

Numerical Modelling of Large Impact Crater Collapse

Gareth S. Collins

Department of Earth Science and Engineering
Imperial College of Science, Technology and Medicine
University of London



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Abstract

To Mum and Dad

Abstract

This thesis addresses the important geologic question of how large impact craters are formed. The approach of the research described in this thesis has been to study impact crater formation using the only feasible method of simulating such a rarely occurring, complicated and highly dynamic process: numerical modelling.

The numerical model described in this thesis simulates the gravitationally driven collapse of a transient cavity, which is generated during the early stages of an impact event. To facilitate collapse, the rheologic properties of the material surrounding the crater must be modified significantly from the typical material properties of rock. This is achieved using acoustic fluidization; where strong vibrations produced by the expanding shock-wave cause extreme pressure fluctuations in the target. At times and positions where the overburden pressure is sufficiently counteracted, the frictional resistance is reduced, enabling the rock debris to flow.

This numerical model of crater collapse is first used to develop a predictive, quantitative model for the collapse of a Chicxulub-scale impact crater. This is achieved by testing and refining the model until the simulations produce a collapsed crater that contains most of the features observed in the seismic data obtained at Chicxulub. The model is also extended to simulate the formation of large impact craters on the Moon.

A generic model for the formation of peak rings in large impact craters is developed. During the simulations material that is originally part of the central uplift collapses outward and is thrust over the inwardly-collapsing transient crater rim to form a concentric, topographic feature inside the final crater rim. This model for peak-ring generation has not been previously demonstrated by numerical simulations and predicts that the peak ring is composed of deeply derived material, and that the stratigraphy within the peak ring is overturned.

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Chapter 1

Introduction

The most beautiful thing we can experience is the mysterious. It is the source of all true art and science.

Albert Einstein

Impact craters are the dominant landform on the majority of the terrestrial planets, moons and asteroids. Over 150 impact craters have now been identified on the Earth and images have returned from spacecraft across the Solar System illustrating the ubiquity of impact craters on solid bodies. As a result, impact cratering has become recognised as an important geological process.

In the last two decades impact cratering has emerged as an influential process in the evolution of both our Solar System and life within it. Impact events have been associated with at least one mass extinction, the possibility of meteorites transporting life across planetary systems and theories for the formation of the Moon. The relevance of impact cratering in such broad terms has attracted greater public awareness; however, the current understanding of impact cratering remains incomplete.

This thesis addresses the important geologic question of how large impact craters on the terrestrial bodies are formed. The specific objectives of this thesis are highlighted at the end of this chapter. Prior to that, however, it is important to provide the motivation for this research. This chapter will, therefore, introduce the curious morphological diversity and complexity of the impact crater population in the Solar System. Furthermore, this chapter will present a brief summary of the current understanding of the impact process and highlight the aspects of the cratering process where current understanding is incomplete. At the end of this chapter the aims of this research and the structure of this thesis will then be discussed.

1.1 Impact Crater Morphology

The population of impact craters within our Solar System is morphologically diverse. The impact craters that speckle the surface of nearly every planet and satellite in the Solar System may be divided into two distinct morphological classifications: *simple* craters and *complex* craters (Dence, 1965). Simple craters, as the name suggests, are circular, bowl-shaped depressions with raised rims; a form that most people associate with the word “crater”. In stark contrast, complex craters exhibit somewhat unintuitive features such as central peaks, or an internal ring of mountains and terraced rim walls. The following discussion describes the distinguishing features of these two classifications of impact craters.

1.1.1 Simple Craters

Figure 1.1 illustrates the typical form of a simple crater; shown is a classic example, the lunar crater Linne. Simple craters are circular in plan; in cross-section they exhibit an approximately parabolic profile with a raised rim. The average slope angle is $\sim 25 - 30^\circ$, depending on the angle of repose of the crater-wall and -floor material. The depth-to-diameter ratio of simple craters is approximately 1:5 (Pike, 1977). There is no theoretical lower limit to the size of simple craters, providing the target has no significant cohesive strength and the impactor is travelling at a velocity greater than a few kilometres per second at the time of impact (Melosh, 1989). In reality, however, the minimum simple crater size is a function of the ability of the atmosphere to absorb small meteors. The upper limit to the size of a simple crater is the point where the crater’s morphology becomes “complex”. This change in structure, termed the simple-to-complex transition, occurs at different crater diameters on different planetary bodies. On the Moon, craters begin to exhibit complex crater characteristics at diameters of approximately 15km. On Earth, the transition depends on the nature of the target rock. Simple craters form in sediments at sizes smaller than 3km; in crystalline rocks the transition diameter is approximately 5km (Grieve et al., 1981).

The best exposed and most thoroughly investigated terrestrial simple crater is Meteor Crater in north-central Arizona, USA (shown in Fig. 1.2). Drilling at Meteor Crater and other similar craters has revealed a lens of *breccia* (broken rock debris) and formerly shock-melted rock underlying the crater floor (Shoemaker, 1960). Below the breccia lens is fractured country rock. The volume of the breccia lens is roughly one-half the volume of the crater itself. The breccia lens at Meteor Crater is a mixture that contains clasts from all of the rock units intersected by the crater (Shoemaker, 1963). This indicates that at least part of the material in the lens has collapsed from higher up the crater wall.

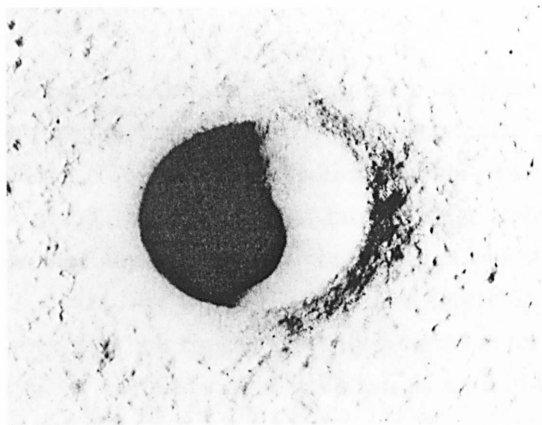


Figure 1.1: A typical simple crater. The 2.5km lunar crater Linne. *Apollo panoramic photo AS15-9353 (P)*.

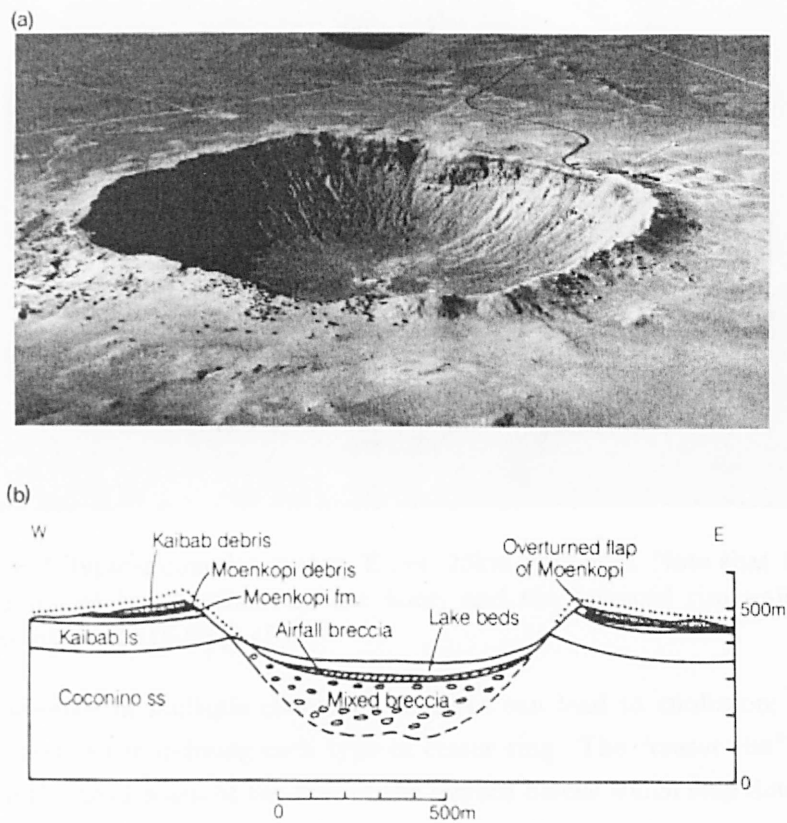


Figure 1.2: (a) Meteor Crater, Arizona. *Roddy and Zeller, U.S.G.S. Courtesy D. Roddy.*) A geologic cross-section of the 1-km diameter crater is shown below in (b). *Cross section is after Shoemaker (1960).*

1.1.2 Complex Craters

At crater diameters larger than the simple-to-complex transition diameter, the morphology of impact craters becomes distinctly more intricate. At crater diameters just above the transition, complex craters exhibit central peaks, terraced rim walls and flat floors (for example, Figure 1.3). At larger crater sizes the central peak is replaced by an internal mountainous ring. This ring of peaks is often referred to as the *peak ring* or *internal ring* which shall be used interchangeably here. At the largest crater sizes an additional concentric “ring” forms external to the main rim of the crater. This type of complex crater is known as a *multi-ring basin*.

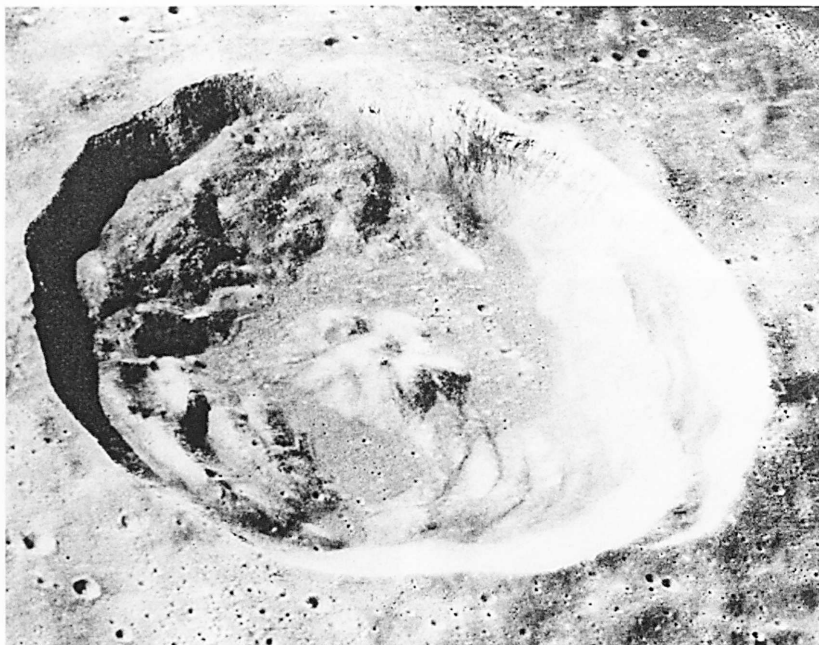


Figure 1.3: A typical complex crater: Euler, 25km diameter. Note that the central peak surrounded by an otherwise flat floor, and the terraced rim walls. *Apollo panoramic photo AS15-0274 (P)*.

The presence of multiple concentric features can lead to confusion; thus, care must be taken when defining each type of crater ring. The “crater rim” is defined here to be the head-scarp of the first of the terrace blocks which step down into the crater (Figure 1.4).

Analysis of images of large impact craters on Venus, which arguably provide the best data-set of pristine complex craters in the Solar System, suggests that there is a continuous morphologic progression between central peak and peak-ring classifications (Alexopoulos and McKinnon, 1994), illustrated in Figure 1.5. At the smallest sizes the central region of the crater is occupied by a peak or mound, as illustrated in Figure 1.5a. At larger crater diameters this central peak becomes a cluster of peaks usually concentric with the crater rim (Fig. 1.5b). This cluster

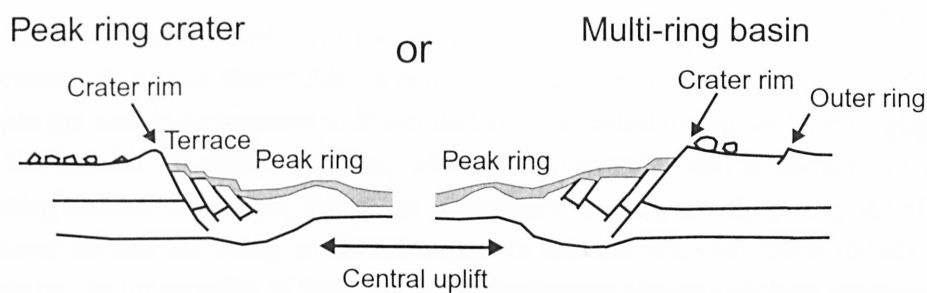


Figure 1.4: Diagram illustrating the nomenclature used during this report for the various crater rings.

gradually becomes more widely separated with increasing crater diameter forming a ring of peaks inside and concentric to the main rim (Fig. 1.5c). There is also evidence that, above a certain crater diameter, the size of the internal ring grows proportionally with the crater itself; the ratio of peak-ring diameter to the final rim diameter is approximately one half (Wood and Head, 1976).

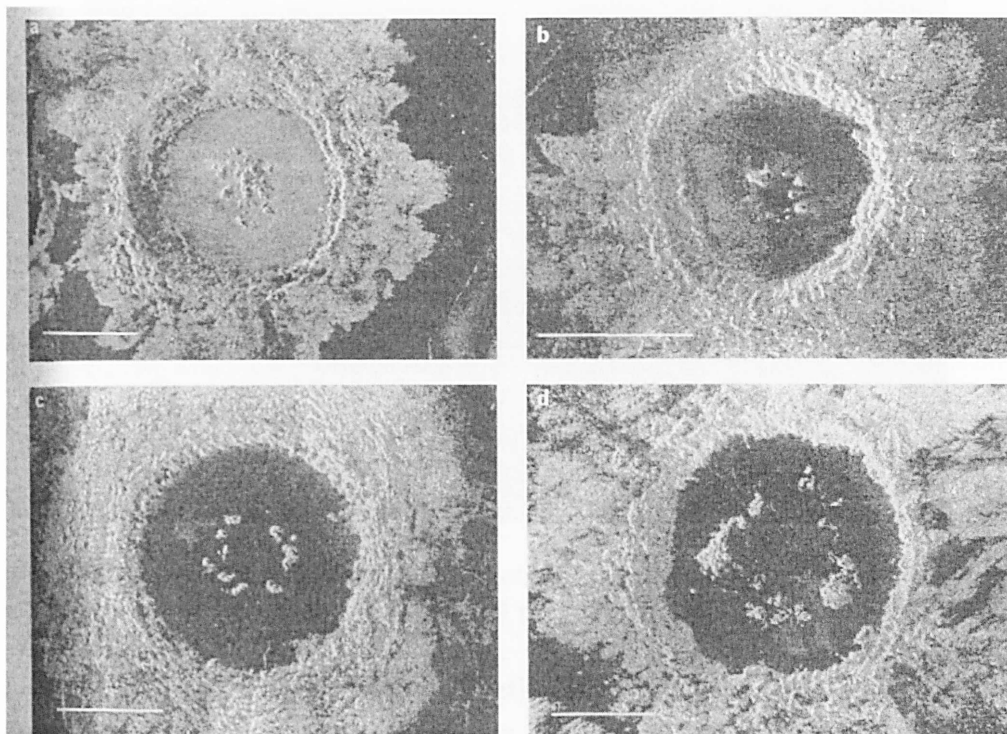


Figure 1.5: Progression of peak-ring crater morphology as seen by Magellan. (a) Montesori, ~ 43 km diameter, crater with a cluster of peaks at centre. (b) Simonenko, ~ 32 km diameter, small diameter peak-ring. (c) Corpman, ~ 45 km, intermediate diameter peak-ring. (d) Cori, ~ 56 km diameter, crater diameter to peak-ring diameter ratio of ~ 2 . *After Alexopoulos and McKinnon (1994)*

On other, more heavily cratered, surfaces in the Solar System this morphologic continuum is not as discernible. On such bodies, the number of complex craters within the size limits required to illustrate the above relationship are far out-weighted by the number of smaller impacts, which act to disguise and obliterate the pre-existing craters. On Venus, the dense atmosphere and apparently young age of the cratered surface has better preserved craters in this size window. Thus, debate still exists on the universality of this apparently continuous size-morphology progression. There is no doubt, however, that the discrete size-morphology progression of simple crater, central peak crater, peak-ring crater exists, as illustrated in Figure 1.6 of three Mercurian craters.

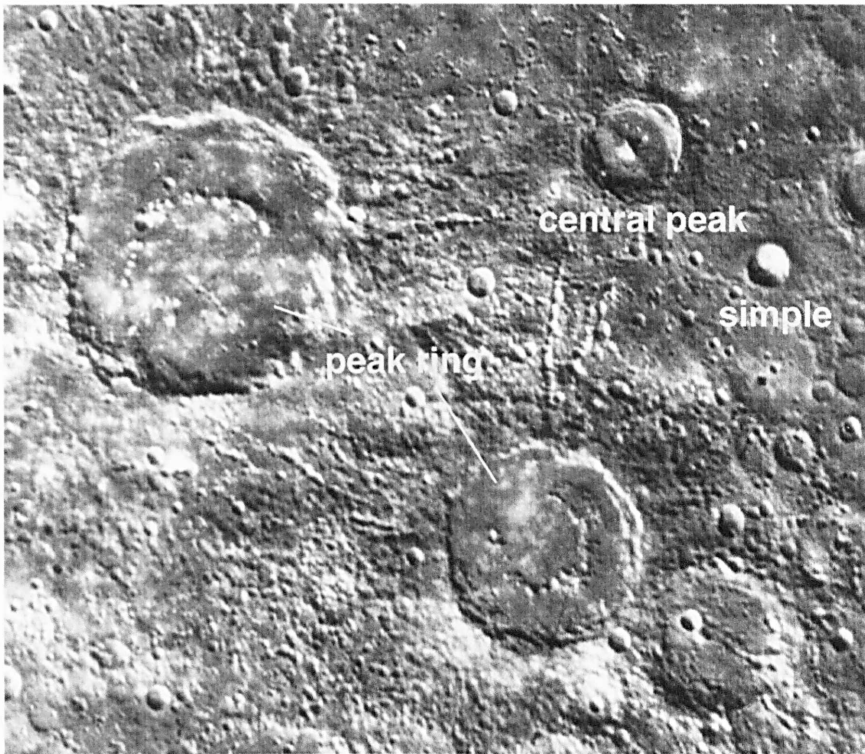


Figure 1.6: Size-morphology progression observed on Mercury. Smallest crater morphology is the simple crater, then central peak crater, then peak-ring crater.

Study of large *simple* craters on Earth suggests that the central mounds that frequently form in these craters are merely an exaggerated development of the debris sliding that creates the breccia lens (Melosh, 1989, p. 136). However, geologic investigation of the central region of terrestrial *complex* craters indicates that in this case the central mound is primarily a result of *uplift*. The central uplifts of central peak craters on Earth are composed of deformed and fractured rock that originally underlay the transient crater. Despite this uplift, the original stratigraphy of the target is usually discernible, distinguishing it from the mixed breccia found beneath

the floors of simple craters (Wilshire and Howard, 1968, for example). Studies of geologic and geophysical data for fourteen terrestrial complex craters ranging between 4 and 100km in diameter (Grieve et al., 1981; Ivanov et al., 1982) have shown that the stratigraphic uplift SU as a function of final crater diameter D may be approximated by:

$$SU = 0.1D. \quad (1.1)$$

Complex crater walls commonly exhibit a wreath of terraces which step down into the crater. The terraces appear to be the head-scarps of landslides that carried large amounts of material from the unstable crater rim into the interior of the crater (Pearce and Melosh, 1986). They are analogous to large terrestrial rotational landslides, sometimes referred to as “Toreva-block” landslides after a locality in Northern Arizona where they are particularly well developed (Reiche, 1937). Large rotational landslides form exclusively where the underlying rock behaves plastically. In the case of the locality at Toreva the plasticity is due to water saturation of the thick clay sediments at depth (Rogers, 1991). The ubiquity of these features in complex craters on bodies throughout the Solar System, however, indicates that the plasticity of the underlying rock cannot be simply attributed to water.

The results of morphometric studies of complex craters on the Moon are summarised in Table 1.1 and Figure 1.7. Clearly illustrated is the change in depth to diameter ratio and the onset of central peaks and terraced rim walls at the simple to complex transition. Note also that the central peak appears to have a maximum height of 3km, which is independent of crater diameter at diameters greater than 70km.

1.1.3 Multi-ring Basins and the Chicxulub Impact Structure

On some planetary bodies and satellites an additional category of impact crater, termed a multi-ring basin, is present at crater diameters larger than the diameter of the largest peak ring crater. Within the innermost asymmetric scarp a multi-ring basin is often morphologically similar to a peak ring crater (Fig. 1.4). Indeed, until recently peak ring craters were classified as a sub-set of multi-ring basins. Figure 1.8 illustrates a classic example of a multi-ring basin; from the crater centre outward a peak ring and two asymmetric scarps are observable.

The “basin” terminology was defined in 1962 by W. K. Hartmann and G. P. Kuiper¹, who were the first to discover the archetypal multi-ring basin on the Moon, Orientale, and wished to separate such features from impact craters. Since their seminal work, the name multi-ring basin has survived but their definition has been modified. For the purposes of this report the classification scheme of Melosh (1989) is adopted: peak ring craters are considered to be the largest type of complex crater;

Table 1.1: Table of best-fit curves for morphometric measurements of fresh lunar craters. *Adapted from Melosh (1989).*

Parameter	Dependence on Rim-to-Rim Diameter (D , km)	Diameter Range (km)
Crater Depth	$H = 0.196D^{1.01}$	<11
	$H = 1.044D^{0.301}$	11–140
	$H^1 = 10^{0.41(\log_{10} D)^{0.57}}$	140–400
Central Peak Diameter ²	$D_{cp} = 0.22D$	20–140
Peak Ring Diameter ³	$D_{pr} = 0.50D$	140–450
Central Peak Height ⁴	$h_{cp} = 0.0006D^{1.97}$	15–80
	$h_{cp} \approx 3$	80–200
Rim Height	$h_R = 0.036D^{1.014}$	<21
	$h_R = 0.236D^{0.399}$	21–400

All measurements from Pike (1977) except

¹ Williams and Zuber (1998)

² Pike (1985)

³ Wood and Head (1976)

⁴ Hale and Grieve (1982)

and the definition of a multi-ring basin is any large circular impact structure that “possess[es] at least two concentric asymmetric scarps, one of which may be the original crater rim.”

There are no unequivocal terrestrial multi-ring basins; although three large craters on the Earth have been discovered that are potential candidates: Vredefort in South Africa, Sudbury in Canada, and Chicxulub in Mexico. Of these only Chicxulub has not been extensively eroded or tectonically deformed. However, although the Chicxulub crater is thought to be almost pristine, it is buried under over a kilometer of Tertiary sediments. In 1996 the Chicxulub Seismic Experiment acquired a series of marine seismic deep-reflection profiles and onshore-offshore wide-angle profiles across the Chicxulub impact structure. The data collected have been instrumental in furthering current understanding of large impact craters.

The central findings from the Chicxulub Seismic Experiment are the identification of three distinct ring types (crater rim, peak ring and outer ring), and the presence of what appear to be massive terraced slump blocks beneath the crater (Figure 1.9). The crater rim (the head scarp of the outermost terrace) has an average diameter of ~ 150 km. The peak ring appears as a topographic high at a diameter of ~ 80 km within the crater rim. Topographic offsets and rings seen outside the crater rim are associated with deeper crustal reflectors. The presence of this third type of ring suggests that the Chicxulub impact structure is analogous to the multi-ring

¹Hartmann and Kuiper’s discovery of multi-ring basins was originally published in a limited-circulation publication in 1962. The interested reader is referred to the much more accessible reference of Hartmann (1981).

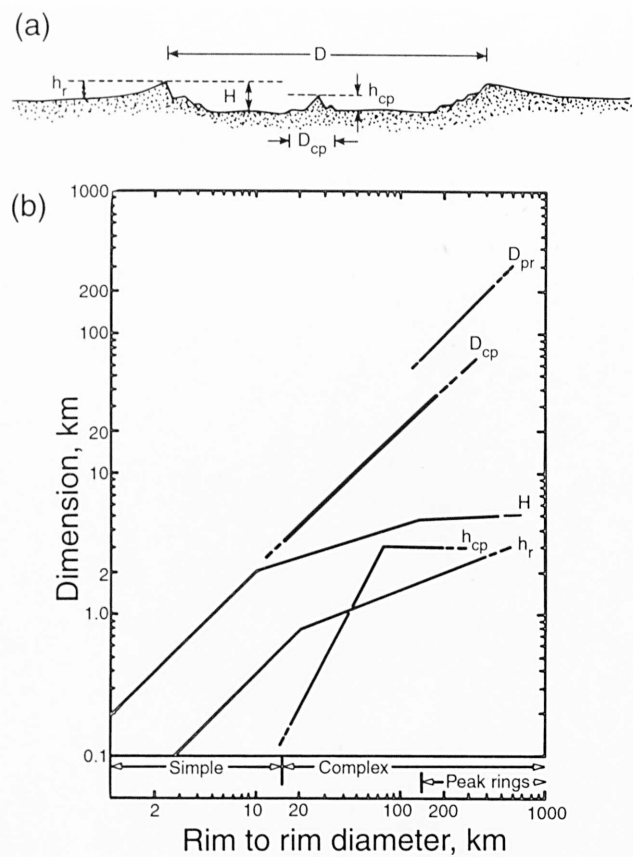


Figure 1.7: Morphology of fresh lunar craters. (a) Morphometric measures of distinguishing features of complex craters; (b) the dependence of the parameters defined in (a) on crater diameter. Curves are based on best fit lines defined in Table 1.1 Adapted from Melosh (1989).

basins observed on other planets. The Venusian multi-ring basin Klenova (shown in Figure 1.8) is, perhaps, the closest in morphology to the Chicxulub structure.

1.2 Mechanics of Impact Crater Formation

The impact cratering process is conventionally divided into three stages: contact and compression, excavation and collapse. Although these three stages are not entirely discrete, each is dominated by different physical processes and are thus separated for convenience and to aid understanding.

1.2.1 Contact and Compression

The early stages of the impact cratering process for hypervelocity impact events can be described as a kind of explosion due to the tremendous kinetic energy supplied by the impactor. Thus, cratering has more in common with high energy explosion



Figure 1.8: Venusian crater Klenova. *After Alexopoulos and McKinnon (1994)*

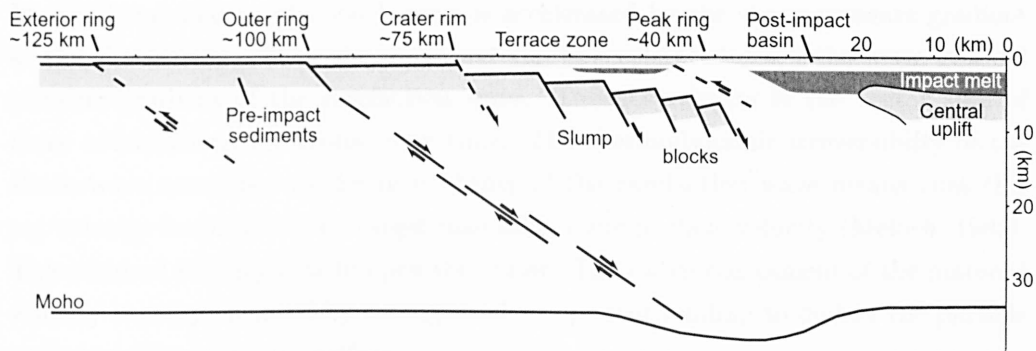


Figure 1.9: Schematic representation of the subsurface structure of the Chicxulub impact crater

physics than momentum driven excavation. As a consequence, the greatest advances in understanding impact crater formation have come in the decades since World War II and from the development of nuclear weapons.

The first stage of the impact crater formation process begins when the impactor first contacts the target surface. Upon impact, the impactor pushes target material out of its path, compressing it and accelerating it to a large fraction of the impactor velocity. At the same time the target acts to decelerate the impactor. These velocity changes result in the formation and propagation of shock waves away from the point (or points) of impact: material in the contact zone between the impactor and target is strongly compressed, creating shock waves between the compressed and uncompressed material.

The principal result of the contact and compression stage is to transfer most of the impactor's kinetic energy into the kinetic and internal energy of the target material. The internal energy heats both the impactor and target, resulting in melting or vaporisation of material in close proximity to the impact site for sufficiently strong shock waves. The residual kinetic energy is spent ejecting target and impactor material and opening the target material to form the *transient cavity*. The opening of the transient cavity is termed the excavation stage.

1.2.2 Excavation

The geometry of the expanding shock wave generated by an impact is that of a sphere centered on a point some distance below the surface (Melosh, 1989, p. 71); thus, the shock waves propagate upward from the impact point toward the free surface. As stress cannot be maintained at free surfaces, such as the original ground surface and the edge of the impactor, a family of rarefaction waves develop and follow the outward moving shock wave, thereby decompressing the target to ambient pressure. Material engulfed by the shock wave is accelerated by the sharp pressure gradient at the shock front. Then the same material is accelerated down the more gradual pressure gradient of the rarefaction wave. The net velocity is the vector sum of these accelerations integrated over time. The thermodynamic irreversibility of the shock wave compared to the reversibility of the rarefaction wave means that this net velocity is non-zero; the target maintains some particle velocity (Melosh, 1985). This residual velocity acts to open the crater. The radial component of the material velocity is complemented by a tangential component tending to deflect the particle trajectories towards the surface.

The excavation flow is retarded by any cohesive strength that the target material may retain plus dry friction (resistance to shear of granular materials) and gravity. In large impacts it is gravity that is most significant; excavation stops when insufficient energy remains to lift the overlying material against the force of its own

weight (Nolan et al., 1996). The culmination of the excavation stage is defined as the moment when the excavation flow ceases everywhere. The form of the transient cavity at this point is called the “transient crater”. Evidence from numerical calculations and laboratory experiments suggests that the transient crater is approximately a paraboloid of revolution with a depth to diameter ratio of between 1:3 and 1:4 (Melosh, 1989, p. 78).

1.2.3 Modification

The subsequent and final stage of the cratering process is known as the collapse or modification stage. In essence, crater collapse is the gravity-driven modification of the unstable cavity formed during excavation. In a recent review of crater collapse, Melosh and Ivanov (1999) report that the detailed morphology of the impact craters in the Solar System is predominantly due to the collapse of the geometrically simple, bowl-shaped “transient crater”. For simple craters the collapse process is intuitive and well understood; however, in larger, morphologically more complex impact structures, collapse is not well understood.

1.3 Motivation and Aims

The motivation behind this thesis is to fully understand the cratering process that produces the diversity and complexity observed in the population of impact craters in the Solar System. The collapse of an impact crater is the final stage of this process and is, perhaps, the most important in shaping the final crater morphology. Although much progress has been made in understanding the processes involved during this stage of crater formation, the questions of how and why complex craters collapse still remain enigmatic. The fundamental goal of this thesis, therefore, is to forward the understanding of the collapse of complex impact craters.

The current understanding of impact cratering as a whole has come from a suite of experimental, morphological, analytical and numerical investigations. However, study of large impact crater collapse has relied on far less diverse resources; abstract computer simulations provide one of the only feasible methods for studying large impact crater collapse. The approach adopted in this research is to use a numerical modelling tool known as a “hydrocode” to simulate the collapse of large impact craters.

The lack of a definitive model for the formation of large impact craters has been due in no small part to the scarcity of large impact craters on the Earth, and to the near impossibility of probing the sub-surface on other planets. This situation changed following the discovery of the buried Chicxulub impact structure in Yucatan, Mexico. The Chicxulub Seismic Experiment obtained data that

clearly image the third dimension of a large complex crater and provide new insight into the kinematics of complex crater collapse (Morgan et al., 1997; Morgan and Warner, 1999; Brittan et al., 1999). Testing dynamical models for the formation of a Chicxulub-scale impact crater is the primary objective of this thesis.

Impact crater morphology is a function of crater size, which shows a first order correlation across solid bodies of differing gravitational accelerations. Any model that attempts to explain crater collapse, therefore, must scale appropriately with crater size and planetary gravity. A secondary goal of this thesis, then, is to derive a model for crater collapse that produces a final crater morphology appropriate for any given crater size on any given body.

1.4 Structure of the Thesis

The chapters in this thesis are designed to stand alone. As such each chapter begins with a brief introduction and ends with a summary of the pertinent points raised in the chapter. However, the thesis does proceed in a logical order and each chapter builds on the previous one.

The next chapter provides a comprehensive review of the current understanding of large impact crater collapse. The chapter includes a discussion of all aspects of the cratering process that have a direct or indirect influence on crater collapse, before presenting the current paradigm for large impact crater collapse. Moreover, the chapter outlines the areas where the current understanding of impact crater formation is poor and poses the fundamental questions that this thesis attempts to answer.

Chapter 3 discusses the application of computer modelling to the problem of crater collapse. In particular, the discussion intimately describes aspects of the computer program used for the work reported here, the SALES_2 hydrocode (Amsden et al., 1980; Collins and Melosh). The chapter outlines the modifications made to the SALES_2 hydrocode for the application of modelling crater collapse. Furthermore, the chapter (and accompanying appendix, B) documents the methodology used to simulate crater collapse.

Chapter 4 describes results of hydrocode simulations of the Chicxulub crater collapse. It also discusses the motivation for the choice of Chicxulub as a case study and provides some comparisons between the results of the numerical models and recent results from the Chicxulub Seismic Experiment. The chapter reports on the verification of a dynamical model for large impact crater collapse and discusses some implications of this model.

Chapter 5 concerns the extension of the model developed in Chapter 4 to craters of different sizes and on different planets. This model is then compared to observations of crater morphometry on the Moon.

Much of the work discussed in this thesis is presented in a number of publications. The hydrocode simulations of the Chicxulub crater collapse are described and discussed in Collins et al. (2000, 2002). The comparisons between the hydrocode simulations results and the seismic data at Chicxulub are also discussed in Morgan et al. (2000). The modelling of complex crater collapse on the Moon is discussed in Collins (2001).

Chapter 2

Impact Crater Collapse: A Geologic Process

Τά πάντα ρει: Everything flows.

Heraclitus, Greek philosopher and first rheologist who flourished circa 500 BC.

It is now believed that the first two stages of impact crater formation are generally independent of crater size. That is, the cratering processes is essentially identical for the formation of all impact craters up to the onset of the modification stage (Melosh and Ivanov, 1999). Thus, the distinct morphological differences between simple and complex craters are now thought to be not a direct result of the crater excavation process, but develop only after most of the material has been expelled from the crater.

The product of crater excavation is a simple bowl-shaped cavity: the transient crater. The form of the transient crater is independent of diameter; however, its size is a function of impactor velocity, the composition of the impactor and target, the angle of impact and gravity. After excavation, differential stresses imposed by gravity tend to return the displaced material to where it came from. This collapse, also known as the *modification stage* of impact crater formation, is the focus of this study.

Impact crater collapse is controlled by the competition between the gravitational forces tending to close the cavity and the inherent material strength properties of the post-shock target. Thus, to understand crater collapse, a detailed knowledge of the strength and rheologic behaviour of the collapsing material is required. Terrestrial impact craters provide limited information regarding the state of the target material during collapse. Furthermore, little is currently known of the behaviour of rock, or rock debris, moving at speeds or in volumes appropriate for complex crater collapse.

It is therefore difficult to formulate anything more than a general paradigm by which the complex crater morphologies introduced in section 1.1 may be produced during the collapse of a bowl-shaped cavity. This chapter discusses the current understanding of the mechanics of impact crater collapse and the current paradigm for large crater collapse. The discussion draws from a concert of morphometric studies, the geologic and geophysical character of terrestrial impact structures, small-scale experimental and high energy explosion craters, numerical simulations and theoretical consideration.

2.1 The Transient Crater

The flow of material as the transient cavity grows is known as the excavation flow. Intuitively, excavation flow velocities are greatest near the impact site and fall off approximately as some inverse power of the distance from it. The excavation flow velocity is never faster than about 6 kilometers per second, even for impactor velocities as high as 40 kilometres per second (Melosh, 1989); thus, the excavation flow is predominantly subsonic and can be treated as almost incompressible.

The excavation flow field is illustrated schematically in Figure 2.1. Velocity vectors in the material surrounding the growing cavity can be connected by “streamlines” representing the paths along which target material moves during the excavation flow. This is a purely two-dimensional analysis of the excavation flow; the actual flow is axisymmetric about a vertical line passing through the impact point, and the streamlines in fact represent “streamsurfaces” in three dimensions. All streamlines will eventually reach the surface of an infinite half space (except the one vertically downward); however, within a region that extends a couple of crater diameters from the point of impact some curve upward quickly enough to intersect the surface while steeper streamlines never reach the surface.

The details of the excavation flow can be determined only by experiment or by elaborate numerical computations, and even these are not without uncertainties. However, a simple kinematic description of the excavation flow was proposed by Maxwell and Seifert (1974). It should be noted, however, that their analytical model is approximate and does not accurately represent the fine details of the excavation flow. Their model, termed the “Z-model”, is based on work simulating high energy explosions but has been shown to be closely analogous to the excavation of high velocity impact craters (Croft, 1980). The central equation in the model relates the radial component of the excavation flow velocity to an inverse power of the radial distance from the explosive charge:

$$u_r = \alpha(\theta, t)/r^Z, \quad (2.1)$$

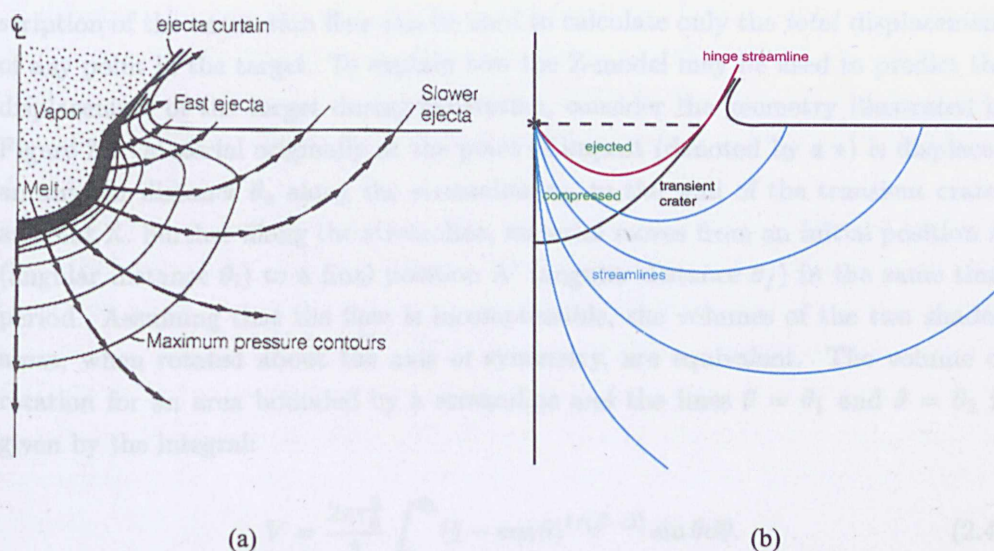


Figure 2.1: Schematic (a) and geometric (b) illustrations of the excavation flow field. Nomenclature in the text is defined in (b). (a) After Melosh (1989).

where the strength of the flow is controlled by $\alpha(\theta, t)$, a function of time t and angular distance from the vertical θ , and Z is the dimensionless power from which the model gets its name.

Assuming that the flow is incompressible, the angular component of the velocity u_θ can be defined (in polar coordinates) as:

$$u_\theta = u_r(Z - 2) \frac{\sin \theta}{(1 + \cos \theta)}. \quad (2.2)$$

The integration of Equation 2.2 gives the equation for all streamlines in polar coordinates:

$$r = r_0(1 - \cos \theta)^{1/(Z-2)}, \quad (2.3)$$

where r_0 is a constant that is different for each streamline, termed the *streamline coefficient*. If the point of explosion is chosen to be at the surface, the streamline coefficient r_0 is equal to the radius at which the streamline emerges from the surface. The streamline that defines the divide between material ejected from the transient cavity and material which is merely displaced downward is called the *hinge streamline*. Roughly equal volumes of material are either ejected from the crater or displaced downward into the crater.

From a temporal perspective, the shape of the transient cavity could, theoretically, be calculated by direct integration of Equation 2.1. However, the dependence of α on t and θ is unknown. Thus, the application of the Z -model must be purely kinematic; assuming an arbitrary shape for the transient crater, the Z -model de-

scription of the excavation flow can be used to calculate only the *total* displacement of any point in the target. To explain how the Z-model may be used to predict the displacement of the target during excavation, consider the geometry illustrated in Figure 2.2. Material originally at the point of impact (denoted by a *) is displaced an angular distance θ_x along the streamline r_0 , to the wall of the transient crater at point X. Further along the streamline, material moves from an initial position A (angular distance θ_i) to a final position A' (angular distance θ_f) in the same time period. Assuming that the flow is incompressible, the volumes of the two shaded areas, when rotated about the axis of symmetry, are equivalent. The volume of rotation for an area bounded by a streamline and the lines $\theta = \theta_1$ and $\theta = \theta_2$ is given by the integral:

$$V = \frac{2\pi r_0^3}{3} \int_{\theta_1}^{\theta_2} (1 - \cos \theta)^{1/(Z-2)} \sin \theta d\theta. \quad (2.4)$$

Thus, solving Equation 2.4 for the two shaded volumes in Figure 2.2 and equating the results gives an equation relating the three angles θ_x , θ_i and θ_f :

$$\theta_f = \cos^{-1} \left(1 - \left[(1 - \cos \theta_x)^{\frac{Z+1}{Z-2}} + (1 - \cos \theta_i)^{\frac{Z+1}{Z-2}} \right]^{\frac{Z-2}{Z+1}} \right). \quad (2.5)$$

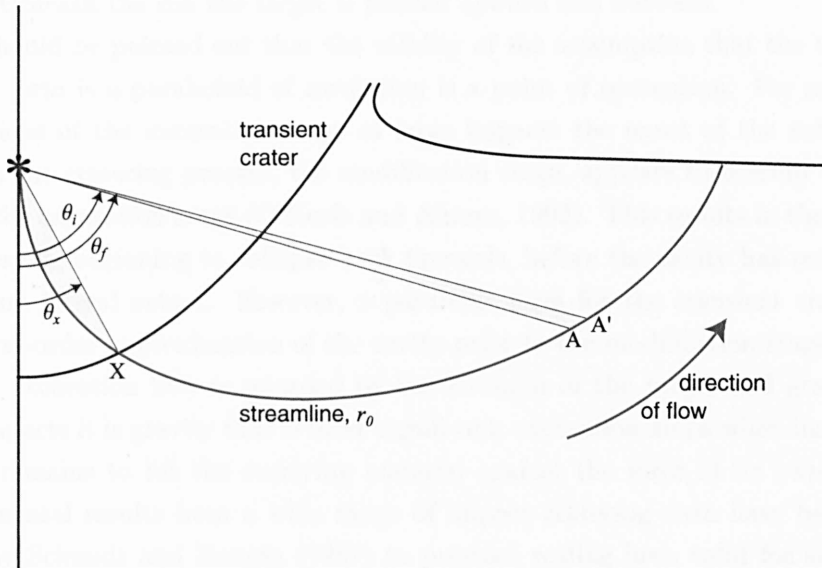


Figure 2.2: Illustration of geometrical parameters used in derivation of Z-model equations for the excavation flow. * denotes the point of impact. Material at * is displaced along the streamline r_0 to point X on the transient crater wall during excavation. During the same period material further along r_0 at A is displaced to A'. For incompressible flow the volumes of the two shaded regions, when rotated about the axis of symmetry, are equal.

Equation 2.5 may be used to map positions in the target before excavation to their corresponding co-ordinates after excavation, provided that θ_x is known. Alternatively, Equation 2.5 may be rearranged to apply the Z-model in reverse and map positions in the target surrounding the transient crater back to their original pre-impact locations.

Numerical calculations suggest that, during the excavation flow, the transient cavity initially grows as a hemisphere. However, due to increased resistance with increasing depth, the cavity's depth stops growing while the cavity radius continues to increase (O'Keefe and Ahrens, 1993). This observation, combined with geological evidence from terrestrial simple craters for the form of the transient crater prior to collapse (Grieve et al., 1977, 1981), suggests that the transient crater may be approximated by a paraboloid of revolution, with a depth-to-diameter ratio of 1:3 (Melosh, 1989, p. 78).

Figure 2.3 shows the displacement of horizontal and vertical layers in the target according to the Z-model approximation of the excavation flow. In the situation illustrated, $Z = 3$ and the point of explosion is at the surface. The transient crater is defined by a parabola, $y = ax^2 + c$. Material above the hinge streamline (green) is not considered by the model. Beneath the floor of the crater, material is squeezed downwards and outwards, thinning the horizontal layers and thickening the vertical layers. Beneath the rim the target is pushed upward and outward.

It should be pointed out that the validity of the assumption that the transient crater's form is a paraboloid of revolution is a point of contention. For numerical simulations of the excavation stage of large impacts the onset of the subsequent stage in the cratering process, the modification stage, appears to overlap with the end of the excavation stage (O'Keefe and Ahrens, 1993). This results in the bottom of the cavity beginning to collapse back upwards, before the cavity has reached its maximum lateral extent. However, a parabolic form for the transient crater is a good first-order approximation of the cavity prior to the modification stage.

The excavation flow is retarded by the strength of the target and gravity. In large impacts it is gravity that is most significant; excavation stops when insufficient energy remains to lift the overlying material against the force of its own weight. Experimental results from a wide range of impact cratering data have been combined by Schmidt and Housen (1987) to produce scaling laws valid for situations where gravity is the dominant arresting force in transient crater formation. Two such relationships relate the relative densities of the impactor and target ρ_i/ρ_t , the impactor diameter D_i , the impact velocity v_i and the acceleration due to gravity g , to the transient crater diameter:

$$D_t = 1.161 \left(\frac{\rho_i}{\rho_t} \right)^{1/3} D_i^{0.78} v_i^{0.44} g^{-0.22}, \quad (2.6)$$

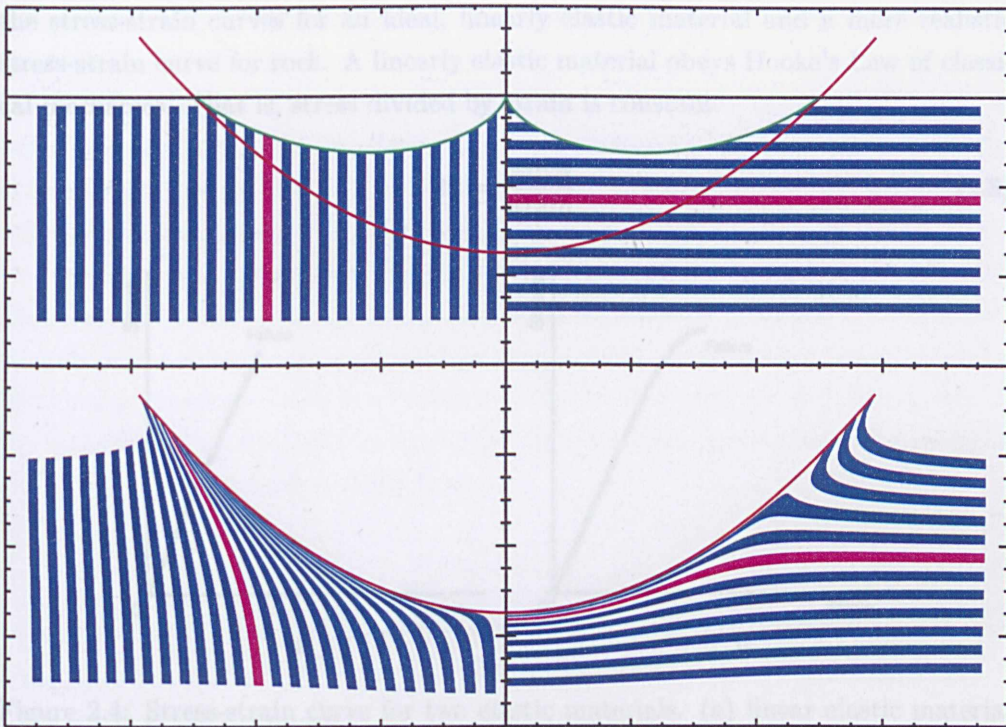


Figure 2.3: Graphical illustration of the Z-model approximation of the excavation flow. Shown are the displacements of originally horizontal and vertical layers regions of the target. Also shown are the hinge streamline (green) and the parabola defining the form of the transient crater (red). The displacements shown are independent of transient crater diameter: the minor ticks on the scale bar represent one-tenth of the transient crater radius.

and the transient crater volume:

$$V_t = 0.28 \frac{\rho_i}{\rho_t} D_i^{2.35} v_i^{1.3} g^{-0.65}, \quad (2.7)$$

(Schmidt and Housen, 1987). Note that these equations are for vertical impacts only. The effect of obliquity may be considered by replacing the impact velocity with its vertical component, $v_i \sin \theta$, where θ is the angle of impact measured from the plane of the target (Chapman and McKinnon, 1986).

2.2 Target Strength: the Enigma of Complex Crater Collapse

Under normal pre-impact conditions and over time-scales appropriate to an impact event (seconds to minutes), the target material on most solid bodies may be considered to be a linearly elastic solid. Linear elasticity is an example of a rheologic model – a concept that will be used repeatedly in this thesis. Figure 2.4 illustrates

the stress-strain curves for an ideal, linearly elastic material and a more realistic stress-strain curve for rock. A linearly elastic material obeys Hooke's Law of classical mechanics. That is, stress divided by strain is constant.

$$2k = \frac{\text{stress}}{\text{strain}} \quad (2.8)$$

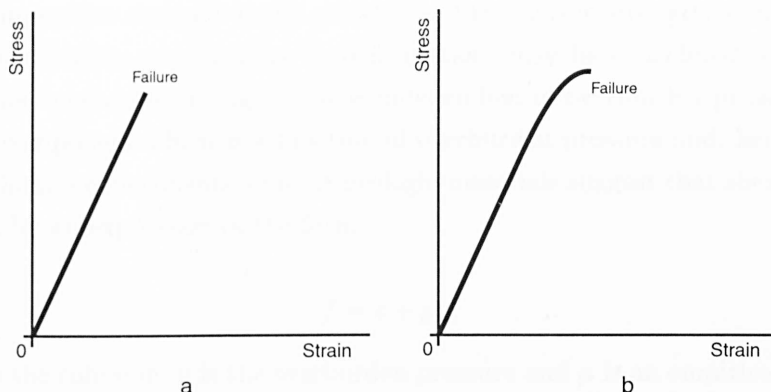


Figure 2.4: Stress-strain curve for two elastic materials. (a) linear elastic material approximation; (b) More realistic approximation of elastic behaviour.

The constant, or modulus, in Equation 2.8 depends on the nature of the stress and strain:

$$\begin{aligned} \text{Young's Modulus (E)} &= \frac{\text{longitudinal applied force per unit area}}{\text{change in length/original length}} \\ \text{Rigidity or Shear Modulus (G)} &= \frac{\text{shear force per unit area}}{\text{angular deformation } \theta} \\ \text{Bulk Modulus (K)} &= \frac{\text{compressive (or tensile) force per unit area}}{\text{change in volume per unit volume}} \end{aligned} \quad (2.9)$$

When a body is stretched, not only does its length increase but its width is also decreased. The lateral change in dimensions is proportional to the longitudinal strain and is a constant, known as Poisson's Ratio:

$$\text{Poisson's Ratio}(\nu) = \frac{\text{change in width/original width}}{\text{change in length/original length}}. \quad (2.10)$$

The difference between the two curves in Figure 2.4 is subtle, implying that the approximation of rock as a linearly elastic material is a good one in most cases. The difference is associated with the behaviour of rock immediately prior to failure. The linear elasticity approximation assumes that the rock behaves elastically up to the point of failure; thus, at stresses up to a critical stress, known as the Hugoniot Elastic Limit, strain is proportional to stress. In such cases the material will return to its original state when the stress is relieved; strain is not permanent. However, in actual rocks the slope of the stress-strain curve decreases steadily to zero with in-

creasing stress. In this regime irreversible changes are induced leading to permanent deformation before failure.

The critical point at which either failure or permanent deformation occurs is defined as the strength. The magnitude of a material's strength depends on the orientation of the applied stress. In other words, the strength of a material may be different when subjected to compression, tension, or shear. In most geologic materials the compressive strength is the greatest and the tensile strength is the smallest. The shear strength, or resistance to deformation, may be considered to have two components, a cohesive strength that is independent of overburden pressure, and a frictional component which is a function of overburden pressure and, hence, depth. Rock mechanics experiments on most geologic materials suggest that shear strength f is given by an expression of the form:

$$f = c + \mu p, \quad (2.11)$$

where c is the cohesion, p is the overburden pressure and μ is an empirically defined constant, termed the coefficient of friction (Jaeger and Cook, 1979, for example).

Comprehensively fracturing a material destroys any cohesive strength the material might have. Experimental measurements of the frictional behaviour of faulted rock materials demonstrate an almost constant ratio of shear strength to confining pressure (Stesky et al., 1974, for example), giving:

$$f = \mu p, \quad (2.12)$$

which is commonly known as Coulomb's law, or the law of "dry friction."

Geological investigation at terrestrial impact sites suggests that, at the time of collapse, the target comprises broken rock debris (section 1.1). It would appear, therefore, that when considering the collapse of any large impact crater, the behaviour of the target is dominated by the physics of granular rock debris and not intact rock. Indeed, the average slope of a simple crater is controlled by the angle of repose of the broken target rock. However, a simple analysis of the competition between the gravitational forces tending to close a cavity and the strength of a granular target (Melosh, 1989, p. 144), reveals a paradox concerning crater collapse.

Consider the negative buoyancy force associated with a hemispherical crater cavity, $\pi/8\rho g H_t D_t^2$, where ρ is the target density, g is the acceleration due to gravity, H_t is the depth of the crater and D_t is the diameter of the crater. Dividing this by the surface area of a hemisphere enclosing the crater, $\pi/2D_t^2$, gives a mean stress acting to close the crater of $1/4\rho g H_t$. The strength of the target is that for a Coulomb material (Equation 2.12), $\mu\rho g H_t$, where μ is the coefficient of friction. Although μ is empirically determined and therefore not uniquely defined, experimental studies have shown that it varies only very slightly for all granular media about a mean

value of 0.85 (Jaeger and Cook, 1979, for example). Thus, it would appear that the medium in which the crater is formed has the “strength” to maintain more than twice the mean stress tending to close the cavity. A fundamental question arises: why do craters collapse at all?

It would appear from the calculation above that, for the transient crater to collapse in such a way as to produce the structures that characterise complex craters, the sub-crater region must undergo some material weakening. The same requirement is not true, however, for simple craters. A more detailed analysis of the stress field due to a cavity in an unconsolidated granular material, shows that there is a gravitationally unstable region at the uppermost part of the transient crater rim (Dent, 1973; Melosh, 1977). In this region the rock debris that defines the wall of the cavity is well above the angle of repose and, hence, collapses. Thus, subsequent to the formation of the transient crater during the excavation stage, the steep rim walls of a simple crater collapse into the crater forming a pool of rock debris at the base of the crater (Grieve et al., 1977). As mentioned in section 1.1.1 such lenses of breccia have been observed beneath all terrestrial simple craters (Grieve et al., 1981). Therefore, debris mechanics would appear to adequately explain the process of simple-crater formation.

2.3 Complex Crater Collapse: Phenomenology

The explanation of simple crater collapse just described does not provide any insight into the simple-to-complex transition. Larger craters should still only be unstable at the steepest part of the transient crater rim, and yet evidence suggests that complex crater collapse is much more spectacular than just failure of the crater wall. Faced with the challenge of explaining the apparent contradiction of the laws of rock and debris mechanics, workers in this field have turned to observational evidence for clues to how a complex crater is formed (Pike, 1988, for example). Attempting to ascertain a process purely from observation is known as a phenomenological approach. The results of many years of study in this area are summarised below.

2.3.1 The Simple-to-Complex Transition

A fundamental change in the collapse process occurs at the simple-to-complex transition. At crater diameters below the transition diameter, collapse is limited to debris sliding; a volume of rock only about half the volume of the crater is displaced during the modification stage. At crater diameters above the transition, however, the volume of rock that must collapse is almost equivalent to the volume of the crater itself. This suggests that a prodigious change in the strength of the material surrounding the crater occurs at the simple-to-complex transition.

The crater diameter and the crater depth at which the simple-to-complex transition occurs are not the same for different Solar System bodies. There is evidence that the transition diameter, D_{coll} , and the transition depth, H_{coll} , are inversely correlated with gravity, shown in Figure 2.5 (Pike, 1988):

$$D_{coll} = \frac{constant_1}{g}, \quad H_{coll} = \frac{constant_2}{g}. \quad (2.13)$$

This relationship implies that, for a crater at the simple-to-complex transition, the stress driving collapse ($\sim \rho g H_{coll}$) is the same, regardless of the body on which it is formed. If the simple-to-complex transition corresponds to a strength threshold, therefore, it would appear that the yield strength is independent of gravity and crater size; that is, the simple-to-complex transition corresponds to some kind of *cohesive* strength threshold. This contradicts the standard strength laws discussed in section 2.2, which reflect the fact that the strength of intact and broken rock material increases with increasing overburden pressure ($\rho g z$).

Evidence from a smaller data set suggests that the diameter at which crater morphology changes from central peak crater to peak ring crater also scales with $1/g$. Thus, it would appear that the onset of collapse to form complex craters, and the change in morphology from central peak to peak-ring both occur when some cohesive strength threshold is exceeded. However, as discussed in section 1.1, complex crater collapse appears to proceed in a cohesionless target; how can a cohesive strength threshold exist in a granular medium?

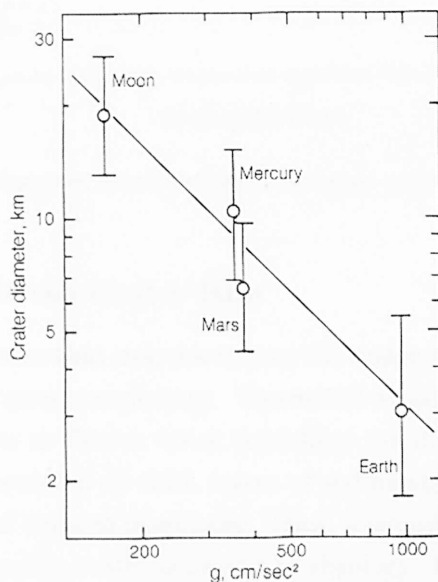


Figure 2.5: Crater diameter at the simple-to-complex transition on the Earth, Moon, Mercury and Mars. The transition is a function of $1/g$. Data from Pike(1988). After Melosh (1989)

Another pertinent observation regarding the simple-to-complex transition is the change in the depth-to-diameter ratio. Figure 2.6 (after Pike, 1974) illustrates that there is a distinct kink in the depth versus diameter curve at a diameter corresponding to the simple-to-complex transition. Similar graphs exist for all the heavily cratered Solar System bodies, which all show the same pattern (Melosh, 1989). The principal information to be extracted from the change in slope in the depth versus diameter curve is that the depth-to-diameter ratio for complex craters is significantly less than for simple craters. The average slope in a complex crater is of the order of a few degrees (Melosh, 1989, p. 145). This is considerably lower than the average angle of repose for unconsolidated rock debris ($\sim 35^\circ$). Thus, there is further evidence that the collapse of a complex crater cannot be explained simply by the frictional sliding of rock debris.

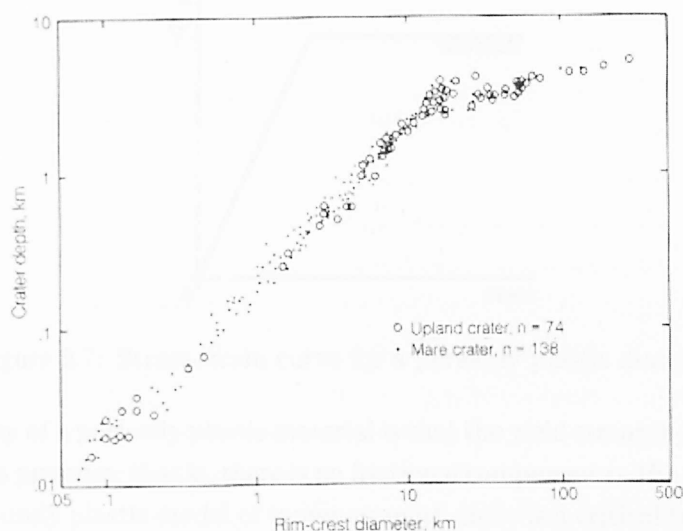


Figure 2.6: Depth/diameter relation for fresh lunar craters. *After Pike (1974).*

2.3.2 Terracing of the Crater Rim

The terraced slump blocks that step down from the crater rim to its floor are another curiosity of complex crater morphology. Terrestrial analogues are the large slump block landslides known as Toreva block landslides (as discussed in section 1.1.2). Such landslides are restricted to thick layers of sediments, or caldera walls, where the underlying material behaves plastically. Thus, it would appear that the material under the rim of a complex crater is plastic in rheology. In sediments, plasticity is attributed to water saturation increasing pore pressure and abrogating the frictional properties of the rock. Under calderas, plasticity is due to high temperature. However, neither of these explanations satisfactorily explains the ubiquity of complex

crater terraces in the Solar System. Consequently, the presence of these features is currently not well understood.

Plasticity is another example of a rheologic model. Figure 2.7 shows the stress-strain curve for a perfectly plastic material. In this case the material behaves elastically up to a certain stress, known as the yield stress σ_Y or the Hugoniot elastic limit Y . A perfectly plastic material can support *no* stress in excess of the yield stress. At stresses beyond the yield point there is no restriction on movement; strain is indefinite until the material fails. If the applied stress is reduced such that it falls below the yield point, then the material ceases to deform plastically and starts to behave elastically again; however, unlike an elastic material, the deformation due to plasticity is permanent.

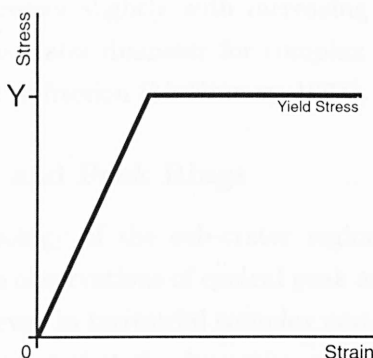


Figure 2.7: Stress-strain curve for a perfectly plastic material.

A property of a perfectly plastic material is that the yield strength is independent of overburden pressure; that is, there is no frictional component to the yield strength. Thus, for a purely plastic model of target strength there is a critical transient crater size at which point the stress surrounding the crater will exceed the yield strength and permit collapse. In other words, craters which form in a plastic material will either be stable (at sizes below the critical crater size), or they will be unstable and collapse (at sizes greater than the critical crater size). Slip line analysis (Melosh, 1977) suggests that the critical depth for collapse in a plastic material is related to the yield strength Y by:

$$\frac{\rho g H_t}{Y} \geq 5. \quad (2.14)$$

This equation implies that, if the yield strength and density are the same for every solid body in the Solar System, then an inverse correlation between the critical depth for collapse and gravity is predicted. Thus, a purely plastic target model explains the observed trend between simple-to-complex transition diameter and gravity.

A corollary to this plastic model for representing the target is that the depth at which a collapsing crater becomes stable – the final crater depth – will be the

same, regardless of crater size. This is because the collapse of a crater in a perfectly plastic material persists until the differential stresses driving the flow drop below the yield criterion defined by Equation 2.14 (Melosh, 1989, p. 145), which occurs at a constant depth.

A purely plastic model for complex crater collapse predicts an inverse correlation between gravity and the size of the smallest crater to collapse in a complex manner; a flat, featureless crater-floor morphology; the terracing of steep rim walls; and that the final depth of all complex craters is constant, regardless of diameter. Thus, several observed features of crater collapse are correctly produced with a target material that behaves approximately plastically. However, complex crater depth is not independent of diameter for a particular body in the Solar System. Instead, complex crater depth increases slightly with increasing diameter (Fig. 1.7). This dependence of depth upon crater diameter for complex craters could be explained by a very small coefficient of friction (McKinnon, 1978).

2.3.3 Central Peaks and Peak Rings

The final clue to the rheology of the sub-crater region during the collapse of a complex crater comes from observations of central peak and peak-ring morphologies. The structural uplift observed in terrestrial complex craters plus the dependence on gravity and crater size suggest that the formation of central peaks and peak-rings is part of the collapse. Other potential mechanisms include erosion and elastic rebound (Baldwin, 1963); however, neither can explain both the stratigraphic uplift and the dependence on gravity and crater size. The presence of terraces and flat crater floors suggest that a good material description of a collapsing complex crater is that of a plastic material. A function of a purely plastic material, however, is that motion is confined to regions of maximum differential stress; in regions of low driving stress the material behaves elastically. Therefore, as the driving stress drops below the threshold stress for the plastic material the material comes to rest. As a consequence, crater collapse in a purely plastic material will never produce a central uplift. In order to overshoot, the material must behave *hydrodynamically*; that is, it must flow as if it were a fluid. The target's fluid-like state must, of course, be ephemeral to allow the peak morphology to be "frozen" in place. This explanation of complex crater formation was first proposed privately by R. B. Baldwin about 1966, then later as the "frozen tsunami" theory (Baldwin, 1972).

In addition to being significantly weaker than competent or uncompetent rock and showing many features characteristic of a plastic material, the collapse of complex craters also appears to be temporarily hydrodynamic. The difference between the "flow" of a plastic material above the yield point and the flow of a fluid is im-

portant but hard to conceptualise. The relationship between stress and strain rate in all fluids is of the form:

$$\dot{\epsilon} = A\sigma^n, \quad (2.15)$$

in which σ is the applied stress, $\dot{\epsilon}$ is the strain rate, A is a constant, and n is a constant with a value of one or more.

A fluid with a direct linear relationship between shear stress and rate of strain ($n = 1$) is said to be Newtonian. In this case A is the ratio of shear stress to the strain rate and is defined as the viscosity, η , with units of Pascal seconds (Pa s). For such a fluid the applied stress σ is commonly related to the *rate* of strain $\dot{\epsilon}$ by:

$$\dot{\epsilon} = \frac{1}{2\eta}\sigma \quad (2.16)$$

For non-Newtonian fluids, however, A does not have units of Pascal seconds and therefore is not a simple measure of the viscosity. In the non-Newtonian case, the ratio of shear stress to the strain rate is called the effective viscosity, η_{eff} , and is defined as:

$$\eta_{\text{eff}} = \frac{A}{\sigma^{n-1}}. \quad (2.17)$$

It is important to note that Newtonian viscosity is independent of the stress, whereas the effective viscosity of a non-Newtonian fluid decreases with increasing stress. The concept of viscosity does not really exist for the flow of a plastic material; plastic flow is non-linear in the sense that the exponent n is infinite. Thus, in regions of high stress the viscosity is effectively zero and deformation is extreme, whereas in regions of low stress the viscosity is infinite and no deformation occurs. Figure 2.8 shows the stress-strain rate curves for a Newtonian fluid, a non-Newtonian fluid and a plastic material. It should be noted that most geological fluids are non-Newtonian with an exponent of not more than 10. However, viscosity is an extremely useful concept and thus Newtonian flow is a commonly used approximation of fluid rheology.

A material that behaves both as a Newtonian fluid, with a viscosity η and has a definite yield strength is known as a *Bingham fluid*. Thus, it has much in common with a plastic material. The distinction is apparent at stresses above the yield stress, where the material rheology of a Bingham fluid is fluid-like; that is, it flows with a finite viscosity, η . Below the yield strength, however, a Bingham material is identical to a plastic material in that deformation is governed by elasticity; no permanent strain is allowed. The Bingham fluid, therefore, seems to provide the best phenomenological model for the rheology of the target during complex crater collapse (Melosh, 1989).

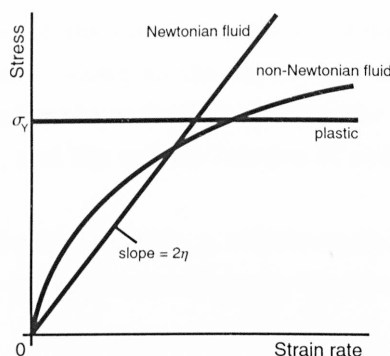


Figure 2.8: Stress-strain rate curve for a Newtonian fluid, a non-Newtonian fluid and a perfectly plastic material.

2.3.4 Time-Scale of Collapse

Limits on the time-scale for complex crater collapse may be derived from geological observations of terrestrial impact craters. In the majority of known terrestrial impact structures, sediments that overlie the crater show no signs of extreme, collapse-related deformation (Melosh and Ivanov, 1999). Thus, the collapse time-scale must be short relative to the accumulation of sediments. Furthermore, two terrestrial impact sites (Moltysh, Ukraine and Manicouagan, Canada) exhibit large, coherent melt sheets, which surround the central uplift (Onorato et al., 1978; Masaitis et al., 1980). This suggests that formation of the central uplift occurs prior to the solidification of the melt sheet. Thus, the time-scale for complex crater collapse must be less than the time it takes an impact melt sheet to solidify.

Unfortunately, the solidification time-scale for an impact melt sheet is difficult to ascertain; impact melt sheets are distinct from more familiar volcanic melts in two important ways. First, an impact melt will have a wide range of initial temperatures, which could bracket the solidus, liquidus and boiling point. A volcanic melt, in contrast, is usually not far from its melting point. Second, an impact melt will contain a large volume of clastic material relative to a volcanic melt. This is a result of extreme mixing during collapse and the fall-back of material thrown out of the crater during excavation.

A large, uniform body of melt cools by conduction. The time-scale for solidification, therefore, is controlled by the thermal diffusivity, κ , and the thickness, h , according to the law $t \sim h^2/\kappa$. For a 1-km thick melt sheet with a thermal diffusivity of order $10^{-6} \text{ m}^2 \text{ s}^{-1}$, $t \sim 30,000$ years. However, the presence of a significant volume of cold rock fragments, termed clasts, has a profound influence on both the cooling rate and the rheology of magma. The clastic material absorbs heat from the melt, resulting in the rapid attainment of a local temperature equilibrium between the melt and the clasts (Onorato et al., 1978). In most cases, this results in

melting of a large fraction of the clastic material, combined with a rapid drop in the mean melt temperature – down to the liquidus in about 100 seconds (Melosh and Ivanov, 1999). Thereafter the melt sheet cools by the more familiar processes of conduction and radiation, and the volume fraction of solid material begins to grow by nucleation.

The viscosity of a subliquidus magma, η , relative to its liquidus viscosity, η_0 , may be roughly approximated by the Roscoe-Einstein equation (McBirney and Murase, 1984):

$$\frac{\eta}{\eta_0} = \left(1 - \frac{\phi}{\phi_m}\right)^{-2.5}, \quad (2.18)$$

where ϕ is the volume fraction of suspended solids (volume occupied by suspended solids / total volume of fluid-solid suspension) and ϕ_m is volume fraction of suspended solids at maximum packing. Equation 2.18 is a reasonably good approximation of the behaviour of solid-fluid suspensions for low values of ϕ . However, when ϕ approaches the maximum volume fraction ϕ_m , the particles become in continuous three-dimensional contact, which effectively inhibits the mobility of the suspending fluid. At the maximum volume fraction the contacts are so numerous and strong that the effective viscosity of the suspension tends to infinity and fluid flow is prevented. Thus, for a volume fraction of solid material approaching ϕ_m Equation 2.18 is not valid.

By considering the packing ratios of crystals in lava, Marsh (1981) concludes that a value of $\phi_m \approx 0.59$ is most appropriate for rock-melt suspensions. Any impact melt with a solid volume fraction exceeding about 0.5, therefore, would not behave in a fluid-like manner over short time-scales. Furthermore, even at lower volume fractions, Equation 2.18 shows that the viscosity ratio has a strong dependence on ϕ ; meaning that the viscosity of the subliquidus melt increases sharply with an increasing fractional volume of suspended solids. For $\phi \geq 0.35$, the viscosity of a subliquidus magma is more than an order of magnitude greater than that of the liquidus melt. Impact melt rocks, whether from the Earth or Moon, typically comprise fine grained glassy matrices surrounding up to 25% mineral clasts (Onorato et al., 1978). Thus, the volume fraction of solid material in an impact melt, after the initial local thermalization, is likely to increase from a minimum of $\phi \approx 0.25$ as crystals form and grow in the cooling melt sheet. The viscosity of the impact melt, therefore, will rise to a level that prohibits rapid flow soon after the initial local thermalization. In other words, the melt solidifies in a time-scale on the order of 100 seconds, which places an upper limit on the time-scale for collapse.

The minimum collapse time-scale is limited by inertia: a crater cannot collapse faster than its free-fall time under gravity, t_{free} . Thus, a lower limit on the collapse time-scale for a crater of depth H_t , is: $t_{free} = \sqrt{2H_t/g}$, where g is the gravitational

acceleration. For a transient crater 10-km deep on Earth, collapse must take longer than 44 seconds.

The time-scale for collapse of a crater in a fluid of viscosity η is given by Melosh (1983):

$$t_{visc} = \frac{1}{\dot{\epsilon}} = \frac{2\eta}{\sigma}. \quad (2.19)$$

Collapse cannot run to completion if t_{visc} is much longer than the free-fall collapse time, t_{free} . Thus, for a driving stress $\sigma \approx \rho g H_t$, there is an upper limit on the effective viscosity of the target, given by:

$$\eta_{crit} \approx \frac{1}{\sqrt{2}} \rho g^{1/2} H_t^{3/2}. \quad (2.20)$$

For a 10-km deep crater on Earth, therefore, the effective viscosity of the target must be less than about 10^{10} Pa s. This is in good agreement with a more sophisticated analytical treatment of a disturbance on the surface of a viscous liquid covered by an elastic layer (Tait, 1890), which predicts that a crater formed in such a substrate will not overshoot for viscosities in excess of $\pi^{3/2} \rho g^{1/2} D^{3/2}$.

The limits discussed above suggest that the time-scale for complex crater collapse is much shorter than most more familiar geologic processes. Only landslides and some volcanic eruptions are as dynamic as crater collapse; however, very few of these events involve volumes of rock comparable to a large impact event. The short time-scale for complex crater collapse implies that the huge volume of rock displaced during the excavation flow must move back into the cavity almost as fast as it was pushed out. Given that the force driving the collapse is significantly smaller than that driving excavation, such rapid movement of rock must require a prodigious change in target rheology. For large impact craters formed on Earth the target must flow with an effective viscosity much less than standard mantle viscosities of $\sim 10^{21}$ Pa s.

2.3.5 The Current Paradigm for Complex Crater Collapse

The Bingham-fluid model for complex crater collapse is, currently, the only model that explains how large impact craters collapse to produce the complex impact structures observed in the Solar System. It reconciles all lines of evidence for the rheological nature of the collapse and avoids a fundamental paradox in the strength of rock materials by allowing the target to regain more familiar strength properties following collapse (Melosh, 1989). For clarity, this paradigm is illustrated schematically in Figure 2.9.

Figure 2.9 shows the collapse of the transient crater proceeding by means of the inward collapse of the transient crater rim, accompanied by the uplift of the floor

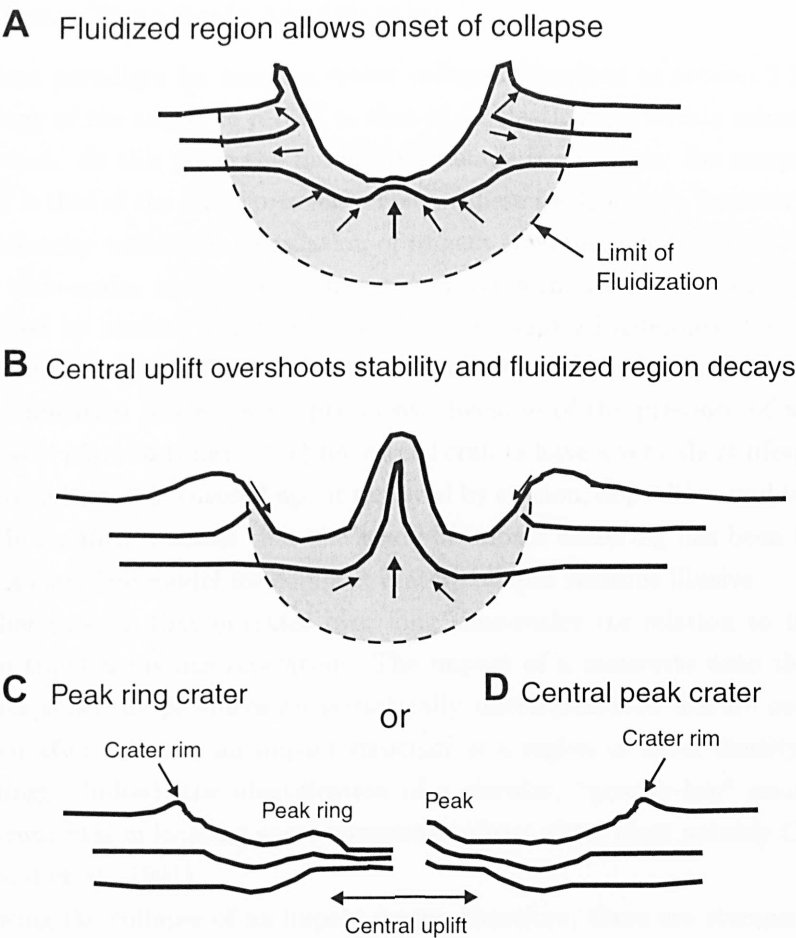


Figure 2.9: A schematic diagram of the current paradigm for complex crater collapse. See text for further explanation. Adapted from Melosh (1989).

of the crater. Such wholesale collapse is permitted by the temporary fluidization of a region of the target surrounding the cavity (shaded). In the final illustration the target is no longer fluidized; the central uplift has collapsed in some way to form the complex crater morphology, which is then “frozen” in place by the halting of the fluidization process.

The details of the collapse process summarized in Figure 2.9 are currently unknown. Moreover, the mechanism by which the temporary fluidization occurs is a point of controversy; no single material weakening mechanism has been widely accepted as providing the missing link in the current paradigm for complex crater collapse. Providing a robust dynamical model to accurately describe and explain the collapse process is a primary objective of this study.

2.3.6 Long Time-Scale Modification

The current paradigm for complex crater collapse described in section 2.3.5 allows the rheology of the target to return to that of an elastic solid within minutes of the impact event. At this point the modification stage is complete; the morphology of the crater is that of the final “pristine” crater. These fresh craters, however, are only a small minority within the population of impact structures in the Solar System.

Over time-scales much longer than the crater formation time, pristine craters are modified by surface processes. On the Moon and all asteroids, the dominant surface process is the collision of other impactors. On Earth, however, far more severe modification processes are prevalent. Because of the presence of water and the process of plate tectonics, most terrestrial craters have a very short lifespan; they are quickly obliterated, covered up, or removed by erosion, deposition and tectonism. It is partly for these reasons that the study of impact cratering has been hindered, and why a complete model for complex crater collapse remains illusive.

Another process that operates over long time-scales (in relation to the crater formation time) is viscous relaxation. The impact of a meteorite onto the surface of a planet generally produces an isostatically uncompensated feature on the surface. Even after collapse, an impact structure is a region of lower density than its surroundings. Indeed, the identification of a circular, “gravity-low” anomaly has been instrumental in locating some terrestrial impact sites, most notably Chicxulub (Hildebrand et al., 1991).

Following the collapse of an impact crater, therefore, there are stresses remnant in the rock surrounding the crater, which tend to compensate isostatically for the loss of material and the reduction in density. The situation is analogous to the rebound of the crust following a period of glaciation: when an ice sheet is removed, the weight of the crust in that region is reduced and the crust slowly rises; a process known as isostatic rebound (Turcotte and Schubert, 1982, for example). The reason for this slow rebound is that no known material is truly “solid”. Over sufficiently long time-scales, all materials behave as a fluid under applied stress; everything flows.

The flow rate $\dot{\epsilon}$ of nearly all studied substances attains a constant value some time after a differential stress τ is applied, given by (Melosh, 1989, p. 154):

$$\dot{\epsilon} = A(\tau)e^{-f\frac{T_m}{T_A}}, \quad (2.21)$$

where $A(\tau)$ is a function of the applied stress, T_m is the melting temperature of the material, T_A is the absolute temperature, and f is a dimensionless constant. For the case of geologic materials f is usually between 25 and 27 and the applied stress is frequently a power law: $A\tau^n$, where n is between 3 and 6 (for example, Ranalli, 1987, p. 75).

Since temperature increases linearly with depth in most planetary bodies, Equation 2.21 predicts that flow rate will increase exponentially with depth. For rocks near the surface the flow could occur so slowly as to be imperceptible. However, flow rates become geologically significant at depths where the absolute temperature approaches one-half of the melting point T_m . Thus, at sufficient depths beneath the surface of the Earth the rheologic behaviour of the mantle becomes fluid over geologic time-scales. This fact has been used to approximately define the base of the mechanical lithosphere as the 600°C isotherm (Melosh, 1989, p. 156).

The Maxwell substance is another rheological model; it has been defined to describe a material that behaves elastically and as a fluid, depending on the period of the applied stress. A Maxwell substance, therefore, combines the concepts of elasticity and viscosity; it may be represented by a system of a dash-pot and a spring in series (Fig. 2.10). Thus, the same stress acts across both the spring and the dash-pot, so that if ϵ_d and ϵ_s are the components of strain, then:

$$\dot{\epsilon}_d = \sigma/2\eta, \quad \epsilon_s = \sigma/2k, \quad (2.22)$$

and the total strain $\epsilon = \epsilon_d + \epsilon_s$ satisfies:

$$\dot{\epsilon} = (\dot{\sigma}/2k) + (\sigma/2\eta). \quad (2.23)$$

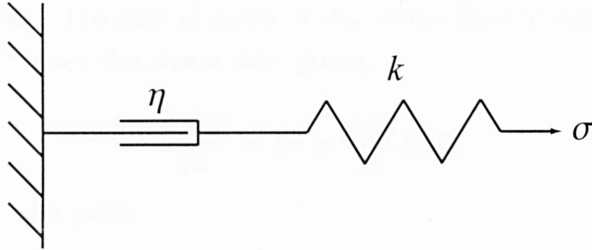


Figure 2.10: Mechanical model of a Maxwell substance

If a Maxwell substance is unstrained at time $t = 0$ and constant stress S is applied, then integrating (2.23) gives:

$$\epsilon = (S/2k) + (St/2\eta) \quad (2.24)$$

Thus, for short time-scales the dominant component of the strain is the elastic part; however, over long time-scales the viscous strain is far more significant. The relative solidity of a Maxwell substance may then be described by the Maxwell time, t_M , which is defined as the length of time for which such a substance must flow under an applied load before the viscous strain $\epsilon_d = \dot{\epsilon}t_M$ equals the elastic strain ϵ_s . The

Maxwell time, then, is simply the ratio of the effective viscosity to the shear modulus of the material G :

$$t_M = \eta/G. \quad (2.25)$$

The Maxwell rheology has been used to describe the Earth's mantle which must behave elastically for short times since it transmits shear waves, but at the same time may flow very slowly under continuously applied stresses (Jaeger, 1969). For upper crustal rocks on Earth the shear modulus is $\sim 10^{10}$ Pa and viscosities exceed 10^{26} Pa s, giving them a Maxwell time in excess of 10^9 years. Thus, surface rocks behave elastically over time-scales as long as the age of the Earth. However, upper mantle viscosities of $< 10^{21}$ Pa s have been determined from measurements of glacial rebound rates (Turcotte and Schubert, 1982, p. 248). The Maxwell time for rocks at this depth, therefore, is only a few thousand years. A logical question arises from this insight with relation to complex crater formation: does viscous relaxation around large impact craters play a significant role?

A simple answer to this question may be obtained by considering a bowl-shaped cavity formed in a constant viscosity material (Melosh, 1989, p. 156), as illustrated in Figure 2.11. The shear stress τ beneath a crater of depth H is roughly $1/4 \rho g H$ (see section 2.2). Assuming that relaxation of the crater is by Newtonian flow, then the strain-rate of material flowing inwards toward the crater is about $\dot{\epsilon} = \rho g H / 8\eta$ (see Equation 2.16). The rate of uplift of the crater floor is roughly equal to the crater diameter D times this strain rate, giving:

$$-\frac{dH(t)}{dt} = D\dot{\epsilon} = \frac{\rho g D}{8\eta} H(t). \quad (2.26)$$

Upon integration, this gives:

$$H(t) = H(0)e^{-\frac{t}{t_R}}, \quad (2.27)$$

where $H(0)$ is the initial crater depth and t_R is the relaxation time:

$$t_R = \frac{8\eta}{\rho g D}. \quad (2.28)$$

Equation 2.28 shows not only the intuitive result that relaxation takes longer for materials with larger viscosities, but also that larger craters relax quicker than smaller ones. For a 100-km diameter terrestrial crater formed in crustal rocks of density $\sim 3 \times 10^3$ kg m $^{-3}$ and viscosity 10^{25} Pa s, Equation 2.28 gives $t_R \sim 10^9$ years.

The relationship between relaxation time and crater diameter, however, is not the simple inverse law suggested by Equation 2.28. Not considered by the rudimen-

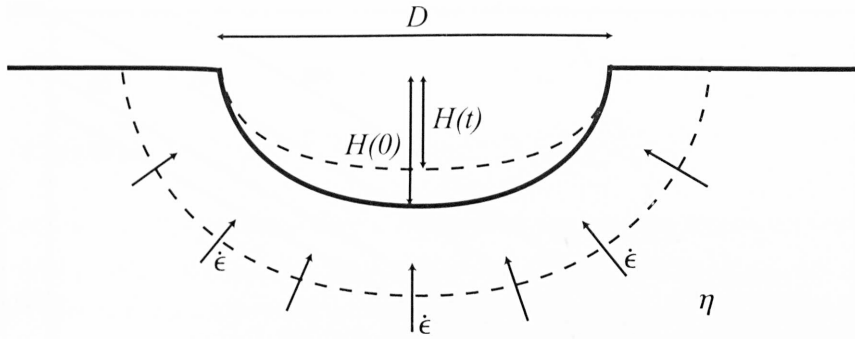


Figure 2.11: Diagram illustrating the parameters used in the text to calculate the relaxation rate of a bowl-shaped cavity in a viscous medium.

tary calculation above is the fact that viscosity is often depth-dependent. A more sophisticated analysis of crater relaxation, first discussed by Scott (1967), provides a relationship between an arbitrary crater profile $h(r, t)$ and a series of J_0 Bessel functions, which all relax self-similarly in a time t_R :

$$h(r, t) = \int_0^{\infty} \kappa(k) J_0(kr) e^{-t/t_R} k dk, \quad (2.29)$$

where $\kappa(k)$ is the Bessel transform of the initial crater profile. In this case, the relaxation time is a function of wavenumber k ($= 2\pi/\lambda$) and viscosity as a function of depth $\eta(z)$. For a material with a viscosity that decreases exponentially with depth the relaxation time is:

$$t_R = \frac{2\eta_0 k}{\rho g} \left(\frac{1 - 2\gamma}{4kd\gamma} [1 + (\gamma/kd)^2 (1 - 2\gamma)] \right), \quad (2.30)$$

after Brennan (1974), where d is the depth over which the viscosity decreases e times and γ is given by:

$$\gamma = 2^{-3/2} [1 + 4(kd)^2 + \sqrt{16(kd)^4 + 24(kd)2 + 1}]^{1/2}. \quad (2.31)$$

For a wavelength of twice the crater diameter $\lambda = 2D$ the quotient outside the brackets in Equation 2.30 is functionally equivalent to Equation 2.28. The effect of the expression in brackets is demonstrated in Figure 2.12. The plot illustrates the relationship between relaxation time and crater diameter for a uniform viscosity half-space ($\eta = 10^{25}$ Pa s, dashed line) and four different viscosity-depth models. Each of these four models is parameterised by the scale-depth Z ; which is defined as the depth at which the viscosity has dropped from 10^{25} Pa s (typical surface rock viscosity) to 10^{21} Pa s. Z is related to d by the expression: $d = Z/5 \ln 10$.

Figure 2.12 demonstrates that the effect of an exponentially decaying viscosity with depth becomes more and more significant with increasing crater diameter. For

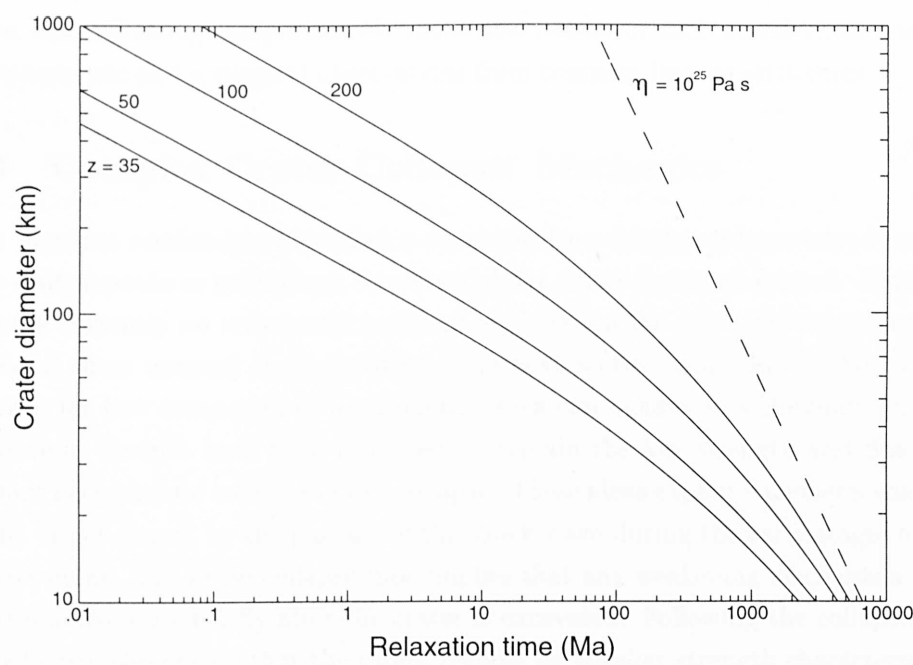


Figure 2.12: Plot of relaxation time versus crater diameter for various viscosity models.

crater diameters less than 10 km the relaxation time is of the order of the relaxation time for a uniform viscosity half-space. For a crater the scale of Chicxulub, however, the relaxation time for an exponentially decreasing viscosity with a scale-depth of 100 km is two orders of magnitude lower than that for the uniform viscosity half-space. Thus significant relaxation of the Chicxulub structure should occur over a period $t_R \sim 10$ Ma. Given that the scale of the lithosphere is also approximately 100 km, this suggests that viscous relaxation of the 65 Ma Chicxulub crater may be significant (20-30%), corresponding to a change in crater depth of about a kilometre.

The previous discussion suggests that the end result of viscous relaxation for all craters is a perfectly level plain. However, this is an over-simplification of the viscous relaxation process beneath craters on bodies like the Earth and the Moon. Such bodies have strong, low density crusts above higher density, lower viscosity mantles. A cratering event on these bodies thins the crust beneath the crater. The subsequent viscous flow in the mantle and crust results in an uplifted mantle plug beneath the thin crust. The higher density of this mantle plug means that isostatic compensation is achieved before the crater has relaxed to a flat surface; a topographic low remains. Thus, the estimate for the amount of relaxation surrounding the Chicxulub impact structure is likely to be an upper limit.

As will be discussed in Chapter 4, numerical simulations of complex crater formation do not consider long-time-scale modification. Thus, it is important to consider the consequences of erosion, deposition, tectonism, bombardment and viscous relax-

ation when drawing comparisons between the results of such simulations and the morphometric and geological observations from complex impact structures.

2.4 Complex Crater Collapse: Mechanics

The previous section has identified a rheological model that adequately describes *how* central peaks or peak-rings, terraces and flat crater floors are formed. However, there is currently no universally accepted explanation for *why* the target material takes on these unusual characteristics. The next section will address the current theories for how competent or incompetent rock can behave as a Bingham fluid.

Several theories have been proposed to explain the low strength and fluid behaviour necessary for complex crater collapse. These ideas exploit ephemeral changes in the target caused by the passage of the shock-wave during the early stages of the impact event. The rapid collapse time implies that any weakening mechanism need remain active only briefly after the crater is excavated. Following the collapse, the model must also ensure that the target regains its familiar strength characteristics, to avoid any fundamental paradox in the strength of rock materials.

2.4.1 The Effect of the Shock Wave

An approximate estimate of the shock pressure experienced by the impactor and target in close proximity to the point of impact is the stagnation pressure:

$$p_{shock} \sim \rho v_i^2, \quad (2.32)$$

where ρ is the smaller of the impactor and target density and v_i is the impact velocity. For impactor speeds above a few kilometres per second, Equation 2.32 yields a peak shock pressure in the range of 100–1000 GPa, which greatly exceeds the strength of any known material. Thus, during this stage of the impact process the strength of the target is of no consequence; the target can be treated as a fluid. Following the release from a high pressure state, however, the strength and rheological behaviour of the target return to a more familiar state for rock material. Theoretical consideration, numerical simulation and geologic evidence from terrestrial craters show that, during the passage of the shock wave, the target is vaporised, melted and fractured. The following discussion describes the effect of the passage of the shock wave on the constitutive properties of the target and its behaviour when subjected to stress.

To determine the post-shock state of the target an accurate prediction of the shock pressures experienced by the target and impactor is required. Some workers, such as Ahrens and O'Keefe (1977) and Pierazzo et al. (1998), have calculated shock pressures experienced by the target in the early stages of an impact using numerical modelling techniques, coupled to sophisticated representations of how

materials respond to high pressures. Their simulations suggest that the peak shock pressure generated by an impact has at least two distinct dependencies on distance from the impact site. The impactor and a region of the target proximal to the impact point (within about one impactor radius) experience approximately the same high shock pressure, p_{max} . This isobaric core shall be referred to here as the “contact zone”. At radial distances greater than one impactor radius a_i , the pressure decays according to some power law:

$$p_{shock} = p_{max} \left(\frac{a_i}{r} \right)^n \quad (2.33)$$

where r is the radial distance from the point of impact. The peak shock pressure attained in the contact zone and the exponent n have been found by detailed computer simulation to be functions of impact velocity, impactor and target density and impact angle (Ahrens and O’Keefe, 1977; Pierazzo et al., 1998; Pierazzo and Melosh, 2000a). However, a first-order approximation for p_{max} and the exponent n may be obtained analytically:

Peak Shock Pressure in the Contact Zone

The quantities of pressure p , density ρ and internal energy per unit mass I before (subscript 0) and after the passage of a shock wave may be obtained using the Hugoniot equations, first derived by P. H. Hugoniot in his 1887 thesis:

$$\rho(U - u_b) = \rho_0 U \quad (2.34)$$

$$p - p_0 = \rho_0 u_b U \quad (2.35)$$

$$I - I_0 = 0.5(p + p_0)(1/\rho_0 - 1/\rho) \quad (2.36)$$

In these equations u_b is the particle velocity behind the shock (the unshocked material is assumed to be at rest), and U is the shock velocity. These three equations represent the conservation of mass, momentum and energy respectively. To complete the set of equations a relationship between pressure, density and internal energy is required, a so-called equation of state, $p = p(\rho, I)$. Alternatively, the Hugoniot equations may be closed by a relationship between the shock velocity and the particle velocity. For many materials this relationship is linear;

$$U = c + S u_b, \quad (2.37)$$

where c and S are empirically defined constants. S is on the order of 1 for most geological materials; c is equivalent to the bulk sound speed.

A solution to this set of equations may be found using the planar impact approximation (Gault and Heitowit, 1963). In this treatment, illustrated in Figure 2.13, the impactor is represented by an infinite sheet with a thickness equal to the impactor diameter. The target is represented by an infinite half-space. The material properties of the target and impactor are labelled on Figure 2.13a; the subscript i denotes the impactor properties and t denotes the target properties. The simulated impact occurs in one-dimension as the sheet impacts vertically with the target. Consequently, although the approximation can be readily solved, it neglects rarefactions that propagate inwards from the edge of the impactor, leading to an overestimation of the actual mean pressure. This approximation, therefore, is only adequate for calculating the shock pressure attained within one impactor radius of the point of impact; the contact zone.

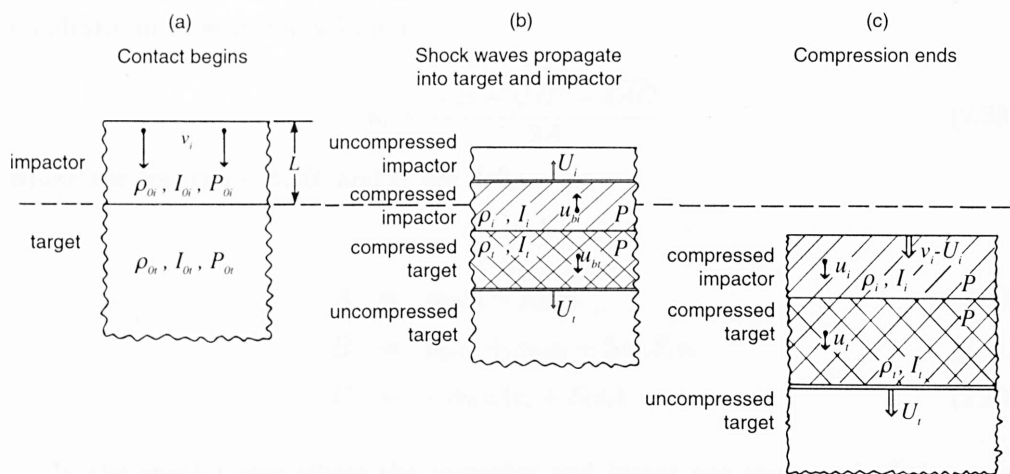


Figure 2.13: The planar impact approximation of the contact and compression stage. (a) The impactor, represented by an infinite slab of thickness L moving at velocity v_i , just contacts the target surface; (b) Strong shocks propagate into both the impactor and the target at different shock velocities. Behind the shock fronts a common pressure P is attained; the variables of particle velocity, density, internal energy and pressure are modified from their initial values according to the Hugoniot equations (see text). Shown in the reference frame of the impactor. (c) To avoid interpenetration or separation the particle velocity of both the impactor and target must also be equivalent, which in the reference frame of the target implies that $u_t = u_i = v_i - u_{bi}$. Compression ends when the shock wave propagating through the impactor reaches the free surface. See text for further discussion. *Adapted from Melosh (1989).*

To calculate the shock pressure within the contact zone using the planar impact approximation, consider two shocks propagating away from the impact interface (between the impactor and target), into both the impactor and target, as viewed from the reference frame of the penetrating impactor (Fig. 2.13b). Between the

shock fronts the target and impactor both experience the same high pressure; thus, if both the target and impactor can be described by the linear relation between shock velocity and particle velocity (Eq. 2.37), then combining Equation 2.37 and the second Hugoniot equation (Eq. 2.35) gives:

$$\rho_{0i}u_{bi}(c_i + S_i u_{bi}) = \rho_{0t}u_{bt}(c_t + S_t u_{bt}), \quad (2.38)$$

provided that the initial pressure in the impactor and target are comparable ($P_{0i} \approx P_{0t}$).

In the reference frame of the target (Fig. 2.13c), the particle velocity in the target is $u_t = u_{bt}$, and the particle velocity in the impactor is $u_i = v_i - u_{bi}$. To avoid separation or interpenetration of the impactor and target material these velocities must be equal; thus, $u_{bi} = v_i - u_t$, and Equation 2.38 can be rearranged to give a quadratic in u_t with the solution:

$$u_t = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad (2.39)$$

where the constants A , B , and C are defined as:

$$A = \rho_{0t}S_t - \rho_{0i}S_i \quad (2.40)$$

$$B = \rho_{0t}c_t + \rho_{0i}c_i + 2\rho_{0i}S_i v_i \quad (2.41)$$

$$C = -\rho_{0i}v_i(c_i + S_i v_i) \quad (2.42)$$

In the special case where the impactor and target are composed of the same material the constant A is zero. Thus, the equation for u_t reduces to $Bu_t + C = 0$, giving $u_t = v_i/2$; if the impactor and target are similar in composition, the particle velocity in the contact zone is approximately one half of the impactor velocity. Substituting this result into a combination of Equation 2.35 and Equation 2.37 gives an equation for the shock pressure experienced by the impactor and a comparable mass of the target:

$$p_{shock} = \rho_{0t}v_i/2(c_t + S_tv_i/2). \quad (2.43)$$

If the shock is strong; that is, the shock velocity is much greater than the bulk sound speed of the target ($U \gg c_t$), and taking $S_t \sim 1.0$ (true for most rocks), then the shock pressure is approximately $0.25\rho_{0t}v_i^2$.

Spatial Pressure Decay Outside the Contact Zone

Beyond a distance of one impactor radius from the impact point, the quantities of pressure, particle velocity, density and internal energy must decay with radial

distance from the point of impact as the shock wave expands into the target. Determining the form of this decay (the exponent in Eq. 2.33) is a problem that lends itself to the approach of self-similarity. This is because the size of the source (approximately one impactor diameter) is small compared to the other dimensions in the problem; and the only natural length-scale in the problem (the radius of the shock front) is constrained by other variables in the problem (shock-front velocity and time). Thus, by normalizing all distances in the problem by the radius of the shock front at a given time $r_f(t)$, a self-similar solution is obtained. For example, the pressure as a function of distance from the impact point is:

$$p(r) = p_f f_1(r/r_f), \quad (2.44)$$

where p_f is the pressure at the shock front and f_1 is some function of r/r_f alone.

The following derivation is based on a seminar by Boris Ivanov. The total energy of the target-impactor system E is given by the integral:

$$E = \int_0^\infty \rho(r) \left[I(r) + \frac{u^2(r)}{2} \right] dV, \quad (2.45)$$

where $I(r)$ is the internal energy per unit mass, $\rho(r)$ is the target density and $u(r)$ is the particle velocity (all as functions of radial distance from the impact point). By replacing $I(r)$ with $u^2(r)/2$ according to the third Hugoniot equation (Eq. 2.36 in the strong-shock limit) and expressing the variables of density and particle velocity in terms of r/r_f ($\rho(r) = \rho_f f_2(r/r_f)$, $u(r) = u_f f_3(r/r_f)$), Equation 2.45 may be re-written as:

$$E = \rho_f u_f^2 r_f^3 \int_0^\infty f_3(r/r_f) f_2^2(r/r_f) d^3(r/r_f). \quad (2.46)$$

Thus, the total energy may be expressed as a product of a set of factors describing the condition of the target at the shock front and an integral that is a function of r/r_f only, and hence constant in time. By the law of conservation of energy the total energy of the system must also remain constant in time, giving:

$$p_f = \rho_f u_f^2 \propto r_f^{-3}. \quad (2.47)$$

Thus, the pressure at the shock front is inversely proportional to radial distance raised to the power 3. Of course, in the case of an impact event the total energy in the target is not conserved because material is ejected through the free surface. This loss of energy may be estimated by integrating the rate at which energy is lost through ejection over the surface area of the target:

$$\frac{dE}{dt} = \int_0^\infty u(r) \rho(r) \left[I(r) + \frac{u(r)}{2} \right] dS. \quad (2.48)$$

Proceeding as before, Equation 2.48 gives a rate of energy loss through the free surface that depends only on the shock-front variables of density, particle velocity and radial distance from the impact point:

$$\frac{dE}{dt} = \alpha \rho_f u_f^3 r_f^2, \quad (2.49)$$

where α is a constant. Replacing the integral in Equation 2.46 with another constant β gives the total energy in the target as:

$$E(t) = \beta \rho_f u_f^2 r_f^3. \quad (2.50)$$

Applying the law of conservation of energy to the entire system, gives:

$$\frac{d}{dt}E(t) + \frac{dE}{dt} = 0. \quad (2.51)$$

Again assuming that the shock is strong; the compressed density remains roughly constant ($d\rho_f/dt = 0$). Thus, solving Equation 2.51 for u_f in terms of r_f by using the fact that $u_f \sim r_f/t$, gives:

$$u_f^2 \propto r_f^{-(3+\alpha/\beta)}, \quad (2.52)$$

implying a relationship between shock pressure and radial distance of:

$$p_f \propto r_f^{-(3+\alpha/\beta)}. \quad (2.53)$$

Thus, the effect of energy loss through the free surface is to increase the rate of decay of both particle velocity and pressure with increasing distance from the impact point, relative to the special case of no ejected material.

Numerical modelling of shock-wave attenuation is in general agreement with the analytical derivation above. Ahrens and O'Keefe (1977) using the Tillotson equation of state and, later, Pierazzo et al. (1997) using the ANEOS (semi-analytic equation of state) and Pierazzo and Melosh (2000a) using the SESAME tabulated equation of state conclude that the exponent n in Equation 2.33 is between ~ 1.5 and 4.0 , depending on impact angle, the composition of the impactor and target, the position in the target and, most significantly, impact velocity. In general, numerical work suggests that the decay exponent, as a function of impactor velocity, takes the form: $n = a + b \log(v_i)$, where a and b are constants and v_i is measured in kilometers per second. Pierazzo et al. (1997) suggest that a is about -1.8 and b about 2.6 ; whereas Ahrens and O'Keefe (1977) derive a similar expression for $-n$ and find that a is about -1.25 and b about -1.6 . For a 20 km s^{-1} impact velocity, these two equations predict an exponent n of 1.6 and 2.1 respectively.

Figure 2.14a illustrates the spatial pressure decay surrounding the contact zone for a high velocity impact taken from the work of Pierazzo and Melosh (2000a). In the case illustrated (their figure 3a), the impactor is 10 km in diameter and has a density of 3320 kg m^{-3} . The impact velocity is 20 km s^{-1} and the density of the target is approximately 2680 kg m^{-3} (granite crust). In this figure the shock pressure falls off as approximately $1/r^2$. For a comparison, Figure 2.14b shows the spatial pressure decay as predicted by the analytical model above, with the exponent in Equation 2.33 taken to be 3 and the same target and impactor properties as used by Pierazzo and Melosh (2000a). The peak shock pressure, p_{max} , used in Equation 2.33 to compute Figure 2.14b, is obtained using equations 2.35, 2.37, and 2.39–2.42, with $S_i = 0.9$, $S_t = 1.24$, $c_i = 6.6 \text{ km s}^{-1}$, $c_t = 3.68 \text{ km s}^{-1}$ (data taken from Melosh, 1989, p 232).

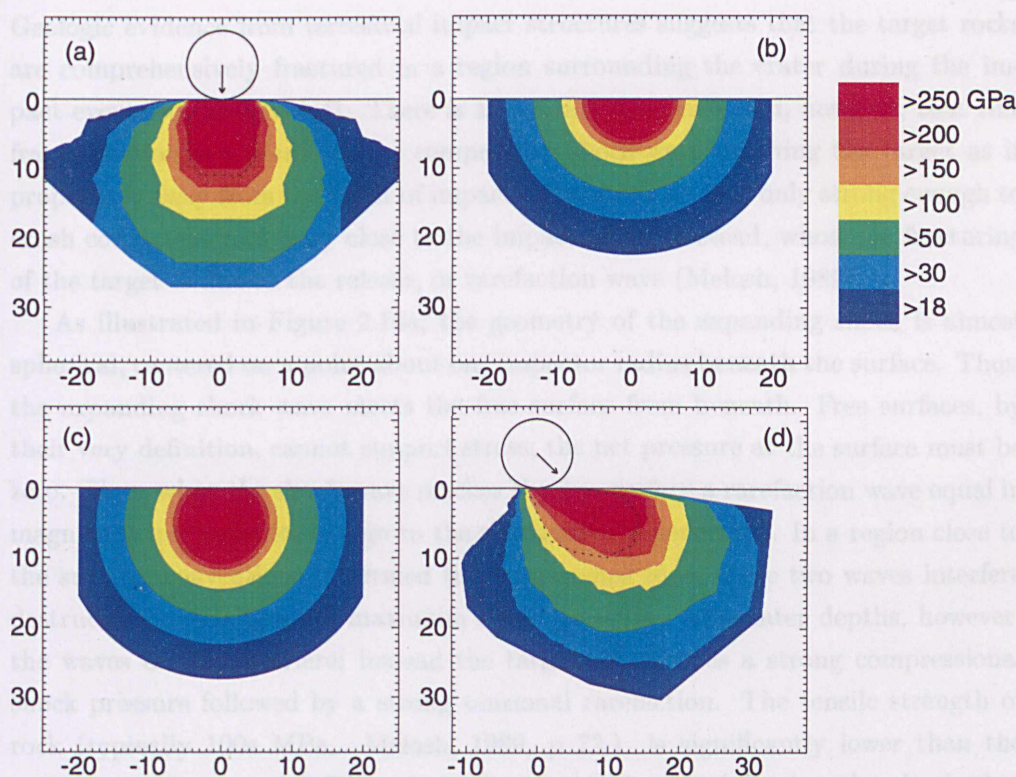


Figure 2.14: Shock-wave attenuation in the target. All contour plots are for a 10-km diameter dunite impactor (density of 3320 kg m^{-3}) impacting into a target of density $\sim 2680 \text{ kg m}^{-3}$ (granite). Figures (a) and (d) depict the results of numerical simulations for a vertical (a) and 45° (d) impact, and are taken from Pierazzo and Melosh (2000a). Figures (b) and (c) are derived from the analytical model of shock-wave attenuation described in the text.

The most substantial difference between the two contour plots is near the target surface. The simple analytical approximation of Equation 2.33 does not take into account the presence of the free surface, at which the pressure must necessarily be

zero. The contour plot derived from the numerical simulations is, therefore, more accurate in this near-surface region. Furthermore, the shock-wave attenuation falls off more rapidly in Figure 2.14b compared to the approximately $1/r^2$ decay determined by Pierazzo and Melosh (2000a). A more accurate approximation to the numerical results of Pierazzo and Melosh (2000a) may be obtained by moving the centre of the contact zone to one impactor radius below the target surface (Fig. 2.14c). It can be shown, therefore, that Equation 2.33 is an adequate first order approximation of shock-wave attenuation following a high velocity impact. However, the exact rate of pressure decay with radial distance outside the contact zone may vary considerably between impact events with different conditions and impact parameters.

Fragmentation

Geologic evidence from terrestrial impact structures suggests that the target rocks are comprehensively fractured in a region surrounding the crater during the impact event (see section 1.1). There is a common misconception, however, that this fragmentation is a result of the compressive shock-wave crushing the target as it propagates away from the point of impact. The shock-wave is only strong enough to crush competent rock very close to the impact point. Instead, wholesale fracturing of the target is due to the release, or rarefaction wave (Melosh, 1989).

As illustrated in Figure 2.14a, the geometry of the expanding shock is almost spherical, centered on a point about one impactor radius beneath the surface. Thus, the expanding shock wave meets the free surface from beneath. Free surfaces, by their very definition, cannot support stress; the net pressure at the surface must be zero. Thus, when the shock-wave reaches the free surface a rarefaction wave equal in magnitude but opposite in sign to the shock-wave is generated. In a region close to the surface, conventionally termed the interference zone, these two waves interfere destructively, reducing the maximum shock pressure. At greater depths, however, the waves do not interfere; instead the target experiences a strong compressional shock pressure followed by a strong tensional rarefaction. The tensile strength of rock (typically 100s MPa, Melosh, 1989, p 72.) is significantly lower than the compressional strength. Thus, it is the tensional wave following the shock that causes fragmentation in large volumes of the target.

Numerical evidence (Asphaug and Melosh, 1993; Nolan et al., 1996) and experimental evidence (Fourney et al., 1984) supports the theoretical argument above; showing that tensile failure occurs in a region of the target proximal to the impact point. Nolan et al. (1996) concludes that for craters larger than a few tens of metres in diameter on large bodies such as Earth, tensile failure occurs within a volume of the target larger than the final crater volume. Thus, in most cases of planetary

interest, a high velocity impact event transforms the intact target rock into a volume of rock fragments, irrevocably altering the tensile and shear strength of the material.

Heating

The consequences of heating in an impact event are governed by the thermodynamics of shock compression and release. Shock compression is a thermodynamically irreversible process represented by the Hugoniot curve, which is defined as the locus of unique states to which a material can be shocked from a given initial state (see Figure 2.15). That is, each point on the Hugoniot curve is the single, unique result of a shock compression to the appropriate pressure (Melosh, 1989, p. 39). The shock front is discontinuous; there are no intermediate states between the shocked and unshocked states. A material's *equation of state* dictates the degree to which the shock can compress and heat the material. It relates changes in pressure to changes in the material properties of internal energy and density.

The release from a high pressure state can be approximated as an isentropic process (thermodynamically reversible). In this case, the material changes state gradually; as the pressure drops back to ambient pressure the specific volume increases. Thus, the shock pressures required for melting and vaporisation of any material are obtained by the intersection of the Hugoniot curve with the corresponding isentropes for melting and vaporisation. Table 2.1 lists the vaporisation and melting pressures for various materials of geologic interest.

Table 2.1: Table of shock pressures for melting and vaporisation.

Material	Incipient Melting Pressure (GPa)	Complete Melting Pressure (GPa)	Incipient Vaporisation Pressure (GPa)	Complete Vaporisation Pressure (GPa)
Water	-	-	4.5	43
Granite	46	56	140	1265
Dunite	135	149	186	2220

All pressures from ANEOS output (Pierazzo et al., 1998)

Vaporisation and melting of the target radically changes the rheologic properties of the target. Any vaporised target material and a fraction of the melt produced will be rapidly expelled from the growing cavity as ejecta and in an expanding vapour plume. This material is of little significance to the process of crater collapse. However, a proportion of the melt produced by the impact will remain within the crater volume and, hence, could influence collapse.

The behaviour of melted rock under applied stress can be approximated by a Bingham substance. As discussed in section 2.3, a Bingham substance, or Bingham fluid, is a fluid with a finite yield strength. The yield strength of molten rock (magma) is a function of the clast content; only completely molten rock behaves as

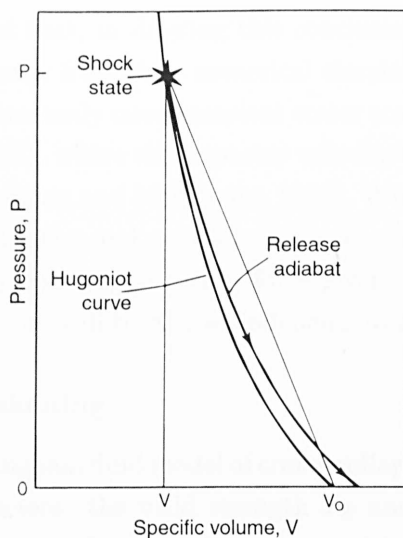


Figure 2.15: Shock compression and decompression. Consider an element of material with an initial specific volume V_0 and pressure $P = 0$. The passage of a shock wave instantaneously changes the pressure and specific volume of this element to P and V respectively (shock state). The relationship between P and V is defined by the Hugoniot curve. The release from high pressure is not instantaneous; the pressure and specific volume change gradually back to ambient. *After Melosh (1989).*

a liquid (yield strength is zero). However, the yield strength of sub-liquidus magma is significantly smaller than that for cold rock. Thus, if enough material is melted during an impact the target's resistance to deformation could be drastically reduced, greatly facilitating collapse of the transient crater.

Effect of Obliquity

The discussion thus far has described only the consequences of an impact perpendicular to the plane of the target. In planetary impacts, however, the most probable angle of impact is 45° to the target plane (Shoemaker, 1962). Recent work by Pierazzo and Melosh (1999, 2000a,b) has focused on the effect of obliquity on shock decay and melt production. They conclude that although an oblique impact generates strong asymmetries in the strength of the pressure contours, the directionally-averaged (or volume) pressure decay agrees with the form of Equation 2.33. For oblique impacts the shock is stronger in the down-range direction, and the isobaric core is displaced down-range. Figure 2.14d illustrates the effect of obliquity on the pressure contours for a 45° impact taken from Pierazzo and Melosh (2000a) (c.f. Figure 2.14a).

Pierazzo and Melosh (2000a) also discuss the effect of impact angle on melt production. They conclude that the volume of melt produced during an oblique impact decreases with impact angle relative to a perpendicular impact; however, the ratio of melt volume to the volume of the excavated transient crater is approximately con-

stant. It should be noted that, in drawing this conclusion, they do not determine the excavated crater volume from their numerical simulations. Instead they use a modified version of the commonly cited transient crater scaling law (Equation 2.6) of Schmidt and Housen (1987), where the impactor velocity is replaced by the vertical velocity component (Chapman and McKinnon, 1986). Whether this scaling relation holds for all impactor velocities and angles is unproven; however, the work of Pierazzo and Melosh (2000a) does suggest that, for a given transient crater diameter, the volume of melt produced will be almost independent of impact angle.

2.4.2 Material Weakening

The phenomenological Bingham fluid model of crater collapse described in section 2.3 is defined by two parameters: the yield strength Y_B and viscosity η . Thus, any potential physical explanation of crater collapse must: (1) explain how competent or incompetent rock can temporarily change its rheology; and (2) predict appropriate values for the yield strength and viscosity of the Bingham fluid.

A commonly used dimensionless strength parameter is: $Y/\rho g H_t$, where Y is the yield strength of the target, ρ is the target density, g is gravity and H_t is the depth of the transient crater. For a granular medium, whose yield strength is given by $Y = \mu p$ (Equation 2.12), this strength parameter is equivalent to the coefficient of friction, μ . The coefficient of friction is typically between 0.5 and 1 for most geological materials. Slip-line analysis and theoretical work applied to the collapse of impact craters (Melosh, 1977; McKinnon, 1978) suggests that the reduction in strength required to facilitate deep-seated collapse is equivalent to reducing the coefficient of friction to less than 0.05. Numerical modelling work by O’Keefe and Ahrens (1999), however, suggests that complex crater collapse occurs for targets with an effective coefficient of friction less than 0.15. The Bingham fluid model for complex crater collapse, therefore, requires that the yield strength is reduced by 1 to 2 orders of magnitude relative to the strength of standard rock materials.

Lowering the strength of a granular medium requires a reduction in the frictional resistance to slippage between grains. This can be achieved by reducing the coefficient of friction or relieving the overburden pressure. Theories for reducing the coefficient of friction between fragments rely on thermal changes induced by the impact process, including: shock heating, frictional melting in regions of faulting and strain localization (pseudotachylite formation), and thermal softening (O’Keefe and Ahrens, 1993, 1999). Generating a reduction in the overburden pressure is achieved by increasing the pore pressure, or via a peculiar mechanical model termed Acoustic Fluidization (Melosh, 1979). The premise of this model is that transient pressures in high-amplitude, high-frequency, random acoustic waves generated by the expand-

ing shock front temporarily relieve the overburden pressure, thus reducing frictional resistance between fragments within the granular medium.

Thermal Softening

Perhaps the most intuitive explanation for the change in rheology of the target is heat. Melted rock, or lava, possesses the correct material properties; it is a Bingham fluid. Moreover, there is direct geologic evidence for melting of the target in terrestrial complex craters and the collapse time-scale is limited by the cooling of a clast-rich impact melt (see section 2.3.4).

Numerical modelling has been used to predict how the amount of melt produced in an impact scales with crater size. Based on computations of melt production in a gabbroic anorthosite target (a rock simulating the lunar crust), O'Keefe and Ahrens (1982) derive a relationship between the mass of the impactor m_i and the mass of melt produced m_m via shock heating:

$$m_m = 0.14m_i \frac{v_i^2}{I_m}, \quad (2.54)$$

where I_m is the specific internal energy of melting. For gabbroic anorthosite $I_m = 3.4 \times 10^6 \text{ J kg}^{-1}$. Combining this with Equation 2.6 and Equation 2.7 for the diameter and volume of the transient crater gives an expression for the mass of melt relative to the mass of target material displaced in forming the transient crater:

$$\frac{m_m}{m_t} = 0.26 \frac{(gD)^{0.83} v_i^{0.33}}{I_m}. \quad (2.55)$$

Figure 2.16 shows a graph of mass of melt relative to the mass of displaced material for terrestrial craters greater than 1 km in diameter, formed by impacts at 15, 20, 35, 40 and 70 km s⁻¹.

The final morphology of a complex crater is much broader and shallower than the supposed dimensions of the transient crater. This suggests that a volume of rock roughly equivalent to the volume of the transient crater is involved in the collapse. In impact events where the amount of melt generated is comparable to the crater volume ($m_m/m_t \sim 1$), therefore, impact melt could easily facilitate collapse. Equation 2.55 and Figure 2.16 suggest that this volume of melt is produced at a transient crater diameter of approximately 200–300 km, for typical asteroidal impacts on Earth. If such a huge volume of melt is produced, however, cooling of the melt is unlikely to be as rapid as that for melt produced in smaller craters, due to the sheer volume of the melt and the reduced clast content. Consequently, collapse facilitated by melt production would not be arrested in the short time-scales predicted by geological observations on Earth (see section 2.3.4); melt fluidization could persist for months to years after the impact. Without the means to arrest

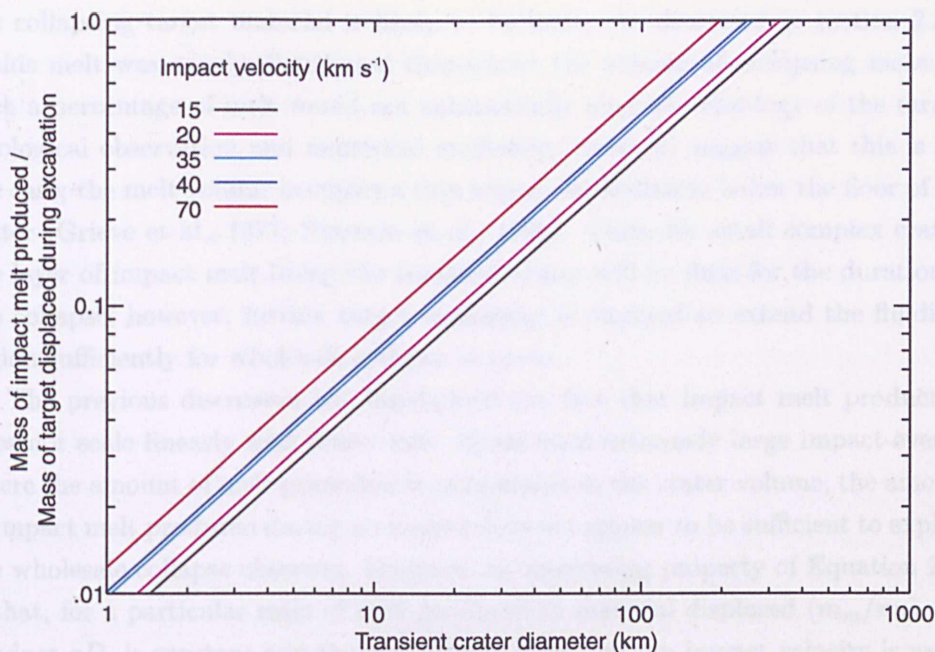


Figure 2.16: Graphical illustration of Equation 2.55 for transient crater diameters from 1-1000 km. The graph is for terrestrial gravity ($g = 9.81 \text{ m s}^{-2}$) and impact velocities of 15, 20, 35, 40 and 70 km s^{-1} .

the collapse none of the characteristic features of complex craters could be formed; crater collapse within a large volume of melt would probably produce a very shallow crater with a flat, featureless floor. Such a postulate is theoretical, however, as no craters of this scale have been observed on Earth.

In smaller craters ($D_t \sim 1\text{--}100 \text{ km}$, the size range observed on Earth) the volume of melt produced is only a small fraction of the volume of rock involved in the collapse process. For the largest transient crater size within these limits ($D_t \sim 100 \text{ km}$), which corresponds to a Chicxulub-scale event (Morgan et al., 1997), Equation 2.55 predicts a melt volume-fraction of about 0.4, for an impact velocity of 20 km s^{-1} . This is in close agreement with hydrocode modelling work by Pierazzo et al. (1998). Their work suggests that, for a Chicxulub-scale impact, the volume of melt produced is of the order of $4 \times 10^4 \text{ km}^3$. Assuming a paraboloid form for the transient crater at Chicxulub, with a radius of 50 km and a depth of 33 km, the volume of material displaced during excavation is $\sim 1.3 \times 10^5 \text{ km}^3$. Thus, the melt volume determined by Pierazzo et al. (1998) accounts for only 30% of the total volume of rock that must be fluidized to facilitate wholesale collapse of the Chicxulub crater. Furthermore, both these estimates include melt that remains in the collapsing cavity and that which is expelled during excavation. The actual percentage of melt that constitutes part of the collapsing material is likely to be around one half of the total melt produced (Melosh, 1989). Thus, for a 100-km diameter transient crater less than 20% of

the collapsing target material is likely to be melt. As discussed in section 2.3.4, if this melt was evenly distributed throughout the volume of collapsing material, such a percentage of melt would not substantially alter the rheology of the target. Geological observation and numerical modelling, however, suggest that this is not the case; the melt volume occupies a thin region, immediately below the floor of the crater (Grieve et al., 1977; Pierazzo et al., 1998). Thus, for small complex craters the layer of impact melt lining the transient crater will be fluid for the duration of the collapse; however, further target weakening is required to extend the fluidized region sufficiently for wholesale collapse to occur.

The previous discussion has highlighted the fact that impact melt production does not scale linearly with crater size. Apart from extremely large impact events, where the amount of melt generated is comparable to the crater volume, the amount of impact melt produced during an impact does not appear to be sufficient to explain the wholesale collapse observed. However, an interesting property of Equation 2.55 is that, for a particular ratio of melt produced to material displaced (m_m/m_t), the product gD_t is constant and the dependence of m_m/m_t on impact velocity is weak. The average asteroidal impact velocities for Mercury, Mars, Earth and the Moon, which are all composed of roughly the same material, are within a factor of 3. Thus, for impacts on the terrestrial planets and the Moon, a particular ratio m_m/m_t is reached at different transient crater diameters on different planetary bodies according to a $1/g$ relationship. The simple-to-complex transition diameter D_{coll} follows a $1/g$ relationship for Mercury, Mars, Earth and the Moon (see Fig. 2.5, section 2.3.1). Furthermore, at the simple-to-complex transition the transient crater diameter and final crater diameter are directly proportional. Thus, if m_m/m_t is the crucial parameter in determining the onset of complex crater collapse, it provides an explanation for the simple-to-complex transition. In addition, it explains why such a relationship would break down for icy satellites, which are made of material with a significantly different specific energy of melting.

O’Keefe and Ahrens (1993) propose a weakening mechanism that relies on temperature, without necessitating the melting of large volumes of the sub-crater region. This process of “thermal softening” is analogous to the effect of temperature on shear strength deep in the Earth, and the ductile behaviour of metals at high temperatures. The principal mechanisms driving thermal softening are shock heating and plastic work. O’Keefe and Ahrens (1999) use a thermal softening model which only applies for temperatures in excess of a certain fraction β of the melt temperature T_m . In this situation, the strength linearly approaches zero as temperature approaches the melt temperature:

$$Y = Y_0 \left(1 - \frac{T - \beta T_m}{T_m - \beta T_m} \right). \quad (2.56)$$

where Y and Y_0 are the degraded and standard yield strength respectively, T is the local temperature and T_m is the melt temperature of the target material. The yield strength is zero when the temperature is above the melt temperature; when the temperature is less than βT_m , the yield strength is not degraded, $Y = Y_0$.

O'Keefe and Ahrens (1999) assume $\beta = 0.8$, which implies that for strength degradation in excess of an order of magnitude the local temperature must be greater than $0.98T_m$. Recent work by Ivanov (2000), however, suggests that even for a Chicxulub-scale event, temperatures within the target are not extreme enough over a large enough volume for thermal softening to be the dominant collapse mechanism.

Shear Heating and Pseudotachylite Formation

Geological investigation of the largest terrestrial impact craters observable at outcrop (Vredefort, South Africa and Sudbury, Canada), have identified narrow zones of frictional melt ubiquitous within the impact structures. These zones, sometimes referred to as pseudotachylites, suggest a possible fluidization mechanism: as rock blocks slip across one another during the crater formation process, shear heating leads to thin melt zones which reduce the coefficient of friction between the blocks.

Spray and Thompson (1995) discuss friction melt as a weakening agent during the formation of multi-ring impact basins, based on observations from the Sudbury impact structure. They state that: "the majority of the Sudbury pseudotachylites possess matrices with micro-igneous textures, indicating that they cooled relatively rapidly from the molten state." Furthermore, Spray (1993) concludes that: "The compositions of the Sudbury pseudotachylites show that they would have possessed very low viscosities when molten," and that: "as clast-melt suspensions, pseudotachylites can undergo shear thinning during slip, resulting in an order of magnitude or more reduction in their viscosity." Evidence from a large terrestrial impact structure, therefore, suggests that the solidification time for a friction melt is short and the effective viscosity of the clast-melt suspension is extremely low ($\eta \sim 10^4$ Pa). Note that the term "clast" in clast-melt suspension refers to the small solid particles within the friction melt bands, not the rock blocks between which the melt is generated.

To facilitate complex-type crater collapse the reduction in the coefficient of friction must be greater than an order of magnitude, $Y/\rho g H_t < 0.1\mu$. Thus, if the resistance to slip of a thin layer of melt is given by $Y = 2\eta v_{slip}/h$, where η is the effective viscosity and v_{slip} is the slip velocity, then the melt thickness, h , required for wholesale collapse is given by:

$$h \approx \frac{20\eta v_{slip}}{\mu \rho g H_t}. \quad (2.57)$$

where, ρ is the target density and g is gravity. For a transient crater 40-km deep, a melt viscosity $\sim 10^4$ Pa s, and a slip velocity between blocks $v_{\text{slip}} \sim 100$ m s $^{-1}$, a minimum melt thickness of only 2 cm is required. Pseudotachylites of this and far greater thicknesses have been observed in abundance in many impact structures. Thus, it would appear that friction melting is a viable target weakening mechanism; the presence of thin melt zones can account for the necessary drop in yield strength. However, frictional melting cannot explain the macroscopic fluid-like behaviour of the target, because the viscosity of the block/friction-melt system is inappropriate for complex crater collapse.

The discussion of the rheology of a solid-fluid suspension in section 2.3.4 highlighted the importance of the volume fraction of solid particles in controlling the viscosity of the suspension. At volume fractions exceeding about 50% the effective viscosity of the suspension rises dramatically, prohibiting rapid flow rates, regardless of the viscosity of the suspending fluid (Marsh, 1981). Detailed mapping along a radial transect of the Sudbury impact structure suggests that pseudotachylites do not occupy more than 1-2% of the total volume (Spray and Thompson, 1995). Therefore, the viscosity of the block/friction-melt system is too high to permit complex crater collapse; the system cannot behave in a fluid manner over the time-scales required for complex crater formation.

There is also a question over what the observed thickness and extent of pseudotachylites demonstrates. Spray and Thompson (1995) use a simple energy balance model to equate the energy required to melt a certain thickness of rock via frictional heating during dry faulting at a depth of 5 km, with the work done in causing displacement along the fault. Such an analysis predicts that to generate a ~ 100 -m thick layer of melt, a displacement along the fault of about 1 km is required. This simple model, however, is flawed; such a rudimentary calculation does not take into account the fact that, after a displacement much smaller than 1 km, the effective coefficient of friction along the fault drops rapidly due to the production of melt. Furthermore, the lubricating melt has a viscosity that is a function of both stress and temperature, tending to reduce the resistance to slip further in regions of greatest stress (McBirney and Murase, 1984).

A more appropriate estimation of melt thickness can be made by assuming that the melt is a Newtonian fluid, and equating the energy generated by viscous dissipation to the energy required to melt the surrounding rock. The energy (per unit area) required to melt a thickness of rock dh is: $L\rho dh$, where L is the latent heat of fusion (about 4×10^4 J kg $^{-1}$ for most geologic materials) and ρ is density. The energy dissipated (per unit area) during viscous flow of constant driving velocity v is given as: σdx , where dx is the incremental displacement and $\sigma = 2\eta v/h$ is the viscous shear stress. Assuming that all the energy dissipated is converted to heat

these equations can be equated, rearranged and integrated to give an estimate for the thickness of the melt produced:

$$h = h_0 + 2\sqrt{\frac{\eta v d}{\rho L}}, \quad (2.58)$$

where h_0 is the initial thickness of the melt layer and d is the total displacement of the fault. Note that this equation ignores the temperature- and stress-dependence of the viscosity η , which will lower the melt thickness. Equation 2.58 is, therefore, an upper limit for the melt thickness.

Equation 2.58, shows that melt thickness is not a linear function of fault displacement, as suggested by Spray and Thompson (1995), but a function of the square root of the fault displacement times the velocity of the slip. Thus, to generate a melt layer 100 m thick along a fault ~ 1 km, where a negligible thickness of melt has already been generated ($h_0 \sim 0$), the slip must occur at a velocity of 30 km s^{-1} , which is clearly implausible. In fact, if the collapsing material behaves as a continuum the actual relative displacement of adjacent blocks must be small, and the slip velocity must be less than the collapse velocity, which is on the order a few 100 s m s^{-1} for a Sudbury-scale impact. Displacements of 6 km have been observed along normal faults between slump blocks in the rims of lunar craters (Melosh, 1989), thus in certain parts of the crater where strain is localized the generation of thick friction melt zones would be expected. However, the pervasive nature of the large Sudbury and Vredefort pseudotachylites is somewhat counter-intuitive. Perhaps, these thick melt bands are regions where friction melt has collected after the cessation of the collapse, forming thicker and fewer pseudotachylites than areas of frictional melting.

Frictional heating is an important process during collapse but it cannot be the sole explanation for complex crater collapse. Frictional sliding between blocks during the early, energetic phases of an impact event generates friction melt, which lowers the effective yield strength of the medium, thus facilitating collapse. In regions of strain localization, melt layers 1-10 metres thick could be generated, lubricating the fault and expediting large displacements. Furthermore, rapid cooling rates, due to the presence of cold rock blocks and the composition of the melt, will arrest the collapse in the appropriate time-scale for complex crater formation. However, from a macroscopic point of view, friction melting does not provide an explanation for the low viscosities required for complex crater collapse.

Fluidization due to the presence of interstitial fluid

Explosion craters on Earth are strongly influenced by the presence of subsurface fluids, namely water. Water saturated silts and clays behave like Bingham fluids and thus it is no surprise that craters produced in such materials on the Earth

resemble extraterrestrial complex craters. The exaggerated development of central peaks observed in many Martian craters may also be due to the presence of near surface water (Melosh, 1989, p. 151). Consequently, models have been proposed for complex crater collapse involving fluidization due to the presence of a fluid. However, fundamental problems exist with this theory: (1) water-mediated fluidization cannot be applicable on the Moon, Mercury, or Venus as there is no current evidence for significant quantities of subsurface fluid; (2) while the presence of pore water has a profound influence on small scale craters formed in saturated materials on Earth, the target material in large terrestrial complex craters is predominantly crystalline rock which has little initial porosity; (3) when the target is fractured by the shock wave any pore pressure will be greatly reduced preventing any vestige of fluidization by water.

Acoustic Fluidization

The final physical mechanism with the potential to explain the temporary change in rheology is known as Acoustic Fluidization (Melosh, 1979). The physical basis for this model is that acoustic vibrations within a granular material, if violent enough, could temporarily relieve the overburden pressure and, hence, abrogate the internal frictional strength of the medium. The theory, therefore, predicts a rheology where stress is supported without appreciable strain unless acoustic vibrations in the medium are strong enough to reduce the material's resistance to shear and let it flow as though it were a fluid.

The acoustic fluidization model for temporary target-weakening during an impact relies on violent but transient pressure fluctuations – seismic, or acoustic, noise – which, for a portion of an oscillation, counteract the overburden pressure. As alluded to earlier, a fraction of the energy transferred from the impactor to the target is released as seismic energy. Despite representing only a small portion of the initial kinetic energy of the impactor, the amount of seismic energy released during a large impact is extremely significant. For a Chicxulub-scale event the seismic energy released would register magnitude 10 on the Richter scale – more violent than any recorded earthquake. Of course, the majority of this seismic energy does not remain in the proximity of the impact; the seismic energy is released during the expansion of the shock wave which soon propagates far away from the point of impact. However, as the shock wave expands away from the point of impact, heterogeneities in the elastic moduli of the target generate a random seismic energy field in its wake, with an amplitude that comfortably exceeds the overburden pressure within one crater diameter from the crater center (Gaffney and Melosh, 1982). Figure 2.17 illustrates the development of random noise behind an initially sharp acoustic wave.

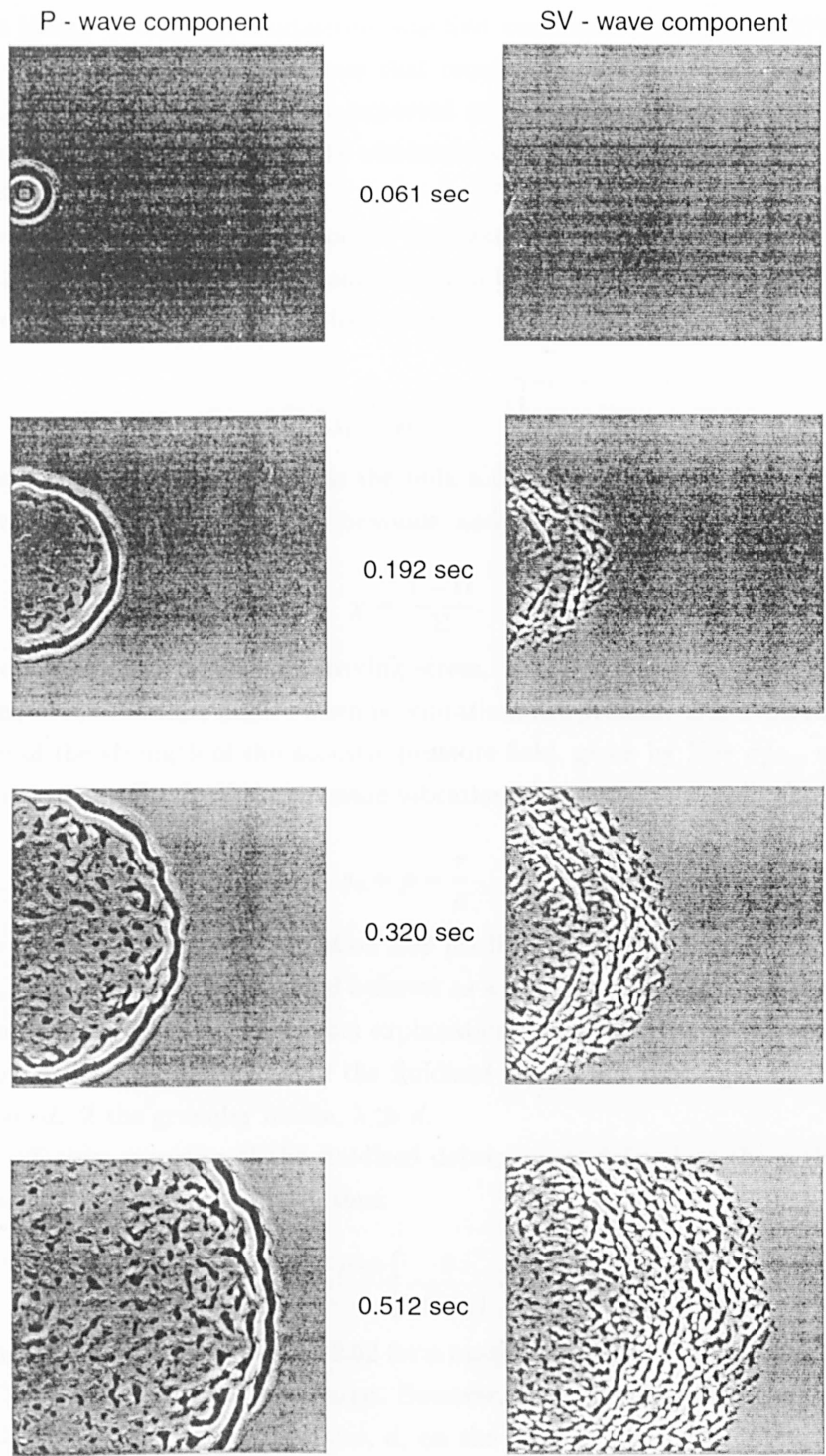


Figure 2.17: The generation of random acoustic energy from the propagation of a strong pulse in a randomly scattering medium. The pulse, originally a pure P-wave, quickly develops S-wave components by mode conversion in addition to generating a random wave field in the wake of the initial pulse. This illustrates how random acoustic energy, required for acoustic fluidization, may arise. *From a numerical experiment by Frankel and Clayton (1986); taken from Melosh (1989).*

The theory of acoustic fluidization was first conceived by Melosh (1979) to explain a number of geologic processes that require the failure of rock debris under driving stresses much smaller than expected on the basis of conventional rock mechanics. These processes include the wholesale collapse of large craters, long run-out landslides, and seismic faulting.

The central equation of the acoustic fluidization model, which is derived in Appendix A.1, describes the relationship between the driving stress, τ and flow rate, $\dot{\epsilon}$, of a volume of acoustically fluidized debris:

$$\dot{\epsilon} \approx \frac{\tau}{\rho \lambda c} \left[\frac{2}{\text{erfc}(\chi)} - 1 \right]^{-1}, \quad (2.59)$$

where ρ is the bulk density and c is the bulk sound speed of the granular debris, λ is the wavelength of the acoustic vibrations, and

$$\chi = \frac{1 - \Omega}{\Sigma}. \quad (2.60)$$

Ω is a dimensionless measure of driving stress, τ/τ_{static} , where $\tau_{static} = \mu p$ is the stress required to initiate failure when no vibrations are present. Σ is a dimensionless measure of the strength of the acoustic pressure field, given by $\Sigma = \sigma/s_c$, where s_c is the critical amplitude of the pressure vibrations necessary to permit slippage:

$$s_c = p - \frac{\tau}{\mu}. \quad (2.61)$$

The flow law governed by Equation 2.59 predicts that, from a macroscopic point of view, a mass of granular material behaves as a fluid when internal pressure vibrations are present. For this continuum explanation to be valid the wavelength of the vibrations λ must be shorter than the fluidized region but much greater than the grain size, d , of the granular media; $\lambda \gg d$.

The effective viscosity of the fluidized debris η_{eff} is defined as the ratio of the shear stress to the net strain rate, thus:

$$\eta_{\text{eff}} \approx \frac{\rho \lambda c}{2} \left[\frac{2}{\text{erfc}(\chi)} - 1 \right]. \quad (2.62)$$

Figure 2.18 illustrates equation 2.62 for a range of values of Σ . In a weak acoustic field ($\Sigma \ll 1$) the viscosity is very large. However, when $\Sigma \approx 1$ the viscosity is of the order $\rho \lambda c$. Assuming a fragment size, d , on the scale of metres, and recalling the fact that $\lambda \gg d$, then the predicted viscosity is of the order of 10^8 Pa s . According to Equation 2.20, a viscosity of 10^8 Pa s would allow wholesale collapse of all transient craters larger than 2 km in diameter on Earth.

The energy balance equation for a unit volume of acoustically fluidized material is presented in Melosh (1996). The rate of change of acoustic energy in the control

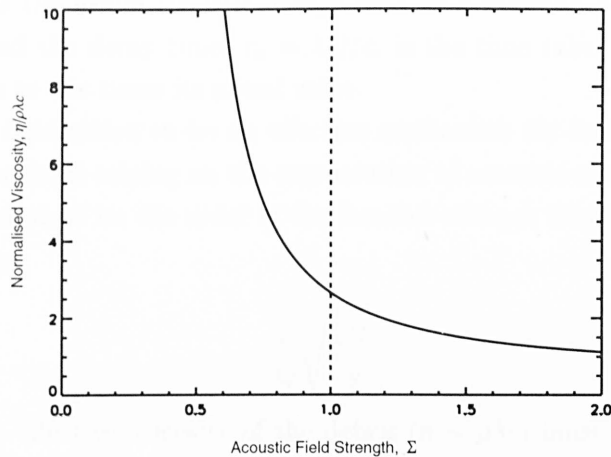


Figure 2.18: Viscosity of acoustically fluidized debris. Graphical representation of Equation 2.62. Normalised viscosity is plotted against the acoustic field strength.

volume $d\mathcal{E}/dt$ is influenced by the processes of scattering into and out of the control volume, absorption as heat and the regeneration of acoustic energy during flow. The full equation linking these phenomena is:

$$\frac{d\mathcal{E}}{dt} = \frac{\xi}{4} \nabla^2 \mathcal{E} - \frac{c}{\lambda Q} \mathcal{E} + e \frac{\dot{\epsilon} \tau}{2}. \quad (2.63)$$

The first term on the right-hand side is the scattering term, where ξ is the scattering diffusivity, which has dimensions of a length times a velocity. The second term parameterizes conversion of elastic energy into heat. Q is defined as the ratio of the energy stored per cycle to the energy dissipated in the same period. The last term quantifies the energy generation during flow, where e is the fraction of the released acoustic energy that is available to regenerate the vibrations. Thus, a potentially important aspect of this full model is that energy dissipated by shear in the flowing debris could itself generate acoustic energy. In static regions the vibrations might dissipate quickly by friction and the effective viscosity in those regions would rapidly rise; however, in flowing regions the regeneration of acoustic energy could permit flow to continue until the driving stresses are relieved.

Direct observation of acoustic noise in the ground surrounding a series of 10-ton TNT explosion tests suggests that in small explosion events the dominant term in Equation 2.63 is the internal dissipation term (Gaffney and Melosh, 1982). Thus, the acoustic energy within the fluidized region decays approximately exponentially, described by the equation:

$$\mathcal{E}(x, \lambda, t) = \mathcal{E}_0(x, \lambda) e^{-\frac{t}{\tau_v}}, \quad (2.64)$$

where $\mathcal{E}_0(x, \lambda)$ is the initial acoustic energy field as a function of position x and wavelength λ , and the decay time, $\tau_v = \lambda Q/c$, is the time taken for the acoustic energy to decline to $1/e$ times its initial value.

For acoustic fluidization to be an effective mechanism for facilitating complex crater collapse, without relying on the regeneration of acoustic energy during flow, the decay time must be on the order of the free-fall collapse time discussed in section 2.3.4, giving:

$$\lambda \approx \frac{c}{Q} \sqrt{\frac{2H_t}{g}}. \quad (2.65)$$

Furthermore, the effective viscosity of the debris ($\eta \sim \rho\lambda c$) must be less than the critical viscosity predicted by Equation 2.20, giving:

$$\lambda \lesssim \frac{gH_t}{2c} \sqrt{\frac{2H_t}{g}}, \quad (2.66)$$

implying that:

$$H_t \gtrsim 2c^2/gQ, \quad (2.67)$$

which suggests that if Q is constant then the depth of the smallest crater for which acoustic fluidization could facilitate collapse is inversely proportional to gravity g . Thus, not only does the theory of acoustic fluidization explain the change in rheology of unconsolidated rock fragments to that of a Bingham fluid and predict appropriate viscosities for the fluidized debris; but it may also explain why the onset of complex crater collapse appears to occur at different crater diameters on different planetary bodies according to a $1/g$ relationship.

A more straightforward analysis of the effect of acoustic vibrations on a mass of rock debris is provided by the block model (Ivanov and Kostuchenko, 1997). The premise of the block model is that the region of the target fractured during an impact comprises a system of large, disconnected rock fragments, surrounded by a matrix of smaller blocks. Acoustic vibrations remnant in the target following the passage of the shock wave force the larger blocks to oscillate about their reference position, modifying the stresses acting along and across the frictional contact between the block and its surrounding matrix. The model is inspired by several sets of field observations and drill cores at terrestrial impact structures, where this situation of large blocks surrounded by smaller matrix material occurs (Ivanov and Kostuchenko, 1997). Melosh and Ivanov (1999) show that the flow law predicted by the block model is very similar to that described by the full theory of acoustic fluidization discussed above. The major advantage that the block model provides stems from

its simplicity; solving the block-model equations is far more straightforward than solving the highly non-linear equations of the full acoustic fluidization model.

The one-dimensional mathematical analysis of the block model presented in Melosh and Ivanov (1999) is discussed in Appendix A.2. The fundamental equation describing the flow of an acoustically fluidized mass of rock debris, according to the block model, is:

$$\dot{\epsilon} = \frac{(\tau - Y_B)T}{2\pi^2\rho h^2}\Phi. \quad (2.68)$$

where τ is the driving stress, T is the period of the block oscillation, h is the block dimension, ρ is the density of the block-matrix system and Y_B is a kind of Bingham yield strength:

$$Y_B = \mu(p - p_v), \quad (2.69)$$

where μ is the coefficient of friction, p is the overburden pressure and p_v is the amplitude of the pressure vibration caused by the oscillating block. Φ is given by:

$$\Phi = \cos^{-1}\chi \left[\sqrt{\frac{1+\chi}{1-\chi}} - \frac{\chi}{1-\chi} \cos^{-1}\chi \right]. \quad (2.70)$$

where χ is defined as before ($\chi = (1 - \Omega/\Sigma)$), but in this case Σ is given by p_v/p .

An interesting property of Equation 2.68 is that it contains a true Bingham threshold (Melosh and Ivanov, 1999). Unlike the full acoustic fluidization model, the block model predicts that flow does not begin until the driving stress τ exceeds Y_B . This fact may provide an alternative physical explanation for the simple to complex transition, because Y_B is a function of the amplitude of the pressure vibrations (see Equation 2.69). The complex collapse threshold could be related to the size of the impact event required for the amplitude of the remnant pressure vibrations to lower the Bingham yield strength enough for wholesale collapse to occur.

The effective viscosity for the fluidized debris in the block model is again defined as the ratio of the stress to strain rate:

$$\eta_{\text{eff}} = \frac{\pi^2\rho h^2}{T\Phi}. \quad (2.71)$$

In a strong acoustic pressure field, $\Sigma \sim 1$, the Bingham yield strength is negligible and $\Phi \sim \pi/2$. In this case, the effective viscosity for the debris is simply:

$$\eta_{\text{eff}} = \frac{2\pi\rho h^2}{T}, \quad (2.72)$$

which is functionally equivalent to the effective viscosity predicted in the full acoustic fluidization model, if it is assumed that the wavelength of the acoustic pressure vibrations λ is proportional to the block size h .

2.5 Summary

Impact crater collapse is an important geological process. It is the final stage in the formation of an impact crater and has the most profound influence on the morphology of the final crater. The stages prior to collapse result in the formation of a bowl-shaped cavity known as the transient crater. The displacement of the target during the formation of this cavity may be calculated approximately by the Z-model approximation of the excavation flow.

The collapse of large impact craters is driven by gravity and resisted by the strength of the target. However, the strength of standard rock material, both competent and completely fractured, is sufficient to prevent all but the steepest part of the transient crater rim from collapsing. Therefore, complex crater collapse must involve significant weakening of a region of the target surrounding the crater.

The size of the smallest impact crater for which collapse results in a complex-style crater follows a first-order correlation between different planetary bodies of $1/g$. This suggests that the simple-to-complex transition corresponds to a cohesive strength threshold. The shallowness of complex craters and the occurrence of terraced rim-walls precludes a simple, debris-sliding explanation for complex crater collapse. Furthermore, the formation of central peaks or rings requires not only weakening of the target, but also temporary fluidization.

A general paradigm for complex crater collapse has been developed: the transient crater collapses to form a complex crater by means of inward slumping of the transient crater rim, accompanied by the uplift of the floor of the crater. The collapse is facilitated by fluidization of the target, which ceases within a few minutes of the impact, effectively freezing the collapse to produce the final, pristine crater morphology.

Crater collapse is a highly dynamic process, involving the rapid transport of extremely large volumes of rock; quite unlike almost all other, more familiar geologic processes. Over very long time-scales, the shape of an impact crater is modified further by more conventional geologic processes: viscous relaxation and other surface processes. Thus, much of the impact crater population in the Solar System, and almost the entire terrestrial crater population, does not reflect the pristine form of a complex crater. The extremely dynamic nature of the terrestrial surface has hindered the development of a definitive explanation of complex crater collapse. An understanding of long time-scale modification is imperative, therefore, when comparing the results of complex crater collapse simulations with geological observation.

The nature of the transient fluidization mechanism still remains equivocal. However, it is widely accepted that the passage of the expanding shock wave irrevocably alters the state of the target rocks. In the wake of the expanding shock front, the target is heated, comprehensively fractured and violently shaken. As a consequence,

several mechanisms could modify the material strength and rheology of the target in the few seconds immediately after a high velocity impact. Of these the two theories to attract the most attention focus on thermal weakening and mechanical weakening via the presence of transient pressure vibrations in the target. From a theoretical point of view, the latter mechanism, termed acoustic fluidization, appears to offer the most robust explanation for temporary fluidization of the target surrounding an impact crater.

The focus of this study is to develop a dynamic model to describe complex crater collapse, and to investigate the potential mechanism by which temporary fluidization may occur. A logical starting point, therefore, is to attempt to simulate the collapse of a large impact crater, as mediated by the acoustic fluidization model of temporary target weakening.

Chapter 3

Modelling Complex Crater Collapse

Things should be made as simple as possible, but not any simpler.

Albert Einstein

The current understanding of impact cratering as a whole has come from a suite of experimental, morphological, analytical and numerical investigations. However, the study of complex crater collapse has relied on far less diverse resources. There has been no direct observation of complex crater collapse in recorded history; large impact events are, perhaps fortunately, too infrequent. In addition, the scale of experimental studies is somewhat inappropriate for drawing conclusions about the collapse of the largest craters in the Solar System. The dominance of gravity in influencing the collapse stage of crater formation implies that the results of the small-scale laboratory collapse experiments cannot be extrapolated meaningfully to the scale of complex craters. Similarly, underground nuclear explosions, although extremely valuable in elucidating the principal features of the excavation stage, are also not really of an applicable scale. Consequently, abstract computer simulations provide one of the only feasible methods for studying complex crater collapse.

One commonly used computational tool for modelling crater formation is the “hydrocode”. A hydrocode, in its purest sense, is a computer code for modelling fluid flow at all speeds. It can, however, be adapted to treat material strength and a range of rheological models for material behaviour. In essence, the code considers the effect of external and internal forces on a predefined “mesh” of cells during a short interval of time. It then adjusts the mesh accordingly and recalculates the forces. This process continues in an iterative fashion for the entire calculation.

The approach adopted in this thesis has been to use such a hydrocode to investigate complex crater collapse. The hydrocode used in this work is SALES₂, a

heavily modified version of the SALE hydrocode (Amsden et al., 1980). SALES.2 has been adapted for the specific purpose of modelling impact crater formation and is freely available for download. Appendix B provides further information on acquiring SALES.2 and full source-code listing of the subroutines modified for the purpose of modelling complex crater collapse.

This chapter has been written to equip the reader with enough insight to fully understand the details and limitations of using a hydrocode to simulate large crater collapse; and to allow the interested reader to reproduce the results described in subsequent chapters. The first section of the chapter discusses the methods for describing continuous media and the use of hydrocodes to model fluid flow. In doing so it describes the fundamental aspects of the original SALE hydrocode that remain in the solution algorithm of the hydrocode used in this work. The second section describes the improvements made to the original SALE equation-of-state and constitutive model, which allow SALES.2 to model impact crater formation. These modifications, which are primarily the result of work by H. J. Melosh and his co-workers, allow for phase changes in the target, the inclusion of more than one material and the application of several rheological models for the target, including that of a Bingham fluid. The third section discusses the more specific issues relating to complex crater collapse. This section will describe modifications made to SALES.2 by the author to facilitate the study of complex crater collapse. Particular areas that will be addressed more thoroughly are the method employed for treating material strength and the algorithm for the inclusion of the predicted effects of acoustic fluidization.

3.1 Modelling Fluid Flows: An Overview

The modelling of fluid flow is the subject of an excellent review by Anderson (1987), in which he states:

The dynamics of a continuous media may be described by a set of differential equations established through the principles of conservation of momentum, mass and energy from a macroscopic point of view.

These differential equations, sometimes called the Navier-Stokes equations, are of the form:

$$\text{Conservation of Momentum} \quad \frac{Dv_i}{Dt} = f_i + \frac{1}{\rho} \frac{\partial \sigma_{ji}}{\partial x_j} \quad (3.1)$$

$$\text{Conservation of Mass} \quad \frac{D\rho}{Dt} + \rho \frac{\partial v_i}{\partial x_i} = 0 \quad (3.2)$$

$$\text{Conservation of Energy} \quad \frac{DI}{Dt} = -\frac{p}{\rho} \frac{\partial v_i}{\partial x_i} + \frac{1}{\rho} \Pi_{ij} \dot{\epsilon}'_{ij} \quad (3.3)$$

where ρ is the material density, v_i is the velocity, I is the specific internal energy, σ_{ij} is the stress tensor, which is composed of a hydrostatic part, the pressure p , and a deviatoric part, Π_{ij} . f_i is the external body forces per unit mass, and $\dot{\epsilon}'_{ij}$ is the deviatoric strainrate. The subscripts represent the standard tensorial notation, and summation is implied by repeated indices.

Two more expressions are required to complete the set of equations; an *equation of state* and a *constitutive model*:

$$\text{Equation of State} \quad p = p(\rho, I) \quad (3.4)$$

$$\text{Constitutive Model} \quad \sigma_{ij} = g(\epsilon_{ij}, \dot{\epsilon}_{ij}, I, D) \quad (3.5)$$

The equation of state accounts for compressibility effects, that is, changes in density and irreversible thermodynamic processes such as shock heating. The constitutive model, relates the stress to a combination of strain ϵ_{ij} , strain rate effects $\dot{\epsilon}_{ij}$, internal energy I , and damage D .

Analytical solutions to Equations 3.1–3.5 above are only obtainable for circumstances where certain simplifying assumptions may be invoked, reducing the number of variables to be considered (Anderson, 1987). In cases of practical interest, where the variables are numerous and the problem is complex, the equations must be solved simultaneously. Numerical techniques, using digital computers, provide the only amenable method to achieve the number of mathematical operations required for the solution.

A computer has a finite memory allocation and, therefore, can only represent a continuous media by dividing it into distinct pieces; a process known as discretisation. Thus, the approach adopted by a computer code for modelling hydrodynamic behaviour (hydrocode) is to define a “mesh” to approximate the geometry of interest, which is divided into manageable divisions, known as cells. Scalar quantities within the continuum are assigned to each cell and are constant within the cell. The cell is usually defined by three or four vertices connected by straight lines to form a triangle or quadrilateral respectively. Vector quantities like velocity are assigned to each vertex.

The differential equations that describe the kinematic deformation of a continuum can, likewise, only be approximated by operations on a computer; these are known as finite difference approximations. At the beginning of the calculation initial values are assigned for each vertex and cell quantity and the spatial derivatives are determined using these finite difference approximations. The solution is then advanced in time by assuming that during a very short interval of time, termed the time step, all quantities in the problem remain constant. The values of the various vector and scalar quantities at the end of the time step can then be determined from the values at the beginning of the time step; a process known as explicit time integration. For example, the change in velocity Δu of a certain vertex during one time step dt , due to an external acceleration a , is given as: $\Delta u = a dt$. At the next time step the time is advanced, the old scalar and vertex quantities are replaced by the new ones and the cycle repeats. Provided the finite difference approximations are valid and the time-step size is appropriate for the problem being studied, the hydrocode simulation will record all the variables of interest for the entire calculation. Thus, as Anderson (1987) points out, “in one sense, hydrocodes are the best instrumented experiments.”

3.1.1 Lagrangian and Eulerian Descriptions

The conservation equations (3.1–3.3) are written with respect to the material; they describe the motion (displacement, velocity and acceleration) from the reference frame of the continuous media itself. This type of description is known as a Lagrangian (or material) description. The equations may also be written down with respect to a fixed reference frame; a formulation termed an Eulerian (or spatial) description.

Intuitively, the two descriptions may be understood by the analogue of an observer watching the flow of a stream, who decides to add some dye to the water to monitor the flow of particles. If the observer follows the path of the dyed particles in the water, the Lagrangian description applies; if the observer remains at a certain point along the river bank and notes which particles pass with time, the Eulerian description applies. From a mathematical point of view the differences are inherent in how the time derivatives are calculated. In the Lagrangian formulation the *material*, or *total time*, derivative, D/Dt , is simply the time derivative of the quantity at a point in the material. In the Eulerian formulation, however, the time derivative of the same quantity is calculated at a point in space, $\partial/\partial t$, known as the *local* derivative. Therefore, the Eulerian formulation must also keep track of

the spatial derivative of that quantity, $v_i \partial / \partial x_i$, also referred to as the convective derivative. The two descriptions are equivalent, giving:

$$D/Dt = \partial/\partial t + v_i \partial/\partial x_i. \quad (3.6)$$

A hydrocode may employ either type of formulation to describe the situation of interest. To obtain a Lagrangian description the computational mesh is defined to represent the geometry of the problem; points within the mesh (vertices) are attached to the material and move with the material. As time progresses, all the variables of interest are computed for each discrete point in the continuum. Cells defined by adjacent vertices become deformed in shape and size due to the forces acting on them and the constitutive relations between force and deformation. From a mathematical point of view the vertices follow the *particle paths* of the material. Mass, momentum and energy are transported by material flow. Mass within a cell is invariant; changes in density are exclusively due to changes in a cell's volume.

In contrast, the Eulerian description relies on material flowing *through* the mesh. In this case the mesh defines the entire space of interest. As time progresses, the variables of interest are calculated at the fixed points of the grid. Thus, mass, momentum and energy must flow across cell boundaries. The amount of flow between cells is used to compute the new variables within each cell. In this formulation it is the volume of the cell that is invariant and changes in density are due to changes in the mass of a particular cell.

The choice of either mode of description depends on the problem under consideration. To illustrate the differences between Lagrangian and Eulerian material descriptions, as well as the inherent advantages and disadvantages of each, an example inspired by lecture notes from a short course on Impact Modelling by H. J. Melosh is presented. Consider the case of a dinosaur plunging to its demise from a "K/T diving board" Figure 3.1. Either the Lagrangian or the Eulerian description may be employed to model this situation with a computer.

In the Lagrangian formulation (Figure 3.2a) the dinosaur is divided into cells, which should be as close as possible to regular quadrilaterals to ensure accuracy from the finite difference approximations used. The cells that make up the dinosaur experience external stresses (gravity) and internal stresses (via the constitutive model) that translate them and deform them. As a consequence, the dinosaur falls, lands and is crushed. The surface of the dinosaur is defined by the edges of the cells, thus the resolution of the surface is a function of the number and size of the cells used. The more cells the better resolved the free-surface. Furthermore, if the problem involves multiple materials, the boundaries between different materials can be accurately defined by cell boundaries. A significant advantage of the Lagrangian description is its ability to follow the history of the material within any cell. Thus,

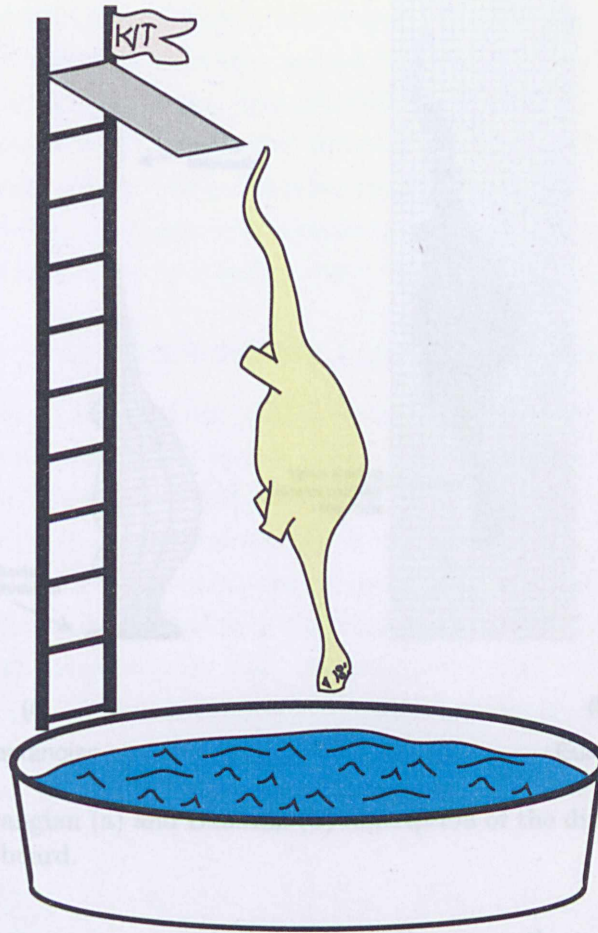


Figure 3.1: Illustration of the a dinosaur diving from the K/T diving board

a material whose properties depend upon its previous history can be modelled; for example, strain hardening or plastic work. Furthermore, it is straightforward to model time-dependent phenomena not included in the solution algorithm implicitly; for example, the temporal decay of acoustic vibrations.

The major weakness with the Lagrangian description comes with extreme deformation. As has just been discussed, the cell quantities of volume, pressure, and so on, are approximated by finite difference relations. These expressions become inaccurate when the form of the cell deviates significantly from a regular quadrilateral. Thus, in cells where deformation is severe (for example in the head of the dinosaur) the calculation will at best produce spurious results and at worst become unstable and stop.

In the Eulerian formulation (Figure 3.2b) the mesh must define the entire space in which the dinosaur's movement is contained. During the calculation the dinosaur will occupy certain cells completely, other cells partially and other cells not at all. This is illustrated by the shading in Figure 3.2b; cells containing a greater con-

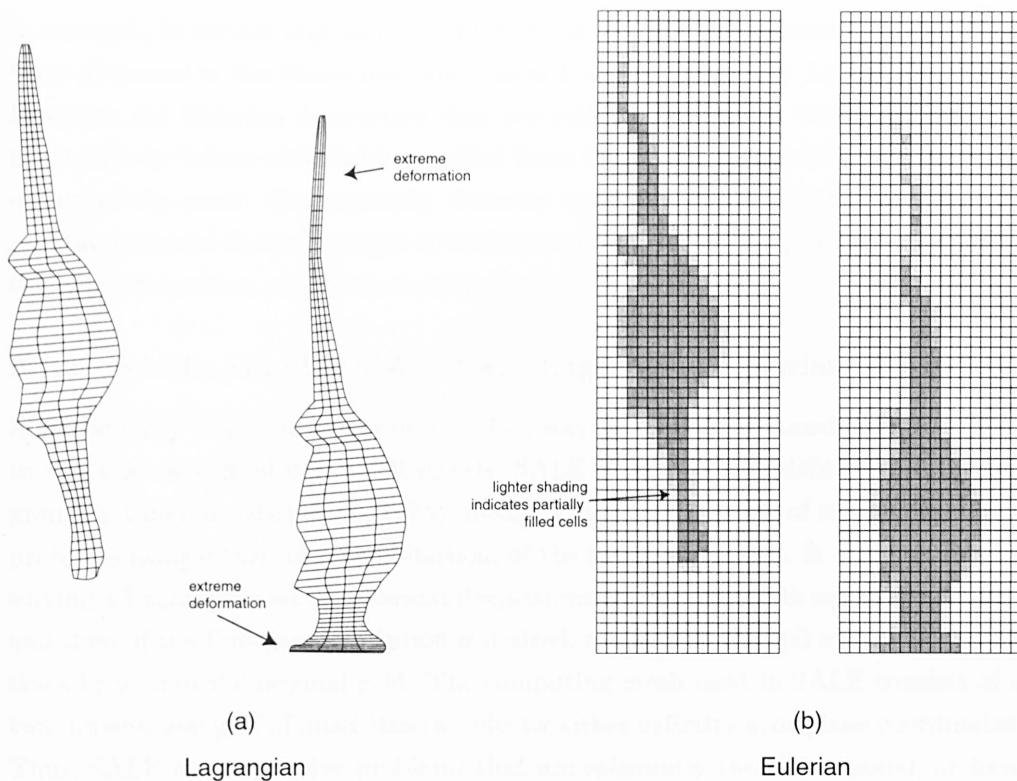


Figure 3.2: Lagrangian (a) and Eulerian (b) description of the dinosaur diving from the K/T diving board.

centration of dinosaur mass are shaded darker. Consequently, to achieve the same spatial resolution over the dinosaur as the Lagrangian description, the number of cells required is significantly greater. Another disadvantage of the Eulerian description is that as the dinosaur moves through the mesh it will only partially occupy some cells; a problem exists of how to represent cells that are a percentage dinosaur and a percentage space. Hence, unlike the Lagrangian description, where the free surface is precisely defined, the free surface is not tracked exactly in the Eulerian description. In other words, the ability to model the free surface in the Eulerian description is a much more sensitive function of the resolution of the mesh than in the Lagrangian description. Furthermore, some cell quantities like viscosity pose a problem in the Eulerian description. Representing a cell that is partially space and partially dinosaur in terms of mass is relatively straight forward. However, defining this cell's viscosity is less intuitive.

When considering just the collapse stage of impact crater formation, where deformation is relatively inextreme and no material is ejected, a purely Lagrangian approach is preferable because (1) it accurately tracks the free surface with a feasible mesh resolution; and (2) it monitors the history of each point and cell in the mesh, facilitating the study of temporal changes to target properties, such as changes

in strength, in certain regions of the problem space. For this reasoning, the calculations discussed in this thesis have been carried out using a purely Lagrangian mode. However, the Eulerian description does not suffer in situations where deformation is extreme or where material is expelled from the problem space, due to the fixed nature of the mesh. Consequently, Eulerian codes are (currently) the only way to accurately model the early stages of the impact crater formation process, or to model the whole formation process in a unified way.

3.1.2 SALE: Simplified Arbitrary Lagrangian Eulerian Hydrocode

A particularly tractable hydrocode, SALE, was developed by Amsden et al. (1980) to study inviscid fluid flow at all speeds. SALE stands for Simplified Arbitrary Lagrangian Eulerian (Hirt et al., 1974); meaning that it is capable of solving fluid-flow problems using either, or a combination, of the two descriptions. It achieves this by solving a Lagrangian set of differential equations corresponding to equations 3.1-3.3, and then, if the Eulerian description is desired, remapping the cell and vertex quantities back onto the original grid. The computing mesh used in SALE consists of a two-dimensional grid of quadrilateral cells for either cylindrical or plane coordinates. Thus, SALE can only solve problems that are inherently two-dimensional, or have a symmetry about an axis. The equations implemented in SALE are:

$$\frac{\partial \rho u}{\partial t} = -\frac{\partial(p+q)}{\partial x} + \frac{1}{r} \frac{\partial r \Pi_{xx}}{\partial x} + \frac{\partial \Pi_{xy}}{\partial y} - \frac{\Pi_{\theta}}{r} + \rho g_x \quad (3.7)$$

$$\frac{\partial \rho v}{\partial t} = -\frac{\partial(p+q)}{\partial y} + \frac{1}{r} \frac{\partial r \Pi_{xy}}{\partial x} + \frac{\partial \Pi_{yy}}{\partial y} + \rho g_y \quad (3.8)$$

and the mass and internal energy equations:

$$\frac{\partial \rho}{\partial t} + \rho \left(\frac{1}{r} \frac{\partial r u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0, \quad (3.9)$$

and

$$\frac{\partial \rho I}{\partial t} = -(p+q) \left(\frac{1}{r} \frac{\partial r u}{\partial x} + \frac{\partial v}{\partial y} \right) + \Pi_{xx} \frac{\partial u}{\partial x} + \Pi_{xy} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) + \Pi_{yy} \frac{\partial v}{\partial y} + \frac{u \Pi_{\theta}}{r}, \quad (3.10)$$

where q is an artificial pressure required for stabilising the calculation when considering shock wave propagation (discussed in section 3.1.6). The subscripts, x , y , and θ are for a cylindrical coordinate system. When Cartesian coordinates are desired, all radii r , which appear in these equations, are set to unity.

Calculations in cylindrical coordinates are scaled to unit azimuthal angle, which allows the equations to be written without any π factors. The radial or horizontal

coordinate is denoted by x , and the axial, or vertical coordinate by y , with the origin located at the lower left corner of the mesh. The quantity r is used to determine the geometry: r is set equal to x for cylindrical coordinates, but the expressions automatically reduce to Cartesian form if all r 's are set to unity. The cells are defined at four corners by "vertices". The quantities of displacement (x and y) and velocity (u and v) are defined at vertices. The quantities of density ρ , internal energy I , volume V , pressure p and viscosity η are cell centered quantities. Mass M is defined for both the cell as a whole and the four surrounding vertices. Figure 3.3 illustrates the index notation for the four vertices that define a cell as well as showing the cell and vertex variables.

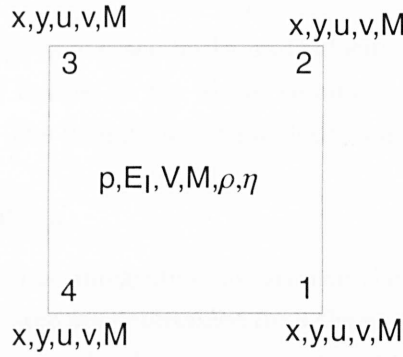


Figure 3.3: The assignment of variables and index notation for a cell in SALE

The stress tensor σ_{ij} is separated into the hydrostatic (p) and deviatoric (Π_{ij}) components. The hydrostatic component influences volumetric changes via the equation of state. The stress deviator terms (Π_{ij}) influence changes in distortion via the constitutive model. A detailed discussion of these material specific relations is reserved for the following section.

3.1.3 Finite Difference Approximation

In a hydrocode the spatial derivatives in the differential equations are replaced by difference equations. For example, for some function F the partial derivative $\partial F / \partial x$ becomes $\Delta F / \Delta x$ where the differences are computed at grid points. The first derivative of F at x_n can be represented by a variety of difference formulae:

$$\begin{aligned} \left. \frac{\partial F}{\partial x} \right|_{x_n} &= \frac{F_{n+1} - F_n}{\Delta x} \\ \left. \frac{\partial F}{\partial x} \right|_{x_n} &= \frac{F_n - F_{n-1}}{\Delta x} \\ \left. \frac{\partial F}{\partial x} \right|_{x_n} &= \frac{F_{n+1} - F_{n-1}}{2(\Delta x)} \end{aligned} \quad (3.11)$$

which correspond to forward, backward and central difference equations, respectively.

The difference approximations used in SALE to calculate spatial derivatives follow a central difference scheme. For example, the velocity gradient across a cell is given by:

$$\frac{du}{dx} = \frac{(u_2 - u_4)(y_3 - y_1) - (u_3 - u_1)(y_2 - y_4)}{(x_2 - x_4)(y_3 - y_1) - (x_1 - x_3)(y_4 - y_2)}, \quad (3.12)$$

which, for a rectangular cell with dimensions dx by dy , reduces to:

$$\frac{du}{dx} = \frac{(u_1 + u_2) - (u_3 + u_4)}{2dx} \quad (3.13)$$

This represents the change in average horizontal velocity component across the cell in the x -direction. The indices in the above equations refer to the four vertices defining each mesh cell. The definition of these indices is illustrated in Figure 3.3.

3.1.4 Time Integration

Hydrocodes use explicit time integration to advance the solution in time; that is, functions at the new time step are determined from the known functions at a previous time step. In SALE, for example, the energy equation (3.10) is advanced using the expression:

$$I' = I - \frac{dt}{\rho} \left[(p + q) \left(\frac{1}{r} \frac{\partial ru}{\partial x} + \frac{\partial v}{\partial y} \right) - \Pi_{ij} \dot{\epsilon}'_{ij} \right]. \quad (3.14)$$

3.1.5 Accuracy and Stability

It is important to understand the limitations and caveats associated with hydrocode modelling. These issues deal with the accuracy of representing a continuum problem on a computer with a finite memory allocation, and numerical instabilities inherent in the finite difference approximations.

Accuracy

Two types of errors occur which impair the accuracy of a finite difference scheme: round-off errors and truncation errors. Round-off errors are associated with the precision of the computer. These errors are only important in the rare situations where a variable changes in a number of orders of magnitude comparable to the number of significant digits retained by the computer. Truncation errors, on the other hand, are a function of the difference scheme used, not the hardware on which the code is run. The essence of this error arises from representing a continuous

variable with a discrete number of points, and the magnitude of the errors depends on the mesh intervals in time and space.

The useful accuracy of a given numerical solution may be difficult to determine analytically (Hirt, 1968). In any case, it is usually insightful to adopt a “brute force” approach and use a spectrum of computer runs with different meshes and time steps to determine if a calculated effect is physical or simply a numerical artifice. In general, the cell size and time step must be fine enough to resolve all spatial and temporal variations of interest. If a solution exhibits large variations over distances comparable to a cell width, or over times comparable to the time step, it is probably not very reliable. The question of spatial and temporal resolution, with regard to the version of SALES.2 used during the work reported in this thesis, is addressed later in this chapter, in section 3.4.

Stability

In addition to concerns over accuracy, hydrocodes suffer from inherent instabilities arising from the difference schemes used. Numerical methods may give solutions that develop large, high-frequency oscillations in space or time (Amsden et al., 1980); if the physical problem being modelled is known not to exhibit such behaviour, the source may be numerical instability.

Many numerical devices have been developed to mitigate against instability. For example, there are well documented limits placed on the size of the time step to ensure that information cannot travel across a full cell width in a single time step. Three such restrictions relevant to the calculations described in this report are discussed here.

Firstly, all hydrocode calculations should require that the Courant condition on sound signal propagation is not violated. That is,

$$dt < \min \left(\frac{dx}{c}, \frac{dy}{c} \right), \quad (3.15)$$

where dt is the time step, dx and dy are the cell dimensions and c is the speed of sound in the material modelled.

Secondly, the time step should be restricted by the very well-verified condition that fluid cannot be moved more than approximately one cell-width per time step; that is,

$$dt < \min \left(\frac{dx}{|u|}, \frac{dy}{|v|} \right), \quad (3.16)$$

which is known as the Gauchy, or convective flux limit. The minimum in the above equations implies that every cell in the mesh must be considered to ensure that the dt satisfies the most restrictive case. Amsden et al. (1980) recommend that the time

step should never exceed one-fifth of the minimum cell transit time, or the minimum sound propagation time.

The final restriction on time step due to numerical stability relates to the stress tensor. When viscous effects are included, the crucial condition to be satisfied in every cell is that

$$dt < \left[\frac{2(\lambda + 2\eta)}{\rho} \left(\frac{1}{dx^2} + \frac{1}{dy^2} \right) \right]^{-1} = \frac{\rho dx^2 dy^2}{2(\lambda + 2\eta)(dx^2 + dy^2)}, \quad (3.17)$$

which roughly states that momentum must diffuse less than one cell-width per time step. λ and η correspond to the bulk and shear viscosities respectively. The SALE hydrocode limits the size of the time step using both the Cauchy and the momentum diffusion conditions; however, the Courant condition must be satisfied by the user when defining the input parameters.

Another numerical consideration is mesh stability. In a Lagrangian calculation of fluid flow using quadrilateral mesh cells, there are certain mesh deformations that do not result in net pressure changes; that is, there is change in cell shape but no change in cell volume. Such nonphysical motion of vertices is sometimes termed “vertex coasting”. Figure 3.4 illustrates two such patterns, the bow-tie pattern and the herringbone pattern. Typically, vertex coasting is associated with the shortest resolvable wavelengths ($2dx$) in the mesh.

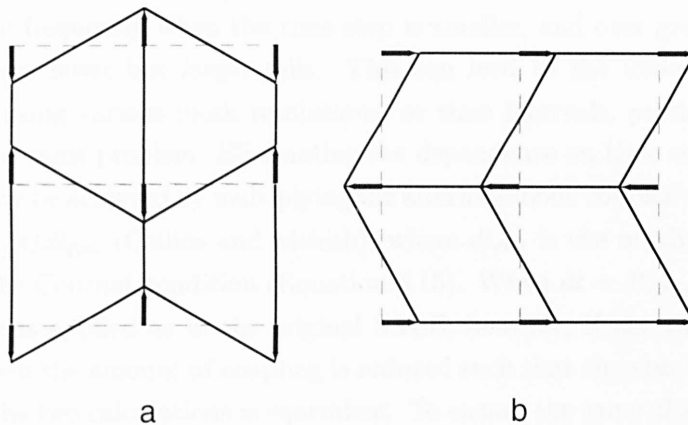


Figure 3.4: Examples of degenerate mesh deformations: (a) the bow-tie pattern; (b) the herringbone pattern (*after Amsden et al. (1980)*).

To prevent such deformations from slowly degrading the solution, alternate mesh nodes can be coupled with a small artificial restoring force (Amsden et al., 1980). Ideally, this force should affect flows only at the $2dx$ wavelength level; to guard against degenerate mesh instabilities, but have no influence on the larger flow variations being studied. In SALE, small accelerations are introduced at each vertex,

which are based on the surrounding velocity field and tend to keep the vertex velocities from deviating too strongly from adjacent node velocities.

As discussed by Amsden et al. (1980), a fourth-order coupling scheme is effective for the bow-tie mode, but a more diffusive second-order scheme must be used for the herringbone pattern. The fourth-order form used in SALE is given by:

$$u_i^j = u_i^j + \frac{a_{nc}}{4} \left[2 \left(u_{i+1}^j + u_i^{j+1} + u_{i-1}^j + u_i^{j-1} \right) - u_{i+1}^{j+1} - u_{i-1}^{j+1} - u_{i-1}^{j-1} - u_{i+1}^{j-1} - 4u_i^j \right], \quad (3.18)$$

in which a_{nc} is a coefficient that governs the amount of coupling and should, therefore, be as small as possible. The second-order form is given by

$$u_i^j = u_i^j + \frac{a_{nc}}{4} \left[\left(u_{i+1}^j + u_i^{j+1} + u_{i-1}^j + u_i^{j-1} \right) - 4u_i^j \right]. \quad (3.19)$$

These equations are implemented in SALE in such a way as to enable both order terms to be applied. A coefficient controls the relative order of the coupling, ranging from 0.0 for second order coupling alone to 1.0 for purely fourth order coupling. Note that the indices in the two equations above correspond to the coordinates of the vertices within the mesh.

A weakness of the alternate-node coupling scheme implemented in SALE is that it is independent of both temporal and spatial resolution. Thus, the velocity is diffused more frequently when the time step is smaller, and over greater distances when there are fewer but larger cells. This can lead to the undesirable trait of calculations using various mesh resolutions, or time intervals, producing different results for the same problem. Eliminating the dependence on time step and spatial resolution may be achieved by multiplying the alternate-node coupler coefficient, a_{nc} , by the ratio dt/dt_{cou} (Collins and Melosh), where dt_{cou} is the maximum time step allowed by the Courant condition (Equation 3.15). When $dt = dt_{cou}$, the alternate-node coupler is applied as in the original SALE; however, if the time step is less than dt_{cou} then the amount of coupling is reduced such that the absolute amount of coupling in the two calculations is equivalent. To ensure the same absolute coupling effect in two calculations of different mesh resolutions, an identical time step size must be used in each run. If this is the case, the factor dt/dt_{cou} will be smaller for the calculation with the larger cell dimension and, hence, the coupling between alternate vertices will be reduced accordingly.

3.1.6 Treating Shock Waves

As discussed in section 1.2, the mathematical idealisation of a shock wave is an instantaneous jump in pressure, velocity, density and internal energy. The shock is actually broadened by rate-dependent physical mechanisms such as viscosity, damage

and pore collapse (Melosh et al., 1992). Nevertheless, rapid changes that occur over shorter distances than the smallest possible cell size are problematic for any numerical treatment using a finite difference representation. A special approach must be adopted, therefore, when modelling the passage of a shock wave with a hydrocode.

One approach for problems involving shock waves, is to apply an artificial viscosity (von Neumann and Richtmyer, 1950), which serves to spread the shock over several cell-widths while preserving the character of the jump in material properties. Without such dissipation, spurious velocity oscillations develop behind the shock, which can mask or swamp the true solution (Anderson, 1987). A widely cited form for the artificial viscosity term is a pressure addition, q given by:

$$\begin{aligned} q &= A\rho\nabla^2.v & \nabla.v < 0, \\ q &= 0 & \nabla.v \geq 0, \end{aligned}$$

(Richtmyer and Morton, 1967), which states that q is proportional to the square of the velocity divergence $\nabla.v$, when $\nabla.v < 0$; that is, in compression only. Thus, the artificial pressure term is only significant close to the shock, and is negligible in the absence of strong shocks. SALE adopts this technique by defining q as:

$$q = \text{artvis } \rho \text{ area } D, \min(0, D), \quad (3.20)$$

where ρ is the cell density, **area** is the cell area, D is the velocity divergence, and the parameter **artvis** is termed the artificial viscosity and should be less than 0.25 to avoid excessive damping. **artvis** = 0.1 has been satisfactory for a wide range of applications (Amsden et al., 1980).

3.2 Material Modelling: SALES_2

Material modelling can be divided into three areas: volumetric response to stress, or resistance to compressibility (equation of state), the resistance to distortion (constitutive model); and the reduction in ability to carry stress as damage accumulates (failure model). The original SALE hydrocode included no failure model and a very simple equation of state and constitutive model, capable of describing only inviscid fluid flow. As has been discussed in section 2.4, complex crater collapse is a phenomenon that is more complicated than a mere fluid flow problem. Collapse involves a complicated rheological response to stress, and a rheological model that changes with time. Thus, to adequately model problems in impact cratering SALE has been extensively modified to include sophisticated methods for modelling material response to stress (for example Melosh et al., 1992; Ivanov et al., 1997). These modifications have led to the development of SALES_2 (Collins and Melosh); a hy-

drocode based on SALE, which retains all its functionality as well as being capable of modelling elasticity, plasticity, multiple materials and rheologies, and strength degradation via fragmentation.

3.2.1 Equation of State

How a material responds to changes in volume and the shock of an impact is governed by its equation of state. An excellent overview of equations of state, with particular emphasis on those pertinent to impact cratering, is presented in Melosh (1989) who states:

An equation of state relates three thermodynamic variables, pressure p , density ρ (or specific volume $V = 1/\rho$), and specific internal energy I (or temperature, T)... The equation of state is different for different materials and is a complex function of the molecular and atomic structure of a given substance

Geological materials are too complex for a complete analytical derivation of their equation of state. Therefore, the few methods that have been developed for describing the equation of state of these materials are based on empirical observations. As a result, there is still considerable uncertainty in the equation of state of geological materials in the velocity range at which most impacts occur in the Solar System. In the velocity regime of crater collapse, however, current equations of state provide an adequate description of compressibility effects.

Numerical hydrocodes, such as SALE, compute the material density ρ and the specific internal energy I every cycle, from which the pressure p must be calculated in the next cycle. Thus, the most convenient form of the equation of state for impact calculations is:

$$p = p(\rho, I).$$

The Liquid Equation of State

The pressure of a liquid subjected to a small compression or expansion is a linear function of the density fluctuation about the uncompressed density ρ_0 :

$$p = c_B^2(\rho - \rho_0) \tag{3.21}$$

where $c_B = \sqrt{K/\rho_0}$ is the bulk sound speed. Although it appears that this equation has no dependence on I , this is a false impression as the bulk modulus, K , is a weak function of temperature. The liquid equation of state is used in the SALE hydrocode.

The perfect gas equation of state

The pressure in a perfect gas is a simple function of density and internal energy:

$$p = (\gamma - 1)\rho I \quad (3.22)$$

where γ is the ratio of the specific heat at a constant pressure to that at constant volume, $\gamma = C_p/C_V$.

The Stiffened Gas Equation of State

The liquid equation of state (3.21) and the perfect gas equation of state (3.22) are sometimes combined to yield a more realistic equation of state for solids called the “stiffened gas” equation of state:

$$P = c_B^2(\rho - \rho_0) + (\gamma - 1)\rho I. \quad (3.23)$$

The stiffened gas equation of state is widely used in problems of geophysics and planetary science. For problems not involving shock waves, such as crater collapse, this equation of state is adequate for describing a materials response to compression. However, to model impact cratering in its entirety, a more sophisticated equation of state is required to accurately describe the effects of shock compression, phase changes and rapid expansion. Consequently, SALES_2 implements the Tillotson equation of state and, although simulations of complex crater collapse do not require an equation of state as sophisticated as the Tillotson equation of state, this formulation is used for the numerical calculations of complex crater collapse discussed in later chapters.

The Tillotson Equation of State

The Tillotson equation of state was specifically designed for high-velocity impact computations, by Tillotson (1962). The formulation was designed such that, at low pressures, it duplicates linear shock-particle velocity relations, which are based on direct results of shock-wave measurements. At high pressures, the Tillotson equation of state extrapolates toward the Thomas-Fermi limit (Melosh, 1989, Appendix II). This point is reached when compression is extreme enough for all the atoms within the material to become ionized, effectively converting the solid to an electron gas. Although this limit is never reached in most meteorite impacts, it does provide a theoretically well-founded upper limit to the response of a material to high pressures. The Tillotson equation of state also provides a description for vapourisation of the shocked material.

The Tillotson equation has two different forms; one for situations where the material is expanded to a lower density than its zero-pressure form and heated to

above the point where the material begins to vapourise, and one for all other situations. The form used in the majority of situations, where the material is compressed ($\rho/\rho_0 \geq 1$) or where the material is expanded but the energy density is less than the energy of incipient vaporisation, $I < I_{iv}$, is the “cold” state:

$$p_{cold} = \left[a + \frac{b}{(I/(E_0(\rho/\rho_0)^2) + 1)} \right] \rho I + A(\rho/\rho_0 - 1) + B(\rho/\rho_0 - 1)^2. \quad (3.24)$$

where a , b , A , B , and E_0 are the Tillotson parameters. It can be shown that A is equivalent to the bulk modulus K . It should be noted that E_0 is not the initial energy density of the substance: it is merely a parameter that is often close to the vaporisation energy. The initial energy density I must actually be zero to ensure that $p_{cold} = 0$ in the initial state (Melosh, 1989, Appendix II).

In hydrocode computations, densities much less than ρ_0 will occur during rapid expansion. Physically, such densities indicate that the material in the computational cell is no longer a continuum, but may be in the form of droplets or solid fragments separated by gas or vacuum instead. Therefore, the pressure computed from this equation of state must be limited to avoid artificially large tensions developing (Melosh, 1989, Appendix II). This is achieved in the Tillotson equation of state, as implemented in SALES-2, by testing the ratio ρ/ρ_0 ; if ρ/ρ_0 is less than some amount, often chosen between 0.8 and 0.95, the pressure is set to zero.

In the “hot” expanded state, when the density is less than the reference density, $\rho/\rho_0 \leq 1$, and the internal energy exceeds the energy of complete vaporisation $I > I_{cv}$, the pressure is given by:

$$p_{hot} = a\rho I + \left[\frac{b\rho I}{(I/(E_0(\rho/\rho_0)^2) + 1)} + A(\rho/\rho_0 - 1)e^{-\beta(\rho/\rho_0 - 1)} \right] e^{-\alpha(\rho/\rho_0 - 1)^2} \quad (3.25)$$

where α and β are constants that control the rate of convergence of this equation to the perfect gas law.

In order to make the transition between the hot and cold regimes smooth, it has been found that in the *partial* vaporisation regime, when $\rho/\rho_0 < 1$ and $I_{iv} < I < I_{cv}$, the pressure may be computed from a hybrid formula (Melosh, 1989, Appendix II):

$$p = \frac{(I - I_{iv})p_{hot} + (I_{cv} - I)p_{cold}}{I_{cv} - I_{iv}}. \quad (3.26)$$

Outside of this regime the pressure is given by either p_{cold} or p_{hot} , depending on the internal energy and the density relative to the zero-pressure density, as discussed above.

3.2.2 The Constitutive Model: Material Strength and Rheology

The constitutive model describes the behaviour of a material under stress. As alluded to earlier, the stress tensor σ_{ij} may be separated into a hydrostatic component, p , and deviatoric components, Π_{ij} . The definition of the deviatoric stress tensor is as follows:

$$\begin{aligned}\Pi_{ij} &= \sigma_{ij} - p && \text{for } i = j \\ \Pi_{ij} &= \sigma_{ij} && \text{for } i \neq j\end{aligned}\tag{3.27}$$

The hydrostatic component (pressure) is used in the equation of state to calculate volumetric changes discussed in the previous section. The deviatoric component is used in the constitutive model to predict changes in distortion.

The strain tensor ϵ_{ij} may be equivalently separated into a volumetric part e and a deviatoric part ($\epsilon'_{ij} = \epsilon_{ij} - e$ for $i = j$, $\epsilon'_{ij} = \epsilon_{ij}$ for $i \neq j$). e is defined as the volumetric strain, or trace, and is given by the expression (in cylindrical coordinates):

$$e = \frac{1}{3}(\epsilon_{xx} + \epsilon_{yy} + \epsilon_{\theta\theta})\tag{3.28}$$

Similar expressions exist for the deviatoric strain rate $\dot{\epsilon}'_{ij}$. The deviatoric stresses may be a function of the deviatoric strain (elasticity) or the deviatoric strain rate (viscosity). The original version of SALE included only viscous effects in its constitutive model. The addition of elasticity and yield strength have been largely the work of Melosh and his co-workers. The following discussion describes the constitutive model incorporated in the version of SALES_2 used for the crater collapse calculations reported in this document.

Rheological Models in SALES_2

As has previously been introduced in Chapter 2, rheological models offer a means of describing material response to stress or deformation. Three simple types of behaviour have been discussed: elasticity, plasticity or yield strength and viscosity. In the aid of understanding, each type of behaviour is conventionally represented by a simple mechanical model. Elasticity is represented by a spring of stiffness k ; viscosity η is represented by a dash-pot; and yield strength is represented by a frictional contact of resistance σ_Y (see Figure 3.5). A rheological model, then, is described by one or a combination of these simple elements.

Section 2.3 discussed the behaviour of the target rocks during crater collapse; the required rheological model is that of a Bingham substance. A Bingham substance is one which behaves elastically for stresses below a finite yield stress (the yield strength or cohesion), and as a fluid at stresses above the yield stress. Thus, to

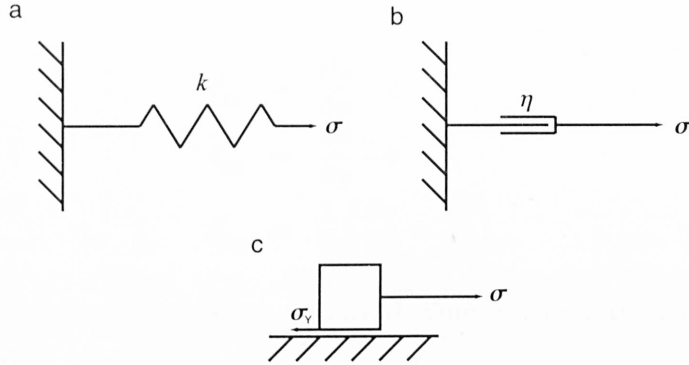


Figure 3.5: Mechanical model analogues for (a) elasticity, (b) viscosity and (c) yield strength.

satisfactorily model crater collapse the constitutive model must include the material properties of viscosity, elasticity and yield strength as well as placing them in the context of the Bingham rheological model.

Viscosity

Although most geologic materials are non-Newtonian, it is difficult to determine the viscous stresses in a non-Newtonian fluid from strain rate information because the parameter relating the two, the effective viscosity, is itself a function of stress. For this reason, hydrocodes such as SALES_2 treat all fluids as Newtonian; that is, the relationship between stress and strain rate is linear. The *viscous* stress tensor in SALES_2 is therefore defined as:

$$\Pi_{ij} \Big|_{viscous} = 2\eta \dot{\epsilon}'_{ij}. \quad (3.29)$$

The viscous stresses resist the forces driving deformation and must therefore be determined from the strain rate tensor. The strain rate is related to the velocity gradient according to the equation:

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left(\frac{du_i}{dx_j} + \frac{du_j}{dx_i} \right), \quad (3.30)$$

The *deviatoric* strain rate may be calculated from the velocity gradient less the volumetric strain, or trace. For example, in two dimensions the deviatoric strain rate is given by:

$$\begin{aligned}
\epsilon'_{xx} &= \frac{\partial u}{\partial x} - e \\
\epsilon'_{yy} &= \frac{\partial v}{\partial y} - e \\
\epsilon'_{xy} &= \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)
\end{aligned} \tag{3.31}$$

If considering three dimensions with axial symmetry then the hoop strain rate ϵ'_θ is defined as:

$$\epsilon'_\theta = \frac{u}{r} - e. \tag{3.32}$$

Therefore the viscous stresses in cylindrical coordinates may be defined as:

$$\begin{aligned}
\Pi_{xx} &= 2\eta \left(\frac{\partial u}{\partial x} - e \right) + \lambda D & \Pi_{yy} &= 2\eta \left(\frac{\partial v}{\partial y} - e \right) + \lambda D \\
\Pi_{xy} &= \eta \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & \Pi_\theta &= 2\eta \left(\frac{u}{r} - e \right) + \lambda D
\end{aligned} \tag{3.33}$$

in which λ is the *bulk* viscosity coefficient and D is the velocity divergence.

Elasticity

The *elastic* deviatoric stress tensor is determined from the shear version of Equation 2.8, namely:

$$\Pi_{ij} \Big|_{elastic} = 2G\epsilon'_{ij} \tag{3.34}$$

where the shear modulus G (Equation 2.9) is used as the constant of proportionality k . The deviatoric strains may be obtained simply by multiplying the deviatoric strain rates by the time step dt :

$$\begin{aligned}
\epsilon'_{xx} &= \left(\frac{\partial u}{\partial x} - e \right) dt & \epsilon'_{yy} &= \left(\frac{\partial v}{\partial y} - e \right) dt \\
\epsilon'_{xy} &= \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) dt & \epsilon'_\theta &= \left(\frac{u}{r} - e \right) dt
\end{aligned} \tag{3.35}$$

Kelvin Substance

In the solution algorithm of SALES_2 the total deviatoric stress tensor is the vector sum of the elastic and viscous contributions:

$$\Pi'_{ij} = 2G\epsilon'_{ij} + 2\eta\epsilon'_{ij}. \tag{3.36}$$

Thus, the SALES_2 hydrocode models a solid material as a spring and a dash pot in parallel, a rheological model known as a Kelvin substance. If a constant devia-

toric stress, S , is applied to a Kelvin substance unstrained at time $t = 0$ then the deviatoric strain at time t is given by:

$$\epsilon' = S/2G \left(1 - e^{-Gt/\eta}\right) \quad (3.37)$$

Thus, if the viscosity η is much less than the stiffness G the substance behaves essentially elastically. For non-zero viscosity, however, the strain will eventually tend to $S/2G$. This model is the simplest which corresponds to a solid in which vibrations are damped by internal friction. Consequently, it has frequently been used to model the behaviour of crustal rocks over short time-scales. The simulations of complex crater collapse discussed in subsequent chapters use this rheological model to represent the target under normal pre-impact conditions.

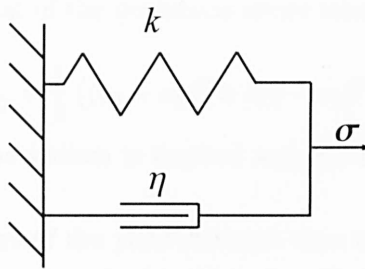


Figure 3.6: Mechanical model of a Kelvin substance

Plasticity, Yield Strength and the Bingham Substance

The Bingham substance (Fig. 3.7) may be represented by a spring, a frictional block and a dash-pot all in series. Thus, it has much in common with a plastic material: The Bingham substance behaves as an elastic body for stresses less than the yield stress; above the yield stress the Bingham substance starts to deform permanently. However, the nature of the flow of a Bingham substance differs to that of a plastic in that flow occurs throughout the material; a Bingham substance flows with a viscosity.

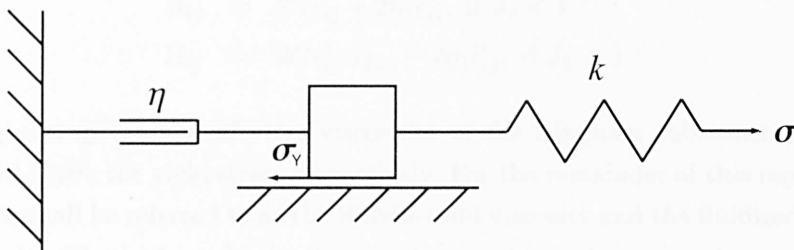


Figure 3.7: Mechanical model of a Bingham substance

The behaviour of a Bingham substance is described by the equations:

$$\sigma_{ij} = 2k\epsilon_{ij}, \text{ if } \sigma < \sigma_Y \quad (3.38)$$

$$\sigma_{ij} = \sigma_Y + 2\eta\dot{\epsilon}_{ij}, \text{ if } \sigma > \sigma_Y. \quad (3.39)$$

To adequately model a Bingham substance, therefore, the constitutive relation in SALES_2 must also incorporate plasticity. The approach adopted in SALES_2 to account for plasticity is to reduce the elastic contribution to the stress tensor when the yield strength of the material is exceeded. Whether or not the yield strength is exceeded in a particular cell is determined by a comparison between the yield strength and an *invariant* measure of the stress in a cell. That is, a function of the stress tensor that is independent of the choice of coordinate axes. The invariant used is the second invariant of the deviatoric stress tensor J_2 defined as:

$$J_2 = \frac{1}{2}\Pi_{ij}\Pi_{ij} = \frac{1}{2}((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_\theta)^2 + (\sigma_\theta - \sigma_1)^2) \quad (3.40)$$

where summation over the indices is implied and σ_1 , σ_2 and σ_θ are the principal stresses.

If J_2 exceeds the square of the yield strength then the elastic stress component is reduced in relation to how much the yield strength has been exceeded by. The exact amount of stress reduction is given by s_{fac} , where

$$s_{fac} = \sqrt{\frac{Y^2}{J_2}}. \quad (3.41)$$

The rheological model available to SALES_2 that most closely analogues a Bingham substance is shown in Figure 3.8. In this rheological model the material behaves as a Kelvin substance at stresses below the yield strength. At stresses above the yield stress the material behaves as a Newtonian fluid. Thus, the deviatoric stress for this pseudo-Bingham rheology is given as:

$$\Pi_{ij} = 2G\epsilon'_{ij} + 2\eta_1\dot{\epsilon}'_{ij}, \text{ if } J_2 < Y^2 \quad (3.42)$$

$$\Pi_{ij} = 2G\epsilon'_{ij} \cdot s_{fac} + 2\eta_2\dot{\epsilon}'_{ij}, \text{ if } J_2 > Y^2 \quad (3.43)$$

where η_1 and η_2 are the effective viscosities of the Bingham substance at stresses below and above the yield stress respectively. For the remainder of this report, these viscosities shall be referred to as the Kelvin-solid viscosity and the fluidized viscosity respectively. The hydrocode simulations discussed in subsequent chapters use this rheological model to represent the target during the peculiar conditions of complex crater collapse.

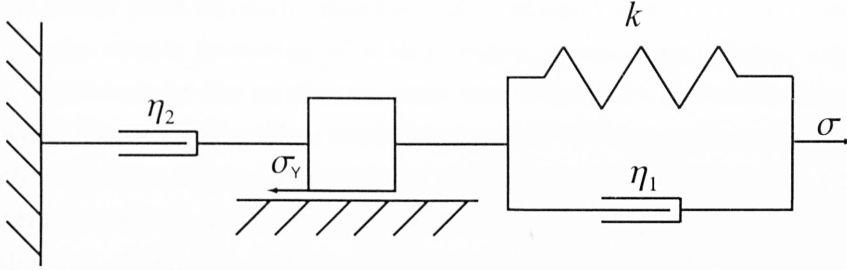


Figure 3.8: Mechanical model of the pseudo-Bingham substance modelled by SALES_2.

The pseudo-Bingham rheology in SALES_2 differs subtly from the classic Bingham fluid model discussed in Chapter 2, in terms of the stress-strain-rate relationship. Figure 3.9 illustrates the distinction graphically. In the classic Bingham fluid model, the strain rate increases from zero at the Bingham yield stress; the strain rate is proportional to the stress in excess of the yield stress Y . In contrast, the pseudo-Bingham rheology implemented in SALES_2 defines the strain rate proportional to the total stress. Thus, at the point of yield the strain rate is non-zero for the pseudo-Bingham rheology. At stresses below the yield criterion the classic Bingham fluid model and that implemented in SALES_2 are equivalent.

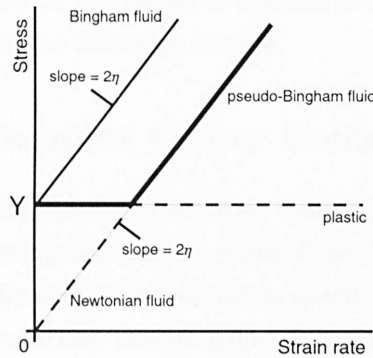


Figure 3.9: Stress versus strain rate relationship of the pseudo-Bingham substance in SALES_2. In the classic Bingham fluid model (thin solid black line), the strain rate increases from zero at the Bingham yield stress; the strain rate is proportional to the stress in excess of the yield stress Y . In contrast, the pseudo-Bingham rheology implemented in SALES_2 defines the strain rate proportional to the total stress (thick solid black line). At stresses below the yield criterion the classic Bingham fluid model and that implemented in SALES_2 are equivalent.

3.2.3 Modelling Tensile Failure

Section 2.2 discussed the different types of material strength; compressive, tensional and shear. The previous section discussed the distortive influence of shear stresses

in excess of the yield strength. However, as mentioned in section 2.4.1, an impact results in the tensile fracturing of a large region of the target during the release from high pressure by the rarefaction wave that follows the outwardly propagating shock wave. Consequently, when modelling impacts it is important to model tensile strength in addition to the deformational yield strength of a material.

Tensile strength is modelled in SALES_2 using the Grady-Kipp-Melosh model (Melosh et al., 1992). In essence the approach is to assess the amount of tensile fracture in a cell and reduce the cohesive strength accordingly. The amount of fracturing in the cell is parameterised by the quantity D , the damage. D is 1.0 for wholesale fracturing in the cell, and $D = 0.0$ corresponds to totally intact material in the cell. The model is implemented in such a way that the amount of damage effects only the cohesive strength of the material. Thus, resistance to compression is unaffected by damage accumulated due to tensile fracture.

Computer simulations of impact crater formation in its entirety require the application of the Grady-Kipp-Melosh tensile strength model. However, when modelling the collapse of large craters as a discrete process it is sufficient to model the target as completely damaged. Therefore, while the algorithm for computing the fractional amount of damage has been omitted from the calculations in this report, the effect of wholesale tensile failure has been investigated. The algorithm for determining the yield strength at a position in the target is specific to the study of impact crater collapse and is discussed in the following section.

3.3 Modelling Complex Crater Collapse

This thesis concerns the simulation of complex crater collapse using a hydrocode. The previous discussion has highlighted the parts of the SALES_2 solution algorithm important to this study. However, there are issues specific to the modelling of impact crater collapse that require further modification of the SALES_2 solution algorithm: the form of the starting mesh; the initial stress state; the static strength conditions; and the implementation of an appropriate material weakening mechanism. These topics, and the modifications entailed, are the subject of this section. The source-code listing for these modifications is presented in Appendix B.

3.3.1 Mesh Geometry

In order to study the modification stage of crater formation, the starting mesh must represent the *transient crater*. In this thesis the transient crater is defined as the form of the cavity immediately subsequent to the excavation stage; the form of the crater after all the ejected material has been expelled and the cavity has reached its maximum dimensions prior to collapse. As mentioned in section 2.1 debate

surrounds the validity of the approximation of the transient crater as a simple bowl-shaped depression with a raised rim and parabolic cross section. However, using this form for the transient crater as a first-order approximation should provide important insight into the collapse process.

The simple bowl-shaped morphology of the transient crater can be calculated using the Z-model, first introduced by Maxwell and Seifert (1974) and discussed in section 2.1. Figure 3.10 illustrates a typical starting mesh geometry for the simulations of crater collapse reported here. As has been previously discussed, it is important for a hydrocode mesh to be made up of quadrilaterals for which the finite difference approximations are accurate. The algorithm for transforming the vertices of a rectangular grid to a regular mesh that mimics the geometry of the transient crater (depicted in Figure 3.10) is summarized in Appendix B.

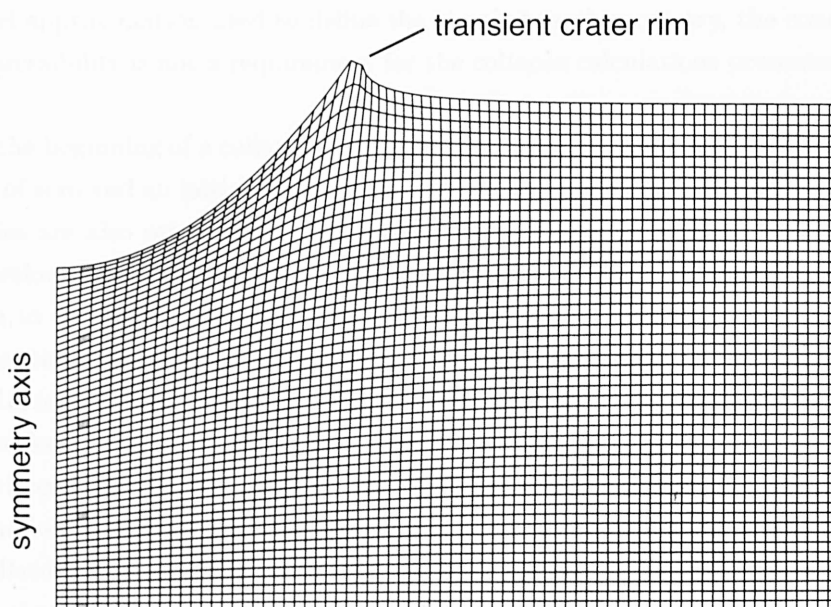


Figure 3.10: Mesh geometry used in complex crater collapse calculations. The five shaded cells represent cells in the mesh where certain mesh-parameters were recorded during example simulations, which are discussed in subsequent sections.

In addition to the geometry of the mesh, the behaviour of the mesh along its four boundaries must also be defined. Along the top boundary (the surface) movement of the vertices is not restricted. This type of boundary is known as a Lagrangian free surface. The other three boundary conditions, that on the axis (left side of the mesh) and on the bottom and right sides of the mesh are all free-slip conditions. Free-slip implies that the velocity component perpendicular to the boundary is zero, while the velocity component parallel to the boundary remains unchanged.

3.3.2 Initial Stress Field

Complex crater collapse is driven by gravity: deviatoric stresses due to the absence of mass within the crater tend to close the cavity such that it becomes more stable in a gravity field. The exact stress field surrounding a cavity cannot be accurately determined analytically and then mapped onto the hydrocode mesh. Instead, the stress field must be determined numerically, by constructing a mesh of the appropriate geometry and target properties, and then introducing gravity. Provided that the constitutive model applied to the mesh does not allow the cavity to start collapsing, the stress field within the mesh will develop to the appropriate steady state.

Typical velocities for the excavation and subsequent collapse flow are generally slow enough for the flow to be approximated as incompressible; however, this assumption is not strictly valid. Hence, despite the assumption being implicit in the Z-model approximation used to define the starting mesh geometry, the condition of incompressibility is not a requirement for the collapse calculations presented in this thesis.

At the beginning of a collapse calculation, an initial pressure and specific internal energy of zero and an initial reference density, ρ_0 , is assigned to every cell. All vertex velocities are also set to zero. The introduction of gravity leads to an increase in vertex velocity, cell pressure and, assuming compressible flow, cell energy and density relative to the reference state. The constitutive model and the equation of state influence the amount, duration and nature of the compression. As mentioned above, SALES_2 models solid materials using a Kelvin rheology; thus, the mesh compression is controlled by the stiffness (shear and bulk modulus) and the viscosity of the mesh. If the viscosity of the mesh is zero, the mesh behaves elastically; the instantaneous introduction of an external force (gravity) compresses and then extends the mesh in an oscillatory manner – the mesh behaves like a spring. A non-zero viscosity in the Kelvin rheology dampens these oscillations, eventually resulting in a compressed mesh with pressure, internal energy and density increasing with depth. A large viscosity dampens the elastic oscillations more quickly than a small viscosity.

Figure 3.11 illustrates the relaxation of various positions in the mesh to a steady state, in terms of vertex velocity in the horizontal and vertical directions and pressure. The relaxation curves for three viscosities are shown. The larger the viscosity the quicker the mesh relaxes to achieve a steady state. This stage of the modelling procedure shall be referred to as the relaxation stage: the mesh is allowed to relax under the force of gravity, without collapsing, until it has achieved a steady state of stress. The collapse calculations, which will be discussed in subsequent chapters, all proceed using mesh quantities recorded at the end of the relaxation stage as initial conditions.

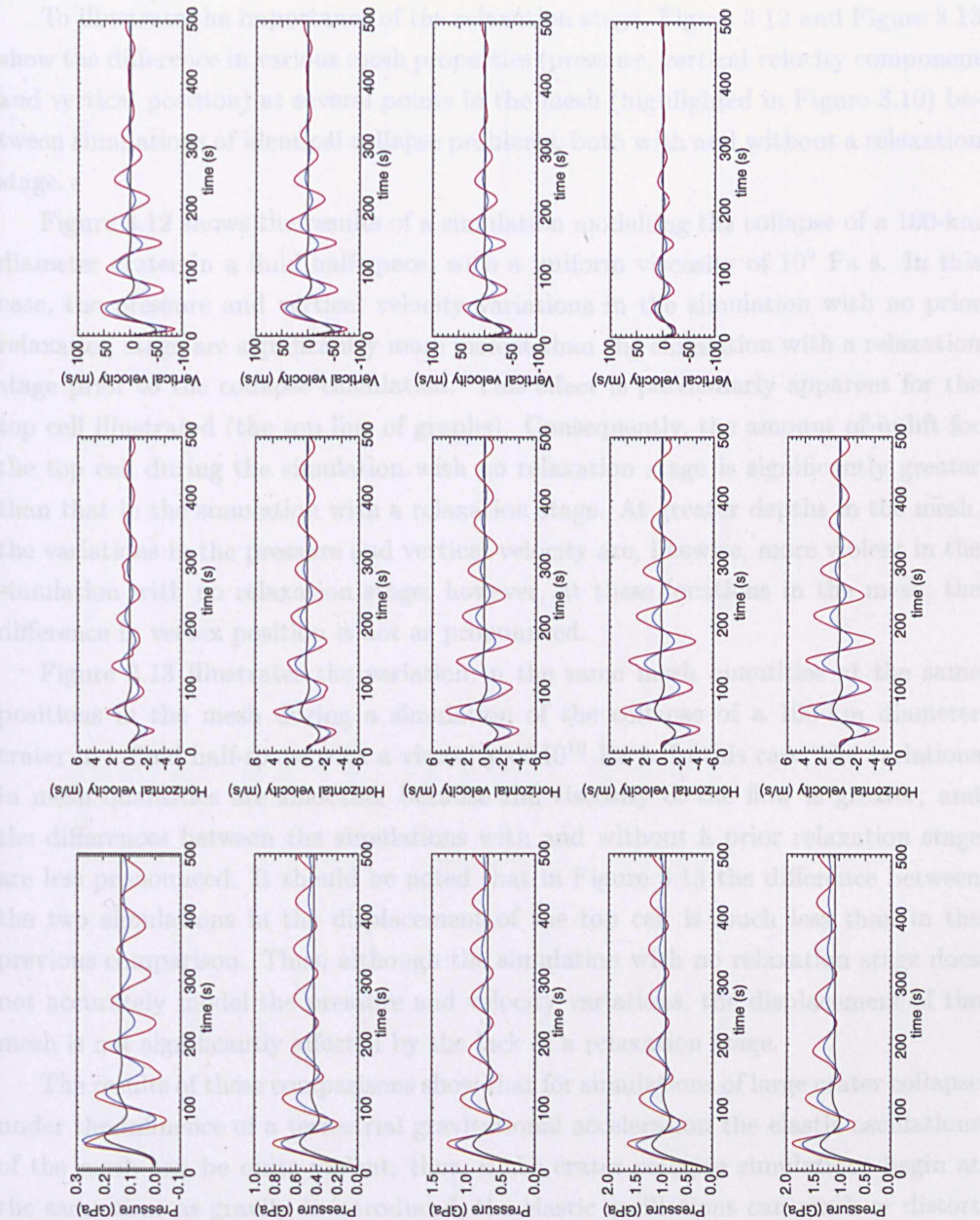


Figure 3.11: Relaxation of mesh properties to a steady state following the introduction of gravity. The graphs show the temporal behaviour of various positions in the mesh in terms of three different mesh properties (pressure, and horizontal and vertical velocity components), during the relaxation stage. In these simulations the target is modelled as a Kelvin substance defined by a stiffness (the bulk modulus) and a viscosity. The three lines illustrate the differences due to the choice of viscosity: the red line depicts the case for a viscosity of 10^{10} Pa s; the blue line is for 3×10^{10} Pa s; and the black line is for 10^{11} Pa s. The five rows (in landscape view) illustrate the change in mesh properties with depth in the mesh. Figure 3.10 shows the cells for which the data were recorded. The three columns are for the three mesh properties of pressure, which is a cell centered quantity, vertex velocity in the horizontal direction (right is positive) and vertex velocity in the vertical direction (up is positive). The vertical velocity component is zero for the cell at the bottom boundary; this graph is omitted.

To illustrate the importance of the relaxation stage, Figure 3.12 and Figure 3.13 show the difference in various mesh properties (pressure, vertical velocity component and vertical position) at several points in the mesh (highlighted in Figure 3.10) between simulations of identical collapse problems, both with and without a relaxation stage.

Figure 3.12 shows the results of a simulation modelling the collapse of a 100-km diameter crater in a fluid half-space, with a uniform viscosity of 10^9 Pa s. In this case, the pressure and vertical velocity variations in the simulation with no prior relaxation stage are significantly more violent than the simulation with a relaxation stage prior to the collapse calculation. This effect is particularly apparent for the top cell illustrated (the top line of graphs). Consequently, the amount of uplift for the top cell during the simulation with no relaxation stage is significantly greater than that in the simulation with a relaxation stage. At greater depths in the mesh, the variations in the pressure and vertical velocity are, likewise, more violent in the simulation with no relaxation stage; however, at these locations in the mesh, the difference in vertex position is not as pronounced.

Figure 3.13 illustrates the variation in the same mesh quantities at the same positions in the mesh during a simulation of the collapse of a 100-km diameter crater in a fluid half-space with a viscosity of 10^{10} Pa s. In this case, the variations in mesh quantities are smoother because the viscosity of the flow is greater; and the differences between the simulations with and without a prior relaxation stage are less pronounced. It should be noted that in Figure 3.13 the difference between the two simulations in the displacement of the top cell is much less than in the previous comparison. Thus, although the simulation with no relaxation stage does not accurately model the pressure and velocity variations, the displacement of the mesh is not significantly affected by the lack of a relaxation stage.

The results of these comparisons show that for simulations of large crater collapse under the influence of a terrestrial gravitational acceleration the elastic oscillations of the mesh can be quite violent; thus, if the crater collapse simulations begin at the same time as gravity is introduced, the elastic oscillations can mask or distort the true collapse solution. It is important, therefore, that the mesh is allowed to adjust to the gravitational forces before the collapse is studied, particularly when the fluid-like collapse flow is characterized by a low effective viscosity.

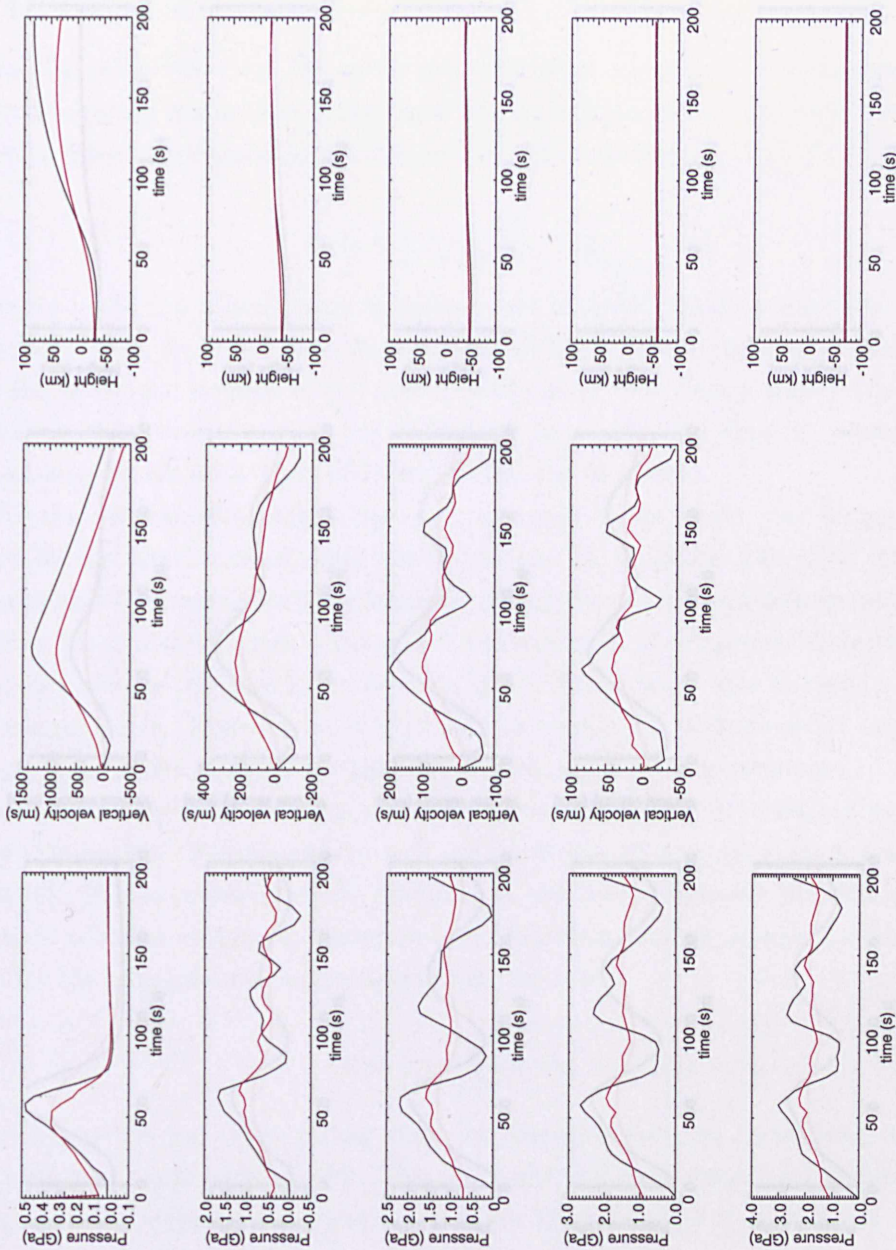


Figure 3.12: Comparison between hydrocode simulation results of a simple collapse in a fluid half-space with and without a relaxation stage prior to the collapse simulation. Results for the collapse of a 100-km diameter crater in a fluid half-space, with a viscosity of 10^9 Pa s. Red line shows the curves for the simulation without a relaxation stage. Black line shows the curves for the simulation with a relaxation stage. The five rows (in landscape view) illustrate the change in mesh properties with depth in the mesh. Figure 3.10 shows the cells for which the data were recorded. The three columns are for the three mesh properties of pressure, which is a cell centered quantity, vertex velocity in the vertical direction (up is positive) and vertical vertex position (above the original fluid surface is positive). The vertical velocity component is zero for the cell at the bottom boundary; this graph is omitted.

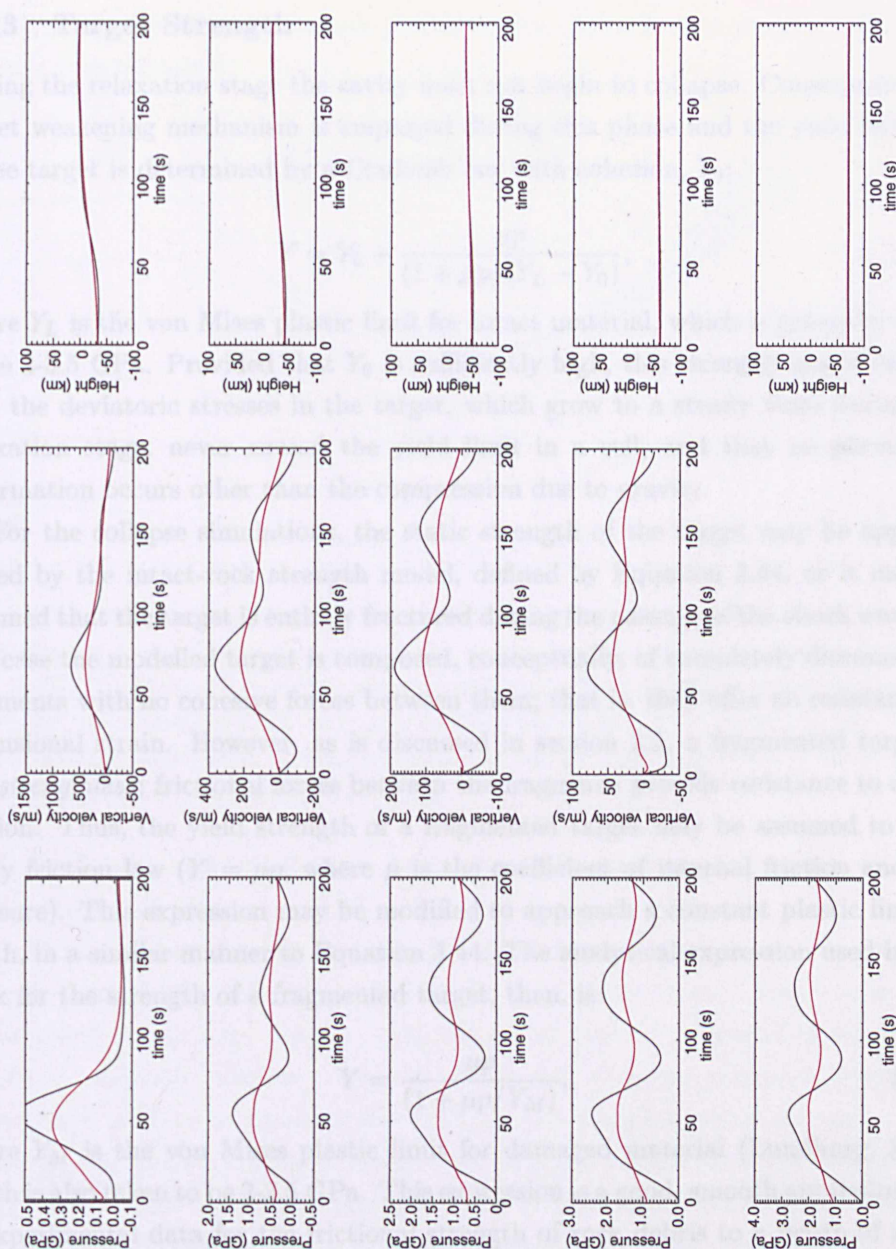


Figure 3.13: Comparison between hydrocode simulation results of a simple collapse in a fluid half-space with and without a relaxation stage prior to the collapse simulation. Results for the collapse of a 100-km diameter crater in a fluid half-space, with a viscosity of 10^{10} Pa s (c.f. Figure 3.12). Red line shows the curves for the simulation without a relaxation stage. Black line shows the curves for the simulation with a relaxation stage. The five rows (in landscape view) illustrate the change in mesh properties with depth in the mesh. Figure 3.10 shows the cells for which the data were recorded. The three columns are for the three mesh properties of pressure, which is a cell centered quantity, vertex velocity in the vertical direction (up is positive) and vertical vertex position (above the original fluid surface is positive). The vertical velocity component is zero for the cell at the bottom boundary; this graph is omitted.

3.3.3 Target Strength

During the relaxation stage the cavity must not begin to collapse. Consequently no target weakening mechanism is employed during this phase and the yield strength of the target is determined by a Coulomb law with cohesion, Y_0 :

$$Y = Y_0 + \frac{\mu p}{(1 + \mu p / (Y_L - Y_0))}, \quad (3.44)$$

where Y_L is the von Mises plastic limit for intact material, which is generally taken to be 2-3.5 GPa. Provided that Y_0 is sufficiently high, this strength model ensures that the deviatoric stresses in the target, which grow to a steady state during the relaxation stage, never exceed the yield limit in a cell, and thus no permanent deformation occurs other than the compression due to gravity.

For the collapse simulations, the static strength of the target may be approximated by the intact-rock strength model, defined by Equation 3.44, or it may be assumed that the target is entirely fractured during the passage of the shock wave. In this case the modelled target is composed, conceptually, of completely disconnected fragments with no cohesive forces between them; that is, they offer no resistance to extensional strain. However, as is discussed in section 2.2, a fragmented target is not *strengthless*; frictional forces between the fragments provide resistance to deformation. Thus, the yield strength of a fragmented target may be assumed to obey a dry friction law ($Y = \mu p$, where μ is the coefficient of internal friction and p is pressure). This expression may be modified to approach a constant plastic limit at depth, in a similar manner to Equation 3.44. The analytical expression used in this work for the strength of a fragmented target, then, is:

$$Y = \frac{\mu p}{(1 + \mu p / Y_M)}, \quad (3.45)$$

where Y_M is the von Mises plastic limit for damaged material (Lundborg, 1968), which is also taken to be 2-3.5 GPa. This expression is a good, smooth approximation of experimental data for the frictional strength of rock debris to a depth of about 60 km on Earth (Lundborg, 1968).

Section 2.3 demonstrated that complex crater collapse cannot occur if the target behaves in a manner described by either of the standard strength models above. This fact may be tested using the SALES_2 hydrocode to simulate collapse governed by either Equation 3.44 or Equation 3.45.

Figure 3.14 shows yield strength variations within the mesh for a 100-km diameter transient crater, using the two standard strength models described above. Figure 3.14a illustrates the case for an “intact rock” strength model, governed by Equation 3.44 with $\mu = 0.85$, $Y_0 = 0.1$ GPa and $Y_L = 2$ GPa. In this model there is no region of the target where stresses exceed the yield strength; confirming that

the use of the intact-rock strength model during the relaxation stage ensures that collapse does not occur.

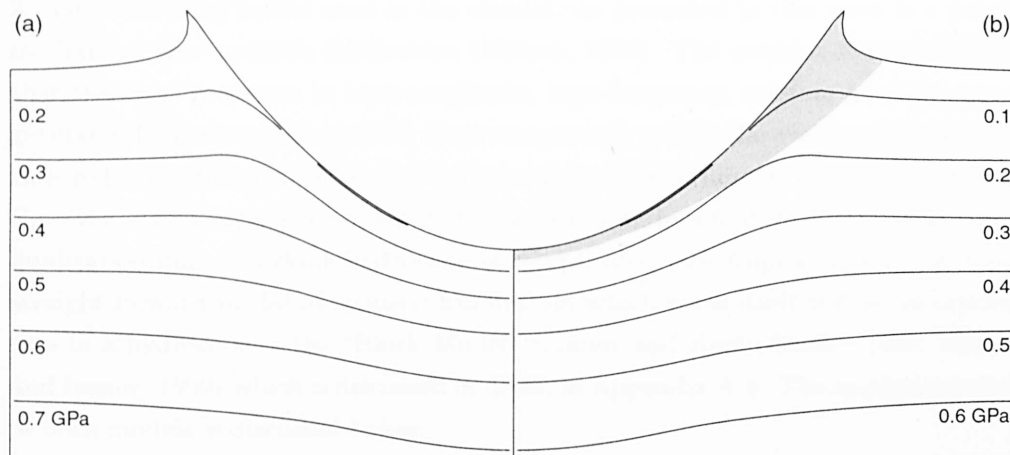


Figure 3.14: Static strength models within the target. (a) Strength contours showing the variation in yield strength around the crater for the intact-rock strength model (Eq. 3.44); (b) Strength contours showing the variation in yield strength around the crater for the damaged-rock strength model (Eq. 3.45). Also shown is the region of the starting mesh that is gravitationally unstable (deviatoric stress exceeds the yield strength) and will collapse (shaded).

Figure 3.14b illustrates the case for a “damaged rock” strength model, governed by Equation 3.45 with $\mu = 0.85$ and $Y_M = 2$ GPa. With this model a small region of the starting mesh is unstable (shaded); deviatoric stress exceeds the yield strength beneath the rim and just below the floor of the crater. However, as shall be discussed in the next chapter, this volume of unstable material is insufficient to permit wholesale collapse; only the crater wall and rim collapses, mimicking the assumed pattern of simple crater collapse, not complex crater collapse. Thus, neither the intact-rock nor the damaged-rock strength models allow significant uplift during the simulated collapse, precluding the formation of central peaks or rings. This result supports the findings of many other workers (Dent, 1973; Melosh, 1977; McKinnon, 1978; Melosh, 1989; O’Keefe and Ahrens, 1993), who conclude that to obtain a dynamical model capable of explaining complex crater collapse, significant weakening of the target surrounding the transient crater must occur.

3.3.4 Implementation of Acoustic Fluidization

Section 3.2.2 described the means by which SALES_2 may be used to model a complicated pseudo-Bingham rheology, which is required when simulating complex crater collapse. To incorporate target weakening and fluidization all that is required is an algorithm to modify or define the yield strength and the fluidized viscosity according

to some physically based mechanism. Potential mechanisms for lowering the strength of a granular medium and permitting fluid-like flow have been discussed in Chapter 2. The weakening model used in the simulations presented in this work is a purely mechanical one: acoustic fluidization (Melosh, 1979). The premise of this model is that transient pressures in high-amplitude, high-frequency, random acoustic waves generated by the expanding shock front temporarily relieve the overburden pressure, thus reducing frictional resistance between fragments within the granular medium. However, the incorporation of the mathematical description of the theory of acoustic fluidization into a working hydrocode is complicated (see Appendix A.1). A more straight forward model of acoustic fluidization which lends itself well to incorporation in a hydrocode is the “Block Model” (Ivanov and Kostuchenko, 1997; Melosh and Ivanov, 1999) which is discussed in detail in Appendix A.2. The implementation of both models is discussed below.

Implementing the Block Model

The theory of acoustic fluidization predicts that at times and positions in the target where the overburden pressure is sufficiently counteracted, the frictional resistance to flow is greatly reduced. This principle is implemented in the hydrocode using the block-model approximation of acoustic fluidization (Ivanov and Kostuchenko, 1997; Melosh and Ivanov, 1999), discussed in section 2.4.2. The block model provides a yield criterion dependent on the presence of vibrational oscillations: $Y = \mu[p - p_v(r, t)]$ (Equation 2.69), which is used to modify the yield strength of the simulated target material. Thus, the greater the amplitude of the pressure vibrations, the lower the frictional resistance to deformation. Equation 2.69 may be incorporated into the constitutive model within SALES.2 and used to determine whether a particular cell in the computational mesh should be fluidized or not. A physical limit is placed on Y , such that if $p_v > p$ then $Y = 0$, to ensure that the yield stress cannot be negative. This reflects the condition that tensile stresses cannot be supported by a granular medium.

The extra mesh-variable required for the implementation of the fluidization algorithm is the amplitude of the pressure vibration p_v , which must be a cell centered quantity. It is thought that the initial vibrational pressure p_{v0} is a fraction f_v of the peak shock pressure at any point in the target ($p_{v0} = f_v p_{shock}$). Thus from Equation 2.33,

$$\begin{aligned} p_{v0} &= f_v p_{max}, & \text{for } r < a_i \\ p_{v0} &= f_v p_{max} \left(\frac{a_i}{r}\right)^n, & \text{for } r > a_i \end{aligned} \quad (3.46)$$

where p_{max} is the maximum shock pressure, a_i is the radius of the impactor and n characterises the spatial decay of the shock wave. Numerical modelling and analyt-

ical work suggests that n is between 1.5 and 4, and may depend on the angle and direction of impact. A simple first-order approximation for p_{max} is discussed in section 2.4.1 and given by Equation 2.43. Thus, assuming $n = 3$, the initial vibrational amplitude as a function of distance from the impact point may be approximated by:

$$\begin{aligned} p_{v0} &= f_v \rho_{0t} v_i / 2 (c_t + S_t v_i / 2), & \text{for } r < a_i \\ p_{v0} &= f_v \rho_{0t} v_i / 2 (c_t + S_t v_i / 2) \cdot \left(\frac{a_i}{r}\right)^3, & \text{for } r > a_i \end{aligned} \quad (3.47)$$

where ρ_{0t} is the target density, v_i is the impactor velocity. c_t and S_t are empirically defined constants that relate the particle velocity behind a shock front to the shock front velocity (see section 2.4.1). The parameter f_v is the first free parameter of the block-model implementation; it is defined as an input parameter.

Equation 3.47 is implemented in SALES_2, allowing each cell in the computational mesh to be assigned an initial vibrational pressure amplitude. For simplicity this maximum vibrational amplitude is assumed to be attained at the same time (during the excavation stage) for all points in the mesh, and that this is the initial condition. The distance r in Equation 3.47 refers to the radial distance from the impact point prior to excavation. Thus, the vibrational pressure assigned to each cell in the transient crater mesh is based on the original position of that point in the target prior to excavation. This is determined using the Z-model approximation of the excavation flow.

The temporal behaviour of impact generated vibrational energy is currently unknown. A recent analytical study by Melosh (1996) provides a rationale for estimating the rate at which pressure vibrations in a mass of rock debris is lost to its surroundings, dissipated by viscous heating and regenerated. The process of regeneration is complex; implementing the required, extremely non-linear equations into the hydrocode have, thus far, been unsuccessful. An attempt to implement these equations is presented in the next section. Furthermore, observational evidence from small explosion craters (Gaffney and Melosh, 1982) suggests that the dominant physical phenomenon controlling the temporal behaviour of the acoustic energy is dissipation not regeneration. Hence, in the simulations discussed here, scattering and regeneration of acoustic energy are neglected, which implies that the vibrational pressure decays exponentially with time. The amplitude of the pressure vibrations p_v in each cell is updated each time-step using the equation:

$$p_v = p_{v0} e^{-t/\tau_v}, \quad (3.48)$$

where τ_v is the *decay time* of the acoustic vibrations; τ_v is the second free parameter of the block-model implementation; it is defined in the input file.

If the Bingham yield criterion is ever satisfied; that is, if the deviatoric stress in the cell exceeds Y_B , then the cell behaves as a fluid with a fluidized viscosity, η . In

other words, while stresses around the transient crater exceed the yield value the code treats the material as a Newtonian fluid, facilitating permanent deformation. The simulated collapse is arrested when the driving stresses fall below the yield strength. η is the third and final free parameter in the block-model implementation of acoustic fluidization used in the work discussed here.

Figure 3.15 illustrates the effect of the acoustic fluidization algorithm on the strength of the target. The left side illustrates target strength without the acoustic fluidization algorithm; the right side illustrates the situation with the acoustic fluidization algorithm applied. The shaded regions represent the initial extent of the unstable region in a typical collapse simulation. The case illustrated is with $f_v = 0.25$; the fluidized region extends over five times as deep as the unstable region determined using the static strength model for rock debris.

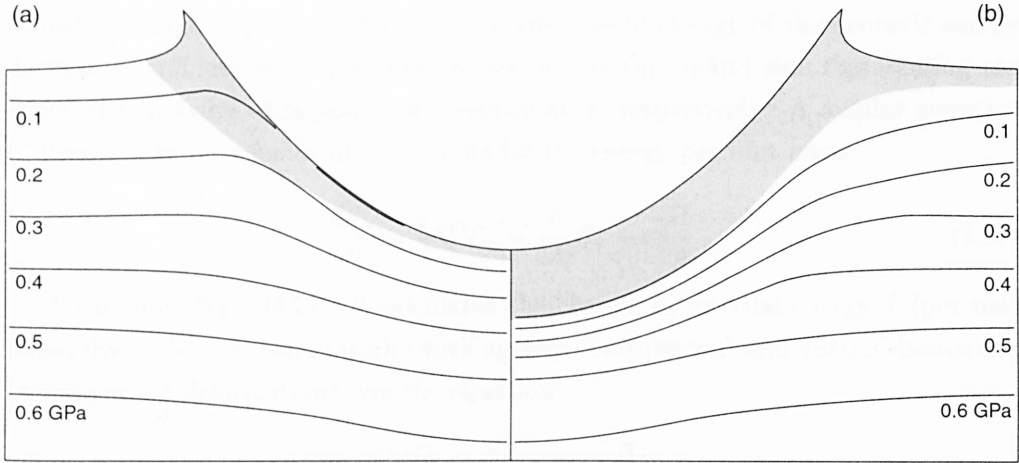


Figure 3.15: The effect of the acoustic fluidization algorithm on the initial strength conditions in the target. (a) Strength contours showing the variation in yield strength around the crater for the damaged-rock strength model (Eq. 3.45); (b) Strength contours showing the variation in yield strength around the crater with the application of the block-model approximation of acoustic fluidization (see text for details). The case illustrated uses a value of 0.25 for f_v . Also shown is the region of the starting mesh that is gravitationally unstable (deviatoric stress exceeds the yield strength) and will collapse (shaded).

Implementing the Original Melosh Model of Acoustic Fluidization

In the original acoustic fluidization model proposed by Melosh (1979), the effective viscosity of the fluidized debris is related to the variance of the acoustic-noise pressure field by Equation 2.62. This variance is a function of the acoustic energy density $\mathcal{E}_v$ (per unit mass):

$$\sigma = c\rho\sqrt{2\mathcal{E}_v}. \quad (3.49)$$

Thus, to implement the full acoustic fluidization formulation into a hydrocode, the acoustic energy per unit mass in each cell must be assigned and monitored. The initial assignment of the acoustic energy per unit mass may be determined in a manner similar to the assignment of the pressure amplitude for the block model. The acoustic energy E_v is related to the amplitude of the pressure vibrations p_v by the equation:

$$E_v = \frac{p_v^2}{2\rho K} \quad (3.50)$$

where K is the bulk modulus and ρ is the density. Thus, the acoustic energy per unit mass $\mathcal{E}_v$ for a cell of mass m_c is simply $\mathcal{E}_v = E_v/m_c$.

The major improvement provided by the implementation of the full treatment of the acoustic fluidization model, is in modelling the temporal behaviour of the acoustic energy. Equation 2.63 describes the rate of change of the acoustic energy density, as influenced by the three terms on the right-hand side representing the effect of scattering, dissipation and regeneration respectively. A similar equation (differing only by a factor of $1/\rho$) exists for the energy per unit mass:

$$\frac{d\mathcal{E}_v}{dt} = \frac{\xi}{4}\nabla^2\mathcal{E}_v - \frac{c}{\lambda Q}\mathcal{E}_v + e\frac{\dot{\epsilon}\Pi}{2\rho}. \quad (3.51)$$

Each time-step, SALES.2 calculates the change in internal energy I (per unit mass) due to both pressure work (work against compression) and viscous dissipation (work against deformation), via the equation:

$$\frac{d(\rho I)}{dt} = \frac{d(p+q)}{dt} + \frac{\Pi\dot{\epsilon}}{2}. \quad (3.52)$$

Thus, it is relatively straight-forward to couple the internal energy equation with the acoustic energy equation: The heat dissipation term in Equation 3.51 must be included in the change in internal energy, and the viscous dissipation term in the internal energy equation (3.52) must be reduced to account for the viscous work that regenerates acoustic energy. Equation 3.52, then becomes:

$$\frac{d(\rho I)}{dt} = \frac{d(p+q)}{dt} + (1-e)\frac{\Pi\dot{\epsilon}}{2} + \frac{c}{\lambda Q}\mathcal{E}_v. \quad (3.53)$$

To complete the implementation of the acoustic energy equation, all that remains is to adapt SALES.2 to compute the scattering of acoustic energy between cells. Because the collapse simulations run with cylindrical symmetry, the scattering in the “hoop” direction cancels out. Thus, the equation for the rate of scattering $d\mathcal{E}_s/dt$, to be implemented in SALES.2 is:

$$\frac{d\mathcal{E}_s}{dt} = \frac{\xi}{4} \left(\frac{d^2\mathcal{E}_v}{dx^2} + \frac{d^2\mathcal{E}_v}{dy^2} \right). \quad (3.54)$$

The modifications made to SALES_2 to incorporate the above equations for the full treatment of the acoustic fluidization model are described in Appendix B.3. It should be noted that this algorithm is not used in any of the subsequent hydrocode simulations of complex crater collapse. Attempts to implement the full acoustic fluidization model into SALES_2 have been unsuccessful during the course of this research due to the extreme non-linearity of the model. The algorithm is presented to aid future work in this direction.

3.4 Summary of the SALES_2 solution algorithm used in this work

The solution algorithm for the hydrocode used during the work discussed in this thesis is summarized in Figure 3.16. The procedure for simulating a particular collapse event is as follows:

1. A mesh of appropriate dimensions and initial conditions is generated. The mesh geometry for a particular transient crater is defined by the diameter and depth of the transient crater to be studied and the desired resolution of the model. The mesh is then exposed to the relevant external force (gravity) for the target surface during the “relaxation stage”. In this phase of the simulation, the internal mesh variables of pressure, density and so on reach a steady state, before being recorded to a dump file. This file is then read by the hydrocode to begin the collapse simulations.
2. Upon reading the dump file and assigning initial values for all variables in the mesh, the hydrocode begins its “Lagrangian cycle”; the vertex velocities are modified sequentially by the effect of: (1) pressure forces, (2) external forces (gravity), (3) the artificial viscosity, (4) the alternate node-coupler, and (5) the constitutive model. At the end of the cycle these velocity contributions are recorded for use in the next cycle and used to reposition each vertex in the mesh by multiplying each velocity by the time step.
3. The pressure in any given cell is defined by the equation-of-state, which relates the density and internal energy to the pressure. The equation of state used in this work is the Tillotson equation of state which is characterized by a number of constant parameters.
4. The rheological model used by the hydrocode to represent the target material is that of a pseudo-Bingham substance. This is characterized by a yield strength Y , a viscosity η and a shear modulus G . The strain is determined for each cell; then, assuming that the cell is elastic, the stress-response to this strain

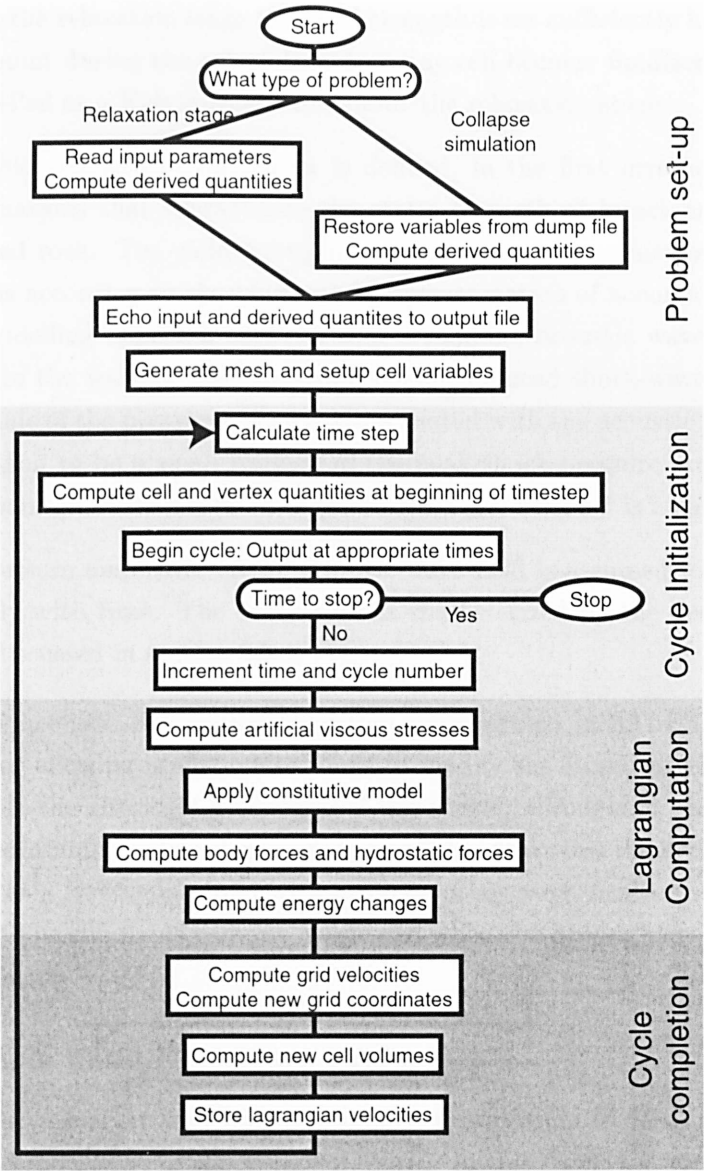


Figure 3.16: Solution algorithm of the hydrocode used to simulate complex crater collapse.

is computed. If an invariant measure of this stress exceeds the yield strength then the cell is assumed to be “fluid”; the elastic stress is reduced accordingly and the viscous stress is determined from the strain rate, assuming a certain fluidized viscosity. If the stress does not exceed the yield strength, the elastic stress remains the same, and is further supplemented by a viscous stress. In this case, however, the viscosity is not the fluidized viscosity but the Kelvin-solid viscosity. The two viscosities, and the shear modulus are defined as input parameters; the yield strength is defined according to a separate algorithm.

During the relaxation stage the yield strength is set sufficiently high such that at no point during the calculation does any cell become fluidized. The target is modelled as a Kelvin-solid throughout the relaxation stage.

5. The yield strength of the target is defined, in the first instance, by one of two equations that characterize the static strength of intact or completely-fractured rock. The yield strength is reduced in the presence of acoustic vibrations according to the block-model approximation of acoustic fluidization. The modelling approach assumes that a random, acoustic wave field is generated in the wake of the expanding impact-induced shock-wave. The initial amplitude of the pressure vibrations associated with the acoustic pressure field p_v is taken to be a small fraction of the peak shock pressure attained during the expansion of the shock wave ($p_v = f(p_{shock})$, where f is a small fraction).
6. The pressure amplitude of the acoustic-wave field is assumed to decay exponentially with time. The decay time is characterized by the decay constant, τ_v , as discussed in section 3.3.4.

Thus, the acoustic fluidization algorithm incorporated in SALES_2 provides a mechanism for allowing acoustic vibrations to modify the Bingham yield strength, which controls the rheologic behaviour of the target, throughout the simulation. While stresses around the transient crater exceed the yield value the hydrocode treats the material as a Newtonian fluid with a constant viscosity, facilitating permanent deformation. The simulated collapse is arrested when the driving stresses fall below the yield strength.

3.4.1 Simple Fluid Half-Space Problem

When using any abstract modelling technique it is important to verify the accuracy, reliability and robustness of the results obtained. In this thesis the specific problem to be simulated is the collapse of a cavity in a media with a complicated rheology. It is thus logical to ensure that, before embarking on complicated crater collapse simulations, the code can produce reasonable results to a much simpler problem.

The simple problem chosen to test the proficiency of the code used in this thesis is the collapse of a crater in a fluid half-space. The procedure for simulating the collapse is to: (1) generate a mesh to represent the transient crater; (2) generate an initial stress state in the mesh during the relaxation stage, where the strength of the target is large enough to prevent collapse; (3) modify the rheology of the mesh by replacing the Kelvin-solid rheology with a Newtonian-fluid rheology.

Figure 3.17 shows eight stages of a test simulation for the collapse of a 100-km diameter crater in a fluid of viscosity 10^9 Pa s. Collapse proceeds via the inward slumping of the crater rim, accompanied by upward collapse of the crater floor.

In the simulation illustrated, the collapsing fluid eventually overshoots the original surface level, before collapsing downwards to generate a secondary cavity. This cycle repeats until the kinetic energy of the collapsing fluid is entirely dissipated and the fluid returns to a flat surface.

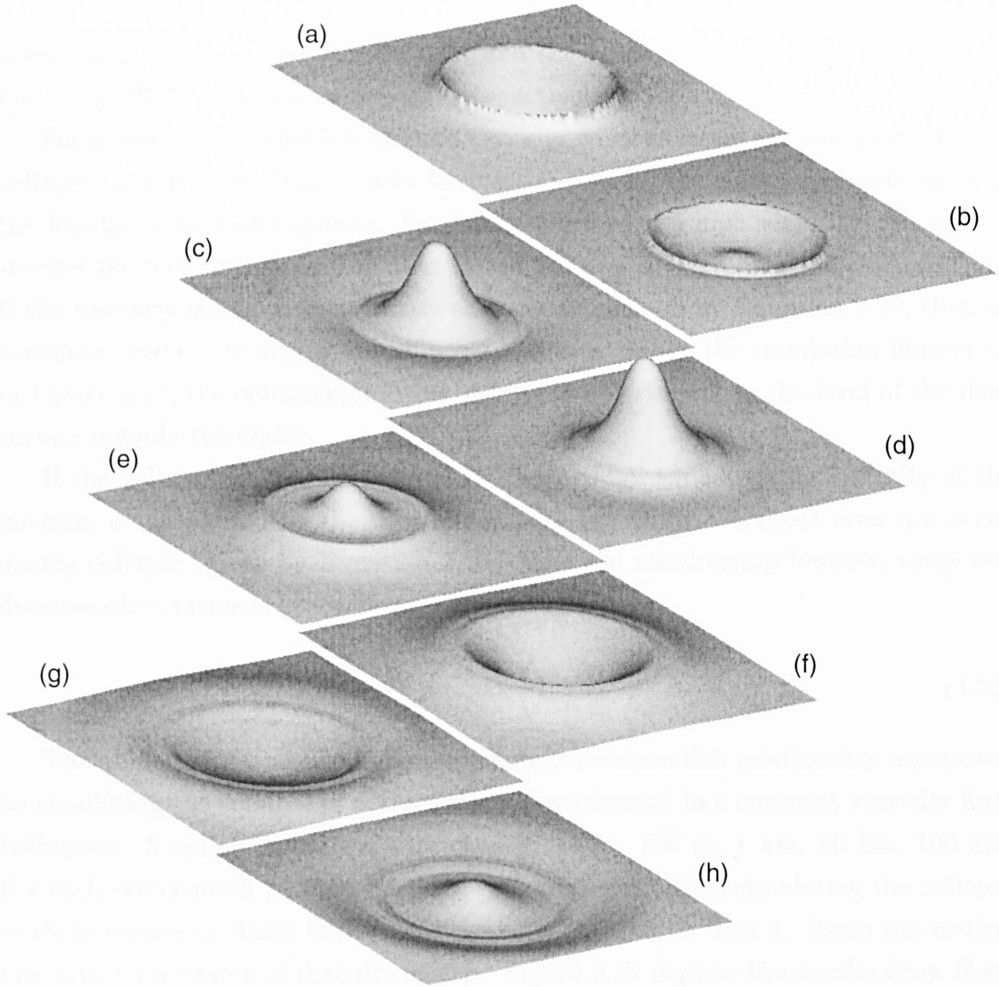


Figure 3.17: Illustration of a hydrocode simulation of the collapse of a bowl-shaped crater in a viscous half-space. Eight stages of an example hydrocode simulation of the test problem detailed in the text. The simulation illustrated is for a 100-km diameter crater, formed in a constant viscosity half-space of viscosity 10^9 Pa s. The surface profile is recorded at intervals during the simulation, then extrapolated round through 360 degrees about the axis of symmetry to produce the illustrations shown. (a) depicts the starting form of the crater; (b) the surface of the mesh after 60 s; (c) 120 s; (d) 180 s; (e) 240 s; (f) 300 s; (g) 360 s; (h) 400 s.

As discussed in section 2.3.4, theory predicts that the collapse time t_{coll} of a crater formed in a constant-viscosity fluid half-space is controlled by either viscous forces or inertia; written formally as:

$$t_{coll} = \max(t_{free}, t_{visc}), \quad (3.55)$$

where t_{free} and t_{visc} , the free-fall time and the viscous collapse time, are given by $t_{free} = \sqrt{2H_t/g}$ and $t_{visc} = 2\eta/\rho g H_t$, respectively.

For a half-space with a low enough viscosity, viscous forces are negligible and the collapse time is controlled entirely by inertia; that is, the collapse time is equal to the free-fall time under gravity. For a half-space with a high viscosity, the viscous stresses induced during fluid motion retard the flow and prolong the collapse time. If the viscosity is higher than a critical viscosity, defined by Equation 2.20, then no overshoot will occur during the collapse. That is, unlike the simulation illustrated in Figure 3.17, the collapsing central uplift will not rise above the level of the fluid surface outside the crater.

If the collapse time is normalised by the free-fall time and the viscosity of the medium is normalised by the critical viscosity for which overshoot does not occur during collapse (given by Equation 2.20), then the relationship between these two dimensionless ratios is given by:

$$\frac{t_{coll}}{t_{free}} = \max\left(1, \frac{4\eta}{\eta_{crit}}\right). \quad (3.56)$$

The ability of the SALES_2 hydrocode to reproduce this relationship was tested by simulating the collapse of various size craters formed in a constant viscosity fluid half-space. 5 crater diameters were chosen: 10 m, 100 m, 1 km, 10 km, 100 km. For each crater-mesh geometry several runs were performed simulating the collapse of these craters in fluids with viscosities of 0, 0.1, 0.5, 1. and 2. times the critical viscosity for a crater of that dimension. Figure 3.18 depicts the results from these simulations. The graph shows the relationship between normalised viscosity and collapse time for each calculation. The normalised collapse time is defined as the time taken for the central uplift to reach within half a cell-width of the level of the fluid surface outside the crater, divided by the theoretical free-fall time under gravity. The theoretical prediction for the collapse time against viscosity (Equation 3.56) is shown as a solid line for comparison.

At viscosities above the critical viscosity, the simulation predictions for the collapse time are not in close agreement with theoretical prediction. Furthermore, at these high viscosities there is a dependence of collapse time on crater diameter, which is particularly pronounced for the largest simulated collapse of a 100-km diameter transient crater. Why the simulation results do not agree well with theoretical pre-

diction for this range of viscosities is unclear; however, the dependence of collapse time on crater diameter is probably due to several factors.

The dependence of collapse time on crater diameter is, in part, due to the criterion for defining when the crater has finished collapsing. This is defined to be one-half of a cell height from the surface. Thus, relatively speaking, the greater the cell dimension the earlier the collapse will terminate. Cell dimension in the simulations is a linear function of crater size; meshes defining the largest crater geometries use the largest cell dimensions and, therefore, would be expected to finish collapsing sooner. However, this effect should become more pronounced for greater viscosities, which it does not. Furthermore, the collapse time predictions for the largest crater diameter are significantly departed from the predictions for the other crater sizes.

The dependence of collapse time on crater size may also be due to differences in initial conditions resulting from varying degrees of compression during the relaxation stage. The amount of compression that accumulates during the relaxation stage is a function of the rigidity and the Kelvin-solid viscosity of the material used to represent the fluid half-space during this stage of the calculation. Scaling these parameters with crater dimension is complicated and inevitably leads to discrepancies between the relative mesh geometries for simulations at different crater dimensions. These differences in crater geometries for different sized craters translate to different normalised collapse times.

The last potential reason for the dependence of collapse time on crater diameter is the fact that at larger crater diameters the viscosity of the fluid half-space becomes high enough to affect the time step (see section 3.1.5), which leads to changes in the application of the alternate-node coupler. For example, in the case of the 100-km diameter crater collapse simulation the time step is restricted by the momentum diffusion criterion (Equation 3.17). As a consequence, the application of the alternate-node coupler is increased in frequency, but reduced in magnitude, such that the overall effect of the velocity diffusion is as close as possible to that for a simulation with an unrestricted time step. However, the difference in overall effect of the alternate-node coupler can never be truly negligible between simulations with both different time-step size and cell dimension. The large discrepancy between the collapse time for the 100-km diameter crater and the collapse time for the other craters is almost certainly due to the effect of the alternate-node coupler as implemented in SALES_2.

The simulation results are in reasonable agreement with the simplified theory of Equation 3.56, for viscosities less than the critical viscosity. There is no distinct transition to the inertial regime predicted by the simulation results; however, this result is, in part, due to the definition used to determine when the crater has collapsed. For low viscosities the central region of the crater crosses the initial fluid-surface level while material outside it is still collapsing upwards. Consequently, the

collapse time for low viscosities is underestimated. The simulation results also show little dependence on crater size for fluid half-space viscosities below the critical viscosity. Thus, within the viscosity range appropriate for the study of complex crater collapse, the SALES.2 hydrocode is capable of accurately simulating the collapse of a crater in a constant viscosity fluid half-space.

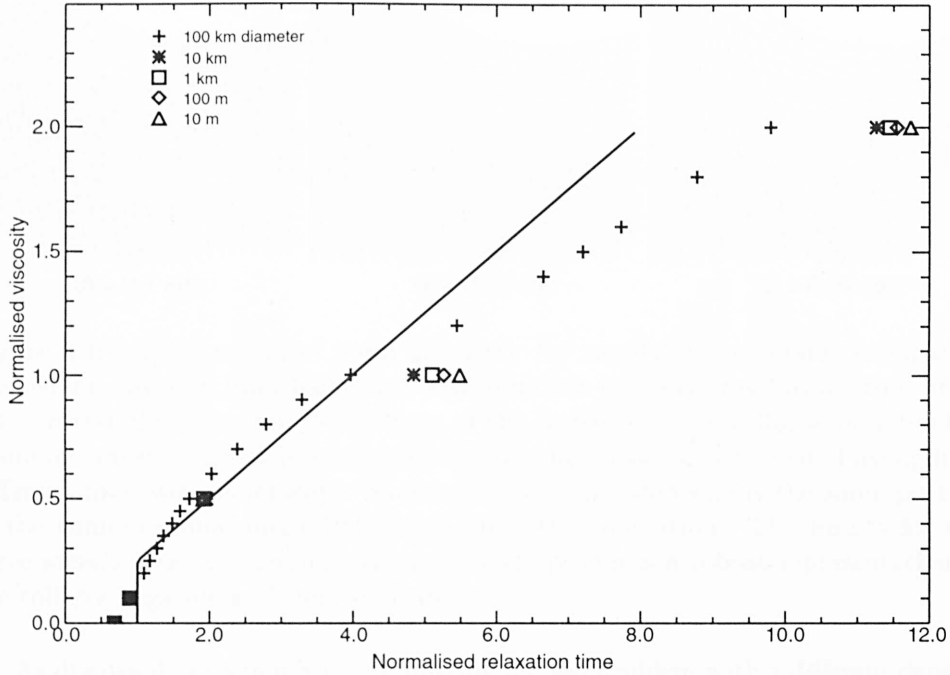


Figure 3.18: Comparison between hydrocode simulation results and theory for crater collapse in a fluid half-space. Graph of normalised viscosity against normalised collapse time. Viscosity of the fluid half-space is divided by the critical viscosity for no overshoot. Collapse time is divided by the free-fall time under gravity. Symbols denote results from five simulation runs with different mesh dimensions. Solid line represents analytical relation between viscosity and collapse time described by Equation 3.56

3.4.2 Resolution and Time-Step Testing

When carrying out numerical analysis of physical phenomena it is common practice to test the effect of the resolution, in space and time, used during the calculation. If an observed result is independent of the density of mesh divisions and the frequency of sampling in time, then it is unlikely to be a numerical artifact. To highlight any resolution effects inherent to the SALES.2 solution algorithm, the test problem of a collapsing crater in a fluid half-space was performed with various time steps and cell densities. Figure 3.19 illustrates the effect of temporal resolution on the simulation results. Shown is the variation in mesh geometry at a given time during the simulation of the collapse of a 100-km diameter transient crater in a constant

viscosity half-space, for which the only variable parameter is the time step increment. The differences in mesh geometry are negligible, implying no dependence of the simulation results on the time step.

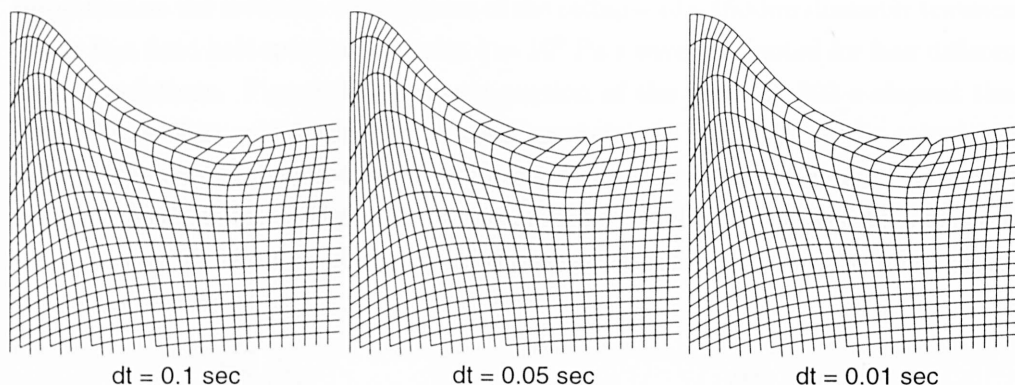


Figure 3.19: Comparison of mesh geometry for simulations of crater collapse in a constant viscosity fluid half-space with different temporal resolution (time step, dt). To test the time step dependence of the hydrocode, the collapse of a 100 km diameter transient crater in a constant viscosity half-space was simulated using three different time step increments. Shown for each time step size is the same portion of the computational mesh 100 seconds into the simulation. The results for the three simulations are identical; the hydrocode produces a robust representation of the collapse regardless of time-step size.

As discussed in section 3.1.5, simulating a given problem with a different density of cells in the mesh using SALES_2 requires care when applying the artificial node-coupler. The artificial node-coupling procedure implemented in SALES_2 ensures that the velocity vectors of adjacent nodes do not differ dramatically in magnitude or direction, which smooths out variations in the velocity field over the smallest mesh-resolvable wavelengths, $2dx$. This guards against degenerate mesh deformations while preserving the character of the longer-wavelength flow variations being modelled. As a consequence, however, there is a problem for calculations using a mesh with a low resolution (a small number of large cells), for which the $2dx$ -wavelength velocity field is close to the velocity field of specific importance to the simulation. In cases such as these, the alternate node-coupler will alter the details of the simulation results more severely than for calculations using a higher resolution mesh. Thus, to produce consistent results between calculations of different resolutions, the use of the alternate node-coupler must be adjusted accordingly. This is achieved by multiplying the alternate node-coupler contribution to the vertex velocities by the factor dt/dt_{cou} , where dt_{cou} is the Courant limit for the time step (Equation 3.15), which is proportional to the cell dimension. Thus, provided the time step, dt , used during each calculation is the same, the amount of coupling in a low-resolution calculation

is reduced compared to that used in simulations with the highest feasible resolution by the ratio of the cell dimensions.

The test problem of section 3.4.1 was used to demonstrate the effect of spatial resolution on the solution. Simulations of the collapse of a 100-km diameter transient crater in a fluid half-space of viscosity $\eta = 10^9$ Pa s were conducted for four different mesh resolutions. Figure 3.20 shows a portion of the mesh at 200 s elapsed time during these four calculations. The mesh geometry at this stage of the calculation differs only very slightly for each resolution. Furthermore, the difference in vertical uplift is due to slightly different initial conditions for each of the different simulations, which is a result of different degrees of compression experienced by the mesh during the relaxation stage.

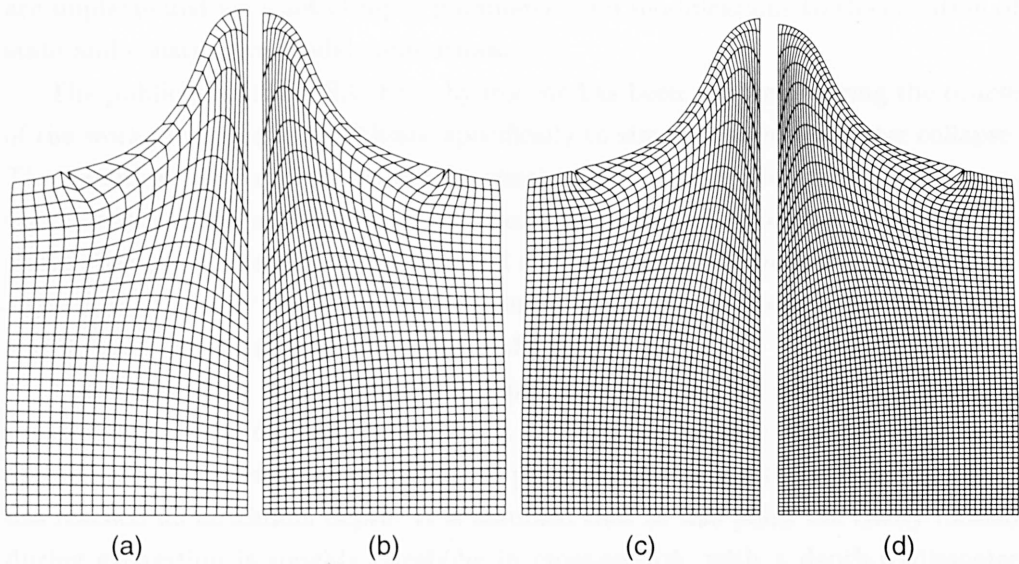


Figure 3.20: Comparison of mesh geometry for simulations of crater collapse in a constant viscosity fluid half-space with different spatial and temporal resolution. To test the spatial-resolution dependence of the hydrocode, the collapse of a 100 km diameter transient crater in a constant viscosity half-space was simulated using four different mesh resolutions. (a) 45 cells by 30 cells; (b) 60 by 40; (c) 75 by 50; (d) 90 x 60.

3.5 Summary

Complex crater collapse is a complicated and highly dynamic process, which appears to involve the macroscopic fluid behaviour of rock debris. As such it must be modelled by a computational technique for describing the dynamics of continuous media: a hydrocode. The hydrocode used in this thesis is based on the SALE hydrocode; a straightforward computer code for modelling fluid flow at all speeds.

SALE stands for Simple Arbitrary Lagrangian Eulerian, because it can model fluid flow using either the Lagrangian or Eulerian material description of continuous media. That is, SALE can model the flow of the material by tracing the position of points within the material with time, or by tracing the flow of material through a fixed grid with time. The Lagrangian description is the preferred method for modelling complex crater collapse, because it accurately tracks the free surface and records the history of points within the material.

SALE has been adapted for the study of impact related phenomena. Its successor, SALES_2, is capable of simulating melting, vaporisation, dynamic fragmentation, multiple material properties and rheologies within the mesh, and the complicated rheology required to best describe complex crater collapse. The additions are implemented via a set of input parameters and modifications to the equation of state and constitutive model subroutines.

The publicly available SALES_2 hydrocode has been adapted, during the course of the work presented in this thesis, specifically to simulate complex crater collapse. The modifications required include the construction of an appropriate mesh geometry to replicate the initial form of the transient crater, an algorithm to predict the static strength properties of the material surrounding the transient crater, and an algorithm to modify these properties using a model for the temporary weakening and fluidization of material surrounding the crater.

The stages of the impact prior to collapse are approximated using the Z-model of the excavation flow, discussed in section 2.1. Thus, the collapse simulations begin at approximately the moment of maximum penetration – when the expanding cavity has reached its maximum depth. It is assumed that at this point the cavity formed during excavation is roughly parabolic in cross-section, with a depth-to-diameter ratio of 1:3. The popular convention is to term the cavity at this point the transient crater.

At the onset of the collapse simulations, the transient over-pressure due to the impact-generated shock wave is assumed to be entirely dissipated and the target material stationary. Thus, the numerical model essentially considers the competition between the gravity driven collapse of the unstable transient crater and the inherent material strength properties of the post-shock target. The yield strength of the target is defined for intact rock or completely broken rock. Thus, the target material may be assumed to have been entirely fractured during the passage of the shock wave and thus have no cohesive strength. In this case the yield strength of the target is assumed to obey a dry friction law ($Y = \mu p$, where μ is the coefficient of internal friction and p is pressure) modified to approach a constant plastic limit at depth. In the case of an intact target, this yield strength is supplemented by an additional cohesive strength.

The yield strength Y is used by the hydrocode to control the elastic stress limit of the modelled target material; that is, at stresses above this limit the hydrocode considers the modelled target material to be fluid-like, with a constant Newtonian viscosity. Below the yield strength, the hydrocode treats the modelled material as elastic; no permanent strain is allowed. In essence, therefore, the simulations consider the target material to be a Bingham fluid, parameterized by the yield strength of the target and the fluidized viscosity.

Hydrocode Simulations of Chicxulub Crater Collapse

Hydrocode simulations of crater collapse have been performed by several authors, including:

• *Chicxulub Crater Collapse* (Chicxulub Crater Collapse Hydrocode Simulations)

A series of hydrocode simulations of the Chicxulub crater collapse have been performed using the hydrocode *SAUL* (Stress Analysis Using Lagrangian) to model the collapse of a crater of diameter 180 km. The simulations were performed using a range of parameters, including different values for the yield strength and viscosity of the target material. The results show that the collapse of the Chicxulub crater is a complex process, involving the formation of a large central peak and the collapse of the surrounding rim. The simulations also show that the collapse of the Chicxulub crater is a highly non-linear process, with the final state of the crater depending on the initial conditions and the parameters of the hydrocode.

The results of the hydrocode simulations show that the collapse of the Chicxulub crater is a complex process, involving the formation of a large central peak and the collapse of the surrounding rim. The simulations also show that the collapse of the Chicxulub crater is a highly non-linear process, with the final state of the crater depending on the initial conditions and the parameters of the hydrocode. The results of the hydrocode simulations show that the collapse of the Chicxulub crater is a complex process, involving the formation of a large central peak and the collapse of the surrounding rim. The simulations also show that the collapse of the Chicxulub crater is a highly non-linear process, with the final state of the crater depending on the initial conditions and the parameters of the hydrocode.

Chapter 4

Hydrocode Simulations of Chicxulub Crater Collapse

Knowledge is proud, that he has learned so much,
Wisdom is humble, that he knows no more.

Winter Walk at Noon, *William Cowper*

A complete understanding of complex crater collapse has long been an enigmatic issue in planetary science. 50 years of study of the impact process have failed to produce a predictive, quantitative model of large impact crater formation (Melosh and Ivanov, 1999). The lack of such a model has been due in no small part to the scarcity of large impact craters on the Earth, and to the near impossibility of probing the subsurface on other planets. This situation changed following the discovery of the buried Chicxulub impact structure in Yucatan, Mexico. The Chicxulub Seismic Experiment obtained data that clearly image the third dimension of a large complex crater and provide new insight into the kinematics of complex crater collapse (Morgan et al., 1997; Morgan and Warner, 1999). Thus, a logical stepping-stone in developing a universal model for large impact crater formation is to produce a predictive dynamical model of a Chicxulub-scale event, for which data exists that can be used to test and refine the model (Collins et al., 2002).

One of the most profound inferences from the seismic data regards the nature and formation of the peak ring (Brittan et al., 1999). Peak rings, like the one observed at Chicxulub, are defined as a concentric ring of rugged hills, peaks and massifs that stand proud of an otherwise flat crater floor, inside the crater rim (Grieve et al., 1981). On extra terrestrial bodies, where erosion rates are extremely slow, countless examples of peak-ring craters exist. On Earth, however, there are very few unequivocal examples; those craters thought to originally possess peak-ring structures have all been eroded to varying degrees and exhibit apparently contradictory evidence

for the nature of peak-ring material at geological outcrop. Some possess a narrow central uplift (Vredefort, Manicouagan – for example), while others show a wider annular ring of uplifted basement (Popigai, Ries). Consequently, there is no single consensual model for the formation of peak rings; however, there is a general agreement that the collapse of the rebounding central uplift plays an important role (Melosh, 1989; Morgan et al., 2000).

Seismic imaging of the Chicxulub impact structure provides a new source of information for understanding peak rings. The peak ring at Chicxulub, although probably modified to some extent, is observable as a topographic feature, making its interpretation more significant than many eroded peak rings found on the surface of the Earth. Furthermore, a model for the formation of peak rings has been inferred from the seismic reflection data below the peak ring (Brittan et al., 1999). The seismic data appear to show downward and inward radial collapse of the transient cavity in the outer crater, and upward and outward collapse within the central structurally uplifted region. This suggests that peak rings are formed by the collision or interference of these two flow regimes.

This chapter reports on the results of hydrocode simulations of the collapse of a Chicxulub-scale crater. In the first section the modelling procedure, discussed in detail in the previous chapter, is summarized. The second part of the chapter discusses the choice of model parameters used for all the simulations, based on the current understanding of the pre-impact target stratigraphy at Chicxulub, the size of the Chicxulub impact and the mechanics of crater collapse on such a scale. In the following section the simulation results are described; a general paradigm for the simulations is presented, which is in accordance with the paradigm for complex crater collapse discussed in section 2.3.5. This section also discusses several suites of model runs to test the influence of the three “free” model parameters, plus the effect of compositionally different target models and different starting conditions. Finally, the model results are compared with seismic data from the Chicxulub impact structure and a dynamical model for the collapse of the Chicxulub crater and the formation of internal (peak) rings is presented.

4.1 Approach

As discussed in Chapters 1 and 2, complex crater formation is a complicated and highly dynamic process. The abrupt deceleration of a large meteorite as it collides with a planetary surface transfers an immense amount of kinetic energy from the impacting body to the target. As a consequence, the target and impactor are rapidly compressed and heated. Between the compressed and uncompressed material, a shock wave is created, which propagates away from the point of impact. In the wake of the expanding shock wave the target is comprehensively fractured, shock-heated,

shaken and set in motion – ultimately leading to the excavation of a cavity many times larger than the impactor itself. However, the processes operating during the halting of the impactor, and the subsequent excavation of the cavity, do not have the most profound influence on the final crater structure. Instead, the final morphology of the crater is believed to be primarily the result of the collapse of a geometrically simple, bowl-shaped cavity produced during excavation (Melosh and Ivanov, 1999). The approach of this research, therefore, has been to model in detail only the collapse stage of the Chicxulub impact event.

The collapse calculations discussed in this chapter have been performed using the methodology outlined in Chapter 3 (specifically section 3.4) and the accompanying Appendix B. The computer code used is based on the SALE hydrocode (Amsden et al., 1980), which was originally developed to study inviscid fluid flow. The version used in this work, SALES_2, has been extensively modified (for example Melosh et al., 1992; Ivanov et al., 1997) to model both visco-elastic and purely viscous rheologies. For the simulations of complex crater collapse discussed in this thesis, SALES_2 is used with a purely Lagrangian method for describing the fluid flow; that is, the reference frame of the calculation moves with the flow of material. This technique allows an accurate representation of the free surface which may become extremely contorted. However, Lagrangian calculations do have the disadvantage of breaking down when material deformation becomes extreme.

SALES_2 is capable of modelling the behaviour of the complicated rheological model that offers the best approximation to that during complex crater collapse: the Bingham substance (Collins et al., 2002). This model is parameterised by the Bingham yield strength, Y_B and the fluidized viscosity η . The acoustic fluidization algorithm incorporated in SALES_2 provides a mechanism for allowing acoustic vibrations to modify the Bingham yield strength. These pressure vibrations are characterized by an initial amplitude p_{v0} (which is a fraction f_v of the peak shock pressure experienced by the target) and a decay time τ_v . Thus, the vibrations control the rheologic behaviour of the target throughout the simulation: while the vibrations are strong enough to lower the yield strength sufficiently that stresses around the transient crater exceed the yield value, the hydrocode treats the material as a Newtonian fluid with a constant viscosity. In this case, permanent deformation is facilitated and collapse proceeds. The simulated collapse is arrested when the yield strength rises due to the decay of the pressure vibrations and the driving stresses fall below the yield strength. The collapse simulations are controlled, therefore, by the free parameters f_v , τ_v and η .

4.2 The Chicxulub Crater

The Chicxulub impact structure, introduced in Chapter 1, was formed 65 Ma ago in a shallow marine environment, which now forms part of the Gulf of Mexico. Since its formation, the crater has been in a stable depositional environment, leading to its burial beneath about one kilometer of Tertiary sediments. The crater lies partly off-shore from the Yucatan Peninsula, Mexico (see Figure 4.1); on-shore its only surface expression is a semi-circular ring of cenotes (sink holes in the Tertiary sediments above the crater, Pope et al., 1996).

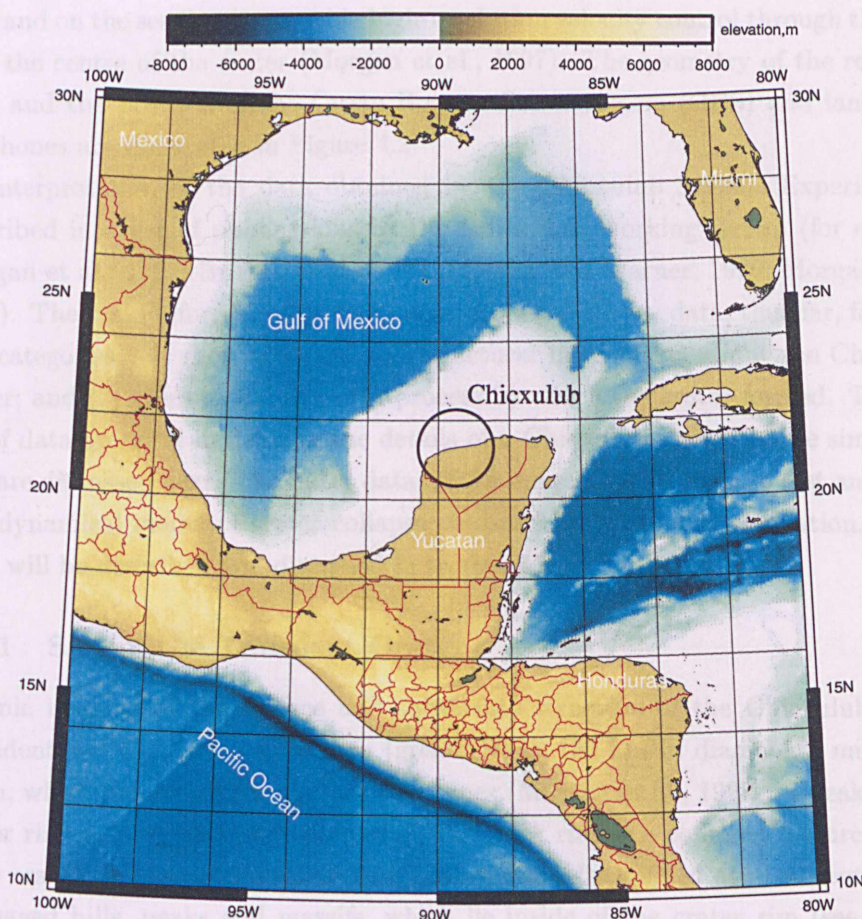


Figure 4.1: Map of the Yucatan Peninsula, Gulf of Mexico. The Chicxulub crater lies partly off-shore and partly on-shore; it is buried beneath about one kilometer of Tertiary sediments.

The quiescence of the geological setting in which Chicxulub exists has prolonged the crater's existence on the Earth, and saved it from extreme tectonic and erosional

degradation. Thus, despite the obvious difficulties in imaging a crater buried nearly a mile below the sea-floor, the Chicxulub impact structure offers a rare opportunity to observe the near-pristine form of a large, terrestrial impact crater. The Chicxulub Seismic Experiment, conducted by the British Institutions Reflection Profiling Syndicate (BIRPS) in October 1996, has provided unique images of the surface morphology, and sub-surface structure of the crater. These images present much information for the impact cratering community to digest.

During the Chicxulub Seismic Experiment about 650 km of marine seismic reflection profile across the crater were recorded to 18 s two-way travel-time, which corresponds to a reflection-depth below the level of the crust-mantle boundary (Moho). The shots fired to generate the seismic data were also recorded at large offsets on land and on the sea floor to provide high-resolution velocity control through the crust over the centre of the crater (Morgan et al., 1997). The geometry of the reflection lines and the position of the Ocean Bottom Seismometers (OBS) and land-based geophones are illustrated in Figure 4.2.

Interpretation of the data obtained by the Chicxulub Seismic Experiment is described in a list of publications by the Chicxulub Working Group (for example Morgan et al., 1997; Brittan et al., 1999; Morgan and Warner, 1999; Morgan et al., 2000). The main information to be gleaned from the survey data, thus far, falls into two categories: (1) morphological and structural information about the Chicxulub crater; and (2) clues to the dynamic process by which the crater formed. The first set of data are vital in defining the details of a Chicxulub-scale collapse simulation and are discussed next. The latter data offer a unique opportunity to test and refine any dynamical model of crater collapse obtained by hydrocode simulation. These data will be described and discussed in section 4.4.

4.2.1 Size of the Transient Crater

Seismic imaging of the surface and subsurface structure of the Chicxulub crater has identified Chicxulub as a large (greater than 150 km in diameter), multi-ring basin, which exhibits three distinct ring types (Morgan et al., 1997): a peak ring; a crater rim; and at least one outer ring. The peak ring is a common feature among large craters in the Solar System and is conventionally defined as a concentric ring of rugged hills, peaks and massifs, which lie inside of the crater rim (see section 1.1.2). At Chicxulub the peak ring is an almost-symmetric topographic feature protruding from the crater floor, with a diameter of about 80 km. The second ring type, the crater rim, is conventionally defined as the head-scarp of the outermost (and invariably widest) terrace of the crater wall. At Chicxulub the crater rim is located at a diameter of about 150 km; it is an asymmetric fault scarp, analogous to a normal fault in standard tectonics. The final ring type, the external or outer

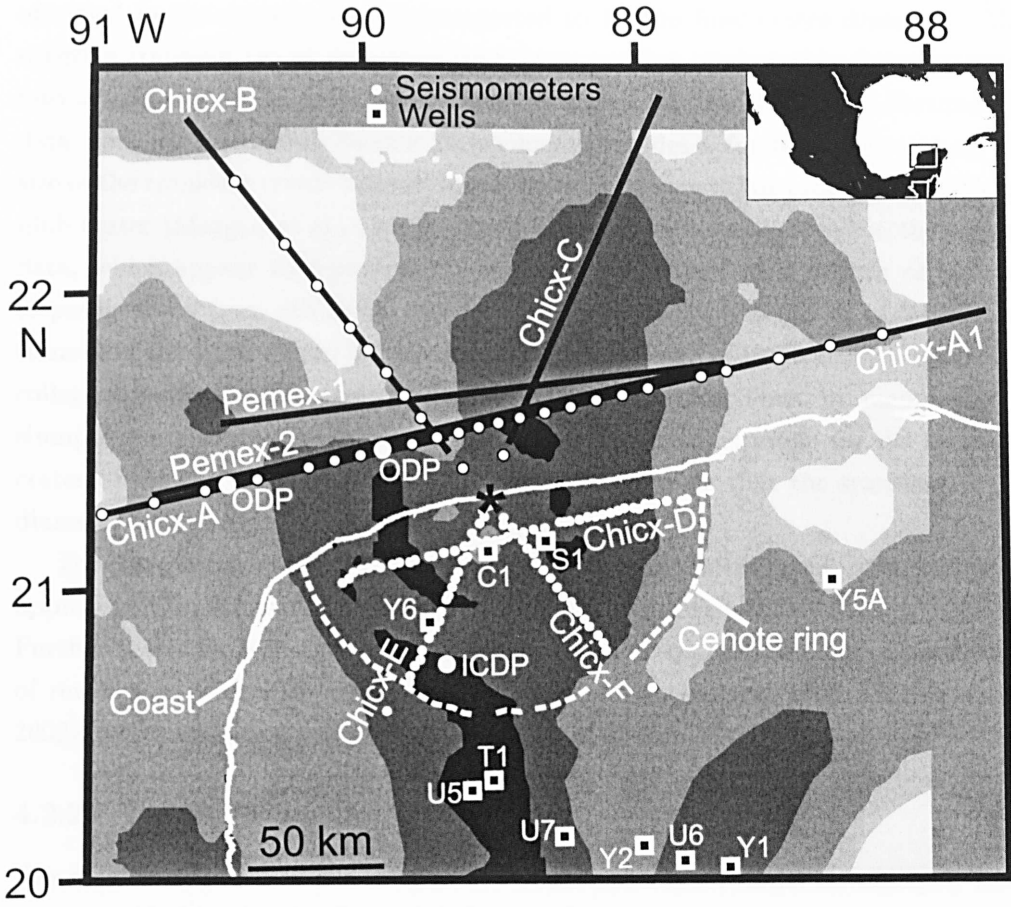


Figure 4.2: Geometry of the Chicxulub Seismic Experiment. Shown are the reflection profile lines, the Ocean Bottom Seismometers and the ground-based geophone stations.

ring, is peculiar to the multi-ring basin classification of impact craters. There is no conventional definition of an external ring, other than that it lies outside of the crater rim. Chicxulub exhibits two external rings. The evidence from Chicxulub suggests that external rings are also asymmetric but, in contrast to the crater rim, they appear to be associated with whole-crustal-scale, low-angle faults, which generally show a thrust fault expression at the surface (Morgan et al., 1997; Morgan and Warner, 1999). In some cases, however, the surface expression is a normal fault signature; not in accordance with the sub-surface fault signature. Consequently, debate still surrounds the interpretation of each ring at Chicxulub and, more significantly, which ring corresponds to the final crater rim.

Croft (1985) suggests a scaling law relating final crater dimension to transient crater size, assuming volume conservation during collapse and a constant final depth for all large impact craters. This gives a rough estimate for the transient crater diameter at Chicxulub of between 90 and 110 km, depending on which ring-feature

observed in the seismic data is interpreted to be the final crater diameter. This range in transient crater diameters introduces another, undesirable, free parameter into attempting to simulate the collapse of a Chicxulub-scale crater. Fortunately, data from the Chicxulub Seismic Experiment provides a further constraint on the size of the transient crater formed during the impact event that produced the Chicxulub crater (Morgan et al., 1997). Slump blocks have been observed in the seismic data, which appear to represent the subsurface signature of a wreath of terraces stepping down from the crater rim to the crater floor. Morgan et al. (1997) assume that the inner slump block originally lined the wall of the transient crater and collapsed inward and downwards during its modification. Thus, by mapping each slump block back to its original location, they derive an estimate for the transient crater diameter (their figure 4). This technique suggests that the transient crater diameter at Chicxulub was approximately 100 km.

For the purposes of this thesis, it is assumed that the transient crater diameter appropriate for simulations of the collapse of a Chicxulub-scale crater is 100 km. Furthermore, the geometry of the transient crater is approximated by a paraboloid of revolution, with a depth-to-diameter ratio of 1:3 (Melosh, 1989; Collins et al., 2002), which implies a transient crater depth of 33 km.

4.2.2 Target Properties

The main constraints on the composition of the pre-impact target stratigraphy have been provided not by the Chicxulub Seismic Experiment but by stratigraphic analysis of onshore well data. Several oil exploration wells have been drilled in the northern region of the Yucatan peninsula, outboard of the assumed crater rim. Interpretation of these data that both predates the discovery of the impact structure (López-Ramos, 1975) and postdates the discovery (Ward et al., 1995) suggest that the target at Chicxulub, from the surface down, comprises 2-3 km of Cretaceous sediments, a granitic crust to a depth of about 33 km, and mantle beneath. The Cretaceous sediments are predominantly evaporites and carbonates, with small amounts of sandstone, dolomite and shale (López-Ramos, 1975).

Given that the transient crater depth was most likely between 30 and 40 km, and that, to produce the final crater observed at Chicxulub, a volume of rock surrounding the crater equivalent to the volume of the transient crater must have been fluidized during the collapse, the bulk of the collapsing material would have been of average crustal composition. A good starting point when modelling the collapse of the Chicxulub crater, therefore, is to assume that the collapsing media is comprehensively fractured crustal material alone (Collins et al., 2002). This model can then be extended to include mantle material below a depth of ~ 33 km and sedimentary material in the top 3 km of the mesh.

The properties of each rock material that are required to model the target at Chicxulub fall into three categories: (1) the equation-of-state parameters; (2) the constitutive model parameters; and (3) the block-model or acoustic fluidization parameters. For simulations of crater collapse, the equation-of-state parameters are of the least significance, as phase changes do not occur during this stage of crater formation and changes in volume are not significant. The equation of state used in the hydrocode modelling work presented here is the Tillotson equation of state, for which there exists data for limestone and granite (Allen, 1967), which may be used to represent the sedimentary material and the bulk of the crust; and a high pressure phase of gabbroic anorthosite (O’Keefe and Ahrens, 1982), which may be used to represent mantle material. These parameters are presented in Table 4.1.

Table 4.1: Table of Tillotson equation-of-state parameters.

Parameter		Limestone ^a	Granite ^a	Anorthosite ^b
density of reference state, kg/m^3	ρ_0	2700	2680	3970
dimensionless Tillotson parameter	a	0.5	0.5	0.5
dimensionless Tillotson parameter	b	0.6	1.3	1.3
linear bulk modulus, GPa	A	40	18	240
coefficient of nonlinear elastic term, GPa	B	67	18	130
parameter categorising energy dependence, MJ/kg	E_0	10	16	1800
constant controlling convergence to perfect gas eos	α	5.0	5.0	5.0
constant controlling convergence to perfect gas eos	β	5.0	5.0	5.0
specific energy of incipient vaporisation, MJ/kg	E_{iv}	2.5	3.5	3.19
specific energy of complete vaporisation, MJ/kg	E_{cv}	18	18	16.8

^a Allen (1967)
^b O’Keefe and Ahrens (1982)

The constitutive model parameters for the three rock types to be considered in the collapse modelling are presented in Table 4.2. It should be noted that the Kelvin-solid viscosities chosen for each material are not accurate; however, for viscosities in excess of these values the time-step restriction imposed by momentum diffusion prohibits the completion of a collapse calculation in a reasonable run-time. Thus, these values are chosen to be as large as possible given the restriction on time step.

The block-model parameters will have the most important influence on the solution to the collapse simulations of the Chicxulub crater. However, defining these parameters for the different rock species is difficult as no specific data of this kind exists. Theory suggests that the viscosity of the fluidized region is a function of density, block size and the period of the block oscillation (see section 2.4.2); however, there is no data to suggest how block size and oscillation period might vary with position in the target, or composition of the target material. Furthermore, the decay time, which defines the length of time during which the amplitude of the acoustic

Table 4.2: Table of constitutive model parameters

Parameter		Limestone	Granite	Anorthosite
bulk modulus, GPa	K	40	18	240
shear modulus, GPa	G	24	11	144
Kelvin-solid bulk viscosity, $GPa\ s$	λ	100	100	100
Kelvin-solid shear viscosity, $GPa\ s$	η	100	100	100
Cohesion at zero confining pressure (intact material), GPa	Y_0	0.1	0.1	0.1
von Mises plastic limit for intact material, GPa	Y_M	3.6	3.6	3.6
Dimensionless coefficient of friction	μ	0.85	0.85	0.85
specific internal energy at melting, MJ/kg	E_m	3.3	3.3	3.3

pressure vibrations decays to $1/e$ times its original value, is a function of two more unknowns: the acoustic vibration wavelength and the quality factor Q .

If it is assumed that the period of the block oscillations and the block size are constant throughout the damaged target, and that the dominant wavelength of the acoustic vibrations is proportional to the block size, then the change in effective viscosity of the fluidized region for different rock species is simply a function of density, and the decay time is controlled by the quality factor Q . The seismic Q varies only slightly for the three different rock types to be considered by the model of the target at Chicxulub, as does the density. To a first order approximation, therefore, the block-model parameters may be considered the same for each rock species, which lends credence to the simplifying assumption of representing the entire target with the bulk properties of the crustal material at Chicxulub. However, the aim of the numerical analysis of the Chicxulub crater collapse is to investigate dynamical models for the formation of Chicxulub. Therefore, a broad region of the parameter-space associated with the block-model has been investigated.

The final parameter required to implement the acoustic fluidization model in simulations of the Chicxulub crater collapse is the initial pressure vibration amplitude in the target. As discussed in section 3.3.4, this pressure amplitude at a point in the target is related to the peak shock pressure experienced at that point. For any impact event the peak shock pressure p_{shock} at a distance r from the impact point may be approximated using the expression:

$$p_{shock} = p_{max} \left(\frac{a_i}{r} \right)^n, \quad (4.1)$$

which is discussed in section 2.4.1. For the simulations discussed here of a Chicxulub-scale event the exponent n is taken to be 3, which is likely to be a conservative estimate. a_i is the impactor radius; the distance r refers to the radial distance from the impact point prior to excavation. Thus, in the calculations the peak shock pressure must be migrated to the appropriate position beneath the transient crater using the Z-model approximation of the excavation flow.

Determining the peak shock pressure p_{max} within the target requires an estimation of the size, velocity and composition of the Chicxulub impactor. Unfortunately, determination of the impactor type has been hindered by the fact that the majority of the meteorite is melted or vaporised during the impact (Pierazzo et al., 1998). Furthermore, the available evidence is predominantly chemical and this is frequently equivocal; for example, chondritic asteroids and Fe-Ni asteroids have similar Platinum Group Element inter-element ratios (Evans et al., 1993). Hence, this attribute cannot be utilised to differentiate between different categories of meteorite (Croskell, 2001).

At present, the most compelling piece of evidence regarding the nature of the Chicxulub impactor is the discovery of a weathered fragment at the K/T boundary in a bore-hole beneath the Pacific Ocean (Kyte, 1998, DSDP 576,). The 2.5mm fossil meteorite possesses some mineralogical, chemical and textural characteristics in common with chondritic meteorites. Kyte (1998) concludes that the fragment might have originated from a CV, CO or CR carbonaceous chondrite; however, this does not exclude the possibility that the Chicxulub impactor was a comet containing primitive chondritic material. Current consensus suggests that the impacting bolide that produced the Chicxulub crater was not an Fe-Ni asteroid or a long period comet, because neither of these impactor types would yield enough iridium to account for the magnitude of the Ir anomaly observed at the K/T boundary (Croskell, 2001). Thus, the impactor was most probably either a short period comet or a chondritic asteroid. It is statistically more likely that the impactor was an asteroid; hence, for the purposes of this thesis, it is assumed that the Chicxulub impactor was a chondritic asteroid. The density of chondritic material is approximately 3320 kg m^{-3} ; however, most asteroids are thought to have a porosity of about 25%. The bulk density of the impactor considered in this thesis, therefore, is taken to be similar to granite ($\sim 2700 \text{ kg m}^{-3}$).

For a Chicxulub-scale impact event the peak shock pressure experienced close to the impact point p_{max} is given approximately as: $p_{max} \sim \rho v_i^2/4$, where v_i is the velocity of the impactor. This expression is valid only for situations where the impactor is compositionally similar to the target material (like the case of Chicxulub as considered in this thesis), and the shock wave velocity is much greater than the bulk sound speed of the target material. Typical velocities for asteroids in Earth-crossing orbits are between 17 and 30 km s^{-1} , with a mean at just over 20 km s^{-1} . These estimates imply an approximate peak shock pressure of $\sim 300 \text{ GPa}$, which is very similar to the peak shock pressure determined by hydrocode simulations of the early stages of the Chicxulub impact (Pierazzo et al., 1998).

Assuming that the Chicxulub crater evolved from a 100-km diameter transient crater, the impactor velocity and density may also be substituted into Equation 2.6 to give an estimate of 12-17 km for the impactor diameter, depending on the assumed

impact velocity. Thus, the Chicxulub crater may have been produced by a relatively fast-moving, small asteroid, or a larger but slower asteroid. The angle of impact will also effect any estimate of impactor size; however, it will not alter the median estimate for the diameter of the impactor (~ 15 km). For the simulations described in this chapter, it is assumed that the impactor velocity is 20 km s^{-1} and the impactor diameter is 15 km. These parameters are summarized in Table 4.3.

Table 4.3: Table of impact parameters

Parameter		Value
Transient crater diameter, km	D_t	100
Impactor radius, km	a_i	7.5
Impactor velocity, $km \text{ s}^{-1}$	v_i	20
Impactor density, $kg \text{ m}^{-3}$	ρ	2700
Peak shock pressure, GPa	p_{max}	270
Shock decay exponent	n	3

4.2.3 Parameter-Space Investigated

The collapse simulations are parameterized by the initial amplitude of the vibrational energy as a fraction of the peak shock pressure f_v , the rate of decay of the acoustic vibrations (characterized by the decay time τ_v) and the effective viscosity of the fluidized debris η . The influence of each of these parameters on the collapse simulations is examined and will be discussed in the next section. Table 4.4 defines the parameter space investigated.

Table 4.4: Table of variable model parameters

Parameter		Range
Amplitude of pressure vibrations / peak shock pressure	f_v	0.01 – 0.36
Pressure vibration decay time	τ_v	25 – 300 s
Effective viscosity of fluidized target	η	$10^7 - 10^{10} \text{ Pa s}$

4.3 Results

4.3.1 General Paradigm for the Simulation Results

A general paradigm for complex crater collapse was presented in section 2.3.5:

The collapse of the transient crater to form a complex crater proceeds by means of the inward slumping of the transient crater rim, accompanied by the uplift of the floor of the crater. The wholesale collapse is permitted by the temporary fluidization of a region of the target surrounding the cavity. By the end of the collapse the target is no longer fluidized; the central uplift has collapsed in some way to form the complex

crater morphology, which is then “frozen” in place by the halting of the fluidization process.

The acoustic fluidization algorithm implemented in the SALES_2 hydrocode provides a physical explanation for the temporary fluidization of the target and, hence, this paradigm. Thus, for a broad range of block-model parameters the collapse simulations of the Chicxulub impact crater follow the pattern of flow described above. Figure 4.3 shows the surface morphology at five stages during a typical collapse simulation for the Chicxulub crater. Figure 4.4 depicts the form of originally flat layers at the same five progressive stages of the collapse. The collapse follows a standard pattern; uplift of the central region of the crater which subsequently collapses downward and outward, accompanied by inward collapse of the transient crater rim (Collins et al., 2002). Animated versions of Figure 4.3 and Figure 4.4 can be found in Appendix C. The crucial difference between these collapse simulations and those involving a fluid half-space (see section 3.4.1) is that the final morphology in Figure 4.3e is “frozen” in place as the fluidized region decays in size (see Figure 4.4). In the fluid half-space simulations, the end result of the collapse is a flat, featureless fluid surface.

All three of the variable parameters listed in Table 4.4 have an important influence on the Chicxulub-scale collapse simulations. The fraction of the peak shock pressure that remains as acoustic noise, f_v , essentially controls the initial size of the fluidized region of the target within which collapse occurs. The decay time of the pressure vibrations (block oscillations) controls the rate at which this fluidized region contracts in size; that is, the length of time before the fluidization process is halted and the crater morphology is “frozen”. The viscosity of the fluidized region, η , dictates how rapidly the fluidized material collapses and rises. The effects of each of these parameters are related; for example, similar results may be obtained from simulations with an initially large fluidized region that decays in size rapidly and a smaller fluidized region that decreases in size less rapidly.

The following discussion describes the influence of each free parameter individually.

4.3.2 Effect of the Decay Time for the Acoustic Vibrations

Figure 4.5 illustrates the effect of block-oscillation decay time on the final simulated crater morphology. As discussed in section 2.3.4, the rate of collapse is limited by inertia: the transient crater cannot collapse faster than its free-fall time under gravity ($t_{free} = \sqrt{2H_t/g}$). For a transient crater depth of ~ 35 km this corresponds to a minimum collapse time of ~ 80 seconds on Earth. For decay times less than the free-fall time the collapse is arrested by frame (b) of Figure 4.4, producing a final morphology containing a central peak or mound. Such central peaks are observed

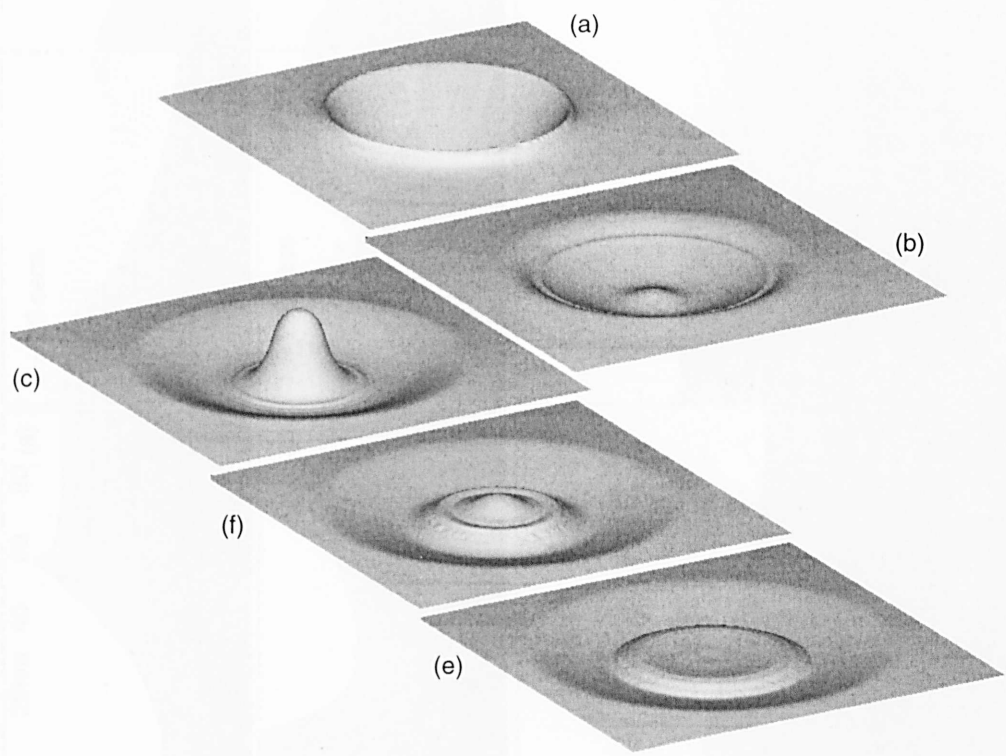


Figure 4.3: The general paradigm of the Chicxulub crater collapse as described by the hydrocode simulations. The central region of the crater collapses upwards forming a central peak as the crater rim collapses inwards (b). For simulations concerning quickly dissipating seismic energy the collapse is arrested at this point. For longer decay times the collapse continues with the central uplift overshooting the original target surface (c), and then collapsing downwards and outwards. The peak ring is formed as this material overrides the collapsed transient crater rim (d and e). The animation from which these frames are taken is presented in Appendix C.

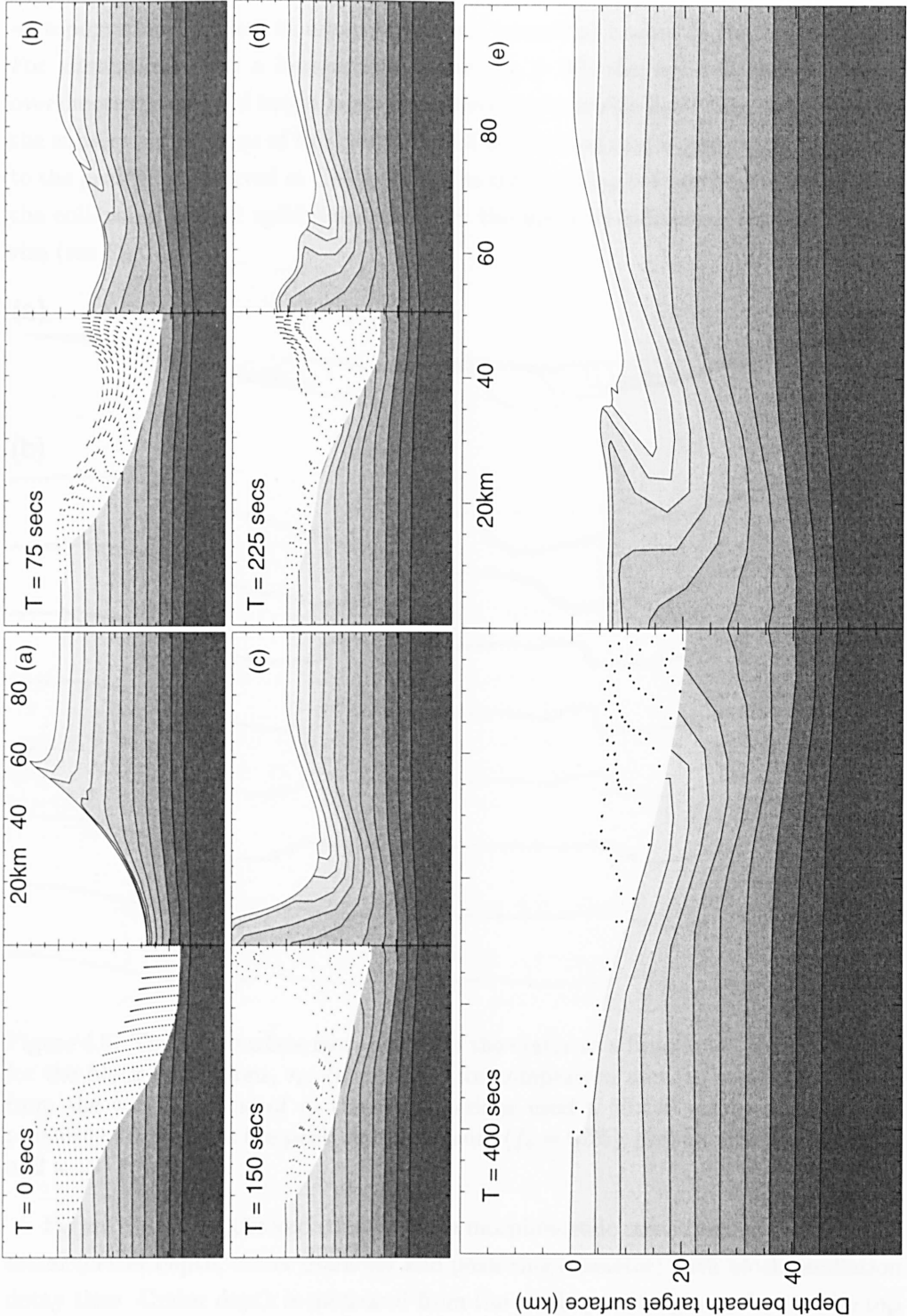


Figure 4.4: An example of results derived from a hydrocode simulation of crater collapse at Chicxulub. The simulation illustrated uses $f_v = 0.25$, $\eta = 2 \times 10^8$ Pa s, $\tau_v = 200$ s. The central region of the crater collapses upwards forming a central peak as the crater rim collapses inwards. For simulations concerning quickly dissipating seismic energy the collapse is arrested at this point. For longer decay times the collapse continues with the central uplift overshooting the original target surface (c), and then collapsing downwards and outwards. The peak ring is formed as this material overrides the collapsed transient crater rim (d and e). An animated version of this figure is presented in Appendix C.

as topographic features in many craters on terrestrial bodies in the Solar system. For simulations with a longer decay time ($\tau_v > 80$) the uplifted central mound overshoots the original target surface and becomes unstable itself (Fig. 4.4c). During the subsequent collapse of this central uplift an internal ring is generated, analogous to the peak ring observed at Chicxulub. The internal ring is formed as material from the collapsing central uplift interacts with the inwardly-collapsing transient crater rim (see Fig. 4.4e).

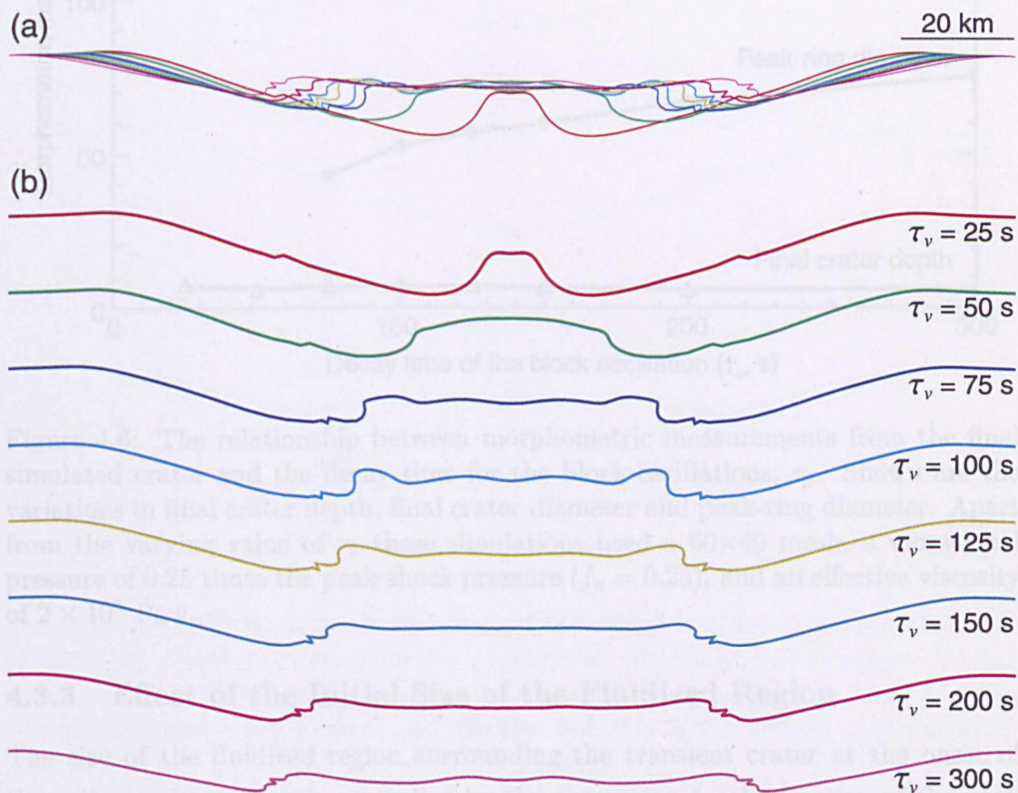


Figure 4.5: The final surface morphology of the crater as a function of the decay time for the block oscillations, τ_v : (a) overlaid for comparison and (b) separated. Apart from the varying value of τ_v these simulations used a 60×40 mesh, a vibrational pressure of 0.25 times the peak shock pressure ($f_v = 0.25$), and an effective viscosity of 2×10^8 Pa s.

Figure 4.6 shows the variation in three morphometric measurements of the final crater (crater depth, crater diameter and peak-ring diameter) with block-oscillation decay time. Crater depth is measured from the pre-impact target surface to the top surface of the mesh in the centre of the crater. For block-oscillation decay times in excess of the free-fall decay time, the final simulated crater diameter and the internal ring diameter are greater for longer decay times; however, the final crater depth and the qualitative morphology of the final simulated crater are independent of decay time.

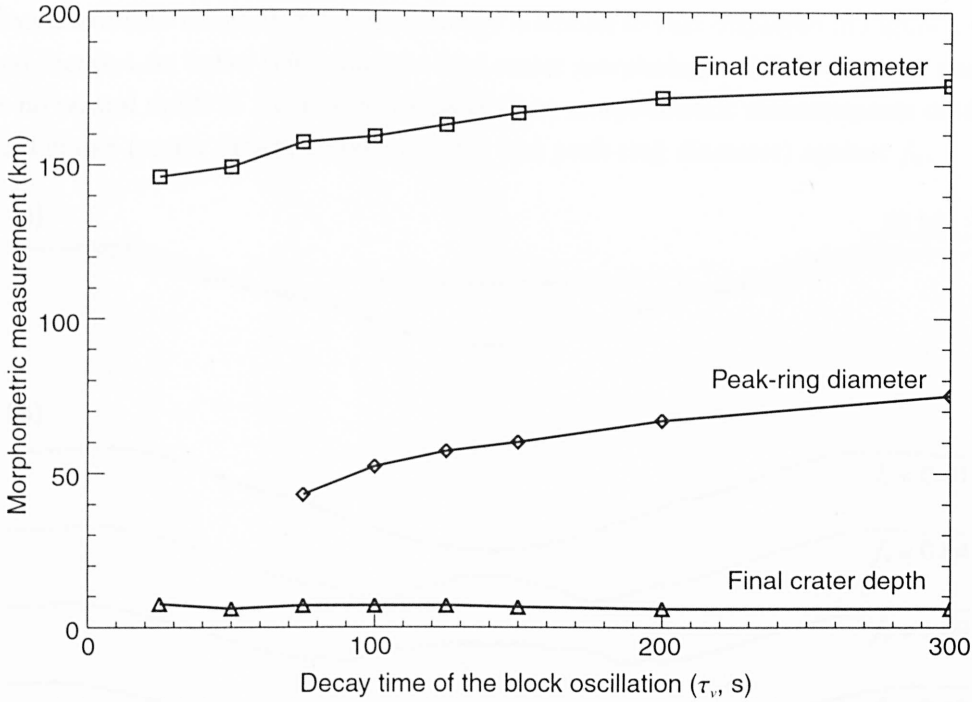


Figure 4.6: The relationship between morphometric measurements from the final simulated crater and the decay time for the block oscillations, τ_v . Shown are the variations in final crater depth, final crater diameter and peak-ring diameter. Apart from the varying value of τ_v these simulations used a 60×40 mesh, a vibrational pressure of 0.25 times the peak shock pressure ($f_v = 0.25$), and an effective viscosity of 2×10^8 Pa s.

4.3.3 Effect of the Initial Size of the Fluidized Region

The size of the fluidized region surrounding the transient crater at the onset of the collapse simulations is controlled by the parameter f_v , the fraction of the peak shock pressure that remains as acoustic noise. A larger value for f_v implies a larger fluidized region. Some experimental evidence of random seismic noise present in the target in the aftermath of shallow-buried chemical explosions (Gaffney and Melosh, 1982) suggests that $f_v \sim 0.1$. Using this value, the fluidized region extends only a small fraction of a transient crater radius away from the crater wall at the start of the collapse. The maximum value of f_v used in the simulations was $f_v = 0.36$; in this case the fluidized region extends nearly one crater radius from the transient crater wall, at the beginning of the calculation.

Figure 4.7 illustrates the dependence of the final simulated crater morphology on the fraction of the peak shock pressure that remains in the target as pressure vibrations, f_v (shown). The larger the initial fluidized region the more significant the collapse and, consequently, the shallower the final crater. For values of f_v greater

than about 0.15 the final crater morphology is similar to that displayed in Figure 4.3e and Figure 4.4e; below this value, the final crater morphology exhibits a central peak or no central uplift at all. Figure 4.8 plots three morphometric measurements of the final crater (crater depth, crater diameter and peak-ring diameter) against f_v .

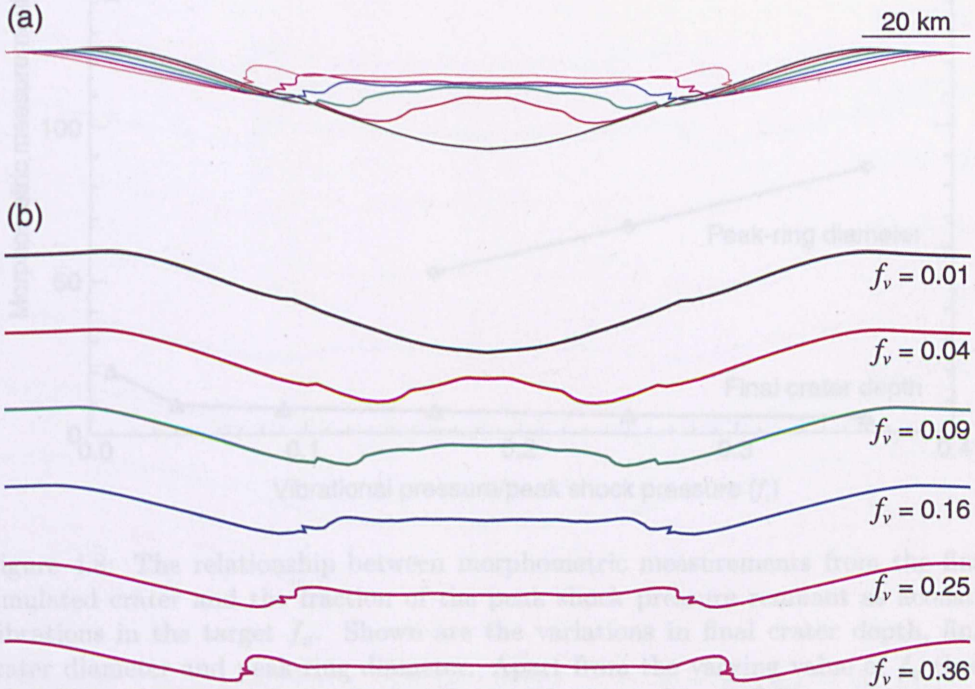


Figure 4.7: The final surface morphology of the crater as a function of the fraction of the peak shock pressure remnant as acoustic vibrations in the target f_v : (a) overlaid for comparison and (b) separated. Apart from the varying value of f_v these simulations used a 60×40 mesh, an effective viscosity of 2×10^8 Pa s, and a decay time of 200 s.

It should be noted that the values of f_v which produce an internal ring crater morphology are all in excess of the experimentally observed estimate of about 0.1. However, the decay law used for the shock pressure is proportional to $1/r^3$. If this law was, instead, $1/r^2$, which is in closer agreement with numerical modelling work of Pierazzo et al. (1998) for the shock pressure decay following an impact event, then the shock pressure farther from the crater would be higher and the fraction f_v need not be as large. With a $1/r^3$ scaling law and $f_v = 0.25$ the vibrational pressure at the rim of the crater ($a_i/r \sim 0.15$) is about 250 MPa at the onset of the collapse calculations. To achieve the same vibrational pressure with a $1/r^2$ scaling law f_v need only be as high as 0.04, which is well below the experimentally observed value of one tenth. Furthermore, although the pressure in the vibrations is assumed to be $1/4$ of the size of the shock itself at this location, the *energy* in the vibrations is only $1/16$ of that in the shock wave, since energy is proportional to the square

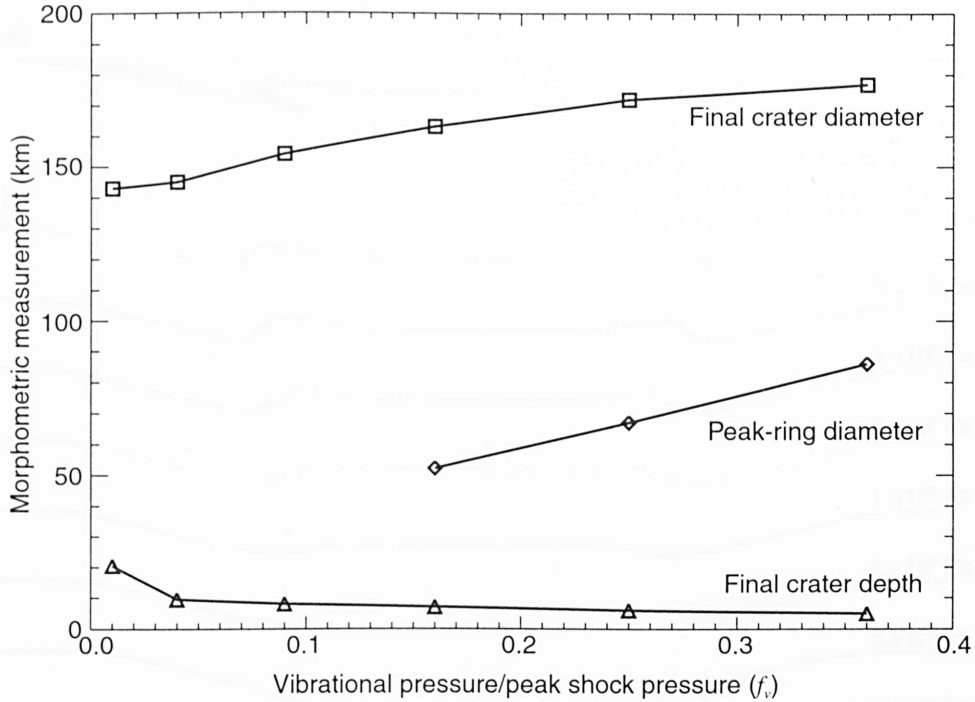


Figure 4.8: The relationship between morphometric measurements from the final simulated crater and the fraction of the peak shock pressure remnant as acoustic vibrations in the target f_v . Shown are the variations in final crater depth, final crater diameter and peak-ring diameter. Apart from the varying value of f_v these simulations used a 60×40 mesh, an effective viscosity of 2×10^8 Pa s, and a decay time of 200 s.

of the peak pressure (Equation 2.36). Consequently, the acoustic energy is not a significant drain on the shock energy.

4.3.4 Effect of the Fluidized Debris Viscosity

The effective viscosity of the fluidized region controls the velocity of the collapse. Figure 4.9 illustrates the final crater morphology for simulations with various effective viscosities for the fluidized region. For viscosities above 10^9 Pa s the final crater profile does not exhibit either a central peak or an internal ring structure because of extreme damping of the collapse. For viscosities below 10^9 Pa s, the final crater morphology is very similar; the only apparent influence of viscosity is a decrease in both final crater and internal-ring diameter with increasing viscosity. The qualitative morphology and the final crater depth are identical for all viscosities below 10^9 Pa s. Figure 4.10 shows the variation in three morphometric measurements of the final crater (crater depth, crater diameter and peak-ring diameter) with the effective viscosity of the fluidized region.

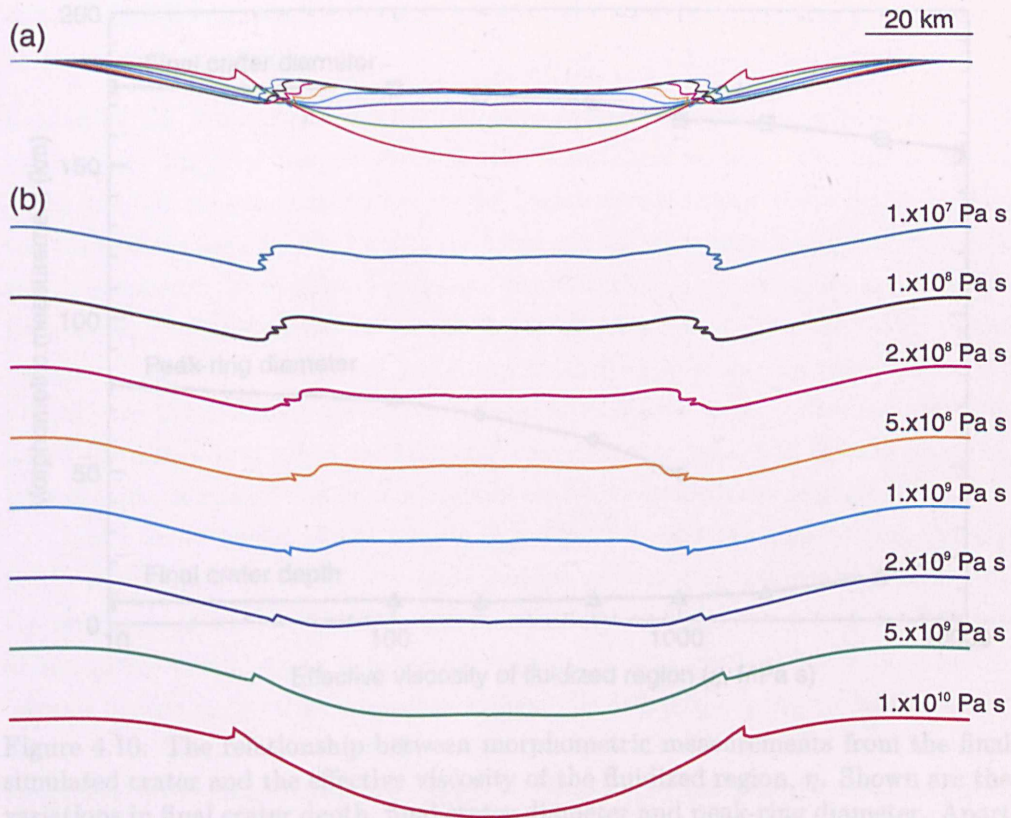


Figure 4.9: The final surface morphology of the crater as a function of the effective viscosity of the fluidized region of the target η : (a) overlaid for comparison and (b) separated. Apart from the varying value of η these simulations used a 60×40 mesh, a vibrational pressure of 0.25 times the peak shock pressure ($f_v = 0.25$), and a decay time of 200 s.

The block model relates the effective viscosity of the fluidized debris, η , to the size of the oscillating blocks, h , and period of block oscillation, T , (Melosh and Ivanov, 1999):

$$\eta = 2\pi\rho h^2/T. \quad (4.2)$$

Assuming a period of a few seconds for the block oscillation, the viscosities for which simulated collapse results in the formation of an internal ring (see Figure 4.9) correspond to a range in block size of ~ 50 – 850 m. Off-shore drilling at Chicxulub in late 2004 will test this prediction and constrain the viscosity of the fluidized region.

The simulations presented in section 3.4.1 illustrated the role of viscosity in influencing the collapse of a cavity in a constant viscosity half-space. For the simulations discussed here, collapse does not occur in a viscous half-space; the fluid region is confined to material within about one transient-crater radius from the crater wall.

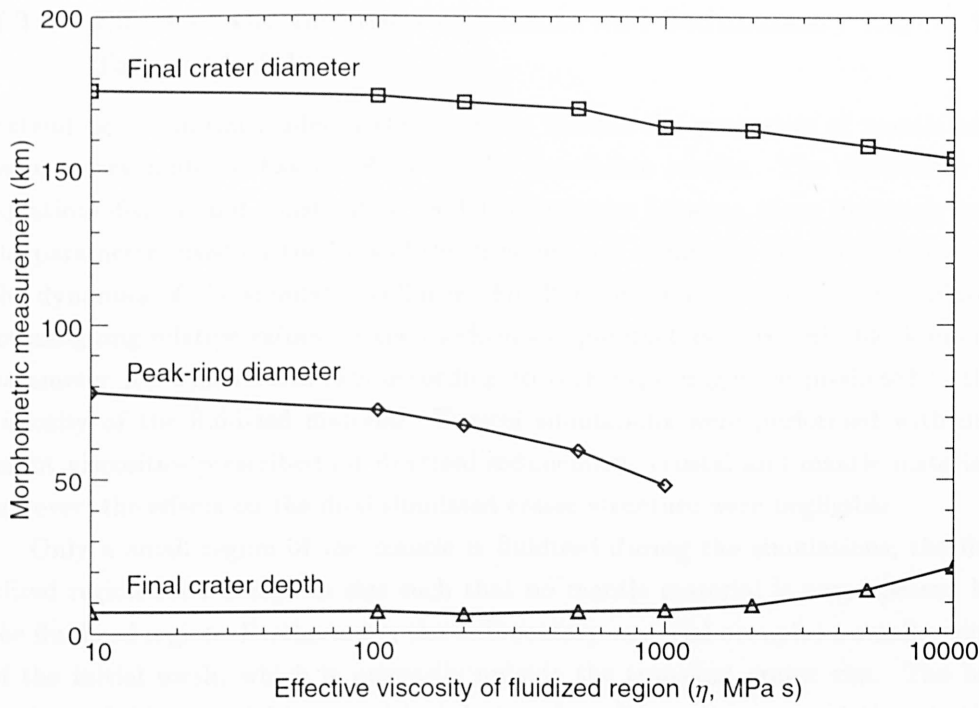


Figure 4.10: The relationship between morphometric measurements from the final simulated crater and the effective viscosity of the fluidized region, η . Shown are the variations in final crater depth, final crater diameter and peak-ring diameter. Apart from the varying value of η these simulations used a 60×40 mesh, a vibrational pressure of 0.25 times the peak shock pressure ($f_v = 0.25$), and a decay time of 200 s.

However, the effect of the viscosity in this region on the collapse is functionally identical to the effect of viscosity in the fluid half-space calculations. That is, for very low viscosities viscous forces are negligible and the fluid motion is controlled by inertia; for higher viscosities the flow is retarded by viscous forces and the fluid velocity is reduced relative to the inertial limit; for viscosities above a critical value the flow of material during collapse is retarded to the point that the collapsing cavity floor never rises above the pre-impact target surface.

In the simulations of the collapse of a Chicxulub-scale crater, the critical viscosity which precludes overshoot is approximately 2×10^9 Pa s. This is significantly smaller than the critical viscosity for a fluid half-space given by Equation 2.20. Hence, although the effect of viscosity on the collapse simulations discussed in this chapter is functionally the same as the effect of viscosity on collapse in a viscous half-space, the critical viscosities are not numerically equivalent because the fluid collapse is confined to a smaller region.

4.3.5 Effect of the Inclusion of Mantle and Sedimentary Layers in Target Model

Extending the initial model of the target to include the properties of mantle and sedimentary material has no effect on the simulation results. The differences in equation-of-state and constitutive model parameters between these materials and the parameters used for the bulk of the crust are not significant enough to influence the dynamics of the simulated collapse. Furthermore, no constraints are available for assigning relative values for the block-model parameters. The only block-model parameter for which variations according to rock type might be predicted is the viscosity of the fluidized material. Several simulations were performed with different viscosities prescribed for fluidized sedimentary, crustal and mantle material; however, the effects on the final simulated crater structure were negligible.

Only a small region of the mantle is fluidized during the simulations; the fluidized region soon decays in size such that no mantle material is encompassed by the fluidized region. Furthermore, the sedimentary material occupies a small region of the initial mesh, which is primarily outside the transient crater rim. The behaviour of this material is not critical during the collapse process; variations in the fluidized viscosity of this material cannot influence the overall collapse. Thus, the simplification of considering the target material to be brecciated crustal material only is justified.

4.3.6 Effect of Dynamic Starting Conditions

The simulations described thus far have all started with the same initial conditions; a static mesh geometry with the associated stress field. However, as discussed in section 2.1, the assumption that the target material is stationary at the onset of the collapse stage is somewhat unrealistic. For example, O’Keefe and Ahrens (1993) report that the kinetic energy of the target does not fall below 25% of the impactor energy until near the end of the collapse process; implying that the rebound of the crater floor begins prior to the collapse of the crater rim.

To examine the effect of rebound of the crater floor prior to rim collapse a suite of simulations were conducted for which an initial velocity field was imposed on the starting mesh (Collins et al., 2002). Figure 4.11 depicts five frames from a typical simulation from this suite of calculations, which shall be referred to as the simulations with dynamic starting conditions. In these simulations, zero time represents the moment when the transient crater rim has reached its maximum lateral extent. Instead of being static at this point, the floor of the crater has already begun to rebound. As with the simulations using static starting conditions, the central region of the crater collapses upwards (Fig. 4.11b), and then downwards and outwards (Fig. 4.11c and d). However, in the simulations with dynamic starting conditions,

the peak ring is formed as this material overrides the still-collapsing transient crater rim (see Fig. 4.11e). As a consequence the free surface becomes extremely contorted during the latter stages of the collapse simulation, which prevents the calculation from running to completion. Nevertheless the dynamic model for the generation of peak rings is evident.

A comparison of Figure 4.4 and Figure 4.11 shows that the final simulated crater structure derived from the simulations with dynamic starting conditions is very similar to that derived from the simulations with static starting conditions, below the upper-most 15 km of the model. Close to the surface, however, there are some significant differences between the two simulation suites. The simulations with dynamic starting conditions produce a central uplift with a more pronounced concave upward top. Furthermore, allowing for the completion of the central-uplift collapse the final crater structure in the second suite of simulations would likely be shallower, particularly around the annular trough. In both suites, quantitative predictions of the amount of structural uplift beneath the simulated crater floor vary with different model parameters; however, the shape of the original-depth contours remain qualitatively similar to those illustrated in Figs. 4.4e and 4.11e.

4.4 Comparison with Observations

Figure 4.12 summarizes the observations to date from the Chicxulub seismic data. The simulated final crater in Figure 4.4e contains most of the features that are observed at Chicxulub (Collins et al., 2002). The transient crater has collapsed to form a broad, shallow final crater. A topographic peak ring is observed. Slumped shallow target material that lay originally outside the transient crater rim now resides beneath the outer part of the peak ring. Furthermore, the model shows a prominent central uplift with a concave upward top; this thins with increasing depth.

4.4.1 Final Crater Diameter

The final crater diameter is not easily constrained by the simulation results. The hydrocode used in this work can only resolve flow variations with a wavelength of twice the cell dimension or above. For the simulations of the collapse of a Chicxulub-scale crater the cell dimension was between 1 and 2 km. Thus, neither localised strain or faulting are predicted by the simulations; in the fluidized region the deformation in each cell is an average for the cell. Consequently, there is no distinct feature that can be easily identified as the crater rim; the surface at this point is smeared out during the simulation. Furthermore, the collapse simulations do not consider ejecta material that is deposited close to the rim of the crater. Some of this material, which lay on top of, and immediately outside of, the transient crater rim in Figure 4.4a, would

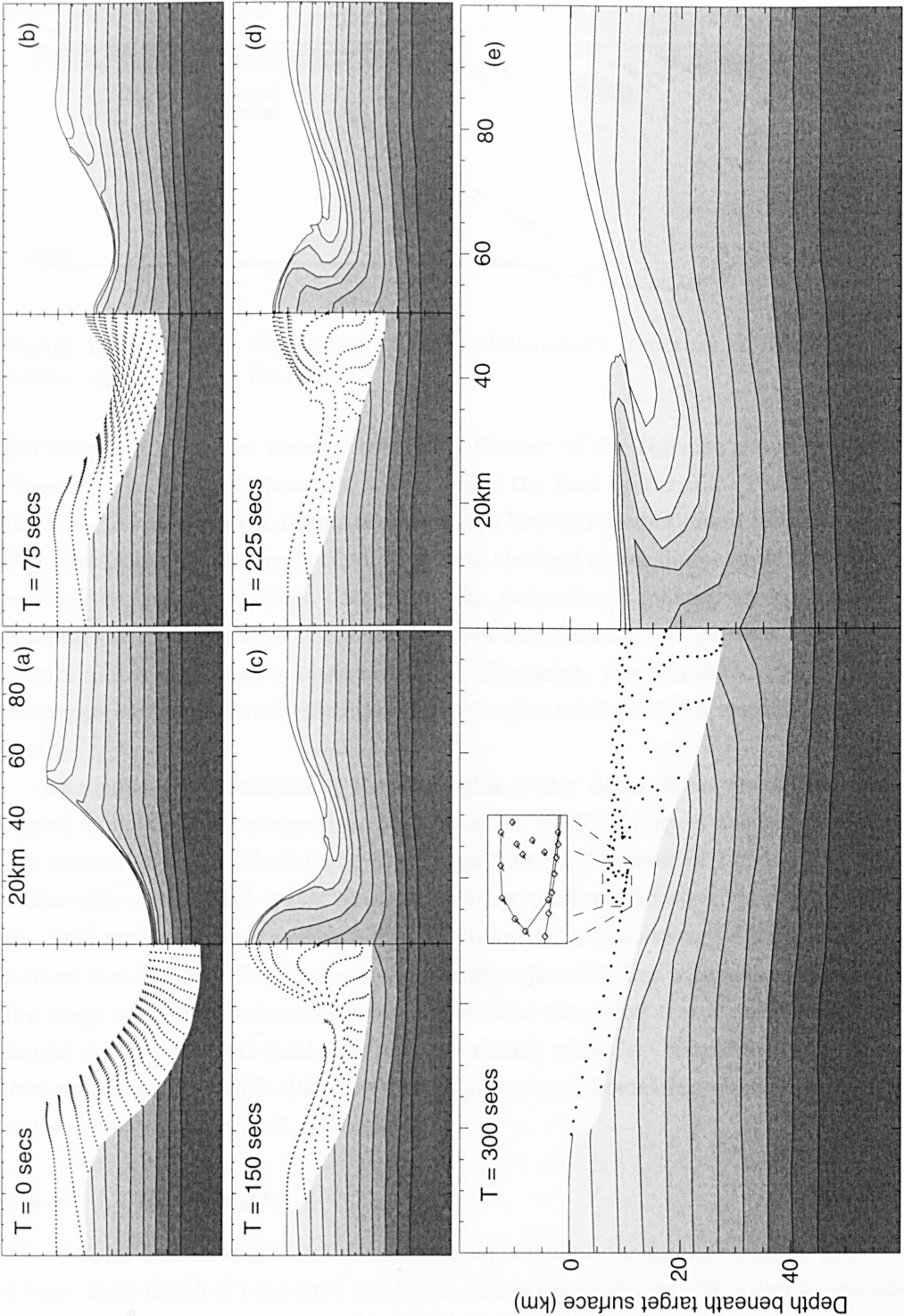


Figure 4.11: An example of results derived from a hydrocode simulation of crater collapse at Chicxulub with dynamic starting conditions. In this simulation zero time represents the moment when the transient crater rim has reached its maximum lateral extent. At this time, the floor of the crater has already begun to rebound. The simulation illustrated uses $f_v = 0.25$, $\eta = 2 \times 10^8$ Pa s, $\tau_v = 150$ s. Extreme contortion of the free surface prevents the simulation from running to completion.

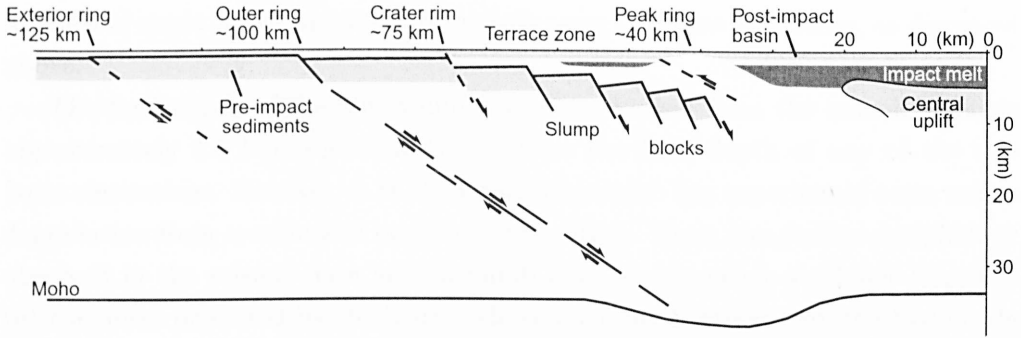


Figure 4.12: To scale illustration of Chicxulub impact structure as inferred from seismic data. See text for details.

probably end up in the annular trough by the end of the collapse simulations (see Figure 4.4e). The rest, however, would define the final crater rim. The absence of this material precludes an accurate comparison between predictions of the final crater diameter from the hydrocode simulations and the final crater diameter at Chicxulub, as derived from the seismic data. For the purposes of making an approximate comparison of final crater diameter between simulations and between simulation results and morphometric observations at Chicxulub, the simulated crater rim is chosen to be the point at which the surface begins to slope down toward the crater centre.

The rim-to-rim diameter of the Chicxulub crater depends on which ring is assigned to be the final crater rim. Morgan et al. (1997) interpret the head-scarp of the outermost slump block to be at an approximate diameter of 140-150 km; they define this as the final crater diameter. Other workers, however, prefer to define the final crater rim as the outer ring in Figure 4.12; the average diameter of this feature is ~ 200 km. The final simulated crater diameter lies within these limits for the range of model parameters investigated. The simulated crater profiles with the largest crater diameters tend to agree more closely with the Chicxulub crater profile derived from the seismic data in terms of other morphometric measurements such as peak-ring diameter and crater depth.

4.4.2 Crater Depth

The simulations for which an internal ring is generated during the collapse, produce a final crater depth of between 5 and 8 km, within the peak ring. This depth depends most strongly on the size of the fluidized region at the onset of collapse and, hence, the initial amplitude of the pressure vibrations. The simulation that produces the shallowest final crater uses an amplitude for the pressure vibrations of 0.36 times the peak shock pressure. Outboard from the internal ring, the crater depth is 1-2 km greater; however, the depth of this annular trough is misleading because of the

absence of ejecta material on top of the collapsing transient crater rim, as discussed above.

The final depth of the Chicxulub crater, as derived from the seismic data, is approximately 1.5 km; significantly less than the final depth of any of the collapse simulations. However, it is likely that Chicxulub has experienced some minor degradation from erosion and or viscous relaxation. Thus, the surface morphology observed in the seismic data at Chicxulub may be somewhat shallower than the pristine form predicted by the hydrocode simulation. Furthermore, the hydrocode simulations do not consider any ejecta material; hence, the fall-back of this material onto the crater floor is ignored. This would reduce the discrepancy between observed crater depth and simulated crater depth further. Nevertheless, the final crater depth is not well predicted by the hydrocode simulations.

4.4.3 Central Uplift

Directly beneath the centre of the final crater the simulation predicts significant uplift of lower-crustal rocks. In Figure 4.4, for example, the stratigraphic uplift of the fifth layer from the top is about 9 km, or 5% of the final simulated crater diameter. No data of this kind exists for the Chicxulub impact structure; however, this prediction is significantly less than the estimate of 15-20 km obtained using Equation 1.1, which is derived from observational evidence collected at several other terrestrial complex craters (Grieve et al., 1981; Ivanov et al., 1982).

The displacement of upper-crustal material near the centre of the crater is more complicated than the simple near-vertical motion of the lower-crustal material. Initially the upper-crustal material is uplifted in a manner identical to the uplift of the lower-crustal material, as the central region of the crater rises above the pre-impact target surface and becomes gravitationally unstable. However, this is followed by downward and outward displacement of the upper-crustal material during the collapse of this central uplift. The downwardly-collapsing upper-crustal material, near the centre of the crater, returns vertically or near-vertically in a relatively simple manner to become part of the central uplift. In contrast, the outwardly-collapsing material then interacts with the inwardly-collapsing crater rim and is pushed up on top of the collapsed or collapsing crater rim. As well as producing a topographic internal-ring feature this process results in large-scale folding of upper-crustal stratigraphy; for example, consider the third layer down in Figure 4.4. From the crater centre outward the stratigraphy is convex upward until beneath the peak ring, at which point the stratigraphy is overturned once and dips back toward the crater centre. The stratigraphy is then overturned again in the other direction and dips shallowly up to its original depth outside the crater.

The central uplift observed in seismic velocity models derived from wide-angle and refraction data that travelled through the central region of the crater is remarkably consistent with the simulation results (Morgan et al., 2000). The seismic velocity model images a central region of higher than ambient P-wave velocity, which is interpreted to represent uplifted crustal material. This central uplift displays a convex top and appears to thin with depth; exactly as predicted by the simulation results.

Although the thermal history of the collapsed central uplift at Chicxulub is not modelled in these simulations, it is likely that upper regions of the central uplift will be melted, in addition to being heavily brecciated. Allogenic impact breccias and melt rocks will lie within the impact basin, and within the thrust zone between the collapsed central uplift and collapsed transient crater rim.

4.4.4 Peak Ring

The simulation results produce a topographic peak ring observable in the final crater profile for a large sub-set of the parameter space investigated. The diameter of the peak ring at Chicxulub is approximately 80 km, or a little over half the final crater diameter as interpreted by Morgan et al. (1997). Only three of the twenty simulations described above produced a peak ring approximately 80 km in diameter. These simulations all correspond to end member models of the parameter space investigated.

Peak-ring generation requires a minimum amplitude for the pressure vibrations of the acoustic noise, a minimum decay time for these vibrations and a maximum viscosity for the fluidized region. Without constraining two of these three free parameters, however, it is impossible to constrain the third parameter. For example, Figure 4.5 suggests that a decay time in excess of 75 seconds is required to generate a peak ring during collapse; however, this result is for a specific viscosity of the fluidized region and amplitude of the pressure vibrations. In other words, the same quantitative simulation results can be achieved for different combinations of the effective viscosity of the fluidized debris and the decay time of the block oscillations. For example, the top surface morphology in Figure 4.9b ($\eta = 10^7$ Pa s) is almost identical to the bottom profile in Figure 4.5b ($\tau_v = 300$ s).

The dynamics of the formation of the peak ring during the simulations appear to be consistent with the kinematic explanation derived from seismic data beneath the peak ring (Brittan et al., 1999). Figure 4.13a shows an unprocessed seismic section beneath the peak ring from one of the reflection profiles. The data extends down to a two-way travel-time of 4 seconds, corresponding to an approximate depth of 10km. Two important observations have been made from these data, which are summarised in Figure 4.13b.

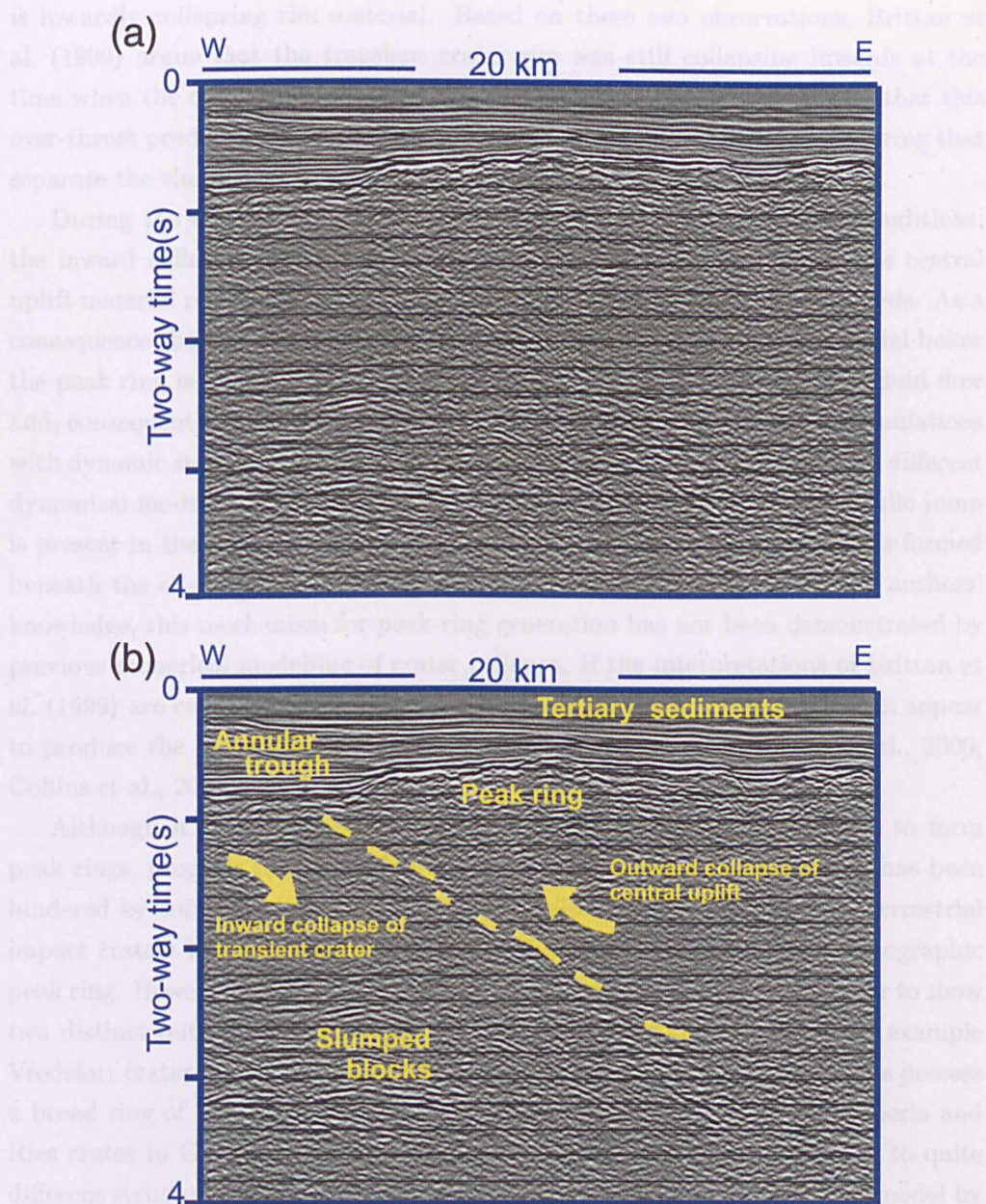


Figure 4.13: (a) Unmigrated seismic section beneath peak-ring; (b) Interpreted seismic section.

Figure 4.13 demonstrates (1) that the innermost slump block that has collapsed down from the transient crater rim lies beneath the interpreted peak-ring, and (2) that there is a weak, shallow dipping reflector beneath the peak-ring and above the slump blocks. It has been postulated that this latter feature represents the interface between two contemporaneous flow regimes. Material above the reflector is assumed to be from the outwardly collapsing central uplift, while material below the reflector

is inwardly collapsing rim material. Based on these two observations, Brittan et al. (1999) argue that the transient crater rim was still collapsing inwards at the time when the outwardly-collapsing central uplift was thrust over it; and that this over-thrust produced the weak shallow-dipping reflectors beneath the peak ring that separate the slumped crater rim wall material from the peak-ring material.

During the simulations discussed here that involved static starting conditions, the inward collapse of the transient crater rim is arrested well before the central uplift material reaches the same point as it collapses downwards and outwards. As a consequence there is no over-thrusting of the peak ring material; the material below the peak ring is merely over-turned. No hydraulic jump is present in the fluid flow and, consequently, no sharp discontinuity between layers is present. The simulations with dynamic starting conditions, however, produce peak rings via a slightly different dynamical model. In these simulations over-thrusting does occur, a hydraulic jump is present in the fluid flow field and a sharp discontinuity between layers is formed beneath the edge of the collapsing central uplift (see Fig. 4.11e). To the authors' knowledge, this mechanism for peak-ring generation has not been demonstrated by previous numerical modelling of crater collapse. If the interpretations of Brittan et al. (1999) are correct, the simulations with the dynamic starting conditions appear to produce the more accurate mode of peak-ring formation (Morgan et al., 2000; Collins et al., 2002).

Although it is widely accepted that centrally uplifted rocks collapse to form peak rings, progress towards a full understanding of peak-ring formation has been hindered by some apparently contradictory field observations. Pristine terrestrial impact craters larger than about 24 km in diameter, should possess a topographic peak ring. However, most large craters on land have been eroded, and appear to show two distinct outcrop patterns. Some contain a narrow central uplift, for example Vredefort crater in South Africa and Manicouagan crater in Canada. Others possess a broad ring of uplifted basement rocks, for example Popigai crater in Siberia and Ries crater in Germany. These contrasting outcrop patterns led, in part, to quite different structural models being proposed for the Chicxulub crater. In a model by Pilkington et al. (1994), the central uplift is 50 km in diameter, whereas the model proposed by Sharpton et al. (1996) contains a broad ring of uplifted basement rocks, ~ 100 km in diameter.

The results of the simulations discussed here, which produce an internal ring during collapse, may explain how such contrasting outcrop patterns occur (Collins et al., 2002). In the final simulated crater model there is significant uplift in the crater center, and horizontal slices through the central uplift mimic observation at eroded terrestrial complex craters. Removing the top 3 km of material from the center of the final simulated crater shown in Figure 4.4e would reveal an annular ring of uplifted material. Removing a further 5 km, however, would uncover a narrow central

uplift. Figure 4.14 illustrates the subsurface structure of a generic peak ring crater, as inferred from these numerical simulations of the collapse of a Chicxulub-scale crater. The solid lines separate originally flat stratigraphic layers that have been deformed by both the excavation and collapse processes. The dashed lines labeled A-D represent possible progressive stages in the erosion of a peak ring crater. If the crater were eroded to a level between line A and B, a broad ring of stratigraphically uplifted rocks would be revealed. This has been observed at Popigai crater, where the depth to the crater floor, the parautochthonous basement, has been mapped from current surface expression, drill holes and gravity data (Masaitis et al., 1999; Visnevsky and Montanari, 1999). Deeper erosion, down to between lines B and C, would reveal a narrower central uplift surrounded by overturned younger stratigraphy. This is similar to the sequence observed at the Vredefort impact structure in South Africa, where the top ~ 6 km of the central uplift, the Vredefort Dome, is thought to have been eroded (Grieve and Therriault, 2000). McCarthy et al. (1990) showed that the Vredefort Dome was surrounded by a synform – this is also predicted by the simulations discussed here. The central uplift at the Manicouagan crater is narrow and surrounded by an annular melt sheet. This outcrop pattern can be obtained by erosion below line C, if a significant fraction of the collapsing central uplift is melted during crater formation. For a particular impact energy, the degree of melting depends on the target chemistry, impactor composition, and the velocity and angle of impact (Pierazzo and Melosh, 2000a); melting and melt distribution is not modelled in the simulations discussed here.

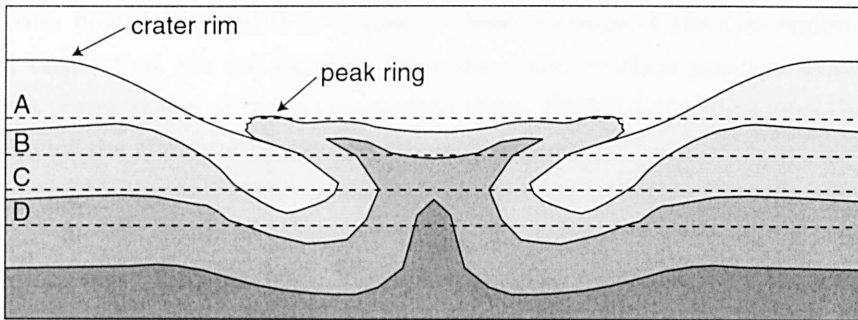


Figure 4.14: Illustration depicting the subsurface structure of a generic peak ring crater as derived from our simulation results. The dashed lines labeled A-D refer to possible stages in the erosion of an initially fresh crater. See text for further explanation. Note that the vertical scale has been exaggerated; the illustration has an aspect ratio of 1:2. Thus, the pre-impact thickness of the stratigraphic layers is on the order of $D/20$, where D is the final crater diameter.

4.4.5 The Fate of the Melt Sheet

Evidence from the seismic velocity data has also been used to constrain the extent and thickness of the melt layer produced during the Chicxulub impact event. The depth and shape of the interpreted central uplift suggests that the melt occupies a thickness of no more than 3.5 km. Furthermore, an abrupt change in seismic-wave traveltime for rays originating at different radial distances from the crater center, and travelling through the central region, limit the presence of significant quantities of melt-rich material to a region of about 35 km radius. An estimate of the melt produced, but not ejected, is therefore on the order of 10^4 km^3 . This is close to the quantity of melt estimated to be produced during the Chicxulub event by the hydrocode modeling work of Pierazzo et al. (1998), assuming roughly half of this melt was ejected.

Although the simulations discussed in this chapter do not consider thermal effects during the cratering process and, hence, ignore the production and movement of impact melt, it is possible to track the likely position of melted target material during collapse by assuming an initial geometry for the melt sheet. Numerical modelling work by Pierazzo et al. (1997, 1998) suggests that a maximum estimate for the volume of the target melted (or vaporised) during a Chicxulub-scale event may be represented by a sphere approximately 37 km across (Pierazzo, personal communication) centered below the target surface, such that the top of the sphere is just tangent to the target surface; a minimum estimate is a sphere approximately 30 km in diameter (Pierazzo et al., 1998). Using the Z-model approximation of the excavation flow it is possible to translate those portions of the two regions of the melted target that are not expelled from the crater to their position beneath the transient crater at the end of the excavation stage. Figure 4.15a illustrates the initial geometry of the transient crater and the melt regions.

Figure 4.15 illustrates the evolution of the melt region within the collapsing crater flow. The motion of the melt regions during the collapse is analysed by prescribing a separate viscosity for the melt. For melt viscosities above 10^7 Pa s the presence of the melt material does not alter the character of the collapse or the final structure of the simulated crater. For melt viscosities lower than this, the only change in the collapse flow is prolonged movement in the very central region of the crater. The melt region is fluid for the entire collapse; thus, for viscosities low enough, the melt continues to flow after the bulk of the acoustically fluidized material has come to rest. The melt forms a secondary uplift on a much smaller scale to the first, which then collapses back to a flat surface again. Even for simulations where the melt region is prescribed low viscosities, the presence of the melt material does not affect the final structure of the simulated crater.

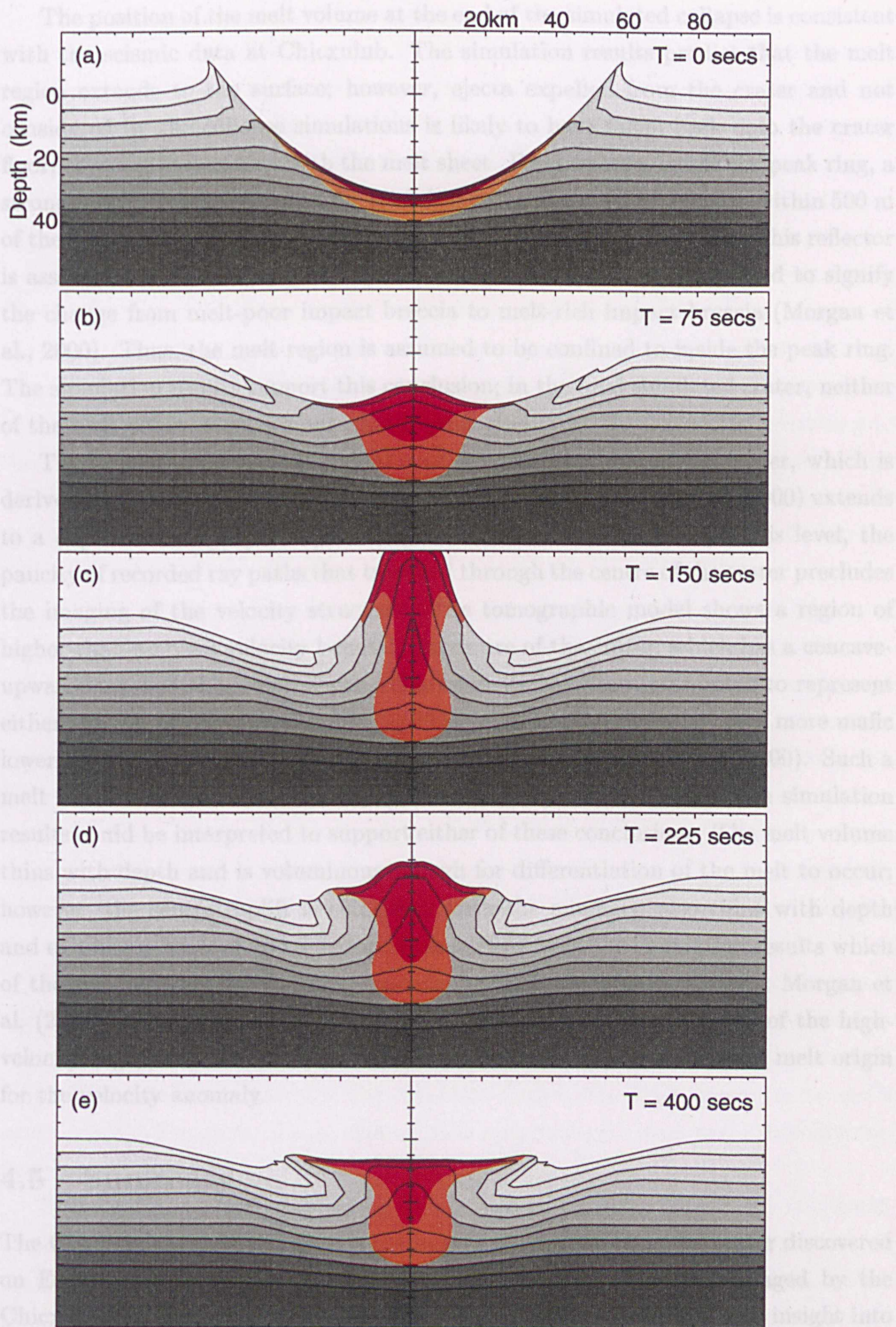


Figure 4.15: An example of results derived from a hydrocode simulation of a Chicxulub-scale crater collapse showing the motion of the predicted melt volume. The simulation illustrated uses $f_v = 0.25$, $\eta = 2 \times 10^8$ Pa s, $\tau_v = 150$ s. The viscosity of the melt is also 2×10^8 Pa s. The initial melt geometry is based on two estimates (minimum red, maximum orange) of the initial melt volume and position by Pierazzo et al. (1998), which have been translated to their position after excavation using the Z-model approximation of the excavation flow. The melt remains inside the peak ring during the simulations.

The position of the melt volume at the end of the simulated collapse is consistent with the seismic data at Chicxulub. The simulation results predict that the melt region extends to the surface; however, ejecta expelled from the crater and not considered by the collapse simulations is likely to have fallen back onto the crater floor, covering and mixing with the melt sheet. Furthermore, inside the peak ring, a strong reflection is observed in the seismic data at about 1.1 s, which is within 500 m of the crater floor; outside the peak ring this reflector is not observed. This reflector is associated with a sharp jump in P-wave velocity, which is interpreted to signify the change from melt-poor impact breccia to melt-rich impact breccia (Morgan et al., 2000). Thus, the melt region is assumed to be confined to inside the peak ring. The simulation results support this conclusion; in the final simulated crater, neither of the melt volumes extend outside the peak ring.

The tomographic velocity model over the central region of the crater, which is derived from wide-angle reflection and refraction data (Morgan et al., 2000) extends to a depth below the pre-impact target surface of ~ 8 km. Below this level, the paucity of recorded ray paths that travelled through the centre of the crater precludes the imaging of the velocity structure. The tomographic model shows a region of higher-than-ambient velocity beneath the centre of the crater, which has a concave-upwards top and thins with depth. This anomaly has been interpreted to represent either the top of the central uplift (uplifted lower-crustal material) or a more mafic lower layer within a differentiated impact melt sheet (Morgan et al., 2000). Such a melt sheet is observed at Sudbury (Grieve and Theriault, 2000). The simulation results could be interpreted to support either of these conclusions. The melt volume thins with depth and is voluminous enough for differentiation of the melt to occur; however, the central uplift is coincident with the anomaly, also thins with depth and exhibits a concave-upwards top. It is unclear from the simulation results which of the two explanations for the high-velocity anomaly is more correct. Morgan et al. (2000) suggest that the absence of seismic reflections from the top of the high-velocity region together with its concavity does not support an impact melt origin for the velocity anomaly.

4.5 Summary

The Chicxulub impact structure is the largest near-pristine impact crater discovered on Earth thus far. Its morphology and subsurface structure, as imaged by the Chicxulub Seismic Experiment, provide unique kinematic and dynamic insight into the process of large impact crater formation and peak-ring formation. Providing a robust, quantitative, predictive model to explain the dynamics of the collapse of a Chicxulub-scale crater, and to reproduce the final crater morphology is the primary task of this thesis.

The simulations described in this chapter show that, for a range of appropriate block-model parameters, temporary fluidization of the target allows the formation of internal peak and ring structures analogous to those observed in terrestrial and extra-terrestrial craters. In the simulations, crater morphology is most sensitive to the decay rate of the fluidized region. For decay times on the order of a few minutes, simulations of a Chicxulub-scale crater collapse produce an internal ring analogous to the peak ring.

In simulating a Chicxulub-scale collapse event, a desired result is the generation of a peak ring, which forms inside the final crater rim as a result of the collapse of the central uplift. A peak ring cannot be generated if the central uplift does not rise above the pre-impact target surface; thus, an upper limit for the viscosity used in the simulations of a Chicxulub-scale collapse is that which precludes overshoot of the central uplift. This limit is significantly below the theoretical prediction of Tait (1890) for the critical viscosity that precludes overshoot in a constant viscosity half-space. Knowledge of the appropriate block-size for the target material at Chicxulub will help constrain the viscosity of the fluidized region.

Analysis of the hydrocode simulation dynamics support the model for peak-ring formation inferred from the Chicxulub seismic data. Peak-ring formation involves the interaction of two flow regimes; the inwardly-collapsing crater rim and the outwardly-collapsing central uplift. Simulations which begin with rebound of the central uplift prior to the commencement of rim collapse show a slightly different dynamic formation mechanism than those simulations which begin with a static transient crater. In the simulations with dynamic starting conditions peak-ring formation involves *collision* between the two flow regimes. To this extent, the simulations with dynamic starting conditions appear to be more consistent with the dynamic model for peak-ring formation inferred from the seismic data.

The subsurface structure predicted by the hydrocode simulations is qualitatively the same for a broad range of model parameters. This structure shows many similarities with subsurface structure inferred from the seismic data at Chicxulub. Furthermore, horizontal slices through the simulated crater structure appear to reconcile contrasting evidence for the nature of peak-ring material from terrestrial outcrop patterns.

Peak-ring formation involves significant lateral, as well as vertical, movement of material near the target surface. As a consequence, deeply derived material is overturned to form the peak ring. The insight into peak-ring formation gleaned from the hydrocode simulations described in this chapter suggests a qualitative generic model for the subsurface structure of peak-ring craters.

Assuming an impact melt volume and initial geometry produced during the early stages of the Chicxulub event, which are within limits determined by the numerical simulations of Pierazzo et al. (1998), the final position of the melt volume, as pre-

dicted by the collapse simulations, is consistent with the seismic data at Chicxulub. Near the surface the melt occupies a broad region within the peak ring and extends only a couple of kilometers below the surface. With increasing depth, the radial extent of the melt region is reduced. The depth of the melt volume at the culmination of the collapse simulation is a function of the assumed melt volume; the larger the assumed melt volume the deeper the base of the final melt volume.

Chapter 5

Scaling the Acoustic Fluidization Model of Crater Collapse with Crater Size and Gravity

To see the world in a grain of sand.

Auguries of Innocence, *William Blake*

Complex craters are ubiquitous features on the solid bodies that populate the Solar System. The morphology of these craters, which is currently believed to be a direct consequence of the collapse process, is a function of both crater size and the gravitational acceleration of the body on which the crater is formed. At the smallest crater diameters within the complex crater classification, immediately above the simple-to-complex transition diameter, complex craters exhibit slumped rim walls and flat floors. At slightly larger crater diameters a peak protrudes from the centre of the crater, and at larger crater diameters still, the central peak is replaced by a concentric ring of peaks. This size-morphology progression is observed on many solid bodies within the Solar System, which span a range in gravitational acceleration of an order of magnitude. The transition diameters between the different types of complex crater, where observable, appear to correlate between the various solid bodies according to an approximate $1/g$ relationship. Any universal, predictive, quantitative model of complex crater collapse, therefore, must scale appropriately with crater diameter and gravity.

Chapter 4 demonstrated that a predictive, quantitative model for the collapse of a Chicxulub-scale crater may be obtained by simulating the collapse of a simple, bowl-shaped cavity under the influence of gravity. This is true provided that the

target surrounding the transient crater is modelled using a simple Bingham rheology, for which the yield strength is reduced in the presence of impact generated seismic noise. The discussion also highlighted the influence of the free model parameters on the collapse process; the effective viscosity of the fluidized debris, η , the decay time of the block oscillations, τ_v and the initial vibrational pressure associated with the block oscillation as a fraction of the peak shock pressure, f_v . Each of these parameters can modify the style of the collapse and the final morphology of the simulated crater. This chapter seeks to extend the model for the collapse of a Chicxulub-scale crater to be valid for craters of all diameters and forming on all the rocky bodies in the Solar System, by developing scaling relationships between the free model parameters, crater diameter and planetary gravity.

5.1 Approach

The object of the hydrocode simulations described in this chapter is to extend the applicability of the dynamical model for complex crater collapse discussed in Chapter 4 to a range of crater dimensions and gravitational accelerations. Thus, relationships between the free parameters from the model of the collapse of a Chicxulub-scale crater (f_v , τ_v and η), and the two new variables of transient crater size D_t and gravity g must be constrained. The next section provides some theoretical arguments for the form of these relationships. The following section will describe the hydrocode simulations performed to test these relationships. The methodology behind these simulations is identical to that discussed previously; the solution algorithm is discussed in detail in Chapter 3 and summarised in section 3.4. The simulations to test the relationships between crater diameter and the model parameters are for craters formed on the Moon. The fresh lunar crater population offers the most extensive morphometric data set with which to compare observation and the model predictions. All the simulations of crater collapse on the Moon assume that the target material is fractured crustal material. Lunar crustal material is assumed to have the same density and Tillotson equation-of-state parameters as the terrestrial crustal material (granite), which was used in the simulations discussed in the previous chapter. Lunar gravity is taken as 1.627 m s^{-2} for all the hydrocode simulations of lunar crater collapse.

5.2 Scaling of Acoustic Fluidization with Crater Size and Gravity

The acoustic fluidization parameters, namely the viscosity of the fluidized region η , the amplitude of the pressure vibrations p_v and the decay time for the pressure

vibrations τ_v are related to: the density of the target material ρ , the bulk sound speed of the target material c , the radius a_i and velocity v_i of the impactor, the characteristic wavelength of the acoustic pressure vibrations λ , the characteristic block size h , the period of the block oscillations T , the initial amplitude of the pressure vibrations as a fraction f_v of the peak shock pressure and the quality factor for the dissipation of wave energy Q . For a first-order model of how the acoustic fluidization parameters relate to gravity and crater diameter, it is reasonable to assume that ρ , c and Q are constant for all crater diameters and gravitational accelerations, and that v_i is the same for impacts on the Moon and Earth.

The theory of acoustic fluidization predicts a complicated relationship between the strain rate $\dot{\epsilon}$ of acoustically fluidized debris and the driving stress, τ (Melosh, 1979). In cases of strong acoustic fields, however, the flow law may be approximated by: $\dot{\epsilon} \approx \tau/\rho\lambda c$, where ρ is the bulk density of the mass of rock debris, λ is the vibration wavelength and c is the bulk sound speed. Thus, strongly fluidized debris behaves as a Newtonian fluid with a viscosity $\eta \approx \rho\lambda c$.

The block-model approximation of acoustic fluidization provides an alternative estimate for the effective viscosity of a granular medium in the presence of strong vibrations; $\eta \approx 2\pi\rho h^2/T$, where h is the characteristic block size and T is the period of one block oscillation. Assuming that the speed of sound in the granular medium and the period of a block oscillation are related by the equation $c = \lambda/T$, then a comparison of the equations for viscosity predicted by the two models of acoustic fluidization provides a relationship between the wavelength of the acoustic noise and the size of the blocks:

$$\lambda \approx h\sqrt{2\pi}. \quad (5.1)$$

From this it follows that the period of the block oscillations may be given by:

$$T = \frac{h\sqrt{2\pi}}{c}. \quad (5.2)$$

Thus, the effective viscosity of the fluidized region may be constrained by knowledge of one of h , λ or T .

In the acoustic fluidization model for the specific case of crater collapse, the acoustic pressure field is generated by the expanding, impact-induced shock front. As the shock wave passes through the target, heterogeneities in the elastic moduli produce a random acoustic energy field in its wake. The initial amplitude of the acoustic pressure field as a function of wavelength λ and position x , $A_0(x, \lambda)$, is therefore related to the impact parameters of size a_i , velocity, v_i and composition (density, ρ_i). Assuming that this acoustic energy is lost by internal dissipation before it has time to radiate away from the system, and that acoustic energy regen-

erated during collapse may be ignored, then the amplitude of the acoustic pressure vibrations at some later time, t , will take the form:

$$A(x, \lambda, t) = A_0(x, \lambda) e^{-\frac{c}{\lambda Q} t} \quad (5.3)$$

(Melosh, 1983) where Q is a parameter describing the rate of internal dissipation. If Q is independent of wavelength, then the shortest wavelength oscillations dissipate most rapidly and the longer wavelengths persist for longer. In general, the amplitude of an acoustic vibration declines by a factor of e after a “decay time” $\tau_v = \lambda Q / c$ (seconds). Thus, for a constant Q -value and bulk sound speed, the decay time is also a linear function of the characteristic block-size and may be uniquely defined by knowledge of one of h , λ or T .

To scale the viscosity of the fluidized region and the decay time for the pressure vibrations with crater diameter a relationship between crater diameter and block size (or wavelength of pressure vibrations, or period of oscillation) is required. Here it is assumed that the characteristic block size is a constant fraction of the transient crater size; that is, $h = f_b D_t$, where f_b is constant for all crater diameters. This is a somewhat arbitrary assumption; there is no reported relationship in the literature relating the mean size of breccia found at terrestrial impact craters to the size of the crater. However, use of this simple relationship should provide a good test of whether the block model of acoustic fluidization may scale appropriately with transient crater diameter at all.

Chapter 4 analysed the specific case of the collapse of a Chicxulub-scale impact crater. The results of several hydrocode simulations suggest that reasonable estimates for the effective viscosity of the fluidized region and the decay time for the pressure vibrations are $\sim 2 \times 10^8$ Pa s and 200 s respectively. Knowledge of the effective viscosity of the fluidized region at Chicxulub permits an estimate of the wavelength of the acoustic vibrations and the associated characteristic block size. Assuming a bulk sound speed within the fluidized debris of 500 m s^{-1} and a density of 2700 kg m^{-3} , the wavelength of the acoustic pressure vibrations is on the order of 150 m; implying a characteristic block size of 60 m, or $0.0006 D_t$. Furthermore, the estimate of the decay time constrains the quality factor Q . For $\tau_v = 200$ s, $\lambda = 150$ m and $c = 500 \text{ m s}^{-1}$ the quality factor $Q \approx 2100$. Using the assumption of a direct linear relationship between block size and transient crater diameter, the results from the hydrocode simulations of the collapse of a Chicxulub-scale crater therefore define $f_b = 0.0006$. From this definition, equations relating the effective viscosity of the fluidized region and the decay time of the acoustic vibrations to crater diameter may be written down.

First, assuming that the bulk sound speed and the density of the fluidized material is constant for all transient crater diameters, the effective viscosity of the fluidized debris is given by:

$$\eta \approx 2000D_t. \quad (5.4)$$

Second, assuming a constant Q -value for all crater diameters, the decay time of the pressure vibrations is:

$$\tau_v \approx 0.002D_t. \quad (5.5)$$

The two equations above are independent of gravitational acceleration; the viscosity of the fluidized region and the decay time for the pressure vibrations is the same for a given transient crater diameter on both the Moon and Earth. Section 2.4.2 demonstrated that the acoustic fluidization model predicts an inverse scaling relationship between gravity and crater depth at the onset of complex crater collapse (the simple-to-complex transition depth), without requiring any dependency of the decay time or viscosity of the fluidized region on gravity. The only model parameters that are related to gravity are the size and the velocity of the impactor required to produce the transient crater, and hence the peak shock pressure experienced by the target. The mean asteroid impact velocities on the Moon and Earth are approximately equivalent; thus, for the simulations discussed here the impact velocity is assumed to be $\sim 20 \text{ km s}^{-1}$, as for the simulations of the Chicxulub-scale collapse.

The size of an impactor that produces a transient crater of diameter D_t may be estimated using Equation 2.6 (Schmidt and Housen, 1987). Assuming that the impactor and target densities are comparable, the impactor radius a_i is constrained by:

$$a_i \approx 0.413D_t^{1.28}v_i^{-0.56}g^{0.28}. \quad (5.6)$$

Thus, for lunar impacts, the impactor radius is related to transient crater diameter by $a_i \approx 0.0018D_t^{1.28}$.

The peak shock pressure experienced by the target may be given approximately by $0.25\rho v_i^2$ (see section 2.4.1). For the simulations discussed here, therefore, it is assumed that the maximum shock pressure experienced by the target within the contact zone is approximately 270 GPa. Hence, the initial amplitude of the pressure vibrations associated with the oscillating blocks is related to transient crater diameter by:

$$\begin{aligned} p_{v0} &= f_v 270 \text{ GPa}, & \text{for } r < 0.0018D_t^{1.28} \\ p_{v0} &= f_v 270 \left(\frac{0.0018D_t^{1.28}}{r} \right)^3, & \text{for } r > 0.0018D_t^{1.28} \end{aligned} \quad (5.7)$$

As a starting point, it is assumed that the factor f_v is constant for all crater diameters; a value of $f_v = 0.25$ is used, which was found to be most appropriate for the Chicxulub-scale crater collapse simulations of Chapter 4. Table 5.1 presents a summary of the scaling relationships used in the lunar crater collapse simulations discussed in the next section.

Table 5.1: Table of impact parameters (D_t in metres)

Parameter		Value
Impactor velocity, $km\ s^{-1}$	v_i	20
Impactor density, $kg\ m^{-3}$	ρ_i	2700
Peak shock pressure, GPa	p_{max}	270
Shock decay exponent	n	3
Lunar gravity, $m\ s^{-2}$	g	1.627
Amplitude of pressure vibrations / peak shock pressure	f_v	0.25
Quality factor	Q	2100
Bulk sound speed, $m\ s^{-1}$	c	500
Acoustic noise wavelength, m	λ	$h\sqrt{2\pi}$
Block size, m	h	$0.0006D_t$
Impactor radius, m	a_i	$0.0018D_t^{1.28}$
Pressure vibration decay time, s	τ_v	$0.002D_t$
Effective viscosity of fluidized target, $Pa\ s$	η	$2000D_t$

5.3 Results

Using the relationships between the various block-model parameters and crater diameter presented in Table 5.1, several simulations were conducted for the collapse of various sizes of complex craters on the Moon. Figures 5.1 to 5.5 illustrate the final simulated crater structure for five different transient crater diameters. The variation in final simulated crater morphology with final crater diameter shows a size-morphology progression qualitatively very similar to that observed on the Moon.

Figure 5.1 illustrates the final simulated crater morphology for a 10-km diameter transient crater. The final crater is bowl-shaped, with a final diameter of ~ 15 km and a depth to diameter ratio of ~ 0.13 . The morphology of the final simulated crater is analogous to a simple crater observed on the Moon; however, the depth to diameter ratio is lower than for an appropriately sized simple crater on the Moon (~ 0.2 , see Table 1.1) and the floor in the centre of the crater is almost flat.

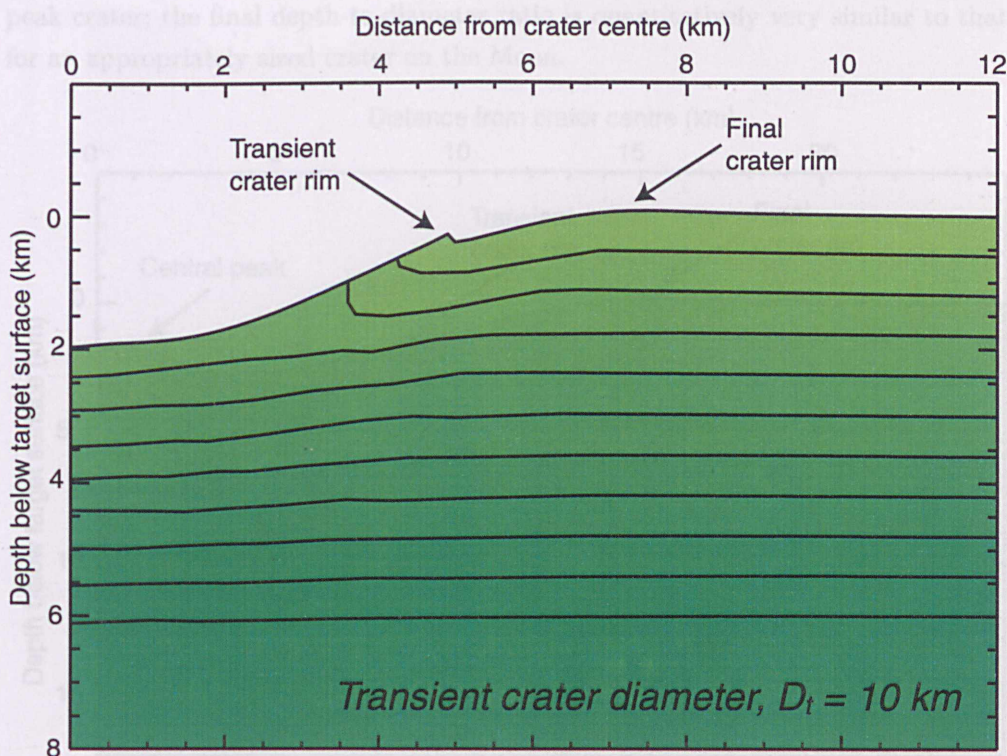


Figure 5.1: Final simulated crater morphology for a 10-km diameter transient crater on the Moon. Hydrocode simulations performed using the model parameters defined in Table 5.1. Colour scale reflects change in depth not composition.

The reason for the shallowness of the final simulated crater is that the simulated collapse involves some uplift of the crater floor; the collapse does not involve just debris-sliding of the crater walls. In other words, the acoustically fluidized region

in this simulation encompasses a sufficient volume of the sub-crater material to permit complex-style collapse. Formation of a central peak is precluded by the rapid decay of the acoustic noise and the viscosity of the fluidized debris. Thus, the scaling relationships defined in Table 5.1 dictate that the collapse of a lunar crater just smaller than the simple-to-complex transition diameter produces a crater qualitatively similar to a simple crater; but formed not just by debris sliding of the crater walls. For simple debris sliding to be the dominant collapse mechanism the initial amplitude of the acoustic pressure vibrations, or the decay time for the pressure vibrations, must be reduced by an order of magnitude relative to that defined in Table 5.1.

Figure 5.2 illustrates the final simulated crater morphology for a 20-km diameter transient crater. In this case, the final simulated crater exhibits a central peak or mound; the final crater diameter is ~ 30 km, with a depth-to-diameter ratio of about 0.1. The morphology of the final simulated crater is analogous to a central peak crater; the final depth-to-diameter ratio is quantitatively very similar to that for an appropriately sized crater on the Moon.

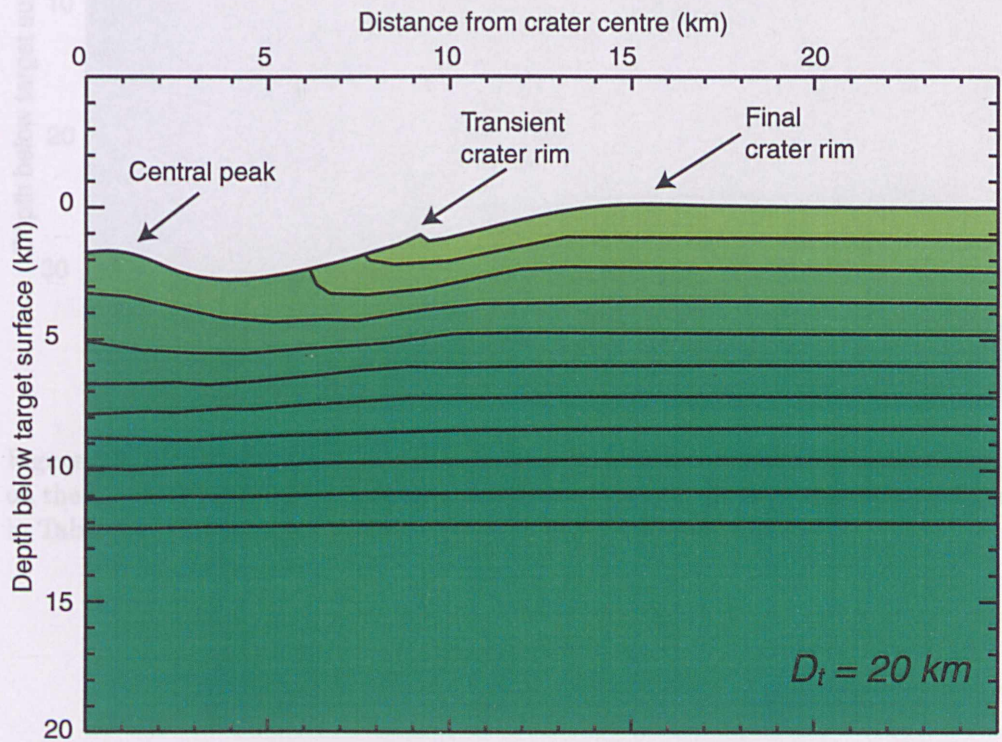


Figure 5.2: Final simulated crater morphology for a 20-km diameter transient crater on the Moon. Hydrocode simulations performed using the model parameters defined in Table 5.1. Colour scale reflects change in depth not composition.

Figure 5.3 illustrates the final simulated crater morphology for a 50-km diameter transient crater. In this case, the central peak or mound within the final simulated crater is more pronounced than for the smaller crater in Figure 5.2. The final crater diameter is ~ 75 km, with a depth-to-diameter ratio of about 0.06. The morphology of the final simulated crater is, again, qualitatively very similar to an appropriately sized crater on the Moon. Furthermore, as with the 20-km diameter transient crater simulation, the depth-to-diameter ratio of the final simulated crater is quantitatively very similar to that for an appropriately sized crater on the Moon.

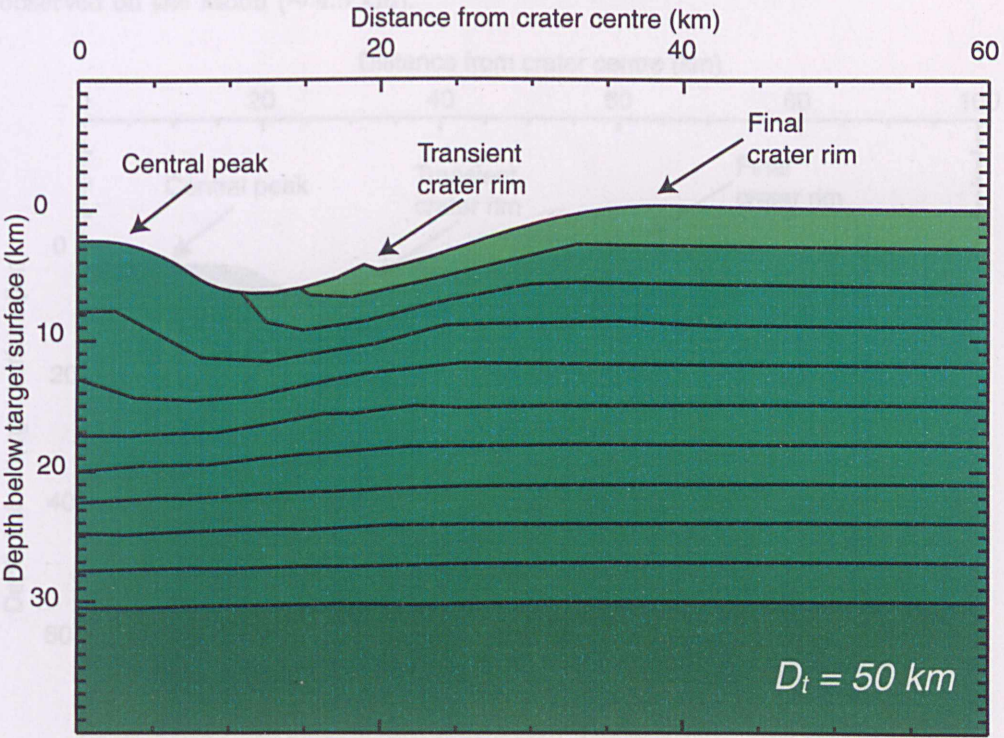


Figure 5.3: Final simulated crater morphology for a 50-km diameter transient crater on the Moon. Hydrocode simulations performed using the model parameters defined in Table 5.1. Colour scale reflects change in depth not composition.

Figure 5.4 illustrates the final simulated crater morphology for a 80-km diameter transient crater. Here the central peak or mound within the final simulated crater is much broader than the central peak within the smaller, simulated central-peak craters in Figure 5.2 and 5.3. The final crater diameter is ~ 135 km, with a depth-to-diameter ratio of about 0.06. The morphology of the final simulated crater is, again, qualitatively very similar to an appropriately sized crater on the Moon. However, in the case of the 80-km diameter transient crater the final simulated crater depth (~ 7.5 km) is significantly deeper than the average depth of a 135-km diameter observed on the Moon (~ 4.5 km).

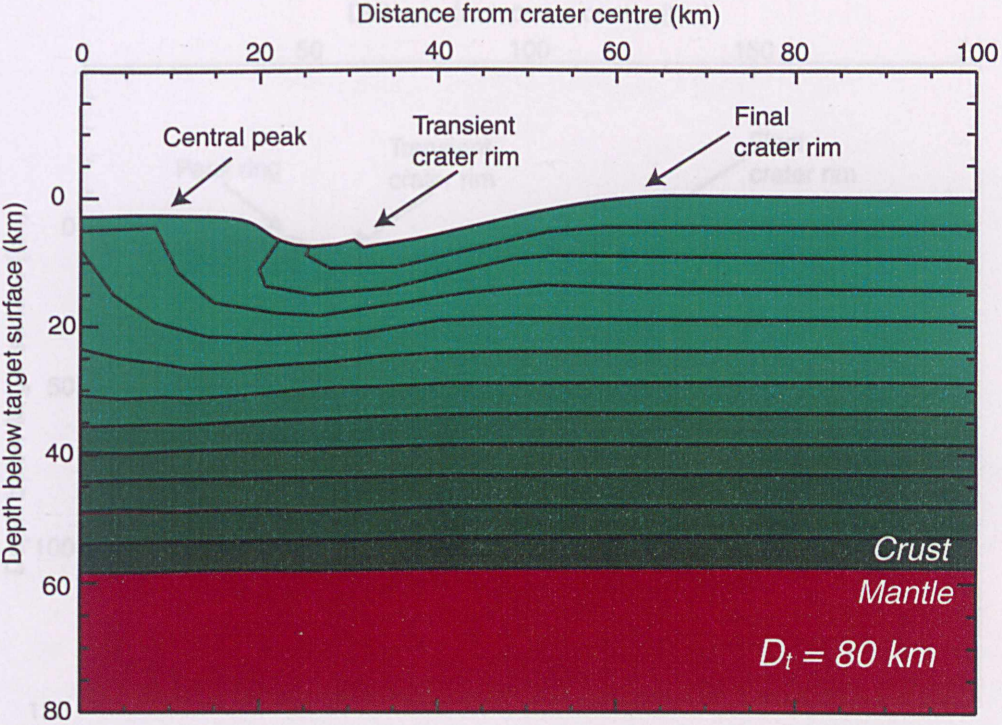


Figure 5.4: Final simulated crater morphology for a 80-km diameter transient crater on the Moon. Hydrocode simulations performed using the model parameters defined in Table 5.1. Colour scale reflects change in depth not composition.

Figure 5.5 illustrates the final simulated crater morphology for a 150-km diameter transient crater. Here the central peak is replaced by a peak ring within the final simulated crater. The final crater diameter is ~ 250 km, with a depth-to-diameter ratio of about 0.05. The morphology of the final simulated crater is qualitatively similar to an appropriately sized crater on the Moon. However, the diameter of the peak ring generated during the simulated collapse of a 150-km diameter transient crater is significantly less than that observed in an appropriately sized crater on the Moon, and the final simulated crater depth (~ 11.5 km) is significantly deeper than the average depth of a ~ 250 -km diameter lunar crater (~ 5.5 km).

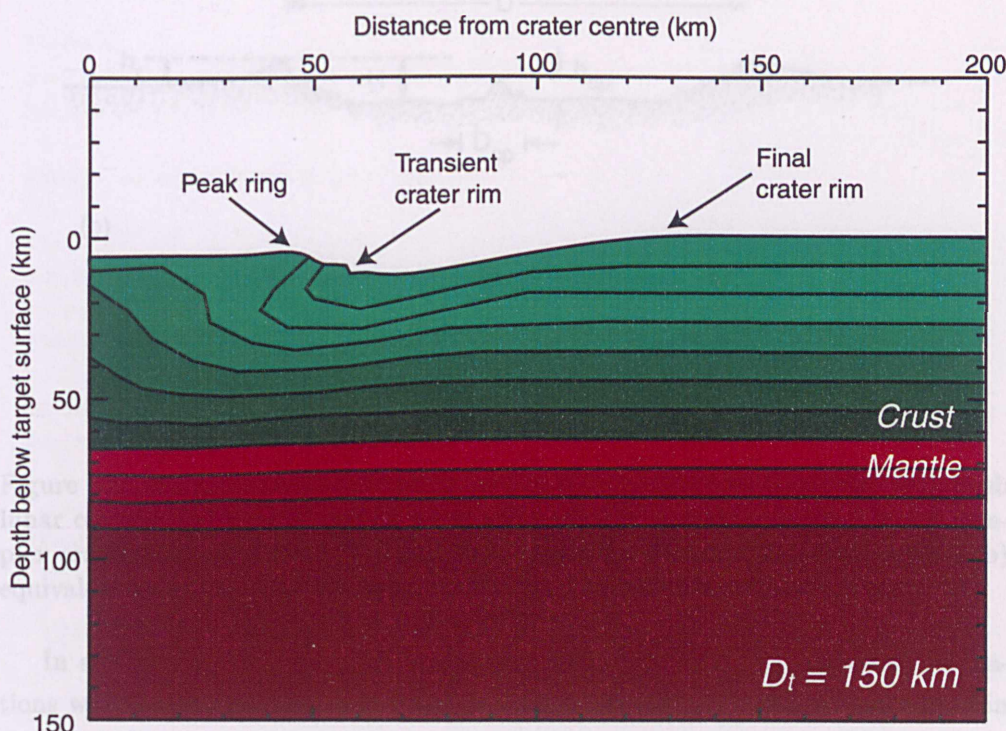


Figure 5.5: Final simulated crater morphology for a 150-km diameter transient crater on the Moon. Hydrocode simulations performed using the model parameters defined in Table 5.1. Colour scale reflects change in depth not composition.

For a 10-km diameter transient crater, therefore, the acoustic fluidization model controlled by the scaling relationships defined in Table 5.1 may over-estimate the effect of acoustic fluidization on complex crater collapse. For 20–50-km diameter transient craters the acoustic fluidization model controlled by the scaling relationships defined in Table 5.1 appears to agree remarkably well with observation. However, the acoustic fluidization model, using the same scaling relationships, does not appear to provide a good quantitative prediction for the collapse of transient craters above about 80 km in diameter.

5.4 Comparison with Observations

To conduct a more thorough investigation of the quantitative similarities between the numerical simulation results and observational data from the population of fresh lunar craters, several morphometric parameters may be compared between observation and simulation. Figure 5.6 defines some morphometric parameters used by previous workers in quantitative studies of fresh lunar craters. It also illustrates the equivalent measurements derived from the final simulated crater structure.

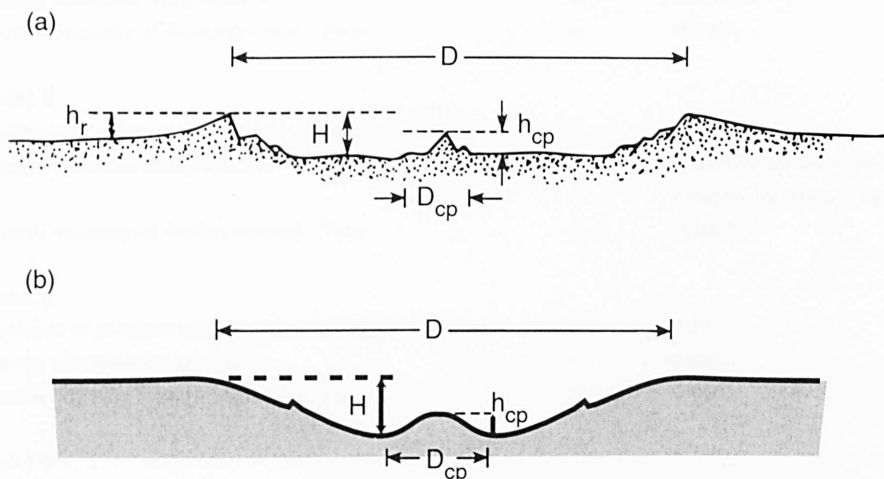


Figure 5.6: Morphometric parameters used to compare simulation results with fresh lunar craters. (a) Morphometric parameters used to measure morphology of complex craters (adapted from Melosh, 1989, peak-ring diameter D_{pr} not shown); (b) equivalent measurements derived from the final simulated crater structure.

In addition to the scaling relationships defined in Table 5.1, suites of simulations were performed with four other scaling relationships, which are presented in Table 5.2. The results of these five suites of hydrocode simulations are illustrated in Figures 5.7–5.10, which present the relationships between the various morphometric parameters defined in Figure 5.6 and final crater diameter (simulated and observed).

5.4.1 Crater Depth

Figure 5.7 illustrates the relationship between final simulated crater diameter and final simulated crater depth, for each of the five suites of simulations, using the scaling laws defined in Table 5.2. Note that here, in contrast to measurements of crater depth reported in Chapter 4, the simulated crater depth is measured outside of the central uplift (see Fig. 5.6). Also shown for comparison are the best-fit curves for crater depth as a function of crater diameter, as observed in the fresh lunar craters (Pike, 1977; Williams and Zuber, 1998, solid black line) and 28 data points representing the depth and diameter of the largest complex craters on the Moon.

Table 5.2: Table of scaling relations

Parameter		Value
Model 1		
Amplitude of pressure vibrations / peak shock pressure	f_v	0.25
Pressure vibration decay time, s	τ_v	$0.002D_t$
Effective viscosity of fluidized target, $Pa\ s$	η	$2000D_t$
Model 2		
Amplitude of pressure vibrations / peak shock pressure	f_v	0.25
Pressure vibration decay time, s	τ_v	$0.0015D_t$
Effective viscosity of fluidized target, $Pa\ s$	η	$2000D_t$
Model 3		
Amplitude of pressure vibrations / peak shock pressure	f_v	0.25
Pressure vibration decay time, s	τ_v	$0.0015D_t$ for $D_t \leq 75\text{ km}$ $0.003D_t$ for $D_t > 75\text{ km}$
Effective viscosity of fluidized target, $Pa\ s$	η	$2000D_t$
Model 4		
Amplitude of pressure vibrations / peak shock pressure	f_v	0.25
Pressure vibration decay time, s	τ_v	$0.04a_i$
Effective viscosity of fluidized target, $Pa\ s$	η	$2000D_t$
Model 5		
Amplitude of pressure vibrations / peak shock pressure	f_v	0.1 for $D_t \leq 75\text{ km}$ 0.25 for $D_t > 75\text{ km}$
Pressure vibration decay time, s	τ_v	$0.0015D_t$
Effective viscosity of fluidized target, $Pa\ s$	η	$2000D_t$

All of the model curves agree qualitatively with the observational data; however, none of the curves are in close quantitative agreement with the fresh lunar crater data over a wide range in crater diameter. At the lower end of the range of crater diameters modelled, the final simulated crater is shallower than that observed in the fresh lunar craters; toward the upper end, the final simulated crater depth becomes greater than the appropriate depth as measured in lunar craters. At large crater diameters, which correspond to the peak-ring crater classification, the depths of the final simulated crater are consistently above that observed in the lunar crater population. The spread of the observed data is insufficient to explain the discrepancy between the observed trend of crater depth versus diameter and that predicted by the simulations of crater collapse. The only models to predict a change in the slope of the depth versus diameter curve at the central peak to peak ring crater transition are those with artificial changes in the block-model parameters at this crater diameter (model 3 and model 5).

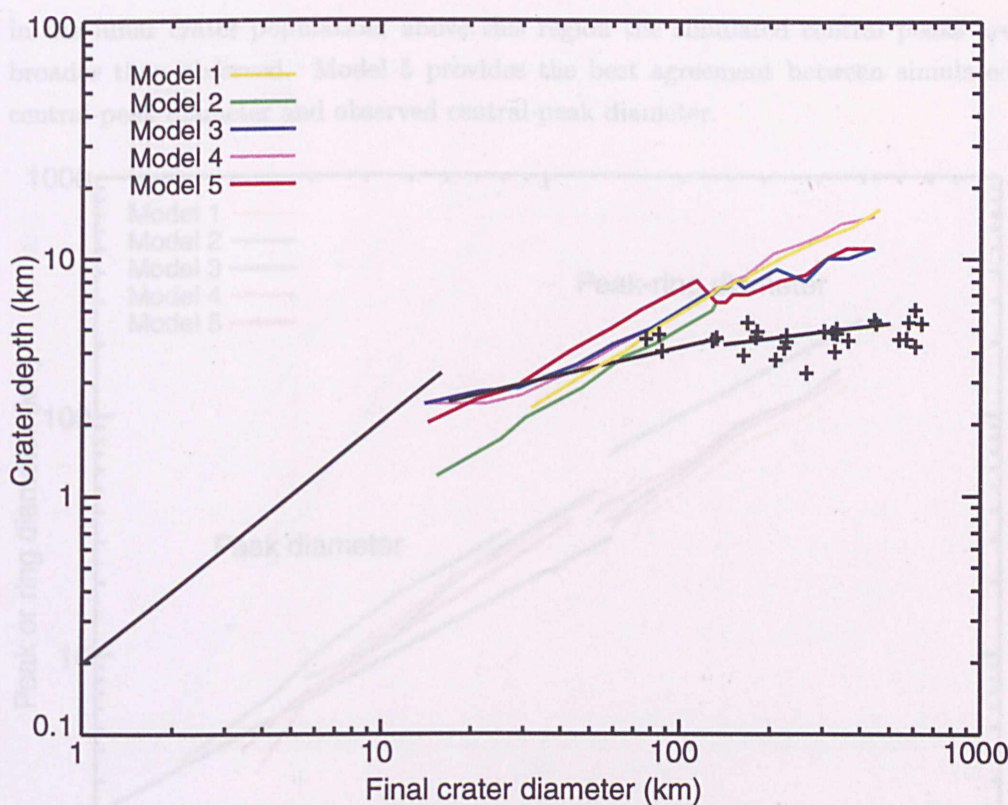


Figure 5.7: Comparison between model predictions of final crater depth and those observed in the fresh lunar craters. Best-fit curves (black solid lines) taken from Pike (1977) and Williams and Zuber (1998). Individual data points for peak ring craters only (crosses) taken from Williams and Zuber (1998).

5.4.2 Central Peak Diameter, Central Peak Height and Peak-Ring Diameter

Figure 5.8 illustrates the relationship between final simulated crater diameter and simulated central peak diameter or peak-ring diameter, for each of the five suites of simulations, using the scaling laws defined in Table 5.2. Figure 5.9 illustrates the relationship between final simulated crater diameter and simulated central-peak height, for each of the five suites of simulations. Also shown for comparison are the best-fit curves for central-peak diameter, peak-ring diameter and central-peak height, as functions of crater diameter, derived from observations of the fresh lunar craters (Pike, 1985; Wood and Head, 1976; Hale and Grieve, 1982).

All of the model curves agree qualitatively with the observational data; however, none of the curves are in close quantitative agreement with the fresh lunar crater data. In the case of central-peak diameter the simulation predictions are in numerical agreement with observation for only a narrow range in crater diameter. Below this region of good agreement, the simulated central peaks are narrower than observed

in the lunar crater population; above this region the simulated central peaks are broader than observed. Model 5 provides the best agreement between simulated central-peak diameter and observed central-peak diameter.

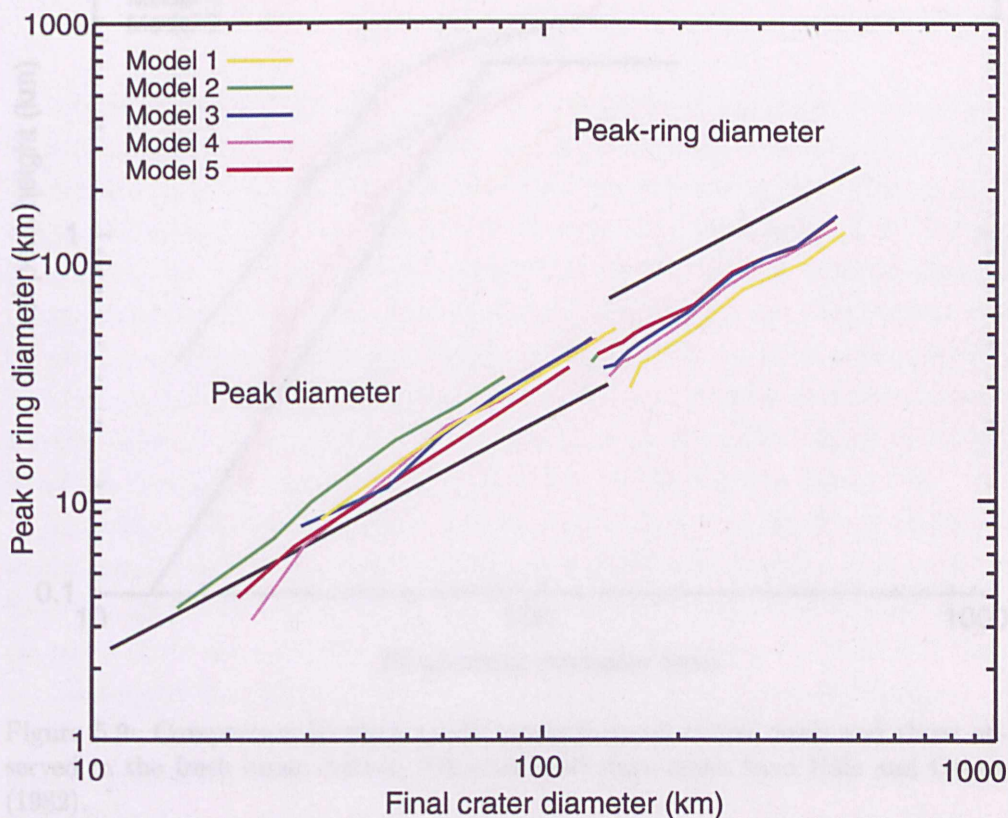


Figure 5.8: Comparison between model predictions of central-peak and peak-ring diameter and those observed in the fresh lunar craters. Observational data taken from Pike (1985); Wood and Head (1976).

In the case of peak-ring diameter, the simulation predictions are consistently below the observed peak-ring diameters; however, the slope of the observed variation and predicted variation appear to be consistent. Model 2 (and 5) provides the closest match between simulated peak-ring diameter and observed peak-ring diameter.

In each of the five suites of simulations central-peak height rises sharply with increasing crater diameter before flattening out to an almost constant level. However, only model 5 predicts a relationship between central-peak height and crater diameter that is in close quantitative agreement with observation. In all cases (excluding model 1), the central peaks are generally more pronounced in the simulations than those observed on the Moon.

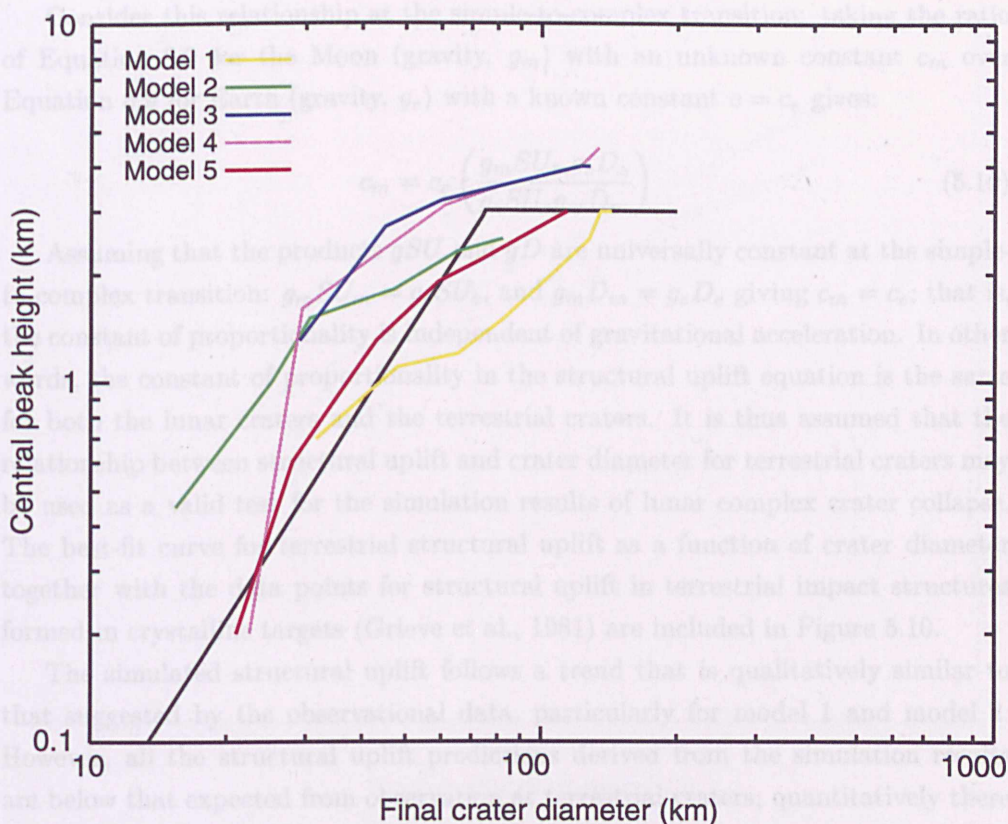


Figure 5.9: Comparison between model predictions of central peak and those observed in the fresh lunar craters. Observational data taken from Hale and Grieve (1982).

5.4.3 Structural Uplift

Figure 5.10 illustrates the relationship between final simulated crater diameter and simulated structural uplift, for each of the five suites of simulations, using the scaling laws defined in Table 5.2. There is no data for the structural uplift beneath lunar craters; however, an estimate may be derived from terrestrial data. Terrestrial structural uplift is given by:

$$SU = cD \quad (5.8)$$

where D is the final crater diameter in kilometres and the constant c is ~ 0.1 . An expression for this constant in the structural uplift relationship for lunar craters, c_m , as a function of the known value of c on Earth, c_e , may be derived by multiplying Equation 5.8 by g and rearranging:

$$c = \frac{gSU}{gD}, \quad (5.9)$$

Consider this relationship at the simple-to-complex transition: taking the ratio of Equation 5.9 for the Moon (gravity, g_m) with an unknown constant c_m over Equation 5.9 for Earth (gravity, g_e) with a known constant $c = c_e$ gives:

$$c_m = c_e \left(\frac{g_m S U_m g_e D_e}{g_e S U_e g_m D_m} \right) \quad (5.10)$$

Assuming that the products gSU and gD are universally constant at the simple-to-complex transition: $g_m S U_m = g_e S U_e$, and $g_m D_m = g_e D_e$ giving $c_m = c_e$; that is, the constant of proportionality is independent of gravitational acceleration. In other words, the constant of proportionality in the structural uplift equation is the same for both the lunar craters and the terrestrial craters. It is thus assumed that the relationship between structural uplift and crater diameter for terrestrial craters may be used as a valid test for the simulation results of lunar complex crater collapse. The best-fit curve for terrestrial structural uplift as a function of crater diameter together with the data points for structural uplift in terrestrial impact structures formed in crystalline targets (Grieve et al., 1981) are included in Figure 5.10.

The simulated structural uplift follows a trend that is qualitatively similar to that suggested by the observational data, particularly for model 1 and model 2. However, all the structural uplift predictions derived from the simulation results are below that expected from observation at terrestrial craters; quantitatively there is considerable disagreement between the simulation results and observation. At smaller crater diameters the disagreement between the curves for structural uplift is more pronounced. The observational data suffers from a paucity of data points above 30 km final crater diameter; however, the spread of the data in the y-direction is insufficient to account for the entire discrepancy between the simulation results and the observational results. Model 5 is particularly poor in terms of its predictions of stratigraphic uplift for lunar complex craters smaller than 100 km.

The comparisons between simulation and observation discussed above suggest that the hydrocode simulation results, which are derived from simple scaling laws representing the change in the nature of acoustic fluidization with increasing impact size, provide a good qualitative model for complex crater collapse. Furthermore, there is reasonable quantitative agreement between simulated crater structure and observation, in terms of some morphometric parameters. However, in general, the quantitative agreement between the simulation results and the observational results is poor. For example, the slope of the peak-ring diameter and the structural uplift curves mimic observation well; however, both curves lie significantly below the observed trends.

The simulation results suggest that no single set of scaling relations, from the 5 tested, provide a good, predictive, quantitative model for complex crater collapse. A set of scaling laws that include an artificial increase in the effect of acoustic flu-

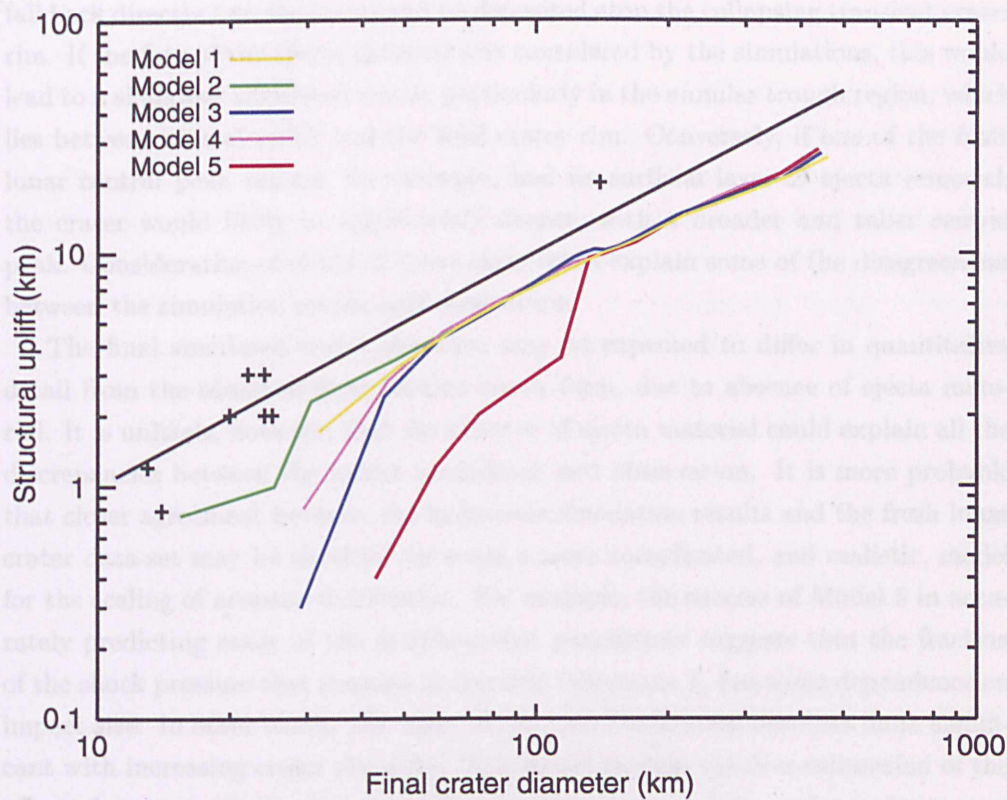


Figure 5.10: Comparison between model predictions of structural uplift of the crater floor and those observed in the fresh lunar craters. Crosses represent observational data from Grieve et al. (1981).

idization at a transient crater diameter of 75 km (model 5), provides the closest match between simulated and observed crater morphology in terms of several morphometric parameters; however, this model produces very poor agreement between simulated crater structure and observation in terms of stratigraphic uplift beneath the crater floor.

Some of the discrepancy between the simulation and observational results may be due to the fact that the morphometric parameters defined in Figure 5.6a and b are not exactly comparable; the final simulated crater structure may not be equivalent to the fresh lunar crater structure. Although the fresh lunar craters are near pristine, they will have been modified in some manner. Furthermore, the lunar craters are covered in a layer of fall-back ejecta, which distorts the majority of the morphometric measurements used to parameterize the shape of an impact crater. Only the quantitative measurements of structural uplift and peak-ring diameter should be unaffected by the presence of impact ejecta material.

The hydrocode simulations discussed here do not consider the fate of material expelled from the crater during the excavation stage. This ejecta material will both

fall back directly into the crater and be deposited atop the collapsing transient crater rim. If the fate of the ejecta material was considered by the simulations, this would lead to a shallower simulated crater, particularly in the annular trough region, which lies between central uplift and the final crater rim. Conversely, if one of the fresh lunar central peak craters, for example, had its surficial layer of ejecta removed, the crater would likely be significantly deeper, with a broader and taller central peak. Consideration of either of these cases could explain some of the disagreement between the simulation results and observation.

The final simulated crater structure may be expected to differ in quantitative detail from the observed final pristine crater form, due to absence of ejecta material. It is unlikely, however, that the absence of ejecta material could explain all the discrepancies between the model predictions and observation. It is more probable that closer agreement between the hydrocode simulation results and the fresh lunar crater data-set may be obtained by using a more complicated, and realistic, model for the scaling of acoustic fluidization. For example, the success of Model 5 in accurately predicting many of the morphometric parameters suggests that the fraction of the shock pressure that remains as acoustic vibrations f_v has some dependence on impact size. In other words, the effect of acoustic fluidization becomes more significant with increasing crater diameter. This would explain the over-estimation of the effect of acoustic fluidization at small crater diameters, and the under-estimation at large crater diameters, when using the simple linear relationships of Model 1. Alternatively, the effect of regeneration of acoustic energy, which has been neglected from the simulations presented here, could become increasingly important at larger crater diameters. A final alternative is that, at an approximate transient crater diameter of 75 km on the Moon, another weakening mechanism becomes important, perhaps thermal softening.

5.5 Summary

A simple analytical model for the scaling of the acoustic fluidization parameters with planetary gravity and crater diameter may be derived by assuming that the characteristic block-size of the fractured target scales linearly with crater diameter. Using this model, hydrocode simulations of lunar complex crater collapse, over a range in crater diameters, produce a size-morphology relationship that is qualitatively very similar to that observed on the Moon.

Quantitatively, this model and a number of similar models, do not accurately mimic observation. In terms of morphometry and predicted structural uplift, no single set of linear block-model scaling laws produces a simulated crater population that is quantitatively very similar to the lunar crater population. In particular, the

simulations need to produce more structural uplift and more lateral expansion of the peak ring to improve agreement with observation.

The discrepancies between observation and simulation results for central-peak height, central-peak diameter and crater depth may be due to the fact that the simulations described here do not consider melt production or ejecta. Ejecta and melt blanket the floors of lunar craters obscuring the true morphology of the crater.

The hydrocode modelling is in good qualitative agreement with observation. However, a more rigorous treatment of the acoustic fluidization theory, which accounts for the regeneration of vibrational energy during deformation, would likely improve the match between the simulation results and observation.

Chapter 6

Conclusions

“But, Eeyore,” said Pooh in distress, “what can we—I mean, how shall we—do you think if we—”

“Yes,” said Eeyore. “One of those would be just the thing. Thank you, Pooh.”

The House at Pooh Corner, *A. A. Milne.*

Impact crater collapse is the gravitationally driven modification of the cavity generated during the early stages of an impact event. It is the last major stage in the formation of an impact crater and has the most profound influence on the final morphology of the crater. Impact craters are ubiquitous features across the Solar System; even the Earth, which shows few surficial scars of extraterrestrial bombardment, has suffered a violent and frequent cratering history. Consequently, impact cratering, and by inference crater collapse, must be considered as important geologic processes.

The population of impact craters in our Solar System contains an intriguing diversity and complexity in morphology and sub-surface structure. The motivation behind this thesis is drawn from the fact that the process that results in this diversity and complexity remains insufficiently understood for a predictive model of impact crater formation to be defined. This chapter will argue that the hydrocode modelling work presented in this thesis provides at least a first-order version of such a model. This chapter will draw together all the issues raised in this thesis regarding the formation of large impact craters, before presenting the pertinent conclusions of this research. Furthermore, the limitations of the work reported here will be discussed and several avenues for future work explored.

6.1 Summary

The approach of this research has been to investigate complex crater collapse using the only feasible method of simulating such a rarely occurring, complicated and highly dynamic process; numerical modelling. Previous research, from a number of related fields, indicates that the morphology of large impact craters is predominantly the result of the collapse of a geometrically simple, bowl-shaped “transient crater,” which forms immediately after the impact (Melosh and Ivanov, 1999). This collapse is driven by gravity and resisted by the strength of the target. However, this research reaffirms that the strength of standard rock material, both competent and completely fractured, is sufficient to prevent all but the steepest part of the transient crater rim from collapsing (Dent, 1973; Melosh, 1977; McKinnon, 1978; O’Keefe and Ahrens, 1993; Collins et al., 2002). Therefore, complex crater collapse must involve significant weakening of a region of the target surrounding the crater; a process that is insufficiently understood for a predictive, quantitative model of complex crater collapse to be defined.

The size of the smallest impact crater for which collapse results in a complex-style crater follows a first order correlation between bodies with different planetary gravities of $1/g$, suggesting that the simple-to-complex transition corresponds to a cohesive strength threshold (Melosh, 1989). The shallowness of complex craters and the occurrence of terraced rim-walls precludes a simple, debris-sliding explanation for complex crater collapse (Melosh, 1977; McKinnon, 1978). Furthermore, the formation of central peaks or rings requires not only weakening of the target, but also temporary fluidization (Baldwin, 1972). Thus, complex crater collapse appears to involve the macroscopic fluid behaviour of rock debris and must, therefore, be simulated using a numerical modelling tool known as a hydrocode.

The hydrocode used for this research has been adapted specifically to simulate complex crater collapse. The modifications required include the construction of an appropriate mesh geometry to replicate the initial form of the transient crater, an algorithm to predict the static strength properties of the material surrounding the transient crater, and an algorithm to modify these properties using a model for the temporary weakening and fluidization of material surrounding the crater. The stages of the impact prior to collapse are approximated using the Z-model of the excavation flow. Thus, the collapse simulations begin at approximately the moment of maximum penetration – when the expanding cavity has reached its maximum depth. It is assumed that at this point the cavity formed during excavation is parabolic in cross-section, with a depth-to-diameter ratio of 1:3. The popular convention is adopted; that is, to term the cavity at this point the transient crater.

At the onset of the collapse simulations, the transient over-pressure due to the impact-generated shock wave is assumed to be entirely dissipated and the target

material stationary. Thus, the numerical model essentially considers the competition between the gravity driven collapse of the unstable transient crater and the inherent material strength properties of the post-shock target. The yield strength Y is used by the hydrocode to control the elastic stress limit of the modelled target material; that is, at stresses above this limit the hydrocode considers the modelled target material to be fluid-like, with a constant Newtonian viscosity. Below the yield strength, the hydrocode treats the modelled material as elastic; no permanent strain is allowed. In essence, therefore, the simulations consider the target material to be a Bingham fluid, parameterized by the yield strength of the target and the fluidized viscosity.

The physical mechanism that lowers the yield strength and permits fluid-like flow of the target during collapse still remains equivocal. However, it is widely accepted that the passage of the expanding shock wave irrevocably alters the state of the target rocks (Melosh, 1989). In the wake of the expanding shock front, the target is heated, comprehensively fractured and violently shaken. As a consequence, several mechanisms could modify the material strength and rheology of the target in the few seconds immediately after a high velocity impact. Of these, the two theories to attract the most attention focus on thermal weakening (O'Keefe and Ahrens, 1993, 1999) and mechanical weakening via the presence of transient pressure vibrations in the target (Melosh, 1979). The theoretical arguments presented here suggest that the latter mechanism, termed acoustic fluidization, appears to offer the most robust explanation for temporary fluidization of the target surrounding an impact crater.

A simple approximation of the acoustic fluidization theory is provided by the Block Model (Ivanov and Kostuchenko, 1997; Melosh and Ivanov, 1999). An implementation of this model into a hydrocode is described and examined in this thesis. The result is a numerical model that provides quantitative predictions of complex crater collapse in accordance with the general paradigm that has emerged over the past twenty years: the transient crater collapses to form a complex crater by means of inward slumping of the transient crater rim, accompanied by the uplift of the floor of the crater. The collapse is facilitated by fluidization of the target, which ceases within a few minutes of the impact, effectively freezing the collapse to produce the final, pristine crater morphology (Melosh, 1989).

The lack of a definitive model for the formation of large impact craters has been due, in no small part, to the scarcity of large impact craters on the Earth, and to the near impossibility of probing the sub-surface on other planets. This situation changed following the discovery of the buried Chicxulub impact structure in Yucatan, Mexico. The Chicxulub impact structure is the largest near-pristine impact crater discovered on Earth to date. Its morphology and subsurface structure, as imaged by the Chicxulub Seismic Experiment, provide unique kinematic and dynamic insight into the process of large impact crater formation and peak-ring formation (Morgan et al., 1997; Morgan and Warner, 1999; Brittan et al., 1999;

Morgan et al., 2000). Thus, the focus of this research was to develop and test dynamical models for the formation of a Chicxulub-scale impact crater.

This research shows that, for a range of appropriate model parameters, temporary fluidization of the target allows the formation of internal peak and ring structures analogous to those observed in terrestrial and extra-terrestrial craters. In the numerical simulations, crater morphology is most sensitive to the decay rate of the fluidized region. For decay times of the order of a few minutes, simulations of a Chicxulub-scale crater collapse produce an internal ring analogous to the peak ring. These simulations thus provide a predictive, quantitative model for the collapse of a Chicxulub-scale impact event.

In simulating a Chicxulub-scale collapse event, a desired result is the generation of a peak ring, which forms inside the final crater rim as a result of the collapse of the central uplift. A peak ring cannot be generated if the central uplift does not rise above the pre-impact target surface; thus, an upper limit for the viscosity used in the simulations of a Chicxulub-scale collapse is that which precludes overshoot of the central uplift. This limit is significantly below the theoretical prediction of Tait (1890) for the critical viscosity that precludes overshoot in a constant viscosity half-space. Knowledge of the appropriate block-size for the brecciated target material at Chicxulub will help constrain the viscosity of the fluidized region.

Analysis of the hydrocode simulation dynamics supports the model for peak-ring formation inferred from the Chicxulub seismic data by Brittan et al. (1999). In the simulations, peak-ring formation involves the interaction of two flow regimes; the inwardly-collapsing crater rim and the outwardly-collapsing central uplift. Simulations which begin with rebound of the central uplift prior to the commencement of rim collapse showed a slightly different dynamic formation mechanism than those simulations which began with a static transient crater. In the simulations with dynamic starting conditions, peak-ring formation involves *collision* between the two flow regimes. To this extent, the simulations with dynamic starting conditions are assumed to be more consistent with the dynamic model for peak-ring formation inferred from the seismic data.

The subsurface structure predicted by the hydrocode simulations is qualitatively the same for a broad range of model parameters. This structure shows many similarities with the subsurface structure inferred from the seismic data at Chicxulub. Furthermore, horizontal slices through the simulated crater structure appear to reconcile contrasting evidence for the nature of peak-ring material from terrestrial outcrop patterns. The hydrocode simulations suggest that peak-ring formation involves significant lateral, as well as vertical, movement of material near the target surface. As a consequence, deeply derived material is overturned to form the peak ring. The insight into peak-ring formation gleaned from the hydrocode simulations of the col-

lapse of a Chicxulub-scale crater may thus be extended to define a qualitative generic model for the subsurface structure of peak-ring craters.

In an extension to the modelling work, estimates of the impact melt volume and initial geometry produced during the early stages of the Chicxulub event, determined by the numerical simulations of Pierazzo et al. (1998), may be used to simulate the fate of the melt-volume during collapse. The results of this exercise are also consistent with the seismic data at Chicxulub. Near the surface of the final simulated crater the melt occupies a broad region within the peak ring, which extends only a couple of kilometers below the surface. The radial extent of this melt region decreases with increasing depth. The depth of the melt volume at the culmination of the collapse simulation is a function of the assumed melt volume; the larger the assumed melt volume the deeper the base of the final melt volume.

An attempt to extend the predictive, quantitative model of the collapse of a Chicxulub-scale crater to all crater diameters, and for craters formed on other planetary bodies, is also discussed here. The approach is to define scaling formulae to relate the acoustic fluidization and block-model parameters to crater diameter and planetary gravity. Hydrocode simulations of crater collapse for a range of crater diameters or on different planetary bodies may then be performed. This thesis discusses an attempt to model the formation of the fresh lunar crater population using such an approach.

The simulations of crater collapse, when applied over a range of crater diameters, produce a size-morphology relationship that is qualitatively very similar to that observed on the Moon for a range of simple laws relating the block-model parameters to crater dimension or projectile size. Quantitatively, however, the model does not accurately mimic observation. In terms of morphometry and predicted structural uplift, no single set of linear block-model scaling laws produce a simulated crater population that is quantitatively very similar to the lunar crater population. The discrepancies between observation and simulation results for central peak height, central peak diameter and crater depth may be due to the fact that the simulations do not consider melt production or ejecta. Ejecta and melt blanket the floors of lunar craters obscuring the true morphology of the crater.

6.2 Conclusions

The major conclusions derived from this unit of research are:

1. Impact crater collapse is an important geological process.
2. The morphology of large impact craters is predominantly the result of the collapse of a geometrically simple bowl-shaped cavity under the force of gravity.

3. Complex crater collapse requires not only the temporary weakening of the target surrounding the transient crater; but also the temporary fluidization of this material. Acoustic Fluidization offers a plausible explanation for this temporary fluidization.
4. A simple hydrodynamic computer code may be used to simulate the collapse of large impact craters. Implementation of the physical laws which govern acoustic fluidization into such a computer code provide a predictive quantitative model for complex crater collapse.
5. This model may be used to reproduce many of the structural and morphologic features present in the Chicxulub impact structure.
6. In particular, the model presented here defines in detail the mechanism by which internal rings form during complex crater formation. Peak-ring formation involves the interaction of two flow regimes; the inwardly-collapsing crater rim collides and overrides the outwardly-collapsing central uplift.
7. In contrast to previous generic models for peak-ring formation, this research suggests that peak-ring generation involves significant lateral movement of the target material immediately beneath the crater. This result leads to a new generic model for peak-ring formation that is capable of reconciling contrasting evidence for the nature of peak-ring material from terrestrial outcrop patterns.
8. Simple scaling laws relating the parameters that constrain this quantitative model of crater collapse to the size of the crater and the gravitational attraction of the planet on which the crater forms provide a reasonable qualitative model for the formation of lunar complex craters over a range of crater diameters.
9. A predictive quantitative model for the formation of all complex craters is not provided by simple scaling laws, and possibly not by considering the effects of acoustic fluidization alone.

6.3 Discussion, Limitations and Future Work

This research has focused on developing a predictive, quantitative model for complex crater collapse. The hydrocode modelling discussed in this thesis provides such a model for the specific case of the Chicxulub impact event. However, this model is non-unique; it depends on several parameters for which a range of feasible values produce a simulated crater structure in good agreement with the inferred structure of the Chicxulub impact crater. Furthermore, the model is, by necessity, oversimplified. To model such a complicated process as crater collapse on a computer with a finite memory capacity and processor speed, appropriate simplifying assumptions must be

invoked. It is therefore remarkable that the hydrocode simulations, when applied over a range of crater diameters, produce a size-morphology relationship that is qualitatively very similar to that observed on the Moon. This result suggests that the simulations developed during this research do provide a first-order, predictive, quantitative model of complex crater collapse.

One limitation of this research is that the block model approximation of acoustic fluidization does not implement the full acoustic fluidization theory of Melosh (1979, 1996) in the sense that the amplitude of the acoustic vibrations does not depend on the flow of the fluidized material. In the full model (which attempts to unify crater collapse, large landslides and seismic faulting) an important aspect of the process is that energy dissipated by shear in the flowing debris itself generates acoustic energy. Thus, in static regions the vibrations soon dissipate by friction and the effective viscosity in those regions rapidly rises. In flowing regions the regeneration of acoustic energy permits flow to continue until the driving stresses are relieved.

Although it would be desirable to implement such a model as part of the crater collapse computations, attempts to insert the extremely nonlinear equations into a working hydrocode have so far been unsuccessful. The methodology for implementing the full acoustic fluidization model is described and discussed in section 3.3.4 and the accompanying Appendix B.3. The most obvious avenue for future work is to complete the implementation of this algorithm and explore the effect of the regeneration of acoustic energy on the collapse process. In the meantime, the block model captures many of the major features of acoustic fluidization and for this reason it is applied to the crater collapse models presented here. The major change that a full treatment of the acoustic fluidization model might make would be to prolong flow in the actively deforming central peak and peak ring region and perhaps allow more complete relaxation of the original crater cavity.

The hydrocode simulations discussed in this thesis are intended to be a first-order assessment of crater collapse upon which, it is hoped, future work will build and refine. Consequently, many simplifying assumptions are applied; namely, that the target is a uniform half-space, with the elastic and Tillotson equation-of-state properties of brecciated crustal material. Preliminary investigation suggests that it is not necessary to consider any compositional changes at the boundary between crust and mantle, nor to consider the contrasting properties of the sedimentary rocks comprising the upper 2 km of the crust. However, little is known about how the acoustic fluidization parameters might vary with composition and pressure. Future work should investigate both the likely effect of pressure and composition on the acoustic fluidization process, and the effect that any such variation might have on the collapse simulations discussed here.

A further simplifying assumption of this research is that the simulations do not model the thermal profile of the excavated crater, thus any thermal weakening of the

target or melt production is not considered. It should be emphasized, therefore, that the calculations discussed in this thesis use a material weakening mechanism that relies exclusively on transient pressure oscillations within the target. The weakening effects are likely to be very similar to temperature dependent weakening mechanisms. Similar modelling results to those presented in this research have been achieved with a thermally influenced weakening mechanism using a modified version of the code discussed in this thesis (Ivanov, 2000) and another modeling package (O’Keefe and Ahrens, 1999). Discerning which, if either, of the two mechanisms is more important is beyond the scope of current numerical techniques and is currently left to theoretical consideration. This research argues that the thermal weakening model is only important for extremely large impact events, which in the terrestrial setting corresponds to transient crater diameters in excess of 200 km.

This research highlights acoustic fluidization as the most plausible target weakening mechanism. However, this phenomenon is an under-studied process; despite some experimental support for acoustic fluidization (Gaffney and Melosh, 1982), it remains to be proven whether seismic energy generated during a large impact event can remain strong enough and last for the required time-scale to facilitate collapse. A strong recommendation of this research, therefore, is that further investigation into the process of the fluidization of a granular media by internal pressure vibrations needs to be conducted. Such research is imperative for discerning the importance of acoustic fluidization not just in impact cratering but also in large-scale mass movements of granular debris and seismic faulting. Research into acoustic fluidization should concentrate on three main aspects:

1. Small-scale analogue experiments of acoustic fluidization to constrain the effect of regeneration and the effect of grain size and strain rate on the wavelength of emitted acoustic noise.
2. Geological field and borehole investigations to constrain the relationship between crater size and characteristic block size.
3. Implementation of the full acoustic fluidization theory and further computer modelling to investigate the effect of scattering and regeneration of acoustic energy on complex crater collapse.

Crater collapse is a highly dynamic process, involving the rapid transport of extremely large volumes of rock; quite unlike almost all other, more familiar geologic processes. Over very long time-scales, the shape of an impact crater is modified further by more conventional geologic processes; for example, viscous relaxation, erosion and tectonism. Thus, much of the impact crater population in the Solar System, and almost the entire terrestrial crater population, does not reflect the

pristine form of a complex crater. The extremely dynamic nature of the terrestrial surface has hindered the development of a definitive explanation of complex crater collapse. An understanding of long time-scale modification is imperative, therefore, when comparing the results of complex crater collapse simulations with geological observation. Another branch of associated further research should be to study long-term relaxation of large impact craters and investigate the correlation between the predictions of numerical models, such as those presented here, and geological/geophysical observation of near-pristine terrestrial craters.

Appendix A

Derivation of the Acoustic Fluidization and Block-Model Equations

A.1 Acoustic Fluidization: Melosh Model

The original acoustic fluidization model was developed by Melosh (1979) to explain a number of phenomena which require temporary weakening of granular debris; crater collapse, large landslides and seismic faulting. The most readable exposition of the theory is presented in Melosh (1983). The derivation below follows the original formulation in Melosh (1979), before discussing the temporal behaviour of acoustic fluidization as presented in Melosh (1996).

Consider a mass of granular rock debris, with a mean particle diameter d , and a bulk density of ρ in a gravitational field g . From Coulomb's law (Eq. 2.12), irreversible sliding at a depth h within such a material occurs for an applied shear stress τ greater than that required to overcome the internal friction, τ_{static} ; that is, when:

$$\tau > \tau_{static} = \mu p, \quad (\text{A.1})$$

where μ is the coefficient of friction and p is the overburden pressure $p = \rho gh$.

The presence of acoustic pressure vibrations within the rock mass will modify the internal friction of the granular media. If the acoustic vibrations have a unique or dominant wavelength λ , and peak rarefaction amplitude s (s positive for tension), then the requirement for inter-granular slip becomes:

$$\tau > \mu(p - s). \quad (\text{A.2})$$

Thus, there is a critical vibrational pressure amplitude that permits slip, s_c , given by:

$$s_c = p - \frac{\tau}{\mu}. \quad (\text{A.3})$$

A schematic diagram of the effect of acoustic pressure vibrations on a mass of rock debris taken from Melosh (1979) is shown in Figure A.1.

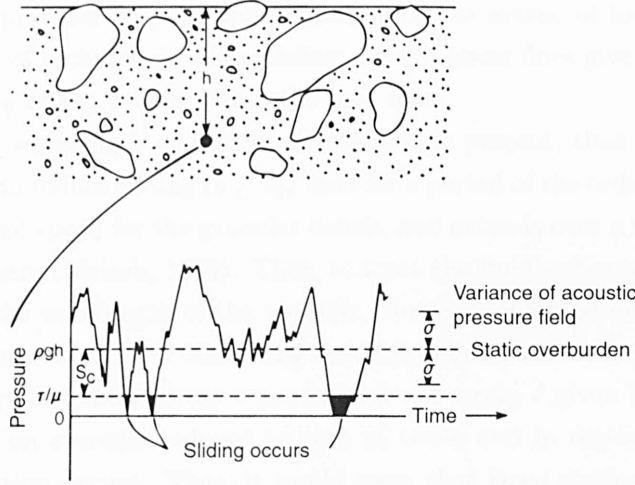


Figure A.1: Schematic illustration of the acoustic fluidization process. Parameters discussed in the text are labelled here. *Taken from Melosh (1989).*

To develop a continuum description for the behaviour of rock debris under the influence of acoustic pressure vibrations it is necessary to characterize the random acoustic noise. Melosh (1979) suggests that, although a complete characterization of a random acoustic field is a complex statistical problem, a useful estimate may be derived by assuming that the acoustic field can be described as a stationary normal random process. In this case, the probability that the rarefaction amplitude lies between s and $s + ds$, at any point within the random acoustic pressure field, is given by:

$$\mathcal{P}(s)ds = \frac{ds}{\sqrt{2\pi}\sigma} e^{-\frac{s^2}{2\sigma^2}} \quad (\text{A.4})$$

where σ is the variance of the pressure field, with dimensions of stress. The fraction of the volume ζ for which $s \geq s_c$, at any given time, is evaluated by the integral of Equation A.4:

$$\zeta(s \geq s_c) = \int_{s_c}^{\infty} \mathcal{P}(s)ds = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{s_c}{\sqrt{2}\sigma} \right) \right]. \quad (\text{A.5})$$

Equation A.5 describes the intuitive result that the greater the variance of the acoustic pressure field the larger the fraction of the volume of rock debris where the rarefaction amplitude exceeds that required for sliding, $s \geq s_c$. In practice, however, σ cannot be larger than some fraction of the overburden since the net pressure of an acoustic rarefaction cannot be less than zero in cohesionless rock debris. A variance substantially in excess of p causes rock contacts to break frequently, implying strong scattering and dissipation of wave energy (Melosh, 1979). As a consequence, Equation A.5 provides only an approximation of the extent of local slip within a vibrating mass of rock debris. Nevertheless, this analysis does give a good estimate of the *frequency* of slip events within the rock mass.

If only one wavelength of pressure oscillation is present, then each rarefaction strong enough to induce sliding ($s \geq s_c$) lasts for a period of the order of λ/c , where c is the bulk sound speed for the granular debris, and extends over a region about one wavelength in size (Melosh, 1979). Thus, to treat the fluidized granular medium as a continuum, the wavelength of the acoustic vibrations must be much greater than the particle diameter, $\lambda \gg d$. Assuming that the debris behaves in a linearly elastic manner, the applied shear stress τ produces a net strain ϵ given by Equation 2.9. This strain is, on average, reduced to zero at times and in regions where a large enough rarefaction occurs. Thus, it would seem that large strains and thus rapid rates of flow could result from a very small shear modulus G . However, Melosh (1979) suggests that if G is much lower than the bulk modulus $K = \rho c^2$, the rock cannot move fast enough to relax the full strain in time λ/c . Thus, taking $G = \rho c^2$, the net *inelastic* strain which occurs during one cycle is:

$$\Delta\epsilon_{net} \approx \frac{\bar{\tau}}{2\rho c^2} \left[1 - \operatorname{erf} \left(\frac{s_c}{\sqrt{2}\sigma} \right) \right] \quad (\text{A.6})$$

where $\bar{\tau}$ is the mean shear stress acting in the medium. This stress is greater than the applied shear stress as the applied shear stress acts only in the fraction of the volume that can support the applied stress ($1 - \zeta(s \geq s_c)$). Thus,

$$\bar{\tau} = \frac{\tau}{1 - \zeta(s \geq s_c)} \quad (\text{A.7})$$

Hence, the net strain *rate* $\dot{\epsilon}$ is:

$$\begin{aligned} \dot{\epsilon} &\approx \Delta\epsilon_{net} \frac{c}{\lambda} \\ &\approx \frac{\tau}{\rho\lambda c} \left[\frac{1 - \operatorname{erf} \left(\frac{s_c}{\sqrt{2}\sigma} \right)}{1 + \operatorname{erf} \left(\frac{s_c}{\sqrt{2}\sigma} \right)} \right]. \end{aligned} \quad (\text{A.8})$$

This is the central equation of the acoustic fluidization model (Melosh, 1979). It can be rewritten in terms of a dimensionless measure of driving stress, $\Omega = \tau/\tau_{static}$; and a dimensionless measure of the strength of the pressure field, $\Sigma = \sigma/s_c$ (Melosh and Ivanov, 1999). $\tau_{static} = \mu p$ is the stress required to initiate failure when no vibrations are present. A somewhat more compact way to represent Equation A.8 is, therefore:

$$\dot{\epsilon} \approx \frac{\tau}{\rho\lambda c} \left[\frac{2}{\text{erfc}(\chi)} - 1 \right]^{-1}, \quad (\text{A.9})$$

where,

$$\chi = \frac{1 - \Omega}{\Sigma}. \quad (\text{A.10})$$

Equation A.9 describes fluid-like behaviour of the mass of rock debris. Thus, from a macroscopic point of view, the granular material behaves as a fluid when vibrations are present. The effective viscosity of the fluidized debris, η_{eff} , is defined as the ratio of the shear stress to the net strain rate:

$$\eta_{\text{eff}} \approx \frac{\rho\lambda c}{2} \left[\frac{2}{\text{erfc}(\chi)} - 1 \right]. \quad (\text{A.11})$$

The mean energy density (energy per unit volume) in an acoustic pressure field is given by:

$$\mathcal{E} = \frac{\sigma^2}{2K} \quad (\text{A.12})$$

where K is the appropriate elastic modulus. The temporal behaviour of this energy is controlled by the differential equation from Melosh (1996):

$$\frac{d\mathcal{E}}{dt} = \frac{\xi}{4} \nabla^2 \mathcal{E} - \frac{c}{\lambda Q} \mathcal{E} + e \frac{\dot{\epsilon} \tau}{2}. \quad (\text{A.13})$$

The first term on the right-hand side is the scattering term, where ξ is the scattering diffusivity, which has dimensions of a length times a velocity. The second term parameterizes conversion of elastic energy into heat. Q is defined as the ratio of the energy stored per cycle to the energy dissipated in the same period. The last term quantifies the energy generation during flow, where e is the fraction of the released acoustic energy that is available to regenerate the vibrations.

Implementing this model of acoustic fluidization as part of the crater collapse computations is a desirable goal; however, inserting the extremely nonlinear equations successfully into a working hydrocode is problematic. Section 3.3.4 and Appendix B.3 discuss the work in this direction undertaken as part of this thesis. In the meantime, a simplified approximation of acoustic fluidization, the block model,

captures many of the major features of acoustic fluidization and for this reason it is applied to the crater collapse models presented here.

A.2 Acoustic Fluidization: Block Model

A simple approximation of acoustic fluidization which lends itself well to incorporation in a hydrocode is the “Block-model” (Ivanov and Kostuchenko, 1997; Melosh and Ivanov, 1999). The premise for this model is that the target region surrounding the transient crater is comprised by a system of rock blocks which oscillate within a matrix of finer material. Bore-hole investigations beneath terrestrial complex craters provide support for this model. The block-model equations derive from a simple, one-dimensional description of this block-matrix system.

Consider a block of length scale h in a small grain matrix subject to a shear stress τ (Fig. A.2a). The block experiences an overburden pressure p ; hence, if the block is at rest the resistance to shear f , is that due to “dry friction”:

$$f = \mu p, \quad (\text{A.14})$$

where μ is the coefficient of friction for the block-matrix surface. If, instead, the block is forced to vibrate with a pressure amplitude of p_v and an angular frequency ω , then the shear strength becomes:

$$f = \mu(p + p_v \sin \omega t). \quad (\text{A.15})$$

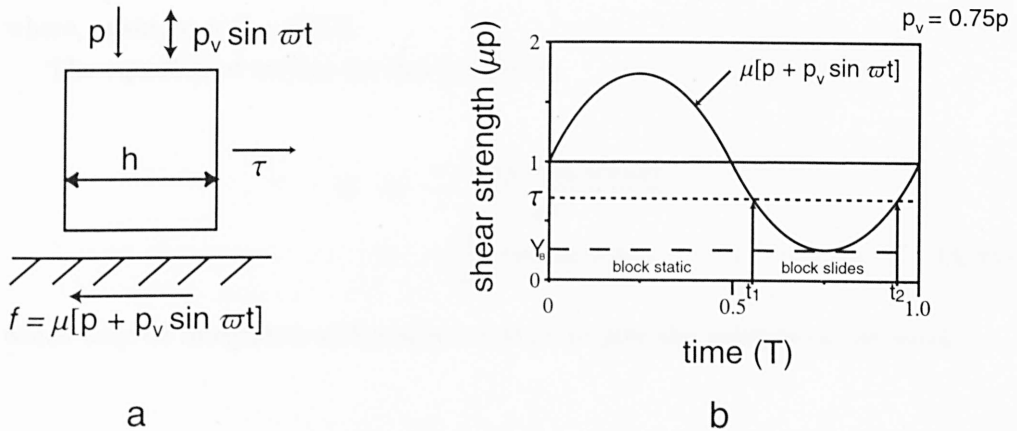


Figure A.2: (a) Diagram showing a block surrounded by a small grained matrix. (b) Graph of resistance to shear on the base of the block against time for one period of oscillation

Figure A.2b shows a graph of the shear strength for one period of oscillation T for the situation where $p_v = 0.75p$. Clearly, sliding occurs where $\tau > f$. Thus, there

is a minimal requirement for motion: $\tau > \mu(p - p_v)$, a condition which defines a kind of Bingham yield stress Y where:

$$Y = \mu(p - p_v). \quad (\text{A.16})$$

It can be seen that, in the illustrated example, the condition for slippage is satisfied between time t_1 and t_2 where:

$$t_1, t_2 = \frac{1}{\omega} \sin^{-1} \left(\frac{\tau/\mu - p}{p_v} \right). \quad (\text{A.17})$$

It is helpful to define the time ΔT such that:

$$t_1 = T/2 + \Delta T, \quad t_2 = T - \Delta T. \quad (\text{A.18})$$

This definition provides the identity:

$$\sin \omega \Delta T = \frac{p - \tau/\mu}{p_v}. \quad (\text{A.19})$$

If, as with the full acoustic fluidization derivation above, the shear stress τ is normalized by the stress required to initiate failure when no vibrations are present, τ_{static} to give a dimensionless measure of the stress, Ω ; and the dimensionless measure of the vibrational amplitude is given by $\Sigma = p_v/p$, then Equation A.19 may be written as:

$$\sin \omega \Delta T = \chi, \quad (\text{A.20})$$

where, again, $\chi = (1 - \Omega)/\Sigma$.

The equation of motion for this system is:

$$\begin{aligned} \ddot{x} &= \frac{\tau - \mu(p + p_v \sin \omega t)}{\rho h}, \\ &= -\frac{\mu p_v}{\rho h} (\sin \omega t + \chi). \end{aligned} \quad (\text{A.21})$$

which may be integrated with respect to time to give the velocity of the block:

$$\begin{aligned} v &= -\frac{\mu}{\rho h p_v} \int_{t_1}^t (\sin \omega t + \chi) dt \\ &= \frac{\mu}{\rho h p_v} \left[\frac{1}{\omega} (\cos \omega t - \cos \omega t_1) - \chi(t - t_1) \right]. \end{aligned} \quad (\text{A.22})$$

Integrating again gives the displacement of the block:

$$\begin{aligned}
\delta &= \frac{\mu p_v}{\rho h} \int_{t_1}^{t_2} \left[\frac{1}{\omega} (\cos \omega t - \cos \omega t_1) - \chi(t - t_1) \right] dt \\
&= \frac{\mu p_v}{\rho h} (t_2 - t_1) \left[-\frac{1}{\omega} \cos \omega t_1 - \frac{\chi}{2} (t_2 - t_1) \right].
\end{aligned} \tag{A.23}$$

The $-\cos \omega t_1$ term may be eliminated from Equation A.23 by recalling the identity, $\cos \theta = \sqrt{1 - \sin^2 \theta}$, giving:

$$-\cos \omega t_1 = \cos \omega \Delta T = \sqrt{1 - \chi^2}. \tag{A.24}$$

Furthermore, to eliminate the $(t_2 - t_1)$ terms, consider equations A.18; combining the two gives:

$$t_2 - t_1 = \frac{T}{2} - 2\Delta T. \tag{A.25}$$

Substituting in ΔT from Equation A.19 gives,

$$t_2 - t_1 = \frac{2}{\omega} \cos^{-1} \chi, \tag{A.26}$$

So, finally, substituting both these results into Equation A.23, gives:

$$\delta = \frac{2\mu p_v}{\rho h \omega^2} \cos^{-1} \chi \left[\sqrt{1 - \chi^2} - \chi \cos^{-1} \chi \right], \tag{A.27}$$

which can be rearranged using the substitution $1 - \chi = (\tau - Y_B)/\mu p_v$ to give,

$$\delta = \frac{2(\tau - Y_B)}{\rho h \omega^2} \Phi, \tag{A.28}$$

where,

$$\Phi = \cos^{-1} \chi \left[\sqrt{\frac{1 + \chi}{1 - \chi}} - \frac{\chi}{1 - \chi} \cos^{-1} \chi \right]. \tag{A.29}$$

The shear strain of the block system *averaged* over time may be defined as $\epsilon = \delta/h$ and the *averaged* strain rate as: $\dot{\epsilon} = \delta/Th$. Thus, the averaged strain rate of the acoustically fluidized debris, according to the block model may be written as:

$$\dot{\epsilon} = \frac{(\tau - Y_B)T}{2\pi^2 \rho h^2} \Phi. \tag{A.30}$$

The effective viscosity of the block-model system, is therefore:

$$\eta_{\text{eff}} = \frac{\pi^2 \rho h^2}{T \Phi}. \tag{A.31}$$

Appendix B

Modifications to the SALES_2 solution algorithm

This section of the thesis discusses the modifications made to the FORTRAN77 source code of the SALES_2 hydrocode to facilitate the simulation of the collapse of large impact craters. The SALES_2 hydrocode is available for download at the following web address:

http://www.lpl.arizona.edu/tekton/sales_2.html

The following three sections describe the modifications required for the construction of an appropriate mesh geometry to replicate the initial form of the transient crater, an algorithm to predict the static strength properties of the material surrounding the transient crater, and an algorithm to modify these properties using a model for the temporary weakening and fluidization of material surrounding the crater. The final section discusses the additions to the common-block and the extra input parameters required. The full source-code listing for the version of SALES_2 used in this research can be found on the CD-ROM included with this thesis.

B.1 Transient Crater Mesh Geometry

The computational mesh for the simulations of complex crater collapse discussed in this thesis represents the transient crater and the target surrounding it. This is achieved by constructing a rectangular grid of cells, n_{xp} vertices wide and n_{yp} vertices tall, which represents the target prior to an impact, and remapping the top and left boundary vertices to represent the form of the transient crater. The original rectangular grid is chosen to be 2-3 times larger than the transient crater dimensions, to avoid any influence of boundary conditions on the collapse. Thus, the right-hand side vertices and the bottom boundary vertices need not be moved during the remapping process. The remaining vertices (those internal to the mesh

edges) are remapped such that they are spread as uniformly as possible throughout the mesh; thus, each cell in the mesh is as close as possible to a regular quadrilateral. The repositioning of the internal vertices in the transient crater geometry is not determined by the excavation flow because some of these vertices would, inevitably, correspond to target material that is ejected. Ejecting material would imply deleting vertices, which is not permitted in the Lagrangian formulation; the SALES_2 hydrocode requires that the mesh is comprised of n_x by n_y cells in a rectangular arrangement. Thus, to assign the cells or vertices any conditions induced at the time of impact, the original location of each cell or vertex in the mesh must be determined. This is discussed in section B.2.

The remapping algorithm is implemented in the subroutine *crater_set*, which is listed below, after a more detailed discussion of the remapping algorithm:

1. The surface morphology of the transient crater inside the crater rim is defined by a parabola of the form $y = ax^2 + c$, where $-c$ is the transient crater depth and $\sqrt{c/a}$ is the transient crater radius. Vertices along the top boundary of the rectangular mesh that lie inside the crater rim are repositioned at regular intervals along this parabolic surface profile (shown in red in Fig. B.1b).
2. Outside the transient crater rim, the surface morphology is determined using the Z-model, with a Z-value of 3. This is achieved by moving the vertex of interest from its initial position (r_0, θ_i) in streamline-angular distance coordinates to its position after excavation (r_0, θ_f) , where θ_f is given by Equation 2.5. θ_x , in Equation 2.5, is the point of intersection between the streamline r_0 and the parabola defining the transient crater wall, $y = ax^2 + c$, and is found numerically by iteration (subroutine *cross*). The final position of these vertices are shown in red in Figure B.1c.
3. Vertices along the left boundary (the axis of symmetry) are repositioned down the left-hand side, beginning at the crater floor and continuing to a given depth below the target surface. The vertices along the bottom and right-hand side of the rectangular mesh need not be repositioned. The final position of these vertices are shown in red in Figure B.1d.
4. The interior vertices are then repositioned using an interpolation scheme (Amsden, 1973). The interpolation algorithm moves each vertex to the average position of its eight surrounding vertices (see Fig. B.1e-h). The boundary vertices are fixed; they are not moved during the interpolation. After several sweeps over all the vertices the interpolation finishes (Fig. B.1h). The result is a mesh where all cells are regular quadrilaterals, for which the finite difference approximations used by SALES_2 will be accurate.

A different, yet still appropriate, mesh geometry may be obtained by following the above algorithm using a "quadrant mesh" instead of a rectangular mesh as the initial grid. Figure B.2 illustrates the stages of the above algorithm in remapping a quadrant grid into the form of the transient crater mesh. Note that, in the case of a quadrant mesh, the i - and j -coordinates refer to the number of vertices from the axis-of-symmetry, along lines of constant distance from the assumed point of impact. Thus, using the terminology employed for the rectangular mesh, the top boundary of the quadrant mesh is inside the crater; the right-hand boundary represents the surface outside the crater rim; the left-hand boundary is the axis of symmetry; and the bottom boundary comprises two sections: the bottom of the mesh, and the right-hand side of the mesh. As a consequence of this change in mesh-coordinates, the algorithm for remapping vertices from a quadrant mesh is slightly different to that for a rectangular mesh.

```

1  *****
2      subroutine crater_set
3  c      =====
4  c      Subroutine to map rectangular mesh into the form of a
5  c      transient crater. Uses the z-model approximation of the
6  c      excavation flow, with z=3. The transient crater has the
7  c      dimensions
8  c
9  c      crater_depth = depth of transient crater
10 c      crater_radius = radius of transient crater
11 c
12 c      which are used to define a parabolic crater wall
13 c
14 c      The routine remaps the top vertices of the mesh to form the
15 c      crater profile. Remaps the other three boundaries. Then
16 c      uses an interpolation scheme to move the interior points
17 c      until a uniform mesh is produced.
18 c
19 c      -----
20 c      INTERNAL VARIABLES:
21 c      a, c      = parabola coefficients describing crater wall
22 c      r0        = streamline coefficient
23 c      r0_hinge  = "hinge streamline" coefficient
24 c      theta_x   = angle subtended between axis and
25 c                  straight line from impact point.
26 c      -----
27 c
28 c      called from celset, (supporting subroutine = cross)
29 c
30 *****
31      include 'sales.comm'
32

```

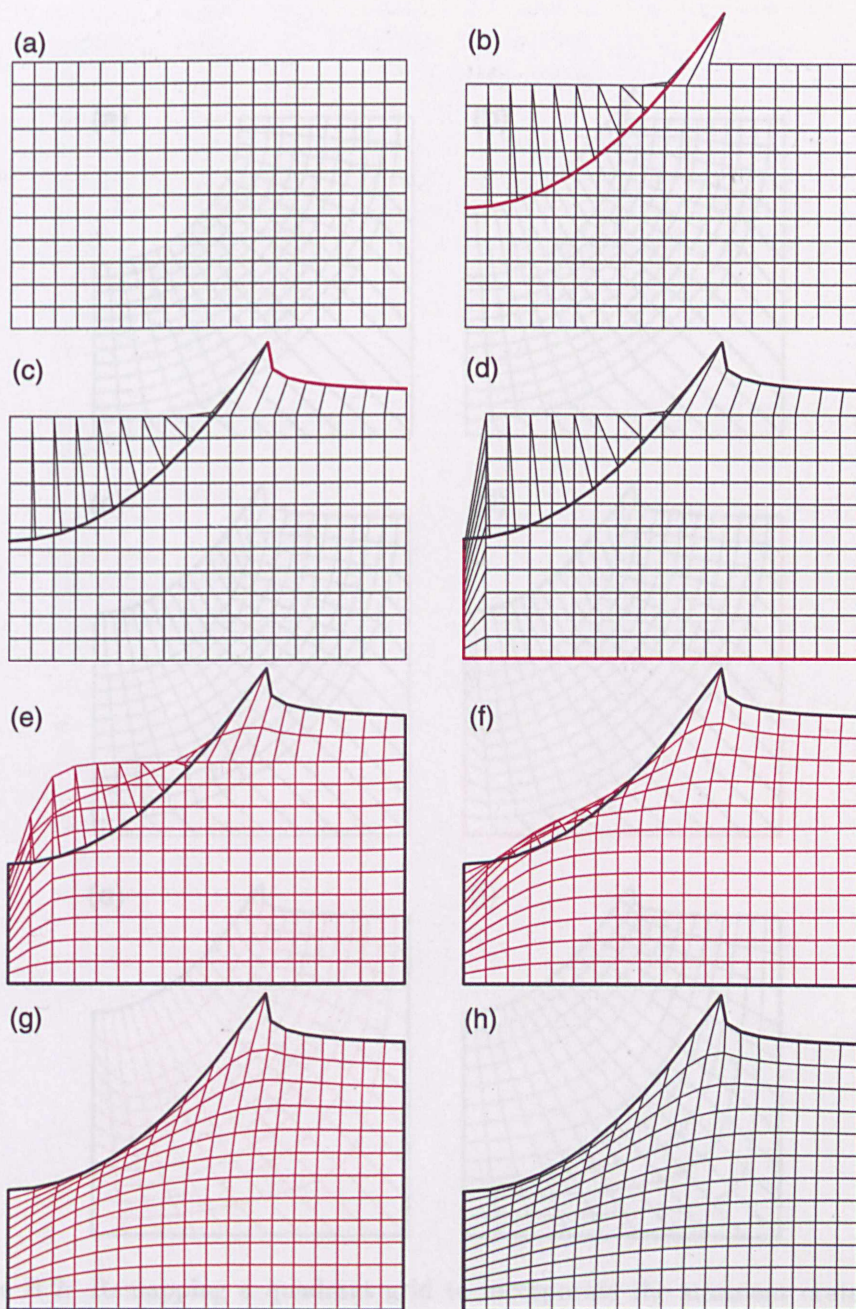


Figure B.1: Remapping a rectangular grid to incorporate the transient crater. (a) Initial rectangular grid 18 cells (19 vertices) $\times$ 12 cells (13 vertices); (b) Vertices inside crater rim on top boundary are moved to positions along the transient crater profile (red); (c) Vertices outside the rim are moved according to the Z-model (red); (d) Left-hand side vertices are positioned beneath crater floor, right-hand side and bottom vertices are not moved (red). (e-h) show the stages of the interpolation technique used to reposition internal vertices (red): (e) shows the mesh after 1 sweep; (f) after 5 sweeps; (g) after 10 sweeps; (h) shows the final mesh after 20 sweeps - each cell in the mesh is as close as possible to a regular quadrilateral.

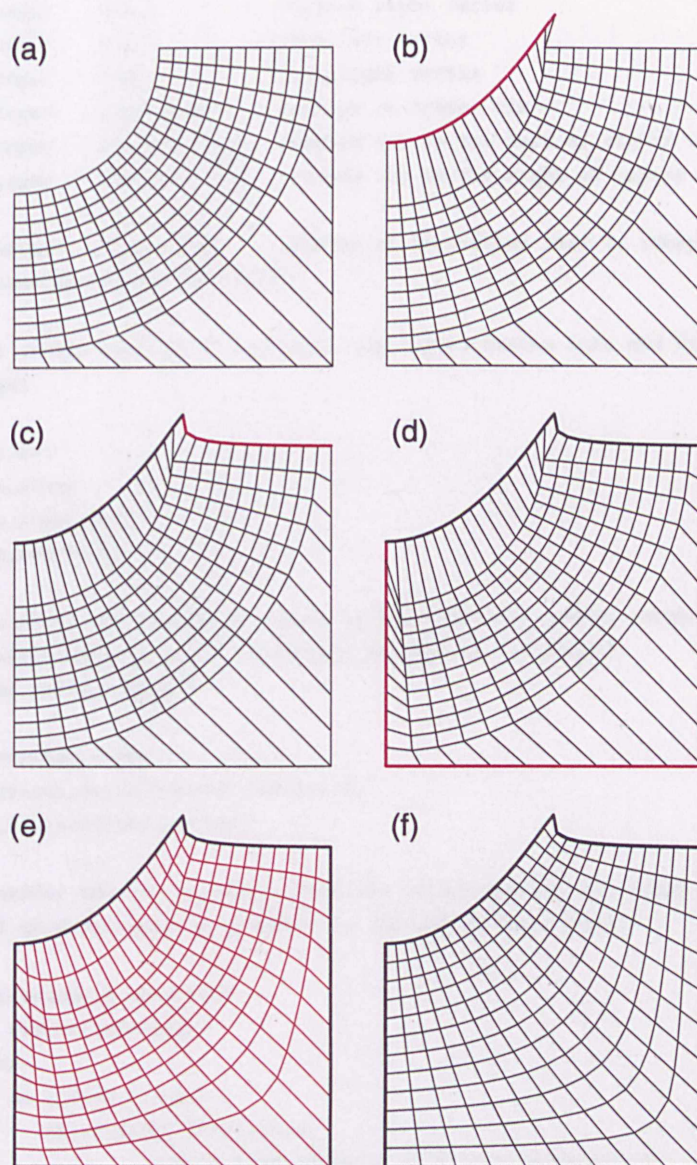


Figure B.2: Remapping a quadrant grid to incorporate the transient crater. (a) Initial quadrant grid 18 cells (19 vertices) $\times$ 12 cells (13 vertices); (b) Vertices inside crater rim (top boundary) are moved to positions along the transient crater profile (red); (c) Vertices outside the rim (right boundary) are moved according to the Z-model (red); (d) Left-hand boundary vertices are positioned beneath crater floor, bottom boundary vertices are not moved (red). (e and f) show two stages of the interpolation technique used to reposition internal vertices (red); (e) shows the mesh after 1 sweep; (f) shows the final mesh after 20 sweeps - each cell in the mesh is as close as possible to a regular quadrilateral.

```

33      integer    fixed(node)    !indicates if vertex is a fixed position
34      integer    bot_l          !bottom left vertex
35      integer    bot_r          !bottom right vertex
36      integer    top_l          !top left vertex
37      integer    top_r          !top right vertex
38      integer    rim_vertex     !vertex of crater rim
39      integer    rim_vertex_m   !vertex one to the left of crater rim
40      integer    rim_vertex_p   !vertex one to the right of crater rim
41
42      integer    iterations     !number of iterations used to remap mesh
43      parameter (iterations=2000)
44  c
45  c      Set vertex numbers of top left, top right, bottom left and bottom
46  c      right.
47  c
48      bot_l=1
49      bot_r=nxp
50      top_l=nxp*ny+1
51      top_r=nxp*nyp
52  c
53  c      Use depth and radius of crater to set transient crater parabola
54  c      coefficients (a and c) and hinge streamline (r0_hinge).
55  c      (For surface burst)
56  c
57      c=-crater_depth
58      a=crater_depth/(crater_radius**2)
59      r0_hinge=crater_radius
60  c
61  c      Consider the top boundary, find the vertex nearest the hinge point
62  c      (If quadrant mesh: rim vertex is defined as top_r vertex.)
63  c
64      if(iquadmsh.eq.1)then
65          rim_vertex=top_r
66      else
67          do ij=top_l,top_r
68              if(r0_hinge-x(ij).lt.0.
69              $          .and.r0_hinge-x(ij-1).ge.0.)rim_vertex=ij-1
70          enddo
71      endif
72  c
73  c      Calculate coordinates of rim point by moving hinge point
74  c      according to the Z-model (Z=3)
75  c
76      r0=r0_hinge
77      theta_old=pi/2.0
78      call cross(r0,a,c,theta_x,ij)
79      if(theta_x.lt.0)theta_x=theta_x+pi
80      theta_new=dacos(1.-
81      $          ( (1.-dcos(theta_old))**4+(1.-dcos(theta_x))**4 )**0.25 )

```



```

82      x(rim_vertex)=r0*dsin(theta_new)*(1-dcos(theta_new))
83      y(rim_vertex)=-r0*dcos(theta_new)*(1-dcos(theta_new))
84      fixed(rim_vertex)=1
85  c
86  c      For all crater wall vertices (those to the left of the rim vertex)
87  c      scale the x-coordinate for an even distribution of nodes and move
88  c      y-coordinate to the crater wall
89  c
90      x_scale_factor=x(rim_vertex)
91      do ij=top_l,rim_vertex-1
92          x(ij)=(ij-top_l)/float(rim_vertex-top_l)*x_scale_factor
93          y(ij)=a*x(ij)**2+c
94          fixed(ij)=1
95      enddo
96  c
97  c      For all vertices on the right boundary (quadrant mesh), or to the
98  c      right of the rim vertex (standard mesh), move vertex according to
99  c      the Z-model
100 c
101      if(iquadmesh.eq.1)then
102          loopbeg=ny*nxp
103          loopend=nxp
104          loopinc=-nxp
105          rim_vertex_m=rim_vertex-1
106          rim_vertex_p=rim_vertex-nxp
107      else
108          loopbeg=rim_vertex+1
109          loopend=top_r
110          loopinc=1
111          rim_vertex_m=rim_vertex-1
112          rim_vertex_p=rim_vertex+1
113      endif
114  c
115      do ij=loopbeg,loopend,loopinc
116          r0=x(ij)
117          theta_old=pi/2.d0
118          call cross(r0,a,c,theta_x,ij)
119          if(theta_x.lt.0)theta_x=theta_x+pi
120          theta_new=dacos(1.-
121      $              ((1.-dcos(theta_old))**4+(1.-dcos(theta_x))**4)**0.25)
122          if(ij.ne.loopend)
123      $              x(ij)=r0*dsin(theta_new)*(1-dcos(theta_new))
124          y(ij)=-r0*dcos(theta_new)*(1.-dcos(theta_new))
125          fixed(ij)=1
126      enddo
127  c
128  c      Now scale and fix the vertices on the left boundary
129  c
130      y_scale_factor=y(top_l)-y(bot_l)

```

```

131      do j=2,ny
132          ij=(j-1)*nxp+1
133          y(ij)=y(bot_l)+(j-1)/(ny*1.)*y_scale_factor
134          fixed(ij)=1
135      enddo
136  c
137  c      Also fix the vertices on the bottom boundary and right if necessary
138  c
139      do ij=bot_l,bot_r
140          fixed(ij)=1
141      enddo
142      if(iquadmesh.ne.1)then
143          y_scale_factor=y(top_r)-y(bot_r)
144          do j=2,ny
145              ij=j*nxp
146              y(ij)=y(bot_r)+(j-1)/(ny*1.)*y_scale_factor
147              fixed(ij)=1
148          enddo
149      endif
150  c
151  c      Fix more vertices for quadmesh
152  c
153      if(iquadmesh.eq.1)then
154          do ij=nxp*15+1,nxp*nyp
155              fixed(ij)=1
156          enddo
157      endif
158  c
159  c      Now iterate over k cycles, relaxing the interior coordinates
160  c      until a uniform mesh is derived
161  c
162      do k=1,iterations
163          do j=jfirst,jlast
164              c
165              c      Assign vertices
166              c
167              c          imjp----ijp----ipjp
168              c          |        |        |
169              c          |        |        |
170              c          imj----ij*----ipj
171              c          |        |        |
172              c          |        |        |
173              c          imjm----ijm----ipjm
174              c
175              c      * represents the vertex being moved
176              c
177              ij=(j-1)*nxp+ifirst
178              ijp=ij+nxp
179              ijm=ij-nxp

```

```

180          do i=ifirst,ilast
181              ipj=ij+1
182              ipjp=ijp+1
183              ipjm=ijm+1
184              imj=ij-1
185              imjp=ijp-1
186              imjm=ijm-1
187  c
188  c      Don't move fixed cells
189  c
190          if(fixed(ij).eq.1)goto 10
191  c
192  c      Calculate new position as average of surrounding eight positions
193  c      Weight positions in favour of top left corner.
194  c
195          x(ij)=0.125*(x(imjm)+x(ijm)+x(ipjm)+x(ipj)
196  $          +x(ipjp)+x(ijp)+x(imjp)+x(imj))
197          y(ij)=0.125*(y(imjm)+y(ijm)+y(ipjm)+y(ipj)
198  $          +y(ipjp)+y(ijp)+y(imjp)+y(imj))
199  c
200  c      increment vertex indices
201  c
202  10          ij=ipj
203              ijp=ipjp
204              ijm=ipjm
205          enddo
206          enddo
207          enddo
208  c
209  c      Finished mesh transformation
210  c
211  20  write(*,100)
212          return
213  c
214  100  format(//
215  $      '====='/
216  $      ' Finished Mesh Transformation...'/
217  $      '=====')//)
218          end
219  *****

```

The subroutine *cross* uses an iteration technique to find the coordinates of the cross-over point between the streamline on which any unejected point in the target lies, and the parabola that defines the shape of the transient crater. The variable returned by the subroutine is θ_x ; the angular distance from the vertical to this point of intersection. This angle is required by Equation 2.5 to calculate the final angular distance along the streamline for a particular position in the target, as determined by the excavation flow.

```

1  *****
2      subroutine cross(r0,a,c,theta_x,ij)
3  c      =====
4  c      Subroutine which calculates the angle subtended by the y-axis
5  c      and the straight line from the origin to the point of
6  c      intersection of the streamline (r0) and the transient crater
7  c      (defined by parabola with coefficients a and c).
8  c
9  c      Note the value of Z for this model is 3.
10 c
11 c      Uses an iteration which in most cases converges.
12 c      When it does not converge the iteration continues 1000
13 c      times such that the value of theta_x is as close as
14 c      possible to the true value.
15 c
16 c      -----
17 c      INPUTS:
18 c          r0      = streamline coefficient of point
19 c          a,c      = coefficients of the parabola defining
20 c                     transient crater shape
21 c
22 c      OUTPUTS:
23 c          theta_x  = value of theta at crossover point
24 c      -----
25 c
26 c      called from subroutine: vibrations
27 c
28  *****
29      implicit none
30      integer          k,ij
31      double precision  r0,a,c
32      double precision  theta_x      !value of theta at crossover point
33      double precision  y_cross      !y co-ordinate of crossover point
34      double precision  temp         !temp. value of theta_x
35      double precision  phi
36  c
37  c      Make initial guess for value of theta_x, and begin iteration
38  c
39      theta_x=0.d0
40      do k=1,1000
41  c
42  c      Calculate y-coord. of crossover pt.
43  c
44      y_cross=-r0*dcos(theta_x)*(1-dcos(theta_x))
45  c
46  c      Calculate the corresponding crossover angle
47  c
48      phi=((y_cross-c)/a+y_cross**2)/(r0**2)
49  c

```


B.2 Target Strength and the Implementation of Acoustic Fluidization

The modifications made to the SALES_2 solution algorithm to model complex crater collapse are incorporated the three subroutines: *vibrations*, *yield_strength* and *stresd*.

B.2.1 Vibrational Pressure Assignment

A new subroutine, *vibrations*, assigns a new cell centered variable, the vibrational pressure (**amp_vib**), to each cell. This pressure is related to the peak shock pressure by Equation 3.47. Thus, the peak shock pressure must be determined for the original location of each cell; that is, the location where the point at the centre of the cell would have been before the Z-model approximation of the excavation flow was applied.

Determining the original location of the point at the centre of each cell is complicated by the fact that, during the mesh-relaxation stage, the original mesh geometry is altered: the mesh is compressed by gravitational forces and the profile of the transient crater deviates slightly from its initial parabolic form. The original position (in terms of streamline coefficient and angular distance along the streamline) of the point of interest may still be calculated using Equation 2.5 and Equation 2.3; however, after the mesh is compressed the angular distance along the streamline to the point where the streamline intersects the profile of the transient crater θ_x cannot be calculated in the same way as in section B.1. Instead, this angle must be determined by finding the intersection point between the streamline along which the point of interest has moved and the straight line connecting two adjacent mesh vertices in the crater wall that lie either side of this streamline. This procedure is illustrated in Figure B.3.

```

1  *****
2      subroutine vibrations
3  c      =====
4  c      Subroutine to calculate the amplitude of the pressure vibrations
5  c      for each cell, amp_vib(ij).
6  c
7  c      r_impactor - is the radius of the impactor
8  c      v_impactor - is the velocity of the impactor
9  c      f_vib      - is the fraction of the peak shock pressure that
10 c                  remains as vibrational oscillation.
11 c
12 c      Called from celset
13 c
14  *****
15      include 'sales.comm'
16

```

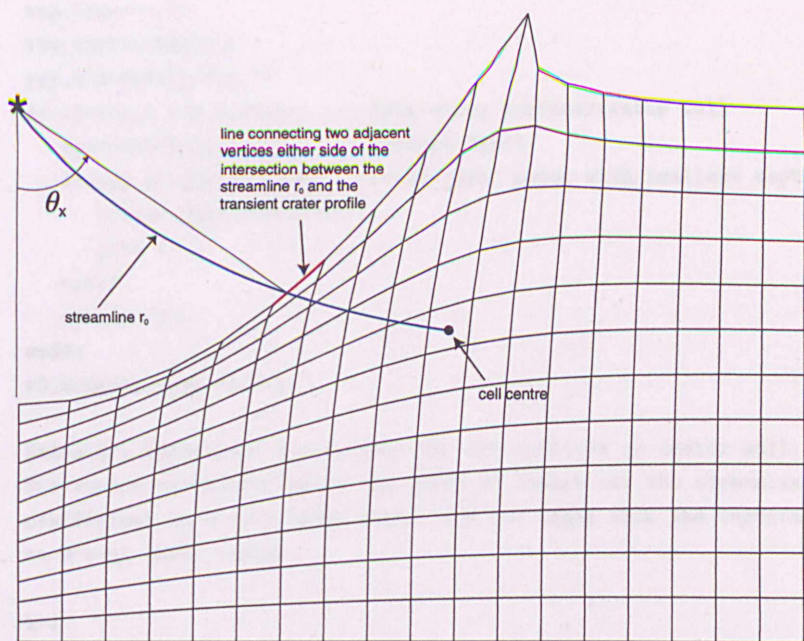


Figure B.3: Schematic illustration of methodology for finding the angular distance of the intersection between a streamline passing through a particular position in the target and the profile of the transient crater, θ_x . The streamline passing through the centre of a particular cell, r_0 (shown in blue), intersects the crater wall between two adjacent vertices along the top boundary of the mesh. These two vertices are connected by a straight line (shown in red).

```

17      integer          top_l          !the top left vertex number
18      integer          rim_vertex     !the number of the rim vertex
19
20      integer          wall_ij(nxp)    !cell number of surface vertices
21      double precision r0_wall(nxp)    !streamline coefficient of s. v.
22      double precision theta_wall(nxp) !angular dist. from vert. of s.v.
23      double precision m                !slope of straight line connecting
24                                      !adjacent surface vertices
25  c
26  c      Find crater radius and hinge streamline coefficient
27  c
28      top_l=nxp*ny+1
29      rim_vertex=nxp*nyp
30      yyy_old=dabs(y(top_l))
31      do ij=top_l,rim_vertex          !run along surface/crater wall
32          yyy=dabs(y(ij))              !record depth
33          if(yyy.gt.yyy_old)then        !find first point with smallest depth
34              crater_radius=x(ij-1)
35              goto 5
36          endif
37          yyy_old=yyy
38      enddo
39  5      r0_hinge=crater_radius
40  c
41  c      Calculate streamline coefficient for all vertices on crater wall:
42  c      For vertex vertically below the point of impact set the streamline
43  c      coefficient to a very large number and the angle from the vertical
44  c      to a very small number.
45  c
46      k=1
47      r0_wall(k)=1.d32
48      wall_ij(k)=top_l
49      theta_wall(k)=1.d-8
50  c
51  c      run along surface vertices, assigning angular distance and
52  c      streamline coefficients to each vertex.
53  c
54      do ij=top_l+1,rim_vertex
55          k=k+1
56          klim=k
57          if(y(ij).eq.0.d0)then
58              theta=pi/2.d0
59          else
60              theta=datan(x(ij)/-y(ij))
61          endif
62          if(theta.lt.0.d0)theta=theta+pi
63          r0_wall(k)=x(ij)/(dsin(theta)*(1-dcos(theta)))
64          wall_ij(k)=ij
65          theta_wall(k)=theta

```

```

66      enddo
67  c
68  c      Now loop over all cells in the mesh
69  c
70      do j=ny,1,-1
71          ij=(j-1)*nxp+ifirst
72          do i=1,nx
73  c
74  c      Need to find where the centre of each cell was before impact...
75  c      So, working backwards, calculate angular distance between vertical
76  c      and cell centre (theta_new).
77  c
78          theta_new=datan(x_cent(ij)/-y_cent(ij))
79          if(theta_new.lt.0.d0)theta_new=theta_new+pi
80          if(y_cent(ij).eq.0.d0)theta_new=pi/2.d0
81  c
82  c      Calculate streamline coefficient for cell centre (r0)
83  c
84          r0=x_cent(ij)/(dsin(theta_new)*(1-dcos(theta_new)))
85  c
86  c      Find the two adjacent vertices on crater wall with closest streamline
87  c      coefficients (one larger one smaller). Determine the slope (m) and
88  c      y-intercept (c) for the straight line connecting these two vertices;
89  c      also record the angular distance from the vertical of the two vertices
90  c      (theta1 and theta2).
91  c
92          do k=1,klim
93              if(r0_wall(k).ge.r0.and.r0_wall(k+1).le.r0)then
94                  m=( y(wall_ij(k+1)) - y(wall_ij(k)) )/
95  $                  ( x(wall_ij(k+1)) - x(wall_ij(k)) )
96                  c=y(wall_ij(k)) -m*x(wall_ij(k))
97                  theta1=theta_wall(k)
98                  theta2=theta_wall(k+1)
99                  goto 10
100             endif
101         enddo
102         write(*,*)'streamline coefficients not found!',i,j
103  c
104  c      Find intersection point between streamline of the cell centre and the
105  c      straight line (y = mx+c) that passes through the two vertices with
106  c      streamline coefficients closest to that of the cell centre.
107  c
108  10      call cross2(r0,m,c,theta1,theta2,theta_x,ij)
109          if(theta_x.lt.0)theta_x=theta_x+pi
110  c
111  c      Calculate old angular distance along streamline
112  c
113          theta_old=dacos(1.-
114  $          ((1.-dcos(theta_new))**4-(1.-dcos(theta_x))**4)**0.25)

```

```

115  c
116  c      Turn this into cartesian form
117  c
118          x_orig=r0*dsin(theta_old)*(1-dcos(theta_old))
119          y_orig=-r0*dcos(theta_old)*(1-dcos(theta_old))
120  c
121  c      Calculate distance from impact point to original position
122  c      of cell centre
123  c
124          radius=dsqrt(x_orig**2+y_orig**2)
125  c
126  c      Calculate the cell-centered maximum particle velocity-squared,
127  c      defined by the cells original position and using a 1/r^3 relationship
128  c      for the particle velocity-squared outside one projectile radius.
129  c      Inside one projectile radius the velocity-squared is constant.
130  c
131          if(radius.le.r_impactor)then
132              vcmax2=v_impactor**2/4.d0
133          else
134              vcmax2=(v_impactor**2/4.d0)*(r_impactor/radius)**3.d0
135          endif
136  c
137  c      Convert velocity-squared to a vibrational pressure amplitude
138  c
139          f_vib=c_v*c_v
140          amp_vib(ij)=f_vib*ro(ij)*vcmax**2
141          ij=ij+1
142      enddo
143  enddo
144  c
145      return
146  end
147  *****

```

B.2.2 Yield Strength

A new subroutine, *yield_strength*, determines the yield strength of each cell every timestep, based on the amount of damage in the cell, the amplitude of the vibrations in the cell and the internal energy in the cell. If the cell is intact (parameter **damag** = 0.) then Equation 3.44 is used to determine the yield strength. If the cell is completely fractured (**damag** = 1.), Equation 3.45 is used to determine the yield strength. The yield strength is then reduced due to (1) the presence of any pressure vibrations using Equation 2.69, and (2) the internal energy I using:

$$Y = Y_0 \left(\frac{I}{I_m} \right)^2, \quad (\text{B.1})$$

where Y_0 is the unmodified yield strength, Y is the thermally weakened yield strength and I_m is the internal energy for melting (**siem**, an input parameter). A flag (**irheology**) records which yield strength criterion is used to calculate the yield strength of the cell.

```

1  *****
2      subroutine yield_strength(yhel,ij)
3      c      =====
4      c      Subroutine to calculate yield strength (Hugoniot Elastic Limit)
5      c      for a cell: yhel. Called from subroutine "stresd".
6      c
7      c      -----
8      c      INPUTS:
9      c      ij    = cell number
10     c      -----
11     c      OUTPUTS:
12     c      yhel = Yield strength (Hugoniot Elastic Limit)
13     c      -----
14     c
15     c      cohesion(mat) is the cohesion at zero pressure for intact material
16     c      lundborg(mat) is the Lundborg plastic limit for intact material
17     c      vonmises(mat) is the von Mises plastic limit for damaged material
18     c      fric_int(mat) is the coefficient of friction for intact material
19     c      fric_dam(mat) is the coefficient of friction for damaged material
20     c      irheology(ij) is a flag for denoting the rheology of the cell
21     c      0 = intact material
22     c      1 = completely damaged material
23     c      2 = partially damaged material
24     c      3 = completely melted material
25     c      4 = thermally softened material (unfluidized)
26     c      5 = acoustically fluidized material (completely damaged)
27     c
28     *****
29     include 'sales.comm'
30
31     double precision yhel           !yield strength (HEL) for cell
32     double precision yheli,yheld    !Intact and damaged limit of HEL
33     double precision ysoft          !Thermal softening coefficient
34     double precision ybing          !Acoustic fluid. yield stress
35     double precision pfric_i,pfric_d !Dry friction components of HEL
36
37     c
38     c      Dry friction components (coefficient of friction * pressure)
39     c
40     mat=imat(ij)
41     pfric_i=dmax1(0.d0,fric_int(mat)*p(ij)) !intact
42     pfric_d=dmax1(0.d0,fric_dam(mat)*p(ij)) !damaged
43     c
44     c      Intact limit (low P: yheli -> cohesion; high P: yheli -> lundborg)

```

```

45  c
46      yheli=cohesion(mat)+pfric_i/
47      $              (1.d0+pfric_i/(lundborg(mat)-cohesion(mat)))
48  c
49  c      Damaged limit (high P: yheld -> vonmises)
50  c
51      yheld=pfric_d/(1.d0+pfric_d/vonmises(mat))
52  c
53  c      Assign appropriate yield strength
54  c
55      if(cohesion(mat).eq.0.d0)damag(ij)=1.d0
56      if(damag(ij).eq.0.d0)then
57          yhel=yheli
58          irheology(ij)=0                      !intact
59      elseif(damag(ij).eq.1.d0)then
60          yhel=yheld
61          irheology(ij)=1                      !totally damaged
62      else
63          yhel=yheli*(1.d0-damag(ij))+yheld*damag(ij)
64          irheology(ij)=2                      !partially damaged
65      endif
66  c
67  c      Modify yield strength due to vibrations =====
68  c
69      if(damag(ij).eq.1.d0.and.iacfl.eq.2)then
70  c
71  c      compute amplitude of decaying oscillation (t_vib is an input
72  c      parameter, representing the decay time of the acoustic vibrations;
73  c      amp_vib is vibrational pressure due to the shock wave).
74  c
75      p_vib(ij)=amp_vib(ij)*exp(-t/t_vib)
76  c
77  c      Ensure never negative net pressure
78  c
79      if(p_vib(ij).gt.p(ij))p_vib(ij)=p(ij)
80  c
81  c      Calculate the Bingham yield strength due to vibrations
82  c
83      ybing=fric_dam(mat)*(p(ij)-p_vib(ij))
84  c
85  c      Modify HEL if required
86  c
87      if(ybing.lt.yhel)then
88          yhel=ybing
89      endif
90  c
91      endif
92  c
93  c      Modify yield strength due to temperature (internal energy) =====

```

```

94  c
95      if(sie(ij).lt.siem(mat))then
96          ysoft=(1.-sie(ij)/siem(mat))**2
97          yhel=yhel*ysoft
98          if(ysoft.lt.0.2d0)irheology(ij)=4 !softened
99      else
100         yhel=0.
101         irheology(ij)=3                !melted
102     endif
103  c
104      return
105  end
106  *****

```

B.2.3 Effective Viscosity

The subroutine *stresd*, which determines the deviatoric stress contributions to the vertex velocities, calls the subroutine *yield_strength* to compute the yield strength of the cell. If the second invariant of the stress tensor in the cell exceeds the square of this yield strength then the elastic component of the stress tensor is reduced by the factor s_{fac} , given by Equation 3.41, and an effective viscosity is assigned to the cell depending on the rheology of the cell. This effective viscosity is represented by η_2 in Figure 3.8. If the yield strength of an intact or damaged cell is exceeded, without requiring the presence of any pressure vibrations, then the effective viscosity of the cell is defined as **visc_debris**. If the yield strength is exceeded because the internal energy is greater than the internal energy of melting, then the effective viscosity is defined as **visc_melt**. If the yield strength is exceeded due to the strength reduction caused by the presence of pressure vibrations, then the effective viscosity is defined as **visc_weak**. If the yield strength is not exceeded, the effective viscosity is set as high as possible. In this case the viscosity corresponds to η_1 in Figure 3.8.

```

1  *****
2      SUBROUTINE STRESD
3  c      =====
4  c      The stress deviator subroutine, in which we calculate the
5  c      shear elastic contribution to the vertex velocities
6  c      (bulk modulus is included in the equation of state).
7  c      the 4 stress terms are saved for later use in subr. energy.
8  c
9  *****
10     include 'sales.comm'
11
12     double precision    lamd
13     double precision    visc_melt    !Melted material viscosity
14     parameter           (visc_melt=1.d5)
15     double precision    visc_debris  !Fractured material viscosity

```

```

16      parameter      (visc_debris=1.d7)
17  c
18  c      for every cell...
19  c
20      dto2=.5*dt
21      do j=jfirst,jlast
22          ij=(j-1)*nxp+ifirst
23          ijp=ij+nxp
24          do i=ifirst,ilast
25              ipj=ij+1
26              ipjp=ijp+1
27  c
28  c      Assign vertex values of x, y, etc. and calculate scalar quantities
29  c
30          mat=imat(ij)
31          x1=x(ipj)
32          r1=r(ipj)
33          y1=y(ipj)
34          u1=u(ipj)
35          v1=v(ipj)
36          x2=x(ipjp)
37          r2=r(ipjp)
38          y2=y(ipjp)
39          u2=u(ipjp)
40          v2=v(ipjp)
41          x3=x(ijp)
42          r3=r(ijp)
43          y3=y(ijp)
44          u3=u(ijp)
45          v3=v(ijp)
46          x4=x(ij)
47          r4=r(ij)
48          y4=y(ij)
49          u4=u(ij)
50          v4=v(ij)
51          x24=x2-x4
52          y24=y2-y4
53          x31=x3-x1
54          y31=y3-y1
55          uor=(u1+u2+u3+u4)*rrsum(ij)*cyl
56          hr13=.5*(r1+r3)
57          hr24=.5*(r2+r4)
58          dto2m1=dto2*rmv(ipj)
59          dto2m2=dto2*rmv(ipjp)
60          dto2m3=dto2*rmv(ijp)
61          dto2m4=dto2*rmv(ij)
62          area=0.5*(x24*y31-x31*y24)
63          area2=area*2.0
64          rarea2=1./area2

```

```

65          u24=u2-u4
66          u31=u3-u1
67          v24=v2-v4
68          v31=v3-v1
69  c
70  c +++ nb: positive stresses and strainrates are tensile
71  c +++ note that *deviatoric* strain rates and stresses
72  c +++ are computed--volume changes are handled in eos.
73  c +++ this is why traces are removed and only the shear
74  c +++ modulus is used. Even in plane strain, where the
75  c +++ total out of plane stress is zero, there is a component
76  c +++ of out of plane deviatoric stress.
77  c
78  c      Calculate strain rates using finite difference relation
79  c
80          dudx=rarea2*(u24*y31-u31*y24)
81          dudy=rarea2*(u31*x24-u24*x31)
82          dvdx=rarea2*(v24*y31-v31*y24)
83          dvdy=rarea2*(v31*x24-v24*x31)
84          trace=(dudx+dvdy+uor)*third
85  c
86  c +++ call the following code only if material has shear strength
87  c
88          if(elastc(mat)) then
89  c
90  c +++ compute rotation angle rot of cell in last time step, and
91  c +++ accessory constants
92  c
93          sinrot = 0.d5*(dvdx-dudy)*dt
94          sin2 = sinrot**2
95          cos2 = 1.d0-sin2
96          cosrot = dsqrt(cos2)
97          sincos =sinrot*cosrot
98  c
99  c +++ rotate elastic strains by this angle, as described in wilkins (1964)
100 c
101          exxrot=eexx(ij)*cos2+eeyy(ij)*sin2+2.d0*eexy(ij)*sincos
102          eyyrot=eexx(ij)*sin2+eeyy(ij)*cos2-2.d0*eexy(ij)*sincos
103          exyrot=-(eexx(ij)-eeyy(ij))*sincos+eexy(ij)*(cos2-sin2)
104 c
105 c +++ update the deviatoric elastic strains
106 c +++ former plane strain bug for eeth fixed 1/21/97 (thanks, Boris)
107 c
108          eexx(ij) = exxrot      + (dudx - trace)*dt
109          eeyy(ij) = eyyrot      + (dvdy - trace)*dt
110          eexy(ij) = exyrot + 0.5*(dudy + dvdx)*dt
111          eeth(ij) = eeth(ij)   + ( uor - trace)*dt
112 c
113 c +++ deviatoric stresses if deformation is entirely elastic

```



```

114  c
115      tixx = 2.0*mu(mat)*eexx(ij)
116      tiyy = 2.0*mu(mat)*eeyy(ij)
117      tixy = 2.0*mu(mat)*eexy(ij)
118      tith = 2.0*mu(mat)*eeth(ij)
119  c
120  c +++ compute the elastic principal stresses and failure criterion
121  c +++ at the Hugoniot elastic limit (hel). Use standard parabolic
122  c +++ stress weakening law as melting energy is approached
123  c
124  c +++ Decrease shear modulus by an amount determined by the
125  c +++ current damage (if such damage is possible) and compute
126  c +++ modifications due to yield at the hugoniot elastic limit
127  c
128      if(gkfrac(mat)) then
129          decrmt = 1.0-damag(ij)
130      else
131          decrmt = 1.0
132      endif
133  c
134  c +++ get principal stresses from elastic stress tensor
135  c
136      shear=decrmt*sqrt(tixy**2+0.25*(tixx-tiyy)**2)
137      diag=.5*(tixx+tiyy)*decrmt-p(ij)
138      s1=diag+shear
139      s2=diag-shear
140      sth=tith*decrmt-p(ij)
141      sm1=dmax1(s1,s2,sth)
142      sm2=dmin1(s1,s2,sth)
143  c
144  c      Compute second invariant of the deviatoric stress tensor.
145  c
146      sdsi=0.5*((s1-s2)**2+(s2-sth)**2+(sth-s1)**2)
147  c
148  c      Calculate shear strength using Coulomb-von Mises model
149  c
150      call yield_strength(yhel,ij)
151      strength(ij)=yhel
152  c
153  c      Test to see if hugoniot elastic limit is exceeded by second
154  c      invariant (J_2 invariant)
155  c
156      sfac=1.0
157      if(sdsi.gt.yhel*yhel)then !plastic limit exceeded
158          sfac=sqrt(yhel**2/sdsi)
159  c
160  c      Modify viscosity of melted cells
161  c
162      if(irheology(ij).eq.3)then

```

```

163             visc_eff(ij)=visc_melt
164 c
165 c     Define shear viscosity of acoustically fluidized cell
166 c
167             elseif(iacfl.ne.0)then           !acoustically fluidized
168                 visc_eff(ij)=visc_weak
169                 irheology(ij)=5
170             elseif(irheology(ij).eq.1)then !damaged rock failure
171                 visc_eff(ij)=visc_debris
172                 irheology(ij)=5
173             else                               !intact rock failure
174                 visc_eff(ij)=visc_debris
175                 irheology(ij)=5
176             endif
177 c
178             else                               !material behaves as a Kelvin substance
179 c
180 c     Define shear viscosity of unfluidized cell
181 c
182             visc_eff(ij)=visc(mat)
183             endif
184 c
185 c +++ Compute the stresses due to bulk and shear viscosity
186 c +++ this is the original code from SALE (with the bug for
187 c +++ pith in plane strain fixed and the trace removed from
188 c +++ the shear viscosity component).
189 c
190 c +++ bulk viscosity coefficient
191 c
192             lamd=3.0*lambda(mat)*trace
193 c
194 c     NB: REPLACED visc(mat) WITH visc_eff(ij)
195 c
196             pixx(ij) = 2.0*visc_eff(ij)*(dudx-trace) + lamd
197             piyy(ij) = 2.0*visc_eff(ij)*(dvdy-trace) + lamd
198             pixy(ij) =      visc_eff(ij)*(dudy+dvdx)
199             pith(ij) = 2.0*visc_eff(ij)*(uor -trace) + lamd
200 c
201 c +++ Add elastic contribution to the deviatoric stress.
202 c +++ Adjust stresses if hel yield envelope is exceeded
203 c +++ note that strain is elastic strain only, so is reduced
204 c +++ when yield occurs--the remainder accumulates as plastic
205 c +++ strain.
206 c
207             pixx(ij) = pixx(ij) + tixx*decrmt*sfac
208             piyy(ij) = piyy(ij) + tiyy*decrmt*sfac
209             pixy(ij) = pixy(ij) + tixy*decrmt*sfac
210             pith(ij) = pith(ij) + tith*decrmt*sfac
211             if(sfac.ne.1.0) then

```

```

212         eexx(ij) = sfac*eexx(ij)
213         eeyy(ij) = sfac*eeyy(ij)
214         eexy(ij) = sfac*eexy(ij)
215         eeth(ij) = sfac*eeth(ij)
216     endif
217 c
218 c     If no shear modulus then compute viscous stress tensor only ----
219 c
220         else
221 c
222 c +++ bulk viscosity coefficient
223 c
224         lamd=3.0*lambda(mat)*trace
225 c
226 c +++ compute the stresses due to bulk and shear viscosity
227 c +++ this is the original code from SALE (with the bug for
228 c +++ pith in plane strain fixed and the trace removed from
229 c +++ the shear viscosity component).
230 c
231 c     NB: REPLACED visc(mat) WITH visc_eff(ij)
232 c
233         pixx(ij) = 2.0*visc_eff(ij)*(dudx-trace) + lamd
234         piyy(ij) = 2.0*visc_eff(ij)*(dvdy-trace) + lamd
235         pixy(ij) =     visc_eff(ij)*(dudy+dvdx)
236         pith(ij) = 2.0*visc_eff(ij)*(uor -trace) + lamd
237 c
238 c     -----
239         endif
240 c +++
241 c +++ record the total (elastic plus viscous) stress extrema
242 c +++
243         shear=sqrt(pixy(ij)**2+0.25*(pixx(ij)-piyy(ij))**2)
244         diag=.5*(pixx(ij)+piyy(ij))-p(ij)
245         s1=diag+shear
246         s2=diag-shear
247         sth=pith(ij)-p(ij)
248         sm1=dmax1(s1,s2,sth)
249         sm2=dmin1(s1,s2,sth)
250         if(sm1.gt.smax(ij)) smax(ij)=sm1
251         if(sm2.lt.smin(ij)) smin(ij)=sm2
252 c +++
253 c +++ adjust velocities for the deviatoric stresses
254 c +++
255         zz=0.5*area*pith(ij)*cyl
256         xx=hr24*(pixy(ij)*x24-pixx(ij)*y24)
257         ul(ipj) =ul(ipj) +dto2m1*(xx-zz)
258         ul(ijp) =ul(ijp) -dto2m3*(xx+zz)
259         xx=hr13*(pixy(ij)*x31-pixx(ij)*y31)
260         ul(ipjp)=ul(ipjp)+dto2m2*(xx-zz)

```

```

261          ul(ij) =ul(ij) -dto2m4*(xx+zz)
262          xx=hr24*(piyy(ij)*x24-pixy(ij)*y24)
263          vl(ipj) =vl(ipj) +dto2m1*xx
264          vl(ijp) =vl(ijp) -dto2m3*xx
265          xx=hr13*(piyy(ij)*x31-pixy(ij)*y31)
266          vl(ipjp)=vl(ipjp)+dto2m2*xx
267          vl(ij) =vl(ij) -dto2m4*xx
268      10      ij=ipj
269          ijp=ipjp
270      enddo
271  enddo
272      call bc(ul,vl)
273  c
274      RETURN
275  100 format(i8,i6,1pe15.5,1pe15.5,1pe15.5)
276      END
277  ****

```

B.3 Full Acoustic Fluidization Treatment

The implementation of the original acoustic fluidization model by Melosh (1979) is incorporated in four subroutines. There is a minor modification to the *vibrations* subroutine, described above, to assign initial values for the acoustic energy per unit mass **aed** to each cell. The original SALES_2 subroutine is modified to calculate both the changes in internal energy and acoustic energy. In doing so a new subroutine *scatter* calculates the scattering between adjacent cells. Finally, the subroutine *stresd* is modified to incorporate the rheological model that results from the acoustic fluidization equations.

The initial acoustic energy per unit mass assigned to each cell is governed by Equation 3.50, divided by the mass of the cell. This assignment is included after line 140 in the subroutine *vibrations*, which determines the amplitude of pressure vibrations for each cell in the mesh.

```

141  c
142  c      Convert pressure amp. into acoustic energy (per unit mass) (aed)
143  c
144          aed(ij)=.5d0*(amp_vib(ij)**2)/(ro(ij)*mc(ij)*atill(mat))

```

The subroutine *energy* from SALES_2 calculates the changes in internal energy (per unit mass) due to pressure work and deviatoric stresses. To incorporate the temporal behaviour of the acoustic energy this subroutine is modified to implement Equation 3.51 and Equation 3.53. Thus, the modified subroutine computes the change in acoustic energy due to scattering between adjacent cells, dissipation to heat, and the regeneration via viscous flow (see lines 96–123). As a consequence the internal energy is reduced because some of the viscous work is expended generating acoustic energy and increased due to the heat generated during the dissipation of acoustic energy (see lines 124–128).

The amount of scattering is computed in the first 44 lines of the *energy* subroutine and the new subroutine *scatter* (called on line 41 of *energy*). *scatter* requires two pieces of information: the cell position and, if the cell lies on a the edge of the mesh or in a corner, which corner or edge.

The amount of dissipation is calculated on line 103 of subroutine *energy* using three input parmaters: **csound**, the speed of sound for the fluidized debris; **wavelength**, the wavelength of the dominant acoustic vibrations; and **q_diss**, the dissipation quality factor (defined as the energy stored per cycle divided by the energy dissipated in the same period).

The amount of regeneration is calculated on line 111 using the original approach in SALES_2 for determining the viscous work, and multiplying this by the fraction of the viscous work that is made available for the regeneration of acoustic energy (*e* in Equation 3.51). This fraction is another input parameter, **regen**.


```

1  *****
2      subroutine energy
3      c      =====
4      c      Subroutine to calculate energy changes due to pdv work and
5      c      deviatoric stresses.
6      c
7      c      Modified to calculate acoustic energy changes. GSC 23/04/2001
8      c
9      c      Called from the main program
10     c
11     *****
12     include 'sales.comm'
13     c
14     if(iacfl.ne.4)goto 10
15     c
16     c      In first loop: Calculate acoustic energy scattering
17     c
18     do j=jfirst,jlast
19         do i=ifirst,ilast
20             ij=(j-1)*nxp+i
21             ipj=ij+1
22             ijp=ij+nxp
23             ipjp=ijp+1
24             ijm=ij-nxp
25             imj=ij-1
26         c
27         c      Set boundary flag
28         c
29         ibc=0
30         if(j.eq.1)ibc=1                !bottom boundary
31         if(i.eq.1)ibc=2                !left boundary
32         if(j.eq.ny)ibc=3               !top boundary
33         if(i.eq.nx)ibc=4               !right boundary
34         if(ij.eq.1)ibc=5               !bottom left corner
35         if(ij.eq.nx)ibc=6              !bottom right corner
36         if(ij.eq.(ny-1)*nxp+1)ibc=7    !top left corner
37         if(ij.eq.ny*nxp-1)ibc=8        !top right corner
38     c
39     c      Calculate scattering between cells
40     c
41         call scatter(ibc,ij,scattering(ij))
42     c
43     enddo
44     enddo
45     c
46     c      In second loop update internal and acoustic energy
47     c
48     10 do j=jfirst,jlast
49         do i=ifirst,ilast

```

```

50         ij=(j-1)*nxp+i
51         ijp=ij+nxp
52         ipj=ij+1
53         ipjp=ijp+1
54         x1=x(ipj)
55         y1=y(ipj)
56         r1=r(ipj)
57         utc1=u(ipj)+ul(ipj)
58         vtc1=v(ipj)+vl(ipj)
59         x2=x(ipjp)
60         y2=y(ipjp)
61         r2=r(ipjp)
62         utc2=u(ipjp)+ul(ipjp)
63         vtc2=v(ipjp)+vl(ipjp)
64         x3=x(ijp)
65         y3=y(ijp)
66         r3=r(ijp)
67         utc3=u(ijp)+ul(ijp)
68         vtc3=v(ijp)+vl(ijp)
69         x4=x(ij)
70         y4=y(ij)
71         r4=r(ij)
72         utc4=u(ij)+ul(ij)
73         vtc4=v(ij)+vl(ij)
74         y24=y2-y4
75         y31=y3-y1
76         x24=x2-x4
77         x31=x3-x1
78         hr24=0.5*(r2+r4)
79         hr13=0.5*(r1+r3)
80         xx1=hr24*(pixy(ij)*x24-pixx(ij)*y24)
81         xx2=hr13*(pixy(ij)*x31-pixx(ij)*y31)
82         xx3=hr24*(piyy(ij)*x24-pixy(ij)*y24)
83         xx4=hr13*(piyy(ij)*x31-pixy(ij)*y31)
84         area=0.5*(x24*y31-x31*y24)
85  c
86  c      Calculate internal energy loss due to pressure work
87  c
88         dv=y24*(utc1*r1-utc3*r3) + y31*(utc2*r2-utc4*r4)
89  1      -hr24*x24*(vtc1-vtc3) - hr13*x31*(vtc2-vtc4)
90  c
91  c      Calculate internal energy loss due to viscous dissipation.
92  c
93         dvis=xx1*(utc1-utc3)+xx2*(utc2-utc4)
94  1      -pith(ij)*(utc1+utc2+utc3+utc4)*0.5*area
95  2      +xx3*(vtc1-vtc3)+xx4*(vtc2-vtc4)
96  c
97  c      ===== Acoustic energy changes =====
98  c

```

```

99         if(iacfl.eq.4)then
100      c
101      c      Calculate acoustic energy dissipation
102      c
103             aed_diss=aed(ij)*pi*csound*dt/(wavelength*q_diss)
104      c
105      c      Calculate acoustic energy scattering (scattering in is positive)
106      c
107             aed_scatt=scattering(ij)*dt
108      c
109      c      Calculate acoustic energy regenerated
110      c
111             aed_regen=regen*dvis*0.25d0*dt/mc(ij)
112             if(aed_regen.lt.0.d0)aed_regen=0.d0
113      c
114      c      Update the new acoustic energy density
115      c
116             aed(ij)=aed(ij)+aed_scatt-aed_diss+aed_regen
117             if(aed(ij).lt.0.d0)aed(ij)=0.d0
118         else
119             regen=0.d0
120             aed_diss=0.d0
121         endif
122      c
123      c      =====
124      c
125      c      Calculate new internal energy
126      c
127             sie(ij)=sie(ij)-((p(ij)+q(ij))*dv+(1.d0-regen)*dvis)
128      $             *0.25*dt/mc(ij)+aed_diss
129      c
130      c      Count the number of occurrences of negative cell energies
131      c
132             if(sie(ij).lt.-0.1)nctsie(ij)=nctsie(ij)+1
133      c
134         enddo
135     enddo
136      c
137         return
138     end
139 *****

```

The scattering into or out of a particular cell in the mesh is determined by the implementation of Equation 3.54. Thus, the second derivative of the acoustic energy across a cell in both the m and n directions must be computed. Figure B.4 illustrates the appropriate cell geometry and labelling.

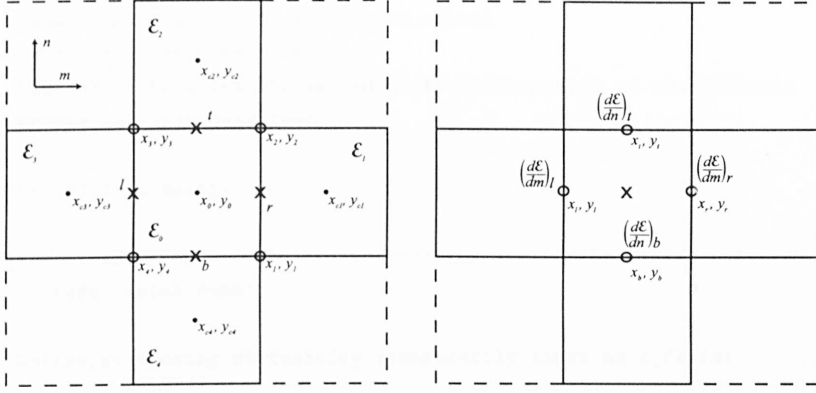


Figure B.4: Diagram showing labelling convention of vertices, cells and other positions in the mesh for the finite difference approximation of the ∇^2 operator.

The second differential in the m -direction is approximated using the scheme:

$$\frac{d^2 \mathcal{E}}{dm^2} = \frac{\left(\frac{d\mathcal{E}}{dm}\right)_r - \left(\frac{d\mathcal{E}}{dm}\right)_l}{\Delta m} \quad (\text{B.2})$$

where the first differential for point r is approximated by:

$$\left(\frac{d\mathcal{E}}{dm}\right)_r = \frac{\mathcal{E}_1 - \mathcal{E}_0}{\sqrt{(x_{c1} - x_0)^2 + (y_{c1} - y_0)^2}}; \quad (\text{B.3})$$

the first differential for point l is approximated by:

$$\left(\frac{d\mathcal{E}}{dm}\right)_l = \frac{\mathcal{E}_0 - \mathcal{E}_3}{\sqrt{(x_0 - x_{c3})^2 + (y_0 - y_{c3})^2}}; \quad (\text{B.4})$$

and Δm is given by:

$$\Delta m = \sqrt{(x_r - x_l)^2 + (y_r - y_l)^2}, \quad (\text{B.5})$$

where,

$$x_r = \frac{x_1 + x_2}{2}, \quad x_l = \frac{x_3 + x_4}{2} \\ y_r = \frac{y_1 + y_2}{2}, \quad y_l = \frac{y_3 + y_4}{2}.$$

Equivalent equations exist for the first order differential in the n -direction.

The above equations are valid provided that (1) the cells do not become extremely deformed, and (2) the cell in question is not at the edge of the mesh. For cells at the edge of the mesh the first differential of the acoustic energy across the boundary of the cell that defines the edge of the mesh is set to zero. If the cell is in the corner of the mesh the first differentials across the two boundaries that define the edge of the mesh are set to zero.

```

2      subroutine scatter(IBC,IJ,scatter_value)
3      c      =====
4      c      Subroutine to calculate second order differential of the acoustic
5      c      energy per unit mass (aed).
6      c
7      c      Called from energy
8      c
9      c      *****
10     include 'sales.comm'
11     c
12     c      Define scattering diffusivity (temporarily input as r_fluid)
13     c
14     c      diffusivity=r_fluid
15     c
16     c      Define cell indices and cell and vertex subscripts
17     c
18     c      -----
19     c      |          |          |          |          |
20     c      |   ijp   |   ipjp   |   c2   |
21     c      |   /    |   /      |          |
22     c      ----- + ----- + -----      -----(3)--t--(2)-----
23     c      |          |          |          |          |          |
24     c      |   imj   |   ij    |   ipj   |          |   c3   1   0   r   c1   |
25     c      |   /    |   /      |   /      |          |          |
26     c      ----- + ----- + -----      -----(4)--b--(1)-----      ^
27     c      |          |          |          |          |          |
28     c      |   ijm   |          |   c4   |          |          |   n   |
29     c      |   /      |          |          |          |          |          |   m
30     c      -----      -----      ----->
31     c
32     ipj=ij+1
33     ijp=ij+nxp
34     ipjp=ijp+1
35     ijm=ij-nxp
36     imj=ij-1
37     x0 =x_cent(ij)
38     x1 =x(ipj)
39     x2 =x(ipjp)
40     x3 =x(ijp)
41     x4 =x(ij)
42     xc1=x_cent(ipj)
43     xc2=x_cent(ijp)
44     xc3=x_cent(imj)
45     xc4=x_cent(ijm)
46     y0 =y_cent(ij)
47     y1 =y(ipj)
48     y2 =y(ipjp)
49     y3 =y(ijp)
50     y4 =y(ij)

```



```

51      yc1=y_cent(ipj)
52      yc2=y_cent(ijp)
53      yc3=y_cent(imj)
54      yc4=y_cent(ijm)
55      c
56      c      compute distances between cell centres
57      c
58      deltam_l=sqrt((x0-xc3)**2+(y0-yc3)**2)
59      deltam_r=sqrt((xc1-x0)**2+(yc1-y0)**2)
60      deltan_b=sqrt((x0-xc4)**2+(y0-yc4)**2)
61      deltan_t=sqrt((xc2-x0)**2+(yc2-y0)**2)
62      c
63      c      Compute first-order difference approximation assuming
64      c      cell is not on boundary.  Note: dx and dy are in the i and j
65      c      direction -- NOT necessarily the x and y direction.
66      c
67      daeddm_l=(aed(ij)-aed(imj))/deltam_l
68      daeddm_r=(aed(ipj)-aed(ij))/deltam_r
69      daeddn_b=(aed(ij)-aed(ijm))/deltan_b
70      daeddn_t=(aed(ijp)-aed(ij))/deltan_t
71      c
72      c      Compute distance across cell (denomx=width, denomy=height)
73      c
74      xml      =x4+x3
75      yml      =y4+y3
76      xmr      =x1+x2
77      ymr      =y1+y2
78      denomm =0.5d0*sqrt((xmr-xml)**2+(ymr-yml)**2)
79      xmb      =x3+x2
80      ymb      =y3+y2
81      xmt      =x4+x1
82      ymt      =y4+y1
83      denomn =0.5d0*sqrt((xmt-xmb)**2+(ymt-ymb)**2)
84      c
85      c      Modify first-order difference terms if *ij* on boundary
86      c      (boundary flag *ibc* defined in subroutine energy)
87      c
88      if(ibc.eq.1)then                                !bottom boundary
89          daeddn_b=0.
90      elseif(ibc.eq.2)then                              !left boundary
91          daeddm_l=0.
92      elseif(ibc.eq.3)then                              !top boundary
93          daeddn_t=0.
94      elseif(ibc.eq.4)then                              !right boundary
95          daeddm_r=0.
96      elseif(ibc.eq.5)then                              !bottom left corner
97          daeddn_b=0.
98          daeddm_l=0.
99      elseif(ibc.eq.6)then                              !bottom right corner

```

```

100      daeddn_b=0.
101      daeddm_r=0.
102      elseif(ibc.eq.7)then          !top left corner
103      daeddn_t=0.
104      daeddm_l=0.
105      elseif(ibc.eq.8)then          !top right corner
106      daeddn_t=0.
107      daeddm_r=0.
108      endif
109  c
110  c      Calculate second-order derivatives of the acoustic energy density
111  c
112      d2aeddm2=(daeddm_r-daeddm_l)/denomm
113      d2aeddn2=(daeddn_t-daeddn_b)/denomn
114  c
115  c      Calculate energy density change due to scattering
116  c
117      scatter_value=diffusivity*.25d0*(d2aeddm2+d2aeddn2)
118  c
119      return
120      end
121  *****

```

At each time-step the amount of acoustic energy in a cell is converted into a stress (the variance of the vibrational pressure amplitude) in a modified part of the subroutine *stresd* (lines 152–191, below). This then allows *stresd* to compute the effective viscosity of the cell using Equation 2.62. Unfortunately, the original acoustic fluidization model does not provide a yield criterion defining the conditions under which fluid behaviour of the debris material would be expected. Instead, the yield strength of the cell is reduced to zero for all cells where the effective viscosity is below a critical viscosity. This critical viscosity should be set to the critical viscosity that precludes overshoot for the transient crater size being modelled. This viscosity is given by Equation 2.20.

```

152  c
153  c      ----- Energy density approach -----
154  c
155      if(iacfl.eq.4)then
156  c
157  c      Compute the shear stress (from the previous time step)
158  c
159      tau=sqrt(sdsi) *.5*(sigma1(ij)-sigma3(ij))
160  c
161  c      Calculate variance from energy density in cell (aed)
162  c
163      variance=csound*ro(ij)*dsqrt(2.*aed(ij))
164  c

```

```

165 c      Calculate critical amplitude of oscillation for sliding to occur
166 c
167           crit_amp_vib=(p(ij)-tau)/fric_dam(mat)
168           if(crit_amp_vib.lt.0.)crit_amp_vib=0.d0
169 c
170 c      Calculate effective viscosity of cell
171 c
172           xi=crit_amp_vib/(variance*dsqrt(2.d0))
173           zed=erf(xi)
174           if(zed.ge.1.)then
175               visc_eff(ij)=visc(mat)
176           else
177               visc_eff(ij)=.5d0*ro(ij)*wavelength*csound*
178 $               (1.d0+zed)/(1.d0-zed)
179           endif
180 c
181 c      if less than cutoff viscosity -- cell is fluidized
182 c
183           if(visc_eff(ij).lt.visc(mat))then
184               yhel=0.d0
185               irheology(ij)=5
186           endif
187 c
188           endif
189 c
190 c      ----- end of acoustic energy density viscosity calculation -----
191 c

```

B.4 Additions to the Common-Block and Extra Input Parameters

The modifications described above require the addition of a number of parameters to the common-block, which is included in the majority of the subroutines in SALES_2. The additional cell and vertex variables, plus the constant parameters included are:

```

36      common /acoustic/ yield_st(node),p_vib(node),amp_vib(node),
37 $      x_cent(node),y_cent(node),irheology(node),strength(node),
38 $      visc_eff(node),tau_max(node),aed(node),scattering(node),
39 $      t_vib,visc_weak,f_v,regen,q_diss,wavelength,csound,
40 $      diffusivity
41      common /materials2/ cohesion(num),lundborg(num),vonmises(num),
42 $      fric_int(num),fric_dam(num)
43      common /crater/ r_impactor,v_impactor,crater_depth,
44 $      crater_radius,x_impact,y_impact,a,c,icrater

```

Of these, the following are input parameters, which are included in the modified input file and read during the modified subroutine *rinput*.

- **cohesion**: The cohesion at zero pressure Y_0 .
- **lundborg**: The von Mises plastic limit for intact material Y_L .
- **vonmises**: The von Mises plastic limit for faulted material Y_M .
- **fric_int**: The coefficient of friction (intact material) μ .
- **fric_dam**: The coefficient of friction (damaged material) μ .
- **t_vib**: The decay time for the pressure vibrations τ_v .
- **visc_weak**: The fluidized viscosity η .
- **f_v**: The fraction of the peak shock pressure that remains as vibration f_v .
- **r_impactor**: The radius of the impactor a_i .
- **v_impactor**: The velocity of the impactor v_i .
- **crater_depth**: The depth of the transient crater H_t .
- **crater_radius**: The radius of the transient crater R_t .
- **regen**: The fraction of the acoustic energy regenerated during flow e .
- **q_diss**: The dissipation factor Q .
- **wavelength**: The wavelength of the acoustic vibrations λ .
- **csound**: The bulk sound speed of the debris c .
- **diffusivity**: The diffusivity parameter ξ .

The input file and subroutine *rinput* are not listed in this appendix; they are included on the CD-ROM accompanying this thesis.

Appendix C

CD-ROM: Example Simulations and Complete Source-Code Listing

A CD-ROM accompanies this thesis. It contains a number of example simulations of complex crater collapse, which are derived from the results of the numerical modelling work described and discussed in this thesis. The animations are included because they offer the best way to illustrate the dynamic results of the modelling work. Also include on the CD-ROM is the complete source-code listing for the code used during this research.

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