

THE PERFORMANCE OF TIME-FREQUENCY CODED SPREAD-SPECTRUM
SYSTEMS

by

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To Rena, Memos, and Eta

Purification through one's own strength,
emancipation through work,
adoration of eternal glorious Nature.

R. Strauss

ABSTRACT

In recent years great interest has been shown in Time-Frequency Coded Spread-Spectrum Systems and particularly in the possibility of employing them as Multiple-Access Communication Systems.

The performance of such systems is limited by other users' interference and by noise. The effect of other users' interference can be partly overcome by successfully selecting the address of each user and by designing a suitable combiner for the receiver. In this thesis Time-Frequency Coded Systems that employ Einarsson's addressing scheme and a hard-limited combiner are studied in detail.

The system is first considered to operate in the synchronous mode and an exact expression for the word-error probability is derived. The probability distribution of the number of occupied time-frequency chips is found and an operational signal-to-noise ratio based on the average number of occupied chips is derived. This is shown to yield a useful design aid when used in conjunction with the exact expression for the word-error probability.

Since in practice synchronization is often difficult, the asynchronous (non-synchronous) operation of the Time-Frequency Coded system is studied and an exact expression for the error probability is derived. This expression is, however, difficult to evaluate and three easily computable upper bounds are derived. The tightest bound is shown to be very close to the exact value of the word-error probability, and the other bounds, although more loose, are sufficiently tight to be of value. All three bounds are tighter than any other known correct bound.

The effect of channel impairments is also studied in detail and an upper bound, which is significantly tighter than all previously known bounds, is derived for the system operating in the presence of noise.

In a new approach to the operation of Time-Frequency Coded systems the use of simple repetition codes and the use of a (7,4) cyclic code is considered. The performance of these systems operating in a variety of channel impairments is considered and compared with that of Einarsson's basic scheme. The comparison of the systems' performance is made both in terms of theoretical calculations and through a simulation of the various systems. The new coding schemes are shown to have performance advantages as compared with the basic Einarsson system. Finally, the effect on the per-

formance of a Time-Frequency Coded system of interference resulting from like Time-Frequency Coded Systems is examined.

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Most of all I would like to thank my parents and sister, without whose love and support this work would never have finished. It is to them that this thesis is gratefully dedicated.

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Spread Spectrum Systems, such as Direct-Sequence Systems and Frequency-Hopped Systems, were originally developed in connection with military communication in which secrecy and the ability to handle high levels of interference (jamming) are of paramount importance.^[1] The advantages of these systems stem from the fact that in order for a receiver to decode a message correctly, it is necessary to know, or to be able to derive, information about the mapping operations involved at the transmitter. For example, in the case of frequency-hopping it is necessary for the receiver to be aware of the sequences of frequencies used and for proper synchronization to be obtained. In order to effect jamming the source of the interference must either provide blanket interference over the entire transmission bandwidth or it must have knowledge of the particular frequencies involved at any time if a narrower band of interfering signal is to be employed.

Following the early development of military systems it became apparent that Spread-Spectrum Systems could be employed in Multiple-Access communication systems, and thereby fulfil a role conventionally filled by Frequency-Division Multiple-Access (FDMA) and Time-Division

Multiple-Access (TDMA) systems.

Another important area in which Spread-Spectrum Systems might be of value is in connection with the question of making more efficient use of limited communication bandwidth. At first sight, this possible application appears to be contradictory to the large bandwidth requirements associated with Spread-Spectrum Systems. In recent years there has been increasing demand for communication capacity so that new services can be accommodated. Two ways in which this demand might be met are:

- (i) To use higher frequencies;
- (ii) To reduce the bandwidth required by services through the use of sophisticated signal-processing techniques, such as bandwidth compression etc.

The use of higher frequencies in the spectrum may provide only a temporary solution on the grounds of limited available bandwidth and the fact that at high frequencies equipment complexity and associated costs may render its use non-economic. Likewise, the use of signal processing in the form of data (bandwidth) compression, can yield only a partial solution to the problem since the per user bandwidth cannot be reduced beyond a certain value. The third possibility is the employment of Spread-Spectrum Systems not as replacements for existing communication systems but rather

as a second layer of systems that share the same bandwidth with those already in operation.

The earliest Spread-Spectrum System known to the author is the Random Access Discrete Address system (RADA). In the original form of the system each user has a time-frequency coded address and marks/spaces (1's or 0's) are transmitted by the respective transmission/non-transmission of the address. Improvements and generalizations to the RADA system were reported by Chesler [2] and Sommer [3]. Chesler's method was particularly interesting in that it pointed to subsequent developments in the field. Briefly, each user is allocated a total of m distinct addresses. Each address serves as an element in the alphabet that the user selects from in order to transmit information. As far as a particular user is concerned, therefore, the system is an M -ary system. Each address consists of a particular fixed k pulses selected from a frequency-time matrix of N pulses. The receiver decides that a particular address (alphabet element) has been transmitted when the particular k pulses associated with that address are received.

The performance of a channel randomly accessed by M users was studied by Cohen et al [4]. He described an asynchronous orthogonal-coding multiple-access system with ON-OFF signalling and neglected all sources of noise except

for other users' interference. As possible areas of application he suggested:

- (i) any line-of-sight communication network with high signal-to-noise ratio and negligible fading and multipath;
- (ii) any hard-wired network with sufficient bandwidth; and
- (iii) linear satellite repeaters.

This work was extended by Viterbi [5]. In his paper Viterbi describes a frequency-hopping (FH) system suitable for use in a processing satellite transponder for multiple-access by low-rate mobile users. In order to provide multiple-access Viterbi suggested that the total available one-way bandwidth be divided into Q frequency-slots (channels), each of which may be randomly accessed by any one of M users. The operation of the transmitter is outlined in Fig.1.1. The m^{th} user has a K -bit input buffer and may, thus, transmit any one of 2^K messages with equal probability. The modulation method employed is MFSK and so $Q \geq 2^K$ channels are required. The m^{th} user is allocated an address, ie. a pseudo-random sequence $\bar{a}_m$ of L frequencies. The address $\bar{a}_m = (a_{m1}, a_{m2}, \dots, a_{mL})$ is such that each a_{mi} $i=1,2,\dots,L$ is an independent random integer number in the range $[0, Q-1]$. The message x_m is combined with the pseudo-random address according to the equation:

$$\bar{Y}_m = \bar{a}_m + x_m \cdot \bar{1} \quad (1.1)$$

where the addition is modulo- Q . The transmission of x_m thus

takes place using L different channels so frequency diversity is thereby achieved. At the the receiver, the message X_m is recovered by simple modulo- Q subtraction of the address $\bar{A}_m$ from the received vector $\bar{Y}_m$. Since one or more users may select the same channel simultaneously, the performance in the absence of noise is limited by other users' interference. It is interesting that frequency-hopping, as described above, is the dual of time-interleaving in the frequency-domain. Contemporary with [5] was the work of Cooper and Nettleton [6] in which a FH scheme was proposed for use in multiple-access Mobile Communication. The basic idea of the Cooper and Nettleton scheme is shown in Fig.1.2.

Any given user out of the set or group of users is assigned an addressing sequence of N different frequencies each lasting $t_1 = T/N$ sec. The entire bandwidth is available for the frequencies of the addressing sequence used by each user. Clearly, it is required that the sets of sequences used differ sufficiently from one another. The information to be transmitted by any given user is imposed on his addressing sequence as follows. The incoming binary data fill an input buffer of length $\log_2 N$, thus forming any one of N possible messages with equal probability. To each of these messages the word-selector assigns one of N orthogonal words using, for example, the rows of an N by N Hadamard matrix. Thus $\log_2 N$ information bits are encoded as N binary digits and the coding scheme thus has rate of $\log_2 N/N$. The ℓ^{th} code bit

($\ell=1,2,\dots,N$) modulates the ℓ^{th} carrier in the address of any given user by differentially shifting its phase (DPSK) with respect to the phase of the corresponding carrier frequency of the previous $\log_2 N$ bits. So if a binary 0 is to be transmitted in the ℓ^{th} time-slot the phase of the carrier frequency in that time-slot is shifted by π relative to the phase of the ℓ^{th} time-slot of the previous sequence. If a binary 1 is to be transmitted no phase-shift takes place. The rate of transmission of information of this scheme is

$$R = \log_2 N / N \quad (\text{bits/sec/user})$$

At the receiver the ℓ^{th} modulated carrier, $\ell=1,2,\dots,L$ is delayed by $(N-1)t_1$ sec so that all carriers will be processed simultaneously (in parallel). The output of the ℓ^{th} product detector is low-pass filtered to remove the second-harmonic terms and then compared with a threshold. The decisions d_ℓ , $\ell=1,2,\dots,L$ are fed into a combiner circuit, the k^{th} output of which is

$$c_k = \sum_{\ell=1}^N h_{k,\ell} \cdot d_\ell$$

where $h_{k,\ell}$ is the ℓ^{th} bit of the k^{th} code-word. In the absence of impairments when the k^{th} word is transmitted the k^{th} output of the combiner is positive and the remaining $N-1$ are zero. Finally, the decoder produces the block of $\log_2 N$ bits that corresponds to the k^{th} code-word.

A variation of the scheme described by Viterbi [5] was developed by Goodman et al [7] with a view to applying it in Mobile Radio communications. Goodman's variation consisted of a modification of the addressing scheme of Viterbi. Comparing the modified Viterbi scheme and the scheme proposed by Cooper and Nettleton, Goodman showed that the MFSK system had a significant advantage over the DPSK system of Cooper and Nettleton in that for the same number of users the error-probability was lower. The analysis of the comparison of the DPSK and MFSK schemes was taken further by Yue [8],[9],[10] who found that the number of users that can be accommodated depends not only on the modulation method but also on the receiver combiner. Prior to the analysis by Yue it had been assumed that the interference caused by other users could be modelled as a Gaussian process and that a linear combiner should, therefore, be used. Yue, however, showed that the use of a Gaussian model was not justified and from a consideration of the statistics associated with the actual operation of the system he argued that a non-linear combiner should be used. Further, he considered the use of linear, hard-limited, and optimum non-linear combiners and demonstrated that the performance of the linear combiner is poor as compared with the optimum non-linear combiner and also with that of the hard-limited combiner. In these cases in which additive Gaussian noise is present at a significantly higher level than the interference aris-

ing from other users, the composite interference is essentially gaussian in nature and the use of a linear combiner, rather than a non-linear device, gives rise to a lower error-probability with the result that an increased number of users can be employed with a linear rather than a non-linear combiner. Yue's optimum non-linear combiner was shown to perform better than the simpler hard-limited combiner but considerations such as the cost and ease of implementation suggest the use of a hard-limited combiner rather than the optimum combiner in other than exceptional circumstances.

From what has been said above it is clear that interference from other users not only limits the performance of the system, but also influences the choice and the design of the receiver as well. Interference from other users arises whenever two, or more, users require (occupy) the same frequency channel simultaneously. It is clear that any addressing scheme should have the property that simultaneous use of frequency channels by more than one users is reduced to a minimum. It was suggested by Viterbi [5] that the components of the address of any given user be randomly selected (Random Addressing - RA). Although this is a satisfactory method when a large number of channels are involved relative to the number of users, it does, however, give rise to significant interference when the number of

channels becomes comparable with the number of users. In [7] the components of the address of the m user were taken to be

$$\bar{a}_m = (m, m+1, \dots, (m+L-1)_Q) \quad (1.2)$$

where $(\cdot)_Q$ denotes modulo- Q , and Q is the number of channels. Although such an addressing scheme yields Q orthogonal vectors (hence FH signals), when a system operates in synchronous mode, that is, when frame synchronism is maintained, it does, however, suffer from the disadvantage that a large level of interference occurs when the system operates in asynchronous mode. The question of addressing was considered further by Einarsson [11] and using the algebra of Galois Fields he derived an addressing scheme which is optimum, that is, has minimum interference for both the synchronous and asynchronous modes of operation. Although the presence of interference between users generally leads to a reduction in system performance, and consequently major effort has been made to reduce the level of such interference, an alternative approach is possible. Haskell [12] and Timor [13], in an effort to make use of any interference present, have developed an algorithm that uses all the information available at the receiver in order to reduce the probability of error, or equivalently, increase the number of users that can be accommodated by the system.

1.2 AIMS OF THE THESIS

This thesis is concerned with the performance of Frequency-Hopping Spread-Spectrum Systems when they are employed and operate in a multiple-access mode. At present, difficulty exists in any attempt to determine the performance of such systems. For example, the only estimators that are available are in the form of upper bounds on the error-probability. These bounds suffer from certain serious disadvantages. In the case of synchronous systems the bounds are so loose so as to be of little real value. In the case of asynchronous operation the only known bound is loose and, as will be shown in the thesis, is incorrect. The question of estimating the performance of Time-Frequency Coded Spread-Spectrum Systems (TFCSSS) using Einarsson's optimum addressing scheme is considered in Chapters 2 and 3 of the thesis. Chapter 2 is concerned with synchronous operation and Chapter 3 with asynchronous operation. In Chapter 2 an exact expression for the word-error probability is derived and this is shown to be relatively easy to compute. The operation of Time-Frequency Coded Spread-Spectrum Systems (TFCSSS) using the Einarsson addressing scheme has been simulated and the performance of such systems has been compared with results derived using the exact expression that has been obtained for the error probability. They are shown to be in excellent agreement. The results obtained

also indicate that the actual performance of a TFCSSS operating in synchronous mode is approximately an order of magnitude better than is suggested by Einarsson's $Pw2$ and $Pw3$ upper bounds [11].

In Chapter 2, in addition to considering the question of the error probability of the system, consideration is also given to questions relating to the design of TFCSSSs. In particular a new relationship is derived for the average transmitter power as a function of the number of available channels and the number of users on the system. The relationship applies to the base-to-mobile case and is of practical importance, since it enables a system designer, through the exact relationship that has been derived for the error probability, to determine for a given average transmitter power, the range of values of number of channels and the number of users that can have a specified error probability.

In Chapter 3 the asynchronous operation of TFCSSS using Einarsson's addressing scheme is considered and here again an exact expression is derived for the word-error probability of the system. This expression is, however, shown to be very difficult to evaluate even with modern computational tools. In view of this three easily computable upper bounds are derived in the chapter. A simulation of asynchronous

systems has been carried out and from this it is seen that the three bounds are all tight, even the weakest which is the most easily computable. The results presented in the chapter show that the tightest bound is almost an exact estimate of the true word-error probability of the system. Also in the chapter, in the course of deriving the bounds some consideration is given to the bound developed by Einarsson [11] and this is shown to be incorrect.

In Chapter 3 some consideration is also given to the question of the relative performance of the Einarsson scheme and the RA scheme proposed by Viterbi [5]. The comparison applies to both the synchronous and the asynchronous case. It is shown that as the number of frequency channels increases with respect to the number of time-slots, the performance of Viterbi's scheme approaches and becomes equivalent to that of the Einarsson scheme.

The operation of TFCSSS in the presence of both other users' interference and various forms of channel impairments and noise is a matter of considerable practical importance. Consideration of this aspect is taken up in Chapter 4 where the operation in the presence of various combinations of additive gaussian noise, and Rayleigh channel fading is considered. A new theoretical upper bound to system performance is derived in terms of false alarm and deletion

probabilities. The bound is shown to be tighter than a similar bound that has been derived by Einarsson [16] and simulation results are presented from which the tightness of the bound can be seen, particularly at low signal-to-noise ratios. It is shown that the bound under these conditions is very close to the system error probability, unlike the Einarsson bound [16] which becomes increasingly poor in this region.

In considering the operation of base-to-mobile systems in the presence of both other users' interference and noise and channel impairments four versions of the operation of the transmitter are considered. In the first version the power that is transmitted within an occupied frequency-time chip is assumed to be fixed irrespective of how many users occupy the chip. In the second version the situation is considered in which the transmitted voltage within the chip is taken as being proportional to the number of users occupying the chip, that is, the transmitted power is taken as being proportional to the square of the number of users occupying the chip. In the third version the transmitted power per chip is made proportional to the number of users occupying the chip. In the fourth version the system is assumed to operate as in Version I but with the power per chip increased so that the average transmitter power is equal to that of Version II. The results obtained in the

chapter show that if the possibility of using an increased transmitter power exists then it is better to increase the power per transmission chip to a higher level and keep it constant irrespective of the number of users occupying the chip, rather than to use a power level that increases with the number of users occupying the chip.

Spread-Spectrum Systems may be visualized as a class of encoders through which the information to be transmitted is encoded in such a way so as to occupy a bandwidth many times wider than that normally required for the direct uncoded transmission of the data. For example, in the TFCSSS introduced by Viterbi [5] and consequently modified and improved by Goodman [7] and Einarsson [11], for each user K information bits at a time are taken from a K -bit input buffer and used to select one of 2^K possible distinct symbols. The selected symbol is then combined with an L -symbol address word which is then used to generate an MFSK signal consisting of L different frequency carriers. In this way the original K -bit information word can be considered as being encoded into L positions (chips) on a frequency-time matrix. Since each entry can appear in any one of the $Q \gg 2^K$ frequency channels, the bandwidth available to each user is Q times larger than that usually required for uncoded transmission of the baseband signal. Since with this system the symbol representing the K -bit word is effectively transmitted L

times, it follows that the rate of transmission of information is K bits/ L time-slots (K/L bits per chip).

In Chapter 5 two new alternatives are presented in which 2^K frequency-slots are distributed among a number of users (usually 2^K) and the performance of the new systems is compared with that of the MFSK-FH system employing Einarsson's addressing scheme. The schemes are put forward with a view to making more efficient use of existing bandwidth. In the first scheme a simple (3,1) repetition code is used and in the second scheme the simple (7,4) cyclic Hamming code is used. With these coding schemes the effective transmission rates are $1/3$ and $4/7$ respectively.

In the scheme in which the (3,1) repetition code is used, each K -bit information word is encoded into a $3K$ -bit code-word by simply repeating each K -bit word twice. The stream of $3K$ code-bits is then transmitted through the channel. Each code-digit is sent either as a burst of frequency or as the absence of such a burst, just as in the schemes proposed by Viterbi, Goodman etc. At the transmitter a protocol is adopted so that with respect to the transmission of each bit of the $3K$ digits comprising the code-word, when m users are involved and m frequency channels are employed, each user occupies a different frequency slot, and the frequency-slot occupied by a given user changes from one of his

bits to the next. The changing pattern of frequencies as described with respect to each user is to mitigate against frequency selective fading or burst noise. With the (7,4) cyclic code, each 4-bit information word is cyclically encoded into a 7-bit code word and the stream of code bits is transmitted with ON-OFF signalling in a similar manner to that adopted with the (3,1) coded scheme. The results contained in Chapter 5 indicate that both of the coded schemes have a performance which is superior to that of the Einarsson scheme. The basic reason for the improved performance of the coded systems is that with random addressing schemes, such as that of Einarsson, the presence of other users' interference is a cause of severe limitation in respect of performance. The extent to which the TFCSSS with Einarsson's addressing scheme is inferior to the simple (3,1) and (7,4) coded systems is examined in detail in Chapter 5, where simulation results are given for operations in the presence of combinations of additive gaussian noise and Rayleigh fading.

It has been suggested in the literature [7] that a TFCSSS may be of importance in Mobile Radio Communication as a multiple-access scheme. One important aspect relative to this application is investigated in Chapter 6. At present, in mobile radio systems, it is current practice to assign two entirely different sets of frequencies to two neighbour-

ing areas (cells) so that co-channel interference will be avoided. In Chapter 6 the possibility of assigning to two neighbouring areas two sets of Q frequencies, such that they have a certain subset of B frequency channels in common, is examined. With regard to the performance of TFCSSS operating with a number of common frequency channels, it is shown in the chapter that the performance of such systems, although inferior to that of a stand-alone system, is not dramatically different, and hence such systems may have advantages in certain applications where economy of bandwidth is of major importance. In the chapter the worst case situation, in which the entries in the frequency-time matrix of the local area are of the same power as those due to the interfering area, is studied. Simulation results are presented and a new theoretical upper-bound to the error probability is derived. This bound is shown to be very tight.

In the final chapter, Chapter 7, the main results contained in this thesis are summarized and some suggestions are made for future work.

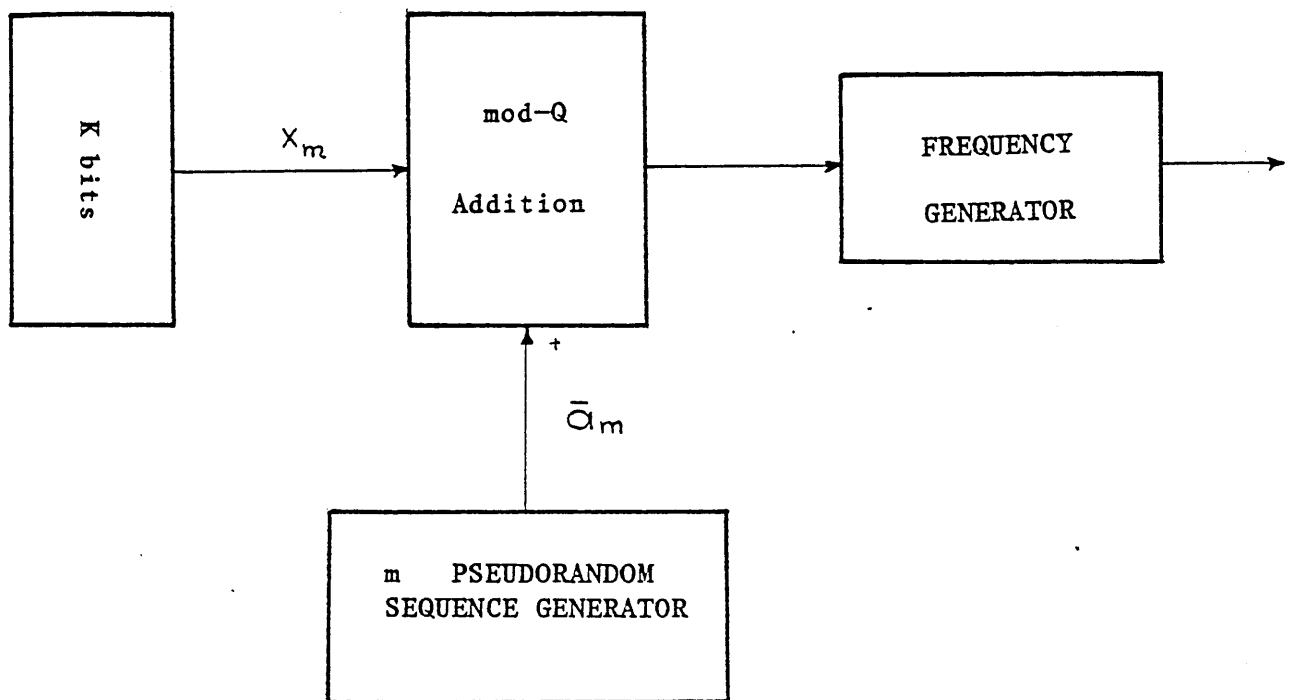


Fig.1.1 Outline of the transmitter of a Viterbi-type system

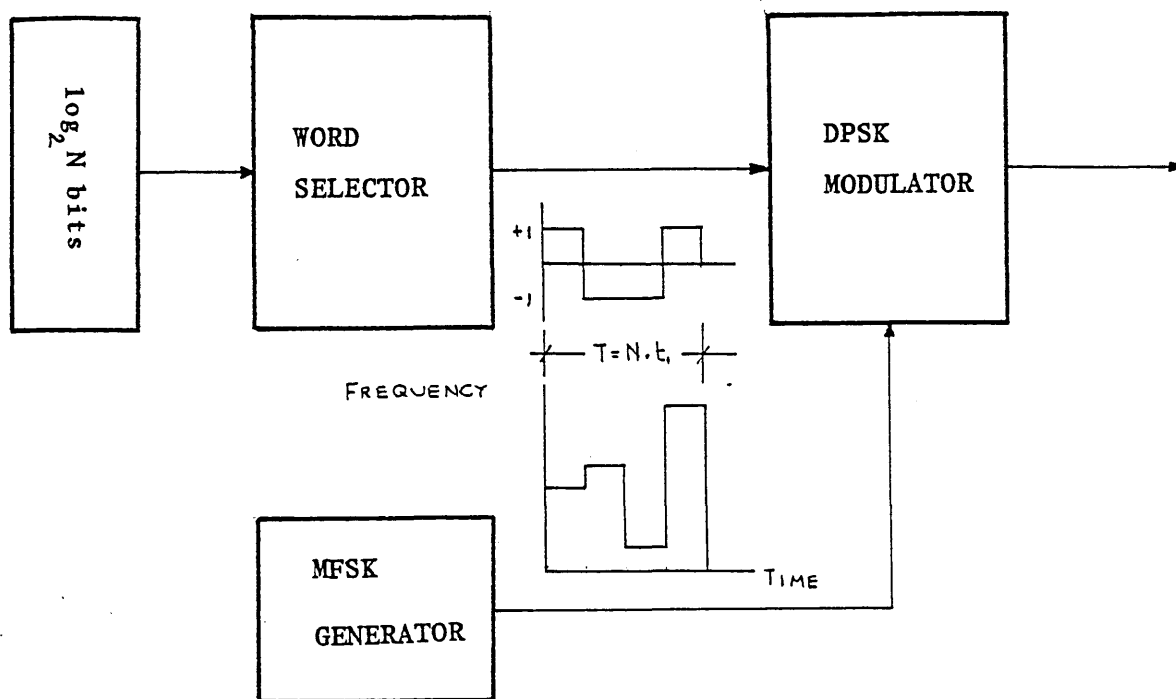


Fig.1.2 Outline of the transmitter of a Cooper and Nettleton-type system

CHAPTER 2

SOME FURTHER RESULTS ON THE PERFORMANCE OF TIME-FREQUENCY CODED SPREAD-SPECTRUM SYSTEMS

2.1 INTRODUCTION

The history of the development of TFCSSS was set out in the previous chapter and it was pointed out that optimum addressing schemes have been developed by Einarsson, for both the synchronous and asynchronous modes of operation. It was also explained that with regard to schemes based on the Einarsson addressing technique the only information available in respect of system performance was in the form of upper bounds to the word-error probability.

In this chapter TFCSSS systems using Einarsson's addressing scheme and operating in synchronous[†] mode are considered. An exact expression is derived for the error probability and the results obtained are compared with the results of a computer simulation of the system and with Einarsson's PW2 and PW3 upper bounds. It is shown that the theoretical results obtained agree with the simulation results and that they are roughly an order of magnitude smaller than the upper bounds. Also, a relationship is derived for the average transmitter power in the base-to-mobile case. This relationship is in the form of a function of the number of available channels and the number

[†] In Chapter 3 the asynchronous (non-synchronous) operation of frequency-hopped systems of the type proposed by Einarsson is considered in detail.

of users on the system. The relationship is of practical importance since it enables a system designer, through the exact relationship that has been derived for the error probability to determine, for a given average transmitter power, the range of values of number of channels and number of users that can be on the system whilst maintaining a specified system word-error probability.

2.2. DESCRIPTION OF EINARSSON'S SYNCHRONOUS ADDRESSING SCHEME

In general the system supports M users, each of which is assigned an address $\bar{a}_m = (a_{m1}, a_{m2}, \dots, a_{mL})$. The address is a set of L tones each τ sec long, chosen in a specified order among the Q available tones, where Q is a prime number or power of a prime number. Each user sends, in a period of $L\tau$ sec, a message $X_i^{(m)}$ selected from a set of 2^K K -bit messages $X_0, X_1, \dots, X_{2^K-1}$, $2^K \ll Q$. The information bearing message is sent by modulating (changing) the address of the user up and down in frequency in a cyclic fashion. Thus in order to send the message $X_i^{(m)}$ the m^{th} user sends $\bar{Y}_m$ where :

$$\bar{Y}_m = \bar{a}_m + X_i^{(m)} \cdot \bar{1} \quad \text{mod-} Q \quad (2.1)$$

$$y_{jm} = a_{jm} + X_i^{(m)} \quad \text{mod-} Q \quad \text{for } j=1,2,\dots,L \quad (2.2)$$

This operation is depicted in Fig2.1 for $Q=7$, $L=4$, $a = (1,3,2,6)$ and $X_i=3$. In the system described, each transmitted message X_i conveys K bits of information and results in an information rate R per user where

$$R = K / L \cdot \tau \quad (2.3)$$

Each receiving user obtains a Q by L frequency-time matrix, which contains occupied chips due to transmissions intended

for him as well as other users' transmissions and noise. The user proceeds by subtracting his address (mod-Q) thus redistributing the occupied chips in the matrix so that the transmitted message X_i intended for him appears as a complete row in the matrix while the other users' transmissions and the noise are dispersed. Einarsson [11] considered the errors arising from the interference caused by other users in the absence of noise and the same situation is considered in this chapter. The system's impairment is caused by other users accidentally giving rise to the formation of a full row, or a number of full rows, in the decoded matrix of the given user. That user is then uncertain as to which row corresponds to the desired message and which are false rows. In the process of making a choice errors may occur. Einarsson studied this system and developed an algebraic method for assigning the address that allows any two messages to coincide in at most one chip. This method is to let the address $\bar{a}_m$ of the m^{th} user be given by

$$\bar{a}_m = (\gamma_m, \gamma_m \beta, \dots, \gamma_m \beta^{L-1}) \quad (2.4)$$

where γ_m is an element of the Galois Field $GF(Q)$ and β is a fixed primitive element. The maximum number of addresses is equal to Q and the maximum value of L is equal to $Q-1$. Einarsson developed a series of upper bounds on the word-error probability, P_w , of this system. He derived a bound denoted by P_{w2} where

$$P_{w2} = (Q-1) \cdot \frac{(M-1) \cdot (M-2) \cdot \dots \cdot (M-L)}{Q^L} \quad (2.5)$$

and an alternative bound PW3 given by

$$P_w < P_{w3} = (Q-1) \cdot \prod_{k=0}^{L-1} \left[1 - \left(1 - \frac{1}{Q-k} \right)^{M-k-1} \right] \quad (2.6)$$

In the next section an exact derivation of the error probability will be presented and compared with a computer simulation of the system. From these results it will be seen that Einarsson's P_{w2} and P_{w3} upper bounds are in fact loose. The results obtained show that the exact error probability is approximately an order of magnitude below Einarsson's bounds.

2.3 ERROR PROBABILITY IN SYNCHRONOUS OPERATION

An exact expression for the word-error probability will now be derived. The probability P of forming a false full in a given channel in the FR -frequency matrix row is given below

$$P_{FR} = P(I_1 \neq 0) \cdot P(I_2 \neq 0 | I_1 \neq 0) \cdots P(I_L \neq 0 | I_1 \neq 0, \dots, I_{L-1} \neq 0) \quad (2.7)$$

where I_k is the number of interferers occupying the k^{th} chip in that false full row.

The following derivation rests upon finding an exact expression for all the conditional probability terms in equation (2.7).

First consider $P(I_1 \neq 0)$ which is given by:

$$P(I_1 \neq 0) = \sum_{v_1=1}^{M-1} P(I_1 = v_1)$$

The probability $P(I_1 = v_1)$, that there are v_1 interferers in the first cell of the false full row is given by

$$P(I_1 = v_1) = \binom{M-1}{v_1} \left(\frac{1}{Q}\right)^{v_1} \left(1 - \frac{1}{Q}\right)^{M-1-v_1} \quad (2.8)$$

where $M-1$ is the total number of possible interferers.

Hence

$$P(I_1 \neq 0) = \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q}\right)^{v_1} \left(1 - \frac{1}{Q}\right)^{M-1-v_1} \quad (2.9)$$

The first two terms of (2.7) may be written as

$$P(I_1 \neq 0) \cdot P(I_2 \neq 0 | I_1 \neq 0) = \sum_{v_1=1}^{M-1} P(I_1 = v_1) \cdot P(I_2 \neq 0 | I_1 = v_1) \quad (2.10)$$

when $I_1 = v_1$ only $M-1-v_1$ interferers may contribute to I_2 , hence

$$P(I_2 \neq 0 | I_1 = v_1) = 1 - \left(1 - \frac{1}{Q-1}\right)^{M-1-v_1} \quad (2.11)$$

and therefore

$$P(I_1 \neq 0) \cdot P(I_2 \neq 0 | I_1 \neq 0) = \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q}\right)^{v_1} \left(1 - \frac{1}{Q}\right)^{M-1-v_1} \left[1 - \left(1 - \frac{1}{Q-1}\right)^{M-1-v_1}\right] \quad (2.12)$$

In general, the j^{th} chip of the false full row may be occupied

by any number of $M-1-\sum_{i=1}^{j-1} v_i$ interferers each having access to $Q-(j-1)$ cells. Thus

$$P(I_j \neq 0 | I_1 \neq 0, \dots, I_{j-1} \neq 0) = 1 - \left(1 - \frac{1}{Q-(j-1)}\right)^{M-1-\sum_{i=1}^{j-1} v_i} \quad (2.13)$$

and therefore

$$\begin{aligned} & P(I_1 \neq 0) \cdot P(I_2 \neq 0 | I_1 \neq 0) \cdots P(I_j \neq 0 | I_1 \neq 0, \dots, I_{j-1} \neq 0) = \\ & \sum_{v_1=1}^{M-1} \sum_{v_2=1}^{M-1-v_1} \cdots \sum_{v_{j-1}=1}^{M-1-\sum_{i=1}^{j-2} v_i} \\ & \left\{ \prod_{k=1}^{j-1} \binom{M-1-\sum_{i=1}^k v_i}{v_k} \left(\frac{1}{Q-(k-1)}\right)^{v_k} \cdot \left(1 - \frac{1}{Q-(k-1)}\right)^{M-1-\sum_{i=1}^k v_i} \right\} \cdot \\ & \left[1 - \left(1 - \frac{1}{Q-(j-1)}\right)^{M-1-\sum_{i=1}^{j-1} v_i} \right] \quad (2.14) \end{aligned}$$

with v_0 defined as zero.

From equation (2.14) it can be seen that the probability P_{FR} of a false full row is given by

$$\begin{aligned}
 P_{FR} &= \sum_{V_1=1}^{M-1} \sum_{V_2=1}^{M-1-V_1} \cdots \sum_{V_{L-1}=1}^{M-1-\sum_{i=1}^{L-2} V_i} \\
 &\left\{ \prod_{k=1}^{L-1} \binom{M-1-\sum_{i=1}^k V_{i-1}}{V_k} \cdot \left(\frac{1}{Q-(k-1)} \right)^{V_k} \cdot \right. \\
 &\quad \left. \left(1 - \frac{1}{Q-(k-1)} \right)^{M-1-\sum_{i=1}^k V_i} \right\} \cdot \\
 &\left[1 - \left(1 - \frac{1}{Q-(L-1)} \right)^{M-1-\sum_{i=1}^{L-1} V_i} \right] \quad (2.15)
 \end{aligned}$$

The probability P_{FR} of a false full row can be used to determine the exact probability, P_w , of word error. The receiving user after decoding the matrix by subtracting his address, will find a full row corresponding to the desired message and possibly a number of other full rows due to other users' interference. The uncertainty involved in which row to select as the desired message is a source of error. Of course, no error is made if only one full row is present. In this case the full row will correspond to the transmitted message and the other users' transmissions will not have conspired so as to form a false full row. When i full rows are present the user can pick one of them at random and ascribe it to the transmitted message. The probability of choosing the wrong row is then

$$P(\text{word error} | i \text{ full rows}) = \frac{i-1}{i} \quad (2.16)$$

Out of i full rows, $i-1$ are false. If the number of rows in the matrix Q is equal to 2^K then any one of them could contain the desired message. If, however, Q is greater than 2^K , there are $Q-2^K$ rows known to both the transmitter and the receiver that cannot contain the desired message. Therefore, the $i-1$ full false rows can be chosen from a total of $2^K - 1$ possibilities in $\binom{2^K - 1}{i - 1}$ ways.

Hence

$$\begin{aligned} P(i \text{ full rows}) &= P(i-1 \text{ full false rows}) = \\ &= \binom{2^K - 1}{i - 1} P_{FR}^{i-1} (1 - P_{FR})^{2^K - i} \end{aligned} \quad (2.17)$$

When the system supports M users, the maximum number of full rows in any user's decoded matrix is M and thus from (2.16) and (2.17), it follows that the word-error probability, P_w , is given by:

$$\begin{aligned}
 P_w &= P(\text{word-error}) = \sum_{i=2}^M P(\text{word-error} | i \text{ full rows}) \cdot P(i \text{ full rows}) \\
 &= \sum_{i=2}^M \frac{i-1}{i} \binom{2^K-1}{i-1} \cdot P_{FR}^{i-1} \cdot (1 - P_{FR})^{2^K-i} \quad (2.18)
 \end{aligned}$$

where P_{FR} is given by equation (2.15).

Equation (2.18) constitutes an exact expression without any approximation or bounding operations, and thus, the word-error probability P_w can be evaluated explicitly for any choice of Q (number of channels), M (number of users), K (number of bits per message) and L (address length). Theoretical results obtained from equation (2.18) are shown in Figs 2.2-2.4 together with Einarsson's $Pw2$ and $Pw3$ bounds and simulation results for various combinations of Q , L and M . The computer simulation was undertaken for the purpose of establishing the operational validity of the analytic expression for Pw given by equation (2.17). It can be seen that there is excellent agreement between the analytical and the simulation results.* It can also be seen that the exact analytical results are roughly an order of magnitude below Einarsson's $Pw2$ and $Pw3$ bounds.

* Conditional on the simulation run-time.

2.4 RELATIONSHIP BETWEEN AVERAGE TRANSMITTER POWER AND WORD-ERROR PROBABILITY

An interesting and important question concerned with the operation of multi-user frequency-hopped systems is the way in which the error probability experienced by users is related to the available average transmitter power. A related important question is concerned with how many users can occupy a system having a fixed number of available channels subject to the conditions that the average transmitter power is fixed and a certain error probability is desired. These aspects are considered in this section.

When more than one user is occupying the system, each contributes an entry per column of the matrix that may coincide with those of other users. Some or all entries may coincide, and the number of occupied cells in any column may vary from 1 to M , where M is the number of users in the system. In the case of communication from base-to-mobile if it is assumed that, irrespective of the number of users requiring it, every occupied chip contains one energy unit whenever it is required, then the required transmitter power is directly related to the number of occupied chips and therefore the energy distribution in the matrix is of importance.

By using combinatorial analysis [15] an expression can be obtained for the probability $P(\ell=i)$, that exactly ℓ out of Q chips are occupied in a given column. The way in which this is done is outlined below :

Assume that M users' entries are distributed among Q chips in Q equiprobable ways then it can be shown[†] (see Appendix 1)

(i) that the number of distributions leaving none of the chips empty is

$$A(M, Q) = \sum_{y=0}^Q (-1)^y \binom{Q}{y} (Q-y)^M \quad (2.19)$$

(ii) that the number of ways in which exactly m chips are left empty is

$$E_m(M, Q) = \binom{Q}{m} \cdot A(M, Q-m) \quad (2.20)$$

and consequently that the probability of finding exactly m empty chips is

$$P_m(M, Q) = Q^{-M} \cdot E_m(M, Q) \quad (2.21)$$

Thus the probability of finding exactly i full chips in the column is given by

$$P(\ell=i) = Q^{-M} \cdot \binom{Q}{Q-i} \cdot \sum_{y=0}^i (-1)^y \binom{i}{y} (i-y)^M \quad (2.22)$$

[†] The results contained in Appendix 1 are based on the solutions to questions posed by Feller [15], p.60

A knowledge of the probability distribution of the number of occupied chips per column can be used in order to find : (i) the average number of occupied chips which is directly related to the average transmitter power and (ii) the variance of the number of occupied chips from which a practical peak power constraint can be established. In the base-to-mobile case each unoccupied chip in the matrix is assumed to contain zero power, while each occupied chip contains $P_1 W$ regardless of the number of users requesting and using it. The maximum average power is needed when there are no coincidences. In this case the number of occupied chips in the column takes on its maximum value M and the power required is $M \cdot P_1$.

The probability distribution of the number of occupied chips in each column has been calculated according to (2.22) for values of $Q=17$ and $M=3,6,10,14$ and 17 . The results are shown in Fig2.5 . The figure also shows the values of the mean and variance of the number of occupied chips per column. It can be seen that the variance is well below the square of the mean and therefore the mean can be safely used as a guide to the required average transmitter power. From the knowledge of $\bar{\ell}_M^Q$, the average number of occupied chips per column of size Q with M users on the system, it can be seen that on average the interference component in each column is $\bar{\ell}_M^Q - 1$ and thus an operational expression for the

signal-to-interference (power or energy) ratio is

$$SIR = \frac{1}{\bar{e}_M^Q - 1} \quad (2.23)$$

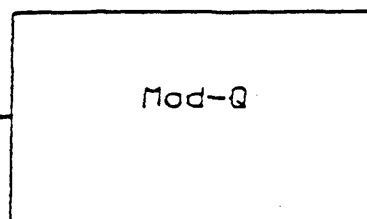
This expression is meaningful for cases in which $M > L+1$.

When $M < L$ the interfering chips cannot conspire to form a false full row and the signal to interference ratio is undefined.

Using the relationship between the signal-to-interference ratio and the parameters Q and M , a set of operating curves relating the word-error probability to the signal-to-interference ratio is plotted as shown in Fig.2.6 for values of $Q = 13, 17$ and 19 .

				6
				5
				4
•	•	•	•	3
				2
				1
				0

Message



			•	6
				5
				4
	•			3
		•		2
•				1
				0

Address

	•			6
		•		5
•				4
				3
			•	2
				1
				0

Transmission matrix

$$Q=7, L=4, K=2$$

Fig.2.1 Addressing scheme

EINARSSON'S SCHEME. $Q=13, L=8, K=3$

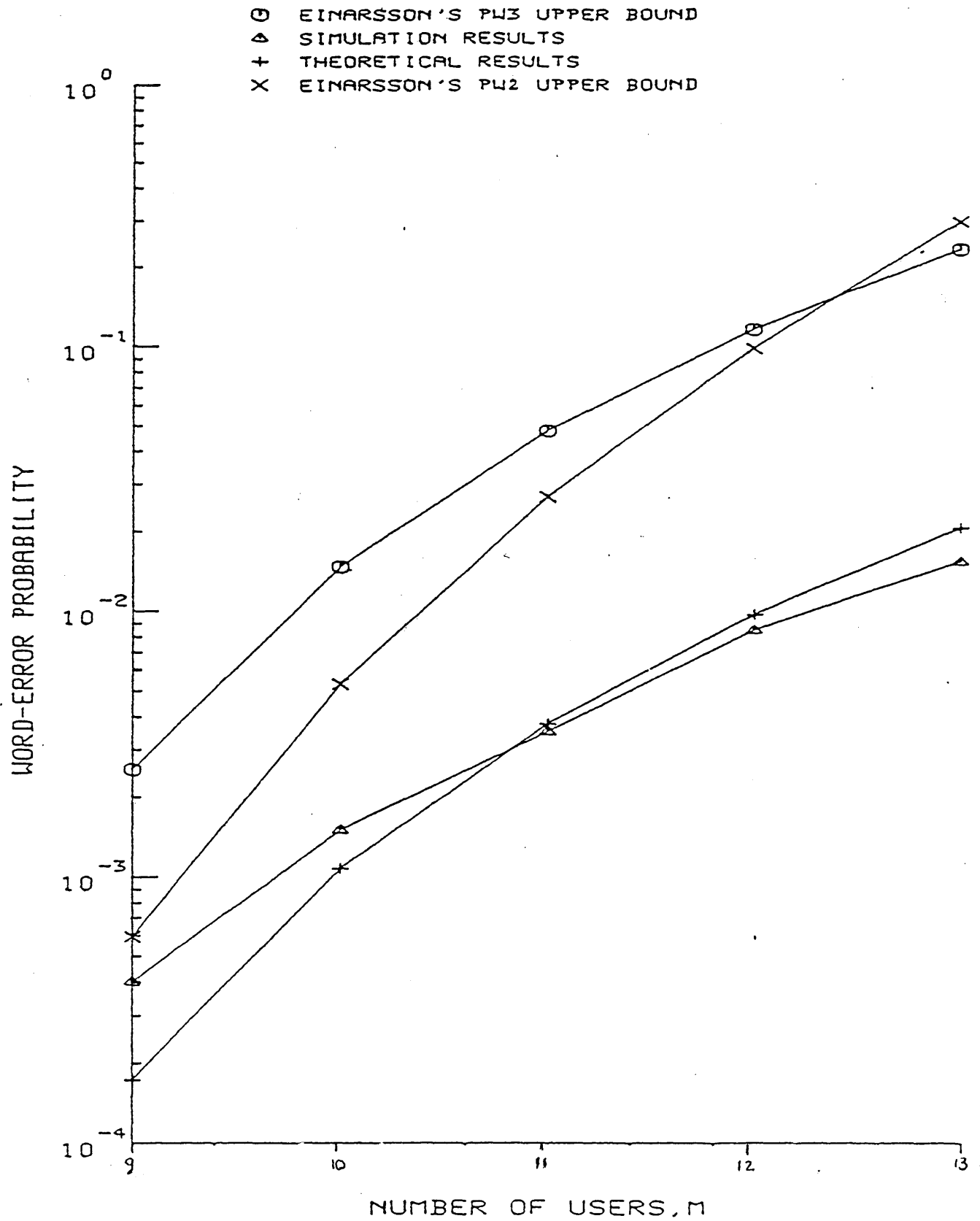


Fig.2.2 Word-error probability .vs. Number of users

EINARSSON'S SCHEME

Q=17,L=8,K=4

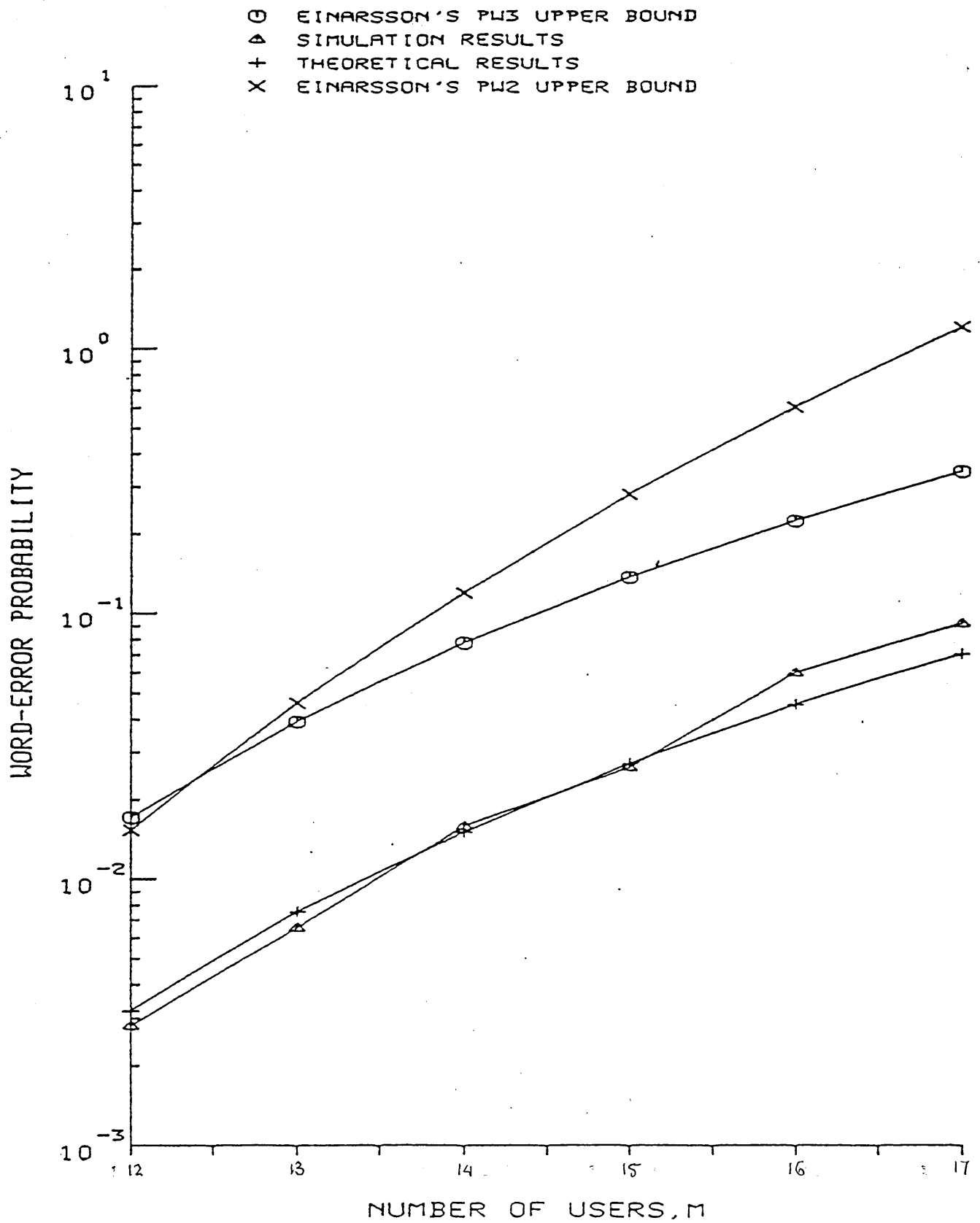


Fig.2.3a Word-error probability .vs. Number of users

EINARSSON'S SCHEME

$Q=17, L=5, K=4$

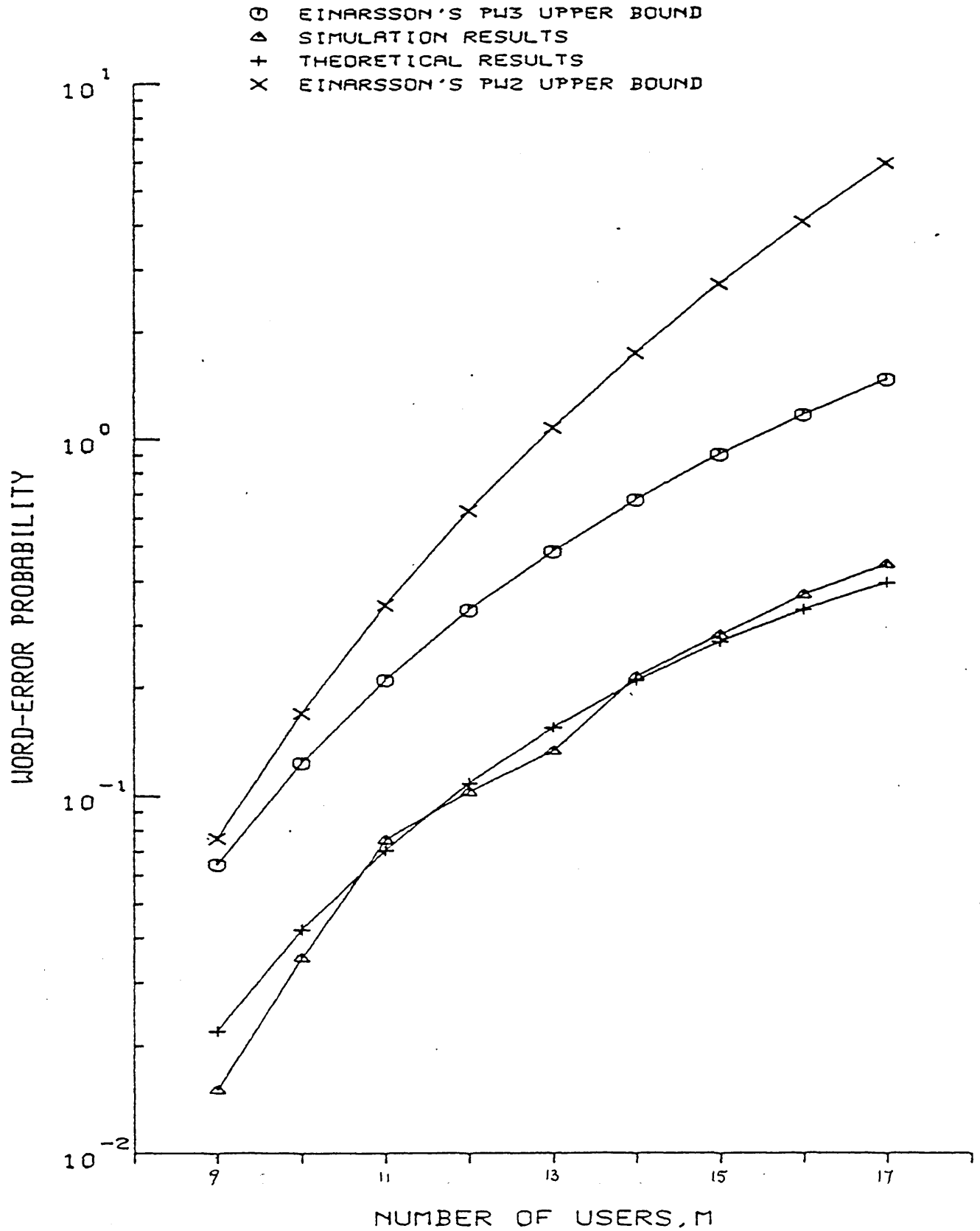


Fig.2.3.b Word-error probability .vs. Number of users

EINARSSON'S SCHEME

$Q=19, L=8, K=4$

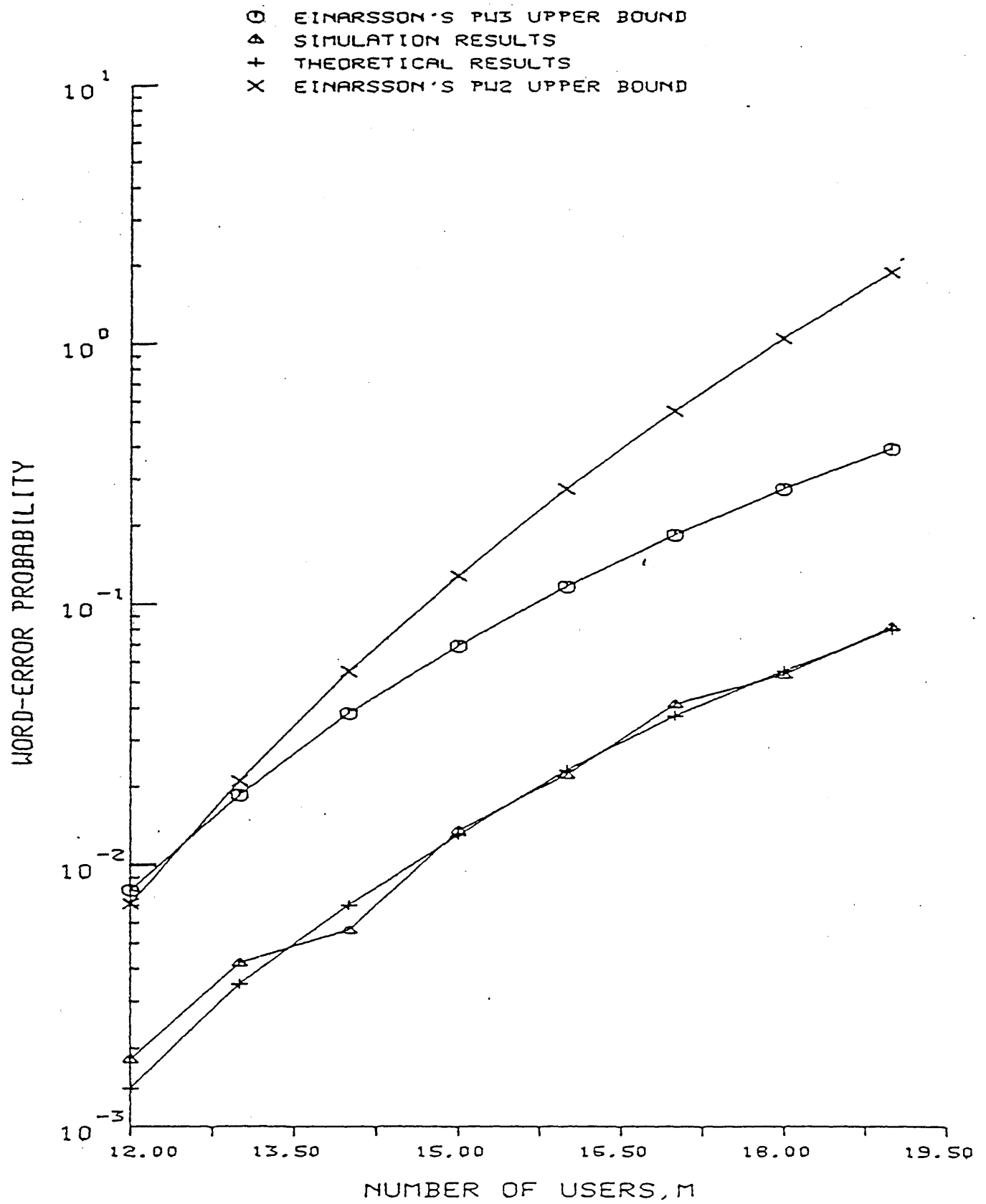


Fig.2.4 Word-error probability .vs. Number of users

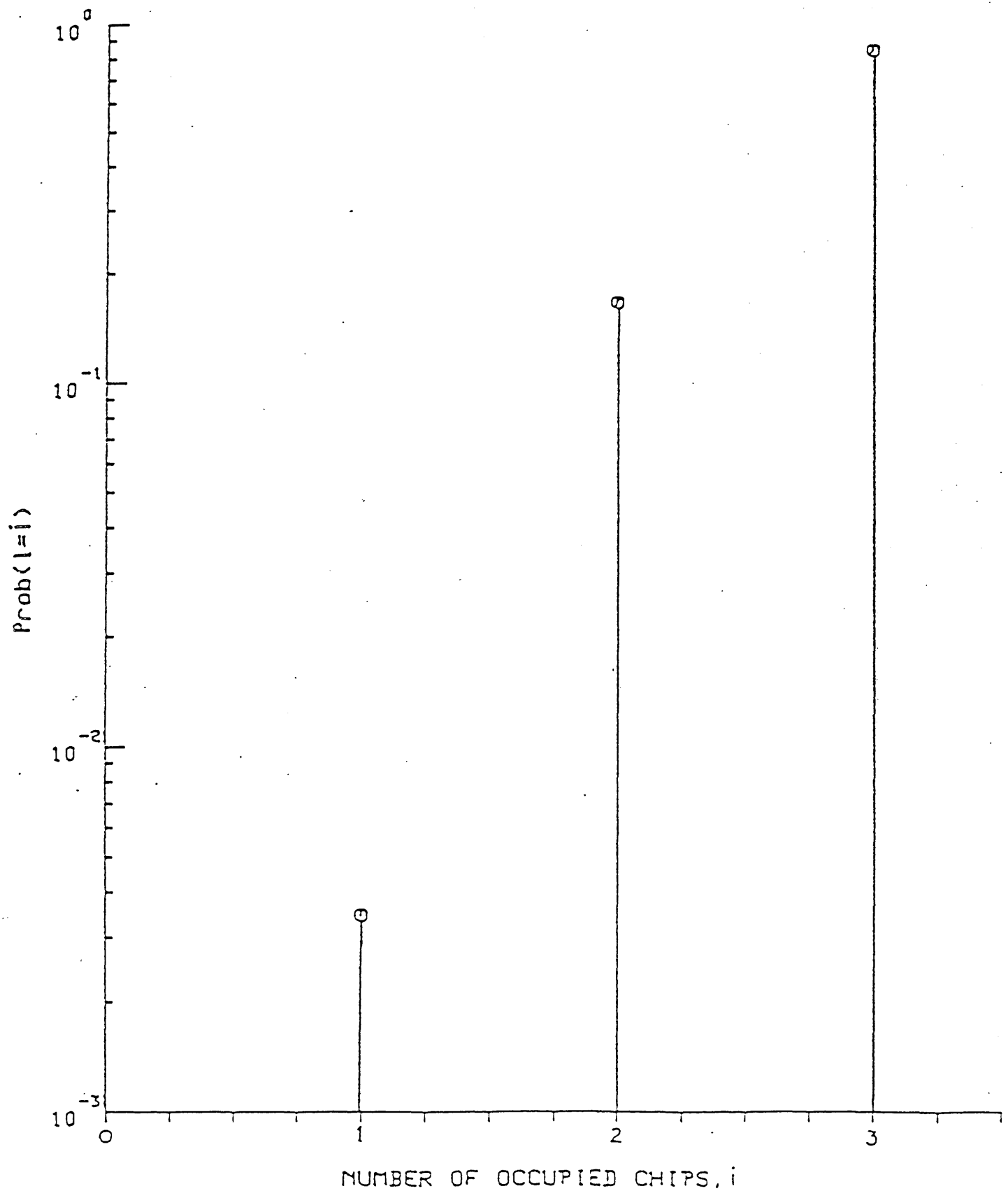


Fig.2.5a Probability density function of the number of occupied chips in a column.

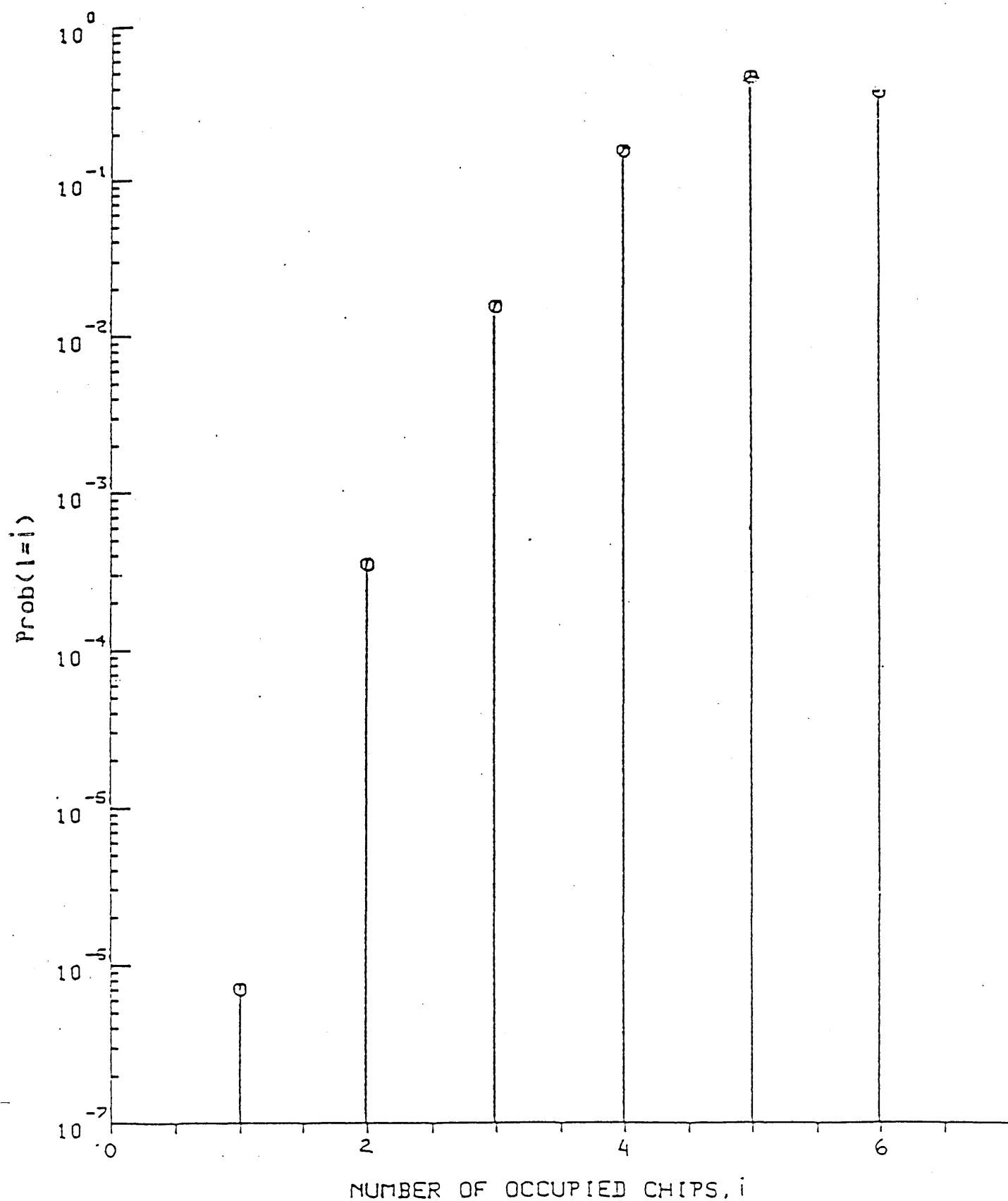


Fig.2.5b Probability density function of the number of occupied chips in a column.

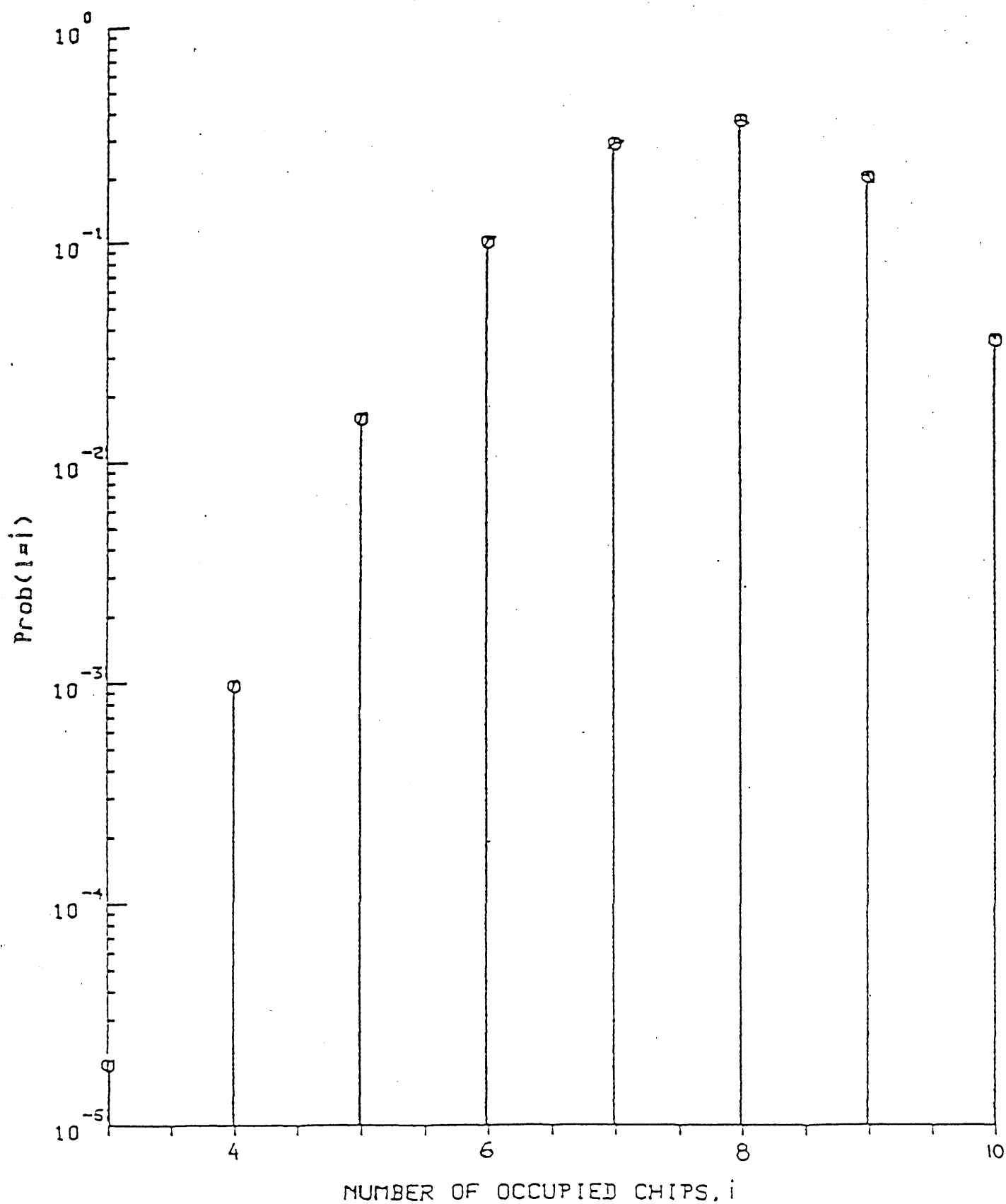


Fig.2.5c Probability density function of the number of occupied chips in a column.

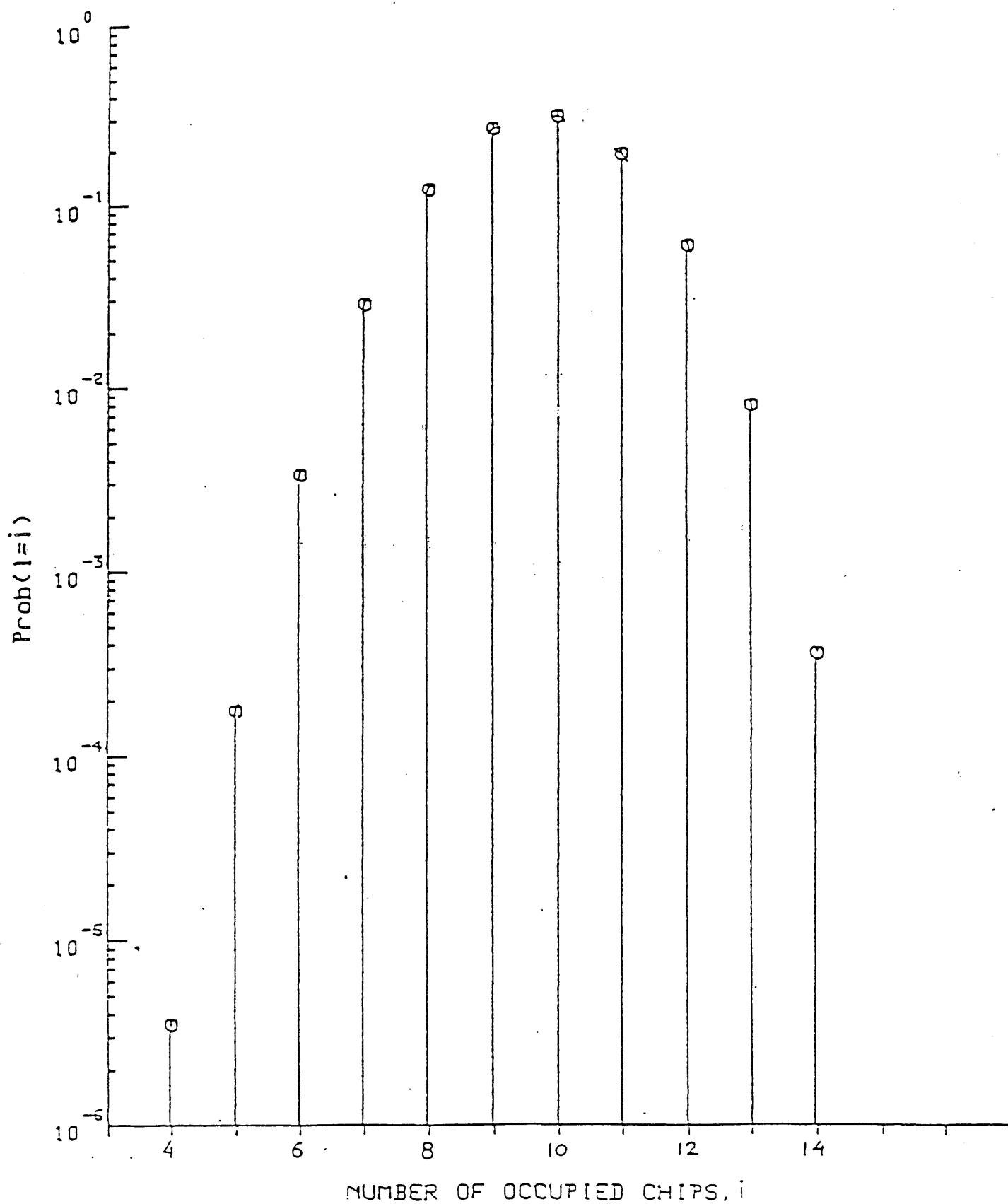


Fig.2.5d Probability density function of the number of occupied chips in a column.

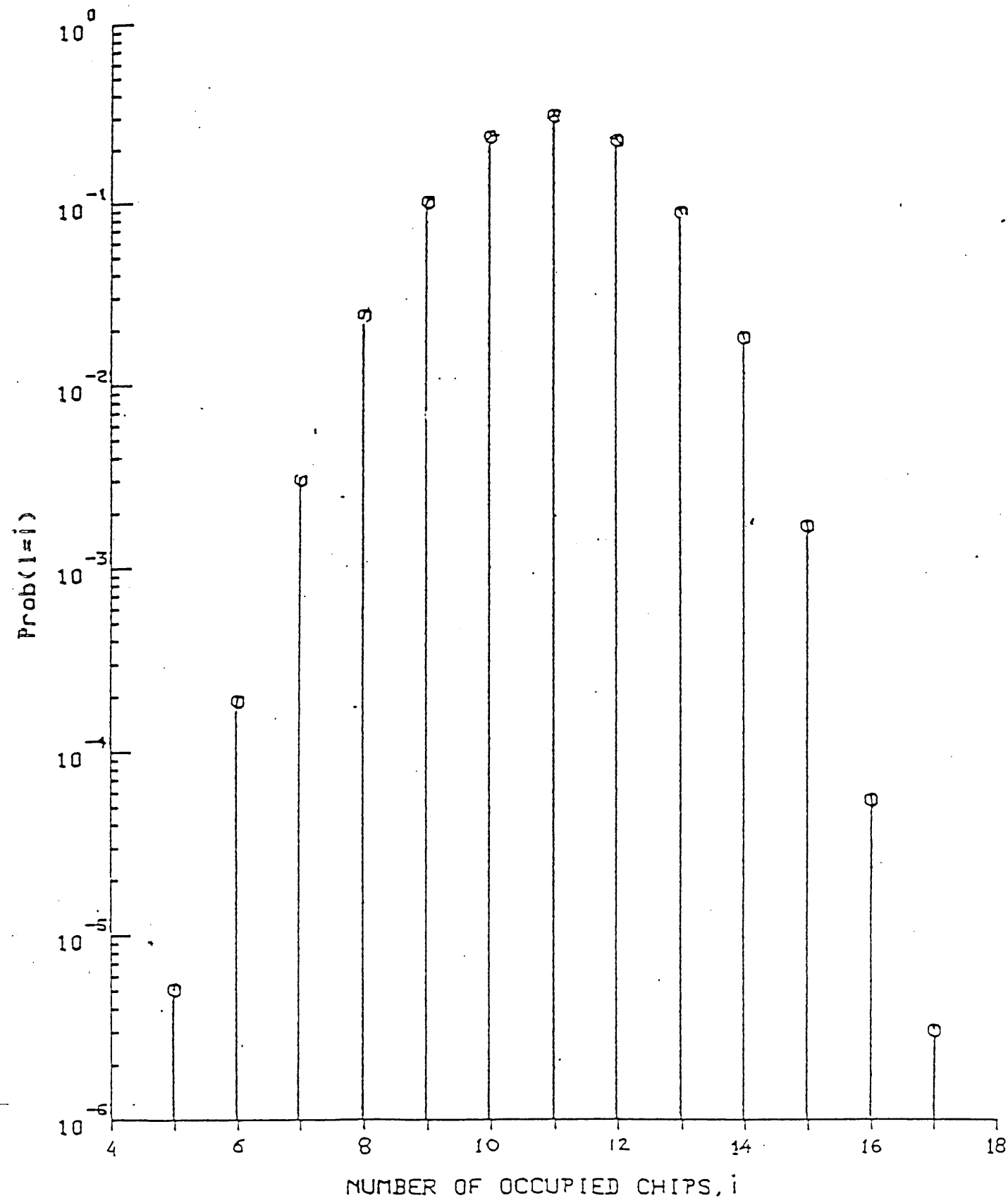


Fig.2.5e Probability density function of the number of occupied chips in a column.

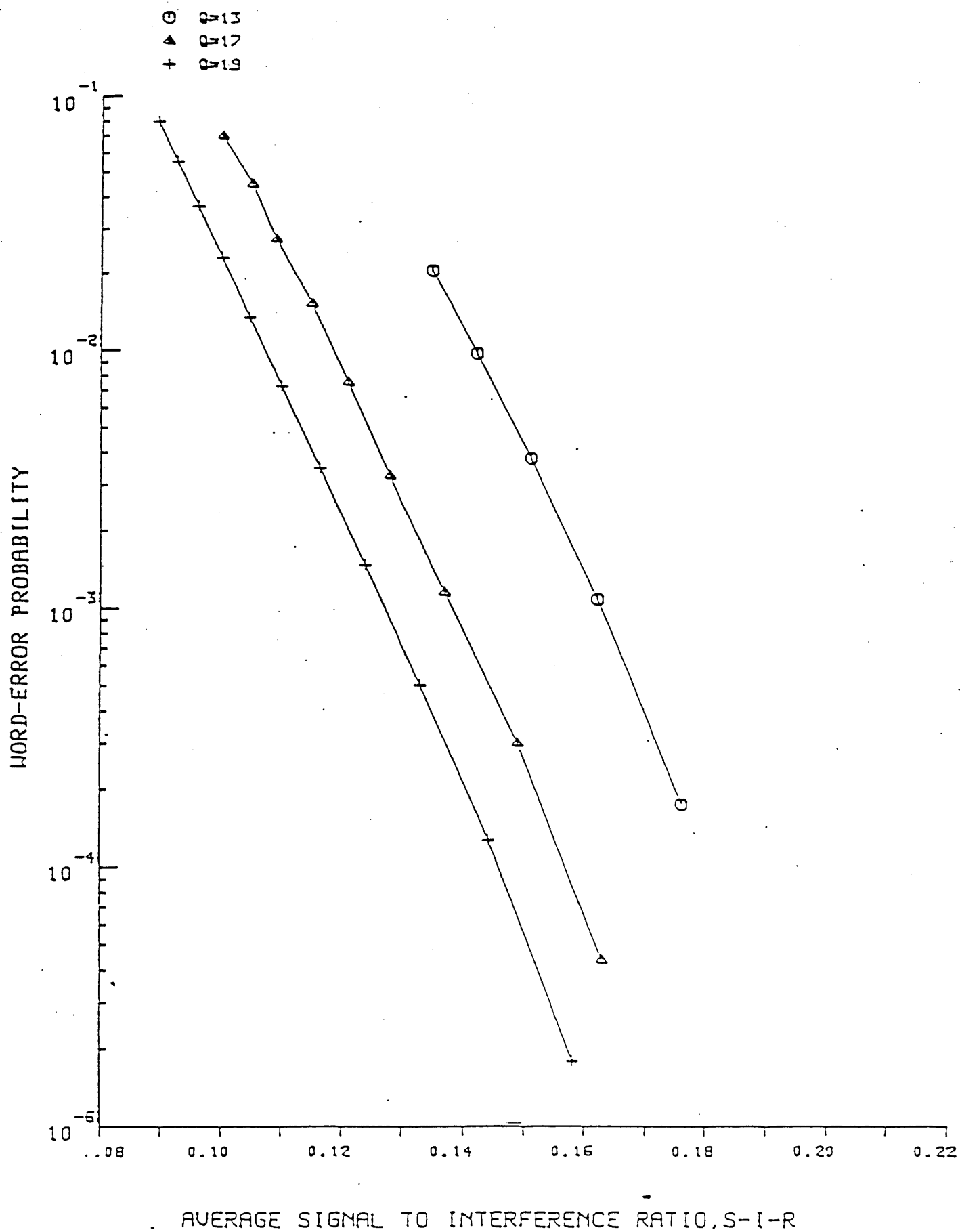


Fig.2.6 Word-error probability .vs. Average Signal to Interference Ratio.

CHAPTER 3

ON THE PERFORMANCE OF ASYNCHRONOUS

TIME-FREQUENCY CODED SPREAD-SPECTRUM SYSTEMS

3.1 INTRODUCTION

In Chapter 2 the performance of time-frequency coded spread spectrum systems operating in a synchronous mode was considered. However, in practice it may be very difficult to maintain synchronism between the transmitter and the receiver associated with each pair of users.

In this chapter the performance of asynchronous (non-synchronous) systems is considered in some detail. An exact closed-form expression has been derived for the error-probability of an asynchronous system. It is found, however, that except in certain simple cases, it is very difficult to actually evaluate the exact error-probability. In view of this a group of three new upper bounds to the error probability are derived. One of these bounds is a correction to the upper bound PW4 provided by Einarsson [11]. The bounds are such that the degree of computational complexity increases with increasing tightness of the bounds. However, all the bounds are relatively easy to evaluate.

3.2 AN EXACT EXPRESSION FOR THE ERROR PROBABILITY OF AN ASYNCHRONOUS SYSTEM.

In practice it may be difficult to achieve and maintain synchronism between users. In the following analysis it is assumed that the transmitted sequences of the $M-1$ interferers are randomly time-shifted by an integer number of time-slots with respect to the desired sequence and to each other, and that full synchronism (chip and frame synchronism) exists in respect of the desired user pair. *This assumption is equivalent to having established bit synchronism; if this is not so, then a more complicated analysis is required.*

Einarsson [11] considered the asynchronous operation of time-frequency coded spread spectrum systems and developed an addressing algorithm that reduces the effect of other users' interference. Einarsson's scheme consisted of letting the transmitted sequence $\bar{Y}_m$ of the m^{th} user be formed according to the rule

$$\begin{aligned}\bar{Y}_m &= x_m \cdot \bar{\beta} + y_m \cdot \bar{1} \\ \bar{\beta} &= (1, \beta, \beta^2, \dots, \beta^{L-1})\end{aligned}\quad (3.1)$$

where $y_m \in GF(Q)$ is the address and x_m is the message symbol of the m^{th} user and β is a fixed primitive element of $GF(Q)$.

In the system operating in asynchronous mode parts of two consecutive messages from the same interferer may dis-

rupt a particular frame of another (the so-called desired) user. If the two messages are identical only one coincidence is possible [11]. If, however, the two messages are different then each of them may result in at most one coincidence, thereby giving a maximum total of two coincidences per frame.

In this section an exact expression for the probability of forming a false full row at a given level in the transmission matrix will be derived. This can then be used in conjunction with the results presented in the previous chapter to obtain an exact expression for the error probability.

It can be assumed that each interferer has a random delay of i slots, $i \in \{0, 1, 2, \dots, L-1\}$ with respect to the desired user and that the delays are independent and uniformly distributed over $[0, L-1]$. Let the random variable U_i denote the number of interferers with delay i . Note that U_i takes on values u_i so as to satisfy

$$\sum_{i=0}^{L-1} u_i = M-1 \quad (3.2)$$

The L -tuple $(u_0, u_1, \dots, u_{L-1})$ defines a unique arrangement of all users for asynchronous operation. Clearly, the synchronous system may be regarded as a special case with

$U_0=M-1$ and $U_1=U_2=\dots=U_{L-1}=0$. The probability of occurrence of $(U_0, U_1, \dots, U_{L-1})$ is given by

$$P(U_0=U_0, \dots, U_{L-1}=U_{L-1}) = \frac{(M-1)!}{U_0! \dots U_{L-1}!} \cdot \frac{1}{L^{M-1}} \quad (3.3)$$

By the total probability theorem [15], the probability $P_{FR-ASYN}$ of a false full row, may be written as

$$P_{FR-ASYN} = \sum_{\forall (U_0, U_1, \dots, U_{L-1})} P(FR | U_0=U_0, \dots, U_{L-1}=U_{L-1}) \cdot P(U_0=U_0, \dots, U_{L-1}=U_{L-1})$$

or, more briefly, as

$$P_{FR-ASYN} = \sum_{\forall (U_0, U_1, \dots, U_{L-1})} P(FR | U_0, U_1, \dots, U_{L-1}) \cdot P(U_0, U_1, \dots, U_{L-1}) \quad (3.4)$$

where $P(FR | U_0=U_0, \dots, U_{L-1}=U_{L-1})$ denotes the probability that a false full row is formed when $U_0=U_0, \dots, U_{L-1}=U_{L-1}$. In (3.4) the summation is taken over all L -tuples $(U_0, U_1, \dots, U_{L-1})$ that satisfy (3.2). Thus equation (3.4) may be rewritten as

$$P_{FR-ASYN} = \sum_{U_0=0}^{M-1} \sum_{U_1=0}^{M-1-U_0} \dots \sum_{U_{L-1}=0}^{M-1-\sum_{i=0}^{L-2} U_i} P(FR | U_0, U_1, \dots, U_{L-1}) \cdot P(U_0, U_1, \dots, U_{L-1}) \cdot \delta_{M-1, \sum_{i=0}^{L-1} U_i} \quad (3.5)$$

and in which the Kronecker delta satisfies

$$\delta_{M-1, \sum_{i=0}^{L-1} u_i} = \begin{cases} 1, & \sum_{i=0}^{L-1} u_i = M-1 \\ 0, & \text{otherwise} \end{cases} \quad (3.6)$$

In fig3.1 a sequence of frames is shown for one of the u_i interfering users with delay i . Let the random variable C_i denote the number of users from the group of u_i who have already occupied chips in the false row in the $L-i$ slots preceding the desired frame. C_i takes on values $C_i \in \{0, 1, \dots, u_i\}$. The probability $P(\text{FR} | u_0, \dots, u_{L-1})$ of a false row conditioned on $(u_0, \dots, u_{L-1})$ also has to be conditioned on the L -tuple $(C_0, C_1, \dots, C_{L-1})$, thus

$$\begin{aligned} P(\text{FR} | u_0, u_1, \dots, u_{L-1}) = \\ \sum_{C_1=0}^{u_1} \sum_{C_2=0}^{u_2} \dots \sum_{C_{L-1}=0}^{u_{L-1}} P(\text{FR} | u_0, u_1, \dots, u_{L-1}, C_1, C_2, \dots, C_{L-1}) \cdot \\ P(C_1, C_2, \dots, C_{L-1} | u_0, u_1, \dots, u_{L-1}) \end{aligned} \quad (3.7)$$

Notice that $C_0 = 0$ and hence it has been omitted in the summation. Taking into account that C_i depends only on u_i for $i=1, 2, \dots, L-1$ it follows from (3.7) that

$$\begin{aligned} P(\text{FR} | u_0, u_1, \dots, u_{L-1}) = \\ \sum_{C_1=0}^{u_1} \sum_{C_2=0}^{u_2} \dots \sum_{C_{L-1}=0}^{u_{L-1}} P(\text{FR} | u_0, u_1, \dots, u_{L-1}, C_1, C_2, \dots, C_{L-1}) \cdot \\ \prod_{i=1}^{L-1} P(C_i | u_i) \end{aligned} \quad (3.8)$$

An expression for $P(C_i | u_i)$ can simply be obtained by noticing that any one of the users occupies one of the $L-i$

chips preceding the desired frame with probability $(L-i)/Q$
 and that each group of users can be selected in
 $\binom{u_i}{c_i}$ ways, thus

$$P(c_i|u_i) = \binom{u_i}{c_i} \left(\frac{L-i}{Q}\right)^{c_i} \left(1 - \frac{L-i}{Q}\right)^{u_i - c_i}, \quad i=1,2,\dots,L-1 \quad (3.9)$$

On combining equations (3.3), (3.5), (3.8) and (3.9) the following result is obtained

$$P_{FR-ASYN} = \sum_{u_0=0}^{M-1} \sum_{u_1=0}^{M-1-u_0} \dots \sum_{u_{L-1}=0}^{M-1-\sum_{i=0}^{L-2} u_i}$$

$$\sum_{c_1=0}^{u_1} \sum_{c_2=0}^{u_2} \dots \sum_{c_{L-1}=0}^{u_{L-1}}$$

$$P(FR | u_0, u_1, \dots, u_{L-1}, c_1, c_2, \dots, c_{L-1}) \cdot$$

$$\left\{ \prod_{i=1}^{L-1} \binom{u_i}{c_i} \left(\frac{L-i}{Q} \right)^{c_i} \left(1 - \frac{L-i}{Q} \right)^{u_i - c_i} \right\} \cdot$$

$$\frac{(M-1)!}{u_0! u_1! \dots u_{L-1}!} \cdot \frac{1}{L^{M-1}} \cdot$$

$$\sum_{M-1, \sum_{i=0}^{L-1} u_i}$$

(3.10)

In order to derive the necessary expression for $P(\text{FR} | u_0, \dots, u_{L-1}, c_1, \dots, c_{L-1})$ let I_n denote the number of interferers that occupy the n^{th} chip of the false row in question. If this is done then it follows that

$$\begin{aligned}
 P(\text{FR} | u_0, u_1, \dots, u_{L-1}, c_1, c_2, \dots, c_{L-1}) = \\
 P(I_1 \neq 0 | u_0, u_1, \dots, u_{L-1}, c_1, c_2, \dots, c_{L-1}) \cdot \\
 P(I_2 \neq 0 | I_1 \neq 0, u_0, u_1, \dots, u_{L-1}, c_1, c_2, \dots, c_{L-1}) \\
 \dots \dots \dots (3.11) \\
 P(I_L \neq 0 | I_1 \neq 0, I_2 \neq 0, \dots, I_{L-1} \neq 0, u_0, \dots, u_{L-1}, c_1, \dots, c_{L-1})
 \end{aligned}$$

or

$$\begin{aligned}
 P(\text{FR} | u_0, u_1, \dots, u_{L-1}, c_1, c_2, \dots, c_{L-1}) = \\
 \prod_{n=1}^L P(I_n \neq 0 | I_1 \neq 0, \dots, I_{n-1} \neq 0, u_0, \dots, u_{L-1}, c_1, \dots, c_{L-1}) \quad (3.12)
 \end{aligned}$$

The factors of (3.12) can be expressed conveniently if the following notation is used

$q_{i,n}$: number of interferers with delay i who have access to (ie. might occupy) chip n of the false row.

$v_{i,n}$: number of interferers with delay i who actually occupy chip n of the false row.

From a consideration of the asynchronous operation of the system it can be seen that the number of interfering users with access to (ie. which might occupy) chip n is given by

$$q_{i,n} = \begin{cases} u_i - \sum_{k=i+1}^{n-1} v_{i,k} & , i+1 < n \\ u_i & , i+1 = n \\ u_i - c_i - \sum_{k=1}^{n-1} v_{i,k} & , i+1 > n \end{cases} \quad (3.13)$$

each with $Q-(n-i-1)_L$ choices, where the notation $(\cdot)_L$ denotes that the number $(n-i-1)$ is expressed modulo- L .

Consider the first factor in the product of (3.12), ie.

$$P(I_1 \neq 0 | c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1})$$

Since each of the $q_{i,1}$ interferers with access to chip $n=1$ has $Q-(L-i)_L$ choices, the number of interferers from this group that actually occupy chip 1 takes on the value $v_{i,1}$ with probability

$$P(v_{i,1} | c_i, u_i) = \binom{q_{i,1}}{v_{i,1}} \cdot \left(\frac{1}{Q-(L-i)_L} \right)^{v_{i,1}} \cdot \left(1 - \frac{1}{Q-(L-i)_L} \right)^{q_{i,1}-v_{i,1}} \quad (3.14)$$

where $i=0,1,2,\dots,L-1$. It follows that

$$P(I, \neq 0 | c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}) =$$

$$= \sum_{v_{0,1}=0}^{u_0} \sum_{v_{1,1}=0}^{u_1-c_1} \dots \dots \sum_{v_{L-1,1}=0}^{u_{L-1}-c_{L-1}}$$

$$P(I, \neq 0 | v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$P(v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1})$$

The value of $v_{i,1}$ depends on C_i and u_i only, so

$$\begin{aligned}
 P(I_1 \neq 0 | C_1, C_2, \dots, C_{L-1}, u_0, u_1, \dots, u_{L-1}) &= \\
 &= \sum_{v_{0,1}=0}^{u_0} \sum_{v_{1,1}=0}^{u_1 - C_1} \dots \sum_{v_{L-1,1}=0}^{u_{L-1} - C_{L-1}} \\
 &\quad P(I_1 \neq 0 | v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, C_1, C_2, \dots, C_{L-1}, u_0, u_1, \dots, u_{L-1}) \cdot \\
 &\quad \left(\prod_{i=0}^{L-1} P(v_{i,1} | C_i, u_i) \right)
 \end{aligned}$$

or,

$$\begin{aligned}
 P(I_1 \neq 0 | C_1, C_2, \dots, C_{L-1}, u_0, u_1, \dots, u_{L-1}) &= \\
 &\sum_{v_{0,1}=0}^{u_0} \sum_{v_{1,1}=0}^{u_1 - C_1} \dots \sum_{v_{L-1,1}=0}^{u_{L-1} - C_{L-1}} \left(1 - \delta_{0, \sum_{i=0}^{L-1} v_{i,1}} \right) \cdot \\
 &\prod_{i=0}^{L-1} \binom{q_{i,1}}{v_{i,1}} \left(\frac{1}{Q - (L-i)_L} \right)^{v_{i,1}} \left(1 - \frac{1}{Q - (L-i)_L} \right)^{q_{i,1} - v_{i,1}} \quad (3.15)
 \end{aligned}$$

because chip 1 is occupied only if $\sum_{i=0}^{L-1} v_{i,1} > 0$. Now consider

$$P(I_2 \neq 0 | I_1 \neq 0, C_1, C_2, \dots, C_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$P(I_1 \neq 0 | C_1, C_2, \dots, C_{L-1}, u_0, u_1, \dots, u_{L-1})$$

A total of $q_{i,2}$ interferers have access to chip 2, each with $Q - (1-i)_L$ choices. The variable $q_{i,2}$ depends on $q_{i,1}$. Similarly, the number of users that actually occupy chip 2 depends on $v_{i,1}, i=0,1,2,\dots,L-1$. Using the above, this expression may be

written

$$P(I_2 \neq 0 | I_1 \neq 0, c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$P(I_1 \neq 0 | c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}) =$$

$$= \sum_{v_{0,1}=0}^{q_{0,1}} \sum_{v_{1,1}=0}^{q_{1,1}} \dots \sum_{v_{L-1,1}=0}^{q_{L-1,1}}$$

$$P(I_2 \neq 0 | v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, I_1 \neq 0, c_1, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$P(I_1 \neq 0 | v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$P(v_{0,1}, v_{1,1}, \dots, v_{L-1,1} | c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1})$$

or

$$P(I_2 \neq 0 | I_1 \neq 0, c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$P(I_1 \neq 0 | c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}) =$$

$$\sum_{v_{0,1}=0}^{q_{0,1}} \sum_{v_{1,1}=0}^{q_{1,1}} \dots \sum_{v_{L-1,1}=0}^{q_{L-1,1}} \sum_{v_{0,2}=0}^{q_{0,2}} \sum_{v_{1,2}=0}^{q_{1,2}} \dots \sum_{v_{L-1,2}=0}^{q_{L-1,2}}$$

$$P(I_2 \neq 0 | v_{0,2}, v_{1,2}, \dots, v_{L-1,2}, v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, I_1 \neq 0, c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$P(v_{0,2}, v_{1,2}, \dots, v_{L-1,2} | v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, I_1 \neq 0, c_1, \dots, c_{L-1}, u_0, \dots, u_{L-1}).$$

$$P(I_1 \neq 0 | v_{0,1}, v_{1,1}, \dots, v_{L-1,L}, c_1, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$P(v_{0,1}, v_{1,1}, \dots, v_{L-1,L} | c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}) \quad (3.16)$$

The first factor may be expressed as

$$\begin{aligned}
 P(I_2 \neq 0 \mid v_{0,2}, v_{1,2}, \dots, v_{L-1,2}, v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, I_1 \neq 0, \\
 c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}) = \\
 = 1 - \delta_0, \sum_{i=0}^{L-1} v_{i,2}
 \end{aligned} \tag{3.17 a}$$

where, of course, $v_{i,2}$ depends on $v_{i,1}$, $I_1 \neq 0$, and u_i .

The second factor may be rewritten as

$$\begin{aligned}
 P(v_{0,2}, v_{1,2}, \dots, v_{L-1,2} \mid v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, I_1 \neq 0, c_1, c_2, \dots, c_{L-1}, \\
 u_0, u_1, \dots, u_{L-1}) = \\
 = \prod_{i=0}^{L-1} P(v_{i,2} \mid v_{i,1}, I_1 \neq 0, c_i, u_i) = \\
 = \prod_{i=0}^{L-1} \left(q_{i,2} \right) \left(\frac{1}{Q - (1-i)_L} \right)^{v_{i,2}} \cdot \left(1 - \frac{1}{Q - (1-i)_L} \right)^{q_{i,2} - v_{i,2}}
 \end{aligned} \tag{3.17 b}$$

where the expression for $q_{i,2}$ includes the dependence on $v_{i,1}$, c_i and u_i (see equation (3.13)).

The third factor has already been found to be

$$\begin{aligned}
 P(I_1 \neq 0 \mid v_{0,1}, v_{1,1}, \dots, v_{L-1,1}, c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}) = \\
 = 1 - \delta_0, \sum_{i=0}^{L-1} v_{i,1}
 \end{aligned} \tag{3.17 c}$$

From (3.16) and (3.17 a-b-c) it follows that

$$P(I_2 \neq 0 | I_1 \neq 0, c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$P(I_1 \neq 0 | c_1, c_2, \dots, c_{L-1}, u_0, u_1, \dots, u_{L-1}) =$$

$$\sum_{v_{0,1}=0}^{q_{0,1}} \sum_{v_{1,1}=0}^{q_{1,1}} \dots \sum_{v_{L-1,1}=0}^{q_{L-1,1}} \sum_{v_{0,2}=0}^{q_{0,2}} \sum_{v_{1,2}=0}^{q_{1,2}} \dots \sum_{v_{L-1,2}=0}^{q_{L-1,2}}$$

$$\cdot \left(1 - \delta_{0, \sum_{i=0}^{L-1} v_{i,2}} \right).$$

$$\prod_{i=0}^{L-1} \binom{q_{i,2}}{v_{i,2}} \left(\frac{1}{Q - (1-i)_L} \right)^{v_{i,2}} \cdot \left(1 - \frac{1}{Q - (1-i)_L} \right)^{q_{i,2} - v_{i,2}}.$$

$$\cdot \left(1 - \delta_{0, \sum_{i=0}^{L-1} v_{i,1}} \right).$$

$$\prod_{i=0}^{L-1} \binom{q_{i,1}}{v_{i,1}} \left(\frac{1}{Q - (L-i)_L} \right)^{v_{i,1}} \cdot \left(1 - \frac{1}{Q - (L-i)_L} \right)^{q_{i,1} - v_{i,1}} \quad (3.18)$$

By rearranging the order of the factors it can be seen that

$$P(I_2 \neq 0 | I_1 \neq 0, C_1, C_2, \dots, C_{L-1}, u_0, u_1, \dots, u_{L-1}).$$

$$\cdot P(I_1 \neq 0 | C_1, C_2, \dots, C_{L-1}, u_0, u_1, \dots, u_{L-1}) =$$

$$= \sum_{V_{0,1}=0}^{q_{0,1}} \sum_{V_{1,1}=0}^{q_{1,1}} \dots \sum_{V_{L-1,1}=0}^{q_{L-1,1}} \sum_{V_{0,2}=0}^{q_{0,2}} \sum_{V_{1,2}=0}^{q_{1,2}} \dots \sum_{V_{L-1,2}=0}^{q_{L-1,2}}$$

$$\left(\prod_{i=0}^{L-1} \binom{q_{i,2}}{V_{i,2}} \left(\frac{1}{Q - (1-i)_L} \right)^{V_{i,2}} \cdot \left(1 - \frac{1}{Q - (1-i)_L} \right)^{q_{i,2} - V_{i,2}} \right).$$

$$\left(\prod_{i=0}^{L-1} \binom{q_{i,1}}{V_{i,1}} \left(\frac{1}{Q - (L-i)_L} \right)^{V_{i,1}} \cdot \left(1 - \frac{1}{Q - (L-i)_L} \right)^{q_{i,1} - V_{i,1}} \right).$$

$$\cdot \left(1 - \delta_{0, \sum_{i=0}^{L-1} V_{i,1}} \right) \cdot \left(1 - \delta_{0, \sum_{i=0}^{L-1} V_{i,2}} \right) \quad (3.19)$$

Following the same procedure and using equation (3.12) results in

$$\begin{aligned}
 & P(FR | u_0, u_1, \dots, u_{L-1}, c_1, c_2, \dots, c_{L-1}) = \\
 & = \sum_{v_{0,1}=0}^{q_{0,1}} \dots \sum_{v_{L-1,1}=0}^{q_{L-1,1}} \sum_{v_{0,2}=0}^{q_{0,2}} \dots \sum_{v_{L-1,2}=0}^{q_{L-1,2}} \sum_{v_{0,L}=0}^{q_{0,L}} \dots \sum_{v_{L-1,L}=0}^{q_{L-1,L}} \\
 & \prod_{\eta=1}^L \left\{ \prod_{i=0}^{L-1} \binom{q_{i,n}}{v_{i,n}} \left(\frac{1}{Q - (n-i-1)_L} \right)^{v_{i,n}} \cdot \left(1 - \frac{1}{Q - (n-i-1)_L} \right)^{q_{i,n} - v_{i,n}} \right\} \\
 & \prod_{\eta=1}^L \left(1 - \delta_{0, \sum_{i=0}^{L-1} v_{i,n}} \right) \tag{3.20}
 \end{aligned}$$

The final expression for the probability of false full row which follows from (3.20) and (3.10) is

$$\begin{aligned}
 P_{\text{FR-ASYNC}} &= \sum_{u_0=0}^{M-1} \sum_{u_1=0}^{M-1-u_0} \cdots \sum_{u_{L-1}=0}^{M-1-\sum_{i=0}^{L-2} u_i} \\
 &\quad \sum_{c_1=0}^{u_1} \sum_{c_2=0}^{u_2} \cdots \sum_{c_{L-1}=0}^{u_{L-1}} \\
 &\quad \sum_{v_{0,1}=0}^{q_{0,1}} \cdots \sum_{v_{L-1,1}=0}^{q_{L-1,1}} \sum_{v_{0,2}=0}^{q_{0,2}} \cdots \sum_{v_{L-1,2}=0}^{q_{L-1,2}} \sum_{v_{0,L}=0}^{q_{0,L}} \cdots \sum_{v_{L-1,L}=0}^{q_{L-1,L}} \\
 &\quad \prod_{n=1}^L \left\{ \prod_{i=0}^{L-1} \binom{q_{i,n}}{v_{i,n}} \left(\frac{1}{Q-(n-i-1)_L} \right)^{v_{i,n}} \cdot \left(1 - \frac{1}{Q-(n-i-1)_L} \right)^{q_{i,n}-v_{i,n}} \right\} \\
 &\quad \prod_{n=1}^L \left(1 - \delta_{0, \sum_{i=0}^{L-1} v_{i,n}} \right) \\
 &\quad \prod_{i=1}^{L-1} \binom{u_i}{c_i} \left(\frac{L-i}{Q} \right)^{c_i} \cdot \left(1 - \frac{L-i}{Q} \right)^{u_i-c_i} \\
 &\quad \frac{(m-1)!}{u_0! u_1! \cdots u_{L-1}!} \cdot \frac{1}{L^{M-1}} \\
 &\quad \delta_{M-1, \sum_{i=0}^{L-1} u_i}
 \end{aligned}
 \tag{3.21}$$

The word-error probability is calculated from (3.21) and the following equation (3.22)

$$P_W = \sum_{i=2}^M \frac{i-1}{i} \binom{2^K-1}{i-1} \cdot P_{FR-ASYN}^{i-1} \cdot (1 - P_{FR-ASYN})^{2^K-i} \quad (3.22)$$

Equation (3.22) is the same as (2.18). In Appendix 2 it is shown that, as expected, when the system operates in a synchronous mode equation (3.21) reduces to equation (2.14) of Chapter 2..

3.3 UPPER BOUNDS FOR THE ASYNCHRONOUS SYSTEM

For realistic values of M and L , evaluation of equation (3.21) requires considerable computation time even when a powerful computer is used. Thus the derivation of a simply computable satisfactory upper bound, or bounds, is desirable.

Einarsson showed [11] that the probability with which a given interferer causes two coincidences in a word of length L is

$$P(N=2) = \frac{i}{Q} \cdot \frac{L-i}{Q} \quad (3.23)$$

where the user in question has delay i with respect to the desired user. By differentiating (3.23) the delay that maximizes $P(N=2)$ is found to be $i=L/2$. It follows that the worst case is for all $M-1$ interferers to have delay $i=L/2$. An upper bound for this case also serves as an upper bound for the system performance in general. In fig 3.3 the frame sequences of $M-1$ interferers with common delay $i=L/2$ are shown together with that of the desired user. It is possible for each interferer to make one contribution in each half of the false full row. The right half of the frame of the desired user may be regarded as a synchronous matrix with Q channels and $L_{\text{Right}} = L/2$ time-slots. Each of the $M-1$

interferers has a maximum of Q choices and according to Einarsson's Pw3 upper bound [11] the probability $P_{FR-Right}$ of forming a false full row of length $L_{Right} = L/2$ is

$$P_{FR-Right} < \prod_{k=0}^{L_{Right}-1} \left[1 - \left(1 - \frac{1}{Q-k} \right)^{M-1-k} \right] \quad (3.24)$$

Similarly, the left half of the same frame may be regarded as a synchronous matrix with Q channels and $L_{Left} = L/2$ time-slots. However, a group of c out of $M-1$ interferers may have already occupied chips of the row in question outside the desired frame, thus leaving $M-1-c$ interferers with a maximum of $Q-L/2$ choices. It follows that the probability of forming a false full row of length $L_{Left} = L/2$ is

$$P_{FR-Left} < \prod_{k=0}^{L_{Left}-1} \left[1 - \left(1 - \frac{1}{Q-L_{Left}-k} \right)^{M-1-c-k} \right]$$

and on setting $c=0$

$$P_{FR-Left} < \prod_{k=0}^{L_{Left}-1} \left[1 - \left(1 - \frac{1}{Q-L_{Left}-k} \right)^{M-1-k} \right] \quad (3.25)$$

The probability of forming a false full row of length L is

then upperbounded by

$$P_{FR-ASYN} < P_{FR-Left} \cdot P_{FR-Right}$$

which from equations (3.24) and (3.25) is seen to be

$$P_{FR-ASYN} < \prod_{k=0}^{L/2-1} \left[1 - \left(1 - \frac{1}{Q-L/2-k} \right)^{M-1-k} \right] \cdot$$

$$\prod_{k=0}^{L/2-1} \left[1 - \left(1 - \frac{1}{Q-k} \right)^{M-1-k} \right]$$

A first upper bound on the word-error probability is given

by UB1 =

$$(Q-1) \cdot \prod_{k=0}^{L/2-1} \left[1 - \left(1 - \frac{1}{Q-L/2-k} \right)^{M-k-1} \right] \cdot \prod_{k=0}^{L/2-1} \left[1 - \left(1 - \frac{1}{Q-k} \right)^{M-1-k} \right] \quad (3.26)$$

It is worth noting that Einarsson [11] ignored that the number of choices in the left half is $Q-L/2$ and he arrived at the upper bound

$$P_{W4} = (Q-1) \cdot \left\{ \prod_{k=0}^{L/2-1} \left[1 - \left(1 - \frac{1}{Q-k} \right)^{M-1-k} \right] \right\}^2 \quad (3.27)$$

Thus although equation (3.27) gives smaller values for the word-error probability upper bound, it cannot be justified. The upper bound UB1 can be tightened by using equation (2.14) of Chapter 2 to calculate $P_{FR-Left}$ for $Q-L/2$ channels and $M-1$ interferers and $P_{FR-Right}$ for Q channels and $M-1$ interferers. If this is done then the probability of a false full row is upperbounded by the product

$P_{FR-Left} \cdot P_{FR-Right}$, that is

$$P_{FR-ASYN} < P_{FR-Left} \cdot P_{FR-Right} \quad (3.28)$$

and the upper bound of the word-error probability is

$$UB2 = \sum_{j=2}^M \frac{j-1}{j} \binom{2^k-1}{j-1} \cdot P_{FR-ASYN}^{j-1} \cdot (1 - P_{FR-ASYN})^{2^k-j} \quad (3.29)$$

where P_{FR} is upperbounded by (3.28). A more sophisticated upper bound can be obtained from equation (3.21). In the worst case, as described above, all $M-1$ interferers have delay $i=L/2$. It follows that

$$u_i = \begin{cases} M-1 & , i = L/2 \\ 0 & , \text{otherwise} \end{cases} \quad (3.30.a)$$

$$\text{and} \quad c_i = \begin{cases} \in [0, u_{L/2}] & , i = L/2 \\ 0 & , \text{otherwise} \end{cases} \quad (3.30.b)$$

Consequently, equation (3.21) is reduced to

$$P_{FR-ASYM} < \sum_{C_{L/2}=0}^{M-1} \sum_{V_{L/2,1}=0}^{q_{L/2,1}} \sum_{V_{L/2,2}=0}^{q_{L/2,2}} \dots \sum_{V_{L/2,L}=0}^{q_{L/2,L}}$$

$$\prod_{n=1}^L \left(\frac{q_{L/2,n}}{v_{L/2,n}} \right) \left(\frac{1}{Q - (n - \frac{L}{2} - 1)_L} \right)^{v_{L/2,n}} \cdot \left(1 - \frac{1}{Q - (n - \frac{L}{2} - 1)_L} \right)^{q_{L/2,n} - v_{L/2,n}}$$

$$\prod_{n=1}^L \left(1 - \delta_{0, v_{L/2,n}} \right).$$

$$\binom{M-1}{C_{L/2}} \cdot \left(\frac{L}{2Q} \right)^{C_{L/2}} \cdot \left(1 - \frac{L}{2Q} \right)^{u_{L/2} - C_{L/2}}$$

By neglecting the subscript $L/2$, the result can be expressed more simply as

$$P_{FR-ASYN} < \sum_{C=0}^{M-1} \sum_{V_1=1}^{q_1} \sum_{V_2=1}^{q_2} \dots \sum_{V_L=1}^{q_L} \prod_{n=1}^L \binom{q_n}{V_n} \left(\frac{1}{Q-(n-L/2-1)_L} \right)^{V_n} \left(1 - \frac{1}{Q-(n-L/2-1)_L} \right)^{q_n-V_n} \binom{M-1}{C} \left(\frac{L}{2Q} \right)^C \left(1 - \frac{L}{2Q} \right)^{M-1-C} \quad (3.31)$$

where

$$q_n = \begin{cases} M-1-C - \sum_{k=1}^{n-1} V_k & , n < L/2+1 \\ M-1 & , n = L/2+1 \\ M-1 - \sum_{k=L/2+1}^{n-1} V_k & , n > L/2+1 \end{cases} \quad (3.32)$$

The upper bound UB3 on the word-error rate is then given by

$$UB3 = \sum_{j=2}^M \frac{j-1}{j} \binom{2^K-1}{j-1} \cdot P_{FR-ASYN}^{j-1} \cdot (1 - P_{FR-ASYN})^{2^K-j} \quad (3.33)$$

It should be noted that in the analysis above L has been assumed to be even. In the case when L is odd an expression for the upper bound on the word-error probability can be obtained by using $(L-1)/2$ instead of $L/2$ in the above derived expression.

3.4 RESULTS

The asynchronous operation of Einarsson's system has been simulated and some of the results obtained are shown in figs.3.4 and 3.5. In fig.3.4 the three new upper bounds are plotted for the case in which $Q=17$ and $L=8$. Also shown in the figure is the simulated performance of the system when operating in asynchronous mode and for the purpose of providing additional information for comparison the exact (calculated) performance of the system operating in synchronous mode is shown. In figs.3.5a-b the cases of $Q=7, L=4$ and $Q=11, L=4$ respectively are considered. In these two figures the UB3 upper bound is plotted together with the exact error-probability for the synchronous case and the simulation results for the asynchronous case. From fig.3.4 it is clear that the bounds UB1, UB2 and UB3 are in fact bounds of increasing tightness, and that the bounds UB2 and UB3 are tighter than the false Pw4 bound proposed by Einarsson. From Figs.3.4 and 3.5 it is clear that the UB3 bound, which as has been pointed out, is relatively easy to compute, is very tight. Figs.3.4 and 3.5 also show that the results relating to synchronous performance may provide a lower bound to the performance of the system when it is operating in asynchronous mode. This intuitive conjecture can be supported by the following simple argument:

The number, N , of contributions that a given interferer makes in the examined false row may be regarded as noise that produces insertions in N chips of the false row. It is not difficult to show (see Appendix 3) that the average value of N for both synchronous and asynchronous operation is

$$E(N)_{\text{SYNC}} = E(N)_{\text{ASYNC}} = L/Q \quad (3.34 a)$$

and that the variances are given by

$$\sigma^2(N)_{\text{SYNC}} = L/Q - (L/Q)^2 \quad (3.34 b)$$

$$\sigma^2(N)_{\text{ASYNC}} = L/Q - (L/Q)^2 + 2 \cdot i \cdot (L-i) \cdot (1-1/Q) / Q^2 \quad (3.34 c)$$

where i is the delay of the given interferer with respect to the desired user. From equations (3.34 a-b-c) it is readily seen that an interferer with non-zero delay corresponds to a noise process with equal mean but greater variance than that of a synchronous interferer and thus causes a higher error probability. Notice that for a large enough value of Q $\sigma^2(N)_{\text{SYNC}} = \sigma^2(N)_{\text{ASYNC}}$ and the two systems are equivalent.

Some insight into the relative performance of the Einarsson system (when it is operating in either the synchronous or asynchronous mode), and the performance of the Random Addressing scheme proposed by Viterbi can be gained by considering the operation of the various systems for increasing Q , with L held fixed.

In Viterbi's Random Addressing (RA) scheme each chip of the false row is available to all $M-1$ interferers, each of which has Q choices. As Q increases with respect to L the probability that any given user contributes more than one entry to a false row decreases until for some value of Q it becomes negligible. When this occurs the Random Addressing scheme of Viterbi is equivalent to Einarsson's scheme with the same Q, L and M . An analytical proof of this is provided in Appendix 4. Supporting results are shown in fig.3.6.

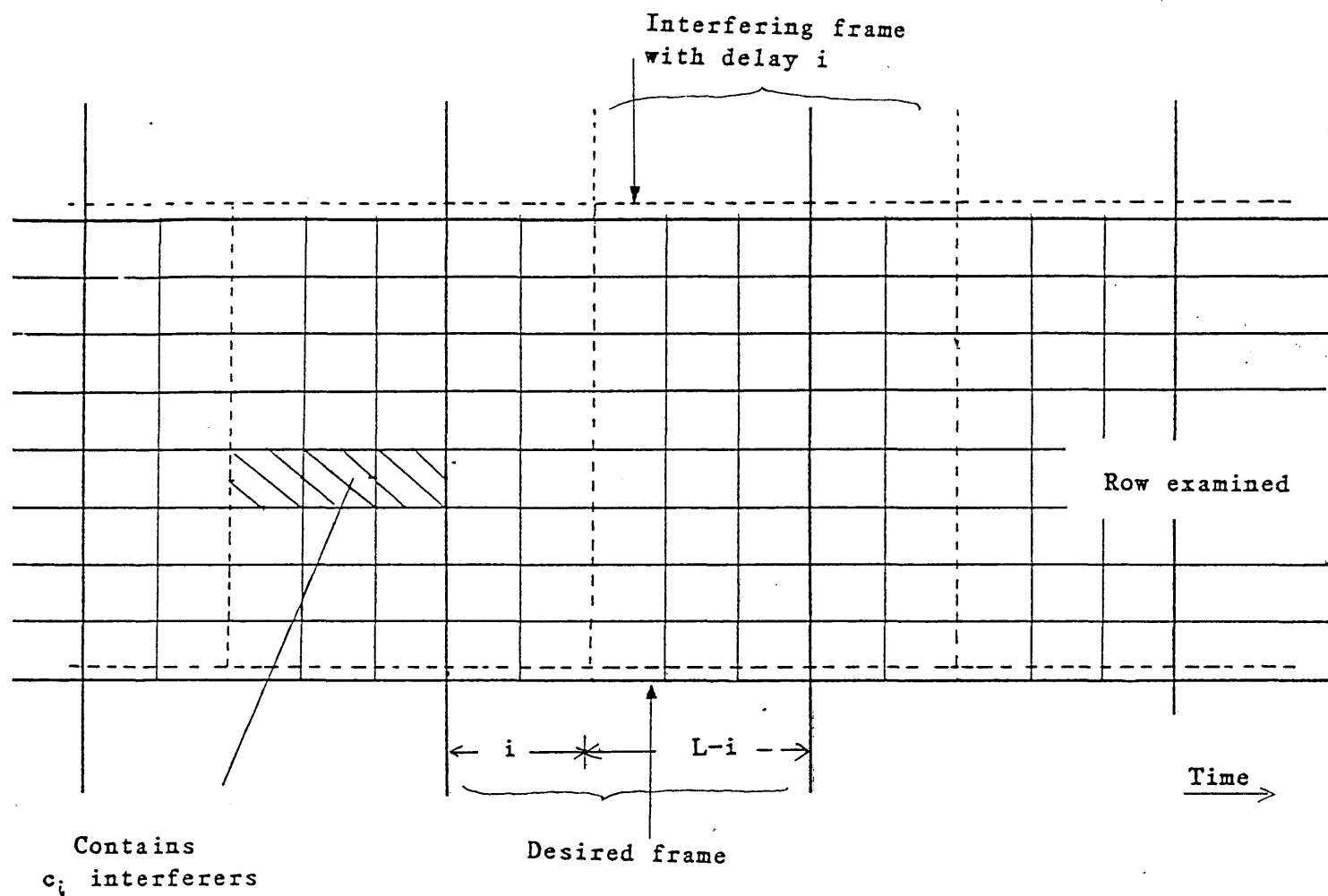


Fig.3.1 Frame of the desired user corrupted by two consecutive frames with delay i

U_0 Q $V_{0,1}$	$U_0 - V_{0,1}$ $Q-1$ $V_{0,2}$	$U_0 - V_{0,1} - V_{0,2}$ $Q-2$ $V_{0,3}$					$U_0 - \sum_{k=1}^{L-1} V_{0,k}$ $Q-(L-1)$ $V_{0,L}$
$U_1 - C_1$ $Q-(L-1)$ $V_{1,L}$	U_1 Q $V_{1,2}$	$U_1 - V_{1,2}$ $Q-1$ $V_{1,3}$	$U_1 - V_{1,2} - V_{1,3}$ $Q-2$ $V_{1,4}$				$U_1 - \sum_{k=2}^{L-1} V_{1,k}$ $Q-(L-2)$ $V_{1,L}$
$U_2 - C_2$ $Q-(L-2)$ $V_{2,1}$	$U_2 - C_2 - V_{2,1}$ $Q-(L-1)$ $V_{2,2}$	U_2 Q $V_{2,3}$	$U_2 - V_{2,3}$ $Q-1$ $V_{2,4}$				$U_2 - \sum_{k=3}^{L-1} V_{2,k}$ $Q-(L-3)$ $V_{2,L}$
$U_3 - C_3$ $Q-(L-3)$ $V_{3,1}$	$U_3 - C_3 - V_{3,1}$ $Q-(L-2)$ $V_{3,2}$	$U_3 - C_3 - V_{3,1} - V_{3,2}$ $Q-(L-1)$ $V_{3,3}$	U_3 Q $V_{3,4}$	$U_3 - V_{3,4}$ $Q-1$ $V_{3,5}$			$U_3 - \sum_{k=4}^{L-1} V_{3,k}$ $Q-(L-4)$ $V_{3,L}$
$U_4 - C_4$ $Q-(L-4)$ $V_{4,1}$							
$U_{L-1} - C_{L-1}$ $Q-1$ $V_{L-1,1}$	$U_{L-1} - C_{L-1} - V_{L-1,1}$ $Q-2$ $V_{L-1,2}$					$U_{L-1} - C_{L-1} - \sum_{k=1}^{L-2} V_{L-1,k}$ $Q-(L-1)$ $V_{L-1,L-1}$	U_{L-1} Q $V_{L-1,L}$

Time-slot ℓ

Fig.3.2 Square (i, ℓ) shows

1. The number of interferers with delay i who have access to chip ℓ of the false row in question
2. The number of choices available to them and
3. The number of interferers with delay i who actually occupy chip ℓ of the false row in question.

Interfering frame
with delay $i=L/2$

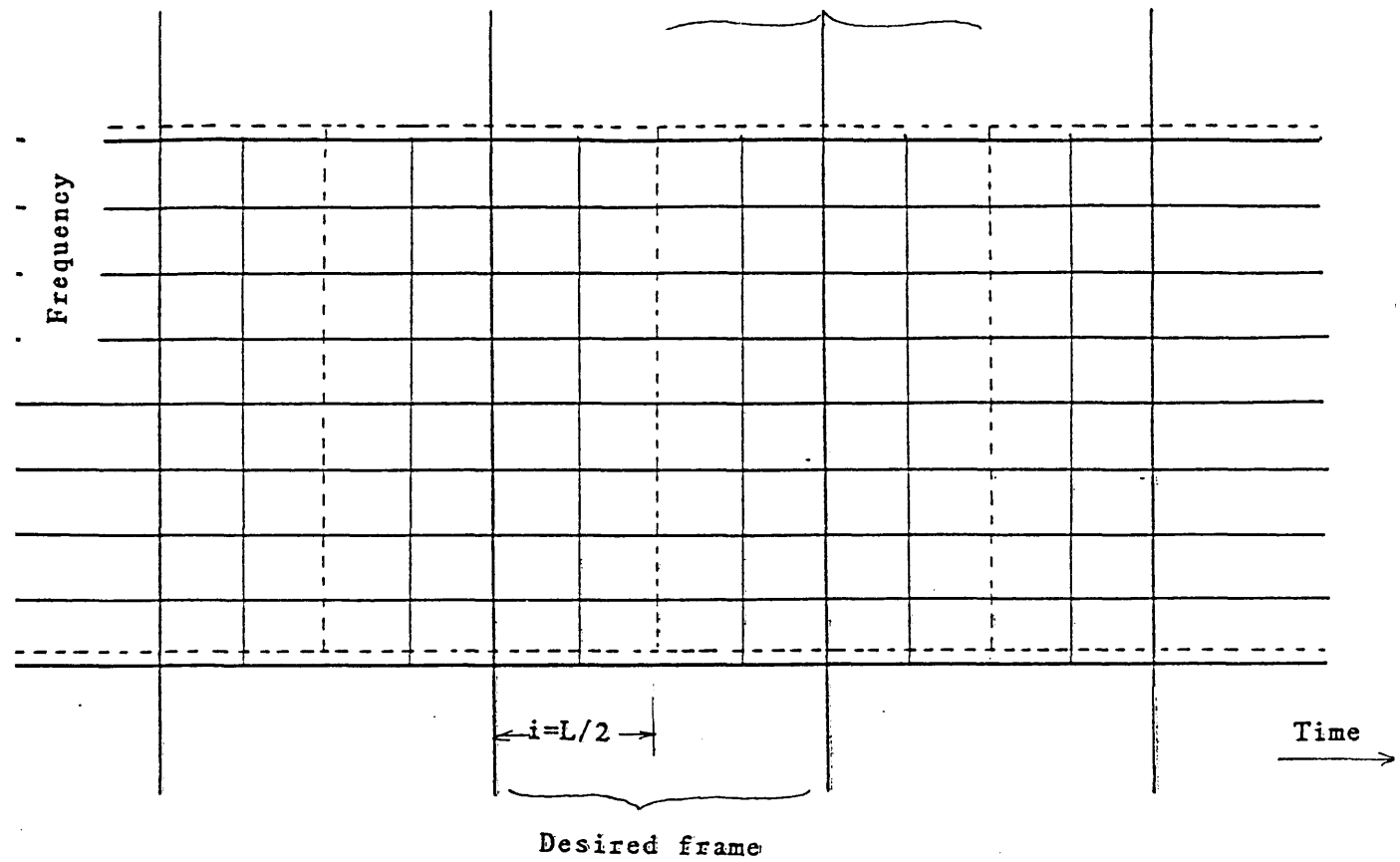


Fig.3.3 Worst case interference

- O PW4 UPPER BOUND
 Δ UB1 UPPER BOUND
 + UB2 UPPER BOUND
 X UB3 UPPER BOUND
 ◇ SYNCHRONOUS OPERATION, THEORETICAL RESULTS
 ▲ ASYNCHRONOUS OPERATION, SIMULATION RESULTS

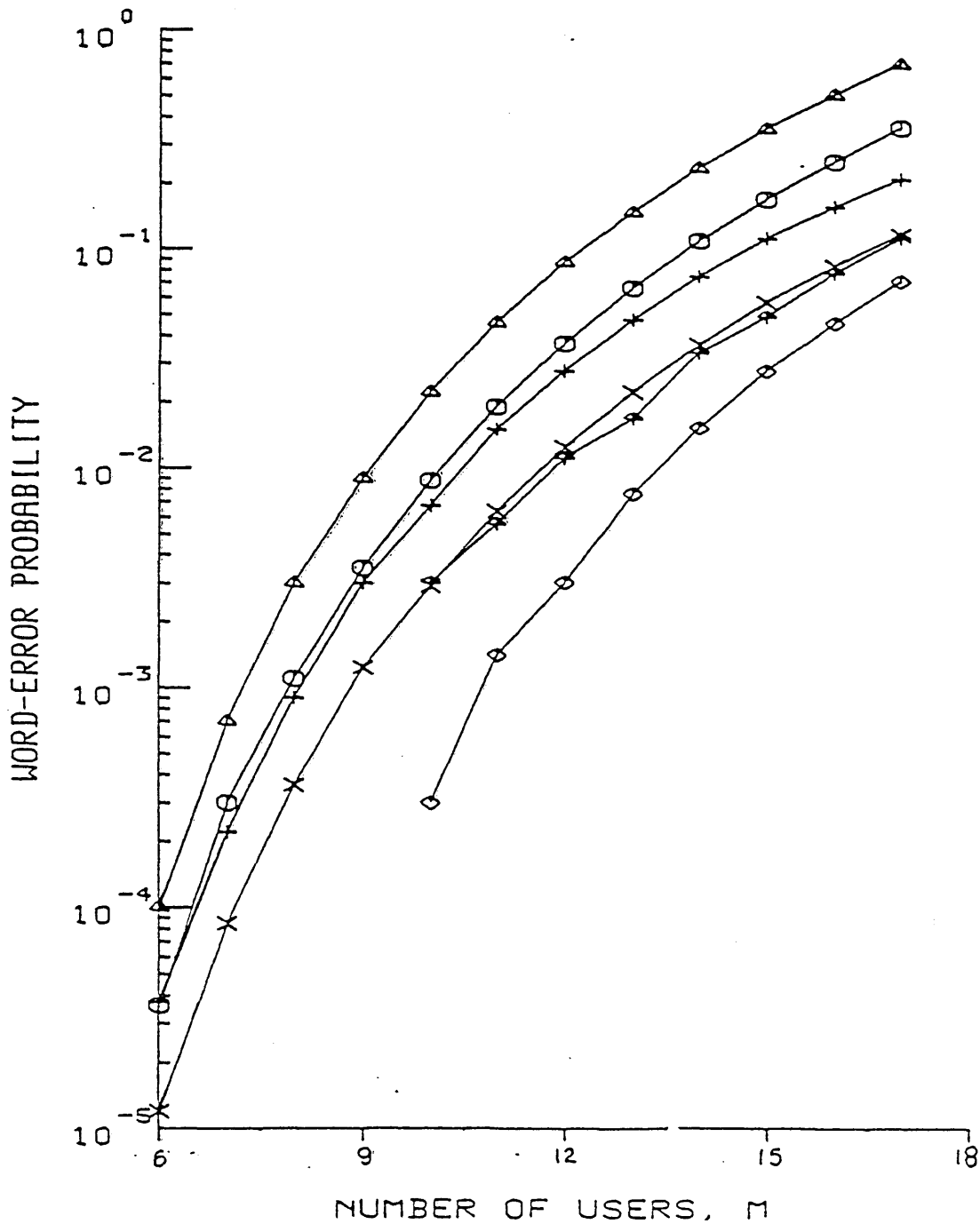


Fig.3.4 Word-error probability .vs. Number of users

EINARSSON'S SCHEME

$Q=7, L=4, k=2$

- SYNCHRONOUS OPERATION, THEORETICAL RESULTS
- △ ASYNCHRONOUS OPERATION, SIMULATION RESULTS
- + ASYNCHRONOUS OPERATION, UB3 UPPER BOUND

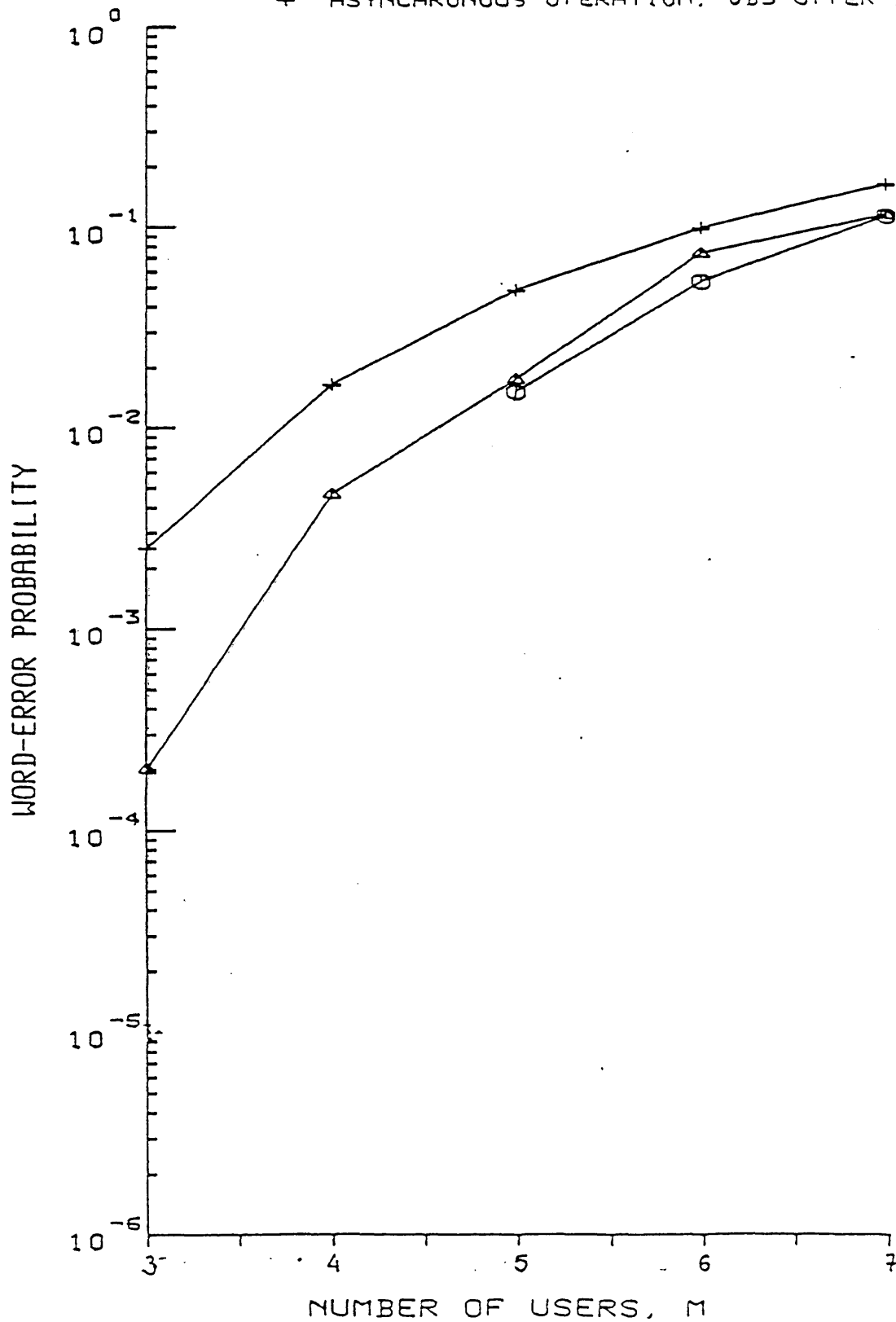


Fig.3.5a Word-error probability .vs. Number of users

EINARSSON'S SCHEME

$Q=11, L=4, K=3$

- SYNCHRONOUS OPERATION, THEORETICAL RESULTS
- △ ASYNCHRONOUS OPERATION, SIMULATION RESULTS
- + ASYNCHRONOUS OPERATION, UB3 UPPER BOUND

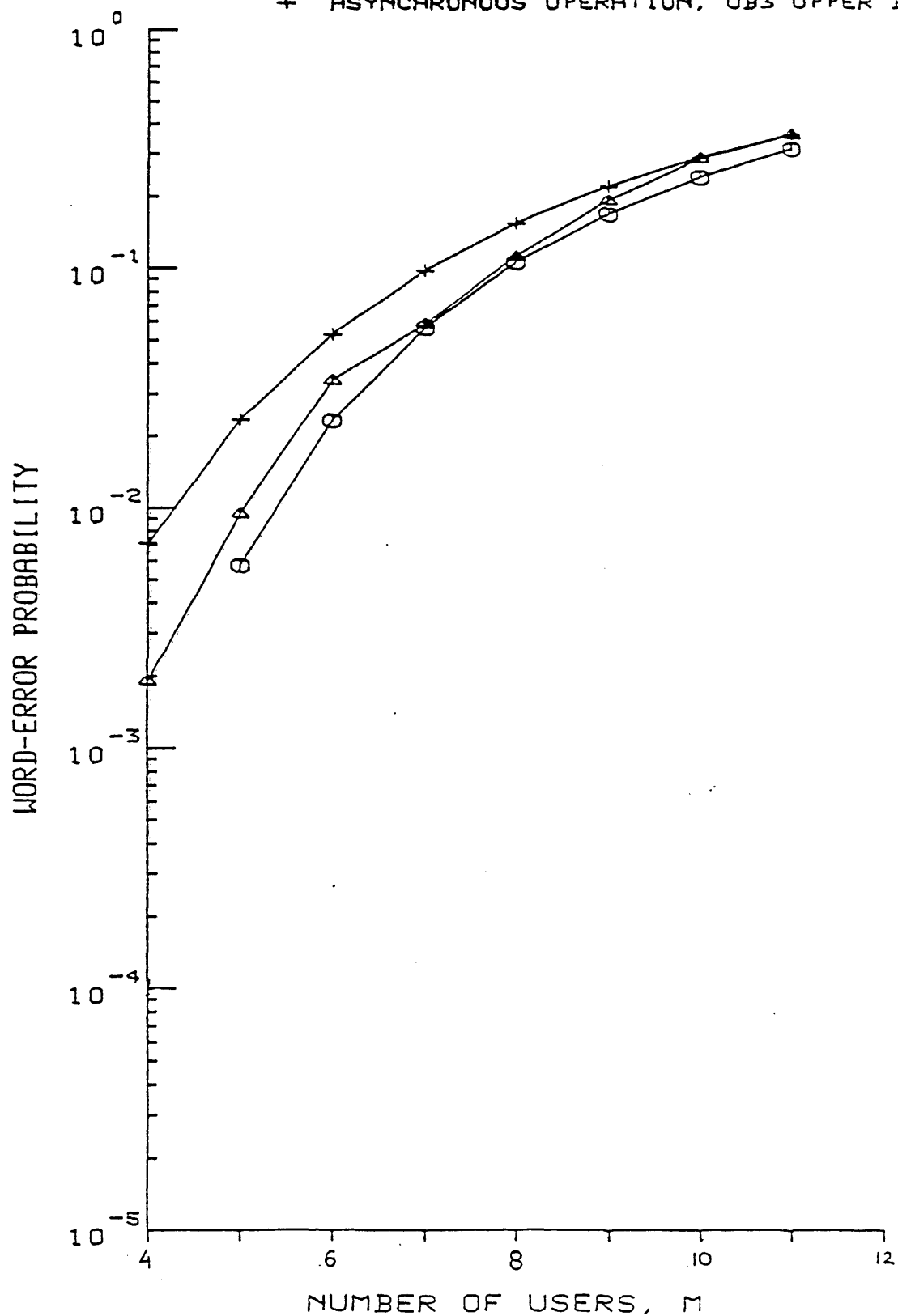


Fig.3.5b Word-error probability .vs. Number of users

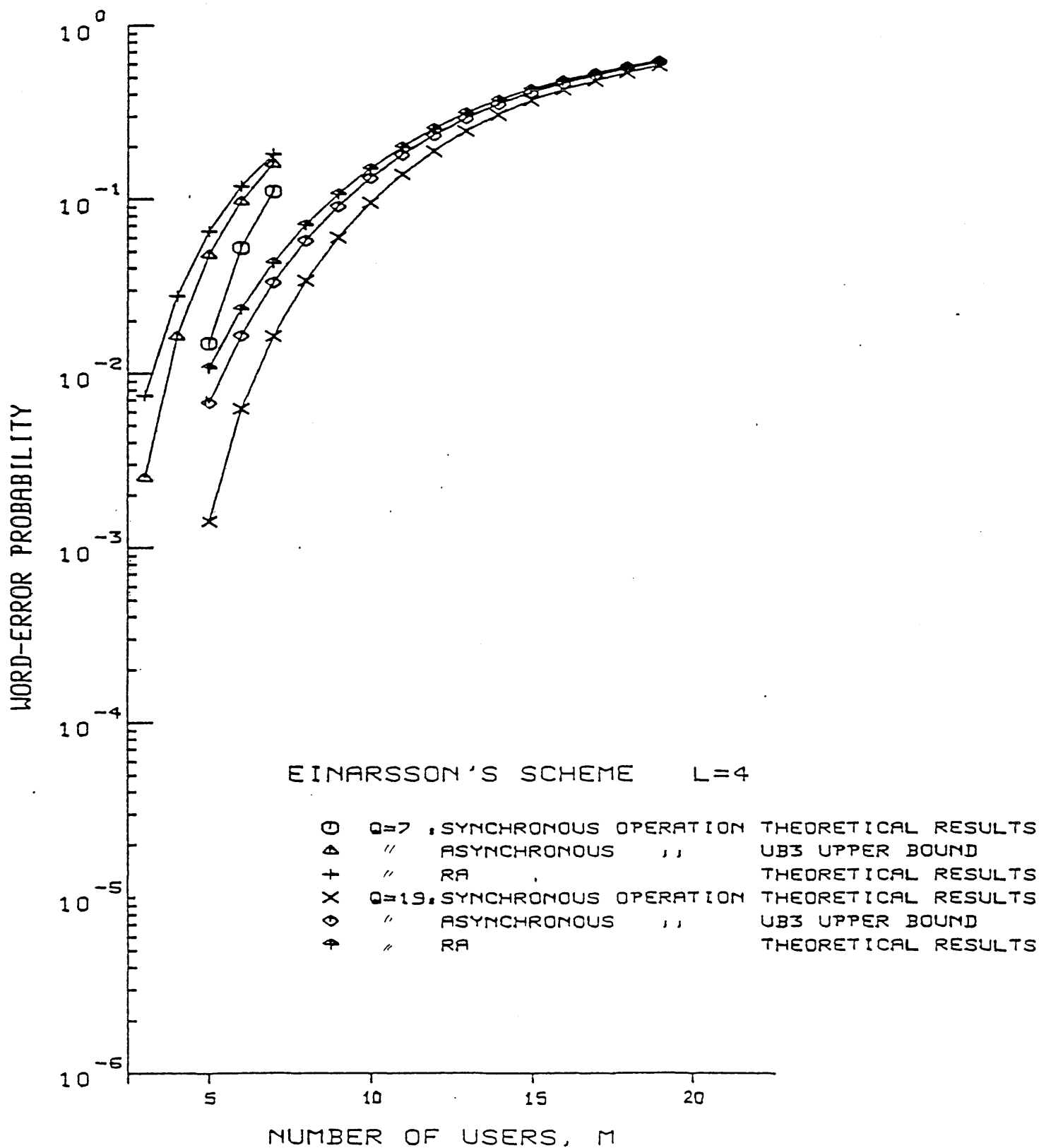


Fig.3.6 Word-error probability .vs. Number of users

CHAPTER 4

THE PERFORMANCE OF TIME-FREQUENCY CODED SPREAD-SPECTRUM SYSTEMS OPERATING OVER NOISY FADING CHANNELS

4.1. INTRODUCTION

In previous chapters the performance of the Einarsson-type TFCSSS has been considered when the sole interference is from other users. In this chapter the performance of the original Viterbi scheme and the scheme proposed by Einarsson are considered when the interference is both from the presence of other users and the presence of additive gaussian noise and fading.

In this chapter a new exact expression is derived for the word-error probability associated with the Viterbi scheme when this is operating over a noisy fading channel. A corresponding approximate expression, based on the exact word-error probability, is derived for the bit-error probability. In addition to providing an exact statement of the performance of the Viterbi Random Addressing (RA) scheme, the exact expression for the word-error probability forms the tightest known bound to the performance of a system using Einarsson's optimum addressing technique. In the case of a system using Einarsson's addressing technique, results obtained from the evaluation of the exact expression for the word-error probability (derived for a Viterbi-type RA scheme) are compared with those obtained from a computer simulation of the system. The comparison confirms that the expression for the word-error probability does in fact con-

stitute an upper bound to the performance of a system using Einarsson's addressing technique. A further comparison shows the bound to be much tighter than that which can be obtained by using Einarsson's bound relating to the performance of Viterbi's original scheme, and the comparison indicates that at low signal-to-noise ratios the bound can be considered as a reliable estimate of the actual error performance of the system.

Also in addition to considering the question of the error probability of system operation in the presence of other users' interference and noise, the question of the way in which transmitter power should be used is considered in this chapter.

The performance of a TFCSSS clearly depends on the actual power transmitted 'per chip' and in this chapter four different versions of a basic time-frequency coded system are considered. In Version I it is assumed that the transmitted power per chip is set equal to unity, irrespective of the number of users which occupy the chip. In Version II it is assumed that the transmitted power per chip is proportional to the square of the number of users occupying the chip, that is, it is assumed that 'voltage coherence' exists between users. In between these two extremes, two other versions are considered. In one of

these, which is referred to as Version III, it is assumed that the transmitted power per chip is proportional to the number of users occupying the chip. In a final version, referred to as Version IV, it is assumed that the system operates as in Version I but with the power per chip being increased so that the overall average transmitted power is the same as that used in connection with Version II. The performance of the various versions is considered and compared and the trade-offs to be obtained in terms of system performance are noted.

4.2. DESCRIPTION OF SYSTEM AND COMMUNICATION CHANNEL

4.2.1 System description

Fig.4.1 shows the essential elements of a basic TFCSSS. In the system, for each of M users K information bits are taken in and these are converted into one of 2^K equiprobable message symbols. These symbols are then combined modulo- Q (Q a prime or a power of a prime, and $Q \gg 2^K$) with an address the L elements of which are generated according to some predetermined rules. In the case of the Viterbi RA scheme the elements of the address are randomly generated integers in the range 0 to $Q-1$. With the more recently developed optimum addressing technique of Einarsson the elements of

the address are generated according to the following rule:

$$\bar{a}_m = (\gamma_m, \gamma_m \beta, \dots, \gamma_m \beta^{L-1})$$

where $\gamma_m \in GF(Q)$ and β is a fixed primitive element of $GF(Q)$.

At the receiver the corresponding user subtracts the address modulo- Q from the incoming signal and this thus results in an output sequence which, ideally, forms a unique full row in the frequency-time matrix (see Fig4.1). In practice, of course, due to various impairments, elements within a desired row may be eliminated, and elements in an unwanted row may be inserted. Yue [8] has considered the question of the optimum way of making decisions and he has shown that although the optimum decision maker is a non-linear device, which is difficult to implement, a good sub-optimum decision maker takes the form of a hard-limited combiner. In the study reported on in this chapter Yue's suboptimum decision maker is assumed.

From the random way in which the addresses are generated, it is clear that, from time to time, more than one user will be assigned to a particular chip in the frequency-time matrix and that some design rule has to be established to determine the power transmitted during such times of coincidence. With respect to the question of transmitted power per chip, various alternatives are available in the

base-to-mobile case, and it is not obvious a priori which rule should be adopted. For example, although it is clear that system performance will improve if the transmitted power per chip is allowed to increase in some way as the number of users wishing to occupy a chip increases, it is not clear how this should be done. Also it is not obvious what degree of improvement can be expected. With a view to answering these and related questions four rules (Versions I-IV of the system) for determining the transmitter power as a function of the number of users wishing to occupy a chip have been investigated. It should be noted that in the mobile-to-base case strict incoherence between users will exist and hence the power per chip will be proportional to the number of users, that is, the situation corresponds to Version III .

4.2.2 Description of the channel

A familiar well-known channel model used to represent a fading channel is that [19] in which the received signal envelope, a , is assumed to have a Rayleigh distribution with $\langle a^2 \rangle = 1$. In the model it is assumed that the fading is independent from chip to chip and that the phase, ϕ , is uniformly distributed over the range $[0, 2\pi]$. This model has been used previously by Einarsson [16] in his consideration of TFCSSS and it is also adopted in this chapter. In addi-

tion to fading, it is assumed that additive gaussian noise is present.

4.3. AN EXACT EXPRESSION FOR THE WORD-ERROR PROBABILITY IN VITERBI'S RANDOM ADDRESSING SCHEME.

In this section the effects of channel impairments on the performance of a TFCSSS will be considered. In particular, the performance of Viterbi's Random Addressing scheme is examined, and an exact expression is derived for the word-error probability. It is shown how this can be used to derive a corresponding approximate expression for the bit-error probability and it is also argued that these expressions form upper bounds to the performance of a system employing Einarsson's optimum addressing technique.

The effects of the impairments, combinations of fading and noise, can be regarded as a process that corrupts the received frequency-time matrix by producing deletions in occupied chips with probability P_D and false alarms in empty chips with probability P_F . As a result of deletions, the row containing the desired message may have a lower number of entries than a false row in which the entries are formed by insertions resulting from a combination of other users' interference and channel impairments.

An exact expression can be derived for the word-error probability in a Viterbi Random Addressing scheme. The expression is in terms of the probabilities P_D and P_F and the system parameters Q , M , and L , where Q is the number of frequency slots, M the number of users on the system and L the number of times a particular message is transmitted. The analysis is general in the sense that it applies to any kind of impairment for which probabilities P_D and P_F can be calculated.

(a) System Version I :

An insertion in any given chip of the received frequency-time matrix arises as a result of other users' interference and noise. The probability that one or more interfering users are present is

$$P = 1 - \left(1 - \frac{1}{Q}\right)^{M-1} \quad (4.1)$$

From the definition of P_D and P_F it follows that an insertion occurs with probability

$$P_I = P \cdot (1 - P_D) + (1 - P) \cdot P_F \quad (4.2)$$

and so the probability, $P_{fr}(n)$, that a false row (fr) has n occupied chips is given by

$$P_{fr}(n) = \binom{L}{n} \cdot P_I^n \cdot (1 - P_I)^{L-n} \quad (4.3)$$

The L occupied chips in a true row (tr) are reduced to some number m by the process of deletion and the probability $P_{tr}^{(m)}$ that a true row contains only m entries is given by

$$P_{tr}^{(m)} = \binom{L}{m} (1 - P_D)^m \cdot P_D^{L-m} \quad (4.4)$$

It should be noted that with a majority decision maker errors will be made with probability 1 when $n > m$, and with some probability less than one when $n = m$. In the case in which $m > n$ no errors are made. The average probability, P_w , of word-error is

$$P_w = \sum_{j=0}^L P_r(\text{error}, TR=j) \quad (4.5)$$

where $TR=j$ denotes that there are j occupied chips in the true row, and $Pr(\text{error}, TR=j)$ is the probability of decoding a true row erroneously when there are j occupied chips in the row.

The term $Pr(\text{error}, TR=j)$ in equation (4.5) can be written more conveniently as

$$P_r(\text{error}, TR=j) = P_r(\text{error}, TR=j, FR_{\max}=j) + P_r(\text{error}, TR=j, FR_{\max} > j) \quad (4.6)$$

where $FR_{\max}=j$ indicates that the false rows with maximum number of occupied chips contains exactly j such occupancies, and $FR_{\max} > j$ indicates that the false row with maximum number of occupied chips contains more than j occupied

chips.

The term $\Pr(\text{error}, TR=j, FR_{\max} > j)$ can be written alternatively as

$$\Pr(\text{error}, TR=j, \exists FR > j) \quad (4.7)$$

where $\exists FR > j$ denotes that there is at least one false row with more than j occupied chips and equation (4.7) can be written as

$$\begin{aligned} \Pr(\text{error}, TR=j, \exists FR > j) &= \Pr(\text{error} | TR=j, \exists FR > j) \cdot \\ &\quad \Pr(TR=j | \exists FR > j) \cdot \Pr(\exists FR > j) \\ &= 1 \cdot \Pr(TR=j | \exists FR > j) \cdot \Pr(\exists FR > j) \\ &= \Pr(TR=j) \cdot [1 - \Pr(\forall FR \leq j)] \quad (4.8) \end{aligned}$$

where $\forall FR \leq j$ denotes that all false rows have occupancy in j or fewer chips.

Since there are a total of $2^K - 1$ possible false rows in the frequency-time matrix it follows that

$$\begin{aligned} &\Pr(TR=j) \cdot [1 - \Pr(\forall FR \leq j)] \\ &= \Pr(TR=j) \cdot [1 - (\Pr(FR \leq j))^{2^K - 1}] \end{aligned}$$

and hence

$$\begin{aligned} &\Pr(\text{error}, TR=j, FR_{\max} > j) \\ &= \Pr(TR=j) \cdot \left[1 - \left(\sum_{k=0}^j P_{fr}(k) \right)^{2^K - 1} \right] \quad (4.9) \end{aligned}$$

Consider now the term $Pr(\text{error}, TR=j, FR=j)$ in equation (4.6). If r out of $2^k - 1$ possible false rows each contain exactly j occupied chips then the remaining $2^k - 1 - r$ false rows contain less than j occupied chips and the term

$$Pr(\text{error}, TR=j, FR_{\max}=j)$$

$$\begin{aligned} & \text{can be written as } Pr(\text{error}, TR=j, FR_{\max}=j) \\ &= \sum_{r=1}^{2^k-1} Pr(\text{error}, TR=j, FR_{(r)}=j, FR_{(2^k-1-r)} < j) \\ &= \sum_{r=1}^{2^k-1} Pr(\text{error} | TR=j, FR_{(r)}=j, FR_{(2^k-1-r)} < j) \\ & \quad \cdot Pr(TR=j | FR_{(r)}=j, FR_{(2^k-1-r)} < j) \\ & \quad \cdot Pr(FR_{(r)}=j, FR_{(2^k-1-r)} < j) \end{aligned} \quad (4.10)$$

where $FR_{(r)}=j$ denotes that r false rows each contain exactly j occupied chips and

$FR_{(2^k-1-r)} < j$ denotes that the remaining $2^k - 1 - r$ false rows each contain less than j occupied chips. It follows from equation (4.10) that

$$\begin{aligned} & Pr(\text{error}, TR=j, FR_{\max}=j) \\ &= \sum_{r=1}^{2^k-1} \left(\frac{r}{r+1} \right) \cdot P_{tr}(j) \cdot \binom{2^k-1}{r} \cdot \left(P_{fr}(j) \right)^r \cdot \left[\sum_{i=0}^{j-1} P_{fr}(i) \right]^{2^k-1-r} \end{aligned} \quad (4.11)$$

On substituting equations (4.9) and (4.11) into equation (4.6), and then substituting the result into equation (4.5),

the following exact expression is obtained for the word-error probability, P_w .

$$P_w = \sum_{j=0}^L \left\{ \sum_{r=1}^{2^K-1} \frac{r}{r+1} \cdot P_{tr}(j) \cdot \binom{2^K-1}{r} \cdot P_{fr}^r(j) \cdot \left[\sum_{i=0}^{j-1} P_{fr}(i) \right]^{2^K-1-r} \right. \\ \left. + P_{tr}(j) \cdot \left[1 - \left(\sum_{k=0}^j P_{fr}(k) \right)^{2^K-1} \right] \right\} \quad (4.12)$$

The bit-error probability, P_b , is given approximately

[5,16] by

$$P_b \approx \frac{2^{k-1}}{2^k - 1} \cdot P_w$$

(b) System Version II

The analysis of the performance of this system is a relatively simple extension of the previous analysis, if it is assumed that when two or more users occupy a chip the resulting signal level is sufficient to prevent a deletion taking place. The probability that a given chip is occupied by only one interfering user is

$$P_i = (M-1) \cdot \frac{1}{Q} \cdot \left(1 - \frac{1}{Q}\right)^{M-2} \quad (4.13)$$

and hence the probability of insertion with respect to a given chip is

$$\begin{aligned} P_I &= P_1 \cdot (1 - P_D) + (P - P_i) + (1 - P) \cdot P_F \\ &= P - P_i \cdot P_D + (1 - P) \cdot P_F \end{aligned} \quad (4.14)$$

If this modified expression for P_i is used in equation (4.3), and the same substitutions are made as in the case of Version I, then an expression corresponding to equation (4.12) is obtained for Version II. Although this is not an exact expression for the word-error probability it is an excellent approximation since the error associated with the assumption regarding deletions is negligible.

The exact expression derived in equation (4.12) for the word-error probability of Version I and the corresponding expression that can be derived for Version II using equation (4.14) can be considered as upper bounds to the corresponding versions of systems employing Einarsson's optimum addressing technique. This conjecture is based on the realisation that with the Einarsson addressing technique the extent of other users' interference is reduced below that associated with the Viterbi RA scheme.

4.4. RESULTS

In order to check the conjecture that the exact expression derived for the word-error probability in a Viterbi-type RA system can be used as a tight upper bound for the performance of a system using Einarsson's optimum addressing technique, a simulation of Einarsson's system was carried out. In the simulation channels perturbed by additive gaussian noise alone and a combination of Rayleigh fading and gaussian noise were considered. The results obtained relating to Version I -type systems are shown in Figs.4.2-4.5. In all the figures shown in this chapter the noise variance is expressed in db relative to unit variance noise. It should be noted that the horizontal axes shown on the figures can be converted to signal-to-noise ratio axes by simply subtracting the given noise level expressed in db

from the average (or peak) signal power computed using the appropriate expressions given in Table 4.1.

Figures 4.2 and 4.3 are for channels that are subject to noise alone and Figs. 4.4 and 4.5 are for channels that are subject to both noise and fading. Comparison of the corresponding figures shows that, as expected, systems that are subject to both fading and noise have a performance inferior to that associated with noise alone.

From the figures it can be seen that the performance estimates based on the newly derived expression for the word-error probability in a Viterbi-type scheme do in fact provide an upper bound to the performance of an Einarsson-type system. Also shown in the figures is the upper bound derived previously by Einarsson [16]. It is clear that the newly derived bound is significantly tighter than Einarsson's bound. At relatively low signal-to noise ratios the newly derived bound is very tight and can in fact be considered as a reliable estimate of system performance, whereas the Einarsson bound becomes increasingly loose and does in fact exceed unity. Figs. 4.6 and 4.7 show simulation results for Version II-type systems. Fig. 4.6 deals with the case in which noise alone is present and Fig. 4.7 deals with the case of combined noise and fading. From the figures it is again clear that the estimates based on the

newly derived expression for the word-error probability constitute an upper bound that is significantly tighter than Einarsson's bound and that, as expected, operation in the presence of combined noise and fading is inferior to the operation in the presence of noise alone.

In addition to the simulation of Versions I and II Versions III and IV of Einarsson's system were also simulated. Some of the results that have been obtained are shown in Figs. 4.8 and 4.9. The figures relate to operation in the presence of noise alone and show how the performance of the system varies from version to version.

When comparing the performance of the various versions of the system it is clear from Figs. 4.8 and 4.9 that for a given noise variance Version II outperforms Version I, except at very low values of noise variance. This improvement is to be expected since a significantly increased level of transmitter power is used. Specifically, M times more peak power is required with Version II than with Version I. For the case shown in Fig. 4.8, for example, this corresponds to an increase in noise variance (relative to unity noise variance) of $10\log_{10} (17)^2 = 24.6$ db. Examination of Figs. 4.8 and 4.9 shows, however, that for operation in the middle of the range of noise variance values, Version II can only handle approximately 5db more noise power than Version

I for a given error probability. This limited improvement in system performance can, however, be explained in terms of the average transmitter power. In Table 4.1 expressions are given for both the peak and average power requirements associated with each version of the system. From this table it can be seen that for the values of Q, M and L being considered Version II uses approximately 5db more average power than Version I.

Figs. 4.8 and 4.9 also show that Version III (in which the transmitted power per chip is made proportional to the number of users occupying the chip) lies between the extremes set by Versions I and II. Examination of the figures shows that in the middle range of noise variance values, Version III can withstand 2 to 3db more noise power than Version I, but approximately 2 to 3db less noise power than Version II. In the case of Version III the peak power requirement (for the case $Q=M=17$) is approximately 10db greater than with Version I. Here again, however, calculation using the formula given in Table 4.1 shows that the average increase of transmitter power over that associated with Version I is approximately 3db which accounts for the observed increased level of immunity to noise.

Results relating to the operation of Version IV are also shown in Figs. 4.8 and 4.9. For the same average transmitter power Version IV requires approximately 20db

less peak power than Version II. From the figure it can be seen that over most of the range of noise variance values Version IV outperforms Version II. The underlying reason for this is that the higher level of transmitter power with Version IV is used more effectively than with Version II. Unlike Version II, with Version IV an increase in transmitter power is made, when only a single user occupies a chip. A consequence of the increased use of power in this situation is that the probability of deletion is significantly reduced and the increased detection threshold associated with the increased transmitter power of Version IV also means that the probability of false alarm is reduced.

Figs. 4.8 and 4.9 show two interesting general trends. These concern the operation at the extremes of very low noise variance (high signal-to-noise ratio) and very high noise variance (low signal-to-noise ratio). The figures show that at these extremes all systems tend to perform like Version I. The reason for this is that for high signal-to-noise ratios the performance of Version I is so good that any increase in transmitter power, irrespective of way it is made, leads to negligible improvement of system performance. At very low signal-to-noise ratios the performance is so poor that any increase in transmitter power has little effect on the error probability.

Two other general points worth noting concern the relative performance of Versions II and IV and the way in which the performance of various versions changes as M is reduced relative to Q . It will be noted from Figs. 4.8 and 4.9 that as the noise variance is increased a point is reached where Version IV becomes inferior to Version II. This cross-over point is associated with the fact that for M close to Q multiple occupancy of chips occurs relatively frequently and as Version II uses more transmitter power than Version IV on these occasions, it is better able to prevent deletions.

When M is significantly less than Q multiple occupations become increasingly unlikely, and all versions of the system use the same average transmitter power (and the same peak power) and hence it follows that all systems will have substantially the same performance.

TABLE 4.1

VERSION	AVERAGE POWER	PEAK POWER
I	$1 - \left(1 - \frac{1}{Q}\right)^M$	1
II	$\frac{M}{Q} \cdot \left(1 - \frac{1}{Q}\right) + \left(\frac{M}{Q}\right)^2$	M^2
III	$\frac{M}{Q}$	M
IV	$A^2 \cdot \left[1 - \left(1 - \frac{1}{Q}\right)^M\right]$	A^2

Q : number of frequency slots

M : number of users

A : amplification factor, $A > 1$.

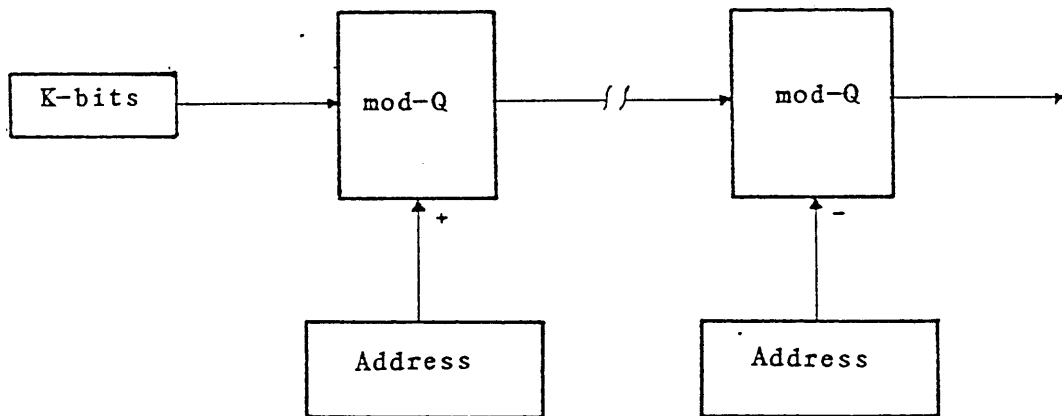


Fig.4.1 Essential elements of a TFCSS System

EINARSSON'S SCHEME VERSION I: $Q=17, L=8, M=17$
 $K=4$

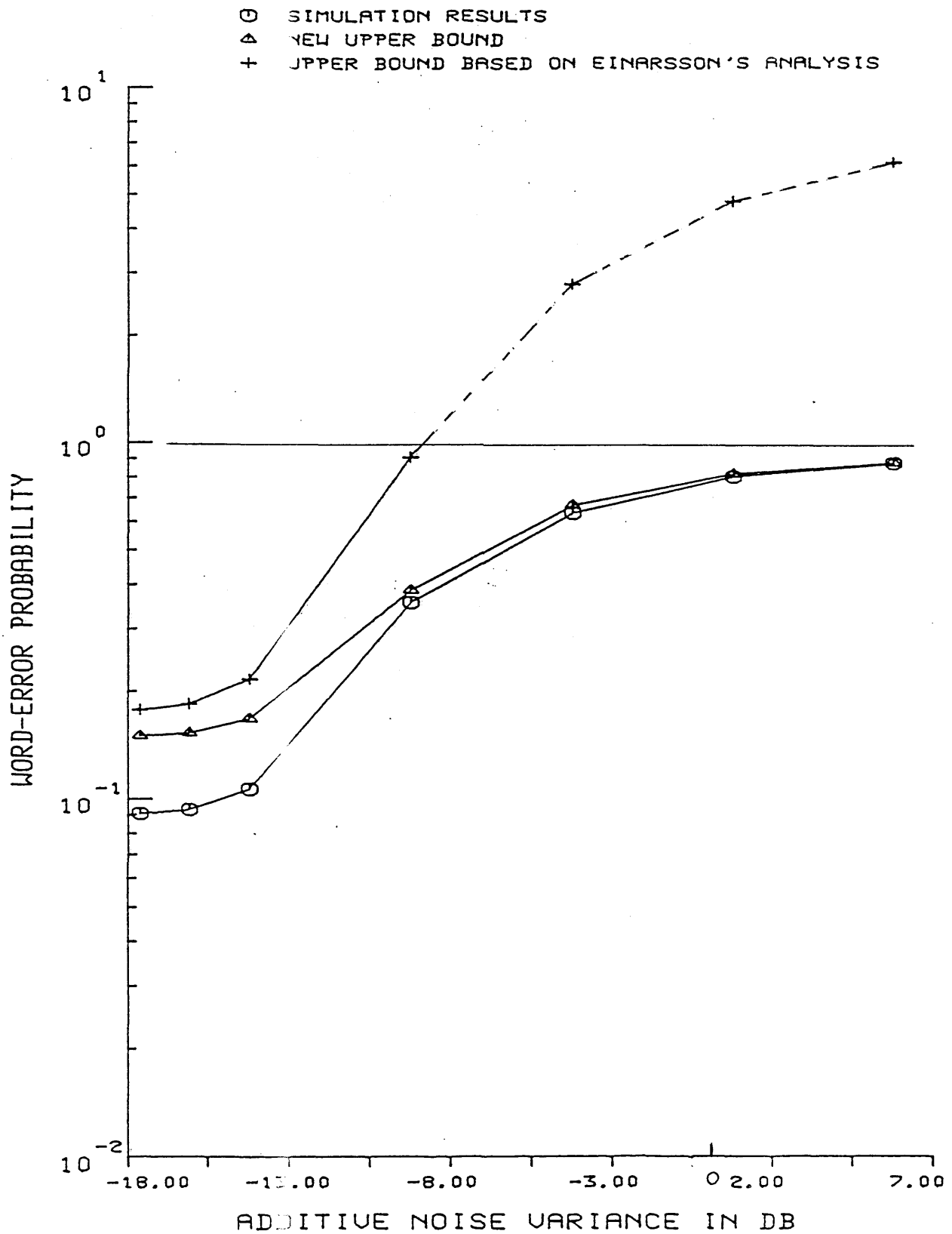


Fig.4.2 Version I: error performance in gaussian noise

EINARSSON'S SCHEME VERSION I, Q=17, L=12, M=17

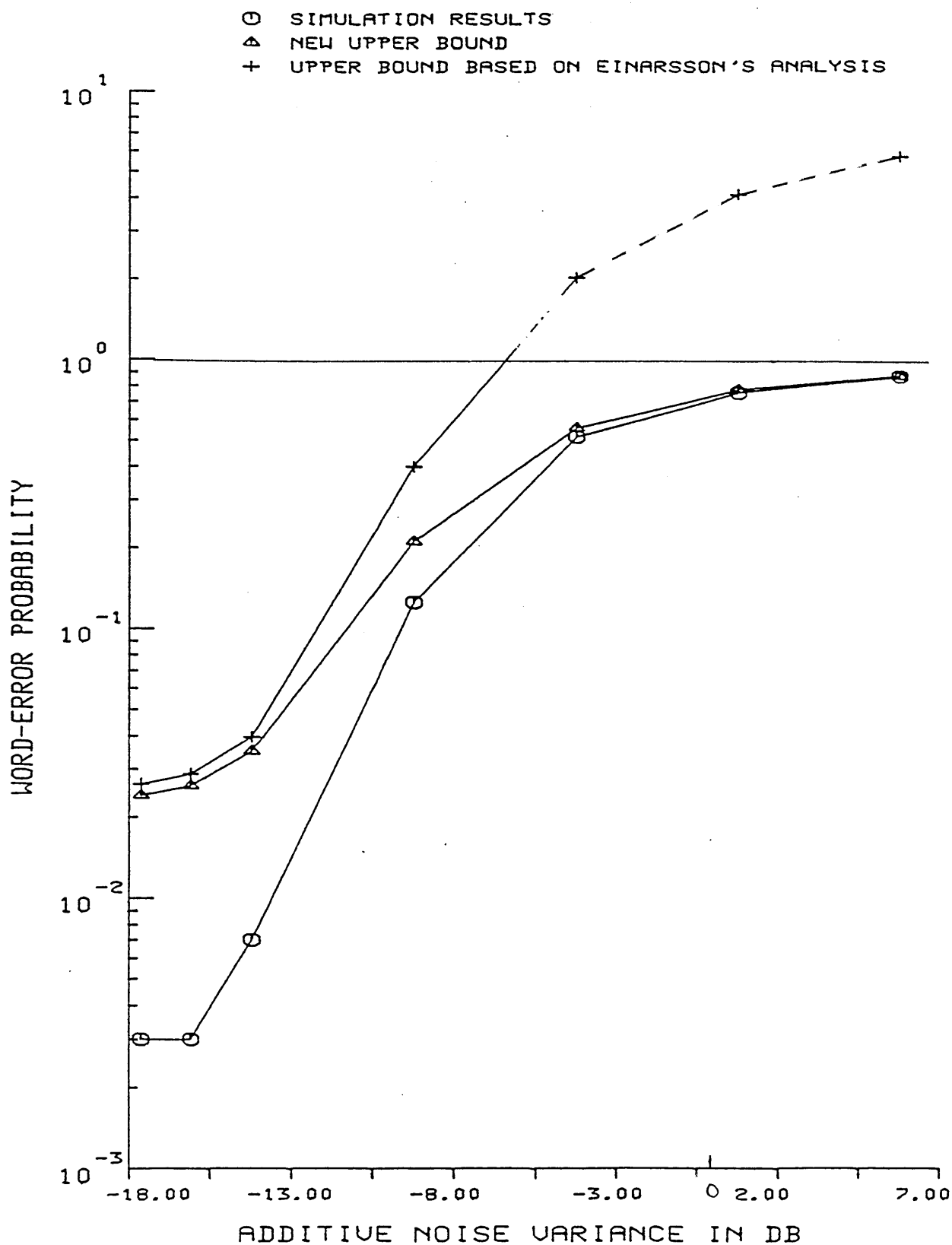


Fig.4.3 Version I: error performance in gaussian noise

EINARSSON'S SCHEME VERSION I: $Q=17, L=12, M=17$

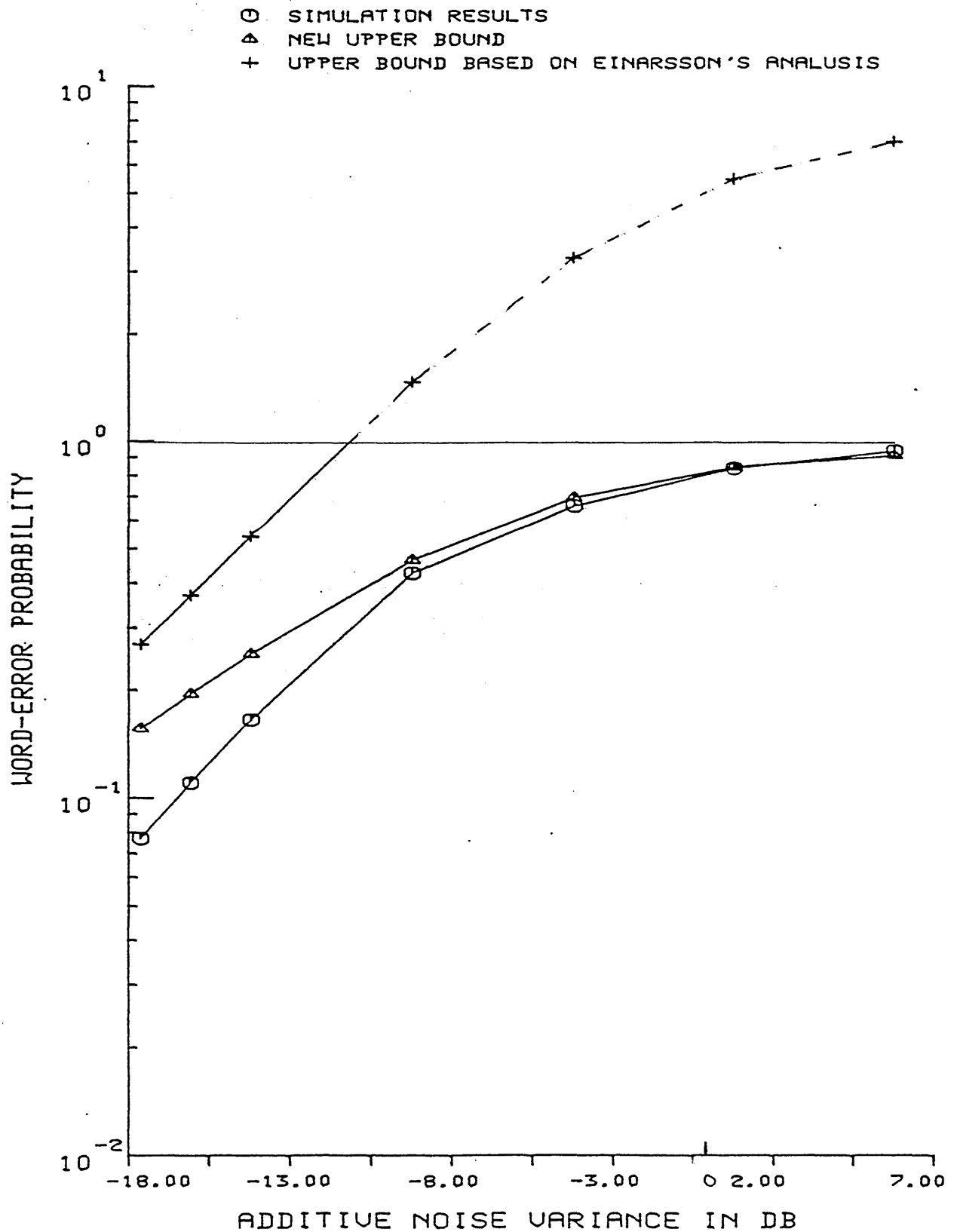


Fig.4.4 Version I: error performance in Rayleigh fading and gaussian noise

EINARSSON'S SCHEME VERSION I, Q=17, L=12, M=15

- SIMULATION RESULTS
- △ NEW UPPER BOUND
- + UPPER BOUND BASED ON EINARSSON'S ANALYSIS

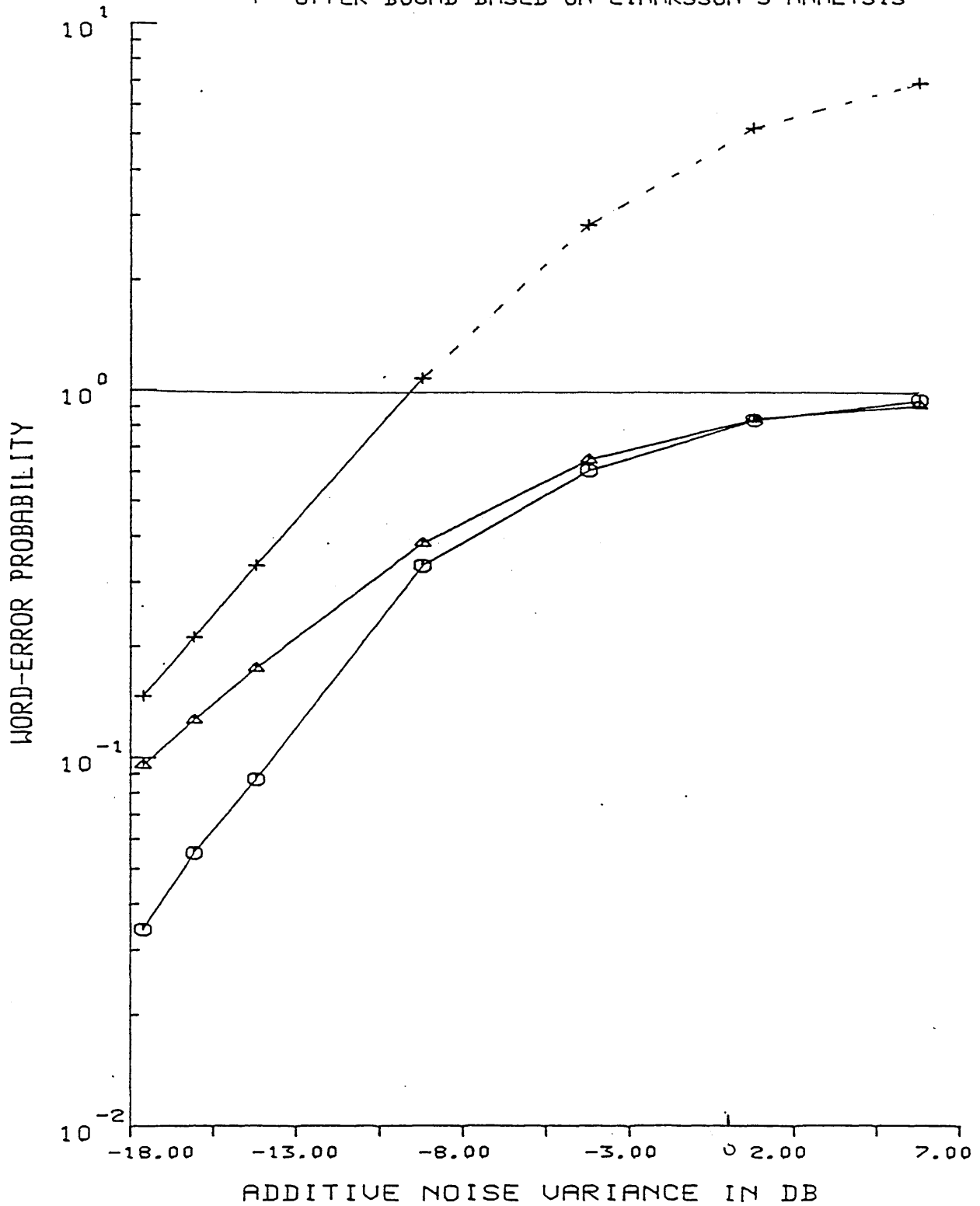


Fig.4.5 Version I: error performance in Rayleigh fading and gaussian noise

EINARSSON'S SCHEME VERSION II, $Q=17, L=12, M=17$

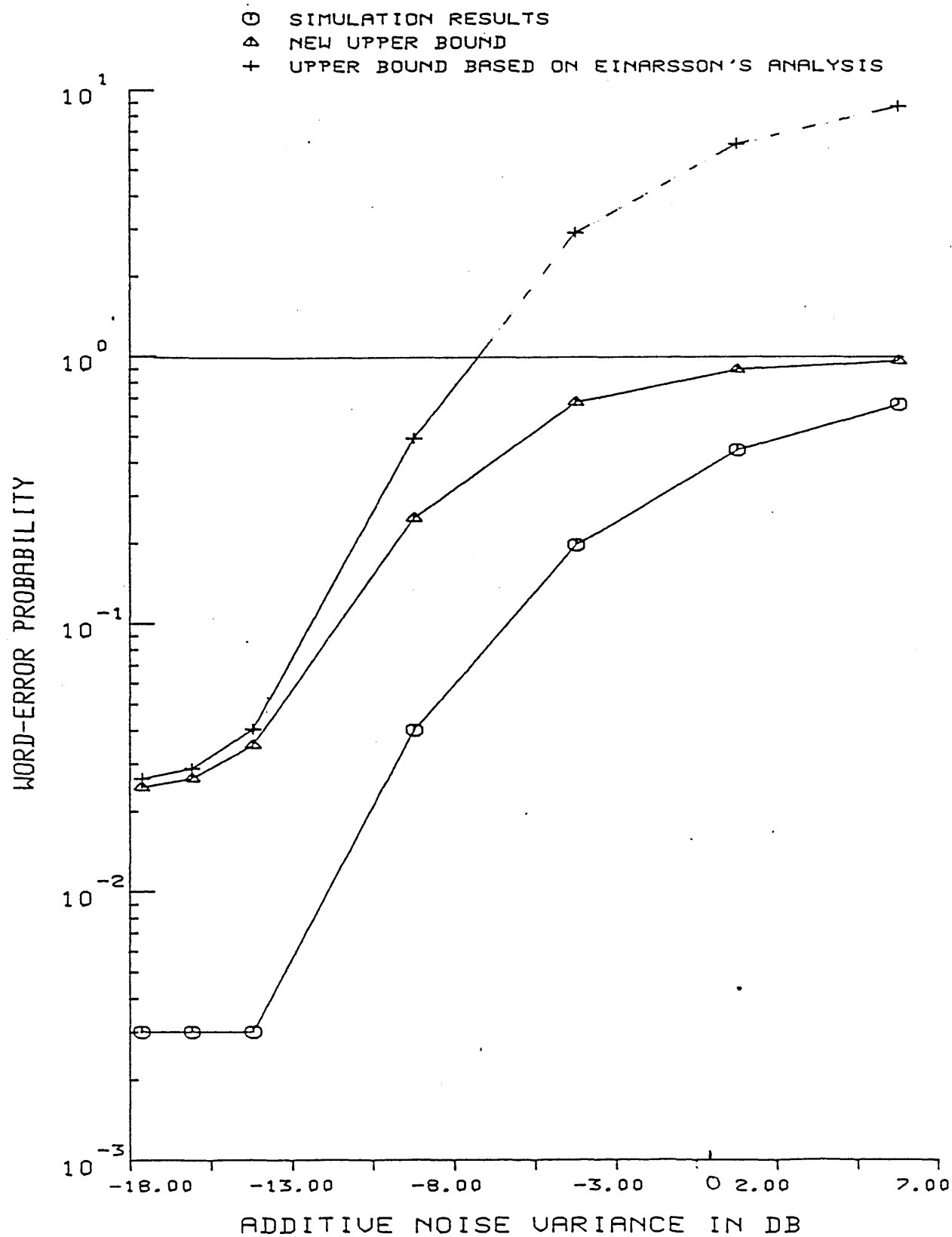


Fig.4.6 Version II: error performance in gaussian noise

EINARSSON'S SCHEME VERSION II: Q=17, L=12, M=17

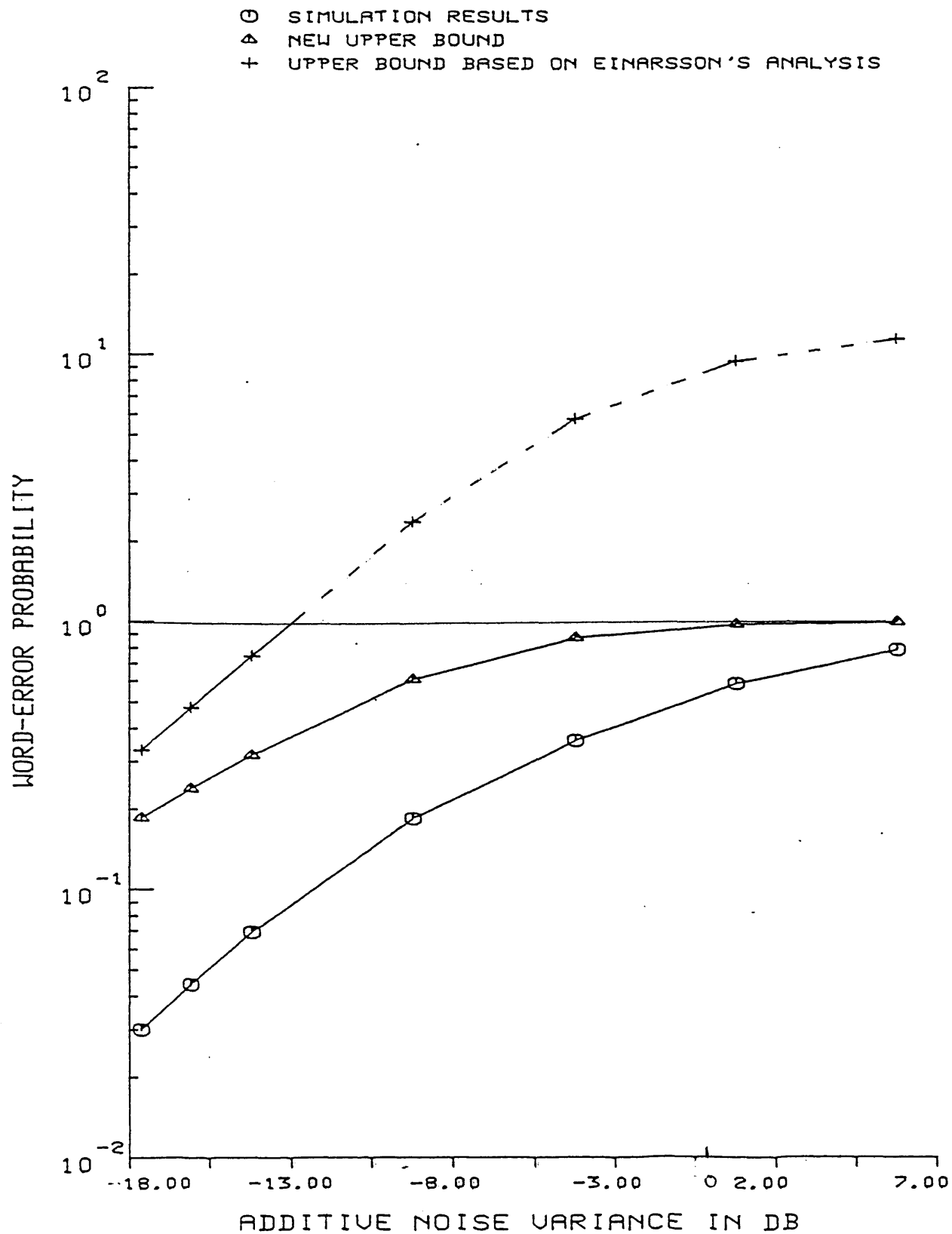


Fig.4.7 Version II: error performance in Rayleigh fading and gaussian noise

EINARSSON'S SCHEME , $Q=17$, $L=12$, $M=17$

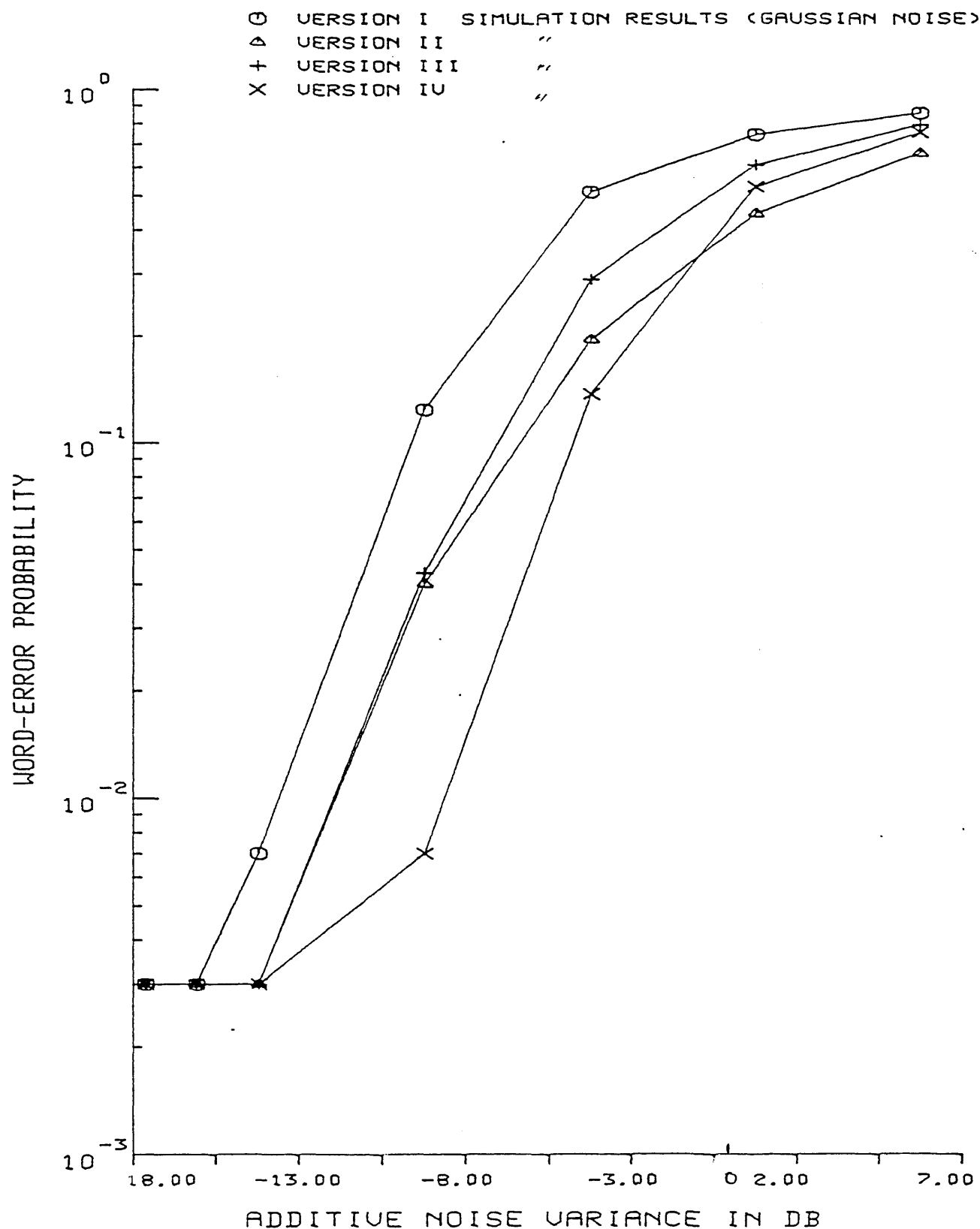


Fig.4.8 Comparison of the error performance of Versions I-IV in gaussian noise

EINARSSON'S SCHEME : Q=17, L=12, M=16

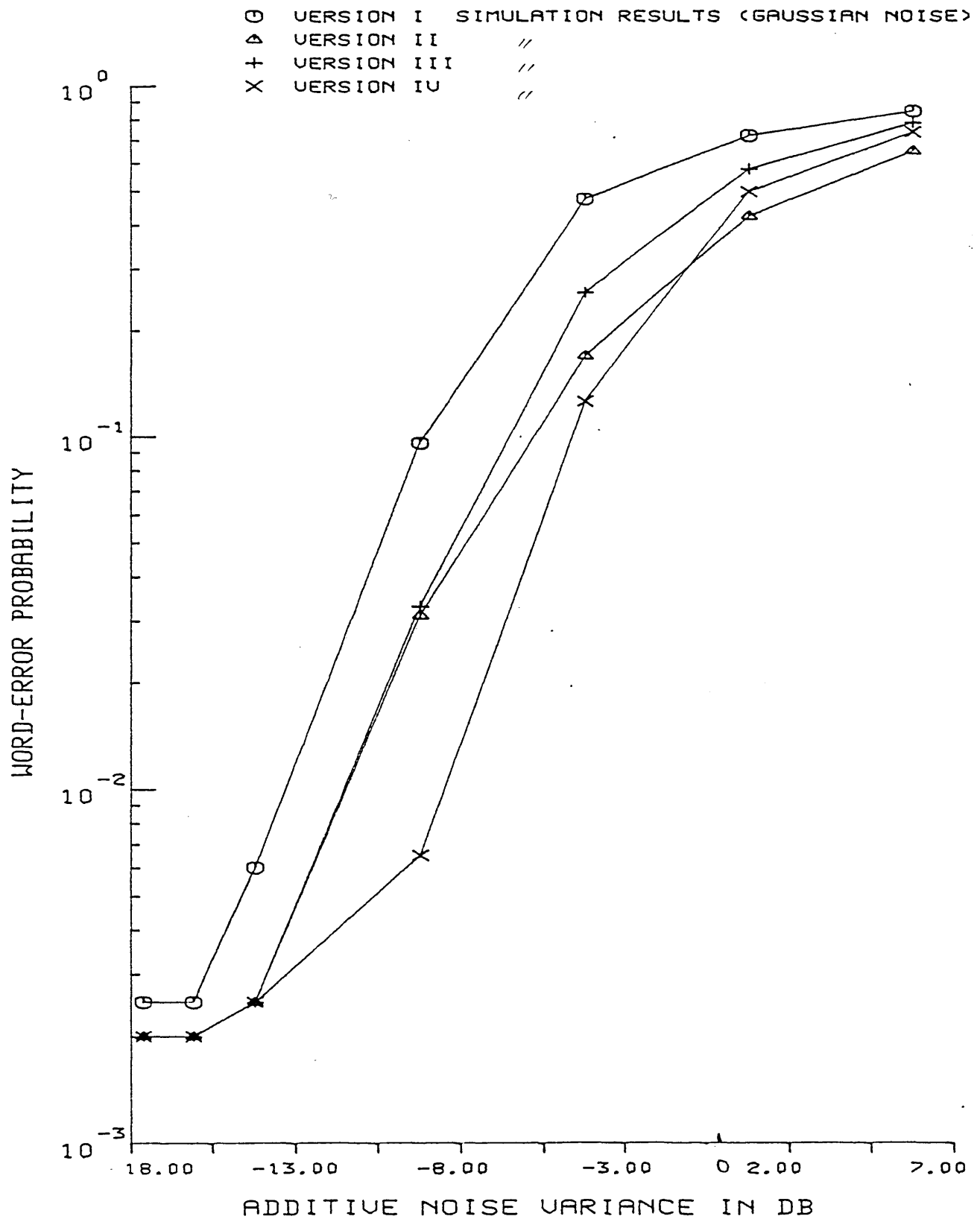


Fig.4.9 Comparison of the error performance of Versions I-IV in gaussian noise

CHAPTER 5

AN APPLICATION OF CHANNEL CODING IN

FREQUENCY-HOPPED MULTI-USER

COMMUNICATION SYSTEMS

5.1. INTRODUCTION

In the previous chapter the use of time and frequency in the manner suggested by Viterbi and Einarsson has been considered at some length. In this chapter an alternative way of using time and frequency is put forward. The alternative involves the combined use of channel coding and frequency hopping. The basic elements of the Viterbi and Einarsson schemes are shown in Fig5.1. In the system there are available Q frequency slots— Q frequency bands each of bandwidth Δf . In the case of an Einarsson-type system up to Q transmitting subscribers can use the system at any one time. Each of M subscribers generates K information bits per frame and these are then mapped into one of 2^K message symbols, with $2^K \ll Q$, and $M \ll Q$. For each symbol generated by a user an L -element addressing word is generated and the message symbol is added modulo- Q to each of the L elements comprising the address. This results in a message word of L components, with each component corresponding to one of Q frequency slots. The message symbol is transmitted by sending the corresponding set of L frequencies.

With the Einarsson scheme the rate of communication per user is K bits/frame or K/L bits/time-slot. As it is possible for there to be up to Q users on the system, the maximum overall communication rate for the system is $Q \cdot K/L$ bits per

time-slot.

It is important to note that with schemes such as those of Viterbi and Einarsson, coincidences, in which two or more users occupy a transmission chip (a frequency-time slot), are inevitable and that these coincidences lead to a degradation in system performance. The basic reason for the existence of coincidences is that in order to make maximum use of the available Q frequency channels (slots) each subscriber is allowed to communicate K information bits/frame-one of the 2^K different message symbols, with K such that $2^K \ll Q$ and as close to Q as possible. As a result of this, each of the Q channels is available to users, irrespective of whether or not it is already occupied.

5.2. AN APPLICATION OF CHANNEL CODING IN FREQUENCY-HOPPED MULTI-USER OPERATION

The realization that other users' interference is a significant factor in limiting system performance, and that such interference is inevitable with the addressing schemes of the Einarsson and Viterbi type, leads to the question whether and to what extent system performance can be improved by developing systems capable of operating without introducing any significant degree of other users' interfer-

ence.

It is established practice that if there are M users and $Q=M$ frequency channels then, by using conventional FDM techniques, it is possible to communicate without there being any significant degree of interference between users (adjacent channel interference). All that is strictly necessary in order for a user to communicate K information bits without other users' interference is for each of the M channels to have associated with it a binary communication link and for the user to employ one of the links K times for the transmission of the K information bits.

With the Viterbi and Einarsson schemes some protection against other users' interference is obtained by using frequency hopping and sending the message symbol associated with the K information bits a total of L times. This procedure also affords some protection against extraneous noise (jamming) and frequency selective channel fading.

An alternative to using time and frequency in the manner suggested by Viterbi and Einarsson is to use FDM in conjunction with error correction and frequency hopping. With FDM used in the manner described above other users' interference is absent and the combined use of frequency hopping and coding provides protection against extraneous

noise and channel fading. Specifically, by using a simple (L,K) error-correcting code some protection can be obtained against noise alone and by frequency hopping applied to each of the L digits comprising the codeword used to transmit the K information digits, some protection can be obtained against frequency-selective fading.

With the simple alternative approach the rate of transmission per user is K/L bits/time-slot and with $M=Q$ users the overall communication rate for the system is $K \cdot Q/L$ bits/time-slot, as before.

It is perhaps worth reemphasizing that the basic distinction between frequency-time coded multi-user systems of the type considered by Viterbi and Einarsson, and the simple alternative being put forward in this thesis is that with the Viterbi and Einarsson schemes the information relating to a K -bit information word is carried by the selection at the transmitter and the subsequent identification at the receiver of one of 2^K allowed transmission symbols, that is, a 2^K -ary system is employed, whereas with the alternative system a binary scheme with L binary digits used to transmit the K -bit information word is employed.

5.3 COMPARISON OF SYSTEM PERFORMANCE

In order to assess the relative performance of systems based on the two approaches outlined above, an extensive simulation of both the Einarsson scheme and schemes employing error correction has been carried out. The performance of the Einarsson system has been compared with that of a system using a simple (3,1) repetition code and a system using a (7,4) cyclic Hamming code. The main features of the systems compared are shown in Fig5.2. From the figure it can be seen that information digits are considered in blocks of $K=4$. This parameter was selected so as to facilitate a sensible comparison in terms of the respective 4-bit word error probabilities associated with the various schemes.

In the case of the (3,1) coded system the 4 information digits are taken in as a block and are repeated twice as shown in Fig.5.2.b with the result that the four information digits are transmitted in coded form as a block of $L=12$ digits. In the case of the Einarsson scheme the same 4-bit information blocks are mapped into one of $2^4 (=16)$ message symbols and are then combined with an address in the manner described by Einarsson. The parameter L is set equal to 12 so that the rate of the Einarsson system is $K/L=1/3$ bits/time-slot/user, the same as that of the system using the repetition code.

In the case of the (7,4) cyclic code, the 4-bit words

are mapped into 7-digit sequences so that the parameter L equals 7. The mapping is illustrated in Fig.5.2.c for the 4-bit word 1100.

In the case of the coded systems the linear addressing technique illustrated in Figs.5.2.b and 5.2.c was adopted, and the energy per transmitted information bit to the noise power ratio (E_b/N_0) was kept the same for all systems being compared. In the simulation of the Einarsson system the scheme in which a fixed level of power is transmitted per occupied chip, irrespective of the number of users occupying the chip, was adopted and for this scheme the average power transmitted per information bit is given by

$$E_b = \frac{\bar{l}_m^Q \cdot L}{M \cdot K} \cdot E$$

where $\bar{l}_m^Q$ is the average number of occupied chips in a column of the frequency-time matrix having Q frequency slots which are available to M users. This equation can be derived using the material contained in Chapter 2.

In the case of the coded systems if ON/OFF keying is used then zero energy is required for the transmission of a binary '0' and some energy E for the transmission of a binary '1'. If it is assumed that '0's and '1's are transmitted with equal probability then the average energy per bit, E_b , for the (3,1) coded system is $E_b = (3/2) \cdot E$ and

the average energy per bit for the (7,4) coded system is $E_b = (7/8) \cdot E$. In order to compare systems properly it is necessary to ensure that the ratio E_b/N_0 is the same for all systems being considered.

Also, for the comparisons to be sensible it is necessary to compare them in terms of the same performance parameter. For convenience the word-error probability was chosen. With the (3,1) code a bit is decoded correctly provided no more than one of the three corresponding transmissions is erroneously decoded. The bit-error probability is then given by

$$P_b = \frac{1}{2} \cdot \sum_{n=2}^3 \binom{3}{n} \cdot P_D^n \cdot (1-P_D)^{3-n} + \frac{1}{2} \cdot \sum_{n=2}^3 \binom{3}{n} \cdot P_F^n \cdot (1-P_F)^{3-n}$$

and the expression for the word-error probability P_w is

$$P_w = 1 - (1 - P_b)^4$$

In the case of the (7,4) code the 4-bit word-error probability associated with the 4 information bits is the same as the probability that the 7-bit transmitted code-word is decoded erroneously. This is due to the particular structure of the (7,4) code, hence

$$P_w = \sum_{n=2}^7 \binom{7}{n} \cdot P_e^n \cdot (1-P_e)^{7-n}$$

where P_e is the probability that a given code-bit is incorrectly decoded.

5.4 COMPARISON OF SYSTEM PERFORMANCE

Some of the results obtained from the simulations are shown in Figs.5.3-5.8. Figs.5.3 and 5.4 show the theoretical and simulation results obtained when using the (3,1) and (7,4) coded systems respectively, when they are operating in the presence of gaussian noise alone. The results are seen to be in excellent agreement, except at high signal-to-noise ratios when errors were so few that the simulation run was not sufficiently long. Figs.5.5 and 5.6 show the same pattern of agreement between theoretical and simulation results for the case when both fading and noise are present.

Figs.5.7 and 5.8 show clearly that the performance of the coded systems is superior to that which can be obtained with the Einarsson scheme, and that the improvement is particularly pronounced at signal-to-noise ratios above 12 db. The basic reason for this improvement is that with the Einarsson-type system the inevitable interference arising from the presence of other users becomes the major factor limiting the performance of the systems as the signal-to-noise ratio increases. No such interference exists, however, with the coded schemes. Figs.5.7 and 5.8 also show that in terms of the word-error probability the performance of the (7,4) coded system is superior to that of the (3,1) coded system. It should be remembered also that this coded system has a higher transmission rate (4/7 bits/time-slot/user) than either the Einarsson scheme or the

(3,1) coded scheme, both of which have a transmission rate of $1/3$ bits/time-slot/user.

From Fig.5.7 it can be seen that at a signal-to-noise ratio (E_b/N_o) of 16db the word-error probability of the (3,1) coded system is approximately two orders of magnitude less than that of the Einarsson scheme, and that the (7,4) coded system is superior to the (3,1) coded system by more than one order of magnitude.

The other important fact that emerges from Figs.5.7 and 5.8 is that in the case of the Einarsson scheme when only $M=15$ users are on a system having $Q=17$ frequency channels, the performance is still significantly inferior to that of the coded systems. Provided that the number of users does not exceed the number of channels, the performance of the coded systems is, of course, independent of the number of users on the system.

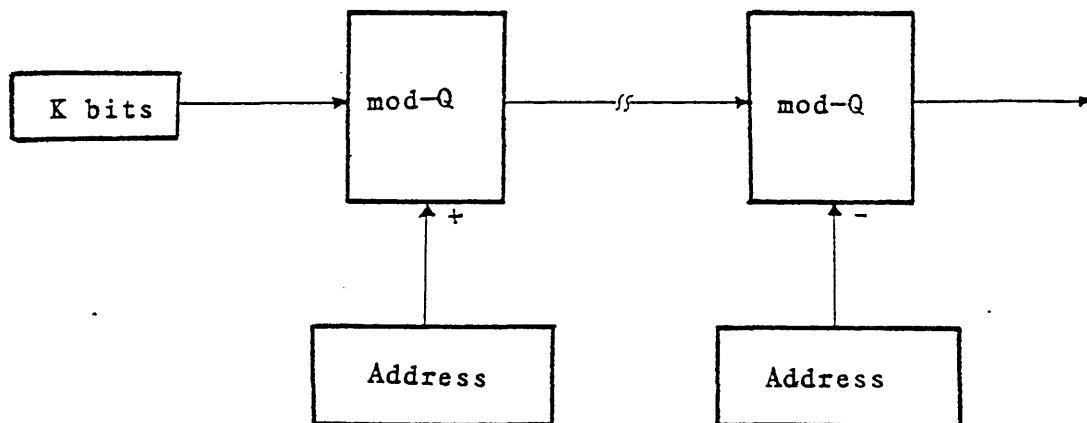
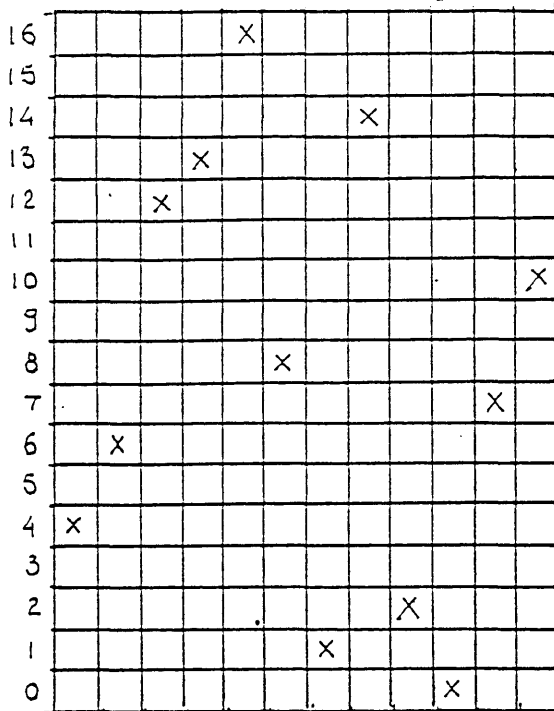
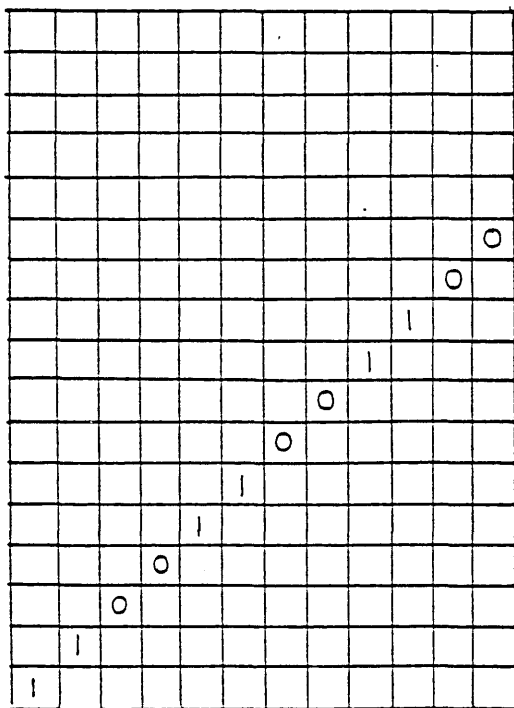


Fig.5.1 Essential elements of a TFCSSS



(a) Einarsson's scheme: $Q=17$ $L=12$

(b) (3,1) Repetition coded scheme



(c) (7,4) Cyclic coded scheme

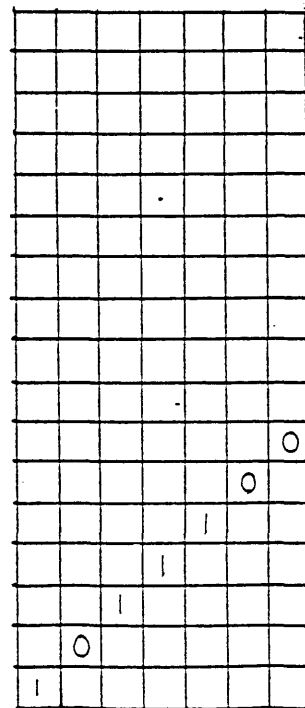


Fig.5.2 Three different ways of utilization of time-frequency for the transmission of the 4-bit word (1100)

(3,1) REPETITION CODE

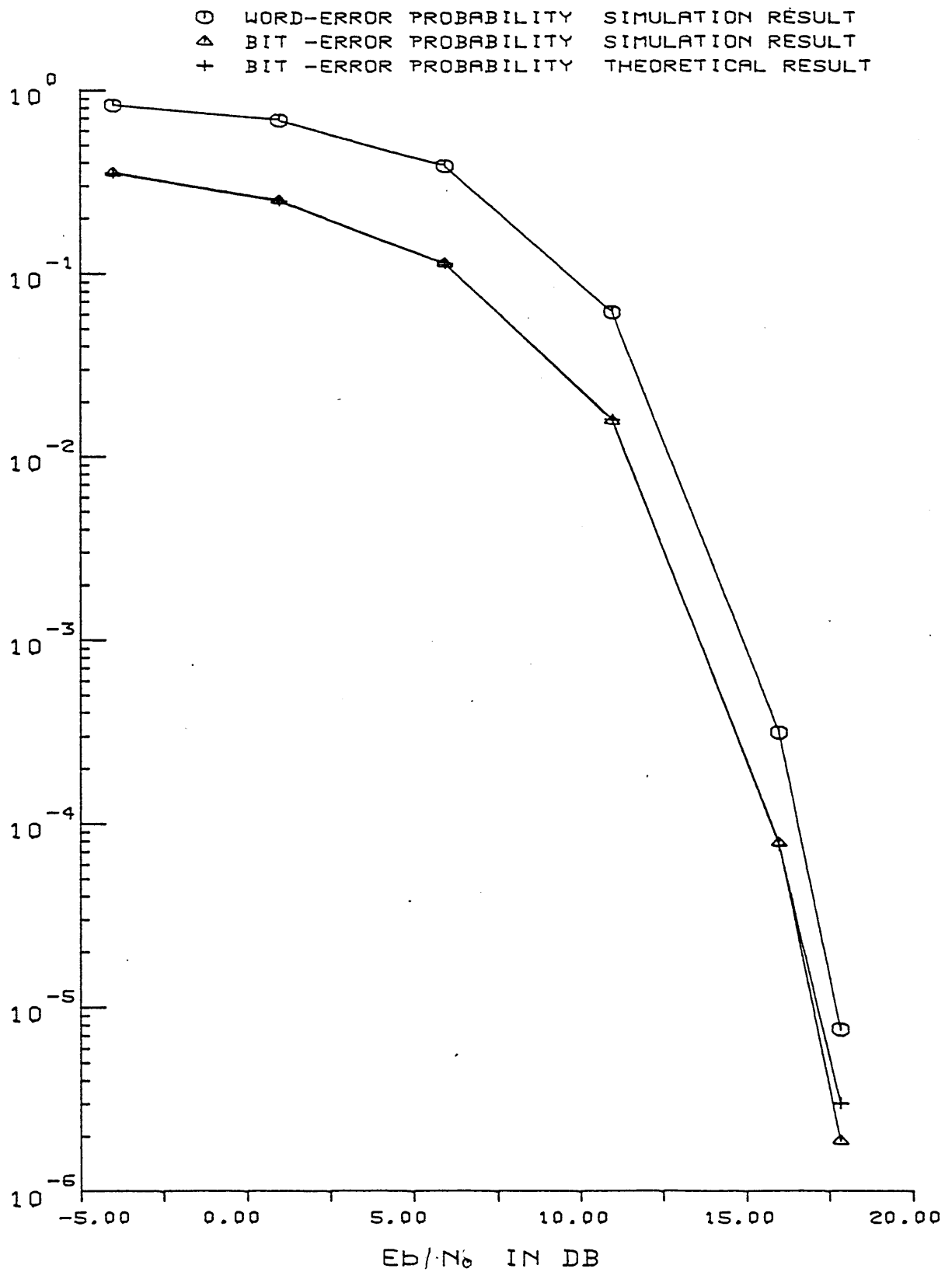


Fig.5.3 The (3,1) coded system in gaussian noise

(7,4) CYCLIC CODE

○ WORD-ERROR PROBABILITY , SIMULATION RESULTS

+ WORD-ERROR PROBABILITY , THEORETICAL , ,

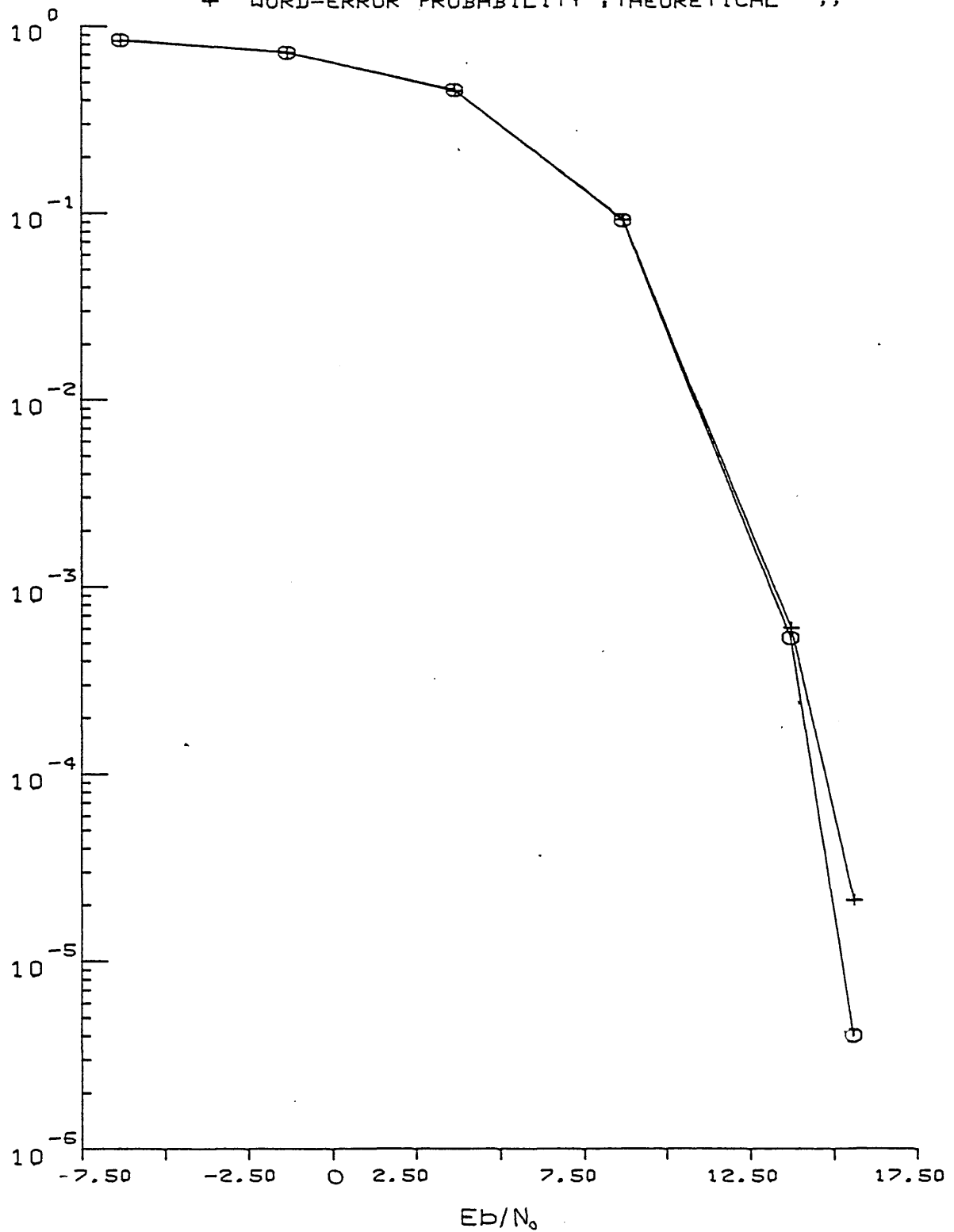


Fig.5.4 The (7,4) coded system in gaussian noise

(3,1) REPETITION CODE

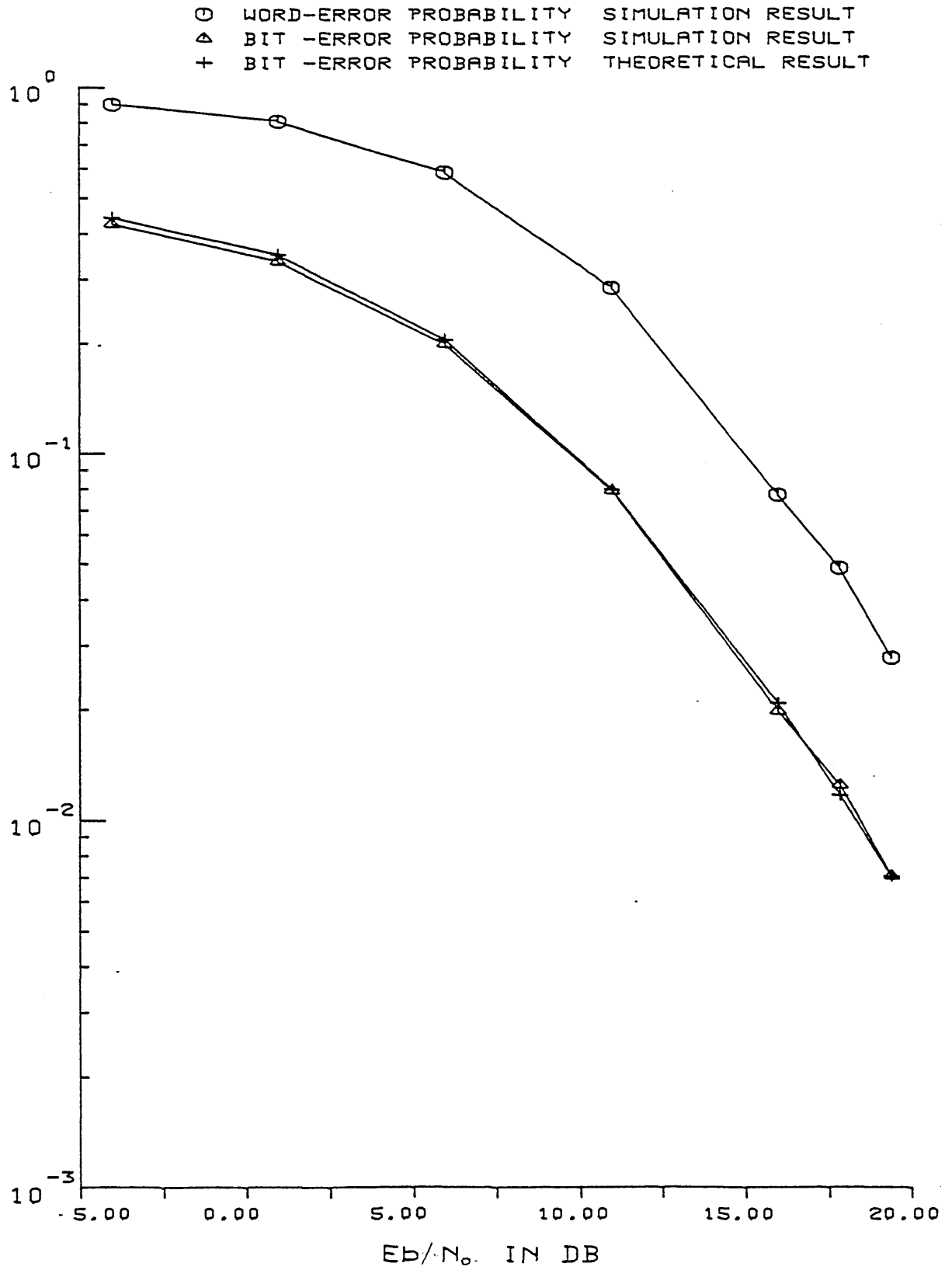


Fig.5.5 The (3,1) coded system in Rayleigh fading and gaussian noise.

(7,4) CYCLIC CODE

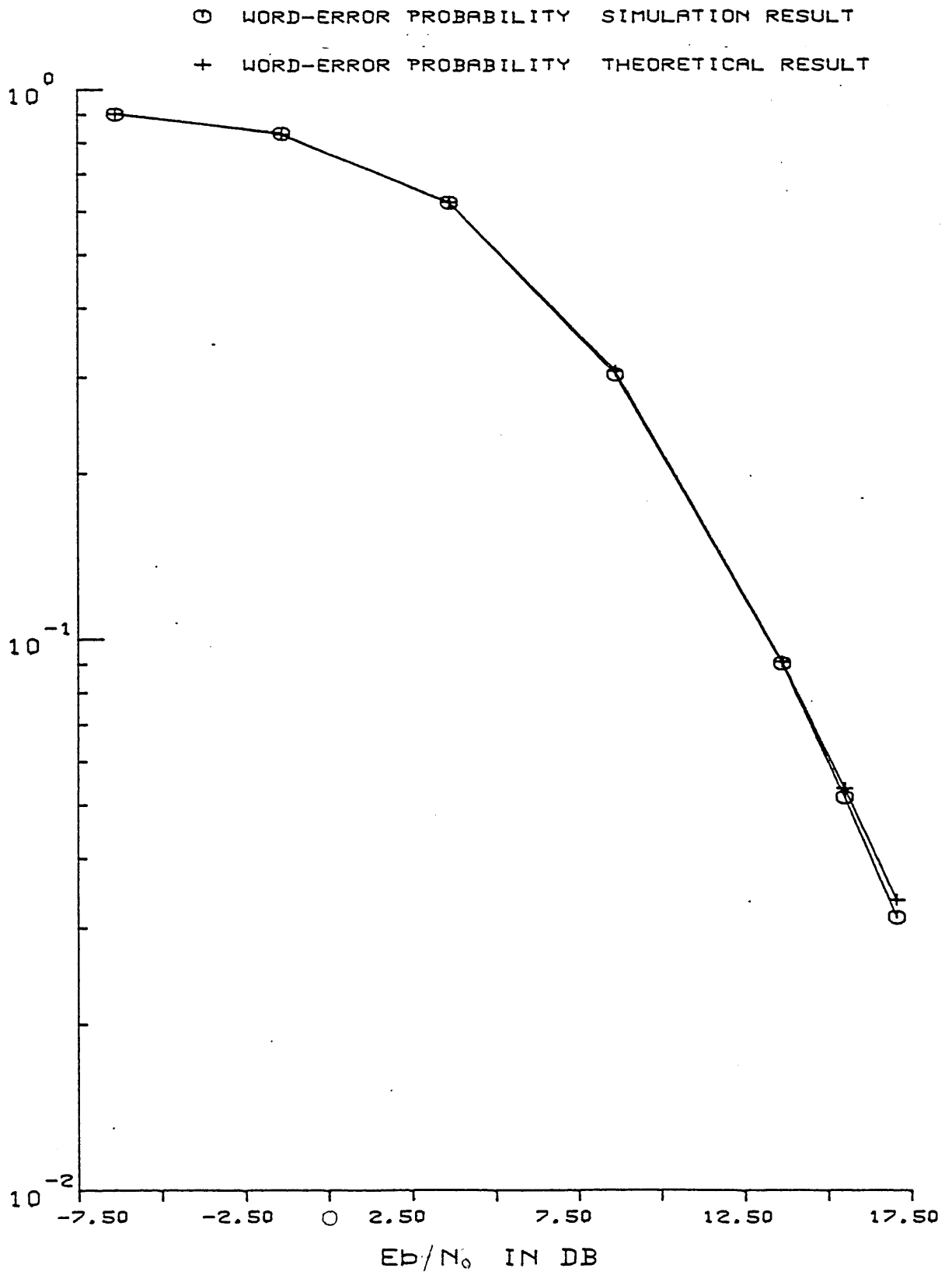
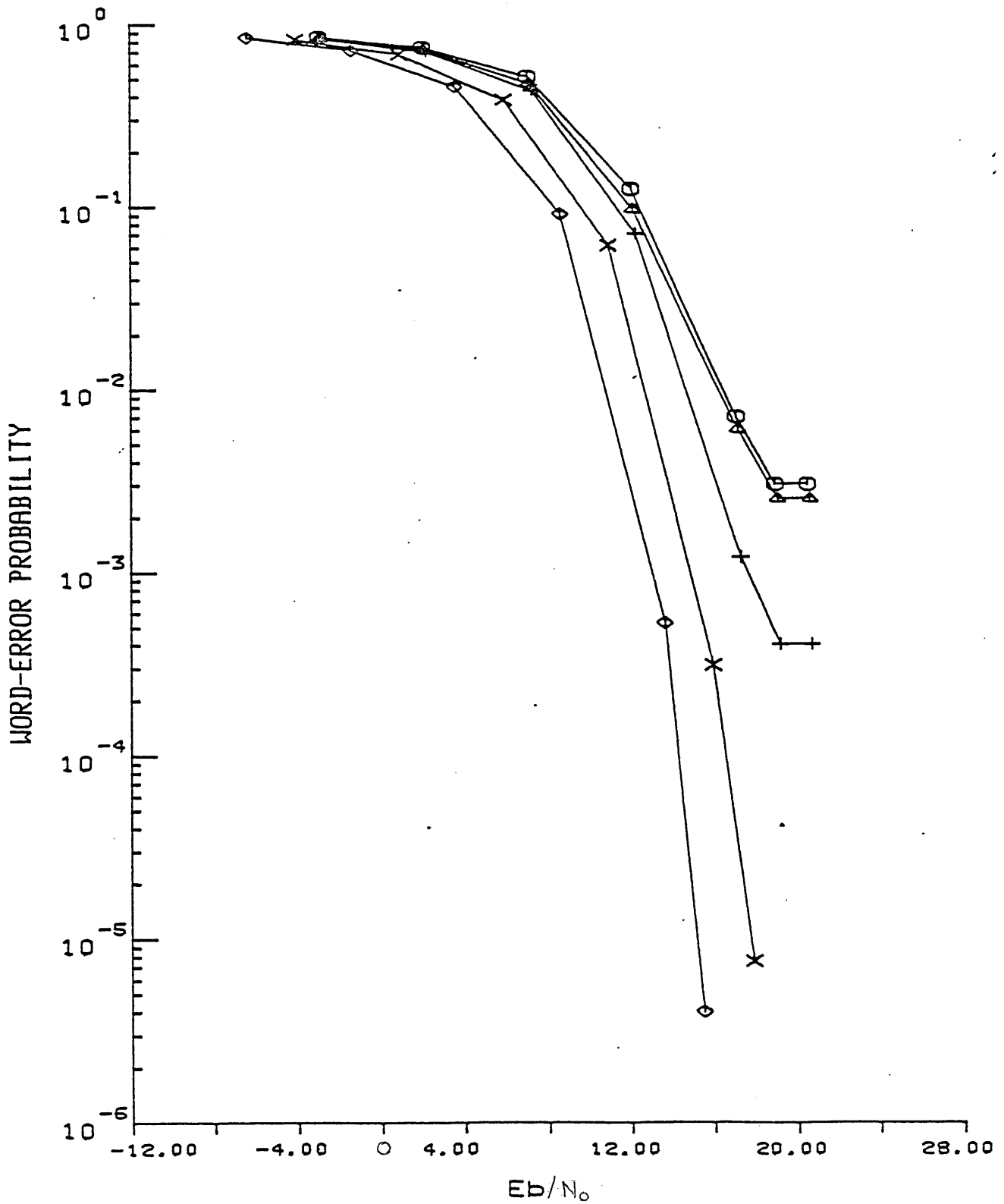


Fig.5.6 The (7,4) coded system in Rayleigh fading and gaussian noise.

: Q=16 M=16



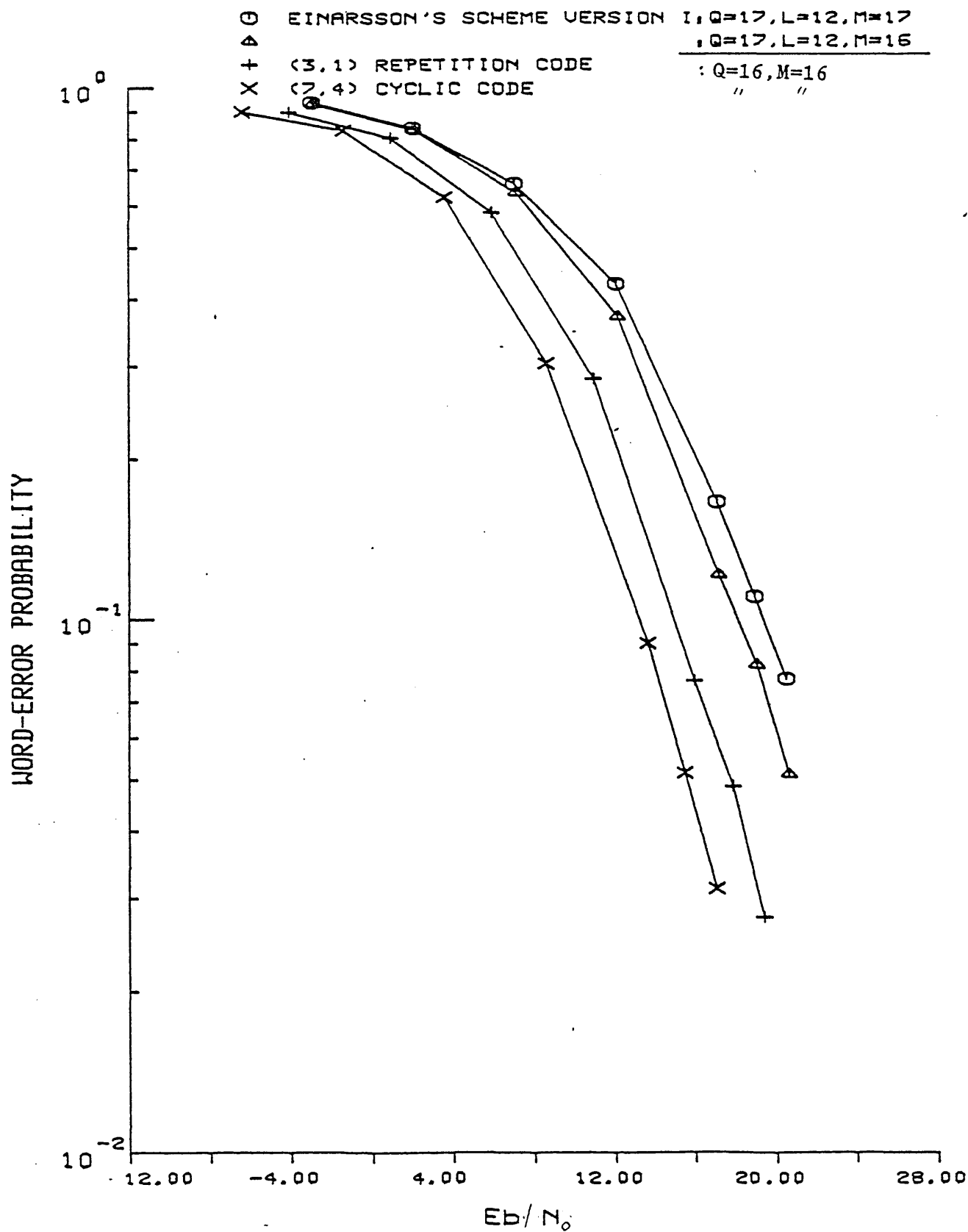


Fig.5.8 Comparison in Rayleigh fading and Gaussian noise

CHAPTER 6

THE EFFECT OF CO-CHANNEL INTERFERENCE ON THE PERFORMANCE OF TIME-FREQUENCY CODED SPREAD-SPECTRUM SYSTEMS

6.1 INTRODUCTION

From the material presented in Chapter 5 it is clear that TFCSSS of the Viterbi and Einarsson type result in an inevitable interference due to the presence of other users, but that despite this they still operate relatively satisfactorily ; even if the performance is somewhat inferior to that of coded systems of the type described in Chapter 5.

In view of this it is natural to ask how interference (defined as co-channel interference) arising from the existence of other similar systems affects the performance of TFCSSS of the Einarsson type and, if they are able to operate satisfactorily in the presence of co-channel interference, to ask whether it might be possible to develop a system in which some degree of bandwidth saving is obtained.

In this chapter the situation is considered in which two TFCSSS of the Einarsson type are operating with a degree of overlap between the sets of frequency slots allocated to the systems. Specifically the situation is considered in which two systems, each using Q frequency channels, operate and in which a number B (less than Q) of the frequency slots are common to the two systems. The situation is illustrated in Fig.6.1. Let it also be assumed that there are M users

on each of the two systems.

In this chapter the worst case operation of the interfering systems is considered both theoretically and through a computer simulation. It is assumed that in the worst case operation, the desired signal and other users' interference (from the same or the interfering system) are of equal power. In addition to the deliberately introduced co-channel interference, arising from the existence of common channels, the systems are considered to be operating over noisy fading channels of the type considered in Chapters 4 and 5. In the chapter the Version I and Version II-type systems, with these being as defined in Chapter 4, are considered.

6.2 ANALYSIS OF VERSION I-TYPE SYSTEMS IN THE PRESENCE OF CO-CHANNEL INTERFERENCE AND CHANNEL IMPAIRMENTS.

Consider two Einarsson-type systems operating as Version I in accordance with the definition given in Chapter 4. Let it be supposed that each system has associated with it a set of Q frequency slots, as before, and that a subset B ($B < Q$) of these slots are common to both systems.

As indicated in subsection 6.1 the situation is illus-

trated in Fig.6.1. The B common frequency slots give rise to an interference between systems which, for convenience, is referred to as co-channel interference.

Clearly, the situation is symmetrical between systems and hence in order to arrive at a statement as to the performance of either of the two systems, it is only necessary to consider one of the systems and the interference it experiences as a result of the existence of the other system.

Let the system being considered be referred to as the 'local' system and the interfering system as the 'neighbouring' system.

In the time-frequency matrix associated with the local systems there are Q-B frequency slots that are free from co-channel interference but in so far as a local user is concerned, there is, of course, the inherent interference due to the presence of the other users. In respect of chips associated with these Q-B frequency slots the probability of a chip being occupied due to interfering users' action is given by

$$P = 1 - \left(1 - \frac{1}{Q}\right)^{M-1} \quad (6.1)$$

With regard to the B channels that are common to the two

systems, and with regard to the use of a chip by a particular user, there exists the possibility of interference from $M-1$ local users and from M users of the neighbouring system. The probability of occupancy due to the local interfering users is

$$P = 1 - \left(1 - \frac{1}{Q}\right)^{M-1} \quad (6.2)$$

and the probability of occupancy due to neighbouring interfering users is

$$P' = 1 - \left(1 - \frac{1}{Q}\right)^M \quad (6.3)$$

It follows that any given chip in the B shared channels is occupied by interfering users with probability

$$P(\text{chip occupied}) = P + P' - P \cdot P' \quad (6.4)$$

In order to take care of channel impairments arising from fading and noise, equations (6.1) and (6.4) have to be appropriately modified. Other users' interference can occur in four possible ways as indicated in Table 6.1. The table also shows the associated probabilities, the number of entries (units of power) associated with the occupancy of the chip and the probability of deletion. In arriving at the statement contained in Table 6.1 it was assumed that deletions cannot occur when two or more users occupy a chip—see Chapter 4 for earlier discussion.

The probability P_{IC} of an insertion in any given chip contained in the B co-channels is

$$P_{IC} = P \cdot P' + [P \cdot (1 - P') + (1 - P) \cdot P'] \cdot (1 - P_D) + (1 - P) \cdot (1 - P') \cdot P_F \quad (6.5)$$

Note that in the absence of channel impairments (noise and fading) $P_D = P_F = 0$ and in this case equation (6.5) reduces to

$$P_{IC} = P + P' - P \cdot P'$$

as expected. If co-channel interference is absent, but channel impairments are present, then

$$P_{IC} = P_I = P \cdot (1 - P_D) + (1 - P) \cdot P_F$$

which, as expected, is the same as equation (4.2) of Chapter 4.

The probability of insertion averaged over all Q channels is

$$\bar{P}_I = \frac{B}{Q} \cdot P_{IC} + \left(1 - \frac{B}{Q}\right) \cdot P_I \quad (6.6)$$

6.3 ANALYSIS OF VERSION II-TYPE SYSTEMS IN THE PRESENCE OF CO-CHANNEL INTERFERENCE AND CHANNEL IMPAIRMENTS.

The analysis of this case is similar to that presented above in connection with Version I-type operation.

The probability that a given chip in the common set of B channels is occupied by local interfering users is again given by

$$P = 1 - \left(1 - \frac{1}{Q}\right)^{M-1} \quad (6.7)$$

and the probability of occupancy due to neighbouring interfering users is

$$P' = 1 - \left(1 - \frac{1}{Q}\right)^M \quad (6.8)$$

Table 6.2 shows the various possible combinations of local and co-channel interference, together with the related probabilities and the number of users occupying any given chip. The probability of deletion is also given in the table. Again it is assumed that deletions cannot take place if a chip is occupied by more than one user.

It follows that the probability of insertion in any given chip in the B channels is given by

$$\begin{aligned} P_{IC} = & (1-P) \cdot (1-P') \cdot P_F \\ & + \left[(1-P) \cdot P_i' + P_i \cdot (1-P') \right] \cdot (1-P_D) \\ & + (1-P) \cdot (P' - P_i') + P_i \cdot P_i' + P_i \cdot (P' - P_i') \\ & + (P - P_i) \cdot (1-P') + (P - P_i) \cdot P_i' \\ & + (P - P_i) \cdot (P' - P_i') \end{aligned} \quad (6.9)$$

Equation (6.9) can be rewritten as

$$\begin{aligned}
 P_{IC} = & (1-P) \cdot (1-P') \cdot P_F \\
 & + \left[(1-P) \cdot P_1' + P_1 \cdot (1-P') \right] \cdot (1-P_D) + (1-P) \cdot (P' - P_1') \\
 & + P_1 \cdot P_1' + P_1 \cdot (P' - P_1') + (P - P_1) \quad (6.10)
 \end{aligned}$$

Again, in the absence of channel impairments, $P_D = P_F = 0$ and hence equation (6.10) reduces to

$$P_{IC} = P + P' - P \cdot P'$$

as expected. With channel impairments but in the absence of co-channel interference, equation (6.10) reduces to

$$P_{IC} = P_I = (1-P) \cdot P_F + P - P_1 + P_1 \cdot (1-P_D)$$

as expected.

As in the case of Version I, an average expression for the insertion probability can be obtained by averaging over all Q channels. If this is done, then

$$\bar{P}_I = \frac{B}{Q} \cdot P_{IC} + \left(1 - \frac{B}{Q} \right) \cdot P_I \quad (6.11)$$

By using equations (6.6) and (6.11) in equations (4.3), (4.4) and (4.12) an exact expression is obtained for the word-error probability of a Viterbi Random Addressing-type scheme. Clearly, as argued previously, these expressions can

be considered as upper bounds to the performance of the Einarsson-type system when it is operating in a frame-synchronous mode.

6.4 RESULTS

Some of the results obtained relating to the performance of Einarsson-type systems operating in the presence of co-channel interference are shown in Figs.6.2-6.11. Figs.6.2-6.9 show the performance of systems for various values of Q, M, L and B . The results of simulation of the system for various combinations of the parameters are shown, together with theoretical performance figures based on equations (6.6) and (6.11). The figures show, as expected, that the performance arrived at from the use of equations (6.6) and (6.11) is in fact an upper bound to system performance. Comparison of the appropriate figures, for example, Figs.6.3 and 6.6 for Version I-type systems and Figs.6.9 and 6.8 for Version II-type systems shows that the performance in the presence of gaussian noise alone is, as expected, better than that to be had when operating in the presence of both gaussian noise and fading.

The feature of major importance to be noted from the figures concerns the way in which the performance derer-

iorates with increasing B for fixed Q, L and M . From Figs.6.4-6.9 it can be seen that for noise variances less than -10db relative to unit noise variance (or values of E_b/NR of 13db and 18db for Versions I and II respectively) the word-error probability is increased by a factor of between 2 and 5 for the values of B, M and Q examined, and that as the noise variance increases the performance approaches that of the system when operating without any co-channel interference. For the values of B, M and Q involved the situation being considered corresponds to approximately a 12% saving in channel bandwidth (30 as opposed to 34 channels).

Another feature can be observed from Figs.6.10 and 6.11. The figures show that even when considerable co-channel interference exists ($B=4$) Version II operates in a manner superior to Version I and that the degree of improvement is approximately 5db. This feature was noted in Chapter 4 when a similar improvement factor was observed to exist. A discussion of this aspect was given in Chapter 4.

LOCAL INTERFERENCE	CO-CHANNEL INTERFERENCE	ASSOCIATED PROBABILITY	NUMBER OF ENTRIES	PROBABILITY OF DELETION
Yes	Yes	$P \cdot P'$	2	0
Yes	No	$P \cdot (1-P')$	1	P_D
No	Yes	$(1-P) \cdot P'$	1	P_D
No	No	$(1-P) \cdot (1-P')$	0	—

TABLE 6.1

NUMBER OF ENTRIES DUE TO :				
LOCAL INTERFERENCE	CO-CHANNEL INTERFERENCE	ASSOCIATED PROBABILITY	TOTAL NUMBER OF ENTRIES	PROBABILITY OF DELETION
0	0	$(1-P) \cdot (1-P')$	0	—
0	1	$(1-P) \cdot P'_1$	1	P_D
0	≥ 2	$(1-P) \cdot (P' - P'_1)$	≥ 2	0
1	0	$P_1 \cdot (1-P')$	1	P_D
1	1	$P_1 \cdot P'_1$	2	0
1	≥ 2	$P_1 \cdot (P' - P'_1)$	≥ 3	0
≥ 2	0	$(P - P_1) \cdot (1-P')$	≥ 2	0
≥ 2	1	$(P - P_1) \cdot P'_1$	≥ 3	0
≥ 2	≥ 2	$(P - P_1) \cdot (P' - P'_1)$	≥ 4	0

$$\therefore P_1 = (M-1) \cdot \frac{1}{Q} \cdot \left(1 - \frac{1}{Q}\right)^{M-2}$$

$$P'_1 = M \cdot \frac{1}{Q} \cdot \left(1 - \frac{1}{Q}\right)^{M-1}$$

TABLE 6.2

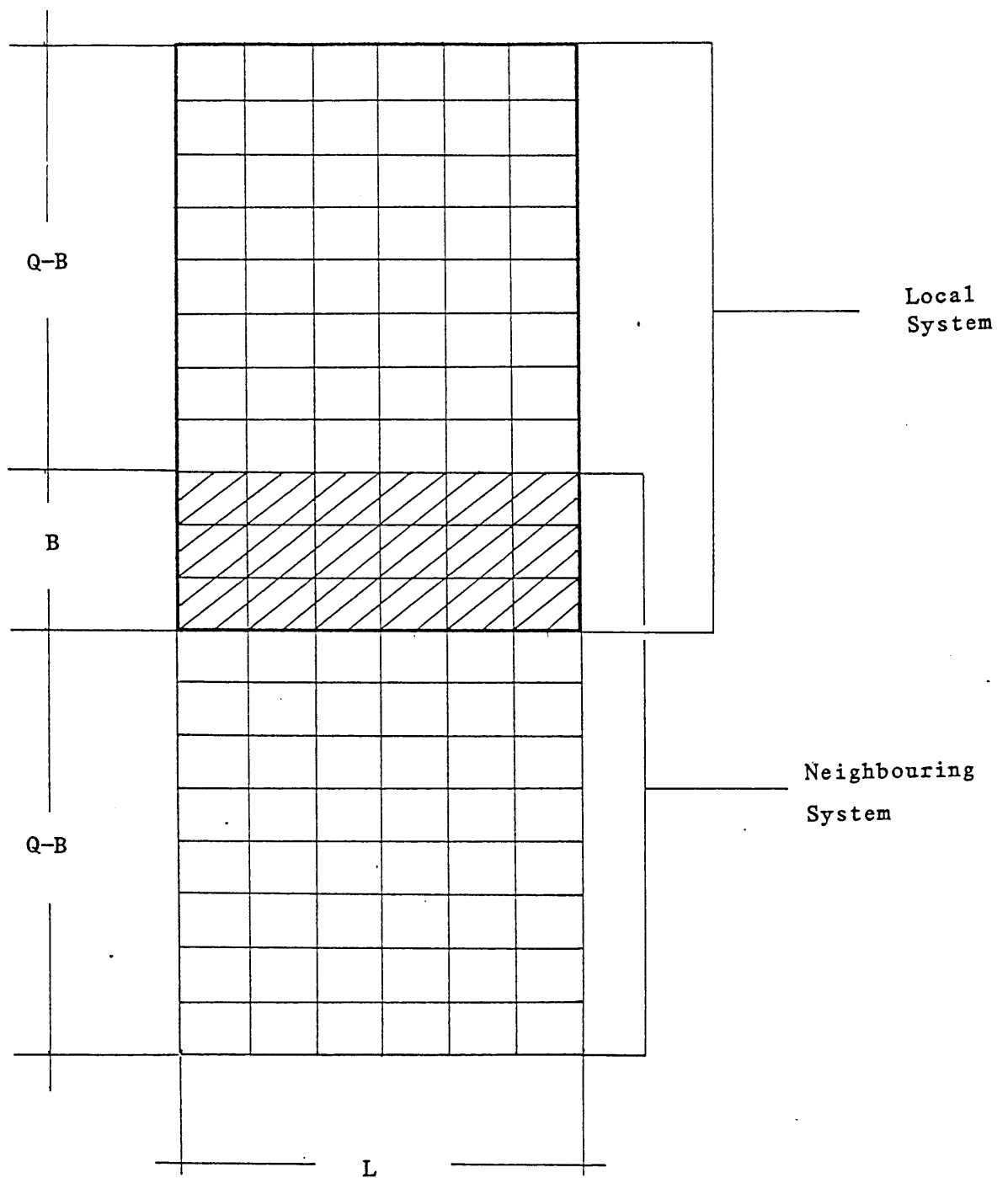


Fig.6.1 Frequency-Time matrices of the local and the neighbouring system.

VERSION I, Q=17, L=12, M=17-GAUSSIAN NOISE

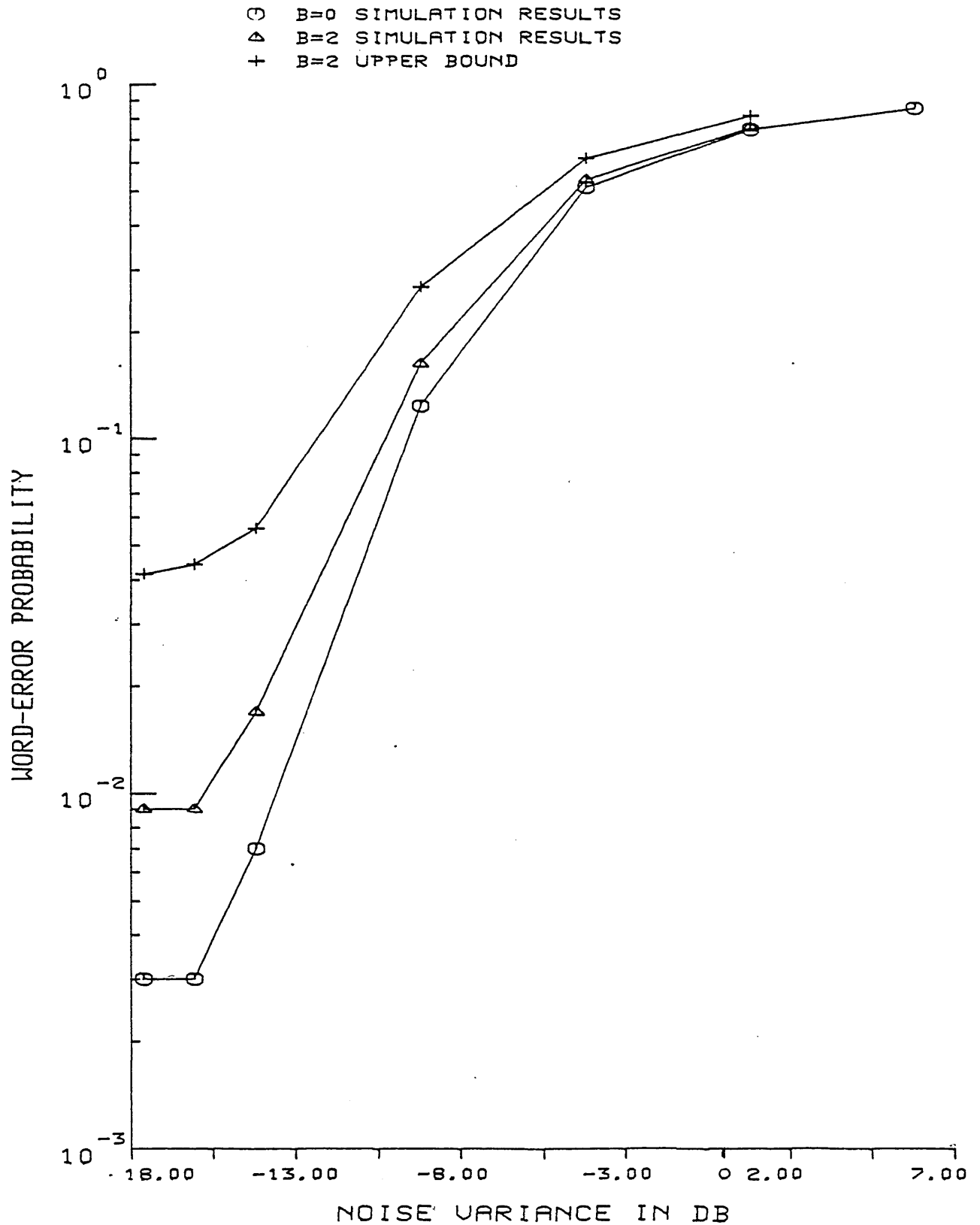


Fig.6.2 Effect of B=2 on system performance

VERSION I, Q=17, L=12, M=17-GAUSSIAN NOISE

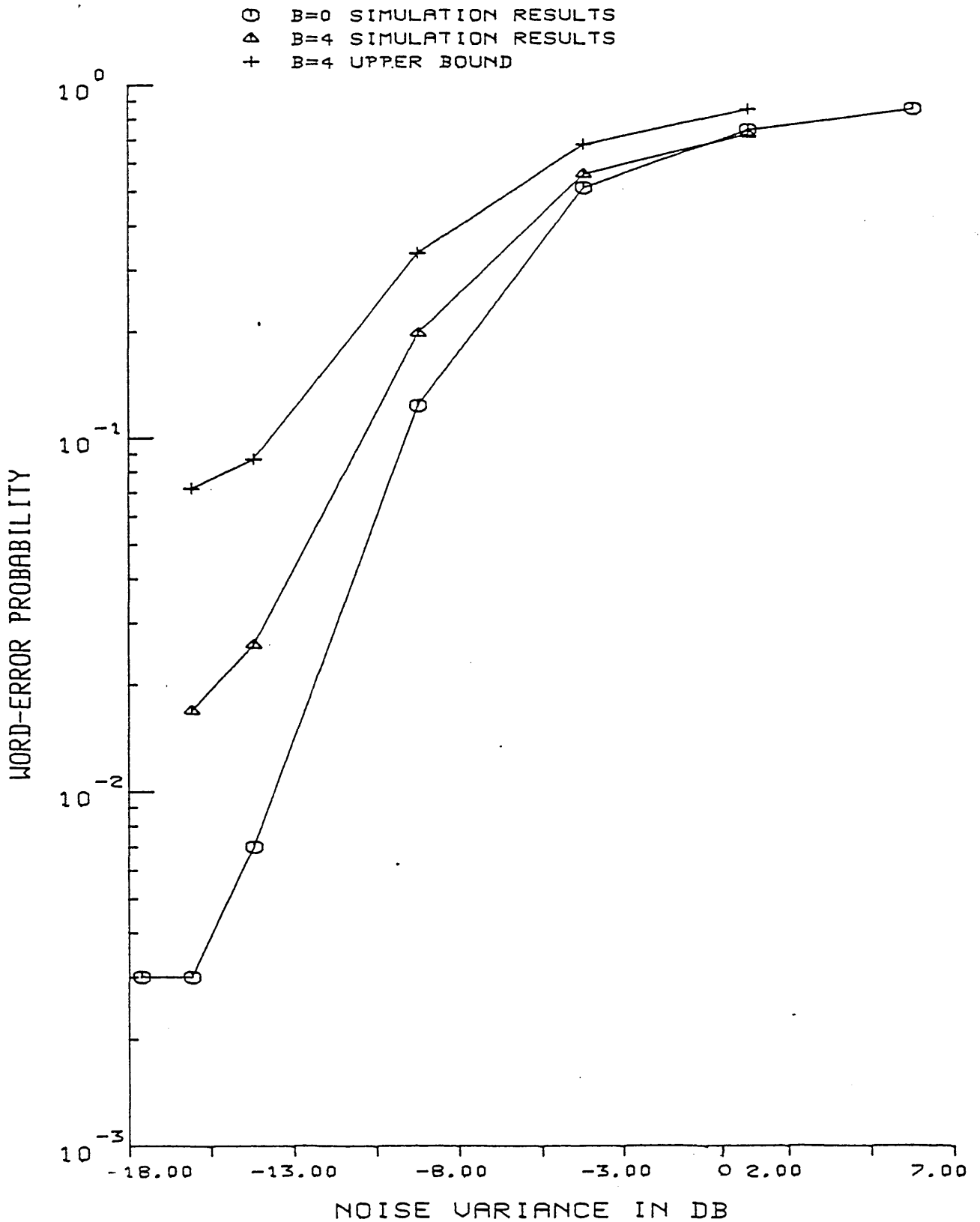


Fig.6.3 Effect of B=4 on system performance

VERSION I; Q=17, L=12, M=16-GAUSSIAN NOISE

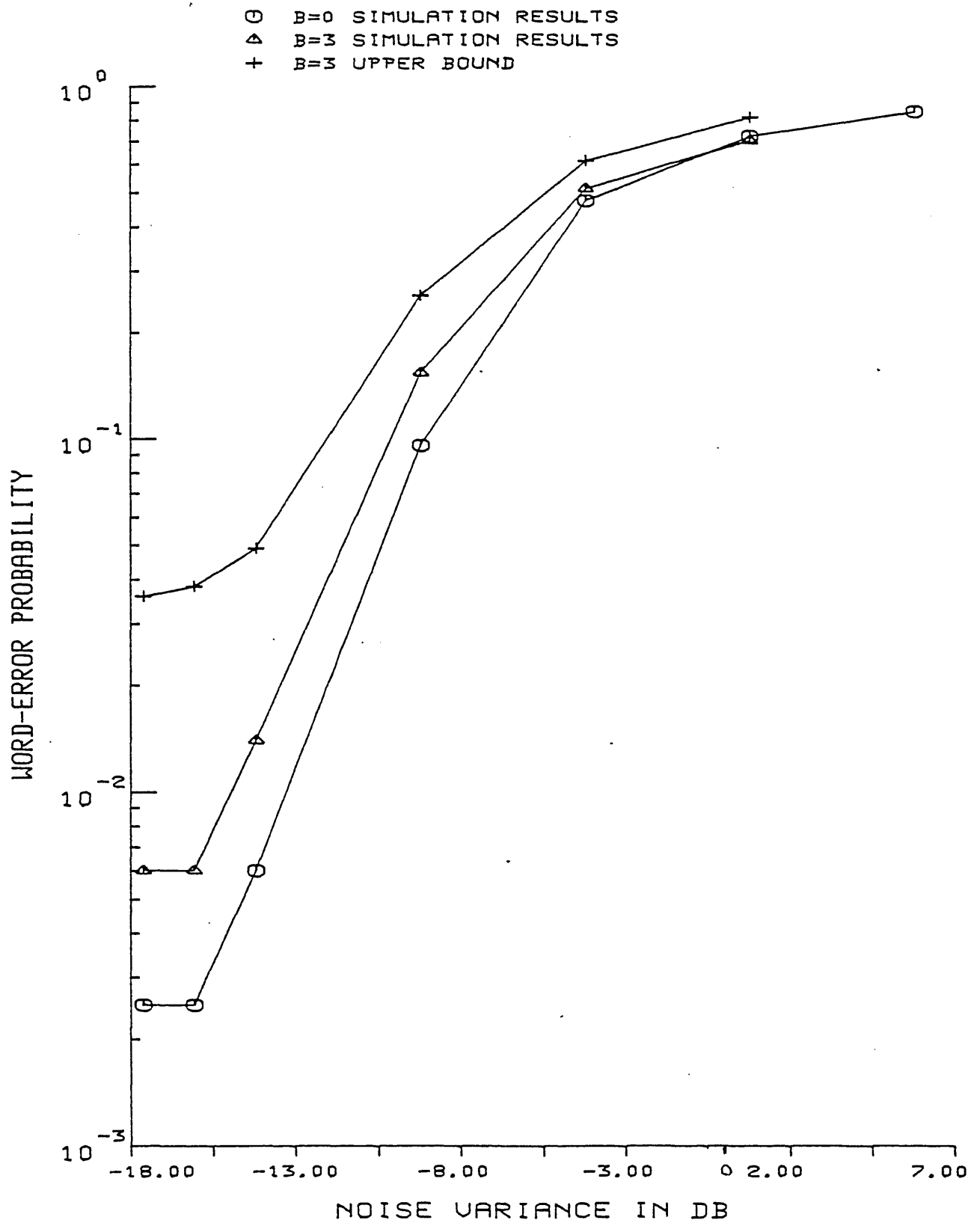


Fig.6.4 Effect of B=3 on system performance.

VERSION I; Q=17, L=12, M=16-GAUSSIAN NOISE

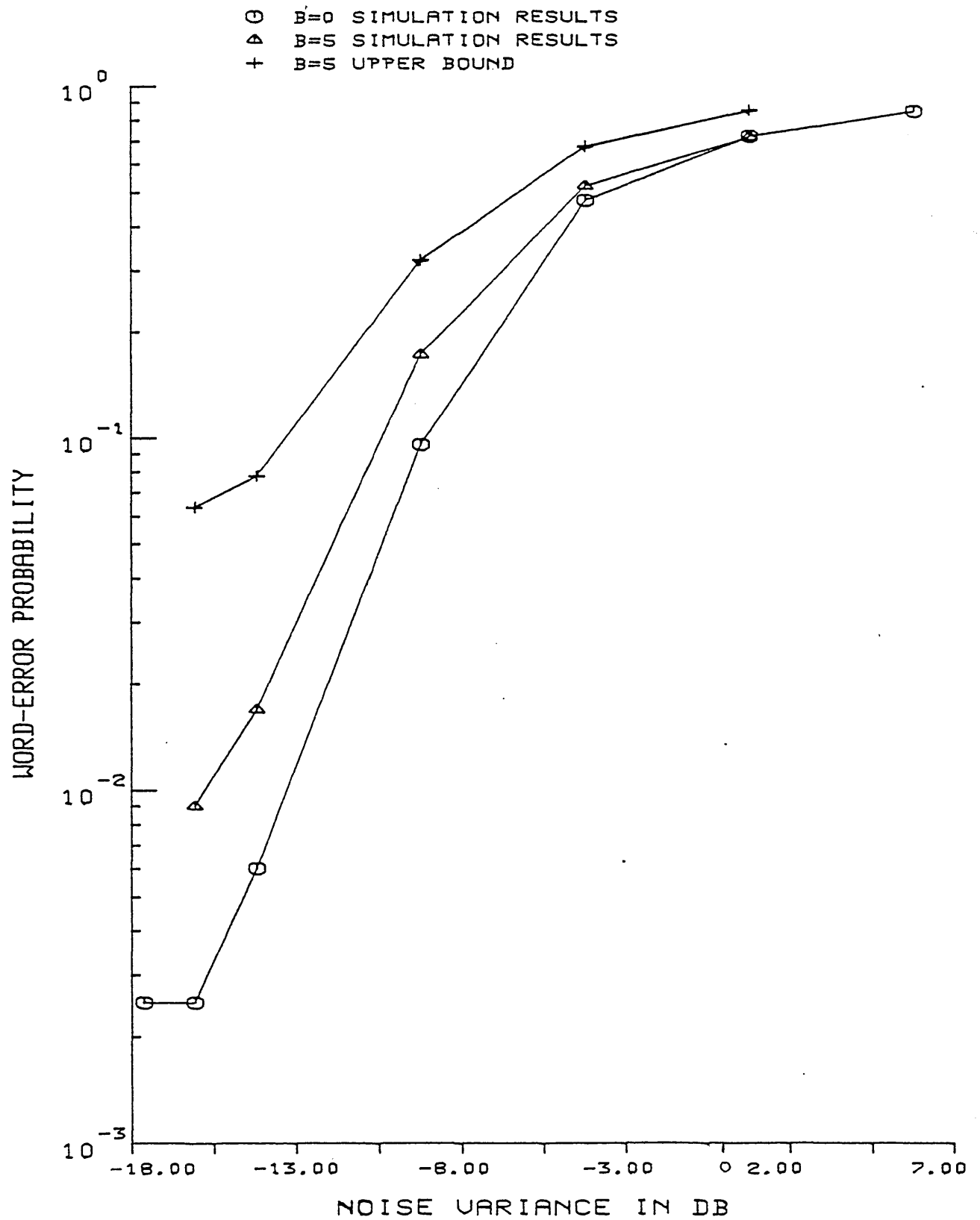


Fig.6.5 Effect of B=5 on system performance.

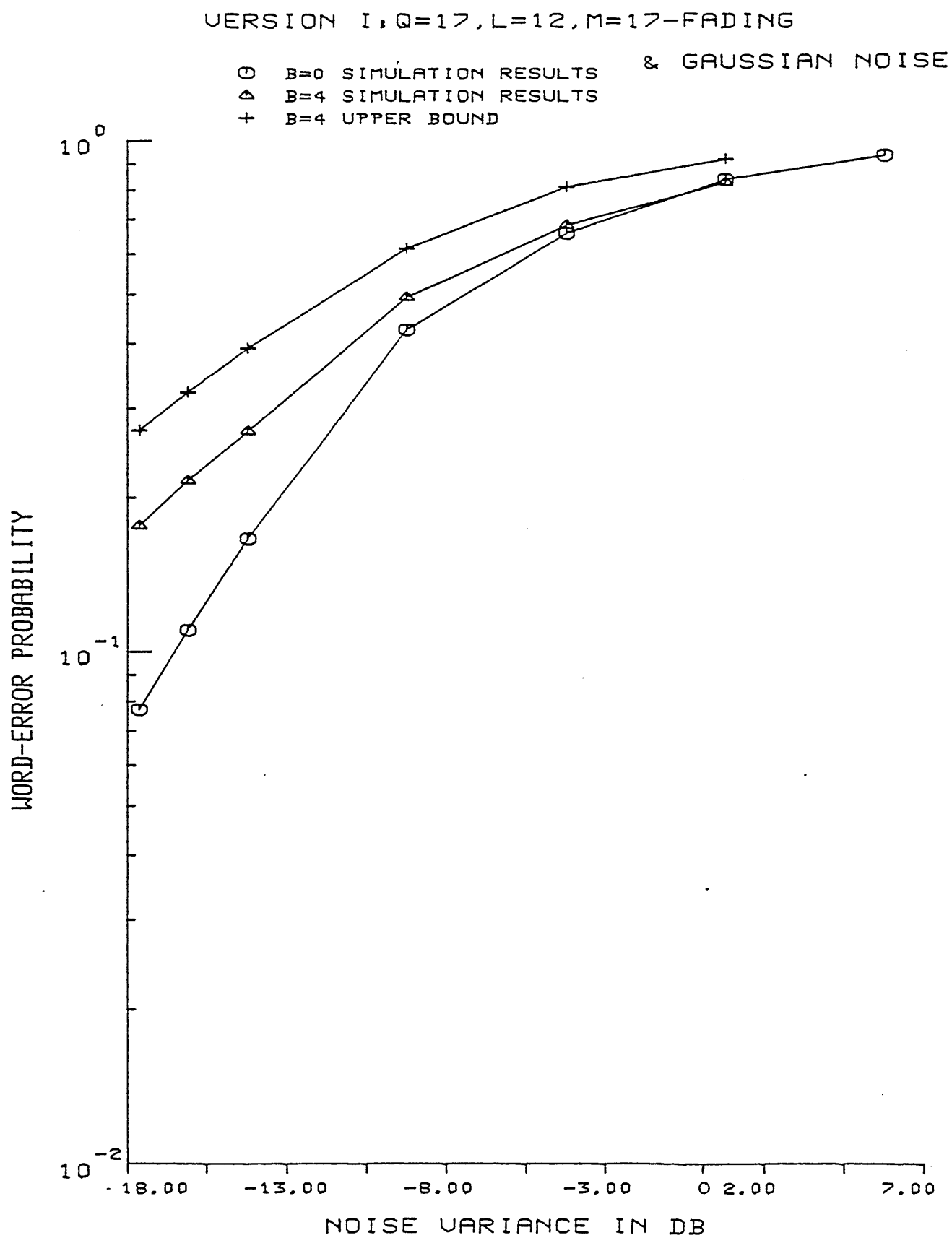


Fig.6.6 Effect of B=4 on system performance.

VERSION I, Q=17, L=12, M=16

FADING

○ B=0 SIMULATION RESULTS
 ▲ B=3 SIMULATION RESULTS
 + B=3 UPPER BOUND

& GAUSSIAN NOISE

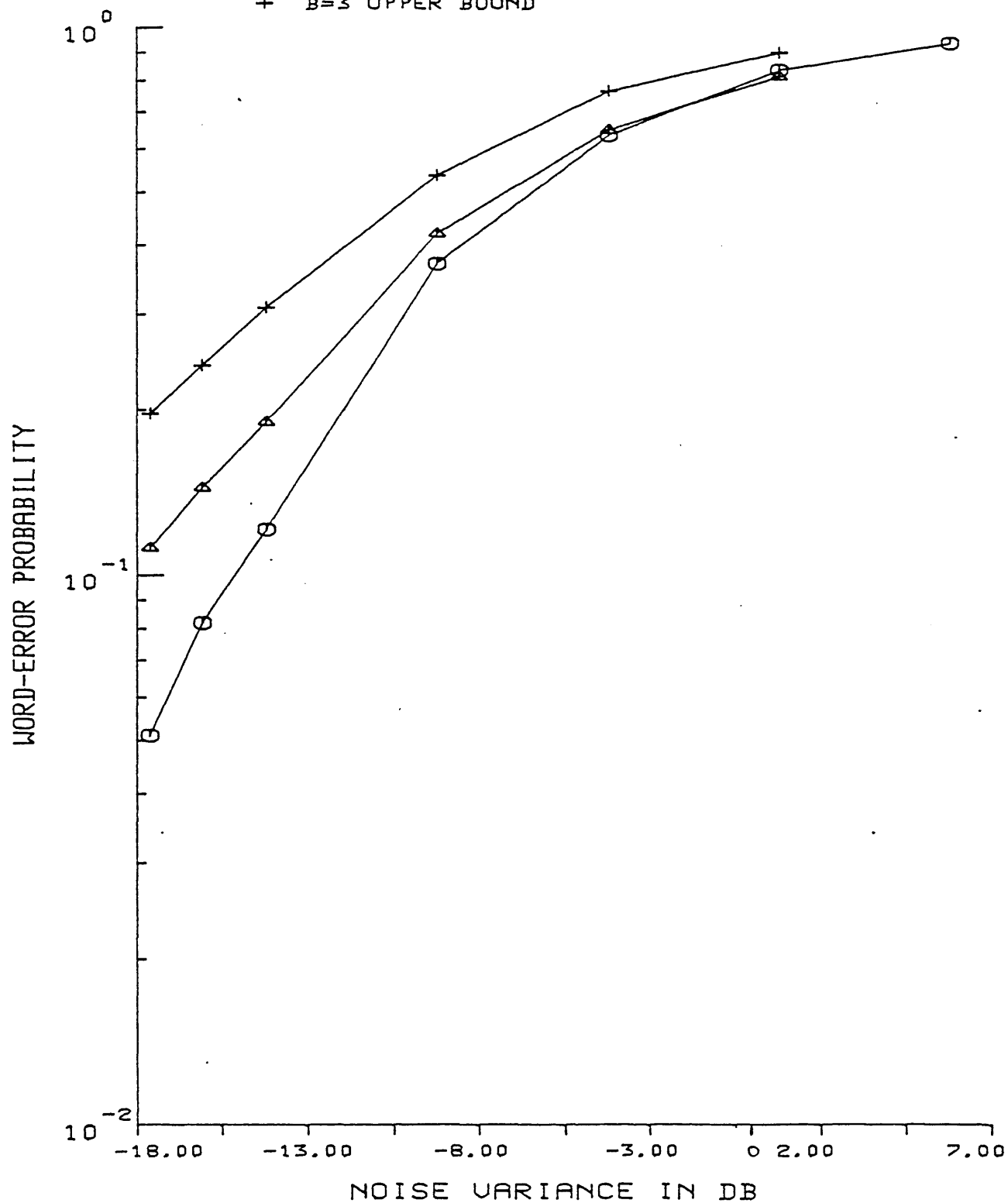


Fig.6.7 Effect of B=3 on system performance.

VERSION II: Q=17, L=12, M=17-GAUSSIAN NOISE

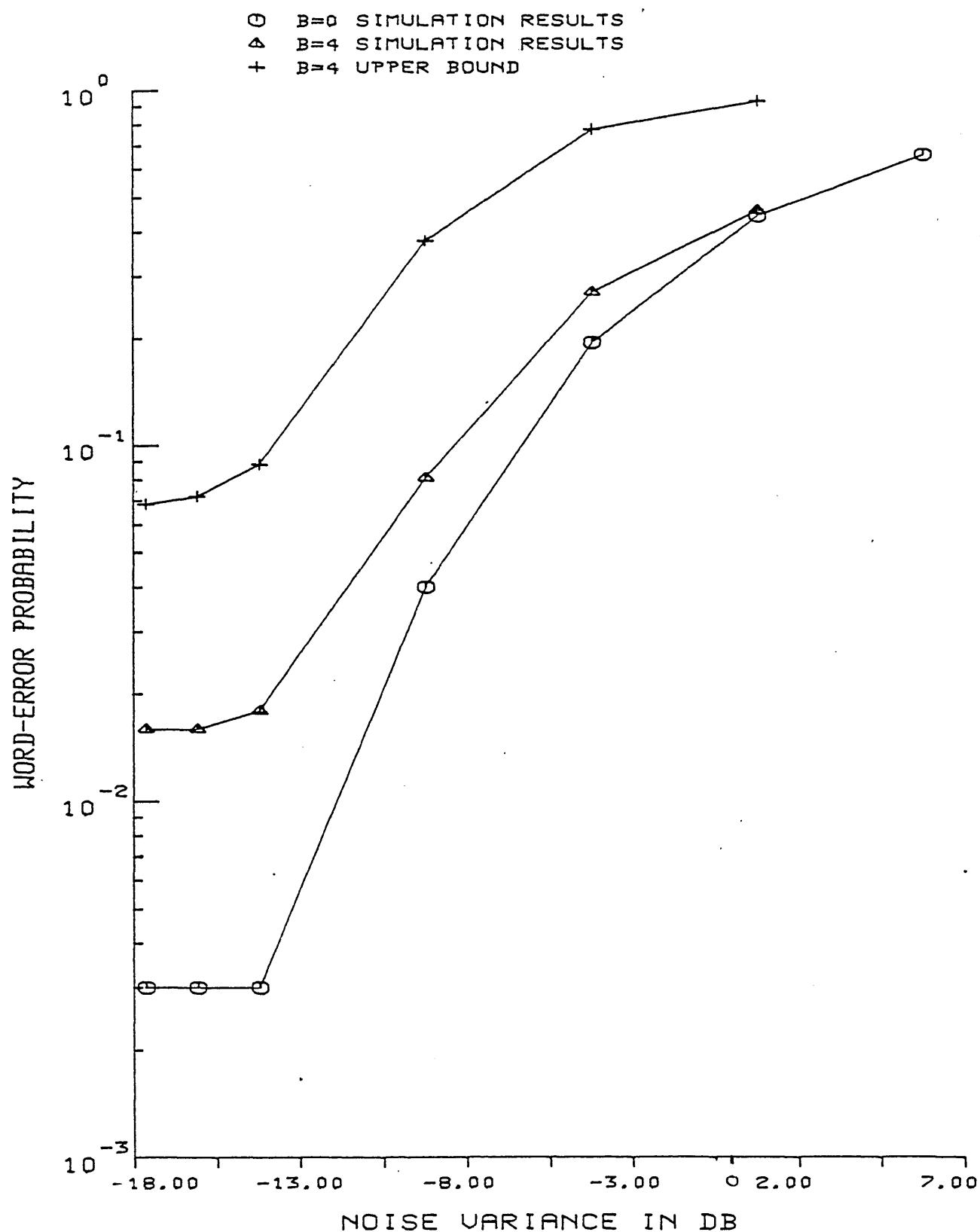


Fig.6.8 Effect of B=4 on system performance

VERSION II: Q=17, L=12, M=17-FADING

& GAUSSIAN NOISE

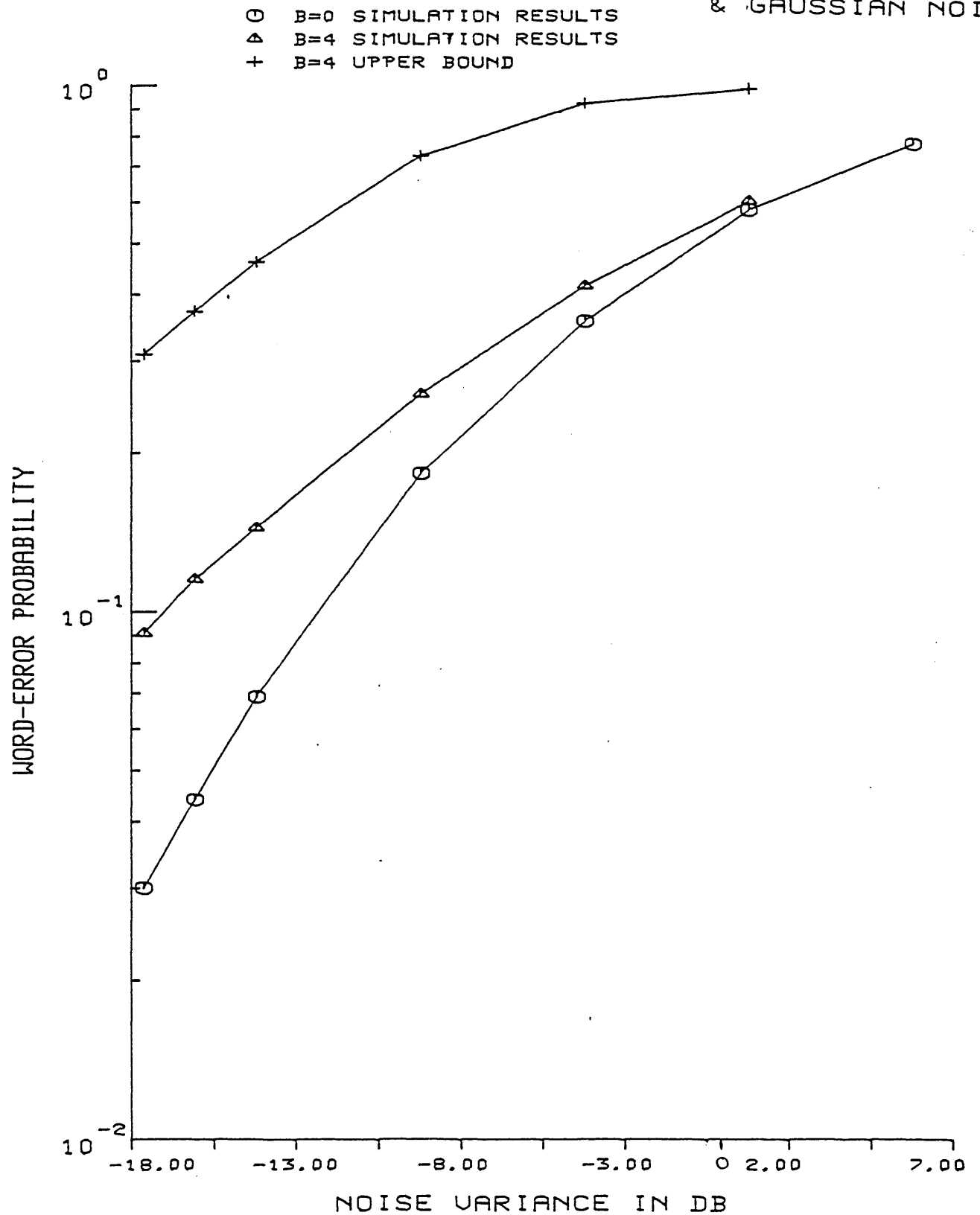


Fig.6.9 Effect of B=4 on system performance.

EINARSSON'S SCHEME, $Q=17, L=12, M=17$

○ VERSION I, SIMULATION RESULTS FOR $B=4$
 ▲ VERSION II, ..

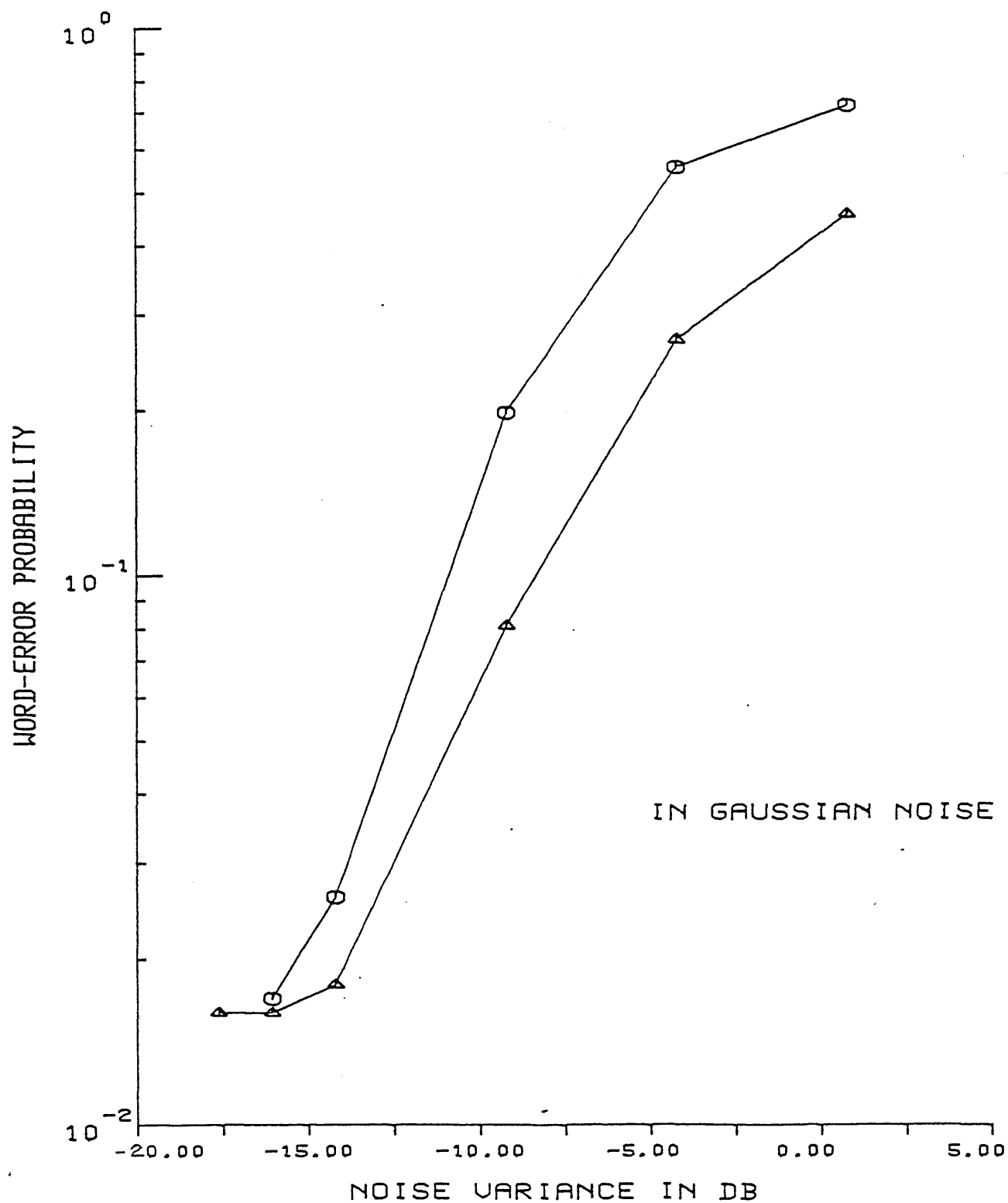


Fig.6.10 Comparison of Versions I and II for $B=4$.

EINARSSON'S SCHEME: $Q=17, L=12, M=17$

○ VERSION I, SIMULATION RESULTS FOR $B=4$
△ VERSION II

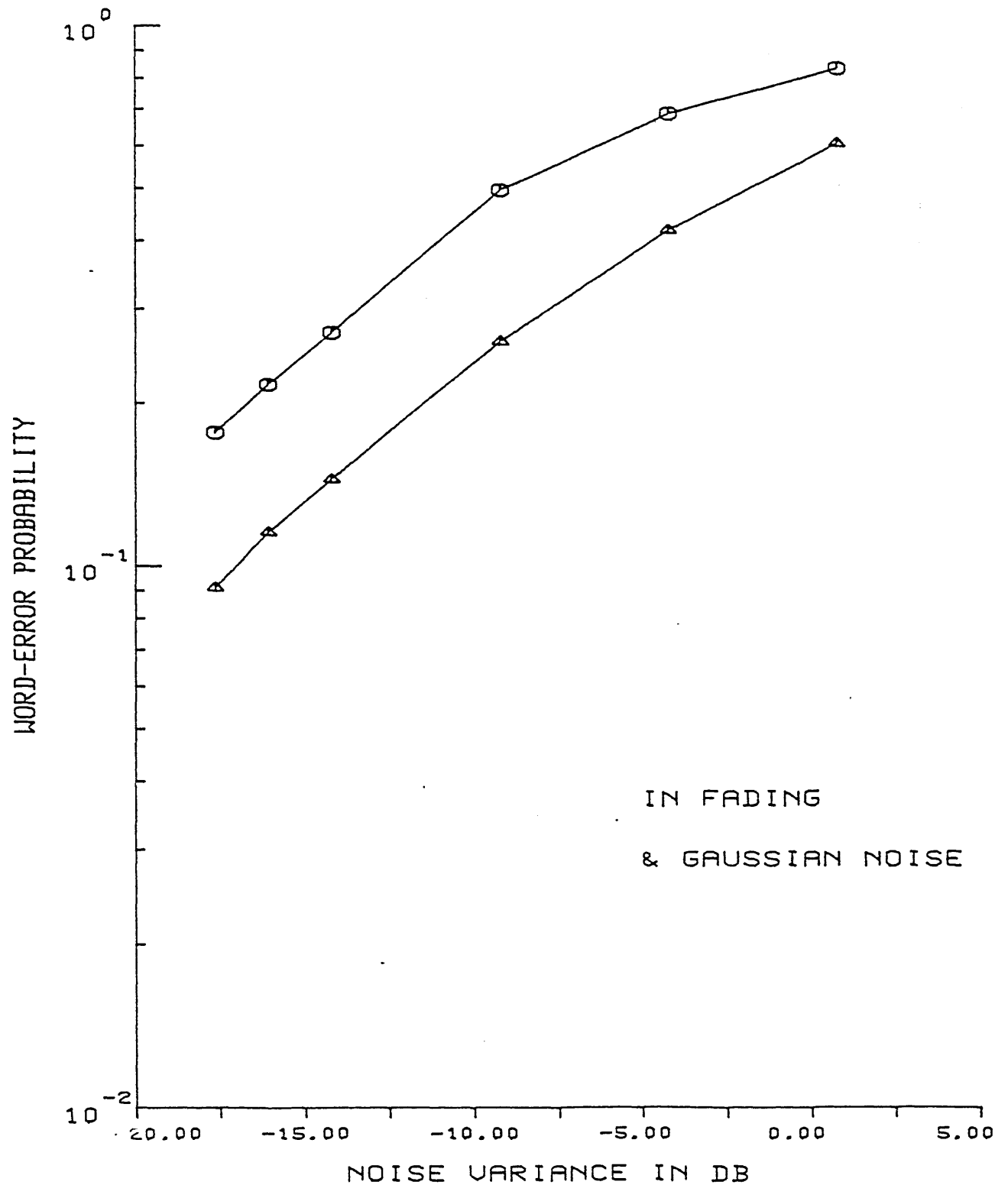


Fig.6.11 Comparison of Versions I and II for $B=4$.

CHAPTER 7

CONCLUSIONS AND SUGGESTIONS

FOR FUTURE WORK

7.1 CONCLUSIONS

The problem of utilizing time and frequency with a view to providing a communication system suitable for multi-user operation is of considerable interest, particularly in respect of Mobile Radio. The use of TFCSSS in this context has been suggested by Goodman et al and the problem of optimum address selection has been solved by Einarsson. However, a rigorous mathematical analysis of the performance of TFCSSS is not available in the literature.

In this thesis the performance of TFCSSS employing Einarsson's optimum addressing scheme has been studied in some detail and theoretical as well as simulation results have been obtained. Exact expressions for the word-error probability have been derived for operation in both the synchronous and asynchronous modes, when system performance is limited only by other users' interference. In the synchronous case the exact expression is found to be approximately an order of magnitude tighter than any known upper bound. The complexity of the word-error probability expression in the asynchronous case is such that its computation is impracticable, and in the thesis a group of three new relatively easily computable upper bounds is derived. The tightest bound is sufficiently tight for it to be used as a reliable estimate of system performance.

The distribution of energy in a frequency-time matrix with Q channels available to M users has been studied in the synchronous case and an exact expression has been derived for the probability density function of the number of occupied chips per column. This is of importance because it facilitates calculation of the energy (power) requirements of the system. A relationship has been derived between the word-error probability and (i) the average transmitter power in the base-to-mobile case and (ii) an operational expression for the signal-to-interference ratio. This relationship can be used as an aid to system design.

In addition to considering the situation in which the performance is limited only by other users' interference, the practical situation has been considered in which operation takes place in the presence of both other users' interference and channel impairments, such as fading and additive noise. A new exact expression for the word-error probability of a TFCSSS with Viterbi's Random Addressing scheme has been derived. This expression provides the tightest known upper bound to the performance of the Einarsson scheme.

The performance of the TFCSSS clearly depends on the energy content of each occupied chip. In the thesis various ways in which transmitter power might be used are consi-

dered. It is shown that if the possibility of using an increased level of transmitter power exists, then it is better to increase the power per occupied chip and keep it constant at the increased level, irrespective of the number of users actually occupying the chip, rather than to use a power level that increases as the number of users occupying the chip increases.

An alternative to the Viterbi and Einarsson schemes of multi-user frequency-hopped communication has been examined. The alternative, which involves the combined use of frequency-hopping and error correction has been shown to have a performance which is significantly superior to that of the Einarsson, and hence that of the Viterbi addressing scheme.

The performance of TFCSSS operating with a number of common channels has been examined in Chapter 6. Simulation results are given for operation in the worst case and a new exact expression for the word-error probability has been derived for the Viterbi addressing scheme operating in the worst case. This expression is shown to be a tight upper bound on the worst-case word-error probability performance of the Einarsson addressing scheme. The performance of such systems, although inferior to that of the stand-alone system, is not dramatically different. Consequently, it may be advantageous to employ them in cases where bandwidth economy

is the chief consideration.

7.2 SUGGESTIONS FOR FUTURE WORK

In this thesis the performance of TFCSSS and of an alternative coded system was studied in some detail in the case of operation over noisy fading channels, and the impairments were assumed to be independent from chip to chip of the transmission matrix. This assumption is unrealistic in many cases where the errors result from fading, and/or bursts of interference noise. In this situation errors tend to occur in bursts. One way of dealing with such errors is to use time-interleaving. An interesting system worthy of investigation would be that which makes combined use of time-interleaving and frequency-hopping as employed in TFCSS systems.

Another area worth considering concerns the combined use of TFCSSS with bandwidth compression techniques. It is known that good speech quality can be obtained at bit rates at and below 32Kbs. Any decrease in user data rate (or equivalent increase of maximum bit-error probability) would admit more simultaneous users on to the system. To facilitate this increase in system efficiency a coder is needed that has (subjective) performance almost independent of the

bit-error probability. For example, a VSDM coder operating at 19.2 Kbs would probably maintain a satisfactory level of intelligibility at bit-error rates of about 0.01. It would be interesting to study the level of system performance that can be obtained through the combined use of source coding and time-frequency coding.

In the TFCSSS, as proposed by Viterbi, ON/OFF signalling is suggested for the occupied/unoccupied chips. It could be interesting to study the performance of the system when bipolar signalling is involved (with the corresponding 3db reduction in power requirements). The possibility of multi-level signalling, is also worth investigating.

The question of increasing the efficiency for TFCSSS has been considered by Haskell and Timur who derived an algorithm that makes use of other users' interference to assist the decision maker at the receiver. Haskell gives simulation results for the noiseless case only. It is, therefore, important to investigate whether and to what extent the performance improvement achieved by his algorithm is maintained when the system operates in noisy environments of the kind considered in this thesis, and also in the presence of burst-type errors.

The practical problem of the design of filters suitable

for the frequency slots of a TFCSSS has not as yet been studied. The type and specifications of these filters are important because they have direct influence on the cost and efficiency of the system.

Finally, the possibility of using TFCSSS so that they occupy simultaneously the same bandwidth as conventional systems such as FDM, is of potential importance since it may be that system development will require TFCSSS to be used as an 'overlay' with existing FDM-type systems. The study of this combined operation is worthy of investigation.

APPENDICES

APPENDIX 1

THE CLASSIC OCCUPANCY PROBLEM

Suppose that r balls are distributed among n cells and each of the n possible distributions has probability n^{-r} . Let $A(r, n)$ be the number of distributions leaving none of the n cells empty. Show that

$$A(r, n+1) = \sum_{k=1}^r \binom{r}{k} \cdot A(r-k, n) \quad (A 1.1)$$

Proof:

r balls occupy $n+1$ cells and no cell is left empty, therefore $r \geq n+1$. If k balls are in a given cell the remaining $r-k$ balls can be distributed over the remaining n full cells in $A(r-k, n)$ ways. The k balls that occupy the given cell can be selected in $\binom{r}{k}$ different ways. It follows that the number of distributions of $r-k$ balls in n cells is

$$\binom{r}{k} \cdot A(r-k, n)$$

Clearly, the number k of balls that occupy the given cell is bound by the double inequality

$$1 \leq k \leq r$$

Hence

$$0 \leq r-k \leq n-1$$

For $r-n+1 \leq k \leq r$ it can be seen that $A(r-k, n)=0$ because there are more cells available than balls and thus we may write

$$A(r, n+1) = \sum_{k=1}^r \binom{r}{k} \cdot A(r-k, n)$$

Using the equation above prove that

$$A(r, n) = \sum_{y=0}^n (-1)^y \binom{n}{y} \cdot (n-y)^r \quad (A.1.2)$$

This can be easily proved by induction:

The case for $n=1$ is trivial.

Assume that the equation holds for some value n , ie.

$$A(r, n) = \sum_{y=0}^n (-1)^y \binom{n}{y} \cdot (n-y)^r$$

then

$$A(r-k, n) = \sum_{y=0}^n (-1)^y \binom{n}{y} \cdot (n-y)^{r-k}$$

This can be used with equation (A1.1) to give

$$A(r, n+1) = \sum_{k=1}^r \binom{r}{k} \cdot \sum_{y=0}^n (-1)^y \binom{n}{y} \cdot (n-y)^{r-k}$$

by rearranging the terms and applying Newton's formula we have that

$$A(r, n+1) = \sum_{y=0}^n (-1)^y \binom{n}{y} \cdot (n-y)^r \cdot \left[\left(1 + \frac{1}{n-y} \right)^r - 1 \right]$$

which after straightforward manipulation may be written as

$$A(r, n+1) = (n+1)^r - \sum_{\ell=1}^m (-1)^{\ell-1} \cdot \left[\binom{n}{\ell} + \binom{m}{\ell-1} \right] \cdot (n-(\ell-1))^r \quad (A.1.3)$$

By using the identity

$$\binom{n}{\ell} + \binom{n}{\ell-1} = \binom{n+1}{\ell}$$

equation (A1.3) may be rewritten as

$$A(r, n+1) = (n+1)^r - \sum_{\ell=1}^n (-1)^{\ell-1} \cdot \binom{n+1}{\ell} \cdot (n-\ell+1)^r$$

or finally

$$A(r, n+1) = \sum_{\ell=0}^{n+1} (-1)^{\ell} \cdot \binom{n+1}{\ell} \cdot (n+1-\ell)^r \quad \text{Q.E.D.}$$

The number of ways in which exactly m empty cells can be chosen from a total of n cells is $\binom{n}{m}$, and the number of distributions leaving none of the remaining $n-m$ cells empty is $A(r, n-m)$, therefore the number of distributions leaving exactly m empty cells is

$$E_m(r, n) = \binom{n}{m} \cdot A(r, n-m)$$

and since all n^r distributions are equiprobable the probability of finding exactly m empty cells satisfies

$$p_m(r, n) = n^{-r} \cdot E_m(r, n)$$

APPENDIX 2

DERIVATION OF $P_{FR-SYNC}$ FROM $P_{FR-ASYNC}$

When all interferers are frame synchronous with the desired user

$$u_0 = M-1, \quad u_i = 0 \quad \text{for } i=1, 2, \dots, L-1$$

Also $c_i = 0$ for $i=1, 2, \dots, L-1$. It follows that $q_{i,n} = 0$ for $i=1, 2, \dots, L-1$ and $q_{0,n} = M-1 - \sum_{k=1}^{n-1} v_{0,k}$

Equation (3.21) may be rewritten as

$$P_{FR-ASYNC} = \sum_{v_{0,1}=0}^{M-1} \sum_{v_{0,2}=0}^{M-1-v_{0,1}} \dots \sum_{v_{0,L-1}=0}^{M-1-\sum_{k=1}^{L-2} v_{0,k}} \sum_{v_{0,L}=0}^{M-1-\sum_{k=1}^{L-1} v_{0,k}}$$

$$\prod_{n=1}^L \left\{ \binom{q_{0,n}}{v_{0,n}} \left(\frac{1}{Q-(n-1)} \right)^{v_{0,n}} \cdot \left(1 - \frac{1}{Q-(n-1)} \right)^{q_{0,n}-v_{0,n}} \right\}.$$

$$\prod_{n=1}^L \left(1 - \delta_{0, v_{0,n}} \right)$$

which, after neglecting the subscript 0 and rearranging the terms becomes

$$P_{FR-ASYN} = \sum_{V_1=1}^{M-1} \sum_{V_2=1}^{M-1-V_1} \dots \sum_{V_{L-1}=1}^{M-1-\sum_{k=1}^{L-2} V_k}$$

$$\prod_{\eta=1}^{L-1} \binom{M-1-\sum_{k=1}^{\eta-1} V_k}{V_\eta} \left(\frac{1}{Q-(\eta-1)} \right)^{V_\eta} \cdot \left(1 - \frac{1}{Q-(\eta-1)} \right)^{M-1-\sum_{k=1}^{\eta} V_k}.$$

$$\sum_{V_L=1}^{M-1-\sum_{k=1}^{L-1} V_k} \binom{M-1-\sum_{k=1}^{L-1} V_k}{V_L} \left(\frac{1}{Q-(L-1)} \right)^{V_L} \cdot \left(1 - \frac{1}{Q-(L-1)} \right)^{M-1-\sum_{k=1}^L V_k}$$

or equivalently,

$$P_{FR-ASYN} = \sum_{V_1=1}^{M-1} \sum_{V_2=1}^{M-1-V_1} \dots \sum_{V_{L-1}=1}^{M-1-\sum_{k=1}^{L-2} V_k}$$

$$\prod_{\eta=1}^{L-1} \binom{M-1-\sum_{k=1}^{\eta-1} V_k}{V_\eta} \left(\frac{1}{Q-(\eta-1)} \right)^{V_\eta} \cdot \left(1 - \frac{1}{Q-(\eta-1)} \right)^{M-1-\sum_{k=1}^{\eta} V_k}.$$

$$\left[1 - \left(1 - \frac{1}{Q-(L-1)} \right)^{M-1-\sum_{k=1}^{L-1} V_k} \right] = P_{FR SYNC}$$

for $u_0 = M-1, u_i = 0 \ i=1, 2, \dots, L-1.$

APPENDIX 3

STATISTICS OF THE NUMBER OF INSERTIONS DUE TO AN INTERFERING USER

Any one of the $M-1$ interferers in a given frame may result in insertions in N chips of the false row. The range of values of N depends on the delay i between the frame of a given interferer and that of the desired user. The statistics of N are derived below.

1. Synchronous operation

According to Einarsson's scheme the probability density of N is

$$P(N) = \begin{cases} 1 - L/Q & , \text{ for } N=0 \\ L/Q & , \text{ for } N=1 \\ 0 & , \text{ for } N \geq 2 \end{cases}$$

The mean and variance can be easily found to be

$$\begin{aligned} \langle N \rangle_{\text{SYNC}} &= L/Q \\ \sigma_{N \text{ SYNC}}^2 &= L/Q - (L/Q)^2 \end{aligned}$$

2. Asynchronous operation

With the Einarsson addressing scheme the number of insertions due to the m^{th} interfering user is limited to a maximum of two. This maximum value is possible only when

the two consecutive messages that interfere with the frame of the desired user are different. It can be easily shown that the probability density of N is given by

$$P(N) = \begin{cases} (1-L/Q) \cdot \frac{1}{Q} + (1-\frac{L}{Q}) \cdot (1-\frac{L-i}{Q}) \cdot (1-\frac{1}{Q}) & \text{for } N=0 \\ \frac{L}{Q} \cdot \frac{1}{Q} + \left[\frac{L}{Q} \cdot (1-\frac{L-i}{Q}) + (1-\frac{L}{Q}) \cdot (\frac{L-i}{Q}) \right] \cdot (1-\frac{1}{Q}) & \text{for } N=1 \\ \frac{L}{Q} \cdot \frac{L-i}{Q} \cdot (1-\frac{1}{Q}) & \text{for } N=2 \\ 0 & \text{for } N \geq 3 \end{cases}$$

The mean and variance are then found to be

$$\langle N \rangle_{\text{ASYN}} = L/Q$$

$$\sigma_N^2 \text{ ASYN} = L/Q - (L/Q)^2 + \frac{2 \cdot L \cdot (L-i)}{Q^2} \cdot (1-\frac{1}{Q})$$

APPENDIX 4

THE BEHAVIOUR OF EINARSSON'S SCHEME FOR LARGE VALUES OF Q

1. Synchronous operation

The probability of forming a full false row in the synchronous case is given by

$$\begin{aligned}
 P_{FR-SYNC} &= \sum_{V_1=1}^{M-1} \sum_{V_2=1}^{M-1-V_1} \cdots \sum_{V_{L-1}=1}^{M-1-\sum_{i=1}^{L-2} V_i} \\
 &\prod_{k=1}^{L-1} \binom{M-1-\sum_{i=1}^k V_{i-1}}{V_k} \left(\frac{1}{Q-(k-1)} \right)^{V_k} \left(1 - \frac{1}{Q-(k-1)} \right)^{M-1-\sum_{i=1}^k V_i} \\
 &\left[1 - \left(1 - \frac{1}{Q-(L-1)} \right)^{M-1-\sum_{i=1}^{L-1} V_i} \right] \quad (A4.1)
 \end{aligned}$$

It will be proved by induction that for $Q \gg L$ equation A(4.1)

reduces to

$$P_{FR SYN} \cong P_{FR ASYN} = \left[1 - \left(1 - \frac{1}{Q} \right)^{M-1} \right]^L \quad (A4.2)$$

For $L=1$ equation (A 4.1) yields the trivial

$$P_{FR SYN} = 1 - \left(1 - \frac{1}{Q} \right)^{M-1}$$

For $L=2$ equation (A 4.1) gives

$$P_{FR SYN} = \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q} \right)^{v_1} \cdot \left(1 - \frac{1}{Q} \right)^{M-1-v_1} \cdot \left[1 - \left(1 - \frac{1}{Q-1} \right)^{M-1-v_1} \right]$$

or

$$\begin{aligned} P_{FR SYN} &= \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q} \right)^{v_1} \cdot \left(1 - \frac{1}{Q} \right)^{M-1-v_1} - \\ &\quad - \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q} \right)^{v_1} \cdot \left(1 - \frac{1}{Q} \right)^{M-1-v_1} \cdot \left(1 - \frac{1}{Q-1} \right)^{M-1-v_1} \end{aligned}$$

which for $Q \gg 2$ may be written as

$$\begin{aligned} P_{FR SYN} &\cong \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q} \right)^{v_1} \cdot \left(1 - \frac{1}{Q} \right)^{M-1-v_1} - \\ &\quad - \left(1 - \frac{1}{Q} \right)^{M-1} \cdot \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q} \right)^{v_1} \cdot \left(1 - \frac{1}{Q} \right)^{M-1-v_1} \cdot \left(1 - \frac{1}{Q} \right)^{-v_1} \quad (A4.3) \end{aligned}$$

For $Q \gg 1$ the factor $(1-1/Q)^{v_1} \cong \exp(v_1/Q)$ and thus its value ranges from 1 for $v_1 \ll Q$ to a maximum value of $\exp((M-1)/Q)$.

The probability with which v_1 takes on the larger values in the range $[1, M-1]$ is very small because $Q \gg 1$, so equation (A4.3)

may be rewritten as

$$P_{FR SYN} \approx \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q}\right)^{v_1} \left(1 - \frac{1}{Q}\right)^{M-1-v_1} - \left(1 - \frac{1}{Q}\right)^{M-1} \cdot \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q}\right)^{v_1} \left(1 - \frac{1}{Q}\right)^{M-1-v_1}$$

or

$$P_{FR SYN} = \left[1 - \left(1 - \frac{1}{Q}\right)^{M-1}\right] \cdot \sum_{v_1=1}^{M-1} \binom{M-1}{v_1} \left(\frac{1}{Q}\right)^{v_1} \left(1 - \frac{1}{Q}\right)^{M-1-v_1}$$

or

$$P_{FR SYN} = \left[1 - \left(1 - \frac{1}{Q}\right)^{M-1}\right]^2$$

and (A 4.2) has been proved for $L=2$.

It will now be assumed that (A 4.2) holds for $L=n$ and using this it will be proved that (A 4.2) is also true for $L=n+1$.

For $L=n+1$ equation (A 4.1) gives

$$P_{FR SYN} = \sum_{V_1=1}^{M-1} \sum_{V_2=1}^{M-1-V_1} \cdots \sum_{V_n=1}^{M-1-\sum_{i=1}^{n-1} V_i}$$

$$\prod_{k=1}^n \binom{M-1-\sum_{i=1}^k V_{i-1}}{V_k} \cdot \left(\frac{1}{Q-(k-1)} \right)^{V_k} \cdot \left(1 - \frac{1}{Q-(k-1)} \right)^{M-1-\sum_{i=1}^k V_i}$$

$$\left[1 - \left(1 - \frac{1}{Q-(L-1)} \right)^{M-1-\sum_{i=1}^n V_i} \right]$$

This may be rewritten as

$$\begin{aligned}
 P_{FR SYN} &= \sum_{V_1=1}^{M-1} \binom{M-1}{V_1} \left(\frac{1}{Q}\right)^{V_1} \left(1 - \frac{1}{Q}\right)^{M-1-V_1} \\
 &\quad \sum_{V_2=1}^{M-1-V_1} \dots \sum_{V_n=1}^{M-1-\sum_{i=1}^{n-1} V_i} \\
 &\quad \prod_{k=2}^n \binom{M-1-\sum_{i=1}^{k-1} V_i}{V_k} \left(\frac{1}{Q-(k-1)}\right)^{V_k} \left(1 - \frac{1}{Q-(k-1)}\right)^{M-1-\sum_{i=1}^k V_i} \\
 &\quad \left[1 - \left(1 - \frac{1}{Q-(L-1)}\right)^{M-1-\sum_{i=1}^n V_i} \right] \quad (A4.4)
 \end{aligned}$$

In equation (A 4.4) the bracketed terms are the probability of forming a false full row in a matrix with Q channels, $M-1-v_i$ interferers and $L=n$ time-slots operating in the synchronous mode. It has been assumed that this is equal to

$$\left[1 - \left(1 - \frac{1}{Q} \right)^{M-1-v_i} \right]^n$$

for very large Q . According to this equation (A 4.4) may be rewritten as

$$P_{FR SYN} \simeq \sum_{v_i=1}^{M-1} \binom{M-1}{v_i} \left(\frac{1}{Q} \right)^{v_i} \left(1 - \frac{1}{Q} \right)^{M-1-v_i} \left[1 - \left(1 - \frac{1}{Q} \right)^{M-1-v_i} \right]^n \quad (A.4.5)$$

Notice that for values of $v_i \ll M-1$ the factor

$$\left[1 - \left(1 - \frac{1}{Q} \right)^{M-1-v_i} \right]^n \simeq \left[1 - \left(1 - \frac{1}{Q} \right)^{M-1} \right]^n$$

Larger values of v_i are taken with negligible probability because $Q \gg L$ and (A 4.5) may be rewritten as

$$P_{FR SYN} \simeq \sum_{v_i=1}^{M-1} \binom{M-1}{v_i} \left(\frac{1}{Q} \right)^{v_i} \left(1 - \frac{1}{Q} \right)^{M-1-v_i} \left[1 - \left(1 - \frac{1}{Q} \right)^{M-1} \right]^n$$

or

$$P_{FR SYN} \simeq \left[1 - \left(1 - \frac{1}{Q} \right)^{M-1} \right]^{n+1}$$

or

$$P_{FR SYN} \simeq P_{FR RA}$$

2. Asynchronous operation

As has already been discussed, in the worst case all $M-1$ interferers have delay $i=L/2$. The matrix of the desired user may then be regarded as split into two synchronous matrices. A right matrix with Q channels and $L_{\text{Right}}=L/2$ time-slots and a left matrix with $Q-L/2$ channels and $L_{\text{Left}}=L/2$ time-slots.

For very large Q the probability of forming a full false row in the right matrix is given by equation (A 4.2).

$$P_{\text{FR Right}} \simeq \left[1 - \left(1 - 1/Q \right)^{M-1} \right]^{L/2} \quad (\text{A4.6 a})$$

and in the case of the left matrix by

$$P_{\text{FR Left}} \simeq \left[1 - \left(1 - \frac{1}{Q-L/2} \right)^{M-1} \right]^{L/2} \simeq \left[1 - \left(1 - 1/Q \right)^{M-1} \right]^{L/2} \quad (\text{A4.6 b})$$

It follows that in the worst case the probability of forming a full false row in the asynchronous matrix is approximated by

$$P_{\text{FR ASYN}} \simeq \left[1 - \left(1 - 1/Q \right)^{M-1} \right]^L \quad (\text{A4.7})$$

It has already been shown that the probability of a false full row in the asynchronous Einarsson system is larger than $P_{\text{FR-SYNC}}$ and smaller than that of the worst case described above. Since for very large Q both upper and lower bounds

approximate the performance of Random Addressing, it follows that

$$P_{FR \text{ ASYN}} \approx \left[1 - (1 - 1/Q)^{M-1} \right]^L \quad (A4.8)$$

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LIST OF SYMBOLS

Q	: number of frequency- slots
Δf	: frequency- slot bandwidth
L	: number of time-slots
τ	: time-slot duration
M	: number of users
ℓ_M^Q	: number of occupied chips in column with Q slots access by M users.
$\bar{a}_m$	: address of m^{th} user
K	: number of bits per message
R	: data rate
P_w	: word-error probability
P_b	: bit-error probability
U_i	: number of users with delay i
C_i	: number of users from the group of u_i who have already occupied chips in the false row in the $L-i$ chips preceding the desired frame
$q_{i,n}$	: number of interferers with delay i who have access to chip n of the false row
$Y_{i,n}$	: number of interferers with delay i who actually occupy chip n of the false row
$P_{FR-ASYN}$	: probability of false row in the case of asynchronous operation
P_{FR-SYN}	: probability of false row in the case of synchronous operation
$\bar{1}$	$= (1 \ 1 \ \dots \ 1)$

P_D :probability of deletion
 P_F :probability of false alarm
 B :number of frequency-slots common to two
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 N_o :variance of additive noise

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