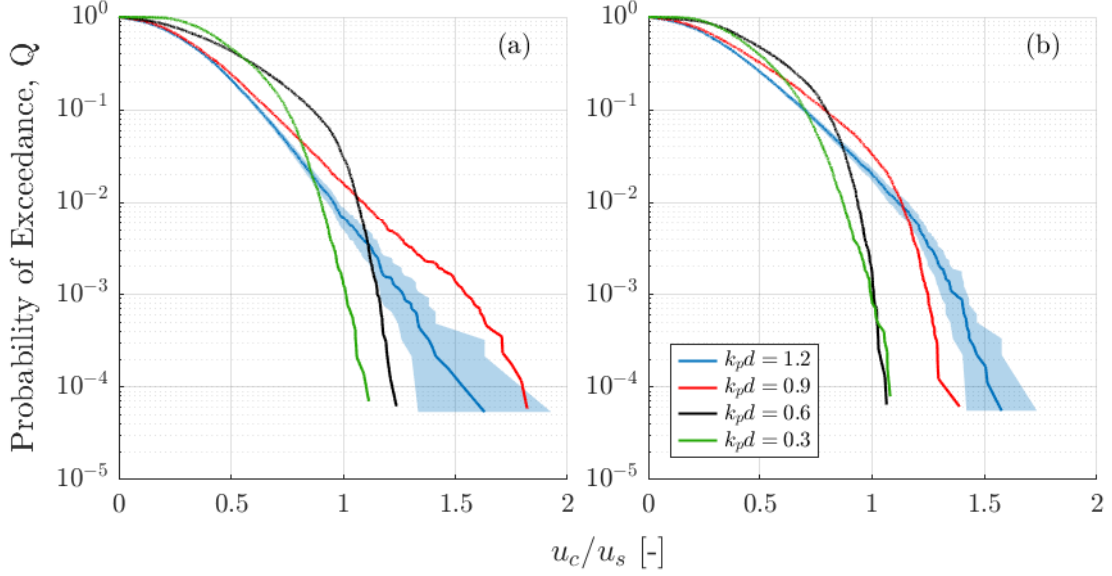


FIGURE 6.4: Probability of exceedance of u_c/u_s for sea-state (a) B3 ($S_p = 0.03$) and (b) B5 ($S_p = 0.05$) propagating over bed slope 1:30. The colour of the line corresponds to the effective water depth, $k_p d$. The 95% confidence intervals are shown by the blue shaded region.

6.3.2 Influence of bed slope gradient

Thus far, the results presented in this work consider a single bathymetry consisting of a uniform bed slope of 1:30. Yet, the gradient of the bed slope controls wave asymmetry and nonlinearity. Figure 6.5 compares the magnitude of u_c across different bed slopes at selected $k_p d$ locations. Each tile corresponds to wave crests associated with a different exceedance probability. This is essentially a horizontal line at Figure 6.4. For clarity, the magnitude of the power of Q defines the rank of a wave: for instance, $Q = 10^{-3}$ defines the 1000th largest wave.

Figure 6.5(a–b) show that while the offshore magnitude of u_c is comparable in $k_p d = 1.2$ for large exceedance probabilities, the effects of the slope gradient become evident at $k_p d = 0.9$. At this location, values of u_c are larger in steeper slopes, indicating a more nonlinear velocity field manifested through a stronger shoaling effect. The greater magnitude of u_c in steeper slopes is observed throughout the shoaling zone. At $k_p d = 0.3$, the evolution is driven by breaking effects, as has previously been established in Section 6.3.1. We observe a smaller magnitude of horizontal crest velocities in milder slopes, which are associated with a wider surf

zone.

Comparing these observations to crest heights associated with smaller probabilities of exceedance, as shown in Figure 6.5(c–d), the discrepancies across slopes within the shoaling region ($k_p d > 0.6$) outlined in Figure 6.5(a–b) are less prominent. This implies that waves associated with larger probabilities of exceedance are more affected by shoaling effects, since those associated with small exceedance probabilities experience competing effects with wave breaking. In shallower water, $k_p d \leq 0.6$, the findings highlighted in Figure 6.5(a–b) continue to hold such that stronger breaking effects are observed in milder slope configurations. The extent of the influence of the bed slope gradient is notable, with differences in the value of u_c across bed slopes of 1:10 and 1:80 of up to 48% in $k_p d = 0.3$. This underscores the significance of considering the characteristics of the underling coastal bathymetry.

It is important to establish to what extent the observations made in Figure 6.5 are dependent on the magnitude of the crest height, η_c . To quantify this, the same format used in Figure 6.5 is adopted in Figure 6.6, but now considering η_c . As a whole, the trends observed in Figure 6.6 are consistent with those described in the context of horizontal crest velocities, manifested as a stronger shoaling behaviour in steep slopes, and smaller crest heights in mild bed slopes in the surf zone. This is in line with the breaker index defined as:

$$H_b = \gamma d \quad (6.4)$$

where H_b is the breaker height, $\gamma = 1.16 N_I^{0.22}$ and $N_I = \frac{\tan \beta}{\sqrt{H_0/\lambda_0}}$ is the surf similarity parameter, commonly referred to as the Iribarren number (Battjes, 1974). As such, for the same offshore conditions ($H_{s,0}, \lambda_{p,0}$), a milder bed slope (smaller $\tan \beta$) will have a smaller value of N_I and consequently smaller γ . Hence, the maximum wave height at a given water depth will be more restricted compared to steeper slopes. Together with the findings of Figure 6.5, it may be stated that the trends in the discrepancies of u_c across bathymetries is largely a consequence of differences in η_c . In other words, if the ratio of the horizontal crest velocity to the crest height is considered, u_c/η_c , the results indicate that this quantity is relatively insensitive to the underlying bed slope (not shown for conciseness).

The previous sections have demonstrated how sea-state steepness and bathymetry influence the statistics of horizontal crest velocities. However, in engineering design

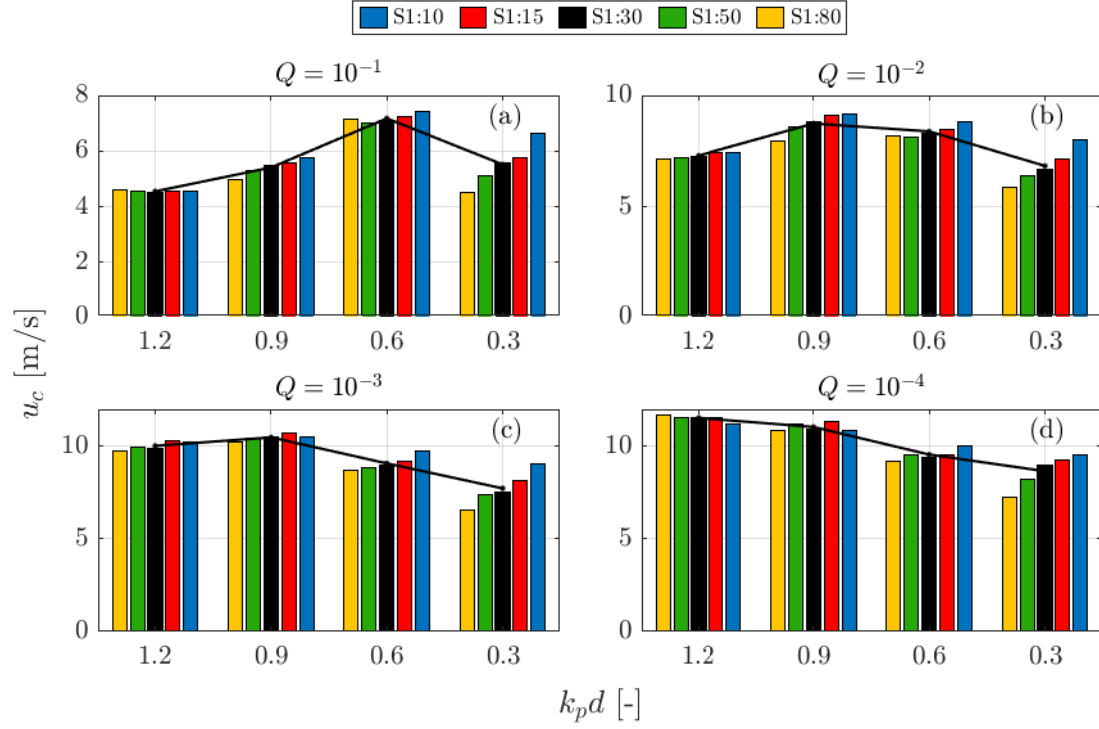


FIGURE 6.5: Variation in u_c associated with probability of exceedance (a) $Q = 10^{-1}$, (b) $Q = 10^{-2}$, (c) $Q = 10^{-3}$ and (d) $Q = 10^{-4}$ for sea-state B4 ($S_p = 0.04$). The colour of the bar corresponds to the bed slope gradient. The black line shows the trend in the average u_c across bed slopes.

practice, these velocities are often estimated using simplified analytical wave theories. The accuracy of such methods, particularly under highly nonlinear and shallow-water conditions, remains uncertain. Therefore, Section 6.4 evaluates the applicability of common regular and irregular wave theories against the numerically simulated velocity fields presented here, in order to provide practical guidance for engineering use.

6.4 Velocity profiles

Typically in the design of ships and offshore structures, wave-induced kinematics are estimated using analytical wave theories (Det Norske Veritas, 2010). Although these formulations provide a computationally efficient means of obtaining velocity profiles, their performance under the nonlinear and depth-limited conditions described in the previous sections is not well established. This section of work provides a detailed assessment of the application of regular (Section 6.4.1) and irregular (Section 6.4.2) methods to model the velocity profile below extreme wave crests in random sea-

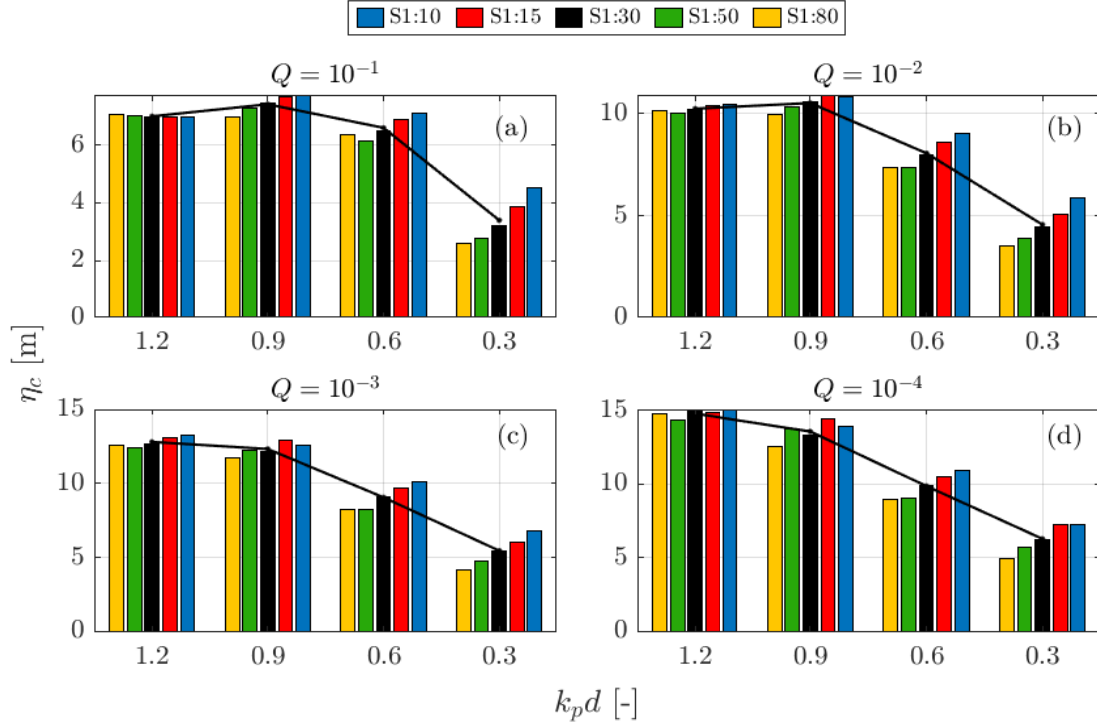


FIGURE 6.6: Variation in η_c associated with probability of exceedance (a) $Q = 10^{-1}$, (b) $Q = 10^{-2}$, (c) $Q = 10^{-3}$ and (d) $Q = 10^{-4}$ for sea-state B4 ($S_p = 0.04$). The colour of the bar corresponds to the bed slope gradient. The black line shows the trend in the average η_c across bed slopes.

states. This is supplemented with a detailed quantification of the error of each method within the parameter space. The findings of this section employ the validated SWASH model, as presented in Section 6.2, as the numerical benchmark for kinematic theory. All the results of Sections 6.4.1 and 6.4.2 are similar across different bed slope configurations. Hence, we focus on results of bed slope 1:30 for clarity and depth. However, any implications of the bed slope are addressed in Section 6.4.3.

6.4.1 Regular wave methods

Utilising regular wave methods to model an extreme wave in a random sea assumes that large waves may be characterised by a representative wave height and period and form part of a regular wave train (Sobey, 1990). The trough-to-trough wave period, T_{tt} , is often regarded as a stable representative wave period measure (Mizuguchi, 1982) and will be adopted in the present work alongside the corresponding trough-to-trough wave number, k_{tt} . Figure 6.7 shows the wave profile and velocity profile below crests associated with a probability of exceedance $Q = 10^{-3}$. Following the

method outlined in Section 6.2, the results shown are an average across 30 wave crests, with the 99% confidence intervals shown by the blue shaded region. We use Stokes and Stream function theory, both of which may be applied by adopting either a crest height or wave height fit to the random wave of interest. We compare these permutations in Figure 6.7. Specifically, in a mild sea-state ($S_p = 0.02$) shown in Figures 6.7(a–b), the crest height match solutions of Stokes and Stream function theory, illustrated by red and black continuous lines, respectively, provide a more accurate prediction of horizontal velocities in the vicinity of the crest compared to wave height match solutions. Closer to the bed, the crest height match Stokes theory overestimates velocities, however, the integrity of the crest height match Stream function continues to hold at this location. Considering a steeper sea-state ($S_p = 0.05$) shown in Figures 6.7(c–d), although the crest height match solution of Stokes and Stream function theory are able to capture the wave profile close to the crest, as evident in Figure 6.7(c), the notable horizontal skewness of the numerical wave profile cannot be matched by regular wave theory (Fenton, 1985). This results in under prediction of velocities above still water level combined with overpredictions lower in the water column. Overall, the applicability of regular wave methods is significantly compromised when considering steeper sea-states (Figure 6.7(d)) compared to milder seas (Figure 6.7(b)). Moreover, with regards to the distribution of velocity profiles around $Q = 10^{-3}$, Figure 6.7(b) exhibits greater variation compared to Figure 6.7(d). This can be explained by the occurrence of wave breaking in steeper sea-states which act to homogenise the wave field. However, across the wave cases considered, the 99% confidence interval remains relatively narrow.

As an alternative, Grue et al. (2003) proposed a new method based on measurements of the kinematics of extreme laboratory waves using particle image velocimetry. This method assumes that the velocity profile under a large wave can be represented by that of a third-order Stokes' wave such that the wave steepness is expressed as:

$$k\eta_c = \epsilon + \frac{1}{2}\epsilon^2 + \frac{1}{2}\epsilon^3, \quad (6.5)$$

following the dispersion relation:

$$\frac{\omega^2}{gk} = 1 + \epsilon^2, \quad (6.6)$$

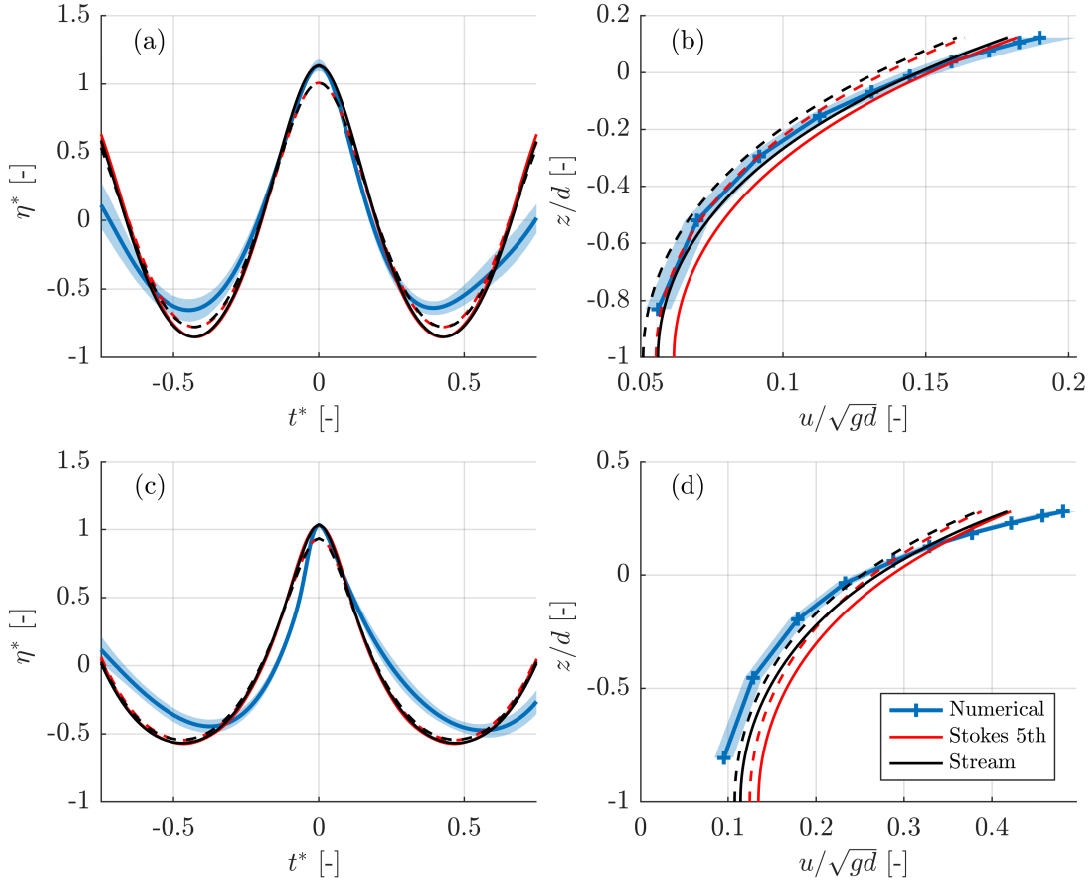


FIGURE 6.7: (a),(c) wave profile and (b), (d) velocity profile below the crest associated with a probability of exceedance $Q = 10^{-3}$ propagating over bed slope 1:30 in $k_p d = 1.22$ for sea-state (a–b) B2 and (c–d) B5. Results of the numerical simulation, Stokes theory and Stream function theory are shown in a blue, red and black, respectively. Crest height and wave height match solutions are shown in solid and dashed lines, respectively. The 99% confidence interval associated with the numerical data is shown by the shaded blue region.

where η_c is the maximum crest height and ϵ is the approximated linear steepness based on the trough-to-trough period and the third order Stokes expansion for steepness. This is unlike the standard linear steepness defined as $Hk/2$. Instead, ϵ is obtained by simultaneously solving for the nonlinear wave steepness (Equation 6.5) and the dispersion relation (Equation 6.6). The vertical velocity, as a function of the vertical coordinate, z , follows an exponential profile defined as:

$$u(z) = \epsilon \sqrt{\frac{g}{k}} \exp(kz). \quad (6.7)$$

Stansberg et al. (2008) has confirmed the effectiveness of this method at capturing

kinematics above the still water level, $z > 0$, in deep water. Following the guidance of Hadjigeorgiou (2019), we modify the method of Grue et al. (2003) for finite water depths. Firstly, the influence of finite water depth on the wave shape is included such that:

$$k\eta_c = \epsilon + B_{22}\epsilon^2, \quad (6.8)$$

where $B_{22} = \coth(kd)(1 + 2S)/[2(1 - S)]$ and $S = \operatorname{sech}(2kd)$, as defined by Fenton (1985). Similarly, the deep water dispersion relation is modified to:

$$\frac{\omega^2}{gk} = C_0(1 + 2C_2\epsilon^2), \quad (6.9)$$

where $C_0 = (\tanh(kd))^{1/2}$ and $C_2 = (\tanh(kd))^{1/2}(2 + 7S^2)/[4(1 - S)^2]$ (Fenton, 1985). This includes a linear finite depth dispersion term represented by C_0 , and a nonlinear correction given by C_2 . Moreover, the assumption of an exponential decay in the vertical velocity profile, as in deep water, is not valid for finite depths. Instead, a hyperbolic function is used:

$$u(z) = \epsilon \sqrt{\frac{g}{k}} \frac{\cosh k(z + d)}{\sinh(kd)}. \quad (6.10)$$

Figure 6.8 assesses the application of the modified Grue model on extreme waves at different effective water depths. The original Grue model, shown in a black dotted line, is comparable to the modified Grue model at the offshore location. However, notable differences in the two formulations are observed in shallower water, with the latter providing enhanced predictions for coastal applications. Strong correspondence between the numerical data (circular markers) and the theoretical predictions (dashed lines) is observed across the water column at the offshore location ($k_p d = 1.2$). As the effective water depth is reduced to $k_p d = 0.9$, the velocity near the crest is still well captured, however overestimations close to the bed are observed by the modified Grue model. These discrepancies become magnified in shallower water ($k_p d = 0.6$), where notable underestimation by the modified Grue model become evident in the vicinity of the crest. Similar results have been shown by Alberello et al. (2018) for focussed waves, who have also demonstrated overestimation of velocities low in the water column in such cases. This is caused by the inability of this formulation to capture the effects associated with multiple frequency wave compo-

nents since the formula uses a single trough-to-trough wave period only. Moreover, in coastal waters, long frequency bound waves exist which are locally out of phase with the wave group. As such, the lack of multiple frequency and phase components in regular wave methods presents important limitations.

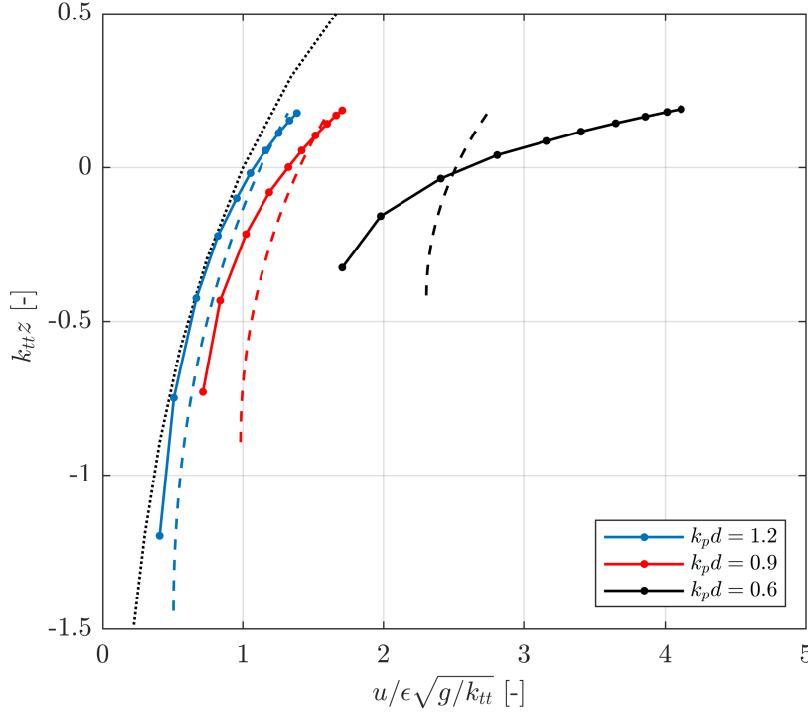


FIGURE 6.8: Wave profile under extreme wave crest propagating over bed slope 1:30 for sea-state B2. Solid and dashed lines correspond to results of the numerical simulation and the modified Grue model, respectively. The original deep water Grue formulation is shown in a black dotted line.

Despite their limitations, regular wave methods can provide reasonable initial estimations of the velocity profile for practical applications. For example, when only the total horizontal force exerted at the base of a coastal structure is of interest, overestimations at the bed combined with underestimations at the crest may provide reasonable total base shear estimates. Here, we present guidance on the application of these methods, by defining an error metric in the horizontal velocity profile prediction as:

$$\varepsilon = \sum_{i=1}^N \frac{u_{th}(z_i^*) - u(z_i^*)}{u(z_i^*)} \Delta z^*, \quad (6.11)$$

where $\Delta z^* = \frac{\eta_c/d+1}{N}$, $u(z^*)$ and $u_{th}(z^*)$ are the horizontal velocity based on numerical and theoretical predictions, respectively, as a function of $z^* = z/d$. This provides

a depth-averaged (from the bed to the wave crest) proxy for the magnitude of the error between numerical and theoretical predictions of horizontal velocity. Values of $\varepsilon > 0$ indicate overestimations by theory, while $\varepsilon < 0$ indicate underestimations.

Adopting the same axes as the validity diagram of Le Méhauté (1976), Figure 6.9 demonstrates the error, ε , associated with modelling wave crests of exceedance probability $Q = 10^{-3}$ in each sea-state at each spatial location using various regular wave theories. Stokes (crest-match) theory shown in Figure 6.9(a), is able to generally provide predictions with low error, $\varepsilon < 0.2$, in deep to intermediate water depths, $d/gT_p^2 > 0.015$. In shallower water, $d/gT_p^2 < 0.015$, the accuracy of the solution begins to break down particularly in steeper sea-states such that errors $\varepsilon < -1$ become evident. The accuracy of the Stream function theory, as shown in Figure 6.9(b), generally remains high even for very steep sea-states in intermediate waters with $\varepsilon < 0.05$. However, given limitations in the formulation, notable overestimations are observed if applied to the shallowest water depths, $d/gT_p^2 < 0.01$. Alternatively, the method of Grue shown in Figure 6.9(c) may be applied across the domain of interest, however larger errors are observed in shallower water depths, particularly for steeper sea-states. These are observed as underestimations primarily due to the underprediction of velocities higher in the water column, as shown previously in Figure 6.8. As such, the application of the modified Grue model in this regime results in non-conservative predictions ($\varepsilon < 0$).

Alternatively, only the velocity of the crest may be of interest in some design applications, such as considerations of impact loads and overtopping. As such, we define a new error metric, ε_c , which only considers the error of modelling the horizontal crest velocity, defined by:

$$\varepsilon_c = \frac{u_{c,th} - u_c}{u_c}, \quad (6.12)$$

where u_c and $u_{c,th}$ is the horizontal velocity at the crest based on numerical and theoretical predictions, respectively. Following the same layout as Figure 6.9, Figure 6.10 demonstrates the error, ε_c , associated with each method. Considering Stokes theory shown in Figure 6.10(a), unlike Figure 6.9(a), at the offshore location, errors are negative in steep sea-states indicating an underestimation of crest velocities. These are in line with the findings presented in Figure 6.7 such that in intermediate water, although underestimations are observed at the crest ($\varepsilon_c < 0$), the overestimations

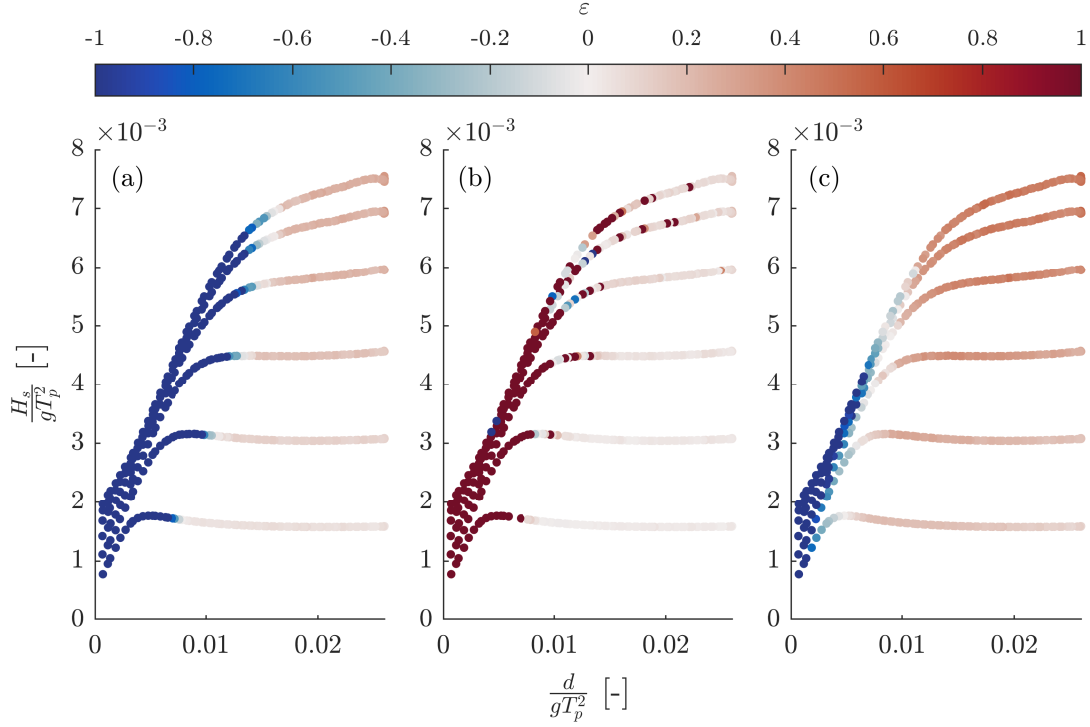


FIGURE 6.9: Error function, ε , associated with modelling the waves associated with a probability of exceedance of $Q = 10^{-3}$ using (a) Stokes 5th, (b) Stream function and (c) modified Grue model, for waves propagating over a bed slope of 1:30. Positive and negative values of ε indicate overestimates and underestimates, respectively.

lower in the water column result in values of $\varepsilon > 0$ at this location. In shallower water, breaking effects result in significant underestimations at the crest. Considering Stream function theory, shown in Figure 6.10(b), similar conclusions can be made where values offshore are $\varepsilon_c \approx -0.05$. Moreover, the overestimations in shallow water are also evident when considering crest velocities alone. In fact, these become notable ($\varepsilon_c > 0.5$) even at $d/gT_p^2 = 0.01$. The underestimations at the offshore location are also observed for the modified Grue model, shown in Figure 6.10(c), and are more notable than other methods in steeper sea-states ($\varepsilon_c \approx -0.15$). Overall, there is a general tendency of all regular wave methods to underestimate crest velocities prior to breaking onset, with the discrepancies being more notable in steeper seas.

Utilising these findings, we define a validity map that provides guidance for the recommended regular wave method to model crests associated with a probability of exceedance of $Q = 10^{-3}$ such that the magnitude of ε is minimised. In cases where the error between methods is comparable, the more conservative method is preferred. This is illustrated in Figure 6.11(a). The marker size is inversely proportional to the

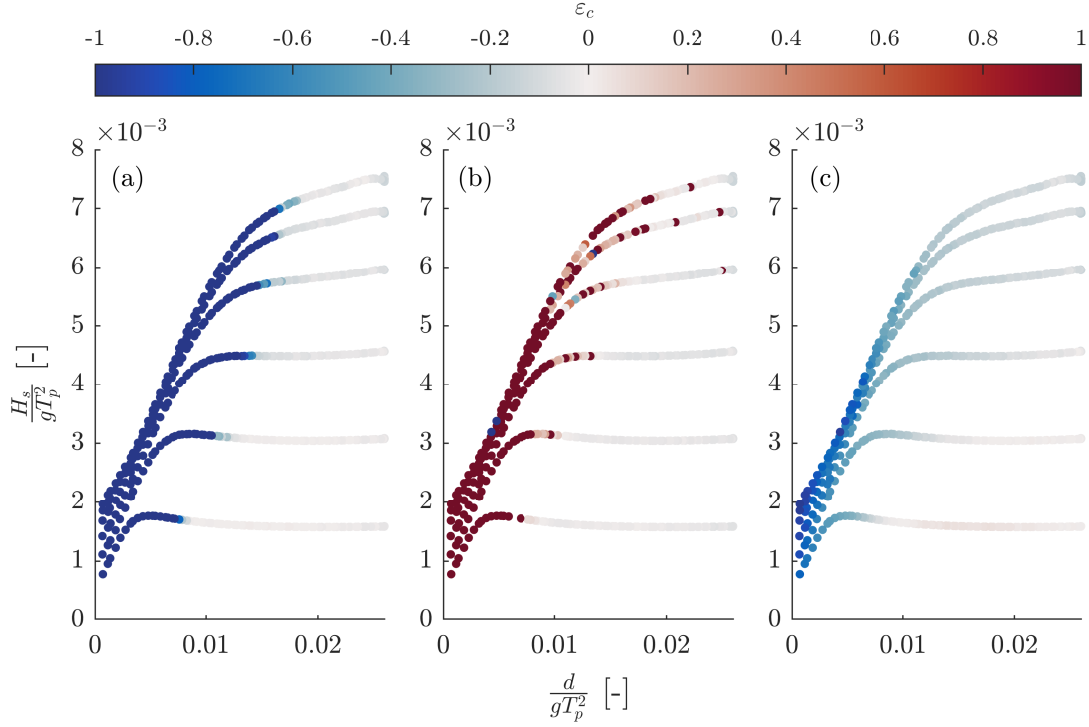


FIGURE 6.10: Error function, ε_c , associated with modelling the waves associated with a probability of exceedance of $Q = 10^{-3}$ using (a) Stokes 5th, (b) Stream function and (c) modified Grue model, for waves propagating over a bed slope of 1:30. Positive and negative values of ε_c indicate overestimates and underestimates, respectively.

magnitude of the error, ε , such that a smaller marker size corresponds to a larger error. In deep to intermediate waters, $d/gT_p^2 > 0.01$, generally the crest height match Stream function performs best. This has previously been shown in Figure 6.7. Although the crest match implementation of Stokes theory may provide good predictions of kinematics above still water level, it tends to overestimate velocities near the bed. Conversely, the Stream function is able to more closely capture the profile of kinematics across the water column. In shallower water, $d/gT_p^2 < 0.01$, particularly for steeper sea-states, the method of Grue appears to perform best. However, it should be noted that in very shallow water, errors associated with this method may still be large, as shown in Figure 6.8, evident as underpredictions near the crest and overpredictions near the bed. Moreover, given the findings highlighted in Figure 6.9, no regular method is able to predict the velocity profile with reasonable error in such water depths. This is shown in Figure 6.11(a) with much smaller markers. Thus, caution should be taken when adopting any of these methods in this region.

The recommended regular wave theory such that ε_c is minimised is shown in

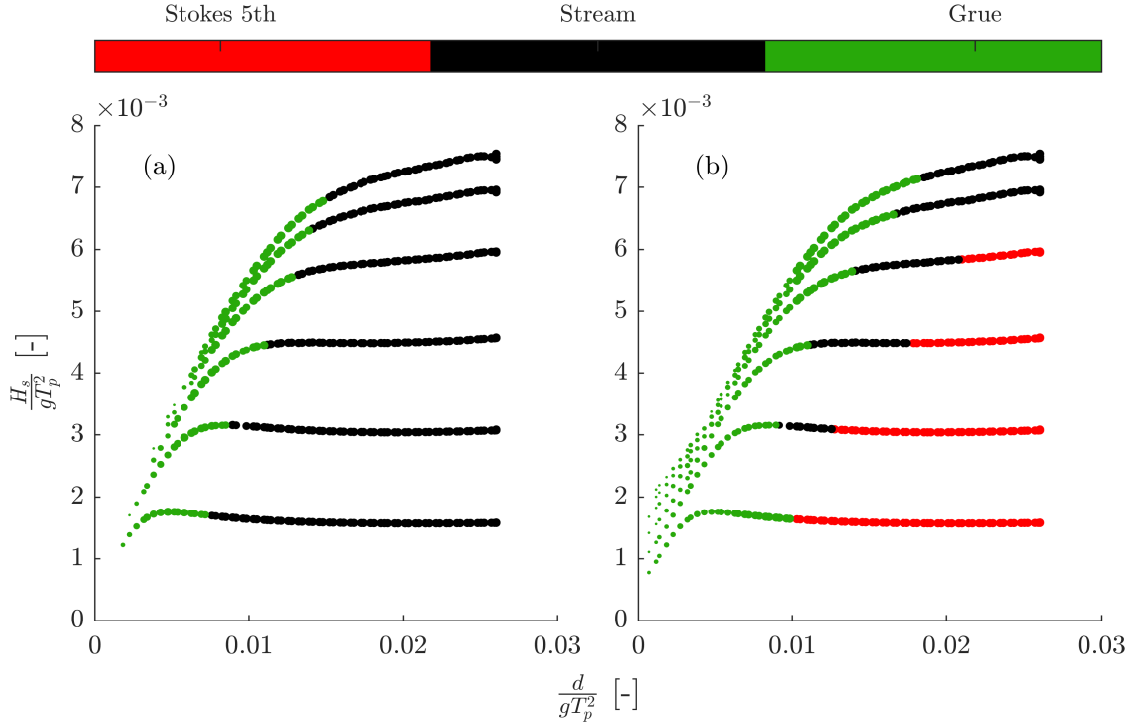


FIGURE 6.11: Recommended regular wave theory to model waves associated with a probability of exceedance of $Q = 10^{-3}$ such that (a) ε is minimised and (b) ε_c is minimised. Results correspond to a bed slope of 1:30. The marker size is inversely proportional to the magnitude of the error.

Figure 6.11(b). Although Figure 6.11(a) has demonstrated that the Stream function generally performs best for $d/gT_p^2 > 0.01$, Figure 6.11(b) suggests that Stokes theory captures the magnitude of velocities at the crest well in smaller steepness waves ($H_s/gT_p^2 \leq 0.005$) further offshore. However, the Stream function is required when modelling more nonlinear waves located further inshore. In shallow water, the limitations of all existing methods continue to be evident in Figure 6.11(b), where despite the notable underestimations at the crest, the modified Grue model still performs better than other regular wave alternatives, though is non-conservative. Once again, caution is required when modelling crest velocities in shallow water using regular wave theories, and should generally be avoided.

6.4.2 Random wave methods

If a more accurate description of the velocity profile is desired, the utilisation of random wave theories is recommended. The horizontal velocity is defined as:

$$u = \frac{\partial \phi}{\partial x}, \quad (6.13)$$

where ϕ is defined by Equation 2.10 and 2.19 for LRWT and SORWT, respectively.

The application of linear (LRWT) or second-order (SORWT) solutions relies on a correct linear signal input, consisting only of freely-propagating wave components. In practice, obtaining such spectra is challenging, with real spectra consisting both of free and bound components. Although this has limitations regardless of the order of the method, SORWT is more sensitive to the presence of bound contributions (Tayfun & Fedele, 2007). Moreover, it is well established that direct application of these solutions results in overestimations near the crest of the wave due to small high frequency waves being displaced vertically by larger long waves. This is commonly referred to as high frequency contamination (Lo & Dean, 1986, Sobey, 1990).

Alternatively, empirically-stretched solutions are often adopted. Commonly, the method of Wheeler (1970) is utilised in coastal engineering practice, which is based on an extrapolation of the profile from the undisturbed domain to the instantaneous free surface. This is given by:

$$u(x, z, t) = \sum_i a_i \omega_i \frac{\cosh\left(k_i d \frac{z+d}{d+\eta}\right)}{\sinh k_i d} \cos(k_i x - \omega_i t + \theta_i), \quad (6.14)$$

where η is the surface elevation. However, Gudmestad (1993) and Stansberg et al. (2008) found that this method underpredicts kinematics under steep irregular waves. Moreover, another limitation of the original Wheeler method is that it applies the same transformation to all frequency components. However, the behaviour of waves is dependent on their frequency. Thus, modifications to the Wheeler solution may be conducted by performing a progressive frequency-dependent vertical stretching using the solution of Molin (2002). In essence, the stretching of the i -th frequency component is based on the sum of the surface elevation due to all lower-frequency components up to i . This accounts for the fact that short waves are more affected

by surface elevation fluctuations compared to long waves. This is expressed as:

$$u(x, z, t) = \sum_i a_i \omega_i \frac{\cosh\left(k_i d \frac{z+d}{d+\eta_I}\right)}{\sinh k_i d} \cos(k_i x - \omega_i t + \theta_i), \quad (6.15)$$

where $\eta_I = \sum_{i=1}^I \eta_i$ and $\eta_i = a_i \cos(k_i x - \omega_i t + \theta_i)$. Alternatively, Chakrabarti (1971) avoids high-frequency contamination by adjusting the water depth such that:

$$u(x, z, t) = \sum_i a_i \omega_i \frac{\cosh k_i(z+d)}{\sinh k_i(d+\eta)} \cos(k_i x - \omega_i t + \theta_i) \quad (6.16)$$

Donelan et al. (1992) assume the superposition of wave components such that spectral wave components are iteratively added to the perturbed solution. This is unlike the traditional linear superposition which has been found to overestimate crest velocities. This formulation defines successive amplitude and velocity increments ($\delta\eta_i$ and δu_i , respectively) as:

$$\delta\eta_i = a_i \cos(k_i x - \omega_i t + \theta_i), \quad (6.17)$$

$$\eta_i = \eta_{i-1} + \delta\eta_i, \quad (6.18)$$

$$\delta u_i = a_i \omega_i \frac{\cosh(k_i(z+d))}{\sinh(k_i(d+\eta_{i-1}))} \cos(k_i x - \omega_i t + \theta_i), \text{ and} \quad (6.19)$$

$$u_i = u_{i-1} + \delta u_i. \quad (6.20)$$

Alberello et al. (2018) has shown good correspondence of this method to experiments of focused waves. However, underestimations of 20% are observed in the breaking region. Such underestimations were also described by Hadjigeorgiou (2019) for irregular directional waves, particularly in shallower water depths.

To assess these methods against the present dataset, we consider the velocity profile under wave crests associated with a probability of exceedance $Q = 10^{-3}$ at each spatial location. The findings are summarised by Figure 6.12. In line with the analysis conducted in Section 6.4.1, 30 wave crests are selected from the concatenated dataset across all 20 realisations whose exceedance probability is closest to the target ($Q = 10^{-3}$). Considering each of the 30 waves independently, the realisation that they appear in is determined. Hence, each wave is associated to a specific measured omni-direction spectrum of the water surface elevation, $S_{\eta\eta}$, based on the realisation in which it appears. The spectra may be nondimensionalised by the peak frequency

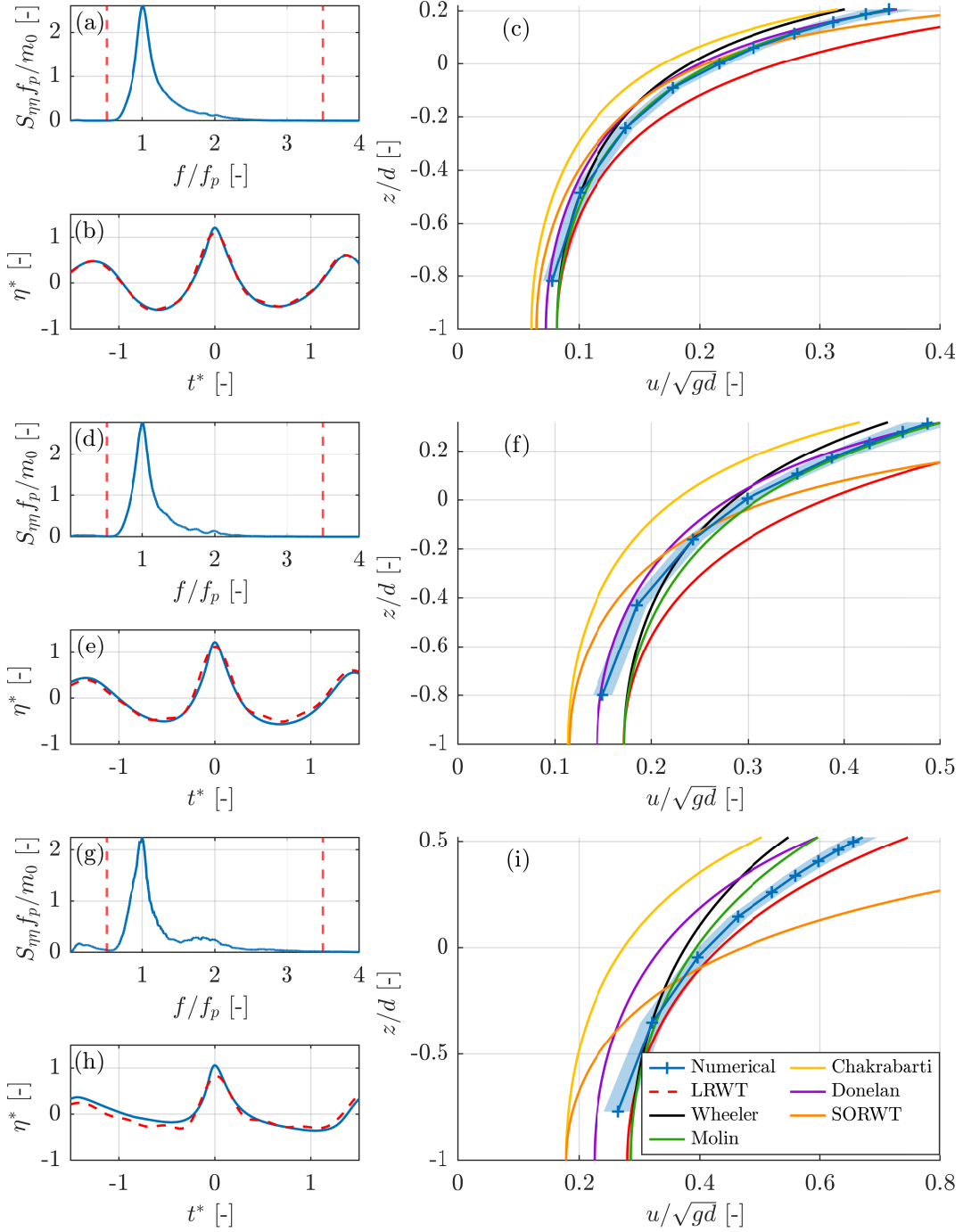


FIGURE 6.12: (a), (d), and (g) amplitude spectra; (b), (e), and (h) mean wave profiles; (c), (f), and (i) horizontal velocity profiles beneath the wave crest. The truncated spectrum used as model input and its associated surface elevation based on LRWT are shown with red dashed lines in the left column. Blue shaded regions in the right column indicate the 99% confidence interval of the velocity profiles. Results correspond to sea-state B3 propagating over a 1:30 bed slope in (a–c) $k_p d = 1.2$, (d–f) $k_p d = 0.9$ and (g–i) $k_p d = 0.6$.

$f_p = 1/T_p$ and the associated zero-th order spectral moment, m_0 . The normalised spectra are shown in Figures 6.12(a), (d) and (g), corresponding to progressively shallower water depths.

The accuracy of these random wave methods is heavily conditioned upon the input spectrum. Ideally, the spectrum should consist entirely of freely propagating wave components. The utilisation of a measured spectrum, particularly from the coastal zone, which contains both free- and bound-wave components will lead to erroneous results. However, direct decomposition of realistic measured spectra into free- and bound-wave components is challenging. As an alternative, the spectrum may be truncated in the frequency domain in an effort to minimise these effects. In doing so, a compromise must be made to ensure sufficient frequency components are included to capture the wave crest with adequate accuracy. In the context of a JONSWAP-type spectral shape, isolation of frequency-difference bound terms from a measured spectrum may be achieved through low-frequency truncation of the spectrum, given that no energy is expected in this region for a linear signal. The lower frequency limit is selected to remove energy associated with the sub-harmonic peak, as is clear in Figure 6.12(g), and hence is set at $f = 0.5f_p$. The selection of the upper frequency limit to isolate the frequency sum terms is more challenging given that they exist in regions also containing notable free wave energy (Lutio, 2017). Commonly in industry, recommendations for the limit range from around $3f_p$ (CEM, 2002) to $4f_p$ (Det Norske Veritas, 2010) are provided. Unfortunately, it is difficult to select an appropriate global upper truncation frequency given that the accuracy of these random wave methods is not known in coastal waters a priori. However, initial investigations have concluded that an upper limit of $3.5f_p$ is able to accurately capture the wave crest across the domain of interest, whilst minimising effects related to high frequency contamination, which become evident when including higher frequency components. This is also consistent with the findings of Lutio (2017). As such, this limit will be adopted in all analysis in this section. It is important to highlight that our investigations have shown high sensitivity of the calculated velocity profiles on the truncation limits adopted. Thus, the selection of appropriate truncation limits in practical design applications, following the guidance provided in this work, is critical to the accurate application of random wave theories.

Figure 6.12(b), (e) and (h) demonstrate the mean wave profile at each location,

where $\eta^* = \eta/H_s$, $t^* = \frac{t-t_c}{T_p}$ and t_c is the time of the crest. We observe that the surface elevation associated with the truncated spectrum calculated using LRWT (red dashed line) is able to match that of the full spectrum (blue solid line), particularly in the vicinity of the crest. Utilising the truncated spectrum, the velocity profile below wave crests associated with a probability of exceedance, $Q = 10^{-3}$, is computed by adopting the same phase as that of the numerical crests. More specifically, numerically, the time of the crest of interest, $t_{c,i}$ is known from the time series analysis. Hence, the phase adopted in the random wave models is $\phi_i = \theta_i - \omega_i t_{c,i}$, where θ_i is the phase according to the spectra of the realisation in which the wave of interest appears. This is used to independently calculate the velocity profile for each of the 30 waves, using the truncated measured spectrum, $S_{\eta\eta}$, associated with the realisation of the time series of the surface elevation in which the wave appears. The horizontal velocity profiles are averaged across the 30 waves, to obtain a single representative profile. This is illustrated in Figure 6.12(c), (f) and (i).

At the offshore location (Figure 6.12(c)), most of the models perform relatively well, with the formulation of Molin and Donelan performing best. Despite the upper truncation limit, overestimations by LRWT are still evident. SORWT experiences overestimations near the crest, however, overall its inclusion of higher order contributions results in a more accurate description of the velocity profile compared to LRWT. These overestimations continue to be evident in Figure 6.12(f) in $k_p d = 0.9$. At this location, the strong performance of the Molin and Donelan methods above still water level, shown in Figure 6.12(c), continues to hold. However, the Molin approach experiences minor overpredictions lower in the water column, with this being within the 99% confidence interval of the numerical model. Baldock et al. (1996) has explained the importance of nonlinear interactions in broad-banded spectra, acting to reduce velocities lower in the water column. This is caused by the formation of long bound waves which produce a reverse flow at the scale of the wave group. This effect is not explicitly included in the linear model formulations, and may explain the overestimations observed by some methods lower in the water column. If shallower water is considered, the mean profile associated with the largest waves changes from a quasi-symmetric profile shown in Figure 6.12(b) to a sawtooth in Figure 6.12(h). At such water depths, common techniques to estimate wave velocity from the surface elevation break down, particularly when breaking effects dominate (Alberello

et al., 2018). This is shown in Figure 6.12(i). Although the Molin approach provides the closest match to numerical observations, notable underestimations are observed across the water column by all methods except LRWT. This highlights limitations in existing methods at fully capturing the complexities of wave kinematics in shallower water. Furthermore, given the sensitivity of SORWT to the inclusion of bound harmonics in the input spectrum, significant reductions in the accuracy of this model is observed in shallower water, where bound wave energy is heightened.

In a similar manner to the guidance outlined in Section 6.4.1, this section will provide insight into the magnitude of uncertainty associated with each method and how this differs within the domain of interest. Figure 6.13 follows a similar format to Figure 6.9 such that the error, ε , associated with various random wave theories is illustrated by adopting the axes in the validity diagram of Le Méhauté (1976). Considering LRWT shown in Figure 6.13(a), overestimation are evident throughout the domain of interest. In intermediate depths, $d/gT_p^2 > 0.01$, these are generally larger in steeper sea-states (greater H_s/gT_p^2) with magnitudes exceeding $\varepsilon = 1$. In shallow water, underestimations of the order of $0.3 < \varepsilon < 0.5$ become evident. In other words, the high frequency contamination which is ubiquitous in $d/gT_p^2 > 0.01$ is reduced in shallow water due to the hyperbolic nature of the velocity profile at this location, compared to the exponential profile in deep water.

Considering the methods of Wheeler, Molin and Donelan, shown in Figure 6.13(b), (c) and (e), the overall trends observed are similar. In intermediate depths, $d/gT_p^2 > 0.01$, the error associated with these methods is very small $\varepsilon \pm 0.05$, indicating their applicability in this region. However, at this location, the methods of Wheeler and Molin tend to overestimate values, whereas underestimations are observed by Donelan. In general, the predictions of Donelan manifest in errors, ε , closer to zero. Conversely, the method of Chakrabarti shown in Figure 6.13(d) exhibits clear underestimations throughout the spatial domain with values as low as $\varepsilon = -0.25$, even for intermediate water depths. Utilising SORWT, shown in Figure 6.13(f), although minor underpredictions are seen at the offshore limit, generally overestimations are observed which are particularly magnified in steep seas. This is due to the inclusion of sum terms in addition to bound terms in the input spectrum. Conversely, similar to LRWT shown in Figure 6.13(a), all other methods also experience notable underestimations in the wave profile in shallow water compared to numerical simulations.

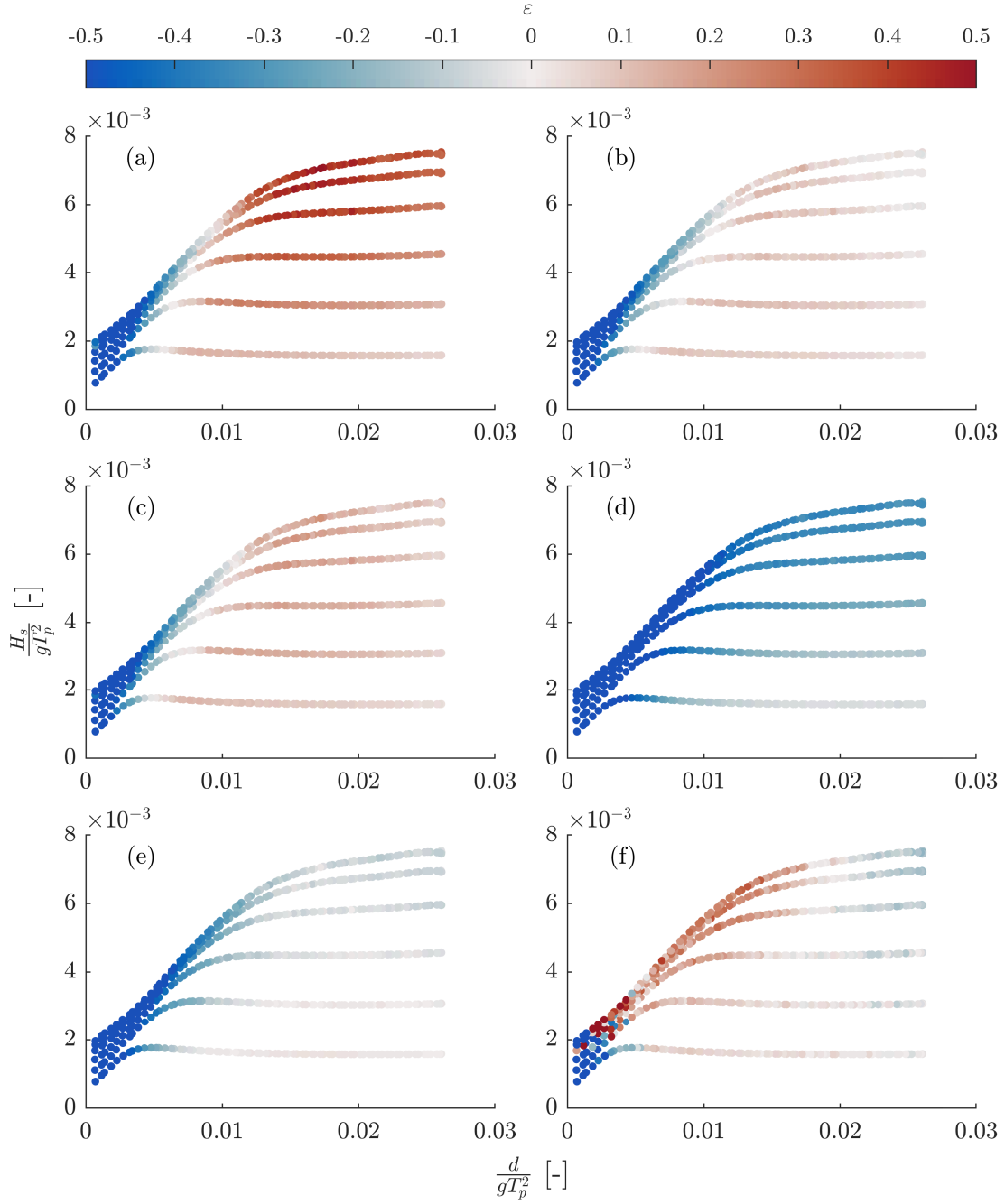


FIGURE 6.13: Error function, ε , associated with modelling the waves associated with a probability of exceedance of $Q = 10^{-3}$ using (a) LRWT, (b) Wheeler, (c) Molin, (d) Chakrabarti, (e) Donelan theory and (f) SORWT, for waves propagating over a bed slope of 1:30. Positive and negative values of ε indicate overestimates and underestimates, respectively.

To summarise these findings, Figure 6.14(a) outlines the recommended random wave theory to adopt such that the error, ε , associated with the velocity profile of wave crests of probability of exceedance $Q = 10^{-3}$ is minimised. As before, where the

error between methods is minimal, the more conservative method is recommended. In intermediate waters, the method of Donelan is found to provide the best accuracy, in line with the observations made in Figure 6.12. However, as the wave steepness increases due to reductions in water depth or increases in the sea-state steepness, then the Molin method is more appropriate. The strong performance of the Donelan and Molin methods emphasises the importance of considering component interaction of short waves riding on long waves. This has also been shown in the early works of Zhang et al. (1992). Following the occurrence of strong breaking, the applicability of stretched methods is reduced. Instead, SORWT is recommended. Given the uncertainties associated with predicting kinematics at this location, SORWT is preferred since it provides more conservative predictions. In the shallowest waters, errors associated with the application of SORWT can be large. Rather, the Molin approach provides more realistic predictions. However, as indicated by the marker size at this location, errors here are generally much higher, so caution should be taken in the implementation.

If only velocities at the wave crest are of interest, then the recommended method is shown in Figure 6.14(b) such that the magnitude of ε_c is minimised. Overall, the findings are comparable with those shown in Figure 6.14(a). This indicates that if the magnitude of the velocity at the wave crest is well captured, then the overall profile is also likely to be well captured. However, there are a few distinctions. Although the Donelan method was best in intermediate waters even for steep seas, as shown in Figure 6.14(a), instead the Molin method is recommended in this region if only the magnitude of the crest velocity is of interest, as shown in Figure 6.14(b). Moreover, in shallow water, although SORWT may match the velocity profile more accurately (Figure 6.14(a)), LRWT provides a higher accuracy for such waves in the vicinity of the crest.

6.4.3 Bed slope effects

This section addresses the role of the bed slope gradient on the magnitude of errors, ε , presented in Sections 6.4.1 and 6.4.2 corresponding to a bed slope of 1:30. Results will be presented for the extremities of the bed slope configurations considered in this study: 1:10 and 1:80. As such, the findings of this section may be used to define upper and lower bounds for the accuracy of applying existing theoretical distributions.

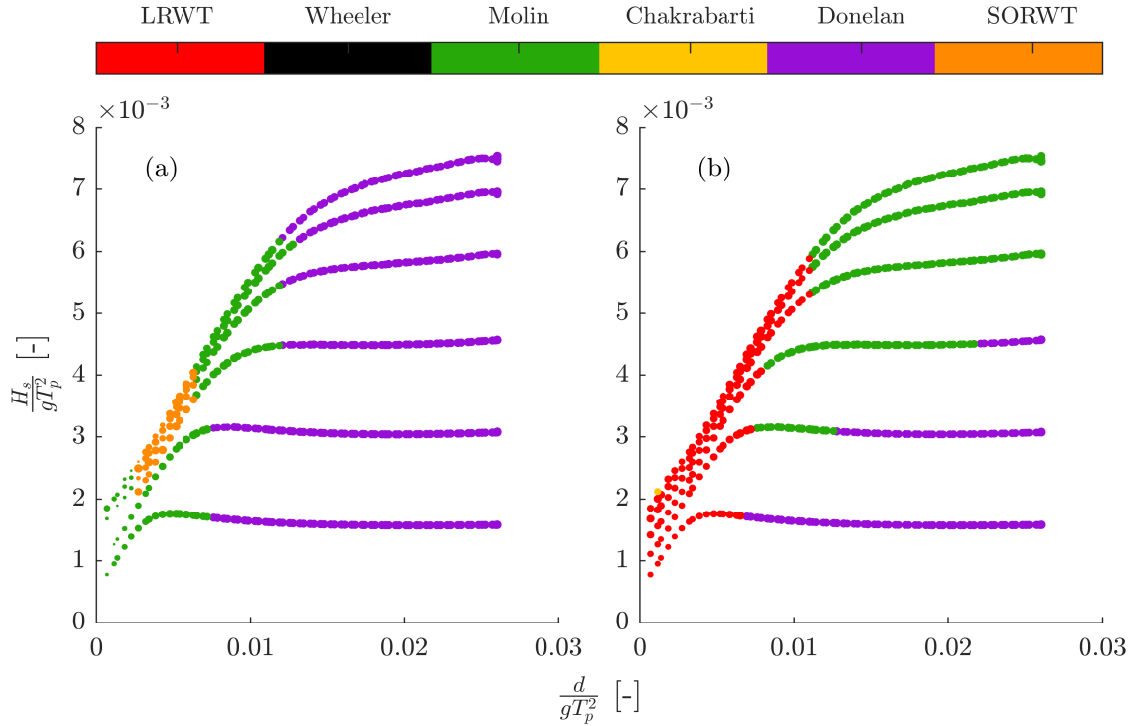


FIGURE 6.14: Recommended random wave theory to model waves associated with a probability of exceedance of $Q = 10^{-3}$ such that (a) ε is minimised and (b) ε_c is minimised. Results correspond to a bed slope of 1:30. The marker size is inversely proportional to the magnitude of the error.

Figure 6.15 considers the variation in ε for various regular wave theories, with the columns from left to right corresponding to bed slopes 1:10, 1:30 and 1:80, respectively. Overall, for all regular wave theories, the magnitude of ε is similar across bathymetries. More specifically, the accuracy of Stokes theory, shown in Figure 6.15(a–c), and the Stream function theory, shown in Figure 6.15(d–f), is relatively insensitive to changes in the bed slope gradient. Conversely, the modified Grue model, shown in Figure 6.15(g–i), demonstrates discrepancies across bed slope configurations in shallow water. The findings indicate a greater accuracy in this region when applied to steeper bed slopes.

Following the same layout as Figure 6.15, Figure 6.16 demonstrates the variation in ε for various irregular wave theories. The trends observed overall are similar across the different theories considered. In slopes steeper than 1:30, shown in the left column, values of ε are generally closer to zero. Conversely, bed slopes milder than 1:30, shown in the right column, exhibit magnitudes of ε greater than what is observed for a bed slope of 1:30. This is true for all irregular wave theories examined,

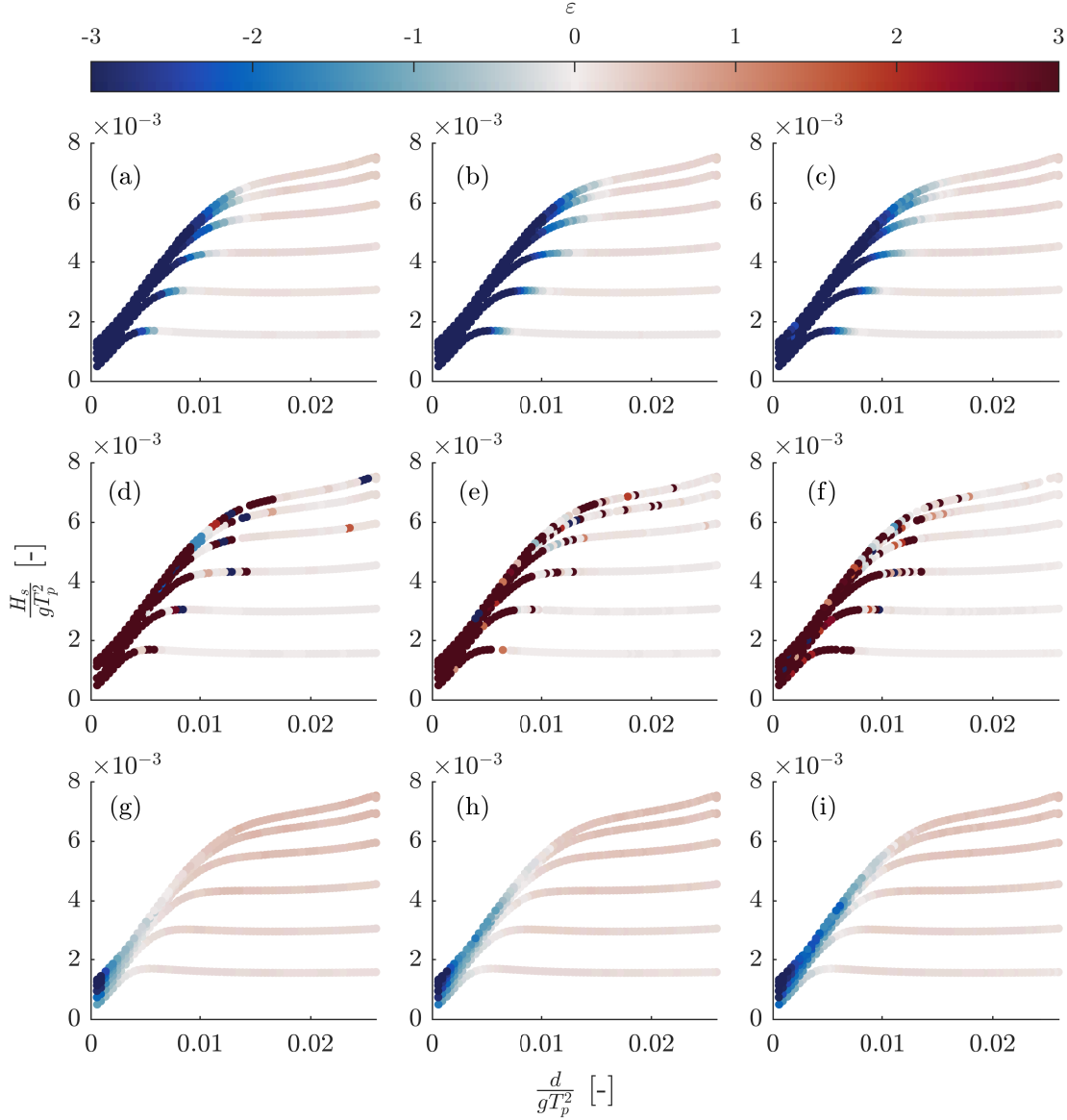


FIGURE 6.15: Error function, ε , associated with modelling the waves associated with a probability of exceedance of $Q = 10^{-3}$ using (a–c) Stokes, (d–f) Stream function, and (g–i) modified Grue model. Columns from left to right correspond to bed slope 1:10, 1:30 and 1:80, respectively. Positive and negative values of ε indicate overestimates and underestimates, respectively.

except SORWT, which generally performs better over milder slopes. For all models, this bed slope dependence is most evident in steeper sea-states (large H_s/gT_p^2) and shallower water (small d/gT_p^2). In these regions, the findings of Figure 6.16 indicate that generally irregular theories provide more accurate predictions in shallow water when applied to steeper bed slopes. This is because wave heights are more restricted in milder slopes, as discussed in Section 6.3.2, and thus greater reductions in wave

heights are observed following breaking onset. Hence, the ability of irregular theories to capture these effects through kinematics is reduced. As a whole, the methods of Chakrabarti and Donelan, are relatively insensitive to such changes. Given the accuracy of the Donelan method exhibited in Section 6.4.2, it is deduced that this method is also the most robust to changes in bathymetry. It should also be noted that given the similarity in the performance of various models across bed slope configurations, the guidance provided in Figures 6.11 and 6.14 is representative of all bed slope configurations used. As such, the recommendations are applicable across the full range of bathymetries investigated.

Comparisons of the errors associated with the implementation of random wave theories, shown in Figure 6.16, to those associated with regular wave theories, shown in Figure 6.15, highlight the general effectiveness of random wave methods over regular wave alternatives. As such, it is generally recommended that random wave methods are utilised, following the guidelines in Figure 6.14. However, in order to apply these methods, spectral information is required, including information regarding the phasing of the wave of interest. This may not be available in some practical contexts. As such, given the good performance of regular wave methods in $d/gT_p^2 > 0.01$, the guidance for the regular wave theory presented in Figure 6.11, may be adopted instead. Nevertheless, it is clear that for $d/gT_p^2 < 0.01$, such methods are not appropriate and should be avoided. In this region, random wave methods must be used following the findings shown in Figure 6.14.

The guidance presented in this section is derived from numerical simulations of long-crested, uni-directional irregular waves propagating over idealised planar bed slopes, without the inclusion of ambient currents or directional spreading. As such, the recommended kinematic formulations are assessed specifically for crest-aligned horizontal orbital velocities and their applicability is limited to similar wave and bathymetric configurations. The results should therefore be applied with caution in conditions involving strongly directional seas, complex or three-dimensional bathymetry, or significant wave-current interaction, where additional physical processes may alter the structure of the velocity field. The simulations cover offshore sea-state steepnesses in the range $0.01 \leq S_p \leq 0.06$, effective water depths approximately $0.16 \leq k_p d \leq 1.22$, relative wave heights of $0.06 \leq H_s/d \leq 2.89$, and uniform bed slopes between 1:10 and 1:80. While the observed trends are physically con-

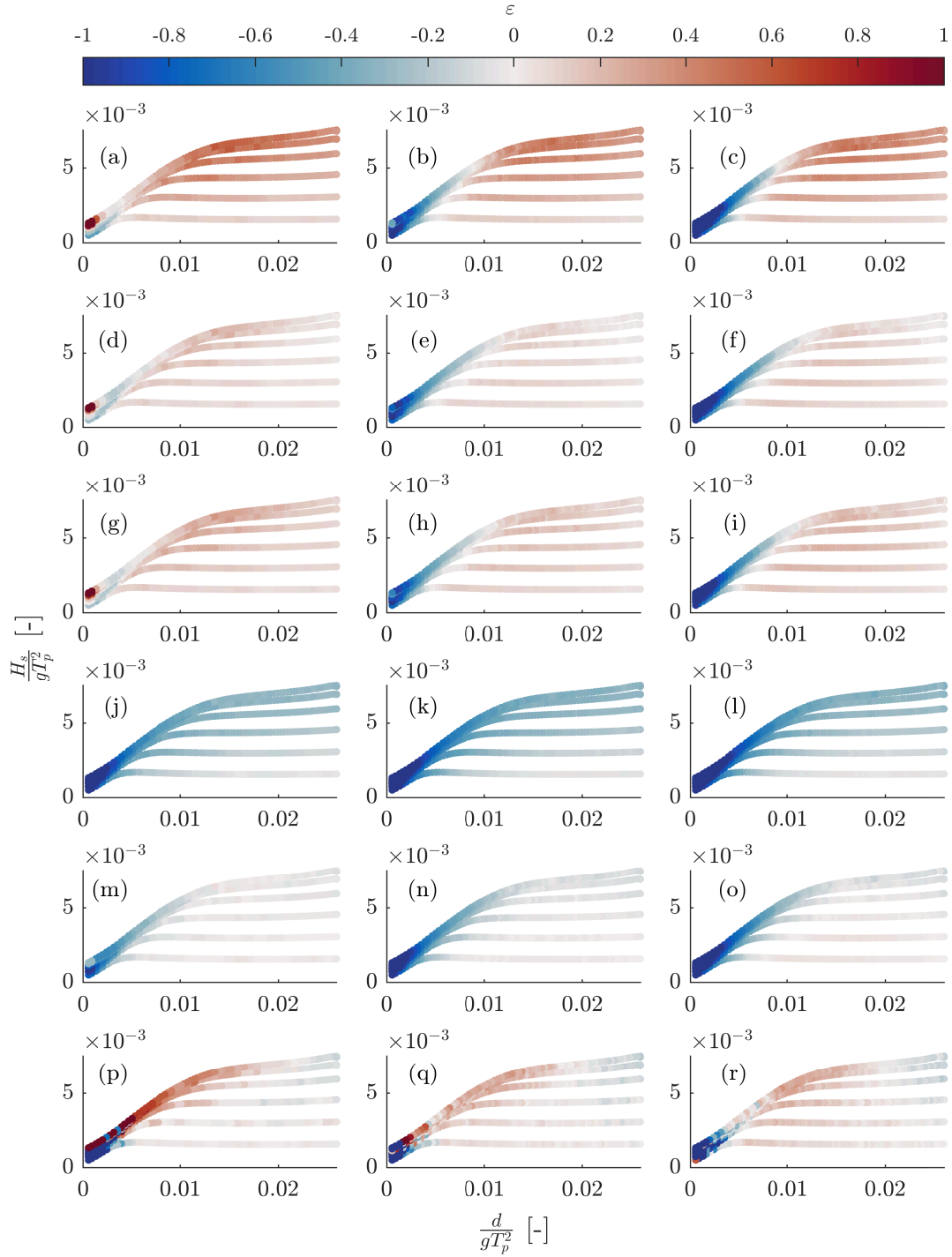


FIGURE 6.16: Error function, ε , associated with modelling the waves associated with a probability of exceedance of $Q = 10^{-3}$ using (a–c) LRWT, (d–f) Wheeler, (g–i) Molin, (j–l) Chakrabarti, (m–o) Donelan theory and (p–r) SORWT. Columns from left to right correspond to bed slope 1:10, 1:30 and 1:80, respectively. Positive and negative values of ε indicate overestimates and underestimates, respectively.

sistent across this parameter space, extrapolation beyond these ranges, particularly into very shallow water, strongly breaking regimes, or environments with complex bathymetric or directional effects, is not advised without further validation.

6.5 Conclusions

Numerical simulations of long-crested irregular waves propagating over a range of planar bed slopes were conducted using SWASH to investigate phase-resolved horizontal velocities beneath extreme crests. Validation against laboratory data confirmed the model's capability to reproduce instantaneous velocity profiles with a non-dimensional root-mean-squared error in $u/\sqrt{gd}$ of up to 0.05, demonstrating its suitability for analysing detailed kinematic behaviour. However, care should be exercised for steep waves in shallow water ($H_s/d > 0.3$), as indicated by Al Khalili et al. (2025). The validated numerical model is utilised to achieve two objectives. Firstly, this work provides understanding on the mechanisms governing phase-resolved crest velocities in the coastal zone. Secondly, the performance of existing kinematic theories is evaluated and practical engineering guidance is provided for their application.

The results reveal that both sea-state steepness and bed slope exert a strong influence on the distribution of crest velocities. Shoaling increases crest velocities relative to the significant value across all exceedance probabilities, while wave breaking in shallow water dissipates energy from the largest waves. Consequently, the tail of the distribution is reduced and a hump emerges at larger exceedance probabilities, reflecting the redistribution of wave probabilities as breaking transfers energy from higher to lower magnitudes. Although similar behaviour has been reported for crest height statistics, the present findings show that wave breaking has a more pronounced effect on crest velocities than on heights. Variations in bed slope were shown to modify shallow-water crest velocities by up to 48% at $k_p d = 0.3$, with steeper slopes accelerating shoaling and milder slopes experiencing stronger breaking-induced reductions.

The evaluation of existing kinematic models demonstrates that regular wave theories, particularly the Stream Function, provide reliable estimates in intermediate depths ($d/gT_p^2 > 0.01$). However, their underlying assumption that large waves can be represented by a characteristic wave height and period forming part of a regular wave train breaks down when applied to highly nonlinear, extreme waves in shallow

water. This limitation arises from their inability to capture wave–wave interactions, which become increasingly significant under such conditions.

To obtain more accurate predictions, the use of irregular wave theories is therefore recommended. The present work provides guidance on the application of these theories, particularly in defining appropriate spectral inputs—a key consideration in practical design scenarios where measured spectra in coastal waters may be complex. Overall, the methods of Molin (2002) and Donelan et al. (1992) perform best for $d/gT_p^2 > 0.01$, owing to their ability to represent spectral superposition effects. In this region, good correspondence with numerical simulations is achieved throughout the water column, with very high accuracy near the crest and non-dimensional depth-averaged errors of approximately $\varepsilon = \pm 0.05$. Despite this, underestimations of up to $\varepsilon = -0.4$ occur in shallower depths ($d/gT_p^2 < 0.01$).

These findings define the range of applicability and expected accuracy of existing velocity prediction methods across a broad spectrum of sea-state and bathymetric conditions. However, the limitations of current formulations in shallow water highlight the need for future research to develop models capable of more accurately representing breaking-induced kinematics in the coastal zone. This chapter fulfils Objective (v) of the thesis and completes the picture of coastal wave field evolution developed in Chapters 4–5 by providing its final component: the detailed kinematics beneath extreme crests.

Chapter 7

Conclusion

Understanding and modelling coastal wave evolution is fundamental to coastal engineering design, hazard assessment, and shoreline management. Given the complexity of coastal processes, numerical models are essential tools for predicting wave transformation and informing robust design decisions. The main aim of this study has been to identify the dominant mechanisms governing the evolution of random waves in coastal regions and to translate these insights into practical guidance for engineering applications. Specifically, the work addressed four objectives across the four results chapters (Chapters 3–6), summarised as follows:

- (i) Validated numerical simulations performed with SWASH against equivalent laboratory measurements.
- (ii) Developed physical insight into the influence of bed slope on the statistics of surface elevations and wave kinematics.
- (iii) Proposed new formulations for the evolution of sea-state parameters in the coastal zone.
- (iv) Established a functional understanding of wave kinematics that is directly applicable to practical design.

To achieve these objectives, the present study performed extensive numerical simulations of unidirectional random sea states using SWASH. The computational efficiency of this model caters to the performance of extensive long duration simulations with reasonable accuracy. This formed a noteworthy dataset modelling

the propagation of over 2 million waves from intermediate water to the shoreline. A broad range of offshore conditions and idealised planar bathymetry were considered, with surface elevations and wave kinematics predictions. The numerical model was validated through comparisons to equivalent sea-states generated experimentally, demonstrating good agreement particularly within the shoaling region. This comprehensive dataset was then used to analyse coastal wave transformation, with particular emphasis on the influence of bed slope effects on wave statistics and kinematics. Building on these insights, practical guidance was distilled for engineering design.

7.1 Key findings

The main findings arising from the present work are:

- (i) Comparisons of numerical simulations of both regular and random, fully non-linear waves propagating over mild bed slopes to equivalent experimental measurements have shown that:
 - (a) SWASH accurately models the mean crest height of regular waves up to moderate steepnesses ($ak = 0.25$), within 5% of experimental measurements.
 - (b) For steeper regular waves ($ak > 0.25$), mean crest heights are underestimated by up to 20%.
 - (c) In random sea states with up to moderate steepness ($H_s/d < 0.3$), SWASH achieves accuracy within 2% of experimentally observed normalised crest heights (η_c/H_s) and wave heights (H/H_s).
 - (d) For steeper random waves, underestimations of up to 15% (for η_c/H_s) and 18% (for H/H_s) are observed relative to experimental measurements.
 - (e) These discrepancies are attributed to the treatment of breaking and/or the absence of higher-order nonlinear effects in the numerical scheme, which are evident experimentally.
- (ii) Analysis of the influence of bed slope on the statistics of waves demonstrates:

- (a) Larger wave heights, crest heights, and near-surface velocities on steeper slopes.
 - (b) These differences arise from stronger shoaling and lower spatially integrated breaking dissipation in the surf zone over steeper bathymetry.
- (iii) Analysis of the influence of bed slope on the energy of waves indicates:
- (a) More abrupt energy transition in steeper bed slopes.
 - (b) Steeper slopes exhibit stronger individual wave-wave interactions.
 - (c) Milder slopes are associated with a greater number of nonlinear coupling modes alongside a wider surf zone.
 - (d) Milder slopes experience greater transfers of energy to sub- and superharmonic regions.
 - (e) Milder slopes have more broad-banded spectra.
 - (f) A downshift of the primary and secondary spectral peaks for surface elevation, horizontal velocity, and vertical velocity are seen on milder slopes due to differences in interaction pathways.
- (iv) Analysis of the influence of bed slope on the shape of waves shows:
- (a) More horizontally asymmetric wave profiles over steeper slopes.
 - (b) Larger skewness and kurtosis of surface elevation and horizontal velocity are observed on milder slopes, indicating more vertically asymmetric wave profiles.
 - (c) Greater skewness and kurtosis of velocity fields compared to surface elevations, underscoring their stronger nonlinear response.
- (v) The physical insights above inform new data-driven parameterisations for the evolution of sea-state parameters used in preliminary engineering design. Through this, it is found that:
- (a) Existing H_{rms} formulations do not provide consistent and accurate descriptions of the numerical evolution across all cases investigated.
 - (b) New formulations for the evolution of H_{rms} are developed, conditional on sea-state steepness and the underlying bed slope gradient.

- (c) Improvements of up to 21% in wave height predictions are obtained compared to existing formulations.
- (d) The commonly adopted T_{m-1} formulation of Hoffland et al. (2017) is shown to be non-conservative, with underestimations of up to half the observed values in shallow water.
- (e) A new parametric relation for the evolution of T_{m-1} is developed, explicitly incorporating additional bed slope effects not considered in existing formulae, with a mean accuracy of 13.8%.
- (f) New formulations for the mean spectral period (T_{m1}) and the mean zero-crossing period ($\overline{T_z}$) are developed, with a mean accuracy of 6.9% and 5.6%, respectively.
- (vi) Investigation of phase-resolved horizontal velocities beneath extreme crests shows:
 - (a) Wave breaking strongly influences crest-velocity magnitudes, leading to a notable redistribution within the broken-wave population.
 - (b) Bed slope effects are significant: stronger shoaling on steeper slopes and stronger breaking-induced reductions on milder slopes.
 - (c) These bed slope differences yield variations in shallow-water crest velocities by up to 48% at $k_p d = 0.3$ across the bathymetries investigated.
- (vii) Evaluation of existing regular wave theories against random sea-states indicates:
 - (a) Reliable estimates in intermediate depths ($d/gT_p^2 > 0.01$), particularly using Stream Function theory.
 - (b) The integrity of regular wave theory is compromised for highly nonlinear, extreme waves in shallow water, with errors of order $\varepsilon = O(1)$ observed in the non-dimensional depth-averaged velocity profile.
 - (c) These limitations are due to the inability of wave-wave interactions to be represented by a single characteristic wave height and period which forms part of a regular wave train.
- (viii) Evaluation of existing random wave theories indicates:

- (a) Improved accuracy relative to regular wave formulations, given their ability to represent the stochasticity and spectral superposition of real seas.
- (b) Practical guidance is provided for applying random wave theory, with emphasis on accurate specification of spectral inputs.
- (c) For intermediate depths ($d/gT_p^2 > 0.01$), the methods of Molin (2002) and Donelan et al. (1992) achieve accuracy of $\varepsilon = \pm 0.05$ relative to numerical predictions.
- (d) Applicability of random wave theories reduces in shallower water ($d/gT_p^2 < 0.01$), where underestimations up to $\varepsilon = -0.4$ are observed.

7.2 Suggestions for future work

- (i) **Improving breaking descriptions in non-hydrostatic, depth-integrated models.** The present work employed the non-hydrostatic, depth-integrated model SWASH, which has demonstrated strong accuracy in the shoaling region but exhibits discrepancies following breaking onset. This is attributable to turbulence production and dissipation, roller dynamics, and higher-order non-linear effects that are only partially captured in current schemes. Future work should refine the breaking parameterisation by either calibrating more closely to experimental data or through improved breaker modelling to better represent breaker-generated turbulence and post-breaking bore dissipation. An alternative to the hydrostatic front approach adopted in SWASH is the inclusion of an additional diffusive-type term in the mass and momentum equations to parametrise a surface-roller (Cienfuegos et al., 2010). Other works have shown the applicability of eddy-viscosity or total kinetic energy closures, particularly in Boussinesq models (Li et al., 2022, Briganti et al., 2004). More recently, Mohanlal et al. (2023) have shown the applicability of the breaking strength parameter (related to crest kinematics) to capture energy dissipation caused by depth-limited wave breaking in Fully Nonlinear Potential Flow (FNPF) models. Where computationally feasible, targeted Reynolds-averaged Navier–Stokes (RANS) simulations with a free-surface method (VOF/SPH) may be used as high-fidelity references to constrain and validate the depth-averaged closures.

- (ii) **Digital twin wave modelling.** Given the flexibility and computational efficiency of the numerical model adopted in this study, it has the potential to serve as a digital twin of both experimental setups and real-world field locations. By integrating location-specific parameters obtained from sensors, radar, and satellite observations, the model can accurately replicate the behaviour and properties of real waves under various conditions. This approach allows for a direct comparison between simulated and observed wave dynamics, enabling the calibration of the model for specific sites. Furthermore, the high efficiency of the code ensures that the digital twin can operate in near-real-time, providing timely predictions and insights. As a result, such a system could be employed as a coastal early warning tool, capable of forecasting hazardous wave events, optimizing operational responses, and enhancing risk management strategies in vulnerable coastal areas.
- (iii) **Extension to short-crested seas.** In coastal waters, it is well-established that the importance of wave directionality is reduced compared to deep water. The work of Karmpadakis et al. (2019a) has shown a reduction in the magnitude of nonlinear crest heights with increased directional spreading in non-breaking seas. Thus, the unidirectional assumption employed in the present work provides a conservative modelling approach. However, due to the earlier breaking occurrence in less-directional seas, more directionally-spread seas may have larger waves in the extremes of the distribution. Thus, it would be interesting to investigate the role of directionality on the findings presented. Future research should extend the analysis to short-crested seas using directional spectral inputs modelled in a three-dimensional numerical domain. Through this, the impact of the directional spreading width on the characteristics of coastal wave evolution can be quantified.
- (iv) **Coupled wind–wave and wave–current dynamics.** The scope of the present study isolates wave dynamics. However, in real seas, additional processes are present, namely wind–wave and wave–current interactions. Wind–wave interactions presents additional energy sources that promote wave growth (Phillips, 1985). Moreover, it modifies spectral inputs, having an influence on both energy growth rates and the directional distribution of waves. Wave–current interaction presents similar complexities. In such cases, currents mod-

ify the intrinsic frequency of waves through the Doppler shift and result in changes in the group velocity (Thomas, 1981). Additionally, through changes in the radiation stresses in the wave momentum balance, modifications in the mean water level occur. This has implications on the breaking location and altering the nearshore energy flux. As such, investigating these effects remains an important avenue for future work. This can be achieved by incorporating wind stress forcing, atmospheric boundary-layer coupling, and ambient currents into the modelling framework, thereby enabling a systematic assessment of their influence on the wave field evolution. The inclusion of these forcings substantially increases the model complexity and introduces additional scales and feedbacks, which require careful consideration.

- (v) **Complex bathymetry, roughness and vegetation effects.** Realistic foreshores often include bars, reefs, and vegetated zones which are far more complex than the idealised planar bathymetries investigated in the present work. In addition, natural coastlines frequently exhibit pronounced three-dimensional features, such as alongshore variability, headlands, and embayments, which can significantly influence wave transformation and run-up processes. Extending the present study to account for three-dimensional shoreline geometries therefore represents an important direction for future work. Further investigation of varying bed shape, building on the work of De Bakker et al. (2016), and incorporating bottom roughness presents additional avenues for research. Moreover, considering the influence of vegetation is a growing area of work in coastal flood-risk reduction and shoreline protection (Mendez & Losada, 2004, Möller et al., 2014, Ondiviela et al., 2014). In recent years, this has drawn particular attention in the field of sustainable nature-based solutions for coastal defences (Narayan et al., 2016).
- (vi) **Physics-informed modelling to accelerate design workflows.** To complement the data-driven parameterisations presented in Chapter 5, physics-informed surrogates can be developed. These are constrained by the governing non-hydrostatic shallow-water equations and can be developed to provide rapid, design-oriented predictions while preserving physical consistency. Such surrogates could embed conservation properties, breaking onset criteria, and slope-conditioned dissipation. Such formulations may be trained on ensembles

of the present SWASH simulations, alongside additional numerical or experimental datasets. These act to provide more physics-informed formulations that can be readily applied in preliminary design stages, as a fast alternative to full numerical modelling.

- (vii) **Distribution of wave kinematics.** The present work in Chapter 6 considers the influence of offshore sea-state conditions and bed slope on the distribution of crest velocities. It was noted that the initiation of wave breaking significantly alters the distribution of crest velocities. This motivates future work for the development of a unified crest velocity distribution that is applicable to coastal waves. Notably, no existing distribution fully captures the nonlinear evolution of coastal waves, including shoaling and breaking, highlighting a clear research gap.

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Appendices

.1 Model formulations

The coefficients for the proposed H_{rms} model are provided in Table 1. These are to be used alongside Equation 5.15. The full model formulations are provided as an addendum to the thesis at https://github.com/ikarmpadakis/HT_coastal_evolution.git.

The transitional effective water depth, $k_p d_{tr}$, increases with increasing sea-state steepness, S_p , and decreasing bed slope, m . This reflects the findings outlined in Section 5.4.2 such that breaking initiation occurs further offshore for steeper sea-states and milder bed slopes. Moreover, in steeper sea-states and steeper bed slopes, the coefficients a_1 , b_1 , c_1 , a_2 , b_2 , c_2 and d_2 indicate a sharper and more rapid evolution.

TABLE 1: Coefficients for the H_{rms} model

Parameter	m	S_p					
		0.01	0.02	0.03	0.04	0.05	0.06
$(k_{pd})_{tr}$	10	0.387	0.523	0.663	0.715	0.765	0.812
	30	0.450	0.580	0.689	0.748	0.779	0.842
	80	0.477	0.599	0.704	0.777	0.796	0.853
a_1	10	0.523	0.207	-0.068	-0.337	-0.624	-0.711
	30	0.315	0.110	-0.069	-0.266	-0.594	-0.679
	80	0.305	0.261	0.110	-0.069	-0.285	-0.595
b_1	10	-1.013	-0.430	0.146	0.756	1.379	1.580
	30	-0.648	-0.207	0.204	0.698	1.441	1.659
	80	-0.595	-0.434	-0.093	0.347	0.860	1.603
c_1	10	1.435	1.171	0.871	0.524	0.189	0.076
	30	1.274	1.035	0.796	0.487	0.068	-0.062
	80	1.217	1.086	0.888	0.617	0.312	-0.126
a_2	10	-14.289	-2.191	0.524	1.272	1.998	2.009
	30	-26.545	-12.053	-7.008	-4.608	-3.456	-2.704
	80	-31.295	-16.002	-8.623	-5.687	-4.709	-2.951
b_2	10	4.016	-1.119	-2.690	-3.242	-3.837	-4.154
	30	19.688	12.128	8.739	6.713	5.366	4.392
	80	26.644	17.213	10.722	7.924	6.898	4.685
c_2	10	3.178	3.178	3.178	3.194	3.203	3.501
	30	-1.938	-1.938	-1.938	-1.840	-1.481	-1.144
	80	-4.427	-3.790	-2.542	-2.069	-1.891	-1.186
d_2	10	0.115	-0.048	-0.131	-0.197	-0.201	-0.287
	30	0.350	0.353	0.386	0.409	0.367	0.329
	80	0.449	0.454	0.360	0.333	0.322	0.274